Amplify Math
TENNESSEE

Teacher Edition

Algebra 1 | Volume 2

FEATURING Amps POWERED BY desmos

Amplify Math

Algebra 1

Volume 2: Units 4–6

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify Math is based on the Illustrative Mathematics (IM) curriculum. IM 6–8 Math™ was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is © 2017–2019 Open Up Resources. Additional adaptations and updates to IM 6–8 Math™ are © 2019 Illustrative Mathematics. IM 9–12 Math™ is © 2019 Illustrative Mathematics. IM 6–8 Math™ and IM 9–12 Math™ are licensed under the Creative Commons Attribution 4.0 International license (CC BY 4.0). Additional modifications contained in Amplify Math are © 2020 Amplify Education, Inc. and its licensors. Amplify is not affiliated with the Illustrative Mathematics organization.

Desmos® is a registered trademark of Desmos, Inc.

English Learners Success Forum is a fiscally sponsored project of the New Venture Fund (NVF), a 501(c)(3) public charity.

Universal Design for Learning Guidelines and framework are developed by the Center for Applied Special Technology. © 2018 CAST.

The Effective Mathematics Teaching Practices are developed by NCTM in Principles to Actions: Ensuring mathematical success for all. © 2014 NCTM.

Notice and Wonder and I Notice/I Wonder are trademarks of NCTM and the Math Forum.

No part of this publication may be reproduced or distributed in its original form, or stored in a database or retrieval system, without the prior written consent of Amplify Education, Inc., except for the classroom use of the worksheets included for students in some lessons.

Cover illustration by Caroline Hadilaksono.

© 2023 by Amplify Education, Inc.
55 Washington Street, Suite 800,
Brooklyn, NY 11201
www.amplify.com

ISBN: 978-1-63643-492-6
Printed in [e.g., the United States of America]
[# of print run] [print vendor] [year of printing]

Acknowledgments

Program Advisors

Amplify gratefully acknowledges the outstanding contributions and work of distinguished program advisors who have been integral to the development of Amplify Math. This product is testimony to their expertise, understanding of student learning needs, and dedication to rigorous and equitable mathematics instruction.

Phil Daro

Board member: Strategic Education Research Partnership (SERP)
Area of focus: Content strategy

Fawn Nguyen

Rio School District, California
Area of focus: Problem solving

Sunil Singh

Educator, author, storyteller
Area of focus: Narrative and storytelling

Paulo Tan, Ph.D.

Johns Hopkins University, School of Education
Area of focus: Meeting the needs of all students

Educator Advisory Board

Amplify gratefully acknowledges the outstanding contributions and work of esteemed members of our Educator Advisory Board. This product exhibits their respected observations and reflections.

Melvin Burnett
Alamance Burlington Schools, North Carolina

Jessica Childers
Putnam County Schools, Tennessee

Brent Christensen
Spokane Public Schools, Washington

Rhonda Creed-Harry
New York City Schools, New York

Jenny Croitoru
Chicago Public Schools, Illinois

Tara DeVaughn
Fairbanks Northstar Borough School District, Alaska

Elizabeth Hailey
Springfield R-XII School District, Missouri

Howie Hua
California State University at Fresno, California

Rachael Jones
Durham Public Schools, North Carolina

Rita Leskovec
Cleveland Metropolitan School District, Ohio

Corey Levin
New York City Schools, New York

Sandhya Raman
Berryessa Union School District, California

Jerry Schmidt
Brentwood School District, Missouri

Deloris Scott
Yazoo County School District, Mississippi

Noah Sharrow
Clarkston Community Schools, Michigan

Myla Simmons
Plainfield Public Schools, New Jersey

Michele Stassfurth
North Plainfield School District, New Jersey

Field Trials

Amplify gratefully acknowledges the time and efforts of educators from the following districts and schools whose participation in field trials provided constructive critiques and resulting improvements. This product reflects their valuable feedback.

Berryessa Union School District, California

Chicago Jesuit Academy, Illinois

Irvine Unified School District, California

Lake Tahoe Unified School District, California

Leadership Learning Academy, Utah

Lusher Charter School, Louisiana

Memphis Grizzlies Preparatory Charter School, Tennessee

Saddleback Valley Unified School District, California

San Juan Unified School District, California

Santa Paula Unified School District, California

Silver Summit Academy, Utah

Streetsboro City Schools, Ohio

West Contra Costa Unified School District, California

Wyoming City Schools, Ohio

Young Women's Leadership School of Brooklyn, New York

Amplify Math Product Development

Product

Molly Batchik
Candy Bratton
Stephanie Cheng
Rebecca Copley
Marsheela Evans
Christina Lee
Brad Shank
Kathleen Sheehy
Jennifer Skelley
Rey Vargas
Allen von Pallandt
Steven Zavari

Curriculum and Editorial

Toni Brokaw
Anna Buchina
Nora Castiglione
Jaclyn Claiborne
Kristina Clayton
Drew Corley
Karen Douglass
Karen Everly
Chris Ignaciuk
Justine Jackson
Louise Jarvis
Brian Kam
Rachel King
Suzanne Magargee
Mark Marzen
Nana Nam
Kim Petersen
Molly Pooler
Elizabeth Re
Allison Shatzman
Kristen Shebek
Ben Simon
Evan Spellman
Amy Sroka
Shelby Strong
Rajan Vedula
Zach Wissner-Gross

Digital Curriculum

Ian Cross
Phil DeOrsey
Ryan de la Garza
Sheila Jaung
Nokware Knight
Michelle Palker
Vincent Panetta
Aaron Robson
Sam Rodriguez
Eileen Rutherford
Elliot Shields
Gabe Turow

Design and Illustration

Amanda Behm
Irene Chan
Tim Chi Ly
Cindy Chung
Caroline Hadilaksono
Justin Moore
Christina Ogbotiti
Renée Park
Eddie Peña
Todd Rawson
Jordan Stine
Veronica Tolentino
J Yang

Narrative Design

Bill Cheng
Gala Mukomolova
Raj Parameswaran

Marketing

Megan Hunter
Zach Slack
Heath Williams

Engineering

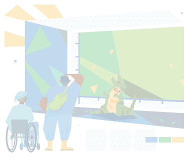
Jessica Graham
Matt Hayes
Bardh Jahjaga
Eduard Korolchuk
Nick Maddalena
Syed Rizvi
Jon Tully

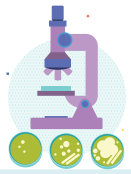
Digital Production

Andrew Avery
Ryan Cooper
Jessica Yin Gerena
Edward Johnson
Charvi Magdaong
Julie Palomba
Heather Ruiz
Ana Zapata

Program Scope and Sequence

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 160 days total	Area and Surface Area 20 Instructional Days 3 Assessment Days 23 days total	Introducing Ratios 20 Instructional Days 2 Assessment Days 22 days total	Rates and Percentages 15 Instructional Days 2 Assessment Days 17 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 153 days total	Scale Drawings 13 Instructional Days 2 Assessment Days 15 days total	Introducing Proportional Relationships 17 Instructional Days 2 Assessment Days 19 days total	Measuring Circles 12 Instructional Days 2 Assessment Days 14 days total	Percentages 13 Instructional Days 2 Assessment Days 15 days total
Grade 8 145 days total	Rigid Transformation and Congruence 18 Instructional Days 3 Assessment Days 21 days total	Dilations and Similarity 12 Instructional Days 2 Assessment Days 14 days total	Linear Relationships 19 Instructional Days 2 Assessment Days 21 days total	Linear Equations and Systems of Linear Equations 17 Instructional Days 2 Assessment Days 19 days total





	Unit 1	Unit 2	Unit 3	Unit 4
Algebra 1 157 days total	Linear Equations, Inequalities, and Systems 26 Instructional Days 3 Assessment Days 29 days total	Data Analysis and Statistics 22 Instructional Days 3 Assessment Days 25 days total	Functions and Their Graphs 22 Instructional Days 3 Assessment Days 25 days total	**Introducing Exponential Functions** **22 Instructional Days** **3 Assessment Days** **25 days total**

Unit 5	Unit 6	Unit 7	Unit 8
Arithmetic in Base Ten 14 Instructional Days 2 Assessment Days 16 days total	Expressions and Equations 19 Instructional Days 2 Assessment Days 21 days total	Rational Numbers 19 Instructional Days 2 Assessment Days 21 days total	Data Sets and Distributions 17 Instructional Days 3 Assessment Days 20 days total
Rational Number Arithmetic 20 Instructional Days 3 Assessment Days 23 days total	Expressions, Equations, and Inequalities 23 Instructional Days 3 Assessment Days 26 days total	Angles, Triangles, and Prisms 18 Instructional Days 3 Assessment Days 21 days total	Probability and Sampling 17 Instructional Days 3 Assessment Days 20 days total
Functions and Volume 21 Instructional Days 3 Assessment Days 24 days total	Exponents and Scientific Notation 15 Instructional Days 2 Assessment Days 17 days total	Irrationals and the Pythagorean Theorem 16 Instructional Days 2 Assessment Days 18 days total	Associations in Data 9 Instructional Days 2 Assessment Days 11 days total
 **Introducing Quadratic Functions** **23 Instructional Days** **3 Assessment Days** **26 days total**	 **Quadratic Equations** **24 Instructional Days** **3 Assessment Days** **27 days total**		

Volume 2

Volume 1

Unit 1 Linear Equations, Inequalities, and Systems

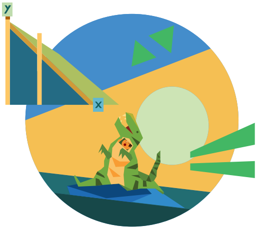
In this unit, students write and solve linear equations and inequalities to model the relationship between different quantities, recalling what is meant by a solution to an equation and interpreting solutions in context. They also solve systems of linear equations by graphing and using substitution and elimination methods.

Unit Narrative:
Adulting (Making Life Decisions)

Sub-Unit Narrative:
How did a tragic accident end a three-month strike?
Revisit how equations and inequalities can be used to model real-world situations, and how they can help you make decisions.

Sub-Unit Narrative:
How do first-gen Americans vault the hurdles of college?
“Solving” an equation doesn’t always mean finding an unknown value — sometimes it can mean changing the equation’s very structure.

Sub-Unit Narrative:
What’s after high school?
Whether you work, intern, attend college, or do something else, inequalities can be used to model your time and money, helping you manage both of them.



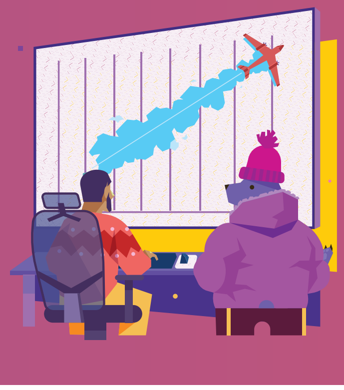
Sub-Unit Narrative:
Are you a “boomerang-er”?
For better or for worse, life is full of constraints. Discover new strategies for solving problems with multiple constraints, which you will see time and again.

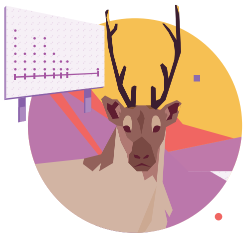
Sub-Unit Narrative:
Is there such a thing as too much choice?
What happens when the decisions become more complicated? Look at the big picture and then fine-tune where the decision overlaps.

Unit 2 Data Analysis and Statistics

Students will explore univariate and bivariate data sets, many of which relate to climate change. Along the way, they will investigate, calculate, and interpret descriptive statistics, including measures of center, variability, and association.

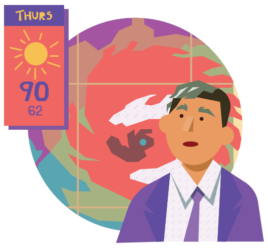
Unit Narrative:
Analyzing
Climate Change





Sub-Unit Narrative:
How can we protect ourselves from a zombie virus?

Remember dot plots, histograms, and box plots? Revisit them through temperature data, while describing the data's center and variability.



Sub-Unit Narrative:
Is Sandy the new normal?

Meet the most commonly used measure of variability: standard deviation.

Sub-Unit Narrative:
What is "Day Zero"?

You have seen linear models before, but now you will (finally!) see how to identify the "best" model, by looking carefully at what are called residuals.

Sub-Unit Narrative:
What makes storms worse and has nothing to do with weather?

Use two-way tables to see how the changing climate has affected marginalized people around the world.

Sub-Unit Narrative:
Who is the "water warrior"?

Finally, you can say just how strong or weak an association is using a new statistical measure: the correlation coefficient.

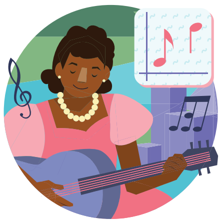
Unit 3 Functions and Their Graphs

Students will expand on their understanding of the key features and multiple representations of functions. Along the way, they will write, graph, and interpret linear functions, piecewise functions, absolute functions, and the inverse of functions.

Unit Narrative:
Artscapes

PRE-UNIT READINESS ASSESSMENT

LAUNCH



Sub-Unit 1 Functions and Their Representations

Sub-Unit Narrative:
How did the blues find a home in Memphis?

Remember representing functions with description, tables, graphs, and equations? Take another look as you visit Memphis, and meet another common tool to represent and interpret functions: *function notation*.

Sub-Unit 2 Analyzing and Creating Graphs of Functions

MID-UNIT ASSESSMENT

Sub-Unit Narrative:
What's the function of a jazz solo?

The way you describe a graph helps you gain insight on the relationship it represents. Average rate of change, domain, and range help to construct and interpret graphs more precisely.

Sub-Unit 3 Piecewise Functions

Sub-Unit Narrative:
Where did the world meet soul?

Piecewise functions allow you to represent a relationship between two quantities as a set of rules. The absolute value function is a specific piecewise function that represents the distance from zero.

Sub-Unit 4 Inverses of Functions

Sub-Unit Narrative:
How do you get Sunday shoppers to hear your song?

What happens if you reverse the process used in a relationship between two quantities? What changes? What stays the same? You can explore these questions using inverses of functions.

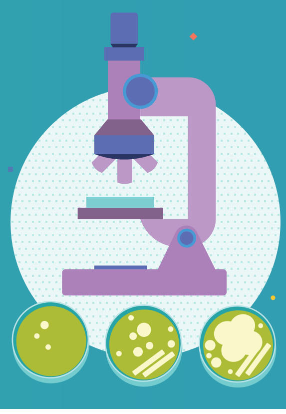
CAPSTONE

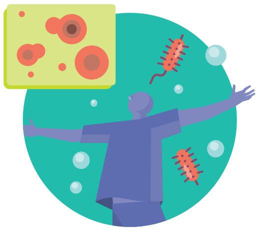
Volume 2

Unit 4 Introducing Exponential Functions

This is a unit of mathematical discovery, where the relationship between quantities is unlike any function students will have seen up to this point. Students encounter the explosiveness of exponential growth and and lingering of exponential decay through applications of infectious disease, vaccination, and prescription drug costs.

Unit Narrative:
Infectious Diseases, Vaccines, and Costs



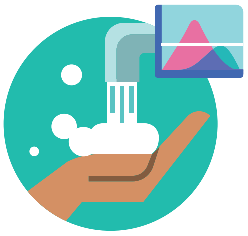


Sub-Unit Narrative:
Where do baby bacteria come from?
Examine nonlinear functions using tables and graphs, before defining an exponential relationship. You'll represent exponential growth using an equation and explore it in context.

Sub-Unit Narrative:
How did an enslaved person save the city of Boston?
Examine growth factors between 0 and 1 as you develop an understanding of exponential decay.

Sub-Unit Narrative:
What does growing and shrinking look like on a graph?
Identify exponential relationships as exponential functions, and determine whether a graph is discrete.

Sub-Unit Narrative:
Want to be CEO for a day?
Make sense of repeated percent increase and see how it relates to compound interest.

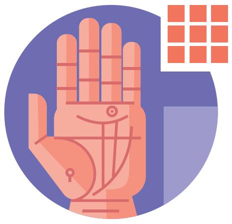


Sub-Unit Narrative:
Does distance make the curve grow flatter?
Compare the growth of different kinds of functions and finish with an exploration of how social distancing can combat the dangers of an epidemic.

Unit 5 Introducing Quadratic Functions

Students study quadratic functions. By analyzing and comparing patterns, tables, graphs, and equations, they gain an appreciation for the special features of quadratic functions and the situations they represent.

Unit Narrative:
Squares in Motion



Sub-Unit Narrative:
What's the best shape for a crystal ball?
Dive into quadratic expression by examining patterns of growth and change.

Sub-Unit Narrative:
What would sports be like without quadratics?
Use quadratic functions to model objects flying through the air or revenues earned by companies.

Sub-Unit Narrative:
How do you put the "quad-" in quadratics?
Use area diagrams and algebra tiles to factor quadratic expressions as you explore equivalent ways to write them.

Sub-Unit Narrative:
Mirror, mirror on the wall, what's the fairest function of them all?

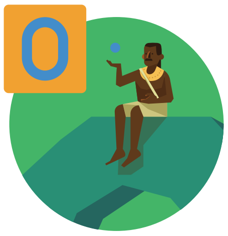
Quadratics have their own beauty, and different forms help you identify features of their graphs.

Unit 6 Quadratic Equations

In this unit, students write and solve quadratic equations and make sense of the solutions. They investigate the structure of quadratic equations and determine efficient strategies for solving them.

Unit Narrative:
The Evolution of Solving Quadratic Equations

Sub-Unit Narrative:
How did the Nile River spur on Egyptian mathematics?
Revisit projectile motion and maximizing revenue as you discover new meanings for the zeros of a quadratic function.



Sub-Unit Narrative:
When is zero more than nothing?
Understand the importance of zero when solving quadratic equations. Then, efficiently factor quadratics using a variety of strategies.

Sub-Unit Narrative:
How many ways can you crack an egg?
Discover the ancient art of taking a quadratic expression and completing the square. It's all about that missing piece.

Sub-Unit Narrative:
Where does a number call its home?

Subtraction and division took you from whole numbers to rationals. Now you must look beyond them as you operate with irrational numbers.

Sub-Unit Narrative:
What was the House of Wisdom?

Discover strategies for solving any quadratic equation. You will also determine which strategies are more efficient.

CAPSTONE

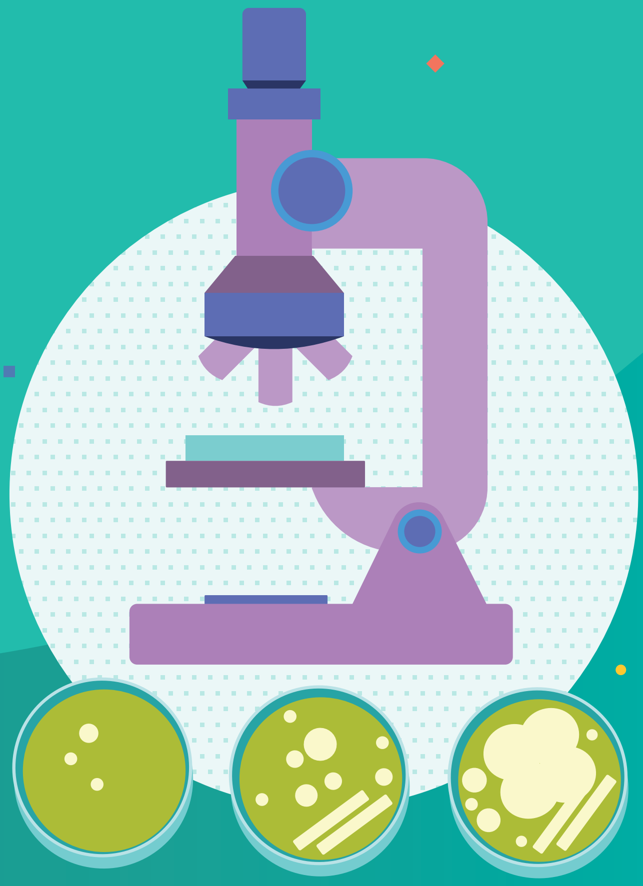
UNIT 4

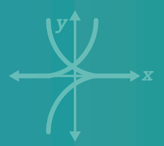
Introducing Exponential Functions

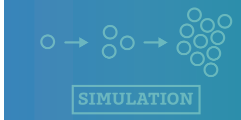
This is a unit of mathematical discovery, where the relationship between quantities is unlike any function students will have seen up to this point. Students encounter the explosiveness of exponential growth and and lingering of exponential decay through applications of infectious disease, vaccination, and prescription drug costs.

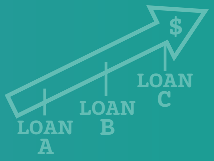
Essential Questions

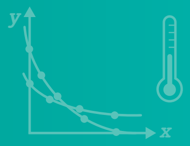
- What characterizes exponential growth and decay?
- What are real-world models of exponential growth and decay?
- How can you differentiate exponential growth from linear growth, given a real-world data set?
- *(By the way, can one person infect the entire world?)*











Key Shifts in Mathematics

Focus

In this unit . . .

Students extend their understanding of functions to exponential functions. They compare exponential functions to linear functions and model exponential growth and decay. They determine how changing different terms affect the graph and interpret what these changes mean in a given context.

Coherence

Previously . . .

In Grade 8, students cemented their understanding of the properties of exponents and were able to apply them across a multitude of contexts and applications.

Earlier in Algebra 1, students further developed their understanding of functions from Grade 8. Students were also introduced to nonlinear functions — absolute value, piecewise, and step functions — with a focus on the graphs of these functions.

Coming soon . . .

In Algebra 2, students will further explore exponential functions by determining their inverse (logarithmic functions), solving exponential equations using algebraic strategies, developing algebraic rules for transformations of exponential graphs, and developing an understanding of the constant e.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Exponential relationships are introduced as nonlinear relationships in Lessons 1–3. Students examine the graphs and tables of exponential relationships and compare them to the graphs and tables of linear relationships to understand how exponential functions grow in Lessons 2–5.

Procedural Fluency

Students develop fluency in graphing and representing exponential relationships and functions in Lessons 2–9. They gain fluency in writing exponential functions in different forms in Lessons 10–18.

Application

Throughout the unit, students explore exponential functions in a variety of applications. In the capstone lesson, they apply their knowledge to model the spread of COVID-19, comparing its spread to smallpox, and they show how hygiene and social distancing slows the spread.

Infectious Diseases, Vaccines, and Costs

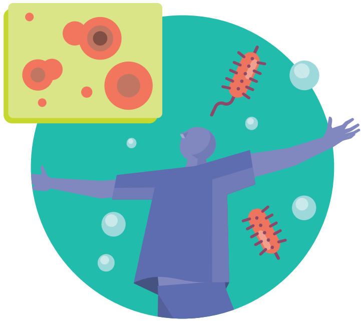
SUB-UNIT

1

Lessons 2–3

Looking at Growth

Students examine ***exponential growth*** — as a type of nonlinear growth — using tables and graphs. They move on to represent ***exponential relationships*** with equations, within a variety of scientific, fictional, and historical contexts.



Narrative: From bacteria growth to social media, explore a special kind of nonlinear growth.

SUB-UNIT

2

Lessons 4–7

A New Kind of Relationship

Students examine ***growth factors*** between 0 and 1 to understand ***exponential decay***. They calculate growth factors and identify them in exponential expressions. Students observe and analyze the numerical and graphical consequences of changing the values of a, b, and x in the expression ab^x.

Narrative: Exponential relationships can help explain how smallpox was eradicated.

SUB-UNIT

3

Lessons 8–14

Exponential Functions

Students identify exponential relationships as functions and explicitly identify the independent and dependent variables in a context using function notation. They determine whether the graph representing a real-world scenario should be continuous or discrete and identify the domain.

Narrative: Exponential functions help us track the spread of disease and effects of medication.

Launch

Lesson 1

What Is an Epidemic?

Students begin the unit by simulating the spread of a disease in their classrooms, using graphs and tables to model the spread, but are left wondering what type of mathematical function models the spread. The terminology used throughout the lesson is ***nonlinear***. What are the global implications of a rapidly spreading disease?

SUB-UNIT

4

Lessons 15–19

Percent Growth and Decay

Students revisit percent change from Grade 7, applying it to exponential expressions to make sense of repeated percent increase — before formally defining it as compound interest. They compare growth factors and ***growth rates*** by studying the expression $a \cdot (1 + r)^x$.

Narrative: The response to medication can be modeled with percent change and exponential expressions.

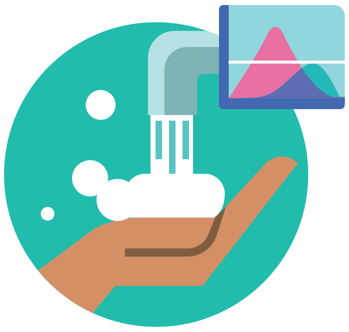
SUB-UNIT

5

Lessons 20–21

Comparing Linear and Exponential Functions

Students build on their understanding of rates of change in linear functions and explore rates of change for exponential functions. Through different contexts students investigate which function grows faster.



Narrative: Exponential functions help us understand the spread of disease — and how to slow it down.

Lesson 22

Capstone

COVID-19

Students explore the spread of COVID-19 on a global level and compare its spread to other infectious diseases. They explore how social distancing can slow the spread of the disease.

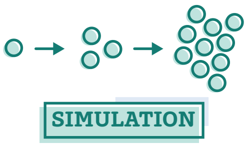
Unit at a Glance

Spoiler Alert: Exponential functions increase at a faster rate than linear functions.

Assessment

A **Pre-Unit Readiness Assessment**

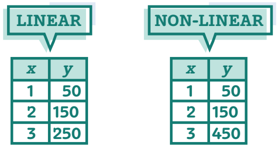
Launch Lesson



1 **What Is an Epidemic?**

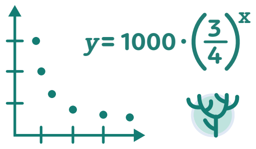
Explore nonlinear functions by simulating an infectious disease and observing its behavior on a graph.

Sub-Unit 1: Looking At Growth



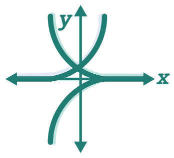
2 **Patterns of Growth** ●

Analyze linear and exponential patterns by examining tables and observing common differences and common factors.



6 **Representing Exponential Decay**

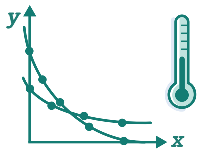
Examine the graphs and equations of scenarios characterized by exponential decay and identify and interpret key features of graphs.



7 **Exploring Parameter Changes of Exponentials** (optional) ●

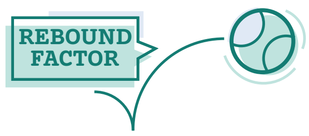
Determine how the graph of an exponential equation of the form $y = ab^x$ is affected as the values of a and b change.

Sub-Unit 3: Exponential Functions



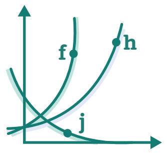
8 **Analyzing Graphs**

Analyze graphs representing depreciation, write equations representing relationships in context, and match scenarios with their graphs representing exponential change.



12 **Modeling Exponential Behavior** ●

Collect real-world data and model the relationship with an exponential function. Use the model to solve the problem and interpret the solution within the context of the problem.



13 **Reasoning About Exponential Graphs**

Analyze the graph of an exponential function $f(x) = a \cdot b^x$ and study the effect of b on the shape of the graph when $b > 1$ and when $0 < b < 1$.

$$\frac{A(2) - A(0)}{2-0} \approx -107$$

14 **Looking At Rates of Change**

Calculate the average rate of change of an exponential function over specified intervals using multiple representations of the function.

Key Concepts

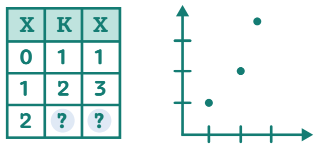
Lesson 4: Meet the exponential equation and interpret what each part of the equation represents.
Lesson 11: Exponential functions represent many different types of real-world phenomena.
Lesson 21: Use rates of change to show how exponential and linear functions change over equal intervals.

Pacing

22 Lessons: 50 min each
3 Assessments: 45 min each
Full Unit: 25 days
● **Modified Unit:** 20 days

Assumes 50-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Sub-Unit 2: A New Kind of Relationship

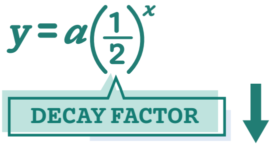


3 Growing and Growing ●

Build a conceptual understanding of linear and exponential relationships by exploring tables and graphs that show different linear and nonlinear growth patterns.

4 Representing Exponential Growth

Introduce the general form of an exponential equation $y = ab^x$ and interpret a and b in context.

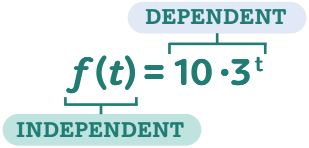


5 Understanding Decay

Examine exponential decay with growth factors greater 0 than and less than 1, write expressions, and make connections between tables and equations.

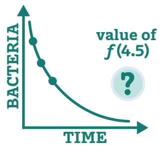
9 Using Negative Exponents

Interpret negative exponents in context and write exponential equations.



10 Exponential Situations as Functions

Determine whether relationships are exponential functions, choose independent and dependent variables, and express relationships using function notation.



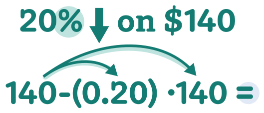
11 Interpreting Exponential Functions

Given a relationship, write one quantity as a function of another, determine reasonable domains, and apply the function to the context.

Assessment

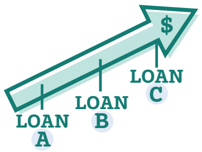
Sub-Unit 4: Percent Growth and Decay

A Mid-Unit Assessment



15 Recalling Percent Change (optional) ●

Use data from the pre-assessment to determine whether this lesson is needed. Write expressions in different forms representing percent increase and decrease.



16 Functions Involving Percent Change ●

Use graphs to illustrate and compare situations with different percent increases and write an expression of the form $(1 + r)^n$ to represent a percent increase applied n times.

Unit at a Glance

Spoiler Alert: Exponential functions increase at a faster rate than linear functions.

‹ continued

MONTHS	$ BALANCE
1	$300 \cdot (1.02)$
2	$300 \cdot (1.02)^2$
3	$300 \cdot (1.02)^3$

17 **Compounding Interest**

Distinguish the effect of compounded percent change from that of simple percent change.

$$P = 4{,}000 \cdot (1.031)^t$$

18 **Expressing Exponentials in Different Ways** ●

Distinguish between growth rate and growth factor (defined earlier in this unit). In functions of the form $a \cdot (1+r)^x$, the growth rate is r and the growth factor is $1+r$.

MONTHS	$
1	$1,000
2	$1,015
6	$1,030

19 **Credit Cards and Exponential Expressions**

Further develop understanding of compound interest in the context of credit card APR, returns on investments, and the rising costs of college tuition.

Assessment

A **End-of-Unit Assessment**

Key Concepts

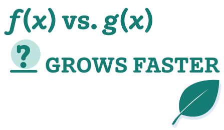
Lesson 4: Meet the exponential equation and interpret what each part of the equation represents.
Lesson 11: Exponential functions represent many different types of real-world phenomena.
Lesson 21: Use rates of change to show how exponential and linear functions change over equal intervals.

Pacing

22 Lessons: 50 min each	**Full Unit:** 25 days
3 Assessments: 45 min each	● **Modified Unit:** 20 days

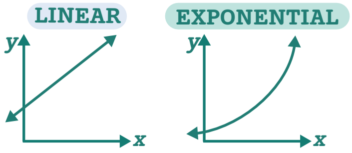
Assumes 50-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Sub-Unit 5: Comparing Linear and Exponential Functions



20 Which One Changes Faster?

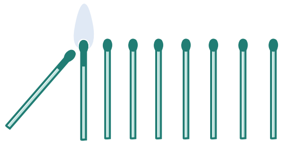
Investigate the fact that exponential functions grow more quickly than linear functions.



21 Changes Over Equal Intervals

Demonstrate that linear functions change by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals.

Capstone Lesson



22 COVID-19

Compare the spread of a recent virus, COVID-19, to an eradicated virus, smallpox. Model the real-world scenarios in functions and graphs, analyze provided data and graphs, and predict the global impact. This lesson may be taught over 2 days.

● Modifications to Pacing

Lessons 2–3: These lessons may be combined as they build conceptual understanding of exponential growth using tables and graphs.

Lessons 7 and 15: These two lessons are optional. Lesson 7 is a digital lesson and Lesson 15 is a review of Grade 7 concepts.

Lessons 12, 16, and 18: These lessons have optional activities which can be omitted, based on student data from the Pre-Unit Readiness Assessment.

Unit Supports

Math Language Development

Lesson	New vocabulary
2	common difference common factor
4	exponential growth growth factor
5	decay factor exponential decay
10	exponential function
17	interest rate principal
18	growth rate

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
1, 12, 19	**MLR1:** Stronger and Clearer Each Time
3, 4, 8, 10, 13, 17, 18, 21	**MLR2:** Collect and Display
4, 11, 21	**MLR3:** Critique, Correct, Clarify
11,	**MLR4:** Information Gap
5, 9	**MLR5:** Co-craft Questions
3, 5, 6, 9, 16, 17, 18, 20, 22	**MLR6:** Three Reads
4, 8, 12, 15, 19, 20	**MLR7:** Compare and Connect
2, 4–19	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Materials
12	balls (three different types)
11	blank sheets of copy paper
8	cell phone advertisements
1	chart paper or graph paper
1–3, 5, 8, 13, 14, 22	colored pencils
4, 6, 7, 10–13, 20	graphing technology
12	measuring tape
1, 4, 7–9, 11, 18, 19, 21	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
18	six-sided dice
1	spreadsheet technology

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routine
15, 22	Gallery Tour
18	Math Talk
1, 5, 14, 22	Notice and Wonder
8, 11	Think-Pair-Share
2	Which One Doesn't Belong?
7	Would You Rather?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 14
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 22

Amps

POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

Measuring Medicine

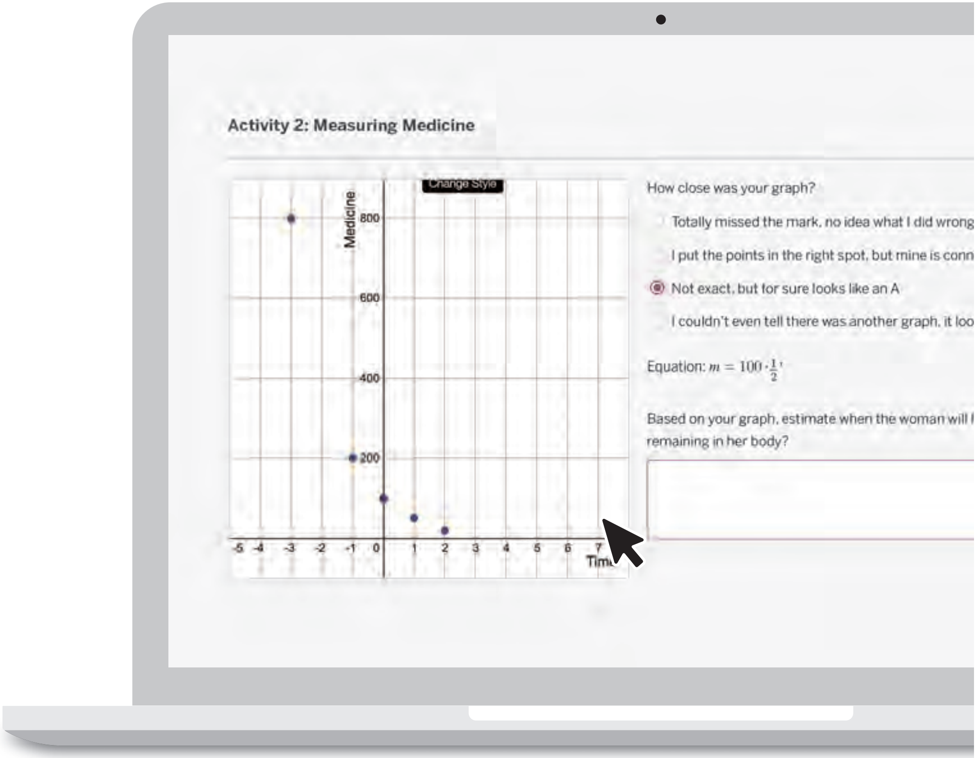
Put on your student hat and work through **Lesson 9, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities

- Notice & Wonder **(Lesson 1)**
- Viral Reproduction Number **(Lesson 3)**
- Insulin in the Body **(Lesson 6)**
- Marble Slides **(Lesson 7)**
- Fever Reducer **(Lesson 8)**
- Beholding Bouncing Balls **(Lesson 12)**
- Modeling an Epidemic **(Lesson 18)**
- Plant Disease **(Lesson 20)**
- Social Distancing **(Lesson 22)**



Activity 2: Measuring Medicine

How close was your graph?

○ Totally missed the mark, no idea what I did wrong

○ I put the points in the right spot, but mine is conn

● Not exact, but for sure looks like an A

○ I couldn't even tell there was another graph, it loo

Equation: $m = 100 \cdot \frac{1}{2}$

Based on your graph, estimate when the woman will

remaining in her body?

Unit Study
Professional Learning

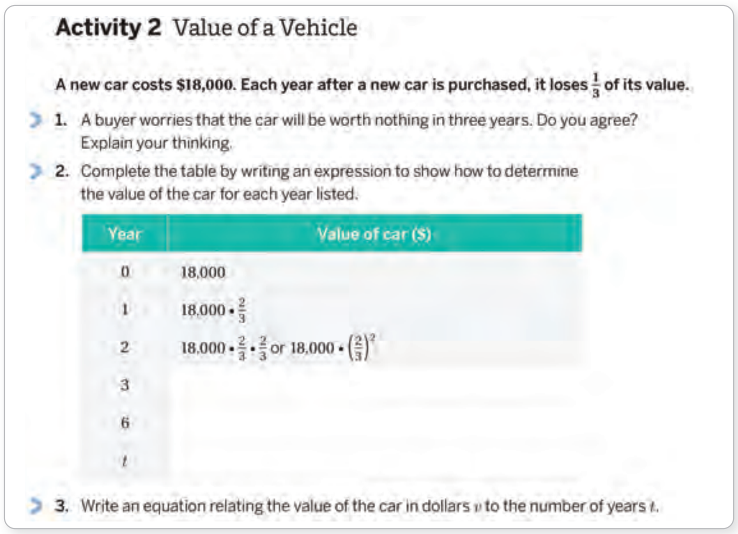
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a brief yet meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces the idea of a rapidly shrinking function. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 5, Activity 2:**



Activity 2 Value of a Vehicle

A new car costs \$18,000. Each year after a new car is purchased, it loses $\frac{1}{3}$ of its value.

1. A buyer worries that the car will be worth nothing in three years. Do you agree? Explain your thinking.
2. Complete the table by writing an expression to show how to determine the value of the car for each year listed.

Year	Value of car (\$)
0	18,000
1	$18{,}000 \cdot \frac{2}{3}$
2	$18{,}000 \cdot \frac{2}{3} \cdot \frac{2}{3}$ or $18{,}000 \cdot \left(\frac{2}{3}\right)^2$
3	
6	
t	

3. Write an equation relating the value of the car in dollars v to the number of years t.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

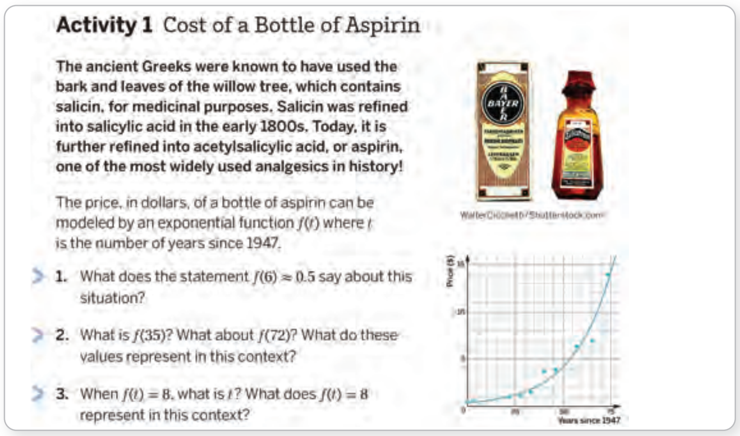
- What was it like to engage in this problem as a learner?
- What approaches might your students take?
- Do any approaches surprise you?
- Do any approaches reveal a misconception that might arise for students?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Think-Pair-Share

Rehearse . . .

How you'll facilitate the *Think-Pair-Share* instructional routine in **Lesson 11, Activity 1:**



Activity 1 Cost of a Bottle of Aspirin

The ancient Greeks were known to have used the bark and leaves of the willow tree, which contains salicin, for medicinal purposes. Salicin was refined into salicylic acid in the early 1800s. Today, it is further refined into acetylsalicylic acid, or aspirin, one of the most widely used analgesics in history!

The price, in dollars, of a bottle of aspirin can be modeled by an exponential function $f(t)$ where t is the number of years since 1947.

1. What does the statement $f(6) \approx 0.5$ say about this situation?
2. What is $f(35)$? What about $f(72)$? What do these values represent in this context?
3. When $f(t) = 8$, what is t? What does $f(t) = 8$ represent in this context?

WalterCicchetti/Shutterstock.com

Points to Ponder . . .

- What are the benefits to having students think quietly about a problem before sharing with a partner?
- What are the benefits to having students share with a partner before engaging in a whole-class discussion?

This routine . . .

- Reduces the pressure that can come with sharing in front of the class – sharing with one person is low-stakes.
- Gives everyone, not just the first person to raise their hand, a chance to come up with a response.
- When it's time to share with the class, students have already had a chance to refine and rehearse their response.
- Opens up the possibility for diverse thinking about the question or problem.

Anticipate . . .

- How might your students think differently about these problems?
- If you *haven't* used this routine before, what classroom management strategies will you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What do you want to refine?

Strengthening your Effective Teaching Practices

Facilitate meaningful mathematical discourse. Pose purposeful questions.

These effective teaching practices . . .

- Ensure that there is a shared understanding of the mathematical concepts presented in each lesson.
- Allow students to listen to and critique the strategies and conclusions of others.
- Help you assess the reasoning behind student responses, and advance their sense-making skills by asking deeper questions about mathematical ideas and relationships.

Points to Ponder . . .

- Some students may not not know how to dive deeper into discussions about mathematics. How can you model these discussions?
- How can you probe for student reasoning to ensure they understand a mathematical concept, beyond just providing a correct response?
- What kinds of questions can you ask that will help advance their understanding to a deeper level?

Math Language Development

MLR8: Discussion Supports

MLR8 appears in Lessons 2, 4–19, and 22.

- Mathematical discourse is a pivotal component throughout the unit. It is best supported by providing students with sentence stems and or graphic organizers to help organize their thoughts. Students are usually asked to present after each activity.

Point to Ponder . . .

- How can you model good mathematical discourse with your students? What does that look like? Do you typically ask questions in which you are looking for answers? Or do you ask questions such as “Why” or “How”?

Differentiated Support

Supporting accessibility for: Conceptual Processing

Support for Conceptual Processing appears in Lessons 2–7, 10, 13, 15, and 17.

- Differentiate the degree of difficulty or complexity in exponential expressions by beginning with patterns, tables, and graphs.
- Connect new concepts to ones with which students have experienced success. For example, students have developed proficiency in understanding linear functions. How might you leverage their understanding of linear functions with exponential functions?

Point to Ponder . . .

- As you look through the unit, which strategies for internalizing the different forms of exponential functions may be most useful for your students?

Unit Assessments

- Use the results of the **Pre-Unit Readiness Assessment** to understand your students’ prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Read through and unpack the **Mid-** and **End-of-Unit Assessments**.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with understanding exponential growth and decay in its multiple representations? Do you think your students will generally:
  - » Miss the underlying concept of the different types of growth?
  - » Interpret what each term means in an algebraic equation or function?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students’ capacity to build strong mathematical habits of mind.

- In this unit, pay particular attention to supporting students in building their self-awareness and self-management skills.

Points to Ponder . . .

- What are their strengths and what do they know about numerical reasoning that they can build upon and use to begin reasoning algebraically?
- Are students able to exercise patience and persist in order to understand the underlying conceptual structure of exponential growth/decay in applications or do they prefer to do problems without context?

What Is an Epidemic?

Let's look at some growth.

Focus

Goals

1. **Language Goal:** Explain and give an example of an epidemic. **(Speaking and Listening)**
2. Recognize the growth pattern of an epidemic, given a table.
3. Recognize a graphical representation of an epidemic model.

Coherence

Today

This unit launches with a close look at epidemics and the spread of disease. Here, students are informally introduced to representations of exponential growth by simulating a viral epidemic.

Previously

In previous units, students were introduced to functions, and they distinguished between linear and nonlinear functions.

Coming Soon

Students will develop the concept of exponential growth and decay, before formally defining exponential functions.

Rigor

- Students build on their **conceptual understanding** of nonlinear growth from Grade 8.
- Students **apply** their understanding of nonlinear growth to real-world situations involving the spread of disease.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Whole Class	Whole Class	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per student
- colored pencils (as needed)
- graph paper/chart paper
- spreadsheet technology

Math Language Development

Review words

- *linear*
- *nonlinear*

Amps | Featured Activity

Warm-up

Animated Notice and Wonder

The digital Warm-up provides the opportunity for students to visually observe a pattern of growth that doubles.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Starting a new unit might raise some students' stress level because they are unsure of the change. Encourage students to use the creativity involved with modeling an epidemic with graphs to be a regulation mechanism for their stress. The models are a visual representation of something that can be stressful, but the mathematical models themselves bring order to the implied chaos. By focusing on the connection between the images and the new mathematics, students can let their stress levels drop.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

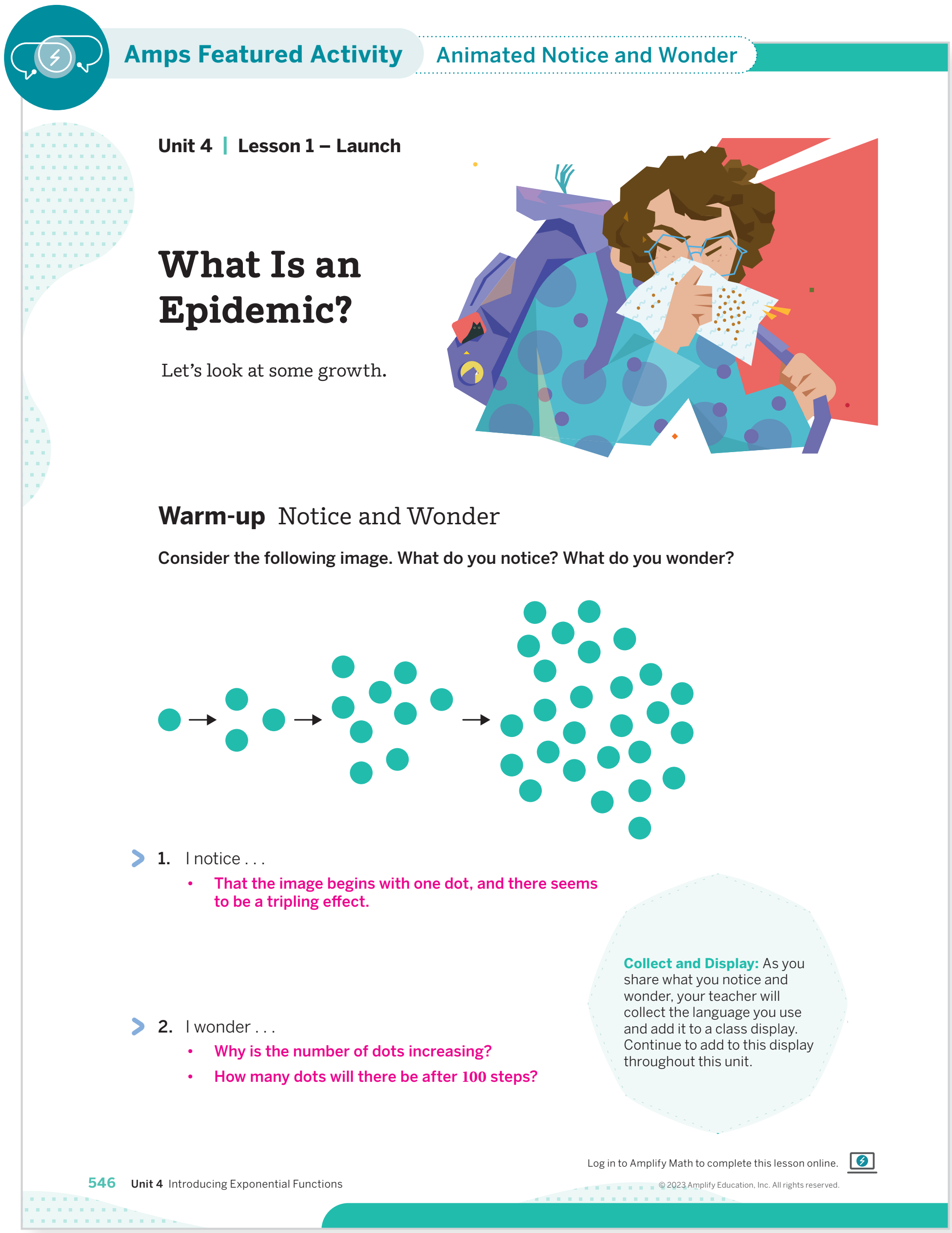
- In **Activity 1**, only conduct two (or even one) of the simulations.
- In **Activity 2**, omit Problem 4.

Independent | 5 min

Warm-up Notice and Wonder

Students examine an image that depicts an increasing pattern. (It's exponential, but we're not calling it that just yet.)

Amps Featured Activity Animated Notice and Wonder



Unit 4 | **Lesson 1 – Launch**

What Is an Epidemic?

Let's look at some growth.

Warm-up Notice and Wonder

Consider the following image. What do you notice? What do you wonder?

1. I notice . . .
   - **That the image begins with one dot, and there seems to be a tripling effect.**

2. I wonder . . .
   - **Why is the number of dots increasing?**
   - **How many dots will there be after 100 steps?**

Collect and Display: As you share what you notice and wonder, your teacher will collect the language you use and add it to a class display. Continue to add to this display throughout this unit.

Log in to Amplify Math to complete this lesson online.

546 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students a minute of think-time to study the image. Conduct the ***Notice and Wonder*** routine. Tell them there are no wrong answers.

2 Monitor

Help students get started by asking what they see or do not see changing within the image.

Look for points of confusion:

- **Misinterpreting the pattern as linear.** Ask, "Does the pattern add a constant number of dots each time?"
- **Not realizing the growth factor is constant.** Ask the students to write the number of dots above each set and consider how they change.

Look for productive strategies:

- Describing the image *qualitatively* (e.g., the image begins with one dot and ends with many dots).
- Describing the image *quantitatively* (e.g., counting to determine how the number of dots is increasing).

3 Connect

Display the visual representation of the (exponential) growth pattern.

Have students share what they notice and wonder about the image. Record and display their thinking. Ask students if they have questions about what is on the list.

Ask, "How does this growth pattern compare to linear growth in earlier units?" It does not have a common difference.

Define the term *nonlinear*, a word students will have encountered before. (Do not introduce the term *exponential* just yet — that will be introduced in Lesson 4.)

Math Language Development

MLR2: Collect and Display

During the Connect, as students share what they notice and wonder about the growth pattern, listen for and amplify language they use to communicate about *tripling*, *rate of change*, and *common factor*. Create a class display with these terms and continue adding to this display throughout the unit.

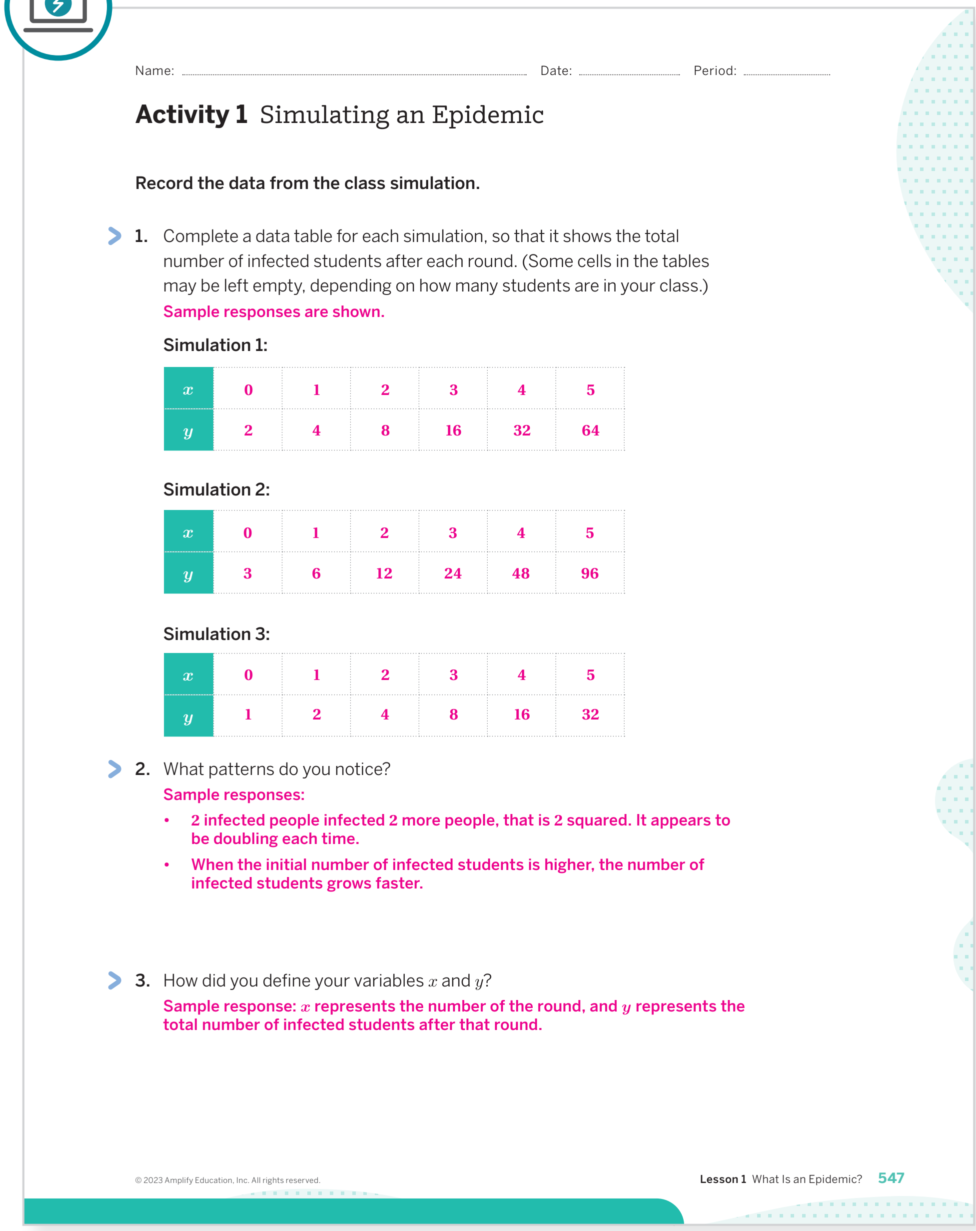
English Learners

Consider including the growth pattern on the display with annotations connecting student language to the diagram.

Whole Class | 20 min

Activity 1 Simulating an Epidemic

Students simulate an epidemic, gather data, define variables, and construct a nonlinear model to represent the simulation.



Name: ______________________ Date: __________ Period: __________

Activity 1 Simulating an Epidemic

Record the data from the class simulation.

1. Complete a data table for each simulation, so that it shows the total number of infected students after each round. (Some cells in the tables may be left empty, depending on how many students are in your class.)
Sample responses are shown.

Simulation 1:

x	0	1	2	3	4	5
y	2	4	8	16	32	64

Simulation 2:

x	0	1	2	3	4	5
y	3	6	12	24	48	96

Simulation 3:

x	0	1	2	3	4	5
y	1	2	4	8	16	32

2. What patterns do you notice?
Sample responses:
   - 2 infected people infected 2 more people, that is 2 squared. It appears to be doubling each time.
   - When the initial number of infected students is higher, the number of infected students grows faster.

3. How did you define your variables x and y?
Sample response: x represents the number of the round, and y represents the total number of infected students after that round.

1 Launch

Explain the following rules for the simulation. For each simulation, each infected student infects one other person each round.

Simulation 1: Two students are initially infected.

Simulation 2: Three students are initially infected.

Simulation 3: One student is initially infected.

Perform multiple simulations of the disease spread, randomly and confidentially selecting a number of students to be initially infected before each simulation. Students should record the data from the spread over several rounds, until the entire class is infected.

2 Monitor

Look for points of confusion:

- **Having difficulty identifying the variables.** Ask, "Which variable is independent: the simulation round or the number of infected students?"
- **Struggling to identify a pattern.** Use arrows, with operations and numbers, to show changes in the tables.
- **Having difficulty choosing a scale for the graphs.** Ask about the smallest and largest values. Say, "Does it make sense to count by 1's, 2's, 5's?"

Look for productive strategies:

- Analyzing the tables to see what is happening from one round to the next.
- Noticing that each person infects another, so that the total number of infected students doubles each round.
- Recognizing that the growth is not linear.

Activity 1 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Visualization and Processing*

Provide students with the first column of values for each simulation and say, "These are the initial number of students who are infected." Consider displaying the rules for each simulation and demonstrate how to determine the values in the second column. Have students complete the rest of the tables.

Math Language Development

MLR7: Compare and Connect

During the Connect, connect similarities and differences between the shapes of the graphs and the rules for each simulation. Ask:

- "How were the rules for each simulation similar? How do you see these similarities in the graphs?" Amplify language, such as *doubling*, *increasing rapidly*, *nonlinear*, *curved*, and connect this to each student infecting one other student each round.
- "How were the rules for each simulation different? How do you see these differences in the graphs?" Listen for students who recognize that the number of students who were initially infected was different for each simulation.

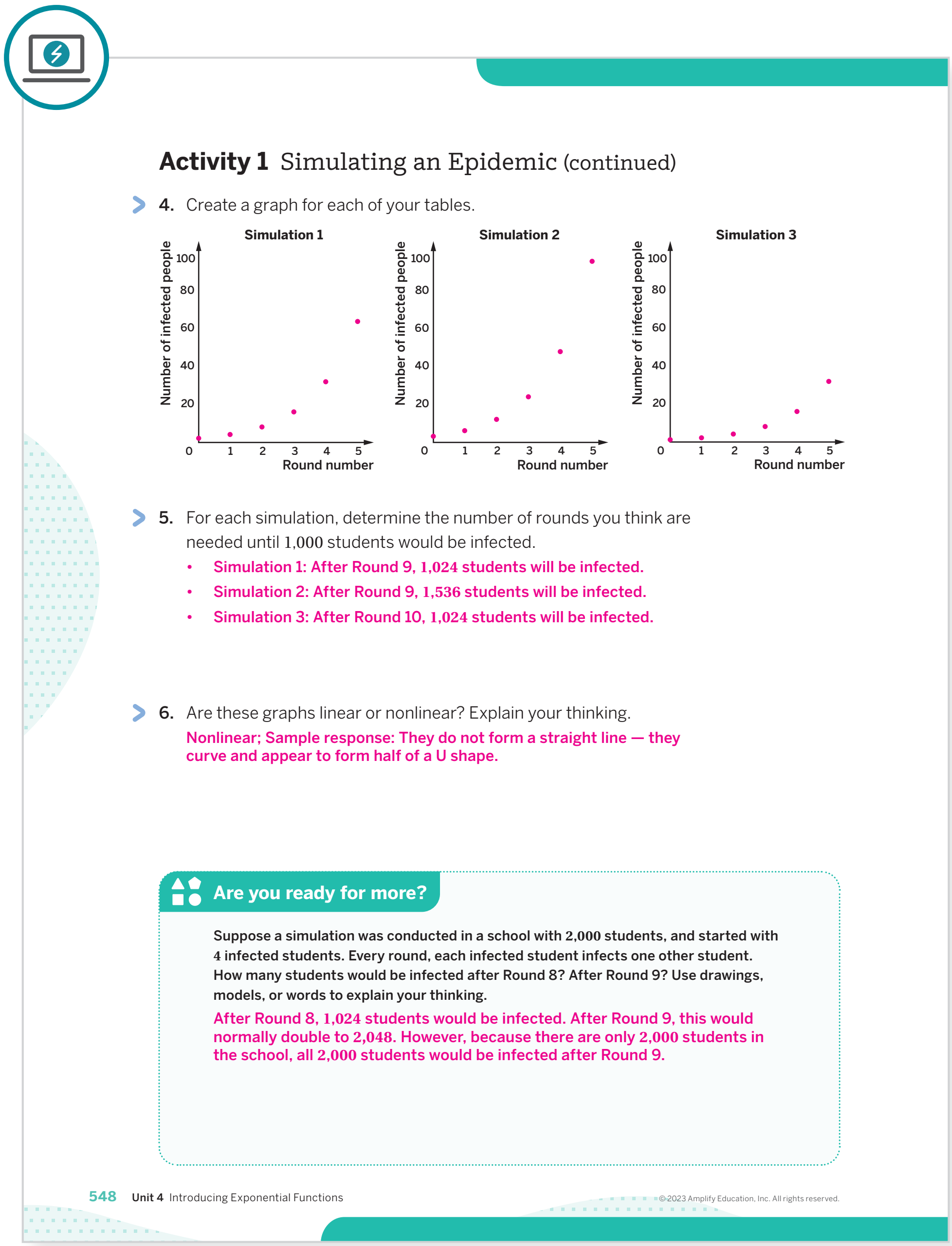
English Learners

Circle the value on each graph that shows the initial number of students infected to help students make this connection.

Whole Class | 20 min

Activity 1 Simulating an Epidemic (continued)

Students simulate an epidemic, gather data, define variables, and construct a nonlinear model to represent the simulation.



Activity 1 Simulating an Epidemic (continued)

4. Create a graph for each of your tables.

5. For each simulation, determine the number of rounds you think are needed until 1,000 students would be infected.
   - Simulation 1: After Round 9, 1,024 students will be infected.
   - Simulation 2: After Round 9, 1,536 students will be infected.
   - Simulation 3: After Round 10, 1,024 students will be infected.

6. Are these graphs linear or nonlinear? Explain your thinking.
   Nonlinear; Sample response: They do not form a straight line — they curve and appear to form half of a U shape.

Are you ready for more?

Suppose a simulation was conducted in a school with 2,000 students, and started with 4 infected students. Every round, each infected student infects one other student. How many students would be infected after Round 8? After Round 9? Use drawings, models, or words to explain your thinking.

After Round 8, 1,024 students would be infected. After Round 9, this would normally double to 2,048. However, because there are only 2,000 students in the school, all 2,000 students would be infected after Round 9.

548 Unit 4 Introducing Exponential Functions

©2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the completed simulation tables.

Have students share patterns they noticed, along with their graphs. Select students that mentioned "doubling." Look for graphs that are exponential, but note that students have not been formally introduced to exponential functions. Have students describe the graphs in their own words.

Ask, "How do the patterns of the epidemic simulation differ from linear patterns?" The patterns of the epidemic simulation do not have a common difference. Their graphs do not form a straight line and instead curve upwards.

Whole Class | 15 min

Activity 2 Real-World Implications

Students predict how the data from their simulations would change, given real-world constraints.

Name: ________________ Date: ________ Period: ________

Activity 2 Real-World Implications

1. Predict how your graphs would change if every sick person infected 2 new people each round (rather than just 1). What do you think would happen to the shape of the graph?
   Sample response: The pattern would triple, rather than double. That means the graph would be steeper, and the number of infected people would increase at a faster rate.

2. In the real world, infected people might come in contact with other infected people (rather than always infecting someone new). How might this change the outcome of the simulation?
   Sample response: As more people get sick, infected people are more likely to be near each other. So, eventually the graph would begin to flatten.

3. In the real world, some people might be immune. Predict how this could change the outcome of the simulation.
   Sample response: The infection will only spread to those who are not immune. The graph would be less steep.

4. Name other real-world phenomena that could result in these types of patterns.
   Sample responses: computer viruses, memes on social media, rumors, credit card interest, population growth.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 What Is an Epidemic? 549

1 Launch

Tell students that they will use their graphs from Activity 1 to complete the problems. Provide students with additional graph paper or chart paper, and give them the option to use spreadsheet technology.

2 Monitor

Help students get started by having them prepare a table or draw models for Problem 1.

Look for points of confusion:

- **Having difficulty visualizing changes to the graph.** Provide students with blank graphs that have different scales to help process each change.
- **Not understanding the term *immune*.** Define this term for students, and provide blank tables if helpful. If needed, help students complete the table or have them draw a model.

3 Connect

Display the table and graph of the number of doubling cases alongside the first simulation graph.

Have students share how they think each constraint would affect the spread of disease.

Highlight that the disease spreads more slowly as more people are ultimately infected, and if more people are naturally immune.

Ask, "How do you think an epidemic can be modeled algebraically?" With an equation.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide blank tables for students to use if they choose to record updated values in the scenario where every sick person infected 2 new people each round.

Provide access to colored pencils and suggest students use a different color to plot the new points if they plot them on the same graphs from Activity 1.

Extension: *Math Enrichment*

Have students make a prediction for how their graphs would change based on each of the following scenarios. Ask them which scenario would have a faster growth as the number of rounds increases. Have them use graphing technology or graph paper to check their predictions.

Scenario 1: 3 people are initially infected and infect 2 more people each round.

Scenario 2: 2 people are initially infected and infect 3 more people each round.

Scenario 2 has a faster growth because the number of people infected triples. By the third round, Scenario 2 has infected more people.

Whole Class | 5 min

Summary Infectious Diseases, Vaccines, and Costs

Review and synthesize the advantages of using tables and graphs to model an epidemic.

Narrative Connections

Unit 4 Introducing Exponential Functions

Infectious Diseases, Vaccines, and Costs

In January of 2020, the first cases of the novel coronavirus, COVID-19, were reported in the United States. Many officials and some of the American public were worried about this disease. The disease appeared to be 20 times more lethal than the seasonal flu, while at the same time being more contagious than the common cold. Worse, there were concerns about it being spread by asymptomatic transmission, meaning if you were infected with COVID-19 but felt healthy, you could still transmit the virus to others.

Amid these concerns, most Americans were *not* worried, because not many people had been infected at the time. On February 1 of 2020, there were only eight confirmed cases in the U.S. By March 1, there were 89. Among a population of 300+ million, how could small numbers like these be so alarming?

As you saw in today's lesson, when the number of infected people keeps doubling, then doubling again, and again and again, things can get overwhelming pretty fast. By April 1, there were more than 200,000 reported cases in the U.S. And a year later, even with protective measures that encouraged social distancing and mask wearing, there were more than 30 million cases and 500,000 dead in the U.S. alone.

This explosive growth is completely unlike the functions you have seen up to this point. In these next lessons, not only will you map out this kind of growth, but you will think about how to manage and predict it. You will hear stories about the toll diseases have taken on us as a species, as well as stories of perseverance and hope — and what we can do to fight back.

Welcome to Unit 4.

550 **Unit 4** Introducing Exponential Functions

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the epidemic simulation graphs from Activity 1.

Have students share what they found most interesting about today's lesson and what more they would like to learn.

Ask, "How do you think the graph would change if everyone who was infected took medicine to prevent the spread of the disease?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

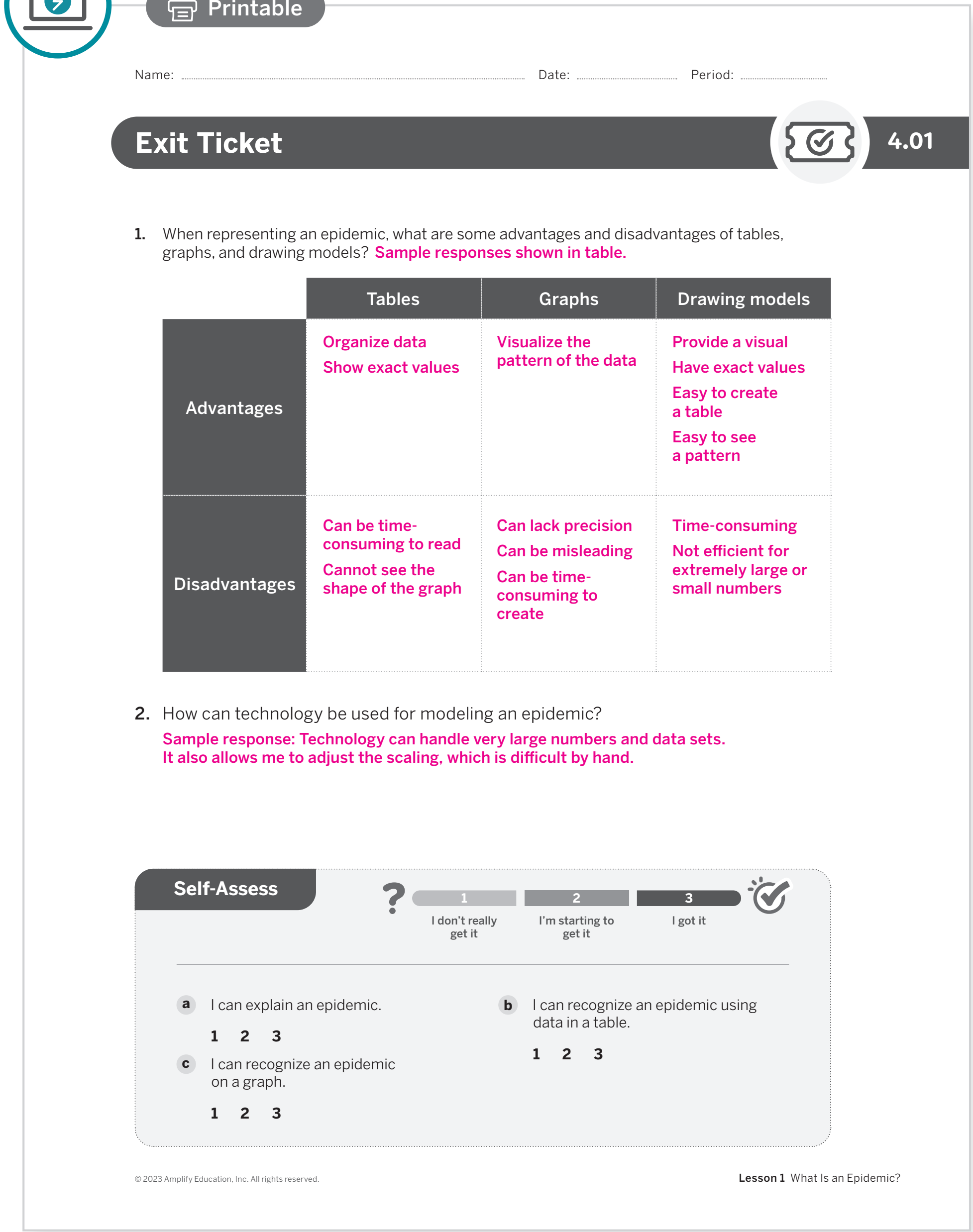
- "How can you describe in your own words how quickly an epidemic can grow or spread?"
- "If a pattern *doubles* or *triples*, why does this not show linear growth?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by explaining the advantages of using tables and graphs to model an epidemic.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 4.01

1. When representing an epidemic, what are some advantages and disadvantages of tables, graphs, and drawing models? Sample responses shown in table.

	Tables	Graphs	Drawing models
Advantages	Organize data Show exact values	Visualize the pattern of the data	Provide a visual Have exact values Easy to create a table Easy to see a pattern
Disadvantages	Can be time-consuming to read Cannot see the shape of the graph	Can lack precision Can be misleading Can be time-consuming to create	Time-consuming Not efficient for extremely large or small numbers

2. How can technology be used for modeling an epidemic?
Sample response: Technology can handle very large numbers and data sets. It also allows me to adjust the scaling, which is difficult by hand.

Self-Assess

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it ✓

a I can explain an epidemic.
1 2 3

b I can recognize an epidemic using data in a table.
1 2 3

c I can recognize an epidemic on a graph.
1 2 3

Success looks like . . .

- **Language Goal:** Explaining and giving examples of an epidemic. **(Speaking and Listening)**
- **Goal:** Recognizing the growth pattern of an epidemic, given a table.
- **Goal:** Recognizing a graphical representation of an epidemic model.
  - » Explaining the advantages and disadvantages of tables, graphs, and drawing models that represent an epidemic.

Suggested next steps

If students struggle to come up with advantages and disadvantages for each representation, consider:

- Asking, "In what way may a table be more useful than a graph? In what way may a graph be more useful than a table?"
- Asking, "How can drawing a model help you in creating a table or graph? When might drawing a model be less helpful?"

Professional Learning

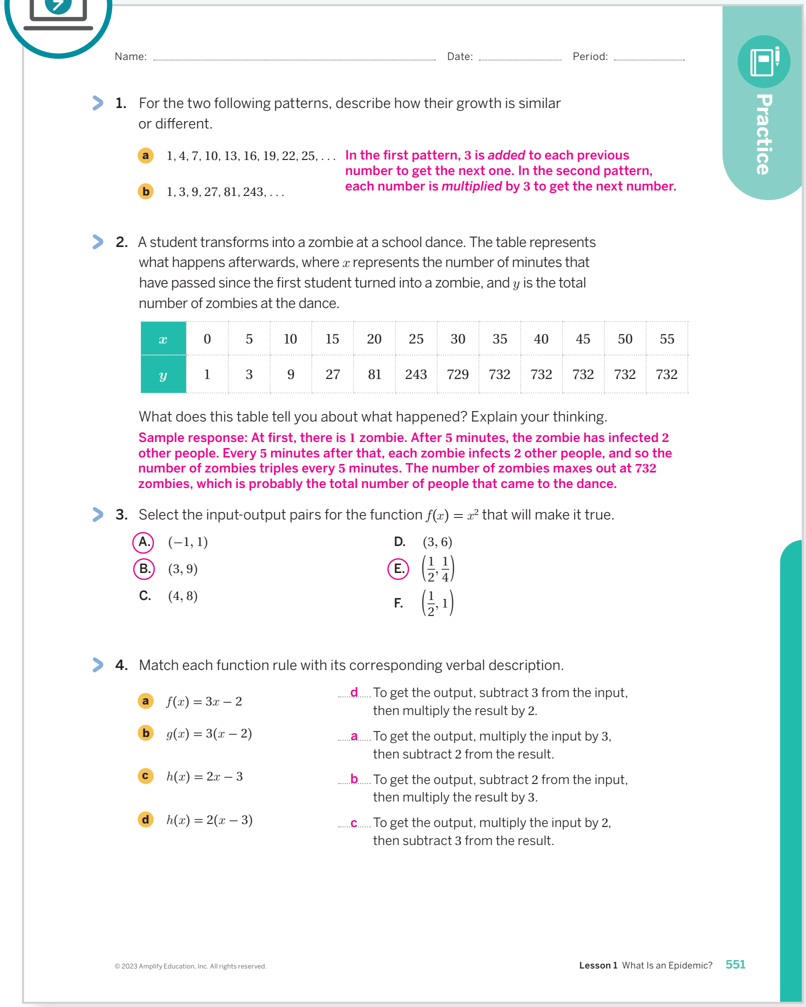
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which students' ideas were you able to highlight during Activity 1 as students described the patterns between the simulation tables?
- In what ways did Activity 2 go as planned, or not go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ____________ Date: ________ Period: ________

Practice

1. For the two following patterns, describe how their growth is similar or different.
   - a $1, 4, 7, 10, 13, 16, 19, 22, 25, \ldots$
   - b $1, 3, 9, 27, 81, 243, \ldots$

   In the first pattern, 3 is *added* to each previous number to get the next one. In the second pattern, each number is *multiplied* by 3 to get the next number.

2. A student transforms into a zombie at a school dance. The table represents what happens afterwards, where x represents the number of minutes that have passed since the first student turned into a zombie, and y is the total number of zombies at the dance.

x	0	5	10	15	20	25	30	35	40	45	50	55
y	1	3	9	27	81	243	729	732	732	732	732	732

   What does this table tell you about what happened? Explain your thinking.

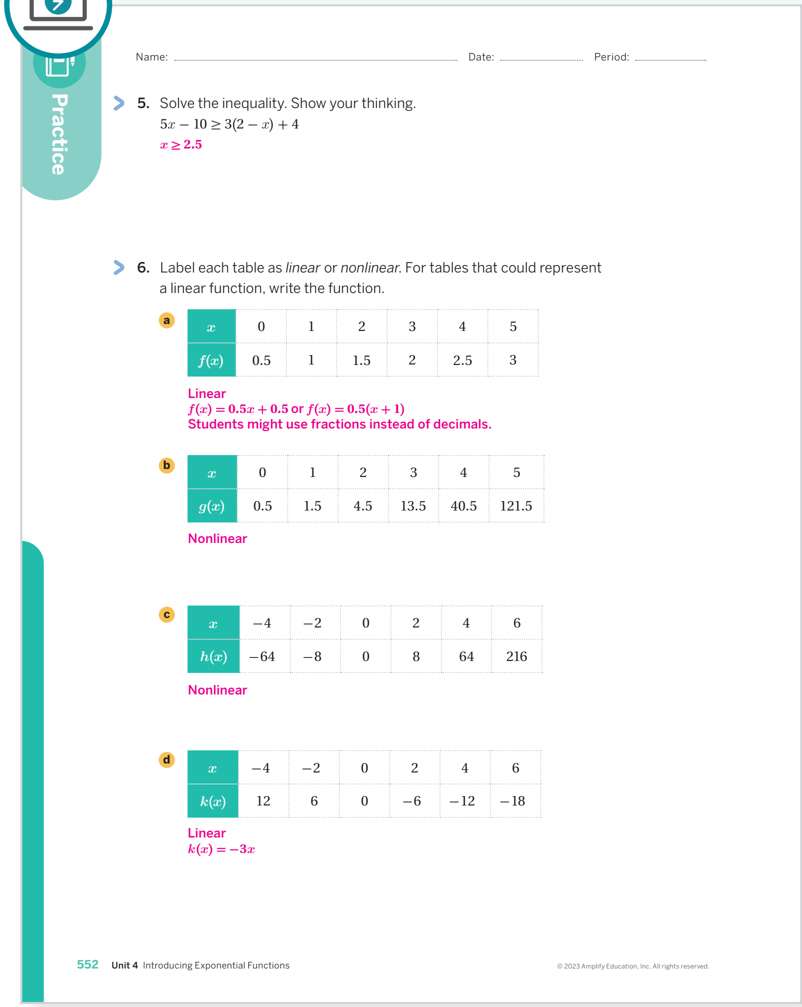
   Sample response: At first, there is 1 zombie. After 5 minutes, the zombie has infected 2 other people. Every 5 minutes after that, each zombie infects 2 other people, and so the number of zombies triples every 5 minutes. The number of zombies maxes out at 732 zombies, which is probably the total number of people that came to the dance.

3. Select the input-output pairs for the function $f(x) = x^2$ that will make it true.

   [✓ A.] $(-1, 1)$
   [✓ B.] $(3, 9)$
   C. $(4, 8)$
   D. $(3, 6)$
   [✓ E.] $\left(\frac{1}{2}, \frac{1}{4}\right)$
   F. $\left(\frac{1}{2}, 1\right)$

4. Match each function rule with its corresponding verbal description.
   - a $f(x) = 3x - 2$
   - b $g(x) = 3(x - 2)$
   - c $h(x) = 2x - 3$
   - d $h(x) = 2(x - 3)$

   d To get the output, subtract 3 from the input, then multiply the result by 2.
   a To get the output, multiply the input by 3, then subtract 2 from the result.
   b To get the output, subtract 2 from the input, then multiply the result by 3.
   c To get the output, multiply the input by 2, then subtract 3 from the result.



Name: ____________ Date: ________ Period: ________

Practice

5. Solve the inequality. Show your thinking.

   $5x - 10 \geq 3(2 - x) + 4$

   $x \geq 2.5$

6. Label each table as *linear* or *nonlinear*. For tables that could represent a linear function, write the function.

   a

x	0	1	2	3	4	5
$f(x)$	0.5	1	1.5	2	2.5	3

   Linear
   $f(x) = 0.5x + 0.5$ or $f(x) = 0.5(x + 1)$
   Students might use fractions instead of decimals.

   b

x	0	1	2	3	4	5
$g(x)$	0.5	1.5	4.5	13.5	40.5	121.5

   Nonlinear

   c

x	-4	-2	0	2	4	6
$h(x)$	-64	-8	0	8	64	216

   Nonlinear

   d

x	-4	-2	0	2	4	6
$k(x)$	12	6	0	-6	-12	-18

   Linear
   $k(x) = -3x$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
Spiral	3	Unit 3 Lesson 11	1
	4	Unit 3 Lesson 4	2
	5	Unit 1 Lesson 19	1
Formative ⚡	6	Unit 4 Lesson 2	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 1

Looking at Growth

Whole Class

In this Sub-Unit, students examine exponential ("nonlinear") functions — particularly looking at growth — using tables and graphs through a variety of scientific, fictional, and historical contexts.

SUB-UNIT

1 Looking at Growth

Narrative Connections

Where do baby bacteria come from?

Picture this:

You're somewhere warm and surrounded by water. You see nothing, you feel nothing. But everything is perfect. The water is perfect, the pH is perfect, and the environment is rich with nutrients.

Inside you, something happens . . .

A great burst of energy, as your DNA uncoils — tearing itself into separate strands, like a strip of leather. They swim apart, pulling and stretching against your insides.

They pull so hard that you feel your midsection tighten, like a rubber band about to snap. And then all of a sudden, *POP* — now there are two of you!

This is the process of "binary fission," and it's how bacteria grow — from one cell into two, two into four, four into eight, on and on, sometimes in a matter of minutes. The process is so dramatic that a single cell of *E. coli* can grow into a million cells in just seven hours.

The speed and magnitude of this kind of growth can be overwhelming at first. It seems different from linear growth, but how? If you take your time and look at the patterns, you can put it into perspective. With the right tools, numbers that once seemed beyond comprehension can become predictable.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore a special kind of nonlinear growth — within medical and social contexts — in the following places:

- **Lesson 2, Activities 1–2:** Pharmacy Expansion, Friends and Followers
- **Lesson 3, Activities 1–2:** Viral Memes, Viral Reproduction Number

Optional

UNIT 4 | **LESSON 2**

Patterns of Growth

Let's compare different patterns of growth.

Focus

Goals

1. **Language Goal:** Describe patterns in tables that represent linear and exponential (here, referred to as "nonlinear") relationships. **(Speaking and Listening, Writing)**
2. Create tables and write expressions given descriptions of linear and exponential (here, referred to as "nonlinear") relationships.

Coherence

Today

Students study linear and exponential patterns by examining tables, observing common differences and common factors between data points. Students match tables with corresponding expressions, and construct a table given a description of an exponential relationship.

Previously

In Grade 8, students were informally introduced to exponential expressions as they explored the rules of exponents.

Coming Soon

Students will continue to build on their conceptual understanding of linear and exponential relationships by exploring tables and graphs showing different growth patterns.

Rigor

- Students further develop their **conceptual understanding** of nonlinear growth by making tables and contrasting with linear growth.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils (as needed)

Math Language Development

New words

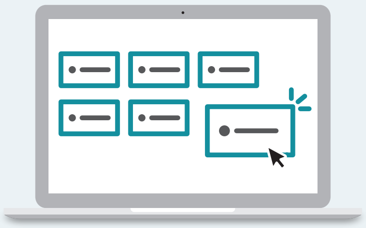
- ***common difference****
- ***common factor***

*Students may confuse the term *difference* with its everyday use, "to be different." Be ready to address that *difference* in this context refers to subtraction.

Amps | Featured Activity

Activity 2
Digital Card Sort

Students match tables and expressions to the scenarios that they represent by dragging and connecting them on screen. Instead of walking from student to student, work can be seen digitally in real-time.



Building Math Identity and Community

Connecting to Mathematical Practices

Working with two different scenarios at the same time might cause some students to become disorganized, either in their work or mentally. In order to stay organized, have students identify the mathematical operation associated with each scenario before working on the rest of the activity. By writing the operation next to the scenario, students can quickly reference their notes to keep the differences in the scenarios straight in their thinking.

Modifications to Pacing

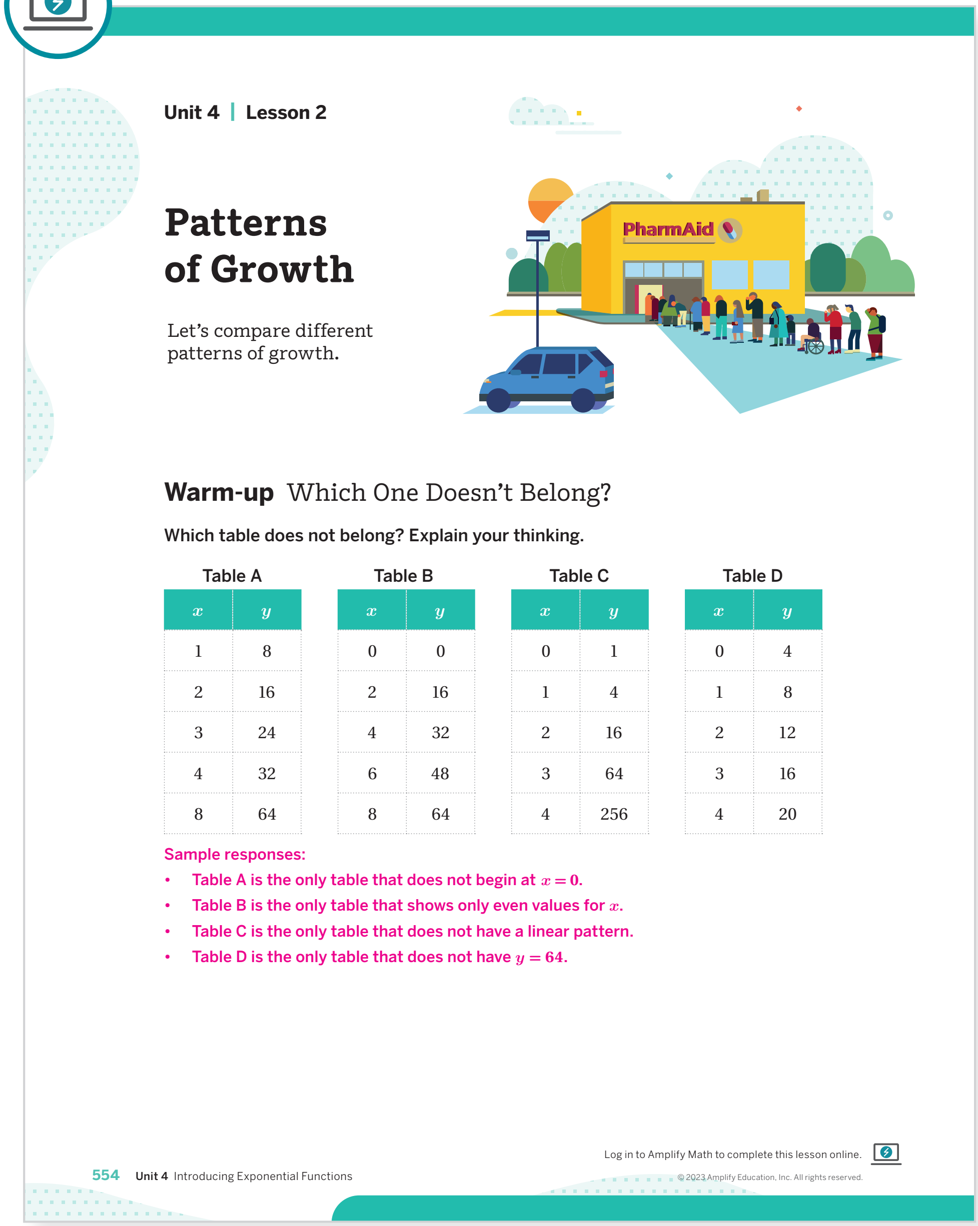
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, have students complete the table to an earlier year, such as Year 7. Modify Problem 6 to ask about the same number of years.

Independent | 10 min

Warm-up Which One Doesn't Belong?

Students compare tables of linear and nonlinear data to determine which table doesn't belong.



Unit 4 | **Lesson 2**

Patterns of Growth

Let's compare different patterns of growth.

Warm-up Which One Doesn't Belong?

Which table does not belong? Explain your thinking.

Table A

x	y
1	8
2	16
3	24
4	32
8	64

Table B

x	y
0	0
2	16
4	32
6	48
8	64

Table C

x	y
0	1
1	4
2	16
3	64
4	256

Table D

x	y
0	4
1	8
2	12
3	16
4	20

Sample responses:

- Table A is the only table that does not begin at $x = 0$.
- Table B is the only table that shows only even values for x.
- Table C is the only table that does not have a linear pattern.
- Table D is the only table that does not have $y = 64$.

Log in to Amplify Math to complete this lesson online.

554 Unit 4 Introducing Exponential Functions

1 Launch

Display the four tables. Conduct the ***Which One Doesn't Belong?*** routine. Give students independent work time before having them share their thinking with a partner.

2 Monitor

Help students get started by prompting them to first determine a pattern for each table.

Look for points of confusion:

- **Struggling to find a pattern.** Encourage students to use addition or multiplication to determine a pattern in the y column.

Look for productive strategies:

- Drawing lines or arcs between consecutive terms in the y column of each table to understand the rate of change. Encourage students to label them with an operation and number to highlight the difference between the patterns.

3 Connect

Have students share one reason why a table does not belong. After each response, ask the class whether they agree or disagree.

Highlight that Tables A, B, and D are linear and Table C is nonlinear. Emphasize how the structure of each table illustrates whether the pattern is linear or nonlinear. Linear patterns have a constant rate of change, or common difference between consecutive terms.

Define the term ***common difference*** as the difference between two consecutive terms in a linear pattern.

Ask, "What pattern do you notice in Table C? How does this differ from the other tables?"

Math Language Development

MLR2: Collect and Display

As students share their partners' choice and reasoning, collect the language they use to describe the tables, such as *linear/nonlinear*. Highlight phrases that describe a common difference between two consecutive terms. Add these terms and phrases, particularly *common difference*, to the class display.

English Learners

Highlight the distinction and connection between difference in the sense of subtraction and difference used in a more general everyday meaning.

Power-up

To power up students' ability to determine if a table is linear or nonlinear, have students complete:

Recall that a table of values is *linear* if it has a constant rate of change for any two coordinate pairs. Determine whether each table is *linear* or *nonlinear*. If it is linear, calculate the slope of the line represented by the pairs of values.

a.

x	0	2	4
y	0	2	8

Nonlinear

b.

x	0	2	4
y	4	6	8

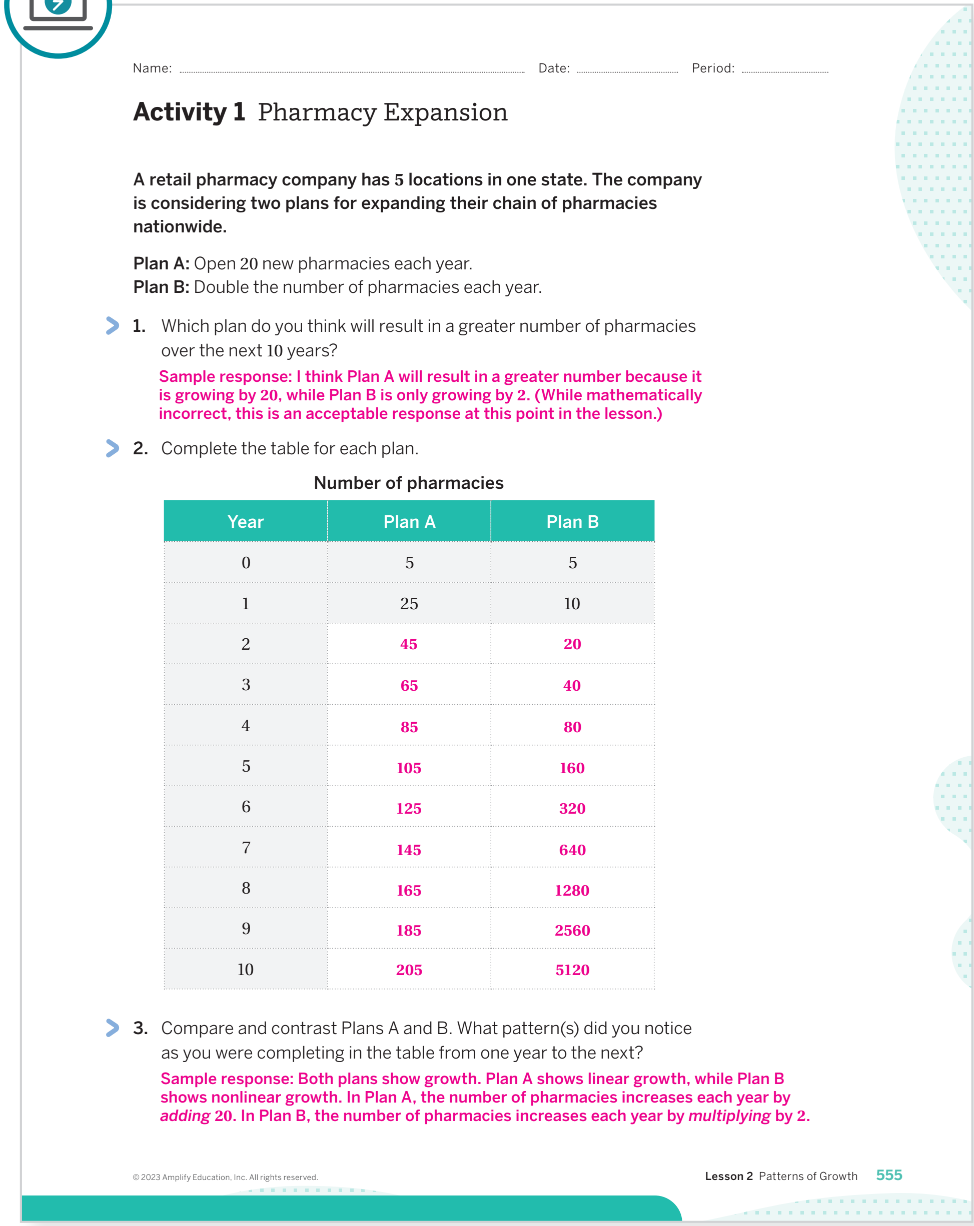
Linear
Slope is 2.

Use: Before the Warm-up
Informed by: Performance on Lesson 1, Practice Problem 6

Pairs | 15 min

Activity 1 Pharmacy Expansion

Students use mathematics to model two plans for the expansion of a pharmacy, determining which results in more growth.



Name: ________ Date: ________ Period: ________

Activity 1 Pharmacy Expansion

A retail pharmacy company has 5 locations in one state. The company is considering two plans for expanding their chain of pharmacies nationwide.

Plan A: Open 20 new pharmacies each year.
Plan B: Double the number of pharmacies each year.

1. Which plan do you think will result in a greater number of pharmacies over the next 10 years?

   Sample response: I think Plan A will result in a greater number because it is growing by 20, while Plan B is only growing by 2. (While mathematically incorrect, this is an acceptable response at this point in the lesson.)

2. Complete the table for each plan.

Number of pharmacies

Year	Plan A	Plan B
0	5	5
1	25	10
2	**45**	**20**
3	**65**	**40**
4	**85**	**80**
5	**105**	**160**
6	**125**	**320**
7	**145**	**640**
8	**165**	**1280**
9	**185**	**2560**
10	**205**	**5120**

3. Compare and contrast Plans A and B. What pattern(s) did you notice as you were completing in the table from one year to the next?

   Sample response: Both plans show growth. Plan A shows linear growth, while Plan B shows nonlinear growth. In Plan A, the number of pharmacies increases each year by *adding* 20. In Plan B, the number of pharmacies increases each year by *multiplying* by 2.

1 Launch

Read the prompt aloud. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them determine the precise number of stores that open each year. Refer them to the strategies they used in the Warm-up.

Look for points of confusion:

- **Struggling to describe Plan B in words.** Encourage students to use the term *nonlinear* to describe patterns that do not have a constant rate of change.

Look for productive strategies:

- Using repeated calculations in tables or written expressions.

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Be sure students understand that each plan begins with 5 locations. Ask them where they see this information in the introductory text.

Accessibility: *Vary Demands to Optimize Challenge*

Have students complete the table in Problem 2 through Year 5. Then provide them with the rest of the values and have them continue the activity with Problem 3.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention between the linear/nonlinear growth patterns and the term *common difference*. Ask:

- "Which plan shows linear growth? Nonlinear growth? Which has a common difference?"
- "What is it about linear relationships that indicates there will be a common difference?"
- "In the description for Plan B, the term *double* was used. Does this term describe a common difference? What does it describe?"

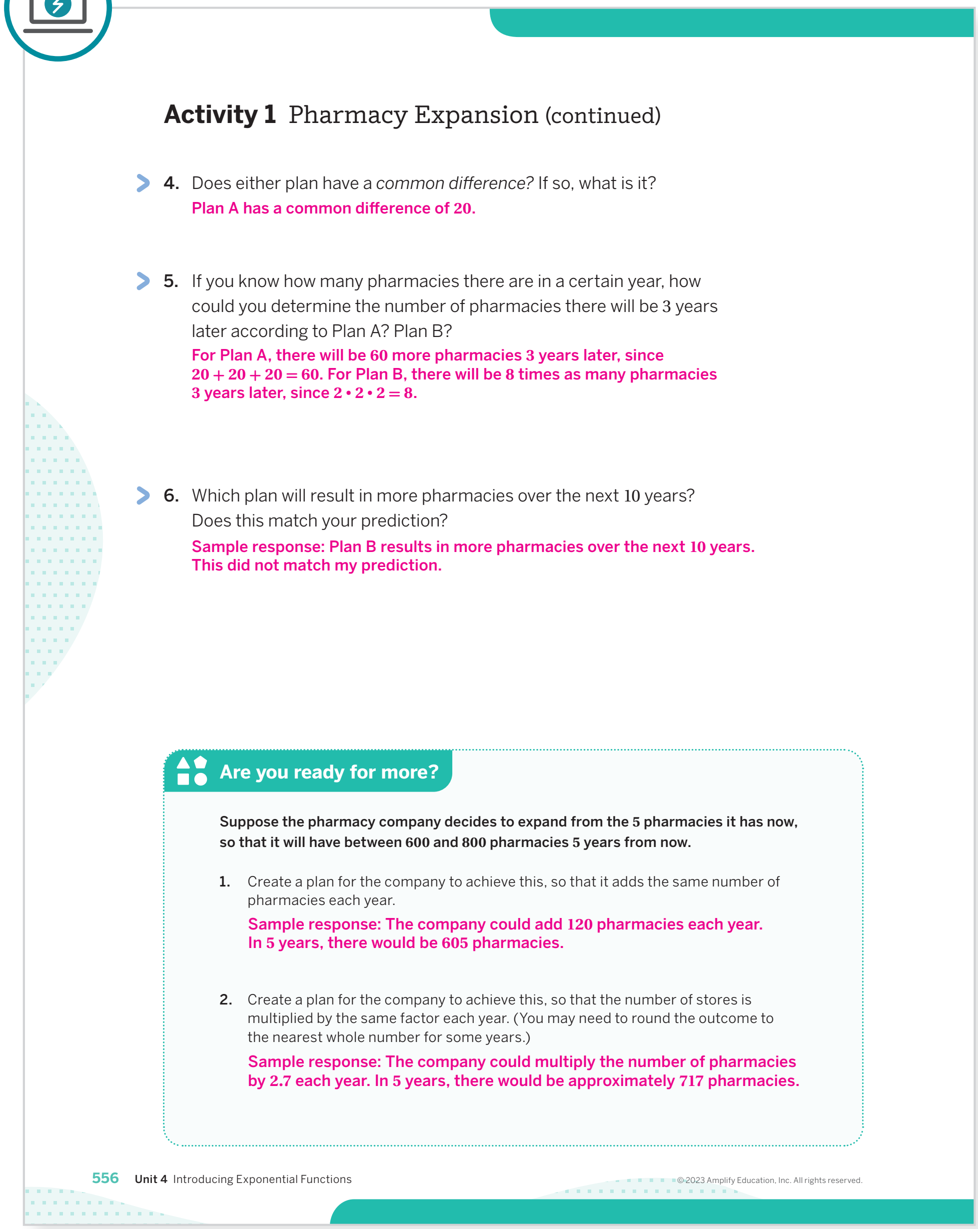
English Learners

Annotate the tables with the terms *linear*, *nonlinear*, and *common difference*. Show how the table for Plan A illustrates the common difference of 20. Highlight this information described in the text for Plan A.

Pairs | 15 min

Activity 1 Pharmacy Expansion (continued)

Students use mathematics to model two plans for the expansion of a pharmacy, determining which results in more growth.



Activity 1 Pharmacy Expansion (continued)

4. Does either plan have a *common difference?* If so, what is it?
   Plan A has a common difference of 20.

5. If you know how many pharmacies there are in a certain year, how could you determine the number of pharmacies there will be 3 years later according to Plan A? Plan B?
   For Plan A, there will be 60 more pharmacies 3 years later, since $20 + 20 + 20 = 60$. For Plan B, there will be 8 times as many pharmacies 3 years later, since $2 \cdot 2 \cdot 2 = 8$.

6. Which plan will result in more pharmacies over the next 10 years? Does this match your prediction?
   Sample response: Plan B results in more pharmacies over the next 10 years. This did not match my prediction.

Are you ready for more?

Suppose the pharmacy company decides to expand from the 5 pharmacies it has now, so that it will have between 600 and 800 pharmacies 5 years from now.

1. Create a plan for the company to achieve this, so that it adds the same number of pharmacies each year.
   Sample response: The company could add 120 pharmacies each year. In 5 years, there would be 605 pharmacies.

2. Create a plan for the company to achieve this, so that the number of stores is multiplied by the same factor each year. (You may need to round the outcome to the nearest whole number for some years.)
   Sample response: The company could multiply the number of pharmacies by 2.7 each year. In 5 years, there would be approximately 717 pharmacies.

3 Connect

Display a completed table for Problem 2.

Have students share how Plan A and Plan B are similar and how they are different. Model how to annotate the table as students describe the patterns they found.

Highlight that a pattern generated by a *common difference* is linear, while a pattern generated by a *common factor* is not linear. The term *common factor* will be formally defined in the next activity. For now, students should recognize that the linear pattern shows additive growth, while the nonlinear pattern in this activity does not show additive growth. Instead, the nonlinear pattern in this activity shows multiplicative growth. Demonstrate the use of precise mathematical vocabulary as you describe the growth patterns by using the terms *common difference*, *additive growth*, and *multiplicative growth*.

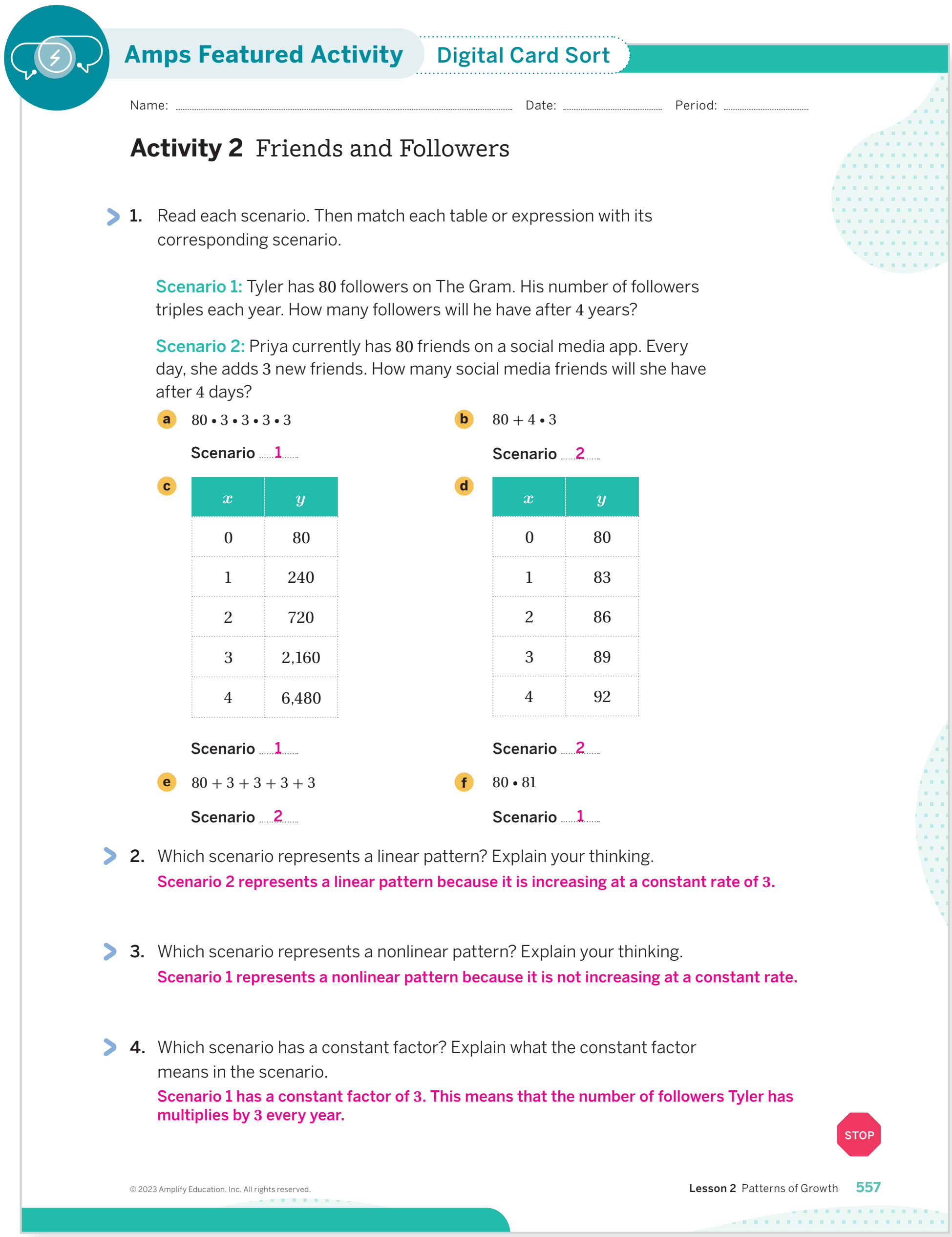
Ask, "Which plan resulted in the greatest number of pharmacies over 10 years? Is this what you predicted? How has your perspective changed?"

Pairs | 15 min

Activity 2 Friends and Followers

Students will read descriptions of two different scenarios and determine which representations (tables and expressions) match each description.

Amps Featured Activity Digital Card Sort



Name: ______ Date: ______ Period: ______

Activity 2 Friends and Followers

1. Read each scenario. Then match each table or expression with its corresponding scenario.

Scenario 1: Tyler has 80 followers on The Gram. His number of followers triples each year. How many followers will he have after 4 years?

Scenario 2: Priya currently has 80 friends on a social media app. Every day, she adds 3 new friends. How many social media friends will she have after 4 days?

a. $80 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Scenario 1

b. $80 + 4 \cdot 3$

Scenario 2

c.

x	y
0	80
1	240
2	720
3	2,160
4	6,480

Scenario 1

d.

x	y
0	80
1	83
2	86
3	89
4	92

Scenario 2

e. $80 + 3 + 3 + 3 + 3$

Scenario 2

f. $80 \cdot 81$

Scenario 1

2. Which scenario represents a linear pattern? Explain your thinking.
Scenario 2 represents a linear pattern because it is increasing at a constant rate of 3.

3. Which scenario represents a nonlinear pattern? Explain your thinking.
Scenario 1 represents a nonlinear pattern because it is not increasing at a constant rate.

4. Which scenario has a constant factor? Explain what the constant factor means in the scenario.
Scenario 1 has a constant factor of 3. This means that the number of followers Tyler has multiplies by 3 every year.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Patterns of Growth 557

1 Launch

Read both scenarios aloud. Draw attention to phrases indicating operations.

2 Monitor

Help students get started by having them generate a table for each scenario and then match these to the given tables.

Look for points of confusion:

- **Not knowing how to evaluate the expressions in a and e.** Ask, "Which scenario describes repeated addition? Which describes repeated multiplication?"

Look for productive strategies:

- Writing equivalent expressions for b and f to determine which is equivalent to a and which is equivalent to e.

3 Connect

Display the two scenarios and the expressions and tables in Problem 1.

Have students share their responses to Problem 1. Then use Problems 2–4 to facilitate a discussion about the patterns described in Scenarios 1 and 2.

Highlight that, in Scenario 1, the constant factor of 3 represents that the number of Tyler's followers becomes 3 times greater every year.

Define the term ***common factor*** as the factor that each term is multiplied by to generate a (type of) nonlinear pattern.

Ask, "What does the common difference represent in Scenario 2?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the quantities in each scenario. Ask:

- "How are these two scenarios similar? Different?"
- "Where can you see these similarities and differences in the expressions or tables?"

Extension: *Math Enrichment*

Have students use graphing technology to graph the tables in Problem 1c and 1d and describe what they notice. Sample response: Scenario 2 is a straight line and Scenario 1 is a curve that is increasing rapidly. They intersect at the point (0, 80), which is also shown in the table.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention between the linear/nonlinear growth patterns and the terms *common difference/factor.* Ask:

- "Where do you see the common difference/common factor in the expressions? In the table?"
- "Can a nonlinear growth pattern show a common difference for all values? Why or why not?"
- "Do you think that all nonlinear growth patterns will always show a common factor? Why or why not?"

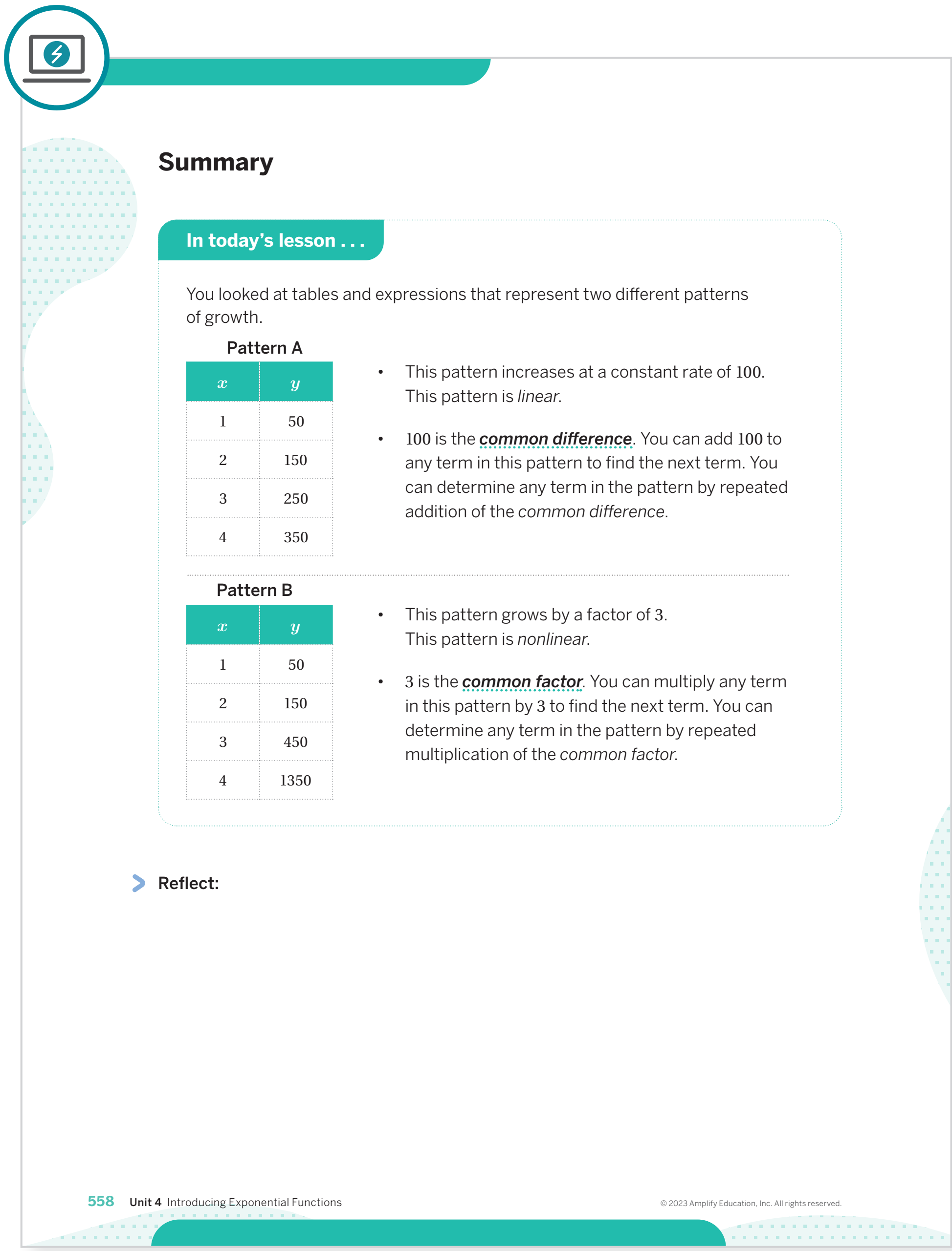
English Learners

Annotate the tables to highlight the common difference/factor. Add these visual examples to the class display.

Whole Class | 5 min

Summary

Review and synthesize how linear patterns grow differently than nonlinear patterns, specifically nonlinear patterns that show repeated multiplication by a common factor.



Summary

In today's lesson . . .

You looked at tables and expressions that represent two different patterns of growth.

Pattern A

x	y
1	50
2	150
3	250
4	350

- This pattern increases at a constant rate of 100. This pattern is *linear*.
- 100 is the ***common difference***. You can add 100 to any term in this pattern to find the next term. You can determine any term in the pattern by repeated addition of the *common difference*.

Pattern B

x	y
1	50
2	150
3	450
4	1350

- This pattern grows by a factor of 3. This pattern is *nonlinear*.
- 3 is the ***common factor***. You can multiply any term in this pattern by 3 to find the next term. You can determine any term in the pattern by repeated multiplication of the *common factor*.

Reflect:

558 **Unit 4** Introducing Exponential Functions

Synthesize

Display the tables for Pattern A and Pattern B.

Have students share the patterns they see in the tables.

Ask:

- "How do the values in each table grow?"
- "Which table of values shows a common difference? Which one shows a common factor?"
- "What expression can you write to determine the 5th term in Pattern A? The 6th term? The 7th term?"
- "What expression can you write to determine the 5th term in Pattern B? The 6th term? The 7th term?"

Highlight that a linear pattern increases at a constant rate and has a common difference and that a nonlinear pattern does not increase at a constant rate and has a common factor. Annotate the tables to help students visually see how the common difference and common factor determine the pattern of growth for each table.

Formalize vocabulary:

- ***common difference***
- ***common factor***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you describe, in your own words, what the terms *common difference* and *common factor* mean?"
- "How does a linear pattern show a common difference? Provide an example not illustrated in this lesson."
- "How might a nonlinear pattern show a common factor? Provide an example not illustrated in this lesson."

Math Language Development

MLR2: Collect and Display

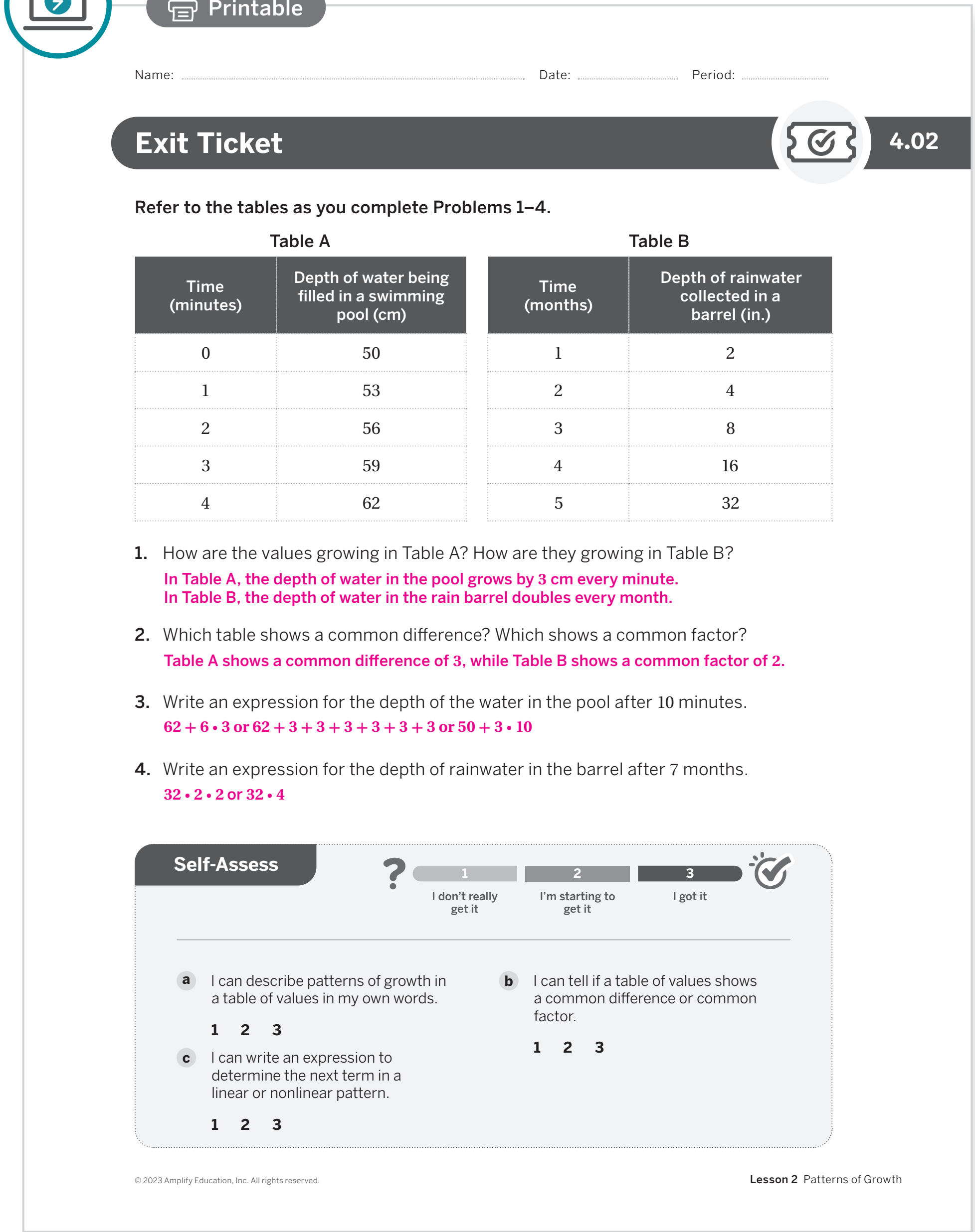
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *common difference* and *common factor* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the growth of two patterns and contextualizing the common difference or common factor.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 4.02

Refer to the tables as you complete Problems 1–4.

Table A

Time (minutes)	Depth of water being filled in a swimming pool (cm)
0	50
1	53
2	56
3	59
4	62

Table B

Time (months)	Depth of rainwater collected in a barrel (in.)
1	2
2	4
3	8
4	16
5	32

1. How are the values growing in Table A? How are they growing in Table B?
   In Table A, the depth of water in the pool grows by 3 cm every minute.
   In Table B, the depth of water in the rain barrel doubles every month.
2. Which table shows a common difference? Which shows a common factor?
   Table A shows a common difference of 3, while Table B shows a common factor of 2.
3. Write an expression for the depth of the water in the pool after 10 minutes.
   $62 + 6 \cdot 3$ or $62 + 3 + 3 + 3 + 3 + 3 + 3$ or $50 + 3 \cdot 10$
4. Write an expression for the depth of rainwater in the barrel after 7 months.
   $32 \cdot 2 \cdot 2$ or $32 \cdot 4$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can describe patterns of growth in a table of values in my own words.
   1 2 3

b. I can tell if a table of values shows a common difference or common factor.
   1 2 3

c. I can write an expression to determine the next term in a linear or nonlinear pattern.
   1 2 3

Success looks like . . .

- **Language Goal:** Describing patterns in the tables that represent linear and exponential (here, referred to as "nonlinear") relationships. **(Speaking and Listening, Writing)**
  - » Describing the growth patterns of Tables A and B in Problem 1.
- **Goal:** Creating tables and writing expressions given descriptions of linear and exponential (here, referred to as "nonlinear") relationships.

Suggested next steps

If students struggle to describe linear and nonlinear patterns of growth in Problem 1, consider:

- Revisiting the Warm-up activity.

If students are unable to identify a common difference or factor from a table of values in Problem 2, consider:

- Reviewing tables of values showing repeated addition and repeated multiplication.
- Revisiting the differences between Plan A and Plan B in Activity 1.

If students write incorrect or incomplete expressions to represent the patterns of growth in Problems 3 and 4, consider:

- Revisiting Activity 2.

Professional Learning

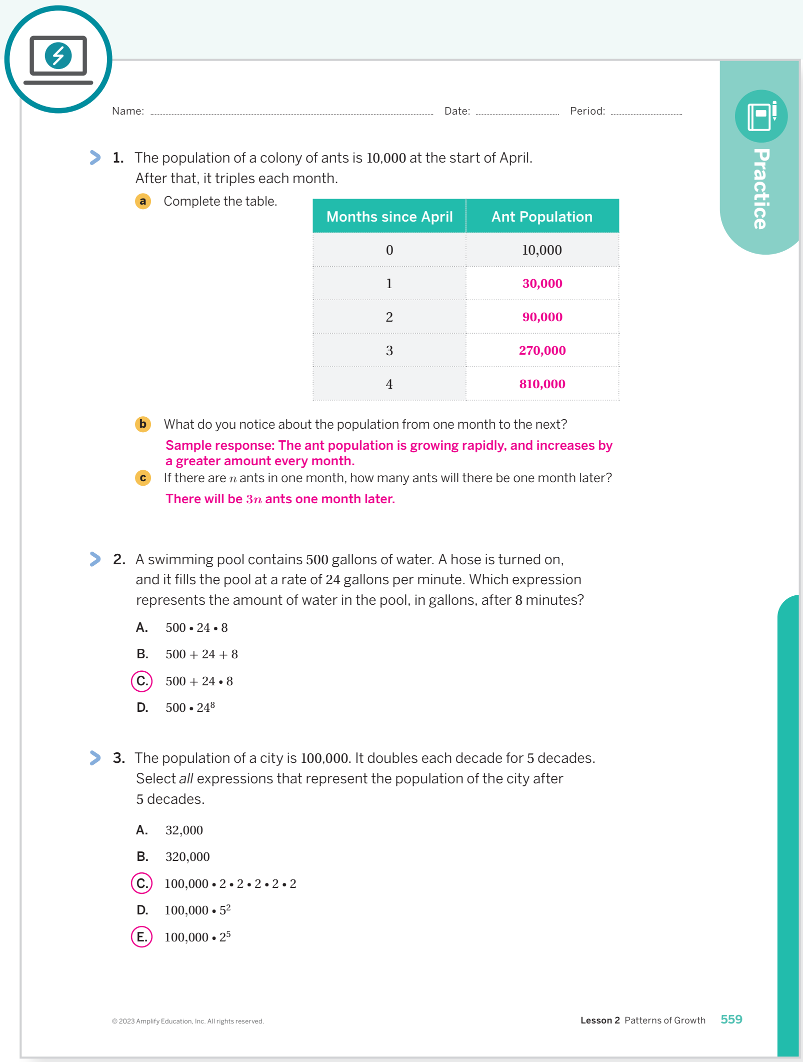
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students explored linear growth with exponential ("nonlinear") growth. How did that build on earlier understandings of linear functions from a prior unit and/or grade?
- During the discussion in Activity 2, how did you encourage each student to share their understandings or ideas about which scenario represented a linear or nonlinear pattern? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. The population of a colony of ants is 10,000 at the start of April. After that, it triples each month.

a Complete the table.

Months since April	Ant Population
0	10,000
1	30,000
2	90,000
3	270,000
4	810,000

b What do you notice about the population from one month to the next?

Sample response: The ant population is growing rapidly, and increases by a greater amount every month.

c If there are n ants in one month, how many ants will there be one month later?

There will be $3n$ ants one month later.

2. A swimming pool contains 500 gallons of water. A hose is turned on, and it fills the pool at a rate of 24 gallons per minute. Which expression represents the amount of water in the pool, in gallons, after 8 minutes?

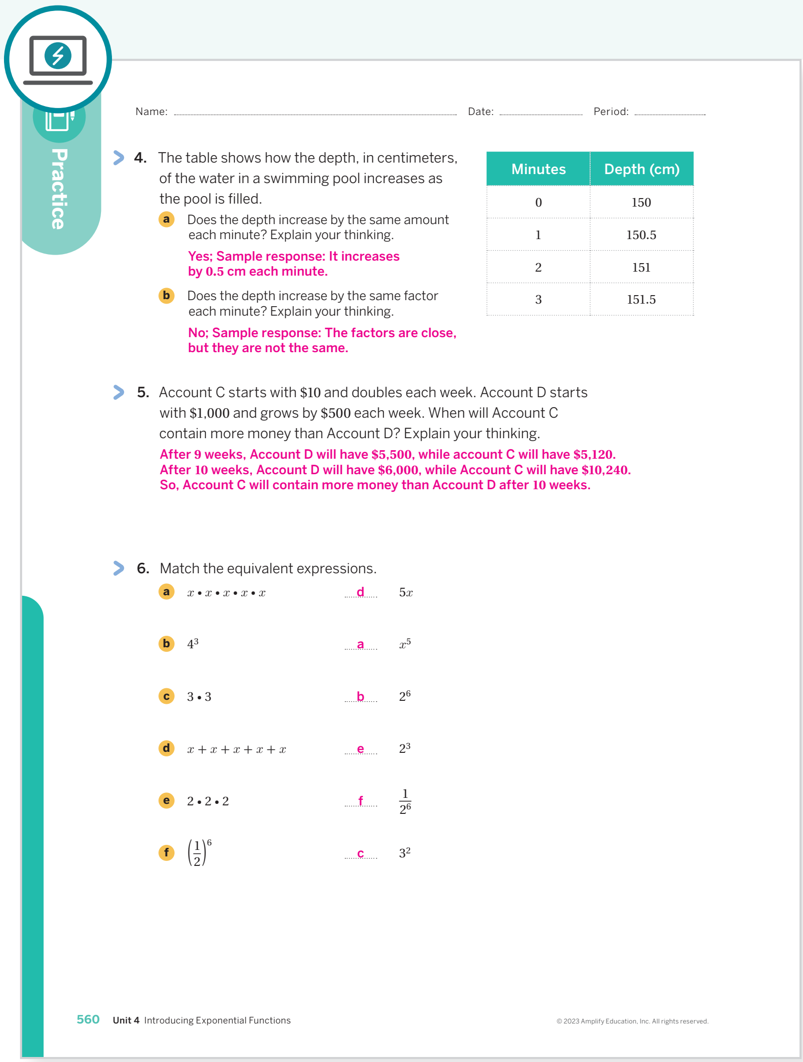
- A. $500 \cdot 24 \cdot 8$
- B. $500 + 24 + 8$
- [✓ C.] $500 + 24 \cdot 8$
- D. $500 \cdot 24^8$

3. The population of a city is 100,000. It doubles each decade for 5 decades. Select *all* expressions that represent the population of the city after 5 decades.

- A. 32,000
- B. 320,000
- [✓ C.] $100{,}000 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
- D. $100{,}000 \cdot 5^2$
- [✓ E.] $100{,}000 \cdot 2^5$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Patterns of Growth 559



Name: ________ Date: ________ Period: ________

Practice

4. The table shows how the depth, in centimeters, of the water in a swimming pool increases as the pool is filled.

Minutes	Depth (cm)
0	150
1	150.5
2	151
3	151.5

a Does the depth increase by the same amount each minute? Explain your thinking.

Yes; Sample response: It increases by 0.5 cm each minute.

b Does the depth increase by the same factor each minute? Explain your thinking.

No; Sample response: The factors are close, but they are not the same.

5. Account C starts with \$10 and doubles each week. Account D starts with \$1,000 and grows by \$500 each week. When will Account C contain more money than Account D? Explain your thinking.

After 9 weeks, Account D will have \$5,500, while account C will have \$5,120. After 10 weeks, Account D will have \$6,000, while Account C will have \$10,240. So, Account C will contain more money than Account D after 10 weeks.

6. Match the equivalent expressions.

a $x \cdot x \cdot x \cdot x \cdot x$ d $5x$

b 4^3 a x^5

c $3 \cdot 3$ b 2^6

d $x + x + x + x + x$ e 2^3

e $2 \cdot 2 \cdot 2$ f $\frac{1}{2^6}$

f $\left(\frac{1}{2}\right)^6$ c 3^2

560 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	1
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 1	2
	5	Unit 4 Lesson 1	3
Formative ⚡	6	Unit 4 Lesson 3	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

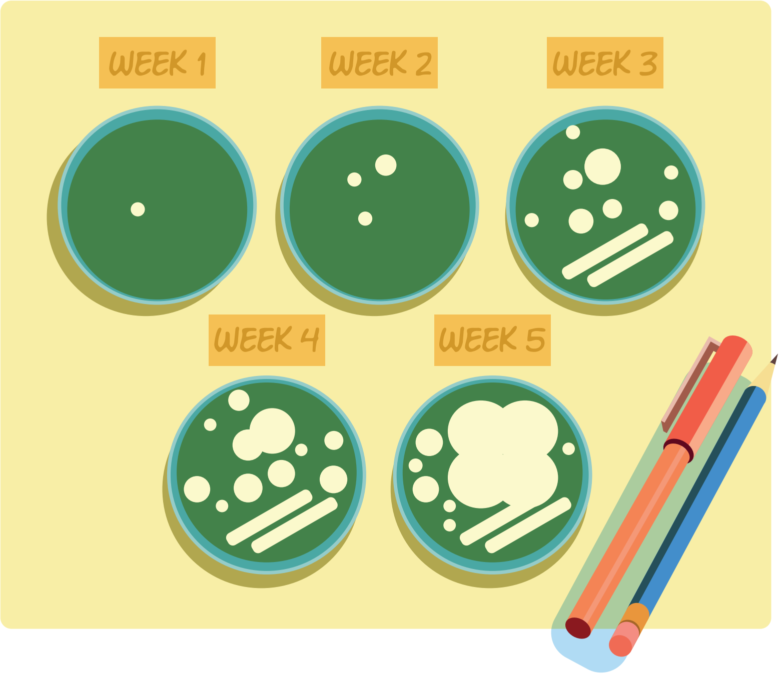
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 3**

Growing and Growing

Let's connect different patterns to their tables and graphs.



Focus

Goal

1. Compare linear and exponential relationships by performing calculations and by interpreting graphs that show two growth patterns.

Coherence

Today

Students continue to build a conceptual understanding of linear and exponential relationships by exploring tables and graphs that show different linear and nonlinear growth patterns. Students increase their understanding of the spread of disease in Activity 2, looking at the spread of the flu.

Previously

Students observed patterns characterized by common differences and factors by examining tables. Students matched tables with expressions and constructed a table of an exponential relationship.

Coming Soon

Students will formally define exponential growth, connecting its equation and graph. They will write and interpret equations representing exponential growth.

Rigor

- Students enhance their **conceptual understanding** of nonlinear growth by analyzing and making their own graphs.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils (as needed)

Math Language Development

Review words

- *common difference*
- *common factor*

Amps | **Featured Activity**

Activity 2

Interactive Graphs

Using tabular data, students move points on an interactive graph to model exponential growth.

Building Math Identity and Community

Connecting to Mathematical Practices

As students evaluate the models for different scenarios, they might feel discouraged or helpless because the numbers increase so quickly. While the numerical data can be overwhelming, explain to students that in such situations, responsible decision making can help prevent such drastic increases. Have students identify ways that they can take their own safety as well as others into play as they make decisions throughout the day.

Modifications to Pacing

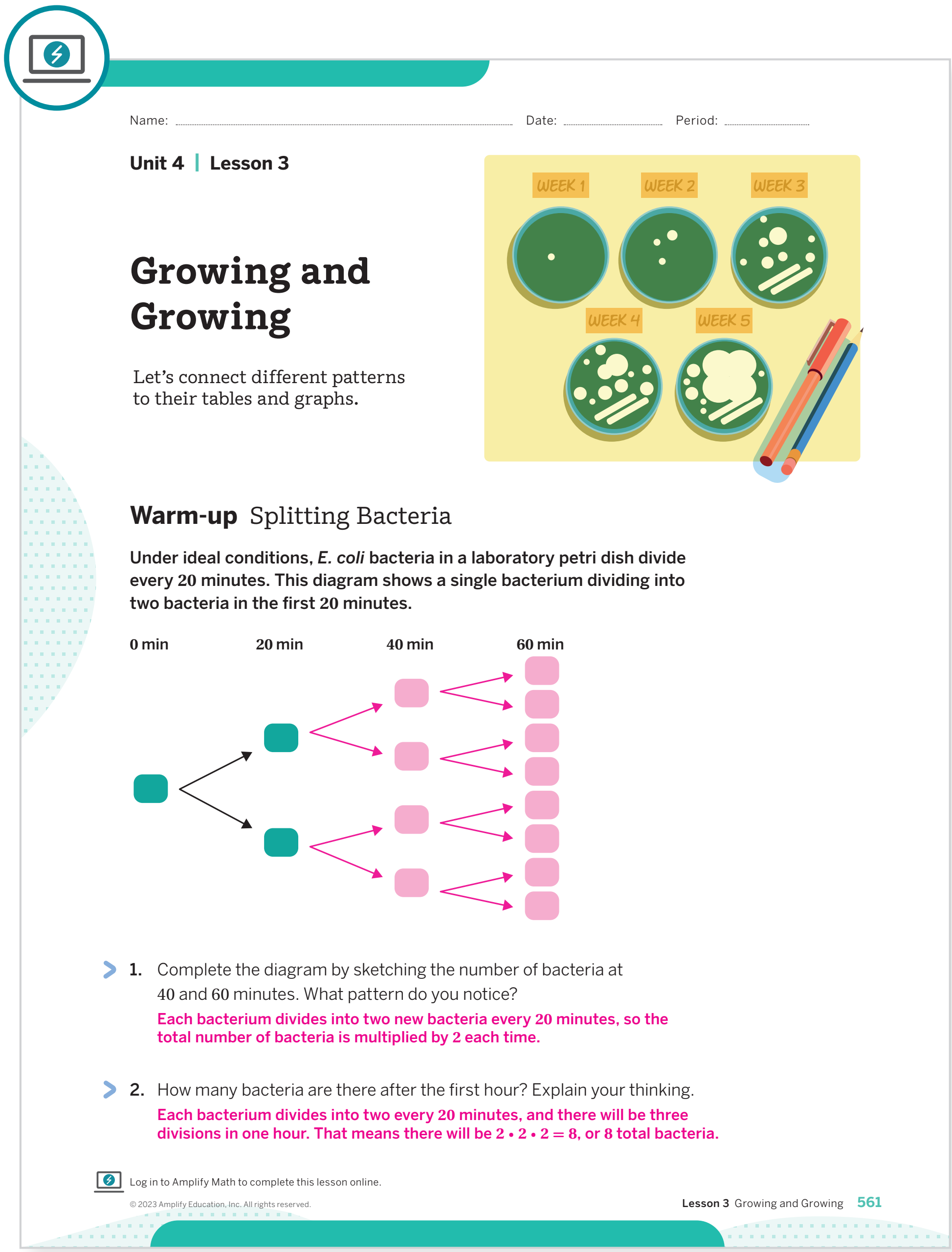
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, limit time spent on Problem 5, as students may try to determine a precise point of intersection from the graph.
- In **Activity 2**, complete Problem 2 with the class, to move through Problems 3 and 4 more quickly.

Independent | 5 min

Warm-up Splitting Bacteria

Students draw a nonlinear growth pattern, connecting the pattern to repeated multiplication.



Name: ____________ Date: ________ Period: ________

Unit 4 | Lesson 3

Growing and Growing

Let's connect different patterns to their tables and graphs.

Warm-up Splitting Bacteria

Under ideal conditions, *E. coli* bacteria in a laboratory petri dish divide every 20 minutes. This diagram shows a single bacterium dividing into two bacteria in the first 20 minutes.

1. Complete the diagram by sketching the number of bacteria at 40 and 60 minutes. What pattern do you notice?

   Each bacterium divides into two new bacteria every 20 minutes, so the total number of bacteria is multiplied by 2 each time.

2. How many bacteria are there after the first hour? Explain your thinking.

   Each bacterium divides into two every 20 minutes, and there will be three divisions in one hour. That means there will be $2 \cdot 2 \cdot 2 = 8$, or 8 total bacteria.

Log in to Amplify Math to complete this lesson online.

Lesson 3 Growing and Growing 561

1 Launch

Display the diagram and read the prompt aloud. Set an expectation for the amount of time students will have to work individually on the Warm-up.

2 Monitor

Help students get started by prompting them to make a tree with two new branches growing from each existing branch.

Look for points of confusion:

- **Misunderstanding the concept of bacterial division.** Remind students that each single bacterium divides into two *new* bacteria.
- **Having difficulty converting from minutes to hours.** Remind students there are 60 minutes in 1 hour.

Look for productive strategies:

- Creating a tree diagram to show that each single bacterium splits into two new bacteria every 20 minutes.

3 Connect

Display the incomplete diagram.

Have students share their completed diagram with their partner. Select student pairs to share their sketches and any patterns that they noticed.

Highlight that the growth pattern is nonlinear because it shows *multiplying* by (not adding) 2 new cells each time.

Ask, "What pattern do you see in the bacteria's growth?"

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to counters, coins, or other concrete objects students can choose to use to model the bacteria growth as an alternative to drawing the diagram.

Extension: *Interdisciplinary Connections*

The study of bacteria is called *bacteriology* and scientists who study bacteria are *bacteriologists*. Students may have studied bacteria growth in their science or biology classes. A bacteria's *generation time* is defined by the bacteria's growth under normal conditions. For *E. coli*, the generation time is about 20 minutes, as shown in the Warm-up. Other bacteria have different generation times. **(Science)**

Power-up

To power up students' ability to understand the relationship between repeated multiplication and exponents, have students complete:

Recall that exponents represent repeated multiplication.

Rewrite each expression using exponent notation.

a. $2 \cdot 2 \cdot 2 = 2^{\boxed{3}}$

b. $3 \cdot 3 \cdot 3 \cdot 3 = 3^{\boxed{4}}$

c. $\frac{2}{3} \cdot \frac{2}{3} = \left(\frac{2}{3}\right)^{\boxed{2}}$

d. $5 = 5^{\boxed{1}}$

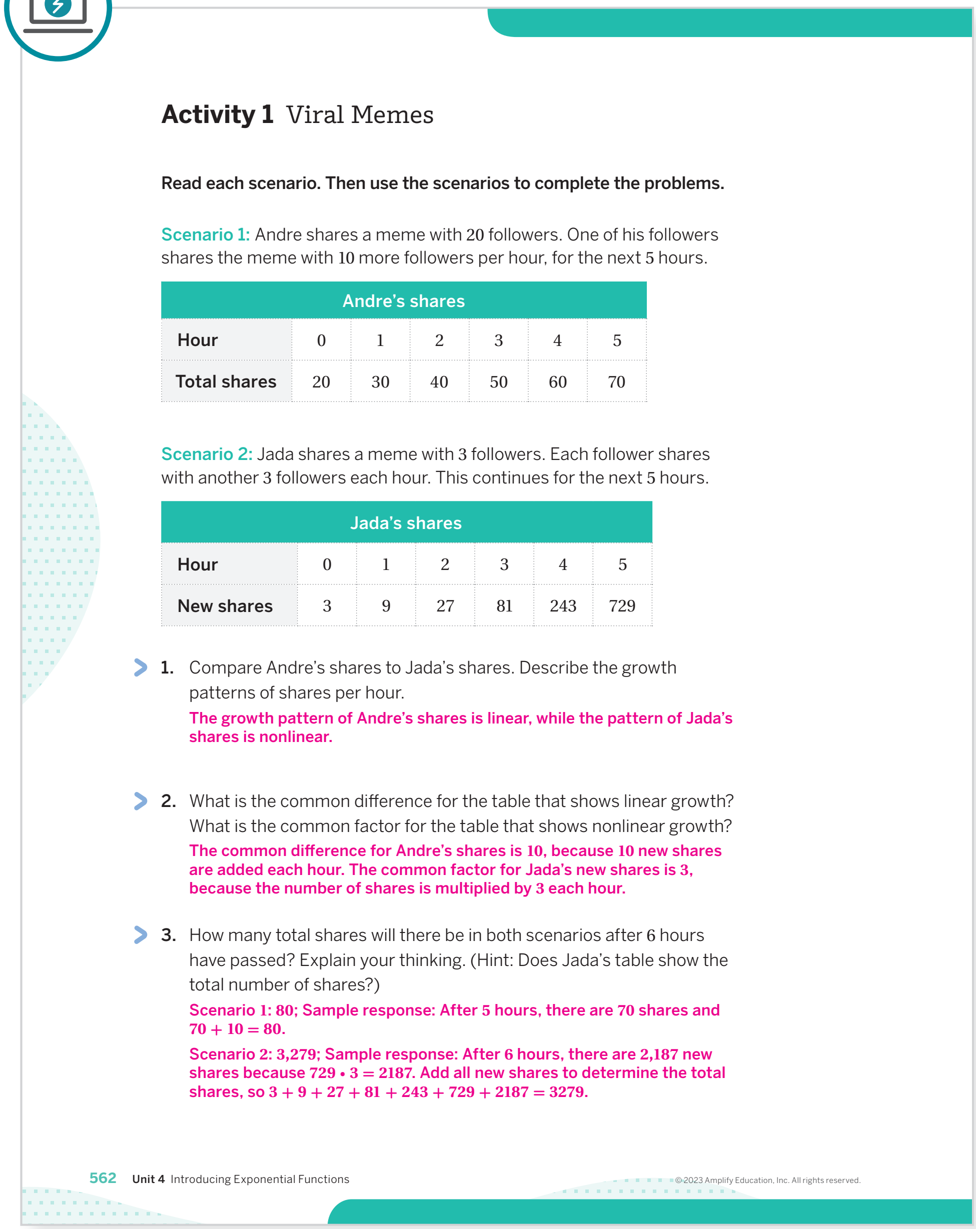
Use: Before the Warm-up

Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 15 min

Activity 1 Viral Memes

Students explore linear and nonlinear relationships by interpreting tables and graphs that show each growth pattern, which helps them understand nonlinear growth.



Activity 1 Viral Memes

Read each scenario. Then use the scenarios to complete the problems.

Scenario 1: Andre shares a meme with 20 followers. One of his followers shares the meme with 10 more followers per hour, for the next 5 hours.

Andre's shares

Hour	0	1	2	3	4	5
Total shares	20	30	40	50	60	70

Scenario 2: Jada shares a meme with 3 followers. Each follower shares with another 3 followers each hour. This continues for the next 5 hours.

Jada's shares

Hour	0	1	2	3	4	5
New shares	3	9	27	81	243	729

1. Compare Andre's shares to Jada's shares. Describe the growth patterns of shares per hour.
The growth pattern of Andre's shares is linear, while the pattern of Jada's shares is nonlinear.

2. What is the common difference for the table that shows linear growth? What is the common factor for the table that shows nonlinear growth?
The common difference for Andre's shares is 10, because 10 new shares are added each hour. The common factor for Jada's new shares is 3, because the number of shares is multiplied by 3 each hour.

3. How many total shares will there be in both scenarios after 6 hours have passed? Explain your thinking. (Hint: Does Jada's table show the total number of shares?)
Scenario 1: 80; Sample response: After 5 hours, there are 70 shares and $70 + 10 = 80$.
Scenario 2: 3,279; Sample response: After 6 hours, there are 2,187 new shares because $729 \cdot 3 = 2187$. Add all new shares to determine the total shares, so $3 + 9 + 27 + 81 + 243 + 729 + 2187 = 3279$.

562 **Unit 4** Introducing Exponential Functions

① Launch

Display each scenario and its table. Have students work independently before discussing their responses with a partner.

② Monitor

Help students get started by modeling how to determine common differences and common factors from a table. Discuss the change in the dependent variable over equal increments of change in the independent variable.

Look for points of confusion:

- **Having difficulty reading the graphs in the coordinate plane.** Review reading a graph in the coordinate plane. Have students demonstrate calculating and describing slopes.
- **Shifting their calculations by one hour ahead or behind.** Have students double-check their work in the table, cell by cell.

Look for productive strategies:

- Looking for common differences and then common factors.
- Identifying ordered pairs from a table of values and a graph.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the initial number of followers in each description and table in one color, and the growth pattern in another color.

Extension: *Math Enrichment*

Ask students to describe when Jada's *total number of shares* will exceed Andre's *total number of shares*. Sample response: Jada's total number of shares for the first 5 hours are: 3, 12, 39, 120, 363, and 1,092. Jada's total number of shares will still exceed Andre's between 2 and 3 hours, but closer to 2 hours.

Math Language Development

MLR6: Three Reads

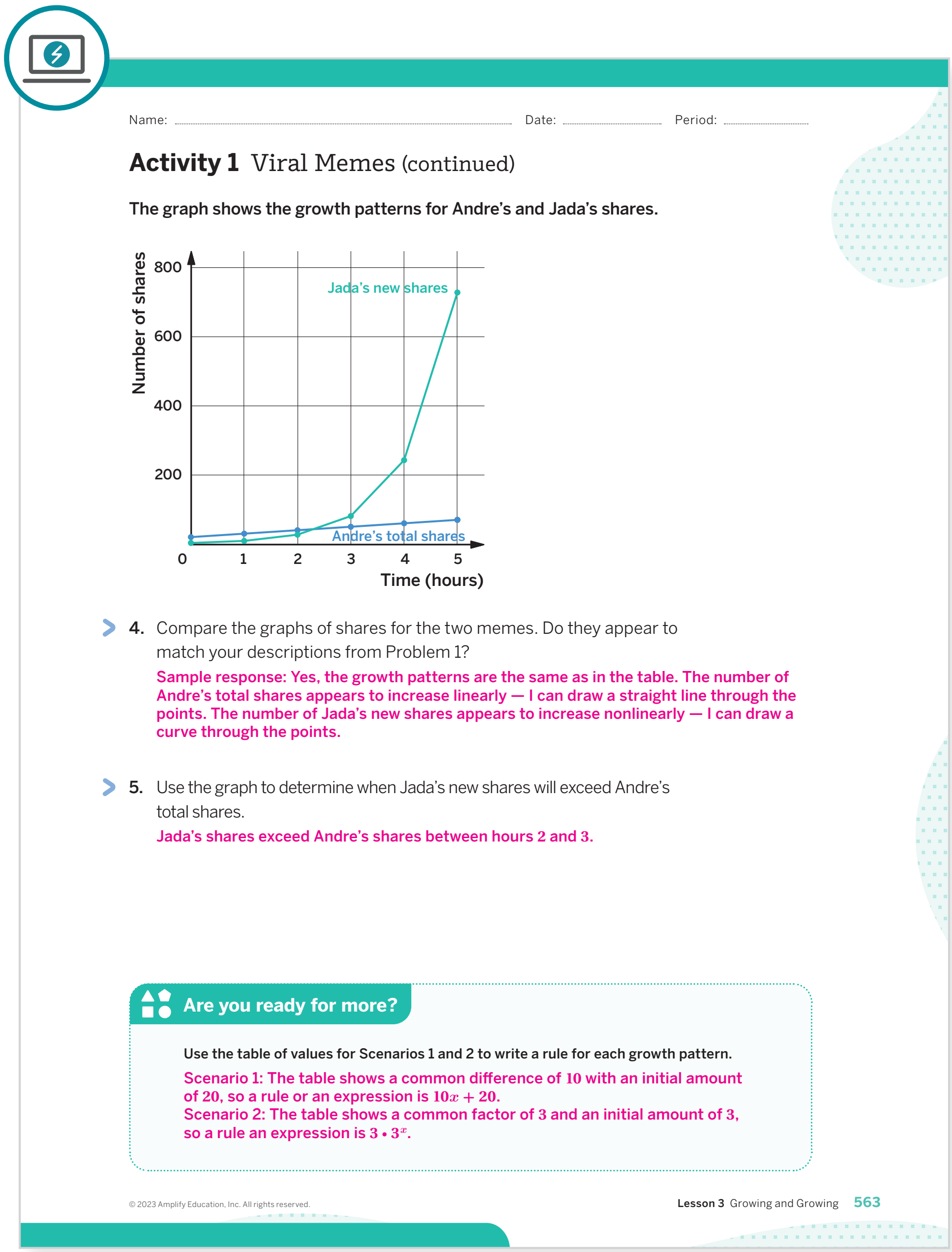
Use this routine to help students make sense of the two scenarios.

- **Read 1:** Students should understand both Andre and Jada share a meme with some of their followers and the meme continues to be shared.
- **Read 2:** Ask students to describe the given quantities and relationships, such as the initial number of followers with which each meme is shared.
- **Read 3:** Ask students to think about whether each growth described is *linear* or *nonlinear*.

Pairs | 15 min

Activity 1 Viral Memes (continued)

Students explore linear and nonlinear relationships by interpreting tables and graphs that show each growth pattern, which helps them understand nonlinear growth.



Name: Date: Period:

Activity 1 Viral Memes (continued)

The graph shows the growth patterns for Andre's and Jada's shares.

4. Compare the graphs of shares for the two memes. Do they appear to match your descriptions from Problem 1?
Sample response: Yes, the growth patterns are the same as in the table. The number of Andre's total shares appears to increase linearly — I can draw a straight line through the points. The number of Jada's new shares appears to increase nonlinearly — I can draw a curve through the points.

5. Use the graph to determine when Jada's new shares will exceed Andre's total shares.
Jada's shares exceed Andre's shares between hours 2 and 3.

Are you ready for more?

Use the table of values for Scenarios 1 and 2 to write a rule for each growth pattern.

Scenario 1: The table shows a common difference of 10 with an initial amount of 20, so a rule or an expression is $10x + 20$.
Scenario 2: The table shows a common factor of 3 and an initial amount of 3, so a rule an expression is $3 \cdot 3^x$.

③ Connect

Display the tables of Andre's and Jada's shares.

Have students share their observations about the relationship between the tables and graphs of each of the meme shares.

Highlight that a common factor can be determined by using a table of values or a graph.

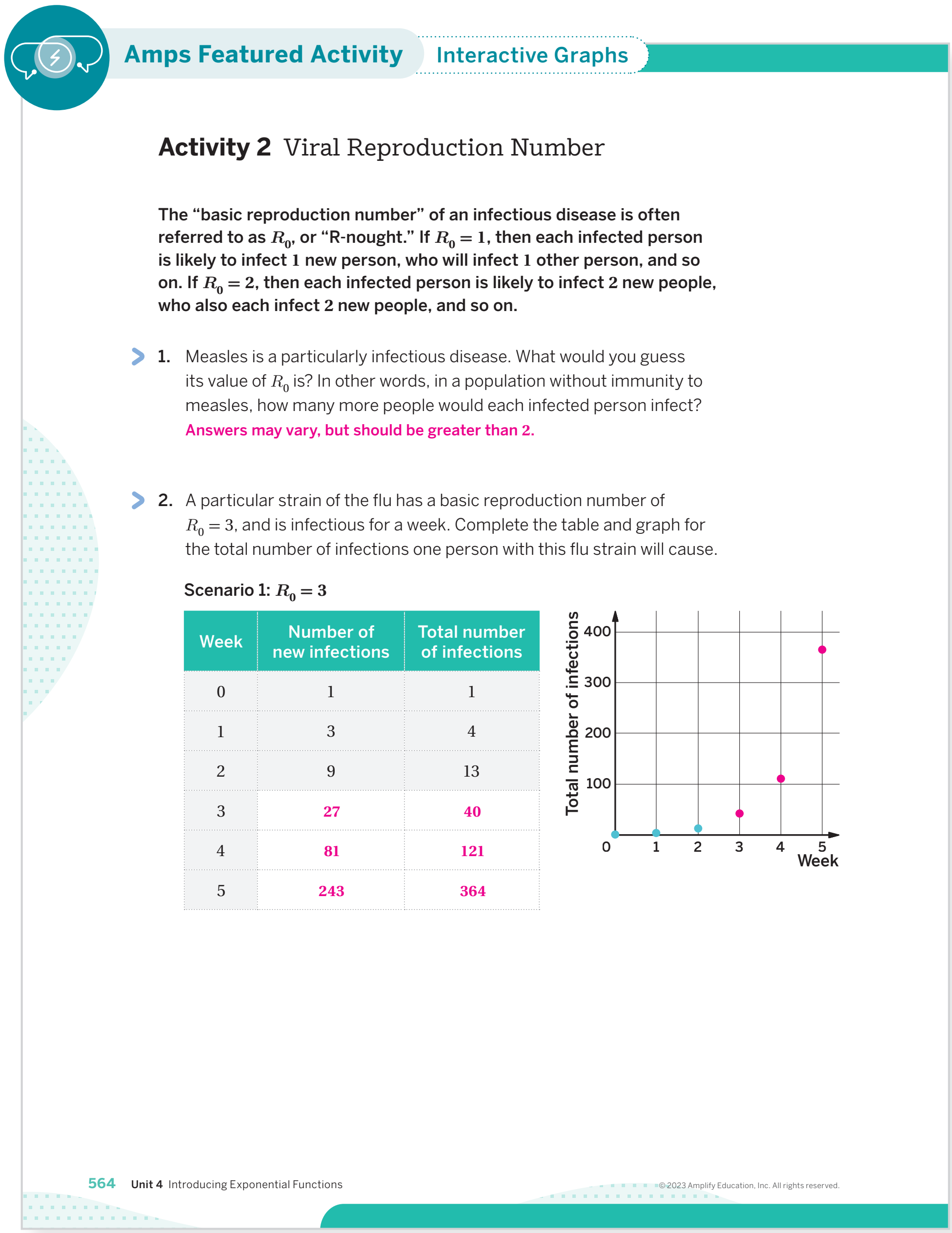
Ask, "How do the growth patterns compare between the scenarios?" Andre's shares are linear and Jada's shares are nonlinear.

Highlight that the tables and graphs show Andre's *total number of shares* and Jada's *new shares*. Ask students what the graphs would look like if they showed Andre's *new shares* to Jada's *new shares*. Sample response: Andre's *new shares are* a constant value, 10 followers per hour, which would be represented by a horizontal line.

Pairs | 20 min

Activity 2 Viral Reproduction Number

Students use the R_0 value to model the basic reproduction number of an infectious disease and recognize when the R_0 value results in a linear or nonlinear growth pattern.



Amps Featured Activity Interactive Graphs

Activity 2 Viral Reproduction Number

The "basic reproduction number" of an infectious disease is often referred to as R_0, or "R-nought." If $R_0 = 1$, then each infected person is likely to infect 1 new person, who will infect 1 other person, and so on. If $R_0 = 2$, then each infected person is likely to infect 2 new people, who also each infect 2 new people, and so on.

1. Measles is a particularly infectious disease. What would you guess its value of R_0 is? In other words, in a population without immunity to measles, how many more people would each infected person infect?
Answers may vary, but should be greater than 2.

2. A particular strain of the flu has a basic reproduction number of $R_0 = 3$, and is infectious for a week. Complete the table and graph for the total number of infections one person with this flu strain will cause.

Scenario 1: $R_0 = 3$

Week	Number of new infections	Total number of infections
0	1	1
1	3	4
2	9	13
3	**27**	**40**
4	**81**	**121**
5	**243**	**364**

564 Unit 4 Introducing Exponential Functions

1 Launch

Read the prompt aloud. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by explaining the subscripts and variables. Help students connect that R_0 values equal to 1 represent linear growth, while R_0 values greater than 1 represent nonlinear growth.

Look for points of confusion:

- **Having difficulty determining the total number of infections.** Draw a tree diagram and count the number of infections. Then connect students' calculations for the table of values.
- **Struggling to label the axes on their graphs.** Label the horizontal axis as *Time* and the vertical axis as the *Total number of infections*.

Look for productive strategies:

- Determining common differences and common factors.
- Using common factors to complete a table of values.
- Identifying ordered pairs from a table of values and graph.

Activity 2 continued ›

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

During the Launch, clarify the meaning of the R_0 notation. Demonstrate the difference between $R_0 = 1$ and $R_0 = 2$ by asking volunteers to stand up, representing the newly infected persons for each.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the connections between the R_0 value and the corresponding growth pattern. After students respond to the Ask questions, consider displaying a table similar to the following (or add one to the class display):

Linear growth	Nonlinear growth	
$R_0 = 1$	$R_0 = 2$	$R_0 = 3$

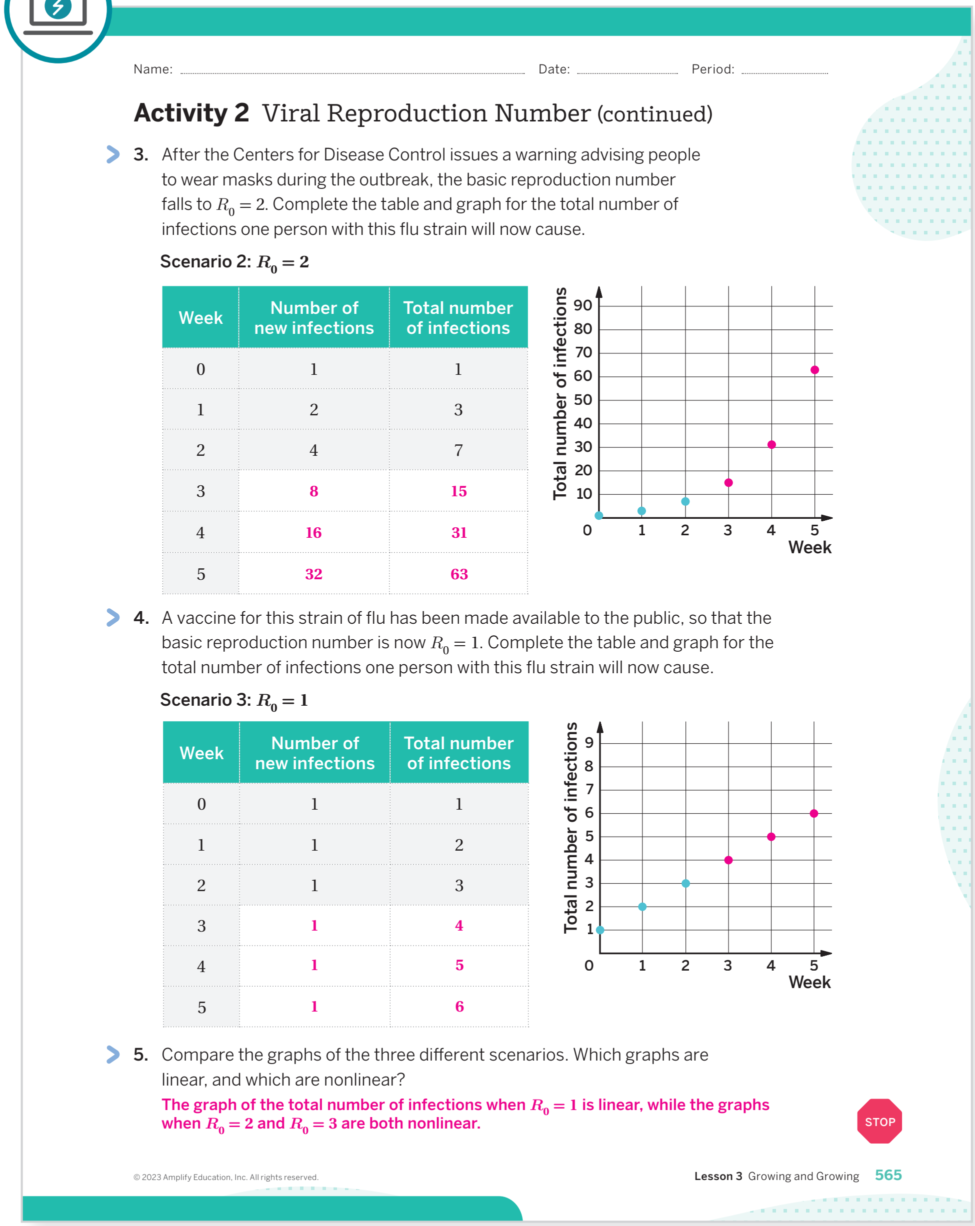
English Learners

Add examples of graphs to the class display to further support this connection.

Pairs | 20 min

Activity 2 Viral Reproduction Number (continued)

Students use the R_0 value to model the basic reproduction number of an infectious disease and recognize when the R_0 value results in a linear or nonlinear growth pattern.



Name: ______ Date: ______ Period: ______

Activity 2 Viral Reproduction Number (continued)

3. After the Centers for Disease Control issues a warning advising people to wear masks during the outbreak, the basic reproduction number falls to $R_0 = 2$. Complete the table and graph for the total number of infections one person with this flu strain will now cause.

Scenario 2: $R_0 = 2$

Week	Number of new infections	Total number of infections
0	1	1
1	2	3
2	4	7
3	8	15
4	16	31
5	32	63

4. A vaccine for this strain of flu has been made available to the public, so that the basic reproduction number is now $R_0 = 1$. Complete the table and graph for the total number of infections one person with this flu strain will now cause.

Scenario 3: $R_0 = 1$

Week	Number of new infections	Total number of infections
0	1	1
1	1	2
2	1	3
3	1	4
4	1	5
5	1	6

5. Compare the graphs of the three different scenarios. Which graphs are linear, and which are nonlinear?

The graph of the total number of infections when $R_0 = 1$ is linear, while the graphs when $R_0 = 2$ and $R_0 = 3$ are both nonlinear.

STOP

Lesson 3 Growing and Growing 565

3 Connect

Display examples of student graphs.

Have students share emerging patterns in the total number of infections depending on the value of R_0.

Highlight the connections between the value of R_0 and its growth pattern shown in the table and graph.

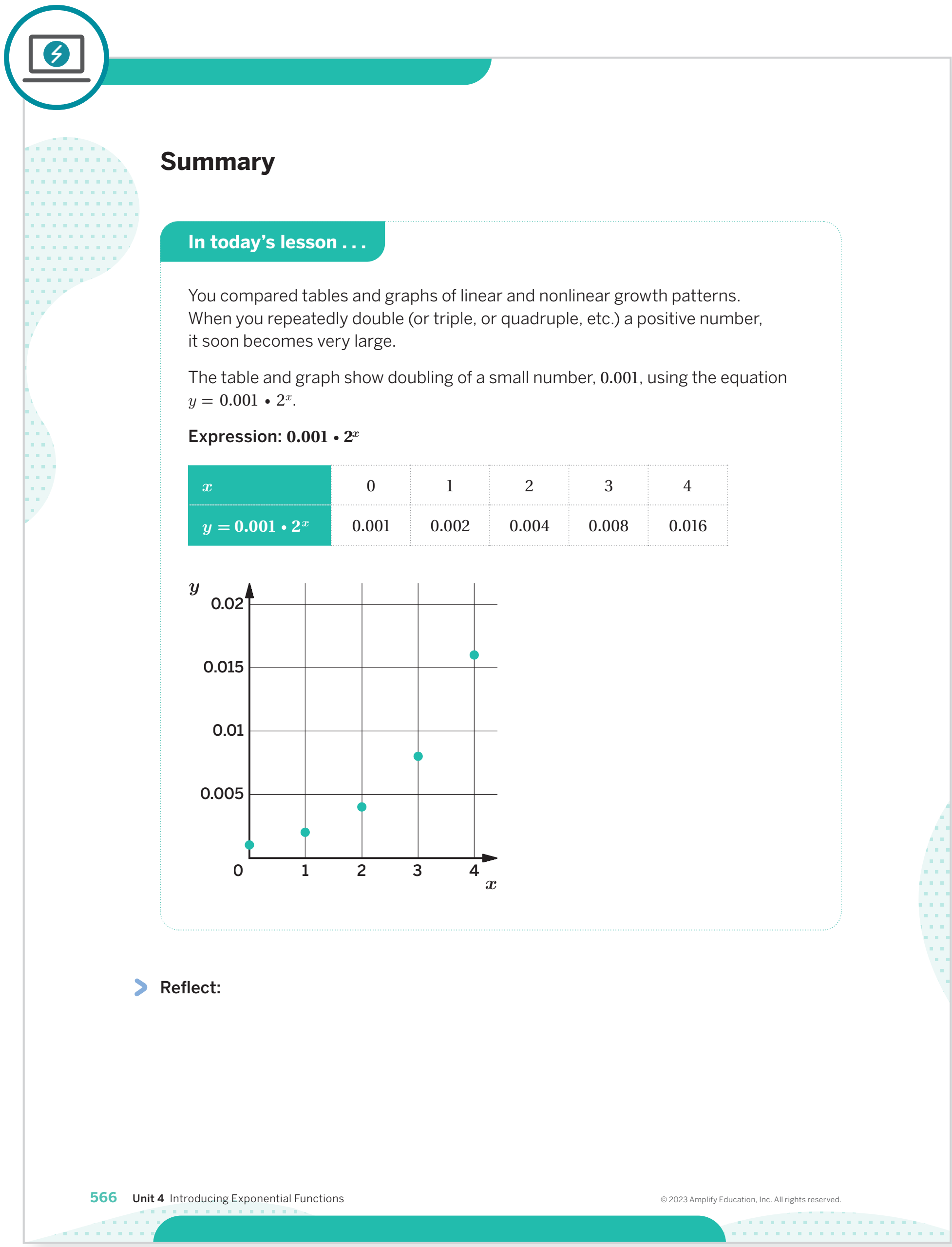
Ask:

- "How do you see nonlinear growth patterns in the tables? In the graphs?" A nonlinear growth pattern has a common factor in the table and the graph. The graph also shows a curve instead of a line.
- "Which basic reproduction number is associated with the least number of infections?" $R_0 = 1$

Whole Class | 5 min

Summary

Review and synthesize the connections between multiple representations of nonlinear growth patterns and exponential expressions.



Summary

In today's lesson . . .

You compared tables and graphs of linear and nonlinear growth patterns. When you repeatedly double (or triple, or quadruple, etc.) a positive number, it soon becomes very large.

The table and graph show doubling of a small number, 0.001, using the equation $y = 0.001 \cdot 2^x$.

Expression: $0.001 \cdot 2^x$

x	0	1	2	3	4
$y = 0.001 \cdot 2^x$	0.001	0.002	0.004	0.008	0.016

Reflect:

566 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the expression, table, and graph from the Summary.

Have students share any patterns or observations that they notice about the table and graph.

Highlight that when a positive number is repeatedly doubled, it represents nonlinear growth and becomes very large.

Ask:

- "Are the values in the table displaying a linear or nonlinear growth pattern?" Nonlinear growth pattern.
- "What is the common factor?" 2
- "How could you determine the next term when $x = 5$?" Multiply $0.016 \cdot 2$ or $0.001 \cdot 2$.
- "Does this expression match the graph?" Yes; As the value of x increases by 1, the value of y doubles.

Reflect

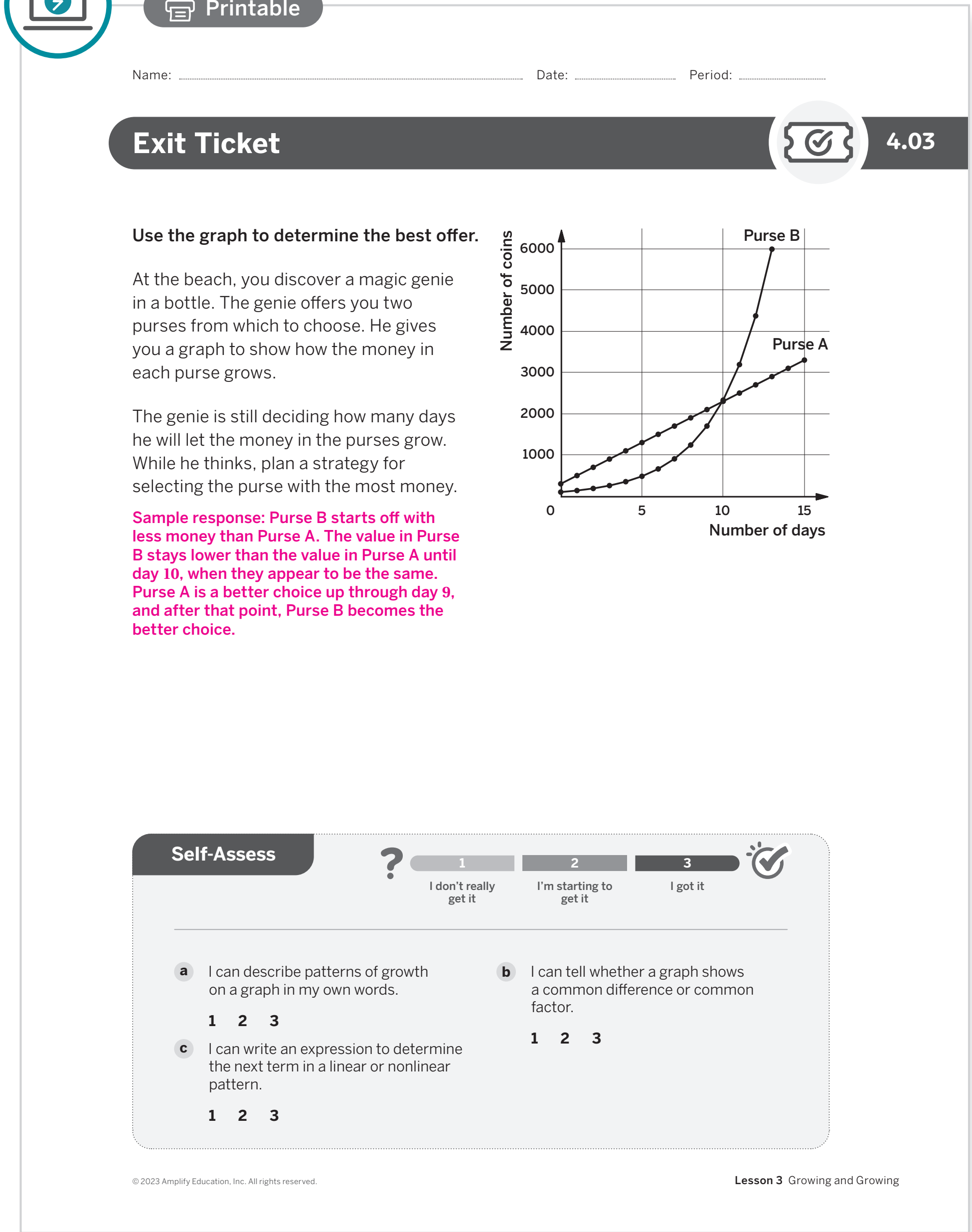
After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does the nonlinear growth you studied in this lesson compare to linear growth? Which grows faster? Why?"
- "How do scientists and researchers use the R_0 value to study the spread of infections? How does it relate to nonlinear growth?"

Exit Ticket

Students demonstrate their understanding of linear and exponential relationships by interpreting graphs that show the two growth patterns.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.03

Use the graph to determine the best offer.

At the beach, you discover a magic genie in a bottle. The genie offers you two purses from which to choose. He gives you a graph to show how the money in each purse grows.

The genie is still deciding how many days he will let the money in the purses grow. While he thinks, plan a strategy for selecting the purse with the most money.

Sample response: Purse B starts off with less money than Purse A. The value in Purse B stays lower than the value in Purse A until day 10, when they appear to be the same. Purse A is a better choice up through day 9, and after that point, Purse B becomes the better choice.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can describe patterns of growth on a graph in my own words.
1 2 3

b I can tell whether a graph shows a common difference or common factor.
1 2 3

c I can write an expression to determine the next term in a linear or nonlinear pattern.
1 2 3

Lesson 3 Growing and Growing

Success looks like . . .

- **Goal:** Comparing linear and exponential relationships by performing calculations and by interpreting graphs that show two growth patterns.
  - » Selecting Purse B as the better choice because of the growth pattern in its graph.

Suggested next steps

If students struggle to understand why Purse B will eventually exceed Purse A (in terms of monetary value), consider:

- Revisiting the differences between the graphs in Activity 2.

If students are unable to discern when Purse A exceeds Purse B, and at what point Purse B exceeds Purse A (in terms of monetary value), consider:

- Revisiting the graph from Problems 4 and 5 in Activity 1.

Professional Learning

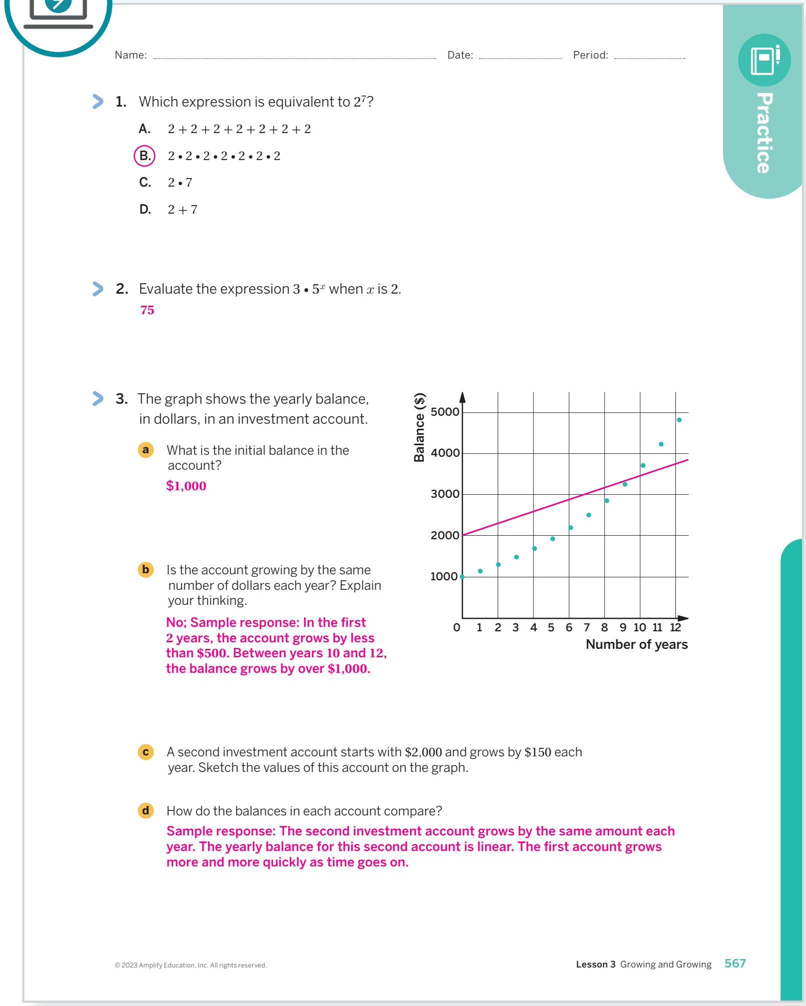
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What did students find challenging learning about the R_0 value? What helped them work through these challenges?
- The instructional goal for this lesson was to compare linear and exponential ("nonlinear") relationships by using tables and graphs, looking for common differences or common factors. How well did students accomplish this goal? What specific types of support(s) did you offer to help them accomplish this goal? What might you change the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Which expression is equivalent to 2^7?
   - A. $2 + 2 + 2 + 2 + 2 + 2 + 2$
   - [✓ B.] $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
   - C. $2 \cdot 7$
   - D. $2 + 7$

2. Evaluate the expression $3 \cdot 5^x$ when x is 2.

   75

3. The graph shows the yearly balance, in dollars, in an investment account.

   a. What is the initial balance in the account?

   $1,000

   b. Is the account growing by the same number of dollars each year? Explain your thinking.

   No; Sample response: In the first 2 years, the account grows by less than $500. Between years 10 and 12, the balance grows by over $1,000.

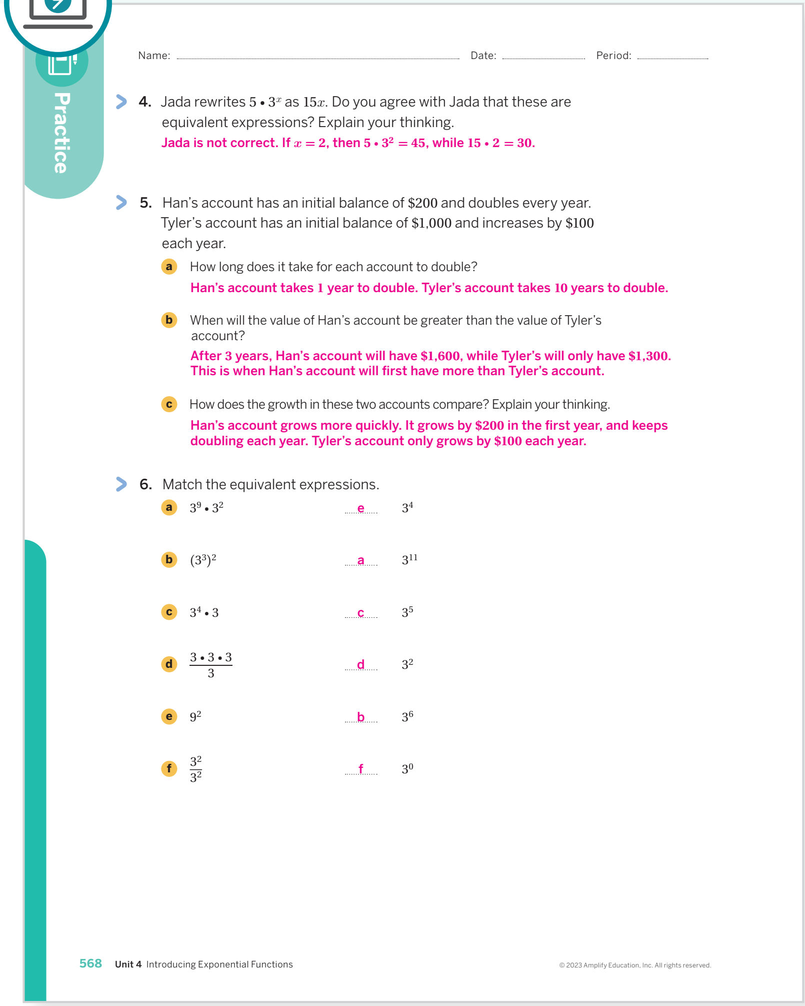
   c. A second investment account starts with $2,000 and grows by $150 each year. Sketch the values of this account on the graph.

   d. How do the balances in each account compare?

   Sample response: The second investment account grows by the same amount each year. The yearly balance for this second account is linear. The first account grows more and more quickly as time goes on.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Growing and Growing 567



Name: Date: Period:

Practice

4. Jada rewrites $5 \cdot 3^x$ as $15x$. Do you agree with Jada that these are equivalent expressions? Explain your thinking.

   Jada is not correct. If $x = 2$, then $5 \cdot 3^2 = 45$, while $15 \cdot 2 = 30$.

5. Han's account has an initial balance of $200 and doubles every year. Tyler's account has an initial balance of $1,000 and increases by $100 each year.

   a. How long does it take for each account to double?

   Han's account takes 1 year to double. Tyler's account takes 10 years to double.

   b. When will the value of Han's account be greater than the value of Tyler's account?

   After 3 years, Han's account will have $1,600, while Tyler's will only have $1,300. This is when Han's account will first have more than Tyler's account.

   c. How does the growth in these two accounts compare? Explain your thinking.

   Han's account grows more quickly. It grows by $200 in the first year, and keeps doubling each year. Tyler's account only grows by $100 each year.

6. Match the equivalent expressions.

a	$3^9 \cdot 3^2$	e	3^4
b	$(3^3)^2$	a	3^{11}
c	$3^4 \cdot 3$	c	3^5
d	$\frac{3 \cdot 3 \cdot 3}{3}$	d	3^2
e	9^2	b	3^6
f	$\frac{3^2}{3^2}$	f	3^0

568 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Grade 6	3
	5	Unit 4 Lesson 2	2
Formative ⚡	6	Unit 4 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 2

A New Kind of Relationship

Whole Class

In this Sub-Unit, students build an understanding of exponential growth and decay, calculate growth factors, and observe what happens to the graph when parameters of an exponential equation are changed.

SUB-UNIT

2 A New Kind of Relationship

Narrative Connections

How did an enslaved man save the city of Boston?

In 1721, the residents of Boston were scared and angry. Smallpox was ripping through the city, killing hundreds of Bostonians. But in the end, thousands of lives were saved by a man who was knowledgeable in the practices of inoculation and also enslaved by an influential minister.

Onesimus, as he had been named, had educated the minister Cotton Mather and others on the process of "variolation," in which pus from someone already infected by smallpox was applied to the open wounds of a healthy patient. By giving the patient a small exposure to the disease, the patient's immune system would create antibodies to protect against a more lethal case of it. This practice was common in Sub-Saharan Africa and parts of Asia. But in the New World, Onesimus' variolation was feared as being too dangerous. It was indeed dangerous, particularly because those who had been inoculated were still contagious, and could further spread the disease throughout the city.

In spite of death threats and mob violence, Dr. Zabdiel Boylston treated 286 people using Oneismus' technique. By the end of the outbreak in 1722, 98% of those treated survived. Onesimus' efforts had dramatically slowed the spread of disease. (The next major outbreak would not occur until 1752.)

In 1980, the World Health Organization declared that smallpox had been completely eradicated, largely due to vaccination efforts. It is the only infectious disease (for humans) to date that has been eradicated. Understanding how diseases grow assist us in knowing how to combat them. Exponential relationships help us do just that.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore exponential growth and decay — within medical and scientific contexts — in the following places:

- **Lesson 4, Activities 2–3:** Multiplying Microbes, Graphing the Microbes
- **Lesson 5, Activity 3:** Exponential Success of the Polio Vaccine
- **Lesson 6, Activities 1–2:** The Algae Bloom, Insulin in the Body

UNIT 4 | **LESSON 4**

Representing Exponential Growth

Let's explore exponential growth.

Focus

Goals

1. **Language Goal:** Explain how a and b are shown on the graph of an equation of the form $y = a \cdot b^x$. **(Speaking and Listening)**
2. **Language Goal:** Interpret a and b in context given an equation of the form $y = a \cdot b^x$. **(Speaking and Listening, Writing)**
3. Write an exponential equation of the form $y = a \cdot b^x$.

Coherence

Today

In this lesson, students are introduced to the general form of an exponential equation $y = a \cdot b^x$ and learn how to interpret a and b in context. Students also see how a and b are represented in a table and a graph, and make connections between these multiple representations. Students are formally introduced to the terms *exponential growth* and *growth factor*.

Previously

Students explored tables and graphs representing growth patterns to build a conceptual understanding of linear and exponential relationships. This will help inform their conjectures as they continue to explore exponential relationships.

Coming Soon

Students will continue to examine quantities that change exponentially, with a focus on quantities that decay or decrease.

Rigor

- Students build their **procedural fluency** by writing expressions for exponential growth and drawing corresponding graphs.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Exponent Rules*
- graphing technology

Math Language Development

New words

- ***exponential growth***
- ***growth factor***

Review words

- *initial value*
- *power*
- *y-intercept*

Amps | Featured Activity

Activity 3
Interactive Graphs

Using tabular data, students move points on an interactive graph to model exponential growth.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack confidence in their ability to work with powers of 0. You can empower them to use their strengths as they look for and build on repeated reasoning. By applying skills that students already know and have worked with, they can derive the meaning of a zero exponent. This achievement will then feed their self-confidence, especially when similar tasks arise in the future.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 3**, choose one of the two functions to graph and analyze, rather than both.

Independent | 5 min

Warm-up Powers of Two

Students review the product rule and quotient rule for exponents.

Unit 4 | Lesson 4

Representing Exponential Growth

Let's explore exponential growth.

Warm-up Powers of Two

Rewrite each expression as a power of 2.

a $2^3 \cdot 2^4 = 2^7$

b $2^5 \cdot 2 = 2^6$

c $2^{10} \div 2^7 = 2^3$

d $2^9 \div 2 = 2^8$

Log in to Amplify Math to complete this lesson online.

570 Unit 4 Introducing Exponential Functions

1 Launch

Ask students what is meant by a "power of 2." Ask students for examples (such as 2^5 and 2^{100}) and incorrect examples (such as 100^2 and $5 \cdot 2$).

2 Monitor

Help students get started by having them write each part of the expression in expanded form first. For example, have them write $(2^3) \cdot (2^4)$ as $(2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$.

Look for points of confusion:

- **Multiplying or dividing the exponents.** Encourage students to write the expression in expanded form before performing any operations.

Look for productive strategies:

- Writing expressions in expanded notation.
- Applying the product rule and quotient rule for exponents.

3 Connect

Have students share their responses for each problem. Select students who used productive strategies.

Highlight that it can be helpful to write each expression in expanded notation before performing any operations, i.e. $2^5 \cdot 2 = (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot 2 = 2^6$. Model how to use the product rule and quotient rule, reminding students that these rules can only be used if the expressions have the same base.

Ask, "How would you rewrite the expression $2^2 \cdot 2^0$ as a power of 2 using the product rule?" $2^{2+0} = 2^2$

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Display the exponent rules students learned in Grade 8, using the Anchor Chart PDF, *Exponent Rules*, for them to reference during this activity. Suggest they first write the powers as repeated multiplication to remind them of the reasoning behind these exponent rules.

Ask students to explain why the bases must be the same in order to add exponents when multiplying powers or subtract exponents when dividing powers. Provide a nonexample, such as $2^4 \cdot 5^3$, to show that the exponents of this expression cannot be added together.

Power-up

To power up students' ability to apply the laws of exponents, have students complete:

Recall that an exponent represents repeated multiplication.

Rewrite each expression using exponent notation.

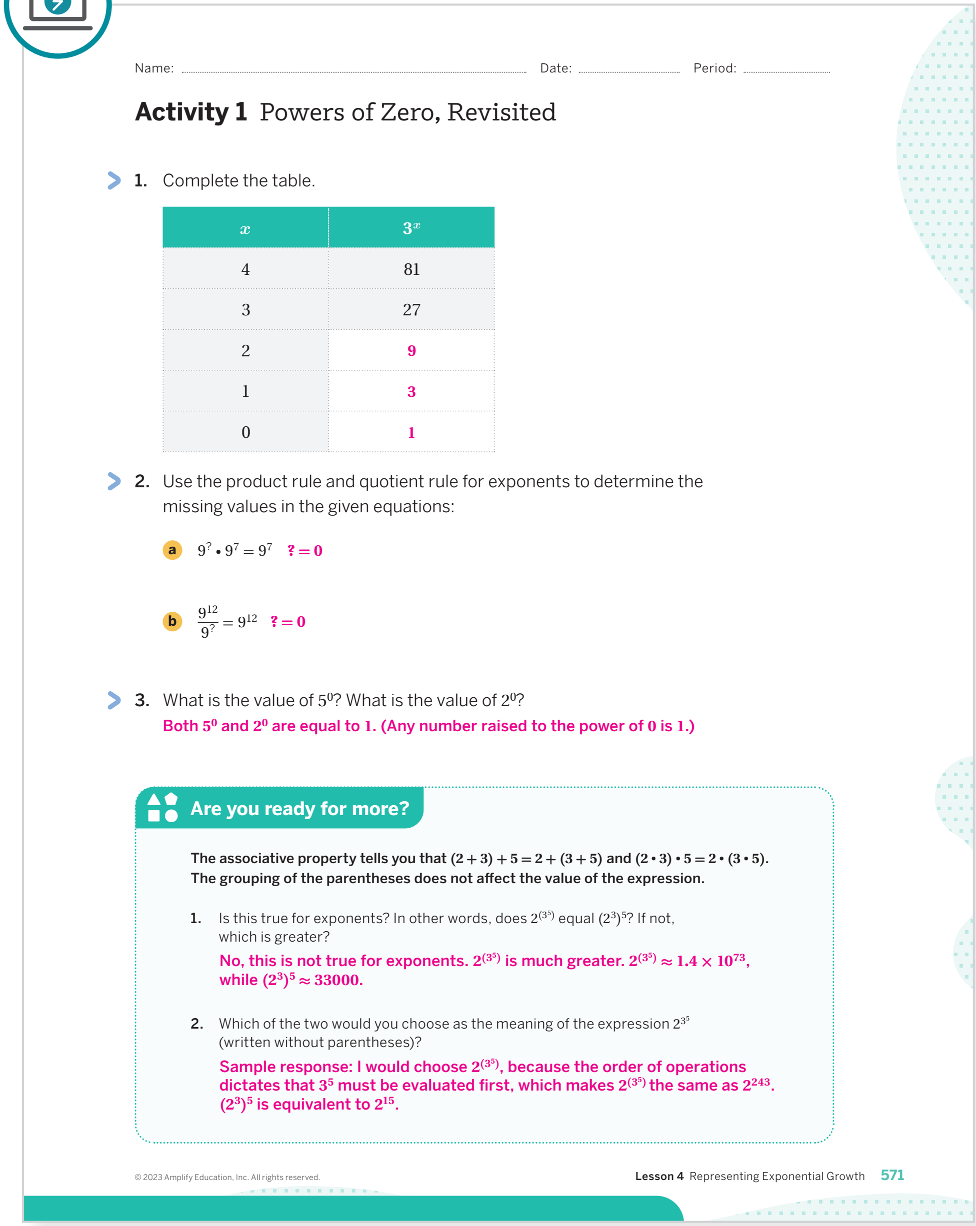
1. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \quad = 2^5$
2. $(2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2) \quad = 2^6$
3. $\dfrac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} \quad = 2^2$
4. $\dfrac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2} \quad = 2^5$

Use: Before the Warm-up
Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

Independent | 10 min

Activity 1 Powers of Zero, Revisited

Students use repeated reasoning to extend an exponential pattern, revisiting the product and quotient rules, to evaluate any number to the power of zero.



Name: ______ Date: ______ Period: ______

Activity 1 Powers of Zero, Revisited

1. Complete the table.

x	3^x
4	81
3	27
2	**9**
1	**3**
0	**1**

2. Use the product rule and quotient rule for exponents to determine the missing values in the given equations:

   a. $9^? \cdot 9^7 = 9^7$ **$? = 0$**

   b. $\frac{9^{12}}{9^?} = 9^{12}$ **$? = 0$**

3. What is the value of 5^0? What is the value of 2^0?

   Both 5^0 and 2^0 are equal to 1. (Any number raised to the power of 0 is 1.)

Are you ready for more?

The associative property tells you that $(2+3)+5=2+(3+5)$ and $(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$. The grouping of the parentheses does not affect the value of the expression.

1. Is this true for exponents? In other words, does $2^{(3^5)}$ equal $(2^3)^5$? If not, which is greater?

   No, this is not true for exponents. $2^{(3^5)}$ is much greater. $2^{(3^5)} \approx 1.4 \times 10^{73}$, while $(2^3)^5 \approx 33000$.

2. Which of the two would you choose as the meaning of the expression 2^{3^5} (written without parentheses)?

   Sample response: I would choose $2^{(3^5)}$, because the order of operations dictates that 3^5 must be evaluated first, which makes $2^{(3^5)}$ the same as 2^{243}. $(2^3)^5$ is equivalent to 2^{15}.

Lesson 4 Representing Exponential Growth 571

1 Launch

Tell students that they are going to explore what happens when they raise a number to the power of zero. Set an expectation for the amount of time students will have to individually work on the activity.

2 Monitor

Have students get started by modeling how to complete the table with the values whose outputs are already given, $x = 4$ and $x = 3$, showing that $3^4 = 81$ and $3^3 = 27$.

Look for points of confusion:

- **Thinking $a^0 = 0$.** Encourage students to extend the pattern in the table to determine the value of 3^x when $x = 0$.
- **Multiplying the base with the exponent.** Remind students that exponents represent repeated multiplication of the base.

Look for productive strategies:

- Testing their responses for Problem 2 by substituting the missing values into the given equations.

3 Connect

Display the table in Problem 1.

Ask, "What can you conclude about a number raised to the 0 power?" Provide time for students to discuss their thinking with a partner, using their responses to justify their position.

Highlight that any number raised to the 0 power equals 1. (If students then ask about 0^0, tell them this is a great question – you can argue why it should be 0 or 1. Mathematicians agree it equals 1, and interested students can research why that is.)

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Display the exponent rules students learned in Grade 8, using the Anchor Chart PDF, *Exponent Rules*, for them to reference during this activity.

Extension: *Math Enrichment*

Have students construct an argument for whether they think 0^0 equals 0 or 1, or whether it is undefined. Have them include tables or diagrams in their argument and share their thinking with another student or the entire class.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement, such as "$5^0 = 0$ because there is no exponent on 5, which means there is no factor of 5." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? What does it mean if there is no exponent on the base?"
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

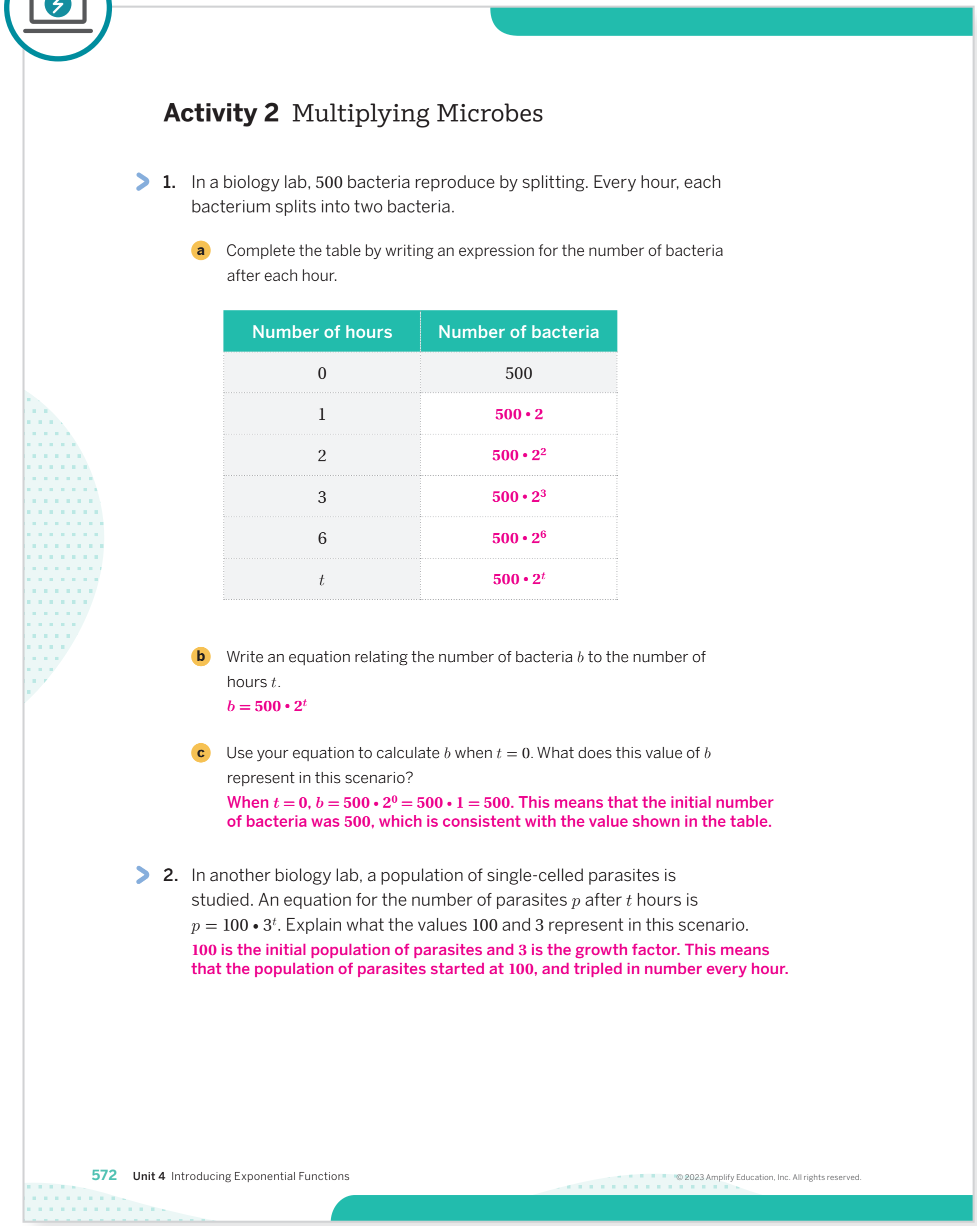
English Learners

Suggest that students use patterns in the table to help them construct their mathematical arguments.

Pairs | 10 min

Activity 2 Multiplying Microbes

Students build expressions of the form $a \cdot b^x$ and identify the pattern they observe as exponential growth.



Activity 2 Multiplying Microbes

1. In a biology lab, 500 bacteria reproduce by splitting. Every hour, each bacterium splits into two bacteria.

   a. Complete the table by writing an expression for the number of bacteria after each hour.

Number of hours	Number of bacteria
0	500
1	$500 \cdot 2$
2	$500 \cdot 2^2$
3	$500 \cdot 2^3$
6	$500 \cdot 2^6$
t	$500 \cdot 2^t$

   b. Write an equation relating the number of bacteria b to the number of hours t.
   $b = 500 \cdot 2^t$

   c. Use your equation to calculate b when $t = 0$. What does this value of b represent in this scenario?
   When $t = 0$, $b = 500 \cdot 2^0 = 500 \cdot 1 = 500$. This means that the initial number of bacteria was 500, which is consistent with the value shown in the table.

2. In another biology lab, a population of single-celled parasites is studied. An equation for the number of parasites p after t hours is $p = 100 \cdot 3^t$. Explain what the values 100 and 3 represent in this scenario.
   100 is the initial population of parasites and 3 is the growth factor. This means that the population of parasites started at 100, and tripled in number every hour.

572 **Unit 4** Introducing Exponential Functions

1 Launch

Read the scenario in Problem 1 aloud. Tell students that they should write expressions in the table, not actual values.

2 Monitor

Help students get started by modeling how to write the number of bacteria as an expression for $x = 1$ (i.e., $500 \cdot 2$ instead of $1{,}000$).

Look for points of confusion:

- **Writing their expressions using addition instead of multiplication.** Encourage students to use multiplication expressions, because it is more straightforward when identifying a pattern.
- **Writing the actual number of bacteria in the table**. Encourage them to use the numbers they found to determine the equivalent expressions in terms of the growth factor, 2.

Look for productive strategies:

- Writing expressions using exponents to represent repeated multiplication.

3 Connect

Display the table from Problem 1.

Have students share their responses to Problem 1 and how the number of bacteria relates to the number of hours.

Highlight that the pattern in the table can be described as changing exponentially. The initial value, 500, is found at $t = 0$ and is the result of evaluating $500 \cdot 2^0$.

Define the term *growth factor*.

Ask, "If the initial population of the parasites is changed to 80, and the population now quadruples every hour, how will the expression change?" It will be $80 \cdot 4^t$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing these supports as students progress through the activity.

- For Problem 1a, suggest that students write the number of bacteria after each hour as an exponential expression. Consider demonstrating how to write the expressions for the first two rows.
- For Problem 1b, ask students which expression in the table gives them the number of bacteria for any number of hours.
- For Problem 2, suggest that students refer back to Problem 1 to color code the initial number of bacteria in the table and equation in one color, and the repeated factor in another color. Have them use this color coding to help them respond to Problem 2.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the connections between the repeated factor of 2 and 3 and the term *growth factor*. Ask:

- "Where do you see the growth factor in the table?"
- "Where do you see the growth factor in the equation?"
- "How do you know this pattern is not linear?"
- "Why do you think this pattern is called growing *exponentially*?"

English Learners

Annotate the growth factor in the equation for Problem 1b as *doubling* and the growth factor in the equation for Problem 2 as *tripling*.

Pairs | 15 min

Activity 3 Graphing the Microbes

Students graph the equations from Activity 2 and use their graphs to interpret the meanings of a and b in context.

Amps Featured Activity Interactive Graphs

Name: ______ Date: ______ Period: ______

Activity 3 Graphing the Microbes

Refer to Activity 2 to complete these problems about the number of bacteria b and the number of single-celled parasites p.

1. Graph (t, b) when t is $0, 1, 2, 3$, and 4.

   a. Rewrite the equation that relates the number of bacteria b to the number of hours t.

   $b = 500 \cdot 2^t$

   b. On the graph of b, where can you see each number that appears in the equation?

   500 is the vertical intercept of the graph and each point is 2 times higher than the previous point.

2. Graph (t, p) when t is $0, 1, 2, 3$, and 4.

   a. Rewrite the equation that relates the number of parasites p to the number of hours t.

   $p = 100 \cdot 3^t$

   b. On the graph of p, where can you see each number that appears in the equation?

   100 is the vertical intercept of the graph and each point is 3 times higher than the previous point.

1 Launch

Display the equations $b = 500 \cdot 2^t$ and $p = 100 \cdot 3^t$. Tell students that these are same equations from Activity 2 and they will be using them to complete Activity 3. Provide access to graphing technology.

2 Monitor

Help students get started by suggesting they use the table from Activity 2 to help them determine the coordinates of the points that need to be plotted in Problem 1.

Look for points of confusion:

- **Having difficulty determining b for different values of t.** Encourage students to create a table like the one they completed in Activity 2.

Look for productive strategies:

- Writing in the scale values along the n and b axes to help estimate the positions of the coordinates.
- Choosing to use graphing technology or creating a table of values to confirm the equations they write are correct.

3 Connect

Display the completed graphs from Problems 1 and 2.

Have students share the connection between the graphs and the numbers in the equations.

Highlight the connection between the initial value of the populations when they were first measured, as represented by the vertical intercept, and the growth factor of the populations, as represented by the rate of increase in the graphs.

Define the term ***exponential growth*** as a change characterized by the repeated multiplication of a growth factor.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can move points on an interactive graph to model exponential growth.

Accessibility: *Guide Processing and Visualization*

Display the equations $b = 500 \cdot 2^t$ and $p = 100 \cdot 3^t$ from Activity 2 so students can record them for part a of each problem. Clarify the meaning of the variables t, b, and p. Suggest students make a table of values to graph $p = 100 \cdot 3^t$.

Math Language Development

MLR7: Compare and Connect

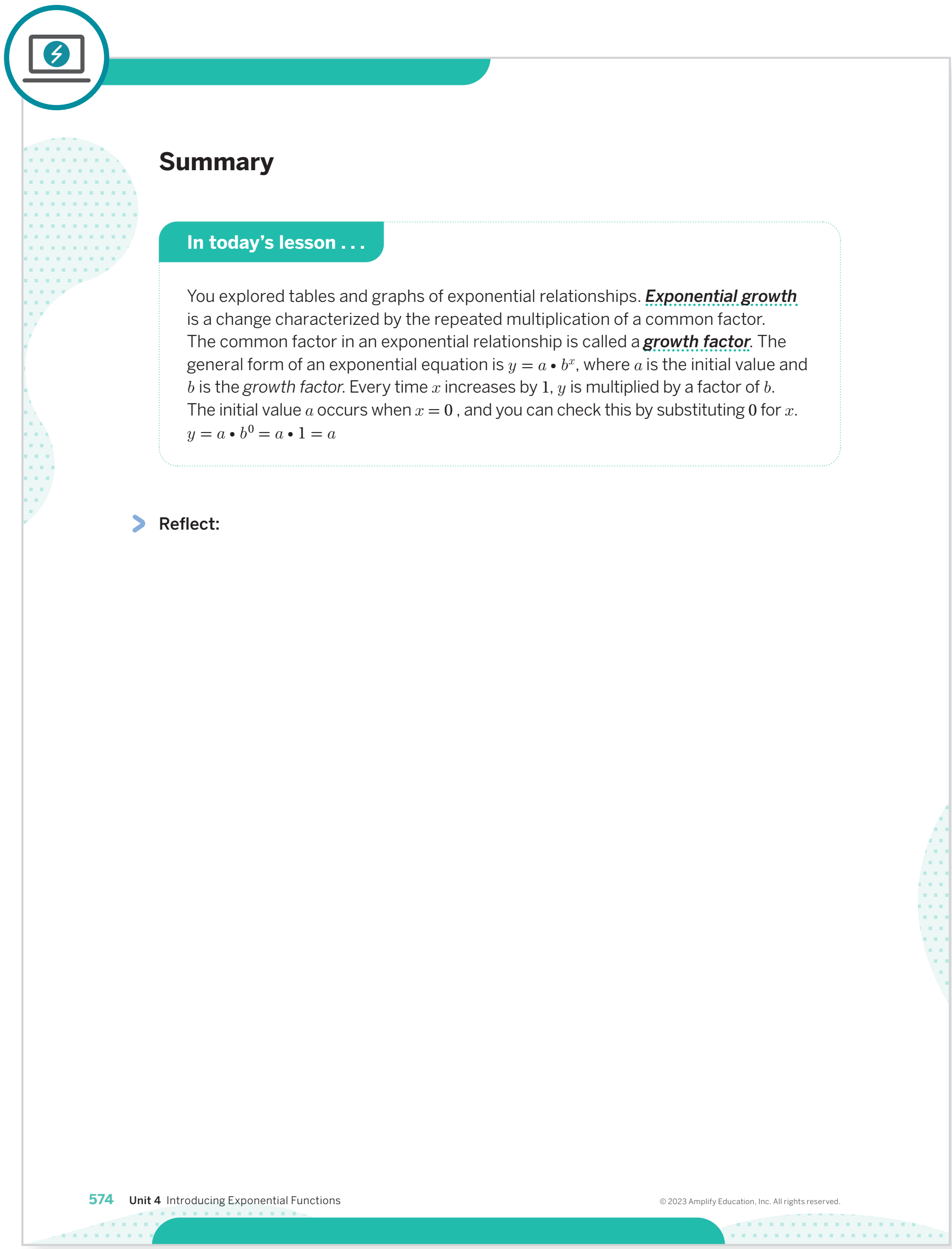
During the Connect, as students share their strategies, use annotations to highlight the connections between how the equations and graphs show the *initial value* and *growth factor.* Consider displaying a table similar to the following:

	Equation	Graph
Initial value	Value multiplied by the power	Vertical intercept
Growth factor	Base of the power	Vertical distance from one point to the next

Whole Class | 5 min

Summary

Review and synthesize how to articulate connections between multiple representations of an exponential pattern.



Summary

In today's lesson . . .

You explored tables and graphs of exponential relationships. ***Exponential growth*** is a change characterized by the repeated multiplication of a common factor. The common factor in an exponential relationship is called a ***growth factor***. The general form of an exponential equation is $y = a \bullet b^x$, where a is the initial value and b is the *growth factor*. Every time x increases by 1, y is multiplied by a factor of b. The initial value a occurs when $x = 0$, and you can check this by substituting 0 for x. $y = a \bullet b^0 = a \bullet 1 = a$

Reflect:

574 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *exponential growth* and *growth factor* that were added to the display during the lesson.

Synthesize

Display the equation $p = 100 \bullet 3^t$ and the completed graph from Activity 3, Problem 2. Allow students 2 minutes to write down how the values in the table are represented in the equation and the graph.

Ask:

- "What is the initial value? How can you identify it in the graph? The equation?"
- "How does the initial value relate to the power of zero?"
- "Describe how y changes with respect to x. How is this change shown in the graph?"
- "Why do you think we refer to this as *exponential growth*?"
- "Describe how you would write an equation to model exponential growth. What information would you need?"

Highlight by means of annotations and explicit labels, the connections between the initial value and *growth factor* of an exponential relationship and how they are represented in its equation, table, and graph. Emphasize that *exponential growth* is characterized by these key features.

Formalize vocabulary:

- ***growth factor***
- ***exponential growth***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

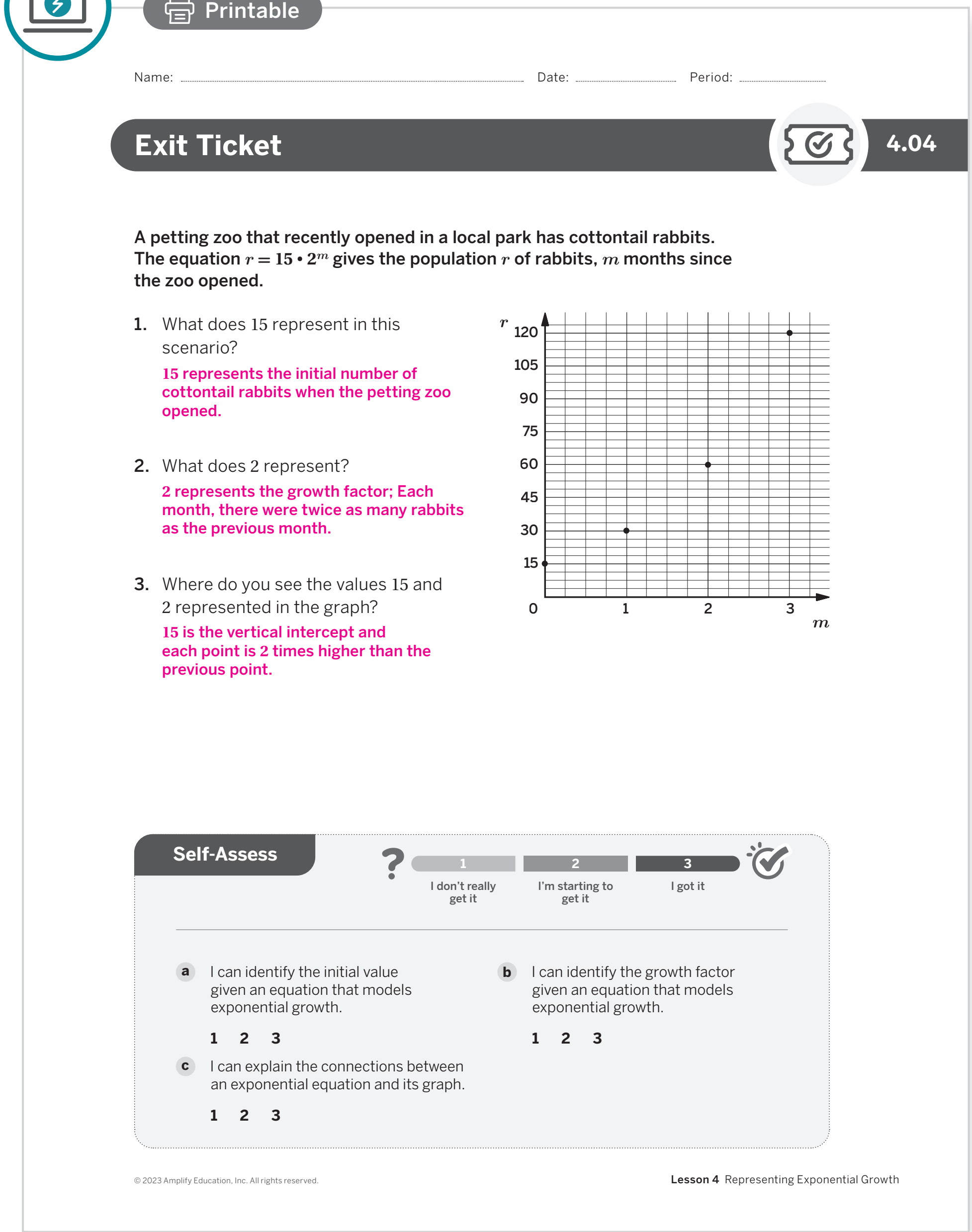
- "What characterizes exponential growth?"
- "What are some real-world examples of exponential growth?"
- "What is the structure of the equation that represents exponential growth? What does each part of the equation mean?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by examining an equation that represents exponential growth, and interpreting its key features in context.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 4.04

A petting zoo that recently opened in a local park has cottontail rabbits. The equation $r = 15 \cdot 2^m$ gives the population r of rabbits, m months since the zoo opened.

1. What does 15 represent in this scenario?

   15 represents the initial number of cottontail rabbits when the petting zoo opened.

2. What does 2 represent?

   2 represents the growth factor; Each month, there were twice as many rabbits as the previous month.

3. Where do you see the values 15 and 2 represented in the graph?

   15 is the vertical intercept and each point is 2 times higher than the previous point.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can identify the initial value given an equation that models exponential growth.
1 2 3

b. I can identify the growth factor given an equation that models exponential growth.
1 2 3

c. I can explain the connections between an exponential equation and its graph.
1 2 3

Lesson 4 Representing Exponential Growth

Success looks like . . .

- **Language Goal:** Explaining how a and b are shown on the graph of an equation of the form $y = a \cdot b^x$. **(Speaking and Listening)**
  - » Identifying what 15 and 2 represent on the graph in Problems 1 and 2.
- **Language Goal:** Interpreting a and b in context given an equation of the form $y = a \cdot b^x$. **(Speaking and Listening, Writing)**
- **Goal:** Writing an exponential equation of the form $y = a \cdot b^x$.

Suggested next steps

If students cannot identify the initial value and growth factor from the exponential growth equation given in Problems 1 and 2, consider:

- Reviewing Activity 1.
- Assigning Practice Problems 3 and 5.

If students are unable to explain and justify connections between the exponential equations and the corresponding graph in Problem 3, consider:

- Reviewing Activity 2, Problems 1c and 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students explored what it means to raise a number to the power of zero. How did that build on earlier understandings of exponent rules from a prior unit and/or grade?
- During the discussion in Activity 2, how did you encourage each student to listen to one another's strategies for how they completed the table, wrote their equation, and calculated the growth factor? What might you change the next time you teach this lesson?

Math Language Development

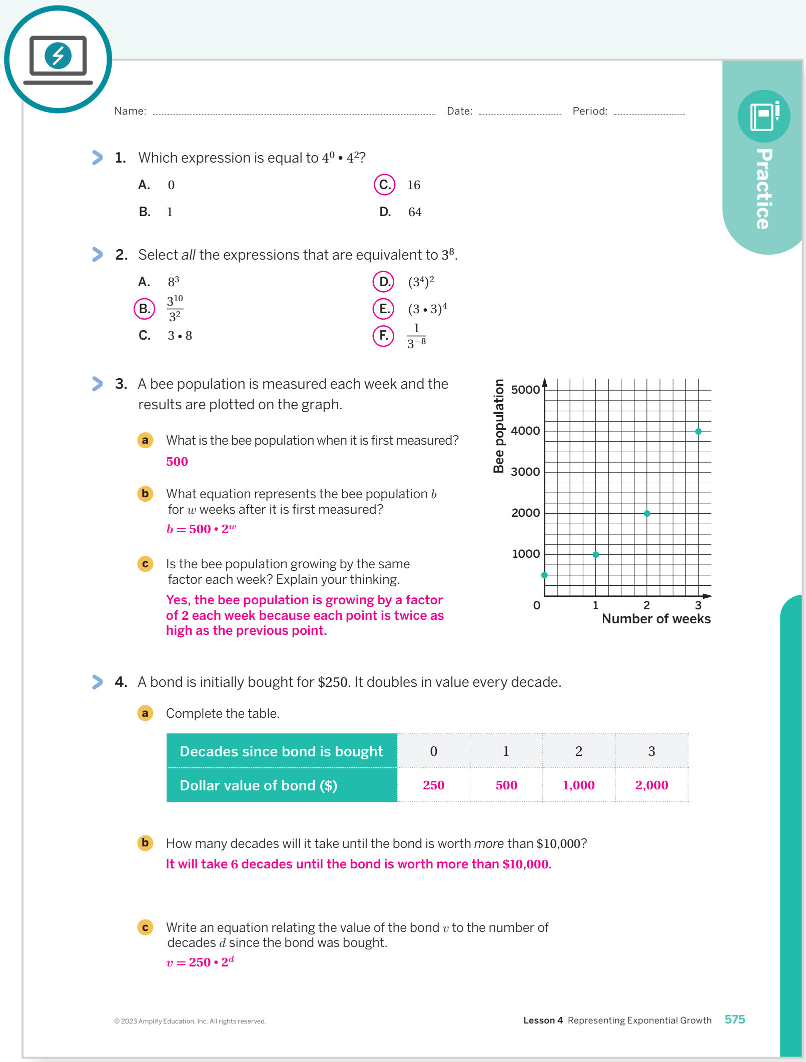
Language Goal: Interpreting a and b in context given an equation of the form $y = a \cdot b^x$.

Reflect on students' language development toward this goal.

- Do students' responses to Problems 1 and 2 of the Exit Ticket demonstrate they understand how to interpret the parameters of the exponential equation within context?
- What math terms are they using to describe what these parameters represent? Are they using terms and phrases such as *initial value, starting value, growth factor, twice as many rabbits each month,* etc.?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Which expression is equal to $4^0 \cdot 4^2$?

   A. 0 [✓ C.] 16

   B. 1 D. 64

2. Select *all* the expressions that are equivalent to 3^8.

   A. 8^3 [✓ D.] $(3^4)^2$

   [✓ B.] $\frac{3^{10}}{3^2}$ [✓ E.] $(3 \cdot 3)^4$

   C. $3 \cdot 8$ [✓ F.] $\frac{1}{3^{-8}}$

3. A bee population is measured each week and the results are plotted on the graph.

   a. What is the bee population when it is first measured?

   500

   b. What equation represents the bee population b for w weeks after it is first measured?

   $b = 500 \cdot 2^w$

   c. Is the bee population growing by the same factor each week? Explain your thinking.

   Yes, the bee population is growing by a factor of 2 each week because each point is twice as high as the previous point.

4. A bond is initially bought for $250. It doubles in value every decade.

   a. Complete the table.

Decades since bond is bought	0	1	2	3
Dollar value of bond ($)	250	500	1,000	2,000

   b. How many decades will it take until the bond is worth *more* than $10,000?

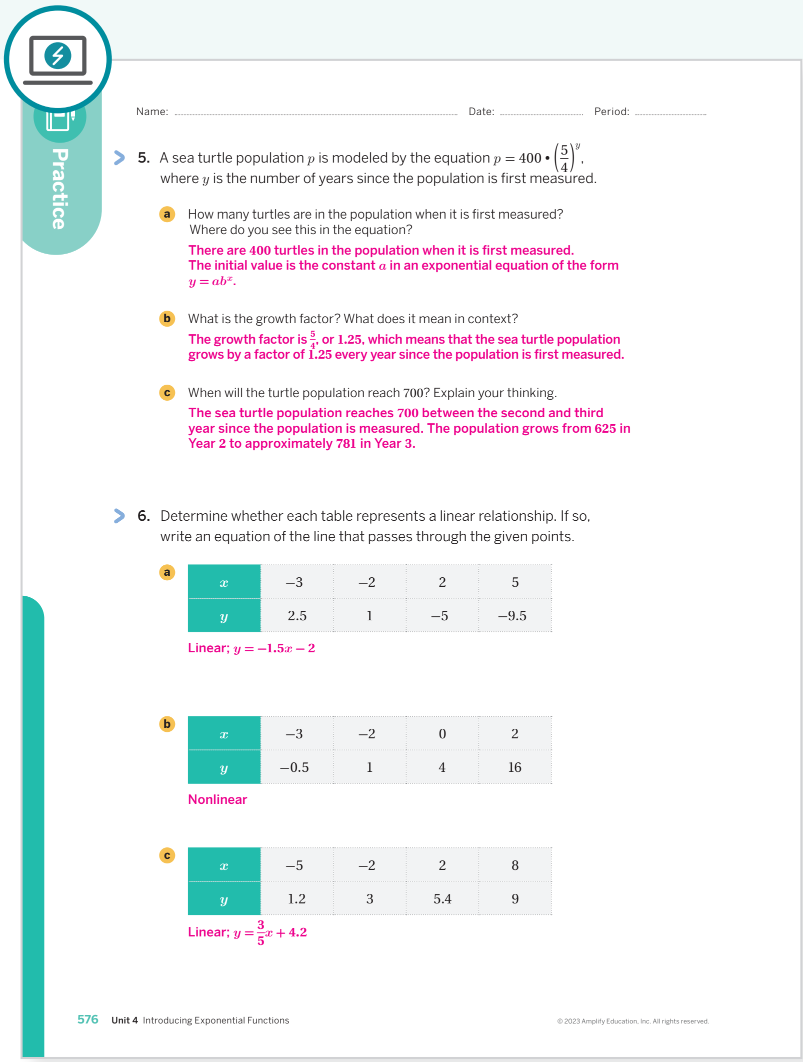
   It will take 6 decades until the bond is worth more than $10,000.

   c. Write an equation relating the value of the bond v to the number of decades d since the bond was bought.

   $v = 250 \cdot 2^d$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Representing Exponential Growth 575



Name: ________ Date: ________ Period: ________

5. A sea turtle population p is modeled by the equation $p = 400 \cdot \left(\frac{5}{4}\right)^y$, where y is the number of years since the population is first measured.

   a. How many turtles are in the population when it is first measured? Where do you see this in the equation?

   There are 400 turtles in the population when it is first measured. The initial value is the constant a in an exponential equation of the form $y = ab^x$.

   b. What is the growth factor? What does it mean in context?

   The growth factor is $\frac{5}{4}$, or 1.25, which means that the sea turtle population grows by a factor of 1.25 every year since the population is first measured.

   c. When will the turtle population reach 700? Explain your thinking.

   The sea turtle population reaches 700 between the second and third year since the population is measured. The population grows from 625 in Year 2 to approximately 781 in Year 3.

6. Determine whether each table represents a linear relationship. If so, write an equation of the line that passes through the given points.

   a.

x	-3	-2	2	5
y	2.5	1	-5	-9.5

   Linear; $y = -1.5x - 2$

   b.

x	-3	-2	0	2
y	-0.5	1	4	16

   Nonlinear

   c.

x	-5	-2	2	8
y	1.2	3	5.4	9

   Linear; $y = \frac{3}{5}x + 4.2$

576 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 3	2
	3	Activity 3	2
	4	Activity 2	2
	5	Activity 2	2
Formative ⚡	6	Unit 4 Lesson 5	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Understanding Decay

Let’s look at exponential decay.

Focus

Goals

1. **Language Goal:** Explain that exponential growth describes a quantity that changes by a growth factor that is greater than 1. **(Speaking and Listening)**
2. **Language Goal:** Explain that exponential decay describes a quantity that changes by a growth factor that is less than 1 but greater than 0. **(Speaking and Listening)**
3. Represent “decreasing a quantity” in terms of multiplying it by some fraction of itself.
4. Write an expression or an equation to represent exponential decay.

Coherence

Today

Students examine exponential decay with growth factors greater than 0 and less than 1, write expressions, and make connections between tables and equations. In Activity 3, students test the validity of an equation modeling the eradication of polio.

Previously

In Lesson 4, students studied exponential growth and were introduced to the term *growth factor.*

Coming Soon

In Lesson 6, students will examine scenarios characterized by exponential decay. They will identify and interpret key features in graphs and equations.

Rigor

- Students further develop their **conceptual understanding** of exponential behavior by exploring tables and expressions that represent exponential decay.
- Students **apply** models of exponential decay to contexts involving depreciation and vaccination.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Pairs	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils (as needed)

Math Language Development

New words

- ***decay factor***
- ***exponential decay***

Review words

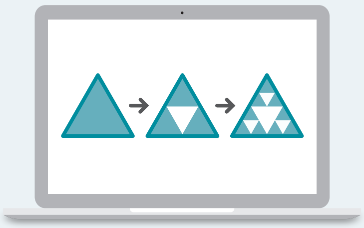
- *common factor*
- *growth factor*

Amps | **Featured Activity**

Activity 2

Digital Sierpiński

If students complete the *Are you ready for more?* activity, they can use digital technology to create a Sierpiński triangle.



Building Math Identity and Community

Connecting to Mathematical Practices

Again students find themselves dealt a new topic that is a twist on a previous topic and might be discouraged that the link between the topics is not immediately obvious. By maintaining a big-picture view of this new kind of exponential growth that decreases, they can apply the regularity of the thought process to consistently evaluate the reasonableness of their results. In fact, a growth mentality can overcome present doubts if they believe that connection can and will become clearer.

Modifications to Pacing

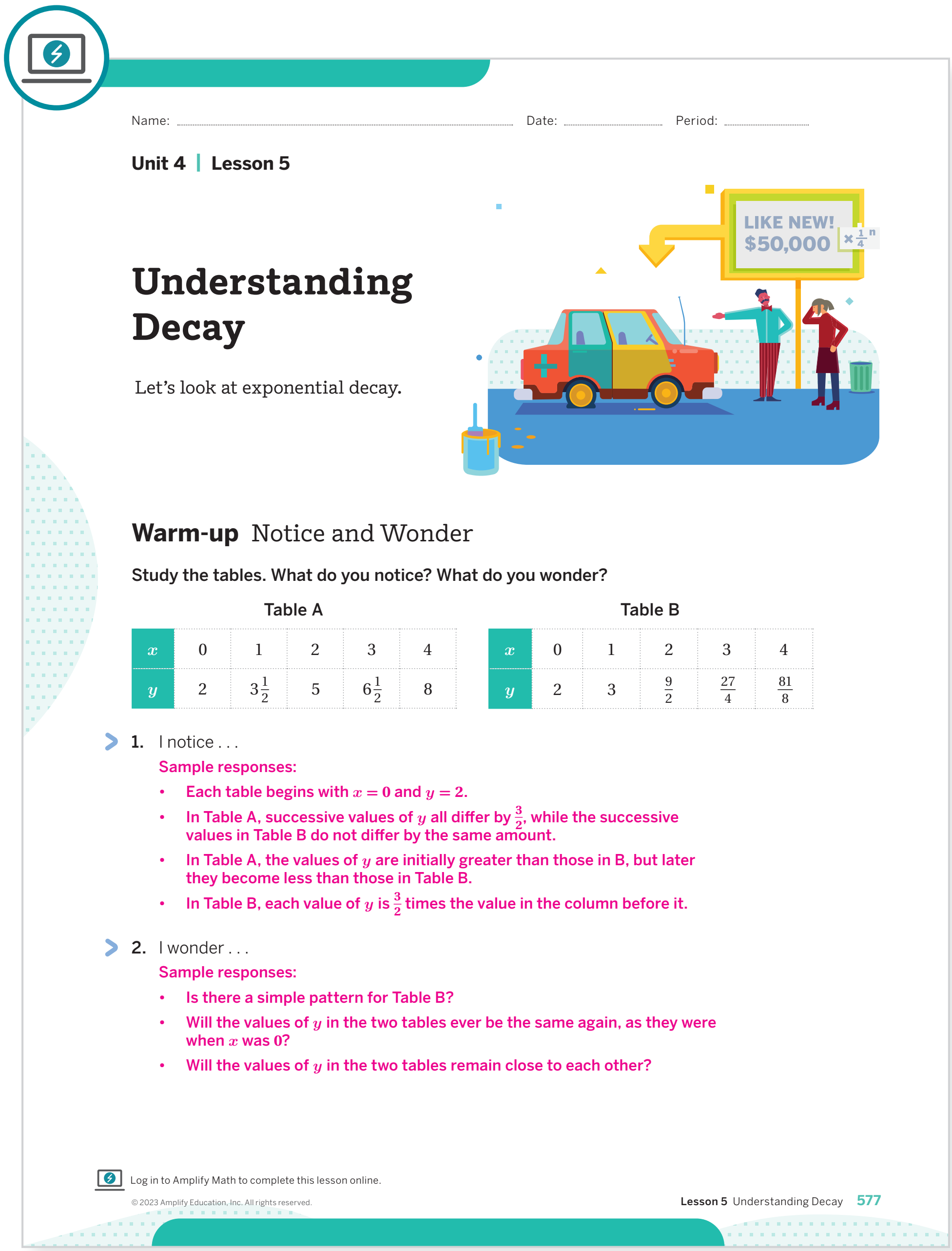
You may want to consider this additional modification if you are short on time.

- If students are comfortable thinking about fractional change, omit **Activity 1**.

Pairs | 5 min

Warm-up Notice and Wonder

Students observe linear and exponential patterns that involve fractions to prepare them for Activity 1.



Name: ______ Date: ______ Period: ______

Unit 4 | **Lesson 5**

Understanding Decay

Let's look at exponential decay.

Warm-up Notice and Wonder

Study the tables. What do you notice? What do you wonder?

Table A

x	0	1	2	3	4
y	2	$3\frac{1}{2}$	5	$6\frac{1}{2}$	8

Table B

x	0	1	2	3	4
y	2	3	$\frac{9}{2}$	$\frac{27}{4}$	$\frac{81}{8}$

1. I notice . . .

   Sample responses:
   - **Each table begins with $x = 0$ and $y = 2$.**
   - **In Table A, successive values of y all differ by $\frac{3}{2}$, while the successive values in Table B do not differ by the same amount.**
   - **In Table A, the values of y are initially greater than those in B, but later they become less than those in Table B.**
   - **In Table B, each value of y is $\frac{3}{2}$ times the value in the column before it.**

2. I wonder . . .

   Sample responses:
   - **Is there a simple pattern for Table B?**
   - **Will the values of y in the two tables ever be the same again, as they were when x was 0?**
   - **Will the values of y in the two tables remain close to each other?**

Log in to Amplify Math to complete this lesson online.

Lesson 5 Understanding Decay 577

1 Launch

Conduct the ***Notice and Wonder*** routine. Prompt students to examine each table individually first. Then have them compare the tables before writing what they notice and wonder.

2 Monitor

Help students get started by modeling operations with fractions.

Look for points of confusion:

- **Having difficulty identifying patterns in each table.** In the first table, have students rewrite fractions as "addition expressions," focusing attention on the changes from the previous values, e.g., $3\frac{1}{2} = 2 + 1\frac{1}{2}$. In the second table, have students write 2 as $\frac{2}{1}$ and 3 as $\frac{6}{2}$ to help them see the pattern.

Look for productive strategies:

- Identifying the common difference in the first table using mental math.
- Identifying the common factor in the second table by comparing $\frac{9}{2}$, $\frac{27}{4}$, and $\frac{81}{8}$.

3 Connect

Display the two tables.

Have students share what they notice and wonder about the tables. Record and display responses shared.

Highlight that Table A shows a common difference, Table B shows a common factor, and both the common difference and the common factor are fractions. Common factors, just like common differences, can be fractions.

Define the term ***decay factor*** as a growth factor between 0 and 1.

Power-up

To power up students' ability to write an equation of a line represented by a table of values, have students complete:

Recall that, in an equation of a line of the form $y = mx + b$, m represents the slope and b represents the y-coordinate of the vertical intercept. The values in the table represent multiple points along the same line when plotted on the coordinate plane.

x	-1	1	2	4
y	6	2	0	-4

a. What is the slope of the line? -2

b. What is the y-intercept of the line? $(0, 4)$

c. What is the equation of the line? $y = -2x + 4$

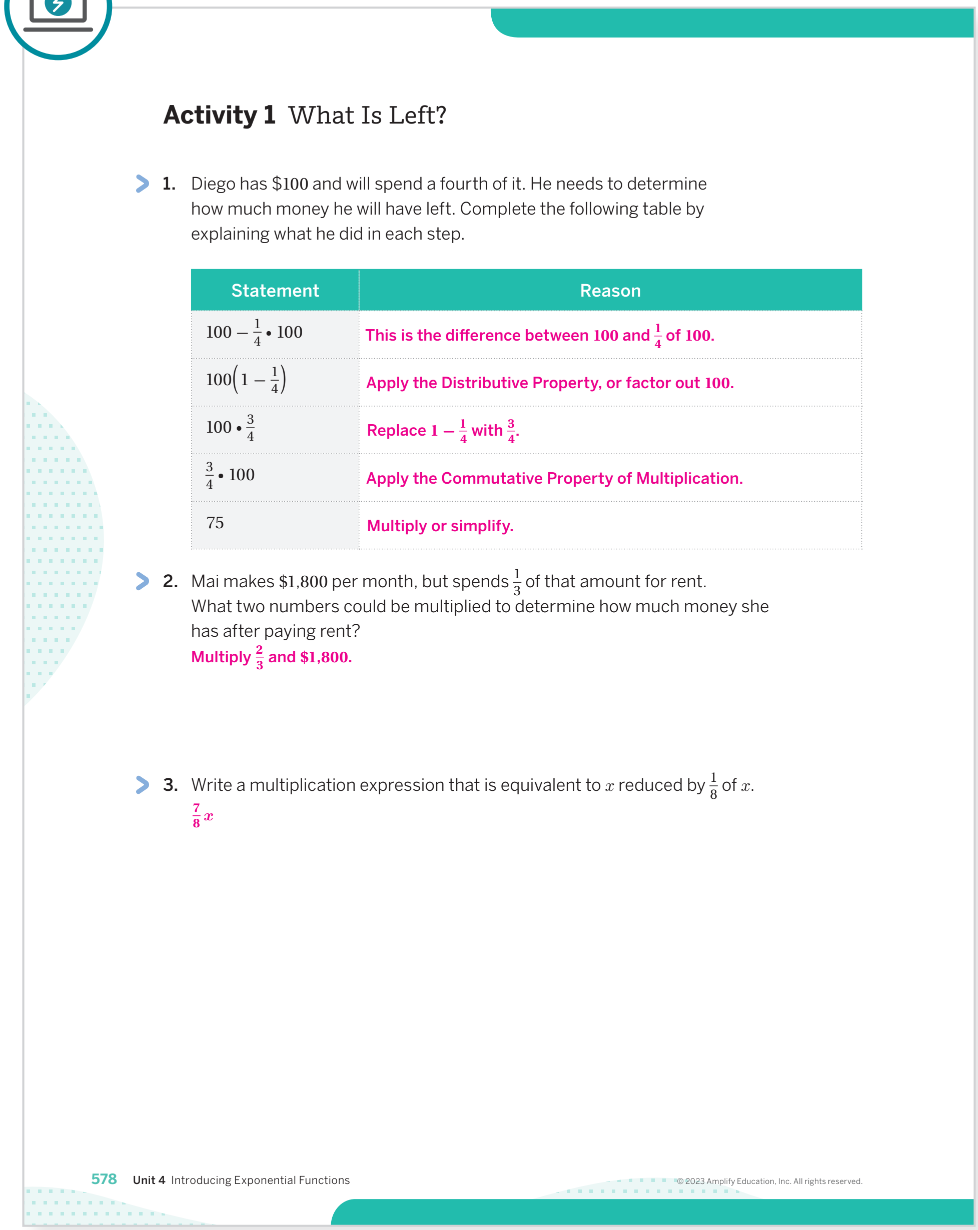
Use: Before the Warm-up

Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5

Independent | 10 min

Activity 1 What Is Left?

Students explore a real-world situation in which the growth factor is less than 1. Multiplying by a growth factor less than 1 means the quantity is decreasing.



Activity 1 What Is Left?

1. Diego has $100 and will spend a fourth of it. He needs to determine how much money he will have left. Complete the following table by explaining what he did in each step.

Statement	Reason
$100 - \frac{1}{4} \bullet 100$	This is the difference between 100 and $\frac{1}{4}$ of 100.
$100\left(1 - \frac{1}{4}\right)$	Apply the Distributive Property, or factor out 100.
$100 \bullet \frac{3}{4}$	Replace $1 - \frac{1}{4}$ with $\frac{3}{4}$.
$\frac{3}{4} \bullet 100$	Apply the Commutative Property of Multiplication.
75	Multiply or simplify.

2. Mai makes $1,800 per month, but spends $\frac{1}{3}$ of that amount for rent. What two numbers could be multiplied to determine how much money she has after paying rent?
Multiply $\frac{2}{3}$ and $1,800.

3. Write a multiplication expression that is equivalent to x reduced by $\frac{1}{8}$ of x.
$\frac{7}{8}x$

578 **Unit 4** Introducing Exponential Functions

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking, "If Diego has $100 and spends $\frac{1}{4}$ of it, how can you determine how much he has left?"

Look for points of confusion:

- **Struggling to connect the fraction being subtracted and the fraction remaining.** Have them identify the common factor in Problem 2; is it $\frac{1}{3}$ or $\frac{2}{3}$?

Look for productive strategies:

- Solving each problem, recognizing distribution as the key to the process.
- Applying a common factor to a new situation and generalizing with an algebraic expression.

3 Connect

Display the table in Problem 1.

Have students share their thinking for what Diego did in each step. Record their responses in the table.

Highlight that decreasing the quantity can be expressed using subtraction *and* multiplication. Using only multiplication (by the common factor) is more efficient.

Define the term ***exponential decay*** as quantities that *decrease* exponentially.

Ask:

- "What is the most efficient way to determine how much Diego has left?" Multiply by the fraction he has left.
- Why can the common factor be called a *growth factor* if the amount is *decreasing*?" It is a repeating factor.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to verbally explain the reasons in Problem 1 to you or a partner. Then during the Connect, display sample correct responses to ensure students understand what is happening at each step.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest students color code what is changing during each step in Problem 1. This will help them focus on the differences in order to think about the mathematical operations occurring and why they can occur.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to how the quantities in each of these problems decreased by a *factor*. Because it is a factor, the expression showing the new amount is a multiplication expression.

Display these expressions and ask the following questions: $\frac{3}{4} \bullet 100$ $\frac{2}{3} \bullet 1800$ $\frac{7}{8} \bullet x$

- "If Diego will spend $\frac{1}{4}$, why is the factor $\frac{3}{4}$?"
- "If Mai spends $\frac{1}{3}$, why is the factor $\frac{2}{3}$?"
- "If x is reduced by $\frac{1}{8}$, why is the factor $\frac{7}{8}$?"

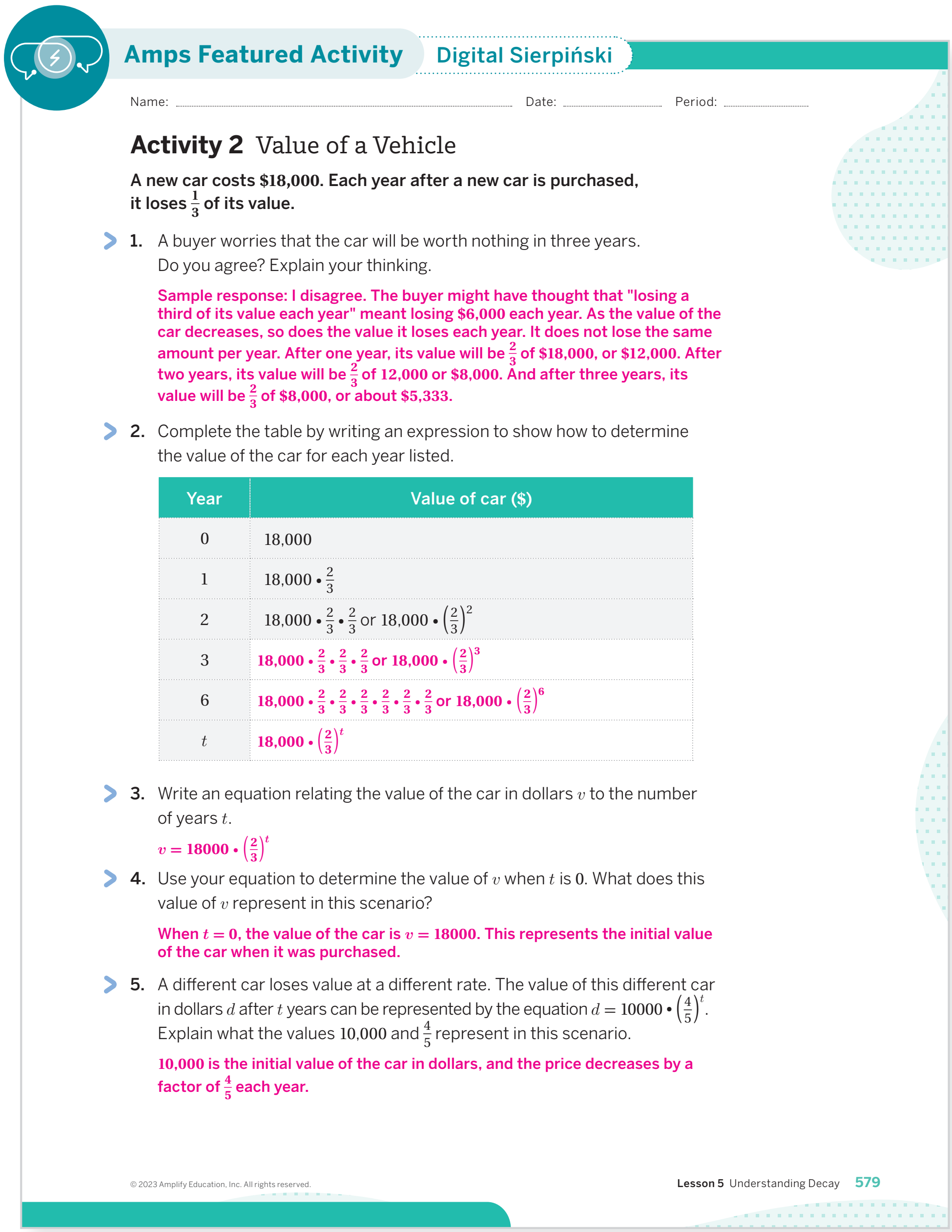
English Learners

Annotate the factors with the words "amount left" or "amount remaining."

Pairs | 15 min

Activity 2 Value of a Vehicle

Students examine a situation where a quantity decreases by repeated multiplication of the same factor.



Amps Featured Activity Digital Sierpiński

Name: Date: Period:

Activity 2 Value of a Vehicle

A new car costs $18,000. Each year after a new car is purchased, it loses $\frac{1}{3}$ of its value.

1. A buyer worries that the car will be worth nothing in three years. Do you agree? Explain your thinking.

   Sample response: I disagree. The buyer might have thought that "losing a third of its value each year" meant losing $6,000 each year. As the value of the car decreases, so does the value it loses each year. It does not lose the same amount per year. After one year, its value will be $\frac{2}{3}$ of $18,000, or $12,000. After two years, its value will be $\frac{2}{3}$ of 12,000 or $8,000. And after three years, its value will be $\frac{2}{3}$ of $8,000, or about $5,333.

2. Complete the table by writing an expression to show how to determine the value of the car for each year listed.

Year	Value of car ($)
0	18,000
1	$18{,}000 \cdot \frac{2}{3}$
2	$18{,}000 \cdot \frac{2}{3} \cdot \frac{2}{3}$ or $18{,}000 \cdot \left(\frac{2}{3}\right)^2$
3	$18{,}000 \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$ or $18{,}000 \cdot \left(\frac{2}{3}\right)^3$
6	$18{,}000 \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$ or $18{,}000 \cdot \left(\frac{2}{3}\right)^6$
t	$18{,}000 \cdot \left(\frac{2}{3}\right)^t$

3. Write an equation relating the value of the car in dollars v to the number of years t.

   $v = 18000 \cdot \left(\frac{2}{3}\right)^t$

4. Use your equation to determine the value of v when t is 0. What does this value of v represent in this scenario?

   When $t = 0$, the value of the car is $v = 18000$. This represents the initial value of the car when it was purchased.

5. A different car loses value at a different rate. The value of this different car in dollars d after t years can be represented by the equation $d = 10000 \cdot \left(\frac{4}{5}\right)^t$. Explain what the values 10,000 and $\frac{4}{5}$ represent in this scenario.

   10,000 is the initial value of the car in dollars, and the price decreases by a factor of $\frac{4}{5}$ each year.

Lesson 5 Understanding Decay 579

1 Launch

Read the prompt aloud. Give students think-time to consider the first problem. Have them discuss their thoughts with their partner before completing the activity in pairs.

2 Monitor

Help students get started by telling them that they are to complete the table using expressions that show *how* to determine the values, rather than determining the value of the car each year.

Look for points of confusion:

- **Dividing by 3.** Remind them multiplying by $\frac{1}{3}$ yields the same result.
- **Not understanding the growth factor is $\frac{2}{3}$.** Ask how much of the car's value will be *retained* each year.
- **Struggling to find products with fractions as factors.** Allow the use of multiplication charts and calculators.

Look for productive strategies:

- Identifying repeated multiplication and writing it in exponential form.
- Applying a common factor to a new situation and generalizing with an algebraic expression.

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge*

Mention that the value of any new car decreases by a significant amount during the first year. To help them visualize what this means, have them determine the value of the new car in this activity after the first year. Allow students to use any strategy they choose, including determining $\frac{1}{3}$ of $18,000 and subtracting that amount from $18,000. $12,000

Extension: *Math Enrichment*

If students complete the *Are you ready for more?* activity, have them use the Amps slides in which they can use digital technology to create a Sierpiński triangle.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory context and have students work with their partner to write 2–3 mathematical questions they could ask about this situation. Sample questions shown.

- What is the value of the car after one year?
- What is the value of the car after 3 years? 10 years?
- Will the car's value ever be $0?

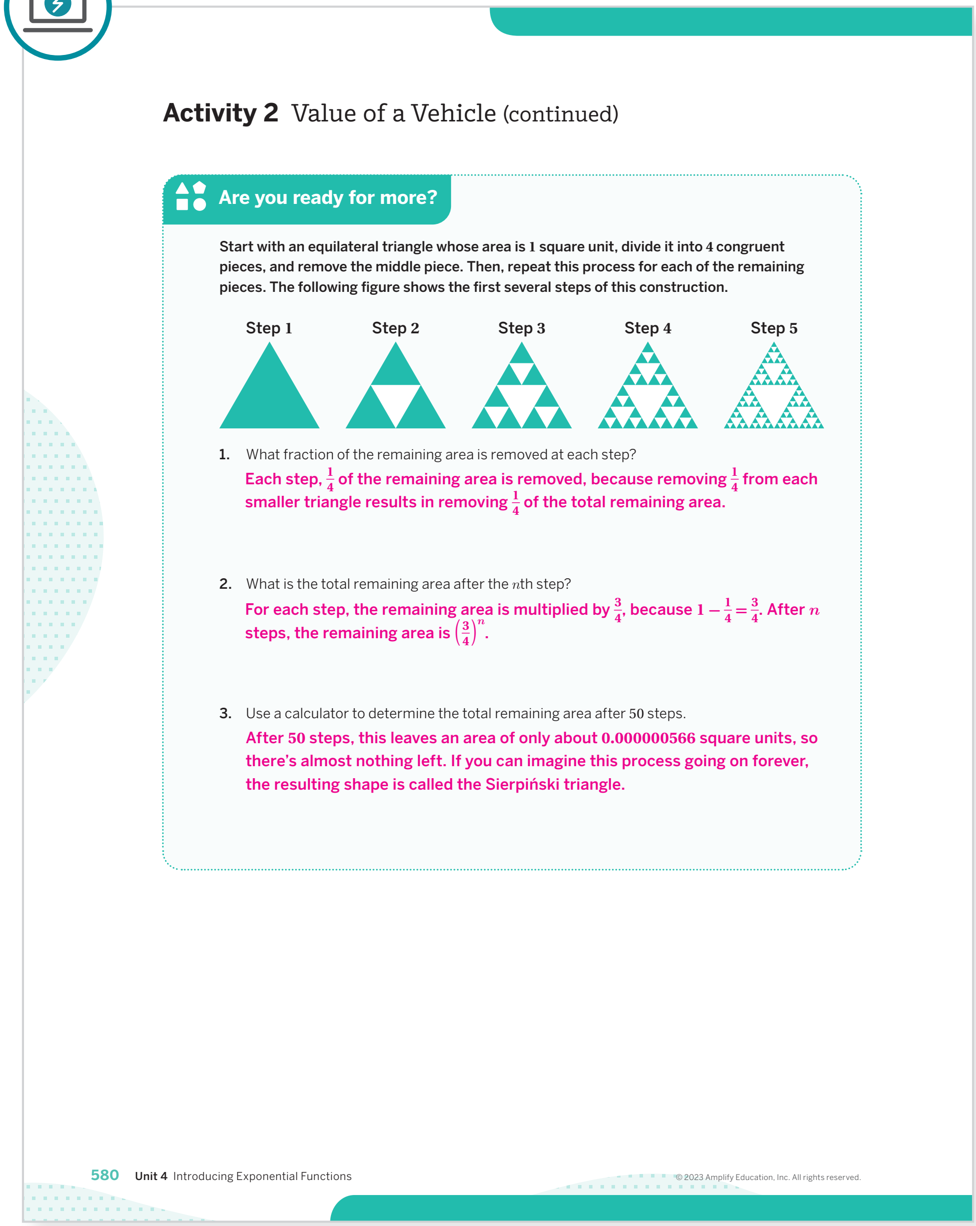
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 15 min

Activity 2 Value of a Vehicle (continued)

Students examine a situation where a quantity decreases by repeated multiplication of the same factor.



Activity 2 Value of a Vehicle (continued)

Are you ready for more?

Start with an equilateral triangle whose area is 1 square unit, divide it into 4 congruent pieces, and remove the middle piece. Then, repeat this process for each of the remaining pieces. The following figure shows the first several steps of this construction.

1. What fraction of the remaining area is removed at each step?
   Each step, $\frac{1}{4}$ of the remaining area is removed, because removing $\frac{1}{4}$ from each smaller triangle results in removing $\frac{1}{4}$ of the total remaining area.

2. What is the total remaining area after the nth step?
   For each step, the remaining area is multiplied by $\frac{3}{4}$, because $1-\frac{1}{4}=\frac{3}{4}$. After n steps, the remaining area is $\left(\frac{3}{4}\right)^n$.

3. Use a calculator to determine the total remaining area after 50 steps.
   After 50 steps, this leaves an area of only about 0.000000566 square units, so there's almost nothing left. If you can imagine this process going on forever, the resulting shape is called the Sierpiński triangle.

580 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the table, explaining each value.

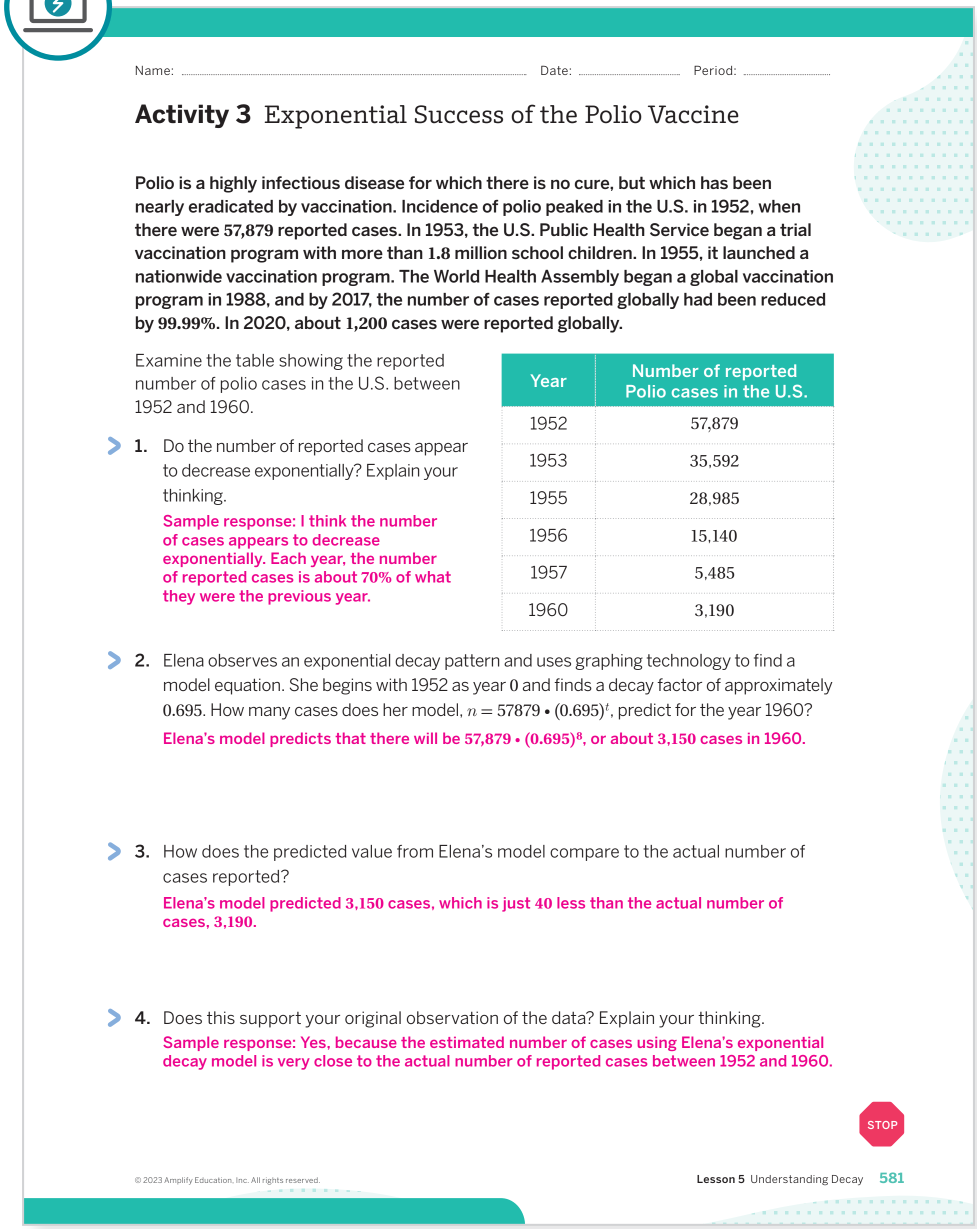
Highlight that the value of the car after t years can be determined by multiplying by $\frac{2}{3}$ repeatedly, t times. This multiplier is still called a *growth factor* even though the value is decreasing. The quantities are decreasing because it is exponential decay. Emphasize that repeated multiplication by this growth factor can be expressed with an exponential expression.

Ask, "Why does it make sense to multiply an entry in one row by $\frac{2}{3}$ to get the entry for the next row?" Each year, only $\frac{2}{3}$ of the car's value remains.

Pairs | 10 min

Activity 3 Exponential Success of the Polio Vaccine

Students utilize real-world data for polio vaccinations to study exponential decay (from a table). They compare results from the exponential equation to the table.



Name: ______ Date: ______ Period: ______

Activity 3 Exponential Success of the Polio Vaccine

Polio is a highly infectious disease for which there is no cure, but which has been nearly eradicated by vaccination. Incidence of polio peaked in the U.S. in 1952, when there were 57,879 reported cases. In 1953, the U.S. Public Health Service began a trial vaccination program with more than 1.8 million school children. In 1955, it launched a nationwide vaccination program. The World Health Assembly began a global vaccination program in 1988, and by 2017, the number of cases reported globally had been reduced by 99.99%. In 2020, about 1,200 cases were reported globally.

Examine the table showing the reported number of polio cases in the U.S. between 1952 and 1960.

Year	Number of reported Polio cases in the U.S.
1952	57,879
1953	35,592
1955	28,985
1956	15,140
1957	5,485
1960	3,190

1. Do the number of reported cases appear to decrease exponentially? Explain your thinking.
   Sample response: I think the number of cases appears to decrease exponentially. Each year, the number of reported cases is about 70% of what they were the previous year.

2. Elena observes an exponential decay pattern and uses graphing technology to find a model equation. She begins with 1952 as year 0 and finds a decay factor of approximately 0.695. How many cases does her model, $n = 57879 \cdot (0.695)^t$, predict for the year 1960?
   Elena's model predicts that there will be $57{,}879 \cdot (0.695)^8$, or about 3,150 cases in 1960.

3. How does the predicted value from Elena's model compare to the actual number of cases reported?
   Elena's model predicted 3,150 cases, which is just 40 less than the actual number of cases, 3,190.

4. Does this support your original observation of the data? Explain your thinking.
   Sample response: Yes, because the estimated number of cases using Elena's exponential decay model is very close to the actual number of reported cases between 1952 and 1960.

STOP

1 Launch

Use the ***Three Reads*** routine to read and help students make sense of the narrative.

2 Monitor

Help students get started by having them study the table, making note of any patterns they notice.

Look for points of confusion:

- **Not understanding the absence of an ambiguous common difference or common factor.** Explain this is real data and focus on how quickly the number of cases decreases over equal periods of time.

Look for productive strategies:

- Identifying nonlinear decay from a table of values.
- Selecting correct values of t, substituting them into the model, and comparing to the real-world data.

3 Connect

Display the table and Elena's model.

Have students share their observations of the patterns shown in the table. Then have them share the results of their predictions of the number of cases in 1960. Finally, have them share whether their predictions support their original observations of the data.

Highlight that exponential decay can be observed given a table of values.

Ask, "What did you discover about growth factors given this real-world data?" They estimate the amount of growth or decay.

MLR Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that the number of cases of polio have been significantly reduced due to the polio vaccination.
- **Read 2:** Ask students to name given quantities, such as the number of cases around the world had been reduced by 99.99% by 2017.
- **Read 3:** Ask students to study the table and brainstorm strategies for how they will respond to Problem 1.

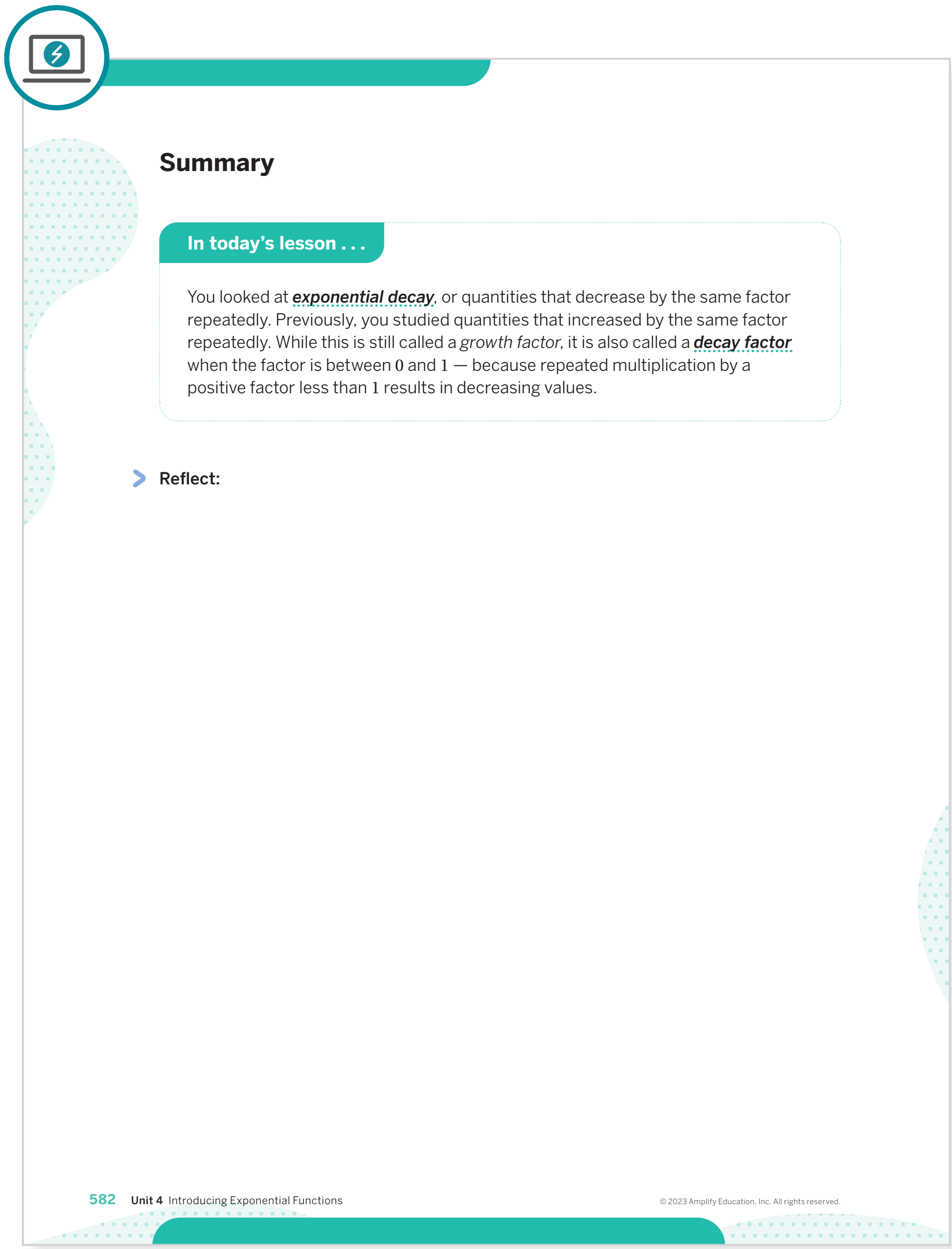
English Learners

Summarize the text for students without using some of the more technical terms.

Whole Class | 5 min

Summary

Review and synthesize that when quantities repeatedly decrease by a constant factor between 0 and 1, that factor is called a decay factor.



Summary

In today's lesson . . .

You looked at ***exponential decay***, or quantities that decrease by the same factor repeatedly. Previously, you studied quantities that increased by the same factor repeatedly. While this is still called a *growth factor*, it is also called a ***decay factor*** when the factor is between 0 and 1 — because repeated multiplication by a positive factor less than 1 results in decreasing values.

Reflect:

582 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the table from Activity 2, Problem 2.

Highlight that a *decay factor* is a growth factor greater than 0 and less than 1. Emphasize that repeated multiplication by a common factor can be expressed using exponents.

Formalize vocabulary:

- *decay factor*
- *exponential decay*

Ask:

- "How can you express the value of the car after one year? After two years?" Multiply 18,000 by $\frac{2}{3}$; by $\left(\frac{2}{3}\right)^2$.
- "Why might it make sense to use only multiplication (instead of subtraction and multiplication) to show the value of the car over time?" It is more efficient to multiply by $\left(\frac{2}{3}\right)^t$.
- "When you use only multiplication, why does the $\frac{1}{3}$ not appear in the expression?" When using only multiplication, the fraction represents the amount *remaining*.
- "What is the value of the car after t years?" $18000 \cdot \left(\frac{2}{3}\right)^t$

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What characterizes exponential decay?"
- "What are some real-world examples of exponential decay?"

Math Language Development

MLR2: Collect and Display

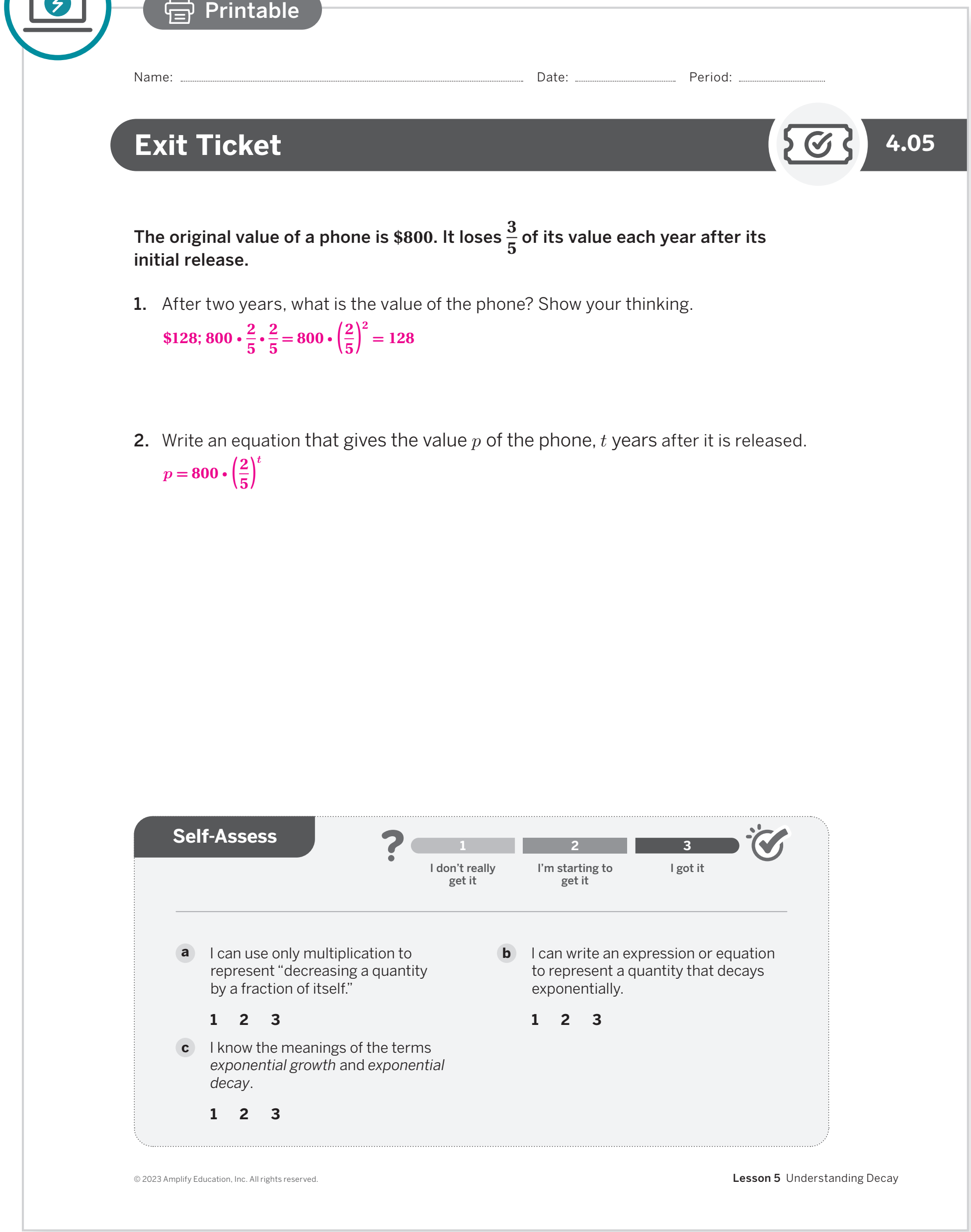
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *decay factor* and *exponential decay* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of exponential decay by using repeated reasoning to write repeated multiplication as an exponential equation with a decay factor.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.05

The original value of a phone is \$800. It loses $\frac{3}{5}$ of its value each year after its initial release.

1. After two years, what is the value of the phone? Show your thinking.

   \$128; $800 \cdot \frac{2}{5} \cdot \frac{2}{5} = 800 \cdot \left(\frac{2}{5}\right)^2 = 128$

2. Write an equation that gives the value p of the phone, t years after it is released.

   $p = 800 \cdot \left(\frac{2}{5}\right)^t$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can use only multiplication to represent "decreasing a quantity by a fraction of itself."

   1 2 3

b. I can write an expression or equation to represent a quantity that decays exponentially.

   1 2 3

c. I know the meanings of the terms *exponential growth* and *exponential decay*.

   1 2 3

Lesson 5 Understanding Decay

Success looks like . . .

- **Language Goal:** Explaining that exponential growth describes a quantity that changes by a growth factor that is greater than 1. **(Speaking and Listening)**
- **Language Goal:** Explaining that exponential decay describes a quantity that changes by a growth factor that is less than 1 but greater than 0. **(Speaking and Listening)**
- **Goal:** Representing "decreasing a quantity" in terms of multiplying it by some fraction of itself.
- **Goal:** Writing an expression or an equation to represent exponential decay.
  - » Writing an equation to represent the value of the phone after a given amount of time in Problem 2.

Suggested next steps

If students are unable to determine the value of the phone after two years in Problem 1, consider:

- Reviewing Activity 1, Problems 2 and 3.
- Reviewing Activity 2, Problem 2.

If students do not correctly write the equation for the value of the phone in Problem 2, consider:

- Reviewing Activity 1, Problem 1.
- Reviewing Activity 2, Problems 2–5.
- Assigning Practice Problem 2.

Professional Learning

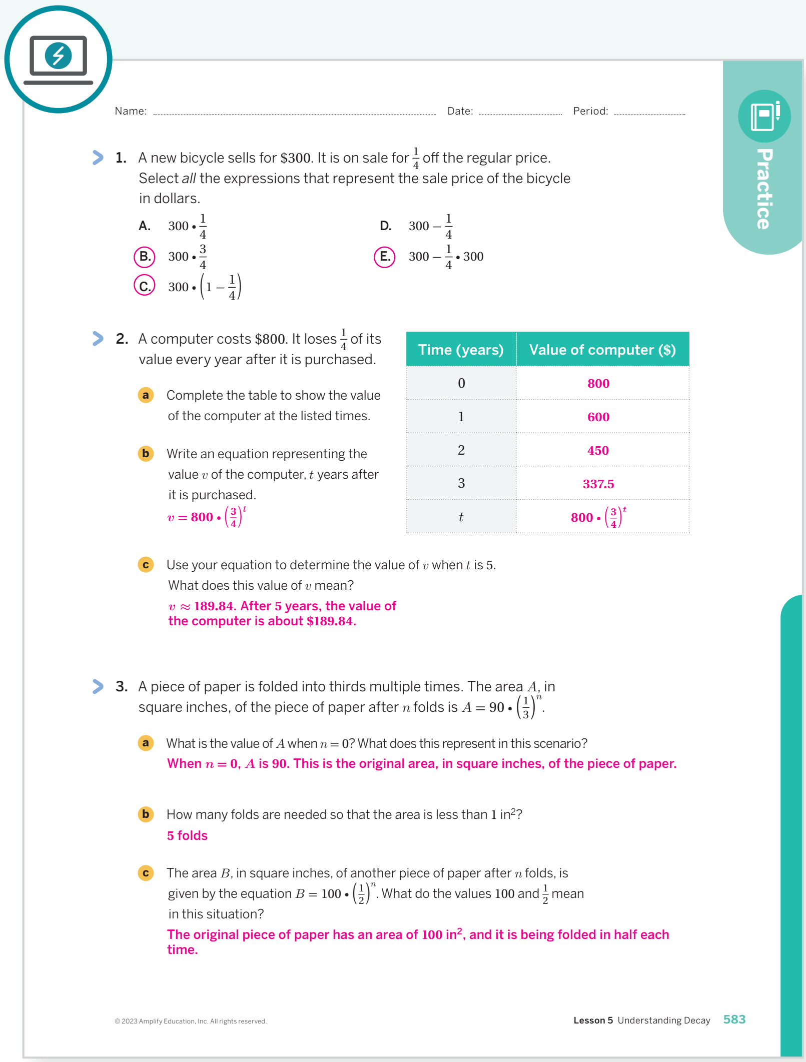
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students explored the concept of "what is left," or how much remains when analyzing an exponential decay relationship. How will this understanding support future work in constructing exponential functions to model real-world data? How well do you think your students understood this concept of 'what is left'?
- Which students or groups of students did and did not have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. A new bicycle sells for $300. It is on sale for $\frac{1}{4}$ off the regular price. Select *all* the expressions that represent the sale price of the bicycle in dollars.

   A. $300 \cdot \frac{1}{4}$
   [✓ B.] $300 \cdot \frac{3}{4}$
   [✓ C.] $300 \cdot \left(1 - \frac{1}{4}\right)$
   D. $300 - \frac{1}{4}$
   [✓ E.] $300 - \frac{1}{4} \cdot 300$

2. A computer costs $800. It loses $\frac{1}{4}$ of its value every year after it is purchased.

Time (years)	Value of computer ($)
0	800
1	600
2	450
3	337.5
t	$800 \cdot \left(\frac{3}{4}\right)^t$

   a. Complete the table to show the value of the computer at the listed times.

   b. Write an equation representing the value v of the computer, t years after it is purchased.
   $v = 800 \cdot \left(\frac{3}{4}\right)^t$

   c. Use your equation to determine the value of v when t is 5. What does this value of v mean?
   $v \approx 189.84$. After 5 years, the value of the computer is about $189.84.

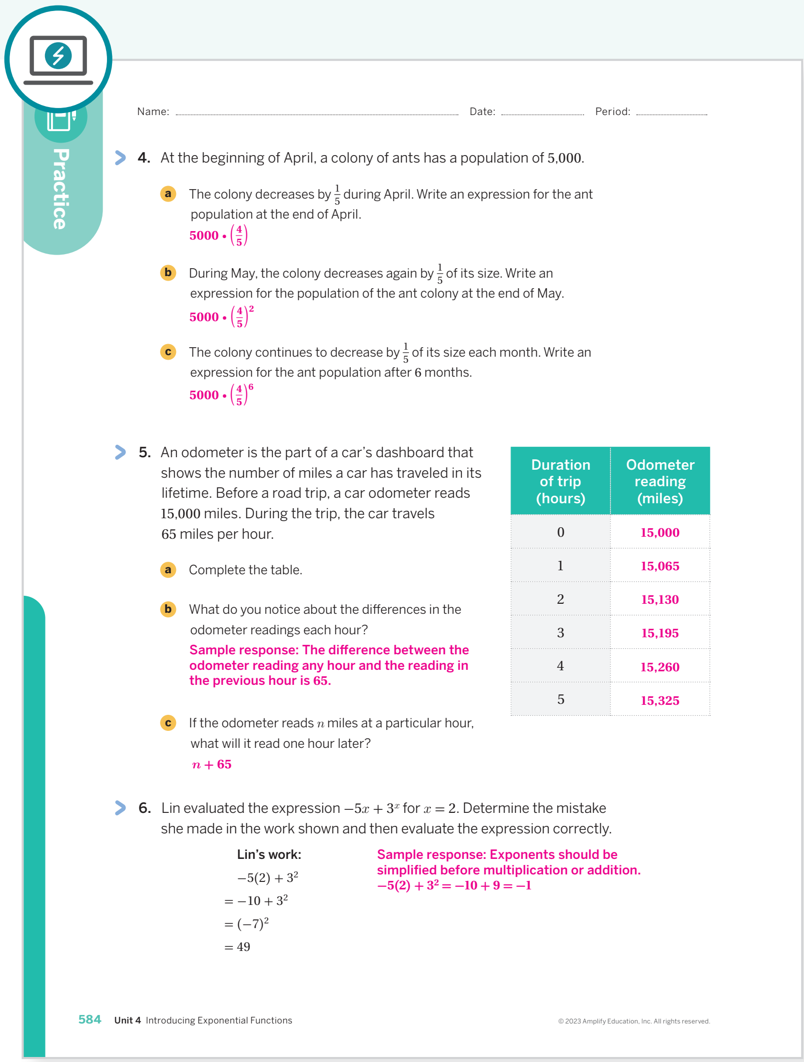
3. A piece of paper is folded into thirds multiple times. The area A, in square inches, of the piece of paper after n folds is $A = 90 \cdot \left(\frac{1}{3}\right)^n$.

   a. What is the value of A when $n = 0$? What does this represent in this scenario?
   When $n = 0$, A is 90. This is the original area, in square inches, of the piece of paper.

   b. How many folds are needed so that the area is less than 1 in^2?
   5 folds

   c. The area B, in square inches, of another piece of paper after n folds, is given by the equation $B = 100 \cdot \left(\frac{1}{2}\right)^n$. What do the values 100 and $\frac{1}{2}$ mean in this situation?
   The original piece of paper has an area of 100 in^2, and it is being folded in half each time.

Lesson 5 Understanding Decay 583



Name: ______ Date: ______ Period: ______

4. At the beginning of April, a colony of ants has a population of 5,000.

   a. The colony decreases by $\frac{1}{5}$ during April. Write an expression for the ant population at the end of April.
   $5000 \cdot \left(\frac{4}{5}\right)$

   b. During May, the colony decreases again by $\frac{1}{5}$ of its size. Write an expression for the population of the ant colony at the end of May.
   $5000 \cdot \left(\frac{4}{5}\right)^2$

   c. The colony continues to decrease by $\frac{1}{5}$ of its size each month. Write an expression for the ant population after 6 months.
   $5000 \cdot \left(\frac{4}{5}\right)^6$

5. An odometer is the part of a car's dashboard that shows the number of miles a car has traveled in its lifetime. Before a road trip, a car odometer reads 15,000 miles. During the trip, the car travels 65 miles per hour.

Duration of trip (hours)	Odometer reading (miles)
0	15,000
1	15,065
2	15,130
3	15,195
4	15,260
5	15,325

   a. Complete the table.

   b. What do you notice about the differences in the odometer readings each hour?
   Sample response: The difference between the odometer reading any hour and the reading in the previous hour is 65.

   c. If the odometer reads n miles at a particular hour, what will it read one hour later?
   $n + 65$

6. Lin evaluated the expression $-5x + 3^x$ for $x = 2$. Determine the mistake she made in the work shown and then evaluate the expression correctly.

Lin's work:

$$-5(2) + 3^2$$
$$= -10 + 3^2$$
$$= (-7)^2$$
$$= 49$$

Sample response: Exponents should be simplified before multiplication or addition.
$-5(2) + 3^2 = -10 + 9 = -1$

584 **Unit 4** Introducing Exponential Functions

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activities 2 and 3	3
	3	Activities 1 and 2	3
	4	Activities 1 and 2	3
Spiral	5	Unit 4 Lesson 2	2
Formative ⚡	6	Unit 4 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 6**

Representing Exponential Decay

Let's think about how to represent exponential decay.

Focus

Goals

1. Determine the growth factor from a graph and write an equation to represent exponential decay.
2. Graph equations that represent quantities that change by growth factors greater than 0 and less than 1.

Rigor

- Students build on their **conceptual understanding** of graphs and expressions for exponential behavior by connecting those showing growth with those showing decay.

Coherence

Today

Students examine the graphs and equations of scenarios characterized by *exponential decay*. They identify key features in the graphs and interpret different parts of the graph and equation in context. Students explain the meaning of a and b in an equation of the form $y = ab^x$ that represents exponential decay.

Previously

Students studied scenarios characterized by exponential growth and made connections between their tables, equations, and graphs.

Coming Soon

Students will explore exponential relationships that have continuous domains, using technology.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- graphing technology

Math Language Development

Review words

- *decay factor*
- *exponential growth*
- *growth factor*
- *linear*

Amps | Featured Activity

Activity 1
Interactive Graphs

Students use interactive tools to construct a graph representing a scenario involving exponential decay.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students seek to interpret exponential decay models for events, they might have to fight an impulsive interpretation of the situation. Explain to students that mathematical tools do not have to be physical tools that they use in their hands. A graph is a type of mathematical tool that can most benefit them as they analyze and interpret the scenarios. The graph visually represents the situation and helps students connect numbers and patterns to the verbal descriptions.

Modifications to Pacing

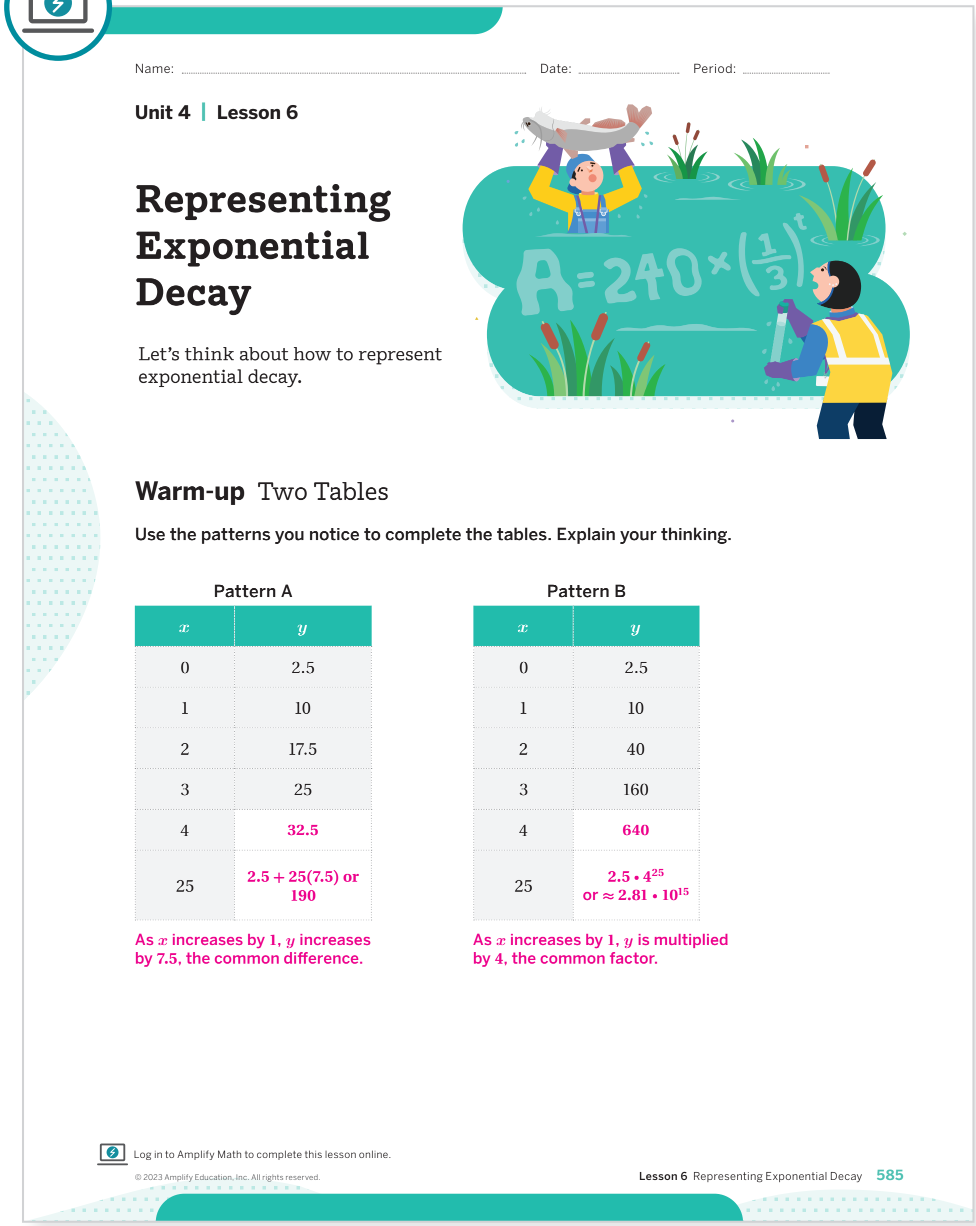
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, omit Problem 5 if students are comfortable calculating values and graphing in Activity 1.

Independent | 5 min

Warm-up Two Tables

Students identify linear and exponential relationships from tables and extend the patterns they see.



Name: ______ Date: ______ Period: ______

Unit 4 | **Lesson 6**

Representing Exponential Decay

Let's think about how to represent exponential decay.

Warm-up Two Tables

Use the patterns you notice to complete the tables. Explain your thinking.

Pattern A

x	y
0	2.5
1	10
2	17.5
3	25
4	**32.5**
25	**$2.5 + 25(7.5)$ or 190**

As x increases by 1, y increases by 7.5, the common difference.

Pattern B

x	y
0	2.5
1	10
2	40
3	160
4	**640**
25	**$2.5 \cdot 4^{25}$ or $\approx 2.81 \cdot 10^{15}$**

As x increases by 1, y is multiplied by 4, the common factor.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work individually on the Warm-up.

2 Monitor

Have students get started by annotating their tables to visually represent the patterns they see.

Look for points of confusion:

- **Having difficulty determining the value of y when $x = 25$.** Have students use the pattern that they notice to write an expression that can be used to determine the value of y for any x-value in the pattern.

Look for productive strategies:

- Using a pattern to generate the expression to determine the value of y when $x = 25$.

3 Connect

Have students share how they used the patterns to complete the tables. Select and sequence students using repeated addition or multiplication first, and then those using an expression.

Highlight that Pattern A is a linear pattern because it is a pattern of repeated addition and Pattern B is an exponential pattern because it is a pattern of repeated multiplication.

Power-up

To power up students' ability to evaluate exponential expressions, have students complete:

Evaluate each expression for $x = 3$.

1. 5^x 125
2. $2 \cdot 5^x$ 250
3. 4^x 64
4. $3 \cdot 4^x$ 192
5. $\left(\frac{1}{2}\right) \cdot 2^x$ 4

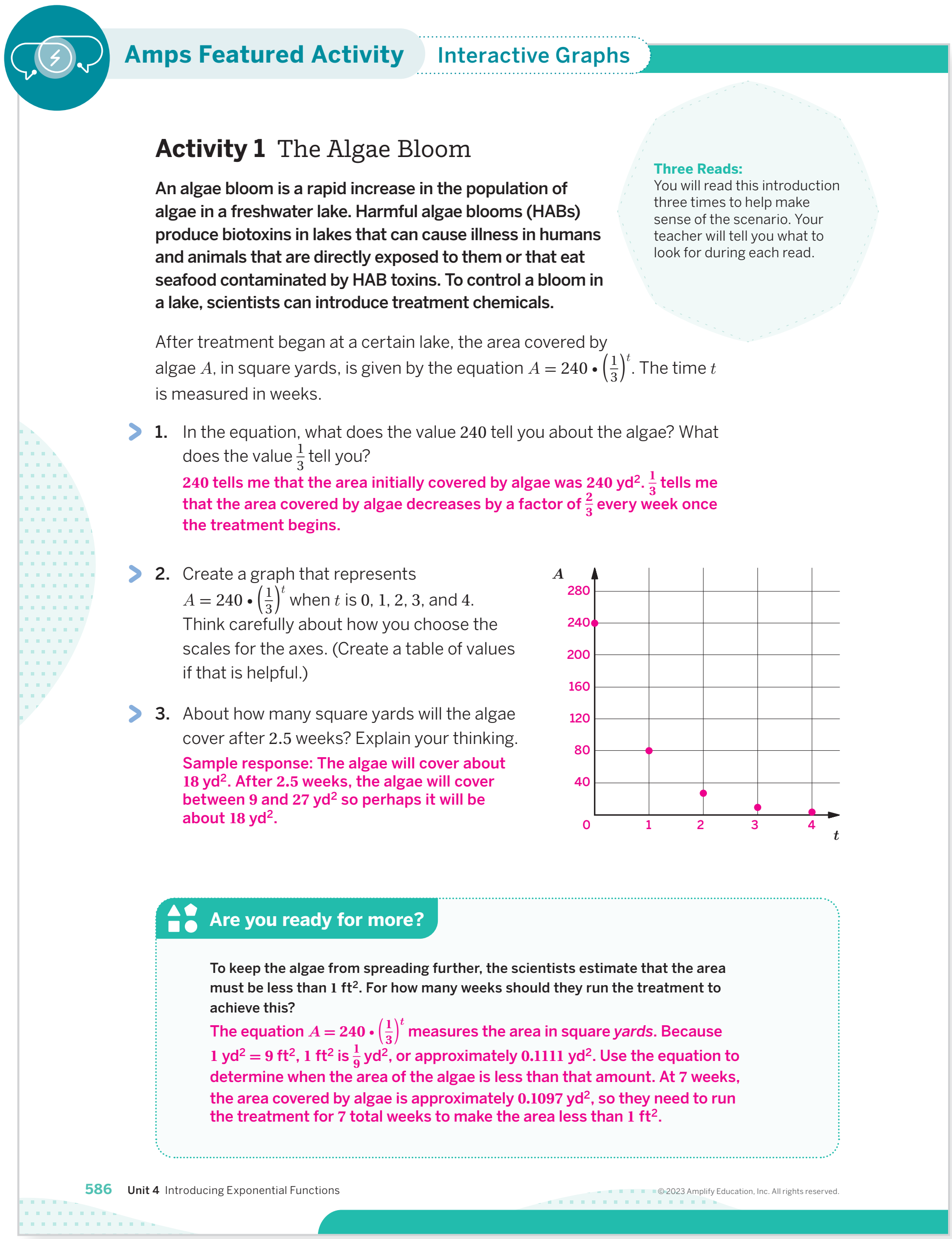
Use: Before the Warm-up

Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2

Independent | 15 min

Activity 1 The Algae Bloom

Students create and interpret a graph given a scenario that represents exponential decay.



Amps Featured Activity Interactive Graphs

Activity 1 The Algae Bloom

Three Reads: You will read this introduction three times to help make sense of the scenario. Your teacher will tell you what to look for during each read.

An algae bloom is a rapid increase in the population of algae in a freshwater lake. Harmful algae blooms (HABs) produce biotoxins in lakes that can cause illness in humans and animals that are directly exposed to them or that eat seafood contaminated by HAB toxins. To control a bloom in a lake, scientists can introduce treatment chemicals.

After treatment began at a certain lake, the area covered by algae A, in square yards, is given by the equation $A = 240 \cdot \left(\frac{1}{3}\right)^t$. The time t is measured in weeks.

1. In the equation, what does the value 240 tell you about the algae? What does the value $\frac{1}{3}$ tell you?
   240 tells me that the area initially covered by algae was 240 yd^2. $\frac{1}{3}$ tells me that the area covered by algae decreases by a factor of $\frac{2}{3}$ every week once the treatment begins.

2. Create a graph that represents $A = 240 \cdot \left(\frac{1}{3}\right)^t$ when t is 0, 1, 2, 3, and 4. Think carefully about how you choose the scales for the axes. (Create a table of values if that is helpful.)

3. About how many square yards will the algae cover after 2.5 weeks? Explain your thinking.
   Sample response: The algae will cover about 18 yd^2. After 2.5 weeks, the algae will cover between 9 and 27 yd^2 so perhaps it will be about 18 yd^2.

Are you ready for more?

To keep the algae from spreading further, the scientists estimate that the area must be less than 1 ft^2. For how many weeks should they run the treatment to achieve this?

The equation $A = 240 \cdot \left(\frac{1}{3}\right)^t$ measures the area in square *yards*. Because 1 yd^2 = 9 ft^2, 1 ft^2 is $\frac{1}{9}$ yd^2, or approximately 0.1111 yd^2. Use the equation to determine when the area of the algae is less than that amount. At 7 weeks, the area covered by algae is approximately 0.1097 yd^2, so they need to run the treatment for 7 total weeks to make the area less than 1 ft^2.

586 **Unit 4** Introducing Exponential Functions

1 Launch

Read the prompt aloud. Set an expectation for the amount of time students will have to work individually on the activity. Provide access to graphing technology.

2 Monitor

Help students get started by asking, “Does the equation display exponential growth or decay?”

Look for points of confusion:

- **Struggling to choose a scale.** Encourage students to think about minimum and maximum values of t and A that must be represented on their graph.

Look for productive strategies:

- Creating a table of coordinates.
- Choosing to use graphing technology or creating a table of values to confirm the equations they write are correct.
- Connecting the values in the equation with what they represent in the given context.

3 Connect

Display the equation and blank graph from Problem 2.

Have students share the values of A they determined that correspond to $t = 0, 1, 2, 3,$ and 4. Record their responses on the graph.

Ask:

- “What does a t-value of 2.5 represent?” 2.5 weeks
- “What is the approximate value of A when $t = 2.5$?” 15.4 yd^2
- “After how many weeks will the area covered by algae be 0?” It will never be 0.

Highlight that as t is increasing, A is getting smaller and smaller, but it will never be equal to 0. Ask students why they think that is.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they have seen an algae bloom or are familiar with how the water becomes discolored because of the increase in algae. Consider showing students photos of algae blooms, such as the Red Tide algae bloom.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive tools to construct a graph representing a scenario involving exponential decay.

Math Language Development

MLR6: Three Reads

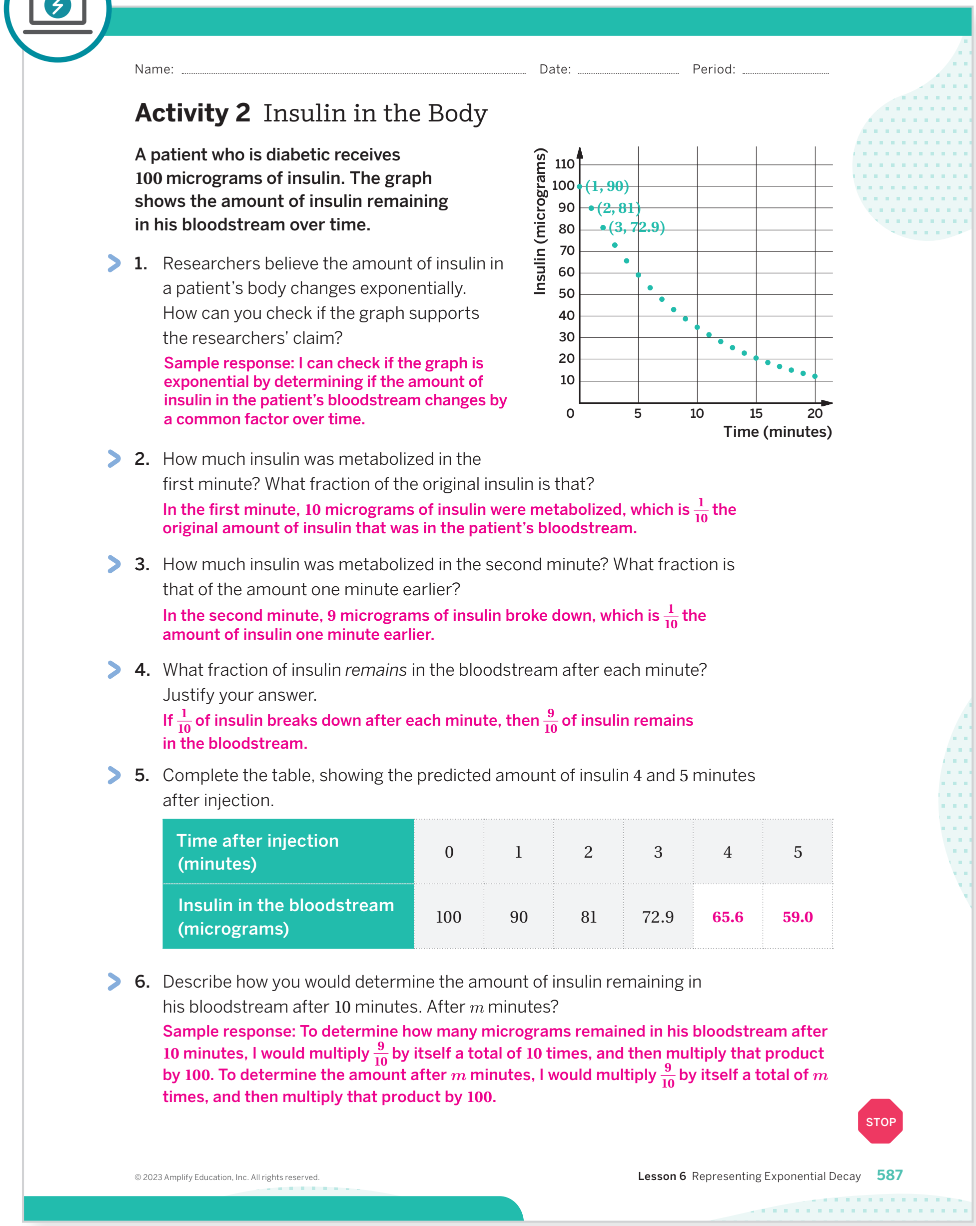
Use this routine to help students make sense of the introductory text.

- **Read 1:** Ask students to describe the effects that harmful algae blooms in lakes have on humans and animals.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the relationship between the area of algae and time represented by the exponential equation.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problems 1 and 2.

Pairs | 20 min

Activity 2 Insulin in the Body

Students will examine and interpret a graph modeling a scenario characterized by exponential decay, and identify its key features.



Name: ______ Date: ______ Period: ______

Activity 2 Insulin in the Body

A patient who is diabetic receives 100 micrograms of insulin. The graph shows the amount of insulin remaining in his bloodstream over time.

1. Researchers believe the amount of insulin in a patient's body changes exponentially. How can you check if the graph supports the researchers' claim?
   Sample response: I can check if the graph is exponential by determining if the amount of insulin in the patient's bloodstream changes by a common factor over time.

2. How much insulin was metabolized in the first minute? What fraction of the original insulin is that?
   In the first minute, 10 micrograms of insulin were metabolized, which is $\frac{1}{10}$ the original amount of insulin that was in the patient's bloodstream.

3. How much insulin was metabolized in the second minute? What fraction is that of the amount one minute earlier?
   In the second minute, 9 micrograms of insulin broke down, which is $\frac{1}{10}$ the amount of insulin one minute earlier.

4. What fraction of insulin *remains* in the bloodstream after each minute? Justify your answer.
   If $\frac{1}{10}$ of insulin breaks down after each minute, then $\frac{9}{10}$ of insulin remains in the bloodstream.

5. Complete the table, showing the predicted amount of insulin 4 and 5 minutes after injection.

Time after injection (minutes)	0	1	2	3	4	5
Insulin in the bloodstream (micrograms)	100	90	81	72.9	65.6	59.0

6. Describe how you would determine the amount of insulin remaining in his bloodstream after 10 minutes. After m minutes?
   Sample response: To determine how many micrograms remained in his bloodstream after 10 minutes, I would multiply $\frac{9}{10}$ by itself a total of 10 times, and then multiply that product by 100. To determine the amount after m minutes, I would multiply $\frac{9}{10}$ by itself a total of m times, and then multiply that product by 100.

1 Launch

Say, "Insulin is a hormone produced in our bodies that processes glucose, or sugar. People who are diabetic sometimes need to get insulin shots to regulate the glucose levels in their body." You may want to clarify what the term "metabolize" means in context.

2 Monitor

Look for points of confusion:

- **Writing 90 and 81 as responses for Problems 2 and 3 (respectively).** Have students compare what the problem is asking to what the graph is measuring.
- **Writing $\frac{1}{10}$ as a response to Problems 2 and 3.** Explain that this is the rate at which the insulin is breaking down. Ask how much is $\frac{1}{10}$ of 100 (or of 90 for Problem 3) to help students determine the actual values.

3 Connect

Display the graph.

Have students share their responses.

Highlight that this scenario represents exponential decay, where b is the decay factor. Exponential decay occurs when b, the growth factor, is a number greater than 0 and less than 1.

Ask:

- "What equation can you use to represent the amount of insulin I in the body after m minutes?" $I = 100 \cdot \left(\frac{9}{10}\right)^m$
- "Where do you see the 100 mg in the graph?" It is the vertical intercept.
- "Where do you see $\frac{9}{10}$ in the graph?" The coordinates show $\frac{1}{10}$ of insulin being metabolized each minute, so $\frac{9}{10}$ is what remains in the body.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Tell students that when Problems 2 and 3 ask about the amount of insulin that was *metabolized* by the body, it refers to the amount of insulin that was *used* by the body. When insulin is metabolized, it breaks down so that the body can use it.

Accessibility: *Guide Processing and Visualization*

Use color coding and annotations on the graph to help students distinguish between the amount of insulin that *metabolizes (breaks down)* every minute and the amount of insulin that *remains* in the body every minute.

Extension: *Math Enrichment*

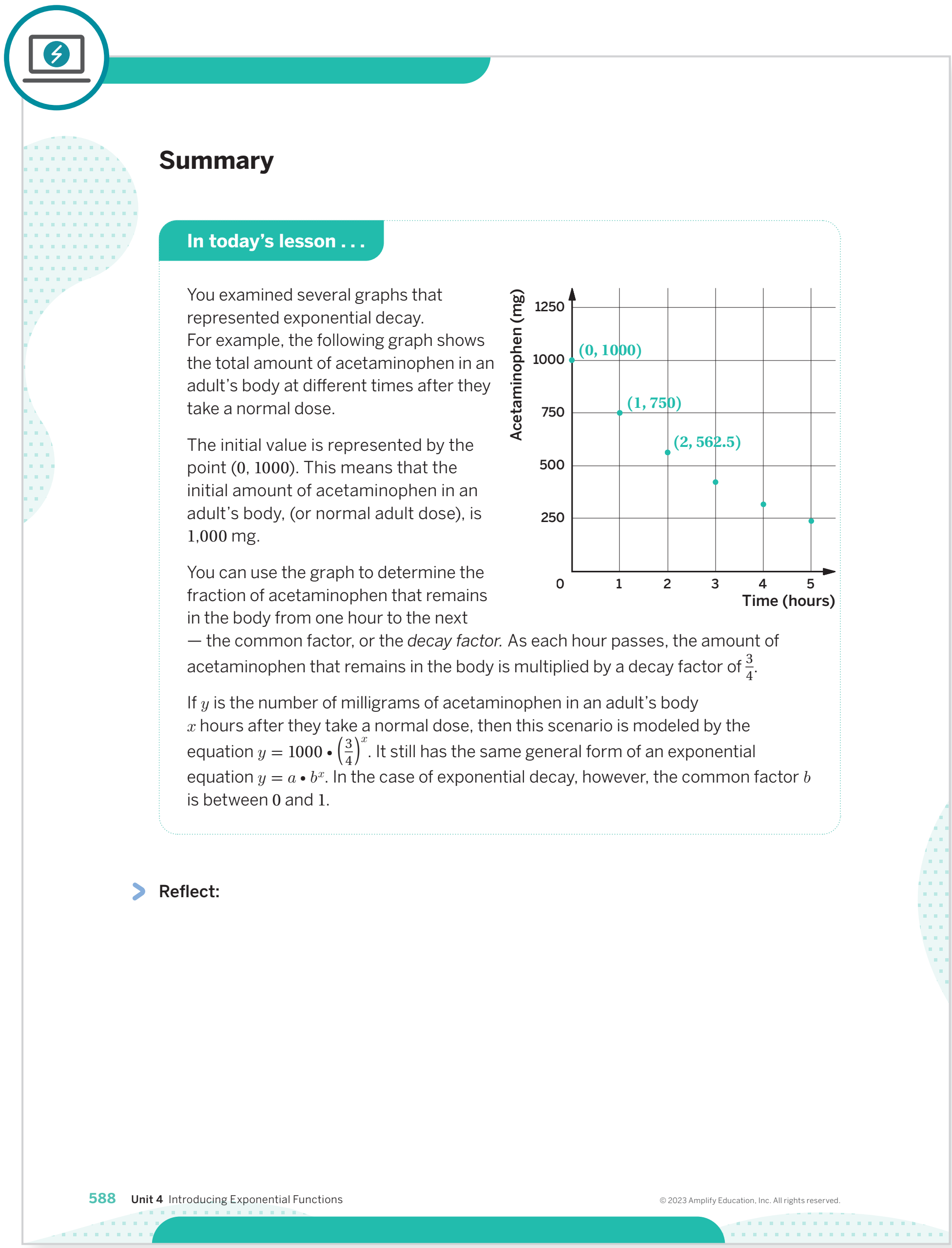
Have students complete the following problems:

- What is the predicted amount of insulin remaining in the bloodstream an hour after the injection? About 0.02 micrograms
- Does the model predict when the amount remaining in the bloodstream will be 0? The graph shows that the amount in the bloodstream is predicted to never reach 0, but it will decrease to an amount that is so small, it is insignificant and no longer helpful to the body.

Whole Class | 5 min

Summary

Review and synthesize how to interpret a graph representing exponential decay.



Summary

In today's lesson . . .

You examined several graphs that represented exponential decay.
For example, the following graph shows the total amount of acetaminophen in an adult's body at different times after they take a normal dose.

The initial value is represented by the point $(0, 1000)$. This means that the initial amount of acetaminophen in an adult's body, (or normal adult dose), is $1{,}000$ mg.

You can use the graph to determine the fraction of acetaminophen that remains in the body from one hour to the next — the common factor, or the *decay factor.* As each hour passes, the amount of acetaminophen that remains in the body is multiplied by a decay factor of $\frac{3}{4}$.

If y is the number of milligrams of acetaminophen in an adult's body x hours after they take a normal dose, then this scenario is modeled by the equation $y = 1000 \bullet \left(\frac{3}{4}\right)^x$. It still has the same general form of an exponential equation $y = a \bullet b^x$. In the case of exponential decay, however, the common factor b is between 0 and 1.

Reflect:

588 Unit 4 Introducing Exponential Functions

Synthesize

Display the graph.

Ask:

- "What is the vertical intercept and what does it mean in context?" $1{,}000$; There are $1{,}000$ mg of medicine in the person's body when they take the medicine.
- "How can you tell from the graph that $\frac{1}{4}$ of the medicine metabolized after one hour?" The amount of medicine in the body decreased by 250 mg, which is $\frac{1}{4}$ of the initial amount.
- "How can you tell from the graph that $\frac{3}{4}$ of the medicine remains in the body after each hour?" 750 is $\frac{3}{4}$ of $1{,}000$, 562.5 is $\frac{3}{4}$ of 750, and so on.
- "What is an equation representing the amount of medicine m, in mg, t hours after taking it?" $m = 1000\left(\frac{3}{4}\right)^t$
- "Why does it make sense to write $\left(\frac{3}{4}\right)^t$?" $\frac{3}{4}$ of the medicine remains in the body each hour.

Highlight how using annotations show the decay factor that is represented in the graph. Emphasize the shape of the graph and how students can identify whether an equation of the form $y = ab^x$ represents exponential decay, if $0 < b < 1$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

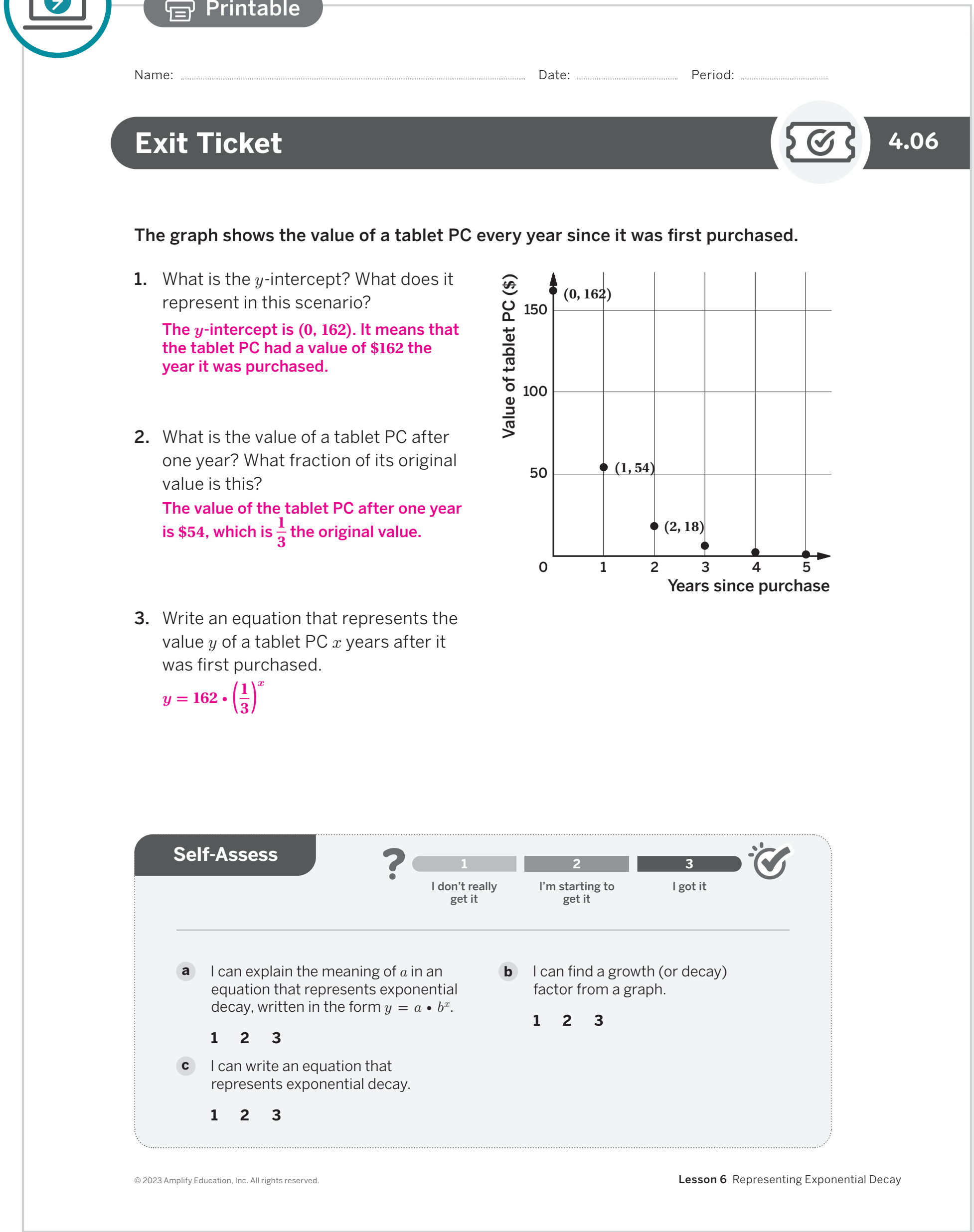
- "What characterizes exponential decay?"
- "What are some real-world examples of exponential decay?"
- "What is the structure of the equation that represents exponential decay? What does each part of the equation mean?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by interpreting key features of a graph representing exponential decay and writing its equation.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 4.06

The graph shows the value of a tablet PC every year since it was first purchased.

1. What is the y-intercept? What does it represent in this scenario?

   The y-intercept is $(0, 162)$. It means that the tablet PC had a value of $162 the year it was purchased.

2. What is the value of a tablet PC after one year? What fraction of its original value is this?

   The value of the tablet PC after one year is $54, which is $\frac{1}{3}$ the original value.

3. Write an equation that represents the value y of a tablet PC x years after it was first purchased.

   $y = 162 \cdot \left(\frac{1}{3}\right)^x$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a) I can explain the meaning of a in an equation that represents exponential decay, written in the form $y = a \cdot b^x$.

1 2 3

b) I can find a growth (or decay) factor from a graph.

1 2 3

c) I can write an equation that represents exponential decay.

1 2 3

Lesson 6 Representing Exponential Decay

Success looks like . . .

- **Goal:** Determining the growth factor from a graph and writing an equation to represent exponential decay.
  - » Determining the decay of the value of the tablet in Problem 2 and writing an equation given the decay in Problem 3.
- **Goal:** Graphing equations that represent quantities that change by growth factors greater than 0 and less than 1.

Suggested next steps

If students are unable to identify and contextualize the y-intercept in Problem 1 consider:

- Reviewing Activity 1, Problem 1.

If students are unable to determine the value of the tablet PC after one year in Problem 2, consider:

- Having students create a table of values to determine the pattern.
- Reviewing Activity 2, Problems 2–4.

If students are unable to write an equation in Problem 2 to represent the graph, consider:

- Reviewing Activity 1, Problem 3. Have students reflect on the connections between the given graph and the equation they created.

Professional Learning

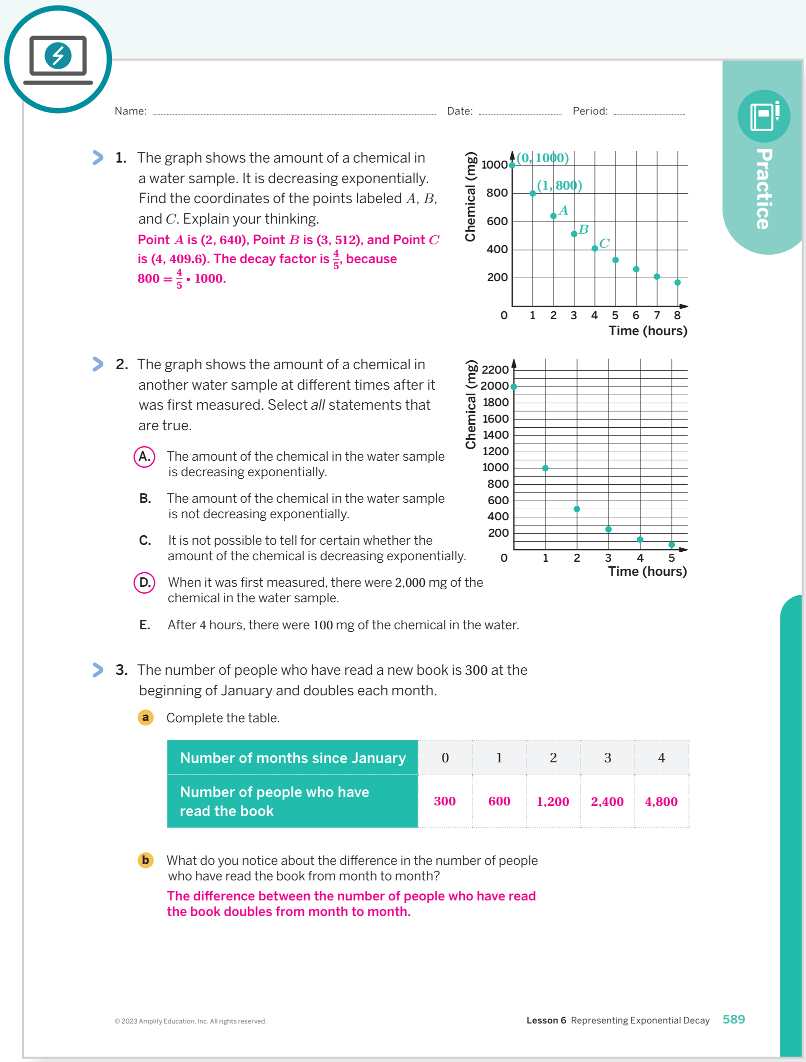
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What challenges did students encounter as they explored exponential decay in this lesson? How did they work through them? What teacher actions did you use and would you use those again?
- Thinking about the questions you asked students today and how students responded, which question(s) were the most efficient? Which questions might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. The graph shows the amount of a chemical in a water sample. It is decreasing exponentially. Find the coordinates of the points labeled A, B, and C. Explain your thinking.

   Point A is (2, 640), Point B is (3, 512), and Point C is (4, 409.6). The decay factor is $\frac{4}{5}$, because $800 = \frac{4}{5} \cdot 1000$.

2. The graph shows the amount of a chemical in another water sample at different times after it was first measured. Select *all* statements that are true.

   [✓ A.] The amount of the chemical in the water sample is decreasing exponentially.

   B. The amount of the chemical in the water sample is not decreasing exponentially.

   C. It is not possible to tell for certain whether the amount of the chemical is decreasing exponentially.

   [✓ D.] When it was first measured, there were 2,000 mg of the chemical in the water sample.

   E. After 4 hours, there were 100 mg of the chemical in the water.

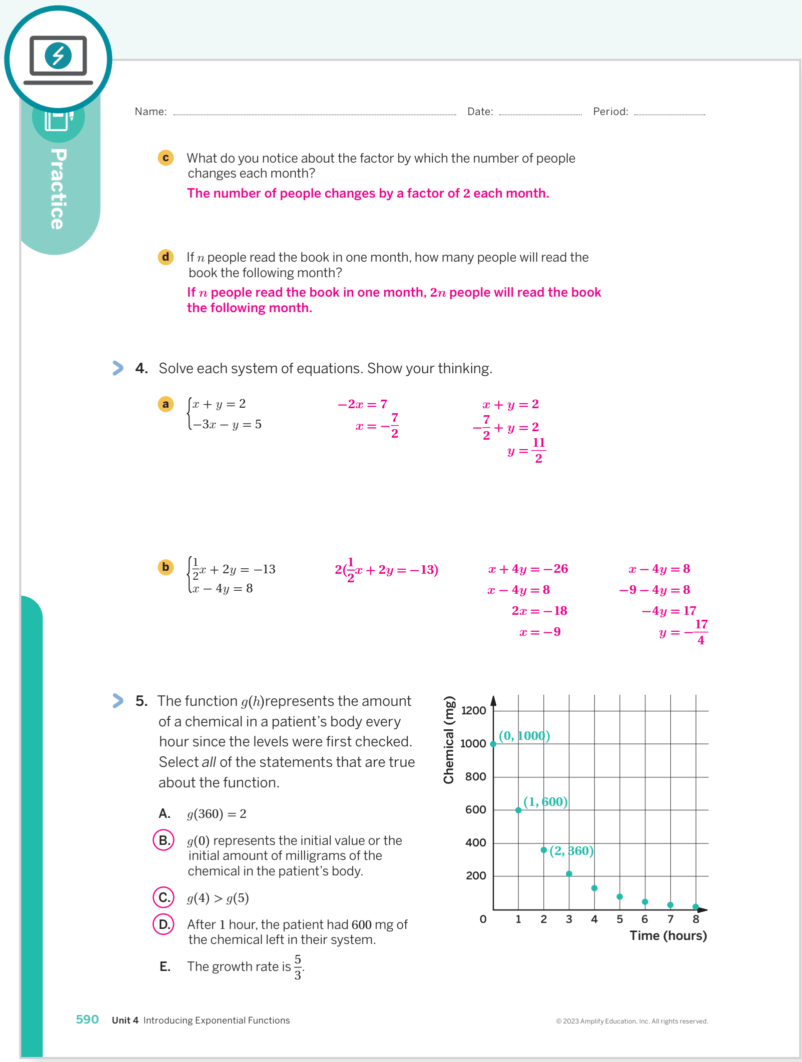
3. The number of people who have read a new book is 300 at the beginning of January and doubles each month.

   a. Complete the table.

Number of months since January	0	1	2	3	4
Number of people who have read the book	**300**	**600**	**1,200**	**2,400**	**4,800**

   b. What do you notice about the difference in the number of people who have read the book from month to month?

   The difference between the number of people who have read the book doubles from month to month.



Practice

Name: Date: Period:

   c. What do you notice about the factor by which the number of people changes each month?

   The number of people changes by a factor of 2 each month.

   d. If n people read the book in one month, how many people will read the book the following month?

   If n people read the book in one month, $2n$ people will read the book the following month.

4. Solve each system of equations. Show your thinking.

   a. $\begin{cases} x + y = 2 \\ -3x - y = 5 \end{cases}$

   $$-2x = 7$$
   $$x = -\frac{7}{2}$$
   $$x + y = 2$$
   $$-\frac{7}{2} + y = 2$$
   $$y = \frac{11}{2}$$

   b. $\begin{cases} \frac{1}{2}x + 2y = -13 \\ x - 4y = 8 \end{cases}$

   $$2(\frac{1}{2}x + 2y = -13)$$
   $$x + 4y = -26$$
   $$x - 4y = 8$$
   $$2x = -18$$
   $$x = -9$$
   $$x - 4y = 8$$
   $$-9 - 4y = 8$$
   $$-4y = 17$$
   $$y = -\frac{17}{4}$$

5. The function $g(h)$represents the amount of a chemical in a patient's body every hour since the levels were first checked. Select *all* of the statements that are true about the function.

   A. $g(360) = 2$

   [✓ B.] $g(0)$ represents the initial value or the initial amount of milligrams of the chemical in the patient's body.

   [✓ C.] $g(4) > g(5)$

   [✓ D.] After 1 hour, the patient had 600 mg of the chemical left in their system.

   E. The growth rate is $\frac{5}{3}$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
Spiral	3	Unit 4 Lesson 3	2
	4	Unit 1 Lesson 21	2
Formative	5	Unit 4 Lesson 7	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

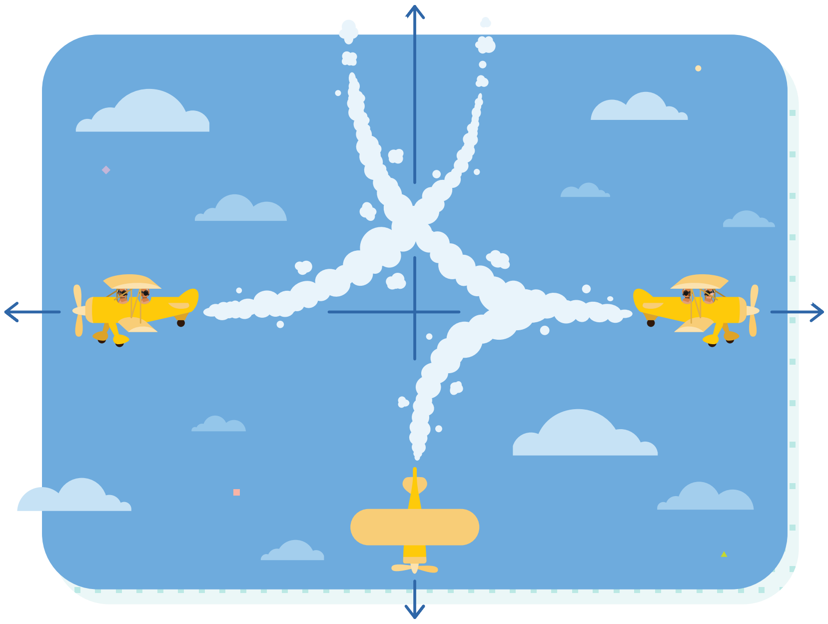
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Optional

UNIT 4 | **LESSON 7**

Exploring Parameter Changes of Exponentials

Let's examine how changing an exponential equation changes its graph.



Focus

Goals

1. **Language Goal:** Explain what happens to the graph of $y = a \cdot b^x$ when b is replaced by its reciprocal. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain what happens to the graph of $y = a \cdot b^x$ when a is negated. **(Speaking and Listening, Writing)**

Coherence

Today

Students use technology to determine how the graph of an exponential equation of the form $y = a \cdot b^x$ is affected when the values of a and b change. Students examine what happens when b is replaced by its reciprocal, when a is negated, and why b must be positive in an exponential relationship.

Previously

Students compared relationships characterized by exponential growth to those characterized by decay, and drew connections between their multiple representations.

Coming Soon

Students will write exponential equations as functions and use them to model real-world contexts.

Rigor

- Students further enhance their **conceptual understanding** of exponential functions by adjusting the functions' parameters and predicting, observing, and understanding the resulting effects on graphs.
- Students build on their **procedural fluency** related to graphing functions and setting appropriate limits for their axes.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	12 min	10 min	5 min	5 min
Independent	Independent	Independent	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activities 1 & 2 PDF, pre-cut, two graphs per student
- graphing technology

Math Language Development

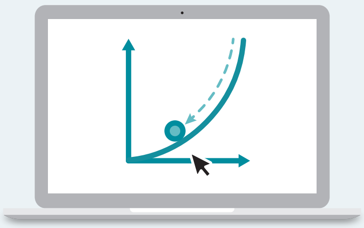
Review words

- *decay*
- *growth*
- *reciprocal*

Amps | Featured Activity

Activity 1
Marbleslides

Students use interactive tools to show what happens to the graph of $y = a \cdot b^x$ when b is replaced with its reciprocal. They use what they observe to play a game of Marbleslides.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might find it difficult to regulate their thoughts and behaviors as they try to make sense of the impact of changing parameters in the exponential equations. Encourage students to use the lesson to guide their solution pathway rather than simply jumping to conclusions based on unsubstantiated assumptions. Changing the parameters very well might not impact the graph in ways students initially think it will, so they need to control their impulsivity to draw valid conclusions.

Modifications to Pacing

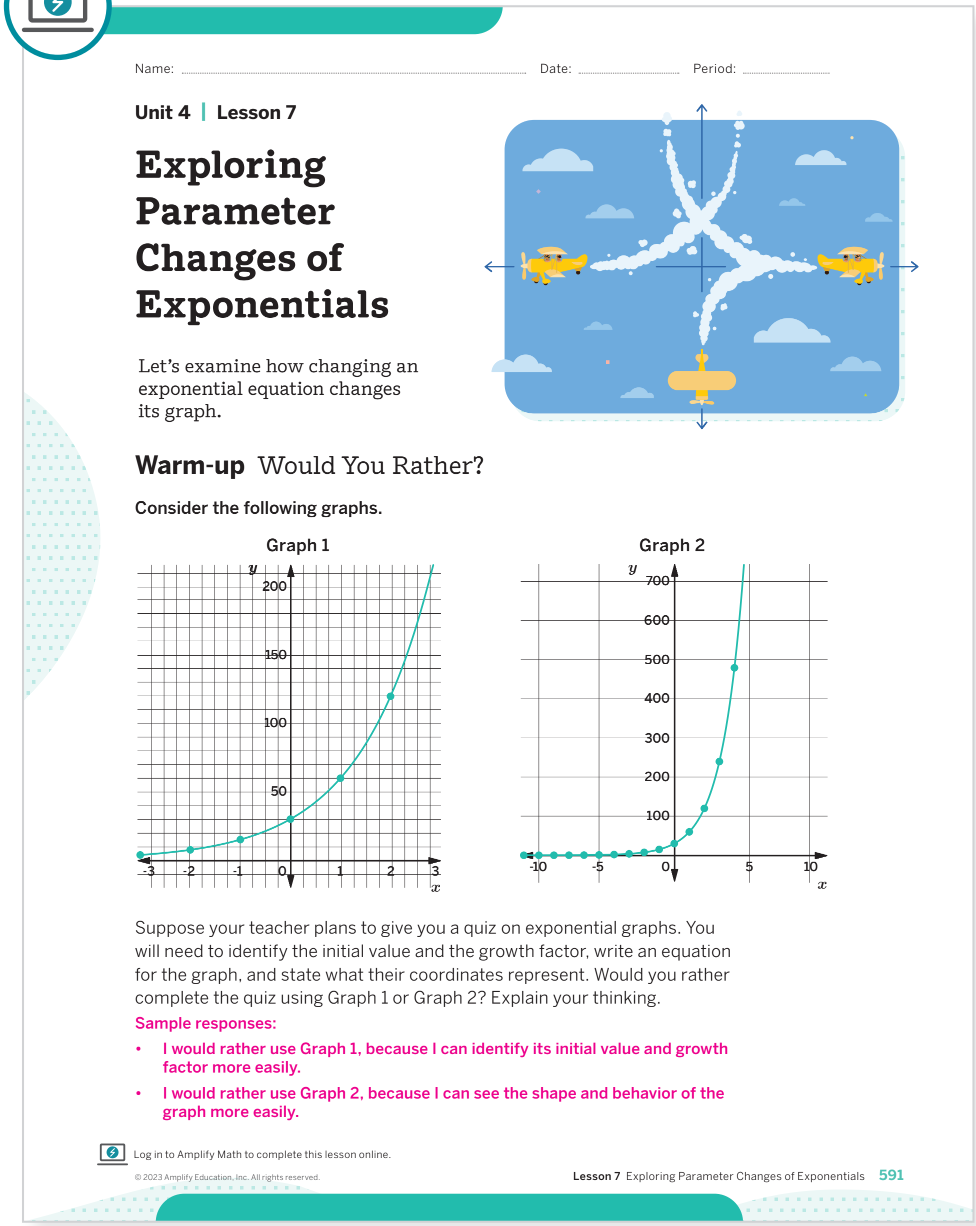
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, have students work with one of the equations rather than both.
- In **Activity 3**, choose one from among Problems 1–3 for students to work on.

Independent | 5 min

Warm-up Would You Rather?

Students observe two graphs with the same data, but with different scales, emphasizing the importance of scaled axes.



Name: ______________ Date: ________ Period: ________

Unit 4 | Lesson 7

Exploring Parameter Changes of Exponentials

Let's examine how changing an exponential equation changes its graph.

Warm-up Would You Rather?

Consider the following graphs.

Suppose your teacher plans to give you a quiz on exponential graphs. You will need to identify the initial value and the growth factor, write an equation for the graph, and state what their coordinates represent. Would you rather complete the quiz using Graph 1 or Graph 2? Explain your thinking.

Sample responses:

- **I would rather use Graph 1, because I can identify its initial value and growth factor more easily.**
- **I would rather use Graph 2, because I can see the shape and behavior of the graph more easily.**

Log in to Amplify Math to complete this lesson online.

Lesson 7 Exploring Parameter Changes of Exponentials 591

1 Launch

Display the graphs and read the prompt aloud. Conduct the ***Would You Rather?*** routine.

2 Monitor

Help students get started by asking them to determine an exact value for the y-intercept in each graph. Prompt them to note which graph was more challenging to use to determine its y-intercept.

Look for points of confusion:

- **Overusing the word "it" to describe parts of the graph or the graph itself (e.g., "it is easier to see").** Tell students to be specific about what they observe in each graph and what they are referring to, encouraging the use of precise math language.

3 Connect

Have students share their selections and reasoning behind them.

Highlight that Graphs 1 and 2 are the same graph, with different axes scales. Graph 2 more readily displays the shape and behavior of the graph as exponential. Graph 1 is beneficial for identifying specific coordinate values and determining the growth factor.

Ask:

- "What is the initial value of Graph 1? Graph 2?" 30; about 30, but it is difficult to determine the exact value.
- "Which graph is increasing at a faster rate?" Graph 1 is doubling. Graph 2 appears to be doubling, but it is challenging to estimate values.
- "For which graph would it be more helpful to use to write an equation? Explain your thinking." Graph 1; The scale allows for better estimates of the values.
- "If you wanted to see the overall shape of the graph, which graph would you select to use?" Graph 2.

Differentiated Support

MLR7: *Compare and Connect*

During the Connect, as pairs of students share their reasons for choosing one of the graphs, ask, "What connections can you make between Graphs 1 and 2? What is similar? What is different?" Emphasize how the grid lines can increase precision.

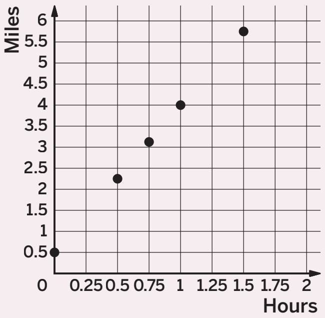
English Learners

Annotate the point $(1, 60)$ on Graph 1 and then use gestures to illustrate how it is challenging to know the coordinates of a similar point on Graph 2.

Power-up

To power up students' ability to analyze graphs of functions, have students complete:

The function $d(t)$ represents the distance in miles Jada is from her home after a certain amount of time. Determine each value based on the graph of the function given.



a The initial value 0.5 miles

b Jada's speed 3.5 mph

c $d(1) = 4$

d $d(\boxed{0.5}) = 2.25$

Use: Before the Warm-up

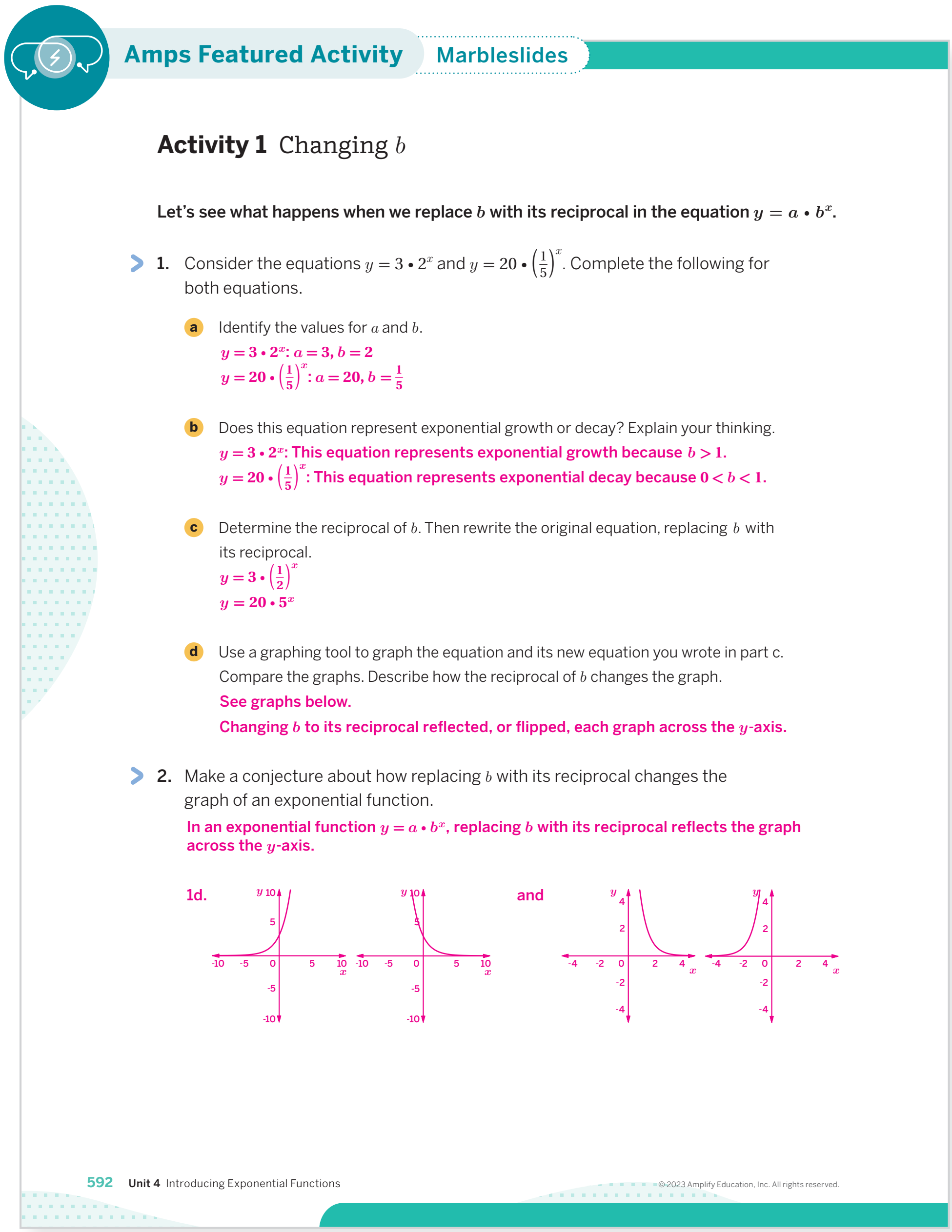
Informed by: Performance on Lesson 6, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 8

Independent | 12 min

Activity 1 Changing b

Students examine the graphs of two exponential equations of the form $y = a \bullet b^x$ and observe what happens when b is replaced with its reciprocal.

Amps Featured Activity Marbleslides



Activity 1 Changing b

Let's see what happens when we replace b with its reciprocal in the equation $y = a \bullet b^x$.

1. Consider the equations $y = 3 \bullet 2^x$ and $y = 20 \bullet \left(\frac{1}{5}\right)^x$. Complete the following for both equations.

   a. Identify the values for a and b.

   $y = 3 \bullet 2^x$: $a = 3, b = 2$
   $y = 20 \bullet \left(\frac{1}{5}\right)^x$: $a = 20, b = \frac{1}{5}$

   b. Does this equation represent exponential growth or decay? Explain your thinking.

   $y = 3 \bullet 2^x$: This equation represents exponential growth because $b > 1$.
   $y = 20 \bullet \left(\frac{1}{5}\right)^x$: This equation represents exponential decay because $0 < b < 1$.

   c. Determine the reciprocal of b. Then rewrite the original equation, replacing b with its reciprocal.

   $y = 3 \bullet \left(\frac{1}{2}\right)^x$
   $y = 20 \bullet 5^x$

   d. Use a graphing tool to graph the equation and its new equation you wrote in part c. Compare the graphs. Describe how the reciprocal of b changes the graph.

   See graphs below.
   Changing b to its reciprocal reflected, or flipped, each graph across the y-axis.

2. Make a conjecture about how replacing b with its reciprocal changes the graph of an exponential function.

   In an exponential function $y = a \bullet b^x$, replacing b with its reciprocal reflects the graph across the y-axis.

592 **Unit 4** Introducing Exponential Functions

1 Launch

Display the numbers $\frac{3}{4}$, $\frac{1}{2}$, and 5. Ask students to determine the reciprocal of each number. Then elicit from students how to determine the reciprocal of any number. Distribute the Activities 1 & 2 PDF to each student.

2 Monitor

Help students get started by displaying the general form of an exponential equation. Circle the term b in the equation and tell students that they will observe how changing b affects the graph.

Look for points of confusion:

- **Switching the values of a and b.** Have students use the general form of an exponential equation when identifying the values of a and b.

Look for productive strategies:

- Graphing the original equation and its reciprocal to notice that replacing b with its reciprocal reflects the graph across the y-axis.

3 Connect

Display the equation $y = 3 \bullet 2^x$ and its graph.

Have students share what they observed when they changed b to its reciprocal in the equation $y = 3 \bullet 2^x$. Then model the change on the graph.

Highlight that for an exponential equation of the form $y = a \bullet b^x$, replacing b with its reciprocal results in a reflection of the graph across the y-axis.

Ask:

- "What does the original graph have in common with its reflected graph?" Sample response: Both graphs have the same y-intercept. They are both exponential.
- "What relationship exists between the growth factor and the behavior of the graph?" Sample response: When $b > 1$, the relationship shows exponential growth. When $0 < b < 1$, the relationship shows exponential decay.

Differentiated Support

Accessibility: *Optimize Access to Technolog*

Have students use the Amps slides for this activity, in which they can use interactive tools to show what happens to the graph of $y = a \bullet b^x$ when b is replaced with its reciprocal. They use what they observe to play a game of Marbleslides.

Extension: Math Enrichment

Have students use graphing technology to graph the equation $y = 3 \bullet 2^{-x}$ and compare it to the graphs they sketched in this activity. Ask them to make a conjecture about how negating x in an exponential equation affects its graph. Negating x reflects the graph across the y-axis.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, after students record their responses for Problem 2, have them meet with 2–3 students to give and receive feedback on their responses. Have reviewers ask these questions:

- "How do you know that your conjecture is true?"
- "What mathematical language can you use in your responses?"

Allow time to complete a final draft based on feedback.

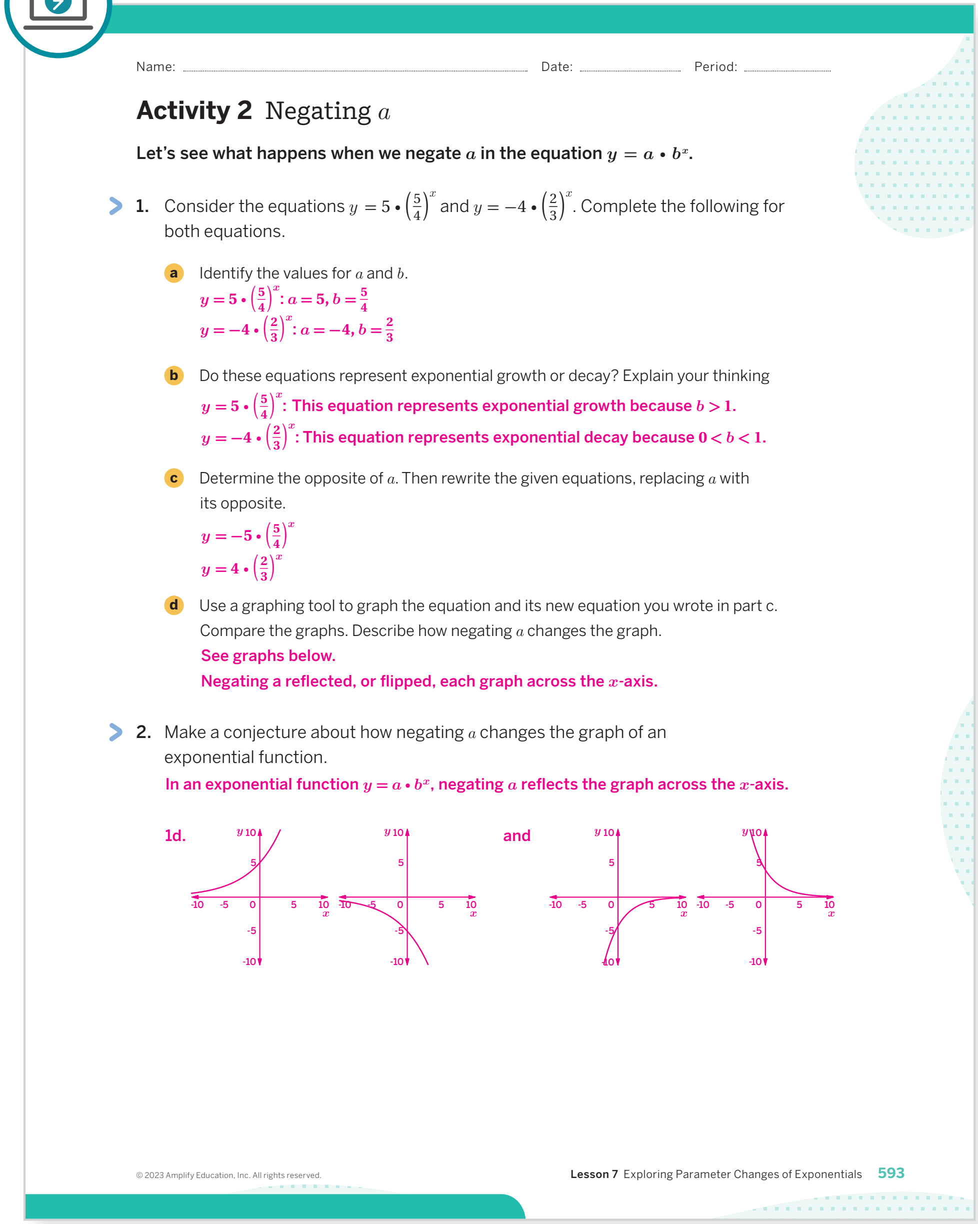
English Learners

Allow students time to formulate with their partner how they will improve their final draft before proceeding with the Connect discussion.

Independent | 12 min

Activity 2 Negating a

Students examine the graphs of two exponential equations of the form $y = a \cdot b^x$ and observe what happens when a is negated.



Name: ________ Date: ________ Period: ________

Activity 2 Negating a

Let's see what happens when we negate a in the equation $y = a \cdot b^x$.

1. Consider the equations $y = 5 \cdot \left(\frac{5}{4}\right)^x$ and $y = -4 \cdot \left(\frac{2}{3}\right)^x$. Complete the following for both equations.

   a. Identify the values for a and b.

   $y = 5 \cdot \left(\frac{5}{4}\right)^x$: $a = 5, b = \frac{5}{4}$

   $y = -4 \cdot \left(\frac{2}{3}\right)^x$: $a = -4, b = \frac{2}{3}$

   b. Do these equations represent exponential growth or decay? Explain your thinking

   $y = 5 \cdot \left(\frac{5}{4}\right)^x$: **This equation represents exponential growth because $b > 1$.**

   $y = -4 \cdot \left(\frac{2}{3}\right)^x$: **This equation represents exponential decay because $0 < b < 1$.**

   c. Determine the opposite of a. Then rewrite the given equations, replacing a with its opposite.

   $y = -5 \cdot \left(\frac{5}{4}\right)^x$

   $y = 4 \cdot \left(\frac{2}{3}\right)^x$

   d. Use a graphing tool to graph the equation and its new equation you wrote in part c. Compare the graphs. Describe how negating a changes the graph.

   See graphs below.

   Negating a reflected, or flipped, each graph across the x-axis.

2. Make a conjecture about how negating a changes the graph of an exponential function.

   In an exponential function $y = a \cdot b^x$, negating a reflects the graph across the x-axis.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Exploring Parameter Changes of Exponentials 593

1 Launch

Display the numbers $\frac{3}{4}$, $\frac{1}{2}$, and 5. Ask students, "What does it mean to negate a number?" Have students negate each of these numbers. Distribute the Activities 1 and 2 PDF to each student.

2 Monitor

Help students get started by displaying the general form of an exponential equation. Place a square around a and tell students that they will observe how changing a affects the graph.

Look for points of confusion:

- **Having difficulty negating a negative number.** Ask, "What happens when you multiply a negative number by another negative number?"

Look for productive strategies:

- Graphing the original equation and the new equation that they wrote to notice that replacing a with its negation reflects the graph across the x-axis.

3 Connect

Display the equation $y = 5 \cdot \left(\frac{5}{4}\right)^x$ and its graph.

Have students share what they observed when they negated a in the equation $y = 5 \cdot \left(\frac{5}{4}\right)^x$. Then model the change on the graph.

Highlight that in an exponential equation of the form $y = a \cdot b^x$, negating a results in a reflection of the graph across the x-axis.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Be sure students understand what negating a means. Negating a number means to determine the opposite of that number. Ask:

- "If you negate the number 3, what is the result?" -3
- "If you negate the number -4, what is the result?" 4
- "If you negate the value a, what is the result?" $-a$

Extension: *Math Enrichment*

Display the equation $y = 4 \cdot 3^x$. Have students make a conjecture as to what the graph of the equation $y = -4 \cdot \left(\frac{1}{3}\right)^x$ will look like, compared to the graph of the original equation. The graph will be reflected across both axes.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, after students record their response for Problem 2, have them meet with 2–3 students to give and receive feedback on their responses. Have reviewers ask these questions:

- "How do you know that your conjecture is true?"
- "What mathematical language can you use in your responses?"

Allow time to complete a final draft based on feedback.

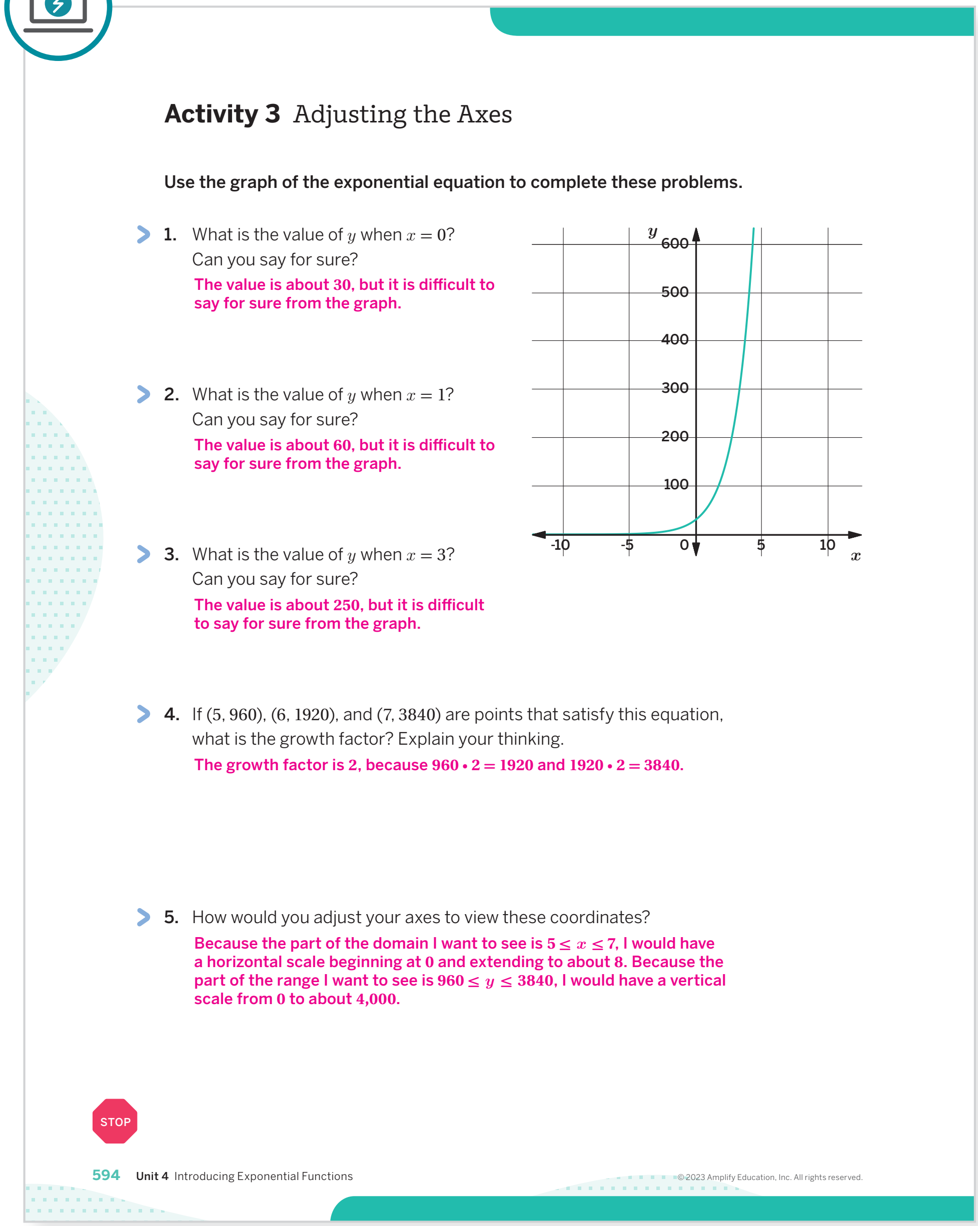
English Learners

Allow students time to formulate with their partner how they will improve their final draft before proceeding with the Connect discussion.

Independent | 10 min

Activity 3 Adjusting the Axes

Students determine appropriate axes limits for displaying the data on a graph representing an exponential equation of the form $y = a \cdot b^x$.



Activity 3 Adjusting the Axes

Use the graph of the exponential equation to complete these problems.

1. What is the value of y when $x = 0$? Can you say for sure?
   The value is about 30, but it is difficult to say for sure from the graph.

2. What is the value of y when $x = 1$? Can you say for sure?
   The value is about 60, but it is difficult to say for sure from the graph.

3. What is the value of y when $x = 3$? Can you say for sure?
   The value is about 250, but it is difficult to say for sure from the graph.

4. If (5, 960), (6, 1920), and (7, 3840) are points that satisfy this equation, what is the growth factor? Explain your thinking.
   The growth factor is 2, because $960 \cdot 2 = 1920$ and $1920 \cdot 2 = 3840$.

5. How would you adjust your axes to view these coordinates?
   Because the part of the domain I want to see is $5 \leq x \leq 7$, I would have a horizontal scale beginning at 0 and extending to about 8. Because the part of the range I want to see is $960 \leq y \leq 3840$, I would have a vertical scale from 0 to about 4,000.

STOP

594 **Unit 4** Introducing Exponential Functions

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity. Provide access to graphing technology.

2 Monitor

Help students get started by plotting the points that correspond to the given values of x in Problems 1–3.

Look for points of confusion:

- **Having difficulty locating specific coordinates.** Prompt students to estimate the coordinates of the points.

Look for productive strategies:

- Plotting coordinates that correspond to the given values of x.
- Labeling minor axes lines.

3 Connect

Display the graph.

Have students share the values of y they found for $x = 1$, 2 and 3. Discuss why it was challenging to determine exact values and how the scaling might be adjusted to better identify these coordinates.

Ask, "Is there enough information to write an equation that could represent the graph?" Model how to use the growth factor found in Problem 4 to determine the initial value, and then have students write an equation.

Highlight how adjusting the scales on the axes helps to identify important information on a graph (such as the corresponding values of x or the initial value), modeling how to do so when discussing Problem 5.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to straightedges and suggest students use them to determine approximate values of y that correspond with the given values of x.

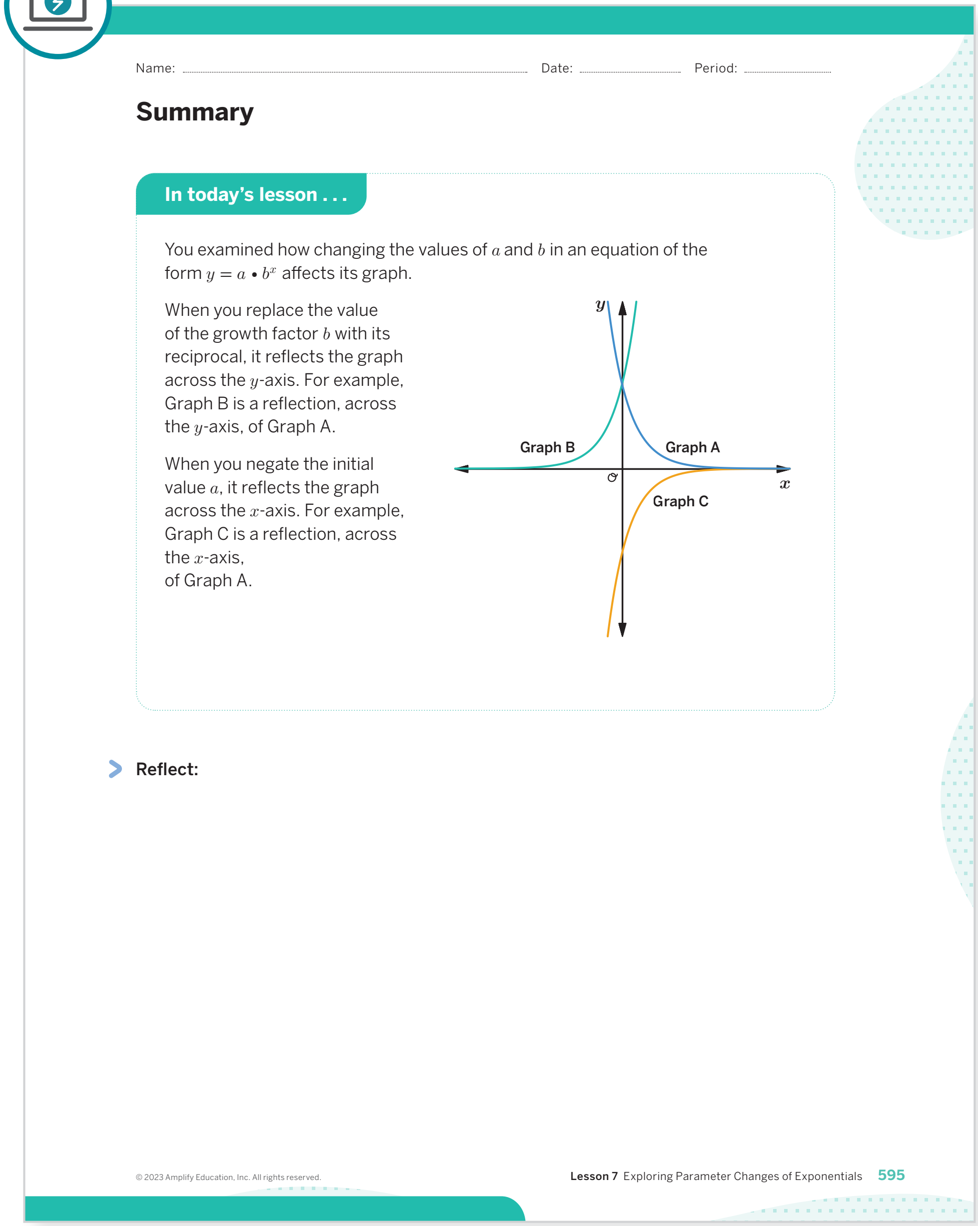
Accessibility: *Optimize Access to Technology*

Provide access to graphing technology and suggest students experiment with creating different axes scales as they respond to Problem 5.

Whole Class | 5 min

Summary

Review and synthesize how the graph of an exponential equation of the form $y = a \cdot b^x$ is affected by changing a and b.



Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You examined how changing the values of a and b in an equation of the form $y = a \cdot b^x$ affects its graph.

When you replace the value of the growth factor b with its reciprocal, it reflects the graph across the y-axis. For example, Graph B is a reflection, across the y-axis, of Graph A.

When you negate the initial value a, it reflects the graph across the x-axis. For example, Graph C is a reflection, across the x-axis, of Graph A.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Exploring Parameter Changes of Exponentials 595

Synthesize

Display the graph.

Ask:

- "What happens to Graph A when b is replaced with its reciprocal?" It is reflected across the y-axis and is in the same location as Graph B.
- "What happens to Graph A when a is negated?" It is reflected across the x-axis and is in the same location as Graph C.

Highlight that in an exponential equation of the form $y = a \cdot b^x$, replacing b with its reciprocal results in a reflection of the graph across the y-axis, and negating a results in a reflection of the graph across the x-axis.

Reflect

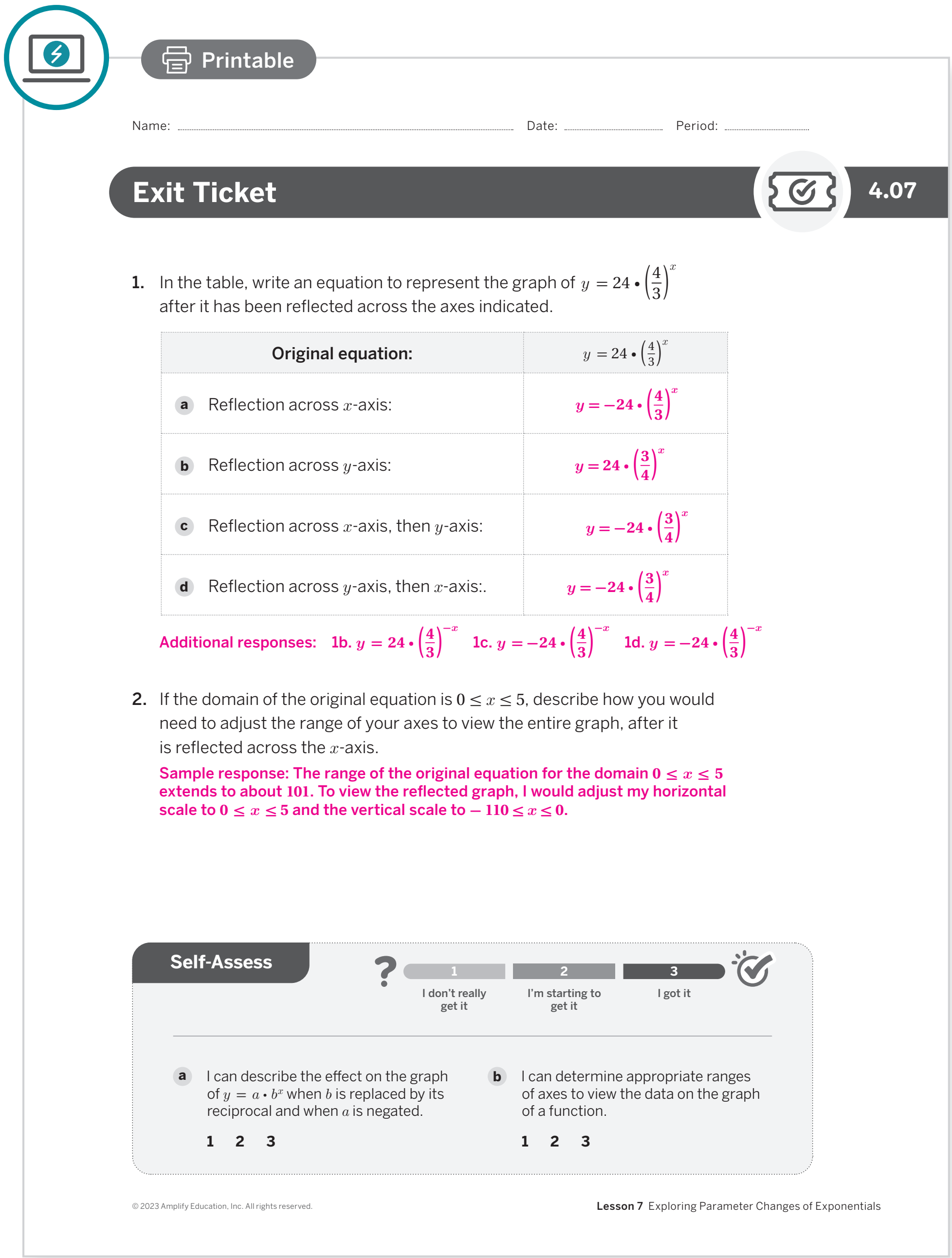
After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean to *negate* a in the exponential equation $y = a \cdot b^x$? Provide an example."
- "What does it mean to *replace* b with its reciprocal in the exponential equation $y = a \cdot b^x$? Provide an example."

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing the effects on the graph when changing a and b in an exponential equation of the form $y = a \bullet b^x$.



Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 4.07

1. In the table, write an equation to represent the graph of $y = 24 \bullet \left(\frac{4}{3}\right)^x$ after it has been reflected across the axes indicated.

Original equation:	$y = 24 \bullet \left(\frac{4}{3}\right)^x$
a Reflection across x-axis:	$y = -24 \bullet \left(\frac{4}{3}\right)^x$
b Reflection across y-axis:	$y = 24 \bullet \left(\frac{3}{4}\right)^x$
c Reflection across x-axis, then y-axis:	$y = -24 \bullet \left(\frac{3}{4}\right)^x$
d Reflection across y-axis, then x-axis:.	$y = -24 \bullet \left(\frac{3}{4}\right)^x$

Additional responses: 1b. $y = 24 \bullet \left(\frac{4}{3}\right)^{-x}$ **1c.** $y = -24 \bullet \left(\frac{4}{3}\right)^{-x}$ **1d.** $y = -24 \bullet \left(\frac{4}{3}\right)^{-x}$

2. If the domain of the original equation is $0 \leq x \leq 5$, describe how you would need to adjust the range of your axes to view the entire graph, after it is reflected across the x-axis.

Sample response: The range of the original equation for the domain $0 \leq x \leq 5$ extends to about 101. To view the reflected graph, I would adjust my horizontal scale to $0 \leq x \leq 5$ and the vertical scale to $-110 \leq x \leq 0$.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can describe the effect on the graph of $y = a \bullet b^x$ when b is replaced by its reciprocal and when a is negated.
1 2 3

b I can determine appropriate ranges of axes to view the data on the graph of a function.
1 2 3

Lesson 7 Exploring Parameter Changes of Exponentials

Success looks like . . .

- **Language Goal:** Explaining what happens to the graph of $y = a \bullet b^x$ when b is replaced by its reciprocal. **(Speaking and Listening, Writing)**
    - » Identifying parts c and d as representing the graph of the function with b replaced as its reciprocal.
- **Language Goal:** Explaining what happens to the graph of $y = a \bullet b^x$ when a is negated. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly interpret the effects of replacing b with its reciprocal or a with its negation in Problem 1, consider:

- Reviewing Activity 1. Have students generate their own pairs of exponential equations of the form $y = a \bullet b^x$ and $y = a \bullet \left(\frac{1}{b}\right)^x$.
- Reviewing Activity 2. Have students generate their own pairs of exponential equations of the form $y = a \bullet b^x$ and $y = -a \bullet b^x$.

If students are unable to accurately describe how to adjust the axes limits in Problem 2, consider:

- Reassigning Activity 3 using the graph of a different exponential function.

Professional Learning

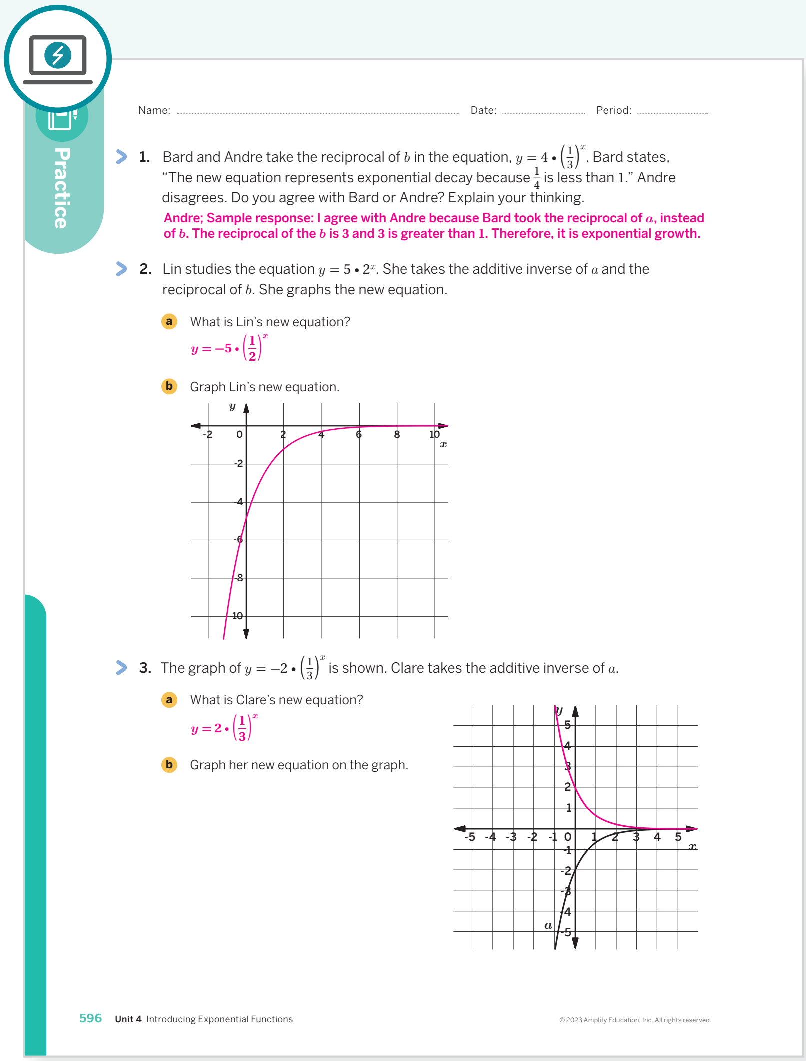
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students explored changing parameters on the exponential equation $y = a \bullet b^x$. How will this understanding support future work in constructing exponential functions to model real-world data? How well do you think your students understood the concepts of negating a or replacing b with its reciprocal?
- During the discussion in Activities 1 and 2, how did you encourage each student to listen to one another's conjectures about the effects of changing these parameters on the graphs of an exponential equation? What might you change the next time you teach this lesson?

Practice

Independent



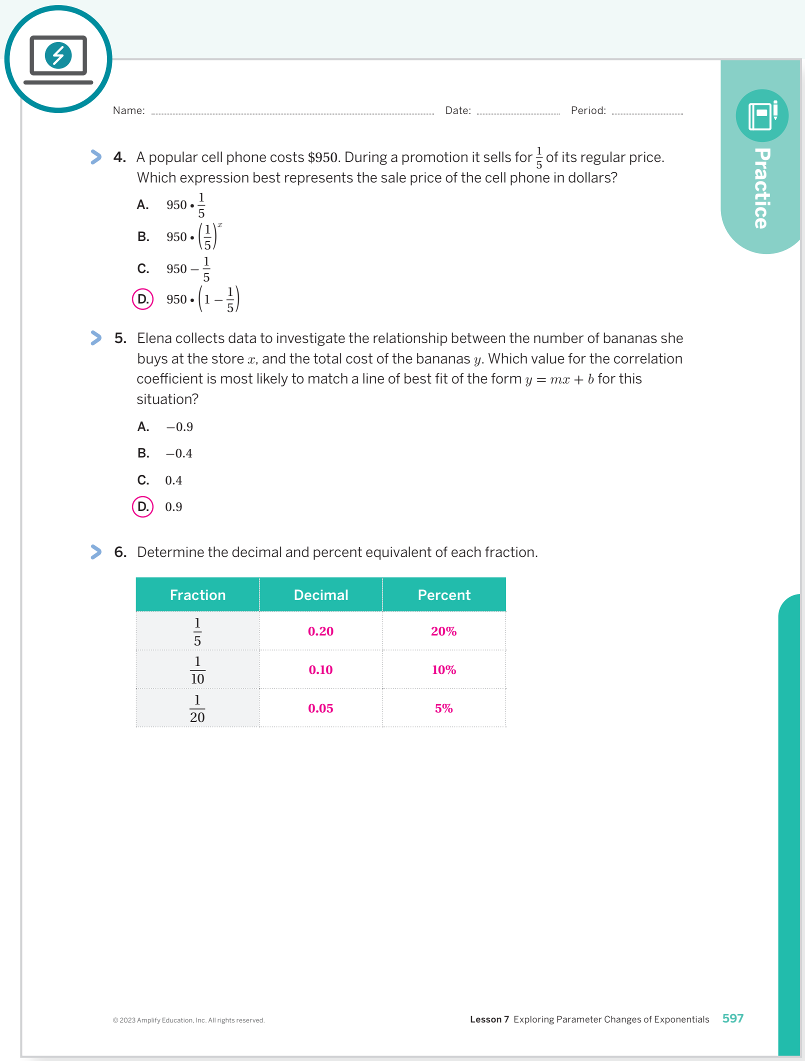
Practice

Name: Date: Period:

1. Bard and Andre take the reciprocal of b in the equation, $y = 4 \cdot \left(\frac{1}{3}\right)^x$. Bard states, “The new equation represents exponential decay because $\frac{1}{4}$ is less than 1.” Andre disagrees. Do you agree with Bard or Andre? Explain your thinking.
   Andre; Sample response: I agree with Andre because Bard took the reciprocal of a, instead of b. The reciprocal of the b is 3 and 3 is greater than 1. Therefore, it is exponential growth.

2. Lin studies the equation $y = 5 \cdot 2^x$. She takes the additive inverse of a and the reciprocal of b. She graphs the new equation.
   - **a** What is Lin’s new equation?
     $y = -5 \cdot \left(\frac{1}{2}\right)^x$
   - **b** Graph Lin’s new equation.

3. The graph of $y = -2 \cdot \left(\frac{1}{3}\right)^x$ is shown. Clare takes the additive inverse of a.
   - **a** What is Clare’s new equation?
     $y = 2 \cdot \left(\frac{1}{3}\right)^x$
   - **b** Graph her new equation on the graph.



Practice

Name: Date: Period:

4. A popular cell phone costs \$950. During a promotion it sells for $\frac{1}{5}$ of its regular price. Which expression best represents the sale price of the cell phone in dollars?
   - A. $950 \cdot \frac{1}{5}$
   - B. $950 \cdot \left(\frac{1}{5}\right)^x$
   - C. $950 - \frac{1}{5}$
   - [✓ D.] $950 \cdot \left(1 - \frac{1}{5}\right)$

5. Elena collects data to investigate the relationship between the number of bananas she buys at the store x, and the total cost of the bananas y. Which value for the correlation coefficient is most likely to match a line of best fit of the form $y = mx + b$ for this situation?
   - A. -0.9
   - B. -0.4
   - C. 0.4
   - [✓ D.] 0.9

6. Determine the decimal and percent equivalent of each fraction.

Fraction	Decimal	Percent
$\frac{1}{5}$	**0.20**	**20%**
$\frac{1}{10}$	**0.10**	**10%**
$\frac{1}{20}$	**0.05**	**5%**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	2
	2	Activities 1 and 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 5	2
	5	Unit 2 Lesson 19	2
Formative ⚡	6	Unit 4 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 3

Exponential Functions

Whole Class

In this Sub-Unit, students identify exponential relationships as functions, consider their domain, and analyze different intervals of an exponential function to determine average rates of change over specified intervals.

SUB-UNIT

3 Exponential Functions

Narrative Connections

What does growing or shrinking look like on a graph?

Math has given us tools that help us understand the meaning behind numbers, from the staggeringly large to the painstakingly minuscule. Representations like tables and graphs let us see things in numbers that we might not fully otherwise grasp.

Think back to Lesson 1, when you simulated the spread of an infection, starting with just one infected person. Could you have imagined how quickly it spread to so many other people? By using a graph, we can see with our own eyes the magnitude of an epidemic. And once we see it, we can be smarter about how we fix it.

With charts and tables, we can see more of the story — from the first outbreak, to the waves of infection, to when a vaccine is developed and introduced. We can see the people who have developed natural immunity, and the effects of treatments, vaccinations, and quarantines. Armed with this information, government officials, scientists, and doctors can make more informed decisions that can save lives.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how exponential functions help model and solve medical decisions in the following places:

- **Lesson 8, Activity 3:** Fever Reduction
- **Lesson 9, Activity 2:** Measuring Medicine
- **Lesson 10, Activities 1 and 3:** Moldy Bread, Choose Limits for Your Axes
- **Lesson 11, Activity 1:** Cost of a Bottle of Aspirin
- **Lesson 13, Activity 2:** Graphs of Exponential Decay
- **Lesson 14, Activity 2:** Further Pharmacy Expansion

UNIT 4 | **LESSON 8**

Analyzing Graphs

Let's explore exponential growth and decay by comparing situations where quantities change exponentially.

Focus

Goals

1. Determine whether situations are characterized by exponential growth or by exponential decay, given descriptions and graphs.
2. **Language Goal:** Use graphs to compare and contrast situations that involve exponential decay. **(Writing)**
3. Use information from a graph to write an equation that represents exponential decay.

Coherence

Today

Students analyze graphs representing depreciation, write equations representing relationships in context, and match scenarios with their graphs representing exponential change.

Previously

Students examined graphs and equations of scenarios characterized by exponential decay, identified key features in the graphs, and interpreted different parts of the graph and equation in context.

Coming Soon

Students will interpret negative exponents in exponential contexts, write equations of exponential form, and determine an appropriate graphing scale.

Rigor

- Students develop their **procedural fluency** with exponential functions by studying and comparing their graphs.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- cell phone advertisements (for display)
- colored pencils (as needed)

Math Language Development

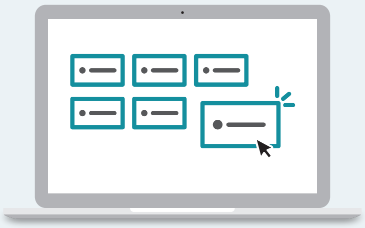
Review words

- *growth factor*

Amps | Featured Activity

Activity 2
Digital Card Sort

Students match scenarios with their graphs representing exponential growth and decay.



Building Math Identity and Community

Connecting to Mathematical Practices

While many mathematical tasks might challenge students to avoid mental impulsivity, working with cards might challenge them more behaviorally. As students try to match a description to its graphical model, encourage them to have a routine with their partner so that both students can stay focused on the task. There might be a temptation to use the cards for something other than the intention, but students should work together to help each other behave in a way that they can achieve their academic goals.

Modifications to Pacing

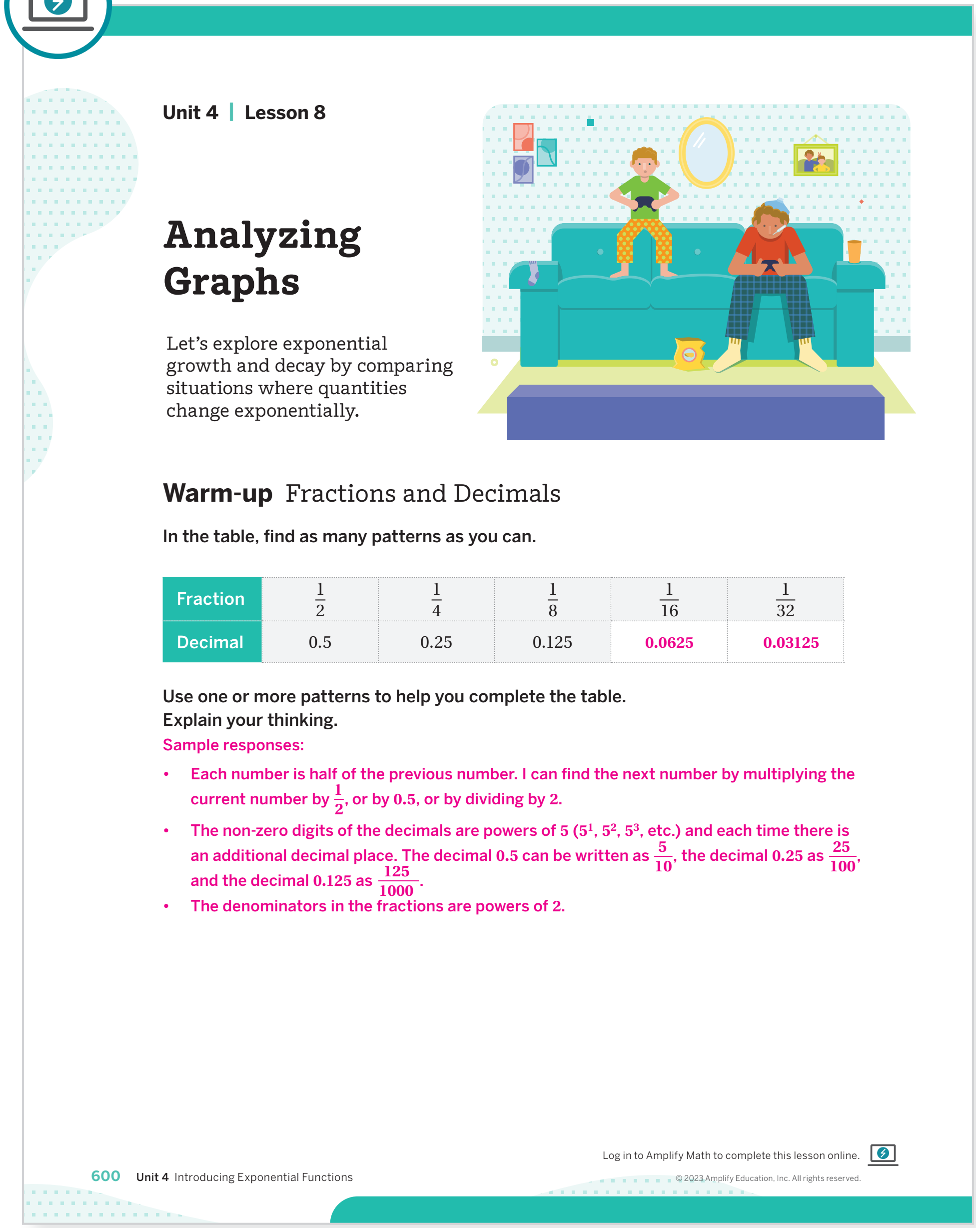
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, work with a subset of cards to sort. Alternatively, project the cards and have students work as a class.

Pairs | 5 min

Warm-up Fractions and Decimals

Students analyze a table to look for patterns that will help them complete the missing values.



Unit 4 | Lesson 8

Analyzing Graphs

Let's explore exponential growth and decay by comparing situations where quantities change exponentially.

Warm-up Fractions and Decimals

In the table, find as many patterns as you can.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$
Decimal	0.5	0.25	0.125	0.0625	0.03125

Use one or more patterns to help you complete the table. Explain your thinking.

Sample responses:

- Each number is half of the previous number. I can find the next number by multiplying the current number by $\frac{1}{2}$, or by 0.5, or by dividing by 2.
- The non-zero digits of the decimals are powers of 5 (5^1, 5^2, 5^3, etc.) and each time there is an additional decimal place. The decimal 0.5 can be written as $\frac{5}{10}$, the decimal 0.25 as $\frac{25}{100}$, and the decimal 0.125 as $\frac{125}{1000}$.
- The denominators in the fractions are powers of 2.

Log in to Amplify Math to complete this lesson online.

600 Unit 4 Introducing Exponential Functions

1 Launch

Have the students use the ***Think-Pair-Share*** routine. Provide them with one minute of individual think time to study the table. Then have them complete the Warm-up with a partner.

2 Monitor

Help students get started by asking, "What do you notice about the denominators of the fractions? What do you notice about the digits in the decimal numbers?" The denominators of the fractions are multiples of 2 and the decimals are multiples of 5.

Look for points of confusion:

- **Struggling with fraction/decimal equivalents.** Have students divide numerators by denominators using a calculator.

Look for productive strategies:

- Dividing the numerator by the denominator to convert fractions to decimals.

3 Connect

Display the table.

Have students share the pattern they used to complete the table.

Highlight that each number is half of the previous number. The next number can be determined by multiplying by $\frac{1}{2}$ or by 0.5, or by dividing by 2.

Ask:

- "Are the successive numbers exhibiting linear or exponential change? Explain your thinking." Exponential because there is a common factor.
- "Are the successive numbers getting larger or smaller? Explain your thinking." Smaller because each number is half the previous number.

Power-up

To power up students' ability to relate fractions and decimals, have students complete:

Recall that to rewrite a fractions as its decimal equivalent, you can divide the value of the numerator by the value of the denominator.

Determine the decimal equivalent of each fraction.

a $\frac{3}{5} = 0.6$

b $\frac{3}{8} = 0.375$

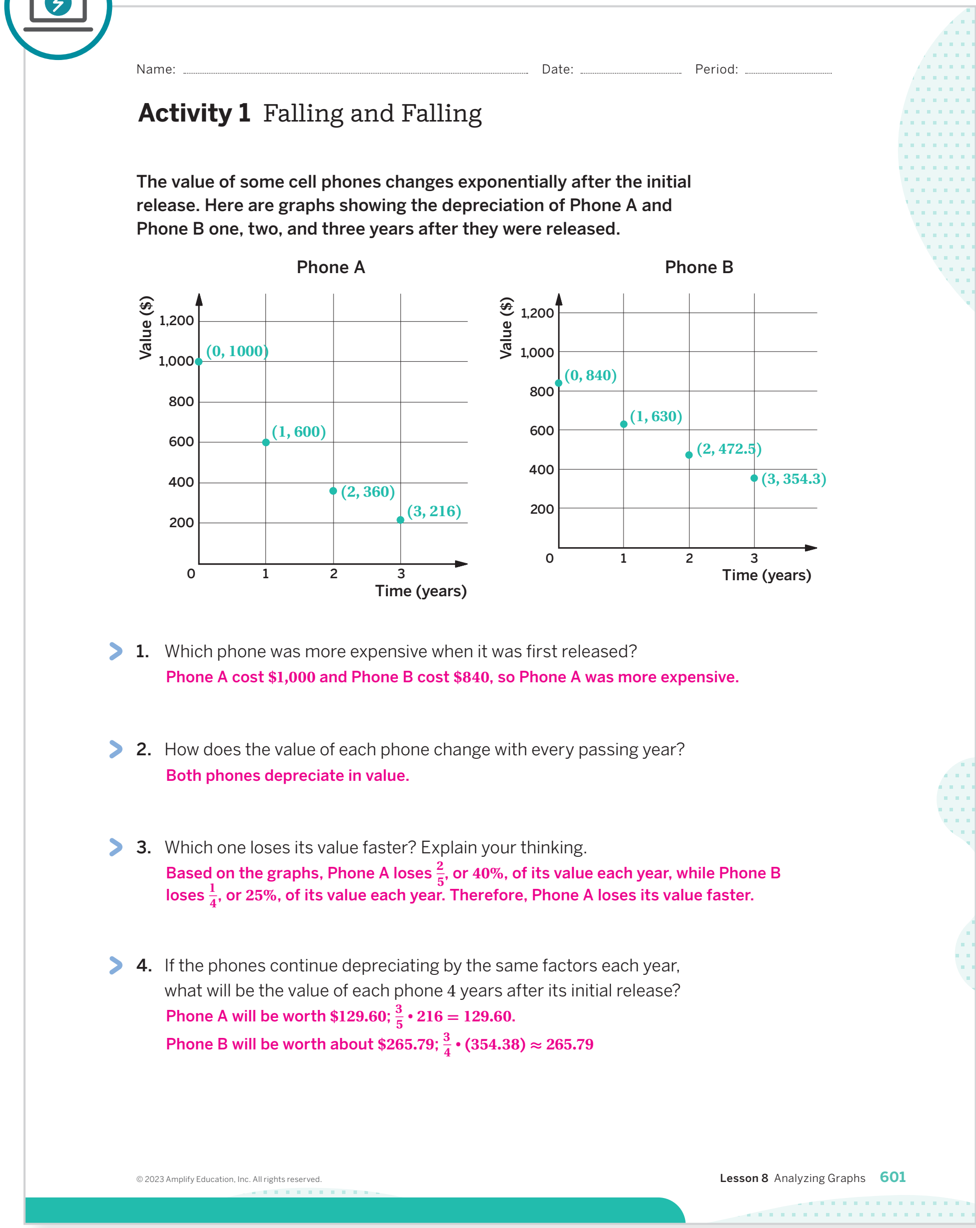
Use: Before the Warm-up

Informed by: Performance on Lesson 7, Practice Problem 6

Pairs | 15 min

Activity 1 Falling and Falling

Students analyze graphs of the values of two cell phones, and then construct equations to model the exponential relationship between value and time.



Name: ______ Date: ______ Period: ______

Activity 1 Falling and Falling

The value of some cell phones changes exponentially after the initial release. Here are graphs showing the depreciation of Phone A and Phone B one, two, and three years after they were released.

1. Which phone was more expensive when it was first released?
   Phone A cost $1,000 and Phone B cost $840, so Phone A was more expensive.

2. How does the value of each phone change with every passing year?
   Both phones depreciate in value.

3. Which one loses its value faster? Explain your thinking.
   Based on the graphs, Phone A loses $\frac{2}{5}$, or 40%, of its value each year, while Phone B loses $\frac{1}{4}$, or 25%, of its value each year. Therefore, Phone A loses its value faster.

4. If the phones continue depreciating by the same factors each year, what will be the value of each phone 4 years after its initial release?
   Phone A will be worth $129.60; $\frac{3}{5} \cdot 216 = 129.60$.
   Phone B will be worth about $265.79; $\frac{3}{4} \cdot (354.38) \approx 265.79$

1 Launch

Display the graphs and read the prompt aloud. Ask, “How does the value of a cell phone change after it is first released?” Consider displaying cell phone advertisements to encourage participation.

2 Monitor

Help students get started by asking them to determine the initial value of each phone.

Look for points of confusion:

- **Not knowing the meaning of the word *depreciate*.** Tell students that when an item depreciates, it loses its value over time.
- **Not recognizing that the relationship is exponential.** Ask, “What is the change in value from the initial release to the first year? From the first year to the second year?”
- **Struggling to identify the vertical intercept as the initial value of the phone.** Have students circle the first point on the graph, and then ask what the x-coordinate represents in context.
- **Using the percent of depreciation (25% and 40%) as the decay factor.** Remind students that losing 40% means the phone is keeping 60% of its value, so the decay factor is 60%, or $\frac{3}{5}$.

Look for productive strategies:

- Identifying the vertical intercept as the initial cost.
- Using each graph to identify the decay factor of each phone.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Be sure students understand that Problem 4 is asking which plan is *losing* its value at a faster rate. If students' responses indicate that the value of Phone A is 60% of its previous year's value, ask them what that means for the value that is *lost* each year.

Math Language Development

MLR8: Discussion Supports

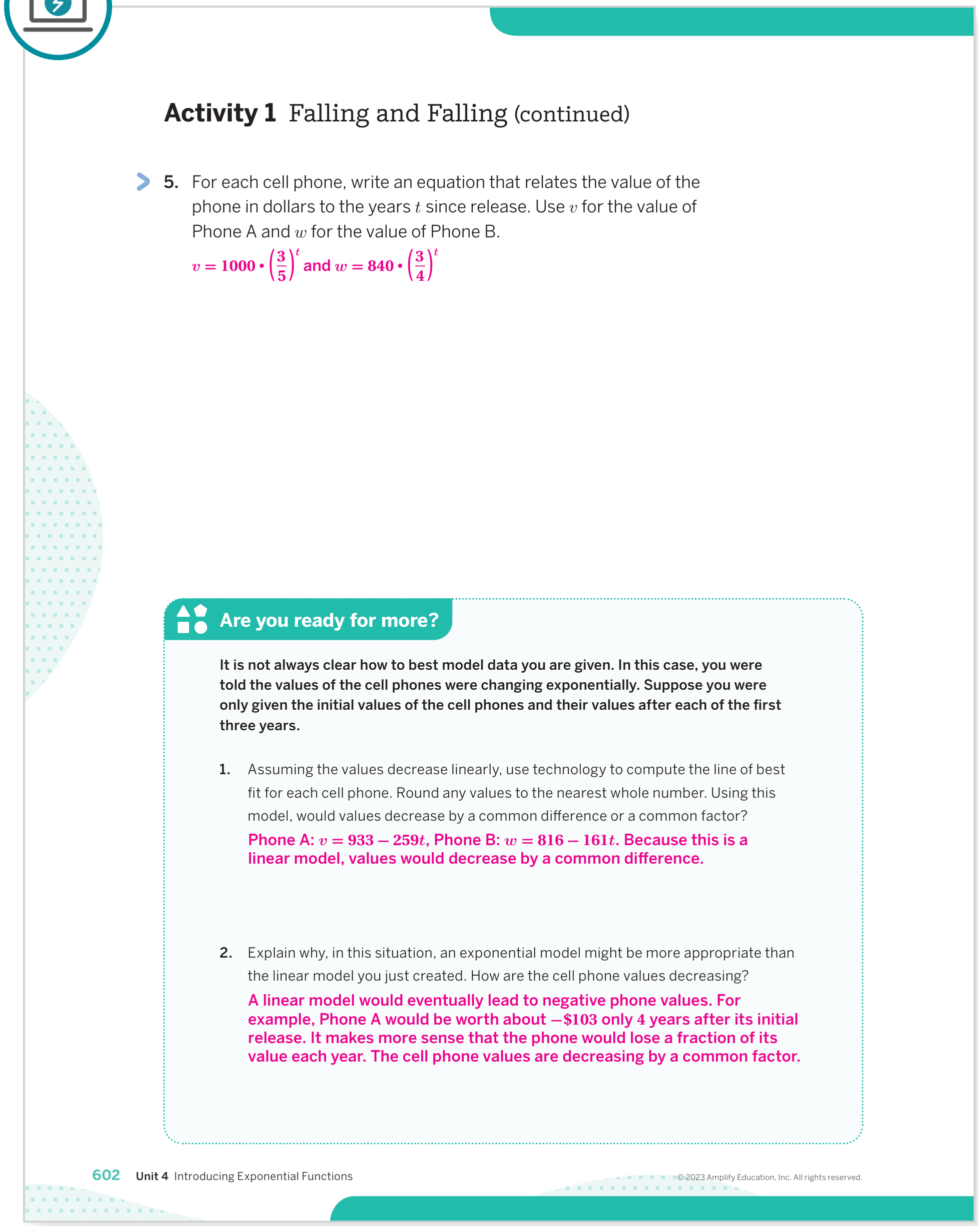
During the Launch, be sure students understand the meaning of the term *depreciation*. Tell them that when an item *depreciates*, it means that the value of that item has been reduced, typically due to normal “wear and tear” over time. To support students in understanding *depreciation*, ask:

- “When Phone A is first available to the public, its value is $1,000. What might happen to the phone over time that would cause its value to decrease?” Sample response: The owner could have dropped it multiple times, causing scratches or dents. Newer phones released will have more updated features than an older phone.
- “Do you think some phones might depreciate at a faster rate than others? Why or why not?” Answers may vary.

Pairs | 15 min

Activity 1 Falling and Falling (continued)

Students analyze graphs of the values of two cell phones, and then construct equations to model the exponential relationship between value and time.



Activity 1 Falling and Falling (continued)

5. For each cell phone, write an equation that relates the value of the phone in dollars to the years t since release. Use v for the value of Phone A and w for the value of Phone B.
   $v = 1000 \cdot \left(\frac{3}{5}\right)^t$ and $w = 840 \cdot \left(\frac{3}{4}\right)^t$

Are you ready for more?

It is not always clear how to best model data you are given. In this case, you were told the values of the cell phones were changing exponentially. Suppose you were only given the initial values of the cell phones and their values after each of the first three years.

1. Assuming the values decrease linearly, use technology to compute the line of best fit for each cell phone. Round any values to the nearest whole number. Using this model, would values decrease by a common difference or a common factor?
   Phone A: $v = 933 - 259t$, Phone B: $w = 816 - 161t$. Because this is a linear model, values would decrease by a common difference.
2. Explain why, in this situation, an exponential model might be more appropriate than the linear model you just created. How are the cell phone values decreasing?
   A linear model would eventually lead to negative phone values. For example, Phone A would be worth about $-\$103$ only 4 years after its initial release. It makes more sense that the phone would lose a fraction of its value each year. The cell phone values are decreasing by a common factor.

602 Unit 4 Introducing Exponential Functions

3 Connect

Display the graphs.

Have students share their strategies or processes for determining which phone depreciated the fastest or how they wrote the equations.

Highlight the process for identifying the decay factor. Make the connection between the initial amount, the decay factor, and the graph to write the equation predicting the value of the phone after t years.

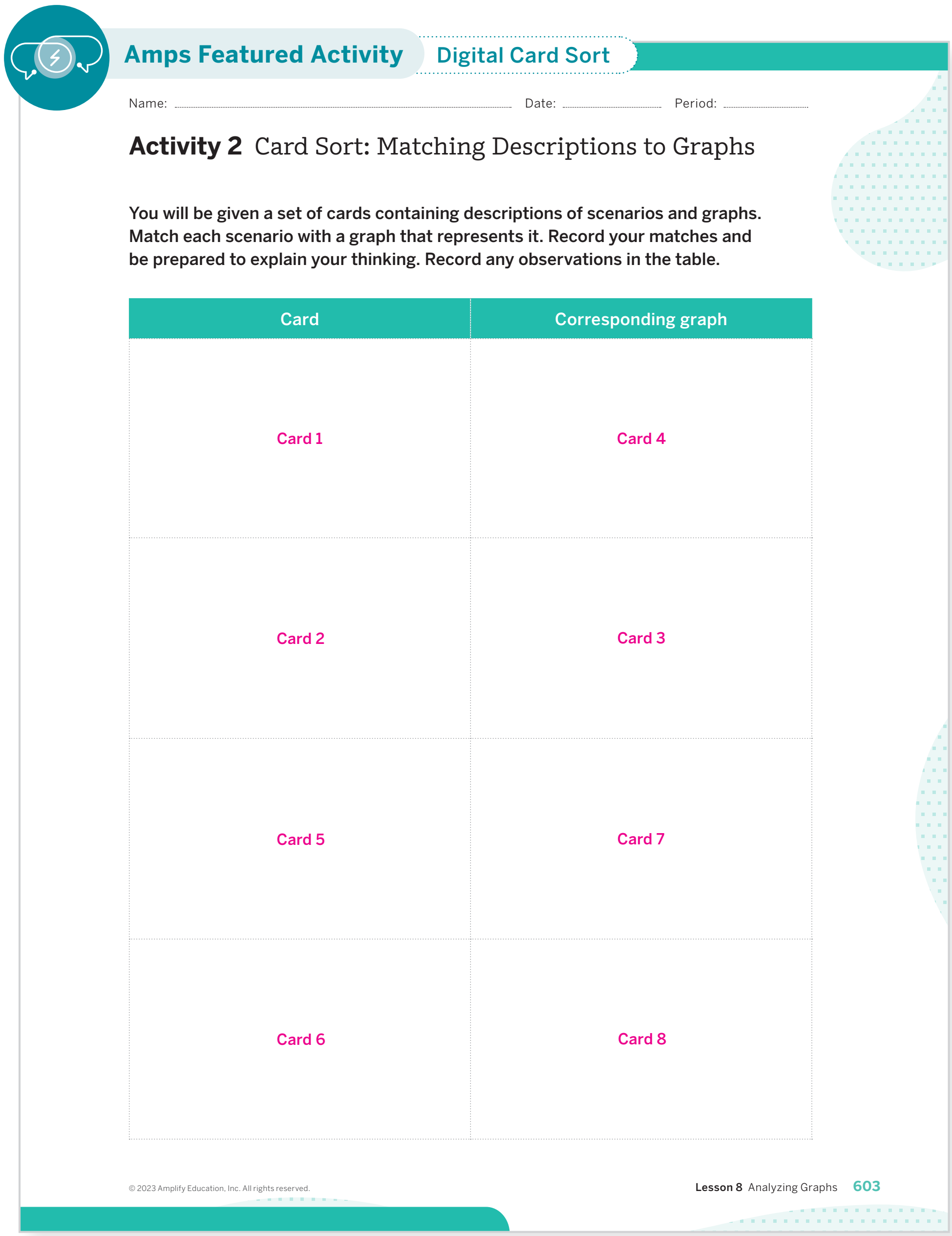
Ask, "Will the value of the phone(s) ever be $\$0$ based on the equation(s) you have written?"

Pairs | 10 min

Activity 2 Card Sort: Matching Descriptions to Graphs

Students match descriptions of real-world situations characterized by exponential change to graphs of equations that model those situations.

Amps Featured Activity Digital Card Sort



Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: Matching Descriptions to Graphs

You will be given a set of cards containing descriptions of scenarios and graphs. Match each scenario with a graph that represents it. Record your matches and be prepared to explain your thinking. Record any observations in the table.

Card	Corresponding graph
Card 1	Card 4
Card 2	Card 3
Card 5	Card 7
Card 6	Card 8

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Analyzing Graphs 603

1 Launch

Distribute a set of cards from the Activity 2 PDF to each pair of students.

2 Monitor

Help students get started by demonstrating the routine. Have students identify the scale of each graph, then ask whether the graph is increasing or decreasing.

Look for points of confusion:

- **Struggling to identify correct graphs.** Tell students to look carefully at the scales of the graphs and notice they are the same.
- **Struggling to identify growth or decay in the descriptions.** Circle the terms which indicate increase or decrease.

Look for productive strategies:

- Identifying the vertical intercept on a graph as the initial value, given a verbal description.
- Identifying the relationship between the domain and range in an ordered pair, given a verbal description.

3 Connect

Have students share their strategies for how they:

- Separated cards into stacks by growth/decay and determined the growth factor (start with cards having growth factors greater than 1).
- Used the information in the descriptions to draw conclusions about the coordinates of a point.

Highlight keywords and graph elements indicating growth and decay.

Ask, "What can you say about the growth factor in each scenario?" The growth happens more quickly in Card 1 than in Card 5. For Card 6, the phone loses $\frac{2}{5}$ of its value each year, while, for Card 2, the car only loses $\frac{1}{4}$ of its value each year.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code key words in the scenarios that help them match with the graphs.

Extension: *Math Enrichment*

Have students examine Cards 1 and 5. Ask them which company's stock will have a greater value after 8 years, 24 years, and 40 years. Tell them to assume the stock value starts at $100 in Year 0.

	Year 0	Year 8	Year 24	Year 40
Card 1	$100	$400	$6,400	$102,400
Card 2	$100	$300	$2,700	$24,300

Math Language Development

MLR7: Compare and Connect

During the Connect, display pairs of matching scenarios and graphs and ask students to share how they used key words from the scenarios to match with key features of the graphs. Ask:

- "How did you know which graphs indicated growth or decay? What words did you use in the scenarios to indicate growth or decay?"
- "For Cards 2 and 6, the word *loses* is used. What does this mean for the value that remains?"

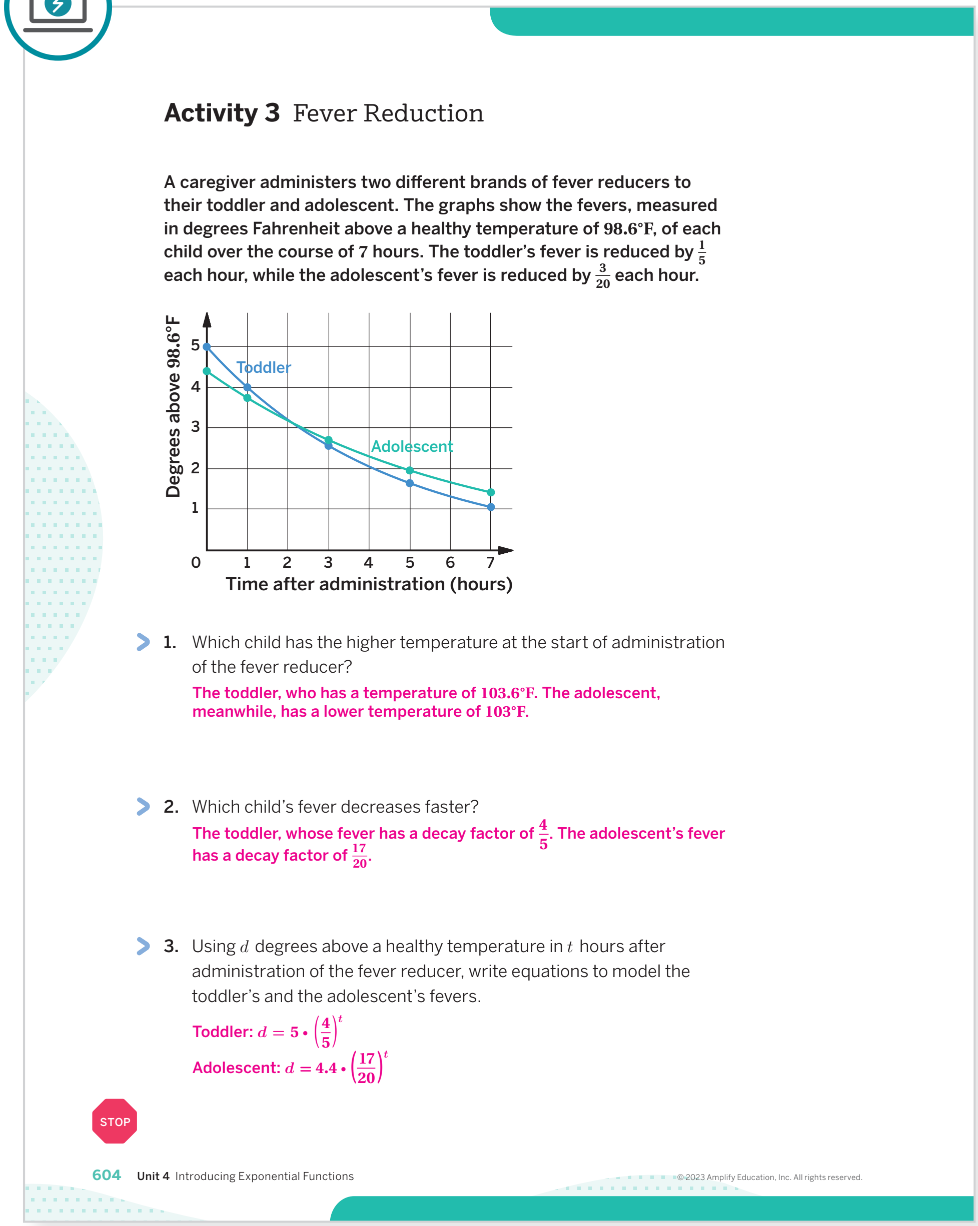
English Learners

Annotate the term *doubles* with a growth factor of 2 and annotate the term *triples* with a growth factor of 3.

Independent | 10 min

Activity 3 Fever Reduction

Students determine the growth factor and construct equations to model real-world situations involving exponential decay.



Activity 3 Fever Reduction

A caregiver administers two different brands of fever reducers to their toddler and adolescent. The graphs show the fevers, measured in degrees Fahrenheit above a healthy temperature of 98.6°F, of each child over the course of 7 hours. The toddler's fever is reduced by $\frac{1}{5}$ each hour, while the adolescent's fever is reduced by $\frac{3}{20}$ each hour.

1. Which child has the higher temperature at the start of administration of the fever reducer?

   The toddler, who has a temperature of 103.6°F. The adolescent, meanwhile, has a lower temperature of 103°F.

2. Which child's fever decreases faster?

   The toddler, whose fever has a decay factor of $\frac{4}{5}$. The adolescent's fever has a decay factor of $\frac{17}{20}$.

3. Using d degrees above a healthy temperature in t hours after administration of the fever reducer, write equations to model the toddler's and the adolescent's fevers.

   Toddler: $d = 5 \cdot \left(\frac{4}{5}\right)^t$

   Adolescent: $d = 4.4 \cdot \left(\frac{17}{20}\right)^t$

STOP

604 Unit 4 Introducing Exponential Functions

1 Launch

Display the graph and read the prompt aloud. Tell students that a fever reducer is a medication that helps cool the body to bring its temperature down to a healthy temperature.

2 Monitor

Help students get started by relating the vertical intercept of a graph to the initial value.

Look for points of confusion:

- **Struggling to identify the initial value.** Review the relationship between the vertical intercept and the initial value.
- **Struggling to identify growth or decay in the graphs.** Select two successive points to determine the decay factor.

Look for productive strategies:

- Identifying the vertical intercept as the initial value.
- Using the graph to determine the decay factor.

3 Connect

Display the graph.

Have students share their responses with a partner. Select students who identified the initial value, determined the decay factor, and wrote an equation for either child.

Highlight the processes for determining which child's temperature reduces faster and how the information is used to write equations.

Ask, "What does the decay factor tell you in this scenario?"

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Have students highlight the introductory text that states the fraction by which each person's fever is *reduced*. Ask:

- "If the toddler's fever is *reduced* by $\frac{1}{5}$ each hour, what fraction *remains*? How do you see this in the graph?" $\frac{4}{5}$ remains
- "If the adolescent's fever is *reduced* by $\frac{3}{20}$ each hour, what fraction *remains*? How do you see this in the graph?" $\frac{17}{20}$ remains

Math Language Development

MLR7: Compare and Connect

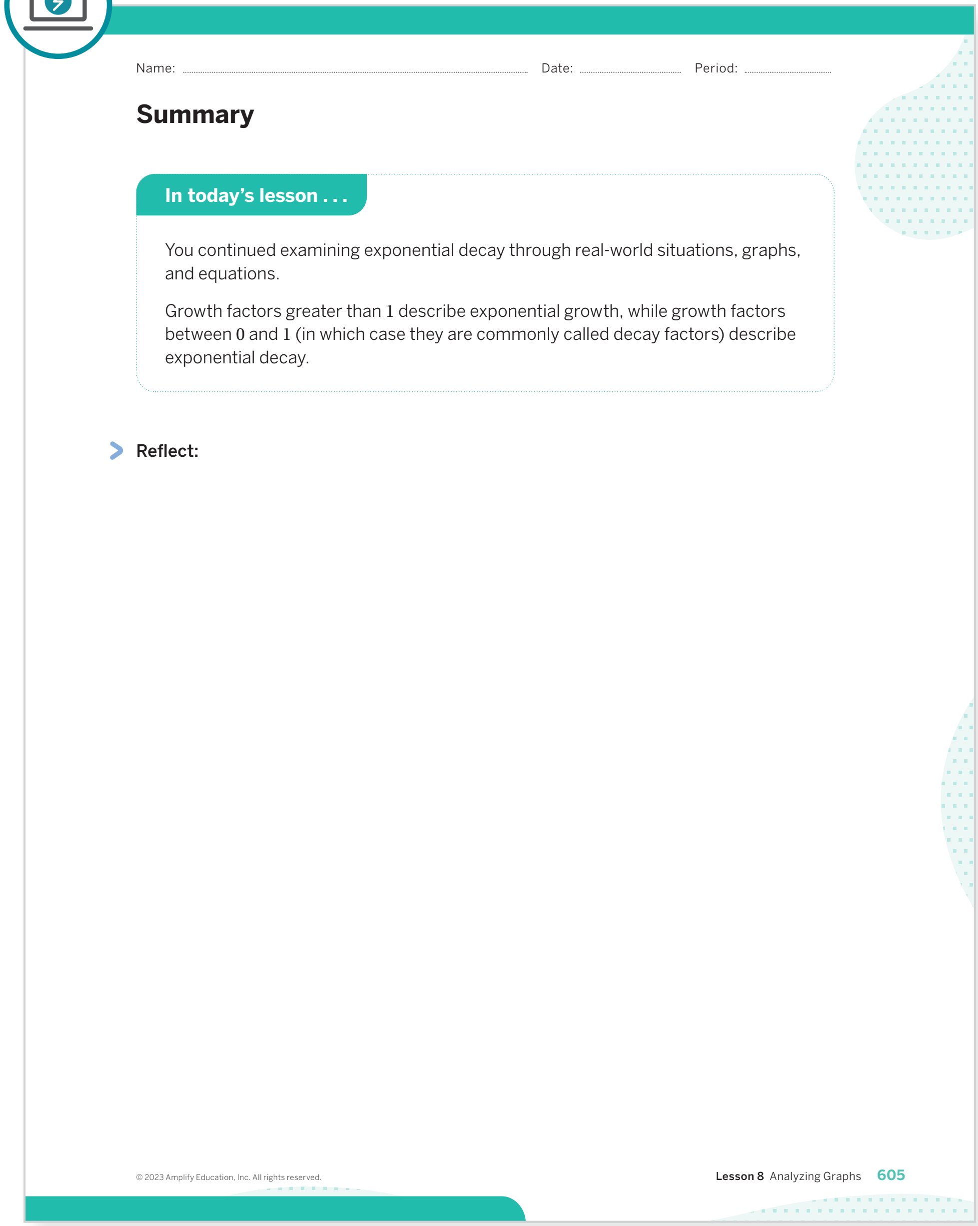
During the Connect, as students share their responses, draw their attention to the connections between the graph and equation for each child. Ask:

- "Which equation shows a greater initial value? Where do you see this on the graph? What does this mean within the context of the scenario?"
- "Which equation shows a greater decay factor? Where do you see this on the graph? What does this mean within the context of the scenario?"

Whole Class | 5 min

Summary

Review and synthesize how key information, such as initial values and growth factors, can be determined in context by examining the structure of the graph of an exponential function.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You continued examining exponential decay through real-world situations, graphs, and equations.

Growth factors greater than 1 describe exponential growth, while growth factors between 0 and 1 (in which case they are commonly called decay factors) describe exponential decay.

Reflect:

Synthesize

Display the graph of Cell Phone B from Activity 1.

Ask:

- "How can you use the graph to determine the initial amount?" Determine the vertical intercept.
- "How can you tell whether the growth factor is greater than 1 or between 0 and 1?" The graph is increasing if the the growth factor is greater than 1 and decreasing if the growth factor is between 0 and 1.
- "How can you use the graph to determine the growth factor?" Determine the ratio of the vertical coordinates of consecutive points by dividing them.
- "Once you know the initial amount and the growth factor, how can you construct an equation to represent the relationship?" Use the general equation $y = a \cdot b^x$, where a is the initial amount and b is the growth factor.

Highlight the correspondence of the initial value with the vertical intercept on a graph. Then highlight how the graph and verbal description of exponential behavior both illustrate the growth factor.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

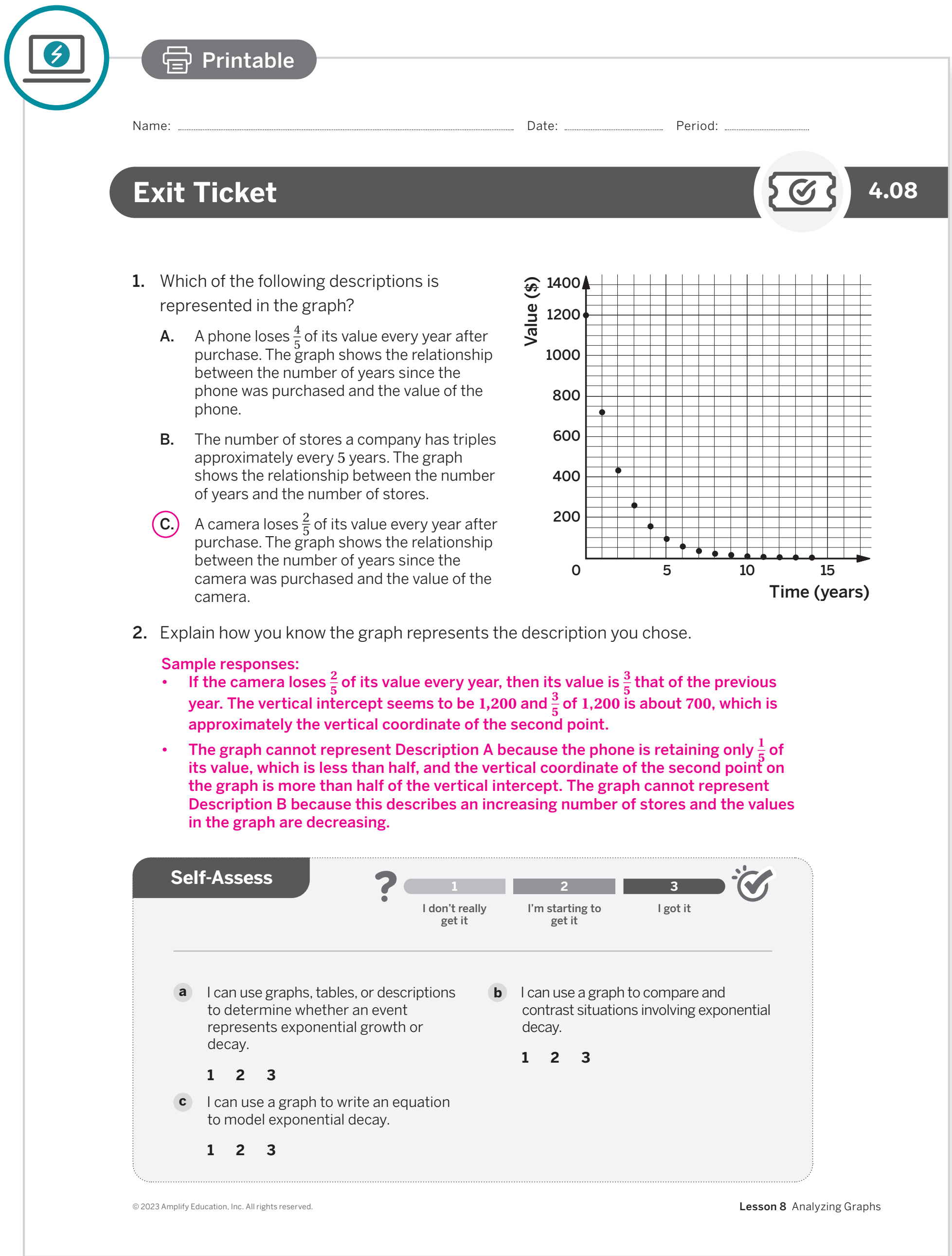
- "How can you determine the initial value of an exponential function by examining the structure of its graph?"
- "How can you determine the growth factor or decay factor of an exponential function by examining the structure of its graph?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing a graph for key characteristics and matching it to the correct scenario.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.08

1. Which of the following descriptions is represented in the graph?

   A. A phone loses $\frac{4}{5}$ of its value every year after purchase. The graph shows the relationship between the number of years since the phone was purchased and the value of the phone.

   B. The number of stores a company has triples approximately every 5 years. The graph shows the relationship between the number of years and the number of stores.

   [✓ C.] A camera loses $\frac{2}{5}$ of its value every year after purchase. The graph shows the relationship between the number of years since the camera was purchased and the value of the camera.

2. Explain how you know the graph represents the description you chose.

   Sample responses:
   - If the camera loses $\frac{2}{5}$ of its value every year, then its value is $\frac{3}{5}$ that of the previous year. The vertical intercept seems to be 1,200 and $\frac{3}{5}$ of 1,200 is about 700, which is approximately the vertical coordinate of the second point.
   - The graph cannot represent Description A because the phone is retaining only $\frac{1}{5}$ of its value, which is less than half, and the vertical coordinate of the second point on the graph is more than half of the vertical intercept. The graph cannot represent Description B because this describes an increasing number of stores and the values in the graph are decreasing.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can use graphs, tables, or descriptions to determine whether an event represents exponential growth or decay.
1 2 3

b. I can use a graph to compare and contrast situations involving exponential decay.
1 2 3

c. I can use a graph to write an equation to model exponential decay.
1 2 3

Lesson 8 Analyzing Graphs

Success looks like . . .

- **Goal:** Determining whether situations are characterized by exponential growth or by exponential decay, given descriptions and graphs.
  - » Selecting answer choice C as the best description of the decaying graph in Problem 1.
- **Language Goal:** Using graphs to compare and contrast situations that involve exponential decay. **(Writing)**
- **Goal:** Using information from a graph to write an equation that represents exponential decay.

Suggested next steps

If students do not select the correct description in Problem 1, consider:

- Reviewing Activity 2.

If students inadequately explain their thinking in Problem 2, consider:

- Reviewing Activity 2.
- Assigning Practice Problems 1 and 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to analyze graphs of exponential decay to highlight key features, such as the initial value and growth (decay) factor. How well did students accomplish this goal? What specific types of support(s) did you offer to help them accomplish this goal? What might you change the next time you teach this lesson?
- Thinking about the questions you asked students today and how students responded, which question(s) were the most efficient? Which questions might you change the next time you teach this lesson?

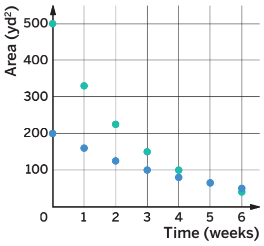
Practice

Independent

Practice

Name: .... Date: .... Period: ....

1. The two graphs show models for the area covered by two algae blooms, in square yards, w weeks after different chemicals were applied.



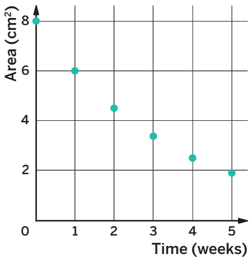
a Which bloom covered a greater area when the chemicals were first applied? Explain your thinking.

The one with the point (0, 500) on the graph. This bloom covered 500 yd^2 when the chemicals were first applied. The other bloom covered only 200 yd^2.

b Which bloom's population is decreasing faster? Explain your thinking.

The one that is initially 500 yd^2 is decreasing faster. The area covered by this population is greater at the beginning, and is less than the area of the other population after 6 weeks.

2. A medicine is applied to a burn on a patient's arm. The area of the burn in square centimeters decreases exponentially and is shown in the graph.



a What fraction of the burn area remains each week?

$\mathbf{\frac{3}{4}}$

b Write an equation representing the area A of the burn, after t weeks?

$A = 8 \cdot \left(\frac{3}{4}\right)^t$

c What is the area of the burn after 7 weeks? Round to three decimal places.

1.068 cm^2

Name: .... Date: .... Period: ....

3. The following problems are about folding pieces of paper.

a The area of a sheet of paper is 100 in^2. Write an equation that gives the area A, in square inches, of a sheet of paper that is folded in half n times.

$A = 100 \cdot \left(\frac{1}{2}\right)^n$

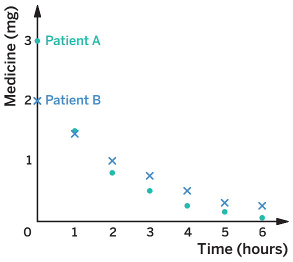
b The area of another sheet of paper is 200 in^2. Write an equation that gives the area B, in square inches, of a sheet of paper that is folded into thirds n times.

$B = 200 \cdot \left(\frac{1}{3}\right)^n$

c Are the areas of the two sheets of paper ever the same after each being folded n times? Show or explain your thinking.

No; Sample response: After one fold, the areas are 50 in^2 and $66\frac{2}{3}$ in^2. After two folds, the areas are 25 in^2 and $22\frac{2}{9}$ in^2. After this, B is always less than A, because the second sheet is folded into more pieces at each step.

4. The graphs show the amount of medicine in two patients after receiving injections. The dots show the medicine in Patient A and the x's show the medicine in Patient B. An equation that gives the amount of medicine m, in milligrams, in Patient A after h hours is $m = 300 \cdot \left(\frac{1}{2}\right)^h$. What is a possible equation for the amount of medicine in Patient B?



A. $m = 500 \cdot \left(\frac{3}{10}\right)^h$

B. $m = 500 \cdot \left(\frac{7}{10}\right)^h$

C. $m = 200 \cdot \left(\frac{3}{10}\right)^h$

[✓ D.] $m = 200 \cdot \left(\frac{7}{10}\right)^h$

5. Select *all* expressions that are equivalent to 3^{-8}.

A. $(3^{-2})^{-4}$

[✓ B.] $\frac{3^4}{3^{12}}$

C. $(3^{-9})^1$

[✓ D.] $(3^4)^{-2}$

E. $3^6 \cdot 3^{-2}$

[✓ F.] $3^{-10} \cdot 3^2$

Practice

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	3
	2	Activity 2	3
	3	Activity 2	3
	4	Activity 1	2
Formative ⚡	5	Unit 4 Lesson 9	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 9**

Using Negative Exponents

Let’s study exponential graphs and equations more closely.

Focus

Goals

1. **Language Goal:** Describe the meaning of a negative exponent in exponential equations. **(Speaking and Listening, Writing)**
2. Write and graph an equation that represents exponential growth and decay to solve problems.

Coherence

Today

Students interpret negative exponents in exponential contexts, such as medicine absorption in the body. They write equations of exponential form. Students determine the appropriate axes limits to create and display exponential graphs.

Previously

Students used graphs to compare and contrast situations that involved exponential decay.

Coming Soon

Students will write exponential equations as exponential functions.

Rigor

- Students enhance their **conceptual understanding** of exponential behavior by closely examining negative exponents, both in terms of exponential growth and decay.
- Students **apply** negative exponents to scenarios involving time, understanding that “negative time” measures time prior to a specific event.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Exponent Rules*

Math Language Development

Review words

- *exponential decay*
- *growth factor*
- *initial amount*

Amps | Featured Activity

Activity 2
Visualize Medicine Absorption

Students visualize the absorption of medicine according to their table, graph, and equation.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

To be successful with finding the pattern or growth factor, students must use great precision. This attention to detail requires a focus that can be challenging at times. Relate the need for precision in the activity to the need for precision when measuring medicine. Discuss why the amount of medicine must be so closely monitored.

Modifications to Pacing

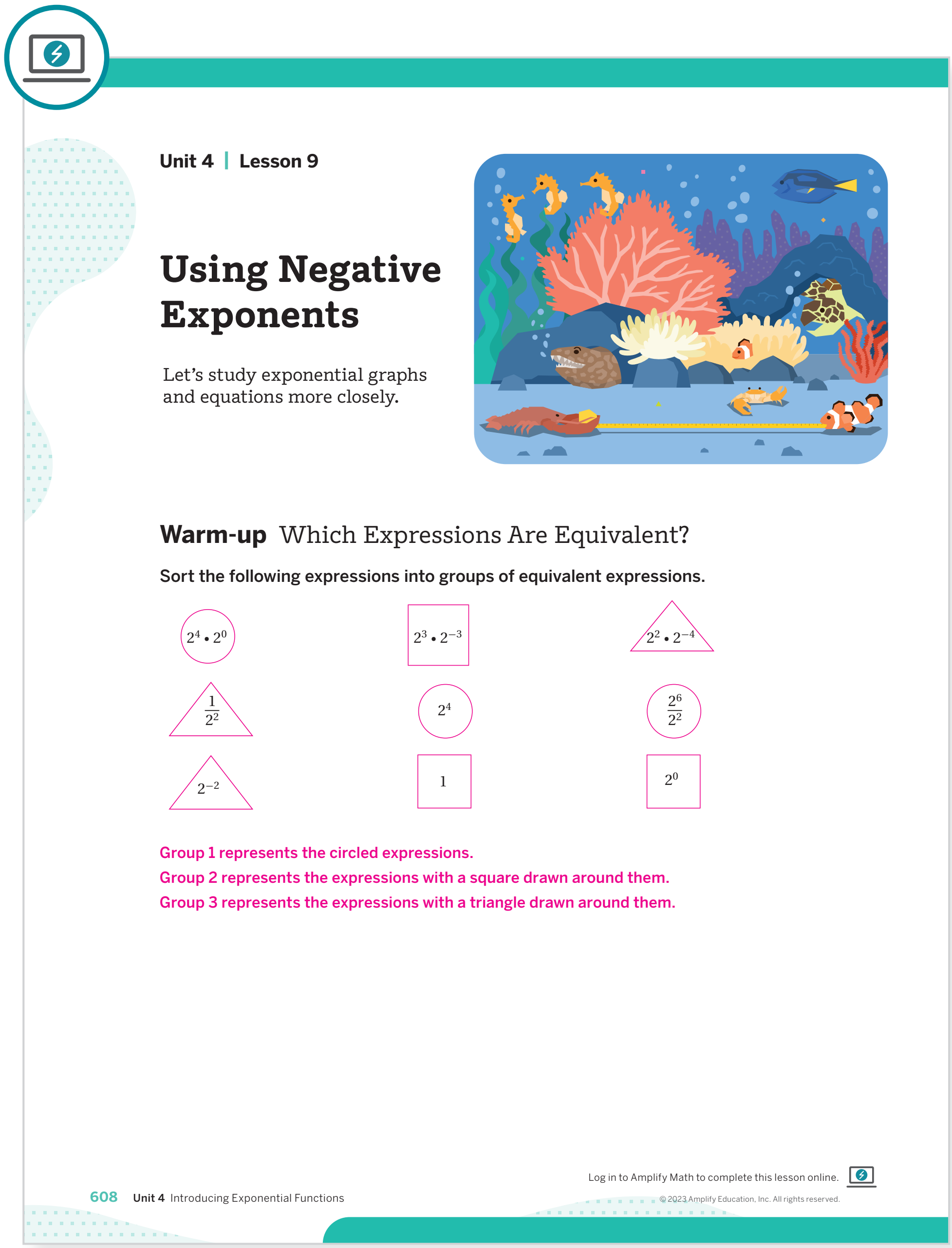
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, omit Problem 6.

Independent | 5 min

Warm-up Which Expressions Are Equivalent?

Students use exponent rules to sort expressions into groups of equivalent expressions.



Unit 4 | Lesson 9

Using Negative Exponents

Let's study exponential graphs and equations more closely.

Warm-up Which Expressions Are Equivalent?

Sort the following expressions into groups of equivalent expressions.

$2^4 \cdot 2^0$	$2^3 \cdot 2^{-3}$	$2^2 \cdot 2^{-4}$
$\frac{1}{2^2}$	2^4	$\frac{2^6}{2^2}$
2^{-2}	1	2^0

Group 1 represents the circled expressions.
Group 2 represents the expressions with a square drawn around them.
Group 3 represents the expressions with a triangle drawn around them.

Log in to Amplify Math to complete this lesson online.

608 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide independent work time before students share their thinking and strategies with a partner.

2 Monitor

Help students get started by identifying the expressions that have an operation, such as division or multiplication, and applying exponent rules to simplify.

Look for points of confusion:

- **Having difficulty applying the rules of exponents.** Display the Anchor Chart PDF, *Exponent Rules*, or distribute copies of the PDF to students.

Look for productive strategies:

- Applying exponent rules to simplify expressions.

3 Connect

Have students share their strategies and responses with a partner. Select student pairs to share their strategies and ways they sorted the expressions with the whole class.

Highlight how to correctly apply the zero and negative exponent rules for like bases.

Ask, "How can you simplify expressions with exponents and like bases using the negative and zero exponent rules?" Have students refer to the Anchor Chart PDF, *Exponent Rules* as needed.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Display the exponent rules students learned in Grade 8, using the Anchor Chart PDF, *Exponent Rules*, for them to reference during the Warm-up.

Power-up

To power up students' ability to evaluate expressions with negative exponents, have students complete:

Recall that $a^{-m} = \frac{1}{a^m}$. Which expression is equivalent to 6^{-3}?

A. $\frac{1}{18}$

[✓ B.] $\frac{1}{216}$

C. -216

D. -18

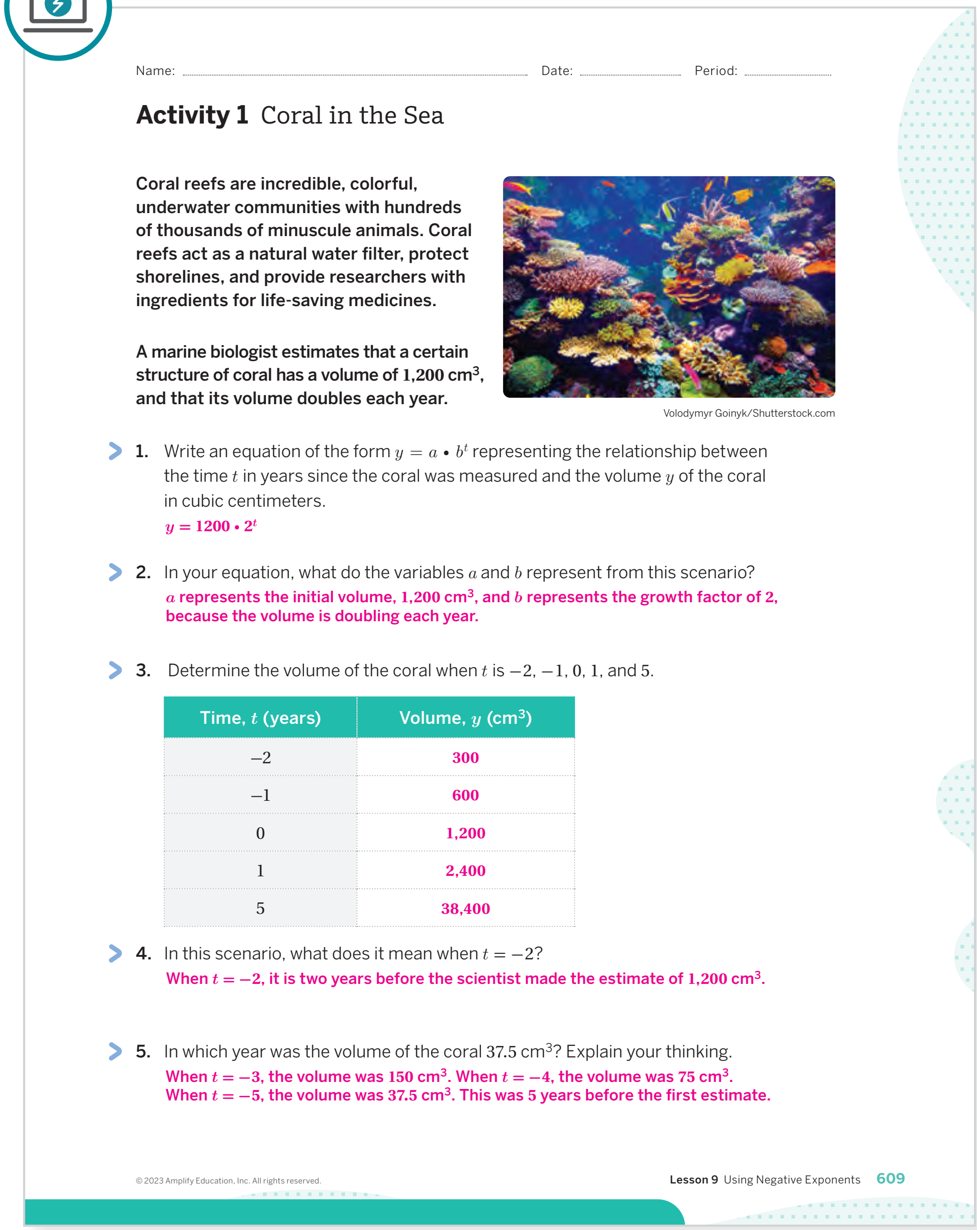
Use: Before the Warm-up

Informed by: Performance on Lesson 8, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 20 min

Activity 1 Coral in the Sea

Students contextualize to interpret the meaning of negative exponents in an exponential equation representing a real-world scenario.



Name: ________ Date: ________ Period: ________

Activity 1 Coral in the Sea

Coral reefs are incredible, colorful, underwater communities with hundreds of thousands of minuscule animals. Coral reefs act as a natural water filter, protect shorelines, and provide researchers with ingredients for life-saving medicines.

A marine biologist estimates that a certain structure of coral has a volume of 1,200 cm^3, and that its volume doubles each year.

Volodymyr Goinyk/Shutterstock.com

1. Write an equation of the form $y = a \cdot b^t$ representing the relationship between the time t in years since the coral was measured and the volume y of the coral in cubic centimeters.
   $y = 1200 \cdot 2^t$

2. In your equation, what do the variables a and b represent from this scenario?
   a represents the initial volume, 1,200 cm^3, and b represents the growth factor of 2, because the volume is doubling each year.

3. Determine the volume of the coral when t is $-2, -1, 0, 1$, and 5.

Time, t (years)	Volume, y (cm^3)
–2	300
–1	600
0	1,200
1	2,400
5	38,400

4. In this scenario, what does it mean when $t = -2$?
   When $t = -2$, it is two years before the scientist made the estimate of 1,200 cm^3.

5. In which year was the volume of the coral 37.5 cm^3? Explain your thinking.
   When $t = -3$, the volume was 150 cm^3. When $t = -4$, the volume was 75 cm^3. When $t = -5$, the volume was 37.5 cm^3. This was 5 years before the first estimate.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Using Negative Exponents 609

1 Launch

Read the prompt aloud. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking them to explain what the quantities a and b represent in the general form of an exponential equation.

Look for points of confusion:

- **Thinking negative exponents mean negative time.** Have students create their own year the study began and write the years next to the values of t to create a timeline.
- **Having difficulty determining the time, given the volume.** Suggest students extend the table, using the pattern to determine the year.

Look for productive strategies:

- Labeling the table with "years before."
- When given the volume, determining the year by extending the table or testing different values of t in the equation.

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students add a column to the left of the Time column in the table for Problem 3 and assign years to each row. For example, have them assign Year 0 as the current year. Ask, "If Year 0 is _____, what does Year –1 mean?"

Extension: *Math Enrichment*

Have students respond to the following question:

"What limitations might this mathematical model have?" Sample response: The growth factor was estimated and even if it was measured exactly, the equation is just a model of the growth. There may be other factors that affect the volume of the coral reef, both in the past and future.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory context and have students work with their partner to write 2–3 mathematical questions they could ask about this situation. Sample questions shown.

- What is the volume of the coral reef after 1 year?
- What is the volume of the coral reef after 3 years? 5 years?
- Does anything slow or prevent the growth of the coral reef?

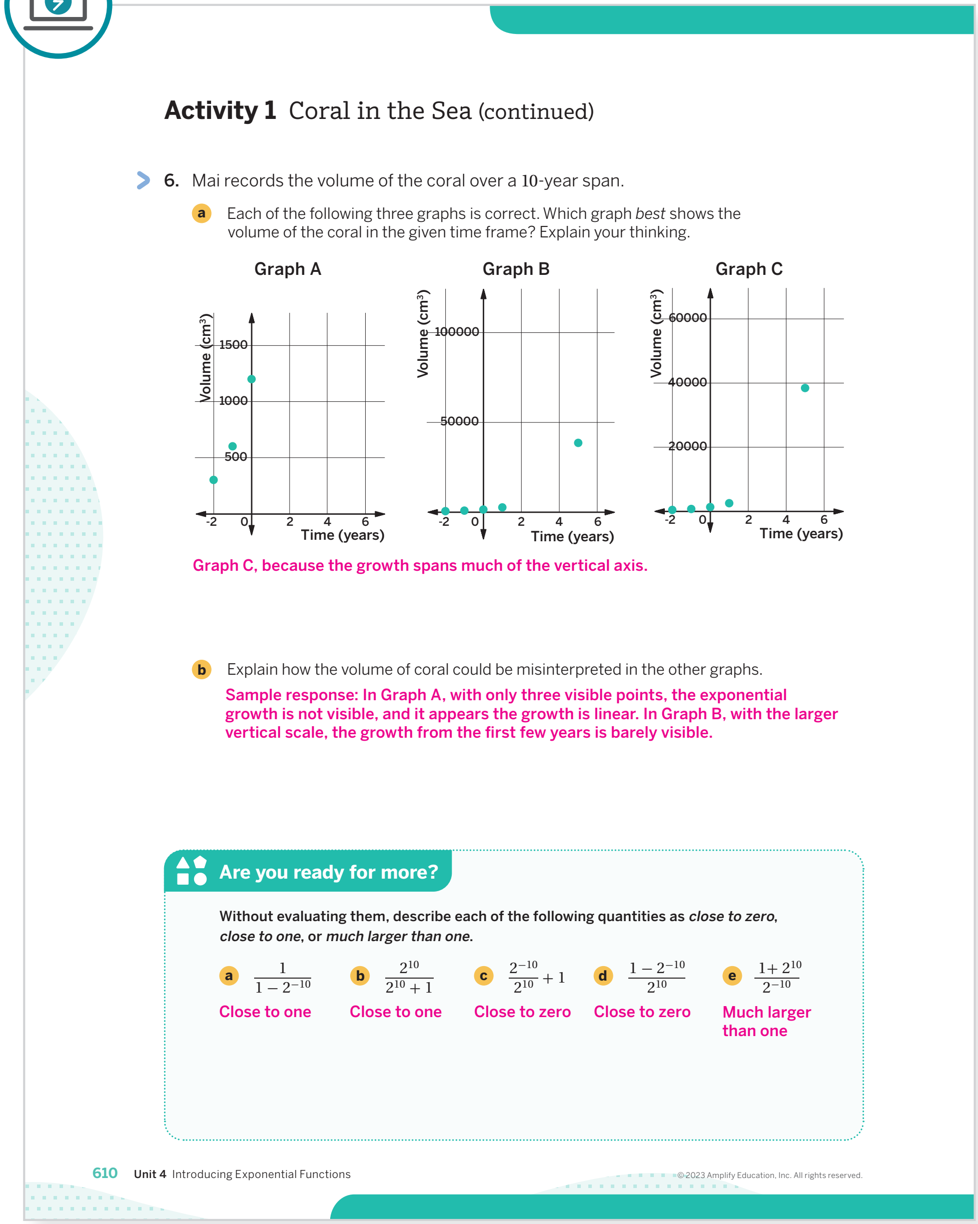
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 20 min

Activity 1 Coral in the Sea (continued)

Students contextualize to interpret the meaning of negative exponents in an exponential equation representing a real-world scenario.



Activity 1 Coral in the Sea (continued)

6. Mai records the volume of the coral over a 10-year span.

a. Each of the following three graphs is correct. Which graph *best* shows the volume of the coral in the given time frame? Explain your thinking.

Graph C, because the growth spans much of the vertical axis.

b. Explain how the volume of coral could be misinterpreted in the other graphs.

Sample response: In Graph A, with only three visible points, the exponential growth is not visible, and it appears the growth is linear. In Graph B, with the larger vertical scale, the growth from the first few years is barely visible.

Are you ready for more?

Without evaluating them, describe each of the following quantities as *close to zero*, *close to one*, or *much larger than one*.

a. $\frac{1}{1-2^{-10}}$ Close to one

b. $\frac{2^{10}}{2^{10}+1}$ Close to one

c. $\frac{2^{-10}}{2^{10}}+1$ Close to zero

d. $\frac{1-2^{-10}}{2^{10}}$ Close to zero

e. $\frac{1+2^{10}}{2^{-10}}$ Much larger than one

610 **Unit 4** Introducing Exponential Functions ©2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the equation $y = 1200 \cdot 2^t$ and the corresponding graph.

Have students share interpretations of the negative values of t and strategies to determine the volume for negative values of t.

Highlight the connections between the context and the interpretation of negative values.

Ask:

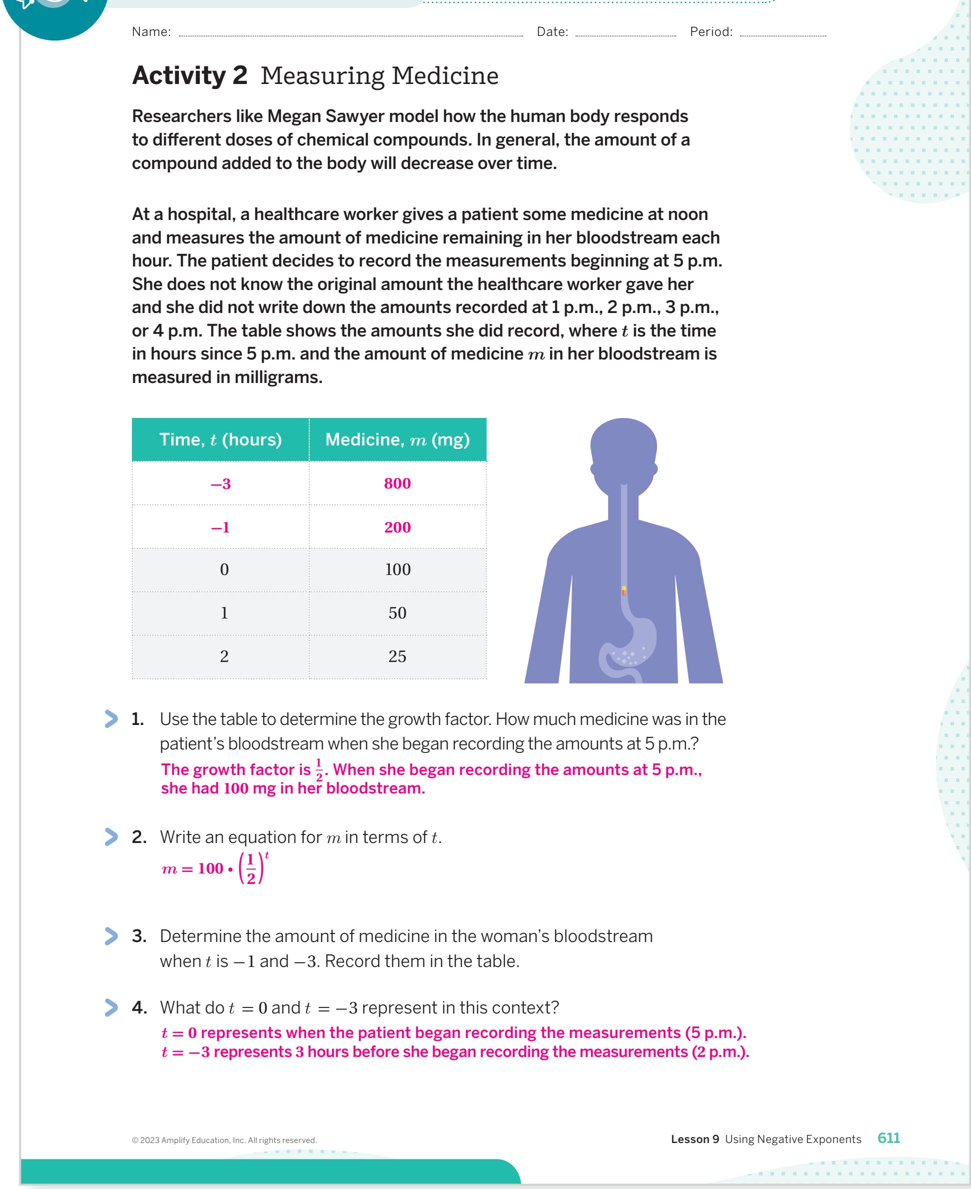
- "In this situation, what does it mean to say that when t is -3, y is 150?" Three years before the biologist estimated the coral volume, the volume was 150 cm^3.
- "How did you determine the year in which the volume of the coral was 37.5 cm^3?" By extending the table and using the growth factor to half each volume for each previous year in the table.

Pairs | 15 min

Activity 2 Measuring Medicine

Students construct an exponential equation to model a real-world scenario and interpret negative exponents in context.

Amps Featured Activity Visualize Medicine Absorption



Name: ______________ Date: ________ Period: ________

Activity 2 Measuring Medicine

Researchers like Megan Sawyer model how the human body responds to different doses of chemical compounds. In general, the amount of a compound added to the body will decrease over time.

At a hospital, a healthcare worker gives a patient some medicine at noon and measures the amount of medicine remaining in her bloodstream each hour. The patient decides to record the measurements beginning at 5 p.m. She does not know the original amount the healthcare worker gave her and she did not write down the amounts recorded at 1 p.m., 2 p.m., 3 p.m., or 4 p.m. The table shows the amounts she did record, where t is the time in hours since 5 p.m. and the amount of medicine m in her bloodstream is measured in milligrams.

Time, t (hours)	Medicine, m (mg)
−3	**800**
−1	**200**
0	100
1	50
2	25

1. Use the table to determine the growth factor. How much medicine was in the patient's bloodstream when she began recording the amounts at 5 p.m.?
   The growth factor is $\frac{1}{2}$. When she began recording the amounts at 5 p.m., she had 100 mg in her bloodstream.

2. Write an equation for m in terms of t.
   $m = 100 \cdot \left(\frac{1}{2}\right)^t$

3. Determine the amount of medicine in the woman's bloodstream when t is -1 and -3. Record them in the table.

4. What do $t = 0$ and $t = -3$ represent in this context?
   $t = 0$ represents when the patient began recording the measurements (5 p.m.).
   $t = -3$ represents 3 hours before she began recording the measurements (2 p.m.).

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Using Negative Exponents 611

1 Launch

Facilitate a brief discussion around students' general experience with medicine and how the amount of medicine decays in the body over time.

2 Monitor

Help students get started by annotating the table to determine the pattern or growth factor.

Look for points of confusion:

- **Struggling to make sense of negative time values.** Provide clock values to the numbers. Say, "The first blood test was done at 5:00 ($t = 0$). What is the value of t at 6:00? 4:00?"
- **Having difficulty writing an equation for m in terms of t.** Remind students to consider the independent and dependent variables and the structure of an exponential equation.

Look for productive strategies:

- Labeling the table with the hours before and after the blood test.
- Using the initial value and decay factor to write an equation for m in terms of t.

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can visualize the absorption of medicine according to their table, graph, and equation.

Accessibility: *Guide Processing and Visualization*

Suggest students annotate their table by writing the time of day next to the number of hours t.

Math Language Development

MLR6: Three Reads

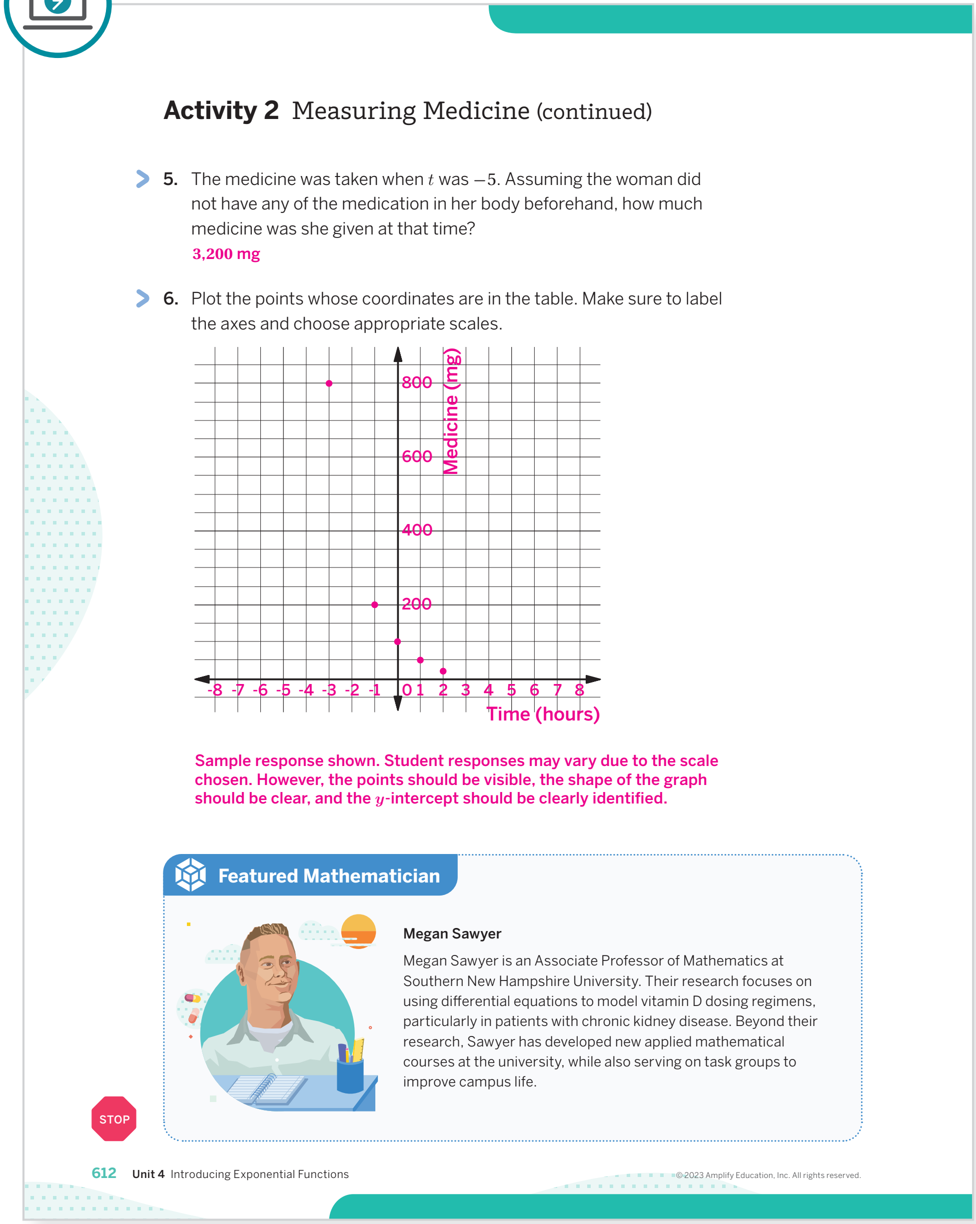
Use this routine to help students make sense of the introductory text and table.

- **Read 1:** Students should understand that a patient was given some medicine and began recording the amounts several hours after the medicine was given to her.
- **Read 2:** Ask students to name or describe given quantities or relationships, such as the patient began recording the measurements at 5 p.m..
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problem 1.

Pairs | 15 min

Activity 2 Measuring Medicine (continued)

Students construct an exponential equation to model a real-world scenario and interpret negative exponents in context.



Activity 2 Measuring Medicine (continued)

5. The medicine was taken when t was -5. Assuming the woman did not have any of the medication in her body beforehand, how much medicine was she given at that time?
3,200 mg

6. Plot the points whose coordinates are in the table. Make sure to label the axes and choose appropriate scales.

Sample response shown. Student responses may vary due to the scale chosen. However, the points should be visible, the shape of the graph should be clear, and the y-intercept should be clearly identified.

Featured Mathematician

Megan Sawyer

Megan Sawyer is an Associate Professor of Mathematics at Southern New Hampshire University. Their research focuses on using differential equations to model vitamin D dosing regimens, particularly in patients with chronic kidney disease. Beyond their research, Sawyer has developed new applied mathematical courses at the university, while also serving on task groups to improve campus life.

STOP

612 Unit 4 Introducing Exponential Functions

3 Connect

Display the equation $m = 100 \cdot \left(\frac{1}{2}\right)^t$ and its graph, which represents a discrete and continuous graph.

Have students share whether they extended the graph to include values that are between integer values.

Highlight negative non-integer values of t as minutes and the type of graph for modeling this scenario.

Ask:

- "Why did you choose to use the discrete graph?" Medicine metabolizes over minutes, not hours.
- "At what point is it no longer reasonable to use this model?" The amount of medicine will eventually reach zero.

Featured Mathematician

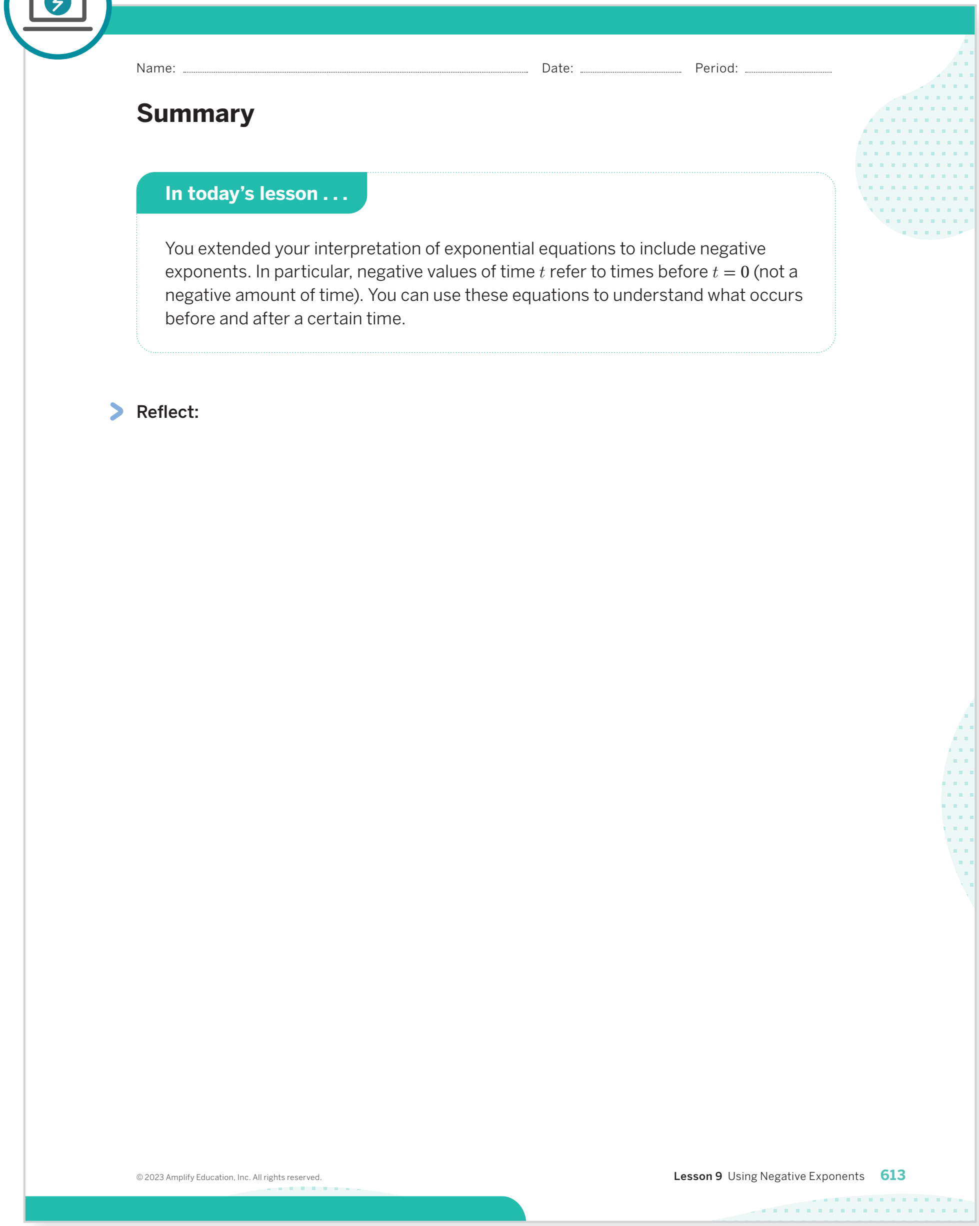
Megan Sawyer

Have students read about Featured Mathematician Megan Sawyer, an assistant professor who studies human dose response of different chemical compounds.

Whole Class | 5 min

Summary

Review and synthesize how to interpret negative exponents in context using an equation, table, and graph.



Name: Date: Period:

Summary

In today's lesson . . .

You extended your interpretation of exponential equations to include negative exponents. In particular, negative values of time t refer to times before $t = 0$ (not a negative amount of time). You can use these equations to understand what occurs before and after a certain time.

Reflect:

Synthesize

Display the equation $m = 1000000 \cdot \left(\frac{3}{2}\right)^t$. Ask students to brainstorm possible scenarios that the equation could represent.

Have students share possible scenarios for the equation.

Highlight that equations are useful for representing relationships that change exponentially, and solving problems about the relationships. When interpreting negative values, use the context to determine the meaning.

Ask:

- "What information can you gather from the exponential equation?" The initial amount is $1{,}000{,}000$.
- "Does the equation represent exponential growth or decay?" The equation represents exponential growth.
- "What is the meaning of $t = 0$ and $t = -4$?" Sample response: $t = 0$ represents the beginning of the experiment, and $t = -4$ could represent 4 hours before the beginning of the experiment.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

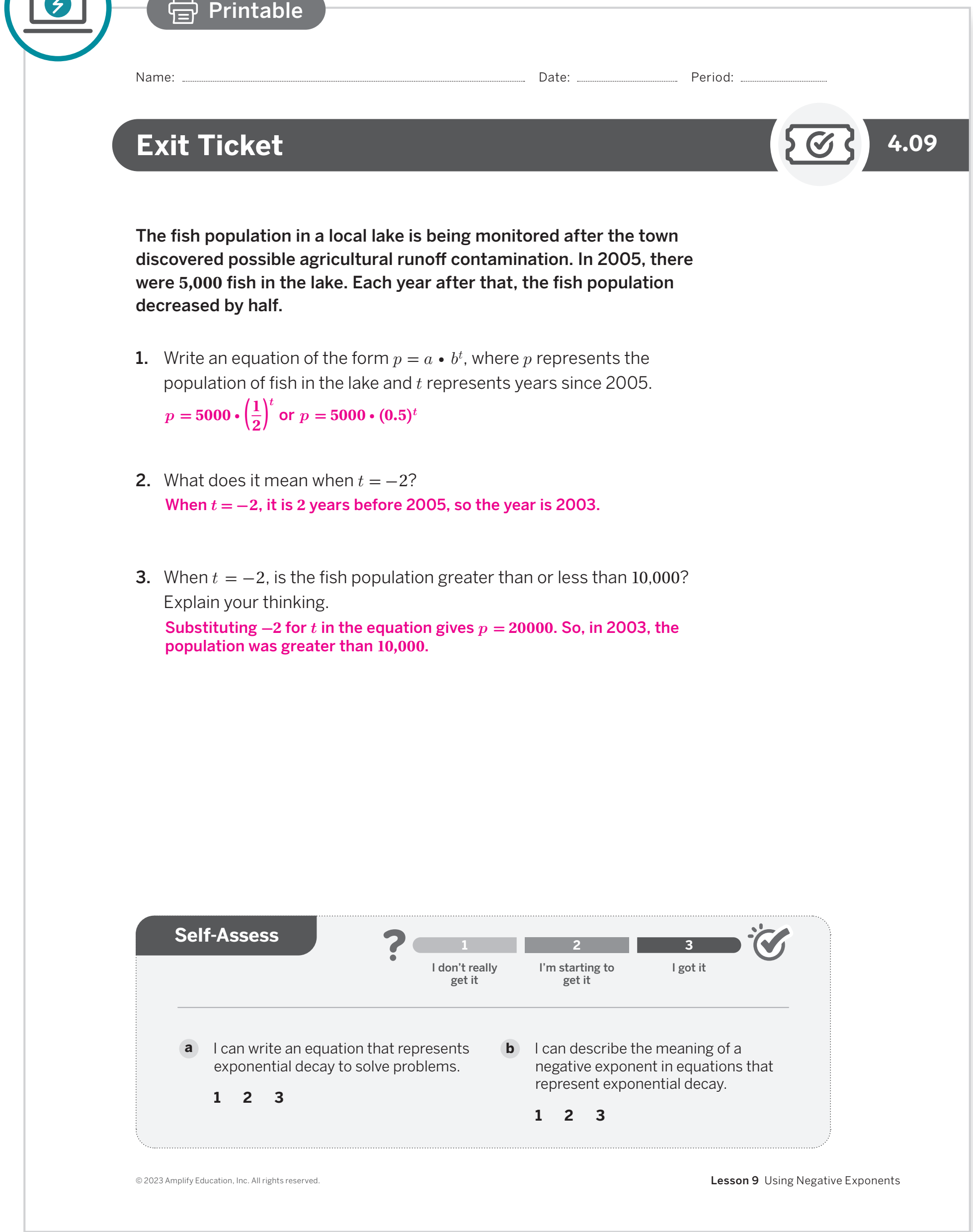
- "What might a negative exponent mean in the context of an exponential function where the independent variable represents time? In which types of real-world scenarios might a negative exponent make sense or not make sense?"
- "How could you use a negative exponent, in the context of an exponential function, to determine an initial value?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing an exponential equation to model a real-world situation and interpreting negative exponent values in context.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 4.09

The fish population in a local lake is being monitored after the town discovered possible agricultural runoff contamination. In 2005, there were 5,000 fish in the lake. Each year after that, the fish population decreased by half.

1. Write an equation of the form $p = a \cdot b^t$, where p represents the population of fish in the lake and t represents years since 2005.
   $p = 5000 \cdot \left(\frac{1}{2}\right)^t$ or $p = 5000 \cdot (0.5)^t$
2. What does it mean when $t = -2$?
   When $t = -2$, it is 2 years before 2005, so the year is 2003.
3. When $t = -2$, is the fish population greater than or less than 10,000? Explain your thinking.
   Substituting -2 for t in the equation gives $p = 20000$. So, in 2003, the population was greater than 10,000.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can write an equation that represents exponential decay to solve problems.
1 2 3

b I can describe the meaning of a negative exponent in equations that represent exponential decay.
1 2 3

Lesson 9 Using Negative Exponents

Success looks like . . .

- **Language Goal:** Describing the meaning of a negative exponent in exponential equations. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of -2 given the scenario in Problem 2.
- **Goal:** Writing and graphing an equation that represents exponential growth and decay to solve problems.
  - » Writing an equation given the scenario in Problem 1.

Suggested next steps

If students incorrectly write the equation in Problem 1, consider:

- Revisiting how to generate an exponential equation given the initial amount and decay factor from Lesson 4, Activity 2 or Lesson 5, Activity 2.
- Assigning Practice Problem 3a.
- Having students create a table for the given decay factor and initial amount to help them generate an equation.

If students incorrectly interpret the meaning of the negative exponent in Problem 2, consider:

- Reviewing Activity 1, Problem 4 and Activity 2, Problem 4.
- Assigning Practice Problem 1a.

If students do not evaluate the equation for $t = -2$ in Problem 3, consider:

- Revisiting Activity 1, Problem 5 or Activity 2, Problem 5.
- Assigning Practice Problems 3b and 3c.
- Having students create a table to see how the values change over time. Have them use the table to determine the value of y.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students explored negative exponents in the context of exponential equations as they model real-world scenarios. How did that build on earlier understandings of negative exponents from Grade 8?
- During the discussion in Activity 1, how did you encourage each student to listen to one another's interpretations of negative values of t? What might you change the next time you teach this lesson?

Math Language Development

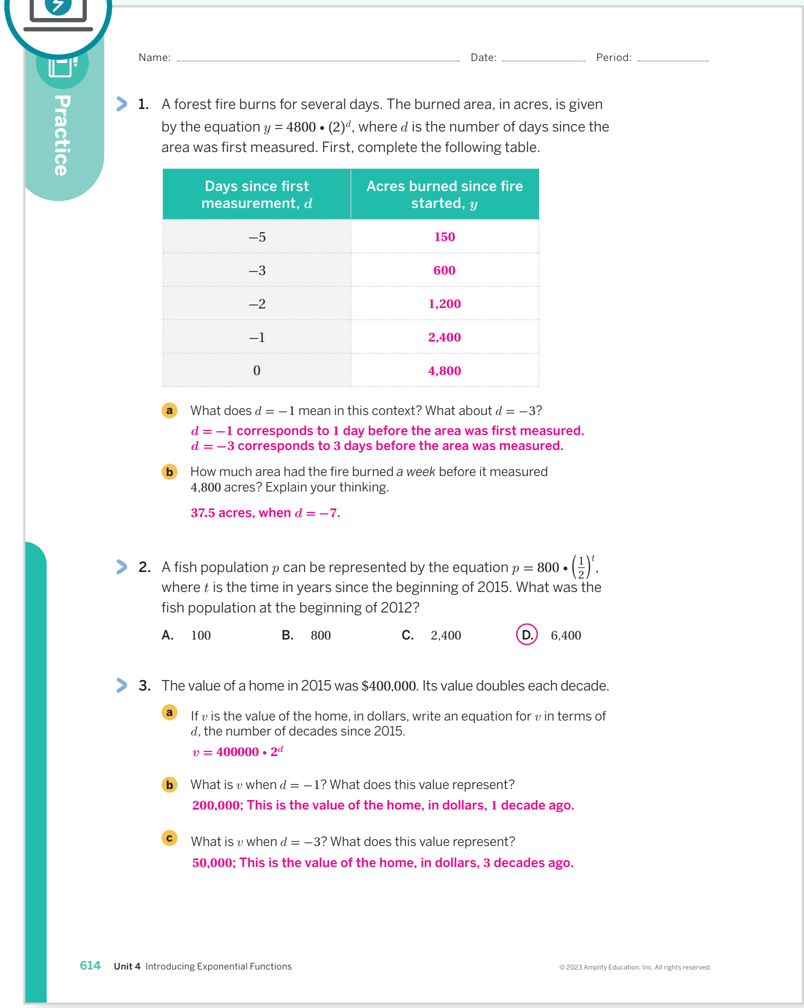
Language Goal: Describing the meaning of a negative exponent in exponential equations.

Reflect on students' language development toward this goal.

- Do students' responses to Problem 3 of the Exit Ticket demonstrate they understand what it means when $t = -2$? Do their explanations include an understanding that t represents the number of years since 2005, so when $t = -2$, the year was 2003?
- How can you help students be more precise in their explanations?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. A forest fire burns for several days. The burned area, in acres, is given by the equation $y = 4800 \cdot (2)^d$, where d is the number of days since the area was first measured. First, complete the following table.

Days since first measurement, d	Acres burned since fire started, y
-5	150
-3	600
-2	1,200
-1	2,400
0	4,800

   a. What does $d = -1$ mean in this context? What about $d = -3$?

   $d = -1$ corresponds to 1 day before the area was first measured.
   $d = -3$ corresponds to 3 days before the area was measured.

   b. How much area had the fire burned *a week* before it measured 4,800 acres? Explain your thinking.

   37.5 acres, when $d = -7$.

2. A fish population p can be represented by the equation $p = 800 \cdot \left(\frac{1}{2}\right)^t$, where t is the time in years since the beginning of 2015. What was the fish population at the beginning of 2012?

A. 100 B. 800 C. 2,400 [✓ D.] 6,400

3. The value of a home in 2015 was $400,000. Its value doubles each decade.

   a. If v is the value of the home, in dollars, write an equation for v in terms of d, the number of decades since 2015.

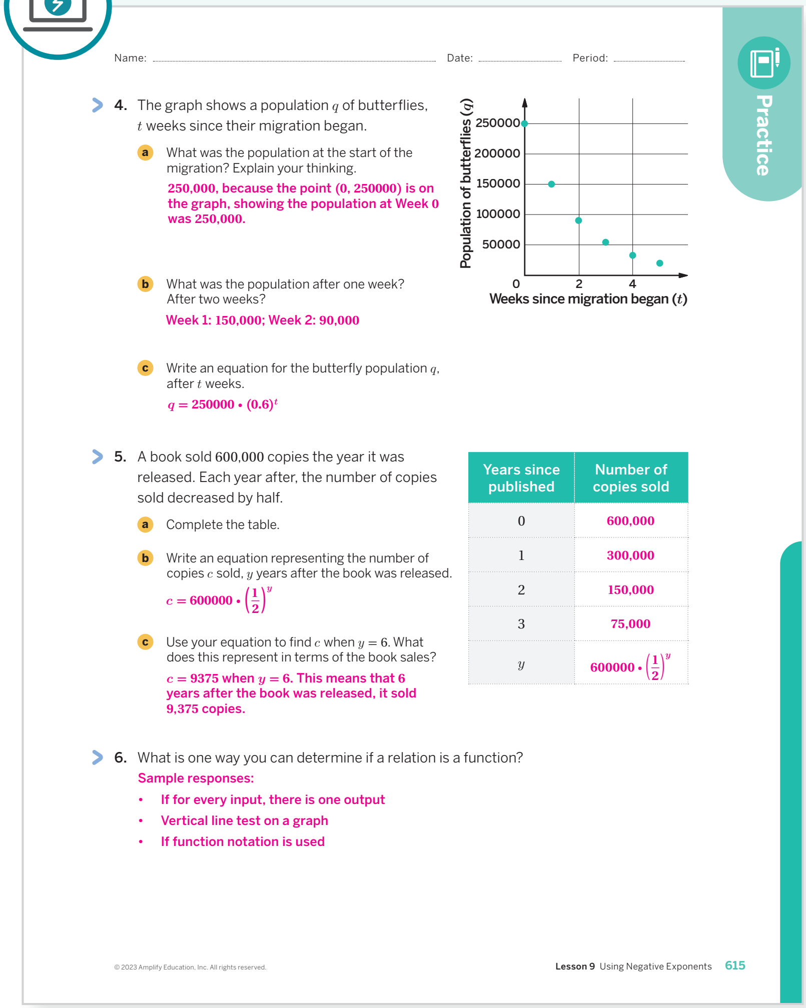
   $v = 400000 \cdot 2^d$

   b. What is v when $d = -1$? What does this value represent?

   200,000; This is the value of the home, in dollars, 1 decade ago.

   c. What is v when $d = -3$? What does this value represent?

   50,000; This is the value of the home, in dollars, 3 decades ago.



Name: ________ Date: ________ Period: ________

4. The graph shows a population q of butterflies, t weeks since their migration began.

   a. What was the population at the start of the migration? Explain your thinking.

   250,000, because the point (0, 250000) is on the graph, showing the population at Week 0 was 250,000.

   b. What was the population after one week? After two weeks?

   Week 1: 150,000; Week 2: 90,000

   c. Write an equation for the butterfly population q, after t weeks.

   $q = 250000 \cdot (0.6)^t$

5. A book sold 600,000 copies the year it was released. Each year after, the number of copies sold decreased by half.

   a. Complete the table.

   b. Write an equation representing the number of copies c sold, y years after the book was released.

   $c = 600000 \cdot \left(\frac{1}{2}\right)^y$

   c. Use your equation to find c when $y = 6$. What does this represent in terms of the book sales?

   $c = 9375$ when $y = 6$. This means that 6 years after the book was released, it sold 9,375 copies.

Years since published	Number of copies sold
0	600,000
1	300,000
2	150,000
3	75,000
y	$600000 \cdot \left(\frac{1}{2}\right)^y$

6. What is one way you can determine if a relation is a function?

Sample responses:

- **If for every input, there is one output**
- **Vertical line test on a graph**
- **If function notation is used**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 6	2
	5	Unit 4 Lesson 6	3
Formative	6	Unit 4 Lesson 10	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 10**

Exponential Situations as Functions

Let's explore exponential functions.

Focus

Goals

1. Use function notation to write equations that represent exponential relationships.
2. Determine whether relationships — represented in descriptions, tables, equations, or graphs — are functions.

Coherence

Today

Students determine whether relationships are exponential functions in the context of mold and its importance in the discovery of penicillin, bacteria growth, and drug trials. They choose independent and dependent variables and express relationships using function notation.

Previously

Students encountered exponential change using descriptions, tables, graphs, and equations.

Coming Soon

Students will study exponential functions in context. Given a relationship, they will write one quantity as a function of another and use the function to solve problems about the context.

Rigor

- Students **apply** exponential models to scenarios involving both exponential growth and decay.
- Students enhance their **procedural fluency** of using tables and graphs to describe the behavior of exponential functions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	15 min	20 min	5 min	5 min
Independent	Independent	Independent	Whole Class	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- graphing technology

Math Language Development

New words

- ***exponential function***

Review words

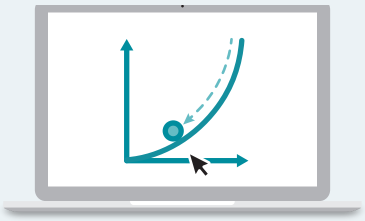
- *dependent variable*
- *independent variable*

Amps | Featured Activity

Activity 2

Marbleslides

Students use Marbleslides to check the accuracy of their function and adjust the domain and range to a more reasonable scale.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

The excitement can build when students are given access to technology to help them model a situation mathematically and they can forget to think about the possible consequences of poor decisions. In this activity, choosing a graphing window can lead to very different results. Without the full view of the graph, it is almost impossible to interpret it correctly. Similarly, without a clear view of an entire situation, it is challenging to know what the best decision is. Constructive choices can be made when all of the information has been gathered and interpreted.

Modifications to Pacing

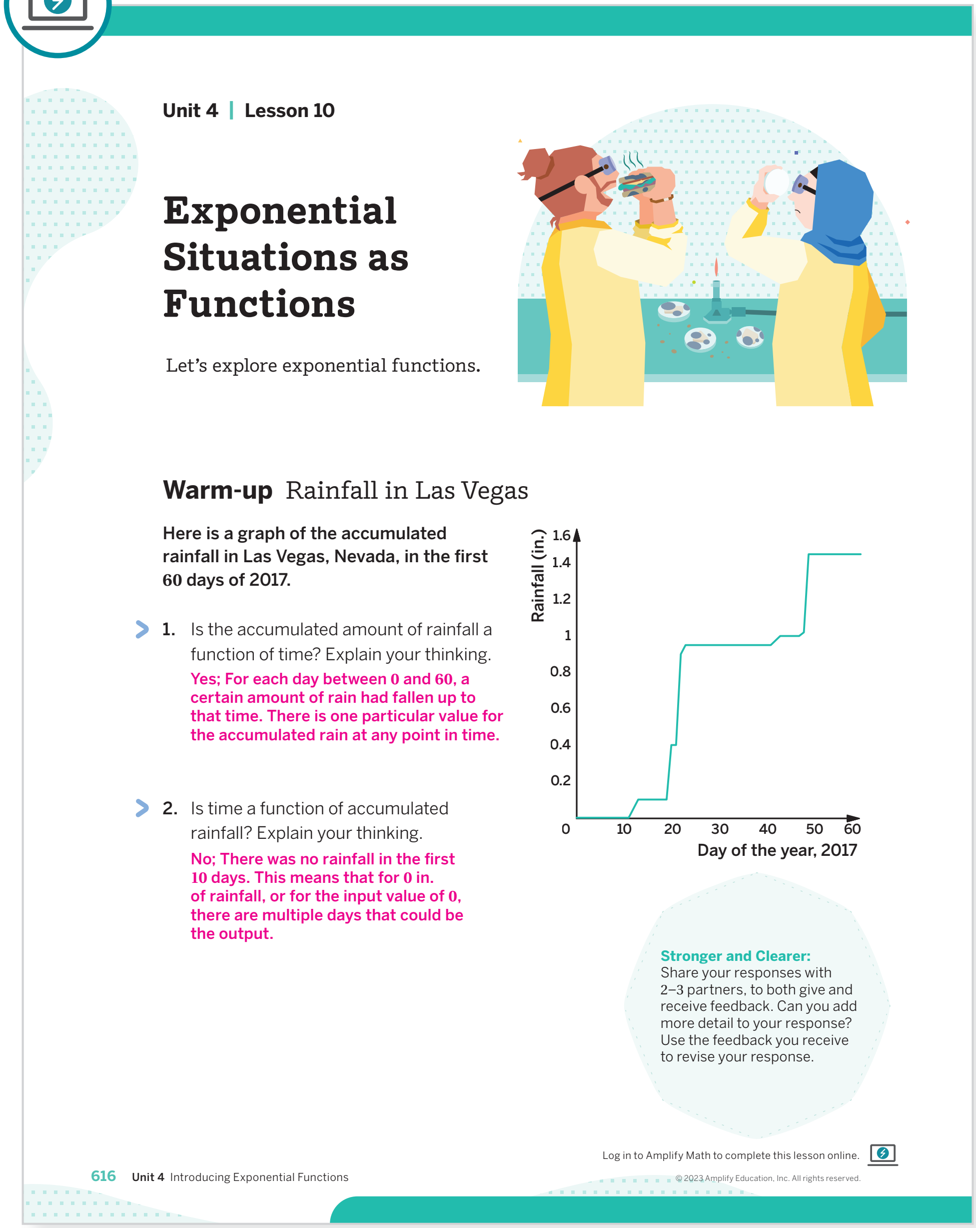
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up** if students are comfortable with the definition of functions and domains.
- Optional **Activity 3** can be omitted.

Independent | 5 min

Warm-up Rainfall in Las Vegas

Students review the meaning of a function presented graphically and interpret the graph in terms of the context.



Unit 4 | Lesson 10

Exponential Situations as Functions

Let's explore exponential functions.

Warm-up Rainfall in Las Vegas

Here is a graph of the accumulated rainfall in Las Vegas, Nevada, in the first 60 days of 2017.

1. Is the accumulated amount of rainfall a function of time? Explain your thinking.
   Yes; For each day between 0 and 60, a certain amount of rain had fallen up to that time. There is one particular value for the accumulated rain at any point in time.
2. Is time a function of accumulated rainfall? Explain your thinking.
   No; There was no rainfall in the first 10 days. This means that for 0 in. of rainfall, or for the input value of 0, there are multiple days that could be the output.

Stronger and Clearer: Share your responses with 2–3 partners, to both give and receive feedback. Can you add more detail to your response? Use the feedback you receive to revise your response.

Log in to Amplify Math to complete this lesson online.

616 Unit 4 Introducing Exponential Functions

1 Launch

Display the graph and read the prompt aloud. Set an expectation for the amount of time students will have to individually work on the Warm-up.

2 Monitor

Help students get started by reminding them that for a relationship to be a function, there must be a unique output (range) for each input (domain).

Look for points of confusion:

- **Mixing up domain and range for each scenario.** Remind students that the independent input values represent the domain.

3 Connect

Have students share their explanations of why accumulated rain is a function of time, but not the other way around.

Highlight the function notation $r(t)$ represents the amount of rainfall as a function of time.

Ask, "How does identifying the independent and dependent variable help create an equation?" The dependent variable will be isolated on one side of the equation, and the independent variable will be part of an expression on the other side.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problems 1 and 2, have them meet with 2–3 other students to share and receive feedback on their responses. Have reviewers ask these questions:

- "Does your response include discussion of what it means for a relationship to be a function?"
- "Can you add more detail to your response?"

Have groups revise their designs, based on the feedback they receive.

English Learners

Let students know that *accumulated rainfall* means the *total amount of rainfall* that has fallen throughout the year.

Power-up

To power up students' ability to identify whether a relationship is a function, have students complete:

Priya is trying to determine whether a relationship is a function. Select *all* the methods that would allow her to decide whether it is a function.

A. Checking that, for every output, there is exactly one input.

[✓ **B.**] Checking that, for every input, there is exactly one output.

[✓ **C.**] Completing the vertical line test on the graph of the relationship.

D. Completing the horizontal line test on the graph of the relationship.

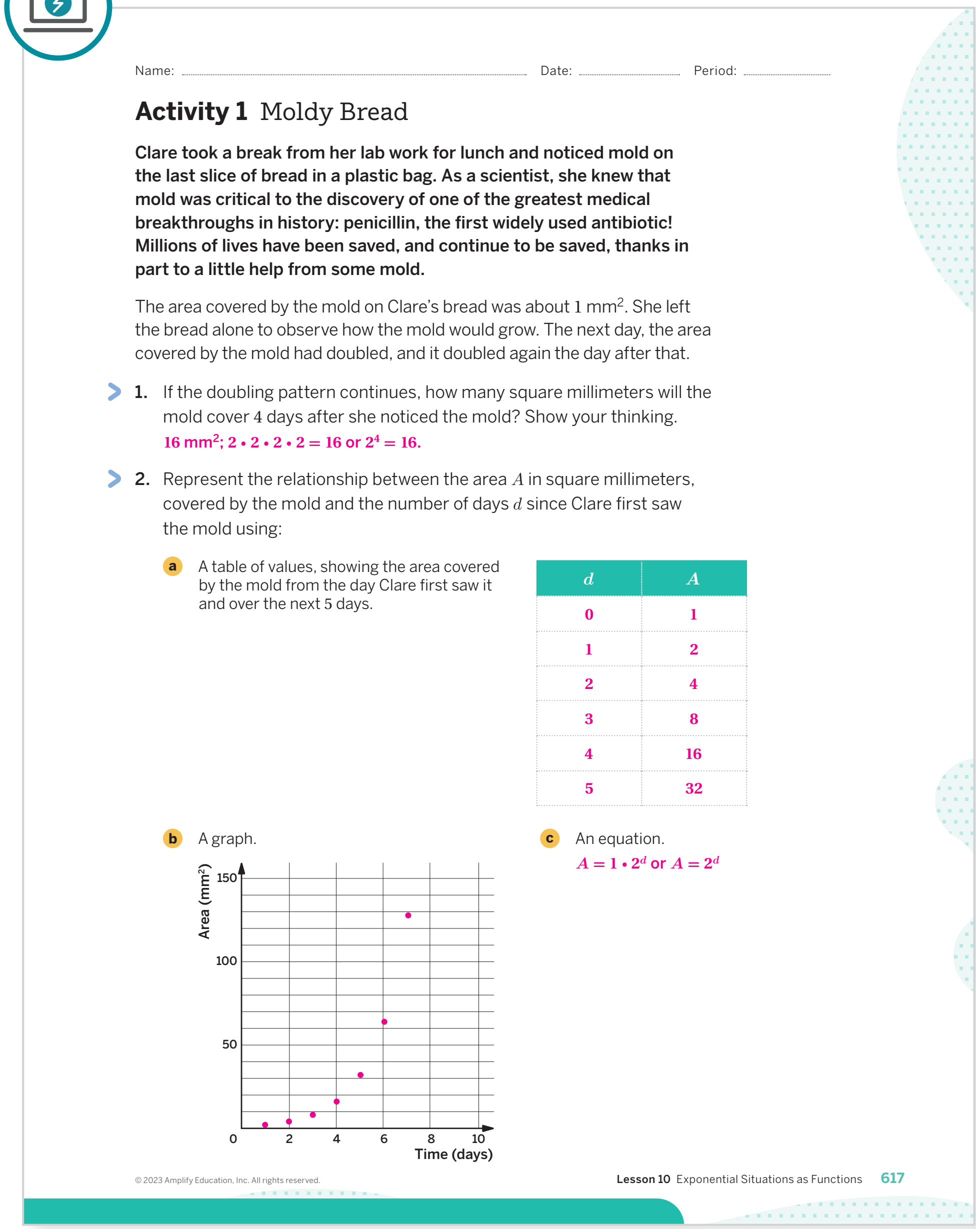
Use: Before the Warm-up

Informed by: Performance on Lesson 9, Practice Problem 6.

Independent | 20 min

Activity 1 Moldy Bread

Students use mathematics to model a real-world situation involving exponential growth and use function language to explain why the relationship is a function.



Name: ______ Date: ______ Period: ______

Activity 1 Moldy Bread

Clare took a break from her lab work for lunch and noticed mold on the last slice of bread in a plastic bag. As a scientist, she knew that mold was critical to the discovery of one of the greatest medical breakthroughs in history: penicillin, the first widely used antibiotic! Millions of lives have been saved, and continue to be saved, thanks in part to a little help from some mold.

The area covered by the mold on Clare's bread was about 1 mm^2. She left the bread alone to observe how the mold would grow. The next day, the area covered by the mold had doubled, and it doubled again the day after that.

1. If the doubling pattern continues, how many square millimeters will the mold cover 4 days after she noticed the mold? Show your thinking.

   16 mm^2; $2 \cdot 2 \cdot 2 \cdot 2 = 16$ or $2^4 = 16$.

2. Represent the relationship between the area A in square millimeters, covered by the mold and the number of days d since Clare first saw the mold using:

   a. A table of values, showing the area covered by the mold from the day Clare first saw it and over the next 5 days.

d	A
0	1
1	2
2	4
3	8
4	16
5	32

   b. A graph.

   c. An equation.

   $A = 1 \cdot 2^d$ or $A = 2^d$

1 Launch

Activate students' background knowledge by asking them if they have ever heard of the antibiotic *penicillin* or how it was discovered. Tell them that, thanks to a little mold, penicillin and other antibiotics have saved millions of lives.

2 Monitor

Help students get started by asking, "What does it mean to double a quantity?" The quantity is multiplied by 2.

Look for points of confusion:

- **Struggling to account for time when responding to Problem 1.** Have students complete the table in Problem 2 first, and then complete Problem 1.
- **Writing an incorrect equation in Problem 2c.** Model the amount of mold on Day 3 as $2 \cdot 2 \cdot 2 = 2^3$ and ask if students can see a connection between the number of days and the exponential expression.

Look for productive strategies:

- Using the table to determine the initial value and growth factor to help them write the equation in Problem 2c.
- Drawing a mapping diagram to show that the relationship is a function.

Activity 1 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students to describe what moldy bread might look like. Consider displaying a photo of mold growing on a slice of bread so that students can grasp the context of this activity.

Extension: *Interdisciplinary Connections*

Have students research how Scottish scientist Sir Alexander Fleming accidentally discovered how mold prevented the growth of a certain strain of bacteria. The mold was covered by a clear substance, later identified as *penicillin* — the first widely used antibiotic. **(Science)**

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that Clare noticed some mold on her slice of bread and left it alone to observe how the mold would grow.
- **Read 2:** Ask students to name or highlight given quantities and relationships, such as the area covered by the mold was 1 mm^2.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problem 1.

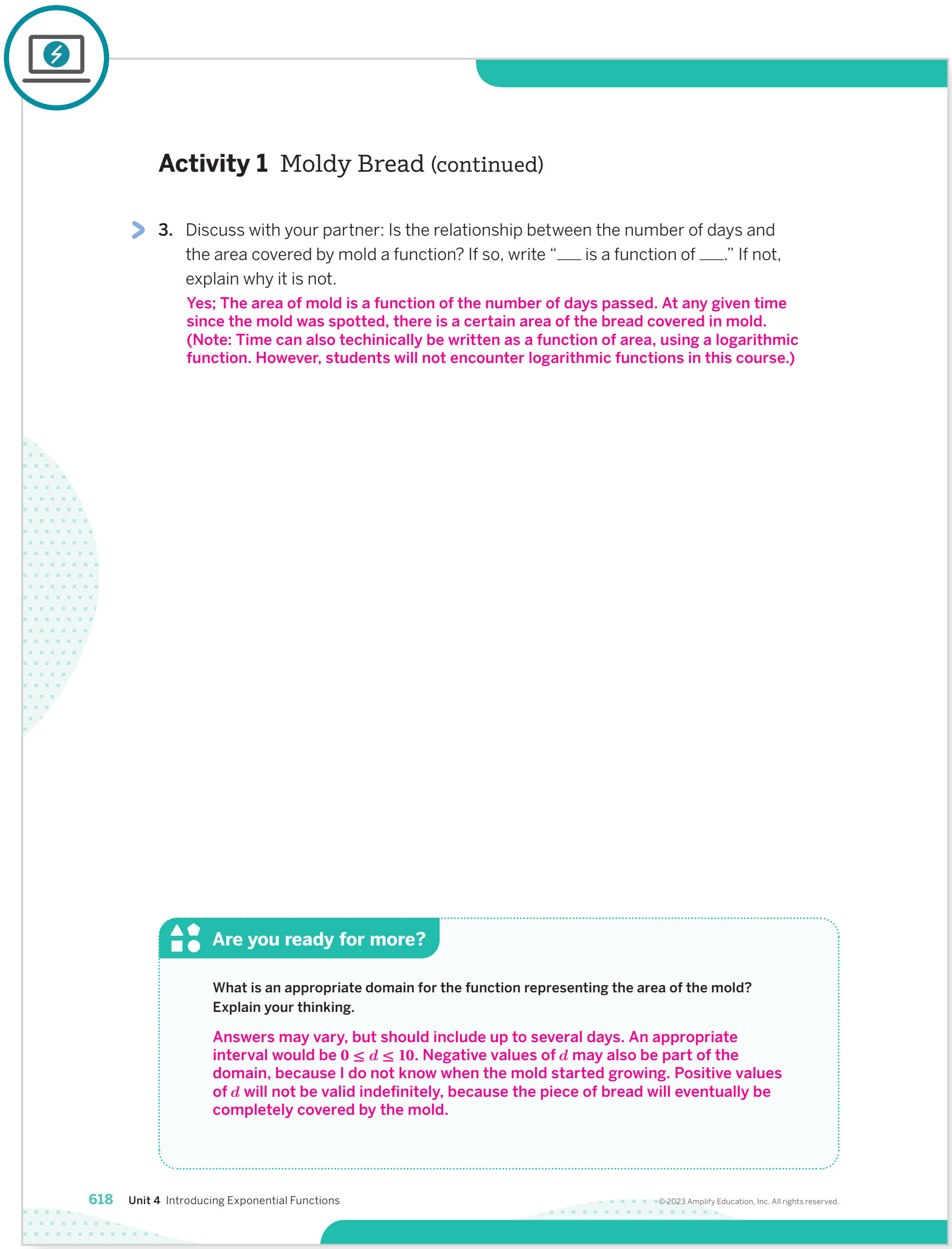
English Learners

Draw a diagram of a rectangular slice of bread and annotate where (and how large) the moldy section might be.

Independent | 20 min

Activity 1 Moldy Bread (continued)

Students use mathematics to model a real-world situation involving exponential growth and use function language to explain why the relationship is a function.



Activity 1 Moldy Bread (continued)

3. Discuss with your partner: Is the relationship between the number of days and the area covered by mold a function? If so, write "___ is a function of ___." If not, explain why it is not.

Yes; The area of mold is a function of the number of days passed. At any given time since the mold was spotted, there is a certain area of the bread covered in mold. (Note: Time can also techinically be written as a function of area, using a logarithmic function. However, students will not encounter logarithmic functions in this course.)

Are you ready for more?

What is an appropriate domain for the function representing the area of the mold? Explain your thinking.

Answers may vary, but should include up to several days. An appropriate interval would be $0 \leq d \leq 10$. Negative values of d may also be part of the domain, because I do not know when the mold started growing. Positive values of d will not be valid indefinitely, because the piece of bread will eventually be completely covered by the mold.

618 Unit 4 Introducing Exponential Functions

3 Connect

Have students share their approaches to writing their equation in Problem 2c.

Highlight that the exponential relationship shown is a function because there is only one output value (area) for each input value (time). Remind students that the output value represents the *dependent variable* and the input value represents the *independent variable*.

Ask:

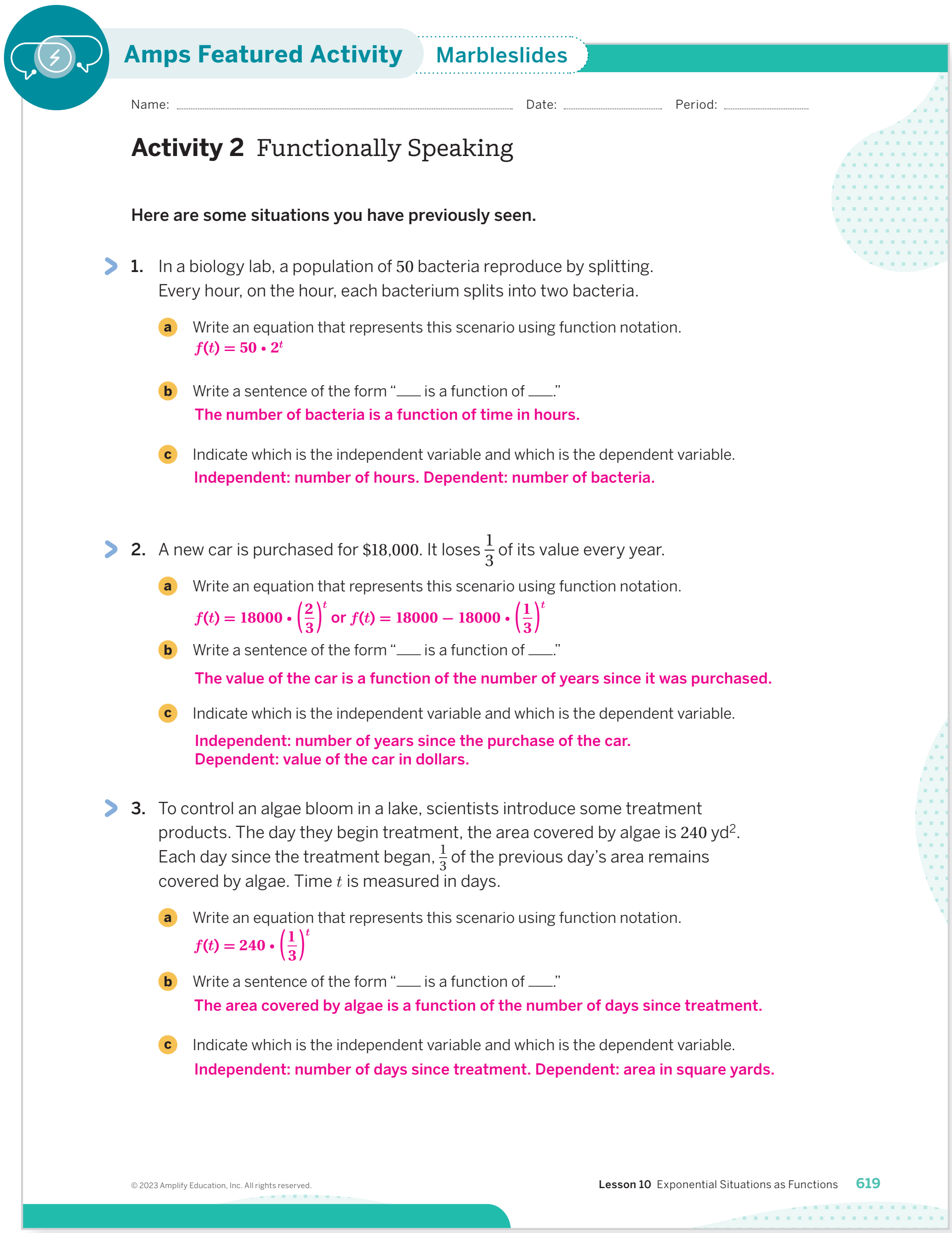
- "Did anyone write their equation without using the initial value? Why can this be done in this case?" Answers may vary. Call students' attention to the fact that the initial value is 1, which means that they can write the equation as $A = 1 \cdot 2^d$ or $A = 2^d$.
- "Will the mold keep growing indefinitely? Why or why not?" Sample response: No, at some point, the mold will overtake the slice of bread and reach a point where it cannot realistically grow anymore.

Independent | 15 min

Activity 2 Functionally Speaking

Students revisit prior contexts to view them as functions, construct functions to model the relationships, and use function notation and language.

Amps Featured Activity Marbleslides



Name: ____________ Date: ________ Period: ________

Activity 2 Functionally Speaking

Here are some situations you have previously seen.

1. In a biology lab, a population of 50 bacteria reproduce by splitting. Every hour, on the hour, each bacterium splits into two bacteria.
   - **a** Write an equation that represents this scenario using function notation.
     $f(t) = 50 \cdot 2^t$
   - **b** Write a sentence of the form "____ is a function of ____."
     The number of bacteria is a function of time in hours.
   - **c** Indicate which is the independent variable and which is the dependent variable.
     Independent: number of hours. Dependent: number of bacteria.

2. A new car is purchased for $\$18{,}000$. It loses $\frac{1}{3}$ of its value every year.
   - **a** Write an equation that represents this scenario using function notation.
     $f(t) = 18000 \cdot \left(\frac{2}{3}\right)^t$ or $f(t) = 18000 - 18000 \cdot \left(\frac{1}{3}\right)^t$
   - **b** Write a sentence of the form "____ is a function of ____."
     The value of the car is a function of the number of years since it was purchased.
   - **c** Indicate which is the independent variable and which is the dependent variable.
     Independent: number of years since the purchase of the car.
     Dependent: value of the car in dollars.

3. To control an algae bloom in a lake, scientists introduce some treatment products. The day they begin treatment, the area covered by algae is 240 yd^2. Each day since the treatment began, $\frac{1}{3}$ of the previous day's area remains covered by algae. Time t is measured in days.
   - **a** Write an equation that represents this scenario using function notation.
     $f(t) = 240 \cdot \left(\frac{1}{3}\right)^t$
   - **b** Write a sentence of the form "____ is a function of ____."
     The area covered by algae is a function of the number of days since treatment.
   - **c** Indicate which is the independent variable and which is the dependent variable.
     Independent: number of days since treatment. Dependent: area in square yards.

Lesson 10 Exponential Situations as Functions 619

1 Launch

Ask students to read the three scenarios. Highlight prior connections. Solicit ideas on why each scenario can be viewed as a function.

2 Monitor

Help students get started by emphasizing function language. Say, "The bacteria growth depends on the time, so the bacteria growth is a function of time."

Look for points of confusion:

- **Confusing *independent* and *dependent*.** Help students by asking them which variable depends on the other.
- **Confusing the initial value with the base in the equation.** Say, "The initial value is multiplied by the exponential growth/decay factor, while the exponential growth/decay factor is the base of the exponent."

Look for productive strategies:

- Using the table in Problem 2 to draw a mapping diagram.

3 Connect

Have students share how they constructed their equations for each problem.

Highlight that exponential functions are similar to linear functions because they are both functions, but different because exponential functions have a constant value that is raised to a variable.

Define *exponential functions* as functions that describe exponential change, whether growth or decay.

Ask, "How does the context of a function help create a relationship in function notation?" Use the context to choose the letter representing the independent variable. The dependent variable is represented by f(independent variable).

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can use Marbleslides to check the accuracy of their function and adjust the domain and range to a more reasonable scale.

Accessibility: *Activate Prior Knowledge*

Display the terms *independent variable* and *dependent variable*. Ask students to use these terms to complete these sentences:

- "The ______ depends on the ______."
- "The ______ is a function of the ______."

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their equations, draw connections between the term *exponential function* and the structure of each function's equation. Ask:

- "In each equation, where do you see the initial value? The growth or decay factor?"
- "Which equation(s) have a growth factor? Decay factor? How can you tell?"
- "Why is the base in the equation in Problem 2 $\frac{2}{3}$ and not $\frac{1}{3}$? Why is the base in the equation in Problem 3 $\frac{1}{3}$ and not $\frac{2}{3}$?"
- "Why do you think these functions are called exponential functions?"

English Learners

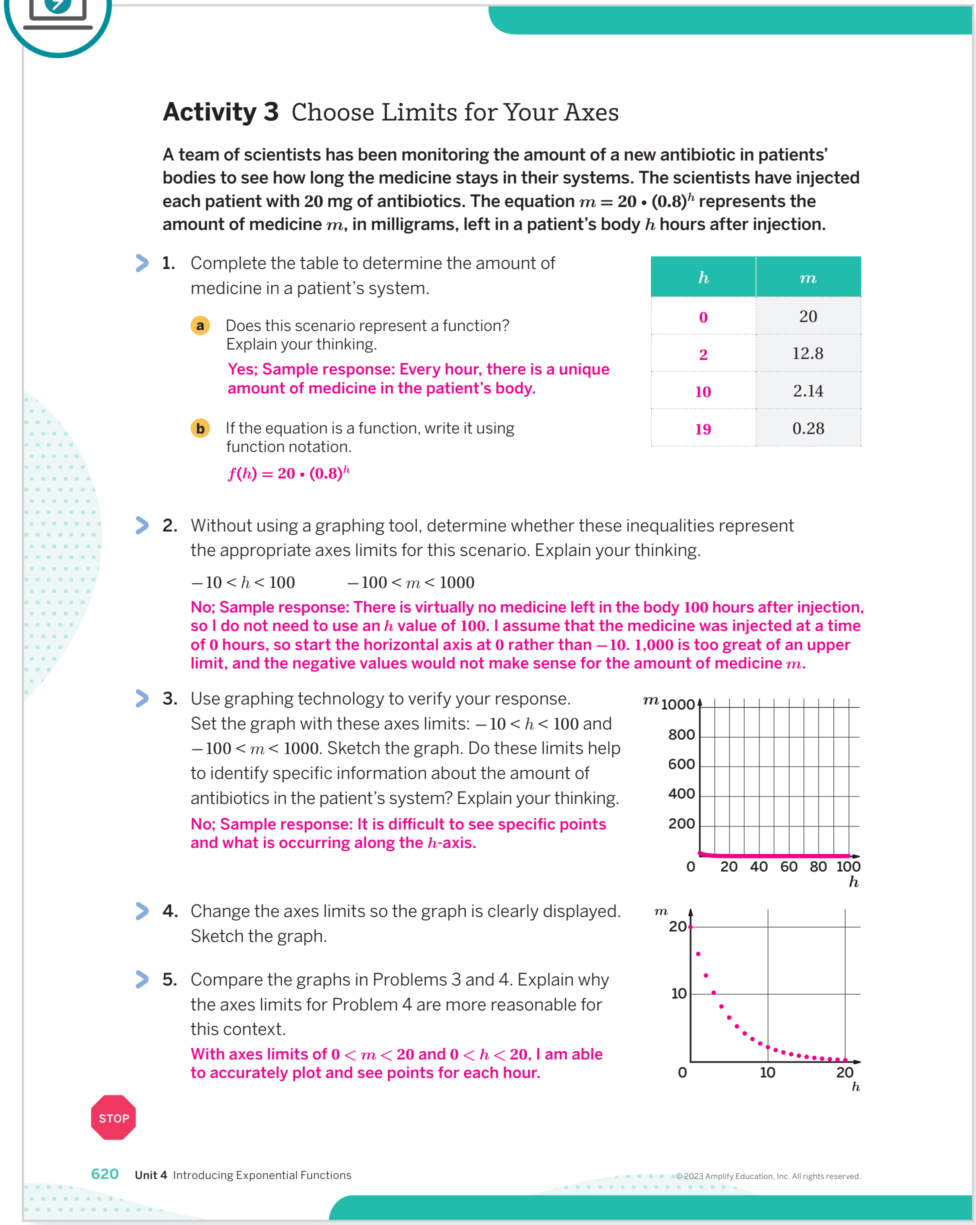
Highlight key words in the text, such as "loses $\frac{1}{3}$" and "$\frac{1}{3}$ . . . remains."

Optional

Whole Class | 20 min

Activity 3 Choose Limits for Your Axes

Students gauge the reasonableness of a graphing window given an equation and a description of a function.



Activity 3 Choose Limits for Your Axes

A team of scientists has been monitoring the amount of a new antibiotic in patients' bodies to see how long the medicine stays in their systems. The scientists have injected each patient with 20 mg of antibiotics. The equation $m = 20 \cdot (0.8)^h$ represents the amount of medicine m, in milligrams, left in a patient's body h hours after injection.

1. Complete the table to determine the amount of medicine in a patient's system.

h	m
0	20
2	12.8
10	2.14
19	0.28

a. Does this scenario represent a function? Explain your thinking.

Yes; Sample response: Every hour, there is a unique amount of medicine in the patient's body.

b. If the equation is a function, write it using function notation.

$f(h) = 20 \cdot (0.8)^h$

2. Without using a graphing tool, determine whether these inequalities represent the appropriate axes limits for this scenario. Explain your thinking.

$-10 < h < 100$ $-100 < m < 1000$

No; Sample response: There is virtually no medicine left in the body 100 hours after injection, so I do not need to use an h value of 100. I assume that the medicine was injected at a time of 0 hours, so start the horizontal axis at 0 rather than -10. 1,000 is too great of an upper limit, and the negative values would not make sense for the amount of medicine m.

3. Use graphing technology to verify your response. Set the graph with these axes limits: $-10 < h < 100$ and $-100 < m < 1000$. Sketch the graph. Do these limits help to identify specific information about the amount of antibiotics in the patient's system? Explain your thinking.

No; Sample response: It is difficult to see specific points and what is occurring along the h-axis.

4. Change the axes limits so the graph is clearly displayed. Sketch the graph.

5. Compare the graphs in Problems 3 and 4. Explain why the axes limits for Problem 4 are more reasonable for this context.

With axes limits of $0 < m < 20$ and $0 < h < 20$, I am able to accurately plot and see points for each hour.

STOP

620 Unit 4 Introducing Exponential Functions

1 Launch

Provide access to graphing technology. Tell students they will explore how to choose limits for the axes when graphing.

2 Monitor

Help students get started by having them use the given equation to complete the table.

Look for points of confusion:

- **Having difficulty determining reasonable axes limits.** Have students connect the table to the scenario to reason about an appropriate interval for the domain and range.

Look for productive strategies:

- Selecting reasonable axes limits by creating a table of values.

3 Connect

Display the equation and the completed table.

Have students share their graphs in Problems 3 and 4. Select students to share how they chose their axes limits to clearly display the graph in Problem 4.

Highlight that zooming in and out of the window may not be helpful. It can be more helpful and efficient for students to set the window themselves.

Ask, "How can the context help you choose an appropriate graphing window?" I can think about the reasonable values I want for the domain and range.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Depending on the type of graphing technology your students use, consider preparing a sheet with step-by-step instructions for how to change the axes limits of a graph.

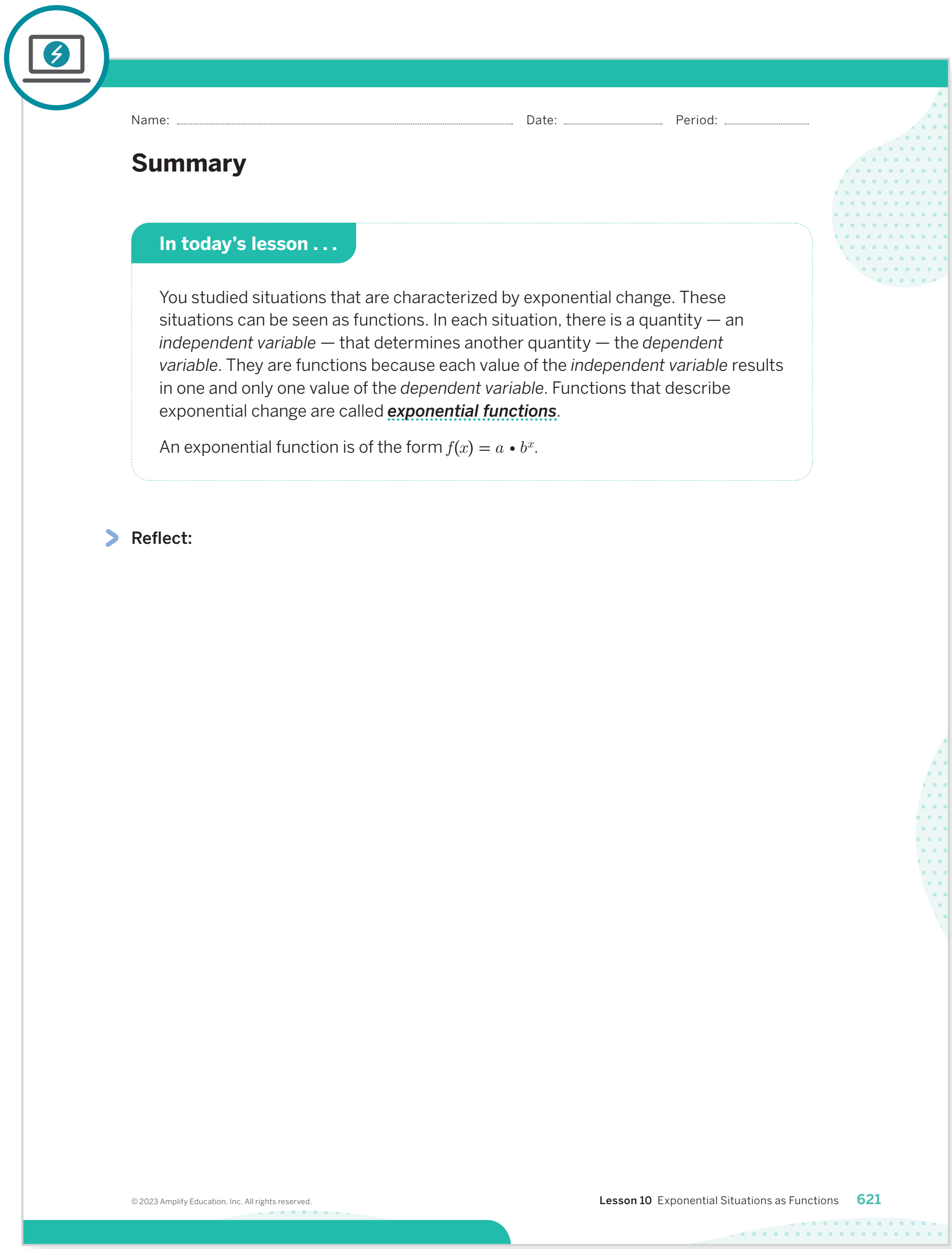
Accessibility: *Vary Demands to Optimize Challenge*

Consider providing students with the table and equation in Problem 1 and have them begin the activity with Problem 2. This will allow them more time to focus on analyzing appropriate axes limits with which to view the function.

Whole Class | 5 min

Summary

Review and synthesize why an exponential relationship is a function and summarize the process of converting an equation to function notation.



Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You studied situations that are characterized by exponential change. These situations can be seen as functions. In each situation, there is a quantity — an *independent variable* — that determines another quantity — the *dependent variable*. They are functions because each value of the *independent variable* results in one and only one value of the *dependent variable*. Functions that describe exponential change are called ***exponential functions***.

An exponential function is of the form $f(x) = a \cdot b^x$.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Exponential Situations as Functions 621

Synthesize

Display the equation $p = 1000 \cdot 2^t$ and its graph. Say, "Consider a bacteria population p, described by the equation $p = 1000 \cdot 2^t$, where t is the number of hours after it is first measured."

Ask, "Is this relationship a function?" Yes; Every hour has exactly one size of bacteria population.

Highlight that this is an exponential relationship because the exponent is a variable. Both linear and exponential functions have independent and dependent variables and a rate of change. The rate of change is *constant* for linear functions, while it is a *multiplier* for exponential functions. An exponential function is of the form $f(x) = a \cdot b^x$.

Formalize vocabulary: *exponential function*

Ask:

- "Which variable is the independent variable? The dependent variable?" The number of hours is the independent variable. The bacteria population is the dependent variable.
- "How do you convert this equation to function notation?" Sample response: Replace the variable p with $f(t)$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you describe in your own words how the rate of change is different between linear and exponential functions?"
- "Why is the rate of change for a linear function considered *constant*, but not for an exponential function?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *exponential functions* that were added to the display during the lesson.

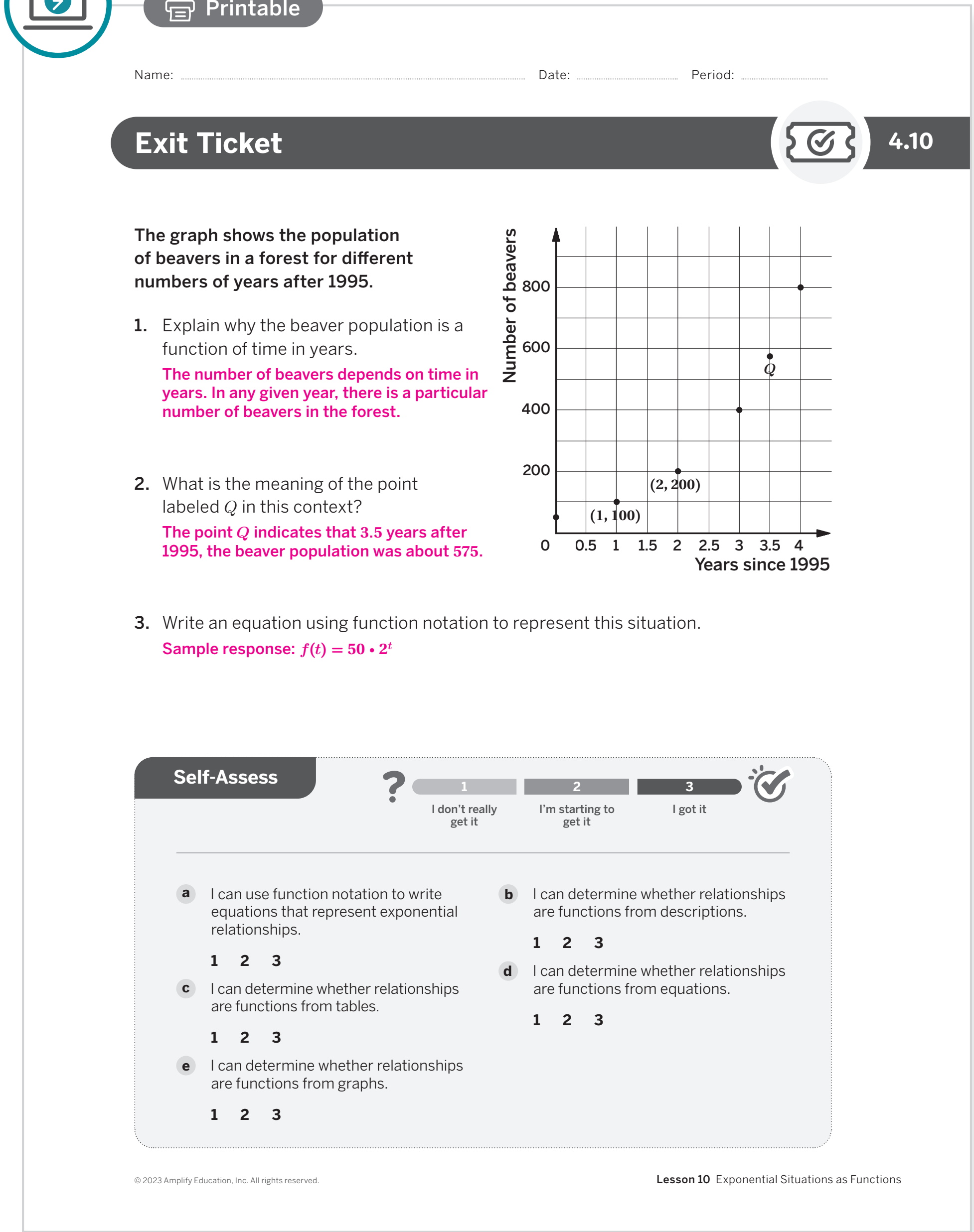
Add visual examples of exponential graphs (one growth and one decay) and annotate them with key terms, such as *initial value* and *growth/decay factor*. Add corresponding equations that represent each graph and annotate the equations with these terms.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by explaining why a relationship is a function and constructing a function to model a given context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.10

The graph shows the population of beavers in a forest for different numbers of years after 1995.

1. Explain why the beaver population is a function of time in years.
   The number of beavers depends on time in years. In any given year, there is a particular number of beavers in the forest.

2. What is the meaning of the point labeled Q in this context?
   The point Q indicates that 3.5 years after 1995, the beaver population was about 575.

3. Write an equation using function notation to represent this situation.
   Sample response: $f(t) = 50 \cdot 2^t$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use function notation to write equations that represent exponential relationships.
   1 2 3

b. I can determine whether relationships are functions from descriptions.
   1 2 3

c. I can determine whether relationships are functions from tables.
   1 2 3

d. I can determine whether relationships are functions from equations.
   1 2 3

e. I can determine whether relationships are functions from graphs.
   1 2 3

Lesson 10 Exponential Situations as Functions

Success looks like . . .

- **Goal:** Using function notation to write equations that represent exponential relationships.
  - » Writing an equation given the situation in Problem 3.
- **Goal:** Determining whether relationships — represented in descriptions, tables, equations, or graphs — are functions.

Suggested next steps

If students provide an unclear response in Problem 1, consider:

- Reviewing the definitions of independent and dependent variables and identifying each.
- Having students check to see if, for every independent input is there only one dependent output. If so, the relationship is a function.
- Reviewing Activity 1, Problem 3.

If students give an unclear or invalid meaning for Problem 2, consider:

- Reviewing how to identify coordinates of points, and using axes labels to identify what each value means in context.
- Reviewing Activity 1.

If students write an incorrect equation for Problem 3, consider:

- Prompting them to determine initial values and multipliers to help construct the equation.
- Assigning Practice Problems 2 and 3.

Professional Learning

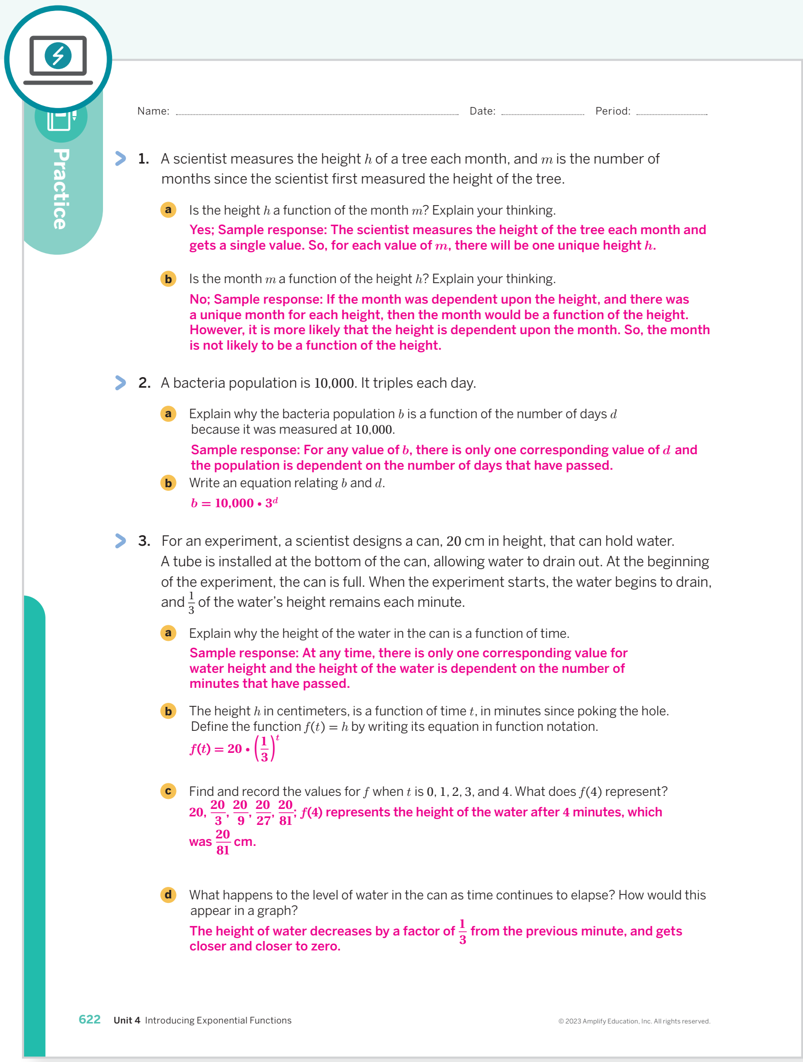
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students recognized exponential growth and decay as exponential functions. How did that build on earlier understandings of functions from a prior grade or unit?
- Which students or groups of students did and did not have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. A scientist measures the height h of a tree each month, and m is the number of months since the scientist first measured the height of the tree.

   a. Is the height h a function of the month m? Explain your thinking.

   Yes; Sample response: The scientist measures the height of the tree each month and gets a single value. So, for each value of m, there will be one unique height h.

   b. Is the month m a function of the height h? Explain your thinking.

   No; Sample response: If the month was dependent upon the height, and there was a unique month for each height, then the month would be a function of the height. However, it is more likely that the height is dependent upon the month. So, the month is not likely to be a function of the height.

2. A bacteria population is $10{,}000$. It triples each day.

   a. Explain why the bacteria population b is a function of the number of days d because it was measured at $10{,}000$.

   Sample response: For any value of b, there is only one corresponding value of d and the population is dependent on the number of days that have passed.

   b. Write an equation relating b and d.

   $b = 10{,}000 \cdot 3^d$

3. For an experiment, a scientist designs a can, 20 cm in height, that can hold water. A tube is installed at the bottom of the can, allowing water to drain out. At the beginning of the experiment, the can is full. When the experiment starts, the water begins to drain, and $\frac{1}{3}$ of the water's height remains each minute.

   a. Explain why the height of the water in the can is a function of time.

   Sample response: At any time, there is only one corresponding value for water height and the height of the water is dependent on the number of minutes that have passed.

   b. The height h in centimeters, is a function of time t, in minutes since poking the hole. Define the function $f(t) = h$ by writing its equation in function notation.

   $f(t) = 20 \cdot \left(\frac{1}{3}\right)^t$

   c. Find and record the values for f when t is $0, 1, 2, 3,$ and 4. What does $f(4)$ represent?

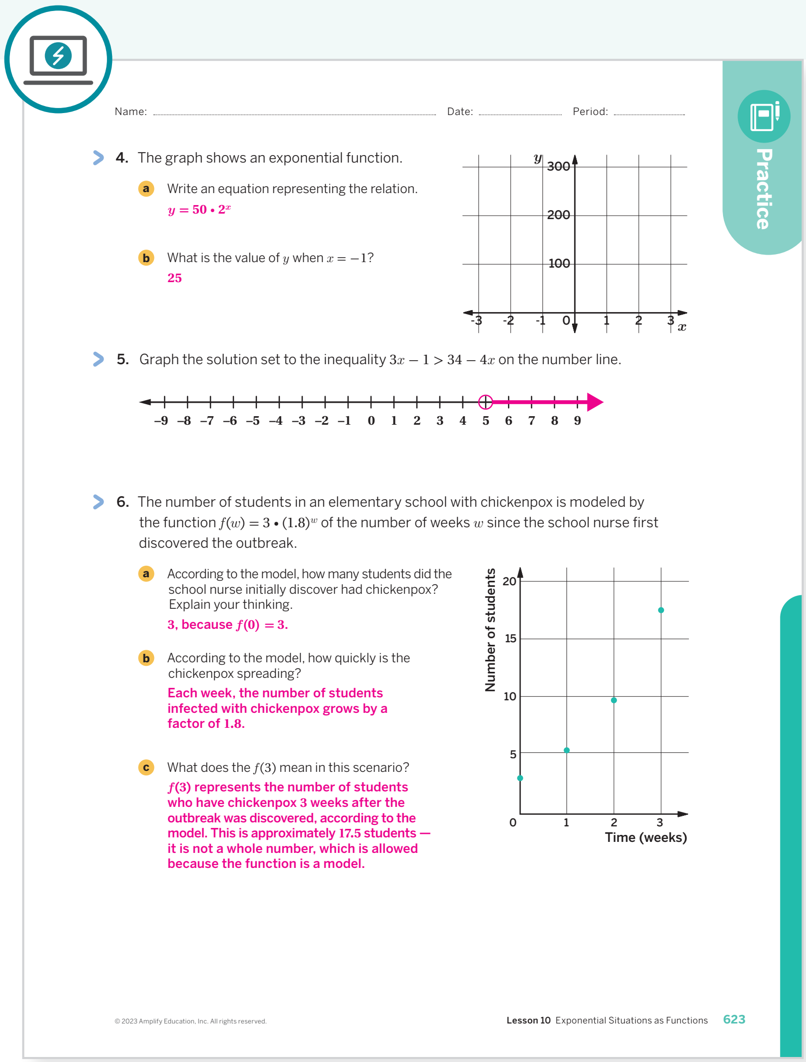
   $20, \frac{20}{3}, \frac{20}{9}, \frac{20}{27}, \frac{20}{81}$; $f(4)$ represents the height of the water after 4 minutes, which was $\frac{20}{81}$ cm.

   d. What happens to the level of water in the can as time continues to elapse? How would this appear in a graph?

   The height of water decreases by a factor of $\frac{1}{3}$ from the previous minute, and gets closer and closer to zero.

622 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ______ Date: ______ Period: ______

4. The graph shows an exponential function.

   a. Write an equation representing the relation.

   $y = 50 \cdot 2^x$

   b. What is the value of y when $x = -1$?

   25

5. Graph the solution set to the inequality $3x - 1 > 34 - 4x$ on the number line.

6. The number of students in an elementary school with chickenpox is modeled by the function $f(w) = 3 \cdot (1.8)^w$ of the number of weeks w since the school nurse first discovered the outbreak.

   a. According to the model, how many students did the school nurse initially discover had chickenpox? Explain your thinking.

   3, because $f(0) = 3$.

   b. According to the model, how quickly is the chickenpox spreading?

   Each week, the number of students infected with chickenpox grows by a factor of 1.8.

   c. What does the $f(3)$ mean in this scenario?

   $f(3)$ represents the number of students who have chickenpox 3 weeks after the outbreak was discovered, according to the model. This is approximately 17.5 students — it is not a whole number, which is allowed because the function is a model.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Exponential Situations as Functions 623

Practice Problem Analysis

Type	Problem	Activity	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 9	1
	5	Unit 1 Lesson 13	1
Formative ⚡	6	Unit 4 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 11**

Interpreting Exponential Functions

Let's find some meaningful ways to represent exponential functions.

Focus

Goals

1. Determine whether a graph that represents a situation should be continuous or discrete.
2. Interpret graphs of exponential functions and equations written in function notation to respond to problems in context.
3. Use function notation to describe an exponential relationship represented by a graph.
4. Use graphing technology to graph exponential functions and analyze their domains.

Coherence

Today

Students continue to write and produce exponential functions and their graphs in context. Given a relationship, they will write one quantity as a function of another, determine reasonable domains, and apply the function to the context.

Previously

Students identified exponential functions from relationships in context, chose independent and dependent variables, and expressed relationships using function notation.

Coming Soon

Students will use real-world data to model relationships using exponential functions and use the model to respond to problems in context.

Rigor

- Students develop their **conceptual understanding**, connecting exponential behavior with exponential functions, which have corresponding independent variables and restricted domains.
- Students continue developing their **procedural fluency** in working with exponentials through expressions, tables, and graphs.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one set per pair
- blank paper, one per student
- graphing technology

Math Language Development

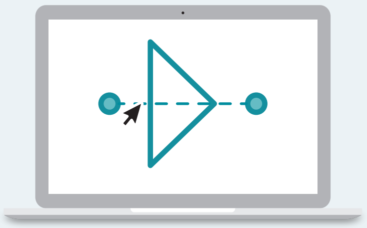
Review words

- *exponential function*

Amps | Featured Activity

Activity 2
Paper Folding

Students can digitally record and graph their findings as they fold a piece of paper as many times as they can.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students use paper folding to establish some fundamental understanding of the exponential functions and they might get lost in the task and forget to consider the mathematics that it represents. Students should be aware of both their strengths and weaknesses as they work with a partner to make some assumptions based on what they see in their models. Explain that it is ok if their results need to be modified later as long as they can explain how they drew their conclusions.

Modifications to Pacing

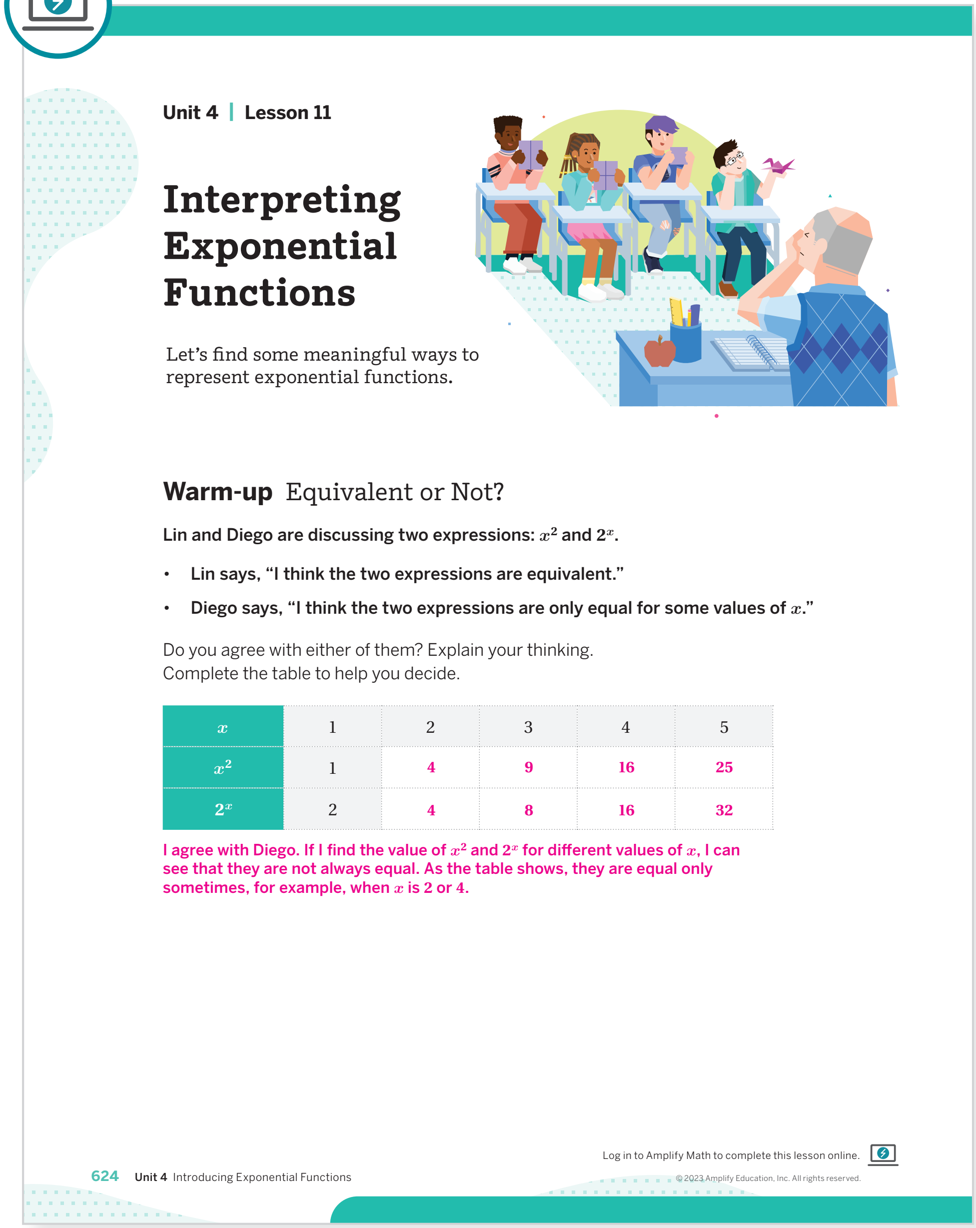
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, omit Problem 1 or Problem 2.
- In **Activity 2**, demonstrate the paper folding at the beginning, rather than have students fold their own paper.

Independent | 5 min

Warm-up Equivalent or Not?

Students compare and contrast the expressions 2^x and x^2 by critiquing student claims to address a common misconception of equivalence.



Unit 4 | **Lesson 11**

Interpreting Exponential Functions

Let's find some meaningful ways to represent exponential functions.

Warm-up Equivalent or Not?

Lin and Diego are discussing two expressions: x^2 and 2^x.

- **Lin says, "I think the two expressions are equivalent."**
- **Diego says, "I think the two expressions are only equal for some values of x."**

Do you agree with either of them? Explain your thinking.
Complete the table to help you decide.

x	1	2	3	4	5
x^2	1	**4**	**9**	**16**	**25**
2^x	2	**4**	**8**	**16**	**32**

I agree with Diego. If I find the value of x^2 and 2^x for different values of x, I can see that they are not always equal. As the table shows, they are equal only sometimes, for example, when x is 2 or 4.

Log in to Amplify Math to complete this lesson online.

624 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Before beginning, ask students what Lin means by *equivalent* to ensure they know *equivalent* means equal for any value of x.

2 Monitor

Help students get started by first completing the table using substitution, then comparing the values in each row.

Look for points of confusion:

- **Multiplying the base by the exponent.** Say, "The base is repeatedly multiplied the number of times indicated by the exponent."
- **Not understanding *equivalence*.** Review that *equivalent* means equal for any value of x.

Look for productive strategies:

- Noticing the expressions are equal only for certain values of x.
- Graphing and comparing each expression.

3 Connect

Display the completed table.

Have students share the approaches they used to determine whether the expressions are equivalent.

Highlight differences in the output values and growth factors.

Ask:

- "For what values of x are the two expressions equal?" 2 and 4
- "Besides substituting different values of x, are there other ways to determine whether the expressions are equal for all values of x?" If x is odd, x^2 will be odd, whereas 2^x is always an even number.
- "Which expression grows more quickly?" For positive values of x, 2^x grows more quickly than x^2.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Students learned about the difference between *equal* and *equivalent* in middle school. Activate their prior knowledge about these terms by reminding them that *equivalent expressions* will always be equal (have the same value) regardless of what value is substituted for the variable. Expressions that are equal to each other (but not equivalent) might only be equal for one particular value of the variable.

Power-up

To power up students' ability to analyze an exponential function in context, have students complete:

Recall that, for an exponential function $f(t) = a \cdot b^t$, a represents the initial value and b represents the growth (or decay) factor.

Elena deposits money into a bank account. Her account balance after t years can be modeled by the function $f(t) = 1000 \cdot 1.002^t$.

a How much money did Elena initially invest? $1,000

b What is the growth factor of the account? 1.002

c How much money will she have in her account after 1 years? $1,002

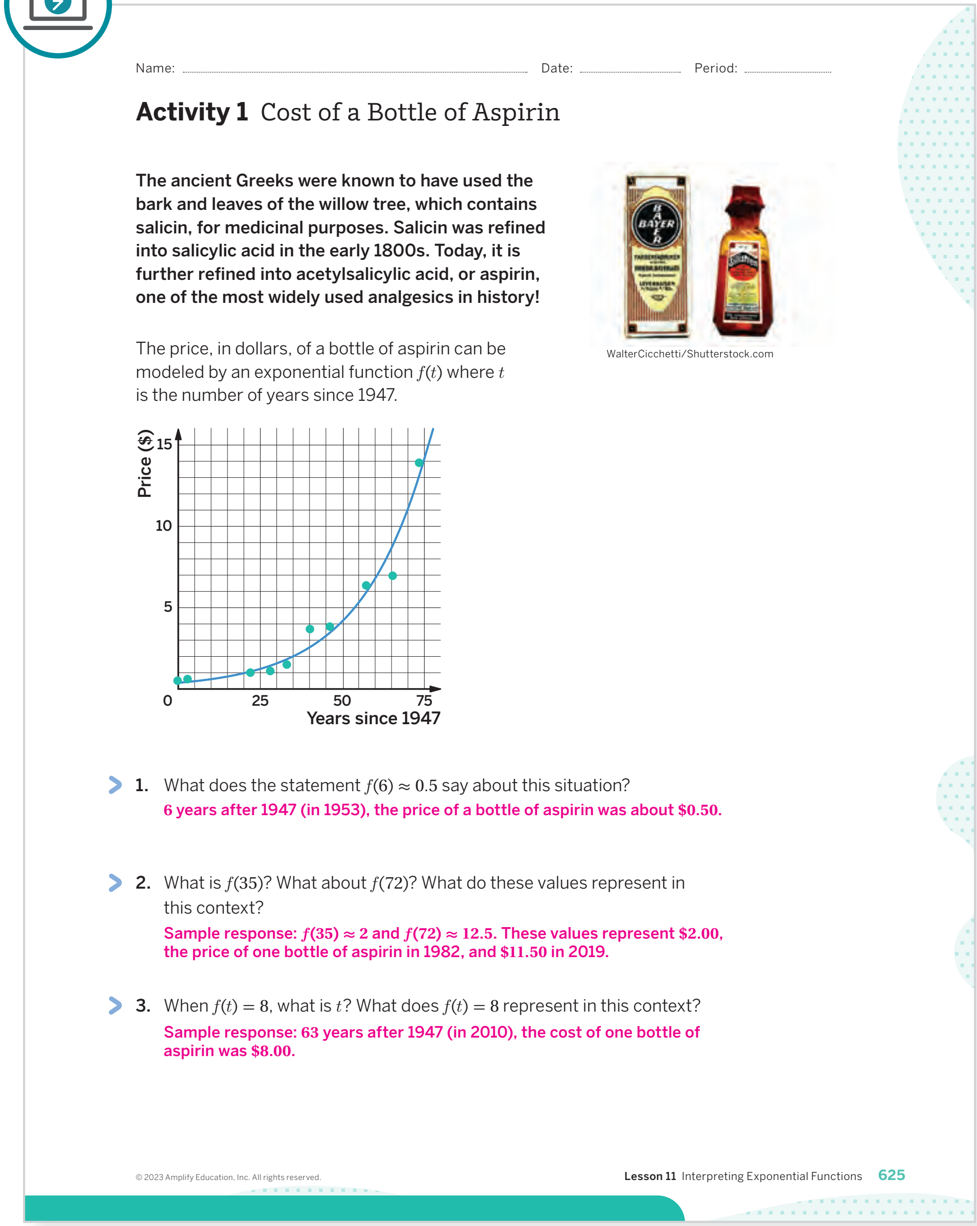
Use: Before Activity 1

Informed by: Performance on Lesson 10, Practice Problem 6

Independent | 10 min

Activity 1 Cost of a Bottle of Aspirin

Students analyze a graph of real-world data in context, so they can understand and interpret statements using function notation within context.



Name: ______ Date: ______ Period: ______

Activity 1 Cost of a Bottle of Aspirin

The ancient Greeks were known to have used the bark and leaves of the willow tree, which contains salicin, for medicinal purposes. Salicin was refined into salicylic acid in the early 1800s. Today, it is further refined into acetylsalicylic acid, or aspirin, one of the most widely used analgesics in history!

WalterCicchetti/Shutterstock.com

The price, in dollars, of a bottle of aspirin can be modeled by an exponential function $f(t)$ where t is the number of years since 1947.

1. What does the statement $f(6) \approx 0.5$ say about this situation?
   6 years after 1947 (in 1953), the price of a bottle of aspirin was about $0.50.

2. What is $f(35)$? What about $f(72)$? What do these values represent in this context?
   Sample response: $f(35) \approx 2$ and $f(72) \approx 12.5$. These values represent $2.00, the price of one bottle of aspirin in 1982, and $11.50 in 2019.

3. When $f(t) = 8$, what is t? What does $f(t) = 8$ represent in this context?
   Sample response: 63 years after 1947 (in 2010), the cost of one bottle of aspirin was $8.00.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Interpreting Exponential Functions 625

1 Launch

Display the graph and read the prompt aloud. Help students understand the relationship between the cost of aspirin and time. Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by modeling how to use the graph to estimate the cost of aspirin in 1947.

Look for points of confusion:

- **Struggling to read or understand function notation.** Using $f(t)$, remind them f is the name of the function and t is the input value.
- **Struggling to determine the year when given an output value.** Add 1947 and the number of years after 1947.

Look for productive strategies:

- Connecting input with domain and output with range on the graph.
- Using $f(\text{input}) = \text{output}$ strategy to identify specific points.

3 Connect

Display the graph.

Have students share their strategies for locating points on the graph and whether or not cost is a function of time.

Highlight the cost as a function of time. Real-world data is not exact and functions can be used to determine approximate fits for the data. Discuss why the graph is continuous.

Ask, "Is there a limit to how high the cost can rise?" Yes, cost is not infinite.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide a table of values of four ordered pairs from the graph.

Extension: *Math Enrichment*

Have students use graphing technology and ordered pairs from the graph to construct an exponential model for the data. Then have them use the same ordered pairs to perform linear regression and ask them to compare the two models. Students' responses may vary.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory text and graph. Ask students to work with one other student to write 2–3 mathematical questions they could ask about this scenario and graph. Sample questions shown.

- What was the price of aspirin in 1947?
- Why did the price increase so rapidly? Is this true for other medicines?
- What will happen to the price of aspirin in the future? Will it still follow this model?

English Learners

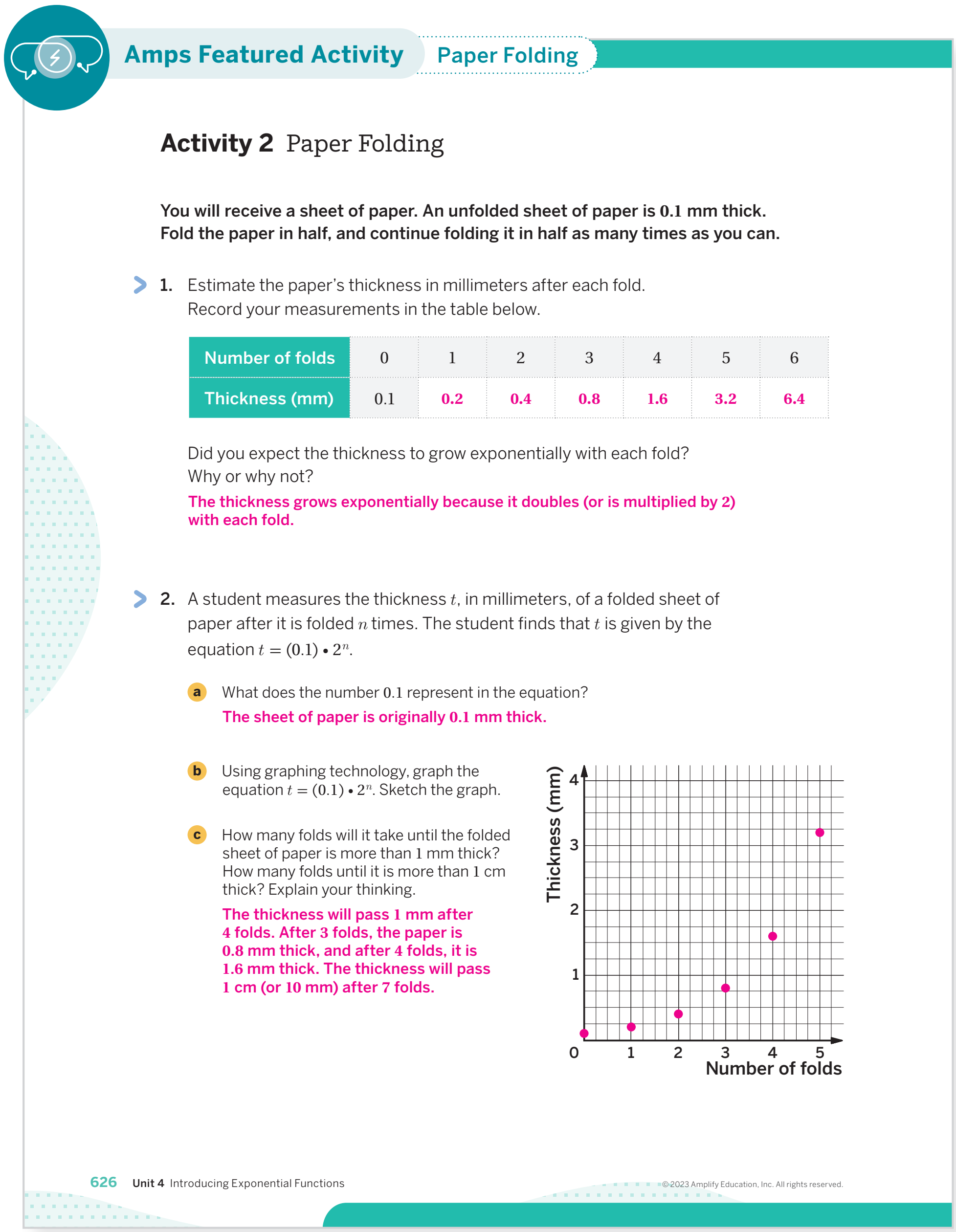
To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 15 min

Activity 2 Paper Folding

Students investigate relationships between paper folds and thickness, construct equations to model those relationships, and consider appropriate domains and ranges given the context.

Amps Featured Activity Paper Folding



Activity 2 Paper Folding

You will receive a sheet of paper. An unfolded sheet of paper is 0.1 mm thick. Fold the paper in half, and continue folding it in half as many times as you can.

1. Estimate the paper's thickness in millimeters after each fold. Record your measurements in the table below.

Number of folds	0	1	2	3	4	5	6
Thickness (mm)	0.1	**0.2**	**0.4**	**0.8**	**1.6**	**3.2**	**6.4**

Did you expect the thickness to grow exponentially with each fold? Why or why not?

The thickness grows exponentially because it doubles (or is multiplied by 2) with each fold.

2. A student measures the thickness t, in millimeters, of a folded sheet of paper after it is folded n times. The student finds that t is given by the equation $t = (0.1) \cdot 2^n$.

   a. What does the number 0.1 represent in the equation?

   The sheet of paper is originally 0.1 mm thick.

   b. Using graphing technology, graph the equation $t = (0.1) \cdot 2^n$. Sketch the graph.

   c. How many folds will it take until the folded sheet of paper is more than 1 mm thick? How many folds until it is more than 1 cm thick? Explain your thinking.

   The thickness will pass 1 mm after 4 folds. After 3 folds, the paper is 0.8 mm thick, and after 4 folds, it is 1.6 mm thick. The thickness will pass 1 cm (or 10 mm) after 7 folds.

626 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the instructions aloud and distribute one piece of blank paper to each student. Tell students they will investigate the relationship between the number of folds and thickness of the paper. Provide access to graphing technology.

2 Monitor

Help students get started by demonstrating how to fold a piece of paper in half several times. Have students complete Problem 1 with a partner before sharing their responses with the whole class.

Look for points of confusion:

- **Struggling to organize data points.** Have students annotate the table writing n by Number of folds and t by Thickness.
- **Having difficulty determining the growth factor.** Remind students the growth factor is repeated multiplication.
- **Connecting the points of each graph.** Ask, "Given the context, what does an x-value between 4 and 5 mean?"

Look for productive strategies:

- Estimating the thickness of the folded paper, by multiplying by 2 for each fold.
- In the equation $t = (0.1) \cdot 2^n$, identifying 0.1 as the thickness of a single sheet of paper and 2 as the common factor.
- Using a table or graph to determine the thickness of the paper after each fold.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can virtually fold a piece of paper and view different perspectives to estimate its thickness using a variety of on-screen objects. The equations they write will dynamically interact with their corresponding graphs.

Extension: *Math Enrichment*

Ask students whether it is possible to graph each equation from this activity on the same coordinate plane and explain their thinking. No, it is not possible because the area and thickness of the paper are measured in different units, so the dependent variables are actually different.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement for either or both of Problems 3d and 3e, such as, "The variable n can have negative values because it is a variable which can represent any number." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Can a variable represent any value? Does the context of the scenario matter?"
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "How do you know that your statement is true?"

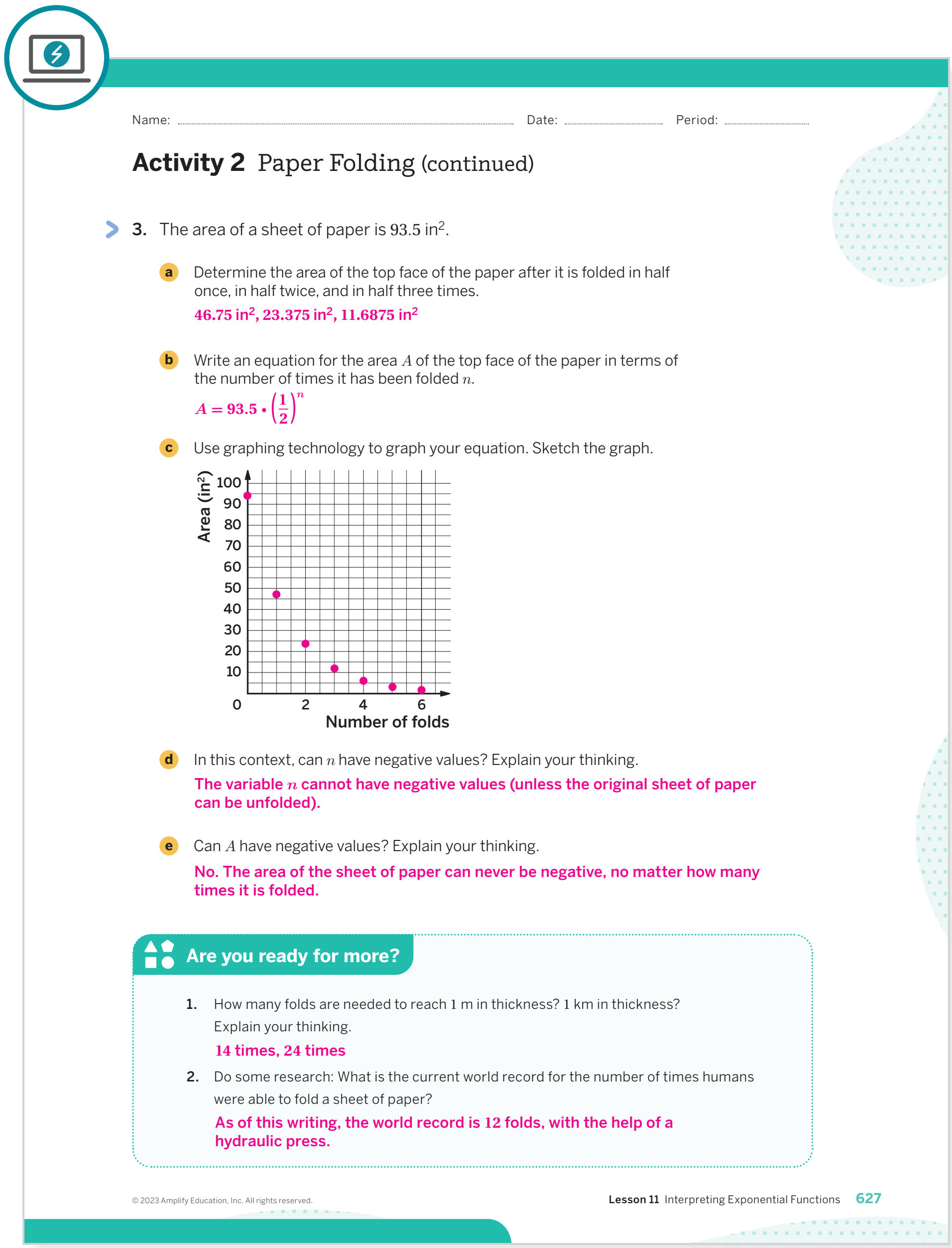
English Learners

Provide a sentence stem, such as, "The variable n can/cannot be negative because . . ."

Pairs | 15 min

Activity 2 Paper Folding (continued)

Students investigate relationships between paper folds and thickness, construct equations to model those relationships, and consider appropriate domains and ranges given the context.



Name: ________ Date: ________ Period: ________

Activity 2 Paper Folding (continued)

3. The area of a sheet of paper is 93.5 in^2.

a Determine the area of the top face of the paper after it is folded in half once, in half twice, and in half three times.

46.75 in^2, 23.375 in^2, 11.6875 in^2

b Write an equation for the area A of the top face of the paper in terms of the number of times it has been folded n.

$A = 93.5 \cdot \left(\frac{1}{2}\right)^n$

c Use graphing technology to graph your equation. Sketch the graph.

d In this context, can n have negative values? Explain your thinking.

The variable n cannot have negative values (unless the original sheet of paper can be unfolded).

e Can A have negative values? Explain your thinking.

No. The area of the sheet of paper can never be negative, no matter how many times it is folded.

Are you ready for more?

1. How many folds are needed to reach 1 m in thickness? 1 km in thickness? Explain your thinking.
   14 times, 24 times
2. Do some research: What is the current world record for the number of times humans were able to fold a sheet of paper?
   As of this writing, the world record is 12 folds, with the help of a hydraulic press.

Lesson 11 Interpreting Exponential Functions 627

3 Connect

Display the completed graphs from Problems 2b and 3c.

Have students share their maximum number of folds and thickness estimates. Select students that used productive strategies to determine the thickness or students who connected the points on the graph.

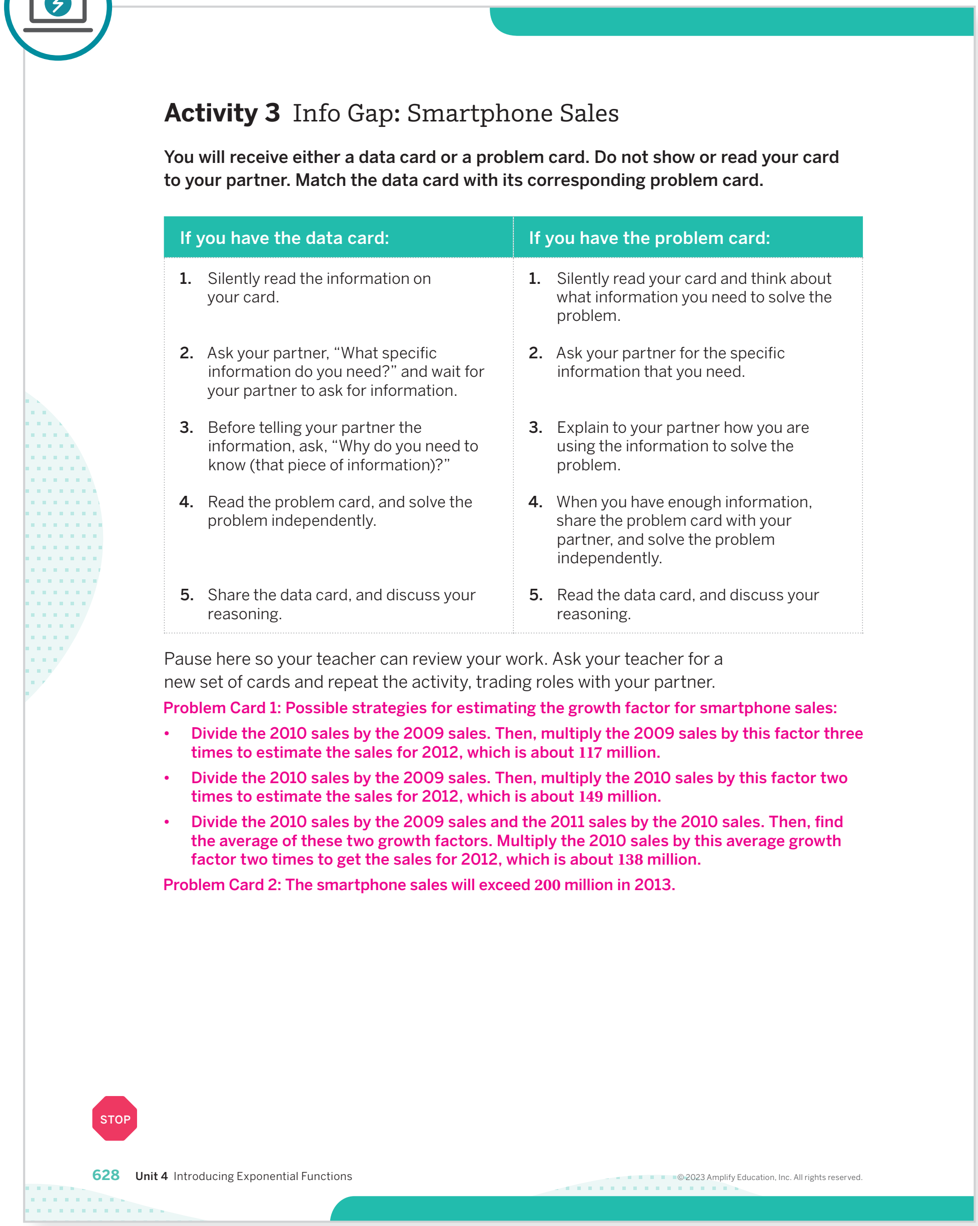
Highlight processes or approaches for identifying the growth factor, in a table or description, for determining the number of folds for paper thickness. Discuss meaningful values of the number of folds to its relationship with a discrete graph.

Ask, "What is the largest number of folds possible?" The world record is 12 folds.

Pairs | 10 min

Activity 3 Info Gap: Smartphone Sales

Students apply their knowledge about key characteristics of exponential functions to real-world data, and then estimate and use strategic thinking to select models.



Activity 3 Info Gap: Smartphone Sales

You will receive either a data card or a problem card. Do not show or read your card to your partner. Match the data card with its corresponding problem card.

If you have the data card:	If you have the problem card:
1. Silently read the information on your card.	**1.** Silently read your card and think about what information you need to solve the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information.	**2.** Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	**3.** Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	**4.** When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your reasoning.	**5.** Read the data card, and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Problem Card 1: Possible strategies for estimating the growth factor for smartphone sales:

- **Divide the 2010 sales by the 2009 sales. Then, multiply the 2009 sales by this factor three times to estimate the sales for 2012, which is about 117 million.**
- **Divide the 2010 sales by the 2009 sales. Then, multiply the 2010 sales by this factor two times to estimate the sales for 2012, which is about 149 million.**
- **Divide the 2010 sales by the 2009 sales and the 2011 sales by the 2010 sales. Then, find the average of these two growth factors. Multiply the 2010 sales by this average growth factor two times to get the sales for 2012, which is about 138 million.**

Problem Card 2: The smartphone sales will exceed 200 million in 2013.

STOP

628 **Unit 4** Introducing Exponential Functions

1 Launch

Explain the ***Info Gap*** instructional routine, and consider demonstrating the routine if students are unfamiliar with it. Distribute the pre-cut cards from the Activity 3 PDF to each student pair.

2 Monitor

Help students get started by modeling the questioning process.

Look for points of confusion:

- **Struggling to identify the data needed to determine the growth factor.** Have students determine the number of sales for consecutive years.
- **Struggling to determine the growth factor.** Use 2, or use sales or average growth factors from two consecutive years. Suggest using a table or graph to determine the growth factor.

Look for productive strategies:

- Noticing key words and values on the cards.
- Creating a graph/table depicting the number of smartphone sales as a function of time.

3 Connect

Have students share their strategies to determine the growth factor.

Highlight the limitations of a real-world model.

Ask:

- "According to your model, how many smartphones were sold worldwide in 2018?" About 5 billion
- "The world population in 2018 was about 7.6 billion. Is the number of smartphones sold based on your model realistic?" No, 66% of the world population buying one brand of smartphone isn't realistic.
- "Could the sales of smartphones continue to grow exponentially?" No, there is a finite number of people.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following questions during the think-aloud.

- "I am given the fact that sales grew exponentially from 2008 to 2011, but I need to know the number sold in 2012. I wonder if the sales continued to grow exponentially."
- "I think I want to know the sales in 2010 and 2011 to help me determine the growth factor."

Math Language Development

MLR4: Information Gap

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

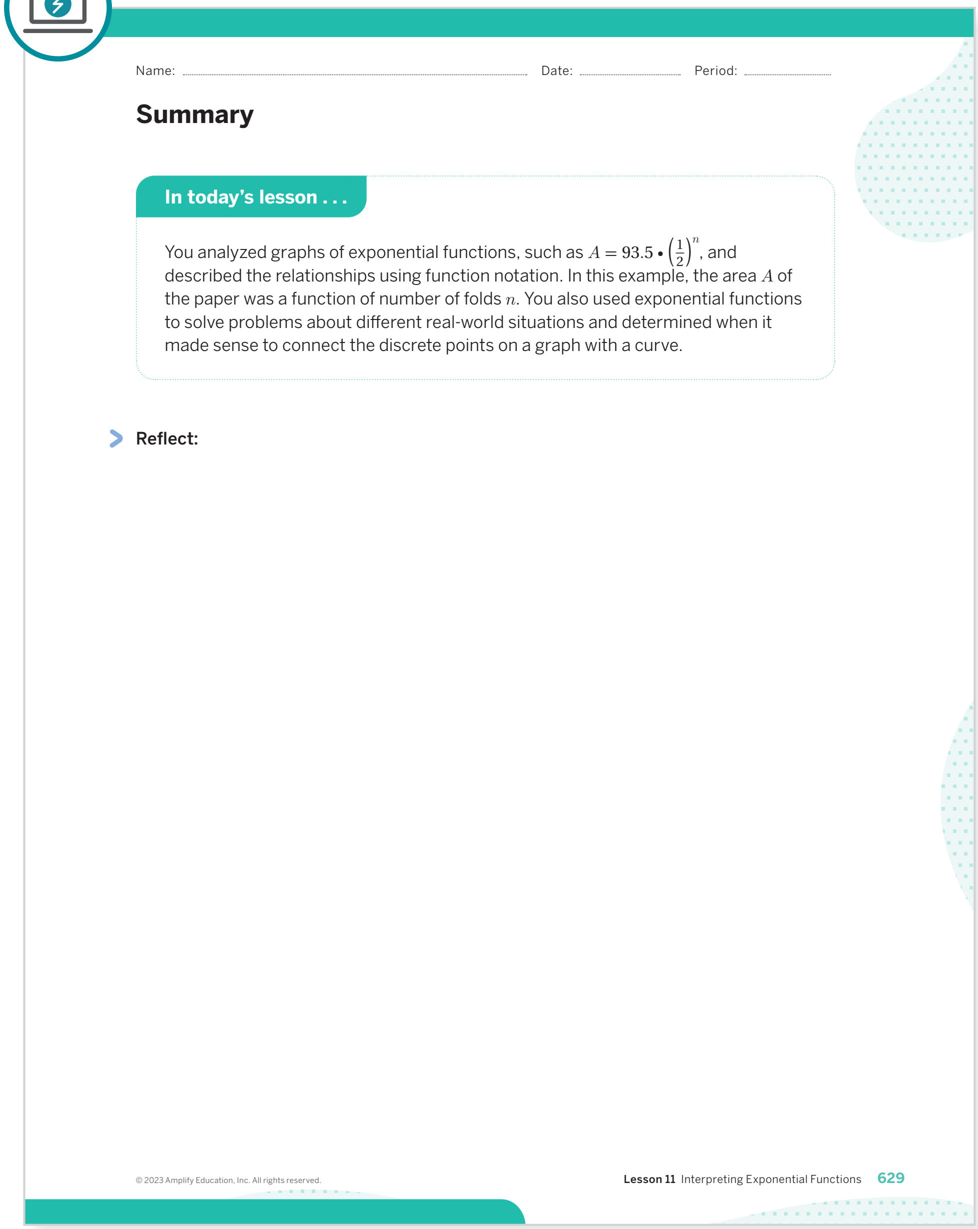
English Learners

Consider providing sample questions students could ask, such as the following for Problem Card 1:

- How many smartphones were sold in 2010?
- How many smartphones were sold in 2011?
- Did the sales continue to increase exponentially?

Summary

Review and synthesize graphs of exponential functions and how to describe the relationship using function notation.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You analyzed graphs of exponential functions, such as $A = 93.5 \cdot \left(\frac{1}{2}\right)^n$, and described the relationships using function notation. In this example, the area A of the paper was a function of number of folds n. You also used exponential functions to solve problems about different real-world situations and determined when it made sense to connect the discrete points on a graph with a curve.

Reflect:

Synthesize

Display the graph of algae area from Lesson 6, Activity 1.

Have students share their strategies for determining the growth factor of an exponential function when given a graph or table.

Ask, "What do $f(1) = 80$ and $f(2) = \frac{80}{3}$ mean in this context?" After 1 week of treatment, 80 yd^2 of algae remained and after 2 weeks of treatment, $\frac{80}{3}$ yd^2 of algae remained.

Highlight the correspondence of values in the domain and range on a graph, determining the growth factor from a graph and/or description, and describing contexts using exponential function notation.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

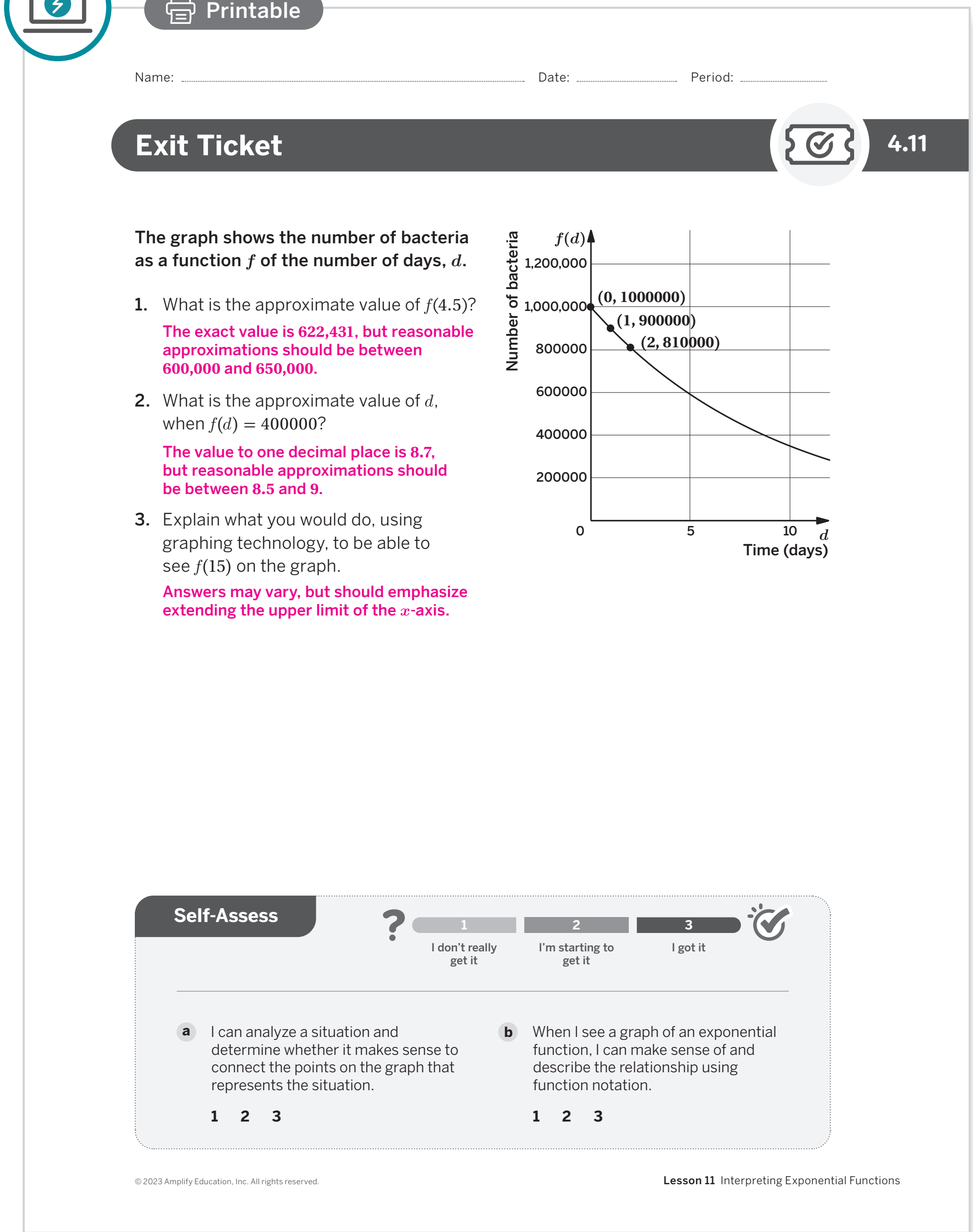
- "What characterizes exponential growth?"
- "What are real-world models of exponential growth?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing the graph of a bacteria population after an antibiotic is applied, interpreting function notation in context.

Printable



Name: Date: Period:

Exit Ticket 4.11

The graph shows the number of bacteria as a function f of the number of days, d.

1. What is the approximate value of $f(4.5)$?
   The exact value is 622,431, but reasonable approximations should be between 600,000 and 650,000.
2. What is the approximate value of d, when $f(d) = 400000$?
   The value to one decimal place is 8.7, but reasonable approximations should be between 8.5 and 9.
3. Explain what you would do, using graphing technology, to be able to see $f(15)$ on the graph.
   Answers may vary, but should emphasize extending the upper limit of the x-axis.

Self-Assess

? 1 I don't really get it 2 I'm starting to get it 3 I got it

a. I can analyze a situation and determine whether it makes sense to connect the points on the graph that represents the situation.
1 2 3

b. When I see a graph of an exponential function, I can make sense of and describe the relationship using function notation.
1 2 3

Lesson 11 Interpreting Exponential Functions

Success looks like . . .

- **Goal:** Determining whether a graph that represents a situation should be continuous or discrete.
- **Goal:** Interpreting graphs of exponential functions and equations written in function notation to respond to problems in context.
  - » Interpreting the value of $f(4.5)$ within context for Problem 1.
- **Goal:** Using function notation to describe an exponential relationship represented by a graph.
  - » Interpreting the function notation statement in Problem 2 to determine the approximate value of d.
- **Goal:** Using graphing technology to graph exponential functions and analyze their domains.

Suggested next steps

If students do not correctly identify the corresponding input value and output value in Problems 1 and 2, consider:

- Reviewing function notation in Activities 1 and 2.
- Assigning Practice Problems 2 and 3.

If students do not select the appropriate axes limits in Problem 3, consider:

- Reviewing function notation in Activities 1 and 2.
- Reviewing strategies for choosing appropriate axes limits.
- Assigning Practice Problem 5.

Professional Learning

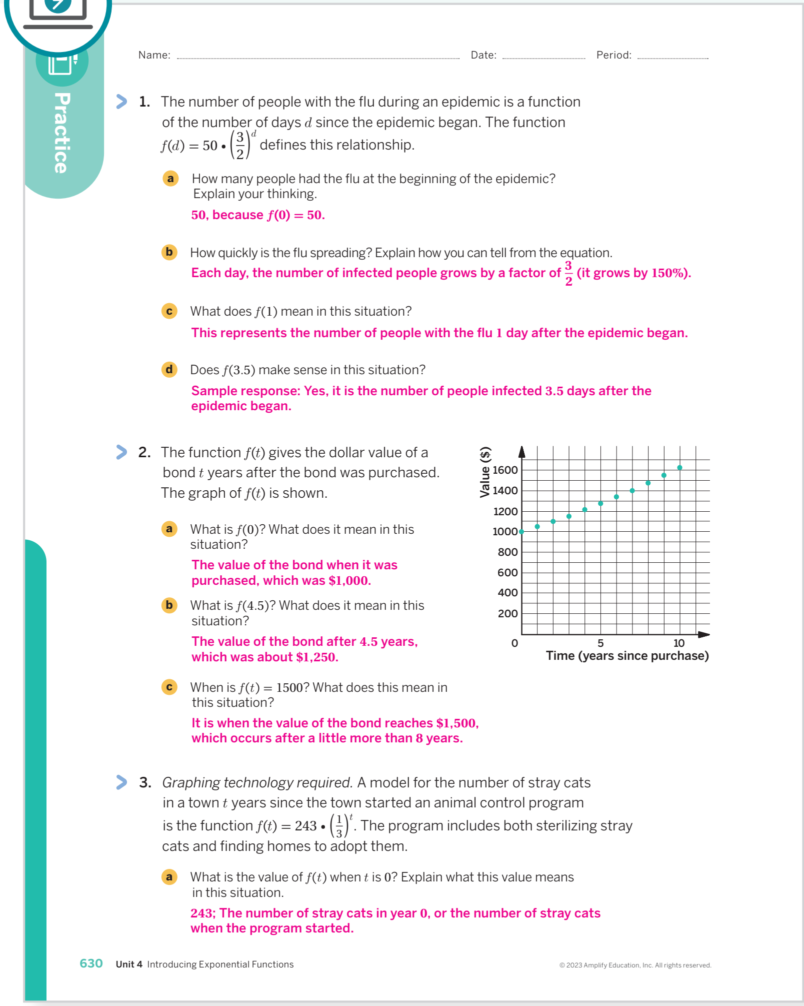
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What challenges did students encounter as they explored reasonable domains for exponential functions in this lesson? How did they work through them? What teacher actions did you use and would you use those again?
- In what ways did Activity 2 go as planned, or not go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. The number of people with the flu during an epidemic is a function of the number of days d since the epidemic began. The function $f(d) = 50 \bullet \left(\frac{3}{2}\right)^d$ defines this relationship.

   a. How many people had the flu at the beginning of the epidemic? Explain your thinking.

   50, because $f(0) = 50$.

   b. How quickly is the flu spreading? Explain how you can tell from the equation.

   Each day, the number of infected people grows by a factor of $\frac{3}{2}$ (it grows by 150%).

   c. What does $f(1)$ mean in this situation?

   This represents the number of people with the flu 1 day after the epidemic began.

   d. Does $f(3.5)$ make sense in this situation?

   Sample response: Yes, it is the number of people infected 3.5 days after the epidemic began.

2. The function $f(t)$ gives the dollar value of a bond t years after the bond was purchased. The graph of $f(t)$ is shown.

   a. What is $f(0)$? What does it mean in this situation?

   The value of the bond when it was purchased, which was $1,000.

   b. What is $f(4.5)$? What does it mean in this situation?

   The value of the bond after 4.5 years, which was about $1,250.

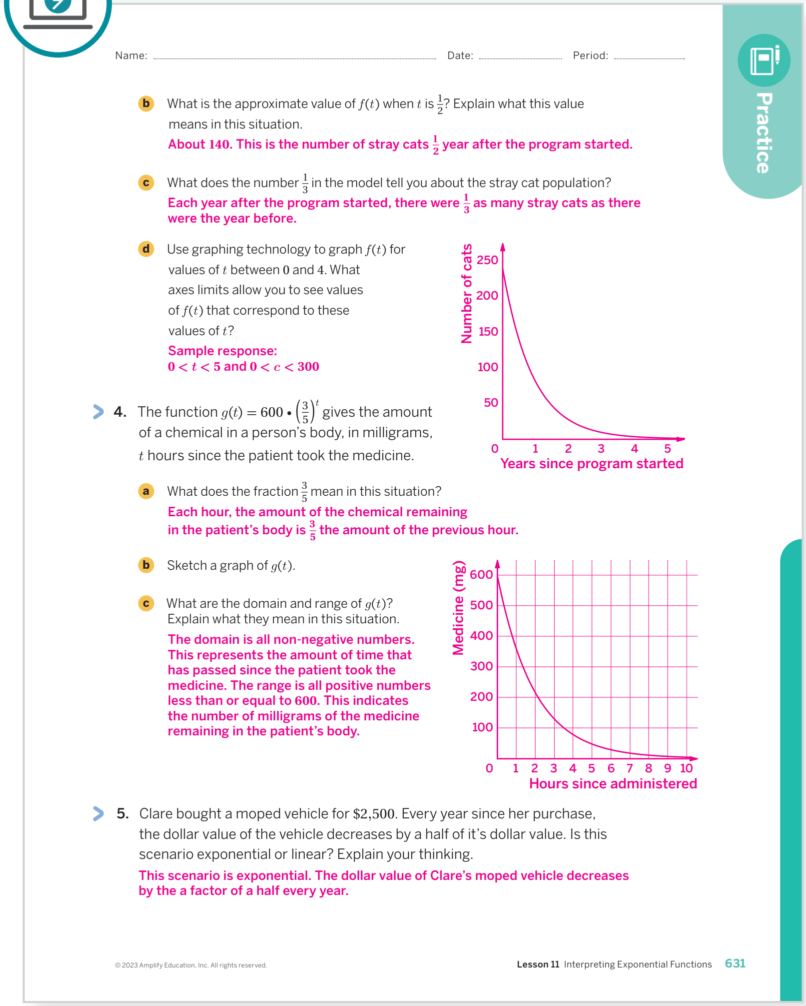
   c. When is $f(t) = 1500$? What does this mean in this situation?

   It is when the value of the bond reaches $1,500, which occurs after a little more than 8 years.

3. *Graphing technology required.* A model for the number of stray cats in a town t years since the town started an animal control program is the function $f(t) = 243 \bullet \left(\frac{1}{3}\right)^t$. The program includes both sterilizing stray cats and finding homes to adopt them.

   a. What is the value of $f(t)$ when t is 0? Explain what this value means in this situation.

   243; The number of stray cats in year 0, or the number of stray cats when the program started.



Name: ______ Date: ______ Period: ______

   b. What is the approximate value of $f(t)$ when t is $\frac{1}{2}$? Explain what this value means in this situation.

   About 140. This is the number of stray cats $\frac{1}{2}$ year after the program started.

   c. What does the number $\frac{1}{3}$ in the model tell you about the stray cat population?

   Each year after the program started, there were $\frac{1}{3}$ as many stray cats as there were the year before.

   d. Use graphing technology to graph $f(t)$ for values of t between 0 and 4. What axes limits allow you to see values of $f(t)$ that correspond to these values of t?

   Sample response:
   $0 < t < 5$ and $0 < c < 300$

4. The function $g(t) = 600 \bullet \left(\frac{3}{5}\right)^t$ gives the amount of a chemical in a person's body, in milligrams, t hours since the patient took the medicine.

   a. What does the fraction $\frac{3}{5}$ mean in this situation?

   Each hour, the amount of the chemical remaining in the patient's body is $\frac{3}{5}$ the amount of the previous hour.

   b. Sketch a graph of $g(t)$.

   c. What are the domain and range of $g(t)$? Explain what they mean in this situation.

   The domain is all non-negative numbers. This represents the amount of time that has passed since the patient took the medicine. The range is all positive numbers less than or equal to 600. This indicates the number of milligrams of the medicine remaining in the patient's body.

5. Clare bought a moped vehicle for $2,500. Every year since her purchase, the dollar value of the vehicle decreases by a half of it's dollar value. Is this scenario exponential or linear? Explain your thinking.

   This scenario is exponential. The dollar value of Clare's moped vehicle decreases by the a factor of a half every year.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activities 2 and 3	3
	4	Activities 2 and 3	3
Formative	5	Unit 4 Lesson 12	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Modeling Exponential Behavior

Let's use exponential functions to model real-world situations.

Focus

Goals

1. Use exponential functions to model situations that involve exponential growth or decay.
2. When given data, determine an appropriate model for the situation described by the data.

Rigor

- Students **apply** exponential functions to real-world, physical scenarios involving bouncing balls.

Coherence

Today

Students study messy real-world data, model the relationship with exponential functions, and use the model to respond to problems based on context. They engage in an optional hands-on experience to collect data from dropping various sizes of balls.

Previously

Students wrote and graphed exponential functions. They determined reasonable domains based on context provided.

Coming Soon

Students will analyze exponential functions and the effect of the growth factor on the shape of the graph.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	15 min/40 min*	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- 3 unique bouncy balls per group (print Activity 3)
- graphing technology
- measuring tapes (print Activity 3)

Math Language Development

Review words

- *decay factor*
- *exponential functions*
- *growth factor*

Amps | Featured Activity

Activity 3
Simulating Bouncing Balls

Students compare ball drops and construct exponential functions and graphs to model the ball drops.

Building Math Identity and Community

Connecting to Mathematical Practices

As students work with others in a small group to determine which ball is the bounciest, conflicts might arise due to poor communication skills. Prior to the activity, have students talk through the task, agreeing on how they will communicate with each other as they collect data. Remind them that clear communication requires a precise choice of words as well as good listening skills.

Modifications to Pacing

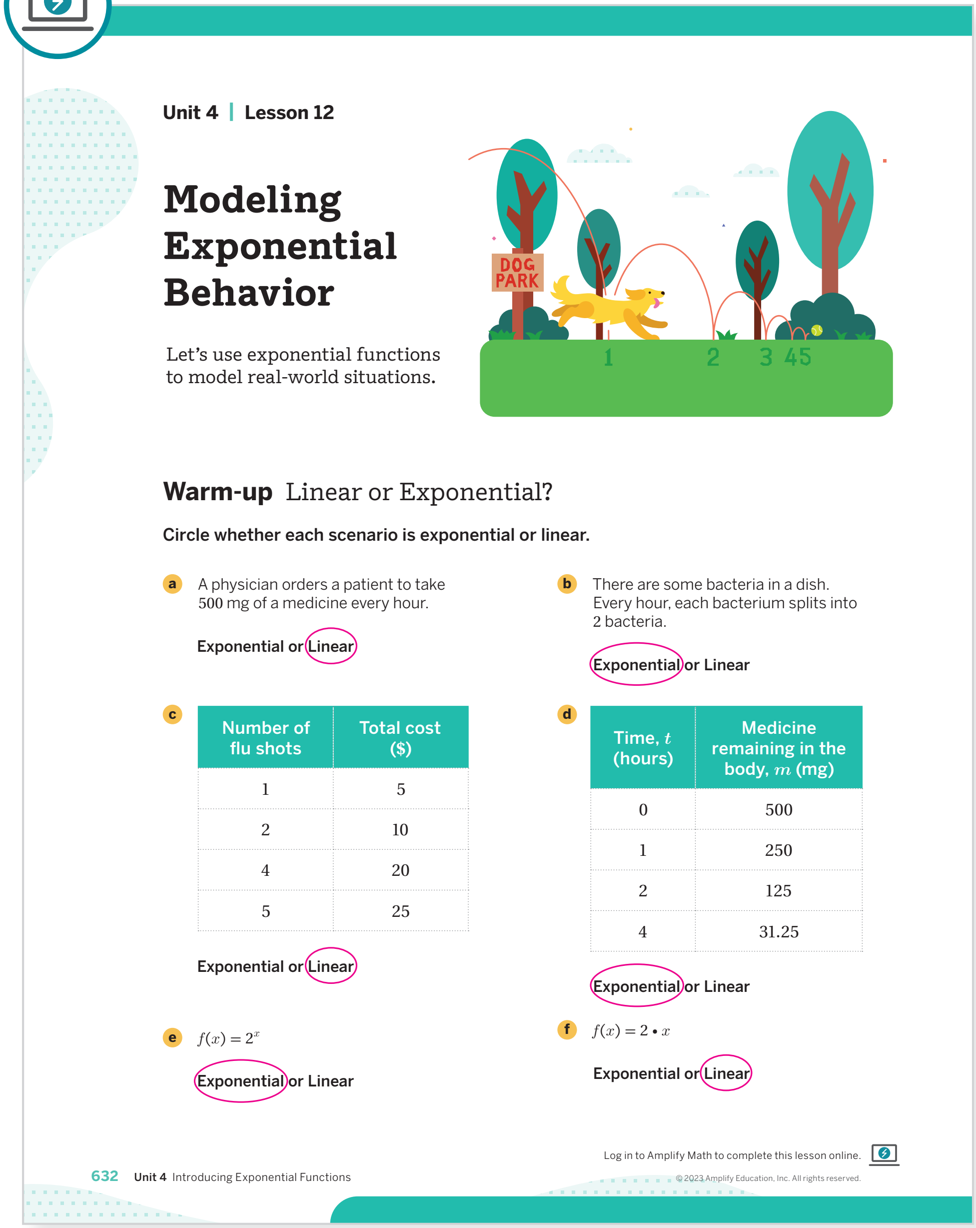
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have some students complete the left column while others complete the right column.
- If you spend time on optional **Activity 3**, prioritize time over **Activity 2**.

Independent | 5 min

Warm-up Linear or Exponential?

Students examine scenarios and the structure of tables and equations to distinguish between exponential and linear functions.



Unit 4 | Lesson 12

Modeling Exponential Behavior

Let's use exponential functions to model real-world situations.

Warm-up Linear or Exponential?

Circle whether each scenario is exponential or linear.

a. A physician orders a patient to take 500 mg of a medicine every hour.

Exponential or [✓ Linear]

b. There are some bacteria in a dish. Every hour, each bacterium splits into 2 bacteria.

[✓ Exponential] or Linear

c.

Number of flu shots	Total cost ($)
1	5
2	10
4	20
5	25

Exponential or [✓ Linear]

d.

Time, t (hours)	Medicine remaining in the body, m (mg)
0	500
1	250
2	125
4	31.25

[✓ Exponential] or Linear

e. $f(x) = 2^x$

[✓ Exponential] or Linear

f. $f(x) = 2 \bullet x$

Exponential or [✓ Linear]

Log in to Amplify Math to complete this lesson online.

632 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students work independently to determine whether each scenario is linear or exponential. Then have them compare their solutions with a partner.

2 Monitor

Help students get started by suggesting they annotate the tables to determine different patterns, or find a common difference or factor for each scenario.

Look for points of confusion:

- **Having difficulty determining whether the real-world scenarios are linear or exponential.** Have students create a table and evaluate at various input values to determine an exponential or linear pattern.

Look for productive strategies:

- Annotating table(s) to find successive quotients or differences.

3 Connect

Have students share their strategies and responses with a partner. Select pairs of students to share strategies and responses with the whole class.

Highlight that exponential functions have a common factor, while linear functions have a common difference.

Ask, "What strategies did you use to determine whether the real-world scenarios were linear or exponential?" Sample responses:

- Look for key words in the text descriptions.
- Analyze the table to determine whether there is a common difference or common factor.
- Study the structure of the equation.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies and responses for determining whether each scenario was linear or exponential, listen for and amplify student responses that indicated examining each representation to determine whether there was a *common difference (linear)* or *common factor (exponential)*. Annotate each representation with how it shows the common difference or common factor.

English Learners

Have students annotate the phrase "splits into 2" as indicating *doubling* or *multiplying by 2*.

Power-up

To power up students' ability to determine whether a scenario is linear or exponential, have students complete:

Recall that linear relationships have a constant rate of change, seen as repeated addition. Exponential relationships have a constant rate of growth, seen as repeated multiplication.

Determine whether each relationship is *linear* or *exponential*.

a. Noah's followers on the DanceOff app increased by 200 people each week. Linear

b. Bard's followers on the DanceOff app doubled each week. Exponential

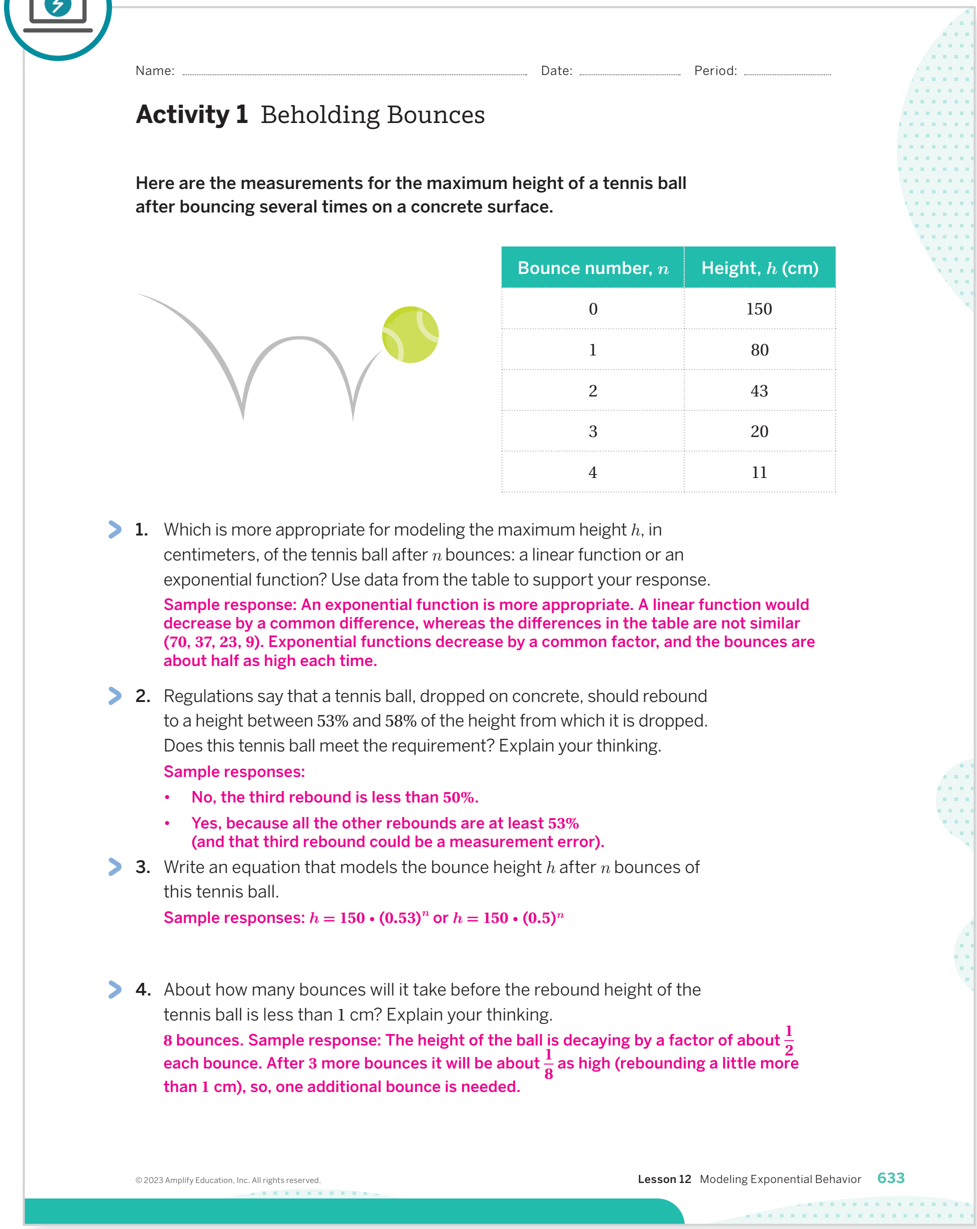
Use: Before the Warm-up

Informed by: Performance on Lesson 11, Practice Problem 5

Small Groups | 15 min

Activity 1 Beholding Bounces

Students construct an exponential function to model real-world data.



Name: ______________ Date: ________ Period: ________

Activity 1 Beholding Bounces

Here are the measurements for the maximum height of a tennis ball after bouncing several times on a concrete surface.

Bounce number, n	Height, h (cm)
0	150
1	80
2	43
3	20
4	11

1. Which is more appropriate for modeling the maximum height h, in centimeters, of the tennis ball after n bounces: a linear function or an exponential function? Use data from the table to support your response.

   Sample response: An exponential function is more appropriate. A linear function would decrease by a common difference, whereas the differences in the table are not similar (70, 37, 23, 9). Exponential functions decrease by a common factor, and the bounces are about half as high each time.

2. Regulations say that a tennis ball, dropped on concrete, should rebound to a height between 53% and 58% of the height from which it is dropped. Does this tennis ball meet the requirement? Explain your thinking.

   Sample responses:

   - **No, the third rebound is less than 50%.**
   - **Yes, because all the other rebounds are at least 53% (and that third rebound could be a measurement error).**

3. Write an equation that models the bounce height h after n bounces of this tennis ball.

   Sample responses: $h = 150 \cdot (0.53)^n$ or $h = 150 \cdot (0.5)^n$

4. About how many bounces will it take before the rebound height of the tennis ball is less than 1 cm? Explain your thinking.

   8 bounces. Sample response: The height of the ball is decaying by a factor of about $\frac{1}{2}$ each bounce. After 3 more bounces it will be about $\frac{1}{8}$ as high (rebounding a little more than 1 cm), so, one additional bounce is needed.

1 Launch

Discuss why real-world data is sometimes messy. Demonstrate a few ball drops and investigate various strategies to model the data. Define *successive difference, successive quotient,* and *rebound factor.* Provide access to graphing technology.

2 Monitor

Help students get started by prompting them to find the successive differences or quotients in the table.

Look for points of confusion:

- **Struggling to determine if data is linear or exponential.** Remind students real-world data may not perfectly align with models.
- **Struggling to calculate the decay factor.** Discuss the variation in the factors, and have students find the average or choose the most frequently occurring factor.

Look for productive strategies:

- Approximating the growth factor: $h = 150 \cdot \left(\frac{1}{2}\right)^n$.
- Using the first two points to determine the growth factor: $h = 150 \cdot \left(\frac{8}{15}\right)^n$.
- Calculating the average of successive quotients to determine the growth factor: $h = 150 \cdot (0.53)^n$.
- Choosing technology to generate the regression: $h = 150.389 \cdot (0.527)^n$.

3 Connect

Display the table and the equation $h = 150 \cdot (0.53)^n$.

Have students share their productive strategies, sequentially, to generate the equation.

Highlight that approximate models can be used with messy real-world data. Connect the data to Problem 2.

Ask, "Given the third bounce, does the data support your conclusion?"

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

Bring in a tennis ball and demonstrate what dropping a tennis ball from a given height looks like. Before doing so, ask students to predict how many times they think the ball will bounce before coming to a rest.

Accessibility: *Clarify Vocabulary and Symbols*

Clarify that the term *rebound* describes the ball bouncing on the ground and back into the air.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 4, have groups meet with one other group to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "Can you explain to me how you arrived at your answer?"
- "Did you use the table or the equation? Was there a reason why you chose the representation that you did?"
- "What mathematical language did you use in your response?"

Have students write a final response, based on the feedback they received.

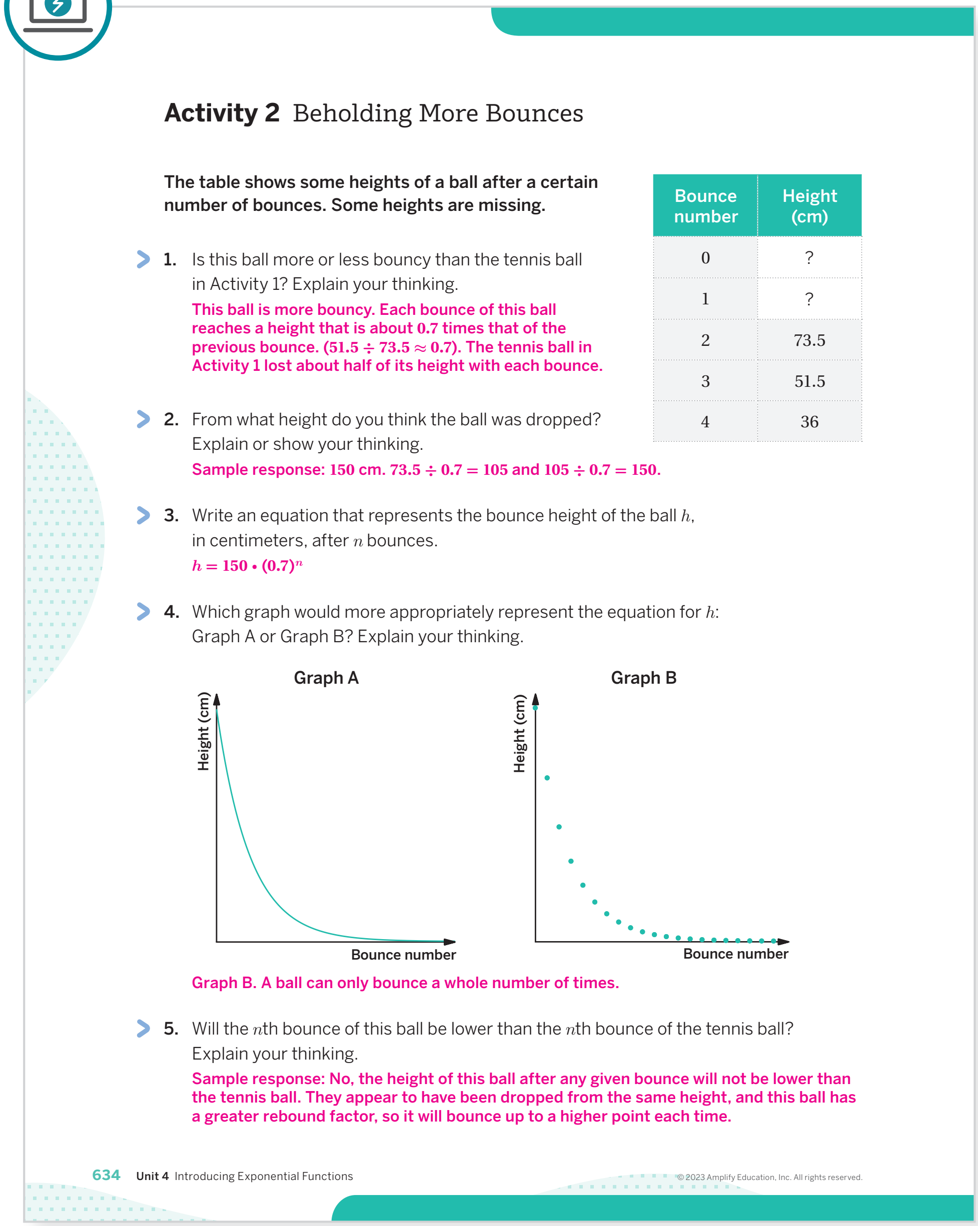
English Learners

Encourage students to use diagrams, tables, or illustrations in their response.

Small Groups | 20 min

Activity 2 Beholding More Bounces

Students write exponential functions to model real-world data, and interpret the parameters of the exponential function in terms of the context.



Activity 2 Beholding More Bounces

The table shows some heights of a ball after a certain number of bounces. Some heights are missing.

Bounce number	Height (cm)
0	?
1	?
2	73.5
3	51.5
4	36

1. Is this ball more or less bouncy than the tennis ball in Activity 1? Explain your thinking.
   This ball is more bouncy. Each bounce of this ball reaches a height that is about 0.7 times that of the previous bounce. ($51.5 \div 73.5 \approx 0.7$). The tennis ball in Activity 1 lost about half of its height with each bounce.
2. From what height do you think the ball was dropped? Explain or show your thinking.
   Sample response: 150 cm. $73.5 \div 0.7 = 105$ and $105 \div 0.7 = 150$.
3. Write an equation that represents the bounce height of the ball h, in centimeters, after n bounces.
   $h = 150 \cdot (0.7)^n$
4. Which graph would more appropriately represent the equation for h: Graph A or Graph B? Explain your thinking.

   Graph B. A ball can only bounce a whole number of times.
5. Will the nth bounce of this ball be lower than the nth bounce of the tennis ball? Explain your thinking.
   Sample response: No, the height of this ball after any given bounce will not be lower than the tennis ball. They appear to have been dropped from the same height, and this ball has a greater rebound factor, so it will bounce up to a higher point each time.

634 Unit 4 Introducing Exponential Functions © 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Tell students they will examine the bounciness of another ball.

2 Monitor

Help students get started by referring to the strategies they used in Activity 1. Have students diagram the ball's bounces using the table of values.

Look for points of confusion:

- **Struggling to relate $n = 0$ to initial height**. Ask students what 0, as a number of bounces, represents.
- **Struggling comparing the nth bounces in Problem 4.** Ask students how initial height and rebound factor compare.

Look for productive strategies:

- Modeling the relationship with an equation using the initial height and rebound factor.

3 Connect

Display Graphs A and B in Problem 4.

Have groups of students share the equations they wrote in Problem 3 and their responses to Problems 4 and 5.

Highlight that the number of bounces can only be whole numbers, so Graph A does not represent this situation. While the heights can have fractional or decimal values, a discrete graph is needed because the independent variable can only have whole number values.

Ask:

- "How do the initial heights of this ball and the tennis ball from Activity 1 compare? How do the rebound factors compare?" The initial heights are the same, 150 cm. The rebound factor of this ball, 0.7, is greater than the rebound factor of the tennis ball from Activity 1.
- "Which ball do you think will stop bouncing first? Why?" The tennis ball will stop bouncing first because it has a lesser rebound height.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Annotate the table by showing how the rebound heights listed are about 0.7 times the previous rebound height. Ask students what this value means and how they can use that to complete the table.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses to Problems 4 and 5, provide the following sentence frames to help them organize their thinking:

- "Graph _____ represents this situation because . . ."
- "The nth bounce of this ball will/will not be lower than the nth bounce of the tennis ball because . . ."
- "This ball has a lesser/greater rebound factor, so . . ."

English Learners

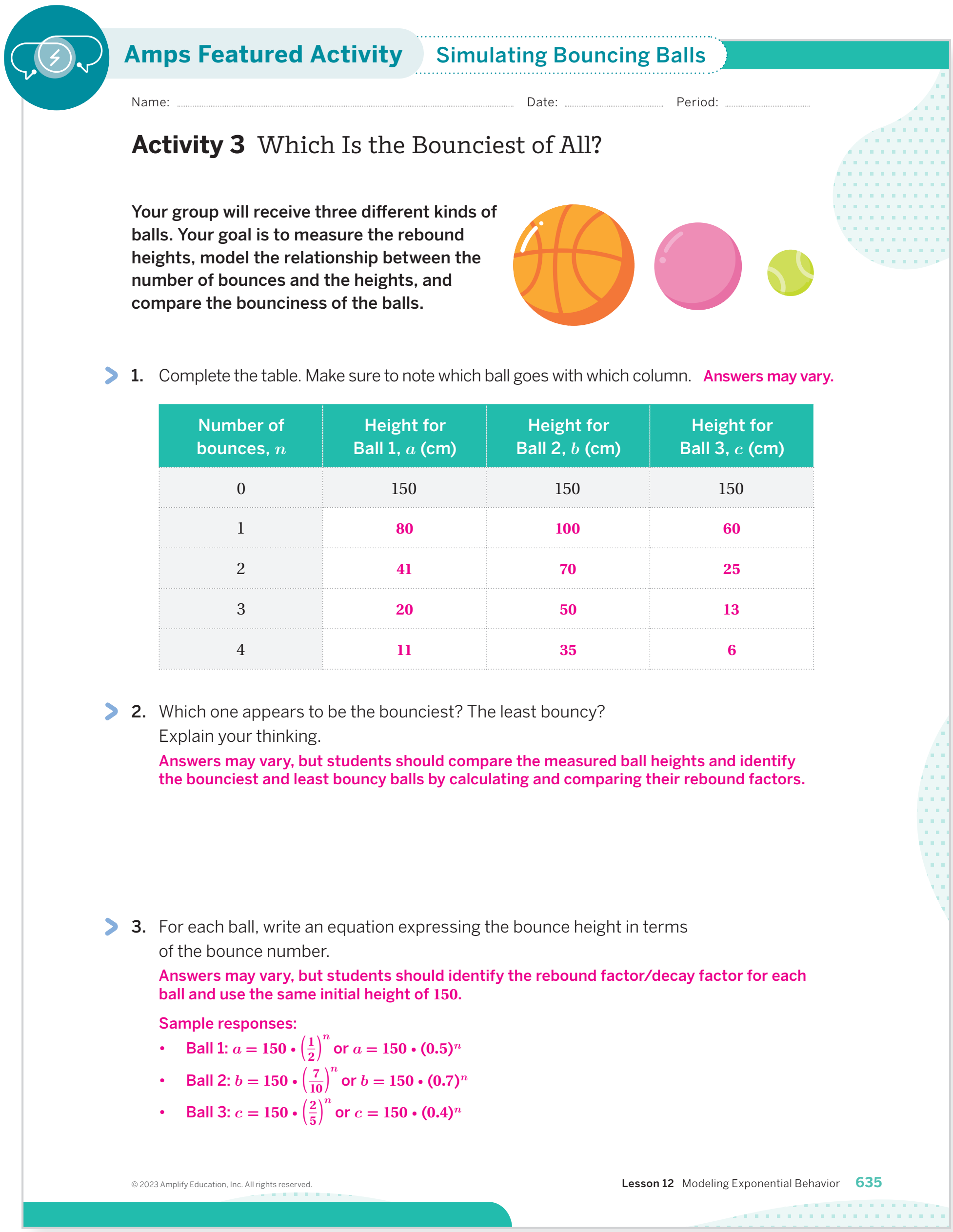
Provide students time to formulate a response before sharing with the class.

Optional

Small Groups | 15 min/40 min

Activity 3 Which Is the Bounciest of All?

Students will gather, analyze, and model real-world data with exponential functions, and interpret the parameters in terms of the context.



Amps Featured Activity Simulating Bouncing Balls

Name: ______ Date: ______ Period: ______

Activity 3 Which Is the Bounciest of All?

Your group will receive three different kinds of balls. Your goal is to measure the rebound heights, model the relationship between the number of bounces and the heights, and compare the bounciness of the balls.

1. Complete the table. Make sure to note which ball goes with which column. **Answers may vary.**

Number of bounces, n	Height for Ball 1, a (cm)	Height for Ball 2, b (cm)	Height for Ball 3, c (cm)
0	150	150	150
1	**80**	**100**	**60**
2	**41**	**70**	**25**
3	**20**	**50**	**13**
4	**11**	**35**	**6**

2. Which one appears to be the bounciest? The least bouncy? Explain your thinking.
Answers may vary, but students should compare the measured ball heights and identify the bounciest and least bouncy balls by calculating and comparing their rebound factors.

3. For each ball, write an equation expressing the bounce height in terms of the bounce number.
Answers may vary, but students should identify the rebound factor/decay factor for each ball and use the same initial height of 150.

Sample responses:
- Ball 1: $a = 150 \cdot \left(\frac{1}{2}\right)^n$ or $a = 150 \cdot (0.5)^n$
- Ball 2: $b = 150 \cdot \left(\frac{7}{10}\right)^n$ or $b = 150 \cdot (0.7)^n$
- Ball 3: $c = 150 \cdot \left(\frac{2}{5}\right)^n$ or $c = 150 \cdot (0.4)^n$

1 Launch

Provide groups of students with measuring tape and 3 different types of balls. Instruct students to find a surface that is hard, flat, and level for bouncing the balls.

Note: You may choose to use the digital version of this activity instead, which should last about 15 minutes.

2 Monitor

Help students get started by demonstrating how to measure the bouncing ball. Provide time for students to practice bouncing and measuring the height of the bounces.

Look for points of confusion:

- **Alternating between units for measurement.** Tell students to use centimeters.
- **Having difficulty with fractional measurements.** Instruct students to measure to the nearest whole centimeter.
- **Struggling to measure the heights of the bounces.** Suggest that one group member drop the ball, while other group members each place their hand where the rebound height is for each bounce. The group member that dropped the ball can measure each rebound height.

Look for productive strategies:

- Approximating the rebound factor.
- Determining the rebound factor from the first two bounces.
- Determining the average of successive quotients.
- Recognizing that the greater the rebound factor, the greater the bounciness of the ball and using mathematical vocabulary to explain their thinking.

Activity 3 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can save time by using technology to compare ball drops and construct exponential functions and graphs to model the ball drops. If you choose to use the digital version of the activity, it should last about 15 minutes.

Accessibility: *Vary Demands to Optimize Challenge*

If you choose not to use the Amps slides for this activity, consider assigning each group one of the balls to measure. Display a class chart of the table in Problem 1 and have groups complete the class chart. After all measurements have been recorded, have groups proceed with Problem 2.

Math Language Development

MLR8: Discussion Supports— Press for Details

During the Connect, ask students to share how they determined which ball is the most bouncy (Problem 4). Press for details in their reasoning by asking:

- "Did you refer to the equations or the table in your response? If you did not refer to the equations, how can you examine the structure of the equations to tell you which ball is the most bouncy?"
- "Are the initial values in your equations the same? Is that important? Explain your thinking."

English Learners

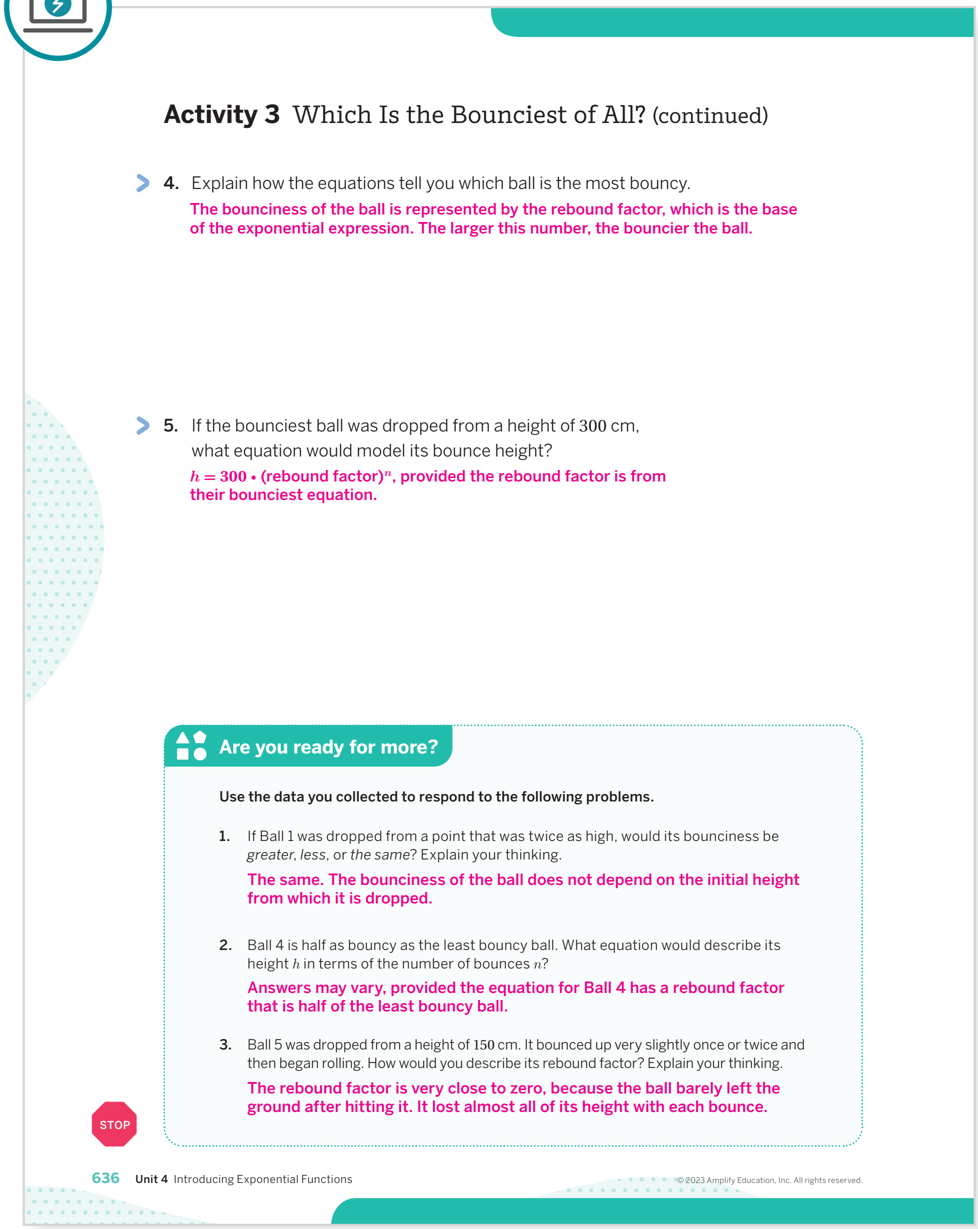
Annotate the equations by selecting the one with the greatest decay factor and writing "most bouncy."

Optional

Small Groups | 15 min/40 min

Activity 3 Which Is the Bounciest of All? (continued)

Students will gather, analyze, and model real-world data with exponential functions, and interpret the parameters in terms of the context.



Activity 3 Which Is the Bounciest of All? (continued)

4. Explain how the equations tell you which ball is the most bouncy.
   The bounciness of the ball is represented by the rebound factor, which is the base of the exponential expression. The larger this number, the bouncier the ball.

5. If the bounciest ball was dropped from a height of 300 cm, what equation would model its bounce height?
   $h = 300 \cdot (\text{rebound factor})^n$, provided the rebound factor is from their bounciest equation.

Are you ready for more?

Use the data you collected to respond to the following problems.

1. If Ball 1 was dropped from a point that was twice as high, would its bounciness be *greater*, *less*, or *the same*? Explain your thinking.
   The same. The bounciness of the ball does not depend on the initial height from which it is dropped.
2. Ball 4 is half as bouncy as the least bouncy ball. What equation would describe its height h in terms of the number of bounces n?
   Answers may vary, provided the equation for Ball 4 has a rebound factor that is half of the least bouncy ball.
3. Ball 5 was dropped from a height of 150 cm. It bounced up very slightly once or twice and then began rolling. How would you describe its rebound factor? Explain your thinking.
   The rebound factor is very close to zero, because the ball barely left the ground after hitting it. It lost almost all of its height with each bounce.

STOP

636 **Unit 4** Introducing Exponential Functions

3 Connect

Display the table and sample equations for each ball.

Have groups of students share the various strategies used to estimate the rebound factor. Select students who realized the ball had to be dropped from the same height to present their strategies and responses to the class.

Highlight that in a situation modeled by exponential decay, a greater decay factor means that the function decays *more slowly*. Point out that the measurements in this activity are likely to vary widely and could be inaccurate to due lack of precision in measuring.

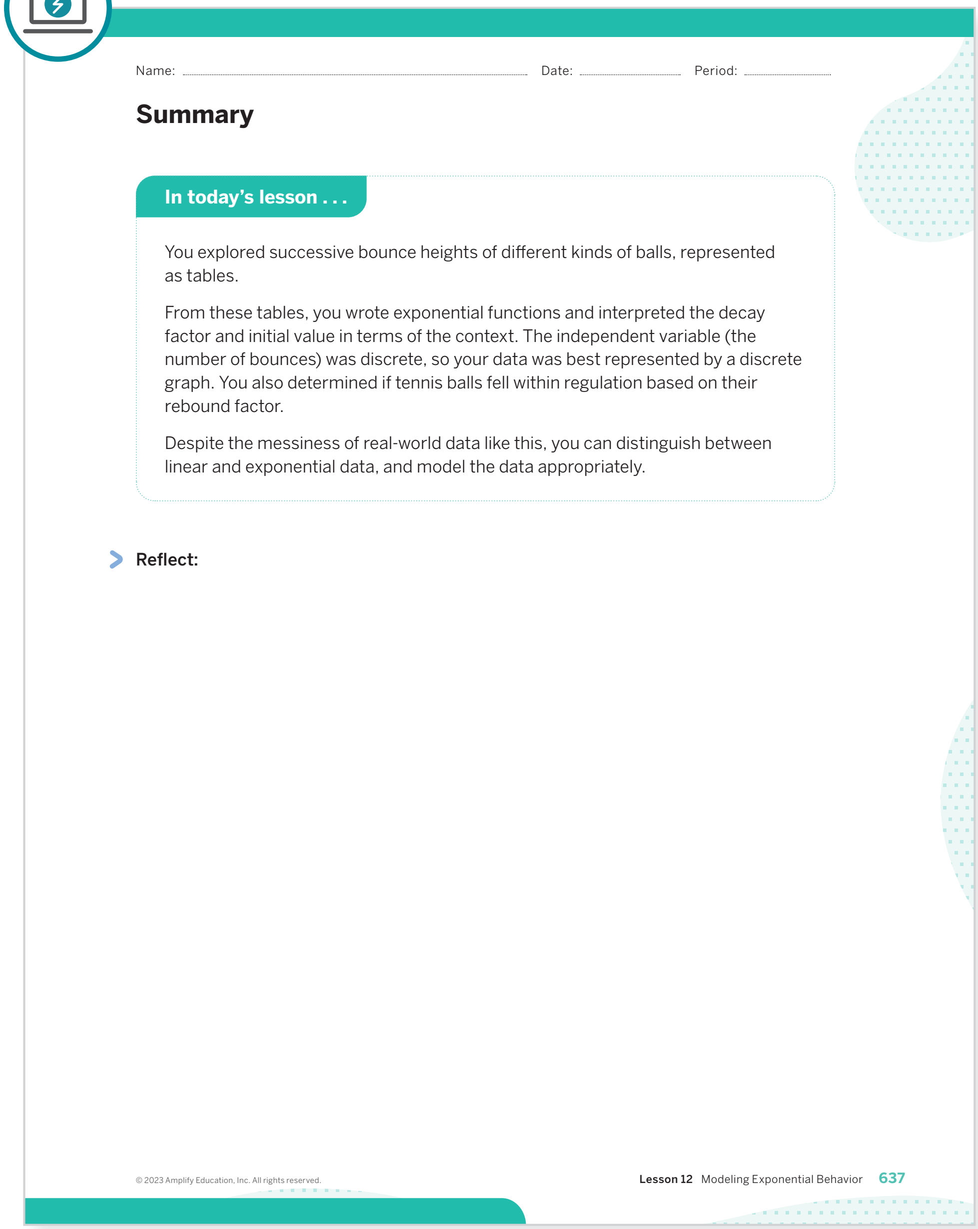
Ask:

- "Does a greater decay factor mean that the ball is more bouncy or less bouncy?" More bouncy.
- "Does a greater decay factor mean that the heights are decreasing more quickly or more slowly?" More slowly.

Whole Class | 5 min

Summary

Review and synthesize how to interpret the parameters of the exponential function in terms of the context.



Name: Date: Period:

Summary

In today's lesson . . .

You explored successive bounce heights of different kinds of balls, represented as tables.

From these tables, you wrote exponential functions and interpreted the decay factor and initial value in terms of the context. The independent variable (the number of bounces) was discrete, so your data was best represented by a discrete graph. You also determined if tennis balls fell within regulation based on their rebound factor.

Despite the messiness of real-world data like this, you can distinguish between linear and exponential data, and model the data appropriately.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Modeling Exponential Behavior 637

Synthesize

Display the table from Activity 1. Remind students that the table shows the measurements for the maximum height of a tennis ball after bouncing several times on a concrete surface.

Have pairs of students share their strategies or process for determining the rebound factor.

Highlight that real-world data are messy. For the ball drop, students must consider varying rebound factors for each ball, determine the most appropriate models, and recognize, if appropriate, factors have been used. In Activity 3, students observed:

- Inaccurate data measurements are inherent.
- Measurement errors of different rebound factors for successive points are small.

Ask:

- "If you were given a different tennis ball, how could you determine if it satisfies the bounce regulation of 53% to 58%?" Sample responses: Record the initial height and 3–4 bounce heights. Use this data to determine the rebound factor and then compare it to the regulation rebound factor.
- "Do the models you produced always work, or only when the rebound height is very small?" No, after a certain point, the ball will stop bouncing and the height would be 0.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

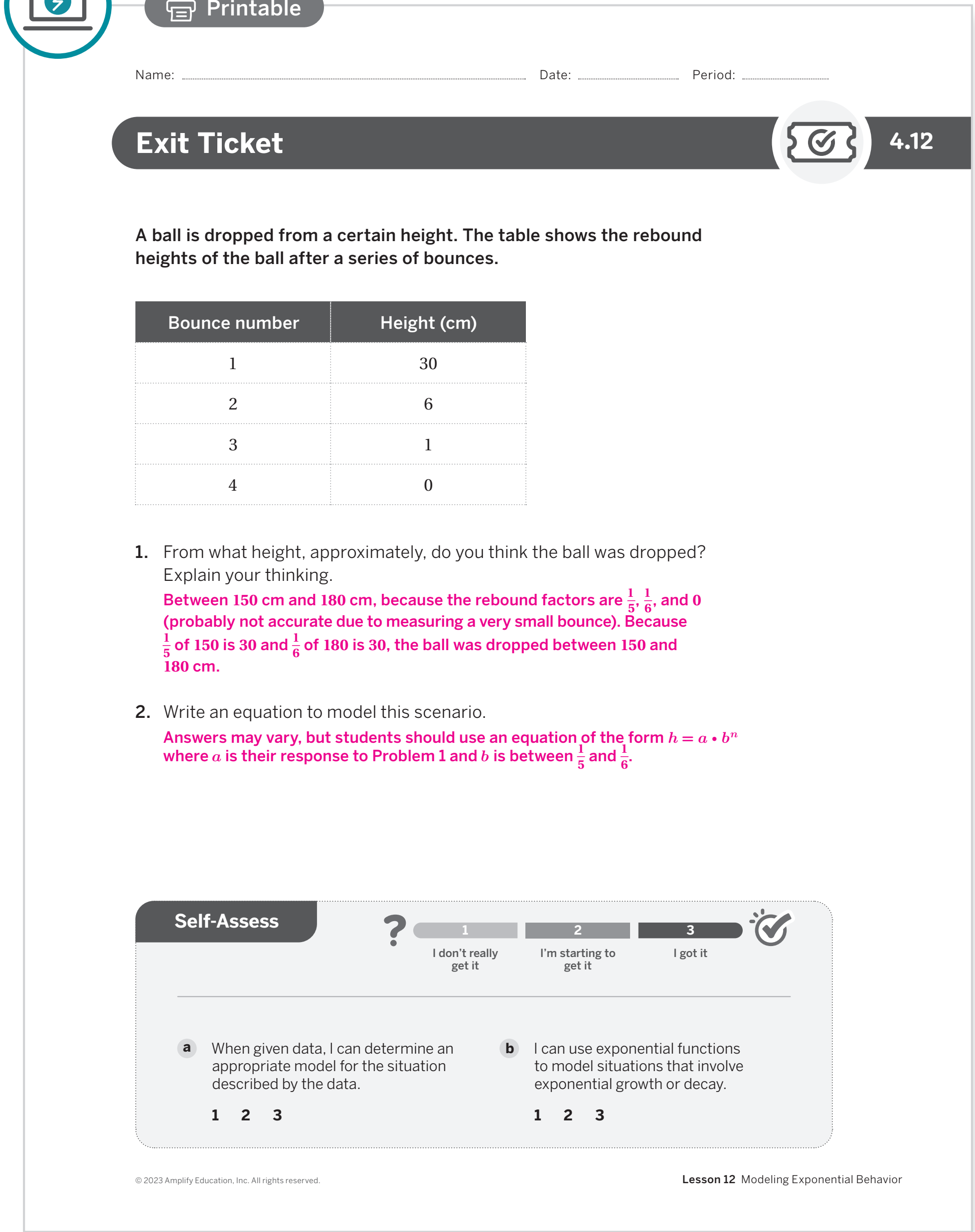
- "How can you differentiate linear growth from exponential growth using real-world scenarios, tables, or equations?"
- "What characterizes exponential decay?"
- "What are real-world models of exponential decay?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by examining real-world data, writing an exponential function, and interpreting it in terms of the context.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 4.12

A ball is dropped from a certain height. The table shows the rebound heights of the ball after a series of bounces.

Bounce number	Height (cm)
1	30
2	6
3	1
4	0

1. From what height, approximately, do you think the ball was dropped? Explain your thinking.
   Between 150 cm and 180 cm, because the rebound factors are $\frac{1}{5}$, $\frac{1}{6}$, and 0 (probably not accurate due to measuring a very small bounce). Because $\frac{1}{5}$ of 150 is 30 and $\frac{1}{6}$ of 180 is 30, the ball was dropped between 150 and 180 cm.

2. Write an equation to model this scenario.
   Answers may vary, but students should use an equation of the form $h = a \bullet b^n$ where a is their response to Problem 1 and b is between $\frac{1}{5}$ and $\frac{1}{6}$.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a When given data, I can determine an appropriate model for the situation described by the data.

1 2 3

b I can use exponential functions to model situations that involve exponential growth or decay.

1 2 3

Lesson 12 Modeling Exponential Behavior

Success looks like . . .

- **Goal:** Using exponential functions to model situations that involve exponential growth or decay.
- **Goal:** When given data, determining an appropriate model for the situation described by the data.
  - » Writing an equation given the scenario in Problem 2.

Suggested next steps

If students incorrectly determine the initial height for Problem 1, consider:

- Revisiting approximating and using real-world decay factors from Activity 2, Problem 2.
- Assigning Practice Problem 1.
- Having students annotate the table to determine and extend the pattern.

If students are unable to write an equation to model the scenario in Problem 2, consider:

- Reviewing the components of an exponential equation (connecting the rebound factor and decay factor, and the initial height and initial value) in Activity 1, Problem 3 or Activity 2, Problem 3.
- Assigning Practice Problem 4.

Professional Learning

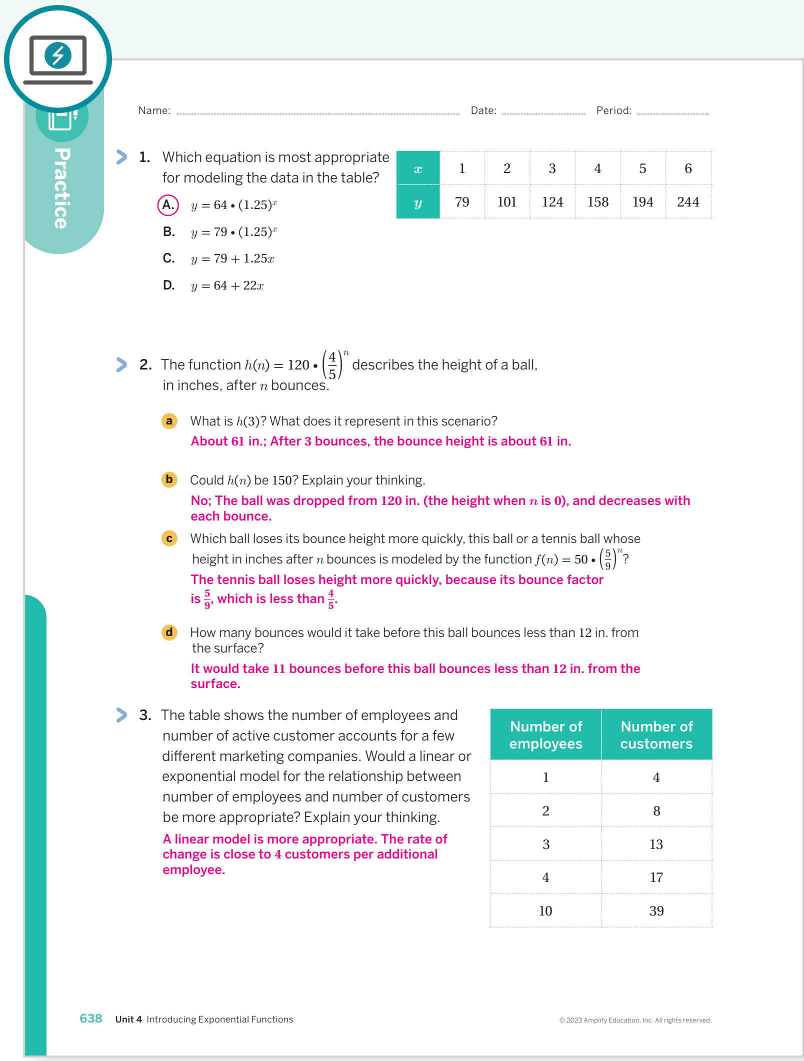
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- This lesson asked students to study messy real-world data and model the data with exponential functions. Where in your students' work today did you see or hear evidence of them interpreting their model and recognizing that models may not fit the entirety of real-world data, or that real-world measurements might be imprecise or inaccurate?
- Who participated and who did not participate in Activity 3 today? What trends do you see in your students' participation or engagement levels? What might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Which equation is most appropriate for modeling the data in the table?

x	1	2	3	4	5	6
y	79	101	124	158	194	244

[✓ A.] $y = 64 \cdot (1.25)^x$

B. $y = 79 \cdot (1.25)^x$

C. $y = 79 + 1.25x$

D. $y = 64 + 22x$

2. The function $h(n) = 120 \cdot \left(\frac{4}{5}\right)^n$ describes the height of a ball, in inches, after n bounces.

   a. What is $h(3)$? What does it represent in this scenario?

   About 61 in.; After 3 bounces, the bounce height is about 61 in.

   b. Could $h(n)$ be 150? Explain your thinking.

   No; The ball was dropped from 120 in. (the height when n is 0), and decreases with each bounce.

   c. Which ball loses its bounce height more quickly, this ball or a tennis ball whose height in inches after n bounces is modeled by the function $f(n) = 50 \cdot \left(\frac{5}{9}\right)^n$?

   The tennis ball loses height more quickly, because its bounce factor is $\frac{5}{9}$, which is less than $\frac{4}{5}$.

   d. How many bounces would it take before this ball bounces less than 12 in. from the surface?

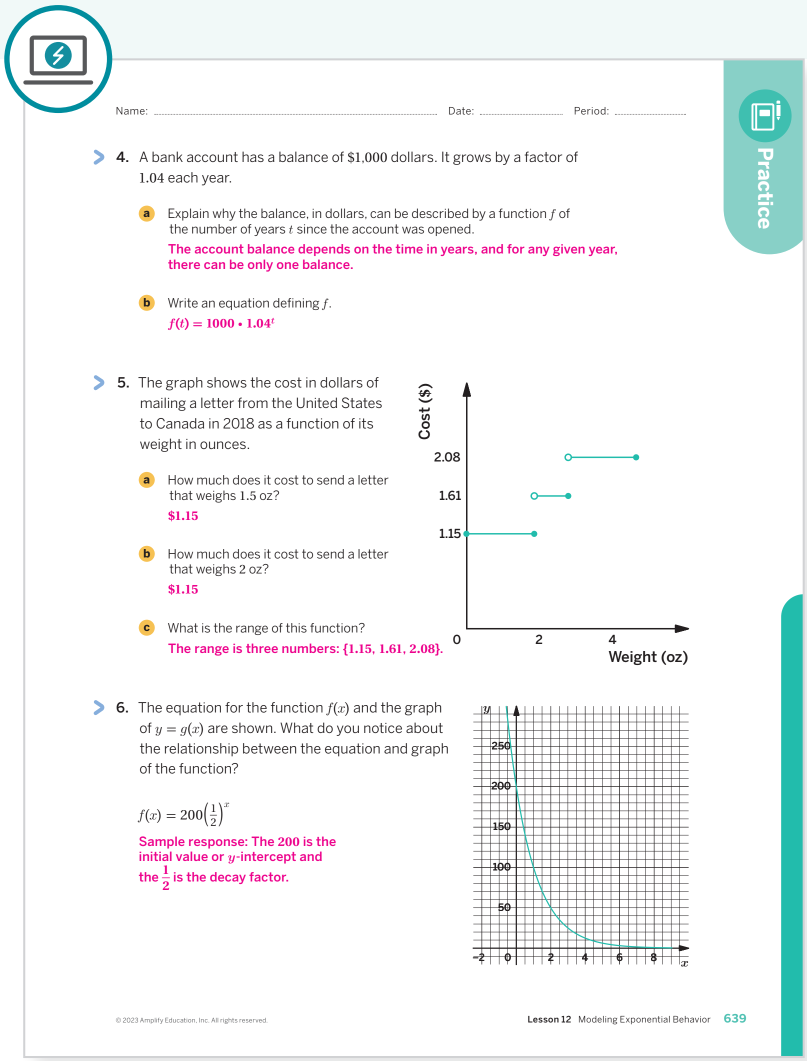
   It would take 11 bounces before this ball bounces less than 12 in. from the surface.

3. The table shows the number of employees and number of active customer accounts for a few different marketing companies. Would a linear or exponential model for the relationship between number of employees and number of customers be more appropriate? Explain your thinking.

A linear model is more appropriate. The rate of change is close to 4 customers per additional employee.

Number of employees	Number of customers
1	4
2	8
3	13
4	17
10	39

638 **Unit 4** Introducing Exponential Functions



Practice

Name: Date: Period:

4. A bank account has a balance of $1,000 dollars. It grows by a factor of 1.04 each year.

   a. Explain why the balance, in dollars, can be described by a function f of the number of years t since the account was opened.

   The account balance depends on the time in years, and for any given year, there can be only one balance.

   b. Write an equation defining f.

   $f(t) = 1000 \cdot 1.04^t$

5. The graph shows the cost in dollars of mailing a letter from the United States to Canada in 2018 as a function of its weight in ounces.

   a. How much does it cost to send a letter that weighs 1.5 oz?

   $1.15

   b. How much does it cost to send a letter that weighs 2 oz?

   $1.15

   c. What is the range of this function?

   The range is three numbers: {1.15, 1.61, 2.08}.

6. The equation for the function $f(x)$ and the graph of $y = g(x)$ are shown. What do you notice about the relationship between the equation and graph of the function?

$f(x) = 200\left(\frac{1}{2}\right)^x$

Sample response: The 200 is the initial value or y-intercept and the $\frac{1}{2}$ is the decay factor.

Lesson 12 Modeling Exponential Behavior 639

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 1	1
Spiral	4	Unit 4 Lesson 10	1
	5	Unit 3 Lesson 14	1
Formative ⚡	6	Unit 4 Lesson 13	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

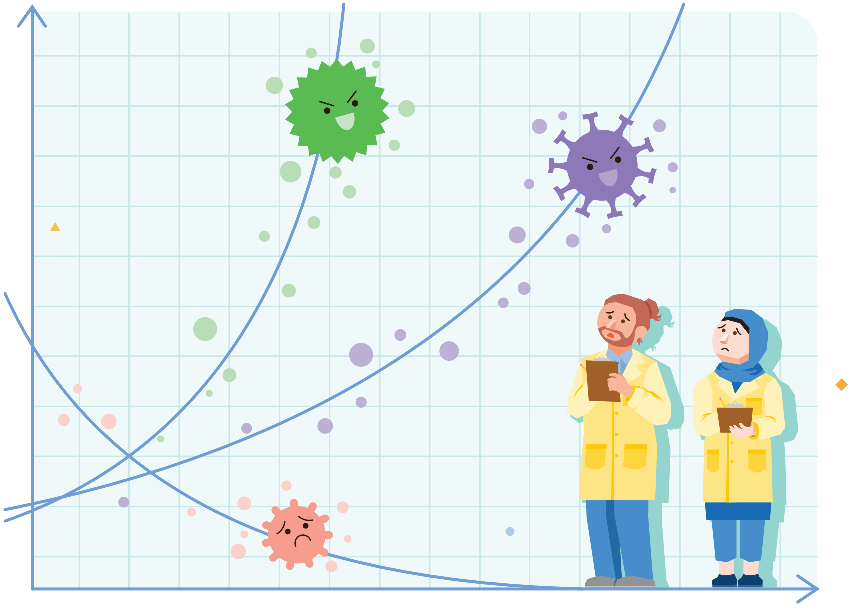
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 13**

Reasoning About Exponential Graphs

Let's study and compare equations and graphs of exponential functions.



Focus

Goals

1. **Language Goal:** Describe how changing the values of a and b affect the graph of $f(x) = a \cdot b^x$. **(Speaking and Listening, Writing)**
2. **Language Goal:** Use equations and graphs to compare exponential functions. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students analyze the graph of an exponential function $f(x) = a \cdot b^x$. They study the effect of b on the shape of the graph when $b > 1$ and when $0 < b < 1$. Students simultaneously examine several functions of this form, all of which have the same value of a or same value of b. They use the structure of the equation to determine the effect on the graph.

Previously

Students created graphs of exponential relationships and expressed the relationships in function notation.

Coming Soon

Students will calculate the average rate of change of an exponential function from a graph, table, and equation.

Rigor

- Students further develop their **procedural fluency** in interpreting graphs of exponential functions, and using the graph to determine specific parameters or properties of the function.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils (as needed)
- graphing technology

Math Language Development

Review words

- *decay factor*
- *exponential functions*
- *exponential decay*
- *growth factor*
- *initial value*

Amps | **Featured Activity**

Activity 1
Changing Parameters

Students can experiment with changing the values of a and b in an exponential function of the form $f(x) = a \cdot b^x$ and see how the graph is affected.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack the motivation to thoroughly interpret a model of an exponential situation. Remind students that using mathematics to model real-world situations is what makes mathematics a powerful tool. As students interpret their results, encourage them to think about whether the results make sense in context and how the model might be improved.

Modifications to Pacing

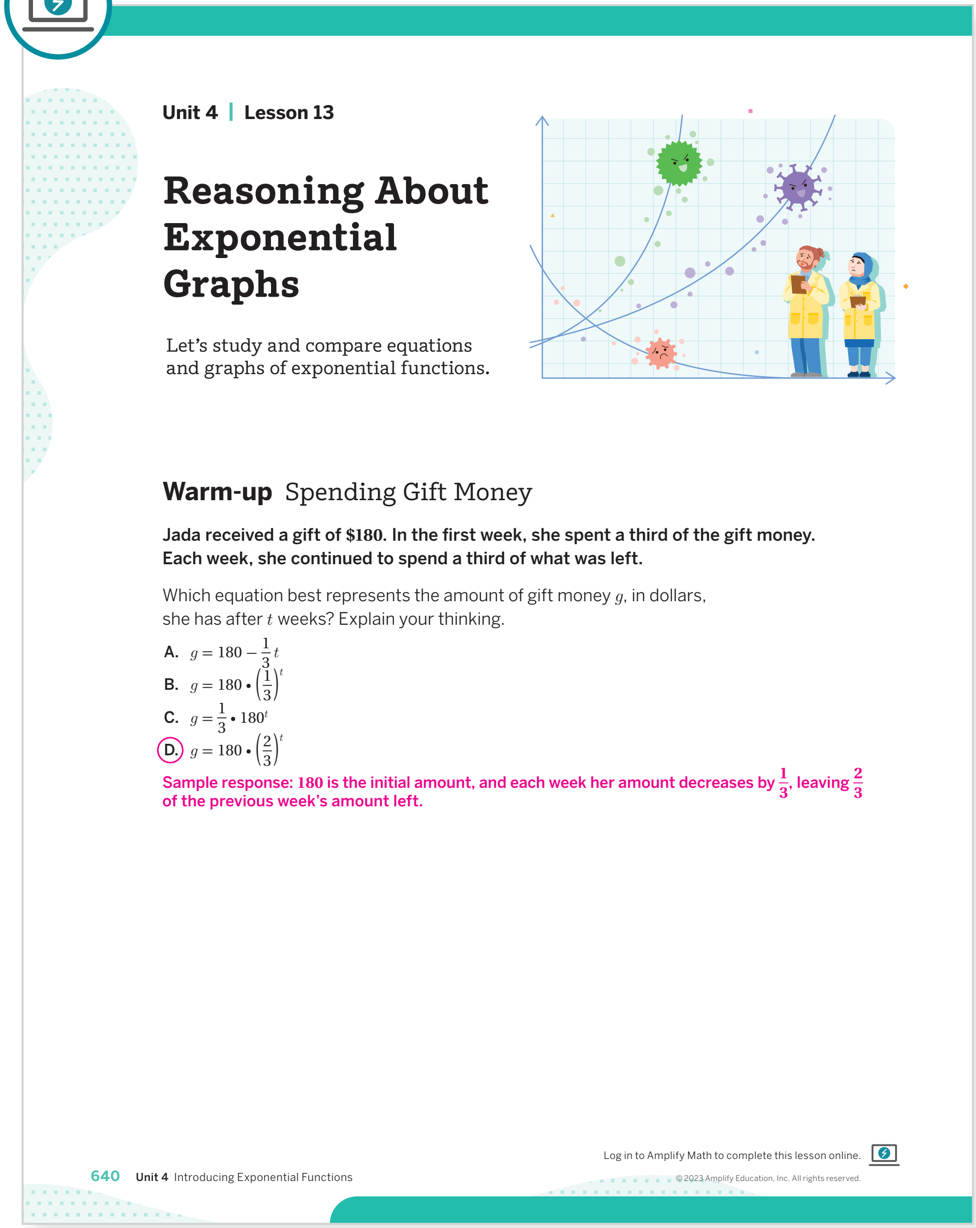
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, select a subset of functions for each problem. Include the last function in Problem 1.

Independent | 10 min

Warm-up Spending Gift Money

Students practice modeling a scenario characterized by exponential decay with an equation.



Unit 4 | Lesson 13

Reasoning About Exponential Graphs

Let's study and compare equations and graphs of exponential functions.

Warm-up Spending Gift Money

Jada received a gift of $180. In the first week, she spent a third of the gift money. Each week, she continued to spend a third of what was left.

Which equation best represents the amount of gift money g, in dollars, she has after t weeks? Explain your thinking.

A. $g = 180 - \frac{1}{3}t$
B. $g = 180 \bullet \left(\frac{1}{3}\right)^t$
C. $g = \frac{1}{3} \bullet 180^t$
[✓ D.] $g = 180 \bullet \left(\frac{2}{3}\right)^t$

Sample response: 180 is the initial amount, and each week her amount decreases by $\frac{1}{3}$, leaving $\frac{2}{3}$ of the previous week's amount left.

Log in to Amplify Math to complete this lesson online.

640 **Unit 4** Introducing Exponential Functions

1 Launch

Say, "Silently read the scenario and underline information that will help you select the equation. Be prepared to explain your thinking."

2 Monitor

Help students get started by saying, "Determine the initial amount and the growth or decay factor to help determine the correct equation."

Look for points of confusion:

- **Selecting B.** Ask, "What fraction *remains* each week?"
- **Selecting A.** Ask, "Is this situation linear?"

Look for productive strategies:

- Recognizing that two thirds of the amount remains each week.

3 Connect

Display the scenario and answer choices.

Have students share their selection and explain their thinking. Begin with those who selected Choice A, then Choice C, followed by Choices B and D.

Highlight that the equation in Choice A is linear. The equation in Choice C has the initial value in the incorrect location and an incorrect decay factor. The equation in Choice B has an incorrect decay factor. Emphasize that *two thirds of the amount remains* each week, not one third.

Ask, "If Choice B was correct, how would the scenario change?" Jada would spend two-thirds each week, leaving her with one third of the previous amount each week.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they have already modeled situations involving exponential decay. Display the general form of an exponential function, $y = a \bullet b^x$ or $f(x) = a \bullet b^x$, and ask students what a and b represent.

a is the initial value and b is the growth/decay factor.

Power-up

To power up students' ability to interpret the decay factor, have students complete:

The value of a car after t years can be modeled by the function $c(t) = 27000 \bullet \left(\frac{9}{10}\right)^t$. Select *all* of the statements that are true about the value of the car.

[✓ A.] The initial value of the car was $27,000.
B. The car decreases in value by 90% each year.
[✓ C.] Each year the car is worth $\frac{9}{10}$ of its value of the previous year.
[✓ D.] The car decreases in value by 10% each year.

Use: Before the Warm-up

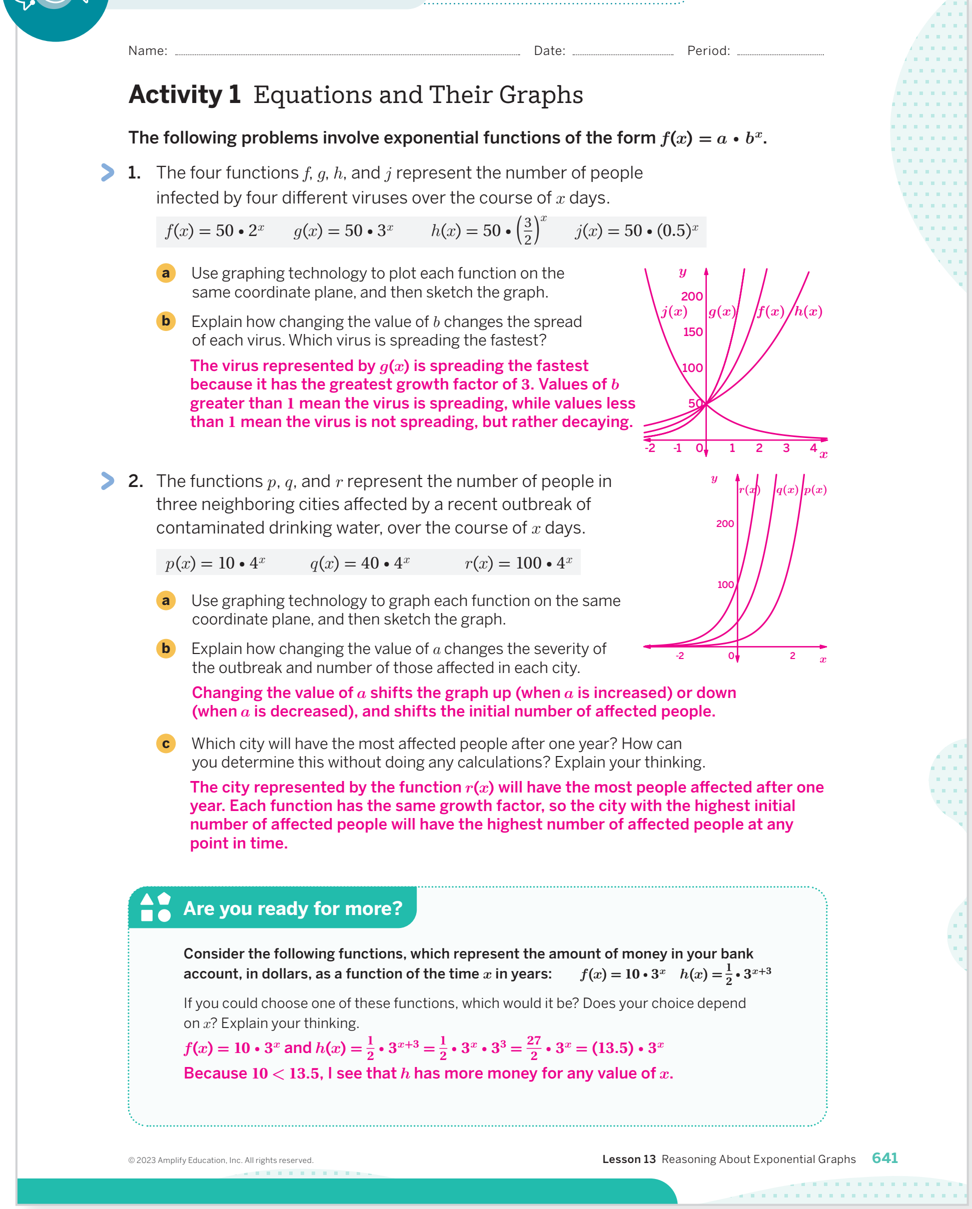
Informed by: Performance on Lesson 12, Practice Problem 6

Pairs | 15 min

Activity 1 Equations and Their Graphs

Students examine how changing the values a and b affect the graph of an exponential function of the form $f(x) = a \cdot b^x$ by studying the structure of the equations and graphs.

Amps Featured Activity Changing Parameters



Name: Date: Period:

Activity 1 Equations and Their Graphs

The following problems involve exponential functions of the form $f(x) = a \cdot b^x$.

1. The four functions f, g, h, and j represent the number of people infected by four different viruses over the course of x days.

$f(x) = 50 \cdot 2^x$ $g(x) = 50 \cdot 3^x$ $h(x) = 50 \cdot \left(\frac{3}{2}\right)^x$ $j(x) = 50 \cdot (0.5)^x$

a. Use graphing technology to plot each function on the same coordinate plane, and then sketch the graph.

b. Explain how changing the value of b changes the spread of each virus. Which virus is spreading the fastest?

The virus represented by $g(x)$ is spreading the fastest because it has the greatest growth factor of 3. Values of b greater than 1 mean the virus is spreading, while values less than 1 mean the virus is not spreading, but rather decaying.

2. The functions p, q, and r represent the number of people in three neighboring cities affected by a recent outbreak of contaminated drinking water, over the course of x days.

$p(x) = 10 \cdot 4^x$ $q(x) = 40 \cdot 4^x$ $r(x) = 100 \cdot 4^x$

a. Use graphing technology to graph each function on the same coordinate plane, and then sketch the graph.

b. Explain how changing the value of a changes the severity of the outbreak and number of those affected in each city.

Changing the value of a shifts the graph up (when a is increased) or down (when a is decreased), and shifts the initial number of affected people.

c. Which city will have the most affected people after one year? How can you determine this without doing any calculations? Explain your thinking.

The city represented by the function $r(x)$ will have the most people affected after one year. Each function has the same growth factor, so the city with the highest initial number of affected people will have the highest number of affected people at any point in time.

Are you ready for more?

Consider the following functions, which represent the amount of money in your bank account, in dollars, as a function of the time x in years: $f(x) = 10 \cdot 3^x$ $h(x) = \frac{1}{2} \cdot 3^{x+3}$

If you could choose one of these functions, which would it be? Does your choice depend on x? Explain your thinking.

$f(x) = 10 \cdot 3^x$ and $h(x) = \frac{1}{2} \cdot 3^{x+3} = \frac{1}{2} \cdot 3^x \cdot 3^3 = \frac{27}{2} \cdot 3^x = (13.5) \cdot 3^x$

Because $10 < 13.5$, I see that h has more money for any value of x.

Lesson 13 Reasoning About Exponential Graphs 641

1 Launch

Display the equation $y = a \cdot b^x$. Tell students that they will examine how changing the values of a and b affect the graph of an exponential function. Provide access to graphing technology.

2 Monitor

Help students get started by asking, "Which virus grows the fastest? How do you know?" $g(x)$ because it has the greatest growth factor of 3. Give students a moment to discuss their choice and rationale with their partner.

Look for points of confusion:

- **Not understanding they can analyze the structure of the equations to match with the graphs.** Ask, "What do you notice about the initial values in each equation? The growth or decay factors? How can this help you identify the graphs?"

Look for productive strategies:

- Using only two graphs at a time to examine the effects of changing a or b.

3 Connect

Display each set of graphs with their labels.

Have students share their graphs of each set of functions.

Highlight that $j(x)$ is the only decaying function. Ask students to use the equations to explain why. Then ask them to use the graphs to explain why.

Ask, "What does the value of a in each function tell you about the situation? How does the size of the values of a and b affect the graph?" The value of a represents the initial number of patients infected or contaminated. The greater the value of a, the greater this initial number. The greater the value of b, the more quickly the virus or the contamination spread.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can experiment with changing the values of a and b in an exponential function of the form $f(x) = a \cdot b^x$ and see how the graph is affected.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code each function in a different color as they create their sketches. Have them color code the changing parameter – either a or b – in each problem to help them see the connections.

Math Language Development

MLR2: Collect and Display

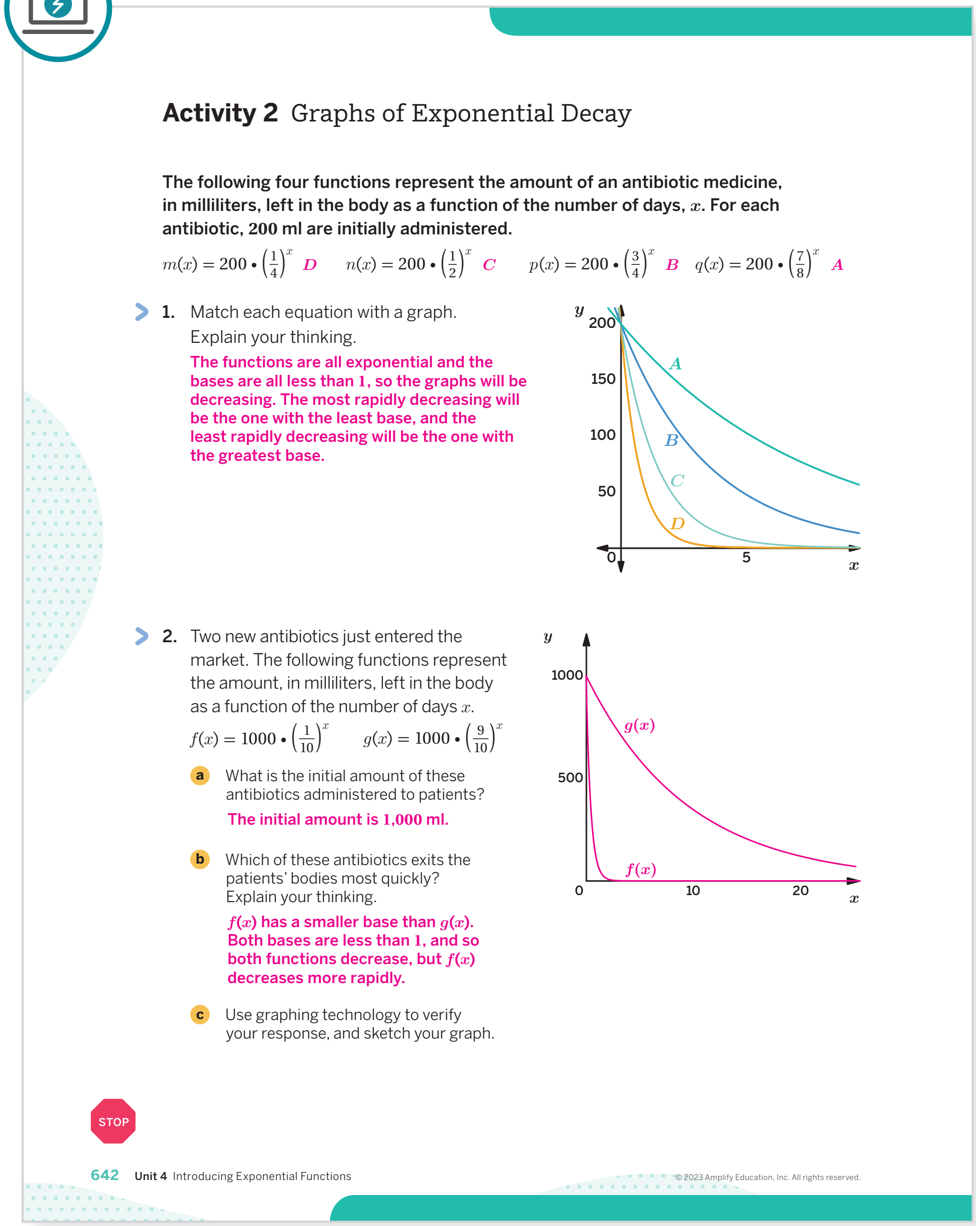
During the Connect, as students share their graphs of each set of functions, add the equations and graphs (on the same coordinate plane) to the class display. Collect the language students use to identify each equation with its graph and add this language to the class display. Students may use language such as:

- "The greatest growth factor has the steepest curve."
- "The greatest initial value has the greatest vertical intercept."

Pairs | 15 min

Activity 2 Graphs of Exponential Decay

Students examine how changing the values a and b affect the graph of an exponential decay function of the form $f(x) = a \cdot b^x$ by studying the structure of the equations and graphs.



Activity 2 Graphs of Exponential Decay

The following four functions represent the amount of an antibiotic medicine, in milliliters, left in the body as a function of the number of days, x. For each antibiotic, 200 ml are initially administered.

$m(x) = 200 \cdot \left(\frac{1}{4}\right)^x$ D $\quad n(x) = 200 \cdot \left(\frac{1}{2}\right)^x$ C $\quad p(x) = 200 \cdot \left(\frac{3}{4}\right)^x$ B $\quad q(x) = 200 \cdot \left(\frac{7}{8}\right)^x$ A

1. Match each equation with a graph. Explain your thinking.

   The functions are all exponential and the bases are all less than 1, so the graphs will be decreasing. The most rapidly decreasing will be the one with the least base, and the least rapidly decreasing will be the one with the greatest base.

2. Two new antibiotics just entered the market. The following functions represent the amount, in milliliters, left in the body as a function of the number of days x.

   $f(x) = 1000 \cdot \left(\frac{1}{10}\right)^x \qquad g(x) = 1000 \cdot \left(\frac{9}{10}\right)^x$

   a. What is the initial amount of these antibiotics administered to patients?

      The initial amount is 1,000 ml.

   b. Which of these antibiotics exits the patients' bodies most quickly? Explain your thinking.

      $f(x)$ has a smaller base than $g(x)$. Both bases are less than 1, and so both functions decrease, but $f(x)$ decreases more rapidly.

   c. Use graphing technology to verify your response, and sketch your graph.

STOP

642 Unit 4 Introducing Exponential Functions

1 Launch

Provide access to graphing technology. Ask, "Which antibiotic exits the body the most quickly? How do you know?" $m(x)$ because it has the least decay factor.

2 Monitor

Help students get started by emphasizing that the greater the decay factor, the greater the amount that *remains*.

Look for points of confusion:

- **Labeling the equations with graphs that are in the opposite order.** Ask students to evaluate each function for $x = 1$ to check their matches. $m(1) = 50$, $n(1) = 100$, $p(1) = 150$, $q(1) = 175$

Look for productive strategies:

- Using only two graphs at a time to examine the effects of changing b.

3 Connect

Display the equations with their matching graphs.

Have pairs of students share how they determined their matches.

Highlight that in exponential decay situations, where b is between 0 and 1, the closer b is to 0, the faster the graph will approach a horizontal line. In these graphs, the horizontal line is the x-axis.

Ask:

- "What would the graph of $v(x) = 200 \cdot \left(\frac{99}{100}\right)^x$ look like?" Very close to a horizontal line.
- "What would the graph of $w(x) = 200 \cdot \left(\frac{1}{100}\right)^x$ look like?" Very close to a vertical line.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For Problem 1, have students compare two graphs and two equations at a time. Provide them with a graph that only shows Graphs A and B first and only provide their corresponding equations. Then introduce the two remaining graphs and equations.

Accessibility: *Guide Processing and Visualization*

If students are struggling to compare the fractions, remind them that they can convert fractions to decimal values. They can also write equivalent fractions using a common denominator, so that they can compare the numerators.

Math Language Development

MLR8: Discussion Supports— Press for Details

During the Connect, as students share how they determined their matches, press them for details in their reasoning. For example, if a student says, "They all start at 200 and then I compared the fractions" for Problem 1, ask, "How did you compare the fractions? How did you know which graph corresponded with the greatest (or least) fraction?"

English Learners

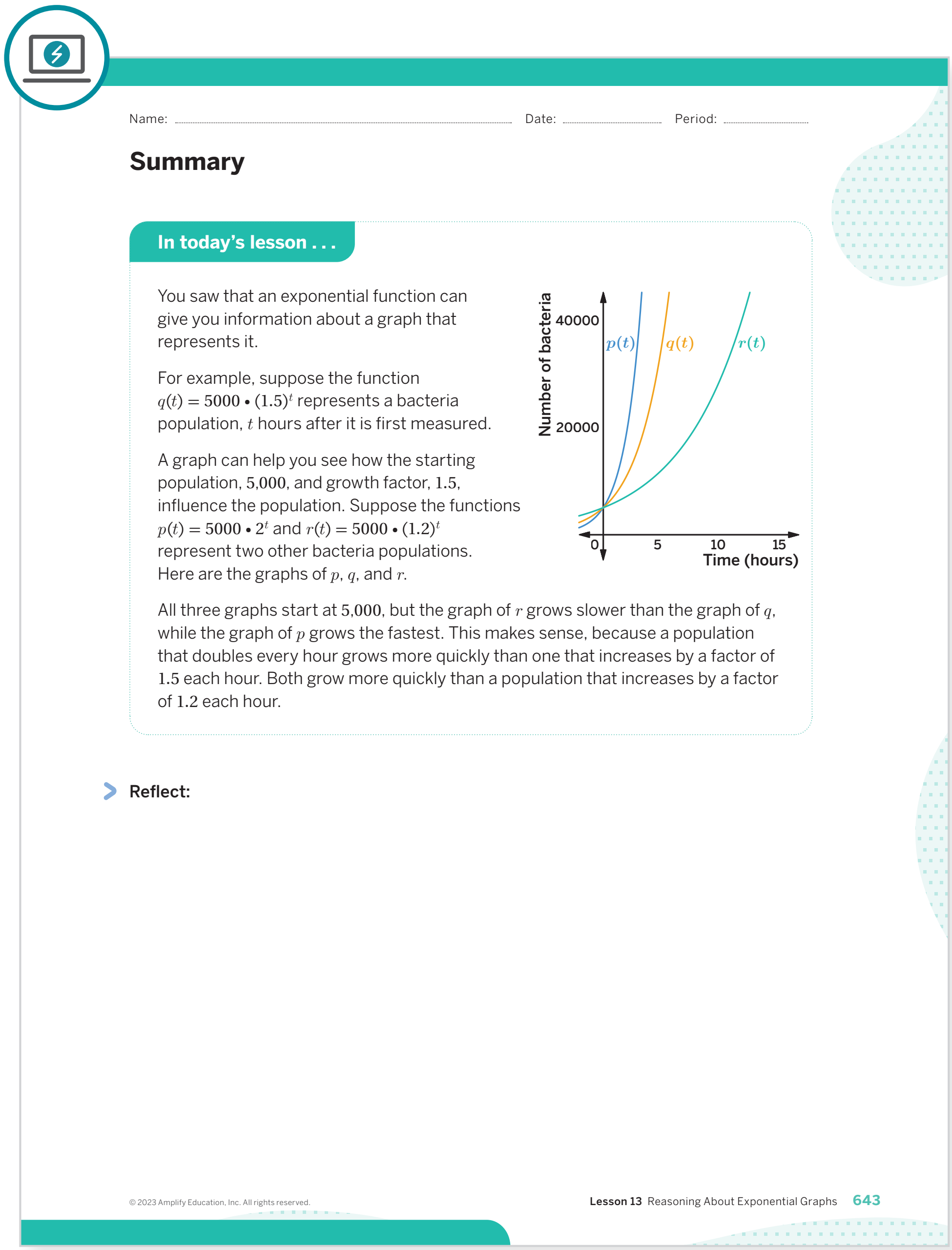
Display these sentence frames to help students organize their thinking:

- "The equation _____ matches graph _____ because . . ."
- "I noticed _____, so I knew that . . ."

Whole Class | 5 min

Summary

Review and synthesize how changing the values a and b affect the graph of an exponential function of the form $f(x) = a \cdot b^x$.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that an exponential function can give you information about a graph that represents it.

For example, suppose the function $q(t) = 5000 \cdot (1.5)^t$ represents a bacteria population, t hours after it is first measured.

A graph can help you see how the starting population, $5{,}000$, and growth factor, 1.5, influence the population. Suppose the functions $p(t) = 5000 \cdot 2^t$ and $r(t) = 5000 \cdot (1.2)^t$ represent two other bacteria populations. Here are the graphs of p, q, and r.

All three graphs start at $5{,}000$, but the graph of r grows slower than the graph of q, while the graph of p grows the fastest. This makes sense, because a population that doubles every hour grows more quickly than one that increases by a factor of 1.5 each hour. Both grow more quickly than a population that increases by a factor of 1.2 each hour.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Reasoning About Exponential Graphs 643

Synthesize

Display the graph of functions $p(t)$, $q(t)$, and $r(t)$ and their equations.

Have students share the initial value and growth or decay factor of each function.

Highlight that the *initial value* is the y-intercept.

Ask:

- "Suppose $5{,}000$ is replaced with $10{,}000$ in each function. How would this change affect the graphs representing these functions?" The initial value would change to $10{,}000$. The y-intercept would now be located at $(0, 10000)$.
- "The functions $q(t)$, $p(t)$, and $r(t)$ have different b-values. How do these values affect the graphs representing these functions?" The greater the growth factor, the steeper the graph.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

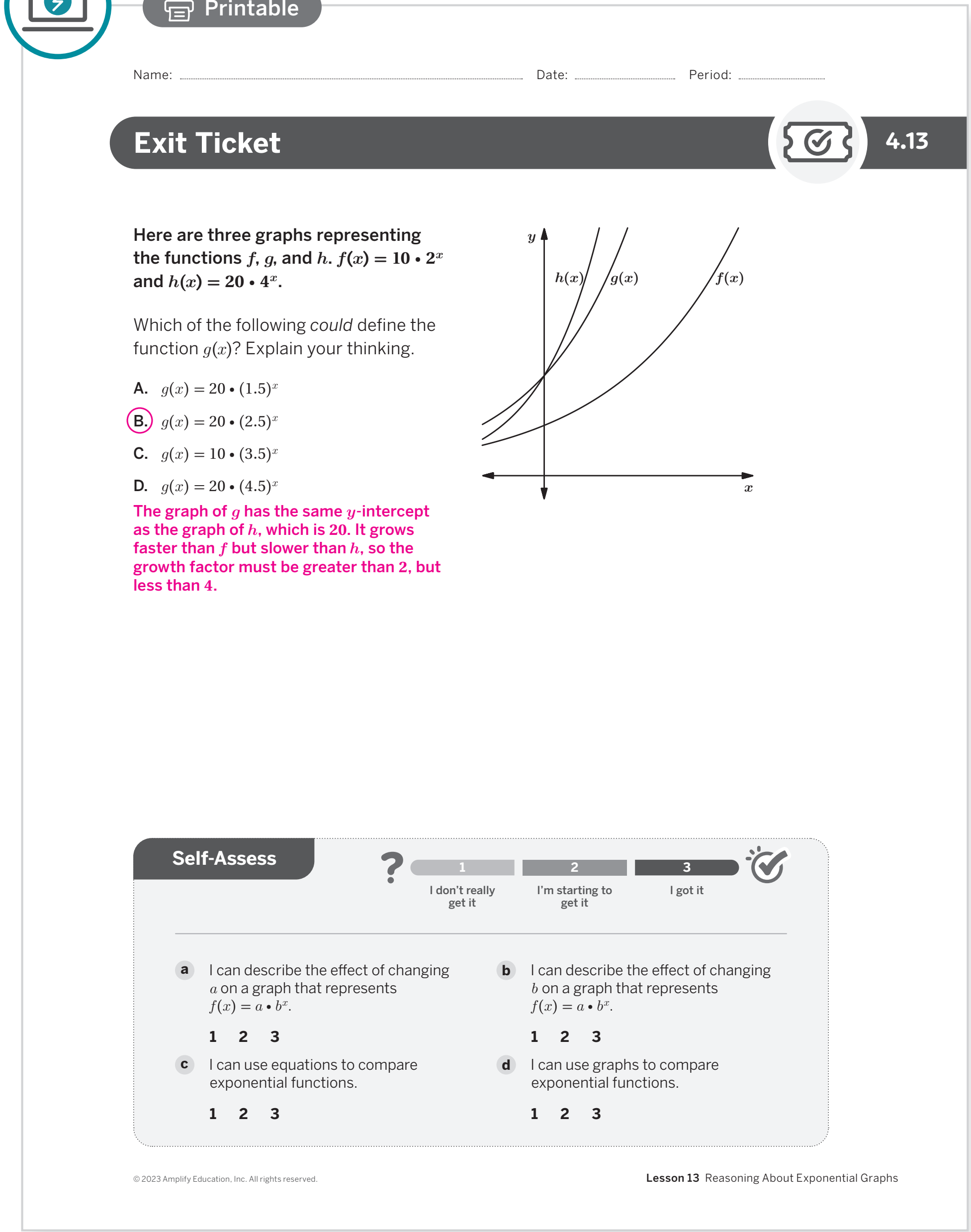
- "How can you describe in your own words the effects of changing a on the graph of an exponential function of the form $f(x) = a \cdot b^x$?"
- "How can you describe in your own words the effects of changing b on the graph of an exponential function of the form $f(x) = a \cdot b^x$?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by explaining how to use the initial value and growth factor of an exponential function to match its equation with a graph.

Printable



Name: Date: Period:

Exit Ticket 4.13

Here are three graphs representing the functions f, g, and h. $f(x) = 10 \cdot 2^x$ and $h(x) = 20 \cdot 4^x$.

Which of the following *could* define the function $g(x)$? Explain your thinking.

A. $g(x) = 20 \cdot (1.5)^x$

[✓ B.] $g(x) = 20 \cdot (2.5)^x$

C. $g(x) = 10 \cdot (3.5)^x$

D. $g(x) = 20 \cdot (4.5)^x$

The graph of g has the same y-intercept as the graph of h, which is 20. It grows faster than f but slower than h, so the growth factor must be greater than 2, but less than 4.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can describe the effect of changing a on a graph that represents $f(x) = a \cdot b^x$.
1 2 3

b. I can describe the effect of changing b on a graph that represents $f(x) = a \cdot b^x$.
1 2 3

c. I can use equations to compare exponential functions.
1 2 3

d. I can use graphs to compare exponential functions.
1 2 3

Lesson 13 Reasoning About Exponential Graphs

Success looks like . . .

- **Language Goal:** Describing how changing the values of a and b affect the graph of $f(x) = a \cdot b^x$. **(Speaking and Listening, Writing)**
- **Language Goal:** Using equations and graphs to compare exponential functions. **(Speaking and Listening, Reading and Writing)**
  - » Comparing graphs of functions f, g, and h, given the equation for f and h.

Suggested next steps

If students select Equations A or D, consider:

- Reviewing comparing growth factors and ordering them from growing least quickly to most quickly.
- Having students evaluate each function at $x = 1$ to verify the matching of functions to graphs.
- Reviewing Activity 1, Problem 1.

If students select Equation C, consider:

- Reviewing how to identify and calculate the initial value and where it is graphed. The initial value is the value of a and can be determined by evaluating the function at 0. It is graphed as the y-intercept.
- Reviewing Activity 1, Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students explored how changing the values of a and b affect the graph of an exponential function. How did that build on earlier understandings of negating a or replacing b with its reciprocal from earlier in this unit?
- Thinking about the questions you asked students today and how students responded, which question(s) were the most efficient? Which questions might you change the next time you teach this lesson?

MLR Math Language Development

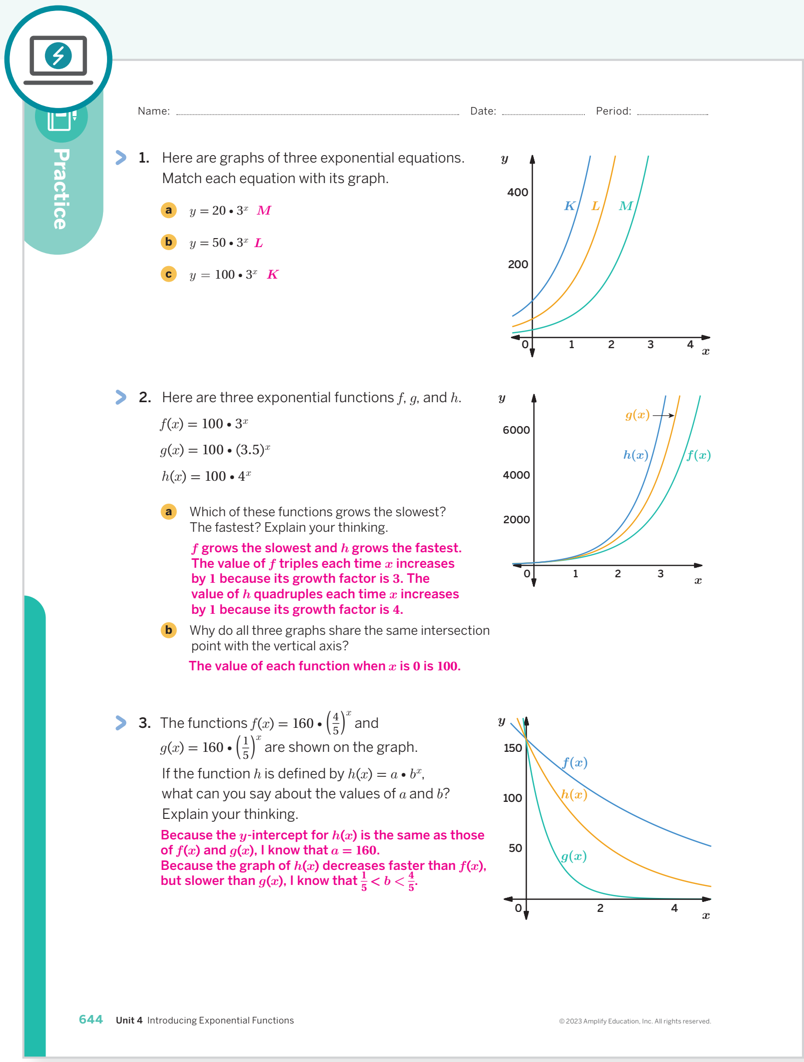
Language Goal: Describing how changing the values of a and b affect the graph of $f(x) = a \cdot b^x$.

Reflect on students' language development toward this goal.

- How have students progressed in their descriptions of how changing the parameters of an exponential function changes the graph? Do they use terms and phrases such as *initial value and growth factor*?
- How did using the language routines in this lesson help students use math language to describe how changing the parameters affect the graph of an exponential function? Would you change anything the next time you use these routines?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Here are graphs of three exponential equations. Match each equation with its graph.

   a. $y = 20 \cdot 3^x$ M

   b. $y = 50 \cdot 3^x$ L

   c. $y = 100 \cdot 3^x$ K

2. Here are three exponential functions f, g, and h.

   $f(x) = 100 \cdot 3^x$

   $g(x) = 100 \cdot (3.5)^x$

   $h(x) = 100 \cdot 4^x$

   a. Which of these functions grows the slowest? The fastest? Explain your thinking.

   f grows the slowest and h grows the fastest. The value of f triples each time x increases by 1 because its growth factor is 3. The value of h quadruples each time x increases by 1 because its growth factor is 4.

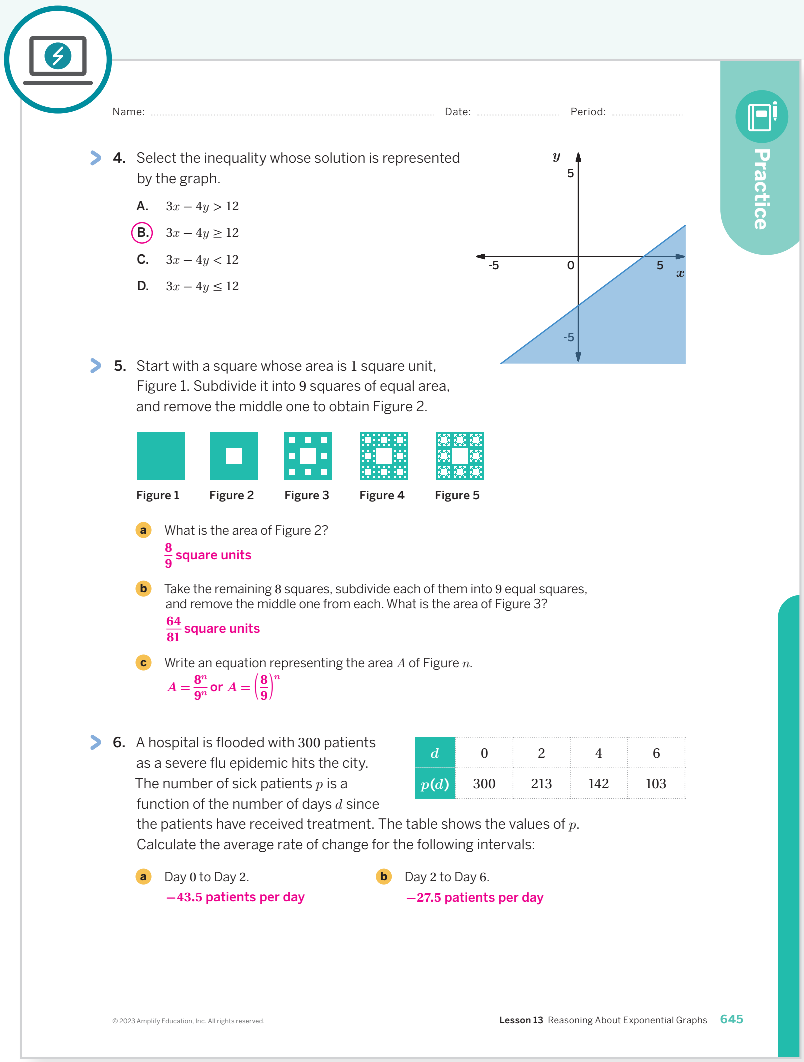
   b. Why do all three graphs share the same intersection point with the vertical axis?

   The value of each function when x is 0 is 100.

3. The functions $f(x) = 160 \cdot \left(\frac{4}{5}\right)^x$ and $g(x) = 160 \cdot \left(\frac{1}{5}\right)^x$ are shown on the graph.

   If the function h is defined by $h(x) = a \cdot b^x$, what can you say about the values of a and b? Explain your thinking.

   Because the y-intercept for $h(x)$ is the same as those of $f(x)$ and $g(x)$, I know that $a = 160$. Because the graph of $h(x)$ decreases faster than $f(x)$, but slower than $g(x)$, I know that $\frac{1}{5} < b < \frac{4}{5}$.



Name: ________ Date: ________ Period: ________

4. Select the inequality whose solution is represented by the graph.

   A. $3x - 4y > 12$

   [✓ B.] $3x - 4y \geq 12$

   C. $3x - 4y < 12$

   D. $3x - 4y \leq 12$

5. Start with a square whose area is 1 square unit, Figure 1. Subdivide it into 9 squares of equal area, and remove the middle one to obtain Figure 2.

   a. What is the area of Figure 2?

   $\frac{8}{9}$ square units

   b. Take the remaining 8 squares, subdivide each of them into 9 equal squares, and remove the middle one from each. What is the area of Figure 3?

   $\frac{64}{81}$ square units

   c. Write an equation representing the area A of Figure n.

   $A = \frac{8^n}{9^n}$ or $A = \left(\frac{8}{9}\right)^n$

6. A hospital is flooded with 300 patients as a severe flu epidemic hits the city. The number of sick patients p is a function of the number of days d since the patients have received treatment. The table shows the values of p. Calculate the average rate of change for the following intervals:

d	0	2	4	6
$p(d)$	300	213	142	103

   a. Day 0 to Day 2.

   -43.5 patients per day

   b. Day 2 to Day 6.

   -27.5 patients per day

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 15	2
	5	Unit 4 Lesson 6	2
Formative ⚡	6	Unit 4 Lesson 14	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 14**

Looking at Rates of Change

Let's calculate average rates of change for exponential functions.

Focus

Goals

1. Calculate the average rate of change of a function over a specified interval.
2. **Language Goal:** Explain how well given rates of change reflect the changes in an exponential function. **(Speaking and Listening, Writing)**
3. Understand that an exponential function has different average rates of change for different intervals.

Coherence

Today

Students practice finding the average rate of change of an exponential function on specific intervals using multiple representations of the function, and explain how well the average rate of change describes the change occuring in the function.

Previously

Students used the slope formula to determine the average rate of change of a linear function.

Coming Soon

In future lessons, students will compare different types of functions, including quadratics, and will be able to calculate and contextualize the average rate of change of different functions over the same intervals.

Rigor

- Students develop their **conceptual understanding** of rate of change, taking what they previously learned about linear functions and using it to better understand exponential functions.
- Students develop their **procedural fluency** in calculating slopes and rates of change, given a graph or table.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1 (optional)	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	25 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils (as needed)
- scientific calculators
- straightedges or rulers

Math Language Development

Review words

- *average rate of change*

Amps | Featured Activity

Activity 2

Interactive Graphs

Students use interactive tools to compare the average rates of change over different intervals of an exponential function.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

While students have used rate of change with other models, the abstract reasoning behind a non-constant rate of change might cause some unease. As students compare the rates of change for different parts of the graph, the quantitative reasoning might connect with its interpretation so that now a non-constant rate of change makes sense. This understanding will better help students see when and how to apply an exponential model.

Modifications to Pacing

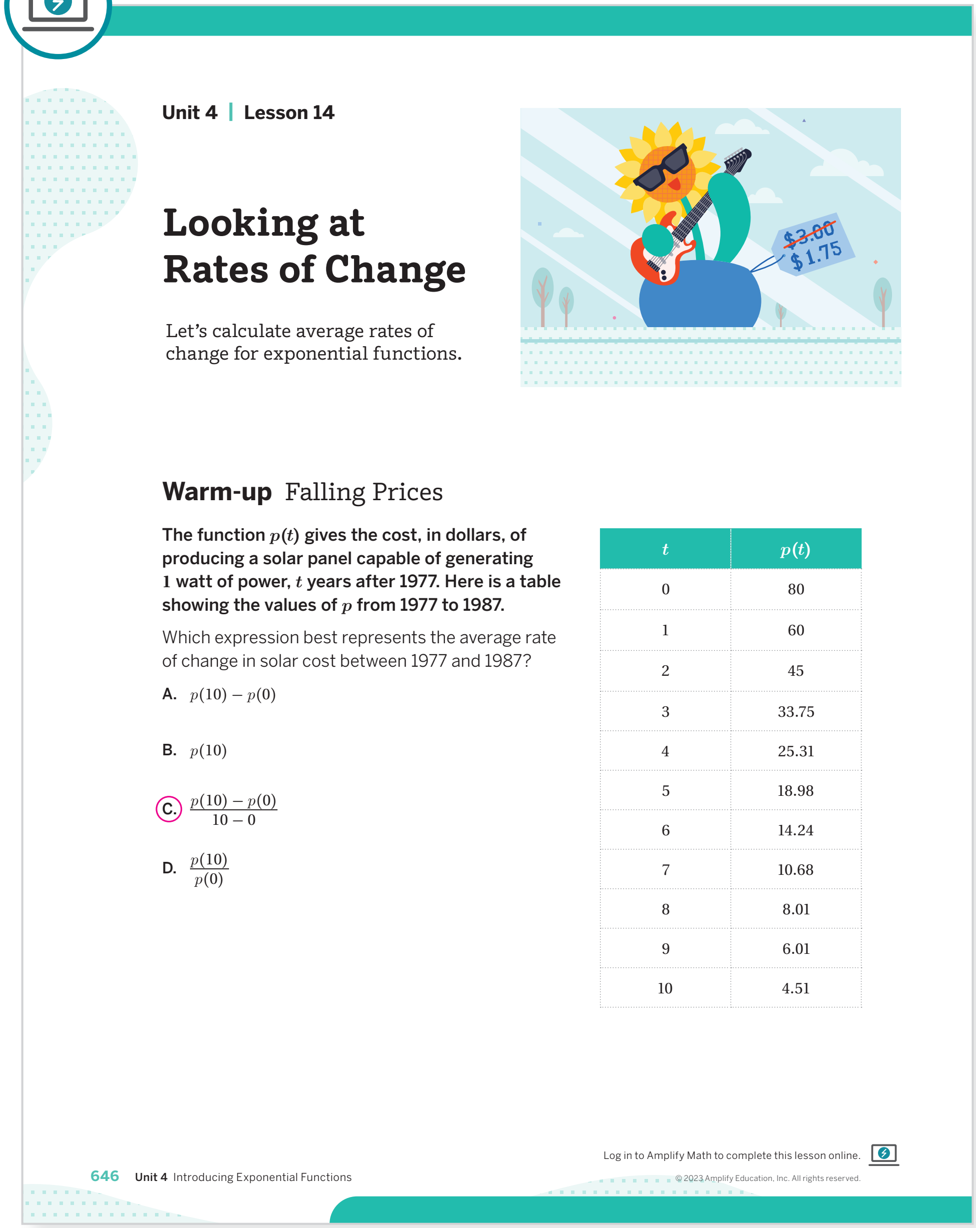
You may want to consider these additional modifications if you are short on time.

- Omit optional **Activity 1**.
- In **Activity 2**, omit Problem 1b and Problem 4.

Independent | 5 min

Warm-up Falling Prices

Students activate prior knowledge by determining how to calculate the average rate of change of a function — over a specified interval — by using a table of values.



Unit 4 | Lesson 14

Looking at Rates of Change

Let's calculate average rates of change for exponential functions.

Warm-up Falling Prices

The function $p(t)$ gives the cost, in dollars, of producing a solar panel capable of generating 1 watt of power, t years after 1977. Here is a table showing the values of p from 1977 to 1987.

Which expression best represents the average rate of change in solar cost between 1977 and 1987?

A. $p(10) - p(0)$

B. $p(10)$

[✓ C.] $\frac{p(10) - p(0)}{10 - 0}$

D. $\frac{p(10)}{p(0)}$

t	$p(t)$
0	80
1	60
2	45
3	33.75
4	25.31
5	18.98
6	14.24
7	10.68
8	8.01
9	6.01
10	4.51

Log in to Amplify Math to complete this lesson online.

646 Unit 4 Introducing Exponential Functions

1 Launch

Conduct the ***Notice and Wonder*** routing using the table of values. Have students describe how the values of $p(t)$ change as the value of t increases.

2 Monitor

Help students get started by asking, "How do you determine the rate of change in a linear function?" Determine the slope.

Look for points of confusion:

- **Mistaking *average* for *average rate of change*.** Remind students that the average rate of change involves comparing how one quantity changes as another quantity increases by 1, (e.g. cost per year).

Look for productive strategies:

- Using the slope formula.

3 Connect

Display the table and the answer choices.

Have students share which answer choice shows the correct expression for determining the average rate of change. Then select students to describe what is meant by each of the other choices in this context.

Highlight that the average rate of change of a function over a specified interval is determined by the slope of a line that passes through the endpoints of the function over that interval. The average rate of change for this function is -7.55, meaning the price decreased $7.55 each year, on average, between 1977 and 1987.

Power-up

To power up students' ability to determine the average rate of change from a table, have students complete:

Recall that to determine the average rate of change between two points you evaluate the ratio $\frac{y_2 - y_1}{x_2 - x_1}$.

The table represents the number of students at study hall each day leading up to a test. Calculate the average rate of change for the given intervals.

a Day 1 to Day 3
4.5 students per day

b Day 3 to Day 4
8 students per day

d	1	3	4
$s(d)$	3	12	20

Use: Before the Warm-up

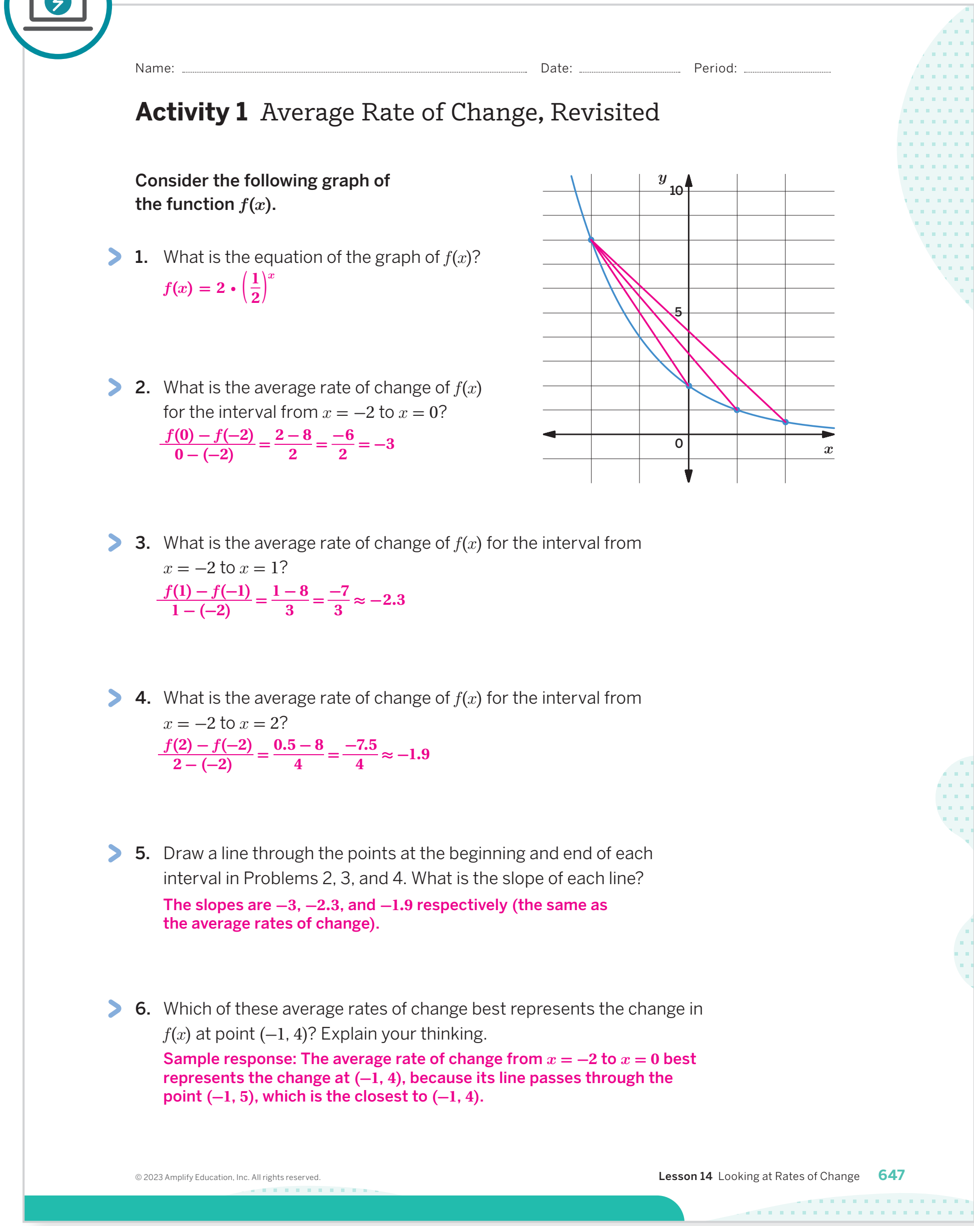
Informed by: Performance on Lesson 13, Practice Problem 6

Optional

Pairs | 10 min

Activity 1 Average Rate of Change, Revisited

Students activate prior knowledge by determining the average rate of change of a function over specific intervals.



Name: Date: Period:

Activity 1 Average Rate of Change, Revisited

Consider the following graph of the function $f(x)$.

1. What is the equation of the graph of $f(x)$?
   $f(x) = 2 \cdot \left(\frac{1}{2}\right)^x$

2. What is the average rate of change of $f(x)$ for the interval from $x = -2$ to $x = 0$?
   $\frac{f(0) - f(-2)}{0 - (-2)} = \frac{2 - 8}{2} = \frac{-6}{2} = -3$

3. What is the average rate of change of $f(x)$ for the interval from $x = -2$ to $x = 1$?
   $\frac{f(1) - f(-1)}{1 - (-2)} = \frac{1 - 8}{3} = \frac{-7}{3} \approx -2.3$

4. What is the average rate of change of $f(x)$ for the interval from $x = -2$ to $x = 2$?
   $\frac{f(2) - f(-2)}{2 - (-2)} = \frac{0.5 - 8}{4} = \frac{-7.5}{4} \approx -1.9$

5. Draw a line through the points at the beginning and end of each interval in Problems 2, 3, and 4. What is the slope of each line?
   The slopes are -3, -2.3, and -1.9 respectively (the same as the average rates of change).

6. Which of these average rates of change best represents the change in $f(x)$ at point $(-1, 4)$? Explain your thinking.
   Sample response: The average rate of change from $x = -2$ to $x = 0$ best represents the change at $(-1, 4)$, because its line passes through the point $(-1, 5)$, which is the closest to $(-1, 4)$.

Lesson 14 Looking at Rates of Change 647

1 Launch

Display the graph. Elicit the initial value and growth factor of the function and the general form of an exponential equation ($y = a \cdot b^x$) from students. Distribute scientific calculators and straightedges.

2 Monitor

Help students get started by reminding them that an interval is a range of numbers.

Look for points of confusion:

- **Calculating the average rate of change incorrectly.** Ask students to determine the slope between the two endpoints of the given intervals.

Look for productive strategies:

- Calculating the slope between the endpoints of the given intervals.

3 Connect

Have students share their responses to Problem 5 by drawing the three lines on the displayed graph. Discuss why the line from $x = -2$ to $x = 0$ best represents the change at point $(-1, 4)$ and which line, if any, can be used to predict other points on the graph of the function.

Highlight that determining the average rate of change of a function over an interval is the same as determining the slope of the line that passes through the two endpoints of the interval. Show how students can use repeated reasoning of their average rate of change calculations to illustrate this point.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously learned how to determine the average rate of change over a specified interval for a nonlinear function. Emphasize that the average rate of change for a nonlinear function over a specified interval is the same as the slope of the line that passes through the endpoints of the function for that interval.

Display the formula for the average rate of change over the domain interval (a, b): $\frac{f(b) - f(a)}{b - a}$

Accessibility: *Guide Processing and Visualization*

Consider providing students with a table, such as the following, to scaffold the steps needed to determine the average rate of change over the domain interval (a, b).

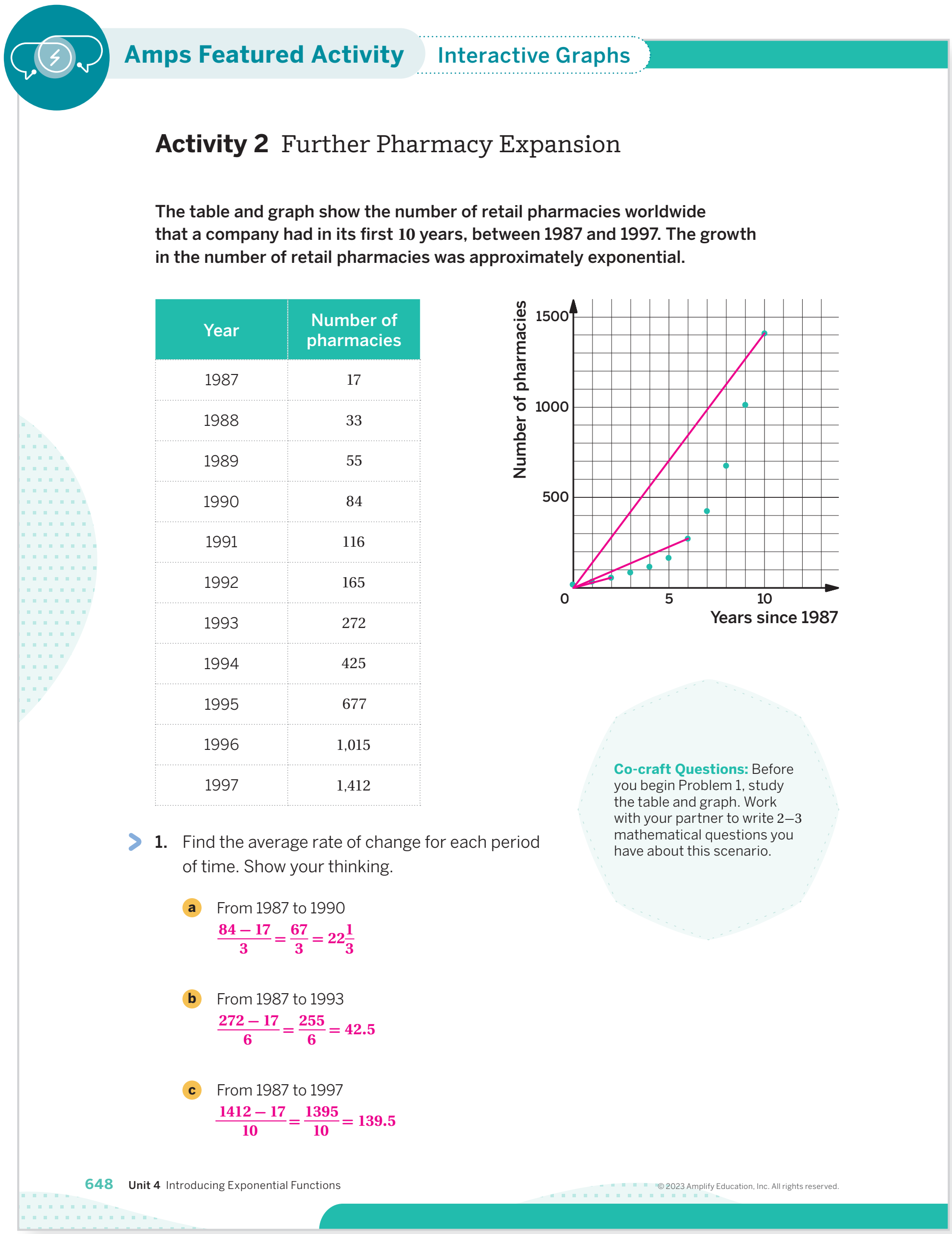
a, b	$f(b)$	$f(a)$	$f(b) - f(a)$	$\frac{f(b) - f(a)}{b - a}$

Pairs | 25 min

Activity 2 Further Pharmacy Expansion

Students explore average rates of change in an exponential growth function to observe how exponential functions have different rates of change over different intervals.

Amps Featured Activity Interactive Graphs



Activity 2 Further Pharmacy Expansion

The table and graph show the number of retail pharmacies worldwide that a company had in its first 10 years, between 1987 and 1997. The growth in the number of retail pharmacies was approximately exponential.

Year	Number of pharmacies
1987	17
1988	33
1989	55
1990	84
1991	116
1992	165
1993	272
1994	425
1995	677
1996	1,015
1997	1,412

Co-craft Questions: Before you begin Problem 1, study the table and graph. Work with your partner to write 2–3 mathematical questions you have about this scenario.

1. Find the average rate of change for each period of time. Show your thinking.

   a. From 1987 to 1990

   $\frac{84-17}{3}=\frac{67}{3}=22\frac{1}{3}$

   b. From 1987 to 1993

   $\frac{272-17}{6}=\frac{255}{6}=42.5$

   c. From 1987 to 1997

   $\frac{1412-17}{10}=\frac{1395}{10}=139.5$

648 Unit 4 Introducing Exponential Functions

1 Launch

Read the scenario aloud. Make sure students understand what is meant by values of x on the graph (years since 1987). Distribute calculators and straightedges.

2 Monitor

Help students get started by asking them to make sense of the table and its corresponding graph.

Look for points of confusion:

- **Struggling to describe their observations for Problems 2–4.** Prompt students to make quantitative comparisons like how much greater or how many times greater one average rate of change is relative to the others.

Look for productive strategies:

- Writing what each average rate of change represents or using other evidence of contextualization.

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive tools to compare the average rates of change over different intervals of an exponential function.

Accessibility: *Guide Processing and Visualization*

Display the formula for the average rate of change over the domain interval (a, b): $\frac{f(b)-f(a)}{b-a}$.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory context, table, and graph. Have students work with their partner to write 2–3 mathematical questions they could ask about this situation. Sample questions shown.

- Why are the dots not connected?
- Why was there such rapid growth in the number of pharmacies?
- Will this growth continue at this rate?

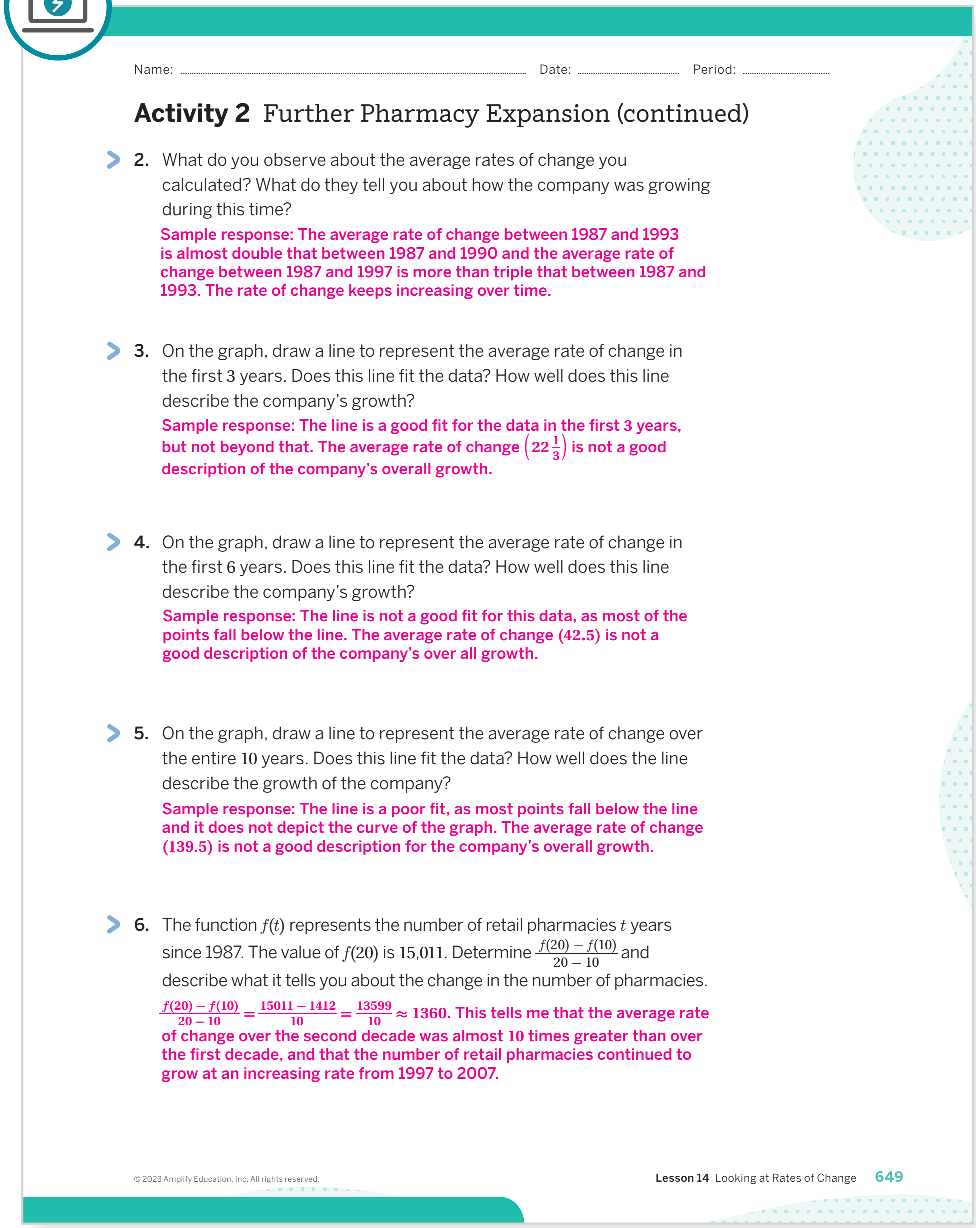
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 25 min

Activity 2 Further Pharmacy Expansion (continued)

Students explore average rates of change in an exponential growth function to observe how exponential functions have different rates of change over different intervals.



Name: ______ Date: ______ Period: ______

Activity 2 Further Pharmacy Expansion (continued)

2. What do you observe about the average rates of change you calculated? What do they tell you about how the company was growing during this time?

Sample response: The average rate of change between 1987 and 1993 is almost double that between 1987 and 1990 and the average rate of change between 1987 and 1997 is more than triple that between 1987 and 1993. The rate of change keeps increasing over time.

3. On the graph, draw a line to represent the average rate of change in the first 3 years. Does this line fit the data? How well does this line describe the company's growth?

Sample response: The line is a good fit for the data in the first 3 years, but not beyond that. The average rate of change $\left(22\frac{1}{3}\right)$ is not a good description of the company's overall growth.

4. On the graph, draw a line to represent the average rate of change in the first 6 years. Does this line fit the data? How well does this line describe the company's growth?

Sample response: The line is not a good fit for this data, as most of the points fall below the line. The average rate of change (42.5) is not a good description of the company's over all growth.

5. On the graph, draw a line to represent the average rate of change over the entire 10 years. Does this line fit the data? How well does the line describe the growth of the company?

Sample response: The line is a poor fit, as most points fall below the line and it does not depict the curve of the graph. The average rate of change (139.5) is not a good description for the company's overall growth.

6. The function $f(t)$ represents the number of retail pharmacies t years since 1987. The value of $f(20)$ is 15,011. Determine $\frac{f(20)-f(10)}{20-10}$ and describe what it tells you about the change in the number of pharmacies.

$\frac{f(20)-f(10)}{20-10}=\frac{15011-1412}{10}=\frac{13599}{10}\approx 1360$. This tells me that the average rate of change over the second decade was almost 10 times greater than over the first decade, and that the number of retail pharmacies continued to grow at an increasing rate from 1997 to 2007.

Lesson 14 Looking at Rates of Change 649

3 Connect

Display the table and graph.

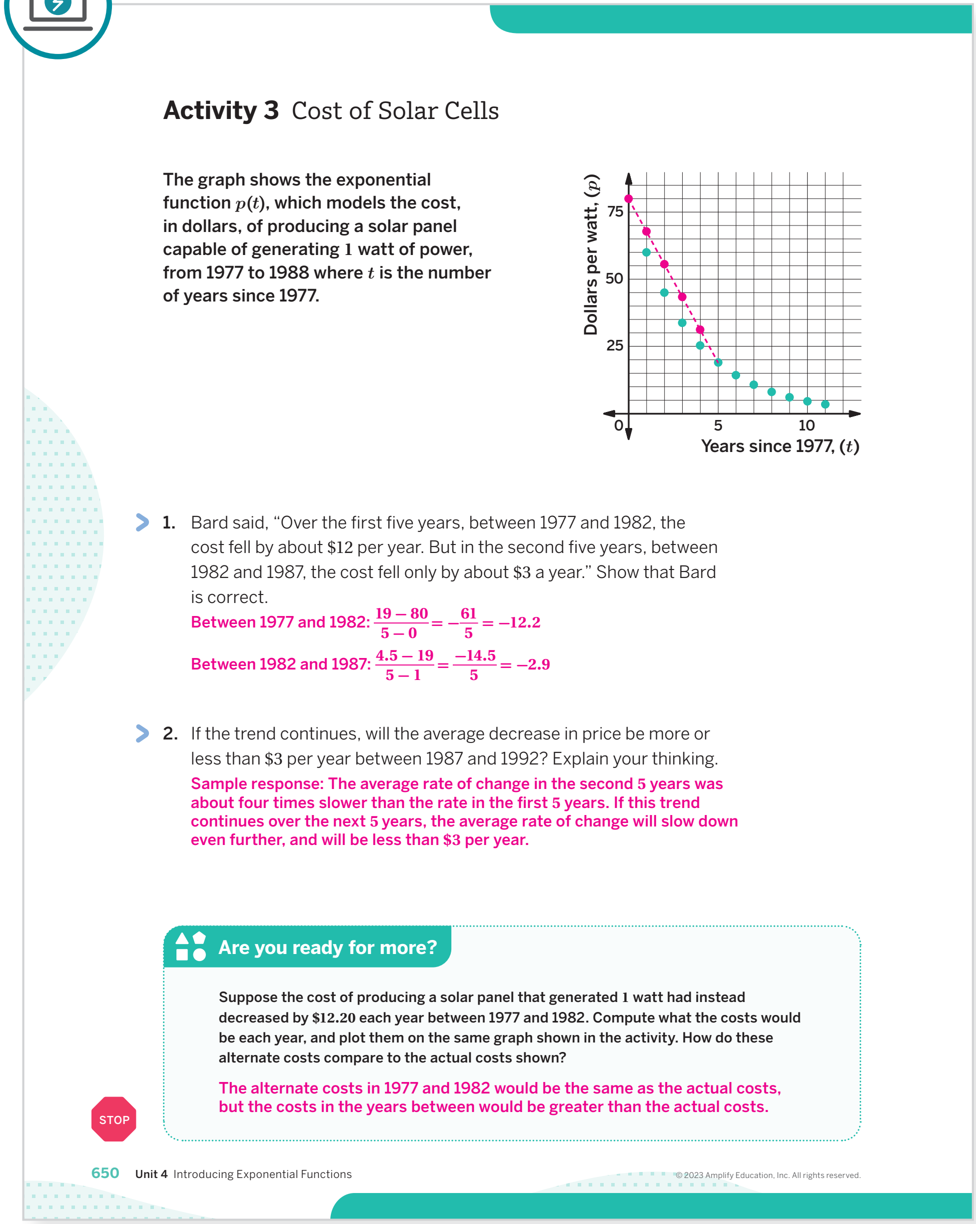
Have students share their responses to Problems 2–4 by drawing the lines on the displayed graph and labeling corresponding average rates of change (slopes). Discuss how well each line represents the points on that specific interval and on the entire 10-year interval of the function.

Ask, "Is there a single period of time whose average rate of change would well summarize how the company was growing from 1987–1992?"
No, the number of pharmacies is growing at an increasing rate.

Independent | 10 min

Activity 3 Cost of Solar Cells

Students calculate the average rate of change over different time intervals from a graph to make predictions about the average rate of change over future intervals.



Activity 3 Cost of Solar Cells

The graph shows the exponential function $p(t)$, which models the cost, in dollars, of producing a solar panel capable of generating 1 watt of power, from 1977 to 1988 where t is the number of years since 1977.

1. Bard said, “Over the first five years, between 1977 and 1982, the cost fell by about \$12 per year. But in the second five years, between 1982 and 1987, the cost fell only by about \$3 a year.” Show that Bard is correct.

   Between 1977 and 1982: $\frac{19-80}{5-0} = -\frac{61}{5} = -12.2$

   Between 1982 and 1987: $\frac{4.5-19}{5-1} = \frac{-14.5}{5} = -2.9$

2. If the trend continues, will the average decrease in price be more or less than \$3 per year between 1987 and 1992? Explain your thinking.

   Sample response: The average rate of change in the second 5 years was about four times slower than the rate in the first 5 years. If this trend continues over the next 5 years, the average rate of change will slow down even further, and will be less than \$3 per year.

Are you ready for more?

Suppose the cost of producing a solar panel that generated 1 watt had instead decreased by \$12.20 each year between 1977 and 1982. Compute what the costs would be each year, and plot them on the same graph shown in the activity. How do these alternate costs compare to the actual costs shown?

The alternate costs in 1977 and 1982 would be the same as the actual costs, but the costs in the years between would be greater than the actual costs.

STOP

650 **Unit 4** Introducing Exponential Functions

1 Launch

Read the scenario aloud. Make sure students understand what year is meant by the values of x on the graph (years since 1977). Distribute calculators and straightedges.

2 Monitor

Help students get started by asking them to make sense of the graph.

Look for points of confusion:

- **Having difficulty determining the average rates of change for each interval.** Have students draw a line representing the average rates of change (slopes) for the given intervals.

Look for productive strategies:

- Drawing lines to visualize the average rate of change for each interval.
- Using the slope formula.

3 Connect

Display the graph.

Have students share their reasoning for why Bard is correct. Invite other students who made slightly different calculations to share their thinking. Select a few students to share and explain their response to Problem 2.

Highlight that if the trend continues, the average change in cost over the next 5 years (1987–1992) will decrease even further than it did in the previous 5 years.

Ask, “If the average rate of change over the second 5 years is approximately $\frac{1}{6}$ that of the first 5 years, how can you predict what the average rate of change is over the next 5 years after that?”

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the formula for the average rate of change over the domain interval (a, b): $\frac{f(b) - f(a)}{b - a}$

Suggest that students use colored pencils to mark the intervals on the graph that correspond with the “first 5 years” and the “second 5 years.”

Math Language Development

MLR8: Discussion Supports— Press for Details

During the Connect, as students share their responses to Problem 2, press for details in their reasoning. For example, if a student says, “It will be less than \$3 per year because it looks that way on the graph”, ask:

- “Without knowing the equation, how can you know what the graph will look like for the next 5 years?”
- “How might the average rate of change help you include more detail in your reasoning and confirm your prediction?”

English Learners

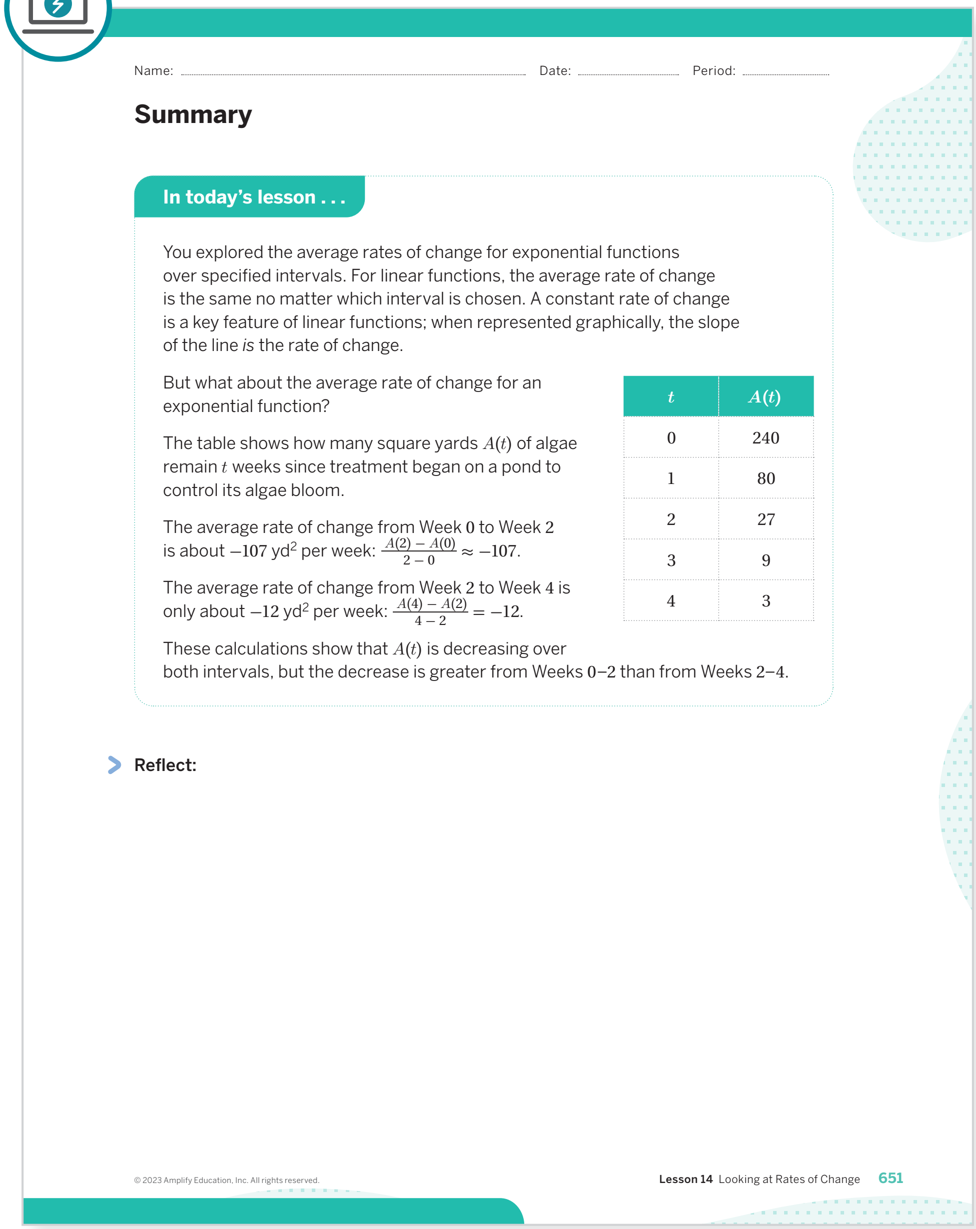
Display the following sentence frame to help students organize their thinking:

“The average decrease in price will be more/less than \$3 per year because . . .”

Whole Class | 5 min

Summary

Students review and synthesize whether the average rate of change of a function, over a specified interval, adequately describes the change occurring in the function.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored the average rates of change for exponential functions over specified intervals. For linear functions, the average rate of change is the same no matter which interval is chosen. A constant rate of change is a key feature of linear functions; when represented graphically, the slope of the line *is* the rate of change.

But what about the average rate of change for an exponential function?

The table shows how many square yards $A(t)$ of algae remain t weeks since treatment began on a pond to control its algae bloom.

t	$A(t)$
0	240
1	80
2	27
3	9
4	3

The average rate of change from Week 0 to Week 2 is about -107 yd^2 per week: $\frac{A(2)-A(0)}{2-0} \approx -107$.

The average rate of change from Week 2 to Week 4 is only about -12 yd^2 per week: $\frac{A(4)-A(2)}{4-2} = -12$.

These calculations show that $A(t)$ is decreasing over both intervals, but the decrease is greater from Weeks 0–2 than from Weeks 2–4.

Reflect:

Synthesize

Display the table.

Ask:

- "About how much was the average rate of change for the 4 week period?" It was about -59.25 yd^2 per week.
- "Does the average rate of change accurately describe how many square yards of algae is removed over the 4 weeks?" No, about 107 yd^2 is removed from Week 0 to Week 2 and only about 12 yd^2 is removed from Week 2 to Week 4.
- "Are there some weeks where the average rate of change shows how the area of the algae is decreasing?" Yes, from Week 1 to 2 it is a reasonable estimate.

Highlight that exponential functions have different rates of change over different intervals that may represent the change occuring over these intervals, but do not adequately describe the *overall change* in the function.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

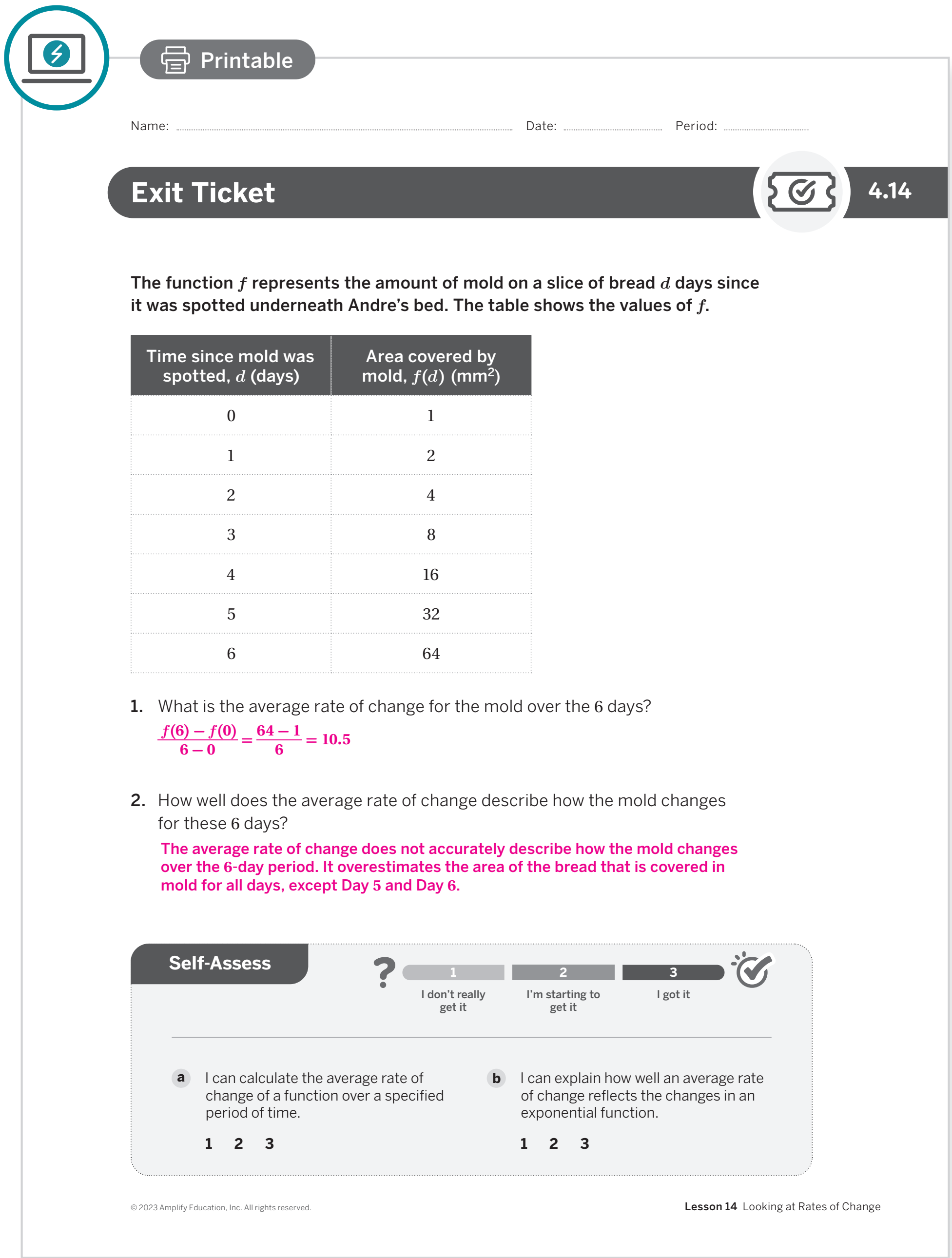
- "How is the average rate of change of an exponential function over a specified interval similar to the slope of a linear function? How is it different?"
- "When might determining the average rate of change of an exponential function over a specified interval be useful? When might it not be useful?"

Independent | 5 min

Exit Ticket

Students revisit the moldy bread scenario (from Lesson 9) and demonstrate their understanding by calculating the average rate of change of the function representing it, over a specified interval.

Printable



Name: Date: Period:

Exit Ticket 4.14

The function f represents the amount of mold on a slice of bread d days since it was spotted underneath Andre's bed. The table shows the values of f.

Time since mold was spotted, d (days)	Area covered by mold, $f(d)$ (mm^2)
0	1
1	2
2	4
3	8
4	16
5	32
6	64

1. What is the average rate of change for the mold over the 6 days?

$$\frac{f(6) - f(0)}{6 - 0} = \frac{64 - 1}{6} = 10.5$$

2. How well does the average rate of change describe how the mold changes for these 6 days?

The average rate of change does not accurately describe how the mold changes over the 6-day period. It overestimates the area of the bread that is covered in mold for all days, except Day 5 and Day 6.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can calculate the average rate of change of a function over a specified period of time.

1 2 3

b I can explain how well an average rate of change reflects the changes in an exponential function.

1 2 3

Success looks like . . .

- **Goal:** Calculating the average rate of change of a function over a specified interval.
  - » Calculating the average rate of change of the mold over 6 days in Problem 1.
- **Language Goal:** Explaining how well given rates of change reflect the changes in an exponential function. **(Speaking and Listening, Writing)**
  - » Explaining how the average rate of change calculated in Problem 1 describes how the mold changes in Problem 2.
- **Goal:** Understanding that an exponential function has different average rates of change for different intervals.

Suggested next steps

If students are unable to determine the average rate of change in Problem 1, consider:

- Having them practice determining the slope between two points, emphasizing that this is the average rate of change over that interval.
- Reviewing Activity 1 and Activity 2.
- Assigning Practice Problem 1.

If students are unable to interpret the meaning of the average rate of change in Problem 2 in context, consider:

- Having students draw the lines representing the average rates of change over different intervals of the graphs of different exponential functions. Then have them compare the points on the function(s) to the points on the line(s).
- Reviewing Activity 1 and Activity 2.
- Assigning Practice Problem 3.

Professional Learning

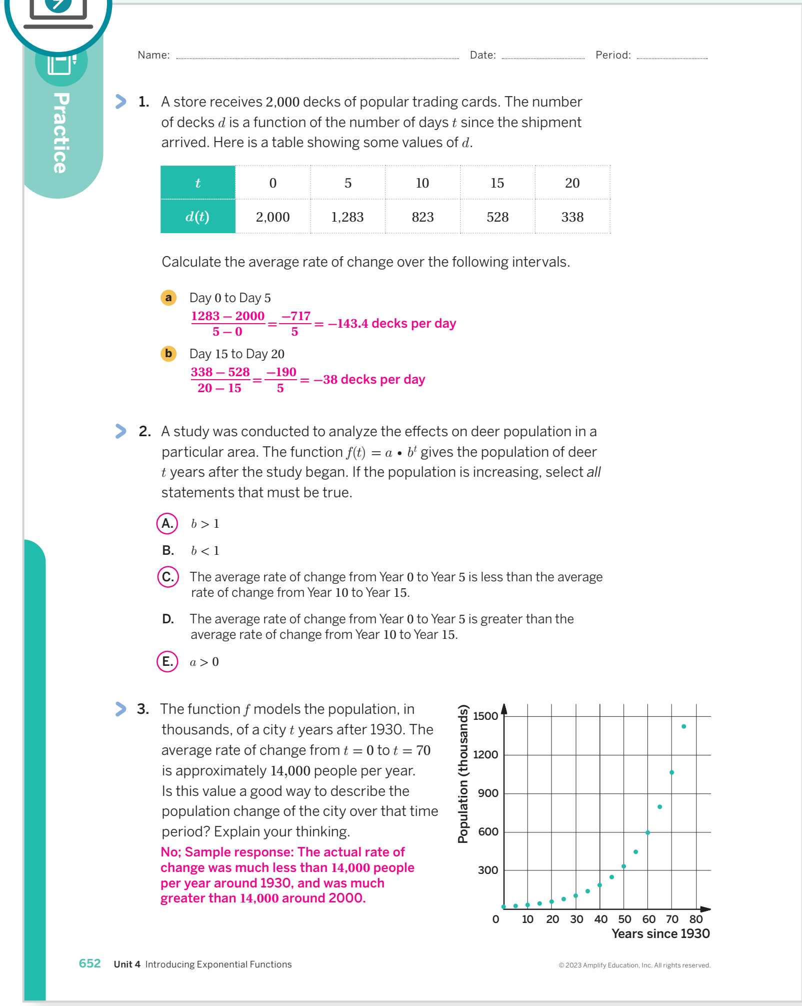
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students determined the average rate of change of an exponential function over specified intervals. How did that build on earlier understandings of the constant of change and slope of a linear function?
- How well do you think your students understand that an exponential function does *not* have a constant rate of change? Which questions might you change the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. A store receives 2,000 decks of popular trading cards. The number of decks d is a function of the number of days t since the shipment arrived. Here is a table showing some values of d.

t	0	5	10	15	20
$d(t)$	2,000	1,283	823	528	338

Calculate the average rate of change over the following intervals.

a. Day 0 to Day 5

$\frac{1283 - 2000}{5 - 0} = \frac{-717}{5} = -143.4$ **decks per day**

b. Day 15 to Day 20

$\frac{338 - 528}{20 - 15} = \frac{-190}{5} = -38$ **decks per day**

2. A study was conducted to analyze the effects on deer population in a particular area. The function $f(t) = a \bullet b^t$ gives the population of deer t years after the study began. If the population is increasing, select *all* statements that must be true.

[✓ A.] $b > 1$

B. $b < 1$

[✓ C.] The average rate of change from Year 0 to Year 5 is less than the average rate of change from Year 10 to Year 15.

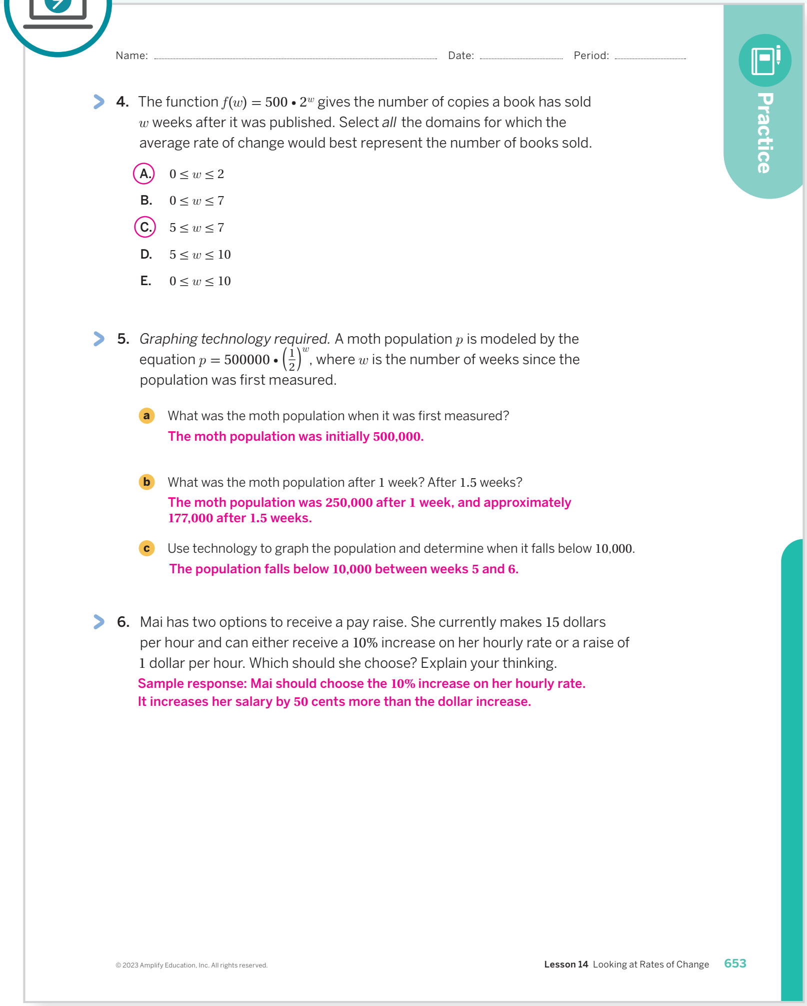
D. The average rate of change from Year 0 to Year 5 is greater than the average rate of change from Year 10 to Year 15.

[✓ E.] $a > 0$

3. The function f models the population, in thousands, of a city t years after 1930. The average rate of change from $t = 0$ to $t = 70$ is approximately 14,000 people per year. Is this value a good way to describe the population change of the city over that time period? Explain your thinking.

No; Sample response: The actual rate of change was much less than 14,000 people per year around 1930, and was much greater than 14,000 around 2000.

652 **Unit 4** Introducing Exponential Functions



Name: ______ Date: ______ Period: ______

4. The function $f(w) = 500 \bullet 2^w$ gives the number of copies a book has sold w weeks after it was published. Select *all* the domains for which the average rate of change would best represent the number of books sold.

[✓ A.] $0 \le w \le 2$

B. $0 \le w \le 7$

[✓ C.] $5 \le w \le 7$

D. $5 \le w \le 10$

E. $0 \le w \le 10$

5. *Graphing technology required.* A moth population p is modeled by the equation $p = 500000 \bullet \left(\frac{1}{2}\right)^w$, where w is the number of weeks since the population was first measured.

a. What was the moth population when it was first measured?

The moth population was initially 500,000.

b. What was the moth population after 1 week? After 1.5 weeks?

The moth population was 250,000 after 1 week, and approximately 177,000 after 1.5 weeks.

c. Use technology to graph the population and determine when it falls below 10,000.

The population falls below 10,000 between weeks 5 and 6.

6. Mai has two options to receive a pay raise. She currently makes 15 dollars per hour and can either receive a 10% increase on her hourly rate or a raise of 1 dollar per hour. Which should she choose? Explain your thinking.

Sample response: Mai should choose the 10% increase on her hourly rate. It increases her salary by 50 cents more than the dollar increase.

Lesson 14 Looking at Rates of Change 653

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 3	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 11	3
	5	Unit 4 Lesson 10	2
Formative	6	Unit 4 Lesson 15	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 4

Percent Growth and Decay

Whole Class

In this Sub-Unit, students build on their understanding of percent change from middle school to apply percent change to exponential functions. They explore repeated percent change, known as *compounding*, and distinguish between growth factors and growth rates.

SUB-UNIT

4 Percent Growth and Decay

Narrative Connections

Want to be CEO for a day?

Imagine being the CEO of a biotech company. Sure, there may be fancy suits, luxurious boardrooms, and a small army of assistants at your beck and call.

But it's also a high-stakes job overseeing the development of new medications, vaccines, and medical devices. To date, the global pharmaceutical industry is worth almost $1 trillion. And when people's lives depend on your company's products, difficult questions can come up.

Which diseases should you target?

How much should a life-saving drug cost?

How do you help your company grow?

What is your responsibility to your shareholders?

These questions can be a heavy burden. Without understanding what the answers might mean — in terms of dollars and cents, market share captured, or lives saved — how would you know you made the best choice?

Whether it is the effect of a new drug on a patient population or the impact on your company's share price, being able to calculate *rates of change* gives you the power to plan ahead and make smart decisions for both your boardroom and your community.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore exponential rates of change — within medical contexts— in the following places:

- **Lesson 15, Activity 2:** The Global Burden of Disease
- **Lesson 16, Activity 1:** Cost of Prescriptions
- **Lesson 17, Activity 2:** Earning Interest
- **Lesson 18, Activity 2:** Simulating an Epidemic

Optional

UNIT 4 | **LESSON 15**

Recalling Percent Change

Let’s see what happens when you change a number by a percentage.

Focus

Goals

1. Calculate a final amount when given a starting amount and a percent increase or decrease.
2. Write expressions using only multiplication to represent a percent increase or decrease.

Coherence

Today

This is an optional lesson; revisit the Pre-Unit Readiness Assessment to determine whether students need this lesson. Students will write expressions in different forms representing percent increase and decrease. They may use a combination of operations while working towards using only multiplication in Activities 1, 2, and 3.

Previously

In Grade 7, students solved multi-step real-world and mathematical problems involving percentages, including expressing percentages as fraction and decimal values.

Coming Soon

Students will calculate the result of applying percent increase repeatedly, use graphs to illustrate and compare situations with different percent increases, and write an expression of the form $(1 + r)^n$ to represent percent increase applied n times.

Rigor

- Students **apply** prior understanding of percent change to scenarios involving taxes and percent growth and decay.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

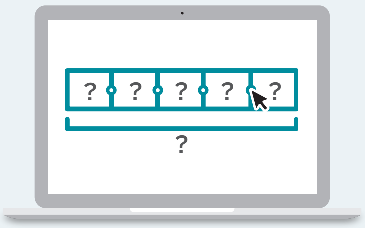
Review words

- *percent*
- *percent change*

Amps | **Featured Activity**

Activity 2
View Work From Previous Slides

As students model percentages that increase or decrease, their work is carried from one slide to the next.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students explore the mathematical concepts of percent increase and decrease through the eyes of global diseases, and, if they do not see themselves as victims of such diseases, they might not show any concern for others who are burdened by it. Encourage students to use their results to support an argument for empathizing with others. The mathematics should support their argument and remind them of the importance of taking on other people's perspectives in order to help them.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, omit Problems 1 and 2.
- In **Activity 3**, have students omit the fifth row of the table.

Independent | 5 min

Warm-up Same Symptoms, Different Prices

Students review strategies for solving percent change problems, preparing them to apply this understanding to situations of exponential change.

Unit 4 | **Lesson 15**

Recalling Percent Change

Let's see what happens when you change a number by a percentage.

Warm-up Same Symptoms, Different Prices

For each problem, show your thinking.

1. The cost of a generic brand of medication is 60% of the cost of a similar commercial brand. If the commercial brand costs \$15.99, what is the price of the generic brand?
   \$9.59, because $0.6 \cdot 15.99 \approx 9.59$.

2. The price of a commercial brand of medication is 30% more than a similar generic brand. If the generic brand costs \$14, what is the price of the commercial brand?
   \$18.20, because $1.3 \cdot 14 = 18.2$.

3. A generic prescription of medication costs 35% less than a similar commercial brand. The commercial brand costs \$49. How much does the generic brand cost?
   \$31.85, because $0.65 \cdot 49 = 31.85$.

Log in to Amplify Math to complete this lesson online.

656 **Unit 4** Introducing Exponential Functions

1 Launch

Have students read each problem and then circle or highlight terms that imply an operation.

2 Monitor

Help students get started by having them identify the *whole*, *part*, and *percent*. Suggest they use a diagram or graphic organizer.

Look for points of confusion:

- **Struggling with operations involving the percent symbol.** Remind them that a percent is a fraction with a denominator of 100 or a decimal to the hundredths place.

Look for productive strategies:

- Expressing the percentages as a fraction or decimal.

3 Connect

Have students share strategies for solving Problems 2 and 3. Select students who used different strategies for expressing the percentages in each problem to share. For example, students may have written $49 \cdot 0.65$ or $49 \cdot \left(\frac{13}{20}\right)$ for Problem 3. Other students may have written $49 \cdot 0.35$ and then subtracted from 49.

Highlight equivalent expressions for "30% more than" and "35% less than" and why they are equivalent. For example, "30% more than" is equivalent to "130% of." If students used tape diagrams, connect the relationship between the diagram and numerical connections.

Display the equations $14 \cdot (1.3) = 14 \cdot \left(\frac{13}{10}\right)$ and $49 \cdot (0.65) = 49 \cdot \left(\frac{13}{20}\right)$ and ask, "Why are these number sentences true?"

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their strategies for solving the problems, ask them how they used key words from the text to help them determine their strategies. Ask, "How does the phrase _____*% of the cost* compare to the phrase _____*% more than* or _____*% less than* for these problems? How did you use these phrases to help you solve the problems?"

Power-up

To power up students' ability to solve problems involving percent change, have students complete:

Recall that, in order to represent a percentage as its decimal equivalent, you divide the percentage by 100. For example, the decimal equivalent of 425% is $425 \div 100 = 4.25$.

Select *all* of the expressions that represent 30% more than x.

[✓ **A.**] $1.3x$
B. $30 + x$
C. $0.30 + x$
[✓ **D.**] $0.3x + x$

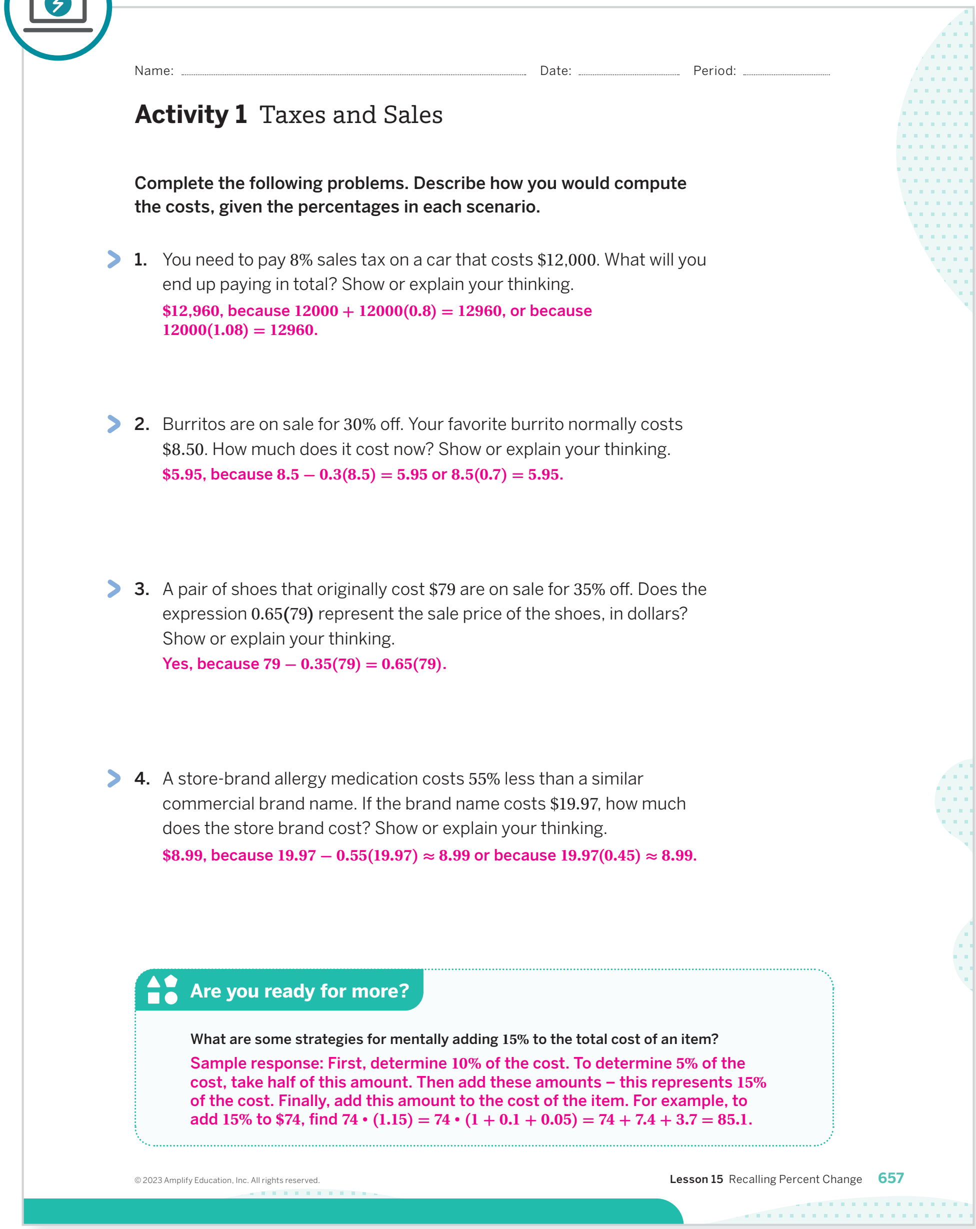
Use: Before the Warm-up

Informed by: Performance on Lesson 14, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 6 and 7

Pairs | 10 min

Activity 1 Taxes and Sales

Students solve problems about percent increase and decrease to further their understanding in how expressions involving percent change can be written in different equivalent forms.



Name: ______ Date: ______ Period: ______

Activity 1 Taxes and Sales

Complete the following problems. Describe how you would compute the costs, given the percentages in each scenario.

1. You need to pay 8% sales tax on a car that costs $12,000. What will you end up paying in total? Show or explain your thinking.
   $12,960, because 12000 + 12000(0.8) = 12960, or because 12000(1.08) = 12960.

2. Burritos are on sale for 30% off. Your favorite burrito normally costs $8.50. How much does it cost now? Show or explain your thinking.
   $5.95, because 8.5 – 0.3(8.5) = 5.95 or 8.5(0.7) = 5.95.

3. A pair of shoes that originally cost $79 are on sale for 35% off. Does the expression 0.65(79) represent the sale price of the shoes, in dollars? Show or explain your thinking.
   Yes, because 79 – 0.35(79) = 0.65(79).

4. A store-brand allergy medication costs 55% less than a similar commercial brand name. If the brand name costs $19.97, how much does the store brand cost? Show or explain your thinking.
   $8.99, because 19.97 – 0.55(19.97) ≈ 8.99 or because 19.97(0.45) ≈ 8.99.

Are you ready for more?

What are some strategies for mentally adding 15% to the total cost of an item?

Sample response: First, determine 10% of the cost. To determine 5% of the cost, take half of this amount. Then add these amounts – this represents 15% of the cost. Finally, add this amount to the cost of the item. For example, to add 15% to $74, find 74 • (1.15) = 74 • (1 + 0.1 + 0.05) = 74 + 7.4 + 3.7 = 85.1.

1 Launch

Have student-pairs discuss each problem before completing independently, comparing solutions upon completion.

2 Monitor

Help students get started by reminding them that a percent represents a relationship to 100, so percentages must be written as decimals or fractions prior to computing with them.

Look for points of confusion:

- **Struggling to determine operations.** Have students connect keywords to the operations they indicate.
- **Struggling to visualize multiple steps.** Have students draw tape diagrams representing their selected operation and compare them to the problem.

Look for productive strategies:

- Multiplying the initial quantity by the percent remaining.

3 Connect

Have groups of students share their work with at least two other groups. Ask them to discuss similarities and differences of strategies used in each group.

Highlight the connections between different forms of equivalent expressions, clarifying them in terms of the properties of operations. Emphasize the efficiency of only using multiplication.

Ask, "How are the following expressions equivalent?"

$79 - 0.35(79)$

$79 \cdot (1 - 0.35)$

$79 \cdot (0.65)$

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they solved problems involving percentages, including sales tax and discounts, in middle school. Refresh their knowledge by reminding them that a percent increase will increase the value of a quantity, while a percent decrease will decrease the value of a quantity. When calculating with percentages, they must be written as fractions or decimals value of a whole.

Math Language Development

MLR7: Compare and Connect

During the Launch, or while students work, support students in developing a problem-solving agency by encouraging them to use the strategy that makes most sense to them.

During the Connect, as students share their work with their classmates, draw their attention to the connections between different forms of equivalent expressions. Sample expressions are shown for Problems 1 and 2.

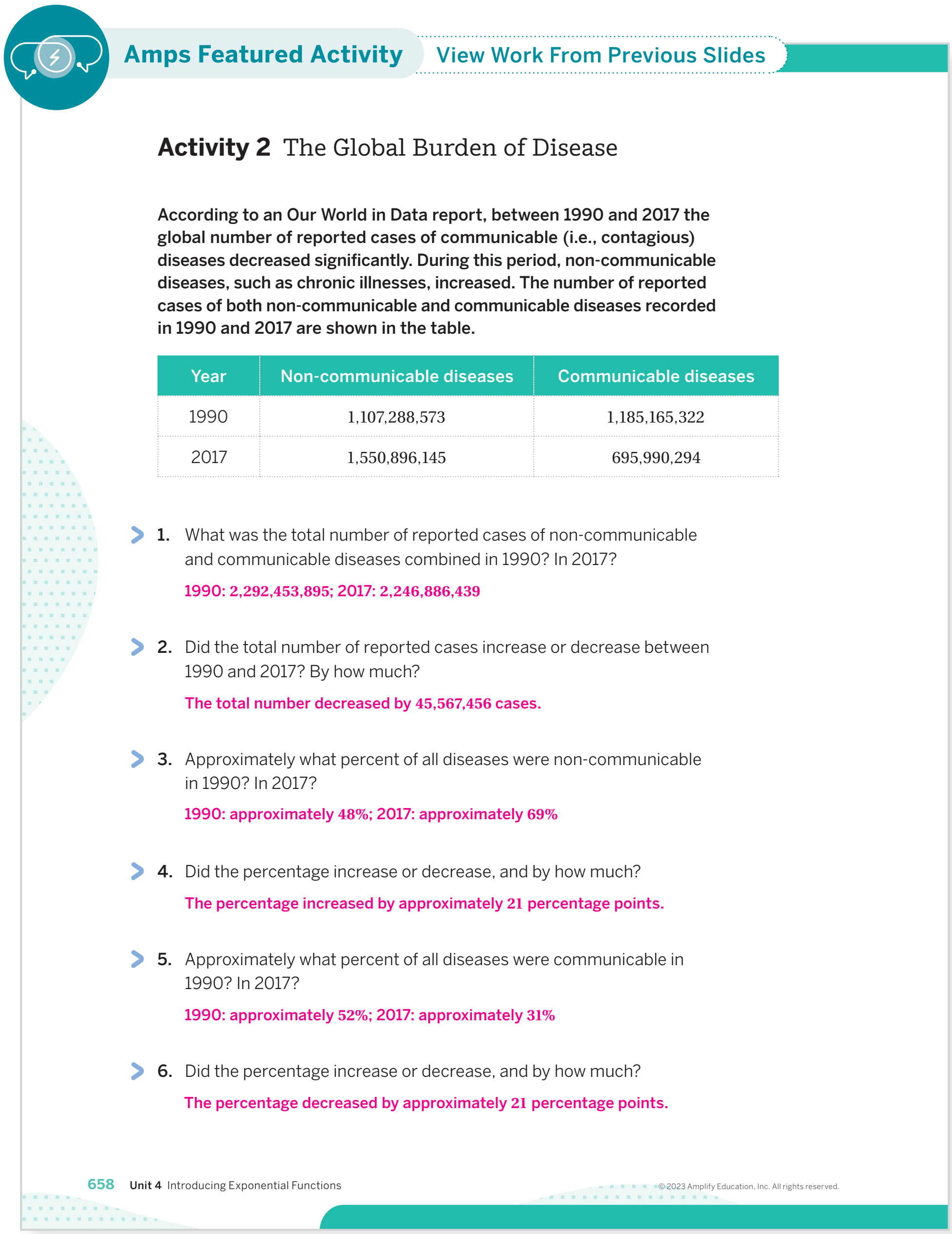
Problem 1 expressions:	Problem 2 expressions:
$12000 + (12000 \cdot 0.8)$	$8.50 - (8.50 \cdot 0.3)$
$12000 \cdot (1 + 0.8)$	$8.50 \cdot (1 - 0.3)$
$12000 \cdot (1.08)$	$8.50 \cdot (0.7)$

Pairs | 10 min

Activity 2 The Global Burden of Disease

Students explore percent increase and decrease by calculating and comparing global percentages of non-communicable and communicable diseases.

Amps Featured Activity View Work From Previous Slides



Activity 2 The Global Burden of Disease

According to an Our World in Data report, between 1990 and 2017 the global number of reported cases of communicable (i.e., contagious) diseases decreased significantly. During this period, non-communicable diseases, such as chronic illnesses, increased. The number of reported cases of both non-communicable and communicable diseases recorded in 1990 and 2017 are shown in the table.

Year	Non-communicable diseases	Communicable diseases
1990	1,107,288,573	1,185,165,322
2017	1,550,896,145	695,990,294

1. What was the total number of reported cases of non-communicable and communicable diseases combined in 1990? In 2017?

   1990: 2,292,453,895; 2017: 2,246,886,439

2. Did the total number of reported cases increase or decrease between 1990 and 2017? By how much?

   The total number decreased by 45,567,456 cases.

3. Approximately what percent of all diseases were non-communicable in 1990? In 2017?

   1990: approximately 48%; 2017: approximately 69%

4. Did the percentage increase or decrease, and by how much?

   The percentage increased by approximately 21 percentage points.

5. Approximately what percent of all diseases were communicable in 1990? In 2017?

   1990: approximately 52%; 2017: approximately 31%

6. Did the percentage increase or decrease, and by how much?

   The percentage decreased by approximately 21 percentage points.

658 Unit 4 Introducing Exponential Functions

1 Launch

Display the table and read the prompt aloud. Set an expectation for the amount of time students have to work in pairs on the activity.

2 Monitor

Help students get started by asking which value they will need to find first before they can calculate a percentage.

Look for points of confusion:

- **Not understanding the desired percent compares a subgroup to the total number of cases.** Have students add the communicable and non-communicable cases for each year.

Look for productive strategies:

- Multiplying a decimal by 100, or moving the decimal two places to the right, to convert the decimal to a percent.

3 Connect

Display the data table.

Have students share strategies they used to find the percentages and determine increase or decrease.

Highlight that students need to first find the combined number of non-communicable and communicable cases before calculating the percentages.

Ask, "What patterns did you notice in the percentages?" The percentages within one year added to 100%.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive tape diagrams to model percent increase or decrease. The tape diagrams provide visual support for writing corresponding expressions.

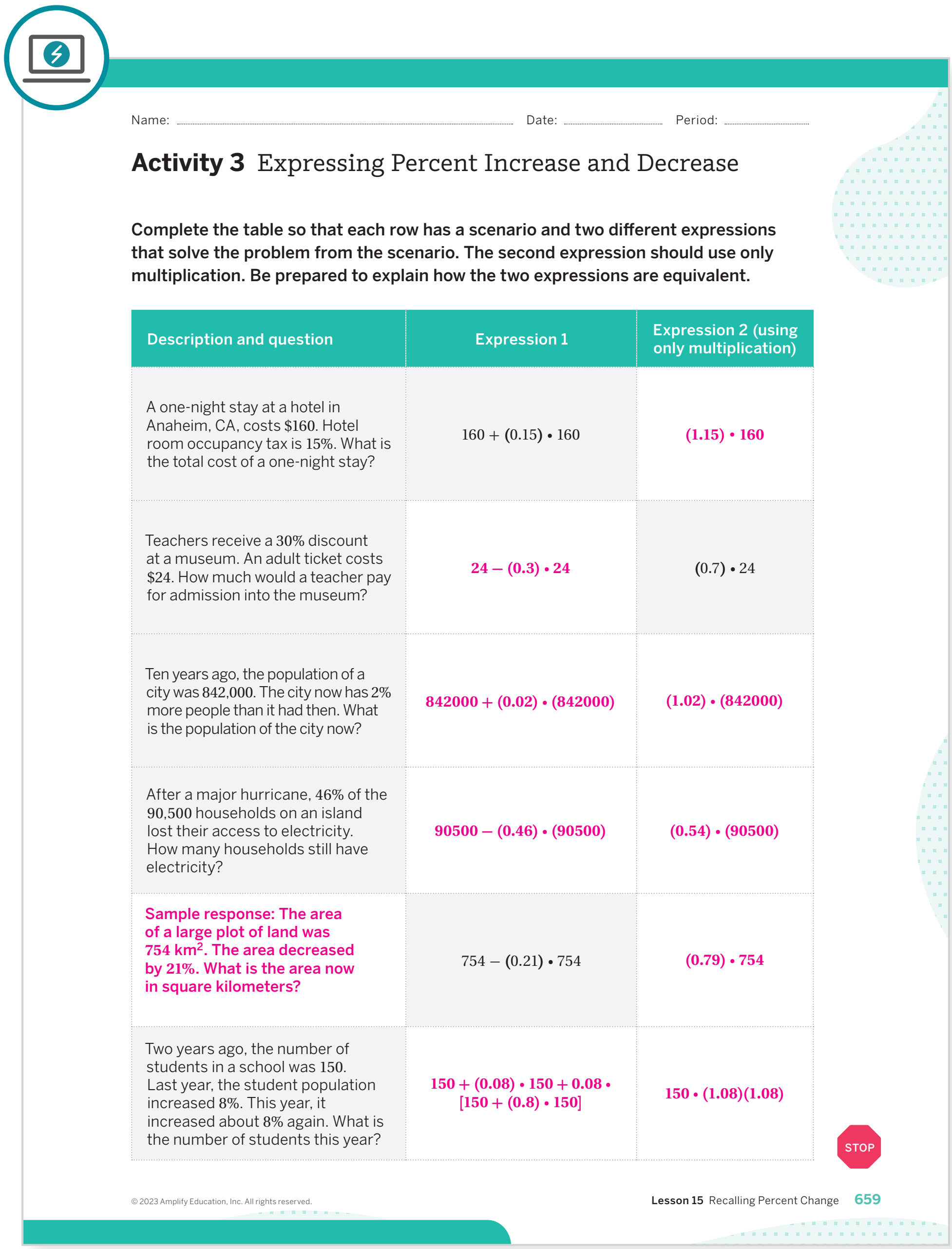
Accessibility: *Clarify Vocabulary and Symbols, Activate Background Knowledge*

Be sure students understand the meaning of *communicable* diseases and *non-communicable* diseases. Communicable diseases are contagious because you can "communicate" or spread them to others. Ask students to think of examples of communicable diseases, such as the flu or a cold. Allergies, for example, are non-communicable.

Pairs | 15 min

Activity 3 Expressing Percent Increase and Decrease

Students continue to represent increase and decrease, using the structure of expressions to write them only using multiplication.



Name: ____________ Date: ____________ Period: ____________

Activity 3 Expressing Percent Increase and Decrease

Complete the table so that each row has a scenario and two different expressions that solve the problem from the scenario. The second expression should use only multiplication. Be prepared to explain how the two expressions are equivalent.

Description and question	Expression 1	Expression 2 (using only multiplication)
A one-night stay at a hotel in Anaheim, CA, costs \$160. Hotel room occupancy tax is 15%. What is the total cost of a one-night stay?	$160 + (0.15) \cdot 160$	$(1.15) \cdot 160$
Teachers receive a 30% discount at a museum. An adult ticket costs \$24. How much would a teacher pay for admission into the museum?	$24 - (0.3) \cdot 24$	$(0.7) \cdot 24$
Ten years ago, the population of a city was 842,000. The city now has 2% more people than it had then. What is the population of the city now?	$842000 + (0.02) \cdot (842000)$	$(1.02) \cdot (842000)$
After a major hurricane, 46% of the 90,500 households on an island lost their access to electricity. How many households still have electricity?	$90500 - (0.46) \cdot (90500)$	$(0.54) \cdot (90500)$
Sample response: The area of a large plot of land was 754 km^2. The area decreased by 21%. What is the area now in square kilometers?	$754 - (0.21) \cdot 754$	$(0.79) \cdot 754$
Two years ago, the number of students in a school was 150. Last year, the student population increased 8%. This year, it increased about 8% again. What is the number of students this year?	$150 + (0.08) \cdot 150 + 0.08 \cdot [150 + (0.8) \cdot 150]$	$150 \cdot (1.08)(1.08)$

STOP

Lesson 15 Recalling Percent Change 659

1 Launch

Set an expectation for the amount of time students will have to work on the activity.

2 Monitor

Help students get started by suggesting they annotate the scenarios as representing percent increase or decrease.

Look for points of confusion:

- **Struggling to use single multiplication.** Remind students how distribution can be used to simplify expressions.

Look for productive strategies:

- Multiplying the initial amount by the percentage remaining or 100% plus the percentage added.

3 Connect

Have pairs of students share their expressions and their strategies for writing them.

Highlight the use of single multiplication via the Distributive Property and demonstrate how the factor was determined. For Row 6, focus on repeated percent increase and guide students towards the expression $150 \cdot (1.08)^2$.

Ask:

- "How do you know the expressions in Rows 2, 4, and 5 are equivalent?" Sample response: In Row 2, $24 - (0.3) \cdot 24$ can be written as $24(1) - 24 \cdot (0.3)$ and then use the Distributive Property.
- "What do you see happening in Row 6? How can you represent that using single multiplication?" There is a repeated percent increase. I can use exponential notation.
- "For Row 6, what expression can you use to represent the population if it grows at the same percentage rate for n years?" $150 \cdot (1.08)^n$
- "Why would you use only multiplication or in some cases, exponents, to represent situations like these?" Sample response: It is more efficient.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them complete the first three rows of the table or allow them to choose three rows of the table to complete. Ask them to choose at least one percent increase scenario and one percent decrease scenario.

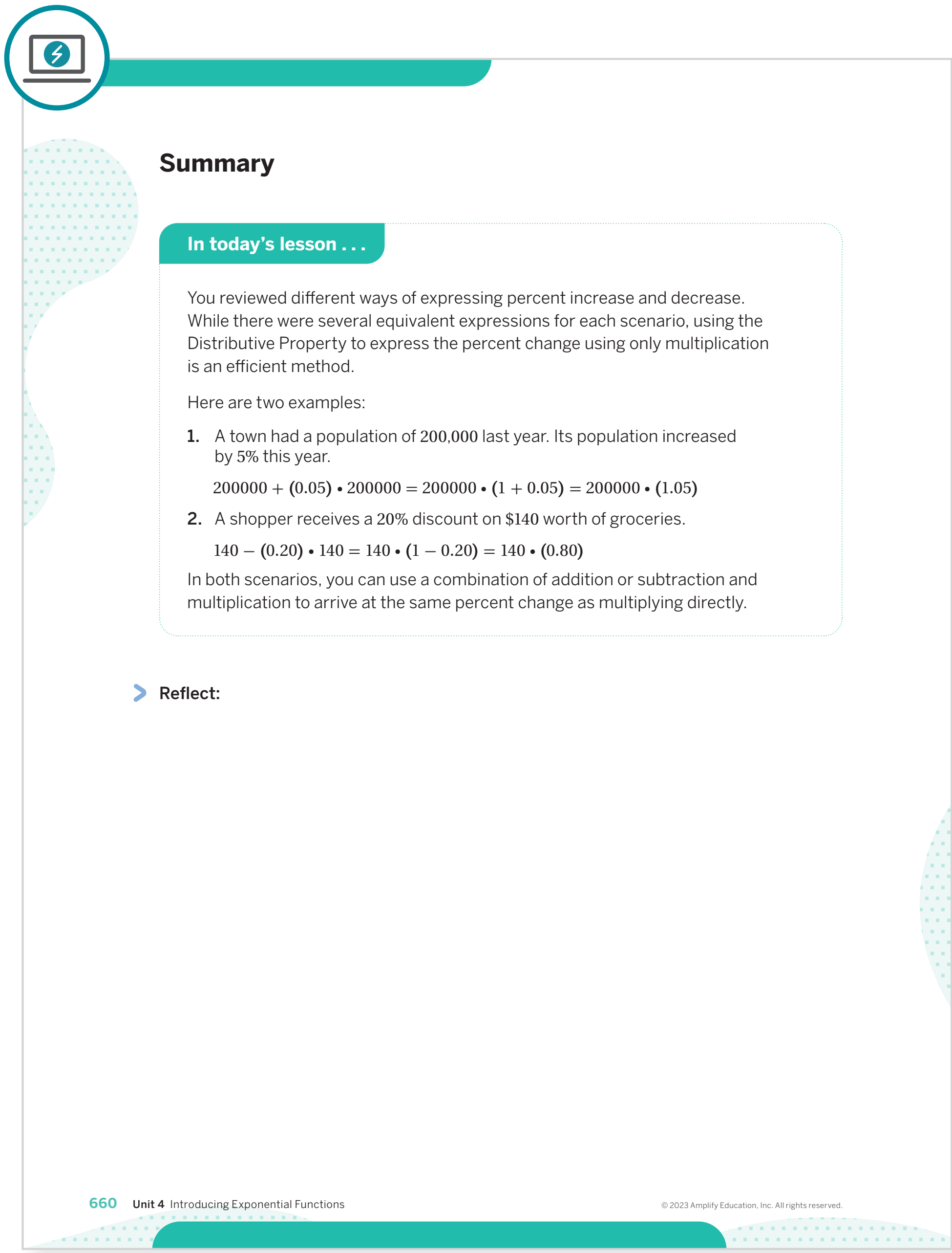
Accessibility: ***Guide Processing and Visualization***

Consider providing an example of one of the rows completed for students to use as a model, and annotate it as *percent increase* or *percent decrease*. Suggest students first determine whether each scenario describes a percent increase or decrease.

Whole Class | 5 min

Summary

Review and synthesize the different ways of expressing percent increase and decrease, while highlighting using only multiplication.



Summary

In today's lesson . . .

You reviewed different ways of expressing percent increase and decrease. While there were several equivalent expressions for each scenario, using the Distributive Property to express the percent change using only multiplication is an efficient method.

Here are two examples:

1. A town had a population of 200,000 last year. Its population increased by 5% this year.

   $200000 + (0.05) \cdot 200000 = 200000 \cdot (1 + 0.05) = 200000 \cdot (1.05)$

2. A shopper receives a 20% discount on $140 worth of groceries.

   $140 - (0.20) \cdot 140 = 140 \cdot (1 - 0.20) = 140 \cdot (0.80)$

In both scenarios, you can use a combination of addition or subtraction and multiplication to arrive at the same percent change as multiplying directly.

> Reflect:

660 Unit 4 Introducing Exponential Functions

Synthesize

Display the two examples of percent increase and percent decrease.

Have students share at least two different ways to express the change in population and the change in the cost of groceries and explain why they are equivalent.

Highlight expressing percent change as single multiplication.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

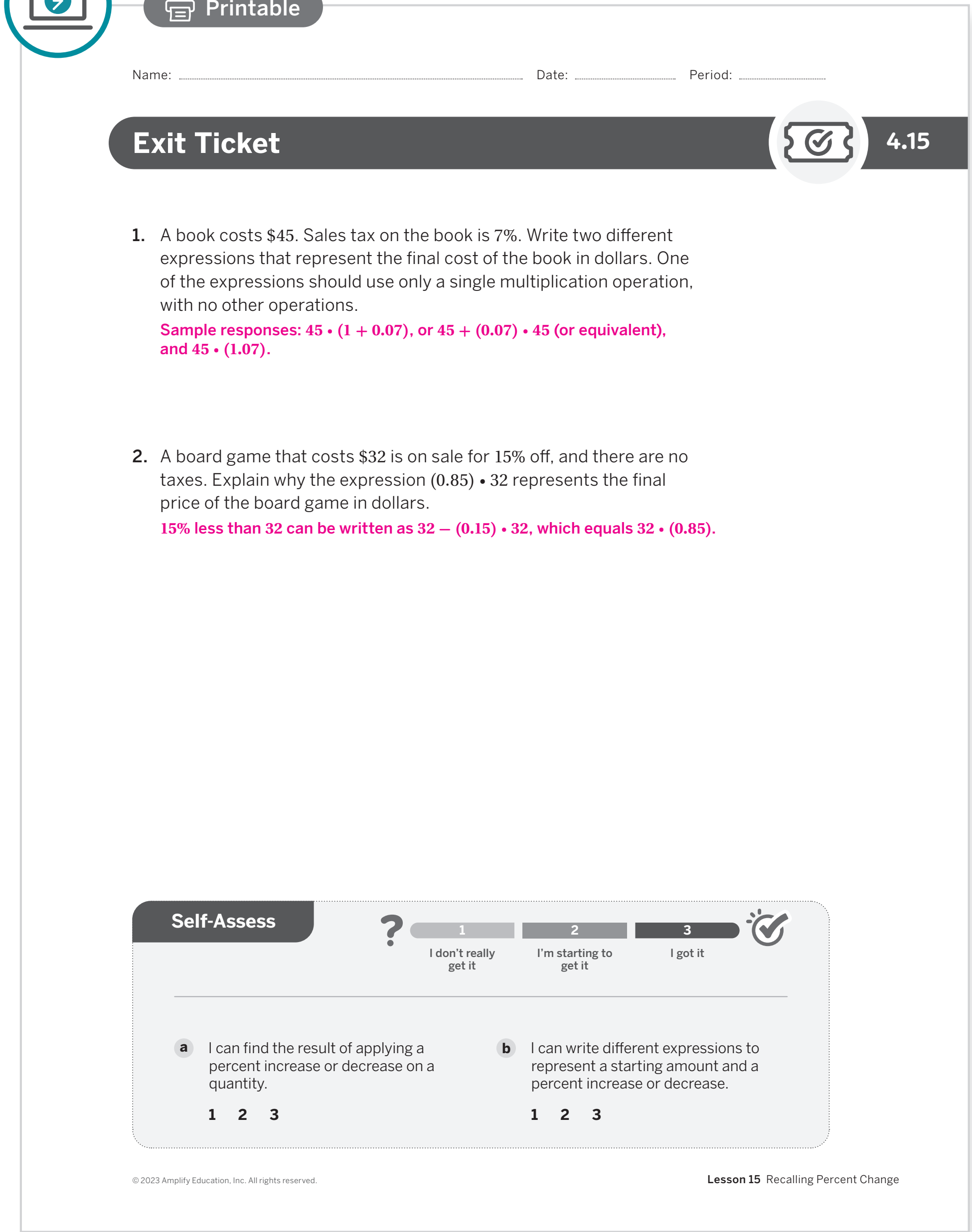
- "Why is expressing percent change using only multiplication considered the most efficient?"
- "Which way of writing these expressions makes the most sense to you?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing expressions representing percent increase and decrease, using only multiplication.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.15

1. A book costs $45. Sales tax on the book is 7%. Write two different expressions that represent the final cost of the book in dollars. One of the expressions should use only a single multiplication operation, with no other operations.

   Sample responses: $45 \cdot (1 + 0.07)$, or $45 + (0.07) \cdot 45$ (or equivalent), and $45 \cdot (1.07)$.

2. A board game that costs $32 is on sale for 15% off, and there are no taxes. Explain why the expression $(0.85) \cdot 32$ represents the final price of the board game in dollars.

   15% less than 32 can be written as $32 - (0.15) \cdot 32$, which equals $32 \cdot (0.85)$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can find the result of applying a percent increase or decrease on a quantity.

1 2 3

b I can write different expressions to represent a starting amount and a percent increase or decrease.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Recalling Percent Change

Success looks like . . .

- **Goal:** Calculating a final amount when given a starting amount and a percent increase or decrease.
- **Goal:** Writing expressions using only multiplication to represent a percent increase or decrease.
  - » Writing one expression in Problem 1 that uses only multiplication.

Suggested next steps

If students cannot write equivalent expressions in Problem 1, consider:

- Reviewing Activities 1 and 2.
- Assigning Practice Problem 3.

If students explain incorrectly or incompletely in Problem 2, consider:

- Reviewing Activity 3.
- Assigning Practice Problems 4 and 5.

Professional Learning

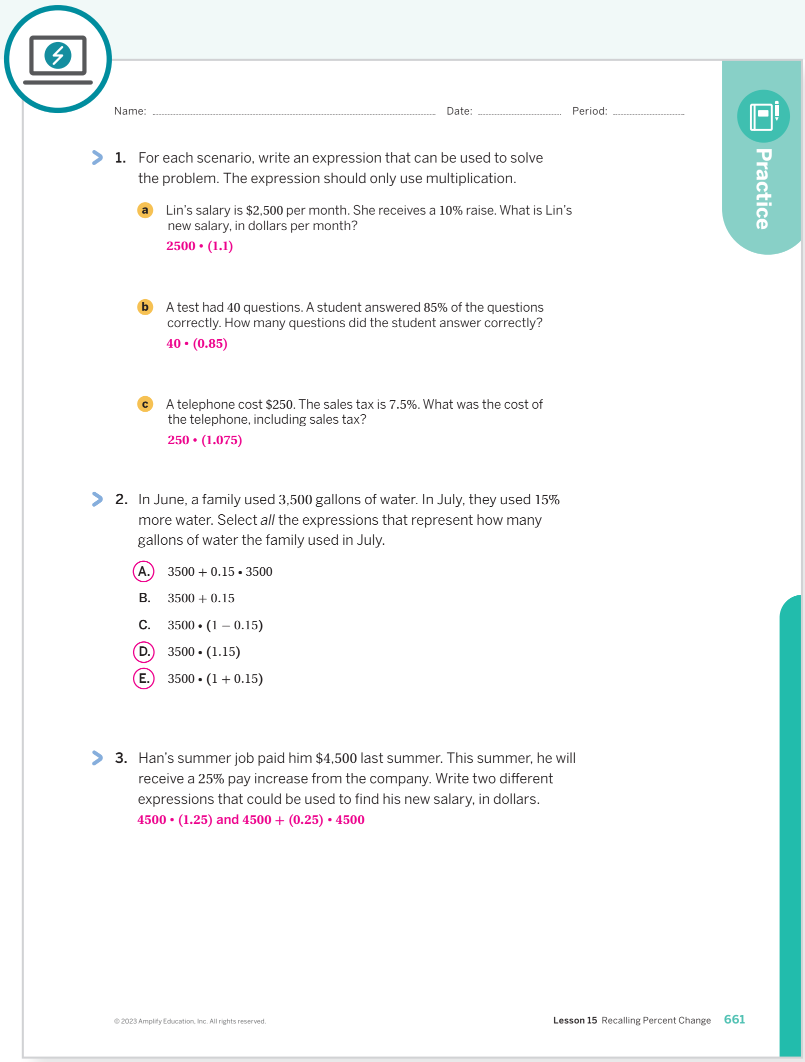
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What different ways did students approach writing equivalent expressions in Activity 1? What does that tell you about the similarities and differences among your students?
- How well do you think your students understand writing an expression using single multiplication or exponents to represent percent change and repeated percent change is the most efficient method? Which questions might you change the next time you teach this lesson?

Practice

Independent



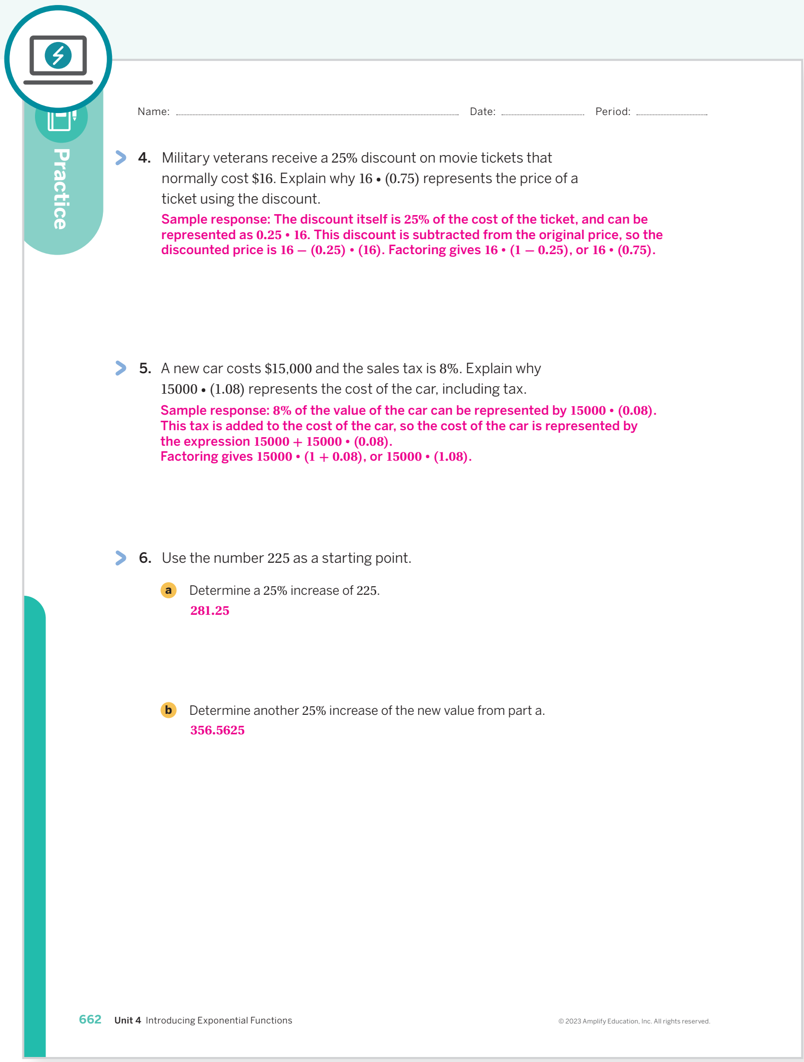
Name: Date: Period:

Practice

1. For each scenario, write an expression that can be used to solve the problem. The expression should only use multiplication.
   - **a** Lin's salary is \$2,500 per month. She receives a 10% raise. What is Lin's new salary, in dollars per month?
     $2500 \cdot (1.1)$
   - **b** A test had 40 questions. A student answered 85% of the questions correctly. How many questions did the student answer correctly?
     $40 \cdot (0.85)$
   - **c** A telephone cost \$250. The sales tax is 7.5%. What was the cost of the telephone, including sales tax?
     $250 \cdot (1.075)$

2. In June, a family used 3,500 gallons of water. In July, they used 15% more water. Select *all* the expressions that represent how many gallons of water the family used in July.
   - [✓ A.] $3500 + 0.15 \cdot 3500$
   - B. $3500 + 0.15$
   - C. $3500 \cdot (1 - 0.15)$
   - [✓ D.] $3500 \cdot (1.15)$
   - [✓ E.] $3500 \cdot (1 + 0.15)$

3. Han's summer job paid him \$4,500 last summer. This summer, he will receive a 25% pay increase from the company. Write two different expressions that could be used to find his new salary, in dollars.
   $4500 \cdot (1.25)$ and $4500 + (0.25) \cdot 4500$



Practice

Name: Date: Period:

4. Military veterans receive a 25% discount on movie tickets that normally cost \$16. Explain why $16 \cdot (0.75)$ represents the price of a ticket using the discount.
   Sample response: The discount itself is 25% of the cost of the ticket, and can be represented as $0.25 \cdot 16$. This discount is subtracted from the original price, so the discounted price is $16 - (0.25) \cdot (16)$. Factoring gives $16 \cdot (1 - 0.25)$, or $16 \cdot (0.75)$.

5. A new car costs \$15,000 and the sales tax is 8%. Explain why $15000 \cdot (1.08)$ represents the cost of the car, including tax.
   Sample response: 8% of the value of the car can be represented by $15000 \cdot (0.08)$. This tax is added to the cost of the car, so the cost of the car is represented by the expression $15000 + 15000 \cdot (0.08)$.
   Factoring gives $15000 \cdot (1 + 0.08)$, or $15000 \cdot (1.08)$.

6. Use the number 225 as a starting point.
   - **a** Determine a 25% increase of 225.
     281.25
   - **b** Determine another 25% increase of the new value from part a.
     356.5625

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	1
	3	Activity 2	2
	4	Activity 2	2
	5	Activity 2	2
Formative ⚡	6	Unit 4 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 16**

Functions Involving Percent Change

Let's investigate what happens when we repeatedly apply a percent increase to a quantity.

Focus

Goals

1. Write a numerical expression or an algebraic expression to represent the result of applying a percent increase repeatedly.
2. Use graphs to illustrate and compare different percent increases.

Coherence

Today

Students use graphs to illustrate and compare situations with different percent increases. Students write an expression of the form $(1 + r)^n$ to represent a percent increase applied n times.

Previously

Students revisited average rate of change and applied the concept to exponential functions.

Coming Soon

Students will distinguish the effect of compound percent change from that of simple percent change.

Rigor

- Students develop their **conceptual understanding** of the connections between percent change and exponential functions by taking scenarios involving iterative percent change and writing them as exponential functions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Independent	Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

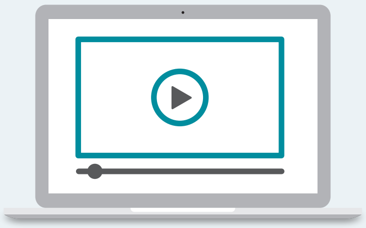
- *average rate of change*

Amps | Featured Activity

Summary

Animated Debt

Students can view an animation of what increasing debt looks like, which illustrates how quickly repeated percent increase grows.



Building Math Identity and Community

Connecting to Mathematical Practices

An incorrect focus might distract students from understanding how the exponential functions and percent increase are related. Because students have covered each of the subjects in previous lessons, the connection should be seen in the repeated reasoning. Have students set a goal of drawing this connection and work with their partner to be prepared to explain how the two topics merge into one.

Modifications to Pacing

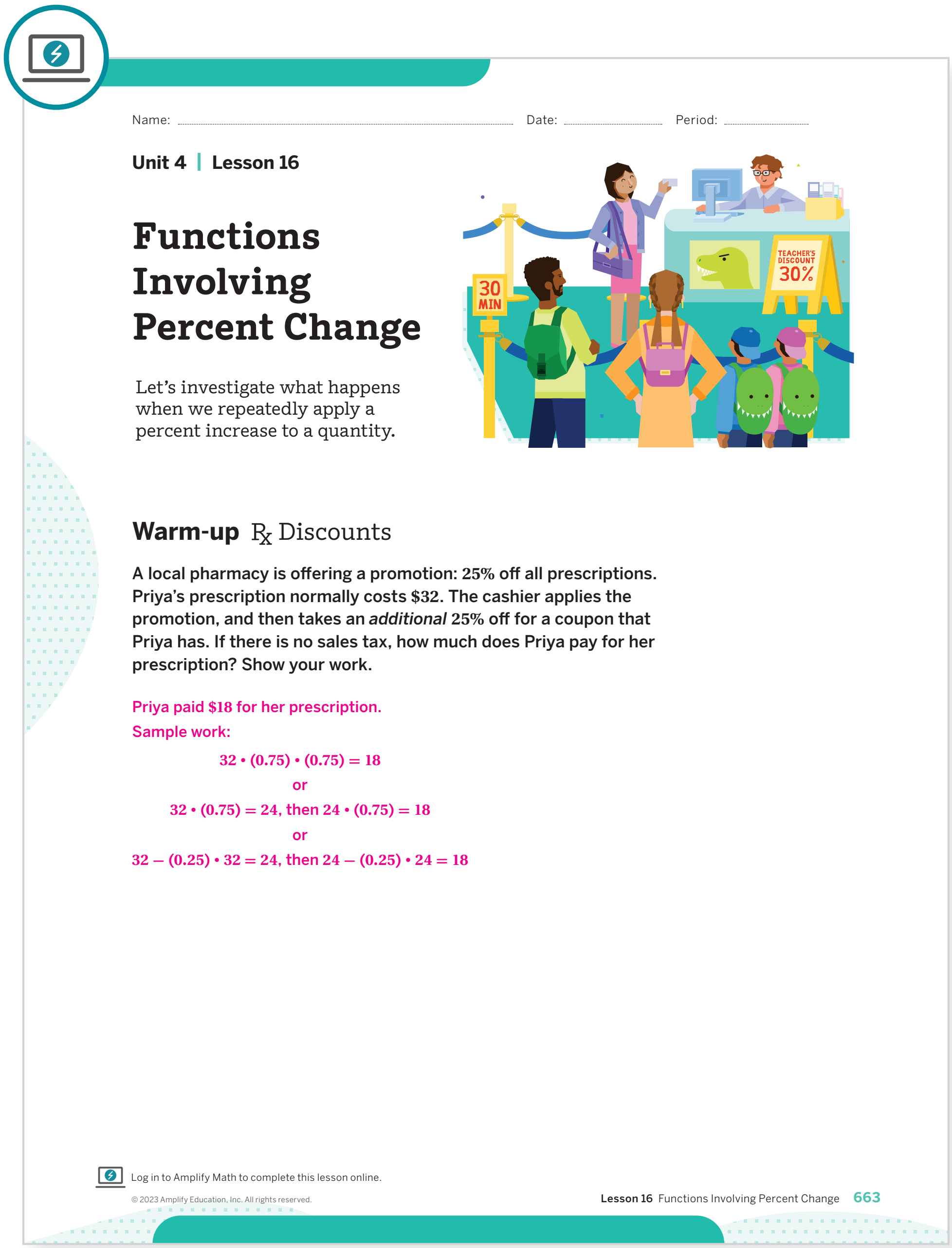
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, have students work with Loans A and B, not Loan C.
- In **Activity 3**, have students work with Loans A and B, not Loan C.

Independent | 5 min

Warm-up ℞ Discounts

Students repeatedly apply a percent decrease.



Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 16

Functions Involving Percent Change

Let's investigate what happens when we repeatedly apply a percent increase to a quantity.

Warm-up ℞ Discounts

A local pharmacy is offering a promotion: 25% off all prescriptions. Priya's prescription normally costs \$32. The cashier applies the promotion, and then takes an *additional* 25% off for a coupon that Priya has. If there is no sales tax, how much does Priya pay for her prescription? Show your work.

Priya paid \$18 for her prescription.

Sample work:

$32 \cdot (0.75) \cdot (0.75) = 18$

or

$32 \cdot (0.75) = 24$, then $24 \cdot (0.75) = 18$

or

$32 - (0.25) \cdot 32 = 24$, then $24 - (0.25) \cdot 24 = 18$

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Functions Involving Percent Change 663

1 Launch

Have students work independently, then have them work in pairs to compare strategies and solutions.

2 Monitor

Help students get started by suggesting they calculate the discount amount for the storewide sale first and then calculate the cost after that discount is applied.

Look for points of confusion:

- **Thinking the prescription is now 50% off.** Remind students to calculate each discount individually.

Look for productive strategies:

- Using repeated multiplication of (0.75).

3 Connect

Have students share their strategies and results with a partner. Select student pairs that calculated the discount separately and used repeated multiplication to share with the whole class.

Highlight that different expressions can be used to efficiently calculate the final cost of the prescription. The percentages are an example of decay, because the values are between 0 and 1.

Ask, "If Priya had a third discount for 25% off, could you use the model $32 \cdot (0.75)^3$ to determine the cost? Would this work if her third discount was for 10% off?"

Power-up

To power up students' ability to apply repeated percent change on a value, have students complete:

Between 2000 and 2010, a town increased its population p by 10%. From 2010 to 2020 the population increased by another 10%.

- Diego says that the expression $1.1(1.1p)$ represents the population in 2020.
- Han says that the expression $1.2p$ represents the population in 2020.

Who is correct? Be prepared to explain your thinking.

Diego; Sample response: $1.1p$ represents the population in 2010. Then the population increased by 10%, so you need to multiply $1.1p$ by 1.1.

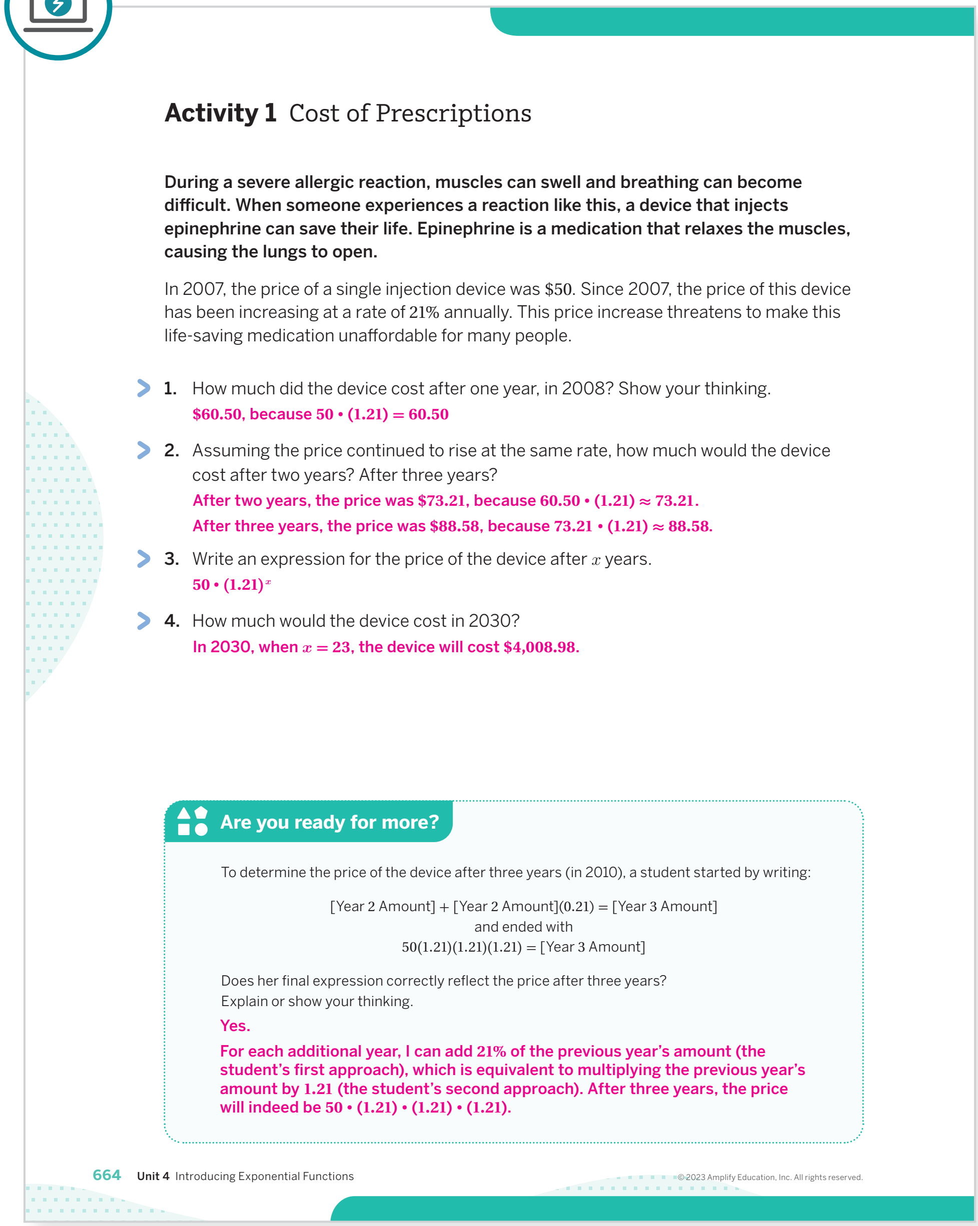
Use: Before the Warm-up

Informed by: Performance on Lesson 15, Practice Problem 6

Pairs | 15 min

Activity 1 Cost of Prescriptions

Students use repeated reasoning and exponential expressions to describe and make sense of repeated percent increase in a context about the cost of prescriptions.



Activity 1 Cost of Prescriptions

During a severe allergic reaction, muscles can swell and breathing can become difficult. When someone experiences a reaction like this, a device that injects epinephrine can save their life. Epinephrine is a medication that relaxes the muscles, causing the lungs to open.

In 2007, the price of a single injection device was \$50. Since 2007, the price of this device has been increasing at a rate of 21% annually. This price increase threatens to make this life-saving medication unaffordable for many people.

1. How much did the device cost after one year, in 2008? Show your thinking.
   \$60.50, because $50 \cdot (1.21) = 60.50$
2. Assuming the price continued to rise at the same rate, how much would the device cost after two years? After three years?
   After two years, the price was \$73.21, because $60.50 \cdot (1.21) \approx 73.21$.
   After three years, the price was \$88.58, because $73.21 \cdot (1.21) \approx 88.58$.
3. Write an expression for the price of the device after x years.
   $50 \cdot (1.21)^x$
4. How much would the device cost in 2030?
   In 2030, when $x = 23$, the device will cost \$4,008.98.

Are you ready for more?

To determine the price of the device after three years (in 2010), a student started by writing:

$$[\text{Year 2 Amount}] + [\text{Year 2 Amount}](0.21) = [\text{Year 3 Amount}]$$
and ended with
$$50(1.21)(1.21)(1.21) = [\text{Year 3 Amount}]$$

Does her final expression correctly reflect the price after three years? Explain or show your thinking.

Yes.

For each additional year, I can add 21% of the previous year's amount (the student's first approach), which is equivalent to multiplying the previous year's amount by 1.21 (the student's second approach). After three years, the price will indeed be $50 \cdot (1.21) \cdot (1.21) \cdot (1.21)$.

664 **Unit 4** Introducing Exponential Functions

1 Launch

Discuss the impacts of medication costs increasing. Say, "Let's see how compounding costs over a period of time can be modeled exponentially."

2 Monitor

Help students get started with Problem 3 by asking, "How did you calculate the amounts for years 1, 2, and 3? Is it possible to determine the cost after 2 years without calculating year 1?"

Look for points of confusion:

- **Struggling to write a general expression for the price of the device after x years.** Suggest creating a table, writing expressions using multiplication until a pattern emerges.
- **Using 0.21 as the common multiplier.** Have students calculate the price of the device after the first year. Ask, "Does it make sense for price after year 1 to be less than the initial price?"

Look for productive strategies:

- Using repeated multiplication and exponents for shortcuts.

3 Connect

Display Problem 3.

Have student pairs share their multiplicative and additive expressions, strategies, and solutions with the whole class.

Highlight the process in creating the expression in Problem 3.

Ask:

- "What is the relationship between the expressions $50(1 + 0.21)$ and $50(1.21)^x$?"
- "In the final expression $50 \cdot (1.21)^x$, what does each part represent?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a three-column table, such as the following:

Years since 2007	Expressions	Calculations
0		
1		
2		
3		
x		

Math Language Development

MLR6: Three Reads

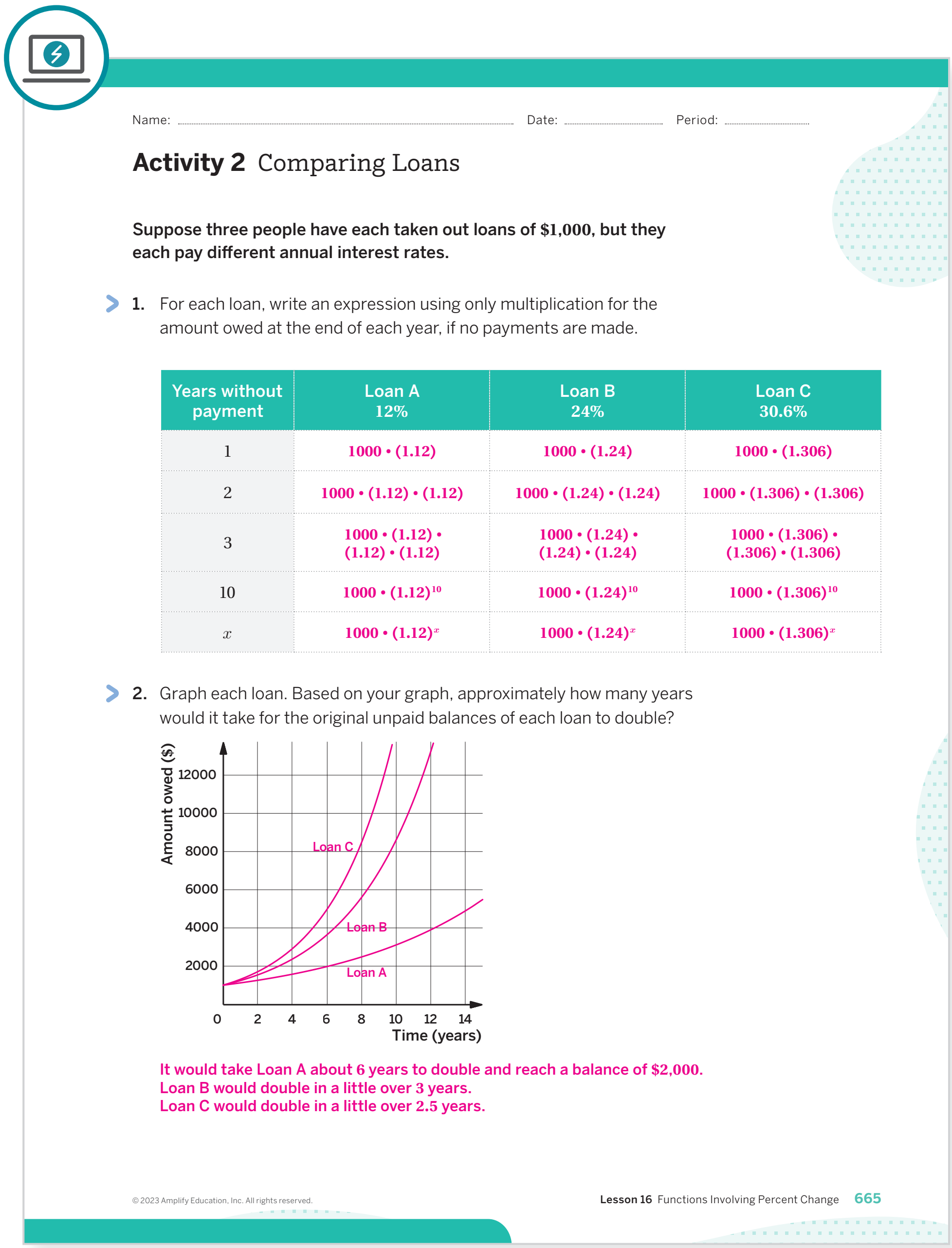
Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that the price of the single-injection device has been increasing at a certain percentage rate each year.
- **Read 2:** Ask students to name or describe the given quantities and relationships, such as the cost of the device in 2007 was \$50.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problem 1.

Small Groups | 10 min

Activity 2 Comparing Loans

Students construct expressions to model the amount owed for different interest rates on a loan and graph the relationships to see the impacts over time.



Name: ________ Date: ________ Period: ________

Activity 2 Comparing Loans

Suppose three people have each taken out loans of $1,000, but they each pay different annual interest rates.

1. For each loan, write an expression using only multiplication for the amount owed at the end of each year, if no payments are made.

Years without payment	Loan A 12%	Loan B 24%	Loan C 30.6%
1	$1000 \cdot (1.12)$	$1000 \cdot (1.24)$	$1000 \cdot (1.306)$
2	$1000 \cdot (1.12) \cdot (1.12)$	$1000 \cdot (1.24) \cdot (1.24)$	$1000 \cdot (1.306) \cdot (1.306)$
3	$1000 \cdot (1.12) \cdot (1.12) \cdot (1.12)$	$1000 \cdot (1.24) \cdot (1.24) \cdot (1.24)$	$1000 \cdot (1.306) \cdot (1.306) \cdot (1.306)$
10	$1000 \cdot (1.12)^{10}$	$1000 \cdot (1.24)^{10}$	$1000 \cdot (1.306)^{10}$
x	$1000 \cdot (1.12)^x$	$1000 \cdot (1.24)^x$	$1000 \cdot (1.306)^x$

2. Graph each loan. Based on your graph, approximately how many years would it take for the original unpaid balances of each loan to double?

It would take Loan A about 6 years to double and reach a balance of $2,000.
Loan B would double in a little over 3 years.
Loan C would double in a little over 2.5 years.

1 Launch

Ask students what they already know about financing and loans. Tell them that they will write expressions using only multiplication, determine a general form for x years without payment, graph each loan, and compare the impact of various interest rates.

2 Monitor

Help students get started by modeling how to complete the first cell of the table, writing the expression for Year 1 of Loan A.

Look for points of confusion:

- **Writing very long expressions for Year 10.** Remind students that exponents represent repeated multiplication by the same value.
- **Struggling to locate the years needed for an unpaid balance to double on the graph.** Have students draw a horizontal line at 2,000 and find the intersection to determine the time associated.

Look for productive strategies:

- Analyzing the graph to determine unknown values.

3 Connect

Display the blank graph.

Have students share their responses and strategies for Problem 2, in order of efficiency.

Highlight the impact of different interest rates, especially when unpaid, over long periods of time. Point out that, for each loan, the initial amount $1,000 is multiplied by $(1 + r)^t$, where r is the interest rate (as a decimal) and t is the number of years without payment.

Ask, "Provide an example of an ideal interest rate for borrowing money. Would this be the same ideal interest rate for saving money? Why or why not?"

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Provide students with a partially-completed table for Problem 1, with the expressions pre-populated for Loan A. Ask, "How many factors of 1.12 do you see in each row? How does this number relate to the number of years without payment?"

Extension: ***Math Enrichment***

Have students research sample loan interest rates at banks in their community, either for auto loans or home loans. Then have them research savings account interest rates for the same bank. Ask them why the savings account interest rates are so much lower than for home loans.

Math Language Development

MLR7: Compare and Connect

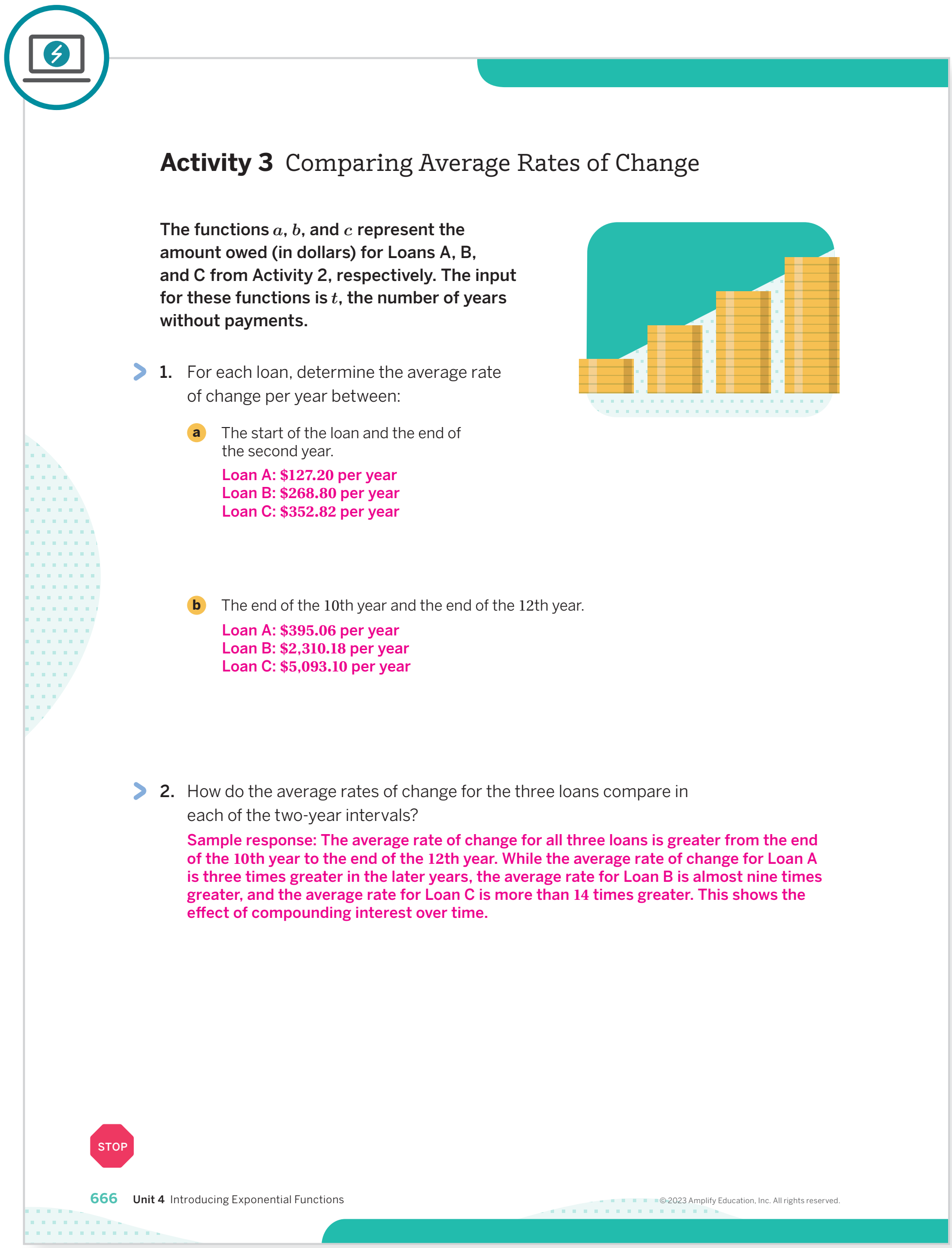
During the Connect, highlight connections between the expressions and the shape of the corresponding graphs. Ask:

- "How are the loans similar? How do you see this in the equations and the graphs?" The loans have the same initial value. This is seen as the value $1,000 in the equations and the vertical intercepts on the graph.
- "How are the loans different? How do you see this in the equations and the graphs?" The loans have different interest rates. This is seen as the base of the powers in the equations and the steepness of the curve of the graph. Greater interest rates have a greater steepness (increasing more rapidly).

Small Groups | 10 min

Activity 3 Comparing Average Rates of Change

Students investigate average rates of change over the same interval of time, but at different points in the lending period.



Activity 3 Comparing Average Rates of Change

The functions a, b, and c represent the amount owed (in dollars) for Loans A, B, and C from Activity 2, respectively. The input for these functions is t, the number of years without payments.

1. For each loan, determine the average rate of change per year between:

   a. The start of the loan and the end of the second year.

   Loan A: $127.20 per year
   Loan B: $268.80 per year
   Loan C: $352.82 per year

   b. The end of the 10th year and the end of the 12th year.

   Loan A: $395.06 per year
   Loan B: $2,310.18 per year
   Loan C: $5,093.10 per year

2. How do the average rates of change for the three loans compare in each of the two-year intervals?

   Sample response: The average rate of change for all three loans is greater from the end of the 10th year to the end of the 12th year. While the average rate of change for Loan A is three times greater in the later years, the average rate for Loan B is almost nine times greater, and the average rate for Loan C is more than 14 times greater. This shows the effect of compounding interest over time.

STOP

666 **Unit 4** Introducing Exponential Functions ©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Tell students they will be using the loans from Activity 2 to examine how each loan changes over different time intervals.

2 Monitor

Help students get started by reminding them how to calculate the average rate of change. Determine the difference in the function (y) values and divide by the difference in years (x).

Look for points of confusion:

- **Having difficulty determining the average rate of change of values not included in the table in Activity 2.** Prompt students to extend or create a new table.
- **Struggling to compare the average rates of change.** Provide graphs to calculate the rate over the given interval.

Look for productive strategies:

- Analyzing the graph to determine unknown values.

3 Connect

Display the completed graph from Activity 2.

Have groups of students share their thinking aloud, then discuss how debt grows over time.

Highlight that as time goes on, the unpaid balance grows more quickly due to compounding. The initial loan of $1,000 grows to a large amount quickly over time.

Ask, "Is it better to pay off a loan earlier or later? Explain your thinking." Earlier is better. As time goes on, you start paying interest on top of interest.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view an animation of what mounting debt looks like, illustrating how quickly repeated percent increase grows.

Extension: *Math Enrichment*

Ask students to make a prediction — without calculating — what the average rate of change per year will be for Loan C between the end of the 20th year and the end of the 22nd year. Then have them perform the calculations and describe how close their prediction was. This will further illustrate to them the effect of compounding interest over time.

Accessibility: *Guide Processing and Visualization*

Provide students with a pre-labeled blank table to use to complete Problems 1a and 1b, such as the following. Suggest that students use the expressions they wrote from Activity 2 to help them complete the table.

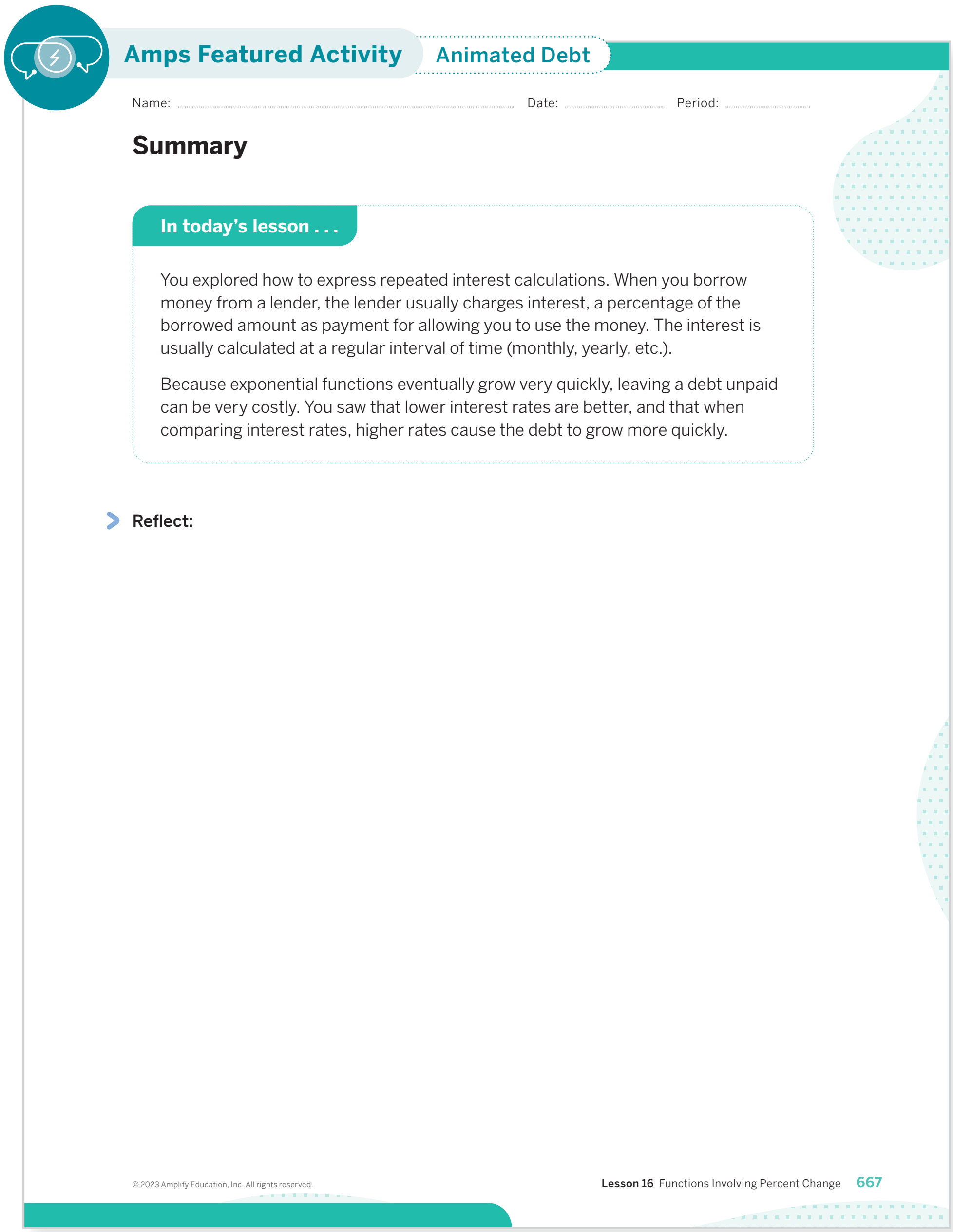
Years since start of loan	Loan A amount owed($)	Loan B amount owed($)	Loan C amount owed($)
0			
2			
10			
12			

Whole Class | 5 min

Summary

Review and synthesize how exponential expressions can be used to represent repeated interest calculations over time.

Amps Featured Activity Animated Debt



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how to express repeated interest calculations. When you borrow money from a lender, the lender usually charges interest, a percentage of the borrowed amount as payment for allowing you to use the money. The interest is usually calculated at a regular interval of time (monthly, yearly, etc.).

Because exponential functions eventually grow very quickly, leaving a debt unpaid can be very costly. You saw that lower interest rates are better, and that when comparing interest rates, higher rates cause the debt to grow more quickly.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Functions Involving Percent Change 667

Synthesize

Display the following prompt:

"Shawn takes out a $2,000 loan from a bank with a 4% annual interest rate."

Highlight that because exponential functions eventually grow very quickly, leaving a debt unpaid can be very costly.

Ask:

- "What expressions can you write to show the amount owed after 1 year if Shawn makes no payments? 2 years? t years?" $2000 \cdot (1.04)$; $2000 \cdot (1.04)^2$; $2000 \cdot (1.04)^t$
- "To determine the interest owed after 3 years if Shawn makes no payments, can you simply triple the interest charged after 1 year? Why or why not?" No, because after each unpaid year, you pay interest on your interest.
- "Why can you represent an amount a increased by 4% as $a \cdot (1.04)$? Where does the number 1.04 come from?" $1.04 = 1 + 0.04$, where 1 represents the unpaid previous balance and 0.04 represents the 4% interest rate charged on that balance.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

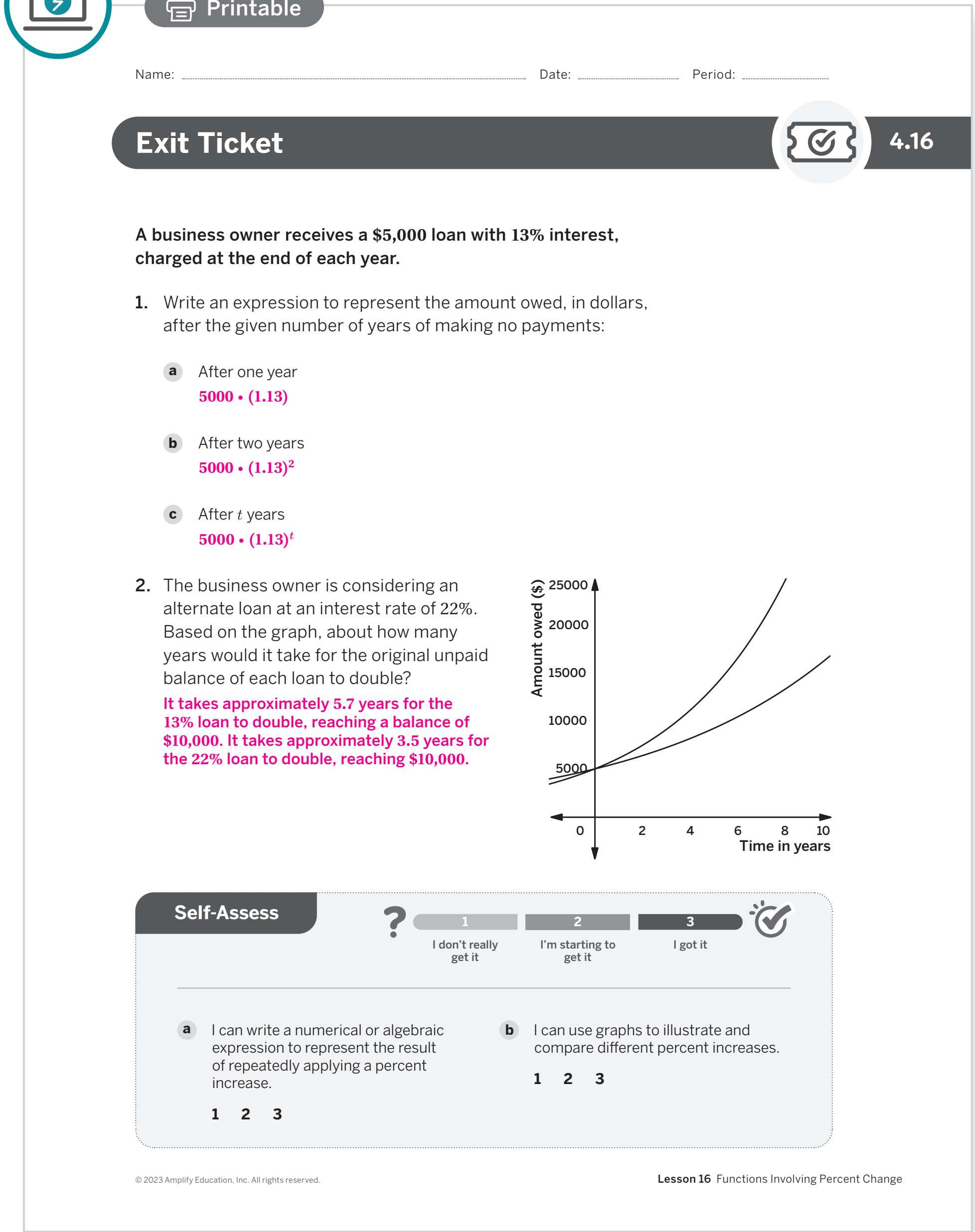
- "What are some ways you explored to write expressions to represent percent increase or repeated percent increase? Which expressions did you find the most efficient? Why?"
- "How will understanding the effects of compounding interest help you navigate loans and credit cards?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing expressions and using graphs to model and compare different percent increases.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 4.16

A business owner receives a $5,000 loan with 13% interest, charged at the end of each year.

1. Write an expression to represent the amount owed, in dollars, after the given number of years of making no payments:

   a. After one year
   $5000 \cdot (1.13)$

   b. After two years
   $5000 \cdot (1.13)^2$

   c. After t years
   $5000 \cdot (1.13)^t$

2. The business owner is considering an alternate loan at an interest rate of 22%. Based on the graph, about how many years would it take for the original unpaid balance of each loan to double?
   It takes approximately 5.7 years for the 13% loan to double, reaching a balance of $10,000. It takes approximately 3.5 years for the 22% loan to double, reaching $10,000.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write a numerical or algebraic expression to represent the result of repeatedly applying a percent increase.
1 2 3

b. I can use graphs to illustrate and compare different percent increases.
1 2 3

Lesson 16 Functions Involving Percent Change

Success Looks Like . . .

- **Goal:** Writing a numerical expression or an algebraic expression to represent the result of applying a percent increase repeatedly.
  - » Writing an expression of the amount owed after 13% interest is applied to a loan after one, two, and three years in Problem 1.
- **Goal:** Graphing to illustrate and compare different percent increases.
  - » Comparing two different interest rates in Problem 2.

Suggested next steps

- If students incorrectly determine the expression in Problem 1c, consider:
  - » Reviewing how to write different expressions to represent percent increase in Activity 1 or Activity 2, Problems 2 and 3.
  - » Assigning Practice Problem 1.
  - » Completing the optional Lesson 15 for additional practice.
- If students incorrectly interpret the graph in Problem 2, consider:
  - » Reviewing how to interpret graphs in Activity 3, Problem 2.
  - » Assigning Practice Problem 2.
  - » Having students draw a horizontal line at the amount owed and see where the graphs intersect to determine the time associated with that value.

Professional Learning

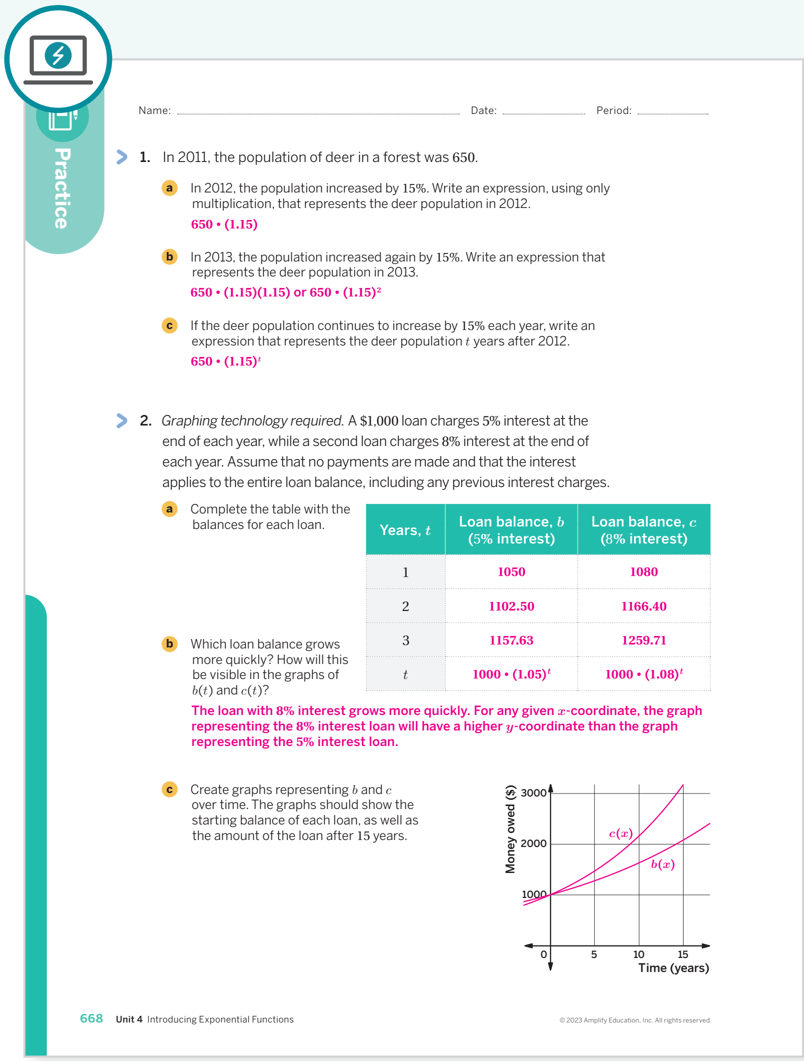
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What different ways did students approach writing expressions using only multiplication in Activity 1? What did your students find challenging? What kind of support did you offer and what might you change the next time you teach this lesson?
- How well do you think your students understand the effects of compounding interest? What questions did you ask students to probe for understanding and how did they respond?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. In 2011, the population of deer in a forest was 650.

   a. In 2012, the population increased by 15%. Write an expression, using only multiplication, that represents the deer population in 2012.

   $650 \cdot (1.15)$

   b. In 2013, the population increased again by 15%. Write an expression that represents the deer population in 2013.

   $650 \cdot (1.15)(1.15)$ or $650 \cdot (1.15)^2$

   c. If the deer population continues to increase by 15% each year, write an expression that represents the deer population t years after 2012.

   $650 \cdot (1.15)^t$

2. *Graphing technology required.* A $1,000 loan charges 5% interest at the end of each year, while a second loan charges 8% interest at the end of each year. Assume that no payments are made and that the interest applies to the entire loan balance, including any previous interest charges.

   a. Complete the table with the balances for each loan.

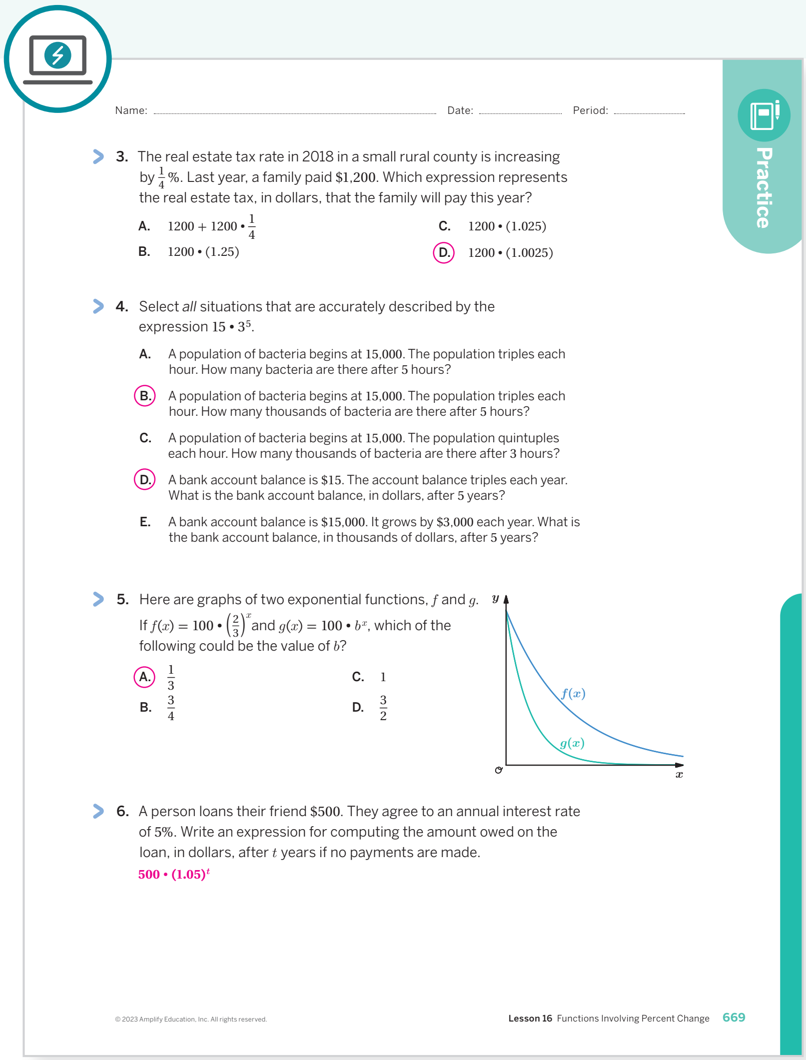
Years, t	Loan balance, b (5% interest)	Loan balance, c (8% interest)
1	**1050**	**1080**
2	**1102.50**	**1166.40**
3	**1157.63**	**1259.71**
t	**$1000 \cdot (1.05)^t$**	**$1000 \cdot (1.08)^t$**

   b. Which loan balance grows more quickly? How will this be visible in the graphs of $b(t)$ and $c(t)$?

   The loan with 8% interest grows more quickly. For any given x-coordinate, the graph representing the 8% interest loan will have a higher y-coordinate than the graph representing the 5% interest loan.

   c. Create graphs representing b and c over time. The graphs should show the starting balance of each loan, as well as the amount of the loan after 15 years.

668 Unit 4 Introducing Exponential Functions



Name: ______ Date: ______ Period: ______

Practice

3. The real estate tax rate in 2018 in a small rural county is increasing by $\frac{1}{4}$%. Last year, a family paid $1,200. Which expression represents the real estate tax, in dollars, that the family will pay this year?

   A. $1200 + 1200 \cdot \frac{1}{4}$

   B. $1200 \cdot (1.25)$

   C. $1200 \cdot (1.025)$

   [✓ D.] $1200 \cdot (1.0025)$

4. Select *all* situations that are accurately described by the expression $15 \cdot 3^5$.

   A. A population of bacteria begins at 15,000. The population triples each hour. How many bacteria are there after 5 hours?

   [✓ B.] A population of bacteria begins at 15,000. The population triples each hour. How many thousands of bacteria are there after 5 hours?

   C. A population of bacteria begins at 15,000. The population quintuples each hour. How many thousands of bacteria are there after 3 hours?

   [✓ D.] A bank account balance is $15. The account balance triples each year. What is the bank account balance, in dollars, after 5 years?

   E. A bank account balance is $15,000. It grows by $3,000 each year. What is the bank account balance, in thousands of dollars, after 5 years?

5. Here are graphs of two exponential functions, f and g. If $f(x) = 100 \cdot \left(\frac{2}{3}\right)^x$ and $g(x) = 100 \cdot b^x$, which of the following could be the value of b?

   [✓ A.] $\frac{1}{3}$

   B. $\frac{3}{4}$

   C. 1

   D. $\frac{3}{2}$

6. A person loans their friend $500. They agree to an annual interest rate of 5%. Write an expression for computing the amount owed on the loan, in dollars, after t years if no payments are made.

   $500 \cdot (1.05)^t$

Lesson 16 Functions Involving Percent Change 669

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 3	2
	3	Activity 1	1
Spiral	4	Unit 4 Lesson 4	1
	5	Unit 4 Lesson 7	2
Formative ⚡	6	Unit 4 Lesson 17	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

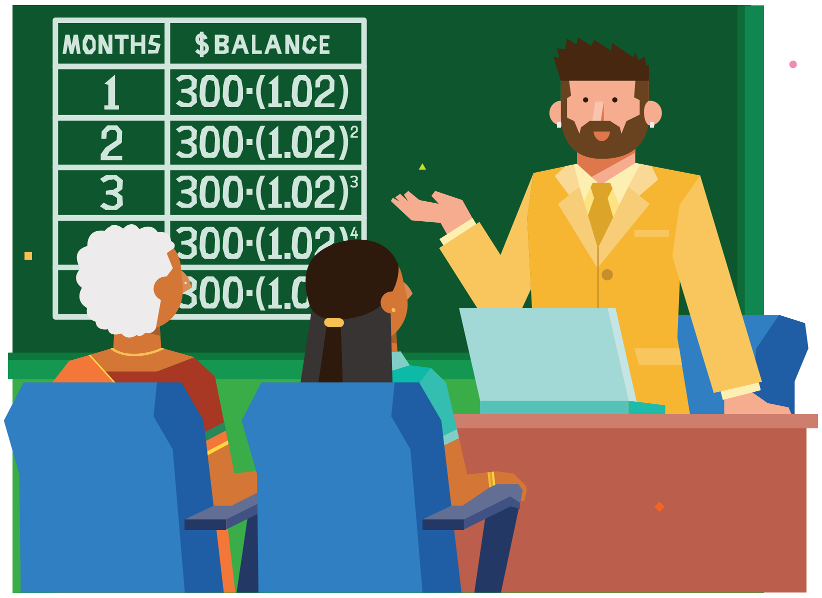
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 17**

Compounding Interest

Let's explore different ways of repeatedly applying a percent increase.



Focus

Goals

1. Calculate compounded percent change.
2. **Language Goal:** Explain why applying a percent increase, p, n times is like or unlike applying the percent increase np. **(Speaking and Listening, Writing)**

Coherence

Today

Students distinguish the effect of compounded percent change from that of simple percent change. They see that the repeated application of a percent change yields a greater final change because, with each iteration, the value that is used to compute the percent increase grows. They learn that this process is called *compounding*.

Previously

Students calculated the effects of high interest rates and compounding percentages.

Coming Soon

Students are asked to choose the better of two investment options with different interest rates and compounding intervals.

Rigor

- Students **apply** their understanding of percent change and exponential functions to explore compounding interest.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New words

- ***interest rate***
- ***principal***

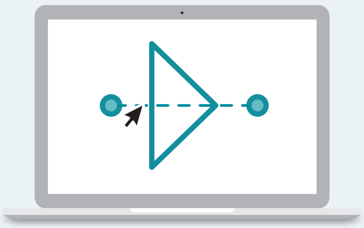
Review words

- *exponential expression*
- *interest*
- *principal*

Amps | Featured Activity

Activity 1
Adjusting Dimensions

Students digitally adjust the dimensions of a rectangle with precision. Students aim for changes of 10% and 20%.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might try to get through this lesson by focusing on the mathematics of repeated percent change rather than reasoning abstractly about the math for the real-word applications. Throughout the calculations, encourage students to continually come back to the conclusions that can be drawn from the examples that they see. While some might find the calculations challenging in and of themselves, help students understand that the mathematics will help their future financial lives.

Modifications to Pacing

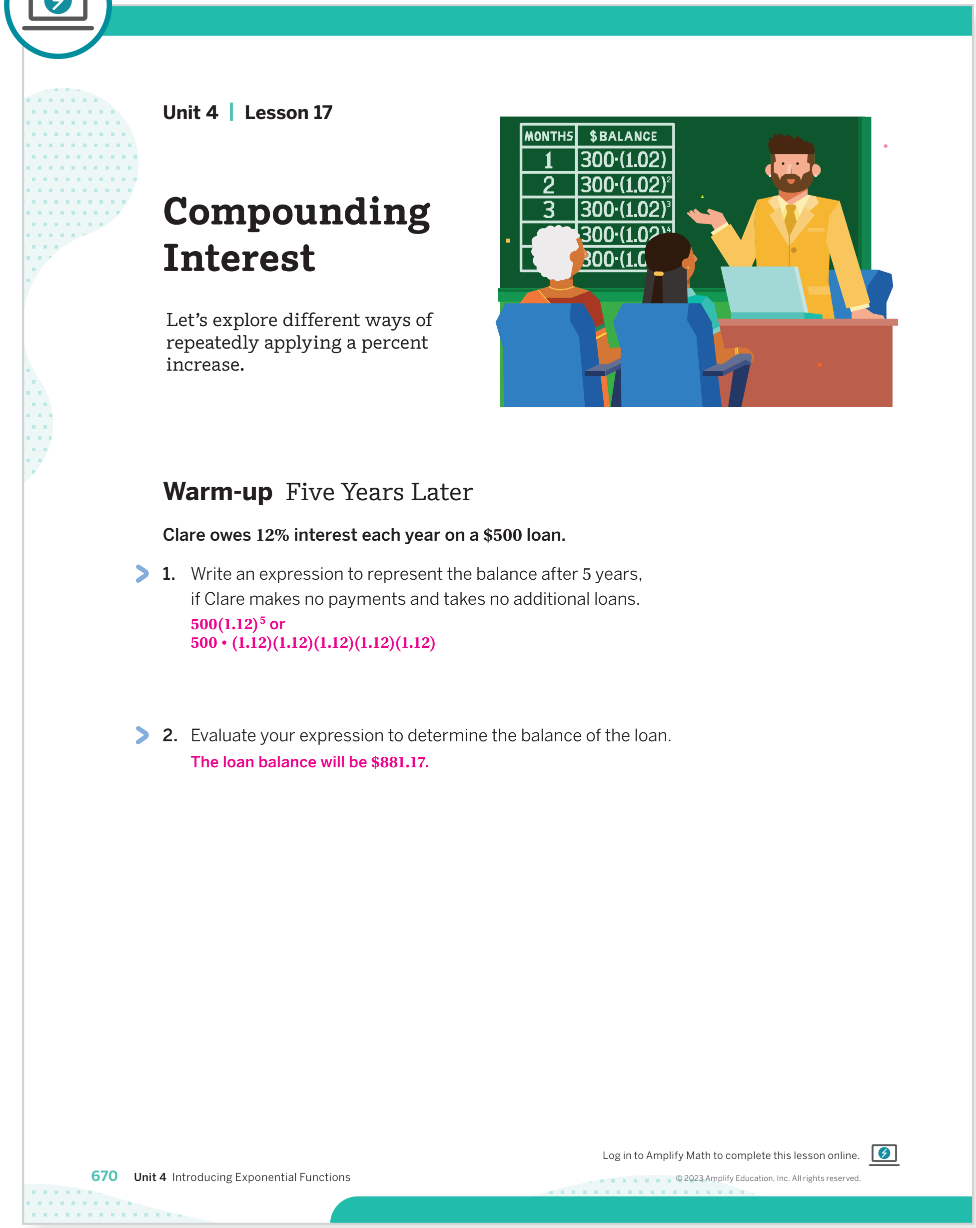
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, adjust the context so that students use a smaller initial amount, making calculations faster.

Independent | 5 min

Warm-up Five Years Later

Students practice writing an exponential expression to represent repeated interest calculations.



Unit 4 | Lesson 17

Compounding Interest

Let's explore different ways of repeatedly applying a percent increase.

Warm-up Five Years Later

Clare owes 12% interest each year on a $500 loan.

1. Write an expression to represent the balance after 5 years, if Clare makes no payments and takes no additional loans.
   $500(1.12)^5$ or
   $500 \cdot (1.12)(1.12)(1.12)(1.12)(1.12)$

2. Evaluate your expression to determine the balance of the loan.
   The loan balance will be $881.17.

Log in to Amplify Math to complete this lesson online.

670 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Tell students that they will explore repeatedly applying interest. Set an expectation for the amount of time students will have to individually work on the Warm-up.

2 Monitor

Help students get started by having them think of each year's balance as a product involving the interest rate and the prior year's balance.

Look for points of confusion:

- **Struggling to write an expression.** Show students that if they have $100, increasing it by 12% would yield a total of $112. Ask, "What would you multiply 100 by to get 112?"

Look for productive strategies:

- Using repeated multiplication or an exponent to write an expression.

3 Connect

Have students share their strategies for writing expressions for Problem 1.

Display the expression after 5 years: $500 \cdot (1.12) \cdot (1.12) \cdot (1.12) \cdot (1.12) \cdot (1.12) = 500(1.12)^5$.

Highlight that the first expression shows 5 years explicitly. In the second expression, the exponent represents the number of times the interest is compounded. *Compounding* is the repeated application of a percent change.

Define the ***interest rate*** as the proportion of the loan that is charged. The ***principal*** is the original amount of the loan.

Ask, "How could you change your expression to use it to calculate the balance for any amount of years?" Replace 5 with x.

Power-up

To power up students' ability to represent compound interest, have students complete:

Determine which expression represents the amount owed on a $300 loan at a 4% interest rate after 5 years if no payments are made.

A. $300(1.04 \cdot 5)$
C. $300(0.04 \cdot 5)$
[✓ B.] $300(1.04)^5$
D. $300(0.04)^5$

Use: Before the Warm-up

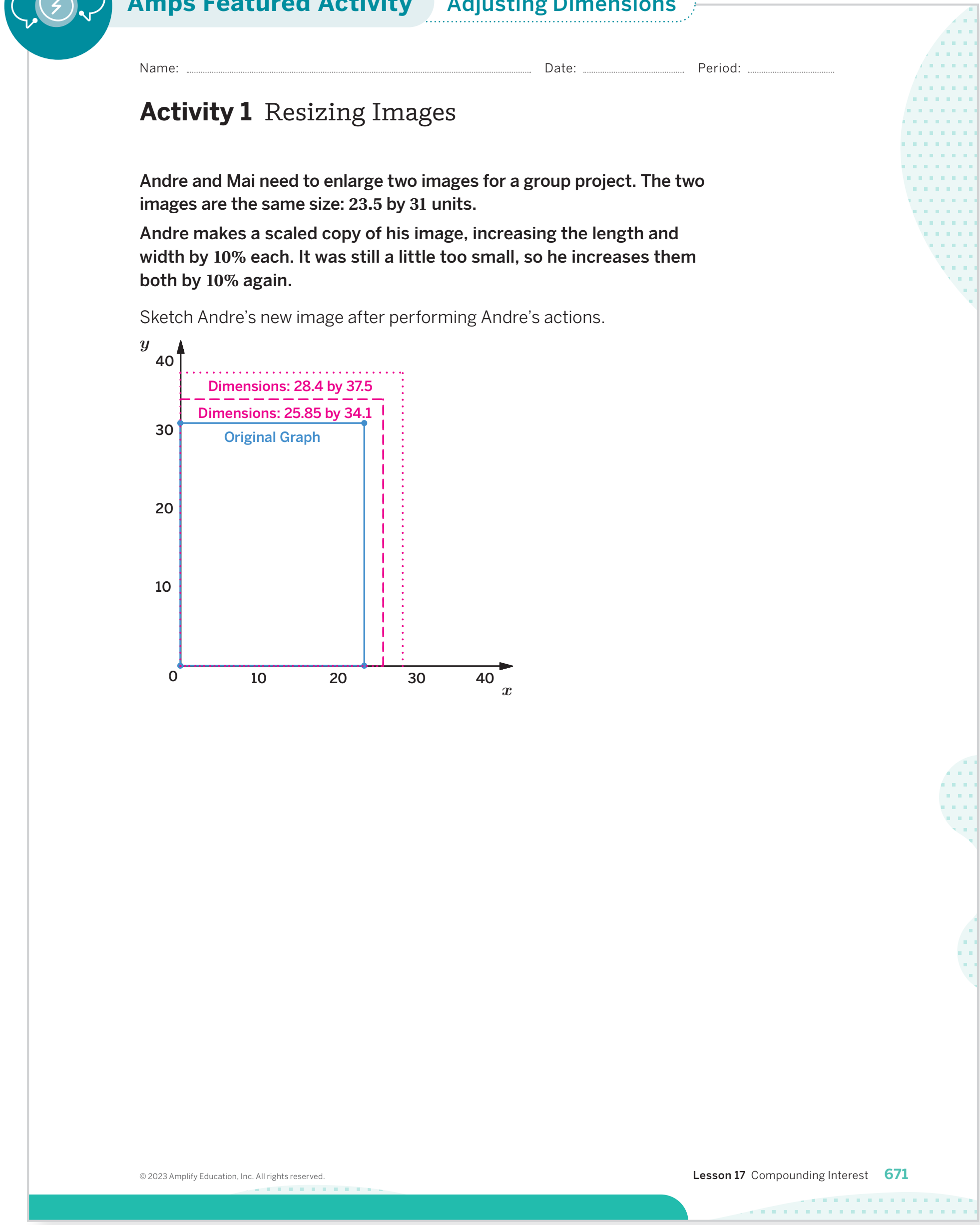
Informed by: Performance on Lesson 16, Practice Problem 6

Pairs | 20 min

Activity 1 Resizing Images

Students use a geometric context to investigate whether increasing an amount by 10% twice is the same as applying a 20% increase once.

Amps Featured Activity Adjusting Dimensions



Name: ______ Date: ______ Period: ______

Activity 1 Resizing Images

Andre and Mai need to enlarge two images for a group project. The two images are the same size: 23.5 by 31 units.

Andre makes a scaled copy of his image, increasing the length and width by 10% each. It was still a little too small, so he increases them both by 10% again.

Sketch Andre's new image after performing Andre's actions.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Compounding Interest 671

1 Launch

Read the prompt aloud. Set an expectation for the amount of time students have to work in pairs on the activity.

2 Monitor

Help students get started by asking them what the dimensions of the rectangle would be after the first percent increase.

Look for points of confusion:

- **Adding 10% and 10% to get 20%.** Ask, "Suppose you had $100 and increased it by 10%, then increased this new amount by 10% again. How much money would you have? Is it the same amount as increasing $100 by 20%?"

Look for productive strategies:

- Annotating the dimensions on the rectangle before and after each percent increase.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use sliders to adjust dimensions of a rectangle. The percentage change from the original dimensions is automatically populated, and students aim for changes for 10% and 20%.

Accessibility: *Vary Demands to Optimize Challenge*

Provide whole number dimensions for the rectangle, so that students can focus on the effects of the repeated percent change.

Accessibility: *Guide Processing and Visualization*

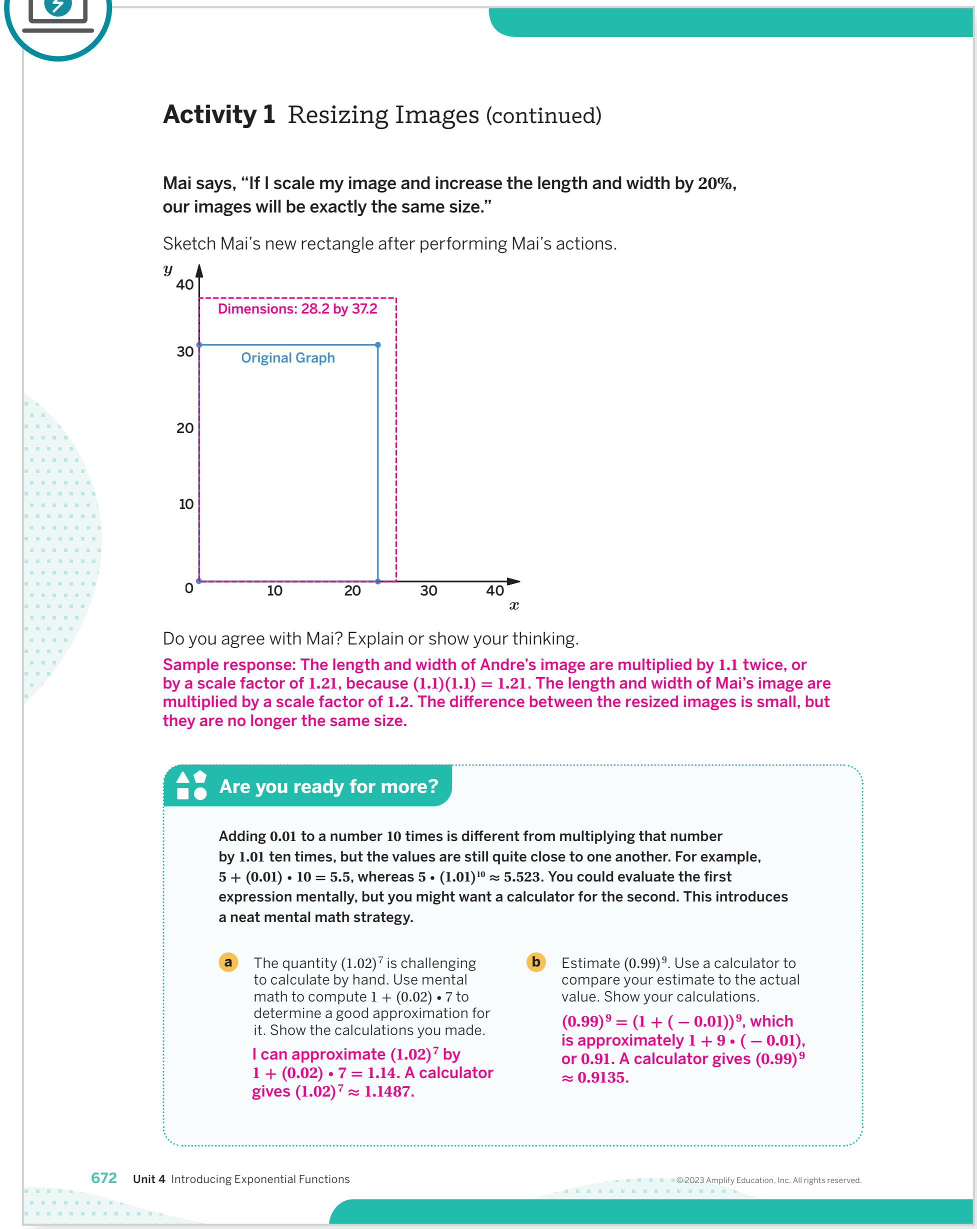
Suggest students create a table that shows the dimensions for each scaled copy before sketching each one.

Original dimensions	Andre's first scaled copy, 10%	Andre's second scaled copy, 10%	Mai's scaled copy, 20%
23.5			
31			

Pairs | 20 min

Activity 1 Resizing Images (continued)

Students use a geometric context to investigate whether increasing an amount by 10% twice is the same as applying a 20% increase once.



Activity 1 Resizing Images (continued)

Mai says, "If I scale my image and increase the length and width by 20%, our images will be exactly the same size."

Sketch Mai's new rectangle after performing Mai's actions.

Do you agree with Mai? Explain or show your thinking.

Sample response: The length and width of Andre's image are multiplied by 1.1 twice, or by a scale factor of 1.21, because $(1.1)(1.1) = 1.21$. The length and width of Mai's image are multiplied by a scale factor of 1.2. The difference between the resized images is small, but they are no longer the same size.

Are you ready for more?

Adding 0.01 to a number 10 times is different from multiplying that number by 1.01 ten times, but the values are still quite close to one another. For example, $5 + (0.01) \cdot 10 = 5.5$, whereas $5 \cdot (1.01)^{10} \approx 5.523$. You could evaluate the first expression mentally, but you might want a calculator for the second. This introduces a neat mental math strategy.

a. The quantity $(1.02)^7$ is challenging to calculate by hand. Use mental math to compute $1 + (0.02) \cdot 7$ to determine a good approximation for it. Show the calculations you made.

I can approximate $(1.02)^7$ by $1 + (0.02) \cdot 7 = 1.14$. A calculator gives $(1.02)^7 \approx 1.1487$.

b. Estimate $(0.99)^9$. Use a calculator to compare your estimate to the actual value. Show your calculations.

$(0.99)^9 = (1 + (-0.01))^9$, which is approximately $1 + 9 \cdot (-0.01)$, or 0.91. A calculator gives $(0.99)^9 \approx 0.9135$.

672 **Unit 4** Introducing Exponential Functions ©2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display students' drawings.

Have students share their explanations regarding Andre's and Mai's drawings.

Highlight that by increasing the dimensions by 10%, this is the same as multiplying the dimensions by a factor of 1.1 twice, or $(1.1)(1.1) = 1.21$. The repeated application of a percent change is known as *compounding*.

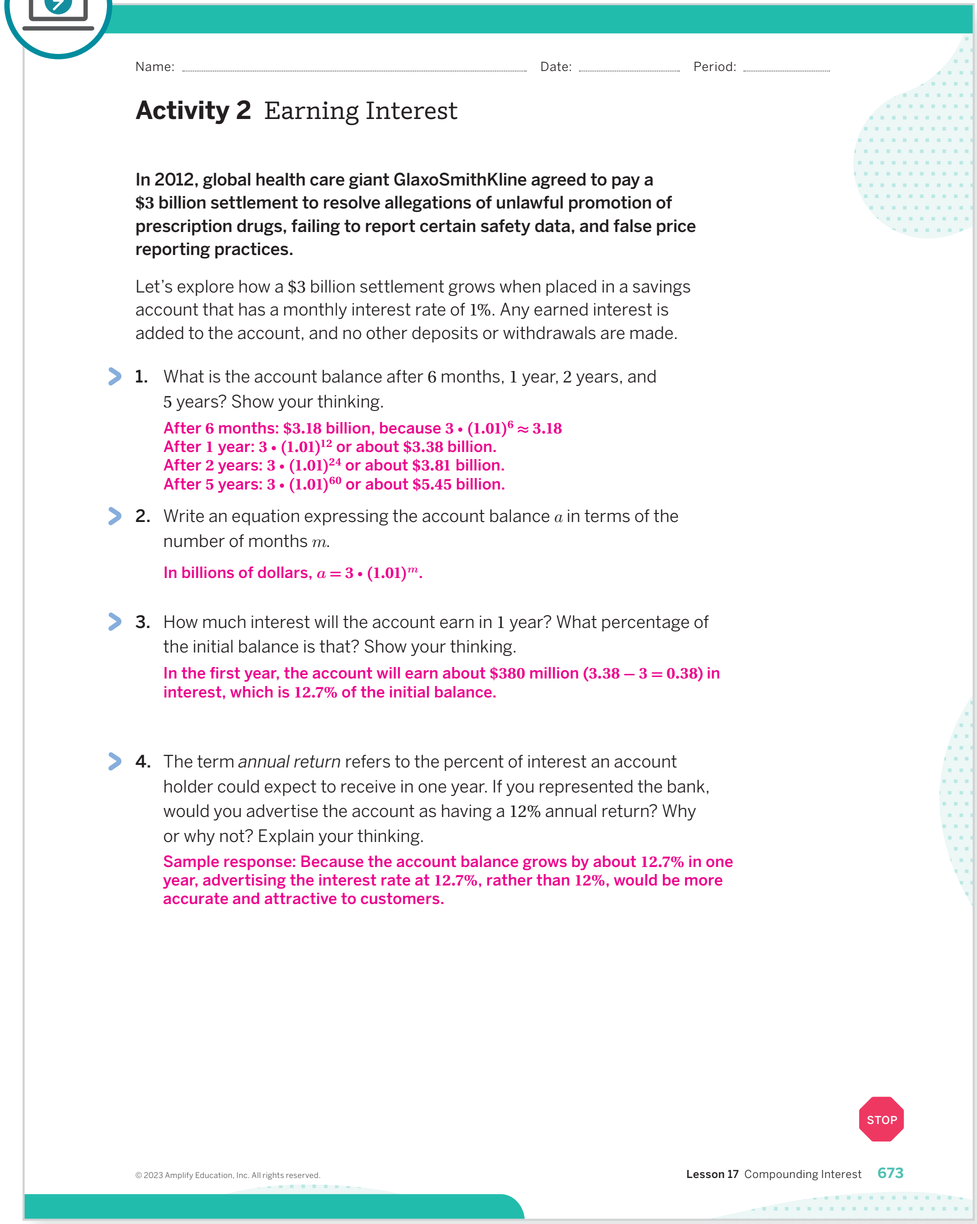
Ask, "What is the percent change that relates Andre's final image with the original?"

$(0.1)(0.1) = 0.21$ or 21%

Pairs | 15 min

Activity 2 Earning Interest

Students explore the idea of repeated percent change in the context of a savings account and use repeated reasoning about percent change calculations to write an exponential equation.



Name: ______ Date: ______ Period: ______

Activity 2 Earning Interest

In 2012, global health care giant GlaxoSmithKline agreed to pay a $3 billion settlement to resolve allegations of unlawful promotion of prescription drugs, failing to report certain safety data, and false price reporting practices.

Let's explore how a $3 billion settlement grows when placed in a savings account that has a monthly interest rate of 1%. Any earned interest is added to the account, and no other deposits or withdrawals are made.

1. What is the account balance after 6 months, 1 year, 2 years, and 5 years? Show your thinking.
   After 6 months: $3.18 billion, because $3 \cdot (1.01)^6 \approx 3.18$
   After 1 year: $3 \cdot (1.01)^{12}$ or about $3.38 billion.
   After 2 years: $3 \cdot (1.01)^{24}$ or about $3.81 billion.
   After 5 years: $3 \cdot (1.01)^{60}$ or about $5.45 billion.
2. Write an equation expressing the account balance a in terms of the number of months m.
   In billions of dollars, $a = 3 \cdot (1.01)^m$.
3. How much interest will the account earn in 1 year? What percentage of the initial balance is that? Show your thinking.
   In the first year, the account will earn about $380 million ($3.38 - 3 = 0.38$) in interest, which is 12.7% of the initial balance.
4. The term *annual return* refers to the percent of interest an account holder could expect to receive in one year. If you represented the bank, would you advertise the account as having a 12% annual return? Why or why not? Explain your thinking.
   Sample response: Because the account balance grows by about 12.7% in one year, advertising the interest rate at 12.7%, rather than 12%, would be more accurate and attractive to customers.

STOP

1 Launch

Say, "Let's take a look at how money grows in a savings account that involves compound interest and grows more rapidly over time. For accounts like credit cards, you have to pay the bank interest, and similarly, the amount grows more rapidly over time."

2 Monitor

Help students get started by saying, "In the Warm-up, the exponent in the expression represented the number of times the interest was compounded."

Look for points of confusion:

- **Applying interest annually, not monthly.** Help students adjust the values to months.
- **Using repeated multiplication to determine the account balance in Problem 1.** Ask, "Is this the most efficient approach for calculating the amount several years into the future?"

Look for productive strategies:

- Writing an exponential equation to express the account balance in terms of the number of months.

3 Connect

Have students share their responses and strategies for Problem 1.

Highlight that 12 successive increases of 1 is greater than a 12% interest. 12% is called the *nominal rate* and 12.7% is called the *effective rate*.

Display the calculations for Problem 1. The effective rate takes into account the monthly interest payment.

Ask, "Will the nominal rate ever be greater than the effective rate?" No, because the effective rate reflects compounding interest.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students create a table that shows the account balance after the various lengths of time. If students do not convert the number of years to months in Problem 1, ask, "Is the interest rate an annual interest rate or a monthly interest rate? How can you express these years in terms of the number of months?"

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they will explore how a monetary settlement grows over time, when it is placed into a savings account.
- **Read 2:** Ask students to name or describe the given quantities and relationships, such as the savings account has a monthly interest rate of 1%.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problem 1.

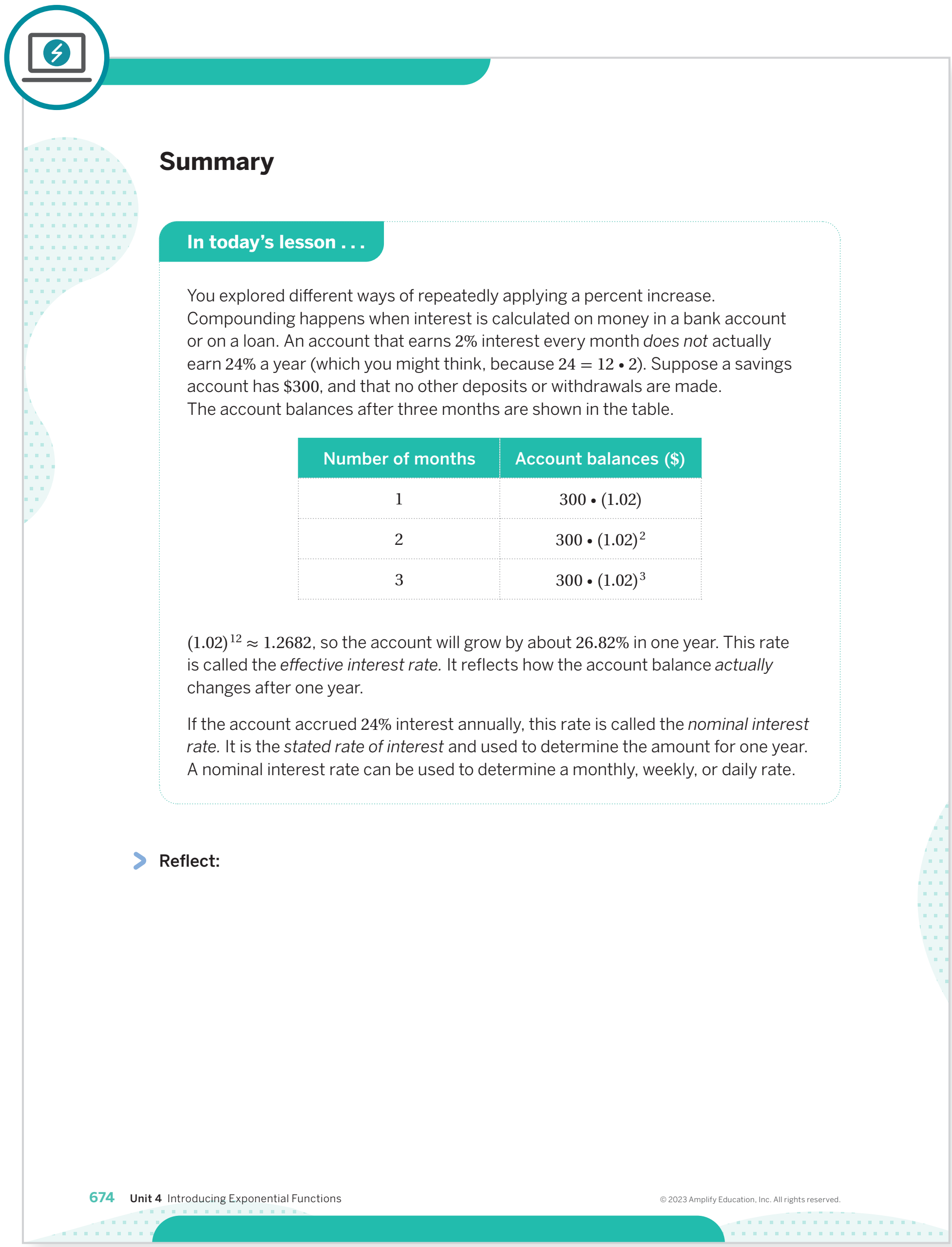
English Learners

Students may be unfamiliar with the term *settlement*. Explain that it is an agreement to pay someone money to settle a lawsuit.

Whole Class | 5 min

Summary

Review and synthesize why applying a percent increase twice is different from doubling the percent and then applying it a single time.



Summary

In today's lesson . . .

You explored different ways of repeatedly applying a percent increase. Compounding happens when interest is calculated on money in a bank account or on a loan. An account that earns 2% interest every month *does not* actually earn 24% a year (which you might think, because $24 = 12 \cdot 2$). Suppose a savings account has $300, and that no other deposits or withdrawals are made. The account balances after three months are shown in the table.

Number of months	Account balances ($)
1	$300 \cdot (1.02)$
2	$300 \cdot (1.02)^2$
3	$300 \cdot (1.02)^3$

$(1.02)^{12} \approx 1.2682$, so the account will grow by about 26.82% in one year. This rate is called the *effective interest rate*. It reflects how the account balance *actually* changes after one year.

If the account accrued 24% interest annually, this rate is called the *nominal interest rate*. It is the *stated rate of interest* and used to determine the amount for one year. A nominal interest rate can be used to determine a monthly, weekly, or daily rate.

Reflect:

674 Unit 4 Introducing Exponential Functions

Synthesize

Display the table of the savings account.

Ask:

- "What is the balance of the savings account after one year?"
- "How much interest does the balance accrue?"
- "Suppose a savings account has an annual interest rate of 24%. What would the balance of the account be after one year?"
- "Which interest rate would you rather have?"

Highlight the differences between *nominal* and *effective interest rates*.

Formalize vocabulary:

- *interest rate*
- *principal*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

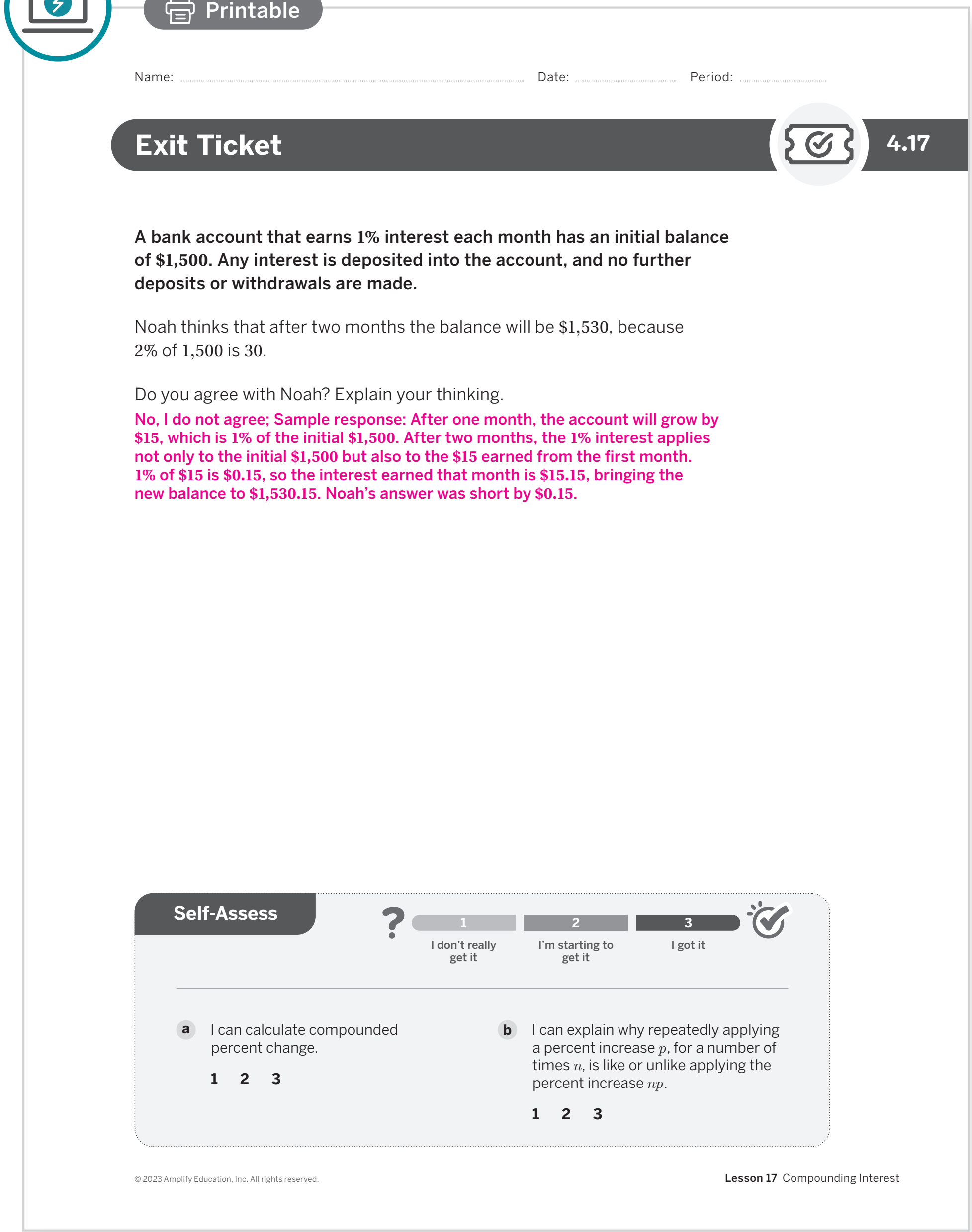
- "Why do you think credit card companies advertise nominal interest rates?"
- "How is repeatedly applying percent increase different from applying the sums of the percent increases a single time?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by critiquing a student's reasoning to explain why applying a percent twice is different from doubling the percent and then applying it.

Printable



Name: .... Date: .... Period: ....

Exit Ticket 4.17

A bank account that earns 1% interest each month has an initial balance of $1,500. Any interest is deposited into the account, and no further deposits or withdrawals are made.

Noah thinks that after two months the balance will be $1,530, because 2% of 1,500 is 30.

Do you agree with Noah? Explain your thinking.

No, I do not agree; Sample response: After one month, the account will grow by $15, which is 1% of the initial $1,500. After two months, the 1% interest applies not only to the initial $1,500 but also to the $15 earned from the first month. 1% of $15 is $0.15, so the interest earned that month is $15.15, bringing the new balance to $1,530.15. Noah's answer was short by $0.15.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can calculate compounded percent change.

1 2 3

b. I can explain why repeatedly applying a percent increase p, for a number of times n, is like or unlike applying the percent increase np.

1 2 3

Lesson 17 Compounding Interest

Success looks like . . .

- **Goal:** Calculating compounded percent change.
  - » Explaining why Noah is not correct by explaining that 1% interest applies to the initial amount and to the interest earned from the first month.
- **Language Goal:** Explaining why applying a percent increase, p, n times is like or unlike applying the percent increase np. **(Speaking and Listening, Writing)**

Suggested next steps

If students agreed and made incorrect calculations or provided an unclear or invalid explanation, consider:

- Revisiting the concept of compounding.
- Reviewing how to develop an equation for scenarios of compound interest in Activities 1 and 2.
- Reviewing Activities 1 and 2.

If students agreed and made correct calculations and a partially clear or valid explanation, consider:

- Revisiting the difference between compounding and simple interest.
- Highlighting that compounding will result in more rapid growth/decay over time.
- Reviewing Activity 1.

If students disagreed and made incorrect calculations or an unclear or invalid explanation, consider:

- Scaffolding the process of creating an exponential function involving compound interest.
- Emphasizing the use of the initial value, the rate of growth/decay, and the number of times the amount is compounded.
- Reviewing Activity 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students explored why repeatedly applying percent increase is not the same as applying the sums of the percent increases a single time. How well do you think your students understood this concept as they completed Activity 1?
- Which students or groups of students did and did not have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Explaining why applying a percent increase, p, n times is like or unlike applying the percent increase np.

Reflect on students' language development toward this goal.

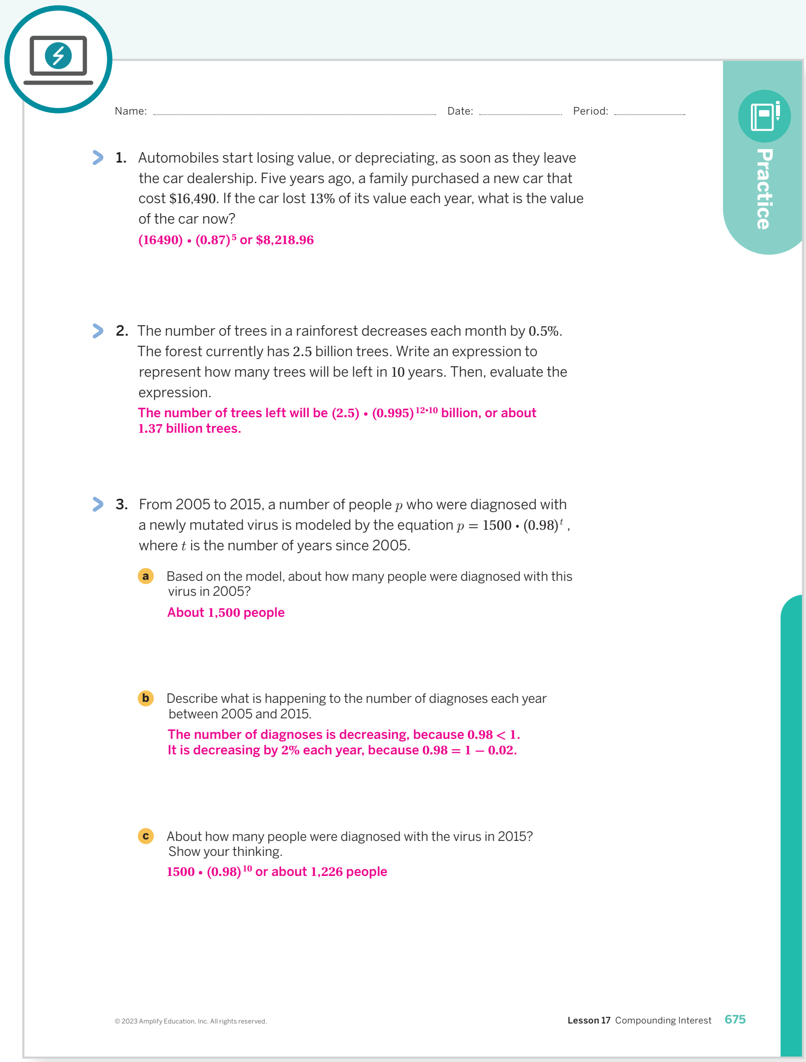
- Do students' responses to the Exit Ticket problem demonstrate they understand why Noah is incorrect in his thinking?
- How can you help them be more precise in their explanations?

Sample explanations:

Emerging	Expanding
The balance changes each month.	After one month, the balance is $1,515. The 1% interest is now applied to this balance for the second month.

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Automobiles start losing value, or depreciating, as soon as they leave the car dealership. Five years ago, a family purchased a new car that cost $16,490. If the car lost 13% of its value each year, what is the value of the car now?

   $(16490) \cdot (0.87)^5$ or $8,218.96

2. The number of trees in a rainforest decreases each month by 0.5%. The forest currently has 2.5 billion trees. Write an expression to represent how many trees will be left in 10 years. Then, evaluate the expression.

   The number of trees left will be $(2.5) \cdot (0.995)^{12 \cdot 10}$ billion, or about 1.37 billion trees.

3. From 2005 to 2015, a number of people p who were diagnosed with a newly mutated virus is modeled by the equation $p = 1500 \cdot (0.98)^t$, where t is the number of years since 2005.

   a. Based on the model, about how many people were diagnosed with this virus in 2005?

      About 1,500 people

   b. Describe what is happening to the number of diagnoses each year between 2005 and 2015.

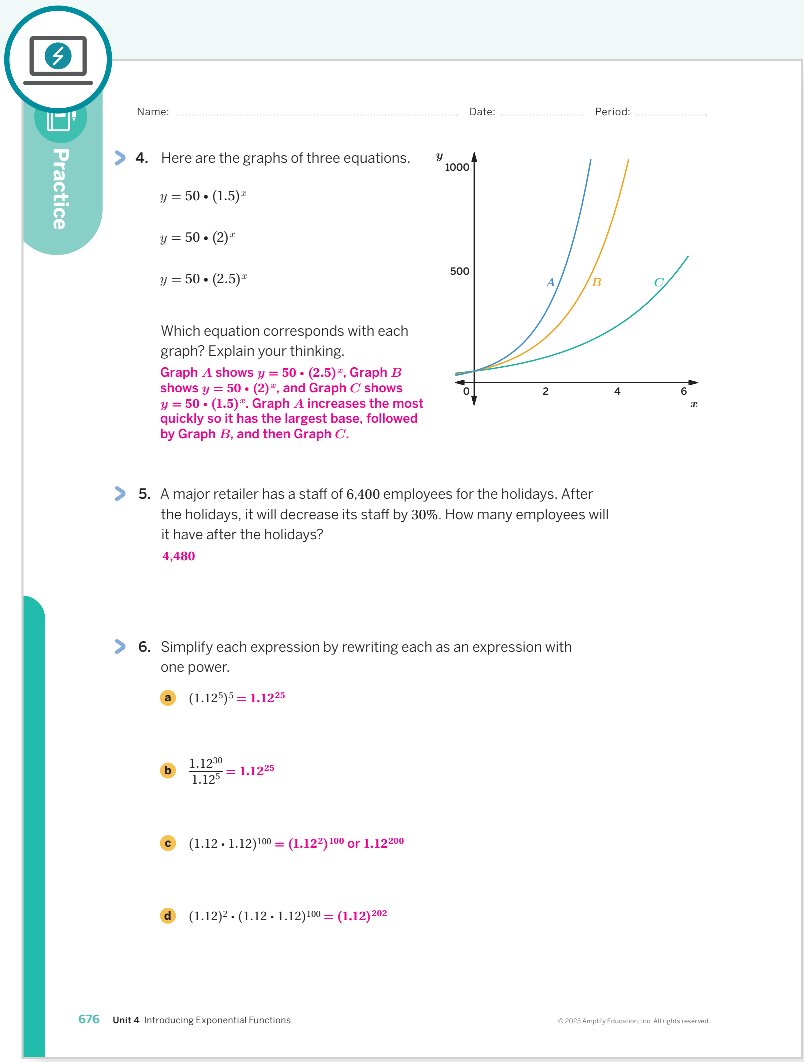
      The number of diagnoses is decreasing, because $0.98 < 1$. It is decreasing by 2% each year, because $0.98 = 1 - 0.02$.

   c. About how many people were diagnosed with the virus in 2015? Show your thinking.

      $1500 \cdot (0.98)^{10}$ or about 1,226 people

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Compounding Interest 675



Name: ______ Date: ______ Period: ______

4. Here are the graphs of three equations.

   $y = 50 \cdot (1.5)^x$

   $y = 50 \cdot (2)^x$

   $y = 50 \cdot (2.5)^x$

   Which equation corresponds with each graph? Explain your thinking.

   Graph A shows $y = 50 \cdot (2.5)^x$, Graph B shows $y = 50 \cdot (2)^x$, and Graph C shows $y = 50 \cdot (1.5)^x$. Graph A increases the most quickly so it has the largest base, followed by Graph B, and then Graph C.

5. A major retailer has a staff of 6,400 employees for the holidays. After the holidays, it will decrease its staff by 30%. How many employees will it have after the holidays?

   4,480

6. Simplify each expression by rewriting each as an expression with one power.

   a. $(1.12^5)^5 = \mathbf{1.12^{25}}$

   b. $\frac{1.12^{30}}{1.12^5} = \mathbf{1.12^{25}}$

   c. $(1.12 \cdot 1.12)^{100} = \mathbf{(1.12^2)^{100}}$ **or** $\mathbf{1.12^{200}}$

   d. $(1.12)^2 \cdot (1.12 \cdot 1.12)^{100} = \mathbf{(1.12)^{202}}$

676 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 14	2
	5	Unit 4 Lesson 15	1
Formative ⚡	6	Unit 4 Lesson 18	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 18**

Expressing Exponentials in Different Ways

Let's write exponential functions in different, yet equivalent ways.

Focus

Goals

1. Interpret and evaluate exponential expressions to solve problems.
2. Write equivalent expressions to highlight different aspects of a situation that involves repeated percent increase or decrease.

Rigor

- Students build their **procedural fluency** in manipulating exponential functions and expressions by writing them in multiple forms that highlight their different key features.

Coherence

Today

Students will distinguish between *growth rate* and *growth factor* (defined earlier in this unit). In functions of the form $a \cdot (1 + r)^x$, the growth rate is r, and the growth factor is $1 + r$.

Previously

Students explored compound interest, choosing the better of two investment options with different interest rates and compounding intervals.

Coming Soon

Students will investigate how exponential functions grow more quickly than linear functions by comparing simple and compound interest, and by examining tables and graphs to determine when an exponential function will overtake the linear function.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Exponent Rules*
- six-sided dice, one per group

Math Language Development

New words

- *growth rate**

Review words

- *growth factor**
- *interest*
- *percent change*

*Students may confuse the terms growth rate and growth factor. Remind them that a factor is a quantity that is multiplied. A rate refers to the percent change of a quantity.

Amps | **Featured Activity**

Activity 2
Modeling Epidemics Exponentially

Students simulate the spread of a disease under different circumstances. As part of the simulation, they roll a digital die and record their results in tables.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not realize that many policy decisions are made based on mathematical models, so they might not see how the activity is useful. Make sure students understand that the purpose of such mathematical models is often to make predictions so that decisions can be made that will help avoid problematic results. Encourage students to use models to help them realistically evaluate the consequences of different choices.

Modifications to Pacing

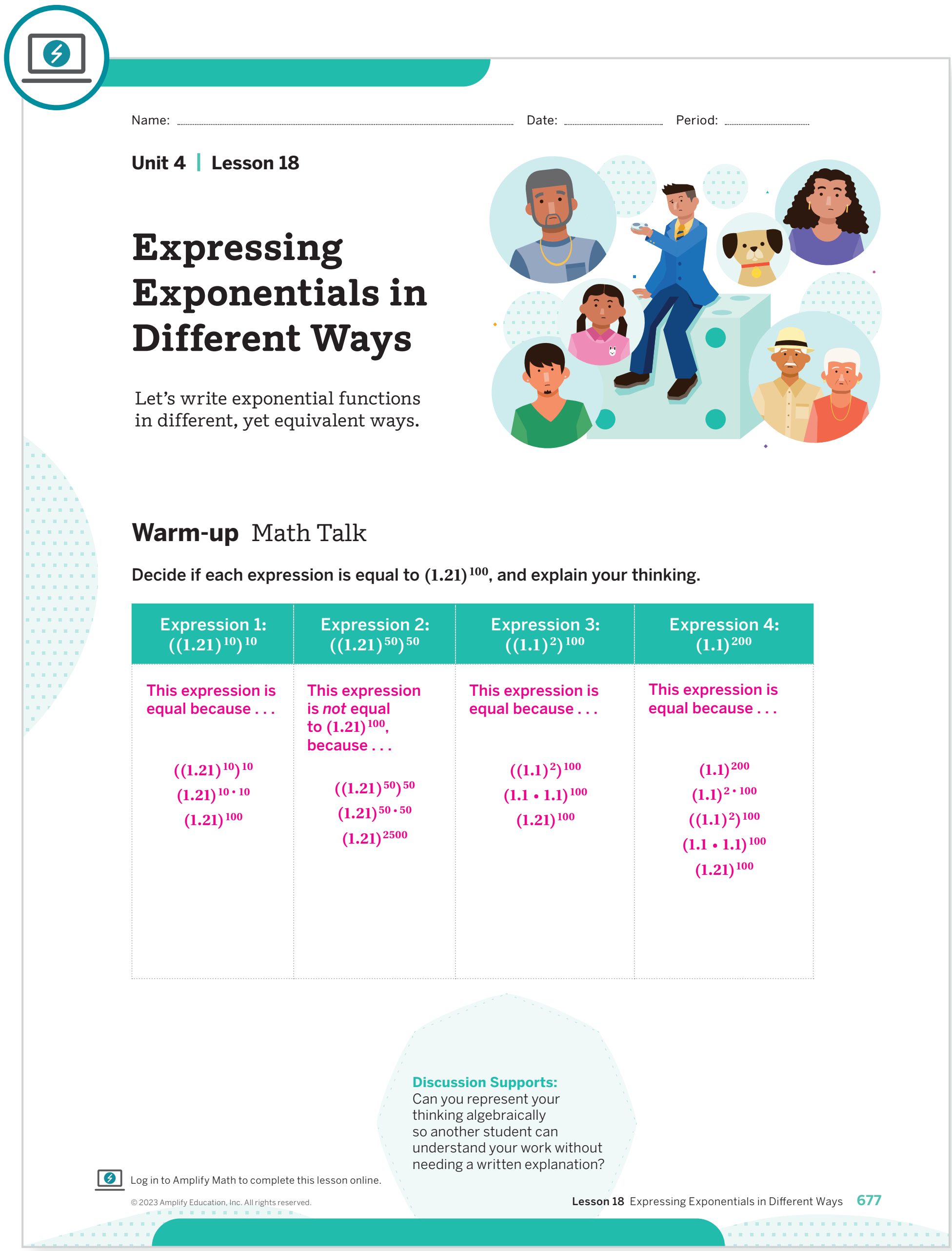
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, omit Scenario 2.

Independent | 5 min

Warm-up Math Talk

Students will use the rules of exponents and the structure of exponential expressions to identify equivalent expressions.



Name: ________ Date: ________ Period: ________

Unit 4 | Lesson 18

Expressing Exponentials in Different Ways

Let's write exponential functions in different, yet equivalent ways.

Warm-up Math Talk

Decide if each expression is equal to $(1.21)^{100}$, and explain your thinking.

Expression 1: $((1.21)^{10})^{10}$	Expression 2: $((1.21)^{50})^{50}$	Expression 3: $((1.1)^{2})^{100}$	Expression 4: $(1.1)^{200}$
This expression is equal because . . . $((1.21)^{10})^{10}$ $(1.21)^{10 \cdot 10}$ $(1.21)^{100}$	This expression is *not* equal to $(1.21)^{100}$, because . . . $((1.21)^{50})^{50}$ $(1.21)^{50 \cdot 50}$ $(1.21)^{2500}$	This expression is equal because . . . $((1.1)^{2})^{100}$ $(1.1 \cdot 1.1)^{100}$ $(1.21)^{100}$	This expression is equal because . . . $(1.1)^{200}$ $(1.1)^{2 \cdot 100}$ $((1.1)^{2})^{100}$ $(1.1 \cdot 1.1)^{100}$ $(1.21)^{100}$

Discussion Supports: Can you represent your thinking algebraically so another student can understand your work without needing a written explanation?

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the ***Math Talk*** routine. Display one expression at a time. Allow students one minute to think and record their responses before displaying the next expression.

2 Monitor

Help students get started by displaying and reviewing exponent rules on the Anchor Chart PDF, *Exponent Rules*.

Look for points of confusion:

- **Having difficulty using exponent rules.** Refer to the rules of exponents. Have students identify which rule is most appropriate.
- **Not recognizing that $(1.1)^2 = 1.21$.** Have students multiply $1.1 \cdot 1.1$.

Look for productive strategies:

- Recognizing $11 \cdot 11 = 121$, writing $1.1 \cdot 1.1 = 1.21$, and then writing $(1.21)^{100}$ as $((1.1)^2)^{100}$.
- Applying the power rule to multiply the exponents.

3 Connect

Display the four expressions.

Have students share their strategies for determining whether the expressions are equivalent.

Highlight that for Expression 4, to determine that $(1.1)^{200}$ is equivalent to $(1.21)^{100}$, write $(1.1)^{200}$ as $((1.1)^2)^{100}$. Similarly, students can write $(1.1)^{200}$ as $(1.1)^{2 \cdot 100}$. This is intended to facilitate structural thinking around the power rule for exponents, $(x^a)^b = x^{ab}$.

Ask, "Why is $((1.21)^{50})^{50}$ not equivalent to $(1.21)^{100}$?" Using the power rule, $((1.21)^{50})^{50} = (1.21)^{50 \cdot 50} = (1.21)^{2500}$.

Math Language Development

MLR8: Discussion Supports

During the Launch, ask students to consider the question posed to them in their Student Edition, "Can you represent your thinking algebraically so another student can understand your work without needing a written explanation?" After they complete each column, ask them to pause and look back at their work to see if it is clear and think of ways to make it clearer.

Power-up

To power up students' ability to simplify expressions using exponent rules, have students complete:

Recall that $(a^m)^n = a^{m \cdot n}$.

Which of the following expressions is *not* equal to $(2)^6$?

A. $((2)^3)^2$

B. $((4)^3)^1$

[✓ **C.**] $((2)^3)^3$

D. $((8)^1)^2$

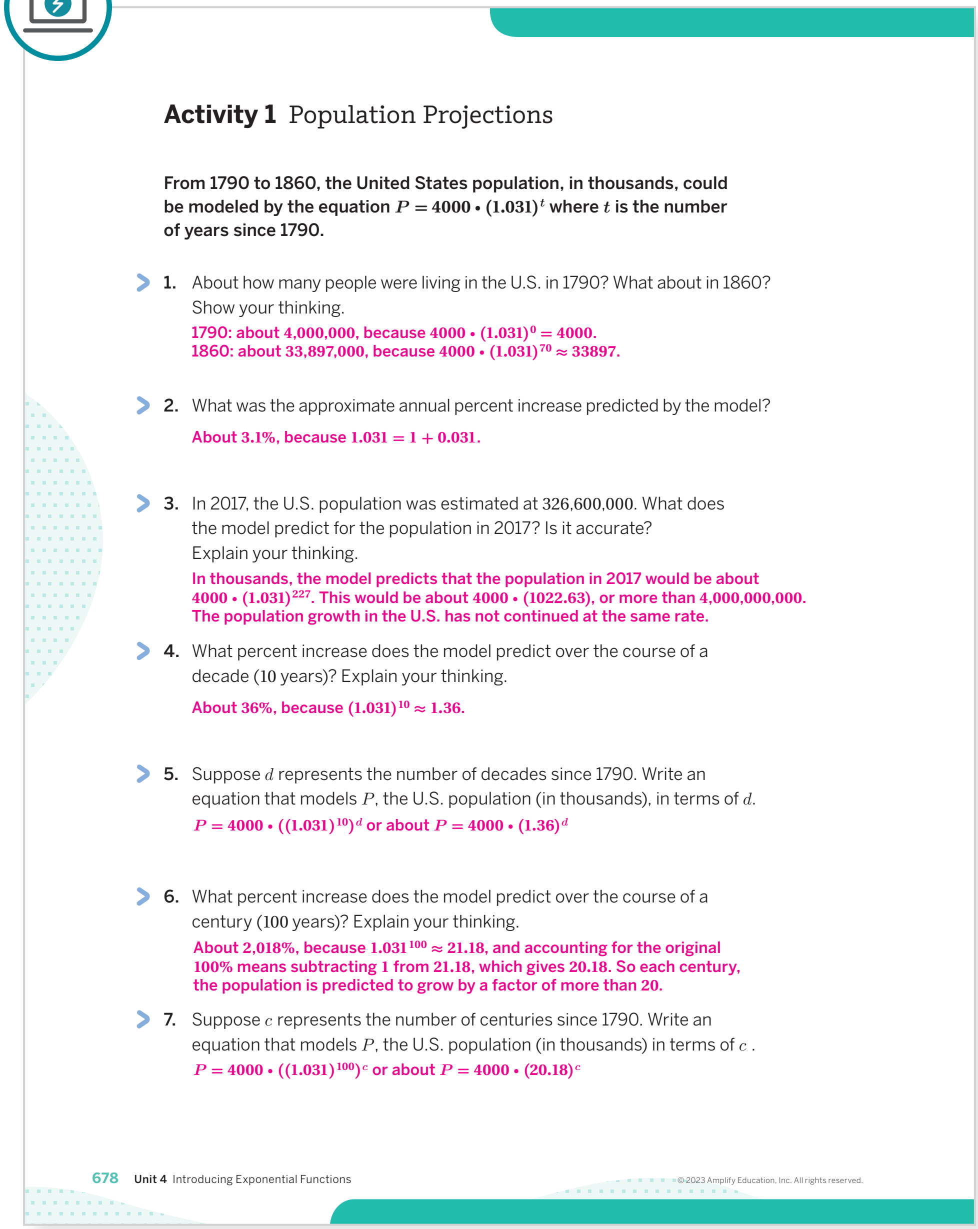
Use: Before the Warm-up

Informed by: Performance on Lesson 17, Practice Problem 6

Pairs | 15 min

Activity 1 Population Projections

Students examine and interpret the structure of an exponential equation that models a population context and construct exponential equations to model the population for different intervals of time.



Activity 1 Population Projections

From 1790 to 1860, the United States population, in thousands, could be modeled by the equation $P = 4000 \cdot (1.031)^t$ where t is the number of years since 1790.

1. About how many people were living in the U.S. in 1790? What about in 1860? Show your thinking.
   1790: about 4,000,000, because $4000 \cdot (1.031)^0 = 4000$.
   1860: about 33,897,000, because $4000 \cdot (1.031)^{70} \approx 33897$.
2. What was the approximate annual percent increase predicted by the model?
   About 3.1%, because $1.031 = 1 + 0.031$.
3. In 2017, the U.S. population was estimated at 326,600,000. What does the model predict for the population in 2017? Is it accurate? Explain your thinking.
   In thousands, the model predicts that the population in 2017 would be about $4000 \cdot (1.031)^{227}$. This would be about $4000 \cdot (1022.63)$, or more than 4,000,000,000. The population growth in the U.S. has not continued at the same rate.
4. What percent increase does the model predict over the course of a decade (10 years)? Explain your thinking.
   About 36%, because $(1.031)^{10} \approx 1.36$.
5. Suppose d represents the number of decades since 1790. Write an equation that models P, the U.S. population (in thousands), in terms of d.
   $P = 4000 \cdot ((1.031)^{10})^d$ or about $P = 4000 \cdot (1.36)^d$
6. What percent increase does the model predict over the course of a century (100 years)? Explain your thinking.
   About 2,018%, because $1.031^{100} \approx 21.18$, and accounting for the original 100% means subtracting 1 from 21.18, which gives 20.18. So each century, the population is predicted to grow by a factor of more than 20.
7. Suppose c represents the number of centuries since 1790. Write an equation that models P, the U.S. population (in thousands) in terms of c .
   $P = 4000 \cdot ((1.031)^{100})^c$ or about $P = 4000 \cdot (20.18)^c$

678 **Unit 4** Introducing Exponential Functions

1 Launch

Have students complete Problems 1–3 with a partner, complete Problems 4 and 5 individually, and discuss Problems 6 and 7 with their partner.

2 Monitor

Help students get started by discussing how they would expect a population to grow and understanding the equation given.

Look for points of confusion:

- **Having difficulty using exponent rules.** Revisit the Anchor Chart PDF, *Exponent Rules* and the power rule.
- **Misinterpreting the value of P in the equation.** Point out the population for the country in 1790 was greater than 4,000.

Look for productive strategies:

- Determining that $t = 1860 - 1790$, or 70, in Problem 1.
- Identifying the growth rate r within the growth factor b as $b = 1 + r$.
- Multiplying P by 1,000 to determine the population.

3 Connect

Have students share strategies for determining the percent increase for the decade and the century.

Highlight the population growth model by decade. Explain that *growth rate* is an exponential change by a percentage. The *growth rate* r can be identified within the *growth factor* b because $b = 1 + r$.

Ask, "Which growth rate is more helpful for understanding the population change in that period?" Growth rate by decade.

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Suggest students create a table that shows the population of the United States for the number of years since 1790.

Year	Number of years since 1790	Population

Math Language Development

MLR2: Collect and Display

While students work, circulate and collect any vocabulary or phrases they use to describe the percent increase in this activity. During the Connect, as you explain the term *growth rate,* connect this term to the *growth factor.* Consider adding something similar to the following to the class display:

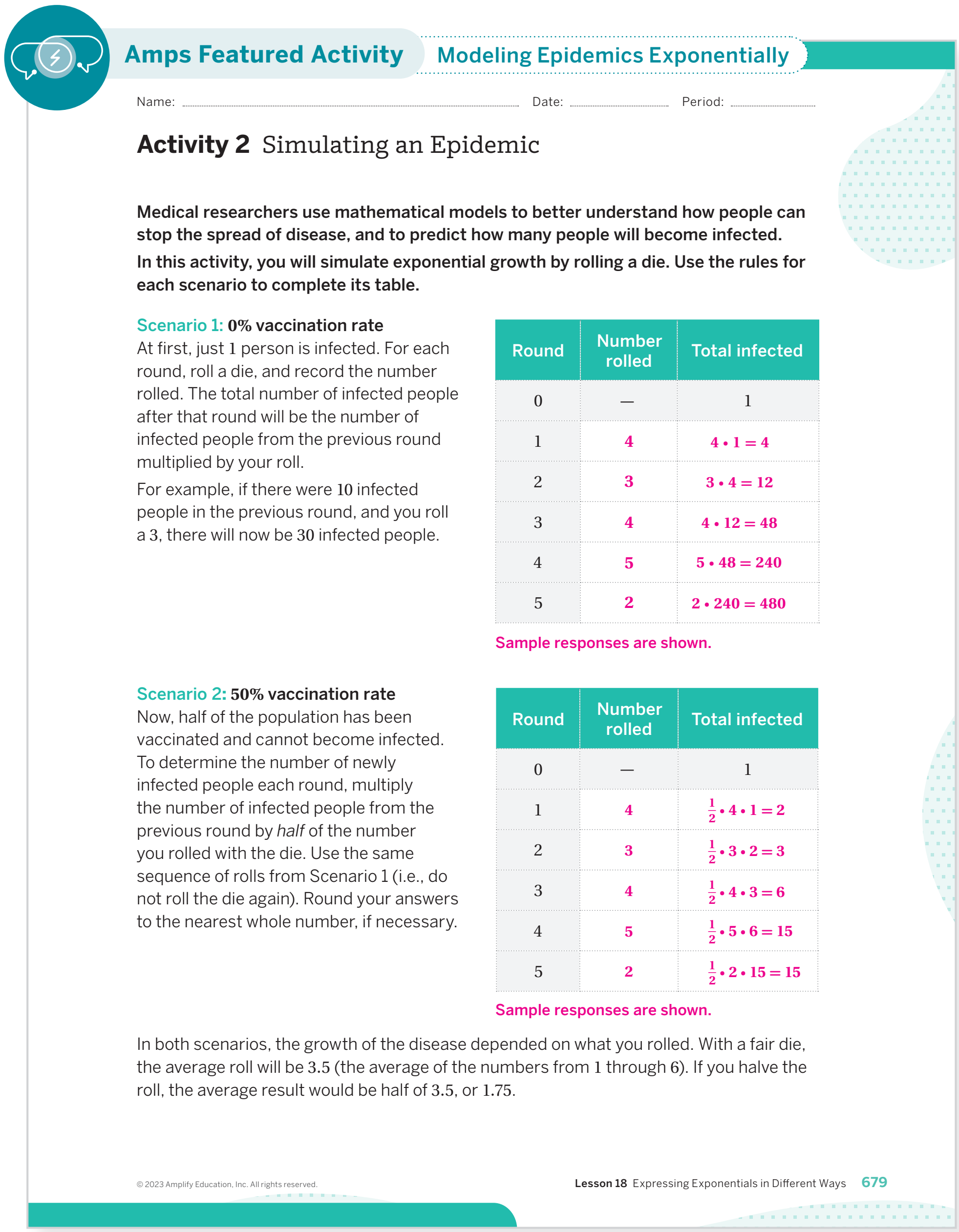
> A population is modeled by the equation $y = 500 \cdot (1.02)^x$, where y is the population and x is the number of years.

Growth factor	Growth rate
1.02	2%
The population is multiplied by a factor of 1.02 each year.	The population grows at a rate of 2% per year.

Small Groups | 20 min

Activity 2 Simulating an Epidemic

Students simulate two scenarios representing exponential growth in the context of the spread of disease, construct functions to model the growth, and compare the models within context.



Amps Featured Activity Modeling Epidemics Exponentially

Name: ______________________ Date: ____________ Period: ____________

Activity 2 Simulating an Epidemic

Medical researchers use mathematical models to better understand how people can stop the spread of disease, and to predict how many people will become infected.

In this activity, you will simulate exponential growth by rolling a die. Use the rules for each scenario to complete its table.

Scenario 1: 0% vaccination rate
At first, just 1 person is infected. For each round, roll a die, and record the number rolled. The total number of infected people after that round will be the number of infected people from the previous round multiplied by your roll.

For example, if there were 10 infected people in the previous round, and you roll a 3, there will now be 30 infected people.

Round	Number rolled	Total infected
0	—	1
1	4	$4 \cdot 1 = 4$
2	3	$3 \cdot 4 = 12$
3	4	$4 \cdot 12 = 48$
4	5	$5 \cdot 48 = 240$
5	2	$2 \cdot 240 = 480$

Sample responses are shown.

Scenario 2: 50% vaccination rate
Now, half of the population has been vaccinated and cannot become infected. To determine the number of newly infected people each round, multiply the number of infected people from the previous round by *half* of the number you rolled with the die. Use the same sequence of rolls from Scenario 1 (i.e., do not roll the die again). Round your answers to the nearest whole number, if necessary.

Round	Number rolled	Total infected
0	—	1
1	4	$\frac{1}{2} \cdot 4 \cdot 1 = 2$
2	3	$\frac{1}{2} \cdot 3 \cdot 2 = 3$
3	4	$\frac{1}{2} \cdot 4 \cdot 3 = 6$
4	5	$\frac{1}{2} \cdot 5 \cdot 6 = 15$
5	2	$\frac{1}{2} \cdot 2 \cdot 15 = 15$

Sample responses are shown.

In both scenarios, the growth of the disease depended on what you rolled. With a fair die, the average roll will be 3.5 (the average of the numbers from 1 through 6). If you halve the roll, the average result would be half of 3.5, or 1.75.

1 Launch

Read and model the rules of the simulation with the class. Distribute the dice to each group. Students may also use technology with a number generator based on numerical values 1–6.

2 Monitor

Help students get started by suggesting they draw a tree diagram representing the growth for each round of each scenario.

Look for points of confusion:

- **Struggling to calculate the total number of cases each round.** Remind students the total is the sum of new cases and total number of cases found in the previous round, including the original case.
- **Struggling to determine the growth pattern.** Have students format their work as shown in the sample response tables.

Look for productive strategies:

- Drawing tree diagrams to organize the growth patterns with each roll.
- Calculating the new total number of cases by first determining the additional number of cases, then adding to the previous sum.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can simulate the spread of a disease under three circumstances:

- Without a vaccine.
- WIth half of the population vaccinated.
- With 95% of the population vaccinated.

Math Language Development

MLR6: Three Reads

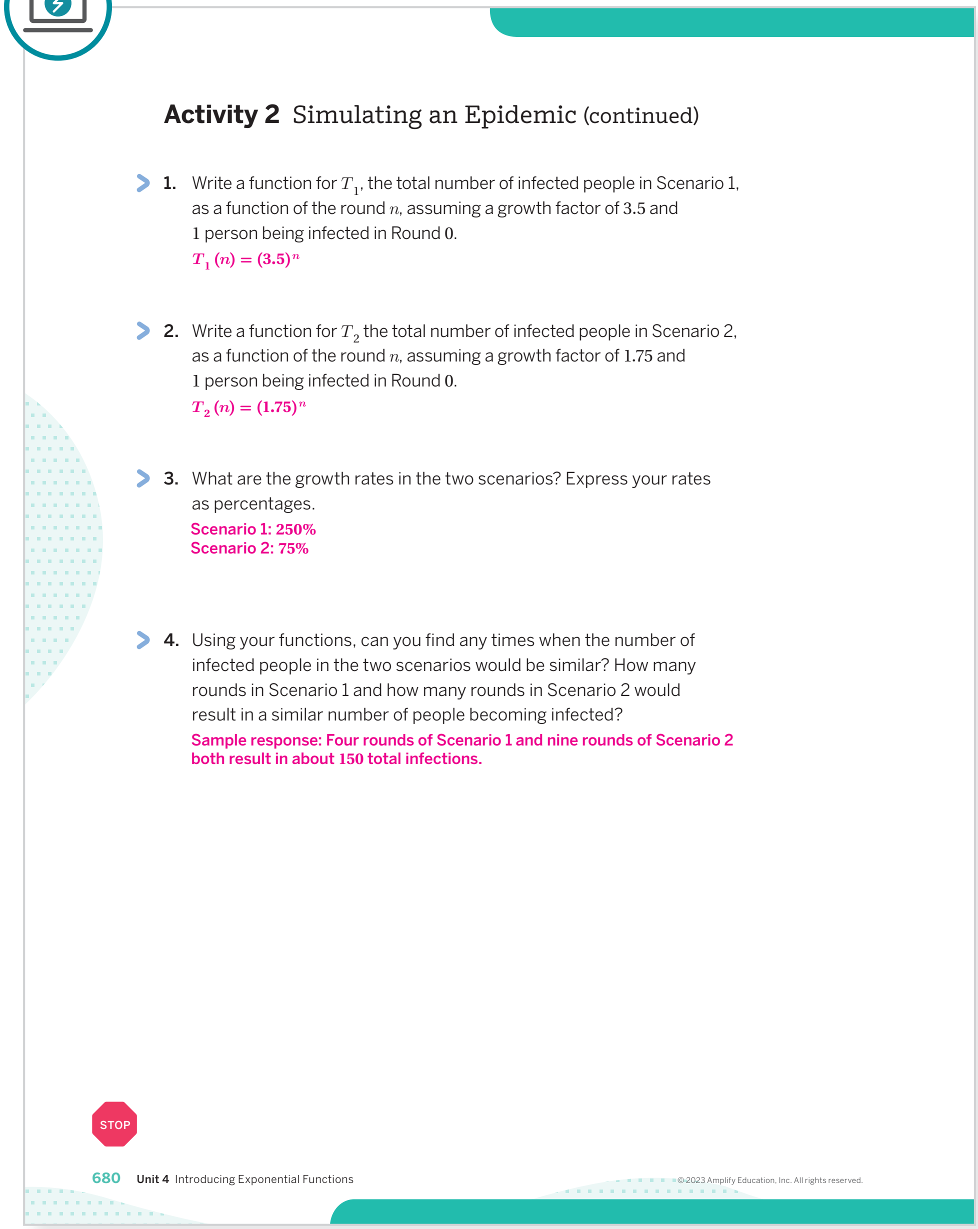
Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they will simulate a random process by rolling a die.
- **Read 2:** Select students to read each scenario aloud to the class. After each scenario is read, ask students to describe the rules for the scenario before simulating the random process for each scenario.
- **Read 3:** Ask groups of students to read Problems 1–4 and plan a strategy for solving the problems, based on their simulations.

Small Groups | 20 min

Activity 2 Simulating an Epidemic (continued)

Students simulate two scenarios representing exponential growth in the context of the spread of disease, construct functions to model the growth, and compare the models within context.



Activity 2 Simulating an Epidemic (continued)

1. Write a function for T_1, the total number of infected people in Scenario 1, as a function of the round n, assuming a growth factor of 3.5 and 1 person being infected in Round 0.
   $T_1(n) = (3.5)^n$

2. Write a function for T_2 the total number of infected people in Scenario 2, as a function of the round n, assuming a growth factor of 1.75 and 1 person being infected in Round 0.
   $T_2(n) = (1.75)^n$

3. What are the growth rates in the two scenarios? Express your rates as percentages.
   Scenario 1: 250%
   Scenario 2: 75%

4. Using your functions, can you find any times when the number of infected people in the two scenarios would be similar? How many rounds in Scenario 1 and how many rounds in Scenario 2 would result in a similar number of people becoming infected?
   Sample response: Four rounds of Scenario 1 and nine rounds of Scenario 2 both result in about 150 total infections.

STOP

680 Unit 4 Introducing Exponential Functions

3 Connect

Display examples of student tables.

Have students share their responses to Problems 1–3.

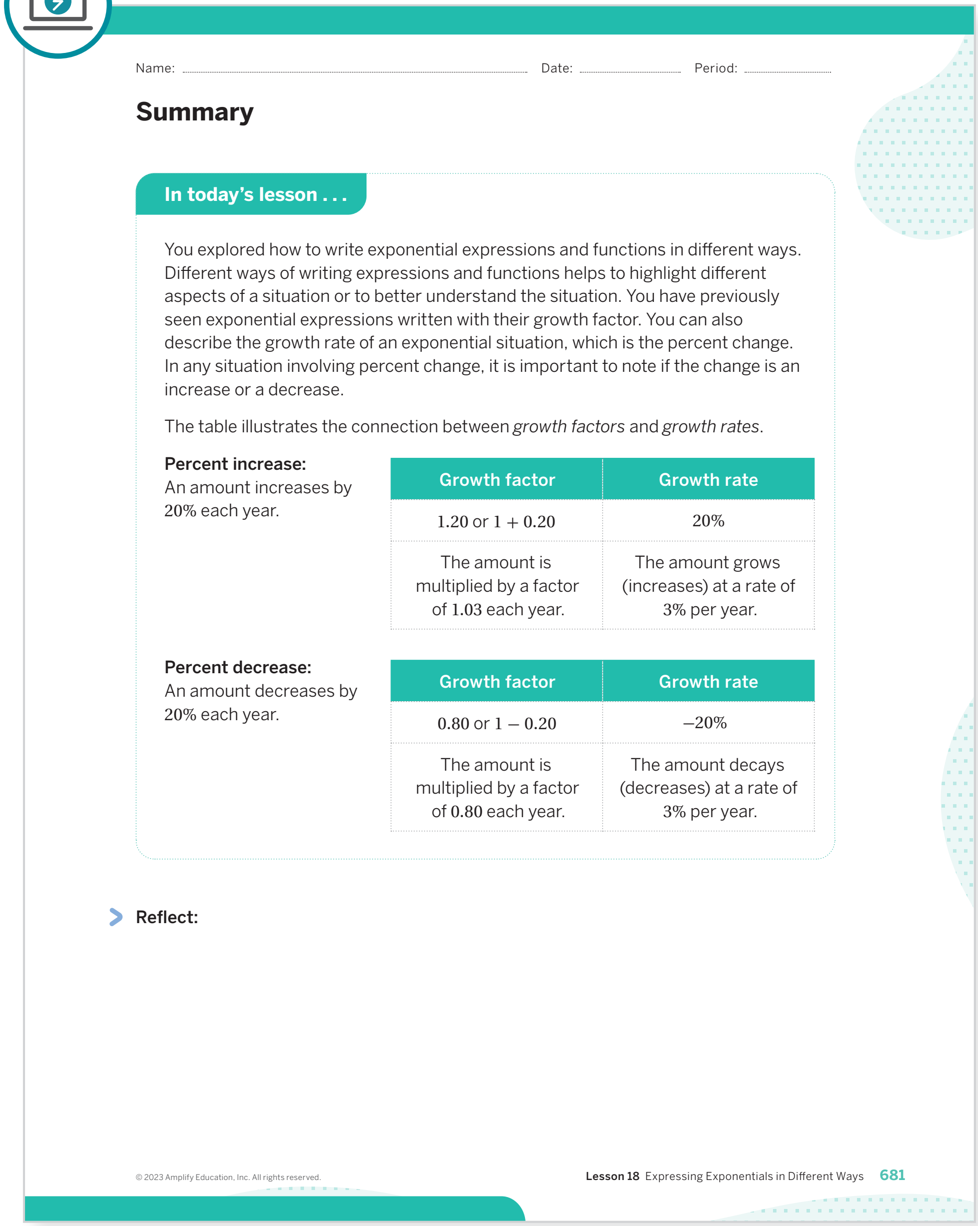
Highlight that the number of people that become infected is less in Scenario 2 than those in Scenario 1 because half of the population has been vaccinated in Scenario 2. The growth factor for Scenario 2 is less than that of Scenario 1.

Ask, "Are there any times when the number of people infected in the two scenarios would be the same? Explain your thinking". Yes; The scenarios will eventually have the same number of people infected. After four rounds of Scenario 1 and nine rounds of Scenario 2, the number of people is the same, 150.

Whole Class | 5 min

Summary

Students review and synthesize the rules of exponents to write exponential functions in different ways, depending on which is more appropriate for a given scenario.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how to write exponential expressions and functions in different ways. Different ways of writing expressions and functions helps to highlight different aspects of a situation or to better understand the situation. You have previously seen exponential expressions written with their growth factor. You can also describe the growth rate of an exponential situation, which is the percent change. In any situation involving percent change, it is important to note if the change is an increase or a decrease.

The table illustrates the connection between *growth factors* and *growth rates*.

Percent increase:
An amount increases by 20% each year.

Growth factor	Growth rate
1.20 or $1 + 0.20$	20%
The amount is multiplied by a factor of 1.03 each year.	The amount grows (increases) at a rate of 3% per year.

Percent decrease:
An amount decreases by 20% each year.

Growth factor	Growth rate
0.80 or $1 - 0.20$	-20%
The amount is multiplied by a factor of 0.80 each year.	The amount decays (decreases) at a rate of 3% per year.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 Expressing Exponentials in Different Ways 681

Synthesize

Display the tables.

Ask, "What is the connection between growth factors and growth rates?"

Have students share how they can use the growth factor to determine the growth rate and vice versa.

Highlight the relationship between *growth rate* and *growth factor*. In functions of the form $a \cdot (1 + r)^x$, the growth rate is r and the growth factor is $(1 + r)$.

Formalize vocabulary:

- *growth rate*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

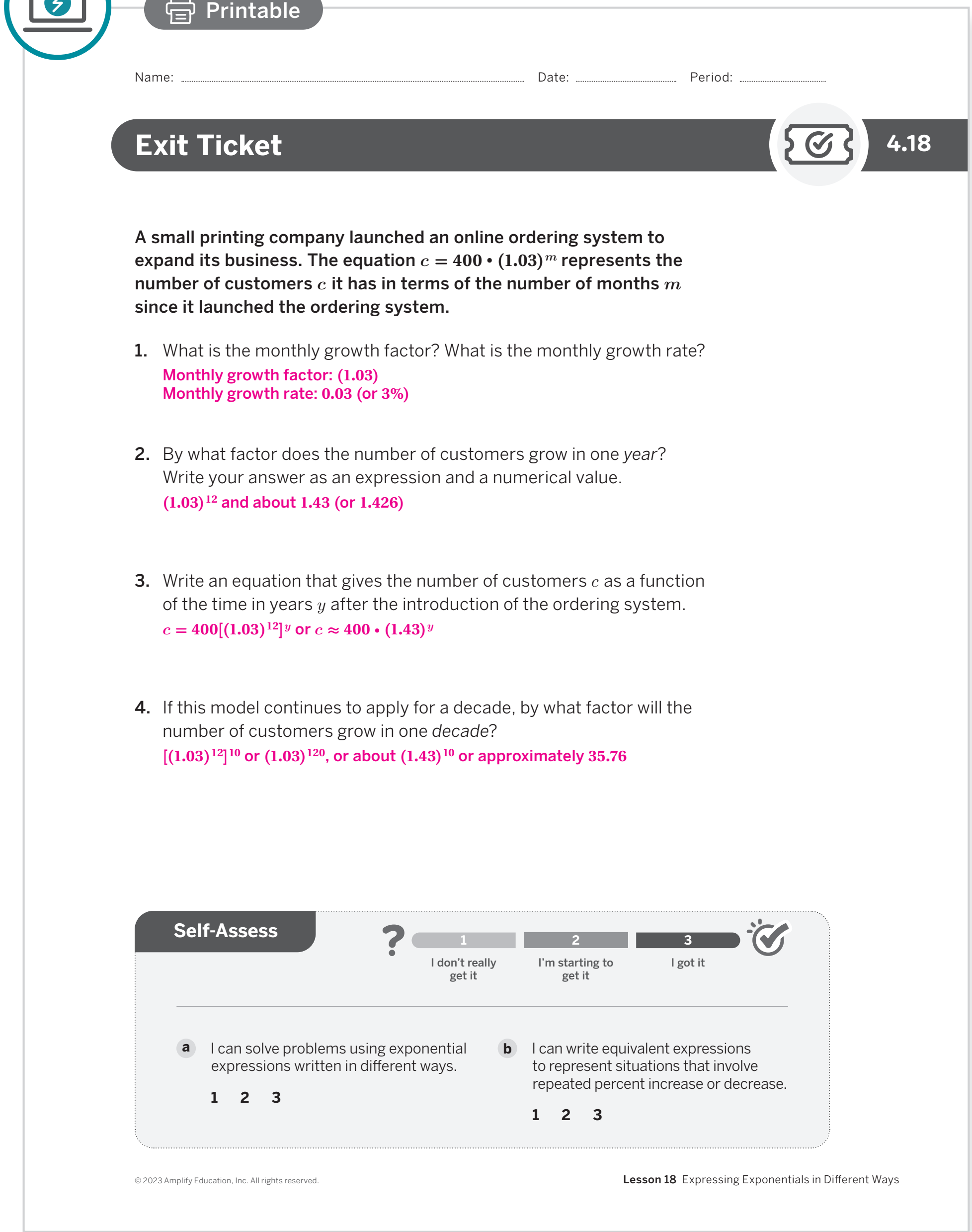
- "How are the terms *growth factor* and *growth rate* similar? How are they different?"
- "How can simulations help you understand and predict exponential growth?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by interpreting the structure of an exponential equation in context and constructing a function to model the relationship for a different period of time.

Printable



Name: Date: Period:

Exit Ticket 4.18

A small printing company launched an online ordering system to expand its business. The equation $c = 400 \cdot (1.03)^m$ represents the number of customers c it has in terms of the number of months m since it launched the ordering system.

1. What is the monthly growth factor? What is the monthly growth rate?
   Monthly growth factor: (1.03)
   Monthly growth rate: 0.03 (or 3%)

2. By what factor does the number of customers grow in one *year*? Write your answer as an expression and a numerical value.
   $(1.03)^{12}$ **and about 1.43 (or 1.426)**

3. Write an equation that gives the number of customers c as a function of the time in years y after the introduction of the ordering system.
   $c = 400[(1.03)^{12}]^y$ **or** $c \approx 400 \cdot (1.43)^y$

4. If this model continues to apply for a decade, by what factor will the number of customers grow in one *decade*?
   $[(1.03)^{12}]^{10}$ **or** $(1.03)^{120}$**, or about** $(1.43)^{10}$ **or approximately 35.76**

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can solve problems using exponential expressions written in different ways.
1 2 3

b. I can write equivalent expressions to represent situations that involve repeated percent increase or decrease.
1 2 3

Lesson 18 Expressing Exponentials in Different Ways

Success looks like . . .

- **Goal:** Interpreting and evaluating exponential expressions to solve problems.
  - » Identifying by what factor customers will grow in one decade in Problem 4.
- **Goal:** Writing equivalent expressions to highlight different aspects of a situation that involves repeated percent increase or decrease.

Suggested next steps

If students cannot identify a growth factor in Problems 1, 2, or 4, consider:

- Revisiting strategies to identify and apply a growth factor and growth rate.
- Reviewing Activity 1, Problems 1–4, 6, and 7.
- Assigning Practice Problem 2.

If students incorrectly write an equation in Problem 3, consider:

- Highlighting that $f(x) = a \cdot b^x$ and $f(x) = a \cdot (1 + r)^x$ both represent the exponential function.
- Reviewing Activity 2, Problems 1 and 2.

Professional Learning

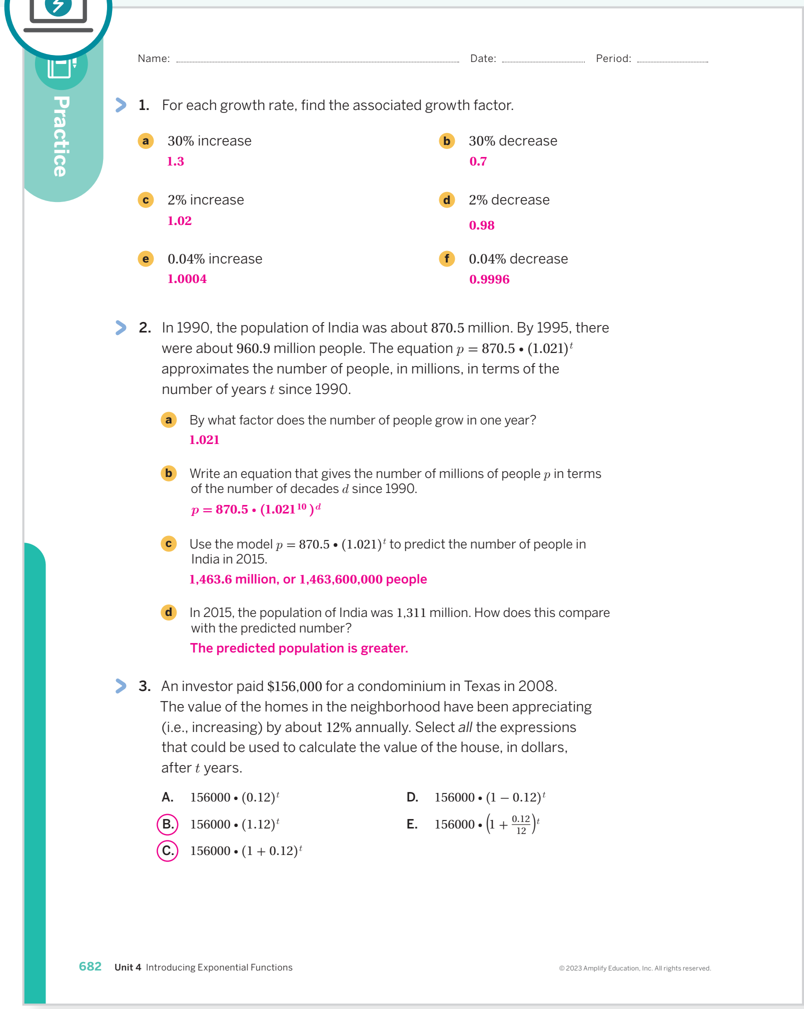
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students explored the difference between growth factors and growth rates. How well do you think your students understood this difference? What might you change the next time you teach this lesson?
- What did students find challenging as they worked through the simulations in Activity 2? What helped them work through these challenges? What teacher actions did you take to help support them? Would you use those again?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. For each growth rate, find the associated growth factor.

a 30% increase **1.3**	b 30% decrease **0.7**
c 2% increase **1.02**	d 2% decrease **0.98**
e 0.04% increase **1.0004**	f 0.04% decrease **0.9996**

2. In 1990, the population of India was about 870.5 million. By 1995, there were about 960.9 million people. The equation $p = 870.5 \bullet (1.021)^t$ approximates the number of people, in millions, in terms of the number of years t since 1990.

a. By what factor does the number of people grow in one year?

1.021

b. Write an equation that gives the number of millions of people p in terms of the number of decades d since 1990.

$p = 870.5 \bullet (1.021^{10})^d$

c. Use the model $p = 870.5 \bullet (1.021)^t$ to predict the number of people in India in 2015.

1,463.6 million, or 1,463,600,000 people

d. In 2015, the population of India was 1,311 million. How does this compare with the predicted number?

The predicted population is greater.

3. An investor paid $156,000 for a condominium in Texas in 2008. The value of the homes in the neighborhood have been appreciating (i.e., increasing) by about 12% annually. Select *all* the expressions that could be used to calculate the value of the house, in dollars, after t years.

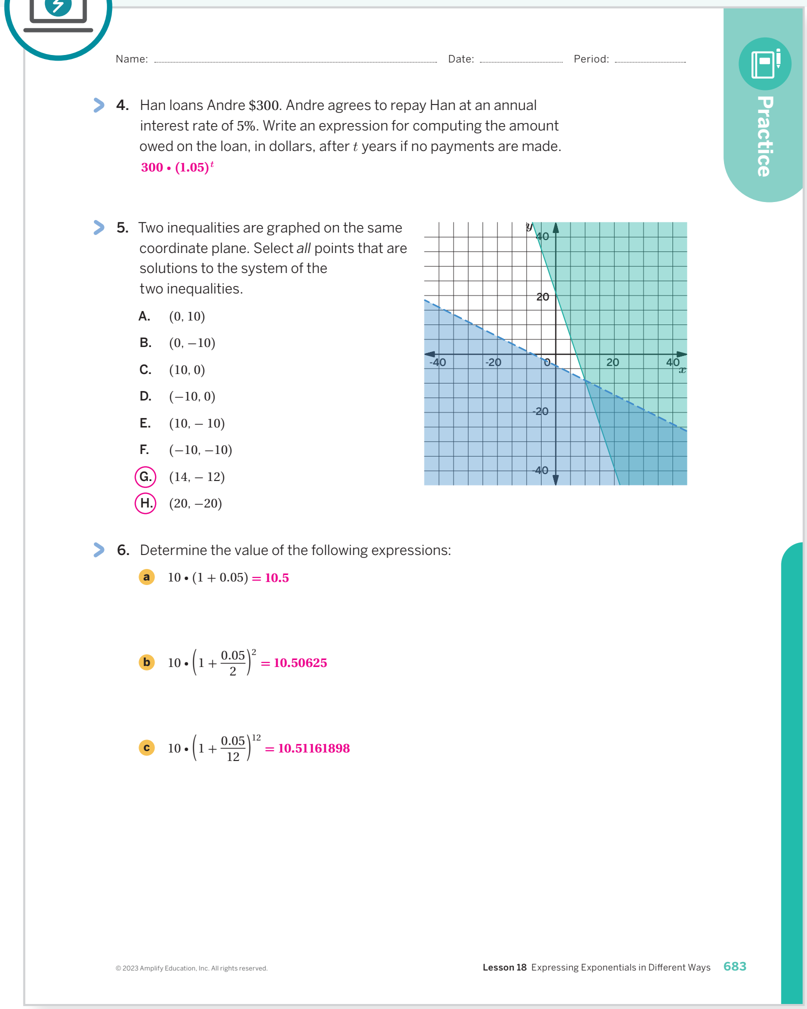
A. $156000 \bullet (0.12)^t$

[✓ B.] $156000 \bullet (1.12)^t$

[✓ C.] $156000 \bullet (1 + 0.12)^t$

D. $156000 \bullet (1 - 0.12)^t$

E. $156000 \bullet \left(1 + \frac{0.12}{12}\right)^t$



Practice

Name: ______ Date: ______ Period: ______

4. Han loans Andre $300. Andre agrees to repay Han at an annual interest rate of 5%. Write an expression for computing the amount owed on the loan, in dollars, after t years if no payments are made.

$300 \bullet (1.05)^t$

5. Two inequalities are graphed on the same coordinate plane. Select *all* points that are solutions to the system of the two inequalities.

A. $(0, 10)$

B. $(0, -10)$

C. $(10, 0)$

D. $(-10, 0)$

E. $(10, -10)$

F. $(-10, -10)$

[✓ G.] $(14, -12)$

[✓ H.] $(20, -20)$

6. Determine the value of the following expressions:

a. $10 \bullet (1 + 0.05) = 10.5$

b. $10 \bullet \left(1 + \frac{0.05}{2}\right)^2 = 10.50625$

c. $10 \bullet \left(1 + \frac{0.05}{12}\right)^{12} = 10.51161898$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	3
	3	Activity 2	3
Spiral	4	Unit 4 Lesson 17	2
	5	Unit 1 Lesson 23	2
Formative	6	Unit 4 Lesson 19	3

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 19**

Credit Cards and Exponential Expressions

Let's find out what happens when we repeatedly apply the same percent increase at different intervals of time.

Focus

Goals

1. Calculate the result of repeated percent increases for the same initial balance and interest rate, but compounded at different intervals.
2. Compare interest rates and compounding intervals to choose the better investment option.

Coherence

Today

Students continue to explore compound interest in the context of credit card APR, returns on investments, and the rising costs of college tuition. They practice calculating interest on different compounding intervals.

Previously

Students explored repeated percent change in a banking context and compared how the increase differs when that value is compounded twice versus once over the same interval.

Coming Soon

Students will learn alternate ways of expressing exponential functions and use what they learn to compare functions that grow linearly and exponentially.

Rigor

- Students **apply** their understanding of compounded growth to situations where the compounding interval can change, particularly in financial scenarios.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Activity 4 (optional)	Summary	Exit Ticket
5 min	10 min	10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Small Groups	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Exponent Rules*
- scientific calculators

Math Language Development

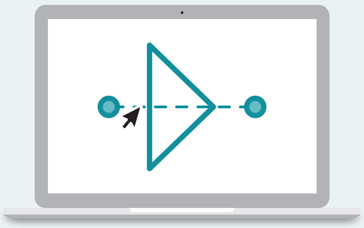
Review words

- *interest*

Amps | Featured Activity

Activity 1
Digital Card Sort

Students match mathematical expressions with statements for different interest calculations.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Because interest is such an important topic for students to understand, any lack of enthusiasm must be overcome. As they approach the activity, look for structures within the activity that can help them later. Explain that by understanding interest rates and payments, they can avoid financial mistakes in the future and challenge them to take this opportunity so that they can manage their personal finances confidently and optimistically.

Modifications to Pacing

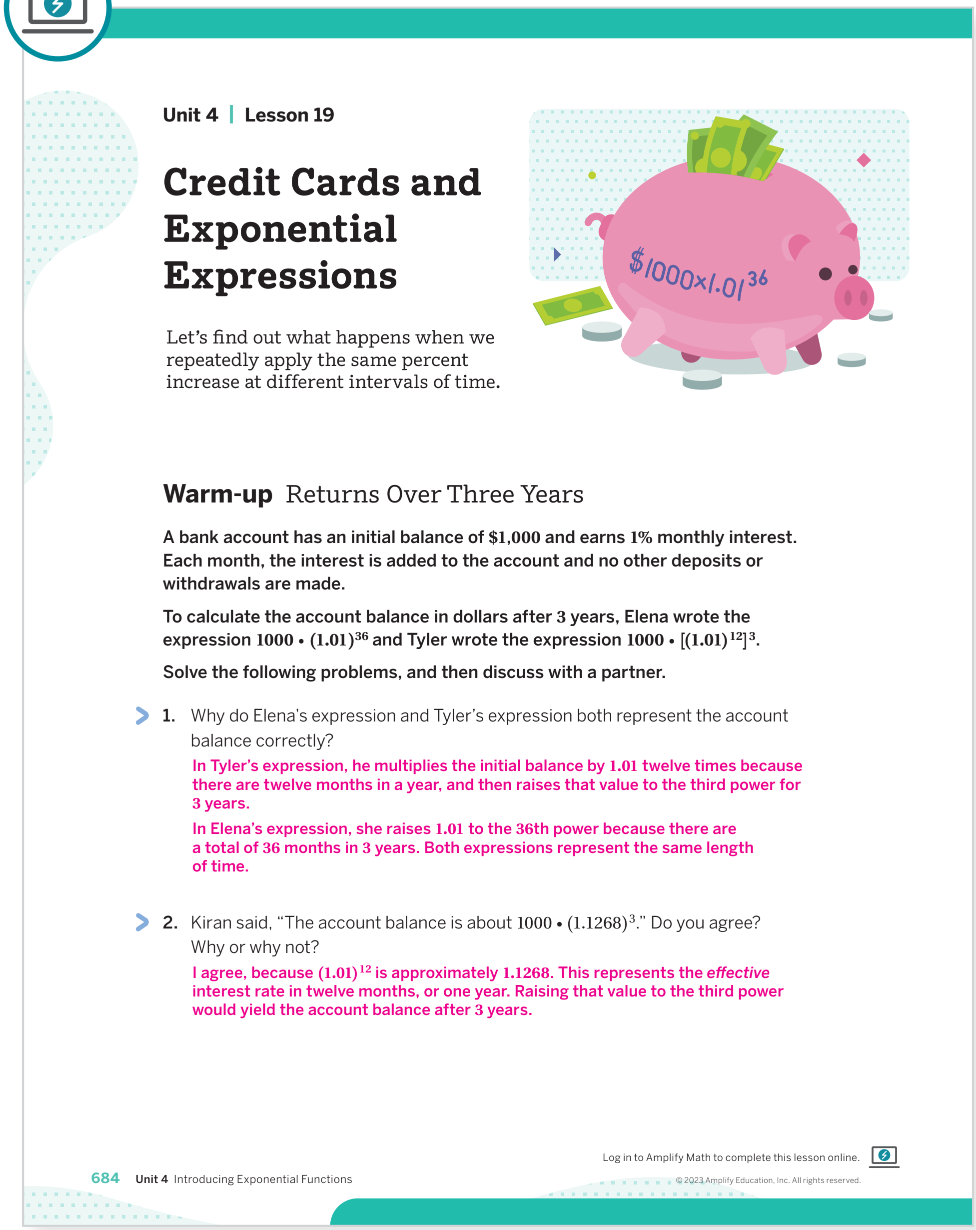
You may want to consider these additional modifications if you are short on time.

- Prioritize **Activity 3** at the expense of **Activity 2**.
- Optional **Activity 4** can be shortened by completing Problems 1 and 2 together as a class.

Independent | 5 min

Warm-up Returns Over Three Years

Students revisit the structure of equivalent expressions for compound interest calculations to prepare them for the consideration of compound interest over different intervals.



Unit 4 | Lesson 19

Credit Cards and Exponential Expressions

Let's find out what happens when we repeatedly apply the same percent increase at different intervals of time.

Warm-up Returns Over Three Years

A bank account has an initial balance of $\$1,000$ and earns 1% monthly interest. Each month, the interest is added to the account and no other deposits or withdrawals are made.

To calculate the account balance in dollars after 3 years, Elena wrote the expression $1000 \cdot (1.01)^{36}$ and Tyler wrote the expression $1000 \cdot [(1.01)^{12}]^3$.

Solve the following problems, and then discuss with a partner.

1. Why do Elena's expression and Tyler's expression both represent the account balance correctly?

   In Tyler's expression, he multiplies the initial balance by 1.01 twelve times because there are twelve months in a year, and then raises that value to the third power for 3 years.

   In Elena's expression, she raises 1.01 to the 36th power because there are a total of 36 months in 3 years. Both expressions represent the same length of time.

2. Kiran said, "The account balance is about $1000 \cdot (1.1268)^3$." Do you agree? Why or why not?

   I agree, because $(1.01)^{12}$ is approximately 1.1268. This represents the *effective* interest rate in twelve months, or one year. Raising that value to the third power would yield the account balance after 3 years.

Log in to Amplify Math to complete this lesson online.

① Launch

Read the prompt aloud. Provide access to scientific calculators.

② Monitor

Have students get started by reminding them of the rules of exponents. Consider displaying the Anchor Chart PDF, *Exponent Rules*. Ask, "What is the relationship between Elena's and Tyler's exponents?"

Look for points of confusion:

- **Disagreeing with Kiran in Problem 2.** Encourage students to use a calculator to show that Kiran's expression is equivalent to both Elena's and Tyler's.

Look for productive strategies:

- Applying properties of exponents to write equivalent expressions.

③ Connect

Have students share their explanations for why Tyler's and Elena's expressions both represent the account balance after 3 years.

Highlight that in Elena's expression, $(1.01)^{36}$ correctly represents the 1% interest compounded every month for 36 months, which is 3 years. In Tyler's expression, $(1.01)^{12}$ represents the 1% interest compounded every month for 12 months, or a year. So for 3 years, students need to multiply the initial balance by that yearly rate 3 times, which is equivalent to $((1.01)^{12})^3$. In Kiran's expression, the growth factor used, 1.1268, is 1.01^{12}, which is equivalent to Tyler's expression.

Power-up

To power up students' ability to apply the order of operations to simplify expressions involving percentages, have students complete:

Is $(3 + 1)^2$ equivalent to $3^2 + 1^2$? Be prepared to explain your thinking.

No; Sample response: When simplifying $(3 + 1)^2$, the addition within the parentheses is completed first, and then the sum is squared. So, $(3 + 1)^2 = 4^2 = 16$. When simplifying $3^2 + 1^2$, the exponents are simplified first, and then the sum $9 + 1$ is calculated. So, $3^2 + 1^2 = 9 + 1 = 10$.

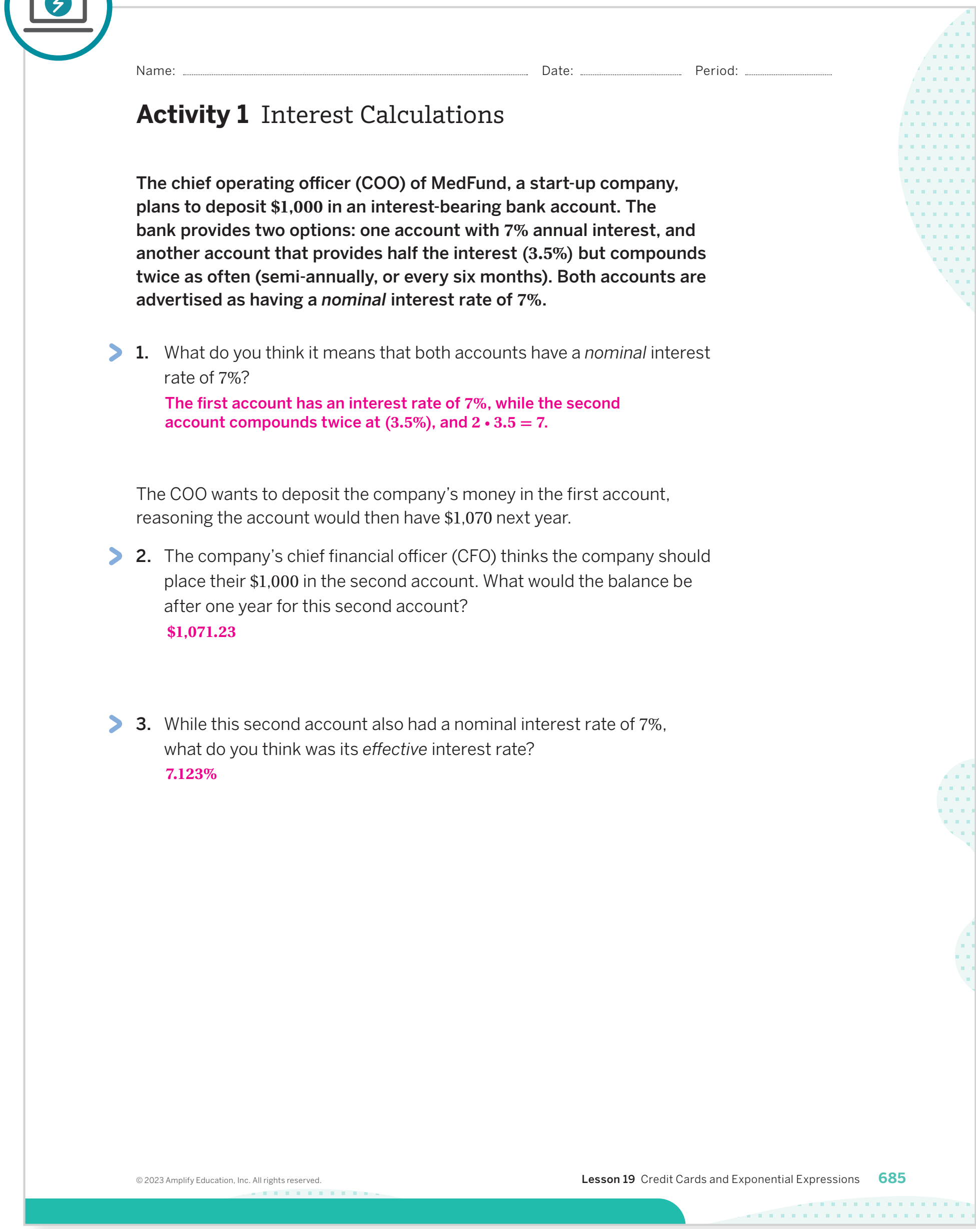
Use: Before Activity 1

Informed by: Performance on Lesson 18, Practice Problem 6

Pairs | 10 min

Activity 1 Interest Calculations

Students examine the structure of expressions representing different compound interest types and use correct terminology to compare and contrast the types of interest rates.



Name: ______ Date: ______ Period: ______

Activity 1 Interest Calculations

The chief operating officer (COO) of MedFund, a start-up company, plans to deposit $1,000 in an interest-bearing bank account. The bank provides two options: one account with 7% annual interest, and another account that provides half the interest (3.5%) but compounds twice as often (semi-annually, or every six months). Both accounts are advertised as having a *nominal* interest rate of 7%.

1. What do you think it means that both accounts have a *nominal* interest rate of 7%?
   The first account has an interest rate of 7%, while the second account compounds twice at (3.5%), and 2 • 3.5 = 7.

The COO wants to deposit the company's money in the first account, reasoning the account would then have $1,070 next year.

2. The company's chief financial officer (CFO) thinks the company should place their $1,000 in the second account. What would the balance be after one year for this second account?
   $1,071.23

3. While this second account also had a nominal interest rate of 7%, what do you think was its *effective* interest rate?
   7.123%

1 Launch

Read the prompt aloud. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started with Problem 4 by identifying parts of the expressions in the table.

Look for points of confusion:

- **Not understanding why the denominator of the fraction is included in the expressions**. The 7% interest for the entire year is divided into smaller percentages calculated more than once per year.

Look for productive strategies:

- Identifying the growth factor, growth rate, compounding period, and exponent in each expression.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Clarify the meanings of the terms nominal interest rate and effective interest rate that students learned about in a prior lesson. Display these terms and their meanings for students to reference during this activity.

Nominal interest rate:	Effective interest rate:
The stated rate of interest used to determine the amount for one year.	How the account balance *actually* changes after one year.

Extension: *Math Enrichment*

Have students write an expression where $1,000 is invested for 6 years into an account that earns 7% nominal annual interest that is compounded every 3 months.

Sample response: $1000 \cdot \left(1+\frac{0.07}{4}\right)^{24}$

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their matches for Problem 4, draw their attention to the structure of each expression and how it connects to how the interest is compounded.

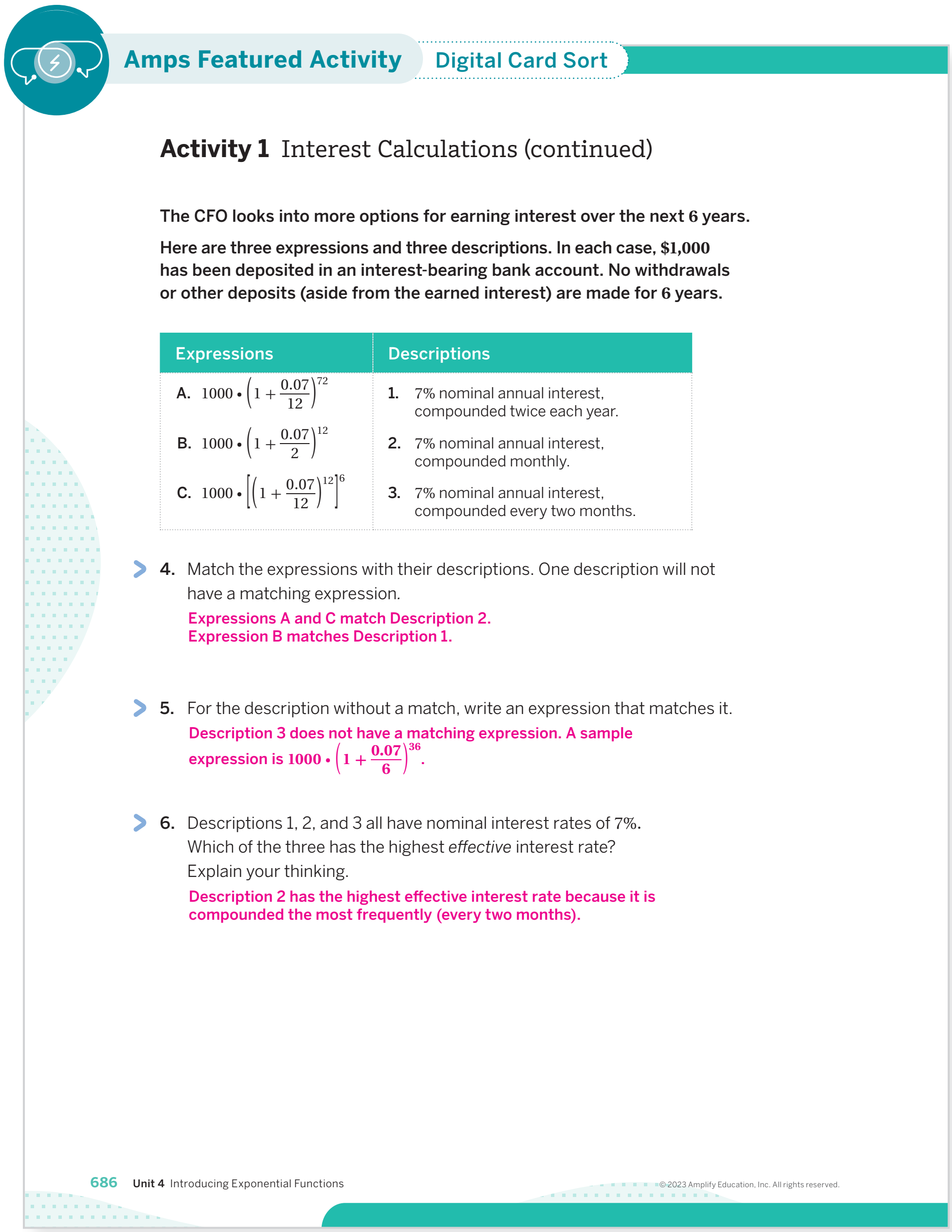
Ask:

- "Where do you see the interest rate? Why is it divided by 12 in two of the expressions, but not the third?"
- "Where do you see *compounded semi-annually* in the expression? *Compounded monthly? Compounded every two months?*"

Pairs | 10 min

Activity 1 Interest Calculations (continued)

Students examine the structure of expressions representing different compound interest types and use correct terminology to compare and contrast the types of interest rates.



Amps Featured Activity Digital Card Sort

Activity 1 Interest Calculations (continued)

The CFO looks into more options for earning interest over the next 6 years.

Here are three expressions and three descriptions. In each case, $1,000 has been deposited in an interest-bearing bank account. No withdrawals or other deposits (aside from the earned interest) are made for 6 years.

Expressions	Descriptions
A. $1000 \cdot \left(1 + \frac{0.07}{12}\right)^{72}$	1. 7% nominal annual interest, compounded twice each year.
B. $1000 \cdot \left(1 + \frac{0.07}{2}\right)^{12}$	2. 7% nominal annual interest, compounded monthly.
C. $1000 \cdot \left[\left(1 + \frac{0.07}{12}\right)^{12}\right]^{6}$	3. 7% nominal annual interest, compounded every two months.

4. Match the expressions with their descriptions. One description will not have a matching expression.
   Expressions A and C match Description 2.
   Expression B matches Description 1.

5. For the description without a match, write an expression that matches it.
   Description 3 does not have a matching expression. A sample expression is $1000 \cdot \left(1 + \frac{0.07}{6}\right)^{36}$.

6. Descriptions 1, 2, and 3 all have nominal interest rates of 7%. Which of the three has the highest *effective* interest rate? Explain your thinking.
   Description 2 has the highest effective interest rate because it is compounded the most frequently (every two months).

686 Unit 4 Introducing Exponential Functions

©2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the table of expressions and descriptions.

Have students share how they determined their matches in Problem 4.

Highlight the structure of each expression.

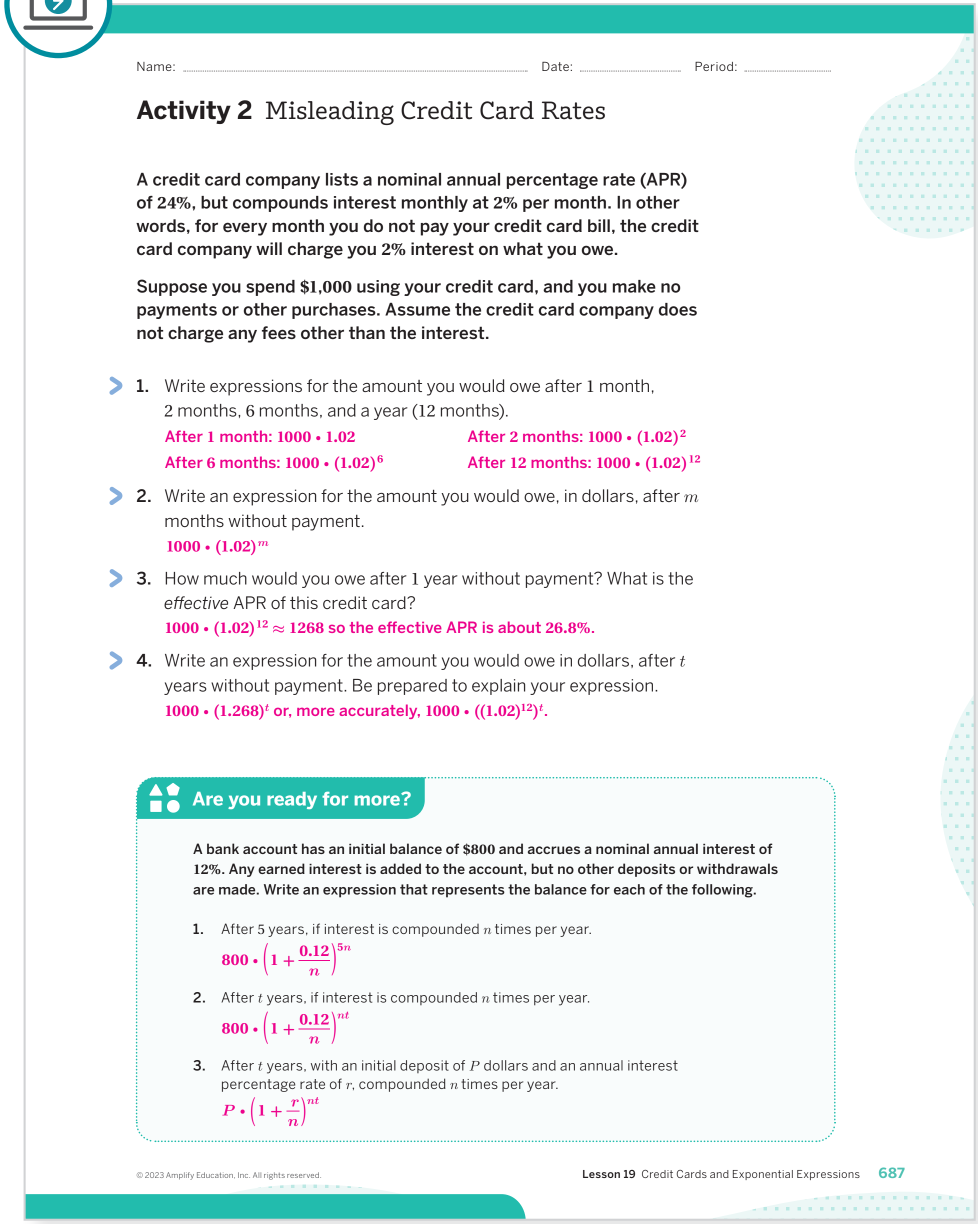
Ask:

- "What does the expression $1 + \frac{0.07}{12}$ mean in context?" The growth factor when the interest rate 7% is applied monthly.
- "What does the expression $\left(1 + \frac{0.07}{12}\right)^{12}$ mean in context?" The growth factor when the interest rate is compounded monthly over 12 months.
- "What does the exponent 6 mean in $\left[\left(1 + \frac{0.07}{12}\right)^{12}\right]^{6}$ in context?" The number of years that the account earns interest, 6.

Small Groups | 10 min

Activity 2 Misleading Credit Card Rates

Students make compound interest calculations in a credit card context and practice writing different exponential expressions to represent the same quantity.



Name: ________ Date: ________ Period: ________

Activity 2 Misleading Credit Card Rates

A credit card company lists a nominal annual percentage rate (APR) of 24%, but compounds interest monthly at 2% per month. In other words, for every month you do not pay your credit card bill, the credit card company will charge you 2% interest on what you owe.

Suppose you spend $1,000 using your credit card, and you make no payments or other purchases. Assume the credit card company does not charge any fees other than the interest.

1. Write expressions for the amount you would owe after 1 month, 2 months, 6 months, and a year (12 months).
   After 1 month: $1000 \cdot 1.02$ **After 2 months: $1000 \cdot (1.02)^2$**
   After 6 months: $1000 \cdot (1.02)^6$ **After 12 months: $1000 \cdot (1.02)^{12}$**
2. Write an expression for the amount you would owe, in dollars, after m months without payment.
   $1000 \cdot (1.02)^m$
3. How much would you owe after 1 year without payment? What is the *effective* APR of this credit card?
   $1000 \cdot (1.02)^{12} \approx 1268$ so the effective APR is about 26.8%.
4. Write an expression for the amount you would owe in dollars, after t years without payment. Be prepared to explain your expression.
   $1000 \cdot (1.268)^t$ or, more accurately, $1000 \cdot ((1.02)^{12})^t$.

Are you ready for more?

A bank account has an initial balance of $800 and accrues a nominal annual interest of 12%. Any earned interest is added to the account, but no other deposits or withdrawals are made. Write an expression that represents the balance for each of the following.

1. After 5 years, if interest is compounded n times per year.
   $800 \cdot \left(1 + \frac{0.12}{n}\right)^{5n}$
2. After t years, if interest is compounded n times per year.
   $800 \cdot \left(1 + \frac{0.12}{n}\right)^{nt}$
3. After t years, with an initial deposit of P dollars and an annual interest percentage rate of r, compounded n times per year.
   $P \cdot \left(1 + \frac{r}{n}\right)^{nt}$

Lesson 19 Credit Cards and Exponential Expressions 687

1 Launch

Explain the overview of credit card rates and APR. Have students provide examples of nominal and effective interest rates.

2 Monitor

Have students get started by reading the prompt aloud, ensuring they understand why the monthly interest rate is 2%.

Look for points of confusion:

- **Having difficulty writing expressions in Problem 1.** Ask students to identify the initial value, interest rate, and time values.
- **Multiplying by a factor of .02 instead of 1.02.** Have students evaluate their expressions to determine whether it is the growth that they intended.
- **Writing an expression using addition to represent the repeated percent change.** Prompt students to write an expression in exponential form ($y = a \cdot b^x$).

Look for productive strategies:

- Using the expression written in Problem 3 to write a new expression representing the balance after t years.

3 Connect

Have students share their expressions for the account balance after 1 month, 2 months, 6 months, and 1 year and explain how they arrived at their solutions. Then have students share their expressions for the balance after t years and how they used the rules of exponents to write them.

Highlight that in Problem 3, the account balance after one year (12 months) is represented by $1000 \cdot (1.02)^{12}$. This expression can be used to write a new expression that represents the balance after t years by raising it to the t power, $1000 \cdot ((1.02)^{12})^t$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the equation $f(x) = a \cdot (1 + r)^x$ that models exponential growth. Ask, "What is the value of r in this context?" $r = 0.02$

Extension: *Math Enrichment*

As a follow-up to Problem 4, ask students to determine the amount they would owe after 2 years without any payments. $1,608.44

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory context and have students work with their partner to write 2–3 mathematical questions they could ask about this situation. Sample questions shown.

- How much will I owe after any number of months?
- Is the amount I would owe after 1 year the same as multiplying $1,000 by 1.24?

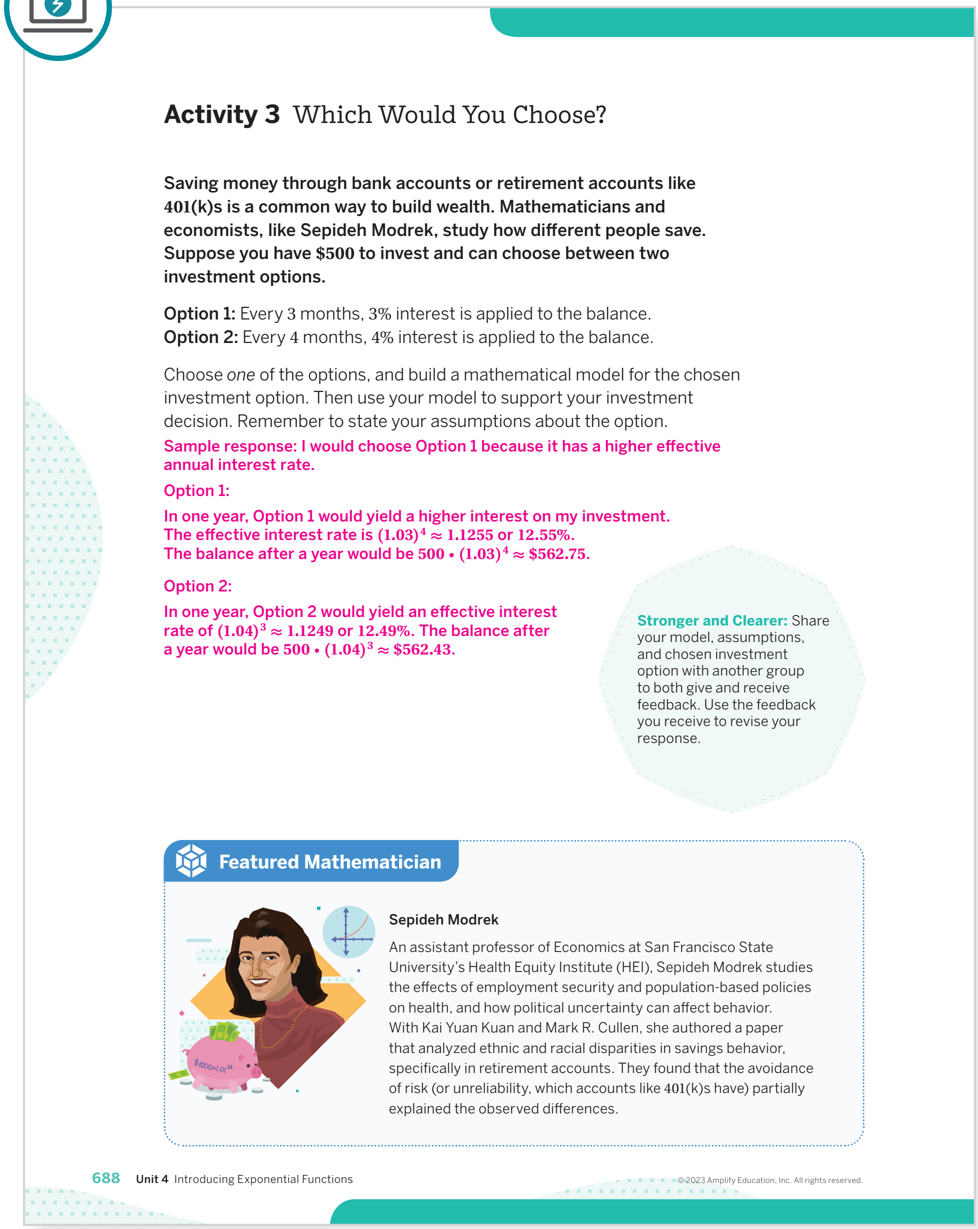
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Small Groups | 15 min

Activity 3 Which Would You Choose?

Students build mathematical models to compare two different investment options, determine which option they would choose, state any assumptions they make, and support their decision.



Activity 3 Which Would You Choose?

Saving money through bank accounts or retirement accounts like 401(k)s is a common way to build wealth. Mathematicians and economists, like Sepideh Modrek, study how different people save. Suppose you have $500 to invest and can choose between two investment options.

Option 1: Every 3 months, 3% interest is applied to the balance.
Option 2: Every 4 months, 4% interest is applied to the balance.

Choose *one* of the options, and build a mathematical model for the chosen investment option. Then use your model to support your investment decision. Remember to state your assumptions about the option.

Sample response: I would choose Option 1 because it has a higher effective annual interest rate.

Option 1:

In one year, Option 1 would yield a higher interest on my investment. The effective interest rate is $(1.03)^4 \approx 1.1255$ or 12.55%. The balance after a year would be $500 \cdot (1.03)^4 \approx \$562.75$.

Option 2:

In one year, Option 2 would yield an effective interest rate of $(1.04)^3 \approx 1.1249$ or 12.49%. The balance after a year would be $500 \cdot (1.04)^3 \approx \$562.43$.

Stronger and Clearer: Share your model, assumptions, and chosen investment option with another group to both give and receive feedback. Use the feedback you receive to revise your response.

Featured Mathematician

Sepideh Modrek

An assistant professor of Economics at San Francisco State University's Health Equity Institute (HEI), Sepideh Modrek studies the effects of employment security and population-based policies on health, and how political uncertainty can affect behavior. With Kai Yuan Kuan and Mark R. Cullen, she authored a paper that analyzed ethnic and racial disparities in savings behavior, specifically in retirement accounts. They found that the avoidance of risk (or unreliability, which accounts like 401(k)s have) partially explained the observed differences.

688 **Unit 4** Introducing Exponential Functions

1 Launch

For each group of students, direct half to complete Option 1 and half to complete Option 2.

2 Monitor

Help students get started by asking them how many times each option would compound annually.

Look for points of confusion:

- **Looking for a given length of time.** Prompt students to generate the account balances for each month for both options. Then ask them to consider what the payout would be if they withdrew money in that specific month.

Look for productive strategies:

- Using diagrams, tables, or illustrations to support their response.

3 Connect

Have students share their models and thinking for each option. Select groups that used productive strategies. Have groups state the nominal and effective annual rates for each option and explain the difference between them.

Ask, "How does the length of investment influence your calculations and investment choice? When will the 4% investment option be the better option?"

Highlight that the interest is calculated (or accrues) every 3 (or 4) months, but is paid at the end of the length of time of the investment. The better investment depends on whether this length of time ends in the same month that interest is accrued.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have groups meet with one other group to share their model, assumptions, and chosen investment option and to give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "Can you explain to me how you selected your option?"
- "How did you construct your mathematical model? What type of function did you choose?"
- "What mathematical language did you use in your response?"

Have students revise their model or chosen investment option, based on the feedback they received.

English Learners

Encourage students to use diagrams, tables, or illustrations in their response.

Featured Mathematician

Sepideh Modrek

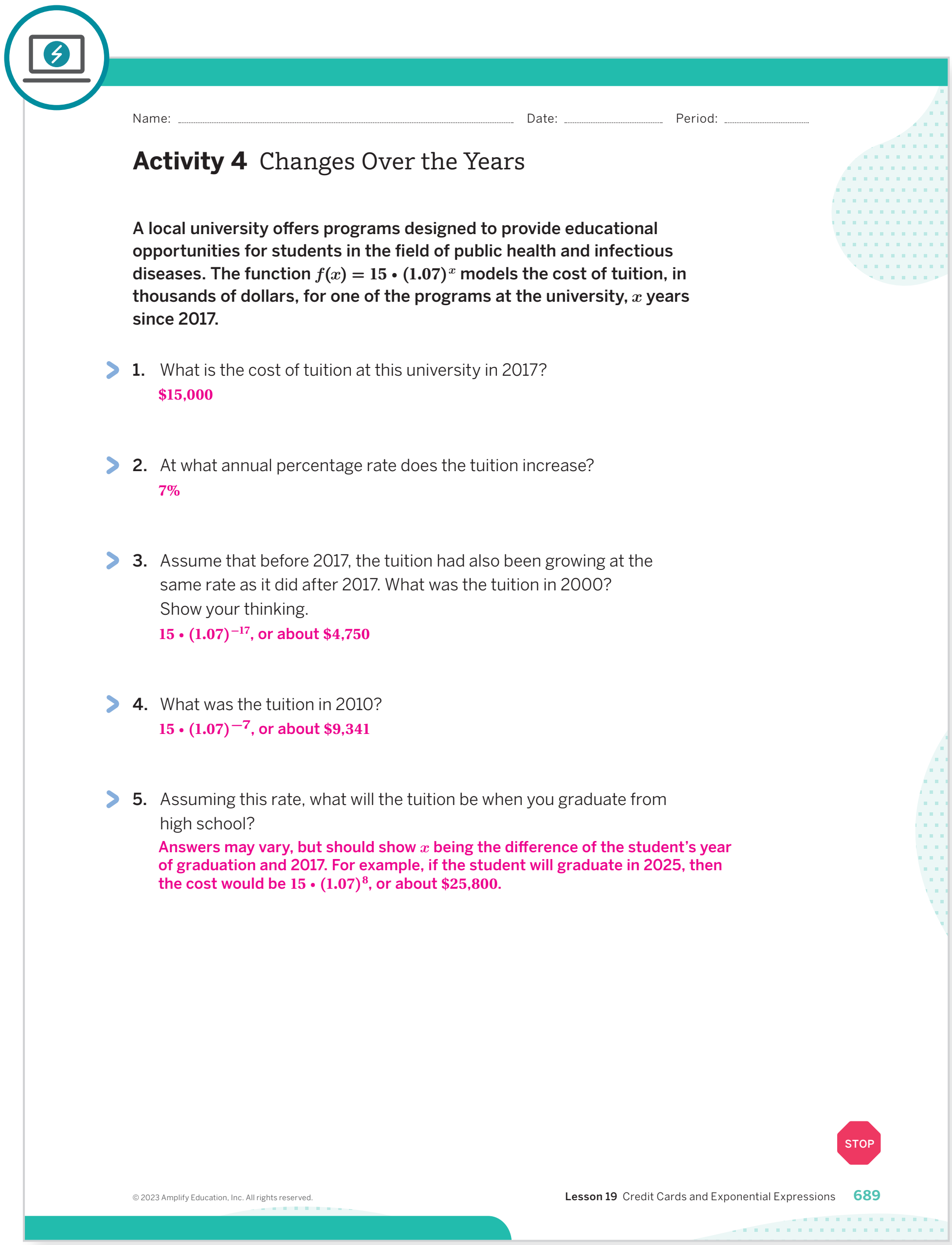
Have students read about Featured Mathematician Sepideh Modrek, an assistant professor who studies how political uncertainty can affect behavior.

Optional

Pairs | 10 min

Activity 4 Changes Over the Years

Students interpret an exponential equation in the context of college tuition cost to examine how the tuition changes over decades.



Name: ______ Date: ______ Period: ______

Activity 4 Changes Over the Years

A local university offers programs designed to provide educational opportunities for students in the field of public health and infectious diseases. The function $f(x) = 15 \cdot (1.07)^x$ models the cost of tuition, in thousands of dollars, for one of the programs at the university, x years since 2017.

1. What is the cost of tuition at this university in 2017?
   $15,000

2. At what annual percentage rate does the tuition increase?
   7%

3. Assume that before 2017, the tuition had also been growing at the same rate as it did after 2017. What was the tuition in 2000? Show your thinking.
   $15 \cdot (1.07)^{-17}$, or about $4,750

4. What was the tuition in 2010?
   $15 \cdot (1.07)^{-7}$, or about $9,341

5. Assuming this rate, what will the tuition be when you graduate from high school?
   Answers may vary, but should show x being the difference of the student's year of graduation and 2017. For example, if the student will graduate in 2025, then the cost would be $15 \cdot (1.07)^8$, or about $25,800.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Credit Cards and Exponential Expressions 689

1 Launch

Read the prompt aloud. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by reminding them of the meaning of x (years since 2017) and $f(x)$ (cost of tuition, in thousands of dollars).

Look for points of confusion:

- **Having difficulty determining the annual percentage rate.** Remind students of the parts of the equation $y = a \cdot (1 + r)^x$.
- **Struggling to determine the value of x for years prior to 2017**. Encourage students to determine the difference between the year and 2017, reminding them 2017 is the initial value when $x = 0$.

3 Connect

Have students share their responses to Problems 1–3. For Problem 3, select students to explain how they determined the exponent.

Highlight that it is important to pay attention to how the variables are defined in the function. The exponent is defined as years since 2017, so in the year 2000, the exponent should be negative.

Ask, "Assuming this trend continues, by what factor will the tuition grow between 2017 and 2037?" 3.87%.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the equation $f(x) = a \cdot (1 + r)^x$ with the variables defined and a table, similar to the following. Include the first entry as shown to support students in determining the value of x to be substituted into the function.

Year	x	$f(x) = a \cdot (1 + r)^x$
2017	0	

Math Language Development

MLR1: Stronger and Clearer Each Time

Give students time to meet with 2–3 partners to share their response to the Ask question, "Assuming this trend continues, by what factor will the tuition grow between 2017 and 2037?" Provide prompts for feedback that will help students strengthen and clarify their ideas. Then give students time to revise their responses based on the feedback they received.

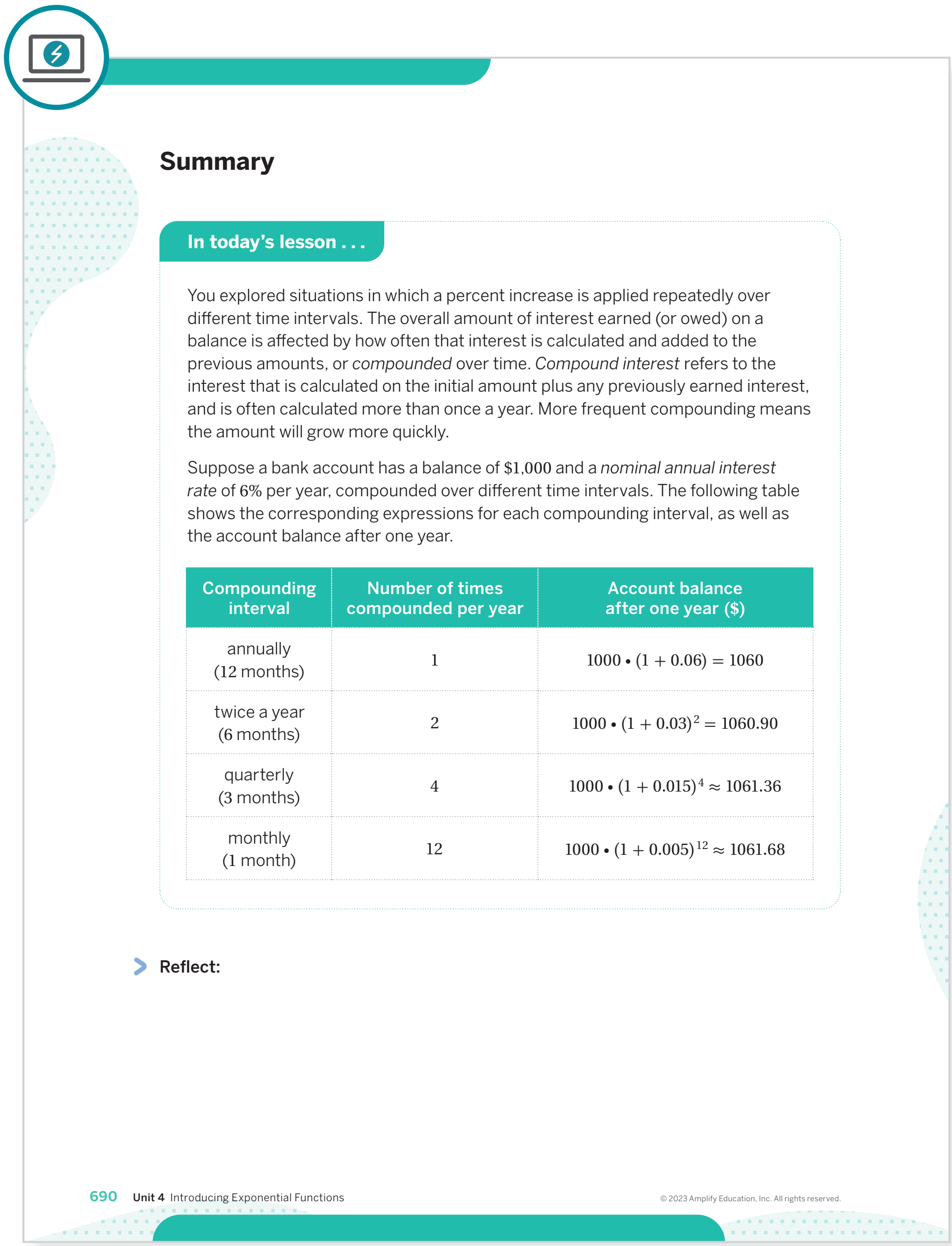
English Learners

Ensure mixed grouping based on differing English language proficiencies so that students have an opportunity to speak and listen to peers with more advanced proficiency levels.

Whole Class | 5 min

Summary

Review and synthesize how interest accrues over intervals of time through various scenarios.



Summary

In today's lesson . . .

You explored situations in which a percent increase is applied repeatedly over different time intervals. The overall amount of interest earned (or owed) on a balance is affected by how often that interest is calculated and added to the previous amounts, or *compounded* over time. *Compound interest* refers to the interest that is calculated on the initial amount plus any previously earned interest, and is often calculated more than once a year. More frequent compounding means the amount will grow more quickly.

Suppose a bank account has a balance of $1,000 and a *nominal annual interest rate* of 6% per year, compounded over different time intervals. The following table shows the corresponding expressions for each compounding interval, as well as the account balance after one year.

Compounding interval	Number of times compounded per year	Account balance after one year ($)
annually (12 months)	1	$1000 \cdot (1 + 0.06) = 1060$
twice a year (6 months)	2	$1000 \cdot (1 + 0.03)^2 = 1060.90$
quarterly (3 months)	4	$1000 \cdot (1 + 0.015)^4 \approx 1061.36$
monthly (1 month)	12	$1000 \cdot (1 + 0.005)^{12} \approx 1061.68$

Reflect:

690 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the table.

Ask:

- "Which of these options for calculating interest would you prefer? Why?" 1% calculated every month, because it will accrue the most interest.
- "If the bank calculates $\frac{1}{12}$ of the interest every month, will you have 6% of $1,000 (or $60) after one year? Why or why not?" You will have slightly more than that ($61.68, to be exact), because the interest is compounded monthly, not annually.

Have students share how compounding intervals affect the quantity being determined and how to represent the different patterns of increase using exponents.

Highlight that interest is often calculated more frequently than once a year. The overall amount of interest earned (or owed) over time is affected by how often the interest is compounded, calculated, and added to the previous amounts.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

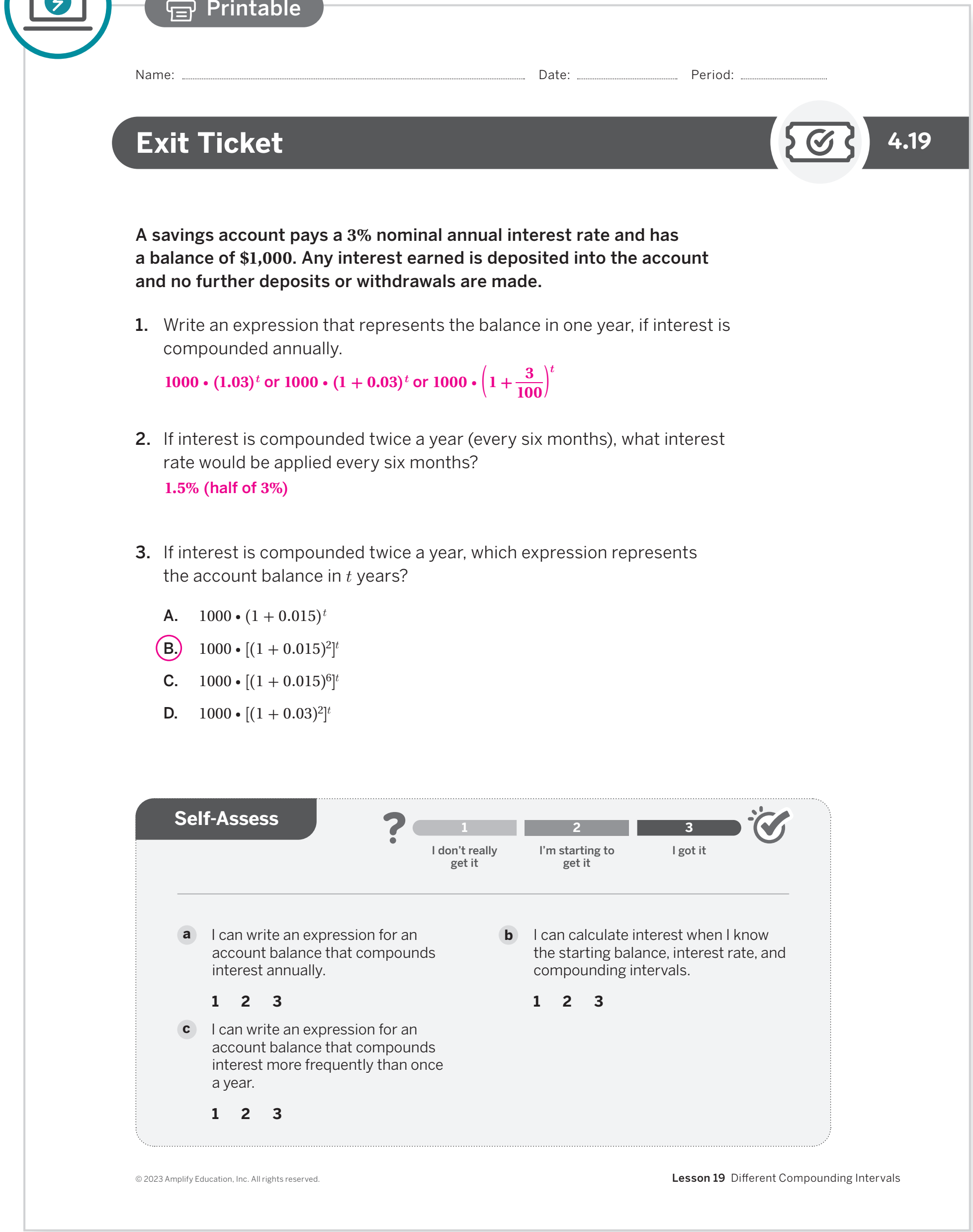
- "How does compounding interest relate to exponential growth?"
- "What might you look for when opening a bank account or applying for a credit card in the future?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing expressions to calculate interest rates over different compounding time intervals.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 4.19

A savings account pays a 3% nominal annual interest rate and has a balance of $1,000. Any interest earned is deposited into the account and no further deposits or withdrawals are made.

1. Write an expression that represents the balance in one year, if interest is compounded annually.

   $1000 \cdot (1.03)^t$ or $1000 \cdot (1 + 0.03)^t$ or $1000 \cdot \left(1 + \frac{3}{100}\right)^t$

2. If interest is compounded twice a year (every six months), what interest rate would be applied every six months?

   1.5% (half of 3%)

3. If interest is compounded twice a year, which expression represents the account balance in t years?

   A. $1000 \cdot (1 + 0.015)^t$
   [✓ B.] $1000 \cdot [(1 + 0.015)^2]^t$
   C. $1000 \cdot [(1 + 0.015)^6]^t$
   D. $1000 \cdot [(1 + 0.03)^2]^t$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can write an expression for an account balance that compounds interest annually.
1 2 3

b. I can calculate interest when I know the starting balance, interest rate, and compounding intervals.
1 2 3

c. I can write an expression for an account balance that compounds interest more frequently than once a year.
1 2 3

Lesson 19 Different Compounding Intervals

Success looks like . . .

- **Goal:** Calculating the result of repeated percent increases for the same initial balance and interest rate, but compounded at different intervals.
- **Goal:** Comparing interest rates and compounding intervals to choose the better investment option.

Suggested next steps

If students incorrectly write an expression in Problem 1, consider:

- Reviewing Activity 3.

If students cannot correctly calculate interest on the compounding interval in Problem 2, consider:

- Reviewing Activity 2.

If students cannot express the balance of the account given in Problem 3 that compounds interest more frequently than once a year, consider:

- Reviewing Activity 3.

Professional Learning

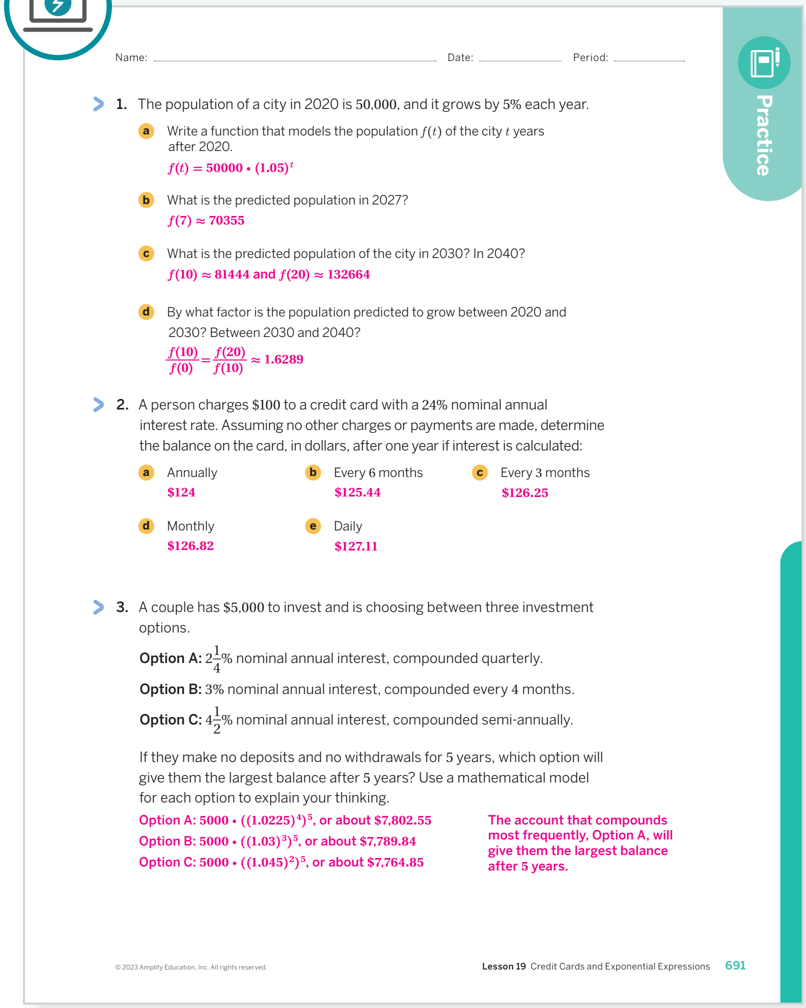
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What different ways did students approach Activity 3? What does that tell you about the similarities and differences among your students?
- Which students or groups of students did and did not have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. The population of a city in 2020 is 50,000, and it grows by 5% each year.
   - a) Write a function that models the population $f(t)$ of the city t years after 2020.
     $f(t) = 50000 \cdot (1.05)^t$
   - b) What is the predicted population in 2027?
     $f(7) \approx 70355$
   - c) What is the predicted population of the city in 2030? In 2040?
     $f(10) \approx 81444$ and $f(20) \approx 132664$
   - d) By what factor is the population predicted to grow between 2020 and 2030? Between 2030 and 2040?
     $\frac{f(10)}{f(0)} = \frac{f(20)}{f(10)} \approx 1.6289$

2. A person charges $100 to a credit card with a 24% nominal annual interest rate. Assuming no other charges or payments are made, determine the balance on the card, in dollars, after one year if interest is calculated:
   - a) Annually
     $124
   - b) Every 6 months
     $125.44
   - c) Every 3 months
     $126.25
   - d) Monthly
     $126.82
   - e) Daily
     $127.11

3. A couple has $5,000 to invest and is choosing between three investment options.

   Option A: $2\frac{1}{4}$% nominal annual interest, compounded quarterly.

   Option B: 3% nominal annual interest, compounded every 4 months.

   Option C: $4\frac{1}{2}$% nominal annual interest, compounded semi-annually.

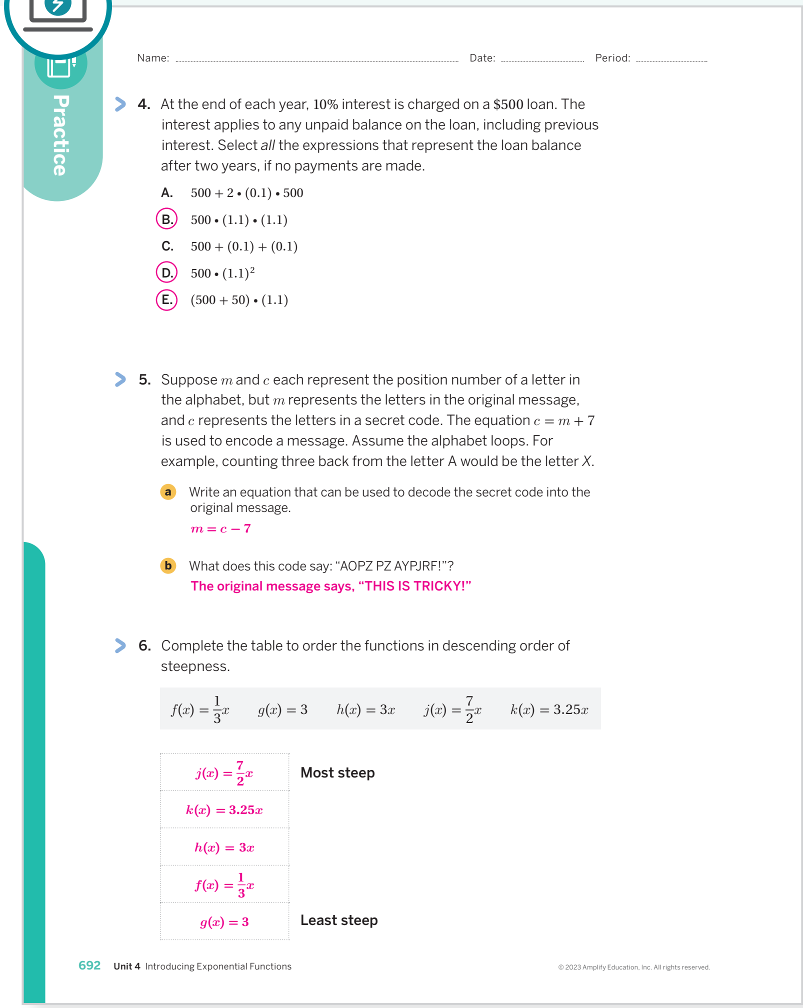
   If they make no deposits and no withdrawals for 5 years, which option will give them the largest balance after 5 years? Use a mathematical model for each option to explain your thinking.

   Option A: $5000 \cdot ((1.0225)^4)^5$, or about $7,802.55
   Option B: $5000 \cdot ((1.03)^3)^5$, or about $7,789.84
   Option C: $5000 \cdot ((1.045)^2)^5$, or about $7,764.85

   The account that compounds most frequently, Option A, will give them the largest balance after 5 years.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Credit Cards and Exponential Expressions 691



Name: ______ Date: ______ Period: ______

Practice

4. At the end of each year, 10% interest is charged on a $500 loan. The interest applies to any unpaid balance on the loan, including previous interest. Select *all* the expressions that represent the loan balance after two years, if no payments are made.
   A. $500 + 2 \cdot (0.1) \cdot 500$
   [✓ B.] $500 \cdot (1.1) \cdot (1.1)$
   C. $500 + (0.1) + (0.1)$
   [✓ D.] $500 \cdot (1.1)^2$
   [✓ E.] $(500 + 50) \cdot (1.1)$

5. Suppose m and c each represent the position number of a letter in the alphabet, but m represents the letters in the original message, and c represents the letters in a secret code. The equation $c = m + 7$ is used to encode a message. Assume the alphabet loops. For example, counting three back from the letter A would be the letter *X*.
   - a) Write an equation that can be used to decode the secret code into the original message.
     $m = c - 7$
   - b) What does this code say: "AOPZ PZ AYPJRF!"?
     The original message says, "THIS IS TRICKY!"

6. Complete the table to order the functions in descending order of steepness.

   $f(x) = \frac{1}{3}x$ $g(x) = 3$ $h(x) = 3x$ $j(x) = \frac{7}{2}x$ $k(x) = 3.25x$

$j(x) = \frac{7}{2}x$	**Most steep**
$k(x) = 3.25x$	
$h(x) = 3x$	
$f(x) = \frac{1}{3}x$	
$g(x) = 3$	**Least steep**

692 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Unit 4 Lesson 17	2
	5	Unit 1 Lesson 9	2
Formative ⚡	6	Unit 4 Lesson 20	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

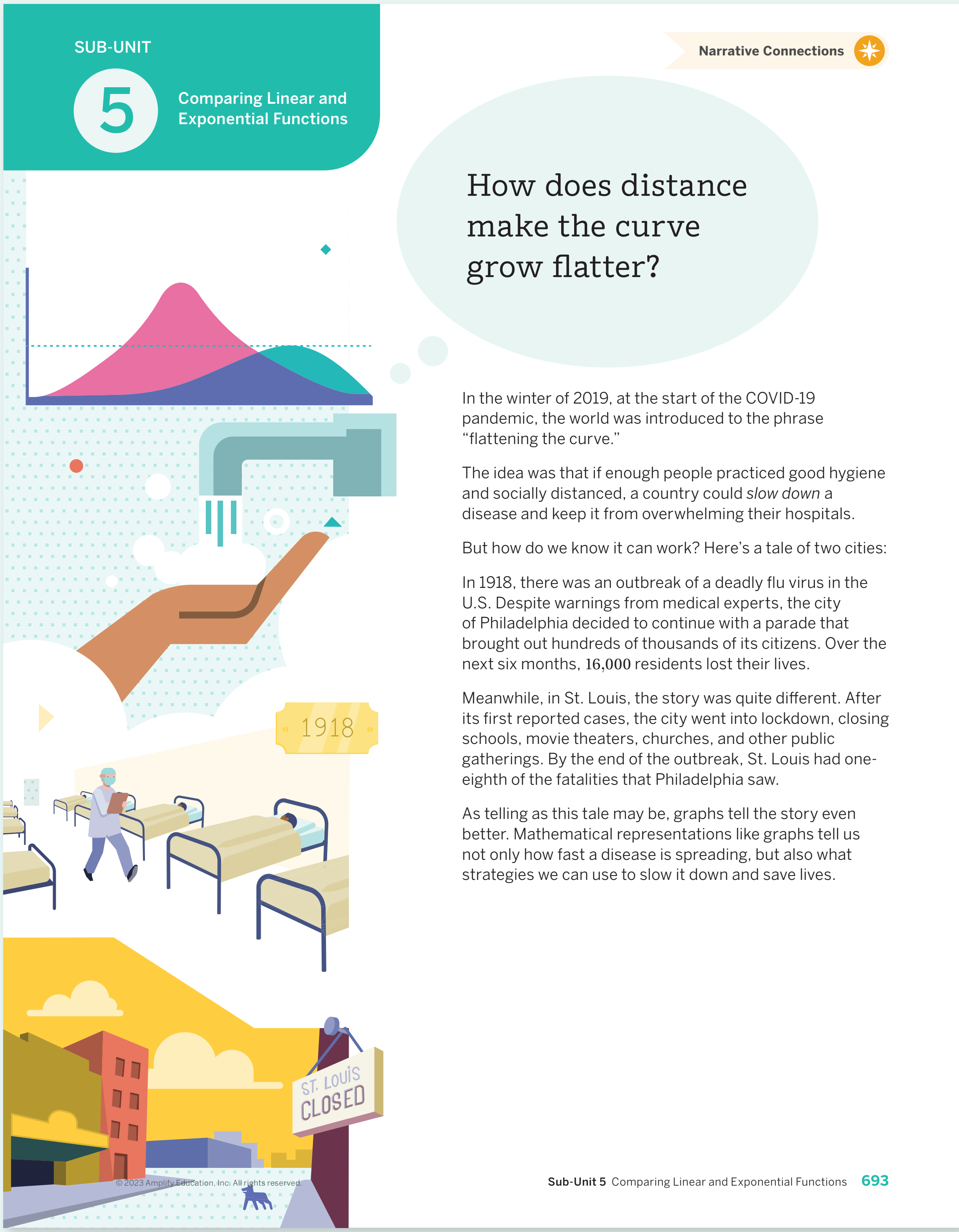
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 5

Comparing Linear and Exponential Functions

Whole Class

In this Sub-Unit, students compare and contrast linear and exponential functions to determine which function changes faster. They recognize that an exponential function will eventually overtake a linear function.



SUB-UNIT

5

Comparing Linear and Exponential Functions

Narrative Connections

How does distance make the curve grow flatter?

In the winter of 2019, at the start of the COVID-19 pandemic, the world was introduced to the phrase "flattening the curve."

The idea was that if enough people practiced good hygiene and socially distanced, a country could *slow down* a disease and keep it from overwhelming their hospitals.

But how do we know it can work? Here's a tale of two cities:

In 1918, there was an outbreak of a deadly flu virus in the U.S. Despite warnings from medical experts, the city of Philadelphia decided to continue with a parade that brought out hundreds of thousands of its citizens. Over the next six months, 16,000 residents lost their lives.

Meanwhile, in St. Louis, the story was quite different. After its first reported cases, the city went into lockdown, closing schools, movie theaters, churches, and other public gatherings. By the end of the outbreak, St. Louis had one-eighth of the fatalities that Philadelphia saw.

As telling as this tale may be, graphs tell the story even better. Mathematical representations like graphs tell us not only how fast a disease is spreading, but also what strategies we can use to slow it down and save lives.

Sub-Unit 5 Comparing Linear and Exponential Functions 693

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to use exponential functions to model the spread of disease in the following places:

- **Lesson 20, Activity 1:** Plant Disease
- **Lesson 21, Activity 3:** Price of Prescription Drugs
- **Lesson 22, Activities 1–3:** COVID-19 Timeline, Tracking the Spread, Vaccine Response

UNIT 4 | **LESSON 20**

Which One Changes Faster?

Let's compare linear and exponential functions as they increase.

Focus

Goals

1. Use graphs and calculations to show that a quantity that increases exponentially will eventually surpass one that increases linearly.
2. Use tables, calculations, and graphs to compare growth rates of linear and exponential functions.

Coherence

Today

Students investigate the fact that exponential functions grow more quickly than linear functions. Students examine tables and graphs over various domains to determine when an exponential function will overtake the linear function.

Previously

Students wrote equivalent expressions to highlight different aspects of a scenario involving repeated percent increase or decrease.

Coming Soon

Students will show that linear functions change by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals.

Rigor

- Students further develop their **conceptual understanding** of exponential functions by comparing their growth to linear functions, recognizing that exponential growth invariably surpasses linear growth.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- graphing technology

Math Language Development

Review words

- *domain*
- *function*
- *interest*
- *range*

Amps | **Featured Activity**

Activity 1

Viewing the Disease Cycle

Students generate expressions for linear and exponential growth and predict how they will compare. When they are ready, they can zoom out on a graph as see how their growth truly compares over time.

Building Math Identity and Community

Connecting to Mathematical Practices

Motivating oneself to see when two graphs will equal each other could be challenging for some students, but remind them that they have tools that can make this task more efficient or even possible. Students use a table to compare the values of the functions, but $2,000$ is not the value of either function for the values in the table. Have students work with a partner to devise a strategic plan for staying motivated by using what information they are provided in the table.

Modifications to Pacing

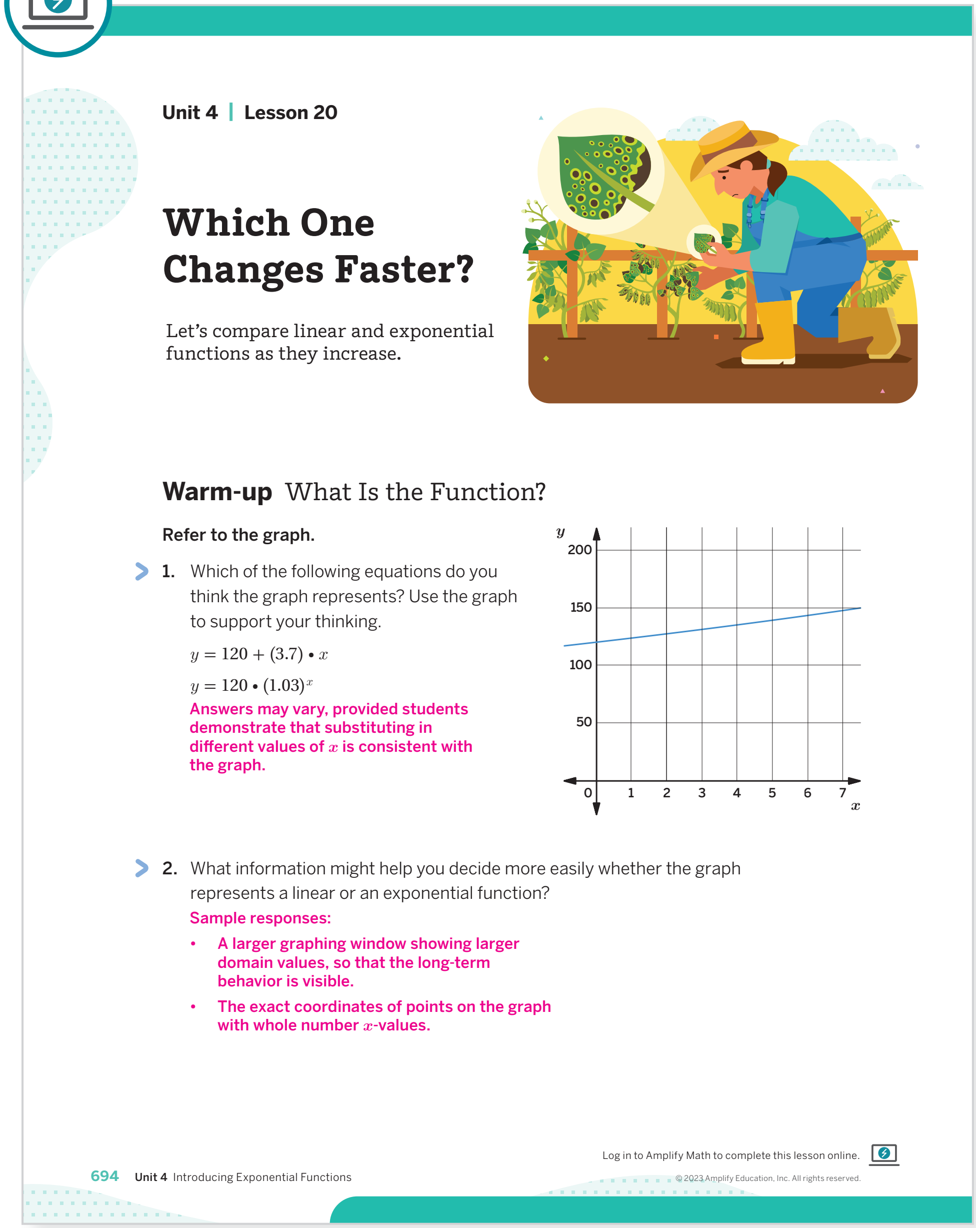
You may want to consider this additional modification if you are short on time.

- Skip to the last paragraph of the **Activity 1** narrative, and then have students proceed to the problems.

Independent | 5 min

Warm-up What Is the Function?

Students compare linear and exponential growth over a portion of the domain to explore how truncated domains affect the appearance of the function.



Unit 4 | Lesson 20

Which One Changes Faster?

Let's compare linear and exponential functions as they increase.

Warm-up What Is the Function?

Refer to the graph.

1. Which of the following equations do you think the graph represents? Use the graph to support your thinking.

   $y = 120 + (3.7) \cdot x$

   $y = 120 \cdot (1.03)^x$

   Answers may vary, provided students demonstrate that substituting in different values of x is consistent with the graph.

2. What information might help you decide more easily whether the graph represents a linear or an exponential function?

   Sample responses:

   - **A larger graphing window showing larger domain values, so that the long-term behavior is visible.**
   - **The exact coordinates of points on the graph with whole number x-values.**

Log in to Amplify Math to complete this lesson online.

694 Unit 4 Introducing Exponential Functions

1 Launch

Say, "Compare each equation to the graph. What information is needed to help you decide?" Allow students think-time before collaborating with a partner.

2 Monitor

Help students get started by estimating points on the graph to the nearest whole number and having them substitute values into each equation.

Look for points of confusion:

- **Struggling to select an equation.** Have students start with the second problem or use the graph to support their indecision.

Look for productive strategies:

- Analyzing the graph and each equation, and justifying their conclusions mathematically.

3 Connect

Have students share the thinking behind their decisions and suggestions with a partner. Select student pairs to share with the whole class and ask them to construct a valid mathematical argument to support their conclusion.

Display the dynamic graph of $y = 120 \cdot (1.03)^x$, using graphing technology. Start with a small window and then zoom out.

Highlight that the graph of a linear function is always a line. An exponential function may appear linear depending on the constraints of the domain and range. Because of this, a graph may be misleading when determining its type of function.

Power-up

To power up students' ability to interpret the slope of linear functions, have students complete:

Consider the functions $f(x) = \frac{2}{3}x$ and $g(x) = \frac{3}{2}x$.

a Which function has a greater value at $x = 1$? $g(x)$

b Which function reaches the value of 12 first? $g(x)$

c Which function is steeper? $g(x)$

Use: Before the Warm-up

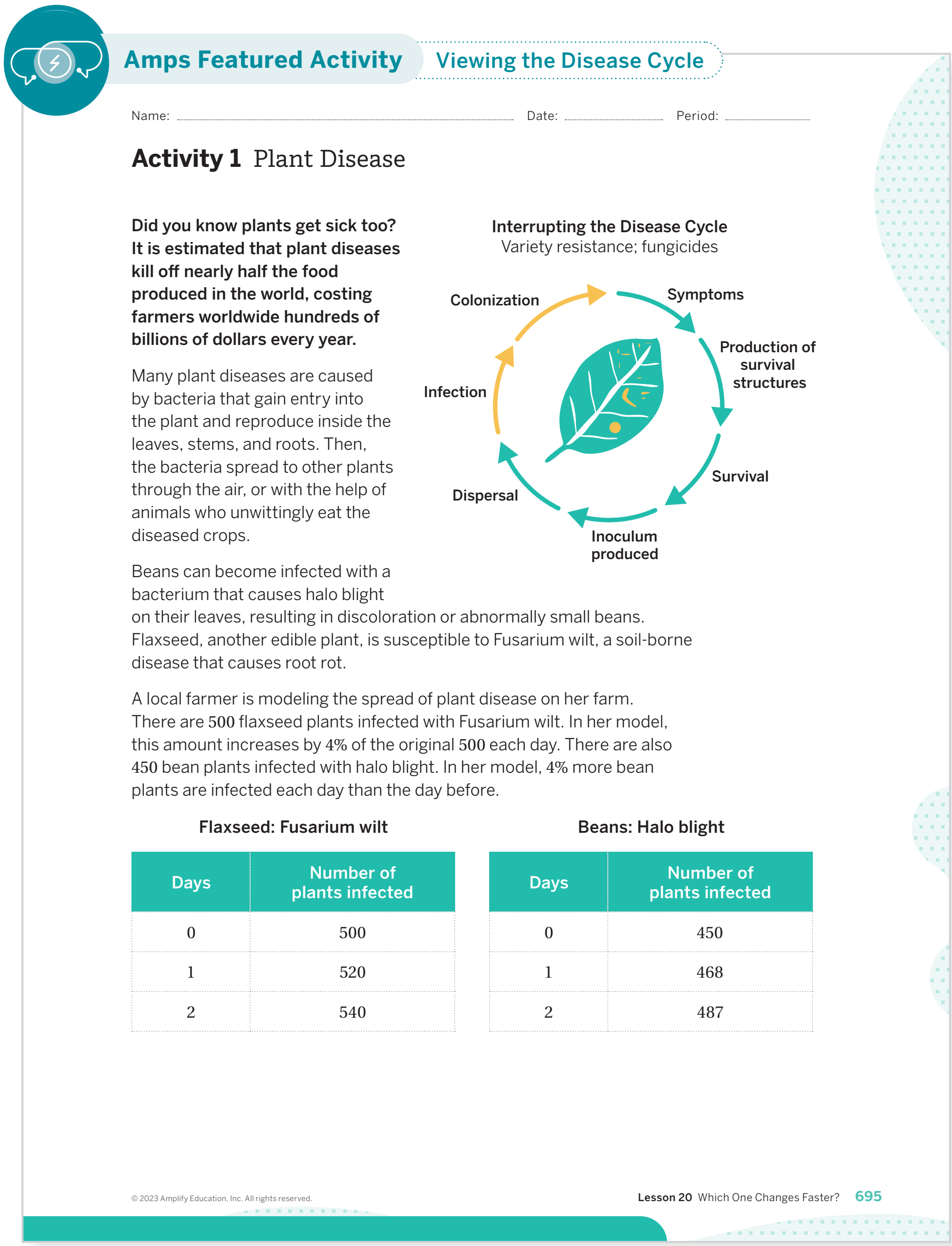
Informed by: Performance on Lesson 19, Practice Problem 6

Pairs | 20 min

Activity 1 Plant Disease

Students construct and compare linear and exponential functions to model the spread of plant disease, use their models to solve problems, and interpret the solution in context.

Amps Featured Activity Viewing the Disease Cycle



Name: ________________ Date: ________ Period: ________

Activity 1 Plant Disease

Did you know plants get sick too? It is estimated that plant diseases kill off nearly half the food produced in the world, costing farmers worldwide hundreds of billions of dollars every year.

Many plant diseases are caused by bacteria that gain entry into the plant and reproduce inside the leaves, stems, and roots. Then, the bacteria spread to other plants through the air, or with the help of animals who unwittingly eat the diseased crops.

Beans can become infected with a bacterium that causes halo blight on their leaves, resulting in discoloration or abnormally small beans. Flaxseed, another edible plant, is susceptible to Fusarium wilt, a soil-borne disease that causes root rot.

A local farmer is modeling the spread of plant disease on her farm. There are 500 flaxseed plants infected with Fusarium wilt. In her model, this amount increases by 4% of the original 500 each day. There are also 450 bean plants infected with halo blight. In her model, 4% more bean plants are infected each day than the day before.

Flaxseed: Fusarium wilt

Days	Number of plants infected
0	500
1	520
2	540

Beans: Halo blight

Days	Number of plants infected
0	450
1	468
2	487

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Which One Changes Faster? 695

1 Launch

Read the prompt aloud. Discuss plant disease, simple interest, and compound interest. Give students one minute of think-time before completing the activity with a partner. Provide access to graphing technology.

2 Monitor

Help students get started by prompting them to extend the tables to determine the pattern.

Look for points of confusion:

- **Not being able to choose which plant should be treated first without additional information.** Ask students to think about what might make sense for the farmer or give students a time frame that would justify a choice.
- **Struggling to read the disease names.** Cross out the disease names and rewrite them using more familiar language such as "wilting" and "shrinking."
- **Writing both equations as linear or as exponential.** Review how to determine whether a table of values represents a linear or exponential function.

Look for productive strategies:

- Making decisions about disease growth based on different time intervals.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can graph the disease cycle while an animation that compares the linear and exponential growth will freeze the graph, before the exponential function overtakes the linear function.

Accessibility: *Vary Demands to Optimize Challenge*

Provide a summary of the introductory text and consider showing photos of flaxseed plants and bean plants for students to visualize the context. Have them study the tables and facilitate a class discussion for Problems 1 and 2. Have students begin the activity with Problem 3.

Math Language Development

MLR6: Three Reads

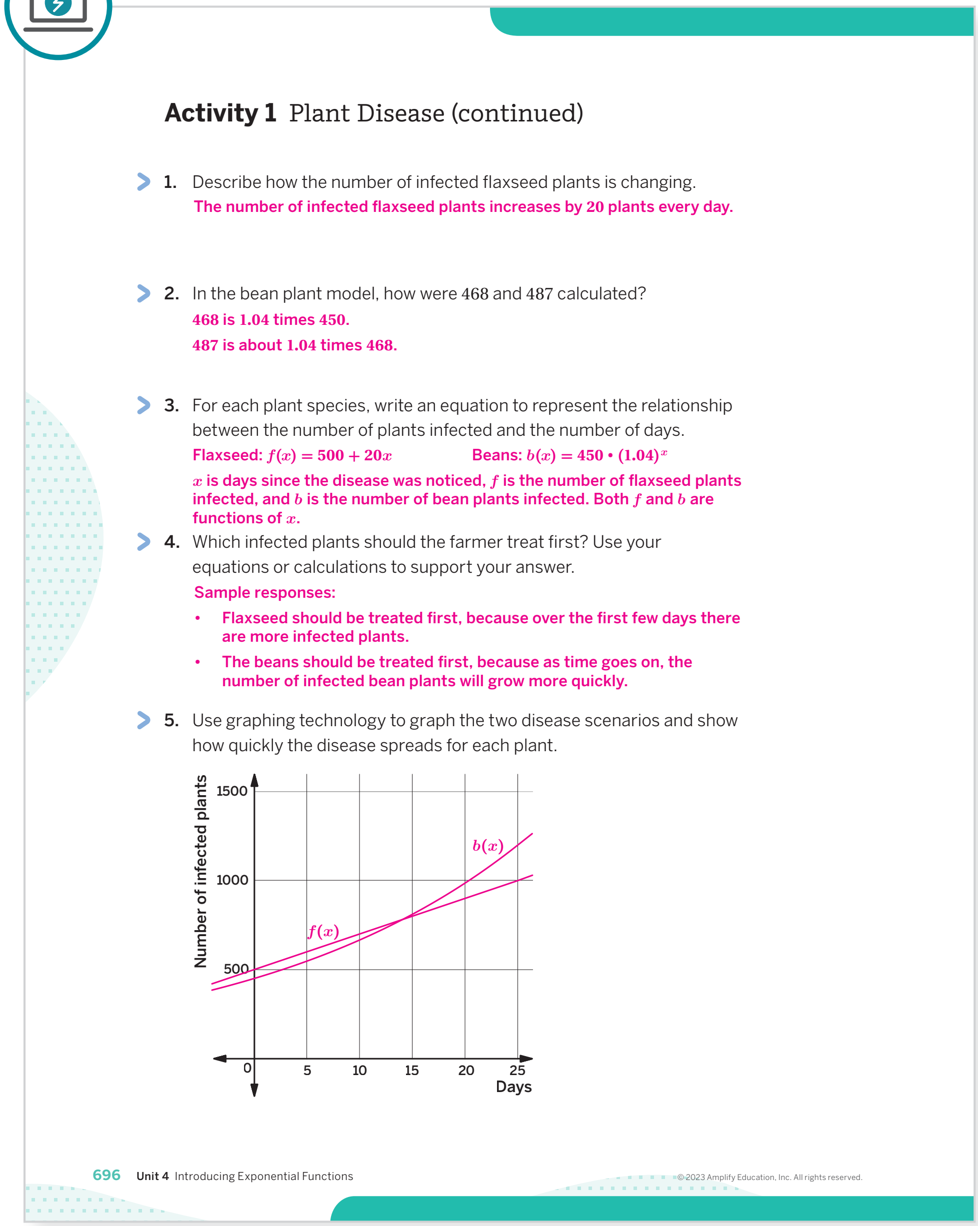
Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they will be considering how many flaxseed plants become infected each day.
- **Read 2:** Ask students to highlight any relevant quantities or relationships, such as the number of flaxseed plants infected increases by 4% of the original 500 each day.
- **Read 3:** Ask groups of students to describe how the number of infected flaxseed plants or beans plants is changing in their own words.

Pairs | 20 min

Activity 1 Plant Disease (continued)

Students construct and compare linear and exponential functions to model the spread of plant disease, use their models to solve problems, and interpret the solution in context.



Activity 1 Plant Disease (continued)

1. Describe how the number of infected flaxseed plants is changing.
   The number of infected flaxseed plants increases by 20 plants every day.

2. In the bean plant model, how were 468 and 487 calculated?
   468 is 1.04 times 450.
   487 is about 1.04 times 468.

3. For each plant species, write an equation to represent the relationship between the number of plants infected and the number of days.
   Flaxseed: $f(x) = 500 + 20x$ Beans: $b(x) = 450 \bullet (1.04)^x$
   x is days since the disease was noticed, f is the number of flaxseed plants infected, and b is the number of bean plants infected. Both f and b are functions of x.

4. Which infected plants should the farmer treat first? Use your equations or calculations to support your answer.
   Sample responses:
   - Flaxseed should be treated first, because over the first few days there are more infected plants.
   - The beans should be treated first, because as time goes on, the number of infected bean plants will grow more quickly.

5. Use graphing technology to graph the two disease scenarios and show how quickly the disease spreads for each plant.

696 **Unit 4** Introducing Exponential Functions

3 **Connect**

Display students' graphs and equations of each plant species.

Have students share the various choices made and explain their responses.

Ask:

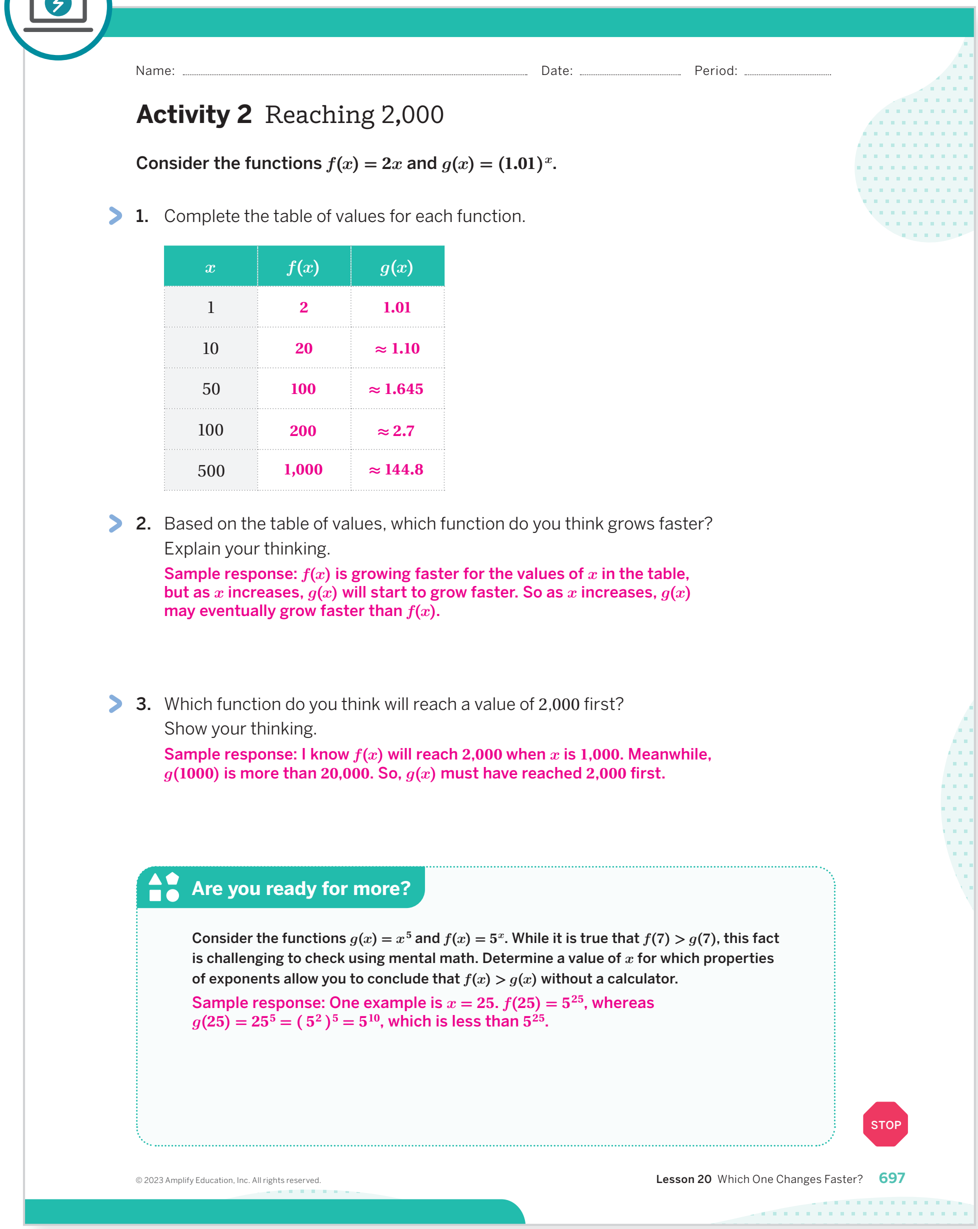
- "How does the initial number of infected plants affect your choice for determining which plant the farmer should treat first?" The linear function started with more infected plants, so I think the flaxseed should be treated first.
- "When might the farmer want to treat the flaxseed first, if ever?" When treating them in a shorter time period, before two weeks, when the infected beans spread quicker.
- "When might the farmer want to treat the beans first, if ever?" After two weeks, when the compounding effects make up for the lower initial number.

Highlight that this exponential function has a relatively slow rate of growth, but it eventually overtakes the linear function.

Pairs | 15 min

Activity 2 Reaching 2,000

Students select their own strategies or tools to compare linear and exponential growth to determine when the exponential function will overtake the linear function.



Name: ______ Date: ______ Period: ______

Activity 2 Reaching 2,000

Consider the functions $f(x) = 2x$ and $g(x) = (1.01)^x$.

1. Complete the table of values for each function.

x	$f(x)$	$g(x)$
1	2	1.01
10	20	≈ 1.10
50	100	≈ 1.645
100	200	≈ 2.7
500	1,000	≈ 144.8

2. Based on the table of values, which function do you think grows faster? Explain your thinking.
Sample response: $f(x)$ is growing faster for the values of x in the table, but as x increases, $g(x)$ will start to grow faster. So as x increases, $g(x)$ may eventually grow faster than $f(x)$.

3. Which function do you think will reach a value of 2,000 first? Show your thinking.
Sample response: I know $f(x)$ will reach 2,000 when x is 1,000. Meanwhile, $g(1000)$ is more than 20,000. So, $g(x)$ must have reached 2,000 first.

Are you ready for more?

Consider the functions $g(x) = x^5$ and $f(x) = 5^x$. While it is true that $f(7) > g(7)$, this fact is challenging to check using mental math. Determine a value of x for which properties of exponents allow you to conclude that $f(x) > g(x)$ without a calculator.

Sample response: One example is $x = 25$. $f(25) = 5^{25}$, whereas $g(25) = 25^5 = (5^2)^5 = 5^{10}$, which is less than 5^{25}.

STOP

1 Launch

Display the two functions $f(x) = 2x$ and $g(x) = (1.01)^x$. Ask, "Which function do you think will reach a value of 2,000 first?" Provide access to graphing technology.

2 Monitor

Help students get started by reminding them how to use function notation to determine the function value, given a value of x.

Look for points of confusion:

- **Struggling to determine which function will reach 2,000 first.** Encourage students to extend the table until a function reaches 2,000.

Look for productive strategies:

- Extending the table, using graphing technology to create a graph, or evaluating the function at various increasing values of x.

3 Connect

Display a dynamic graph of both equations.

Have students share the strategies they used to determine which function grows faster.

Highlight that the table values and axes limits need to be chosen carefully to identify when the values of g become greater than those of f.

Ask, "What method(s) might be the best to determine which of two functions will reach a large value first?" Adjusting the axes limits on a graph and evaluating a large value of x.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with an extended table with x-values of 600, 700, 800, 900, and 1,000.

Accessibility: *Optimize Access to Technology*

Provide access to graphing technology should students choose to graph the functions to respond to Problem 3.

Math Language Development

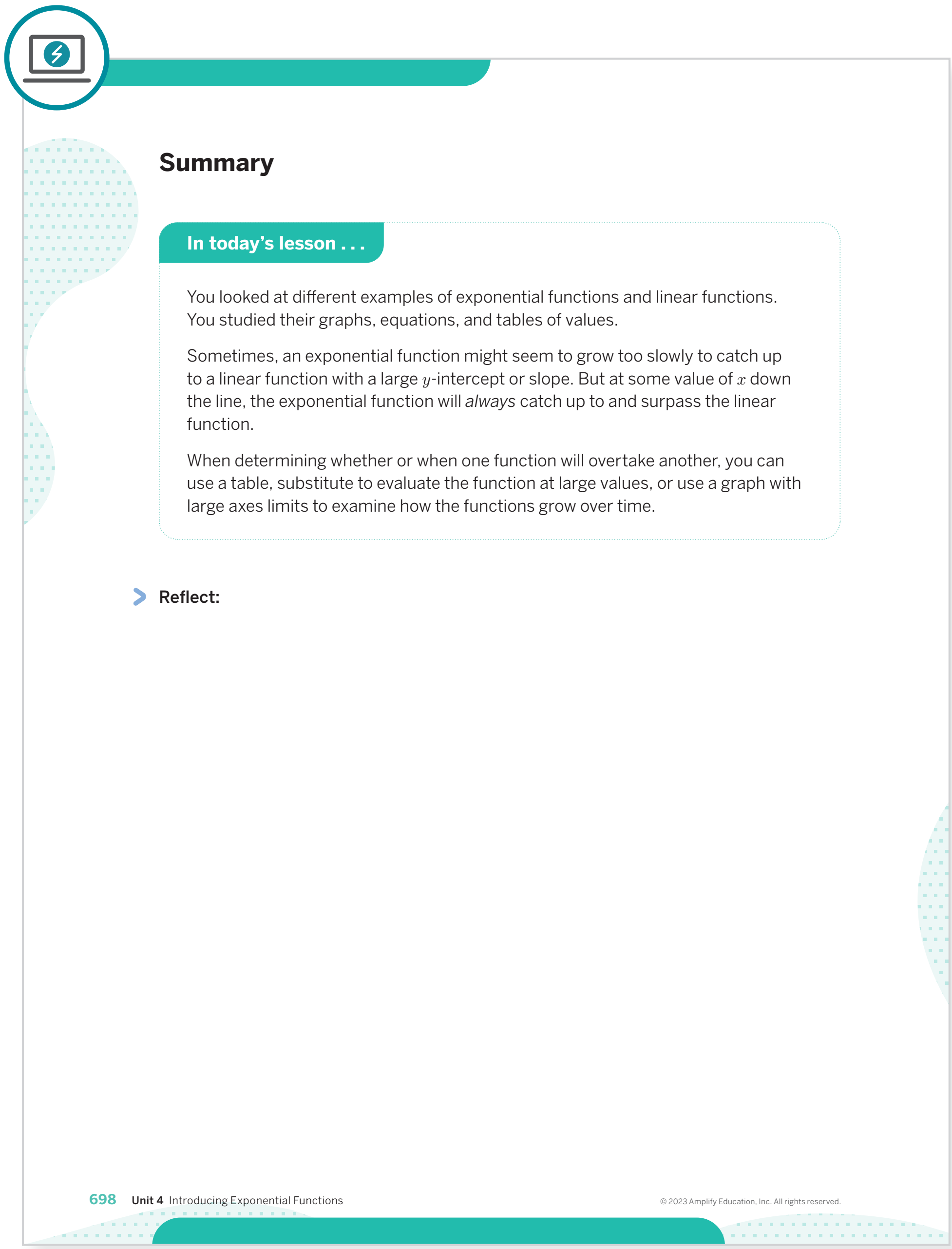
MLR7: Compare and Connect

Have students create a visual display showcasing the question posed in Problem 3. Their displays should include the question posed, each function, and the strategy they used to determine the answer to the question. Consider conducting a ***Gallery Tour*** and ask students to compare each other's strategies. Ask, "Which strategy do you think is the most efficient in determining a solution?" Listen for observations of advantages and disadvantages of different approaches, such as using a particular approach for a certain value of x.

Whole Class | 5 min

Summary

Review and synthesize the growth rates of linear and exponential functions, noting that while exponential functions may seem to grow slowly at first, they will eventually overtake linear functions.



Summary

In today's lesson . . .

You looked at different examples of exponential functions and linear functions. You studied their graphs, equations, and tables of values.

Sometimes, an exponential function might seem to grow too slowly to catch up to a linear function with a large y-intercept or slope. But at some value of x down the line, the exponential function will *always* catch up to and surpass the linear function.

When determining whether or when one function will overtake another, you can use a table, substitute to evaluate the function at large values, or use a graph with large axes limits to examine how the functions grow over time.

Reflect:

698 Unit 4 Introducing Exponential Functions

Synthesize

Display the completed graph from Activity 2, Problem 5. Label each graph.

Have students share with a partner, after some individual think-time first. Select two pairs for a whole-class discussion.

Ask:

- "How can you tell which function grows faster?" Sample responses: compare the slopes, initial values, and types of functions.
- "Will the function that started out with the lower initial value catch up with the function that starts with a greater initial value?" Yes, because the rate of change is greater.
- "What are some ways to check if one function will overtake another?" Substitute a large value into each function, create a table or graph.

Highlight that even though an exponential function might seem to be growing too slowly to catch up to a linear function, at some value of x, the exponential function will catch up with, and overtake, the linear function.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

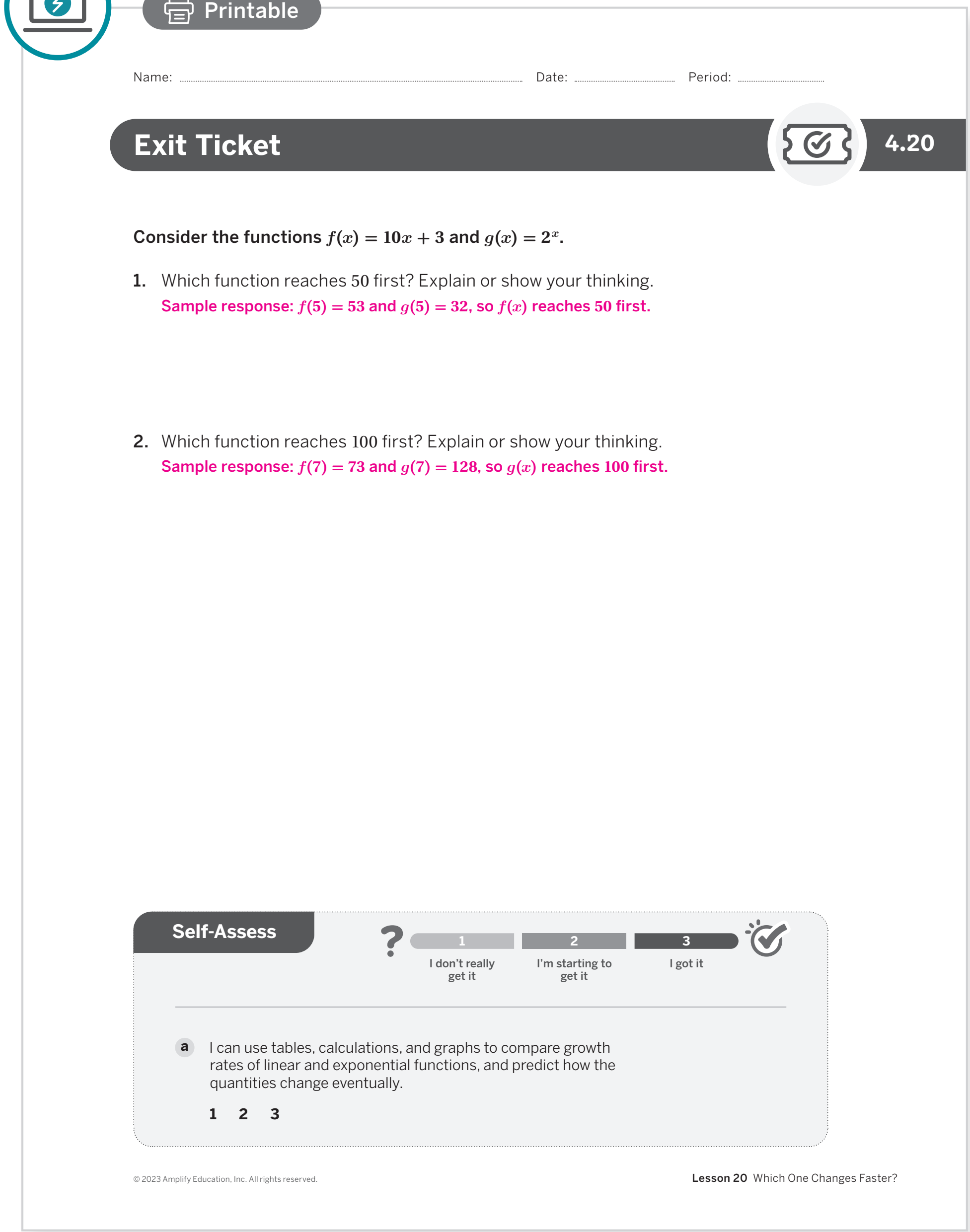
- "How can you differentiate exponential growth from linear growth, given a real-world data set?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by selecting strategies or tools to compare growth rates of linear and exponential functions.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 4.20

Consider the functions $f(x) = 10x + 3$ and $g(x) = 2^x$.

1. Which function reaches 50 first? Explain or show your thinking.
   Sample response: $f(5) = 53$ and $g(5) = 32$, so $f(x)$ reaches 50 first.

2. Which function reaches 100 first? Explain or show your thinking.
   Sample response: $f(7) = 73$ and $g(7) = 128$, so $g(x)$ reaches 100 first.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use tables, calculations, and graphs to compare growth rates of linear and exponential functions, and predict how the quantities change eventually.

1 2 3

Lesson 20 Which One Changes Faster?

Success looks like . . .

- **Goal:** Using graphs and calculations to show that a quantity that increases exponentially will eventually surpass one that increases linearly.
- **Goal:** Using tables, calculations, and graphs to compare growth rates of linear and exponential functions.
  - » Identifying the function that reaches the given value first in Problems 1 and 2 by comparing functions.

Suggested next steps

If students incorrectly predict which function reaches the value first in Problems 1 and 2, consider:

- Reviewing strategies to determine how function quantities change over time in Activity 2, Problem 3.
- Assigning Practice Problem 3.
- Having students create a table and graph and explain which strategy works best under what circumstances.

Professional Learning

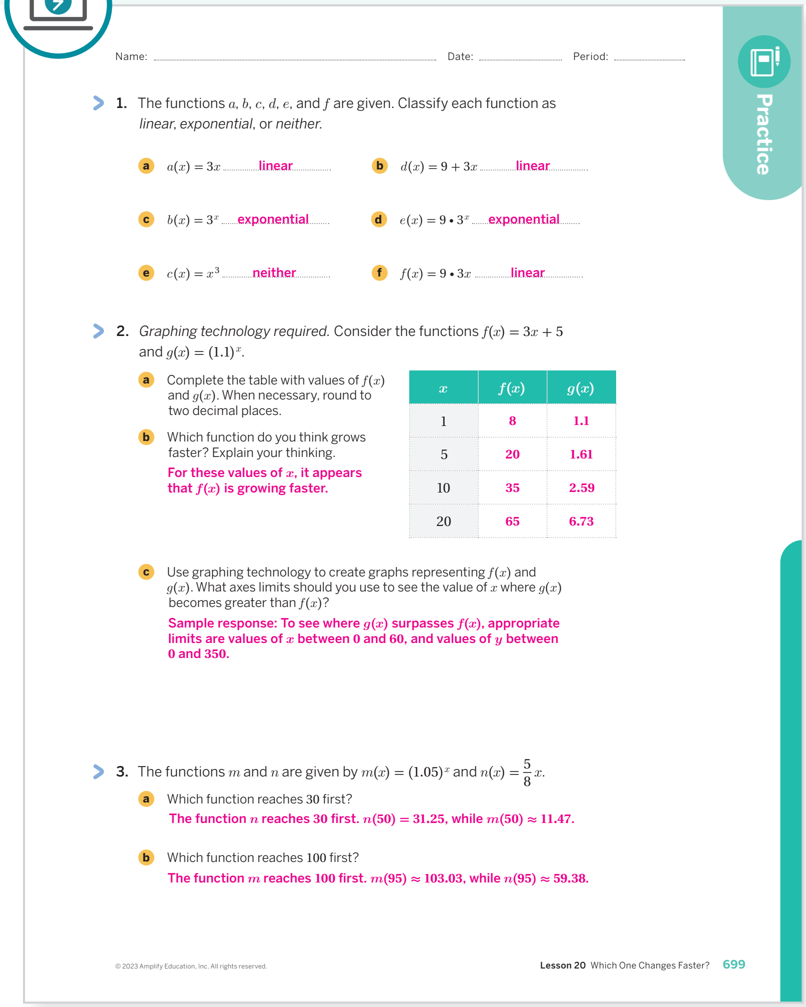
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students compared linear functions to exponential functions to determine which changes faster. How does this concept build on their earlier work with analyzing linear and exponential patterns and rates of change.
- Thinking about the questions you asked students today and how students responded, which question(s) were the most efficient? Which questions might you change the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

Practice

1. The functions a, b, c, d, e, and f are given. Classify each function as *linear*, *exponential*, or *neither*.
   - a $a(x) = 3x$ ....linear....
   - b $d(x) = 9 + 3x$ ....linear....
   - c $b(x) = 3^x$ ....exponential....
   - d $e(x) = 9 \bullet 3^x$ ....exponential....
   - e $c(x) = x^3$ ....neither....
   - f $f(x) = 9 \bullet 3x$ ....linear....

2. *Graphing technology required.* Consider the functions $f(x) = 3x + 5$ and $g(x) = (1.1)^x$.
   - a Complete the table with values of $f(x)$ and $g(x)$. When necessary, round to two decimal places.
   - b Which function do you think grows faster? Explain your thinking.

     For these values of x, it appears that $f(x)$ is growing faster.

x	$f(x)$	$g(x)$
1	8	1.1
5	20	1.61
10	35	2.59
20	65	6.73

   - c Use graphing technology to create graphs representing $f(x)$ and $g(x)$. What axes limits should you use to see the value of x where $g(x)$ becomes greater than $f(x)$?

     Sample response: To see where $g(x)$ surpasses $f(x)$, appropriate limits are values of x between 0 and 60, and values of y between 0 and 350.

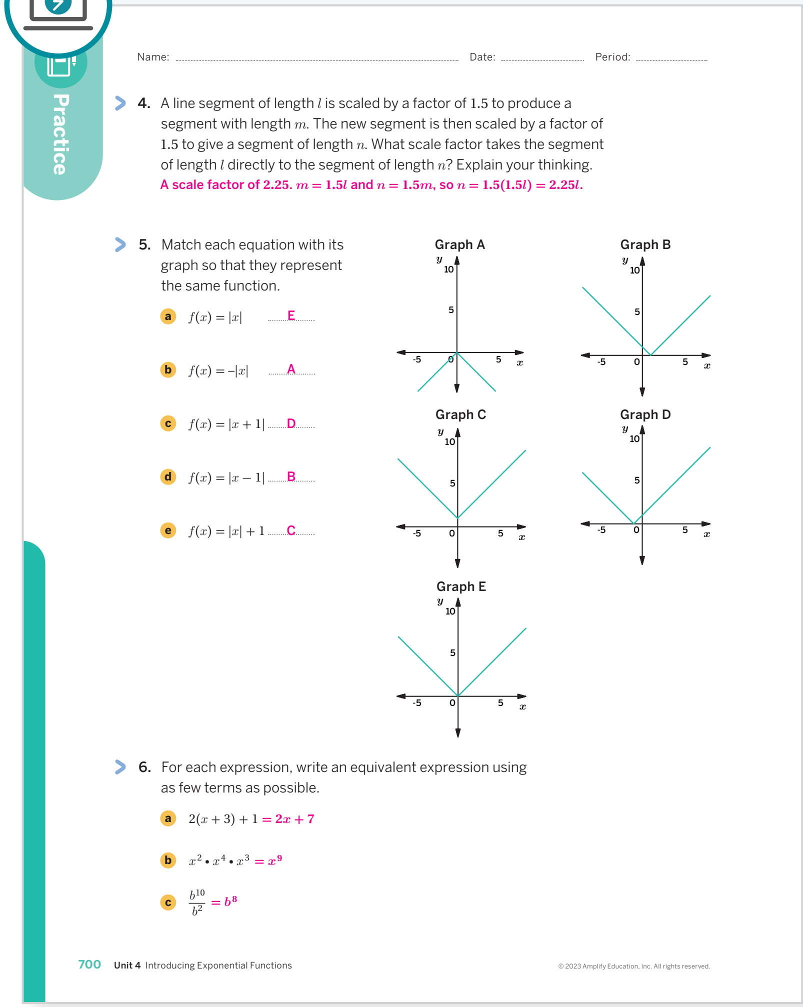
3. The functions m and n are given by $m(x) = (1.05)^x$ and $n(x) = \frac{5}{8}x$.
   - a Which function reaches 30 first?

     The function n reaches 30 first. $n(50) = 31.25$, while $m(50) \approx 11.47$.
   - b Which function reaches 100 first?

     The function m reaches 100 first. $m(95) \approx 103.03$, while $n(95) \approx 59.38$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Which One Changes Faster? 699



Name: .... Date: .... Period: ....

Practice

4. A line segment of length l is scaled by a factor of 1.5 to produce a segment with length m. The new segment is then scaled by a factor of 1.5 to give a segment of length n. What scale factor takes the segment of length l directly to the segment of length n? Explain your thinking.

   A scale factor of 2.25. $m = 1.5l$ and $n = 1.5m$, so $n = 1.5(1.5l) = 2.25l$.

5. Match each equation with its graph so that they represent the same function.
   - a $f(x) = |x|$ ....E....
   - b $f(x) = -|x|$ ....A....
   - c $f(x) = |x + 1|$ ....D....
   - d $f(x) = |x - 1|$ ....B....
   - e $f(x) = |x| + 1$ ....C....

6. For each expression, write an equivalent expression using as few terms as possible.
   - a $2(x + 3) + 1 = 2x + 7$
   - b $x^2 \bullet x^4 \bullet x^3 = x^9$
   - c $\frac{b^{10}}{b^2} = b^8$

700 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 17	1
	5	Unit 3 Lesson 17	1
Formative ⚡	6	Unit 4 Lesson 21	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Mathematical Modeling Prompt

At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's Mathematical Modeling Prompt, *Critically Examining the National Debt*, which is available in the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 21**

Changes Over Equal Intervals

Let’s explore how linear and exponential functions change over equal intervals.

Focus

Goals

1. Calculate rates of change of functions given graphs, equations, or tables.
2. **Language Goal:** Use rates of change to describe how a linear function and an exponential function change over equal intervals. **(Speaking and Listening, Writing)**

Coherence

Today

Students show that linear functions change by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. Students observe this structure repeatedly for specific intervals and then generalize this reasoning to apply to all intervals.

Previously

Students examined tables and graphs and saw that exponential functions grow more quickly, eventually, than linear functions.

Coming Soon

Students will use their knowledge of linear and exponential functions to model data.

Rigor

- Students enhance their **conceptual understanding** of exponential versus linear behavior by exploring changes in both over equal intervals.
- Students enhance their **procedural fluency** in manipulating expressions that are products or quotients of exponential terms.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Independent	Independent	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Exponent Rules*
- scientific calculators

Math Language Development

Review words

- *commutative property*
- *exponential function*
- *independent variable*
- *rate of change*

Amps ⋮ **Featured Activity**

Activity 2
Interactive Table

Students use an interactive table to explore the change of an exponential function over equal intervals.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students use the structure of exponential functions to examine and interpret the models for the price of prescription drugs, some emotions of panic or concern might be stirred. Allow students time to reflect on those emotions and how they are influencing their current behaviors. Encourage students to adjust and approach the entire situation with a growth mindset. For example, they might conclude that they do not know how they will be able to afford medications . . . yet.

Modifications to Pacing

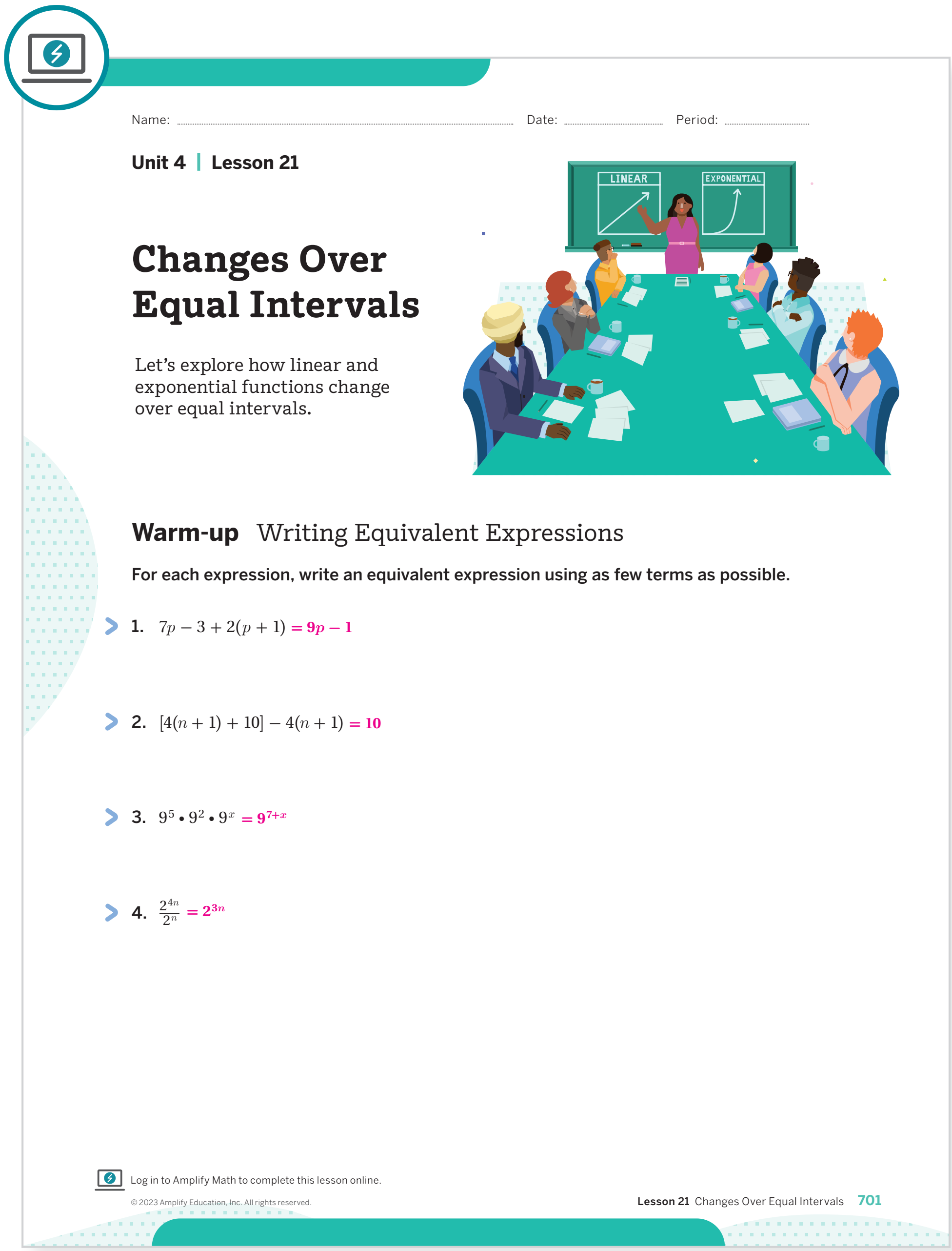
You may want to consider this additional modification if you are short on time.

- Omit **Activity 3**, which compares the growth of two exponential functions.

Independent | 5 min

Warm-up Writing Equivalent Expressions

Students revisit using the structure of expressions to write equivalent expressions to prepare them for working with complex expressions arising from linear and exponential functions.



Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 21

Changes Over Equal Intervals

Let's explore how linear and exponential functions change over equal intervals.

Warm-up Writing Equivalent Expressions

For each expression, write an equivalent expression using as few terms as possible.

1. $7p - 3 + 2(p + 1)$ $= 9p - 1$
2. $[4(n + 1) + 10] - 4(n + 1)$ $= 10$
3. $9^5 \bullet 9^2 \bullet 9^x$ $= 9^{7+x}$
4. $\frac{2^{4n}}{2^n}$ $= 2^{3n}$

Log in to Amplify Math to complete this lesson online.

Lesson 21 Changes Over Equal Intervals 701

1 Launch

Say, "Let's explore how linear and exponential functions change over equal intervals."

2 Monitor

Help students get started by displaying and reviewing the Anchor Chart PDF, *Exponent Rules* to help them simplify each expression.

Look for points of confusion:

- **Multiplying the exponents in Problem 3.** Write 9^3 as $9 \bullet 9 \bullet 9$ and 9^2 as $9 \bullet 9$. Say, "If you multiply the expressions together, how many 9s are being multiplied? Is there a more efficient way to write this using the original exponents?" Five 9s. Add the exponents.

Look for productive strategies:

- Using arrows to show the Distributive Property.

3 Connect

Have students share strategies for simplifying the expressions.

Highlight that the commutative property, Distributive Property, and properties of exponents are used to produce equivalent expressions.

Ask, "What differences do you notice between the first two and last two problems?" The first two problems involve the Distributive Property and combining like terms. The variable is in the exponent for the last two problems which use the properties of exponents to simplify.

Power-up

To power up students' ability to write equivalent expressions in fewer terms, have students complete:

Recall that an expression has as few terms as possible when all like terms have been combined and all possible operations have been completed.

Rewrite each expression with as few terms as possible.

1. $7p - 3 + 2p$ $= 9p - 3$
2. $[4n + 4 + 10] - 4(n + 1)$ $= 10$
3. $9^5 \bullet 9^x$ $= 9^{5+x}$
4. $\frac{2^4}{2^1}$ $= 2^3$

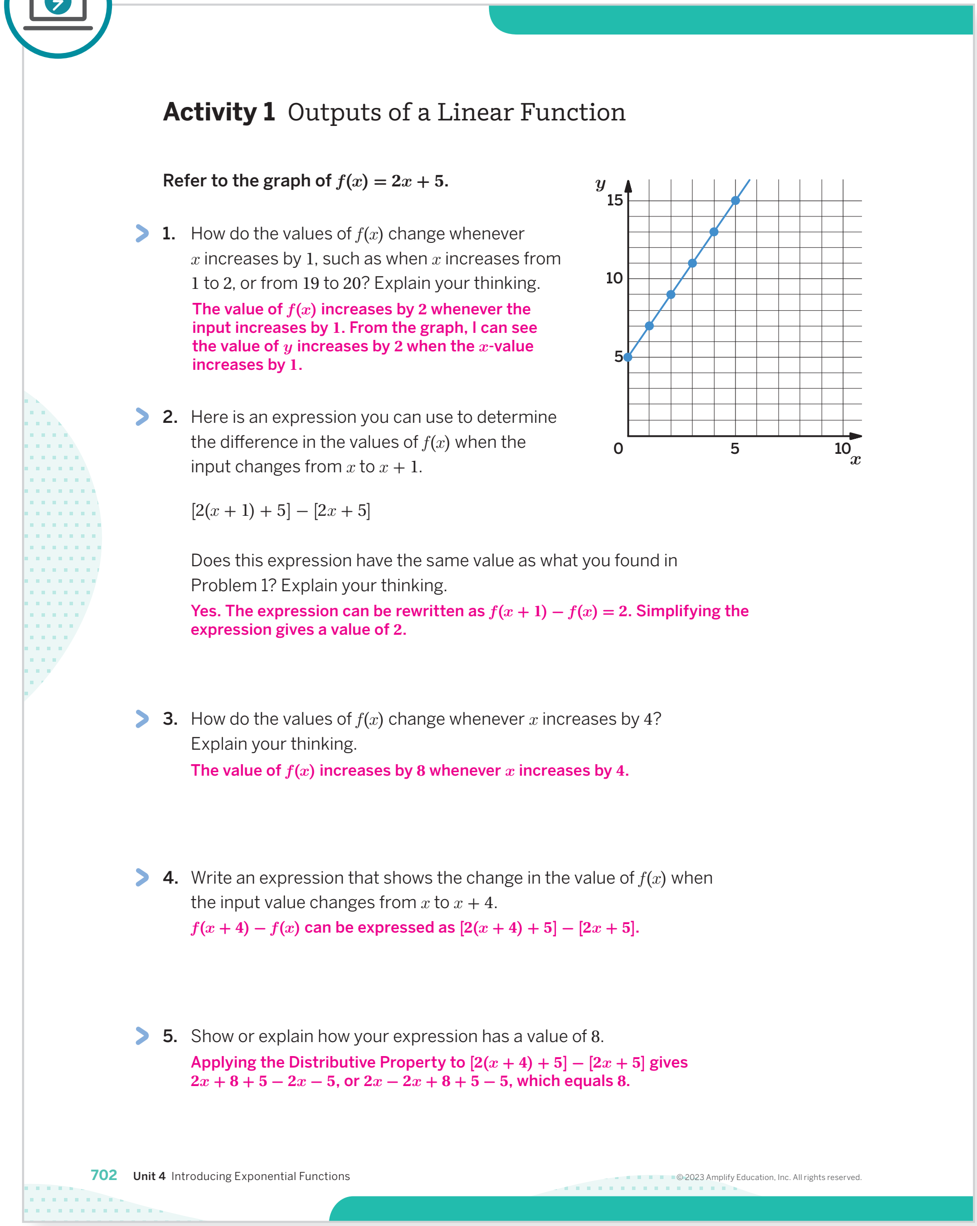
Use: Before the Warm-up

Informed by: Performance on Lesson 20, Practice Problem 6

Pairs | 10 min

Activity 1 Outputs of a Linear Function

Students extend their understanding of the rate of change for a linear function and use the structure of expressions to show that $f(x+1)-f(x)$ has the same value as the slope of the function.



Activity 1 Outputs of a Linear Function

Refer to the graph of $f(x) = 2x + 5$.

1. How do the values of $f(x)$ change whenever x increases by 1, such as when x increases from 1 to 2, or from 19 to 20? Explain your thinking.
   The value of $f(x)$ increases by 2 whenever the input increases by 1. From the graph, I can see the value of y increases by 2 when the x-value increases by 1.

2. Here is an expression you can use to determine the difference in the values of $f(x)$ when the input changes from x to $x+1$.

   $[2(x+1)+5]-[2x+5]$

   Does this expression have the same value as what you found in Problem 1? Explain your thinking.
   Yes. The expression can be rewritten as $f(x+1)-f(x)=2$. Simplifying the expression gives a value of 2.

3. How do the values of $f(x)$ change whenever x increases by 4? Explain your thinking.
   The value of $f(x)$ increases by 8 whenever x increases by 4.

4. Write an expression that shows the change in the value of $f(x)$ when the input value changes from x to $x+4$.
   $f(x+4)-f(x)$ can be expressed as $[2(x+4)+5]-[2x+5]$.

5. Show or explain how your expression has a value of 8.
   Applying the Distributive Property to $[2(x+4)+5]-[2x+5]$ gives $2x+8+5-2x-5$, or $2x-2x+8+5-5$, which equals 8.

702 **Unit 4** Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students complete Problem 1 with a partner. Then facilitate a class discussion about Problem 1 before having pairs continue with the activity.

2 Monitor

Help students get started by asking, "What are some ways to determine the slope of a line?"

Look for points of confusion:

- **Struggling to make sense of the expression in Problem 2.** Have students substitute the expressions x and $x+1$ with numerical expressions.
- **Having difficulty determining how the values of $f(x)$ change whenever x increases by 4 in Problem 3.** Have students try several pairs of values to determine a pattern.

Look for productive strategies:

- Using slope triangles for Problem 1.

3 Connect

Have students share their responses to the remaining problems.

Ask:

- "Can you find an example when the output of $f(x)$ does not increase by 2 when x increases by 1?" No, linear graphs have a constant rate of change.
- "Can you find an example where the input increases by 4, but the output $f(x)$ does not increase by 8? Explain your thinking." No, if increasing the input by 1 always changes the output by 2, then increasing by 1 four times always changes the output by $4 \cdot 2 = 8$.

Highlight that for any linear function, when x increases by an equal amount, the output changes by an equal amount.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students color code the expression in Problem 2 so that x is in one color and $(x+1)$ is in another color. This should help them view $(x+1)$ as one entity that is substituted for x in the expression $2x+5$.

Extension: *Math Enrichment*

Have students complete the following problem:

For $g(x) = hx + 5$, where h is a constant, how do the values of $g(x)$ change whenever x increases by p, where p is a number? $h \cdot p$

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses and respond to the Ask questions, draw their attention to the connections between the slope of a linear function and simplifying the expressions that contain successive differences.

Linear function: $f(x) = mx + b$

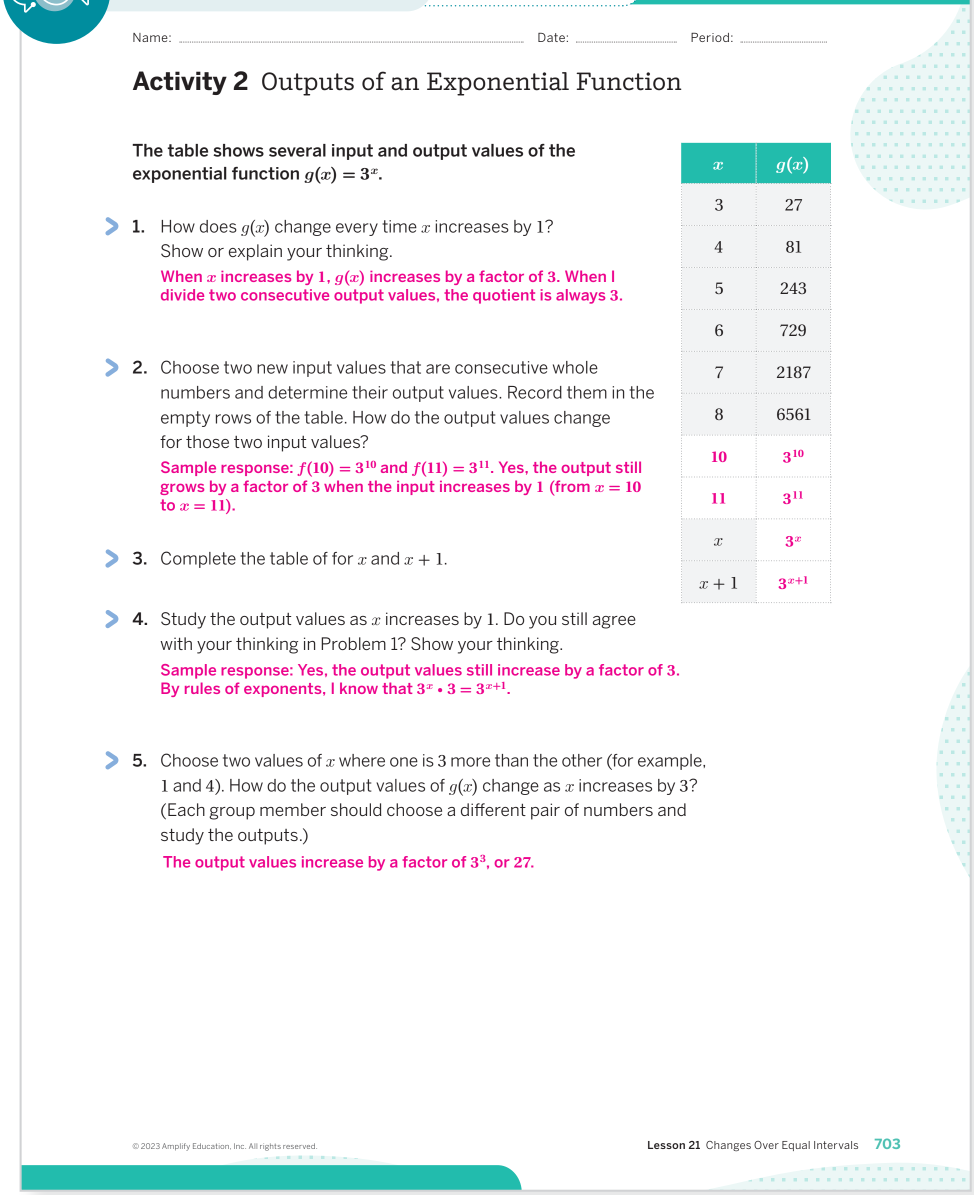
Slope:	Expression containing successive differences:
m	$[m(x+1)+b]-\lvert mx+b\rvert$ $= mx + m + b - mx - b$ $= m$

Independent | 15 min

Activity 2 Outputs of an Exponential Function

Students extend their understanding of the growth factor of an exponential function and use the structure of expressions to show the ratio of $f(x+1)$ to $f(x)$ has the same value as the growth factor.

Amps Featured Activity Interactive Table



Name: ________ Date: ________ Period: ________

Activity 2 Outputs of an Exponential Function

The table shows several input and output values of the exponential function $g(x) = 3^x$.

x	$g(x)$
3	27
4	81
5	243
6	729
7	2187
8	6561
10	$\mathbf{3^{10}}$
11	$\mathbf{3^{11}}$
x	$\mathbf{3^x}$
$x+1$	$\mathbf{3^{x+1}}$

1. How does $g(x)$ change every time x increases by 1? Show or explain your thinking.

When x increases by 1, $g(x)$ increases by a factor of 3. When I divide two consecutive output values, the quotient is always 3.

2. Choose two new input values that are consecutive whole numbers and determine their output values. Record them in the empty rows of the table. How do the output values change for those two input values?

Sample response: $f(10) = 3^{10}$ and $f(11) = 3^{11}$. Yes, the output still grows by a factor of 3 when the input increases by 1 (from $x = 10$ to $x = 11$).

3. Complete the table of for x and $x + 1$.

4. Study the output values as x increases by 1. Do you still agree with your thinking in Problem 1? Show your thinking.

Sample response: Yes, the output values still increase by a factor of 3. By rules of exponents, I know that $3^x \cdot 3 = 3^{x+1}$.

5. Choose two values of x where one is 3 more than the other (for example, 1 and 4). How do the output values of $g(x)$ change as x increases by 3? (Each group member should choose a different pair of numbers and study the outputs.)

The output values increase by a factor of 3^3, or 27.

Lesson 21 Changes Over Equal Intervals 703

1 Launch

Have students work independently on Problems 1–4 and then in pairs for Problems 5 and 6.

2 Monitor

Help students get started by asking, "How could you determine what each output value is multiplied by to get the next output value?" Divide an output value by the previous output value in the table.

Look for points of confusion:

- **Treating the table as a linear relationship.** Have students use the *guess-and-check* strategy to determine a common factor.
- **Struggling to simplify $\frac{3^{x+1}}{3^x}$.** Remind students that when dividing powers with the same base, they subtract the exponents.

Look for productive strategies:

- Dividing consecutive $g(x)$ terms to determine the growth factor.

Activity 2 continued >

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Display the expression $b^x \cdot b = ?$ to activate students' prior knowledge about the product rule for exponents and to support their information processing for Problem 4.

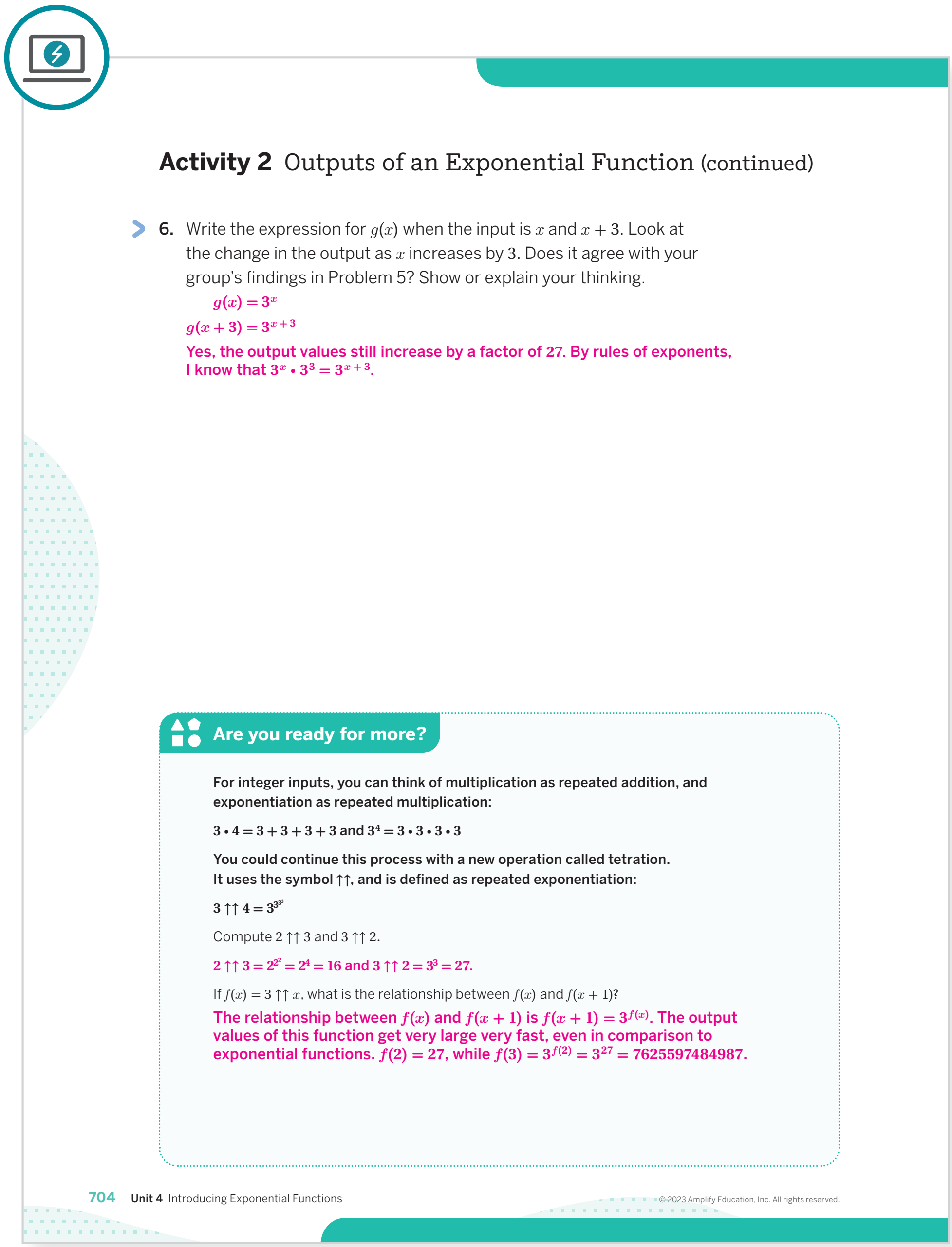
Ask:

- "Is there an exponent on the second factor?"
- "What does the product rule for exponents tell you about the powers that are being multiplied?"

Independent | 15 min

Activity 2 Outputs of an Exponential Function (continued)

Students extend their understanding of the growth factor of an exponential function and use the structure of expressions to show the ratio of $f(x+1)$ to $f(x)$ has the same value as the growth factor.



Activity 2 Outputs of an Exponential Function (continued)

6. Write the expression for $g(x)$ when the input is x and $x+3$. Look at the change in the output as x increases by 3. Does it agree with your group's findings in Problem 5? Show or explain your thinking.

$g(x) = 3^x$

$g(x+3) = 3^{x+3}$

Yes, the output values still increase by a factor of 27. By rules of exponents, I know that $3^x \cdot 3^3 = 3^{x+3}$.

Are you ready for more?

For integer inputs, you can think of multiplication as repeated addition, and exponentiation as repeated multiplication:

$3 \cdot 4 = 3+3+3+3$ **and** $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$

You could continue this process with a new operation called tetration.
It uses the symbol ↑↑, and is defined as repeated exponentiation:

$3 \uparrow\uparrow 4 = 3^{3^{3^3}}$

Compute 2 ↑↑ 3 and 3 ↑↑ 2.

$2 \uparrow\uparrow 3 = 2^{2^2} = 2^4 = 16$ and $3 \uparrow\uparrow 2 = 3^3 = 27$.

If $f(x) = 3 \uparrow\uparrow x$, what is the relationship between $f(x)$ and $f(x+1)$?

The relationship between $f(x)$ and $f(x+1)$ is $f(x+1) = 3^{f(x)}$. The output values of this function get very large very fast, even in comparison to exponential functions. $f(2) = 27$, while $f(3) = 3^{f(2)} = 3^{27} = 7625597484987$.

3 Connect

Display the completed table to help students with Problem 6.

Have students share what they noticed about the values of g for consecutive whole numbers.

Ask:

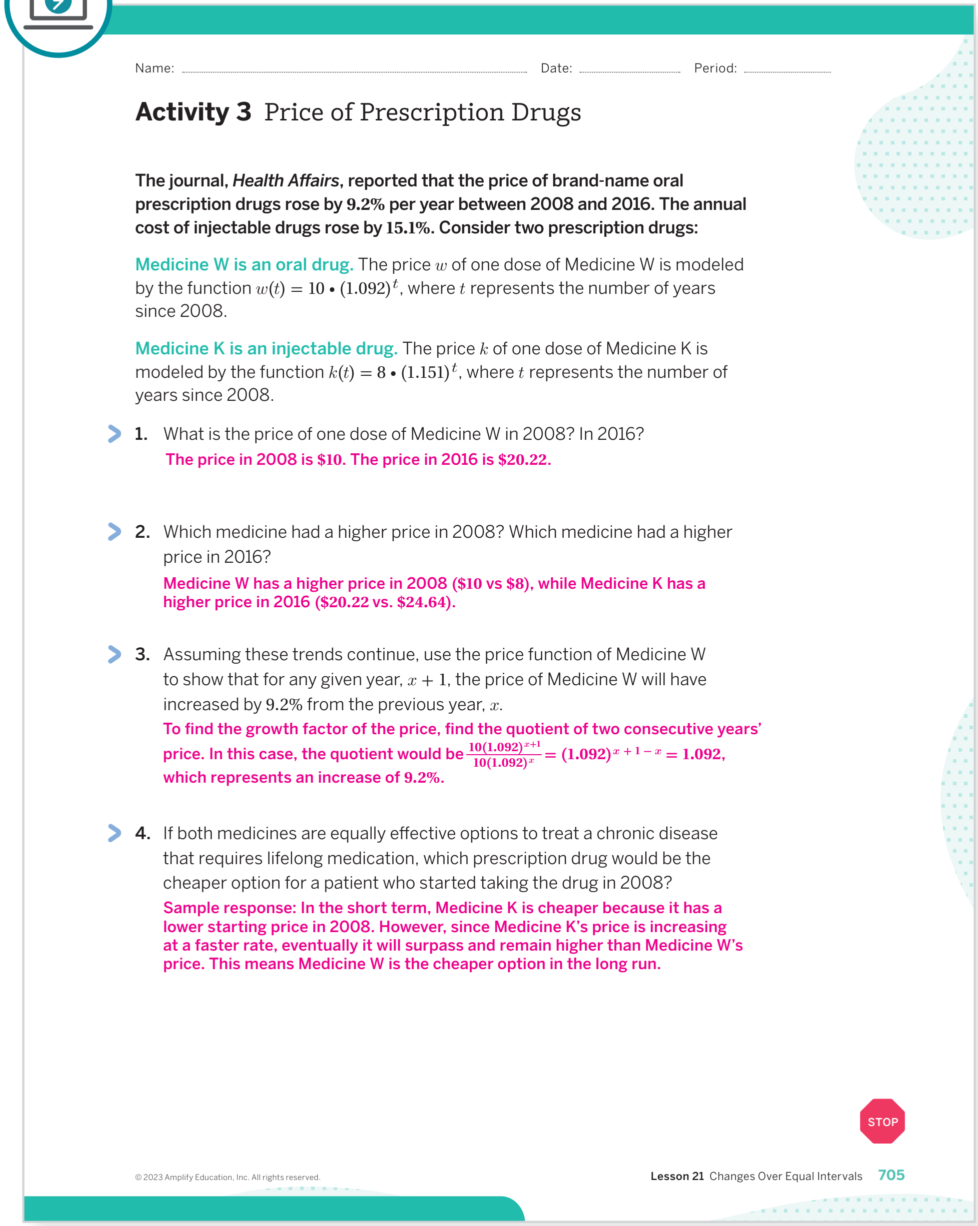
- "How are the output values of $g(x+1)$ and $g(x)$ related?" $g(x+1)$ is 3 times $g(x)$.
- "What happens to the output value of g when x increases by 2?" $g(x)$ increases by a factor of 3 twice, or 3^2.

Highlight that as the value of x increases by 1, the output value changes by a factor of 3.

Independent | 10 min

Activity 3 Price of Prescription Drugs

Students apply their understandings of the structure of expressions from prior activities to examine increasing drug prices, and decide which of two drugs is the cheaper option for lifelong treatment.



Name: ________ Date: ________ Period: ________

Activity 3 Price of Prescription Drugs

The journal, *Health Affairs*, reported that the price of brand-name oral prescription drugs rose by 9.2% per year between 2008 and 2016. The annual cost of injectable drugs rose by 15.1%. Consider two prescription drugs:

Medicine W is an oral drug. The price w of one dose of Medicine W is modeled by the function $w(t) = 10 \cdot (1.092)^t$, where t represents the number of years since 2008.

Medicine K is an injectable drug. The price k of one dose of Medicine K is modeled by the function $k(t) = 8 \cdot (1.151)^t$, where t represents the number of years since 2008.

1. What is the price of one dose of Medicine W in 2008? In 2016?
   The price in 2008 is \$10. The price in 2016 is \$20.22.

2. Which medicine had a higher price in 2008? Which medicine had a higher price in 2016?
   Medicine W has a higher price in 2008 (\$10 vs \$8), while Medicine K has a higher price in 2016 (\$20.22 vs. \$24.64).

3. Assuming these trends continue, use the price function of Medicine W to show that for any given year, $x + 1$, the price of Medicine W will have increased by 9.2% from the previous year, x.
   To find the growth factor of the price, find the quotient of two consecutive years' price. In this case, the quotient would be $\frac{10(1.092)^{x+1}}{10(1.092)^x} = (1.092)^{x+1-x} = 1.092$, which represents an increase of 9.2%.

4. If both medicines are equally effective options to treat a chronic disease that requires lifelong medication, which prescription drug would be the cheaper option for a patient who started taking the drug in 2008?
   Sample response: In the short term, Medicine K is cheaper because it has a lower starting price in 2008. However, since Medicine K's price is increasing at a faster rate, eventually it will surpass and remain higher than Medicine W's price. This means Medicine W is the cheaper option in the long run.

STOP

Lesson 21 Changes Over Equal Intervals 705

1 Launch

Say, "Health care costs and prescription drug prices continue to skyrocket in the U.S. You are going to examine the recent price increases of two types of prescription drugs." Provide access to scientific calculators.

2 Monitor

Help students get started by asking, "What is the price of each medicine in 2008?"

Look for points of confusion:

- **Substituting the year for t.** Remind students that t represents the number of years since 2008.
- **Struggling to set up and simplify the expression for Problem 3.** Refer students back to Activity 2, Problem 6 to see how they created the quotient and used the rules of exponents.

Look for productive strategies:

- Calculating medicine prices for several years to help find trends.

3 Connect

Display the description of Medicine W and Medicine K.

Have students share their responses to Problems 3 and 4.

Highlight that in Problem 3, the numerator and denominator of $\frac{1.092^{x+1}}{1.092^x}$ have the same base, so students should subtract the exponents.

Ask:

- "What should you consider when selecting the cheaper option?" The initial price, rate of growth, and length of treatment.
- "What could change and make Medicine W the cheaper option?" If it grew at a lower rate, or the treatment was for a short period of time.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students create a table to help them with Problems 1 and 2. Consider providing them with a table, similar to the following:

Year	t	Medicine W	Medicine K
2008			
2016			

Accessibility: *Activate Prior Knowledge*

Display the expression shown to activate students' prior knowledge about the quotient rule for exponents and to support their information processing for Problem 3.

$$\frac{a \cdot b^{x+1}}{a \cdot b^x} = ?$$

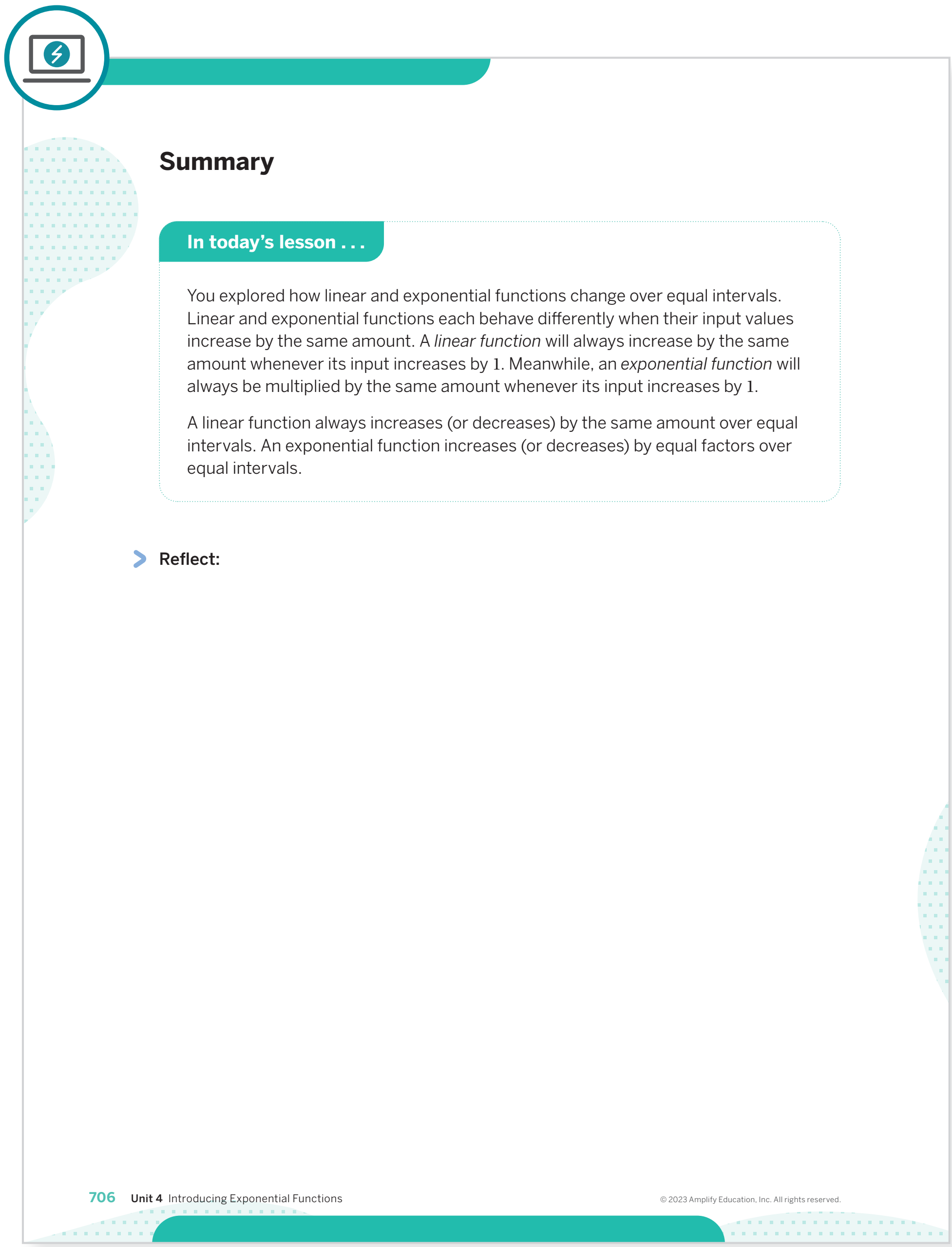
Ask:

- "What happens to a during this division process?"
- "What does the quotient rule for exponents tell you about the powers that are being divided?"

Whole Class | 5 min

Summary

Review and synthesize how linear and exponential functions compare, as the input changes over equal intervals.



Summary

In today's lesson . . .

You explored how linear and exponential functions change over equal intervals. Linear and exponential functions each behave differently when their input values increase by the same amount. A *linear function* will always increase by the same amount whenever its input increases by 1. Meanwhile, an *exponential function* will always be multiplied by the same amount whenever its input increases by 1.

A linear function always increases (or decreases) by the same amount over equal intervals. An exponential function increases (or decreases) by equal factors over equal intervals.

Reflect:

706 Unit 4 Introducing Exponential Functions

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the functions $f(x) = 3x + 2$ and $g(x) = 2 \cdot 3^x$ and their graphs. Say, "Previously, you learned that an exponential function eventually overtakes a linear function. Let's examine how these functions change when their input values change. Let's look at two functions, $f(x) = 3x + 2$ and $g(x) = 2 \cdot 3^x$."

Have students share the difference and similarities they notice between determining the change in linear functions versus exponential functions.

Highlight that students can use the slope of linear functions and the growth factor of exponential functions to determine the growth of functions over a given interval.

Ask, "How does each function $f(x)$ and $g(x)$ change when x increases by 2? By 5? By 10? When x increases by 2, f grows by $3 \cdot 2$ or 6, and g grows by a factor 3^2 or 9. Any time x increases by 5, the value of f grows by $3 \cdot 5$ or 15, but the value of g grows by a factor of 3^5 or 243. When x increases by 10, f increases by 30 and g increases by 3^{10}, or $59{,}049$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

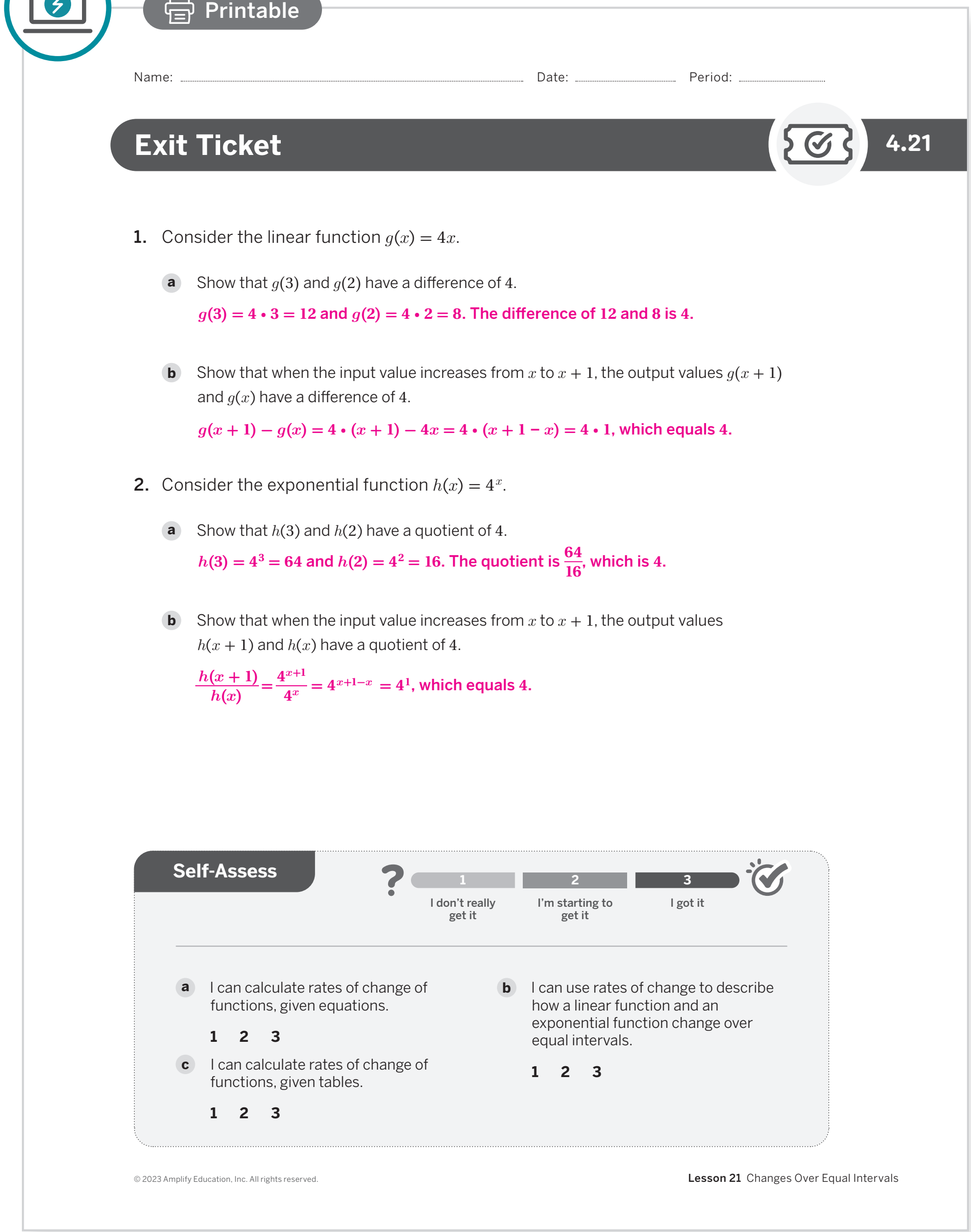
- "How can you differentiate exponential growth from linear growth, given a real-world data set?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using the structure of expressions to show how linear and exponential functions change as the input value changes over equal intervals.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.21

1. Consider the linear function $g(x) = 4x$.

a Show that $g(3)$ and $g(2)$ have a difference of 4.

$g(3) = 4 \cdot 3 = 12$ and $g(2) = 4 \cdot 2 = 8$. The difference of 12 and 8 is 4.

b Show that when the input value increases from x to $x + 1$, the output values $g(x + 1)$ and $g(x)$ have a difference of 4.

$g(x + 1) - g(x) = 4 \cdot (x + 1) - 4x = 4 \cdot (x + 1 - x) = 4 \cdot 1$, which equals 4.

2. Consider the exponential function $h(x) = 4^x$.

a Show that $h(3)$ and $h(2)$ have a quotient of 4.

$h(3) = 4^3 = 64$ and $h(2) = 4^2 = 16$. The quotient is $\frac{64}{16}$, which is 4.

b Show that when the input value increases from x to $x + 1$, the output values $h(x + 1)$ and $h(x)$ have a quotient of 4.

$\frac{h(x+1)}{h(x)} = \frac{4^{x+1}}{4^x} = 4^{x+1-x} = 4^1$, which equals 4.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can calculate rates of change of functions, given equations.

1 2 3

b I can use rates of change to describe how a linear function and an exponential function change over equal intervals.

1 2 3

c I can calculate rates of change of functions, given tables.

1 2 3

Lesson 21 Changes Over Equal Intervals

Success looks like . . .

- **Goal:** Calculating rates of change of functions given graphs, equations, or tables.
  - » Calculating the rates of change for both functions in Problems 1 and 2.
- **Language Goal:** Using rates of change to describe how a linear function and an exponential function change over equal intervals. **(Speaking and Listening, Writing)**
  - » Calculating rates of change from x to $x + 1$ for a linear function in Problem 1b and for an exponential function in Problem 2b.

Suggested next steps

If students make a procedural error in Problem 1a and/or 2a, consider:

- Reviewing calculations and the order of operations in Activity 1, Problem 1.

If students incorrectly show that $g(x + 1)$ and $g(x)$ have a difference of 4 for Problem 1b, consider:

- Asking, "What does the function notation $g(x + 1)$ represent?"
- Reviewing how to substitute an expression in for x in Activity 1, Problem 3.

If students incorrectly show that $h(x + 1)$ and $h(x)$ have a quotient of 4 for Problem 2b, consider:

- Asking, "What does the function notation $h(x + 1)$ represent?"
- Reviewing how to substitute an expression in for x in Activity 1, Problem 3.
- Reviewing Activity 2, Problems 3 and 4.

Professional Learning

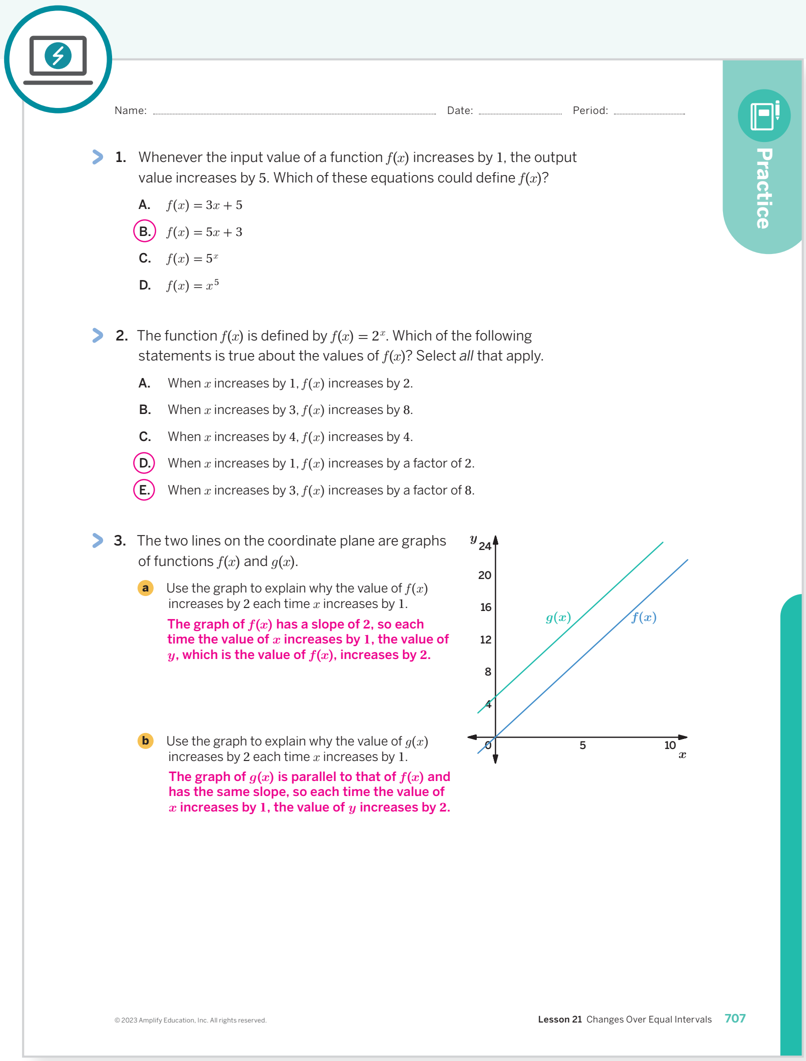
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students examine how linear functions change by equal differences over equal intervals and exponential functions change by equal factors over equal intervals. How does this build on their understanding of common differences and common factors from the beginning of the unit, and their understanding of growth factors of exponential functions they explored throughout this unit?
- How well do you think your students understood how to relate the value of a linear or exponential function when the input value changes from x to $x + 1$ in Activities 1 and 2 to the slope (linear) or growth factor (exponential)? Which questions might you change the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Whenever the input value of a function $f(x)$ increases by 1, the output value increases by 5. Which of these equations could define $f(x)$?
   - A. $f(x) = 3x + 5$
   - [✓ B.] $f(x) = 5x + 3$
   - C. $f(x) = 5^x$
   - D. $f(x) = x^5$

2. The function $f(x)$ is defined by $f(x) = 2^x$. Which of the following statements is true about the values of $f(x)$? Select *all* that apply.
   - A. When x increases by 1, $f(x)$ increases by 2.
   - B. When x increases by 3, $f(x)$ increases by 8.
   - C. When x increases by 4, $f(x)$ increases by 4.
   - [✓ D.] When x increases by 1, $f(x)$ increases by a factor of 2.
   - [✓ E.] When x increases by 3, $f(x)$ increases by a factor of 8.

3. The two lines on the coordinate plane are graphs of functions $f(x)$ and $g(x)$.

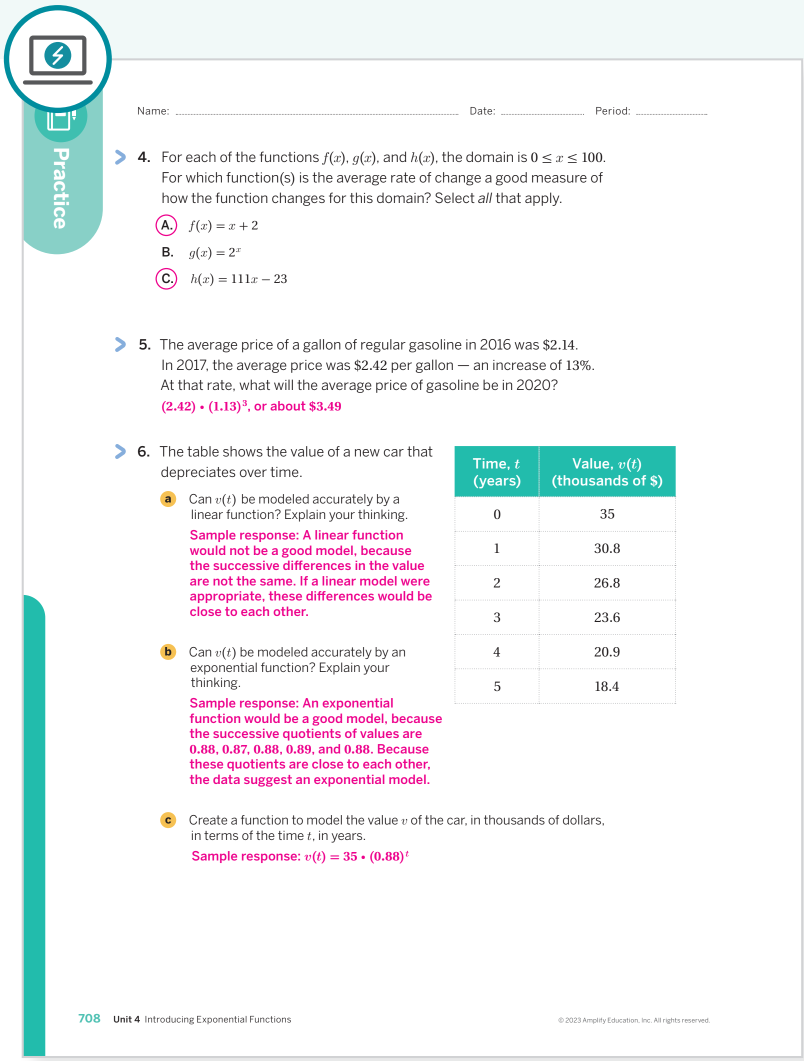
   a. Use the graph to explain why the value of $f(x)$ increases by 2 each time x increases by 1.

   The graph of $f(x)$ has a slope of 2, so each time the value of x increases by 1, the value of y, which is the value of $f(x)$, increases by 2.

   b. Use the graph to explain why the value of $g(x)$ increases by 2 each time x increases by 1.

   The graph of $g(x)$ is parallel to that of $f(x)$ and has the same slope, so each time the value of x increases by 1, the value of y increases by 2.

Lesson 21 Changes Over Equal Intervals 707



Name: Date: Period:

4. For each of the functions $f(x)$, $g(x)$, and $h(x)$, the domain is $0 \le x \le 100$. For which function(s) is the average rate of change a good measure of how the function changes for this domain? Select *all* that apply.
   - [✓ A.] $f(x) = x + 2$
   - B. $g(x) = 2^x$
   - [✓ C.] $h(x) = 111x - 23$

5. The average price of a gallon of regular gasoline in 2016 was $2.14. In 2017, the average price was $2.42 per gallon — an increase of 13%. At that rate, what will the average price of gasoline be in 2020?

   $(2.42) \cdot (1.13)^3$, or about $3.49

6. The table shows the value of a new car that depreciates over time.

Time, t (years)	Value, $v(t)$ (thousands of $)
0	35
1	30.8
2	26.8
3	23.6
4	20.9
5	18.4

   a. Can $v(t)$ be modeled accurately by a linear function? Explain your thinking.

   Sample response: A linear function would not be a good model, because the successive differences in the value are not the same. If a linear model were appropriate, these differences would be close to each other.

   b. Can $v(t)$ be modeled accurately by an exponential function? Explain your thinking.

   Sample response: An exponential function would be a good model, because the successive quotients of values are 0.88, 0.87, 0.88, 0.89, and 0.88. Because these quotients are close to each other, the data suggest an exponential model.

   c. Create a function to model the value v of the car, in thousands of dollars, in terms of the time t, in years.

   Sample response: $v(t) = 35 \cdot (0.88)^t$

708 **Unit 4** Introducing Exponential Functions

Practice Problem Analysis

Type	Problem	Activity	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 14	2
	5	Unit 4 Lesson 16	1
Formative	6	Unit 4 Lesson 22	3

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 4 | **LESSON 22 – CAPSTONE**

COVID-19

Let's construct a linear or exponential model to represent the spread of COVID-19 and use our model to make predictions.

Focus

Goals

1. **Language Goal:** Determine whether a linear or exponential model is most appropriate for a set of real-world data and justify the selection. **(Speaking and Listening, Writing)**
2. Construct a function to model a set of real-world data and use the function to make predictions.

Coherence

Today

In this capstone lesson, students will apply the skills and concepts they have learned about exponential functions in this unit to analyze data sets related to the spread of COVID-19. They will determine whether a linear or exponential function models the data, construct a function, and use it to make predictions.

Previously

Students have studied exponential functions throughout the course of this unit.

Coming Soon

In the next two units, students will study another non-linear function, the quadratic function. They will compare quadratic growth to linear and exponential growth, construct quadratic functions to model real-world phenomena, describe key features of the graphs and equations of quadratic functions, and explore different ways of solving quadratic equations.

Rigor

- Students **apply** what they have learned in this unit to the context of the COVID-19 pandemic, modeling the spread of and vaccination against the disease.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils (as needed)

Math Language Development

Review words

- *exponential function*
- *average rate of change*

Amps | **Featured Activity**

Activity 1
Modeling a Pandemic

Students fit an exponential curve to real data related to the spread of COVID-19 in the U.S. From this, they can infer the growth rate and make projections.

Building Math Identity and Community

Connecting to Mathematical Practices

While interpreting the data about the global spread of disease, some students might be focused on the data and the mathematics and say something that is callous or inconsiderate of others. Before the activity, remind students to always be sensitive to those around them. They need to take on the perspective of someone who has had a different experience with the global spread of disease. Explain that behind the data are people who deserve empathy and respect.

Modifications to Pacing

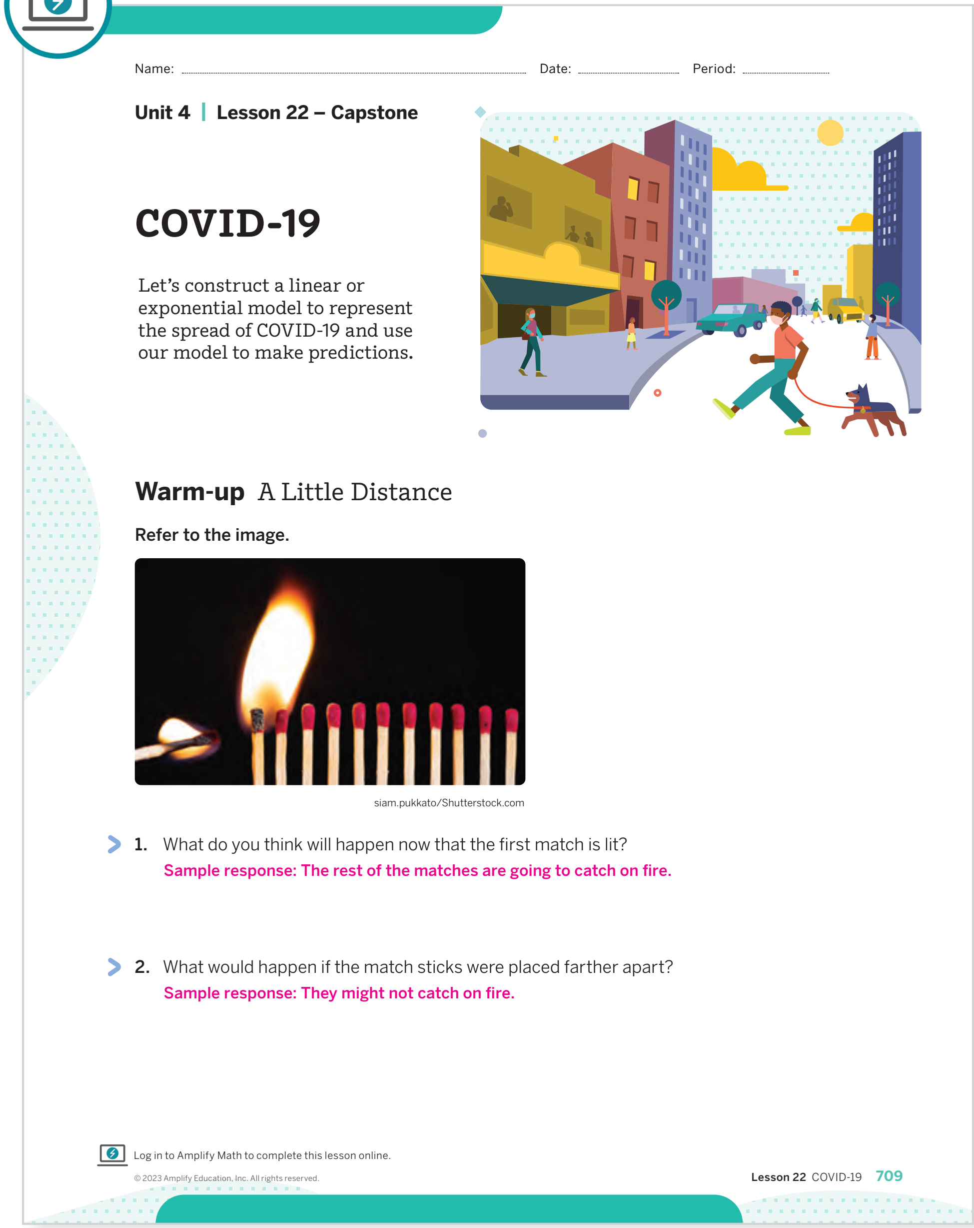
You may want to consider this additional modification if you are short on time.

- Omit **Activity 2**, in which students analyze the initial spread of COVID-19 by race and ethnicity.

Independent | 5 min

Warm-up A Little Distance

Students consider a photo of matches where one match is lit as a metaphor of how distance can lessen the spread of the fire. This will prepare them for studying the spread of COVID-19 in the upcoming activities.



Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 22 – Capstone

COVID-19

Let's construct a linear or exponential model to represent the spread of COVID-19 and use our model to make predictions.

Warm-up A Little Distance

Refer to the image.

siam.pukkato/Shutterstock.com

1. What do you think will happen now that the first match is lit?
   Sample response: The rest of the matches are going to catch on fire.

2. What would happen if the match sticks were placed farther apart?
   Sample response: They might not catch on fire.

Log in to Amplify Math to complete this lesson online.

1 Launch

Display the photo and conduct the ***Notice and Wonder*** routine before displaying Problems 1 and 2.

2 Monitor

Help students get started by having them describe the photo in their own words.

Look for points of confusion:

- **Not connecting distance with the spread of the fire in Problem 2.** Have students draw a sketch of match sticks that are placed much farther apart. Ask, "If the first match is lit, how close do you think the next match needs to be in order to catch on fire?"

Look for productive strategies

- Recognizing that if the match sticks were placed farther apart, they might not all catch on fire.

3 Connect

Highlight how the photo illustrates the concept of how social distancing can lessen the spread of a virus, a situation with which students are likely to be familiar due to COVID-19.

Ask:

- "What does the fire represent in relation to the spread of COVID-19?" The virus.
- "What do the match sticks represent?" People.
- "How does placing the match sticks farther apart illustrate the effects of social distancing on the spread of the virus?" Sample response: Placing the match sticks farther apart (social distancing) lessens the likelihood that they will catch on fire (catch the virus) from the previous lit match.

Power-up

To power up students' ability to identify whether a table of values is best represented by a linear or exponential function, have students complete:

Recall that linear relationships have a constant rate of change, seen in repeated addition. Exponential relationships have a constant rate of growth, seen as repeated multiplication.

Identify which relationship is linear and which is exponential.

a

x	1	2	3
y	4	8	16

Exponential

b

x	1	2	3
y	4	8	12

Linear

Use: Before Activity 1

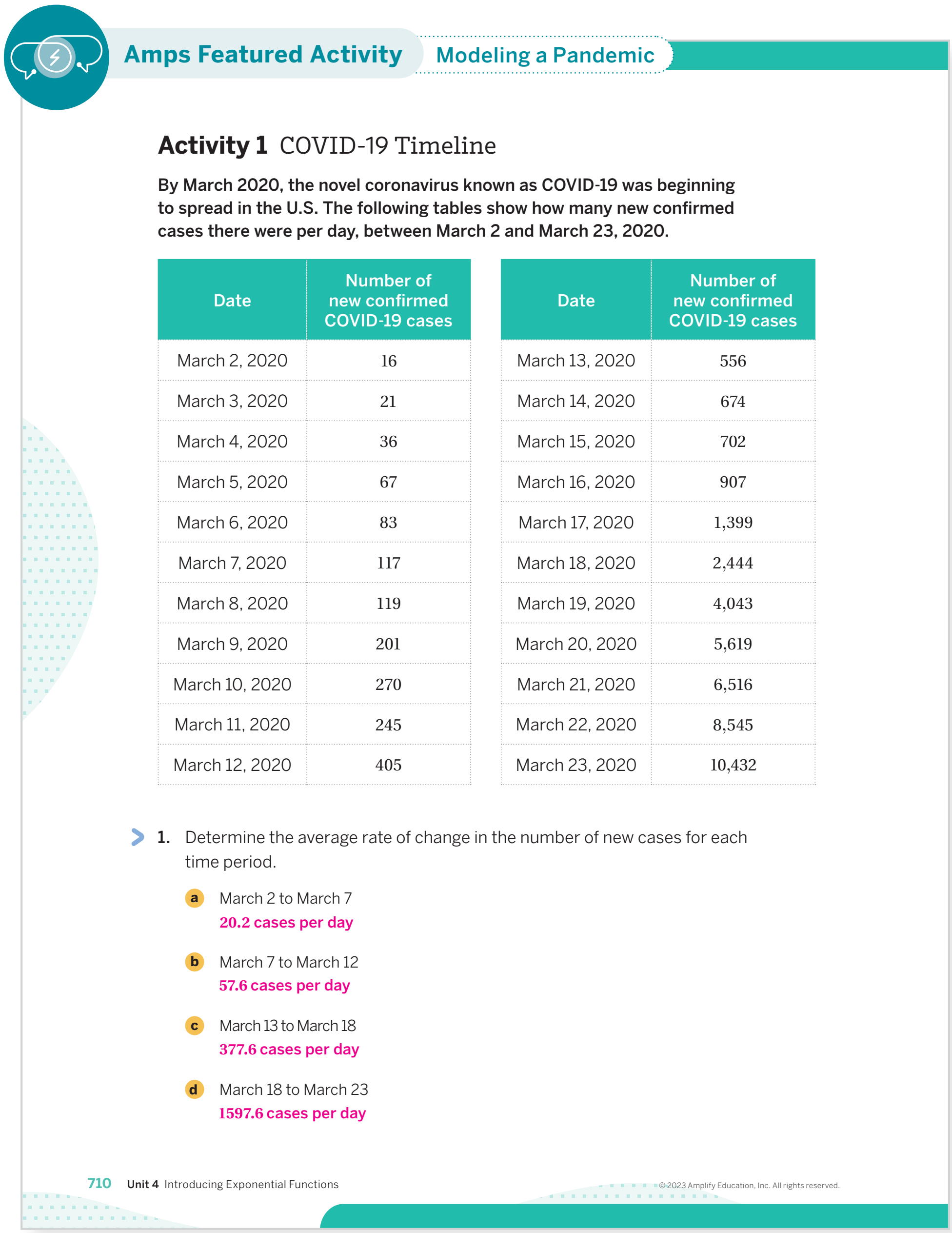
Informed by: Performance on Lesson 20, Practice Problem 6

Small Groups | 15 min

Activity 1 COVID-19 Timeline

Students apply the concepts of this unit to determine and construct an appropriate model to represent the spread of COVID-19, and use their model to make predictions.

Amps Featured Activity Modeling a Pandemic



Activity 1 COVID-19 Timeline

By March 2020, the novel coronavirus known as COVID-19 was beginning to spread in the U.S. The following tables show how many new confirmed cases there were per day, between March 2 and March 23, 2020.

Date	Number of new confirmed COVID-19 cases
March 2, 2020	16
March 3, 2020	21
March 4, 2020	36
March 5, 2020	67
March 6, 2020	83
March 7, 2020	117
March 8, 2020	119
March 9, 2020	201
March 10, 2020	270
March 11, 2020	245
March 12, 2020	405

Date	Number of new confirmed COVID-19 cases
March 13, 2020	556
March 14, 2020	674
March 15, 2020	702
March 16, 2020	907
March 17, 2020	1,399
March 18, 2020	2,444
March 19, 2020	4,043
March 20, 2020	5,619
March 21, 2020	6,516
March 22, 2020	8,545
March 23, 2020	10,432

1. Determine the average rate of change in the number of new cases for each time period.

   a. March 2 to March 7
   20.2 cases per day

   b. March 7 to March 12
   57.6 cases per day

   c. March 13 to March 18
   377.6 cases per day

   d. March 18 to March 23
   1597.6 cases per day

710 **Unit 4** Introducing Exponential Functions

1 Launch

Display the introductory text and table. Conduct the *Notice and Wonder* routine, asking students to describe what they notice about the data shown in the table, and what questions they have.

2 Monitor

Help students get started by revisiting how to calculate the average rate of change over a specified interval.

Look for points of confusion:

- **Having difficulty calculating the average rate of change.** Display the formula for the average rate of change over the domain interval (a, b): $\frac{f(b) - f(a)}{b - a}$
- **Thinking they should use a linear function to model the data because they determined the average rates of change.** Ask, "How is the average rate of change over a specified interval similar to and different from the slope of a line? If you determine an average rate of change, does this mean the data must be linear?"
- **Recognizing they should use an exponential function, but struggling to construct the function.** Ask, "What is the general form of an exponential function and what information does it convey? Can you use the table to determine that information?"

Look for productive strategies:

- Recognizing that an exponential function is more appropriate to model the data and determining the growth factor and initial value from the table.
- Using the number of new cases on March 2, 2020 as the initial value.
- Recognizing that the exponential model is a model and that there are limitations to any model (Problem 5).

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the data for each interval to help them respond to Problem 1. For example, for Problem 1a, suggest they color code the data for March 2 and March 7 with the same color.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 5, have groups meet with one other group to share their responses and to give and receive feedback. Encourage reviewers to ask clarifying questions such as:

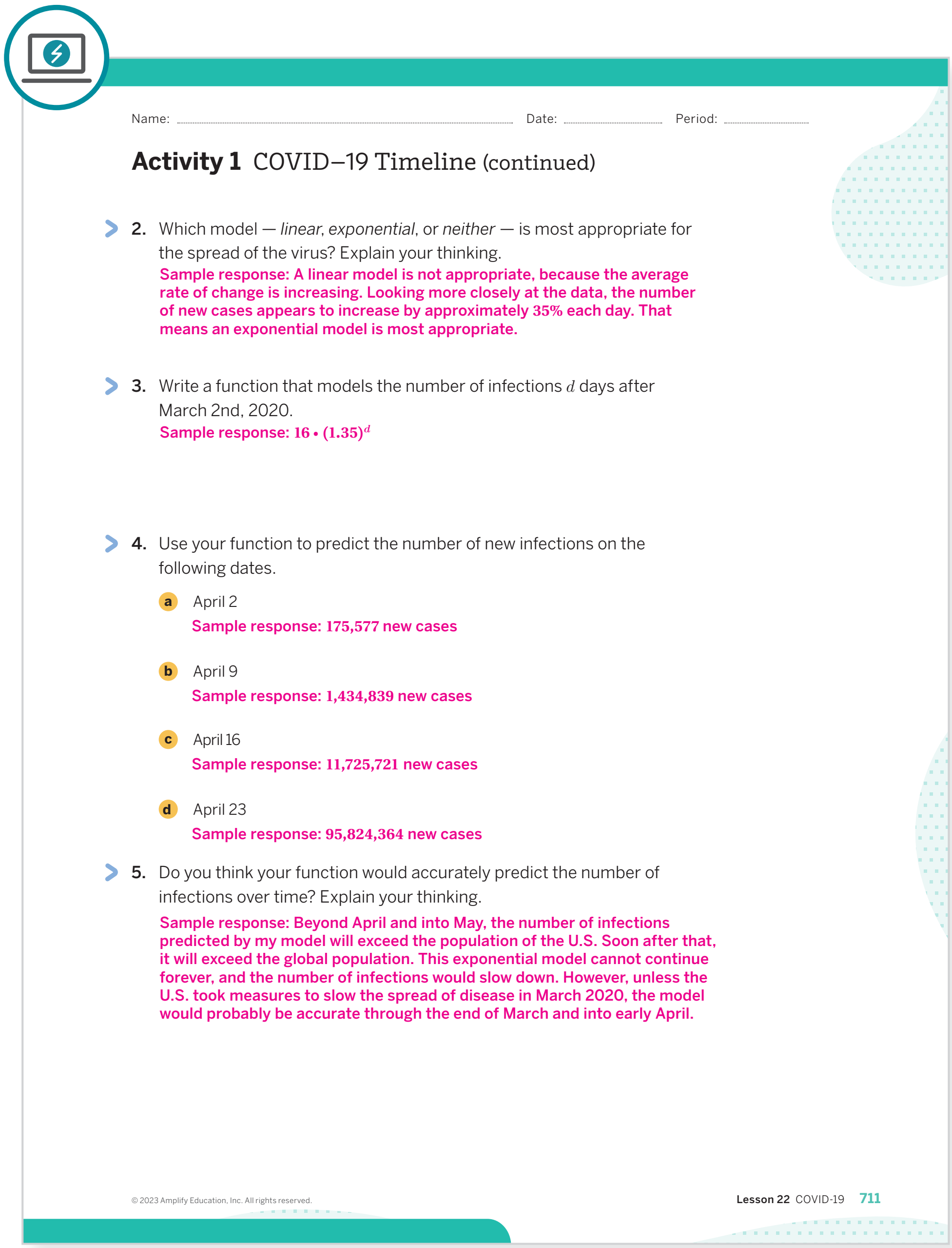
- "How did you construct your mathematical model? What type of function did you choose?"
- "What calculations did you use to arrive at your response?"
- "What mathematical language did you use in your response?"

Have students revise their response, based on the feedback they received.

Small Groups | 15 min

Activity 1 COVID-19 Timeline (continued)

Students apply the concepts of this unit to determine and construct an appropriate model to represent the spread of COVID-19, and use their model to make predictions.



Name: .... Date: .... Period: ....

Activity 1 COVID–19 Timeline (continued)

2. Which model — *linear*, *exponential*, or *neither* — is most appropriate for the spread of the virus? Explain your thinking.
   Sample response: A linear model is not appropriate, because the average rate of change is increasing. Looking more closely at the data, the number of new cases appears to increase by approximately 35% each day. That means an exponential model is most appropriate.

3. Write a function that models the number of infections d days after March 2nd, 2020.
   Sample response: $16 \cdot (1.35)^d$

4. Use your function to predict the number of new infections on the following dates.
   - a April 2
     Sample response: 175,577 new cases
   - b April 9
     Sample response: 1,434,839 new cases
   - c April 16
     Sample response: 11,725,721 new cases
   - d April 23
     Sample response: 95,824,364 new cases

5. Do you think your function would accurately predict the number of infections over time? Explain your thinking.
   Sample response: Beyond April and into May, the number of infections predicted by my model will exceed the population of the U.S. Soon after that, it will exceed the global population. This exponential model cannot continue forever, and the number of infections would slow down. However, unless the U.S. took measures to slow the spread of disease in March 2020, the model would probably be accurate through the end of March and into early April.

3 Connect

Have groups of students share whether they constructed a linear or exponential function and explain their thinking. Sequence responses by asking students who constructed linear functions to share first, followed by students who constructed exponential functions.

Display the different functions students constructed and the different predictions they made for the April dates.

Ask:

- "How are the functions that your classmates made similar? How are they different?"
- "How are the predictions that your classmates made similar? How are they different?"
- "What do you notice about the predictions made for April 23? Do you think this is reasonable? What might be other factors that affect the spread of the disease?"

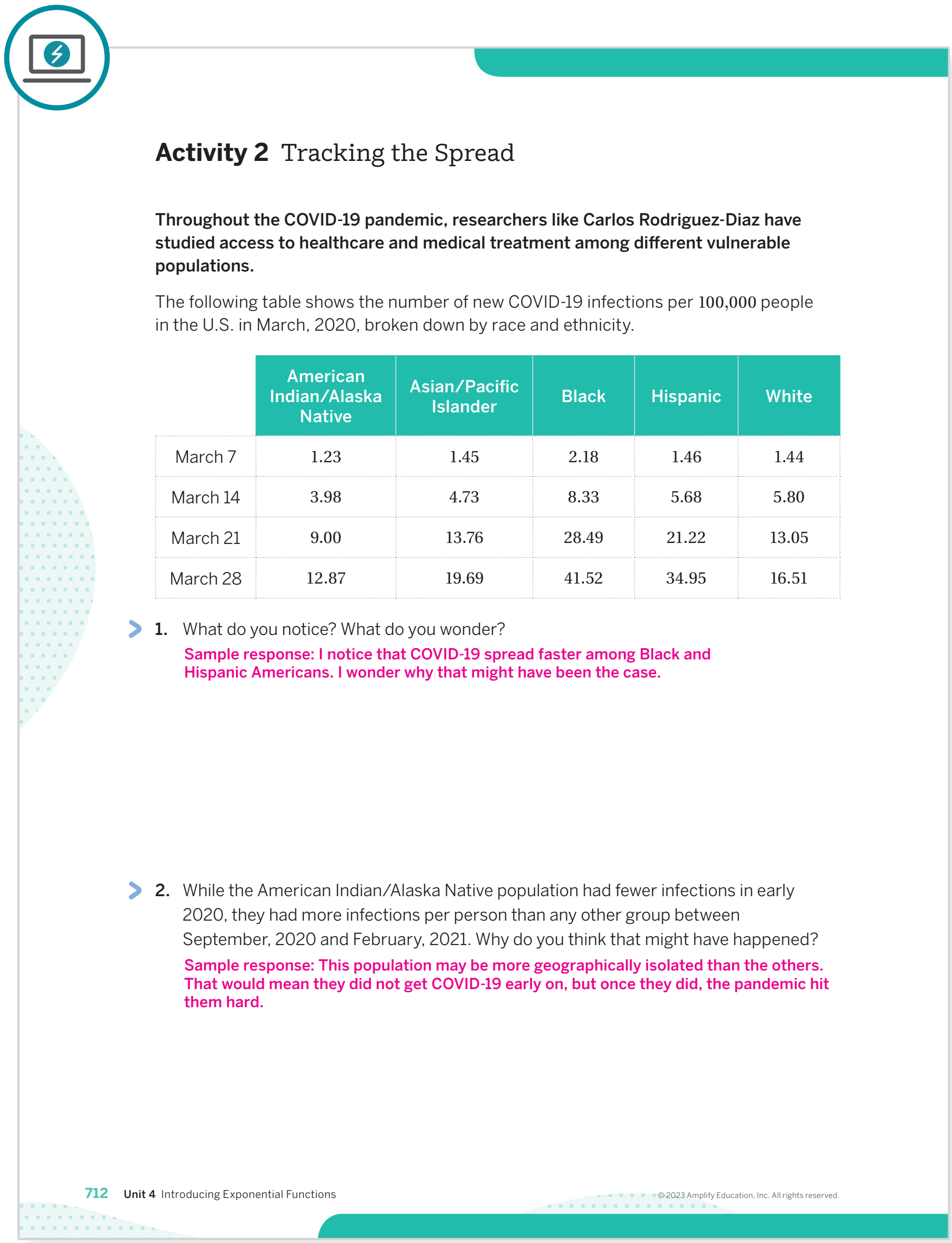
Have student volunteers share their responses to Problem 5.

Highlight that while mathematical modeling is a valuable tool to make predictions and help solve problems, real-world data is often messy. There are often other factors at play that affect the data, which may impose limits or assumptions on the mathematical model. Mathematicians, scientists, and researchers ask questions and study data to help determine these other factors.

Small Groups | 10 min

Activity 2 Tracking the Spread

Students study a set of data showing the spread of COVID-19 by race and ethnicity to recognize that there may be other factors at play that affect the spread of the virus.



Activity 2 Tracking the Spread

Throughout the COVID-19 pandemic, researchers like Carlos Rodriguez-Diaz have studied access to healthcare and medical treatment among different vulnerable populations.

The following table shows the number of new COVID-19 infections per 100,000 people in the U.S. in March, 2020, broken down by race and ethnicity.

	American Indian/Alaska Native	Asian/Pacific Islander	Black	Hispanic	White
March 7	1.23	1.45	2.18	1.46	1.44
March 14	3.98	4.73	8.33	5.68	5.80
March 21	9.00	13.76	28.49	21.22	13.05
March 28	12.87	19.69	41.52	34.95	16.51

1. What do you notice? What do you wonder?

   Sample response: I notice that COVID-19 spread faster among Black and Hispanic Americans. I wonder why that might have been the case.

2. While the American Indian/Alaska Native population had fewer infections in early 2020, they had more infections per person than any other group between September, 2020 and February, 2021. Why do you think that might have happened?

   Sample response: This population may be more geographically isolated than the others. That would mean they did not get COVID-19 early on, but once they did, the pandemic hit them hard.

1 Launch

Read the introductory text aloud and let students know they will analyze the data shown.

2 Monitor

Help students get started by asking them to study two of the columns at a time.

Look for points of confusion:

- **Struggling to note observations by looking at the data values.** Ask, "Look at the data values for March 28 for each race or ethnicity, compared to the data values for March 7. What do you notice?"
- **Thinking the data values represent the total number of new infections on each date.** Ask, "Look back at the sentence before the table. What does 'per 100,000 people' mean?"

Look for productive strategies:

- Calculating average rates of change from March 7 to March 28 for each column.
- Recognizing that the number of new infections for Blacks and Hispanics were significantly higher each week than the other columns.

3 Connect

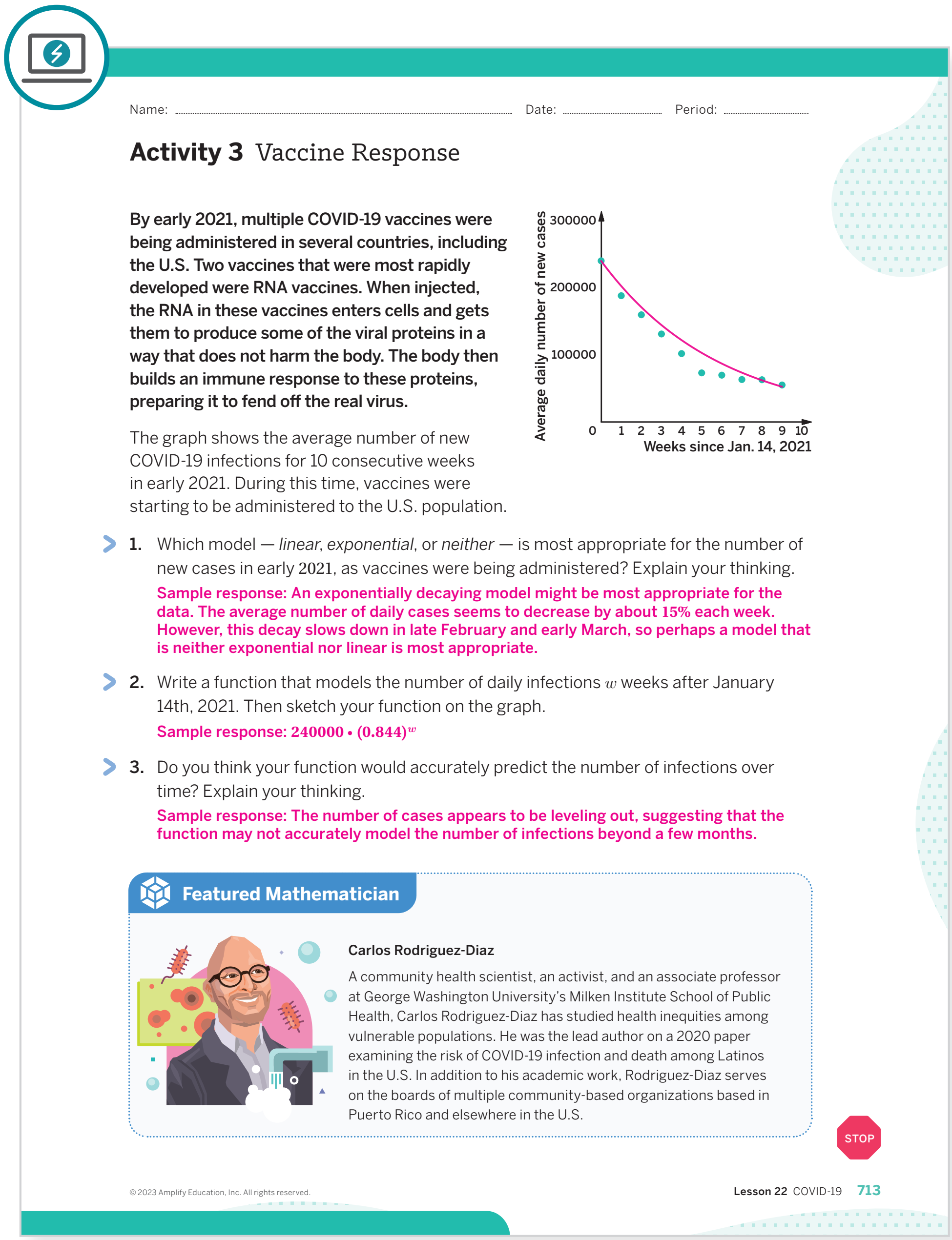
Have groups of students share what they noticed and wondered and their responses to Problem 2.

Highlight that scientists and researchers, such as Carlos Rodriguez-Diaz, study data sets similar to the one shown in the table to learn more about health inequities and how the spread of disease affects vulnerable populations.

Small Groups | 10 min

Activity 3 Vaccine Response

Students examine the effects of vaccines and construct a function to model the number of new infected cases over time.



Name: ________ Date: ________ Period: ________

Activity 3 Vaccine Response

By early 2021, multiple COVID-19 vaccines were being administered in several countries, including the U.S. Two vaccines that were most rapidly developed were RNA vaccines. When injected, the RNA in these vaccines enters cells and gets them to produce some of the viral proteins in a way that does not harm the body. The body then builds an immune response to these proteins, preparing it to fend off the real virus.

The graph shows the average number of new COVID-19 infections for 10 consecutive weeks in early 2021. During this time, vaccines were starting to be administered to the U.S. population.

1. Which model — *linear*, *exponential*, or *neither* — is most appropriate for the number of new cases in early 2021, as vaccines were being administered? Explain your thinking.

   Sample response: An exponentially decaying model might be most appropriate for the data. The average number of daily cases seems to decrease by about 15% each week. However, this decay slows down in late February and early March, so perhaps a model that is neither exponential nor linear is most appropriate.

2. Write a function that models the number of daily infections w weeks after January 14th, 2021. Then sketch your function on the graph.

   Sample response: $240000 \cdot (0.844)^w$

3. Do you think your function would accurately predict the number of infections over time? Explain your thinking.

   Sample response: The number of cases appears to be leveling out, suggesting that the function may not accurately model the number of infections beyond a few months.

Featured Mathematician

Carlos Rodriguez-Diaz

A community health scientist, an activist, and an associate professor at George Washington University's Milken Institute School of Public Health, Carlos Rodriguez-Diaz has studied health inequities among vulnerable populations. He was the lead author on a 2020 paper examining the risk of COVID-19 infection and death among Latinos in the U.S. In addition to his academic work, Rodriguez-Diaz serves on the boards of multiple community-based organizations based in Puerto Rico and elsewhere in the U.S.

STOP

Lesson 22 COVID-19 713

1 Launch

Display the introductory text and graph. Have students use the ***Co-craft Questions*** routine described in the Math Language Development section to help make sense of the information.

2 Monitor

Help students get started by asking them to describe what they think is happening to the data points on the graph as time progresses.

Look for points of confusion:

- **Thinking they should use a linear function to model the data because the first five data points seem to fall on a line or close to on a line.** Allow students to construct their linear model and then ask them how well they think their linear model fits all of the data shown.
- **Recognizing they should use an exponential function, but struggling to construct the function.** Ask, "Where do you think the initial value is shown on the graph? Does the graph show growth or decay?"

Look for productive strategies:

- Recognizing that an exponential function is more appropriate to model the data and determining the decay factor and initial value from the graph by examining selected points.
- Recognizing that the exponential model is a model and that there are limitations to any model (Problem 3).

3 Connect

Have groups of students share their models and explain their thinking behind their selections. Then have the groups share their responses to Problem 3.

Ask, "How are the functions that your classmates made similar? How are they different?"

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory text and graph and have students work with their groups to write 2–3 mathematical questions they could ask about the image. Sample questions shown.

- At what rate is the number of new infections decreasing?
- Can I use a linear or exponential model to represent this data?
- Will the number of new infections continue to decrease at this same rate indefinitely?

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Featured Mathematician

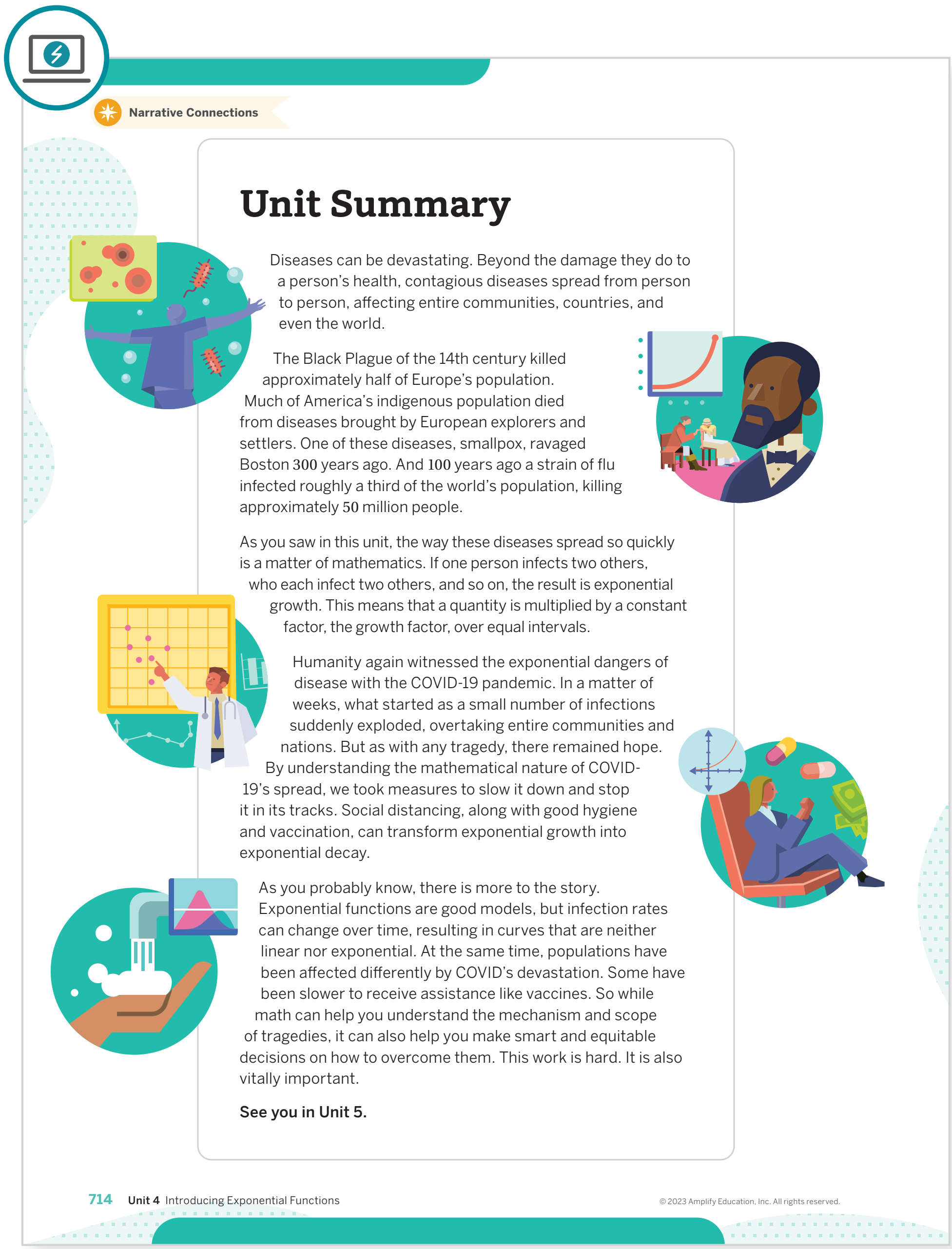
Carlos Rodriguez-Diaz

Have students read about Featured Mathematician Carlos Rodriguez-Diaz, a community health scientist, activist, and associate professor who has studied health inequities among vulnerable populations.

Whole Class | 5 min

Unit Summary

Review and synthesize the big ideas of this unit, discussing the world impacts of epidemics and vaccines.



Narrative Connections

Unit Summary

Diseases can be devastating. Beyond the damage they do to a person's health, contagious diseases spread from person to person, affecting entire communities, countries, and even the world.

The Black Plague of the 14th century killed approximately half of Europe's population. Much of America's indigenous population died from diseases brought by European explorers and settlers. One of these diseases, smallpox, ravaged Boston 300 years ago. And 100 years ago a strain of flu infected roughly a third of the world's population, killing approximately 50 million people.

As you saw in this unit, the way these diseases spread so quickly is a matter of mathematics. If one person infects two others, who each infect two others, and so on, the result is exponential growth. This means that a quantity is multiplied by a constant factor, the growth factor, over equal intervals.

Humanity again witnessed the exponential dangers of disease with the COVID-19 pandemic. In a matter of weeks, what started as a small number of infections suddenly exploded, overtaking entire communities and nations. But as with any tragedy, there remained hope. By understanding the mathematical nature of COVID-19's spread, we took measures to slow it down and stop it in its tracks. Social distancing, along with good hygiene and vaccination, can transform exponential growth into exponential decay.

As you probably know, there is more to the story. Exponential functions are good models, but infection rates can change over time, resulting in curves that are neither linear nor exponential. At the same time, populations have been affected differently by COVID's devastation. Some have been slower to receive assistance like vaccines. So while math can help you understand the mechanism and scope of tragedies, it can also help you make smart and equitable decisions on how to overcome them. This work is hard. It is also vitally important.

See you in Unit 5.

714 **Unit 4** Introducing Exponential Functions

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the vaccine response graph from Activity 3 and a few of the exponential functions students created to model the data.

Highlight how while the spread of a virus can be modeled with an exponential growth function, the vaccine response can be modeled with an exponential decay function.

Have students share how exponential growth or decay is different from linear change. Ask them to use the mathematical vocabulary they studied in this unit in their response. Consider having them create a visual display that compares linear functions with exponential functions that includes the following terms or representations.

- Common difference
- Common ratio
- Rate of change
- Average rate of change
- Initial value
- Growth/decay factor
- Vertical intercept
- Slope
- Equations
- Graphs
- Tables
- Real-world phenomena that can be modeled by each type of function

Then conduct a ***Gallery Tour*** so that students can examine their classmates' work, ask any clarifying questions, and add any new ideas to their own displays.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

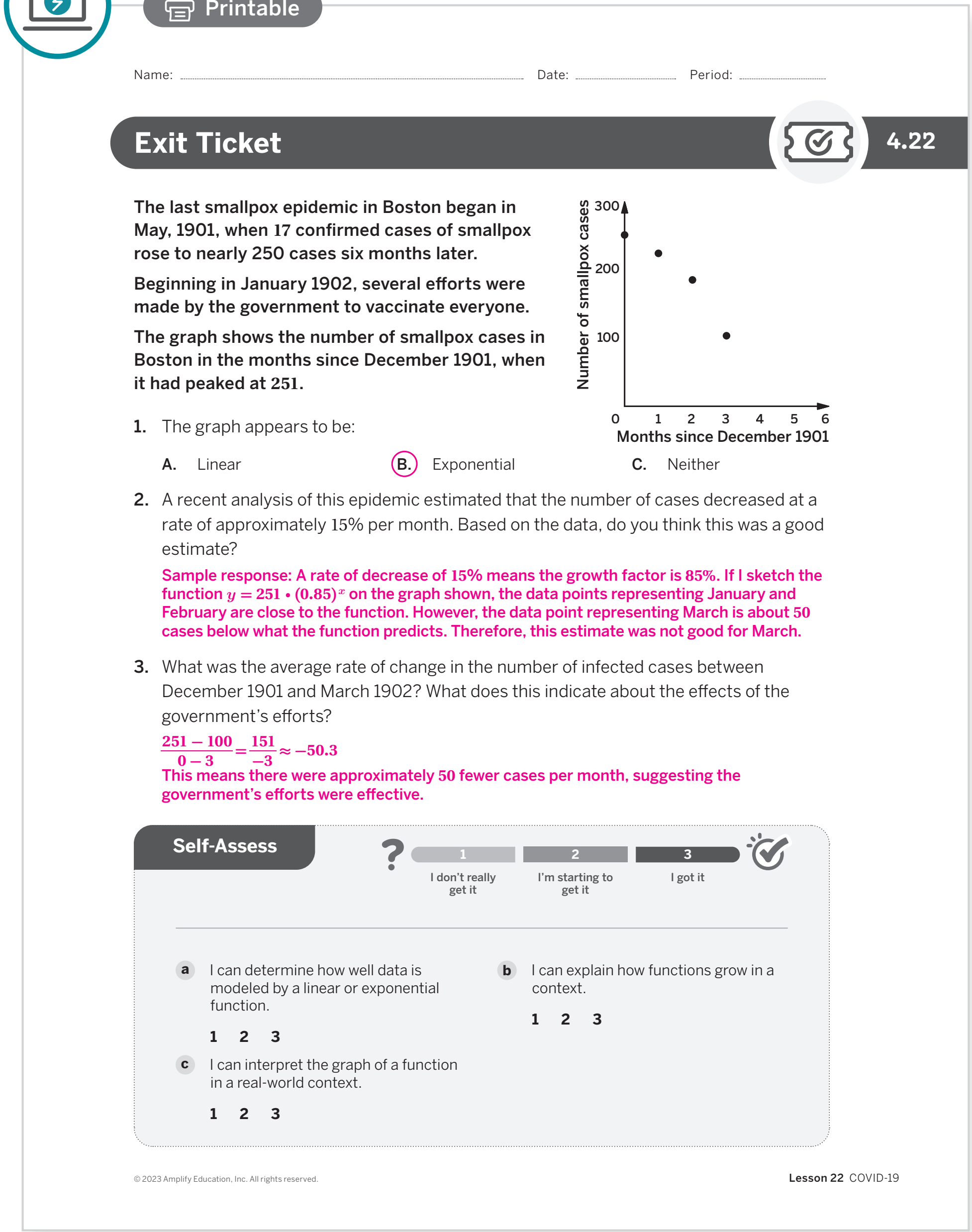
- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by interpreting the graph of a function in a real world context and making predictions.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 4.22

The last smallpox epidemic in Boston began in May, 1901, when 17 confirmed cases of smallpox rose to nearly 250 cases six months later.

Beginning in January 1902, several efforts were made by the government to vaccinate everyone.

The graph shows the number of smallpox cases in Boston in the months since December 1901, when it had peaked at 251.

1. The graph appears to be:

   A. Linear [✓ B. Exponential] C. Neither

2. A recent analysis of this epidemic estimated that the number of cases decreased at a rate of approximately 15% per month. Based on the data, do you think this was a good estimate?

   Sample response: A rate of decrease of 15% means the growth factor is 85%. If I sketch the function $y = 251 \cdot (0.85)^x$ on the graph shown, the data points representing January and February are close to the function. However, the data point representing March is about 50 cases below what the function predicts. Therefore, this estimate was not good for March.

3. What was the average rate of change in the number of infected cases between December 1901 and March 1902? What does this indicate about the effects of the government's efforts?

   $\frac{251 - 100}{0 - 3} = \frac{151}{-3} \approx -50.3$

   This means there were approximately 50 fewer cases per month, suggesting the government's efforts were effective.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can determine how well data is modeled by a linear or exponential function.

1 2 3

b I can explain how functions grow in a context.

1 2 3

c I can interpret the graph of a function in a real-world context.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 COVID-19

Success looks like . . .

- **Language Goal:** Determining whether a linear or exponential model is most appropriate for a set of real-world data and justify the selection. **(Speaking and Listening, Writing)**
  - » Determining whether a linear function or exponential function fits the graphed data in Problem 1.
- **Goal:** Constructing a function to model a set of real-world data and use the function to make predictions.

Suggested next steps

If students do not select exponential in Problem 1, consider:

- Reviewing modeling functions given data in Activity 1, Problem 2 or Activity 2, Problem 1.
- Assigning Unit 4, Lesson 2 Additional Practice.
- Having students practice comparing linear and exponential written scenarios, tables, and graphs.

If students struggle to respond to Problem 2, consider:

- Revisiting how to determine and interpret growth rates when $0 < r < 1$.
- Assigning Unit 4, Lesson 6 Additional Practice.

If students incorrectly interpret the average rate of change in Problem 3, consider:

- Reviewing calculating average rate of change given a graph or table in Activity 1, Problem 1.
- Assigning Unit 4, Lesson 14 Additional Practice.
- Having students determine the slope between the two points, emphasizing that this is the average rate of change over that interval.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students applied their understanding of exponential functions to study the spread of COVID-19 and the vaccine response. What did students find engaging or surprising about the activities in this lesson?
- How well do you think your students understand exponential growth and decay and how exponential functions are different from linear functions? How well do you think they are able to construct exponential models to analyze real-world data and use those models to make predictions? What might you change the next time you teach this unit?

Practice

Independent

Name: ______ Date: ______ Period: ______

The table gives the population of three cities over several decades. Refer to the table as you complete Problems 1–2.

City	1950	1960	1970	1980	1990	2000
Paris	6,300,000	7,400,000	8,200,000	8,700,000	9,300,000	9,700,000
Austin	132,000	187,000	254,000	346,000	466,000	657,000
Chicago	3,600,000	3,550,000	3,400,000	3,000,000	2,800,000	2,900,000

1. How would you describe the population change in each city?

Sample response:

Paris' population increased steadily, but the increase slowed over time.

Austin's population increased more and more rapidly as time went on.

Chicago's population decreased until 1990, when it increased.

2. What kind of model — *linear*, *exponential*, or *neither* — do you think is most appropriate for each city population?

Sample response:

For Paris, a linear model might be appropriate. While the successive differences are not constant, they show a general upward trend of a little more than 500,000 per year.

For Austin, an exponential model would be appropriate because the growth factor lies within a narrow range between 1.36 and 1.42.

For Chicago, the population was decreasing for most of the period, but then it increased slightly at the end. It was neither linear nor exponential.

Refer to Problem 2.

a Write an equation for each population that you think can be modeled by a linear or exponential function.

Sample response:

For Paris, the population (in millions), d decades after 1950, could be modeled by $f(d) = 0.7d + 6.3$.

For Austin, the population (in hundred thousands) d, decades after 1950, could modeled by $g(d) = 1.32 \cdot (1.38)^d$.

b Compare the graphs of your functions with the actual population data. How well do your models fit the data?

Sample response:

The model for the Paris data is not perfect. The line consistently underestimates the population for earlier decades and then overestimates the population at the very end.

The model for Austin fits the data very well.

For Chicago, I can see that the data does not have a linear or exponential shape.

Name: ______ Date: ______ Period: ______

Refer to the table as you complete Problems 3 and 4.

Year	1804	1927	1960	1974	1987	1999	2007
World population, in billions	1	2	3	4	5	6	7

3. Would a linear or exponential function be more appropriate for modeling the world population growth over the last 200 years? Explain your thinking.

Sample response: The world population has been growing more and more rapidly since 1800, indicating that its growth might be exponential, rather than linear. The number of years needed for the population to increase by a billion keeps decreasing.

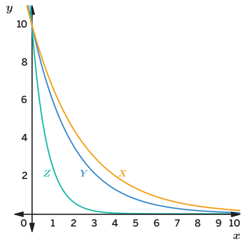
4. How many years did it take for the population to grow from 1 billion to 2 billion? From 6 billion to 7 billion? Why do you think that was that the case?

It took 123 years for the population to grow from 1 billion to 2 billion, but only 8 years to grow from 6 billion to 7 billion.

Sample response: If the population is growing exponentially, then over equal time intervals it will grow by a constant factor (rather than by a constant amount). In going from 1 billion to 2 billion, the population increased by a factor of 2. However, in growing from 6 billion to 7 billion, the population only increased by a factor of $\frac{7}{6}$, or about 1.17. Therefore, it makes sense that growing from 6 billion to 7 billion took far less time.

5. Which function in the graph decays the fastest? Which function decays the slowest?

The function represented by Graph Z decays the fastest. The function represented by Graph X decays the slowest.



Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 3	2
	3	Activity 3	1
	4	Activity 1	1
Spiral	5	Unit 4 Lesson 13	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

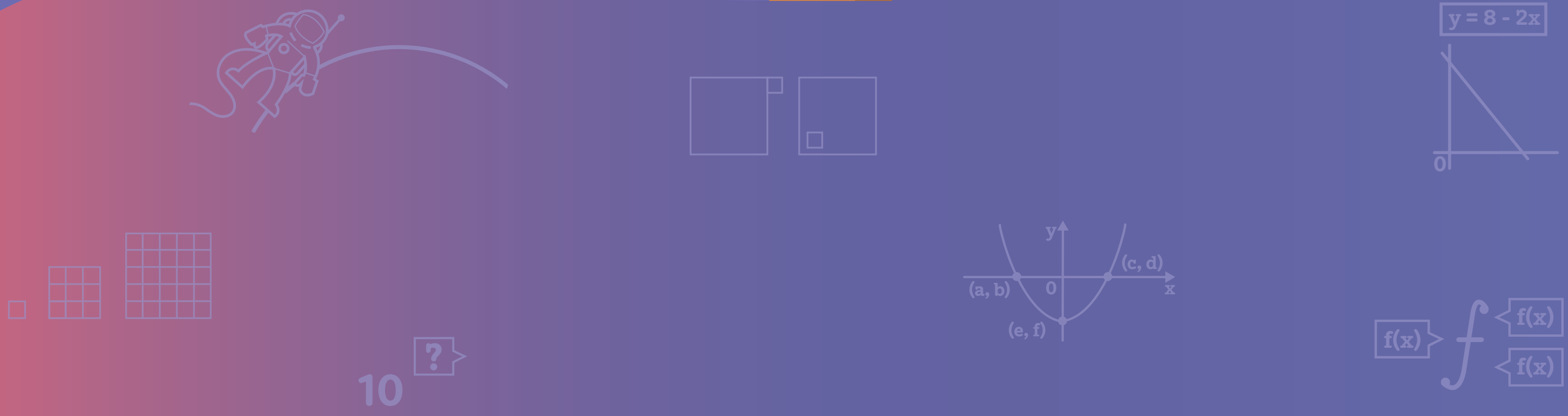
UNIT 5

Introducing Quadratic Functions

Students study quadratic functions. By analyzing and comparing patterns, tables, graphs, and equations, they gain an appreciation for the special features of quadratic functions and the situations they represent.

Essential Questions

- How are quadratic functions used to model, analyze, and interpret mathematical relationships?
- What characteristics of the graph of a quadratic function distinguish it from a linear function? An exponential function?
- What are the advantages of writing a quadratic function in vertex form? In standard form? In factored form?
- *(By the way, should Martians have to study quadratic functions?)*



Key Shifts in Mathematics

Focus

In this unit . . .

Students are introduced to a variety of quadratic functions through rectangular area models, patterns of growth, and projectile motion. They will study different functional forms and graphs of quadratics in context.

Coherence

Previously . . .

In Unit 3, students explored various functions and explored their graphs. In Unit 4, they focused on exponential functions, another broad class of functions akin to quadratics and other polynomials.

Coming soon . . .

After exploring quadratic functions throughout this unit, students will move on to solve quadratic equations in Unit 6.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students build their conceptual understanding of quadratics by first analyzing square patterns. They connect multiple representations of quadratics — tables, equations, and graphs.

Procedural Fluency

Students have ample opportunities to develop proficiency by beginning to write quadratic expressions and progressing to write quadratic equations and functions in various forms.

Application

Throughout the unit, students have opportunities to apply quadratics in relevant contexts such as free fall, projectile motion, and revenue.

Squares in Motion

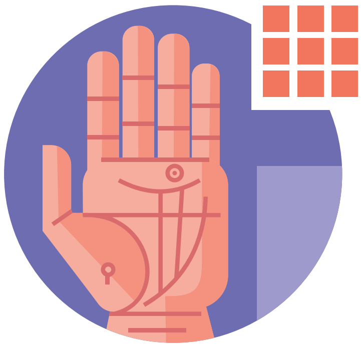
SUB-UNIT

1

Lessons 2–5

A Different Kind of Change

Students build a conceptual understanding of ***quadratic expressions*** and ***quadratic functions*** by exploring and building patterns involving squared terms. They connect the term ***quadratic*** with the four sides of a rectangle (or square) and determine whether a relationship is quadratic by determining the first and second differences, given a table or a list of ordered pairs.



Narrative: A square has more to offer than just four sides.

SUB-UNIT

2

Lessons 6–9

Quadratic Functions

Students discover that exponential growth eventually overtakes quadratic growth. They explore how quadratics can model free-falling objects, projectile motion, and revenue and make mathematical observations about the quadratic functions that represent these real-world contexts.

Narrative: From sports to freefall and earning revenue, quadratics model them all.

SUB-UNIT

3

Lessons 10–13

Quadratic Expressions

Students explore quadratic expressions using ***area diagrams*** and algebra tiles and visualize the multiplication of two linear terms. Through these visual models, they are introduced to the linear factors of quadratic expressions. The ***factored forms*** and ***standard forms*** of quadratic expressions are formally defined.

Narrative: Discover where the *quad-* in *quadratic* comes from.

Launch

Lesson 1

The Perfect Shot

Students explore projectile motion by tossing balls into buckets. They try out different throwing techniques, and ultimately consider how they can best model the trajectory of the ball.

SUB-UNIT

4

Lessons 14–22

Features of Graphs of Quadratic Functions

Students explore graphs of quadratic functions, including symmetry and key features that connect factored form and standard form equations with their graphs. They meet a new form — ***vertex form*** — and use technology to explore the effects of changing parameters.

Narrative: Discover the elegance and symmetry of the graph of a quadratic function.

Capstone

Lesson 23

Monster Ball

Students apply their understanding of quadratic functions and projectile motion to play a game of Monster Ball. They determine where to place players on the court, what types of balls to choose, and what type of throw to use.

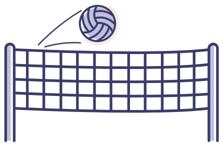
Unit at a Glance

Spoiler Alert: This unit is all about quadratic functions and their graphs — not solving equations. There is much to see and understand about quadratics before it is time to solve their equations.

Assessment

A **Pre-Unit Readiness Assessment**

Launch



1 **The Perfect Shot**

By analyzing sports and tossing balls into buckets, make conjectures about the path of an object.

Sub-Unit 1: A Different Kind of Change

x	y
1	0
2	5

2 **A Different Kind of Change**

Given a fixed perimeter, determine the maximum area of a field. The term *quadratic relationship* is not defined yet.

Sub-Unit 2: Quadratic Functions

?

$x \cdot 10$

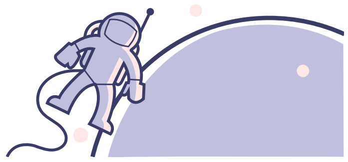
6 **Comparing Functions**

By examining successive quotients in tables, discover that exponential growth eventually overtakes quadratic growth.

x	0	1	2
y	0	16	64

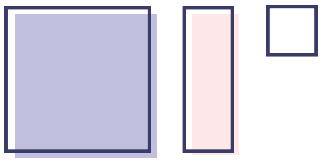
7 **Building Quadratic Functions to Describe Falling Objects**

Analyze the vertical distance traveled by free-falling objects over a period of time.



8 **Building Quadratic Functions to Describe Projectile Motions**

Explore the effects of gravity on an object by comparing a launch on Earth to a launch on the moon.



11 **Equivalent Quadratic Expressions** (Part 2)

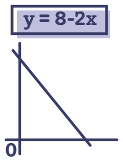
Use algebra tiles to visualize the multiplication of two linear terms.

$16t^2+48t-80$

$(t+16)(t+5)$

12 **Standard Form and Factored Form**

Factored and standard forms are defined for quadratics. Transition from multiplication with diagrams to using the Distributive Property.



13 **Graphs of Functions in Standard and Factored Forms**

Explore connections between quadratic forms and their graphs. Factored form reveals x-intercepts, while standard form reveals the y-intercept.

Key Concepts

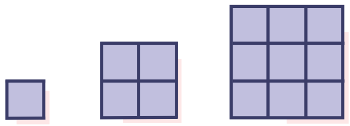
Lesson 2: Quadratics are introduced by determining the maximum area of a rectangle given a fixed perimeter.
Lessons 7–8: The paths of falling and launched objects are described using quadratic functions.
Lessons 11–13: The factored and standard forms of quadratics are explored and defined.

Pacing

23 Lessons: 50 min each
3 Assessments: 45 min each

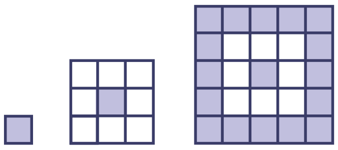
Full Unit: 26 days
● **Modified Unit:** 19 days

Assumes 50 minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



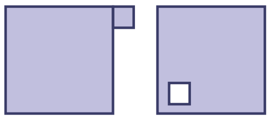
3 How Does It Change? ●

Given patterns of figures, discover the square of each figure number represents the number of objects in the figure. The term *quadratic expression* is formally defined.



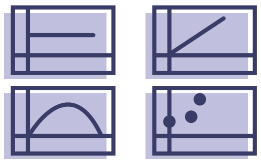
4 Squares ●

Quadratic relationships are further explored by looking closely at *first* and *second differences*. A quadratic relationship is defined in terms of x^2.



5 Seeing Squares as Functions

By exploring more visual patterns, write quadratic relationships as quadratic functions.



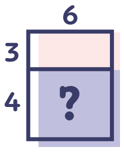
9 Building Quadratic Functions to Maximize Revenue ●

Quadratic functions are used to explain revenue for a fictional sports network.

Assessment

A Mid-Unit Assessment

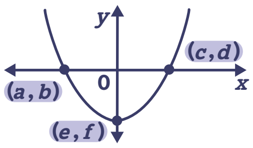
Sub-Unit 3: Quadratic Expressions



10 Equivalent Quadratic Expressions (Part 1) ●

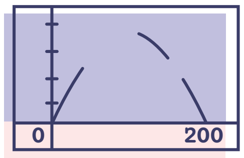
Revisit decomposing figures to calculate the area of parts of a rectangle with unknown dimensions.

Sub-Unit 4: Features of Graphs of Quadratic Functions



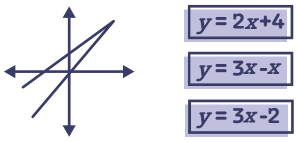
14 Graphing Quadratics Using Points of Symmetry

Discover the symmetrical relationship of the ordered pairs of quadratic functions.



15 Interpreting Quadratics in Factored Form

Examine different points on the graph of quadratic functions in factored form through various contexts.



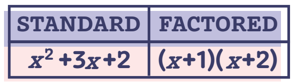
16 Graphing With the Standard Form (Part 1)

Identify equivalent forms of the same quadratic function. Explore the relationships between the terms of each form and its graph.

Unit at a Glance

Spoiler Alert: This unit is all about quadratic functions and their graphs — not solving equations. There is much to see and understand about quadratics before it is time to solve their equations.

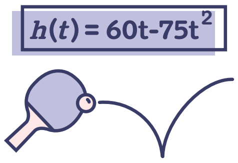
‹ continued



STANDARD	FACTORED
x^2+3x+2	$(x+1)(x+2)$

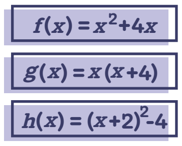
17 **Graphing With the Standard Form** (Part 2) ●

Use factored and standard form to graph quadratic functions.



18 **Graphs That Represent Scenarios** ●

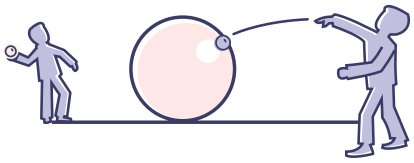
Interpret graphs and equations of quadratic functions in context.



19 **Vertex Form**

Introduce vertex form of quadratic functions, and use technology to explore the effects of changing this form's parameters.

Capstone



23 **Monster Ball**

Get your students moving as they create teams, develop a game plan, and execute their strategy on a field.

Assessment

A **End-of-Unit Assessment**

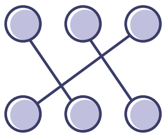
Key Concepts

Lesson 2: Quadratics are introduced by determining the maximum area of a rectangle given a fixed perimeter.
Lessons 7–8: The paths of falling and launched objects are described using quadratic functions.
Lessons 11–13: The factored and standard forms of quadratics are explored and defined.

Pacing

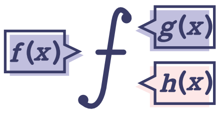
23 Lessons: 50 min each
3 Assessments: 45 min each
Full Unit: 26 days
● **Modified Unit:** 19 days

Assumes 50 minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



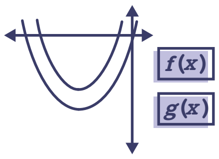
20 Graphing With the Vertex Form

Use the structure of vertex form to determine whether its graph has a maximum or minimum.



21 Changing Parameters and Choosing a Form

Upon changing the parameters of the different quadratic forms, choose the best form to use for different situations.



22 Changing the Vertex

Understand how changing the values of h and k on a graph changes its meaning given a context.

● Modifications to Pacing

Lessons 3–4: These lessons can be combined. Lesson 3 introduces first and second differences and Lesson 4 provides more practice and opportunities to write more complex quadratic expressions representing patterns of growth.

Lessons 9–10: These lessons may be omitted. Consider using Lesson 10 for students who are struggling with exponent rules.

Lessons 17–18: Lesson 17 may be omitted, depending upon students' results from Lesson 16's Exit Ticket. Lesson 18 may be omitted and replaced with Lesson 21.

Unit Supports

Math Language Development

Lesson	New vocabulary	
1	projectile	
3	quadratic	quadratic expression
5	quadratic function	
8	vertex	zero
10	area diagram	
12	factored form (of a quadratic expression) standard form (of a quadratic expression)	
19	vertex form	

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
15, 21	**MLR1:** Stronger and Clearer Each Time
2, 3, 5, 8, 10, 12, 16, 19	**MLR2:** Collect and Display
10, 23	**MLR3:** Critique, Correct, Clarify
18	**MLR4:** Information Gap
4, 6, 12, 19, 20	**MLR5:** Co-craft Questions
9, 13, 18, 22	**MLR6:** Three Reads
1, 3, 4, 7–9, 11–17, 20–22	**MLR7:** Compare and Connect
2, 4, 5, 8, 9, 10, 12, 15, 16, 20, 23	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket Additional Practice

Lesson(s)	Additional Required Materials	
11	algebra tiles	
7	aluminum foil (6.5 ft)	
23	basketball court (optional) large exercise ball	soccer balls, basketballs, tennis balls, kickballs
1	buckets or large cups	colored pencils/pens
14, 20	graph paper	
6	graphing or spreadsheet technology	
8, 9, 14–19, 21–23	graphing technology	
6	headphones	
7	hard-boiled eggs	measuring tape
1, 2, 4–12, 16–20, 23	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.	
3, 4	snap cubes	
7	stopwatches	
1	table tennis balls	
20	tracing paper	

Instructional Routines

Activities throughout this unit include these routines:

Lesson(s)	Instructional Routines
16, 20	Card Sort
20	Connecting Representations
10, 12	I Have, Who Has?
18	Info Gap
12	Math Talk
2, 3, 5, 7, 8, 12, 14, 19	Notice and Wonder
9	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 11
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 23

Amps
POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

First and Second Differences

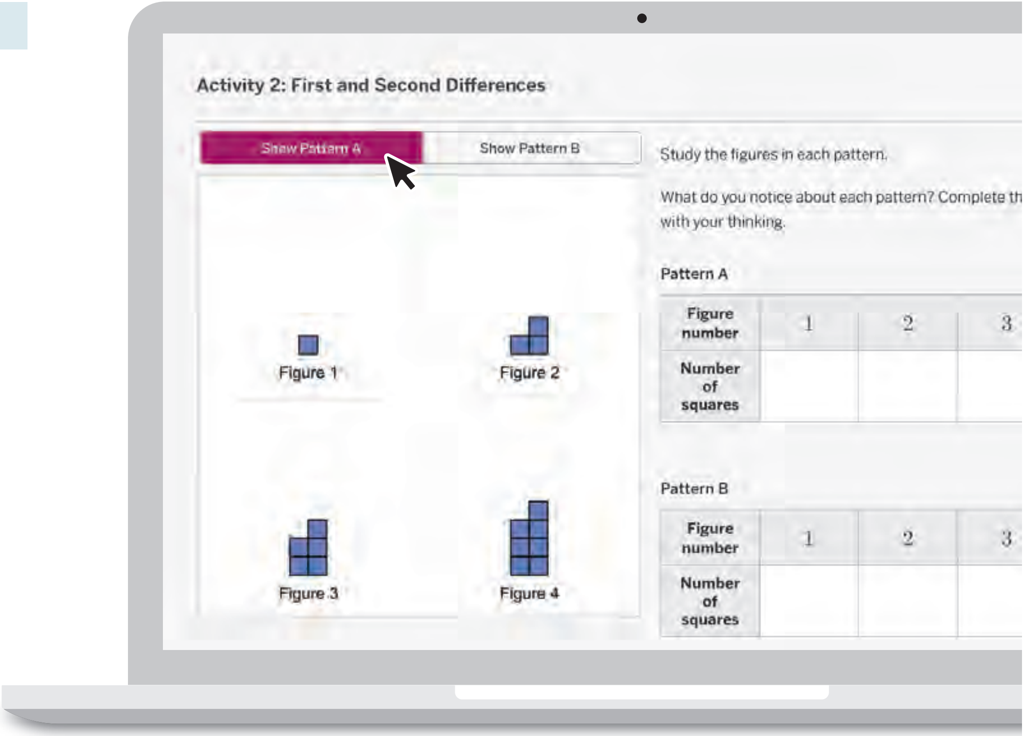
Put on your student hat and work through **Lesson 3, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities

- Functions Have Sound **(Lesson 6)**
- Egg Drop **(Lesson 7)**
- Using Algebra Tiles to Find Equivalent Quadratic Expressions **(Lesson 11)**
- The Cow Jumped Over the Moon **(Lesson 22)**



Activity 2: First and Second Differences

Study the figures in each pattern.

What do you notice about each pattern? Complete th
with your thinking.

Pattern A

Figure number	1	2	3
Number of squares			

Pattern B

Figure number	1	2	3
Number of squares			

Unit Study
Professional Learning

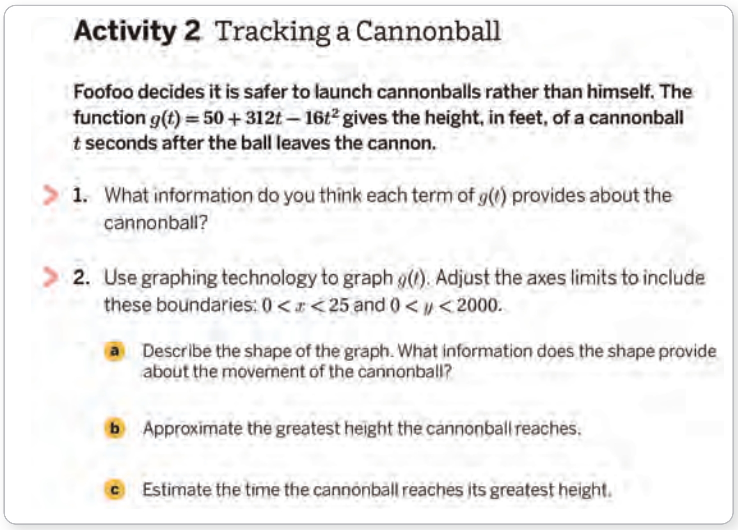
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces the idea of using quadratic functions to describe projectile motion. Students become familiar with the context of a, b, and c in the equation $y = ax^2 + bx + c$ as it relates to the path of falling objects. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 8, Activity 2:



Activity 2 Tracking a Cannonball

Foofoo decides it is safer to launch cannonballs rather than himself. The function $g(t) = 50 + 312t - 16t^2$ gives the height, in feet, of a cannonball t seconds after the ball leaves the cannon.

1. What information do you think each term of $g(t)$ provides about the cannonball?
2. Use graphing technology to graph $g(t)$. Adjust the axes limits to include these boundaries: $0 < x < 25$ and $0 < y < 2000$.
   - a Describe the shape of the graph. What information does the shape provide about the movement of the cannonball?
   - b Approximate the greatest height the cannonball reaches.
   - c Estimate the time the cannonball reaches its greatest height.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

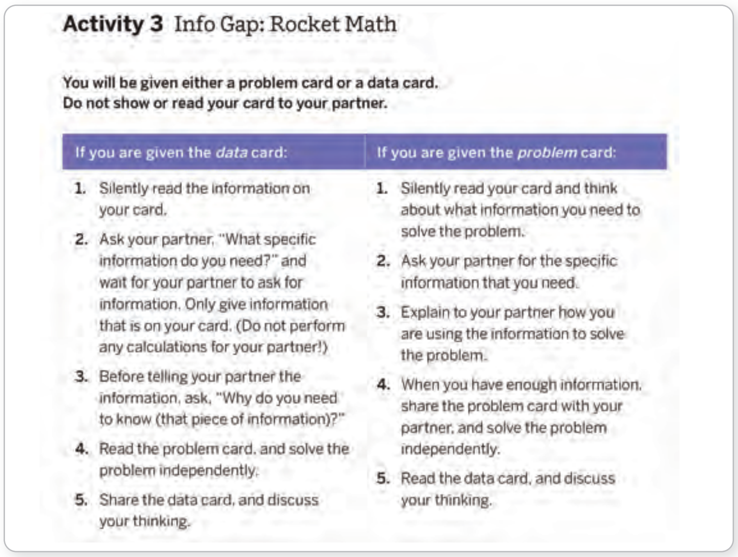
- What was it like to engage in this problem as a learner?
- Problem 2 suggests limiting the boundaries of the axes when using a graphing tool. Do you think your students would come up with this on their own if the suggestion was removed?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Information Gap (Info Gap)

Rehearse . . .

How you'll facilitate the ***Info Gap*** instructional routine in Lesson 18, Activity 3:



Activity 3 Info Gap: Rocket Math

You will be given either a problem card or a data card.
Do not show or read your card to your partner.

If you are given the *data* card:	If you are given the *problem* card:
1. Silently read the information on your card. 2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not perform any calculations for your partner!) 3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?" 4. Read the problem card, and solve the problem independently. 5. Share the data card, and discuss your thinking.	1. Silently read your card and think about what information you need to solve the problem. 2. Ask your partner for the specific information that you need. 3. Explain to your partner how you are using the information to solve the problem. 4. When you have enough information, share the problem card with your partner, and solve the problem independently. 5. Read the data card, and discuss your thinking.

Point to Ponder . . .

- Am I a model for asking good questions? Do I tend to ask questions that elicit straightforward responses, such as process questions or solution-oriented questions? How can I be more intentional about probing for student understanding using open-ended questioning techniques?

This routine . . .

- Encompasses **MLR5** Co-craft Questions.
- Encompasses a need for graphic organizers and sentence stems for students with disabilities.
- Allows students the opportunity to practice using precise mathematical language.
- Requires time to plan, depending on class size and time constraints.

Anticipate . . .

- Intentional pairing of students with certain cards.
- Preparing scaffolds or questions to help students get started.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthen Your Effective Teaching Practices

Use and connect mathematical representations.

This effective teaching practice . . .

- Deepens student understanding of mathematical concepts and procedures and builds a toolkit for problem solving.
- Facilitates meaningful mathematical discourse by drawing connections between the different representations used and how each one illustrates the same mathematical ideas.

Points to Ponder . . .

- What representations will be presented in this unit?
- Where do you see opportunities to make connections among the different representations used, and when should students be allowed choice of the representation used?

Math Language Development

MLR5: Co-craft Questions

MLR5 appears in Lessons 4, 6, 12, 19, and 20.

- In Lesson 4, ask students to work with their partner to co-craft questions they have about the figures shown, how they are growing, and what the next figures in the pattern will look like. Sample questions are provided.
- In Lesson 19, reveal the functions in Sets 1 and 2 before having students begin the activity. Generating their own questions about the functions will help them make sense of their structure.
- **English Learners:** Display 1 or 2 sample questions and allow students to say their questions aloud as opposed to writing them, or vice versa.

Point to Ponder . . .

- As students work with each other to create mathematical questions they have about graphs or scenarios, how can you model for them how to use their developing mathematical vocabulary?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with understanding quadratic relationships throughout the unit? Do you think your students will generally:
  - Miss the underlying concept that multiplying two linear expressions produces a quadratic expression?
  - Struggle to determine the most appropriate form of a quadratic expression, equation, or function?
  - Be prepared to solve procedurally and efficiently, but unable to apply a skill to problems given a context?

Differentiated Support

Accessibility: *Optimize Access to Technology*

Throughout this unit, have students use the Amps slides. Specific suggested opportunities to have students use technology to deepen their conceptual understanding appear in Lessons 1–17, 19, 21–23.

- In Lesson 6, students can listen to the sound of linear, quadratic, and exponential functions and create their own function to produce a particular sound.
- In Lesson 8, students can use an interactive graph to track Foofoo's flight with and without gravity to see how its influence changes the equation that models Foofoo's motion.
- In Lesson 21, students can use digital tools to explore the effects of changing parameters on the graph of a quadratic function written in vertex form.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology to optimize student understanding?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-management and self-awareness skills.

Points to Ponder . . .

- Do students take into consideration the context of the problem when setting up their models and interpreting their answers to determine if their model helped them reach the goals of the problem?
- Can students confidently work within the structure of quadratic functions to model, interpret, and predict within real-world scenarios?

UNIT 5 | **LESSON 1 – LAUNCH**

The Perfect Shot

Let's explore the mechanics of throwing a ball.

Focus

Goals

1. Sketch the trajectory of a projectile.
2. **Language Goal:** Describe the type of function that models projectile motion. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Explain the effects of gravity on the trajectory of a projectile. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students explore projectile motion in sports, which can be modeled by quadratic functions, though this terminology is not yet introduced. They take turns launching projectiles and observing each other's strategies for making a shot, and make conjectures about what would happen if a projectile were launched in space where there is little gravity.

Previously

In Unit 4, students examined exponential functions in contexts that involved rapid spread or growth, including disease and infections, population growth, and credit card interest rates.

Coming Soon

In this unit, students will explore quadratic functions in the context of projectile motion, learn its historic Babylonian origins, and understand the connection between its linear factors and its curve.

Rigor

- Students launch projectiles and observe their trajectories to develop a basic **conceptual understanding** of projectile motion.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (instructions)
- buckets, pitchers, or large cups, 3 per group
- colored pencils/pens
- table tennis balls, 3 per group

Math Language Development

New words

- *projectile*

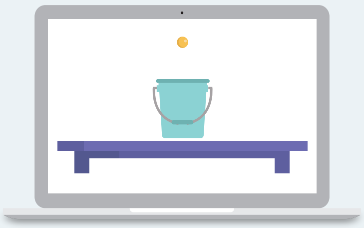
Review words

- *linear*

Amps | **Featured Activity**

Activity 1
Foofoo's Shots

Students are informally introduced to the general paths of projectiles by means of the game, Foofoo's Shots.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might be uncomfortable at first because the mathematics of the situation in Activity 1 is not immediately obvious. Challenge students to use mathematical terms when analyzing the shooting techniques and to think about how mathematics could be used to model the shooting techniques.

Modifications to Pacing

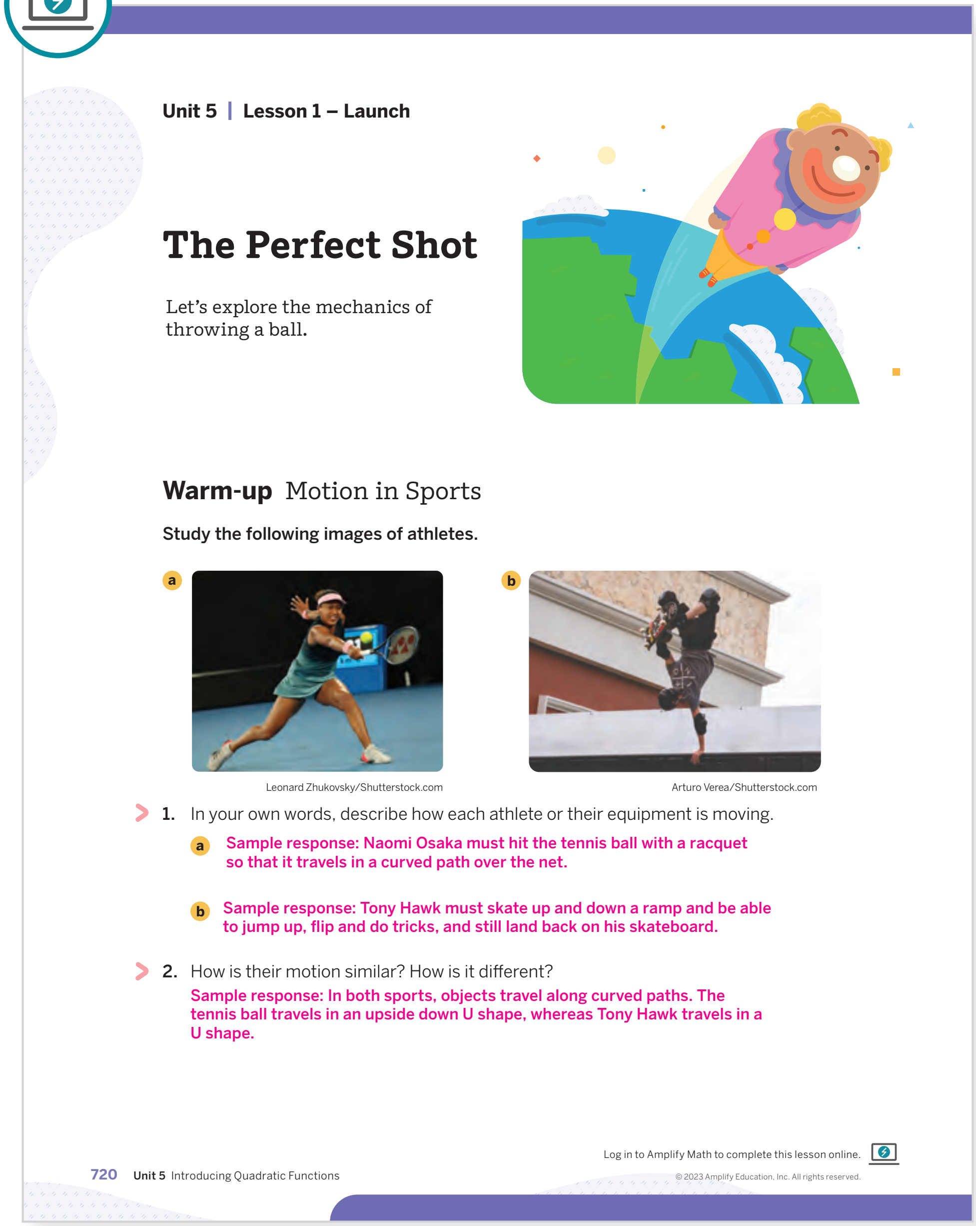
You may want to consider these additional modifications if you are short on time.

- Reduce the time allotted for **Activity 1** to 20 minutes.
- In **Activity 2**, Problems 4 and 5 may be omitted.

Independent | 5 min

Warm-up Motion in Sports

Students describe the motion exhibited in two different sports to prepare them for developing an understanding of projectile motion.



Unit 5 | Lesson 1 – Launch

The Perfect Shot

Let's explore the mechanics of throwing a ball.

Warm-up Motion in Sports

Study the following images of athletes.

a

Leonard Zhukovsky/Shutterstock.com

b

Arturo Verea/Shutterstock.com

1. In your own words, describe how each athlete or their equipment is moving.
   - a Sample response: Naomi Osaka must hit the tennis ball with a racquet so that it travels in a curved path over the net.
   - b Sample response: Tony Hawk must skate up and down a ramp and be able to jump up, flip and do tricks, and still land back on his skateboard.
2. How is their motion similar? How is it different?
   Sample response: In both sports, objects travel along curved paths. The tennis ball travels in an upside down U shape, whereas Tony Hawk travels in a U shape.

Log in to Amplify Math to complete this lesson online.

720 **Unit 5** Introducing Quadratic Functions

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask students to name some of their favorite athletes. Set an expectation for the amount of time students will have to work independently on the activity.

2 Monitor

Help students get started by activating their background knowledge. Ask, "What is the objective of the motion in each sport?"

Look for productive strategies:

- Sketching models or diagrams of the motion in each sport.

3 Connect

Display the images from the Warm-up.

Have several students share their descriptions of the motion that must occur in each image for the athlete shown to be successful in their sport. If any students diagrammed the motion involved in either of the images, have them share, highlighting the path of the moving object.

Ask, "What motion occurs in both of these sports?"

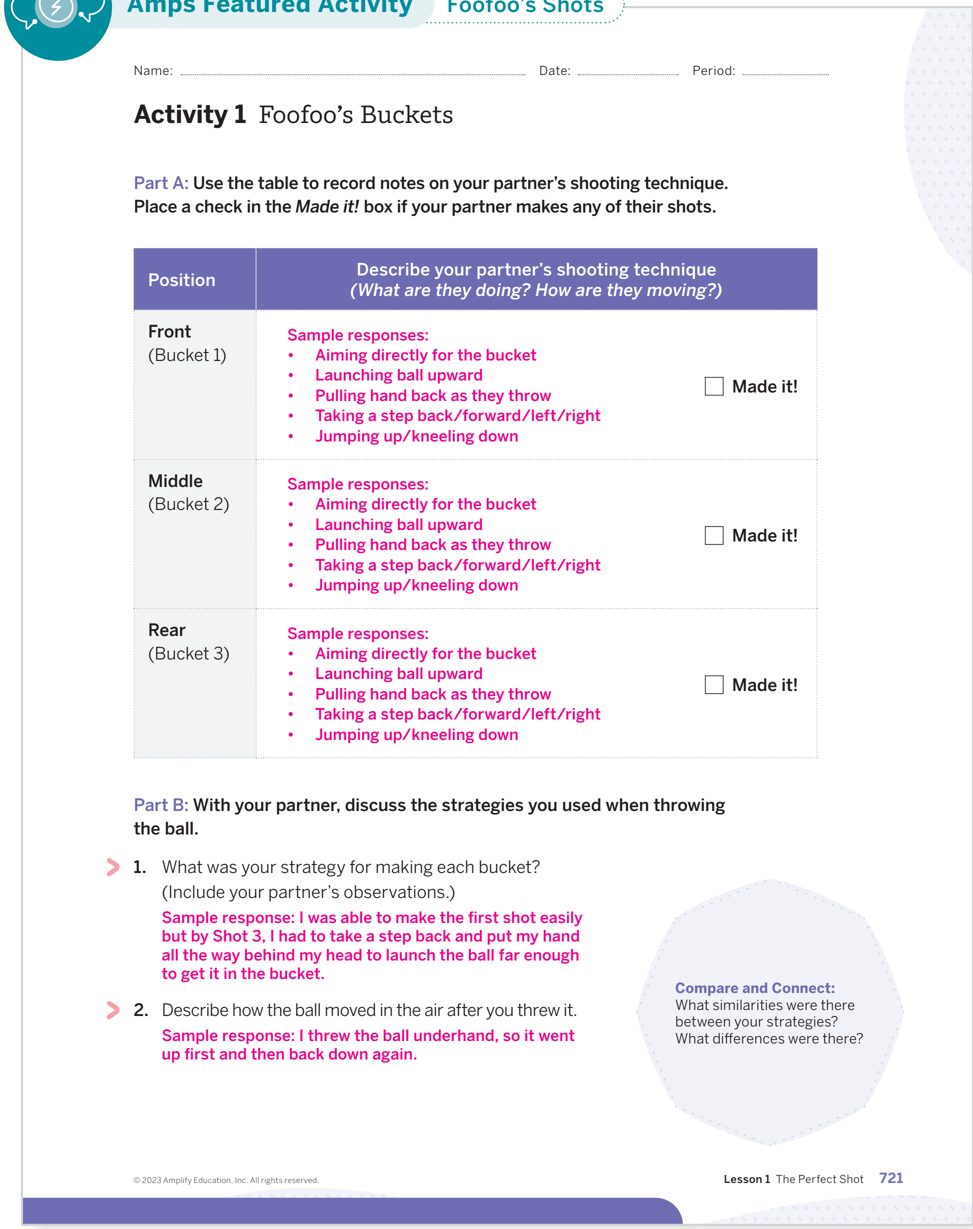
Highlight that the paths in both of these instances are curved, as are the paths of the object that is hit or thrown in several other sports.

Pairs | 20 min

Activity 1 Foofoo's Buckets

Students play a game tossing balls into buckets at varying distances and observe one another's shooting techniques to further develop their understanding of projectile motion.

Amps Featured Activity Foofoo's Shots



Name: ______________ Date: ______ Period: ______

Activity 1 Foofoo's Buckets

Part A: Use the table to record notes on your partner's shooting technique. Place a check in the *Made it!* box if your partner makes any of their shots.

Position	Describe your partner's shooting technique *(What are they doing? How are they moving?)*	
Front (Bucket 1)	Sample responses: • Aiming directly for the bucket • Launching ball upward • Pulling hand back as they throw • Taking a step back/forward/left/right • Jumping up/kneeling down	☐ Made it!
Middle (Bucket 2)	Sample responses: • Aiming directly for the bucket • Launching ball upward • Pulling hand back as they throw • Taking a step back/forward/left/right • Jumping up/kneeling down	☐ Made it!
Rear (Bucket 3)	Sample responses: • Aiming directly for the bucket • Launching ball upward • Pulling hand back as they throw • Taking a step back/forward/left/right • Jumping up/kneeling down	☐ Made it!

Part B: With your partner, discuss the strategies you used when throwing the ball.

1. What was your strategy for making each bucket? (Include your partner's observations.)
   Sample response: I was able to make the first shot easily but by Shot 3, I had to take a step back and put my hand all the way behind my head to launch the ball far enough to get it in the bucket.
2. Describe how the ball moved in the air after you threw it.
   Sample response: I threw the ball underhand, so it went up first and then back down again.

Compare and Connect: What similarities were there between your strategies? What differences were there?

1 Launch

Display the Activity 1 PDF to review the instructions for playing the game. Model the difference between an overhand and an underhand throw.

2 Monitor

Look for productive strategies: Taking note of what their partner is physically doing when taking each shot, paying attention to any similar techniques used by students who are throwing underhand and those used by students who are throwing overhand, including:

- Leaning forward instead of stepping back.
- Kneeling down instead of jumping up.
- Using a quick release instead of a follow-through.
- Launching the ball forward instead of upward.

3 Connect

Have students share the strategies their partners used to make each bucket and discuss why those strategies might have been used, eliciting responses from the students who used them.

Ask:

- "Is it easier to throw the ball using an underhand or overhand technique? Why do you think so?"
- "Which bucket was the easiest to make? Which was the most difficult?"
- "How would your technique change if you were throwing a tennis ball instead? Would it be harder or easier to throw?"

Highlight the shapes of the paths determined by an overhand throw and an underhand throw by drawing a sketch. Note the curved path in both instances.

Define the term ***projectile*** as an object launched into the air or space and affected by gravity.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they are informally introduced to the general paths of projectiles by means of the game, Foofoo's Shots. The digital version of this activity allows students to see the motion, rather than play the game themselves.

Extension: *Math Enrichment*

Have students name other sports or other examples of projectiles that are launched into the air or space and are affected by gravity. Sample responses: A football, a baseball/softball, a tennis ball, a basketball, a golf ball

Math Language Development

MLR7: Compare and Connect

During the Connect, have students compare strategies, responding to the questions posed to them in their Student Edition, "What similarities were there between your strategies? What differences were there?"

Provide these sentence frames to help students organize their thinking:

- "My strategy was to . . ."
- "To launch the ball, I . . ."
- "After I threw the ball, it . . ."

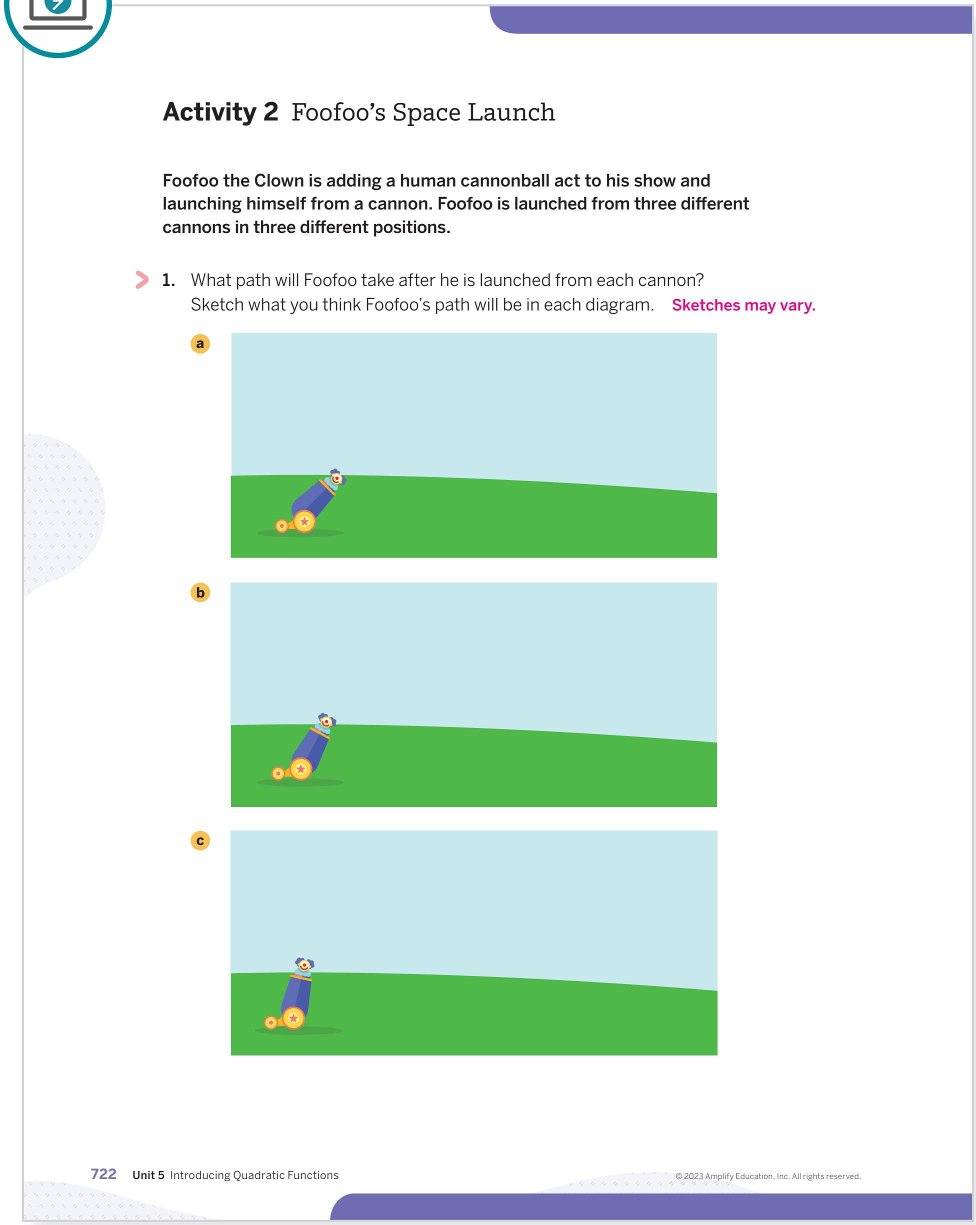
English Learners

As students describe their strategies, encourage them to model the motion they use when launching the ball and the motion by which the ball moved through the air.

Pairs | 15 min

Activity 2 Foofoo's Space Launch

Students consider how Foofoo's path will be altered after being launched from different cannon positions to study the impacts of launch angles and gravity on his path.



Activity 2 Foofoo's Space Launch

Foofoo the Clown is adding a human cannonball act to his show and launching himself from a cannon. Foofoo is launched from three different cannons in three different positions.

1. What path will Foofoo take after he is launched from each cannon? Sketch what you think Foofoo's path will be in each diagram. **Sketches may vary.**

1 Launch

Display the three illustrations in Problem 1 and draw students' attention to the different positions of each cannon. Let students know that a "sketch" is a rough drawing and does not need to be accurate.

2 Monitor

Help students get started by prompting them to sketch what they *think* the path will be, letting them know there are no incorrect answers.

Look for points of confusion:

- **Sketching a path beyond the constraints of the diagram in Problem 1.** Allow students to do so.
- **Sketching a line to model the path on Earth.** Ask, "Where or when do you think Foofoo will land?"

Look for productive strategies:

- Sketching a curved line or set of points that opens downward to model Foofoo's path after being launched on Earth.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1–3, eliminating Problem 4.

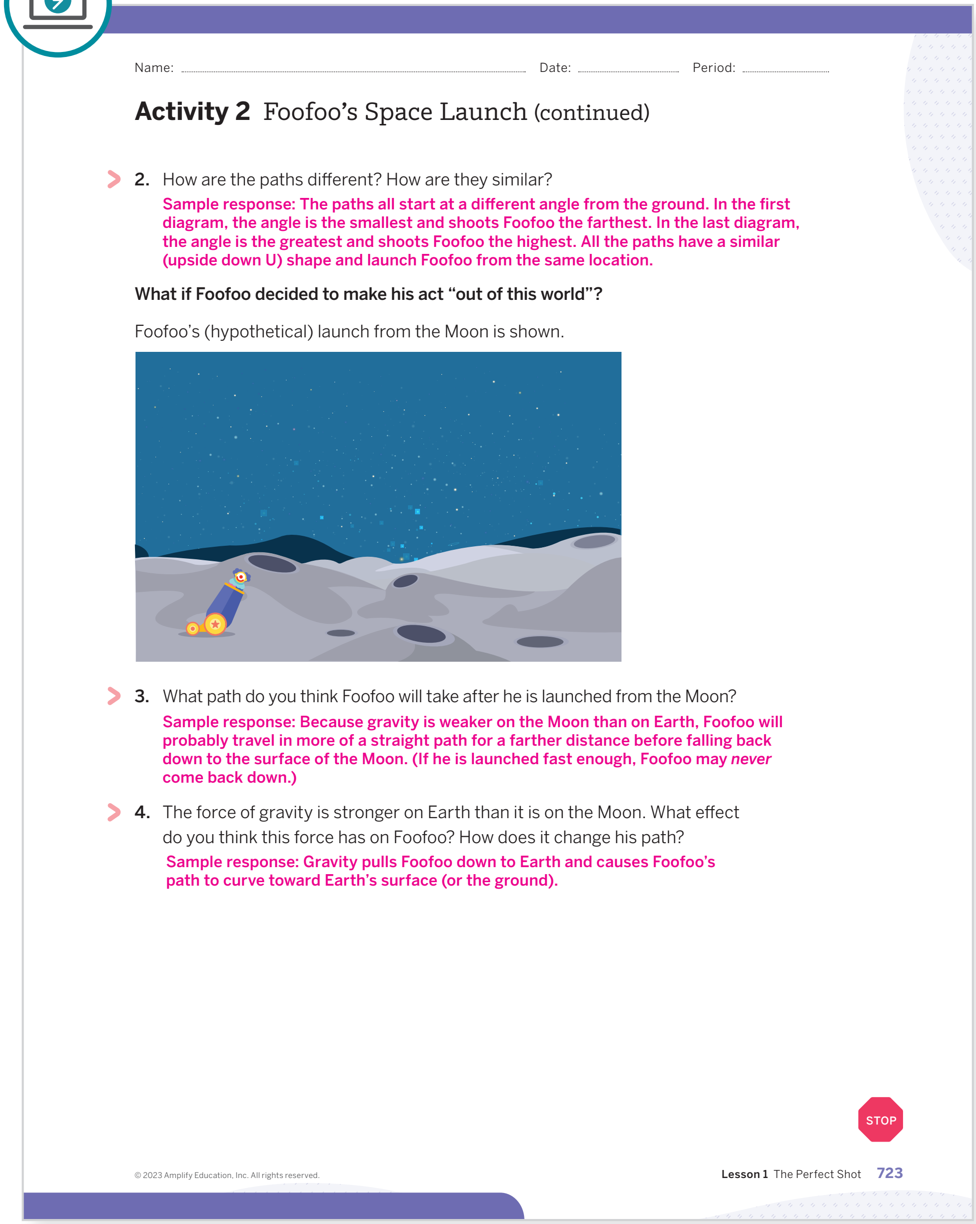
Extension: *Math Enrichment, Interdisciplinary Connections*

After students complete Problems 3 and 4, ask them if they know how gravity on the Moon compares to gravity on Earth. Tell them that the Moon's surface gravity is about $\frac{1}{6}$ times the gravity on Earth's surface. An object falling near the surface of Earth would accelerate toward Earth at a rate of about 9.8 m per second^2, while an object falling near the surface of the Moon would accelerate toward the Moon at a much slower rate, about 1.62 m per second^2. Ask students to determine the ratio of these accelerations and what they notice. **(Science)**

Pairs | 15 min

Activity 2 Foofoo's Space Launch (continued)

Students consider how Foofoo's path will be altered after being launched from different cannon positions to study the impacts of launch angles and gravity on his path.



Name: ______________________ Date: __________ Period: __________

Activity 2 Foofoo's Space Launch (continued)

2. How are the paths different? How are they similar?
   Sample response: The paths all start at a different angle from the ground. In the first diagram, the angle is the smallest and shoots Foofoo the farthest. In the last diagram, the angle is the greatest and shoots Foofoo the highest. All the paths have a similar (upside down U) shape and launch Foofoo from the same location.

What if Foofoo decided to make his act "out of this world"?

Foofoo's (hypothetical) launch from the Moon is shown.

3. What path do you think Foofoo will take after he is launched from the Moon?
   Sample response: Because gravity is weaker on the Moon than on Earth, Foofoo will probably travel in more of a straight path for a farther distance before falling back down to the surface of the Moon. (If he is launched fast enough, Foofoo may *never* come back down.)

4. The force of gravity is stronger on Earth than it is on the Moon. What effect do you think this force has on Foofoo? How does it change his path?
   Sample response: Gravity pulls Foofoo down to Earth and causes Foofoo's path to curve toward Earth's surface (or the ground).

STOP

Lesson 1 The Perfect Shot 723

3 Connect

Have three students share their sketches and ask the rest of the class if they drew something similar. Compare and contrast the paths drawn and discuss how a launch from the Moon would be different due to minimal gravity.

Ask:

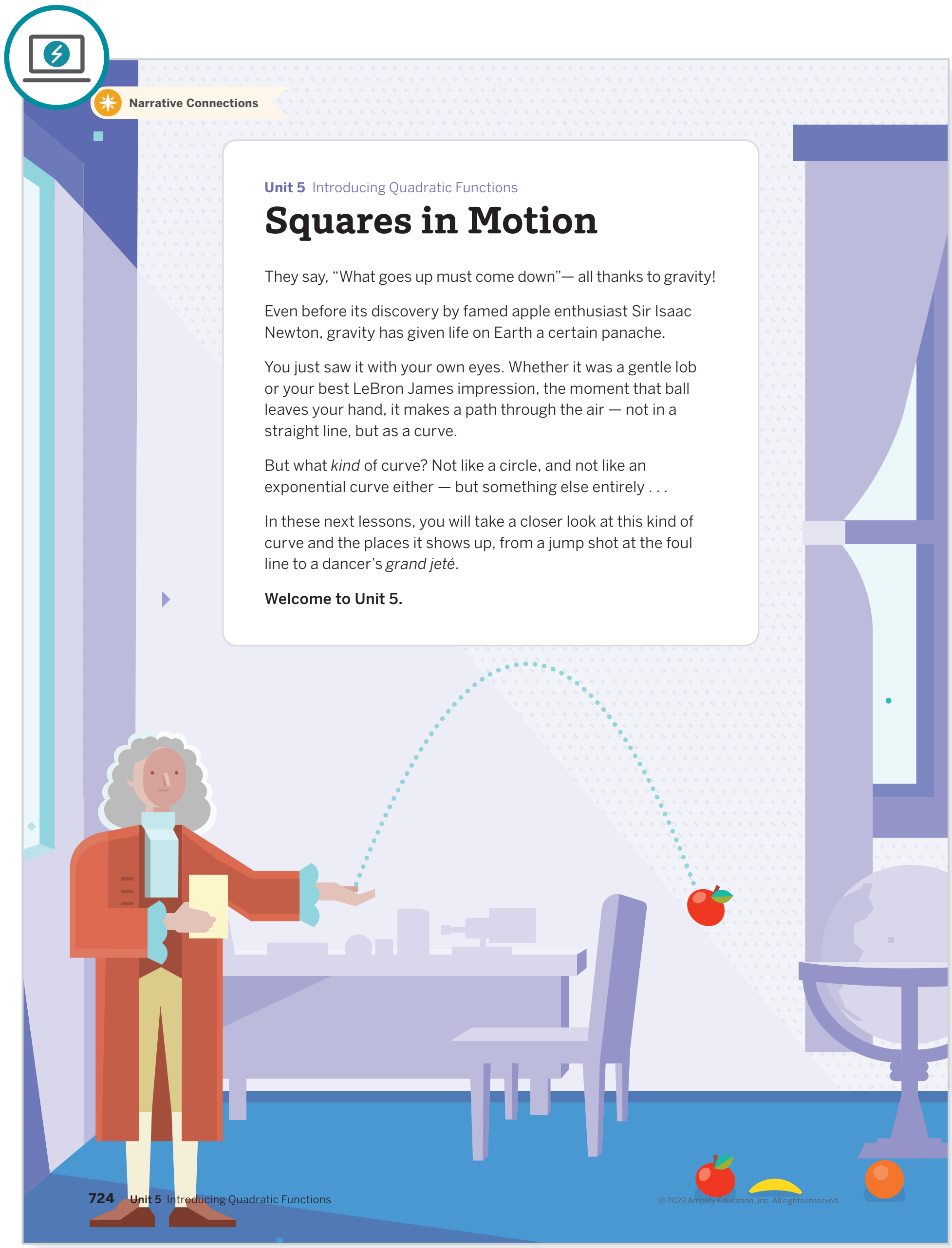
- "How did the position of the cannon affect Foofoo's path?" Answers may vary.
- "In which diagram was Foofoo launched the farthest? The highest?" Answers may vary.
- "How is Foofoo's path different when launched from the Moon?" Answers may vary.

Highlight that the presence of gravity on Earth causes Foofoo to fall down, no matter how far or from what height he is launched. If launched from the Moon at a fast enough speed, Foofoo might never land.

Independent | 5 min

Summary Squares in Motion

Review and synthesize how the general motion of a projectile is curved in nature due to gravity.



Narrative Connections

Unit 5 Introducing Quadratic Functions

Squares in Motion

They say, "What goes up must come down"— all thanks to gravity!

Even before its discovery by famed apple enthusiast Sir Isaac Newton, gravity has given life on Earth a certain panache.

You just saw it with your own eyes. Whether it was a gentle lob or your best LeBron James impression, the moment that ball leaves your hand, it makes a path through the air — not in a straight line, but as a curve.

But what *kind* of curve? Not like a circle, and not like an exponential curve either — but something else entirely . . .

In these next lessons, you will take a closer look at this kind of curve and the places it shows up, from a jump shot at the foul line to a dancer's *grand jeté*.

Welcome to Unit 5.

724 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display Foofoo's path on AMPs so that student's can see Foofoo's launch on Earth and the Moon.

Ask:

- "How are the paths of Foofoo's launches on Earth different from each other? What factors influence these differences?"
- "All the paths on Earth seem to have a similar shape. Why do you think this is so?"
- "What factors influenced where Foofoo landed after he was launched from the cannon?"
- "What effect does the absence of gravity have on Foofoo's launch in space?"

Highlight that when an object is launched, it follows a path that curves toward the Earth due to the force of gravity.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

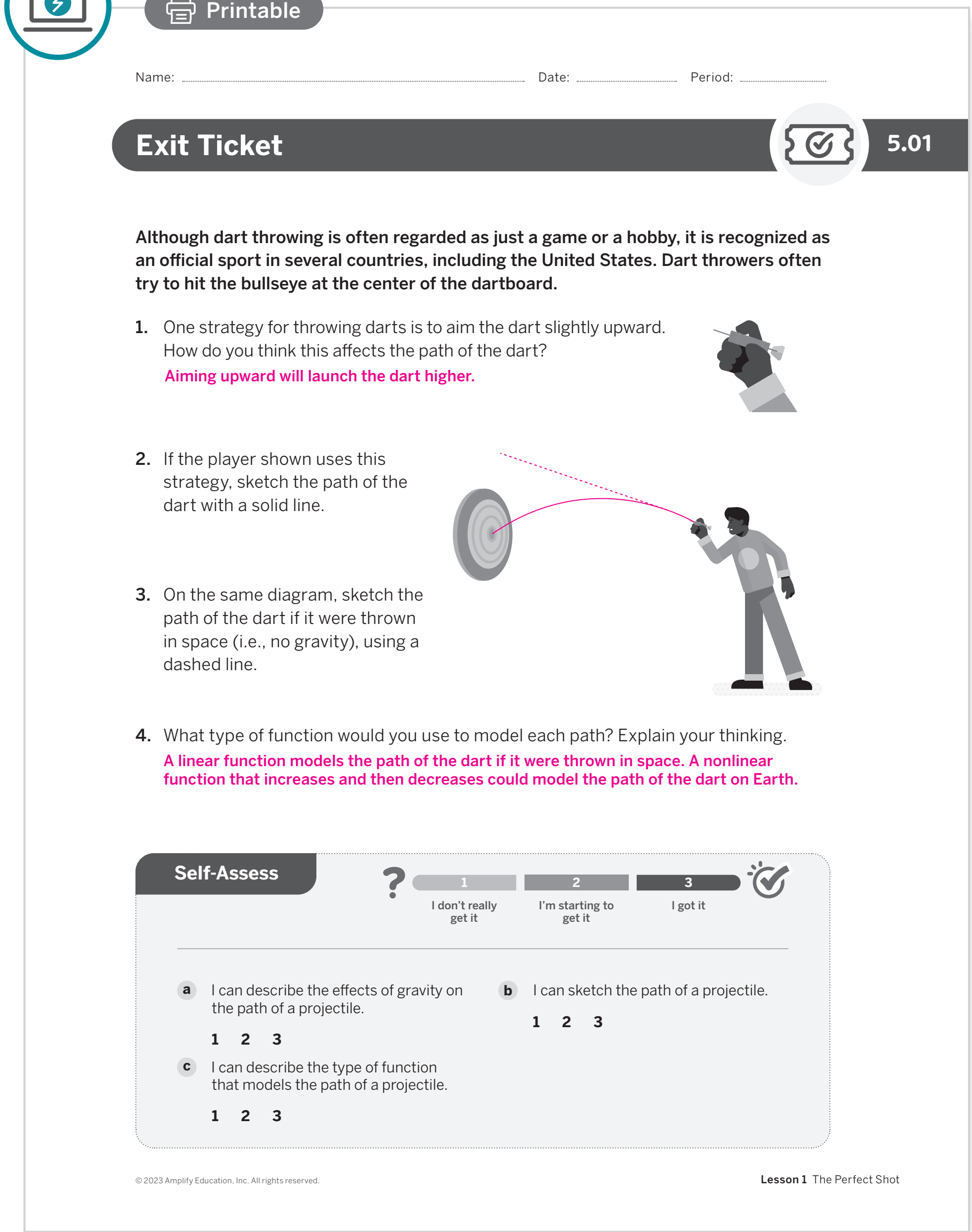
- "What factors affect the path of a projectile?"
- "Why does the path of a projectile curve downward?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by sketching the possible trajectory of a thrown dart and describing a function to represent it.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 5.01

Although dart throwing is often regarded as just a game or a hobby, it is recognized as an official sport in several countries, including the United States. Dart throwers often try to hit the bullseye at the center of the dartboard.

1. One strategy for throwing darts is to aim the dart slightly upward. How do you think this affects the path of the dart?
   Aiming upward will launch the dart higher.

2. If the player shown uses this strategy, sketch the path of the dart with a solid line.

3. On the same diagram, sketch the path of the dart if it were thrown in space (i.e., no gravity), using a dashed line.

4. What type of function would you use to model each path? Explain your thinking.
   A linear function models the path of the dart if it were thrown in space. A nonlinear function that increases and then decreases could model the path of the dart on Earth.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can describe the effects of gravity on the path of a projectile.
1 2 3

b I can sketch the path of a projectile.
1 2 3

c I can describe the type of function that models the path of a projectile.
1 2 3

Lesson 1 The Perfect Shot

Success looks like . . .

- **Goal:** Sketching the trajectory of a projectile.
  - » Sketching the path of the dart in Problem 2.
- **Language Goal:** Describing the type of function that models projectile motion. **(Speaking and Listening, Reading and Writing)**
  - » Explaining which type of function models each path in Problem 3.
- **Language Goal:** Explaining the effects of gravity on the trajectory of a projectile. **(Speaking and Listening, Reading and Writing)**
  - » Sketching the path of the dart as if it were thrown in space in Problem 3.

Suggested next steps

If students are unable to describe the effect of the given strategy in Problem 1, consider:

- Reviewing Activity 2, Problem 1.

If students are unable to sketch the paths of the dart in Problems 2 and 3, consider:

- Reviewing Activity 2, Problems 3 and 4.

If students are unable to describe the function that models the path of the dart in Problem 4, consider:

- Reviewing Activity 2, Problems 3 and 4.

Professional Learning

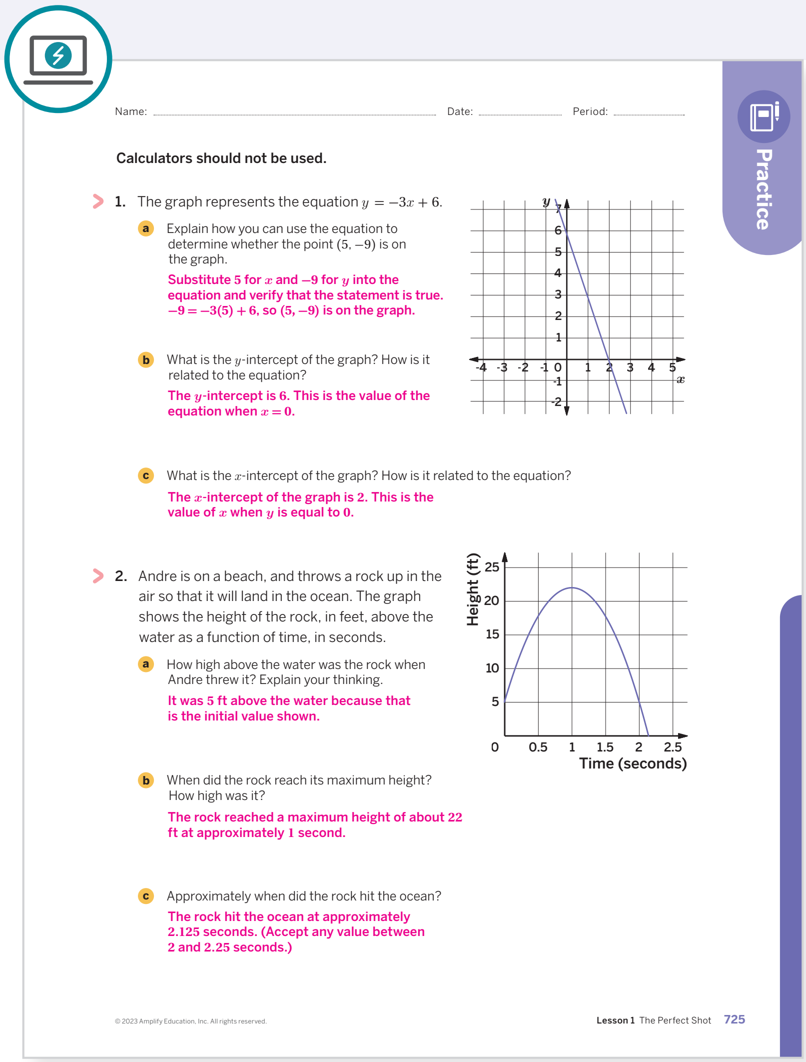
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In what ways did using different shooting techniques in Activity 1 go as planned?
- In this lesson, students explore the path of a projectile. How will that support their understanding of key features of quadratic functions?

Practice

Independent



Name: Date: Period:

Calculators should not be used.

1. The graph represents the equation $y = -3x + 6$.

   a. Explain how you can use the equation to determine whether the point $(5, -9)$ is on the graph.

   Substitute 5 for x and −9 for y into the equation and verify that the statement is true. $-9 = -3(5) + 6$, so $(5, -9)$ is on the graph.

   b. What is the y-intercept of the graph? How is it related to the equation?

   The y-intercept is 6. This is the value of the equation when $x = 0$.

   c. What is the x-intercept of the graph? How is it related to the equation?

   The x-intercept of the graph is 2. This is the value of x when y is equal to 0.

2. Andre is on a beach, and throws a rock up in the air so that it will land in the ocean. The graph shows the height of the rock, in feet, above the water as a function of time, in seconds.

   a. How high above the water was the rock when Andre threw it? Explain your thinking.

   It was 5 ft above the water because that is the initial value shown.

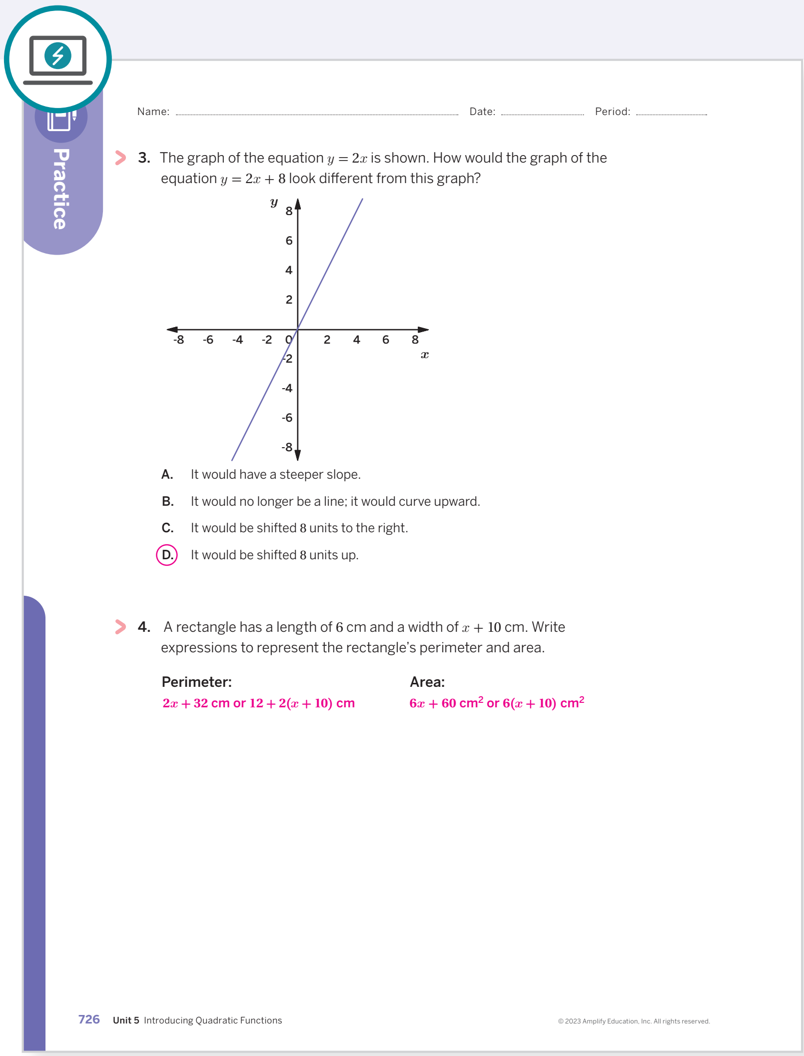
   b. When did the rock reach its maximum height? How high was it?

   The rock reached a maximum height of about 22 ft at approximately 1 second.

   c. Approximately when did the rock hit the ocean?

   The rock hit the ocean at approximately 2.125 seconds. (Accept any value between 2 and 2.25 seconds.)

Lesson 1 The Perfect Shot 725



Name: Date: Period:

3. The graph of the equation $y = 2x$ is shown. How would the graph of the equation $y = 2x + 8$ look different from this graph?

   A. It would have a steeper slope.

   B. It would no longer be a line; it would curve upward.

   C. It would be shifted 8 units to the right.

   [✓ D.] It would be shifted 8 units up.

4. A rectangle has a length of 6 cm and a width of $x + 10$ cm. Write expressions to represent the rectangle's perimeter and area.

Perimeter:	**Area:**
$2x + 32$ cm or $12 + 2(x + 10)$ cm	**$6x + 60$ cm^2 or $6(x + 10)$ cm^2**

726 **Unit 5** Introducing Quadratic Functions

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 1 Lesson 6	2
	2	Unit 3 Lesson 8	2
	3	Unit 1 Lesson 6	1
Formative ⚡	4	Unit 5 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

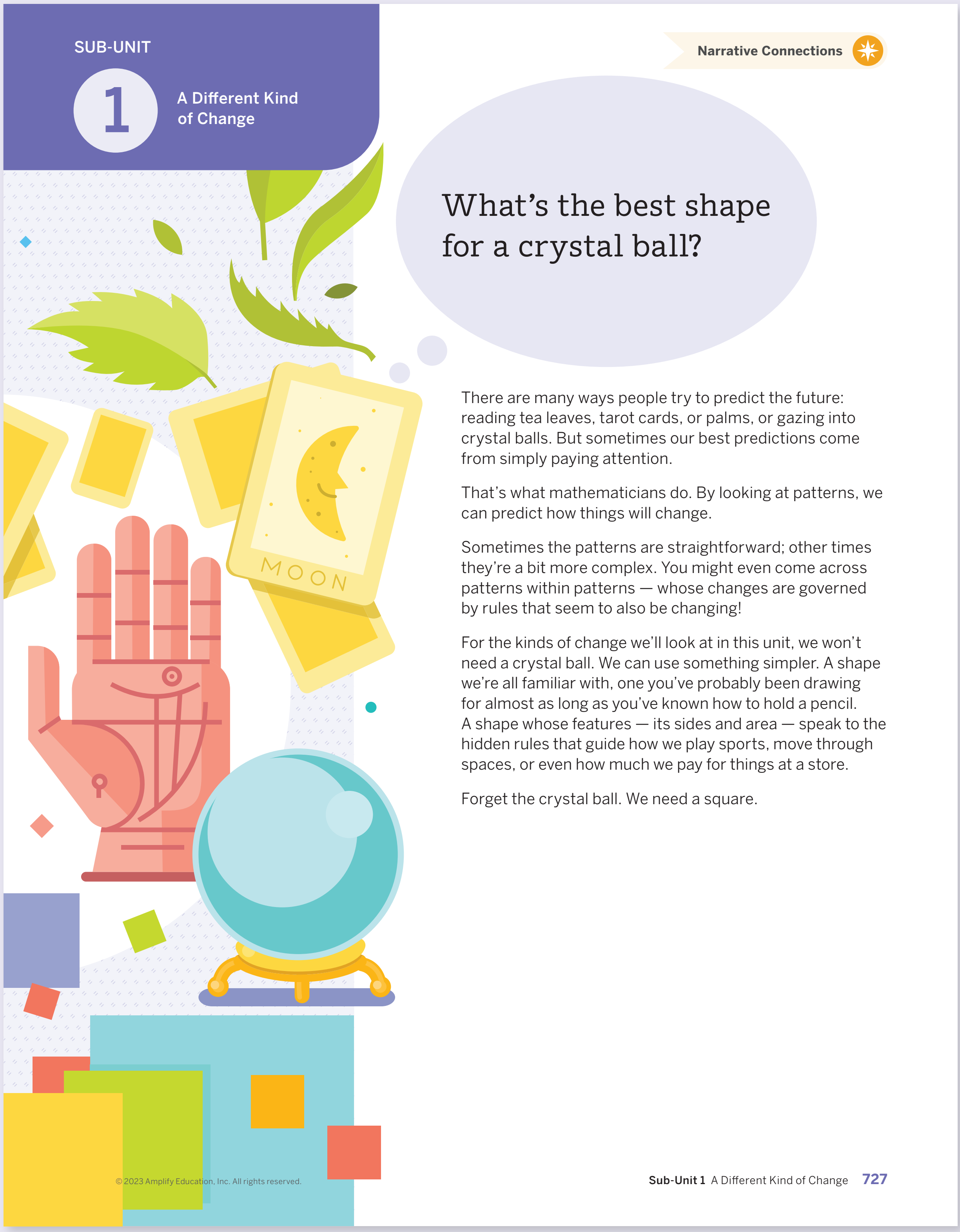
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 1

A Different Kind of Change

Whole Class

In this Sub-Unit, students build a conceptual understanding of quadratic expressions and functions by exploring and building patterns involving squared terms.



SUB-UNIT

1 A Different Kind of Change

Narrative Connections

What's the best shape for a crystal ball?

There are many ways people try to predict the future: reading tea leaves, tarot cards, or palms, or gazing into crystal balls. But sometimes our best predictions come from simply paying attention.

That's what mathematicians do. By looking at patterns, we can predict how things will change.

Sometimes the patterns are straightforward; other times they're a bit more complex. You might even come across patterns within patterns — whose changes are governed by rules that seem to also be changing!

For the kinds of change we'll look at in this unit, we won't need a crystal ball. We can use something simpler. A shape we're all familiar with, one you've probably been drawing for almost as long as you've known how to hold a pencil. A shape whose features — its sides and area — speak to the hidden rules that guide how we play sports, move through spaces, or even how much we pay for things at a store.

Forget the crystal ball. We need a square.

Sub-Unit 1 A Different Kind of Change 727

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how squares illustrate a new, special kind of relationship in the following places:

- **Lesson 3, Activities 1–2:** Growing Squares, First and Second Differences
- **Lesson 4, Activities 1–3:** Racing Boards, Color Squares, Checkerboards
- **Lesson 5, Activities 1–3:** Squares on Squares, Expanding Squares, Expressing Quadratic Patterns

UNIT 5 | **LESSON 2**

A Different Kind of Change

Let's determine the rectangle with the greatest area.

Focus

Goals

1. Create drawings, tables, and graphs that represent the area of a garden.
2. **Language Goal:** Recognize a situation represented by a graph that increases then decreases. **(Speaking and Listening, Reading and Writing)**

Rigor

- Students build **conceptual understanding** of nonlinear relationships.
- Students **apply** nonlinear relationships in the context of maximizing the area of a rectangle with a fixed perimeter.

Coherence

Today

Students encounter a scenario where a quantity increases, then decreases. They do not yet have a name for this new pattern of change, but they recognize that it is nonlinear, and is unlike an exponential function.

Previously

In Lesson 1, the unit launch, students explored projectile motion, took turns launching projectiles, and observed strategies for making a shot.

Coming Soon

In Lessons 4 and 5, students will identify this new type of change as a quadratic relationship, and write quadratic expressions to model patterns of growth.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min	5 min
Independent	Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*

Math Language Development

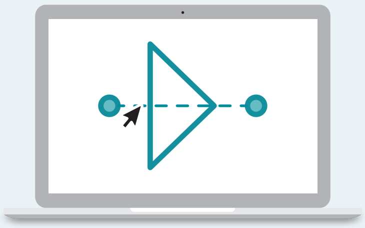
Review words

- *exponential function*
- *growth factor*
- *nonlinear relationship*

Amps | **Featured Activity**

Activity 1
Digital Recreational Field

Students drag a point on the perimeter of a rectangular multi-purpose recreational field to compile possible dimensions of a field with a fixed perimeter.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might resist thinking metacognitively as they work with the quantitative measures length, width, and area. Ask students to make sense of the quantities and the relationships among them as they analyze the situation. After organizing the results, students need to find the motivation to probe into the more abstract conclusion about how to maximize the area.

Modifications to Pacing

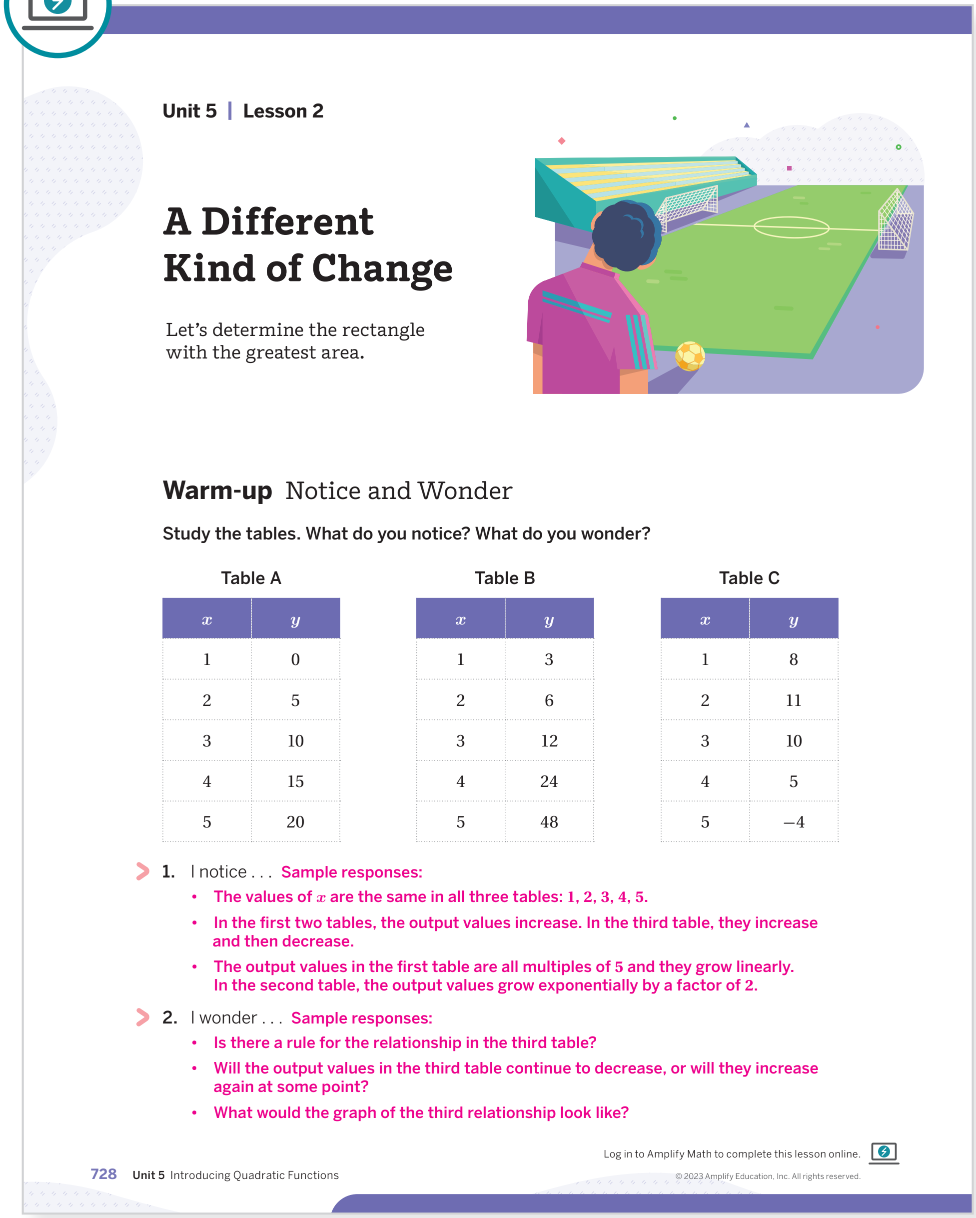
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Table B may be omitted.
- In **Activity 1**, Problem 3, three rows of the table may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students observe and make use of the structure to contrast a quadratic pattern of change with familiar linear and exponential patterns. The term *quadratic* has not been introduced yet.



Unit 5 | Lesson 2

A Different Kind of Change

Let's determine the rectangle with the greatest area.

Warm-up Notice and Wonder

Study the tables. What do you notice? What do you wonder?

Table A

x	y
1	0
2	5
3	10
4	15
5	20

Table B

x	y
1	3
2	6
3	12
4	24
5	48

Table C

x	y
1	8
2	11
3	10
4	5
5	−4

1. I notice . . . **Sample responses:**
   - **The values of x are the same in all three tables: 1, 2, 3, 4, 5.**
   - **In the first two tables, the output values increase. In the third table, they increase and then decrease.**
   - **The output values in the first table are all multiples of 5 and they grow linearly. In the second table, the output values grow exponentially by a factor of 2.**
2. I wonder . . . **Sample responses:**
   - **Is there a rule for the relationship in the third table?**
   - **Will the output values in the third table continue to decrease, or will they increase again at some point?**
   - **What would the graph of the third relationship look like?**

Log in to Amplify Math to complete this lesson online.

728 **Unit 5** Introducing Quadratic Functions

1 Launch

Conduct the *Notice and Wonder* routine. Prompt students to examine each table individually first. Then have them compare the tables before they write what they notice and wonder.

2 Monitor

Help students get started by having them determine how consecutive values of y change.

Look for points of confusion:

- **Trying to determine a constant rate of change for Table B.** Ask, "Where have you seen a function similar to this one before?"
- **Trying to determine a constant rate of change for Table C.** Have students record why their method is not working.

Look for productive strategies:

- Drawing arrows alongside consecutive values of y to determine the first differences in Table A.
- Dividing consecutive values of y to calculate the growth factor in Table B.

3 Connect

Have students share what they notice and wonder. Record and display some of the responses.

Ask, "Is there anything on these lists that you are still wondering about?" Encourage students to respectfully disagree, ask for clarification, point out contradicting information, etc.

Highlight that Table C shows a different relationship between the values of x and y than Tables A and B.

Math Language Development

MLR2: Collect and Display

During the Connect, display or provide access to the Anchor Chart PDF, *Sentence Stems, Describing My Thinking* to support students as they share what they noticed and wondered about the tables of values. As they share their responses, display the language they use on a visual display. Continue adding to this display throughout the unit and invite students to borrow language from the display during class discussions.

Power-up

To power up students' ability to write algebraic expressions to represent the perimeter and area of rectangles, have students complete:

Recall that to determine the *perimeter* of a rectangle, you can use the formula $P = 2(\ell + w)$. To determine the rectangle's area, you can use the formula $A = \ell \cdot w$, where ℓ represents the length and w represents the width. A rectangle has a length of 3 and a width of $x + 1$. Write expressions to represent its perimeter and area.

Perimeter:	**Area:**
$2(3 + x + 1)$ or $2x + 8$	$3(x + 1)$ or $3x + 3$

Use: Before Activity 1

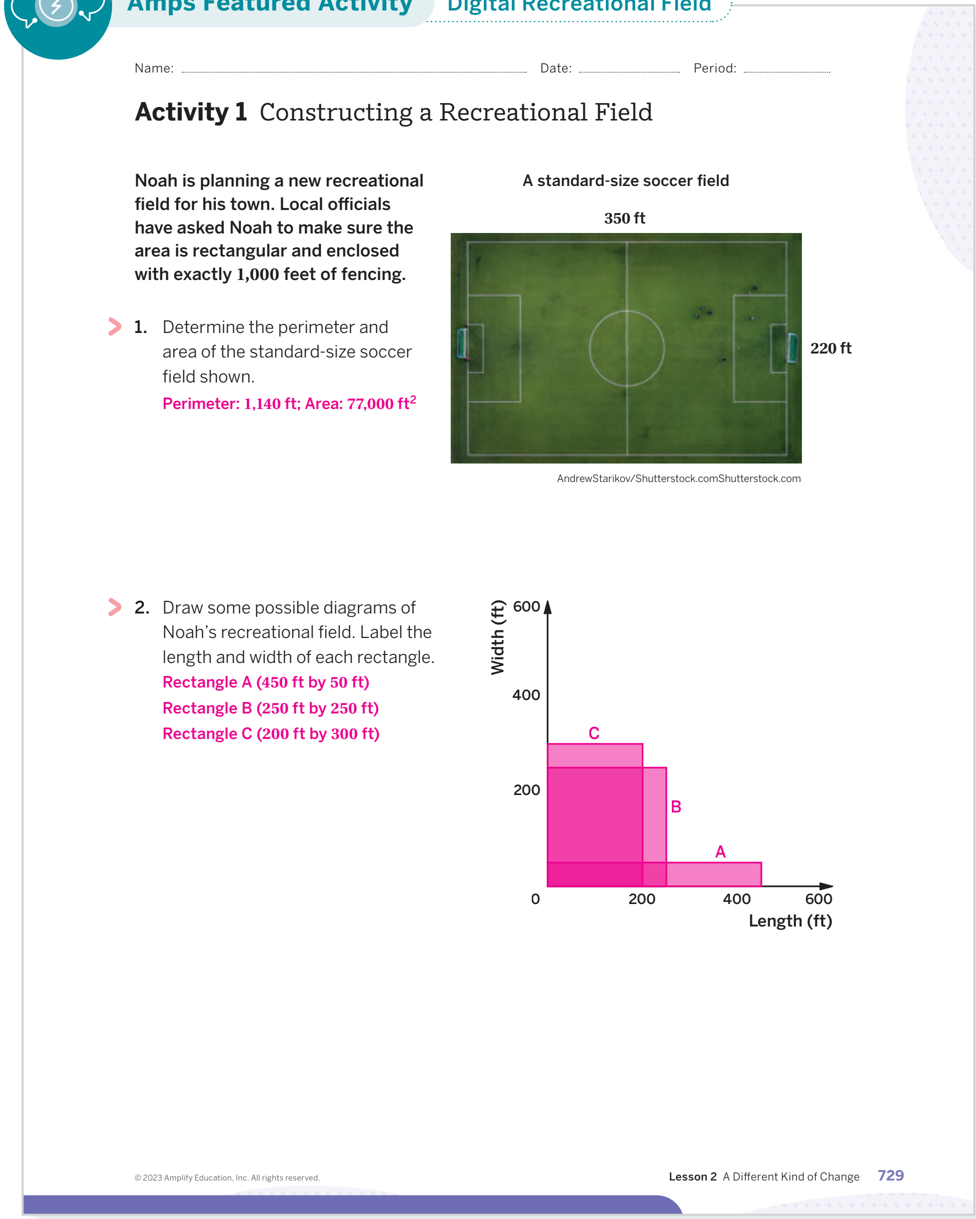
Informed by: Performance on Lesson 1, Practice Problem 4

Independent | 20 min

Activity 1 Constructing a Recreational Field

Given a rectangle with a fixed perimeter, students experiment how changing one dimension of the rectangle affects its area.

Amps Featured Activity Digital Recreational Field



Name: ______ Date: ______ Period: ______

Activity 1 Constructing a Recreational Field

Noah is planning a new recreational field for his town. Local officials have asked Noah to make sure the area is rectangular and enclosed with exactly 1,000 feet of fencing.

1. Determine the perimeter and area of the standard-size soccer field shown.
   Perimeter: 1,140 ft; Area: 77,000 ft^2

A standard-size soccer field

AndrewStarikov/Shutterstock.comShutterstock.com

2. Draw some possible diagrams of Noah's recreational field. Label the length and width of each rectangle.
   Rectangle A (450 ft by 50 ft)
   Rectangle B (250 ft by 250 ft)
   Rectangle C (200 ft by 300 ft)

1 Launch

Activate students' prior knowledge by asking them how to calculate the perimeter and area of the soccer field.

2 Monitor

Help students get started by providing the formulas for perimeter and area of a rectangle.

Look for points of confusion:

- **Excluding a square as a rectangle.** Ask a series of questions that lead students to the definition of a rectangle.

Look for productive strategies:

- Using the same values in the beginning and end of the table in Problem 3, but in reverse order.
- Using the formula for perimeter, the fixed perimeter of 1,000, and a chosen length to calculate the width.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can drag a point on the perimeter of a rectangular multi-purpose recreational field to compile possible dimensions of a field with a fixed perimeter.

Extension: *Math Enrichment*

Ask students to determine the length and width of a rectangular field that would produce the greatest possible area if the perimeter must remain fixed at 1,200 ft, 1,400 ft, or n ft. 300 ft; 350 ft; $\frac{n}{4}$ ft

Math Language Development

MLR8: Discussion Supports — Restate It!

During the Connect, as students share their observations, ask the class to listen carefully and ask volunteers to restate what they hear their classmate say using their developing mathematical language. Ask the original speaker if their peer was accurately able to restate their thinking. For example:

If a student says . . .	A classmate could say . . .
"The field gets bigger and bigger until a certain point, when it becomes smaller."	"I hear you saying the field gets bigger — do you mean the area? At that 'certain point' when it changes — do you mean when the length and width are the same?"

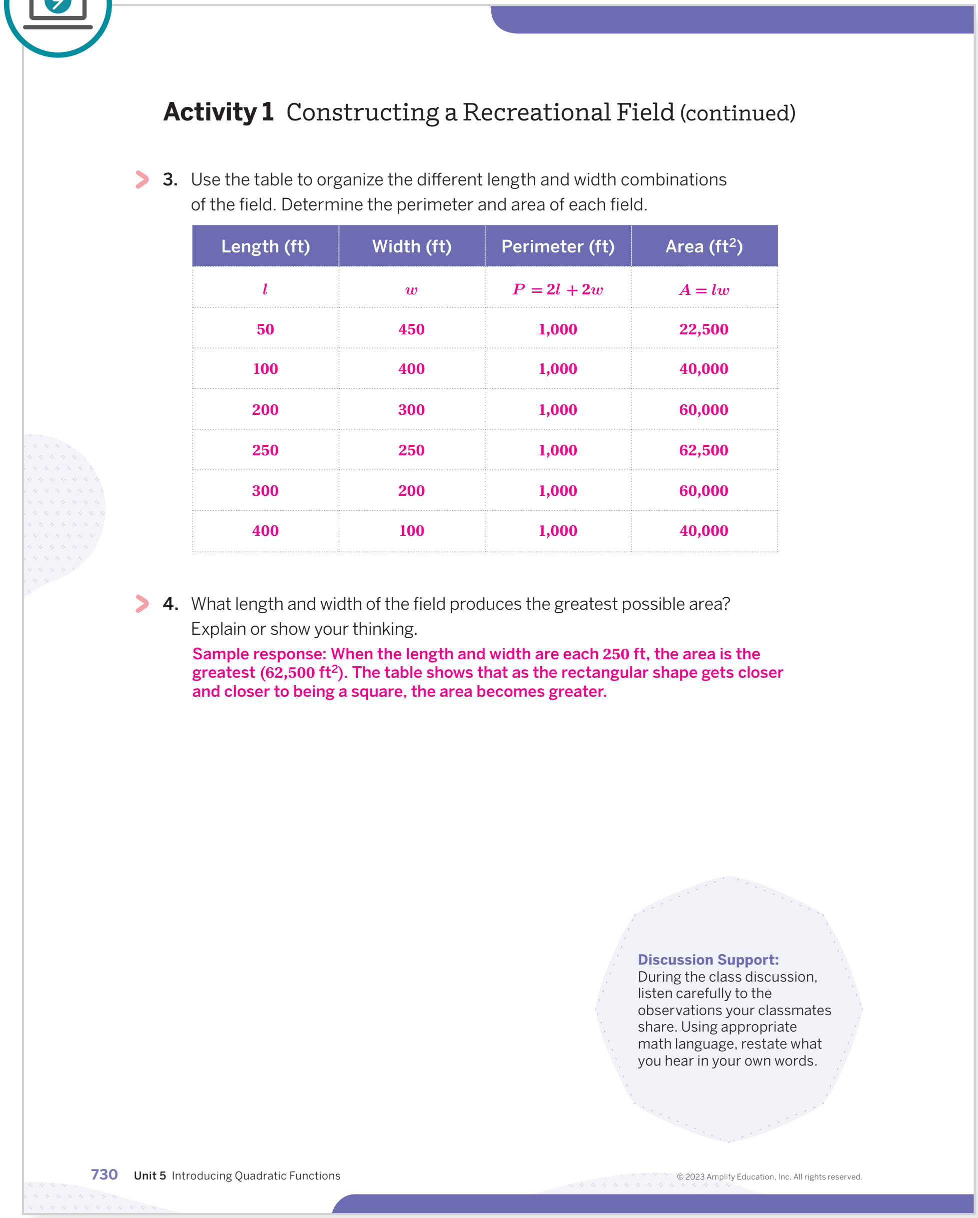
English Learners

Encourage students to use language from the class display as they restate their thinking.

Independent | 20 min

Activity 1 Constructing a Recreational Field (continued)

Given a rectangle with a fixed perimeter, students experiment how changing one dimension of the rectangle affects its area.



Activity 1 Constructing a Recreational Field (continued)

3. Use the table to organize the different length and width combinations of the field. Determine the perimeter and area of each field.

Length (ft)	Width (ft)	Perimeter (ft)	Area (ft^2)
l	w	$P = 2l + 2w$	$A = lw$
50	450	1,000	22,500
100	400	1,000	40,000
200	300	1,000	60,000
250	250	1,000	62,500
300	200	1,000	60,000
400	100	1,000	40,000

4. What length and width of the field produces the greatest possible area? Explain or show your thinking.
Sample response: When the length and width are each 250 ft, the area is the greatest (62,500 ft^2). The table shows that as the rectangular shape gets closer and closer to being a square, the area becomes greater.

Discussion Support:
During the class discussion, listen carefully to the observations your classmates share. Using appropriate math language, restate what you hear in your own words.

730 **Unit 5** Introducing Quadratic Functions

3 Connect

Display possible dimensions for the field.

Have students share their observations about how changing the lengths and widths affected the area of the field.

Ask:

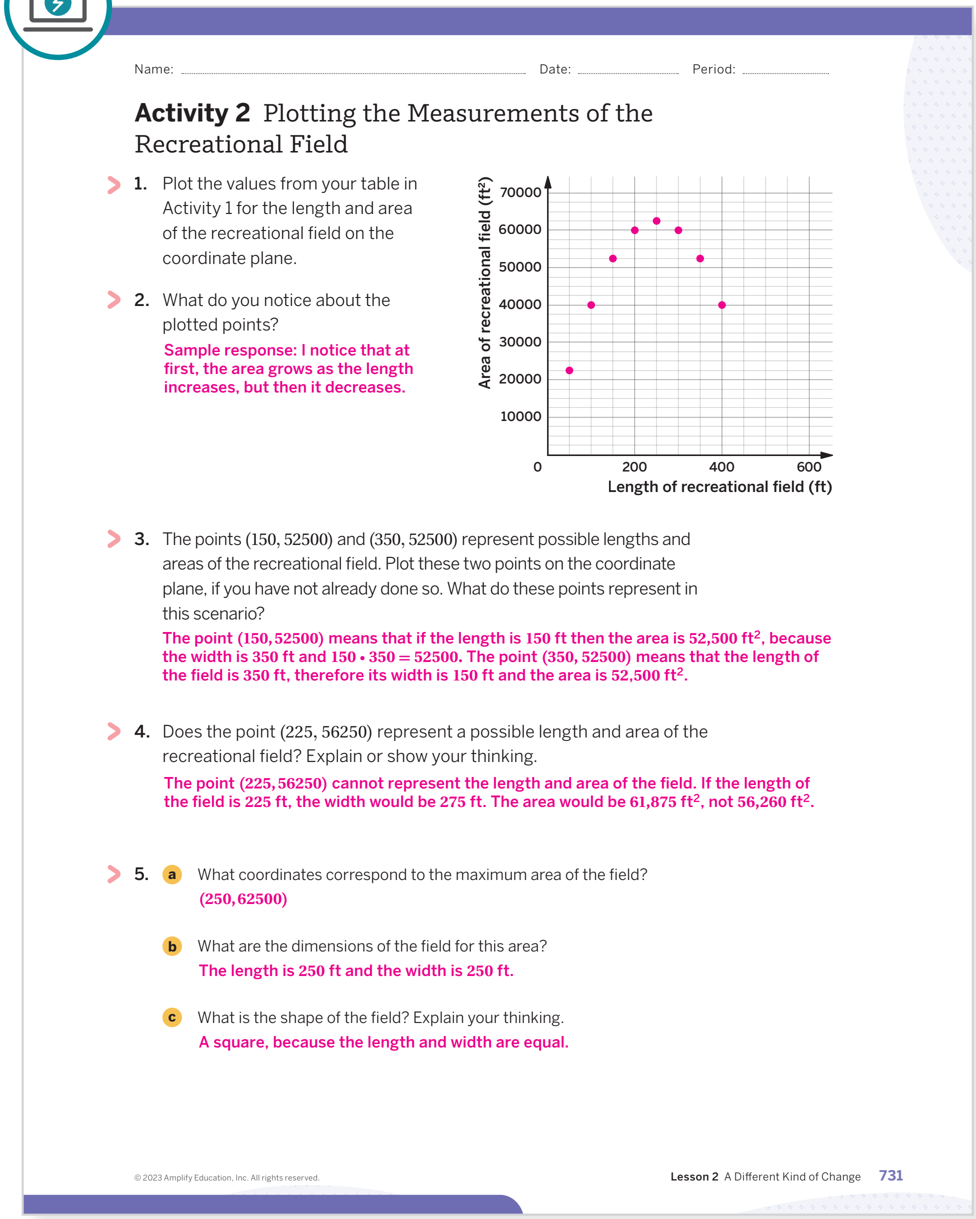
- "What type of relationship is there between length and width?" There is a linear relationship.
- "As the length of the rectangle increases, does the area increase?" Yes, the area increases up until a certain length, and then the area decreases as the length continues to increase.

Highlight that the relationship between length and area is neither linear nor exponential. For now, students should call this a *nonlinear relationship*.

Independent | 10 min

Activity 2 Plotting the Measurements of the Recreational Field

Students plot points that represent the relationship between a side length and the area of a rectangle with fixed perimeter, and encounter a quadratic (nonlinear) graph.



Name: Date: Period:

Activity 2 Plotting the Measurements of the Recreational Field

1. Plot the values from your table in Activity 1 for the length and area of the recreational field on the coordinate plane.

2. What do you notice about the plotted points?

   Sample response: I notice that at first, the area grows as the length increases, but then it decreases.

3. The points $(150, 52500)$ and $(350, 52500)$ represent possible lengths and areas of the recreational field. Plot these two points on the coordinate plane, if you have not already done so. What do these points represent in this scenario?

   The point $(150, 52500)$ means that if the length is 150 ft then the area is $52{,}500$ ft^2, because the width is 350 ft and $150 \cdot 350 = 52500$. The point $(350, 52500)$ means that the length of the field is 350 ft, therefore its width is 150 ft and the area is $52{,}500$ ft^2.

4. Does the point $(225, 56250)$ represent a possible length and area of the recreational field? Explain or show your thinking.

   The point $(225, 56250)$ cannot represent the length and area of the field. If the length of the field is 225 ft, the width would be 275 ft. The area would be $61{,}875$ ft^2, not $56{,}260$ ft^2.

5. a. What coordinates correspond to the maximum area of the field?

   $(250, 62500)$

   b. What are the dimensions of the field for this area?

   The length is 250 ft and the width is 250 ft.

   c. What is the shape of the field? Explain your thinking.

   A square, because the length and width are equal.

1 Launch

Display the completed table from Activity 1. Say, "Let's observe the shape of the graph of this nonlinear relationship between length and area."

2 Monitor

Help students get started by highlighting the two columns in the table that will be used to plot the relationships.

Look for points of confusion:

- **Thinking that the point $(225, 56250)$ represents a possible length and area of the field in Problem 4.** Ask students whether the width needed for an area of $56{,}250$ would give a perimeter of $1{,}000$.

Look for productive strategies:

- Using the formula $w = 500 - l$ to calculate the width, given the length.
- Plotting additional points by using the symmetry of the graph.

3 Connect

Display a graph with some of the points plotted and amend it with additional points that students provide.

Highlight that the relationship between length and area is neither linear nor exponential, and that students should have observed that the output increases and decreases in a non-random way.

Ask, "If you plot points for every whole-number length between 0 and 250, what do you think the graph would look like?"
An upside down U or an arch.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Consider providing a graph with pre-labeled points so students can focus on analyzing them and have them begin the activity with Problem 2. As students complete Problems 3 and 4, consider displaying the formulas for the perimeter and area of a rectangle. You may wish to display the formula $w = 500 - \ell$, but tell students this formula only works for this scenario because the sum of the length and width must be 500 ft.

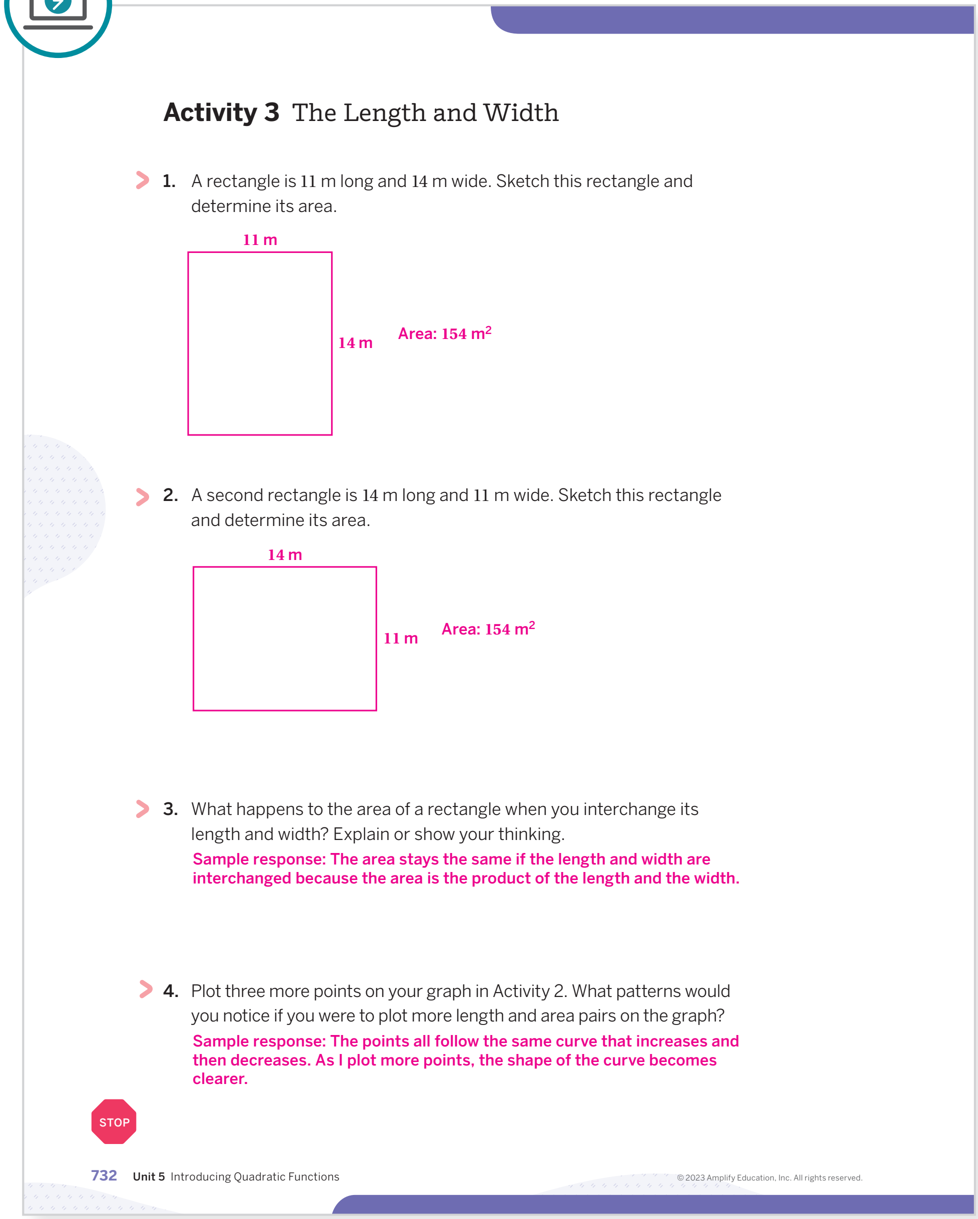
Extension: ***Math Enrichment***

Have students determine whether the point $(0, 0)$ makes sense in this context and explain their thinking. The point $(0, 0)$ would mean that the length and width would both be 0, which is not realistic for a field.

Pairs | 5 min

Activity 3 The Length and Width

Students determine the area does not change when interchanging the length and width of a rectangle.



Activity 3 The Length and Width

1. A rectangle is 11 m long and 14 m wide. Sketch this rectangle and determine its area.

11 m

14 m

Area: 154 m^2

2. A second rectangle is 14 m long and 11 m wide. Sketch this rectangle and determine its area.

14 m

11 m

Area: 154 m^2

3. What happens to the area of a rectangle when you interchange its length and width? Explain or show your thinking.
Sample response: The area stays the same if the length and width are interchanged because the area is the product of the length and the width.

4. Plot three more points on your graph in Activity 2. What patterns would you notice if you were to plot more length and area pairs on the graph?
Sample response: The points all follow the same curve that increases and then decreases. As I plot more points, the shape of the curve becomes clearer.

STOP

732 Unit 5 Introducing Quadratic Functions

1 Launch

Have students complete Problems 1 and 2 independently before sharing their responses with a partner.

2 Monitor

Help students get started by providing them with the formula for area of a rectangle.

Look for points of confusion:

- **Plotting points that do not have a perimeter of $1{,}000$ ft in Problem 4.** Remind students that the perimeter of the recreational field is a fixed perimeter of $1{,}000$ ft.

Look for productive strategies:

- Recognizing that they can create two new points by using the length and width of one rectangle as two different values of x, and the rectangle's area as the two values of y.

3 Connect

Have students share the three points they plot in Problem 4.

Highlight that even though the output value of the function increases and then decreases, it does so in a way that is not random. Some output values are the same for different input values.

Ask, "Can you determine whether a point represents the length and area of the field by plotting it?" If it is far away from other points, then it likely does not represent the length and area of the field. If it follows the general trend, then I need to verify by determining whether the perimeter is $1{,}000$ ft.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students draw their own rectangles for Problems 1 and 2, provide pre-drawn and pre-labeled rectangles and have students describe what they notice. Then have them begin the activity with Problem 3.

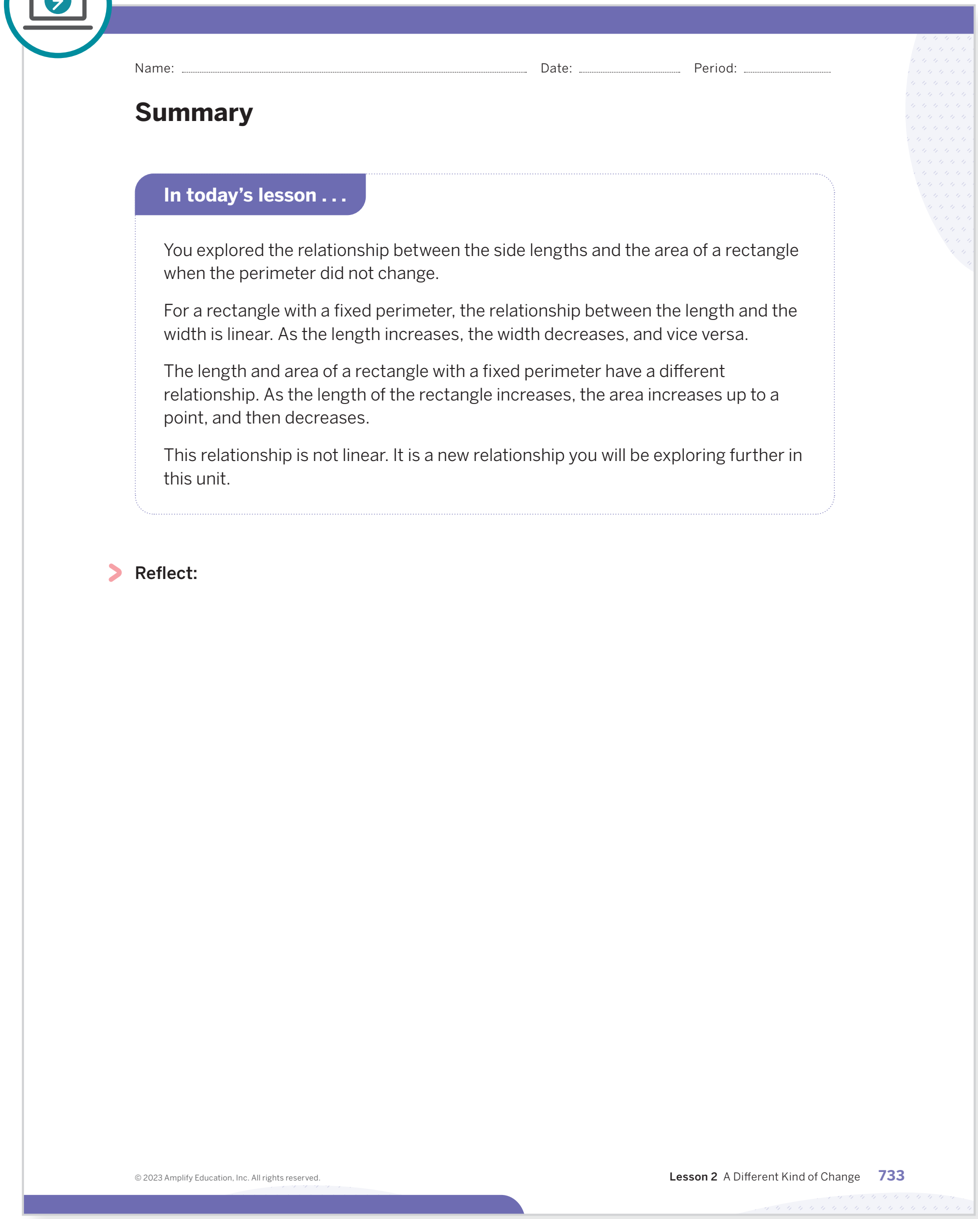
Extension: *Math Enrichment*

Have students determine the domain of the relationship graphed in Activity 2 and explain their thinking. 0 to 500; Sample response: After 250, the length begins to decrease.

Whole Class | 5 min

Summary

Review and synthesize the relationship between the length and area of rectangles that have a fixed perimeter.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored the relationship between the side lengths and the area of a rectangle when the perimeter did not change.

For a rectangle with a fixed perimeter, the relationship between the length and the width is linear. As the length increases, the width decreases, and vice versa.

The length and area of a rectangle with a fixed perimeter have a different relationship. As the length of the rectangle increases, the area increases up to a point, and then decreases.

This relationship is not linear. It is a new relationship you will be exploring further in this unit.

Reflect:

Lesson 2 A Different Kind of Change 733

Synthesize

Display the completed graph from Activity 2.

Highlight that, initially, as the length of the rectangle increases, the area also increases. However, at some point, as the length continues to increase, the area begins to decrease. Point out that this is a nonlinear relationship, yet it is unlike an exponential relationship.

Ask:

- "How would you describe the relationship between the side length and area?" Sample response: The graph makes an arch. The relationship is nonlinear. The change is positive in the beginning, but then is negative later.
- "What other unique features do you notice about the relationship's graph or table?" Sample response: There seems to be a maximum point where the graph switches from increasing to decreasing. If you draw a vertical line down the middle of the graph, the graph is symmetrical over this line.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

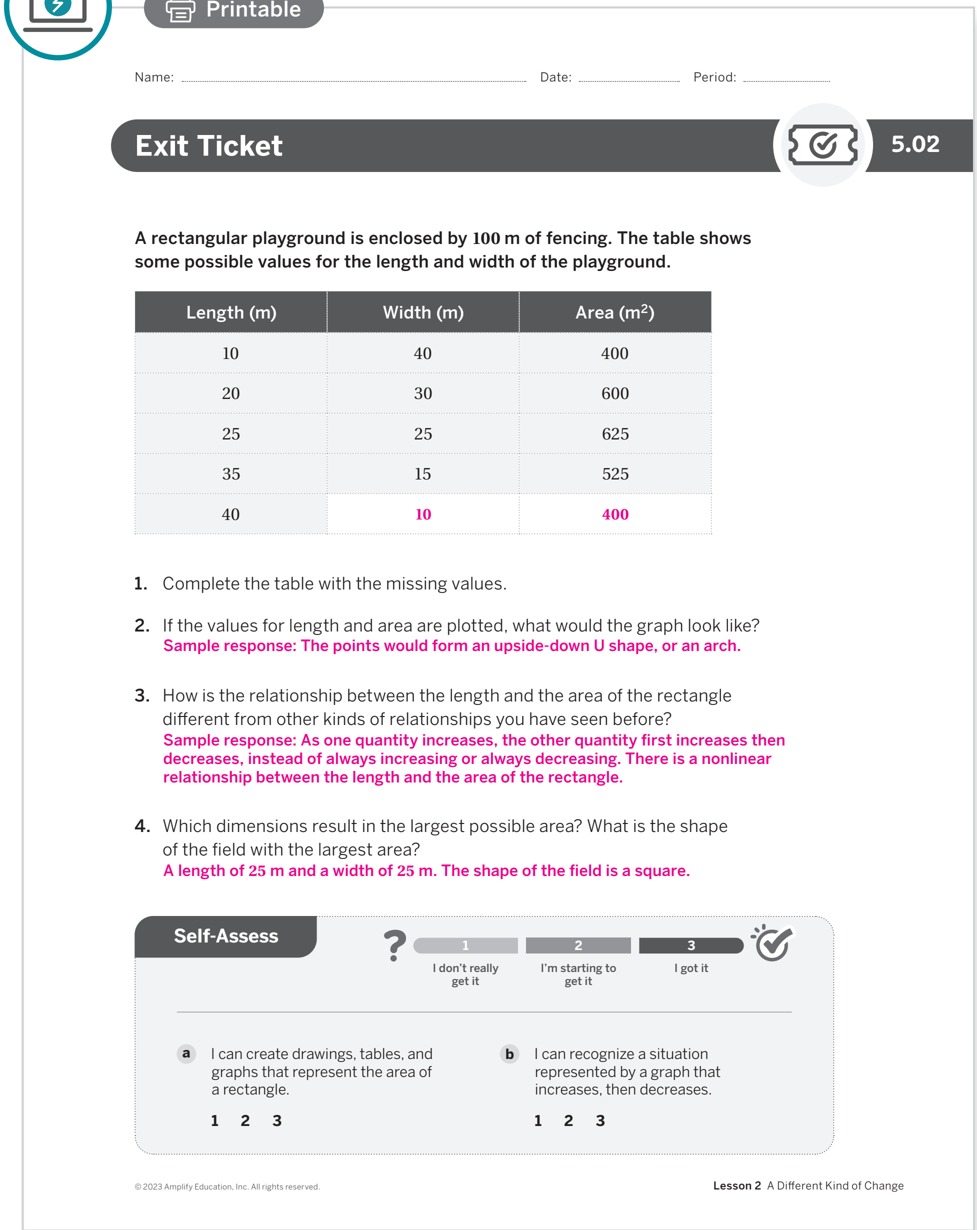
- "What strategies were helpful today when identifying a relationship as nonlinear?"
- "What strategies were helpful today when determining the greatest possible area of a rectangle with a fixed perimeter?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing the relationship between the length and area of rectangles that have a fixed perimeter.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 5.02

A rectangular playground is enclosed by 100 m of fencing. The table shows some possible values for the length and width of the playground.

Length (m)	Width (m)	Area (m^2)
10	40	400
20	30	600
25	25	625
35	15	525
40	10	400

1. Complete the table with the missing values.
2. If the values for length and area are plotted, what would the graph look like?
   Sample response: The points would form an upside-down U shape, or an arch.
3. How is the relationship between the length and the area of the rectangle different from other kinds of relationships you have seen before?
   Sample response: As one quantity increases, the other quantity first increases then decreases, instead of always increasing or always decreasing. There is a nonlinear relationship between the length and the area of the rectangle.
4. Which dimensions result in the largest possible area? What is the shape of the field with the largest area?
   A length of 25 m and a width of 25 m. The shape of the field is a square.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can create drawings, tables, and graphs that represent the area of a rectangle.
1 2 3

b I can recognize a situation represented by a graph that increases, then decreases.
1 2 3

Lesson 2 A Different Kind of Change

Success looks like . . .

- **Goal:** Creating drawings, tables, and graphs that represent the area of a rectangle.
  - » Completing the table related to length, width, and area in Problem 1.
- **Language Goal:** Recognizing a situation represented by a graph that increases then decreases. **(Speaking and Listening, Reading and Writing)**
  - » Explaining that as the length or area of the rectangle increases, the other quantity increases first and then decreases in Problem 3.

Suggested next steps

If students' explanation to Problem 2 is vague or incorrect, consider:

- Providing a blank graph to plot points.
- Referring to Activity 2 and its completed graph.
- Assigning Practice Problem 2.

If students' explanation to Problem 3 is vague or incorrect, consider:

- Having them draw quick sketches of a linear graph and an exponential graph, and then list the properties of each type of relationship.
- Reviewing Activity 2, Problem 2.
- Assigning Practice Problem 2.

If students do not identify the correct dimensions and shape in Problem 4, consider:

- Reviewing Activity 1, Problem 4.
- Drawing the rectangle with the dimensions they identified and asking them whether this shape shows the greatest possible area.
- Assigning Practice Problem 2.

Professional Learning

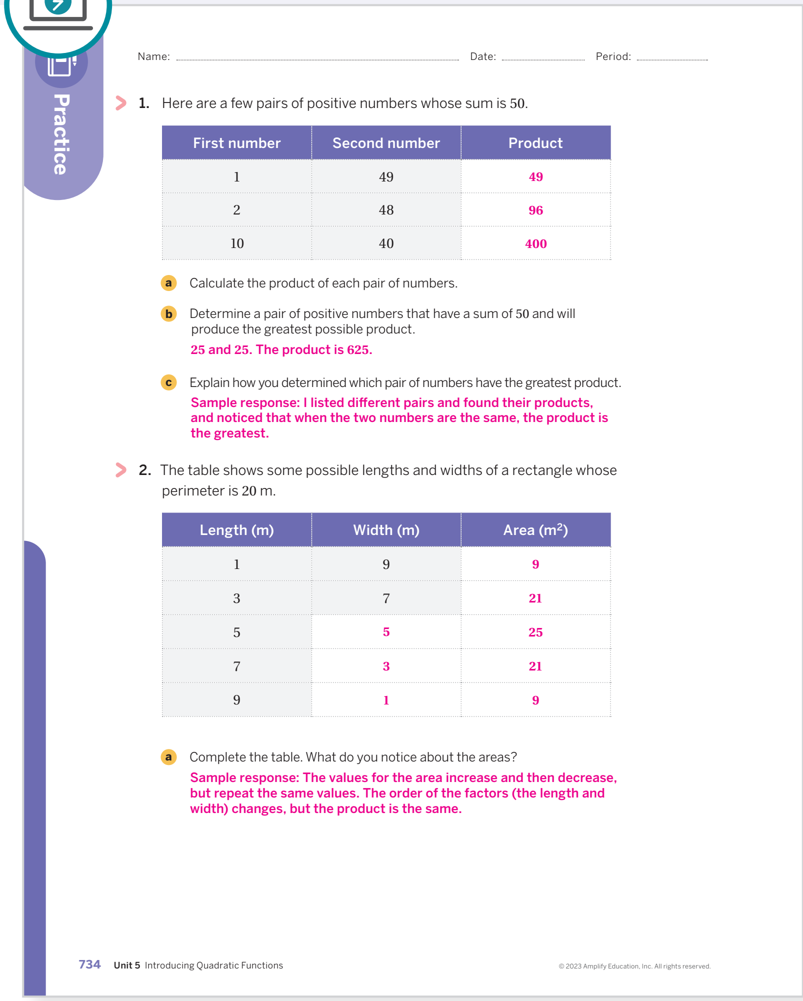
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was to recognize a nonlinear pattern represented by a graph that increases then decreases. How well did students accomplish this? What did you specifically do to help students accomplish it?
- How did determining the greatest possible area of a rectangle with fixed perimeter set students up to develop understanding of nonlinear relationships? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Here are a few pairs of positive numbers whose sum is 50.

First number	Second number	Product
1	49	49
2	48	96
10	40	400

a. Calculate the product of each pair of numbers.

b. Determine a pair of positive numbers that have a sum of 50 and will produce the greatest possible product.

25 and 25. The product is 625.

c. Explain how you determined which pair of numbers have the greatest product.

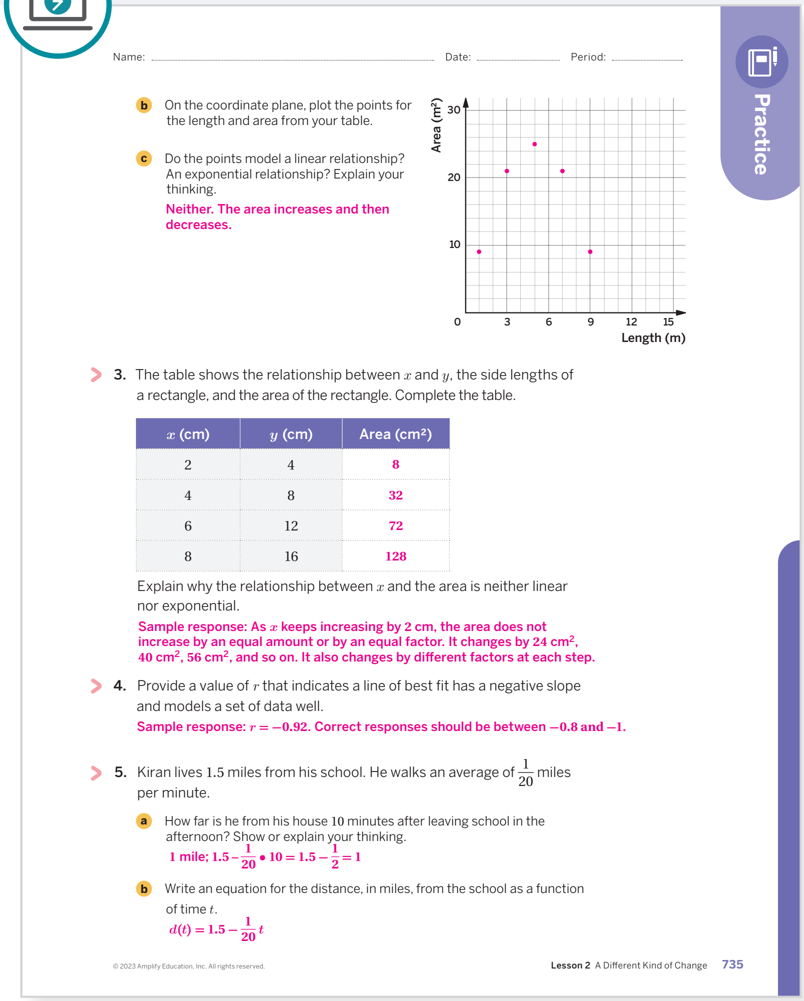
Sample response: I listed different pairs and found their products, and noticed that when the two numbers are the same, the product is the greatest.

2. The table shows some possible lengths and widths of a rectangle whose perimeter is 20 m.

Length (m)	Width (m)	Area (m^2)
1	9	9
3	7	21
5	5	25
7	3	21
9	1	9

a. Complete the table. What do you notice about the areas?

Sample response: The values for the area increase and then decrease, but repeat the same values. The order of the factors (the length and width) changes, but the product is the same.



Name: Date: Period:

Practice

b. On the coordinate plane, plot the points for the length and area from your table.

c. Do the points model a linear relationship? An exponential relationship? Explain your thinking.

Neither. The area increases and then decreases.

3. The table shows the relationship between x and y, the side lengths of a rectangle, and the area of the rectangle. Complete the table.

x (cm)	y (cm)	Area (cm^2)
2	4	8
4	8	32
6	12	72
8	16	128

Explain why the relationship between x and the area is neither linear nor exponential.

Sample response: As x keeps increasing by 2 cm, the area does not increase by an equal amount or by an equal factor. It changes by 24 cm^2, 40 cm^2, 56 cm^2, and so on. It also changes by different factors at each step.

4. Provide a value of r that indicates a line of best fit has a negative slope and models a set of data well.

Sample response: $r = -0.92$. Correct responses should be between -0.8 and -1.

5. Kiran lives 1.5 miles from his school. He walks an average of $\frac{1}{20}$ miles per minute.

a. How far is he from his house 10 minutes after leaving school in the afternoon? Show or explain your thinking.

1 mile; $1.5 - \frac{1}{20} \bullet 10 = 1.5 - \frac{1}{2} = 1$

b. Write an equation for the distance, in miles, from the school as a function of time t.

$d(t) = 1.5 - \frac{1}{20}t$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 19	1
Formative ⚡	5	Unit 5 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

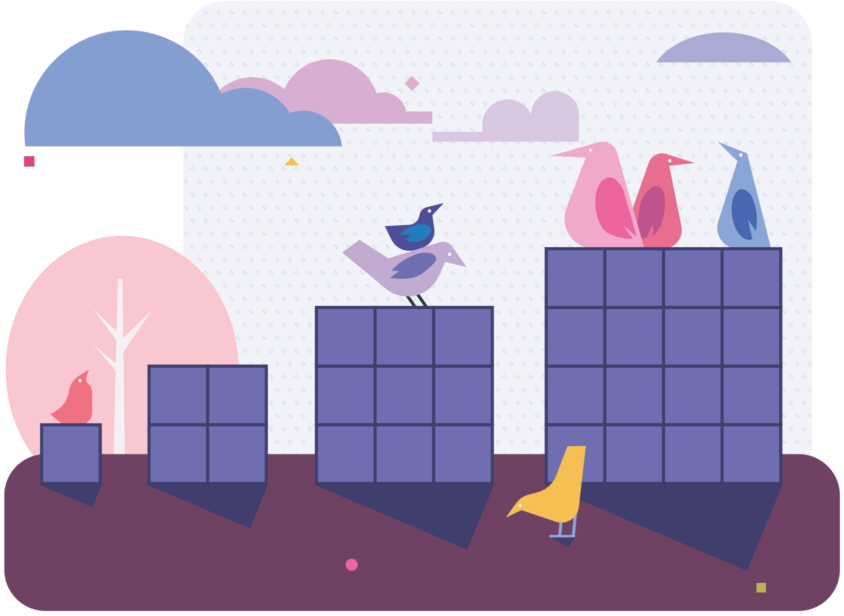
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 3**

How Does It Change?

Let's describe some patterns of change.



Focus

Goals

1. Understand that a quadratic relationship can be expressed with a squared term.
2. **Language Goal:** Describe a pattern of change associated with a quadratic relationship. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Determine and explain whether a visual pattern represents a linear, exponential, or quadratic relationship. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

In this lesson, students strengthen their conceptual understanding of the relationship between squares, squared numbers, and quadratics by analyzing patterns, generating tables, and creating graphs. They also encounter the term *quadratic expression*, learning that quadratic relationships can be written using an expression with a squared term.

Previously

In Lesson 2, students encountered situations in which a quantity increases and then decreases, recognizing that it is nonlinear, and unlike an exponential function.

Coming Soon

Students will continue building their understanding of quadratic relationships and write quadratic expressions, describing them in Lesson 4.

Rigor

- Students strengthen **conceptual understanding** of quadratic expressions and patterns by creating tables and graphs.
- Students are introduced to quadratic patterns to build **procedural skills**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- snap cubes

Math Language Development

New words

- *quadratic*
- *quadratic expression*

Amps | **Featured Activity**

Activity 1
Virtual Patterns

Students observe patterns in the number of squares to determine the type of growth. Same concept, no cleanup!

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to make sense of the patterns independently and may doubt themselves when they do draw some conclusions. As students share their work with partners in Activity 3, ask them to encourage each other, recognizing any level of success in the activity. Then, have them work through any incomplete or incorrect part of the activity together.

Modifications to Pacing

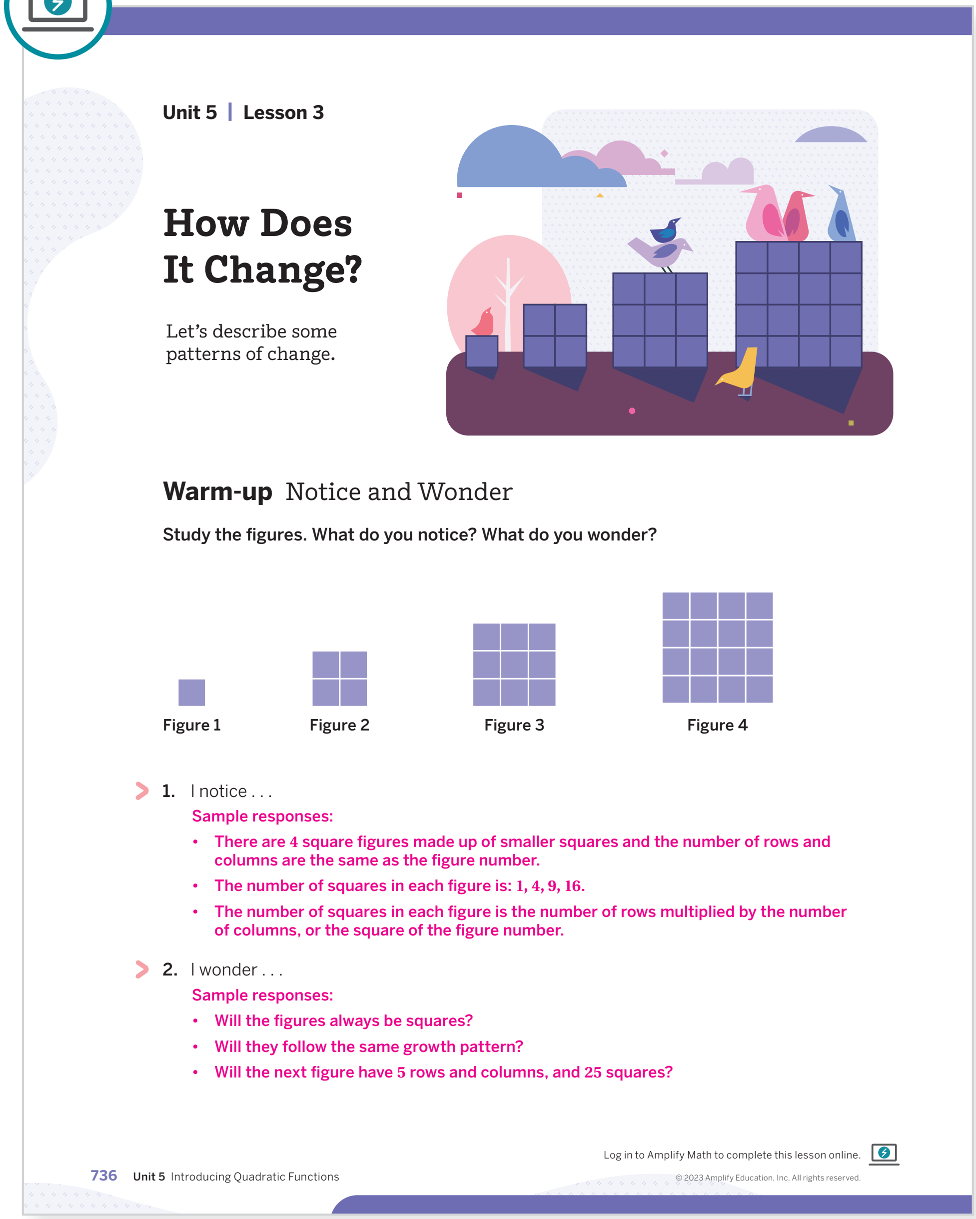
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 3 may be omitted and in Problem 5, two rows may be omitted in the table.
- In **Activity 3**, Problem 1, two rows in the table may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students examine a quadratic growth pattern presented in square arrays to prepare them for understanding patterns that involve squaring a number. The term *quadratic* is not introduced yet.



Unit 5 | Lesson 3

How Does It Change?

Let's describe some patterns of change.

Warm-up Notice and Wonder

Study the figures. What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - There are 4 square figures made up of smaller squares and the number of rows and columns are the same as the figure number.
   - The number of squares in each figure is: 1, 4, 9, 16.
   - The number of squares in each figure is the number of rows multiplied by the number of columns, or the square of the figure number.
2. I wonder . . .
   Sample responses:
   - Will the figures always be squares?
   - Will they follow the same growth pattern?
   - Will the next figure have 5 rows and columns, and 25 squares?

Log in to Amplify Math to complete this lesson online.

736 Unit 5 Introducing Quadratic Functions

1 Launch

Give students one minute of think-time to study the figures. Conduct the ***Notice and Wonder*** routine. Tell them there are no wrong answers.

2 Monitor

Help students get started by having them count and record the number of small squares in each figure.

Look for points of confusion:

- **Viewing the pattern independent of the figure number.** Ask, "What pattern do you notice from one figure to the next?"

Look for productive strategies:

- Extending the pattern.
- Recognizing the figures represent perfect square numbers.

3 Connect

Display the figures.

Have students share what they notice and wonder about the figures. Record and display their thinking. Ask students if they have any questions about what is on the list.

Highlight the connection between the area of the figure to the number of squares in the figure.

Ask, "Is there a relationship between the shape and the figure number? If so, what is that relationship?" Yes, the figure is a square. By raising the figure number to the second power, or squaring it, I get the total number of small squares.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw connections between the figure number, the number of squares in the figure, and the area of the figure. Consider displaying these sentence frames and have students complete them.

- "The number of squares in each figure is the ________ of the figure number."
- "The ________ of each figure is the square of the figure number."
- "The ________ of each figure is the number of squares in each figure."

Power-up

To power up students' ability to write a function to represent a linear relationship, have students complete:

Study the table of values.

a. How many squares are added as the figure number increases? 2

b. Write an equation that gives the number of squares S as a function of the figure number f. $S(f) = 2f + 1$

Figure	Number of squares
1	3
2	5
3	7
4	9

Use: Before Activity 1

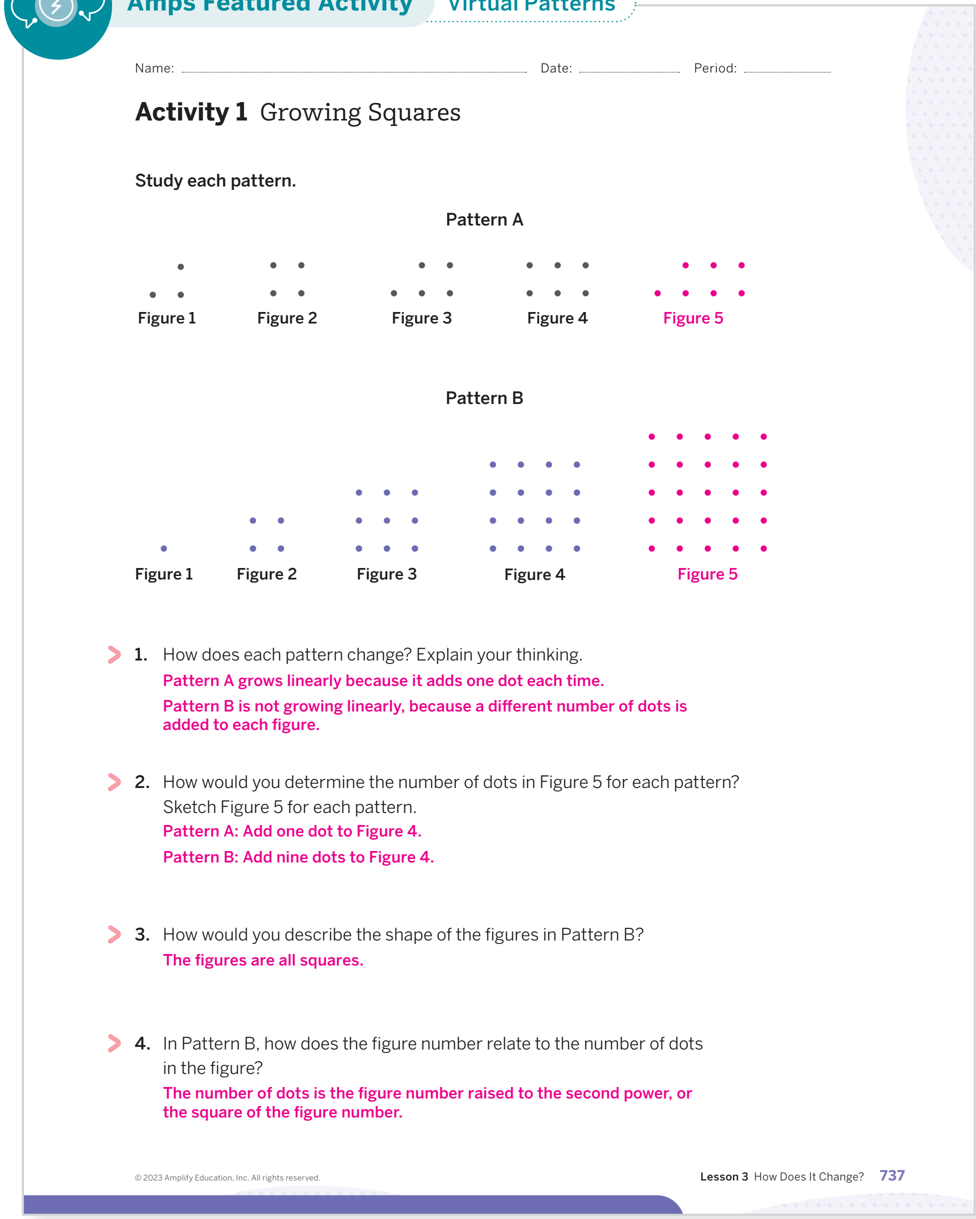
Informed by: Performance on Lesson 3, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 6

Pairs | 10 min

Activity 1 Growing Squares

Students investigate a pattern that grows by squaring a number and compare this growth to linear growth. The term *quadratic* is introduced at the end of this activity.

Amps Featured Activity Virtual Patterns



Name: ______ Date: ______ Period: ______

Activity 1 Growing Squares

Study each pattern.

Pattern A

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

Pattern B

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

1. How does each pattern change? Explain your thinking.
   Pattern A grows linearly because it adds one dot each time.
   Pattern B is not growing linearly, because a different number of dots is added to each figure.
2. How would you determine the number of dots in Figure 5 for each pattern? Sketch Figure 5 for each pattern.
   Pattern A: Add one dot to Figure 4.
   Pattern B: Add nine dots to Figure 4.
3. How would you describe the shape of the figures in Pattern B?
   The figures are all squares.
4. In Pattern B, how does the figure number relate to the number of dots in the figure?
   The number of dots is the figure number raised to the second power, or the square of the figure number.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 How Does It Change? 737

1 Launch

Activate students' prior knowledge by asking them what strategies they can use to notice patterns. Have students work individually to complete the problems, then have them share their strategies and solutions with their partner. As a whole class, discuss Problems 5 and 6.

2 Monitor

Help students get started by suggesting they annotate the number of dots by which each figure increases in the table in Problem 5.

Look for points of confusion:

- **Forgetting that adding the same number of dots to each consecutive figure represents linear growth (Pattern A).** Remind them that linear growth has a constant rate of change.
- **Thinking that all nonlinear change(s) are exponential (Pattern B)**. Ask, "Does each figure increase by a common factor?"

Look for productive strategies:

- Recognizing the array of dots represents a square number in Pattern B.
- Connecting the square of the figure number to the number of dots in each figure in Pattern B.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can build virtual models of the figures and observe patterns in the number of squares added to determine the type of growth.

Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Tools*

Provide students with snap cubes or other manipulatives they could use to build Figure 5 for each pattern. Consider providing them with a pre-completed table for Problem 5 that shows the number of dots in each pattern for Figures 1–4 and have students complete the rest of the table.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to how the growth patterns are similar and different. If students have not used the term *linear* to describe the growth of Pattern A, ask:

- "Do either of these patterns show linear growth? Nonlinear growth? Explain your thinking."
- "What do you notice about the expressions for the number of dots in Figure n for each pattern?"

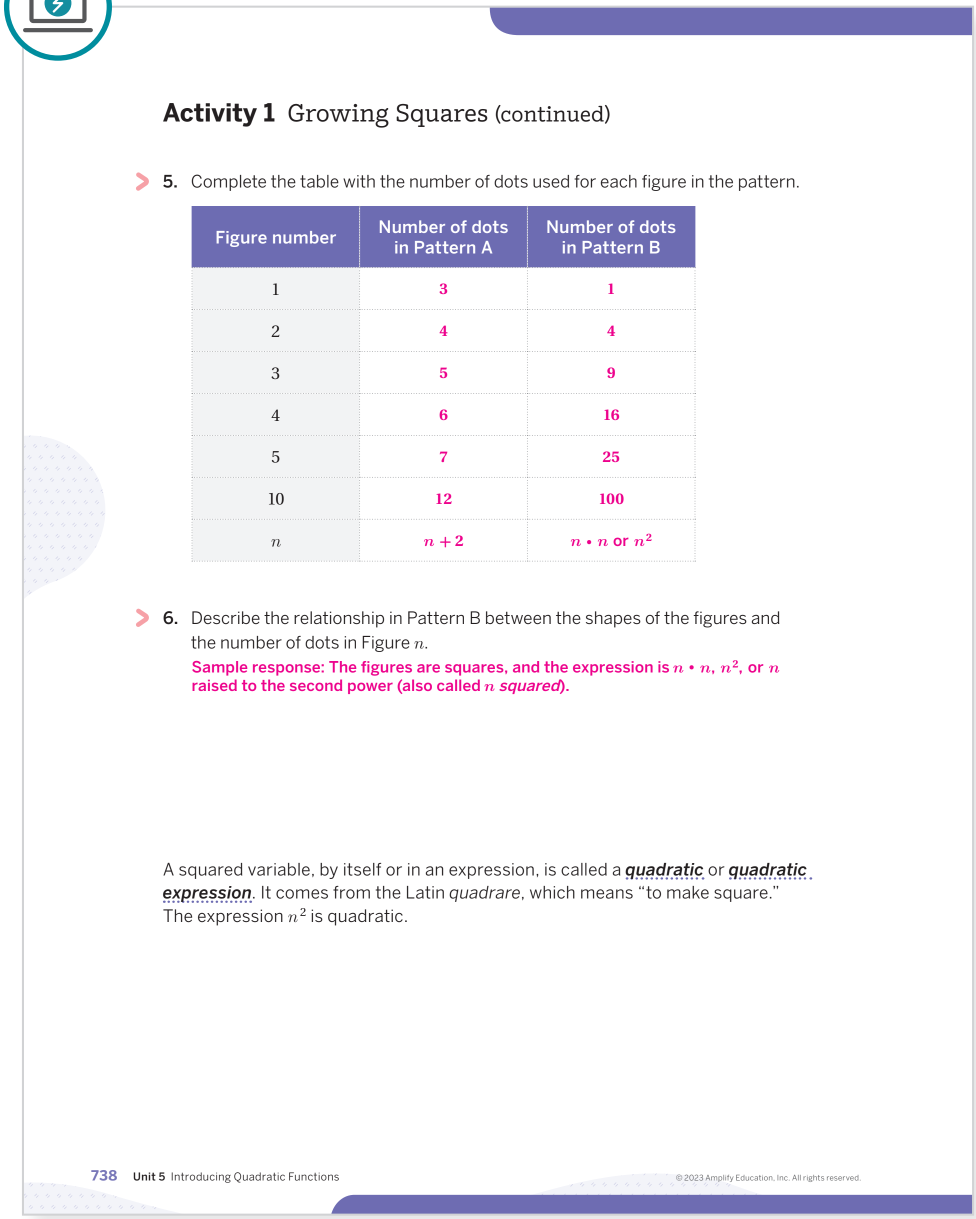
English Learners

Annotate the expressions for each pattern as *linear* and *quadratic*. Show how the linear pattern increases by a constant value each time, yet the quadratic pattern does not.

Pairs | 10 min

Activity 1 Growing Squares (continued)

Students investigate a pattern that grows by squaring a number and compare this growth to linear growth. The term *quadratic* is introduced at the end of this activity.



Activity 1 Growing Squares (continued)

5. Complete the table with the number of dots used for each figure in the pattern.

Figure number	Number of dots in Pattern A	Number of dots in Pattern B
1	3	1
2	4	4
3	5	9
4	6	16
5	7	25
10	12	100
n	$n + 2$	$n \cdot n$ or n^2

6. Describe the relationship in Pattern B between the shapes of the figures and the number of dots in Figure n.
Sample response: The figures are squares, and the expression is $n \cdot n$, n^2, or n raised to the second power (also called n *squared*).

A squared variable, by itself or in an expression, is called a ***quadratic*** or ***quadratic expression***. It comes from the Latin *quadrare*, which means "to make square." The expression n^2 is quadratic.

738 **Unit 5** Introducing Quadratic Functions

3 Connect

Display the figures and table in Problem 5.

Have pairs of students share their strategies for completing the table and identifying the growth patterns.

Highlight the perfect square numbers in Pattern B, if not mentioned by students.

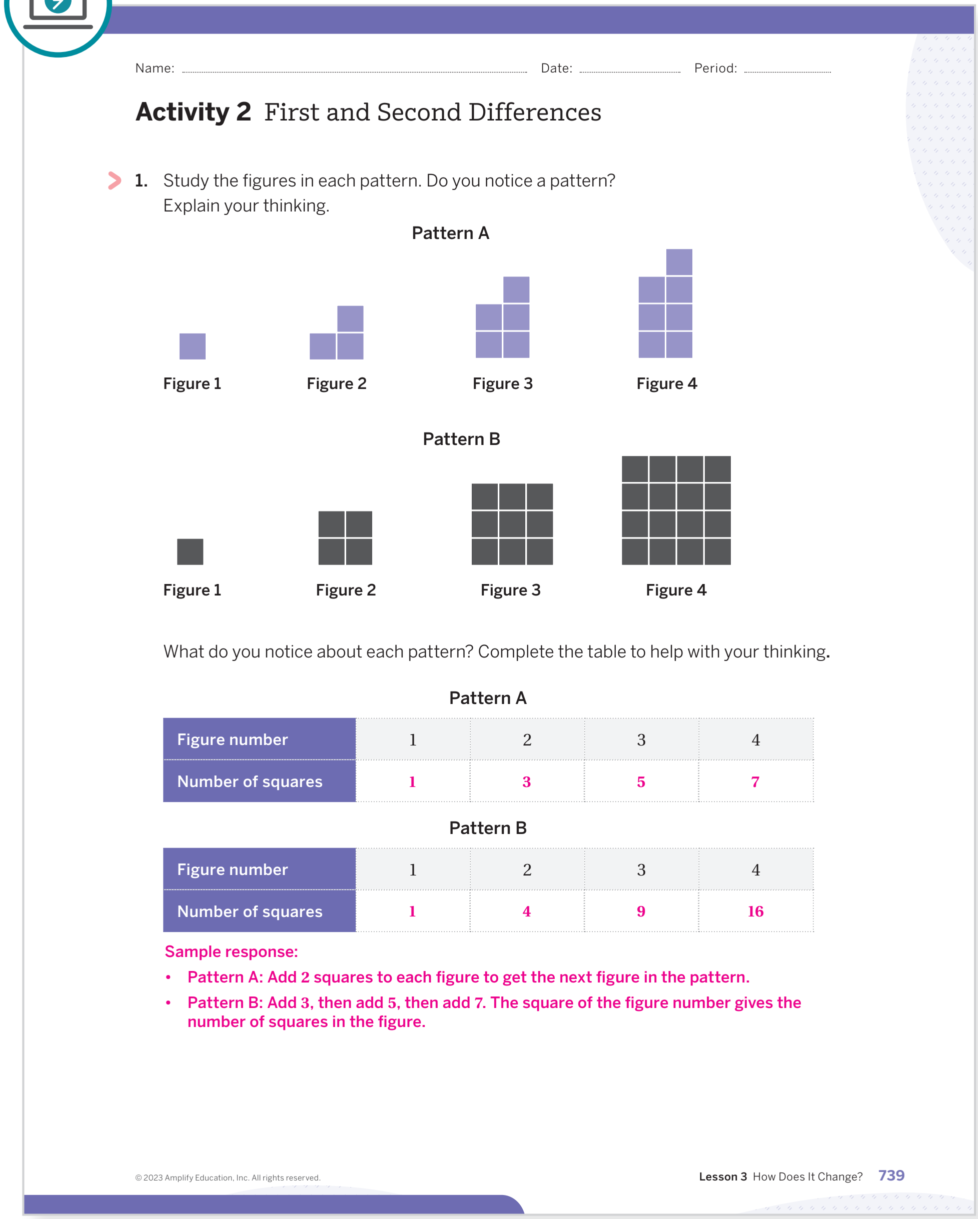
Define the term ***quadratic expression***. Connect raising the figure number to the second power, or "squaring" them, to the perfect squares.

Ask:

- "How could you identify quadratic growth using diagrams?" Pattern B grows by squaring each figure number. I could use square diagrams to represent quadratic growth.
- "How could you identify quadratic growth patterns using tables of values?" Pattern A grows by adding the same amount each time, while Pattern B grows by squaring the figure number. I could see the output values in a table are generated by squaring a number.

Activity 2 First and Second Differences

Students further explore quadratic growth by calculating first and second differences to uncover a special relationship about these differences for linear and quadratic growth.



Name: ______ Date: ______ Period: ______

Activity 2 First and Second Differences

1. Study the figures in each pattern. Do you notice a pattern? Explain your thinking.

Pattern A

Figure 1 Figure 2 Figure 3 Figure 4

Pattern B

Figure 1 Figure 2 Figure 3 Figure 4

What do you notice about each pattern? Complete the table to help with your thinking.

Pattern A

Figure number	1	2	3	4
Number of squares	1	3	5	7

Pattern B

Figure number	1	2	3	4
Number of squares	1	4	9	16

Sample response:
- Pattern A: Add 2 squares to each figure to get the next figure in the pattern.
- Pattern B: Add 3, then add 5, then add 7. The square of the figure number gives the number of squares in the figure.

1 Launch

Have students study Pattern A and complete Problem 1 independently before sharing their thinking with a partner.

2 Monitor

Help students get started by having them count and record the number of squares added for each successive figure in each pattern.

Look for points of confusion:

- **Having difficulty calculating the differences in Problem 2.** Ask, "From Figure 1 to Figure 2, how many squares were added to each pattern? From Figure 2 to Figure 3? From Figure 3 to Figure 4?"

Look for productive strategies:

- Classifying each pattern as linear or quadratic.
- Recognizing that the first difference is the rate of change or the number of squares added to each figure.

Activity 2 continued ›

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols, Guide Processing and Visualization*

Be sure students understand the meaning of the terms *first differences* and *second differences*. Annotate the write-in boxes in the tables for Problem 2 as *first differences* and say, "These are first differences because this is the first time we are calculating the differences." Then consider drawing write-in boxes below those first differences to show the second differences and say, "These are second differences because this is the second time we are calculating differences for each pattern."

Math Language Development

MLR7: Compare and Connect

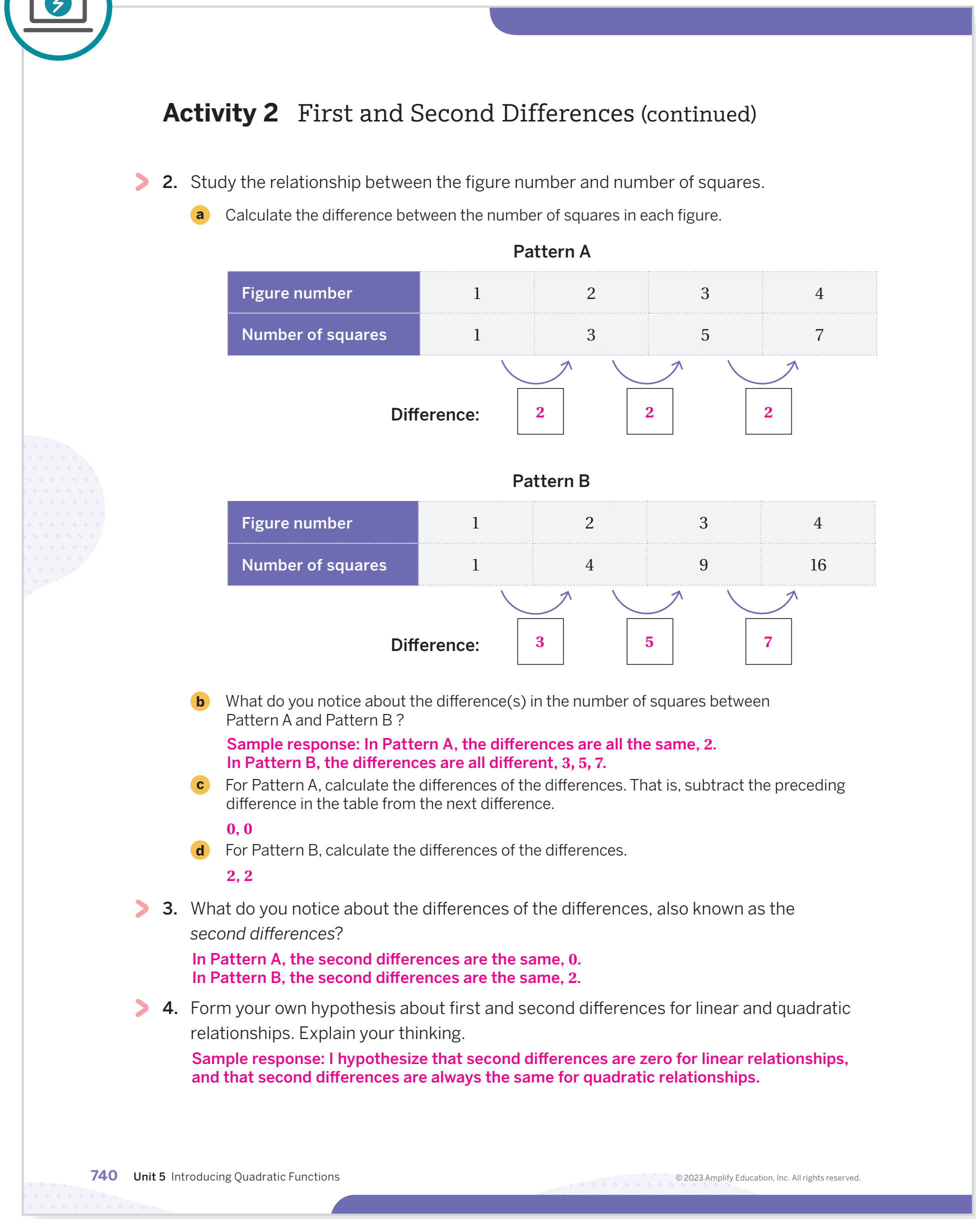
During the Connect, consider displaying a table similar to the following, or add it to the class display. Highlight the mathematical language used.

	Linear growth	Quadratic growth
First differences	The same value. (There is a common difference.)	Not the same value. (There is no common difference.)
Second differences	0	The same value.

Pairs | 15 min

Activity 2 First and Second Differences (continued)

Students further explore quadratic growth by calculating first and second differences to uncover a special relationship about these differences for linear and quadratic growth.



Activity 2 First and Second Differences (continued)

2. Study the relationship between the figure number and number of squares.

a) Calculate the difference between the number of squares in each figure.

Pattern A

Figure number	1	2	3	4
Number of squares	1	3	5	7

Difference: 2 2 2

Pattern B

Figure number	1	2	3	4
Number of squares	1	4	9	16

Difference: 3 5 7

b) What do you notice about the difference(s) in the number of squares between Pattern A and Pattern B ?

Sample response: In Pattern A, the differences are all the same, 2.
In Pattern B, the differences are all different, 3, 5, 7.

c) For Pattern A, calculate the differences of the differences. That is, subtract the preceding difference in the table from the next difference.

0, 0

d) For Pattern B, calculate the differences of the differences.

2, 2

3. What do you notice about the differences of the differences, also known as the *second differences*?

In Pattern A, the second differences are the same, 0.
In Pattern B, the second differences are the same, 2.

4. Form your own hypothesis about first and second differences for linear and quadratic relationships. Explain your thinking.

Sample response: I hypothesize that second differences are zero for linear relationships, and that second differences are always the same for quadratic relationships.

3 Connect

Display the figures and completed tables in Problem 1.

Have pairs of students share their responses and any growth patterns they recognize as linear or quadratic.

Highlight that the second differences are zero for linear relationships, and the second differences are always the same for quadratic relationships.

Ask:

- "Why are the first differences equal in a linear growth pattern?" Because there is a constant rate of change, they are all equal to each other.
- "Why are the second differences in a linear growth pattern equal to 0?" Because the first differences are equal and subtracting them yields 0.
- "How do you know the number of squares grows quadratically in Pattern B?" The number of squares is equal to the square of the figure number.

Differentiated Support

Accessibility: *Math Enrichment*

Provide students with the table of values shown for the side length and volume of a cube. Have them determine the first, second, and third differences. Ask them to describe what they notice, and to explain whether they think the relationship is linear, quadratic, or neither. Ask them to explain their thinking.

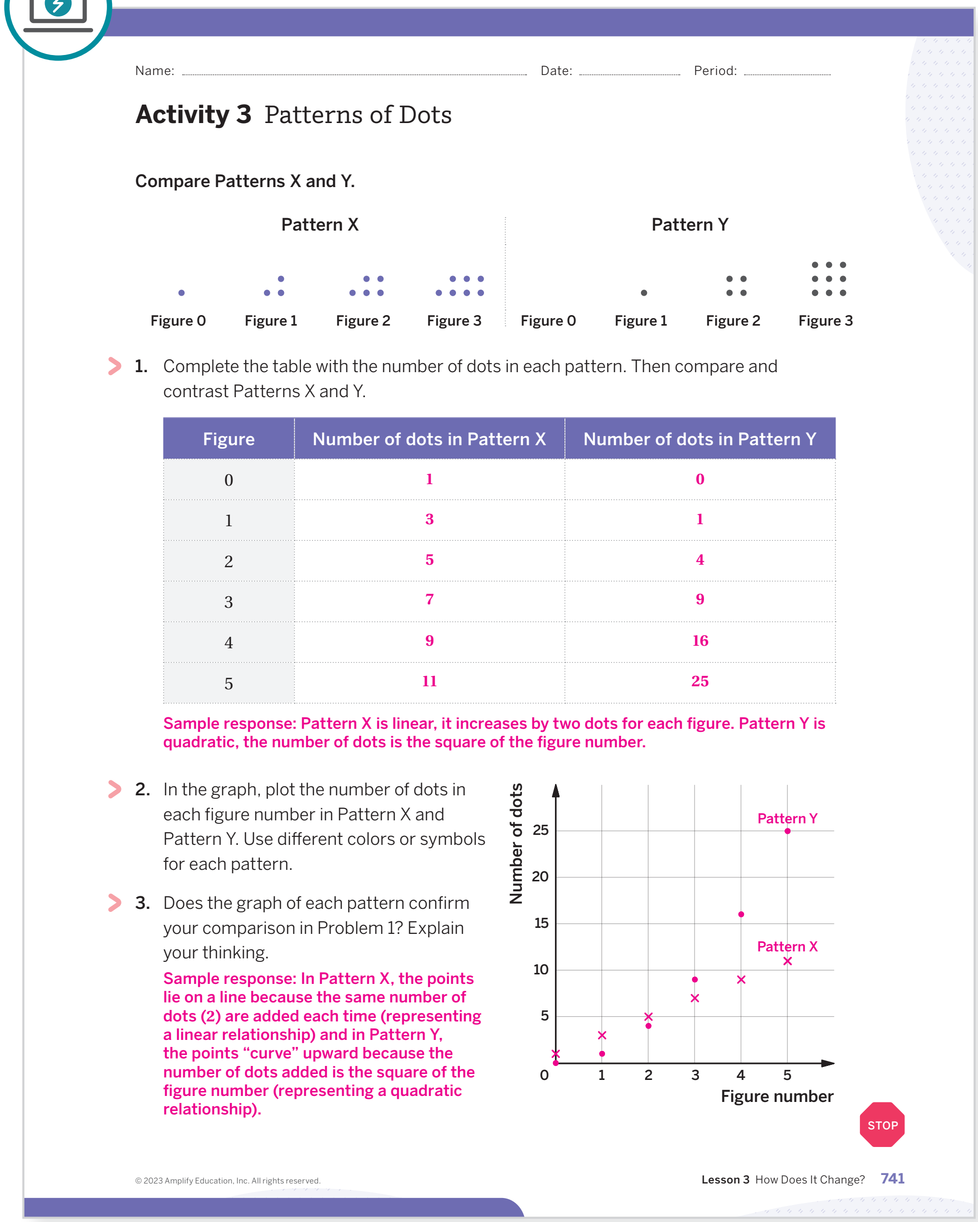
First differences: 7, 19, 37, 61, 91
Second differences: 12, 18, 24, 30
Third differences: 6, 6, 6
The relationship is neither linear nor quadratic because neither of the first nor second differences are constant.

Side length	Volume
1	1
2	8
3	27
4	64
5	125
6	216

Independent | 10 min

Activity 3 Patterns of Dots

Students compare the tables and graphs of linear and quadratic growth patterns to see that the graph of a quadratic growth pattern is a curve.



Name: ______________ Date: ________ Period: ________

Activity 3 Patterns of Dots

Compare Patterns X and Y.

1. Complete the table with the number of dots in each pattern. Then compare and contrast Patterns X and Y.

Figure	Number of dots in Pattern X	Number of dots in Pattern Y
0	1	0
1	3	1
2	5	4
3	7	9
4	9	16
5	11	25

Sample response: Pattern X is linear, it increases by two dots for each figure. Pattern Y is quadratic, the number of dots is the square of the figure number.

2. In the graph, plot the number of dots in each figure number in Pattern X and Pattern Y. Use different colors or symbols for each pattern.

3. Does the graph of each pattern confirm your comparison in Problem 1? Explain your thinking.

Sample response: In Pattern X, the points lie on a line because the same number of dots (2) are added each time (representing a linear relationship) and in Pattern Y, the points "curve" upward because the number of dots added is the square of the figure number (representing a quadratic relationship).

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 How Does It Change? 741

1 Launch

Have students work on the activity independently before sharing their responses with a partner.

2 Monitor

Help students get started by having them extend each pattern. Suggest they annotate the number of dots by which each figure increases in each pattern.

Look for points of confusion:

- **Mistaking quadratic relationships for exponential (Problem 3).** Ask how exponential and quadratic relationships differ.

Look for productive strategies:

- Using first and second differences to confirm whether a pattern is linear or quadratic.
- Connecting the square of the figure number with the number of dots in the array and the vertical coordinates of points in the graph for Pattern Y.

3 Connect

Have students share their strategies for determining whether each pattern shows linear or quadratic growth.

Highlight the connection between input and output values in the graph of Pattern Y. The output value is the square of the input value.

Ask, "How could you tell the difference between quadratic and exponential growth on a graph? How does this help you in knowing whether Pattern Y exhibits quadratic growth or exponential growth?" In quadratic growth, the vertical coordinate is the square of the horizontal coordinate. In exponential growth, vertical coordinates are multiplied by a common factor each time. Because the vertical coordinates in Pattern Y are squares of the horizontal coordinates, Pattern Y exhibits quadratic growth.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide a partially-completed table that shows the number of dots for Figures 1–3 for each pattern. Consider providing a pre-completed graph for Problem 2, so that students can spend more time analyzing the patterns and their graphs.

Math Language Development

MLR7: Co-craft Questions

During the Launch, reveal Patterns X and Y. Have students work with a partner to write 2–3 mathematical questions they could ask about the figures shown in each pattern. Have volunteers share their questions with the class. Sample questions shown.

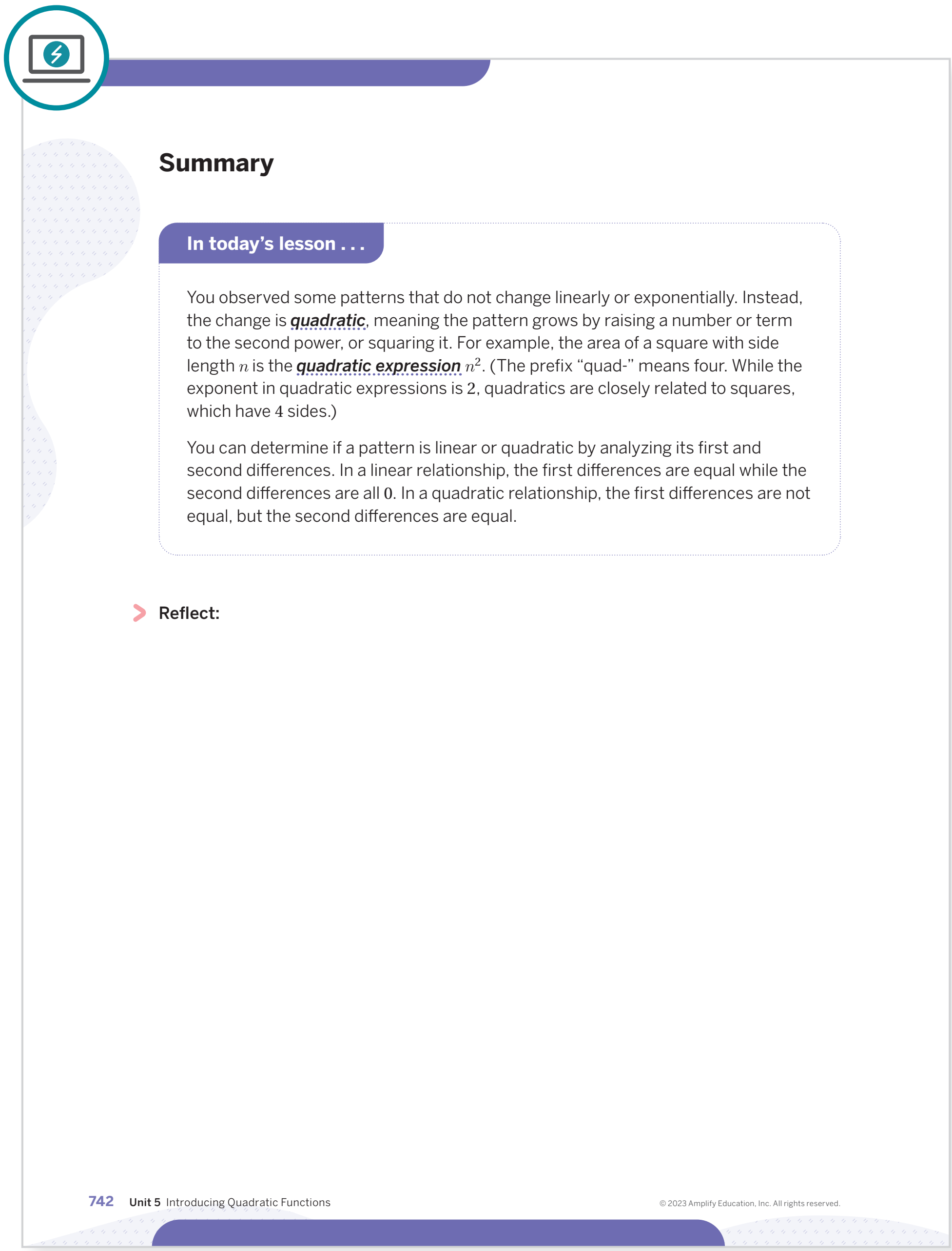
- How is each pattern growing?
- What will Figure 4 look like for each pattern?
- Are any of these patterns linear? Exponential? Quadratic?

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Summary

Review and synthesize how quadratic growth patterns differ from linear and exponential growth patterns.



Summary

In today's lesson . . .

You observed some patterns that do not change linearly or exponentially. Instead, the change is ***quadratic***, meaning the pattern grows by raising a number or term to the second power, or squaring it. For example, the area of a square with side length n is the ***quadratic expression*** n^2. (The prefix "quad-" means four. While the exponent in quadratic expressions is 2, quadratics are closely related to squares, which have 4 sides.)

You can determine if a pattern is linear or quadratic by analyzing its first and second differences. In a linear relationship, the first differences are equal while the second differences are all 0. In a quadratic relationship, the first differences are not equal, but the second differences are equal.

> **Reflect:**

742 **Unit 5** Introducing Quadratic Functions

Synthesize

Highlight that students can determine whether a relationship is quadratic by studying patterns in figures and by calculating first and second differences.

Formalize vocabulary:

- ***quadratic***
- ***quadratic expression***

Ask:

- "In what ways are quadratic relationships different from linear relationships you have seen so far?" In linear relationships, there is a constant rate of change. In quadratic relationships, the rates of change are not constant. As one quantity increases by the same amount, the other quantity increases (or decreases) by different amounts. The graph of a linear relationship is a straight line. The graph of a quadratic relationship is a curve.
- "In what ways is a quadratic relationship different from an exponential relationship?" Exponential relationships grow by a constant factor. Quadratic relationships do not grow by a constant factor.
- "How would you describe a quadratic relationship to someone who is unfamiliar with it?" Sample response: Think about a pattern in which the output value is the square of the input value. This is a quadratic relationship.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you identify a quadratic relationship in a table?"
- "How can you identify a quadratic relationship in a pattern?"

Differentiated Support

Extension: *Math Around the World*

A rectangular plot of land has an area of 400 square units and it is 15 units longer than it is wide. What are the dimensions of the plot?

Ancient Babylonians lived in a region called Mesopotamia around 2000–1600 BCE. Understanding mathematics concepts such as measurement and geometry were very important aspects of their society. While the term *quadratic expression* had not yet been introduced, ancient Babylonians used reasoning involving the concept of quadratics to solve area problems. For example, to solve the problem shown, they would . . .

- Determine half of the difference between the length and the width.
- Square this value and add it to the area of the plot.
- Take the square root. Add half of the difference of the square root to determine the length. Subtract half of the difference of the square root to determine the width.

Have students verify that this method gives the correct solution to the problem.

Math Language Development

MLR7: Collect and Display

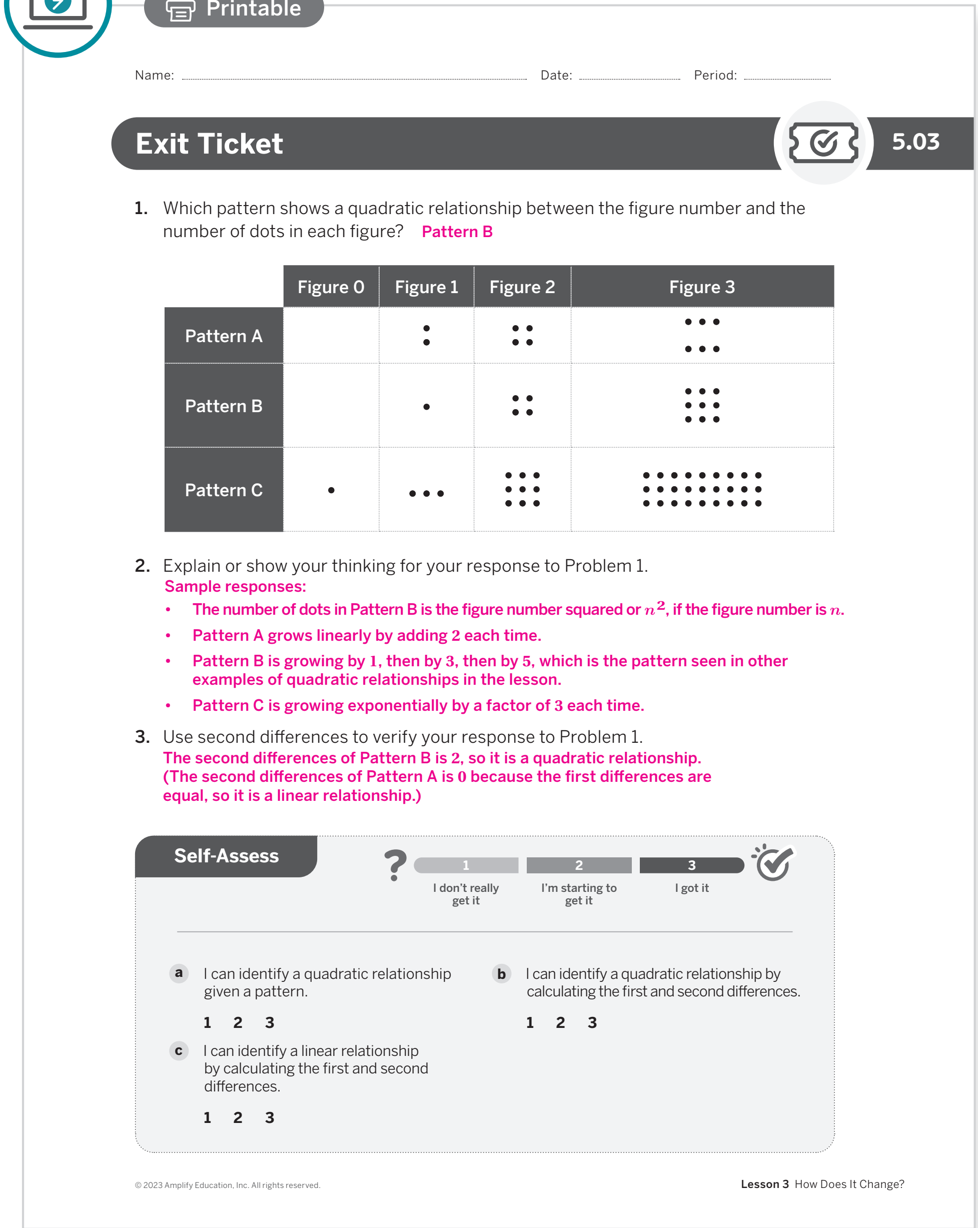
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *quadratic* and *quadratic expression* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying a quadratic relationship among dot patterns and justifying their response.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 5.03

1. Which pattern shows a quadratic relationship between the figure number and the number of dots in each figure? **Pattern B**

	Figure 0	Figure 1	Figure 2	Figure 3
Pattern A		• •	• • • •	• • • • • •
Pattern B		•	• • • •	• • • • • • • • •
Pattern C	•	• • •	• • • • • • • • •	• •

2. Explain or show your thinking for your response to Problem 1.
   Sample responses:
   - **The number of dots in Pattern B is the figure number squared or n^2, if the figure number is n.**
   - **Pattern A grows linearly by adding 2 each time.**
   - **Pattern B is growing by 1, then by 3, then by 5, which is the pattern seen in other examples of quadratic relationships in the lesson.**
   - **Pattern C is growing exponentially by a factor of 3 each time.**
3. Use second differences to verify your response to Problem 1.
   The second differences of Pattern B is 2, so it is a quadratic relationship. (The second differences of Pattern A is 0 because the first differences are equal, so it is a linear relationship.)

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can identify a quadratic relationship given a pattern. 1 2 3

b. I can identify a quadratic relationship by calculating the first and second differences. 1 2 3

c. I can identify a linear relationship by calculating the first and second differences. 1 2 3

Success looks like . . .

- **Goal:** Understanding that a quadratic relationship can be expressed with a squared term.
  - » Representing the appropriate pattern in the table with a squared term in Problem 2.
- **Language Goal:** Describing a pattern of change associated with a quadratic relationship. **(Speaking and Listening, Reading and Writing)**
  - » Explaining how the number of dots in the selected pattern changes by squaring the figure number in Problem 2.
- **Language Goal:** Determining and explaining whether a visual pattern represents a linear, exponential, or quadratic relationship. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students do not identify the quadratic pattern in Problem 1, consider:

- Reviewing Activity 1, Problems 1 and 2.
- Having them sketch the next figure for each pattern.
- Assigning Practice Problem 1.

If students do not accurately describe the growth patterns in Problem 2 or do not find the second differences in Problem 3, consider:

- Reviewing Activity 2, Problems 2–4.
- Having them calculate the first and second differences for each pattern.
- Assigning Practice Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What did students' descriptions of the relationship in the patterns reveal about your students as learners?
- How did using patterns to identify quadratic relationships set students up to develop writing quadratic functions?

Math Language Development

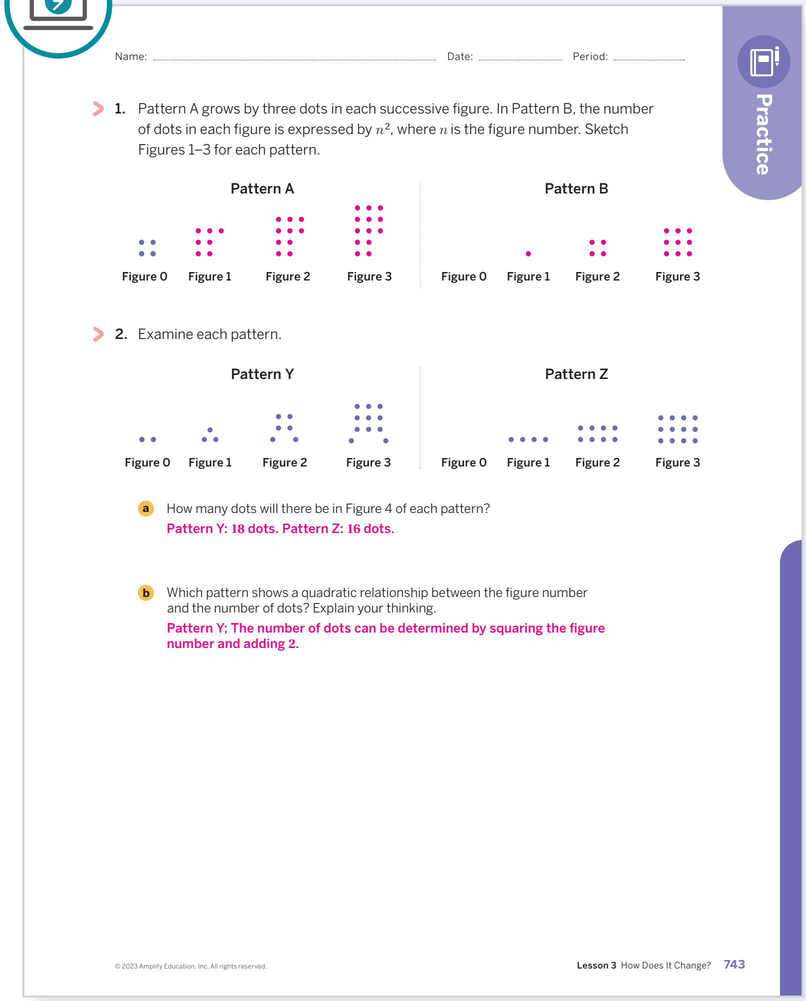
Language Goal: Describing a pattern of change associated with a quadratic relationship.

Reflect on students' language development toward this goal.

- How have students progressed in their descriptions of patterns of change that show quadratic relationships? What math language do they use to describe this change?
- How have students progressed in their comfort using terms and phrases such as *square of the figure number, grows linearly, grows quadratically, grows exponentially, first differences, second differences, and constant?*

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Pattern A grows by three dots in each successive figure. In Pattern B, the number of dots in each figure is expressed by n^2, where n is the figure number. Sketch Figures 1–3 for each pattern.

2. Examine each pattern.

a. How many dots will there be in Figure 4 of each pattern?

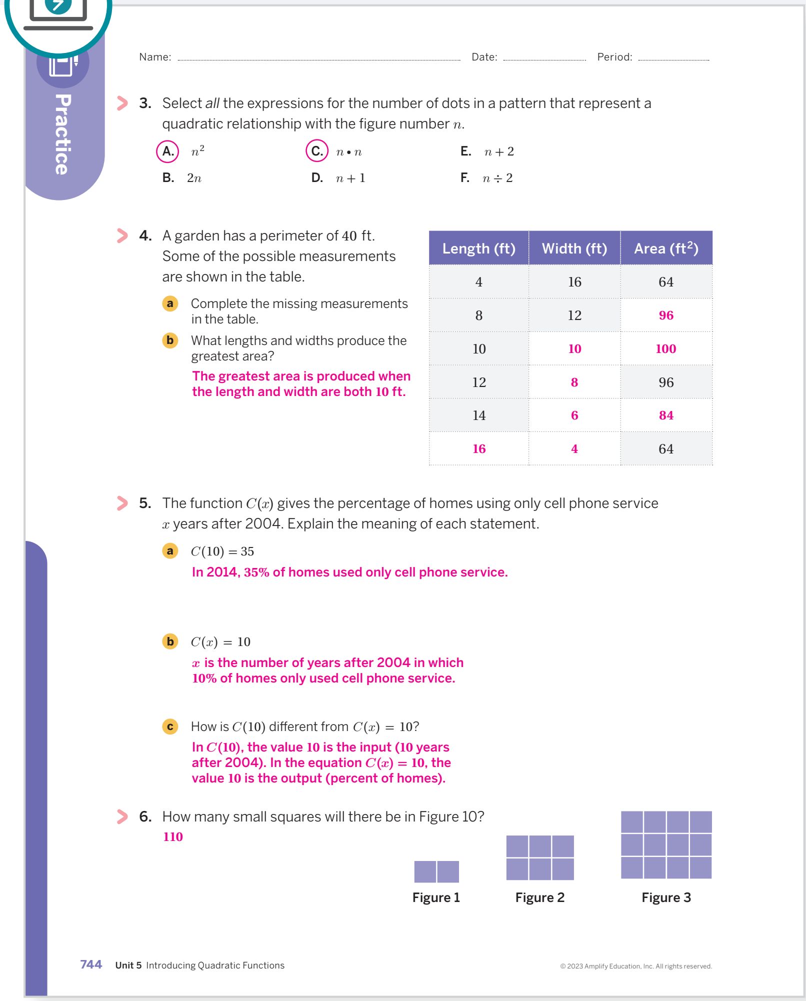
Pattern Y: 18 dots. Pattern Z: 16 dots.

b. Which pattern shows a quadratic relationship between the figure number and the number of dots? Explain your thinking.

Pattern Y; The number of dots can be determined by squaring the figure number and adding 2.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 How Does It Change? 743



Practice

Name: ________ Date: ________ Period: ________

3. Select *all* the expressions for the number of dots in a pattern that represent a quadratic relationship with the figure number n.

[✓ A.] n^2
B. $2n$
[✓ C.] $n \bullet n$
D. $n + 1$
E. $n + 2$
F. $n \div 2$

4. A garden has a perimeter of 40 ft. Some of the possible measurements are shown in the table.

a. Complete the missing measurements in the table.

b. What lengths and widths produce the greatest area?

The greatest area is produced when the length and width are both 10 ft.

Length (ft)	Width (ft)	Area (ft^2)
4	16	64
8	12	**96**
10	**10**	**100**
12	**8**	96
14	**6**	**84**
16	**4**	64

5. The function $C(x)$ gives the percentage of homes using only cell phone service x years after 2004. Explain the meaning of each statement.

a. $C(10) = 35$

In 2014, 35% of homes used only cell phone service.

b. $C(x) = 10$

x is the number of years after 2004 in which 10% of homes only used cell phone service.

c. How is $C(10)$ different from $C(x) = 10$?

In $C(10)$, the value 10 is the input (10 years after 2004). In the equation $C(x) = 10$, the value 10 is the output (percent of homes).

6. How many small squares will there be in Figure 10?

110

744 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 3	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 2	2
	5	Unit 3 Lesson 3	2
Formative	6	Unit 5 Lesson 4	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 4**

Squares

Let's write new quadratic expressions.

Focus

Goals

1. Understand that a quadratic relationship can be expressed with a squared term.
2. **Language Goal:** Describe a pattern of change associated with a quadratic relationship. **(Speaking and Listening, Writing)**
3. Write expressions describing quadratic relationships.

Coherence

Today

In this lesson, students continue to build upon their understanding of quadratics as squares and extend their understanding to more complex quadratic expressions. They analyze patterns and write quadratic expressions describing quadratic growth.

Previously

In Lesson 3, students began building a conceptual understanding relating the abstract notion of *quadratic* to concrete representations of squares, comparing this new type of growth to their understanding of linear and exponential growth.

Coming Soon

In Lesson 5, students will continue to build on their understanding of quadratic expressions and patterns, as well as writing functions defining them.

Rigor

- Students build **conceptual understanding** of quadratic expressions by analyzing more complex expressions.
- Students write quadratic expressions from patterns to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- snap cubes

Math Language Development

Review words

- *quadratic*
- *quadratic expression*

Amps | **Featured Activity**

Activity 3
Square Models

Students explore quadratic growth patterns by observing square models and writing numeric quadratic expressions.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might resist using snap cubes or grid paper to represent the figures and find the regularity in quadratic patterns in Activity 2. Ask students to choose to use all tools available to them to properly analyze the situation. Explain that using these physical models can help lead to success.

Modifications to Pacing

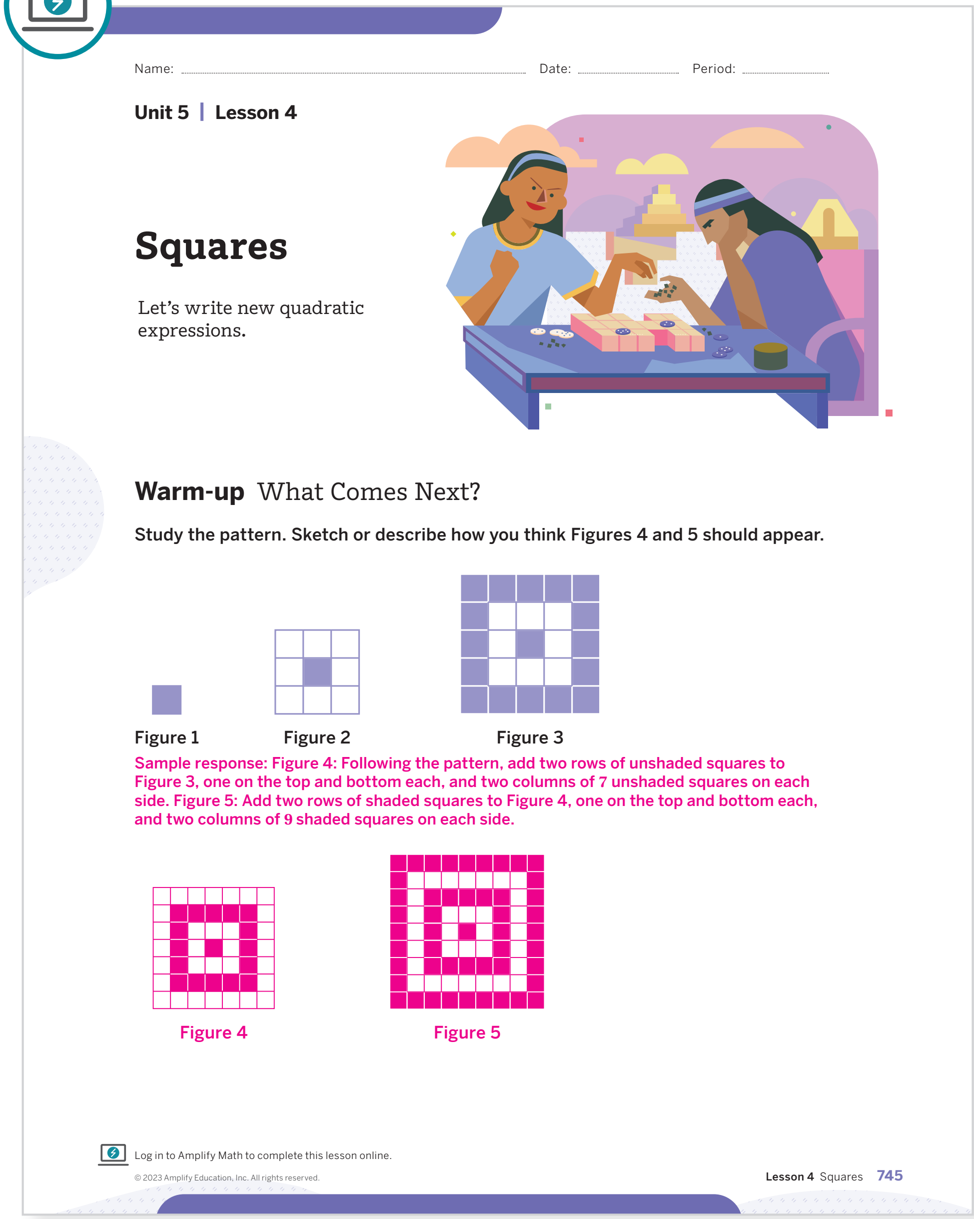
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2, students may omit two rows in the table.
- In **Activity 3**, Problem 2, have students only complete the first four rows of the table.

Independent | 5 min

Warm-up What Comes Next?

Students examine the structure of a quadratic pattern involving squares to discover that, in some quadratic patterns, the number of squares is not simply the square of the figure number.



Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 4

Squares

Let's write new quadratic expressions.

Warm-up What Comes Next?

Study the pattern. Sketch or describe how you think Figures 4 and 5 should appear.

Sample response: Figure 4: Following the pattern, add two rows of unshaded squares to Figure 3, one on the top and bottom each, and two columns of 7 unshaded squares on each side. Figure 5: Add two rows of shaded squares to Figure 4, one on the top and bottom each, and two columns of 9 shaded squares on each side.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Squares 745

1 Launch

Set an expectation for the amount of time students will have to work independently on the activity.

2 Monitor

Help students get started by asking what they notice about the three figures in the pattern.

Look for productive strategies:

- Calculating the difference between each successive figure.
- Noting how the shading alternates for the number of new squares that are added to each figure.
- Sketching several figures beyond Figure 3.
- Describing the pattern using the term *quadratic*.

3 Connect

Have students share what they noticed about the patterns of shading and the number of new squares added to each figure.

Display students' figures. Provide time for students to copy, or make note of, the correct figures.

Highlight that the patterns in shading can help them determine the first differences in the quadratic growth pattern. For example, the number of new squares in each figure, represented by the alternate shading of the outside perimeter, represent the first differences. The first differences are 8, 16, 24, and 32.

Ask, "How is this pattern different than the other square patterns you have seen in previous lessons? How is it similar?" Sample response: The number of squares in this pattern is not equal to the square of the figure number, like the other quadratic patterns in earlier lessons in this unit. However, because the first differences are 8, 16, 24, and 32, the second differences are all equal (8). So, this does represent a quadratic relationship.

Math Language Development

MLR8: Discussion Supports

During the Connect, display or provide access to the Anchor Chart PDF, *Sentence Stems, Describing My Thinking* to support students as they share what they noticed about the patterns of shading and the number of new squares added to each figure.

Power-up

To power up students' ability to recognize quadratic growth:

Provide students with a copy of the Power-up PDF.

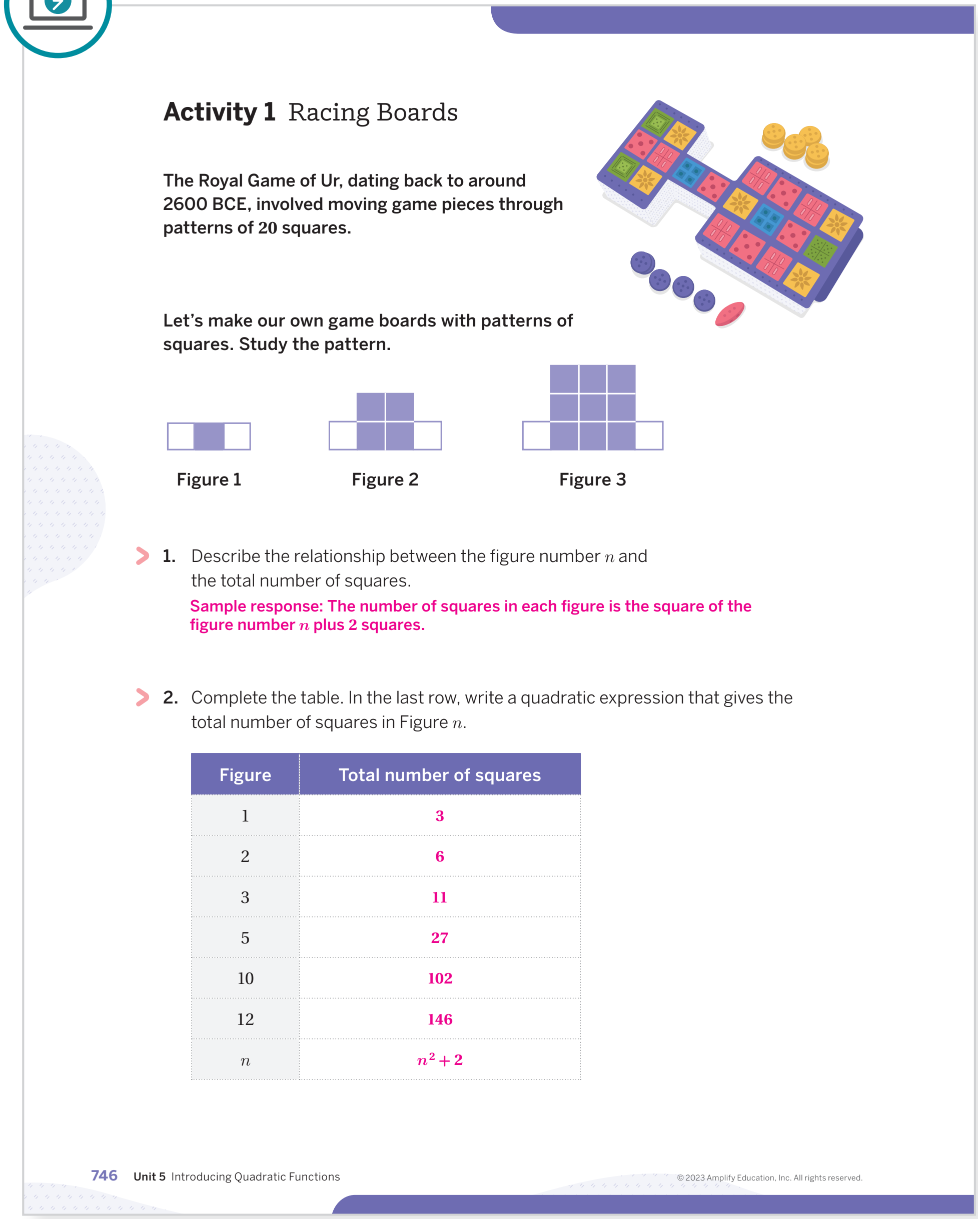
Use: Before the Warm-up

Informed by: Performance on Lesson 3, Practice Problem 6

Pairs | 10 min

Activity 1 Racing Boards

Students examine a quadratic pattern involving squares to understand that in some quadratic patterns, part of the pattern may remain unchanged (constant).



Activity 1 Racing Boards

The Royal Game of Ur, dating back to around 2600 BCE, involved moving game pieces through patterns of 20 squares.

Let's make our own game boards with patterns of squares. Study the pattern.

Figure 1 Figure 2 Figure 3

1. Describe the relationship between the figure number n and the total number of squares.
Sample response: The number of squares in each figure is the square of the figure number n plus 2 squares.

2. Complete the table. In the last row, write a quadratic expression that gives the total number of squares in Figure n.

Figure	Total number of squares
1	3
2	6
3	11
5	27
10	102
12	146
n	$n^2 + 2$

746 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask students if they have heard of the Royal Game of Ur. Consider providing a brief description of the game if students are unfamiliar.

2 Monitor

Help students get started by asking what changes, and what stays the same, from figure to figure.

Look for points of confusion:

- **Thinking the two unshaded squares indicate linear growth.** Ask, "If the two unshaded squares represent linear growth, how are they growing? Do you know another term that describes a value that does not change?"

Look for productive strategies:

- Recognizing the two unshaded squares are constant in each figure and the quadratic growth is represented by the shaded squares.

3 Connect

Display Figures 1–3.

Have students share patterns they see in the number of unshaded and shaded squares, and the strategies they used for writing their expressions in Problem 2.

Highlight that the two unshaded squares remain constant in each figure. The number of shaded squares is the square of the figure number, n^2.

Ask:

- "What stays the same from figure to figure? How is this represented in your expression?" The number of unshaded squares. It is the constant 2.
- "What changes from figure to figure? How is this represented in your expression?" The number of shaded squares. They are represented by the expression n^2.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students study what stays the same in each figure and what changes. Have them write a number or expression that represents what stays the same, e.g., 2 unshaded squares. Have them do the same for the quantities that change, e.g., n^2 shaded squares if the figure number is n.

Extension: *Math Enrichment*

Ask students whether the expression $n^2 + 2$ would change if both unshaded squares were on the left side of each figure. Have them explain their thinking. No; Sample response: The 2 just represents the number of unshaded squares that is constant, not the location of those squares.

Math Language Development

MLR5: Co-craft Questions

During the Launch, reveal the introductory text and Figures 1–3. Have students work with their partner to write 2–3 mathematical questions they could ask about the figures. Have volunteers share their questions with the class. Sample questions shown.

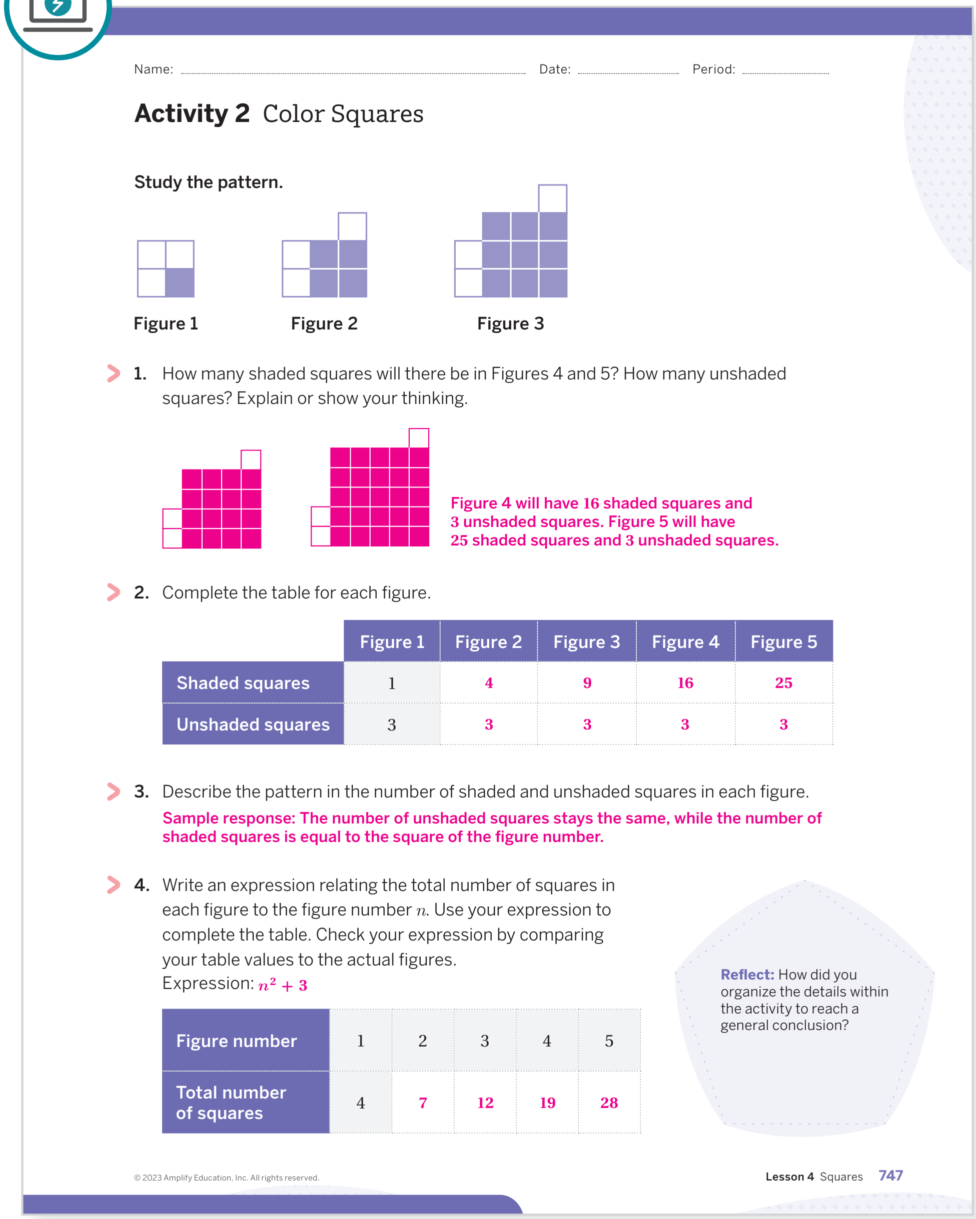
- How is this pattern growing? Is it linear? Quadratic?
- What will Figure 4 look like? Figure n?
- How many squares will there be in Figure 4? Figure n?

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Activity 2 Color Squares

Students study another quadratic pattern in which part of the pattern remains constant to make connections between the pattern and the quadratic expression that represents it.



Name: ______ Date: ______ Period: ______

Activity 2 Color Squares

Study the pattern.

Figure 1 Figure 2 Figure 3

1. How many shaded squares will there be in Figures 4 and 5? How many unshaded squares? Explain or show your thinking.

Figure 4 will have 16 shaded squares and 3 unshaded squares. Figure 5 will have 25 shaded squares and 3 unshaded squares.

2. Complete the table for each figure.

	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5
Shaded squares	1	4	9	16	25
Unshaded squares	3	3	3	3	3

3. Describe the pattern in the number of shaded and unshaded squares in each figure.

Sample response: The number of unshaded squares stays the same, while the number of shaded squares is equal to the square of the figure number.

4. Write an expression relating the total number of squares in each figure to the figure number n. Use your expression to complete the table. Check your expression by comparing your table values to the actual figures.
Expression: $n^2 + 3$

Figure number	1	2	3	4	5
Total number of squares	4	7	12	19	28

Reflect: How did you organize the details within the activity to reach a general conclusion?

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Squares 747

1 Launch

Provide student pairs with snap cubes to build the figures, or grid paper to sketch the figures.

2 Monitor

Help students get started having them build or sketch the next figure in the pattern. Ask, "How is the pattern growing?"

Look for points of confusion:

- **Not representing the number of unshaded squares that remain unchanged in each figure as a constant in the quadratic expression.** Draw Figures 1–3 without the unshaded squares and ask students to write a quadratic expression that represents the pattern.

Look for productive strategies:

- Connecting the dimensions of the larger shaded square to the figure number, and squaring the figure number to calculate the total number of small shaded squares.
- Connecting the three unshaded squares that remain unchanged in each figure to the constant in the quadratic expression $n^2 + 3$.

3 Connect

Display the figures.

Have students share their reasoning or thinking for Problem 4.

Highlight that the total number of squares in the table for Problem 4 could be written as $1 + 3$, $4 + 3$, $9 + 3$, $16 + 3$, and $25 + 3$ to see more clearly the pattern of square numbers and the added constant.

Ask, "What relationship do you see between the figure number and the dimensions of the shaded square in each figure?" The dimension is the same as the figure number.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students study what stays the same in each figure and what changes. Have them write a number or expression that represents what stays the same, e.g., 3 unshaded squares. Have them do the same for the quantities that change, e.g., n^2 shaded squares if the figure number is n.

Extension: *Math Enrichment*

Tell students that the quadratic expression $n^2 + n^2$, or $2n^2$, represents a quadratic pattern, where n represents the figure number. Have them draw or describe what the first three figures in the pattern look like. Answers may vary, but students should describe a pattern where the total number of squares in the first three figures is 2, 8, and 18.

Math Language Development

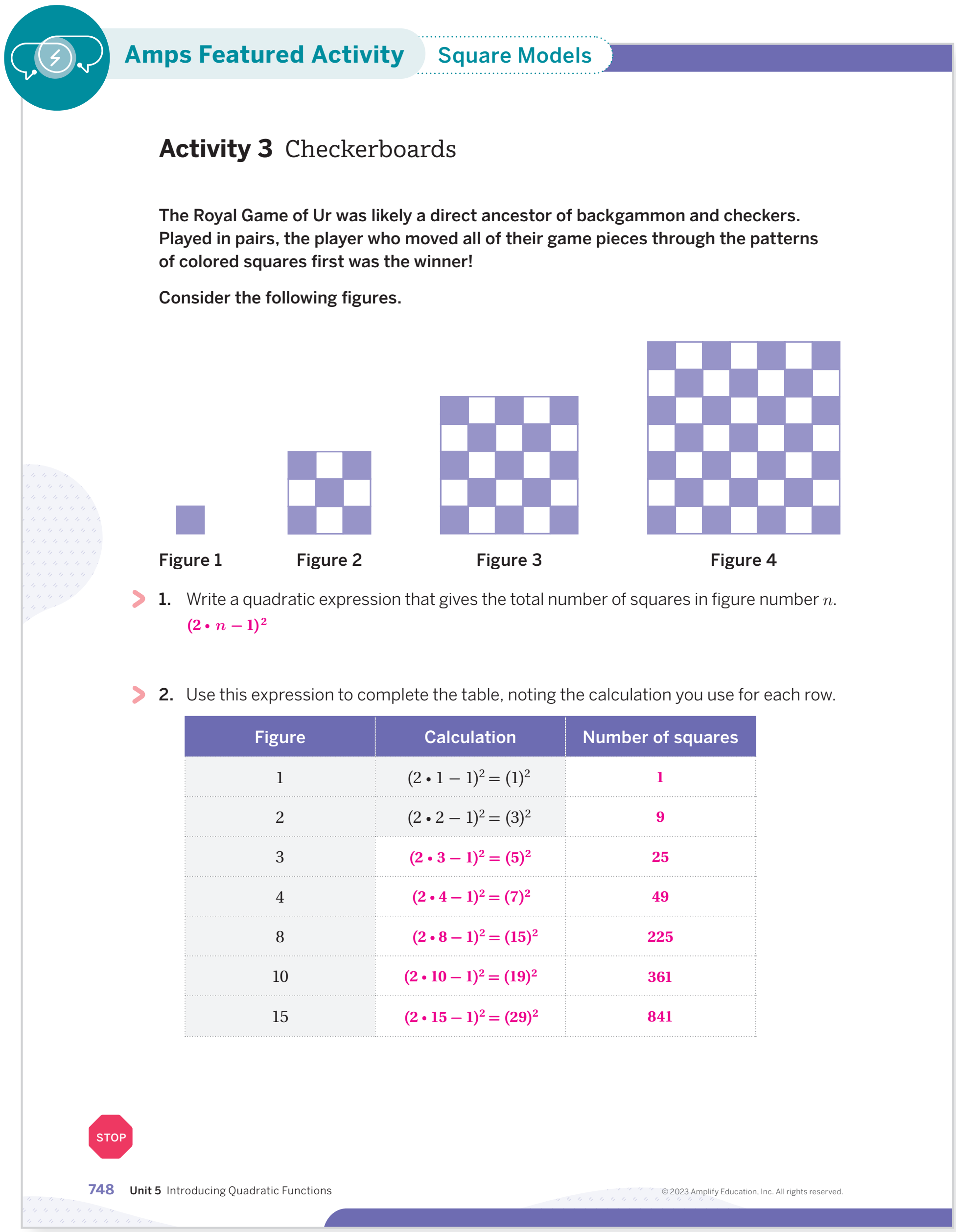
MLR7: Compare and Connect

During the Connect, as students respond to the Ask question, tell them that in this activity, the figures showed very clearly the square pattern by the fact that the square pattern was shaded and the shaded squares formed a larger square. Ask students, "Would the growth pattern still be quadratic if the number of squares in each pattern remained the same, but were arranged differently?" Consider drawing an example or asking students to draw an example. Model continued use of mathematical language, such as *square pattern, quadratic growth, constant*, and highlight how the *square of the figure number* persists in the expression.

Pairs | 15 min

Activity 3 Checkerboards

Students encounter a more complicated quadratic pattern involving squares to see that quadratic expressions can represent more complex patterns.



Amps Featured Activity | Square Models

Activity 3 Checkerboards

The Royal Game of Ur was likely a direct ancestor of backgammon and checkers. Played in pairs, the player who moved all of their game pieces through the patterns of colored squares first was the winner!

Consider the following figures.

1. Write a quadratic expression that gives the total number of squares in figure number n.
   $(2 \bullet n - 1)^2$

2. Use this expression to complete the table, noting the calculation you use for each row.

Figure	Calculation	Number of squares
1	$(2 \bullet 1 - 1)^2 = (1)^2$	1
2	$(2 \bullet 2 - 1)^2 = (3)^2$	9
3	$(2 \bullet 3 - 1)^2 = (5)^2$	25
4	$(2 \bullet 4 - 1)^2 = (7)^2$	49
8	$(2 \bullet 8 - 1)^2 = (15)^2$	225
10	$(2 \bullet 10 - 1)^2 = (19)^2$	361
15	$(2 \bullet 15 - 1)^2 = (29)^2$	841

STOP

748 **Unit 5** Introducing Quadratic Functions

1 Launch

Read the prompt aloud. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking them to describe what is changing from one figure to the next.

Look for points of confusion:

- **Understanding there is a squared term but unable to determine a pattern.** Have students write the number of squares as squared terms, 1^2, 3^2, 5^2, and 7^2, and then continue the pattern to determine the number of squares in Figure 5 to help them see the pattern.

Look for productive strategies:

- Creating a table of figure numbers and number of squares, and calculating the first and second differences.
- Connecting the dimensions of the square to the figure number, noting that the side length of the square is 1 less than twice the figure number.

3 Connect

Display the table in Problem 2.

Have students share their strategies for writing their quadratic expressions.

Highlight that expressions can be used to model quadratic relationships. The figure number is connected to the expression. The dimension of each square is 1 *less than twice the figure number.* The nth figure is a square with side length $2n - 1$.

Ask, "How could you use the area of a square to determine the number of squares in the nth figure?" Calculate the square of its dimensions, $(2n - 1)^2$.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital tools to explore quadratic growth patterns by building square models and writing numeric quadratic expressions.

Accessibility: *Guide Processing and Visualization*

Suggest that students annotate the side lengths for each figure. Ask:

- "Is there a relationship between the figure number and the figure's side length? Is it the same relationship you saw in the prior activities?"
- "Is there a constant term that does not change, as you saw in the prior activities?"

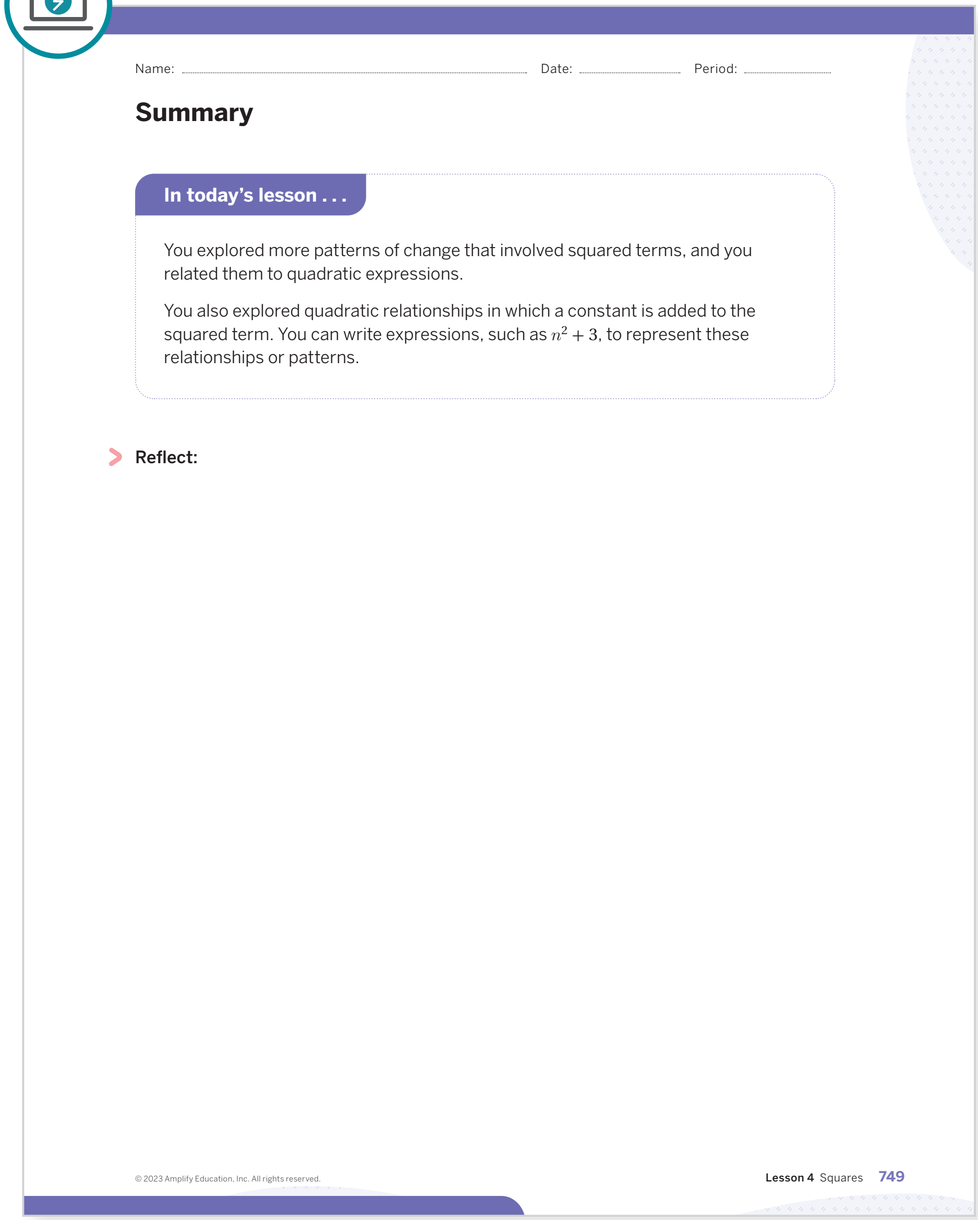
Extension: *Math Enrichment*

Tell students that the quadratic expression $(2n - 1)^2 + 3$ represents a quadratic pattern, where n represents the figure number. Have them draw or describe what the first four figures in the pattern look like. Have students calculate the first and second differences of their pattern to confirm the pattern is quadratic. Answers may vary, but students should describe a pattern where the total number of squares in the first four figures is 4, 12, 28, and 52.

Whole Class | 5 min

Summary

Review and synthesize how a quadratic relationship can be seen in a pattern of squares and how a quadratic expression can be written, using a squared term such as n^2, to represent the pattern.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored more patterns of change that involved squared terms, and you related them to quadratic expressions.

You also explored quadratic relationships in which a constant is added to the squared term. You can write expressions, such as $n^2 + 3$, to represent these relationships or patterns.

Reflect:

Synthesize

Display the Figures 1–3 from Activity 2.

Have students share how they wrote a quadratic expression to represent the growth pattern, and how they knew the expression had a constant term.

Display Figures 1–4 from Activity 3.

Have students share how they wrote a quadratic expression to represent the pattern and how they knew the expression did *not* have a constant term.

Highlight the connections between quadratic expressions and the visual patterns that they represent. Emphasize that some quadratic expressions involve a constant term and this is seen by an unchanging value in the pattern. Other quadratic expressions do not have a constant term. All quadratic expressions have a squared term, such as n^2. Sometimes the number of squares in the pattern is simply the square of the figure number, and other times — as in Activity 3 — the squared expression is more complex.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

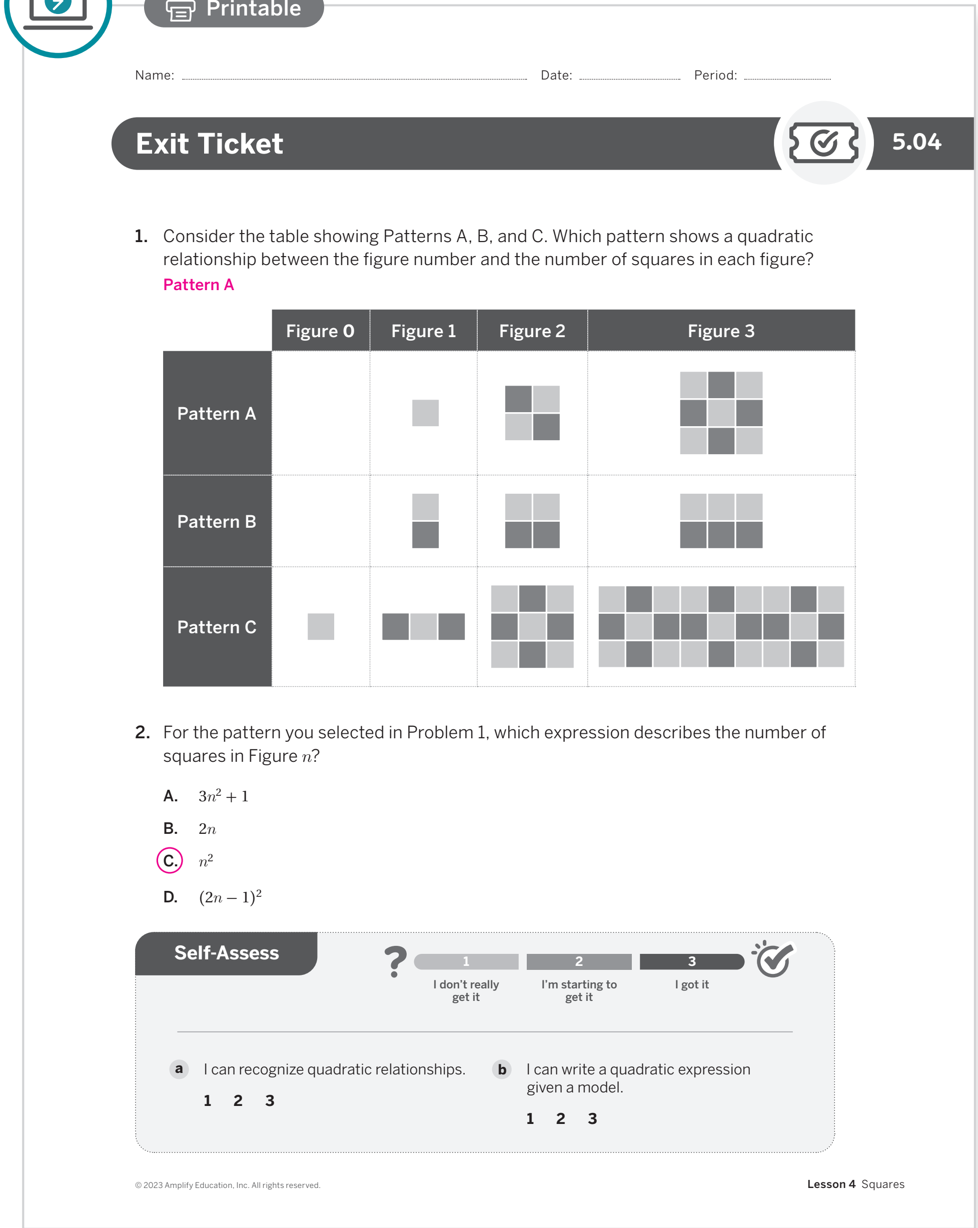
- "What does it mean for a relationship to be quadratic?"
- "How do you use a table of values to write a quadratic expression?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying quadratic growth patterns and writing quadratic expressions to represent them.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.04

1. Consider the table showing Patterns A, B, and C. Which pattern shows a quadratic relationship between the figure number and the number of squares in each figure?
   Pattern A

	Figure 0	Figure 1	Figure 2	Figure 3
Pattern A				
Pattern B				
Pattern C				

2. For the pattern you selected in Problem 1, which expression describes the number of squares in Figure n?
   - A. $3n^2 + 1$
   - B. $2n$
   - [✓ C.] n^2
   - D. $(2n - 1)^2$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can recognize quadratic relationships. 1 2 3

b I can write a quadratic expression given a model. 1 2 3

Lesson 4 Squares

Success looks like . . .

- **Goal:** Understanding that a quadratic relationship can be expressed with a squared term.
- **Language Goal:** Describing a pattern of change associated with a quadratic relationship. **(Speaking and Listening, Writing)**
  - » Identifying the pattern that represents a quadratic relationship in Problem 1.
- **Goal:** Writing expressions that describe quadratic relationships.
  - » Selecting the correct expression that describes the number of squares in Problem 2.

Suggested next steps

If students do not identify Pattern A as quadratic growth in Problem 1, consider:

- Reviewing how to use first and second differences to identify quadratic relationships.

If students do not select the correct quadratic expression in Problem 2, consider:

- Having them first relate the figure number to the number of rows/columns or squares in each figure. Then have them relate the figure number to the total number of squares in the figure.
- Reviewing Activity 1, Problem 2 and Activity 3, Problem 1.

Professional Learning

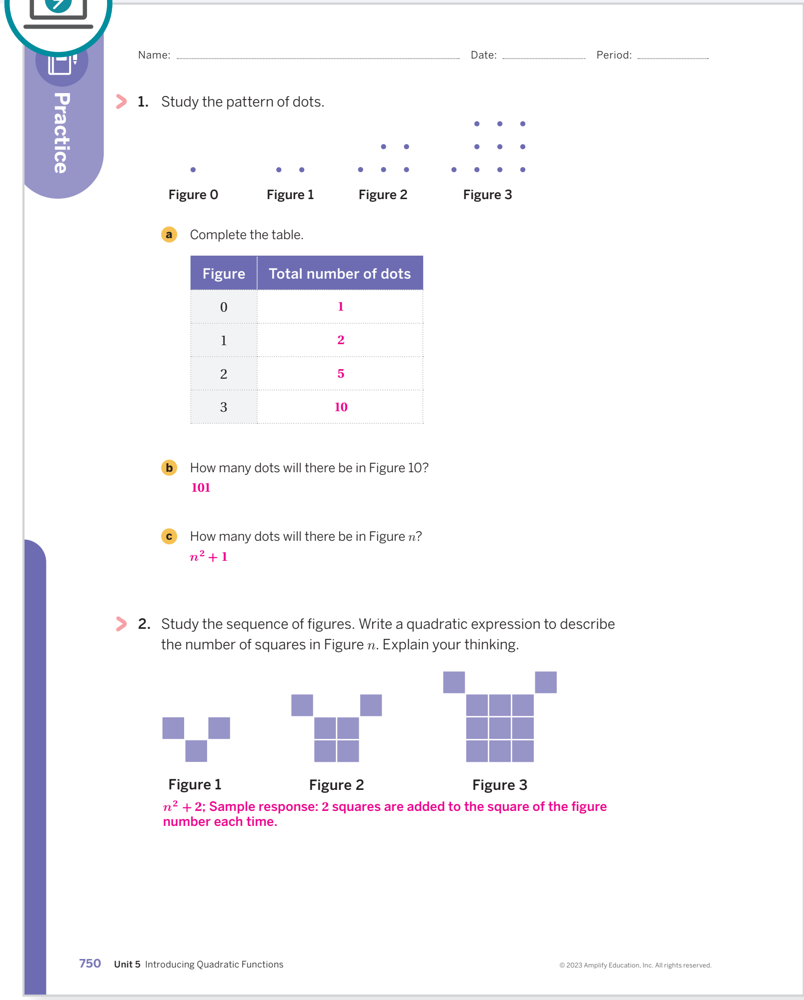
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How was describing and using patterns similar to or different from Lesson 3's pattern activities?
- What did students find frustrating about writing quadratic expressions? What helped them work through this frustration?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Study the pattern of dots.

Figure 0 Figure 1 Figure 2 Figure 3

a Complete the table.

Figure	Total number of dots
0	1
1	2
2	5
3	10

b How many dots will there be in Figure 10?

101

c How many dots will there be in Figure n?

$n^2 + 1$

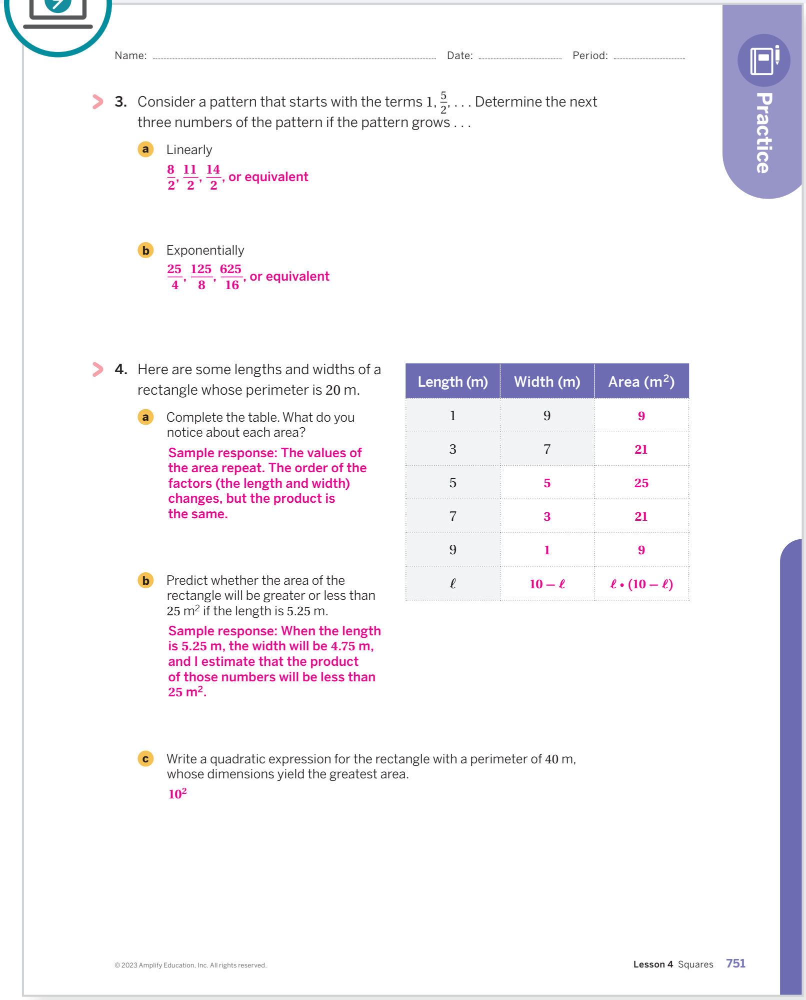
2. Study the sequence of figures. Write a quadratic expression to describe the number of squares in Figure n. Explain your thinking.

Figure 1 Figure 2 Figure 3

$n^2 + 2$; Sample response: 2 squares are added to the square of the figure number each time.

750 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ________ Date: ________ Period: ________

3. Consider a pattern that starts with the terms $1, \frac{5}{2}, \ldots$ Determine the next three numbers of the pattern if the pattern grows . . .

a Linearly

$\frac{8}{2}, \frac{11}{2}, \frac{14}{2}$, or equivalent

b Exponentially

$\frac{25}{4}, \frac{125}{8}, \frac{625}{16}$, or equivalent

4. Here are some lengths and widths of a rectangle whose perimeter is 20 m.

Length (m)	Width (m)	Area (m^2)
1	9	9
3	7	21
5	5	25
7	3	21
9	1	9
ℓ	$10 - \ell$	$\ell \cdot (10 - \ell)$

a Complete the table. What do you notice about each area?

Sample response: The values of the area repeat. The order of the factors (the length and width) changes, but the product is the same.

b Predict whether the area of the rectangle will be greater or less than 25 m^2 if the length is 5.25 m.

Sample response: When the length is 5.25 m, the width will be 4.75 m, and I estimate that the product of those numbers will be less than 25 m^2.

c Write a quadratic expression for the rectangle with a perimeter of 40 m, whose dimensions yield the greatest area.

10^2

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Squares 751

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	3
	2	Activity 1	2
Spiral	3	Unit 4 Lesson 4	2
Formative	4	Unit 5 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 5**

Seeing Squares as Functions

Let's describe some other geometric patterns.

Focus

Goals

1. **Language Goal:** Interpret (using multiple representations) the quadratic relationships in growing patterns as functions, where each input gives a particular output. **(Speaking and Listening, Writing)**
2. Write expressions that define quadratic functions.
3. Understand that the same quadratic function can be expressed symbolically in different ways.

Coherence

Today

Students continue to explore the connection between quadratic expressions and squares. They use the patterns in sequences of figures to visualize and write quadratic expressions that will be formally defined as quadratic functions.

Previously

In previous lessons, students used visual patterns to build their conceptual understanding of squared terms and their relationship with quadratics. They wrote quadratic expressions to describe those patterns and began building their capacity to understand the differences between linear and quadratic change.

Coming Soon

Students will compare quadratic functions to exponential functions in subsequent lessons and explore real-world scenarios modeled by quadratic functions.

Rigor

- Students build **conceptual understanding** of quadratic functions.
- Students write quadratic functions to represent the area of a figure to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	15 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Critiquing*

Math Language Development

New words

- ***quadratic function***

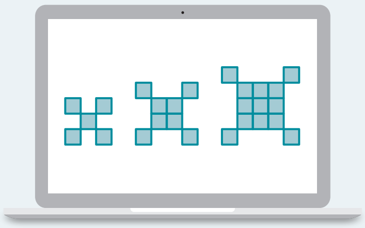
Review words

- *quadratic expression*

Amps | Featured Activity

Activity 2
Growing Patterns

Students use interactive tools to create sketches of figures that represent quadratic expressions and extend quadratic patterns.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 3, students are asked to share their thinking about the pattern and they might feel inhibited because of previous poor responses from others when they did so. Prior to the discussion, establish some guidelines for the conversation, ensuring that all students understand what proper social interactions look like. Reinforce positive and successful communications.

Modifications to Pacing

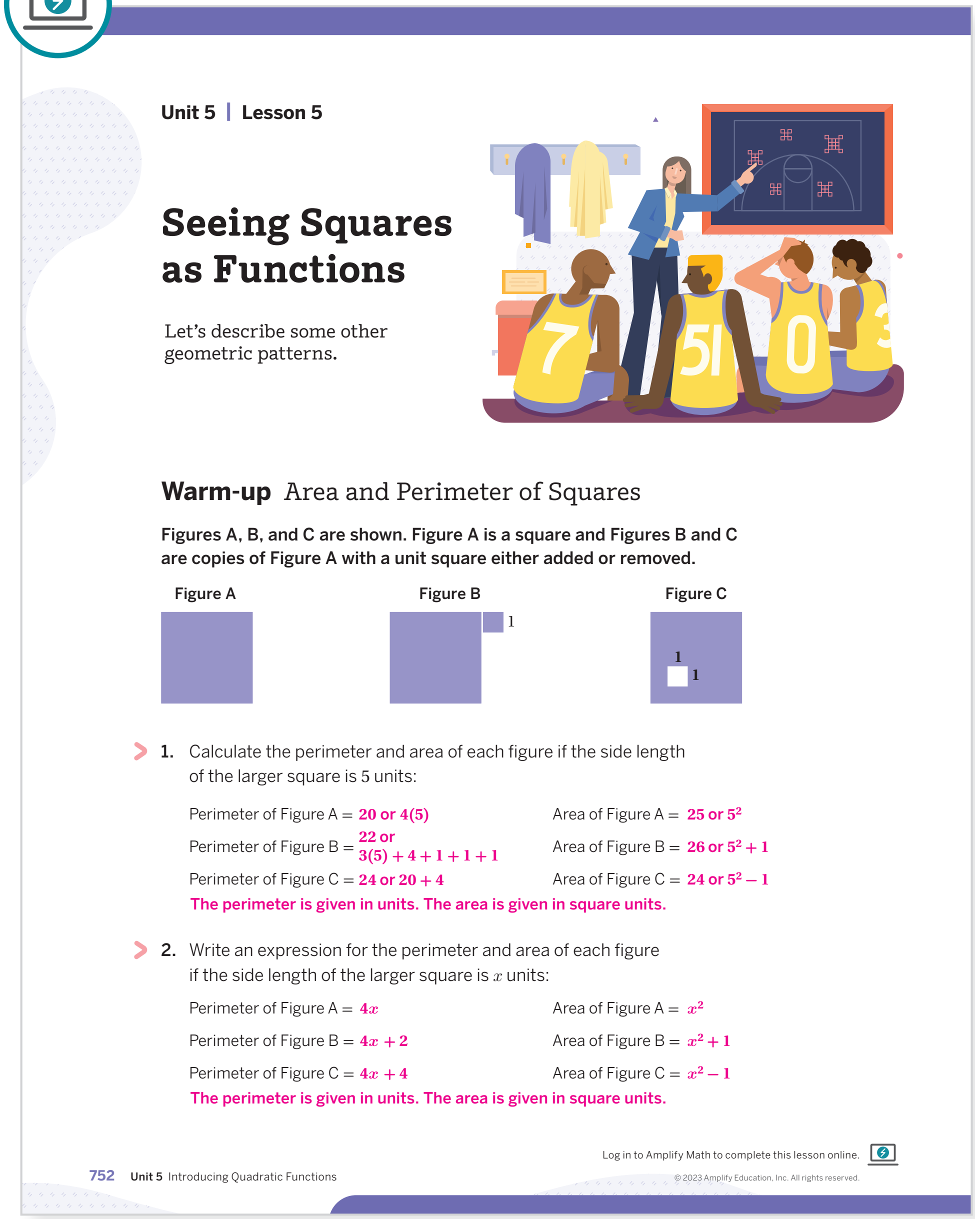
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problem 5 may be omitted.
- Optional **Activity 3** may be omitted.

Independent | 10 min

Warm-up Area and Perimeter of Squares

Students write expressions to represent the area of figures — composed of squares — to connect algebraic and geometric representations of quadratic patterns.



Unit 5 | Lesson 5

Seeing Squares as Functions

Let's describe some other geometric patterns.

Warm-up Area and Perimeter of Squares

Figures A, B, and C are shown. Figure A is a square and Figures B and C are copies of Figure A with a unit square either added or removed.

1. Calculate the perimeter and area of each figure if the side length of the larger square is 5 units:

   Perimeter of Figure A = **20 or $4(5)$**
   Area of Figure A = **25 or 5^2**
   Perimeter of Figure B = **22 or $3(5)+4+1+1+1$**
   Area of Figure B = **26 or 5^2+1**
   Perimeter of Figure C = **24 or $20+4$**
   Area of Figure C = **24 or 5^2-1**
   The perimeter is given in units. The area is given in square units.

2. Write an expression for the perimeter and area of each figure if the side length of the larger square is x units:

   Perimeter of Figure A = $4x$
   Area of Figure A = x^2
   Perimeter of Figure B = $4x+2$
   Area of Figure B = x^2+1
   Perimeter of Figure C = $4x+4$
   Area of Figure C = x^2-1
   The perimeter is given in units. The area is given in square units.

Log in to Amplify Math to complete this lesson online.

752 **Unit 5** Introducing Quadratic Functions

1 Launch

Activate students' prior knowledge by asking them how to calculate the perimeter and area of a square. Students are not required to complete all parts of the Warm-up, but should make an attempt.

2 Monitor

Help students get started by prompting them to label the sides of the figures with the given side length in Problems 1 and 2.

Look for points of confusion:

- **Miscalculating the perimeter of Figure B.** Prompt students to trace or highlight the edges of the figure and label their lengths.
- **Miscalculating the perimeter of Figure C.** Ask, "How much fencing would you need if you wanted to enclose the entire shaded area?"

Look for productive strategies:

- Labeling or tracing the sides of the figures.
- Using the numerical expressions in Problem 1 to write algebraic expressions in Problem 2.

3 Connect

Display Figures A, B, and C.

Have students share their strategies for determining the area and perimeter. Select students who used productive strategies to share how they determined the perimeter and area for each figure when the side length is 5 and when the side length is x.

Highlight that each expression representing the area is a *quadratic expression*. (All expressions have a variable or a term that is squared because they correspond to the geometric area of a square.)

Power-up

To power up students' ability to relate quadratic expressions to area, have students complete:

Write an expression to represent the area of each rectangle with the following dimensions.

1. Length: x, Width: $(x+2)$ $x(x+2)$, or x^2+2x
2. Length: a, Width: $(a-3)$ $a(a-3)$, or a^2-3a
3. Length: $2.4x$, Width: $(0.5x+6)$ $2.4x(0.5x+6)$, or $1.2x^2+14.4x$

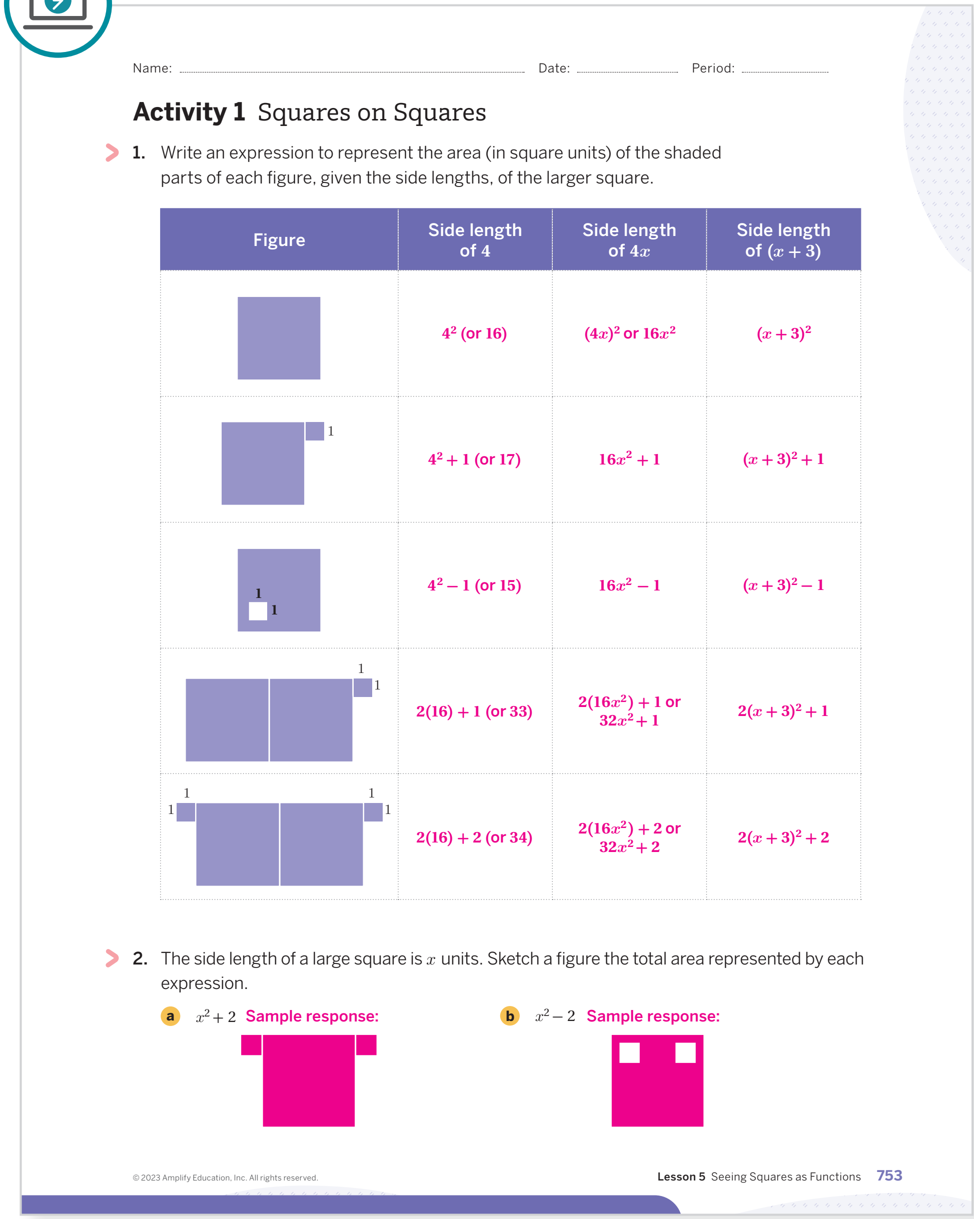
Use: Before the Warm-up

Informed by: Performance on Lesson 4, Practice Problem 4 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 15 min

Activity 1 Squares on Squares

Students extend the figures they encountered in the Warm-up to write more complicated quadratic expressions representing their areas.



Name: ______ Date: ______ Period: ______

Activity 1 Squares on Squares

1. Write an expression to represent the area (in square units) of the shaded parts of each figure, given the side lengths, of the larger square.

Figure	Side length of 4	Side length of $4x$	Side length of $(x+3)$
	4^2 (or 16)	$(4x)^2$ or $16x^2$	$(x+3)^2$
	4^2+1 (or 17)	$16x^2+1$	$(x+3)^2+1$
	4^2-1 (or 15)	$16x^2-1$	$(x+3)^2-1$
	$2(16)+1$ (or 33)	$2(16x^2)+1$ or $32x^2+1$	$2(x+3)^2+1$
	$2(16)+2$ (or 34)	$2(16x^2)+2$ or $32x^2+2$	$2(x+3)^2+2$

2. The side length of a large square is x units. Sketch a figure the total area represented by each expression.

a x^2+2 **Sample response:**

b x^2-2 **Sample response:**

1 Launch

Encourage students to refer to their responses in the Warm-up, particularly for the first three figures in the table.

2 Monitor

Help students get started by having them use the given side lengths to label the side lengths of each figure.

Look for points of confusion:

- **Squaring $4x$ incorrectly.** Remind students of what it means to "square" a quantity. Each part, 4 and x, of the quantity should be squared.
- **Struggling to square $(x+3)$.** Tell students that they may express the area as a product of two factors, $(x+3)(x+3)$, or by using an exponent, $(x+3)^2$.

Look for productive strategies:

- Using parentheses when squaring the side lengths.
- Connecting two large squares as two of the squared quantities 4, $4x$, or $(x+3)$.
- Connecting the addition or removal of one unit square to adding or subtracting 1 in the expression.

3 Connect

Display the table.

Have students share their responses for the table. Select pairs of students to share different equivalent expressions. Include students who made use of structure to complete Problem 2.

Highlight that all the expressions in the table are quadratic because there is a squared term, even if that squared term is simplified (as in the first column). For Problem 2, point out the connection between the two small squares and the constant terms in the figures represented by x^2+2 and x^2-2, if not previously discussed.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them complete the first two rows of the table. Then have them complete the diagonal cells from top to bottom right for the remaining figures. They will have exposure to other quadratic expressions and the opportunity to make connections during the Connect.

Extension: *Math Enrichment*

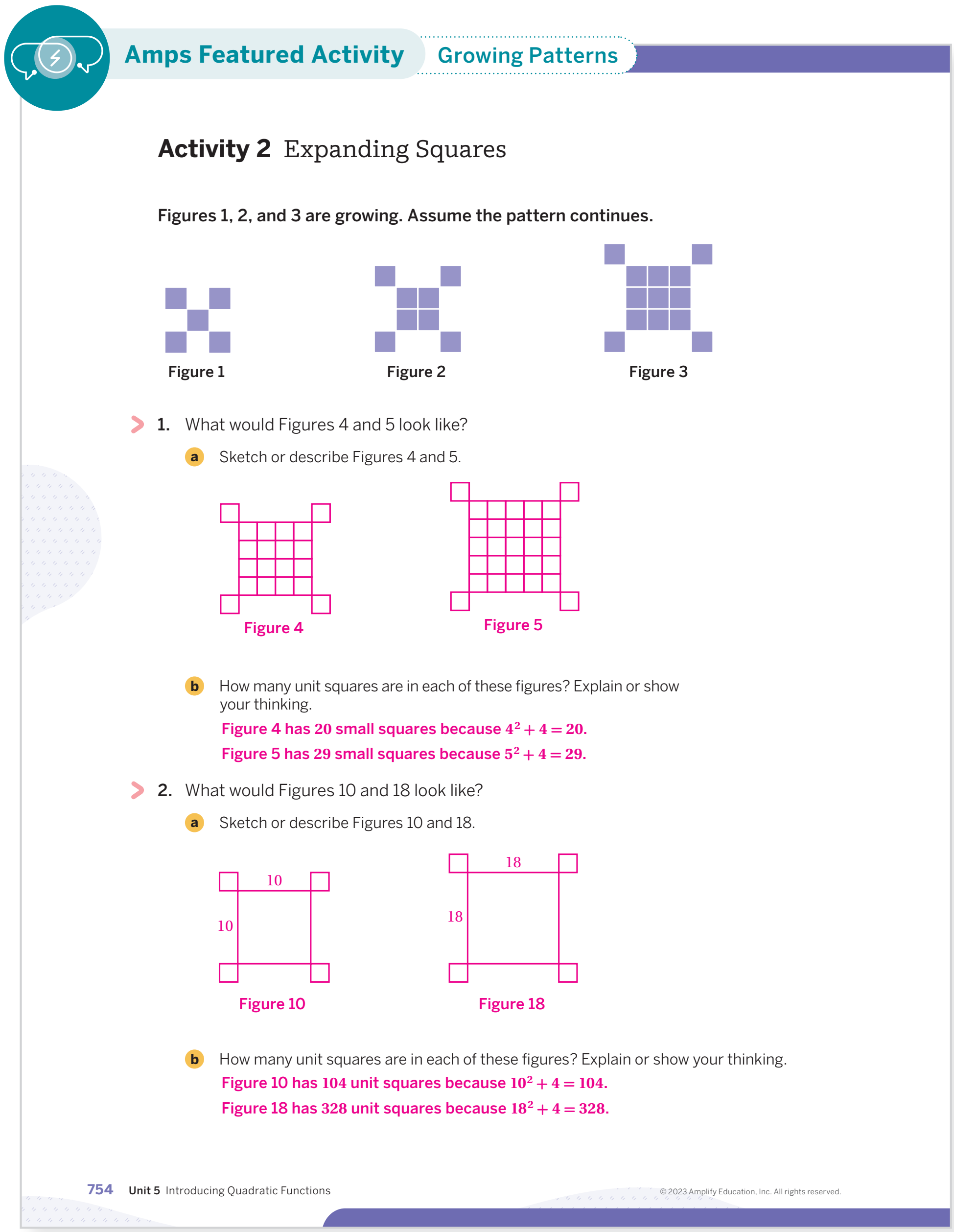
Have students use the structure of the expressions they have written to write an expression for the area of the last figure in the table in Problem 1 if the side length of one of the large squares is (x^2+1). Ask them if they think this expression is a quadratic expression and have them explain their thinking. $2(x^2+1)^2+2$; Sample response: This expression is not quadratic because the second differences are not the same. For values of x from 1–4, the values of the expression are 10, 52, 202, and 580. The first differences are 42, 150, and 378. The second differences are 108 and 228.

Pairs | 15 min

Activity 2 Expanding Squares

Students build a quadratic function to represent a geometric pattern involving a squared term and a constant, without using a table of values.

Amps Featured Activity Growing Patterns



Activity 2 Expanding Squares

Figures 1, 2, and 3 are growing. Assume the pattern continues.

Figure 1 Figure 2 Figure 3

1. What would Figures 4 and 5 look like?

   a Sketch or describe Figures 4 and 5.

   Figure 4 Figure 5

   b How many unit squares are in each of these figures? Explain or show your thinking.

   Figure 4 has 20 small squares because $4^2 + 4 = 20$.

   Figure 5 has 29 small squares because $5^2 + 4 = 29$.

2. What would Figures 10 and 18 look like?

   a Sketch or describe Figures 10 and 18.

   Figure 10 Figure 18

   b How many unit squares are in each of these figures? Explain or show your thinking.

   Figure 10 has 104 unit squares because $10^2 + 4 = 104$.

   Figure 18 has 328 unit squares because $18^2 + 4 = 328$.

754 **Unit 5** Introducing Quadratic Functions

1 Launch

Display Figures 1–3. Conduct the ***Notice and Wonder*** routine, asking students to describe what they notice about the pattern and what questions they have.

2 Monitor

Help students get started by examining the growth pattern and having them sketch or describe the next figure in the pattern.

Look for points of confusion:

- **Having difficulty sketching Figures 10 and 18 in Problem 2.** Encourage students to label the dimensions or write a description, instead of drawing individual squares.
- **Having difficulty sketching the first figure represented by $V(n)$ in Problem 5.** Prompt students to begin with Figure 2 or 3 and work backward to Figure 1.

Look for productive strategies:

- Labeling the dimensions or area of each figure, or showing other evidence of counting the number of squares.
- Making a table of values to organize input and output values.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive tools to create sketches of figures that represent quadratic expressions and extend quadratic patterns.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them sketch the figure and determine the number of squares in Figure 4 only for Problem 1, and in Figure 10 only for Problem 2.

Accessibility: *Activate Prior Knowledge*

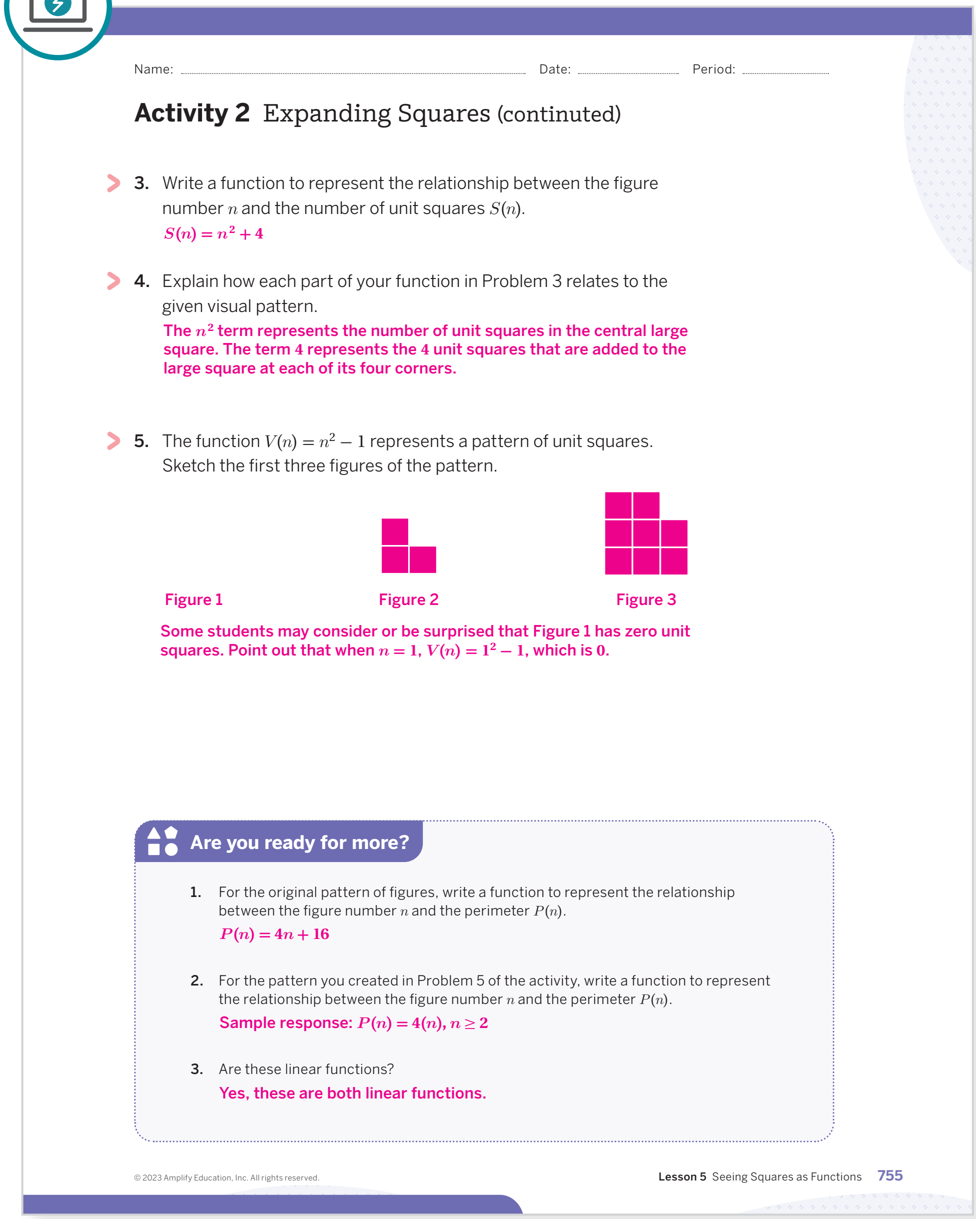
Remind students they previously learned about functions and function notation. Have them preview Problem 5 and ask, "Is the statement $V(n) = n^2 - 1$ written in function notation? Why or why not?" Then ask them to explain in their own words what makes a relationship a function. Consider displaying the following sentence frame.

"A relationship is a function if . . ." There is exactly one output value for every input value.

Pairs | 15 min

Activity 2 Expanding Squares (continued)

Students build a quadratic function to represent a geometric pattern involving a squared term and a constant, without using a table of values.



Name: ______ Date: ______ Period: ______

Activity 2 Expanding Squares (continued)

3. Write a function to represent the relationship between the figure number n and the number of unit squares $S(n)$.
   $S(n) = n^2 + 4$

4. Explain how each part of your function in Problem 3 relates to the given visual pattern.
   The n^2 term represents the number of unit squares in the central large square. The term 4 represents the 4 unit squares that are added to the large square at each of its four corners.

5. The function $V(n) = n^2 - 1$ represents a pattern of unit squares. Sketch the first three figures of the pattern.

Some students may consider or be surprised that Figure 1 has zero unit squares. Point out that when $n = 1$, $V(n) = 1^2 - 1$, which is 0.

Are you ready for more?

1. For the original pattern of figures, write a function to represent the relationship between the figure number n and the perimeter $P(n)$.
   $P(n) = 4n + 16$
2. For the pattern you created in Problem 5 of the activity, write a function to represent the relationship between the figure number n and the perimeter $P(n)$.
   Sample response: $P(n) = 4(n), n \geq 2$
3. Are these linear functions?
   Yes, these are both linear functions.

3 Connect

Display student sketches for Figures 4 and 5.

Have students share their processes for sketching Figures 10 and 18 of the pattern. Select students who used productive strategies to discern a pattern from the given figures.

Highlight the connection between $S(n) = n^2 + 4$ and the composition of the squares in the pattern, namely that the figure for term n is an n-by-n square that has four unit squares at each corner.

Have students share their sketches for Problem 5. Compare sketches and have them explain why any n-by-n array with a unit square missing is an appropriate sketch. Point out that Figure 1 is composed of 0 unit squares because when $n = 1$, $V(n) = 1^2 - 1$, which is 0.

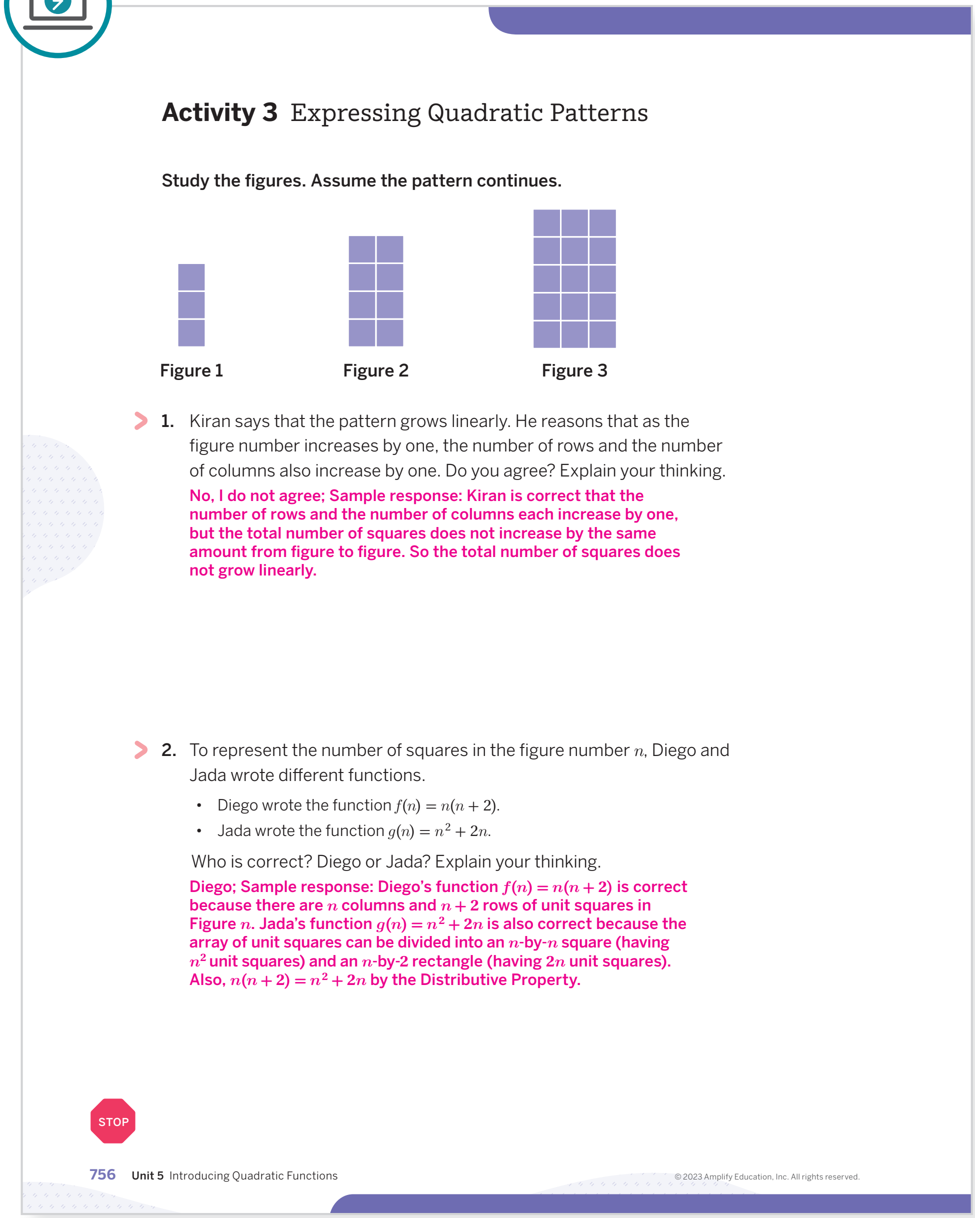
Ask, "Why are each of these patterns a function?" For every figure number, there is a unique number of squares.

Optional

Pairs | 10 min

Activity 3 Expressing Quadratic Patterns

Students activate their prior knowledge of linear growth by critiquing the reasoning of others as they determine the type of growth and the function that defines it.



Activity 3 Expressing Quadratic Patterns

Study the figures. Assume the pattern continues.

Figure 1

Figure 2

Figure 3

1. Kiran says that the pattern grows linearly. He reasons that as the figure number increases by one, the number of rows and the number of columns also increase by one. Do you agree? Explain your thinking.
   No, I do not agree; Sample response: Kiran is correct that the number of rows and the number of columns each increase by one, but the total number of squares does not increase by the same amount from figure to figure. So the total number of squares does not grow linearly.

2. To represent the number of squares in the figure number n, Diego and Jada wrote different functions.
   - Diego wrote the function $f(n) = n(n + 2)$.
   - Jada wrote the function $g(n) = n^2 + 2n$.

   Who is correct? Diego or Jada? Explain your thinking.
   Diego; Sample response: Diego's function $f(n) = n(n + 2)$ is correct because there are n columns and $n + 2$ rows of unit squares in Figure n. Jada's function $g(n) = n^2 + 2n$ is also correct because the array of unit squares can be divided into an n-by-n square (having n^2 unit squares) and an n-by-2 rectangle (having $2n$ unit squares). Also, $n(n + 2) = n^2 + 2n$ by the Distributive Property.

STOP

1 Launch

Have students first complete the activity independently before sharing their responses with a partner.

2 Monitor

Help students get started by asking them to sketch the next figure in the pattern. Ask, "How does the pattern grow?"

Look for points of confusion:

- **Agreeing with Kiran**. Probe student thinking regarding linear growth over equal intervals.
- **Distributing n in Diego's expression incorrectly**. Make sure students distribute the variable n across the sum.

Look for productive strategies:

- Labeling the dimensions or area of each figure or showing other evidence of counting the number of squares.
- Applying the Distributive Property to show that $n(n + 2) = n^2 + 2n$.

3 Connect

Display the figures. Using the ***Poll the Class*** routine, ask students whether they agree or disagree with Kiran.

Have students share their thinking for Problems 1 and 2. Select students who calculated first and second differences in Problem 1, and students who used the Distributive Property in Problem 2.

Define the term ***quadratic function*.**

Highlight both expressions $n(n + 2)$ and $n^2 + 2n$, and explain that they define the same quadratic function. Sketch a figure to show their equivalence.

Ask, "How can a function be quadratic if it is written without a squared term, e.g., $n(n + 2)$?" Discuss the Distributive Property.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Ask students to use the Distributive Property to write an equivalent expression for the expression $3(n + 2)$. $3n + 6$ Ask, "How is Diego's expression similar to and different from this expression?"

Extension: *Math Enrichment*

Have students determine whether the growth pattern would be linear if only the number of columns increased by 1 for each figure in Problem 1 and the number of rows remained the same. Ask them to explain their thinking. Yes; Sample response: If only the number of columns changed, then the number of squares in each new figure would increase by 3, which demonstrates a constant rate of change.

Math Language Development

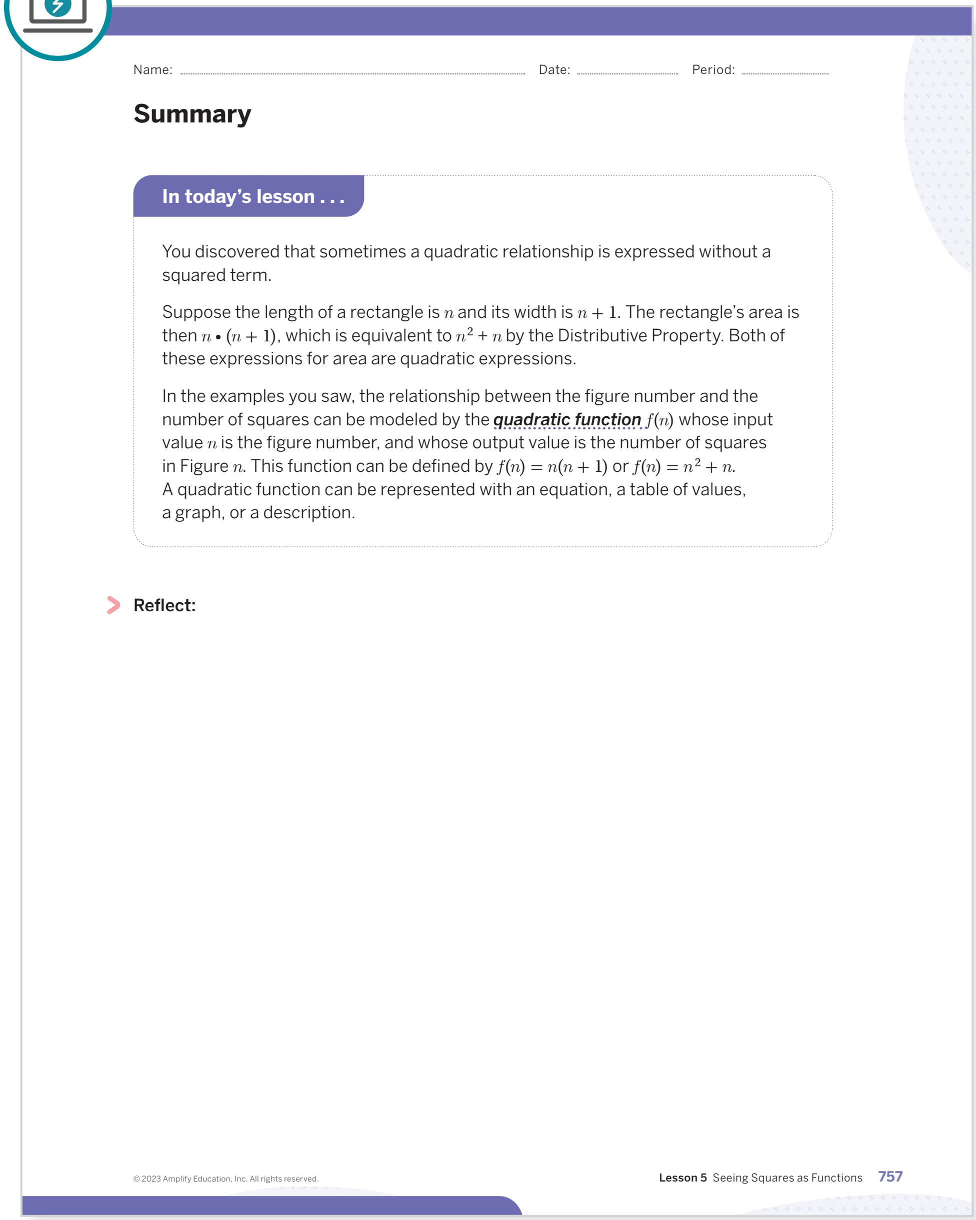
MLR8: Discussion Supports—Press for Details

During the Connect, as students share their thinking and responses for Problems 1 and 2, ask their classmates to press each other for details in their reasoning. Display or provide access to the Anchor Chart PDF, *Sentence Stems*, *Critiquing* to support students as they share and analyze each others' responses. For example:

If a student says . . .	A classmate could ask . . .
"Kiran's reasoning is correct in Problem 1, so it must be linear."	"If the number of rows and columns each increase by 1, does this mean there is a constant rate of change? How many squares were added each time? Is it a constant?"

Summary

Review and synthesize how a quadratic function can be written with or without a squared term, due to the Distributive Property.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You discovered that sometimes a quadratic relationship is expressed without a squared term.

Suppose the length of a rectangle is n and its width is $n + 1$. The rectangle's area is then $n \cdot (n + 1)$, which is equivalent to $n^2 + n$ by the Distributive Property. Both of these expressions for area are quadratic expressions.

In the examples you saw, the relationship between the figure number and the number of squares can be modeled by the ***quadratic function*** $f(n)$ whose input value n is the figure number, and whose output value is the number of squares in Figure n. This function can be defined by $f(n) = n(n + 1)$ or $f(n) = n^2 + n$. A quadratic function can be represented with an equation, a table of values, a graph, or a description.

Reflect:

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *quadratic function* that were added to the display during the lesson.

Synthesize

Display a rectangle with length n and width $n + 1$.

Ask:

- "What is the area of this rectangle? Is there another way you can represent the area?" $n(n + 1)$ or $n^2 + n$
- "What does it mean for two expressions to define the same relationship?" The two expressions describe the same relationship between two quantities.
- "Do $n(n + 1)$ and $n^2 + n$ represent the same relationship? Explain your thinking." Yes; Use the Distributive Property, sketch an n-by-n square with an n-by-1 row added to it, or form an n-by-$(n + 1)$ rectangle.
- "How do you know whether the relationship between two quantities represents a quadratic function?"
  1. If there is only one output for every input, then the relationship is a function.
  2. If one quantity is, in some way, squared or multiplied by itself to obtain a second quantity, the relationship is quadratic.

Formalize vocabulary: ***quadratic function***

Highlight the connections between quadratic expressions (expressed with and without a squared term) and the visual patterns that represent them. Refer to the figure number and number of squares using function language. Emphasize that the functions $f(n) = n(n + 1)$ and $f(n) = n^2 + n$ define the same quadratic function.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

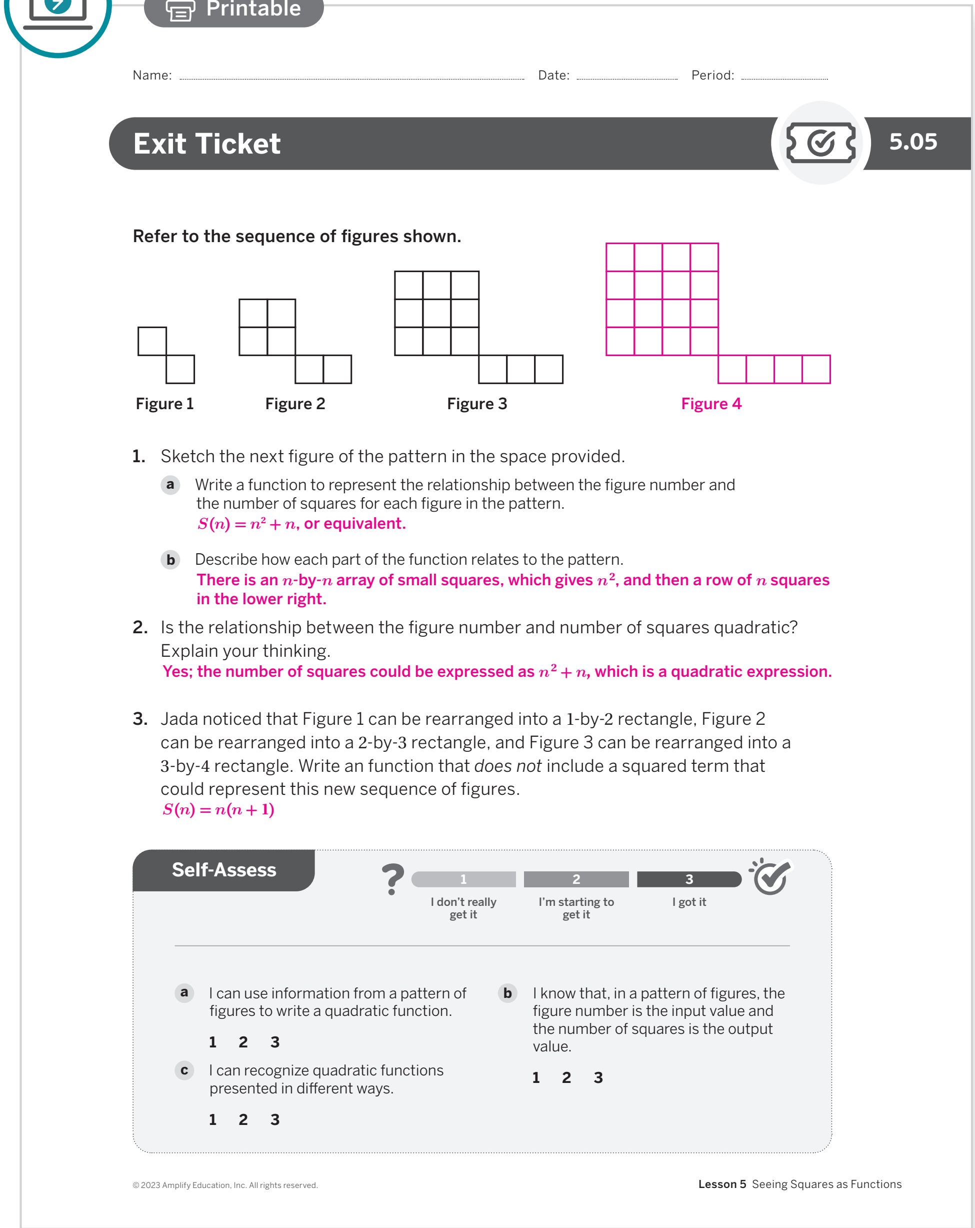
- "What strategies did you find helpful today when writing a quadratic function to express the area of a figure? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing a quadratic function to represent a pattern of shapes and by recognizing equivalent quadratic expressions.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 5.05

Refer to the sequence of figures shown.

Figure 1 Figure 2 Figure 3 Figure 4

1. Sketch the next figure of the pattern in the space provided.
   - **a** Write a function to represent the relationship between the figure number and the number of squares for each figure in the pattern.
     $S(n) = n^2 + n$, or equivalent.
   - **b** Describe how each part of the function relates to the pattern.
     There is an n-by-n array of small squares, which gives n^2, and then a row of n squares in the lower right.
2. Is the relationship between the figure number and number of squares quadratic? Explain your thinking.
   Yes; the number of squares could be expressed as $n^2 + n$, which is a quadratic expression.
3. Jada noticed that Figure 1 can be rearranged into a 1-by-2 rectangle, Figure 2 can be rearranged into a 2-by-3 rectangle, and Figure 3 can be rearranged into a 3-by-4 rectangle. Write an function that *does not* include a squared term that could represent this new sequence of figures.
   $S(n) = n(n + 1)$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use information from a pattern of figures to write a quadratic function.

1 2 3

b I know that, in a pattern of figures, the figure number is the input value and the number of squares is the output value.

1 2 3

c I can recognize quadratic functions presented in different ways.

1 2 3

Lesson 5 Seeing Squares as Functions

Success looks like . . .

- **Language Goal:** Interpreting (using multiple representation) the quadratic relationships in growing patterns as functions, where each input gives a particular output. **(Speaking and Listening, Writing)**
  - » Relating a quadratic function to the pattern of figures in Problems 1 and 2.
- **Goal:** Writing expressions that define quadratic functions.
- **Goal:** Understanding that the same quadratic function can be expressed symbolically in different ways.

Suggested next steps

If students are unable to write a function for the pattern in Problem 1, consider:

- Reviewing Activity 2, Problem 3. Ask, "How does each part of the function $S(n)$ relate to its pattern?"
- Assigning Practice Problems 1–4.

If students are unable to determine whether the pattern is quadratic in Problem 2, consider:

- Prompting students to rearrange squares into patterns that include an n-by-n array, whose area can be represented with a quadratic expression.
- Reviewing Activity 3.

If students are unable to write a quadratic equation without a squared term in Problem 3, consider:

- Having students sketch the visual pattern, as described by Jada in Problem 3, and label the dimensions of the figures in each term. Ask, "If the term number n represents the width of each figure, how could you represent the length?"
- Reviewing Activity 3, Problem 2.

Professional Learning

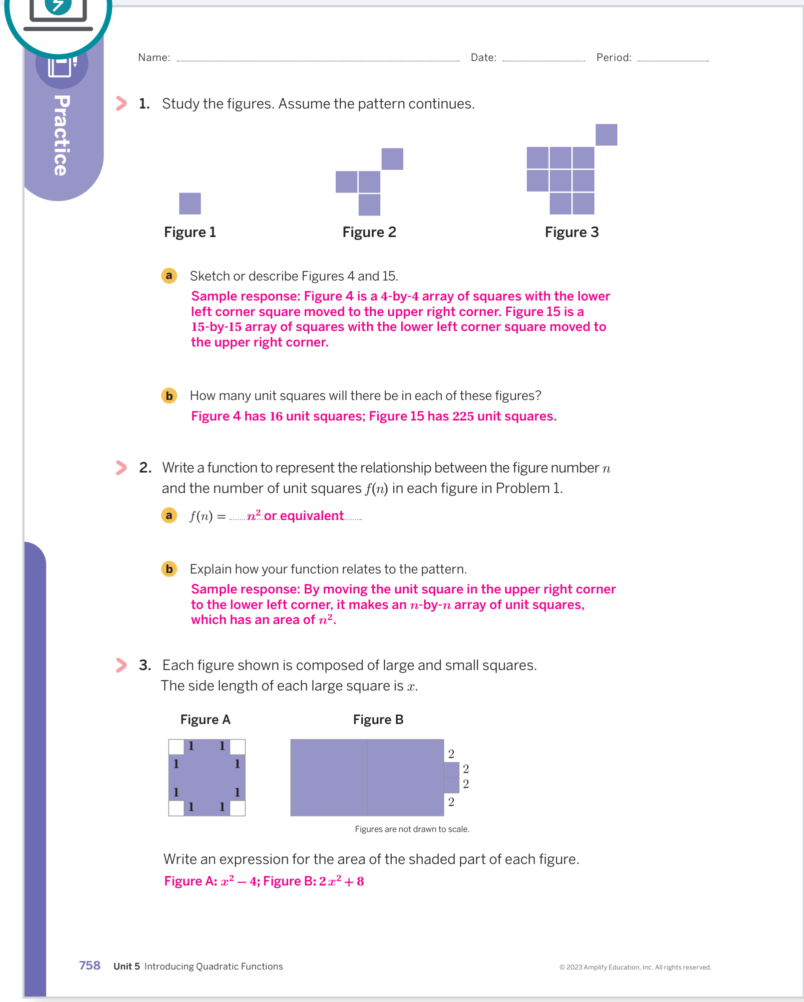
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did drawing and describing figures in a pattern set students up to develop writing quadratic functions?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?

Practice

Independent



Name: Date: Period:

1. Study the figures. Assume the pattern continues.

a. Sketch or describe Figures 4 and 15.
Sample response: Figure 4 is a 4-by-4 array of squares with the lower left corner square moved to the upper right corner. Figure 15 is a 15-by-15 array of squares with the lower left corner square moved to the upper right corner.

b. How many unit squares will there be in each of these figures?
Figure 4 has 16 unit squares; Figure 15 has 225 unit squares.

2. Write a function to represent the relationship between the figure number n and the number of unit squares $f(n)$ in each figure in Problem 1.

a. $f(n) =$ **n^2 or equivalent**

b. Explain how your function relates to the pattern.
Sample response: By moving the unit square in the upper right corner to the lower left corner, it makes an n-by-n array of unit squares, which has an area of n^2.

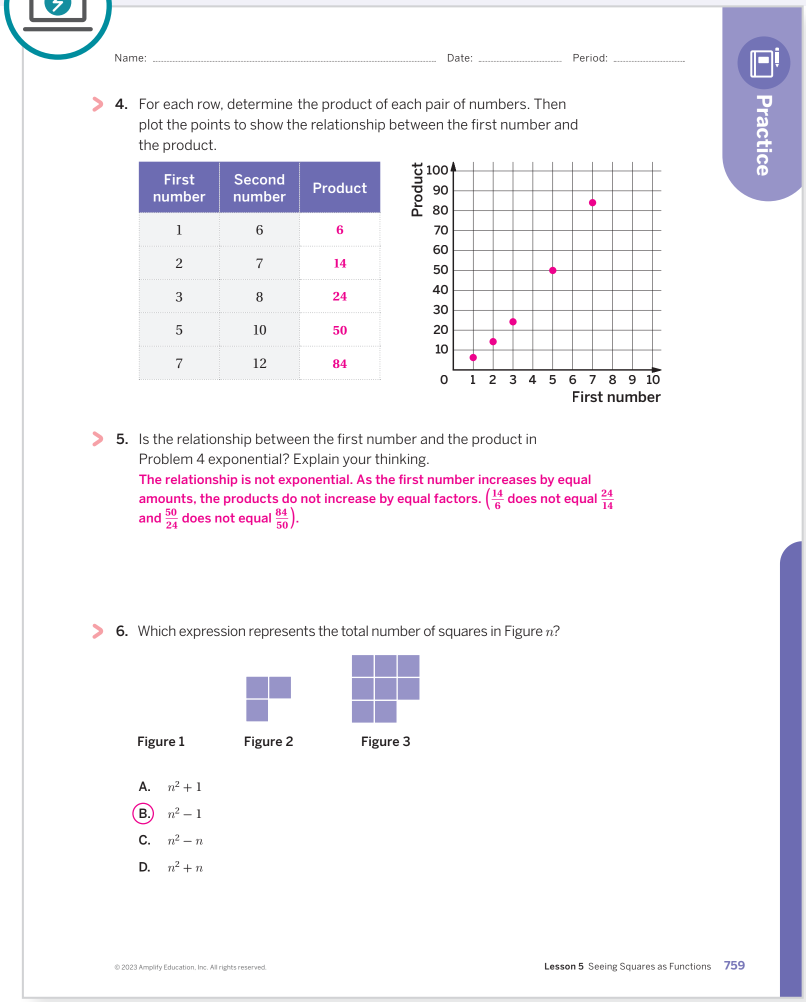
3. Each figure shown is composed of large and small squares. The side length of each large square is x.

Figures are not drawn to scale.

Write an expression for the area of the shaded part of each figure.
Figure A: $x^2 - 4$; Figure B: $2x^2 + 8$

758 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

4. For each row, determine the product of each pair of numbers. Then plot the points to show the relationship between the first number and the product.

First number	Second number	Product
1	6	6
2	7	14
3	8	24
5	10	50
7	12	84

5. Is the relationship between the first number and the product in Problem 4 exponential? Explain your thinking.
The relationship is not exponential. As the first number increases by equal amounts, the products do not increase by equal factors. $\left(\frac{14}{6}\right.$ does not equal $\frac{24}{14}$ and $\frac{50}{24}$ does not equal $\left.\frac{84}{50}\right)$.

6. Which expression represents the total number of squares in Figure n?

A. $n^2 + 1$
[✓ B.] $n^2 - 1$
C. $n^2 - n$
D. $n^2 + n$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Seeing Squares as Functions 759

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 3	2
	3	Activity 1	2
Spiral	4	Unit 5 Lesson 2	1
	5	Unit 5 Lesson 2	2
Formative ⚡	6	Unit 5 Lesson 6	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 2

Quadratic Functions

Whole Class

In this Sub-Unit, students explore how quadratics can be used to model free-falling objects, projectile motion, and revenue.

SUB-UNIT

2 Quadratic Functions

Narrative Connections

What would sports be like without quadratics?

In a word: *dull.*

Imagine a quarterback throwing a pass, only to have it sail serenely over the receiver's head in a straight line. Or a basketball shot from half-court that hopelessly deflects off the backboard.

Shot puts would not land. Olympic divers would be awkwardly stranded on their diving boards. Racers would always be on cruise control.

Imagine all the nice arcs we are used to seeing replaced by hard, straight lines. Sure, it might be funny the first few times, but pretty soon the fans would be asleep in their seats.

Quadratics bring spice to every sport.

It is what allows balls to land, and bodies to accelerate during freefall. Without quadratics, there is no excitement, no mystery. Will a free throw swish or miss? Can a batter clear the fence for a homerun?

Quadratics (and an insane amount of skill) let record-shattering gymnast Simone Biles land her triple-twisting double backflip. They're what allowed legendary shot putter Randy Barnes to hurl a 16-pound ball more than 23 meters.

All the excitement and tension we feel in these great moments of sports history are thanks in part to quadratics. While most of us probably do not watch sports with pencil and paper in our laps, we still experience these quadratic relationships, from something as simple as a game of catch to the breaking of a world record.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore applications of quadratics in the following places:

- **Lesson 7, Activities 1–3:** Falling From the Sky, Egg Drop, Galileo and Gravity
- **Lesson 8, Activities 1–2:** Tracking Foofoo's Flight, Tracking a Cannonball
- **Lesson 9, Activities 1–3:** The Rise of Streaming, What Price to Charge, Domain, Vertex and Zeros

UNIT 5 | **LESSON 6**

Comparing Functions

Let's compare quadratic and exponential relationships to see which one grows faster.

Focus

Goal

1. **Language Goal:** Use graphs, tables, and calculations to show that exponential functions eventually grow faster than quadratic functions. **(Reading and Writing)**

Coherence

Today

Students investigate how quantities that grow quadratically compare to quantities that grow exponentially by examining the successive quotients of each function within a table. They discover that exponential functions that increase will eventually surpass quadratic functions that increase — which the Featured Mathematician, Tibor Radó, explored further when he introduced the world to the "busy beaver function."

Previously

In Unit 4, students compared linear and exponential growth and observed that exponential growth eventually overtakes linear growth.

Coming Soon

In Lessons 7 and 8, students will apply their knowledge of quadratic change to understand free-falling objects and projectile motion.

Rigor

- Students build **conceptual understanding** of how exponential functions will eventually surpass quadratic functions.
- Students strengthen their **fluency** in writing quadratic functions from a pattern and table.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	15 min	10 min	5 min	5 min
Pairs	Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Comparing and Contrasting*
- graphing or spreadsheet technology
- headphones

Math Language Development

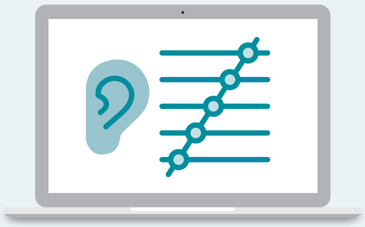
Review words

- *exponential function*
- *first difference*
- *growth factor*
- *linear function*
- *quadratic function*

Amps | Featured Activity

Activity 3
Function Sounds

Students listen to the sound of linear, quadratic, and exponential functions, match the audio to the type of function, and create their own function to produce a sound.



Building Math Identity and Community

Connecting to Mathematical Practices

While most likely competent with many technologies, students might get frustrated as they attempt to use graphing technologies to compare different growth patterns in Activity 3. Ask students to set goals about what they want to be able to do independently using the graphing technology, and provide step-by-step directions they can follow to be successful.

Modifications to Pacing

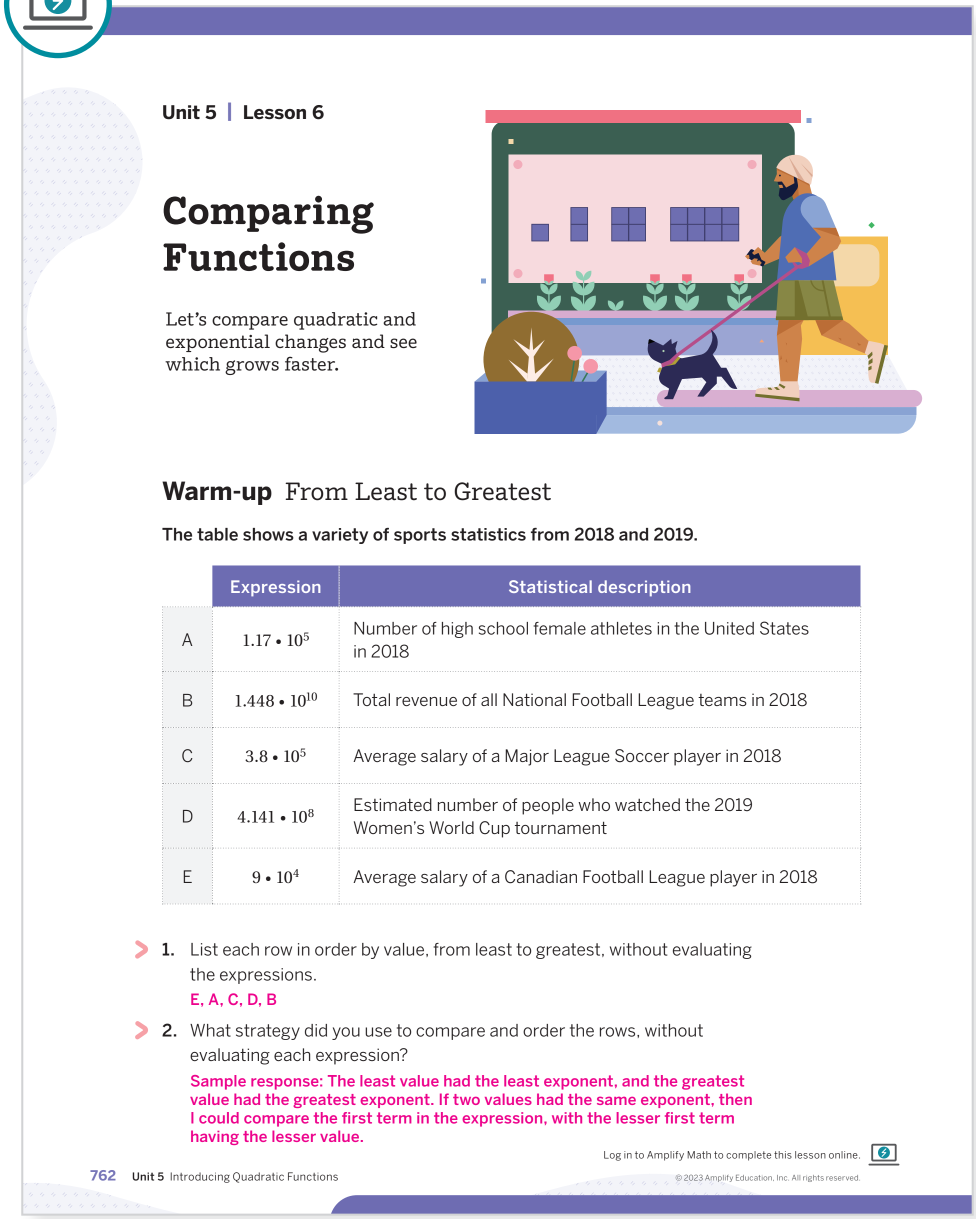
You may want to consider this additional modification if you are short on time.

- Optional **Activity 3** may be omitted.

Pairs | 5 min

Warm-up From Least to Greatest

Students order the values of exponential expressions to review the structure of exponential expressions and the meaning of exponents.



Unit 5 | Lesson 6

Comparing Functions

Let's compare quadratic and exponential changes and see which grows faster.

Warm-up From Least to Greatest

The table shows a variety of sports statistics from 2018 and 2019.

	Expression	Statistical description
A	$1.17 \cdot 10^5$	Number of high school female athletes in the United States in 2018
B	$1.448 \cdot 10^{10}$	Total revenue of all National Football League teams in 2018
C	$3.8 \cdot 10^5$	Average salary of a Major League Soccer player in 2018
D	$4.141 \cdot 10^8$	Estimated number of people who watched the 2019 Women's World Cup tournament
E	$9 \cdot 10^4$	Average salary of a Canadian Football League player in 2018

1. List each row in order by value, from least to greatest, without evaluating the expressions.
   E, A, C, D, B
2. What strategy did you use to compare and order the rows, without evaluating each expression?
   Sample response: The least value had the least exponent, and the greatest value had the greatest exponent. If two values had the same exponent, then I could compare the first term in the expression, with the lesser first term having the lesser value.

Log in to Amplify Math to complete this lesson online.

762 Unit 5 Introducing Quadratic Functions

1 Launch

Give students think-time to consider the first problem. Have them discuss possible strategies with a partner before completing the problems. Calculators should not be used.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "How can you use the exponent and the first term in the expression to help order the descriptions?"

Look for points of confusion:

- **Reversing the order of the values.** Have students compare the values of 10^4 and 10^5.
- **Attempting to evaluate each expression.** Have students compare the first factor of each expression before comparing the base 10 factors.

Look for productive strategies:

- Writing each product in standard form.
- Reasoning by using the properties of exponents.

3 Connect

Have students share strategies for ordering the expressions.

Highlight that expressions can be compared by analyzing their structure, and it is not always necessary to determine their exact value. If the exponents are the same for like bases, then the expressions can be ordered by comparing the first factor.

Ask, "Which is greater, 10^2 or 2^{10}? How can you be sure without using a calculator?" Sample response: 10^2 is equal to 100, but I also know that 2^{10} grows more rapidly, because 2 is raised to a greater exponent. 2^{10} is greater than 2^7, and 2^7 is greater than 100, so 2^{10} is greater than 10^2.

Power-up

To power up students' ability to write quadratic expressions to represent a pattern:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

Informed by: Performance on Lesson 3, Practice Problem 6

Pairs | 20 min

Activity 1 Which One Grows Faster?

Students activate their prior knowledge of exponential functions by studying quadratic and exponential patterns to discover that the exponential pattern eventually grows faster.

Name: ______ Date: ______ Period: ______

Activity 1 Which One Grows Faster?

In Pattern A, the lengths and widths of the figures each grow by one unit square from one figure to the next. In Pattern B, the number of unit squares doubles from each figure to the next. In each pattern, the number of unit squares is a function of the figure number n.

Co-craft Questions: Before you begin this activity, study these patterns. Work with your partner to write 2–3 mathematical questions you have about this scenario.

Pattern A

Figure 1 Figure 2 Figure 3

1. Sketch Figures 4 and 5 of Pattern A.

Figure 4

Figure 5

a. Write a function $f(n)$ to represent the number of unit squares in Figure n of Pattern A.
$f(n) = n^2$

b. Is the function linear, quadratic, or exponential?
Quadratic

Pattern B

Figure 1 Figure 2 Figure 3

2. Sketch Figures 4 and 5 of Pattern B.

Figure 4

Figure 5

a. Write a function $g(n)$ to represent the number of unit squares in Figure n of Pattern B.
$g(n) = 2^n$

b. Is the function linear, quadratic, or exponential?
Exponential

Lesson 6 Comparing Functions 763

1 Launch

Read the prompt aloud. Ask, "In which pattern will Figure 5 have the most squares?" In each student pair, have one partner complete Problem 1 and the other partner complete Problem 2. Then have students share their responses with their partner before completing the rest of the activity.

2 Monitor

Help students get started by suggesting that after sketching Figures 4 and 5, they complete the table in Problem 3 before writing the function.

Look for points of confusion:

- **Writing the equation as $2n$ for Pattern B in Problem 2a.** Ask, "How is Pattern B changing? Is it a linear pattern?" Then have students test their function by substituting different values of n.

Look for productive strategies:

- Calculating the first differences, for Pattern A, and growth factor, for Pattern B, in the tables to help write the functions.
- Graphing each function to compare them.
- Sketching additional figures.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Prior Knowledge*

Suggest that students write the total number of squares in each figure above or below the figure number. As they do so, ask them what they notice about the total number of squares in each figure for Pattern B. Remind them they have previously studied this type of relationship. Ask:

- "Is there a common difference? What do you notice about the first and second differences?"
- "Is there a common factor?"
- "What is the name of the type of relationship in which there is a common factor?"

Math Language Development

MLR5: Co-Craft Questions

During the Launch, display the introductory scenario and Patterns A and B. Have students work with their partner to write 2–3 mathematical questions they could ask about this scenario. Have volunteers share their questions with the class. Sample questions shown.

- Is each pattern linear? Quadratic? Exponential? None of these?
- What will Figure 4 look like? Figure n?
- How many squares will there be in Figure 4? Figure n?

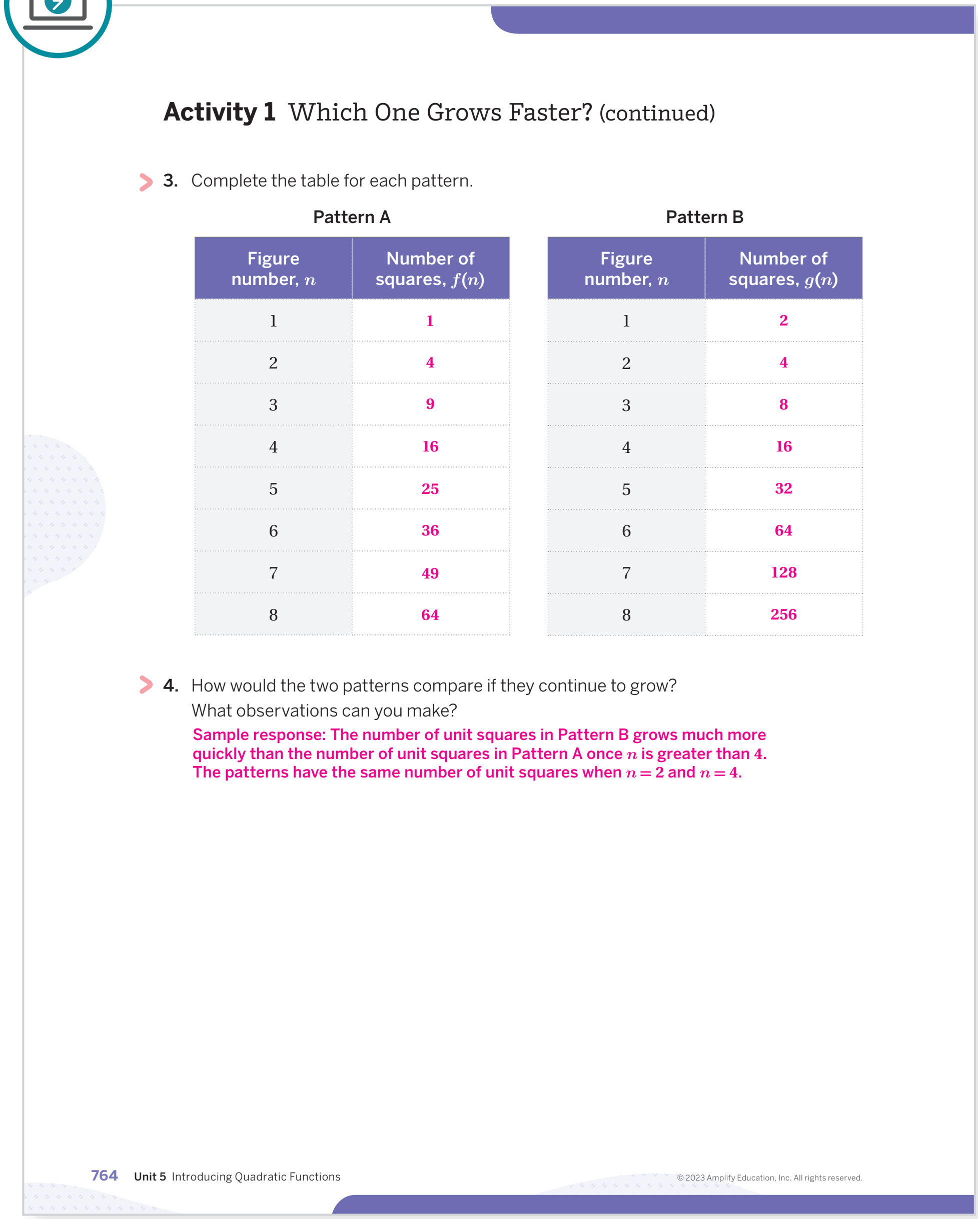
English Learners

Clarify the meaning of the term doubles by annotating the number of squares for each figure in Pattern A and saying, "To double a value means to multiply the value by 2."

Pairs | 20 min

Activity 1 Which One Grows Faster? (continued)

Students activate their prior knowledge of exponential functions by studying quadratic and exponential patterns to discover that the exponential pattern eventually grows faster.



Activity 1 Which One Grows Faster? (continued)

3. Complete the table for each pattern.

Pattern A

Figure number, n	Number of squares, $f(n)$
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64

Pattern B

Figure number, n	Number of squares, $g(n)$
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256

4. How would the two patterns compare if they continue to grow? What observations can you make?

Sample response: The number of unit squares in Pattern B grows much more quickly than the number of unit squares in Pattern A once n is greater than 4. The patterns have the same number of unit squares when $n = 2$ and $n = 4$.

3 Connect

Display Pattern A and Pattern B.

Have students share their sketches, functions, and tables. Ask, "How could you use the tables to determine the function type that each pattern represents?"

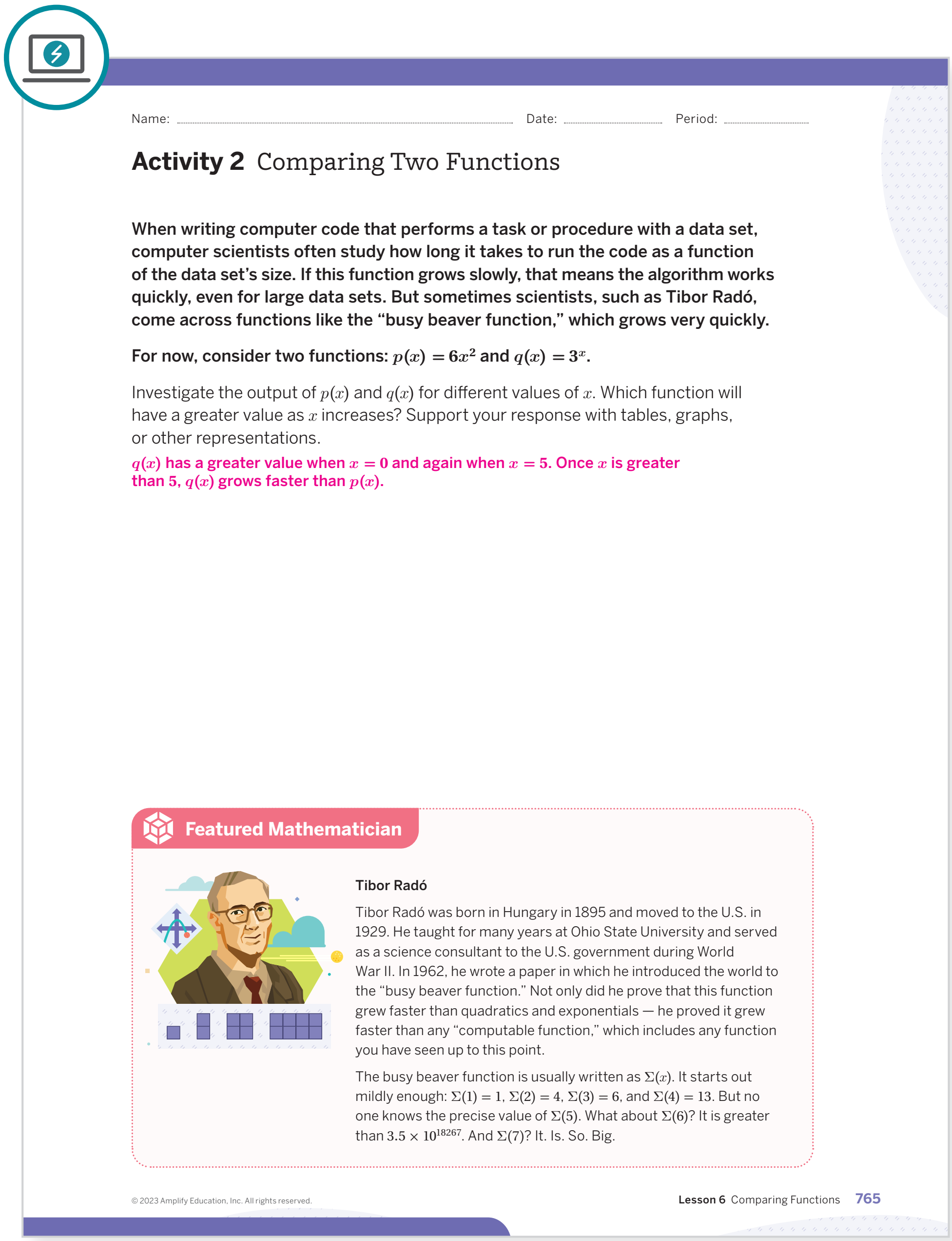
Highlight that Pattern B represents an exponential function because the growth factor is constant, 2. Pattern A does not show a constant growth factor, but it does show equal second differences, so it represents a quadratic function. The values in Pattern A (quadratic function) are growing slower than the values in Pattern B (exponential function). Remind them that they previously learned that *exponential functions* change by equal factors over equal intervals.

Ask, "How would the growth of these two functions compare to linear growth?"
Linear growth would grow the slowest.

Independent | 15 min

Activity 2 Comparing Two Functions

Students choose and create their own representations to make an argument for why an exponential function will eventually overtake a quadratic function.



Name: Date: Period:

Activity 2 Comparing Two Functions

When writing computer code that performs a task or procedure with a data set, computer scientists often study how long it takes to run the code as a function of the data set's size. If this function grows slowly, that means the algorithm works quickly, even for large data sets. But sometimes scientists, such as Tibor Radó, come across functions like the "busy beaver function," which grows very quickly.

For now, consider two functions: $p(x) = 6x^2$ and $q(x) = 3^x$.

Investigate the output of $p(x)$ and $q(x)$ for different values of x. Which function will have a greater value as x increases? Support your response with tables, graphs, or other representations.

$q(x)$ has a greater value when $x = 0$ and again when $x = 5$. Once x is greater than 5, $q(x)$ grows faster than $p(x)$.

Featured Mathematician

Tibor Radó

Tibor Radó was born in Hungary in 1895 and moved to the U.S. in 1929. He taught for many years at Ohio State University and served as a science consultant to the U.S. government during World War II. In 1962, he wrote a paper in which he introduced the world to the "busy beaver function." Not only did he prove that this function grew faster than quadratics and exponentials — he proved it grew faster than any "computable function," which includes any function you have seen up to this point.

The busy beaver function is usually written as $\Sigma(x)$. It starts out mildly enough: $\Sigma(1) = 1$, $\Sigma(2) = 4$, $\Sigma(3) = 6$, and $\Sigma(4) = 13$. But no one knows the precise value of $\Sigma(5)$. What about $\Sigma(6)$? It is greater than 3.5×10^{18267}. And $\Sigma(7)$? It. Is. So. Big.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Comparing Functions 765

1 Launch

Read the prompt aloud. Provide an expectation for the amount of time students will have to work independently on the activity. Provide access to graphing or spreadsheet technology.

2 Monitor

Help students get started by asking, "What are some strategies that you can use to investigate the output values of the quadratic and exponential function?"

Look for points of confusion:

- **Attempting to sketch figures for each function.** Ask, "Is this the most efficient method for comparing the growth?"

Look for productive strategies:

- Using graphing technology to compare large values of each function by table or graph.
- Using spreadsheet technology to compare large values of each function.

3 Connect

Have students share the strategies they used to compare the two functions.

Display graphs of the two functions on one coordinate plane.

Highlight that a quadratic function may have greater output values than an exponential function for many input values, but output values of the exponential function will eventually overtake the quadratic.

Ask, "When will the exponential function be greater than the quadratic function?" After the graphs intersect.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Provide access to graphing technology or spreadsheet technology that students can choose to use to compare the values of the two functions.

Accessibility: *Guide Processing and Visualization*

If technology is unavailable, provide, or suggest that students create, a table in which they can record the values of each function. Consider pre-populating the table with given values of x, such as the values shown here.

x	$p(x) = 6x^2$	$q(x) = 3^x$
0		
1		
3		
5		
8		
10		

Featured Mathematician

Tibor Radô

Have students read about featured mathematician Tibor Radó who introduced the world to the "busy beaver function."

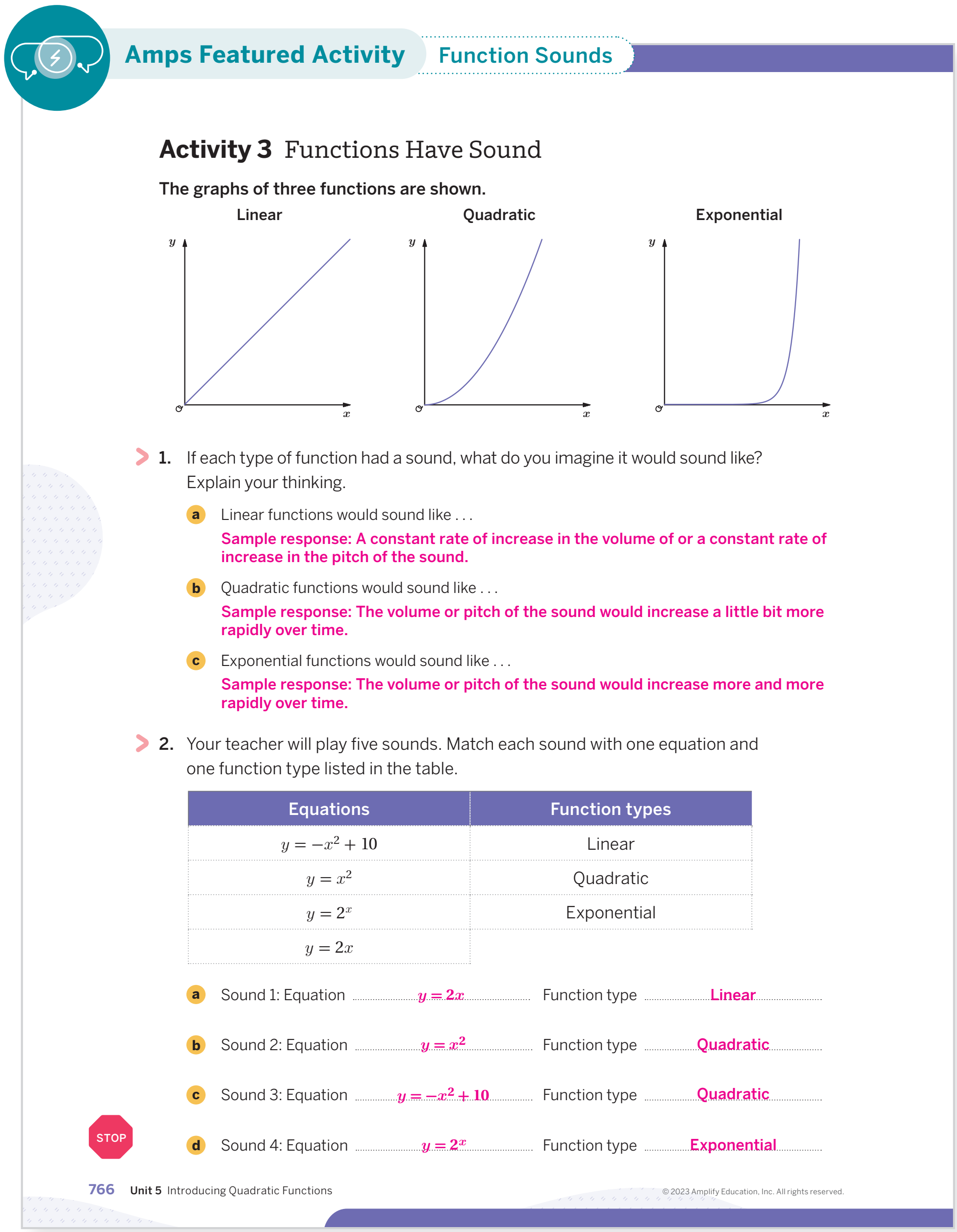
Optional

Pairs | 10 min

Activity 3 Functions Have Sound

Students contrast linear, quadratic, and exponential growth through sound using graphing technology to strengthen their understanding that exponential functions grow the fastest.

Amps Featured Activity Function Sounds



Activity 3 Functions Have Sound

The graphs of three functions are shown.

Linear | Quadratic | Exponential

1. If each type of function had a sound, what do you imagine it would sound like? Explain your thinking.
   - a Linear functions would sound like . . .
     Sample response: A constant rate of increase in the volume of or a constant rate of increase in the pitch of the sound.
   - b Quadratic functions would sound like . . .
     Sample response: The volume or pitch of the sound would increase a little bit more rapidly over time.
   - c Exponential functions would sound like . . .
     Sample response: The volume or pitch of the sound would increase more and more rapidly over time.

2. Your teacher will play five sounds. Match each sound with one equation and one function type listed in the table.

Equations	Function types
$y = -x^2 + 10$	Linear
$y = x^2$	Quadratic
$y = 2^x$	Exponential
$y = 2x$	

a Sound 1: Equation $y = 2x$ Function type **Linear**

b Sound 2: Equation $y = x^2$ Function type **Quadratic**

c Sound 3: Equation $y = -x^2 + 10$ Function type **Quadratic**

d Sound 4: Equation $y = 2^x$ Function type **Exponential**

STOP

766 **Unit 5** Introducing Quadratic Functions

1 Launch

Give students one minute of think time to study the graphs. Provide access to headphones.

2 Monitor

Help students get started by having them hum what they think each function might sound like.

Look for points of confusion:

- **Struggling to connect the sounds to each function.** Direct students to listen to the speed of each sound and think about how each type of function grows.

Look for productive strategies:

- Concluding that all functions start off by sounding similar, but then the change is distinguishably different for each.
- Using the image of a function's graph to connect to its sound.

3 Connect

Display the graphs while students listen to the sounds.

Have students share how they think each function sounds — by humming — before playing the recording.

Highlight how exponential functions would have an increasingly rapid sound change over time.

Ask, "What would a horizontal line sound like? What would a vertical line sound like?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can listen to the sound of linear, quadratic, and exponential functions, match the audio to the type of function, and create their own function to produce a particular sound.

Accessibility: *Guide Processing and Visualization*

Play each sound, one at a time, repeatedly so that students can process the auditory information and make strong connections. Ask them to sketch the graph of the sound as it is being played.

Accessibility: *Vary Demands to Optimize Challenge*

This is an optional activity that involves the use of students' auditory functions. If there are students in your class who have auditory impairments, consider omitting this activity or alter it by having students walk at a rate, or different rates, that they think would match each graph.

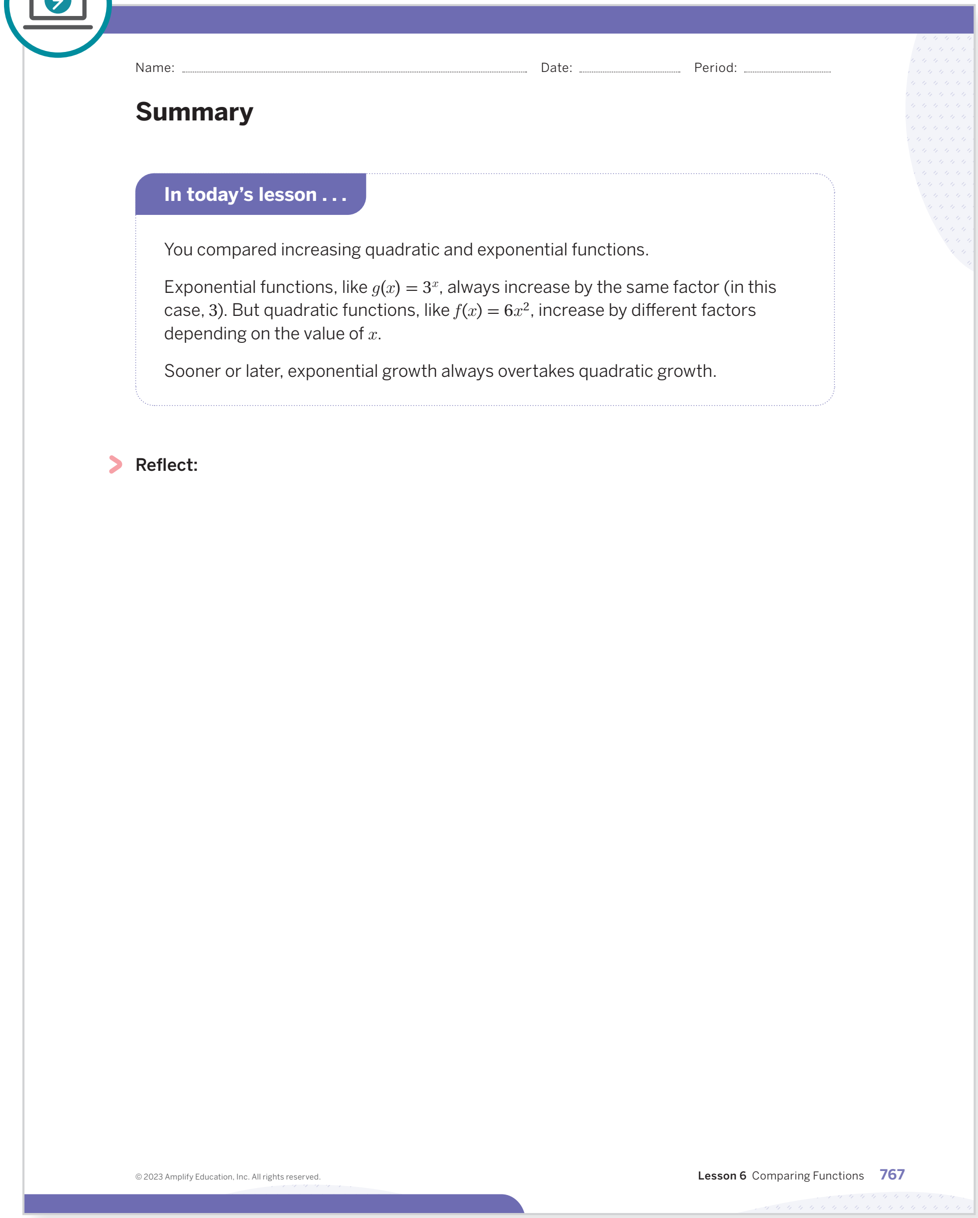
Extension: *Math Enrichment*

Display other graphs, such as sine and cosine, and have students hum or describe what they think the sound of these graphs would be. You do not need to tell them the names of the types of graphs, just reveal their overall shape.

Whole Class | 5 min

Summary

Review and synthesize how exponential functions will always eventually grow faster than — and overtake — quadratic functions.



Name: Date: Period:

Summary

In today's lesson . . .

You compared increasing quadratic and exponential functions.

Exponential functions, like $g(x) = 3^x$, always increase by the same factor (in this case, 3). But quadratic functions, like $f(x) = 6x^2$, increase by different factors depending on the value of x.

Sooner or later, exponential growth always overtakes quadratic growth.

Reflect:

©2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Comparing Functions 767

Synthesize

Display a the graph of $g(x) = 3^x$ and $f(x) = 6x^2$ on the same coordinate plane.

Have students share ways to compare the growth of exponential and quadratic functions.

Ask:

- "What are some ways for comparing quadratic growth to exponential growth?" By comparing their values in a table, by graphing the equations that represent them, or by comparing how the output values change as the input value increases.
- "When you compared n^2 and 2^n, you saw the value of 2^n become greater than n^2 at $n = 5$. If you compare, for example, $100000n^2$ and 2^n, will the exponential still overtake the quadratic? Why or why not?" Yes; This will occur at a greater value than $n = 5$, but exponential functions will always eventually overtake quadratic functions.

Highlight that there could be a large domain of values where the quadratic function is greater than the exponential function. Similarly, a linear function could also be greater over a domain of values, but any exponential function will eventually always overtake both linear and quadratic functions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

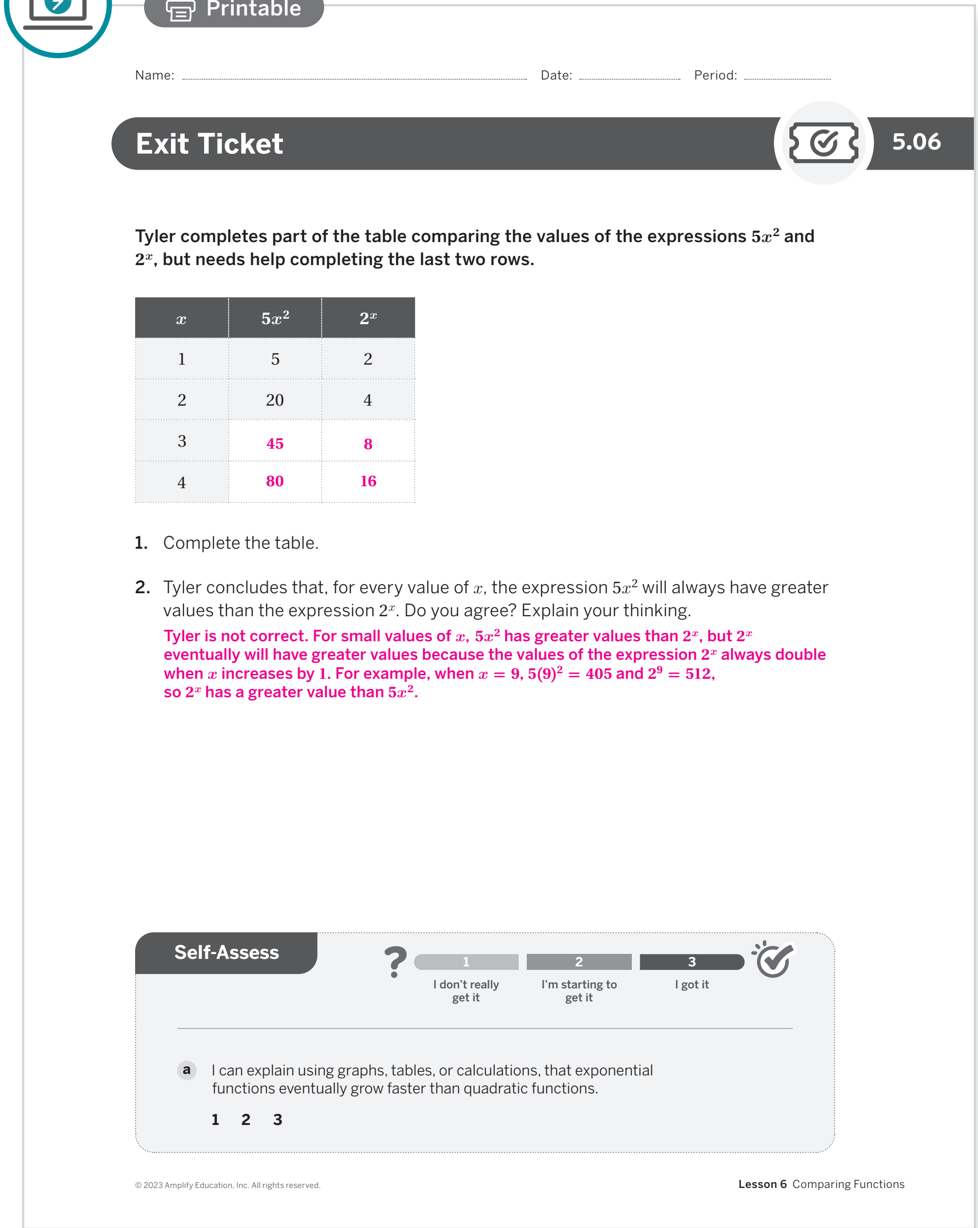
- "What characteristics of the graph of a quadratic function distinguish it from a linear function? An exponential function?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how quadratic and exponential functions compare by completing a table and critiquing a student's reasoning.

Printable



Name: Date: Period:

Exit Ticket 5.06

Tyler completes part of the table comparing the values of the expressions $5x^2$ and 2^x, but needs help completing the last two rows.

x	$5x^2$	2^x
1	5	2
2	20	4
3	**45**	**8**
4	**80**	**16**

1. Complete the table.
2. Tyler concludes that, for every value of x, the expression $5x^2$ will always have greater values than the expression 2^x. Do you agree? Explain your thinking.
   Tyler is not correct. For small values of x, $5x^2$ has greater values than 2^x, but 2^x eventually will have greater values because the values of the expression 2^x always double when x increases by 1. For example, when $x = 9$, $5(9)^2 = 405$ and $2^9 = 512$, so 2^x has a greater value than $5x^2$.

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a. I can explain using graphs, tables, or calculations, that exponential functions eventually grow faster than quadratic functions.
1 2 3

Lesson 6 Comparing Functions

Success looks like . . .

- **Language Goal:** Using graphs, tables, and calculations to show that exponential functions eventually grow faster than quadratic functions. **(Reading and Writing)**
  - » Explaining why $2x$ will eventually grow faster than $5x^2$ in Problem 2.

Suggested next steps

If students incorrectly complete the table, consider:

- Providing them with a calculator to help with calculations.

If students agree with Tyler in Problem 2, consider:

- Reviewing the table of values in Activity 1 and highlighting the row where the quadratic function is greater, and then when the exponential function is greater.
- Reviewing Activity 2.
- Assigning Practice Problems 2 and 3.

Professional Learning

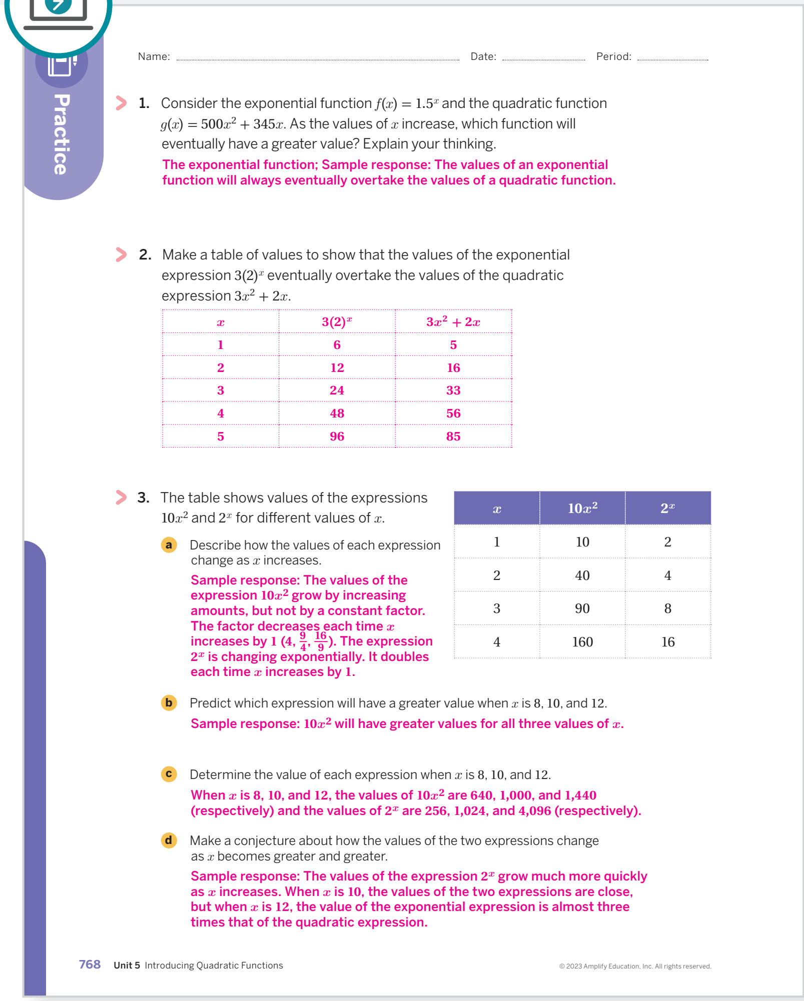
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was to compare quadratic growth to exponential growth. How did this comparison go?
- How did comparing the sounds of graphs set students up to develop an understanding of the differences between linear, exponential, and quadratic growth?

Practice

Independent



Name: Date: Period:

1. Consider the exponential function $f(x) = 1.5^x$ and the quadratic function $g(x) = 500x^2 + 345x$. As the values of x increase, which function will eventually have a greater value? Explain your thinking.

The exponential function; Sample response: The values of an exponential function will always eventually overtake the values of a quadratic function.

2. Make a table of values to show that the values of the exponential expression $3(2)^x$ eventually overtake the values of the quadratic expression $3x^2 + 2x$.

x	$3(2)^x$	$3x^2 + 2x$
1	6	5
2	12	16
3	24	33
4	48	56
5	96	85

3. The table shows values of the expressions $10x^2$ and 2^x for different values of x.

x	$10x^2$	2^x
1	10	2
2	40	4
3	90	8
4	160	16

a. Describe how the values of each expression change as x increases.

Sample response: The values of the expression $10x^2$ grow by increasing amounts, but not by a constant factor. The factor decreases each time x increases by 1 $(4, \frac{9}{4}, \frac{16}{9})$. The expression 2^x is changing exponentially. It doubles each time x increases by 1.

b. Predict which expression will have a greater value when x is 8, 10, and 12.

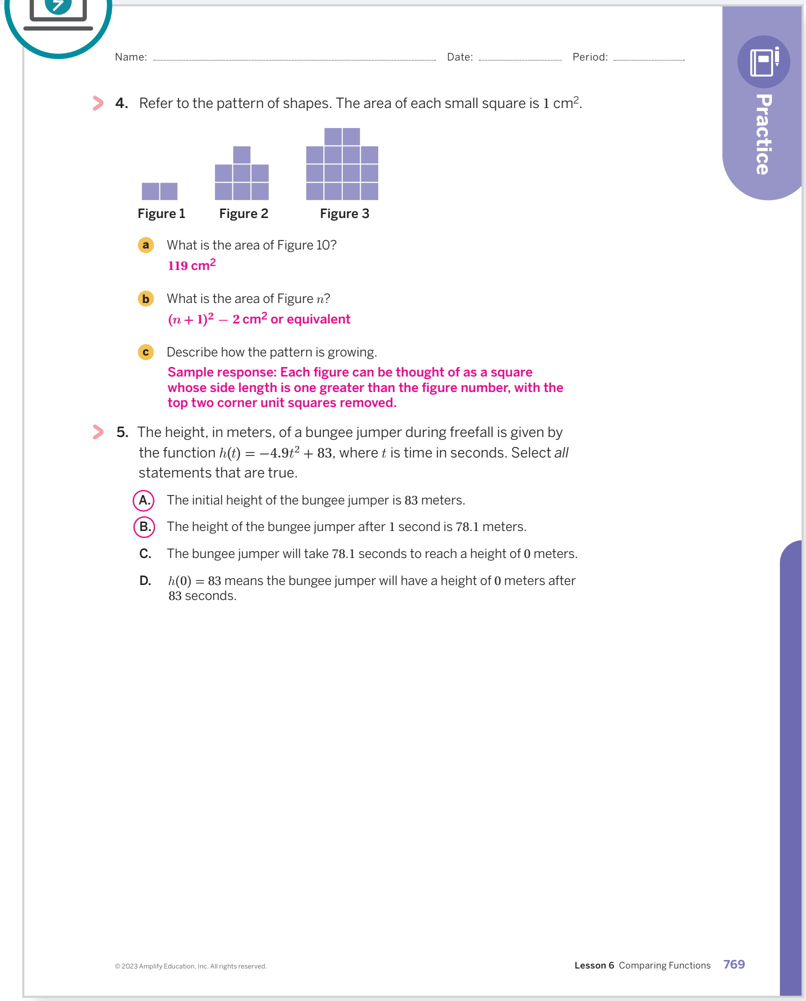
Sample response: $10x^2$ will have greater values for all three values of x.

c. Determine the value of each expression when x is 8, 10, and 12.

When x is 8, 10, and 12, the values of $10x^2$ are 640, 1,000, and 1,440 (respectively) and the values of 2^x are 256, 1,024, and 4,096 (respectively).

d. Make a conjecture about how the values of the two expressions change as x becomes greater and greater.

Sample response: The values of the expression 2^x grow much more quickly as x increases. When x is 10, the values of the two expressions are close, but when x is 12, the value of the exponential expression is almost three times that of the quadratic expression.



Name: Date: Period:

4. Refer to the pattern of shapes. The area of each small square is 1 cm^2.

a. What is the area of Figure 10?

119 cm^2

b. What is the area of Figure n?

$(n + 1)^2 - 2$ cm^2 or equivalent

c. Describe how the pattern is growing.

Sample response: Each figure can be thought of as a square whose side length is one greater than the figure number, with the top two corner unit squares removed.

5. The height, in meters, of a bungee jumper during freefall is given by the function $h(t) = -4.9t^2 + 83$, where t is time in seconds. Select *all* statements that are true.

[✓ A.] The initial height of the bungee jumper is 83 meters.
[✓ B.] The height of the bungee jumper after 1 second is 78.1 meters.
C. The bungee jumper will take 78.1 seconds to reach a height of 0 meters.
D. $h(0) = 83$ means the bungee jumper will have a height of 0 meters after 83 seconds.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 3	2
Formative ⚡	5	Unit 5 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 7**

Building Quadratic Functions to Describe Falling Objects

Let's measure falling objects.

Focus

Goals

1. **Language Goal:** Explain the meaning of the terms in a quadratic expression that represent the height of a falling object. **(Speaking and Listening)**
2. Use tables, and equations to represent the height of a falling object.
3. Write quadratic functions to represent the height of an object falling due to gravity.

Coherence

Today

Students analyze the vertical distances that free-falling objects travel over time to understand that they are described by quadratic functions. This is the first lesson in which students explore quadratic relationships without the use of visual models. They use tables, and equations to represent and interpret functions.

Previously

Students compared quadratic and exponential change in tables and patterns in Lesson 6.

Coming Soon

In Lessons 8 and 9, students will examine projectile motion and revenue and price relationships, to develop understanding of the zeros, vertex, and domain of quadratic functions.

Rigor

- Students build **conceptual understanding** of how quadratic functions can be used to model falling objects.
- Students **apply** their understanding of quadratic functions to study the height of falling objects.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	25 min	5 min	5 min
Whole Class	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Graphs of Free Falling Objects*
- aluminum foil, 6.5 ft per group
- hard-boiled eggs, two per group
- measuring tape, one per group
- stopwatch, one per group

Math Language Development

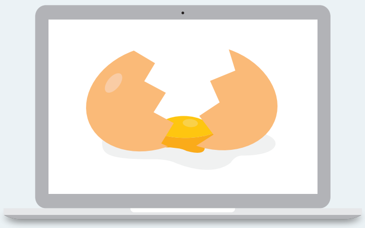
Review words

- *exponential function*
- *first difference*
- *growth factor*
- *linear function*
- *quadratic function*
- *second difference*

Amps | Featured Activity

Activity 2
Virtual Egg Drop

Students virtually test eggs of different sizes, dropping from different heights, to see whether they will break.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel confused about how the three scenarios in this lesson are related. The variation in the scenarios might prevent students from being able to recognize the repeated reasoning that is used with falling object models. Encourage students to identify how they feel as they start Activity 3, and then have them identify their own strengths and recognize why they are capable of successfully completing the activity.

Modifications to Pacing

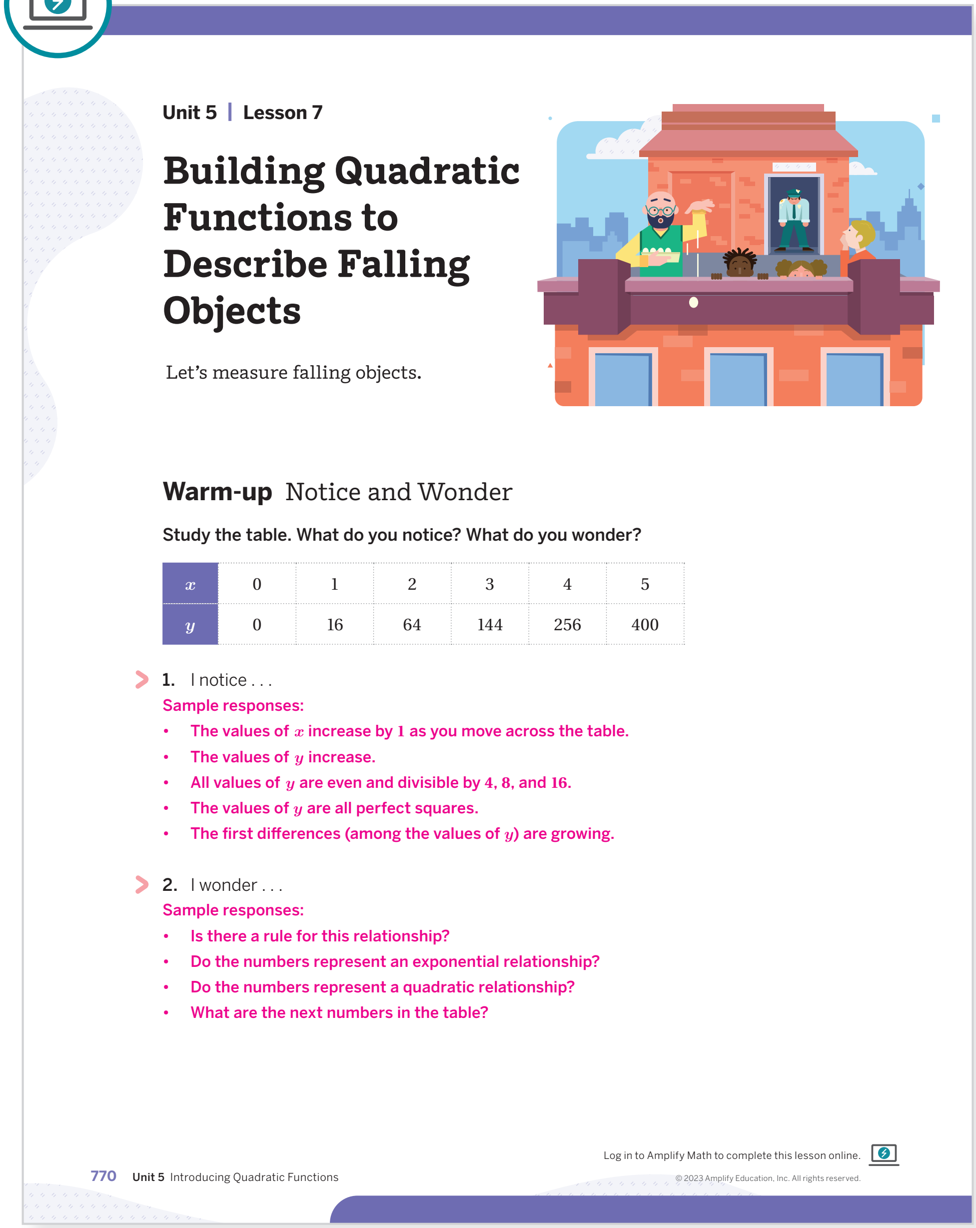
You may want to consider this additional modification if you are short on time.

- Optional **Activity 2** may be omitted.

Whole Class | 5 min

Warm-up Notice and Wonder

Students study the structure of a quadratic pattern represented in a table to prepare them to explore quadratic relationships that represent falling objects.



Unit 5 | Lesson 7

Building Quadratic Functions to Describe Falling Objects

Let's measure falling objects.

Warm-up Notice and Wonder

Study the table. What do you notice? What do you wonder?

x	0	1	2	3	4	5
y	0	16	64	144	256	400

1. I notice . . .

Sample responses:

- The values of x increase by 1 as you move across the table.
- The values of y increase.
- All values of y are even and divisible by 4, 8, and 16.
- The values of y are all perfect squares.
- The first differences (among the values of y) are growing.

2. I wonder . . .

Sample responses:

- Is there a rule for this relationship?
- Do the numbers represent an exponential relationship?
- Do the numbers represent a quadratic relationship?
- What are the next numbers in the table?

Log in to Amplify Math to complete this lesson online.

770 Unit 5 Introducing Quadratic Functions

1 Launch

Conduct the ***Notice and Wonder*** routine. Give students one minute of think-time to study the table. Ask, "Do the values in the table show a pattern?"

2 Monitor

Help students get started by asking them to determine the greatest common factor for the values of y.

Look for points of confusion:

- **Checking only for exponential or linear growth.** Ask, "Did you try using a table to show first and second differences?"

Look for productive strategies:

- Using the first and second differences to determine whether the relationship is quadratic.
- Determining a relationship for the first few terms, then applying that relationship to the remainder of the table.

3 Connect

Have students share what they notice and wonder. Record and display their thinking.

Highlight interesting questions recorded and anything students noticed that address them (e.g., all the values of y are multiples of 16 and perfect squares, and second differences are equal).

Ask, "How do the values of y (perfect squares) relate to the values of x?" Each value of y is found by multiplying the square of its corresponding value of x by 16.

Power-up

To power up students' ability to evaluate a function and describe what it means in context, have students complete:

The height in meters of an apple falling off a branch of a tree can be modeled by the equation $h(t) = -4.9t^2 + 5$, where t is time in seconds.

1. Evaluate the function for the values given.

   a. $f(0)$ $= 5$ b. $f(1)$ $= 0.1$

2. What does the value of $f(0)$ represent in context? Select *all* that apply.

   [✓ A.] The initial height of the apple.
   B. The number of seconds it takes the apple to hit the ground.
   C. The number of seconds it will take the apple to have a height of 0 m.
   [✓ D.] The height of the apple at 0 seconds.

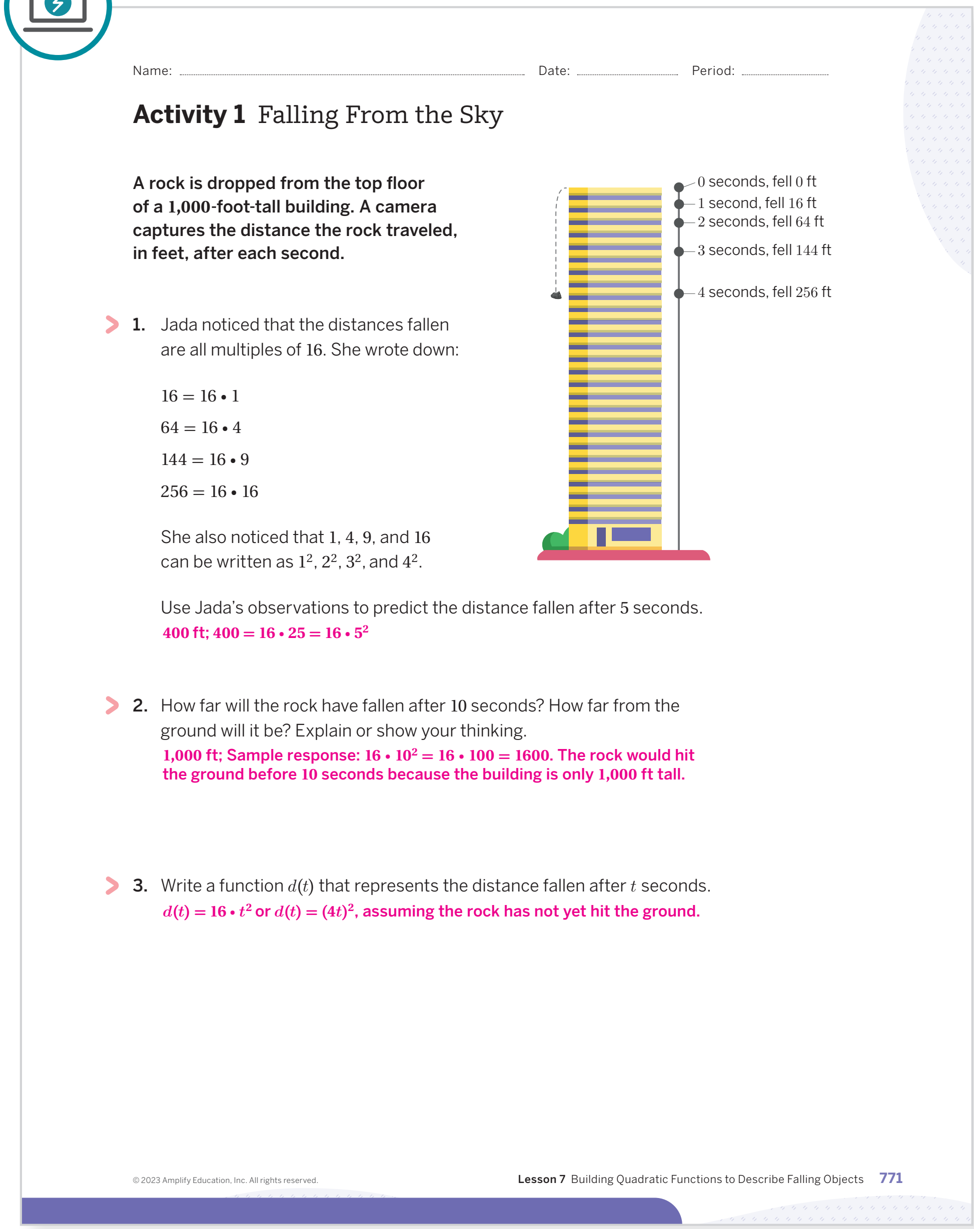
Use: Before Activity 3

Informed by: Performance on Lesson 6, Practice Problem 5

Pairs | 10 min

Activity 1 Falling From the Sky

Students create a simple quadratic model using time-distance data of a free-falling object to understand how the expression $16t^2$ represents gravity.



Name: ______ Date: ______ Period: ______

Activity 1 Falling From the Sky

A rock is dropped from the top floor of a 1,000-foot-tall building. A camera captures the distance the rock traveled, in feet, after each second.

0 seconds, fell 0 ft
1 second, fell 16 ft
2 seconds, fell 64 ft
3 seconds, fell 144 ft
4 seconds, fell 256 ft

1. Jada noticed that the distances fallen are all multiples of 16. She wrote down:

   $16 = 16 \bullet 1$

   $64 = 16 \bullet 4$

   $144 = 16 \bullet 9$

   $256 = 16 \bullet 16$

   She also noticed that 1, 4, 9, and 16 can be written as 1^2, 2^2, 3^2, and 4^2.

   Use Jada's observations to predict the distance fallen after 5 seconds.
   400 ft; $400 = 16 \bullet 25 = 16 \bullet 5^2$

2. How far will the rock have fallen after 10 seconds? How far from the ground will it be? Explain or show your thinking.
   1,000 ft; Sample response: $16 \bullet 10^2 = 16 \bullet 100 = 1600$. The rock would hit the ground before 10 seconds because the building is only 1,000 ft tall.

3. Write a function $d(t)$ that represents the distance fallen after t seconds.
   $d(t) = 16 \bullet t^2$ or $d(t) = (4t)^2$, assuming the rock has not yet hit the ground.

Lesson 7 Building Quadratic Functions to Describe Falling Objects 771

1 Launch

Display the image and then read the prompt aloud. Conduct the ***Notice and Wonder*** routine. Discuss students' observations, and then ask, "What do you think the numbers represent? Does the object fall the same distance every successive second?"

2 Monitor

Help students get started by connecting the numbers in the image to the table from the Warm-up. Discuss Problem 1 together as a class before having students complete the activity with a partner.

Look for points of confusion:

- **Thinking that the distance value should decrease.** Discuss how the distance from the top of the building increases as the object falls.

Look for productive strategies:

- Utilizing the table from the Warm-up.
- Using the structure of "multiplication of 16 by the next perfect square" to extend the distance calculations in Problem 2; $16 \bullet 10^2 = 16 \bullet 100 = 1600$.

3 Connect

Highlight that the distance traveled by a falling object is modeled by $d = \frac{1}{2}gt^2$ where $g = 32\frac{\text{ft}}{\text{sec}^2}$. Because half of 32 is 16, the coefficient of 16 represents the effects of gravity.

Ask:

- "How can you confirm that the rock will travel 600 ft in less than 6 seconds?" Substitute 6 for t.
- "If the same rock was dropped from a building twice its height, how does the distance traveled in the first 5 seconds change?" The distance traveled would be the same.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide, or suggest that students create, a table that they can use to organize their thinking about the scenario in this activity. A sample table is shown.

	Distances fallen (ft)		
Number of seconds	Value	Product of 16 and a number	Product of 16 and a squared number
1	16	$16 \bullet 1$	$16 \bullet 1^2$
2	64	$16 \bullet 4$	$16 \bullet 2^2$
3	144	$16 \bullet 9$	$16 \bullet 3^2$
4	256	$16 \bullet 16$	$16 \bullet 4^2$

Extension: *Math Enrichment*

Have students determine how long it would take for the rock to hit the ground if it was released from a height of 300 ft above the ground. Ask them to explain their thinking. About 4.3 seconds; Sample response: Determine when the distance fallen, $d(t)$, equals 300; $300 = 16 \bullet t^2$, Divide both sides by 16 and take the positive square root.

Extension: *Interdisciplinary Connections*

Tell students that the 32 in the equation $d = \frac{1}{2}gt^2$ represents the effects of Earth's gravity. A free-falling object, such as the one in this activity, will *accelerate* towards Earth at a rate of 32 ft per second, per second. This means that each second, the object falls at a faster rate. **(Science)**

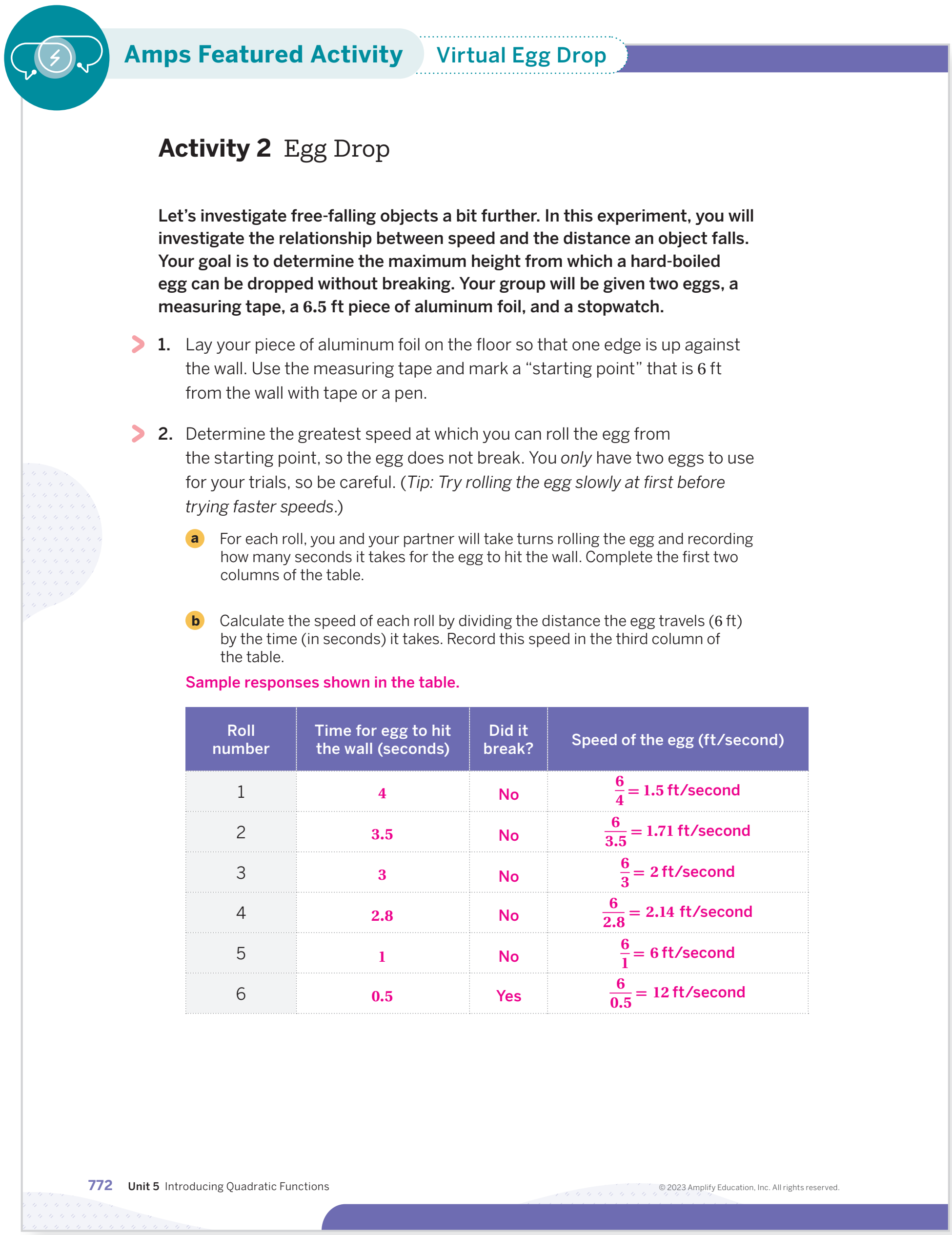
Optional

Pairs | 15 min

Activity 2 Egg Drop

Students measure the free fall of objects by rolling and dropping eggs to investigate the relationship between speed and free fall.

Amps Featured Activity Virtual Egg Drop



Activity 2 Egg Drop

Let's investigate free-falling objects a bit further. In this experiment, you will investigate the relationship between speed and the distance an object falls. Your goal is to determine the maximum height from which a hard-boiled egg can be dropped without breaking. Your group will be given two eggs, a measuring tape, a 6.5 ft piece of aluminum foil, and a stopwatch.

1. Lay your piece of aluminum foil on the floor so that one edge is up against the wall. Use the measuring tape and mark a "starting point" that is 6 ft from the wall with tape or a pen.
2. Determine the greatest speed at which you can roll the egg from the starting point, so the egg does not break. You *only* have two eggs to use for your trials, so be careful. (*Tip: Try rolling the egg slowly at first before trying faster speeds.*)
   - a For each roll, you and your partner will take turns rolling the egg and recording how many seconds it takes for the egg to hit the wall. Complete the first two columns of the table.
   - b Calculate the speed of each roll by dividing the distance the egg travels (6 ft) by the time (in seconds) it takes. Record this speed in the third column of the table.

Sample responses shown in the table.

Roll number	Time for egg to hit the wall (seconds)	Did it break?	Speed of the egg (ft/second)
1	4	No	$\frac{6}{4}$ = 1.5 ft/second
2	3.5	No	$\frac{6}{3.5}$ = 1.71 ft/second
3	3	No	$\frac{6}{3}$ = 2 ft/second
4	2.8	No	$\frac{6}{2.8}$ = 2.14 ft/second
5	1	No	$\frac{6}{1}$ = 6 ft/second
6	0.5	Yes	$\frac{6}{0.5}$ = 12 ft/second

772 **Unit 5** Introducing Quadratic Functions

1 Launch

Read the prompt aloud. Provide each group with 2 hard-boiled eggs, measuring tape, 6.5 ft of aluminum foil, and a stopwatch. Ask students if they have any questions about the activity.

2 Monitor

Help students get started by demonstrating how to set up the activity and how to roll an egg.

Look for points of confusion:

- **Struggling with team roles.** Assign these roles: roller, measurer, dropper, time-keeper, and recorder. Some students will need to have multiple roles.
- **Breaking eggs during the first rounds.** Estimate a lower rate based on the severity of the egg break. Compare with other groups.

Look for productive strategies:

- Gradually increasing the speed of each roll.
- Making the connection between rolling the egg and dropping it.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can virtually test eggs of different sizes dropping from different heights to see whether they will break.

Accessibility: *Guide Processing and Visualization*

Chunk this activity into smaller, more manageable parts with shorter time limits. For example:

- Give students 5 minutes to experiment with rolling the egg.
- Give students 5–8 minutes to make calculations and discuss what their calculations mean.
- Give students 3–5 minutes to measure and complete the egg drop.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses to the Ask question, consider displaying the two quadratic equations they have explored so far in this lesson: $d = \frac{1}{2}gt^2$ (or $d = 16t^2$) and $d = \frac{v^2}{64}$. Ask these additional follow-up questions:

- "For a relationship to be *quadratic*, what must be true? Where do you see this in each of these equations?"
- "Where do you see constant terms in each of these equations?"

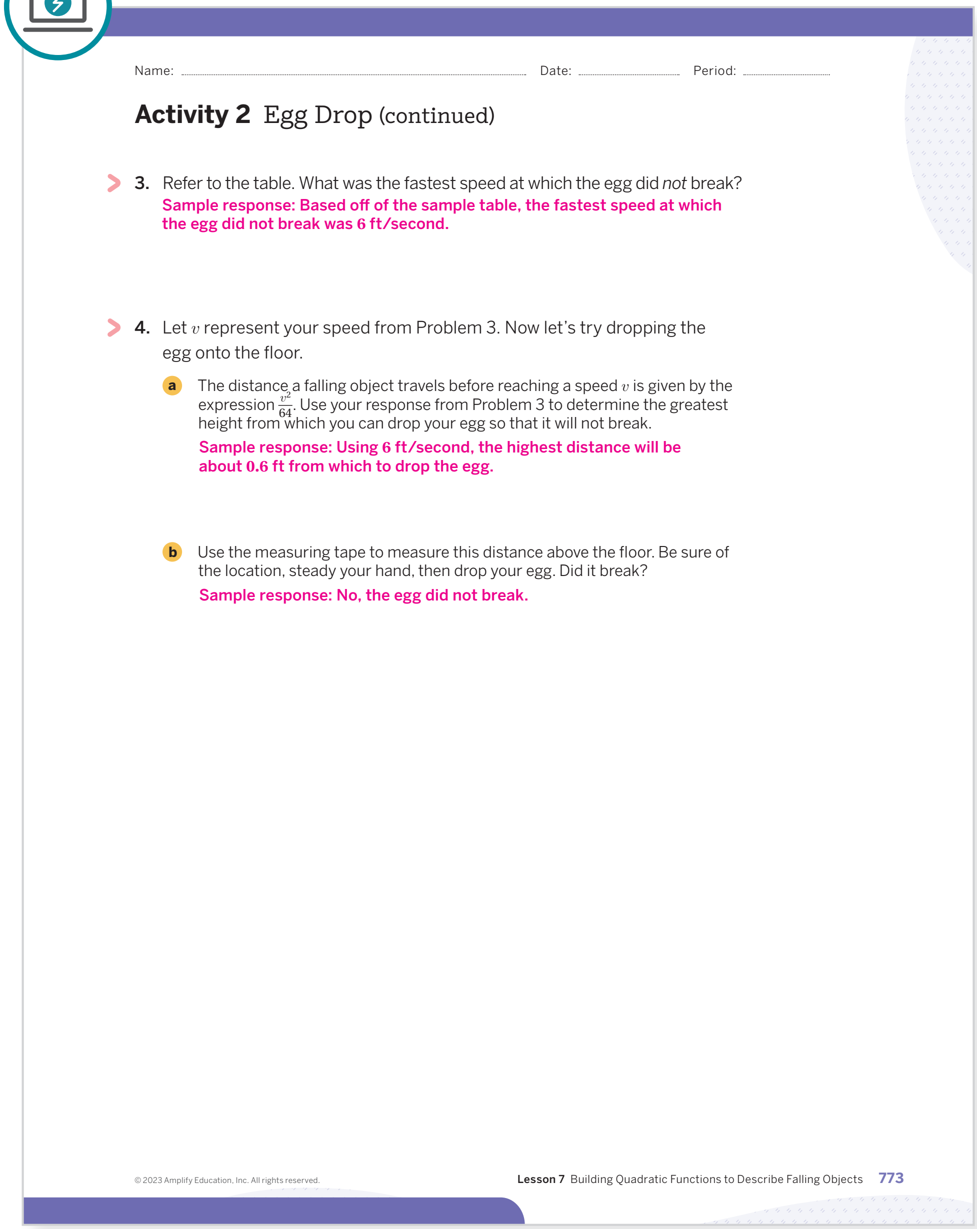
English Learners

Annotate the equations to identify the squared variable term and the constant.

Pairs | 15 min

Activity 2 Egg Drop (continued)

Students measure the free fall of objects by rolling and dropping eggs to investigate the relationship between speed and free fall.



Name: Date: Period:

Activity 2 Egg Drop (continued)

3. Refer to the table. What was the fastest speed at which the egg did *not* break?
Sample response: Based off of the sample table, the fastest speed at which the egg did not break was 6 ft/second.

4. Let v represent your speed from Problem 3. Now let's try dropping the egg onto the floor.

- **a** The distance a falling object travels before reaching a speed v is given by the expression $\frac{v^2}{64}$. Use your response from Problem 3 to determine the greatest height from which you can drop your egg so that it will not break.
  Sample response: Using 6 ft/second, the highest distance will be about 0.6 ft from which to drop the egg.

- **b** Use the measuring tape to measure this distance above the floor. Be sure of the location, steady your hand, then drop your egg. Did it break?
  Sample response: No, the egg did not break.

3 Connect

Have groups of students share their strategies for working together and rolling their eggs without breaking them.

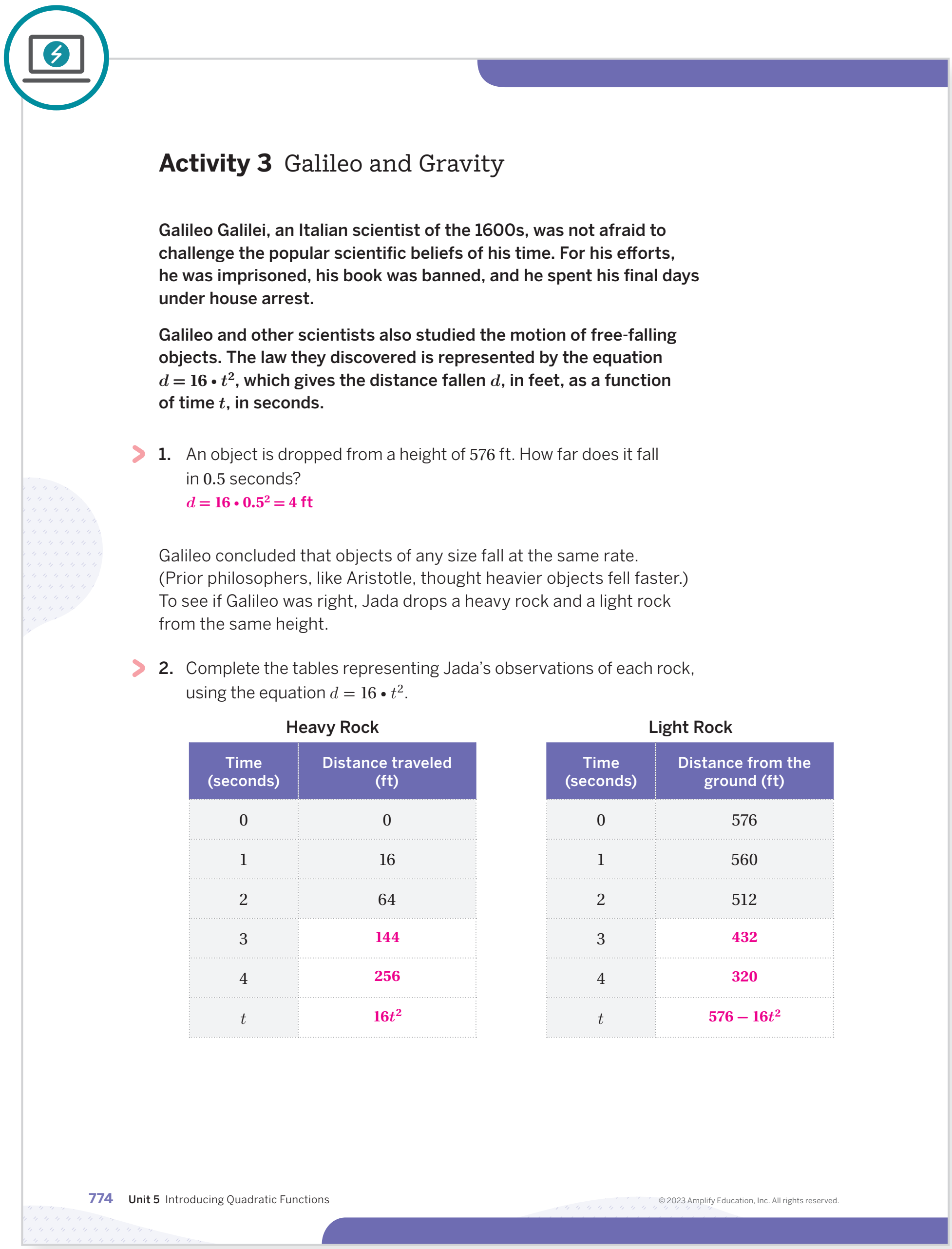
Highlight that the speed of the rolled egg can be used to determine the height at which the egg should be dropped with the equation $d = \frac{v^2}{64}$. The egg will hit the floor at the same speed as it did when it hit the wall.

Ask, "How is the relationship between the drop distance and the speed a quadratic relationship?" The variable speed v is squared in the distance equation.

Small Groups | 25 min

Activity 3 Galileo and Gravity

Students measure a free-falling object from two points of view, using a historical context.



Activity 3 Galileo and Gravity

Galileo Galilei, an Italian scientist of the 1600s, was not afraid to challenge the popular scientific beliefs of his time. For his efforts, he was imprisoned, his book was banned, and he spent his final days under house arrest.

Galileo and other scientists also studied the motion of free-falling objects. The law they discovered is represented by the equation $d = 16 \cdot t^2$, which gives the distance fallen d, in feet, as a function of time t, in seconds.

1. An object is dropped from a height of 576 ft. How far does it fall in 0.5 seconds?
   $d = 16 \cdot 0.5^2 = 4$ ft

Galileo concluded that objects of any size fall at the same rate. (Prior philosophers, like Aristotle, thought heavier objects fell faster.) To see if Galileo was right, Jada drops a heavy rock and a light rock from the same height.

2. Complete the tables representing Jada's observations of each rock, using the equation $d = 16 \cdot t^2$.

Heavy Rock

Time (seconds)	Distance traveled (ft)
0	0
1	16
2	64
3	144
4	256
t	$16t^2$

Light Rock

Time (seconds)	Distance from the ground (ft)
0	576
1	560
2	512
3	432
4	320
t	$576 - 16t^2$

774 Unit 5 Introducing Quadratic Functions

1 Launch

Read and discuss the prompt as a class. Activate students' background knowledge by asking for other similar examples. Have students work in groups on the activity.

2 Monitor

Help students get started by asking "Do you recognize the values in the heavy rock table?" The values were the same that were used in the Warm-up and Activity 1.

Look for points of confusion:

- **Using a growth rate of 4 to calculate the heavy rock's distance.** Verify this conjecture with the same values in Activity 1.
- **Struggling to write an expression for t.** Rewrite the previous distances in terms of 16 multiplied by a perfect square.

Look for productive strategies:

- Using Jada's repeated reasoning from Activity 1.
- Calculating the values in the light rock table by subtracting the values in the heavy rock table from 576.

Activity 3 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Have students circle or highlight the given equation in the introductory text, $d = 16 \cdot t^2$, and remind them that this is the same equation they used in Activity 1 to describe the free-falling rock dropped from the height of the building.

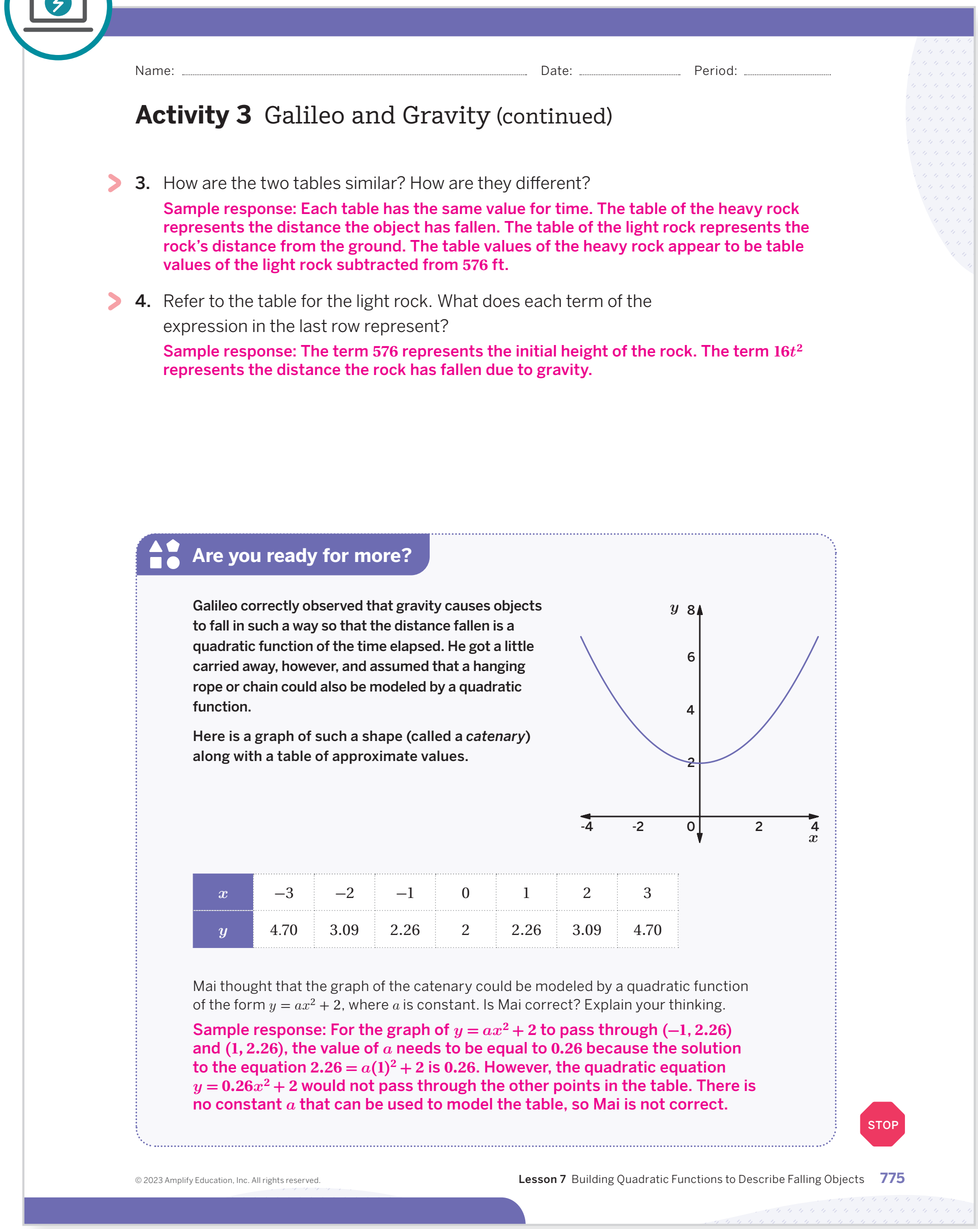
Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students color code t in the equation with the values in the first column of the tables in Problem 2. To help students understand the difference between the two tables, annotate "Distance traveled" and "Distance from the ground" with sketches that show the heavy rock's distance traveled increasing, while the light rock's distance from the ground decreases.

Small Groups | 25 min

Activity 3 Galileo and Gravity (continued)

Students measure a free-falling object from two points of view, using a historical context.



Name: ______ Date: ______ Period: ______

Activity 3 Galileo and Gravity (continued)

3. How are the two tables similar? How are they different?

Sample response: Each table has the same value for time. The table of the heavy rock represents the distance the object has fallen. The table of the light rock represents the rock's distance from the ground. The table values of the heavy rock appear to be table values of the light rock subtracted from 576 ft.

4. Refer to the table for the light rock. What does each term of the expression in the last row represent?

Sample response: The term 576 represents the initial height of the rock. The term $16t^2$ represents the distance the rock has fallen due to gravity.

Are you ready for more?

Galileo correctly observed that gravity causes objects to fall in such a way so that the distance fallen is a quadratic function of the time elapsed. He got a little carried away, however, and assumed that a hanging rope or chain could also be modeled by a quadratic function.

Here is a graph of such a shape (called a *catenary*) along with a table of approximate values.

x	-3	-2	-1	0	1	2	3
y	4.70	3.09	2.26	2	2.26	3.09	4.70

Mai thought that the graph of the catenary could be modeled by a quadratic function of the form $y = ax^2 + 2$, where a is constant. Is Mai correct? Explain your thinking.

Sample response: For the graph of $y = ax^2 + 2$ to pass through $(-1, 2.26)$ and $(1, 2.26)$, the value of a needs to be equal to 0.26 because the solution to the equation $2.26 = a(1)^2 + 2$ is 0.26. However, the quadratic equation $y = 0.26x^2 + 2$ would not pass through the other points in the table. There is no constant a that can be used to model the table, so Mai is not correct.

STOP

3 Connect

Have individual students share their strategies for completing the tables.

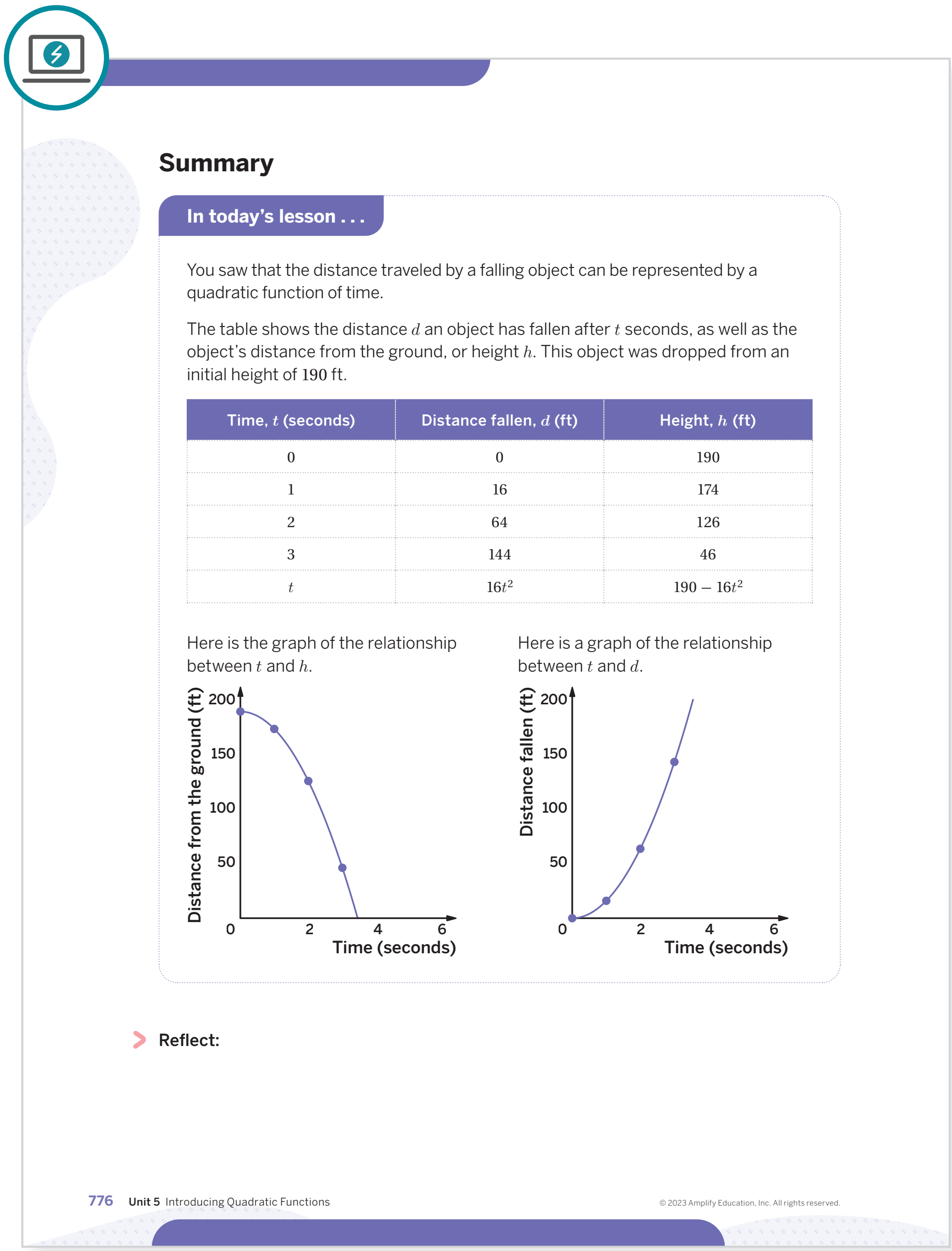
Highlight that the distance of falling objects is measured in two ways: "top-down" by calculating the distance traveled using $16t^2$, and "ground-up" by calculating the distance from the ground by subtracting $16t^2$ from the initial height.

Ask, "Suppose you graphed each expression? How would the graphs compare?" The graphs would curve in opposite directions.

Whole Class | 5 min

Summary

Review and synthesize how the distance traveled of a falling object over time is modeled by a quadratic function.



Summary

In today's lesson . . .

You saw that the distance traveled by a falling object can be represented by a quadratic function of time.

The table shows the distance d an object has fallen after t seconds, as well as the object's distance from the ground, or height h. This object was dropped from an initial height of 190 ft.

Time, t (seconds)	Distance fallen, d (ft)	Height, h (ft)
0	0	190
1	16	174
2	64	126
3	144	46
t	$16t^2$	$190 - 16t^2$

Here is the graph of the relationship between t and h.

Here is a graph of the relationship between t and d.

Reflect:

Synthesize

Display the Anchor Chart PDF, *Graphs of Free Falling Objects*.

Highlight that the distance an object falls increases each second. The average rate of change also increases each second, which means that the object speeds up over time as it moves closer to the ground. Students can use two different functions to describe the movement of a falling object. One function measures the distance the object traveled from its starting point, and the other measures its distance from the ground.

Ask:

- "How are the representations of these functions similar? How are they different?" The equations both include $16t^2$, but one is positive and the other is negative because it is subtracted from an initial height.
- "How are these functions similar or dissimilar to those representing visual patterns in earlier lessons?" They can all be represented by quadratic functions. For these functions, it is helpful to create a table of values, check for *second differences*, and try to determine patterns in the table first.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

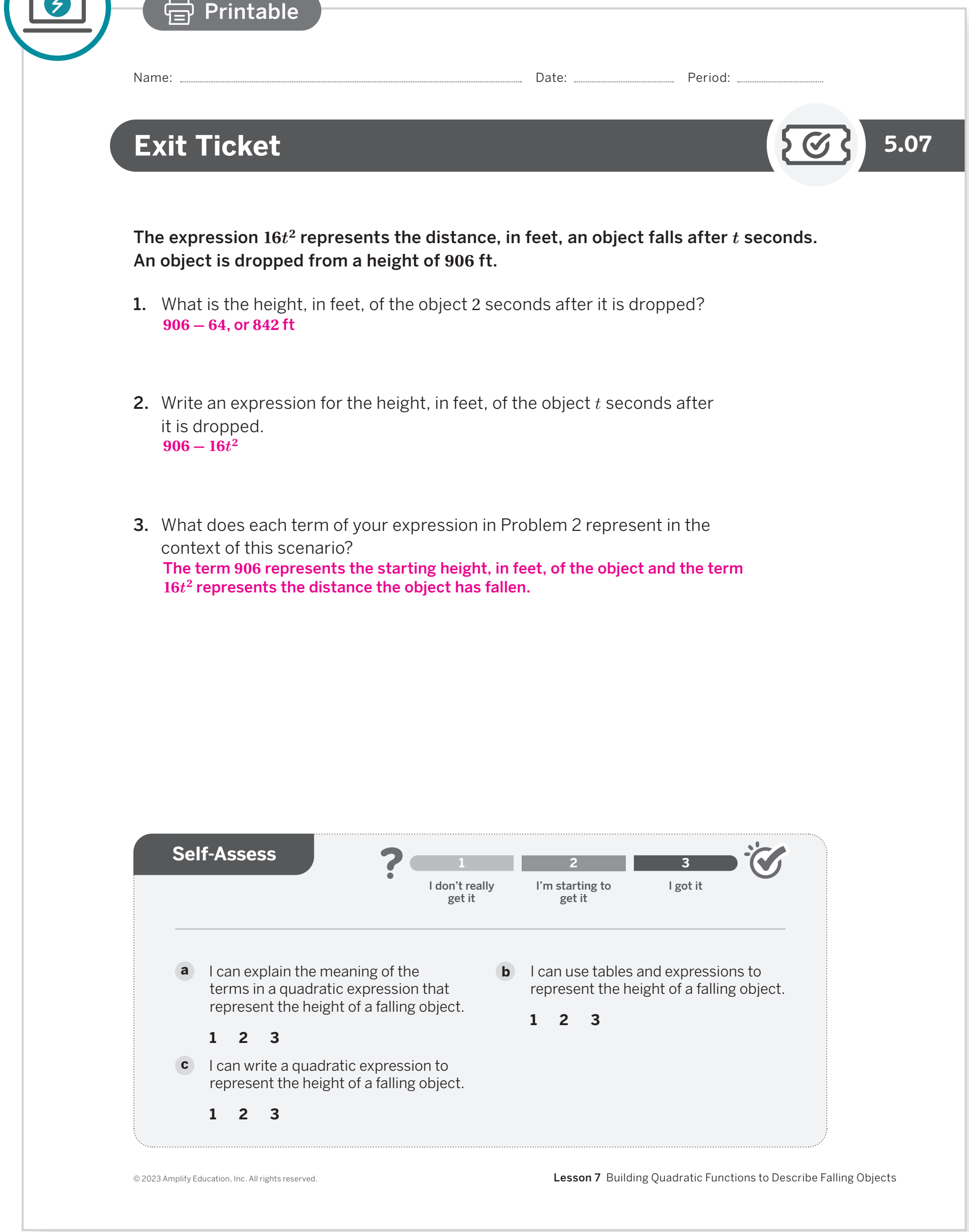
- "How are quadratic functions used to model, analyze, and interpret mathematical relationships?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing a quadratic expression to represent the height of a falling object and interpreting each term of their expression in context.

Printable



Name: ______________________ Date: ____________ Period: ____________

Exit Ticket 5.07

The expression $16t^2$ represents the distance, in feet, an object falls after t seconds. An object is dropped from a height of 906 ft.

1. What is the height, in feet, of the object 2 seconds after it is dropped?
   906 − 64, or 842 ft

2. Write an expression for the height, in feet, of the object t seconds after it is dropped.
   $906 - 16t^2$

3. What does each term of your expression in Problem 2 represent in the context of this scenario?
   The term 906 represents the starting height, in feet, of the object and the term $16t^2$ represents the distance the object has fallen.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain the meaning of the terms in a quadratic expression that represent the height of a falling object.
   1 2 3

b. I can use tables and expressions to represent the height of a falling object.
   1 2 3

c. I can write a quadratic expression to represent the height of a falling object.
   1 2 3

Success looks like . . .

- **Language Goal:** Explaining the meaning of the terms in a quadratic expression that represents the height of a falling object. **(Speaking and Listening)**
  - » Explaining what is represented by each term in the context of an object falling in Problem 3.
- **Goal:** Using tables and equations to represent the height of a falling object.
- **Goal:** Writing quadratic functions to represent the height of an object falling due to gravity.

Suggested next steps

If students substitute 2 for t, but do not subtract from 906 in Problem 1, consider:

- Asking, "What does your value represent?"
- Reviewing the difference between the tables in Activity 3.

If students provide an incorrect expression for Problem 2, consider:

- Asking, "How would you determine the height of an object dropped from 906 ft?"
- Reviewing how they found the expressions for the heavy rock and light rock tables in Activity 3.
- Assigning Practice Problems 2 and 3.

If students provide an incorrect or vague explanation for Problem 3, consider:

- Asking, "Where in the expression do you see the value 906? Why would the next term be negative?"
- Reviewing Activity 3, Problem 4.
- Assigning Practice Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In earlier lessons, students wrote quadratic expressions and functions to model patterns. How did that support writing quadratic functions to represent the height of falling objects?
- Knowing where students need to be by the end of this unit, how did comparing the quadratic expressions representing the distance fallen and representing the distance from the ground influence that future goal?

Math Language Development

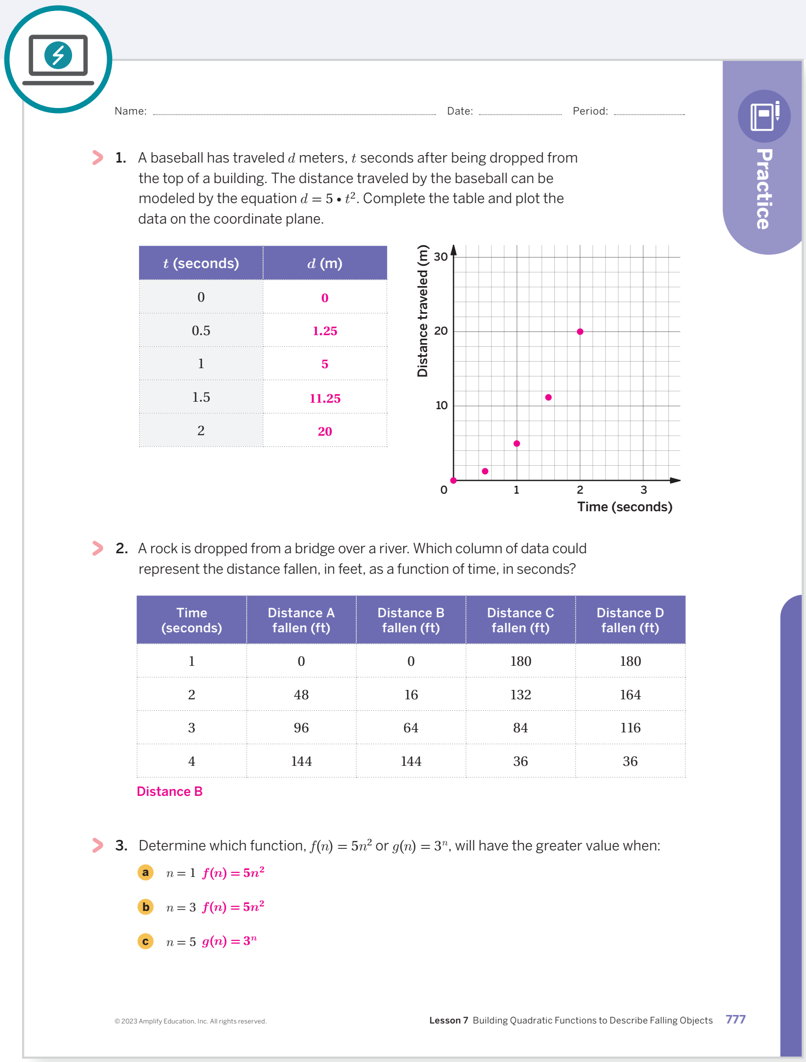
Language Goal: Explaining the meaning of the terms in a quadratic expression that represents the height of a falling object.

Reflect on students' language development toward this goal.

- Do students' responses to Problem 3 of the Exit Ticket demonstrate they understand what each term of their expression represents?
- Are they using terms and phrases such as *starting/initial height* and *distance fallen*?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. A baseball has traveled d meters, t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5 \cdot t^2$. Complete the table and plot the data on the coordinate plane.

t (seconds)	d (m)
0	0
0.5	1.25
1	5
1.5	11.25
2	20

2. A rock is dropped from a bridge over a river. Which column of data could represent the distance fallen, in feet, as a function of time, in seconds?

Time (seconds)	Distance A fallen (ft)	Distance B fallen (ft)	Distance C fallen (ft)	Distance D fallen (ft)
1	0	0	180	180
2	48	16	132	164
3	96	64	84	116
4	144	144	36	36

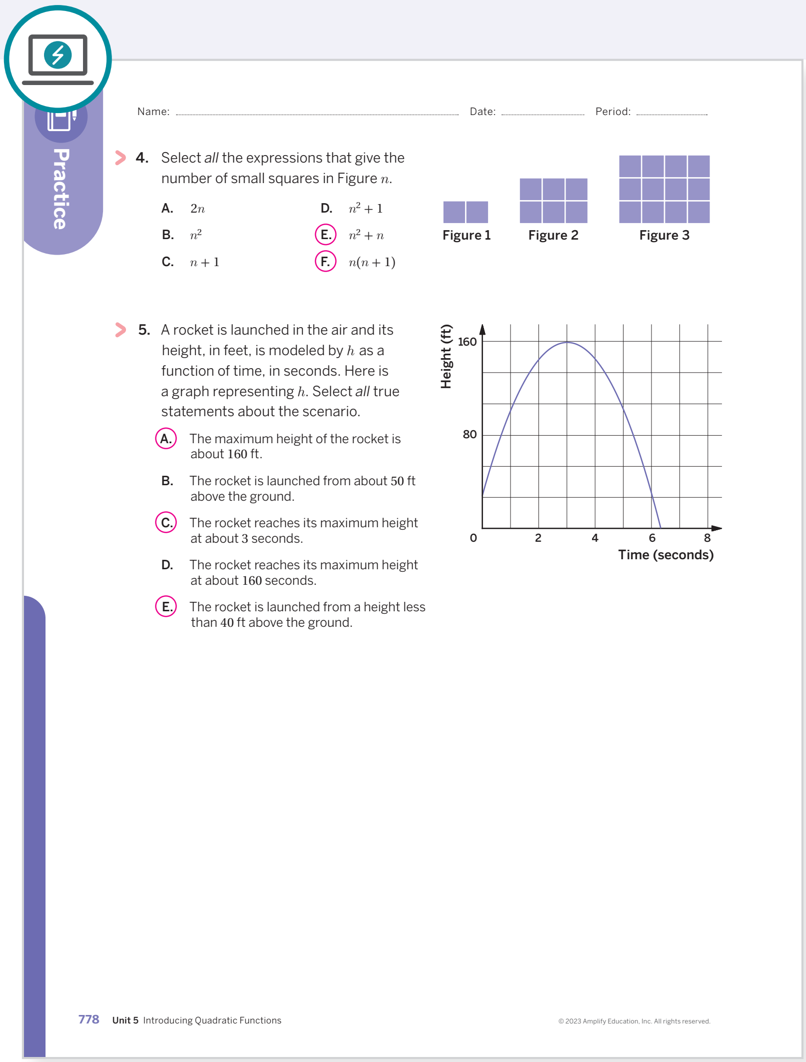
Distance B

3. Determine which function, $f(n) = 5n^2$ or $g(n) = 3^n$, will have the greater value when:

a. $n = 1$ $f(n) = 5n^2$

b. $n = 3$ $f(n) = 5n^2$

c. $n = 5$ $g(n) = 3^n$



Name: ________ Date: ________ Period: ________

Practice

4. Select *all* the expressions that give the number of small squares in Figure n.

A. $2n$

B. n^2

C. $n + 1$

D. $n^2 + 1$

[✓ E.] $n^2 + n$

[✓ F.] $n(n + 1)$

5. A rocket is launched in the air and its height, in feet, is modeled by h as a function of time, in seconds. Here is a graph representing h. Select *all* true statements about the scenario.

[✓ A.] The maximum height of the rocket is about 160 ft.

B. The rocket is launched from about 50 ft above the ground.

[✓ C.] The rocket reaches its maximum height at about 3 seconds.

D. The rocket reaches its maximum height at about 160 seconds.

[✓ E.] The rocket is launched from a height less than 40 ft above the ground.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
Spiral	3	Unit 5 Lesson 6	1
	4	Unit 5 Lesson 4	2
Formative	5	Unit 5 Lesson 8	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 8**

Building Quadratic Functions to Describe Projectile Motion

Let's study objects being launched in the air.

Focus

Goals

1. Create graphs of quadratic functions that represent a physical phenomenon and determine an appropriate domain when graphing.
2. **Language Goal:** Identify and interpret the meaning of the vertex of a graph and the zeros of a function represented in tables and graphs. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Write and interpret quadratic functions that represent a physical phenomenon. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students continue to build their understanding of quadratics by examining the effect of gravity on projectiles. They construct quadratic functions to model projectile motion in a context. Students are introduced to the vertex and the zero of a quadratic function by using a graph to understand and consider an appropriate domain given a context.

Previously

In the previous lesson, students used quadratic functions to describe free-falling objects over time and as a tool to determine its distance from the ground.

Coming Soon

In the next lesson, students will explore quadratic functions in an economic context.

Rigor

- Students build **conceptual understanding** of quadratic functions by modeling projectile motion.
- Students **apply** their understanding of quadratic functions to study projectile motion.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	25 min	10 min	5 min	5 min
Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Representing Projectile Motion*
- graphing technology

Math Language Development

New words

- *vertex*
- *zero*

Review words

- *parabola*
- *projectile*

Amps ⋮ **Featured Activity**

Activity 1

Foofoo's Flight

Students use an interactive graph to track Foofoo's flight with and without gravity to see how its influence changes the equation that models Foofoo's motion.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be discouraged that the answer to projectile-motion problems do not necessarily come quickly. The solution process often requires abstract thinking and reasoning. Encourage students to identify how they feel as they start Activity 2, and then have them identify their own strengths and recognize why they are capable of successfully completing the activity.

Modifications to Pacing

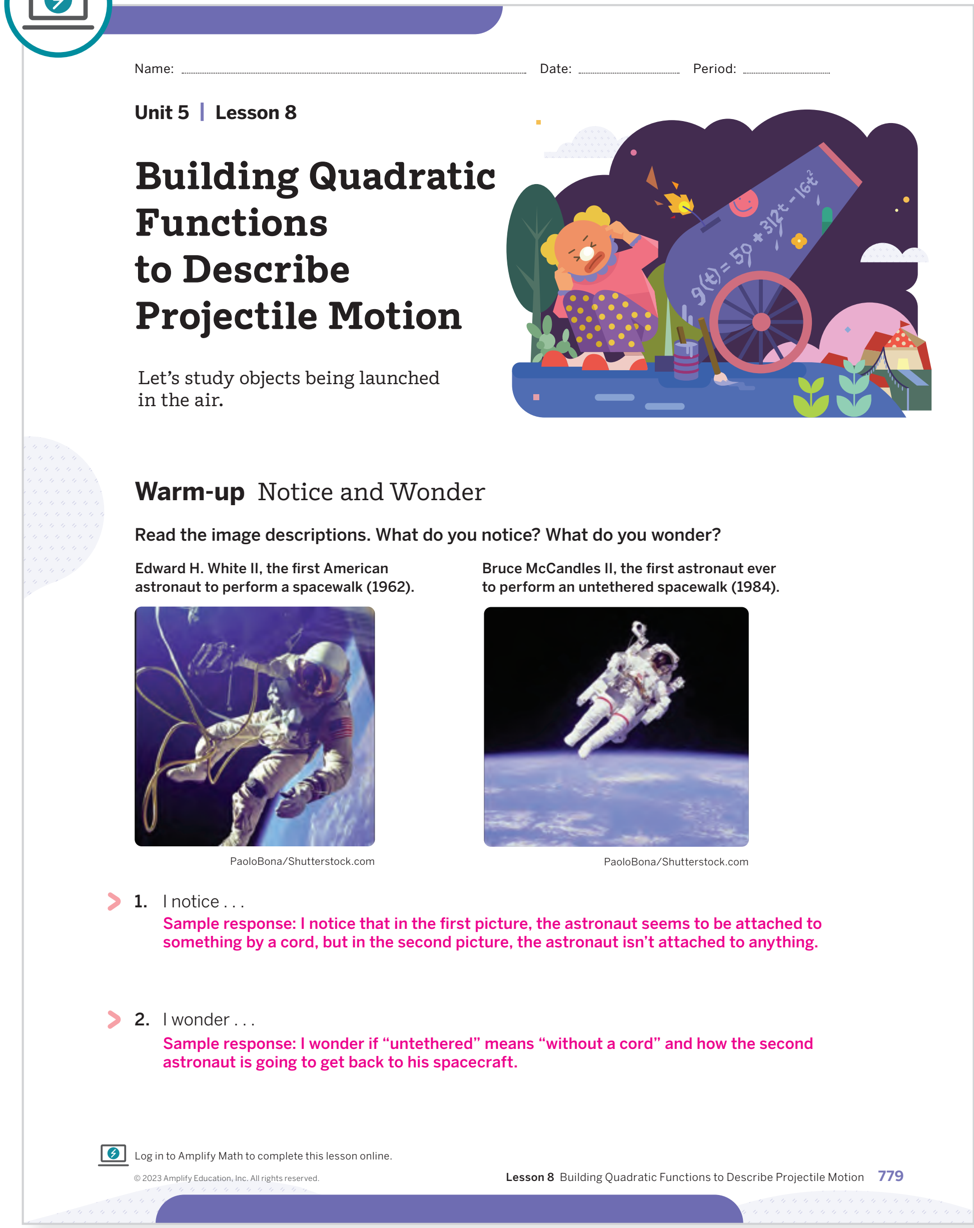
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Problem 1, have students only complete the first four columns of the table.

Independent | 5 min

Warm-up Notice and Wonder

Students activate their background knowledge of gravity by observing images of astronauts in space to formulate ideas about the influence of gravity.



Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 8

Building Quadratic Functions to Describe Projectile Motion

Let's study objects being launched in the air.

Warm-up Notice and Wonder

Read the image descriptions. What do you notice? What do you wonder?

Edward H. White II, the first American astronaut to perform a spacewalk (1962).

PaoloBona/Shutterstock.com

Bruce McCandles II, the first astronaut ever to perform an untethered spacewalk (1984).

PaoloBona/Shutterstock.com

1. I notice . . .
   Sample response: I notice that in the first picture, the astronaut seems to be attached to something by a cord, but in the second picture, the astronaut isn't attached to anything.
2. I wonder . . .
   Sample response: I wonder if "untethered" means "without a cord" and how the second astronaut is going to get back to his spacecraft.

Log in to Amplify Math to complete this lesson online.

Lesson 8 Building Quadratic Functions to Describe Projectile Motion 779

1 Launch

Display the images along with their descriptions. Conduct the *Notice and Wonder* routine.

2 Monitor

Help students get started by having them read the corresponding descriptions for each image.

Look for productive strategies:

- Circling the rope in one image.
- Annotating "untethered" in one of the image descriptions.
- Noticing that the second astronaut appears to have a jet-pack.

3 Connect

Have students share what they notice and wonder about the images. Discuss how the tether in the first image is used to pull the astronaut back to the spacecraft and how the astronaut in the second image used an MMU (jetpack) to propel himself, and why these methods are necessary in space (due to lack of gravity).

Define the term *spacewalk*.

Ask, "Why do you think the first spacewalk was tethered?" So the astronaut would not float away into space!

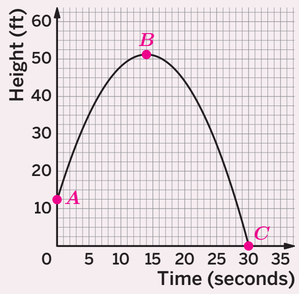
Highlight that without the influence of gravity, the astronauts would travel in a straight line forever, if they were in motion. Ask students to think about any popular movies, or other references, that involve spacewalks or objects floating in space.

Power-up

To power up students' ability to identify key features of a function from a graph, have students complete:

The graph represents the trajectory of a pebble launched in the air by a slingshot.

1. Add a point to the graph at the initial height of the pebble. Label the point A.
2. Add a point to the graph at the maximum height of the pebble. Label the point B.
3. Add a point to the graph at the time the pebble hits the ground. Label the point C.



Use: Before Activity 2

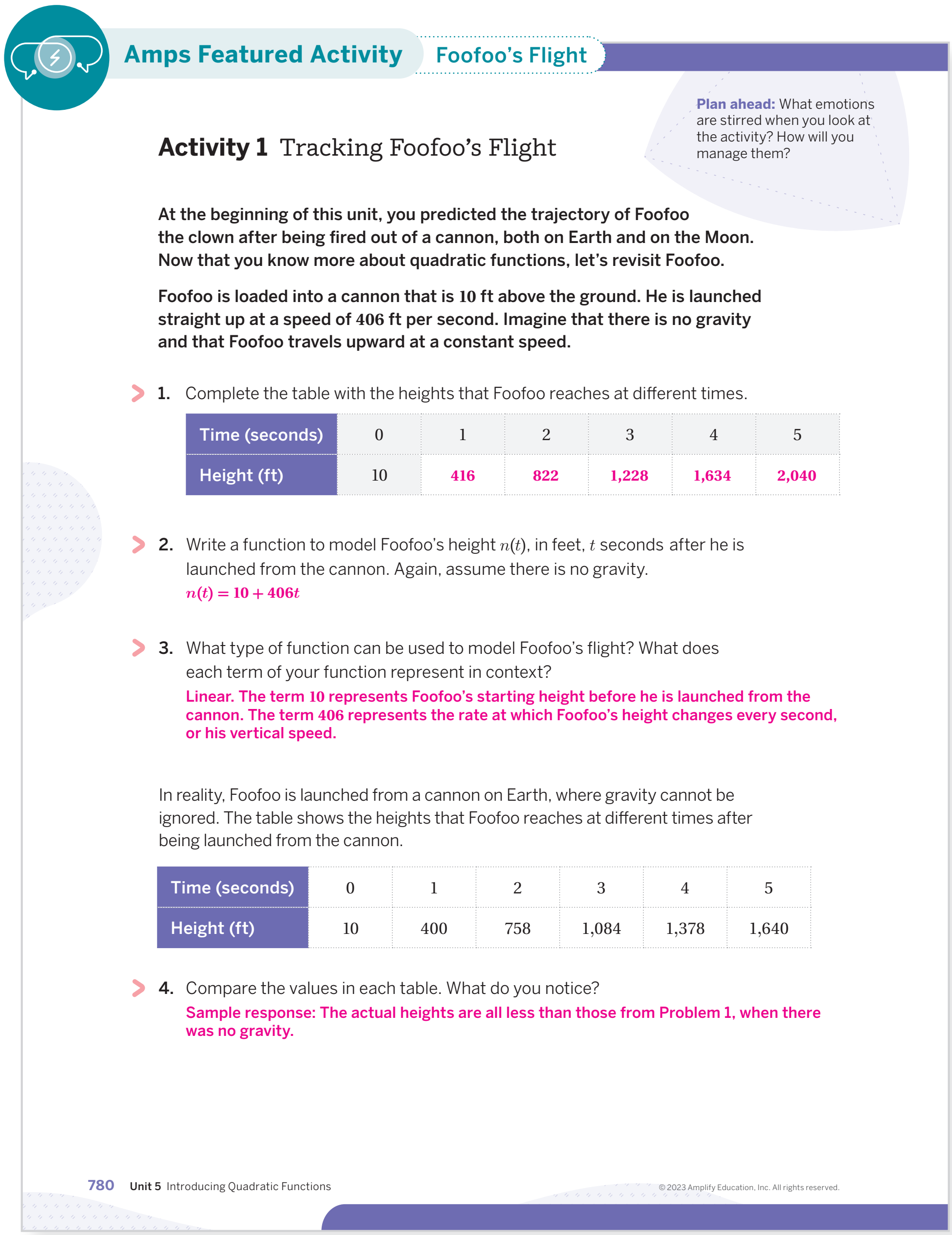
Informed by: Performance on Lesson 6, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 25 min

Activity 1 Tracking Foofoo's Flight

Students graph and compare functions representing the height of Foofoo over time to determine the effects with and without gravity.

Amps Featured Activity Foofoo's Flight



Plan ahead: What emotions are stirred when you look at the activity? How will you manage them?

Activity 1 Tracking Foofoo's Flight

At the beginning of this unit, you predicted the trajectory of Foofoo the clown after being fired out of a cannon, both on Earth and on the Moon. Now that you know more about quadratic functions, let's revisit Foofoo.

Foofoo is loaded into a cannon that is 10 ft above the ground. He is launched straight up at a speed of 406 ft per second. Imagine that there is no gravity and that Foofoo travels upward at a constant speed.

1. Complete the table with the heights that Foofoo reaches at different times.

Time (seconds)	0	1	2	3	4	5
Height (ft)	10	416	822	1,228	1,634	2,040

2. Write a function to model Foofoo's height $n(t)$, in feet, t seconds after he is launched from the cannon. Again, assume there is no gravity.
$n(t) = 10 + 406t$

3. What type of function can be used to model Foofoo's flight? What does each term of your function represent in context?
Linear. The term 10 represents Foofoo's starting height before he is launched from the cannon. The term 406 represents the rate at which Foofoo's height changes every second, or his vertical speed.

In reality, Foofoo is launched from a cannon on Earth, where gravity cannot be ignored. The table shows the heights that Foofoo reaches at different times after being launched from the cannon.

Time (seconds)	0	1	2	3	4	5
Height (ft)	10	400	758	1,084	1,378	1,640

4. Compare the values in each table. What do you notice?
Sample response: The actual heights are all less than those from Problem 1, when there was no gravity.

780 **Unit 5** Introducing Quadratic Functions

1 Launch

Read the prompt aloud. Remind students of the unit launch when they compared Foofoo's Earth and Moon launch. Connect this back to the Warm-up and ask, "What would happen if Foofoo was launched into space? Why?"

2 Monitor

Help students get started by having them annotate the given information from the introduction.

Look for points of confusion:

- **Struggling to complete the table in Problem 5.** Ask students which table (from Problems 1 and 3) represents Foofoo's path with gravity and which table represents it without gravity.
- **Not using the table to determine the differences in the heights in Problem 6.** Have students circle each column of heights. Allow them the opportunity to subtract the heights in any order. The absolute value of the difference can be used to determine the distance.
- **Struggling to differentiate between data points on the graph in Problem 7.** Have students use different colors to plot the different functions.

Look for productive strategies:

- Annotating tables with first and second differences.
- Determining the difference of the heights *with* and *without* gravity in the last column of the table in Problem 5.
- Referring to the tables from the previous lesson (or some other indication that they recognize the pattern).
- Generalizing the relationship between time and distance through repeated reasoning.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive graph to track Foofoo's flight with and without gravity to see how its influence changes the equation that models Foofoo's motion.

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing a pre-populated graph for students to analyze in Problem 7 so that they can spend more time comparing and describing the graphs.

Math Language Development

MLR8: Discussion Supports

During the Connect, as you highlight the differences with and without gravity, display these sentence frames and ask students to complete them.

- Without gravity, Foofoo's height will continue to ___ and the equation ___ represents his height over time. This type of relationship is ___." increase; $n(t) = 10 + 406t$; linear
- "With gravity, Foofoo's height will ___ and then ___ and the equation ___ represents his height over time. This type of relationship is ___." increase; decrease; $n(t) = 10 + 406t - 16t^2$; quadratic

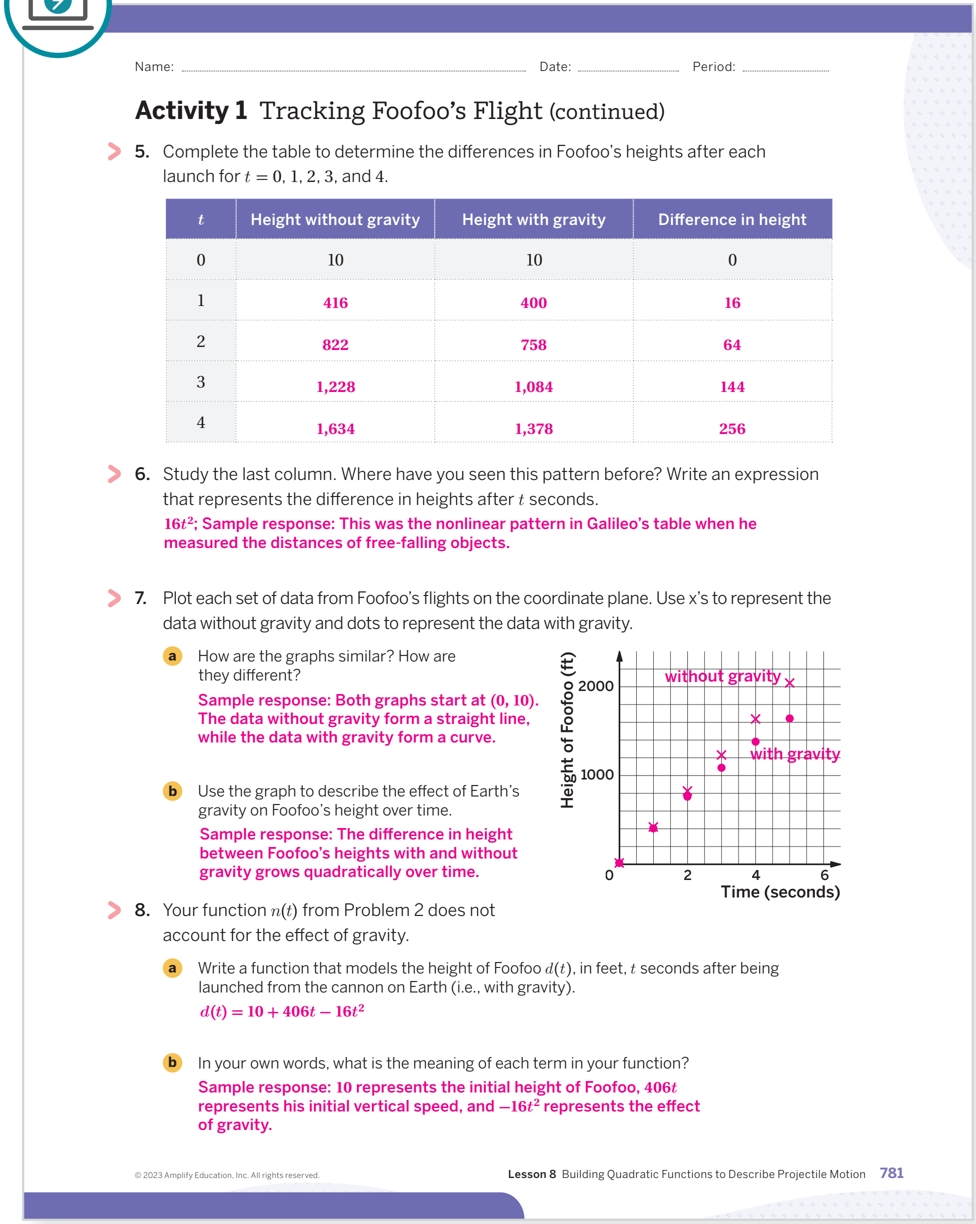
English Learners

Provide students with wait time and allow them to rehearse what they will say with a partner before sharing with the class.

Pairs | 25 min

Activity 1 Tracking Foofoo's flight (continued)

Students graph and compare functions representing the height of Foofoo over time to determine the effects with and without gravity.



Name: ______ Date: ______ Period: ______

Activity 1 Tracking Foofoo's Flight (continued)

5. Complete the table to determine the differences in Foofoo's heights after each launch for $t = 0, 1, 2, 3,$ and 4.

t	Height without gravity	Height with gravity	Difference in height
0	10	10	0
1	416	400	16
2	822	758	64
3	1,228	1,084	144
4	1,634	1,378	256

6. Study the last column. Where have you seen this pattern before? Write an expression that represents the difference in heights after t seconds.
$16t^2$; Sample response: This was the nonlinear pattern in Galileo's table when he measured the distances of free-falling objects.

7. Plot each set of data from Foofoo's flights on the coordinate plane. Use x's to represent the data without gravity and dots to represent the data with gravity.

a. How are the graphs similar? How are they different?
Sample response: Both graphs start at $(0, 10)$. The data without gravity form a straight line, while the data with gravity form a curve.

b. Use the graph to describe the effect of Earth's gravity on Foofoo's height over time.
Sample response: The difference in height between Foofoo's heights with and without gravity grows quadratically over time.

8. Your function $n(t)$ from Problem 2 does not account for the effect of gravity.

a. Write a function that models the height of Foofoo $d(t)$, in feet, t seconds after being launched from the cannon on Earth (i.e., with gravity).
$d(t) = 10 + 406t - 16t^2$

b. In your own words, what is the meaning of each term in your function?
Sample response: 10 represents the initial height of Foofoo, $406t$ represents his initial vertical speed, and $-16t^2$ represents the effect of gravity.

3 Connect

Have groups of students share their responses. Select one group to discuss Problem 2 and one group to discuss Problem 5. Probe the first group to identify the function type (linear) and why the function is this type (there is no gravity). Probe the second group to compare the values in the first and second columns in the table for Problem 5. Probe both groups regarding the last column of the table in Problem 5, asking them to make connections to the previous lesson ($16t^2$) and how the function $d(t)$ includes the effect of gravity in Problem 8.

Display the completed graph in Problem 7.

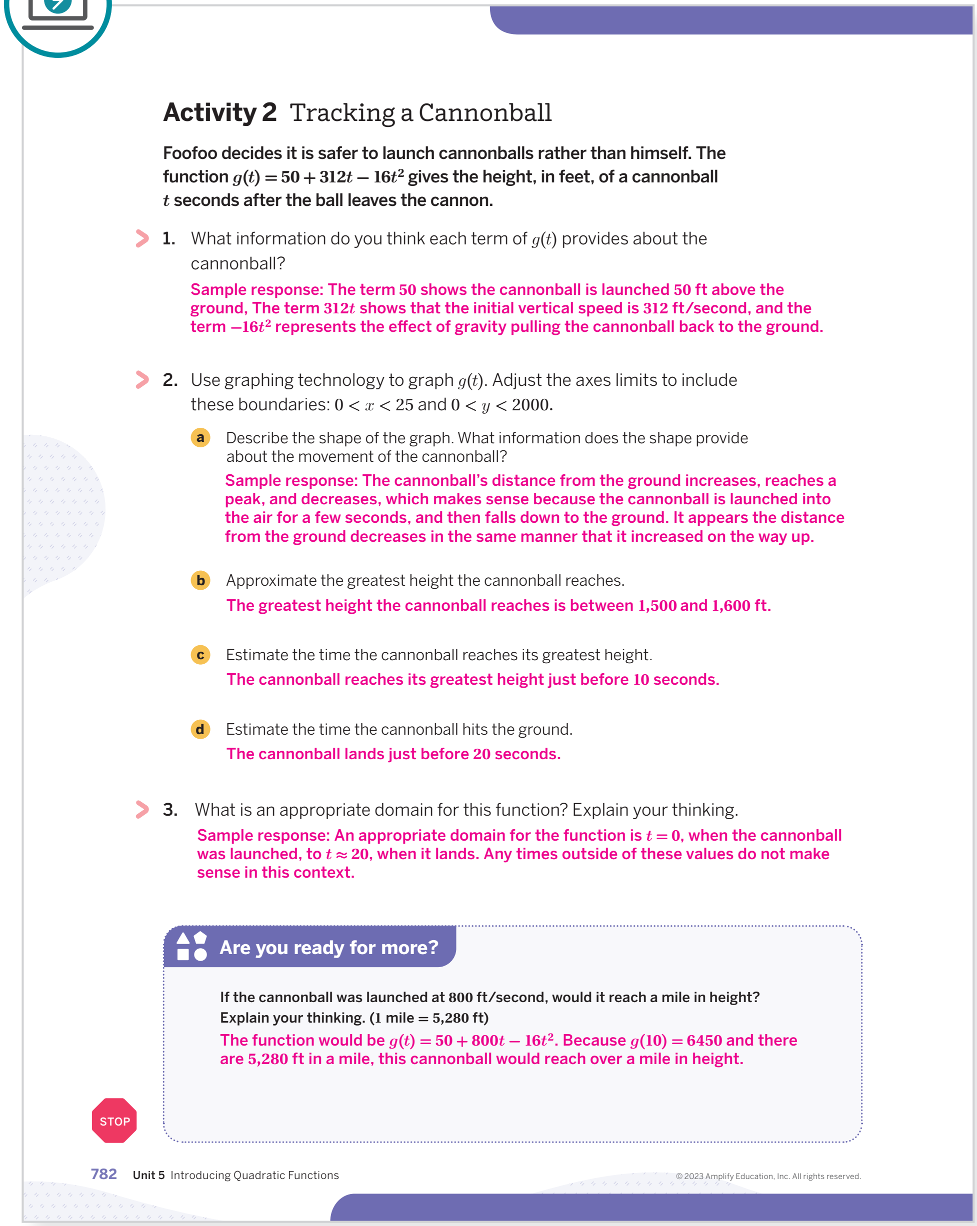
Ask, "What function models the height of Foofoo after being launched on Earth? What does each term in the function represent?"
$d(t) = 10 + 406t - 16t^2$ where 10 is the initial height of Foofoo, $406t$ is his initial speed, and $-16t^2$ is the effect of gravity.

Highlight how in the absence of gravity, Foofoo's height changes at a rate of 406 ft per second (as indicated by the table in Problem 1). Gravity, whose effect is represented by $-16t^2$, causes the straight line, $n(t)$, to curve or bend. To determine a function $d(t)$, subtract $16t^2$ from $n(t)$ to account for the effect of gravity.

Independent | 10 min

Activity 2 Tracking a Cannonball

Students use graphing technology and determine an appropriate domain to show how the graph of a quadratic function illustrates the path of the cannonball.



Activity 2 Tracking a Cannonball

Foofoo decides it is safer to launch cannonballs rather than himself. The function $g(t) = 50 + 312t - 16t^2$ gives the height, in feet, of a cannonball t seconds after the ball leaves the cannon.

1. What information do you think each term of $g(t)$ provides about the cannonball?

   Sample response: The term 50 shows the cannonball is launched 50 ft above the ground, The term $312t$ shows that the initial vertical speed is 312 ft/second, and the term $-16t^2$ represents the effect of gravity pulling the cannonball back to the ground.

2. Use graphing technology to graph $g(t)$. Adjust the axes limits to include these boundaries: $0 < x < 25$ and $0 < y < 2000$.

   a. Describe the shape of the graph. What information does the shape provide about the movement of the cannonball?

   Sample response: The cannonball's distance from the ground increases, reaches a peak, and decreases, which makes sense because the cannonball is launched into the air for a few seconds, and then falls down to the ground. It appears the distance from the ground decreases in the same manner that it increased on the way up.

   b. Approximate the greatest height the cannonball reaches.

   The greatest height the cannonball reaches is between 1,500 and 1,600 ft.

   c. Estimate the time the cannonball reaches its greatest height.

   The cannonball reaches its greatest height just before 10 seconds.

   d. Estimate the time the cannonball hits the ground.

   The cannonball lands just before 20 seconds.

3. What is an appropriate domain for this function? Explain your thinking.

   Sample response: An appropriate domain for the function is $t = 0$, when the cannonball was launched, to $t \approx 20$, when it lands. Any times outside of these values do not make sense in this context.

Are you ready for more?

If the cannonball was launched at 800 ft/second, would it reach a mile in height? Explain your thinking. (1 mile = 5,280 ft)

The function would be $g(t) = 50 + 800t - 16t^2$. Because $g(10) = 6450$ and there are 5,280 ft in a mile, this cannonball would reach over a mile in height.

STOP

782 **Unit 5** Introducing Quadratic Functions

1 Launch

Read the prompt aloud. Provide access to graphing technology. Consider reviewing how to adjust the axes limits.

2 Monitor

Help students get started by reviewing the function $d(t)$ and the meaning of each of its terms from Activity 1 as a reference.

Look for points of confusion:

- **Having difficulty interpreting the graph.** Remind students that the input value represents the time in seconds and the output value represents the height in feet.
- **Struggling to interpret the domain.** Ask, "Is the function a good model for predicting the height of the cannonball 10 seconds after it is fired? What about 1 minute after it is fired?"

Look for productive strategies:

- Sketching and labeling the curve in context, such as when the cannonball reaches its greatest height and when it hits the ground.
- Using the trace function on the graphing tool.

3 Connect

Display the function $g(t)$ and ask students to identify the terms in the equation that represent the initial height, initial vertical speed, and the effect of gravity. Then display the function's graph on the given domain.

Have students share how the movement of the cannonball is represented by the graph. Ask students to share where they see the initial height, the height over time, the maximum height, and when the cannonball hits the ground.

Define the terms ***vertex*** and ***zero***.

Highlight that any output values greater than the horizontal intercept do not make sense in this context.

Ask, "Why do values less than $t = 0$ not make sense in this context?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Discuss Problem 1 together as a class. Provide access to colored pencils and have students annotate what each term of the function $g(t)$ represents. Have students sketch the graph of the function for Problem 2 in the margin of their Student Edition, or on a separate piece of paper. Have them label each of the following parts of the graph.

- The initial height of the cannonball.
- Where the cannonball is ascending.
- The highest point.
- Where the cannonball is descending.
- When the cannonball will hit the ground.

Math Language Development

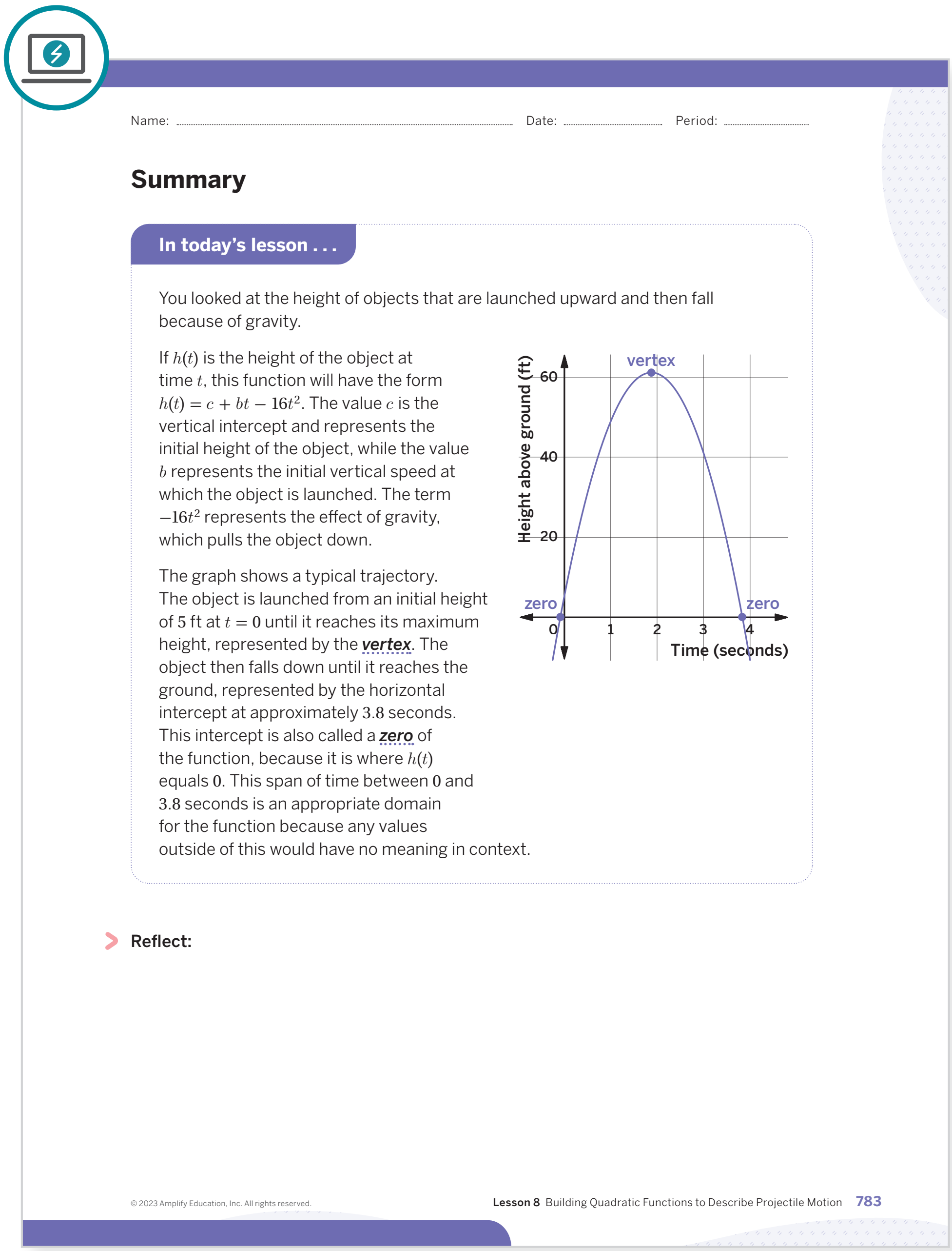
MLR7: Compare and Connect

During the Connect, add the graph of the function $g(t)$ to the class display. As students discuss where they see each of the following represented in the graph, annotate the graph with these terms.

- The initial height of the cannonball. *Initial height*, *Vertical intercept*
- Where the cannonball is ascending. *Increasing interval*
- The highest point. *Vertex*, *Maximum*
- Where the cannonball is descending. *Decreasing interval*
- When the cannonball will hit the ground. *Zero*, *Horizontal intercept*

Summary

Review and synthesize the connections between the terms of a quadratic function, the situation it represents, and the function's graph — highlighting what the vertex and zeros mean in context.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You looked at the height of objects that are launched upward and then fall because of gravity.

If $h(t)$ is the height of the object at time t, this function will have the form $h(t) = c + bt - 16t^2$. The value c is the vertical intercept and represents the initial height of the object, while the value b represents the initial vertical speed at which the object is launched. The term $-16t^2$ represents the effect of gravity, which pulls the object down.

The graph shows a typical trajectory. The object is launched from an initial height of 5 ft at $t = 0$ until it reaches its maximum height, represented by the ***vertex***. The object then falls down until it reaches the ground, represented by the horizontal intercept at approximately 3.8 seconds. This intercept is also called a ***zero*** of the function, because it is where $h(t)$ equals 0. This span of time between 0 and 3.8 seconds is an appropriate domain for the function because any values outside of this would have no meaning in context.

Reflect:

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *vertex* and *zero* that were added to the display during the lesson.

Synthesize

Display the function $h(t) = c + bt - 16t^2$ and the graph.

Have students share what the term $-16t^2$ means in the function $h(t)$.

Highlight that $h(t) = c + bt - 16t^2$ represents the height of an object launched upward, where:

- c represents the initial height of the object. On the graph, this is the vertical intercept of the function.
- b represents the initial vertical speed at which the object is launched.
- $-16t^2$ represents the effect of gravity, which pulls the object down.

On the graph, point out that one of the function's two zeros, when $h = 0$, is located before the object is launched at $t = 0$. This is because the object is launched from an initial height that is not zero. The other function's zero is located at the time t when the object falls back to the ground. The vertex of the function represents the maximum height of the object. An appropriate domain for this function is the span of time between when the object is launched and when it hits the ground.

Formalize vocabulary:

- ***vertex***
- ***zero***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

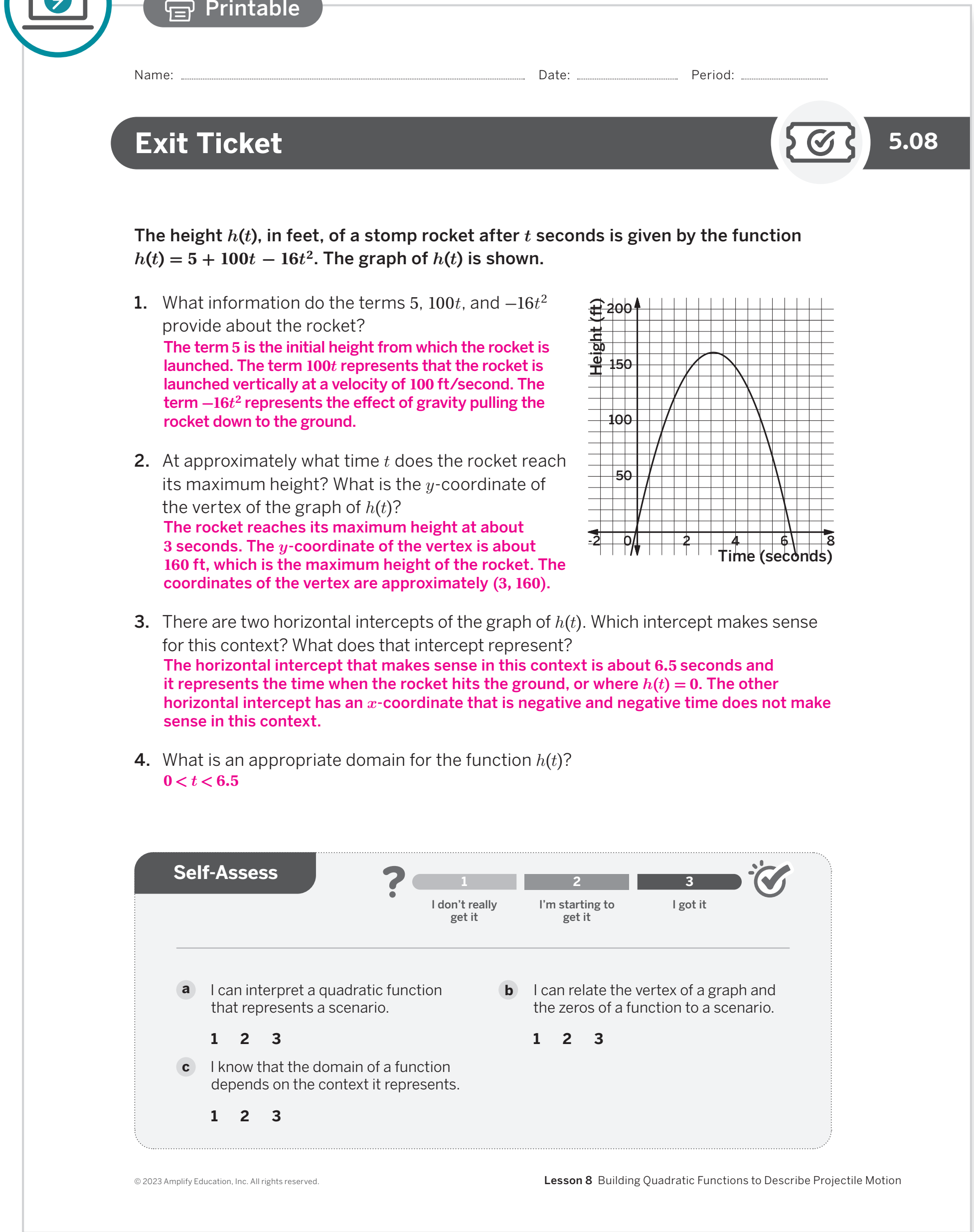
- "How did the initial speed and characteristics of the projectile's launch help you write a quadratic function to model it's height?"
- "What does it mean for the term that represents the effects of gravity to be negative?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by interpreting the parameters and graph of a quadratic function in context, including an appropriate domain.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.08

The height $h(t)$, in feet, of a stomp rocket after t seconds is given by the function $h(t) = 5 + 100t - 16t^2$. The graph of $h(t)$ is shown.

1. What information do the terms 5, $100t$, and $-16t^2$ provide about the rocket?
   The term 5 is the initial height from which the rocket is launched. The term $100t$ represents that the rocket is launched vertically at a velocity of 100 ft/second. The term $-16t^2$ represents the effect of gravity pulling the rocket down to the ground.
2. At approximately what time t does the rocket reach its maximum height? What is the y-coordinate of the vertex of the graph of $h(t)$?
   The rocket reaches its maximum height at about 3 seconds. The y-coordinate of the vertex is about 160 ft, which is the maximum height of the rocket. The coordinates of the vertex are approximately (3, 160).
3. There are two horizontal intercepts of the graph of $h(t)$. Which intercept makes sense for this context? What does that intercept represent?
   The horizontal intercept that makes sense in this context is about 6.5 seconds and it represents the time when the rocket hits the ground, or where $h(t) = 0$. The other horizontal intercept has an x-coordinate that is negative and negative time does not make sense in this context.
4. What is an appropriate domain for the function $h(t)$?
   $0 < t < 6.5$

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can interpret a quadratic function that represents a scenario.

1 2 3

b I can relate the vertex of a graph and the zeros of a function to a scenario.

1 2 3

c I know that the domain of a function depends on the context it represents.

1 2 3

Lesson 8 Building Quadratic Functions to Describe Projectile Motion

Success looks like . . .

- **Goal:** Creating graphs of quadratic functions that represent a physical phenomenon and determining an appropriate domain when graphing.
- **Language Goal:** Identifying and interpreting the meaning of the vertex of a graph and the zeros of a function represented in tables and graphs. **(Speaking and Listening, Reading and Writing)**
  - » Interpreting the vertex of the quadratic function in Problem 2 and determining the horizontal intercept that makes sense for the context in Problem 3.
- **Language Goal:** Writing and interpreting quadratic functions that represent a physical phenomenon. **(Speaking and Listening, Reading and Writing)**
  - » Interpreting the terms of the quadratic function in Problem 1.

Suggested next steps

If students do not correctly interpret the given parameters in Problem 1, consider:

- Assigning Practice Problems 1 and 3.

If students incorrectly identify the y-coordinate of the vertex in Problem 2 or the intercept that makes sense in context in Problem 3, consider:

- Reviewing Activity 2, Problem 2.
- Assigning Practice Problems 2 and 3.

If students do not give an appropriate domain for Problem 4, consider:

- Reviewing Activity 2, Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was writing and graphing quadratic functions to model projectile motion. How did writing and graphing these quadratic functions go?
- How did comparing the height of a projectile affected by gravity and the height of a projectile not affected by gravity help students develop a method to write functions that model projectile motion?

Math Language Development

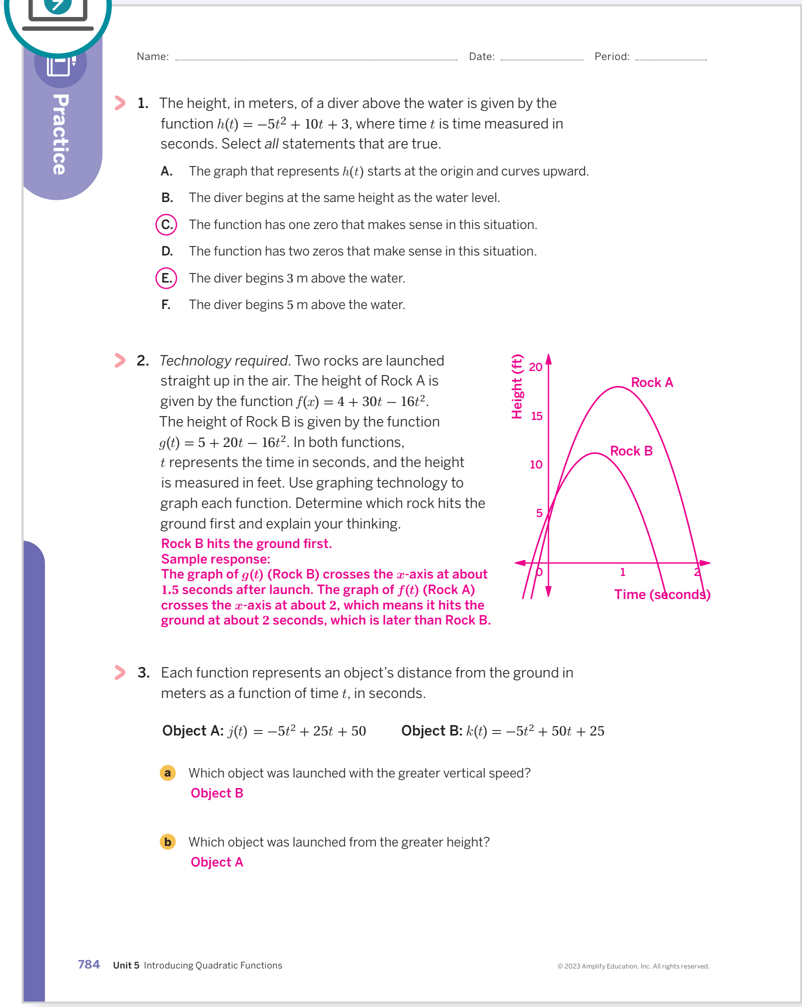
Language Goal: Identifying and interpreting the meaning of the vertex of a graph and the zeros of a function represented in tables and graphs.

Reflect on students' language development toward this goal.

- Do students' responses to Problems 2 and 3 of the Exit Ticket demonstrate they understand the meaning of the vertex and zeros (horizontal intercepts) of the graph, within context?
- Do students' responses to Problem 3 of the Exit Ticket demonstrate they understand that negative time does not make sense within this context? Are they connecting the negative x-coordinate with negative time?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. The height, in meters, of a diver above the water is given by the function $h(t) = -5t^2 + 10t + 3$, where time t is time measured in seconds. Select *all* statements that are true.
   - A. The graph that represents $h(t)$ starts at the origin and curves upward.
   - B. The diver begins at the same height as the water level.
   - [✓ C.] The function has one zero that makes sense in this situation.
   - D. The function has two zeros that make sense in this situation.
   - [✓ E.] The diver begins 3 m above the water.
   - F. The diver begins 5 m above the water.

2. *Technology required*. Two rocks are launched straight up in the air. The height of Rock A is given by the function $f(x) = 4 + 30t - 16t^2$. The height of Rock B is given by the function $g(t) = 5 + 20t - 16t^2$. In both functions, t represents the time in seconds, and the height is measured in feet. Use graphing technology to graph each function. Determine which rock hits the ground first and explain your thinking.

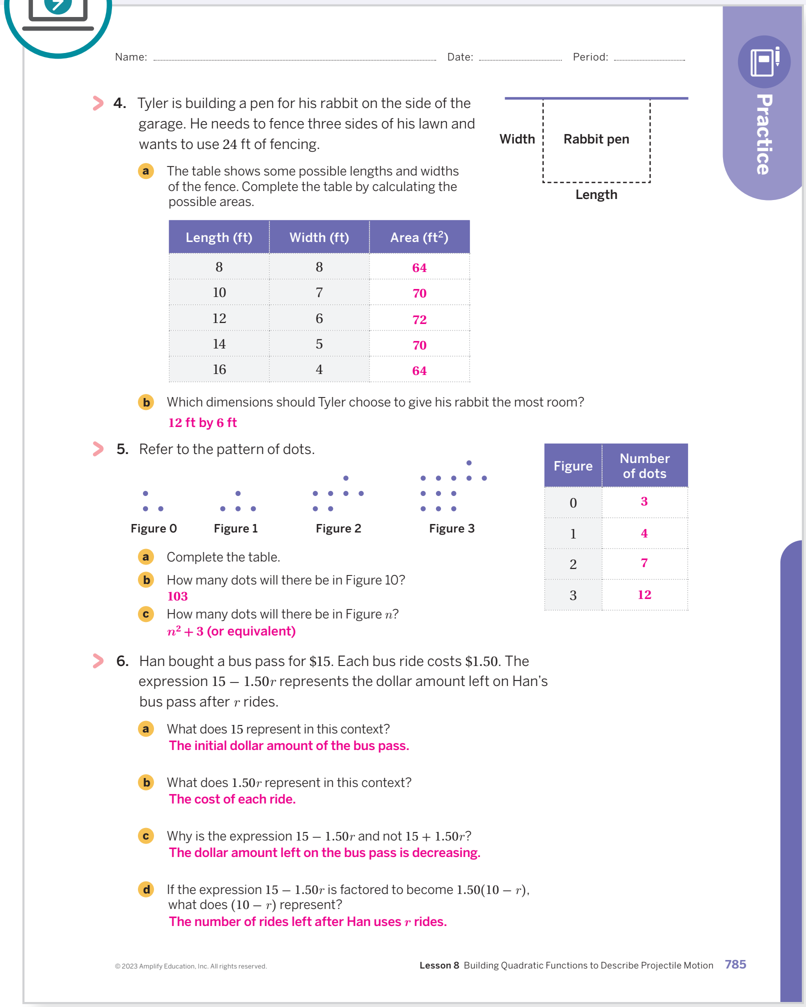
   Rock B hits the ground first.
   Sample response:
   The graph of $g(t)$ (Rock B) crosses the x-axis at about 1.5 seconds after launch. The graph of $f(t)$ (Rock A) crosses the x-axis at about 2, which means it hits the ground at about 2 seconds, which is later than Rock B.

3. Each function represents an object's distance from the ground in meters as a function of time t, in seconds.

   Object A: $j(t) = -5t^2 + 25t + 50$ **Object B:** $k(t) = -5t^2 + 50t + 25$

   - a Which object was launched with the greater vertical speed?
     Object B
   - b Which object was launched from the greater height?
     Object A

784 **Unit 5** Introducing Quadratic Functions



Name: ______ Date: ______ Period: ______

4. Tyler is building a pen for his rabbit on the side of the garage. He needs to fence three sides of his lawn and wants to use 24 ft of fencing.
   - a The table shows some possible lengths and widths of the fence. Complete the table by calculating the possible areas.

Length (ft)	Width (ft)	Area (ft^2)
8	8	**64**
10	7	**70**
12	6	**72**
14	5	**70**
16	4	**64**

   - b Which dimensions should Tyler choose to give his rabbit the most room?
     12 ft by 6 ft

5. Refer to the pattern of dots.
   - a Complete the table.
   - b How many dots will there be in Figure 10?
     103
   - c How many dots will there be in Figure n?
     $n^2 + 3$ (or equivalent)

Figure	Number of dots
0	**3**
1	**4**
2	**7**
3	**12**

6. Han bought a bus pass for \$15. Each bus ride costs \$1.50. The expression $15 - 1.50r$ represents the dollar amount left on Han's bus pass after r rides.
   - a What does 15 represent in this context?
     The initial dollar amount of the bus pass.
   - b What does $1.50r$ represent in this context?
     The cost of each ride.
   - c Why is the expression $15 - 1.50r$ and not $15 + 1.50r$?
     The dollar amount left on the bus pass is decreasing.
   - d If the expression $15 - 1.50r$ is factored to become $1.50(10 - r)$, what does $(10 - r)$ represent?
     The number of rides left after Han uses r rides.

Lesson 8 Building Quadratic Functions to Describe Projectile Motion 785

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 2	1
	5	Unit 5 Lesson 3	2
Formative ⚡	6	Unit 5 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 9**

Building Quadratic Functions to Maximize Revenue

Let's study how to maximize revenue.

Focus

Goals

1. **Language Goal:** Determine a domain that makes sense for the given context. **(Reading and Writing)**
2. Model revenue with quadratic functions and graphs.
3. **Language Goal:** Relate key features of a quadratic function, (vertex, zeros, domain), to a revenue context. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students continue to build their understanding of quadratic functions by applying a revenue context. They use tables and equations and consider graphs to understand and model revenue. They interpret key features of a quadratic graph in the context of revenue.

Previously

In Lessons 7 and 8, students developed quadratic functions to model and understand falling objects and projectile motion.

Coming Soon

In Lesson 10, students will extend their understanding of equivalent quadratics by relating each factor to the side lengths of a rectangle.

Rigor

- Students strengthen their **fluency** in identifying key features of quadratic functions.
- Students **apply** their understanding of quadratic functions to study maximum revenue.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	12 min	12 min	5 min	5 min
Independent	Independent	Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Graph of Revenue vs. Price*
- graphing technology (as needed)

Math Language Development

Review words

- *discrete*
- *horizontal intercept*
- *revenue*
- *vertex*
- *vertical intercept*

Amps | Featured Activity

Activity 1
Maximizing Revenue

To track their work and determine the maximum revenue for a company.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel ill-equipped to work on solutions for business challenges and that might raise their stress levels. Ask students to identify how they can make constructive decisions about their behaviors that can lead to a better understanding of the mathematics behind business practices.

Modifications to Pacing

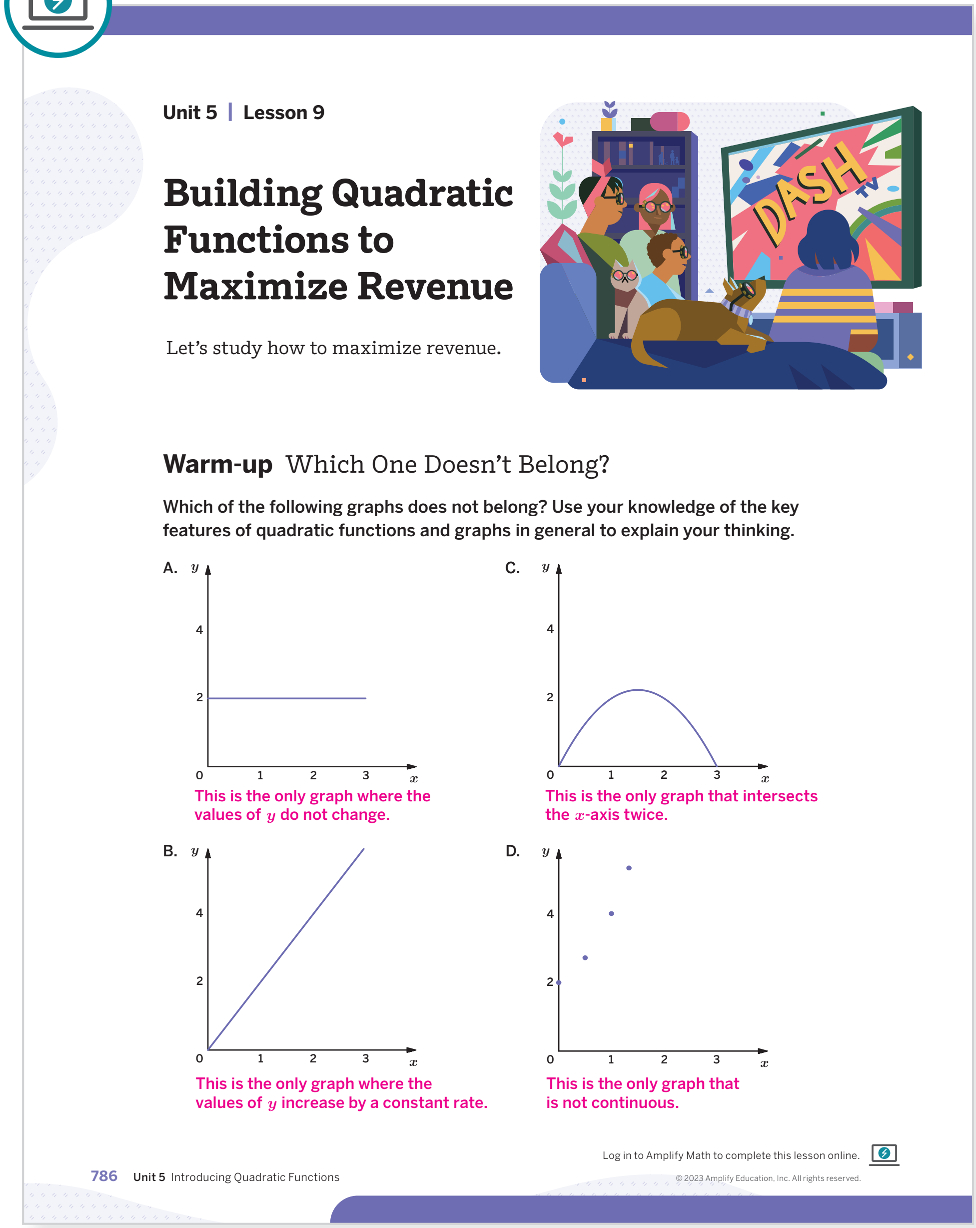
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Graph D may be omitted.
- In **Activity 2**, students may omit three rows of the table.
- In **Activity 3**, Problem 3 may be omitted.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students analyze and compare features of graphs to build language capacity using mathematical terminology and to prepare them for studying quadratics in the context of revenue.



Unit 5 | Lesson 9

Building Quadratic Functions to Maximize Revenue

Let's study how to maximize revenue.

Warm-up Which One Doesn't Belong?

Which of the following graphs does not belong? Use your knowledge of the key features of quadratic functions and graphs in general to explain your thinking.

A. This is the only graph where the values of y do not change.

C. This is the only graph that intersects the x-axis twice.

B. This is the only graph where the values of y increase by a constant rate.

D. This is the only graph that is not continuous.

Log in to Amplify Math to complete this lesson online.

786 Unit 5 Introducing Quadratic Functions

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the *Which One Doesn't Belong?* routine. Provide one minute of think-time before students share their thinking with a partner.

2 Monitor

Help students get started by displaying the terms *rate of change*, *intercepts*, *continuous*, and *discrete*. Encourage students to use precise mathematical vocabulary to describe why each graph doesn't belong.

Look for points of confusion:

- **Identifying Choice D as exponential.** Although the graph may appear exponential, students would need a table of values or the function to confirm it.
- **Making baseless claims.** Challenge students to provide more information to support their claim.

Look for productive strategies:

- Using the intercepts to support their claim.
- Using the terms *quadratic, exponential,* and *linear growth* in their explanation.
- Talking about how the functions change, such as increasing, no change, increasing and then decreasing.

3 Connect

Display the four graphs.

Have individual students share why each graph doesn't belong. After each response, ask the class whether they agree or disagree.

Highlight that Graph C is the only graph that has a maximum value and two x-intercepts.

Ask, "Which graph would you choose to model money earned? Why?"

Power-up

To power up students' ability to analyze a linear expression in context, have students complete:

Diego has a gift card worth \$25. He decides to use it to pay for a monthly music subscription that costs \$2 per month. The total on his gift card can be represented by the expression $25 - 2m$. Match each part of the expression with what it represents.

a. 25	d	The total amount of money remaining on his gift card.
b. 2	c	The number of months he used his gift card.
c. m	a	The original value of the gift card.
d. $25 - 2m$	b	The amount he spends per month.

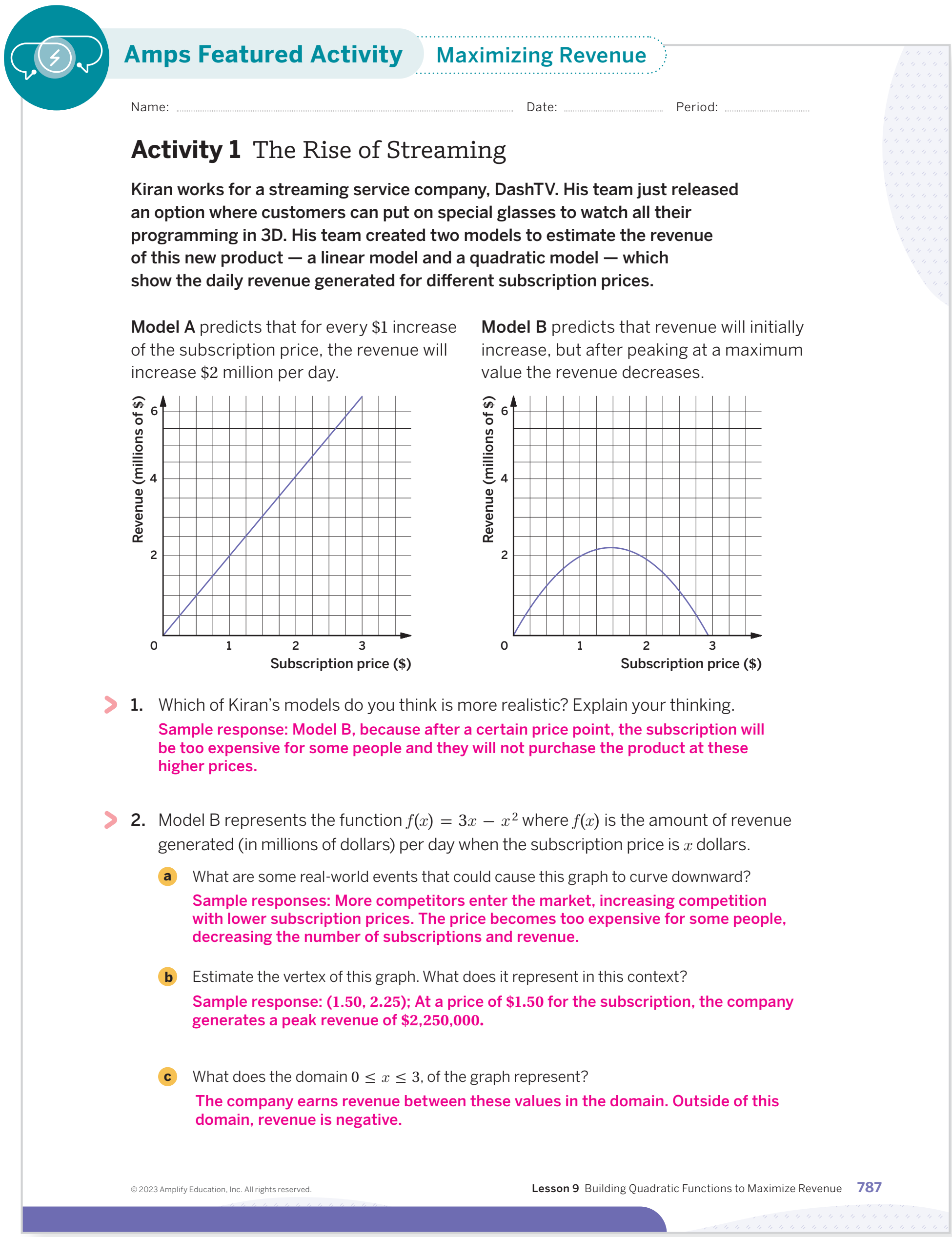
Use: Before Activity 1

Informed by: Performance on Lesson 9, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Independent | 10 min

Activity 1 The Rise of Streaming

Students analyze two functions to understand that revenue is modeled using quadratic functions and engage in modeling to make a business recommendation.



Amps Featured Activity Maximizing Revenue

Name: ________ Date: ________ Period: ________

Activity 1 The Rise of Streaming

Kiran works for a streaming service company, DashTV. His team just released an option where customers can put on special glasses to watch all their programming in 3D. His team created two models to estimate the revenue of this new product — a linear model and a quadratic model — which show the daily revenue generated for different subscription prices.

Model A predicts that for every $1 increase of the subscription price, the revenue will increase $2 million per day.

Model B predicts that revenue will initially increase, but after peaking at a maximum value the revenue decreases.

1. Which of Kiran's models do you think is more realistic? Explain your thinking.
   Sample response: Model B, because after a certain price point, the subscription will be too expensive for some people and they will not purchase the product at these higher prices.

2. Model B represents the function $f(x) = 3x - x^2$ where $f(x)$ is the amount of revenue generated (in millions of dollars) per day when the subscription price is x dollars.
   - a What are some real-world events that could cause this graph to curve downward?
     Sample responses: More competitors enter the market, increasing competition with lower subscription prices. The price becomes too expensive for some people, decreasing the number of subscriptions and revenue.
   - b Estimate the vertex of this graph. What does it represent in this context?
     Sample response: (1.50, 2.25); At a price of $1.50 for the subscription, the company generates a peak revenue of $2,250,000.
   - c What does the domain $0 \leq x \leq 3$, of the graph represent?
     The company earns revenue between these values in the domain. Outside of this domain, revenue is negative.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Building Quadratic Functions to Maximize Revenue 787

1 Launch

Read the narrative aloud. Tell students that revenue is the amount of money that is received from selling goods or services over a period of time. Have students discuss Problem 1 with a partner before completing the activity independently.

2 Monitor

Help students get started by asking, "What are some reasons a company loses revenue?"

Look for points of confusion:

- **Interpreting the linear model as revenue.** Point out that the horizontal axis represents the price of any item. Ask, "Do you think customers would (or could) buy a specific item at any price?"

Look for productive strategies:

- Using the vertex and zeros of Model B to support the decrease in revenue.

3 Connect

Display each model. Use the ***Poll the Class*** routine to determine which model students think is realistic.

Have individual students share the model they chose and why. Choose supporters of each model to share with the class.

Highlight that for any product, there is a selling price where the product supply meets the product demand. Past this point, the selling price is too high for some consumers, effectively decreasing product demand. The revenue decreases past this point.

Ask, "Think of some similar real-world examples. What price for cars, food, or clothing would you consider to be so expensive, that no one (or very few customers) would buy them?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use tables and digital sketches to determine the maximum revenue for the company.

Extension: *Math Enrichment*

Have students explain what they think an exponential decay model would imply about the revenue in this context. Sample response: For every increase in price, the revenue would decrease by a given percent. The company will always make less and less revenue as the price increases, although never reaching zero revenue.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that there are two revenue models that show the estimated revenue earned for different subscription prices of the new product.
- **Read 2:** Ask students to study the descriptions and graphs for each model and identify the given quantities or relationships.
- **Read 3:** Ask students to describe what each graph shows in terms of the change in revenue.

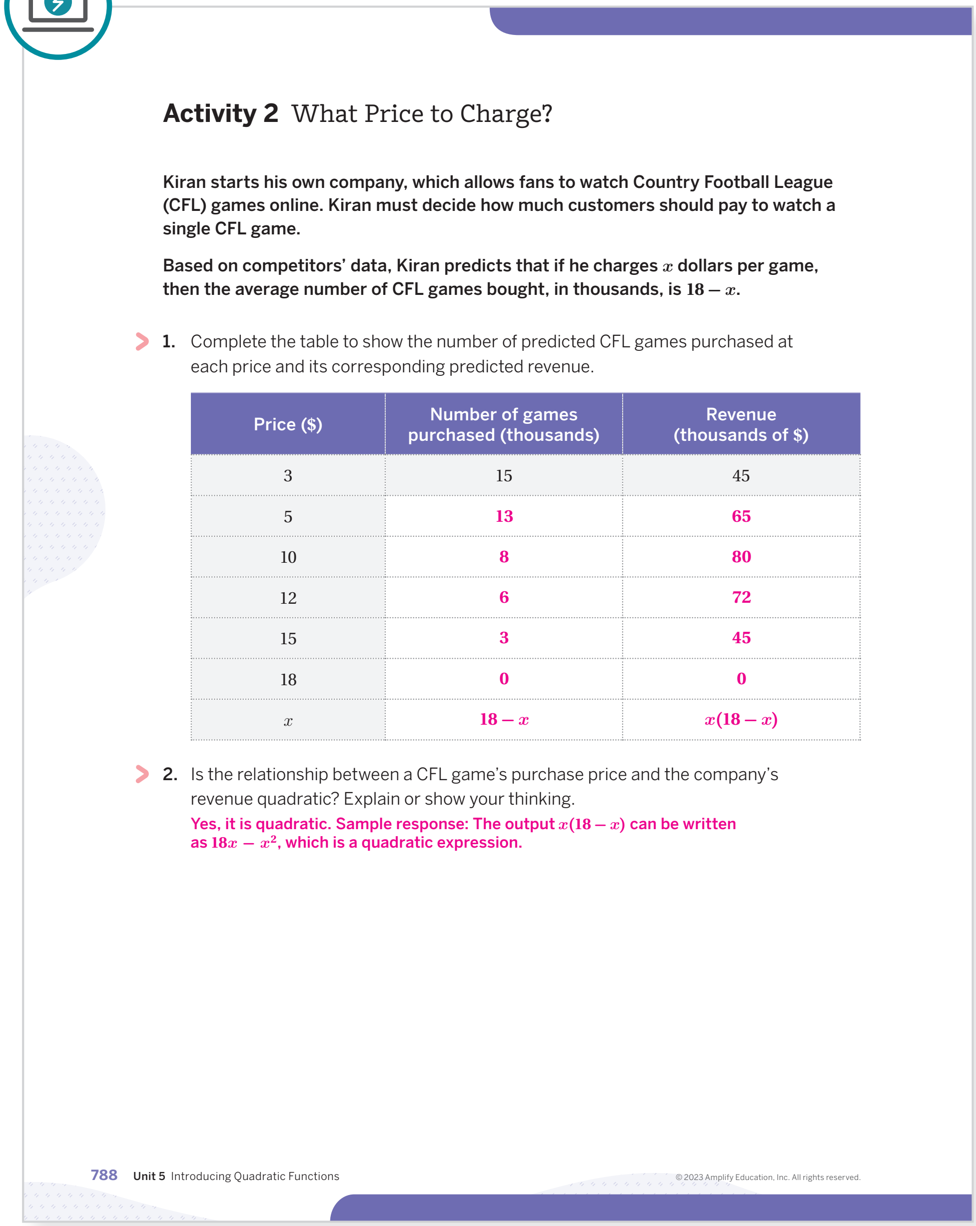
English Learners

Clarify the meaning of the term *revenue*. Let students know it means the amount of money earned by the sale of the product.

Independent | 12 min

Activity 2 What Price to Charge?

Students study a table of values to build a model, create a graph to further understand the relationship, and use their model to make a business recommendation.



Activity 2 What Price to Charge?

Kiran starts his own company, which allows fans to watch Country Football League (CFL) games online. Kiran must decide how much customers should pay to watch a single CFL game.

Based on competitors' data, Kiran predicts that if he charges x dollars per game, then the average number of CFL games bought, in thousands, is $18 - x$.

1. Complete the table to show the number of predicted CFL games purchased at each price and its corresponding predicted revenue.

Price ($)	Number of games purchased (thousands)	Revenue (thousands of $)
3	15	45
5	**13**	**65**
10	**8**	**80**
12	**6**	**72**
15	**3**	**45**
18	**0**	**0**
x	**$18 - x$**	**$x(18 - x)$**

2. Is the relationship between a CFL game's purchase price and the company's revenue quadratic? Explain or show your thinking.
   Yes, it is quadratic. Sample response: The output $x(18 - x)$ can be written as $18x - x^2$, which is a quadratic expression.

1 Launch

Activate students' background knowledge by discussing business earnings and profits. Ask students to predict the number of games purchased as the selling price increases or decreases.

2 Monitor

Help students get started by displaying, *revenue = price • number of games purchased.*

Look for points of confusion:

- **Considering each expression separately and determining the revenue relationship is linear.** Have students substitute the price into the revenue expression, and then distribute.

Look for productive strategies:

- Interpreting the revenue relationship as quadratic.
- Using the Distributive Property to show a squared term in the expression.
- Calculating the second differences to show the relationship is quadratic.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Prior Knowledge*

Remind students that *revenue* means the money earned, typically from the sale of a product or service. Use a think-aloud to demonstrate how the first row of the table was completed. For example:

- "If the price is $3 per game, then the average number of games purchased is $18 - 3$, or 15."
- "If 15 thousand games are purchased at $3 per game, I can multiply these to determine the revenue."

Accessibility: *Optimize Access to Technology*

Have students use graphing technology to graph the points in Problem 3.

Math Language Development

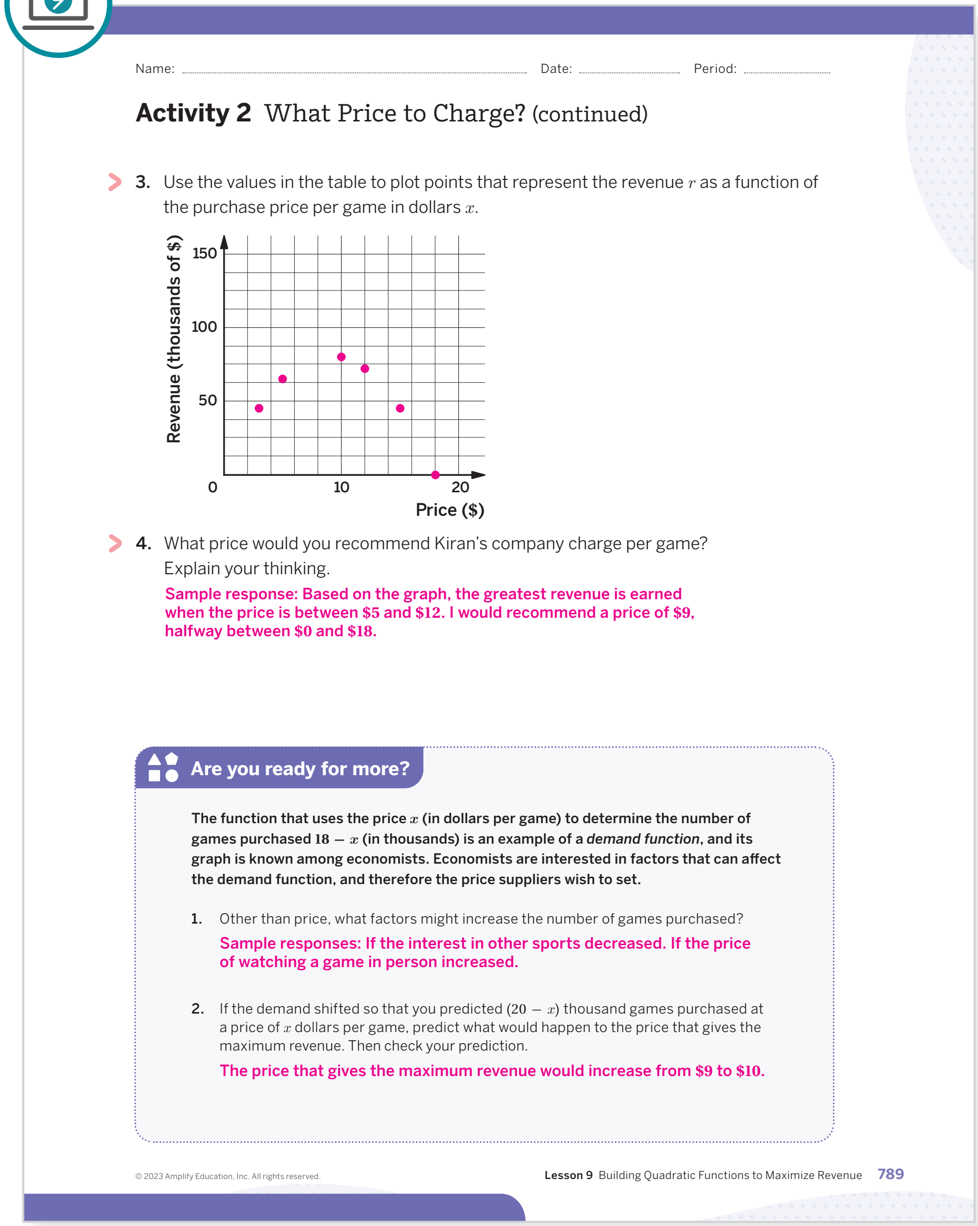
MLR8: Discussion Supports — Press for Details

During the Connect, as students share how they determined whether the relationship was quadratic, press them for details in their reasoning. For example:

If a student says . . .	Ask . . .
"There are two xs in the expression for the last row in the table, so it is a quadratic expression."	"When you say there are two xs, what do you mean? Would the expression $x + x$ be a quadratic expression? Why or why not?"

Activity 2 What Price to Charge? (continued)

Students study a table of values to build a model, create a graph to further understand the relationship, and use their model to make a business recommendation.



Name: ______ Date: ______ Period: ______

Activity 2 What Price to Charge? (continued)

3. Use the values in the table to plot points that represent the revenue r as a function of the purchase price per game in dollars x.

4. What price would you recommend Kiran's company charge per game? Explain your thinking.

   Sample response: Based on the graph, the greatest revenue is earned when the price is between \$5 and \$12. I would recommend a price of \$9, halfway between \$0 and \$18.

Are you ready for more?

The function that uses the price x (in dollars per game) to determine the number of games purchased $18 - x$ (in thousands) is an example of a *demand function*, and its graph is known among economists. Economists are interested in factors that can affect the demand function, and therefore the price suppliers wish to set.

1. Other than price, what factors might increase the number of games purchased?

   Sample responses: If the interest in other sports decreased. If the price of watching a game in person increased.

2. If the demand shifted so that you predicted $(20 - x)$ thousand games purchased at a price of x dollars per game, predict what would happen to the price that gives the maximum revenue. Then check your prediction.

   The price that gives the maximum revenue would increase from \$9 to \$10.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Building Quadratic Functions to Maximize Revenue 789

3 Connect

Display the completed table.

Have individual students share how they determined whether the relationship was quadratic. Select and sequence by productive strategies. If any students sketched continuous graphs, have them share their graphs.

Highlight that while students plotted a series of points from the table, the revenue curve is continuous, not discrete, because the price can be any amount between \$0 and \$18.

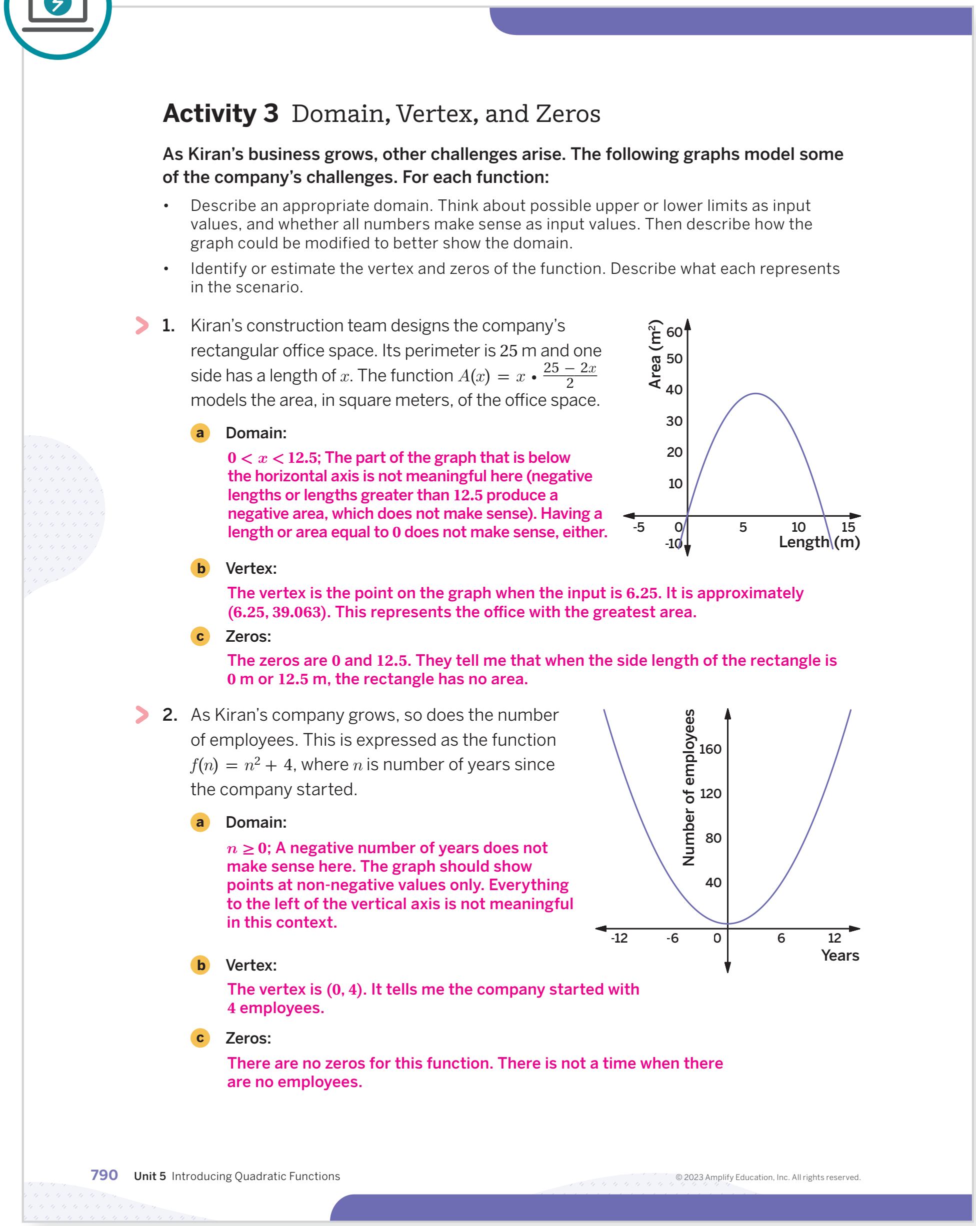
Ask:

- "Is it possible for Kiran's company to not make any money? How do you know?" Yes, it is possible for Kiran's company to not make any money. I can tell this by looking at the graph and identifying the x-intercepts, where the revenue is zero.
- "How can you determine at what price Kiran's company makes the greatest revenue?" Sample response: I can calculate the revenue at other prices that were not originally in the table and see which price gives the greatest revenue. The graph also gives a hint, but I may need to plot a few more points to estimate the maximum point from the graph.

Small Groups | 12 min

Activity 3 Domain, Vertex, and Zeros

Students examine four business challenges, determine an appropriate domain, vertex. and zeros for each — to better understand key features of the graphs of quadratic functions.



Activity 3 Domain, Vertex, and Zeros

As Kiran's business grows, other challenges arise. The following graphs model some of the company's challenges. For each function:

- Describe an appropriate domain. Think about possible upper or lower limits as input values, and whether all numbers make sense as input values. Then describe how the graph could be modified to better show the domain.
- Identify or estimate the vertex and zeros of the function. Describe what each represents in the scenario.

1. Kiran's construction team designs the company's rectangular office space. Its perimeter is 25 m and one side has a length of x. The function $A(x) = x \bullet \frac{25 - 2x}{2}$ models the area, in square meters, of the office space.

a Domain:

$0 < x < 12.5$; The part of the graph that is below the horizontal axis is not meaningful here (negative lengths or lengths greater than 12.5 produce a negative area, which does not make sense). Having a length or area equal to 0 does not make sense, either.

b Vertex:

The vertex is the point on the graph when the input is 6.25. It is approximately $(6.25, 39.063)$. This represents the office with the greatest area.

c Zeros:

The zeros are 0 and 12.5. They tell me that when the side length of the rectangle is 0 m or 12.5 m, the rectangle has no area.

2. As Kiran's company grows, so does the number of employees. This is expressed as the function $f(n) = n^2 + 4$, where n is number of years since the company started.

a Domain:

$n \geq 0$; A negative number of years does not make sense here. The graph should show points at non-negative values only. Everything to the left of the vertical axis is not meaningful in this context.

b Vertex:

The vertex is $(0, 4)$. It tells me the company started with 4 employees.

c Zeros:

There are no zeros for this function. There is not a time when there are no employees.

790 Unit 5 Introducing Quadratic Functions

1 Launch

Have students first read the introduction independently, then read together as a class. Call on students to summarize the directions.

2 Monitor

Help students get started by suggesting they use the horizontal intercepts to help determine the domain of the graph.

Look for points of confusion:

- **Interpreting zeros as ordered pairs.** Point out that a zero is a value. It is the x-coordinate of the ordered pair where the function crosses the x-axis.
- **Struggling to adjust the domain.** Ask, "Do negative values make sense for the domain in this scenario? Why or why not? For what values of the domain would the scenario make sense?"
- **Misinterpreting the context's graph.** Highlight what each variable represents as well as the labels on the axes. Select 1 or 2 points on the function and ask students to interpret those points within the context.

Look for productive strategies:

- Labeling the x-intercept and vertex on each graph.
- Creating a table of values to better understand the function.
- Highlighting or circling the part of the graph that determines the appropriate domain, in context.

Activity 3 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

Remind students they previously learned about the domain of a function. Display the graph in Problem 1 and ask, "Can the values along the x-axis take on any value within this context? Why or why not? What values would be meaningful? What values would not be meaningful?"

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose two of the four problems to complete. Allowing them the power of choice can increase their engagement and ownership of the task.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the four graphs, along with their respective functions. Draw student's attention to how the graphs compare, particularly in how some graphs open upward and other graphs open downward. Ask:

- "What do you notice about the equations of the functions whose graphs open upward? Downward?"
- "Why does it make sense that the graph in Problem 4 would open downward? Where have you seen this squared term before?"

Model the use of precise mathematical language by using language, such as "the sign of the coefficient."

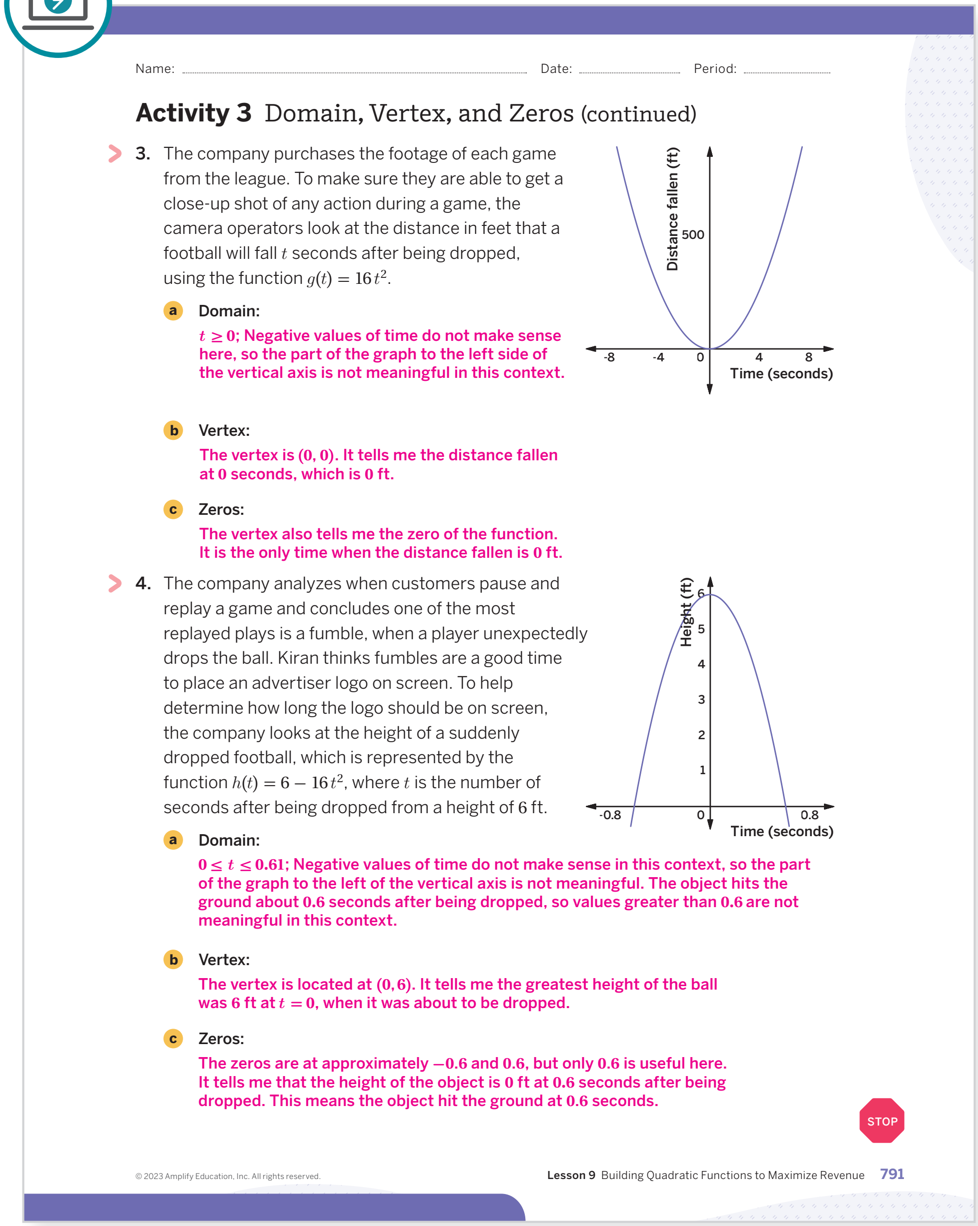
English Learners

Annotate the graphs and functions with "opens upward/downward" and "positive/negative coefficient."

Small Groups | 12 min

Activity 3 Domain, Vertex, and Zeros (continued)

Students examine four business challenges, determine an appropriate domain, vertex. and zeros for each — to better understand key features of the graphs of quadratic functions.



Name: ______ Date: ______ Period: ______

Activity 3 Domain, Vertex, and Zeros (continued)

3. The company purchases the footage of each game from the league. To make sure they are able to get a close-up shot of any action during a game, the camera operators look at the distance in feet that a football will fall t seconds after being dropped, using the function $g(t) = 16t^2$.
   - a Domain:
     $t \geq 0$; Negative values of time do not make sense here, so the part of the graph to the left side of the vertical axis is not meaningful in this context.
   - b Vertex:
     The vertex is $(0, 0)$. It tells me the distance fallen at 0 seconds, which is 0 ft.
   - c Zeros:
     The vertex also tells me the zero of the function. It is the only time when the distance fallen is 0 ft.

4. The company analyzes when customers pause and replay a game and concludes one of the most replayed plays is a fumble, when a player unexpectedly drops the ball. Kiran thinks fumbles are a good time to place an advertiser logo on screen. To help determine how long the logo should be on screen, the company looks at the height of a suddenly dropped football, which is represented by the function $h(t) = 6 - 16t^2$, where t is the number of seconds after being dropped from a height of 6 ft.
   - a Domain:
     $0 \leq t \leq 0.61$; Negative values of time do not make sense in this context, so the part of the graph to the left of the vertical axis is not meaningful. The object hits the ground about 0.6 seconds after being dropped, so values greater than 0.6 are not meaningful in this context.
   - b Vertex:
     The vertex is located at $(0, 6)$. It tells me the greatest height of the ball was 6 ft at $t = 0$, when it was about to be dropped.
   - c Zeros:
     The zeros are at approximately -0.6 and 0.6, but only 0.6 is useful here. It tells me that the height of the object is 0 ft at 0.6 seconds after being dropped. This means the object hit the ground at 0.6 seconds.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Building Quadratic Functions to Maximize Revenue 791

3 Connect

Display each scenario.

Have groups of students share how they determined the domain for each scenario.

Highlight that the graphs of quadratic functions may or may not show the vertex and other key features, depending on the domain.

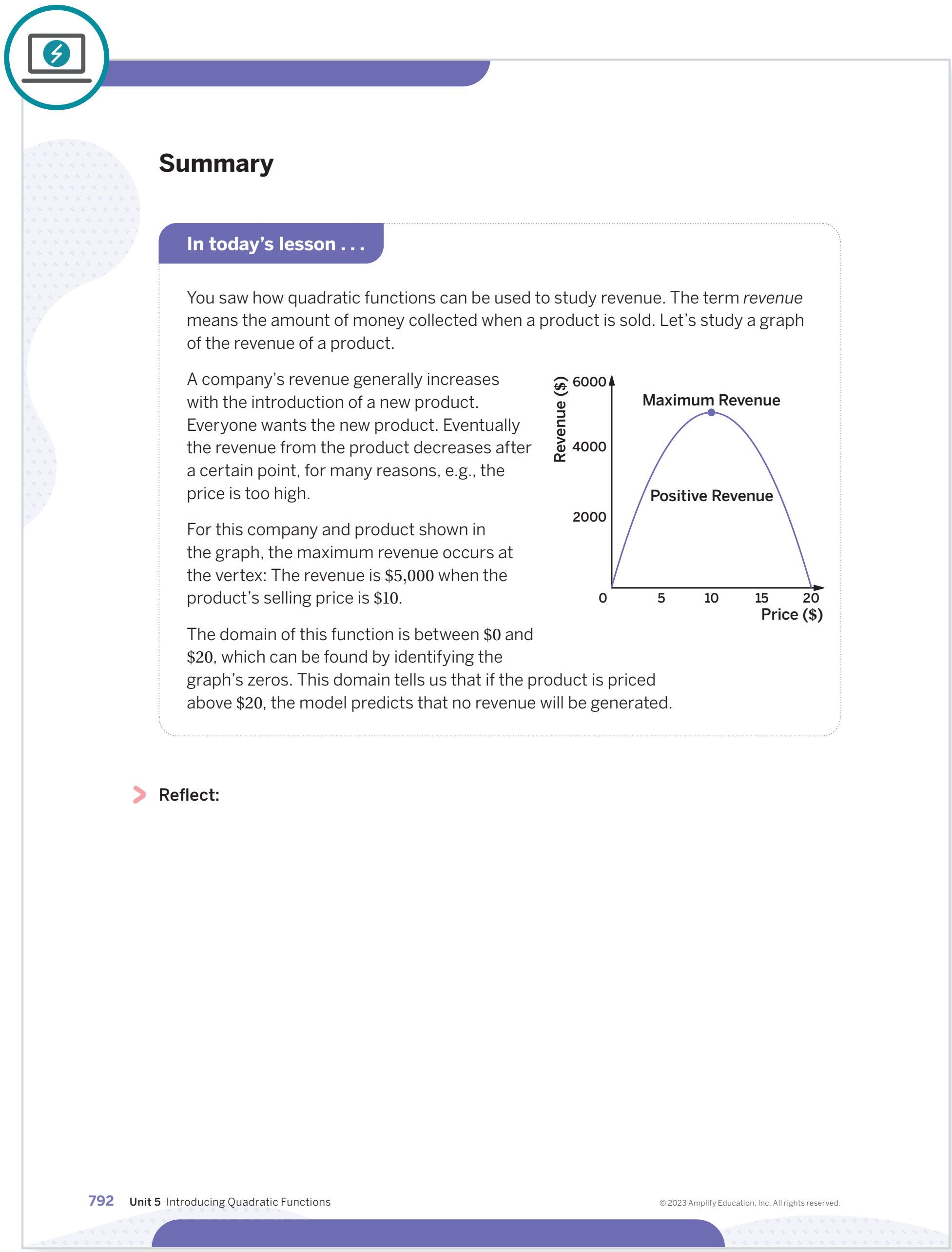
Ask:

- "Will the graph of a quadratic function always have an x-intercept?" No, for example, an upward-facing graph could have a vertex (minimum) above the horizontal axis, and therefore not have an x-intercept.
- "What does the vertex represent in each scenario?" The maximum or minimum function value.
- "Problems 3 and 4 both involve falling objects. Why do the graphs open in opposite directions?" The function in Problem 3 represents the distance a falling object has traveled, which will increase as the object falls. The function in Problem 4 represents the height of a falling object, or the distance from the ground, which will decrease as the object falls.

Whole Class | 5 min

Summary

Review and synthesize how quadratic functions are used to study revenue.



Summary

In today's lesson . . .

You saw how quadratic functions can be used to study revenue. The term *revenue* means the amount of money collected when a product is sold. Let's study a graph of the revenue of a product.

A company's revenue generally increases with the introduction of a new product. Everyone wants the new product. Eventually the revenue from the product decreases after a certain point, for many reasons, e.g., the price is too high.

For this company and product shown in the graph, the maximum revenue occurs at the vertex: The revenue is $5,000 when the product's selling price is $10.

The domain of this function is between $0 and $20, which can be found by identifying the graph's zeros. This domain tells us that if the product is priced above $20, the model predicts that no revenue will be generated.

Reflect:

792 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the Anchor Chart PDF, *Graph of Revenue vs. Price*.

Have students share the domain, vertex, and zeros of the graph.

Highlight that the revenue of a product will increase until the price hits a value that is too high and begins to deter people from purchasing the product. As the price increases past this point, less and less people are willing to purchase the product.

Ask:

- "What do the zeros tell you about this scenario?" The price where no revenue is generated from selling the item or service.

  "What does the vertex tell you about this scenario?" The maximum revenue that can be generated is $500 when the price is $5.
- "How can you determine an appropriate domain?" Use the zeros of the function and the context of the scenario. It does not make sense to have the price of a product be negative dollars, nor will the price increase past $10.
- "What are some other real-world contexts in which it does not make sense to have negative values in the domain?" Sample response: If the age of a person is the domain, it would not make sense to have negative age values in the domain.

Reflect

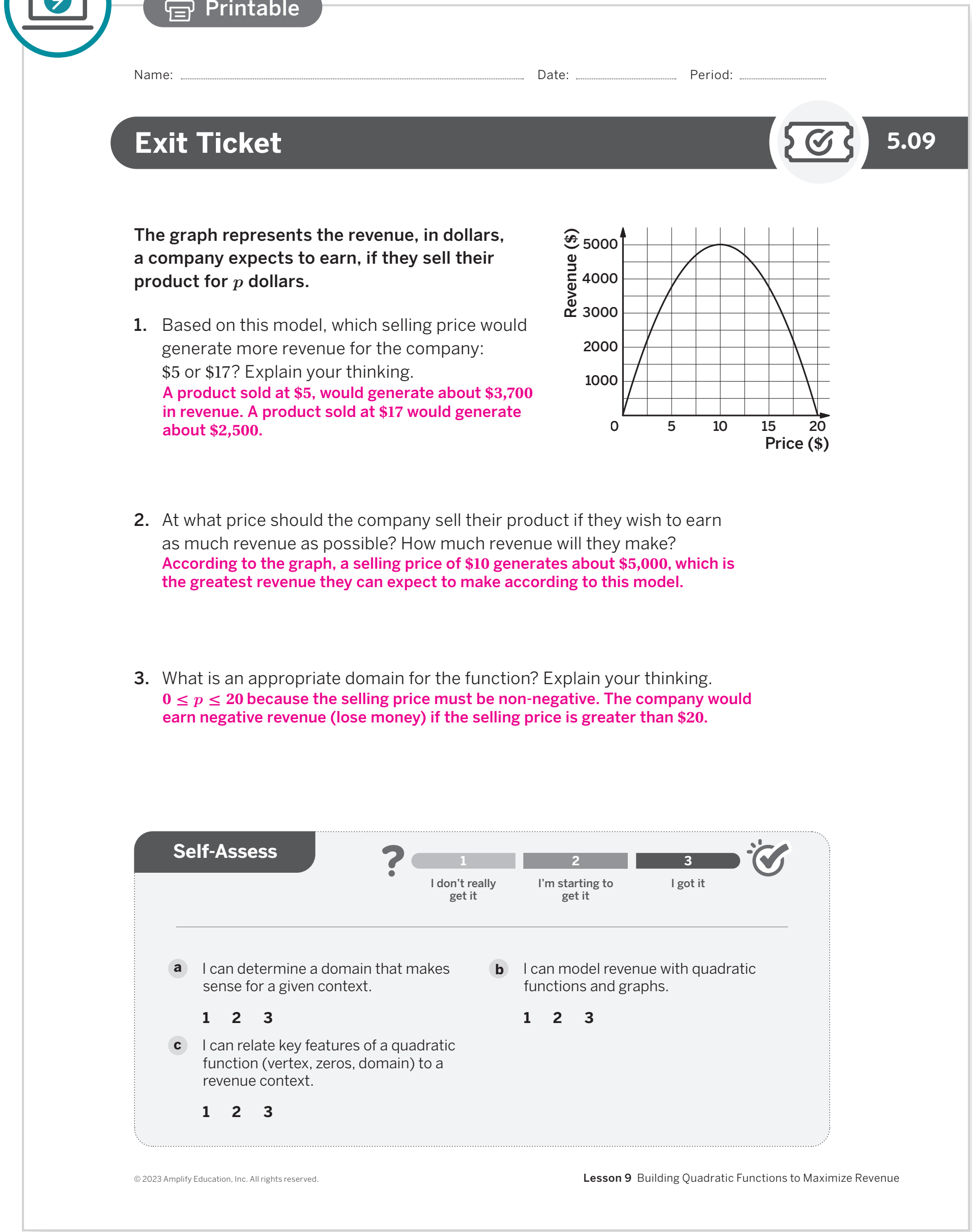
After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are quadratic functions used to model, analyze, and interpret mathematical relationships?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using a quadratic model to interpret the meaning of the vertex and zeros in a revenue context and to make a business recommendation.



Name: ________ Date: ________ Period: ________

Exit Ticket 5.09

The graph represents the revenue, in dollars, a company expects to earn, if they sell their product for p dollars.

1. Based on this model, which selling price would generate more revenue for the company: $5 or $17? Explain your thinking.
   A product sold at $5, would generate about $3,700 in revenue. A product sold at $17 would generate about $2,500.

2. At what price should the company sell their product if they wish to earn as much revenue as possible? How much revenue will they make?
   According to the graph, a selling price of $10 generates about $5,000, which is the greatest revenue they can expect to make according to this model.

3. What is an appropriate domain for the function? Explain your thinking.
   $0 \leq p \leq 20$ because the selling price must be non-negative. The company would earn negative revenue (lose money) if the selling price is greater than $20.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can determine a domain that makes sense for a given context.

1 2 3

b I can model revenue with quadratic functions and graphs.

1 2 3

c I can relate key features of a quadratic function (vertex, zeros, domain) to a revenue context.

1 2 3

Lesson 9 Building Quadratic Functions to Maximize Revenue

Success looks like . . .

- **Language Goal:** Determining a domain that makes sense for the given context. **(Reading and Writing)**
- **Goal:** Modeling revenue with quadratic functions and graphs.
- **Language Goal:** Relating key features of a quadratic function, (vertex, zeros, domain), to a revenue context. **(Speaking and Listening, Reading and Writing)**
  - » Determining the price for the maximum revenue from the graph of the quadratic function in Problem 2.

Suggested next steps

If students choose $17 for Problem 1, consider:

- Reviewing plotting table values in Activity 1.
- Assigning Practice Problems 1 and 2.
- Having students sketch on the graph where the price is $5 and $17, and writing a sentence using these points in context.

If students misidentify the vertex for Problem 2, consider:

- Reviewing determining the vertex of each scenario in Activity 2.
- Assigning Practice Problems 1 and 2.
- Having students mark the maximum revenue on the graph and label its coordinates in context.

If students do not use zeros to identify an appropriate domain for Problem 3, consider:

- Reviewing how to determine the domain of each scenario in Activity 2.
- Assigning Practice Problems 1 and 2.
- Having students identify the zeros on the graph, and asking, "At which prices will the company make no revenue?"

Professional Learning

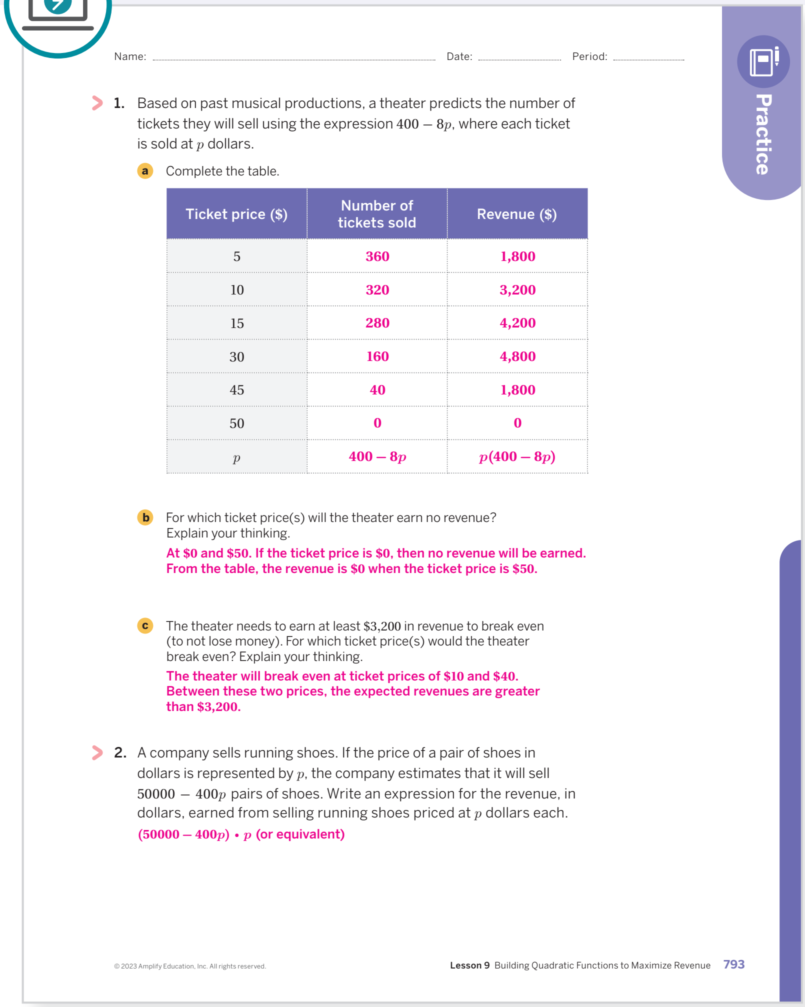
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students modeled revenue with quadratic functions and graphs. How did that build on the earlier work students did with quadratic relationships?
- How were students' interpretations of key features of quadratic graphs similar to or different from their graphing of quadratic functions in previous lessons?

Practice

Independent



Name: Date: Period:

Practice

1. Based on past musical productions, a theater predicts the number of tickets they will sell using the expression $400 - 8p$, where each ticket is sold at p dollars.

a Complete the table.

Ticket price ($)	Number of tickets sold	Revenue ($)
5	**360**	**1,800**
10	**320**	**3,200**
15	**280**	**4,200**
30	**160**	**4,800**
45	**40**	**1,800**
50	**0**	**0**
p	$\mathbf{400 - 8p}$	$\mathbf{p(400 - 8p)}$

b For which ticket price(s) will the theater earn no revenue? Explain your thinking.

At $0 and $50. If the ticket price is $0, then no revenue will be earned. From the table, the revenue is $0 when the ticket price is $50.

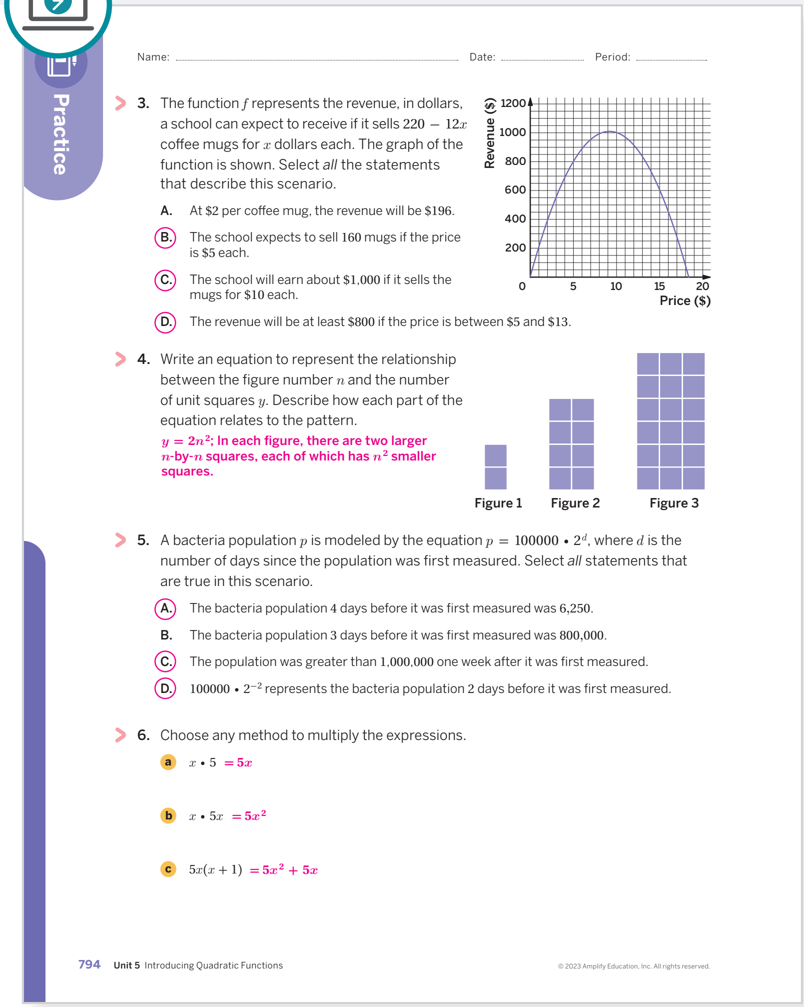
c The theater needs to earn at least $3,200 in revenue to break even (to not lose money). For which ticket price(s) would the theater break even? Explain your thinking.

The theater will break even at ticket prices of $10 and $40. Between these two prices, the expected revenues are greater than $3,200.

2. A company sells running shoes. If the price of a pair of shoes in dollars is represented by p, the company estimates that it will sell $50000 - 400p$ pairs of shoes. Write an expression for the revenue, in dollars, earned from selling running shoes priced at p dollars each.

$\mathbf{(50000 - 400p) \bullet p}$ **(or equivalent)**

Lesson 9 Building Quadratic Functions to Maximize Revenue 793



Name: Date: Period:

3. The function f represents the revenue, in dollars, a school can expect to receive if it sells $220 - 12x$ coffee mugs for x dollars each. The graph of the function is shown. Select *all* the statements that describe this scenario.

A. At $2 per coffee mug, the revenue will be $196.

[✓ **B.**] The school expects to sell 160 mugs if the price is $5 each.

[✓ **C.**] The school will earn about $1,000 if it sells the mugs for $10 each.

[✓ **D.**] The revenue will be at least $800 if the price is between $5 and $13.

4. Write an equation to represent the relationship between the figure number n and the number of unit squares y. Describe how each part of the equation relates to the pattern.

$\mathbf{y = 2n^2}$**; In each figure, there are two larger** $\mathbf{n}$**-by-**$\mathbf{n}$ **squares, each of which has** $\mathbf{n^2}$ **smaller squares.**

5. A bacteria population p is modeled by the equation $p = 100000 \bullet 2^d$, where d is the number of days since the population was first measured. Select *all* statements that are true in this scenario.

[✓ **A.**] The bacteria population 4 days before it was first measured was 6,250.

B. The bacteria population 3 days before it was first measured was 800,000.

[✓ **C.**] The population was greater than 1,000,000 one week after it was first measured.

[✓ **D.**] $100000 \bullet 2^{-2}$ represents the bacteria population 2 days before it was first measured.

6. Choose any method to multiply the expressions.

a $x \bullet 5$ $\mathbf{= 5x}$

b $x \bullet 5x$ $\mathbf{= 5x^2}$

c $5x(x + 1)$ $\mathbf{= 5x^2 + 5x}$

794 **Unit 5** Introducing Quadratic Functions

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	1
Spiral	4	Unit 5 Lesson 5	2
	5	Unit 4 Lesson 4	2
Formative ⚡	6	Unit 5 Lesson 10	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

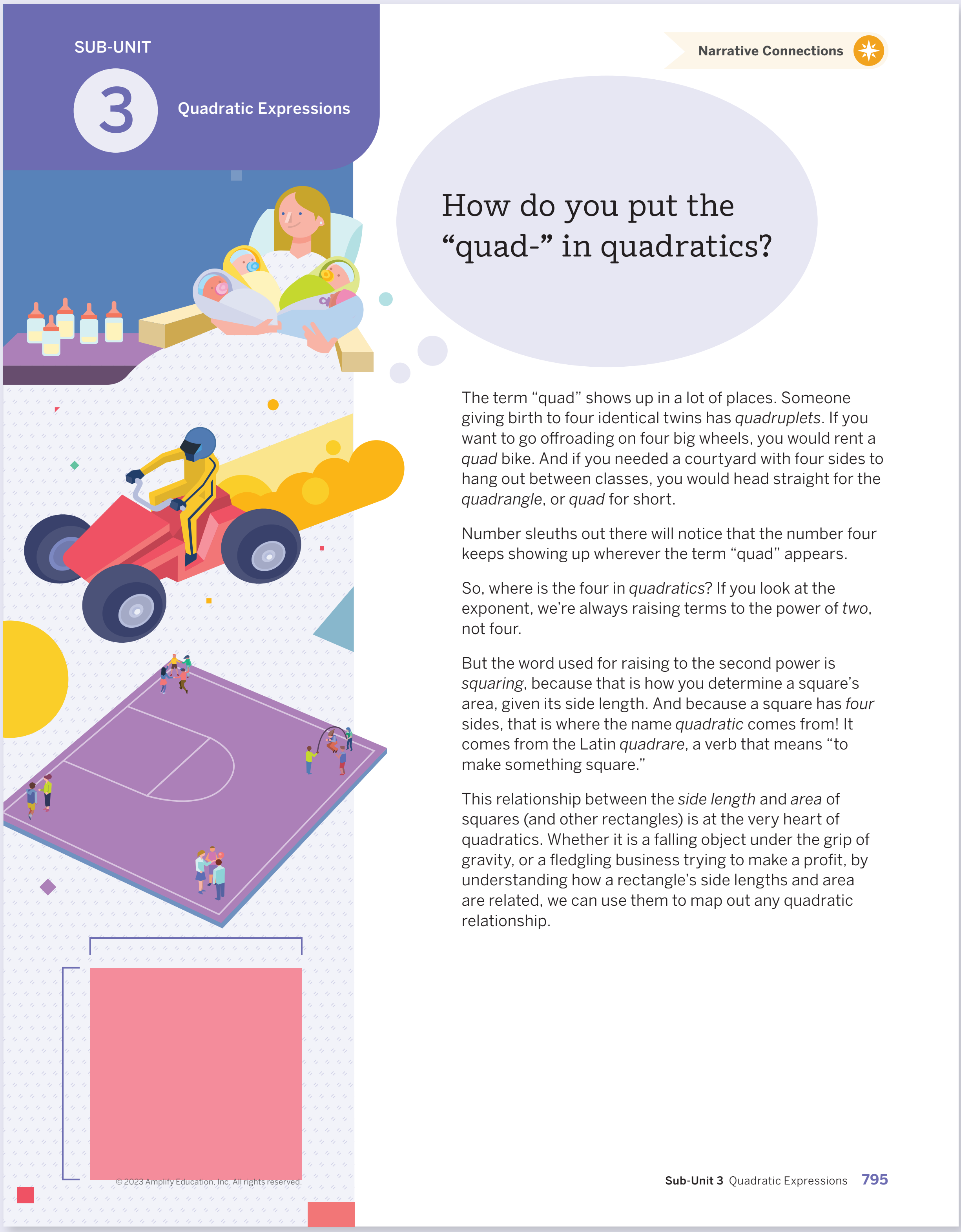
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 3

Quadratic Expressions

Whole Class

In this Sub-Unit, students are introduced to factoring quadratic expressions using area diagrams and algebra tiles.



SUB-UNIT

3 Quadratic Expressions

Narrative Connections

How do you put the "quad-" in quadratics?

The term "quad" shows up in a lot of places. Someone giving birth to four identical twins has *quadruplets*. If you want to go offroading on four big wheels, you would rent a *quad* bike. And if you needed a courtyard with four sides to hang out between classes, you would head straight for the *quadrangle*, or *quad* for short.

Number sleuths out there will notice that the number four keeps showing up wherever the term "quad" appears.

So, where is the four in *quadratics*? If you look at the exponent, we're always raising terms to the power of *two*, not four.

But the word used for raising to the second power is *squaring*, because that is how you determine a square's area, given its side length. And because a square has *four* sides, that is where the name *quadratic* comes from! It comes from the Latin *quadrare*, a verb that means "to make something square."

This relationship between the *side length* and *area* of squares (and other rectangles) is at the very heart of quadratics. Whether it is a falling object under the grip of gravity, or a fledgling business trying to make a profit, by understanding how a rectangle's side lengths and area are related, we can use them to map out any quadratic relationship.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore the relationship of quadratics to rectangles (or squares) in the following places:

- **Lesson 10, Activities 1–2:** What Property Is It?, Determining Area
- **Lesson 11, Activities 1–2:** Using Tiles to Find Equivalent Quadratic Expressions, Using Diagrams to Determine Products
- **Lesson 12, Activity 1:** Finding Products of Differences

UNIT 5 | **LESSON 10**

Equivalent Quadratic Expressions (Part 1)

Let's use diagrams to write quadratic expressions.

Focus

Goals

1. **Language Goal:** Use area diagrams to reason about the product of a monomial and a sum (binomial). **(Speaking and Listening, Reading and Writing)**
2. Use area diagrams to write equivalent quadratic expressions.
3. Use the Distributive Property to write equivalent quadratic expressions.

Coherence

Today

Students use their understanding of decomposing figures to calculate the area of a whole figure, by relating side lengths of rectangles to binomial and monomial linear expressions. They represent the rectangle's area using a quadratic expression that is equivalent to its linear factors.

Previously

In Lesson 9, students used quadratic functions to model revenue. They determined an appropriate domain, and made sense of the vertex and horizontal intercepts of the graph in each context.

Coming Soon

In Lesson 11, students will calculate the product of two linear binomials using manipulatives, models, and the Distributive Property.

Rigor

- Students build **conceptual understanding** of the product of a monomial and a binomial.
- Students use area diagrams to build **procedural skills** in writing quadratic expressions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (Optional)	Summary	Exit Ticket
5 min	15 min	20 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF (answers)
- Activity 3 PDF (instructions)
- Activity 3 PDF, pre-cut cards
- Anchor Chart PDF, *Multiplying a Monomial by a Binomial*

Math Language Development

New words

- ***area diagram***

Review words

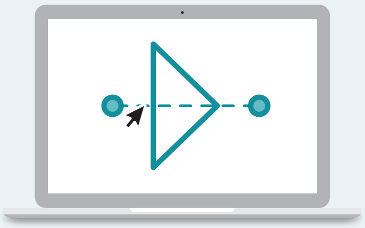
- *commutative property*
- *Distributive Property*
- *equivalent expression*

Amps | **Featured Activity**

Activity 1

Digital Area Diagrams

Students use digital area diagrams to multiply a monomial and binomial. Through the diagrams, they model the Distributive Property, calculating and adding partial areas to write equivalent expressions.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively complete the area diagrams in Activity 2, which probably look familiar, without paying attention to the details involved with the operations with expressions. Ask students to think about how they can control their impulse to rush through each of the expressions. Remind students that they are not only using the geometric structure for the area of a rectangle but also the structures involved with multiplying expressions to find equivalent expressions.

Modifications to Pacing

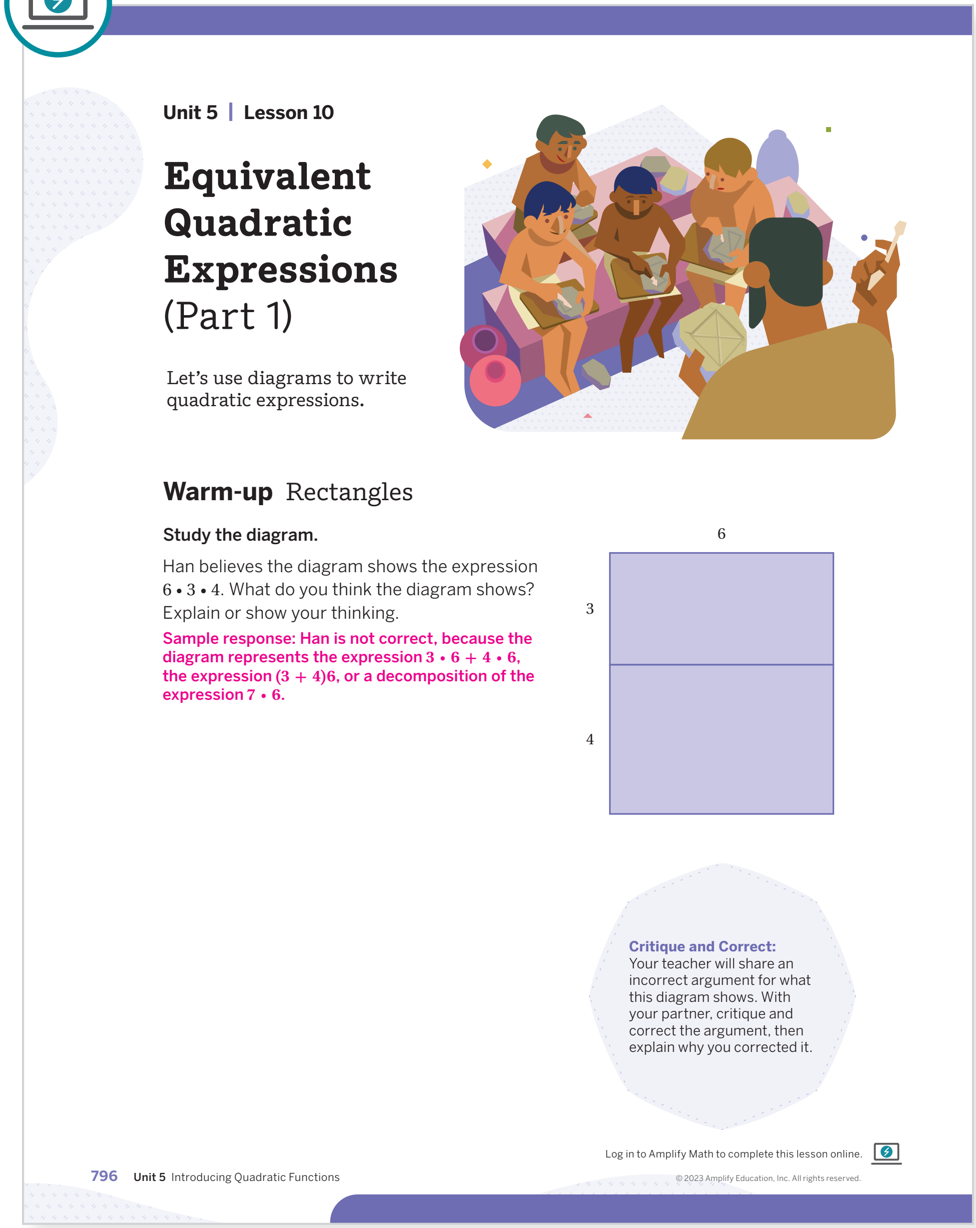
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 5 may be omitted.
- In **Activity 2**, have students only complete the first three rows of the table.
- In **Activity 3**, the number of game cards may be reduced.

Pairs | 5 min

Warm-up Rectangles

Students critique a diagram to determine the validity of a numerical expression, preparing them to reason about the product of linear expressions.



Unit 5 | Lesson 10

Equivalent Quadratic Expressions (Part 1)

Let's use diagrams to write quadratic expressions.

Warm-up Rectangles

Study the diagram.

Han believes the diagram shows the expression $6 \cdot 3 \cdot 4$. What do you think the diagram shows? Explain or show your thinking.

Sample response: Han is not correct, because the diagram represents the expression $3 \cdot 6 + 4 \cdot 6$, the expression $(3 + 4)6$, or a decomposition of the expression $7 \cdot 6$.

Critique and Correct: Your teacher will share an incorrect argument for what this diagram shows. With your partner, critique and correct the argument, then explain why you corrected it.

Log in to Amplify Math to complete this lesson online.

796 **Unit 5** Introducing Quadratic Functions

1 Launch

Provide students one minute of quiet think-time before they complete the Warm-up with a partner.

2 Monitor

Help students get started by asking how they can use a rectangle to help them determine a product.

Look for points of confusion:

- **Agreeing with Han's statement.** Ask, "What is the area of *each* rectangle? What is the sum of those areas?"
- **Disagreeing with Han, but only providing a partial reason.** Ask, "What expression does the diagram show and how is that different from Han's expression?"

Look for productive strategies:

- Recognizing Han's mistake — he multiplied all three numbers together.
- Calculating the areas of each smaller rectangle, then adding those areas to determine the area of the larger rectangle.

3 Connect

Display the diagram, then use the ***Poll the Class*** routine to determine who agrees or disagrees with Han.

Have students share their reasons for agreeing with Han, then select and sequence students who disagreed and used any of the productive strategies.

Ask, "How should Han determine the product?"

Highlight the two ways the area of the rectangle can be represented, as a sum of the areas of the two smaller rectangles, $6 \cdot 3 + 6 \cdot 4$, and as a product of its two side lengths, $6(3 + 4)$.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect argument, such as "The diagram represents the expression $6 \cdot 3 \cdot 4$ because the dimensions are 6, 3, and 4 and you multiply the dimensions to determine the area." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
- ***Correct and Clarify:*** "How would you correct this statement? What does this diagram show?" Listen for and amplify language that describes the Distributive Property.

Power-up

To power up students' ability to apply the Distributive Property, have students complete:

Recall that you can use the Distributive Property to say $a(b \pm c)$ is equivalent to $a \cdot b \pm a \cdot c$.

Match the equivalent expressions.

a. $2(x - 4)$ ....b.... $2x^2 - 8x$

b. $2x(x - 4)$ ....c.... $-2x - 8$

c. $-2(x + 4)$ ....a.... $2x - 8$

Use: Before the Warm-up

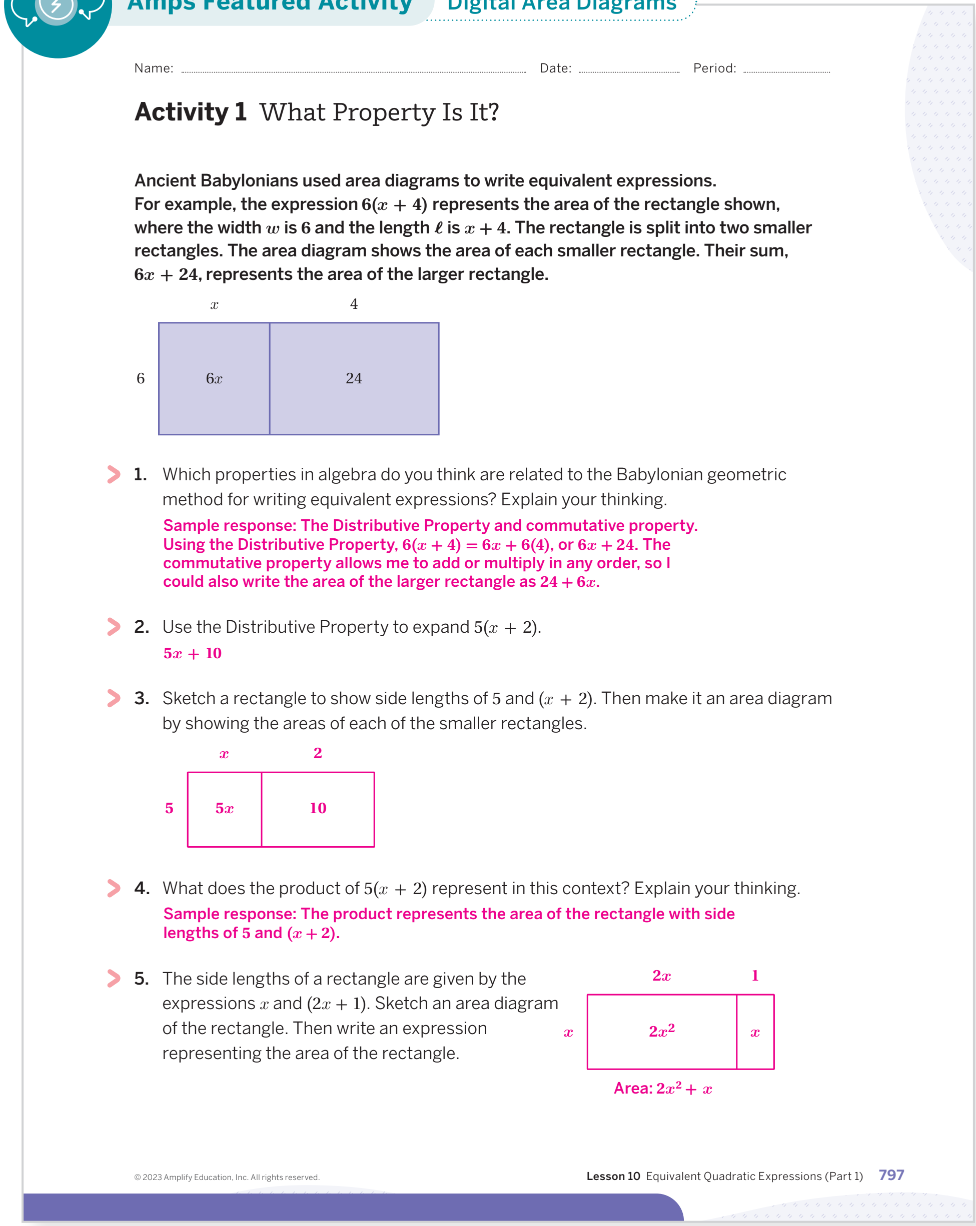
Informed by: Performance on Lesson 9, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2

Pairs | 15 min

Activity 1 What Property Is It?

Students study ancient Babylonian area diagrams to connect the Distributive Property to equivalent linear and quadratic expressions.

Amps Featured Activity Digital Area Diagrams



Name: ________ Date: ________ Period: ________

Activity 1 What Property Is It?

Ancient Babylonians used area diagrams to write equivalent expressions. For example, the expression $6(x + 4)$ represents the area of the rectangle shown, where the width w is 6 and the length ℓ is $x + 4$. The rectangle is split into two smaller rectangles. The area diagram shows the area of each smaller rectangle. Their sum, $6x + 24$, represents the area of the larger rectangle.

1. Which properties in algebra do you think are related to the Babylonian geometric method for writing equivalent expressions? Explain your thinking.

   Sample response: The Distributive Property and commutative property. Using the Distributive Property, $6(x + 4) = 6x + 6(4)$, or $6x + 24$. The commutative property allows me to add or multiply in any order, so I could also write the area of the larger rectangle as $24 + 6x$.

2. Use the Distributive Property to expand $5(x + 2)$.

   $5x + 10$

3. Sketch a rectangle to show side lengths of 5 and $(x + 2)$. Then make it an area diagram by showing the areas of each of the smaller rectangles.

4. What does the product of $5(x + 2)$ represent in this context? Explain your thinking.

   Sample response: The product represents the area of the rectangle with side lengths of 5 and $(x + 2)$.

5. The side lengths of a rectangle are given by the expressions x and $(2x + 1)$. Sketch an area diagram of the rectangle. Then write an expression representing the area of the rectangle.

Area: $2x^2 + x$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Equivalent Quadratic Expressions (Part 1) 797

1 Launch

Display the figure and then read the prompt aloud. Ask students what process was used to determine the area of the larger rectangle.

2 Monitor

Help students get started by color-coding the area diagram.

Look for points of confusion:

- **Not connecting the terms of the expression to the dimensions of the rectangle.** Have students label the dimensions of the smaller rectangles.
- **Calculating only the areas of the smaller rectangles.** Ask students how to calculate the total area of the large rectangle.
- **Multiplying all the terms.** Have students show what each term represents in the small rectangles.

Look for productive strategies:

- Labeling the factors as side lengths.
- Recognizing the sum of the smaller rectangles' area is the area of the larger (whole) rectangle.
- Using the Distributive Property to check expressions.

3 Connect

Have students share their strategies for decomposing the rectangles and relating the linear expressions to the side lengths.

Define the term ***area diagram***.

Highlight that the sides of an area diagram help students multiply the individual terms on the sides of the smaller rectangles, yielding partial areas, whose sum is an equivalent expression to the total area.

Ask, "How could you write equivalent expressions using area diagrams?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital area diagrams to multiply a monomial by a binomial.

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Consider providing blank area diagrams for students to complete for Problems 3 and 5. Provide access to colored pencils and suggest they color code the factors and partial products. For example, in the diagram at the top of the page, they could color 6, x, and $6x$ with one color, and 6, 4, and 24 in another color. The 6 would be shaded twice because it is distributed to both x and 4.

Extension: *Math Around the World*

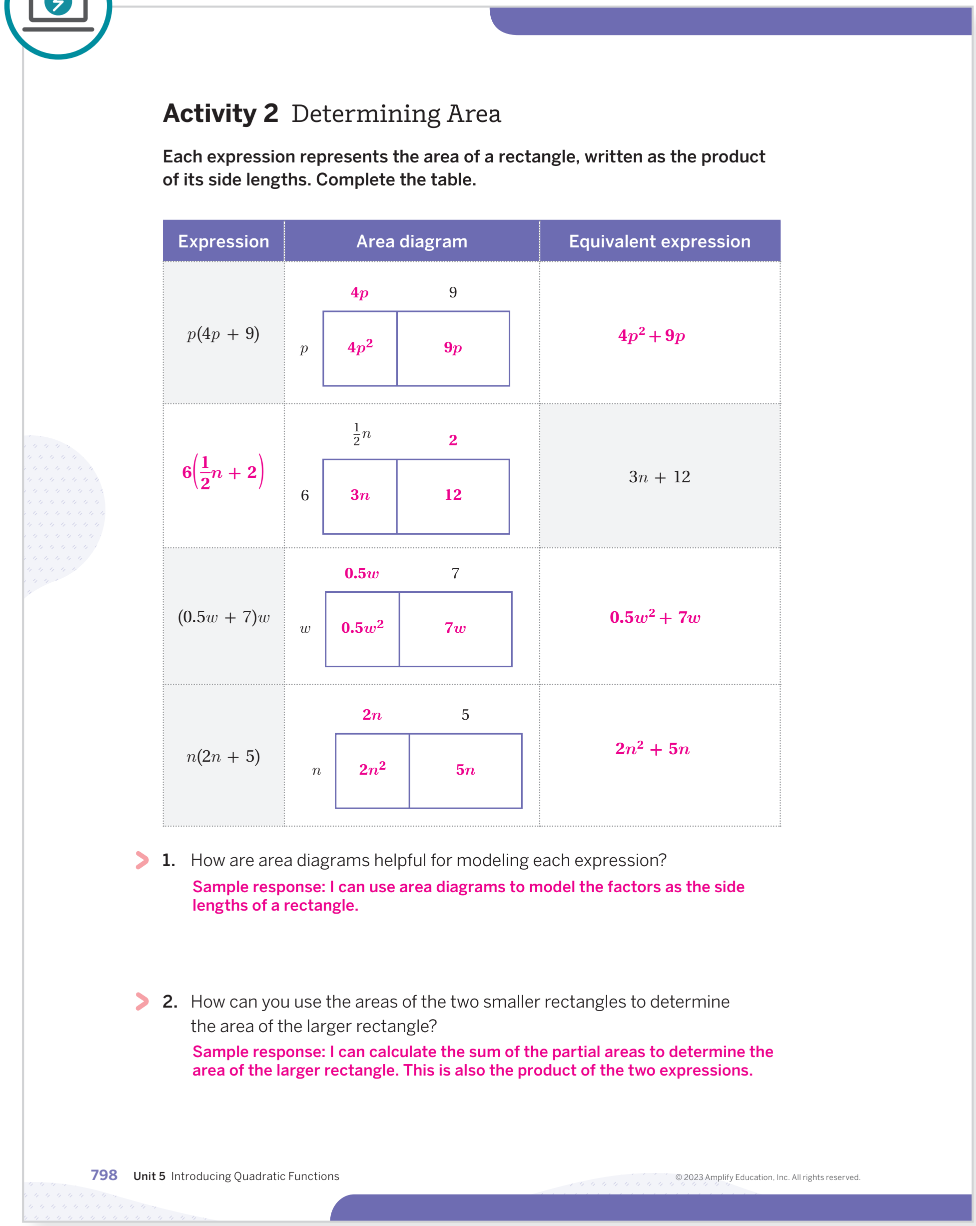
Tell students that ancient Babylonians used area diagrams and tablets, often containing multiples, to perform multiplication. While they used a base 60 number system instead of the base 10 number system we use, their multiplication methods demonstrated understanding of the Distributive Property — although the property was not named as such during this time. To multiply two numbers, such as 43 • 37 . . .

- Ancient Babylonian mathematicians would have used an area diagram to write 37 as the sum 30 + 7.
- They then would have used their tablet of multiples to determine 30 multiples of 43 and 7 multiples of 43.

Pairs | 20 min

Activity 2 Determining Area

Students activate their prior knowledge of the Distributive Property and area by using area diagrams to write equivalent expressions.



Activity 2 Determining Area

Each expression represents the area of a rectangle, written as the product of its side lengths. Complete the table.

Expression	Area diagram	Equivalent expression
$p(4p + 9)$	$4p$, 9 p $4p^2$, $9p$	$4p^2 + 9p$
$6\left(\frac{1}{2}n + 2\right)$	$\frac{1}{2}n$, 2 6 $3n$, 12	$3n + 12$
$(0.5w + 7)w$	$0.5w$, 7 w $0.5w^2$, $7w$	$0.5w^2 + 7w$
$n(2n + 5)$	$2n$, 5 n $2n^2$, $5n$	$2n^2 + 5n$

1. How are area diagrams helpful for modeling each expression?
   Sample response: I can use area diagrams to model the factors as the side lengths of a rectangle.

2. How can you use the areas of the two smaller rectangles to determine the area of the larger rectangle?
   Sample response: I can calculate the sum of the partial areas to determine the area of the larger rectangle. This is also the product of the two expressions.

798 **Unit 5** Introducing Quadratic Functions

1 Launch

Remind students that area diagrams can be used to multiply linear expressions.

2 Monitor

Help students get started by suggesting they label the sides of each smaller rectangle with the corresponding terms from each factor in the expression.

Look for points of confusion:

- **Struggling to decompose the rectangle into two smaller rectangles.** Ask students what the dimensions of the smaller rectangles should be.

Look for productive strategies:

- Writing the areas of the smaller rectangles, then adding them to determine the total area.
- Using the Distributive Property to expand each expression to write an equivalent expression.

3 Connect

Have students share their area diagrams and their strategies for determining the equivalent expressions.

Display an area diagram showing $\frac{5}{2}r(r+3) = \frac{5}{2}r^2 + \frac{15}{2}r$.

Highlight that students can write an equivalent expression using the area diagram. First, calculate the areas of the smaller rectangles. Then, write the equivalent expression as the sum of these areas.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Prior Knowledge*

Display or provide copies of the Anchor Chart PDF, *Multiplying a Monomial by a Binomial*, for students to reference as they complete this activity.

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Connect, as students share their area diagrams and strategies for determining the equivalent expressions, ask the class to critique each other's reasoning. Display or provide access to the Anchor Chart PDF, *Sentence Stems*, *Critiquing* to support students as they analyze each other's reasoning. Revoice student ideas by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.

English Learners

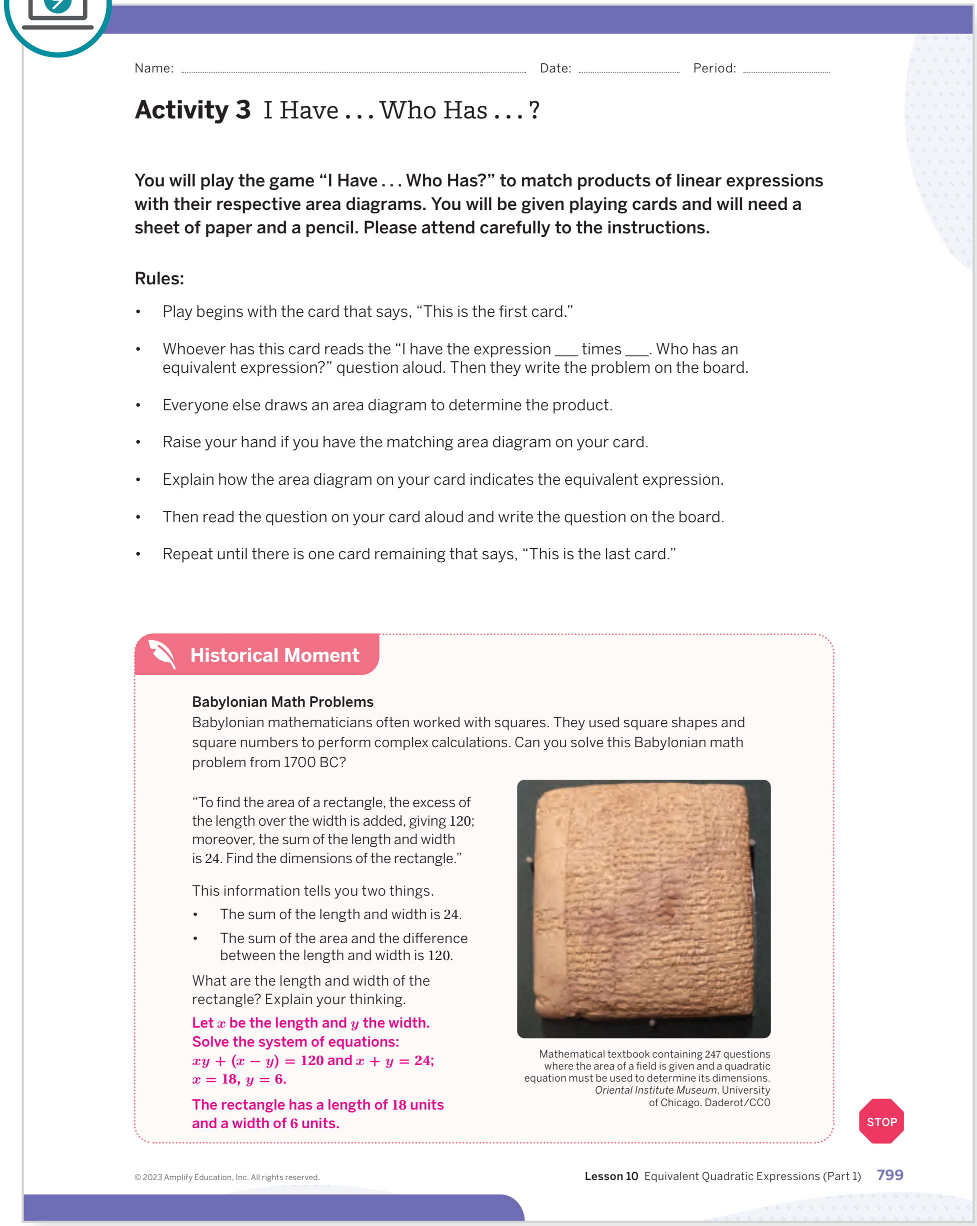
Allow students to rehearse with a partner what they will say, before sharing with the entire class.

Optional

Whole Class | 15 min

Activity 3 I Have . . . Who Has . . . ?

Students play a matching game to strengthen the connections between area diagrams and equivalent quadratic expressions.



Name: ______ Date: ______ Period: ______

Activity 3 I Have . . . Who Has . . . ?

You will play the game "I Have . . . Who Has?" to match products of linear expressions with their respective area diagrams. You will be given playing cards and will need a sheet of paper and a pencil. Please attend carefully to the instructions.

Rules:

- Play begins with the card that says, "This is the first card."
- Whoever has this card reads the "I have the expression ___ times ___. Who has an equivalent expression?" question aloud. Then they write the problem on the board.
- Everyone else draws an area diagram to determine the product.
- Raise your hand if you have the matching area diagram on your card.
- Explain how the area diagram on your card indicates the equivalent expression.
- Then read the question on your card aloud and write the question on the board.
- Repeat until there is one card remaining that says, "This is the last card."

Historical Moment

Babylonian Math Problems
Babylonian mathematicians often worked with squares. They used square shapes and square numbers to perform complex calculations. Can you solve this Babylonian math problem from 1700 BC?

"To find the area of a rectangle, the excess of the length over the width is added, giving 120; moreover, the sum of the length and width is 24. Find the dimensions of the rectangle."

This information tells you two things.

- The sum of the length and width is 24.
- The sum of the area and the difference between the length and width is 120.

What are the length and width of the rectangle? Explain your thinking.

Let x be the length and y the width.
Solve the system of equations:
$xy + (x - y) = 120$ and $x + y = 24$;
$x = 18$, $y = 6$.

The rectangle has a length of 18 units and a width of 6 units.

Mathematical textbook containing 247 questions where the area of a field is given and a quadratic equation must be used to determine its dimensions. *Oriental Institute Museum*, University of Chicago. Daderot/CC0

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Equivalent Quadratic Expressions (Part 1) 799

1 Launch

Display the Activity 3 PDF (instructions). Shuffle and distribute the pre-cut cards from the Activity 3 PDF. Read the instructions and demonstrate how to play the game. Conduct the *I Have, Who Has?* routine.

2 Monitor

Help students get started by having them create blank area diagrams on a separate sheet of paper.

Look for points of confusion:

- **Not relating the terms to the dimensions of the smaller rectangles.** Ask students what the dimensions of the smaller rectangles should be.

Look for productive strategies:

- Modeling the expression written on the board with an area diagram, and comparing it to the partially-completed diagram on their card.

3 Connect

Have individual students share their strategies for determining the matching area diagrams.

Highlight the connection between the area diagrams and distribution, partial areas, and the equivalent expressions.

Ask, "How could you use an area diagram to write equivalent expressions when both factors have two terms?"

Historical Moment

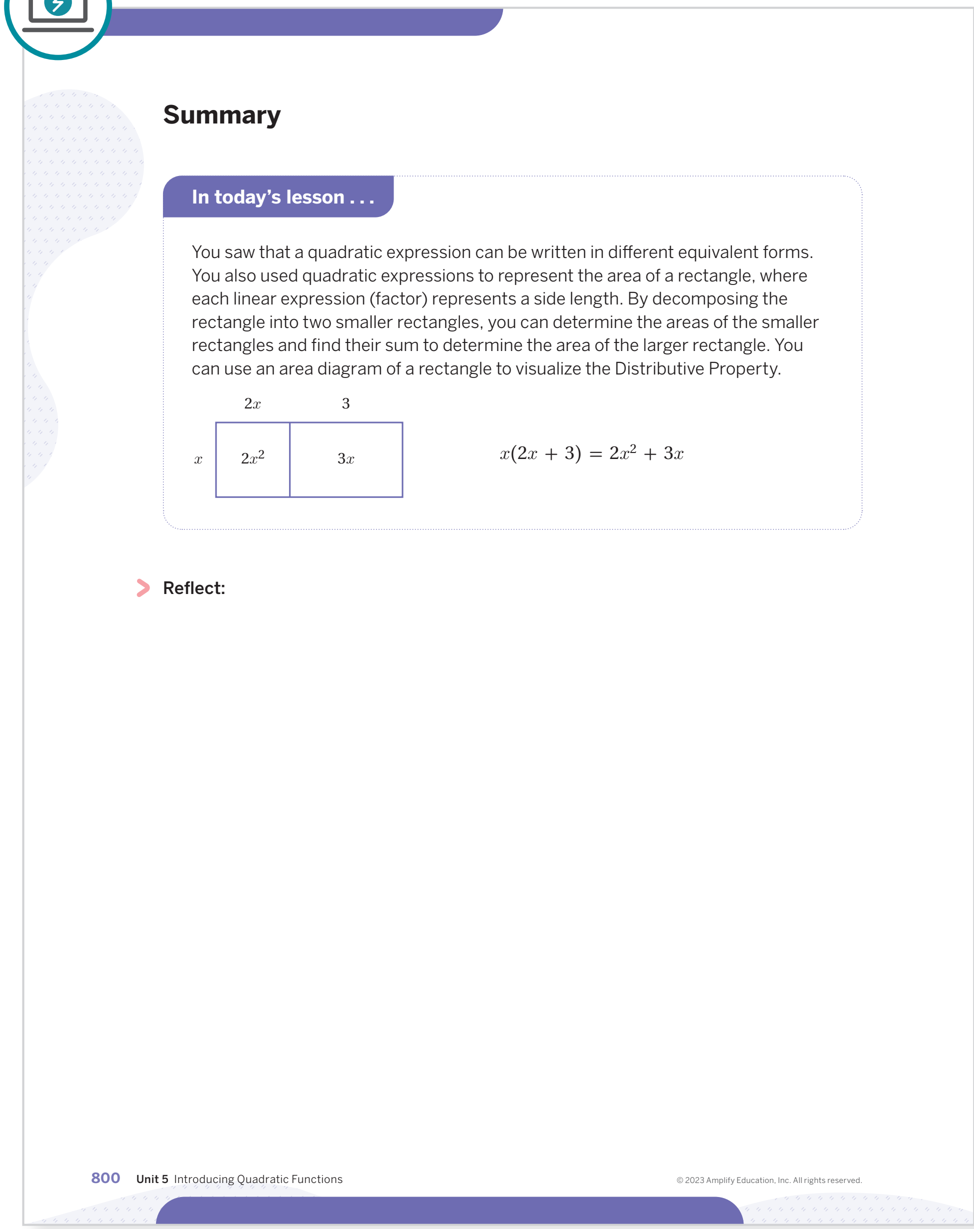
Babylonian Math Problems

Have students complete the *Historical Moment* activity to solve an ancient Babylonian math problem involving the area of a rectangle.

Whole Class | 5 min

Summary

Review and synthesize how equivalent quadratic expressions can represent the area of rectangles that are decomposed into smaller rectangles.



Summary

In today's lesson . . .

You saw that a quadratic expression can be written in different equivalent forms. You also used quadratic expressions to represent the area of a rectangle, where each linear expression (factor) represents a side length. By decomposing the rectangle into two smaller rectangles, you can determine the areas of the smaller rectangles and find their sum to determine the area of the larger rectangle. You can use an area diagram of a rectangle to visualize the Distributive Property.

	$2x$	3
x	$2x^2$	$3x$

$x(2x + 3) = 2x^2 + 3x$

> Reflect:

800 Unit 5 Introducing Quadratic Functions

Synthesize

Display the area diagram.

Have students share strategies for multiplying the terms and how the sum of the partial areas yields two equivalent expressions.

Highlight that an area diagram is a tool used to visualize the Distributive Property when writing expressions. A quadratic function can be represented by an area diagram of a rectangle, where each linear expression represents the side lengths of the rectangle. By decomposing the rectangle into two smaller rectangles, you can relate the areas of the smaller rectangles to the Distributive Property. Then, you can find the sum to calculate the area of the larger rectangle.

Formalize vocabulary:

- *area diagram*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does an area diagram help to determine the product of two expressions?"
- "How could you determine two expressions that represent the dimensions of an area diagram if you were given the area of each rectangle?"

Math Language Development

MLR2: Collect and Display

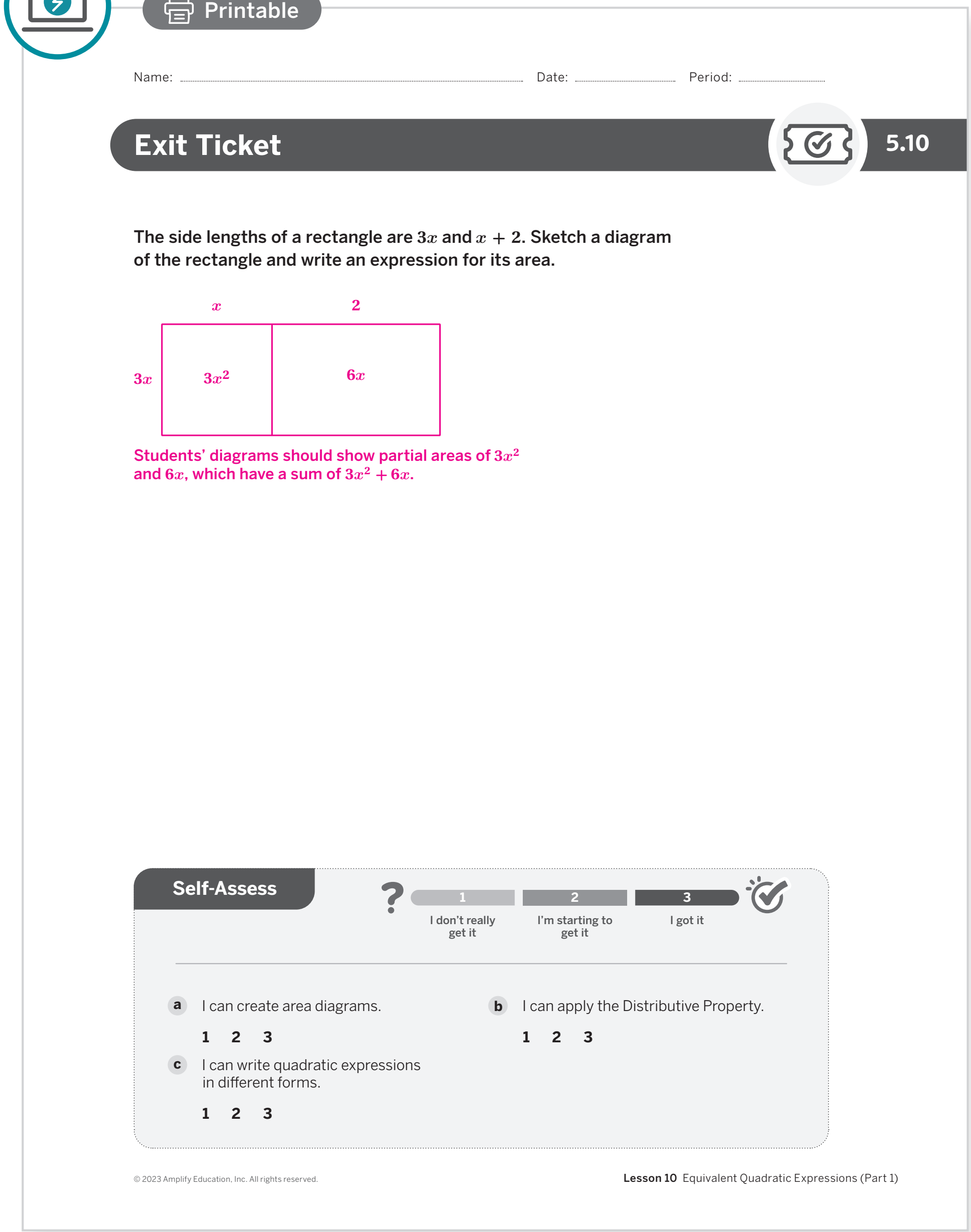
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *area diagram* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of equivalent quadratic expressions by using area diagrams and applying the Distributive Property.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

5.10

The side lengths of a rectangle are $3x$ and $x + 2$. Sketch a diagram of the rectangle and write an expression for its area.

Students' diagrams should show partial areas of $3x^2$ and $6x$, which have a sum of $3x^2 + 6x$.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can create area diagrams.
1 2 3

b I can apply the Distributive Property.
1 2 3

c I can write quadratic expressions in different forms.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Equivalent Quadratic Expressions (Part 1)

Success looks like . . .

- **Language Goal:** Using area diagrams to reason about the product of a monomial and a sum (binomial). **(Speaking and Listening, Reading and Writing)**
- **Goal:** Using area diagrams to write equivalent quadratic expressions.
    - » Sketching a diagram of the rectangle and writing an expression for its area.
- **Goal:** Using the Distributive Property to write equivalent quadratic expressions.

Suggested next steps

If students do not sketch and label the area diagram correctly, consider:

- Reviewing area diagrams from Activities 1 and 2.
- Reminding them that the side lengths of the smaller rectangles are represented by the terms in the expressions.

If students do not multiply the variable terms correctly, consider:

- Reminding them some variables have a coefficient of 1 that is usually not written.
- Reviewing the product of powers rule for exponents when multiplying variables (coefficients are multiplied and exponents are added).

If students do not multiply the variable and constant terms correctly, consider:

- Reminding them to multiply the coefficients and the constants.
- Reminding them that the constant term can be considered as being a coefficient for the same variable, where the variable is raised to the 0 power. Then apply the product of powers rule for exponents.

Professional Learning

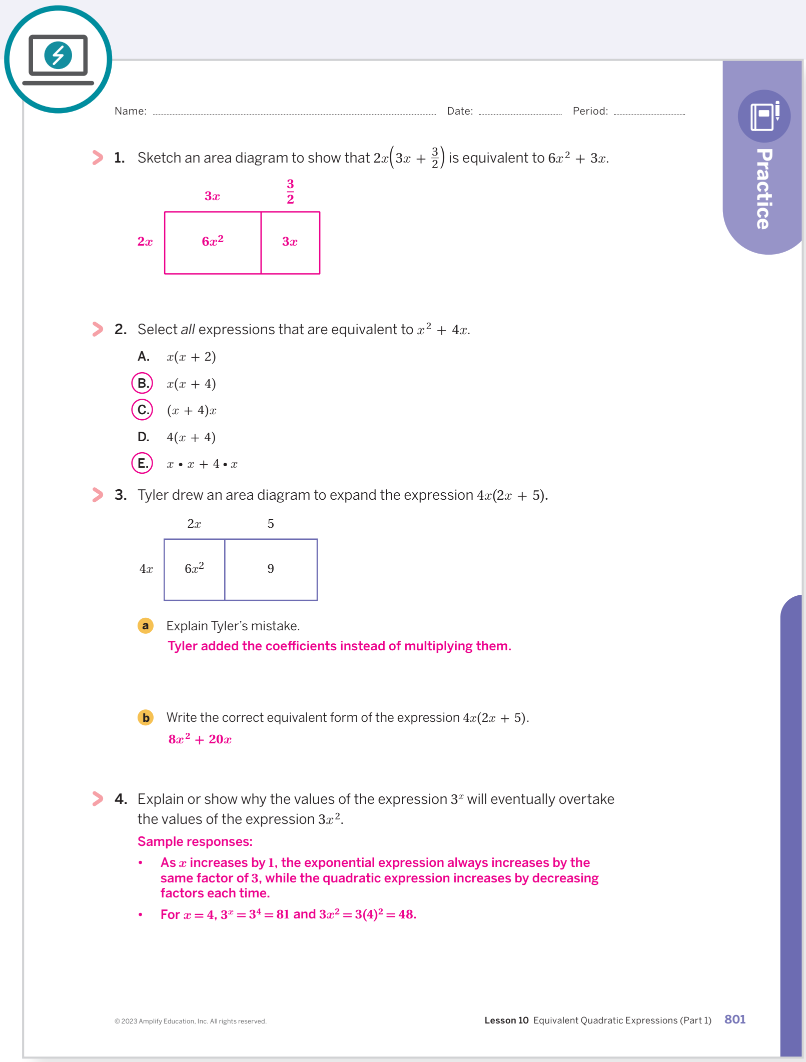
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What did students' use of the area diagram reveal about your students as learners?
- What did you see in the way some students approached multiplying a monomial and a binomial that you would like other students to try?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Sketch an area diagram to show that $2x\left(3x + \frac{3}{2}\right)$ is equivalent to $6x^2 + 3x$.

	$3x$	$\frac{3}{2}$
$2x$	$6x^2$	$3x$

2. Select *all* expressions that are equivalent to $x^2 + 4x$.
   - A. $x(x + 2)$
   - [✓ B.] $x(x + 4)$
   - [✓ C.] $(x + 4)x$
   - D. $4(x + 4)$
   - [✓ E.] $x \bullet x + 4 \bullet x$

3. Tyler drew an area diagram to expand the expression $4x(2x + 5)$.

	$2x$	5
$4x$	$6x^2$	9

a. Explain Tyler's mistake.

Tyler added the coefficients instead of multiplying them.

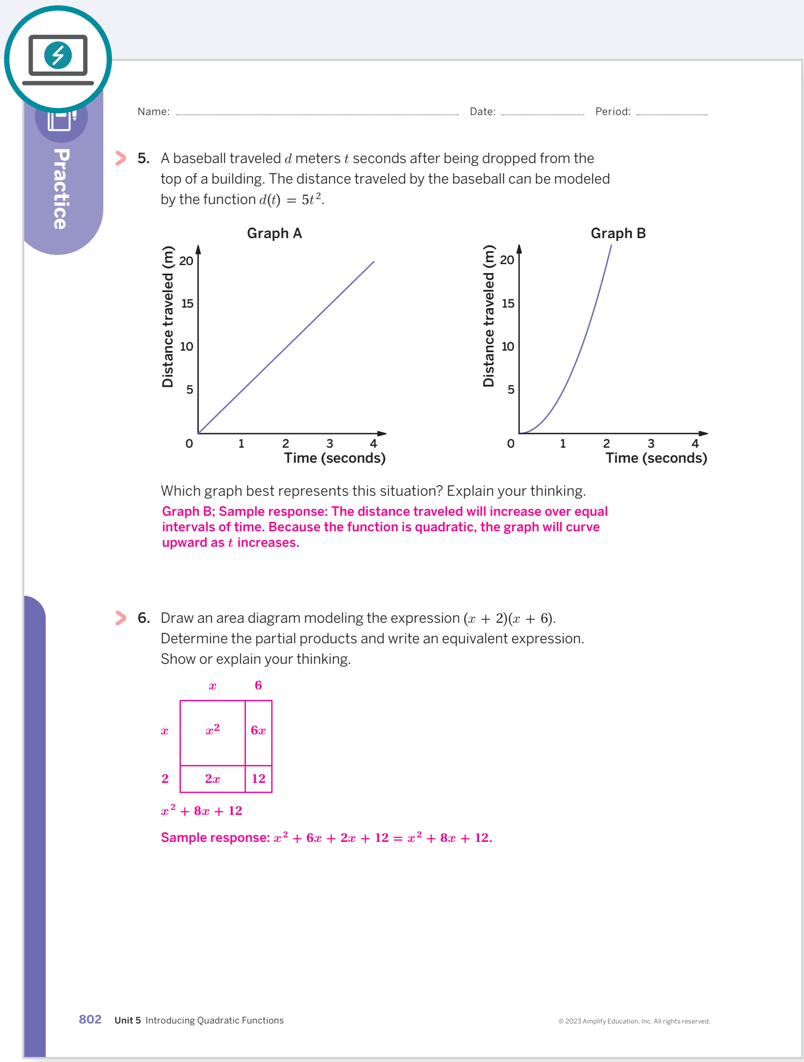
b. Write the correct equivalent form of the expression $4x(2x + 5)$.

$\mathbf{8x^2 + 20x}$

4. Explain or show why the values of the expression 3^x will eventually overtake the values of the expression $3x^2$.

Sample responses:

- **As x increases by 1, the exponential expression always increases by the same factor of 3, while the quadratic expression increases by decreasing factors each time.**
- **For $x = 4$, $3^x = 3^4 = 81$ and $3x^2 = 3(4)^2 = 48$.**



Name: ______ Date: ______ Period: ______

5. A baseball traveled d meters t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the function $d(t) = 5t^2$.

Which graph best represents this situation? Explain your thinking.

Graph B; Sample response: The distance traveled will increase over equal intervals of time. Because the function is quadratic, the graph will curve upward as t increases.

6. Draw an area diagram modeling the expression $(x + 2)(x + 6)$. Determine the partial products and write an equivalent expression. Show or explain your thinking.

	x	6
x	x^2	$6x$
2	$2x$	12

$\mathbf{x^2 + 8x + 12}$

Sample response: $x^2 + 6x + 2x + 12 = x^2 + 8x + 12$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	3
	2	Activity 1	3
	3	Activity 2	3
Spiral	4	Unit 5 Lesson 6	2
	5	Unit 5 Lesson 7	2
Formative ⚡	6	Unit 5 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 11**

Equivalent Quadratic Expressions (Part 2)

Let's examine how the product of two binomial factors can be expressed as an equivalent quadratic expression.

Focus

Goals

1. **Language Goal:** Use algebra tiles to reason about the product of two sums (two binomials). **(Speaking and Listening, Reading and Writing)**
2. Use algebra tiles and area diagrams to write equivalent quadratic expressions that represent the product of two binomial factors.
3. Extend understanding of the Distributive Property to multiply two binomial linear expressions without the use of diagrams or algebra tiles.

Coherence

Today

In this lesson, students create models with algebra tiles and use the structure of their models to write equivalent quadratic expressions. Students also revisit area diagrams, this time multiplying pairs of linear expressions with two terms.

Previously

In the previous lesson, students used area diagrams to write equivalent quadratic expressions and related this to the Distributive Property.

Coming Soon

In subsequent lessons, students will formally define the different forms of a quadratic expression and identify them based on the parameters they are given.

Rigor

- Students build **conceptual understanding** of the product of two binomials.
- Students use algebra tiles to build **procedural skills** in writing quadratic expressions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Multiplying Linear Expressions Using the Distributive Property*
- algebra tiles

Math Language Development

Review words

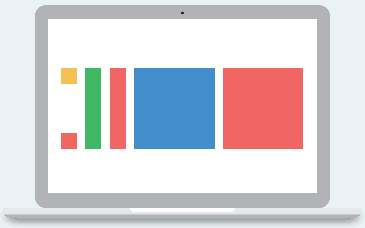
- *area diagram*
- *equivalent expression*

Amps | **Featured Activity**

Activity 1

Digital Algebra Tiles

Students create models with digital algebra tiles to visualize the different ways quadratic expressions can be written.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel stressed about drawing the diagrams to help multiply binomials in Activity 2. Ask students to look at the sample diagram and discuss its structure. Help students find the similarities in the structure of both the products and the diagrams that represent them. By relying on the structure of the diagram, students can monitor their own progress and set themselves up for success.

Modifications to Pacing

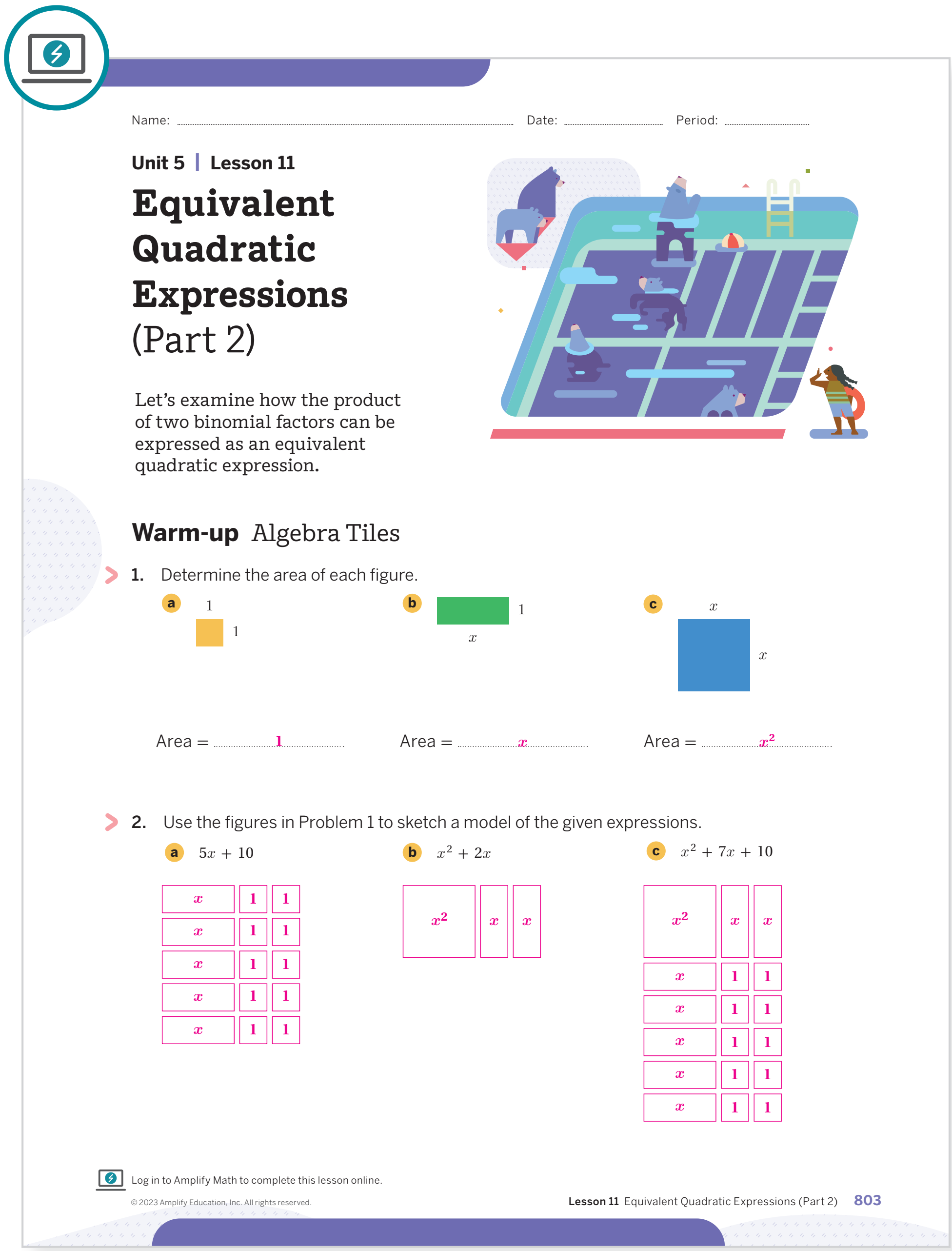
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 3 may be omitted.
- In **Activity 2**, Problem 2, have students only complete three models.

Pairs | 10 min

Warm-up Algebra Tiles

Students model expressions with algebra tiles to prepare them for the next activity in which they will use algebra tiles to find equivalent quadratic expressions.



Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 11

Equivalent Quadratic Expressions
(Part 2)

Let's examine how the product of two binomial factors can be expressed as an equivalent quadratic expression.

Warm-up Algebra Tiles

1. Determine the area of each figure.

Area = 1 Area = x Area = x^2

2. Use the figures in Problem 1 to sketch a model of the given expressions.

a $5x + 10$ b $x^2 + 2x$ c $x^2 + 7x + 10$

Log in to Amplify Math to complete this lesson online.

Lesson 11 Equivalent Quadratic Expressions (Part 2) 803

1 Launch

Provide a set of algebra tiles to each pair of students. Display each tile, demonstrating its side lengths.

2 Monitor

Help students get started by prompting them to refer to each tile by its area (for example, "the 1-tile, the x-tile, and the x^2-tile")

Look for points of confusion:

- **Having difficulty building a rectangle to model each expression.** Tell students to only gather the algebra tiles needed to represent the expression they are working on.

Look for productive strategies:

- Counting the tiles to match the coefficients of the terms of the given expressions.

3 Connect

Have student pairs share their models for each given expression, having them show the tiles and number of tiles used to represent the expression.

Highlight that the number of tiles used to represent each term is determined by the coefficient of that term in the quadratic expression. For example, in the expression $x^2 + 2x$, there should be one x^2-tile and two x-tiles.

Power-up

To power up students' ability to connect area diagrams to the product of two expressions, have students complete:

Complete the area diagram to determine the product of $(x + 2)$ and $(2 + 1)$.

	2	1
x	$2x$	x
2	4	2

$2x + x + 4 + 2$ or $3x + 6$

Use: Before the Warm-up

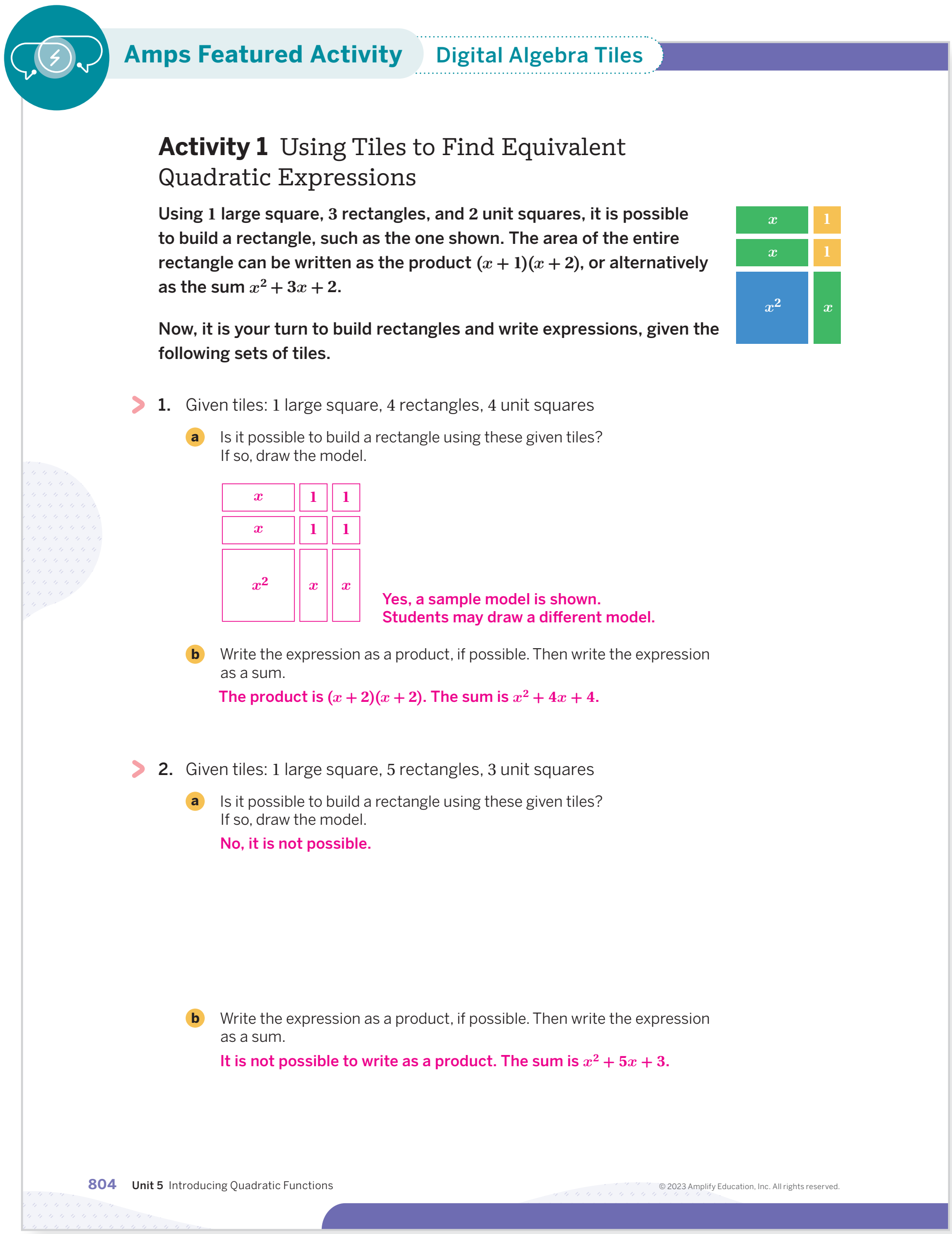
Informed by: Performance on Lesson 10, Practice Problem 6

Pairs | 15 min

Activity 1 Using Tiles to Find Equivalent Quadratic Expressions

Students use algebra tiles to build rectangular area models to understand that two equivalent quadratic expressions can be written to represent the area, when a rectangle can be formed.

Amps Featured Activity Digital Algebra Tiles



Activity 1 Using Tiles to Find Equivalent Quadratic Expressions

Using 1 large square, 3 rectangles, and 2 unit squares, it is possible to build a rectangle, such as the one shown. The area of the entire rectangle can be written as the product $(x+1)(x+2)$, or alternatively as the sum x^2+3x+2.

Now, it is your turn to build rectangles and write expressions, given the following sets of tiles.

1. Given tiles: 1 large square, 4 rectangles, 4 unit squares

   a. Is it possible to build a rectangle using these given tiles? If so, draw the model.

   Yes, a sample model is shown. Students may draw a different model.

   b. Write the expression as a product, if possible. Then write the expression as a sum.

   The product is $(x+2)(x+2)$. The sum is x^2+4x+4.

2. Given tiles: 1 large square, 5 rectangles, 3 unit squares

   a. Is it possible to build a rectangle using these given tiles? If so, draw the model.

   No, it is not possible.

   b. Write the expression as a product, if possible. Then write the expression as a sum.

   It is not possible to write as a product. The sum is x^2+5x+3.

1 Launch

Use the given example to model the instructions for the activity. Advise that some models cannot make a rectangle and will not have equivalent expressions in factored form.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "How can you model area as a product?"

Look for points of confusion:

- **Struggling to build a rectangle for Problems 1 and 4.** Prompt students to begin with the x^2-tile for each problem and add tiles accordingly.

Look for productive strategies:

- Writing an expression as a sum based on the given tiles (without the use of algebra tiles).
- Writing or labeling the side lengths of the rectangle models.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create models with digital algebra tiles to visualize the different ways quadratic expressions can be written.

Extension: *Math Enrichment*

Challenge students to come up with their own two sets of tiles, one for which it is possible to build a rectangle, and one in which it is not possible. Have them write each expression as a product, if possible, and as a sum. Answers may vary.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight the expressions in Problems 1 and 4, draw students' attention to the expression written as a product and the expression written as a sum. Consider displaying blank expressions, such as the ones shown here, to illustrate how each is considered a product or a sum.

Product	Sum
(□)(□)	□+□+□

Emphasize that it was only possible to build a rectangle when the expression can also be written as a product.

Activity 1 Using Tiles to Find Equivalent Quadratic Expressions (continued)

Pairs | 15 min

Students use algebra tiles to build rectangular area models to understand that two equivalent quadratic expressions can be written to represent the area, when a rectangle can be formed.

Name: ______ Date: ______ Period: ______

Activity 1 Using Tiles to Find Equivalent Quadratic Expressions (continued)

3. Given tiles: 1 large square, 3 rectangles, 9 unit squares

a Is it possible to build a rectangle using these given tiles? If so, draw the model.

No, it is not possible.

b Write the expression as a product, if possible. Then write the expression as a sum.

It is not possible to write as a product. The sum is $x^2 + 3x + 9$.

4. Given tiles: 1 large square, 5 rectangles, 6 unit squares

a Is it possible to build a rectangle using these given tiles? If so, draw the model.

Yes, a sample model is shown. Students may draw a different model.

b Write the expression as a product, if possible. Then write the expression as a sum.

The product is $(x + 2)(x + 3)$. The sum is $x^2 + 5x + 6$.

③ Connect

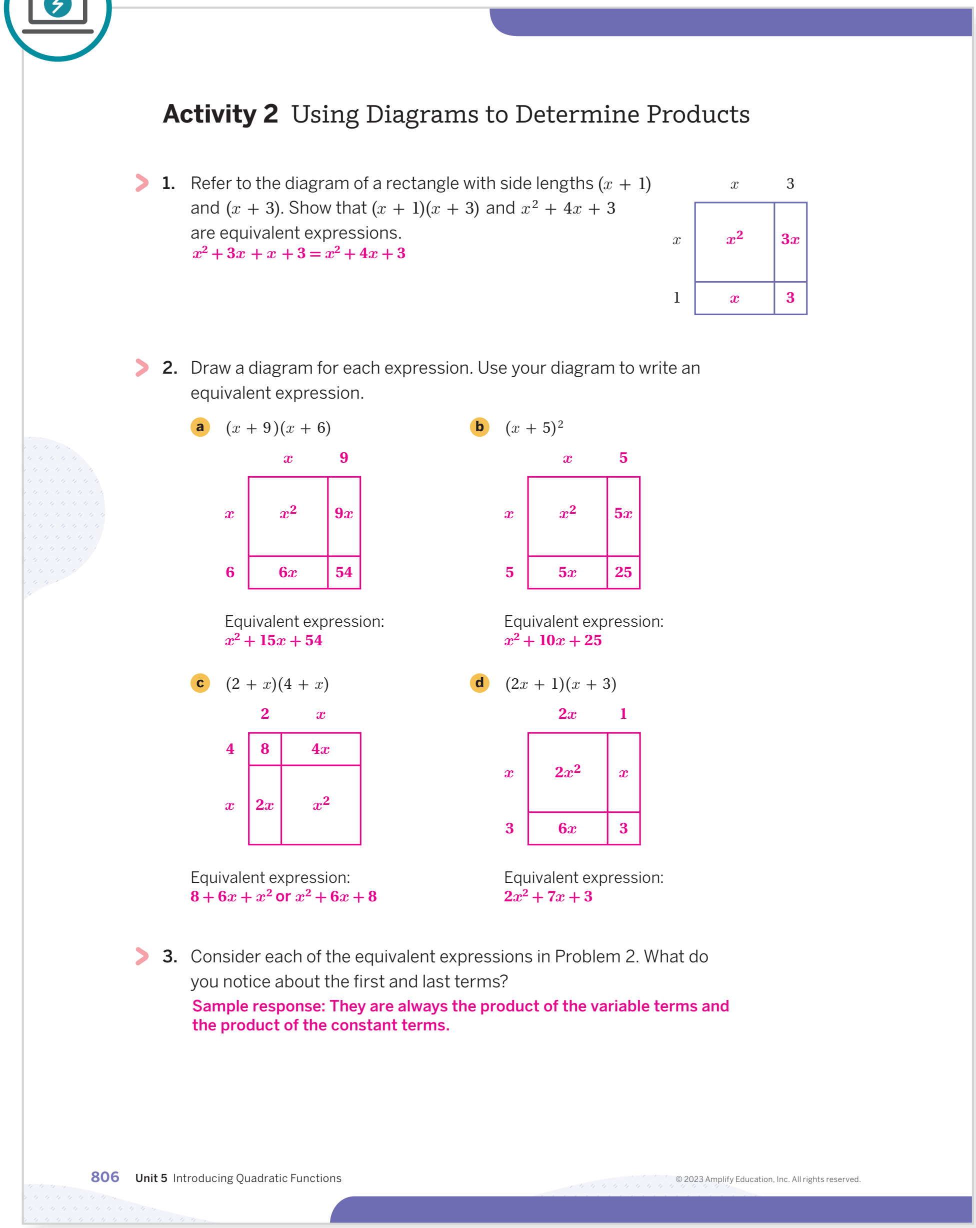
Have pairs of students share whether it was possible to build a rectangle for each problem. If yes, have them share the expression written as a product and as a sum. If no, have them share the expression written as a sum.

Highlight the expressions found in Problems 1 and 4, writing an equation on the board to show the equivalence between the expression given as a product and the expression given as a sum. Multiplying the side lengths of the rectangle yields an equivalent quadratic expression.

Independent | 15 min

Activity 2 Using Diagrams to Determine Products

Students study the structure of equivalent expressions written using area diagrams to prepare to write equivalent expressions without the use of area diagrams.



Activity 2 Using Diagrams to Determine Products

1. Refer to the diagram of a rectangle with side lengths $(x + 1)$ and $(x + 3)$. Show that $(x + 1)(x + 3)$ and $x^2 + 4x + 3$ are equivalent expressions.
$x^2 + 3x + x + 3 = x^2 + 4x + 3$

	x	3
x	x^2	$3x$
1	x	3

2. Draw a diagram for each expression. Use your diagram to write an equivalent expression.

a $(x + 9)(x + 6)$

	x	9
x	x^2	$9x$
6	$6x$	54

Equivalent expression:
$x^2 + 15x + 54$

b $(x + 5)^2$

	x	5
x	x^2	$5x$
5	$5x$	25

Equivalent expression:
$x^2 + 10x + 25$

c $(2 + x)(4 + x)$

	2	x
4	8	$4x$
x	$2x$	x^2

Equivalent expression:
$8 + 6x + x^2$ or $x^2 + 6x + 8$

d $(2x + 1)(x + 3)$

	$2x$	1
x	$2x^2$	x
3	$6x$	3

Equivalent expression:
$2x^2 + 7x + 3$

3. Consider each of the equivalent expressions in Problem 2. What do you notice about the first and last terms?
Sample response: They are always the product of the variable terms and the product of the constant terms.

1 Launch

Give students two minutes to complete Problem 1 individually. Display the diagram and ask a student to explain their process for determining the product. Ask them to write an equation to show that the expressions are equivalent.

2 Monitor

Help students get started by prompting them to visualize the algebra tiles they would use to represent each expression.

Look for points of confusion:

- **Struggling to articulate how to determine an equivalent expression without using a diagram in Problem 5.** Prompt students to describe the steps they took to write an equivalent expression in Problem 2.
- **Not understanding why there are three terms in each equivalent expression, yet four boxes (Problem 4).** Ask students to study the four terms in each box to see if there is a simpler way to write the sum of those four terms.

Look for productive strategies:

- Labeling the side lengths of each area diagram in Problem 2.
- Writing the area inside of each sub-rectangle in Problem 2.
- Connecting the squared variable term in the equivalent expression to the product of the variable terms in each factor.
- Connecting the constant term in the equivalent expression to the product of the constant terms in each factor.
- Recognizing there is a middle term and connecting it to the sum of the two like terms in the diagram.

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Provide continued access to algebra tiles should students choose to use them to model each expression, similarly to Activity 1. Then they can draw an area diagram, based on the arrangement of their algebra tiles. Consider also providing blank area diagrams that students can use to complete Problem 2.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, draw their attention to the connections between the area diagrams and the structure of their corresponding equivalent expressions.

English Learners

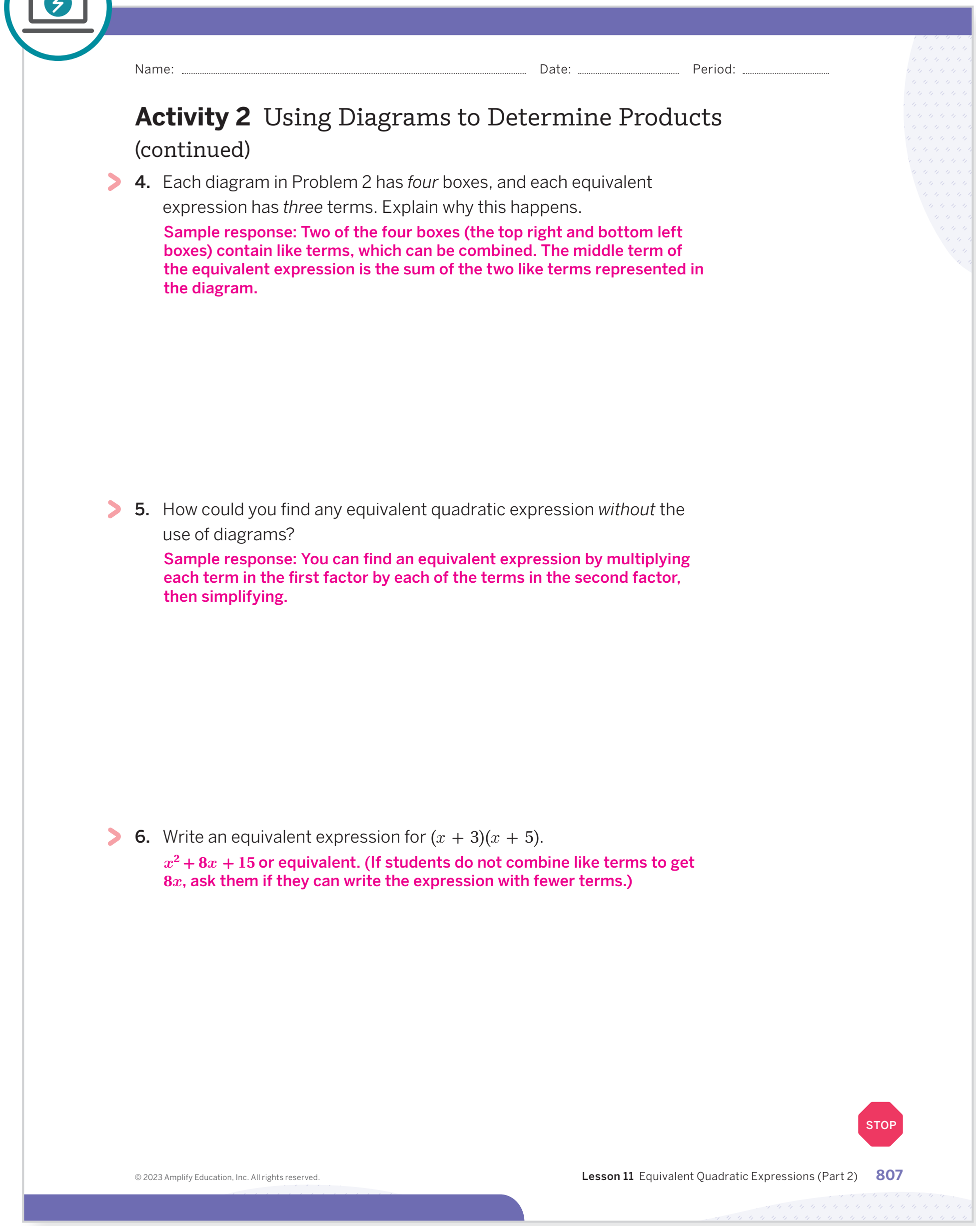
Provide sentence frames for students to help them organize their thinking, such as:

- "The first term in the equivalent expression represents the ___ of the ___ terms." product/variable (x)
- "The last term in the equivalent expression represents the ___ of the ___ terms." product/constant
- "The middle term in the equivalent expression represents the ___ of the ___ terms." sum/like

Independent | 15 min

Activity 2 Using Diagrams to Determine Products (continued)

Students study the structure of equivalent expressions written using area diagrams to prepare to write equivalent expressions without the use of area diagrams.



Name: ________ Date: ________ Period: ________

Activity 2 Using Diagrams to Determine Products (continued)

4. Each diagram in Problem 2 has *four* boxes, and each equivalent expression has *three* terms. Explain why this happens.

Sample response: Two of the four boxes (the top right and bottom left boxes) contain like terms, which can be combined. The middle term of the equivalent expression is the sum of the two like terms represented in the diagram.

5. How could you find any equivalent quadratic expression *without* the use of diagrams?

Sample response: You can find an equivalent expression by multiplying each term in the first factor by each of the terms in the second factor, then simplifying.

6. Write an equivalent expression for $(x + 3)(x + 5)$.

$x^2 + 8x + 15$ or equivalent. (If students do not combine like terms to get $8x$, ask them if they can write the expression with fewer terms.)

STOP

3 Connect

Have students share their diagrams and equivalent expressions from Problem 2. Discuss what operations are performed on the given factors to arrive at each of the terms in the equivalent expression.

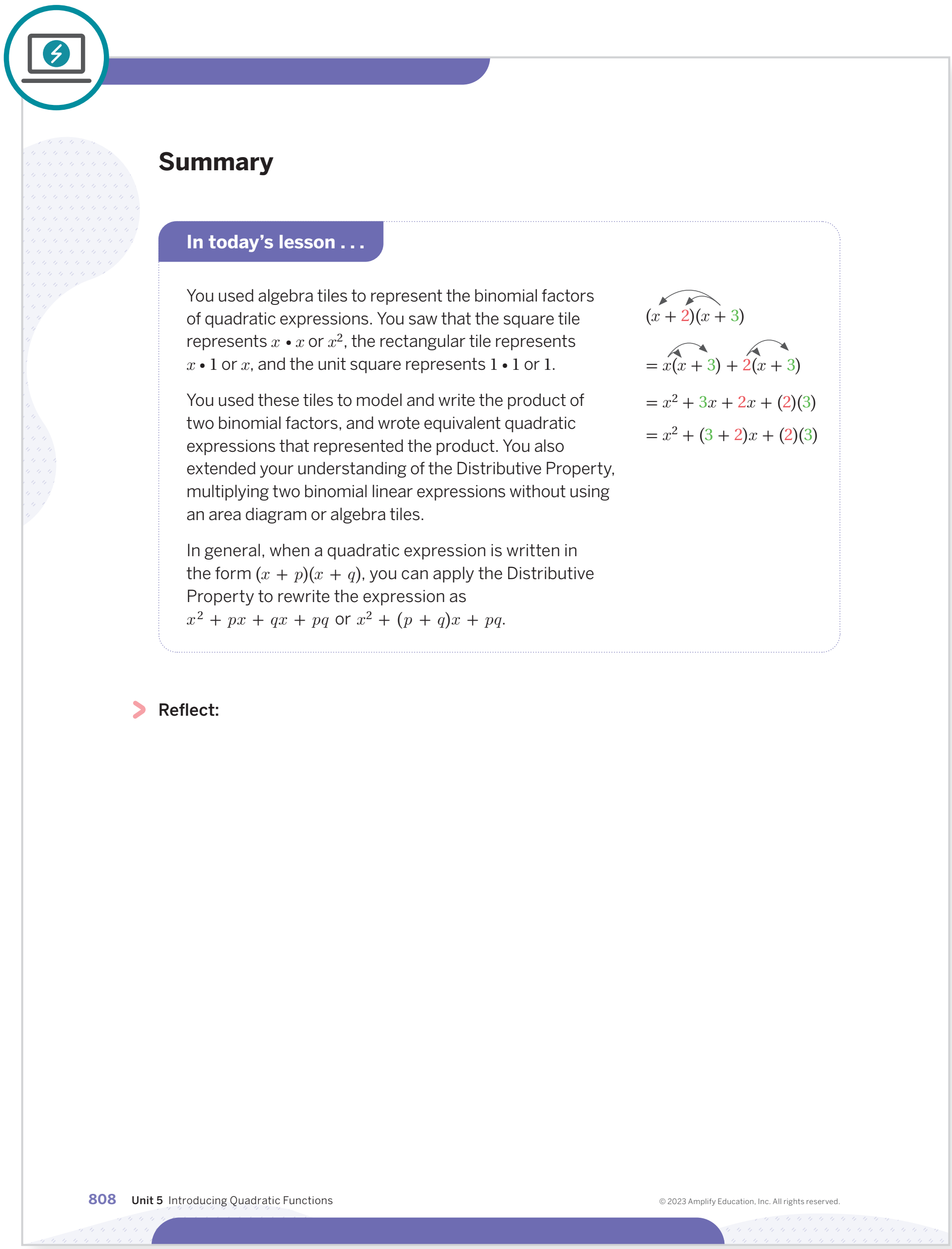
Ask, “What property allows you to do this?”

Highlight that the area diagrams students have been using are a visual representation of the Distributive Property. In the case of multiplying linear expressions, each term in the first factor is multiplied by each term in the second. Then, after eliciting a response to Problem 6, model how to multiply $(x + 3)(x + 5)$ without a diagram.

Whole Class | 5 min

Summary

Review and synthesize how some quadratic expressions can be written in two equivalent ways, as a sum, and as a product.



Summary

In today's lesson . . .

You used algebra tiles to represent the binomial factors of quadratic expressions. You saw that the square tile represents $x \cdot x$ or x^2, the rectangular tile represents $x \cdot 1$ or x, and the unit square represents $1 \cdot 1$ or 1.

You used these tiles to model and write the product of two binomial factors, and wrote equivalent quadratic expressions that represented the product. You also extended your understanding of the Distributive Property, multiplying two binomial linear expressions without using an area diagram or algebra tiles.

In general, when a quadratic expression is written in the form $(x + p)(x + q)$, you can apply the Distributive Property to rewrite the expression as $x^2 + px + qx + pq$ or $x^2 + (p + q)x + pq$.

$$(x + 2)(x + 3)$$
$$= x(x + 3) + 2(x + 3)$$
$$= x^2 + 3x + 2x + (2)(3)$$
$$= x^2 + (3 + 2)x + (2)(3)$$

Reflect:

808 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Have students share the benefits of using algebra tiles, area diagrams, or the Distributive Property to illustrate the two equivalent ways to write quadratic expressions.

Ask:

- "In what ways are area diagrams useful for expanding expressions such as $(x + 4)(x + 9)$? Are there any drawbacks to using area diagrams?" The diagrams can help me see and keep track of the different parts of the factors that need to be multiplied, but drawing area diagrams can be time consuming.
- "In what ways is using the Distributive Property helpful? Are there any drawbacks?" Using the Distributive Property to expand an expression is a quicker method than using an area diagram, but I need to mentally keep track of all the terms that are being multiplied.
- "Which strategy would you choose to expand $(x + 11)(2x + 3)$? Why?" Answers may vary.

Highlight that no matter what strategy students use, they should keep track of the factors multiplied, so that no term is missed. Display the Anchor Chart PDF, *Multiplying Linear Expressions Using the Distributive Property* as a reminder for how to keep track of terms.

Reflect

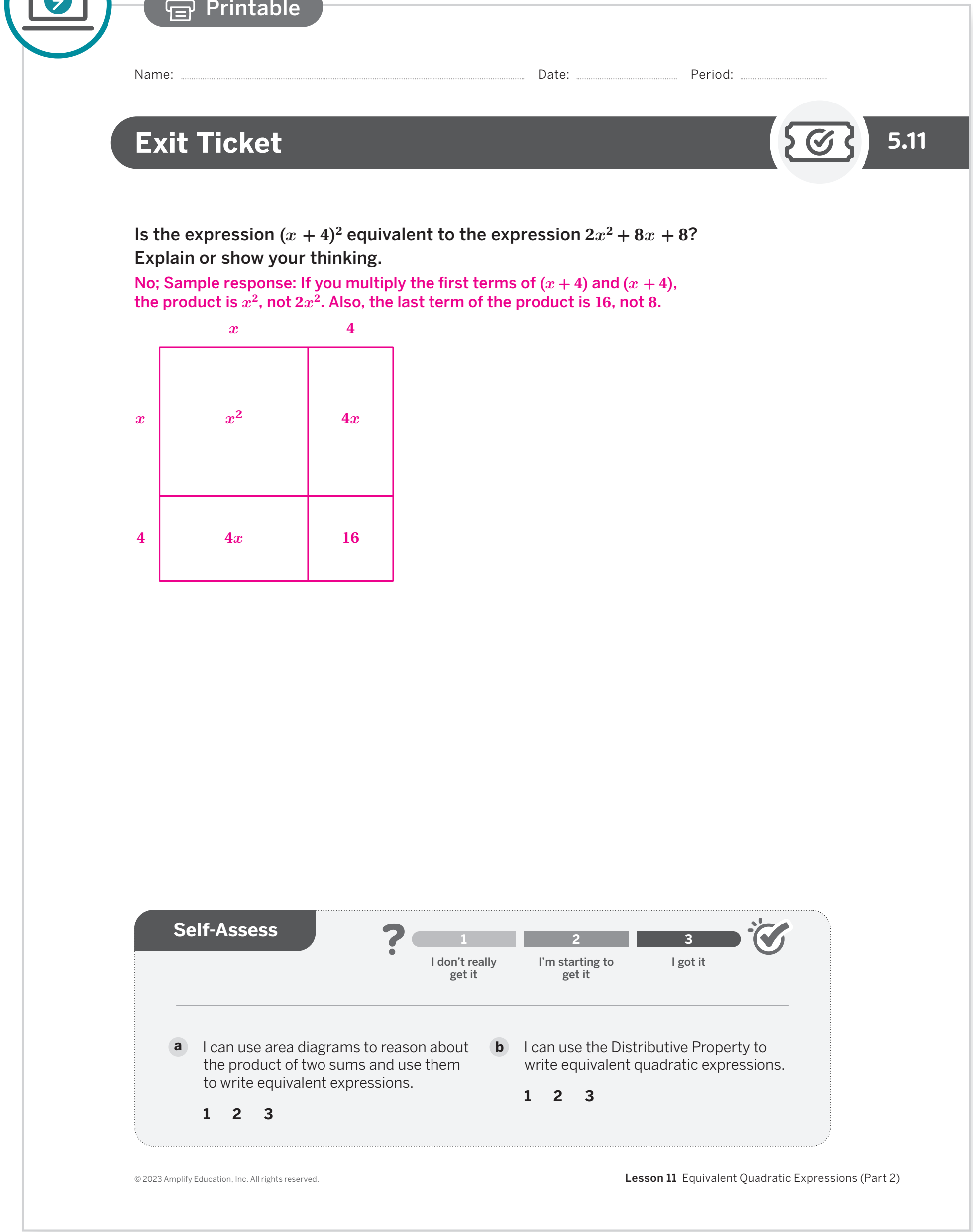
After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is the product of two binomials modeled by the area of a rectangle?"
- "Which strategy and tools were helpful today? Which were not? Why?"

Exit Ticket

Students demonstrate their understanding by determining whether two quadratic expressions — one expressed as a product (square) and the other expressed as a sum — are equivalent.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 5.11

Is the expression $(x + 4)^2$ equivalent to the expression $2x^2 + 8x + 8$? Explain or show your thinking.

No; Sample response: If you multiply the first terms of $(x + 4)$ and $(x + 4)$, the product is x^2, not $2x^2$. Also, the last term of the product is 16, not 8.

	x	4
x	x^2	$4x$
4	$4x$	16

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use area diagrams to reason about the product of two sums and use them to write equivalent expressions.

1 2 3

b I can use the Distributive Property to write equivalent quadratic expressions.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Equivalent Quadratic Expressions (Part 2)

Success looks like . . .

- **Language Goal:** Using algebra tiles to reason about the product of two sums (two binomials). **(Speaking and Listening, Reading and Writing)**
- **Goal:** Using algebra tiles and area diagrams to write equivalent quadratic expressions that represent the product of two binomial factors.
- **Goal:** Extending understanding of the Distributive Property to multiply two binomial linear expressions without the use of diagrams or algebra tiles.

Suggested next steps

If students conclude the given expressions are equivalent, consider:

- Allowing them to use algebra tiles to create a rectangle model. Have them write equivalent quadratic expressions for their model.
- Reviewing Activity 2, Problem 2.
- Assigning Practice Problems 1 and 2.

Professional Learning

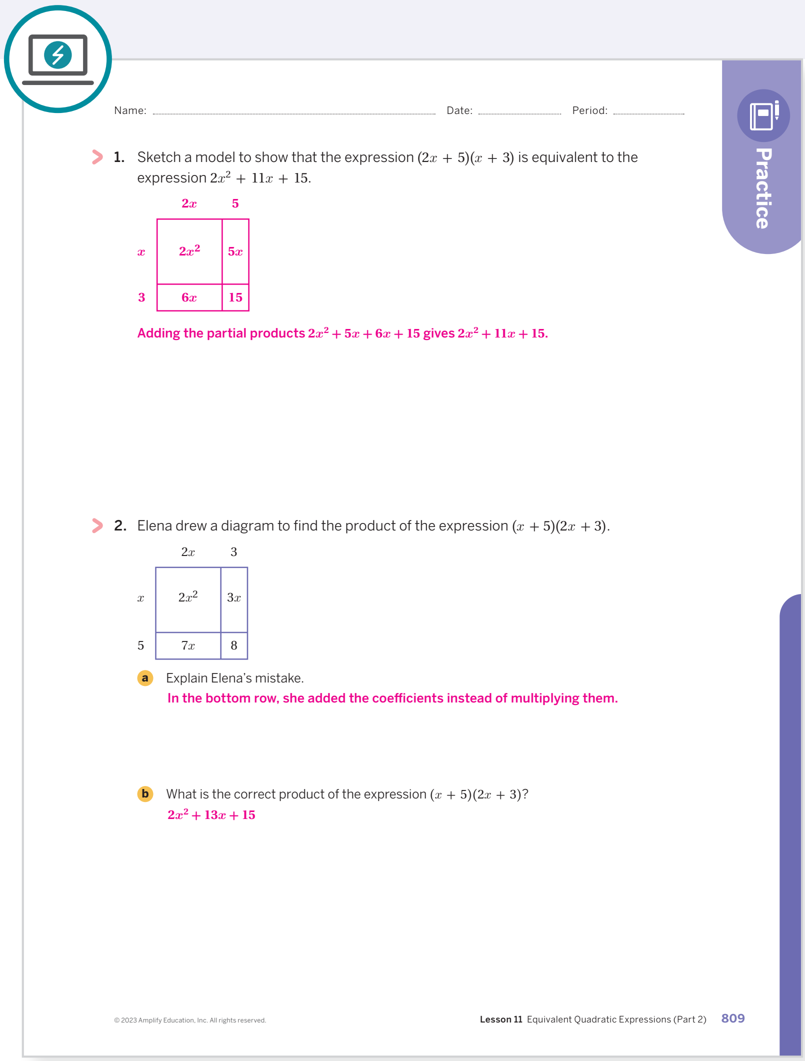
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students used algebra tiles to represent the product of two binomials. How did that build on the earlier work students did with area diagrams?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Sketch a model to show that the expression $(2x + 5)(x + 3)$ is equivalent to the expression $2x^2 + 11x + 15$.

	$2x$	5
x	$2x^2$	$5x$
3	$6x$	15

Adding the partial products $2x^2 + 5x + 6x + 15$ gives $2x^2 + 11x + 15$.

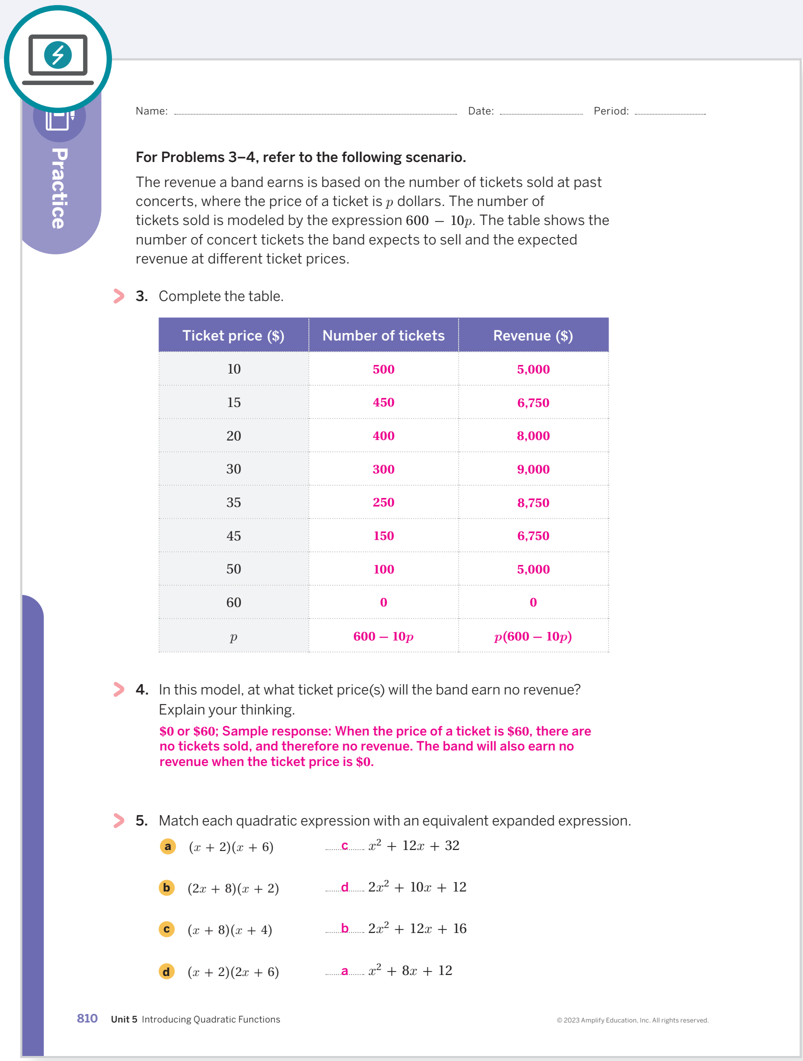
2. Elena drew a diagram to find the product of the expression $(x + 5)(2x + 3)$.

	$2x$	3
x	$2x^2$	$3x$
5	$7x$	8

a Explain Elena's mistake.
In the bottom row, she added the coefficients instead of multiplying them.

b What is the correct product of the expression $(x + 5)(2x + 3)$?
$2x^2 + 13x + 15$

Practice



Practice

Name: ______ Date: ______ Period: ______

For Problems 3–4, refer to the following scenario.

The revenue a band earns is based on the number of tickets sold at past concerts, where the price of a ticket is p dollars. The number of tickets sold is modeled by the expression $600 - 10p$. The table shows the number of concert tickets the band expects to sell and the expected revenue at different ticket prices.

3. Complete the table.

Ticket price ($)	Number of tickets	Revenue ($)
10	500	5,000
15	450	6,750
20	400	8,000
30	300	9,000
35	250	8,750
45	150	6,750
50	100	5,000
60	0	0
p	$600 - 10p$	$p(600 - 10p)$

4. In this model, at what ticket price(s) will the band earn no revenue? Explain your thinking.
$0 or $60; Sample response: When the price of a ticket is $60, there are no tickets sold, and therefore no revenue. The band will also earn no revenue when the ticket price is $0.

5. Match each quadratic expression with an equivalent expanded expression.

a	$(x + 2)(x + 6)$	c	$x^2 + 12x + 32$
b	$(2x + 8)(x + 2)$	d	$2x^2 + 10x + 12$
c	$(x + 8)(x + 4)$	b	$2x^2 + 12x + 16$
d	$(x + 2)(2x + 6)$	a	$x^2 + 8x + 12$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
Spiral	3	Unit 5 Lesson 7	1
	4	Unit 5 Lesson 7	2
Formative ⚡	5	Unit 5 Lesson 12	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

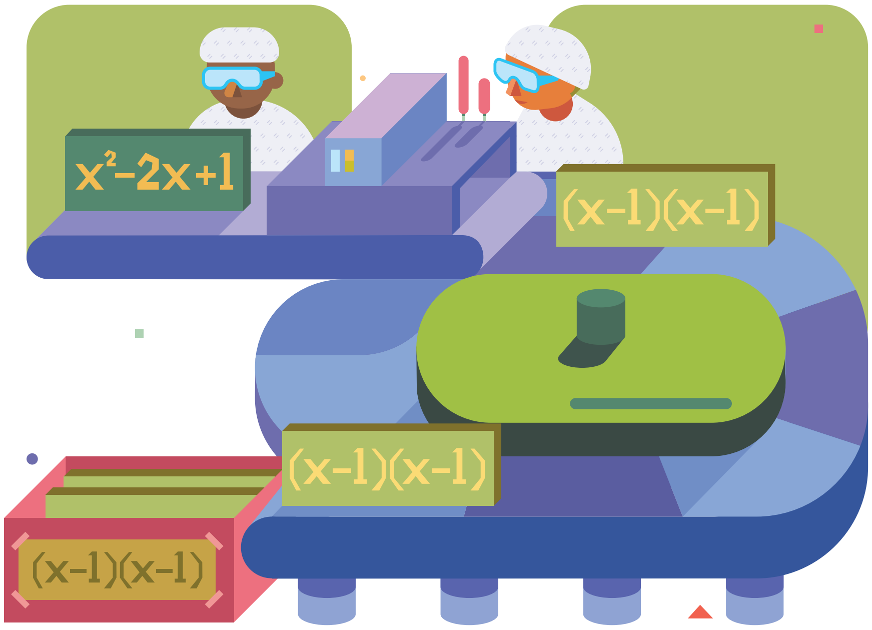
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 12**

Standard Form and Factored Form

Let's write quadratic expressions in different forms.



Focus

Goals

1. **Language Goal:** Understand and use the terms *standard form* and *factored form* when describing quadratic expressions. **(Speaking and Listening, Reading and Writing)**
2. Use rectangular diagrams to reason about the product of two differences or of a sum and difference, and to write equivalent quadratic expressions.
3. Use the Distributive Property to write equivalent quadratic expressions that are given in either factored form or standard form.

Coherence

Today

Students expand expressions in factored form that contain a sum or a difference, and formally define two forms of quadratics, *standard form* and *factored form*. They transition from thinking about rectangular diagrams in terms of area to multiplying the terms in each factor.

Previously

In Lesson 11, students used area diagrams and algebra tiles to expand expressions in the form $(x + p)(x + q)$ to $x^2 + (p + q)x + pq$.

Coming Soon

In Lesson 13, students will graph quadratic functions and study how features of the graphs relate to factored and standard form.

Rigor

- Students build **conceptual understanding** of equivalent quadratic expressions.
- Students are introduced to standard and factored form to build **procedural skills** in writing quadratic expressions in different forms.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF (answers)
- Activity 3 PDF (instructions)
- Activity 3 PDF, pre-cut cards
- Anchor Chart PDF, *Factored and Standard Forms of Quadratic Expressions*
- Anchor Chart PDF, *Sentence Stems, Critiquing*

Math Language Development

New words

- ***factored form***
- ***standard form***

Review words

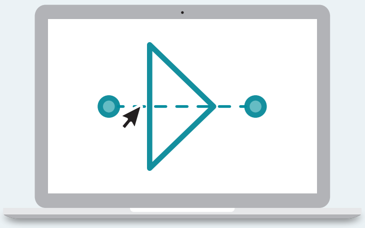
- *equivalent expression*

Amps ⋮ Featured Activity

Activity 1

Digital Area Diagrams

Students use digital diagrams to multiply binomial sums and differences or two differences. Using diagrams, they model the Distributive Property, finding and adding partial products to write equivalent expressions.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students may not cooperate with others as they discuss the similarities and differences in each form. Remind students that, while they initially worked independently, they should take advantage of discussing the results with a partner. Throughout the interaction, the students should seek and offer help, if needed, as they work together to form a primitive, yet precise, definition of each term.

Modifications to Pacing

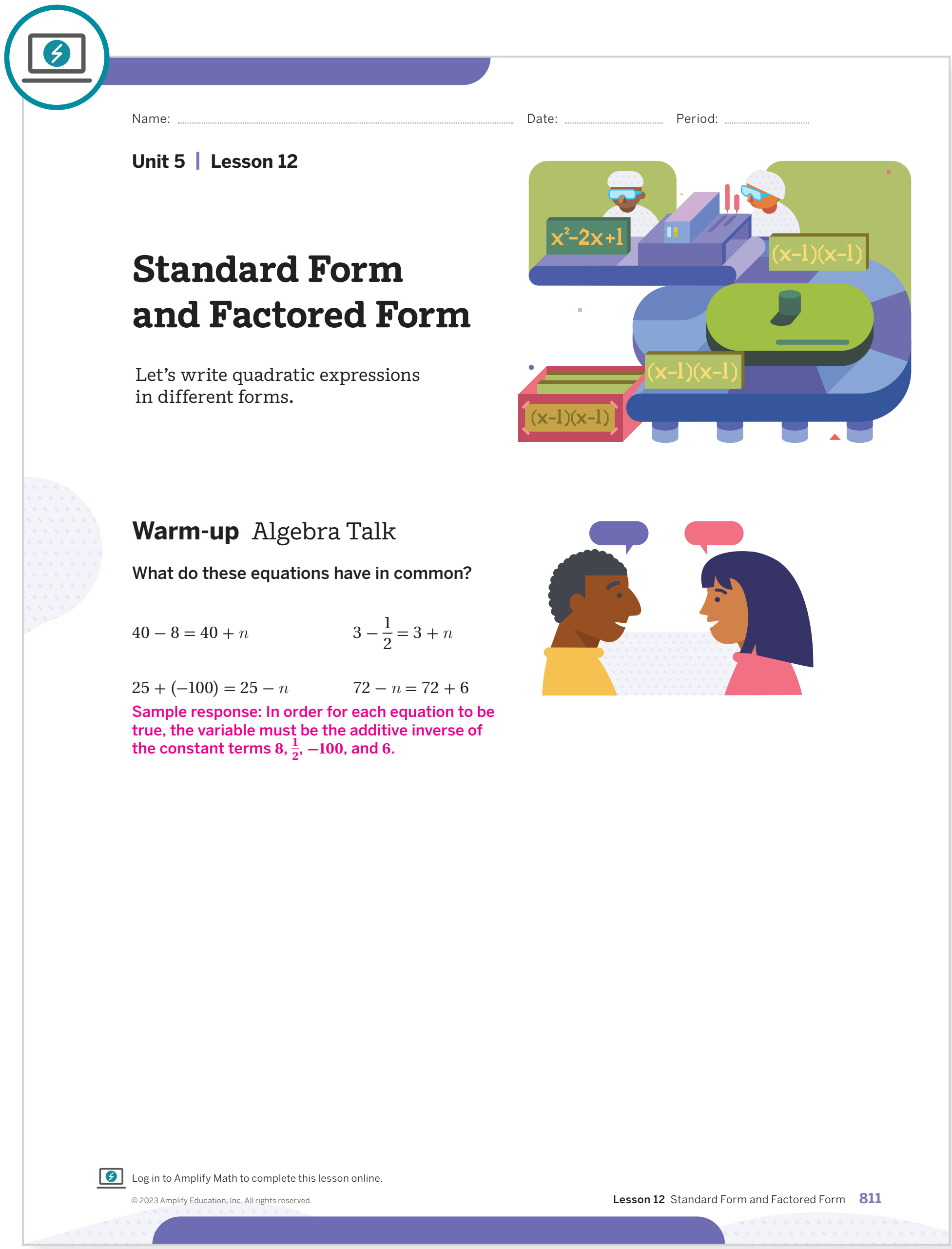
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students only consider two equations.
- In **Activity 1**, have students only complete the first three rows of the table.
- In **Activity 3**, the number of game cards may be reduced.

Independent | 5 min

Warm-up Algebra Talk

Students activate their prior knowledge of operations with numbers by studying four equations to review that subtracting a number is equivalent to adding the opposite of that number.



Name: ______________ Date: ________ Period: ________

Unit 5 | Lesson 12

Standard Form and Factored Form

Let's write quadratic expressions in different forms.

Warm-up Algebra Talk

What do these equations have in common?

$40 - 8 = 40 + n$

$3 - \frac{1}{2} = 3 + n$

$25 + (-100) = 25 - n$

$72 - n = 72 + 6$

Sample response: In order for each equation to be true, the variable must be the additive inverse of the constant terms 8, $\frac{1}{2}$, −100, and 6.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Standard Form and Factored Form 811

1 Launch

Conduct the ***Math Talk*** routine. Display two equations at a time, giving students time to think. Ask them to give a signal when they have a response. Stop revealing equations when students see the pattern of adding the opposite.

2 Monitor

Help students get started by asking, "What is the opposite of 100? 25? $\frac{1}{2}$? $25x$?"

Look for points of confusion:

- **Not understanding that subtraction is the same as adding a negative number.** Give them the expressions $40 - 8$ and $40 + (-8)$ and ask them to evaluate the expressions.

Look for productive strategies:

- Recognizing the pattern of adding the opposite.
- Rewriting subtraction as addition of the opposite.

3 Connect

Display the four equations.

Have individual students share their strategies or processes for thinking about each equation.

Ask, "Did anyone have the same strategy, but would like to explain it differently? Does anyone want to add on to ___'s strategy?"

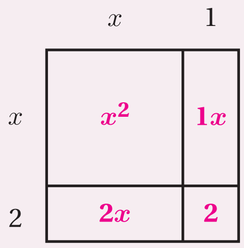
Highlight that subtracting a number provides the same outcome as adding that number's opposite. Thinking of subtraction in terms of addition can help students rewrite quadratic expressions such as $(x - 5)(x + 2)$ or $(x - 9)(x - 3)$, where one or both factors are differences.

Power-up

To power up students' ability to expand quadratic expressions written in factored form, have students complete:

Complete the area diagram to determine the product of $(x + 2)$ and $(x + 1)$.

$x^2 + 1x + 2x + 2$ or $x^2 + 3x + 2$



	x	1
x	x^2	$1x$
2	$2x$	2

Use: Before the Warm-up

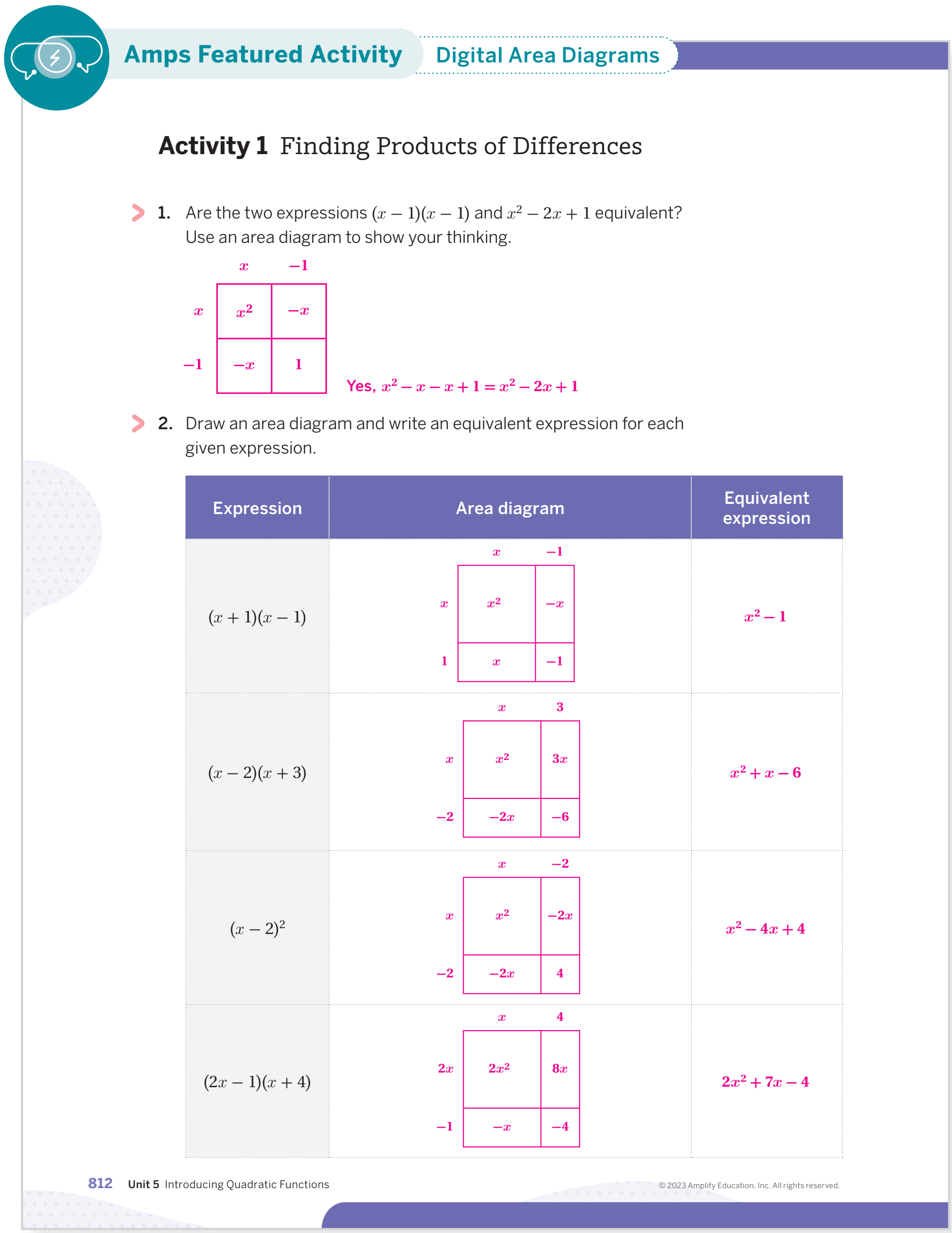
Informed by: Performance on Lesson 11, Practice Problem 5

Pairs | 20 min

Activity 1 Finding Products of Differences

Students return to the area diagram model, this time using area diagrams to model quadratic expressions in which at least one of the factors represents a difference.

Amps Featured Activity Digital Area Diagrams



Activity 1 Finding Products of Differences

1. Are the two expressions $(x-1)(x-1)$ and x^2-2x+1 equivalent? Use an area diagram to show your thinking.

$$\begin{array}{ccc} & x & -1 \\ x & x^2 & -x \\ -1 & -x & 1 \end{array}$$

Yes, $x^2-x-x+1=x^2-2x+1$

2. Draw an area diagram and write an equivalent expression for each given expression.

Expression	Area diagram	Equivalent expression
$(x+1)(x-1)$	$\begin{array}{ccc} & x & -1 \\ x & x^2 & -x \\ 1 & x & -1 \end{array}$	x^2-1
$(x-2)(x+3)$	$\begin{array}{ccc} & x & 3 \\ x & x^2 & 3x \\ -2 & -2x & -6 \end{array}$	x^2+x-6
$(x-2)^2$	$\begin{array}{ccc} & x & -2 \\ x & x^2 & -2x \\ -2 & -2x & 4 \end{array}$	x^2-4x+4
$(2x-1)(x+4)$	$\begin{array}{ccc} & x & 4 \\ 2x & 2x^2 & 8x \\ -1 & -x & -4 \end{array}$	$2x^2+7x-4$

812 Unit 5 Introducing Quadratic Functions

1 Launch

Have students complete Problem 1 individually before discussing with a partner. Then set an expectation for the amount of time students will have to complete Problem 2 with a partner.

2 Monitor

Help students get started by reminding them that area diagrams can be used to help write equivalent expressions.

Look for points of confusion:

- **Thinking that $(x-2)^2$ is equivalent to x^2-2^2.** Remind them that just as x^2 means $x \cdot x$, the expression $(x-2)^2$ means $(x-2)(x-2)$.

Look for productive strategies:

- Recognizing that $(x+p)(x+q)$ is equivalent to $x^2+(p+q)x+pq$ and using negative numbers for p and q, where applicable.

3 Connect

Display each expression with its corresponding area diagram and equivalent expression.

Have students share how they used the diagrams to write the equivalent expressions.

Highlight the area diagrams as a way to organize the terms of the factors by directly applying the Distributive Property.

Ask:

- "What if both p and q are negative? Can they be written as a sum?" Yes; Both differences can be rewritten as a sum.
- "Can you use the expressions from the activity to support your answer?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital diagrams to model binomial sums and differences or two differences. Using diagrams, they model the Distributive Property, finding and adding partial products to write equivalent expressions.

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Consider providing blank area diagrams for students to complete. Provide access to colored pencils and suggest they color code the factors and partial products.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share the area diagrams they drew, ask their classmates to critique and provide feedback. For example:

If a student says . . .	Their classmate could ask . . .
"My area diagram shows 4 terms, so my expression has 4 terms."	"Can you combine any like terms? How do you know which terms you can combine?"

Draw connections between each diagram and the structure of the corresponding equivalent expression.

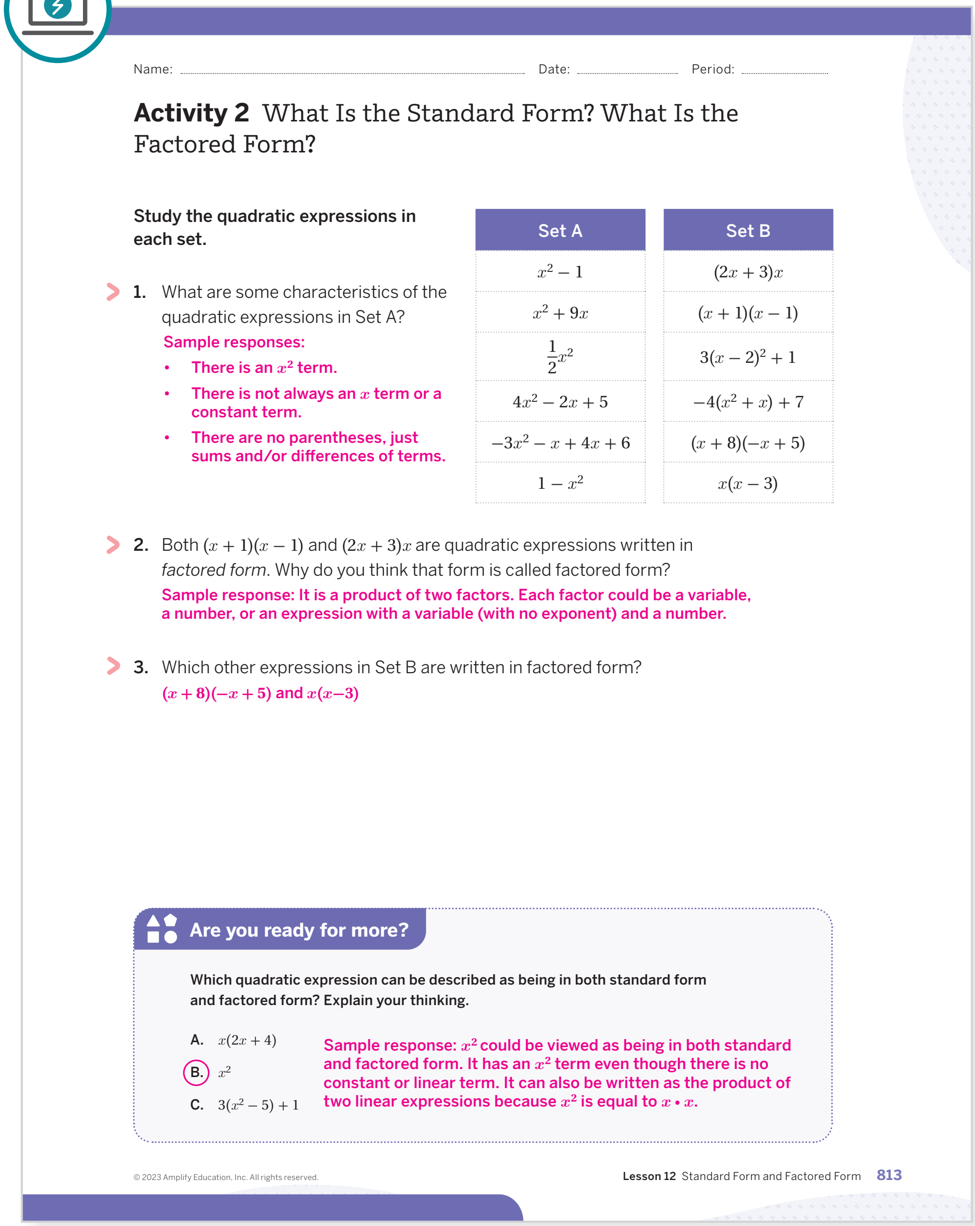
English Learners

Display or provide access to the Anchor Chart PDF, *Sentence Stems, Critiquing.*

Pairs | 15 min

Activity 2 What Is the Standard Form? What Is the Factored Form?

Students learn to distinguish the expressions by their forms and to refer to each form by its formal name.



Name: ________ Date: ________ Period: ________

Activity 2 What Is the Standard Form? What Is the Factored Form?

Study the quadratic expressions in each set.

Set A	Set B
$x^2 - 1$	$(2x + 3)x$
$x^2 + 9x$	$(x + 1)(x - 1)$
$\frac{1}{2}x^2$	$3(x - 2)^2 + 1$
$4x^2 - 2x + 5$	$-4(x^2 + x) + 7$
$-3x^2 - x + 4x + 6$	$(x + 8)(-x + 5)$
$1 - x^2$	$x(x - 3)$

1. What are some characteristics of the quadratic expressions in Set A?

   Sample responses:
   - There is an x^2 term.
   - There is not always an x term or a constant term.
   - There are no parentheses, just sums and/or differences of terms.

2. Both $(x + 1)(x - 1)$ and $(2x + 3)x$ are quadratic expressions written in *factored form*. Why do you think that form is called factored form?

   Sample response: It is a product of two factors. Each factor could be a variable, a number, or an expression with a variable (with no exponent) and a number.

3. Which other expressions in Set B are written in factored form?

   $(x + 8)(-x + 5)$ and $x(x–3)$

Are you ready for more?

Which quadratic expression can be described as being in both standard form and factored form? Explain your thinking.

A. $x(2x + 4)$

[✓ B.] x^2

C. $3(x^2 - 5) + 1$

Sample response: x^2 could be viewed as being in both standard and factored form. It has an x^2 term even though there is no constant or linear term. It can also be written as the product of two linear expressions because x^2 is equal to $x \cdot x$.

1 Launch

Display the tables and give students one minute to study the quadratic expressions in each set. Have students work individually, before sharing their responses with a partner.

2 Monitor

Help students get started by conducting the ***Notice and Wonder*** routine using the tables. Have students note any similarities and differences.

Look for points of confusion:

- **Thinking expressions in standard form must contain a constant term.** Remind them the constant could be equal to 0.

Look for productive strategies:

- Looking for visual cues relating expressions in standard form, such as the lack of parentheses with a coefficient in front.

3 Connect

Display the two sets of expressions.

Have students share the similarities and differences noticed in each form.

Highlight that each form gives useful information, which students will explore further in the next lesson.

Define *standard form* explicitly as $ax^2 + bx + c$, where a is the non-zero coefficient of the squared term x^2, b is the coefficient of the linear term x, and c is the constant term. The ***factored form*** is a product of two linear expressions.

Ask:

- "How would you write $(2x + 3)x$ in standard form?" $2x^2 + 3x$
- "What are the values of the coefficients a and b?" 2 and 3
- "What is the constant term?" 0

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students color code the factors of a product that are shown in the expressions in Set B. For example, in the expression $3(x + 2)^2 + 1$, have them color code 3 in one color and color code $(x + 2)^2$ in another color. Ask:

- "Why are these factors?"
- "Why is + 7 not a factor?"
- "Which expressions in Set B consist only of factors? Which expressions in Set B have additional terms added that are not factors?"

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the table showing the expressions in Sets A and B. Have students work with their partner to write 2–3 mathematical questions they could ask about the expressions in each set. Have volunteers share their questions with the class. Sample questions shown.

- What do the expressions in Set A have in common? Set B?
- Why are there no parentheses in Set A?
- Can I write another expression to place in each set?

English Learners

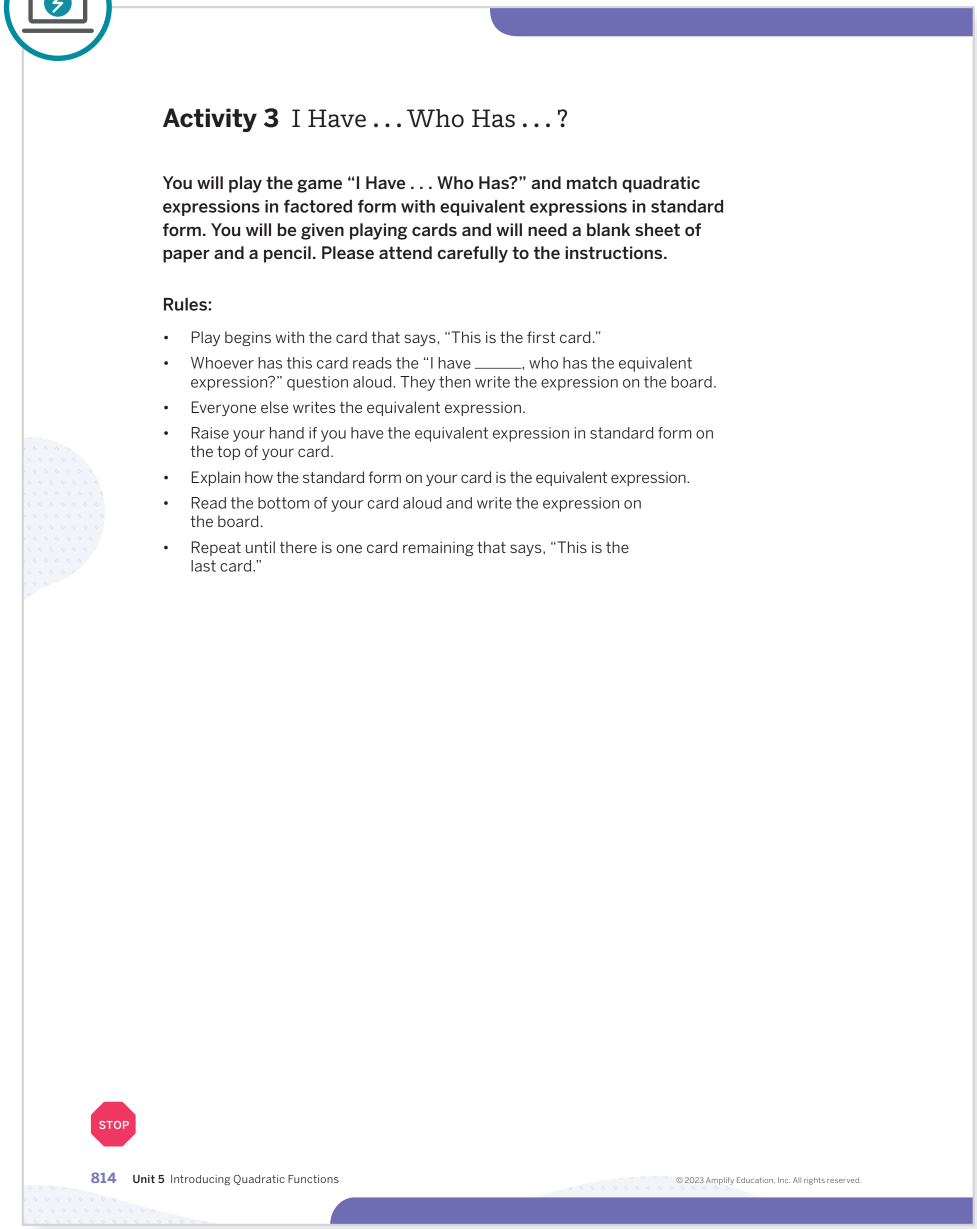
Model how to craft a mathematical question. Display one of the sample questions, using a sentence frame for students to complete.

Optional

Whole Class | 15 min

Activity 3 I Have . . . Who Has . . .?

Students play a matching game to strengthen the connections between quadratic expressions written in factored and standard forms.



Activity 3 I Have . . . Who Has . . . ?

You will play the game "I Have . . . Who Has?" and match quadratic expressions in factored form with equivalent expressions in standard form. You will be given playing cards and will need a blank sheet of paper and a pencil. Please attend carefully to the instructions.

Rules:

- Play begins with the card that says, "This is the first card."
- Whoever has this card reads the "I have ______, who has the equivalent expression?" question aloud. They then write the expression on the board.
- Everyone else writes the equivalent expression.
- Raise your hand if you have the equivalent expression in standard form on the top of your card.
- Explain how the standard form on your card is the equivalent expression.
- Read the bottom of your card aloud and write the expression on the board.
- Repeat until there is one card remaining that says, "This is the last card."

STOP

814 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the Activity 3 PDF (instructions). Shuffle and distribute the pre-cut from the Activity 3 PDF. Read the instructions and demonstrate, if needed, how to play the game. Conduct the *I Have, Who Has?* routine.

2 Monitor

Help students get started by having them create blank area diagrams on a separate sheet of paper.

Look for points of confusion:

- **Forgetting to apply the properties of exponents.** Remind students that exponents are added when multiplying powers that have the same base.

Look for productive strategies:

- Multiplying the coefficients and adding the powers of variable terms that have the same base.

3 Connect

Have students share their responses and strategies for writing the standard form, given the factored form.

Highlight how a diagram can help to expand an expression from factored form into standard form.

Ask:

- "What does it mean to expand a factored expression?" To multiply each term in one factor by each term in the other factor.
- "Show how a diagram can help you expand $(x + 4)(x - 10)$ and write an equivalent expression."

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Provide the Anchor Chart PDF, *Factored and Standard Forms of Quadratic Expressions* for students to use as a reference in this activity.

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Consider providing blank area diagrams for students to complete should they choose to do so. Provide access to colored pencils and suggest they color code the factors and partial products.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their strategies, demonstrate and model the use of their developing math language around the terms *factored form*. Ask:

- "Why do you think the factored form of a quadratic expression is called the *factored form*? What are factors?"
- "Give an example of a quadratic expression in factored form. What is an example of a quadratic expression that is *not* in factored form?" Draw connections between each diagram and the structure of the corresponding equivalent expression.

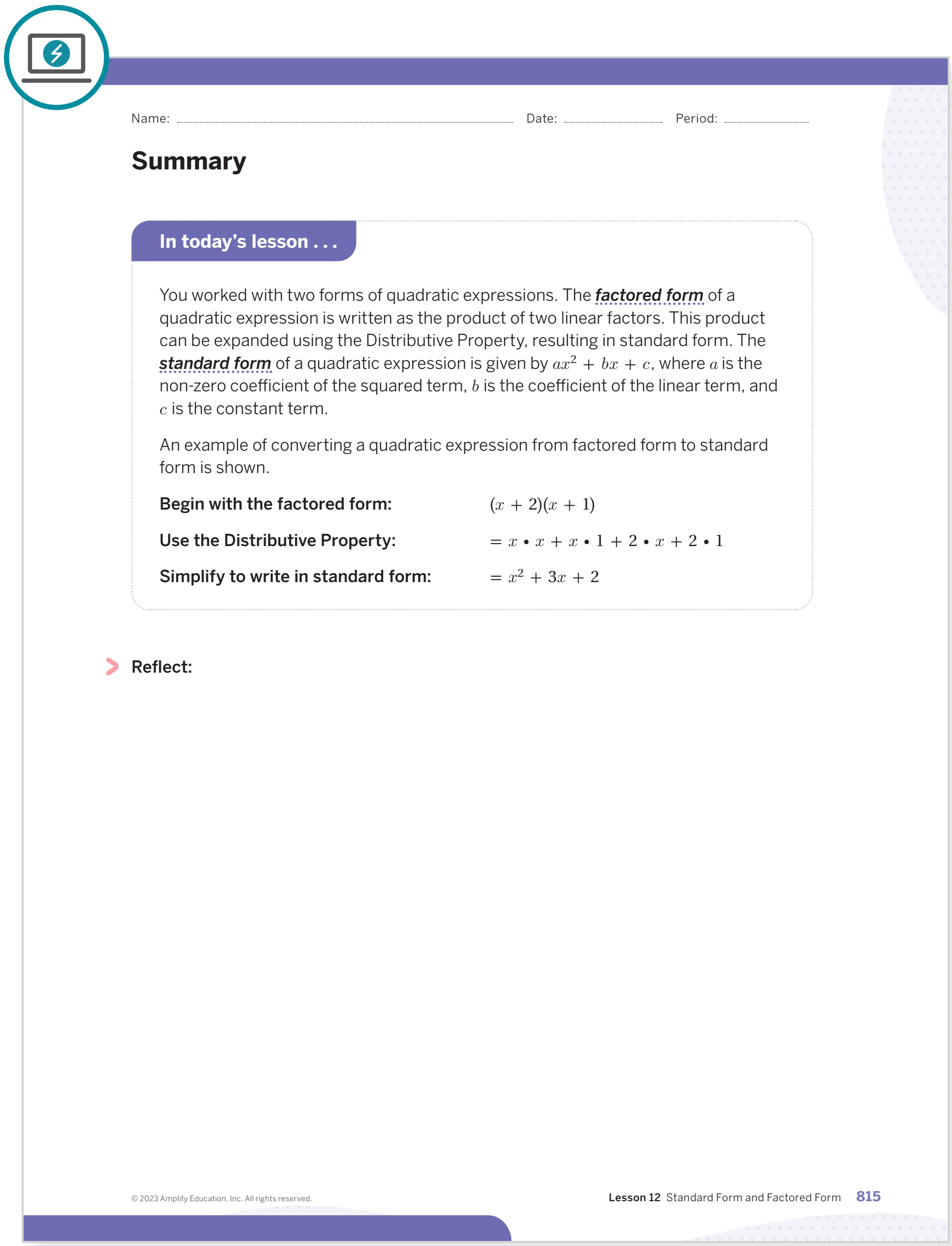
English Learners

Add examples of quadratic expressions in factored form to the class display.

Whole Class | 5 min

Summary

Review and synthesize how quadratic expressions can be written in standard form and factored form.



Name: Date: Period:

Summary

In today's lesson . . .

You worked with two forms of quadratic expressions. The ***factored form*** of a quadratic expression is written as the product of two linear factors. This product can be expanded using the Distributive Property, resulting in standard form. The ***standard form*** of a quadratic expression is given by $ax^2 + bx + c$, where a is the non-zero coefficient of the squared term, b is the coefficient of the linear term, and c is the constant term.

An example of converting a quadratic expression from factored form to standard form is shown.

Begin with the factored form:	$(x + 2)(x + 1)$
Use the Distributive Property:	$= x \cdot x + x \cdot 1 + 2 \cdot x + 2 \cdot 1$
Simplify to write in standard form:	$= x^2 + 3x + 2$

Reflect:

Lesson 12 Standard Form and Factored Form 815

Synthesize

Display the Anchor Chart PDF, *Factored and Standard Forms of Quadratic Expressions*.

Highlight the quadratic expression $x^2 + 3x + 2$ is written in standard form, in this case it is the sum of a multiple of x^2 and a linear expression $3x + 2$. In general, the standard form of a quadratic expression is $ax^2 + bx + c$. When the quadratic expression is a product of two factors — where each one is a linear expression — this is called the factored form. An expression in factored form can be rewritten in standard form by applying the Distributive Property.

Formalize vocabulary:

- *factored form*
- *standard form*

Ask:

- "How would you explain to a friend — who is absent today — how to write an equivalent expression for $(x - 10)(x - 5)$? What strategy (or strategies) would you suggest?"
- "Give a few different examples of quadratic expressions in standard form and a few in factored form. Ask a partner if they agree that your examples are indeed in those forms."

Have students share their strategies for rewriting $(x - 10)(x - 5)$ in standard form.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What are the differences and similarities between standard and factored form?"
- "Without using multiplication, how might you show that two quadratic expressions are equivalent?"

Math Language Development

MLR2: Collect and Display

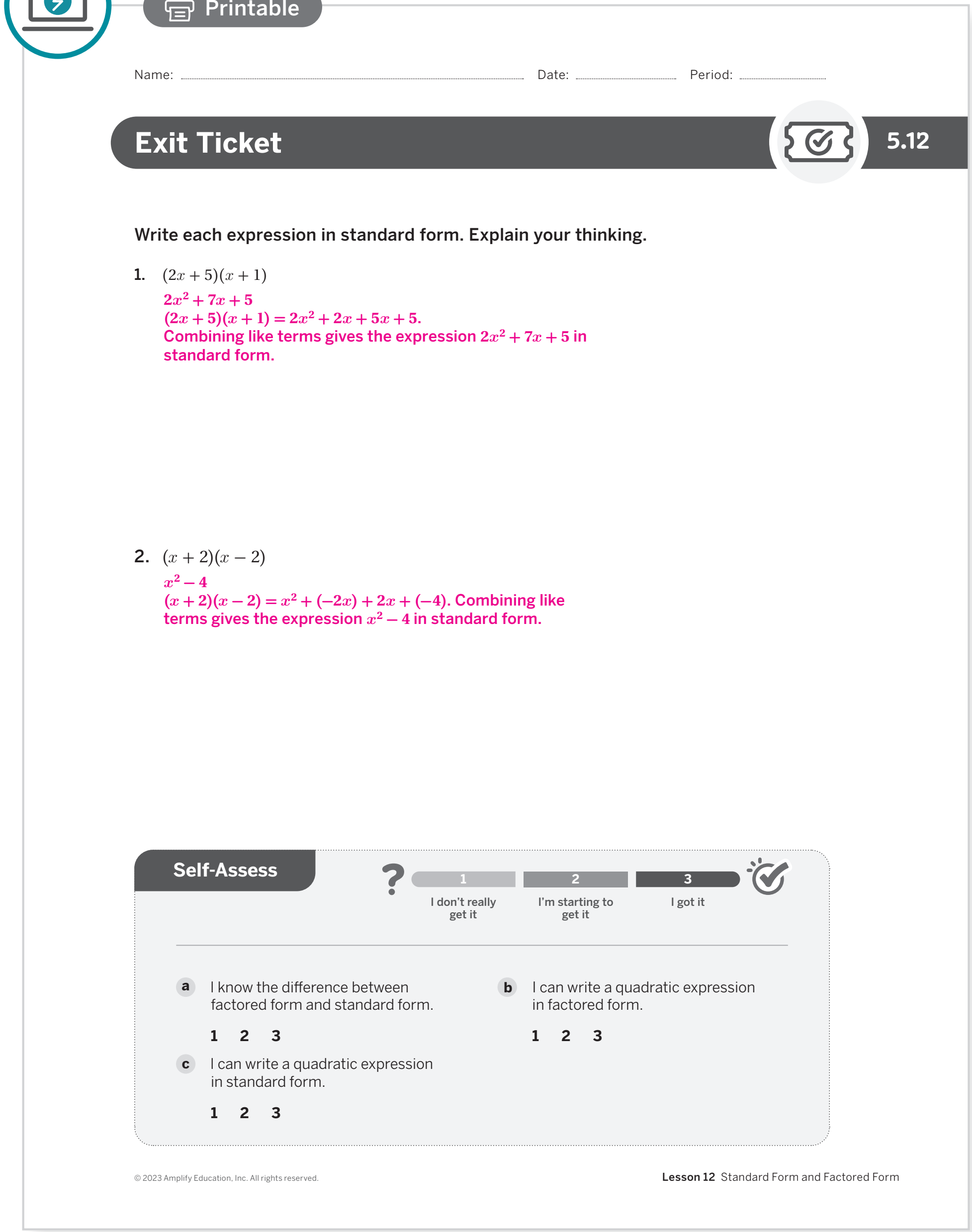
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *factored form* and *standard form* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing quadratic expressions — that are given in factored form — into standard form.

Printable



Name: Date: Period:

Exit Ticket 5.12

Write each expression in standard form. Explain your thinking.

1. $(2x + 5)(x + 1)$

   $2x^2 + 7x + 5$

   $(2x + 5)(x + 1) = 2x^2 + 2x + 5x + 5.$

   Combining like terms gives the expression $2x^2 + 7x + 5$ in standard form.

2. $(x + 2)(x - 2)$

   $x^2 - 4$

   $(x + 2)(x - 2) = x^2 + (-2x) + 2x + (-4)$. Combining like terms gives the expression $x^2 - 4$ in standard form.

Self-Assess

a I know the difference between factored form and standard form.
1 2 3

b I can write a quadratic expression in factored form.
1 2 3

c I can write a quadratic expression in standard form.
1 2 3

Lesson 12 Standard Form and Factored Form

Success looks like . . .

- **Language Goal:** Understanding and using the terms *standard form* and *factored form* when describing quadratic expressions. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Using rectangular diagrams to reason about the product of two differences or of a sum and difference, and to write equivalent equivalent expressions.
- **Goal:** Using the Distributive Property to write equivalent quadratic expressions that are given in either factored form or standard form.
  - » Using the Distributive Property to write the expressions in standard form in Problems 1 and 2.

Suggested next steps

If students struggle to multiply two linear factors consider:

- Reviewing multiplicative strategies from Activity 1 and Activity 2.
- Assigning Practice Problem 1.
- Asking, "How would you rewrite this expression if, instead of multiplying $(2x + 5)$ $(x + 1)$, you were multiplying $2x(x + 1)$ and $5(x + 1)$?" Then have them determine the sum of the products.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did the Warm-up support students in writing equivalent quadratic expressions?
- When you compare and contrast today's work with work students did earlier this unit on determining the product of two expressions, what similarities and differences do you see?

Math Language Development

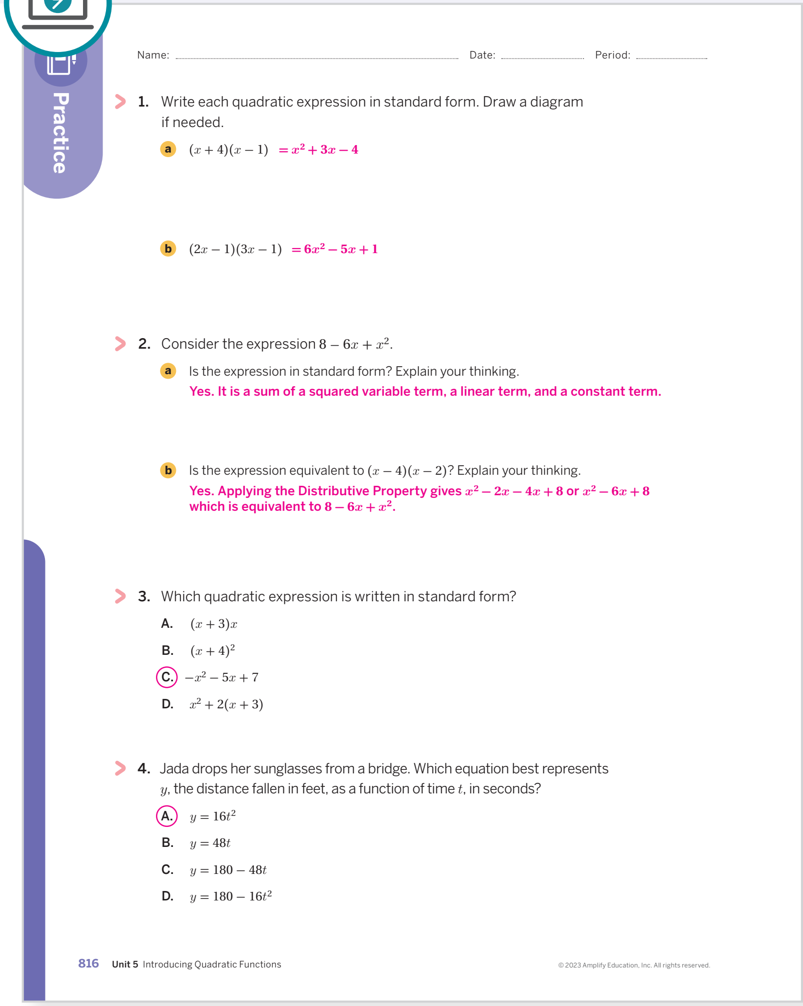
Language Goal: Understanding and using the terms *standard form* and *factored form* when describing quadratic expressions.

Reflect on students' language development toward this goal.

- How have students progressed in their comfort using these terms as they identify quadratic expressions?
- How did using the language routines in this lesson help students deepen their understanding of these two different forms of quadratic expressions? Would you change anything the next time you use these routines?

Practice

Independent



Practice

Name: Date: Period:

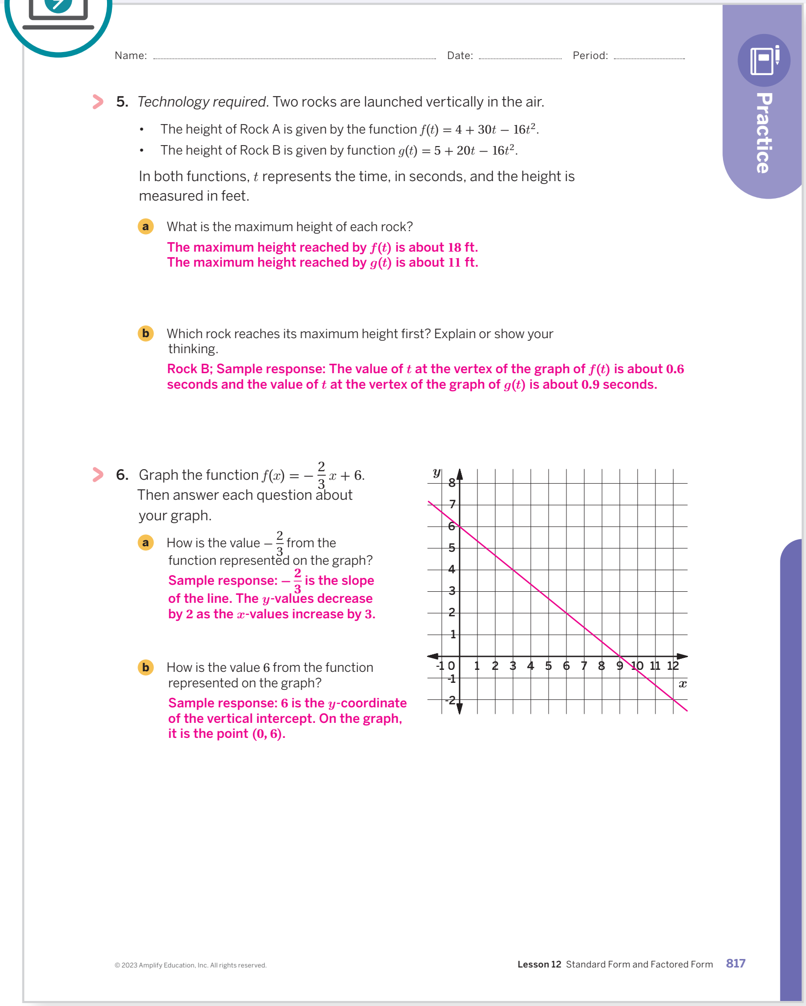
1. Write each quadratic expression in standard form. Draw a diagram if needed.
   - a $(x+4)(x-1)$ $= x^2 + 3x - 4$
   - b $(2x-1)(3x-1)$ $= 6x^2 - 5x + 1$

2. Consider the expression $8 - 6x + x^2$.
   - a Is the expression in standard form? Explain your thinking.
     Yes. It is a sum of a squared variable term, a linear term, and a constant term.
   - b Is the expression equivalent to $(x-4)(x-2)$? Explain your thinking.
     Yes. Applying the Distributive Property gives $x^2 - 2x - 4x + 8$ or $x^2 - 6x + 8$ which is equivalent to $8 - 6x + x^2$.

3. Which quadratic expression is written in standard form?
   - A. $(x+3)x$
   - B. $(x+4)^2$
   - [✓ C.] $-x^2 - 5x + 7$
   - D. $x^2 + 2(x+3)$

4. Jada drops her sunglasses from a bridge. Which equation best represents y, the distance fallen in feet, as a function of time t, in seconds?
   - [✓ A.] $y = 16t^2$
   - B. $y = 48t$
   - C. $y = 180 - 48t$
   - D. $y = 180 - 16t^2$

816 **Unit 5** Introducing Quadratic Functions © 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

5. *Technology required.* Two rocks are launched vertically in the air.
   - The height of Rock A is given by the function $f(t) = 4 + 30t - 16t^2$.
   - The height of Rock B is given by function $g(t) = 5 + 20t - 16t^2$.

   In both functions, t represents the time, in seconds, and the height is measured in feet.
   - a What is the maximum height of each rock?
     The maximum height reached by $f(t)$ is about 18 ft.
     The maximum height reached by $g(t)$ is about 11 ft.
   - b Which rock reaches its maximum height first? Explain or show your thinking.
     Rock B; Sample response: The value of t at the vertex of the graph of $f(t)$ is about 0.6 seconds and the value of t at the vertex of the graph of $g(t)$ is about 0.9 seconds.

6. Graph the function $f(x) = -\frac{2}{3}x + 6$. Then answer each question about your graph.
   - a How is the value $-\frac{2}{3}$ from the function represented on the graph?
     Sample response: $-\frac{2}{3}$ is the slope of the line. The y-values decrease by 2 as the x-values increase by 3.
   - b How is the value 6 from the function represented on the graph?
     Sample response: 6 is the y-coordinate of the vertical intercept. On the graph, it is the point $(0, 6)$.

© 2023 Amplify Education, Inc. All rights reserved. **Lesson 12** Standard Form and Factored Form 817

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 3	2
	2	Activity 2	3
	3	Activities 2 and 3	1
Spiral	4	Unit 5 Lesson 7	2
	5	Unit 5 Lesson 8	3
Formative	6	Unit 5 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

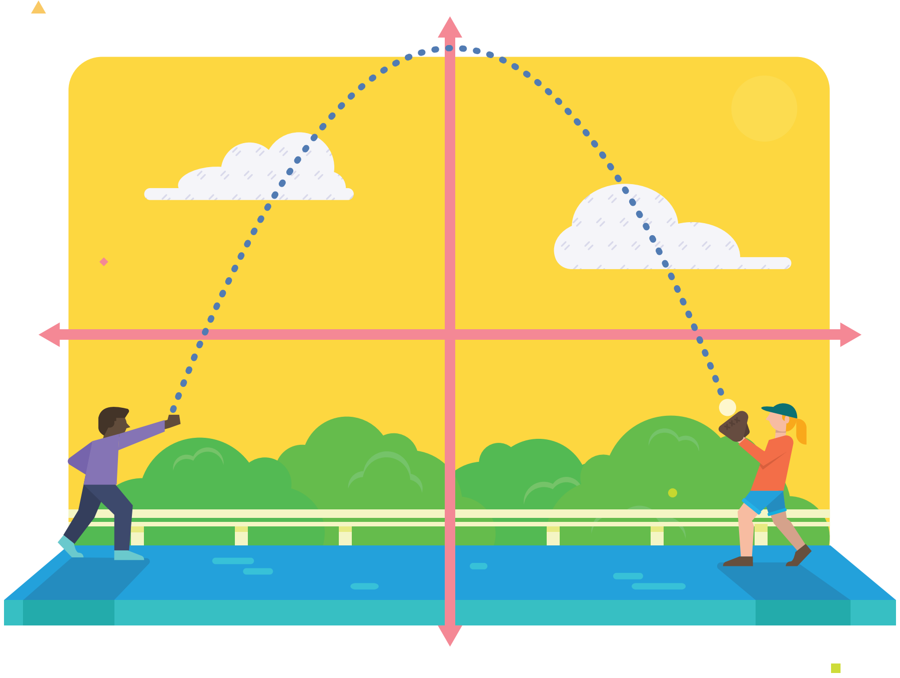
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 13**

Graphs of Functions in Standard and Factored Forms

Let's explore what information each quadratic form reveals about the properties of their graphs.



Focus

Goals

1. Identify the intercepts on a graph of a quadratic function.
2. **Language Goal:** Relate the value(s) in the factored form of a quadratic function to the intercepts of its graph. **(Speaking and Listening, Writing)**
3. **Language Goal:** Relate the value(s) in the standard form of a quadratic function to the intercepts of the graph. **(Speaking and Listening, Writing)**

Coherence

Today

Students make connections between the standard and factored forms of quadratic expressions and features of the graphs that represent them. They identify the x- and y-intercepts of graphs and observe that some of the values in the standard or factored forms are related to the intercepts.

Previously

In Lesson 12, students expanded expressions in factored form, and formally defined two forms of quadratics: *standard form* and *factored form*.

Coming Soon

In Lesson 14, students will use a quadratic function expressed in factored form to identify key coordinates of the graph of the function.

Rigor

- Students graph quadratics in standard and factored form to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *factored form*
- *standard form*

Amps | **Featured Activity**

Activity 2

Interactive Graph

Students can use interactive graphs to identify x- and y-intercepts of quadratics.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students explore the connections between the functions and their graphs in Activity 1, they may feel defeated if they cannot easily see the relationships. Remind students to share their thinking with the partner. As pairs process the problem together looking for how the structure of the function relates to the corresponding graph, they will build their self-confidence leading to increased self-efficacy with quadratic graphs.

Modifications to Pacing

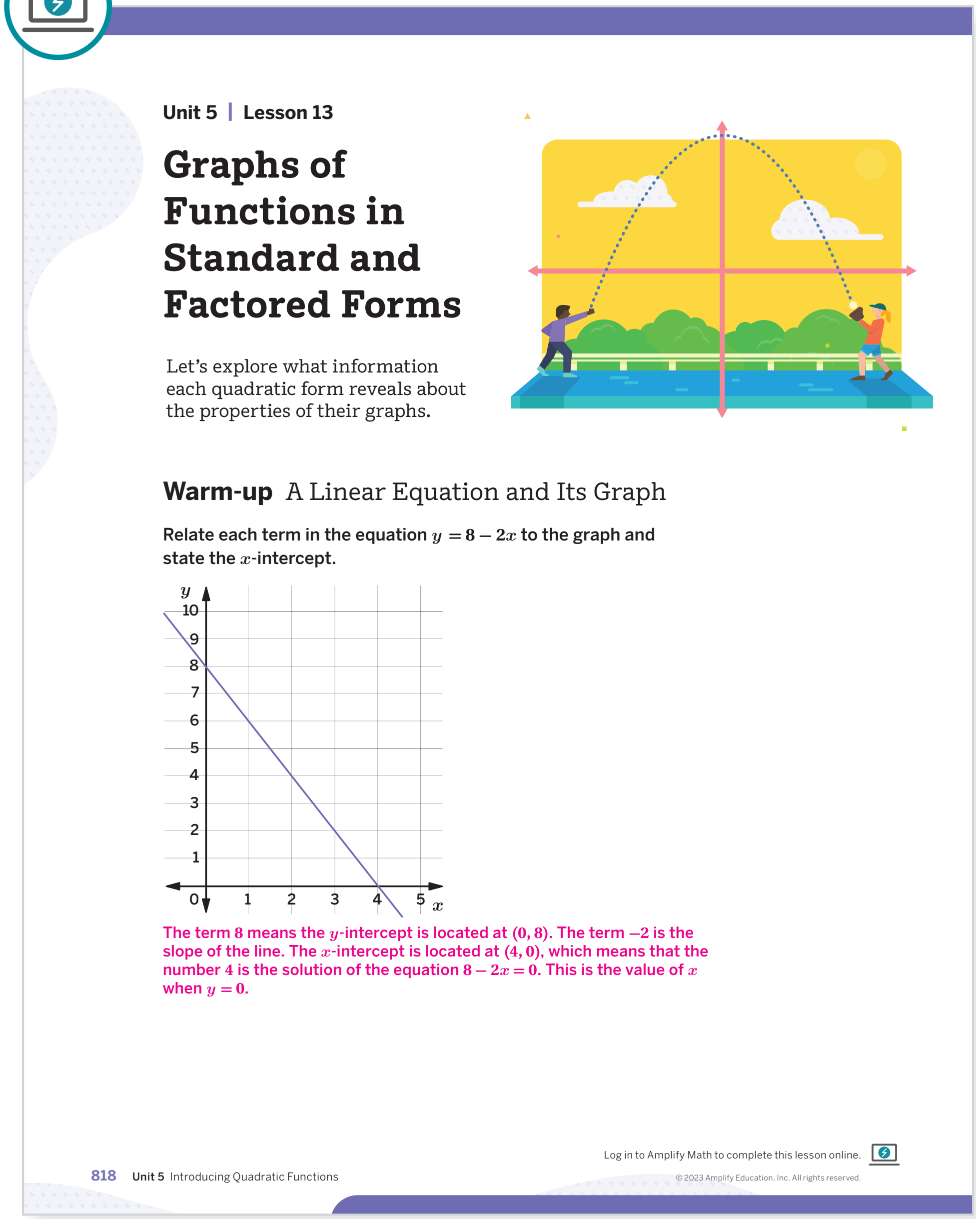
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students complete only Problems 4, 5, and 6.

Independent | 5 min

Warm-up A Linear Equation and Its Graph

Students revisit slope-intercept form of a linear function to prepare them for understanding how the forms of quadratic functions can tell them information about their graphs.



Unit 5 | Lesson 13

Graphs of Functions in Standard and Factored Forms

Let's explore what information each quadratic form reveals about the properties of their graphs.

Warm-up A Linear Equation and Its Graph

Relate each term in the equation $y = 8 - 2x$ to the graph and state the x-intercept.

The term 8 means the y-intercept is located at $(0, 8)$. The term -2 is the slope of the line. The x-intercept is located at $(4, 0)$, which means that the number 4 is the solution of the equation $8 - 2x = 0$. This is the value of x when $y = 0$.

Log in to Amplify Math to complete this lesson online.

818 **Unit 5** Introducing Quadratic Functions

1 Launch

Say, "Recall that an equation can tell you something about the graph that represents it, and vice versa."

2 Monitor

Help students get started by activiting their prior knowledge by asking, "What does the slope-intercept form of a linear equation tell you about the graph?"

Look for points of confusion:

- **Not recognizing 8 as the y-intercept b, or not recognizing -2 as the slope m in the equation.** Have students label the y-intercept on the graph and determine the slope of the line.

Look for productive strategies:

- Recognizing that 8 is the y-intercept and -2 is the slope in the equation $y = 8 - 2x$.
- Identifying the x- and y-intercepts on the graph.

3 Connect

Display the graph and the equation $y = 8 - 2x$.

Have students share their strategies for identifying the slope and intercepts from the equation and graph.

Ask, "What does the equation tell you?" The slope and the y-intercept.

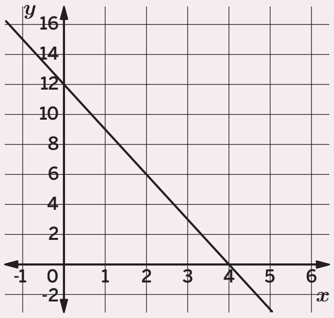
Highlight that equations provide information about their graphs and the graphs provide information about their equations. In a linear equation of the form $y = mx + b$, the y-intercept is b, given by the ordered pair $(0, b)$. Ask, "How would you locate the x-intercept given the equation $y = mx + b$?" Let $y = 0$ and solve for x.

Power-up

To power up students' ability to connect the structure of a linear equation to its graph, have students complete:

Recall that, for linear equations of the form $f(x) = mx + b$, m represents the slope and b represents the y-coordinate of the vertical intercept, $(0, b)$.

The graph of $f(x) = -3x + 12$ is shown.



1. What is the slope of the line? -3

2. What are the coordinates of the y-intercept of the line? $(0, 12)$

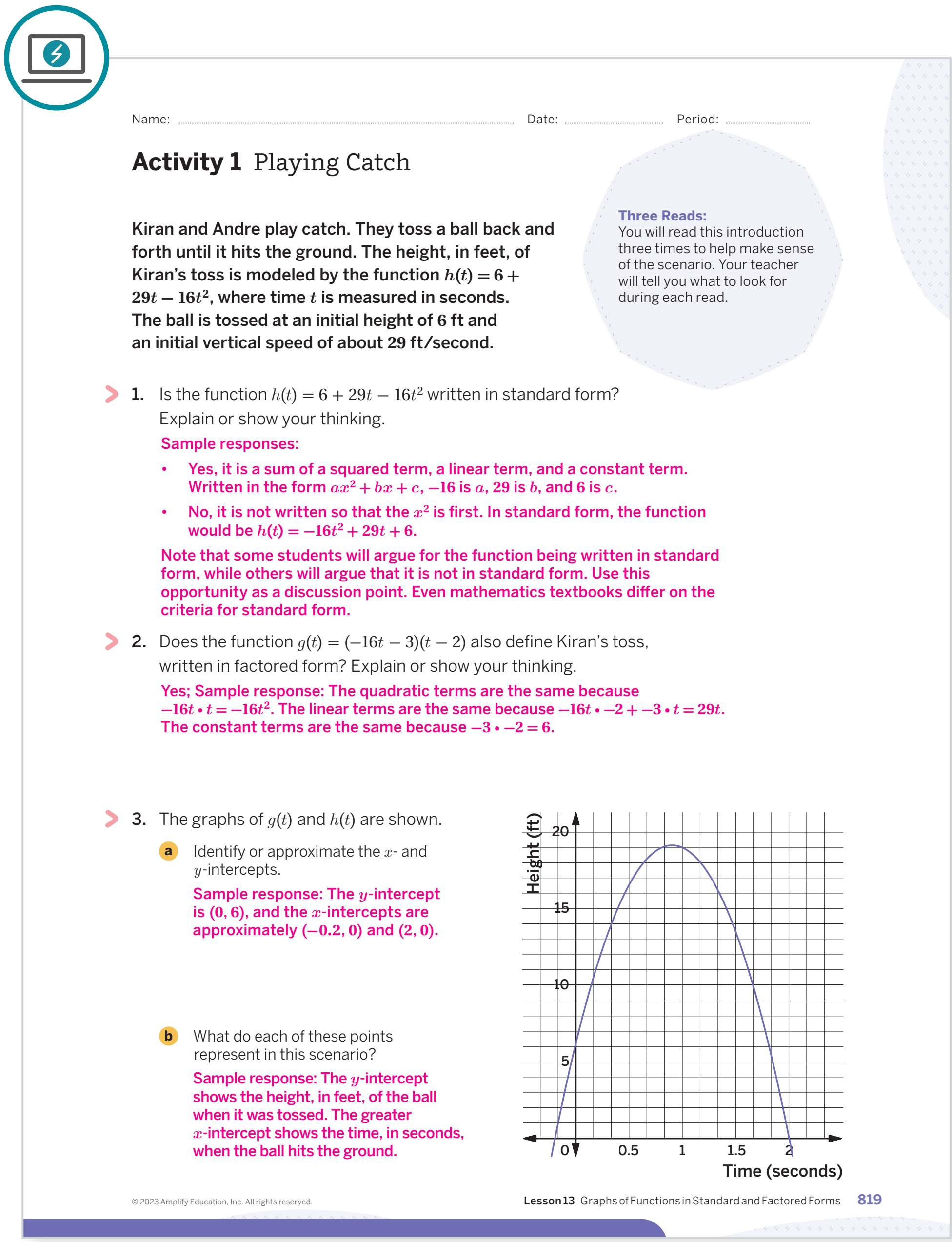
Use: Before the Warm-up

Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 15 min

Activity 1 Playing Catch

Students study two equivalent quadratic functions and their graph to understand how the standard and factored forms reveal key features of the graph.



Name: ________ Date: ________ Period: ________

Activity 1 Playing Catch

Kiran and Andre play catch. They toss a ball back and forth until it hits the ground. The height, in feet, of Kiran's toss is modeled by the function $h(t) = 6 + 29t - 16t^2$, where time t is measured in seconds. The ball is tossed at an initial height of 6 ft and an initial vertical speed of about 29 ft/second.

Three Reads: You will read this introduction three times to help make sense of the scenario. Your teacher will tell you what to look for during each read.

1. Is the function $h(t) = 6 + 29t - 16t^2$ written in standard form? Explain or show your thinking.

   Sample responses:
   - **Yes, it is a sum of a squared term, a linear term, and a constant term. Written in the form $ax^2 + bx + c$, -16 is a, 29 is b, and 6 is c.**
   - **No, it is not written so that the x^2 is first. In standard form, the function would be $h(t) = -16t^2 + 29t + 6$.**

   Note that some students will argue for the function being written in standard form, while others will argue that it is not in standard form. Use this opportunity as a discussion point. Even mathematics textbooks differ on the criteria for standard form.

2. Does the function $g(t) = (-16t - 3)(t - 2)$ also define Kiran's toss, written in factored form? Explain or show your thinking.

   Yes; Sample response: The quadratic terms are the same because $-16t \cdot t = -16t^2$. The linear terms are the same because $-16t \cdot -2 + -3 \cdot t = 29t$. The constant terms are the same because $-3 \cdot -2 = 6$.

3. The graphs of $g(t)$ and $h(t)$ are shown.

   a. Identify or approximate the x- and y-intercepts.

   Sample response: The y-intercept is $(0, 6)$, and the x-intercepts are approximately $(-0.2, 0)$ and $(2, 0)$.

   b. What do each of these points represent in this scenario?

   Sample response: The y-intercept shows the height, in feet, of the ball when it was tossed. The greater x-intercept shows the time, in seconds, when the ball hits the ground.

Lesson 13 Graphs of Functions in Standard and Factored Forms 819

1 Launch

Have students complete Problems 1 and 2 individually. Then have them share responses with a partner before completing Problem 3.

2 Monitor

Help students get started by reviewing standard and factored forms of quadratic expressions.

Look for points of confusion:

- **Not recognizing the squared variable term because it is being subtracted and placed at the end of the expression in Problem 1.** Show them a simpler expression, such as $5 - 10$, and rewrite it as $5 + (-10)$. Then, ask them to rewrite the function $h(t)$ in a similar way.

Look for productive strategies:

- Recognizing the terms in the standard form are commutative and rewriting the function in Problem 1 as $h(t) = -16t^2 + 29t + 6$.
- Converting factored form to standard form by multiplying the factors in Problem 2.

3 Connect

Display the functions $h(t)$ and $g(t)$ and the graph.

Have students share strategies used to show that $h(t)$ and $g(t)$ are equivalent.

Highlight that the function $h(t) = 6 + 29t - 16t^2$ is in standard form, even though it is written as $c + bx + ax^2$. -16 is a (the coefficient of the squared variable term), 29 is b (the coefficient of the linear term), and 6 is c (the constant term).

Ask, "Do you notice any connections between the two functions and the features of the graph?" Sample response: The number 6 in $6 + 29t - 16t^2$ is the vertical intercept.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Display or provide copies of the Anchor Chart PDF, *Factored and Standard Forms of Quadratic Expressions* for students to use as a reference as they determine whether the function $g(t)$ is written in factored form.

Extension: *Math Enrichment*

Have students explain whether the negative x-intercept makes sense within the context of this scenario. No; It would represent negative time, which does not make sense in this scenario.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand Kiran tossed a ball into the air and the ball will eventually hit the ground.
- **Read 2:** Ask students to identify the given quantities or relationships, such as the initial height of the ball when it was tossed was 6 ft above the ground.
- **Read 3:** Ask students to study the structure of the function $h(t)$ and think about whether it is in factored form or standard form.

English Learners

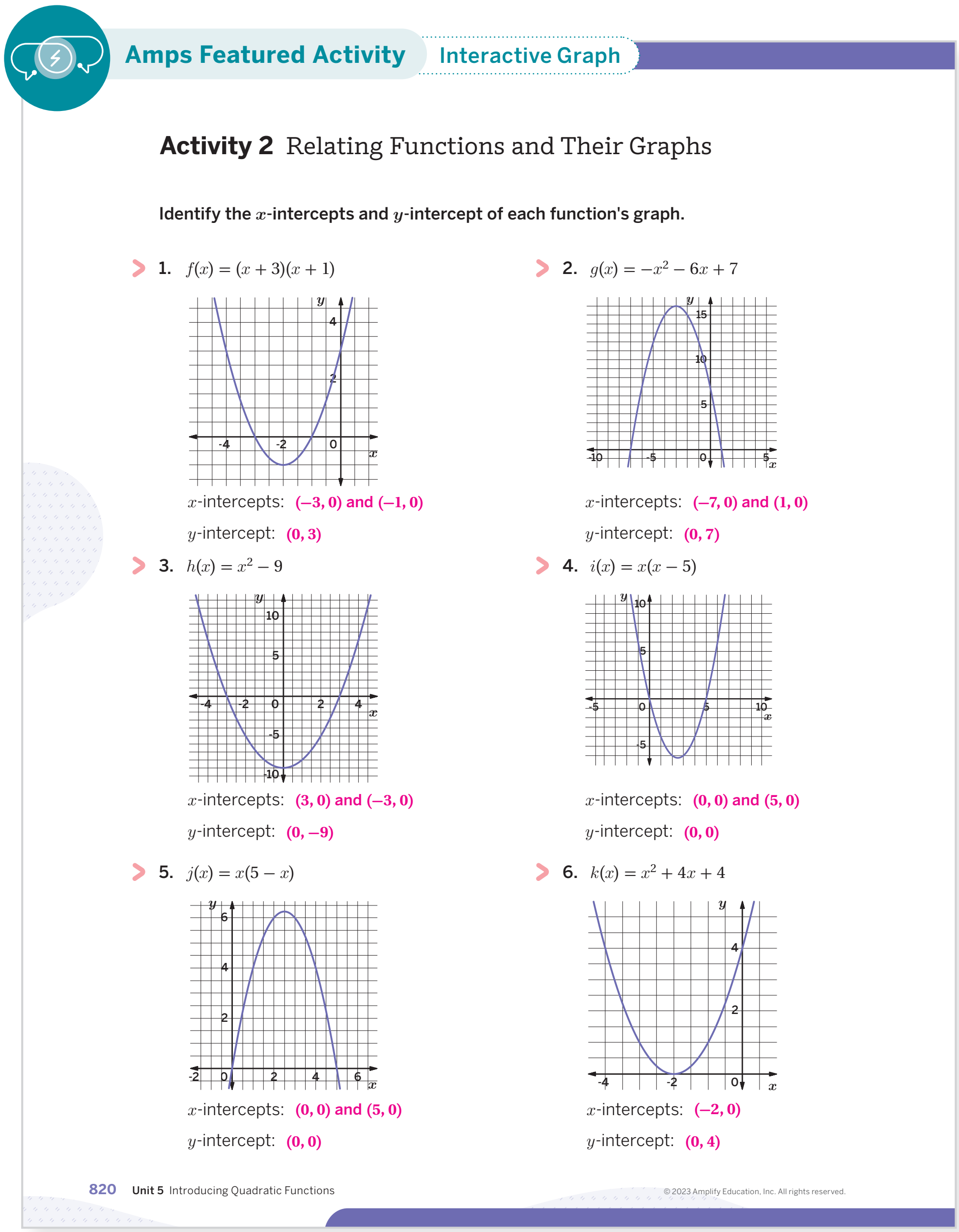
Draw a quick sketch of what Kiran's toss might look like and ask students what the term 6 represents.

Pairs | 20 min

Activity 2 Relating Functions and Their Graphs

Students activate their prior knowledge of x- and y-intercepts by connecting them to the parameters of quadratic expressions written in both standard and factored form.

Amps Featured Activity Interactive Graph



Activity 2 Relating Functions and Their Graphs

Identify the x-intercepts and y-intercept of each function's graph.

1. $f(x) = (x + 3)(x + 1)$

x-intercepts: $(-3, 0)$ and $(-1, 0)$

y-intercept: $(0, 3)$

2. $g(x) = -x^2 - 6x + 7$

x-intercepts: $(-7, 0)$ and $(1, 0)$

y-intercept: $(0, 7)$

3. $h(x) = x^2 - 9$

x-intercepts: $(3, 0)$ and $(-3, 0)$

y-intercept: $(0, -9)$

4. $i(x) = x(x - 5)$

x-intercepts: $(0, 0)$ and $(5, 0)$

y-intercept: $(0, 0)$

5. $j(x) = x(5 - x)$

x-intercepts: $(0, 0)$ and $(5, 0)$

y-intercept: $(0, 0)$

6. $k(x) = x^2 + 4x + 4$

x-intercepts: $(-2, 0)$

y-intercept: $(0, 4)$

820 Unit 5 Introducing Quadratic Functions

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students work on Problems 1–6 independently before discussing their responses with a partner.

2 Monitor

Help students get started by reminding them that the x-intercepts are located where the graph intersects the horizontal axis and the y-intercept is located where the graph intersects the vertical axis.

Look for points of confusion:

- **Switching the x- and y-intercepts.** Remind students that the x-intercepts are located where the graph intersects the x-axis and the y-intercept is located where the graph intersects the y-axis.
- **Thinking that each graph has to have two x-intercepts (Problem 6).** Remind students that if the vertex is located on the horizontal axis, there is only one x-intercept.

Look for productive strategies:

- Recognizing that at the x-intercept, $y = 0$, and at the y-intercept, $x = 0$.
- Connecting the constant term in standard form to the y-intercept of the graph.
- Connecting the factored form to the x-intercepts on the graph.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive graphs to identify x- and y-intercepts of quadratics.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest they color code the x-intercepts in one color and the y-intercepts in another color.

Extension: *Math Enrichment*

Ask students if the function $w(x) = (x + 1)(x + 2)(x + 3)$ is written in factored form and whether it is a quadratic function. It is written in factored form, but it is not quadratic because there will be an x^3 term.

Math Language Development

MLR7: Compare and Connect

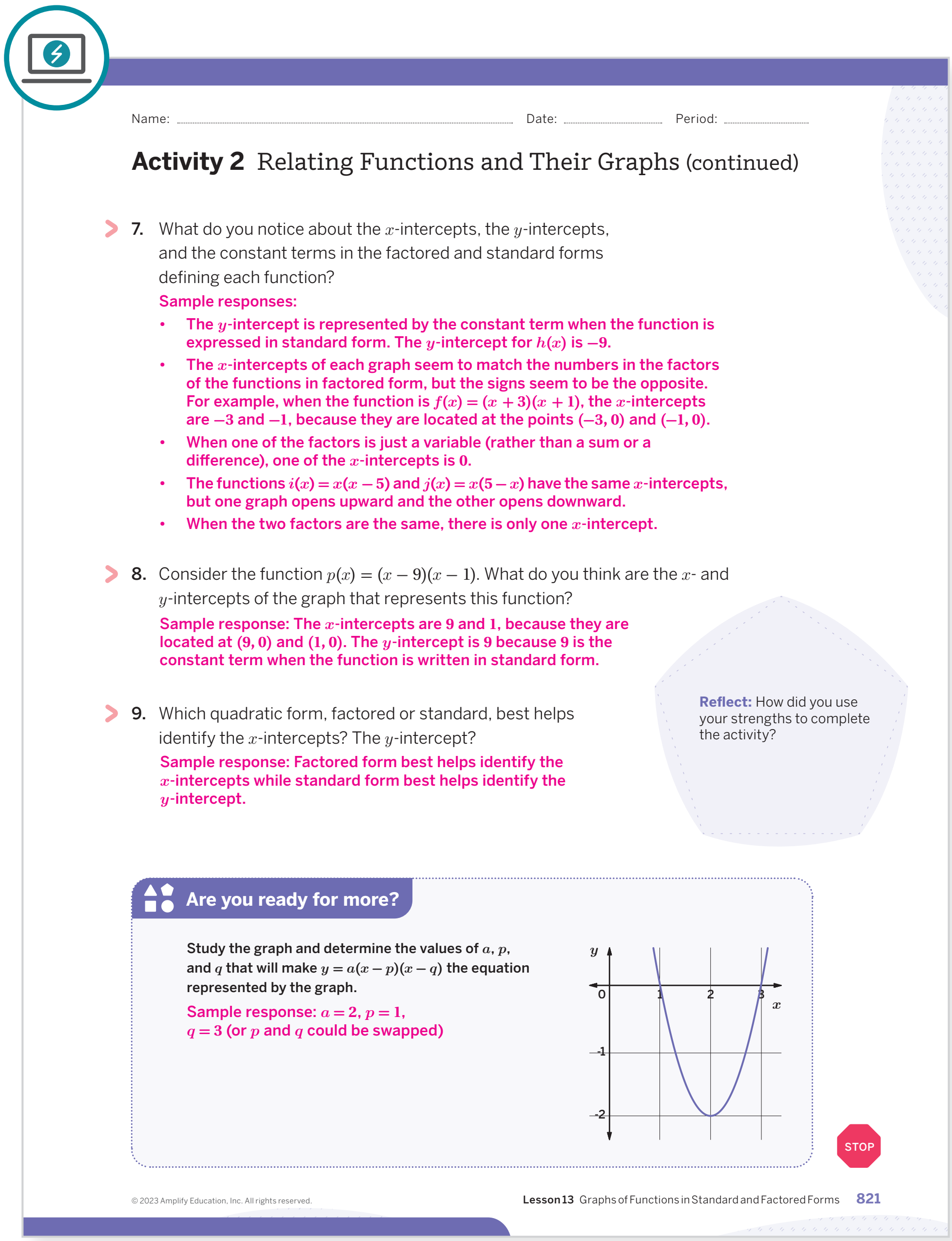
During the Connect, as you highlight the features that a function written in factored form or standard form illustrates, add the following to the class display. Have students help you complete the table as you add it to the display.

Factored form	Standard form
Indicates the x-intercepts (the zeros of the function). $f(x) = (x + 2)(x - 5)$ x-intercepts: -2 and 5	Indicates the y-intercept. $f(x) = x^2 - 3x - 10$ y-intercept: -10

Pairs | 20 min

Activity 2 Relating Functions and Their Graphs (continued)

Students activate their prior knowledge of x- and y-intercepts by connecting them to the parameters of quadratic expressions written in both standard and factored form.



Name: Date: Period:

Activity 2 Relating Functions and Their Graphs (continued)

7. What do you notice about the x-intercepts, the y-intercepts, and the constant terms in the factored and standard forms defining each function?

   Sample responses:
   - **The y-intercept is represented by the constant term when the function is expressed in standard form. The y-intercept for $h(x)$ is -9.**
   - **The x-intercepts of each graph seem to match the numbers in the factors of the functions in factored form, but the signs seem to be the opposite. For example, when the function is $f(x) = (x + 3)(x + 1)$, the x-intercepts are -3 and -1, because they are located at the points $(-3, 0)$ and $(-1, 0)$.**
   - **When one of the factors is just a variable (rather than a sum or a difference), one of the x-intercepts is 0.**
   - **The functions $i(x) = x(x - 5)$ and $j(x) = x(5 - x)$ have the same x-intercepts, but one graph opens upward and the other opens downward.**
   - **When the two factors are the same, there is only one x-intercept.**

8. Consider the function $p(x) = (x - 9)(x - 1)$. What do you think are the x- and y-intercepts of the graph that represents this function?

   Sample response: The x-intercepts are 9 and 1, because they are located at $(9, 0)$ and $(1, 0)$. The y-intercept is 9 because 9 is the constant term when the function is written in standard form.

9. Which quadratic form, factored or standard, best helps identify the x-intercepts? The y-intercept?

   Sample response: Factored form best helps identify the x-intercepts while standard form best helps identify the y-intercept.

Reflect: How did you use your strengths to complete the activity?

Are you ready for more?

Study the graph and determine the values of a, p, and q that will make $y = a(x - p)(x - q)$ the equation represented by the graph.

Sample response: $a = 2$, $p = 1$, $q = 3$ (or p and q could be swapped)

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Graphs of Functions in Standard and Factored Forms 821

③ Connect

Display the functions and their graphs.

Have students share what they notice about the x-intercepts, y-intercepts, and the constant terms in the factored and standard forms defining each function.

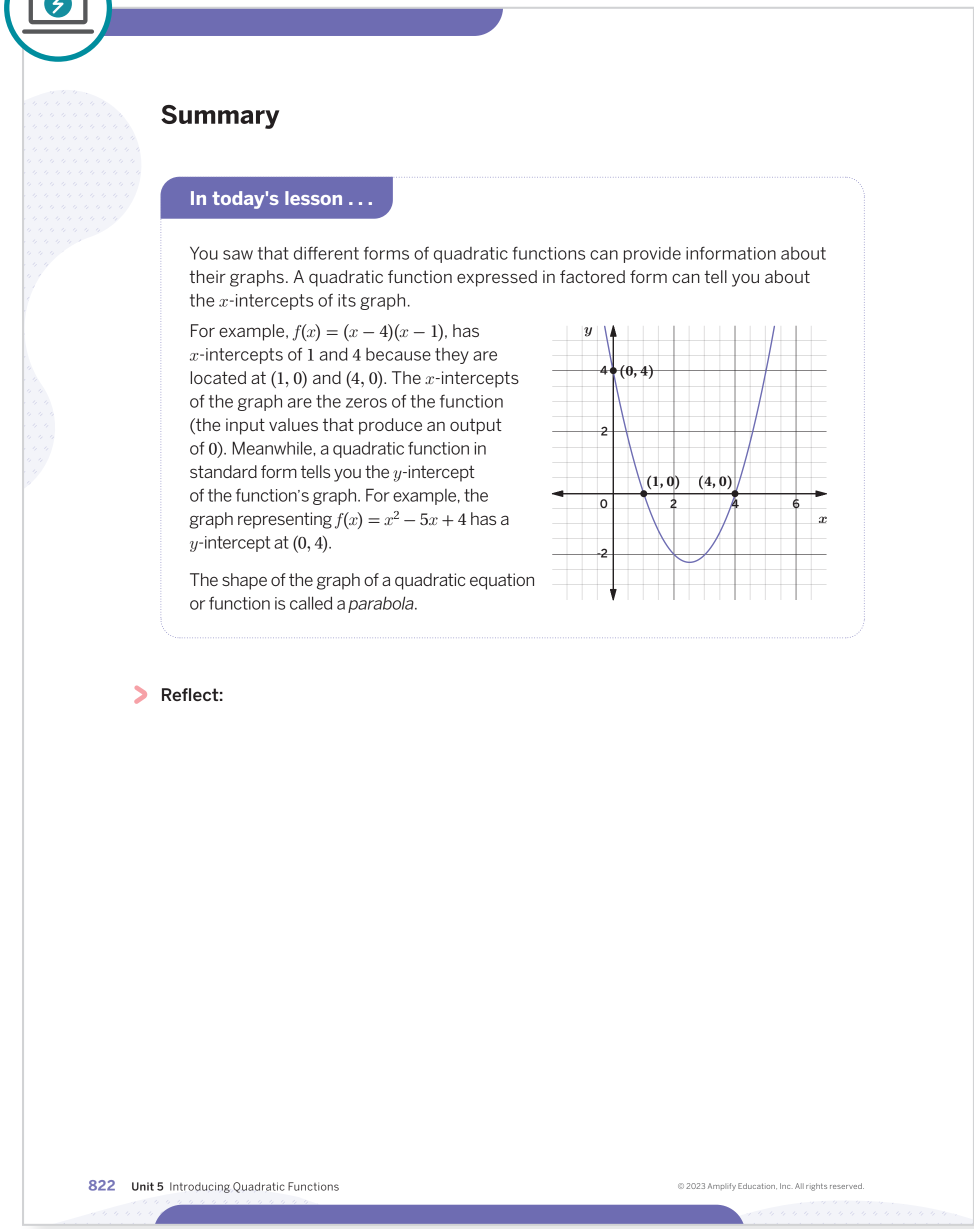
Highlight that a function in factored form gives the x-intercept(s) of the graph, which are also the zeros of the function. A function in standard form gives the y-intercept of the graph.

Ask:
- "How could you find the x-intercept of the graph of function $f(x)$ without graphing?" By looking at the linear factors in the factored form. The x-intercepts are the opposite of the constant term in the linear factors.
- "How could you find the y-intercept?" By writing the expression in standard form and finding the constant.

Whole Class | 5 min

Summary

Review and synthesize how standard and factored forms of quadratic functions or equations provides key information about the intercepts of their graphs.



Summary

In today's lesson . . .

You saw that different forms of quadratic functions can provide information about their graphs. A quadratic function expressed in factored form can tell you about the x-intercepts of its graph.

For example, $f(x) = (x - 4)(x - 1)$, has x-intercepts of 1 and 4 because they are located at $(1, 0)$ and $(4, 0)$. The x-intercepts of the graph are the zeros of the function (the input values that produce an output of 0). Meanwhile, a quadratic function in standard form tells you the y-intercept of the function's graph. For example, the graph representing $f(x) = x^2 - 5x + 4$ has a y-intercept at $(0, 4)$.

The shape of the graph of a quadratic equation or function is called a *parabola*.

Reflect:

Synthesize

Display the graph.

Ask:

- "If you graph $f(x) = (x - 4)(x - 1)$ and $g(x) = x^2 - 5x + 4$, will you end up with the same graph? How do you know?" Yes, $f(x)$ and $g(x)$ are equivalent. Expanding $(x - 4)(x - 1)$ gives $x^2 - x - 4x + 4$, or $x^2 - 5x + 4$.
- "Without graphing, where do you think the x- and y-intercepts are located? Explain your thinking." The y-intercept will be located at $(0, 4)$ and the x-intercepts will be located at $(4, 0)$ and $(1, 0)$. The factored form gives a clue about the x-intercepts, and the constant term in the standard form gives a clue about the y-intercept. Also, evaluating y when $x = 0$ gives the y-intercept.

Highlight that different forms of quadratic functions can provide information about the function's graph. When a quadratic function is expressed in standard form, it provides the y-intercept of the graph representing the function. The connection between the factored form and the x-intercepts of the graph provide information about the zeros of the function (the input values that produce an output value of 0).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

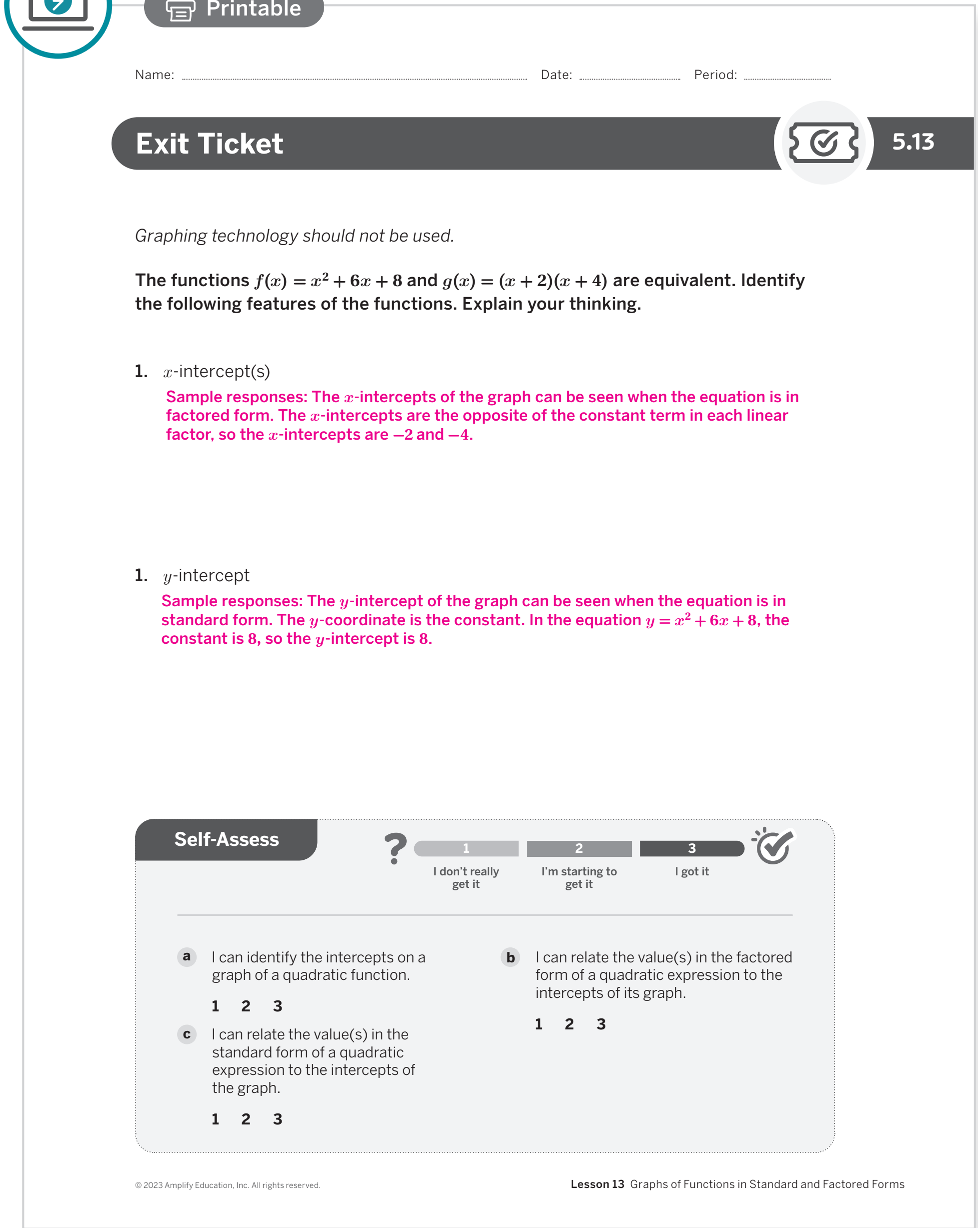
- "How are the standard and factor forms useful when graphing a quadratic function?"
- "What are the disadvantages of each form?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying the x- and y-intercepts from the factored and standard forms of quadratic functions.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 5.13

Graphing technology should not be used.

The functions $f(x) = x^2 + 6x + 8$ and $g(x) = (x + 2)(x + 4)$ are equivalent. Identify the following features of the functions. Explain your thinking.

1. x-intercept(s)

   Sample responses: The x-intercepts of the graph can be seen when the equation is in factored form. The x-intercepts are the opposite of the constant term in each linear factor, so the x-intercepts are -2 and -4.

1. y-intercept

   Sample responses: The y-intercept of the graph can be seen when the equation is in standard form. The y-coordinate is the constant. In the equation $y = x^2 + 6x + 8$, the constant is 8, so the y-intercept is 8.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can identify the intercepts on a graph of a quadratic function.

1 2 3

b I can relate the value(s) in the factored form of a quadratic expression to the intercepts of its graph.

1 2 3

c I can relate the value(s) in the standard form of a quadratic expression to the intercepts of the graph.

1 2 3

Success looks like . . .

- **Goal:** Identifying the intercepts on a graph of a quadratic function.
- **Language Goal:** Relating the value(s) in the factored form of a quadratic function to the intercepts of its graph. **(Speaking and Listening, Writing)**
  - » Relating $g(x)$ to the intercepts of its graph in Problems 1 and 2.
- **Language Goal:** Relating the value(s) in the standard form of a quadratic function to the intercepts of the graph. **(Speaking and Listening, Writing)**
  - » Relating $f(x)$ to the intercepts of its graph in Problems 1 and 2.

Suggested next steps

If students cannot identify the x-intercepts from the factored form, consider:

- Reviewing Activity 1, Problem 2, and the graphs in Activity 2.
- Assigning Practice Problems 1 and 2.
- Asking, "How can you determine the value of x when $y = 0$?"

If students cannot identify the y-intercept from the standard form, consider:

- Reviewing Activity 1, Problem 1, and the graphs in Activity 2.
- Assigning Practice Problem 6.
- Asking, "How can you determine the value of y when $x = 0$?"

Professional Learning

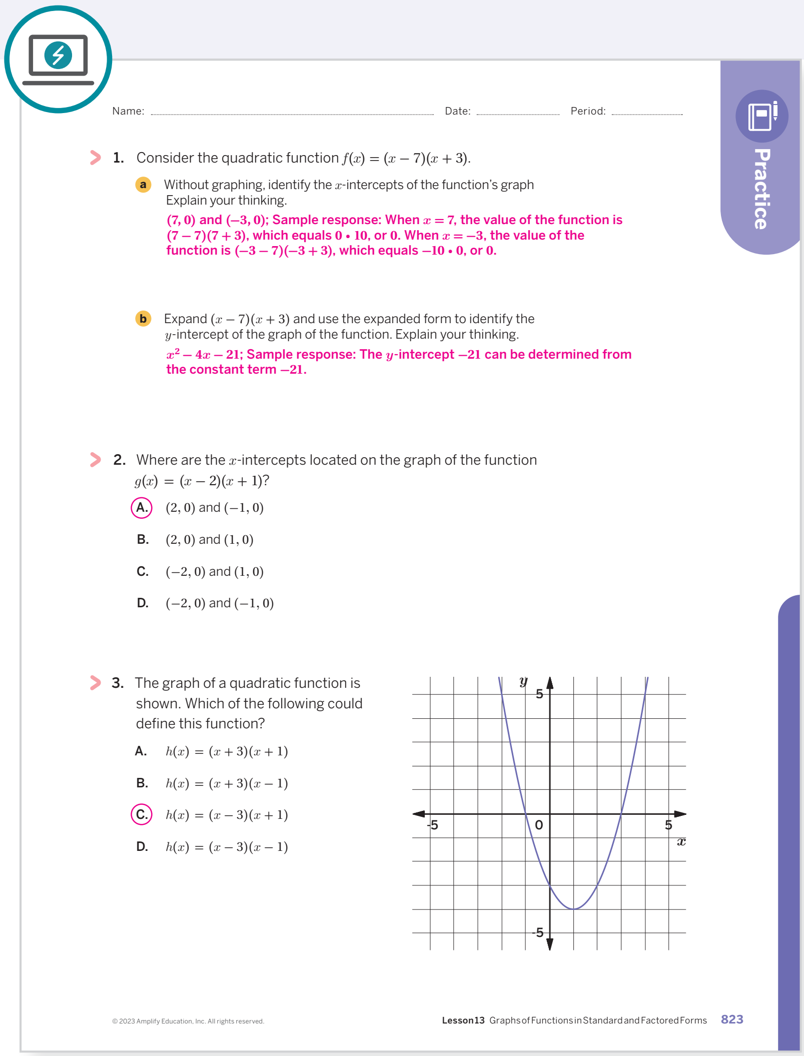
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did identifying the differences between the two forms set students up to develop the skills to graph any quadratic function?
- How did students make use of structure today? How are you helping students become aware of how they are progressing in this area?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. Consider the quadratic function $f(x) = (x - 7)(x + 3)$.

   a. Without graphing, identify the x-intercepts of the function's graph Explain your thinking.

      $(7, 0)$ and $(-3, 0)$; Sample response: When $x = 7$, the value of the function is $(7 - 7)(7 + 3)$, which equals $0 \cdot 10$, or 0. When $x = -3$, the value of the function is $(-3 - 7)(-3 + 3)$, which equals $-10 \cdot 0$, or 0.

   b. Expand $(x - 7)(x + 3)$ and use the expanded form to identify the y-intercept of the graph of the function. Explain your thinking.

      $x^2 - 4x - 21$; Sample response: The y-intercept -21 can be determined from the constant term -21.

2. Where are the x-intercepts located on the graph of the function $g(x) = (x - 2)(x + 1)$?

   [✓ A.] $(2, 0)$ and $(-1, 0)$

   B. $(2, 0)$ and $(1, 0)$

   C. $(-2, 0)$ and $(1, 0)$

   D. $(-2, 0)$ and $(-1, 0)$

3. The graph of a quadratic function is shown. Which of the following could define this function?

   A. $h(x) = (x + 3)(x + 1)$

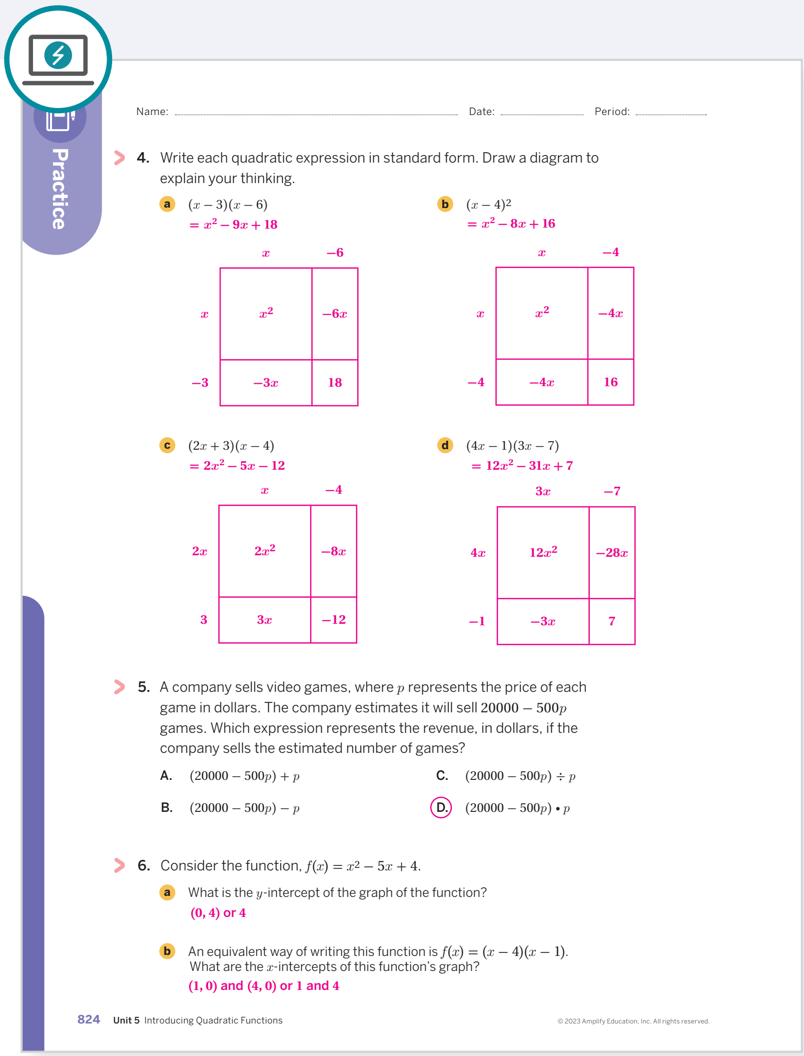
   B. $h(x) = (x + 3)(x - 1)$

   [✓ C.] $h(x) = (x - 3)(x + 1)$

   D. $h(x) = (x - 3)(x - 1)$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Graphs of Functions in Standard and Factored Forms 823



Name: ________ Date: ________ Period: ________

Practice

4. Write each quadratic expression in standard form. Draw a diagram to explain your thinking.

   a. $(x - 3)(x - 6)$

      $= x^2 - 9x + 18$

	x	-6
x	x^2	$-6x$
-3	$-3x$	18

   b. $(x - 4)^2$

      $= x^2 - 8x + 16$

	x	-4
x	x^2	$-4x$
-4	$-4x$	16

   c. $(2x + 3)(x - 4)$

      $= 2x^2 - 5x - 12$

	x	-4
$2x$	$2x^2$	$-8x$
3	$3x$	-12

   d. $(4x - 1)(3x - 7)$

      $= 12x^2 - 31x + 7$

	$3x$	-7
$4x$	$12x^2$	$-28x$
-1	$-3x$	7

5. A company sells video games, where p represents the price of each game in dollars. The company estimates it will sell $20000 - 500p$ games. Which expression represents the revenue, in dollars, if the company sells the estimated number of games?

   A. $(20000 - 500p) + p$

   B. $(20000 - 500p) - p$

   C. $(20000 - 500p) \div p$

   [✓ D.] $(20000 - 500p) \cdot p$

6. Consider the function, $f(x) = x^2 - 5x + 4$.

   a. What is the y-intercept of the graph of the function?

      $(0, 4)$ or 4

   b. An equivalent way of writing this function is $f(x) = (x - 4)(x - 1)$. What are the x-intercepts of this function's graph?

      $(1, 0)$ and $(4, 0)$ or 1 and 4

824 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	2
	2	Activities 1 and 2	1
	3	Activity 2	1
Spiral	4	Unit 5 Lesson 12	2
	5	Unit 5 Lesson 9	1
Formative ⚡	6	Unit 5 Lesson 14	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

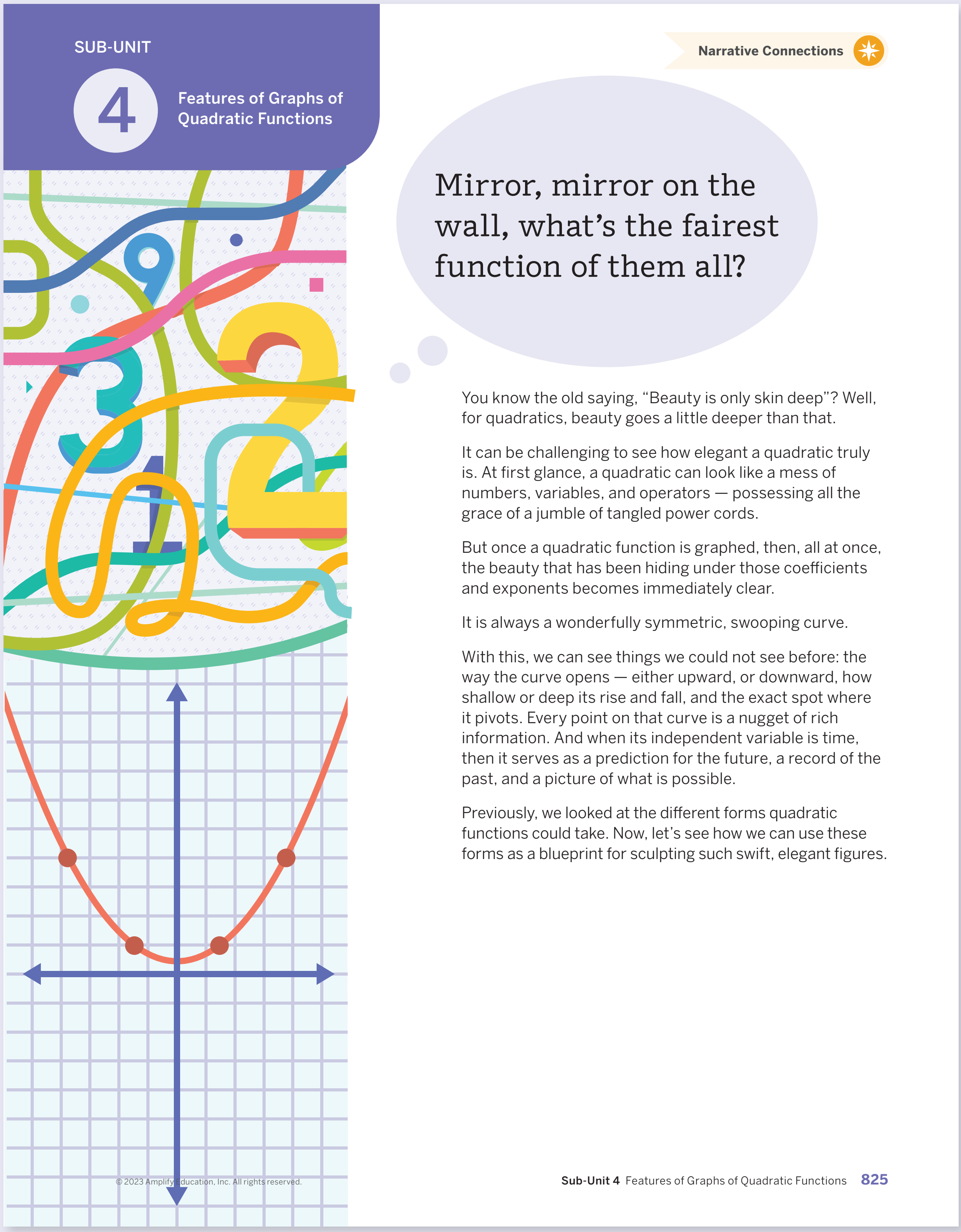
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 4

Features of Graphs of Quadratic Functions

Whole Class

In this Sub-Unit, students develop an understanding of the key features of the graphs of quadratic functions for each form. Students examine the symmetry in graphs of quadratic functions.



SUB-UNIT

4 Features of Graphs of Quadratic Functions

Narrative Connections

Mirror, mirror on the wall, what's the fairest function of them all?

You know the old saying, "Beauty is only skin deep"? Well, for quadratics, beauty goes a little deeper than that.

It can be challenging to see how elegant a quadratic truly is. At first glance, a quadratic can look like a mess of numbers, variables, and operators — possessing all the grace of a jumble of tangled power cords.

But once a quadratic function is graphed, then, all at once, the beauty that has been hiding under those coefficients and exponents becomes immediately clear.

It is always a wonderfully symmetric, swooping curve.

With this, we can see things we could not see before: the way the curve opens — either upward, or downward, how shallow or deep its rise and fall, and the exact spot where it pivots. Every point on that curve is a nugget of rich information. And when its independent variable is time, then it serves as a prediction for the future, a record of the past, and a picture of what is possible.

Previously, we looked at the different forms quadratic functions could take. Now, let's see how we can use these forms as a blueprint for sculpting such swift, elegant figures.

© 2023 Amplify Education, Inc. All rights reserved.

Sub-Unit 4 Features of Graphs of Quadratic Functions 825

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore the elegance and symmetry of quadratic functions in the following places:

- **Lesson 14, Activities 1–2:** Comparing Two Graphs, The Axis of Symmetry
- **Lesson 15, Activities 1–2:** The Shot Put World Record, The Perfect Synchronized Dive
- **Lesson 18, Activities 1–2:** The Water Catapult, Flight of Two Baseballs
- **Lesson 19, Activity 2:** Half-Pipe
- **Lesson 20, Activity 1:** Sharing a Vertex

UNIT 5 | **LESSON 14**

Graphing Quadratics Using Points of Symmetry

Let’s graph some quadratic functions using factored form.

Focus

Goals

1. Graph a quadratic function given in factored form.
2. Identify the axis of symmetry of a parabola.
3. Without graphing, identify the vertex and y-intercept of the graph of a quadratic function expressed in factored form.

Coherence

Today

In today’s lesson, students deepen the connections they made between quadratic functions in factored form and the x-intercepts of their graphs. Students also identify key coordinates, including the vertex of the graph, and use these values to graph a quadratic function.

Previously

In the previous lesson, students explored quadratic functions in standard and factored form and made some preliminary observations about the connection between equations given in these forms and their graphs.

Coming Soon

In the subsequent lessons, students will revisit the standard form of a quadratic function and explore how to graph a function given in that form.

Rigor

- Students build **conceptual understanding** of the axis of symmetry for a quadratic function.
- Students strengthen their **fluency** in graphing quadratics in factored form.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	12 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- graph paper
- graphing technology

Math Language Development

New words

- ***axis of symmetry***

Review words

- *factored form*
- *standard form*
- *vertex*
- x*-intercept*
- y*-intercept*
- *zeros of a function*

Amps | Featured Activity

Activity 2

Interactive Graph

Students draw the axis of symmetry of a partial graph of a parabola and use it to help determine missing points of symmetry on the graph.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might become too focused on the details of Activity 2, forgetting to oversee the process and how it will be used to graph the parabola. Encourage students to remember their goal, which is to determine and use the axis of symmetry for a parabola. Explain that the process is the same for all parabolas because all parabolas are symmetric.

Modifications to Pacing

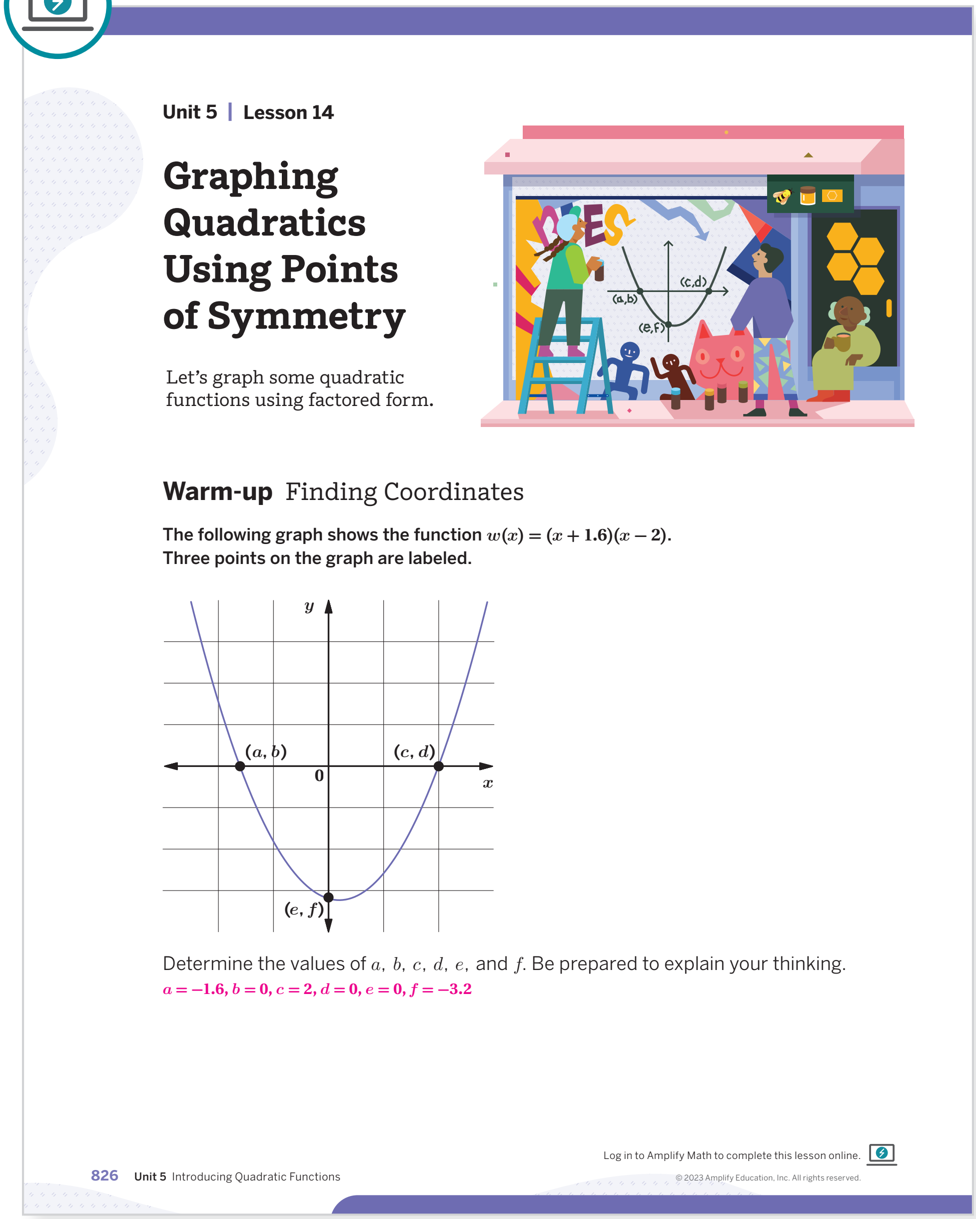
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Problem 1, have students omit the first two and last two rows of the table.

Pairs | 5 min

Warm-up Finding Coordinates

Students analyze the graph of a quadratic function in factored form to make connections between the factors and the x-intercepts, and how to use the equation to find the y-intercept.



Unit 5 | Lesson 14

Graphing Quadratics Using Points of Symmetry

Let's graph some quadratic functions using factored form.

Warm-up Finding Coordinates

The following graph shows the function $w(x) = (x + 1.6)(x - 2)$. Three points on the graph are labeled.

Determine the values of a, b, c, d, e, and f. Be prepared to explain your thinking.

$a = -1.6, b = 0, c = 2, d = 0, e = 0, f = -3.2$

Log in to Amplify Math to complete this lesson online.

826 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate students' prior knowledge by asking, "What information is provided from a quadratic function written in factored form?" The x-intercepts or zeros. Then ask, "What is the form of the coordinates of an x-intercept?" $(n, 0)$

2 Monitor

Help students get started by referring to the graph and having them point out the x- and y-intercepts that are plotted on the graph.

Look for points of confusion:

- **Thinking that the numerical values in $w(x)$ correspond directly to the x-intercepts in the graph.** Have students verify whether their values of x satisfy $w(x) = 0$.
- **Transposing the x- and y-coordinates.** Have students verify whether substituting their values of x into $w(x)$ results in their values of y.

Look for productive strategies:

- Indicating on paper that c is farther away from 0 than a is from 0.
- Negating 1.6 and -2 when writing a and c.

3 Connect

Have three students share the coordinates of (a, b), (c, d), and (e, f).

Highlight that $b = 0$ and $d = 0$ because they are y-coordinates of the x-intercepts and $e = 0$ because it is the x-coordinate of the y-intercept. Draw attention to the fact that the x-intercepts have the opposite sign of the constant term of the given linear factors of $w(x)$.

Power-up

To power up students' ability to describe key features of a quadratic function, have students complete:

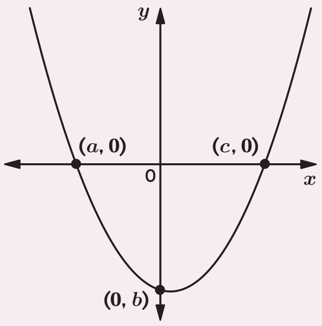
The graph represents the function $w(x) = (x + 4)(x - 3)$.

1. What is the value of the function when $x = -4$? 0
2. What is the value of the function when $x = 0$? -12
3. What is the value of the function when $x = 3$. 0
4. What are the values of a, b, and c on the graph?

   $a = -4$

   $b = -12$

   $c = 3$



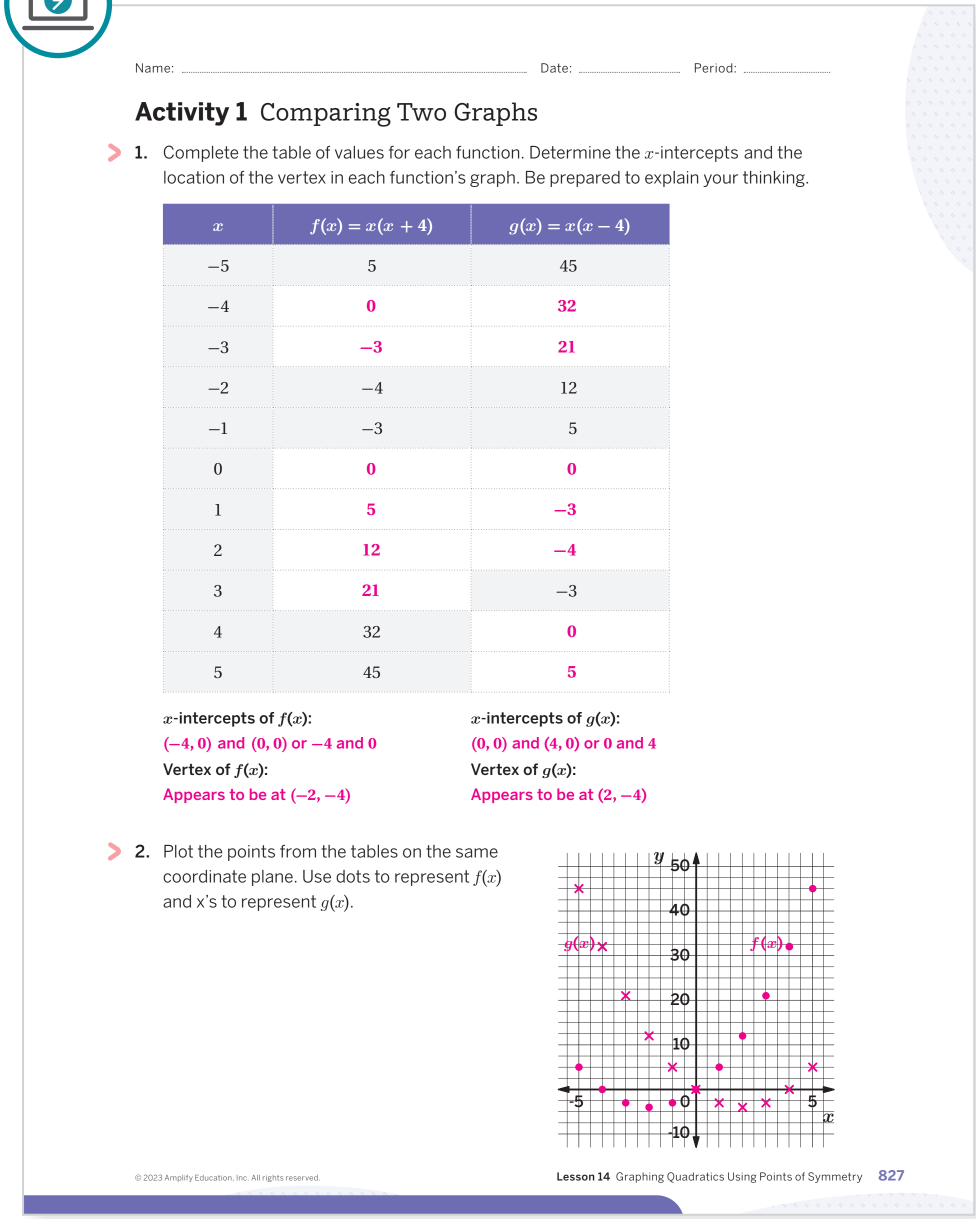
Use: Before the Warm-up

Informed by: Performance on Lesson 13, Practice Problem 6

Pairs | 12 min

Activity 1 Comparing Two Graphs

Students examine the tables and graphs of two quadratic functions in factored form to understand that the vertex is located halfway between each pair of x-intercepts.



Name: ______ Date: ______ Period: ______

Activity 1 Comparing Two Graphs

1. Complete the table of values for each function. Determine the x-intercepts and the location of the vertex in each function's graph. Be prepared to explain your thinking.

x	$f(x) = x(x + 4)$	$g(x) = x(x - 4)$
–5	5	45
–4	**0**	**32**
–3	**–3**	**21**
–2	–4	12
–1	–3	5
0	**0**	**0**
1	**5**	**–3**
2	**12**	**–4**
3	**21**	–3
4	32	**0**
5	45	**5**

x-intercepts of $f(x)$:
$(-4, 0)$ and $(0, 0)$ or -4 and 0
Vertex of $f(x)$:
Appears to be at $(-2, -4)$

x-intercepts of $g(x)$:
$(0, 0)$ and $(4, 0)$ or 0 and 4
Vertex of $g(x)$:
Appears to be at $(2, -4)$

2. Plot the points from the tables on the same coordinate plane. Use dots to represent $f(x)$ and x's to represent $g(x)$.

Lesson 14 Graphing Quadratics Using Points of Symmetry 827

1 Launch

Display the functions $f(x)$ and $g(x)$. Conduct a ***Notice and Wonder*** routine. Ask, "How are they similar? How are they different?" Record students' responses. Arrange students in pairs and divide the task so that one partner considers $f(x)$ and the other considers $g(x)$.

2 Monitor

Help students get started by having them define the term *vertex* in their own words.

Look for points of confusion:

- **Struggling to locate the vertex in the table in Problem 1.** Remind students that the vertex is where the graph changes from increasing to decreasing, or decreasing to increasing. It is also either the minimum or maximum value of the function.

Look for productive strategies:

- Circling or highlighting the zeros in the table.
- Making use of the increasing/decreasing structure of quadratics to identify the vertex in the table.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing a scaffolded table so that students can first substitute the given values of x into each expression before evaluating. Provide access to colored pencils and have students color code the values substituted into each expression. An example is shown here.

x	$x(x+4)$	$f(x) = x(x+4)$	$x(x-4)$	$g(x) = x(x-4)$
–5	**–5**$(-5 + 4)$	5	**–5**$(-5 - 4)$	45
–4	**–4**$(-4(+4)$	0	**–4**$(-4 - 4)$	32

Math Language Development

MLR7: Compare and Connect

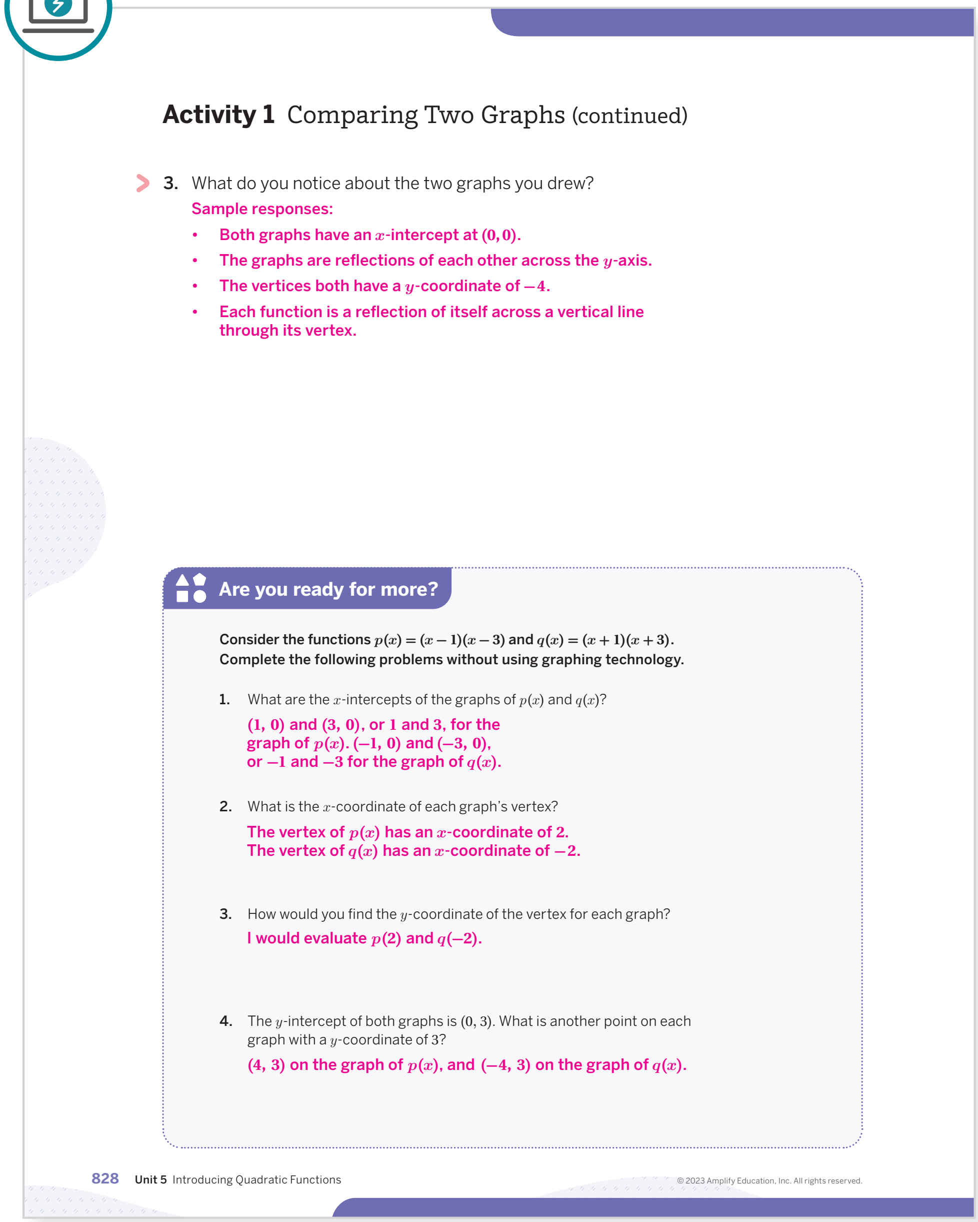
During the Connect, as you highlight how the zeros and vertex are represented in the graphs of the functions, add the following to the class display. Have students help you complete the table as you add it to the display.

Zeros of the function	Vertex of the function
Indicates the input values of x for which the output value of the function is 0. **Graph:** Where the function intersects the x-axis.	Indicates the minimum or maximum value of the function. **Graph:** The highest or lowest point of the function.

Pairs | 12 min

Activity 1 Comparing Two Graphs (continued)

Students examine the tables and graphs of two quadratic functions in factored form to understand that the vertex is located halfway between each pair of x-intercepts.



Activity 1 Comparing Two Graphs (continued)

3. What do you notice about the two graphs you drew?

   Sample responses:

   - Both graphs have an x-intercept at $(0, 0)$.
   - The graphs are reflections of each other across the y-axis.
   - The vertices both have a y-coordinate of -4.
   - Each function is a reflection of itself across a vertical line through its vertex.

Are you ready for more?

Consider the functions $p(x) = (x-1)(x-3)$ and $q(x) = (x+1)(x+3)$. Complete the following problems without using graphing technology.

1. What are the x-intercepts of the graphs of $p(x)$ and $q(x)$?

   $(1, 0)$ and $(3, 0)$, or 1 and 3, for the graph of $p(x)$. $(-1, 0)$ and $(-3, 0)$, or -1 and -3 for the graph of $q(x)$.

2. What is the x-coordinate of each graph's vertex?

   The vertex of $p(x)$ has an x-coordinate of 2.
   The vertex of $q(x)$ has an x-coordinate of -2.

3. How would you find the y-coordinate of the vertex for each graph?

   I would evaluate $p(2)$ and $q(-2)$.

4. The y-intercept of both graphs is $(0, 3)$. What is another point on each graph with a y-coordinate of 3?

   $(4, 3)$ on the graph of $p(x)$, and $(-4, 3)$ on the graph of $q(x)$.

828 **Unit 5** Introducing Quadratic Functions

3 Connect

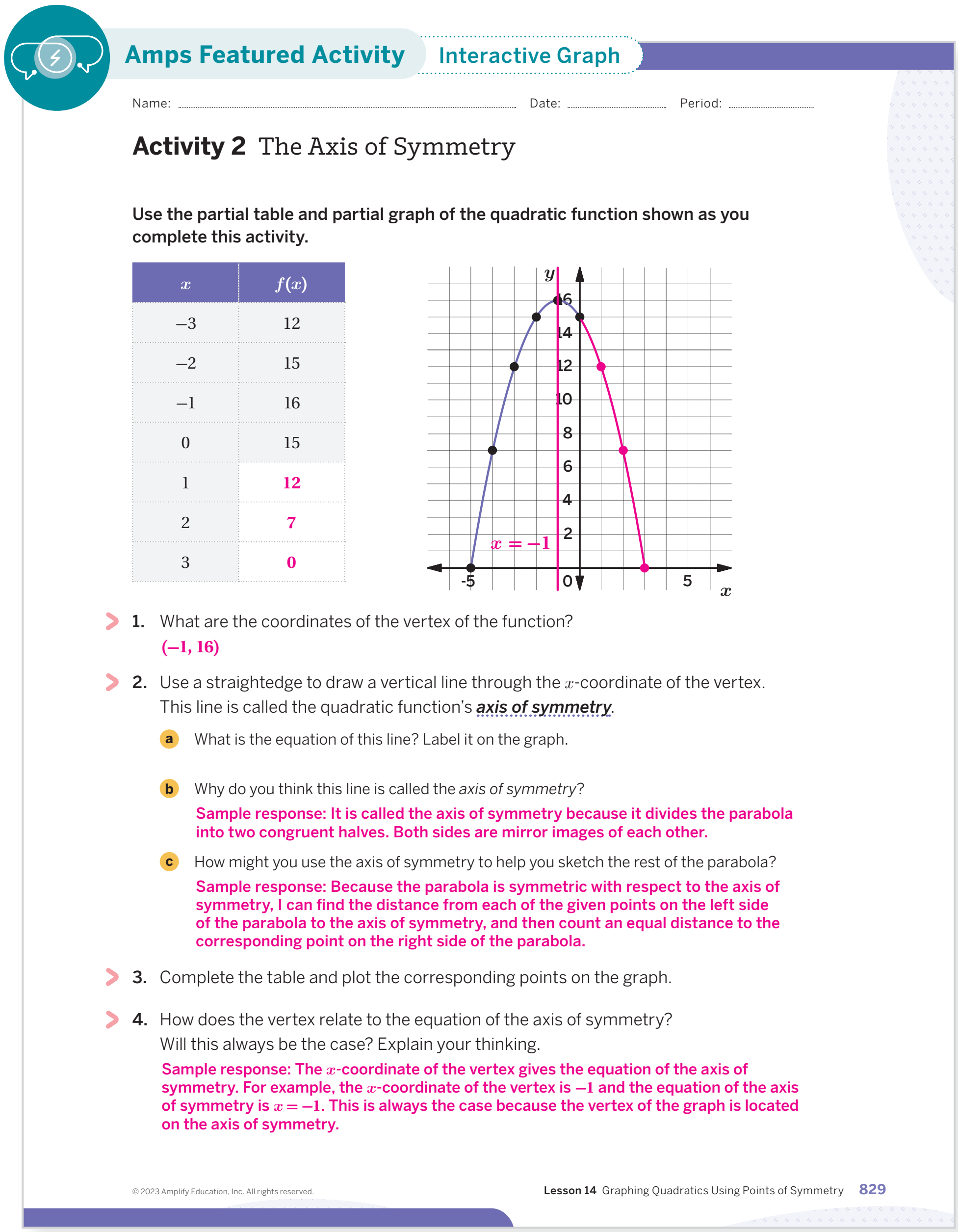
Have pairs of students share their strategies or processes for using the table to determine the x-intercepts and vertex for each graph. Be sure to discuss how key features of the graphs of each function compare.

Highlight that *zeros* represent the input values of x for which the output value of the function is 0. On a graph, the zeros are located along the x-axis where the function intersects this axis. The vertex of the function represents the minimum or maximum value of the function and its x-coordinate is located halfway between the function's two x-intercepts. **Note:** This statement is true when the function has two x-intercepts. Students will learn more about the vertex and the symmetry of a quadratic function in the next activity.

Pairs | 12 min

Activity 2 The Axis of Symmetry

Students identify the axis of symmetry of a parabola and use it to determine the missing points of symmetry in a table and on a partial graph.



Amps Featured Activity Interactive Graph

Name: ________ Date: ________ Period: ________

Activity 2 The Axis of Symmetry

Use the partial table and partial graph of the quadratic function shown as you complete this activity.

x	$f(x)$
−3	12
−2	15
−1	16
0	15
1	**12**
2	**7**
3	**0**

1. What are the coordinates of the vertex of the function?
   (−1, 16)
2. Use a straightedge to draw a vertical line through the x-coordinate of the vertex. This line is called the quadratic function's ***axis of symmetry***.
   - a. What is the equation of this line? Label it on the graph.
   - b. Why do you think this line is called the *axis of symmetry*?
     Sample response: It is called the axis of symmetry because it divides the parabola into two congruent halves. Both sides are mirror images of each other.
   - c. How might you use the axis of symmetry to help you sketch the rest of the parabola?
     Sample response: Because the parabola is symmetric with respect to the axis of symmetry, I can find the distance from each of the given points on the left side of the parabola to the axis of symmetry, and then count an equal distance to the corresponding point on the right side of the parabola.
3. Complete the table and plot the corresponding points on the graph.
4. How does the vertex relate to the equation of the axis of symmetry? Will this always be the case? Explain your thinking.
   Sample response: The x-coordinate of the vertex gives the equation of the axis of symmetry. For example, the x-coordinate of the vertex is −1 and the equation of the axis of symmetry is $x = -1$. This is always the case because the vertex of the graph is located on the axis of symmetry.

Lesson 14 Graphing Quadratics Using Points of Symmetry 829

1 Launch

Display the graph of $f(x)$ from Activity 1 and have students identify the vertex. Ask what they notice about the points to the left of the vertex and those to the right. They are symmetrical. Invite a student to draw the line of symmetry and remind students of what symmetry means.

2 Monitor

Help students get started by asking them what the graph of a quadratic function looks like.

Look for points of confusion:

- **Forgetting how to write the equation of a vertical line in Problem 2a.** Have students explicitly write several coordinate points on the axis of symmetry and state what is similar about those points.
- **Having difficulty articulating how the axis of symmetry helps to graph a parabola in Problem 1c.** Ask students what the distance is to the axis of symmetry from one of the plotted points and how they can use that to find the corresponding point of symmetry.

Look for productive strategies:

- Annotating the table to highlight the points of symmetry on either side of the vertex.
- Annotating the graph to show the distance from corresponding points of symmetry to the axis of symmetry.
- Applying the Distributive Property to convert the function $f(x)$ written in factored form to standard form.
- Labeling the values of a, b, and c in the function $f(x)$ in standard form.

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital graphs to draw the axis of symmetry of a partial graph of a parabola and use it to help them determine missing points of symmetry on the graph.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students color code the vertex of the function and using the same color, color code the vertical line that represents the axis of symmetry. This will help them visualize how the two sides of the parabola look the same.

Math Language Development

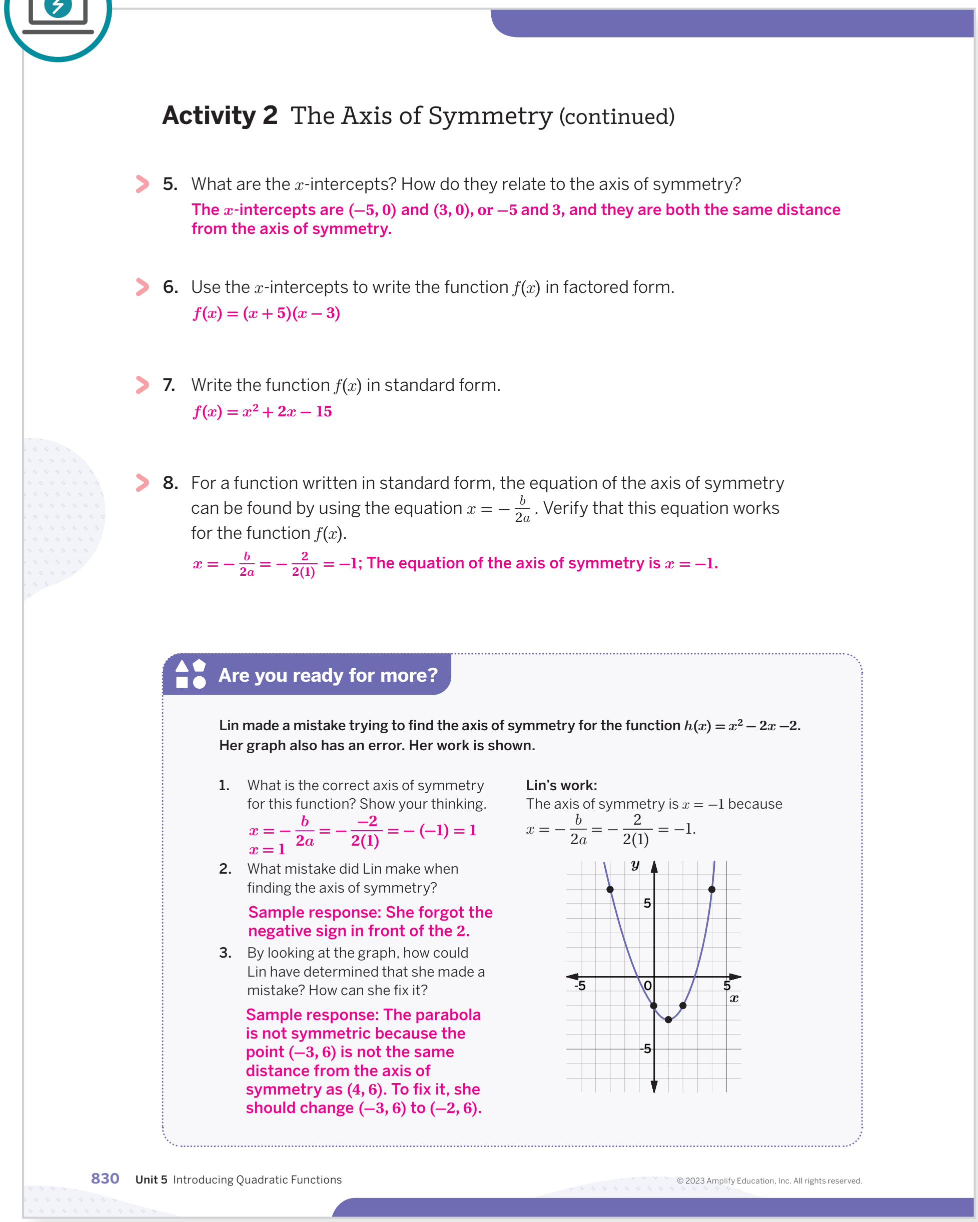
MLR7: Compare and Connect

During the Connect, clarify the meaning of the word *axis* as it is used in the term *axis of symmetry*. It is similar to the x- and y-axes in that it is either a horizontal or vertical line, but it is different from the x- and y-axes because the axis of symmetry is not intended to divide the coordinate plane into four quadrants. Instead, it divides the parabola into two equal halves.

Pairs | 12 min

Activity 2 The Axis of Symmetry (continued)

Students identify the axis of symmetry of a parabola and use it to determine the missing points of symmetry in a table and on a partial graph.



Activity 2 The Axis of Symmetry (continued)

5. What are the x-intercepts? How do they relate to the axis of symmetry?

   The x-intercepts are $(-5, 0)$ and $(3, 0)$, or -5 and 3, and they are both the same distance from the axis of symmetry.

6. Use the x-intercepts to write the function $f(x)$ in factored form.

   $f(x) = (x + 5)(x - 3)$

7. Write the function $f(x)$ in standard form.

   $f(x) = x^2 + 2x - 15$

8. For a function written in standard form, the equation of the axis of symmetry can be found by using the equation $x = -\frac{b}{2a}$. Verify that this equation works for the function $f(x)$.

   $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$; **The equation of the axis of symmetry is $x = -1$.**

Are you ready for more?

Lin made a mistake trying to find the axis of symmetry for the function $h(x) = x^2 - 2x - 2$. Her graph also has an error. Her work is shown.

1. What is the correct axis of symmetry for this function? Show your thinking.

   $x = -\frac{b}{2a} = -\frac{-2}{2(1)} = -(-1) = 1$
   $x = 1$

2. What mistake did Lin make when finding the axis of symmetry?

   Sample response: She forgot the negative sign in front of the 2.

3. By looking at the graph, how could Lin have determined that she made a mistake? How can she fix it?

   Sample response: The parabola is not symmetric because the point $(-3, 6)$ is not the same distance from the axis of symmetry as $(4, 6)$. To fix it, she should change $(-3, 6)$ to $(-2, 6)$.

Lin's work:

The axis of symmetry is $x = -1$ because $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$.

830 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the table and graph.

Have students share their responses by completing the table, drawing the axis of symmetry, and plotting the missing points. Select students who were using productive strategies to explain their strategies for determining the missing points and how knowing the axis of symmetry helped them to do so. Then select a student to share how they verified that the equation of the axis of symmetry works for the function $f(x)$.

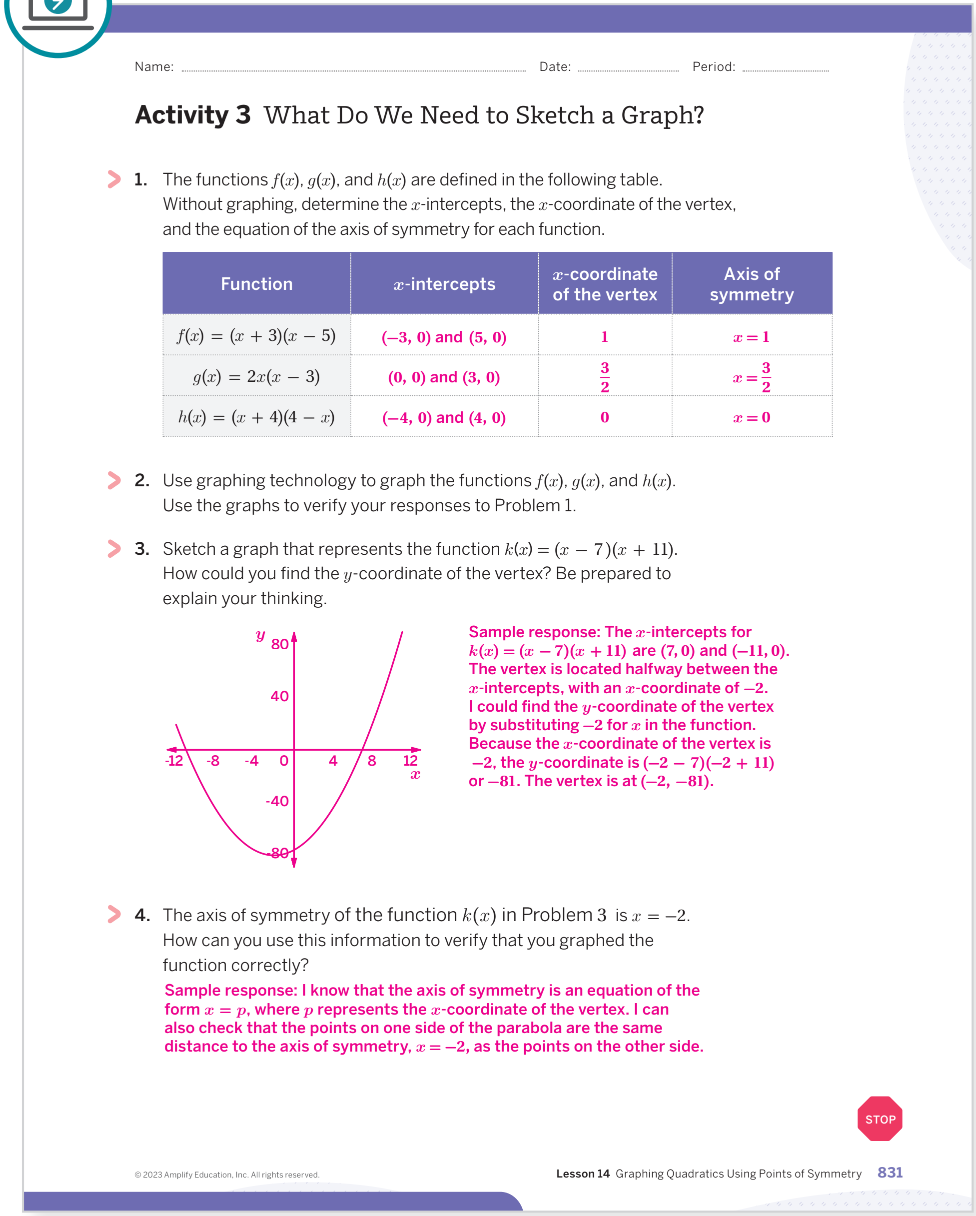
Highlight that a parabola is always symmetrical and the axis of symmetry divides the parabola into two equal halves. If there is a point on one side of the parabola, the corresponding point of symmetry can be found by counting the distance between the first point and the axis of symmetry. Then count the same distance on the opposite side of the axis of symmetry. Also highlight the formula to determine the axis of symmetry, $x = -\frac{b}{2a}$, emphasizing that the quadratic function must be in standard form to use it. If time permits, give students another quadratic function in standard form (without a graph) and have them use the formula to determine the axis of symmetry.

Define the term ***axis of symmetry***.

Small Groups | 10 min

Activity 3 What Do We Need to Sketch a Graph?

Students examine three functions written in factored form to determine whether they have all the necessary information to sketch the graphs that represent them.



Name: ______ Date: ______ Period: ______

Activity 3 What Do We Need to Sketch a Graph?

1. The functions $f(x)$, $g(x)$, and $h(x)$ are defined in the following table. Without graphing, determine the x-intercepts, the x-coordinate of the vertex, and the equation of the axis of symmetry for each function.

Function	x-intercepts	x-coordinate of the vertex	Axis of symmetry
$f(x) = (x + 3)(x - 5)$	$(-3, 0)$ and $(5, 0)$	1	$x = 1$
$g(x) = 2x(x - 3)$	$(0, 0)$ and $(3, 0)$	$\frac{3}{2}$	$x = \frac{3}{2}$
$h(x) = (x + 4)(4 - x)$	$(-4, 0)$ and $(4, 0)$	0	$x = 0$

2. Use graphing technology to graph the functions $f(x)$, $g(x)$, and $h(x)$. Use the graphs to verify your responses to Problem 1.

3. Sketch a graph that represents the function $k(x) = (x - 7)(x + 11)$. How could you find the y-coordinate of the vertex? Be prepared to explain your thinking.

Sample response: The x-intercepts for $k(x) = (x - 7)(x + 11)$ are $(7, 0)$ and $(-11, 0)$. The vertex is located halfway between the x-intercepts, with an x-coordinate of -2. I could find the y-coordinate of the vertex by substituting -2 for x in the function. Because the x-coordinate of the vertex is -2, the y-coordinate is $(-2 - 7)(-2 + 11)$ or -81. The vertex is at $(-2, -81)$.

4. The axis of symmetry of the function $k(x)$ in Problem 3 is $x = -2$. How can you use this information to verify that you graphed the function correctly?

Sample response: I know that the axis of symmetry is an equation of the form $x = p$, where p represents the x-coordinate of the vertex. I can also check that the points on one side of the parabola are the same distance to the axis of symmetry, $x = -2$, as the points on the other side.

STOP

1 Launch

Provide access to graph paper and graphing technology. Graphing technology should *not* be used until students have completed Problem 1.

2 Monitor

Help students get started by prompting them to think about the relationship between the given factor and the x-intercept it represents.

Look for points of confusion:

- **Expanding the equations in factored form to use the formula for axis of symmetry. Although this is not incorrect, it is not necessary here.** Remind students that the axis of symmetry is the same value as the x-coordinate of the vertex (which they found in Problem 1).

Look for productive strategies:

- Writing the axis of symmetry as an equation.
- Calculating the average of the x-coordinates of the x-intercepts of each of the given functions to determine the x-coordinate of the vertex.
- Tracing a function using graphing technology to locate specific coordinate points.

3 Connect

Have groups of students share the strategies they used to find the x-intercepts and x-coordinate of the vertex for each function. Then discuss how to use these values to sketch a graph.

Highlight that to find an x-intercept of a quadratic function in factored form, students need to find the values at which the graph of the function crosses the x-axis. The x-coordinate of the vertex is the midpoint of the x-intercepts, and the y-coordinate of the vertex is the value of the function at that x-coordinate.

Ask, "What is the y-intercept of $p(x) = (x - 7)(x + 11)$?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students color code the factors of a product that are shown in each of the functions's equations in Problem 1. Ask:

- "Do factors have to be contained inside parentheses? What are the factors of $g(x)$?"
- "What do you notice about the structure of the function $h(x)$ that is different compared to $f(x)$ or $g(x)$?"

Accessibility: *Activate Prior Knowledge*

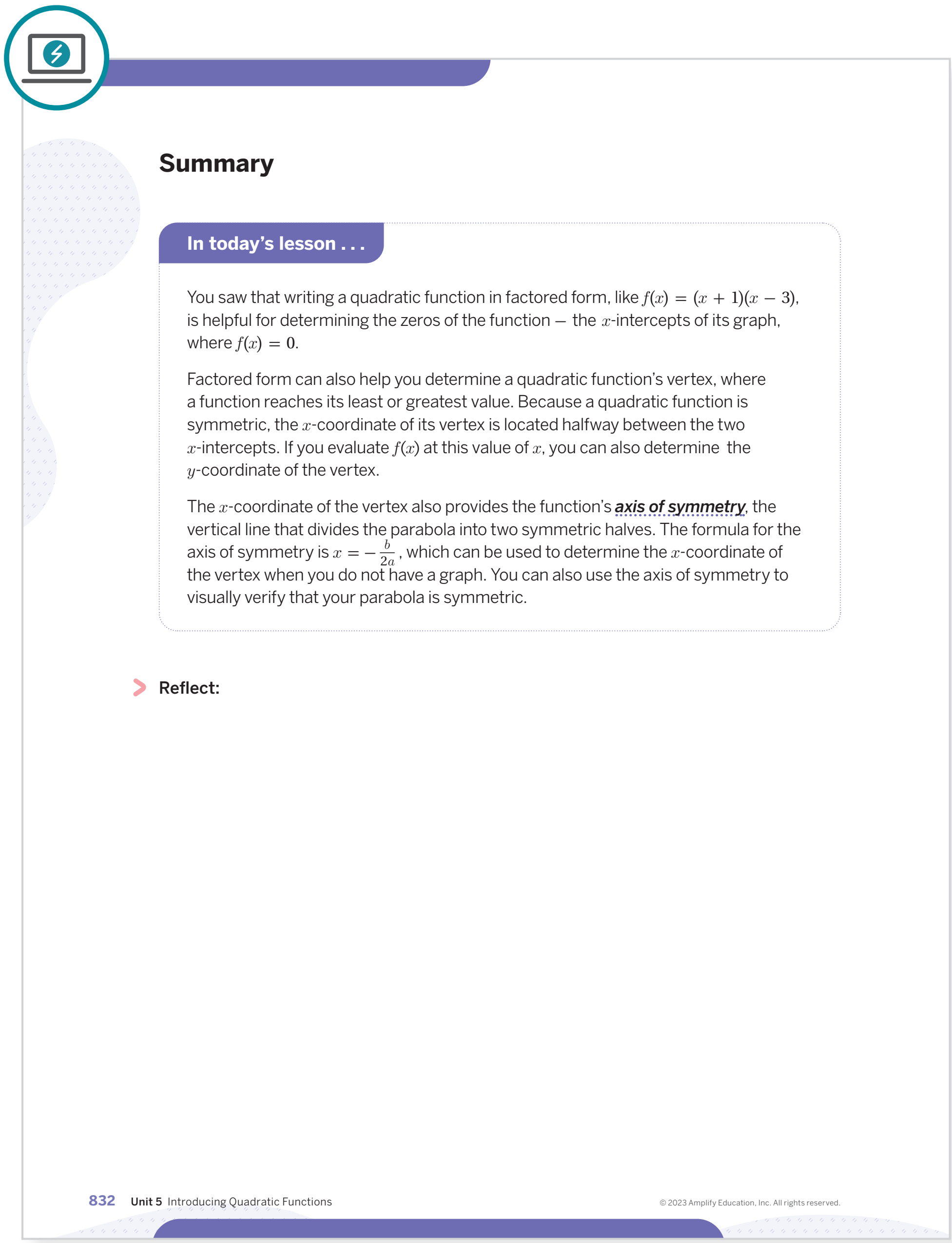
Remind students they previously learned how to determine the vertex and the equation of the axis of symmetry for a parabola. Ask:

- "There are two x-intercepts for each function. What must be true about the x-coordinate of the vertex?" The x-coordinate of the vertex is located halfway between the function's two x-intercepts.
- "What is the general equation for the axis of symmetry?" $x = -\frac{b}{2a}$
- "What does b represent? What does a represent?" b is the coefficient of the linear term when the function is in standard form. a is the coefficient of the squared variable term.

Whole Class | 5 min

Summary

Review and synthesize how to sketch the graph of a quadratic function given in factored form.



Summary

In today's lesson . . .

You saw that writing a quadratic function in factored form, like $f(x) = (x + 1)(x - 3)$, is helpful for determining the zeros of the function — the x-intercepts of its graph, where $f(x) = 0$.

Factored form can also help you determine a quadratic function's vertex, where a function reaches its least or greatest value. Because a quadratic function is symmetric, the x-coordinate of its vertex is located halfway between the two x-intercepts. If you evaluate $f(x)$ at this value of x, you can also determine the y-coordinate of the vertex.

The x-coordinate of the vertex also provides the function's ***axis of symmetry***, the vertical line that divides the parabola into two symmetric halves. The formula for the axis of symmetry is $x = -\frac{b}{2a}$, which can be used to determine the x-coordinate of the vertex when you do not have a graph. You can also use the axis of symmetry to visually verify that your parabola is symmetric.

Reflect:

832 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Ask, "How would you explain to a friend — who is absent today — how to use the factored form of the equation to:"

- "Determine the zeros of the function?"
- "Determine the x-intercepts of the graph?"
- "Determine the equation of the axis of symmetry?"
- "Determine the x-coordinate of the vertex without graphing?"
- "Determine the y-coordinate of the vertex without graphing?"
- "Determine the y-intercept of the graph?"

Highlight how to use the x-intercepts and vertex to graph a quadratic function when it is in factored form.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

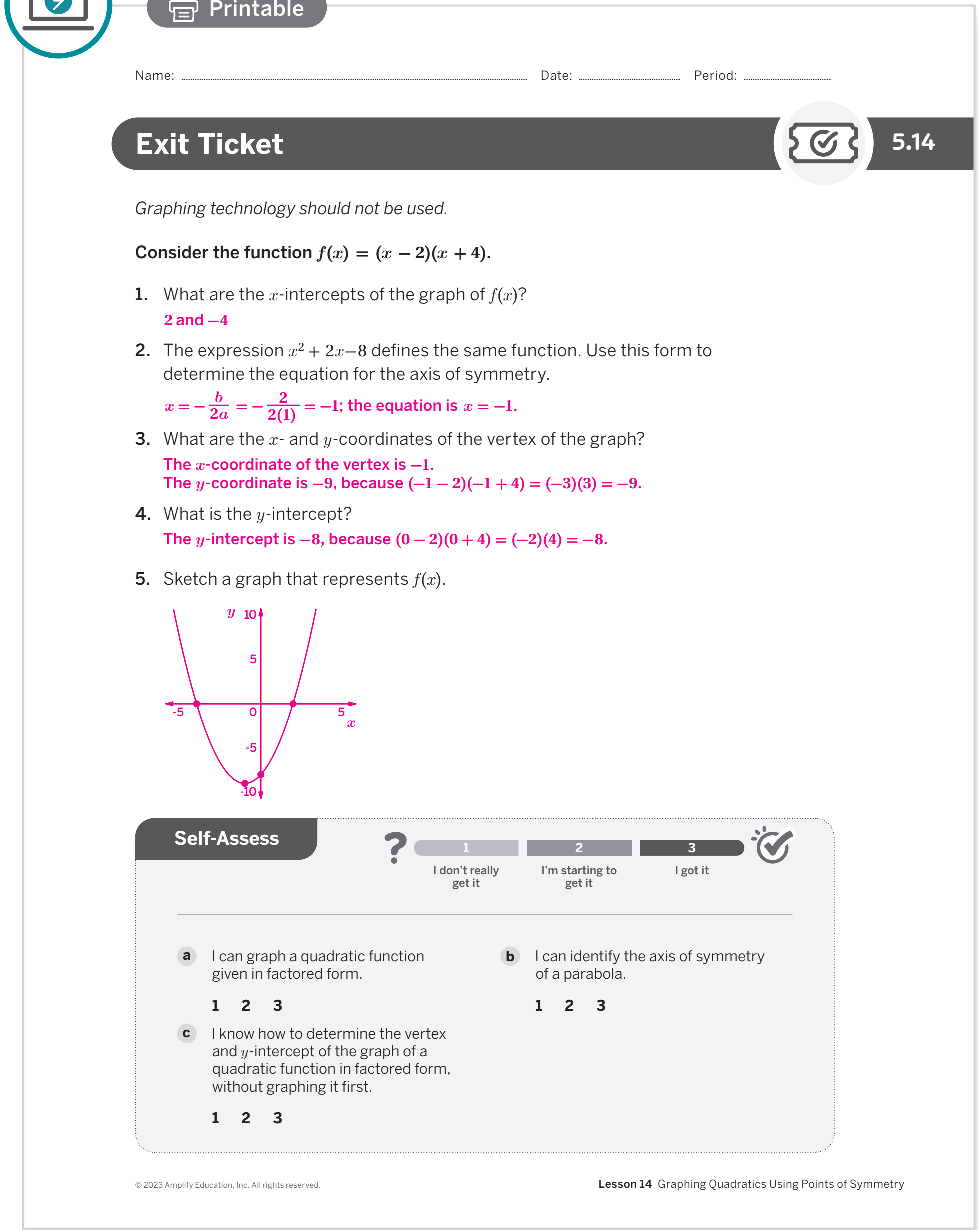
- "How could the axis of symmetry be useful in graphing a quadratic function that is not in factored form?"
- "Why is this line that intersects the vertex referred to as the axis of symmetry?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using the structure of a quadratic function written in factored form to identify the x-intercepts, vertex, and y-intercept — without using graphing technology.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.14

Graphing technology should not be used.

Consider the function $f(x) = (x - 2)(x + 4)$.

1. What are the x-intercepts of the graph of $f(x)$?

   2 and -4

2. The expression $x^2 + 2x - 8$ defines the same function. Use this form to determine the equation for the axis of symmetry.

   $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$; the equation is $x = -1$.

3. What are the x- and y-coordinates of the vertex of the graph?

   The x-coordinate of the vertex is -1.
   The y-coordinate is -9, because $(-1 - 2)(-1 + 4) = (-3)(3) = -9$.

4. What is the y-intercept?

   The y-intercept is -8, because $(0 - 2)(0 + 4) = (-2)(4) = -8$.

5. Sketch a graph that represents $f(x)$.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can graph a quadratic function given in factored form.

1 2 3

b. I can identify the axis of symmetry of a parabola.

1 2 3

c. I know how to determine the vertex and y-intercept of the graph of a quadratic function in factored form, without graphing it first.

1 2 3

Lesson 14 Graphing Quadratics Using Points of Symmetry

Success looks like . . .

- **Goal:** Graphing a quadratic function given in factored form.
- **Goal:** Identify the axis of symmetry of a parabola.
- **Goal:** Without graphing, identifying the vertex and y-intercept of the graph of a quadratic function expressed in factored form.
  - » Identifying the vertex and y-intercept of the graph from the function in Problems 3 and 4.

Suggested next steps

If students provide incorrect x-intercepts in Problem 1, consider:

- Assigning the graphs of different quadratic functions and having students locate the x-intercepts of the graphs.
- Reviewing Activity 3, Problem 1.

If students are unable to determine the axis of symmetry in Problem 2, consider:

- Reviewing Activity 2, Problem 8.
- Having students practice identifying a, b, and c in quadratic expressions of form $ax^2 + bx + c$.

If students are unable to determine the coordinates of the vertex in Problem 3, consider:

- Having students determine the x-coordinate of the vertex given the graph of a quadratic function.
- Reviewing Activity 1, Problem 1.

If students are unable to determine the y-intercept in Problem 4, consider:

- Having students practice substituting 0 into different quadratic functions to determine the y-intercept.

If students incorrectly sketch the graph of $f(x)$ in Problem 5, consider:

- Having students practice identifying the x-intercepts, y-intercept, and vertex given different quadratic functions in factored form.

Professional Learning

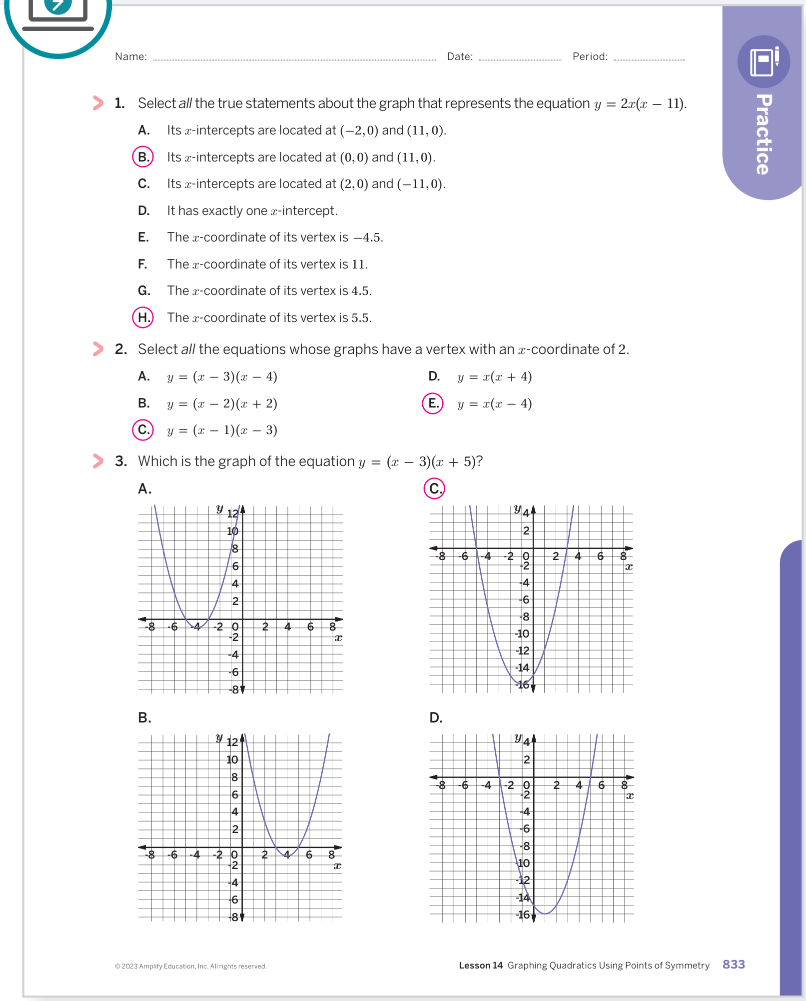
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was graphing a quadratic in factored form. How did graphing go?
- Knowing where students need to be by the end of this unit, how did using the horizontal intercepts to determine the vertex influence that future goal?

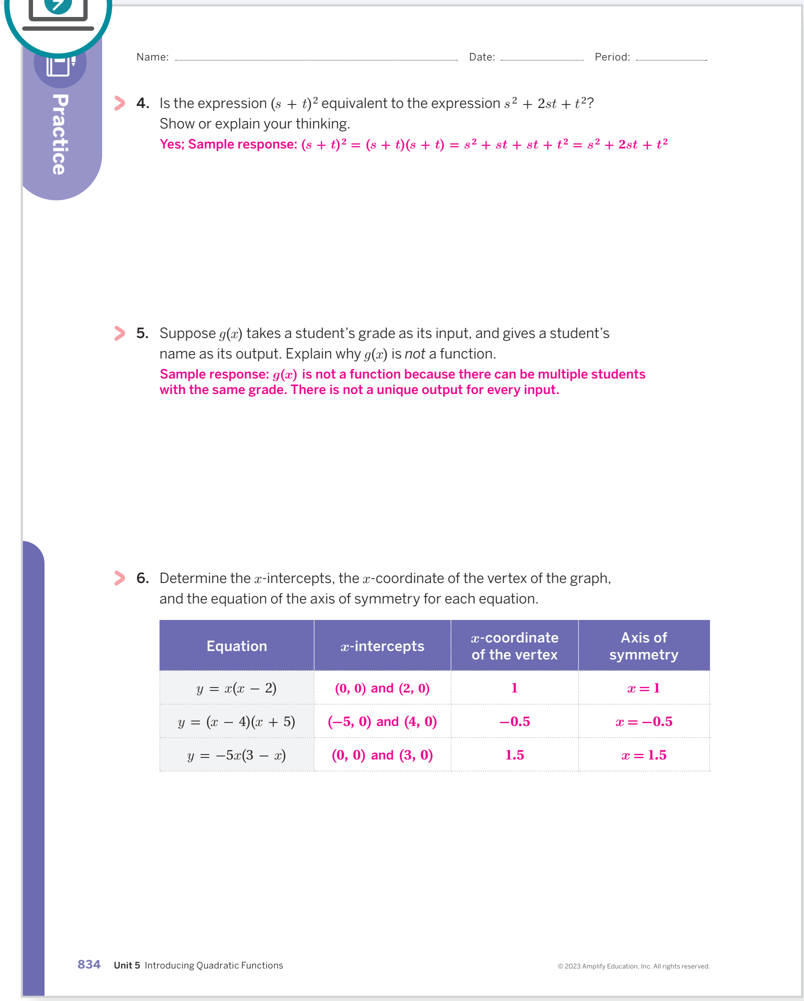
Practice

Independent



Name: Date: Period:

1. Select *all* the true statements about the graph that represents the equation $y = 2x(x - 11)$.
   - A. Its x-intercepts are located at $(-2, 0)$ and $(11, 0)$.
   - [✓ B.] Its x-intercepts are located at $(0, 0)$ and $(11, 0)$.
   - C. Its x-intercepts are located at $(2, 0)$ and $(-11, 0)$.
   - D. It has exactly one x-intercept.
   - E. The x-coordinate of its vertex is -4.5.
   - F. The x-coordinate of its vertex is 11.
   - G. The x-coordinate of its vertex is 4.5.
   - [✓ H.] The x-coordinate of its vertex is 5.5.
2. Select *all* the equations whose graphs have a vertex with an x-coordinate of 2.
   - A. $y = (x - 3)(x - 4)$
   - B. $y = (x - 2)(x + 2)$
   - [✓ C.] $y = (x - 1)(x - 3)$
   - D. $y = x(x + 4)$
   - [✓ E.] $y = x(x - 4)$
3. Which is the graph of the equation $y = (x - 3)(x + 5)$?
   - A.
   - [✓ C.]
   - B.
   - D.



Name: Date: Period:

4. Is the expression $(s + t)^2$ equivalent to the expression $s^2 + 2st + t^2$? Show or explain your thinking.
   Yes; Sample response: $(s + t)^2 = (s + t)(s + t) = s^2 + st + st + t^2 = s^2 + 2st + t^2$
5. Suppose $g(x)$ takes a student's grade as its input, and gives a student's name as its output. Explain why $g(x)$ is *not* a function.
   Sample response: $g(x)$ is not a function because there can be multiple students with the same grade. There is not a unique output for every input.
6. Determine the x-intercepts, the x-coordinate of the vertex of the graph, and the equation of the axis of symmetry for each equation.

Equation	x-intercepts	x-coordinate of the vertex	Axis of symmetry
$y = x(x - 2)$	**(0, 0) and (2, 0)**	**1**	**$x = 1$**
$y = (x - 4)(x + 5)$	**(−5, 0) and (4, 0)**	**−0.5**	**$x = -0.5$**
$y = -5x(3 - x)$	**(0, 0) and (3, 0)**	**1.5**	**$x = 1.5$**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	1
Spiral	4	Unit 5 Lesson 11	2
	5	Unit 3 Lesson 3	1
Formative ⚡	6	Unit 5 Lesson 15	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

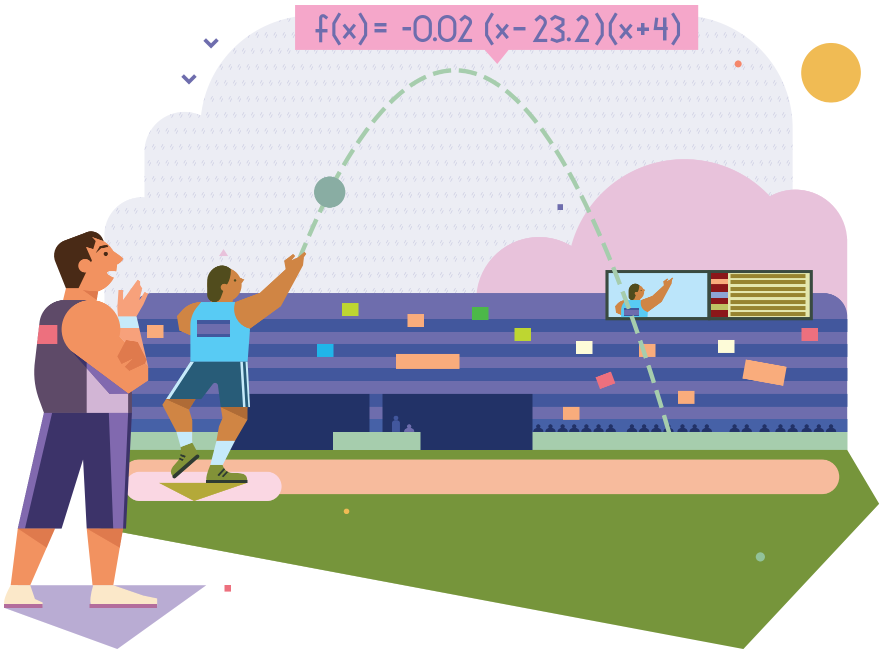
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 15**

Interpreting Quadratics in Factored Form

Let's see what information the factored form provides, given a context.



Focus

Goals

1. **Language Goal:** Determine the zeros of quadratic functions in factored form and interpret their meaning in context. **(Reading and Writing)**
2. **Language Goal:** Determine the x-coordinate of the vertex of a quadratic function $f(x)$ in factored form and interpret its meaning in context. **(Reading and Writing)**

Coherence

Today

Students interpret the coordinates of varying contexts modeled by the factored form. Students interpret the effects on key features of a graph by changing values in a context.

Previously

In Lesson 14, students learned that the graphs of quadratics are symmetrical. They used that knowledge to explain how to determine the vertex and y-intercept of a graph, given a quadratic in factored form.

Coming Soon

In Lessons 16 and 17, students will graph quadratic functions that are written in standard form.

Rigor

- Students **apply** their understanding of the factored form of quadratic functions to study sports competitions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- graphing technology

Math Language Development

Review words

- *horizontal intercept*
- *vertex*
- *vertical intercept*
- *zeros*

Amps | Featured Activity

Activity 3

Interactive Graph

Students use an interactive graph to represent the possible dimensions of a wheelchair basketball court.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not be comfortable making assumptions as they consider how a function in equation and graphical form models a real-life scenario in Activity 1. Encourage students to take mathematical risks, accepting that their thinking might need to be adjusted during the activity. Remind them that throughout the learning process, correcting flawed thinking can lead to revisions and positive results.

Modifications to Pacing

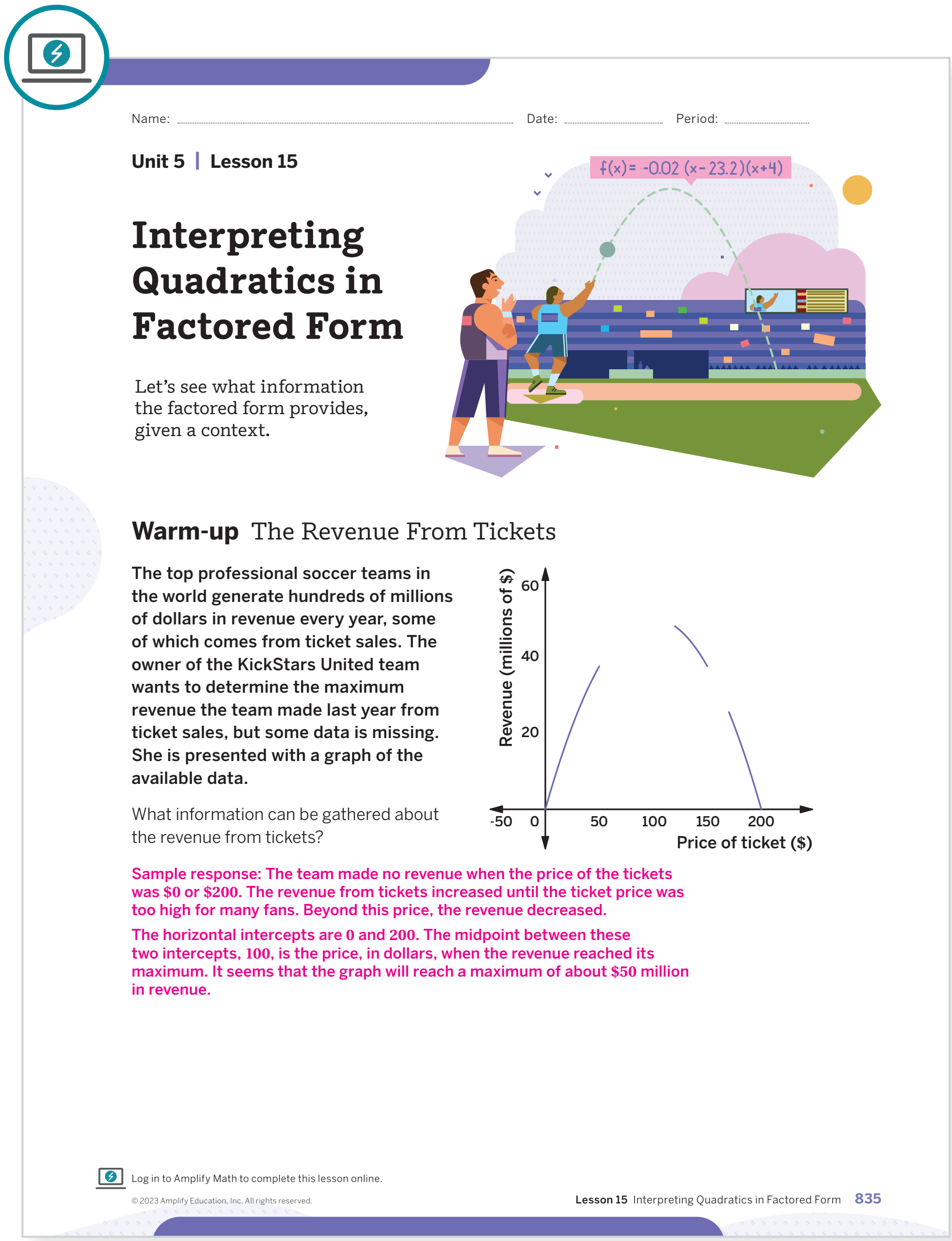
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students only complete Problems 2–4.
- In **Activity 2**, Problem 3 may be omitted.

Independent | 5 min

Warm-up The Revenue From Tickets

Students use the symmetry of the graph of a quadratic function to determine the missing information that can be found in context.



Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 15

Interpreting Quadratics in Factored Form

Let's see what information the factored form provides, given a context.

Warm-up The Revenue From Tickets

The top professional soccer teams in the world generate hundreds of millions of dollars in revenue every year, some of which comes from ticket sales. The owner of the KickStars United team wants to determine the maximum revenue the team made last year from ticket sales, but some data is missing. She is presented with a graph of the available data.

What information can be gathered about the revenue from tickets?

Sample response: The team made no revenue when the price of the tickets was $0 or $200. The revenue from tickets increased until the ticket price was too high for many fans. Beyond this price, the revenue decreased.

The horizontal intercepts are 0 and 200. The midpoint between these two intercepts, 100, is the price, in dollars, when the revenue reached its maximum. It seems that the graph will reach a maximum of about $50 million in revenue.

Log in to Amplify Math to complete this lesson online.

① Launch

Display the graph and then read the prompt aloud. Activate students' background knowledge by asking them about revenue from sporting events and explain the context of the graph.

② Monitor

Help students get started by suggesting they connect the parts of the graph to create a continuous graph.

Look for points of confusion:

- **Identifying the x-coordinate of the vertex by estimating.** Ask, "How could you use the horizontal intercepts of the graph to determine this coordinate?"

Look for productive strategies:

- Using the line of symmetry to construct the rest of the graph.
- Determining the midpoint between the two horizontal intercepts to plot the x-coordinate of the vertex.

③ Connect

Have individual students share their responses to the problem.

Highlight that the symmetry of quadratic graphs can be used to help construct the missing pieces of the graph. The x-coordinate of the vertex is the midpoint of the horizontal intercepts.

Ask, "If one of the horizontal intercepts was missing, what key features would you need to determine it?" I could use the other horizontal intercept and the x-coordinate of the vertex, because the horizontal intercepts are symmetric about the vertex.

MLR Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete the Warm-up, have them meet with 1–2 other students to share their responses and both give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "How do you know you can gather this information?"
- "Where do you see this information on the graph?"
- "What math language can you use in your response?"

Have students revise their responses, as needed.

English Learners

Display or provide access to the Anchor Chart PDF, *Sentence Stems, Describing My Thinking.*

Power-up

To power up students' ability to determine key features of a quadratic function written in factored form:

Provide students with a copy of the Power-up PDF.

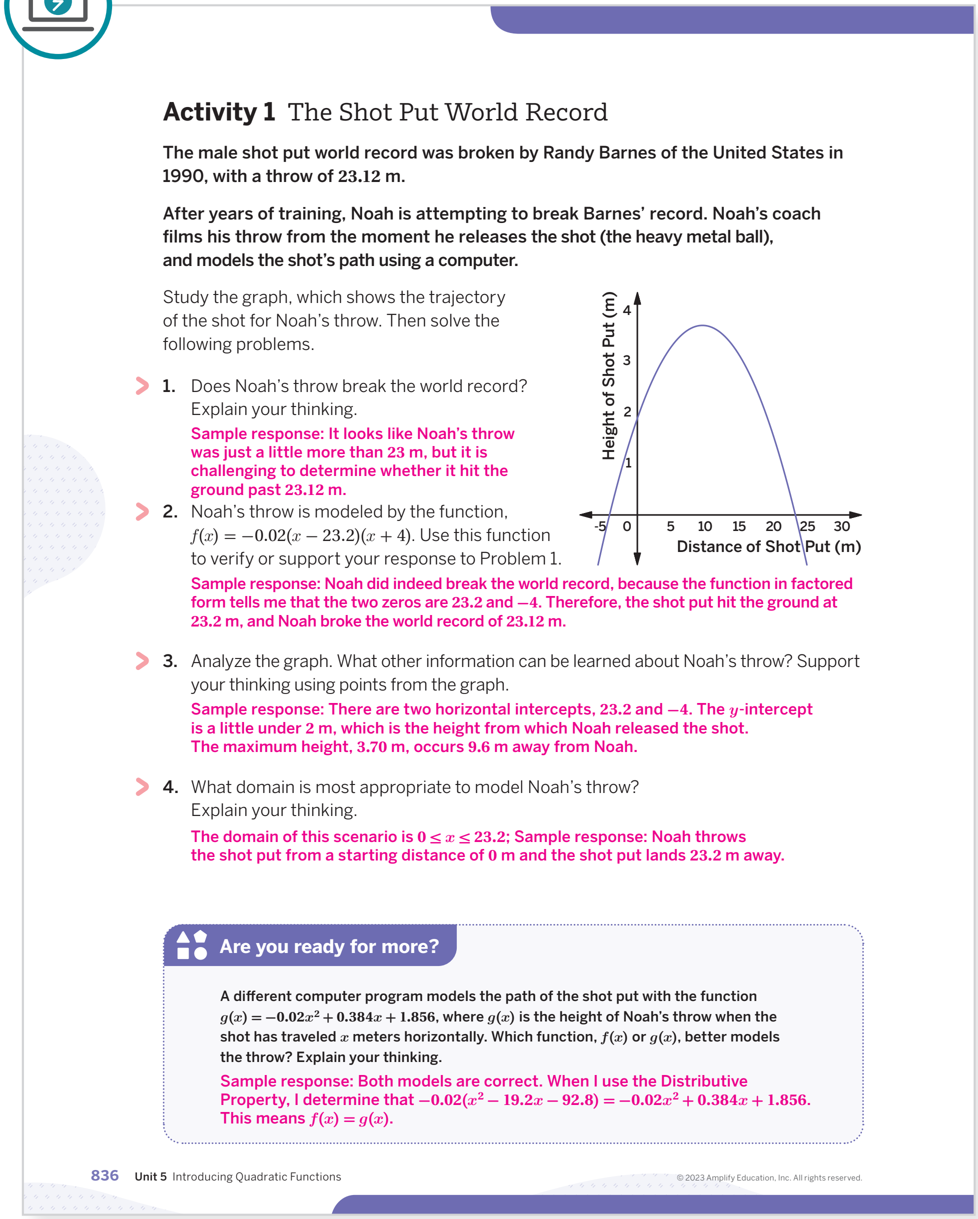
Use: Before Activity 1

Informed by: Performance on Lesson 14, Practice Problem 6

Independent | 10 min

Activity 1 The Shot Put World Record

Students use the factored form of a quadratic function to interpret key features of the graph within the context of a shot putter's throw.



Activity 1 The Shot Put World Record

The male shot put world record was broken by Randy Barnes of the United States in 1990, with a throw of 23.12 m.

After years of training, Noah is attempting to break Barnes' record. Noah's coach films his throw from the moment he releases the shot (the heavy metal ball), and models the shot's path using a computer.

Study the graph, which shows the trajectory of the shot for Noah's throw. Then solve the following problems.

1. Does Noah's throw break the world record? Explain your thinking.
   Sample response: It looks like Noah's throw was just a little more than 23 m, but it is challenging to determine whether it hit the ground past 23.12 m.
2. Noah's throw is modeled by the function, $f(x) = -0.02(x - 23.2)(x + 4)$. Use this function to verify or support your response to Problem 1.
   Sample response: Noah did indeed break the world record, because the function in factored form tells me that the two zeros are 23.2 and -4. Therefore, the shot put hit the ground at 23.2 m, and Noah broke the world record of 23.12 m.
3. Analyze the graph. What other information can be learned about Noah's throw? Support your thinking using points from the graph.
   Sample response: There are two horizontal intercepts, 23.2 and -4. The y-intercept is a little under 2 m, which is the height from which Noah released the shot. The maximum height, 3.70 m, occurs 9.6 m away from Noah.
4. What domain is most appropriate to model Noah's throw? Explain your thinking.
   The domain of this scenario is $0 \le x \le 23.2$; Sample response: Noah throws the shot put from a starting distance of 0 m and the shot put lands 23.2 m away.

Are you ready for more?

A different computer program models the path of the shot put with the function $g(x) = -0.02x^2 + 0.384x + 1.856$, where $g(x)$ is the height of Noah's throw when the shot has traveled x meters horizontally. Which function, $f(x)$ or $g(x)$, better models the throw? Explain your thinking.

Sample response: Both models are correct. When I use the Distributive Property, I determine that $-0.02(x^2 - 19.2x - 92.8) = -0.02x^2 + 0.384x + 1.856$. This means $f(x) = g(x)$.

836 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask, "What are some track and field events? Is anyone familiar with the shot put?" Consider showing a brief video of the shot put event so that students can see what a shot putter's throw looks like.

2 Monitor

Help students get started by saying, "Use the graph to help estimate where you think the shot put hit the ground."

Look for points of confusion:

- **Using both zeros for the domain.** Ask, "What does the negative zero represent in this context? Does it make sense to have a negative distance?"
- **Thinking that the equation in Problem 2 is not in factored form because there is a number outside the parentheses.** Point out that there are three factors and the number outside the parentheses is one of the factors.

Look for productive strategies:

- Checking the zeros by substituting their values into the function for x to see if the function value is 0.

3 Connect

Have individual students share their evidence from Problem 2, supporting or disproving their Problem 1 responses.

Highlight that students can determine the zeros by substituting values in for x to see if this gives a function value of 0, or by finding the opposite of the constant term in each linear factor.

Ask, "Would this function still represent the scenario if the coefficient outside the parentheses changed to a positive value?" No, the zeros would remain the same, but the graph would open upward and the vertical intercept would be a negative value.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the x-intercepts of the graph in one color, the y-intercept in a second color, and the vertex in a third color.

Extension: *Math Enrichment*

Have students study the structure of the function's equation in Problem 2 and ask them to explain why there is no zero at 0.02. In order to be a zero, substituting that value into the function would make the function's value equal to zero, but that is not true for this function because the term -0.02 is not added to or subtracted from x. It just represents a constant multiplied by the rest of the expression.

Math Language Development

MLR8: Discussion Supports

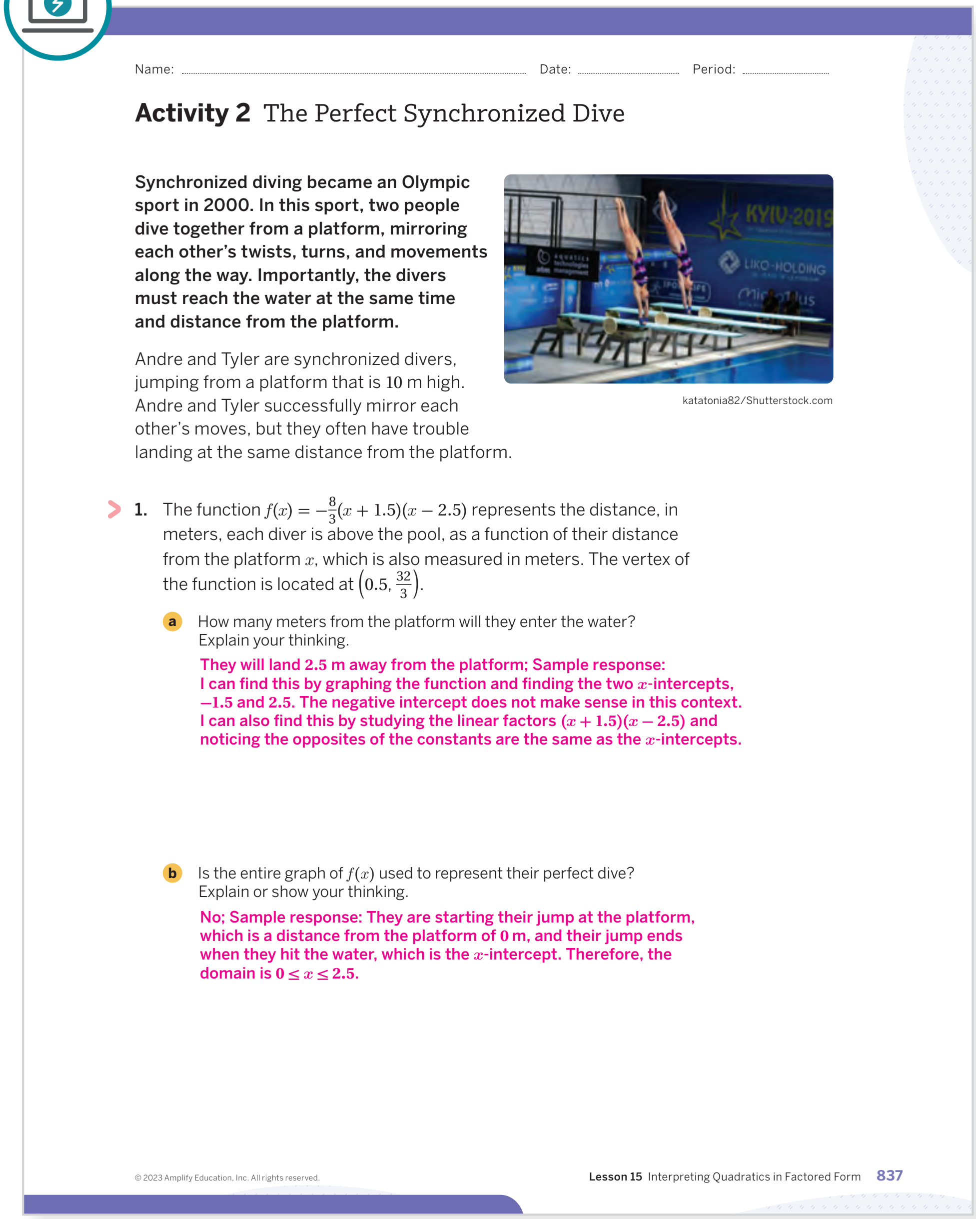
During the Connect, as students share their responses, display these sentence frames to help them organize their thinking.

- "Noah did/did not break the world record because . . ."
- "The shot put hit the ground at _____, which is the _____ of the graph."
- "There are _____ horizontal intercepts, which tell me that . . ."
- "The vertical intercept _____ tells me that . . ."
- "The vertex of the function tells me that . . ."
- "Negative values along the horizontal axis do/do not make sense in this situation because . . ."

Independent | 10 min

Activity 2 The Perfect Synchronized Dive

Students compare two functions modeling the path of synchronized divers to interpret the effects of changing the value outside the parentheses when the equation is written in factored form.



Name: ______ Date: ______ Period: ______

Activity 2 The Perfect Synchronized Dive

Synchronized diving became an Olympic sport in 2000. In this sport, two people dive together from a platform, mirroring each other's twists, turns, and movements along the way. Importantly, the divers must reach the water at the same time and distance from the platform.

Andre and Tyler are synchronized divers, jumping from a platform that is 10 m high. Andre and Tyler successfully mirror each other's moves, but they often have trouble landing at the same distance from the platform.

katatonia82/Shutterstock.com

1. The function $f(x) = -\frac{8}{3}(x + 1.5)(x - 2.5)$ represents the distance, in meters, each diver is above the pool, as a function of their distance from the platform x, which is also measured in meters. The vertex of the function is located at $\left(0.5, \frac{32}{3}\right)$.

   a. How many meters from the platform will they enter the water? Explain your thinking.

   They will land 2.5 m away from the platform; Sample response: I can find this by graphing the function and finding the two x-intercepts, -1.5 and 2.5. The negative intercept does not make sense in this context. I can also find this by studying the linear factors $(x + 1.5)(x - 2.5)$ and noticing the opposites of the constants are the same as the x-intercepts.

   b. Is the entire graph of $f(x)$ used to represent their perfect dive? Explain or show your thinking.

   No; Sample response: They are starting their jump at the platform, which is a distance from the platform of 0 m, and their jump ends when they hit the water, which is the x-intercept. Therefore, the domain is $0 \leq x \leq 2.5$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Interpreting Quadratics in Factored Form 837

1 Launch

Read the prompt aloud. Ask students to sketch what they think the graph of a dive might look like. Provide access to graphing technology.

2 Monitor

Help students get started by suggesting they use graphing technology to graph the function given in Problem 1.

Look for points of confusion:

- **Thinking the horizontal axis represents time.** Ask, "What does x represent in this function?"
- **Not considering the context of the scenario when interpreting the domain of the function in Problem 1b.** Ask, "Where do the divers start, related to the platform? Where will they finish the dive, related to the platform?"
- **Thinking that Andre's path will be the same as Tyler's because the zeros are the same.** Ask, "What is the same and what is different about their functions? What happens when you graph both functions?"

Look for productive strategies:

- Using either graphing technology or the factored form of the function to determine the zeros in Problem 1a, recognizing that the negative intercept does not make sense in this context.
- Interpreting the domain of the function within the context of the scenario in Problem 1b.
- Using graphing technology to compare the graphs of Tyler's and Andre's dives in Problem 2.

Activity 2 continued ›

Differentiated Support

Extension: ***Math Enrichment***

Ask students to write both Andre's and Tyler's functions in standard form. Then have them explain what information standard form gives them and where they see this information on the graphs.

Andre: $f(x) = -\frac{8}{3}x^2 + \frac{8}{3}x + 10$; The vertical intercept is 10, which means that Andre starts his dive 10 m above the pool.

Tyler: $f(x) = -\frac{7}{3}x^2 + \frac{7}{3}x + 8.75$; The vertical intercept is 8.75, which means that Tyler thinks his dive would start at 8.75 m above the pool. Students may notice that the platform is 10 m above the pool, so Tyler could not even realistically begin his dive at 8.75 m above the pool.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses and reasoning for Problem 2a, draw their attention to how the graphs and function equations are similar and different. Ask:

- "What is the same about both function equations? Where do you see these similarities on the graph?"
- "What is different about the graphs of the functions? Where do you see this on the graph?"

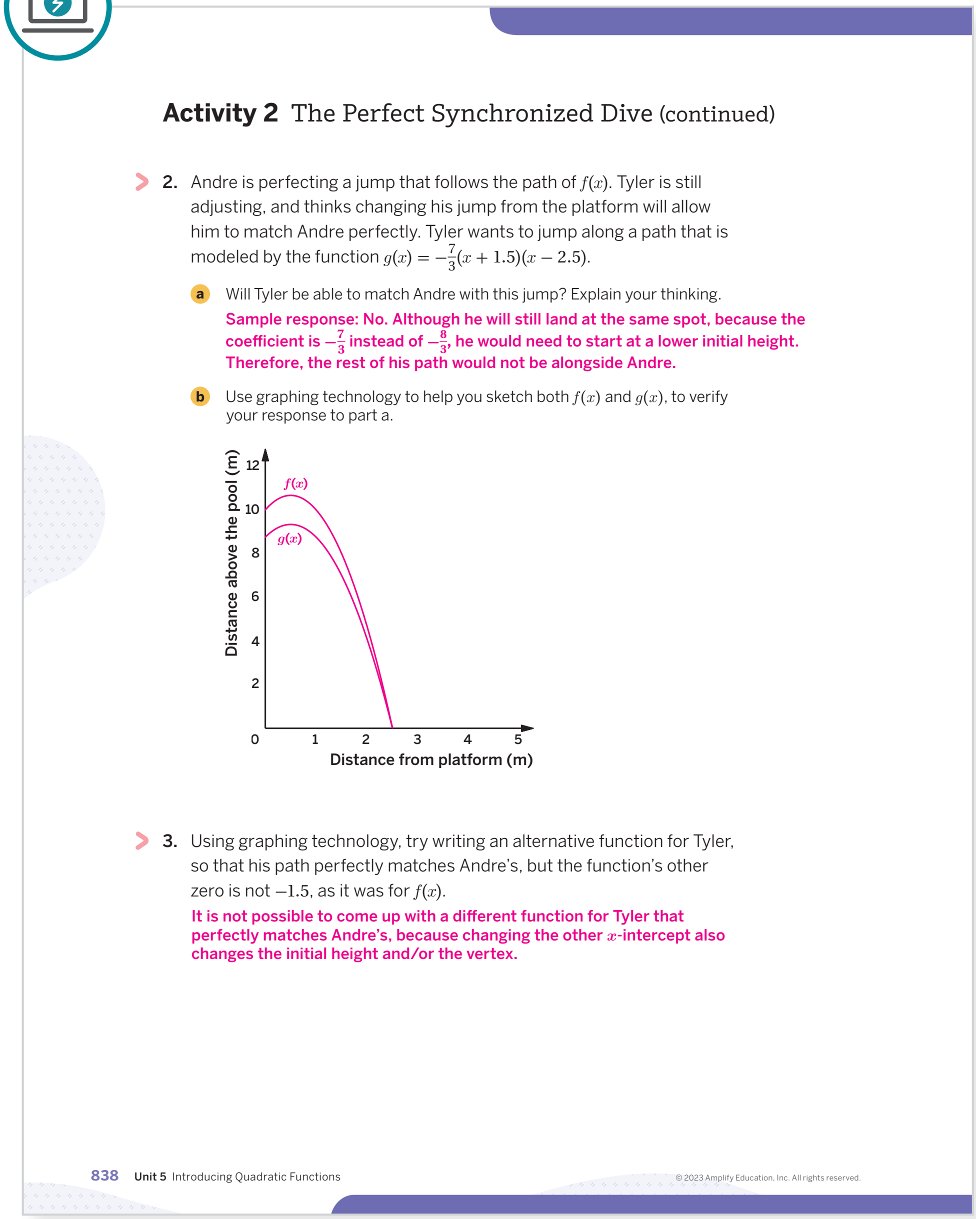
English Learners

Annotate the graph with where the zeros and vertical intercepts are located.

Independent | 10 min

Activity 2 The Perfect Synchronized Dive (continued)

Students compare two functions modeling the path of synchronized divers to interpret the effects of changing the value outside the parentheses when the equation is written in factored form.



Activity 2 The Perfect Synchronized Dive (continued)

2. Andre is perfecting a jump that follows the path of $f(x)$. Tyler is still adjusting, and thinks changing his jump from the platform will allow him to match Andre perfectly. Tyler wants to jump along a path that is modeled by the function $g(x) = -\frac{7}{3}(x + 1.5)(x - 2.5)$.

   a. Will Tyler be able to match Andre with this jump? Explain your thinking.

   Sample response: No. Although he will still land at the same spot, because the coefficient is $-\frac{7}{3}$ instead of $-\frac{8}{3}$, he would need to start at a lower initial height. Therefore, the rest of his path would not be alongside Andre.

   b. Use graphing technology to help you sketch both $f(x)$ and $g(x)$, to verify your response to part a.

3. Using graphing technology, try writing an alternative function for Tyler, so that his path perfectly matches Andre's, but the function's other zero is not −1.5, as it was for $f(x)$.

   It is not possible to come up with a different function for Tyler that perfectly matches Andre's, because changing the other x-intercept also changes the initial height and/or the vertex.

838 **Unit 5** Introducing Quadratic Functions

3 Connect

Have students share whether they think Tyler will be able to match Andre with his jump. Ask them to share their reasoning.

Display the graphs of functions $f(x)$ and $g(x)$ on the same coordinate plane.

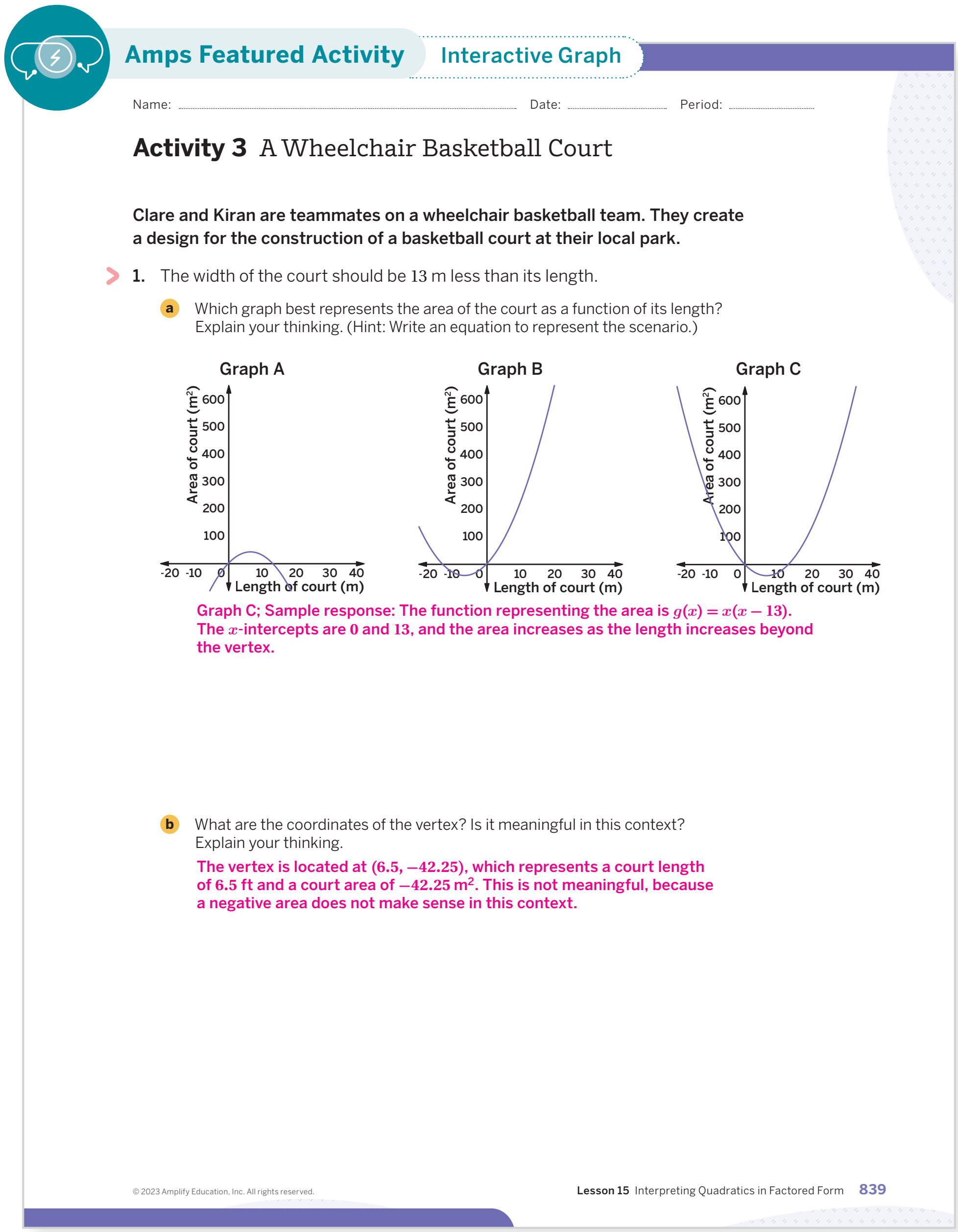
Highlight that quadratic graphs are symmetric across the vertex, so the part of the graph to the left of the vertex has to be symmetrical with the part to the right of the vertex.

Ask, "How does changing the coefficient in front of the factors affect the shape of the graph? If the coefficient increases, the vertex moves up. If the coefficient decreases, the vertex moves down.

Pairs | 15 min

Activity 3 A Wheelchair Basketball Court

Students plan the size of a basketball court using key features of a quadratic graph to understand that sometimes the vertex may not represent realistic minimum or maximum values, given the context.



Amps Featured Activity Interactive Graph

Name: Date: Period:

Activity 3 A Wheelchair Basketball Court

Clare and Kiran are teammates on a wheelchair basketball team. They create a design for the construction of a basketball court at their local park.

1. The width of the court should be 13 m less than its length.

   a. Which graph best represents the area of the court as a function of its length? Explain your thinking. (Hint: Write an equation to represent the scenario.)

   Graph C; Sample response: The function representing the area is $g(x) = x(x - 13)$. The x-intercepts are 0 and 13, and the area increases as the length increases beyond the vertex.

   b. What are the coordinates of the vertex? Is it meaningful in this context? Explain your thinking.

   The vertex is located at (6.5, −42.25), which represents a court length of 6.5 ft and a court area of −42.25 m². This is not meaningful, because a negative area does not make sense in this context.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Interpreting Quadratics in Factored Form 839

1 Launch

Have a brief discussion about the Paralympics. Consider displaying an image of wheelchair basketball.

2 Monitor

Help students get started by activating students' prior knowledge by asking, "How could you test points on each curve to determine which graph represents the function?"

Look for points of confusion:

- **Thinking the graph displays width versus length.** Have students study the axes labels carefully. Remind them that they can calculate the width at any given length by dividing the area by the corresponding length.

Look for productive strategies:

- Creating the function and then choosing a graph.
- Sketching the courts in Problem 4 to help make comparisons.

Activity 3 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive graph to represent the possible dimensions of a wheelchair basketball court.

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

Remind students they previously used quadratic expressions to model the area of a rectangle with a given width or length. Ask:

"If the width of the court is 13 m less than its length, what are some possible dimensions for the length and the width?" Sample responses: 14 by 1, 20 by 7, 57 by 44

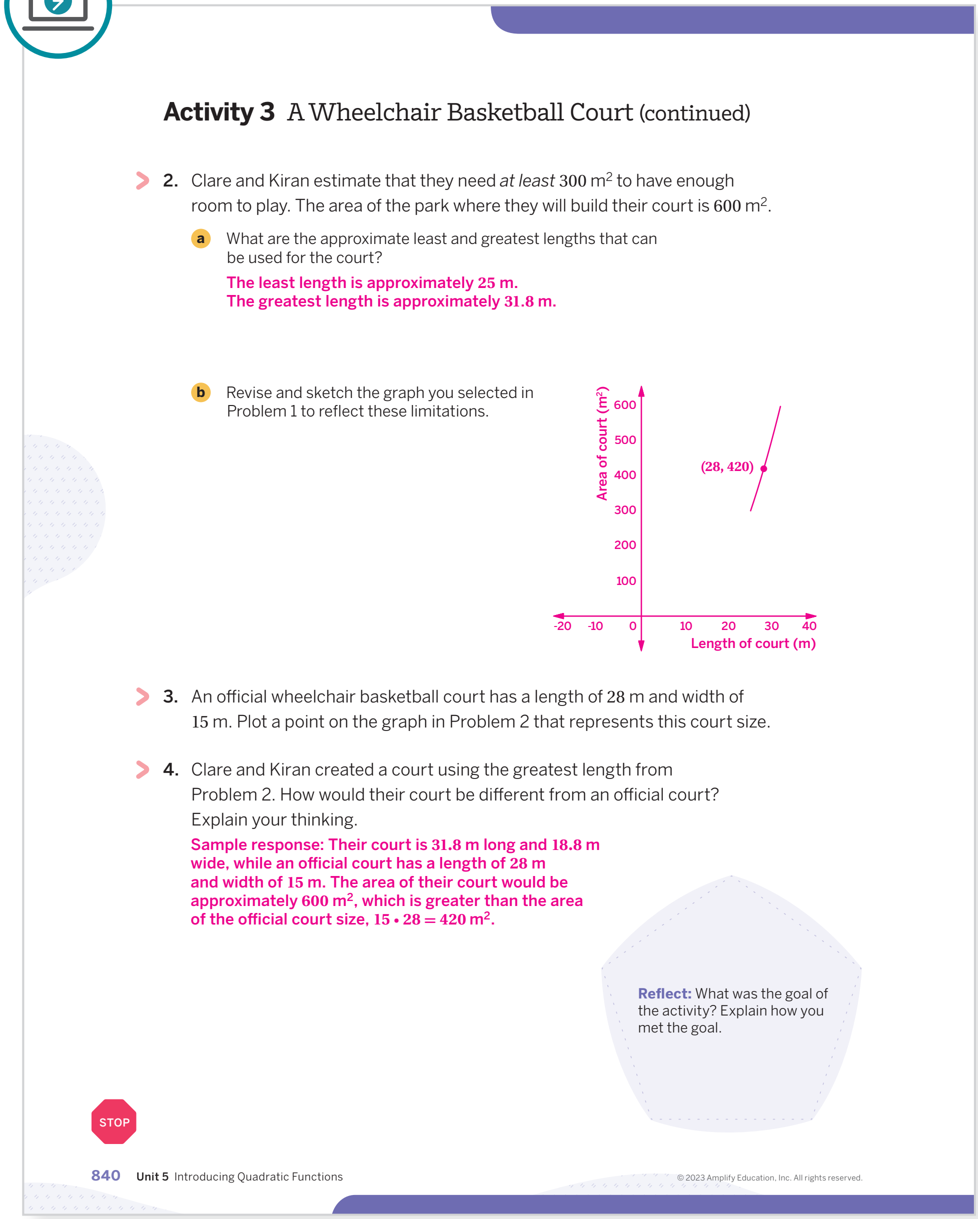
"What are the areas of rectangles with these possible dimensions?" Sample responses: 14, 140, 2,508

"What expression could you write to represent the area of a rectangle with length x?" $x(x - 3)$

Pairs | 15 min

Activity 3 A Wheelchair Basketball Court (continued)

Students plan the size of a basketball court using key features of a quadratic graph to understand that sometimes the vertex may not represent realistic minimum or maximum values, given the context.



Activity 3 A Wheelchair Basketball Court (continued)

2. Clare and Kiran estimate that they need *at least* 300 m^2 to have enough room to play. The area of the park where they will build their court is 600 m^2.

   a. What are the approximate least and greatest lengths that can be used for the court?

   The least length is approximately 25 m.
   The greatest length is approximately 31.8 m.

   b. Revise and sketch the graph you selected in Problem 1 to reflect these limitations.

3. An official wheelchair basketball court has a length of 28 m and width of 15 m. Plot a point on the graph in Problem 2 that represents this court size.

4. Clare and Kiran created a court using the greatest length from Problem 2. How would their court be different from an official court? Explain your thinking.

   Sample response: Their court is 31.8 m long and 18.8 m wide, while an official court has a length of 28 m and width of 15 m. The area of their court would be approximately 600 m^2, which is greater than the area of the official court size, $15 \cdot 28 = 420$ m^2.

Reflect: What was the goal of the activity? Explain how you met the goal.

STOP

3 Connect

Display the three graphs.

Have individual students share which graph they think best represents the area of the court as a function of its length and why.

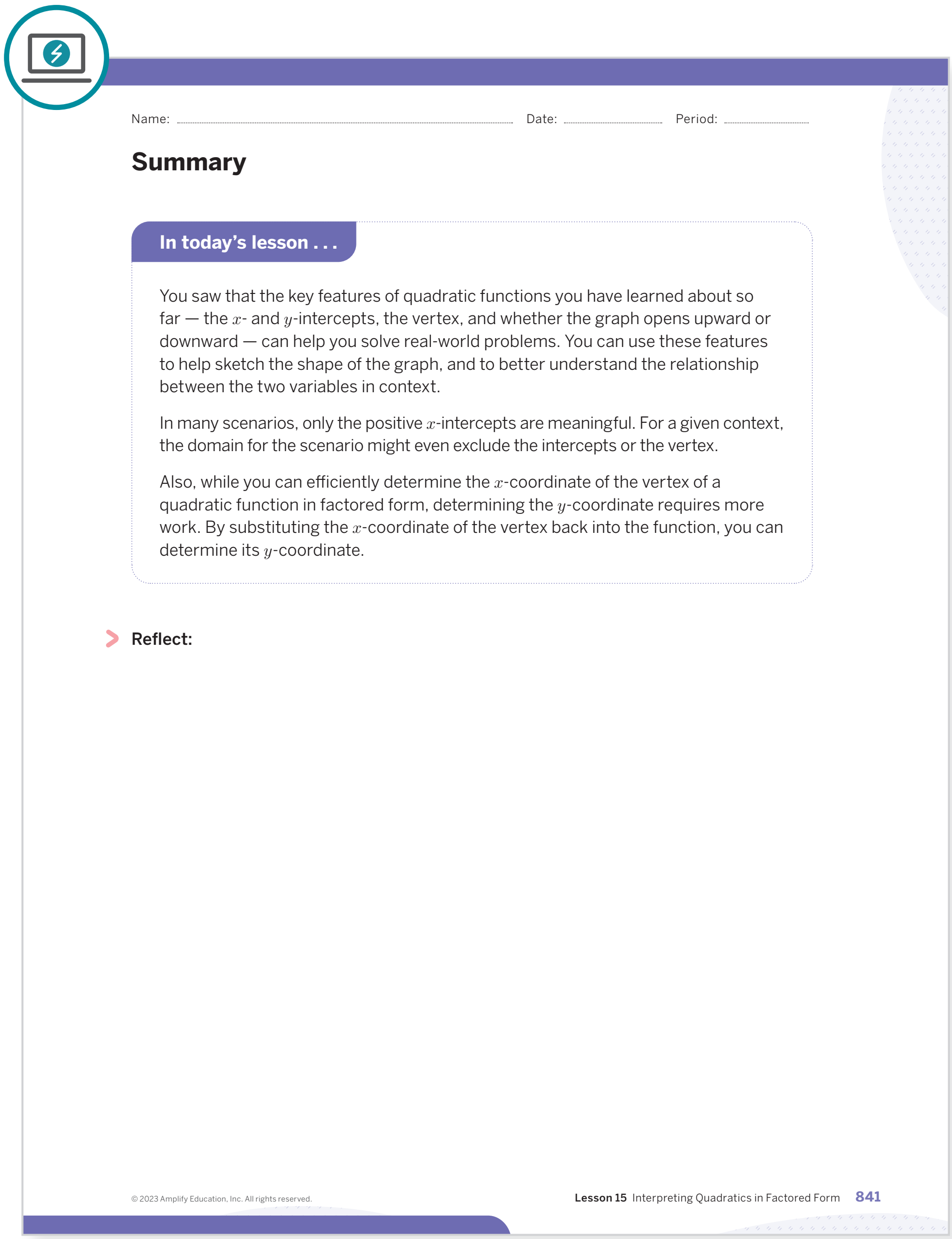
Highlight that the vertex does not represent the minimum possible area, given the context. Depending whether a scenario has further restrictions, the vertex may or may not represent the maximum or minimum function value.

Ask, "Could there be different restrictions so the vertex would represent the minimum court area?" No, the function value is negative here, which does not make sense in this scenario.

Whole Class | 5 min

Summary

Review and synthesize the information a quadratic function provides when written in factored form.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that the key features of quadratic functions you have learned about so far — the x- and y-intercepts, the vertex, and whether the graph opens upward or downward — can help you solve real-world problems. You can use these features to help sketch the shape of the graph, and to better understand the relationship between the two variables in context.

In many scenarios, only the positive x-intercepts are meaningful. For a given context, the domain for the scenario might even exclude the intercepts or the vertex.

Also, while you can efficiently determine the x-coordinate of the vertex of a quadratic function in factored form, determining the y-coordinate requires more work. By substituting the x-coordinate of the vertex back into the function, you can determine its y-coordinate.

Reflect:

Synthesize

Display the function $f(x) = -3(x - 1)(x + 5)$ which models the path of a table tennis ball located x ft from a player.

Have students share the information that they can determine from this function, without graphing it.

Highlight that the maximum of the ball's path can be found by determining the midpoint between the two zeros, which is due to the symmetry of the quadratic graph over the vertex.

Ask, "If one of the zeros shifted 1 unit, how would this affect the location of the vertex?" The vertex would move in the same direction as half of a unit because the vertex is still located at the midpoint of the two horizontal intercepts.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

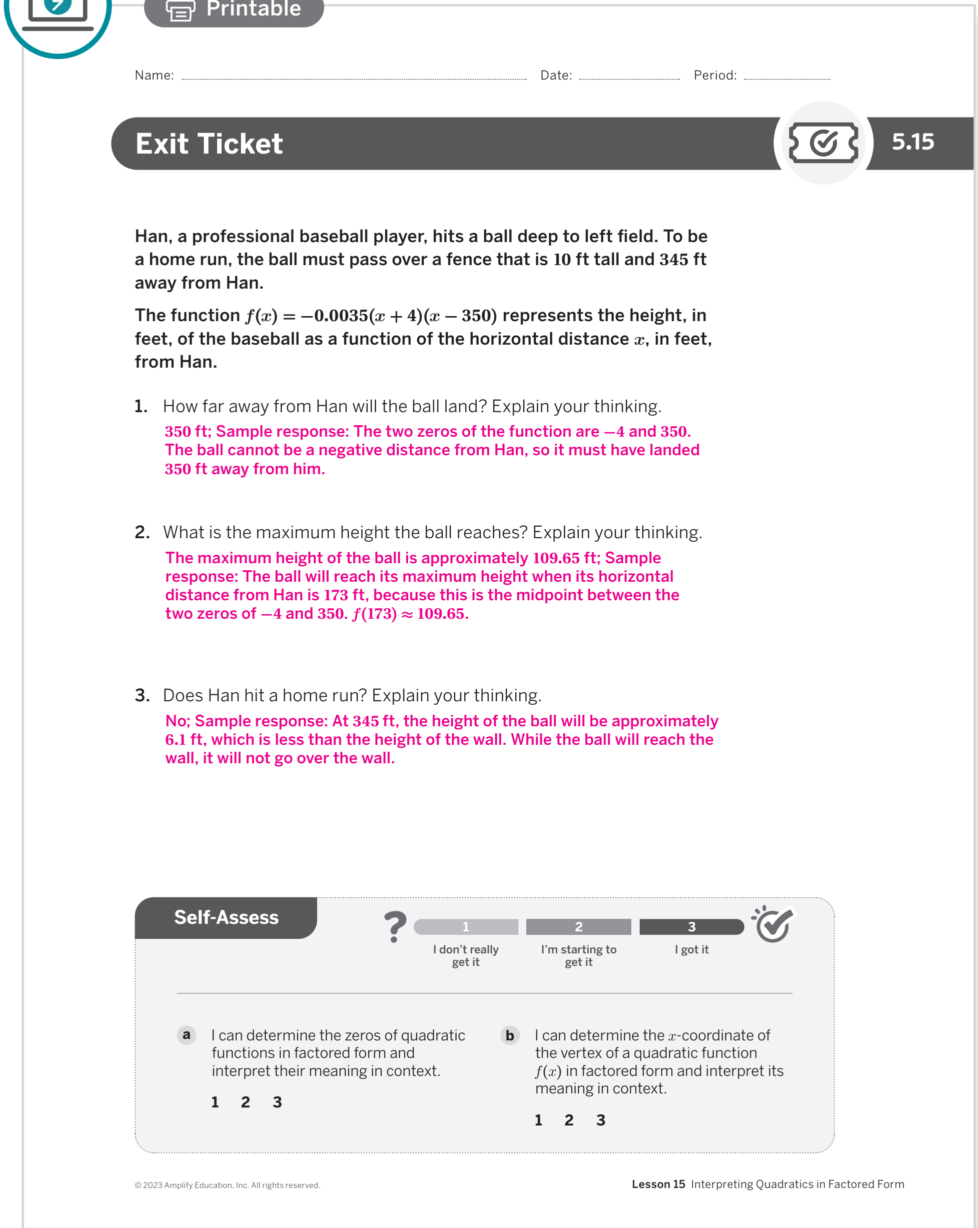
- "How are quadratic functions used to model, analyze, and interpret mathematical relationships?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the zeros and vertex of a quadratic function and interpreting their meanings in context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

5.15

Han, a professional baseball player, hits a ball deep to left field. To be a home run, the ball must pass over a fence that is 10 ft tall and 345 ft away from Han.

The function $f(x) = -0.0035(x + 4)(x - 350)$ represents the height, in feet, of the baseball as a function of the horizontal distance x, in feet, from Han.

1. How far away from Han will the ball land? Explain your thinking.
   350 ft; Sample response: The two zeros of the function are -4 and 350. The ball cannot be a negative distance from Han, so it must have landed 350 ft away from him.

2. What is the maximum height the ball reaches? Explain your thinking.
   The maximum height of the ball is approximately 109.65 ft; Sample response: The ball will reach its maximum height when its horizontal distance from Han is 173 ft, because this is the midpoint between the two zeros of -4 and 350. $f(173) \approx 109.65$.

3. Does Han hit a home run? Explain your thinking.
   No; Sample response: At 345 ft, the height of the ball will be approximately 6.1 ft, which is less than the height of the wall. While the ball will reach the wall, it will not go over the wall.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine the zeros of quadratic functions in factored form and interpret their meaning in context.
1 2 3

b. I can determine the x-coordinate of the vertex of a quadratic function $f(x)$ in factored form and interpret its meaning in context.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Interpreting Quadratics in Factored Form

Success looks like . . .

- **Language Goal:** Determining the zeros of quadratic functions in factored form and interpret their meaning in context. **(Reading and Writing)**
- **Language Goal:** Determining the x-coordinate of the vertex of a quadratic function $f(x)$ in factored form and interpreting its meaning in context. **(Reading and Writing)**
  - » Determining the maximum height of the ball in Problem 2.

Suggested next steps

If students cannot determine where the ball lands in Problem 1, consider:

- Reviewing strategies to determine zeros from Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "How can you use the linear factors to determine the horizontal intercepts?"

If students cannot or incorrectly determine the x-coordinate of the vertex in Problem 2, consider:

- Reviewing using the midpoint of the zeros to determine the x-coordinate of the vertex from Activity 2, and drawing a sketch of a parabola, highlighting the symmetry over the vertex.
- Assigning Practice Problem 3.

If students incorrectly or vaguely respond to Problem 3, consider:

- Suggesting that students draw a sketch of the quadratic graph and include the location of where the ball hits the ground, the maximum height of the ball, and the approximate location of the fence.
- Assigning Practice Problem 3.

Professional Learning

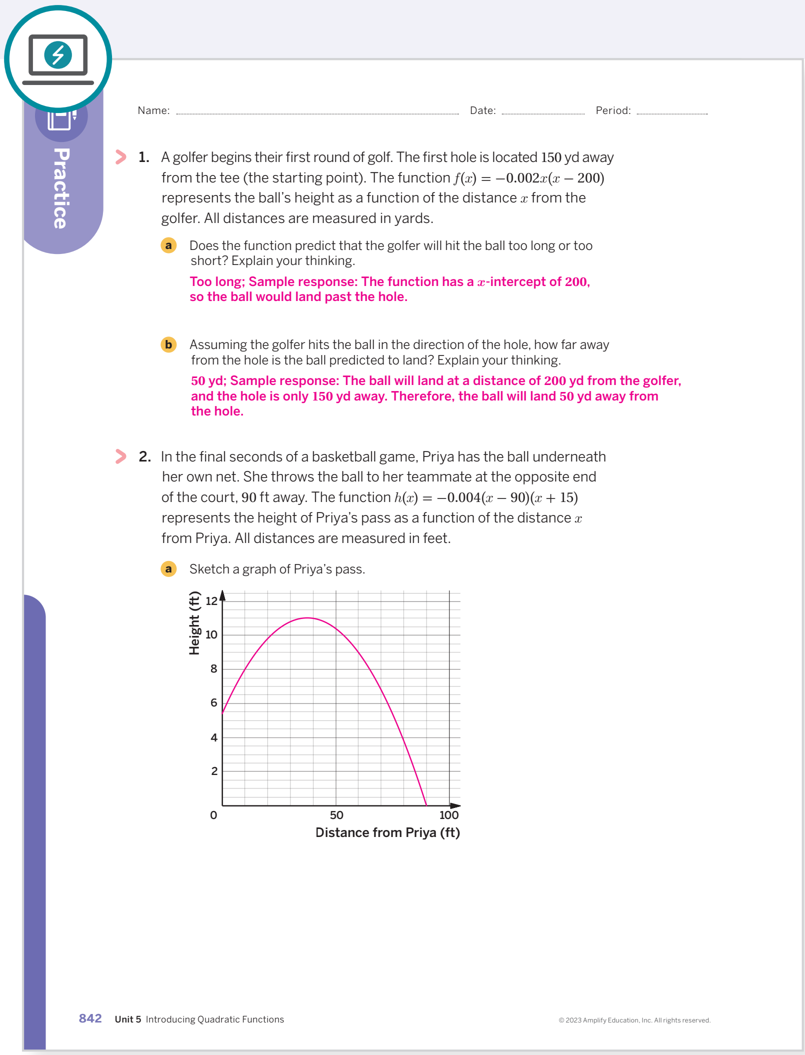
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students interpret quadratic functions in context. How did that build on the earlier work students did with factored form?
- How did students look for and express regularity in repeated reasoning today? How are you helping students become aware of how they are progressing in this area?

Practice

Independent



Practice

Name: Date: Period:

1. A golfer begins their first round of golf. The first hole is located 150 yd away from the tee (the starting point). The function $f(x) = -0.002x(x - 200)$ represents the ball's height as a function of the distance x from the golfer. All distances are measured in yards.

   a. Does the function predict that the golfer will hit the ball too long or too short? Explain your thinking.

   Too long; Sample response: The function has a x-intercept of 200, so the ball would land past the hole.

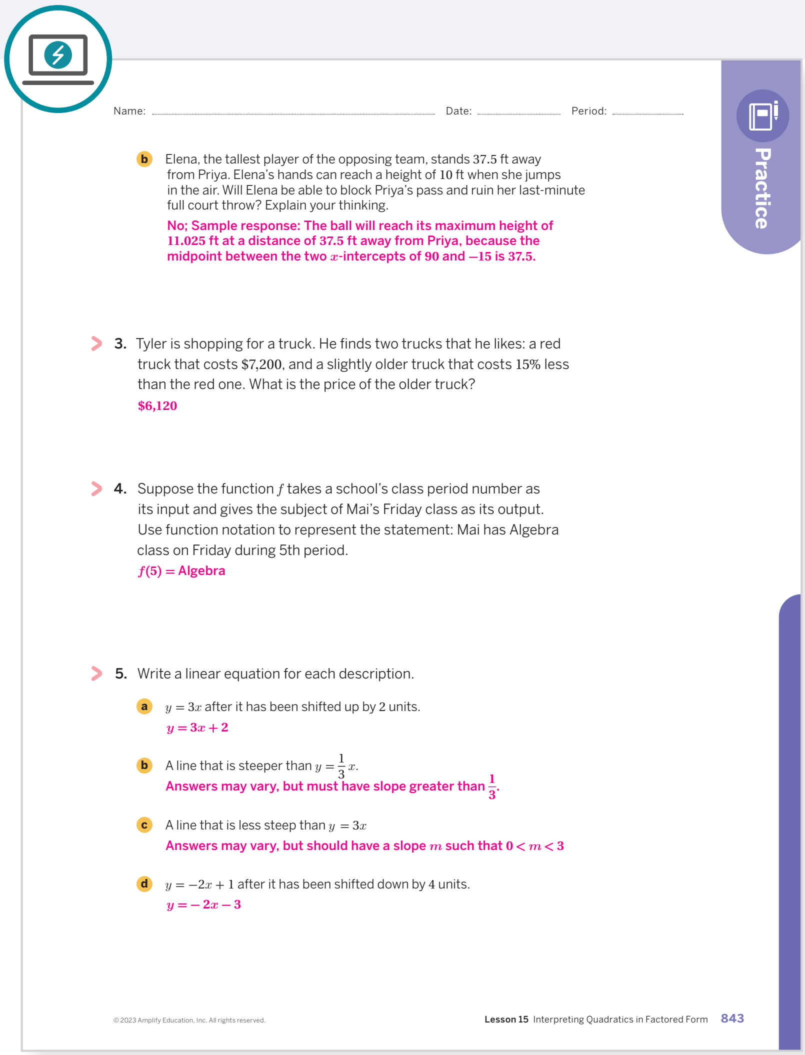
   b. Assuming the golfer hits the ball in the direction of the hole, how far away from the hole is the ball predicted to land? Explain your thinking.

   50 yd; Sample response: The ball will land at a distance of 200 yd from the golfer, and the hole is only 150 yd away. Therefore, the ball will land 50 yd away from the hole.

2. In the final seconds of a basketball game, Priya has the ball underneath her own net. She throws the ball to her teammate at the opposite end of the court, 90 ft away. The function $h(x) = -0.004(x - 90)(x + 15)$ represents the height of Priya's pass as a function of the distance x from Priya. All distances are measured in feet.

   a. Sketch a graph of Priya's pass.

842 Unit 5 Introducing Quadratic Functions



Practice

Name: Date: Period:

   b. Elena, the tallest player of the opposing team, stands 37.5 ft away from Priya. Elena's hands can reach a height of 10 ft when she jumps in the air. Will Elena be able to block Priya's pass and ruin her last-minute full court throw? Explain your thinking.

   No; Sample response: The ball will reach its maximum height of 11.025 ft at a distance of 37.5 ft away from Priya, because the midpoint between the two x-intercepts of 90 and –15 is 37.5.

3. Tyler is shopping for a truck. He finds two trucks that he likes: a red truck that costs $7,200, and a slightly older truck that costs 15% less than the red one. What is the price of the older truck?

   $6,120

4. Suppose the function f takes a school's class period number as its input and gives the subject of Mai's Friday class as its output. Use function notation to represent the statement: Mai has Algebra class on Friday during 5th period.

   $f(5) =$ Algebra

5. Write a linear equation for each description.

   a. $y = 3x$ after it has been shifted up by 2 units.

   $y = 3x + 2$

   b. A line that is steeper than $y = \frac{1}{3}x$.

   Answers may vary, but must have slope greater than $\frac{1}{3}$.

   c. A line that is less steep than $y = 3x$

   Answers may vary, but should have a slope m such that $0 < m < 3$

   d. $y = -2x + 1$ after it has been shifted down by 4 units.

   $y = -2x - 3$

Lesson 15 Interpreting Quadratics in Factored Form 843

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
Spiral	3	Unit 4 Lesson 15	1
	4	Unit 3 Lesson 3	2
Formative ⚡	5	Unit 5 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

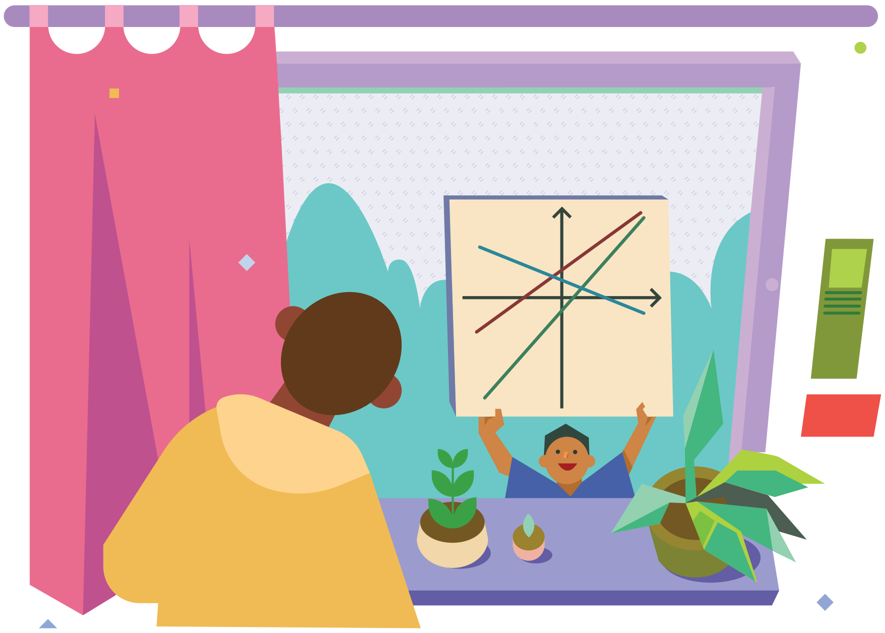
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 16**

Graphing With the Standard Form (Part 1)

Let's see how changing the values of the coefficients in quadratic functions affect their graphs.



Focus

Goals

1. **Language Goal:** Comprehend how the values of a and c in $y = ax^2 + bx + c$ are visible on the graph. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Coordinate different representations of quadratic functions (expressions, tables, and graphs). **(Speaking and Listening, Reading and Writing)**
3. Use technology to explore how the parameters of quadratic equations in standard form are visible on the graph.

Coherence

Today

Students further their understanding of the standard form for quadratics and explore how the coefficient of the squared variable and constant terms relate to features of the graphs. They build on their understanding of identifying equivalent quadratic expressions in standard and factored form and their graphs.

Previously

In Lesson 15, students interpreted key features of expressions written in factored form and studied how changing values in context changed the graph of the function.

Coming Soon

In Lesson 17, students will focus on the coefficient of the linear term, how it affects the graph, and writing equations using the structure in quadratic expressions.

Rigor

- Students build **conceptual understanding** of the graphs of quadratic functions in standard form.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards
- Anchor Chart PDF, *Sentence Stems, Matching Prompts*
- graphing technology

Math Language Development

Review words

- *equivalent expressions*
- *factored form*
- *intercepts*
- *standard form*
- *vertex*

Amps | Featured Activity

Activity 2
Interactive Graph

Students use graphing technology to explore changes to the graph of graph $y = x^2$ due to changing the coefficient of the quadratic term and the constant term.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

If students are not familiar with transformations of functions, they might doubt their own abilities to work with quadratic graphs in Activity 2. By adopting a positive attitude and making observations using graphing technology, students can use the regularity of transforming the parent function to further their understanding of different representations of the same function, an equation and a graph.

Modifications to Pacing

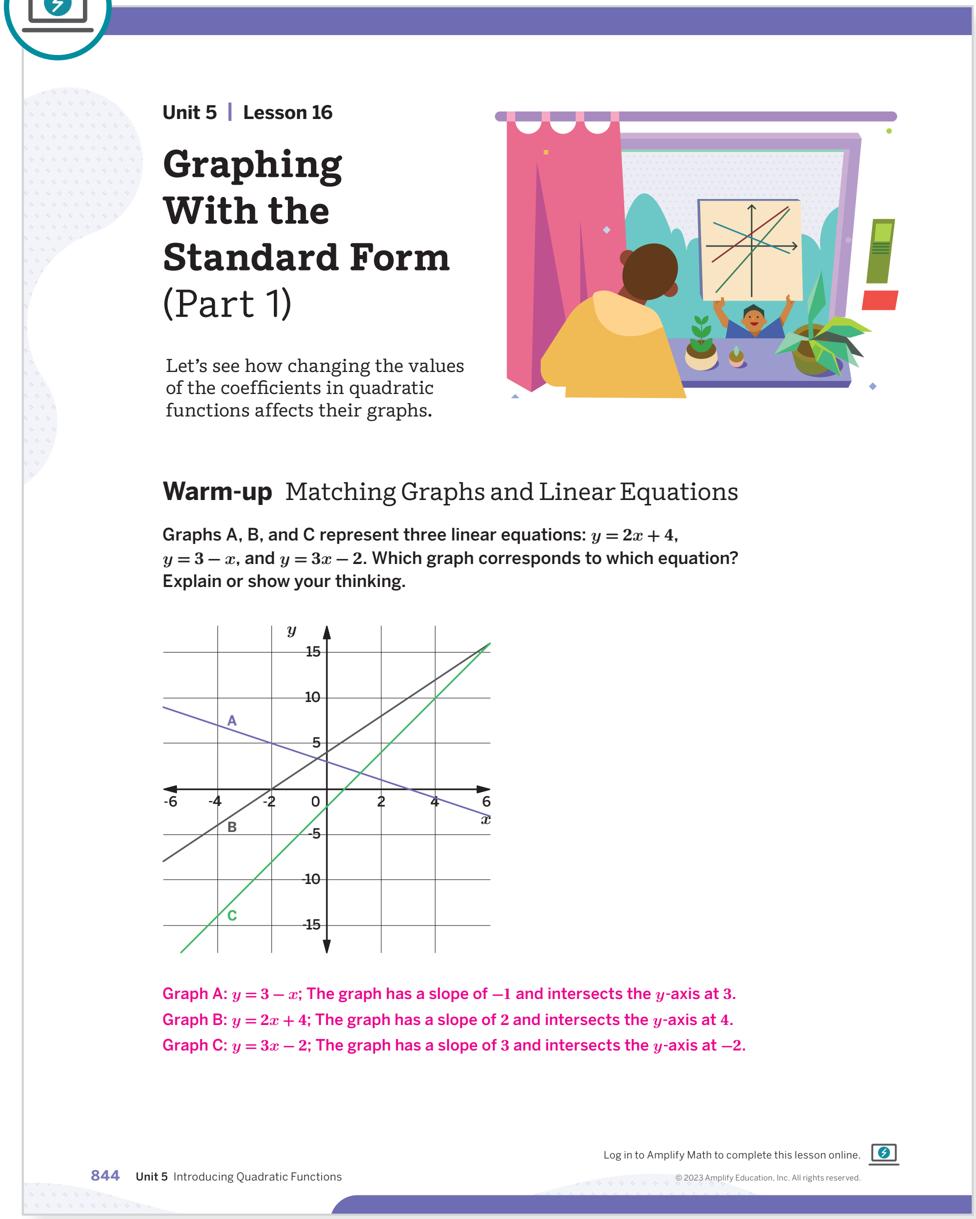
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Graph C may be omitted.
- In **Activity 1**, Problems 1 and 2, the first and last columns in each table may be omitted.
- In **Activity 3**, Set 3 of the Card Sort may be omitted.

Independent | 5 min

Warm-up Matching Graphs and Linear Equations

Students activate their prior knowledge by recalling how parameters of linear equations affect their graphs, preparing them to study graphs of quadratic functions.



Unit 5 | Lesson 16

Graphing With the Standard Form (Part 1)

Let's see how changing the values of the coefficients in quadratic functions affects their graphs.

Warm-up Matching Graphs and Linear Equations

Graphs A, B, and C represent three linear equations: $y = 2x + 4$, $y = 3 - x$, and $y = 3x - 2$. Which graph corresponds to which equation? Explain or show your thinking.

Graph A: $y = 3 - x$; The graph has a slope of -1 and intersects the y-axis at 3.
Graph B: $y = 2x + 4$; The graph has a slope of 2 and intersects the y-axis at 4.
Graph C: $y = 3x - 2$; The graph has a slope of 3 and intersects the y-axis at -2.

Log in to Amplify Math to complete this lesson online.

844 Unit 5 Introducing Quadratic Functions

1 Launch

Have students look for connections between the linear equations and the graphs that represent them.

2 Monitor

Help students get started by having them identify the slope and y-intercept of the graphs.

Look for points of confusion:

- **Not connecting the negative slope of Graph A to one of the equations.** Ask them to find the coefficient of the variable term in each equation.
- **Not recognizing the constant term in each equation.** Remind them the y-intercept is b in the equation $y = mx + b$.

Look for productive strategies:

- Studying the graphs and thinking about their corresponding equations.
- Starting with the equations and then visualizing the graphs.

3 Connect

Display the equations, identifying the coefficient and constant terms.

Have students share their different approaches to matching the equations and the graphs.

Highlight that the equations and the graphs are connected in more than one way. There are different ways to know what a graph would look like, given its equation, or vice versa. Tell students that they will study the connections between the equations and graphs that represent quadratic functions.

Power-up

To power up students' ability to compare linear functions, using their slopes and y-intercepts, have students complete:

Determine whether each statement is *true* or *false*.

1. The graph of $y = 3x - 5$ is shifted 3 units up from the graph of $y = 3x$. False
2. The graph of $y = 3x - 5$ is shifted 5 units down from the graph of $y = 3x$. True
3. The graph of $y = \frac{1}{4}x + 1$ is shifted 1 unit up from the graph of $y = \frac{1}{4}x$. True
4. The graph of $y = \frac{1}{4}x + 1$ is steeper than the graph of $y = 3x$. False

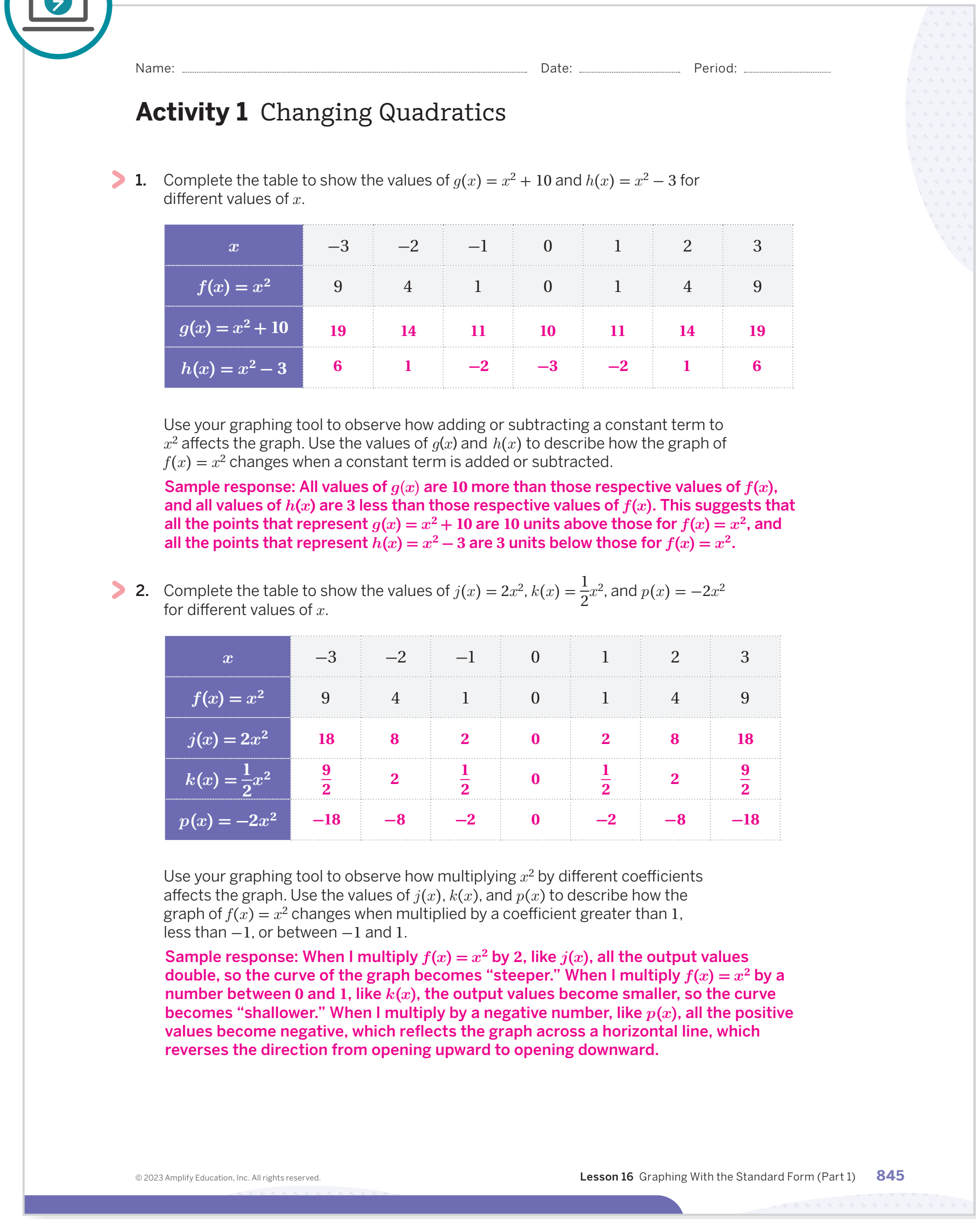
Use: Before Activity 1

Informed by: Performance on Lesson 15, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 7

Pairs | 10 min

Activity 1 Changing Quadratics

Students compare the values of quadratic functions, with different coefficients and constant terms, to the values of $f(x) = x^2$, to see the effects of adding a constant or multiplying the x^2 term by a coefficient.



Name: ________ Date: ________ Period: ________

Activity 1 Changing Quadratics

1. Complete the table to show the values of $g(x) = x^2 + 10$ and $h(x) = x^2 - 3$ for different values of x.

x	–3	–2	–1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9
$g(x) = x^2 + 10$	**19**	**14**	**11**	**10**	**11**	**14**	**19**
$h(x) = x^2 - 3$	**6**	**1**	**–2**	**–3**	**–2**	**1**	**6**

Use your graphing tool to observe how adding or subtracting a constant term to x^2 affects the graph. Use the values of $g(x)$ and $h(x)$ to describe how the graph of $f(x) = x^2$ changes when a constant term is added or subtracted.

Sample response: All values of $g(x)$ are 10 more than those respective values of $f(x)$, and all values of $h(x)$ are 3 less than those respective values of $f(x)$. This suggests that all the points that represent $g(x) = x^2 + 10$ are 10 units above those for $f(x) = x^2$, and all the points that represent $h(x) = x^2 - 3$ are 3 units below those for $f(x) = x^2$.

2. Complete the table to show the values of $j(x) = 2x^2$, $k(x) = \frac{1}{2}x^2$, and $p(x) = -2x^2$ for different values of x.

x	–3	–2	–1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9
$j(x) = 2x^2$	**18**	**8**	**2**	**0**	**2**	**8**	**18**
$k(x) = \frac{1}{2}x^2$	$\frac{9}{2}$	**2**	$\frac{1}{2}$	**0**	$\frac{1}{2}$	**2**	$\frac{9}{2}$
$p(x) = -2x^2$	**–18**	**–8**	**–2**	**0**	**–2**	**–8**	**–18**

Use your graphing tool to observe how multiplying x^2 by different coefficients affects the graph. Use the values of $j(x)$, $k(x)$, and $p(x)$ to describe how the graph of $f(x) = x^2$ changes when multiplied by a coefficient greater than 1, less than –1, or between –1 and 1.

Sample response: When I multiply $f(x) = x^2$ by 2, like $j(x)$, all the output values double, so the curve of the graph becomes "steeper." When I multiply $f(x) = x^2$ by a number between 0 and 1, like $k(x)$, the output values become smaller, so the curve becomes "shallower." When I multiply by a negative number, like $p(x)$, all the positive values become negative, which reflects the graph across a horizontal line, which reverses the direction from opening upward to opening downward.

1 Launch

Provide access to graphing technology.

2 Monitor

Help students get started by comparing the values of y when the constant term c is changed.

Look for points of confusion:

- **Not recognizing the connection between adding or subtracting the constant terms to the y-intercept in Problem 1.** Ask, "What do you notice about the constant term and the output value when $x = 0$?"
- **Not recognizing the connection between multiplying the x^2 term by a coefficient other than 1 in Problem 2.** Have them write the input and output values as ordered pairs.

Look for productive strategies:

- Recognizing that adding a constant term increases the output value of $f(x)$ by that number.
- Recognizing that multiplying x^2 by a value changes the output values by the same factor.

3 Connect

Display the graphs of each set of functions and ask students how they compare to the graph of $f(x) = x^2$.

Have students share how the graphs compare, focusing their attention on the structure of the equations for each function.

Highlight that adding a constant term increases the value of x^2 by that number, shifting the points on the graph up by that amount. Multiplying x^2 by 2 or $\frac{1}{2}$ changes the output values by the same factor, shifts the points for x^2 by that same factor (twice the value or half the value).

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with pre-completed tables for Problems 1 and 2 and have them focus their time on graphing the functions and analyzing how their graphs compare to the graph of $f(x) = x^2$.

Extension: *Math Enrichment*

Display the following functions. Have students make a prediction about how their graphs will compare to the graph of $f(x) = x^2$. Then have them use graphing technology to test their predictions.

$f(x) = 5x^2 + 3$ $\quad$ $f(x) = \frac{1}{3}x^2 - 1$ $\quad$ $f(x) = -x^2 + 4$

Math Language Development

MLR7: Compare and Connect

During the Connect, as you display the graphs of each set of functions, draw students' attention to the connections between the structure of the equations in each set and how their graphs compare to the graph of $f(x) = x^2$. Ask:

- "How does adding or subtracting a constant term to x^2 affect the graph of the function?"
- "How does multiplying the x^2 term by a constant greater than 1 affect the graph of the function? Less than –1? Between –1 and 1?"

English Learners

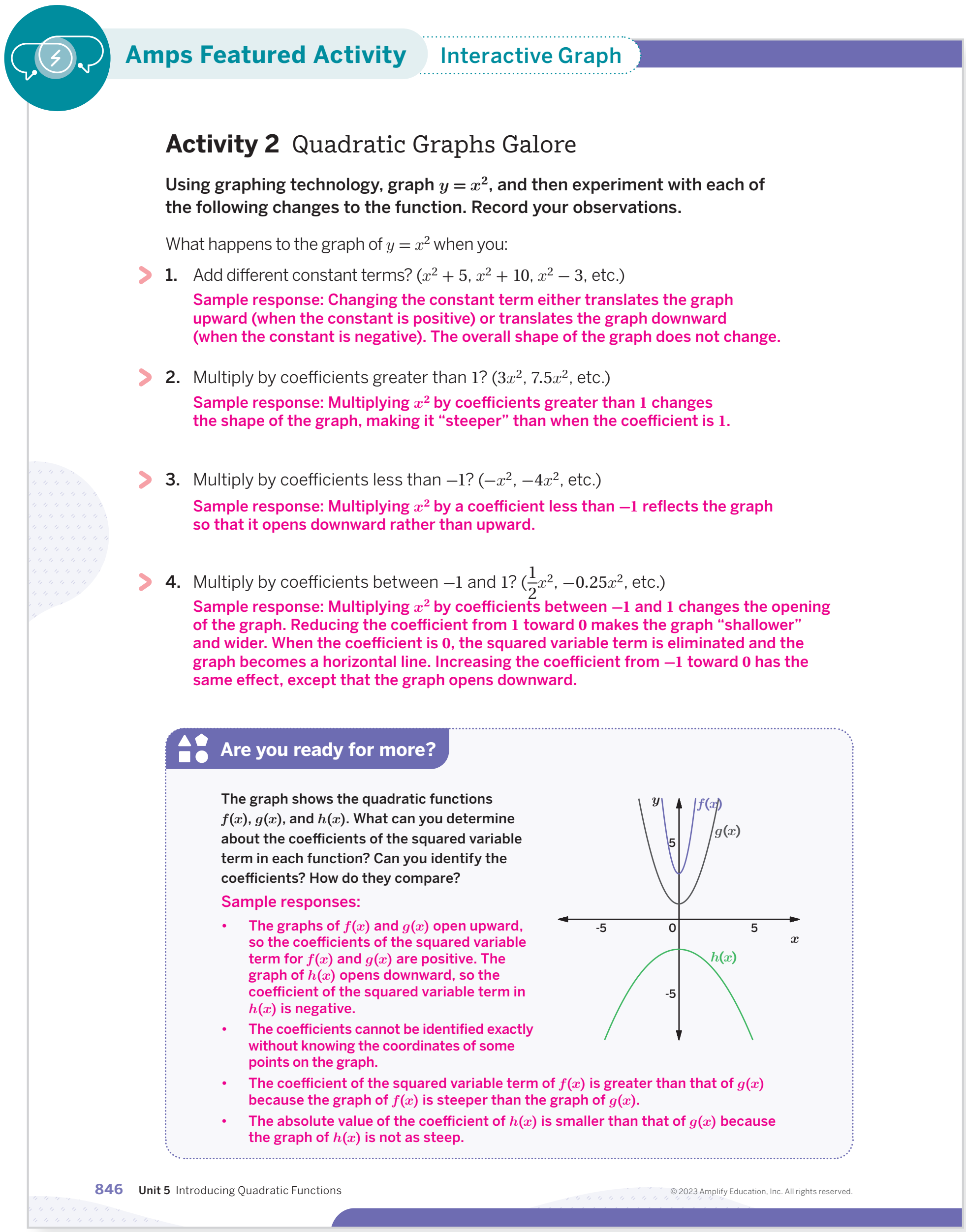
Annotate the graphs with their equations, color coding each function and its graph in a different color.

Pairs | 15 min

Activity 2 Quadratic Graphs Galore

Students experiment with changing the constant term or the coefficient of the x^2 term, and note the effects on the graphs.

Amps Featured Activity Interactive Graph



Activity 2 Quadratic Graphs Galore

Using graphing technology, graph $y = x^2$, and then experiment with each of the following changes to the function. Record your observations.

What happens to the graph of $y = x^2$ when you:

1. Add different constant terms? ($x^2 + 5$, $x^2 + 10$, $x^2 - 3$, etc.)
   Sample response: Changing the constant term either translates the graph upward (when the constant is positive) or translates the graph downward (when the constant is negative). The overall shape of the graph does not change.
2. Multiply by coefficients greater than 1? ($3x^2$, $7.5x^2$, etc.)
   Sample response: Multiplying x^2 by coefficients greater than 1 changes the shape of the graph, making it "steeper" than when the coefficient is 1.
3. Multiply by coefficients less than –1? ($-x^2$, $-4x^2$, etc.)
   Sample response: Multiplying x^2 by a coefficient less than –1 reflects the graph so that it opens downward rather than upward.
4. Multiply by coefficients between –1 and 1? ($\frac{1}{2}x^2$, $-0.25x^2$, etc.)
   Sample response: Multiplying x^2 by coefficients between –1 and 1 changes the opening of the graph. Reducing the coefficient from 1 toward 0 makes the graph "shallower" and wider. When the coefficient is 0, the squared variable term is eliminated and the graph becomes a horizontal line. Increasing the coefficient from –1 toward 0 has the same effect, except that the graph opens downward.

Are you ready for more?

The graph shows the quadratic functions $f(x)$, $g(x)$, and $h(x)$. What can you determine about the coefficients of the squared variable term in each function? Can you identify the coefficients? How do they compare?

Sample responses:

- **The graphs of $f(x)$ and $g(x)$ open upward, so the coefficients of the squared variable term for $f(x)$ and $g(x)$ are positive. The graph of $h(x)$ opens downward, so the coefficient of the squared variable term in $h(x)$ is negative.**
- **The coefficients cannot be identified exactly without knowing the coordinates of some points on the graph.**
- **The coefficient of the squared variable term of $f(x)$ is greater than that of $g(x)$ because the graph of $f(x)$ is steeper than the graph of $g(x)$.**
- **The absolute value of the coefficient of $h(x)$ is smaller than that of $g(x)$ because the graph of $h(x)$ is not as steep.**

846 **Unit 5** Introducing Quadratic Functions

1 Launch

Provide access to graphing technology. Have each pair take turns using graphing technology and recording their observations.

2 Monitor

Help students get started by having them predict what they think will happen to the graph of $y = x^2$ with each change.

Look for points of confusion:

- **Thinking that a in $ax^2 + bx + c$ describes the slope of the graph.** Review that slope is a linear concept.

Look for productive strategies:

- Comparing the graph of $y = x^2$ to the graphs using different values of a.
- Noting how values of $a < 0$, $0 < a < 1$, and $a > 1$ affect the graph.
- Connecting changes in the constant c to shifting the graph up or down along the y-axis.

3 Connect

Display the graph of $y = x^2$.

Have students share their observations when changing the values and why they think the graphs changed.

Highlight that when $a < 0$, the graph opens down. When $-1 < a < 1$, the graph appears wider, and when $a > 1$, the graph appears narrower. Adding or subtracting values from x^2 shifts the graph up or down the y-axis.

Ask:

- "What happens to the values of y when adding constants to or subtracting constants from x^2?" The values of y increase or decrease, respectively.
- "How do the values of y change when you multiply x^2 by a positive number?" The values of y for x^2 are multiplied by that number.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use graphing technology to graph $y = x^2$ and explore the effects on the graph due to changing the coefficient of the x^2-term and the constant.

Math Language Development

MLR2: Collect and Display

During the Connect, have students share what they notice about how the graphs compare to the graph of $y = x^2$ as they alter the equation. Add the following table to the class display and ask students to help you complete it using their developing math language.

How does the graph of $y = ax^2$ compare to $y = x^2$ when . . .		
$a < 0$	$-1 < a < 1$	$a > 1$

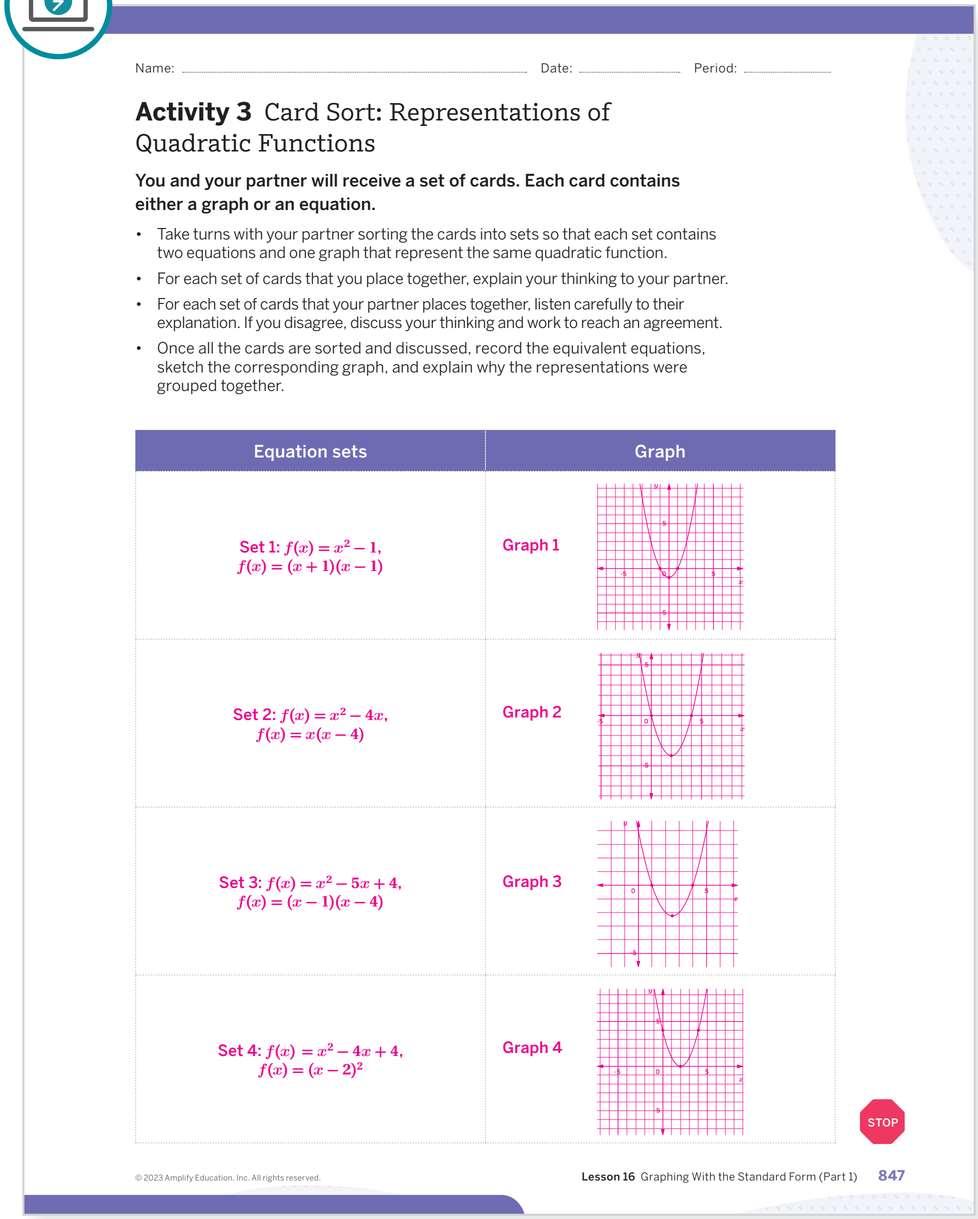
English Learners

Clarify that the language "a is between –1 and 1" can be represented by the inequality $-1 < a < 1$.

Pairs | 10 min

Activity 3 Card Sort: Representations of Quadratic Functions

Students participate in a card sort activity to build fluency in recognizing different representations of the same quadratic equation: standard form, factored form, graph.



Name: ………… Date: ………… Period: …………

Activity 3 Card Sort: Representations of Quadratic Functions

You and your partner will receive a set of cards. Each card contains either a graph or an equation.

- Take turns with your partner sorting the cards into sets so that each set contains two equations and one graph that represent the same quadratic function.
- For each set of cards that you place together, explain your thinking to your partner.
- For each set of cards that your partner places together, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
- Once all the cards are sorted and discussed, record the equivalent equations, sketch the corresponding graph, and explain why the representations were grouped together.

Equation sets	Graph
Set 1: $f(x) = x^2 - 1$, $f(x) = (x + 1)(x - 1)$	Graph 1
Set 2: $f(x) = x^2 - 4x$, $f(x) = x(x - 4)$	Graph 2
Set 3: $f(x) = x^2 - 5x + 4$, $f(x) = (x - 1)(x - 4)$	Graph 3
Set 4: $f(x) = x^2 - 4x + 4$, $f(x) = (x - 2)^2$	Graph 4

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Graphing With the Standard Form (Part 1) 847

1 Launch

Distribute a set of the pre-cut cards from the Activity 3 PDF to each student pair. Conduct the *Card Sort* routine.

2 Monitor

Help students get started by reviewing the directions for the activity and demonstrating how to sort the cards, if necessary.

Look for points of confusion:

- **Thinking a factor such as $(x - 1)$ relates to an x-intercept of $(-1, 0)$.** Remind students that the x-intercept is the opposite of the constant term in the linear factor

Look for productive strategies:

- Connecting the y-intercept of the graph to the standard form.
- Connecting the x-intercepts to the factored form.

3 Connect

Display graphs and equations on the card set.

Have students share how they analyzed the structure of the different equations and graphs to make connections to match the cards. Ask them to justify their matches as they practice constructing logical arguments.

Highlight that the intercepts of the graph help to find the equation in standard or factored forms. The quadratic expression in factored form can be expanded to standard form.

Ask:

- "What information can you obtain from an equation in standard form?" Whether the graph opens upward or downward and the y-intercept.
- "From the factored form?" The x-intercepts and the x-coordinate of the vertex.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, reduce the number of equations and corresponding graphs they need to sort. Alternatively, you can pair both forms of the equations together, or pair one form of the equation with the correct graph, and have students determine the missing match.

Math Language Development

MLR8: Discussion Supports

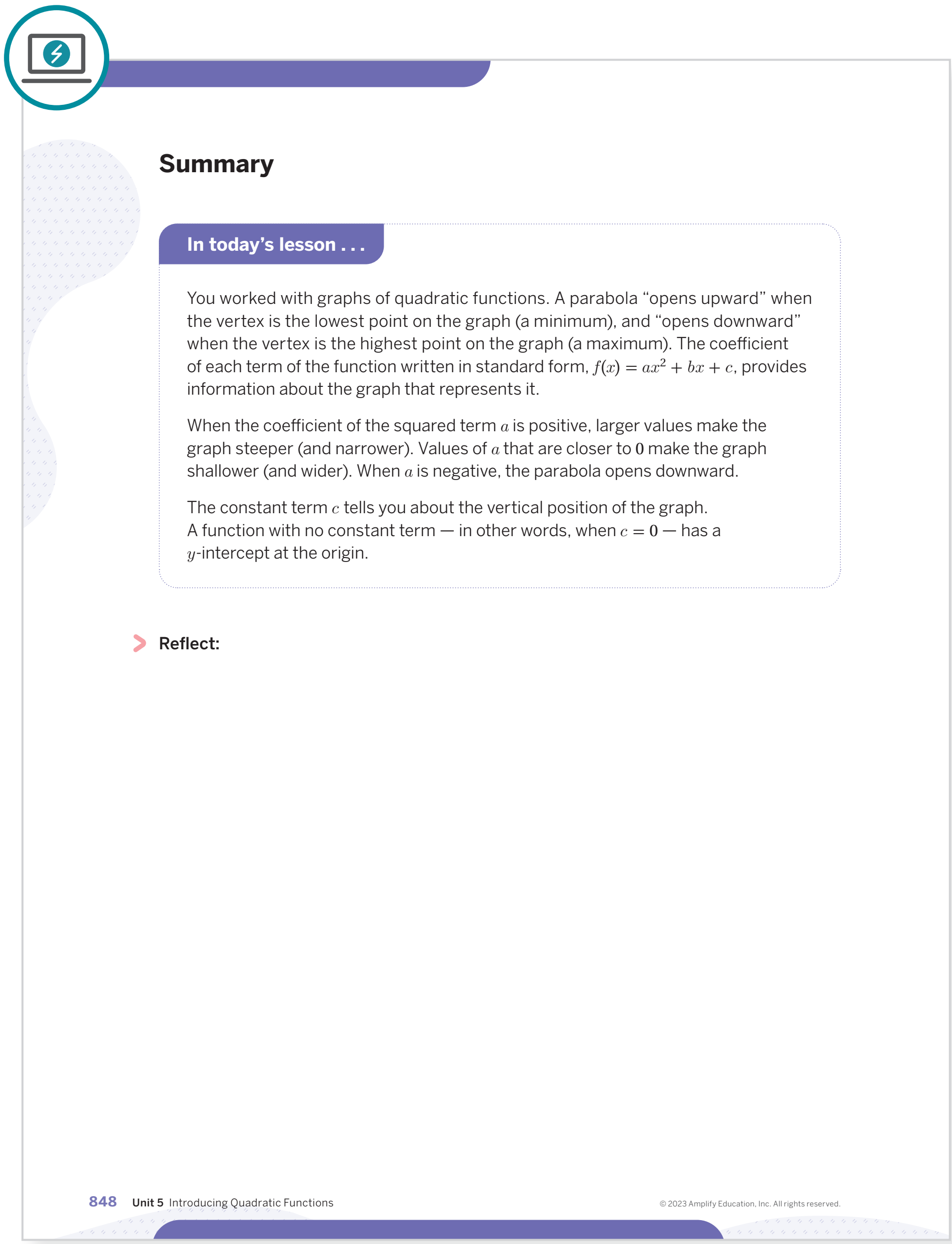
During the Connect, display or provide access to the Anchor Chart PDF, *Sentence Stems, Matching Prompts* for students to use to help them organize their thinking as they explain how they determined their matches. The PDF uses the following sentence frames.

- "………. and ………. match because . . ."
- "I noticed ………., so I matched . . ."

Whole Class | 5 min

Summary

Review and synthesize how key features of standard and factored forms of quadratic functions are visible on their graphs.



Summary

In today's lesson . . .

You worked with graphs of quadratic functions. A parabola "opens upward" when the vertex is the lowest point on the graph (a minimum), and "opens downward" when the vertex is the highest point on the graph (a maximum). The coefficient of each term of the function written in standard form, $f(x) = ax^2 + bx + c$, provides information about the graph that represents it.

When the coefficient of the squared term a is positive, larger values make the graph steeper (and narrower). Values of a that are closer to 0 make the graph shallower (and wider). When a is negative, the parabola opens downward.

The constant term c tells you about the vertical position of the graph. A function with no constant term — in other words, when $c = 0$ — has a y-intercept at the origin.

Reflect:

848 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the graphs from the Exit Ticket.

Have students share connections between the slope and y-intercept in a linear equation, and the coefficient of the x^2 term and the constant term in a quadratic function.

Highlight the advantages of using functions in different forms to anticipate the graphs representing quadratic functions.

Ask:

- "What information about the graph can you obtain from a quadratic function in standard form?" Determining whether the graph opens upward or downward based on the coefficient a, and determining the y-intercept based on the constant term.
- "What information can you obtain from the factored form?" The x-intercepts can be identified in each linear factor, as well as the x-coordinate of the vertex.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

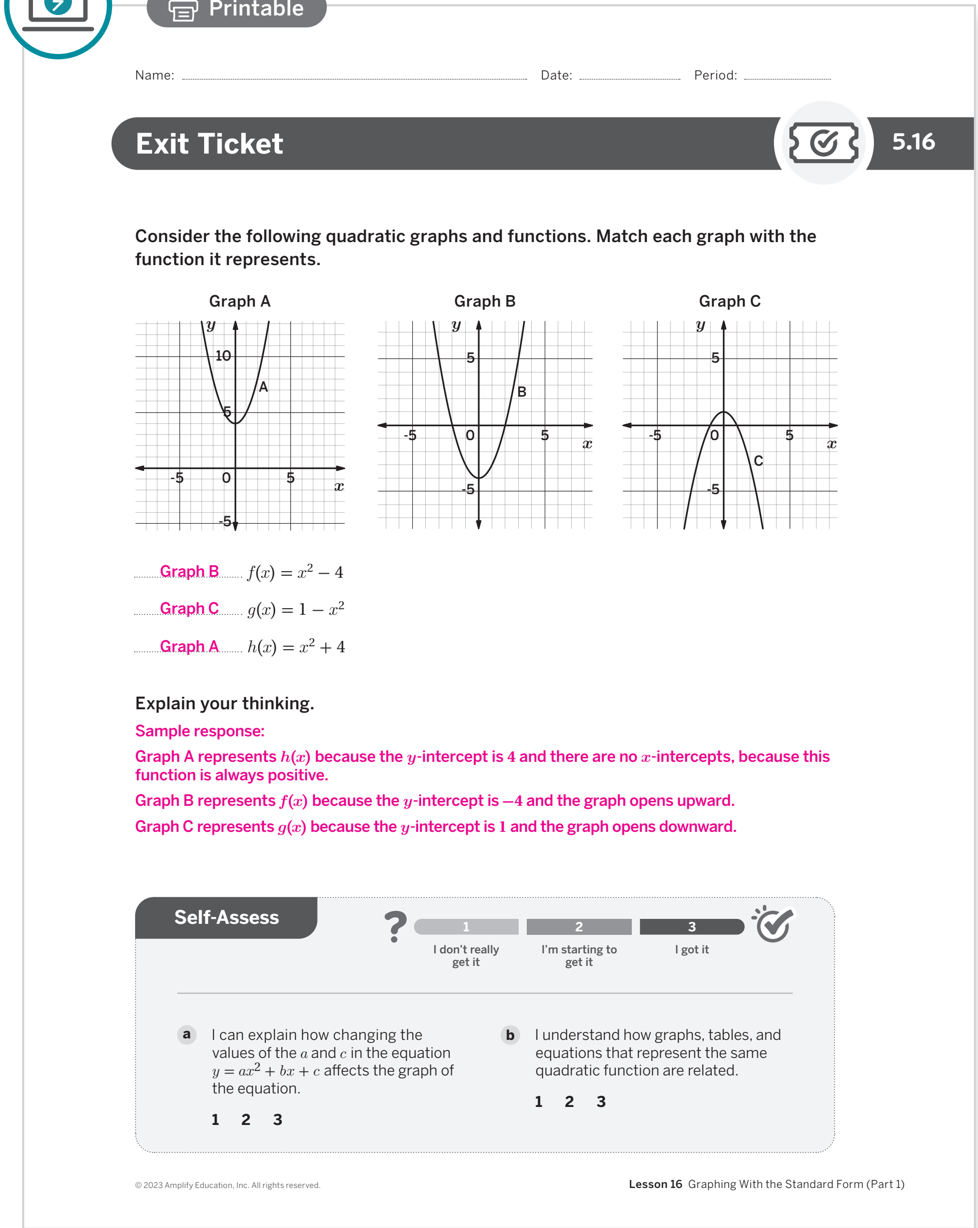
- "How does the coefficient of the squared variable term and the constant term of a quadratic function affect its graph?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of key features of quadratic functions by matching equations to the graphs representing them.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.16

Consider the following quadratic graphs and functions. Match each graph with the function it represents.

Graph B $f(x) = x^2 - 4$

Graph C $g(x) = 1 - x^2$

Graph A $h(x) = x^2 + 4$

Explain your thinking.

Sample response:

Graph A represents $h(x)$ because the y-intercept is 4 and there are no x-intercepts, because this function is always positive.

Graph B represents $f(x)$ because the y-intercept is —4 and the graph opens upward.

Graph C represents $g(x)$ because the y-intercept is 1 and the graph opens downward.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can explain how changing the values of the a and c in the equation $y = ax^2 + bx + c$ affects the graph of the equation.

1 2 3

b I understand how graphs, tables, and equations that represent the same quadratic function are related.

1 2 3

Lesson 16 Graphing With the Standard Form (Part 1)

Success looks like . . .

- **Language Goal:** Comprehending how the values of a and c in $y = ax^2 + bx + c$ are visible on the graph. **(Speaking and Listening, Writing)**
  - » Matching each graph with its equation using the values of a and c.
- **Language Goal:** Coordinating different representations of quadratic functions (expressions, tables, and graphs). **(Speaking and Listening, Writing)**
- **Goal:** Using technology to explore how the parameters of quadratic equations in standard form are visible on the graph.

Suggested next steps

If students do not recognize the effects of the constant terms on the graphs for $f(x)$ and $h(x)$, consider:

- Reviewing Activity 2 and Activity 3.
- Assigning Practice Problem 1.

If students do not recognize that a negative coefficient of x^2 makes the graph open downward for $g(x)$, consider:

- Reviewing Activity 2.
- Assigning Practice Problem 3.

Professional Learning

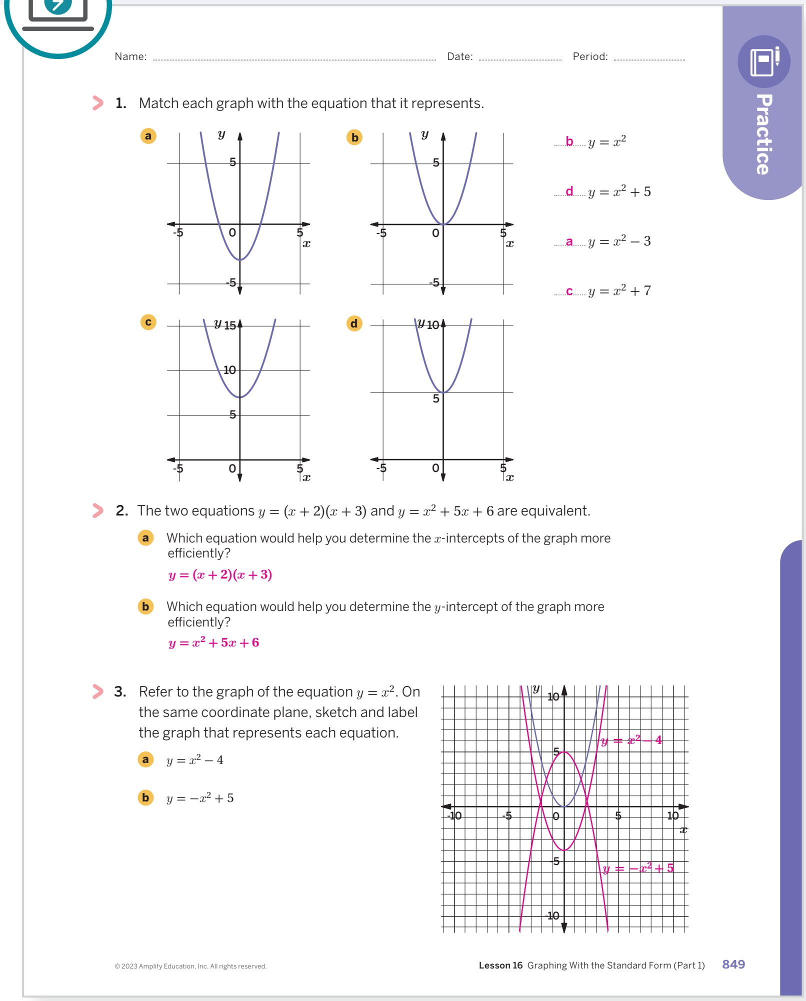
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students used the coefficient of the squared variable term and the constant term to graph a quadratic function. How did that build on the earlier work students did with slope intercept form?
- The focus of this lesson was to explain how the a and c in $y = ax^2 + bx + c$ affect the graph of the equation. How did the explanations go?

Practice

Independent



Name: .... Date: .... Period: ....

1. Match each graph with the equation that it represents.

b $y = x^2$

d $y = x^2 + 5$

a $y = x^2 - 3$

c $y = x^2 + 7$

2. The two equations $y = (x + 2)(x + 3)$ and $y = x^2 + 5x + 6$ are equivalent.

   a. Which equation would help you determine the x-intercepts of the graph more efficiently?

   $y = (x + 2)(x + 3)$

   b. Which equation would help you determine the y-intercept of the graph more efficiently?

   $y = x^2 + 5x + 6$

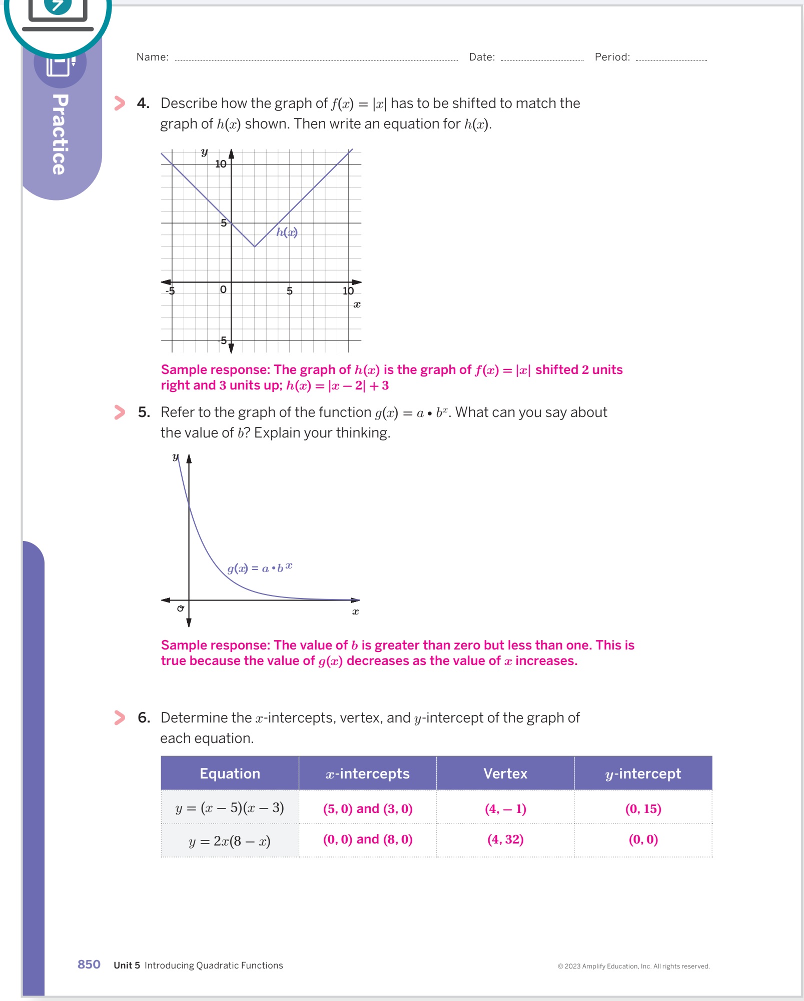
3. Refer to the graph of the equation $y = x^2$. On the same coordinate plane, sketch and label the graph that represents each equation.

   a. $y = x^2 - 4$

   b. $y = -x^2 + 5$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Graphing With the Standard Form (Part 1) 849



Name: .... Date: .... Period: ....

4. Describe how the graph of $f(x) = |x|$ has to be shifted to match the graph of $h(x)$ shown. Then write an equation for $h(x)$.

Sample response: The graph of $h(x)$ is the graph of $f(x) = |x|$ shifted 2 units right and 3 units up; $h(x) = |x - 2| + 3$

5. Refer to the graph of the function $g(x) = a \bullet b^x$. What can you say about the value of b? Explain your thinking.

Sample response: The value of b is greater than zero but less than one. This is true because the value of $g(x)$ decreases as the value of x increases.

6. Determine the x-intercepts, vertex, and y-intercept of the graph of each equation.

Equation	x-intercepts	Vertex	y-intercept
$y = (x - 5)(x - 3)$	(5, 0) and (3, 0)	$(4, -1)$	$(0, 15)$
$y = 2x(8 - x)$	(0, 0) and (8, 0)	$(4, 32)$	$(0, 0)$

850 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 3	2
	3	Activity 3	2
Spiral	4	Unit 3 Lesson 17	1
	5	Unit 4 Lesson 6	1
Formative ⚡	6	Unit 5 Lesson 17	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 17**

Graphing With the Standard Form (Part 2)

Let's see how changing other values in quadratic functions affect their graphs.

Focus

Goals

1. **Language Goal:** Describe how changing the value of b in $y = ax^2 + bx + c$ affects the graph. **(Listening and Speaking, Reading and Writing)**
2. Write quadratic equations in standard and factored forms that match given graphs.

Coherence

Today

In this lesson, students examine the effect of the linear term (bx) on the graph of a quadratic function and write equivalent quadratic expressions in standard and factored form. They compare the structures of these forms and use them to identify key features of the graph. Students learn the featured mathematician, Katherine Johnson, solved complicated quadratics without graphing technology.

Previously

Students have spent the last several lessons in this unit examining the structure of quadratic expressions in standard and factored form, and have made connections between these different forms and specific features of the graph representing quadratic functions.

Coming Soon

In the next lesson, students continue exploring quadratic functions in the context of real-world situations and interpret the equations and graphs in terms of the situations they represent.

Rigor

- Students build **conceptual understanding** of the graphs of quadratic functions in standard form.
- Students strengthen their **fluency** in graphing quadratic functions in standard form.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1 (optional)	Activity 2	Summary	Exit Ticket
10 min	20 min	25 min	5 min	10 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, Sentence Stems, *Matching Prompts*
- graphing technology

Math Language Development

Review words

- *equivalent expressions*
- *factored form*
- *linear*
- *standard form*
- *vertex*
- *x-intercepts*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students write quadratic functions to match the graphs of given parabolas. The digital technology checks their responses in real time.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel overwhelmed as they begin to work in reverse, writing the equation from its graph in Activity 2. By identifying and controlling their emotions, students will be able to look for and make use of the structure they previously learned about transformations of functions. They can confidently analyze the key features of the graphs and represent those features by writing the equation of the function.

Modifications to Pacing

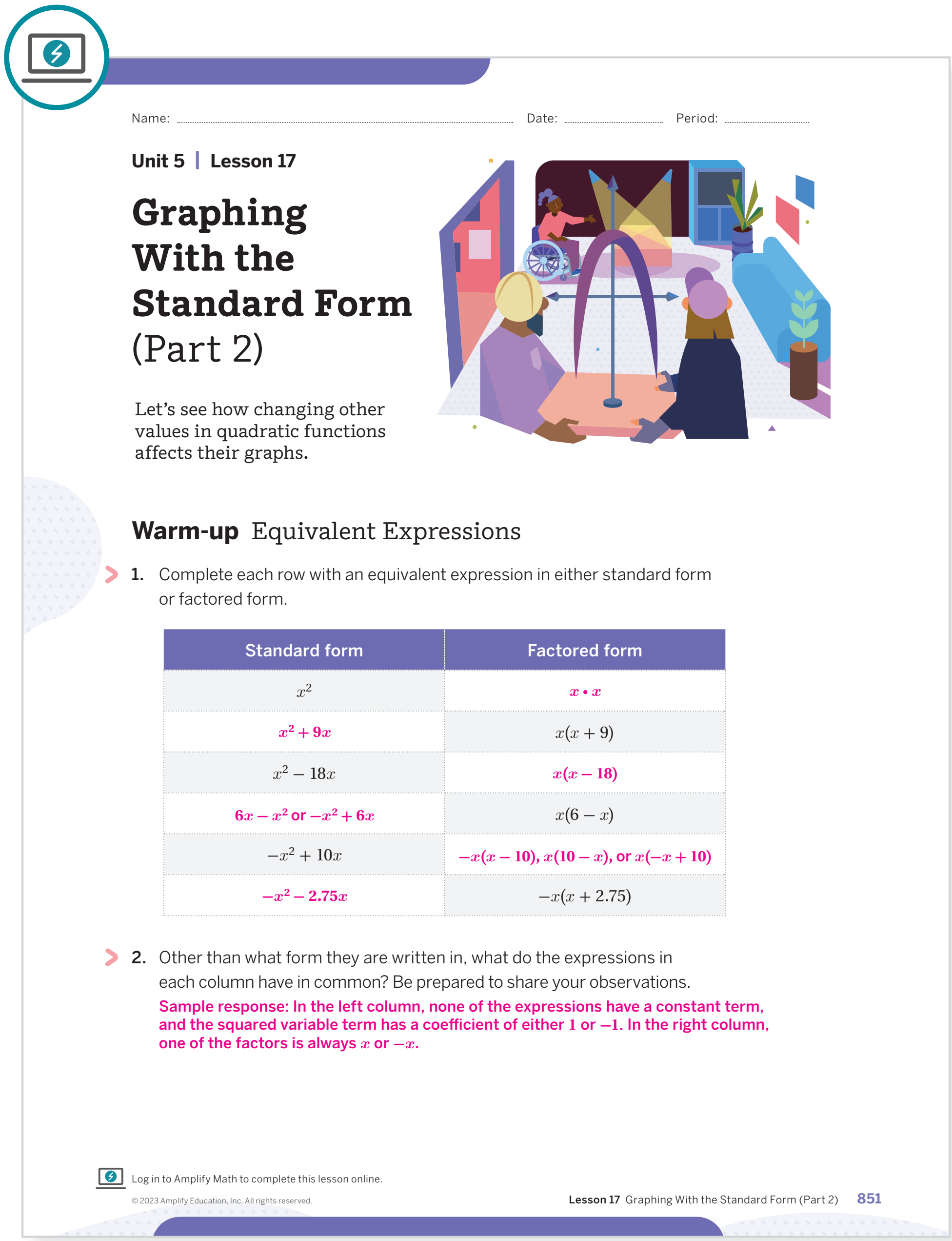
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students only complete the first four rows of the table.
- Optional **Activity 1** may be omitted.
- In **Activity 2**, have students only complete the first six graphs.

Pairs | 10 min

Warm-up Equivalent Expressions

Students write equivalent quadratic expressions in standard and factored form and examine their structure to prepare them for graphing quadratic functions from standard form.



Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 17

Graphing With the Standard Form (Part 2)

Let's see how changing other values in quadratic functions affects their graphs.

Warm-up Equivalent Expressions

1. Complete each row with an equivalent expression in either standard form or factored form.

Standard form	Factored form
x^2	$x \cdot x$
$x^2 + 9x$	$x(x + 9)$
$x^2 - 18x$	$x(x - 18)$
$6x - x^2$ or $-x^2 + 6x$	$x(6 - x)$
$-x^2 + 10x$	$-x(x - 10)$, $x(10 - x)$, or $x(-x + 10)$
$-x^2 - 2.75x$	$-x(x + 2.75)$

2. Other than what form they are written in, what do the expressions in each column have in common? Be prepared to share your observations.
Sample response: In the left column, none of the expressions have a constant term, and the squared variable term has a coefficient of either 1 or –1. In the right column, one of the factors is always x or $-x$.

Log in to Amplify Math to complete this lesson online.

① Launch

Have students work independently before sharing their responses with a partner.

② Monitor

Help students get started by reviewing standard form and factored form of quadratic expressions.

Look for points of confusion:

- **Struggling to write an equivalent expression in standard form.** Remind students that they can use the Distributive Property to rewrite the expression in standard form.
- **Struggling to write an equivalent expression in factored form.** Prompt students to begin with the expressions written in factored form first. Then, have them think about how they could work backward to find the factored form, given the standard form.

Look for productive strategies:

- Annotating expressions to visually show the Distributive Property.

③ Connect

Display the table and select students to complete it with their responses.

Have students share what all the quadratic expressions in standard form have in common (a squared variable term) and what those in factored form have in common ($\pm x$ multiplied by a linear term).

Highlight that students can tell from the expressions given in standard form that the y-intercept of each of them is $(0, 0)$. Students can also tell from the expressions given in factored form what the x-intercepts are.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Display or provide copies of the Anchor Chart PDF, *Factored and Standard Forms of Quadratic Expressions* for students to use as a reference as they complete the table.

Power-up

To power up students' ability to interpret the features of a quadratic function in factored form, have students complete:

Complete each problem for the equation $y = (x + 1)(x - 3)$.

1. One of the x-intercepts is –1. What is the other x-intercept? 3
2. The x-coordinate of the vertex is the average of the x-intercepts. What is the x-coordinate of the vertex? 1
3. What is the y-coordinate of the vertex? –4
4. Write the equation in standard form. What is the y-intercept? $y = x^2 - 2x - 3$; The y-intercept is –3.

Use: Before Activity 1

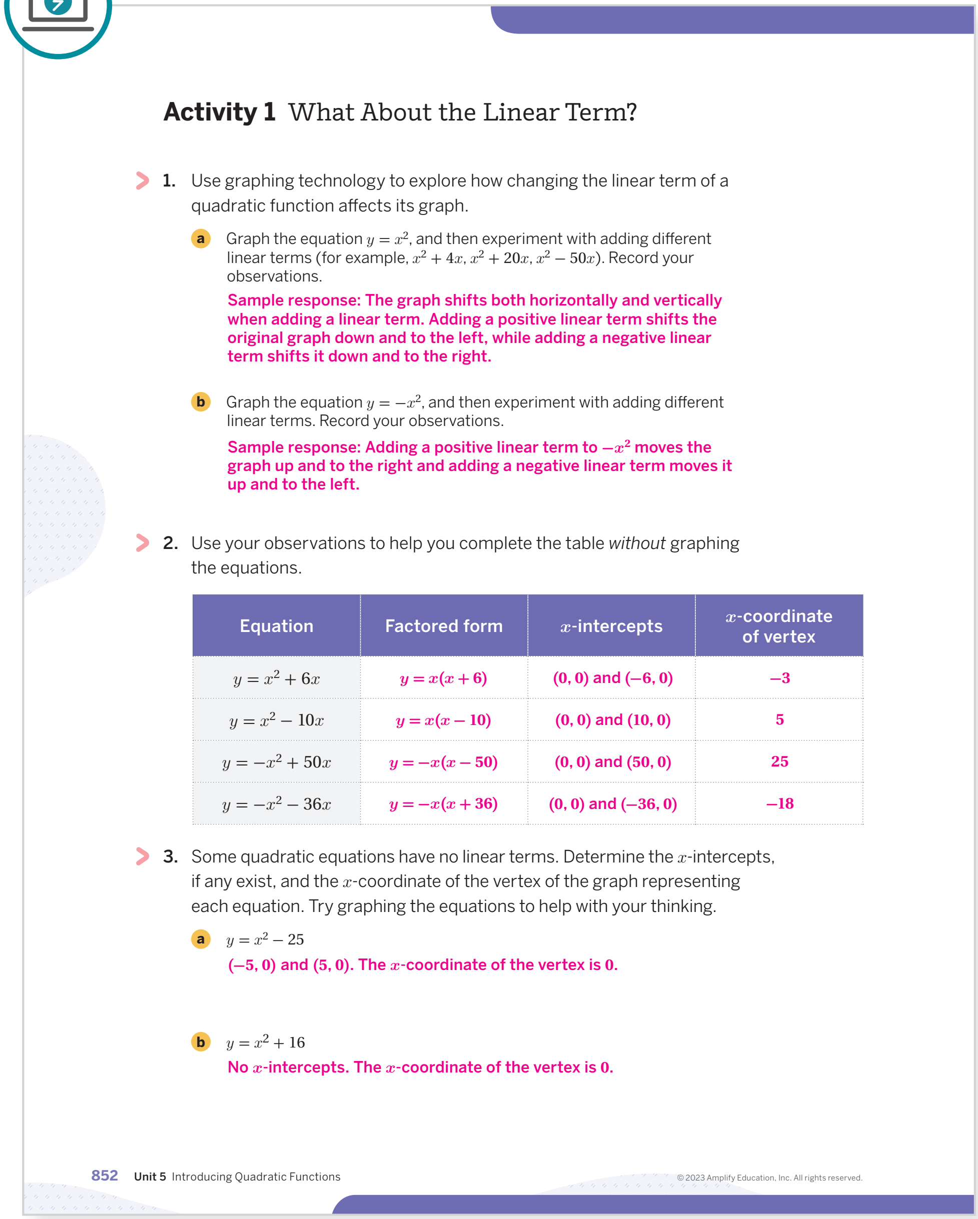
Informed by: Performance on Lesson 16, Practice Problem 6

Optional

Pairs | 20 min

Activity 1 What About the Linear Term?

Students use technology to experiment with graphing quadratic functions with varying linear terms, to understand how the linear term of the equation affects the graph.



Activity 1 What About the Linear Term?

1. Use graphing technology to explore how changing the linear term of a quadratic function affects its graph.

   a Graph the equation $y = x^2$, and then experiment with adding different linear terms (for example, $x^2 + 4x$, $x^2 + 20x$, $x^2 - 50x$). Record your observations.

   Sample response: The graph shifts both horizontally and vertically when adding a linear term. Adding a positive linear term shifts the original graph down and to the left, while adding a negative linear term shifts it down and to the right.

   b Graph the equation $y = -x^2$, and then experiment with adding different linear terms. Record your observations.

   Sample response: Adding a positive linear term to $-x^2$ moves the graph up and to the right and adding a negative linear term moves it up and to the left.

2. Use your observations to help you complete the table *without* graphing the equations.

Equation	Factored form	x-intercepts	x-coordinate of vertex
$y = x^2 + 6x$	$y = x(x + 6)$	(0, 0) and (–6, 0)	–3
$y = x^2 - 10x$	$y = x(x - 10)$	(0, 0) and (10, 0)	5
$y = -x^2 + 50x$	$y = -x(x - 50)$	(0, 0) and (50, 0)	25
$y = -x^2 - 36x$	$y = -x(x + 36)$	(0, 0) and (–36, 0)	–18

3. Some quadratic equations have no linear terms. Determine the x-intercepts, if any exist, and the x-coordinate of the vertex of the graph representing each equation. Try graphing the equations to help with your thinking.

   a $y = x^2 - 25$

   (–5, 0) and (5, 0). The x-coordinate of the vertex is 0.

   b $y = x^2 + 16$

   No x-intercepts. The x-coordinate of the vertex is 0.

1 Launch

Provide access to graphing technology. Have one partner operate the graphing tool, while the other partner records their observations. Switch roles halfway through the activity.

2 Monitor

Help students get started by prompting them to graph $y = x^2$ and letting it remain as a reference while they experiment with the other quadratic equations.

Look for points of confusion:

- **Referring to a diagonal shift in Problem 1.** Prompt students to examine horizontal and vertical shifts separately.
- **Struggling to determine the x-intercepts of $y = -x^2 + 50x$ and $y = -x^2 - 36x$ in Problem 2.** Prompt students to factor $-x$ instead of x from any equations with a negative squared variable term.

Look for productive strategies:

- Graphing successively larger linear terms in Problem 1.
- Averaging the x-intercepts to find the x-coordinate of the vertex.

3 Connect

Have students share their observations and predictions about how adding (or subtracting) a linear term affects the graph, prompting them to give examples of repeated calculations. Discuss why it is more beneficial to factor $-x$ from a quadratic equation with a negative squared variable term.

Highlight that writing an expression of the form $x^2 + bx$ as $x(x + b)$ allows students to identify the zeros of the equation. If the squared variable term is negative, factor $-x$ instead of x.

Ask, "Suppose there is no linear term, but there is a constant term. How would you find the x-intercepts without graphing? Would you be able to write the equation in factored form?"

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

During the Launch, display a few of the equations given in the table for Problem 2. Clarify the meaning of the linear term by asking:

- "Which term is the *squared term*? How do you know?"
- "Which term is the *linear term*? How do you know?"

Then have students preview Problem 3 and ask, "Why do these equations not have a linear term?"

Extension: *Math Enrichment*

After students complete Problem 3a, display the equation $y = (x + 5)(x - 5)$ and ask them to graph the equation and describe what they notice. The graph of the equation $y = (x + 5)(x - 5)$ is the same as the graph of the equation $y = x^2 - 25$.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their observations about how adding or subtracting a linear term affects the graph of $y = x^2$, consider adding the following table to the class display and have students help you complete it. Draw students' attention to the connection between the linear term and the impact on the graph of $y = x^2$. Include a graph of $y = x^2$ and graphs of the examples listed.

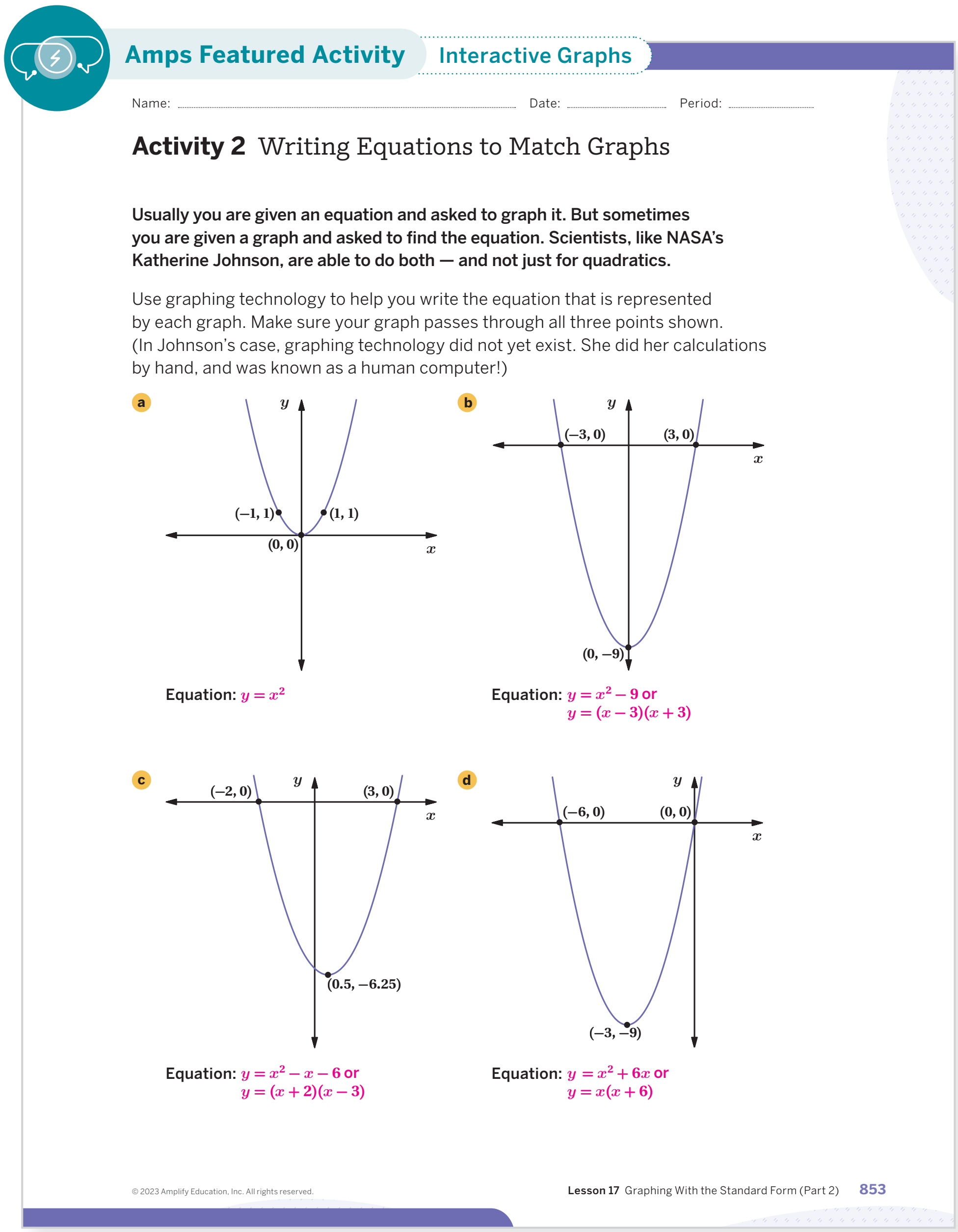
Adding a positive linear term	Adding a negative linear term
The graph shifts down and to the left. **Example:** $y = x^2 + 3x$	The graph shifts down and to the right. **Example:** $y = x^2 + (-3)x$, or $y = x^2 - 3x$

Pairs | 25 min

Activity 2 Writing Equations to Match Graphs

Students write equations to match the graphs of quadratic functions to build fluency with the different representations of quadratic equations: standard form, factored form, graph.

Amps Featured Activity Interactive Graphs



Name: ______ Date: ______ Period: ______

Activity 2 Writing Equations to Match Graphs

Usually you are given an equation and asked to graph it. But sometimes you are given a graph and asked to find the equation. Scientists, like NASA's Katherine Johnson, are able to do both — and not just for quadratics.

Use graphing technology to help you write the equation that is represented by each graph. Make sure your graph passes through all three points shown. (In Johnson's case, graphing technology did not yet exist. She did her calculations by hand, and was known as a human computer!)

a. Equation: $y = x^2$

b. Equation: $y = x^2 - 9$ or $y = (x - 3)(x + 3)$

c. Equation: $y = x^2 - x - 6$ or $y = (x + 2)(x - 3)$

d. Equation: $y = x^2 + 6x$ or $y = x(x + 6)$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Graphing With the Standard Form (Part 2) 853

1 Launch

Activate students' prior knowledge by asking them to explain the different forms of quadratic functions and what information it reveals about their graphs. Provide access to graphing technology.

2 Monitor

Help students get started by drawing their attention to the labeled points on each of the given graphs.

Look for points of confusion:

- **Struggling to determine when to use factored or standard form.** Prompt students to use factored form when they are given the x-intercepts, and standard form when they are given the y-intercepts.
- **Struggling to find a second factor for part f.** Point out that the second factor is the same as the first factor when there is only one x-intercept. Prompt students to expand the square of the first factor, $(x - 2)$.

Look for productive strategies:

- Negating the given x-intercepts when writing a quadratic equation in factored form.
- Writing an equation in standard form for the graphs whose vertices are centered on the y-axis.
- Making a connection between the structure of quadratic expressions written in different forms and the graphs that represent them.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can write quadratic functions to match the graphs of given parabolas. The digital technology checks their responses in real time.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose to complete six of the eight problems. Allowing them the power of choice can increase their engagement and ownership in the task.

Math Language Development

MLR7: Compare and Connect

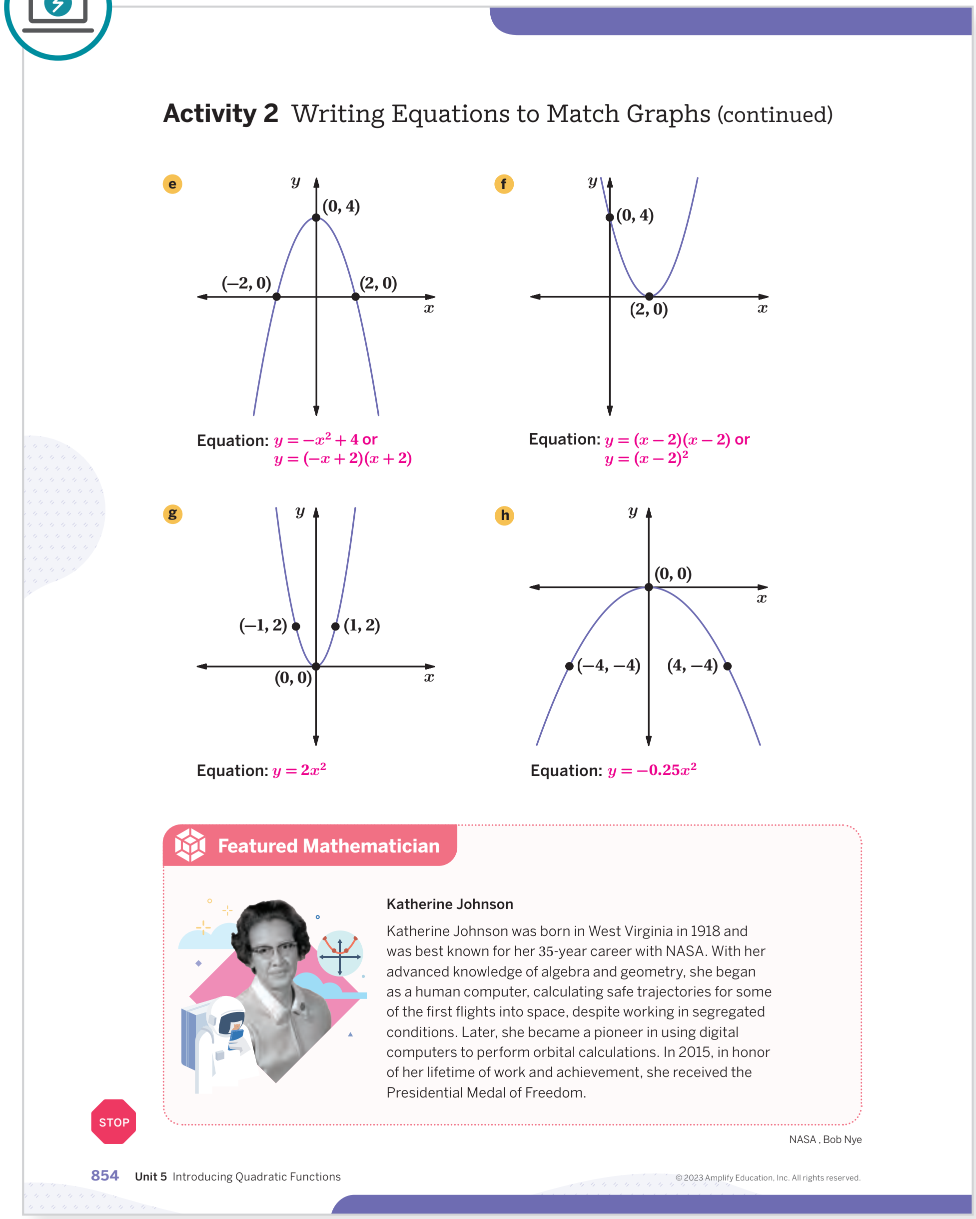
During the Connect, ask students:

- "Which graphs have one x-intercept?" a, f, g, h
- "Which graphs have two x-intercepts?" b, c, d, e
- "What do you notice about the equations of the graphs with one x-intercept?" a, g, and h are written in the form $y = ax^2$ and f is written in factored form.
- "What do you notice about the equations of the graphs with two x-intercepts?" They can be written in factored form. When written in standard form, sometimes there is a linear term and sometimes there isn't.

Pairs | 25 min

Activity 2 Writing Equations to Match Graphs (continued)

Students write equations to match the graphs of quadratic functions to build fluency with the different representations of quadratic equations: standard form, factored form, graph.



Activity 2 Writing Equations to Match Graphs (continued)

e Equation: $y=-x^2+4$ or $y=(-x+2)(x+2)$

f Equation: $y=(x-2)(x-2)$ or $y=(x-2)^2$

g Equation: $y=2x^2$

h Equation: $y=-0.25x^2$

Featured Mathematician

Katherine Johnson

Katherine Johnson was born in West Virginia in 1918 and was best known for her 35-year career with NASA. With her advanced knowledge of algebra and geometry, she began as a human computer, calculating safe trajectories for some of the first flights into space, despite working in segregated conditions. Later, she became a pioneer in using digital computers to perform orbital calculations. In 2015, in honor of her lifetime of work and achievement, she received the Presidential Medal of Freedom.

NASA , Bob Nye

STOP

3 Connect

Have students share their strategies for writing the equations for each graph.

Highlight that the graph of a quadratic equation opens upward when x^2 has a positive coefficient and opens downward when it has a negative coefficient. Any graph with two x-intercepts can be written in factored form, and sometimes graphs with one x-intercept can be written in factored form, such as in the graph for part f. Any graph whose vertex is centered on the y-axis can be written in the form of $y=ax^2+c$, where $(0, c)$ is the y-intercept. If $(0, 0)$ is the y-intercept, the vertex, and the only x-intercept, then the equation of the graph has the form $y=ax^2$.

Featured Mathematician

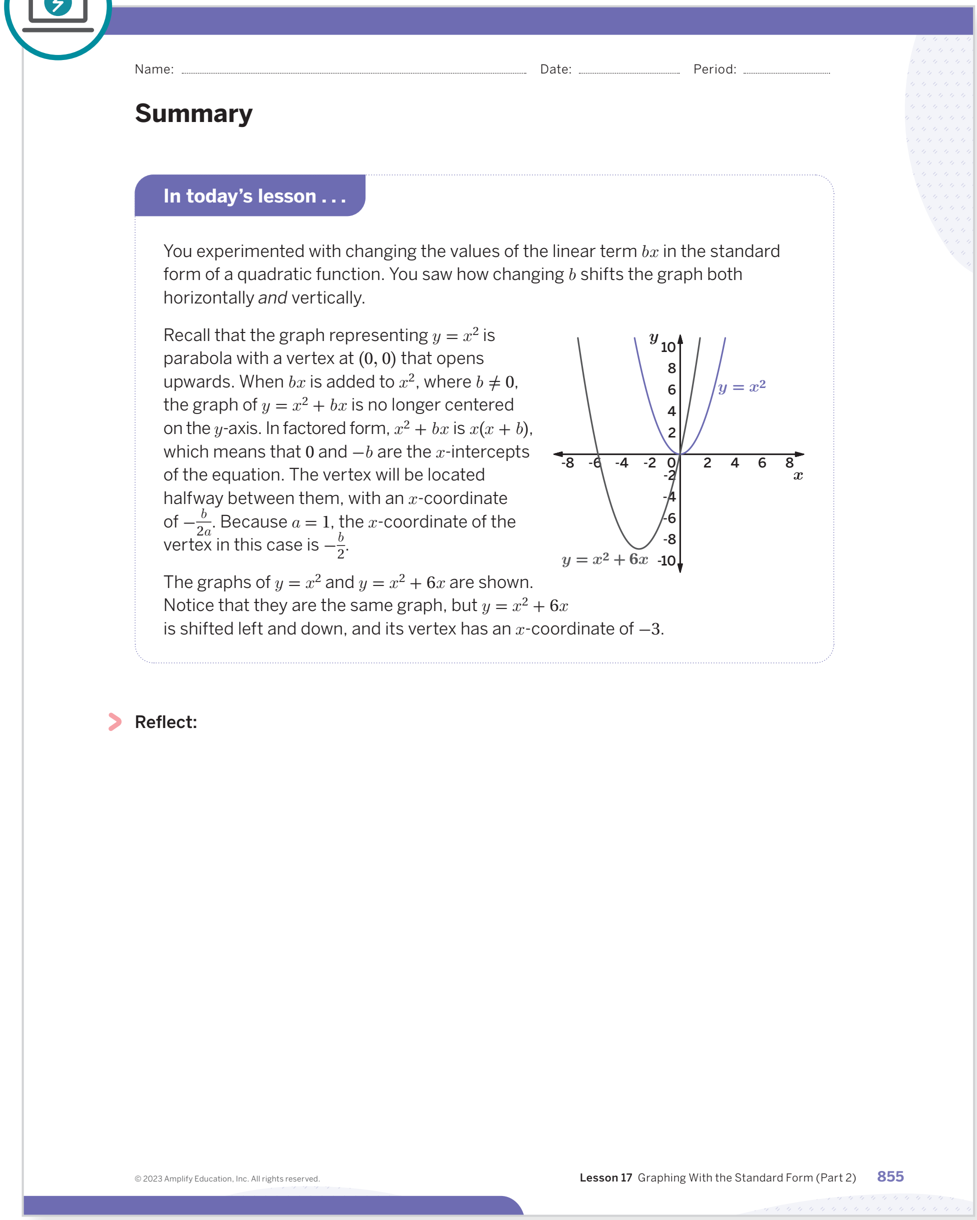
Katherine Johnson

Have students read about the featured mathematician, Katherine Johnson, a human computer for NASA whose calculations helped to put the first man on the moon.

Whole Class | 5 min

Summary

Review and synthesize how adjusting the linear term in the standard form of a quadratic equation shifts the graph both horizontally and vertically.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You experimented with changing the values of the linear term bx in the standard form of a quadratic function. You saw how changing b shifts the graph both horizontally *and* vertically.

Recall that the graph representing $y = x^2$ is parabola with a vertex at $(0, 0)$ that opens upwards. When bx is added to x^2, where $b \neq 0$, the graph of $y = x^2 + bx$ is no longer centered on the y-axis. In factored form, $x^2 + bx$ is $x(x + b)$, which means that 0 and $-b$ are the x-intercepts of the equation. The vertex will be located halfway between them, with an x-coordinate of $-\frac{b}{2a}$. Because $a = 1$, the x-coordinate of the vertex in this case is $-\frac{b}{2}$.

The graphs of $y = x^2$ and $y = x^2 + 6x$ are shown. Notice that they are the same graph, but $y = x^2 + 6x$ is shifted left and down, and its vertex has an x-coordinate of -3.

Reflect:

Synthesize

Display the graph.

Ask students to compare the features of the graphs of $y = x^2$ and $y = x^2 + 6x$. Ask them what they notice about the location of the vertex for each graph and the general location of each parabola.

Highlight that adding the linear term $6x$ to the equation $y = x^2$ shifts the graph down and to the left. The vertex is no longer at $(0, 0)$. Ask students what would happen to the graph of $y = x^2$ if the linear term $-6x$ was added to the equation. The graph would shift down and to the right.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

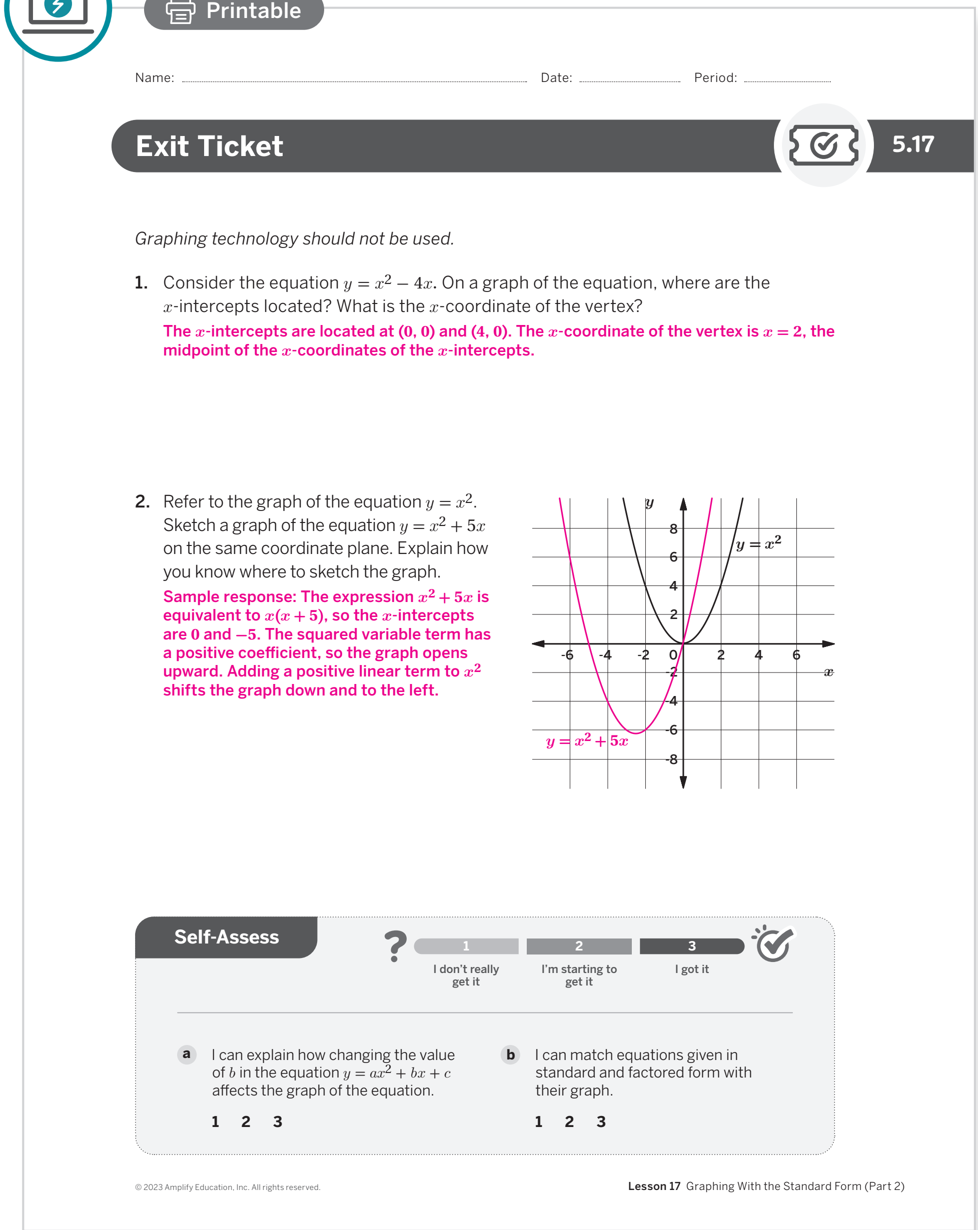
- "How does the coefficient of the linear term of a quadratic function affect its graph?"
- "What strategies did you find helpful today when graphing a quadratic function in standard form?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of quadratic functions of the form $y = ax^2 + bx$ by describing how the linear term affects the graph of the function.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.17

Graphing technology should not be used.

1. Consider the equation $y = x^2 - 4x$. On a graph of the equation, where are the x-intercepts located? What is the x-coordinate of the vertex?

   The x-intercepts are located at $(0, 0)$ and $(4, 0)$. The x-coordinate of the vertex is $x = 2$, the midpoint of the x-coordinates of the x-intercepts.

2. Refer to the graph of the equation $y = x^2$. Sketch a graph of the equation $y = x^2 + 5x$ on the same coordinate plane. Explain how you know where to sketch the graph.

   Sample response: The expression $x^2 + 5x$ is equivalent to $x(x + 5)$, so the x-intercepts are 0 and –5. The squared variable term has a positive coefficient, so the graph opens upward. Adding a positive linear term to x^2 shifts the graph down and to the left.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can explain how changing the value of b in the equation $y = ax^2 + bx + c$ affects the graph of the equation.

1 2 3

b I can match equations given in standard and factored form with their graph.

1 2 3

Lesson 17 Graphing With the Standard Form (Part 2)

Success looks like . . .

- **Language Goal:** Describing how changing the value of b in $y = ax^2 + bx + c$ affects the graph. **(Listening and Speaking, Reading and Writing)**
  - » Graphing $x^2 + 5x$ using the graph of x^2.
- **Goal:** Writing quadratic equations in standard and factored forms that match given graphs.

Suggested next steps

If students are unable to determine the x-intercepts or the x-coordinate of the vertex in Problem 1, consider:

- Having them practice rewriting quadratic functions of the form x^2+ bx as $x(x + b)$ and identifying the x-intercepts from the factored form.
- Revisiting Activity 1, Problem 2.
- Assigning Practice Problems 1 and 2.

If students sketch an inaccurate graph of $y = x^2 + 5x$ for Problem 2, or fail to sketch one at all, consider:

- Reviewing Activity 2 or generating similar graphs for which students can write an equation.
- Assigning Practice Problem 6.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In earlier lessons, students graphed quadratics in factored form. How did that support describing how the coefficient of the linear term affects the graph of a quadratic function?
- The focus of this lesson was to explain how the value of b in $y = ax^2 + bx + c$ affects the graph of the equation. How did the explanations go?

MLR Math Language Development

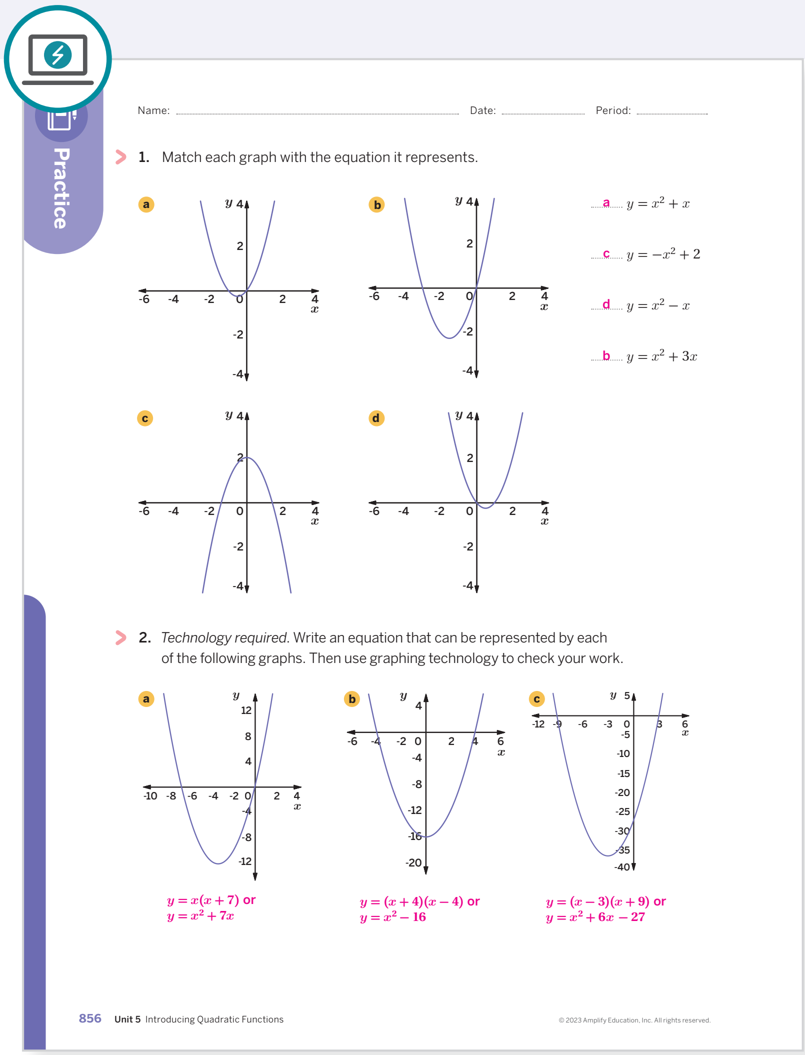
Language Goal: Describing how changing the value of b in $y = ax^2 + bx + c$ affects the graph.

Reflect on students' language development toward this goal.

- Do students' responses to Problem 2 of the Exit Ticket demonstrate they understand how adding $5x$ affects the graph of $y = x^2$?
- Do they use terms and phrases such as *squared variable term, positive coefficient, opens upward, positive linear term, shifts the graph down, and/or shifts the graph to the left?*

Practice

Independent



Practice

Name: Date: Period:

1. Match each graph with the equation it represents.

a $y = x^2 + x$

c $y = -x^2 + 2$

d $y = x^2 - x$

b $y = x^2 + 3x$

2. *Technology required*. Write an equation that can be represented by each of the following graphs. Then use graphing technology to check your work.

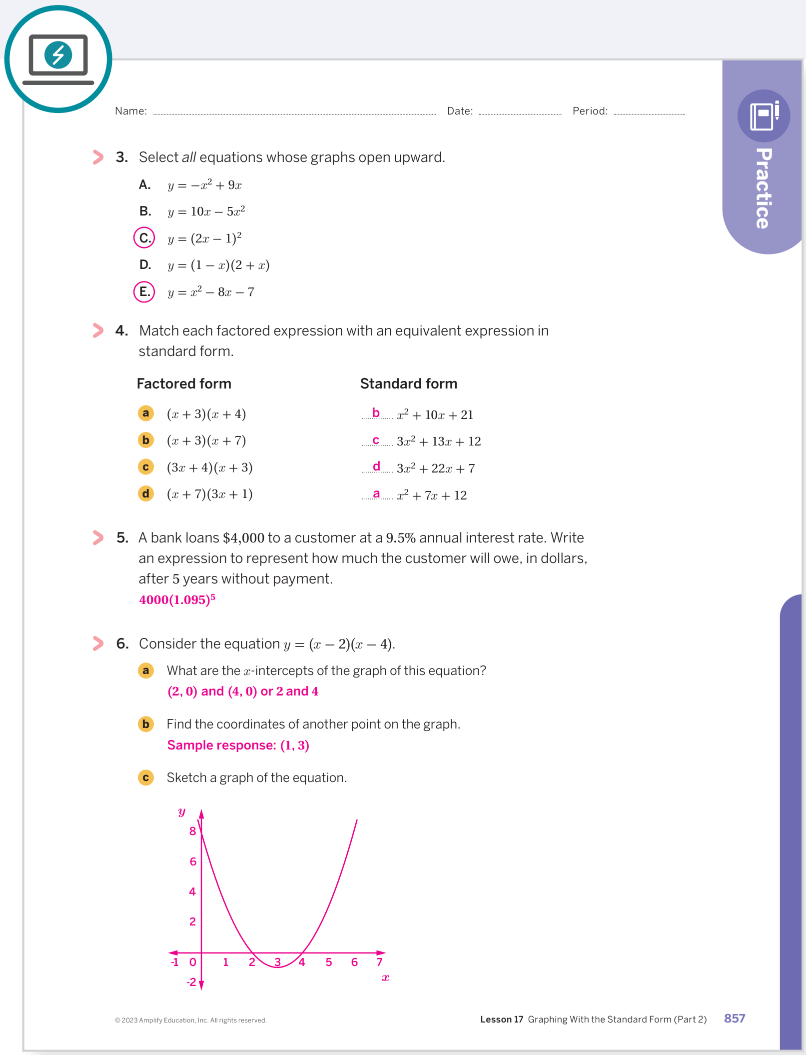
a $y = x(x + 7)$ or $y = x^2 + 7x$

b $y = (x + 4)(x - 4)$ or $y = x^2 - 16$

c $y = (x - 3)(x + 9)$ or $y = x^2 + 6x - 27$

856 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

3. Select *all* equations whose graphs open upward.
   - A. $y = -x^2 + 9x$
   - B. $y = 10x - 5x^2$
   - [✓ C.] $y = (2x - 1)^2$
   - D. $y = (1 - x)(2 + x)$
   - [✓ E.] $y = x^2 - 8x - 7$

4. Match each factored expression with an equivalent expression in standard form.

Factored form		Standard form
a	$(x + 3)(x + 4)$	b $x^2 + 10x + 21$
b	$(x + 3)(x + 7)$	c $3x^2 + 13x + 12$
c	$(3x + 4)(x + 3)$	d $3x^2 + 22x + 7$
d	$(x + 7)(3x + 1)$	a $x^2 + 7x + 12$

5. A bank loans $4,000 to a customer at a 9.5% annual interest rate. Write an expression to represent how much the customer will owe, in dollars, after 5 years without payment.

$4000(1.095)^5$

6. Consider the equation $y = (x - 2)(x - 4)$.
   - a What are the x-intercepts of the graph of this equation?
     (2, 0) and (4, 0) or 2 and 4
   - b Find the coordinates of another point on the graph.
     Sample response: (1, 3)
   - c Sketch a graph of the equation.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Graphing With the Standard Form (Part 2) 857

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1	1
Spiral	4	Unit 5 Lesson 8	2
	5	Unit 4 Lesson 16	1
Formative ⚡	6	Unit 5 Lesson 18	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

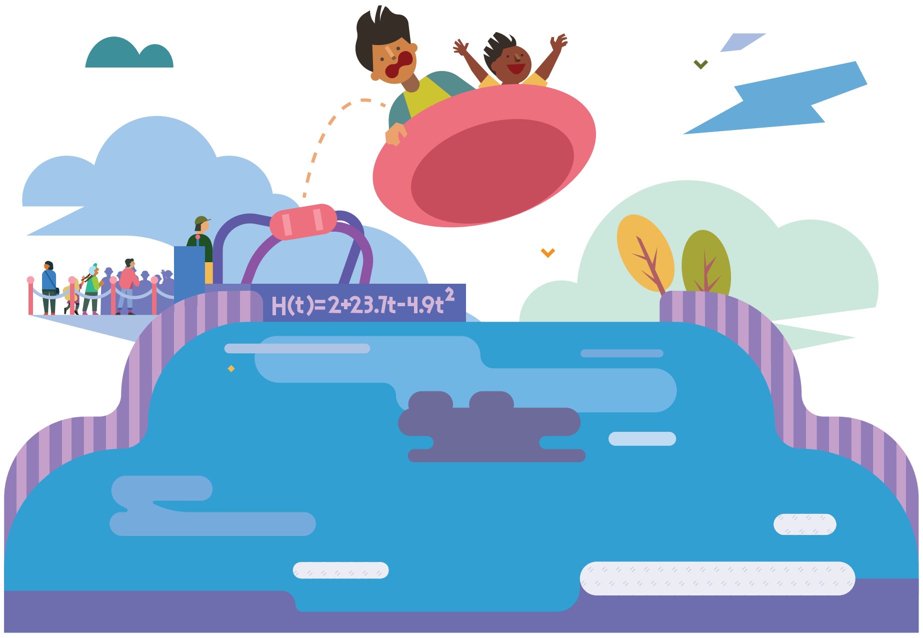
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 18**

Graphs That Represent Scenarios

Let's examine graphs that represent the paths of objects being launched in the air.



Focus

Goals

1. **Language Goal:** Explain how key features, such as vertex, domain, and intercepts of the graph of a quadratic function relate to a scenario. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students interpret equations and graphs of quadratic functions in context. They determine vertical and horizontal intercepts, the vertex, and the domain of graphs, and use their analysis of the functions to solve problems and to compare quadratic functions given in different representations.

Previously

Students used factored form (Lessons 14 and 15) and standard form (Lessons 16 and 17) to graph quadratic functions.

Coming Soon

Students will examine and use vertex form to graph and interpret quadratic functions in context in Lessons 19 and 20.

Rigor

- Students **apply** their understanding of the graphs of quadratic functions to study projectiles in context.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards
- Instructional Routine PDF, *Info Gap: Instructions*
- graphing technology
- scientific calculators

Math Language Development

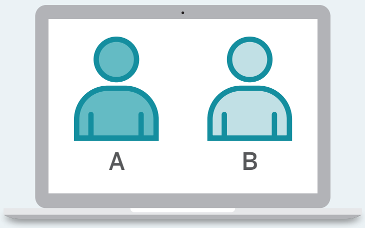
Review words

- *domain*
- *horizontal intercept*
- *vertex*
- *vertical intercept*
- *zeros*

Amps | Featured Activity

Activity 3
Digital Collaboration

Students are presented with a familiar situation and need to consider what relevant details are missing



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not choose to follow the directions for their role given in Activity 3, resulting in not being able to make sense of the given problem. Remind students that the instructions are given to provide a path to success. Have them reflect on their choices of behavior and how it affected the outcome of the activity.

Modifications to Pacing

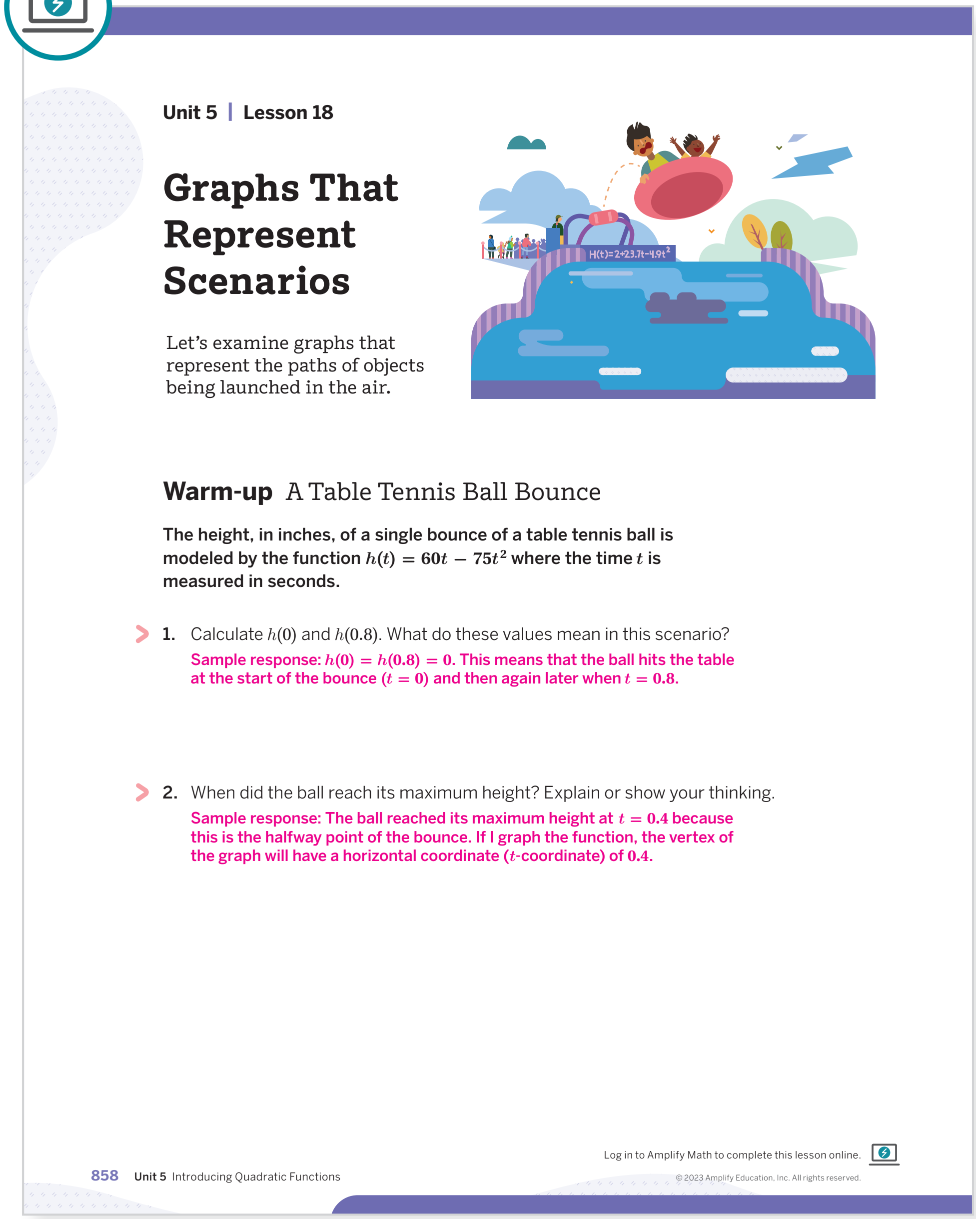
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, omit Problem 3.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up A Table Tennis Ball Bounce

Students evaluate a quadratic function of a the bounce of a table tennis ball and determine its maximum to interpret in context.



Unit 5 | Lesson 18

Graphs That Represent Scenarios

Let's examine graphs that represent the paths of objects being launched in the air.

Warm-up A Table Tennis Ball Bounce

The height, in inches, of a single bounce of a table tennis ball is modeled by the function $h(t) = 60t - 75t^2$ where the time t is measured in seconds.

1. Calculate $h(0)$ and $h(0.8)$. What do these values mean in this scenario?
   Sample response: $h(0) = h(0.8) = 0$. This means that the ball hits the table at the start of the bounce ($t = 0$) and then again later when $t = 0.8$.

2. When did the ball reach its maximum height? Explain or show your thinking.
   Sample response: The ball reached its maximum height at $t = 0.4$ because this is the halfway point of the bounce. If I graph the function, the vertex of the graph will have a horizontal coordinate (t-coordinate) of 0.4.

Log in to Amplify Math to complete this lesson online.

858 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Tell students that they will examine a function representing a single bounce of a table tennis ball. Provide access to scientific calculators.

2 Monitor

Help students get started by asking, "What does $h(0)$ mean in context?" The height of the ball at a time of 0 seconds.

Look for points of *confusion*:

- **Having difficulty determining when the ball reaches its maximum height.** Ask, "How could you use the symmetry of the graph and the horizontal intercepts to determine the time the ball reached its maximum height without estimating?"

Look for productive strategies:

- Graphing the function and using its horizontal intercepts to respond to Problem 2.

3 Connect

Ask, "What does the graph look like? Where are the intercepts and vertex located?" The graph opens downward. The horizontal intercepts are located at $(0, 0)$ and $(0.8, 0)$ and the t-coordinate of the vertex is located at $t = 0.4$.

Display the graph of the function $h(t) = 60t - 75t^2$.

Have individual students share their method for determining the time the table tennis ball reached its maximum height.

Highlight that the graph opens downward because the coefficient of the squared variable term is negative.

Power-up

To power up students' ability to sketch a graph of a quadratic function in factored form, have students complete:

Complete each problem for the equation $y = (x - 2)(x - 4)$.

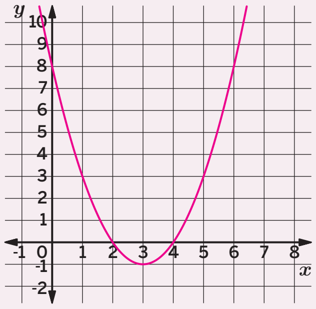
1. What are the x-intercepts of the graph of the equation? $(2, 0)$ and $(4, 0)$

2. What is the x-coordinate of the vertex? 3

3. What are the coordinates of the vertex? $(3, -1)$

4. What is the y-intercept of the graph? $(0, 8)$

5. Sketch the graph.



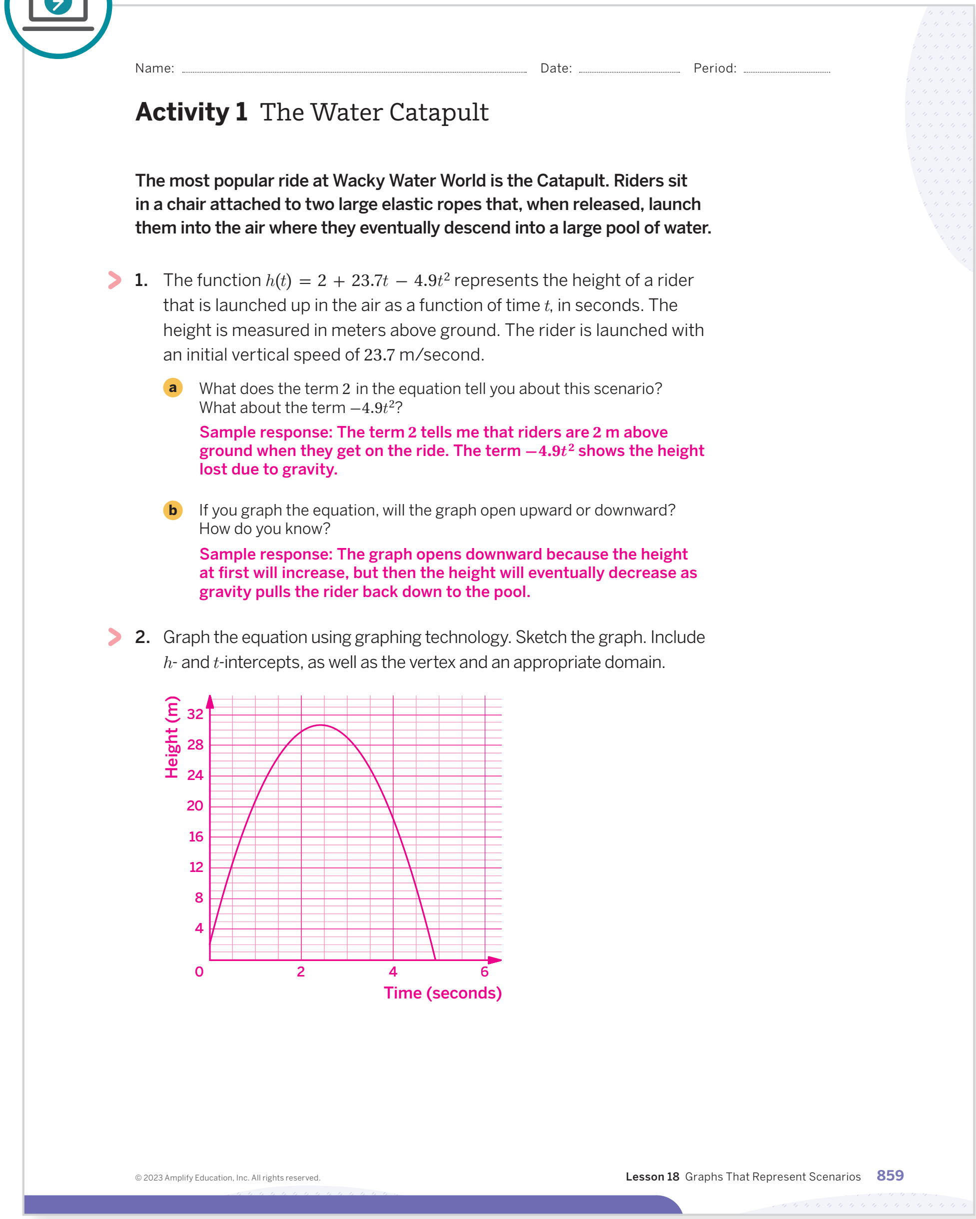
Use: Before the Warm-up

Informed by: Performance on Lesson 17, Practice Problem 6

Pairs | 20 min

Activity 1 The Water Catapult

Students graph and interpret key features of a quadratic equation that represents the path of a rider at a water park.



Name: ______ Date: ______ Period: ______

Activity 1 The Water Catapult

The most popular ride at Wacky Water World is the Catapult. Riders sit in a chair attached to two large elastic ropes that, when released, launch them into the air where they eventually descend into a large pool of water.

1. The function $h(t) = 2 + 23.7t - 4.9t^2$ represents the height of a rider that is launched up in the air as a function of time t, in seconds. The height is measured in meters above ground. The rider is launched with an initial vertical speed of 23.7 m/second.

   a. What does the term 2 in the equation tell you about this scenario? What about the term $-4.9t^2$?

   Sample response: The term 2 tells me that riders are 2 m above ground when they get on the ride. The term $-4.9t^2$ shows the height lost due to gravity.

   b. If you graph the equation, will the graph open upward or downward? How do you know?

   Sample response: The graph opens downward because the height at first will increase, but then the height will eventually decrease as gravity pulls the rider back down to the pool.

2. Graph the equation using graphing technology. Sketch the graph. Include h- and t-intercepts, as well as the vertex and an appropriate domain.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 Graphs That Represent Scenarios 859

① Launch

Give students a few minutes of quiet think-time to read the first problem and think about their responses. Ask them to share their thoughts with a partner before proceeding.

② Monitor

Help students get started by having them sketch the path of a rider.

Look for points of confusion:

- **Confusing the vertical and horizontal intercept.** The vertical intercept is another name for the h-intercept and the horizontal intercept is another name for the t-intercept.
- **Misidentifying the horizontal intercept from the graphing technology.** Say, “Substitute the value of t that appears to be the horizontal intercept into the equation to see if the height is 0. Adjust your value of t and try again if the height is not approximately 0.”

Look for productive strategies:

- Plotting both horizontal intercepts and vertical intercepts, and using the midpoint between the two to help plot the vertex.
- Using a table or list of values on the calculator to check possible coordinates of the vertex.

Activity 1 continued >

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

Ask students if they have ever seen a ride similar to the one mentioned in this activity. Consider showing images or a video of a similar ride so that students can visualize the motion. Ask, “Do you think this motion can be modeled with a quadratic function? Explain your thinking.”

Accessibility: *Guide Processing and Visualization*

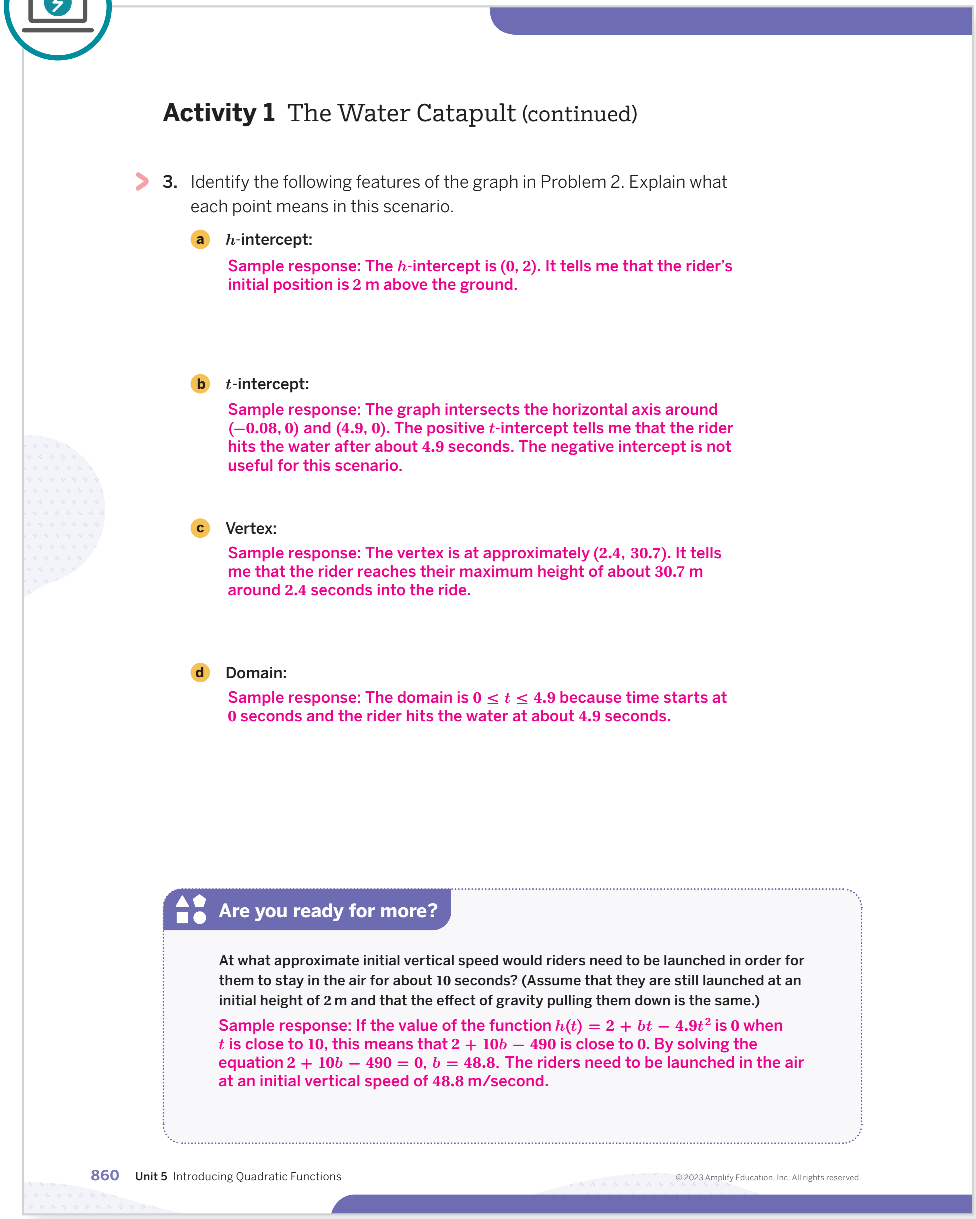
Provide access to colored pencils and have students color code the terms in the function given in Problem 1 with how they are represented on the graph in Problem 2. Ask them to annotate the graph with what these terms represent in the scenario. Consider displaying a table like the one shown.

$h(t) = 2 + 23.7t - 4.9t^2$		
2	$23.7t$	$-4.9t^2$
The initial height.	The initial vertical speed.	The effects due to gravity.

Pairs | 20 min

Activity 1 The Water Catapult (continued)

Students graph and interpret key features of a quadratic equation that represents the path of a rider at a water park.



Activity 1 The Water Catapult (continued)

3. Identify the following features of the graph in Problem 2. Explain what each point means in this scenario.

   a. h-intercept:

   Sample response: The h-intercept is $(0, 2)$. It tells me that the rider's initial position is 2 m above the ground.

   b. t-intercept:

   Sample response: The graph intersects the horizontal axis around $(-0.08, 0)$ and $(4.9, 0)$. The positive t-intercept tells me that the rider hits the water after about 4.9 seconds. The negative intercept is not useful for this scenario.

   c. Vertex:

   Sample response: The vertex is at approximately $(2.4, 30.7)$. It tells me that the rider reaches their maximum height of about 30.7 m around 2.4 seconds into the ride.

   d. Domain:

   Sample response: The domain is $0 \le t \le 4.9$ because time starts at 0 seconds and the rider hits the water at about 4.9 seconds.

Are you ready for more?

At what approximate initial vertical speed would riders need to be launched in order for them to stay in the air for about 10 seconds? (Assume that they are still launched at an initial height of 2 m and that the effect of gravity pulling them down is the same.)

Sample response: If the value of the function $h(t) = 2 + bt - 4.9t^2$ is 0 when t is close to 10, this means that $2 + 10b - 490$ is close to 0. By solving the equation $2 + 10b - 490 = 0$, $b = 48.8$. The riders need to be launched in the air at an initial vertical speed of 48.8 m/second.

860 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share their graph and interpretations of the features of the graph.

Display the graph of the equation.

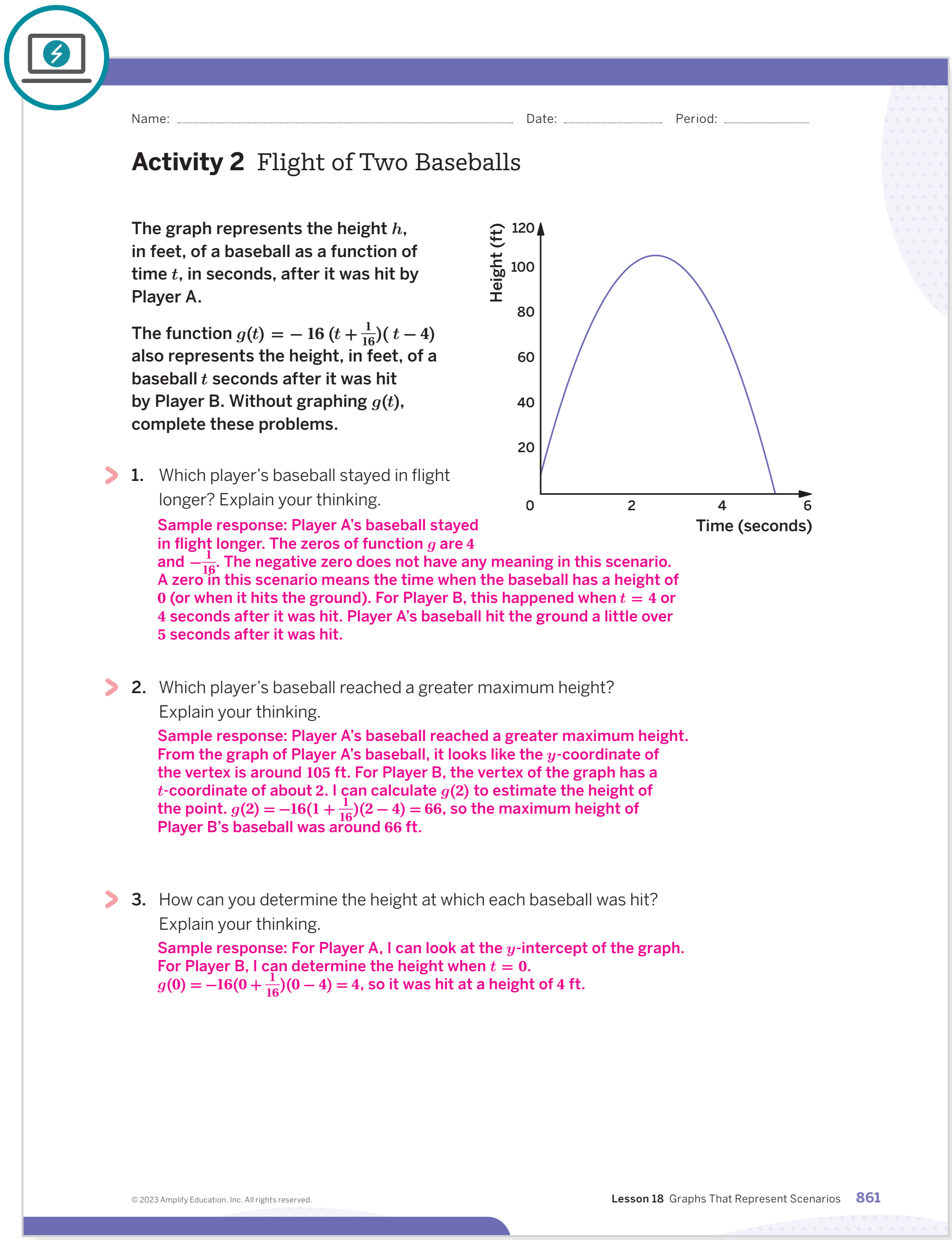
Highlight how to use graphing technology to identify the coordinates of points on a graph. The term $-4.9t^2$ represents the effect of gravity because the force of gravity is $-9.8\,\frac{\text{m}}{\text{sec}^2}$, and half of -9.8 is -4.9.

Ask, "The graph shows two horizontal intercepts, one with a positive t-coordinate and the other with a negative t-coordinate. How do you make sense of the negative t-coordinate?" The negative t-coordinate does not make sense for this scenario. The domain is restricted to positive values of t because the time is restricted to positive values.

Pairs | 15 min

Activity 2 Flight of Two Baseballs

Students analyze two quadratic functions, one represented by a graph and the other by an equation, to compare the key features of their graphs within context.



Name: Date: Period:

Activity 2 Flight of Two Baseballs

The graph represents the height h, in feet, of a baseball as a function of time t, in seconds, after it was hit by Player A.

The function $g(t) = -16\left(t + \frac{1}{16}\right)(t - 4)$ also represents the height, in feet, of a baseball t seconds after it was hit by Player B. Without graphing $g(t)$, complete these problems.

1. Which player's baseball stayed in flight longer? Explain your thinking.

   Sample response: Player A's baseball stayed in flight longer. The zeros of function g are 4 and $-\frac{1}{16}$. The negative zero does not have any meaning in this scenario. A zero in this scenario means the time when the baseball has a height of 0 (or when it hits the ground). For Player B, this happened when $t = 4$ or 4 seconds after it was hit. Player A's baseball hit the ground a little over 5 seconds after it was hit.

2. Which player's baseball reached a greater maximum height? Explain your thinking.

   Sample response: Player A's baseball reached a greater maximum height. From the graph of Player A's baseball, it looks like the y-coordinate of the vertex is around 105 ft. For Player B, the vertex of the graph has a t-coordinate of about 2. I can calculate $g(2)$ to estimate the height of the point. $g(2) = -16\left(1 + \frac{1}{16}\right)(2 - 4) = 66$, so the maximum height of Player B's baseball was around 66 ft.

3. How can you determine the height at which each baseball was hit? Explain your thinking.

   Sample response: For Player A, I can look at the y-intercept of the graph. For Player B, I can determine the height when $t = 0$. $g(0) = -16\left(0 + \frac{1}{16}\right)(0 - 4) = 4$, so it was hit at a height of 4 ft.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 Graphs That Represent Scenarios 861

1 Launch

Have students think quietly about the scenario and problems, before discussing with a partner. Activate students' background knowledge by asking them what it means for a baseball to stay in flight.

2 Monitor

Help students get started by saying, "Determine the horizontal intercepts of both functions to help you complete the problems."

Look for points of confusion:

- **Thinking the zero from the factor $\left(t + \frac{1}{16}\right)$ is $\frac{1}{16}$.** Have students check their zeros by substituting these values into the function to see if they make the function equal to zero.

Look for productive strategies:

- Calculating the average of the two zeros to determine the t-coordinate of the vertex.

3 Connect

Display the graph of $g(t)$.

Highlight that to determine which baseball stayed in flight longer, students can compare the horizontal intercepts, which tell them when the baseball hits the ground.

Ask, "How could you determine the initial height of the ball for $g(t)$ by using the equation?" Expand and simplify the expression on the right side of the equation, using the area diagram method, and then identify the constant term.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students complete Problems 1–3 while first just analyzing the graph for Player B. Have them annotate the graph with their estimation for when the baseball will hit the ground (Problem 1), the vertex (Problem 2), and the initial height (Problem 3). Then have them return to analyze the function for $g(t)$. Ask:

- "Is the function written in standard form or factored form? What information does this form tell you?"
- "What strategies can you use to determine the vertex of the function, without graphing it?"
- "What strategies can you use to determine the initial height of the function, without graphing it? Will one of the intercepts help you?"

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that Players A and B each hit a baseball into the air.
- **Read 2:** Ask students to identify the given quantities or relationships, such as the baseball hit by Player B is given by the equation and the baseball hit by Player A is given by the graph.
- **Read 3:** Ask students to brainstorm strategies for how they can compare these two functions, without graphing Player B's function.

English Learners

Annotate the graph with "Player A."

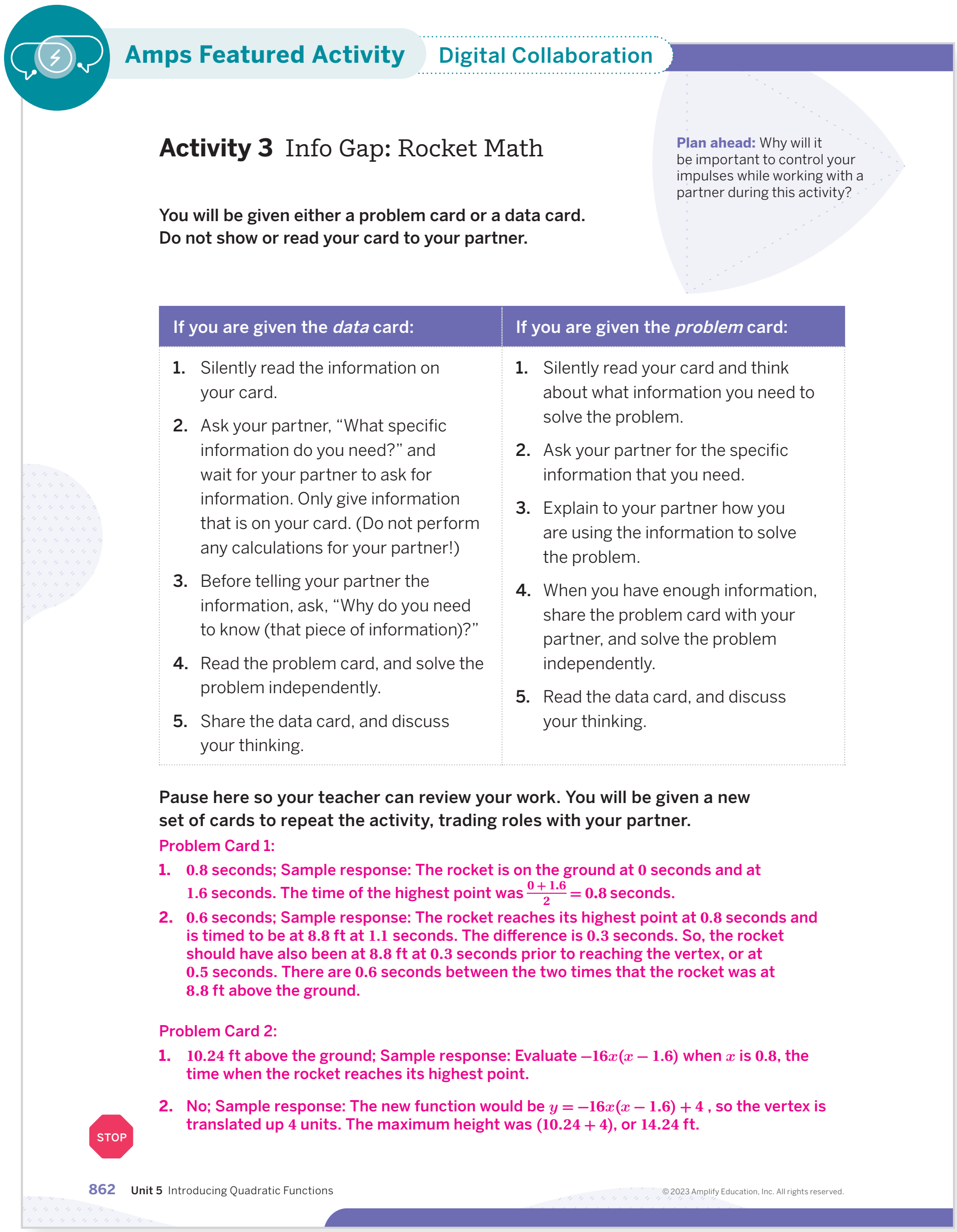
Optional

Pairs | 15 min

Activity 3 Info Gap: Rocket Math

Students are presented with scenarios and consider any missing relevant details as they apply their knowledge of key features of quadratics.

Amps Featured Activity Digital Collaboration



Activity 3 Info Gap: Rocket Math

Plan ahead: Why will it be important to control your impulses while working with a partner during this activity?

You will be given either a problem card or a data card. Do not show or read your card to your partner.

If you are given the *data* card:	If you are given the *problem* card:
1. Silently read the information on your card.	1. Silently read your card and think about what information you need to solve the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not perform any calculations for your partner!)	2. Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	3. Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	5. Read the data card, and discuss your thinking.

Pause here so your teacher can review your work. You will be given a new set of cards to repeat the activity, trading roles with your partner.

Problem Card 1:

1. **0.8 seconds; Sample response: The rocket is on the ground at 0 seconds and at 1.6 seconds. The time of the highest point was $\frac{0+1.6}{2} = 0.8$ seconds.**
2. **0.6 seconds; Sample response: The rocket reaches its highest point at 0.8 seconds and is timed to be at 8.8 ft at 1.1 seconds. The difference is 0.3 seconds. So, the rocket should have also been at 8.8 ft at 0.3 seconds prior to reaching the vertex, or at 0.5 seconds. There are 0.6 seconds between the two times that the rocket was at 8.8 ft above the ground.**

Problem Card 2:

1. **10.24 ft above the ground; Sample response: Evaluate $-16x(x-1.6)$ when x is 0.8, the time when the rocket reaches its highest point.**
2. **No; Sample response: The new function would be $y = -16x(x-1.6)+4$, so the vertex is translated up 4 units. The maximum height was $(10.24+4)$, or 14.24 ft.**

STOP

862 **Unit 5** Introducing Quadratic Functions

1 Launch

Explain the ***Info Gap*** instructional routine and display the Instructional Routine PDF, *Info Gap: Instructions*. Consider demonstrating the routine if students are unfamiliar with it. Provide a problem and data card to each pair of students from the Activity 3 PDF.

2 Monitor

Help students get started by modeling the questioning process. Encourage students to use precise language when asking questions to get the information they need.

Look for points of confusion:

- **Struggling to find the point of symmetry to respond to Problem 2 on Problem Card 1.** Have students sketch the graph, and use the symmetry of the graph to determine when the rocket reached 8.8 ft the first time.

Look for productive strategies:

- Listing the key features given from each data card, or needed for each problem card.

3 Connect

Have pairs of students share the key features of each graph and how they found them.

Highlight that in Data Card 1, the zeros are given. Once students know both zeros, they can determine the x-coordinate of the vertex.

Ask, "How did you determine the maximum height of the rocket?" The maximum height is at $x = 0.8$. If I replace x in the expression with 0.8, then $-16(0.8)(0.8-1.6) = 10.24$.

Math Language Development

MLR4: Information Gap

During the Launch, display Problem Card 1, without revealing any of the information on Data Card 1. Ask students to work with their partner to write questions they could ask that might help them determine the answer to the first question, "How many seconds after launch did the rocket reach its highest point?" Sample questions are shown.

- "Can you tell me at what time the rocket was launched?"
- "Can you tell me at what time the rocket landed?"
- "Can you tell me the maximum height of the rocket?"

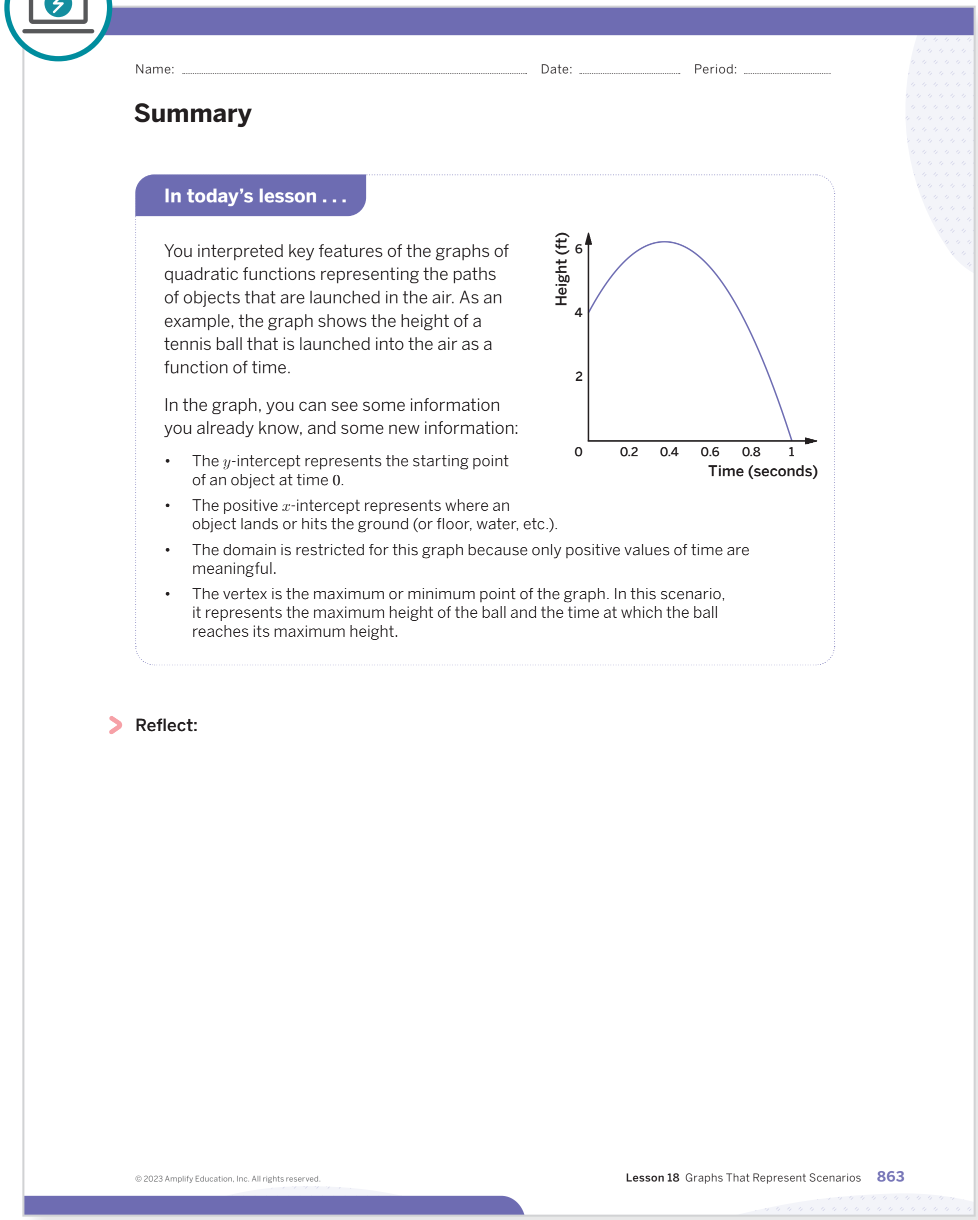
English Learners

Consider displaying one of the sample questions to help students craft their own questions.

Whole Class | 5 min

Summary

Review and synthesize the key features of quadratic graphs and their functions, within the context of launching objects into the air.



Name: Date: Period:

Summary

In today's lesson . . .

You interpreted key features of the graphs of quadratic functions representing the paths of objects that are launched in the air. As an example, the graph shows the height of a tennis ball that is launched into the air as a function of time.

In the graph, you can see some information you already know, and some new information:

- The y-intercept represents the starting point of an object at time 0.
- The positive x-intercept represents where an object lands or hits the ground (or floor, water, etc.).
- The domain is restricted for this graph because only positive values of time are meaningful.
- The vertex is the maximum or minimum point of the graph. In this scenario, it represents the maximum height of the ball and the time at which the ball reaches its maximum height.

Reflect:

Synthesize

Display the graph.

Have students share key features they can determine from the graph.

Highlight the key information the graph shows, such as:

- The starting height of the tennis ball.
- The maximum height of the ball.
- When the ball hit the ground.

Ask:

- "Why is the domain restricted?" Only positive values of time make sense in this context.
- "Why is there only one meaningful horizontal intercept?" Because only positive values of time make sense, only the positive horizontal intercept is meaningful in this context.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

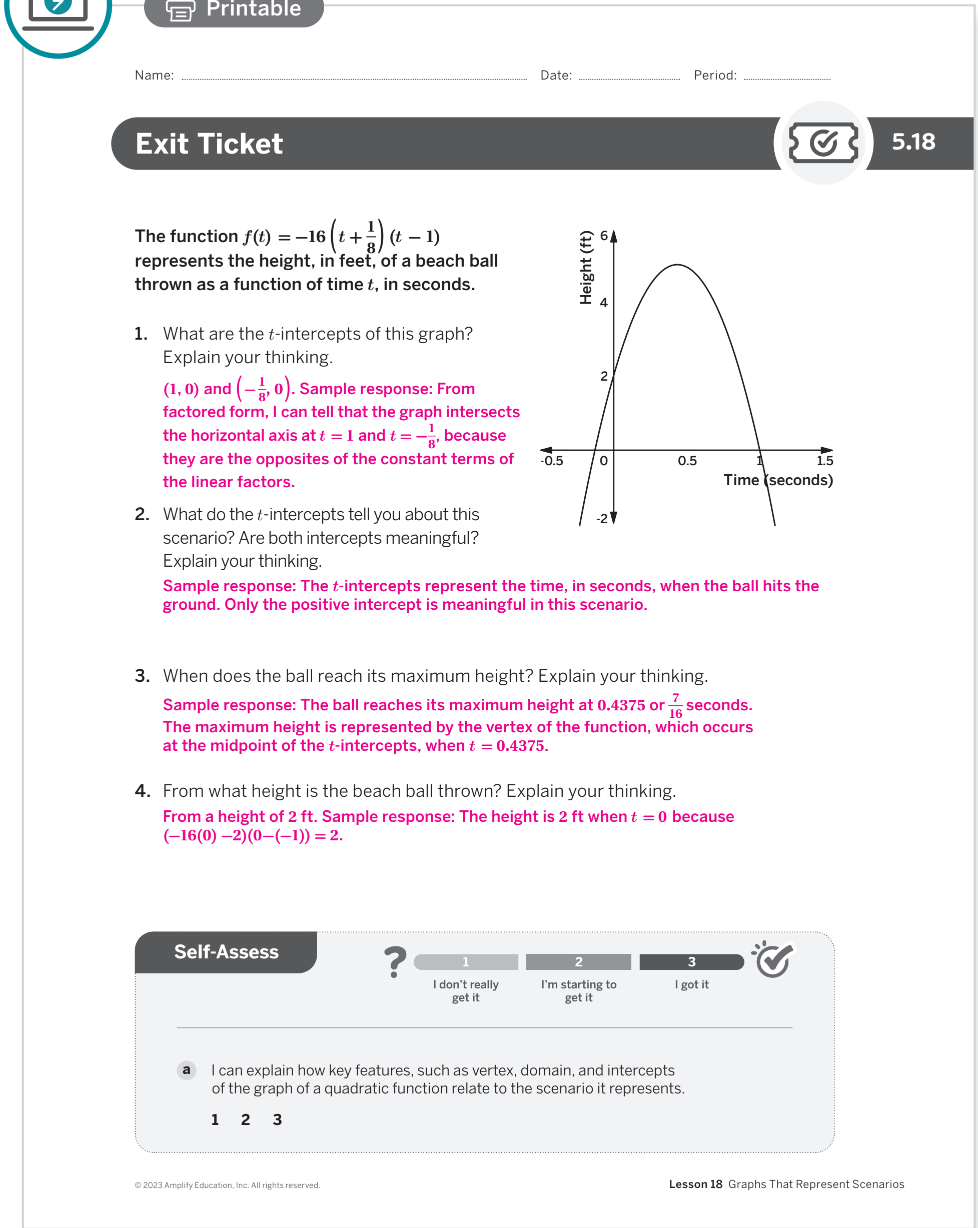
- "How are quadratic functions used to model, analyze, and interpret mathematical relationships?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining and interpreting key features of a quadratic function in context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

5.18

The function $f(t) = -16\left(t + \frac{1}{8}\right)(t - 1)$ represents the height, in feet, of a beach ball thrown as a function of time t, in seconds.

1. What are the t-intercepts of this graph? Explain your thinking.

   $(1, 0)$ and $\left(-\frac{1}{8}, 0\right)$. Sample response: From factored form, I can tell that the graph intersects the horizontal axis at $t = 1$ and $t = -\frac{1}{8}$, because they are the opposites of the constant terms of the linear factors.

2. What do the t-intercepts tell you about this scenario? Are both intercepts meaningful? Explain your thinking.

   Sample response: The t-intercepts represent the time, in seconds, when the ball hits the ground. Only the positive intercept is meaningful in this scenario.

3. When does the ball reach its maximum height? Explain your thinking.

   Sample response: The ball reaches its maximum height at 0.4375 or $\frac{7}{16}$ seconds. The maximum height is represented by the vertex of the function, which occurs at the midpoint of the t-intercepts, when $t = 0.4375$.

4. From what height is the beach ball thrown? Explain your thinking.

   From a height of 2 ft. Sample response: The height is 2 ft when $t = 0$ because $(-16(0) - 2)(0-(-1)) = 2$.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain how key features, such as vertex, domain, and intercepts of the graph of a quadratic function relate to the scenario it represents.

1 2 3

Lesson 18 Graphs That Represent Scenarios

Success looks like . . .

- **Language Goal:** Explaining how key features, such as vertex, domain, and intercepts of the graph of a quadratic function relate to a scenario. **(Speaking and Listening, Writing)**
  - » Determining the t-intercepts, the maximum height, and the height from which the beach ball is thrown in Problems 1–4.

Suggested next steps

If students use the constant term as their zero from the first factor in Problem 1, consider:

- Reviewing how to determine the zeros in Activity 2, Problem 1.
- Assigning Practice Problem 2.
- Asking, "What does it mean for a value to be a zero?"

If students are unclear or vague in their explanations for Problem 2, consider:

- Reviewing the meaning of the zeros in Activities 1 and 2.
- Assigning Practice Problems 1 and 2.
- Asking, "What does the horizontal axis represent in the graph of this function?"

If students cannot determine the time the ball reaches its maximum or the initial height in Problems 3 and 4, consider:

- Reviewing the methods for determining the vertex and vertical intercept in Activity 1, Problem 3.
- Assigning Practice Problems 1 and 2.
- Asking, How could you use the zeros of the function to determine the vertex?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students used key features of quadratic functions to interpret a scenario. How did that build on the earlier work students did with quadratic functions in standard and factored form?
- When you compare and contrast today's work with work students did earlier this year on graphing quadratic functions, what similarities and differences do you see?

Math Language Development

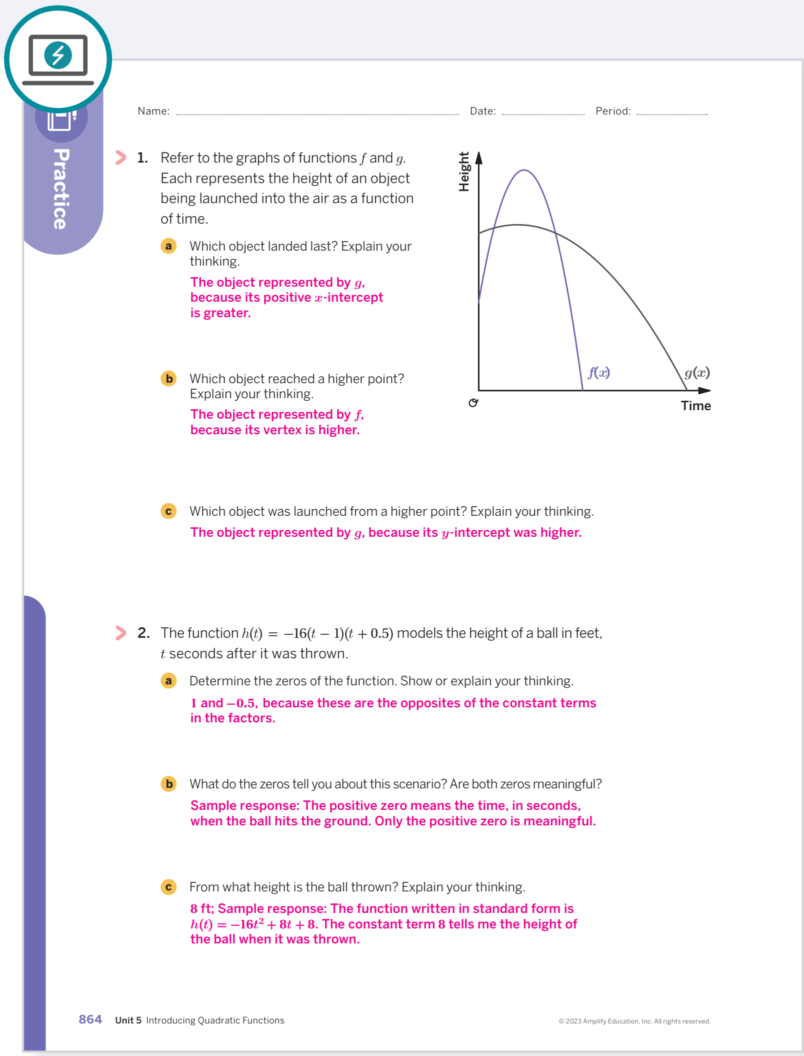
Language Goal: Explaining how key features, such as vertex, domain, and intercepts of the graph of a quadratic function relate to a scenario.

Reflect on students' language development toward this goal.

- How have students progressed in their descriptions of key features of the graphs of quadratic functions and interpreting them within context?
- Do students' responses to Problem 2 of the Exit Ticket indicate they understand the meaning of the x-coordinate of the intercepts?
- Do students' responses to Problems 2 and 3 of the Exit Ticket indicate they understand how to interpret each coordinate of the graph's vertex?

Practice

Independent



Name: .... Date: .... Period: ....

1. Refer to the graphs of functions f and g. Each represents the height of an object being launched into the air as a function of time.

   a. Which object landed last? Explain your thinking.
   The object represented by g, because its positive x-intercept is greater.

   b. Which object reached a higher point? Explain your thinking.
   The object represented by f, because its vertex is higher.

   c. Which object was launched from a higher point? Explain your thinking.
   The object represented by g, because its y-intercept was higher.

2. The function $h(t) = -16(t - 1)(t + 0.5)$ models the height of a ball in feet, t seconds after it was thrown.

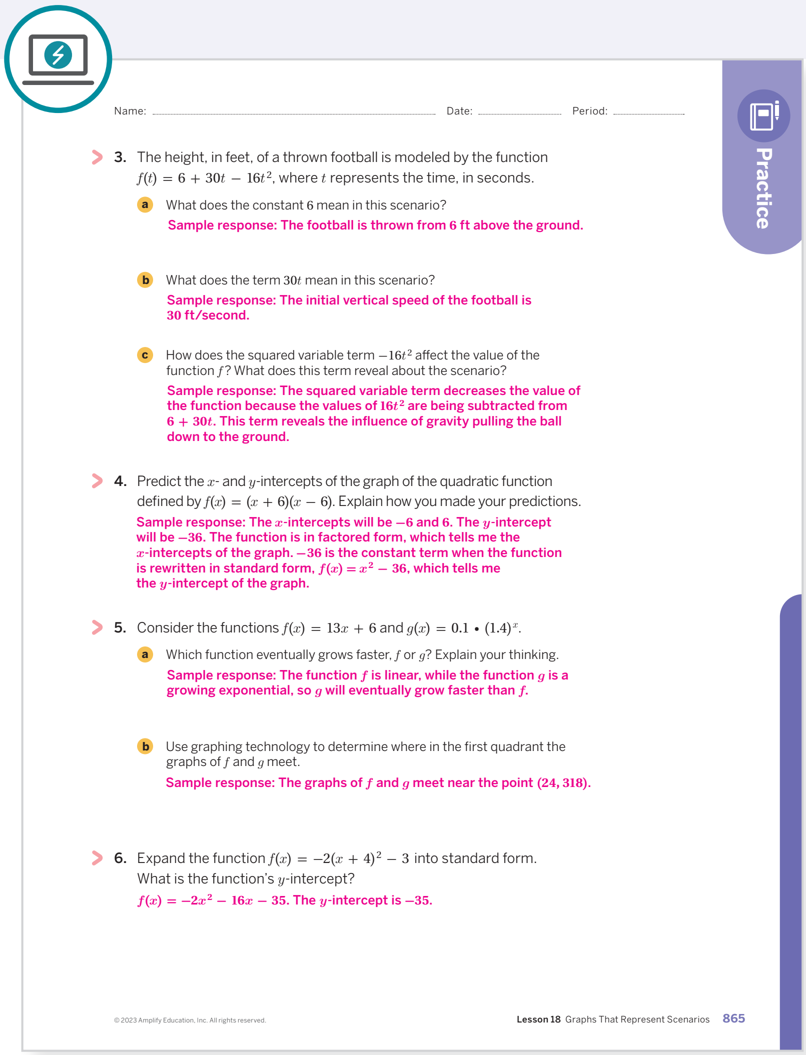
   a. Determine the zeros of the function. Show or explain your thinking.
   1 and -0.5, because these are the opposites of the constant terms in the factors.

   b. What do the zeros tell you about this scenario? Are both zeros meaningful?
   Sample response: The positive zero means the time, in seconds, when the ball hits the ground. Only the positive zero is meaningful.

   c. From what height is the ball thrown? Explain your thinking.
   8 ft; Sample response: The function written in standard form is $h(t) = -16t^2 + 8t + 8$. The constant term 8 tells me the height of the ball when it was thrown.

864 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.



Name: .... Date: .... Period: ....

3. The height, in feet, of a thrown football is modeled by the function $f(t) = 6 + 30t - 16t^2$, where t represents the time, in seconds.

   a. What does the constant 6 mean in this scenario?
   Sample response: The football is thrown from 6 ft above the ground.

   b. What does the term $30t$ mean in this scenario?
   Sample response: The initial vertical speed of the football is 30 ft/second.

   c. How does the squared variable term $-16t^2$ affect the value of the function f? What does this term reveal about the scenario?
   Sample response: The squared variable term decreases the value of the function because the values of $16t^2$ are being subtracted from $6 + 30t$. This term reveals the influence of gravity pulling the ball down to the ground.

4. Predict the x- and y-intercepts of the graph of the quadratic function defined by $f(x) = (x + 6)(x - 6)$. Explain how you made your predictions.
Sample response: The x-intercepts will be -6 and 6. The y-intercept will be -36. The function is in factored form, which tells me the x-intercepts of the graph. -36 is the constant term when the function is rewritten in standard form, $f(x) = x^2 - 36$, which tells me the y-intercept of the graph.

5. Consider the functions $f(x) = 13x + 6$ and $g(x) = 0.1 \cdot (1.4)^x$.

   a. Which function eventually grows faster, f or g? Explain your thinking.
   Sample response: The function f is linear, while the function g is a growing exponential, so g will eventually grow faster than f.

   b. Use graphing technology to determine where in the first quadrant the graphs of f and g meet.
   Sample response: The graphs of f and g meet near the point (24, 318).

6. Expand the function $f(x) = -2(x + 4)^2 - 3$ into standard form. What is the function's y-intercept?
$f(x) = -2x^2 - 16x - 35$. The y-intercept is -35.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 Graphs That Represent Scenarios 865

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 5 Lesson 13	2
	5	Unit 4 Lesson 20	2
Formative ⚡	6	Unit 5 Lesson 19	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 19**

Vertex Form

Let's find out about the vertex form.

Focus

Goals

1. **Language Goal:** Comprehend quadratic functions in vertex form by seeing the form as a constant plus a coefficient times a squared term. **(Speaking and Listening, Writing)**
2. **Language Goal:** Coordinate (using multiple representations) the parameters of a quadratic function in vertex form and the graph that represents it. **(Speaking and Listening, Writing)**

Coherence

Today

Students use technology to experiment with the parameters of quadratic functions written in vertex form, examine how they are visible on the graphs, and articulate their observations. They also attend to precision identifying parameters in context.

Previously

Students interpreted equations and graphs of quadratic functions in context in Lesson 18, comparing quadratic functions given in different representations.

Coming Soon

In Lesson 20, students will graph in vertex form, showing a maximum or minimum and the y-intercept.

Rigor

- Students build **conceptual understanding** of vertex form of a quadratic function.
- Students identify the vertex of a quadratic function in vertex form to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	10 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Describing my Thinking*
- graphing technology (as needed)

Math Language Development

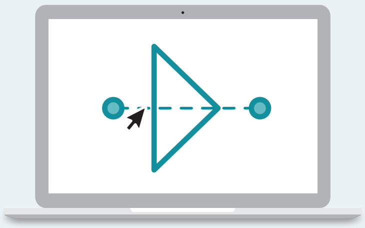
New word

- *vertex form*

Amps | **Featured Activity**

Activity 2
Digital Half-Pipe

Students use a digital skateboard ramp to explore key features of quadratic equations, written in standard and vertex form.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to communicate clearly about their observations with their partner, resulting in a rough beginning to Activity 2. Remind students that clear communication requires precision of language. As they evaluate the different forms of the equation and how they relate to the graph, students must determine the benefits or consequences for each form in the real-world scenario.

Modifications to Pacing

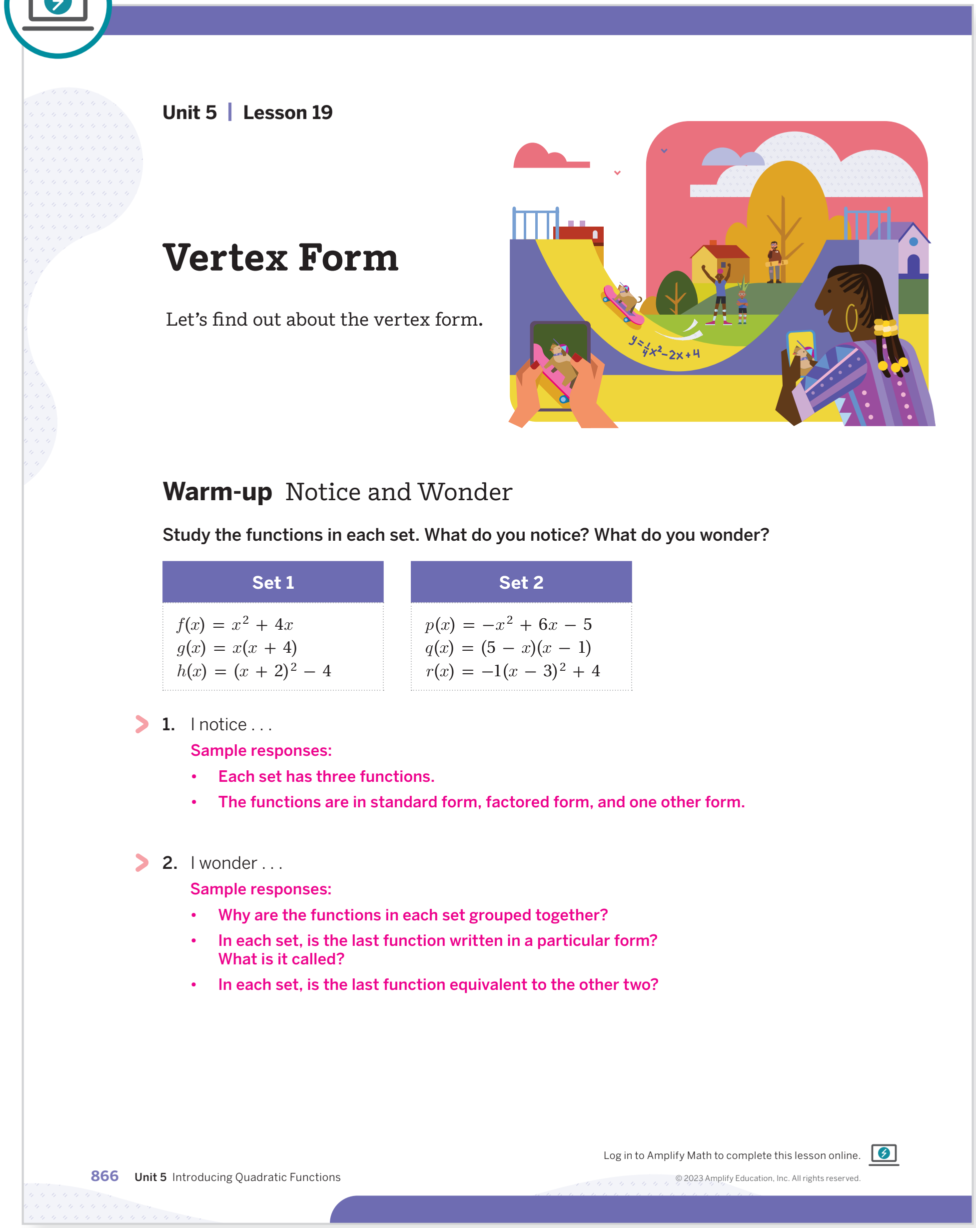
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 3 may be omitted.
- In **Activity 2**, Problem 5 may be omitted.

Pairs | 5 min

Warm-up Notice and Wonder

Students analyze two sets of functions, preparing them to reason later that the expressions defining each output are equivalent.



Unit 5 | **Lesson 19**

Vertex Form

Let's find out about the vertex form.

Warm-up Notice and Wonder

Study the functions in each set. What do you notice? What do you wonder?

Set 1	Set 2
$f(x) = x^2 + 4x$	$p(x) = -x^2 + 6x - 5$
$g(x) = x(x + 4)$	$q(x) = (5 - x)(x - 1)$
$h(x) = (x + 2)^2 - 4$	$r(x) = -1(x - 3)^2 + 4$

1. I notice . . .

   Sample responses:
   - Each set has three functions.
   - The functions are in standard form, factored form, and one other form.

2. I wonder . . .

   Sample responses:
   - Why are the functions in each set grouped together?
   - In each set, is the last function written in a particular form? What is it called?
   - In each set, is the last function equivalent to the other two?

Log in to Amplify Math to complete this lesson online.

866 **Unit 5** Introducing Quadratic Functions

1 Launch

Display the two sets of functions. Give students one minute to study the functions in each set. Conduct the ***Notice and Wonder*** routine. Tell students there are no wrong answers.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "What are key features of the standard and factored forms of quadratic functions?"

Look for points of confusion:

- **Not differentiating between standard and factored form.** Ask, "Which form provides the x-intercepts? The y-intercepts?"
- **Not identifying that a function in each set is in neither form.** Ask whether every function in each set provides an intercept. Have students explain why or why not.

Look for productive strategies:

- Identifying both the standard and factored forms.
- Recognizing that one function in each set is neither in standard nor factored form.

3 Connect

Have students share what they notice and wonder. Record and display their thinking. Ask students if they have questions about anything on the list.

Highlight that $h(x)$ in Set 1 and $r(x)$ in Set 2 are not in factored or standard form. Students will explore this new form of a quadratic function today.

Ask, "Is there anything else that you are wondering about these functions?"

Power-up

To power up students' ability to expand quadratic functions, have students complete:

Follow the steps to apply the order of operations in order to expand the function $g(x) = 3(x + 1)^2 - 4$.

Step 1 Square the expression $(x + 1)$. $f(x) = 3(x^2 + 2x + 2) - 4$

Step 2 Multiply the result by 3. $f(x) = 3x^2 + 6x + 6 - 4$

Step 3 Subtract 4 by combining like terms. $f(x) = 3x^2 + 6x - 2$

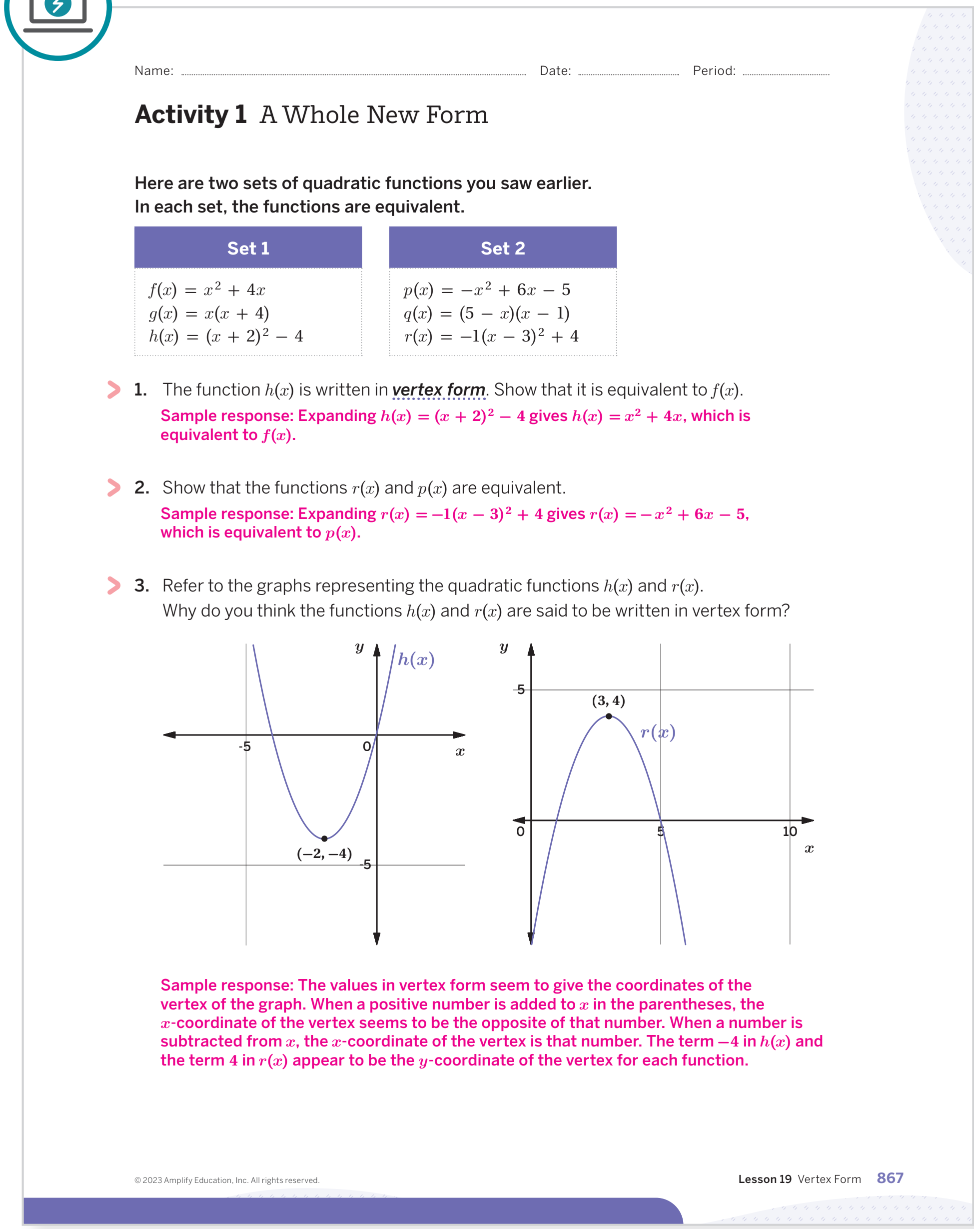
Use: Before the Warm-up

Informed by: Performance on Lesson 18, Practice Problem 6

Pairs | 15 min

Activity 1 A Whole New Form

Students look for structure in given quadratic functions written in vertex form to notice there is a connection between each expression and the vertex of the corresponding graph.



Name: ______ Date: ______ Period: ______

Activity 1 A Whole New Form

Here are two sets of quadratic functions you saw earlier. In each set, the functions are equivalent.

Set 1	Set 2
$f(x) = x^2 + 4x$	$p(x) = -x^2 + 6x - 5$
$g(x) = x(x + 4)$	$q(x) = (5 - x)(x - 1)$
$h(x) = (x + 2)^2 - 4$	$r(x) = -1(x - 3)^2 + 4$

1. The function $h(x)$ is written in ***vertex form***. Show that it is equivalent to $f(x)$.
   Sample response: Expanding $h(x) = (x + 2)^2 - 4$ gives $h(x) = x^2 + 4x$, which is equivalent to $f(x)$.

2. Show that the functions $r(x)$ and $p(x)$ are equivalent.
   Sample response: Expanding $r(x) = -1(x - 3)^2 + 4$ gives $r(x) = -x^2 + 6x - 5$, which is equivalent to $p(x)$.

3. Refer to the graphs representing the quadratic functions $h(x)$ and $r(x)$. Why do you think the functions $h(x)$ and $r(x)$ are said to be written in vertex form?

Sample response: The values in vertex form seem to give the coordinates of the vertex of the graph. When a positive number is added to x in the parentheses, the x-coordinate of the vertex seems to be the opposite of that number. When a number is subtracted from x, the x-coordinate of the vertex is that number. The term -4 in $h(x)$ and the term 4 in $r(x)$ appear to be the y-coordinate of the vertex for each function.

1 Launch

Tell students that they will be further examining the function sets that they saw in the Warm-up. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by completing Problem 1 together, illustrating the equivalence of the functions h and f.

Look for points of confusion:

- **Thinking that $(x - 3)^2$ is $(x^2 - 3^2)$.** Remind them that $(x - 3)^2$ means $(x - 3)(x - 3)$.

Look for productive strategies:

- Expanding $h(x)$ and $r(x)$, showing the functions are equivalent to $f(x)$ and $p(x)$, respectively.

3 Connect

Display the functions and graphs.

Have students share strategies they used to show that the functions $r(x)$ and $p(x)$ are equivalent.

Highlight that vertex form reveals the coordinates of the vertex and is used to easily determine the maximum or the minimum of a function.

Define the term ***vertex form***.

Ask, "Can you give an example of when it might be useful to have the relationship presented using the vertex form?" Sample response: When I want to know the maximum height of an object in projectile motion.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students focus on the functions in Set 1. Ask them to annotate which functions are written in standard form or factored form, and what information it tells them about the function. Tell them the third function is written in vertex form. Ask, "What information do you think it will tell you?"

Extension: *Math Enrichment*

Have students use graphing technology to graph the following functions. Ask them to explain why the vertex of the function $z(x)$ is $(-1, -3)$ instead of $(1, 3)$. Sample response: Adding 1 to x before squaring the term shifts the function to the left, not the right.

$w(x) = x^2 - 3$ $\quad$ $z(x) = (x + 1)^2 - 3$

Math Language Development

MLR5: Co-craft Questions

During the Launch, reveal the functions in Sets 1 and 2. Have students work with their partner to write 2–3 mathematical questions they could ask about the functions. Have volunteers share their questions with the class. Amplify questions that ask about the structure of the equations. Sample questions shown.

- Which functions are written in standard form?
- Which functions are written in factored form?
- What is the third form shown? Does it mean anything?

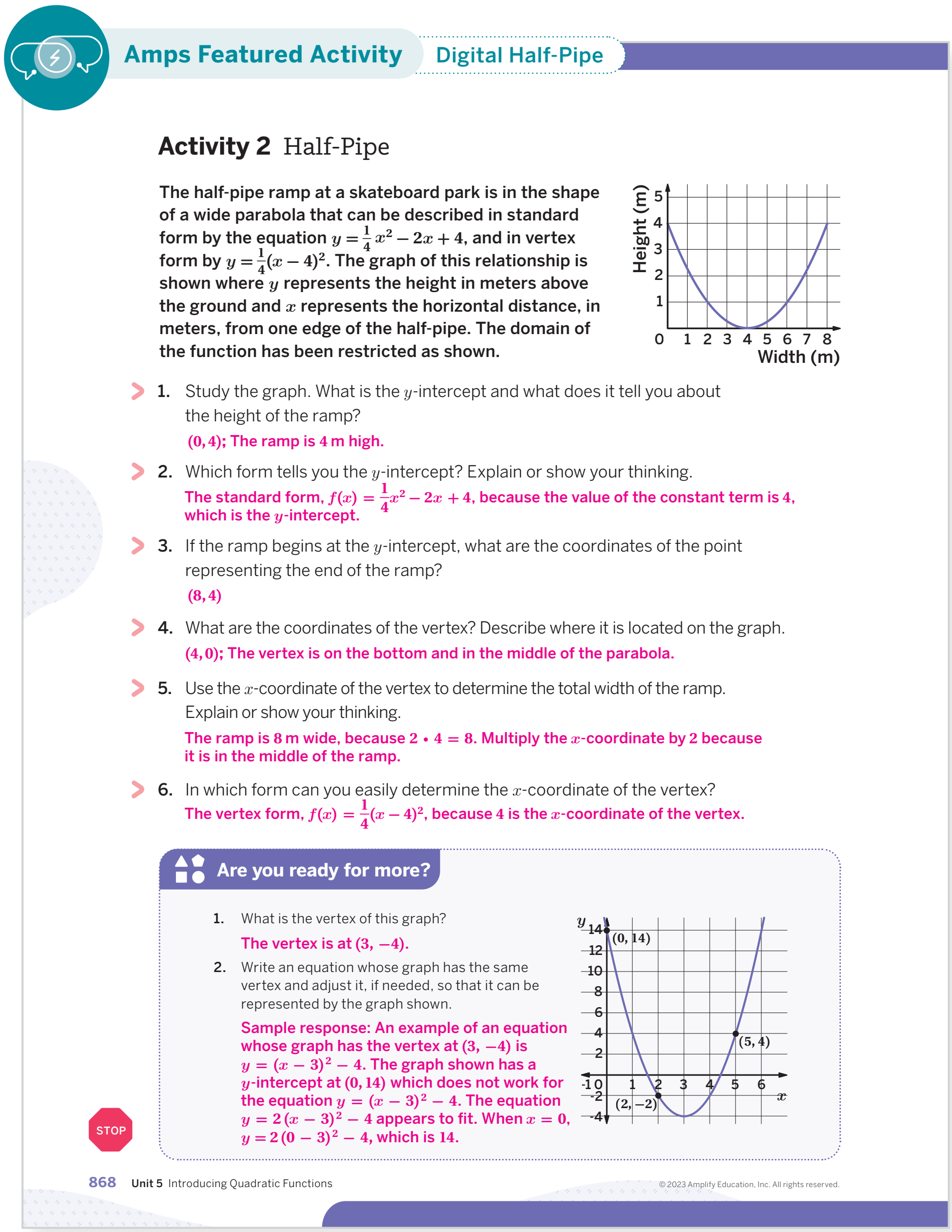
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 15 min

Activity 2 Half-Pipe

Students analyze the graph of a quadratic function, given in standard form and vertex form, to understand the key information that vertex form provides about the graph.



Amps Featured Activity Digital Half-Pipe

Activity 2 Half-Pipe

The half-pipe ramp at a skateboard park is in the shape of a wide parabola that can be described in standard form by the equation $y = \frac{1}{4}x^2 - 2x + 4$, and in vertex form by $y = \frac{1}{4}(x - 4)^2$. The graph of this relationship is shown where y represents the height in meters above the ground and x represents the horizontal distance, in meters, from one edge of the half-pipe. The domain of the function has been restricted as shown.

1. Study the graph. What is the y-intercept and what does it tell you about the height of the ramp?

   $(0, 4)$; The ramp is 4 m high.

2. Which form tells you the y-intercept? Explain or show your thinking.

   The standard form, $f(x) = \frac{1}{4}x^2 - 2x + 4$, because the value of the constant term is 4, which is the y-intercept.

3. If the ramp begins at the y-intercept, what are the coordinates of the point representing the end of the ramp?

   $(8, 4)$

4. What are the coordinates of the vertex? Describe where it is located on the graph.

   $(4, 0)$; The vertex is on the bottom and in the middle of the parabola.

5. Use the x-coordinate of the vertex to determine the total width of the ramp. Explain or show your thinking.

   The ramp is 8 m wide, because $2 \cdot 4 = 8$. Multiply the x-coordinate by 2 because it is in the middle of the ramp.

6. In which form can you easily determine the x-coordinate of the vertex?

   The vertex form, $f(x) = \frac{1}{4}(x - 4)^2$, because 4 is the x-coordinate of the vertex.

Are you ready for more?

1. What is the vertex of this graph?

   The vertex is at $(3, -4)$.

2. Write an equation whose graph has the same vertex and adjust it, if needed, so that it can be represented by the graph shown.

   Sample response: An example of an equation whose graph has the vertex at $(3, -4)$ is $y = (x - 3)^2 - 4$. The graph shown has a y-intercept at $(0, 14)$ which does not work for the equation $y = (x - 3)^2 - 4$. The equation $y = 2(x - 3)^2 - 4$ appears to fit. When $x = 0$, $y = 2(0 - 3)^2 - 4$, which is 14.

STOP

868 Unit 5 Introducing Quadratic Functions

1 Launch

Provide students with quiet time to read the passage and study the graph. Then have them discuss with a partner their observations and strategies for connecting the function to the graph before beginning the activity.

2 Monitor

Help students get started by prompting them to annotate and label the graph with any important features they notice.

Look for points of confusion:

- **Thinking the vertex form gives the y-intercept in Problem 2.** Have students evaluate each form when $x = 0$.
- **Not relating the vertex to the middle of the ramp in Problem 5.** Have students determine the width without the x-coordinate and then draw the line of symmetry to notice any shortcuts.

Look for productive strategies:

- Noticing and applying how each form provides different information about the parabola.

3 Connect

Display the graph.

Have students share their strategies for determining the dimensions of the ramp.

Highlight that vertex form and standard form provide different information about the parabola.

Ask, “What are the advantages and drawbacks, if any, to using each form? Explain your thinking.”

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital skateboard ramp to explore key features of quadratic equations, written in standard and vertex form.

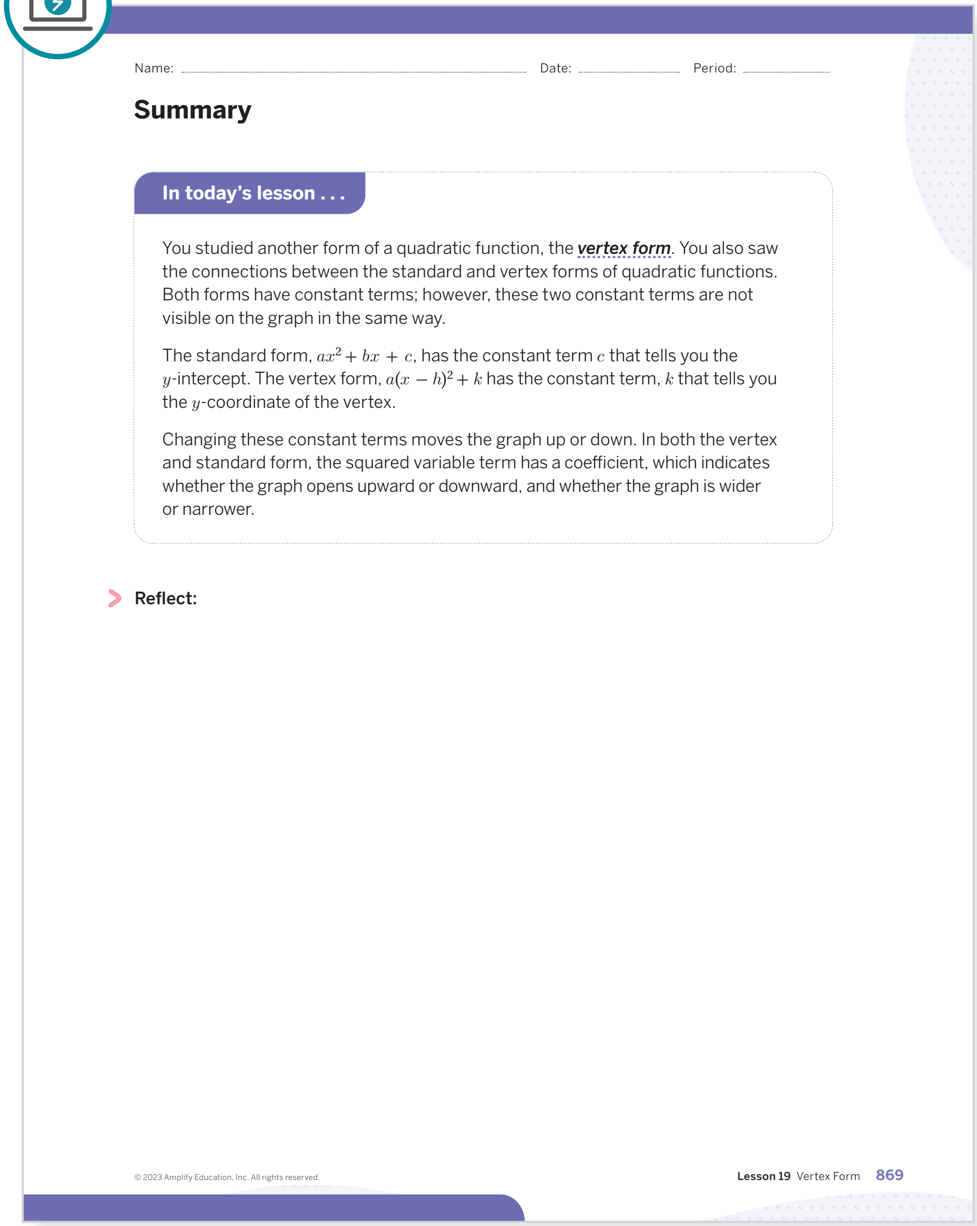
Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

Ask students if they have ever seen a half-pipe ramp at a skateboard park. Consider showing images or a video of such a ramp so that students can visualize its shape in context. Provide access to colored pencils and have students annotate the intercepts and vertex of the graph.

Whole Class | 10 min

Summary

Review and synthesize connections between the three different forms of quadratic functions: standard form, factored form, and vertex form.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You studied another form of a quadratic function, the ***vertex form***. You also saw the connections between the standard and vertex forms of quadratic functions. Both forms have constant terms; however, these two constant terms are not visible on the graph in the same way.

The standard form, $ax^2 + bx + c$, has the constant term c that tells you the y-intercept. The vertex form, $a(x - h)^2 + k$ has the constant term, k that tells you the y-coordinate of the vertex.

Changing these constant terms moves the graph up or down. In both the vertex and standard form, the squared variable term has a coefficient, which indicates whether the graph opens upward or downward, and whether the graph is wider or narrower.

Reflect:

Synthesize

Display the three forms of quadratic functions: standard form, factored form, and vertex form.

Highlight that the constant terms in standard and vertex form are not visible on the graph in the same way. The k in vertex form gives the y-coordinate of the vertex. The c in standard form gives the y-intercept. However, changing these parameters has the same effect of shifting the graph upward or downward.

Formalize vocabulary: ***vertex form***.

Ask, "In both vertex and standard form, the squared variable term x^2 has a coefficient (which could be 1). Does this coefficient affect the graph in similar ways?" Yes, they both influence the direction and width of the opening of the graph.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What are advantages and disadvantages of graphing a quadratic function in vertex form?"
- "How could you determine key features, besides the vertex, of a quadratic function in vertex form?"

Math Language Development

MLR2: Collect and Display

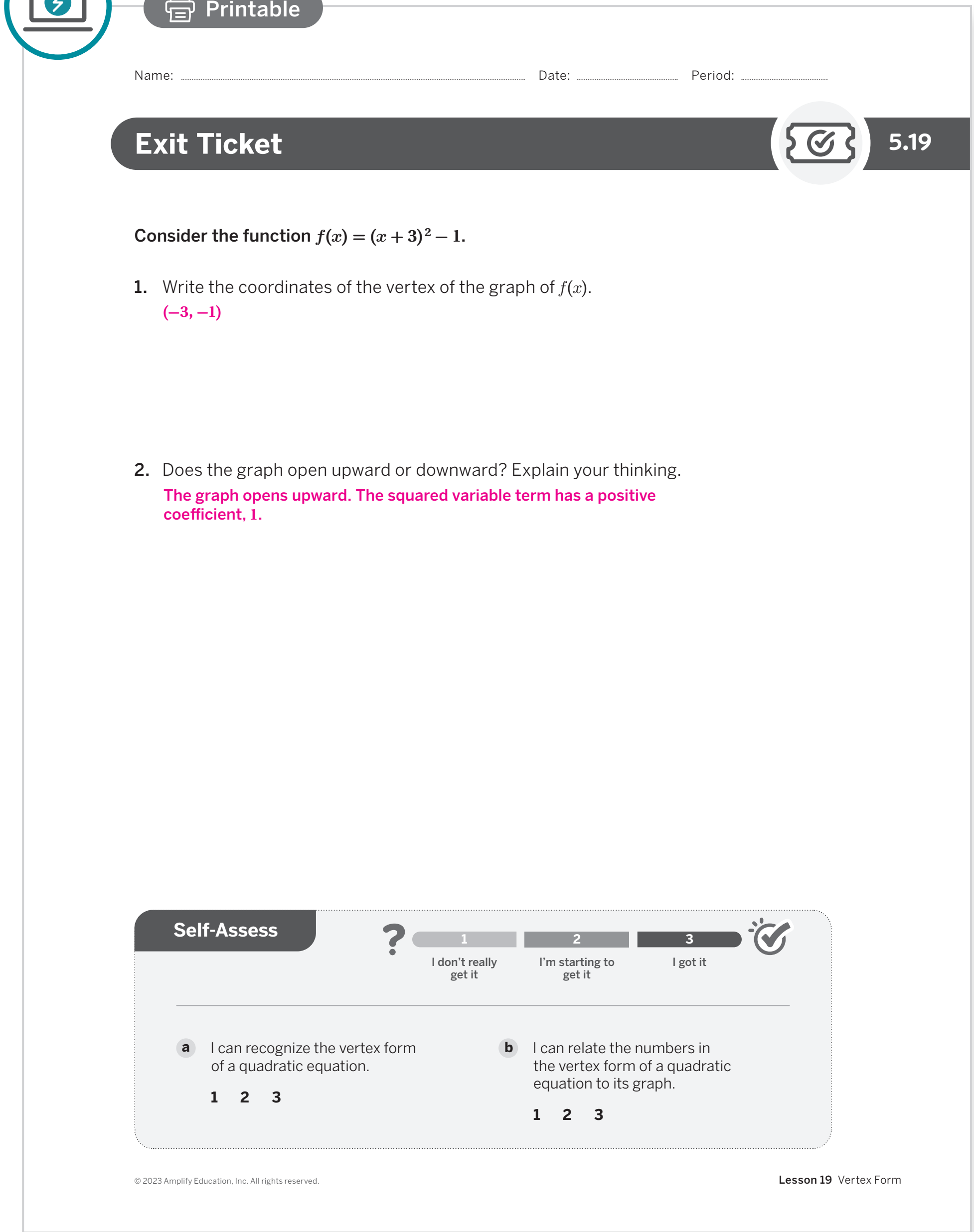
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *vertex form* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing the coordinates of the vertex and determining whether the graph opens upward or downward — given the vertex form of a quadratic function.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.19

Consider the function $f(x) = (x + 3)^2 - 1$.

1. Write the coordinates of the vertex of the graph of $f(x)$.
   $(-3, -1)$
2. Does the graph open upward or downward? Explain your thinking.
   The graph opens upward. The squared variable term has a positive coefficient, 1.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can recognize the vertex form of a quadratic equation.
1 2 3

b. I can relate the numbers in the vertex form of a quadratic equation to its graph.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Vertex Form

Success looks like . . .

- **Language Goal:** Comprehending quadratic functions in vertex form by seeing the form as a constant plus a coefficient times a squared term. **(Speaking and Listening, Writing)**
  - » Determining the vertex and whether the graph of the quadratic function opens upward or downward from the vertex form of a quadratic function in Problems 1–2.
- **Language Goal:** Coordinating (using multiple representations) the parameters of a quadratic function in vertex form and the graph that represents it. **(Speaking and Listening, Writing)**

Suggested next steps

If students do not correctly identify the vertex of $f(x)$ from its equation in Problem 1, consider:

- Reviewing the connection of h and k in the function to the x- and y-coordinates of the vertex in the graph in Activity 1.
- Assigning Practice Problem 3.
- Asking, "What do the values of h and k represent in the graph of the function?"

If students write that the graph opens downward in Problem 2, consider:

- Reviewing Problem 2 in Activity 1.
- Assigning Practice Problem 3.
- Asking, "How did a negative coefficient of the x^2 term affect the graph?"

Professional Learning

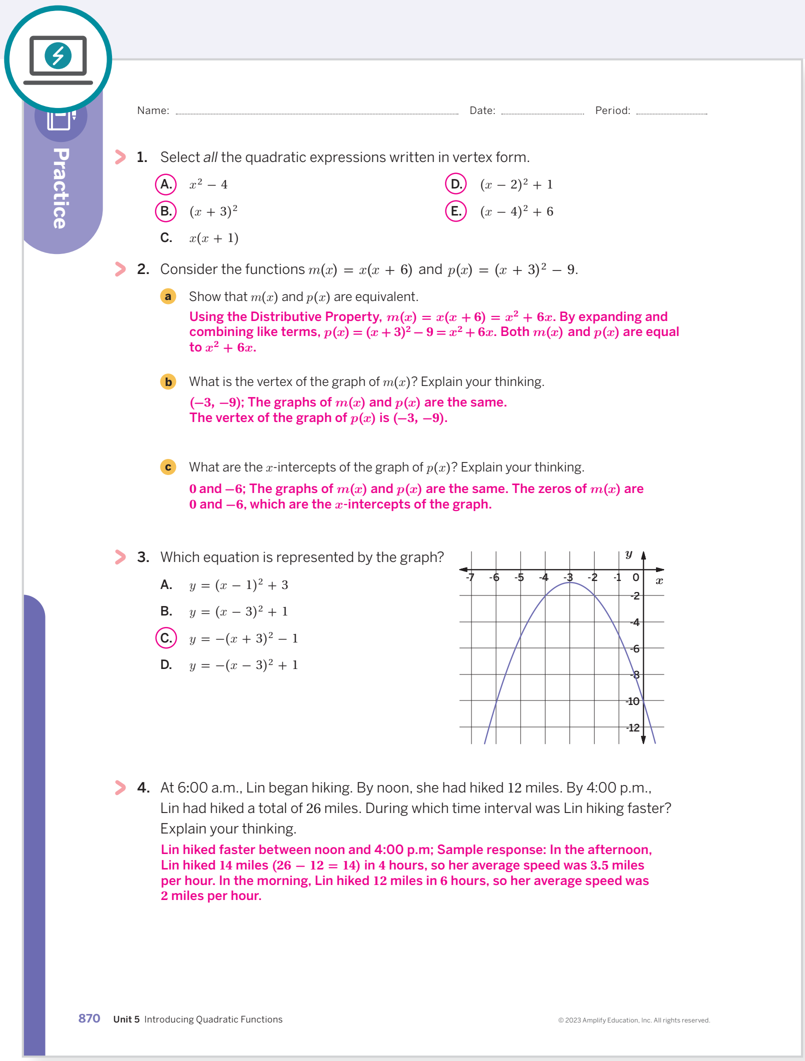
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did ***Notice and Wonder*** support students in recognizing the vertex form of a quadratic equation?
- In this lesson, students matched quadratic functions in vertex form with an equivalent quadratic function in standard form. How will that support determining the advantages of each type of form of quadratic functions?

Practice

Independent



Name: .... Date: .... Period: ....

1. Select *all* the quadratic expressions written in vertex form.
   - [✓ A.] $x^2 - 4$
   - [✓ B.] $(x + 3)^2$
   - C. $x(x + 1)$
   - [✓ D.] $(x - 2)^2 + 1$
   - [✓ E.] $(x - 4)^2 + 6$

2. Consider the functions $m(x) = x(x + 6)$ and $p(x) = (x + 3)^2 - 9$.
   - a Show that $m(x)$ and $p(x)$ are equivalent.

     Using the Distributive Property, $m(x) = x(x + 6) = x^2 + 6x$. By expanding and combining like terms, $p(x) = (x + 3)^2 - 9 = x^2 + 6x$. Both $m(x)$ and $p(x)$ are equal to $x^2 + 6x$.
   - b What is the vertex of the graph of $m(x)$? Explain your thinking.

     (–3, –9); The graphs of $m(x)$ and $p(x)$ are the same. The vertex of the graph of $p(x)$ is (–3, –9).
   - c What are the x-intercepts of the graph of $p(x)$? Explain your thinking.

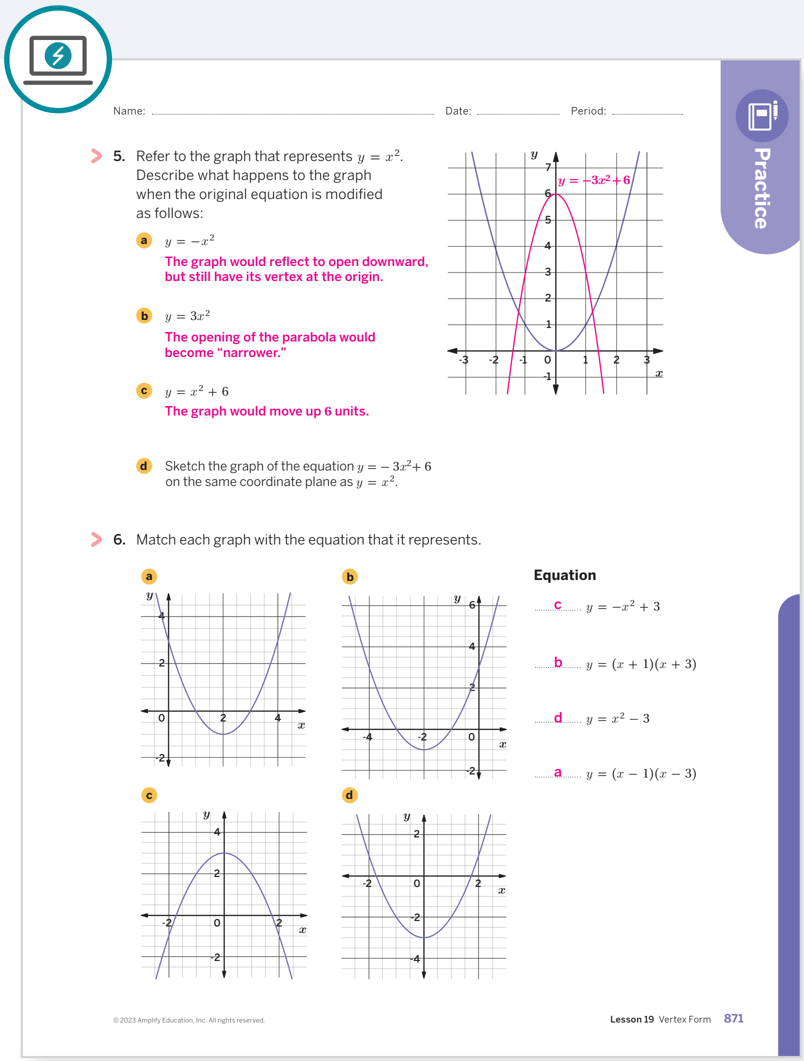
     0 and –6; The graphs of $m(x)$ and $p(x)$ are the same. The zeros of $m(x)$ are 0 and –6, which are the x-intercepts of the graph.

3. Which equation is represented by the graph?
   - A. $y = (x - 1)^2 + 3$
   - B. $y = (x - 3)^2 + 1$
   - [✓ C.] $y = -(x + 3)^2 - 1$
   - D. $y = -(x - 3)^2 + 1$

4. At 6:00 a.m., Lin began hiking. By noon, she had hiked 12 miles. By 4:00 p.m., Lin had hiked a total of 26 miles. During which time interval was Lin hiking faster? Explain your thinking.

   Lin hiked faster between noon and 4:00 p.m; Sample response: In the afternoon, Lin hiked 14 miles ($26 - 12 = 14$) in 4 hours, so her average speed was 3.5 miles per hour. In the morning, Lin hiked 12 miles in 6 hours, so her average speed was 2 miles per hour.

870 **Unit 5** Introducing Quadratic Functions



Name: .... Date: .... Period: ....

5. Refer to the graph that represents $y = x^2$. Describe what happens to the graph when the original equation is modified as follows:
   - a $y = -x^2$

     The graph would reflect to open downward, but still have its vertex at the origin.
   - b $y = 3x^2$

     The opening of the parabola would become “narrower.”
   - c $y = x^2 + 6$

     The graph would move up 6 units.
   - d Sketch the graph of the equation $y = -3x^2 + 6$ on the same coordinate plane as $y = x^2$.

6. Match each graph with the equation that it represents.

Equation

c .... $y = -x^2 + 3$

b .... $y = (x + 1)(x + 3)$

d .... $y = x^2 - 3$

a .... $y = (x - 1)(x - 3)$

Lesson 19 Vertex Form 871

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	3
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 21	2
	5	Unit 5 Lesson 16	2
Formative ⚡	6	Unit 5 Lesson 20	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

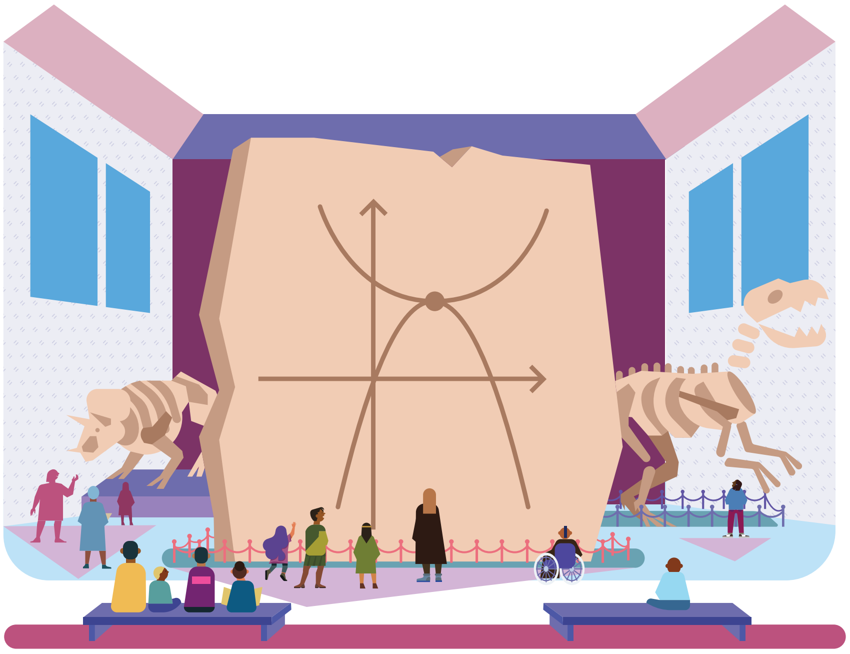
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 20**

Graphing With the Vertex Form

Let's graph functions using vertex form.



Focus

Goals

1. Graph a quadratic function given in vertex form, showing a maximum or minimum and the y-intercept.
2. Know how to find a maximum or minimum of a quadratic function given in vertex form, without first graphing it.

Coherence

Today

Students continue to explore quadratic functions in vertex form and think about how the structure of the form shows whether the vertex is the maximum or minimum value of the function. Students then use this knowledge to graph a quadratic function given in vertex form without using graphing technology.

Previously

In the previous lesson, students were introduced to the vertex form of a quadratic function and saw how it revealed the location of the vertex of the graph of the function.

Coming Soon

In the next lesson, students further explore the effects of changing the parameters of a quadratic function given in vertex form and how it affects the graph.

Rigor

- Students graph quadratic functions in vertex form to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	15 min	10 min	5 min	5 min
Independent	Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards
- Anchor Chart PDF, *Sentence Stems*, *Matching Prompts*
- graph paper
- tracing paper

Math Language Development

Review words

- *factored form*
- *vertex form*
- *vertex*

Amps | Featured Activity

Activity 3
Interactive Graph

Students write quadratic functions in different forms, so that they pass through specific coordinates of an interactive graph.

Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students work in pairs for Activity 2, they might be tempted to provide too much help or to allow their partner to do the majority of the work. Encourage students to appreciate the different talents that each person has by guiding a partner to the correct answer or accepting help while striving to understand for themselves.

Modifications to Pacing

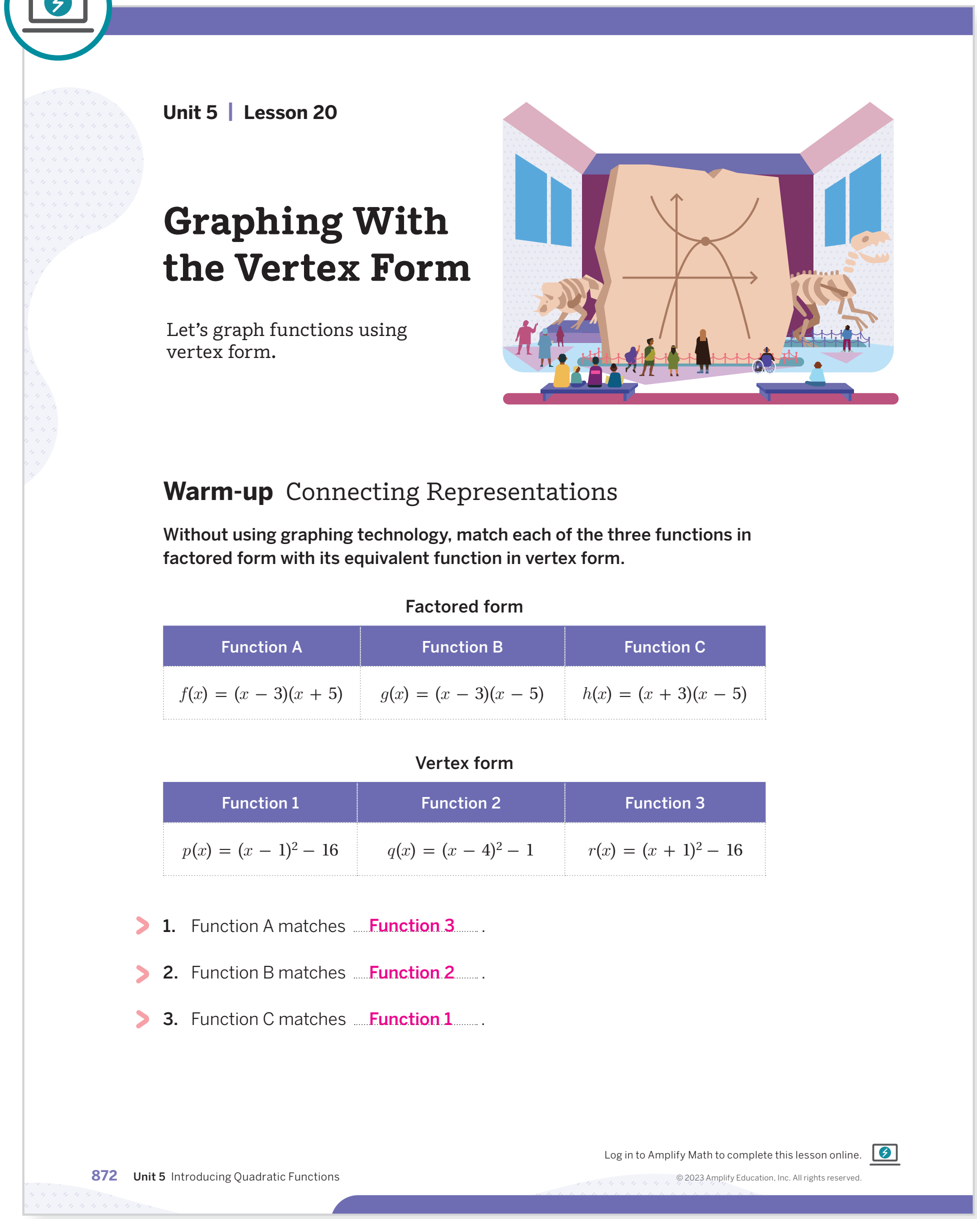
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 3 may be omitted.
- In **Activity 2**, have students only complete three sets of cards.

Independent | 5 min

Warm-up Connecting Representations

Students match functions in factored form with an equivalent function in vertex form to discover a special relationship between the forms.



Unit 5 | Lesson 20

Graphing With the Vertex Form

Let's graph functions using vertex form.

Warm-up Connecting Representations

Without using graphing technology, match each of the three functions in factored form with its equivalent function in vertex form.

Factored form

Function A	Function B	Function C
$f(x) = (x - 3)(x + 5)$	$g(x) = (x - 3)(x - 5)$	$h(x) = (x + 3)(x - 5)$

Vertex form

Function 1	Function 2	Function 3
$p(x) = (x - 1)^2 - 16$	$q(x) = (x - 4)^2 - 1$	$r(x) = (x + 1)^2 - 16$

1. Function A matches Function 3.
2. Function B matches Function 2.
3. Function C matches Function 1.

Log in to Amplify Math to complete this lesson online.

872 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the standard form and vertex form for a quadratic function. Remind students the information each form provides.

2 Monitor

Help students get started by asking them what information each form provides. Provide access to graph paper.

Look for productive strategies:

- Writing the functions in standard form to see that they are equivalent.
- Determining the x-coordinate of the vertex from the vertex form and comparing it to the x-coordinate of the vertex by averaging the zeros of the function from the factored form (because there are two zeros).
- Recognizing that the x-coordinate of the vertex has the opposite sign as the number added to x inside the parentheses of the vertex form.

3 Connect

Display the functions in factored form and select three students who used productive strategies to share their matches.

Have students share the strategies they used to match the functions.

Highlight that the x-coordinate of the vertex is determined from a function in vertex form; the vertex is located halfway between the x-intercepts on the axis of symmetry. The x-intercepts can be determined by a function in factored form (assuming there are two x-intercepts).

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share the different strategies they used to match the functions, ask:

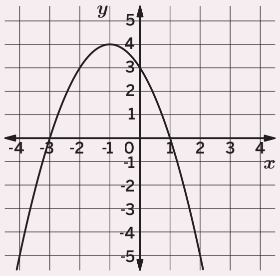
- "What information does the factored form of a quadratic provide? How did you use this information to determine the matches?"
- "What information does the vertex form of a quadratic provide? How did you use this information to determine the matches?"
- "Did anyone use the standard form? How could writing the functions in standard form help you?"

English Learners

Display or provide access to the Anchor Chart PDF, *Sentence Stems*, *Matching Prompts* to help students explain their thinking.

Power-up

To power up students' ability to match graphs of quadratic equations to their equations in factored or standard forms, have students complete:



Which functions match the graph? Select *all* that apply.

A. $f(x) = -(x + 1)(x - 3)$

[✓ B.] $g(x) = -(x - 1)(x + 3)$

C. $h(x) = -x^2 + 2x - 3$

[✓ D.] $j(x) = -x^2 - 2x + 3$

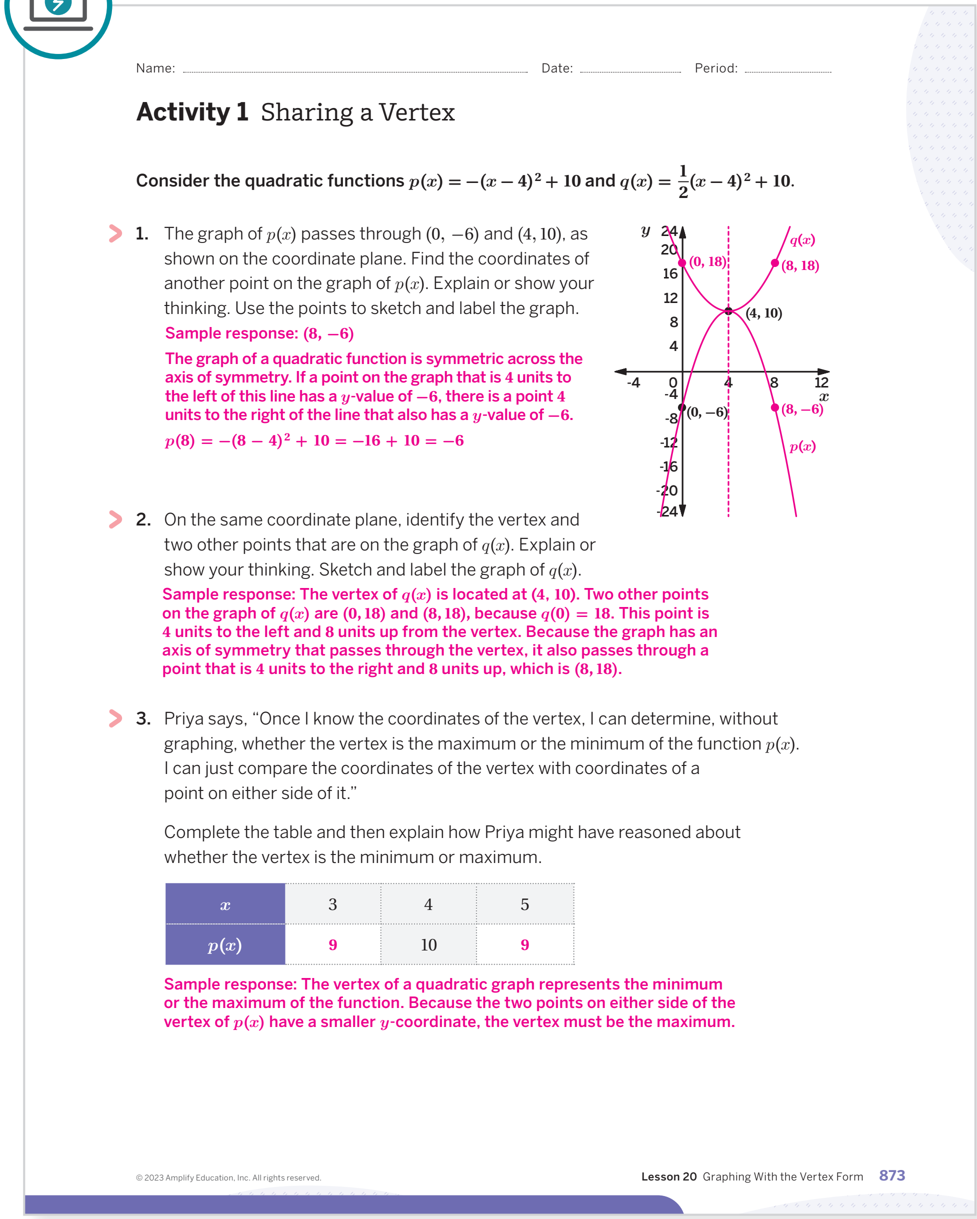
Use: Before Activity 2

Informed by: Performance on Lesson 19, Practice Problem 6

Independent | 20 min

Activity 1 Sharing a Vertex

Students study the structure and graph two quadratic functions that share the same vertex to reason about why one parabola opens upward and the other opens downward.



Name: Date: Period:

Activity 1 Sharing a Vertex

Consider the quadratic functions $p(x) = -(x - 4)^2 + 10$ and $q(x) = \frac{1}{2}(x - 4)^2 + 10$.

1. The graph of $p(x)$ passes through $(0, -6)$ and $(4, 10)$, as shown on the coordinate plane. Find the coordinates of another point on the graph of $p(x)$. Explain or show your thinking. Use the points to sketch and label the graph.
   Sample response: $(8, -6)$

   The graph of a quadratic function is symmetric across the axis of symmetry. If a point on the graph that is 4 units to the left of this line has a y-value of -6, there is a point 4 units to the right of the line that also has a y-value of -6.

   $p(8) = -(8 - 4)^2 + 10 = -16 + 10 = -6$

2. On the same coordinate plane, identify the vertex and two other points that are on the graph of $q(x)$. Explain or show your thinking. Sketch and label the graph of $q(x)$.
   Sample response: The vertex of $q(x)$ is located at $(4, 10)$. Two other points on the graph of $q(x)$ are $(0, 18)$ and $(8, 18)$, because $q(0) = 18$. This point is 4 units to the left and 8 units up from the vertex. Because the graph has an axis of symmetry that passes through the vertex, it also passes through a point that is 4 units to the right and 8 units up, which is $(8, 18)$.

3. Priya says, "Once I know the coordinates of the vertex, I can determine, without graphing, whether the vertex is the maximum or the minimum of the function $p(x)$. I can just compare the coordinates of the vertex with coordinates of a point on either side of it."

   Complete the table and then explain how Priya might have reasoned about whether the vertex is the minimum or maximum.

x	3	4	5
$p(x)$	9	10	9

Sample response: The vertex of a quadratic graph represents the minimum or the maximum of the function. Because the two points on either side of the vertex of $p(x)$ have a smaller y-coordinate, the vertex must be the maximum.

1 Launch

Display $p(x)$ and $q(x)$ and ask students how the functions are alike and how they are different. Activate students' prior knowledge by asking, "Where are the vertices of each graph located?" $(4, 10)$ for both $p(x)$ and $q(x)$.

2 Monitor

Help students get started by prompting them to draw the axis of symmetry and to locate the image of $(0, -6)$.

Look for points of confusion:

- **Struggling to graph $q(x)$.** Prompt students to substitute values of x that are straightforward to evaluate.

Look for productive strategies:

- Drawing the axis of symmetry or some other indication of using symmetry.

3 Connect

Have students share their graphs and how they determined the coordinates of one other point on the graph of $p(x)$, and two other points on the graph of $q(x)$. Select students to explain their analysis of Priya's reasoning.

Ask, "How can you determine whether the vertex of the graph represents a maximum or a minimum?"

Highlight that the vertex of $p(x)$ cannot be a minimum value, because there are no other values of $p(x)$ that are less than 10. Similarly, because $(0, -6)$ is on the graph and its y-coordinate is less than that of the vertex, the vertex is a maximum.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the information in each function that gives them the coordinates of the vertex. Ask:

- "Is the vertex the same for each function? How do you know?"
- "Why are the coordinates of the vertex $(4, 10)$ and not $(-4, 10)$?"
- "Without sketching these graphs, how do you know whether they are the same function or different functions? Can different functions pass through the same vertex?"

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the two functions $p(x)$ and $q(x)$ Have students work with a partner to write 2–3 mathematical questions they could ask about the functions. Have volunteers share their questions with the class. Sample questions shown.

- What do the graphs of these functions look like? How do they compare to the graph of $f(x) = x^2$?
- How does subtracting 4 from x before squaring it affect the graph?
- How are these graphs different from each other?

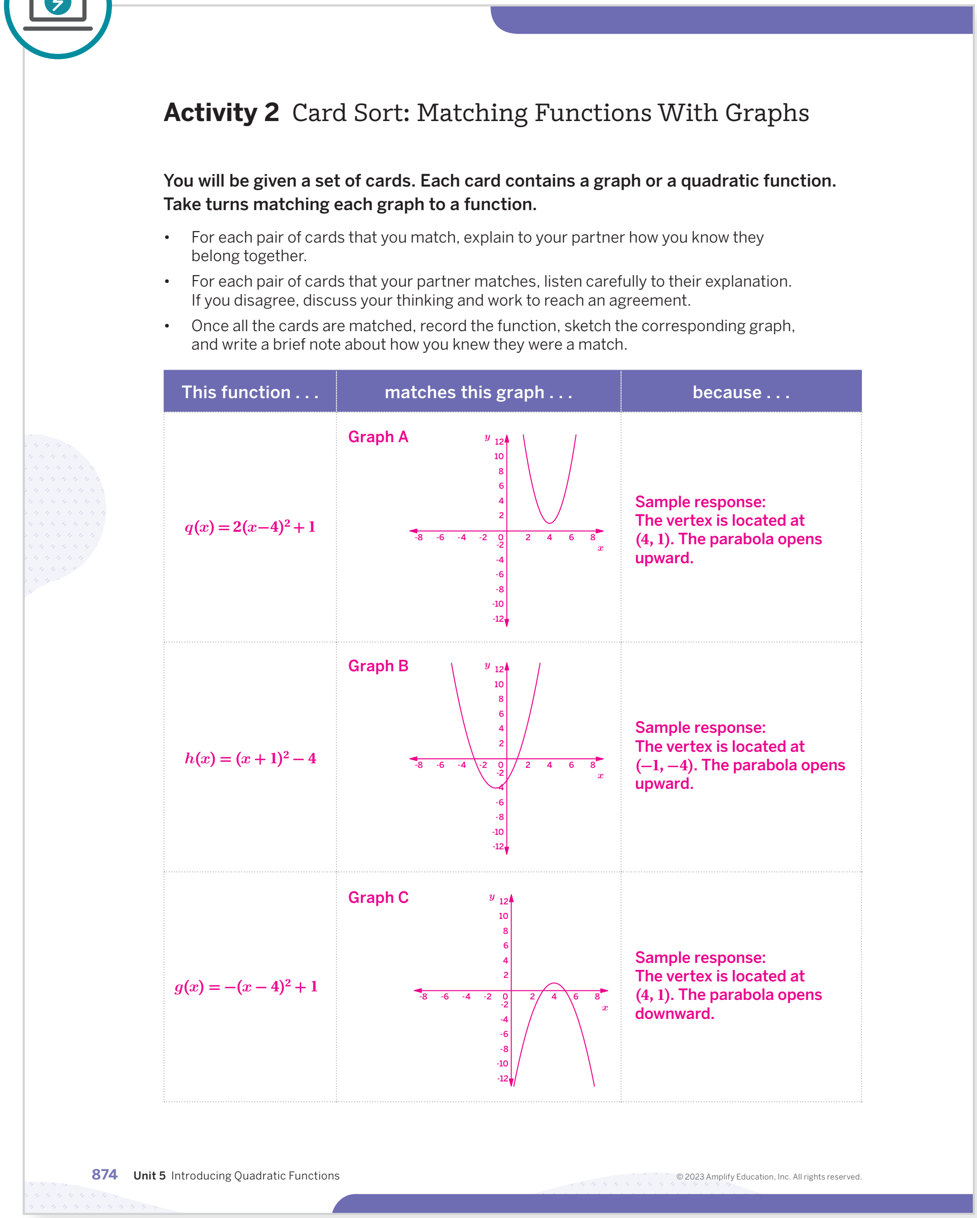
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 15 min

Activity 2 Card Sort: Matching Functions With Graphs

Students match quadratic functions given in vertex form to their corresponding graphs to build fluency in connecting representations: functions and graphs.



Activity 2 Card Sort: Matching Functions With Graphs

You will be given a set of cards. Each card contains a graph or a quadratic function. Take turns matching each graph to a function.

- For each pair of cards that you match, explain to your partner how you know they belong together.
- For each pair of cards that your partner matches, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
- Once all the cards are matched, record the function, sketch the corresponding graph, and write a brief note about how you knew they were a match.

This function . . .	matches this graph . . .	because . . .
$q(x) = 2(x-4)^2 + 1$	Graph A	Sample response: The vertex is located at $(4, 1)$. The parabola opens upward.
$h(x) = (x + 1)^2 - 4$	Graph B	Sample response: The vertex is located at $(-1, -4)$. The parabola opens upward.
$g(x) = -(x - 4)^2 + 1$	Graph C	Sample response: The vertex is located at $(4, 1)$. The parabola opens downward.

874 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Distribute the pre-cut cards from the Activity 2 PDF to each student pair. Read and review the directions aloud. Conduct the ***Card Sort*** routine. If time is limited, have students record the card label instead of sketching the graph.

2 Monitor

Help students get started by prompting them to first determine whether the functions represent a parabola that opens upward or downward.

Look for points of confusion:

- **Struggling to identify the vertex on a graph.** Prompt students to identify any intercepts they see on the given graphs.
- **Struggling to determine whether one graph is "steeper" than another.** Have students trace and compare graphs using tracing paper.

Look for *productive* strategies:

- Writing the vertex for each of the given functions.
- Substituting the values of x into the given functions.
- Using precise language and mathematical terms to explain why each equation and graph is a match.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with a subset of the cards to match. After they have completed their initial matches, provide them with the remaining cards.

Extension: *Math Enrichment*

Have students sort the cards into different categories, such as:

- Functions whose graphs open upward vs. functions whose graphs open downward.
- Functions that are shifted up from $y = x^2$ vs. functions that are shifted down.
- Functions that are shifted to the left from $y = x^2$ vs. functions that are shifted to the right.

Math Language Development

MLR8: Discussion Supports—Press for Details

Provide Partner A with the Anchor Chart PDF, *Sentence Stems, Matching Prompts* to initiate partner discussion. Prompt Partner B to ask questions to determine additional details by referring to specific features of the graphs and equations, such as *vertex*, *wider*, *narrower*, and the *direction of the opening*. Students should switch roles after each match.

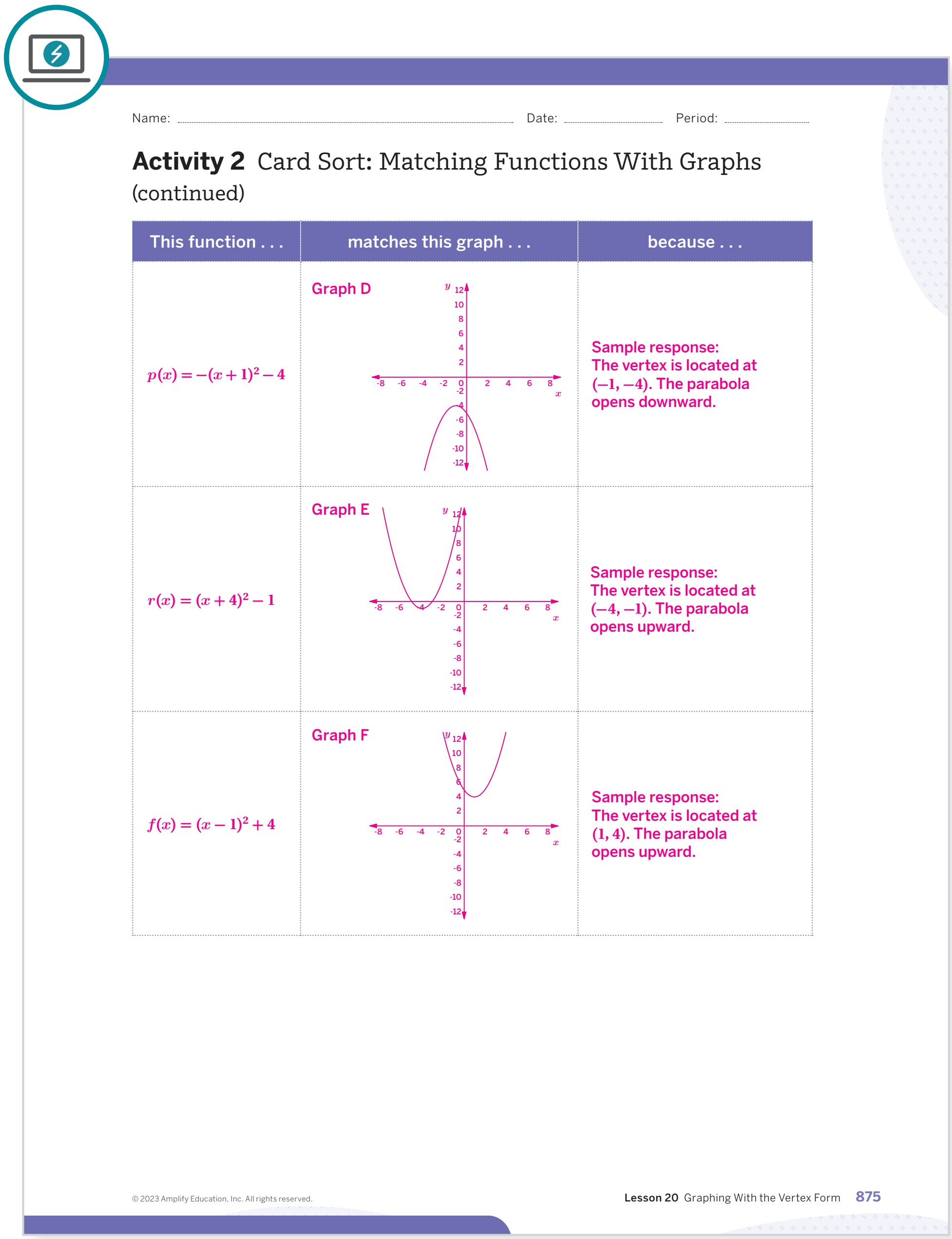
English Learners

Have students annotate their graphs with the terms *vertex*, *opens upward*, and *opens downward*. Display a sample graph already annotated for them to use as a reference.

Activity 2 Card Sort: Matching Functions With Graphs (continued)

Pairs | 15 min

Students match quadratic functions given in vertex form to their corresponding graphs to build fluency in connecting representations: functions and graphs.



Name: ______________________ Date: __________ Period: __________

Activity 2 Card Sort: Matching Functions With Graphs (continued)

This function . . .	matches this graph . . .	because . . .
$p(x) = -(x+1)^2 - 4$	Graph D	Sample response: The vertex is located at $(-1, -4)$. The parabola opens downward.
$r(x) = (x+4)^2 - 1$	Graph E	Sample response: The vertex is located at $(-4, -1)$. The parabola opens upward.
$f(x) = (x-1)^2 + 4$	Graph F	Sample response: The vertex is located at $(1, 4)$. The parabola opens upward.

3 Connect

Have students share aspects of the functions and graphs they found helpful when forming matches.

Highlight that when h is subtracted from x, $(x-h)^2$, the x-coordinate of the vertex is positive. When h is added to x, $(x+h)^2$, the x-coordinate of the vertex is negative. When the constant term k is positive, the y-coordinate of the vertex is also positive. When k is negative, the y-coordinate of the vertex is negative. When a is negative, the graph of the function opens downward and the vertex is a maximum. When a is positive, the graph opens upward and the vertex is a minimum. The larger the value of $|a|$, the narrower the opening of the graph.

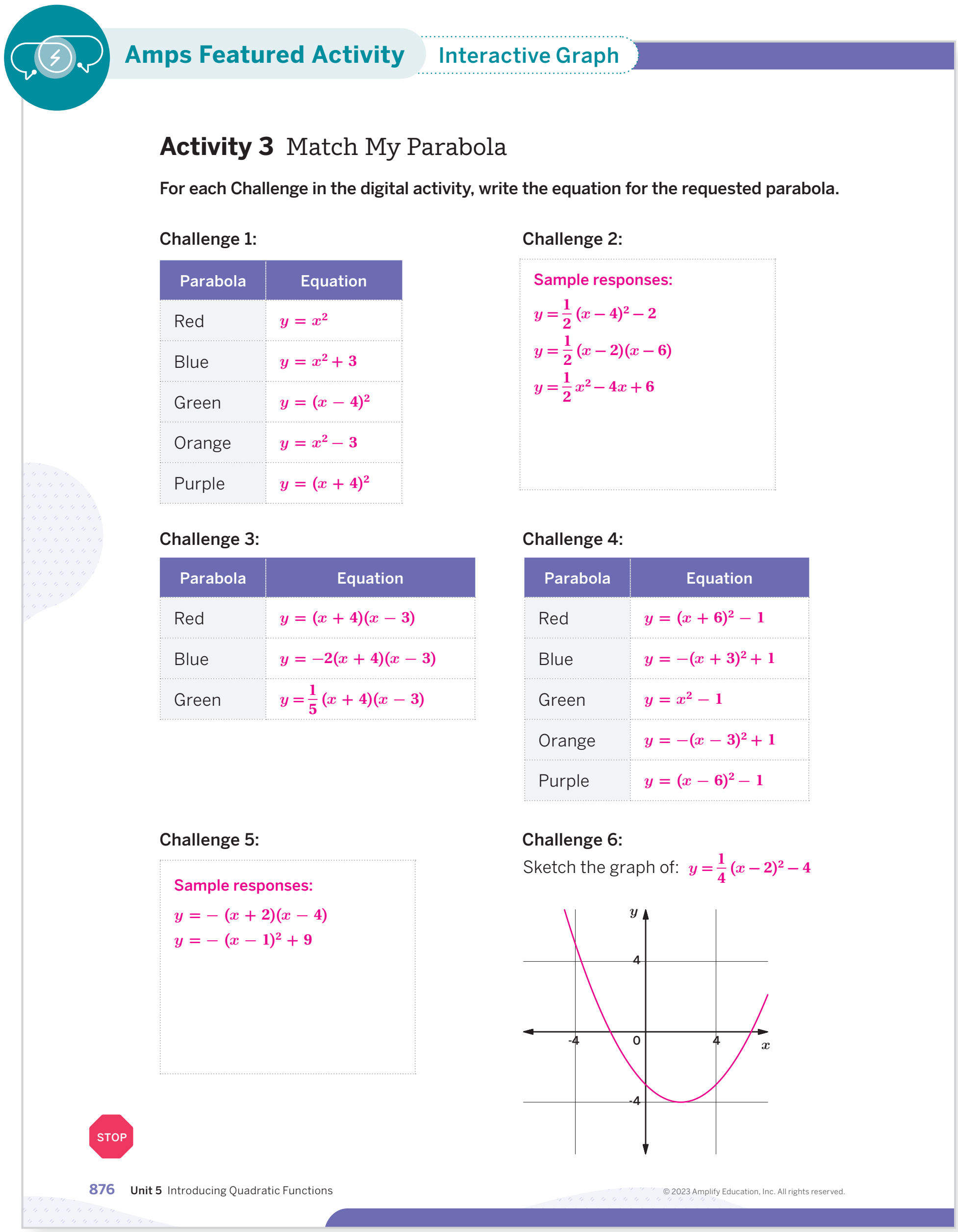
Optional

Independent | 10 min

Activity 3 Match My Parabola

Students work through a series of challenges writing quadratic equations to match a set of given parameters.

Amps Featured Activity Interactive Graph



Activity 3 Match My Parabola

For each Challenge in the digital activity, write the equation for the requested parabola.

Challenge 1:

Parabola	Equation
Red	$y = x^2$
Blue	$y = x^2 + 3$
Green	$y = (x - 4)^2$
Orange	$y = x^2 - 3$
Purple	$y = (x + 4)^2$

Challenge 2:

Sample responses:

$y = \frac{1}{2}(x - 4)^2 - 2$

$y = \frac{1}{2}(x - 2)(x - 6)$

$y = \frac{1}{2}x^2 - 4x + 6$

Challenge 3:

Parabola	Equation
Red	$y = (x + 4)(x - 3)$
Blue	$y = -2(x + 4)(x - 3)$
Green	$y = \frac{1}{5}(x + 4)(x - 3)$

Challenge 4:

Parabola	Equation
Red	$y = (x + 6)^2 - 1$
Blue	$y = -(x + 3)^2 + 1$
Green	$y = x^2 - 1$
Orange	$y = -(x - 3)^2 + 1$
Purple	$y = (x - 6)^2 - 1$

Challenge 5:

Sample responses:

$y = -(x + 2)(x - 4)$

$y = -(x - 1)^2 + 9$

Challenge 6:

Sketch the graph of: $y = \frac{1}{4}(x - 2)^2 - 4$

876 Unit 5 Introducing Quadratic Functions

1 Launch

Write one of the quadratic equations from Activity 2 on the board. Ask, "How does this function compare to the graph of $y = x^2$?" Elicit from students that the number inside the parentheses shifts the graph horizontally and the number on the outside shifts it vertically.

2 Monitor

Help students get started on Challenge 1 by prompting them to begin with $y = x^2$.

Look for points of confusion:

- **Writing a quadratic equation, but not knowing what to write for a in Challenge 2.** Ask students how the equation needs to change to make the graph wider.
- **Not knowing what form to use in Challenge 4.** Prompt students to begin with the green graph and to refer back to Challenge 1.

Look for productive strategies:

- Using vertex form when the vertex is given.
- Using factored form when the horizontal intercepts are given.

3 Connect

Display each challenge, one at a time, allowing students to share their strategies for writing equations. Highlight the equations written by students who used productive strategies.

Have students share how they decided on which form of a quadratic equation to use, paying particular attention to students who may have written different equations for the same parabola.

Highlight Challenge 6 by having students state what each part of the equation tells them about its graph. Then display the work of students who were able to accurately sketch it.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can write quadratic functions in different forms, so that they pass through specific coordinates of an interactive graph.

Accessibility: *Activate Prior Knowledge*

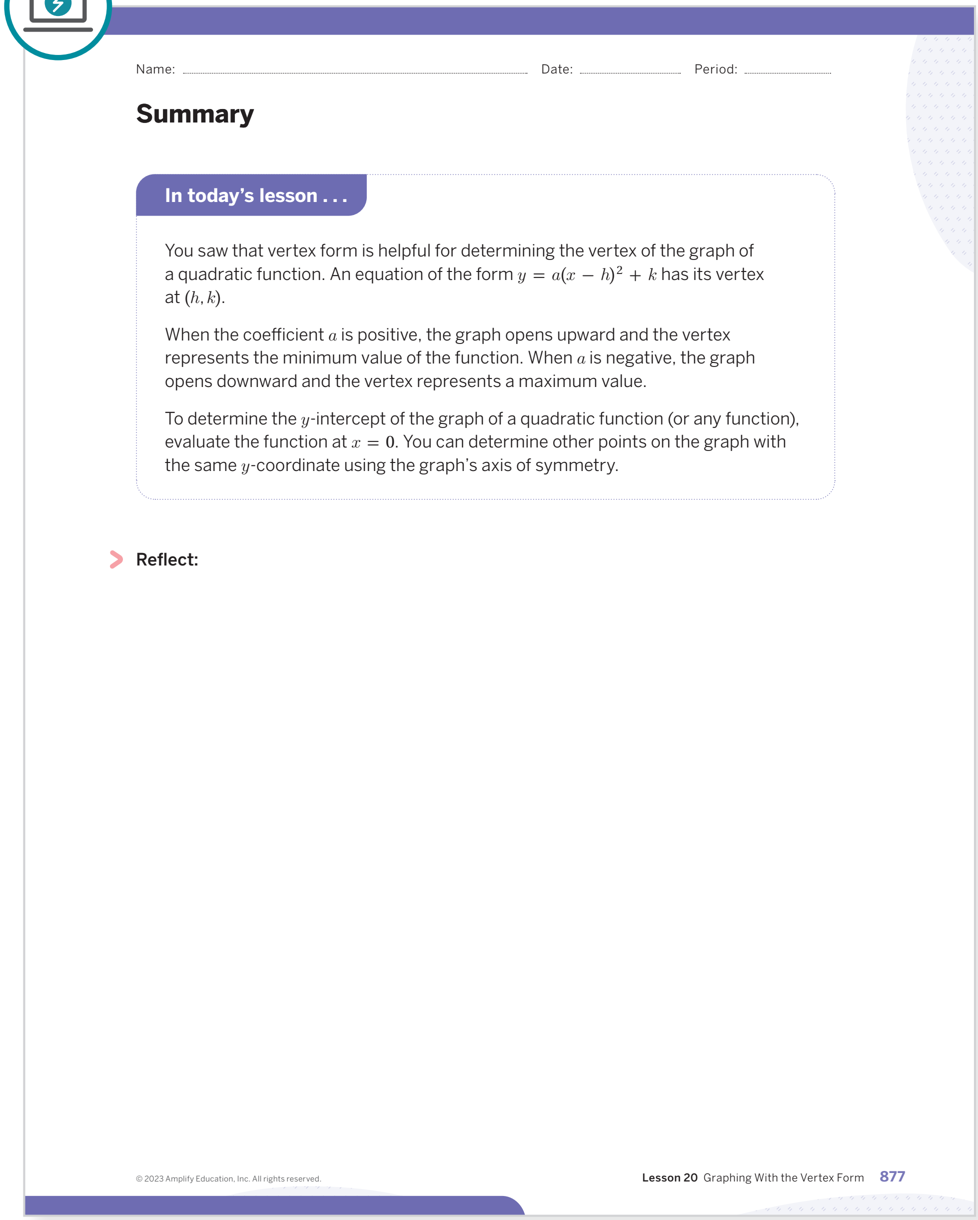
Have students help you add the three forms of quadratic functions to the class display, if they are not already added. Include examples and the information each form provides. Have students reference the display during the activity. For example:

Forms of quadratic functions		
Standard form	**Factored form**	**Vertex form**
$f(x) = ax^2 + bx + c$ Indicates: y-intercept c	$f(x) = (x - m)(x - n)$ Indicates: x-intercepts m and n (also called zeros)	$f(x) = a(x - h)^2 + k$ Indicates: vertex (h, k)

Whole Class | 5 min

Summary

Review and synthesize how to graph a quadratic function, given in vertex form.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that vertex form is helpful for determining the vertex of the graph of a quadratic function. An equation of the form $y = a(x - h)^2 + k$ has its vertex at (h, k).

When the coefficient a is positive, the graph opens upward and the vertex represents the minimum value of the function. When a is negative, the graph opens downward and the vertex represents a maximum value.

To determine the y-intercept of the graph of a quadratic function (or any function), evaluate the function at $x = 0$. You can determine other points on the graph with the same y-coordinate using the graph's axis of symmetry.

Reflect:

Lesson 20 Graphing With the Vertex Form 877

Synthesize

Display the function $f(x) = -2(x - 7)^2 + 5$.

Ask:

- "What is the vertex of the graph?" $(7, 5)$
- "Is it a maximum or a minimum? How do you know?" Maximum; the parabola opens downward because the coefficient of the x^2 term is negative.
- "How can you verify that 5 is the y-coordinate of the vertex?" Evaluate the function at $x = 7$.
- "Is knowing the location of the vertex and whether the graph opens upward or downward sufficient information for sketching the graph? If not, what else is needed?" I still need to know how "wide" or "narrow" the opening of the graph is, so it helps to know another point.

Highlight that the vertex, found from the vertex form of a quadratic function, can represent the maximum or minimum value of the function. When the x^2 term has a positive coefficient, the graph opens upward and the vertex is the minimum value. When it has a negative coefficient, the graph opens downward and the vertex is the maximum value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

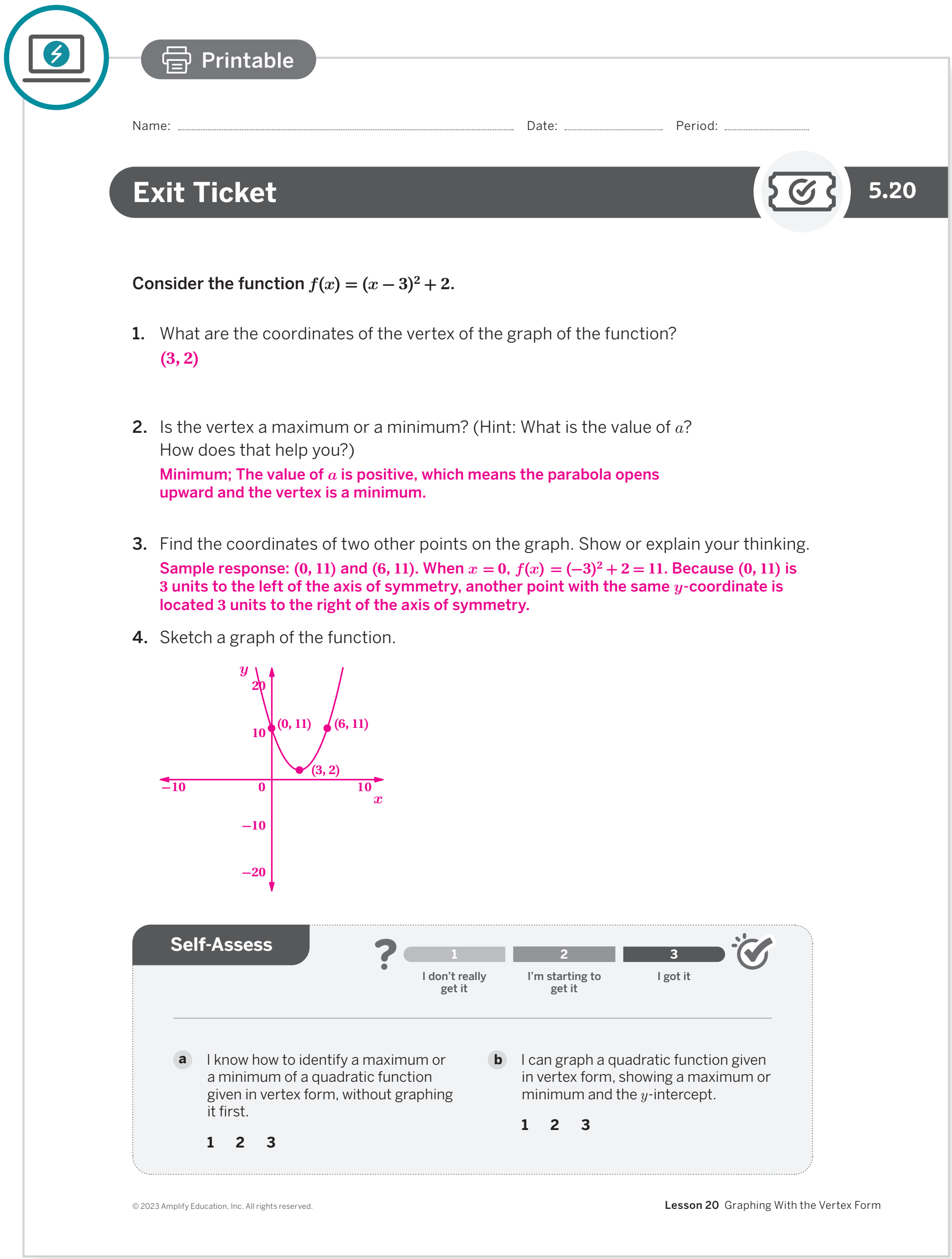
- "What other key features can you determine from a quadratic function in vertex form?"
- "How could you determine the minimum or maximum of a quadratic function that is not in vertex form?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by graphing a quadratic function given in vertex form, and determining whether the vertex is a maximum or minimum.

Printable



Name: Date: Period:

Exit Ticket 5.20

Consider the function $f(x) = (x - 3)^2 + 2$.

1. What are the coordinates of the vertex of the graph of the function?
   (3, 2)

2. Is the vertex a maximum or a minimum? (Hint: What is the value of a? How does that help you?)
   Minimum; The value of a is positive, which means the parabola opens upward and the vertex is a minimum.

3. Find the coordinates of two other points on the graph. Show or explain your thinking.
   Sample response: (0, 11) and (6, 11). When $x = 0$, $f(x) = (-3)^2 + 2 = 11$. Because (0, 11) is 3 units to the left of the axis of symmetry, another point with the same y-coordinate is located 3 units to the right of the axis of symmetry.

4. Sketch a graph of the function.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I know how to identify a maximum or a minimum of a quadratic function given in vertex form, without graphing it first.
1 2 3

b I can graph a quadratic function given in vertex form, showing a maximum or minimum and the y-intercept.
1 2 3

Lesson 20 Graphing With the Vertex Form

Success looks like . . .

- **Goal:** Graphing a quadratic function given in vertex form, showing a maximum or minimum and the y-intercept.
  - » Sketching a quadratic function in Problem 4.
- **Goal:** Knowing how to find a maximum or minimum of a quadratic function given in vertex form, without first graphing it.
  - » Determine whether the vertex is a maximum or a minimum in Problem 2.

Suggested next steps

If students are unable to determine the coordinates of the vertex in Problem 1, consider:

- Reviewing vertex form.
- Assigning Practice Problems 1 and 2.

If students are unable to determine whether the vertex is a maximum or a minimum in Problem 2, consider:

- Revisiting Activity 1, Problem 3.
- Assigning Practice Problem 6.

If students are unable to find two points on the graph in Problem 3, consider:

- Revisiting Activity 1, Problems 1 and 2.

If students are unable to sketch a graph in Problem 4, consider:

- Reviewing how to sketch the graph of a function given in vertex form.
- Revisiting Activity 2.
- Assigning Practice Problems 2 and 4.

Professional Learning

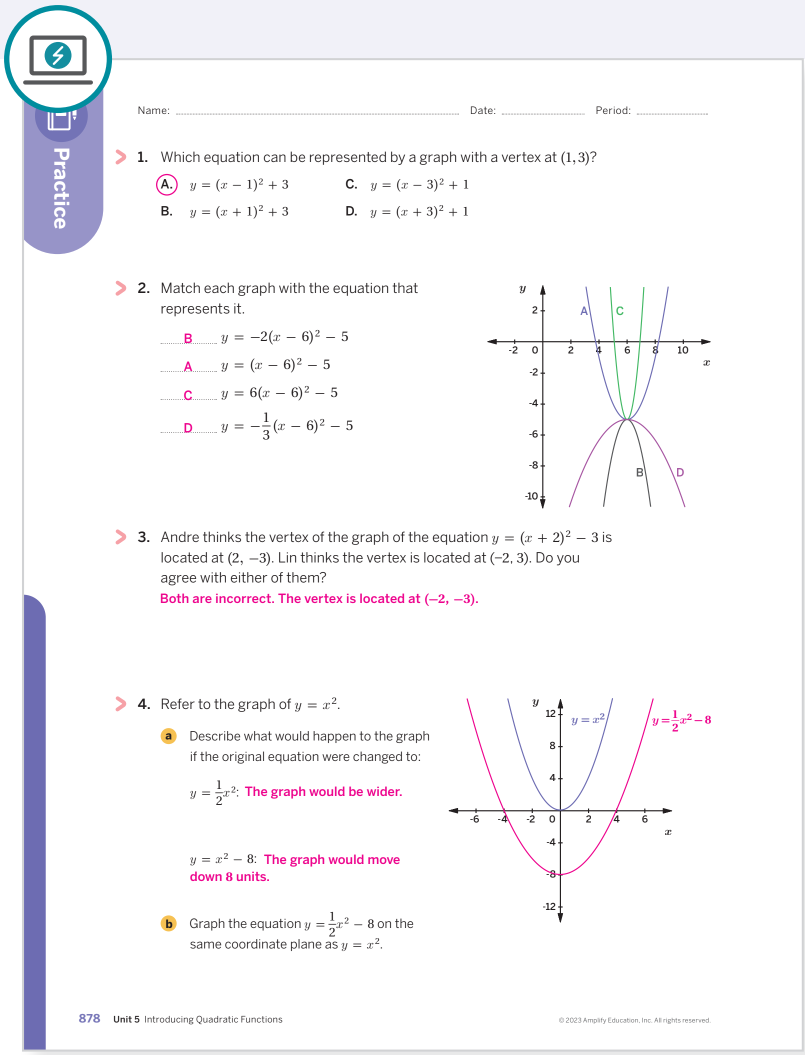
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- How did students attend to precision today? How are you helping students become aware of how they are progressing in this area?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Which equation can be represented by a graph with a vertex at $(1, 3)$?

   [✓ A.] $y = (x - 1)^2 + 3$
   B. $y = (x + 1)^2 + 3$
   C. $y = (x - 3)^2 + 1$
   D. $y = (x + 3)^2 + 1$

2. Match each graph with the equation that represents it.

   B $y = -2(x - 6)^2 - 5$
   A $y = (x - 6)^2 - 5$
   C $y = 6(x - 6)^2 - 5$
   D $y = -\frac{1}{3}(x - 6)^2 - 5$

3. Andre thinks the vertex of the graph of the equation $y = (x + 2)^2 - 3$ is located at (2, –3). Lin thinks the vertex is located at (–2, 3). Do you agree with either of them?

   Both are incorrect. The vertex is located at (–2, –3).

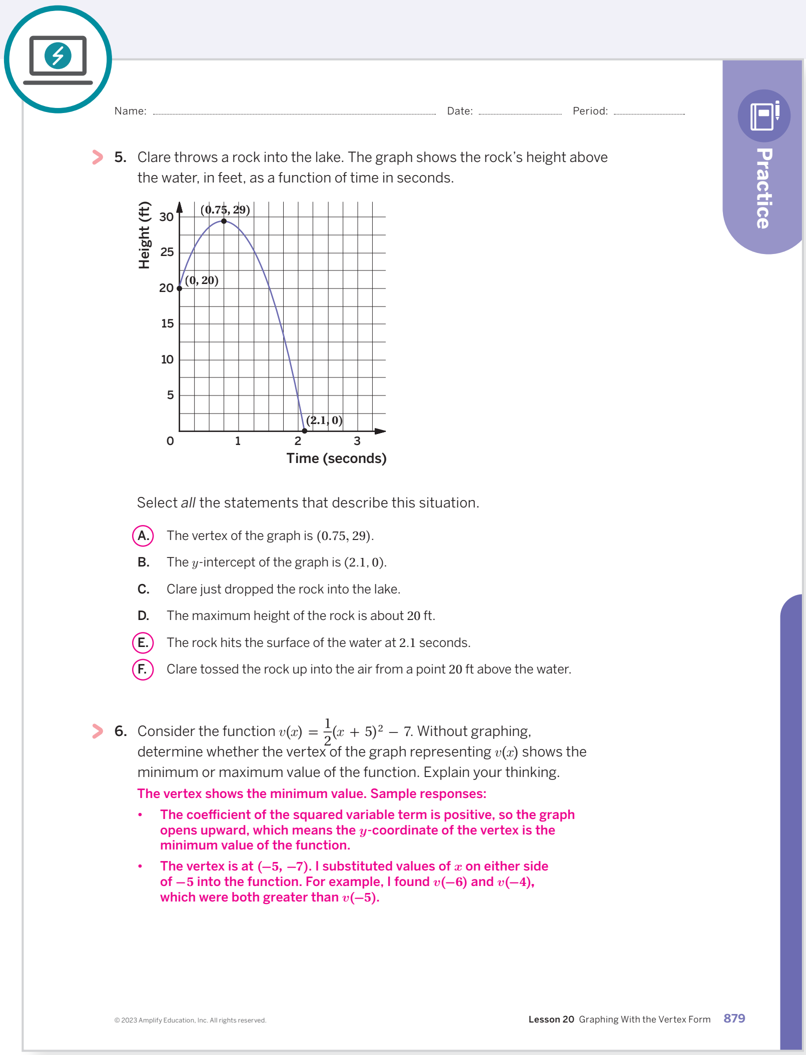
4. Refer to the graph of $y = x^2$.

   a. Describe what would happen to the graph if the original equation were changed to:

   $y = \frac{1}{2}x^2$: **The graph would be wider.**

   $y = x^2 - 8$: **The graph would move down 8 units.**

   b. Graph the equation $y = \frac{1}{2}x^2 - 8$ on the same coordinate plane as $y = x^2$.



Practice

Name: ________ Date: ________ Period: ________

5. Clare throws a rock into the lake. The graph shows the rock's height above the water, in feet, as a function of time in seconds.

   Select *all* the statements that describe this situation.

   [✓ A.] The vertex of the graph is $(0.75, 29)$.
   B. The y-intercept of the graph is $(2.1, 0)$.
   C. Clare just dropped the rock into the lake.
   D. The maximum height of the rock is about 20 ft.
   [✓ E.] The rock hits the surface of the water at 2.1 seconds.
   [✓ F.] Clare tossed the rock up into the air from a point 20 ft above the water.

6. Consider the function $v(x) = \frac{1}{2}(x + 5)^2 - 7$. Without graphing, determine whether the vertex of the graph representing $v(x)$ shows the minimum or maximum value of the function. Explain your thinking.

   The vertex shows the minimum value. Sample responses:

   - **The coefficient of the squared variable term is positive, so the graph opens upward, which means the y-coordinate of the vertex is the minimum value of the function.**
   - **The vertex is at (–5, –7). I substituted values of x on either side of –5 into the function. For example, I found $v(-6)$ and $v(-4)$, which were both greater than $v(-5)$.**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 16	2
	5	Unit 5 Lesson 18	2
Formative ⚡	6	Unit 5 Lesson 21	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

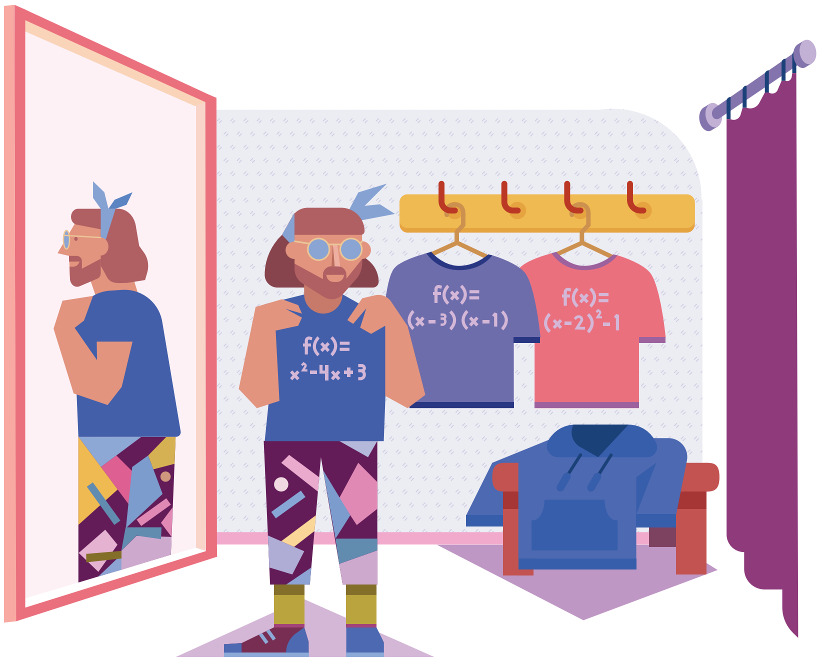
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 21**

Changing Parameters and Choosing a Form

Let's change the parameters of quadratic functions and examine the usefulness of different forms.



Focus

Goals

1. **Language Goal:** Describe how changing a number in vertex form changes the shape of a graph. **(Speaking and Listening, Writing)**
2. Create a quadratic equation by shifting the quadratic function horizontally and vertically.
3. **Language Goal:** Choose one of the forms of quadratic functions to use to create an equation of a quadratic graph (factored form, standard form, vertex form) based on information given in the graph. **(Reading and Writing)**

Coherence

Today

Students examine the effects of changing parameters of a quadratic function, create a function after $y = x^2$ is shifted, and determine which form of quadratic functions would be best to use to create a function from key features in a graph.

Previously

In the previous several lessons, students examined how the forms of quadratic functions reveal different key features, and used these forms to graph quadratic functions.

Coming Soon

In Lesson 22, students will create equations of quadratic functions in vertex form, after the vertex is changed.

Rigor

- Students strengthen their **fluency** in creating an equation of quadratic graphs.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- graphing technology

Math Language Development

Review words

- *factored form*
- *horizontal intercept*
- *standard form*
- *vertex*
- *vertex form*
- *vertical intercept*

Amps | Featured Activity

Activity 1
Interactive Graph

Students explore the effects of changing parameters on the graph of a quadratic function, written in vertex form.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 3, students might lack the motivation to apply what they know about the key features of a graph to write the equation. Ask students to monitor their own progress by recognizing all that they do know about the graphs and acknowledging the gains that they have made during this unit.

Modifications to Pacing

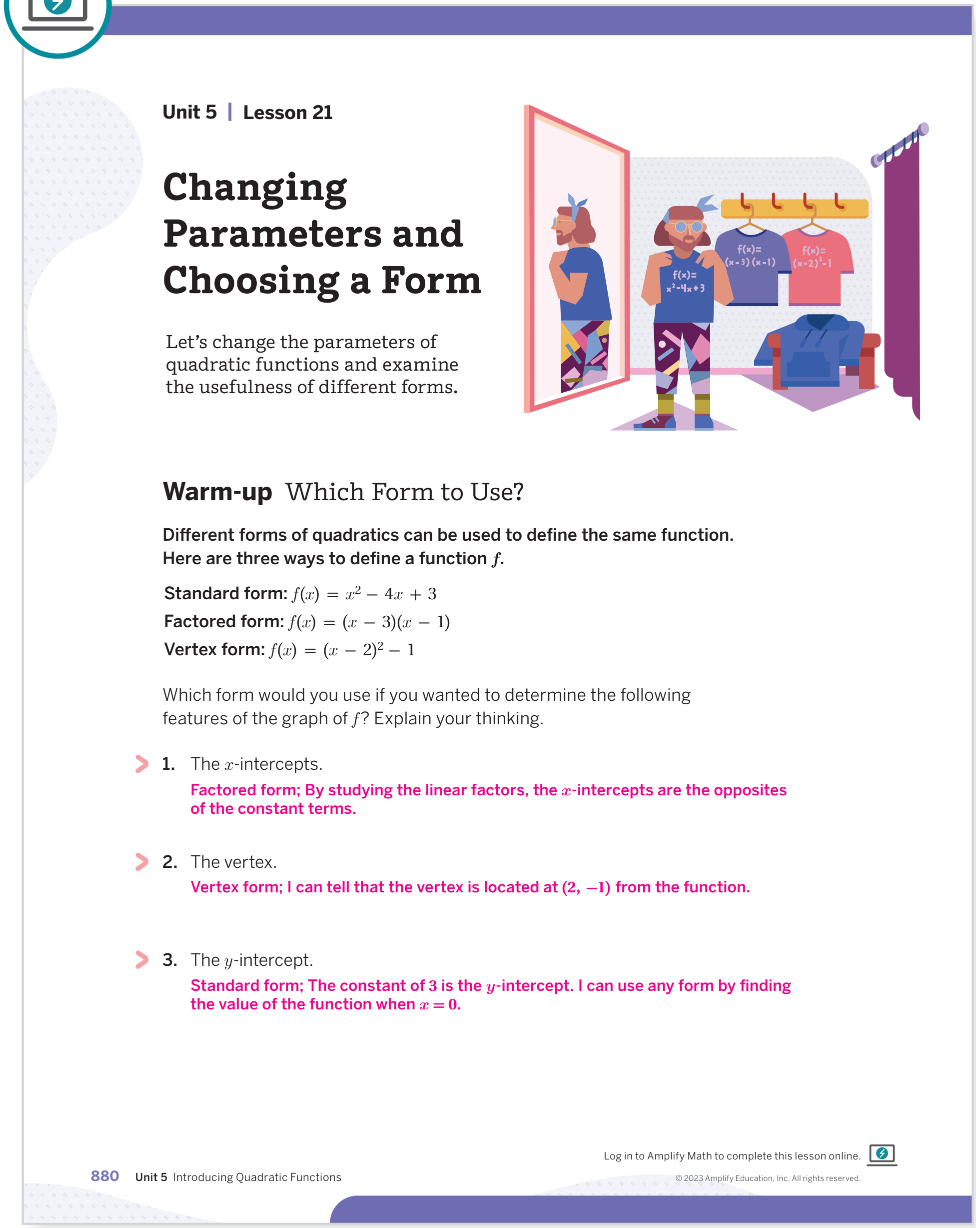
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 3, have students only complete the first three rows of the table.
- In **Activity 2**, Problem 2 may be omitted.

Independent | 5 min

Warm-up Which Form to Use?

Students choose which form of a quadratic to use, based on the insights each form reveals about its graph to prepare them for changing parameters later in this lesson.



Unit 5 | Lesson 21

Changing Parameters and Choosing a Form

Let's change the parameters of quadratic functions and examine the usefulness of different forms.

Warm-up Which Form to Use?

Different forms of quadratics can be used to define the same function. Here are three ways to define a function f.

Standard form: $f(x) = x^2 - 4x + 3$
Factored form: $f(x) = (x - 3)(x - 1)$
Vertex form: $f(x) = (x - 2)^2 - 1$

Which form would you use if you wanted to determine the following features of the graph of f? Explain your thinking.

1. The x-intercepts.
   Factored form; By studying the linear factors, the x-intercepts are the opposites of the constant terms.
2. The vertex.
   Vertex form; I can tell that the vertex is located at $(2, -1)$ from the function.
3. The y-intercept.
   Standard form; The constant of 3 is the y-intercept. I can use any form by finding the value of the function when $x = 0$.

Log in to Amplify Math to complete this lesson online.

880 **Unit 5** Introducing Quadratic Functions

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by activating prior knowledge and asking them to identify key features from each form.

Look for points of confusion:

- **Identifying -1 from the function in vertex form as the y-intercept.** Have students check if this is correct by evaluating the function for $x = 0$, and then revising their selection.

Look for productive strategies:

- Listing all key features that can be found for each function.
- Using precise language and mathematical terms in their explanations.

3 Connect

Have individual students share their responses and explanations.

Highlight that standard form allows students to determine the vertical y-intercept quickly. Students can calculate the y-intercept in any form by evaluating the function at $x = 0$.

Ask, "If you did not have vertex form, which function would you choose to determine the vertex? Why?" Factored form; The midpoint of the zeros is the x-coordinate of the vertex. I can substitute this value into the function to determine the y-coordinate of the vertex.

Power-up

To power up students' ability to identify whether a vertex is a maximum or a minimum from an equation of a quadratic function, have students complete:

Determine the coordinates of the vertex of the function and whether it is a maximum or a minimum. Be prepared to explain your thinking.

- $f(x) = (x - 1)^2 + 4$ $(1, 4)$; Minimum because the coefficient of the x^2 term will be positive. The parabola opens upward.
- $g(x) = -(x + 0.5)^2 - 6$ $(-0.5, -6)$; Maximum because the coefficient of the x^2 term will be negative. The parabola opens downward.

Use: Before the Warm-up

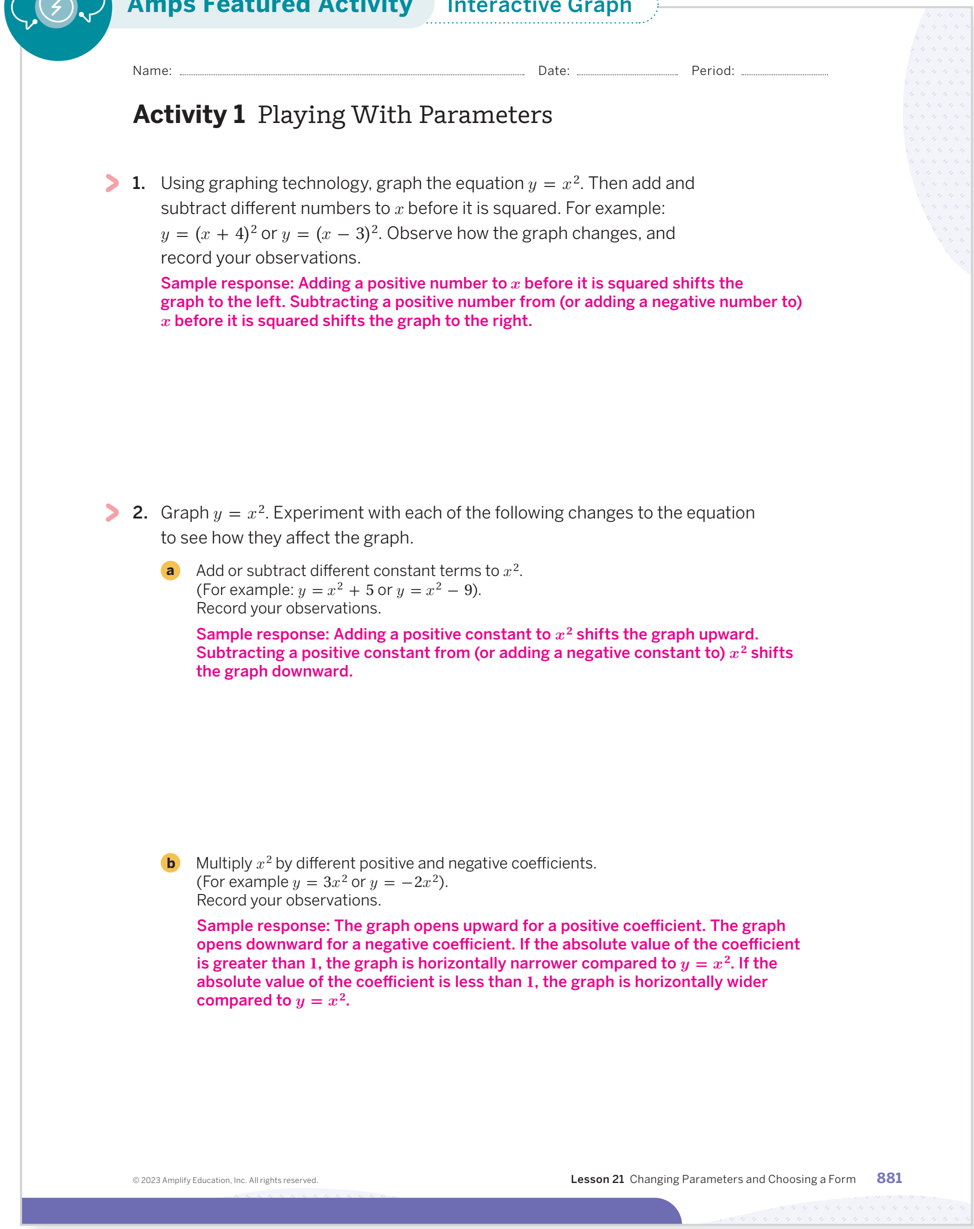
Informed by: Performance on Lesson 20, Practice Problem 6

Pairs | 15 min

Activity 1 Playing With Parameters

Students use technology to experiment with each parameter of quadratic equations, written in vertex form, and study the effects on the graphs.

Amps Featured Activity Interactive Graph



Name: ________ Date: ________ Period: ________

Activity 1 Playing With Parameters

1. Using graphing technology, graph the equation $y = x^2$. Then add and subtract different numbers to x before it is squared. For example: $y = (x + 4)^2$ or $y = (x - 3)^2$. Observe how the graph changes, and record your observations.

   Sample response: Adding a positive number to x before it is squared shifts the graph to the left. Subtracting a positive number from (or adding a negative number to) x before it is squared shifts the graph to the right.

2. Graph $y = x^2$. Experiment with each of the following changes to the equation to see how they affect the graph.

   a. Add or subtract different constant terms to x^2.
   (For example: $y = x^2 + 5$ or $y = x^2 - 9$).
   Record your observations.

   Sample response: Adding a positive constant to x^2 shifts the graph upward. Subtracting a positive constant from (or adding a negative constant to) x^2 shifts the graph downward.

   b. Multiply x^2 by different positive and negative coefficients.
   (For example $y = 3x^2$ or $y = -2x^2$).
   Record your observations.

   Sample response: The graph opens upward for a positive coefficient. The graph opens downward for a negative coefficient. If the absolute value of the coefficient is greater than 1, the graph is horizontally narrower compared to $y = x^2$. If the absolute value of the coefficient is less than 1, the graph is horizontally wider compared to $y = x^2$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Changing Parameters and Choosing a Form 881

1 Launch

Have students complete Problems 1 and 2 independently before sharing their responses with a partner. Provide access to graphing technology.

2 Monitor

Help students get started by saying, "Try changing the values you use by increasing and decreasing the numbers by 1."

Look for points of confusion:

- **Using the value added to x as the x-coordinate of the vertex.** Have students verify their coordinates of the vertex with a calculator.
- **Reversing horizontally wider and narrower.** Have students graph the equation $y = x^2$ to help more easily compare the graphs.

Look for productive strategies:

- Using the equal increments to compare values in Problems 1 and 2 to help build conclusions.
- Checking to see if their conclusions for Problems 1 and 2 are correct by using larger values.
- Rewriting squared expressions to help reveal the x-coordinate of the vertex.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can explore the effects of changing parameters on the graph of a quadratic function, written in vertex form.

Extension: *Math Enrichment*

Have students write the equation of a function whose graph is wider horizontally than the graph of $y = x^2$, shifted down 3 units, shifted to the left 2 units, and opens downward.
Sample response: $y = -0.25\,(x + 2)^2 - 3$

Math Language Development

MLR7: Compare and Connect

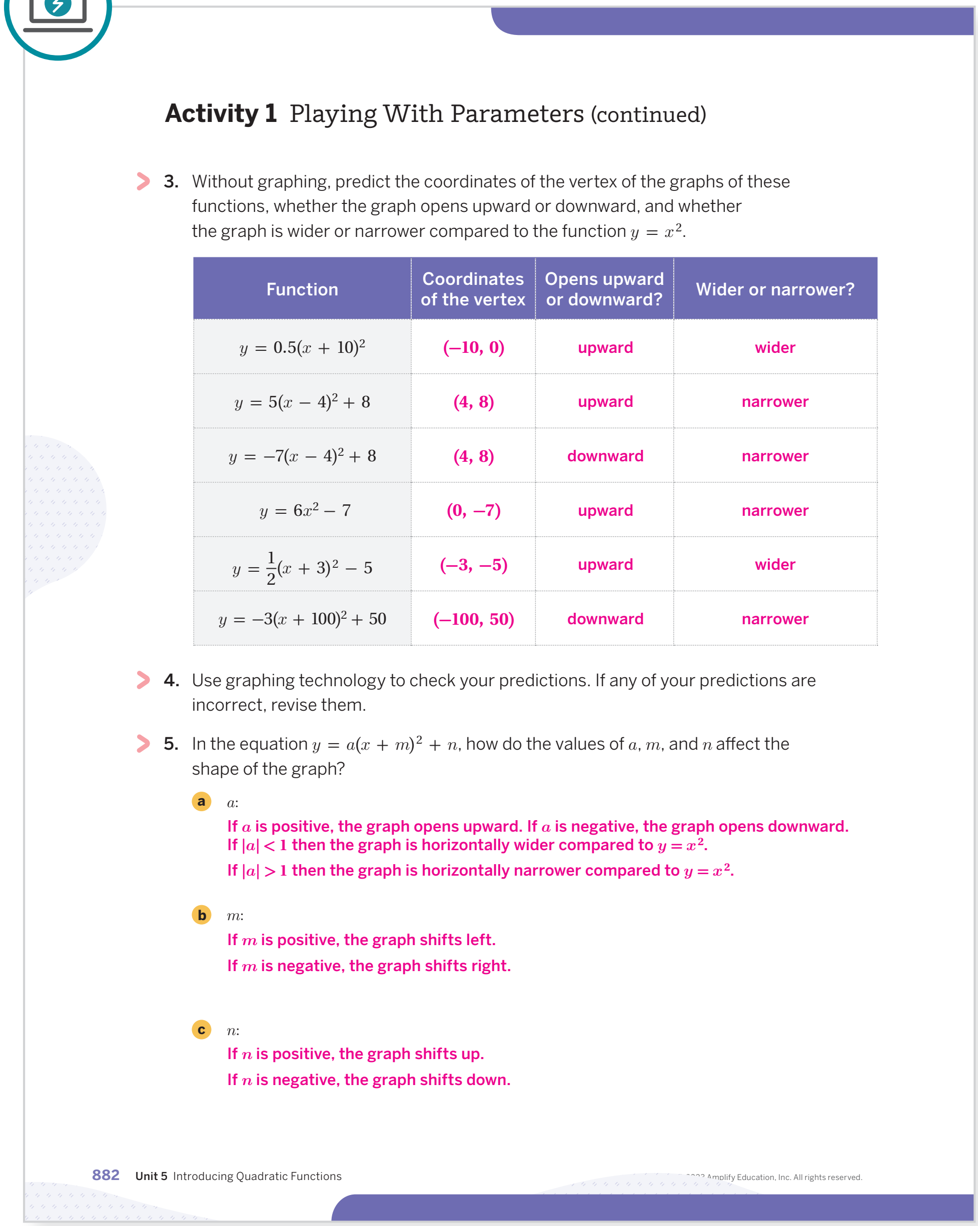
While students complete Problems 1 and 2, display these questions that partners can ask each other to connect the structure of their equations and their corresponding graphs:
"How can you adjust the equation of a quadratic function so that its graph . . .
- "Shifts up? Shifts down?"
- "Shifts to the left? To the right?"
- "Reflects across the x-axis?"
- "Becomes wider? Becomes narrower?"

English Learners

Before students complete Problem 3, clarify the meaning of the terms *wider* and *narrower* by sketching examples of graphs that are wider than $y = x^2$ and examples of graphs that are narrower than $y = x^2$.

Activity 1 Playing With Parameters (continued)

Students use technology to experiment with each parameter of quadratic equations, written in vertex form, and study the effects on the graphs.



Activity 1 Playing With Parameters (continued)

3. Without graphing, predict the coordinates of the vertex of the graphs of these functions, whether the graph opens upward or downward, and whether the graph is wider or narrower compared to the function $y = x^2$.

Function	Coordinates of the vertex	Opens upward or downward?	Wider or narrower?
$y = 0.5(x + 10)^2$	(–10, 0)	upward	wider
$y = 5(x - 4)^2 + 8$	(4, 8)	upward	narrower
$y = -7(x - 4)^2 + 8$	(4, 8)	downward	narrower
$y = 6x^2 - 7$	(0, –7)	upward	narrower
$y = \frac{1}{2}(x + 3)^2 - 5$	(–3, –5)	upward	wider
$y = -3(x + 100)^2 + 50$	(–100, 50)	downward	narrower

4. Use graphing technology to check your predictions. If any of your predictions are incorrect, revise them.

5. In the equation $y = a(x + m)^2 + n$, how do the values of a, m, and n affect the shape of the graph?

   a. a:
   If a is positive, the graph opens upward. If a is negative, the graph opens downward.
   If $|a| < 1$ then the graph is horizontally wider compared to $y = x^2$.
   If $|a| > 1$ then the graph is horizontally narrower compared to $y = x^2$.

   b. m:
   If m is positive, the graph shifts left.
   If m is negative, the graph shifts right.

   c. n:
   If n is positive, the graph shifts up.
   If n is negative, the graph shifts down.

882 **Unit 5** Introducing Quadratic Functions

3 Connect

Display the incomplete table.

Have individual students share their responses and strategies used to complete the table. Record their responses in the table.

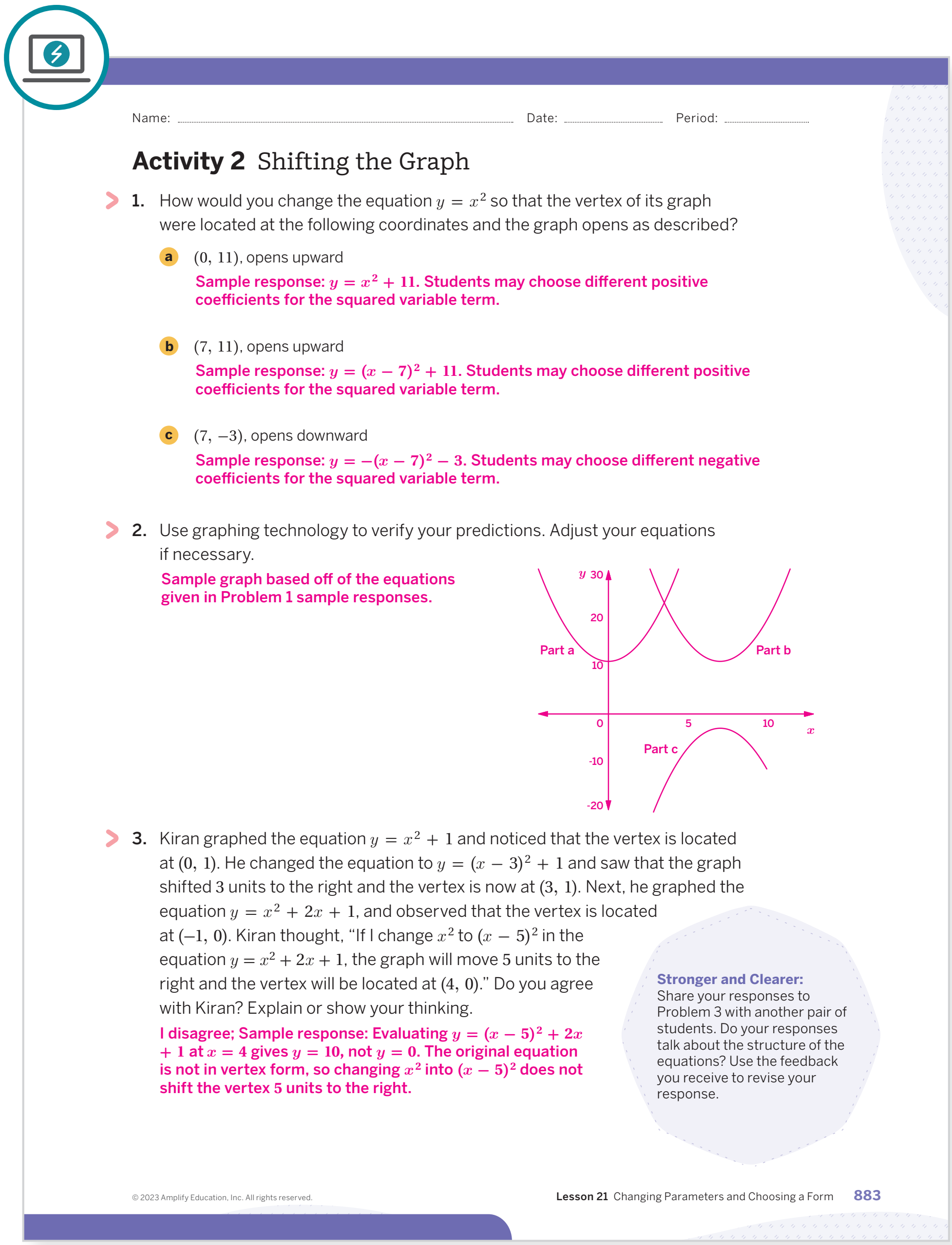
Highlight that most of the squared terms contain a sum or a difference of x and a number. For the example where the squared term is not a sum or a difference, $k(x) = 6x^2 - 7$, students can think of it as containing a sum of x and 0. The squared term's coefficient, which can be positive or negative, determines whether the graph opens upward or downward, and if it is horizontally wider or narrower compared to $f(x) = x^2$.

Ask, "How do you know whether the graph is horizontally wider or narrower compared to $f(x) = x^2$?" Look at the absolute value of the coefficient on the squared variable term. If it is less than 1, then the graph is horizontally wider. If it is greater than 1, then the graph is horizontally narrower.

Pairs | 10 min

Activity 2 Shifting the Graph

Students use the structure of a quadratic equation written in vertex form to translate the graph it represents.



Name: ______ Date: ______ Period: ______

Activity 2 Shifting the Graph

1. How would you change the equation $y = x^2$ so that the vertex of its graph were located at the following coordinates and the graph opens as described?

   a. $(0, 11)$, opens upward

   Sample response: $y = x^2 + 11$. Students may choose different positive coefficients for the squared variable term.

   b. $(7, 11)$, opens upward

   Sample response: $y = (x - 7)^2 + 11$. Students may choose different positive coefficients for the squared variable term.

   c. $(7, -3)$, opens downward

   Sample response: $y = -(x - 7)^2 - 3$. Students may choose different negative coefficients for the squared variable term.

2. Use graphing technology to verify your predictions. Adjust your equations if necessary.

   Sample graph based off of the equations given in Problem 1 sample responses.

3. Kiran graphed the equation $y = x^2 + 1$ and noticed that the vertex is located at $(0, 1)$. He changed the equation to $y = (x - 3)^2 + 1$ and saw that the graph shifted 3 units to the right and the vertex is now at $(3, 1)$. Next, he graphed the equation $y = x^2 + 2x + 1$, and observed that the vertex is located at $(-1, 0)$. Kiran thought, "If I change x^2 to $(x - 5)^2$ in the equation $y = x^2 + 2x + 1$, the graph will move 5 units to the right and the vertex will be located at $(4, 0)$." Do you agree with Kiran? Explain or show your thinking.

   I disagree; Sample response: Evaluating $y = (x - 5)^2 + 2x + 1$ at $x = 4$ gives $y = 10$, not $y = 0$. The original equation is not in vertex form, so changing x^2 into $(x - 5)^2$ does not shift the vertex 5 units to the right.

Stronger and Clearer: Share your responses to Problem 3 with another pair of students. Do your responses talk about the structure of the equations? Use the feedback you receive to revise your response.

Lesson 21 Changing Parameters and Choosing a Form 883

1 Launch

Have students complete Problem 1 independently before discussing their responses with a partner. Provide access to graphing technology after students have completed Problem 1.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "What does vertex form tell you about the graph of a quadratic equation?"

Look for points of confusion:

- **Reversing the sign of the y-coordinate of the vertex to create a quadratic equation in vertex form.** Highlight that the constant outside the squared variable term shifts the graph vertically.

Look for productive strategies:

- Leaving x^2 as is when the x-coordinate of the vertex is 0.
- Noting that Kiran's equation is in standard form.

3 Connect

Have individual students share their responses to Problem 1 and whether they agree with Kiran in Problem 3. Ask students to explain their thinking.

Ask, "What would the equation need to look like for Kiran to be correct?" The equation would need to be $y = (x - 5)^2 + 2(x - 5) + 1$.

Highlight that if 5 is subtracted from both x's in Kiran's problem, to result in the equation $y = (x - 5)^2 + 2(x - 5) + 1$, the graph does shift 5 units to the right.

Differentiated Support

Extension: *Math Enrichment*

Display the following equations and ask students to determine whether these equations are written in more than one of these forms: standard form, factored form, and vertex form. Have them explain their thinking.

$y = x^2$ Yes; this equation is written in standard form (without a linear term or a constant term) and it is also written in vertex form, where the vertex is located at $(0, 0)$.

$y = x^2 - 5$ Yes; this equation is written in standard form (without a linear term) and it is also written in vertex form, where the vertex is located at $(0, -5)$.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "Do the responses talk about the structure of the equations?"
- "Does it matter which form the equation is written in?"
- "Is the equation $y = x^2 + 1$ in vertex form? Why or why not?"

Have students revise their responses, as needed.

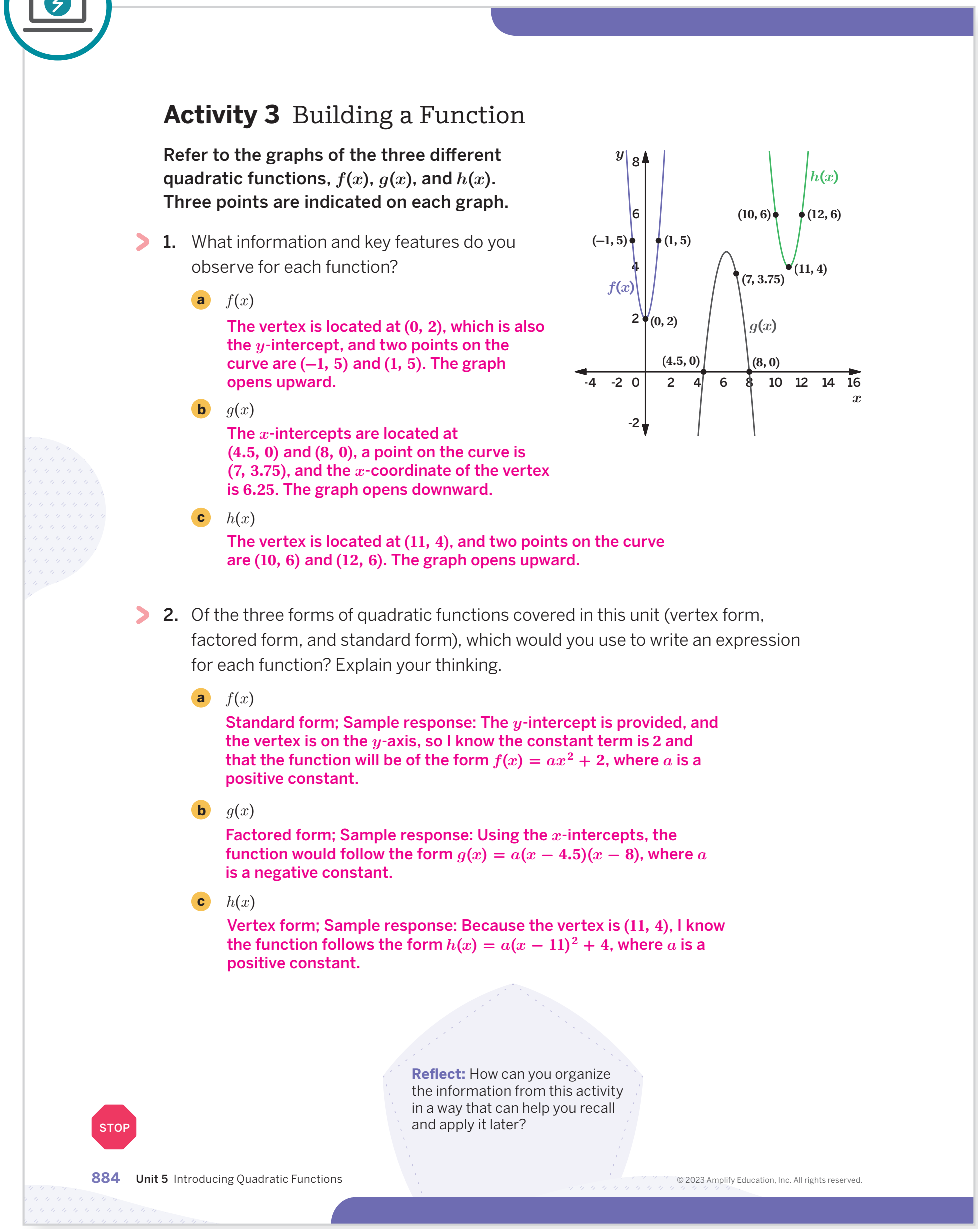
English Learners

Use intentional grouping so that students with different English language proficiency levels can interact and have an opportunity to listen to peers with more advanced proficiency.

Independent | 10 min

Activity 3 Building a Function

Students use key features to determine which form of a quadratic to use to build a quadratic function.



Activity 3 Building a Function

Refer to the graphs of the three different quadratic functions, $f(x)$, $g(x)$, and $h(x)$. Three points are indicated on each graph.

1. What information and key features do you observe for each function?
   - a $f(x)$

     The vertex is located at $(0, 2)$, which is also the y-intercept, and two points on the curve are $(–1, 5)$ and $(1, 5)$. The graph opens upward.
   - b $g(x)$

     The x-intercepts are located at $(4.5, 0)$ and $(8, 0)$, a point on the curve is $(7, 3.75)$, and the x-coordinate of the vertex is 6.25. The graph opens downward.
   - c $h(x)$

     The vertex is located at $(11, 4)$, and two points on the curve are $(10, 6)$ and $(12, 6)$. The graph opens upward.

2. Of the three forms of quadratic functions covered in this unit (vertex form, factored form, and standard form), which would you use to write an expression for each function? Explain your thinking.
   - a $f(x)$

     Standard form; Sample response: The y-intercept is provided, and the vertex is on the y-axis, so I know the constant term is 2 and that the function will be of the form $f(x) = ax^2 + 2$, where a is a positive constant.
   - b $g(x)$

     Factored form; Sample response: Using the x-intercepts, the function would follow the form $g(x) = a(x - 4.5)(x - 8)$, where a is a negative constant.
   - c $h(x)$

     Vertex form; Sample response: Because the vertex is $(11, 4)$, I know the function follows the form $h(x) = a(x - 11)^2 + 4$, where a is a positive constant.

Reflect: How can you organize the information from this activity in a way that can help you recall and apply it later?

884 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by activating their prior knowledge. Have them list the key features on each graph, and list the key features revealed by each form.

Look for points of confusion:

- **Estimating the vertex to be able to use vertex form.** Remind them to only utilize the points given for each curve to determine which form to use.

Look for productive strategies:

- Attempting to create functions for each graph as a method to determine which form to use.

3 Connect

Display the graphs.

Have individual students share which form they chose for each function, and their thinking behind their choices.

Highlight that for each form, there is a coefficient, called a, that students need to calculate to build the actual function for each graph. Students can use a point on the graph to substitute for x and y, and then solve for a.

Ask, "Is there a way you could use factored form for $f(x)$ and $h(x)$?" No, there are no horizontal intercepts for either function.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide an enlarged copy of the graph and suggest that students annotate the graph of each function with its key features: x-intercept(s) and y-intercept (if any) and the vertex. Ask:

- "Which graphs have two x-intercepts? Do any graphs have 1 or 0 x-intercepts?" Only $g(x)$ has two x-intercepts. None of the graphs have 1 x-intercept. Both $f(x)$ and $h(x)$ have 0 x-intercepts.
- "On this graph, $h(x)$ and $g(x)$ are not shown to intersect the y-axis. Does this mean they do not have a y-intercept?" No, they have y-intercepts, but they are not shown on this graph because the axes scales are limited.

Math Language Development

MLR7: Compare and Connect

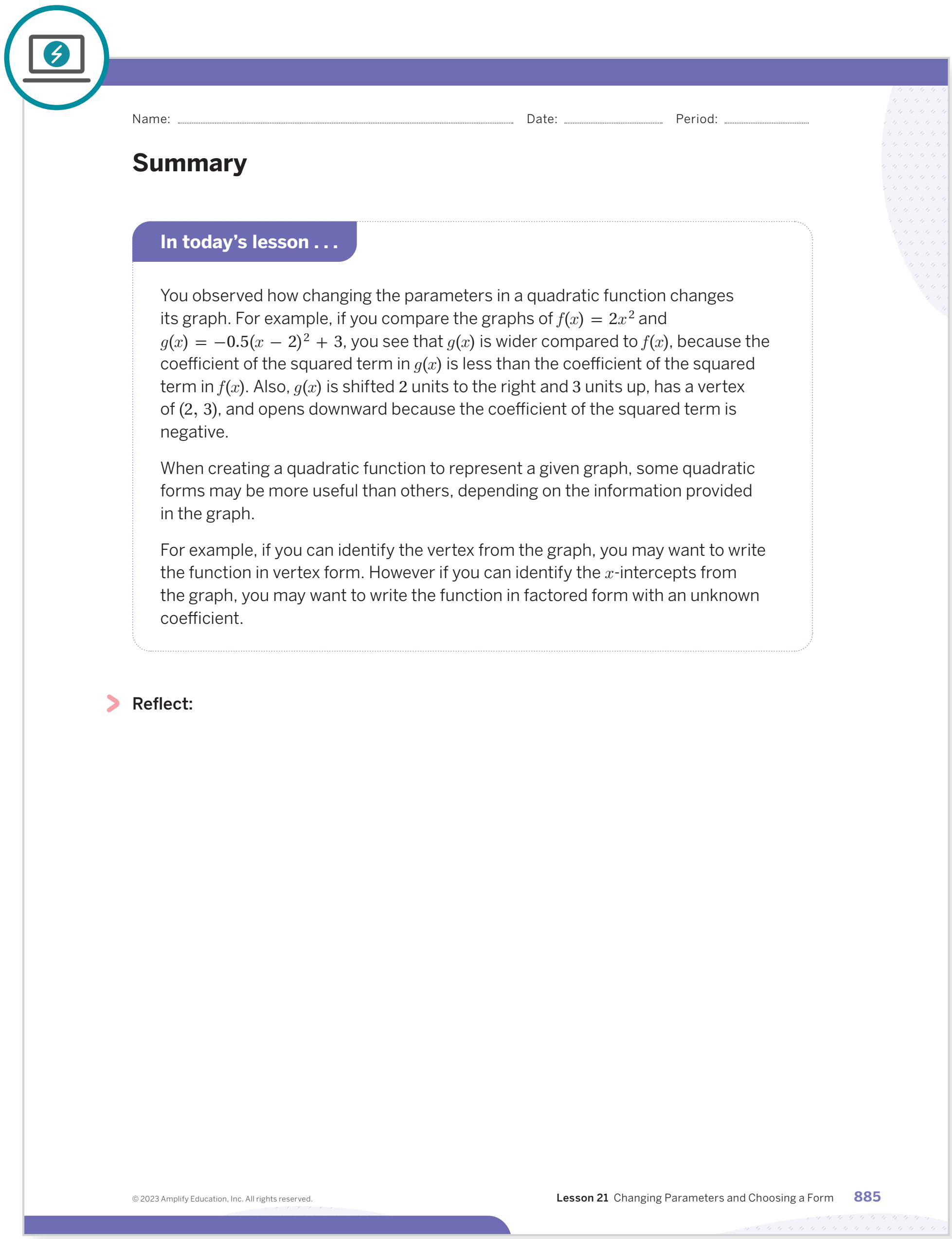
During the Connect, as you highlight the coefficient a that needs to be determined, annotate each graph with the equation from Problem 2. Ask these questions and use revoicing to model the use of mathematical language, such as *substitution*, *coordinates*, x-*intercept*, *zero*, and *vertex* as students respond.

- "For $f(x) = ax^2 + 2$, how can you use the graph to help you determine the value of a?"
- "For $g(x) = a(x - 4.5)(x - 8)$, could you use either of the points $(8, 0)$ or $(4.5, 0)$? Why or why not?"
- "For $h(x) = a(x - 11)^2 + 4$, could you use the point $(11, 4)$? Why or why not?"

Whole Class | 5 min

Summary

Review and synthesize the effects of changing parameters for quadratic functions, and the utility of each form for the equations of quadratic functions.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You observed how changing the parameters in a quadratic function changes its graph. For example, if you compare the graphs of $f(x) = 2x^2$ and $g(x) = -0.5(x - 2)^2 + 3$, you see that $g(x)$ is wider compared to $f(x)$, because the coefficient of the squared term in $g(x)$ is less than the coefficient of the squared term in $f(x)$. Also, $g(x)$ is shifted 2 units to the right and 3 units up, has a vertex of $(2, 3)$, and opens downward because the coefficient of the squared term is negative.

When creating a quadratic function to represent a given graph, some quadratic forms may be more useful than others, depending on the information provided in the graph.

For example, if you can identify the vertex from the graph, you may want to write the function in vertex form. However if you can identify the x-intercepts from the graph, you may want to write the function in factored form with an unknown coefficient.

> **Reflect:**

Synthesize

Display the functions $h(x) = (x - 4)^2 + 1$, $j(x) = 2x^2 - 3x + 2$, and $k(x) = 1.5(x - 2)(x - 4)$.

Have students share what information each function reveals about the graph of the function.

Highlight that $h(x)$ is in vertex form, revealing a vertex located at $(4, 1)$, and the graph opens upward. The function $j(x)$ is in standard form, revealing a vertical intercept of 2, and the graph opens upward. The function $k(x)$ is in factored form, revealing horizontal intercepts of 2 and 4, and the graph opens upward.

Ask, "What key feature could you determine from all three forms?" The vertical intercept can be found by evaluating each function when $x = 0$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

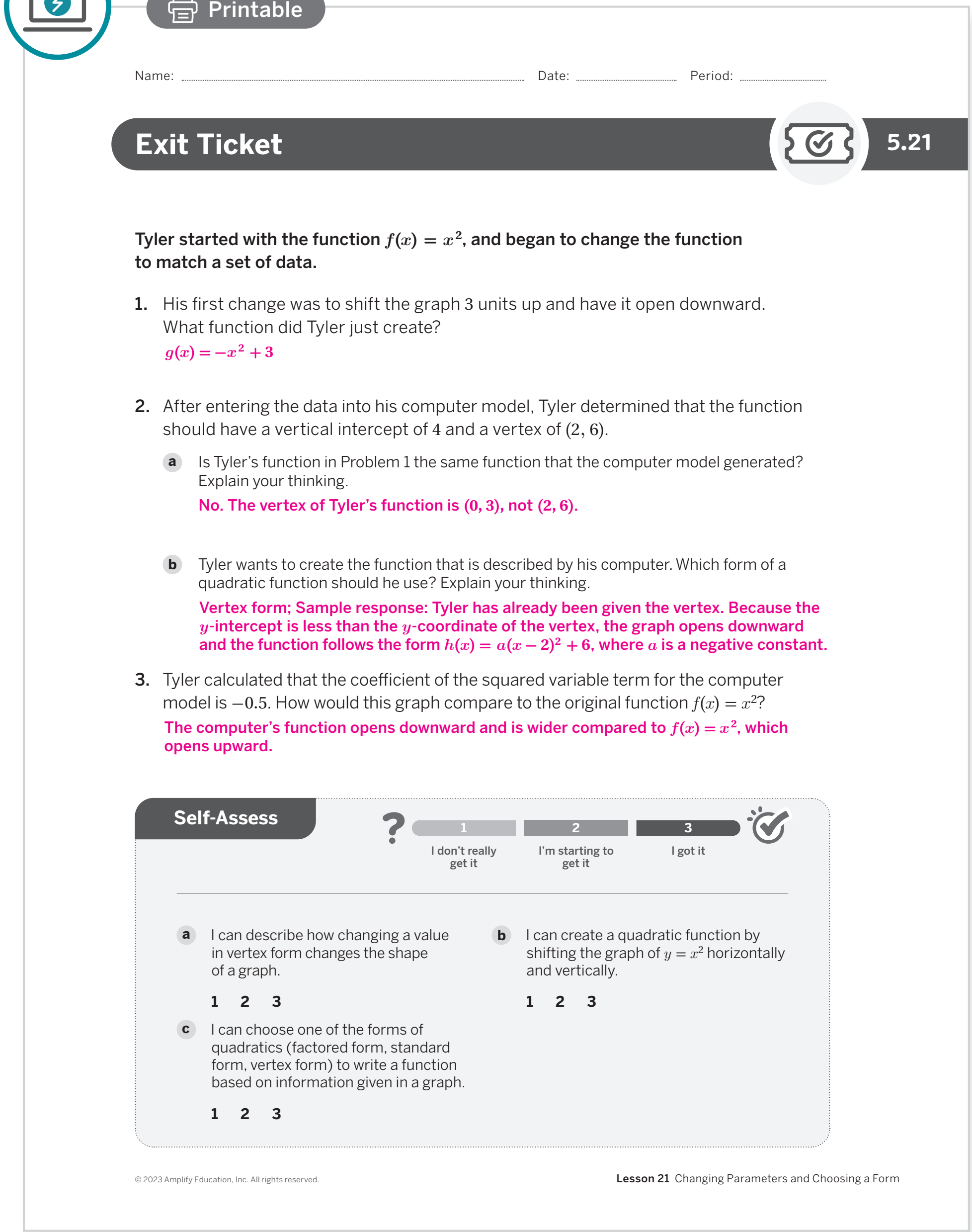
- "What are the advantages of writing a quadratic function in vertex form? In standard form? In factored form?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing the effects of changing parameters and the uses of different forms of quadratic functions.

Printable



Name: ______________________ Date: ________ Period: ________

Exit Ticket 5.21

Tyler started with the function $f(x) = x^2$, and began to change the function to match a set of data.

1. His first change was to shift the graph 3 units up and have it open downward. What function did Tyler just create?
   $g(x) = -x^2 + 3$

2. After entering the data into his computer model, Tyler determined that the function should have a vertical intercept of 4 and a vertex of $(2, 6)$.
   - **a** Is Tyler's function in Problem 1 the same function that the computer model generated? Explain your thinking.
     No. The vertex of Tyler's function is $(0, 3)$, not $(2, 6)$.
   - **b** Tyler wants to create the function that is described by his computer. Which form of a quadratic function should he use? Explain your thinking.
     Vertex form; Sample response: Tyler has already been given the vertex. Because the y-intercept is less than the y-coordinate of the vertex, the graph opens downward and the function follows the form $h(x) = a(x - 2)^2 + 6$, where a is a negative constant.

3. Tyler calculated that the coefficient of the squared variable term for the computer model is -0.5. How would this graph compare to the original function $f(x) = x^2$?
   The computer's function opens downward and is wider compared to $f(x) = x^2$, which opens upward.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can describe how changing a value in vertex form changes the shape of a graph.

1 2 3

b I can create a quadratic function by shifting the graph of $y = x^2$ horizontally and vertically.

1 2 3

c I can choose one of the forms of quadratics (factored form, standard form, vertex form) to write a function based on information given in a graph.

1 2 3

Lesson 21 Changing Parameters and Choosing a Form

Success looks like . . .

- **Language Goal:** Describing how changing a number in vertex form changes the shape of a graph. **(Speaking and Listening, Writing)**
- **Goal:** Creating a quadratic equation by shifting the quadratic function horizontally and vertically.
  - » Writing the function for a shifted quadratic function in Problem 1.
- **Language Goal:** Choosing one of the forms of quadratic functions to use to create an equation of a quadratic graph (factored form, standard form, vertex form) based on information given in the graph. **(Reading and Writing)**

Suggested next steps

If students create an inaccurate function for Problem 1, consider:

- Reviewing the table from Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "What does the coefficient of the squared variable term tell you about the graph?"

If students choose standard or factored form for Problem 2b, without sufficient explanation, consider:

- Reviewing why vertex form was chosen for $h(x)$ in Activity 3.
- Assigning Practice Problem 3.
- Asking, "What does each form reveal about the graph?"

If students have a vague or inaccurate explanation for Problem 3, consider:

- Reviewing the table from Activity 1.
- Assigning Practice Problems 1 and 2.

Professional Learning

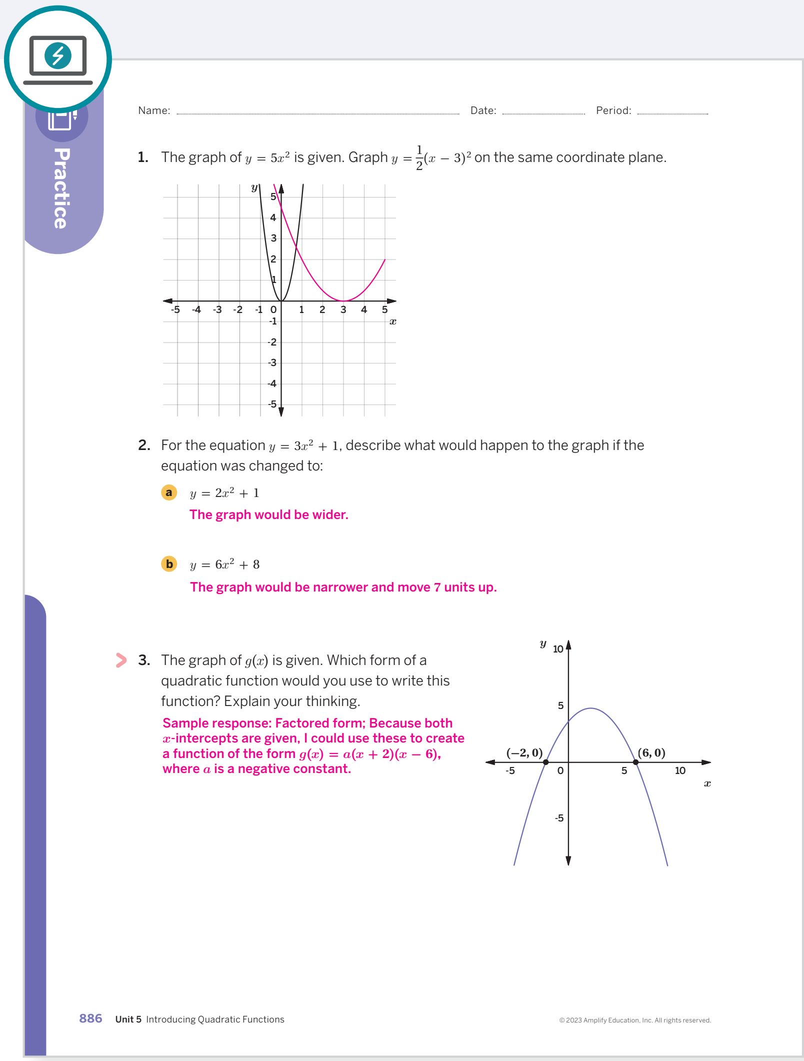
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students chose which form of a quadratic function to use to create an equation from a graph. How did that build on the earlier work students did with each form of quadratic functions?
- What different ways did students approach choosing which form of a quadratic function to use? What does that tell you about similarities and differences among your students?

Practice

Independent



Practice

Name: Date: Period:

1. The graph of $y = 5x^2$ is given. Graph $y = \frac{1}{2}(x - 3)^2$ on the same coordinate plane.

2. For the equation $y = 3x^2 + 1$, describe what would happen to the graph if the equation was changed to:

   a. $y = 2x^2 + 1$

   The graph would be wider.

   b. $y = 6x^2 + 8$

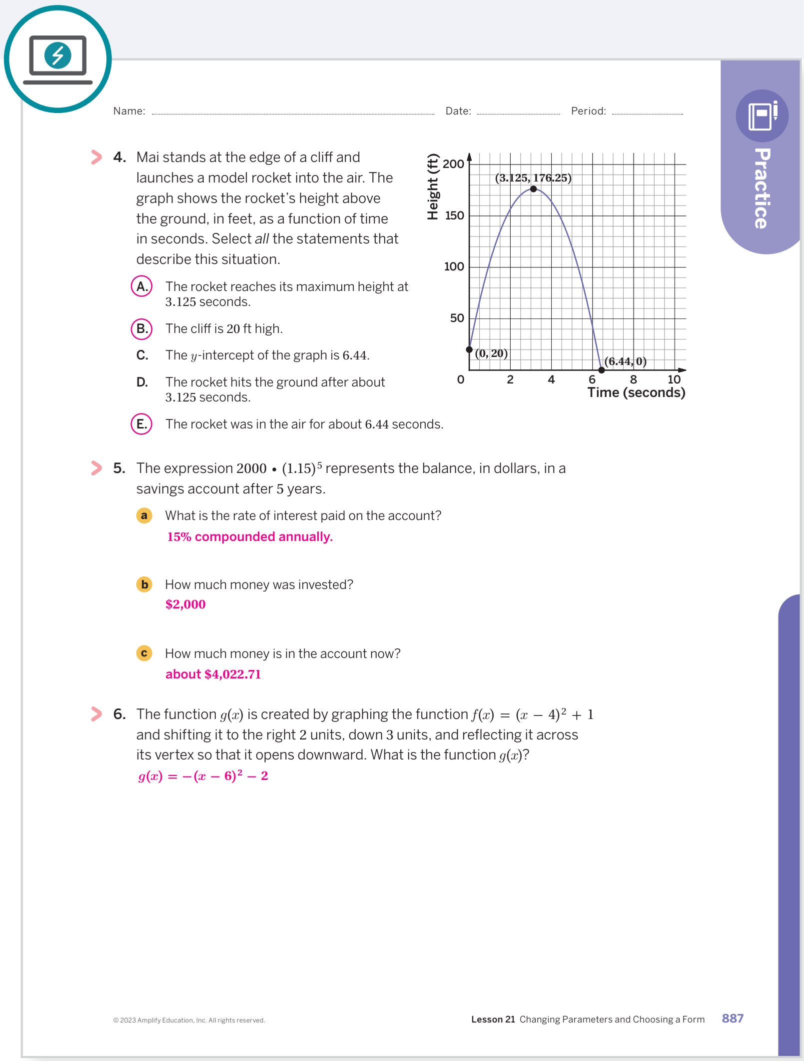
   The graph would be narrower and move 7 units up.

3. The graph of $g(x)$ is given. Which form of a quadratic function would you use to write this function? Explain your thinking.

   Sample response: Factored form; Because both x-intercepts are given, I could use these to create a function of the form $g(x) = a(x + 2)(x - 6)$, where a is a negative constant.

886 **Unit 5** Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

4. Mai stands at the edge of a cliff and launches a model rocket into the air. The graph shows the rocket's height above the ground, in feet, as a function of time in seconds. Select *all* the statements that describe this situation.

   [✓ A.] The rocket reaches its maximum height at 3.125 seconds.

   [✓ B.] The cliff is 20 ft high.

   C. The y-intercept of the graph is 6.44.

   D. The rocket hits the ground after about 3.125 seconds.

   [✓ E.] The rocket was in the air for about 6.44 seconds.

5. The expression $2000 \cdot (1.15)^5$ represents the balance, in dollars, in a savings account after 5 years.

   a. What is the rate of interest paid on the account?

   15% compounded annually.

   b. How much money was invested?

   $2,000

   c. How much money is in the account now?

   about $4,022.71

6. The function $g(x)$ is created by graphing the function $f(x) = (x - 4)^2 + 1$ and shifting it to the right 2 units, down 3 units, and reflecting it across its vertex so that it opens downward. What is the function $g(x)$?

   $g(x) = -(x - 6)^2 - 2$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Changing Parameters and Choosing a Form 887

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 3	2
Spiral	4	Unit 5 Lesson 18	2
	5	Unit 4 Lesson 16	1
Formative	6	Unit 5 Lesson 22	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 5 | **LESSON 22**

Changing the Vertex

Let’s write new quadratic functions in vertex form to make specific graphs.

Focus

Goals

1. Create a quadratic function by changing the vertex of an existing function given its equation, graph, and a description.
2. **Language Goal:** Describe informally the effect on the graph of a quadratic function when performing simple algebraic transformations. **(Speaking and Listening, Writing)**

Coherence

Today

Students look at how changing the values of h and the k in a quadratic function, written in vertex form, translates on the graph. They reason abstractly with functions and graphs in context.

Previously

In Lesson 21, students were given quadratic functions in vertex form and asked to visualize the location of the vertex and the direction of the opening of the graph.

Coming Soon

In Lesson 23, students will summarize Unit 5 skills and concepts by modeling a quadratic function and choosing a ball and type of throw using their knowledge of projectiles.

Rigor

- Students build **conceptual understanding** of algebraic transformations.
- Students **apply** quadratic functions in vertex form to study the path of objects in flight.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

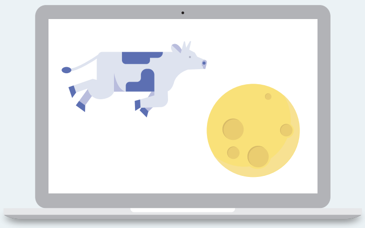
- *vertex form*

Amps | Featured Activity

Activity 1

Interactive Graph

Students use an interactive graph to write an equation in vertex form, adjust the vertex, and identify key features of the function within context.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not be able to decide how their mathematical skills apply to the scenario in Activity 2. Ask students what responsible decisions they can make to help themselves not only get started on this task, but also complete it successfully.

Modifications to Pacing

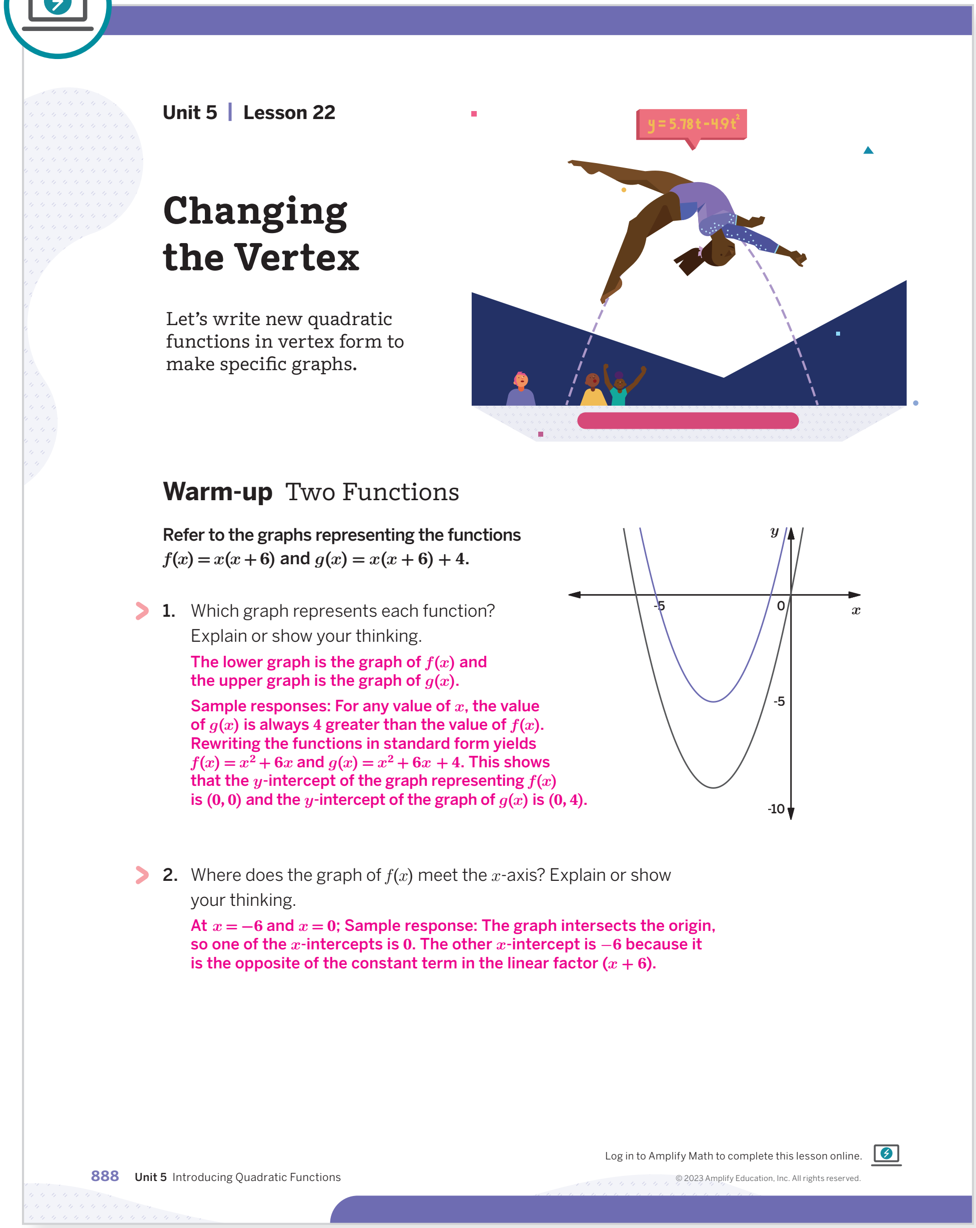
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 2**, Problem 5 may be omitted.

Pairs | 5 min

Warm-up Two Functions

Students match equations and graphs representing two quadratic functions.



Unit 5 | Lesson 22

Changing the Vertex

Let's write new quadratic functions in vertex form to make specific graphs.

Warm-up Two Functions

Refer to the graphs representing the functions $f(x) = x(x + 6)$ and $g(x) = x(x + 6) + 4$.

1. Which graph represents each function? Explain or show your thinking.

   The lower graph is the graph of $f(x)$ and the upper graph is the graph of $g(x)$.

   Sample responses: For any value of x, the value of $g(x)$ is always 4 greater than the value of $f(x)$. Rewriting the functions in standard form yields $f(x) = x^2 + 6x$ and $g(x) = x^2 + 6x + 4$. This shows that the y-intercept of the graph representing $f(x)$ is $(0, 0)$ and the y-intercept of the graph of $g(x)$ is $(0, 4)$.

2. Where does the graph of $f(x)$ meet the x-axis? Explain or show your thinking.

   At $x = -6$ and $x = 0$; Sample response: The graph intersects the origin, so one of the x-intercepts is 0. The other x-intercept is -6 because it is the opposite of the constant term in the linear factor $(x + 6)$.

Log in to Amplify Math to complete this lesson online.

888 Unit 5 Introducing Quadratic Functions

1 Launch

Have students study the structure of the given quadratic functions and identify key features. Then have them match the key features to the graph.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "Which form of the quadratic function gives the x-intercepts and which form gives the y-intercept?"

Look for points of confusion:

- **Not identifying the y-intercept of either function.** Have them rewrite both functions in standard form.

Look for productive strategies:

- Rewriting both functions in standard form and identifying the y-intercepts on the graph.
- Determining the x-intercepts of $f(x)$ and identifying each in the graph.

3 Connect

Display the graphs and functions.

Have students share their matches and explanations.

Highlight that the y-intercepts of $f(x)$ and $g(x)$ are $(0, 0)$ and $(0, 4)$, respectively. $f(x)$ is in factored form and the x-intercepts of $f(x)$ are $(0, 0)$ and $(-6, 0)$. The value of $g(x)$ is always 4 greater than $f(x)$.

Power-up

To power up students' ability to understand how changing the vertex affects the structure of a quadratic function, have students complete:

Complete each problem for the function $p(x) = (x + 3)^2 - 4$.

1. What are the coordinates of the vertex? $(-3, -4)$
2. If the graph of $p(x)$ is shifted up 5 units to become $q(x)$, what are the coordinates of the vertex of $q(x)$? What is the equation of $q(x)$? $(-3, 1)$; $q(x) = (x + 3)^2 + 1$
3. If the graph of $p(x)$ is shifted left 6 units left to become $r(x)$, what are the coordinates of the vertex of $r(x)$? What is the equation of $r(x)$? $(-9, -4)$; $r(x) = (x + 9)^2 - 4$

Use: Before the Warm-up

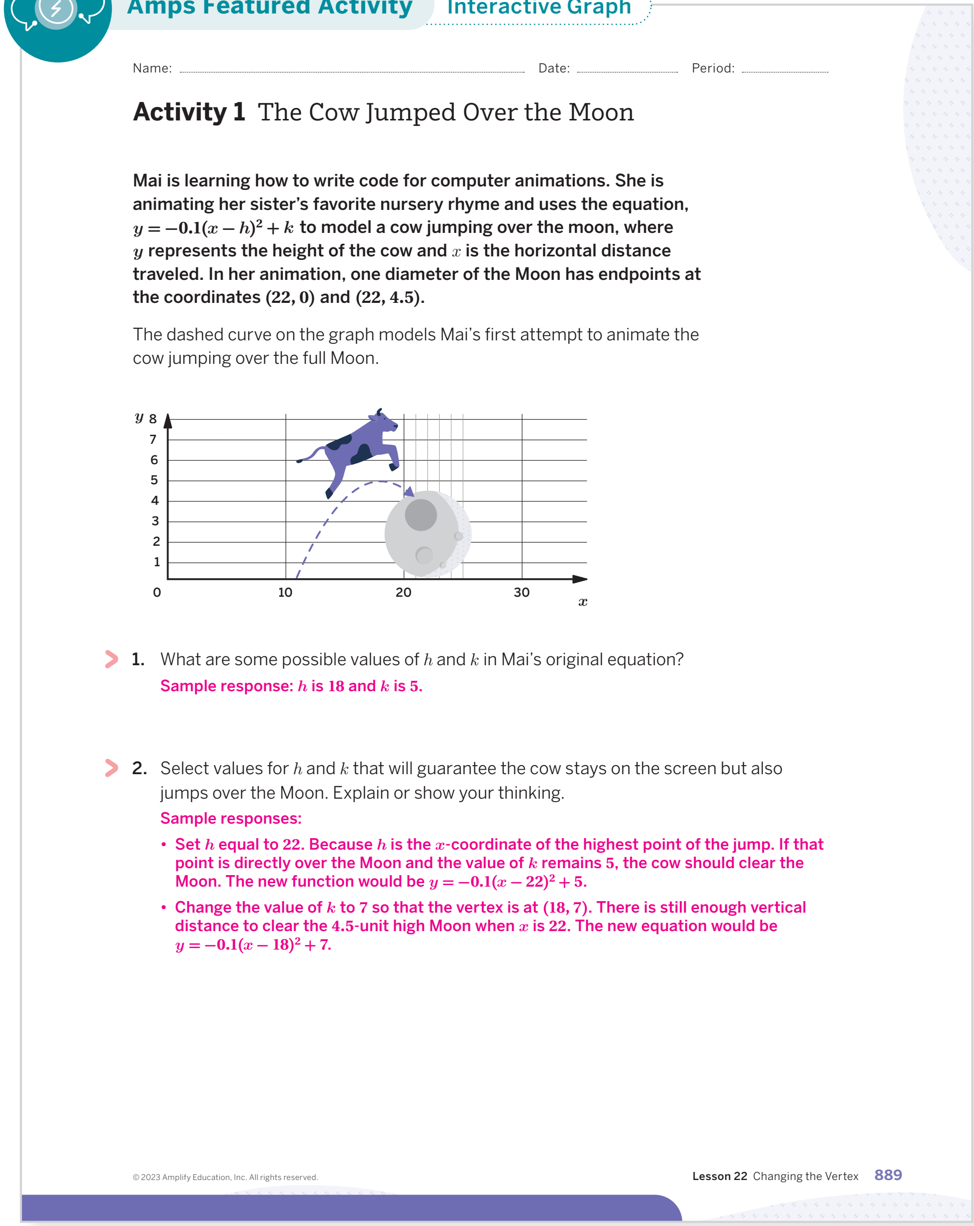
Informed by: Performance on Lesson 21, Practice Problem 6

Pairs | 15 min

Activity 1 The Cow Jumped Over the Moon

Students change the parameters of a quadratic function modeling a parabolic path while meeting certain restrictions.

Amps Featured Activity Interactive Graph



Name: ________ Date: ________ Period: ________

Activity 1 The Cow Jumped Over the Moon

Mai is learning how to write code for computer animations. She is animating her sister's favorite nursery rhyme and uses the equation, $y = -0.1(x - h)^2 + k$ to model a cow jumping over the moon, where y represents the height of the cow and x is the horizontal distance traveled. In her animation, one diameter of the Moon has endpoints at the coordinates $(22, 0)$ and $(22, 4.5)$.

The dashed curve on the graph models Mai's first attempt to animate the cow jumping over the full Moon.

1. What are some possible values of h and k in Mai's original equation?
   Sample response: h is 18 and k is 5.

2. Select values for h and k that will guarantee the cow stays on the screen but also jumps over the Moon. Explain or show your thinking.
   Sample responses:
   - Set h equal to 22. Because h is the x-coordinate of the highest point of the jump. If that point is directly over the Moon and the value of k remains 5, the cow should clear the Moon. The new function would be $y = -0.1(x - 22)^2 + 5$.
   - Change the value of k to 7 so that the vertex is at $(18, 7)$. There is still enough vertical distance to clear the 4.5-unit high Moon when x is 22. The new equation would be $y = -0.1(x - 18)^2 + 7$.

Lesson 22 Changing the Vertex 889

1 Launch

Read the prompt aloud. Activate students' background knowledge by asking them if they are familiar with how games or shows are animated.

2 Monitor

Help students get started by making sure they understand the function $f(x) = -0.1(x - h)^2 + k$ is the path of the cow's jump.

Look for points of confusion:

- **Randomly choosing values for h and k.** Have students reason about the values of h and k using vertex form.
- **Using the coordinate $(22, 4.5)$ of the diameter of the Moon as the vertex.** Ask how high the cow must jump in order to clear the Moon.
- **Not understanding how the function changes when changing the values of h and k.** Point out that (h, k) represents the coordinates of the vertex.

Look for productive strategies:

- Shifting the starting position of the point closer to the moon.
- Shifting the vertex to a point closer to the moon.
- Shifting both the cow and the vertex of the parabolic path.

3 Connect

Display the graph.

Have students share contrasting strategies for how they chose values for h and k and how they wrote their functions.

Highlight the connection between the parameters in the function and the vertex of the graph. Here, students see a model where the height of an object is a function of horizontal distance.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive graph to write an equation in vertex form, adjust the vertex, and identify key features of the function within context.

Math Language Development

MLR6: Three Reads

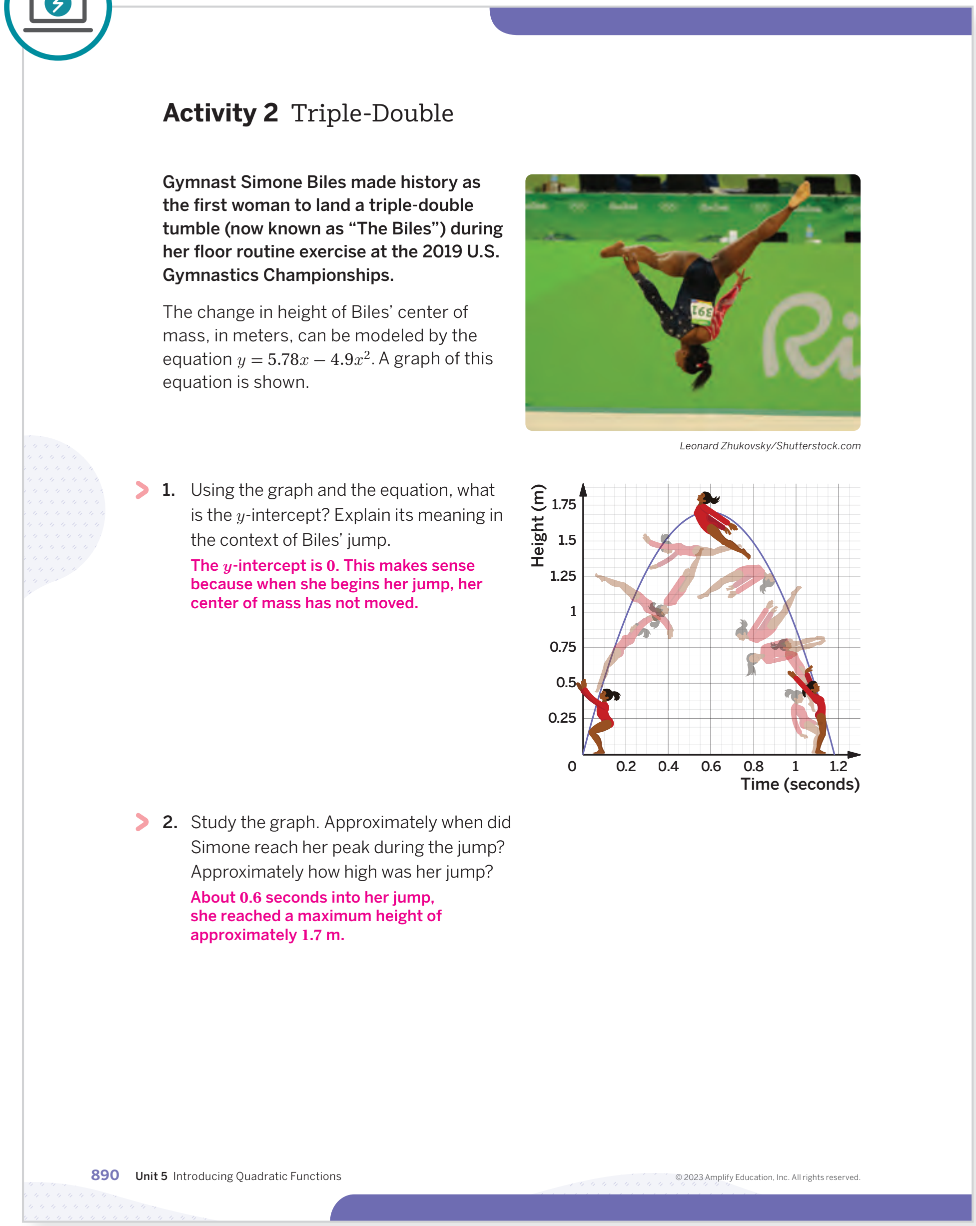
Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that a quadratic equation can model the path a cow takes if it were to jump over the Moon.
- **Read 2:** Ask students to identify the given quantities or relationships, such as the diameter of the Moon has endpoints at the given coordinates.
- **Read 3:** Ask students to brainstorm strategies for how they can determine the coordinates of the vertex of the function.

Pairs | 20 min

Activity 2 Triple-Double

Students apply what they learned about quadratic expressions and their graphs to solve a problem in context.



Activity 2 Triple-Double

Gymnast Simone Biles made history as the first woman to land a triple-double tumble (now known as "The Biles") during her floor routine exercise at the 2019 U.S. Gymnastics Championships.

The change in height of Biles' center of mass, in meters, can be modeled by the equation $y = 5.78x - 4.9x^2$. A graph of this equation is shown.

Leonard Zhukovsky/Shutterstock.com

1. Using the graph and the equation, what is the y-intercept? Explain its meaning in the context of Biles' jump.
   The y-intercept is 0. This makes sense because when she begins her jump, her center of mass has not moved.

2. Study the graph. Approximately when did Simone reach her peak during the jump? Approximately how high was her jump?
   About 0.6 seconds into her jump, she reached a maximum height of approximately 1.7 m.

890 **Unit 5** Introducing Quadratic Functions

1 Launch

Have students read the introductory text and study the equation and graph. Point out that the quadratic equation looks different from the ones they have previously studied. The squared term representing the effects of gravity in this equation is $-4.9x^2$ instead of $-16x^2$. Ask, "Are the units in feet or some other unit?" Mention that the effects of gravity when the units are in meters is represented by $-4.9x^2$.

2 Monitor

Help students get started by prompting them to annotate the ordered pair with its appropriate unit, and then verbally explain what the ordered pair represents.

Look for points of confusion:

- **Guessing the coordinates in Problem 2.** Ask, "What does the vertex represent in this scenario?"
- **Writing $(x + 0.6)$ in Problem 4.** Have students evaluate for $x = 0.6$ to check if their equation results in the vertex.

Look for productive strategies:

- Relating the equation, the prompt, and the graph to make sense of the problem.
- Noticing how the vertex changes in each scenario.

Activity 2 continued >

Differentiated Support

Accessibility: *Activate Background Knowledge*

Consider playing a video of the moment when Simone Biles completed the triple-double routine to help students visualize and connect to the task.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students annotate the graph with estimates for the horizontal intercepts and vertex and what they represent within the context of the scenario.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight how the graphs of the equations from Problems 4 and 5 compare, draw students' attention to the connections between the structure of the vertex form of the equations and their corresponding graphs. Ask:

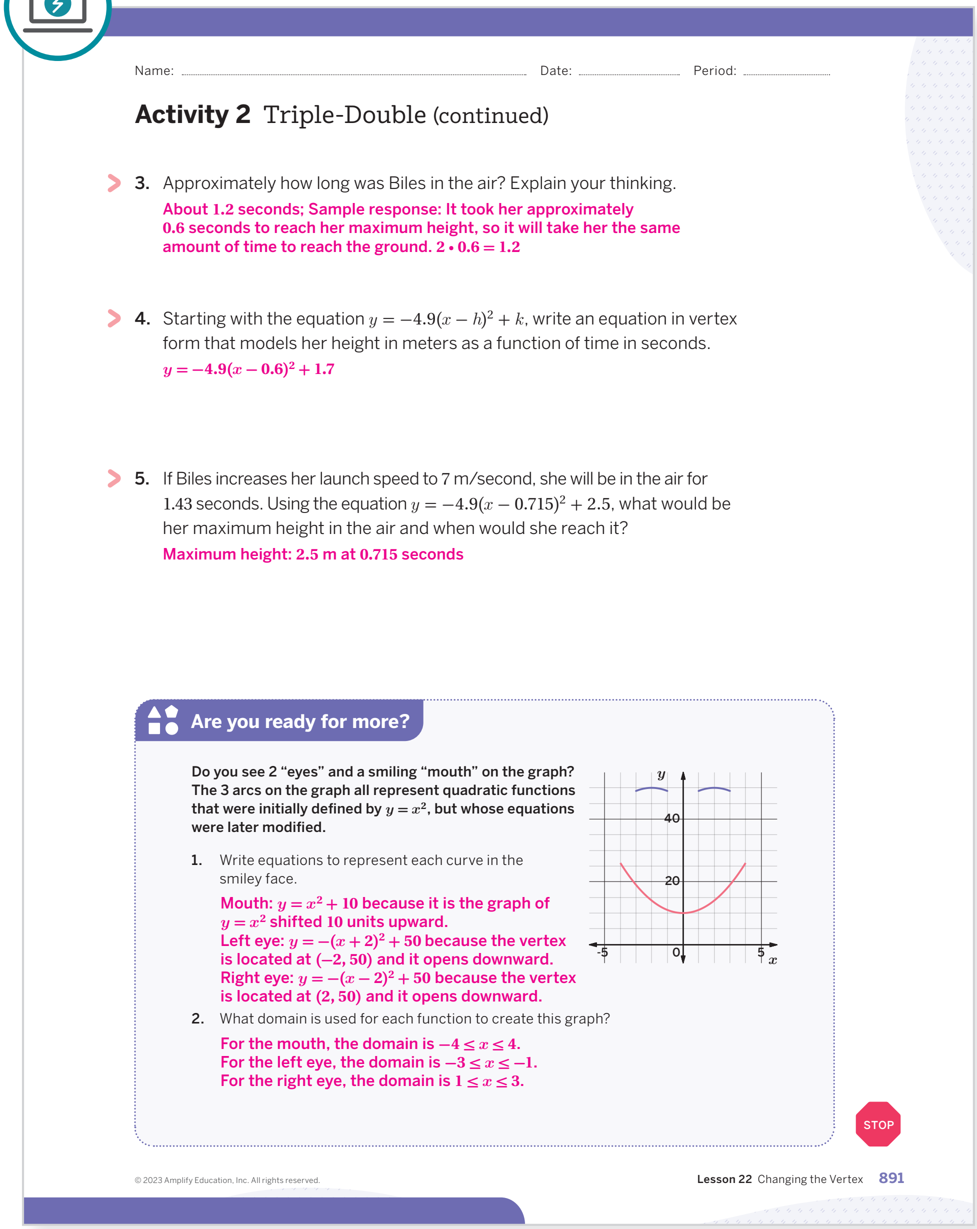
- "What mathematical terms can you use to compare the two graphs? The two equations?"
- "It looks like the graph of the equation in Problem 5 is wider than the graph of the equation in Problem 4. Is it really wider? How do you know?" Listen for students who reason that the equation was not altered to show that the parabola would become wider or narrower because the coefficient on x^2 did not change.

While it may look like the graph of the equation in Problem 5 is wider, it only appears that way because the parabola shifted up and to the right, so students see more of the parabola's opening.

Pairs | 20 min

Activity 2 Triple-Double (continued)

Students apply what they learned about quadratic expressions and their graphs to solve a problem in context.



Name: Date: Period:

Activity 2 Triple-Double (continued)

3. Approximately how long was Biles in the air? Explain your thinking.
   About 1.2 seconds; Sample response: It took her approximately 0.6 seconds to reach her maximum height, so it will take her the same amount of time to reach the ground. $2 \cdot 0.6 = 1.2$

4. Starting with the equation $y = -4.9(x - h)^2 + k$, write an equation in vertex form that models her height in meters as a function of time in seconds.
   $y = -4.9(x - 0.6)^2 + 1.7$

5. If Biles increases her launch speed to 7 m/second, she will be in the air for 1.43 seconds. Using the equation $y = -4.9(x - 0.715)^2 + 2.5$, what would be her maximum height in the air and when would she reach it?
   Maximum height: 2.5 m at 0.715 seconds

Are you ready for more?

Do you see 2 "eyes" and a smiling "mouth" on the graph? The 3 arcs on the graph all represent quadratic functions that were initially defined by $y = x^2$, but whose equations were later modified.

1. Write equations to represent each curve in the smiley face.
   Mouth: $y = x^2 + 10$ because it is the graph of $y = x^2$ shifted 10 units upward.
   Left eye: $y = -(x + 2)^2 + 50$ because the vertex is located at $(-2, 50)$ and it opens downward.
   Right eye: $y = -(x - 2)^2 + 50$ because the vertex is located at $(2, 50)$ and it opens downward.
2. What domain is used for each function to create this graph?
   For the mouth, the domain is $-4 \leq x \leq 4$.
   For the left eye, the domain is $-3 \leq x \leq -1$.
   For the right eye, the domain is $1 \leq x \leq 3$.

STOP

3 Connect

Display the graph.

Have students share strategies for determining the intercepts, the vertex, and what the values represent in context.

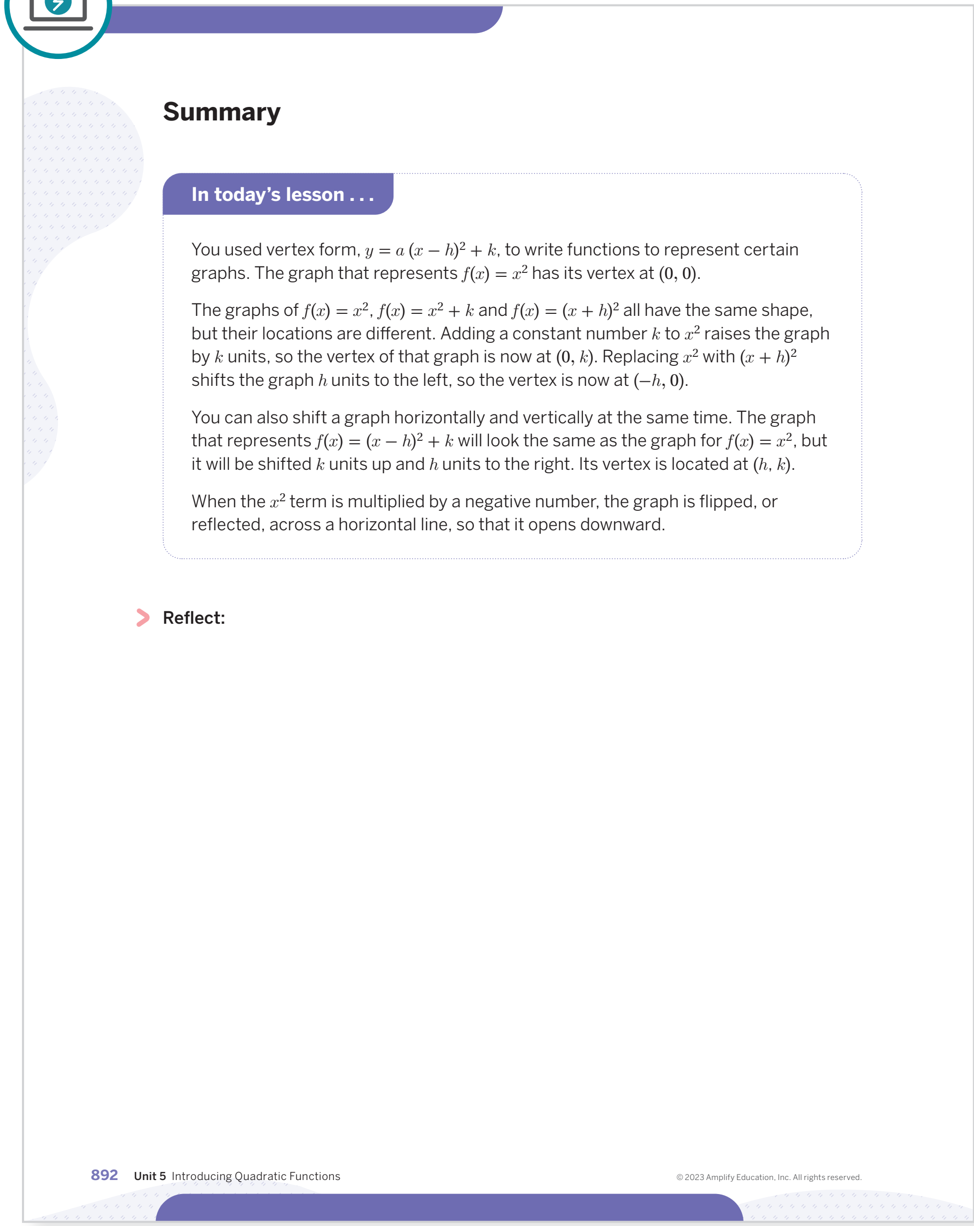
Highlight that changing the vertex, as in Problem 5, alters the scenario and translates the graph. Use graphing technology to show how the two graphs compare. Ask students to describe the shift in the graph of the equation in Problem 5 with how it compares to the graph of the equation in Problem 4. The graph is moved to the right 0.115 units (0.715 – 0.6) and up 0.8 units (2.5 – 1.7).

Ask, "Write an equation to represent a triple-double that you think Biles would love to land." Answers will vary, but should include a large y-coordinate for height, and a small x-coordinate for time.

Whole Class | 5 min

Summary

Review and synthesize how changing the values of h and k in the vertex form of a quadratic function changes its graph.



Summary

In today's lesson . . .

You used vertex form, $y = a\,(x - h)^2 + k$, to write functions to represent certain graphs. The graph that represents $f(x) = x^2$ has its vertex at $(0, 0)$.

The graphs of $f(x) = x^2$, $f(x) = x^2 + k$ and $f(x) = (x + h)^2$ all have the same shape, but their locations are different. Adding a constant number k to x^2 raises the graph by k units, so the vertex of that graph is now at $(0, k)$. Replacing x^2 with $(x + h)^2$ shifts the graph h units to the left, so the vertex is now at $(-h, 0)$.

You can also shift a graph horizontally and vertically at the same time. The graph that represents $f(x) = (x - h)^2 + k$ will look the same as the graph for $f(x) = x^2$, but it will be shifted k units up and h units to the right. Its vertex is located at (h, k).

When the x^2 term is multiplied by a negative number, the graph is flipped, or reflected, across a horizontal line, so that it opens downward.

Reflect:

892 **Unit 5** Introducing Quadratic Functions

Synthesize

Display the vertex form of a quadratic function, along with the equations $y = x^2$, $y = x^2 + 12$, and $y = (x + 3)^2$.

Have students share how changing the values of h and k in the vertex form of the quadratic function would affect the graph of the function.

Highlight that the graphs of $y = x^2$, $y = x^2 + 12$, and $y = (x + 3)^2$ all have the same shape but their locations are different. Have students graph these functions or display the graphs of these functions to the class. Highlight that:

- The graph that represents $y = x^2$ has its vertex at $(0, 0)$.
- Adding 12 to x^2 shifts the graph up by 12 units, so the vertex of that graph is now at $(0, 12)$.
- Replacing x^2 with $(x + 3)^2$ shifts the graph 3 units to the left, so the vertex is now at $(-3, 0)$.
- The graph that represents $y = (x + 3)^2 + 12$ will be shifted 12 units up and 3 units to the left, with its vertex now at $(-3, 12)$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

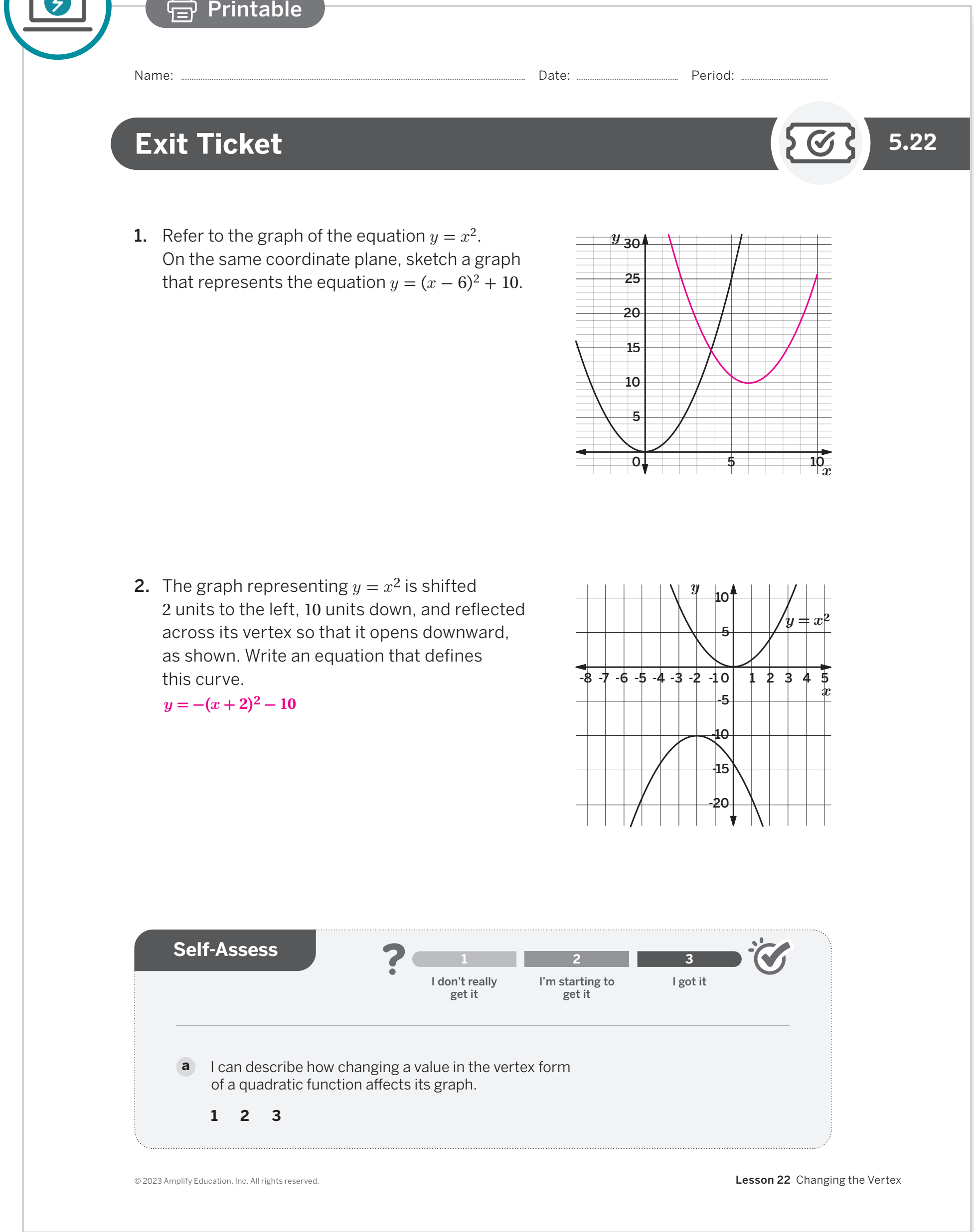
- "How does a quadratic function in vertex form reflect horizontal and vertical shifts?"
- "Why does a positive value added to x in vertex form represent a shift to the left?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of transformations by changing the vertex of a quadratic function and describing the effects on the functions graph or equation.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.22

1. Refer to the graph of the equation $y = x^2$. On the same coordinate plane, sketch a graph that represents the equation $y = (x - 6)^2 + 10$.

2. The graph representing $y = x^2$ is shifted 2 units to the left, 10 units down, and reflected across its vertex so that it opens downward, as shown. Write an equation that defines this curve.
$y = -(x + 2)^2 - 10$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can describe how changing a value in the vertex form of a quadratic function affects its graph.
1 2 3

Lesson 22 Changing the Vertex

Success looks like . . .

- **Goal:** Creating a quadratic function by changing the vertex of an existing function given its equation, graph, and a description.
  - » Graphing the equation $y = (x - 6)^2 + 10$ using the graph of $y = x^2$ in Problem 1.
- **Language Goal:** Describing informally the effect on the graph of a quadratic function when performing simple algebraic transformations. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly sketch the graph the equation in Problem 1, consider:

- Reviewing graphing strategies from Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, “How would you find the vertex on the graph and what values do its coordinates represent in the vertex form of the equation?”

If students incorrectly write the equation that defines the curve in Problem 2, consider:

- Reviewing graphing strategies from Activity 1 and locating the vertex from Activity 2.
- Assigning Practice Problems 2 and 3.
- Asking, “How would you use the coordinates of the vertex to write the vertex form of the equation?”

Professional Learning

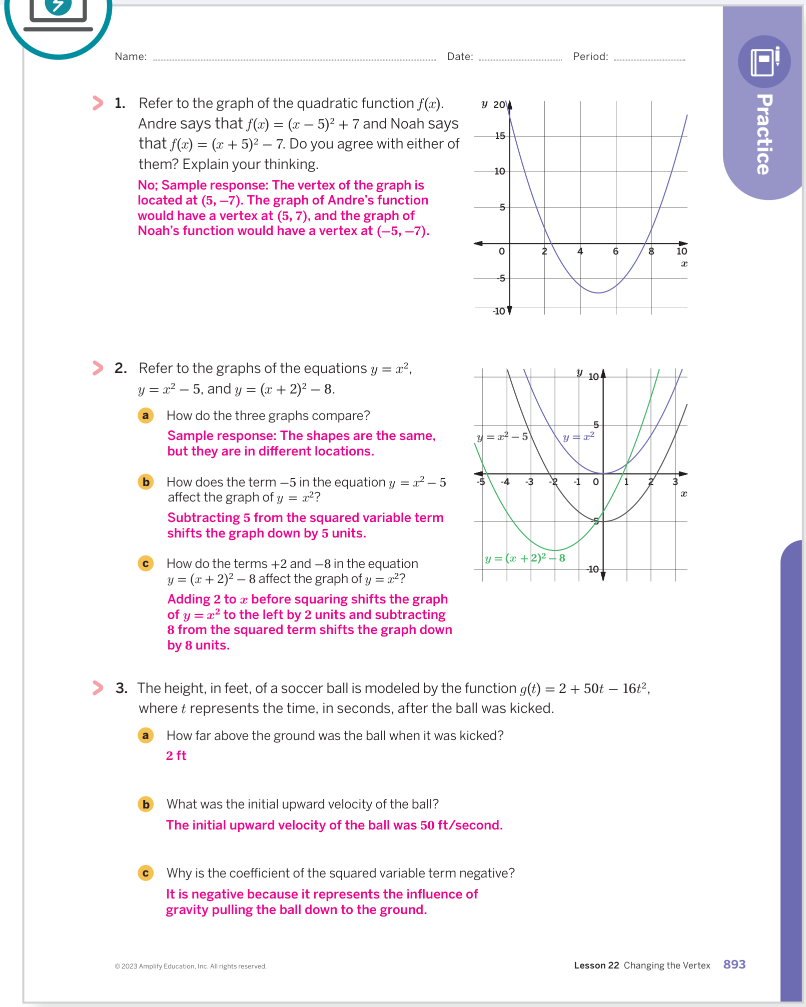
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to describe how changing a number in the vertex form of a quadratic function affects its graph. How well did students accomplish this? What did you specifically do to help students accomplish it?
- In what ways have your students gotten better at using key features of quadratic functions to interpret scenarios?

Practice

Independent



Name: Date: Period:

1. Refer to the graph of the quadratic function $f(x)$. Andre says that $f(x) = (x - 5)^2 + 7$ and Noah says that $f(x) = (x + 5)^2 - 7$. Do you agree with either of them? Explain your thinking.

   No; Sample response: The vertex of the graph is located at (5, –7). The graph of Andre's function would have a vertex at (5, 7), and the graph of Noah's function would have a vertex at (–5, –7).

2. Refer to the graphs of the equations $y = x^2$, $y = x^2 - 5$, and $y = (x + 2)^2 - 8$.

   a. How do the three graphs compare?

   Sample response: The shapes are the same, but they are in different locations.

   b. How does the term –5 in the equation $y = x^2 - 5$ affect the graph of $y = x^2$?

   Subtracting 5 from the squared variable term shifts the graph down by 5 units.

   c. How do the terms +2 and –8 in the equation $y = (x + 2)^2 - 8$ affect the graph of $y = x^2$?

   Adding 2 to x before squaring shifts the graph of $y = x^2$ to the left by 2 units and subtracting 8 from the squared term shifts the graph down by 8 units.

3. The height, in feet, of a soccer ball is modeled by the function $g(t) = 2 + 50t - 16t^2$, where t represents the time, in seconds, after the ball was kicked.

   a. How far above the ground was the ball when it was kicked?

   2 ft

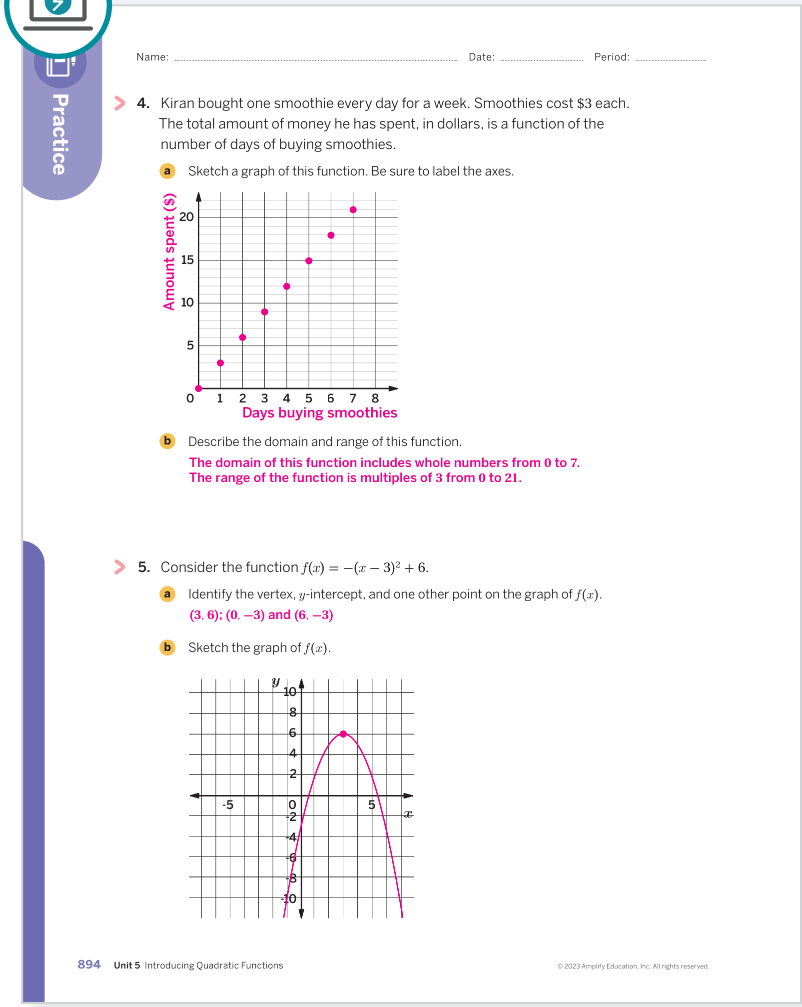
   b. What was the initial upward velocity of the ball?

   The initial upward velocity of the ball was 50 ft/second.

   c. Why is the coefficient of the squared variable term negative?

   It is negative because it represents the influence of gravity pulling the ball down to the ground.

Lesson 22 Changing the Vertex 893



Name: Date: Period:

4. Kiran bought one smoothie every day for a week. Smoothies cost $3 each. The total amount of money he has spent, in dollars, is a function of the number of days of buying smoothies.

   a. Sketch a graph of this function. Be sure to label the axes.

   b. Describe the domain and range of this function.

   The domain of this function includes whole numbers from 0 to 7. The range of the function is multiples of 3 from 0 to 21.

5. Consider the function $f(x) = -(x - 3)^2 + 6$.

   a. Identify the vertex, y-intercept, and one other point on the graph of $f(x)$.

   (3, 6); (0, –3) and (6, –3)

   b. Sketch the graph of $f(x)$.

894 **Unit 5** Introducing Quadratic Functions

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 11	2
Formative	5	Unit 5 Lesson 23	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Mathematical Modeling Prompt

At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's Mathematical Modeling Prompt, Critically Examining the National Debt, which is available in the **Algebra 1 Additional Practice**.

Monster Ball

Let's use our knowledge of quadratic functions and their graphs to play monster ball!

Focus

Goals

1. **Language Goal:** Write a quadratic function that represents a scenario. **(Reading and Writing)**
2. **Language Goal:** Interpret how a quadratic function and its graph relate to a scenario. **(Reading and Writing)**
3. **Language Goal:** Relate key features of a quadratic function, (vertex, zeros, domain) to a scenario **(Speaking and Listening, Writing)**

Coherence

Today

Students summarize Unit 5 skills and concepts by modeling; they develop a game plan to play Monster Ball. They focus on placement of players on a court, ball selection, and type of throw using their knowledge of quadratic functions as projectiles.

Previously

In Lesson 2, students found the maximum area of a field with fixed perimeter by interpreting a quadratic function. In Lesson 8, students interpreted and created quadratic functions of projectiles.

Coming Soon

In Unit 6, students will solve quadratic equations by using the Zero Product Principle, completing the square, and applying the quadratic formula.

Rigor

- Students **apply** their understanding of quadratic functions to analyze and play the game of Monster Ball.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	30 min	5 min	5 min
Whole Class	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Critiquing*
- basketball court
- graphing technology
- large exercise ball
- variety of different-sized balls (basketballs, kickballs, soccer balls, tennis balls)

Math Language Development

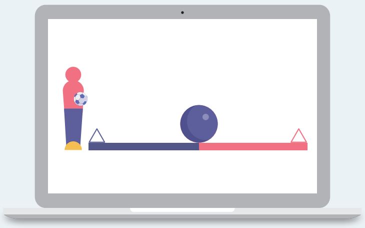
Review words

- *horizontal intercepts*
- *maximum/minimum*
- *quadratic function*
- *vertex*
- *vertical intercepts*
- *zeros of a function*

Amps | Featured Activity

Activity 3
Digital Monster Ball

Students can play a digital version of Monster Ball. Their calculations will be evaluated in real time to check against their predictions.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

The word perfect in the title of Activity 2 might trigger stress in some students. Have students take a few deep breaths and read the activity, noting that the perfection is not referring to them. If needed, students can even cross out the word so that they can focus on reasoning through the quantitative measures in the activity.

Modifications to Pacing

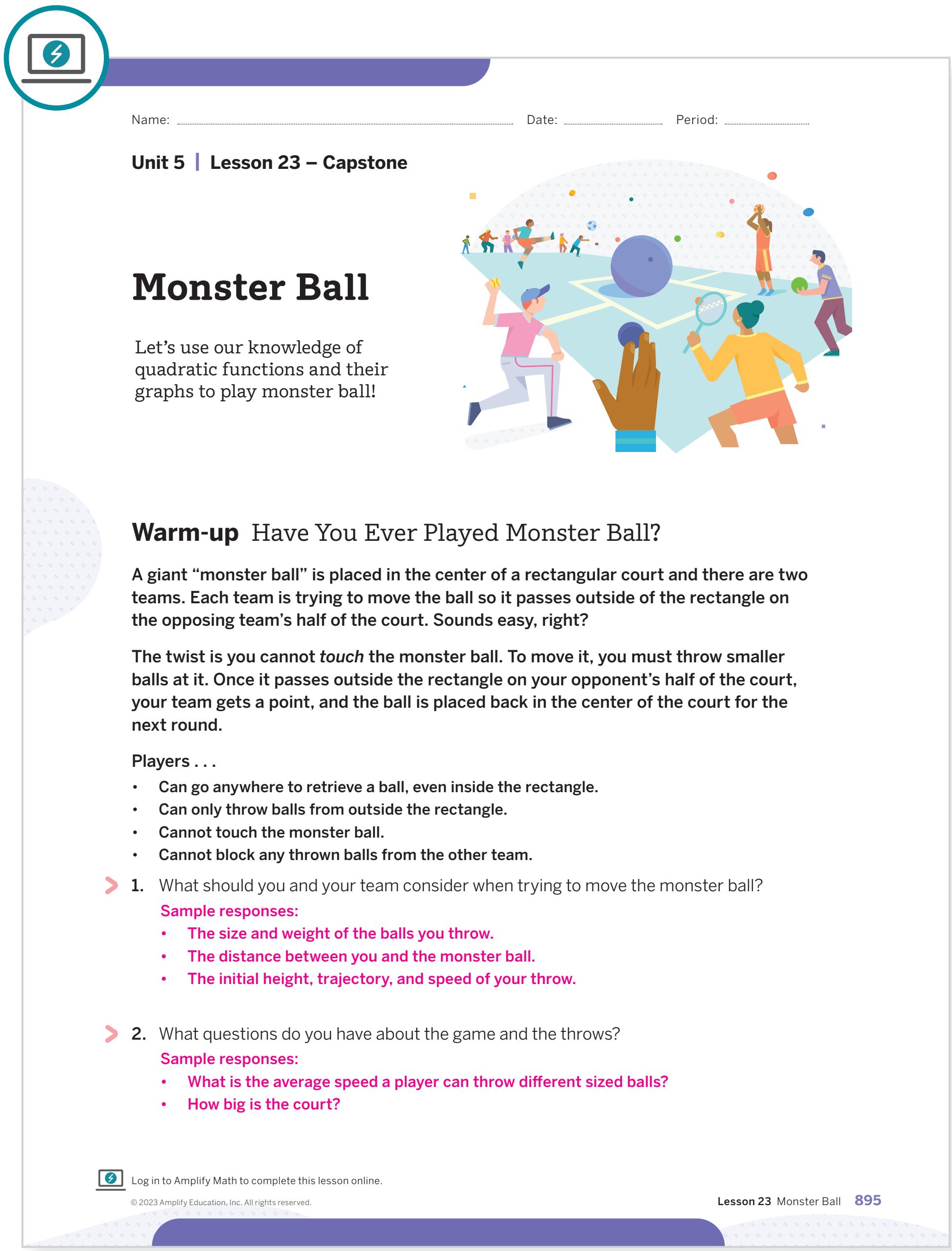
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, reduce the number of balls to three.
- Consider conducting the lesson over two days with **Activity 3** on the second day.

Whole Class | 5 min

Warm-up Have You Ever Played Monster Ball?

Students learn the game of Monster Ball and generate a list of things to consider and questions about the game.



Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 23 – Capstone

Monster Ball

Let's use our knowledge of quadratic functions and their graphs to play monster ball!

Warm-up Have You Ever Played Monster Ball?

A giant "monster ball" is placed in the center of a rectangular court and there are two teams. Each team is trying to move the ball so it passes outside of the rectangle on the opposing team's half of the court. Sounds easy, right?

The twist is you cannot *touch* the monster ball. To move it, you must throw smaller balls at it. Once it passes outside the rectangle on your opponent's half of the court, your team gets a point, and the ball is placed back in the center of the court for the next round.

Players . . .

- **Can go anywhere to retrieve a ball, even inside the rectangle.**
- **Can only throw balls from outside the rectangle.**
- **Cannot touch the monster ball.**
- **Cannot block any thrown balls from the other team.**

1. What should you and your team consider when trying to move the monster ball?

Sample responses:
- **The size and weight of the balls you throw.**
- **The distance between you and the monster ball.**
- **The initial height, trajectory, and speed of your throw.**

2. What questions do you have about the game and the throws?

Sample responses:
- **What is the average speed a player can throw different sized balls?**
- **How big is the court?**

Log in to Amplify Math to complete this lesson online.

Lesson 23 Monster Ball 895

① Launch

Read the prompt aloud. Consider showing a short clip of Monster Ball to familiarize students with the game. Have students complete Problems 1 and 2 independently before sharing their thinking with a partner.

② Monitor

Help students get started by asking, "How would you organize your team and choose which ball to throw?"

Look for points of confusion:

- **Discounting ball size.** Ask, "If you were far away from the ball, how would this impact your throw and ball choice compared to being close to the ball?"

Look for productive strategies:

- Using key features from quadratic functions in their considerations and questions.
- Drawing a diagram of a possible court.

③ Connect

Have individual students share their considerations and questions.

Highlight that another way to explain a throw is describing it as a projectile, as students did in Lesson 7.

Ask:

- "What information do you need to construct the height of a projectile?" Initial height, initial vertical speed, and effects of gravity.
- "What types of sports balls would you choose if you were far away versus close to the monster ball?"

Power-up

To power up students' ability to identify the coordinates of the vertex from a quadratic function written in vertex form, have students complete:

Recall that, for quadratic functions written of the form $y = a(x - h)^2 + k$, the vertex is (h, k). Determine the coordinates of the vertex for each quadratic function.

1. $f(x) = \left(x + \frac{3}{2}\right)^2 - 1$ $\left(-\frac{3}{2}, -1\right)$
2. $g(x) = (x - 1.44)^2 + 0.8$ $(1.44, 0.8)$
3. $h(x) = \left(x - \frac{2}{5}\right)^2 - 5$ $\left(\frac{2}{5}, -5\right)$
4. $k(x) = (x + 17)^2 + 11$ $(-17, 11)$

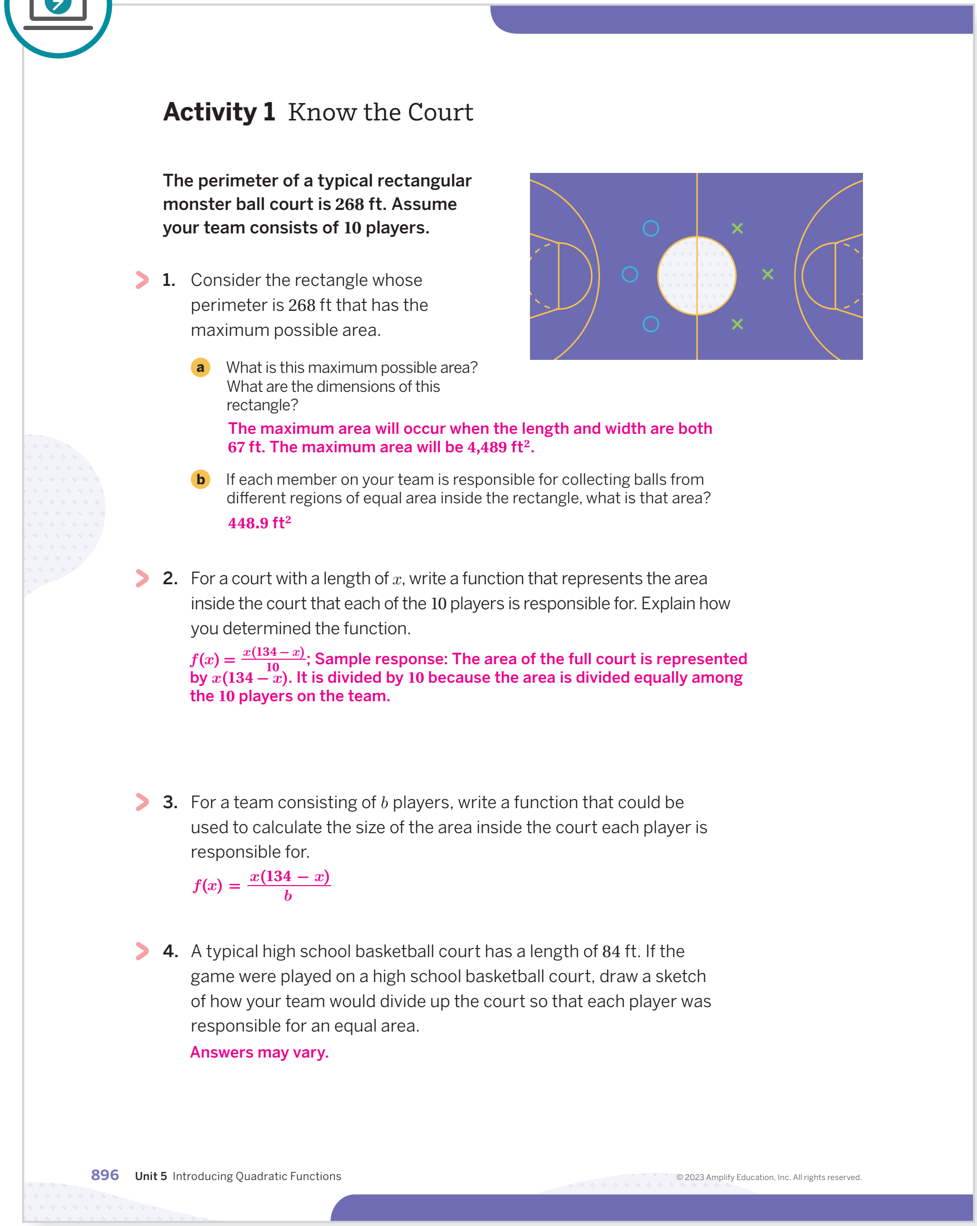
Use: Before Activity 1

Informed by: Performance on Lesson 23, Practice Problem 5

Pairs | 15 min

Activity 1 Know the Court

Students write a function to model possible sizes of a Monster Ball court and to determine players' starting locations.



Activity 1 Know the Court

The perimeter of a typical rectangular monster ball court is 268 ft. Assume your team consists of 10 players.

1. Consider the rectangle whose perimeter is 268 ft that has the maximum possible area.
   - a What is this maximum possible area? What are the dimensions of this rectangle?
     The maximum area will occur when the length and width are both 67 ft. The maximum area will be 4,489 ft^2.
   - b If each member on your team is responsible for collecting balls from different regions of equal area inside the rectangle, what is that area?
     448.9 ft^2
2. For a court with a length of x, write a function that represents the area inside the court that each of the 10 players is responsible for. Explain how you determined the function.
   $f(x) = \frac{x(134-x)}{10}$; Sample response: The area of the full court is represented by $x(134-x)$. It is divided by 10 because the area is divided equally among the 10 players on the team.
3. For a team consisting of b players, write a function that could be used to calculate the size of the area inside the court each player is responsible for.
   $f(x) = \frac{x(134-x)}{b}$
4. A typical high school basketball court has a length of 84 ft. If the game were played on a high school basketball court, draw a sketch of how your team would divide up the court so that each player was responsible for an equal area.
   Answers may vary.

896 **Unit 5** Introducing Quadratic Functions

1 Launch

Read the description of the court aloud. Activate students' background knowledge by asking them about possible starting positions of players on a team. Have pairs of students brainstorm and then share with the class.

2 Monitor

Help students get started by saying, "Try sketching a few possible court sizes with different dimensions that fit the perimeter."

Look for points of confusion:

- **Representing width with $268 - x$.** Let x represent the length. Use the formula of the perimeter of a rectangle to determine an expression for the width.

Look for productive strategies:

- Using functions to calculate the area each player is responsible for on their sketch.

3 Connect

Have individual students share their sketches and thinking behind their sketch.

Highlight that for a rectangular court with a fixed perimeter, the maximum area will occur when the court is a square, when each side has a length of 67 ft.

Ask, "How could you use your function to calculate the maximum possible area?" I could locate the vertex on the graph, or evaluate the function at the midpoint of the zeros.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

Remind students they have previously written quadratic expressions that represent the area of a rectangle with a given (fixed) perimeter. Ask:

- "What is the formula for the area of a rectangle?"
- "How can you write an equation for the area using only one variable? How can you write the length in terms of the width, or vice versa?"
- "What strategies can you use to determine the maximum possible area?"

Extension: *Math Enrichment*

Have students write their function in Problem 2 if the number of players is n and the perimeter of the court is P. $f(x) = \frac{x\left(\frac{P}{2}-x\right)}{n}$

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement, such as "The maximum area for the rectangular court whose perimeter is 268 ft is 17,956 ft^2 because I halved the perimeter and then squared it." Ask:

- ***Critique:*** "Do you agree with this statement? Explain your thinking."
- ***Correct and Clarify:*** "Write a corrected statement. What strategies and mathematical language can you use to verify your statement is correct?"

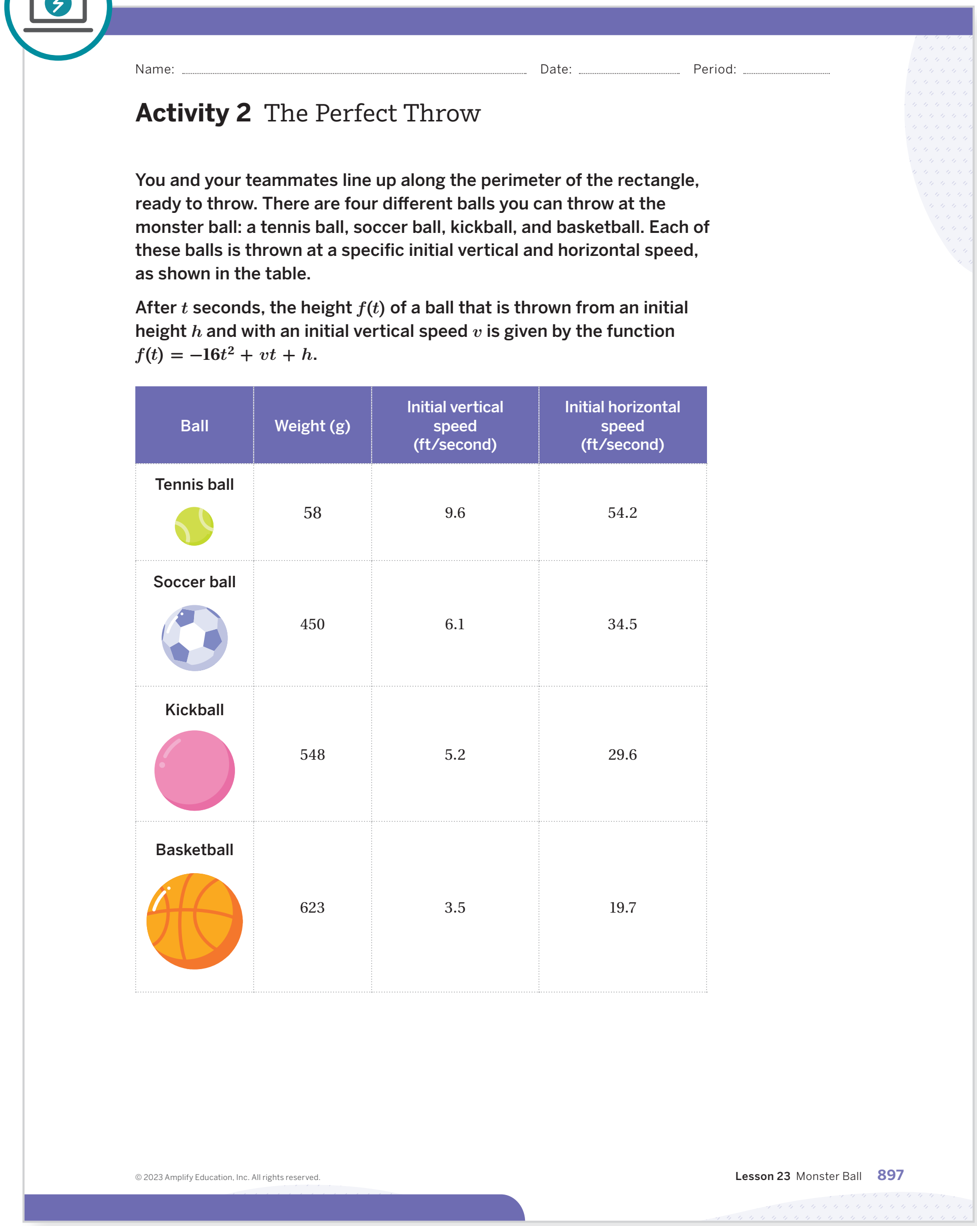
English Learners

Display or provide access to the Anchor Chart PDF, *Sentence Stems*, *Critiquing* for students to refer to as they critique the statement.

Pairs | 20 min

Activity 2 The Perfect Throw

Students write equations to represent throws of different balls, and use these equations to determine whether a throw hits or misses the monster ball.



Name: ______________ Date: ________ Period: ________

Activity 2 The Perfect Throw

You and your teammates line up along the perimeter of the rectangle, ready to throw. There are four different balls you can throw at the monster ball: a tennis ball, soccer ball, kickball, and basketball. Each of these balls is thrown at a specific initial vertical and horizontal speed, as shown in the table.

After t seconds, the height $f(t)$ of a ball that is thrown from an initial height h and with an initial vertical speed v is given by the function $f(t) = -16t^2 + vt + h$.

Ball	Weight (g)	Initial vertical speed (ft/second)	Initial horizontal speed (ft/second)
Tennis ball	58	9.6	54.2
Soccer ball	450	6.1	34.5
Kickball	548	5.2	29.6
Basketball	623	3.5	19.7

1 Launch

Read the prompt aloud. Review the different size balls and types of throws. Provide students with graphing technology.

2 Monitor

Help students get started by modeling different types of throws and having them share their thoughts on which ball would be best for each throw.

Look for points of confusion:

- **Mixing the use of horizontal and vertical speed.** Say, "The function for projectiles that you know how to create is a function that models height. Which speed relates to height?"
- **Using the horizontal intercept of the graph of $f(t)$.** Ask, "What is the height of the ball as it travels along its path, assuming it hits the center of the monster ball?" 1 ft Review how to estimate the time when the function has a value of 1.

Look for productive strategies:

- Using the tracing tool on the graphing technology to determine the time that $f(t)$ has a value of 1.
- Verifying the times found in Problem 2 by evaluating each function at these times, to verify each function has a value of 1.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the equation $f(x) = -16t^2 + 9.6t + 4$ that represents the height $f(t)$ of the tennis ball, if the ball is thrown from a height of 4 ft. Ask:

- "Why was the initial vertical speed used in the function and not the initial horizontal speed?"
- "Why is the weight of the ball not represented in the equation?"

Extension: *Math Enrichment*

Ask students to explain why the standard form of the quadratic function is the most appropriate form to use in this activity. Sample response: It gives the initial height, initial vertical speed, and effects of gravity.

Math Language Development

MLR8: Discussion Supports

While students work, display these frames for Problem 2 that they could refer to as they construct their responses and organize their thinking.

- "She should stand about ___ ft away from the monster ball."
- "The function ___ models the height of ___ from an initial height of ___ ft."
- "The ___ will reach a height of 1 ft at about ___ seconds."
- "The distance can be found by . . ."

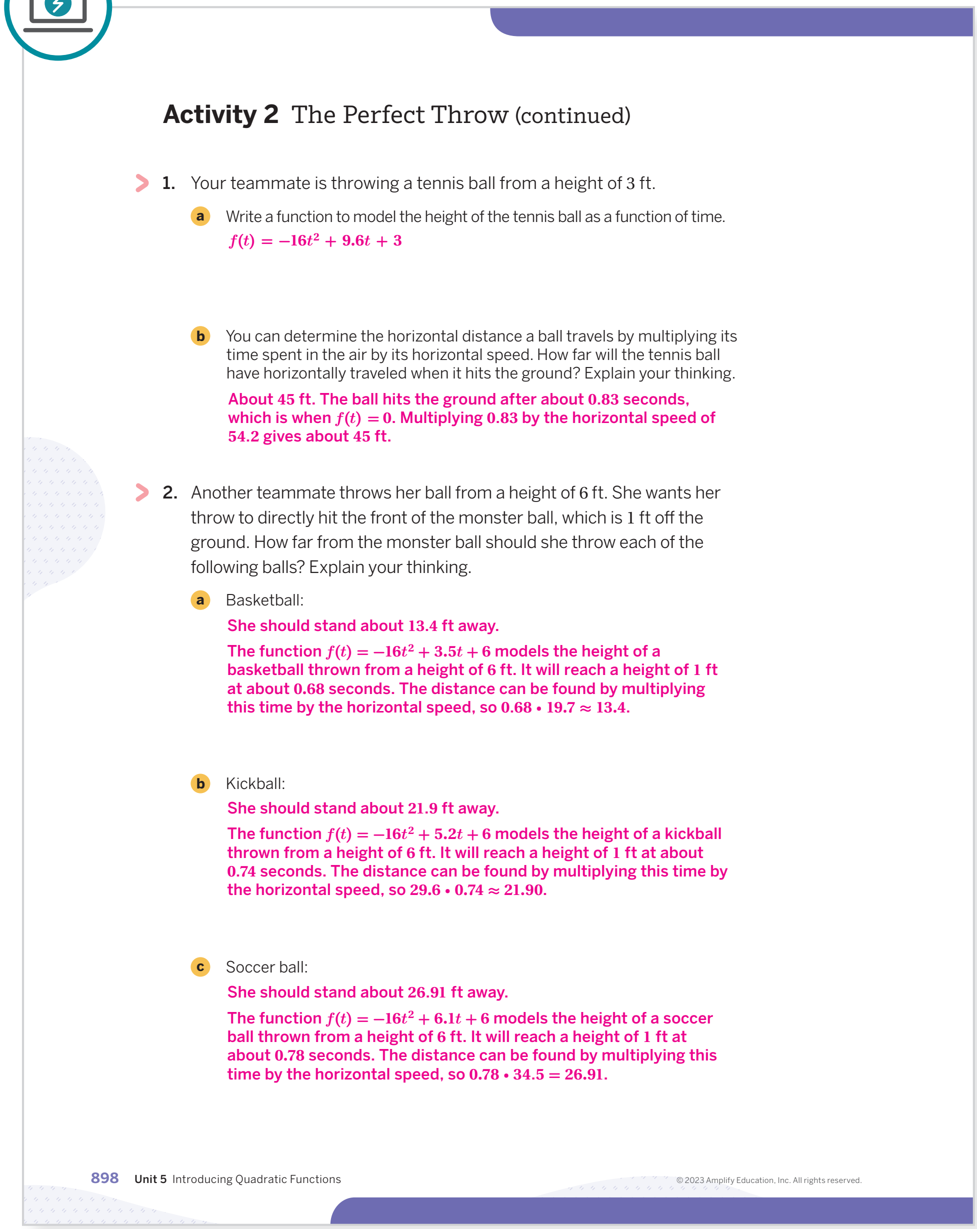
English Learners

Demonstrate how to draw a quick sketch to illustrate a ball hitting the front of the monster ball at a height of 1 ft.

Pairs | 20 min

Activity 2 The Perfect Throw (continued)

Students write equations to represent throws of different balls, and use these equations to determine whether a throw hits or misses the monster ball.



Activity 2 The Perfect Throw (continued)

1. Your teammate is throwing a tennis ball from a height of 3 ft.
   - a Write a function to model the height of the tennis ball as a function of time.

     $f(t) = -16t^2 + 9.6t + 3$
   - b You can determine the horizontal distance a ball travels by multiplying its time spent in the air by its horizontal speed. How far will the tennis ball have horizontally traveled when it hits the ground? Explain your thinking.

     About 45 ft. The ball hits the ground after about 0.83 seconds, which is when $f(t) = 0$. Multiplying 0.83 by the horizontal speed of 54.2 gives about 45 ft.
2. Another teammate throws her ball from a height of 6 ft. She wants her throw to directly hit the front of the monster ball, which is 1 ft off the ground. How far from the monster ball should she throw each of the following balls? Explain your thinking.
   - a Basketball:

     She should stand about 13.4 ft away.

     The function $f(t) = -16t^2 + 3.5t + 6$ models the height of a basketball thrown from a height of 6 ft. It will reach a height of 1 ft at about 0.68 seconds. The distance can be found by multiplying this time by the horizontal speed, so $0.68 \cdot 19.7 \approx 13.4$.
   - b Kickball:

     She should stand about 21.9 ft away.

     The function $f(t) = -16t^2 + 5.2t + 6$ models the height of a kickball thrown from a height of 6 ft. It will reach a height of 1 ft at about 0.74 seconds. The distance can be found by multiplying this time by the horizontal speed, so $29.6 \cdot 0.74 \approx 21.90$.
   - c Soccer ball:

     She should stand about 26.91 ft away.

     The function $f(t) = -16t^2 + 6.1t + 6$ models the height of a soccer ball thrown from a height of 6 ft. It will reach a height of 1 ft at about 0.78 seconds. The distance can be found by multiplying this time by the horizontal speed, so $0.78 \cdot 34.5 = 26.91$.

898 **Unit 5** Introducing Quadratic Functions

3 Connect

Display the function $f(t) = -16t^2 + vt + 6$.

Have individual students share how they determined the value of v and calculated the distance for each ball for Problem 2.

Highlight that students are aiming for the center of the monster ball. The radius of the monster ball is 1 ft, which is why students need to determine the time the throw reaches a height of 1.

Ask, "What is the range of heights that your throw could be so that it still hits the monster ball? What would the contact with the monster ball be like at the boundaries of this range?" From 0 ft to 2 ft; At 0 ft, the throw would just barely touch the bottom of the monster ball. At 2 ft, the throw would touch the very top of the monster ball.

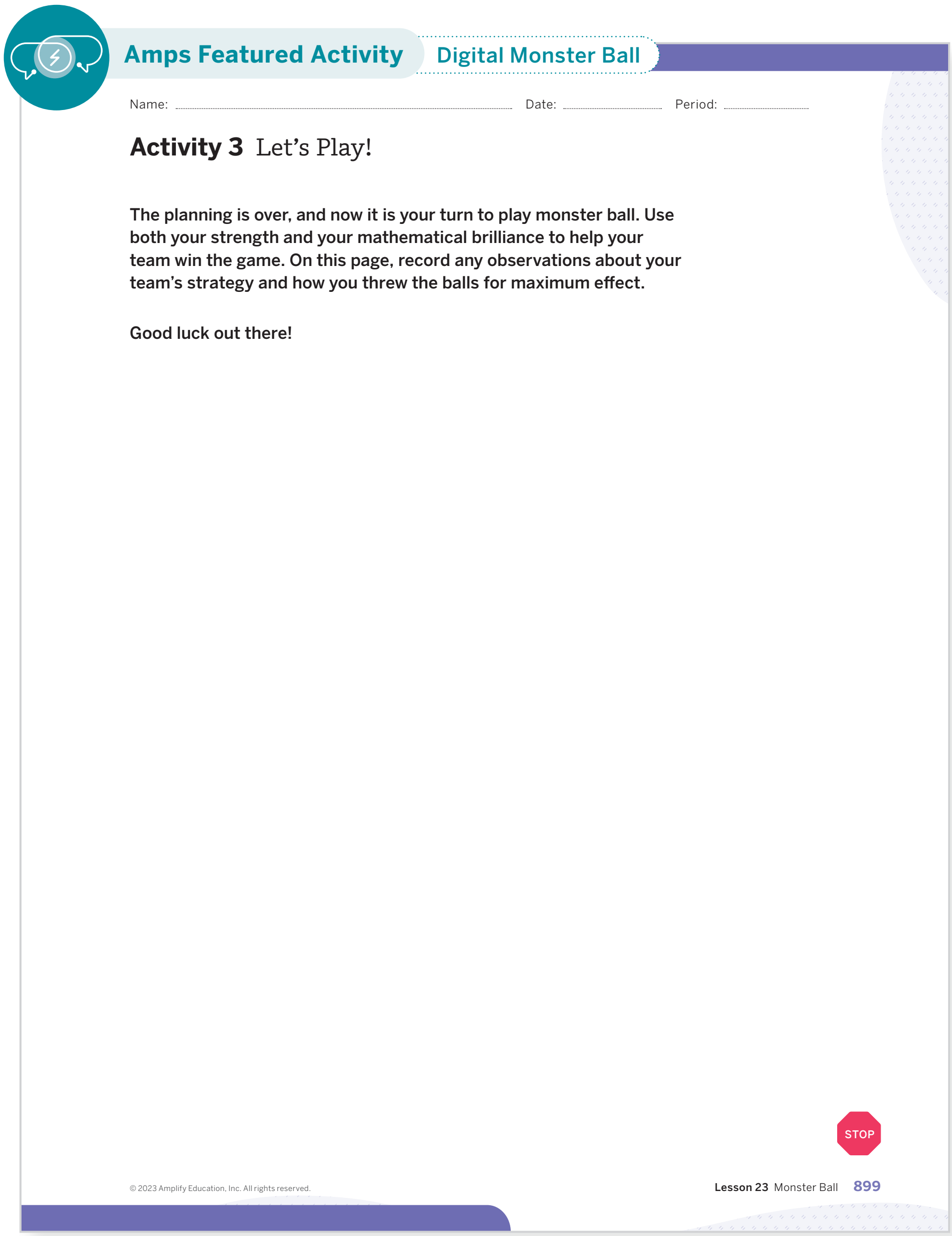
Optional

Small Groups | 30 min

Activity 3 Let's Play!

Students use their equations and approaches to determine the starting location of their team and to choose a ball.

Amps Featured Activity Digital Monster Ball



Name: ______ Date: ______ Period: ______

Activity 3 Let's Play!

The planning is over, and now it is your turn to play monster ball. Use both your strength and your mathematical brilliance to help your team win the game. On this page, record any observations about your team's strategy and how you threw the balls for maximum effect.

Good luck out there!

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 23 Monster Ball 899

1 Launch

This activity is for classes that are able to play a game of Monster Ball. Say, "Now it is time to use your insights from today to get ready to play a game of Monster Ball." Students will meet within their team.

2 Monitor

Help students get started by highlighting key information of different throws.

Look for points of confusion:

- **Thinking a different throw is necessary to hit the monster ball.** Ask students to consider if throwing the ball up higher or lower will cause the ball to hit the monster ball. If so, have them explain these adjustments.

Look for productive strategies:

- Teams assigning a captain to make starting location choices.
- Asking questions regarding the size of the field.
- Adjusting players' spacing similarly to Activity 1.

3 Connect

Have individual students share how their team decided on starting locations and why they chose their ball.

Highlight that the speed in the table is the average speed of overhand and underhand throws launched along the normal trajectory of an overhand and underhand throw. When it is time to throw, they may have to adjust their release.

Ask, "Who revised their ball choice? Why did you decide to revise your choice, and what is the new ball and throw style you chose?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can play a digital version of Monster Ball. Their calculations will be evaluated in real time to check against their predictions.

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

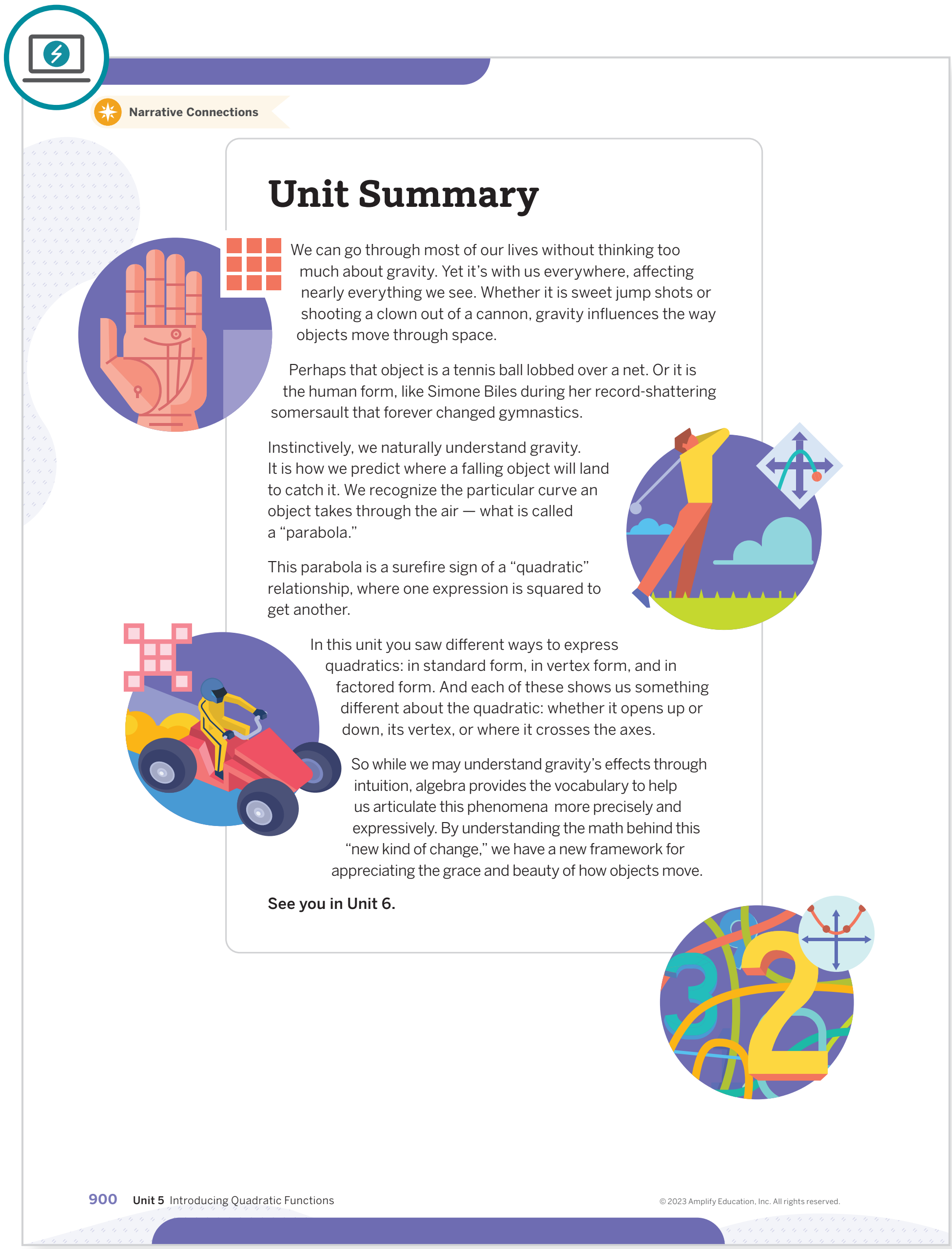
Once a student decides on a starting location and a ball to use . . .

- Provide them with three options for throwing the ball. If they need to revise their choice, have them continue to choose from these options.
- Display the following function that they can use to complete. $f(x) = -16x^2 +$ [initial vertical speed] $\cdot\, x +$ [initial height]

Whole Class | 5 min

Unit Summary

Review and synthesize how quadratic functions can be used to model relationships in the real world.



Narrative Connections

Unit Summary

We can go through most of our lives without thinking too much about gravity. Yet it's with us everywhere, affecting nearly everything we see. Whether it is sweet jump shots or shooting a clown out of a cannon, gravity influences the way objects move through space.

Perhaps that object is a tennis ball lobbed over a net. Or it is the human form, like Simone Biles during her record-shattering somersault that forever changed gymnastics.

Instinctively, we naturally understand gravity. It is how we predict where a falling object will land to catch it. We recognize the particular curve an object takes through the air — what is called a "parabola."

This parabola is a surefire sign of a "quadratic" relationship, where one expression is squared to get another.

In this unit you saw different ways to express quadratics: in standard form, in vertex form, and in factored form. And each of these shows us something different about the quadratic: whether it opens up or down, its vertex, or where it crosses the axes.

So while we may understand gravity's effects through intuition, algebra provides the vocabulary to help us articulate this phenomena more precisely and expressively. By understanding the math behind this "new kind of change," we have a new framework for appreciating the grace and beauty of how objects move.

See you in Unit 6.

900 Unit 5 Introducing Quadratic Functions

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display two functions which represent the height of each throw, in feet, as a function of time in seconds: $h(t) = -16t^2 + 40t + 7$ and $f(t) = -16\left(t + \frac{1}{4}\right)(t - 2)$.

Have students share how they could determine whether the player's throw reaches the monster ball.

Highlight that if one of the pieces of information provided was missing, it would not be possible to construct a function to model the height of the throw or distance.

Ask,

- "How do you determine the domain of each throw?" The domain is from 0 to the time it takes the ball to hit the ground.
- "How could you compare other key features of each throw?" I could expand $f(t)$ and rewrite it in standard form to compare the initial vertical velocity and initial height of the throw.
- "How could you use the graphs of the functions to determine whether the balls hit the monster ball?" I can determine when the balls reach the height of the monster ball, and multiply this time by the horizontal speed to determine the horizontal distance the ball travels.

Reflect

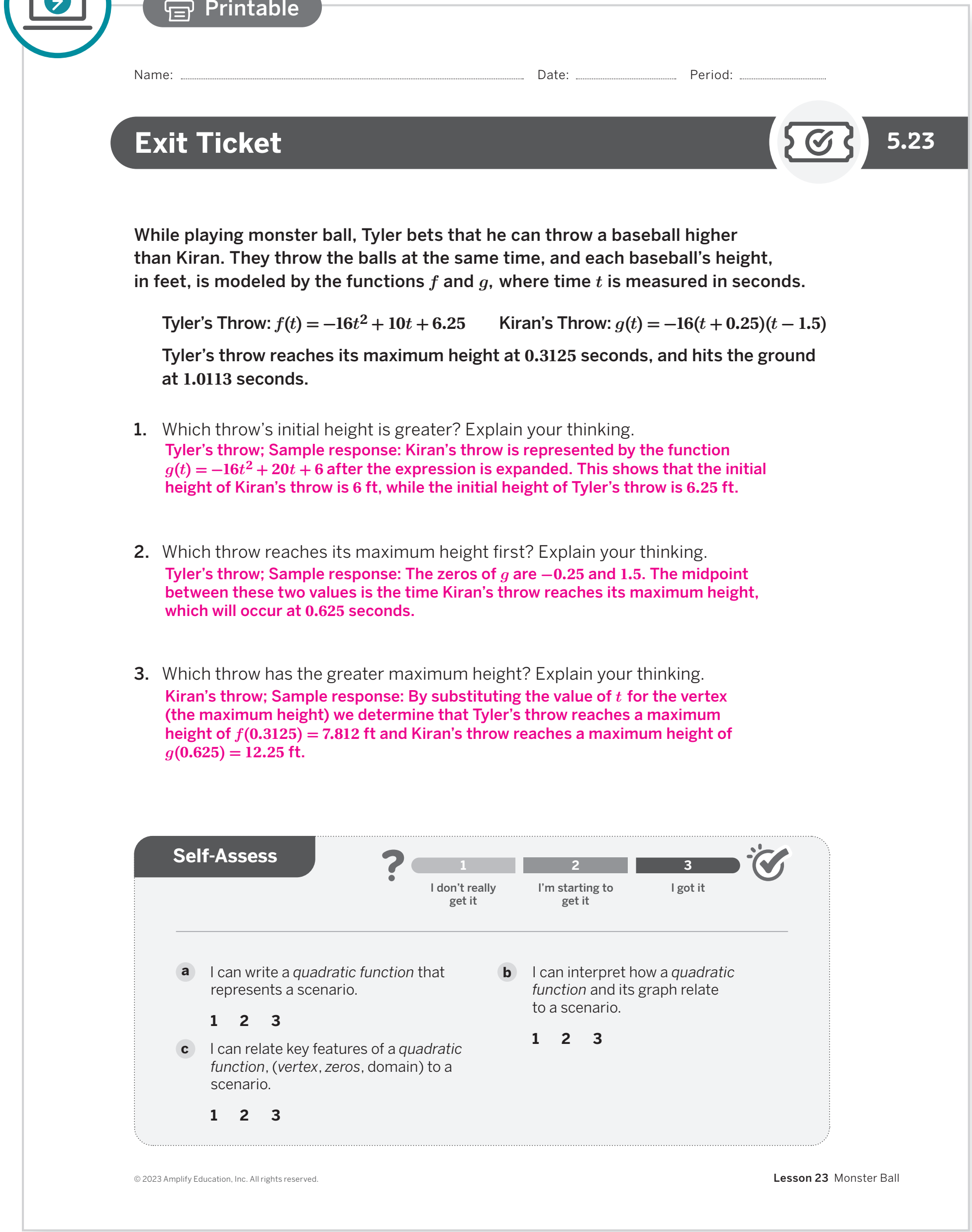
After synthesizing the concepts of this unit, allow students a few moments for reflection around the unit narratives. To help them engage in meaningful reflection, consider asking:

- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Exit Ticket

Students demonstrate their understanding by writing quadratics from a scenario and interpreting key features in context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.23

While playing monster ball, Tyler bets that he can throw a baseball higher than Kiran. They throw the balls at the same time, and each baseball's height, in feet, is modeled by the functions f and g, where time t is measured in seconds.

Tyler's Throw: $f(t) = -16t^2 + 10t + 6.25$ **Kiran's Throw: $g(t) = -16(t + 0.25)(t - 1.5)$**

Tyler's throw reaches its maximum height at 0.3125 seconds, and hits the ground at 1.0113 seconds.

1. Which throw's initial height is greater? Explain your thinking.
   Tyler's throw; Sample response: Kiran's throw is represented by the function $g(t) = -16t^2 + 20t + 6$ after the expression is expanded. This shows that the initial height of Kiran's throw is 6 ft, while the initial height of Tyler's throw is 6.25 ft.

2. Which throw reaches its maximum height first? Explain your thinking.
   Tyler's throw; Sample response: The zeros of g are -0.25 and 1.5. The midpoint between these two values is the time Kiran's throw reaches its maximum height, which will occur at 0.625 seconds.

3. Which throw has the greater maximum height? Explain your thinking.
   Kiran's throw; Sample response: By substituting the value of t for the vertex (the maximum height) we determine that Tyler's throw reaches a maximum height of $f(0.3125) = 7.812$ ft and Kiran's throw reaches a maximum height of $g(0.625) = 12.25$ ft.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can write a *quadratic function* that represents a scenario.
1 2 3

b I can interpret how a *quadratic function* and its graph relate to a scenario.
1 2 3

c I can relate key features of a *quadratic function*, (*vertex*, *zeros*, domain) to a scenario.
1 2 3

Lesson 23 Monster Ball

Success looks like . . .

- **Language Goal:** Writing a quadratic function that represents a scenario. **(Reading and Writing)**
- **Language Goal:** Interpreting how a quadratic function and its graph relate to a scenario. **(Reading and Writing)**
- **Language Goal:** Relating key features of a quadratic function, (vertex, zeros, domain) to a scenario. **(Speaking and Listening, Writing)**
  - » Determining the throw with the greater maximum height in Problem 3.

Suggested next steps

If students incorrectly identify the initial heights for Problem 1, consider:

- Reviewing the use of initial height in the functions in Activity 2.
- Assigning Practice Problem 1.
- Asking, "How is the initial height represented in the function of a projectile?"

If students cannot identify which throw reaches its maximum height first in Problem 2, consider:

- Reviewing how to use the zeros of functions to maximize the court area in Activity 1.
- Assigning Practice Problems 5 and 6.
- Asking, "How could you determine the zeros of the Kiran's function? How could you use the zeros to identify the time his throw reaches its maximum height?"

If students cannot identify which throw has the greatest maximum height in Problem 3, consider:

- Review determining the maximum height in Activity 2.
- Assigning Practice Problems 5 and 6.

Professional Learning

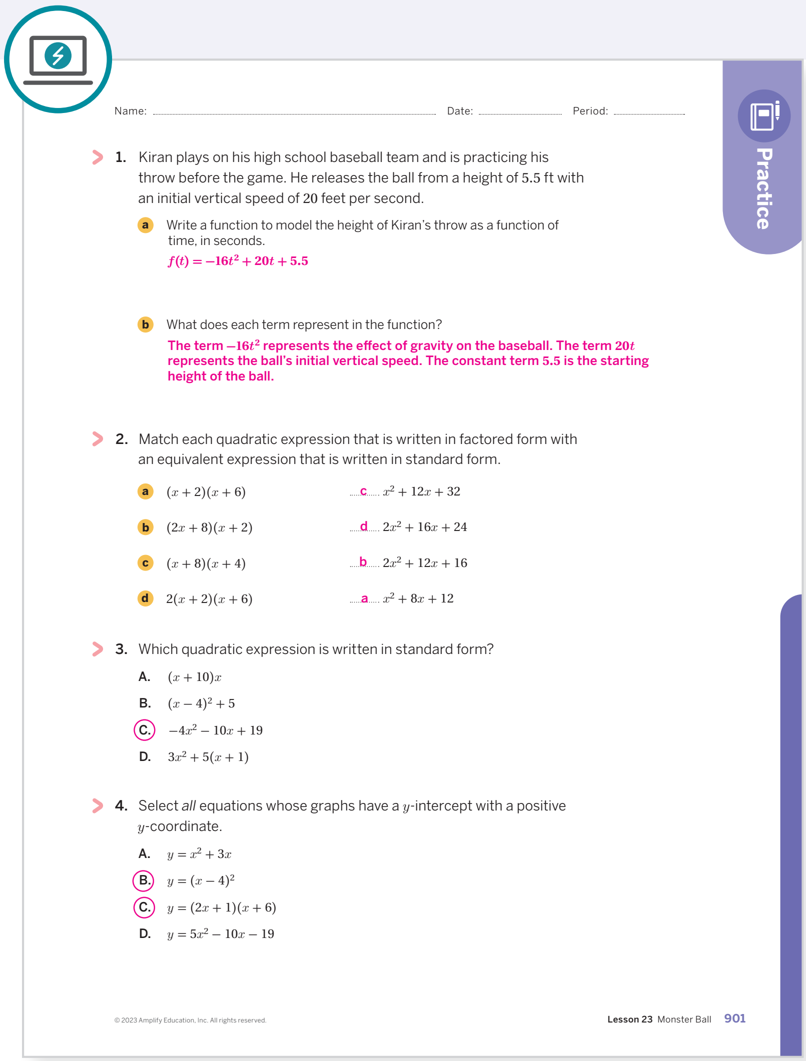
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What did students find frustrating about writing a quadratic function to represent the court size? What helped them work through this frustration?
- What different ways did students approach developing a team strategy to plan monster ball? What does that tell you about similarities and differences among your students?

Practice

Independent



Name: Date: Period:

Practice

1. Kiran plays on his high school baseball team and is practicing his throw before the game. He releases the ball from a height of 5.5 ft with an initial vertical speed of 20 feet per second.
   - a Write a function to model the height of Kiran's throw as a function of time, in seconds.
     $f(t) = -16t^2 + 20t + 5.5$
   - b What does each term represent in the function?
     The term $-16t^2$ represents the effect of gravity on the baseball. The term $20t$ represents the ball's initial vertical speed. The constant term 5.5 is the starting height of the ball.

2. Match each quadratic expression that is written in factored form with an equivalent expression that is written in standard form.

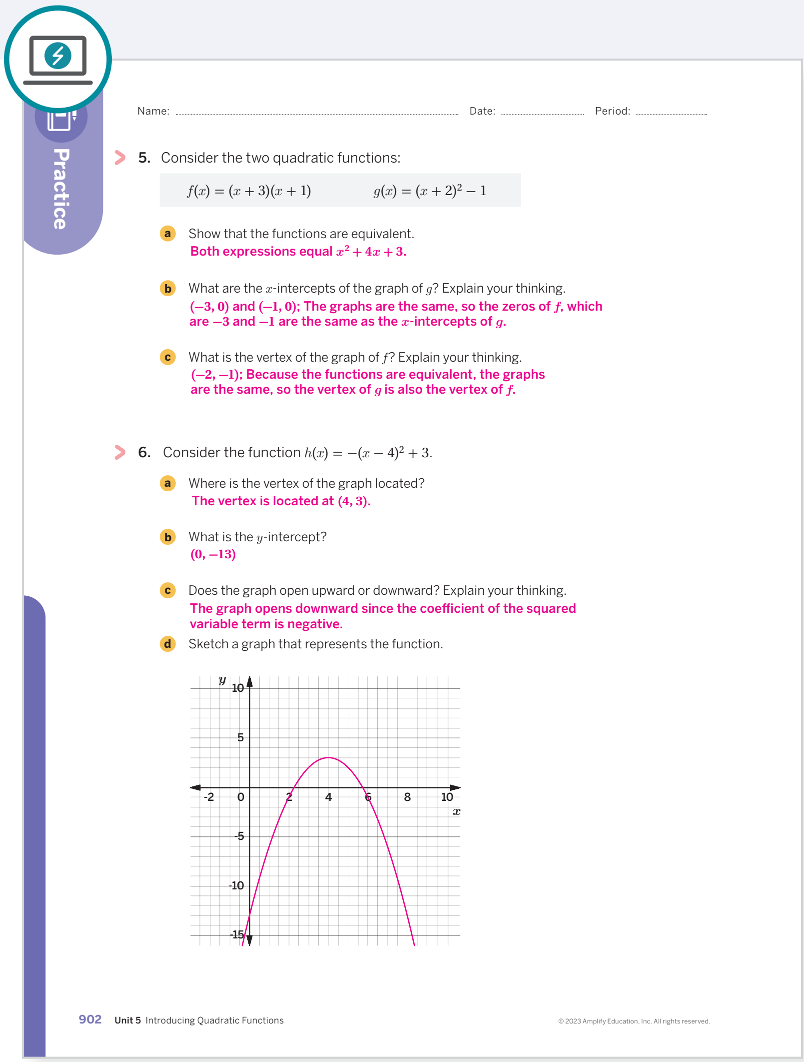
a	$(x+2)(x+6)$	c	$x^2 + 12x + 32$
b	$(2x+8)(x+2)$	d	$2x^2 + 16x + 24$
c	$(x+8)(x+4)$	b	$2x^2 + 12x + 16$
d	$2(x+2)(x+6)$	a	$x^2 + 8x + 12$

3. Which quadratic expression is written in standard form?
   - A. $(x+10)x$
   - B. $(x-4)^2+5$
   - [✓ C.] $-4x^2 - 10x + 19$
   - D. $3x^2 + 5(x+1)$

4. Select *all* equations whose graphs have a y-intercept with a positive y-coordinate.
   - A. $y = x^2 + 3x$
   - [✓ B.] $y = (x-4)^2$
   - [✓ C.] $y = (2x+1)(x+6)$
   - D. $y = 5x^2 - 10x - 19$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 23 Monster Ball 901



Name: Date: Period:

Practice

5. Consider the two quadratic functions:

   $f(x) = (x+3)(x+1)$ $g(x) = (x+2)^2 - 1$

   - a Show that the functions are equivalent.
     Both expressions equal $x^2 + 4x + 3$.
   - b What are the x-intercepts of the graph of g? Explain your thinking.
     $(-3, 0)$ and $(-1, 0)$; The graphs are the same, so the zeros of f, which are -3 and -1 are the same as the x-intercepts of g.
   - c What is the vertex of the graph of f? Explain your thinking.
     $(-2, -1)$; Because the functions are equivalent, the graphs are the same, so the vertex of g is also the vertex of f.

6. Consider the function $h(x) = -(x-4)^2 + 3$.
   - a Where is the vertex of the graph located?
     The vertex is located at $(4, 3)$.
   - b What is the y-intercept?
     $(0, -13)$
   - c Does the graph open upward or downward? Explain your thinking.
     The graph opens downward since the coefficient of the squared variable term is negative.
   - d Sketch a graph that represents the function.

902 Unit 5 Introducing Quadratic Functions

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
Spiral	2	Unit 5 Lesson 10	2
	3	Unit 5 Lesson 12	1
	4	Unit 5 Lesson 16	2
	5	Unit 5 Lesson 19	2
	6	Unit 5 Lesson 20	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6

Quadratic Equations

In this unit, students write and solve quadratic equations and make sense of the solutions. They investigate the structure of quadratic equations and determine efficient strategies for solving them.

Essential Questions

- How does solving quadratic equations compare to solving linear equations?
- How does the structure of a quadratic equation determine efficient strategies for solving it algebraically?
- How are quadratic equations used to solve real-world problems?
- *(By the way, in which year was a new way to solve quadratic equations discovered: 628 CE or 2019 CE?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students are introduced to a variety of algebraic strategies for solving quadratic equations including factoring, completing the square, and the quadratic formula. They will study the structure of monic and non-monic quadratic equations and determine which strategies are more efficient for certain equations and explain why. Students will also be exposed to the history of how quadratic equations were studied over time.

Coherence

Previously . . .

In Unit 5, students examined the graphs and the different algebraic forms of quadratic functions. They also wrote, graphed, and understood quadratic functions in real-world scenarios.

Coming soon . . .

In Algebra 2, students will further their understanding of quadratic equations and their solutions by also exploring their complex solutions. They will see that quadratics are a type of polynomial and relate its degree to the number of roots.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students conceptualize the solutions of quadratic equations in Lessons 2–5, visualize completing the square in Lesson 12, rewrite forms of expressions in Lesson 14, and derive the quadratic formula in Lesson 19.

Procedural Fluency

Students practice factoring and solving by factoring in Lessons 6–11. They practice completing the square in Lessons 13 and 15, and they use the quadratic formula in Lessons 19–21.

Application

Throughout the unit, students have opportunities to consider and solve problems as ancient mathematicians would. Students reason and determine efficient strategies for solving different quadratic equations in Lessons 19–23.

The Evolution of Solving Quadratic Equations

SUB-UNIT

1

Lessons 2–5

Connecting Quadratic Functions to Their Equations

Students revisit projectile motion and maximizing revenue as they recall the meaning of a quadratic function's zeros. They examine quadratic expressions in standard and factored form and determine which form indicates the solutions to ***quadratic equations***.

Narrative: The story of solving quadratic equations began with the need to calculate area.

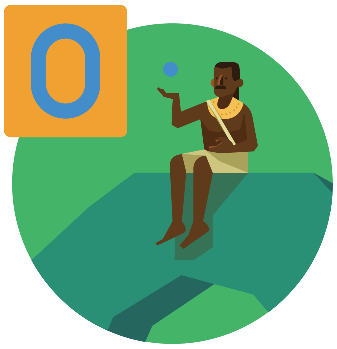
SUB-UNIT

2

Lessons 6–10

Factoring Quadratic Expressions and Equations

Students are introduced to several strategies for factoring quadratic expressions. By setting quadratic expressions equal to zero, they examine the structure of different quadratic equations, considering their ***coefficients***, ***constant terms***, and ***linear terms***.



Narrative: Discover what happens when you set a quadratic expression equal to zero.

SUB-UNIT

3

Lessons 11–15

Completing the Square

Students analyze the structure of ***square expressions***. They visualize ***completing the square*** using algebra tiles and area diagrams and determine the steps to complete the square for monic and non-monic quadratic expressions. They apply this method of completing the square to solve quadratic equations.

Narrative: Turn to geometry and your old friend, the square, to solve quadratic equations.

Launch

Lesson 1

Determining Unknown Inputs

Students attempt to frame a picture — physically *and* algebraically — modeling the problem using a quadratic equation. They will recognize the limitations of the strategies they have used up to this point, as they engage in productive struggle and motivate the need for strategies to solve quadratic equations.

SUB-UNIT

4

Lessons 16–18

Roots and Irrationals

Students express irrational solutions (and their rational approximations) to quadratic equations by completing the square and using graphing technology. They classify the sums and products of rational and irrational numbers and make sense of irrational numbers through the process of ***rationalizing denominators***.

Narrative: Explore how the world beyond rational numbers relates to quadratic equations.

SUB-UNIT

5

Lessons 19–23

The Quadratic Formula

Students derive the ***quadratic formula*** and verify that it produces the same solutions as factoring and completing the square. They examine the efficiency of different strategies learned in the unit and are introduced to the latest way to solve quadratic questions, discovered in 2019.

Narrative: Al-khwarizmi documented a way to solve *any* quadratic equation.

Lesson 24

Capstone

The Latest Way to Solve Quadratic Equations

Students investigate the relationship between a quadratic equation and its solutions. They use this knowledge to reconstruct Professor Po-shen Loh's strategy for solving quadratic equations, which was discovered in 2019.

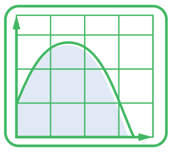
Unit at a Glance

Spoiler Alert: While the quadratic formula can solve any quadratic equation, students should work through the other methods first. They will not formally encounter the quadratic formula until Lesson 19.

Assessment

A Pre-Unit Readiness Assessment

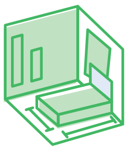
Launch Lesson



1 Determining Unknown Inputs

Frame pictures to motivate the need for solving quadratic equations. The term quadratic equation is informally used, but not formally defined.

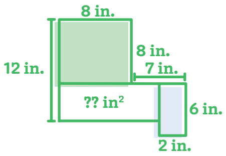
Sub-Unit 1:



2 When and Why Do We Write Quadratic Equations? ●

Consider the different forms of quadratic equations and possible strategies for solving them.

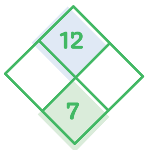
Sub-Unit 2: Factoring Quadratic Expressions and Equations



6 Writing Quadratic Expressions in Factored Form (Part 1) ●

🔑

Use area diagrams to model the factoring of monic quadratic expressions. Define the terms coefficient, constant term, and linear term.



7 Writing Quadratic Expressions in Factored Form (Part 2) ●

🔑

Use the Distributive Property to develop strategies for factoring monic quadratic expressions.

$$\frac{(54^2 - 29^2)}{54 + 29}$$

8 Special Types of Factors

🔑

Analyze and apply the structure of quadratic expressions that are in the form of difference of squares.

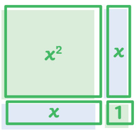
Sub-Unit 3: Completing the Square

$$(x + 1)^2$$

11 Square Expressions

🔑

Analyze the structure of square expressions by factoring and expanding. Use structure to help solve quadratic equations.



12 Completing the Square

🔑

Use algebra tiles, area diagrams, and algebraic steps to complete the square for monic quadratic expressions.

$$x^2 + 10x + \square$$

13 Solving Quadratic Equations by Completing the Square

🔑

Solve monic quadratic equations, including those with non-integer coefficients, by completing the square. Then analyze common errors.

Key Concepts

Lessons 6–12: Different factoring strategies are introduced for solving quadratic equations.
Lessons 13–15: Completing the square is visualized, defined, and applied.
Lessons 19–24: The quadratic formula is derived and applied to real-world scenarios.

Pacing

24 Lessons: 50 min each
3 Assessments: 45 min each

Full Unit: 27 days
● **Modified Unit:** 22 days

Assumes 50 minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Connecting Quadratic Functions to Their Equations

HOW MANY SOLUTIONS?

0 1 2

3 Solving Quadratic Equations by Reasoning ●

Use reasoning to determine the values needed to make quadratic equations true, and strategize about how to solve them.

$$a \cdot b = \boxed{0}$$

4 The Zero Product Principle ●

Examine how zero was used by an Egyptian scribe. Solve factored quadratic equations by applying the Zero Product Principle.

TRUE / FALSE

$x^2 - 9 = 0$ $x = 3$

$x^2 + 25 = 0$ $x = -5$

5 How Many Solutions?

Make connections between the solutions of a quadratic equation and the horizontal intercepts of its graph.

FIND THAT NUMBER

$x^2 - 2x - 35 = 0$

9 Solving Quadratic Equations by Factoring

Solve monic quadratic equations by factoring and applying the Zero Product Principle. Then, examine when there are one vs. two solutions.

$(3x - 5)(x - 1)$

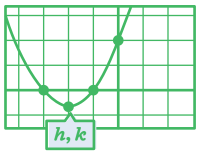
$3x^2 - 8x + 5$

10 Writing Non-Monic Quadratic Expressions in Factored Form

Factor non-monic expressions and recognize the limitations of solving quadratic equations using this strategy.

Assessment

A Mid-Unit Assessment



14 Writing Quadratic Expressions in Vertex Form

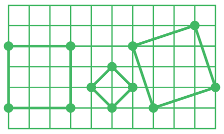
Review different forms of quadratic functions, then complete the square to convert quadratic expressions from standard form to vertex form.

$$(2x + 3)^2 = 4x^2 + 6x + \square$$

15 Solving Non-Monic Quadratic Equations by Completing the Square

Examine the structure of expanding and factoring square expressions to solve non-monic quadratic equations by completing the square.

Sub-Unit 4: Roots and Irrationals

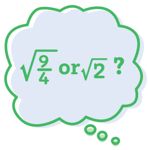


16 Quadratic Equations With Irrational Solutions

Express irrational solutions (and rational approximations) to quadratic equations by completing the square and using graphing technology.

Unit at a Glance

Spoiler Alert: While the quadratic formula can solve any quadratic equation, students should work through the other methods first. They will not formally encounter the quadratic formula until Lesson 19.



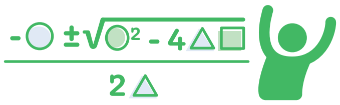
17 Rational and Irrational Numbers

Determine and classify the sums and products of rational and irrational numbers.

18 Rational and Irrational Solutions

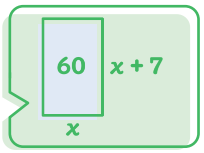
Make sense of irrational numbers through the process of rationalizing denominators.

Sub-Unit 5: The Quadratic Formula



19 A Formula for Any Quadratic

Derive the quadratic formula using area models, difference of squares, and completing the square.



23 Systems of Linear and Quadratic Equations

Given different linear and quadratic scenarios, write, solve, and interpret mixed systems of linear and quadratic equations.

Capstone Lesson

$$x^2 - 2x - 15 = 0$$

$$(1 + u)(1 - u) = -15$$

24 The Latest Way to Solve Quadratic Equations

Examine a recently discovered strategy for solving quadratic equations derived from ancient knowledge.

Assessment

A End-of-Unit Assessment

Key Concepts

Lessons 6–12: Different factoring strategies are introduced for solving quadratic equations.
Lessons 13–15: Completing the square is visualized, defined, and applied.
Lessons 19–24: The quadratic formula is derived and applied to real-world scenarios.

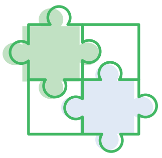
Pacing

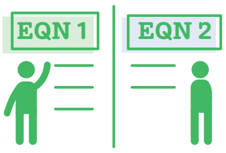
24 Lessons: 50 min each
3 Assessments: 45 min each

Full Unit: 27 days
● **Modified Unit:** 22 days

Assumes 50 minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$





20 The Quadratic Formula ●

Verify that the quadratic formula produces the same solutions as those found using other strategies, and examine the efficiency of different strategies.

21 Error Analysis: Quadratic Formula ●

Attend to precision while analyzing and classifying common errors made when applying the quadratic formula.

22 Applying the Quadratic Formula

Write quadratic equations to represent scenarios and realize the efficiency of the quadratic formula.

● Modifications to Pacing

Lesson 2: This lesson revisits concepts from Unit 5 and may be omitted, depending on student results from Unit 5 assessments.

Lessons 3–4: These lessons can be combined. Lesson 3 introduces the Zero Product Principle as a strategy for finding solutions to quadratic equations, while Lesson 4 provides additional practice.

Lessons 6–7: These lessons can be combined. Lesson 6 is a visual introduction to factoring using area diagrams, while Lesson 7 introduces strategies for factoring.

Lessons 20–21: These lessons can be combined. Lesson 20 is a procedural introduction to the quadratic formula, while Lesson 21 provides additional error analysis and an application problem.

Lesson 24: This lesson examines additional strategies for solving quadratic equations and may be omitted.

Unit Supports

Math Language Development

Lesson	New vocabulary
2	quadratic equation
4	Zero Product Principle
6	coefficient constant term linear term
8	difference of squares
9	monic expression
10	non-monic expression
11	square expression
12	completing the square
18	rationalizing the denominator
19	quadratic formula

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
5, 7, 19, 23	**MLR1:** Stronger and Clearer Each Time
1–4, 8–12, 18, 19	**MLR2:** Collect and Display
8, 13, 15, 16, 18, 22, 24	**MLR3:** Critique, Correct, Clarify
14	**MLR4:** Information Gap
1, 14, 19, 23	**MLR5:** Co-craft Questions
1, 9, 10, 21, 23	**MLR6**: Three Reads
1–7, 10–12, 14, 18	**MLR7:** Compare and Connect
3, 5, 6, 8–13, 15–18, 20, 23	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket   Additional Practice

Lesson(s)	Additional Required Materials
12–15, 19	algebra tiles
16	colored pencils
5	graph paper
2–5, 9, 10, 16, 18	graphing technology
1–5, 8–24	PDFs are required for these lessons. Refer to each lesson to see which activities require PDFs.
1	ruler
16, 17, 20–23	scientific calculator
1, 16	scissors
23	sticky notes

Instructional Routines

Activities throughout this unit include these routines:

Lesson(s)	Instructional Routines
5, 13, 15, 20, 21	Find and Fix
21	Gallery Tour
3	I Have, Who Has?
14	Info Gap
20	Jigsaw
3, 8, 13, 23	Math Talk
7, 9, 11, 13, 19	Notice and Wonder
2, 11, 12, 20, 24	Poll the Class
9, 22	Turn and Talk
10, 18, 20	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 10
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 24

Amps

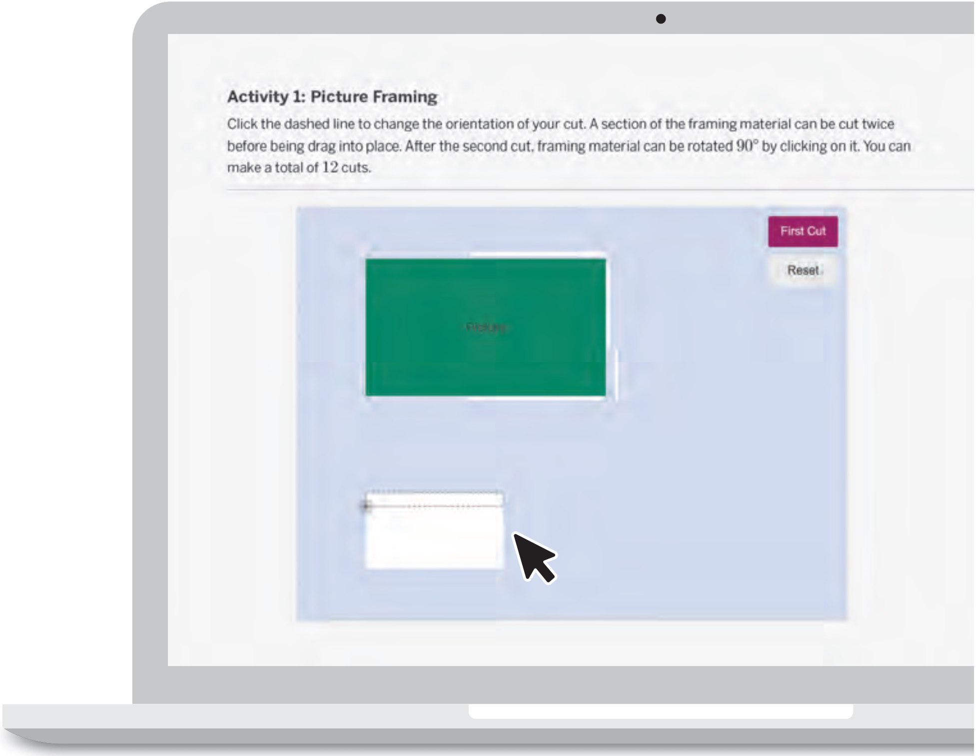
POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

A Trip to the Frame Shop

Put on your student hat and work through **Lesson 1, Activity 1:**



Activity 1: Picture Framing

Click the dashed line to change the orientation of your cut. A section of the framing material can be cut twice before being drag into place. After the second cut, framing material can be rotated 90° by clicking on it. You can make a total of 12 cuts.

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Diamond Puzzles **(Lesson 7)**
- Deriving Difference of Squares **(Lesson 8)**
- Building Complete Squares with Digital Algebra Tiles **(Lesson 12)**
- The Cannonball and the Pumpkin **(Lesson 22)**

Unit Study
Professional Learning

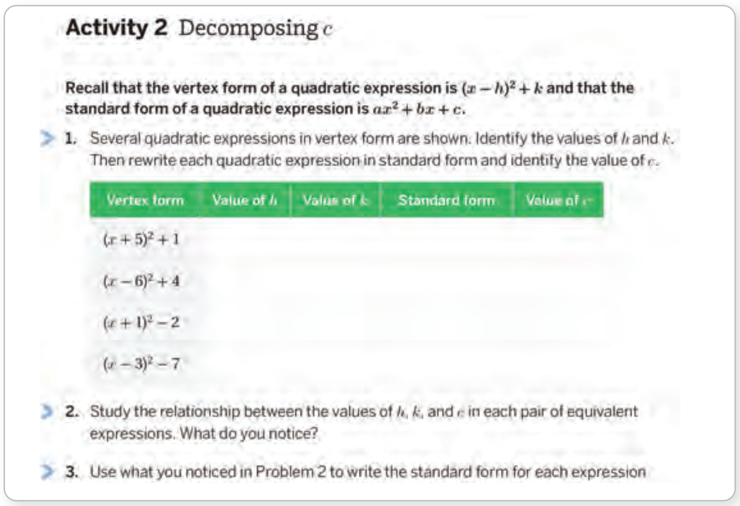
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 3 introduces square expressions and how to solve quadratic equations by completing the square. Students examine this strategy visually (via algebra tiles) and algebraically. They learn to write quadratic equations in vertex form from either the factored form or standard form. They extend the strategy of completing the square to solve for non-monic quadratic equations. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 14, Activity 2:



Activity 2 Decomposing c

Recall that the vertex form of a quadratic expression is $(x-h)^2+k$ and that the standard form of a quadratic expression is ax^2+bx+c.

1. Several quadratic expressions in vertex form are shown. Identify the values of h and k. Then rewrite each quadratic expression in standard form and identify the value of c.

Vertex form	Value of h	Value of k	Standard form	Value of c
$(x+5)^2+1$				
$(x-6)^2+4$				
$(x+1)^2-2$				
$(x-3)^2-7$				

2. Study the relationship between the values of h, k, and c in each pair of equivalent expressions. What do you notice?
3. Use what you noticed in Problem 2 to write the standard form for each expression

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

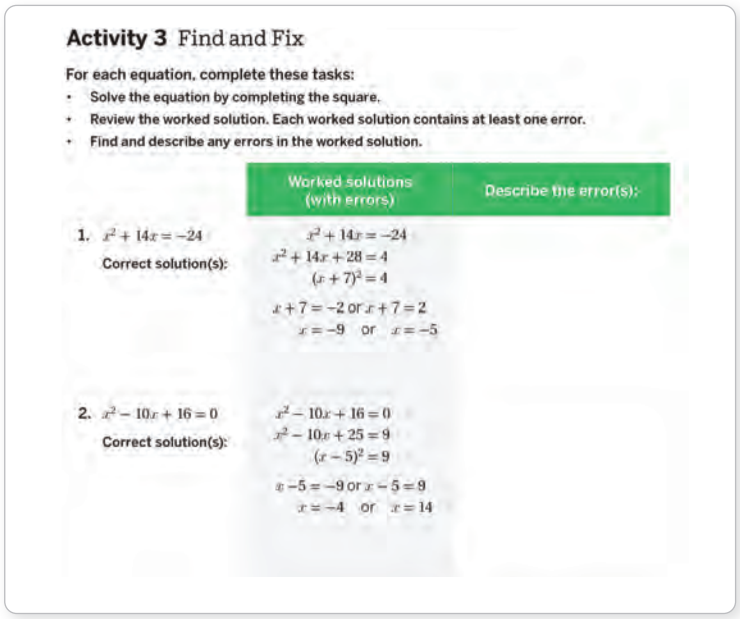
- What was it like to engage in this problem as a learner?
- This activity challenges students to go from vertex form to standard form, and vice versa. How does knowing the different forms help students make a quick sketch of a quadratic function?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Find and Fix

Rehearse . . .

How you'll facilitate the ***Find and Fix*** instructional routine in Lesson 13, Activity 3:



Activity 3 Find and Fix

For each equation, complete these tasks:

- **Solve the equation by completing the square.**
- **Review the worked solution. Each worked solution contains at least one error.**
- **Find and describe any errors in the worked solution.**

	Worked solutions (with errors)	Describe the error(s):
1. $x^2+14x=-24$ Correct solution(s):	$x^2+14x=-24$ $x^2+14x+28=4$ $(x+7)^2=4$ $x+7=-2$ or $x+7=2$ $x=-9$ or $x=-5$	
2. $x^2-10x+16=0$ Correct solution(s):	$x^2-10x+16=0$ $x^2-10x+25=9$ $(x-5)^2=9$ $x-5=-9$ or $x-5=9$ $x=-4$ or $x=14$	

Points to Ponder . . .

- Am I a model for the process of analyzing completed work and thinking aloud? Do I tend to find and fix student errors without explaining my process? Or do I ask questions to help students analyze their own work, take ownership of their errors, and correct them? How can I be more intentional about thinking out loud when correcting student errors?

This routine . . .

- Encompasses MLR8 Discussion Supports.
- Requires chunking tasks and checking in with students who would benefit from it.
- Uses elements of MP3, with students constructing viable arguments and critique reasoning of group members.
- Uses elements of MP6, with students needing to use precise mathematical language.

Anticipate . . .

- Intentional grouping of students to best support dialogue and focus.
- Modeling the think-aloud process for finding and fixing errors.
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Elicit and use evidence of student thinking.

This effective teaching practice . . .

- Helps you assess student progress toward the mathematical goals and objectives of the lessons and units. By knowing where your students are at, you can help them get to where they need to be!
- Allows you to adjust your instruction, based upon student responses, so that you can support your students and extend their learning.

Points to Ponder . . .

- How and when will I use student responses from this unit's assessments to help adjust my instruction?
- Where are there moments during the lessons' activities in which I can informally elicit evidence of my students' thinking? How will I approach these moments?

Math Language Development

MLR3: Critique, Correct, Clarify

MLR3 appears in Lessons 8, 13, 15, 16, 18, 22, and 24.

- In Lesson 8, students analyze several worked solutions (with errors) involving completing the square to identify and correct the errors. They are asked how they would convince someone that their solutions are correct.
- In Lesson 15, students may have misconceptions about squaring a linear expression, thinking they can just square the variable term and the constant term. This is a good opportunity to present the misconception as a statement and have students critique it.
- **English Learners:** Provide students time to formulate their responses and allow them to rehearse what they will say with a partner before sharing with the class.

Point to Ponder . . .

- In this routine, students analyze incorrect statements and work to correct them. How can you model what an effective and respectful critique looks like?

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools, Optimize Access to Technology*

Opportunities to provide visual support, physical manipulatives, or the use of technology appear in Lessons 1–15, 17–24.

- Throughout the unit, display or provide access to Anchor Chart PDFs and Graphic Organizer PDFs, such as *Solving Monic Quadratic Equations by Factoring, Solving Non-monic Quadratic Equations by Factoring, Completing the Square, The Quadratic Formula, and Different Forms of Quadratic Expressions*.
- Throughout the unit, provide access to algebra tiles, blank area diagrams, and colored pencils that students can use to visualize relationships and make connections.
- In Lesson 9, students use technology to explore the zeros of quadratic functions, relating them to the solutions of quadratic equations.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology or tools, or when to suggest students use color coding to help them make sense of the different strategies they can use to solve quadratic equations?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving quadratic equations throughout the unit? Do you think your students will generally:
  - Miss the underlying connections between the structures of equations and strategies for solving them?
  - Struggle with the procedures of different strategies?
  - Be ready to solve procedurally and efficiently, but unable to determine efficient strategies?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self awareness and self-management skills.

Points to Ponder . . .

- Are students able to recognize their emotions and use them to influence their behaviors in a positive way? Do students know how to use their strengths to provide confidence when approaching tasks? Can students remind themselves to have a growth mindset?
- Are students able to control their impulses and remain focused on the task at hand? Are they able to regulate their thoughts and emotions so that they can work towards and achieve their goals? Do they know how to manage their stress levels? Do students recognize the value in organization?

UNIT 6 | **LESSON 1 – LAUNCH**

Determining Unknown Inputs

Let's frame some pictures.

Focus

Goals

1. **Language Goal:** Explain the meaning of the solution to a quadratic equation in terms of a situation. **(Speaking and Listening, Writing)**
2. Write a quadratic equation that represents geometric constraints.

Coherence

Today

In this lesson, students encounter a problem that cannot be easily solved by familiar strategies, which gives them a chance to persevere in problem solving. They write a quadratic equation to model a situation and interpret what the solution means within that context. This work motivates the need to solve quadratic equations algebraically. **Note:** The term *quadratic equation* is not formally defined in this lesson.

Previously

In the previous unit, students were introduced to the different forms of quadratic functions: factored form, standard form, and vertex form. They learned how each form is useful for understanding and determining the key features of respective graphs.

Coming Soon

In the next lesson, students will work to solve quadratic equations that model situations encountered in the previous unit and discover the limitations of certain strategies.

Rigor

- Students develop their **conceptual understanding** of solving quadratic equations by reasoning through previously used strategies for determining solutions of quadratic functions.
- Students improve their **fluency** in writing simple quadratic expressions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Framing Material*, one per student
- Activity 1 PDF, *Picture*, one per student
- scissors
- rulers

Math Language Development

New words

- ***quadratic equation***

Note: An informal definition is provided in this lesson. The formal definition will be introduced in Lesson 2.

Review words

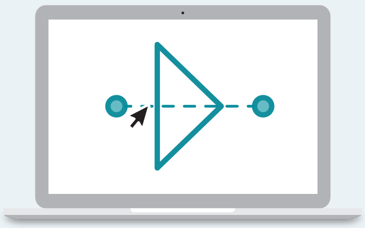
- *quadratic expression*

Amps | Featured Activity

Activity 1

See Student Thinking

Students explain their thinking as they solve a problem involving picture frames and quadratics. You are able to see their thinking in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated or overwhelmed in Activity 2 as they make attempts and fail to create a model that satisfies the given criteria. Encourage students to note what further information they obtained from performing each trial and give authentic feedback when students demonstrate perseverance (e.g., "I noticed you asked a peer for help and tried their suggestion.")

Modifications to Pacing

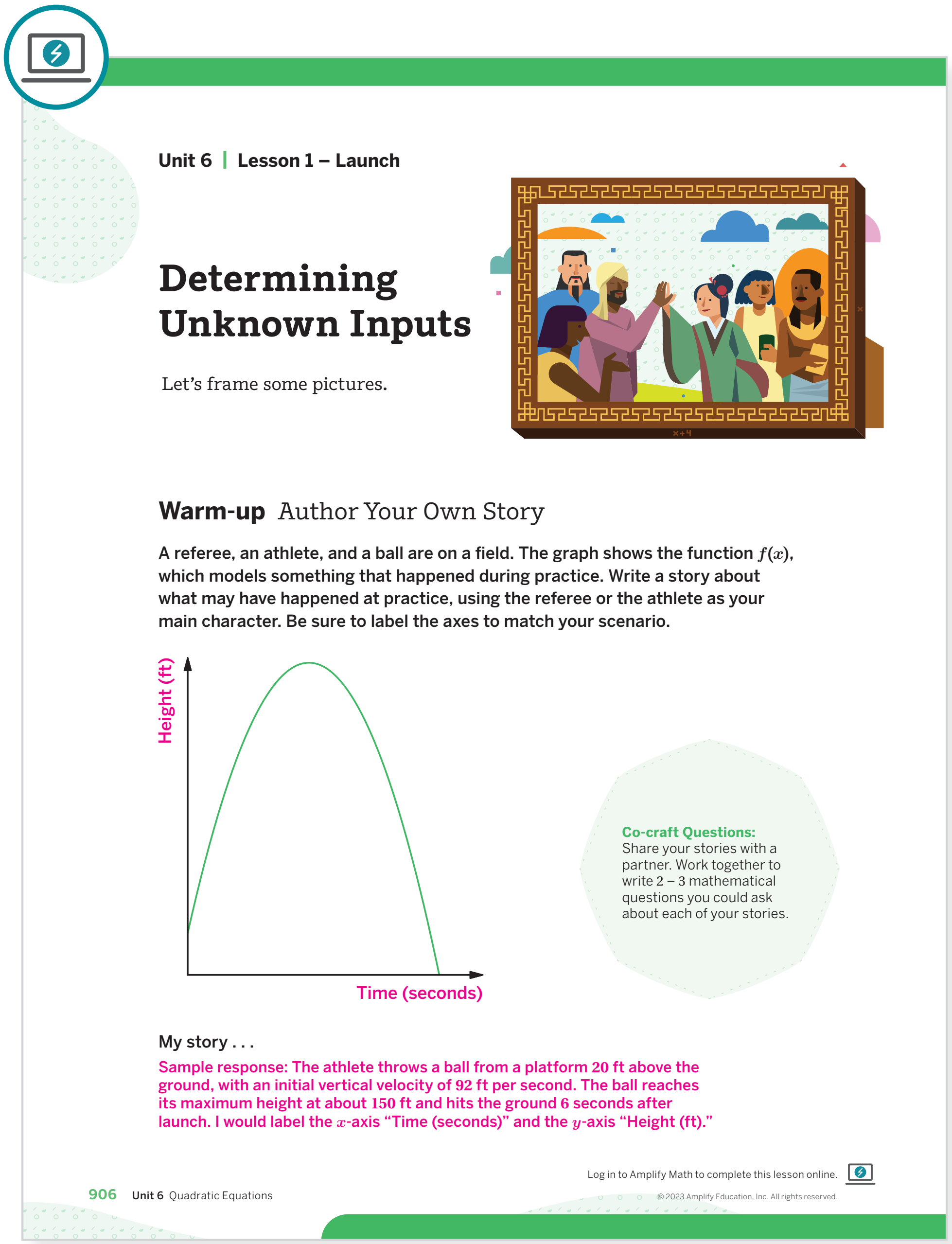
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, reduce time to 10 minutes.
- In **Activity 2**, reduce time to 10 minutes.

Independent | 5 min

Warm-up Author Your Own Story

Students examine the graph of a quadratic function and create a story using its key features to activate prior knowledge they learned about quadratics in the previous unit.



Unit 6 | Lesson 1 – Launch

Determining Unknown Inputs

Let's frame some pictures.

Warm-up Author Your Own Story

A referee, an athlete, and a ball are on a field. The graph shows the function $f(x)$, which models something that happened during practice. Write a story about what may have happened at practice, using the referee or the athlete as your main character. Be sure to label the axes to match your scenario.

Co-craft Questions: Share your stories with a partner. Work together to write 2 – 3 mathematical questions you could ask about each of your stories.

My story . . .

Sample response: The athlete throws a ball from a platform 20 ft above the ground, with an initial vertical velocity of 92 ft per second. The ball reaches its maximum height at about 150 ft and hits the ground 6 seconds after launch. I would label the x-axis "Time (seconds)" and the y-axis "Height (ft)."

Log in to Amplify Math to complete this lesson online.

906 **Unit 6** Quadratic Equations

1 Launch

Display the graph and say, "You saw several graphs that looked like this one in the last unit. What type of motion is modeled by this type of graph?" Projectile motion.

2 Monitor

Help students get started by prompting them to label the axes.

Look for points of confusion:

- **Describing the relationship between x and $f(x)$ only.** Prompt students to label the graph with actual values for key coordinate points.
- **Writing a story that does not correspond to the graph.** Prompt students to write a story about projectile motion.

Look for productive strategies:

- Labeling the x-axis with time and the y-axis with height and providing an appropriate scale for each.
- Using key features of a quadratic function to model a real-world situation.

3 Connect

Have students share the stories they created. Select students whose stories included actual values and have them explain the connections between those values and specific coordinates on their graph.

Highlight key features of the graph, namely, the vertical intercept, vertex, and positive horizontal intercept of the parabola, and explain how they represent the initial value, maximum height, and time when the ball hits the ground (respectively).

Math Language Development

MLR5: Co-craft Questions

Before the Connect, ask students to share their stories with a partner. Have students work together to write 2–3 mathematical questions they could ask about each of their stories. Have volunteers share their questions with the class.

MLR2: Collect and Display

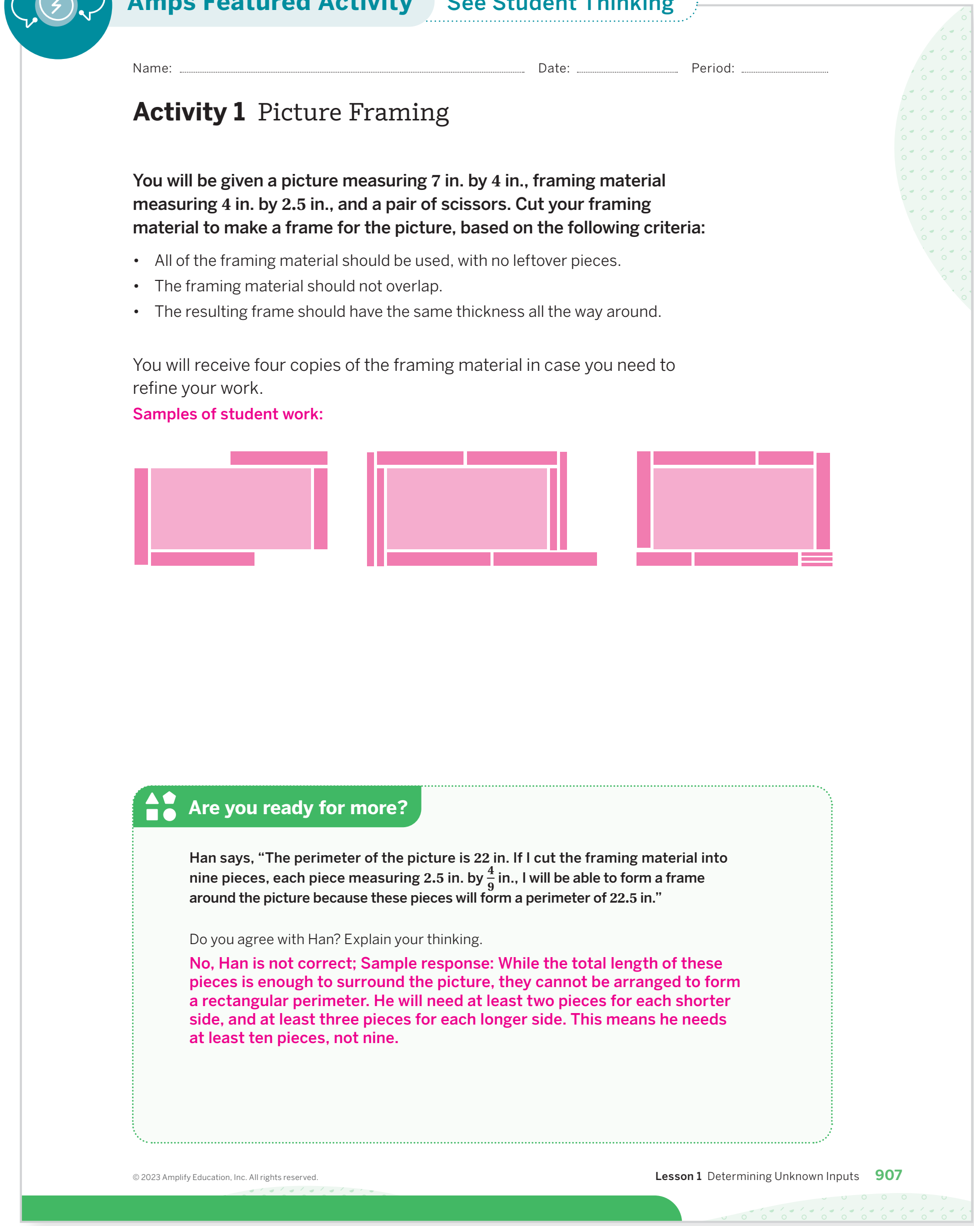
Begin a class display for this unit. As students share their stories with the class during the Connect, add the mathematical language they use to describe key features of the graph to the class display, such as *initial height, initial vertical speed, maximum height*, and the *time at which the ball hits the ground*.

Independent | 20 min

Activity 1 Picture Framing

Students create a frame by arranging “framing material” around a picture, which motivates the need for writing and solving a quadratic equation.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 1 Picture Framing

You will be given a picture measuring 7 in. by 4 in., framing material measuring 4 in. by 2.5 in., and a pair of scissors. Cut your framing material to make a frame for the picture, based on the following criteria:

- All of the framing material should be used, with no leftover pieces.
- The framing material should not overlap.
- The resulting frame should have the same thickness all the way around.

You will receive four copies of the framing material in case you need to refine your work.

Samples of student work:

Are you ready for more?

Han says, “The perimeter of the picture is 22 in. If I cut the framing material into nine pieces, each piece measuring 2.5 in. by $\frac{4}{9}$ in., I will be able to form a frame around the picture because these pieces will form a perimeter of 22.5 in.”

Do you agree with Han? Explain your thinking.

No, Han is not correct; Sample response: While the total length of these pieces is enough to surround the picture, they cannot be arranged to form a rectangular perimeter. He will need at least two pieces for each shorter side, and at least three pieces for each longer side. This means he needs at least ten pieces, not nine.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 Determining Unknown Inputs 907

1 Launch

Introduce the scenario with a story about framing. Distribute the Activity 1 PDFs, *Framing Material* and *Picture*. **Note:** The framing material is intentionally smaller than the picture. Students should realize that the *guess-and-check* strategy is inefficient to create a frame that satisfies the criteria.

2 Monitor

Help students get started by focusing on framing one pair of opposite side lengths at a time.

Look for points of confusion:

- **Using strips from more than one sheet of framing material.** Remind students that they should only use the framing material that was given to them.

Look for productive strategies:

- Using a ruler to mark and measure the framing material.
- Attempting to write an equation to determine the exact measurements.
- Persevering through the activity and refining their strategy as needed.

3 Connect

Display students’ frames. Consider taking snapshots while monitoring for ease of display.

Have students share their strategies and challenges they encountered along the way.

Ask:

- “How did you decide on the thickness of the frame?” Answers may vary.
- “Were you able to use all of the framing materials so that the widths were the same?” Answers may vary.
- “What was your final frame width?” Answers may vary.

Highlight that there are strategies that are more effective than the *guess-and-check* strategy for solving problems like this one.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time or have difficulty measuring or using scissors, have them work with a partner on this activity. One partner could measure and cut, while the other partner verifies the given criteria are satisfied.

Math Language Development

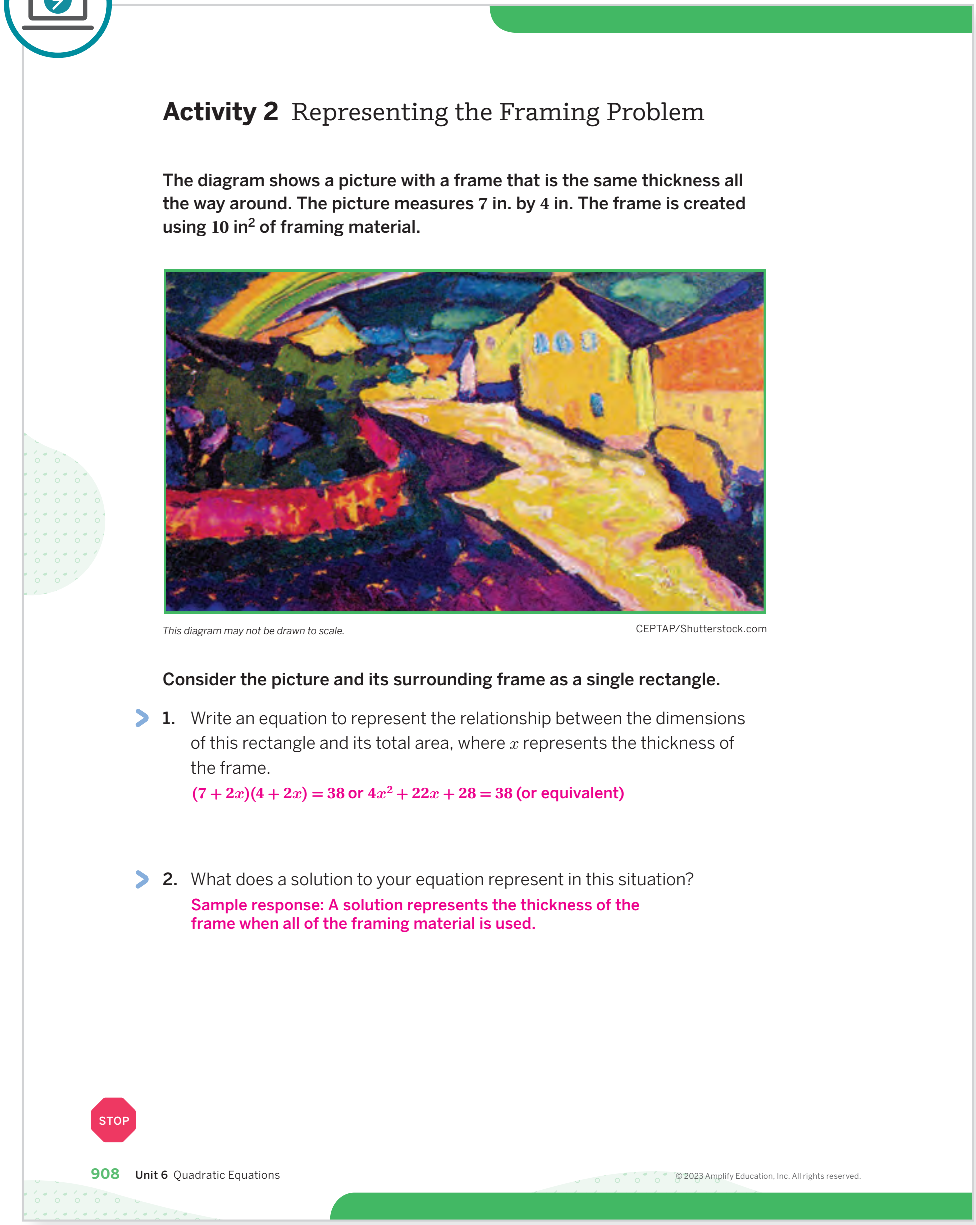
MLR7: Compare and Connect

Before or during the Connect, invite students to circulate and examine at least two other “frames” created by other students. Ask them to record any observations and comparisons they noted and share them during the Connect. During the Connect, restate the challenges students faced and amplify the need for more efficient strategies for solving this problem.

Pairs | 15 min

Activity 2 Representing the Framing Problem

Students write a quadratic equation to represent the framing problem from Activity 1 and consider what its solutions represent in context.



Activity 2 Representing the Framing Problem

The diagram shows a picture with a frame that is the same thickness all the way around. The picture measures 7 in. by 4 in. The frame is created using 10 in^2 of framing material.

This diagram may not be drawn to scale.

CEPTAP/Shutterstock.com

Consider the picture and its surrounding frame as a single rectangle.

1. Write an equation to represent the relationship between the dimensions of this rectangle and its total area, where x represents the thickness of the frame.

   $(7 + 2x)(4 + 2x) = 38$ or $4x^2 + 22x + 28 = 38$ (or equivalent)

2. What does a solution to your equation represent in this situation?

   Sample response: A solution represents the thickness of the frame when all of the framing material is used.

STOP

908 Unit 6 Quadratic Equations

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students a minute of think-time before discussing their thinking with a partner.

2 Monitor

Help students get started by prompting them to label the framed picture with relevant lengths.

Look for points of confusion:

- **Expressing the incorrect dimensions of the framed picture algebraically.** Suggest substituting numerical values for the frame's width to start.

Look for productive strategies:

- Formulating a mathematical model for the framing task.
- Writing numerical or algebraic expressions to reason about the length and width of the frame.
- Determining the combined area of the picture and the framing material.

3 Connect

Have pairs of students share their equations and reasoning. Select students who used productive strategies.

Highlight that the total area of the framed picture is 38 in^2, which equals the product of the side lengths, $(7 + 2x)(4 + 2x)$. The area can also be found by decomposing the frame into squares and rectangles, resulting in the equation $4x^2 + 22x + 28 = 38$.

Ask:

- "Are the equations $(7 + 2x)(4 + 2x) = 38$ and $4x^2 + 22x + 28 = 38$ equivalent?" Yes.
- "What would a solution to these equations represent?" The thickness of the frame.
- "How could you solve these equations?" Sample response: Using the *guess-and-check* strategy.

Define the term ***quadratic equation*** informally (for now) as an equation with a quadratic expression.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Ask, "How could you annotate this picture to show that the frame surrounding it must be the same thickness all the way around?" Once students understand that the same variable, e.g., x, can be used to represent the thickness all the way around, have them annotate the picture accordingly.

Extension: *Math Enrichment*

Ask students how the equation would be altered if the frame is created using 14 in^2 of framing material and the dimensions of the picture are 10 in. by 6 in. $(10 + 2x)(6 + 2x) = 72$

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand the concept of a picture having a frame around it.
- **Read 2:** Ask students to identify the given quantities and relationships, such as the frame is created using 10 in^2 of framing material.
- **Read 3:** Ask students to brainstorm possible strategies for writing an equation to represent the total area of the picture and its frame.

Whole Class | 5 min

Summary The Evolution of Solving Quadratic Equations

Review and synthesize how quadratic equations are used to solve a variety of real-world problems.

Narrative Connections

Unit 6 Quadratic Equations

The Evolution of Solving Quadratic Equations

There is a lot we take for granted. Even something as ordinary as the border of a picture frame can require careful thought and planning. And lurking inside what might at first appear to be a straightforward problem about area, we find our old pal, a quadratic.

In the previous unit, you learned that quadratics can take on different forms and represent different kinds of relationships. They can represent how objects fall through the air, how dancers twirl their bodies, or even how businesses generate profit. But humankind's pursuit of quadratics has its roots, if you will pardon the pun, in *area*.

The study of quadratics is an old story — thousands of years old — spanning civilizations across three continents. It had day-to-day implications on how land was divided and how buildings were planned out. In this unit, we will look at this story more closely. We will see how these societies tackled these problems, often using mathematics quite different from what we are used to today.

We will look at key moments across this wide swath of history, where problems of area inspired flashes of genius and innovation, giving birth to algebra as we know it.

Welcome to Unit 6.

Lesson 1 Determining Unknown Inputs 909

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *quadratic equation* that were added to the display during the lesson.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Ask:

- "In the framing problem, what does the output value of the equation represent? What output value did you want the equation to produce?" The total area of the framed picture. I wanted the output value to be 38.
- "What does the input value x represent?" The thickness of the frame.
- "Why might it be helpful to write an equation to represent a problem such as the one used in the framing scenario?" Sample response: An equation helps me see the relationship between the quantities in the problem, including unknown values. If I can solve the equation, I can determine the unknown values.
- "In the framing scenario, if x represents the thickness of the frame, in inches, what would a solution to the equation $(2x + 7)(2x + 4) = 50$ mean?" The thickness of the frame that would produce a total area of 50 in^2.
- "Equations such as $(2x + 7)(2x + 4) = 50$ and $4x^2 + 22x + 28 = 38$ are called quadratic equations. Why do you think equations like these are described as quadratic?" Sample response: Each equation is composed of a quadratic expression set equal to another value. In this case, one quadratic expression is in factored form, and the other is in standard form.

Highlight that students will be learning algebraic techniques for solving quadratic equations in this unit, which will be helpful for when they cannot find an exact solution from a graph. The algebraic techniques are more straightforward (and less time-consuming) than using the *guess-and-check* strategy with input values.

Formalize vocabulary: *quadratic equation*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

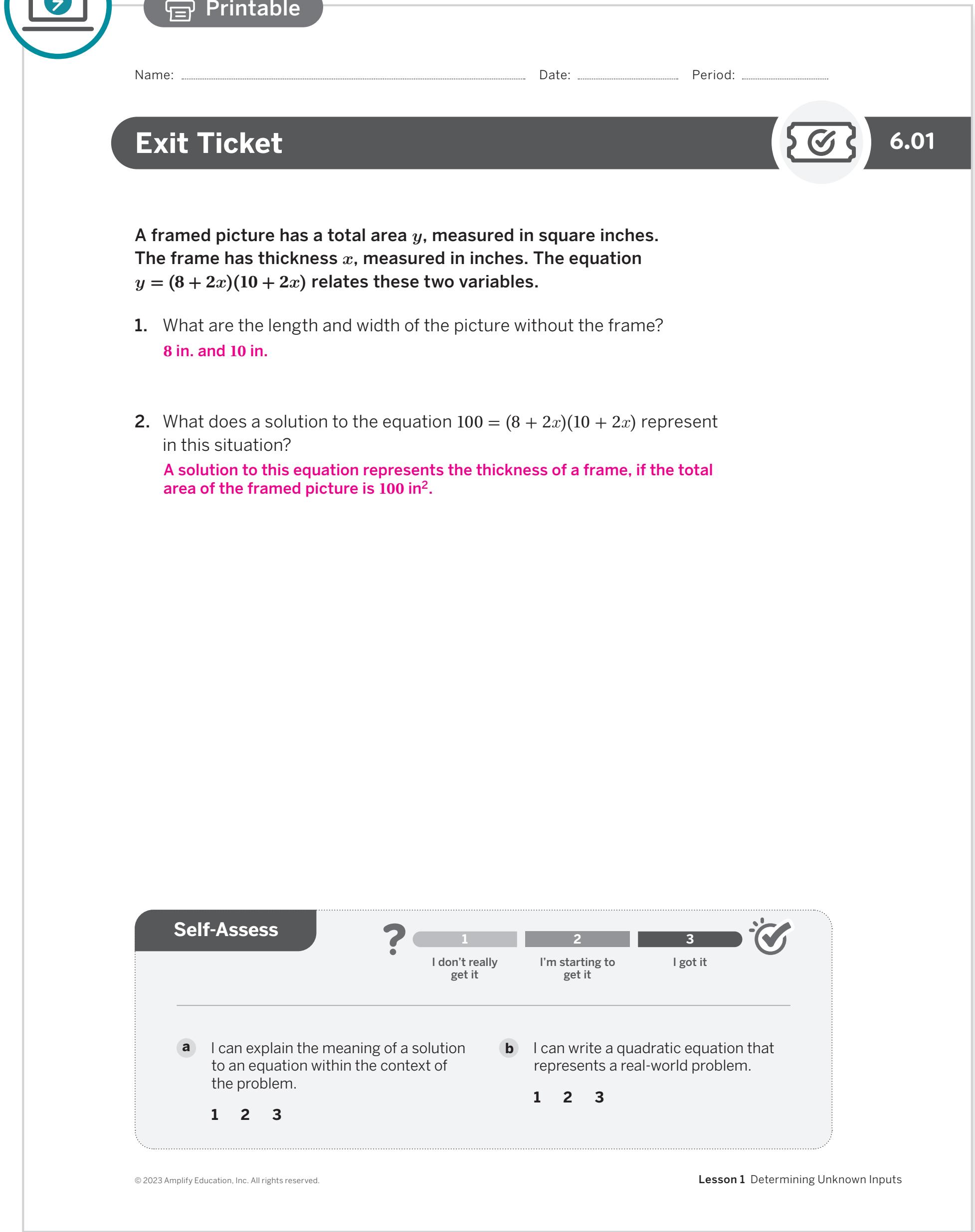
- "What makes an equation quadratic?"
- "What does a solution to a quadratic equation represent?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing a quadratic equation that represents a situation and interpreting what a solution would represent in context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.01

A framed picture has a total area y, measured in square inches. The frame has thickness x, measured in inches. The equation $y = (8 + 2x)(10 + 2x)$ relates these two variables.

1. What are the length and width of the picture without the frame?
   8 in. and 10 in.

2. What does a solution to the equation $100 = (8 + 2x)(10 + 2x)$ represent in this situation?
   A solution to this equation represents the thickness of a frame, if the total area of the framed picture is 100 in^2.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain the meaning of a solution to an equation within the context of the problem.
1 2 3

b. I can write a quadratic equation that represents a real-world problem.
1 2 3

Lesson 1 Determining Unknown Inputs

Success looks like . . .

- **Language Goal:** Explaining the meaning of the solution to a quadratic equation in terms of a situation. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of the solution in the context of the framed picture in Problem 2.
- **Goal:** Writing a quadratic equation that represents geometric constraints.

Suggested next steps

If students are unable to state the length and width in Problem 1, consider:

- Reassigning Activity 2, Problem 1, and highlighting connections between the quadratic equation in factored form and the dimensions of the framed picture.
- Having students draw and label a diagram of a framed picture.
- Assigning Practice Problem 2.

If students are unable to interpret what a solution to the equation would represent in Problem 2, consider:

- Reassigning Activity 2, Problem 2.
- Assigning Practice Problem 1.

Professional Learning

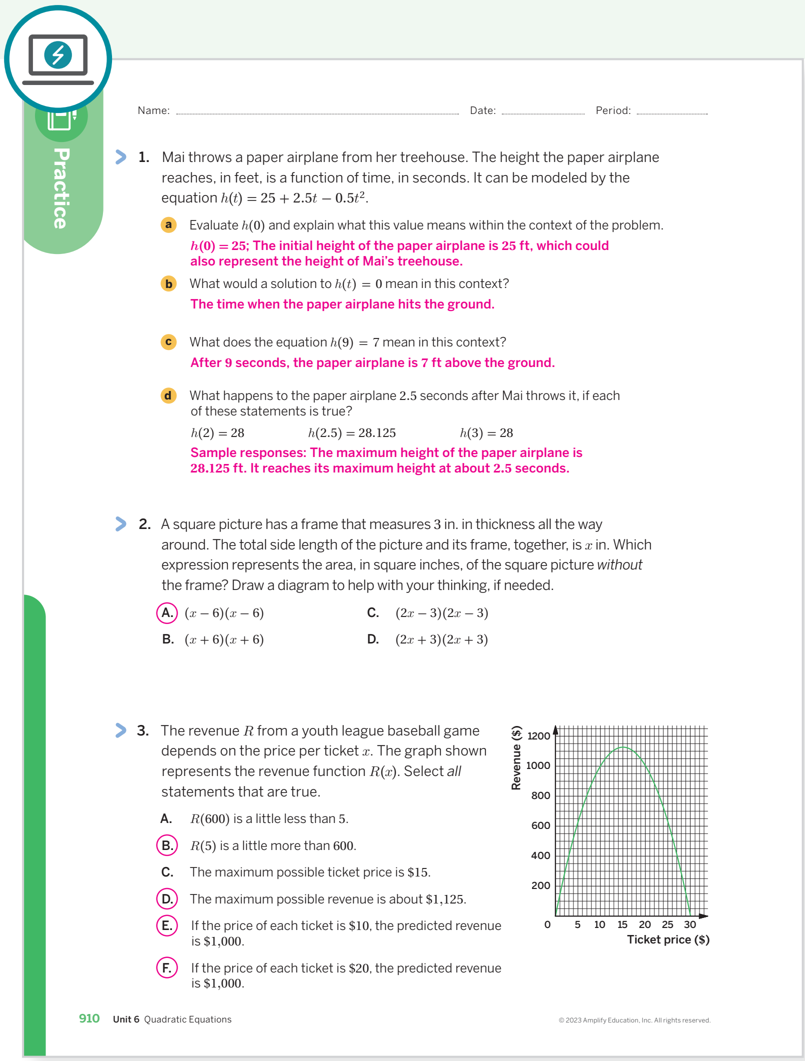
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What different ways did students approach framing the picture? What does that tell you about similarities and differences among your students?
- How did students face the challenges in Activity 1? Were they able to recognize the need for a more efficient way to solve quadratic equations? What might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Mai throws a paper airplane from her treehouse. The height the paper airplane reaches, in feet, is a function of time, in seconds. It can be modeled by the equation $h(t) = 25 + 2.5t - 0.5t^2$.

   a. Evaluate $h(0)$ and explain what this value means within the context of the problem.

   $h(0) = 25$; The initial height of the paper airplane is 25 ft, which could also represent the height of Mai's treehouse.

   b. What would a solution to $h(t) = 0$ mean in this context?

   The time when the paper airplane hits the ground.

   c. What does the equation $h(9) = 7$ mean in this context?

   After 9 seconds, the paper airplane is 7 ft above the ground.

   d. What happens to the paper airplane 2.5 seconds after Mai throws it, if each of these statements is true?

   $h(2) = 28$ $h(2.5) = 28.125$ $h(3) = 28$

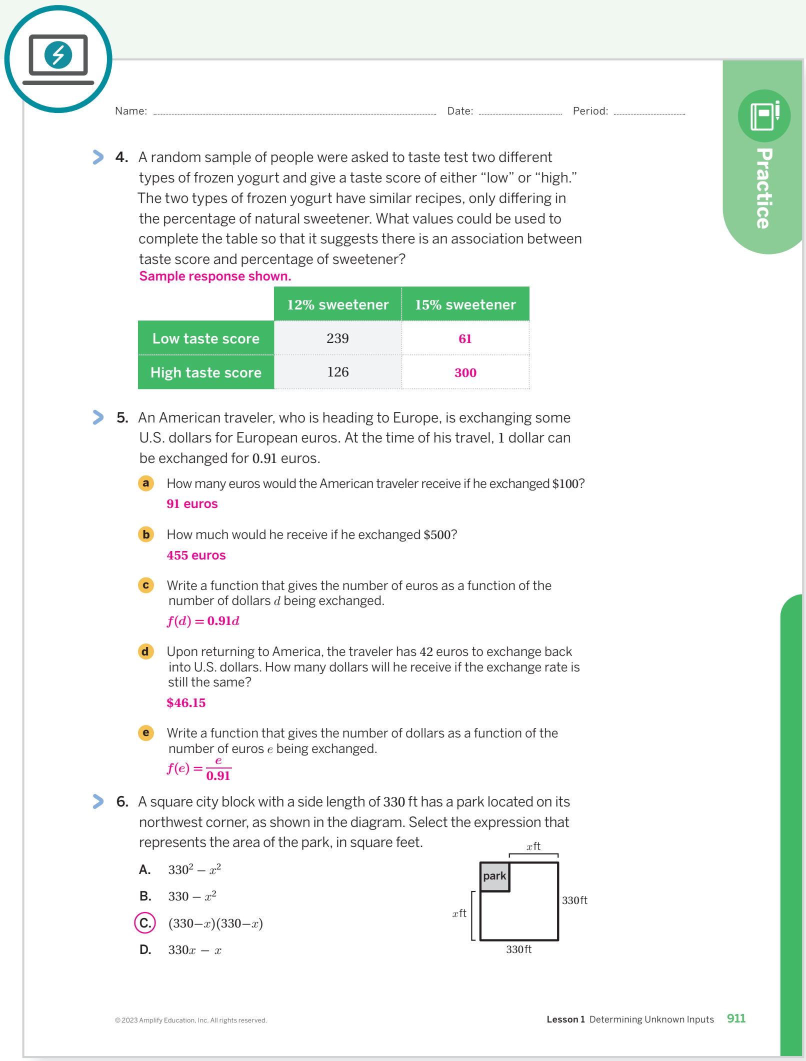
   Sample responses: The maximum height of the paper airplane is 28.125 ft. It reaches its maximum height at about 2.5 seconds.

2. A square picture has a frame that measures 3 in. in thickness all the way around. The total side length of the picture and its frame, together, is x in. Which expression represents the area, in square inches, of the square picture *without* the frame? Draw a diagram to help with your thinking, if needed.

   [✓ A.] $(x-6)(x-6)$
   B. $(x+6)(x+6)$
   C. $(2x-3)(2x-3)$
   D. $(2x+3)(2x+3)$

3. The revenue R from a youth league baseball game depends on the price per ticket x. The graph shown represents the revenue function $R(x)$. Select *all* statements that are true.

   A. $R(600)$ is a little less than 5.
   [✓ B.] $R(5)$ is a little more than 600.
   C. The maximum possible ticket price is $15.
   [✓ D.] The maximum possible revenue is about $1,125.
   [✓ E.] If the price of each ticket is $10, the predicted revenue is $1,000.
   [✓ F.] If the price of each ticket is $20, the predicted revenue is $1,000.



Practice

Name: ______ Date: ______ Period: ______

4. A random sample of people were asked to taste test two different types of frozen yogurt and give a taste score of either "low" or "high." The two types of frozen yogurt have similar recipes, only differing in the percentage of natural sweetener. What values could be used to complete the table so that it suggests there is an association between taste score and percentage of sweetener?

   Sample response shown.

	12% sweetener	15% sweetener
Low taste score	239	**61**
High taste score	126	**300**

5. An American traveler, who is heading to Europe, is exchanging some U.S. dollars for European euros. At the time of his travel, 1 dollar can be exchanged for 0.91 euros.

   a. How many euros would the American traveler receive if he exchanged $100?

   91 euros

   b. How much would he receive if he exchanged $500?

   455 euros

   c. Write a function that gives the number of euros as a function of the number of dollars d being exchanged.

   $f(d) = 0.91d$

   d. Upon returning to America, the traveler has 42 euros to exchange back into U.S. dollars. How many dollars will he receive if the exchange rate is still the same?

   $46.15

   e. Write a function that gives the number of dollars as a function of the number of euros e being exchanged.

   $f(e) = \frac{e}{0.91}$

6. A square city block with a side length of 330 ft has a park located on its northwest corner, as shown in the diagram. Select the expression that represents the area of the park, in square feet.

   A. $330^2 - x^2$
   B. $330 - x^2$
   [✓ C.] $(330-x)(330-x)$
   D. $330x - x$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 15	1
	5	Unit 3 Lesson 19	2
Formative ⚡	6	Unit 6 Lesson 2	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

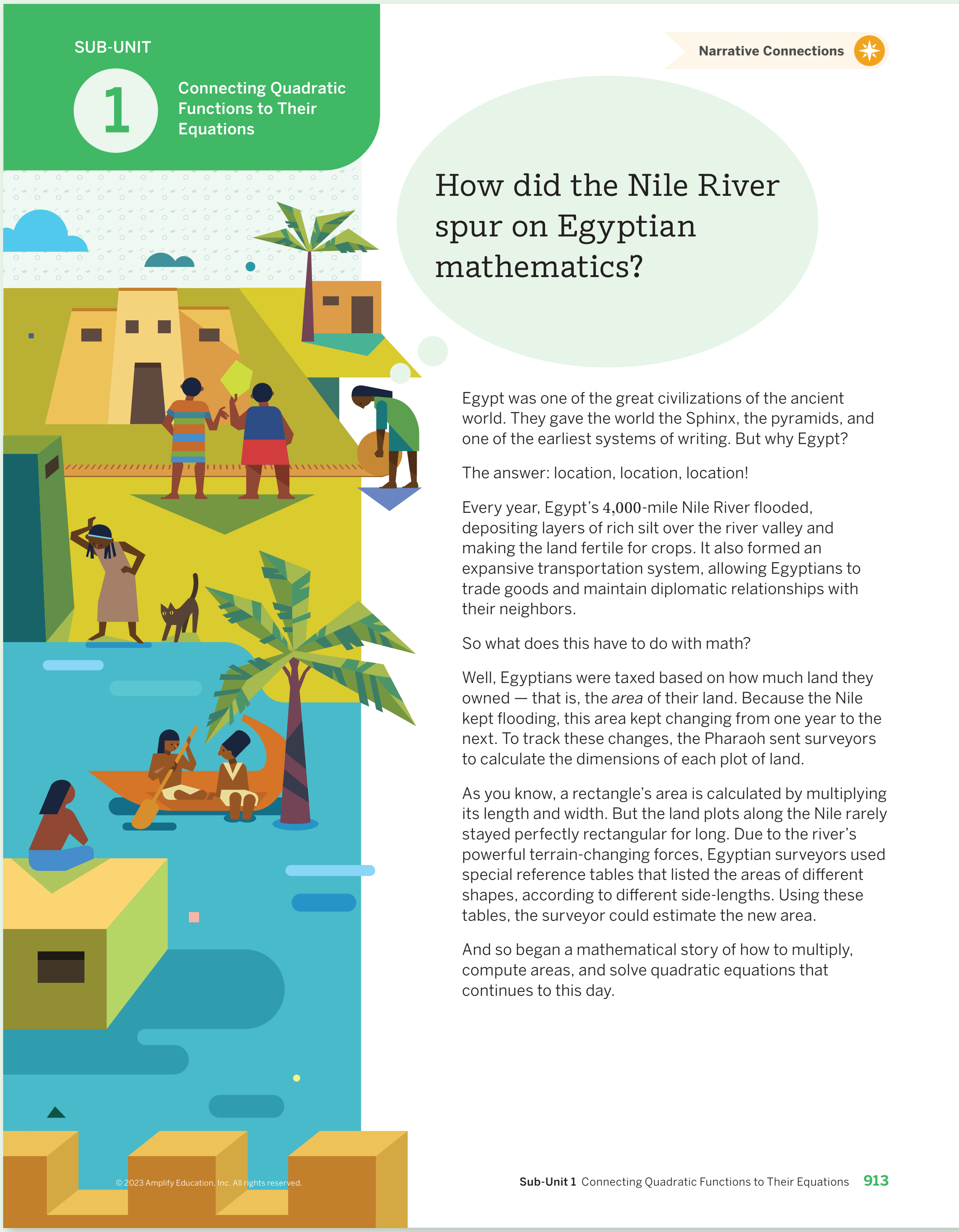
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 1

Whole Class

Connecting Quadratic Functions to Their Equations

In this Sub-Unit, students revisit strategies for determining the zeros of a quadratic function and determine that they are not efficient for solving quadratic equations.



SUB-UNIT

1 Connecting Quadratic Functions to Their Equations

Narrative Connections

How did the Nile River spur on Egyptian mathematics?

Egypt was one of the great civilizations of the ancient world. They gave the world the Sphinx, the pyramids, and one of the earliest systems of writing. But why Egypt?

The answer: location, location, location!

Every year, Egypt's 4,000-mile Nile River flooded, depositing layers of rich silt over the river valley and making the land fertile for crops. It also formed an expansive transportation system, allowing Egyptians to trade goods and maintain diplomatic relationships with their neighbors.

So what does this have to do with math?

Well, Egyptians were taxed based on how much land they owned — that is, the *area* of their land. Because the Nile kept flooding, this area kept changing from one year to the next. To track these changes, the Pharaoh sent surveyors to calculate the dimensions of each plot of land.

As you know, a rectangle's area is calculated by multiplying its length and width. But the land plots along the Nile rarely stayed perfectly rectangular for long. Due to the river's powerful terrain-changing forces, Egyptian surveyors used special reference tables that listed the areas of different shapes, according to different side-lengths. Using these tables, the surveyor could estimate the new area.

And so began a mathematical story of how to multiply, compute areas, and solve quadratic equations that continues to this day.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue the mathematical story of solving quadratic equations in the following places:

- **Lesson 2, Activities 1–2:** The Perfect Shot, Revisited, Revenue From Ticket Sales
- **Lesson 3, Activities 1–2:** How Many Solutions?, Determining Pairs of Solutions
- **Lesson 4, Activities 1-2:** Solving Quadratics Algebraically, Revisiting Projectiles
- **Lesson 5, Activities 1–3:** Solving by Graphing, Determining All the Solutions, Find and Fix

UNIT 6 | **LESSON 2**

When and Why Do We Write Quadratic Equations?

Let's solve some quadratic equations.

Focus

Goals

1. **Language Goal:** Recognize the limitations of certain strategies used to solve a quadratic equation. **(Speaking and Listening)**
2. Understand that the factored form of a quadratic expression can help determine the zeros and solve its related quadratic equation.
3. **Language Goal:** Write quadratic equations and reason about their solutions in terms of a situation. **(Reading and Writing)**

Coherence

Today

Students continue to explore strategies for solving quadratic equations that model real-world situations. They examine quadratic equations in factored and standard form and determine which form is more efficient to solve when applying their knowledge about zeros of functions from Unit 5. Students review the Zero Property of Multiplication. **Note:** The term *quadratic equation* is formally defined in Activity 2.

Previously

In the previous lesson, students persevered to determine solutions to a quadratic equation using strategies that proved inadequate, prompting the need for a more efficient method. Students used an informal definition of the term *quadratic equation*.

Coming Soon

In the next lesson, students begin to solve quadratic equations by reasoning about what values make the equations true and learn that some quadratic equations can have two solutions. **Note:** In this course, the solutions to quadratic equations are limited to real numbers. In Algebra 2, students will come to understand that a quadratic equation can have imaginary solutions.

Rigor

- Students build **conceptual understanding** by reasoning on when it is appropriate to solve a quadratic equation.
- Students improve their **fluency** of factoring quadratic expressions to determine the zeros of the function.
- Students **apply** their skills when solving quadratic equations that model projectile motion and revenue.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems*, *Critiquing*
- Anchor Chart PDF, *Sentence Stems*, *Explaining My Steps*
- graphing technology

Math Language Development

New words

- ***quadratic equation***

Review words

- *factored form*
- *quadratic expression*
- *standard form*
- *zero*
- *Zero Product Principle*

Amps | **Featured Activity**

Warm-up
See Student Thinking

Students work through a warm-up problem that involves quadratic thinking. See what they have to say in real time.

Building Math Identity and Community

Connecting to Mathematical Practices

If students have not made sense of the zeros of a function in the given context in Activity 2, they may not be able to envision a path for solving the quadratic equation. Encourage them to visit other student pairs to observe other avenues of thinking, particularly those that chose factored form instead of standard form, and to record any strategies that they find helpful.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

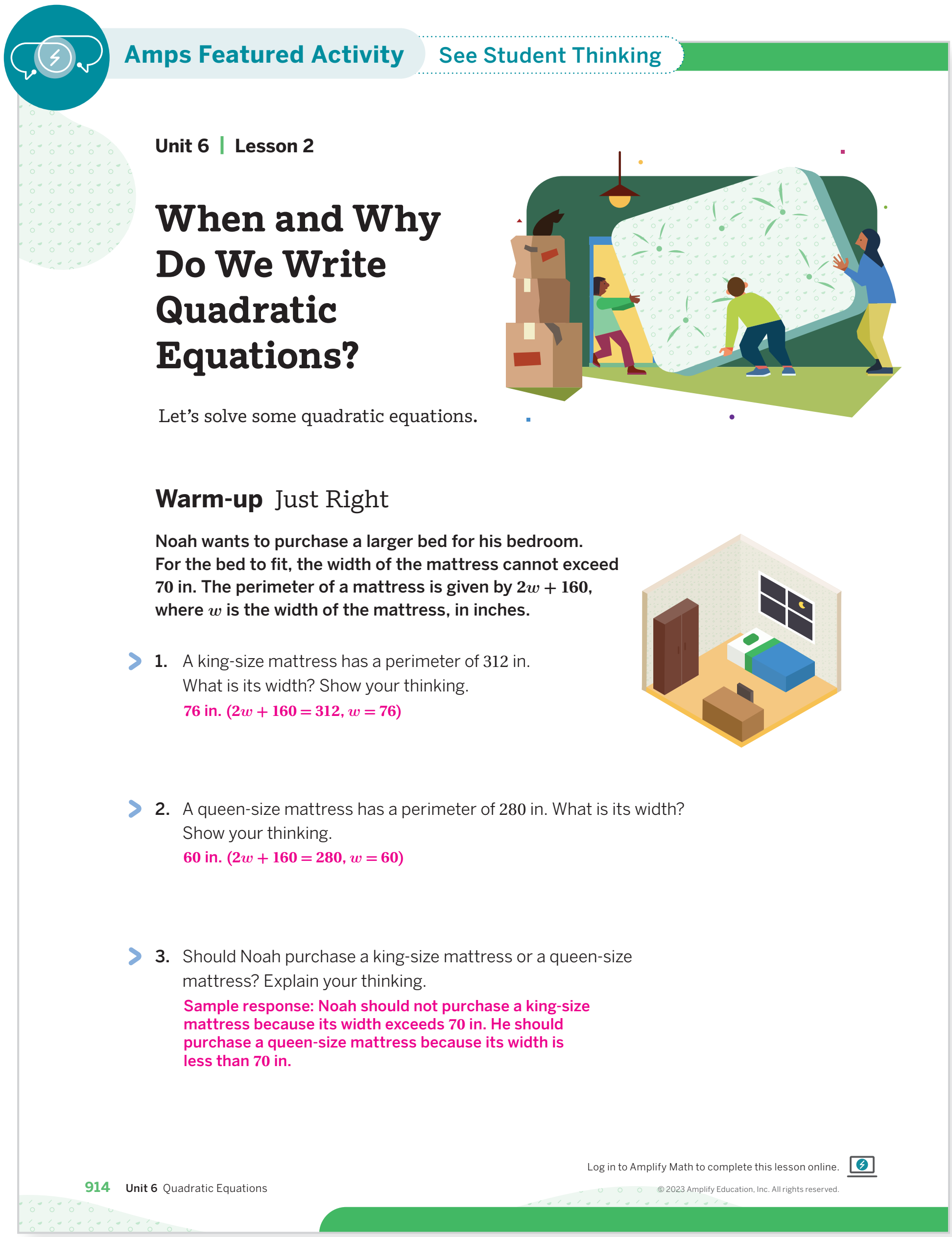
- In **Activity 1**, instead of having students complete this activity, display the graph of $f(t)$ and show that it does not yield an exact value for t when $f(t) = 0$. Then discuss the limitations or shortcomings of solving the quadratic equation by graphing, by using the *guess-and-check* strategy, or by attempting to isolate the variable.

Independent | 5 min

Warm-up Just Right

Students solve for the input of a linear equation, given its output, to activate prior knowledge about solving equations and contextualizing the solutions.

Amps Featured Activity See Student Thinking



Unit 6 | Lesson 2

When and Why Do We Write Quadratic Equations?

Let's solve some quadratic equations.

Warm-up Just Right

Noah wants to purchase a larger bed for his bedroom. For the bed to fit, the width of the mattress cannot exceed 70 in. The perimeter of a mattress is given by $2w + 160$, where w is the width of the mattress, in inches.

1. A king-size mattress has a perimeter of 312 in. What is its width? Show your thinking.
   76 in. ($2w + 160 = 312$, $w = 76$)

2. A queen-size mattress has a perimeter of 280 in. What is its width? Show your thinking.
   60 in. ($2w + 160 = 280$, $w = 60$)

3. Should Noah purchase a king-size mattress or a queen-size mattress? Explain your thinking.
   Sample response: Noah should not purchase a king-size mattress because its width exceeds 70 in. He should purchase a queen-size mattress because its width is less than 70 in.

Log in to Amplify Math to complete this lesson online.

914 Unit 6 Quadratic Equations

1 Launch

Ask, "What is the smallest mattress you have seen? The largest? Does the size of a room influence the choice of mattress size?" Explain when purchasing a bed for a room, the width of the mattress must often be taken into consideration.

2 Monitor

Help students get started by prompting them to draw and label a diagram of the mattress for Problem 1.

Look for points of confusion:

- **Trying to include 70 in. into calculations for Problems 1 or 2.** Ask students if this information is useful for these problems.
- **Confusing area with perimeter.** Remind students that perimeter is the distance around a figure.

Look for productive strategies:

- Writing and solving $2w + 160 = 312$ for Problem 1.
- Writing and solving $2w + 160 = 280$ for Problem 2.
- Substituting different values of w to make 312.

3 Connect

Have students share their approaches to solving this problem. Select students who used only a diagram to display their solution first. Then select students who wrote and solved an equation.

Ask:

- "What does a solution represent in this context?" The width of the mattress.
- "What does 160 represent in the expression?" Twice the length of the mattress.
- "What does this tell you about the lengths of a king- and queen-size mattress?" They are the same.

Highlight the steps taken to isolate the variable in the linear equations determined by Problems 1 and 2.

Differentiated Support

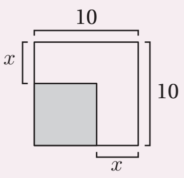
Accessibility: *Guide Processing and Visualization*

Display the formula for the perimeter of a rectangle, $P = 2w + 2\ell$, next to the expression $2w + 160$ and ask:

- "What does 160 represent?" Twice the length of the mattress.
- "What would the length of the mattress be?" 80 in.
- "If the perimeter is 312 in., what equation can you write? How can you solve this equation? What strategies can you use?" $2w + \ell = 312$; I can use inverse operations to solve this equation, or I can create a table of values and use the guess-and-check strategy.

Power-up

To power up students' ability to identify quadratic expressions that represent geometric relationships, have students complete:



1. Determine the length and width of the shaded region.
   Length: $10 - x$
   Width: $10 - x$
2. Which expression represents the area of the shaded region?
   A. $100 - x^2$
   B. $100 - x$
   C. $100x - x$
   [✓ D.] $(10 - x)(10 - x)$

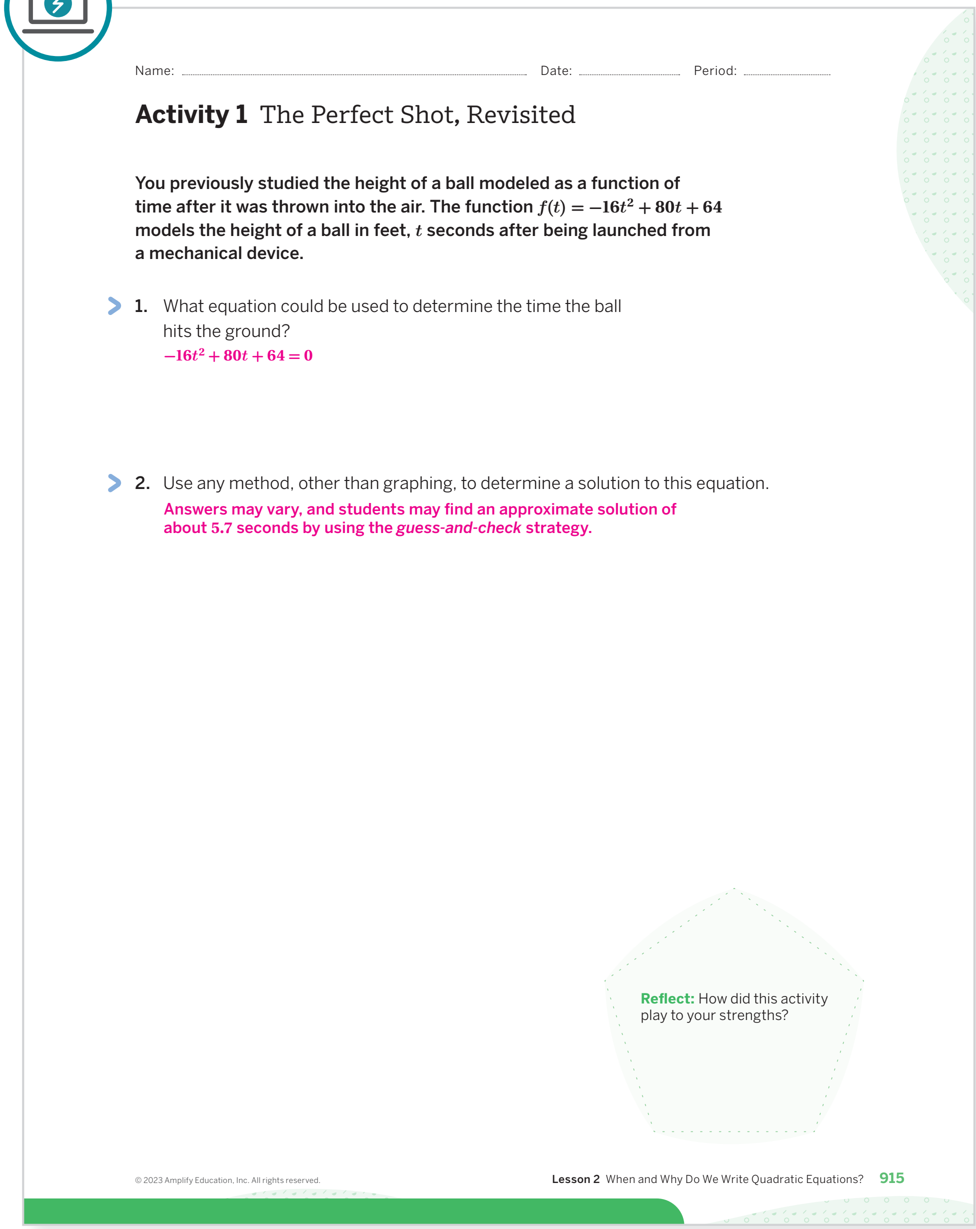
Use: Before the Warm-up

Informed by: Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 15 min

Activity 1 The Perfect Shot, Revisited

Students write and attempt to solve a quadratic equation from a previous context using known strategies, realizing the limitations of those strategies.



Name: ______ Date: ______ Period: ______

Activity 1 The Perfect Shot, Revisited

You previously studied the height of a ball modeled as a function of time after it was thrown into the air. The function $f(t) = -16t^2 + 80t + 64$ models the height of a ball in feet, t seconds after being launched from a mechanical device.

1. What equation could be used to determine the time the ball hits the ground?
   $-16t^2 + 80t + 64 = 0$

2. Use any method, other than graphing, to determine a solution to this equation.
   Answers may vary, and students may find an approximate solution of about 5.7 seconds by using the *guess-and-check* strategy.

Reflect: How did this activity play to your strengths?

1 Launch

Tell students they should not use graphing or graphing technology in this activity.

2 Monitor

Help students get started by asking, "What is the height of the ball when it hits the ground?" 0 ft

Look for points of confusion:

- **Thinking that they cannot do anything without graphing.** Ask, "How might a table of values of help you?"
- **Struggling to isolate t.** Have students articulate why they cannot get t by itself. Ask, "Can you move any of the terms to the other side to help you? Do you see any common factors with the terms that remain?"

Look for productive strategies:

- Using the *guess-and-check* strategy to determine values of t.

3 Connect

Have students share their strategies for solving the equation and any associated challenges. ***Poll the class*** on their solutions to the equation and display them.

Display a graph of the equation $y = f(t)$ to show that solving by graphing only gives an approximate solution. Substitute 5.7 into the equation to show that the result is not exactly 0.

Ask, "The quadratic expression that models this function is given in what form?" Standard form.

Define the term ***quadratic equation***.

Highlight that solving a quadratic equation by isolating the variable can be challenging, solving using the *guess-and-check* strategy is time-consuming, and solving by graphing may not always yield exact values. Students will learn additional strategies that may be more efficient than these for solving quadratic equations.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they have seen similar functions in the previous unit. Display the function $f(t) = -16t^2 + 80t + 64$ and ask:

- "What type of function is this?" Quadratic
- "What does the term $-16t^2$ represent?" The effects due to gravity.
- "What does the term $80t$ represent?" The initial vertical speed.
- "What does the term 64 represent?" The initial height of the ball.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the connections between the various strategies they used to try to solve the equation. Mention these strategies if no one used them. Ask:

- "How is the *guess-and-check strategy* similar to creating a table of values?"
- "If you subtract 64 from both sides and then factor out the greatest common factor from the terms that remain on the right side, what is the result?" $-64 = -16t(t - 5)$
- "What could you do next?" Use the *guess-and-check* strategy or create a table of values to determine when the right side equals -64 for different values of t.

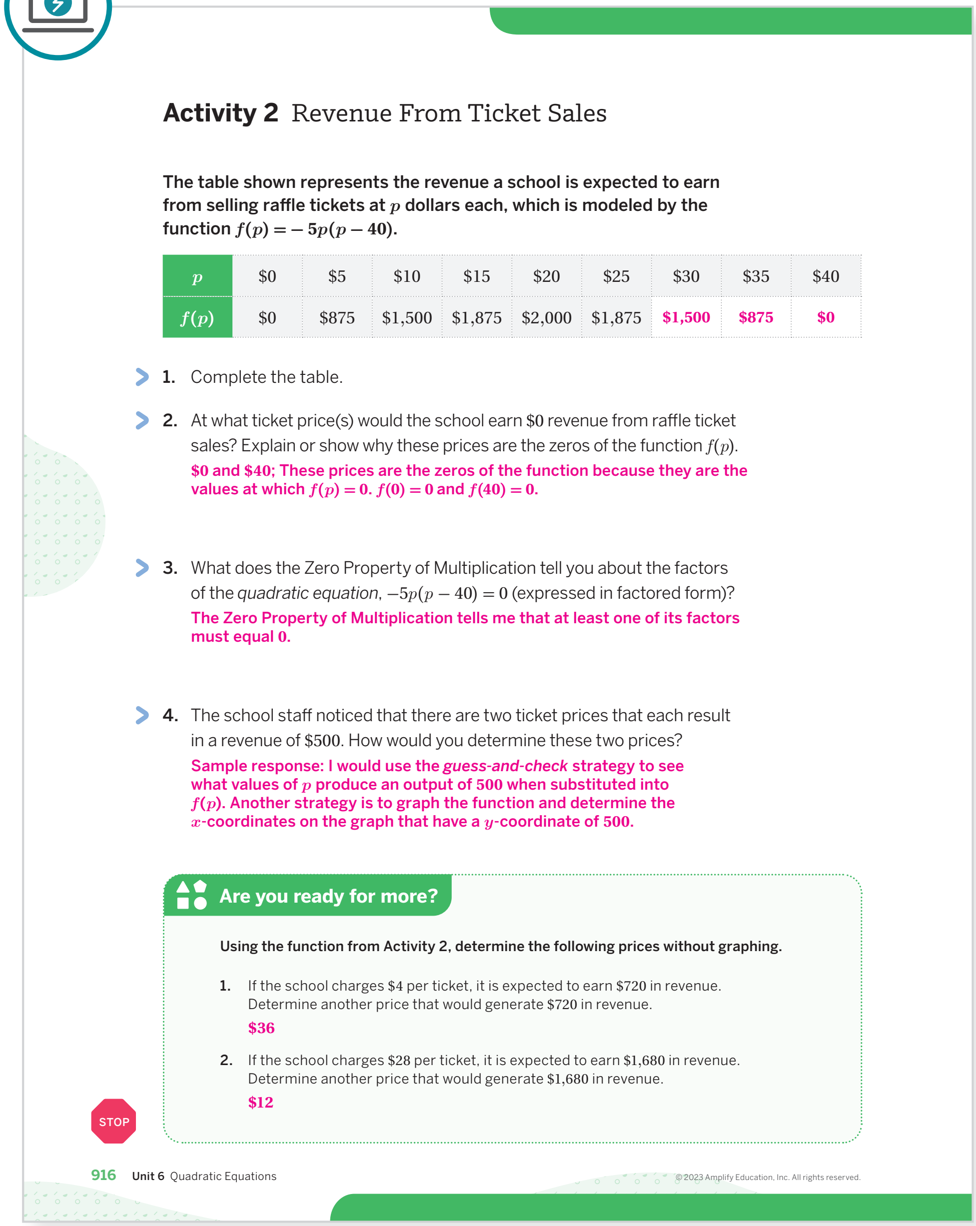
English Learners

Display or provide the Anchor Chart PDF, *Sentence Stems*, *Explaining My Steps* to support students when they explain their strategy.

Pairs | 20 min

Activity 2 Revenue From Ticket Sales

Students apply their previous understanding about zeros of a function to solve a quadratic equation in factored form that is equal to zero.



Activity 2 Revenue From Ticket Sales

The table shown represents the revenue a school is expected to earn from selling raffle tickets at p dollars each, which is modeled by the function $f(p) = -5p(p - 40)$.

p	\$0	\$5	\$10	\$15	\$20	\$25	\$30	\$35	\$40
$f(p)$	\$0	\$875	\$1,500	\$1,875	\$2,000	\$1,875	**\$1,500**	**\$875**	**\$0**

1. Complete the table.
2. At what ticket price(s) would the school earn \$0 revenue from raffle ticket sales? Explain or show why these prices are the zeros of the function $f(p)$.
   \$0 and \$40; These prices are the zeros of the function because they are the values at which $f(p) = 0$. $f(0) = 0$ and $f(40) = 0$.
3. What does the Zero Property of Multiplication tell you about the factors of the *quadratic equation*, $-5p(p - 40) = 0$ (expressed in factored form)?
   The Zero Property of Multiplication tells me that at least one of its factors must equal 0.
4. The school staff noticed that there are two ticket prices that each result in a revenue of \$500. How would you determine these two prices?
   Sample response: I would use the *guess-and-check* strategy to see what values of p produce an output of 500 when substituted into $f(p)$. Another strategy is to graph the function and determine the x-coordinates on the graph that have a y-coordinate of 500.

Are you ready for more?

Using the function from Activity 2, determine the following prices without graphing.

1. If the school charges \$4 per ticket, it is expected to earn \$720 in revenue. Determine another price that would generate \$720 in revenue.
   \$36
2. If the school charges \$28 per ticket, it is expected to earn \$1,680 in revenue. Determine another price that would generate \$1,680 in revenue.
   \$12

STOP

916 Unit 6 Quadratic Equations

1 Launch

Display $x \cdot y = 0$ and ask, "What can you conclude about the values of x and y?" Elicit from students that either x or y must be 0 for their product to be 0. Students may use graphing technology for this activity.

2 Monitor

Help students get started by asking, "What type of function is $f(p)$? How can you tell?"

Look for points of confusion:

- **Having difficulty connecting the expressions that define the function.** Ask, "What does each input and output value represent?" The input value represents the raffle ticket price and the output value represents the revenue from raffle tickets purchased.
- **Writing \$0 as the only answer in Problem 2.** Ask students if there is another value at which the function may equal 0. Yes, 40.

Look for productive strategies:

- Annotating the table to show the quadratic pattern.
- Substituting values from the table into the function to see if it equals 0.
- Graphing the function using graphing technology.

3 Connect

Display the table in the activity and select a student to complete it.

Have students share their responses to the remaining problems, and discuss their thinking.

Ask, "Did the table help you find the values of p that produced \$0 in revenue? \$500 in revenue?"

Highlight that students can find the solutions to quadratic equations in factored form, such as $p(200 - p) = 0$, by using the factors to help determine the zeros. This is not the case with the equation $p(200 - p) = 500$, because there are many pairs of factors that have 500 as a product.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they have seen similar functions in the previous unit. Display the function $f(p) = -5p(p - 40)$ and ask:

- "What type of function is this?" Quadratic
- "In what form is it written in and what information does that form tell you about the function?" Factored form; It tells me the factors, which I can use to find the zeros.

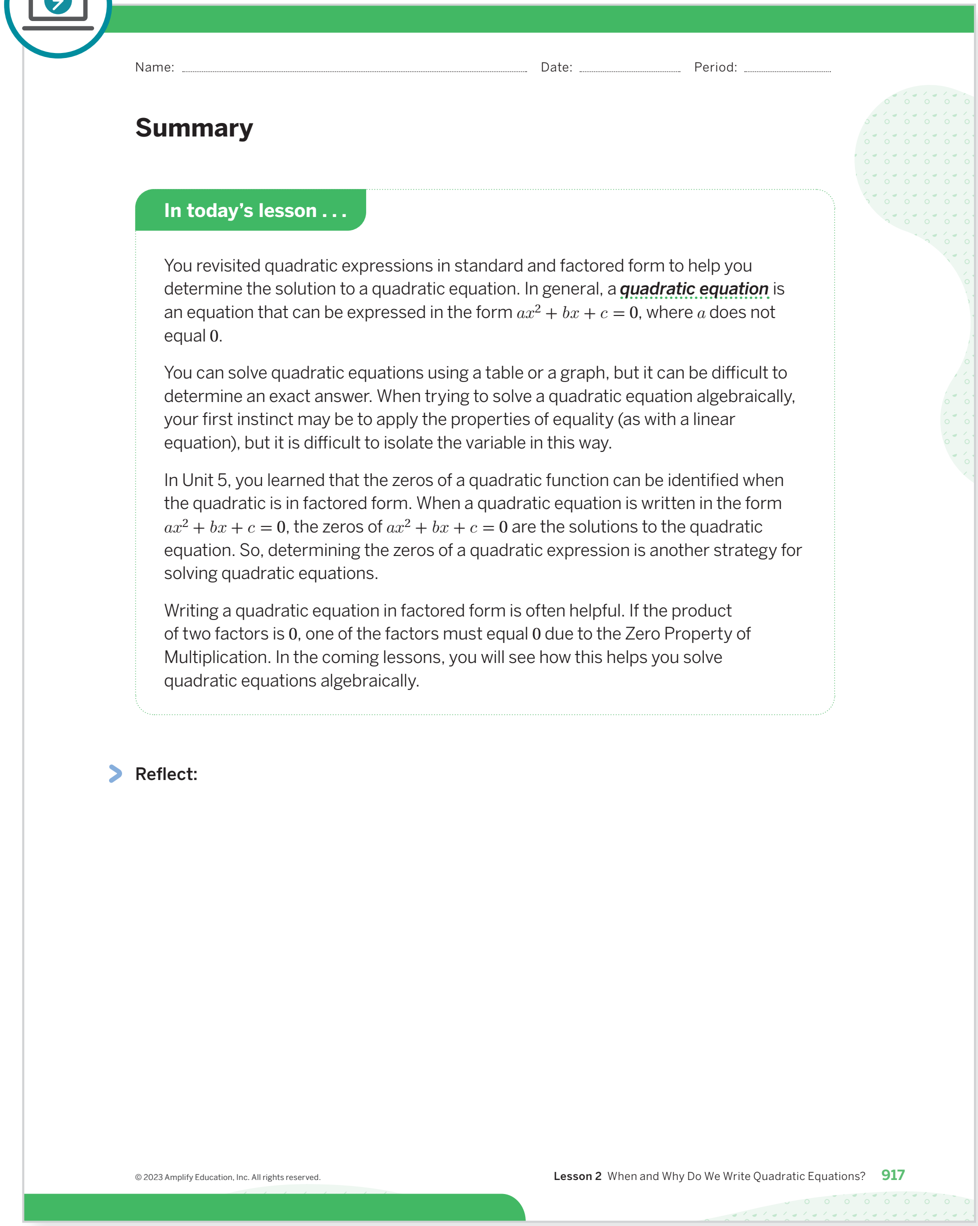
Accessibility: *Guide Processing and Visualization*

As students complete Problem 3, have them cover up the expression $p - 40$ and ask, "What number multiplied by $-5p$ would give a product of 0? What does this tell you about the value of p in the expression $p - 40$?" Repeat by having students cover up the expression $-5p$ and asking similar questions.

Whole Class | 5 min

Summary

Review and synthesize the limitations of solving quadratic equations by graphing, using tables, or the *guess-and-check* strategy.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You revisited quadratic expressions in standard and factored form to help you determine the solution to a quadratic equation. In general, a ***quadratic equation*** is an equation that can be expressed in the form $ax^2 + bx + c = 0$, where a does not equal 0.

You can solve quadratic equations using a table or a graph, but it can be difficult to determine an exact answer. When trying to solve a quadratic equation algebraically, your first instinct may be to apply the properties of equality (as with a linear equation), but it is difficult to isolate the variable in this way.

In Unit 5, you learned that the zeros of a quadratic function can be identified when the quadratic is in factored form. When a quadratic equation is written in the form $ax^2 + bx + c = 0$, the zeros of $ax^2 + bx + c = 0$ are the solutions to the quadratic equation. So, determining the zeros of a quadratic expression is another strategy for solving quadratic equations.

Writing a quadratic equation in factored form is often helpful. If the product of two factors is 0, one of the factors must equal 0 due to the Zero Property of Multiplication. In the coming lessons, you will see how this helps you solve quadratic equations algebraically.

Reflect:

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *quadratic equation* that were added to the display during the lesson.

Synthesize

Formalize vocabulary: *quadratic equation*

Ask:

- "What are some limitations of solving a quadratic equation by graphing? The *guess-and-check* strategy? Using a table of values?"

  Sample responses:

  - Graphing can be challenging when the solutions are not integer values.
  - The *guess-and-check* strategy can be time consuming. If the factors are not whole numbers they can be challenging to find.
  - A table of values may not include the specific values I am looking for.

- "How can you use the factored form of a quadratic function to determine the solutions of its equation?"

  I can determine the solutions by identifying the zeros of the function if the equation is expressed in factored form.

Highlight that solving a quadratic equation in factored form (set equal to zero) is a more straightforward way of determining its solutions than using the *guess-and-check* strategy and can be more accurate than graphing or using a table. In upcoming lessons, students will learn even more ways to solve quadratic equations.

Note: In a future lesson, students will formally define the Zero Product Principle. For now, help them build their conceptual understanding leading up to this principle by having them focus on the Zero Property of Multiplication and analyzing the factors of a quadratic equation written in factored form, set equal to 0.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

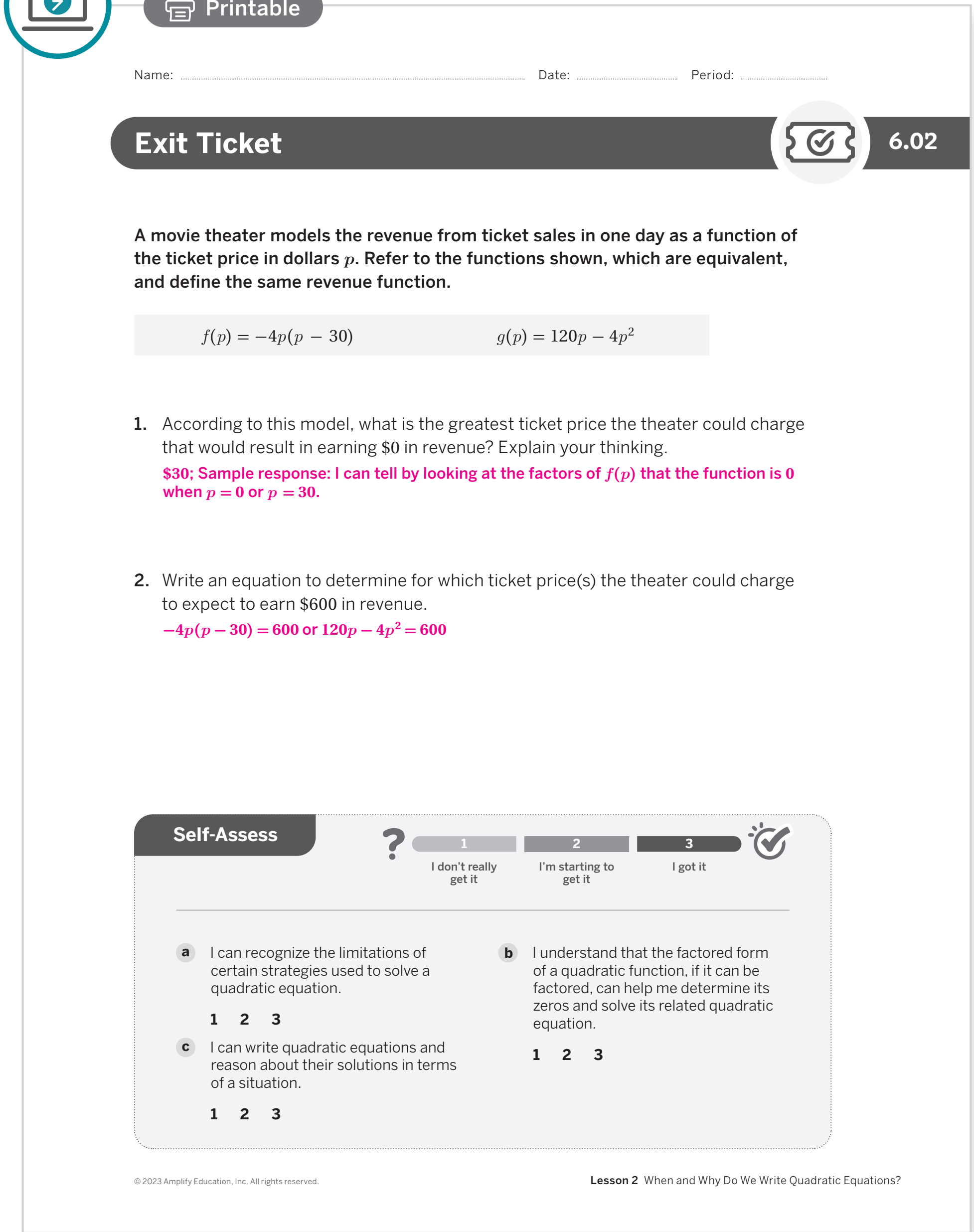
- "What strategies could you use to solve a quadratic equation?"
- "Which strategies do you find more helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the solutions to quadratic equations in factored form, based on a given context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.02

A movie theater models the revenue from ticket sales in one day as a function of the ticket price in dollars p. Refer to the functions shown, which are equivalent, and define the same revenue function.

$f(p) = -4p(p - 30)$ $g(p) = 120p - 4p^2$

1. According to this model, what is the greatest ticket price the theater could charge that would result in earning \$0 in revenue? Explain your thinking.

   \$30; Sample response: I can tell by looking at the factors of $f(p)$ that the function is 0 when $p = 0$ or $p = 30$.

2. Write an equation to determine for which ticket price(s) the theater could charge to expect to earn \$600 in revenue.

   $-4p(p - 30) = 600$ or $120p - 4p^2 = 600$

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can recognize the limitations of certain strategies used to solve a quadratic equation.

1 2 3

b I understand that the factored form of a quadratic function, if it can be factored, can help me determine its zeros and solve its related quadratic equation.

1 2 3

c I can write quadratic equations and reason about their solutions in terms of a situation.

1 2 3

Lesson 2 When and Why Do We Write Quadratic Equations?

Success looks like . . .

- **Language Goal:** Recognizing the limitations of certain strategies used to solve a quadratic equation. **(Speaking and Listening)**
- **Goal:** Understanding that the factored form of a quadratic function can help determine the zeros and solve its related quadratic equation.
- **Language Goal:** Writing quadratic equations and reasoning about their solutions in terms of a situation. **(Reading and Writing)**
  - » Writing an equation that determines the ticket price for expected revenue in Problems 1–2.

Suggested next steps

If students determine an inaccurate value in Problem 1, consider:

- Reviewing Activity 2, Problem 2.
- Assigning Practice Problem 3.

If students are unable to write an equation to determine a possible value of p in Problem 3, consider:

- Reviewing writing an equation in Activity 2, Problem 3.
- Reviewing Activity 2, Problem 2
- Assigning Practice Problem 2.

Professional Learning

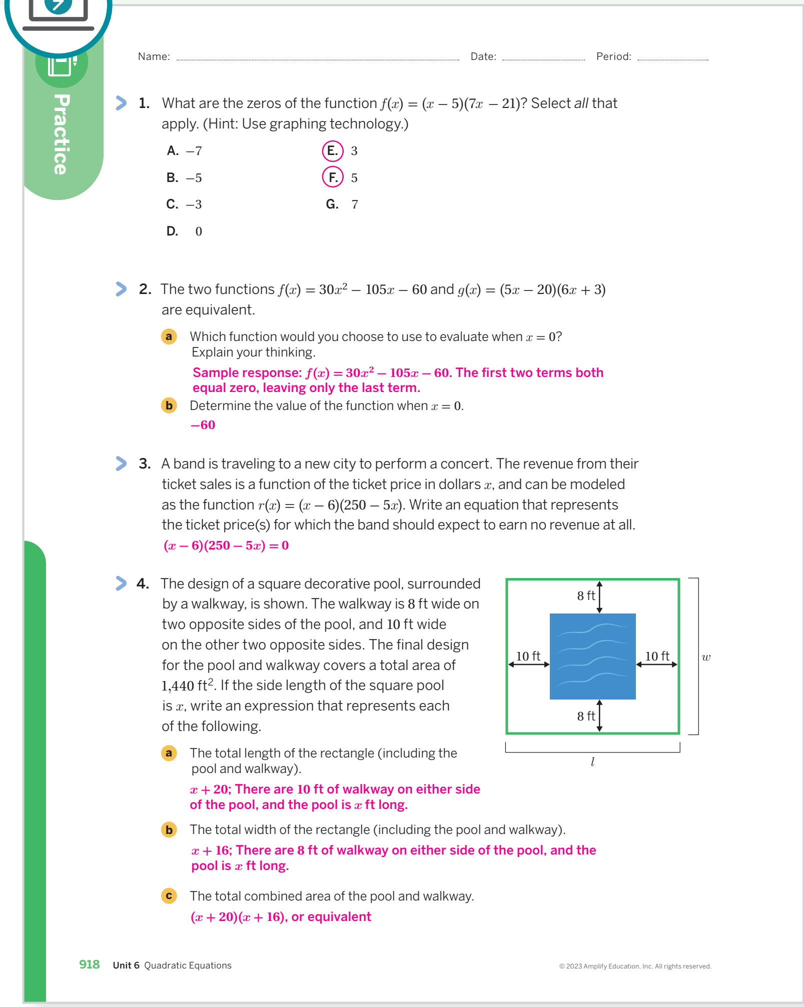
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did you see in the way some students approached solving the equation in Activity 1 that you would like other students to try?
- In what ways did Activity 2 go as planned, or not go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. What are the zeros of the function $f(x) = (x - 5)(7x - 21)$? Select *all* that apply. (Hint: Use graphing technology.)

   A. -7
   B. -5
   C. -3
   D. 0
   [✓ E.] 3
   [✓ F.] 5
   G. 7

2. The two functions $f(x) = 30x^2 - 105x - 60$ and $g(x) = (5x - 20)(6x + 3)$ are equivalent.

   a. Which function would you choose to use to evaluate when $x = 0$? Explain your thinking.

   Sample response: $f(x) = 30x^2 - 105x - 60$. The first two terms both equal zero, leaving only the last term.

   b. Determine the value of the function when $x = 0$.

   -60

3. A band is traveling to a new city to perform a concert. The revenue from their ticket sales is a function of the ticket price in dollars x, and can be modeled as the function $r(x) = (x - 6)(250 - 5x)$. Write an equation that represents the ticket price(s) for which the band should expect to earn no revenue at all.

   $(x - 6)(250 - 5x) = 0$

4. The design of a square decorative pool, surrounded by a walkway, is shown. The walkway is 8 ft wide on two opposite sides of the pool, and 10 ft wide on the other two opposite sides. The final design for the pool and walkway covers a total area of 1,440 ft². If the side length of the square pool is x, write an expression that represents each of the following.

   a. The total length of the rectangle (including the pool and walkway).

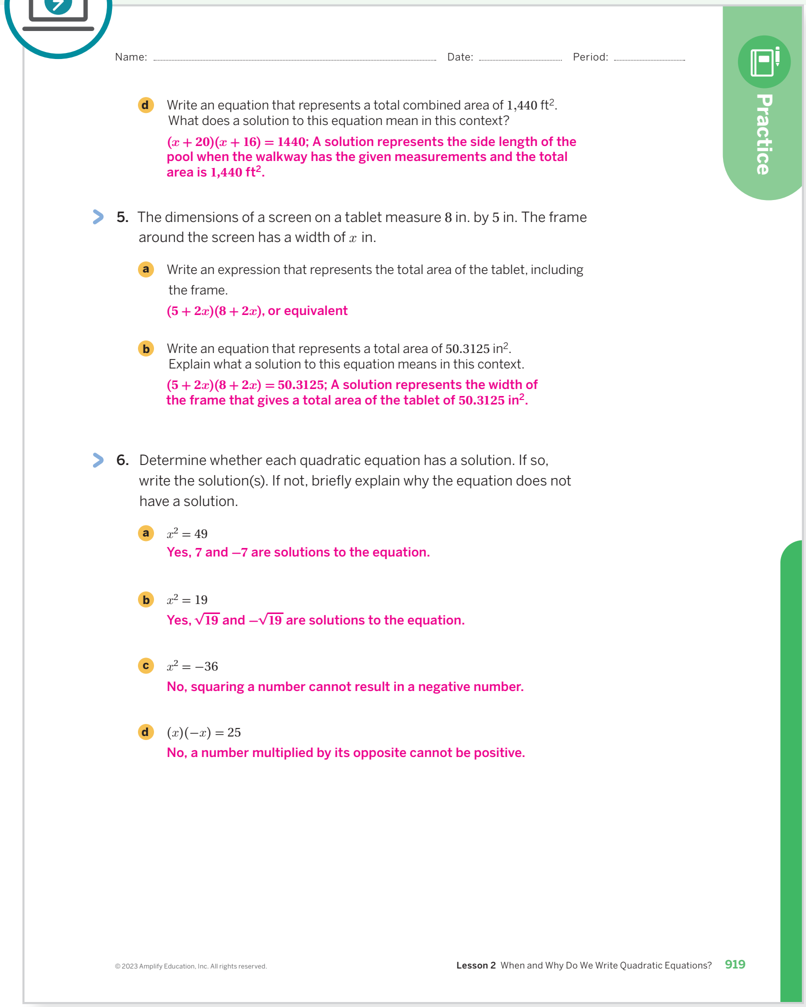
   $x + 20$; There are 10 ft of walkway on either side of the pool, and the pool is x ft long.

   b. The total width of the rectangle (including the pool and walkway).

   $x + 16$; There are 8 ft of walkway on either side of the pool, and the pool is x ft long.

   c. The total combined area of the pool and walkway.

   $(x + 20)(x + 16)$, or equivalent



Practice

Name: .... Date: .... Period: ....

   d. Write an equation that represents a total combined area of 1,440 ft². What does a solution to this equation mean in this context?

   $(x + 20)(x + 16) = 1440$; A solution represents the side length of the pool when the walkway has the given measurements and the total area is 1,440 ft².

5. The dimensions of a screen on a tablet measure 8 in. by 5 in. The frame around the screen has a width of x in.

   a. Write an expression that represents the total area of the tablet, including the frame.

   $(5 + 2x)(8 + 2x)$, or equivalent

   b. Write an equation that represents a total area of 50.3125 in². Explain what a solution to this equation means in this context.

   $(5 + 2x)(8 + 2x) = 50.3125$; A solution represents the width of the frame that gives a total area of the tablet of 50.3125 in².

6. Determine whether each quadratic equation has a solution. If so, write the solution(s). If not, briefly explain why the equation does not have a solution.

   a. $x^2 = 49$

   Yes, 7 and -7 are solutions to the equation.

   b. $x^2 = 19$

   Yes, $\sqrt{19}$ and $-\sqrt{19}$ are solutions to the equation.

   c. $x^2 = -36$

   No, squaring a number cannot result in a negative number.

   d. $(x)(-x) = 25$

   No, a number multiplied by its opposite cannot be positive.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 1	2
	5	Unit 6 Lesson 1	1
Formative ⚡	6	Unit 6 Lesson 3	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Mathematical Modeling Prompt

At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's Mathematical Modeling Prompt, *Planning a Fundraiser*, which is available in the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 3**

Solving Quadratic Equations by Reasoning

Let’s use square roots to solve some quadratic equations.

Focus

Goals

1. **Language Goal:** Determine the solutions to simple quadratic equations and justify the reasoning that leads to the solutions. **(Speaking and Listening, Writing)**
2. **Language Goal:** Understand that a quadratic equation may have two solutions. **(Speaking and Listening, Writing)**

Coherence

Today

Students build on their Grade 8 knowledge of solving equations of the form $x^2 = a$ to quadratic equations of the form $(x-c)^2 = a^2$ and use this structure to manipulate simple quadratic equations to solve. The idea that some quadratic equations have two (real) solutions is also made explicit, without focusing on the term *real number*. Solving the problems in this lesson gives students many opportunities to engage in sense-making, perseverance, and abstract reasoning.

Previously

Students observed the limitations of determining solutions of quadratic equations by graphing and using tables.

Coming Soon

In Lesson 4, students are formally introduced to the Zero Product Principle, solidifying the connection between a quadratic function’s factored form and x-intercepts, and the solutions of a quadratic equation in factored form.

Rigor

- Students build **conceptual understanding** of solving quadratic equations, extending their Grade 8 knowledge from $x^2 = a$ to $(x + c)^2 = a$.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Pairs	Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF (answers)
- Activity 3 PDF (instructions)
- Activity 3 PDF, pre-cut cards, one set per group
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- graphing technology
- scientific calculators

Math Language Development

New words

- *plus-or-minus* (±)

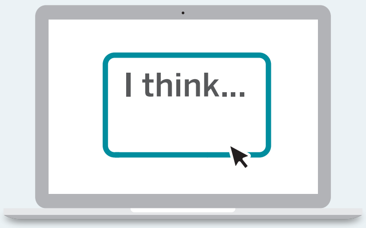
Review words

- *factored form*
- *quadratic equation*

Amps | **Featured Activity**

Activity 2
See Student Thinking

Students are asked to explain their thinking as they reason about the solutions to quadratic equations, and these explanations are available to you digitally, in real time.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may become confused if they have determined a particular structure that helps them to solve one problem that does not necessarily work for another. Prompt students to use that approach on all problems where it applies first in order to build confidence in their ability, then to shift their perspective as they attempt to solve the other problems, or circulate the room to examine approaches taken by their peers.

Modifications to Pacing

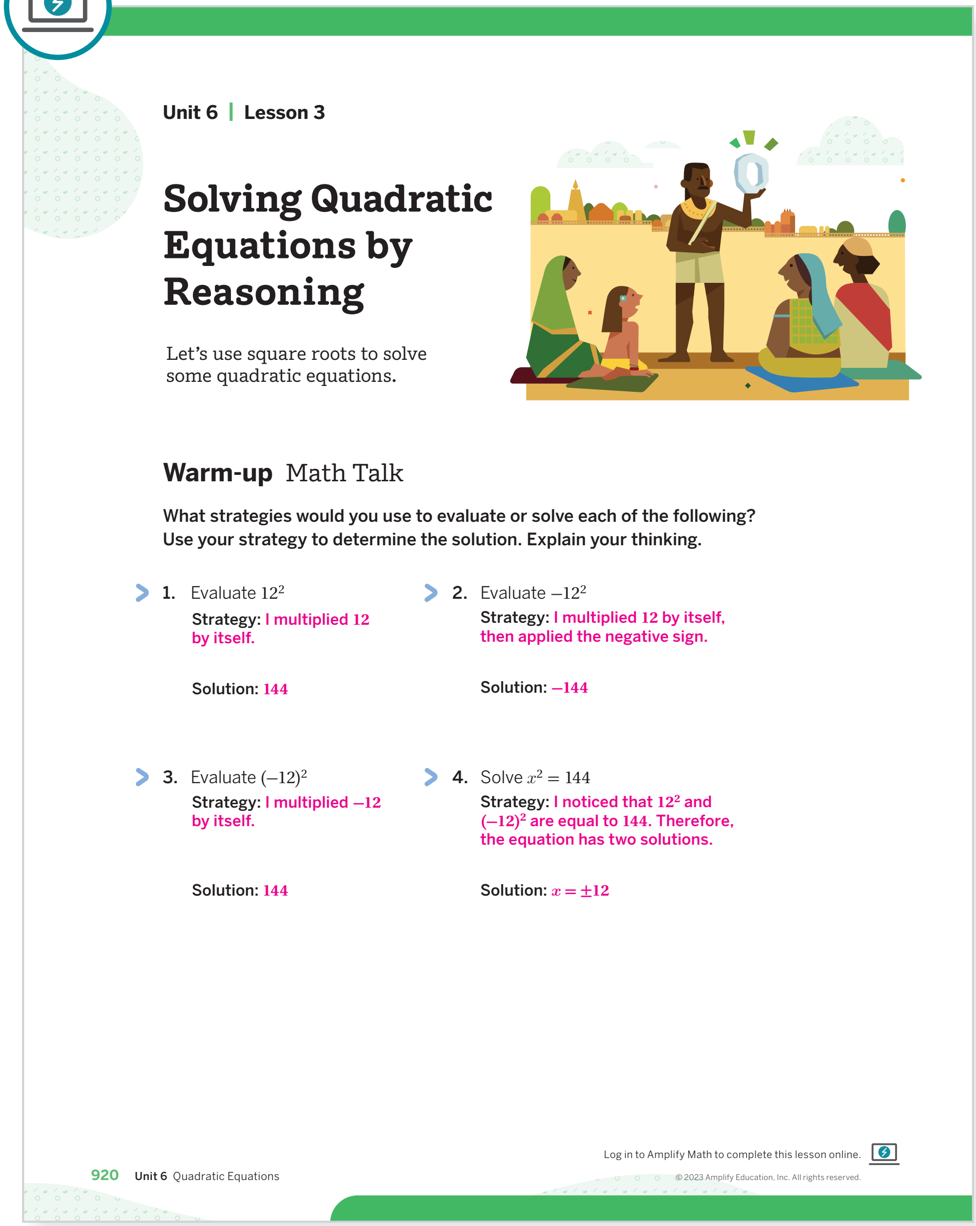
You may want to consider this additional modification if you are short on time.

- **Activity 3** may be omitted.

Pairs | 5 min

Warm-up Math Talk

Students use repeated reasoning to notice that a number and its opposite are solutions when solving square root equations.



Unit 6 | Lesson 3

Solving Quadratic Equations by Reasoning

Let's use square roots to solve some quadratic equations.

Warm-up Math Talk

What strategies would you use to evaluate or solve each of the following? Use your strategy to determine the solution. Explain your thinking.

1. Evaluate 12^2
   Strategy: I multiplied 12 by itself.
   Solution: 144
2. Evaluate -12^2
   Strategy: I multiplied 12 by itself, then applied the negative sign.
   Solution: -144
3. Evaluate $(-12)^2$
   Strategy: I multiplied -12 by itself.
   Solution: 144
4. Solve $x^2 = 144$
   Strategy: I noticed that 12^2 and $(-12)^2$ are equal to 144. Therefore, the equation has two solutions.
   Solution: $x = \pm 12$

Log in to Amplify Math to complete this lesson online.

920 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the *Math Talk* routine. Allow students individual work time. Then have them discuss their strategies with a partner.

2 Monitor

Help students get started by reminding them that squaring a negative value results in a positive value because the product of two negative values is positive.

Look for points of confusion:

- **Thinking $(-12)^2$ produces a negative value.** Have students write the expression in expanded form and ask, "What is -12 times -12?"
- **Forgetting the negative square root.** Have students refer to their solutions for 12^2 and $(-12)^2$.
- **Not seeing a difference between $(-12)^2$ and -12^2.** Explain that a negative sign is a shorthand of multiplying by -1 and encourage them to use the order of operations.

Look for productive strategies:

- Expanding the given expressions.
- Recognizing that a square root has two solutions.

3 Connect

Have pairs of students share the strategies they used to evaluate each expression.

Ask, "What is the difference between $(-12)^2$ with parentheses and -12^2 without parentheses?"

Highlight that the order of operations performs exponents before multiplication, which is why $-12^2 = -144$. Then draw students' attention to the last problem and highlight that 12^2 and $(-12)^2$ yield 144, so the square root of 144 includes both values. The square root of a number will always have a positive and a negative solution.

Math Language Development

MLR8: Discussion Supports

Before the Connect, have students use the *Think-Pair-Share* routine to respond to the question, "How did evaluating 12^2 and $(-12)^2$ help you solve the equation $x^2 = 144$?" Then ask these students to share their responses to this question during the Connect.

English Learners

To help students explain their thinking, display or provide the Anchor Chart PDF, *Sentence Stems, Explaining My Steps*.

Power-up

To power up students' ability to determine solutions to equations of the form $x^2 = p$, have students complete:

Recall that the product of two factors with the same signs is positive. The product of two factors with different signs is negative. For each equation, determine whether it has a solution. Be prepared to explain your thinking.

1. $x^2 = -100$ No, squaring a number always results in a positive value.
2. $x^2 = 100$ Yes, there are two solutions, 10 and -10.
3. $(x)(-x) = 100$ No, one of these values will be negative, resulting in a negative product.

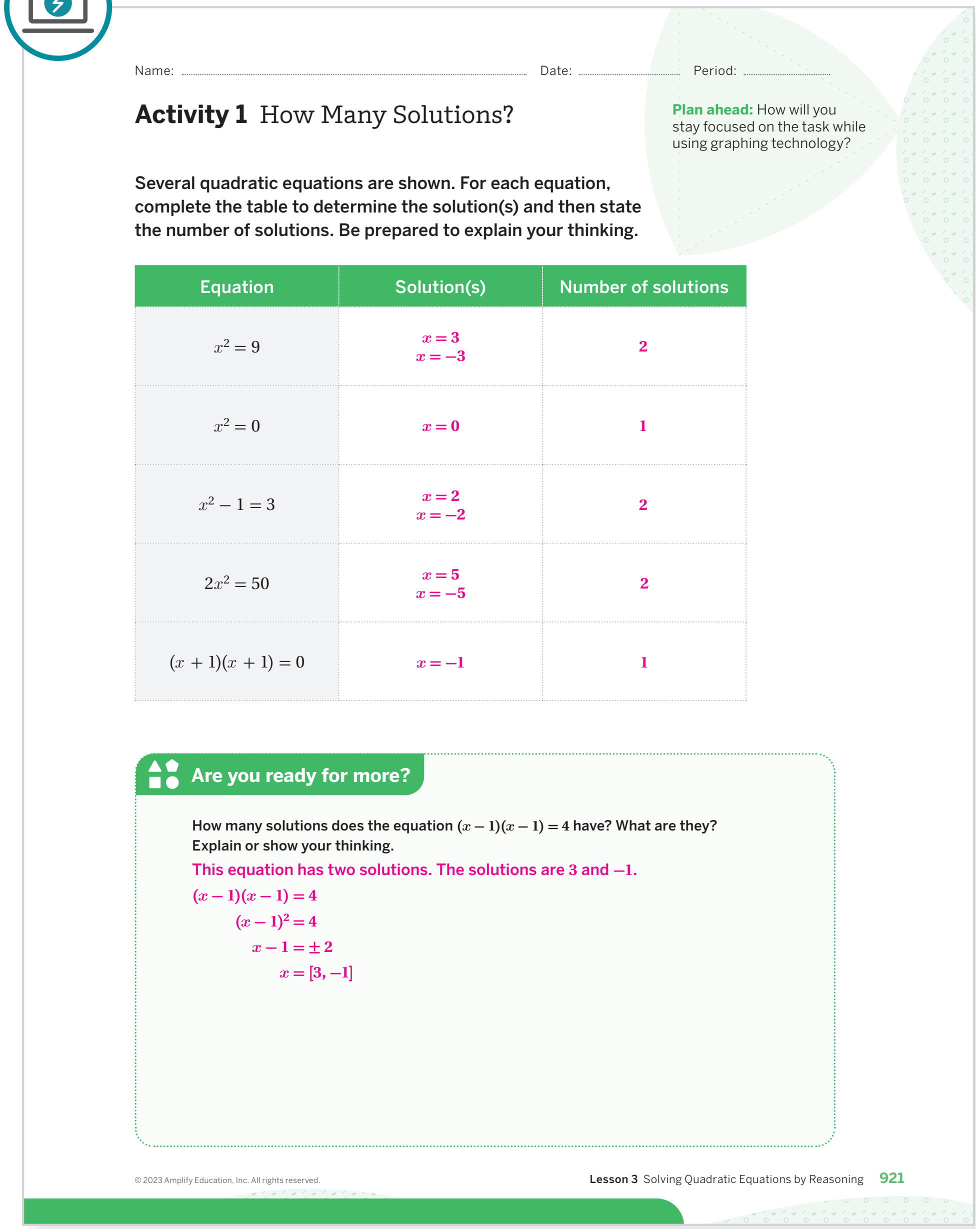
Use: Before the Warm-up

Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Independent | 10 min

Activity 1 How Many Solutions?

Students determine the solutions and number of solutions to simple quadratic equations by reasoning about the values that would make the equations true.



Name: Date: Period:

Activity 1 How Many Solutions?

Plan ahead: How will you stay focused on the task while using graphing technology?

Several quadratic equations are shown. For each equation, complete the table to determine the solution(s) and then state the number of solutions. Be prepared to explain your thinking.

Equation	Solution(s)	Number of solutions
$x^2 = 9$	$x = 3$ $x = -3$	2
$x^2 = 0$	$x = 0$	1
$x^2 - 1 = 3$	$x = 2$ $x = -2$	2
$2x^2 = 50$	$x = 5$ $x = -5$	2
$(x + 1)(x + 1) = 0$	$x = -1$	1

Are you ready for more?

How many solutions does the equation $(x - 1)(x - 1) = 4$ have? What are they? Explain or show your thinking.

This equation has two solutions. The solutions are 3 and −1.

$$\begin{aligned} (x-1)(x-1) &= 4 \\ (x-1)^2 &= 4 \\ x - 1 &= \pm 2 \\ x &= [3, -1] \end{aligned}$$

1 Launch

Provide access to graphing technology. Ask, "When will an equation of the form $x^2 = b$ have two integer solutions?" When b is a square number.

2 Monitor

Help students get started by asking, "What is the square root of 144?" 12 and −12

Look for points of confusion:

- **Writing $\pm\frac{\sqrt{50}}{2}$ as a solution for the equation $2x^2 = 50$.** Explain that only x is squared, not $2x^2$.

Look for productive strategies:

- Identifying square numbers.
- Manipulating the equation to include a square number.

3 Connect

Have students share their responses and strategies for solving each equation.

Ask:

- "Which equations only have one solution?" $x^2 = 0$ and $(x + 1)^2 = 0$.
- "Why do you think that is the case?" Both equations are set equal to 0, which has only one square root, 0. The other numbers had a positive and a negative square root.

Highlight that a square number or expression has two roots — a number and its opposite.

Define the term ***plus-or-minus*** as a number and its opposite. Introduce its notation, ±, if students are unfamiliar with it.

Note: In this course, students are determining only the real solutions to quadratic equations. Unless otherwise specified, the number of solutions to a quadratic equation is limited to the number of real solutions. Students will learn about non-real solutions in Algebra 2.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Have students complete the first two rows of the table. Then display the equations $x^2 = 4$ and $x^2 - 1 = 3$ and ask:

- "Are these equations equivalent? Why or why not?"
- "Can you use the first equation to determine the solution(s)?"
- "How could you use a similar strategy to determine the solution(s) to the equation $2x^2 = 50$?"
- "Does the equation in the last row look familiar to you? Think about the forms of quadratic equations you studied in the previous unit."

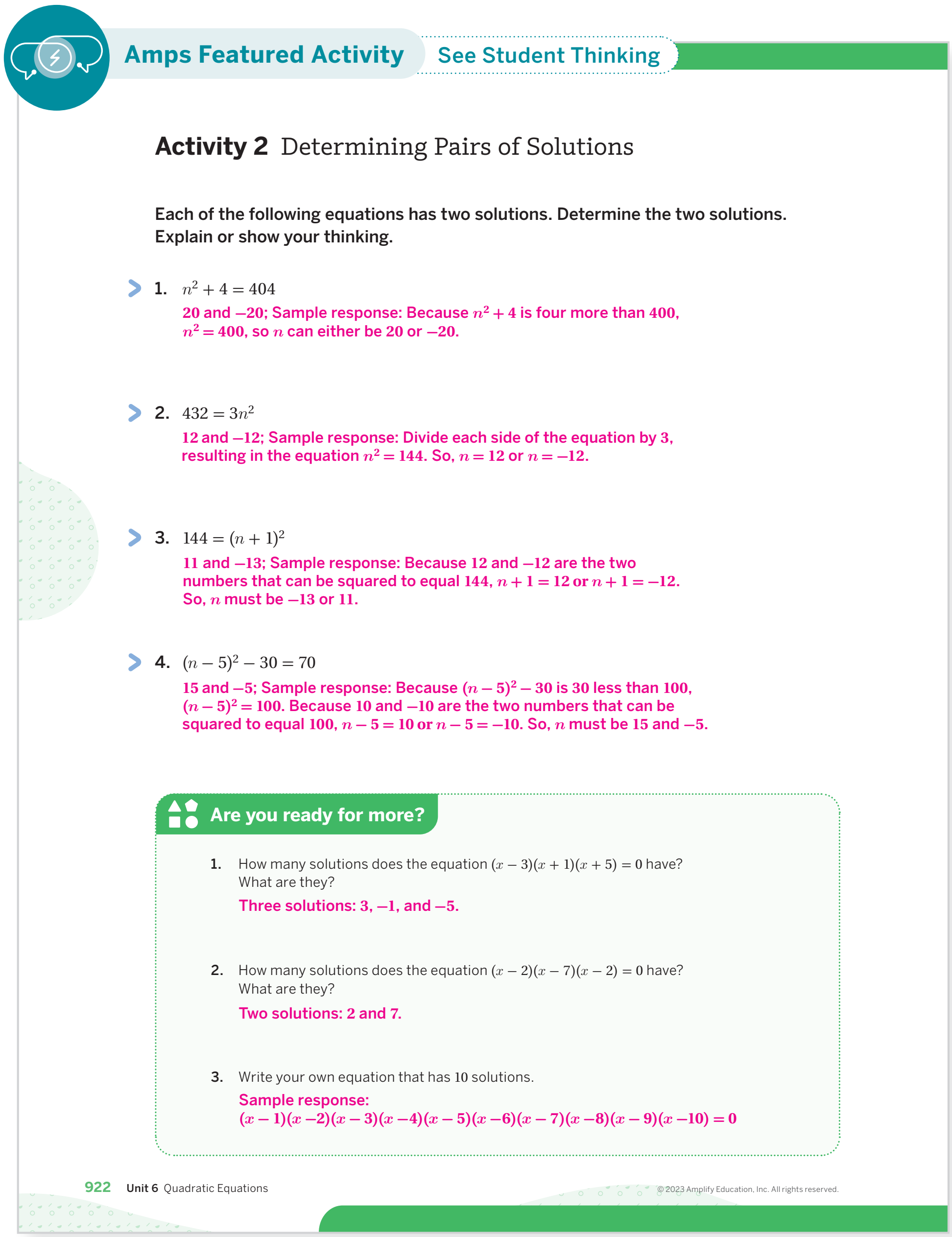
Extension: *Math Enrichment*

Tell students that, in this course, they are determining the number of *real* solutions to a quadratic equation, even though the term *real solution* is not stated. Mention that every quadratic equation actually has two solutions. Sometimes, these solutions are real numbers and sometimes they are not real numbers. For example, there are actually two solutions to the equation $2x^2 = -50$. Because evaluating the square root of a negative number does not give a real solution, the two solutions are not real. Students will learn about the solutions that are not real in future mathematics courses.

Pairs | 10 min

Activity 2 Determining Pairs of Solutions

Students solve quadratic equations algebraically by isolating square numbers.



Amps Featured Activity See Student Thinking

Activity 2 Determining Pairs of Solutions

Each of the following equations has two solutions. Determine the two solutions. Explain or show your thinking.

1. $n^2 + 4 = 404$

   20 and –20; Sample response: Because $n^2 + 4$ is four more than 400, $n^2 = 400$, so n can either be 20 or –20.

2. $432 = 3n^2$

   12 and –12; Sample response: Divide each side of the equation by 3, resulting in the equation $n^2 = 144$. So, $n = 12$ or $n = -12$.

3. $144 = (n + 1)^2$

   11 and –13; Sample response: Because 12 and –12 are the two numbers that can be squared to equal 144, $n + 1 = 12$ or $n + 1 = -12$. So, n must be –13 or 11.

4. $(n - 5)^2 - 30 = 70$

   15 and –5; Sample response: Because $(n - 5)^2 - 30$ is 30 less than 100, $(n - 5)^2 = 100$. Because 10 and –10 are the two numbers that can be squared to equal 100, $n - 5 = 10$ or $n - 5 = -10$. So, n must be 15 and –5.

Are you ready for more?

1. How many solutions does the equation $(x - 3)(x + 1)(x + 5) = 0$ have? What are they?

   Three solutions: 3, –1, and –5.

2. How many solutions does the equation $(x - 2)(x - 7)(x - 2) = 0$ have? What are they?

   Two solutions: 2 and 7.

3. Write your own equation that has 10 solutions.

   Sample response:
   $(x - 1)(x - 2)(x - 3)(x - 4)(x - 5)(x - 6)(x - 7)(x - 8)(x - 9)(x - 10) = 0$

922 Unit 6 Quadratic Equations

1 Launch

Arrange students in pairs. Have them work independently for a few minutes before discussing their thinking with a partner. Provide access to graphing technology, if requested.

2 Monitor

Help students get started by prompting them to solve the equations by applying the properties of equality and looking for perfect squares.

Look for points of confusion:

- **Writing one root for positive square numbers.** Ask, "Is this the only solution?"
- **Having difficulty deriving a square number.** Prompt students to isolate the variable.

Look for productive strategies:

- Substituting different values for n.
- Using graphing technology to make a table and look for the target value.
- Reasoning and making use of the structure of the equations.
- Solving the equations algebraically.

3 Connect

Have pairs of students share their strategies in order of their efficiency, as listed in the productive strategies. Help students to make connections between the different strategies.

Highlight how to reason about the structure of each equation and how the properties of equality can be used to isolate the quantity that is squared. Be sure students understand the difference between expressions such as $x^2 - 5$ and $(x - 5)^2$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students color code the quantity that is squared in each equation. If there are numbers on the same side of the squared quantity, ask them how they can move those numbers to the other side. For example:

$n^2 + 4 = 404$ Color code the squared quantity.

Ask:

- "What number is on the same side of the squared quantity?" 4
- "How can you move 4 to the other side?" Subtract 4 from each side.
- "What equation is the result?" $n^2 = 400$

Math Language Development

MLR7: Compare and Connect

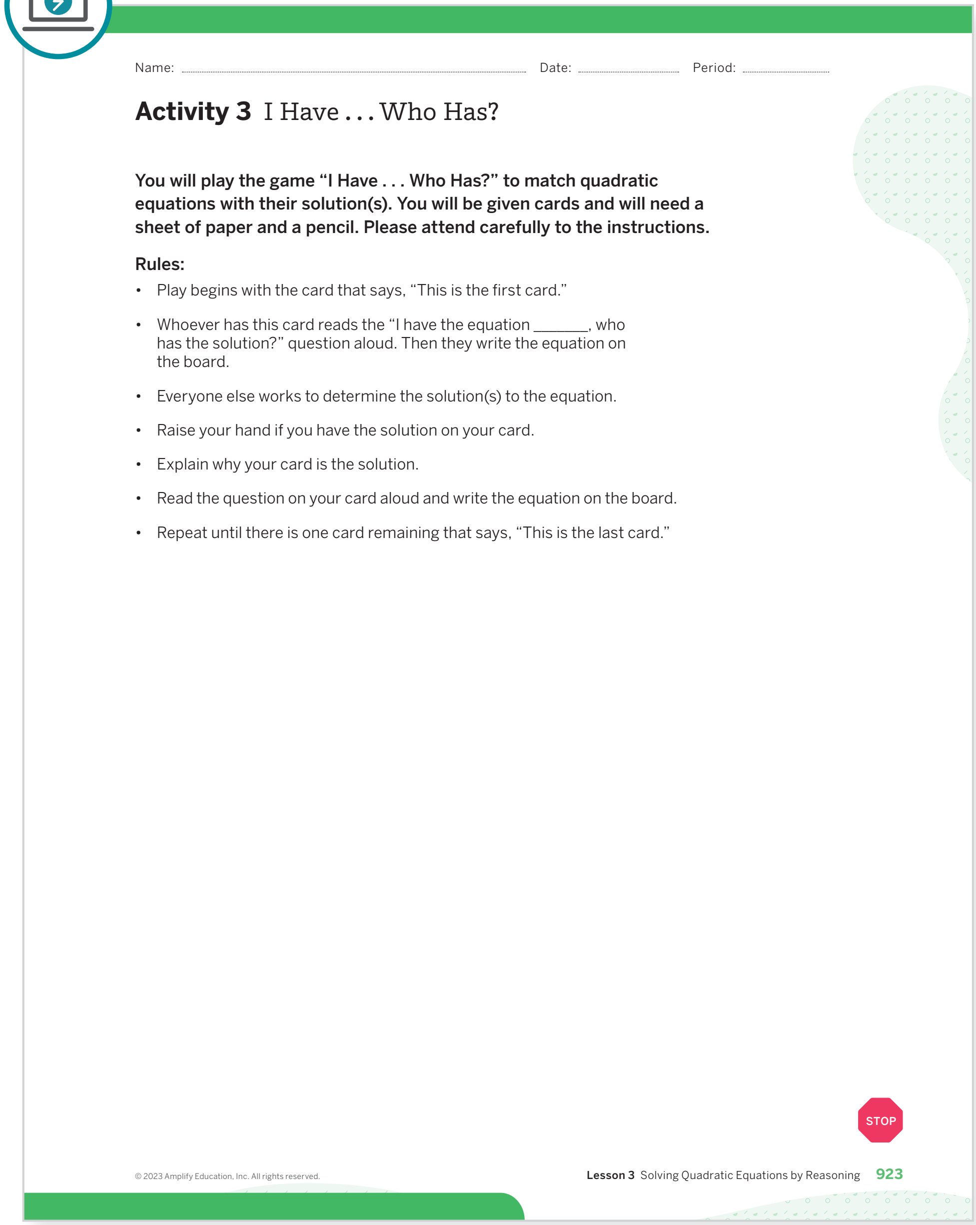
During the Connect, display the equation from Problem 4 and the new equation $n^2 - 5 - 30 = 70$. Ask:

- "What is different about the structure of these equations?" In the equation in Problem 4, 5 is subtracted from n before it is squared. In the new equation, 5 is subtracted from n^2 (after n is squared).
- "To solve the equation in Problem 4, how can you isolate the squared quantity?" Add 30 to each side.
- "To solve the equation $n^2 - 5 - 30 = 70$, how can you isolate the squared quantity?" Combine the like terms –5 and –30. Then add 35 to each side.

Small Groups | 15 min

Activity 3 I Have . . . Who Has?

Students determine the solutions that match given quadratic equations by reasoning about the values that make them true.



Name: ____________ Date: ________ Period: ________

Activity 3 I Have . . . Who Has?

You will play the game "I Have . . . Who Has?" to match quadratic equations with their solution(s). You will be given cards and will need a sheet of paper and a pencil. Please attend carefully to the instructions.

Rules:

- Play begins with the card that says, "This is the first card."
- Whoever has this card reads the "I have the equation _______, who has the solution?" question aloud. Then they write the equation on the board.
- Everyone else works to determine the solution(s) to the equation.
- Raise your hand if you have the solution on your card.
- Explain why your card is the solution.
- Read the question on your card aloud and write the equation on the board.
- Repeat until there is one card remaining that says, "This is the last card."

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Solving Quadratic Equations by Reasoning 923

1 Launch

Display the Activity 3 PDF (instructions). Shuffle and distribute the pre-cut cards from the Activity 3 PDF to each group. Conduct the ***I Have, Who Has?*** routine. Read the directions and demonstrate how to play the game. Consider providing a whiteboard to each group for students to write quadratic equations.

2 Monitor

Help students get started by prompting them to look for or rearrange quadratic equations so that they are equal to a square integer.

Look for productive strategies:

- Substituting the values on their cards for x.
- Using graphing technology to make a table and look for the target value.
- Using graphing technology to graph the given quadratic equation.
- Solving the equation algebraically.

3 Connect

Have students share the reasoning they used to determine their matches after a solution is found for each card.

Highlight the structure of each equation, and how the properties of equality can help isolate the squared quantity.

Ask:

- "Why does $(x - 7)^2 = -100$ have no solution?" No number multiplied by itself will result in a negative number.
- "What do you notice about the quadratic equations with only one solution?" The two linear factors of the equation are the same.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

For students that receive a card with a quadratic equation on it, ask them to analyze the structure of their equation prior to beginning the game. Ask:

- "Without performing the operation(s), what number(s) can you move to the other side to help you isolate the squared quantity?"
- "Are there any numbers added to or subtracted from x before that value is squared?"

Math Language Development

MLR8: Discussion Supports— Revoicing

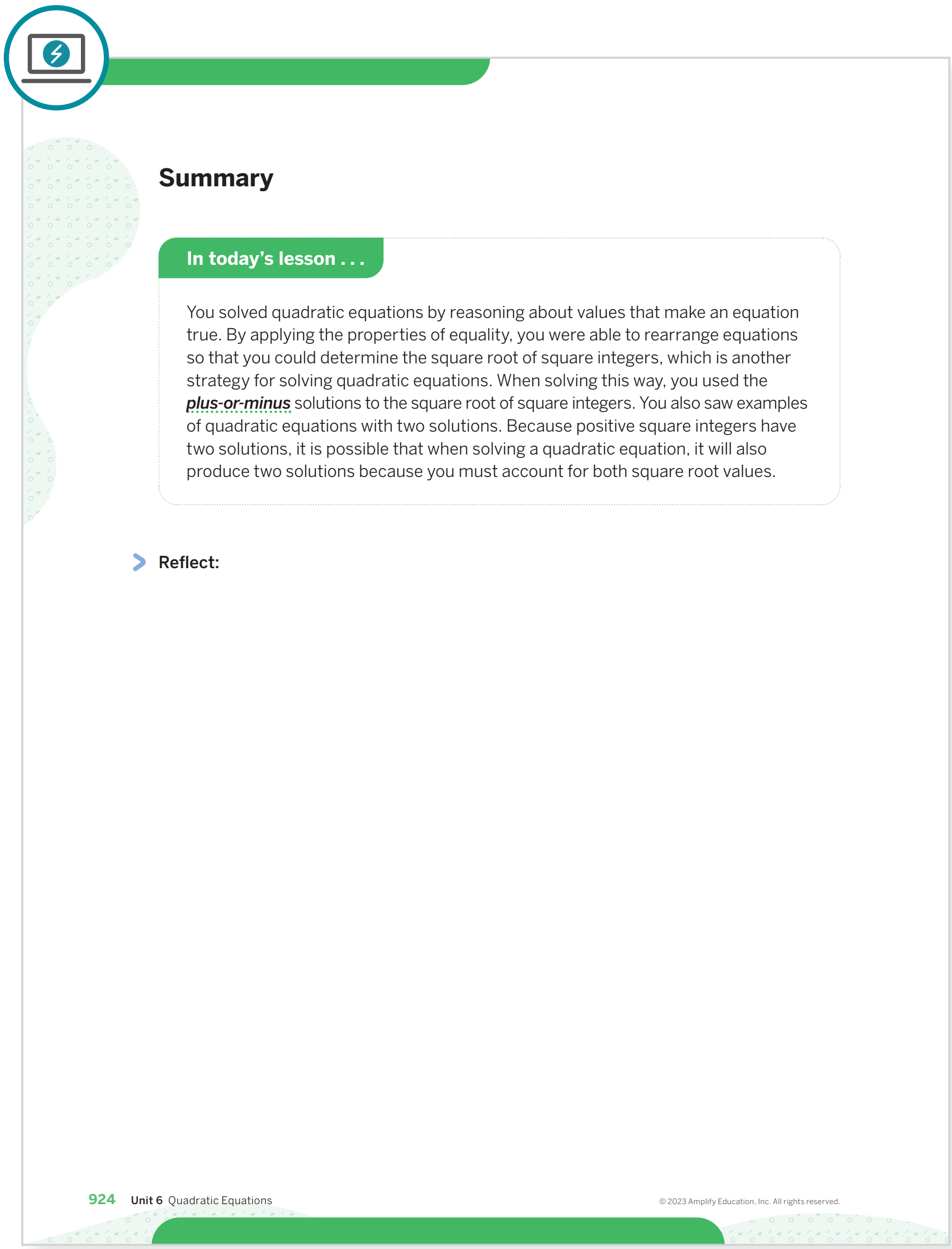
As students play the game, listen for and amplify the language they use to determine their matches. Revoice or restate the language they use to demonstrate precise mathematical vocabulary and press them for details in their reasoning. For example, for the equation $(x - 2)^2 - 3 = 6$:

If a student says . . .	Revoice their ideas by asking . . .
"I wrote the $(x - 2)^2 = 9$. Then I took the square root of 9, and added 2."	"What property allows you to add 3? How did you know you needed to add 3 first? Why did you add 2 after taking the square root? How many solutions are there?"

Whole Class | 5 min

Summary

Review and synthesize how the structure of a quadratic equation can help students reason about its solution(s).



Summary

In today's lesson . . .

You solved quadratic equations by reasoning about values that make an equation true. By applying the properties of equality, you were able to rearrange equations so that you could determine the square root of square integers, which is another strategy for solving quadratic equations. When solving this way, you used the ***plus-or-minus*** solutions to the square root of square integers. You also saw examples of quadratic equations with two solutions. Because positive square integers have two solutions, it is possible that when solving a quadratic equation, it will also produce two solutions because you must account for both square root values.

Reflect:

924 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *plus-or-minus* that were added to the display during the lesson.

Synthesize

Display the equations from Activity 1.

Have students share their strategies for solving each equation. Ask, "Did you use the same strategy or different strategies for each equation?" Be sure to hone in on why students chose to use a different strategy, if they mention using different strategies.

Display the equations from Activity 2.

Ask, "Without referring to the solutions in Activity 2, can you determine which equations have two solutions that are opposites (additive inverses)? Which equations have two solutions that are not opposites? How can you tell?" Equations that have two solutions that are opposites are equations with an isolated squared variable term. Equations that do not have two solutions that are opposites are equations that have a linear expression being squared.

Highlight that students can determine the solutions to some quadratic equations by performing the same operation on both sides of the equation first and then reasoning about the values of the variable that make the equation true. Manipulating the equation often results in a square number, which enables them to solve it by taking square roots.

Formalize vocabulary: *plus-or-minus* (±)

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

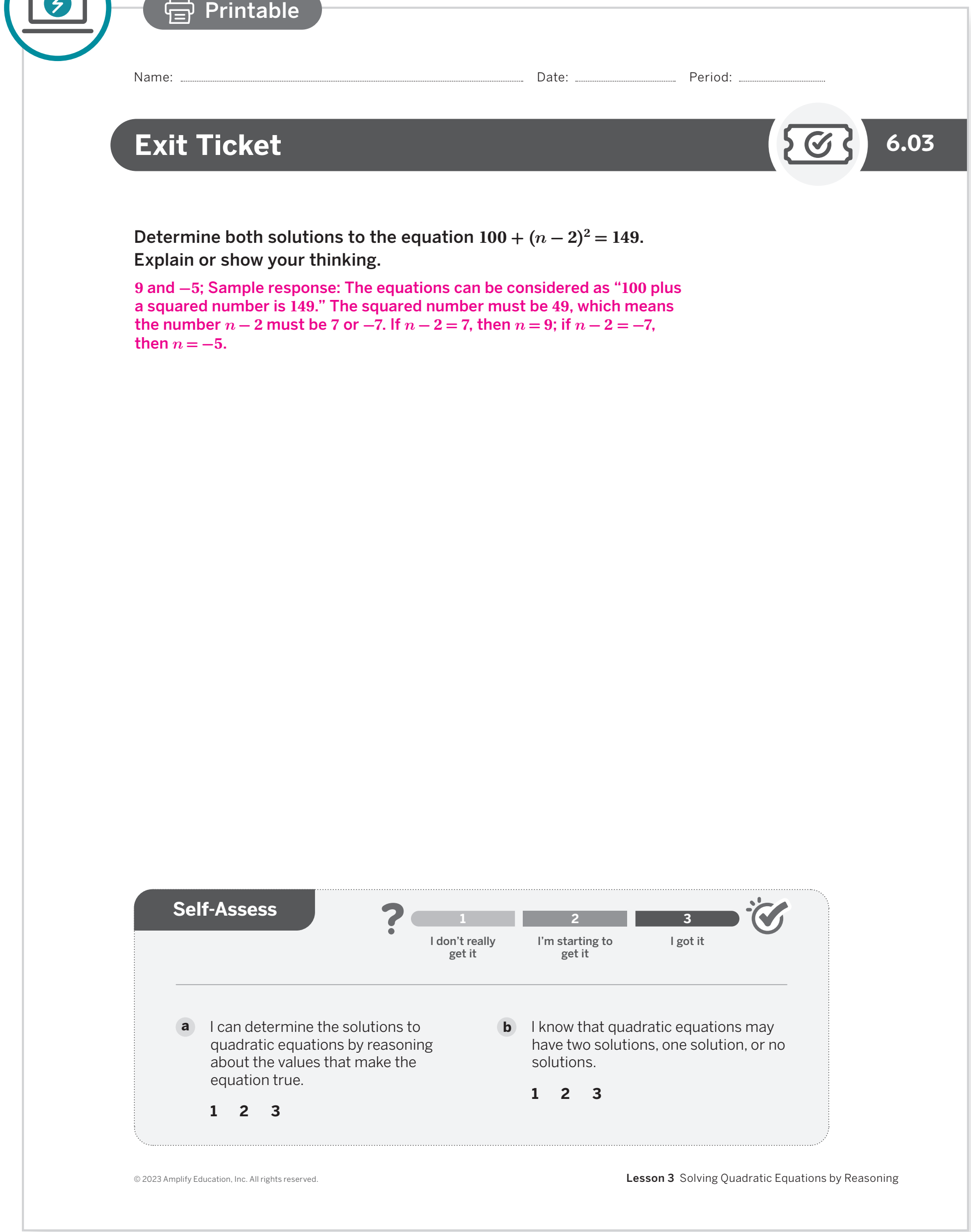
- "When is the plus-or-minus symbol used in solving a quadratic equation?"
- "What can you tell about the graph of a quadratic equation if you know the number of solutions it has?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reasoning about the values that make a quadratic equation true.

Printable



Name: Date: Period:

Exit Ticket 6.03

Determine both solutions to the equation $100 + (n - 2)^2 = 149$. Explain or show your thinking.

9 and –5; Sample response: The equations can be considered as "100 plus a squared number is 149." The squared number must be 49, which means the number $n - 2$ must be 7 or –7. If $n - 2 = 7$, then $n = 9$; if $n - 2 = -7$, then $n = -5$.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can determine the solutions to quadratic equations by reasoning about the values that make the equation true.
1 2 3

b I know that quadratic equations may have two solutions, one solution, or no solutions.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Solving Quadratic Equations by Reasoning

Success looks like . . .

- **Language Goal:** Determining the solutions to simple quadratic equations and justifying the reasoning that leads to the solutions. **(Speaking and Listening, Writing)**
  - » Determining the solutions to a quadratic equation and explaining how to determine it.
- **Language Goal:** Understanding that a quadratic equation may have two solutions. **(Speaking and Listening, Writing)**

Suggested next steps

If students are unable to determine both solutions to the equation, consider:

- Prompting students to change the equation to include a perfect square.
- Reviewing Activity 2.
- Assigning Practice Problems 2 and 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students determined the number of solutions of quadratic equations. How will that support students future use of the Zero Product Principle?
- Who participated and who didn't participate in the "I Have . . . Who Has?" activity today? What might you change for the next time you teach this lesson?

Math Language Development

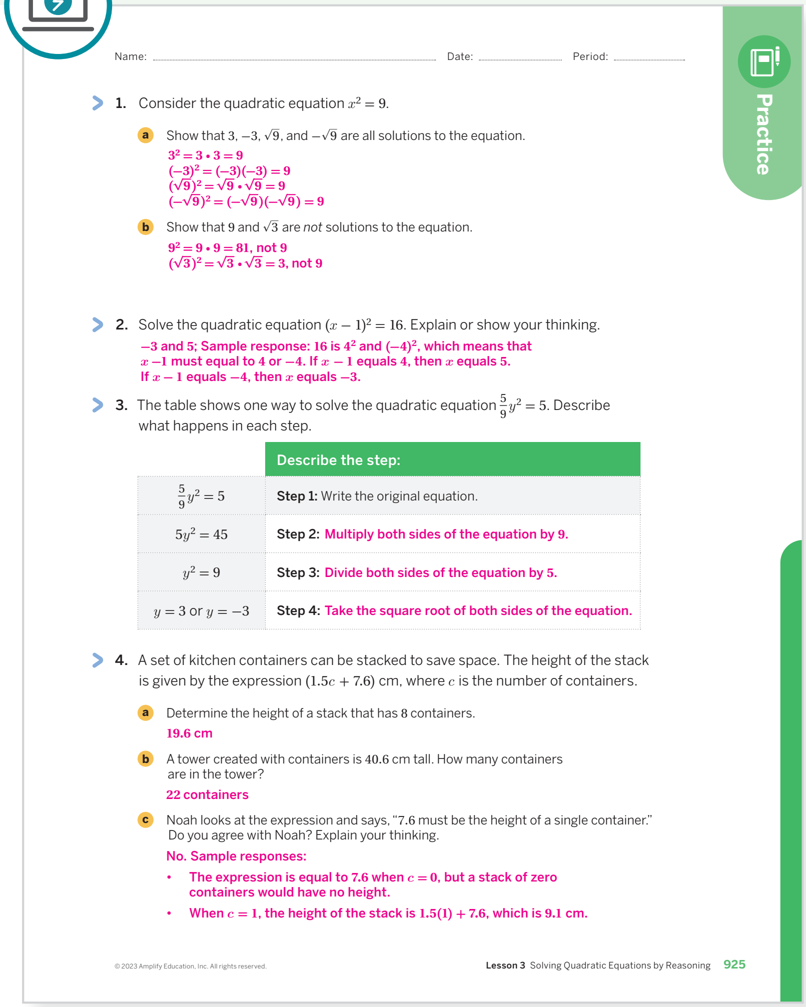
Language Goal: Determining the solutions to simple quadratic equations and justifying the reasoning that leads to the solutions.

Reflect on students' language development toward this goal.

- How did using the *Compare* and Connect routine in Activity 2 help students see the structure of simple quadratic equations?
- How did using the *Discussion Supports* routine in Activity 3 help students use math language to describe their reasoning for solving simple quadratic equations?

Practice

Independent



Name: Date: Period:

1. Consider the quadratic equation $x^2 = 9$.

   a Show that 3, –3, $\sqrt{9}$, and $-\sqrt{9}$ are all solutions to the equation.

   $3^2 = 3 \cdot 3 = 9$
   $(-3)^2 = (-3)(-3) = 9$
   $(\sqrt{9})^2 = \sqrt{9} \cdot \sqrt{9} = 9$
   $(-\sqrt{9})^2 = (-\sqrt{9})(-\sqrt{9}) = 9$

   b Show that 9 and $\sqrt{3}$ are *not* solutions to the equation.

   $9^2 = 9 \cdot 9 = 81$, not 9
   $(\sqrt{3})^2 = \sqrt{3} \cdot \sqrt{3} = 3$, not 9

2. Solve the quadratic equation $(x - 1)^2 = 16$. Explain or show your thinking.

   –3 and 5; Sample response: 16 is 4^2 and $(-4)^2$, which means that $x - 1$ must equal to 4 or –4. If $x - 1$ equals 4, then x equals 5. If $x - 1$ equals –4, then x equals –3.

3. The table shows one way to solve the quadratic equation $\frac{5}{9}y^2 = 5$. Describe what happens in each step.

	Describe the step:
$\frac{5}{9}y^2 = 5$	**Step 1:** Write the original equation.
$5y^2 = 45$	**Step 2:** Multiply both sides of the equation by 9.
$y^2 = 9$	**Step 3:** Divide both sides of the equation by 5.
$y = 3$ or $y = -3$	**Step 4:** Take the square root of both sides of the equation.

4. A set of kitchen containers can be stacked to save space. The height of the stack is given by the expression $(1.5c + 7.6)$ cm, where c is the number of containers.

   a Determine the height of a stack that has 8 containers.

   19.6 cm

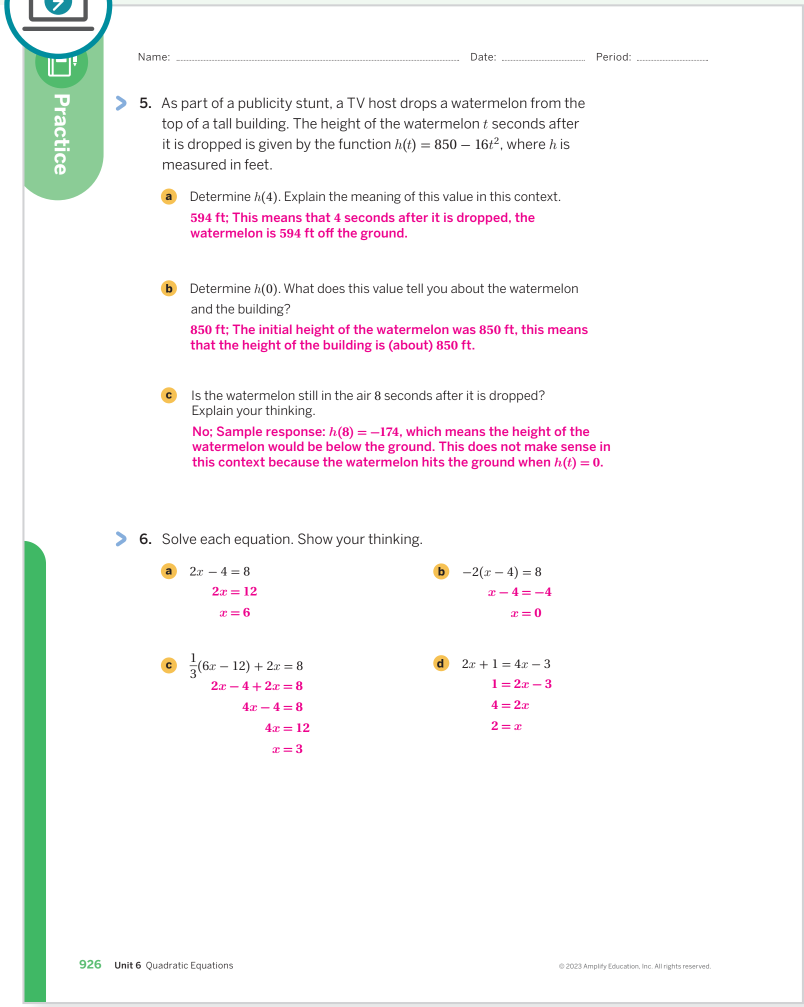
   b A tower created with containers is 40.6 cm tall. How many containers are in the tower?

   22 containers

   c Noah looks at the expression and says, "7.6 must be the height of a single container." Do you agree with Noah? Explain your thinking.

   No. Sample responses:

   - The expression is equal to 7.6 when $c = 0$, but a stack of zero containers would have no height.
   - When $c = 1$, the height of the stack is $1.5(1) + 7.6$, which is 9.1 cm.



Name: Date: Period:

5. As part of a publicity stunt, a TV host drops a watermelon from the top of a tall building. The height of the watermelon t seconds after it is dropped is given by the function $h(t) = 850 - 16t^2$, where h is measured in feet.

   a Determine $h(4)$. Explain the meaning of this value in this context.

   594 ft; This means that 4 seconds after it is dropped, the watermelon is 594 ft off the ground.

   b Determine $h(0)$. What does this value tell you about the watermelon and the building?

   850 ft; The initial height of the watermelon was 850 ft, this means that the height of the building is (about) 850 ft.

   c Is the watermelon still in the air 8 seconds after it is dropped? Explain your thinking.

   No; Sample response: $h(8) = -174$, which means the height of the watermelon would be below the ground. This does not make sense in this context because the watermelon hits the ground when $h(t) = 0$.

6. Solve each equation. Show your thinking.

   a $2x - 4 = 8$
   $2x = 12$
   $x = 6$

   b $-2(x - 4) = 8$
   $x - 4 = -4$
   $x = 0$

   c $\frac{1}{3}(6x - 12) + 2x = 8$
   $2x - 4 + 2x = 8$
   $4x - 4 = 8$
   $4x = 12$
   $x = 3$

   d $2x + 1 = 4x - 3$
   $1 = 2x - 3$
   $4 = 2x$
   $2 = x$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 2	2
	5	Unit 6 Lesson 2	2
Formative ⚡	6	Unit 6 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 4**

The Zero Product Principle

Let's determine solutions of equations when products equal 0.

Focus

Goals

1. **Language Goal:** Given a quadratic equation consisting of a product of factors set equal to 0, determine the solution(s) and explain why the solution(s) make the equation true. **(Speaking and Listening, Writing)**
2. Understand the Zero Product Principle: If the product of two numbers is 0, then one or both of the factors must be 0.

Rigor

- Students build **conceptual understanding** of the Zero Product Principle by making connections between the linear factors of quadratic expressions to familiar numeric properties.

Coherence

Today

Students are formally introduced to the Zero Product Principle after engaging in a Math Talk in which they transition from quantitiative to abstract reasoning. They solve quadratic equations in factored form by setting each factor equal to 0, or by determining the values needed to make them true. Students revisit a real-world context modeled by a quadratic equation and apply the Zero Product Principle to solve the equation.

Previously

In the previous lesson, students solved quadratic equations by reasoning about values that made them true. They learned that quadratic equations can have as many as two solutions.

Coming Soon

In Lesson 5, students revisit how to determine the zeros of a function to help them determine solutions to quadratic equations that are set equal to 0.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, Sentence *Stems, Explaining My Steps*
- Anchor Chart PDF, *Zero Product Principle*
- graphing technology

Math Language Development

New words

- ***Zero Product Principle***

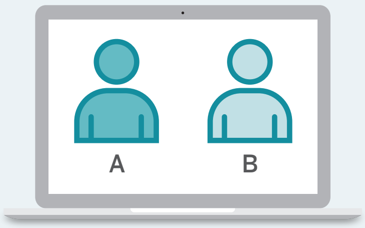
Review words

- *quadratic equation*

Amps | Featured Activity

Activity 1
Comparing Solutions

Students solve a set of quadratic equations set equal to zero, and compare these with a partner's solutions to make conclusions about the Zero Product Principle.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may lose motivation or focus if they do not immediately see how to make use of the structure of a function in factored form when it is given a context in Activity 2. Help them practice taking control of these impulses by suggesting they use their peers as a resource and asking them who they think might be able to help them with this Activity.

Modifications to Pacing

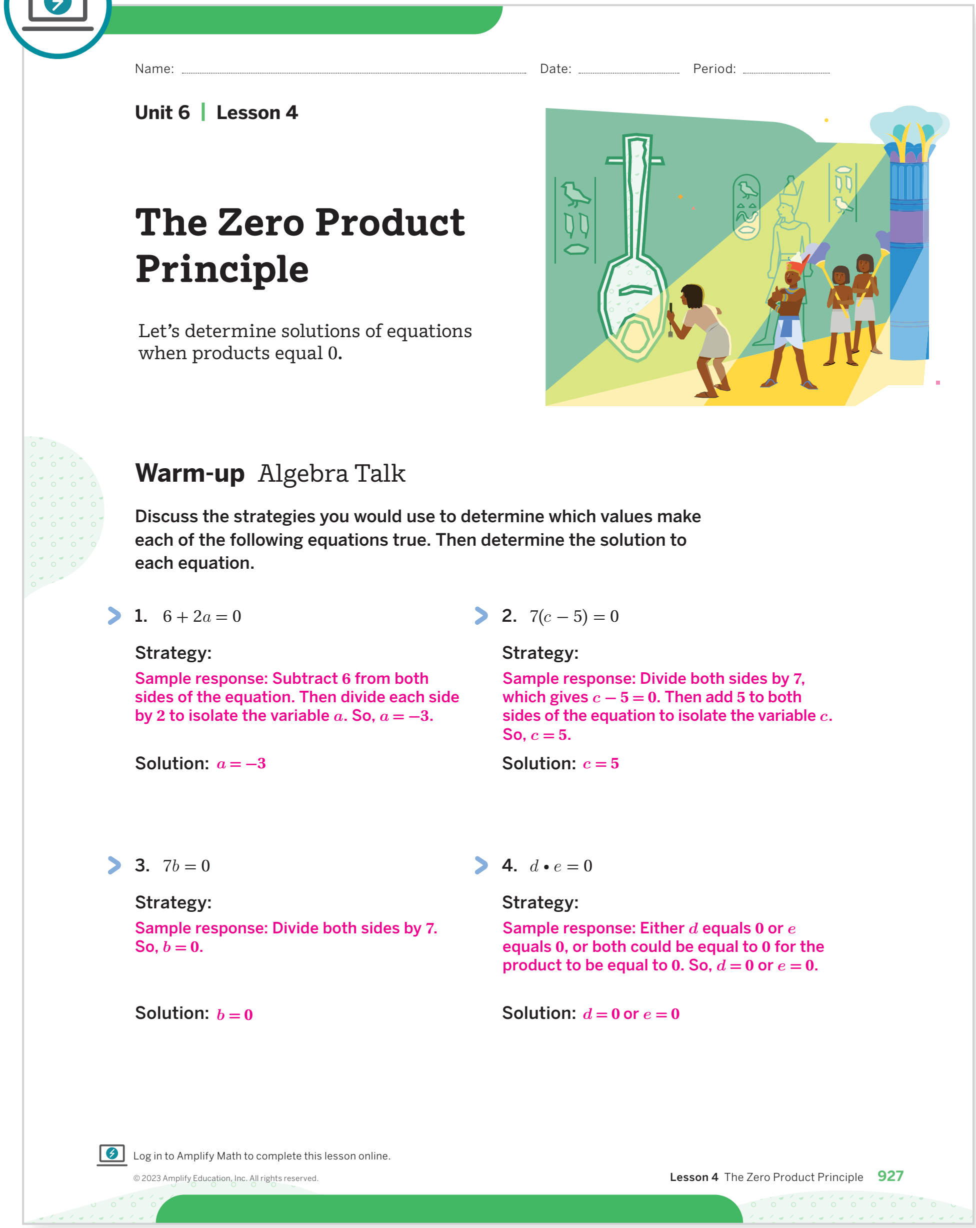
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, omit the partner problems and have students complete Column B independently.

Independent | 10 min

Warm-up Algebra Talk

Students determine the values needed to make an equation equal zero, formally introducing them to the Zero Product Principle.



Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 4

The Zero Product Principle

Let's determine solutions of equations when products equal 0.

Warm-up Algebra Talk

Discuss the strategies you would use to determine which values make each of the following equations true. Then determine the solution to each equation.

1. $6 + 2a = 0$

Strategy:
Sample response: Subtract 6 from both sides of the equation. Then divide each side by 2 to isolate the variable a. So, $a = -3$.

Solution: $a = -3$

2. $7(c - 5) = 0$

Strategy:
Sample response: Divide both sides by 7, which gives $c - 5 = 0$. Then add 5 to both sides of the equation to isolate the variable c. So, $c = 5$.

Solution: $c = 5$

3. $7b = 0$

Strategy:
Sample response: Divide both sides by 7. So, $b = 0$.

Solution: $b = 0$

4. $d \cdot e = 0$

Strategy:
Sample response: Either d equals 0 or e equals 0, or both could be equal to 0 for the product to be equal to 0. So, $d = 0$ or $e = 0$.

Solution: $d = 0$ or $e = 0$

Log in to Amplify Math to complete this lesson online.

Lesson 4 The Zero Product Principle 927

1 Launch

Display one problem at a time and ask students to respond without writing anything down. Give students think-time for each problem and then have them share their strategy and solution. Keep all problems displayed throughout the activity.

2 Monitor

Help students get started by asking, "What types of numbers are added to yield a sum of 0?" Any number and its additive inverse. "What numbers must be multiplied to yield a product of 0?" Any number and 0.

Look for points of confusion:

- **Struggling to find a value for d and e in Problem 4.** Prompt students to compare this problem to Problem 3.

Look for productive strategies:

- Applying the properties of equality, including the Distributive Property.
- Substituting numbers into the equations and evaluating.
- Utilizing the Zero Property of Multiplication to solve Problems 2–4.

3 Connect

Have students share the strategy they used for each problem.

Highlight explanations that state that any number multiplied by 0 is 0. Then review the Zero Property of Multiplication, which states that if the product of two numbers is 0, then at least one of the numbers is 0.

Display the Anchor Chart PDF, *Zero Product Principle*.

Define the ***Zero Product Principle***, $a \cdot b = 0$ if and only if $a = 0$, $b = 0$, or a and b equal 0.

Math Language Development

MLR7: Compare and Connect

During the Connect, connect the Zero Property of Multiplication to the Zero Product Principle. Ask:

- "What is the product of any number and 0?" 0. Emphasize this is the Zero Property of Multiplication.
- "How can you use the Zero Property of Multiplication to solve the equation in Problem 3? Problem 4?" In Problem 3, b must equal 0. In Problem 4, either d or e (or both) must equal zero. Emphasize that when the Zero Property of Multiplication is used to solve equations set equal to 0, this illustrates the Zero Product Principle.

English Learners

To help students explain their thinking, display or provide the Anchor Chart PDF, *Sentence Stems, Explaining My Steps*.

Power-up

To power up students' ability to solve equations using the properties of equality, have students complete:

Solve each equation. Show your thinking.

a. $6x + 2 = 14$
$6x = 12$
$x = 2$

b. $4(x + 1) = -2(x - 5)$
$4x + 4 = -2x + 10$
$6x + 4 = 10$
$6x = 6$
$x = 1$

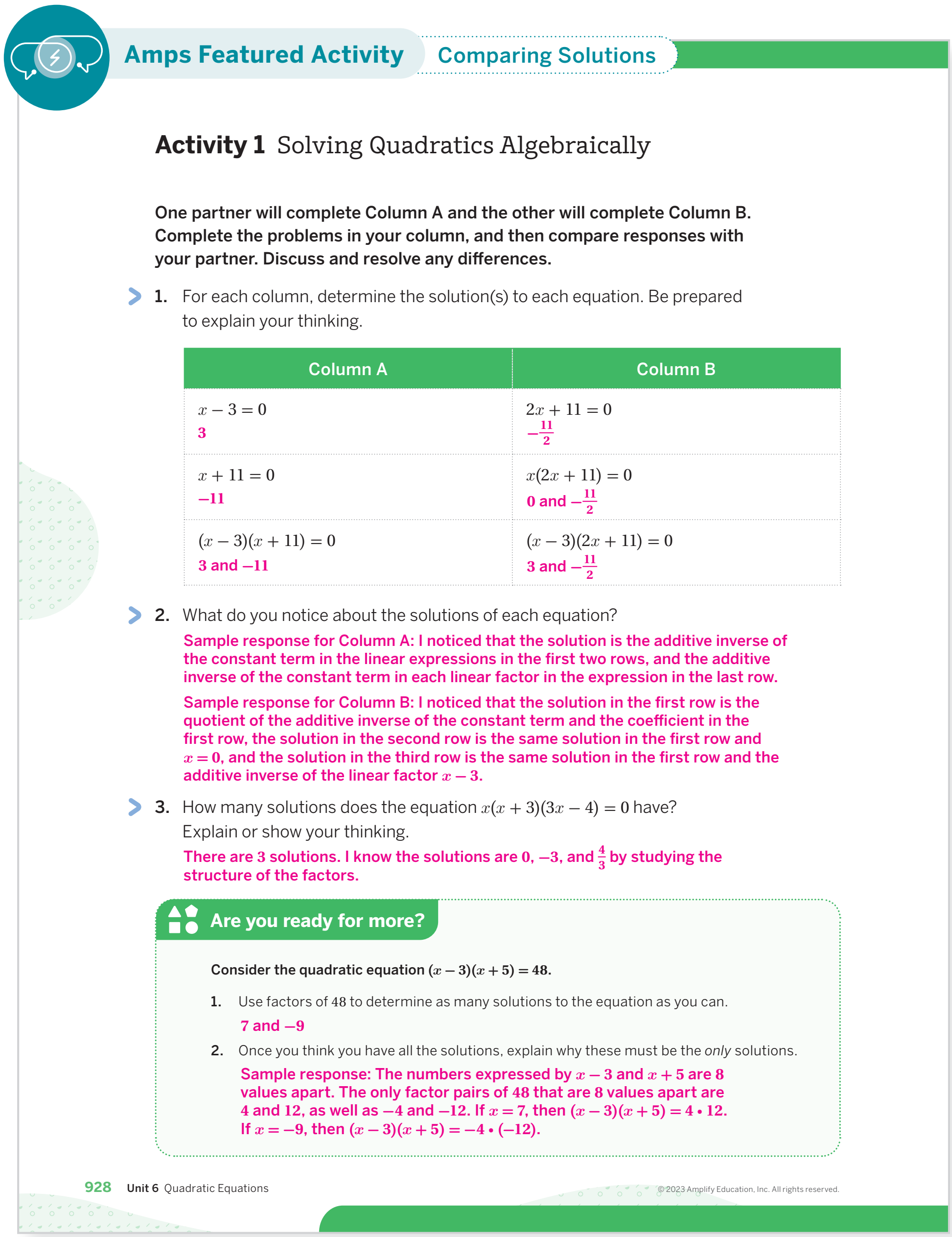
Use: Before the Warm-up

Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

Pairs | 15 min

Activity 1 Solving Quadratics Algebraically

Students solve equations of increasing complexity by reasoning about the structure of the expressions needed to make the equation equal to zero.



Amps Featured Activity Comparing Solutions

Activity 1 Solving Quadratics Algebraically

One partner will complete Column A and the other will complete Column B. Complete the problems in your column, and then compare responses with your partner. Discuss and resolve any differences.

1. For each column, determine the solution(s) to each equation. Be prepared to explain your thinking.

Column A	Column B
$x - 3 = 0$ 3	$2x + 11 = 0$ $-\frac{11}{2}$
$x + 11 = 0$ -11	$x(2x + 11) = 0$ 0 and $-\frac{11}{2}$
$(x - 3)(x + 11) = 0$ 3 and -11	$(x - 3)(2x + 11) = 0$ 3 and $-\frac{11}{2}$

2. What do you notice about the solutions of each equation?

   Sample response for Column A: I noticed that the solution is the additive inverse of the constant term in the linear expressions in the first two rows, and the additive inverse of the constant term in each linear factor in the expression in the last row.

   Sample response for Column B: I noticed that the solution in the first row is the quotient of the additive inverse of the constant term and the coefficient in the first row, the solution in the second row is the same solution in the first row and $x = 0$, and the solution in the third row is the same solution in the first row and the additive inverse of the linear factor $x - 3$.

3. How many solutions does the equation $x(x + 3)(3x - 4) = 0$ have? Explain or show your thinking.

   There are 3 solutions. I know the solutions are 0, -3, and $\frac{4}{3}$ by studying the structure of the factors.

Are you ready for more?

Consider the quadratic equation $(x - 3)(x + 5) = 48$.

1. Use factors of 48 to determine as many solutions to the equation as you can.

   7 and -9

2. Once you think you have all the solutions, explain why these must be the *only* solutions.

   Sample response: The numbers expressed by $x - 3$ and $x + 5$ are 8 values apart. The only factor pairs of 48 that are 8 values apart are 4 and 12, as well as -4 and -12. If $x = 7$, then $(x - 3)(x + 5) = 4 \bullet 12$. If $x = -9$, then $(x - 3)(x + 5) = -4 \bullet (-12)$.

928 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs and assign each student to Column A or Colum B. Students should work independently on their assigned problems before discussing with their partner.

2 Monitor

Help students get started by reminding them that some equations may have more than one solution.

Look for points of confusion:

- **Substituting two different values of x in the same equation.** Remind students that only one input value can be entered into an equation at a time.

Look for productive strategies:

- Applying the properties of equality.
- Substituting values into the equations by means of the *guess-and-check* strategy, and evaluating.

3 Connect

Display the problems and their solutions.

Have pairs of students share the strategies they used to solve each of the equations, selecting partner A to discuss problems from Column A and partner B to discuss problems from Column B. Record and organize their explanations and reasoning processes for the class to view.

Highlight that at least one factor must be 0 if the product is 0, by the Zero Product Principle. By setting each factor equal to 0 and solving each equation separately, they can determine solutions that make the equation true. When checking their solutions, use one value at a time to substitute into the original equation.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Have pairs work together to complete both columns. Have them solve the equations in Column A first, study those solutions, and then solve the equations in Column B. Provide access to colored pencils and have them color code the expression $x - 3$ in one color, $x + 11$ in a second color, and then use those same colors to color code the expressions in the third row. Have them repeat for Column B.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share the strategies they used, display the equations in the third row of each column. Draw students' attention to the structure of each equation. Ask:

- "How does each equation show a product of factors? What are the factors?"
- "By the Zero Product Principle, what must be true about one or both of the factors?"
- "Would this be true if the equation was not set equal to 0?"

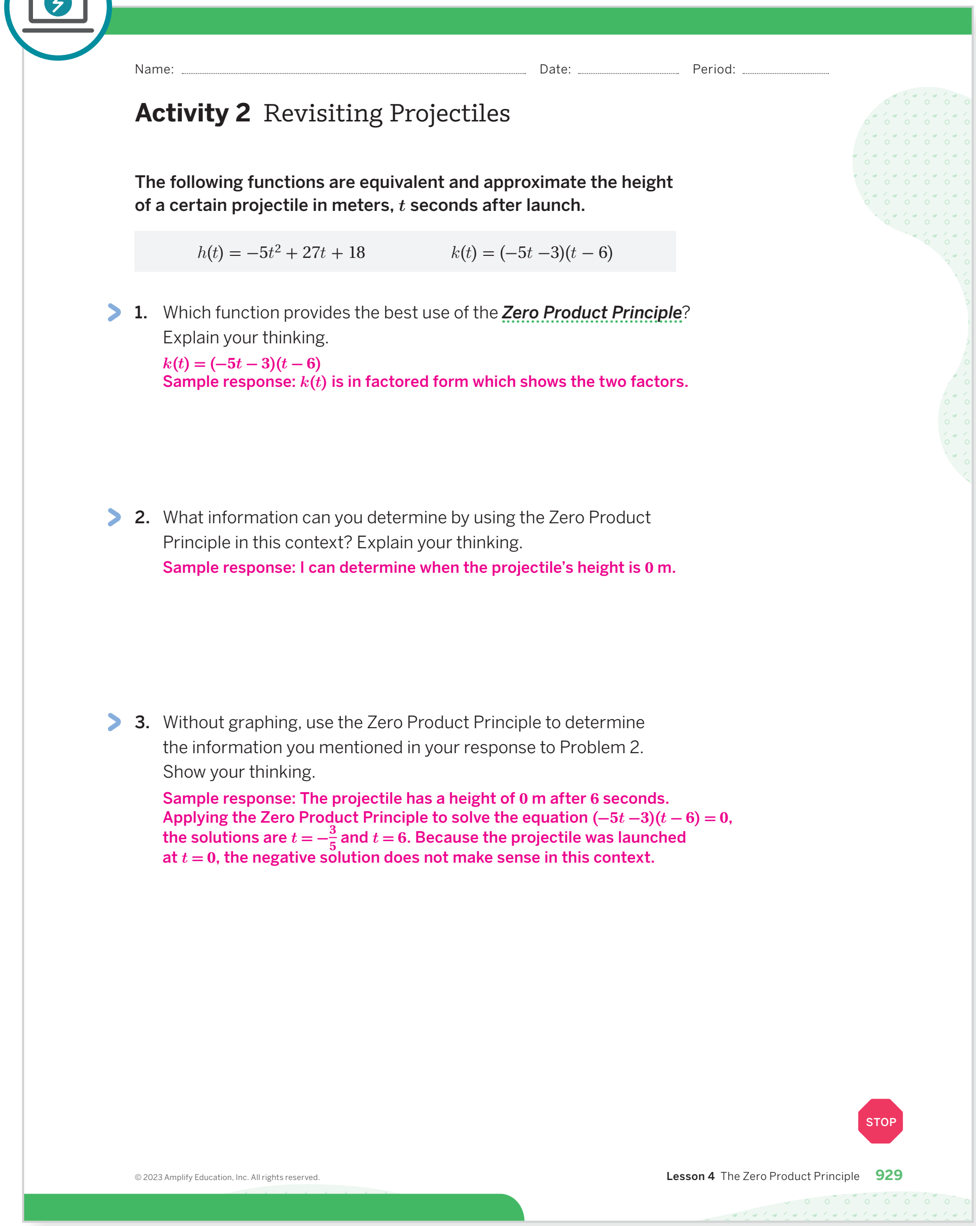
English Learners

Annotate the factors of each equation by writing the term *factor* underneath each set of parentheses.

Pairs | 15 min

Activity 2 Revisiting Projectiles

Students revisit a projectile motion and apply the Zero Product Principle to solve a projectile motion problem in context.



Name: ______ Date: ______ Period: ______

Activity 2 Revisiting Projectiles

The following functions are equivalent and approximate the height of a certain projectile in meters, t seconds after launch.

$h(t) = -5t^2 + 27t + 18$ $k(t) = (-5t - 3)(t - 6)$

1. Which function provides the best use of the ***Zero Product Principle***? Explain your thinking.
   $k(t) = (-5t - 3)(t - 6)$
   Sample response: $k(t)$ is in factored form which shows the two factors.

2. What information can you determine by using the Zero Product Principle in this context? Explain your thinking.
   Sample response: I can determine when the projectile's height is 0 m.

3. Without graphing, use the Zero Product Principle to determine the information you mentioned in your response to Problem 2. Show your thinking.
   Sample response: The projectile has a height of 0 m after 6 seconds. Applying the Zero Product Principle to solve the equation $(-5t - 3)(t - 6) = 0$, the solutions are $t = -\frac{3}{5}$ and $t = 6$. Because the projectile was launched at $t = 0$, the negative solution does not make sense in this context.

STOP

Lesson 4 The Zero Product Principle 929

1 Launch

Display the two functions and ask, "How do you know the functions are equivalent?" By graphing, using a table, or applying the Distributive Property. Give students think-time to determine a strategy and test it, and then discuss with their partner.

2 Monitor

Help students get started by providing access to graphing technology.

Look for points of confusion:

- **Choosing the function defined in standard form in Problem 1.** Ask students to articulate what factors they would multiply to produce a zero product. $(-5t - 3)$ and $(t - 6)$
- **Having difficulty applying the Zero Product Principle in Problem 3.** Prompt students to rewrite each factor as an equation equal to 0.

Look for productive strategies:

- Graphing both functions on the same coordinate plane.
- Applying the Distributive Property to multiply the function in factored form.

3 Connect

Have pairs of students share their responses and reasoning.

Ask, "Why is the factored form more helpful for determining the time(s) when the projectile has a height of 0 meters?" When $k(t)$ is in factored form, I can apply the Zero Product Principle to determine the values of t that make $k(t) = 0$.

Highlight the connection between an equation in factored form and its x-intercepts when graphed. Solving for x using the factored form tells them where the graph intersects the x-axis.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they worked with quadratic functions that model projectile motion in the previous unit. Display the two functions and ask:

- "What form is the first function written in? The second function?" The first function is written in standard form. The second function is written in factored form.
- "What information does the first function tell you? The second function?" The first function tells me the initial height of the projectile. The second function tells me the zeros of the function, at what times the projectile has a height of 0 m.

Math Language Development

MLR7: Compare and Connect

During the Connect, display a graph of $k(t)$ to connect the solutions to the equation $(-5t - 3)(t - 6) = 0$ to the zeros of the function. Ask:

- "Why does the equation have to be set equal to 0 to determine the zeros?" The zeros are when the function has a value of 0.
- "Why is factored form more helpful when solving using the Zero Product Principle than standard form?" The equation needs to consist of factors.

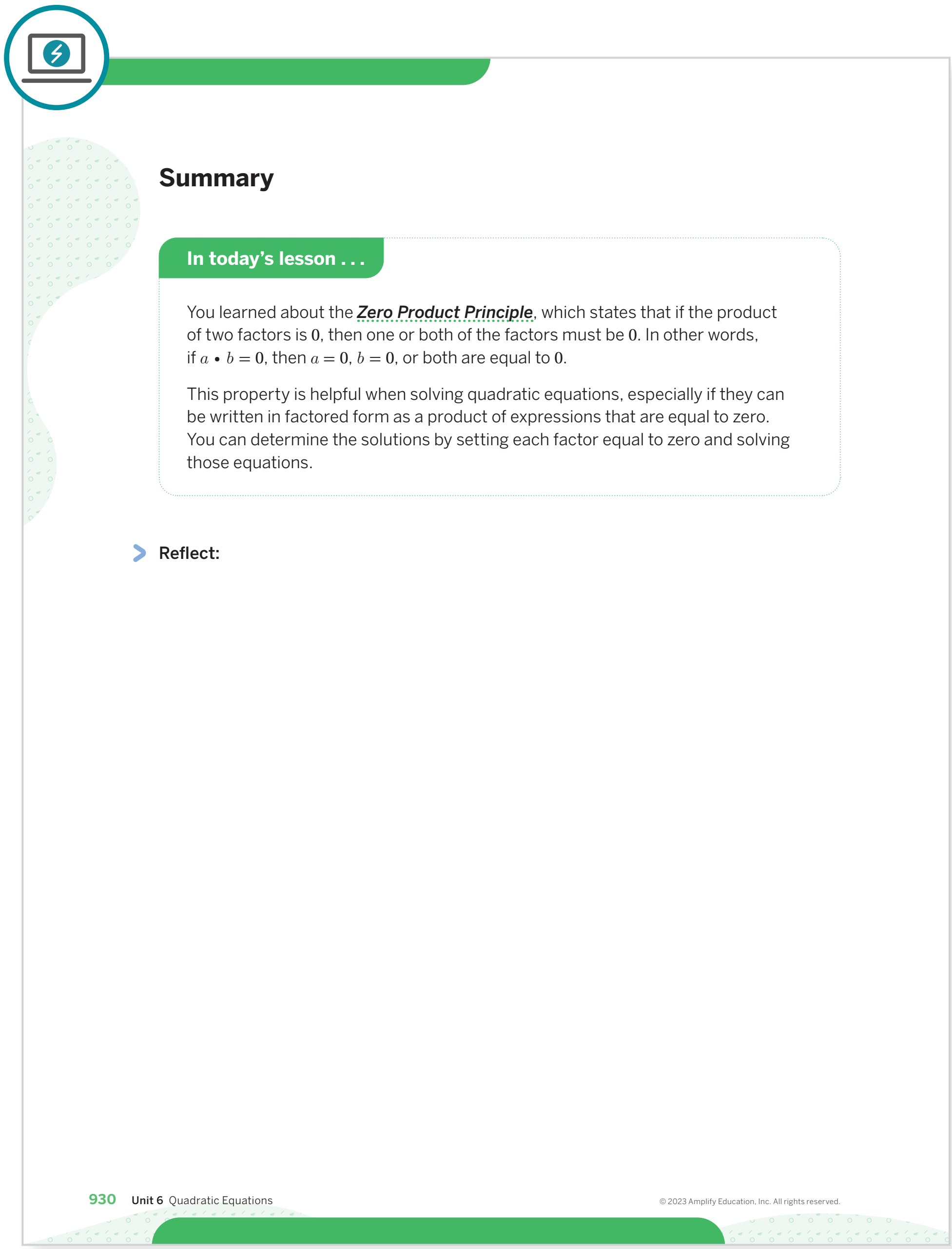
English Learners

Add a sketch of the graph of $k(t)$ to the class display. Annotate the zeros of the function along with the equation $(-5t - 3)(t - 6) = 0$ and its solutions.

Whole Class | 5 min

Summary

Review and synthesize how the Zero Product Principle is used to determine solutions of quadratic equations written in factored form.



Summary

In today's lesson . . .

You learned about the ***Zero Product Principle***, which states that if the product of two factors is 0, then one or both of the factors must be 0. In other words, if $a \bullet b = 0$, then $a = 0$, $b = 0$, or both are equal to 0.

This property is helpful when solving quadratic equations, especially if they can be written in factored form as a product of expressions that are equal to zero. You can determine the solutions by setting each factor equal to zero and solving those equations.

Reflect:

930 **Unit 6** Quadratic Equations

Synthesize

Display the expression $(x - 3)(x + 4)$.

Ask:

- "How does the Zero Product Principle help you determine the solutions to $(x - 3)(x + 4) = 0$?" It tells me that either $x - 3 = 0$ or $x + 4 = 0$. Model how to set each factor equal to zero by solving $(x - 3)(x + 4) = 0$.
- "Why are the solutions to $(x - 3)(x + 4) = 8$ *not* 3 and -4?" The Zero Product Principle only works when the product of the factors is equal to zero.
- "The expression $x^2 + x - 12$ is equivalent to $(x + 3)(x + 4)$. Can you apply the Zero Product Principle to solve $x^2 - x - 12 = 0$?" Only if I rewrite the equation in factored form first. The Zero Product Principle cannot be used when the equation is not written as a product of factors set equal to 0.

Highlight that the Zero Product Principle is useful for solving quadratic equations written in factored form. If the product of the factors are equal to 0, then one or more of the factors must be equal to 0.

Formalize vocabulary: ***Zero Product Principle***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does the structure of a quadratic equation help you determine efficient strategies for solving it algebraically?"

Math Language Development

MLR2: Collect and Display

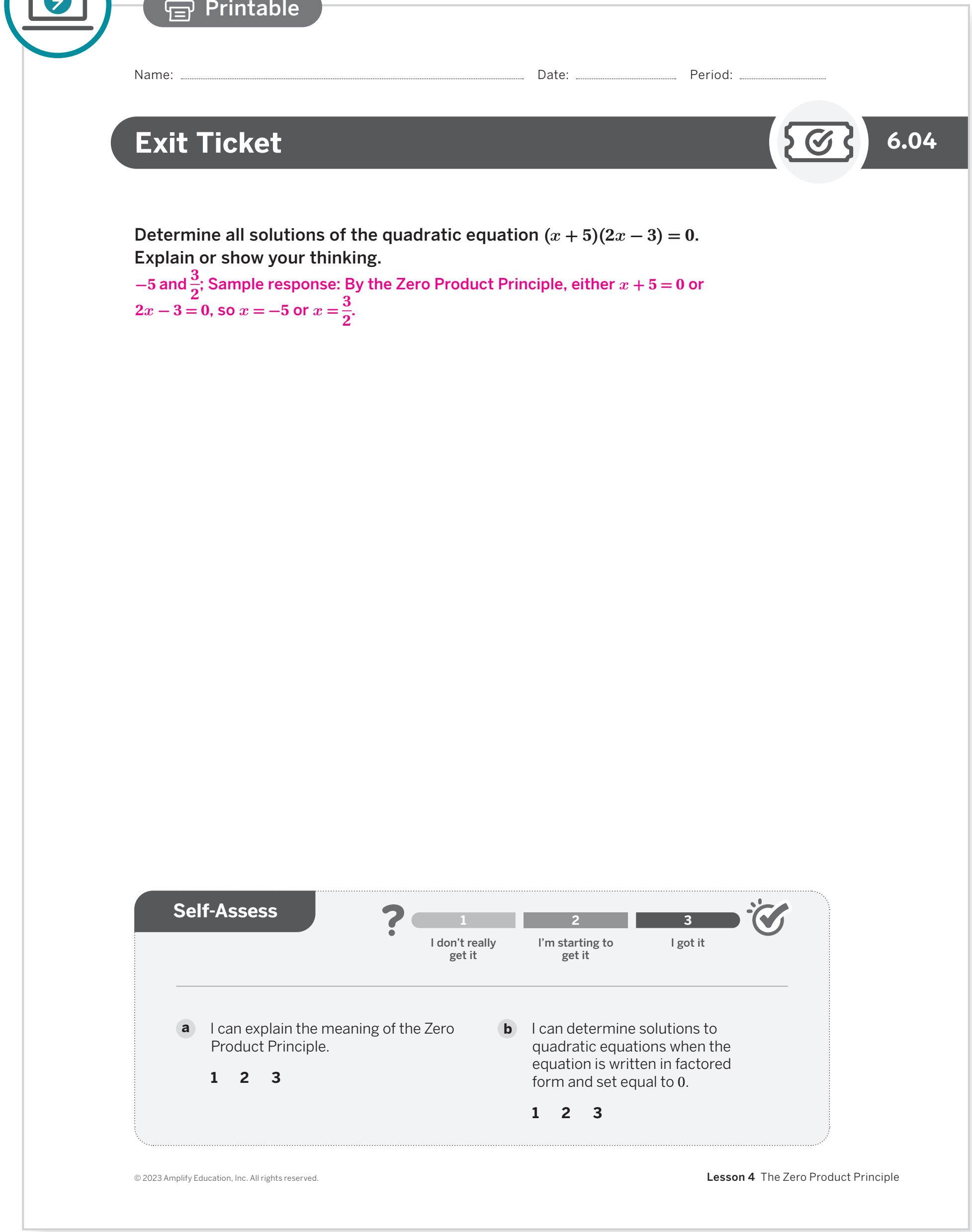
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *Zero Product Principle* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the Zero Product Principle by solving a quadratic equation written in factored form.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.04

Determine all solutions of the quadratic equation $(x + 5)(2x - 3) = 0$. Explain or show your thinking.

-5 and $\frac{3}{2}$; Sample response: By the Zero Product Principle, either $x + 5 = 0$ or $2x - 3 = 0$, so $x = -5$ or $x = \frac{3}{2}$.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain the meaning of the Zero Product Principle.

1 2 3

b I can determine solutions to quadratic equations when the equation is written in factored form and set equal to 0.

1 2 3

Success looks like . . .

- **Language Goal:** Given a quadratic equation consisting of a product of factors set equal to 0, determining the solution(s) and explaining why the solution(s) make the equation true. **(Speaking and Listening, Writing)**
  - » Determining the solution to $(x + 2)(2x - 3) = 0$.
- **Goal:** Understanding the Zero Product Principle: If the product of two numbers is 0, then one or both the factors must be 0.
  - » Explaining how to determine the solution to the quadratic equation using the Zero Product Principle.

Suggested next steps

If students do not apply the Zero Product Principle correctly, consider:

- Asking, "What value of x would make $x + 5 = 0$? What value of x would make $2x - 3 = 0$?" -5, $\frac{3}{2}$
- Reviewing the problems in Column B in Activity 1.
- Assigning Practice Problems 1–3.

Professional Learning

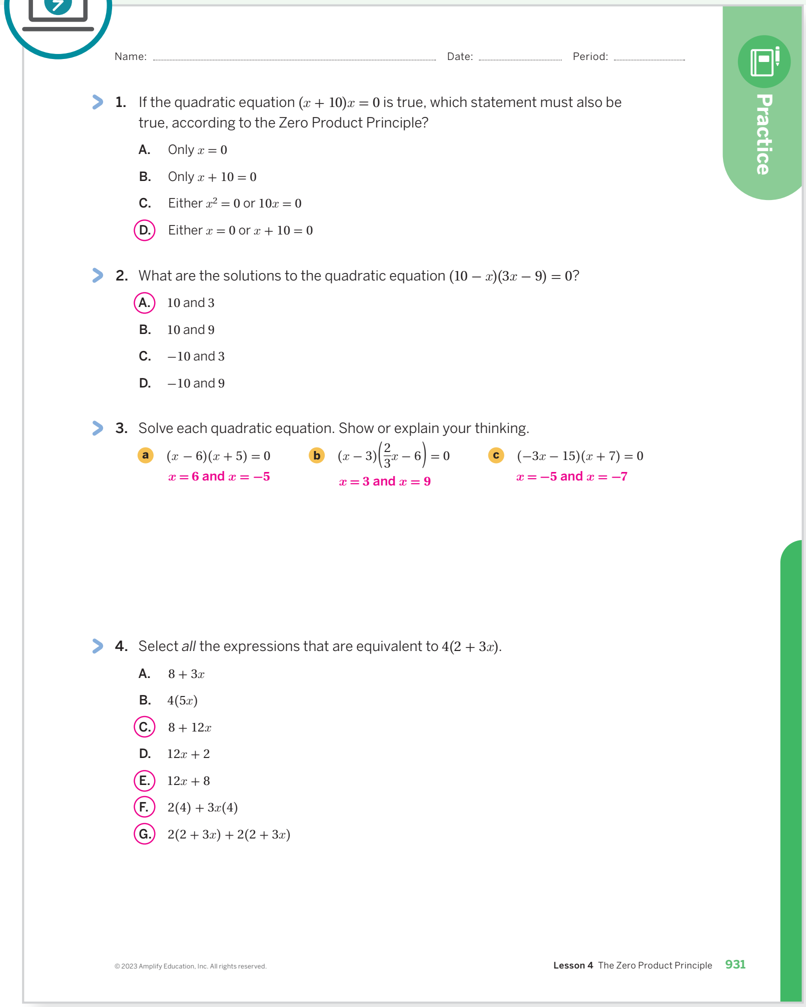
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was using the Zero Product Principle to solve quadratic equations similar to or different from using reasoning in the previous lesson?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



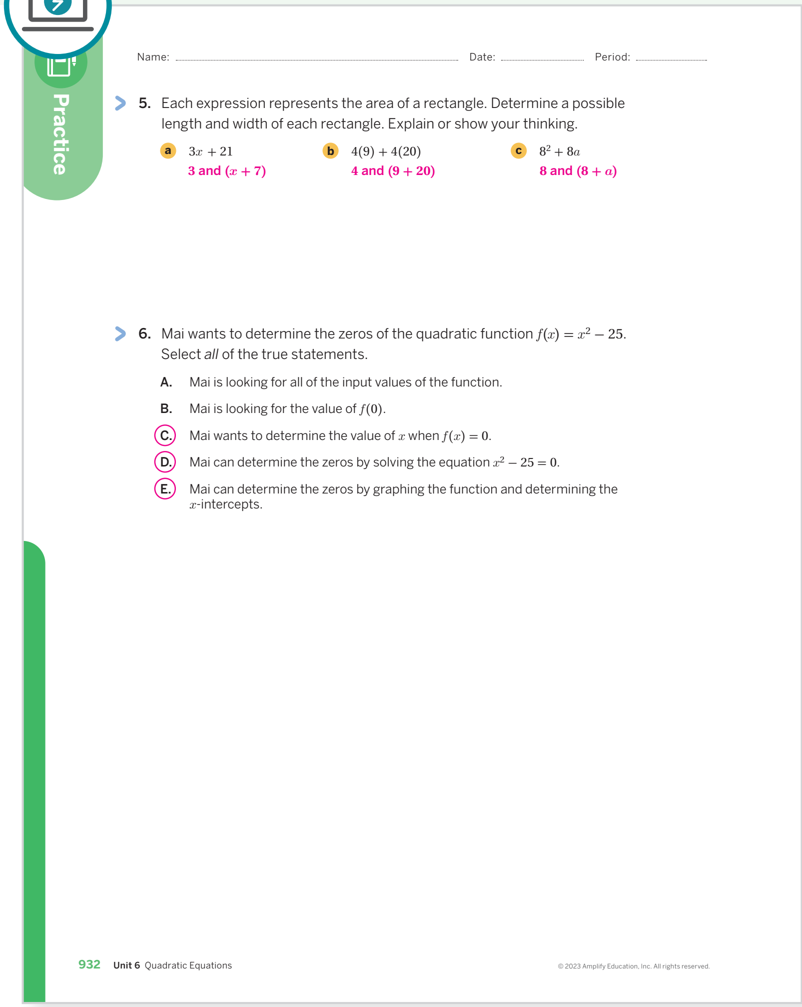
Name: ______ Date: ______ Period: ______

1. If the quadratic equation $(x + 10)x = 0$ is true, which statement must also be true, according to the Zero Product Principle?
   - A. Only $x = 0$
   - B. Only $x + 10 = 0$
   - C. Either $x^2 = 0$ or $10x = 0$
   - [✓ D.] Either $x = 0$ or $x + 10 = 0$

2. What are the solutions to the quadratic equation $(10 - x)(3x - 9) = 0$?
   - [✓ A.] 10 and 3
   - B. 10 and 9
   - C. -10 and 3
   - D. -10 and 9

3. Solve each quadratic equation. Show or explain your thinking.
   - a $(x - 6)(x + 5) = 0$
     $x = 6$ and $x = -5$
   - b $(x - 3)\left(\frac{2}{3}x - 6\right) = 0$
     $x = 3$ and $x = 9$
   - c $(-3x - 15)(x + 7) = 0$
     $x = -5$ and $x = -7$

4. Select *all* the expressions that are equivalent to $4(2 + 3x)$.
   - A. $8 + 3x$
   - B. $4(5x)$
   - [✓ C.] $8 + 12x$
   - D. $12x + 2$
   - [✓ E.] $12x + 8$
   - [✓ F.] $2(4) + 3x(4)$
   - [✓ G.] $2(2 + 3x) + 2(2 + 3x)$



Name: ______ Date: ______ Period: ______

5. Each expression represents the area of a rectangle. Determine a possible length and width of each rectangle. Explain or show your thinking.
   - a $3x + 21$
     3 and $(x + 7)$
   - b $4(9) + 4(20)$
     4 and $(9 + 20)$
   - c $8^2 + 8a$
     8 and $(8 + a)$

6. Mai wants to determine the zeros of the quadratic function $f(x) = x^2 - 25$. Select *all* of the true statements.
   - A. Mai is looking for all of the input values of the function.
   - B. Mai is looking for the value of $f(0)$.
   - [✓ C.] Mai wants to determine the value of x when $f(x) = 0$.
   - [✓ D.] Mai can determine the zeros by solving the equation $x^2 - 25 = 0$.
   - [✓ E.] Mai can determine the zeros by graphing the function and determining the x-intercepts.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 10	2
	5	Unit 5 Lesson 10	2
Formative ⚡	6	Unit 6 Lesson 5	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

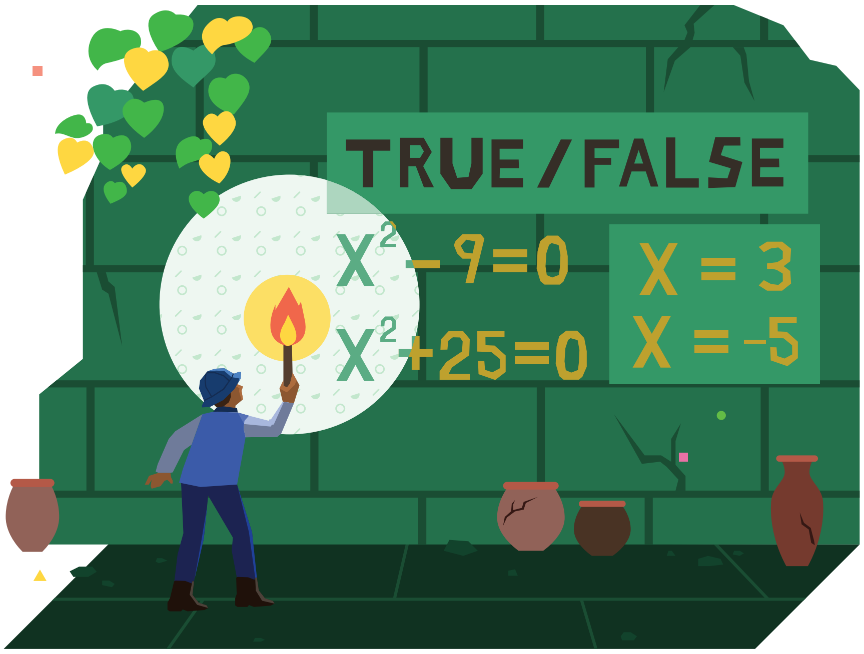
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 5**

How Many Solutions?

Let's use graphs to investigate the number of solutions to a quadratic equation.



Focus

Goals

1. Coordinate between graphs with no horizontal intercepts, quadratic functions with no zeros, and quadratic equations with no solutions.
2. **Language Goal:** Describe the relationship between the solutions to quadratic equations set equal to 0 and the horizontal intercepts of the graph of the related function. **(Speaking and Listening, Writing)**
3. **Language Goal:** Explain why dividing each side of a quadratic equation by a variable is not a reliable way to solve the equation. **(Speaking and Listening, Writing)**

Coherence

Today

In this lesson, students use quadratic equations set equal to 0 to graph the related quadratic function as a strategy for determining its solutions. They analyze and critique the strategies used by other students. They activate prior knowledge about how to determine the zeros of a quadratic function and the horizontal intercepts of its graph and build on this knowledge to determine the number of solutions to a quadratic equation. In Algebra 1, students do not yet encounter imaginary numbers, so they are expected to state that a quadratic equation can have 0, 1, or 2 solutions. They are not expected to use the term *real* when describing these solutions.

Previously

In the previous lesson, students were formally introduced to the Zero Product Principle, which they used as a strategy for solving quadratic equations set equal to 0, where the other side of the equation is expressed in factored form.

Coming Soon

In the next lesson, students will learn how to use algebraic manipulation to factor quadratic expressions given in standard form, thus rewriting them in factored form.

Rigor

- Students build **conceptual understanding** of how the number of solutions to a quadratic equation relate to the zeros of the graph of the related function.
- Students further their **fluency** in solving quadratic equations by graphing.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
10 min	15 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- graph paper
- graphing technology

Math Language Development

Review words

- *factored form*
- *quadratic equation*
- *zero*
- *Zero Product Principle*

Amps | **Featured Activity**

Activity 1
Interactive Graphs

Students can explore — in real time — how different quadratic equations have different numbers of solutions.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may have conflicting ideas when critiquing the arguments posed in Activity 3 or each other's arguments. Establish protocols for students to use when they disagree, allowing them to take turns speaking and actively listen to one another. Give students authentic feedback anytime they work well with others and thank them when they listen well and interact respectfully, particularly when they disagree with their peers.

Modifications to Pacing

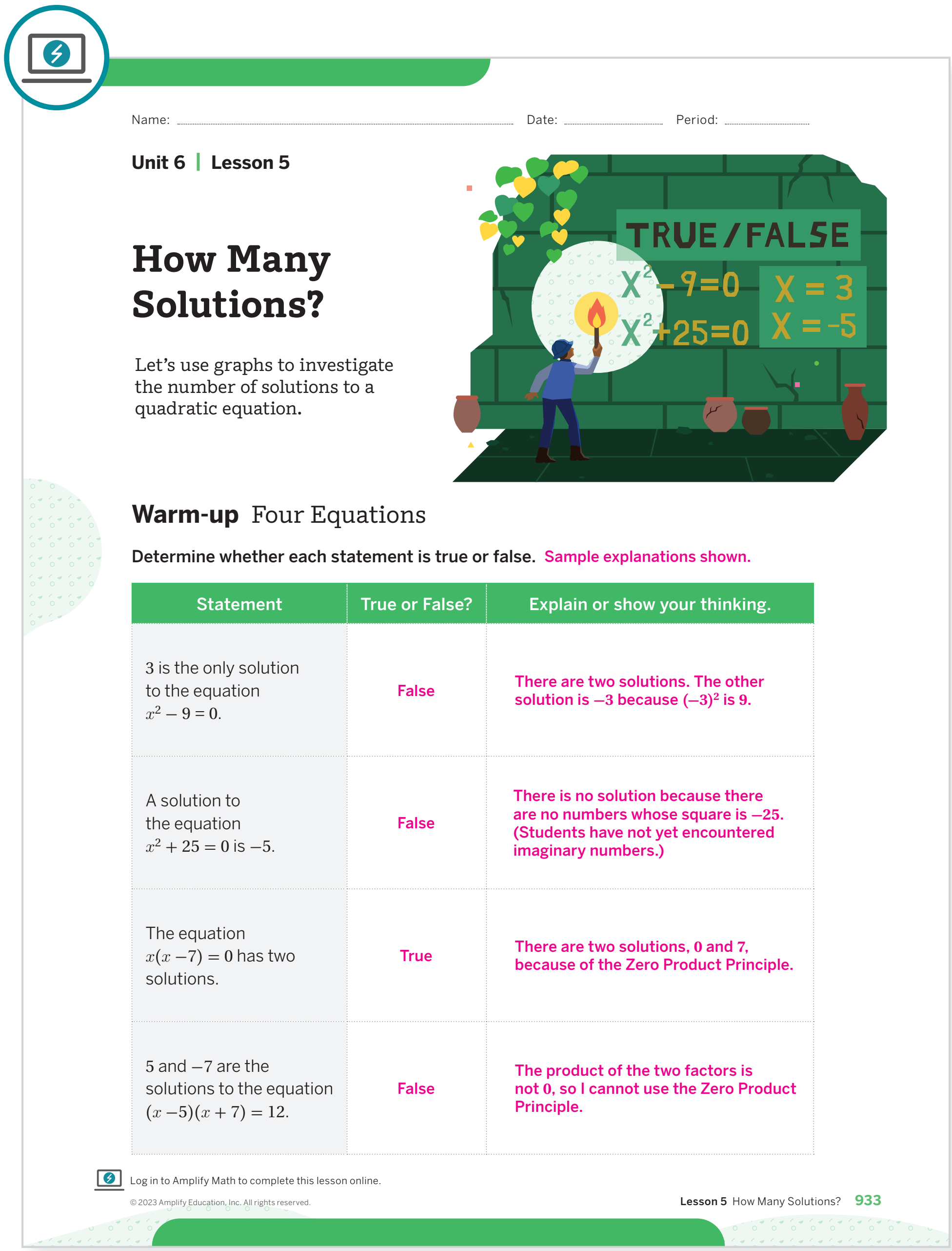
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, reduce the number of tasks by having students complete only Problems 1a–1e.

Independent | 10 min

Warm-up Four Equations

Students reason about the number of solutions each equation has, constructing logical arguments to explain why they think each statement is true or false.



Name: ________ Date: ________ Period: ________

Unit 6 | Lesson 5

How Many Solutions?

Let's use graphs to investigate the number of solutions to a quadratic equation.

Warm-up Four Equations

Determine whether each statement is true or false. **Sample explanations shown.**

Statement	True or False?	Explain or show your thinking.
3 is the only solution to the equation $x^2 - 9 = 0$.	**False**	**There are two solutions. The other solution is –3 because $(–3)^2$ is 9.**
A solution to the equation $x^2 + 25 = 0$ is -5.	**False**	**There is no solution because there are no numbers whose square is –25. (Students have not yet encountered imaginary numbers.)**
The equation $x(x - 7) = 0$ has two solutions.	**True**	**There are two solutions, 0 and 7, because of the Zero Product Principle.**
5 and -7 are the solutions to the equation $(x - 5)(x + 7) = 12$.	**False**	**The product of the two factors is not 0, so I cannot use the Zero Product Principle.**

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 How Many Solutions? 933

1 Launch

Display each problem one at time and provide students with think-time to respond to each problem. Have students give a signal when they have a response and an explanation.

2 Monitor

Help students get started by prompting them to solve the given equation on their own before determining whether the statement is true or false.

Look for points of confusion:

- **Only substituting values into the equation that are given in each statement.** Remind students that quadratic equations can have 0, 1, or 2 solutions.
- **Applying the Zero Product Principle to a quadratic equation not set equal to 0.** Ask, "What rule are you using? Why do you think it has this name?"

Look for productive strategies:

- Substituting the given solutions into each of the equations.
- Using the Zero Product Principle for equations in factored form set equal to 0.

3 Connect

Have students share their responses and explanations for each problem. After each explanation, give the class a chance to agree or disagree.

Ask:

- "Did anyone solve the problem a different way?"
- "Does anyone want to add on to _______'s strategy?"

Highlight the rationale that makes each statement true or false. Emphasize that there is no number that can be squared to obtain a negative product.

Math Language Development

MLR8: Discussion Supports — Press for Details

During the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Explaining My Steps* to support students as they share their responses and explanations for each problem. Ask classmates to press each other for details in their reasoning. For example:

If a student says . . .	Their classmates could ask . . .
"The equation $x(x - 7) = 0$ has one solution, $x = 7$."	"How many factors do you see in this equation? What does it mean if x is by itself as a factor?"

English Learners

Provide students time to rehearse what they will say with a partner before sharing with the whole class.

Power-up

To power up students' ability to relate the meaning of the zeros of a function to its equation and graph, have students complete:

Recall that the zeros of a function $f(x)$ are the input values that result in $f(x) = 0$.

Determine which of the following statements is true about the zeros of the function $f(x) = x^2 - 16$. Select all that apply.

[✓ **A.**] The zeros are $x = 4$ and $x = -4$ because $f(4) = f(-4) = 0$.

B. The zero is -16 because $f(0) = -16$.

[✓ **C.**] The zeros are $x = 4$ and $x = -4$ because the x-intercepts are $(4, 0)$ and $(-4, 0)$.

D. The zero is -16 because the y-intercept of the function is $(0, -16)$.

Use: Before the Warm-up

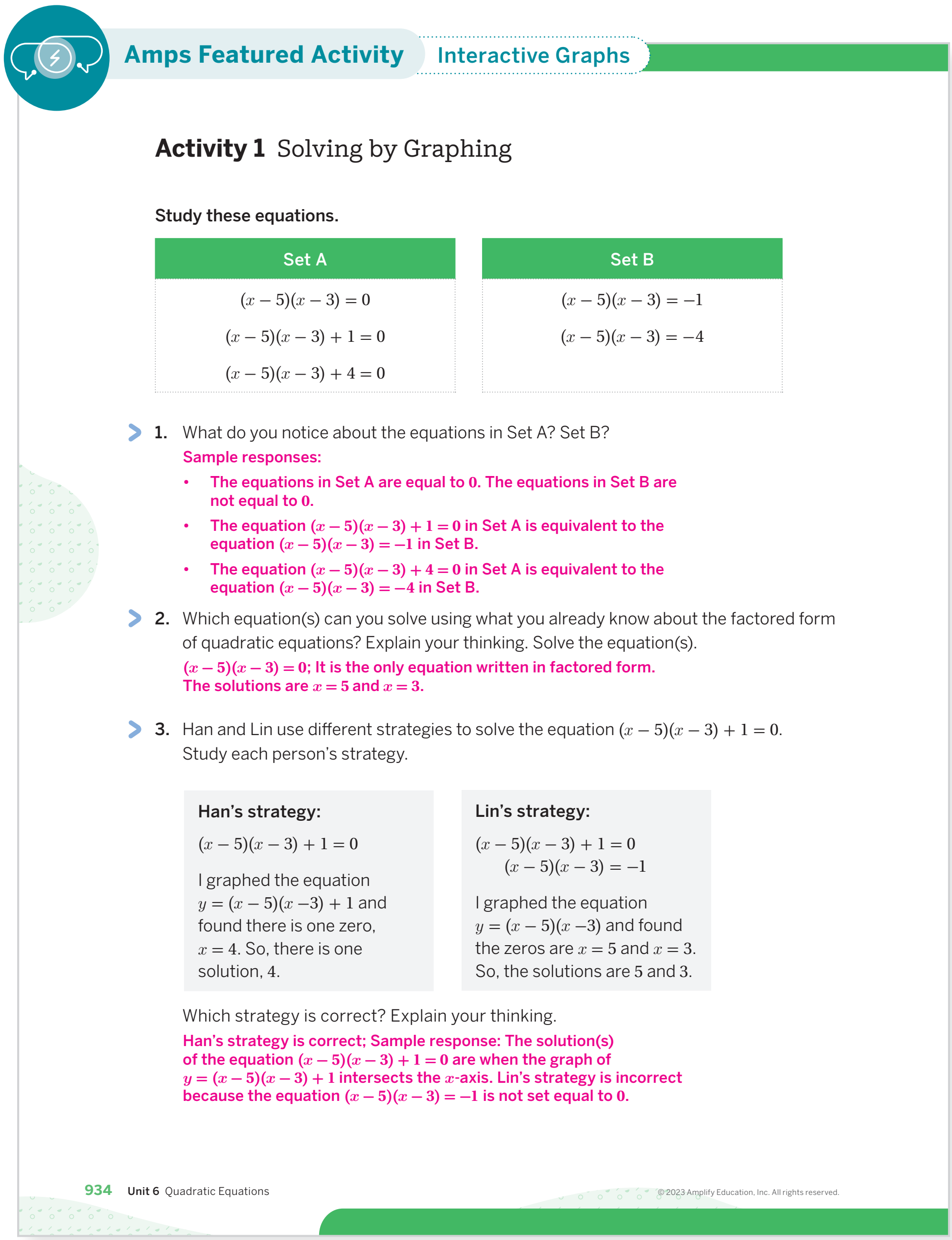
Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2

Pairs | 15 min

Activity 1 Solving by Graphing

Students activate prior knowledge about how to determine the zeros of functions by graphing, applying this knowledge to determine the solutions to quadratic equations set equal to 0.

Amps Featured Activity Interactive Graphs



Activity 1 Solving by Graphing

Study these equations.

Set A	Set B
$(x-5)(x-3)=0$	$(x-5)(x-3)=-1$
$(x-5)(x-3)+1=0$	$(x-5)(x-3)=-4$
$(x-5)(x-3)+4=0$	

1. What do you notice about the equations in Set A? Set B?
   Sample responses:
   - **The equations in Set A are equal to 0. The equations in Set B are not equal to 0.**
   - **The equation $(x-5)(x-3)+1=0$ in Set A is equivalent to the equation $(x-5)(x-3)=-1$ in Set B.**
   - **The equation $(x-5)(x-3)+4=0$ in Set A is equivalent to the equation $(x-5)(x-3)=-4$ in Set B.**
2. Which equation(s) can you solve using what you already know about the factored form of quadratic equations? Explain your thinking. Solve the equation(s).
   $(x-5)(x-3)=0$; It is the only equation written in factored form. The solutions are $x=5$ and $x=3$.
3. Han and Lin use different strategies to solve the equation $(x-5)(x-3)+1=0$. Study each person's strategy.

Han's strategy:

$(x-5)(x-3)+1=0$

I graphed the equation $y=(x-5)(x-3)+1$ and found there is one zero, $x=4$. So, there is one solution, 4.

Lin's strategy:

$(x-5)(x-3)+1=0$
$(x-5)(x-3)=-1$

I graphed the equation $y=(x-5)(x-3)$ and found the zeros are $x=5$ and $x=3$. So, the solutions are 5 and 3.

Which strategy is correct? Explain your thinking.

Han's strategy is correct; Sample response: The solution(s) of the equation $(x-5)(x-3)+1=0$ are when the graph of $y=(x-5)(x-3)+1$ intersects the x-axis. Lin's strategy is incorrect because the equation $(x-5)(x-3)=-1$ is not set equal to 0.

934 **Unit 6** Quadratic Equations

1 Launch

Give students think-time to respond to the first problem independently and then have them discuss their response with their partner before continuing with the rest of the activity. Provide access to graphing technology.

2 Monitor

Help students get started by asking "Which equation would be the most straightforward to solve and why?" $(x-5)(x-3)=0$ because I can use the Zero Product Principle to set each linear factor equal to zero and then solve each linear equation.

Look for points of confusion:

- **Selecting Lin's strategy as the correct strategy in Problem 2.** Remind students that to use the Zero Product Principle, the equation needs to be in factored form and set equal to 0.

Look for productive strategies:

- Studying the structures of the equations in Sets A and B in Problems 1 and 2 to recognize which equations are written in factored form and set equal to 0.
- Substituting the x-coordinate of the x-intercepts of the related graph into the given equation in Problem 4.
- Labeling the graphs of each equation with 0, 1, or 2 solutions in Problems 4 and 5.
- Articulating the solutions of each equation in Problems 4 and 5.
- Utilizing the structure of the equation to critique and identify the error in Lin's strategy in Problem 3.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view — in real time — how different quadratic equations have different numbers of solutions.

Extension: *Math Enrichment*

Have students draw three different quadratic graphs, one in which its corresponding equation would have exactly two (real) solutions, one in which its corresponding equation would have exactly one (real) solution, and one in which its corresponding equation would have no (real) solutions. Answers may vary.

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to articulate the connection between the solutions to the equation $(x-5)(x-3)=0$ and the x-intercepts of its corresponding graph, $y=(x-5)(x-3)$. Invite students to compare this equation to the equations not in factored form or not set equal to 0 and the x-intercepts of their related graphs. Highlight that they are looking for the same information from the graph, even though the equation is structured differently.

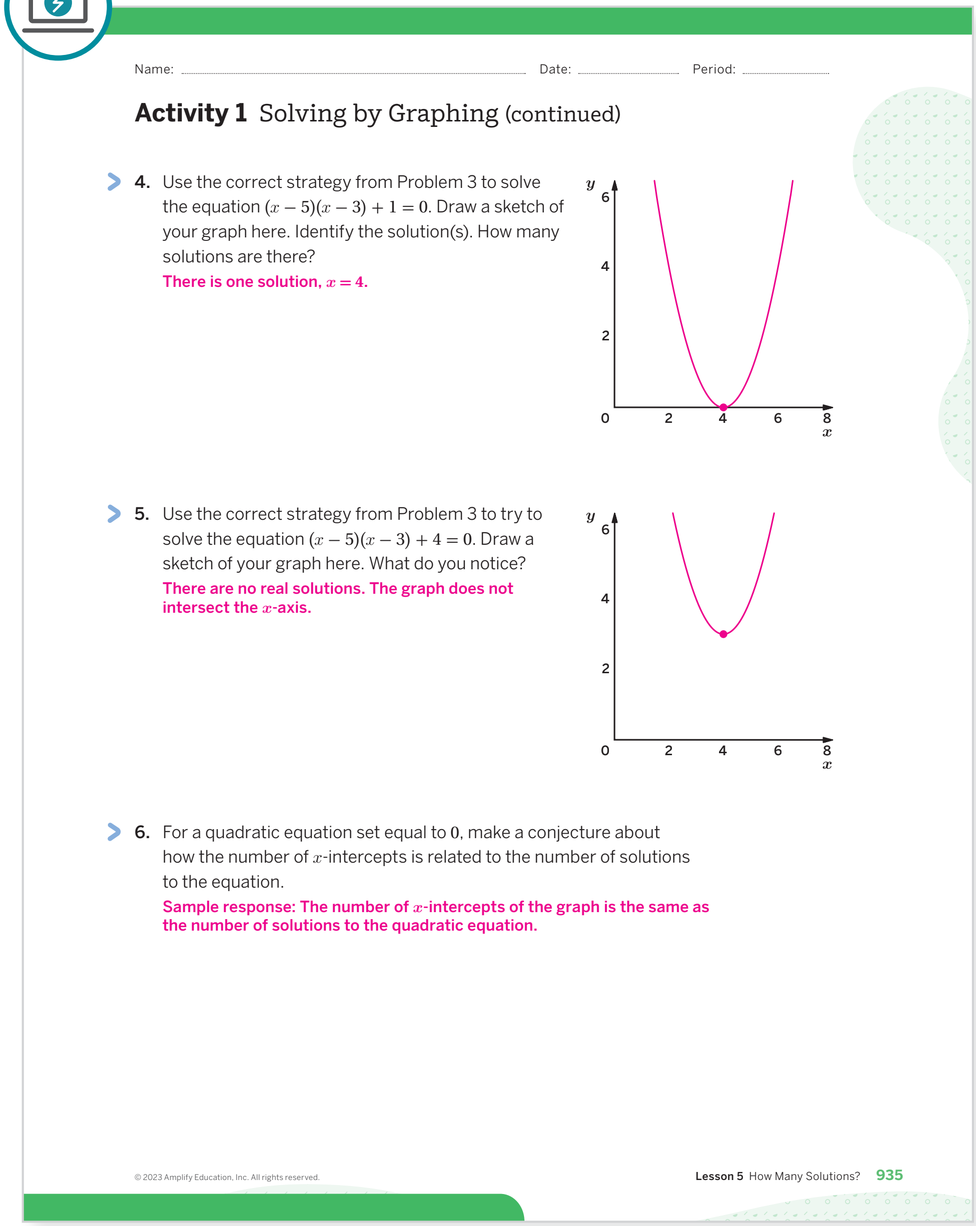
English Learners

Add a sketch of each graph from Problems 4 and 5 to the class display. Annotate the graph with the number of x-intercepts and solutions.

Pairs | 15 min

Activity 1 Solving by Graphing (continued)

Students activate prior knowledge about how to determine the zeros of functions by graphing, applying this knowledge to determine the solutions to quadratic equations set equal to 0.



Name: ______ Date: ______ Period: ______

Activity 1 Solving by Graphing (continued)

4. Use the correct strategy from Problem 3 to solve the equation $(x - 5)(x - 3) + 1 = 0$. Draw a sketch of your graph here. Identify the solution(s). How many solutions are there?
There is one solution, $x = 4$.

5. Use the correct strategy from Problem 3 to try to solve the equation $(x - 5)(x - 3) + 4 = 0$. Draw a sketch of your graph here. What do you notice?
There are no real solutions. The graph does not intersect the x-axis.

6. For a quadratic equation set equal to 0, make a conjecture about how the number of x-intercepts is related to the number of solutions to the equation.
Sample response: The number of x-intercepts of the graph is the same as the number of solutions to the quadratic equation.

3 Connect

Have pairs of students share their responses and explanations for how they used the graphs to solve the equations.

Ask:

- "Are the equations $(x - 5)(x - 3) = -1$ and $(x - 5)(x - 3) + 1 = 0$ equivalent? How do you know?" Yes, they are equivalent. Sample response: The value 1 is added to both sides of the equation $(x - 5)(x - 3) = -1$ to yield the equation $(x - 5)(x - 3) + 1 = 0$.
- "Why might it be helpful to rearrange the equation so that one side is set equal to 0?" Sample response: It allows me to determine the zeros of the related function. The zeros correspond to the x-intercepts of the graph.
- "What equation would you choose to graph to solve the equation $(x - 4)(x - 6) = 15$? What about $(x + 3)^2 - 1 = 5$?" $y = (x - 4)(x - 6) - 15$ and $y = (x + 3)^2 - 6$, respectively.

Highlight that some quadratic functions have two zeros, some have one zero, and some have no zeros, so their respective graphs will have two, one, or no horizontal intercepts, respectively. The number of horizontal intercepts correspond to the number of solutions of the quadratic equation.

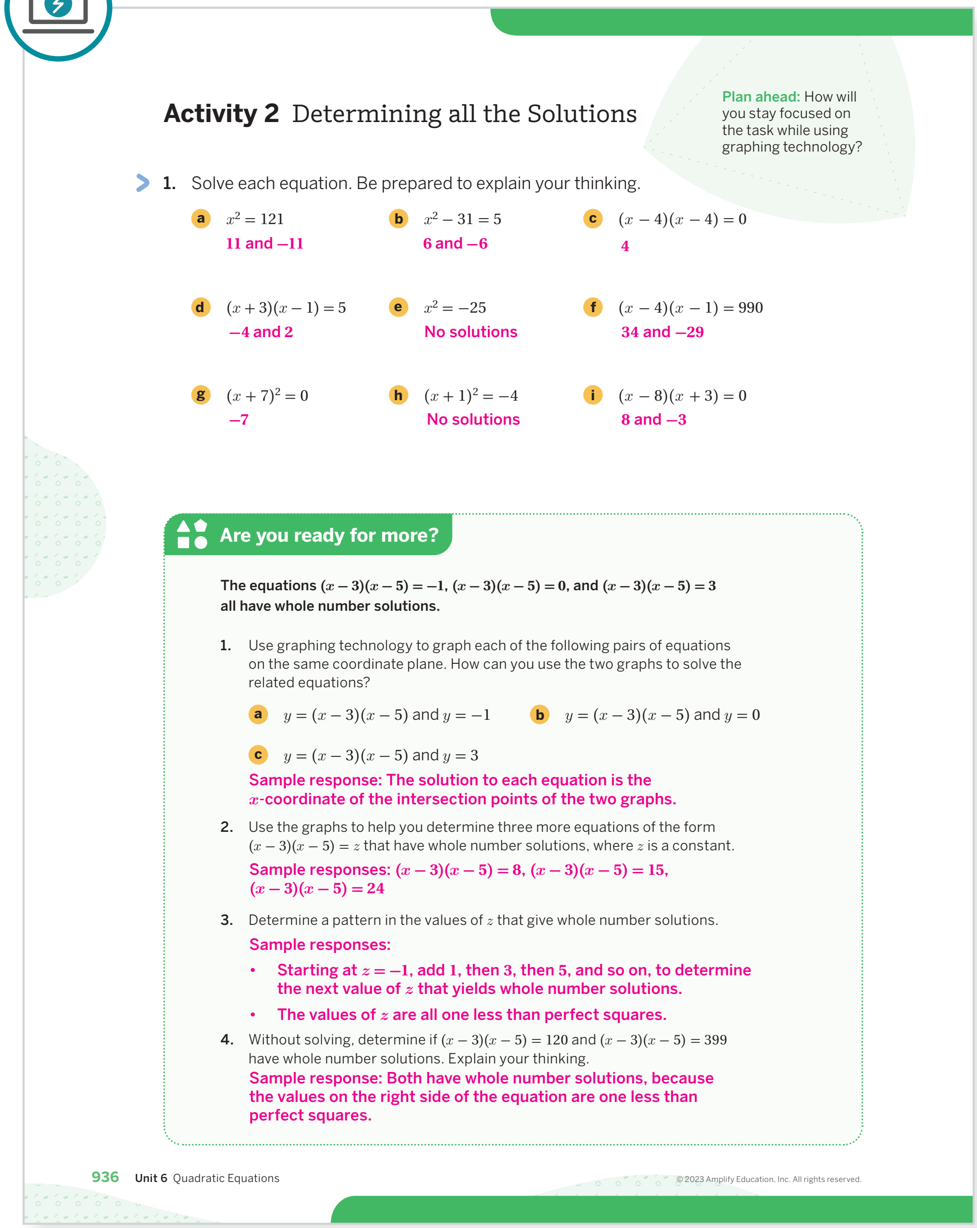
Note: In this course, the solutions to quadratic equations are limited to real numbers. In Algebra 2, students will come to understand that a quadratic equation can have imaginary solutions.

Optional

Pairs | 15 min

Activity 2 Determining all the Solutions

Students solve quadratic equations by reasoning or by graphing to strengthen their understanding of the relationship between the solutions to the equations and the zeros of the related function.



Activity 2 Determining all the Solutions

Plan ahead: How will you stay focused on the task while using graphing technology?

1. Solve each equation. Be prepared to explain your thinking.

a $x^2 = 121$
11 and –11

b $x^2 - 31 = 5$
6 and –6

c $(x - 4)(x - 4) = 0$
4

d $(x + 3)(x - 1) = 5$
–4 and 2

e $x^2 = -25$
No solutions

f $(x - 4)(x - 1) = 990$
34 and –29

g $(x + 7)^2 = 0$
–7

h $(x + 1)^2 = -4$
No solutions

i $(x - 8)(x + 3) = 0$
8 and –3

Are you ready for more?

The equations $(x - 3)(x - 5) = -1$, $(x - 3)(x - 5) = 0$, and $(x - 3)(x - 5) = 3$ all have whole number solutions.

1. Use graphing technology to graph each of the following pairs of equations on the same coordinate plane. How can you use the two graphs to solve the related equations?

   a $y = (x - 3)(x - 5)$ and $y = -1$

   b $y = (x - 3)(x - 5)$ and $y = 0$

   c $y = (x - 3)(x - 5)$ and $y = 3$

   Sample response: The solution to each equation is the x-coordinate of the intersection points of the two graphs.

2. Use the graphs to help you determine three more equations of the form $(x - 3)(x - 5) = z$ that have whole number solutions, where z is a constant.

   Sample responses: $(x - 3)(x - 5) = 8$, $(x - 3)(x - 5) = 15$, $(x - 3)(x - 5) = 24$

3. Determine a pattern in the values of z that give whole number solutions.

   Sample responses:

   - **Starting at $z = -1$, add 1, then 3, then 5, and so on, to determine the next value of z that yields whole number solutions.**
   - **The values of z are all one less than perfect squares.**

4. Without solving, determine if $(x - 3)(x - 5) = 120$ and $(x - 3)(x - 5) = 399$ have whole number solutions. Explain your thinking.

   Sample response: Both have whole number solutions, because the values on the right side of the equation are one less than perfect squares.

936 **Unit 6** Quadratic Equations

1 Launch

Provide access to graph paper and graphing technology.

2 Monitor

Help students get started by prompting them to begin with the equations where they can apply the Zero Product Principle.

Look for points of confusion:

- **Graphing every single equation.** This is a time consuming strategy. Prompt students to use reasoning in parts a–c and e and then to verify their conclusions using graphing technology.

Look for productive strategies:

- Rearranging the equation to set it equal to a perfect square and determining the square root.
- Applying the Zero Product Principle to factored equations set equal to 0.
- Recognizing there are no solutions for squared equations equal to a negative number.

3 Connect

Display parts a–f, one at a time.

Have pairs of students share their solutions and strategies. Select students first who attempted to use graphing, then select students who used other strategies.

Highlight that some strategies are more efficient to use than others, based on the quadratic equations given. Model how to use reasoning by taking square roots in parts a and b, how to apply the Zero Product Principle in part c, and how to use graphing in parts d–f. Emphasize that there are no solutions for parts e and h, because there is no number that can be squared that yields a negative product.

Ask, "What new strategy did you learn that you can add to your toolbox of strategies for solving quadratic equations?" Answers may vary.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide a bank of strategies that students can choose to use, such as the following:

- Taking the square root of each side.
- Using the Zero Product Principle.
- Rearranging the equation to set it equal to 0.
- Using graphing or graphing technology.
- Analyzing the structure of the equation to determine there are no solutions.

Consider providing an example of each strategy for students to use as a reference.

Math Language Development

MLR7: Compare and Connect

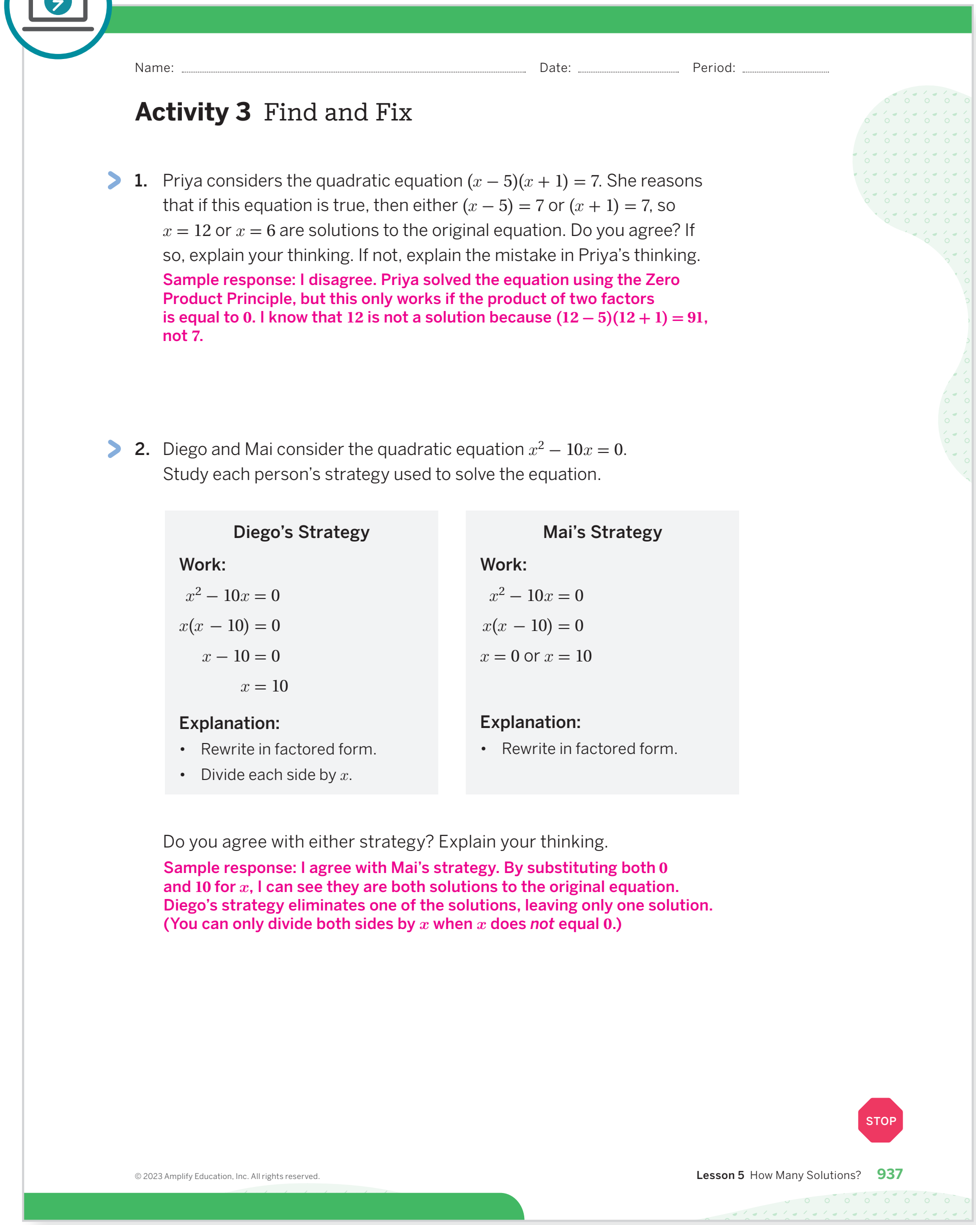
During the Connect, draw students' attention to the connections between the structure of each equation and an efficient solution strategy. Ask: "Which equations can you solve by . . ."

- "Simply taking the square root of each side? What is it about the structure of these equations that indicates this is an efficient strategy?"
- "Using the Zero Product Principle, without manipulating the equation? What is it about the structure of these equations that indicates this is an efficient strategy?"
- "Using graphing? What is it about the structure of these equations that indicates this is an efficient strategy?"

Pairs | 15 min

Activity 3 Find and Fix

Students examine and critique the arguments given by students on how to solve quadratic equations to uncover ineffective strategies and common points of confusion.



Name: Date: Period:

Activity 3 Find and Fix

1. Priya considers the quadratic equation $(x-5)(x+1)=7$. She reasons that if this equation is true, then either $(x-5)=7$ or $(x+1)=7$, so $x=12$ or $x=6$ are solutions to the original equation. Do you agree? If so, explain your thinking. If not, explain the mistake in Priya's thinking.
   Sample response: I disagree. Priya solved the equation using the Zero Product Principle, but this only works if the product of two factors is equal to 0. I know that 12 is not a solution because $(12-5)(12+1)=91$, not 7.

2. Diego and Mai consider the quadratic equation $x^2-10x=0$. Study each person's strategy used to solve the equation.

Diego's Strategy

Work:

$$\begin{aligned} x^2-10x&=0\\ x(x-10)&=0\\ x-10&=0\\ x&=10 \end{aligned}$$

Explanation:

- Rewrite in factored form.
- Divide each side by x.

Mai's Strategy

Work:

$$\begin{aligned} x^2-10x&=0\\ x(x-10)&=0\\ x=0 \text{ or } x&=10 \end{aligned}$$

Explanation:

- Rewrite in factored form.

Do you agree with either strategy? Explain your thinking.

Sample response: I agree with Mai's strategy. By substituting both 0 and 10 for x, I can see they are both solutions to the original equation. Diego's strategy eliminates one of the solutions, leaving only one solution. (You can only divide both sides by x when x does *not* equal 0.)

STOP

1 Launch

Conduct the ***Find and Fix*** routine. Give students think-time to work independently on both problems first before discussing their responses with their partner.

2 Monitor

Help students get started by asking, "What rule is Priya applying? What conditions are needed to apply this rule?" The Zero Product Principle. The product of the factors needs to equal 0.

Look for points of confusion:

- **Automatically concluding Diego is correct because his solution works out.** Prompt students to compare his strategy to Mai's strategy and articulate what they did differently.

Look for productive strategies:

- Indicating that Priya used the Zero Product Principle incorrectly and that Mai used it correctly.
- Noticing that Diego disregarded the second solution.

3 Connect

Display the equation that Priya solved in Problem 1. Then display the equation that Diego and Mai solved in Problem 2.

Have pairs of students share their responses and reasoning, specifically selecting students who used productive strategies.

Highlight that the Zero Product Principle only works when the product of the factors is 0. Rewriting x^2-10x in factored form, as Mai did, enables students to determine both solutions. However, dividing by a variable on both sides of the equation as Diego did is not a valid strategy, as it eliminates a solution.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with graphing technology or graph paper should they choose to graph the equations as they consider each person's solution strategy. For the equation in Problem 1, ask, "What equation would you graph? What do you need to do to the equation before you can graph it?"

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problems 1 and 2, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "How could you convince Priya that her solution method does not work?"
- "In Diego's strategy, 10 is a solution. What is incorrect about his strategy?"

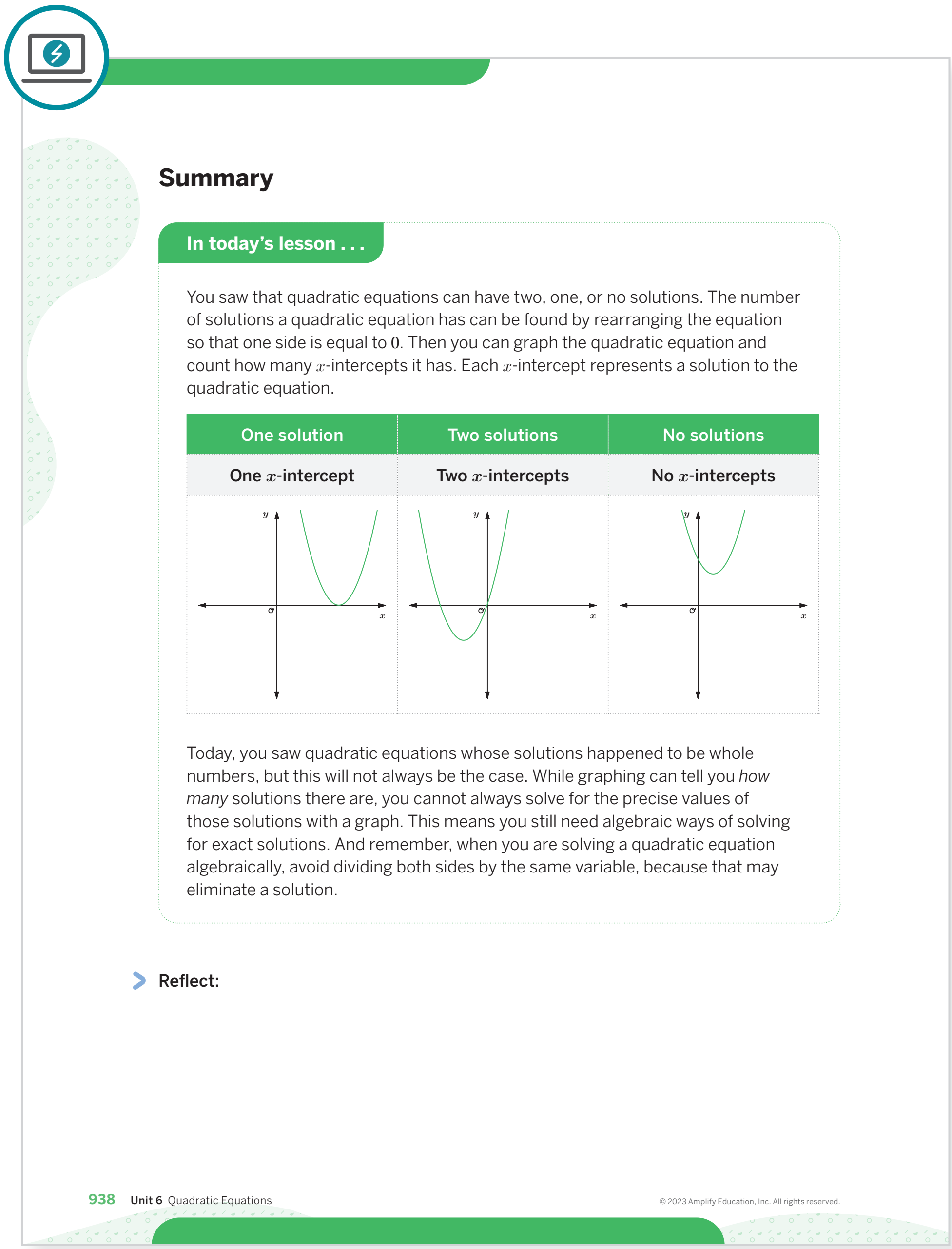
Have students revise their responses, as needed.

English Learners

Use intentional grouping so that students with different English language proficiency levels can interact and have an opportunity to listen to peers with more advanced proficiency.

Summary

Review and synthesize that quadratic equations can have two, one, or no solutions.



Summary

In today's lesson . . .

You saw that quadratic equations can have two, one, or no solutions. The number of solutions a quadratic equation has can be found by rearranging the equation so that one side is equal to 0. Then you can graph the quadratic equation and count how many x-intercepts it has. Each x-intercept represents a solution to the quadratic equation.

One solution	Two solutions	No solutions
One x-intercept	Two x-intercepts	No x-intercepts

Today, you saw quadratic equations whose solutions happened to be whole numbers, but this will not always be the case. While graphing can tell you *how many* solutions there are, you cannot always solve for the precise values of those solutions with a graph. This means you still need algebraic ways of solving for exact solutions. And remember, when you are solving a quadratic equation algebraically, avoid dividing both sides by the same variable, because that may eliminate a solution.

Reflect:

938 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the three equations: $x^2 = 5x$, $(x - 3)^2 = -4$, and $(x - 6)(x - 4) = -1$.

Ask:

- "How can you determine the solutions to the first two equations without graphing? What are the solutions?" Sample response: I can move the values from the right side of the equations to the left to set the equations equal to zero. I could attempt to factor the equations and then use the Zero Product Principle to find the solutions. The solutions to the equation $x^2 = 5x$ are $x = 0$ and $x = 5$. There are no solutions to $(x - 3)^2 = -4$, because it cannot be factored.
- "How many x-intercepts do the graphs of these equations have? How do you know?" Two, zero, and one, respectively. The number of x-intercepts of the graph of the related function, when the equation is set equal to 0, is the same as the number of solutions to the equation.
- "How can you rewrite each of these three equations to determine the x-intercepts to their graphs?" $x^2 - 5x = 0$, $(x - 3)^2 + 4 = 0$, and $(x - 6)(x - 4) + 1 = 0$.
- "What are the solution(s) to the equation $(x - 6)(x - 4) = -1$?" $x = 5$

Have students share strategies for solving the given equations with and without graphing.

Highlight that if a quadratic equation is set equal to 0, students can rewrite the equation in the form $y =$ [expression], graph the equation, and determine the number of solutions by determining the number of x-intercepts.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

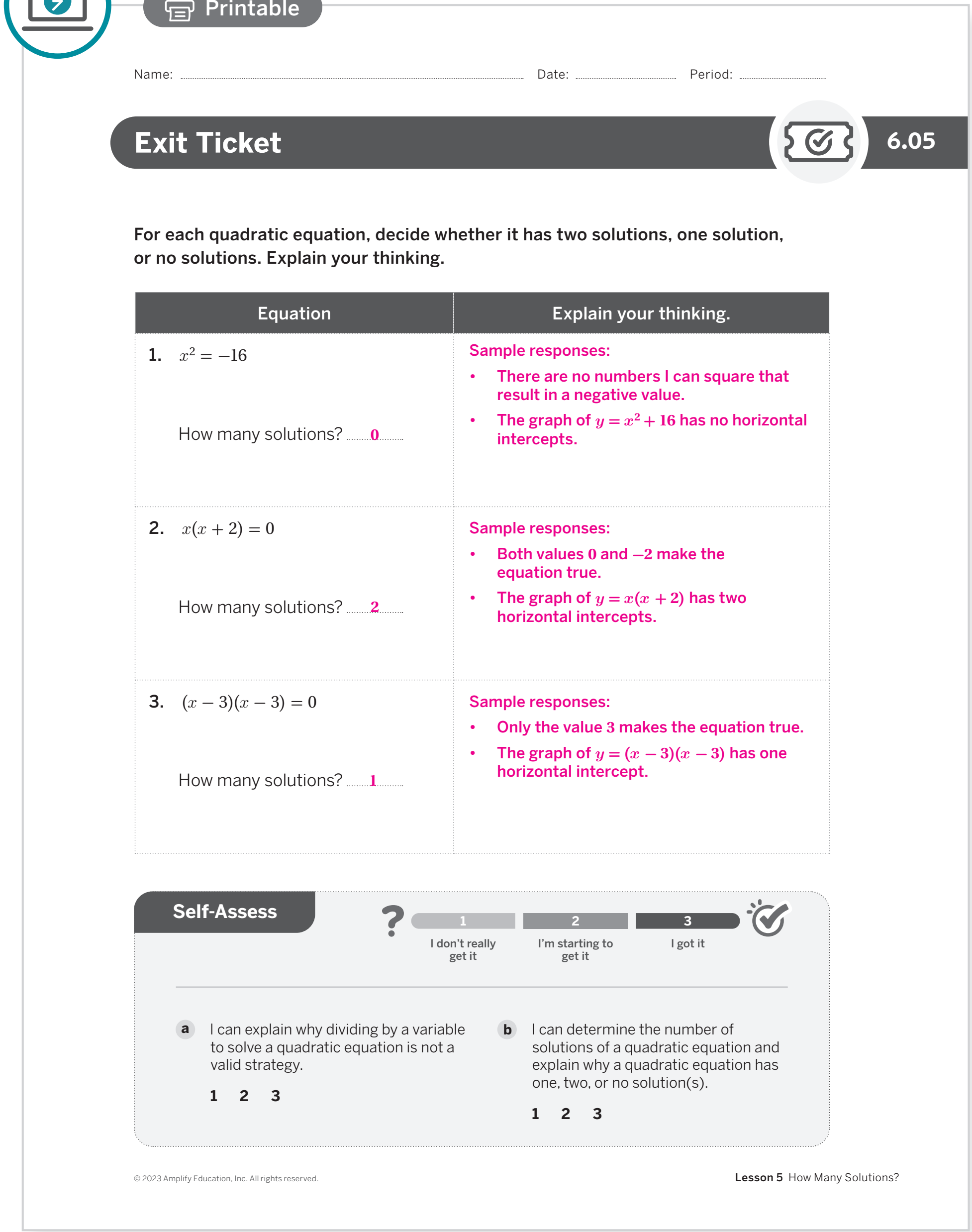
- "Why does a quadratic equation need to be set equal to zero to use the Zero Product Principle?"
- "How are solutions to quadratic functions represented graphically?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the number of solutions to a quadratic equation, without the use of a calculator.

Printable



Name: Date: Period:

Exit Ticket 6.05

For each quadratic equation, decide whether it has two solutions, one solution, or no solutions. Explain your thinking.

Equation	Explain your thinking.
1. $x^2 = -16$ How many solutions? 0	Sample responses: • There are no numbers I can square that result in a negative value. • The graph of $y = x^2 + 16$ has no horizontal intercepts.
2. $x(x + 2) = 0$ How many solutions? 2	Sample responses: • Both values 0 and −2 make the equation true. • The graph of $y = x(x + 2)$ has two horizontal intercepts.
3. $(x - 3)(x - 3) = 0$ How many solutions? 1	Sample responses: • Only the value 3 makes the equation true. • The graph of $y = (x - 3)(x - 3)$ has one horizontal intercept.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain why dividing by a variable to solve a quadratic equation is not a valid strategy.

1 2 3

b. I can determine the number of solutions of a quadratic equation and explain why a quadratic equation has one, two, or no solution(s).

1 2 3

Lesson 5 How Many Solutions?

Success looks like . . .

- **Goal:** Coordinating between graphs with no horizontal intercepts, quadratic functions with no zeros, and quadratic equations with no solutions.
- **Language Goal:** Describing the relationship between the solutions to quadratic equations set equal to 0 and the horizontal intercepts of the graph of the related function. **(Speaking and Listening, Writing)**
  - » Determining the number of solutions of each quadratic equation.
- **Language Goal:** Explaining why dividing each side of a quadratic equation by a variable is not a reliable way to solve the equation. **(Speaking and Listening, Writing)**

Suggested next steps

If students determine the incorrect number of solutions for Problem 1, consider:

- Reviewing Activity 2.
- Asking students to substitute the value they believe to be a solution into the equation.

If students determine the incorrect number of solutions for Problem 2, consider:

- Revisiting the Zero Product Principle.
- Reviewing Priya's and Mai's strategy in Activity 3.
- Having students graph the equation using graphing technology.
- Assigning Practice Problem 2.

If students determine the incorrect number of solutions for Problem 3, consider:

- Revisiting the Zero Product Principle.
- Having students graph the equation using graphing technology.
- Assigning Practice Problem 2.

Professional Learning

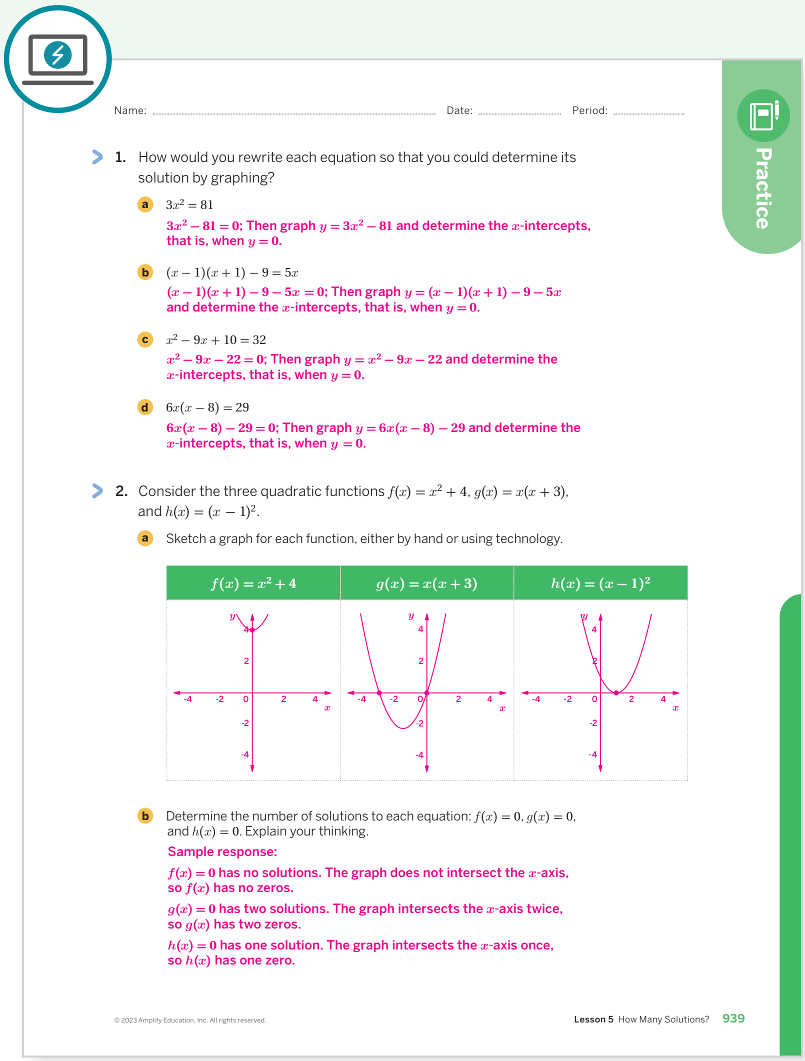
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students critique others' arguments and receive feedback today? How are you helping students become aware of how they are progressing in this area?
- During the discussion about determining the number of solutions, how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. How would you rewrite each equation so that you could determine its solution by graphing?

   a $3x^2 = 81$

   $3x^2 - 81 = 0$; Then graph $y = 3x^2 - 81$ and determine the x-intercepts, that is, when $y = 0$.

   b $(x - 1)(x + 1) - 9 = 5x$

   $(x - 1)(x + 1) - 9 - 5x = 0$; Then graph $y = (x - 1)(x + 1) - 9 - 5x$ and determine the x-intercepts, that is, when $y = 0$.

   c $x^2 - 9x + 10 = 32$

   $x^2 - 9x - 22 = 0$; Then graph $y = x^2 - 9x - 22$ and determine the x-intercepts, that is, when $y = 0$.

   d $6x(x - 8) = 29$

   $6x(x - 8) - 29 = 0$; Then graph $y = 6x(x - 8) - 29$ and determine the x-intercepts, that is, when $y = 0$.

2. Consider the three quadratic functions $f(x) = x^2 + 4$, $g(x) = x(x + 3)$, and $h(x) = (x - 1)^2$.

   a Sketch a graph for each function, either by hand or using technology.

$f(x) = x^2 + 4$	$g(x) = x(x + 3)$	$h(x) = (x - 1)^2$

   b Determine the number of solutions to each equation: $f(x) = 0$, $g(x) = 0$, and $h(x) = 0$. Explain your thinking.

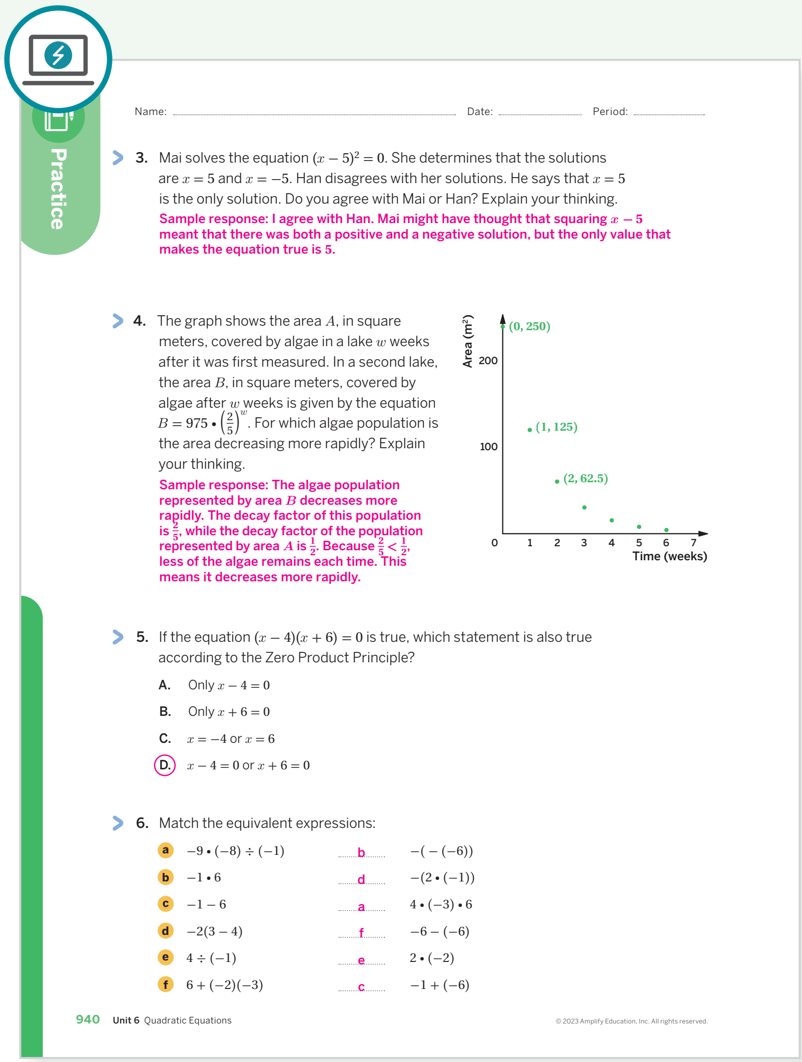
   Sample response:

   $f(x) = 0$ has no solutions. The graph does not intersect the x-axis, so $f(x)$ has no zeros.

   $g(x) = 0$ has two solutions. The graph intersects the x-axis twice, so $g(x)$ has two zeros.

   $h(x) = 0$ has one solution. The graph intersects the x-axis once, so $h(x)$ has one zero.

Lesson 5 How Many Solutions? 939



Name: Date: Period:

Practice

3. Mai solves the equation $(x - 5)^2 = 0$. She determines that the solutions are $x = 5$ and $x = -5$. Han disagrees with her solutions. He says that $x = 5$ is the only solution. Do you agree with Mai or Han? Explain your thinking.

   Sample response: I agree with Han. Mai might have thought that squaring $x - 5$ meant that there was both a positive and a negative solution, but the only value that makes the equation true is 5.

4. The graph shows the area A, in square meters, covered by algae in a lake w weeks after it was first measured. In a second lake, the area B, in square meters, covered by algae after w weeks is given by the equation $B = 975 \bullet \left(\frac{2}{5}\right)^w$. For which algae population is the area decreasing more rapidly? Explain your thinking.

   Sample response: The algae population represented by area B decreases more rapidly. The decay factor of this population is $\frac{2}{5}$, while the decay factor of the population represented by area A is $\frac{1}{2}$. Because $\frac{2}{5} < \frac{1}{2}$, less of the algae remains each time. This means it decreases more rapidly.

5. If the equation $(x - 4)(x + 6) = 0$ is true, which statement is also true according to the Zero Product Principle?

   A. Only $x - 4 = 0$

   B. Only $x + 6 = 0$

   C. $x = -4$ or $x = 6$

   [✓ D.] $x - 4 = 0$ or $x + 6 = 0$

6. Match the equivalent expressions:

a	$-9 \bullet (-8) \div (-1)$	b	$-(-(-6))$
b	$-1 \bullet 6$	d	$-(2 \bullet (-1))$
c	$-1 - 6$	a	$4 \bullet (-3) \bullet 6$
d	$-2(3 - 4)$	f	$-6 - (-6)$
e	$4 \div (-1)$	e	$2 \bullet (-2)$
f	$6 + (-2)(-3)$	c	$-1 + (-6)$

940 **Unit 6** Quadratic Equations

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 3	2
Spiral	4	Unit 4 Lesson 6	2
	5	Unit 6 Lesson 4	2
Formative ⚡	6	Unit 6 Lesson 6	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 2

Whole Class

Factoring Quadratic Expressions and Equations

In this Sub-Unit, students make connections between zeros of quadratic functions and how zero is used when solving quadratic equations.

SUB-UNIT

2 Factoring Quadratic Expressions and Equations

Narrative Connections

When is zero more than nothing?

There are few numbers as elegant as the humble zero — that unique number you can add to any number without changing its value. And yet, it took humanity a long time to even imagine such a number could exist!

Several ancient civilizations, including the Mesopotamians and the Mayans, had symbols that were *kind of like* a zero. These symbols were used as placeholders, making it easy to distinguish 102 from 12, for example.

But to see how zero became an actual *number*, we journey to 7th century India. Here, the mathematician Brahmagupta laid out the mathematical properties of zero, noting that a positive number subtracted from itself equaled this new special value!

There's debate as to precisely how zero emerged in India. Some scholars think it's related to Hindu and Buddhist philosophy, where the idea of spiritual "nothingness" was easier to grasp. Whatever the case may be, the idea took hold and spread to China and the Middle East. But as Arab traders traveled into Europe, the idea was met with strenuous resistance.

Medieval European thinkers, ironically, equated zero with chaos. In 1299, the city of Florence even went so far as to ban the number (along with other Indo-Arabic numerals), claiming it was too easy to doctor into other numbers. It wouldn't be until the 16th century that Europe finally embraced the goose egg we all know and love today.

We know now that zero is more than, well, nothing. Seeing where a parabola's y-coordinate equals zero can tell you when a ball will hit the ground, when profits turn into losses, and when rectangular gardens cannot hold any flowers. What's more, writing a quadratic expression so that it is *equal to* zero can be a powerful first step toward *solving* it.

Sub-Unit 2 Factoring Quadratic Expressions and Equations 941

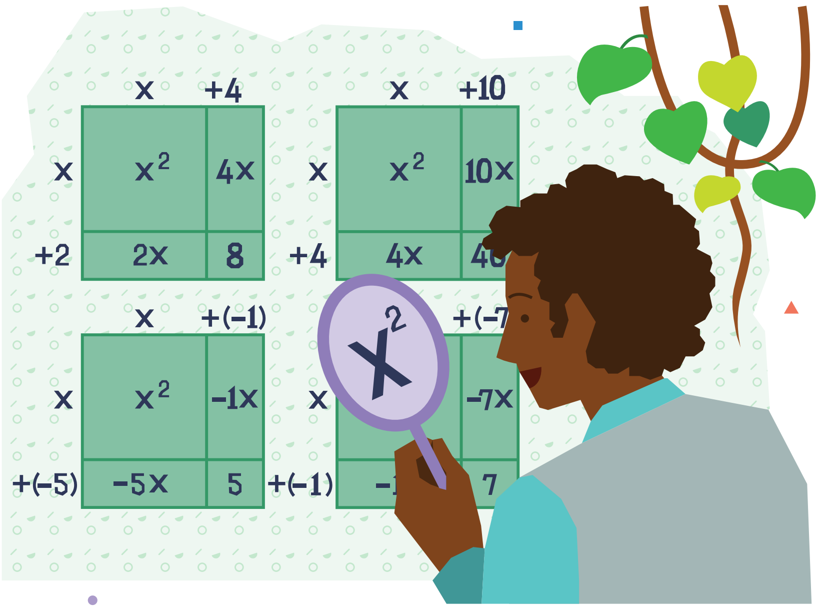
Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how setting a quadratic expression equal to zero relates to solving quadratic equations in the following places:

- **Lesson 9, Activities 1–2:** Applying the Zero Product Principle, Revisiting the Zeros of Quadratic Functions
- **Lesson 10, Activity 3:** Timing a Drop of Water

Writing Quadratic Expressions in Factored Form (Part 1)

Let's write quadratic expressions in factored form.



Focus

Goals

1. Apply the Distributive Property to multiply two sums or two differences, using a rectangular diagram to illustrate the distribution as needed.
2. **Language Goal:** Generalize the relationship between equivalent quadratic expressions in standard form and factored form, and use the generalization to transform expressions from one form to the other. **(Speaking and Listening)**
3. **Language Goal:** Use a diagram to represent quadratic expressions in different forms and explain how the numbers in the factors relate to the numbers in the product. **(Speaking and Listening, Writing)**

Coherence

Today

In Unit 5, students learned to expand quadratic expressions in factored form and rewrite them in standard form. The attention to structure continues in this lesson. Students relate the numbers in factored form to the coefficients of the terms in standard form, looking for structure that can be used to go in reverse — from standard form to factored form.

Previously

Previously, students learned that a quadratic expression in factored form can be quite handy in revealing the zeros of a function and the x-intercepts of its graph. They also observed that the factored form can help them solve quadratic equations algebraically.

Coming Soon

Students will work on transforming quadratic expressions. They make use of structure as they take this insight to transform factorable quadratic expressions into factored form.

Rigor

- Students further their **conceptual understanding** of the structure of quadratic expressions written in standard and factored form.
- Students continue to build their **fluency** skills by writing quadratic equations in different forms.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	10 min	5 min
Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

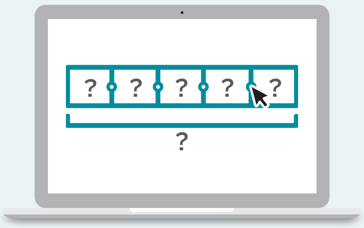
- *coefficient*
- *constant term*
- *factors*
- *linear term*
- *quadratic equations*
- *quadratic expressions*

Amps | Featured Activity

Activity 1

Digital Area Diagrams

Students can use digital area diagrams to show equivalent quadratic expressions.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel discontented in Activities 1 and 2 when discerning the structure to rewrite expressions given in standard form in factored form. Remind students that they have the underlying skills needed to complete the task and that they should rely on their strengths to complete them. They may not understand the purpose of the factoring yet, but assure them that it is a skill they will be using in the future.

Modifications to Pacing

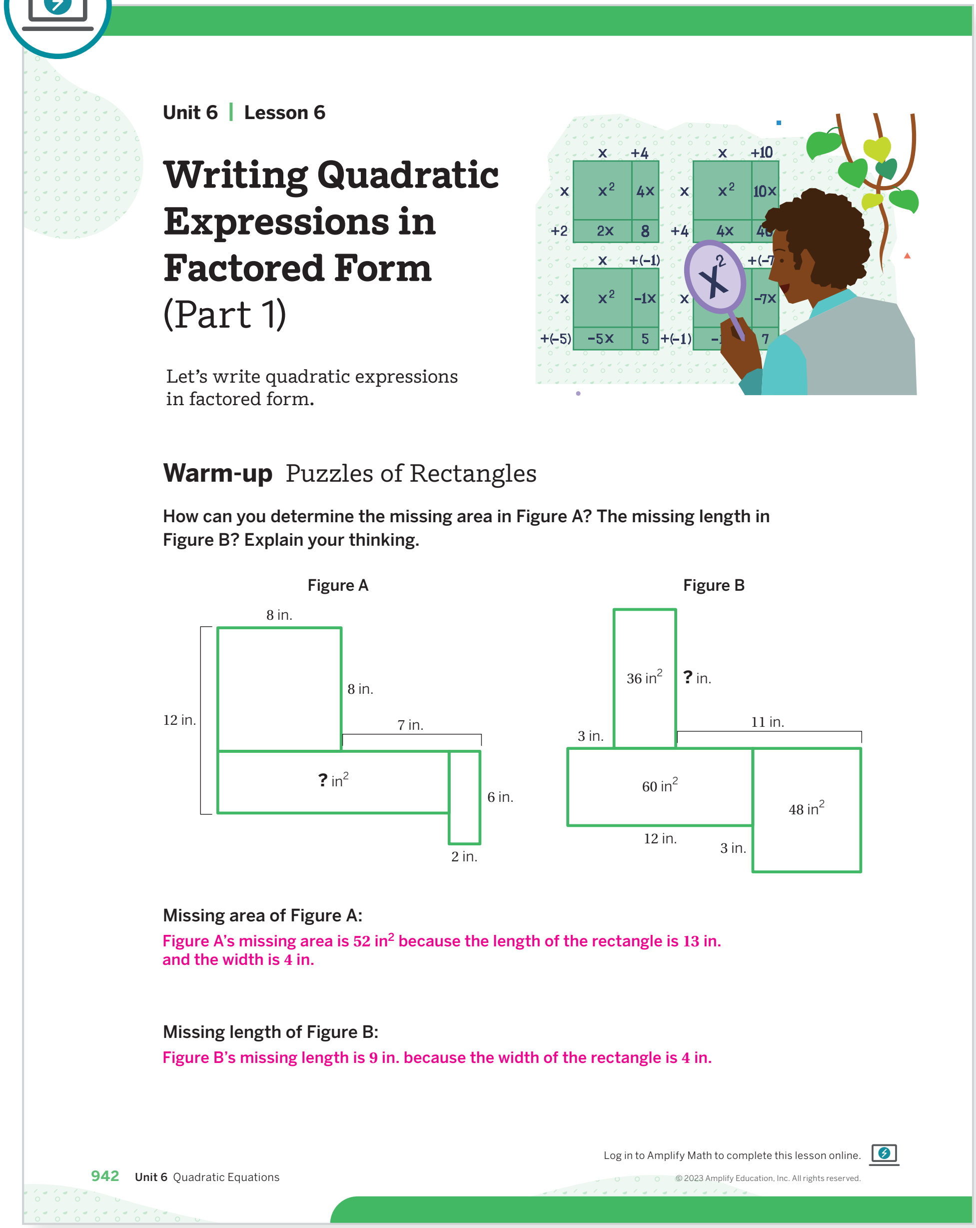
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Figure B may be omitted.
- In **Activity 1**, have students only complete Problems 1, 3, and 5.
- In **Activity 2**, have students only complete the first four rows in the table.

Pairs | 10 min

Warm-up Puzzles of Rectangles

Students use reasoning to determine unknown areas and side lengths to prepare them for using area diagrams to write quadratic expressions in factored form, given standard form.



Unit 6 | **Lesson 6**

Writing Quadratic Expressions in Factored Form (Part 1)

Let's write quadratic expressions in factored form.

Warm-up Puzzles of Rectangles

How can you determine the missing area in Figure A? The missing length in Figure B? Explain your thinking.

Missing area of Figure A:

Figure A's missing area is 52 in^2 because the length of the rectangle is 13 in. and the width is 4 in.

Missing length of Figure B:

Figure B's missing length is 9 in. because the width of the rectangle is 4 in.

Log in to Amplify Math to complete this lesson online.

942 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the figures and provide a few minutes of think-time before students share their thoughts with a partner.

2 Monitor

Help students get started by identifying the location of the missing area or side length in each figure.

Look for points of confusion:

- **Solving for the area without determining the missing side lengths.** Have students locate and label sides needed before attempting to calculate the area.
- **Using incorrect factors of 36 as side lengths in Figure B.** Ask, "What is the width of the rectangle? How can you determine the width of the rectangle?" 4 in.; I can use the area of the other rectangles to determine their lengths and widths and reason that the total width of the entire figure is 18 in. This means that the missing width of the rectangle is 4 in., making the missing length 9 in.

Look for productive strategies:

- Decomposing the figures into individual rectangles to determine the side lengths.
- Locating and determining unknown side lengths before determining an unknown area.

3 Connect

Display each figure.

Have student pairs share their processes and strategies for determining the missing values.

Highlight that students can use reasoning about the other rectangles shown in each diagram to determine other lengths or areas before they can determine the missing area and side length indicated.

Math Language Development

MLR8: Discussion Supports — Press for Details

During the Connect, give pairs of students 2–3 minutes to plan what they will say when they share how they determined the missing area or side length. Display these questions for them to think about as they plan what they will say:

- "What details are important to share?"
- "What math language can you use to show your thinking?"

English Learners

Display these sentence frames for students to complete as they share their strategies:

- "The area of Figure A/Figure B is ___ because . . ."
- "I noticed that ___, so I . . ."

Power-up

To power up students' ability to evaluate expressions with positive and negative numbers, have students complete:

Evaluate each of the following expressions.

1. $8 \cdot (-3) = -24$

2. $8 + (-3) = 5$

3. $(-8) + (-3) = -11$

4. $(-8) \cdot (-3) = 24$

5. $(-8)^2 = 64$

Use: Before Activity 1

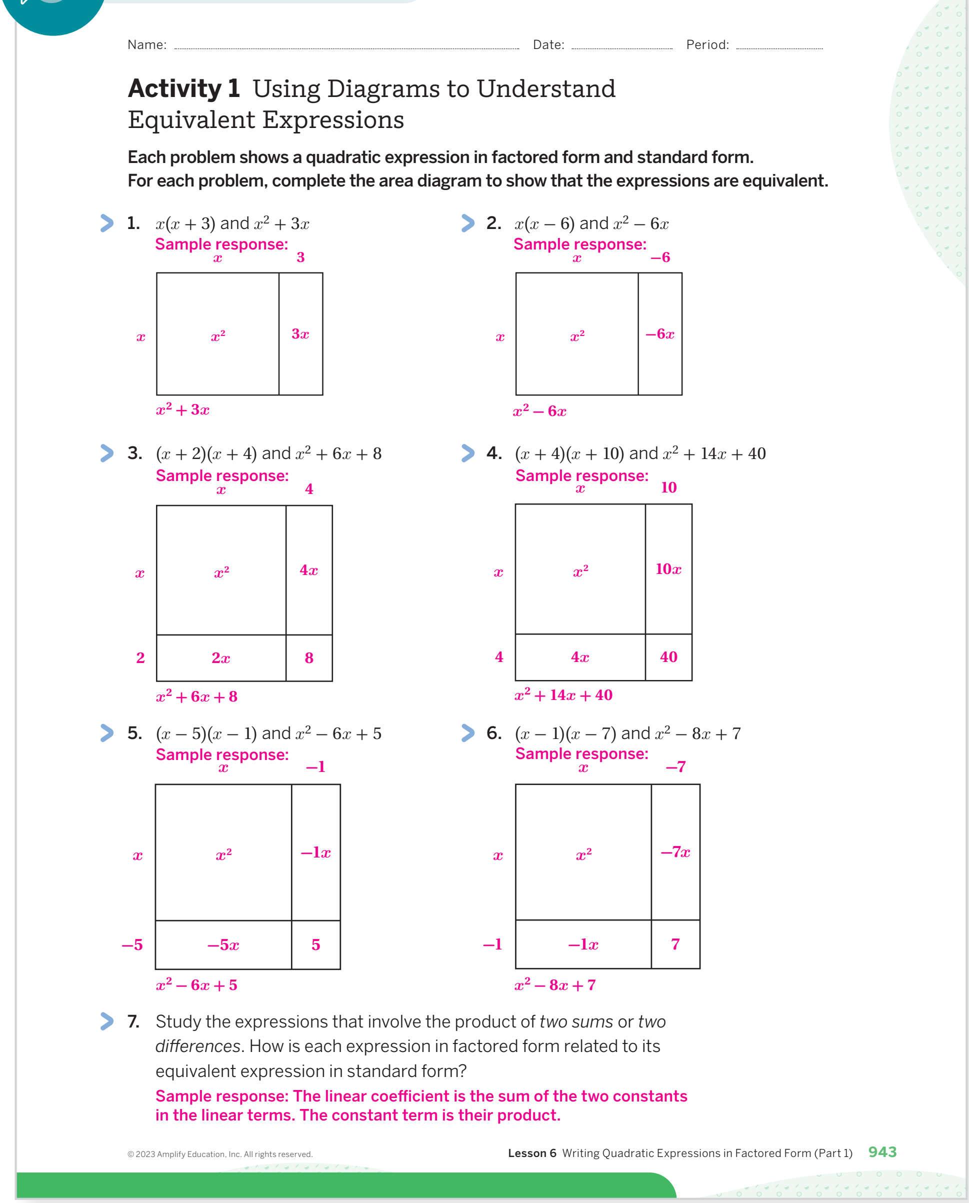
Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1

Activity 1 Using Diagrams to Understand Equivalent Expressions

Independent | 15 min

Students notice the structure that relates quadratic expressions to factored form and their equivalent counterparts in standard form.

Amps Featured Activity Digital Area Diagrams



Name: Date: Period:

Activity 1 Using Diagrams to Understand Equivalent Expressions

Each problem shows a quadratic expression in factored form and standard form. For each problem, complete the area diagram to show that the expressions are equivalent.

1. $x(x + 3)$ and $x^2 + 3x$
Sample response:

	x	3
x	x^2	$3x$

$x^2 + 3x$

2. $x(x - 6)$ and $x^2 - 6x$
Sample response:

	x	-6
x	x^2	$-6x$

$x^2 - 6x$

3. $(x + 2)(x + 4)$ and $x^2 + 6x + 8$
Sample response:

	x	4
x	x^2	$4x$
2	$2x$	8

$x^2 + 6x + 8$

4. $(x + 4)(x + 10)$ and $x^2 + 14x + 40$
Sample response:

	x	10
x	x^2	$10x$
4	$4x$	40

$x^2 + 14x + 40$

5. $(x - 5)(x - 1)$ and $x^2 - 6x + 5$
Sample response:

	x	-1
x	x^2	$-1x$
-5	$-5x$	5

$x^2 - 6x + 5$

6. $(x - 1)(x - 7)$ and $x^2 - 8x + 7$
Sample response:

	x	-7
x	x^2	$-7x$
-1	$-1x$	7

$x^2 - 8x + 7$

7. Study the expressions that involve the product of *two sums* or *two differences*. How is each expression in factored form related to its equivalent expression in standard form?
Sample response: The linear coefficient is the sum of the two constants in the linear terms. The constant term is their product.

Lesson 6 Writing Quadratic Expressions in Factored Form (Part 1) 943

1 Launch

Draw an area diagram that represents the expression $(x + 2)(x + 3)$. Review how to use an area diagram to write the equivalent expression $x^2 + 5x + 6$.

2 Monitor

Help students get started by pointing out they need to complete area diagrams to show that the two expressions in each problem are equivalent.

Look for points of confusion:

- **Not including the negative sign in Problems 2, 5, and 6.** Remind students that subtracting a number is the same as adding its opposite.
- **Not combining like terms.** Remind students to use the commutative property to combine like terms.
- **Struggling to make a connection between factored form and standard form.** Ask, "In Problem 2, what is another way to write $x \bullet x$? $x \bullet -6$?" x^2, $-6x$

Look for productive strategies:

- Writing the corresponding sign with its terms in the diagram.
- Expressing subtraction as the additive inverse.
- Noticing patterns among the coefficients and constants of the terms in standard form to the constants in factored form.

3 Connect

Have individual students share their diagrams and strategies with the class. Select and sequence students who used productive strategies.

Highlight that the linear coefficient in standard form of a quadratic is the sum of the two constants in the linear terms in factored form. The constant term in standard form is the product of the two constants in factored form.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can use digital area diagrams to show equivalent quadratic expressions.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose three of the six area diagrams to complete. Allowing them to choose can result in increased ownership of and engagement in the task.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses to Problem 7, use color coding and model using precise mathematical language to highlight how the **linear coefficient** in standard form is the sum of the two constants in the sum or difference from the factored form. The **constant term** in standard form is their product. For example, $(x + a)(x + b) = x^2 + (a + b)x + ab$. When a and b are subtracted, the sign of $(a + b)$ is negative, yet the product ab is positive.

English Learners

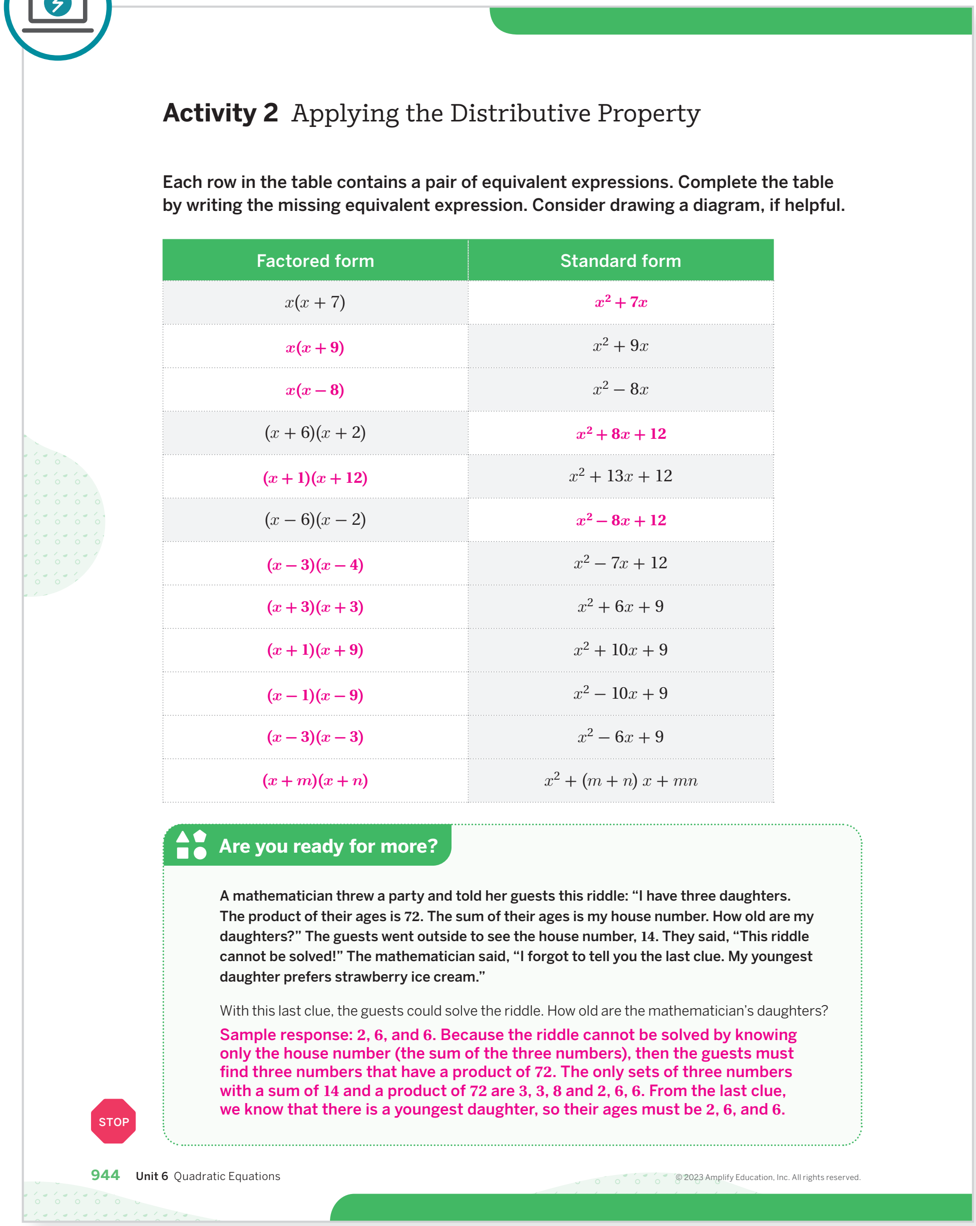
Consider adding the table shown to the class display so that students understand the phrasing "product of two sums" or "product of two differences."

Product of two sums	Product of two differences
(sum)(sum)	(difference)(difference)
$(x + 2)(x + 4)$	$(x - 5)(x - 1)$

Pairs | 10 min

Activity 2 Applying the Distributive Property

Students practice rewriting quadratic expressions in standard and factored form by using the strategies they used in Activity 1.



Activity 2 Applying the Distributive Property

Each row in the table contains a pair of equivalent expressions. Complete the table by writing the missing equivalent expression. Consider drawing a diagram, if helpful.

Factored form	Standard form
$x(x+7)$	x^2+7x
$x(x+9)$	x^2+9x
$x(x-8)$	x^2-8x
$(x+6)(x+2)$	$x^2+8x+12$
$(x+1)(x+12)$	$x^2+13x+12$
$(x-6)(x-2)$	$x^2-8x+12$
$(x-3)(x-4)$	$x^2-7x+12$
$(x+3)(x+3)$	x^2+6x+9
$(x+1)(x+9)$	$x^2+10x+9$
$(x-1)(x-9)$	$x^2-10x+9$
$(x-3)(x-3)$	x^2-6x+9
$(x+m)(x+n)$	$x^2+(m+n)\,x+mn$

Are you ready for more?

A mathematician threw a party and told her guests this riddle: "I have three daughters. The product of their ages is 72. The sum of their ages is my house number. How old are my daughters?" The guests went outside to see the house number, 14. They said, "This riddle cannot be solved!" The mathematician said, "I forgot to tell you the last clue. My youngest daughter prefers strawberry ice cream."

With this last clue, the guests could solve the riddle. How old are the mathematician's daughters?

Sample response: 2, 6, and 6. Because the riddle cannot be solved by knowing only the house number (the sum of the three numbers), then the guests must find three numbers that have a product of 72. The only sets of three numbers with a sum of 14 and a product of 72 are 3, 3, 8 and 2, 6, 6. From the last clue, we know that there is a youngest daughter, so their ages must be 2, 6, and 6.

STOP

944 Unit 6 Quadratic Equations

1 Launch

Provide students with individual think time before they share their thoughts with a partner.

2 Monitor

Help students get started by modeling how to complete the first two rows of the table.

Look for points of confusion:

- **Trying to use the Distributive Property beginning in the fourth row and only distributing x to each term in the second factor.** Have students draw an area diagram to illustrate that both x and the constant in the first factor are multiplied by each term in the second factor.
- **Struggling to write expressions in factored form, given standard form.** Ask them to draw an area diagram and determine the missing terms that would result in each standard form.

Look for productive strategies:

- Writing equivalent expressions without using an area diagram, paying attention to the signs of each term.

3 Connect

Display the table.

Have pairs of students share how they arrived at their equivalent expressions for each row. Select students who drew area diagrams to share their strategies first. Then select students who used the patterns noticed in Activity 1 to share.

Highlight that area diagrams could be drawn to help determine the equivalent expressions.

Ask, "How did you determine the equivalent expression for the last row in the table?" Sample response: I used the patterns I noticed in Activity 1. The linear coefficient in standard form is the sum of the two constants in factored form. The constant term in standard form is their product.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose six of the twelve rows in the table to complete. Allowing them to choose can result in increased ownership and engagement of the task. Alternatively, have them complete the three rows in the table where the factored form is given first. Pause to go over their responses and then have them choose to complete three additional rows.

Accessibility: *Optimize Access to Tools*

Provide access to blank area diagrams that students can use to complete if they choose to do so.

Math Language Development

MLR7: Compare and Connect

While students work, consider displaying the following from the Math Language Development feature from Activity 1.

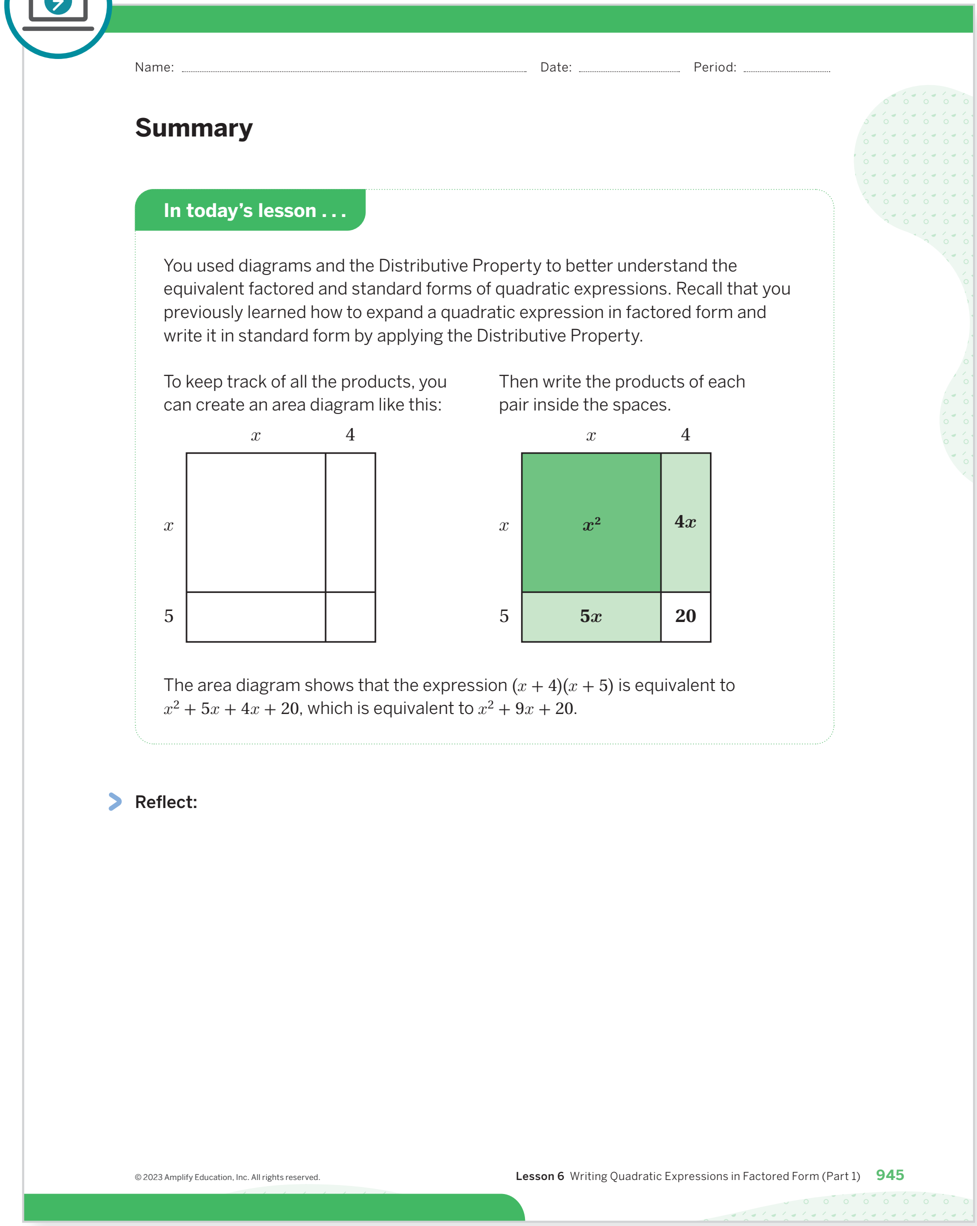
The **linear coefficient** in standard form is the sum of the two constants in the sum or difference from the factored form. The **constant term** in standard form is their product.

For example, $(x+a)(x+b)=x^2+(a+b)x+ab$.

Whole Class | 10 min

Summary

Review and synthesize how area diagrams and reasoning can help rewrite quadratic expressions in factored form, given standard form.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used diagrams and the Distributive Property to better understand the equivalent factored and standard forms of quadratic expressions. Recall that you previously learned how to expand a quadratic expression in factored form and write it in standard form by applying the Distributive Property.

To keep track of all the products, you can create an area diagram like this:

	x	4
x		
5		

Then write the products of each pair inside the spaces.

	x	4
x	x^2	$4x$
5	$5x$	20

The area diagram shows that the expression $(x + 4)(x + 5)$ is equivalent to $x^2 + 5x + 4x + 20$, which is equivalent to $x^2 + 9x + 20$.

Reflect:

Synthesize

Display the expressions $x^2 + 8x + 15$ and $x^2 + 11x + 28$.

Ask, "How can you transform each expression into factored form?"
Sample responses:

- I can draw an area diagram and determine the missing terms (in factored form) that would result in each standard form.
- I can use the patterns that I noticed in Activity 1 to determine the factored form.

Have students share the strategies they would use to rewrite each expression in factored form.

Highlight that using the review vocabulary words *coefficient*, *constant term*, and *linear term* when describing how they would rewrite each expression are helpful so that it is clear which term or value is being referenced.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

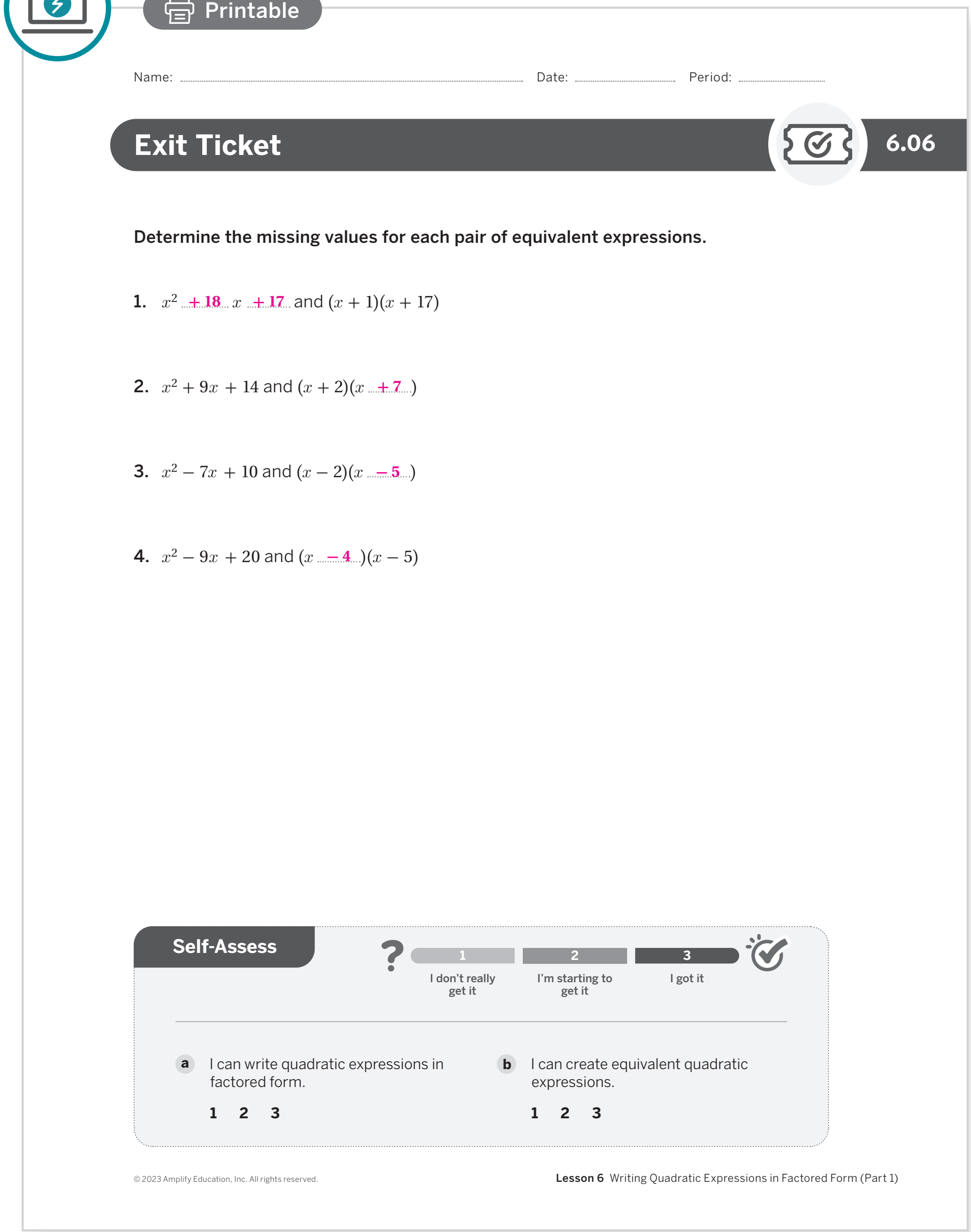
- "How is an area diagram helpful in transforming a quadratic expression into factored form?"
- "How could rewriting a quadratic equation into factored form be helpful in determining its solutions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by transforming between the standard and factored forms of quadratic expressions.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

6.06

Determine the missing values for each pair of equivalent expressions.

1. $x^2 \underline{\;+18\;}\,x \underline{\;+17\;}$ and $(x+1)(x+17)$
2. $x^2+9x+14$ and $(x+2)(x \underline{\;+7\;})$
3. $x^2-7x+10$ and $(x-2)(x \underline{\;-5\;})$
4. $x^2-9x+20$ and $(x \underline{\;-4\;})(x-5)$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can write quadratic expressions in factored form.
1 2 3

b. I can create equivalent quadratic expressions.
1 2 3

Lesson 6 Writing Quadratic Expressions in Factored Form (Part 1)

Success looks like . . .

- **Goal:** Applying the Distributive Property to multiply two sums or two differences, using a rectangular diagram to illustrate the distribution as needed.
- **Language Goal:** Generalizing the relationship between equivalent quadratic expressions in standard form and factored form, and using the generalization to transform expressions from one form to the other. **(Speaking and Listening)**
  - » Completing the expressions to show that the expressions are equivalent in Problems 1–4.
- **Language Goal:** Using a diagram to represent quadratic expressions in different forms and explain how the numbers in the factors relate to the numbers in the product. **(Speaking and Listening, Writing)**

Suggested next steps

If students struggle to determine the correct factors for the expressions, consider:

- Reviewing how to determine a missing factor from the Warm-up.
- Reviewing Activity 1, Problem 1, using area diagrams.
- Assigning Practice Problem 2.

Professional Learning

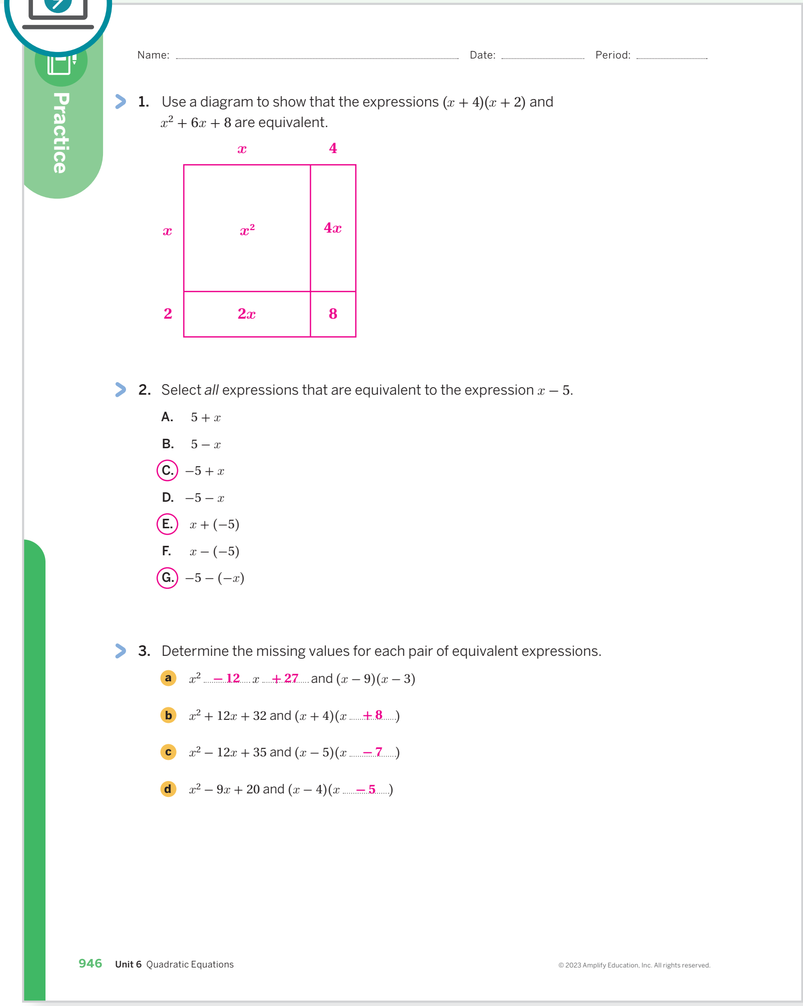
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier this year on rewriting quadratic expressions in standard and factored form, what similarities and differences do you see?
- What did you see in the way some students approached rewriting quadratic expressions in factored or standard form that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

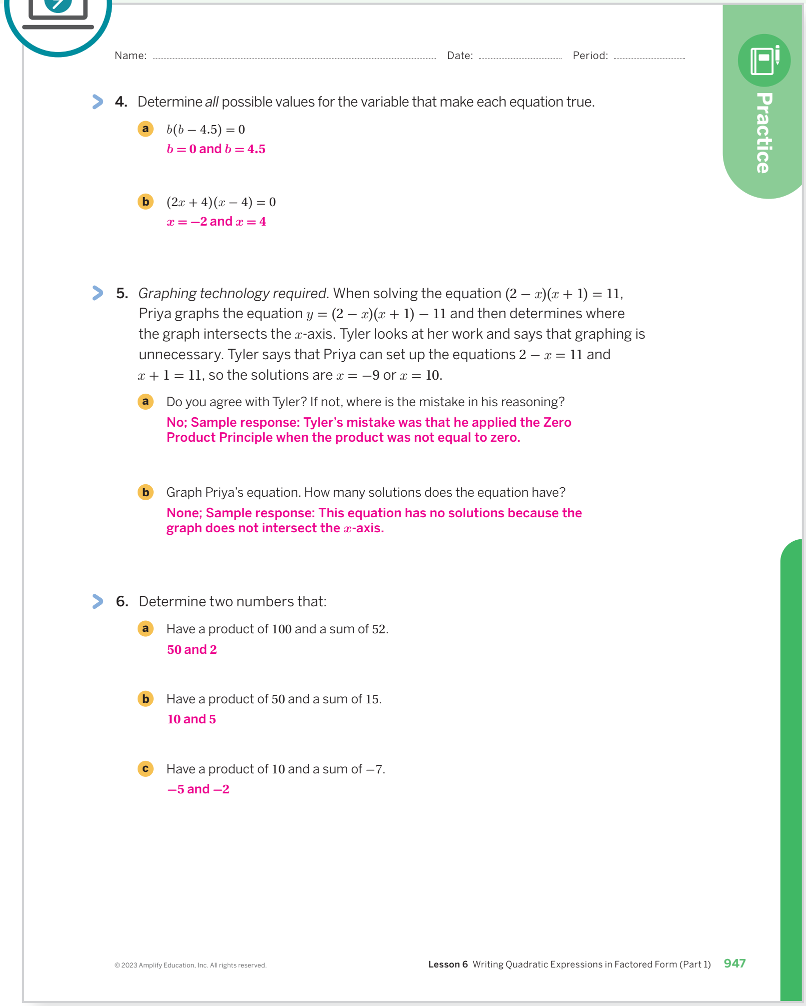
Name: ________ Date: ________ Period: ________

1. Use a diagram to show that the expressions $(x+4)(x+2)$ and x^2+6x+8 are equivalent.

2. Select *all* expressions that are equivalent to the expression $x-5$.
   - A. $5+x$
   - B. $5-x$
   - [✓ C.] $-5+x$
   - D. $-5-x$
   - [✓ E.] $x+(-5)$
   - F. $x-(-5)$
   - [✓ G.] $-5-(-x)$

3. Determine the missing values for each pair of equivalent expressions.
   - a x^2 **-12** x **$+27$** and $(x-9)(x-3)$
   - b $x^2+12x+32$ and $(x+4)(x$ **$+8$**$)$
   - c $x^2-12x+35$ and $(x-5)(x$ **-7**$)$
   - d $x^2-9x+20$ and $(x-4)(x$ **-5**$)$

946 **Unit 6** Quadratic Equations



Practice

Name: ________ Date: ________ Period: ________

4. Determine *all* possible values for the variable that make each equation true.
   - a $b(b-4.5)=0$
     $b=0$ and $b=4.5$
   - b $(2x+4)(x-4)=0$
     $x=-2$ and $x=4$

5. *Graphing technology required.* When solving the equation $(2-x)(x+1)=11$, Priya graphs the equation $y=(2-x)(x+1)-11$ and then determines where the graph intersects the x-axis. Tyler looks at her work and says that graphing is unnecessary. Tyler says that Priya can set up the equations $2-x=11$ and $x+1=11$, so the solutions are $x=-9$ or $x=10$.
   - a Do you agree with Tyler? If not, where is the mistake in his reasoning?
     No; Sample response: Tyler's mistake was that he applied the Zero Product Principle when the product was not equal to zero.
   - b Graph Priya's equation. How many solutions does the equation have?
     None; Sample response: This equation has no solutions because the graph does not intersect the x-axis.

6. Determine two numbers that:
   - a Have a product of 100 and a sum of 52.
     50 and 2
   - b Have a product of 50 and a sum of 15.
     10 and 5
   - c Have a product of 10 and a sum of -7.
     -5 and -2

Lesson 6 Writing Quadratic Expressions in Factored Form (Part 1) 947

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	1
Spiral	4	Unit 6 Lesson 4	2
	5	Unit 6 Lesson 4	3
Formative	6	Unit 6 Lesson 7	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

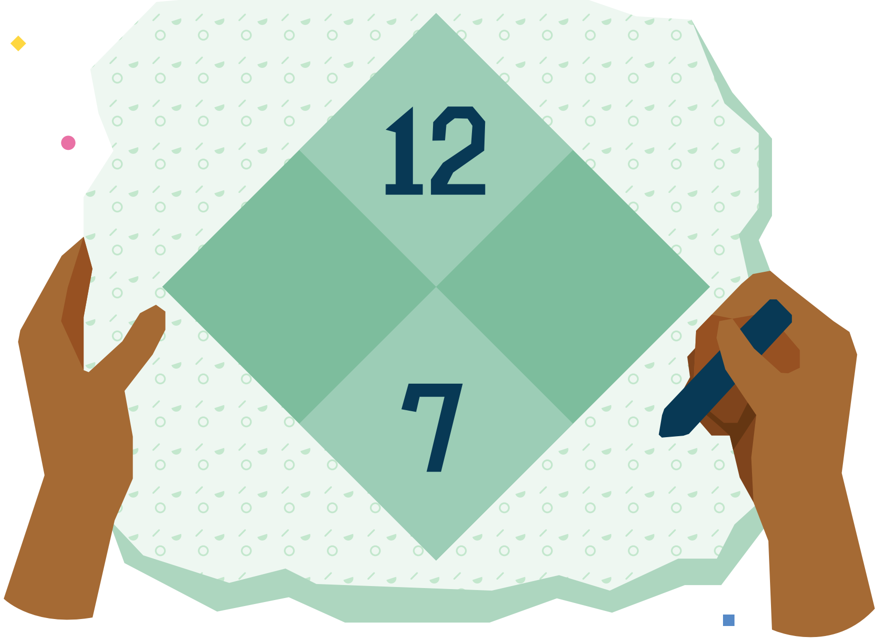
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 7**

Writing Quadratic Expressions in Factored Form
(Part 2)

Let's write more quadratic expressions in factored form.



Focus

Goals

1. Apply the Distributive Property to multiply a sum and a difference, using a diamond puzzle or area diagram to illustrate the distribution, as needed.
2. Given a factorable quadratic expression of the form $x^2 + bx + c$, where c is negative, write an equivalent expression in factored form.
3. **Language Goal:** When multiplying a sum and a difference, explain how the numbers and signs of the factors relate to the numbers in the product. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue to develop their capacity for rewriting factorable quadratic expressions into factored form, when given in standard form. They notice that when applying the Distributive Property to multiply a sum or difference, the product has a negative constant term, but the linear term can be negative or positive. Students make use of structure as they take this insight to write factorable quadratic expressions in factored form.

Previously

In Lesson 6, students used area diagrams to write factorable quadratic expressions in factored form, when given in standard form. They related the constants in factored form to the coefficients and constants of the terms in standard form.

Coming Soon

In the next lesson, students will encounter factorable quadratic expressions, written in standard form and without a linear term, and consider how to write them in factored form.

Rigor

- Students build **conceptual understanding** of what a negative constant in the standard form of a quadratic expression tells them about the factored form.
- Students develop **fluency** with writing quadratic expressions in factored form, given standard form.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1 (optional)	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

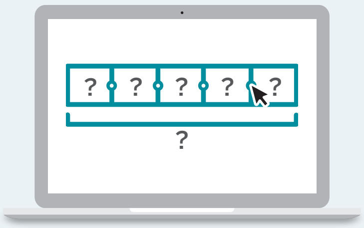
Review words

- *coefficient*
- *constant term*
- *Distributive Property*
- *factored form*
- *linear term*
- *plus-or-minus* (±)
- *standard form*

Amps | **Featured Activity**

Activity 1
Interactive Diamond Puzzles

Students use interactive diamond puzzles to determine products and sums, leading them to write the factored form of quadratic expressions.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may struggle to look for and make use of structure as they develop a strategy to determine how to write expressions in factored form. Motivate students to seek clues from the problem itself and use any connections or patterns they notice between problems already completed as a class to help develop a strategy.

Modifications to Pacing

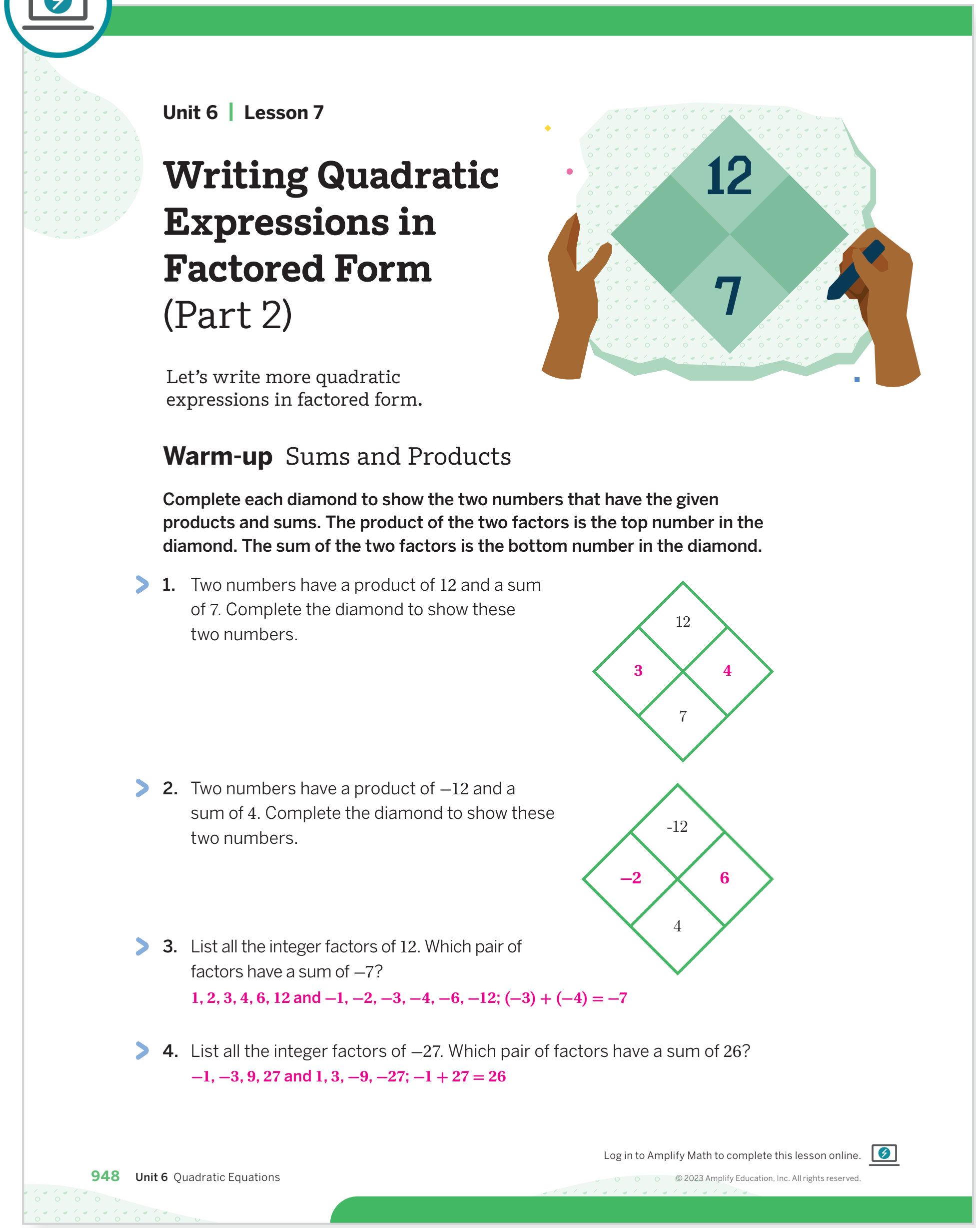
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 4 may be omitted.
- In **Activity 1**, have students only complete Problems 1, 3, and 5.
- In **Activity 3**, Problems 3c and 3d may be omitted.

Independent | 5 min

Warm-up Sums and Products

Students complete a diamond puzzle relating sums and products, preparing them for the kind of reasoning needed to write quadratics in factored form in the upcoming activities.



Unit 6 | Lesson 7

Writing Quadratic Expressions in Factored Form (Part 2)

Let's write more quadratic expressions in factored form.

Warm-up Sums and Products

Complete each diamond to show the two numbers that have the given products and sums. The product of the two factors is the top number in the diamond. The sum of the two factors is the bottom number in the diamond.

1. Two numbers have a product of 12 and a sum of 7. Complete the diamond to show these two numbers.

2. Two numbers have a product of –12 and a sum of 4. Complete the diamond to show these two numbers.

3. List all the integer factors of 12. Which pair of factors have a sum of –7?
   1, 2, 3, 4, 6, 12 and –1, –2, –3, –4, –6, –12; $(-3) + (-4) = -7$

4. List all the integer factors of –27. Which pair of factors have a sum of 26?
   –1, –3, 9, 27 and 1, 3, –9, –27; $-1 + 27 = 26$

Log in to Amplify Math to complete this lesson online.

948 **Unit 6** Quadratic Equations

1 Launch

Have students work independently to complete the diamond puzzles. Then have them share their responses with a partner.

2 Monitor

Help students get started by asking, "What are the integer factors of 12? 1, 2, 3, 4, 6, 12 and –1, –2, –3, –4, –6, –12.

Look for points of confusion:

- **Interchanging the signs of the factor pairs to produce a sum of –4 in Problem 2.** Ask, "Do the factors chosen have a sum of 4?" Have students make a list of all factors of –12 and then determine which factors have a sum of 4.

Look for productive strategies:

- Organizing the factor pairs in a list and crossing off pairs that do not have a sum of 7 or 4.

3 Connect

Display the diamond puzzles.

Have individual students share their strategies for determining which factors were needed to complete each diamond puzzle. Select and sequence students from those who did not create an organized list and to those who did.

Ask, "Consider a new diamond puzzle. What are all the factors of 10? Which of these factor pairs sum to 7?" 5 and 2

Highlight that the sign of the product is important and affects the signs of the factors. If the product is negative, then one factor must be negative and the other must be positive.

Power-up

To power up students' ability to determine the integer factors of a number, have students complete:

1. Determine all of the *positive* and *negative* factor pairs of 6. The first one is given to you.
   –1 and –6 1 and 6, –3 and –2, and 2 and 3.
2. Are the factors of –6 the same or different from the factors of 6? Be prepared to explain your thinking. The same; Sample response: The factors listed in Problem 1 include all of the factors of –6, but the factor pairs would have one positive and one negative value.
3. Which pairs of factors have a sum of 5? 2 and 3 or –1 and 6.

Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 6

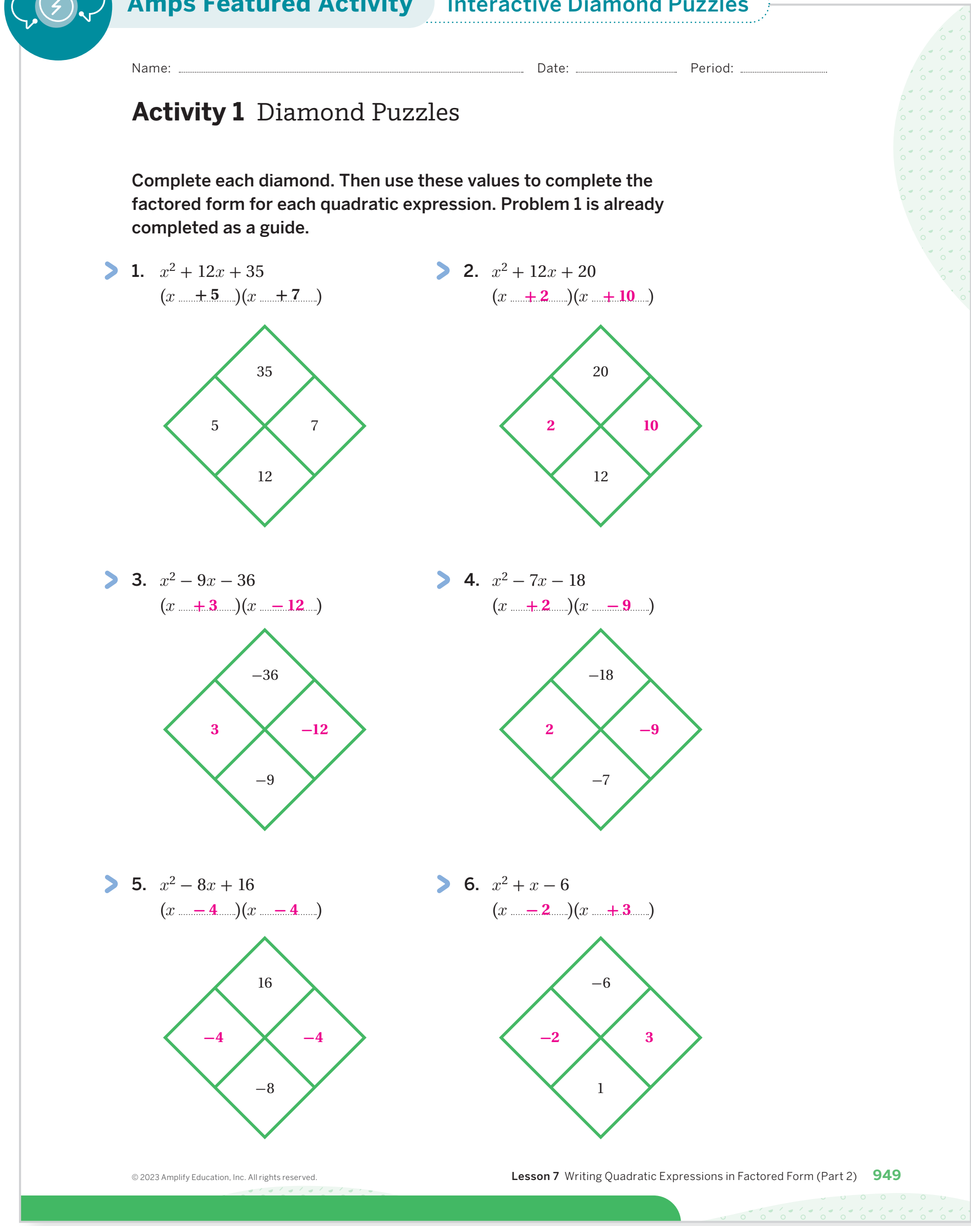
Optional

Pairs | 15 min

Activity 1 Diamond Puzzles

Students use diamond puzzles to write the factored form of quadratic expressions.

Amps Featured Activity Interactive Diamond Puzzles



Name: ________ Date: ________ Period: ________

Activity 1 Diamond Puzzles

Complete each diamond. Then use these values to complete the factored form for each quadratic expression. Problem 1 is already completed as a guide.

1. $x^2 + 12x + 35$
   $(x \underline{+5})(x \underline{+7})$

2. $x^2 + 12x + 20$
   $(x \underline{+2})(x \underline{+10})$

3. $x^2 - 9x - 36$
   $(x \underline{+3})(x \underline{-12})$

4. $x^2 - 7x - 18$
   $(x \underline{+2})(x \underline{-9})$

5. $x^2 - 8x + 16$
   $(x \underline{-4})(x \underline{-4})$

6. $x^2 + x - 6$
   $(x \underline{-2})(x \underline{+3})$

Lesson 7 Writing Quadratic Expressions in Factored Form (Part 2) 949

1 Launch

Provide students one minute of think time to study the first problem. Pause for a class discussion before having pairs work together to complete the rest of the activity.

2 Monitor

Help students get started by having them list the factors of each constant, c.

Look for points of confusion:

- **Using incorrect signs for the factors.** Have students list the factors of c. Ask, "If b and c are positive, which factors of c sum to b?" Repeat the questions based on student error.

Look for productive strategies:

- Writing the corresponding sign with its term in the diagram.
- Relating constants of the expression in factored form to the coefficient and constant of the expression in standard form.

3 Connect

Display the diamond puzzles to the class.

Have pairs of students share strategies for how they determined the two missing factors for each puzzle.

Highlight that the diamond puzzles can be used to help determine the factored form for quadratic expressions, given the standard form. It is important to pay attention to the signs of the terms to ensure the factors yield the correct sum and product.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive diamond puzzles to determine products and sums, leading them to write the factored form of quadratic expressions.

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to use the patterns they noticed in Lesson 6, and highlighted in the Math Language Development feature for this activity, as opposed to completing the diamond puzzles. The diamond puzzles can be used as a tool, but are not necessary in understanding the concept.

Math Language Development

MLR7: Compare and Connect

While students work, consider displaying the following from the Math Language Development features from Lesson 6.

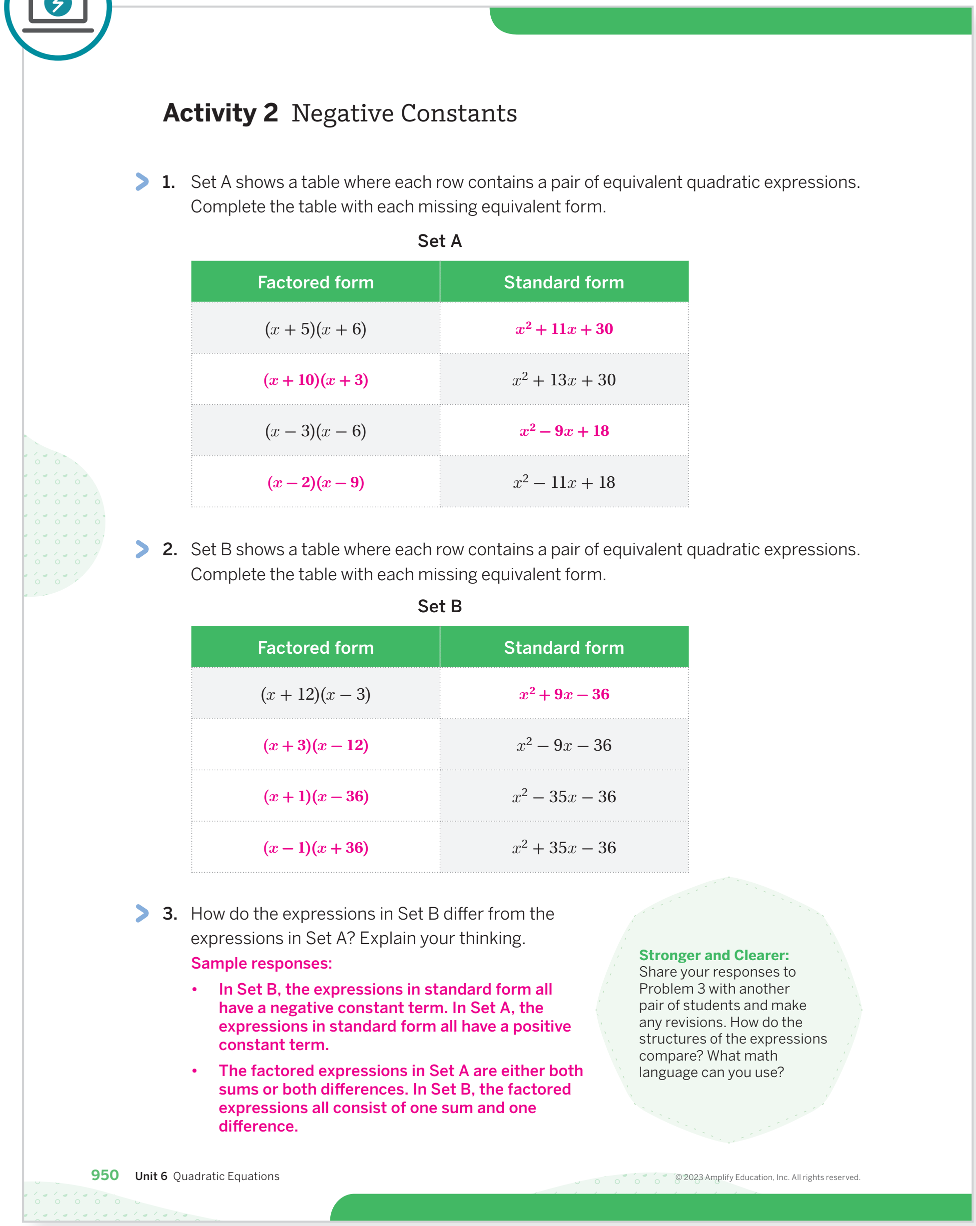
The **linear coefficient** in standard form is the sum of the two constants in the sum or difference from the factored form. The **constant term** in standard form is their product.

For example, $(x + a)(x + b) = x^2 + (a + b)x + ab$.

Pairs | 15 min

Activity 2 Negative Constants

Students study two sets of quadratic expressions written in equivalent forms to understand what a negative constant in standard form tells them about the factored form.



Activity 2 Negative Constants

1. Set A shows a table where each row contains a pair of equivalent quadratic expressions. Complete the table with each missing equivalent form.

Set A

Factored form	Standard form
$(x+5)(x+6)$	$\mathbf{x^2+11x+30}$
$\mathbf{(x+10)(x+3)}$	$x^2+13x+30$
$(x-3)(x-6)$	$\mathbf{x^2-9x+18}$
$\mathbf{(x-2)(x-9)}$	$x^2-11x+18$

2. Set B shows a table where each row contains a pair of equivalent quadratic expressions. Complete the table with each missing equivalent form.

Set B

Factored form	Standard form
$(x+12)(x-3)$	$\mathbf{x^2+9x-36}$
$\mathbf{(x+3)(x-12)}$	$x^2-9x-36$
$\mathbf{(x+1)(x-36)}$	$x^2-35x-36$
$\mathbf{(x-1)(x+36)}$	$x^2+35x-36$

3. How do the expressions in Set B differ from the expressions in Set A? Explain your thinking.

Sample responses:

- **In Set B, the expressions in standard form all have a negative constant term. In Set A, the expressions in standard form all have a positive constant term.**
- **The factored expressions in Set A are either both sums or both differences. In Set B, the factored expressions all consist of one sum and one difference.**

Stronger and Clearer: Share your responses to Problem 3 with another pair of students and make any revisions. How do the structures of the expressions compare? What math language can you use?

950 **Unit 6** Quadratic Equations

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking them to create a diamond puzzle by drawing an "X" as a shortcut to drawing the entire diamond. This is also known as an "X" diagram.

Look for points of confusion:

- **Using an incorrect sign for either the factored form or standard form.** Have students list the factor pairs with signs before writing the factored or standard form.

Look for productive strategies:

- Writing the corresponding sign with its term in the table.
- Expressing subtraction as the additive inverse.

3 Connect

Display the completed tables. Conduct the ***Notice and Wonder*** routine using the tables for Set A and Set B.

Ask, "What do you notice? What do you wonder?" Record students' responses. Answers may vary.

Have pairs of students share their strategies or processes for completing each table.

Highlight that in Set A, the factors in factored form are either both sums or both differences. The constant terms in standard form are all positive. In Set B, each factored form contains one factor that is a sum and one that is a difference. The constants in standard form are all negative. Both tables illustrate the rules for multiplying signed numbers.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose two of the four rows in each table to complete. Allowing them to choose can result in increased ownership of and engagement in the task.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the constants (along with their signs) in factored form. Have them use one color for positive constants and another color for negative constants. Then have them study the corresponding standard form expressions.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

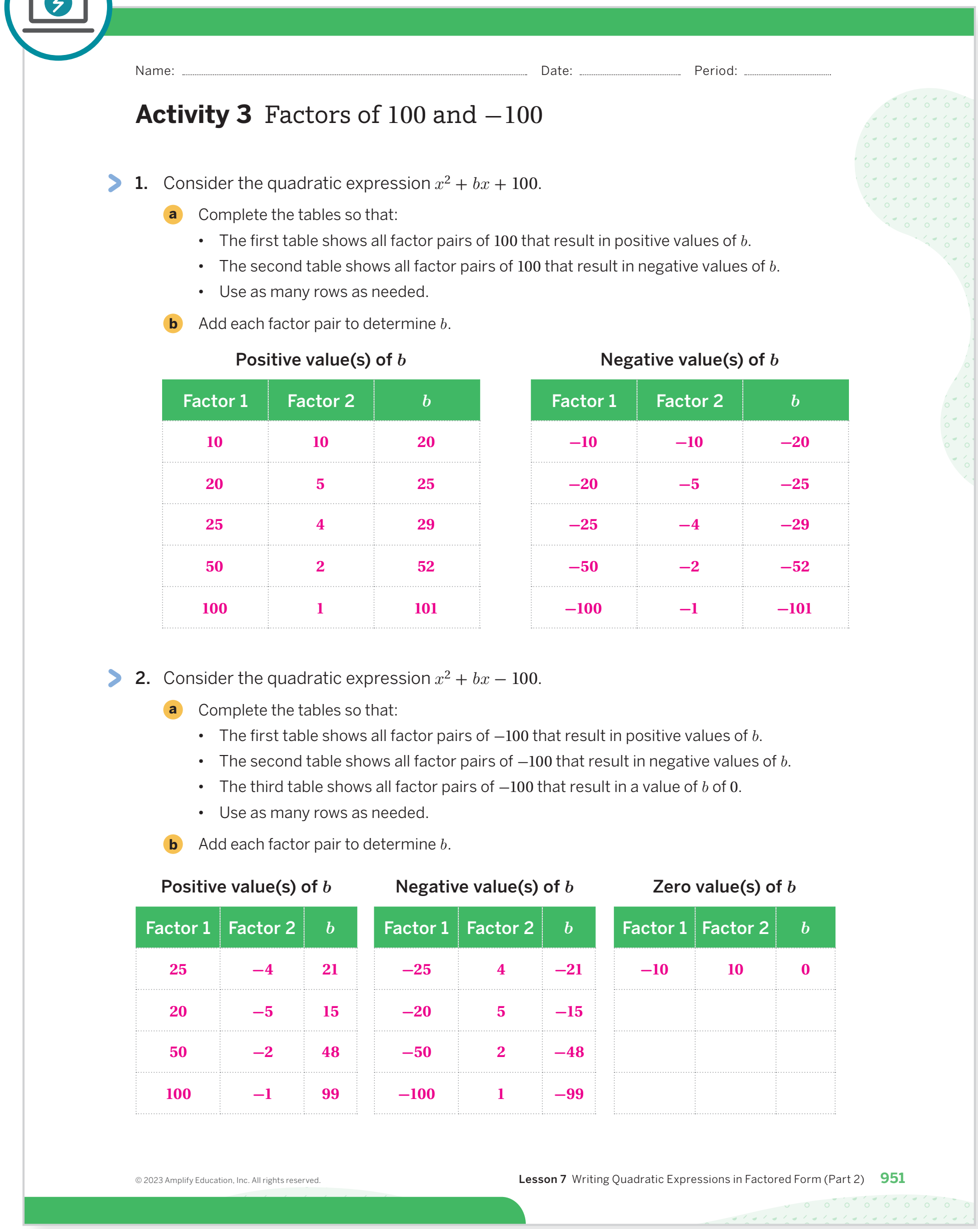
- "How do the structures of the expressions compare?"
- "How can you explain the pattern among the signs of each term in standard form, based on the corresponding factored term?"
- "What math language can you use in your response?"

Have students revise their responses, as needed.

Independent | 20 min

Activity 3 Factors of 100 and -100

Students will solidify their observations about the structure of standard form and how the signed coefficients and constants connect to the factors in factored form.



Name: ________ Date: ________ Period: ________

Activity 3 Factors of 100 and -100

1. Consider the quadratic expression $x^2 + bx + 100$.

   a Complete the tables so that:
   - The first table shows all factor pairs of 100 that result in positive values of b.
   - The second table shows all factor pairs of 100 that result in negative values of b.
   - Use as many rows as needed.

   b Add each factor pair to determine b.

Positive value(s) of b

Factor 1	Factor 2	b
10	10	20
20	5	25
25	4	29
50	2	52
100	1	101

Negative value(s) of b

Factor 1	Factor 2	b
-10	-10	-20
-20	-5	-25
-25	-4	-29
-50	-2	-52
-100	-1	-101

2. Consider the quadratic expression $x^2 + bx - 100$.

   a Complete the tables so that:
   - The first table shows all factor pairs of -100 that result in positive values of b.
   - The second table shows all factor pairs of -100 that result in negative values of b.
   - The third table shows all factor pairs of -100 that result in a value of b of 0.
   - Use as many rows as needed.

   b Add each factor pair to determine b.

Positive value(s) of b

Factor 1	Factor 2	b
25	-4	21
20	-5	15
50	-2	48
100	-1	99

Negative value(s) of b

Factor 1	Factor 2	b
-25	4	-21
-20	5	-15
-50	2	-48
-100	1	-99

Zero value(s) of b

Factor 1	Factor 2	b
-10	10	0

1 Launch

Tell students they will analyze the quadratic expressions $x^2 + bx + 100$ and $x^2 + bx - 100$ in this activity.

2 Monitor

Help students get started by asking, "What are the factors of 100?" $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm25, \pm50, \pm100$

Look for points of confusion:

- **Multiplying factors to determine b, rather than adding them.** Consider showing an area diagram that shows how the factors in factored form are added to determine the coefficient of the linear term bx.

Look for productive strategies:

- Listing the factors and then adding each pair to determine the value of b.
- Writing the corresponding sign with its term.
- Expressing subtraction as the additive inverse.

Activity 3 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students study each expression before beginning this activity. Ask, "What is similar about these expressions? What is different about them?" They are both written in standard form and have an unknown linear coefficient represented by b. In the first expression, the constant term is positive. In the second expression, the constant term is negative.

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with Factor 1 for each table and have them determine Factor 2 and the value of b. Consider demonstrating how to complete the first row of each table.

Extension: *Math Enrichment*

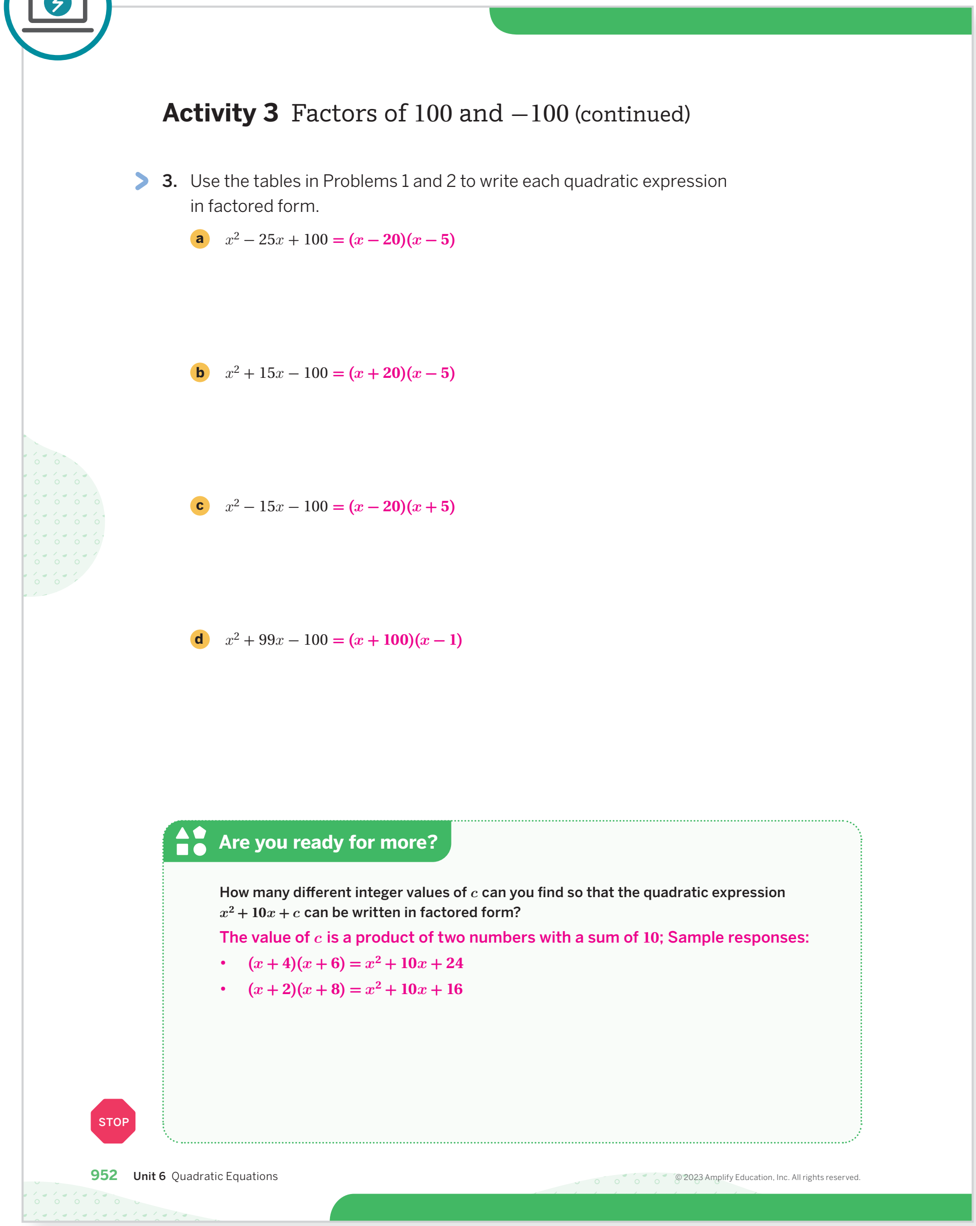
Ask students how many possible factored expressions they could write for the standard form expression $x^2 + bx - 16$. Have them explain their thinking. Sample response: 5 expressions, because the factors of 16 that result in a product of -16 are -1 and 16, -2 and 8, -4 and 4 (or 4 and -4), 1 and -16, and 2 and -8.

Then ask them to write the factored form of the expression $x^2 + 6x - 16$. $(x - 2)(x + 8)$

Independent | 20 min

Activity 3 Factors of 100 and −100 (continued)

Students will solidify their observations about the structure of standard form and how the signed coefficients and constants connect to the factors in factored form.



Activity 3 Factors of 100 and −100 (continued)

3. Use the tables in Problems 1 and 2 to write each quadratic expression in factored form.

- a $x^2 - 25x + 100 = (x - 20)(x - 5)$
- b $x^2 + 15x - 100 = (x + 20)(x - 5)$
- c $x^2 - 15x - 100 = (x - 20)(x + 5)$
- d $x^2 + 99x - 100 = (x + 100)(x - 1)$

Are you ready for more?

How many different integer values of c can you find so that the quadratic expression $x^2 + 10x + c$ can be written in factored form?

The value of c is a product of two numbers with a sum of 10; Sample responses:

- $(x + 4)(x + 6) = x^2 + 10x + 24$
- $(x + 2)(x + 8) = x^2 + 10x + 16$

STOP

952 **Unit 6** Quadratic Equations

3 Connect

Display the completed tables to the class.

Ask:

- "In Problem 1, what do you notice about the factor pairs that yield positive values of b? Negative values of b?" They are both positive; They are both negative.
- "In Problem 2, what do you notice about the factor pairs that yield positive values of b? Negative values of b?" For each table, one factor is positive and the other is negative.

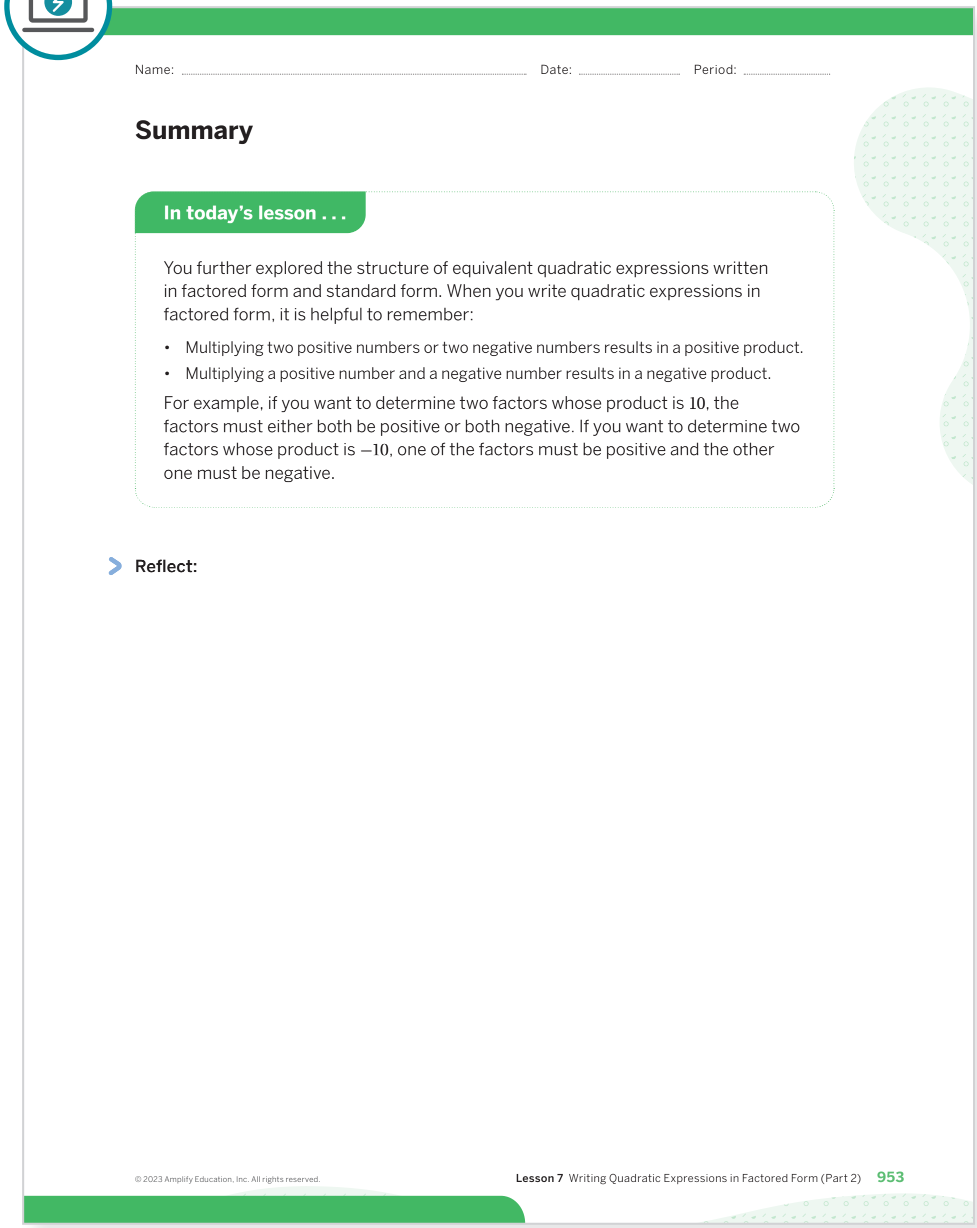
Have individual students share the strategies they used to complete the tables.

Highlight that if the linear term bx is positive, then the factors could be either positive or negative. If the linear term is negative, the factors could be either positive or negative. The sign of the constant term — in this case 100 or −100 — helps determine the sign of the factors.

Whole Class | 5 min

Summary

Review and synthesize the relationships between the coefficients and constants of terms in standard form and the constants of the equivalent expression in factored form.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You further explored the structure of equivalent quadratic expressions written in factored form and standard form. When you write quadratic expressions in factored form, it is helpful to remember:

- Multiplying two positive numbers or two negative numbers results in a positive product.
- Multiplying a positive number and a negative number results in a negative product.

For example, if you want to determine two factors whose product is 10, the factors must either both be positive or both negative. If you want to determine two factors whose product is -10, one of the factors must be positive and the other one must be negative.

Reflect:

Synthesize

Ask:

- "How would you explain to a classmate who is absent today how to rewrite the expression $x^2 + 16x - 36$ in factored form?" Sample response: I would show them how to factor using a diamond puzzle or "X" diagram. I would ask them to determine factors of -36 that produce a sum of 16.
- "How would you explain how to rewrite $x^2 - 5x - 24$ in factored form?" Sample response: I would show them how to factor using a diamond puzzle or "X" diagram. I would ask them to determine factors of -24 that produce a sum of -5.

Have students share their strategy to rewrite a quadratic expression written in standard form, such as $x^2 + 16x - 36$ or $x^2 - 5x - 24$, into factored form.

Highlight that when students apply the Distributive Property to multiply two linear expressions in which one is a sum and one is a difference, the product has a negative constant term, but the linear term can be negative or positive.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

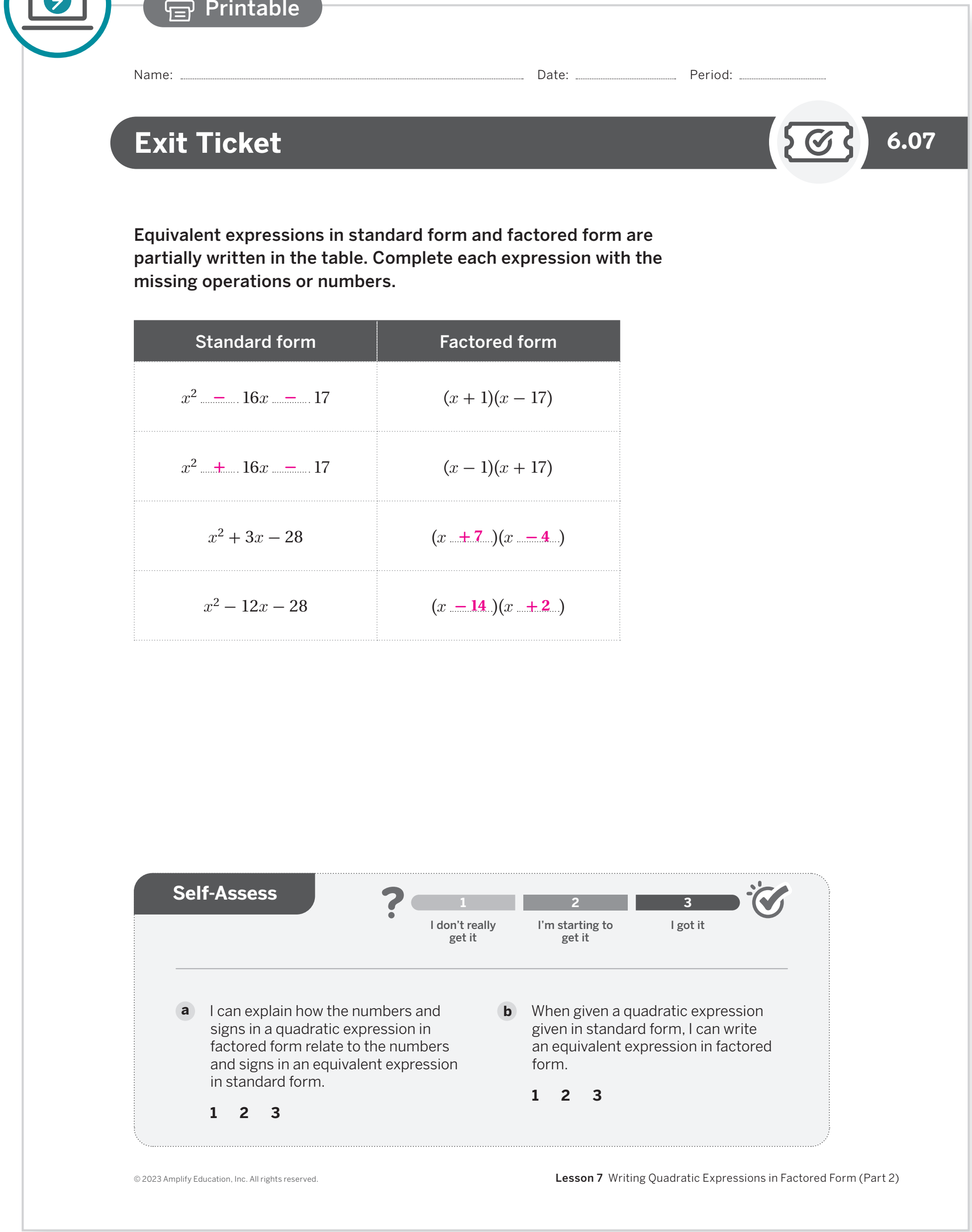
- "What does each value in a diamond puzzle diagram represent?"
- "How is a diamond puzzle diagram helpful in transforming a quadratic expression into factored form?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing equivalent expressions in standard and factored form.

Printable



Name: Date: Period:

Exit Ticket

6.07

Equivalent expressions in standard form and factored form are partially written in the table. Complete each expression with the missing operations or numbers.

Standard form	Factored form
$x^2 - 16x - 17$	$(x + 1)(x - 17)$
$x^2 + 16x - 17$	$(x - 1)(x + 17)$
$x^2 + 3x - 28$	$(x + 7)(x - 4)$
$x^2 - 12x - 28$	$(x - 14)(x + 2)$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain how the numbers and signs in a quadratic expression in factored form relate to the numbers and signs in an equivalent expression in standard form.

1 2 3

b When given a quadratic expression given in standard form, I can write an equivalent expression in factored form.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Writing Quadratic Expressions in Factored Form (Part 2)

Success looks like . . .

- **Goal:** Applying the Distributive Property to multiply a sum and a difference, using a diamond puzzle or area diagram to illustrate the distribution, as needed.
- **Goal:** Given a factorable quadratic expression of the form $x^2 + bx + c$, where c is negative, writing an equivalent expression in factored form.
  - » Writing the equivalent factored form in the last two rows of the table.
- **Language Goal:** When multiplying a sum and a difference, explaining how the numbers and signs of the factors relate to the numbers in the product. **(Speaking and Listening, Writing)**

Suggested next steps

If students use incorrect signs for the first two rows in the table, consider:

- Reviewing using an area diagram to multiply the factors, and using diamond puzzles to connect the signs of the factors with the signs of the terms.
- Reviewing or assigning optional Activity 1.
- Assigning Practice Problem 1.

If students use incorrect factors for the second two rows in the table, consider:

- Reviewing using diamond puzzles from the Warm-up.
- Reviewing or assigning optional Activity 1.
- Assigning Practice Problems 2–4.

Professional Learning

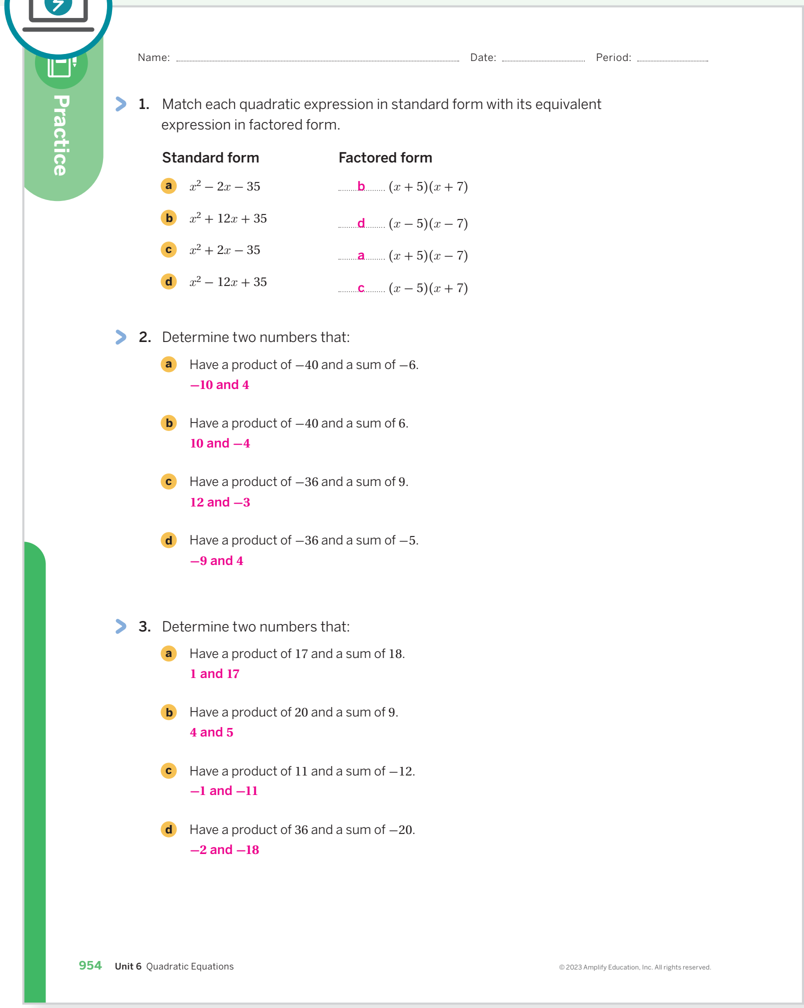
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did the use of the diamond puzzle diagram support students in transforming a quadratic expression into factored form?
- What different ways did students approach determining the factors to use in the diamond puzzle diagram? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Match each quadratic expression in standard form with its equivalent expression in factored form.

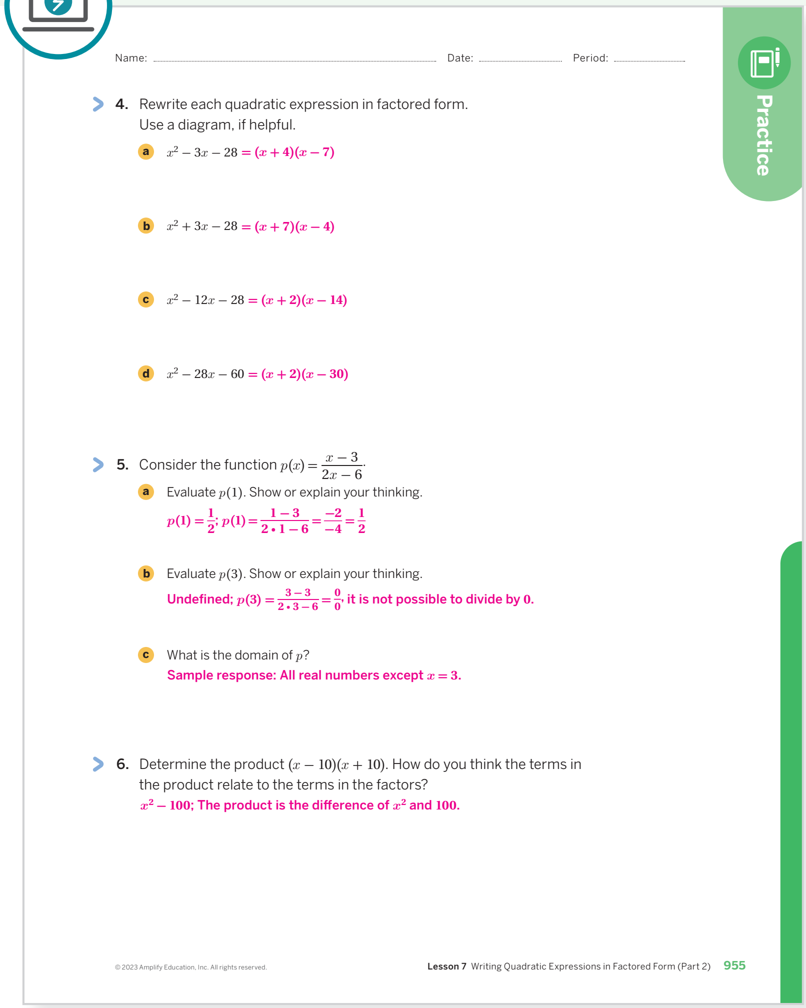
Standard form	Factored form
a $x^2 - 2x - 35$	b $(x + 5)(x + 7)$
b $x^2 + 12x + 35$	d $(x - 5)(x - 7)$
c $x^2 + 2x - 35$	a $(x + 5)(x - 7)$
d $x^2 - 12x + 35$	c $(x - 5)(x + 7)$

2. Determine two numbers that:
   - a Have a product of -40 and a sum of -6.
     -10 and 4
   - b Have a product of -40 and a sum of 6.
     10 and -4
   - c Have a product of -36 and a sum of 9.
     12 and -3
   - d Have a product of -36 and a sum of -5.
     -9 and 4

3. Determine two numbers that:
   - a Have a product of 17 and a sum of 18.
     1 and 17
   - b Have a product of 20 and a sum of 9.
     4 and 5
   - c Have a product of 11 and a sum of -12.
     -1 and -11
   - d Have a product of 36 and a sum of -20.
     -2 and -18

954 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ______ Date: ______ Period: ______

4. Rewrite each quadratic expression in factored form. Use a diagram, if helpful.
   - a $x^2 - 3x - 28 = (x + 4)(x - 7)$
   - b $x^2 + 3x - 28 = (x + 7)(x - 4)$
   - c $x^2 - 12x - 28 = (x + 2)(x - 14)$
   - d $x^2 - 28x - 60 = (x + 2)(x - 30)$

5. Consider the function $p(x) = \frac{x-3}{2x-6}$.
   - a Evaluate $p(1)$. Show or explain your thinking.
     $p(1) = \frac{1}{2}$; $p(1) = \frac{1-3}{2 \cdot 1 - 6} = \frac{-2}{-4} = \frac{1}{2}$
   - b Evaluate $p(3)$. Show or explain your thinking.
     Undefined; $p(3) = \frac{3-3}{2 \cdot 3 - 6} = \frac{0}{0}$, it is not possible to divide by 0.
   - c What is the domain of p?
     Sample response: All real numbers except $x = 3$.

6. Determine the product $(x - 10)(x + 10)$. How do you think the terms in the product relate to the terms in the factors?
   $x^2 - 100$; The product is the difference of x^2 and 100.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Writing Quadratic Expressions in Factored Form (Part 2) 955

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 6	2
	5	Unit 3 Lesson 10	2
Formative ⚡	6	Unit 6 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

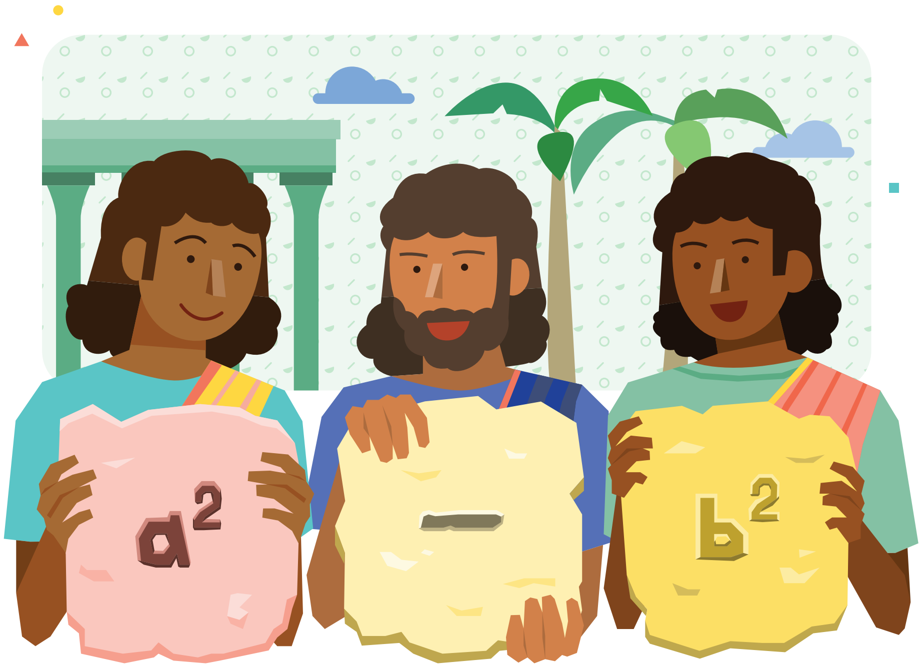
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 8**

Special Types of Factors

Let's study special types of factors.



Focus

Goals

1. **Language Goal:** Understand that multiplying a sum and a difference, $(x+m)(x-m)$, results in a quadratic expression with no linear term and explain why. **(Speaking and Listening)**
2. When given factorable quadratic expressions with no linear term, write equivalent expressions in factored form.

Rigor

- Students build **conceptual understanding** of why some factored expressions result in a standard form with no linear term.

Coherence

Today

Students encounter factorable quadratic expressions without a linear term and consider how to write them in factored form. Students come to understand that an expression in standard form that is a difference of two squares can be written in factored form in which one factor is a sum and the other factor is a difference. The constants in each factor are the same number. Through repeated reasoning, students generalize the equivalence of these two forms as $(x+m)(x-m)=x^2-m^2$. Then they make use of the structure relating the two expressions to rewrite expressions from standard form to factored form, and vice versa.

Previously

Students transformed factorable quadratic expressions from standard form to factored form, in which the expressions in standard form were of the forms x^2+bx+c or x^2+bx.

Coming Soon

After this lesson, students will have the tools they need to solve factorable quadratic equations given in standard form by first rewriting them in factored form. This work begins in the next lesson.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
10 min	10 min	10 min	10 min	5 min	5 min
Whole Class	Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems*, *Explaining My Steps*
- calculators

Math Language Development

New words

- ***difference of squares***

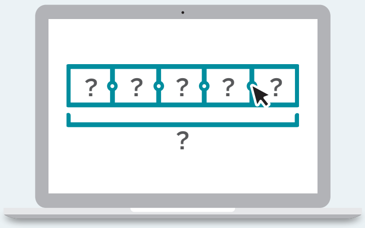
Review words

- *coefficient*
- *constant term*
- *linear term*

Amps | Featured Activity

Activity 1
Interactive Area Diagrams

Students use interactive area diagrams to derive the general formula for the difference of squares, $a^2 - b^2 = (a + b)(a - b)$.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may have difficulty managing stress and self-motivation when deriving the difference of squares in Activity 1. Lead a discussion on barriers students may encounter and have them think and discuss about ways they could overcome them. Have students consider who might be able to help or what other resources might be available.

Modifications to Pacing

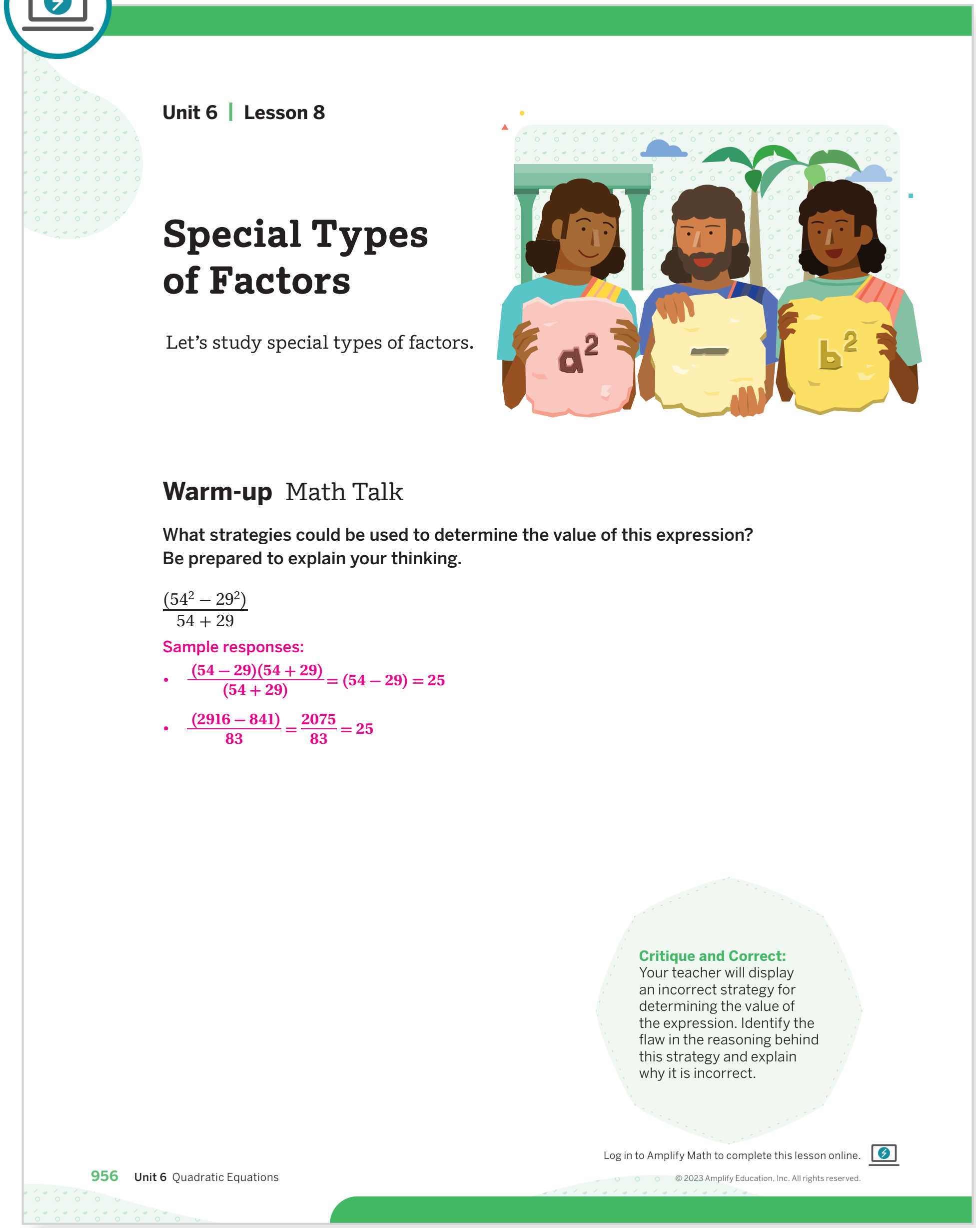
You may want to consider this additional modification if you are short on time.

- In **Activity 3**, have students only complete the first four rows of the table.

Whole Class | 10 min

Warm-up Math Talk

Students engage in a Math Talk to talk through strategies used to determine the value of an expression, preparing them for upcoming work regarding the difference of squares.



Unit 6 | Lesson 8

Special Types of Factors

Let's study special types of factors.

Warm-up Math Talk

What strategies could be used to determine the value of this expression? Be prepared to explain your thinking.

$$\frac{(54^2 - 29^2)}{54 + 29}$$

Sample responses:

- $\frac{(54-29)(54+29)}{(54+29)} = (54-29) = 25$
- $\frac{(2916-841)}{83} = \frac{2075}{83} = 25$

Critique and Correct: Your teacher will display an incorrect strategy for determining the value of the expression. Identify the flaw in the reasoning behind this strategy and explain why it is incorrect.

Log in to Amplify Math to complete this lesson online.

956 Unit 6 Quadratic Equations

① Launch

Conduct the ***Math Talk*** routine. Give students one minute of independent think-time before facilitating a discussion with the entire class.

② Monitor

Help students get started by asking, "What are some strategies that you could use to determine the value of the expression?" Answers may vary.

Look for points of confusion:

- **Thinking that canceling factors results in a value of 0.** Have students apply this reasoning to a fraction, such as $\frac{5}{5}$, which equals 1, not 0.

Look for productive strategies:

- Evaluating each power in the numerator, then subtracting, and finally dividing by the sum of the numbers in the denominator.
- Using strategies from prior lessons to write the numerator in factored form.

③ Connect

Display the expression to the class.

Have individual students share how they determined the value of the expression.

Highlight that students could factor the numerator to show that cancellation could occur between the numerator and denominator. Point out that the strategies they used in prior lessons can help them factor the numerator. Students can reason that the first number in each factor should be 54 and the last number in each factor should be 29. In order for there to be no term in the middle, one factor should be a sum and the other factor should be a difference.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect strategy for evaluating the expression, such as the one shown. Then ask the following questions.

$$\frac{(54^2 - 29^2)}{54 + 29} = \frac{54^2 - 29^2}{54^1 + 29^1}$$

$$= 54^1 - 29^1 \qquad \text{Subtract the exponents.}$$

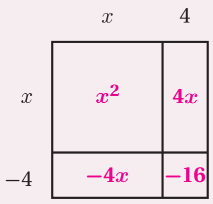
- ***Critique:*** "Do you agree or disagree with this strategy? Explain your thinking." Listen for students who reason that the quotient rule cannot be applied for this expression.
- ***Correct and Clarify:*** "What strategies can you use to evaluate this expression? Why are these strategies valid?"

Power-up

To power up students' ability to reason about quadratic expressions in factored form to lead to writing the standard form with no linear term, have students complete:

Complete the area diagram to rewrite the expression $(x-4)(x+4)$ in standard form.

$x^2 - 16$



Use: Before Activity 1

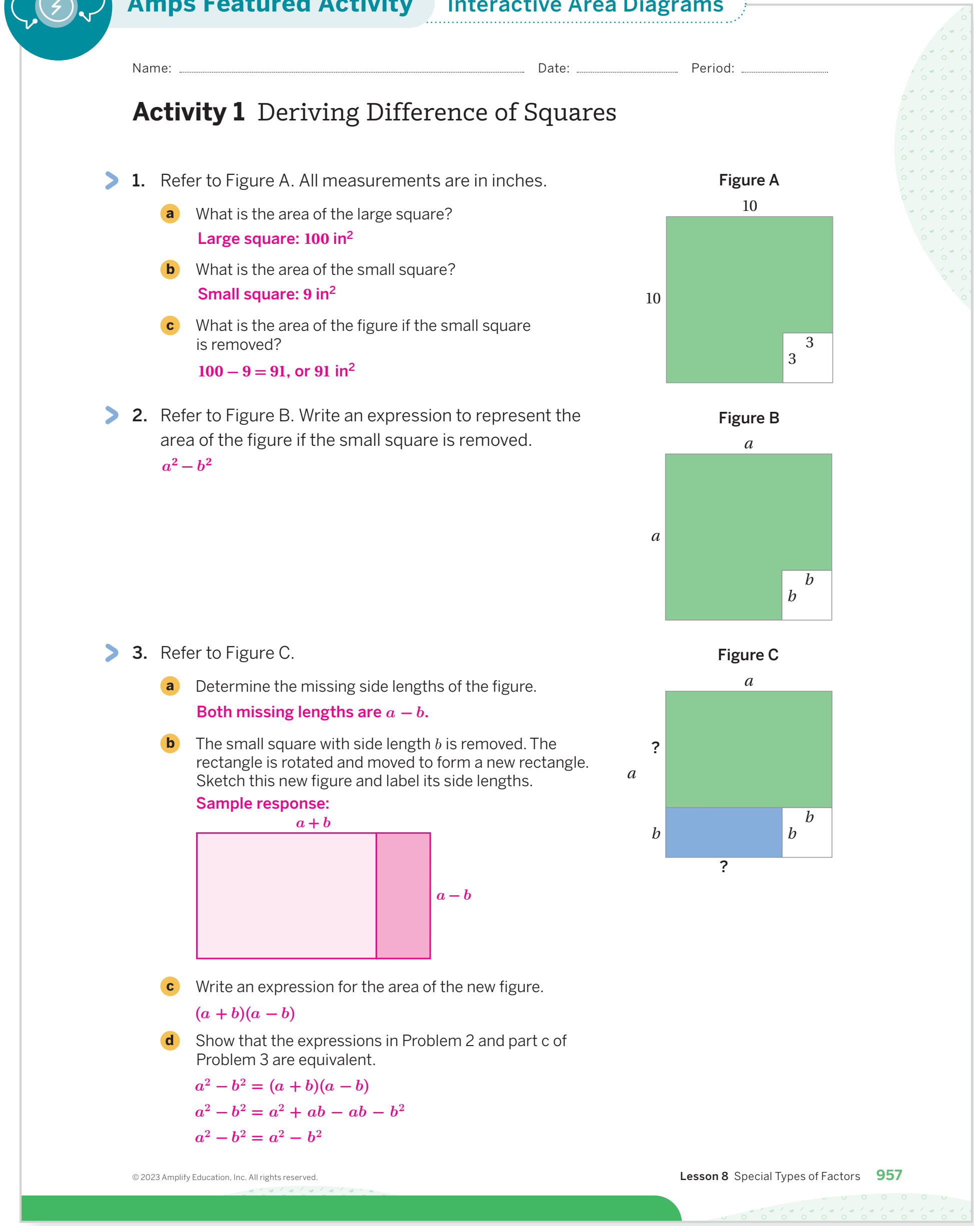
Informed by: Performance on Lesson 7, Practice Problem 6

Pairs | 10 min

Activity 1 Deriving the Difference of Squares

Students will use area diagrams to understand and derive the difference of squares.

Amps Featured Activity Interactive Area Diagrams



Name: ________ Date: ________ Period: ________

Activity 1 Deriving Difference of Squares

1. Refer to Figure A. All measurements are in inches.
   - a What is the area of the large square?
     Large square: 100 in^2
   - b What is the area of the small square?
     Small square: 9 in^2
   - c What is the area of the figure if the small square is removed?
     $100 - 9 = 91$, or 91 in^2
2. Refer to Figure B. Write an expression to represent the area of the figure if the small square is removed.
   $a^2 - b^2$
3. Refer to Figure C.
   - a Determine the missing side lengths of the figure.
     Both missing lengths are $a - b$.
   - b The small square with side length b is removed. The rectangle is rotated and moved to form a new rectangle. Sketch this new figure and label its side lengths.
     Sample response:
   - c Write an expression for the area of the new figure.
     $(a + b)(a - b)$
   - d Show that the expressions in Problem 2 and part c of Problem 3 are equivalent.
     $a^2 - b^2 = (a + b)(a - b)$
     $a^2 - b^2 = a^2 + ab - ab - b^2$
     $a^2 - b^2 = a^2 - b^2$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Special Types of Factors 957

1 Launch

Allow students two minutes of think-time before discussing and completing the activity with a partner.

2 Monitor

Help students get started by asking, "How would you determine the area of the figure?" Determine the product of the length and width.

Look for points of confusion:

- **Forgetting to subtract the area of the smaller square from the larger square in Problems 1c or 2.** Ask, "In Problem 1c, is the area of the figure with the small square removed 100, or less than 100? In Problem 2, is the area of the figure with the small square removed a^2, or less than a^2? What operation can you use to represent this area?"
- **Struggling to sketch the new figure in Problem 3b.** Suggest students trace the original figure onto a separate sheet of paper, cut it out, and physically transform the shapes.

Look for productive strategies:

- Writing the expression for the larger square minus the smaller square.

3 Connect

Display the final rectangle to the class.

Have individual students share what they notice about the rectangle and the expressions that they wrote in Problem 2 and Problem 3c.

Highlight that students have derived the formula for the difference of squares.

Define the term ***difference of squares*** as a squared number subtracted from another squared number, or $a^2 - b^2$, which is equivalent to $(a + b)(a - b)$.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive area diagrams to derive the general formula for the difference of squares.

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by having students complete each of Problems 1, 2, and 3, one at a time, separated by a brief class discussion before moving to the next problem.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share what they notice about the rectangle they sketched in Problem 3b and the expressions they wrote in Problem 2 and Problem 3c, collect and display the language they use that leads to the definition of the term *difference of squares*. For example, they may use language such as "large square minus small square" in Problem 2.

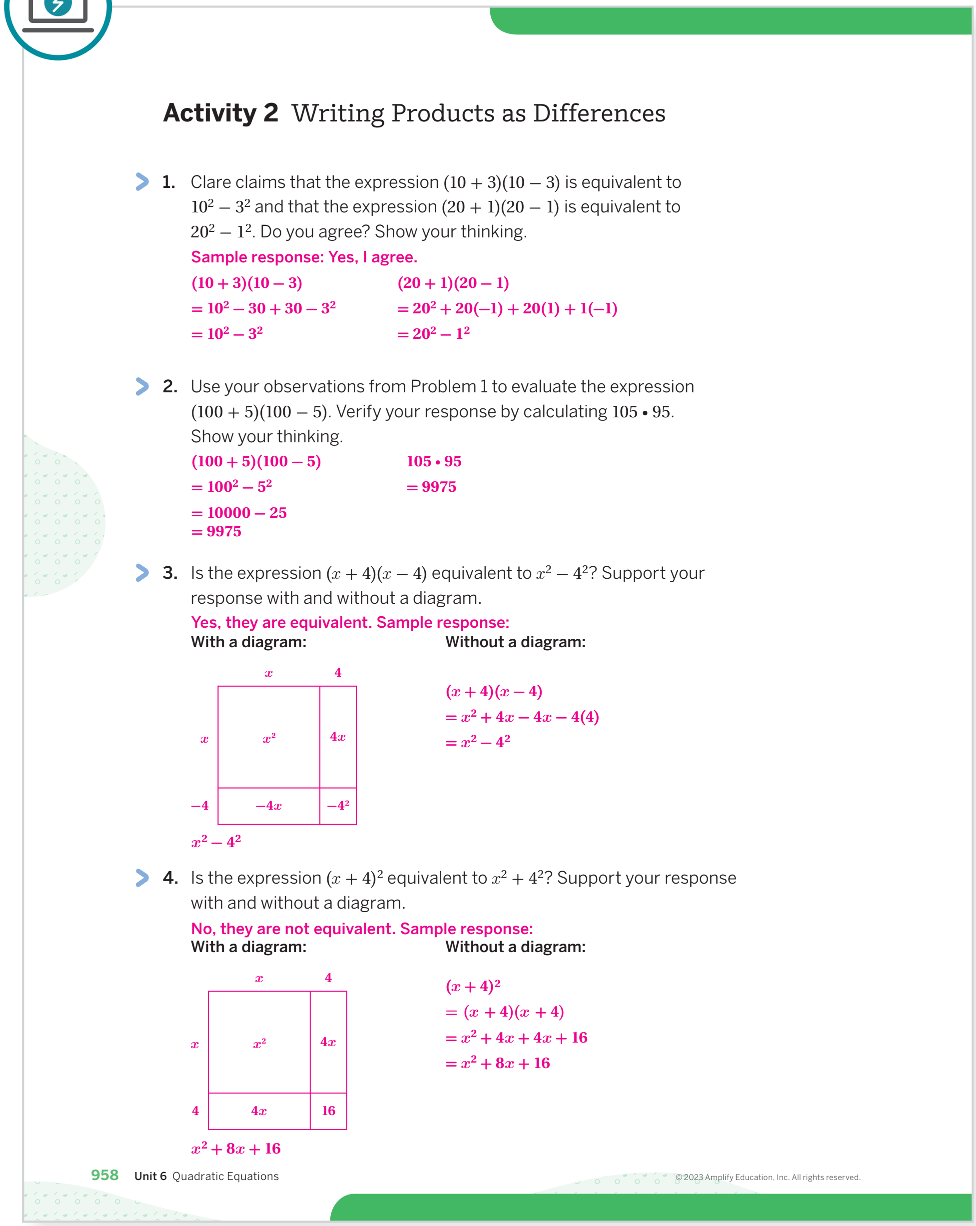
English Learners

Add annotated diagrams and symbols to the class display so students have an opportunity to connect their expressions and definition to visual models.

Independent | 10 min

Activity 2 Writing Products as Differences

Students use diagrams and the Distributive Property to multiply two expressions of the forms $(x + m)$ and $(x - m)$.



Activity 2 Writing Products as Differences

1. Clare claims that the expression $(10 + 3)(10 - 3)$ is equivalent to $10^2 - 3^2$ and that the expression $(20 + 1)(20 - 1)$ is equivalent to $20^2 - 1^2$. Do you agree? Show your thinking.

Sample response: Yes, I agree.

$(10 + 3)(10 - 3)$	$(20 + 1)(20 - 1)$
$= 10^2 - 30 + 30 - 3^2$	$= 20^2 + 20(-1) + 20(1) + 1(-1)$
$= 10^2 - 3^2$	$= 20^2 - 1^2$

2. Use your observations from Problem 1 to evaluate the expression $(100 + 5)(100 - 5)$. Verify your response by calculating $105 \bullet 95$. Show your thinking.

$(100 + 5)(100 - 5)$	$105 \bullet 95$
$= 100^2 - 5^2$	$= 9975$
$= 10000 - 25$	
$= 9975$	

3. Is the expression $(x + 4)(x - 4)$ equivalent to $x^2 - 4^2$? Support your response with and without a diagram.

Yes, they are equivalent. Sample response:

With a diagram:

	x	4
x	x^2	$4x$
-4	$-4x$	-4^2

$x^2 - 4^2$

Without a diagram:

$(x + 4)(x - 4)$
$= x^2 + 4x - 4x - 4(4)$
$= x^2 - 4^2$

4. Is the expression $(x + 4)^2$ equivalent to $x^2 + 4^2$? Support your response with and without a diagram.

No, they are not equivalent. Sample response:

With a diagram:

	x	4
x	x^2	$4x$
4	$4x$	16

$x^2 + 8x + 16$

Without a diagram:

$(x + 4)^2$
$= (x + 4)(x + 4)$
$= x^2 + 4x + 4x + 16$
$= x^2 + 8x + 16$

1 Launch

Provide access to calculators.

2 Monitor

Help students get started by pointing out they need to expand the expression in Problem 1 to determine whether the expressions are equivalent.

Look for points of confusion:

- **Struggling to generalize the pattern after a few examples.** Provide additional factored expressions for students to expand.

Look for productive strategies:

- Expanding the factored expressions.
- Using the difference of squares to factor $x^2 - 4^2$ in Problem 3.
- Realizing that the expression $x^2 + 4^2$ in Problem 4 is not a difference of squares.

3 Connect

Display a blank area diagram to the class.

Ask, "What expressions should be placed in each rectangle of the area diagrams?"

Have individual students share their observations about the area diagrams and the expanded expressions.

Highlight that knowing this structure allows students to write the factored form of any quadratic expression with no linear term and that this form is a difference of a squared variable and a squared constant. Point out that the expression $x^2 + 4^2$ in Problem 4 cannot be factored using the difference of squares because it is not a *difference* of two squares. It is actually a sum of two squares.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

As students approach Problem 3, suggest they use the patterns they discovered in an earlier lesson. Ask:

- "Is $(x + 4)(x - 4)$ in factored form? How can you write it in standard form without using an area diagram?"
- "What is true about the linear coefficient in standard form? What is true about the constant in standard form?"

Accessibility: *Optimize Access to Tools*

Provide access to blank area diagrams that students can use to complete if they choose to do so.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present an incorrect response and explanation, such as "$(x + 4)^2$ is equal to $x^2 + 4^2$ because you distribute the power of 2 to both terms." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking." Listen for students who reason that the exponent cannot be distributed because $(x + 4)^2$ is equal to $(x + 4)(x + 4)$ and there will be a linear term of $8x$.
- ***Correct and Clarify:*** "What strategies can you use to evaluate this expression? Why are these strategies valid?"

Pairs | 10 min

Activity 3 When There Is No Linear Term

Students apply previous understanding about standard and factored form of quadratic expressions and difference of squares to write equivalent expressions.

Name: Date: Period:

Activity 3 When There Is No Linear Term

Complete the table to show an equivalent expression for each form. One row does not have an equivalent form.

Factored form	Standard form
$(x-10)(x+10)$	x^2-100
$(2x+1)(2x-1)$	$4x^2-1$
$(4-x)(4+x)$	$16-x^2$
$(x+9)(x-9)$	x^2-81
$(7+y)(7-y)$	$49-y^2$
$(3z+4)(3z-4)$	$9z^2-16$
$(5t+9)(5t-9)$	$25t^2-81$
$\left(c+\frac{2}{5}\right)\left(c-\frac{2}{5}\right)$	$c^2-\frac{4}{25}$
$\left(\frac{7}{4}+d\right)\left(\frac{7}{4}-d\right)$	$\frac{49}{16}-d^2$
$(x+5)(x+5)$	$x^2+10x+25$
$(x+\sqrt{6})(x-\sqrt{6})$	x^2-6
not possible	x^2+100

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Special Types of Factors 959

1 Launch

Ask students to complete as many rows of the table as time permits.

2 Monitor

Help students get started by asking them to study the structure of the expressions and describe what they notice.

Look for points of confusion:

- **Struggling to see the perfect squares.** Prompt them to create a list or table of square numbers to have as a reference. Others may benefit by rewriting both terms as squares before writing the factored form.

Look for productive strategies:

- Rewriting the terms in standard form as squared terms.
- Recognizing that x^2+100 is not a difference of squares.

3 Connect

Display the incomplete table to the class.

Have individual students share the expressions they wrote and any disagreements or questions that they have about them.

Highlight that students can check their work by expanding the factored expression using the Distributive Property.

Ask:

- "What if the constant is not a perfect square, as in the expression x^2-6?" Sample response: I can still write the expression in factored form by taking the square root, so $(x+\sqrt{6})(x-\sqrt{6})$.
- "Why is it not possible to factor the expression x^2+100?" Sample response: It is not a difference of squares. Instead, it is a sum of two squares.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the variable terms in one color and the constants in another color. Have them include the signs of any terms being added or subtracted. For example, they could color code the first row of the table as:

$(x-10)(x+10)$ and x^2-100.

Math Language Development

MLR8: Discussion Supports

During the Connect, display or provide students the Anchor chart PDF, *Sentence Stems, Explaining My Steps* to support students as they explain the strategies they used to complete the table.

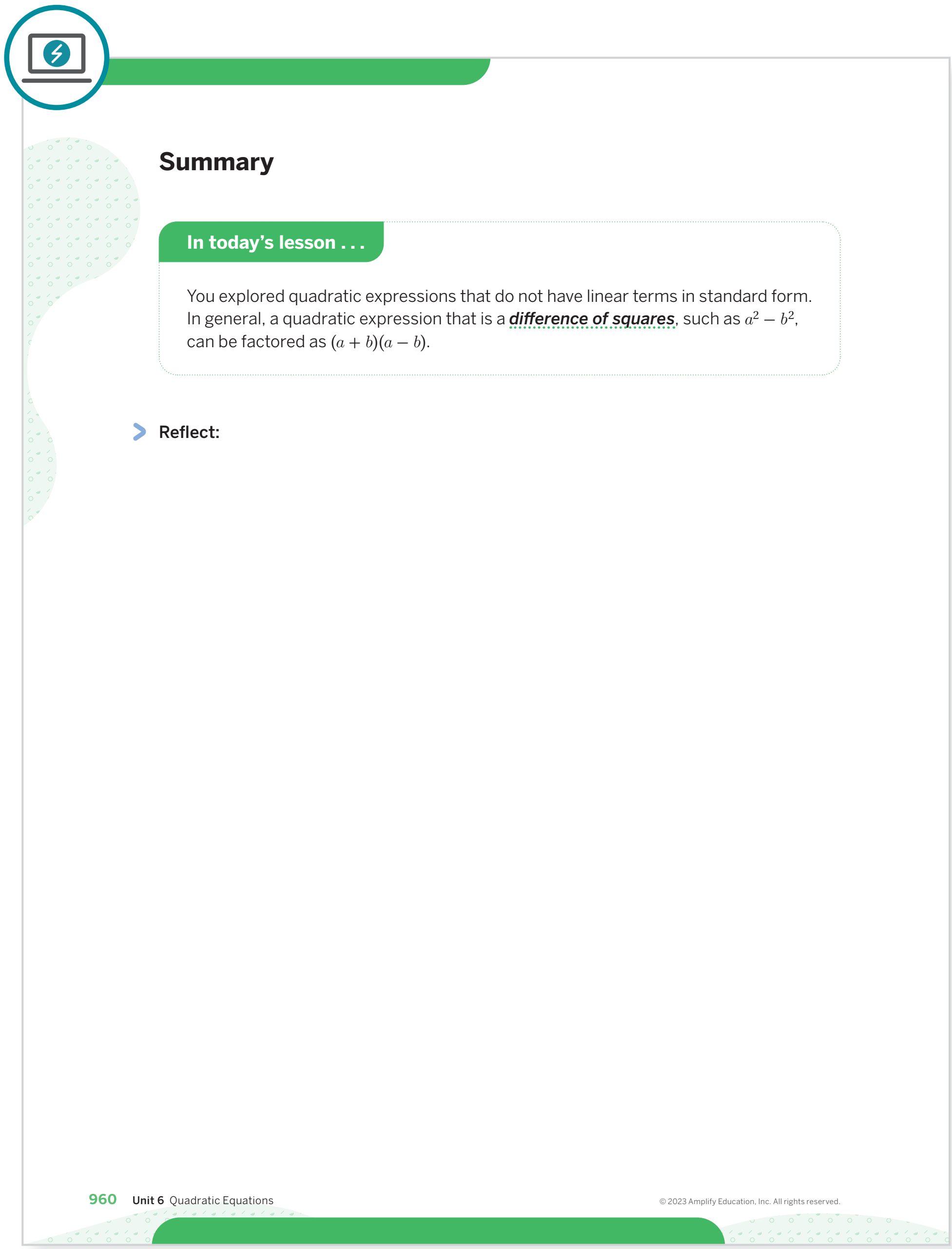
English Learners

Allow students to rehearse what they will say before sharing with the whole class.

Whole Class | 5 min

Summary

Review and synthesize how the difference of squares can help to factor some quadratic expressions without using area diagrams.



Summary

In today's lesson . . .

You explored quadratic expressions that do not have linear terms in standard form. In general, a quadratic expression that is a ***difference of squares***, such as $a^2 - b^2$, can be factored as $(a + b)(a - b)$.

Reflect:

960 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the expressions $(x + 4)(x - 4)$ and $(x + 8)(x - 2)$ and point out that each expression has two factors, one of which is a sum and the other a difference.

Ask, "When expanded into standard form, why does one expression have a linear term, but not the other?" Sample response: Expanding the expression $(x + 4)(x - 4)$ yields two linear terms that are opposites, $4x$ and $-4x$, which when added together have a sum of 0, so the linear term disappears. Expanding the expression $(x + 8)(x - 2)$ also yields two linear terms, $8x$ and $-2x$. Because they are not opposites, their sum is $6x$, which is not 0, so there will be a linear term.

Highlight that the difference of squares can be written in factored and standard form and when in standard form, there will be no linear term because the two linear terms are opposites and have a sum of 0.

Formalize vocabulary: *difference of squares*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did seeing area diagrams help you visualize the difference of squares?"
- "What patterns did you notice when comparing factored form to standard form?"

Math Language Development

MLR2: Collect and Display

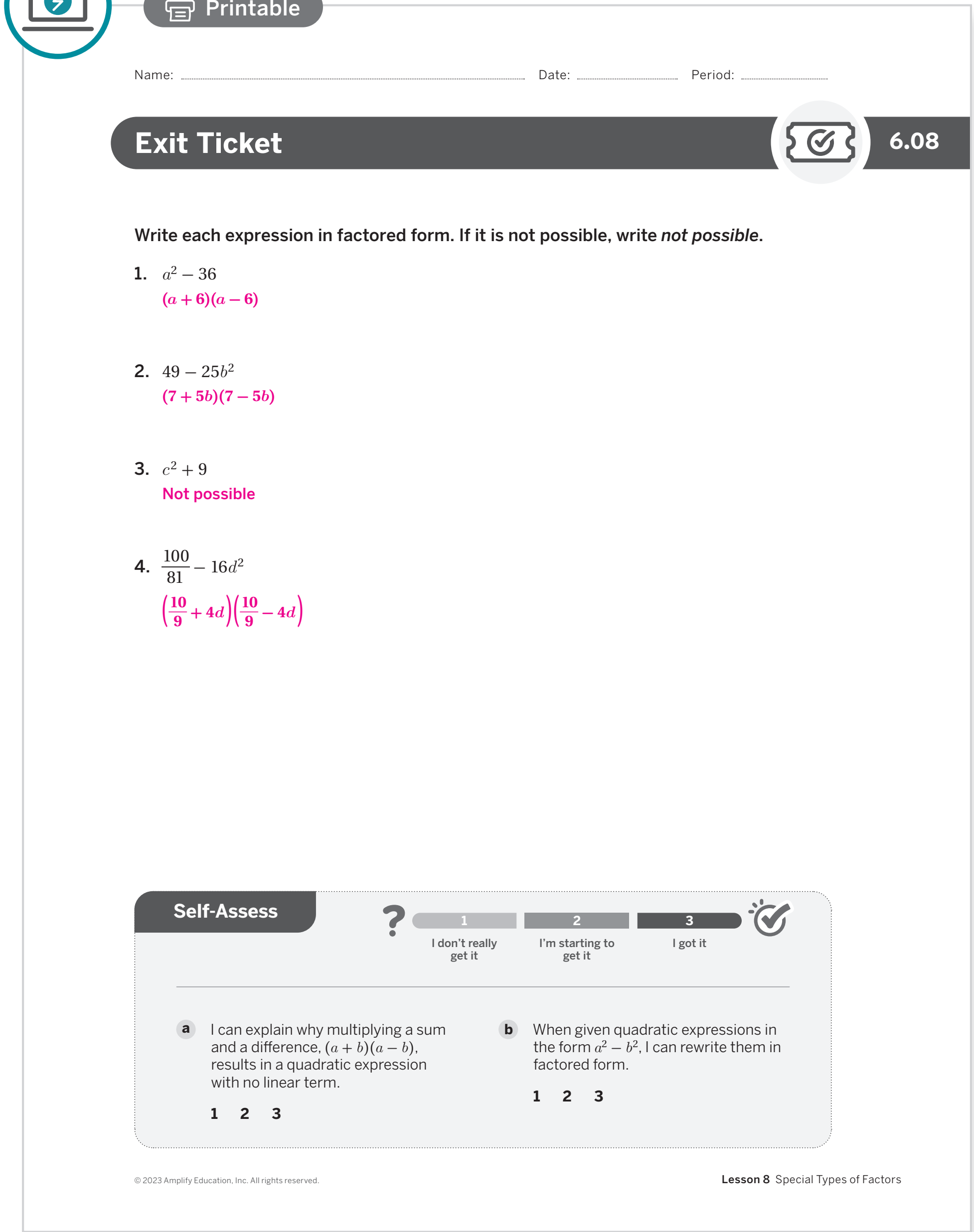
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *difference of squares* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using the difference of squares to rewrite expressions in factored form, given standard form.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.08

Write each expression in factored form. If it is not possible, write *not possible*.

1. $a^2 - 36$
   $(a + 6)(a - 6)$

2. $49 - 25b^2$
   $(7 + 5b)(7 - 5b)$

3. $c^2 + 9$
   Not possible

4. $\frac{100}{81} - 16d^2$
   $\left(\frac{10}{9} + 4d\right)\left(\frac{10}{9} - 4d\right)$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain why multiplying a sum and a difference, $(a + b)(a - b)$, results in a quadratic expression with no linear term.

1 2 3

b When given quadratic expressions in the form $a^2 - b^2$, I can rewrite them in factored form.

1 2 3

Lesson 8 Special Types of Factors

Success looks like . . .

- **Language Goal:** Understanding that multiplying a sum and a difference, $(x + m)(x - m)$, results in a quadratic expression with no linear term and explaining why. **(Speaking and Listening)**
- **Goal:** When given factorable quadratic expressions with no linear term, writing equivalent expressions in factored form.
  - » Factoring quadratic expressions with no linear term in Problems 1–4.

Suggested next steps

If students do not correctly write the expression in factored form in Problems 1 and 2, consider:

- Making a table of squares.
- Assigning Practice Problem 1.

If students do not correctly identify the expression $c^2 + 9$ in Problem 3 as not possible, consider:

- Reviewing Activity 2, Problem 4.
- Reviewing the last row in the table from Activity 3.
- Having them expand the expression they wrote in this problem to see that it is not equivalent to $c^2 + 9$.

If students do not correctly factor the expression in Problem 4, consider:

- Asking them to evaluate the square root of $\frac{100}{81}$ and $16d^2$.
- Reviewing the expressions containing fractions from Activity 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did the use of area diagrams support students in understanding the difference of squares?
- Which students' ideas were you able to highlight during Activity 3? What might you change for the next time you teach this lesson?

Math Language Development

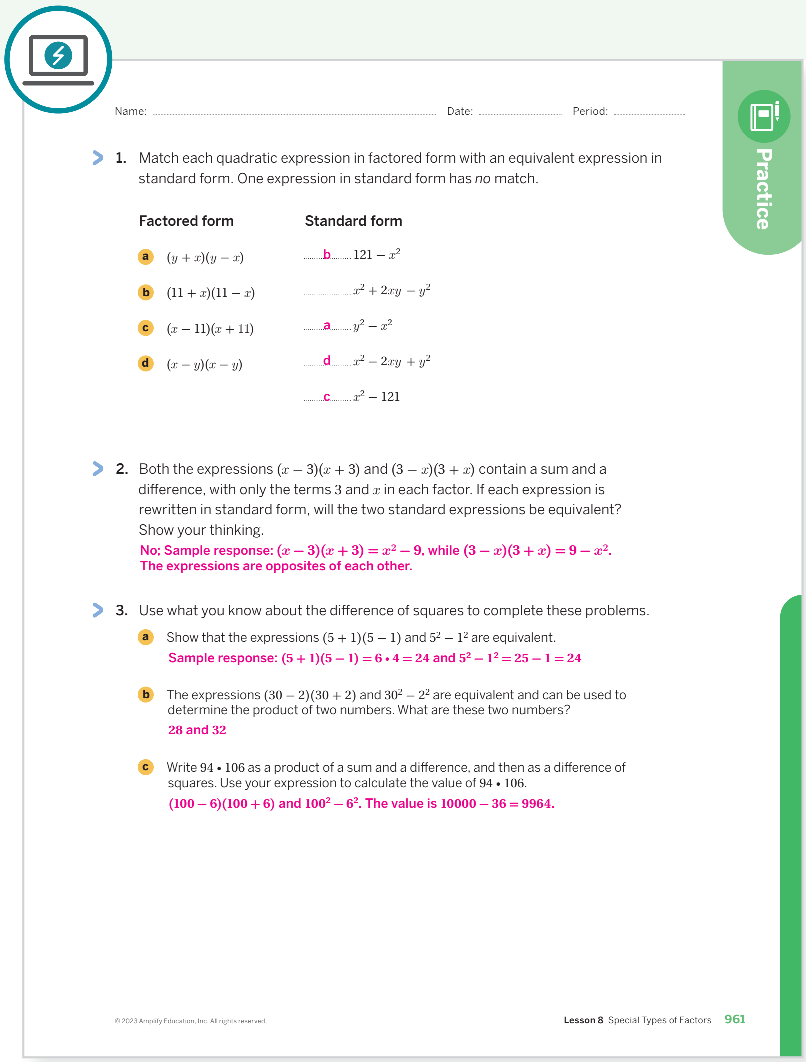
Language Goal: Understanding that multiplying a sum and a difference, $(x + m)(x - m)$, results in a quadratic expression with no linear term and explaining why.

Reflect on students' language development toward this goal.

- How did using the Critique, Correct, Clarify routine in Activity 2 help students use math language to explain why there is a linear term when multiplying $(x + 4)$ by $(x + 4)$?
- How are students describing the difference between the expressions $(x + 4)(x - 4)$ and $(x + 4)(x + 4)$? How can you help them be more precise in their explanations?

Practice

Independent


Name: ________ Date: ________ Period: ________

Practice

1. Match each quadratic expression in factored form with an equivalent expression in standard form. One expression in standard form has *no* match.

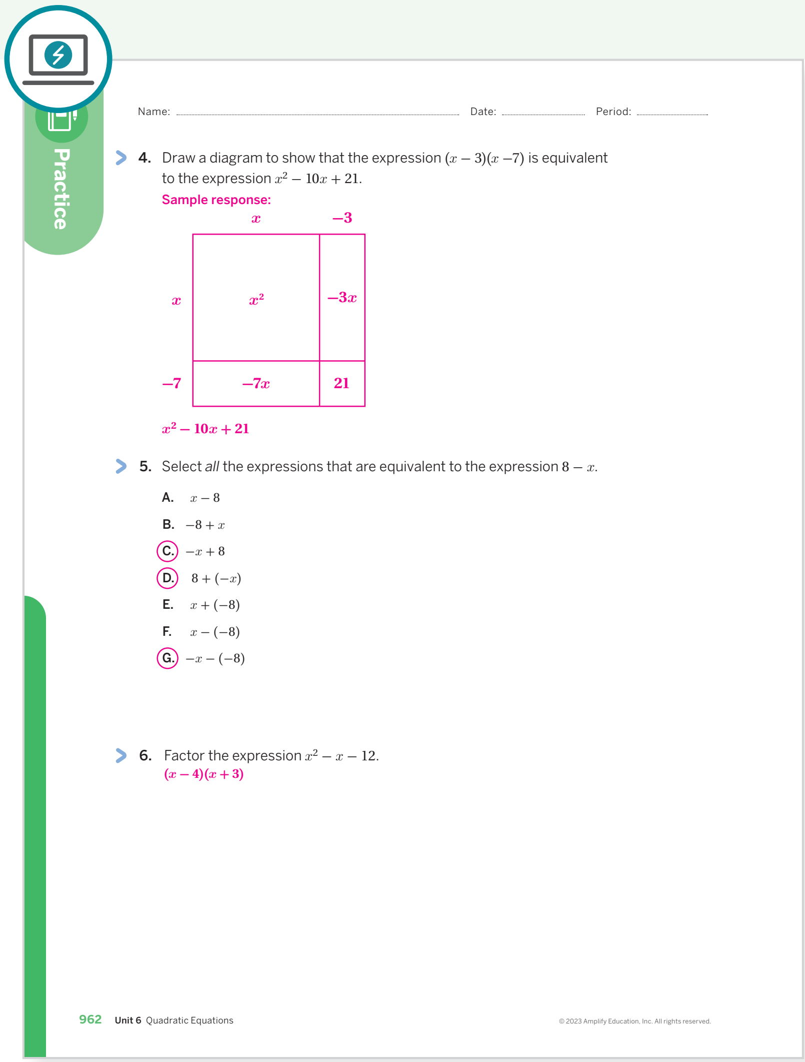
Factored form	Standard form
a $(y+x)(y-x)$	...b... $121-x^2$
b $(11+x)(11-x)$	...... $x^2+2xy-y^2$
c $(x-11)(x+11)$	...a... y^2-x^2
d $(x-y)(x-y)$	...d... $x^2-2xy+y^2$
	...c... x^2-121

2. Both the expressions $(x-3)(x+3)$ and $(3-x)(3+x)$ contain a sum and a difference, with only the terms 3 and x in each factor. If each expression is rewritten in standard form, will the two standard expressions be equivalent? Show your thinking.
No; Sample response: $(x-3)(x+3)=x^2-9$, while $(3-x)(3+x)=9-x^2$. The expressions are opposites of each other.

3. Use what you know about the difference of squares to complete these problems.
   - a Show that the expressions $(5+1)(5-1)$ and 5^2-1^2 are equivalent.
     Sample response: $(5+1)(5-1)=6 \bullet 4=24$ and $5^2-1^2=25-1=24$
   - b The expressions $(30-2)(30+2)$ and 30^2-2^2 are equivalent and can be used to determine the product of two numbers. What are these two numbers?
     28 and 32
   - c Write $94 \bullet 106$ as a product of a sum and a difference, and then as a difference of squares. Use your expression to calculate the value of $94 \bullet 106$.
     $(100-6)(100+6)$ and 100^2-6^2. The value is $10000-36=9964$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Special Types of Factors 961


Name: ________ Date: ________ Period: ________

Practice

4. Draw a diagram to show that the expression $(x-3)(x-7)$ is equivalent to the expression $x^2-10x+21$.
Sample response:

	x	-3
x	x^2	$-3x$
-7	$-7x$	21

$x^2-10x+21$

5. Select *all* the expressions that are equivalent to the expression $8-x$.
   - A. $x-8$
   - B. $-8+x$
   - [✓ C.] $-x+8$
   - [✓ D.] $8+(-x)$
   - E. $x+(-8)$
   - F. $x-(-8)$
   - [✓ G.] $-x-(-8)$

6. Factor the expression x^2-x-12.
$(x-4)(x+3)$

962 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 6	2
	5	Unit 6 Lesson 6	2
Formative ⚡	6	Unit 6 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 9**

Solving Quadratic Equations by Factoring

Let's solve some quadratic equations without graphing.

Focus

Goals

1. Recognize that the number of solutions to a factorable quadratic equation can be revealed when the equation is written in factored form and set equal to 0.
2. Use factored form and the Zero Product Principle to solve quadratic equations.

Coherence

Today

Students transform factorable quadratic equations that are given in standard form into factored form to make sense of quadratic equations and persevere in solving them. They rearrange quadratic equations so that one side — an expression in factored form — is set equal to 0, and use the Zero Product Principle to solve equations that previously could only be solved by graphing.

Previously

Students encountered factorable quadratic expressions without a linear term and considered how to write them in factored form. They derived the difference of squares as $a^2 - b^2 = (a + b)(a - b)$.

Coming Soon

Students will rewrite non-monic quadratic expressions, i.e., when $a \neq 1$ in $ax^2 + bx + c$ — that are not differences of squares — in standard form.

Rigor

- Students build **conceptual understanding** of how factoring and using the Zero Product Principle can be used to solve quadratic equations.
- Students build **procedural skills** by rewriting expressions in factored form.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	20 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor chart PDF, *Solving Monic Quadratic Equations by Factoring*
- Anchor chart PDF, *Sentence Stems, Explaining My Steps*
- graphing technology

Math Language Development

New words

- ***monic quadratic equation* (or expression)**

Review words

- *coefficient*
- *constant term*
- *linear term*
- *Zero Product Principle*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students explore the zeros of quadratic functions using graphing technology and see the relationship between the zeros of the function and the solutions to its related equation.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overwhelmed making sense of and applying the strategies in Activity 2. Ask students how they are feeling, listening deeply and reflecting on what you heard. For example, "It sounds like you are feeling very frustrated right now . . ." Then have students describe other challenging lessons or concepts in which they have persevered and succeeded.

Modifications to Pacing

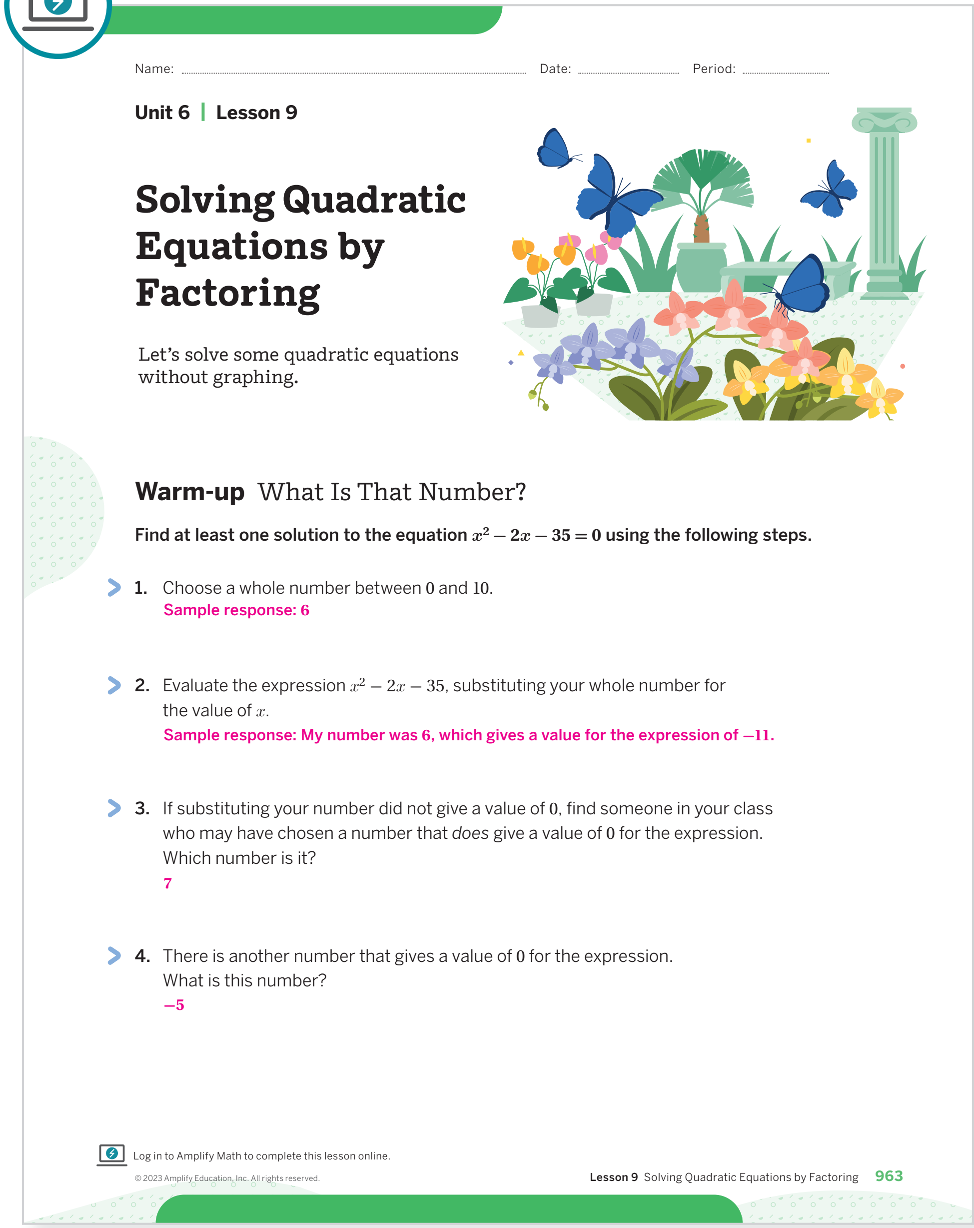
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2, have students complete only the first two rows in the table.
- **Activity 3** may be omitted. It serves as an application of solving quadratic equations in factored form.

Independent | 10 min

Warm-up What Is That Number?

Students determine a solution to a quadratic equation using an inefficient strategy, which will challenge them to seek out more efficient strategies.



Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 9

Solving Quadratic Equations by Factoring

Let's solve some quadratic equations without graphing.

Warm-up What Is That Number?

Find at least one solution to the equation $x^2 - 2x - 35 = 0$ using the following steps.

1. Choose a whole number between 0 and 10.
   Sample response: 6
2. Evaluate the expression $x^2 - 2x - 35$, substituting your whole number for the value of x.
   Sample response: My number was 6, which gives a value for the expression of -11.
3. If substituting your number did not give a value of 0, find someone in your class who may have chosen a number that *does* give a value of 0 for the expression. Which number is it?
   7
4. There is another number that gives a value of 0 for the expression. What is this number?
   -5

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Solving Quadratic Equations by Factoring 963

1 Launch

Have students complete Problems 1 and 2 independently. Provide time for them to circulate around the class to complete Problem 3.

2 Monitor

Help students get started by asking them to list possible input values they can choose in Problem 1.

Look for points of confusion:

- **Not considering negative values in Problem 4.** Encourage students to substitute any value of x, including negative values.

Look for productive strategies:

- Factoring the expression to determine the zeros of the related equation $y = x^2 - 2x - 35$.
- Listing the factors of -35 that have a sum of -2.
- Organizing their work in a table or list.

3 Connect

Display an organized way for keeping track of the input values and the value of the expression, such as a table or organized list.

Have individual students share their values chosen for x. Select students by those who did not find a solution, found solutions by talking to their classmates, and found solutions using the *guess-and-check* strategy and/or factoring.

Highlight that factoring is more efficient for solving quadratic equations than using the *guess-and-check* strategy. Show the factored form of the expression, $(x - 7)(x + 5)$, if no students mentioned using it.

Power-up

To power up students' ability to factor quadratic expressions, have students complete:

Complete each problem to factor the expression $x^2 - 6x + 8$.

a. What are the positive and negative factor pairs of 8?
   -8 and -1, 8 and 1, -4 and -2, and 4 and 4.

b. Which of those factor pairs have a sum of -6 and a product of 8?
   -4 and -2

c. Rewrite $x^2 - 6x + 8$ in factored form using the values from part c.
   $(x - 4)(x - 2)$

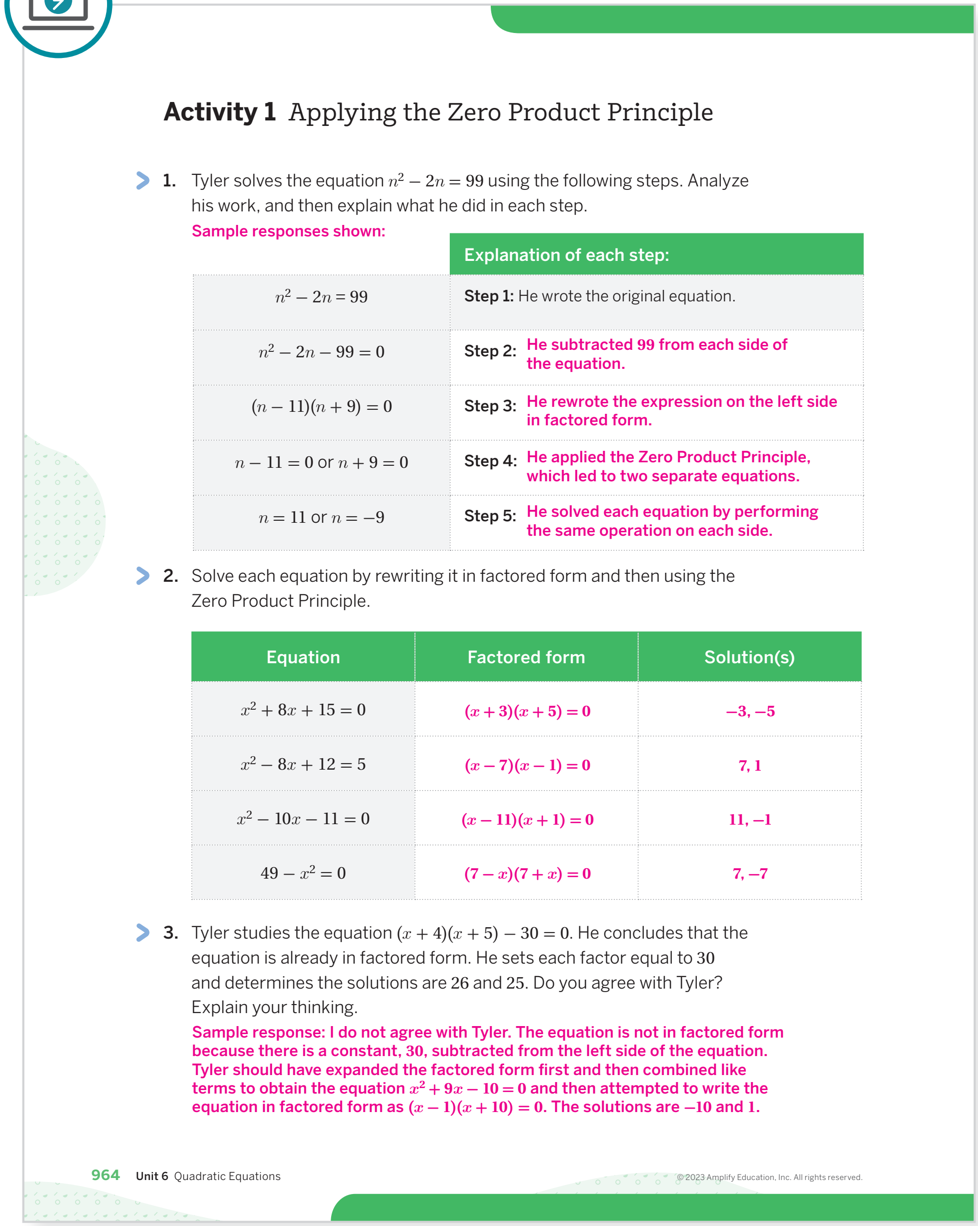
Use: Before Activity 1

Informed by: Performance on Lesson 8, Practice Problem 6

Pairs | 20 min

Activity 1 Applying the Zero Product Principle

Students solve quadratic equations by integrating how to rewrite quadratic expressions in factored form using their understanding of the Zero Product Principle.



Activity 1 Applying the Zero Product Principle

1. Tyler solves the equation $n^2 - 2n = 99$ using the following steps. Analyze his work, and then explain what he did in each step.

Sample responses shown:

	Explanation of each step:
$n^2 - 2n = 99$	**Step 1:** He wrote the original equation.
$n^2 - 2n - 99 = 0$	**Step 2:** He subtracted 99 from each side of the equation.
$(n - 11)(n + 9) = 0$	**Step 3:** He rewrote the expression on the left side in factored form.
$n - 11 = 0$ or $n + 9 = 0$	**Step 4:** He applied the Zero Product Principle, which led to two separate equations.
$n = 11$ or $n = -9$	**Step 5:** He solved each equation by performing the same operation on each side.

2. Solve each equation by rewriting it in factored form and then using the Zero Product Principle.

Equation	Factored form	Solution(s)
$x^2 + 8x + 15 = 0$	$(x + 3)(x + 5) = 0$	$-3, -5$
$x^2 - 8x + 12 = 5$	$(x - 7)(x - 1) = 0$	$7, 1$
$x^2 - 10x - 11 = 0$	$(x - 11)(x + 1) = 0$	$11, -1$
$49 - x^2 = 0$	$(7 - x)(7 + x) = 0$	$7, -7$

3. Tyler studies the equation $(x + 4)(x + 5) - 30 = 0$. He concludes that the equation is already in factored form. He sets each factor equal to 30 and determines the solutions are 26 and 25. Do you agree with Tyler? Explain your thinking.

Sample response: I do not agree with Tyler. The equation is not in factored form because there is a constant, 30, subtracted from the left side of the equation. Tyler should have expanded the factored form first and then combined like terms to obtain the equation $x^2 + 9x - 10 = 0$ and then attempted to write the equation in factored form as $(x - 1)(x + 10) = 0$. The solutions are -10 and 1.

964 Unit 6 Quadratic Equations

1 Launch

Conduct the *Notice and Wonder* routine with Problem 1. Give students one minute of think-time before they share. Record their responses.

2 Monitor

Help students get started by having them complete Problem 1 with their partner.

Look for points of confusion:

- **Misunderstanding why Tyler subtracts 99 from each side in Problem 1.** Say, "The Zero Product Principle requires an equation set equal to 0."
- **Struggling to determine the factor pairs for the constant terms.** Have students list the factor pairs for each constant.

Look for productive strategies:

- Listing all factoring possibilities and determining the correct factored form.
- Applying the Zero Product Principle when determining the solutions.

3 Connect

Have pairs of students share their strategies for solving each equation in Problem 2. Select and sequence students who attempted Problem 3.

Ask, "In Problem 3, can you use the Zero Product Principle to write the equations $x + 4 = 0$ and $x + 5 = 0$? Explain." No; The Zero Product Principle can only be applied to products that are set equal to 0.

Highlight that the factored form and the Zero Product Principle are needed to solve some quadratic equations.

Define the term ***monic quadratic equation***.

Display the Anchor Chart PDF, *Solving Monic Quadratic Equations by Factoring*.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For Problem 1, provide the explanations for each step on index cards and have students sort the cards in order. In Problem 2, have students choose to complete three of the four rows, where one of their chosen rows is the last row.

Extension: *Math Enrichment*

Have students study the equation in the second row in the table and explain how they can solve it, even though it is not set equal to 0. Then have them determine whether they can use a similar method to solve the equation $x^2 - 8x + 12 = 4$ and explain their thinking. By setting the equation $x^2 - 8x + 12 = 5$ equal to 0, the left side becomes $x^2 - 8x + 7$ which can be factored as $(x - 7)(x - 1)$. But in setting the equation $x^2 - 8x + 12 = 4$ equal to 0, it becomes $x^2 - 8x + 8 = 0$, which cannot be factored.

Math Language Development

MLR8: Discussion Supports

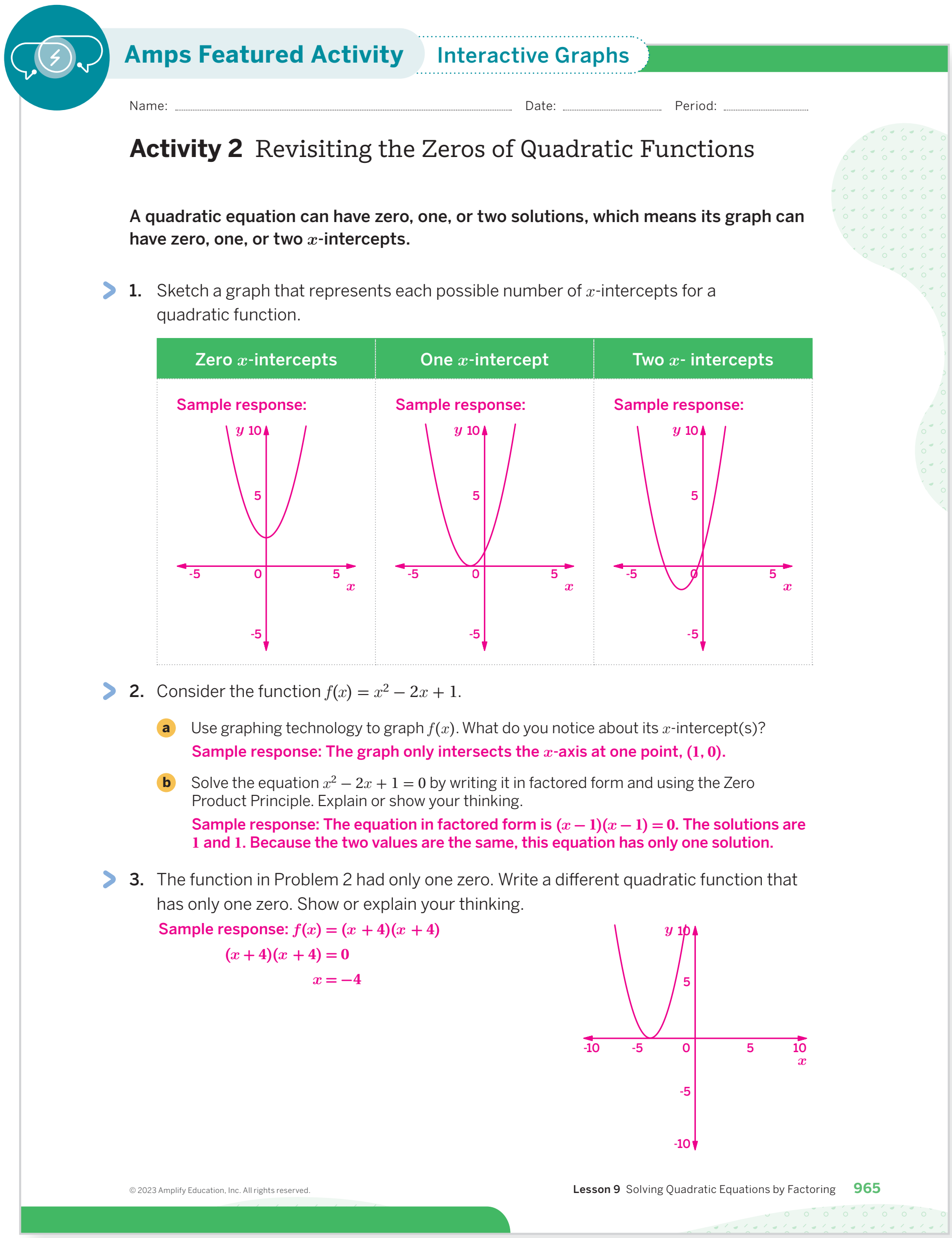
During the Connect, as you define the term *monic quadratic equation*, tell students that all of the quadratic equations in this activity are monic because the coefficient on the x^2 term is 1. Ask students to think of other terms that begin with the prefix "mono-" or "mon-," that also mean 1, to help them make sense of this term. For example: *monarchy, monochrome, monocle, monologue, monopoly, monorail,* and *monosyllable*.

Pairs | 15 min

Activity 2 Revisiting the Zeros of Quadratic Functions

Students relate the number of zeros of a quadratic function to the number of solutions to its related equation, noting that when the two factors are the same, there is 1 solution.

Amps Featured Activity Interactive Graphs



Name: ______ Date: ______ Period: ______

Activity 2 Revisiting the Zeros of Quadratic Functions

A quadratic equation can have zero, one, or two solutions, which means its graph can have zero, one, or two x-intercepts.

1. Sketch a graph that represents each possible number of x-intercepts for a quadratic function.

Zero x-intercepts	One x-intercept	Two x- intercepts
Sample response:	Sample response:	Sample response:

2. Consider the function $f(x) = x^2 - 2x + 1$.

   a. Use graphing technology to graph $f(x)$. What do you notice about its x-intercept(s)?

   Sample response: The graph only intersects the x-axis at one point, $(1, 0)$.

   b. Solve the equation $x^2 - 2x + 1 = 0$ by writing it in factored form and using the Zero Product Principle. Explain or show your thinking.

   Sample response: The equation in factored form is $(x - 1)(x - 1) = 0$. The solutions are 1 and 1. Because the two values are the same, this equation has only one solution.

3. The function in Problem 2 had only one zero. Write a different quadratic function that has only one zero. Show or explain your thinking.

   Sample response: $f(x) = (x + 4)(x + 4)$

   $(x + 4)(x + 4) = 0$

   $x = -4$

1 Launch

Provide access to graphing technology. Activate prior knowledge by asking, "What information can be determined from the graph of a quadratic function?" Have students ***Turn and Talk*** to discuss the question.

2 Monitor

Help students get started by asking, "What does the graph of a parabola look like if it has no x-intercepts?"

Look for points of confusion:

- **Confusing the y-intercept for a solution to a quadratic function.** Remind students that zeros mean where the graph crosses the x-axis, not the y-axis.
- **Not relating the number of solutions to the equations with the number of zeros of the related functions.** Allow this for now, and make note to revisit during the Connect.

Look for productive strategies:

- Making use of the structure of the factored form and applying the Zero Product Principle in Problem 3.
- Noticing the number of solutions to the equations corresponds to the number of zeros of the related functions.

3 Connect

Display the graph of $f(x) = x^2 - 2x + 1$.

Have pairs of students share their strategies for solving the related equation in Problem 2b.

Ask, "How does the number of solutions to each equation relate to the number of zeros in its corresponding function?" There is 1 solution, because the factors are the same. There is 1 zero.

Highlight that the factored form $(x + m)^2$ or $(x - m)^2$ will have one solution, which implies that the graph of the related function will have only one zero or x-intercept.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can explore the zeros of quadratic functions using graphing technology and see the relationship between the zeros of the function and the solutions to its related equation.

Accessibility: *Vary Demands to Optimize Challenge*

Provide pre-created sketches of graphs for Problem 1 and have students sort them as to the number of x-intercepts they have. This will allow students to spend more time making connections.

Extension: *Math Enrichment*

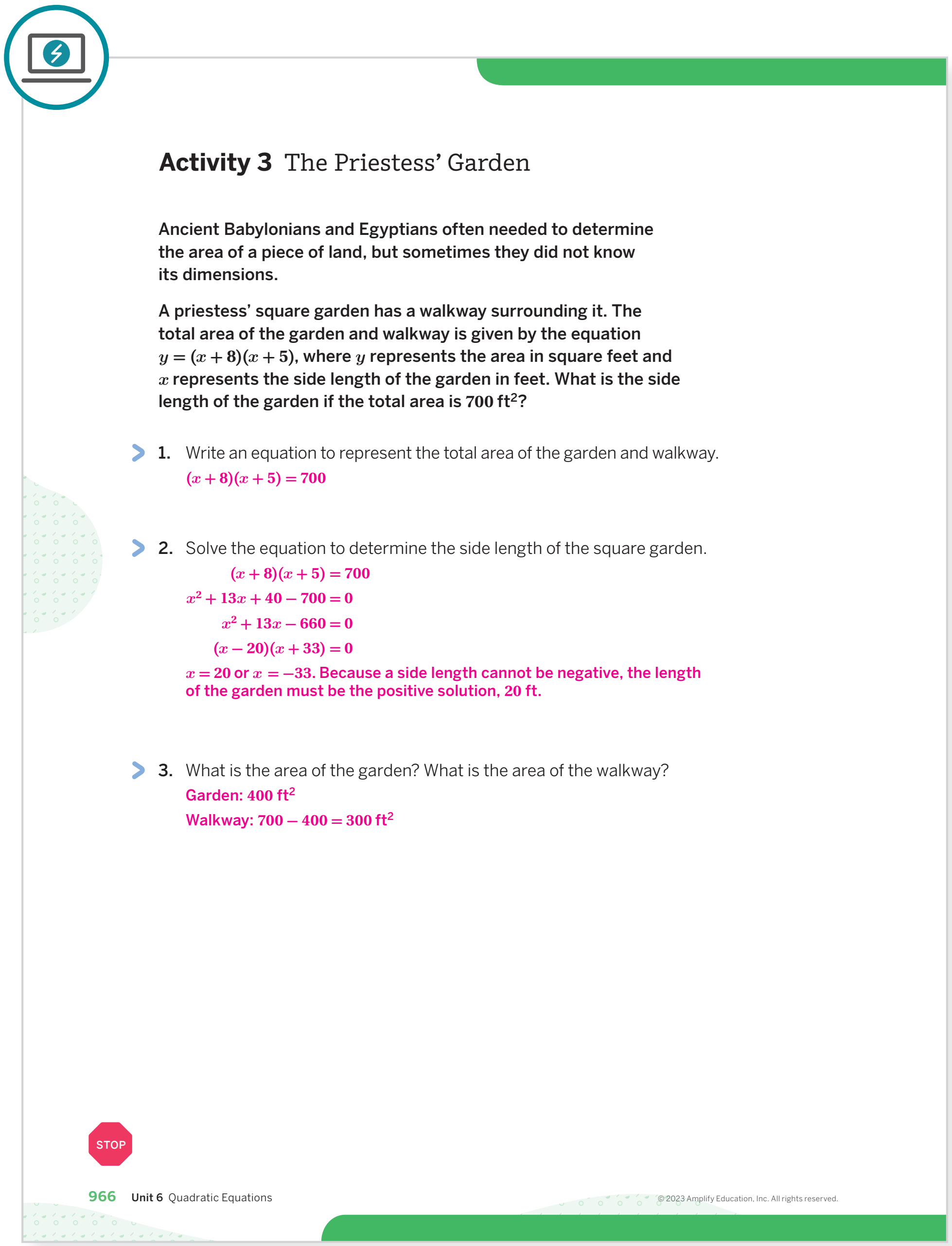
Ask students to determine the number of solutions to any quadratic equation that can be represented as a difference of squares set equal to 0, and explain their thinking. There will always be 2 solutions; Sample response: For any quadratic equation of the form $a^2 - b^2 = 0$, $a^2 - b^2 = (a + b)(a - b)$. This means that $(a + b)(a - b) = 0$. Either $a + b = 0$ or $a - b = 0$, which means that either $a = -b$ or $a = b$.

Optional

Pairs | 15 min

Activity 3 The Priestess' Garden

Students apply factoring and the Zero Product Principle to determine the length of a garden in a historical context.



Activity 3 The Priestess' Garden

Ancient Babylonians and Egyptians often needed to determine the area of a piece of land, but sometimes they did not know its dimensions.

A priestess' square garden has a walkway surrounding it. The total area of the garden and walkway is given by the equation $y = (x + 8)(x + 5)$, where y represents the area in square feet and x represents the side length of the garden in feet. What is the side length of the garden if the total area is 700 ft^2?

1. Write an equation to represent the total area of the garden and walkway.
   $(x + 8)(x + 5) = 700$

2. Solve the equation to determine the side length of the square garden.
   $$(x + 8)(x + 5) = 700$$
   $$x^2 + 13x + 40 - 700 = 0$$
   $$x^2 + 13x - 660 = 0$$
   $$(x - 20)(x + 33) = 0$$
   $x = 20$ or $x = -33$. Because a side length cannot be negative, the length of the garden must be the positive solution, 20 ft.

3. What is the area of the garden? What is the area of the walkway?
   Garden: 400 ft^2
   Walkway: $700 - 400 = 300$ ft^2

STOP

966 **Unit 6** Quadratic Equations

1 Launch

Use the *Three Reads* routine to discuss the passage.

2 Monitor

Help students get started by asking them to sketch a drawing of the square garden and walkway and label what they know.

Look for points of confusion:

- **Writing the equation $(x + 8)(x + 5) = 0$.** Remind students that they need to set up an equation that represents the area of the garden and walkway, and that the total area is 700 ft^2, not 0 ft^2.
- **Thinking the negative solution of the equation makes sense in this context.** Remind students that the garden cannot have a negative length, so the negative solution does not make sense in this context.

Look for productive strategies:

- Expanding the expression, and then subtracting 700 in order to set the equation equal to 0.
- Factoring the expression to determine the garden's side length.

3 Connect

Display the equation $(x - 20)(x + 33) = 0$.

Ask, "How do you know which factor to use to determine the side length?" I should use the expression $(x - 20)$ because the equation $x - 20 = 0$ has a positive solution, $x = 20$. The equation $x + 33 = 0$ has a negative solution, which does not make sense in this context.

Highlight that sometimes only one solution to a quadratic equation makes sense, given a context.

Differentiated Support

Extension: ***Math Enrichment***

Have students determine the width of the walkway along each side of the garden. 4 ft along one side and 2.5 ft along the other side.

Then ask them to write an equation representing the area of the garden and walkway if the priestess wanted to increase the width of the garden by 2 ft along each side of the garden. $y = (x + 12)(x + 9)$

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that there is a square garden surrounded by a walkway.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as "the area of the garden is 700 ft^2."
- **Read 3:** Ask students to plan their solution strategy as to how they will write and solve an equation to determine the side length of the garden.

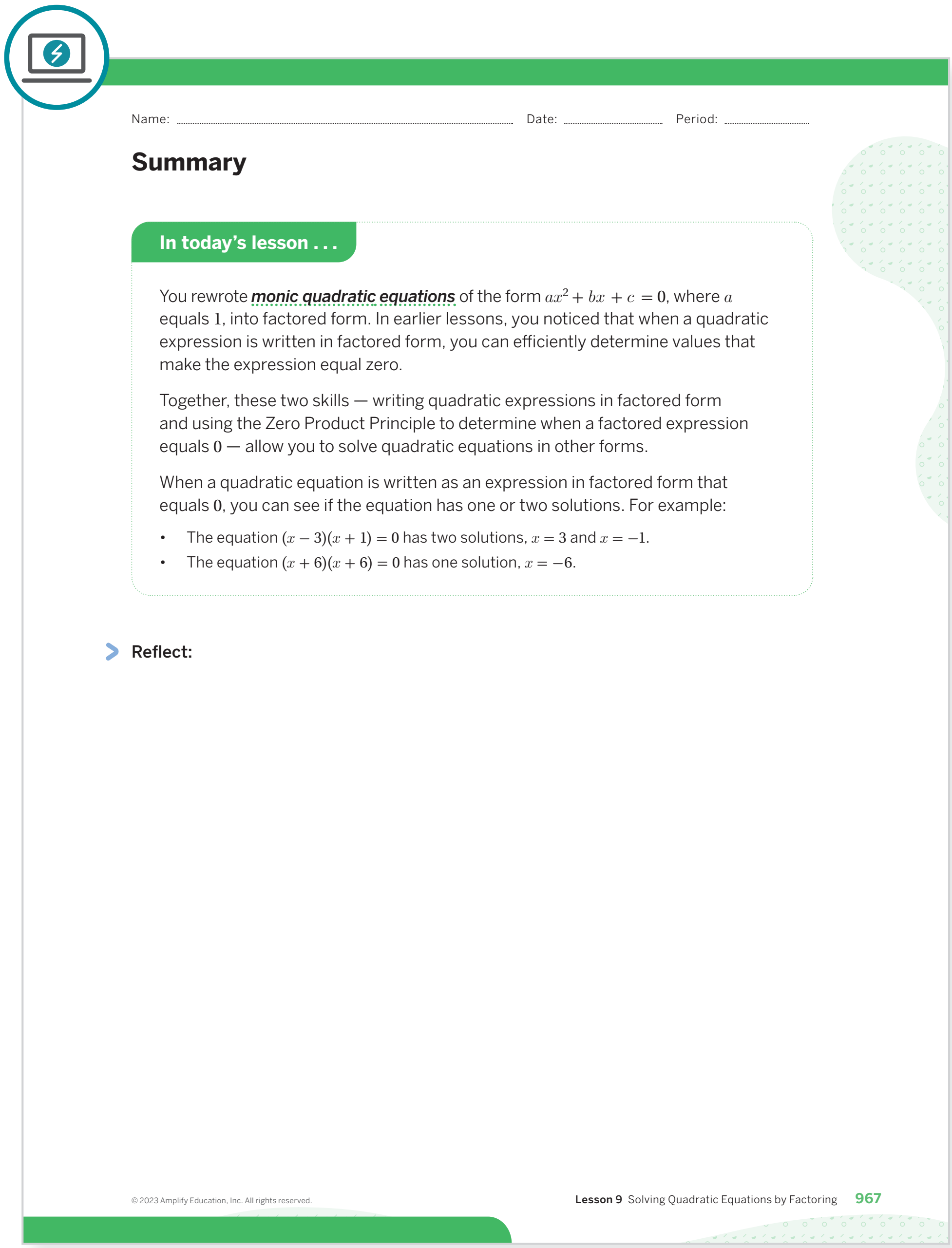
English Learners

If students are unfamiliar with the term *walkway*, draw a quick sketch of the square garden surrounded by a walkway and annotate the walkway with its term.

Whole Class | 5 min

Summary

Review and synthesize how writing quadratic equations in factored form and using the Zero Product Principle provides an efficient way for solving factorable quadratic equations.



Name: Date: Period:

Summary

In today's lesson . . .

You rewrote ***monic quadratic equations*** of the form $ax^2 + bx + c = 0$, where a equals 1, into factored form. In earlier lessons, you noticed that when a quadratic expression is written in factored form, you can efficiently determine values that make the expression equal zero.

Together, these two skills — writing quadratic expressions in factored form and using the Zero Product Principle to determine when a factored expression equals 0 — allow you to solve quadratic equations in other forms.

When a quadratic equation is written as an expression in factored form that equals 0, you can see if the equation has one or two solutions. For example:

- The equation $(x - 3)(x + 1) = 0$ has two solutions, $x = 3$ and $x = -1$.
- The equation $(x + 6)(x + 6) = 0$ has one solution, $x = -6$.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Solving Quadratic Equations by Factoring 967

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *monic quadratic* that were added to the display during the lesson. Include examples of equations that are monic quadratic equations and examples that are not, such as the following:

Monic quadratic equations	Non-monic quadratic equations
$y = x^2 + 6x + 8$ $y = x(x + 1)$ $y = (x - 2)(x + 7)$	$y = 3x^2 - 3x + 2$ $y = 2x(x + 4)$ $y = (x - 1)(5x + 3)$

Synthesize

Display the following equations:

Equation 1: $x^2 + 3x - 18 = 0$

Equation 2: $x(x - 7) = -6$

Equation 3: $2x^2 - 9x + 10 = 0$

Equation 4: $(x - 6)(x - 6) = 11$

Ask, "Which equation(s) can be solved without graphing? Explain." $x^2 + 3x - 18 = 0$ and $x(x - 7) = -6$; Sample response: I can write the factored form of Equation 1 as $(x - 3)(x + 6) = 0$, and use the Zero Product Principle. I can write Equation 2 in standard form and set it equal to 0, $x^2 - 7x + 6 = 0$. Then I can write it in factored form, $(x - 6)(x - 1) = 0$, and use the Zero Product Principle.

Have students share their thinking about which equations could be solved without graphing.

Highlight that it is more efficient to compare the equations when they are in the same form. The factored form of quadratic equations set equal to zero is helpful for determining the solutions without graphing. Point out that some equations are not factorable.

Formalize vocabulary: *monic quadratic equation*

Ask, "Do any of the equations appear to be unsolvable (or challenging to solve) without graphing? Why?"

Yes; Sample responses:

- $2x^2 - 9x + 10 = 0$ because the coefficient of x^2 is not 1.
- $(x - 6)(x - 6) = 11$ because when written in standard form and set equal to 0, the quadratic does not appear to be factorable.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

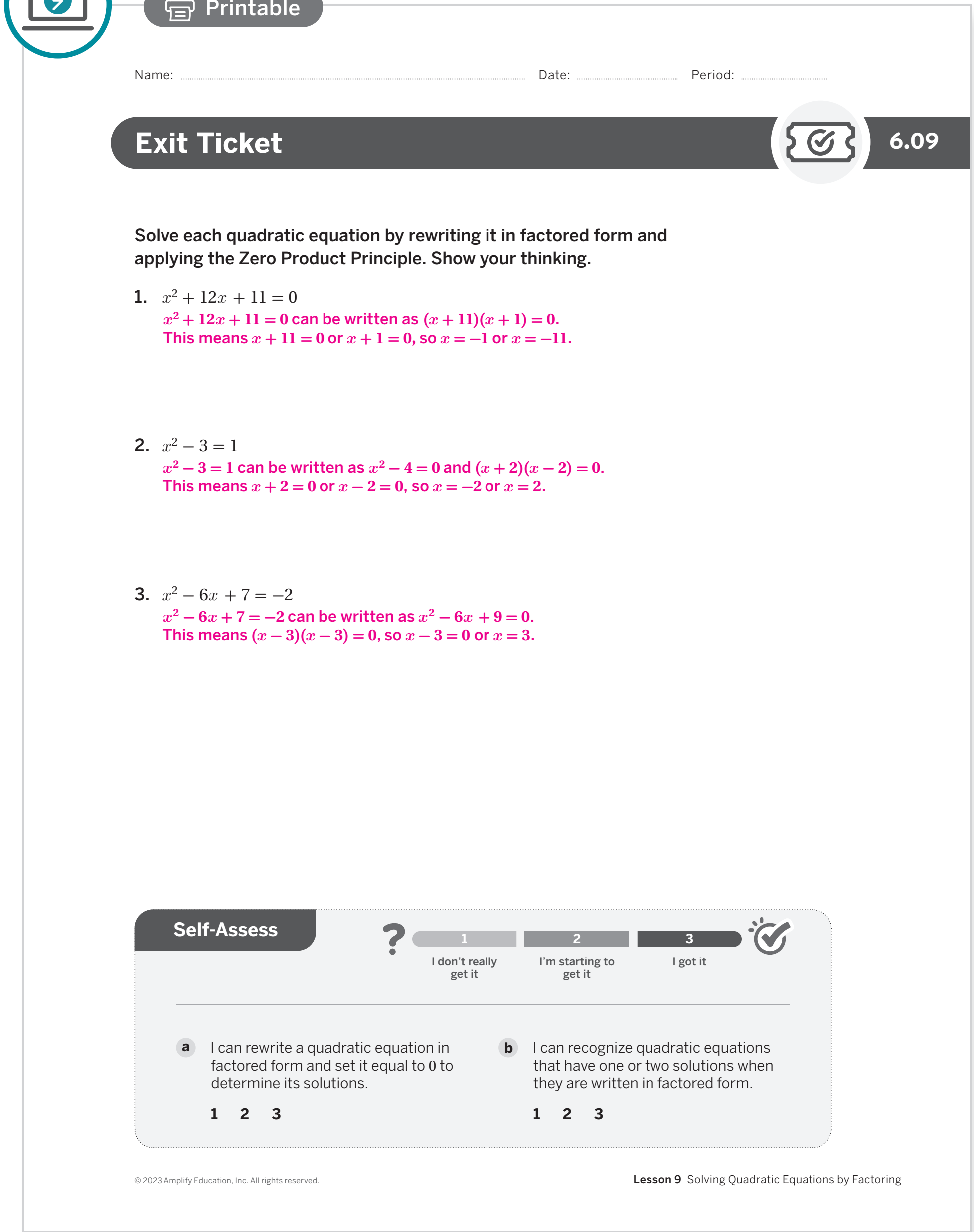
- "How does using the Zero Product Principle help solve quadratic equations?"
- "How are the zeros of a quadratic function related to the number of solutions to the related equation?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by rewriting quadratic equations in factored form and applying the Zero Product Principle to solve the equations.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

6.09

Solve each quadratic equation by rewriting it in factored form and applying the Zero Product Principle. Show your thinking.

1. $x^2 + 12x + 11 = 0$

   $x^2 + 12x + 11 = 0$ can be written as $(x + 11)(x + 1) = 0$.
   This means $x + 11 = 0$ or $x + 1 = 0$, so $x = -1$ or $x = -11$.

2. $x^2 - 3 = 1$

   $x^2 - 3 = 1$ can be written as $x^2 - 4 = 0$ and $(x + 2)(x - 2) = 0$.
   This means $x + 2 = 0$ or $x - 2 = 0$, so $x = -2$ or $x = 2$.

3. $x^2 - 6x + 7 = -2$

   $x^2 - 6x + 7 = -2$ can be written as $x^2 - 6x + 9 = 0$.
   This means $(x - 3)(x - 3) = 0$, so $x - 3 = 0$ or $x = 3$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can rewrite a quadratic equation in factored form and set it equal to 0 to determine its solutions.

1 2 3

b. I can recognize quadratic equations that have one or two solutions when they are written in factored form.

1 2 3

Success looks like . . .

- **Goal:** Recognizing that the number of solutions to a factorable quadratic equation can be revealed when the equation is written in factored form and set equal to 0.
- **Goal:** Using factored form and the Zero Product Principle to solve quadratic equations.
  - » Solving quadratic equations in Problems 1–3.

Suggested next steps

If students struggled with rewriting the equations in Problems 2 and 3 in factored form, consider:

- Reviewing the Zero Product Principle in Activity 1.
- Assigning Practice Problem 5.
- Asking, "How would you solve these equations if they were set equal to 0?"

Professional Learning

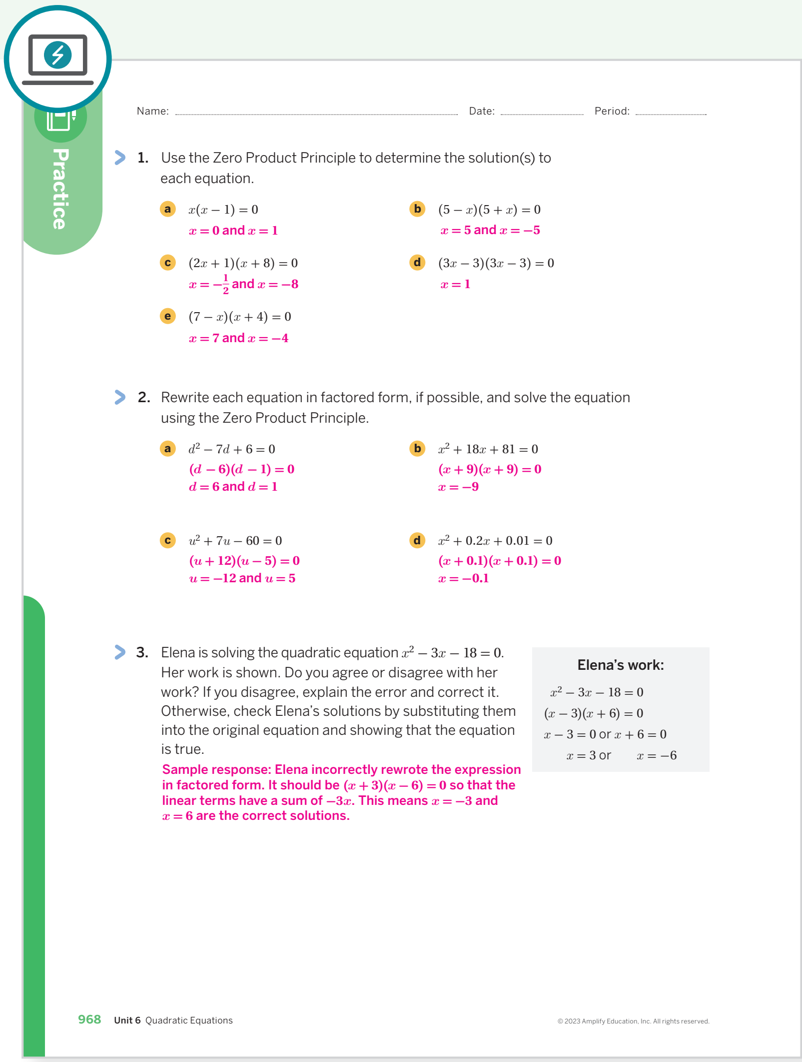
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students graphed quadratic functions. How did that support students making connections between the zeros of a quadratic function and the number of solutions?
- What surprised you as your students worked on using the Zero Product Principle? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

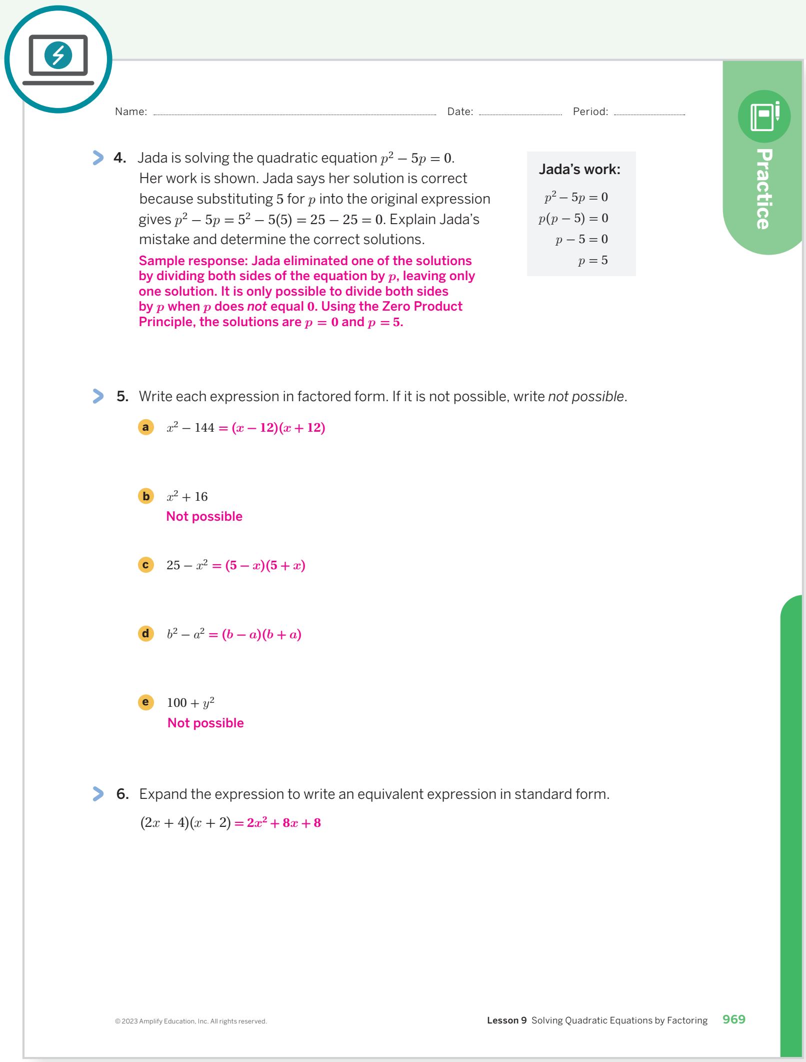
1. Use the Zero Product Principle to determine the solution(s) to each equation.
   - a $x(x-1)=0$
     $x=0$ and $x=1$
   - b $(5-x)(5+x)=0$
     $x=5$ and $x=-5$
   - c $(2x+1)(x+8)=0$
     $x=-\frac{1}{2}$ and $x=-8$
   - d $(3x-3)(3x-3)=0$
     $x=1$
   - e $(7-x)(x+4)=0$
     $x=7$ and $x=-4$

2. Rewrite each equation in factored form, if possible, and solve the equation using the Zero Product Principle.
   - a $d^2-7d+6=0$
     $(d-6)(d-1)=0$
     $d=6$ and $d=1$
   - b $x^2+18x+81=0$
     $(x+9)(x+9)=0$
     $x=-9$
   - c $u^2+7u-60=0$
     $(u+12)(u-5)=0$
     $u=-12$ and $u=5$
   - d $x^2+0.2x+0.01=0$
     $(x+0.1)(x+0.1)=0$
     $x=-0.1$

3. Elena is solving the quadratic equation $x^2-3x-18=0$. Her work is shown. Do you agree or disagree with her work? If you disagree, explain the error and correct it. Otherwise, check Elena's solutions by substituting them into the original equation and showing that the equation is true.
   Sample response: Elena incorrectly rewrote the expression in factored form. It should be $(x+3)(x-6)=0$ so that the linear terms have a sum of $-3x$. This means $x=-3$ and $x=6$ are the correct solutions.

Elena's work:

$x^2-3x-18=0$
$(x-3)(x+6)=0$
$x-3=0$ or $x+6=0$
$x=3$ or $x=-6$



Name: Date: Period:

4. Jada is solving the quadratic equation $p^2-5p=0$. Her work is shown. Jada says her solution is correct because substituting 5 for p into the original expression gives $p^2-5p=5^2-5(5)=25-25=0$. Explain Jada's mistake and determine the correct solutions.
   Sample response: Jada eliminated one of the solutions by dividing both sides of the equation by p, leaving only one solution. It is only possible to divide both sides by p when p does *not* equal 0. Using the Zero Product Principle, the solutions are $p=0$ and $p=5$.

Jada's work:

$p^2-5p=0$
$p(p-5)=0$
$p-5=0$
$p=5$

5. Write each expression in factored form. If it is not possible, write *not possible*.
   - a $x^2-144=(x-12)(x+12)$
   - b x^2+16
     Not possible
   - c $25-x^2=(5-x)(5+x)$
   - d $b^2-a^2=(b-a)(b+a)$
   - e $100+y^2$
     Not possible

6. Expand the expression to write an equivalent expression in standard form.
   $(2x+4)(x+2)=2x^2+8x+8$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 8	3
	5	Unit 6 Lesson 8	3
Formative ⚡	6	Unit 6 Lesson 10	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 10**

Writing Non-Monic Quadratic Expressions in Factored Form

Let's write non-monic quadratic expressions in factored form.

Focus

Goals

1. Given a factorable quadratic expression of the form $ax^2 + bx + c$ where $a \neq 1$, write an equivalent expression in factored form.
2. **Language Goal:** Write a quadratic equation that represents a context, consider different methods for solving it, and describe the limitations of each method. **(Speaking and Listening, Writing)**

Coherence

Today

Students rewrite non-monic factorable quadratic expressions — that are not a difference of squares — in factored form. Students notice the structure $(mx + p)(nx + q)$ is similar to $(x + p)(x + q)$ from earlier lessons. They engage with more complicated quadratic expressions, providing many opportunities to look for and make use of structure, which motivates them to look for more efficient strategies for solving these equations.

Previously

In Lesson 9, students integrated their understanding of factored form, the Zero Product Principle, and the zeros of a quadratic function to solve monic quadratic equations by factoring. They noted the relationship between the number of solutions to a quadratic equation and the number of zeros of its related function.

Coming Soon

In the next lesson, students will study the structure of square expressions.

Rigor

- Students develop **procedural skills** for writing factorable non-monic quadratic expressions in factored form.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Solving Non-Monic Quadratic Equations*
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- calculators
- graphing technology

Math Language Development

New words

- ***non-monic quadratic equation*** **(or expression)**

Review words

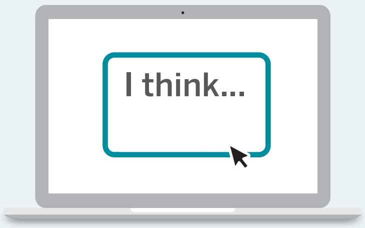
- *coefficient*
- *constant term*
- *linear term*
- *Zero Product Principle*

Amps | Featured Activity

Activity 1

See Student Thinking

Students explore how to factor when the coefficient of the quadratic term is greater than 1, as they previously only knew how to factor with a coefficient of 1. They'll share with you what they notice about such expressions.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated in Activity 2 when discerning the structure to rewrite factorable non-monic quadratics in factored form. Help them practice taking control of their own impulses by suggesting they seek out support from 2–3 sources, such as other students or you, as a general guideline when they feel frustrated.

Modifications to Pacing

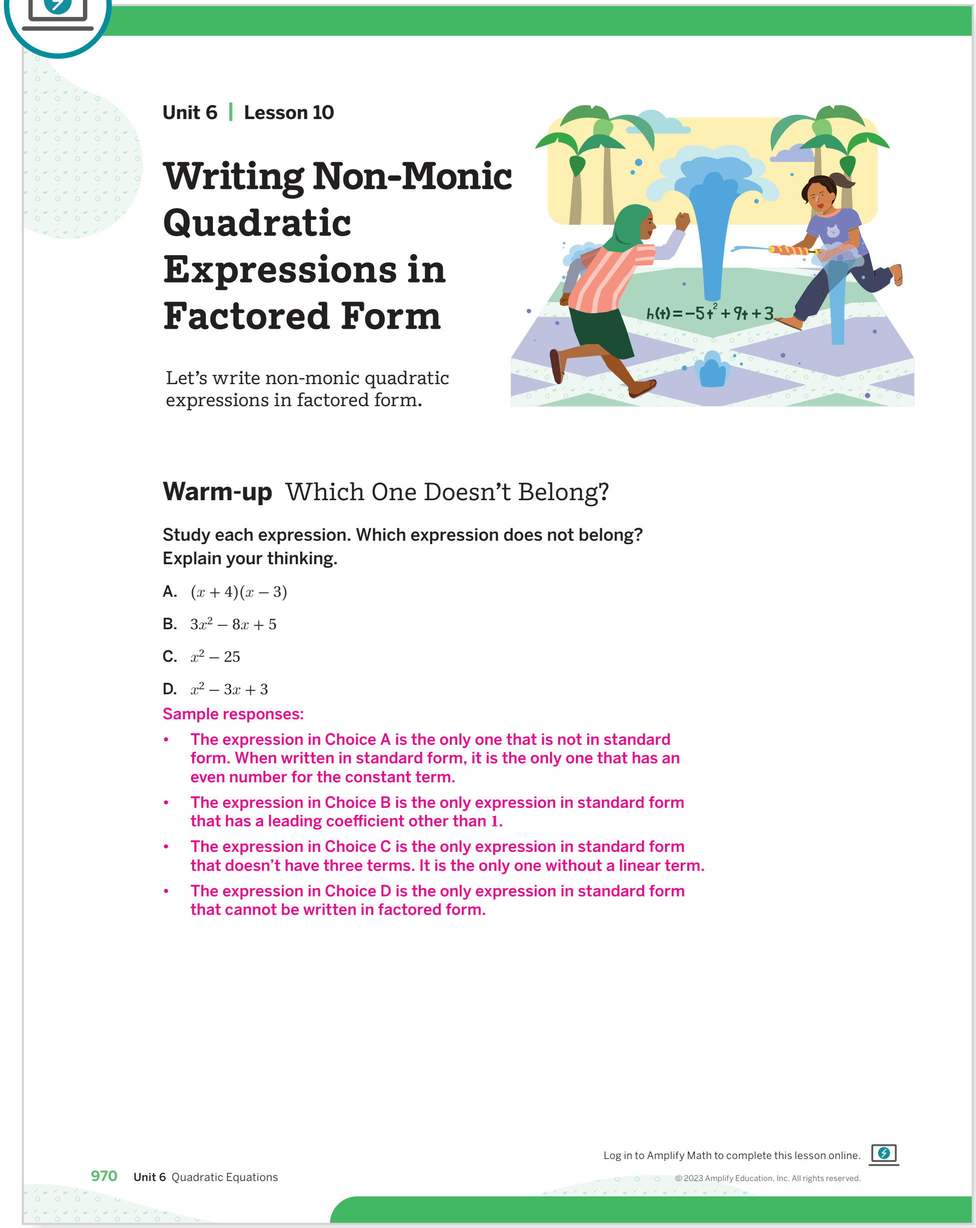
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, omit Problem 2 as students will learn how to factor non-monic quadratics in Activity 2.
- In **Activity 2**, omit Problem 3 as long as students have a solid understanding of Jada's and Clare's strategies from Problems 1 and 2.

Pairs | 5 min

Warm-up Which One Doesn't Belong?

Students analyze and compare quadratic expressions to look for common structures.



Unit 6 | Lesson 10

Writing Non-Monic Quadratic Expressions in Factored Form

Let's write non-monic quadratic expressions in factored form.

Warm-up Which One Doesn't Belong?

Study each expression. Which expression does not belong? Explain your thinking.

A. $(x + 4)(x - 3)$

B. $3x^2 - 8x + 5$

C. $x^2 - 25$

D. $x^2 - 3x + 3$

Sample responses:

- **The expression in Choice A is the only one that is not in standard form. When written in standard form, it is the only one that has an even number for the constant term.**
- **The expression in Choice B is the only expression in standard form that has a leading coefficient other than 1.**
- **The expression in Choice C is the only expression in standard form that doesn't have three terms. It is the only one without a linear term.**
- **The expression in Choice D is the only expression in standard form that cannot be written in factored form.**

Log in to Amplify Math to complete this lesson online.

970 **Unit 6** Quadratic Equations

1 Launch

Conduct the ***Which One Doesn't Belong?*** routine. Provide students one minute of think-time and two minutes to share their thoughts with a partner. Remind students that there is no single correct response.

2 Monitor

Help students get started by asking, "How are these expressions similar or different?"

Look for points of confusion:

- **Not noticing that Choice A is in factored form.** Ask students to identify whether each expression is in factored form or standard form.

Look for productive strategies:

- Expanding the expression in Choice A.
- Noticing that the expression in Choice C does not have a linear term.

3 Connect

Have students share their strategies for thinking about the chosen expression. Select students who chose each expression, sequencing those who selected Choice B to share last.

Ask, "Do you agree or disagree?" Listen for mathematical language as students critique or defend each response.

Highlight that Choice B is unlike any of the quadratic expressions students have seen yet in this unit.

Define the term ***non-monic quadratic equation*** (or expression) as a quadratic equation (or expression) in which the coefficient of the squared variable term is not equal to 1. This is often called the *leading coefficient* when written in standard form, because it is the coefficient of the first term.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their strategies, collect and display the language students use as they describe which choice might not belong with the others. Emphasize the language students use to describe Choice B and define this expression as a *non-monic quadratic*. Ask:

- "Are the expressions in Choices A, C, and D monic quadratic expressions? Explain your thinking."
- "Give another example of a non-monic quadratic expression and explain why it is non-monic."

English Learners

Annotate the coefficient in front of x^2 in the expression in Choice B and write "non-monic because $3 \neq 1$."

Power-up

To power up students' ability to expand non-monic quadratic expressions in factored form, have students complete:

Complete the area diagram to rewrite the expression $(2x + 3)(3x + 1)$ in standard form.

$6x^2 + 11x + 3$

	$3x$	1
$2x$	$6x^2$	$2x$
3	$9x$	3

Use: Before the Warm-up

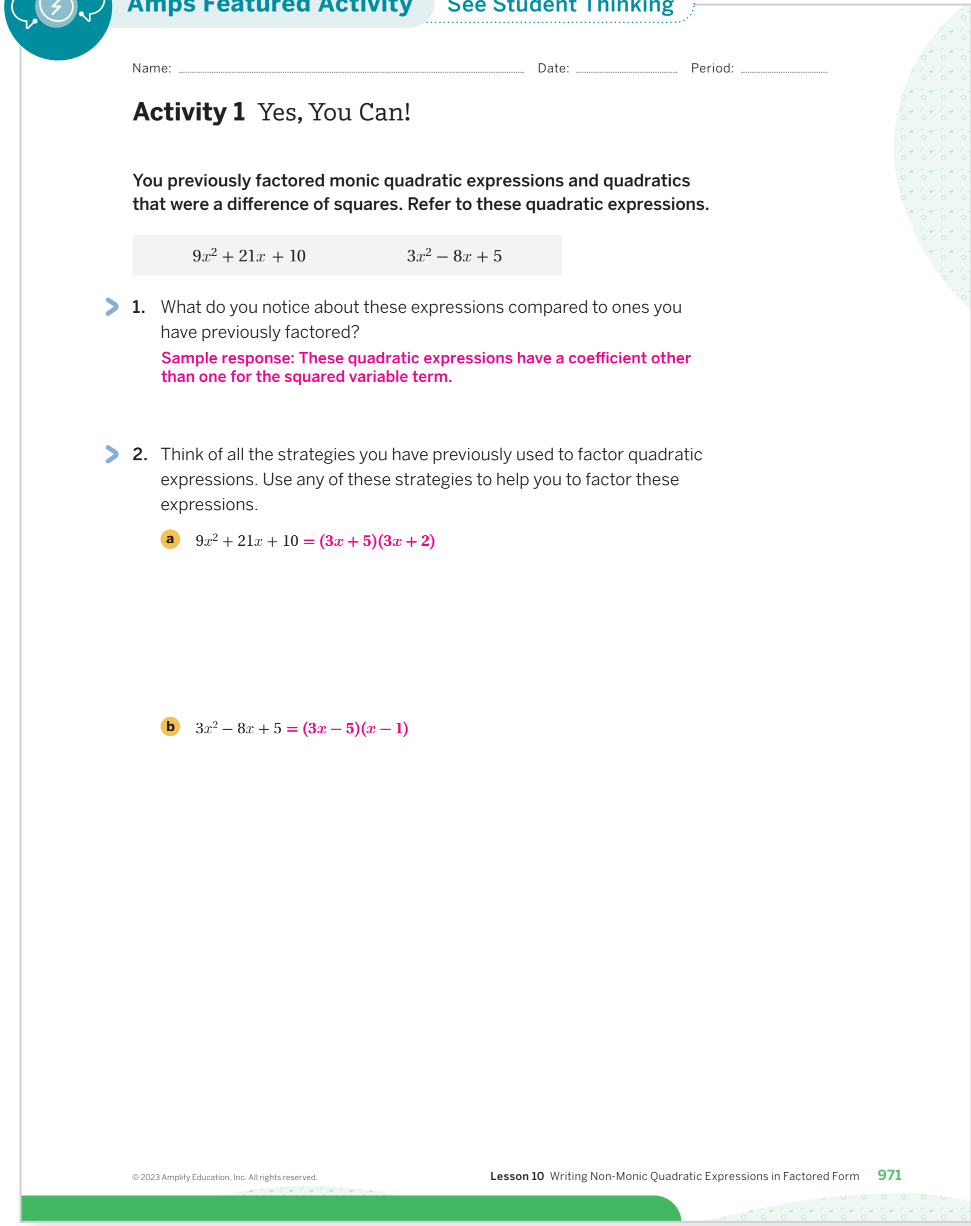
Use: Performance on Lesson 9, Practice Problem 6

Pairs | 15 min

Activity 1 Yes, You Can!

Students apply prior knowledge and strategies of factoring monic quadratic expressions to non-monic quadratic expressions, realizing the limitations of those strategies.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 1 Yes, You Can!

You previously factored monic quadratic expressions and quadratics that were a difference of squares. Refer to these quadratic expressions.

$9x^2 + 21x + 10$ $3x^2 - 8x + 5$

1. What do you notice about these expressions compared to ones you have previously factored?
   Sample response: These quadratic expressions have a coefficient other than one for the squared variable term.

2. Think of all the strategies you have previously used to factor quadratic expressions. Use any of these strategies to help you to factor these expressions.
   - a $9x^2 + 21x + 10 = (3x + 5)(3x + 2)$
   - b $3x^2 - 8x + 5 = (3x - 5)(x - 1)$

1 Launch

Display the expressions, then ask students to think independently about Problem 1 before discussing with a partner.

2 Monitor

Help students get started by asking, “What are the factors of 9 you could use to begin to factor the expression in Problem 2a?”

Look for points of confusion:

- **Struggling to determine factor pairs that work for each expression.** Encourage students to list factor pairs for each part of the expression and then use the *guess-and-check* strategy to determine the ones that will work.

Look for productive strategies:

- Listing factors of two terms and using the *guess-and-check* strategy to determine which factor pairs will yield the correct standard form.
- Drawing area diagrams to help factor the expressions.

3 Connect

Display both expressions.

Have individual students share their attempts at rewriting the expressions in factored form.

Ask, “What do you need to think about when trying to factor a non-monic quadratic expression?” Sample response: I cannot simply find the factors of the constant term that add up to the linear term. I also need to take into account the factors of the coefficient of the squared variable term.

Highlight that previous methods to determine the factored expression will still work here. Students will need to look at two factor pairs instead of one, and then use the *guess-and-check* strategy to determine the equivalent factored expressions.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they have previously factored monic quadratic expressions in this unit. Have them refer back to prior lessons and activities if they need a refresher on the strategies they have used.

Accessibility: *Guide Processing and Visualization*

Display a template students can use to write the expressions in factored form, such as the one shown. Remind them to check the signs of the constants and coefficients and adapt, as needed.

$(\_\_\, x + \_\_)(\_\_\, x + \_\_)$

Math Language Development

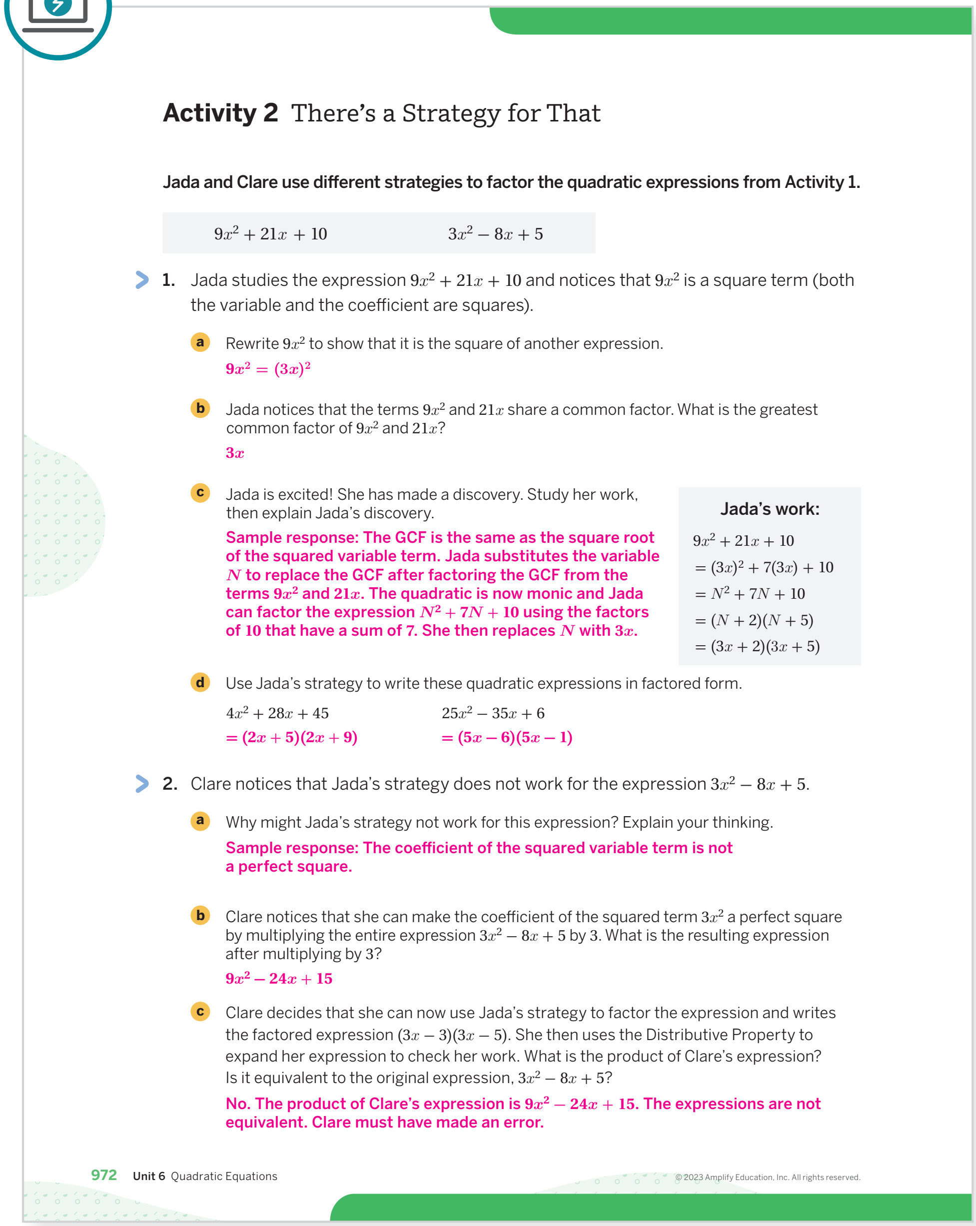
MLR8: Discussion Supports—Press for Reasoning

Display or provide students Anchor chart PDF, *Sentence Frames, Explaining My Steps* to support students as they explain the strategies they tried to use. Allow students to rehearse what they will say before sharing with the whole class.

Pairs | 20 min

Activity 2 There's A Strategy for That

Students investigate two strategies for writing non-monic quadratic expressions in factored form, noting the limitations of one of the strategies.



Activity 2 There's a Strategy for That

Jada and Clare use different strategies to factor the quadratic expressions from Activity 1.

$9x^2 + 21x + 10$	$3x^2 - 8x + 5$

1. Jada studies the expression $9x^2 + 21x + 10$ and notices that $9x^2$ is a square term (both the variable and the coefficient are squares).

   a. Rewrite $9x^2$ to show that it is the square of another expression.

   $\mathbf{9x^2 = (3x)^2}$

   b. Jada notices that the terms $9x^2$ and $21x$ share a common factor. What is the greatest common factor of $9x^2$ and $21x$?

   $\mathbf{3x}$

   c. Jada is excited! She has made a discovery. Study her work, then explain Jada's discovery.

   Jada's work:

   $9x^2 + 21x + 10$
   $= (3x)^2 + 7(3x) + 10$
   $= N^2 + 7N + 10$
   $= (N + 2)(N + 5)$
   $= (3x + 2)(3x + 5)$

   Sample response: The GCF is the same as the square root of the squared variable term. Jada substitutes the variable N to replace the GCF after factoring the GCF from the terms $9x^2$ and $21x$. The quadratic is now monic and Jada can factor the expression $N^2 + 7N + 10$ using the factors of 10 that have a sum of 7. She then replaces N with $3x$.

   d. Use Jada's strategy to write these quadratic expressions in factored form.

   $4x^2 + 28x + 45$
   $\mathbf{= (2x + 5)(2x + 9)}$

   $25x^2 - 35x + 6$
   $\mathbf{= (5x - 6)(5x - 1)}$

2. Clare notices that Jada's strategy does not work for the expression $3x^2 - 8x + 5$.

   a. Why might Jada's strategy not work for this expression? Explain your thinking.

   Sample response: The coefficient of the squared variable term is not a perfect square.

   b. Clare notices that she can make the coefficient of the squared term $3x^2$ a perfect square by multiplying the entire expression $3x^2 - 8x + 5$ by 3. What is the resulting expression after multiplying by 3?

   $\mathbf{9x^2 - 24x + 15}$

   c. Clare decides that she can now use Jada's strategy to factor the expression and writes the factored expression $(3x - 3)(3x - 5)$. She then uses the Distributive Property to expand her expression to check her work. What is the product of Clare's expression? Is it equivalent to the original expression, $3x^2 - 8x + 5$?

   No. The product of Clare's expression is $9x^2 - 24x + 15$. The expressions are not equivalent. Clare must have made an error.

972 Unit 6 Quadratic Equations

1 Launch

Have students work in pairs to complete Problems 1a, 1b, and 1c. Then pause for a class discussion about Jada's discovery before having students complete the rest of the activity with their partner.

2 Monitor

Help students get started by asking what they noticed about each of the two given quadratic expressions. Sample responses:

- Both expressions are in standard form.
- Both expressions are non-monic quadratics.
- In the first expression, the first term is a square term. This is not true of the second expression.

Look for points of confusion:

- **Struggling to understand Jada's discovery in Problem 1c.** Have students use colored pencils or highlighters to show how the expression $3x$ can be thought of as its own entity, such as N. Then ask them whether the expression $N^2 + 7N + 10$ can be factored.
- **Not understanding why Jada's strategy does not work for the expression given in Problem 2a.** Ask, "In your own words, can you explain what the first step is in Jada's strategy?"
- **Not understanding why Clare made a mistake in Problem 2c.** Ask students whether the original expression is equivalent to Clare's expression if it is multiplied by 3.

Look for productive strategies:

- Realizing that Clare changed the value of the expression when she multiplied the expression by 3 in Problem 2b.
- Understanding Jada's and Clare's strategies and being able to explain why Jada's strategy does not always work.

Activity 2 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider completing Problem 1 together as a class to ensure students understand Jada's strategy. Provide access to colored pencils and color code how N is substituted for $3x$ in the table showing Jada's work. Discuss Problem 2a together as a class, and then have students work in pairs to complete the rest of the activity.

Math Language Development

MLR7: Compare and Connect

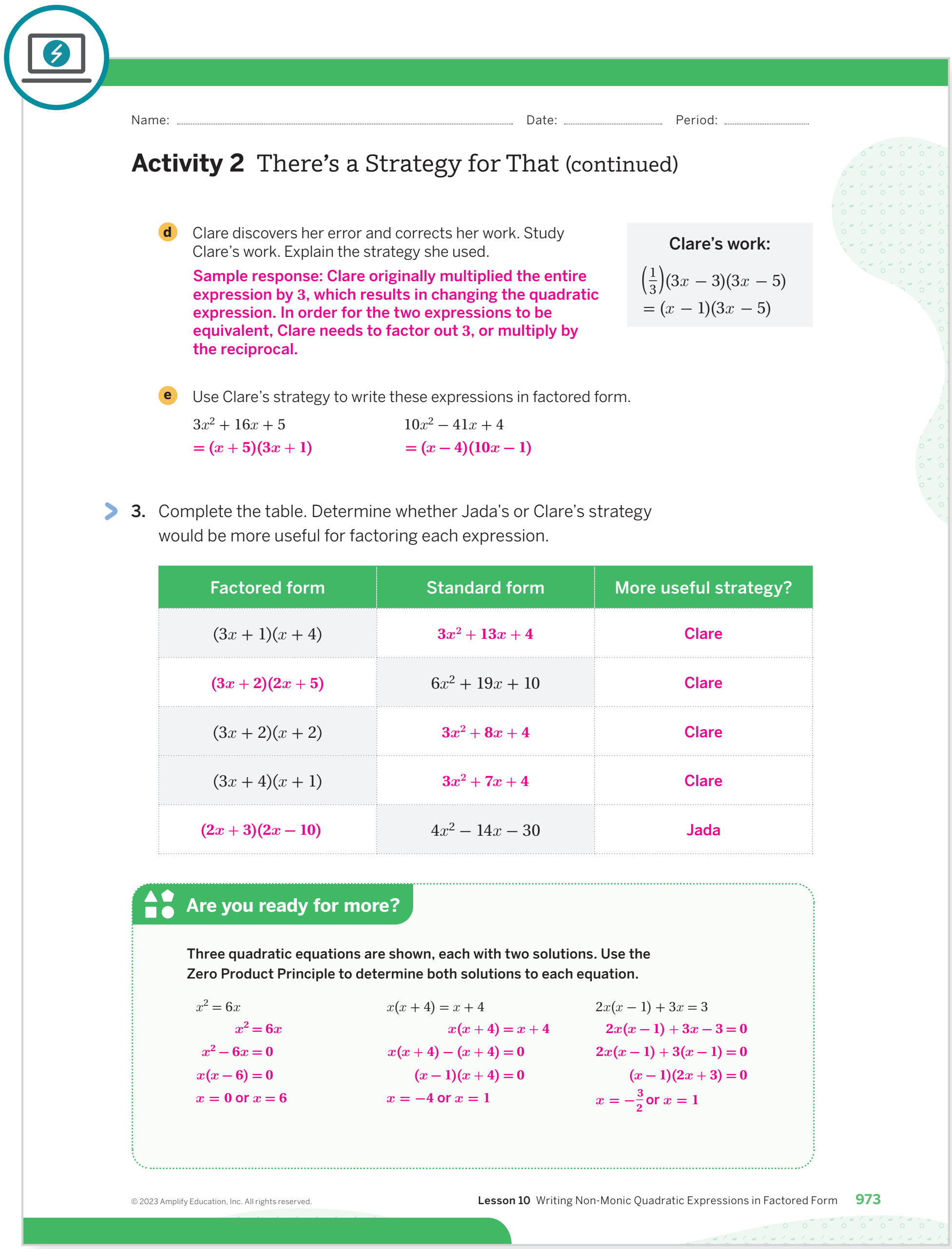
During the Connect, have partners share what they notice that is similar and different between Clare's and Jada's strategies. Ask:

- "In Problem 2b, why did Clare multiply the entire expression by 3? Is the resulting expression equivalent to the original? What does it mean for two expressions to be equivalent?"
- "In Problem 2d, why did Clare multiply the expression by $\frac{1}{3}$? Is this resulting expression equivalent to the original?"
- "In Problem 3, why does Jada's strategy only work for the last expression in the table?"

Pairs | 20 min

Activity 2 There's A Strategy for That (continued)

Students investigate two strategies for writing non-monic quadratic expressions in factored form, noting the limitations of one of the strategies.



Name: ________ Date: ________ Period: ________

Activity 2 There's a Strategy for That (continued)

d Clare discovers her error and corrects her work. Study Clare's work. Explain the strategy she used.

Clare's work:

$\left(\frac{1}{3}\right)(3x-3)(3x-5)$

$= (x-1)(3x-5)$

Sample response: Clare originally multiplied the entire expression by 3, which results in changing the quadratic expression. In order for the two expressions to be equivalent, Clare needs to factor out 3, or multiply by the reciprocal.

e Use Clare's strategy to write these expressions in factored form.

$3x^2 + 16x + 5$
$= (x+5)(3x+1)$

$10x^2 - 41x + 4$
$= (x-4)(10x-1)$

3. Complete the table. Determine whether Jada's or Clare's strategy would be more useful for factoring each expression.

Factored form	Standard form	More useful strategy?
$(3x+1)(x+4)$	$3x^2 + 13x + 4$	Clare
$(3x+2)(2x+5)$	$6x^2 + 19x + 10$	Clare
$(3x+2)(x+2)$	$3x^2 + 8x + 4$	Clare
$(3x+4)(x+1)$	$3x^2 + 7x + 4$	Clare
$(2x+3)(2x-10)$	$4x^2 - 14x - 30$	Jada

Are you ready for more?

Three quadratic equations are shown, each with two solutions. Use the Zero Product Principle to determine both solutions to each equation.

$x^2 = 6x$
$x^2 = 6x$
$x^2 - 6x = 0$
$x(x-6) = 0$
$x = 0$ or $x = 6$

$x(x+4) = x + 4$
$x(x+4) = x + 4$
$x(x+4) - (x+4) = 0$
$(x-1)(x+4) = 0$
$x = -4$ or $x = 1$

$2x(x-1) + 3x = 3$
$2x(x-1) + 3x - 3 = 0$
$2x(x-1) + 3(x-1) = 0$
$(x-1)(2x+3) = 0$
$x = -\frac{3}{2}$ or $x = 1$

Lesson 10 Writing Non-Monic Quadratic Expressions in Factored Form 973

3 Connect

Display the incomplete table from Problem 3.

Have individual students share the standard form for each expression and whether Jada's or Clare's strategy would be more efficient to use to factor each expression. Record their responses in the table.

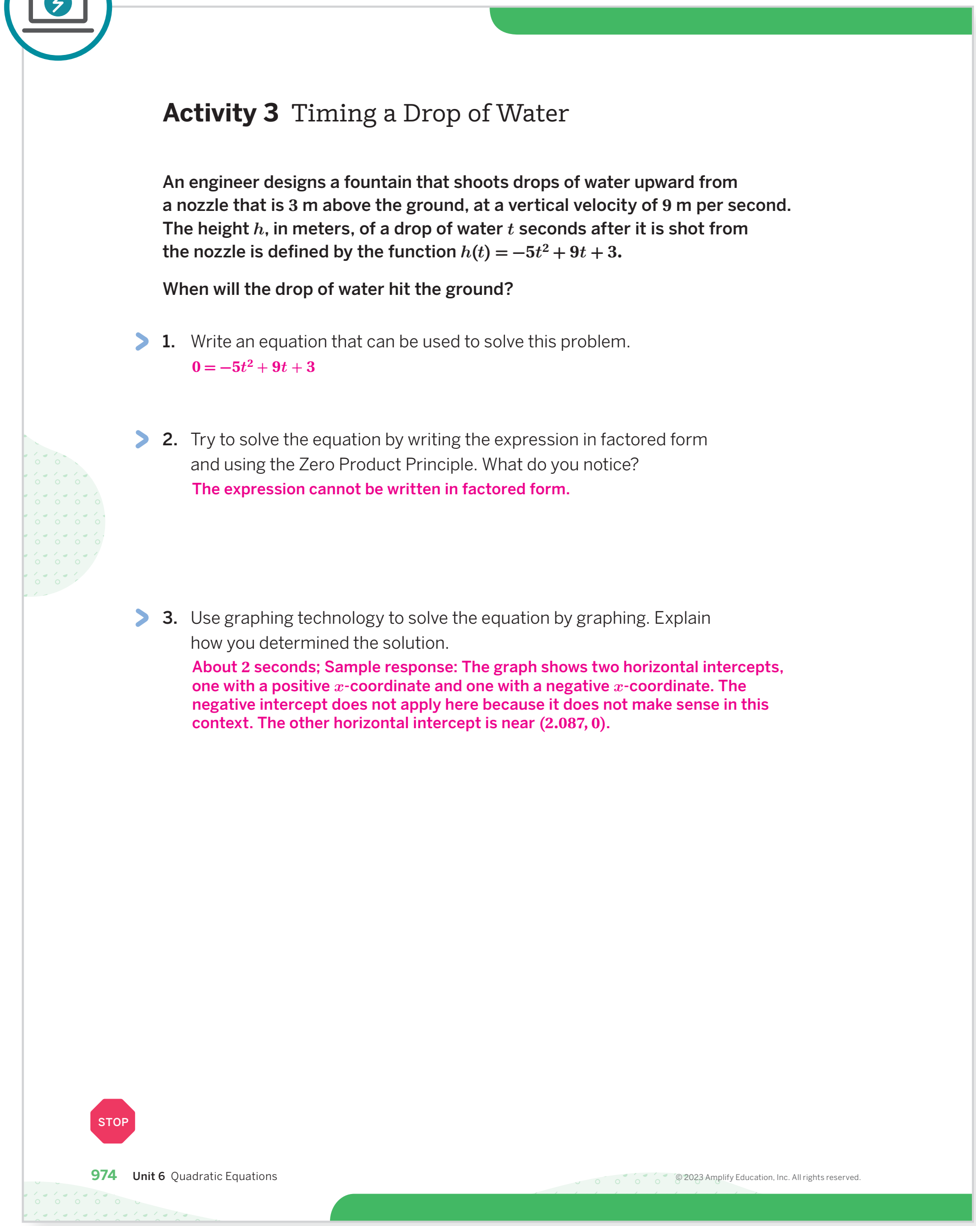
Highlight that Jada's strategy only works for the last expression in the table because the coefficient of the squared variable term is a perfect square.

Optional

Pairs | 15 min

Activity 3 Timing a Drop of Water

Students write and attempt to solve a quadratic equation in context using known strategies, soon realizing the limitations of those strategies.



Activity 3 Timing a Drop of Water

An engineer designs a fountain that shoots drops of water upward from a nozzle that is 3 m above the ground, at a vertical velocity of 9 m per second. The height h, in meters, of a drop of water t seconds after it is shot from the nozzle is defined by the function $h(t) = -5t^2 + 9t + 3$.

When will the drop of water hit the ground?

1. Write an equation that can be used to solve this problem.
   $0 = -5t^2 + 9t + 3$
2. Try to solve the equation by writing the expression in factored form and using the Zero Product Principle. What do you notice?
   The expression cannot be written in factored form.
3. Use graphing technology to solve the equation by graphing. Explain how you determined the solution.
   About 2 seconds; Sample response: The graph shows two horizontal intercepts, one with a positive x-coordinate and one with a negative x-coordinate. The negative intercept does not apply here because it does not make sense in this context. The other horizontal intercept is near $(2.087, 0)$.

STOP

974 **Unit 6** Quadratic Equations

1 Launch

Allow students to productively struggle in their attempts to solve the equation by factoring. Provide access to graphing technology for use in Problem 3.

2 Monitor

Help students get started by asking what it means, in terms of the equation, for the drop of water to hit the ground. The equation will be equal to 0.

Look for points of confusion:

- **Struggling to find a way to factor the given equation in Problem 2.** Allow students to productively struggle for about 5 minutes. If students show signs of unproductive struggle, have them move on to Problem 3.

Look for productive strategies:

- Multiplying the expression $-5t^2 + 9t + 3$ by -5, so that the coefficient of t^2 is a perfect square.
- Attempting to create an area diagram.

3 Connect

Display the equation $0 = -5t^2 + 9t + 3$.

Have individual students share what they noticed as they attempted to solve the equation by factoring and using the Zero Product Principle.

Highlight that the expression $-5t^2 + 9t + 3$ cannot be factored, which is why students were encouraged to use graphing technology to graph the equation in Problem 3.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

The intent of this activity is for students to realize that factoring techniques do not work with this equation. Allow students to struggle with Problem 2 before intervening and asking them to move on to Problem 3.

Extension: *Math Enrichment*

Challenge students to determine the maximum height of the water drop using graphing technology.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

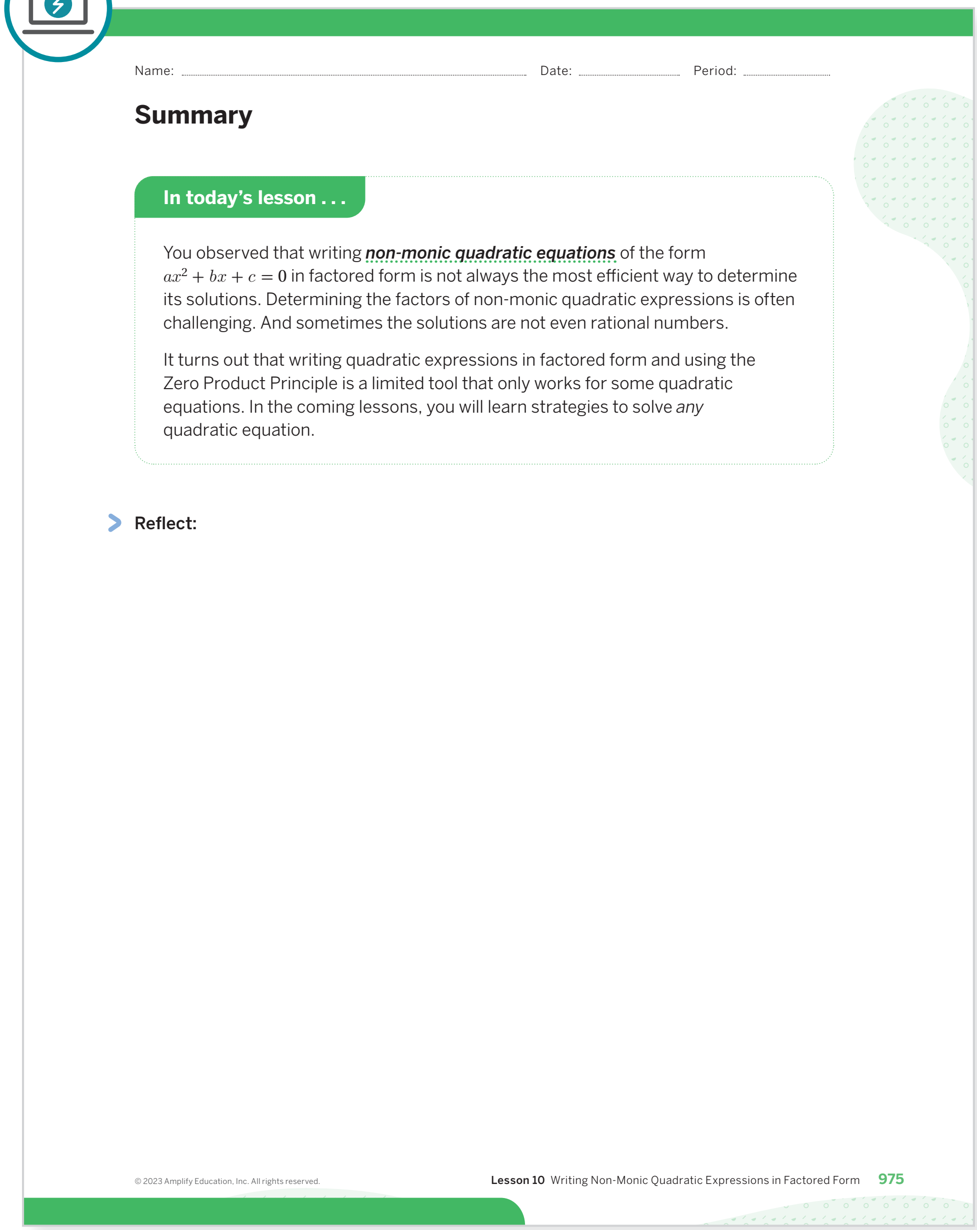
- **Read 1:** Students should understand that there is a fountain shooting drops of water up into the air.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as “the nozzle is 3 m above the ground.”
- **Read 3:** Ask students to plan their solution strategy as to how they will write and solve an equation to determine when the drop of water will hit the ground.

English Learners

Draw a sketch of a drop of water shooting out of the nozzle to illustrate the quadratic motion.

Summary

Review and synthesize how rewriting quadratic equations in factored form and using the Zero Product Principle only works for some quadratic equations.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You observed that writing ***non-monic quadratic equations*** of the form $ax^2 + bx + c = 0$ in factored form is not always the most efficient way to determine its solutions. Determining the factors of non-monic quadratic expressions is often challenging. And sometimes the solutions are not even rational numbers.

It turns out that writing quadratic expressions in factored form and using the Zero Product Principle is a limited tool that only works for some quadratic equations. In the coming lessons, you will learn strategies to solve *any* quadratic equation.

Reflect:

Synthesize

Display the equation $ax^2 + bx + c = 0$.

Have students share how they would generalize the relationship between the expression $ax^2 + bx + c$ and its factored form.

Highlight that for some quadratic expressions, the right pairs of factors might be immediately spotted, but for others, the process of the *guess-and-check* strategy can be cumbersome, especially if both a and c have many pairs of factors. There is also no guarantee that they will find a combination that works because some equations do not have rational solutions, i.e., they cannot find factors using rational numbers. This means that if students rely on writing an equation in factored form to solve, they may get stuck. If students rely on graphing, the solutions may not be exact. There needs to be another way!

Formalize vocabulary: ***non-monic quadratic equation***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What was the same or different about factoring non-monic quadratic expressions?"
- "What did you find helpful about Clare and Jada's strategies in Activity 2?"

Math Language Development

MLR2: Collect and Display

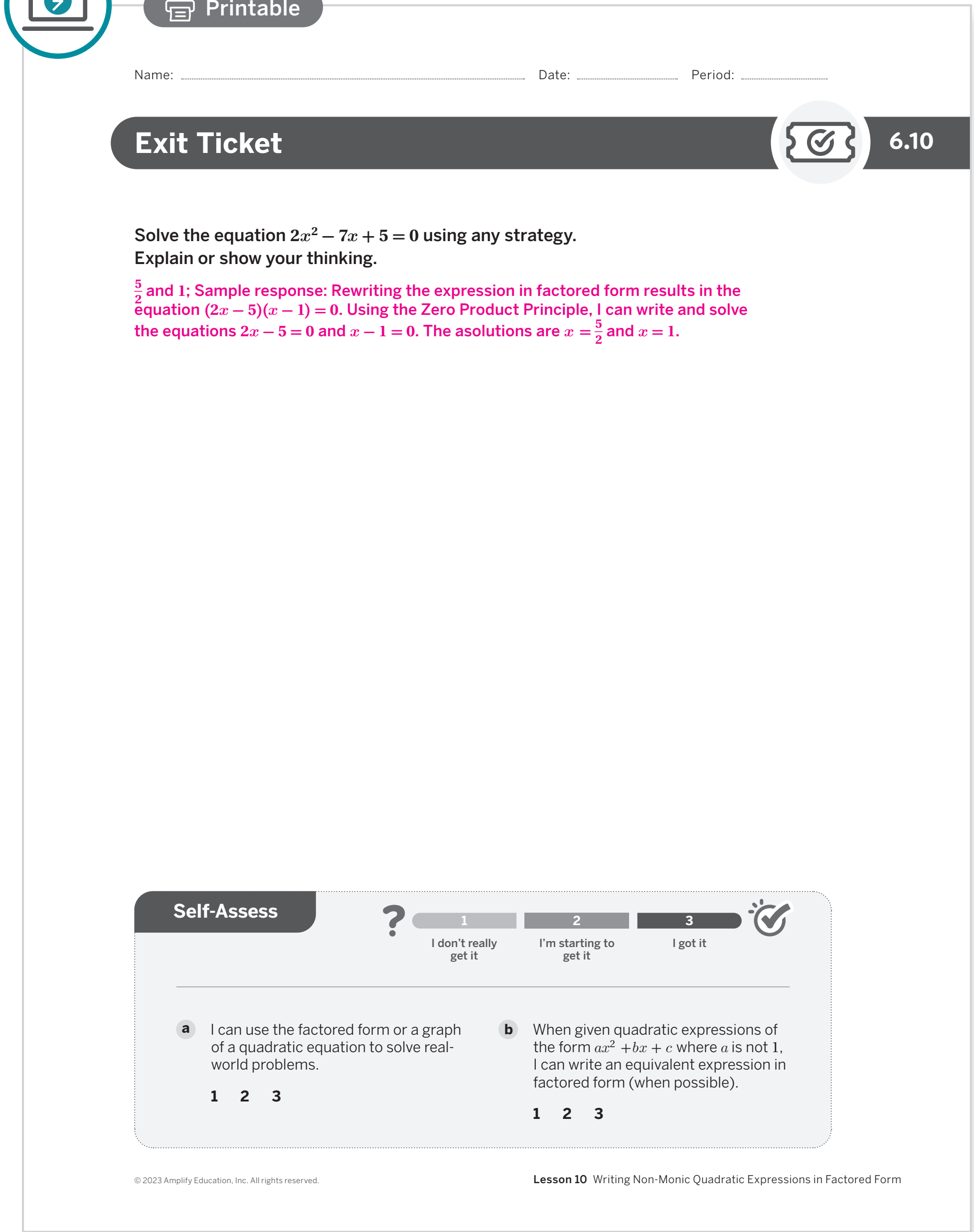
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *non-monic quadratic equation* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by rewriting a non-monic quadratic equation in factored form.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 6.10

Solve the equation $2x^2 - 7x + 5 = 0$ using any strategy.
Explain or show your thinking.

$\frac{5}{2}$ and 1; Sample response: Rewriting the expression in factored form results in the equation $(2x - 5)(x - 1) = 0$. Using the Zero Product Principle, I can write and solve the equations $2x - 5 = 0$ and $x - 1 = 0$. The asolutions are $x = \frac{5}{2}$ and $x = 1$.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can use the factored form or a graph of a quadratic equation to solve real-world problems.

1 2 3

b. When given quadratic expressions of the form $ax^2 + bx + c$ where a is not 1, I can write an equivalent expression in factored form (when possible).

1 2 3

Lesson 10 Writing Non-Monic Quadratic Expressions in Factored Form

Success looks like . . .

- **Goal:** Given a factorable quadratic expression of the form $ax^2 + bx + c$ where $a \neq 1$, writing an equivalent expression in factored form.
  - » Factoring the quadratic expression in the left-hand side to solve the equation.
- **Language Goal:** Writing a quadratic equation that represents a context, considering different methods for solving it, and describing the limitations of each method. **(Speaking and Listening, Writing)**

Suggested next steps

If students struggle factoring the quadratic equation, consider:

- Suggesting they list the possible factors of 2 and 5.
- Reviewing Jada's and Clare's strategies from Activity 2.
- Offering graphing technology to assist with solving.

If students factor correctly, but do not identify the correct solutions, consider:

- Reviewing the Zero Product Principle and how it applies to non-monic quadratic equations.

Professional Learning

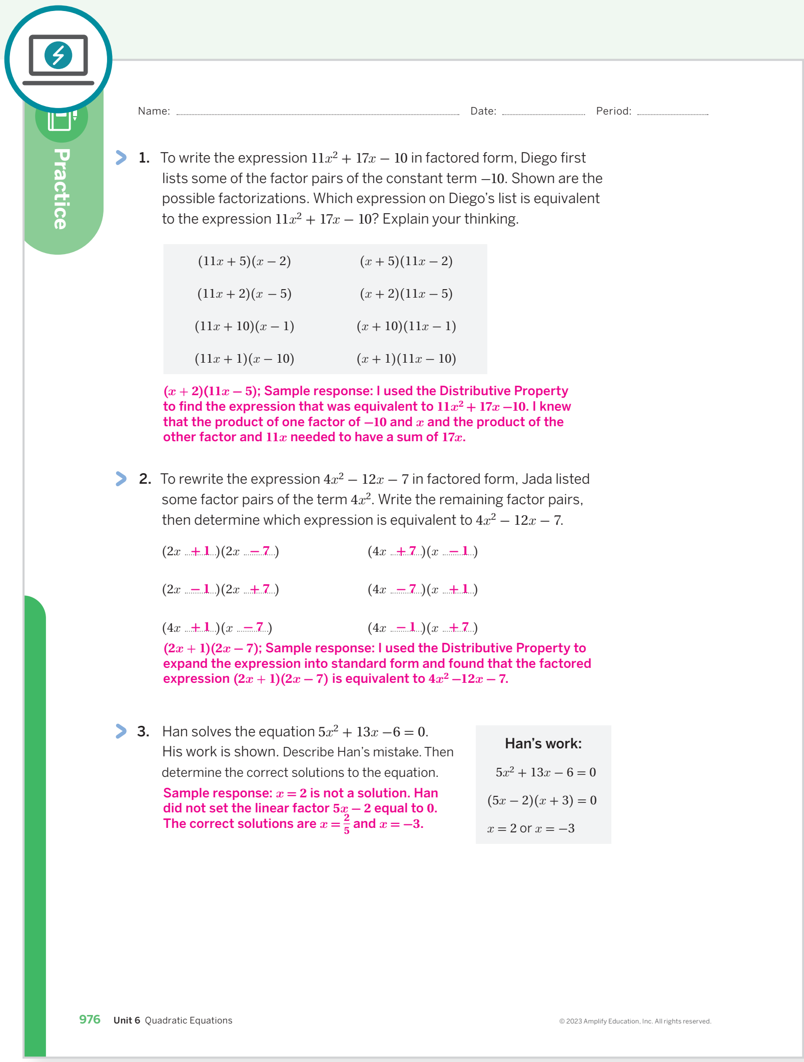
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on Activity 1?
- What challenges did students encounter as they worked on Activity 2? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. To write the expression $11x^2 + 17x - 10$ in factored form, Diego first lists some of the factor pairs of the constant term -10. Shown are the possible factorizations. Which expression on Diego's list is equivalent to the expression $11x^2 + 17x - 10$? Explain your thinking.

$(11x + 5)(x - 2)$	$(x + 5)(11x - 2)$
$(11x + 2)(x - 5)$	$(x + 2)(11x - 5)$
$(11x + 10)(x - 1)$	$(x + 10)(11x - 1)$
$(11x + 1)(x - 10)$	$(x + 1)(11x - 10)$

$(x + 2)(11x - 5)$; Sample response: I used the Distributive Property to find the expression that was equivalent to $11x^2 + 17x - 10$. I knew that the product of one factor of -10 and x and the product of the other factor and $11x$ needed to have a sum of $17x$.

2. To rewrite the expression $4x^2 - 12x - 7$ in factored form, Jada listed some factor pairs of the term $4x^2$. Write the remaining factor pairs, then determine which expression is equivalent to $4x^2 - 12x - 7$.

$(2x + 1)(2x - 7)$	$(4x + 7)(x - 1)$
$(2x - 1)(2x + 7)$	$(4x - 7)(x + 1)$
$(4x + 1)(x - 7)$	$(4x - 1)(x + 7)$

$(2x + 1)(2x - 7)$; Sample response: I used the Distributive Property to expand the expression into standard form and found that the factored expression $(2x + 1)(2x - 7)$ is equivalent to $4x^2 - 12x - 7$.

3. Han solves the equation $5x^2 + 13x - 6 = 0$. His work is shown. Describe Han's mistake. Then determine the correct solutions to the equation.

Han's work:

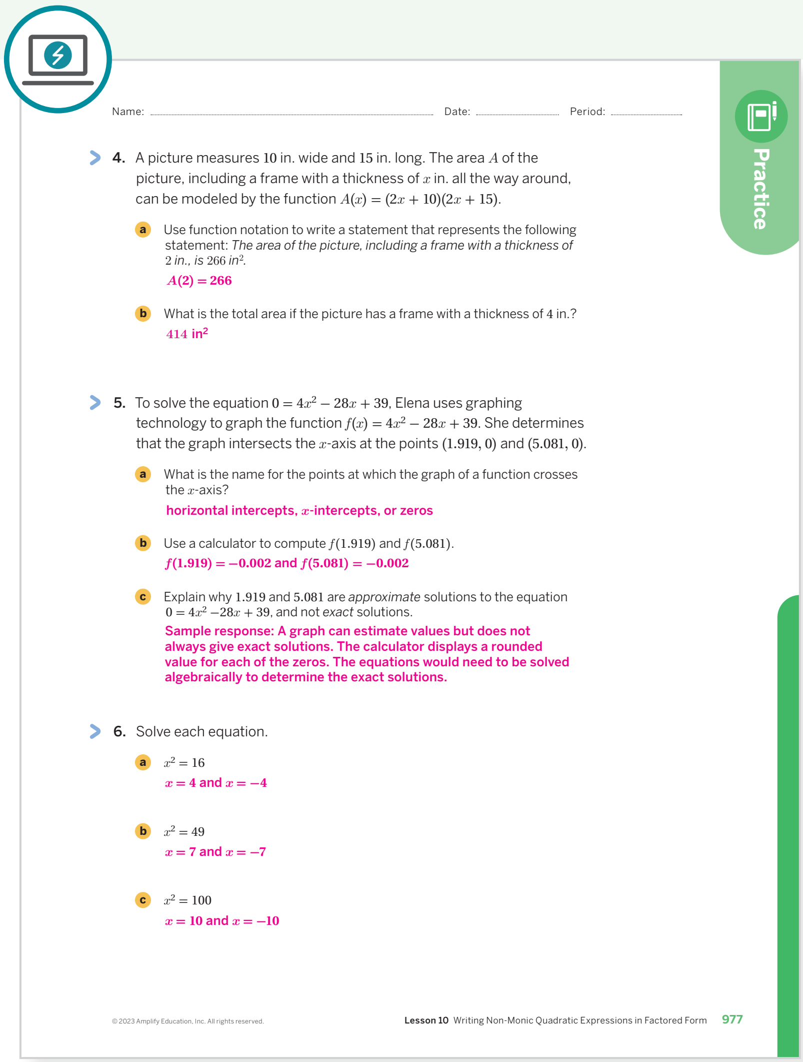
$5x^2 + 13x - 6 = 0$

$(5x - 2)(x + 3) = 0$

$x = 2$ or $x = -3$

Sample response: $x = 2$ is not a solution. Han did not set the linear factor $5x - 2$ equal to 0. The correct solutions are $x = \frac{2}{5}$ and $x = -3$.

976 **Unit 6** Quadratic Equations



Practice

Name: Date: Period:

4. A picture measures 10 in. wide and 15 in. long. The area A of the picture, including a frame with a thickness of x in. all the way around, can be modeled by the function $A(x) = (2x + 10)(2x + 15)$.
   - a) Use function notation to write a statement that represents the following statement: *The area of the picture, including a frame with a thickness of 2 in., is 266 in*^{2}.
     $A(2) = 266$
   - b) What is the total area if the picture has a frame with a thickness of 4 in.?
     414 in^2

5. To solve the equation $0 = 4x^2 - 28x + 39$, Elena uses graphing technology to graph the function $f(x) = 4x^2 - 28x + 39$. She determines that the graph intersects the x-axis at the points $(1.919, 0)$ and $(5.081, 0)$.
   - a) What is the name for the points at which the graph of a function crosses the x-axis?
     horizontal intercepts, x-intercepts, or zeros
   - b) Use a calculator to compute $f(1.919)$ and $f(5.081)$.
     $f(1.919) = -0.002$ and $f(5.081) = -0.002$
   - c) Explain why 1.919 and 5.081 are *approximate* solutions to the equation $0 = 4x^2 - 28x + 39$, and not *exact* solutions.
     Sample response: A graph can estimate values but does not always give exact solutions. The calculator displays a rounded value for each of the zeros. The equations would need to be solved algebraically to determine the exact solutions.

6. Solve each equation.
   - a) $x^2 = 16$
     $x = 4$ and $x = -4$
   - b) $x^2 = 49$
     $x = 7$ and $x = -7$
   - c) $x^2 = 100$
     $x = 10$ and $x = -10$

Lesson 10 Writing Non-Monic Quadratic Expressions in Factored Form 977

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 1	2
	5	Unit 6 Lesson 2	3
Formative ⚡	6	Unit 6 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 3

Completing the Square

Whole Class

In this Sub-Unit, students discover another strategy for solving quadratic equations. They examine the historical roots of “completing the square.”

SUB-UNIT

3 Completing the Square

Narrative Connections

How many ways can you crack an egg?

When Lemuel Gulliver washed onto the island of Lilliput, he found a nation of six-inch tall people at war with their neighbors. The source of their conflict? They couldn’t agree which was the right end of the egg to crack.

So, what can we learn from Jonathan Swift’s smirking satire, *Gulliver’s Travels*?

Like Gulliver, we know there’s more than one way to crack an egg. And so too, for our math problems.

At its core, the history of math is the history of how people solve problems. All developing civilizations faced the same kinds of problems, like how to grow sufficient numbers of crops, build storehouses, and collect taxes. And to solve their problems efficiently, they had to develop their own mathematical methods.

You can see this in the variety of ways different societies approached the problem of quadratic equations.

Building on their discovery of zero and negative numbers, the Indians and Persians in the 7th century tackled these problems *algebraically* — laying out specific procedures for multiplying and taking the square roots of different terms.

But more than a thousand years earlier, ancient Babylonian mathematicians, along with ancient Chinese mathematicians, approached the problem *geometrically*, making use of everyone’s favorite quadrilateral — the square!

Despite being scattered across the Asian continent and separated by millennia, these societies drew on their individual creativity, history, and learning to arrive at the same answer, through solutions that were uniquely their own.

Sub-Unit 3 Completing the Square 979

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how geometry can help solve quadratic equations in the following places:

- **Lesson 11, Activities 1–2:** Squares in Different Forms, Two Strategies
- **Lesson 12, Activities 1–3:** Is It Square?, Building Complete Squares, Algebraically Building Complete Squares
- **Lesson 13, Activity 1:** Solving by Completing the Square
- **Lesson 14, Activity 1:** Adding and Subtracting
- **Lesson 15, Activity 1:** Square in a Different Way

UNIT 6 | **LESSON 11**

Square Expressions

Let's examine how perfect squares can help us solve some quadratic equations more efficiently.

Focus

Goals

1. Comprehend that equations containing a square expression on both sides of the equal sign can be solved by finding square roots.
2. Comprehend that square expressions of the form $(x + n)^2$ are equivalent to the square expression $x^2 + 2nx + n^2$.
3. Use the structure of expressions to identify them as square expressions.

Coherence

Today

In this lesson, students analyze various examples of perfect squares. They apply the Distributive Property repeatedly to expand square expressions given in factored form. Students observe that square expressions are useful for solving equations because the solutions are found by taking their square roots. They use the plus-or-minus symbol (±) as a way to express both positive and negative solutions.

Previously

In Lesson 10, students rewrote factorable non-monic quadratic expressions in factored form using different strategies.

Coming Soon

Students will derive the formula for completing the square using manipulatives.

Rigor

- Students develop **conceptual understanding** of the structure of square expressions and how it can be used to solve quadratic equations that include square expressions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- Anchor Chart PDF, *Square Expressions*

Math Language Development

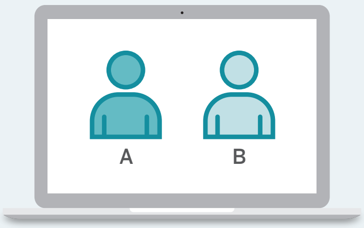
New words

- ***square expression***

Amps | Featured Activity

Activity 2
Step-by-Step Solving

Students observe and use two different methods for solving the same quadratic equation. Along the way, they document their steps in a dynamic table.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel insecure in Activities 1 and 2 when discerning the structure to rewrite expressions in factored form. Help them set goals for understanding the factoring so that they can motivate themselves to persevere and feel accomplished as they begin to see the patterns and recognize how to use them to factor.

Modifications to Pacing

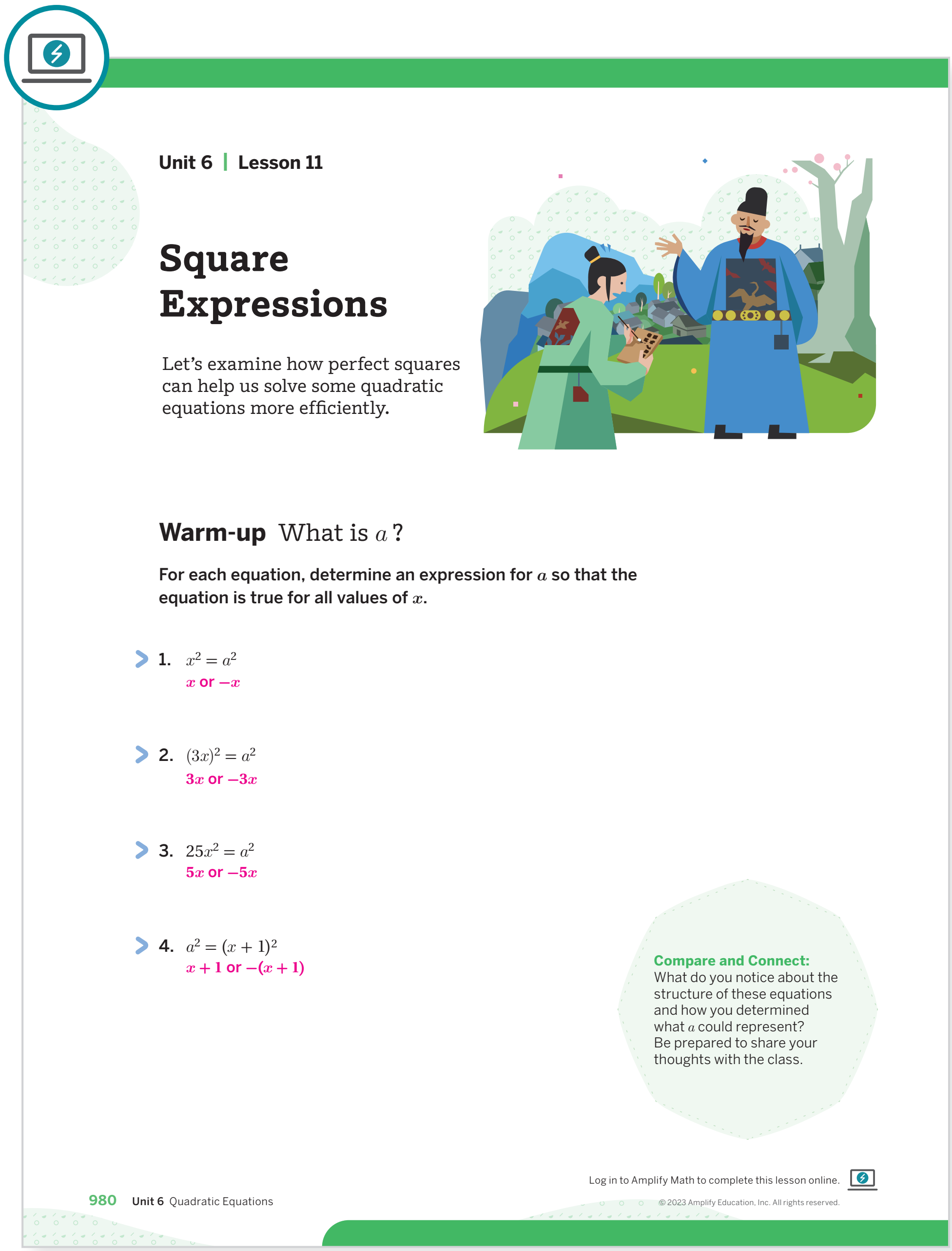
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 3 and 4 may be omitted.
- In **Activity 1**, have students only write equivalent expressions for the first three expressions in Problem 1.

Independent | 5 min

Warm-up What is a?

Students reason about equations containing quadratic expressions on both sides of the equal sign to prepare for their understanding of completing the square.



Unit 6 | Lesson 11

Square Expressions

Let's examine how perfect squares can help us solve some quadratic equations more efficiently.

Warm-up What is a?

For each equation, determine an expression for a so that the equation is true for all values of x.

1. $x^2 = a^2$
   x or $-x$
2. $(3x)^2 = a^2$
   $3x$ or $-3x$
3. $25x^2 = a^2$
   $5x$ or $-5x$
4. $a^2 = (x + 1)^2$
   $x + 1$ or $-(x + 1)$

Compare and Connect: What do you notice about the structure of these equations and how you determined what a could represent? Be prepared to share your thoughts with the class.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

980 Unit 6 Quadratic Equations

1 Launch

Provide students one minute of think time to study the equations before sharing their thinking with a partner.

2 Monitor

Help students get started by asking what they notice about each equation. Sample response: Both sides of the equations are square expressions.

Look for points of confusion:

- **Attempting to solve for each variable rather than analyzing the structure of each equation.** Ask, "In Problem 2, think of $3x$ as its own entity. What must it be equal to?"

Look for productive strategies:

- Thinking of $3x$ in Problem 2 and $x + 1$ in Problem 4 as a single entity.
- Realizing that the values of a could be positive or negative.

3 Connect

Have individual students share their strategies for determining the values of a. Select and sequence students using the structure of x^2 to determine a, or those that recognize that the opposite value of x can be substituted for a.

Highlight that each equation contains expressions that are square expressions.

Define the term ***square expression*** as the product of a linear expression and itself.

Display the equation $(2x - 9)(2x - 9) = a^2$.

Ask:

- "What expression could a be equal to?"
  $(2x - 9)$ or $-(2x - 9)$
- "How does this equation differ from Problem 4?"
  Sample response: In this equation, the coefficient of x is not 1, as it is in Problem 4.

MLR Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the connections between the structure of the equations and the strategies students used. Ask, "Which equation(s) could you solve by each of the following? How did the structure of the equation indicate this strategy?"

- "Only taking the square root of each side?"
- "First undoing another operation before taking the square root?"
- "Taking the square root and then undoing another operation?"

Power-up

To power up students' ability to determine the solutions of equations of the form $x^2 = p$, have students complete:

Diego and Han both solved the equation $x^2 = 49$. Diego says the solution is $x = 7$ while Han says the solution is $x = -7$. Noah says that both values, 7 and –7, are solutions of the equation. Who is correct? Be prepared to explain your thinking.

Noah; Sample response: $7^2 = 49$ and $(-7)^2 = 49$, so both values are solutions to the equation.

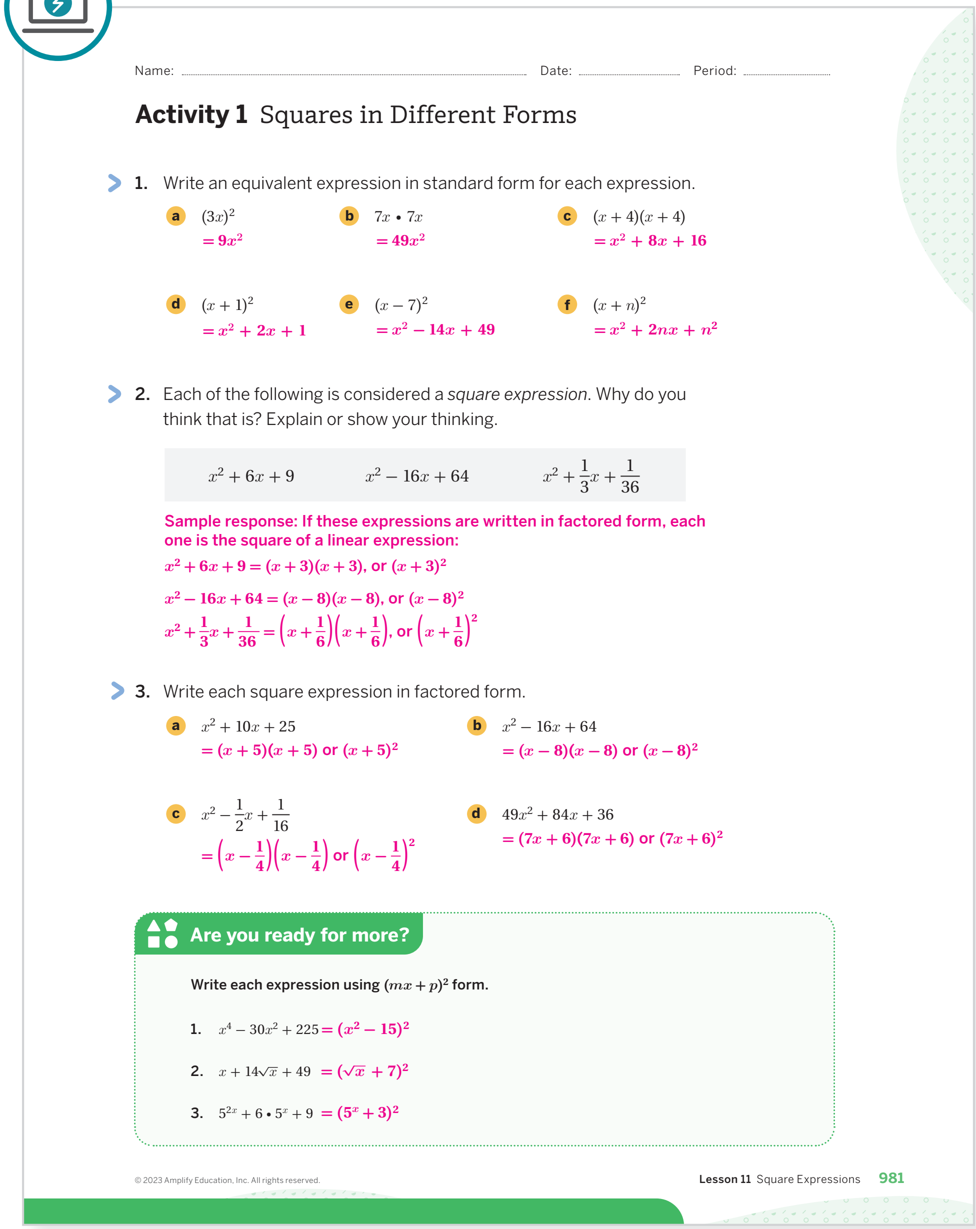
Use: Before the Warm-up

Informed by: Performance on Lesson 10, Practice Problem 6

Pairs | 15 min

Activity 1 Squares in Different Forms

Students observe square expressions in different forms to recognize their structure as $(ax \pm b)^2 = a^2x^2 \pm 2abx + b^2$.



Name: ______ Date: ______ Period: ______

Activity 1 Squares in Different Forms

1. Write an equivalent expression in standard form for each expression.

a $(3x)^2$ $= 9x^2$

b $7x \cdot 7x$ $= 49x^2$

c $(x+4)(x+4)$ $= x^2 + 8x + 16$

d $(x+1)^2$ $= x^2 + 2x + 1$

e $(x-7)^2$ $= x^2 - 14x + 49$

f $(x+n)^2$ $= x^2 + 2nx + n^2$

2. Each of the following is considered a *square expression*. Why do you think that is? Explain or show your thinking.

$x^2 + 6x + 9$ $x^2 - 16x + 64$ $x^2 + \frac{1}{3}x + \frac{1}{36}$

Sample response: If these expressions are written in factored form, each one is the square of a linear expression:

$x^2 + 6x + 9 = (x+3)(x+3)$, or $(x+3)^2$

$x^2 - 16x + 64 = (x-8)(x-8)$, or $(x-8)^2$

$x^2 + \frac{1}{3}x + \frac{1}{36} = \left(x + \frac{1}{6}\right)\left(x + \frac{1}{6}\right)$, or $\left(x + \frac{1}{6}\right)^2$

3. Write each square expression in factored form.

a $x^2 + 10x + 25$ $= (x+5)(x+5)$ or $(x+5)^2$

b $x^2 - 16x + 64$ $= (x-8)(x-8)$ or $(x-8)^2$

c $x^2 - \frac{1}{2}x + \frac{1}{16}$ $= \left(x - \frac{1}{4}\right)\left(x - \frac{1}{4}\right)$ or $\left(x - \frac{1}{4}\right)^2$

d $49x^2 + 84x + 36$ $= (7x+6)(7x+6)$ or $(7x+6)^2$

Are you ready for more?

Write each expression using $(mx+p)^2$ form.

1. $x^4 - 30x^2 + 225 = (x^2 - 15)^2$
2. $x + 14\sqrt{x} + 49 = (\sqrt{x} + 7)^2$
3. $5^{2x} + 6 \cdot 5^x + 9 = (5^x + 3)^2$

Lesson 11 Square Expressions 981

1 Launch

Have students work in pairs to complete Problem 1. Pause for a class discussion of Problem 2, then have students work independently on the rest of the activity.

2 Monitor

Help students get started by asking how they would expand the expressions in Problem 1.

Look for points of confusion:

- **Struggling to expand the expression in Problem 1f.** Have students use an area diagram or replace the variables with concrete values.
- **Struggling to determine why the expressions in Problem 2 are square expressions.** Have students rewrite the problems in factored form so that they notice the structure.

Look for productive strategies:

- Recognizing a pattern among the expressions after repeated use of the Distributive Property.
- Using an area diagram to write equivalent expressions.

3 Connect

Have individual students share their equivalent expressions. Select and sequence students who notice a pattern between the standard form and factored form of the expressions.

Highlight that each expression is considered a square expression. The squared variable term and the constant term in each expression are squares. The coefficient of the linear term is twice the product of the constant term and the coefficient of the squared variable term. Square expressions have a general form of $(ax + b)^2 = a^2x^2 + 2abx + b^2$ or $(ax - b)^2 = a^2x^2 - 2abx + b^2$.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide blank area diagrams for students to use if they choose as they complete Problem 1.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the expression that is being squared in each of the expressions given in Problem 1. For example, they could color code the following:

$(3x)^2$ $7x \cdot 7x$ $(x+4)(x+4)$

Then have them do the same for the factored forms of the expressions throughout the rest of the activity.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight how each of the expressions in this activity are considered square expressions, display the following two expressions and ask students to determine whether each expression is a square expression and explain their thinking.

Expression A	Expression B
$x^2 - 12x + 36$	$x^2 - 36$

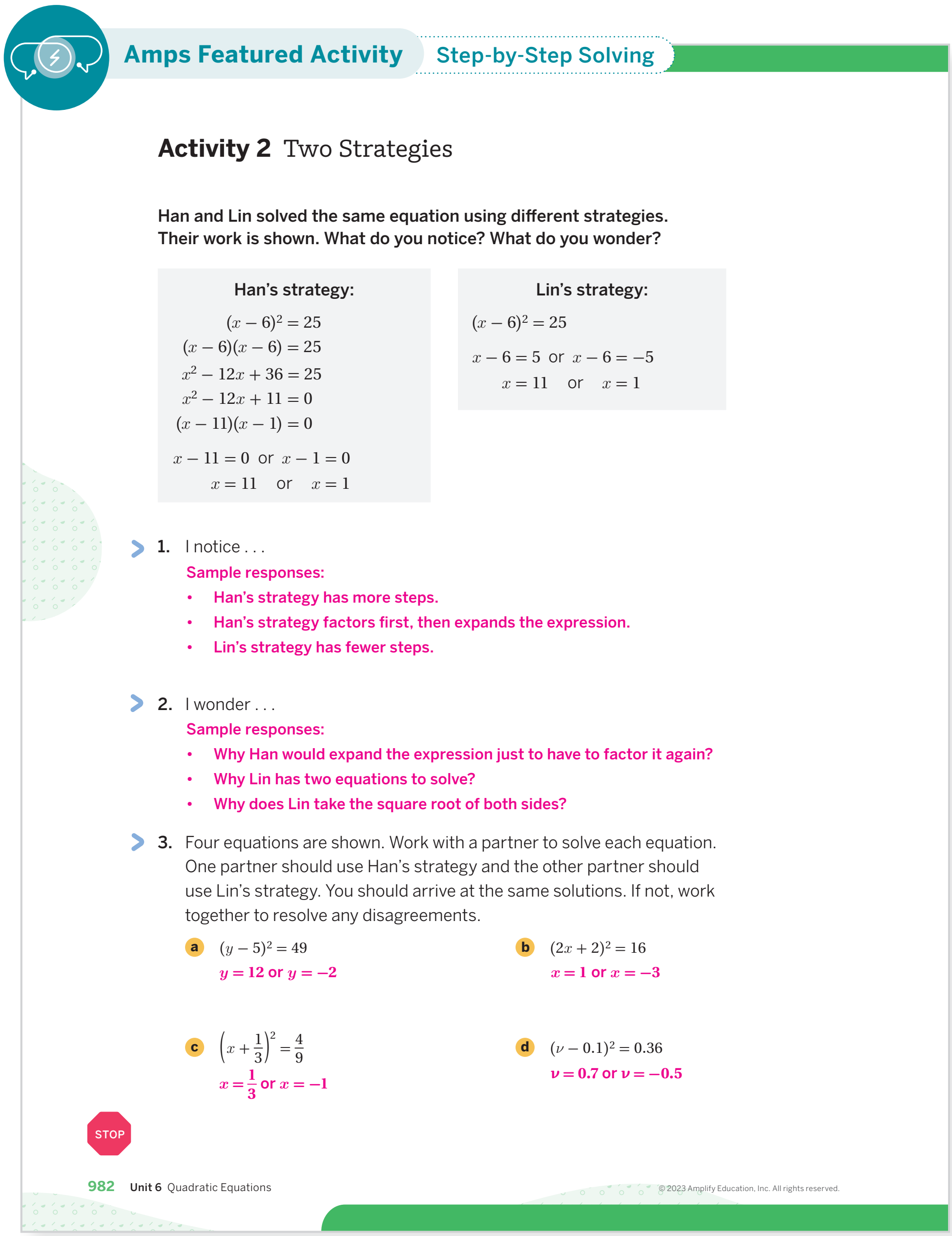
Listen for, and amplify, student reasoning that only Expression A is a square expression because its factored form consists of an expression multiplied by itself, $(x-6)(x-6)$. Expression B is a difference of squares and its factored form does not consist of an expression multiplied by itself. The two factors are different, $(x+6)(x-6)$.

Pairs | 20 min

Activity 2 Two Strategies

Students analyze two strategies to solve quadratic equations, connecting their previous understanding of solving equations with square roots to square expressions.

Amps Featured Activity Step-by-Step Solving



Activity 2 Two Strategies

Han and Lin solved the same equation using different strategies. Their work is shown. What do you notice? What do you wonder?

Han's strategy:

$(x-6)^2 = 25$
$(x-6)(x-6) = 25$
$x^2 - 12x + 36 = 25$
$x^2 - 12x + 11 = 0$
$(x-11)(x-1) = 0$
$x - 11 = 0$ or $x - 1 = 0$
$x = 11$ or $x = 1$

Lin's strategy:

$(x-6)^2 = 25$
$x - 6 = 5$ or $x - 6 = -5$
$x = 11$ or $x = 1$

1. I notice . . .
   Sample responses:
   - Han's strategy has more steps.
   - Han's strategy factors first, then expands the expression.
   - Lin's strategy has fewer steps.
2. I wonder . . .
   Sample responses:
   - Why Han would expand the expression just to have to factor it again?
   - Why Lin has two equations to solve?
   - Why does Lin take the square root of both sides?
3. Four equations are shown. Work with a partner to solve each equation. One partner should use Han's strategy and the other partner should use Lin's strategy. You should arrive at the same solutions. If not, work together to resolve any disagreements.

a. $(y-5)^2 = 49$
$y = 12$ or $y = -2$

b. $(2x+2)^2 = 16$
$x = 1$ or $x = -3$

c. $\left(x + \frac{1}{3}\right)^2 = \frac{4}{9}$
$x = \frac{1}{3}$ or $x = -1$

d. $(v - 0.1)^2 = 0.36$
$v = 0.7$ or $v = -0.5$

STOP

982 Unit 6 Quadratic Equations

1 Launch

Display Han's and Lin's strategies. Conduct the *Notice and Wonder* routine. Record student responses. Have students discuss the steps taken in each strategy.

2 Monitor

Help students get started by displaying the equation $x^2 = 25$, then have them solve it by taking the square root of each side. Make sure students provide both solutions, 5 and −5.

Look for points of confusion:

- **Thinking they need to add or subtract before taking the square root.** Point out the structure of each equation, emphasizing they need to undo the squaring before they can add or subtract to isolate the variable.

Look for productive strategies:

- Noticing that each side of each equation is either a square expression or perfect square.

3 Connect

Ask, "Who's strategy do you prefer, Han's or Lin's? Why?" Use the *Poll the Class* routine to determine which students prefer each strategy.

Display student responses.

Have students share their responses. Select and sequence students preferring Lin's strategy to Han's strategy.

Highlight that each equation includes a square expression and a square number. Emphasize that when taking the square root, they should consider both the positive solution and the negative solution.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Allow pairs to work together to solve each equation using both strategies, as opposed to each partner using either Han's strategy or Lin's strategy.

Extension: *Math Enrichment*

Have students determine whether they could use Lin's strategy to solve the equation $(x-6)(x+6) = 25$ and explain their thinking.
No; Sample response: The expression $(x-6)(x+6)$ is not a square expression because the factors are not the same. You can only take the square root of each side if the expression is a square expression.

Math Language Development

MLR8: Discussion Supports

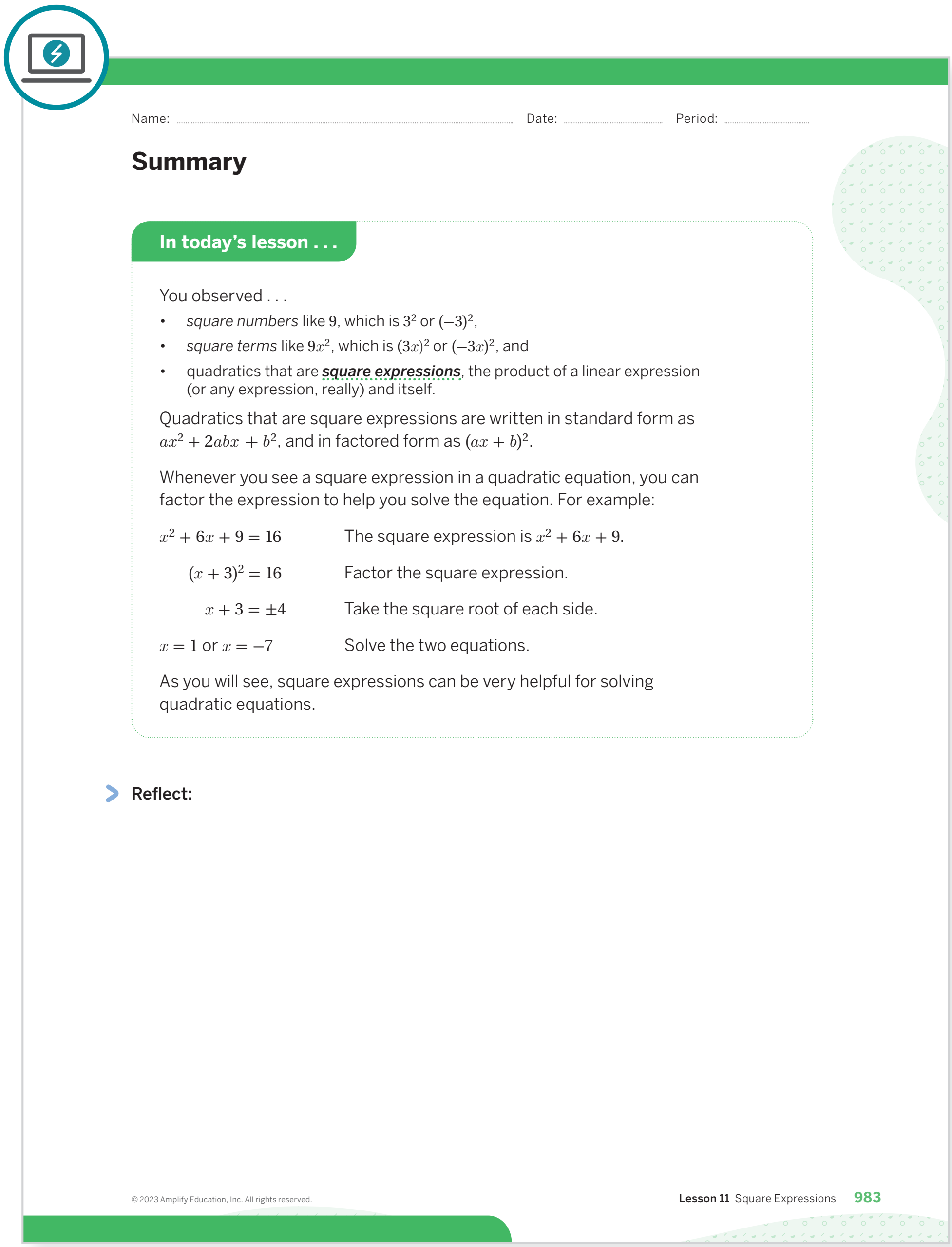
While students complete Problem 3, display or provide access to the Anchor Chart PDF, *Partner and Group Questioning*, for students to refer to as they compare solutions and discuss and resolve any disagreements. After they have agreed on the solutions, have them compare strategies and discuss these questions with their partner.

- "Are both strategies valid?"
- "Which strategy seems more efficient? Why do you think so?"
- "Are there any limitations to either of Han's or Lin's strategies?"

Whole Class | 5 min

Summary

Review and synthesize how the structure of square expressions can be used to factor them.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You observed . . .

- *square numbers* like 9, which is 3^2 or $(-3)^2$,
- *square terms* like $9x^2$, which is $(3x)^2$ or $(-3x)^2$, and
- quadratics that are ***square expressions***, the product of a linear expression (or any expression, really) and itself.

Quadratics that are square expressions are written in standard form as $ax^2 + 2abx + b^2$, and in factored form as $(ax + b)^2$.

Whenever you see a square expression in a quadratic equation, you can factor the expression to help you solve the equation. For example:

$x^2 + 6x + 9 = 16$	The square expression is $x^2 + 6x + 9$.
$(x + 3)^2 = 16$	Factor the square expression.
$x + 3 = \pm 4$	Take the square root of each side.
$x = 1$ or $x = -7$	Solve the two equations.

As you will see, square expressions can be very helpful for solving quadratic equations.

Reflect:

Synthesize

Display the following expressions:

Expression 1: $x^2 + 4x + 8$

Expression 2: $x^2 + 24x + 144$

Expression 3: $x^2 + 6x + 16$

Expression 4: $x^2 - 40x + 400$

Ask, "Which of the expressions are square expressions?" Expressions 2 and 4.

Have students share their strategies and thinking for determining which expressions are square expressions. Listen for students who can articulate why the first and third expressions are not square expressions.

Highlight that the third expression is not a square expression because the coefficient of the linear term is *not* twice the product of the square root of the constant term and the coefficient of the squared variable term. In other words, $6 \neq 2 \cdot 4 \cdot 1$.

Formalize vocabulary: ***square expression***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does recognizing quadratics that are square expressions help solve quadratic equations?"

Math Language Development

MLR2: Collect and Display

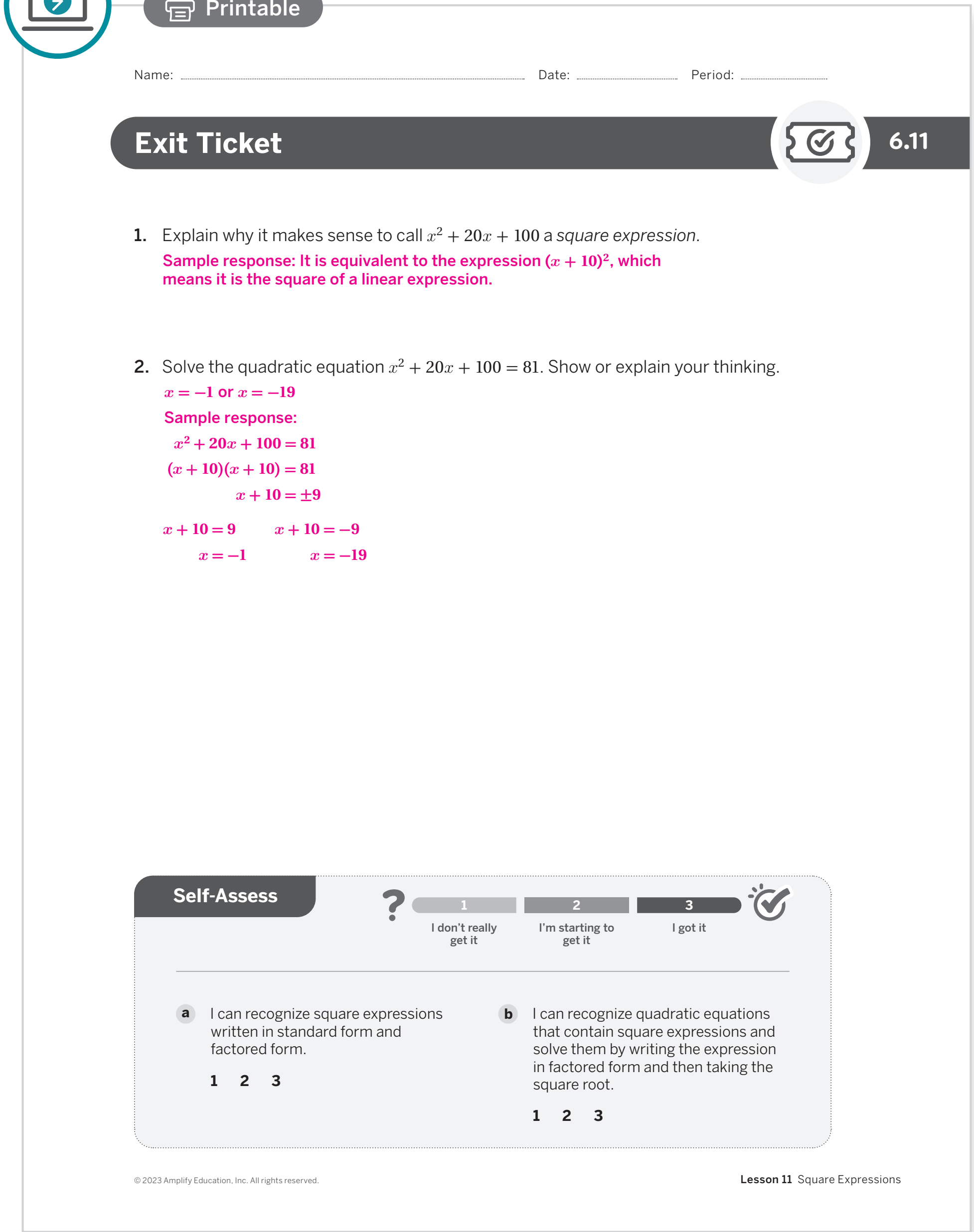
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *square expression* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of square expressions by solving a quadratic equation that contains a square expression.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 6.11

1. Explain why it makes sense to call $x^2 + 20x + 100$ a *square expression*.

   Sample response: It is equivalent to the expression $(x + 10)^2$, which means it is the square of a linear expression.

2. Solve the quadratic equation $x^2 + 20x + 100 = 81$. Show or explain your thinking.

   $x = -1$ or $x = -19$

   Sample response:

   $$x^2 + 20x + 100 = 81$$
   $$(x + 10)(x + 10) = 81$$
   $$x + 10 = \pm 9$$
   $$x + 10 = 9 \qquad x + 10 = -9$$
   $$x = -1 \qquad x = -19$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can recognize square expressions written in standard form and factored form.

1 2 3

b. I can recognize quadratic equations that contain square expressions and solve them by writing the expression in factored form and then taking the square root.

1 2 3

Lesson 11 Square Expressions

Success looks like . . .

- **Goal:** Comprehending that equations containing a square expression on both sides of the equal sign can be solved by finding square roots.
- **Goal:** Comprehending that square expressions of the form $(x + n)^2$ are equivalent to the square expression $x^2 + 2nx + n^2$.
- Using the structure of expressions to identify them as square expressions.
  - » Explaining why the given expression is a square expression in Problem 1.

Suggested next steps

If students struggle with explaining why the expression in Problem 1 is a square expression, consider:

- Reviewing the expressions in the Warm-up.
- Asking students to write the expression in factored form and then asking them what they notice.

If students solve the equation incorrectly in Problem 2, consider:

- Reviewing the expressions in Activity 1.
- Reviewing Han's and Lin's methods in Activity 2.
- Reviewing rewriting expressions in factored form.

If students do not state both solutions in Problem 2, consider:

- Reviewing why taking the square root of a number has both a positive solution and a negative solution.

Professional Learning

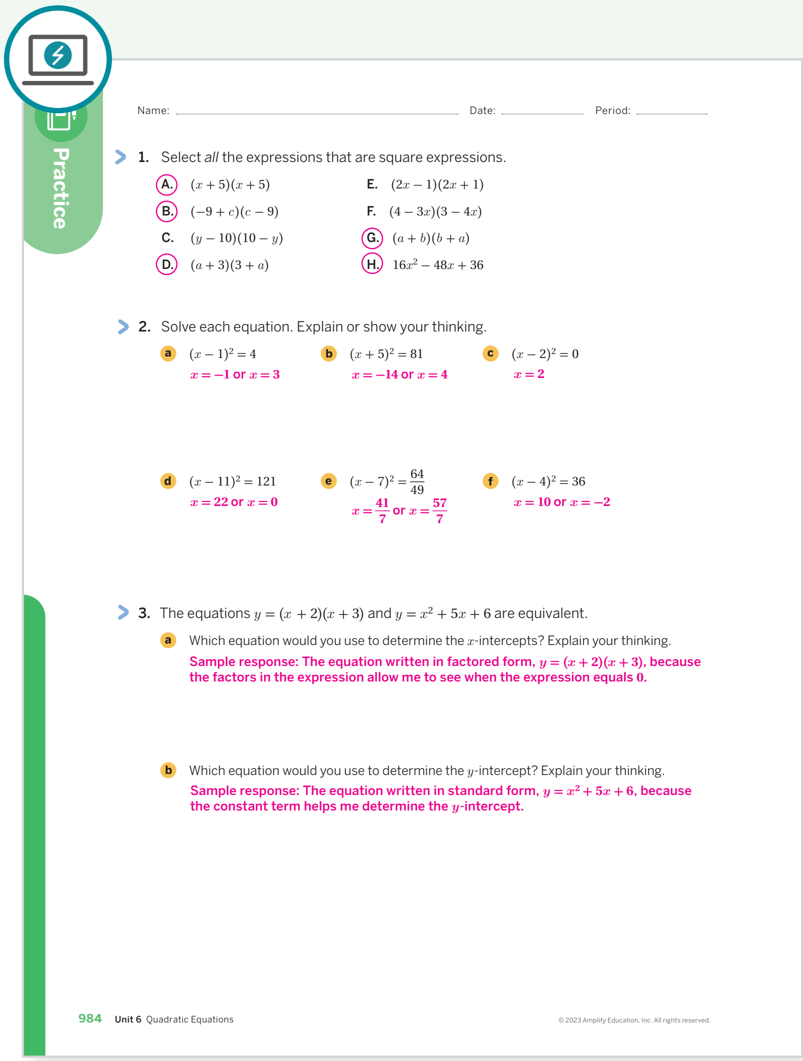
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students solved quadratics with square expressions. How will that support their understanding when completing the square?
- The focus of this lesson was solving quadratic equations with square expressions. How did solving quadratic equations with square expressions go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Select *all* the expressions that are square expressions.

[✓ A.] $(x+5)(x+5)$
[✓ B.] $(-9+c)(c-9)$
C. $(y-10)(10-y)$
[✓ D.] $(a+3)(3+a)$
E. $(2x-1)(2x+1)$
F. $(4-3x)(3-4x)$
[✓ G.] $(a+b)(b+a)$
[✓ H.] $16x^2-48x+36$

2. Solve each equation. Explain or show your thinking.

a. $(x-1)^2=4$
$x=-1$ or $x=3$

b. $(x+5)^2=81$
$x=-14$ or $x=4$

c. $(x-2)^2=0$
$x=2$

d. $(x-11)^2=121$
$x=22$ or $x=0$

e. $(x-7)^2=\frac{64}{49}$
$x=\frac{41}{7}$ or $x=\frac{57}{7}$

f. $(x-4)^2=36$
$x=10$ or $x=-2$

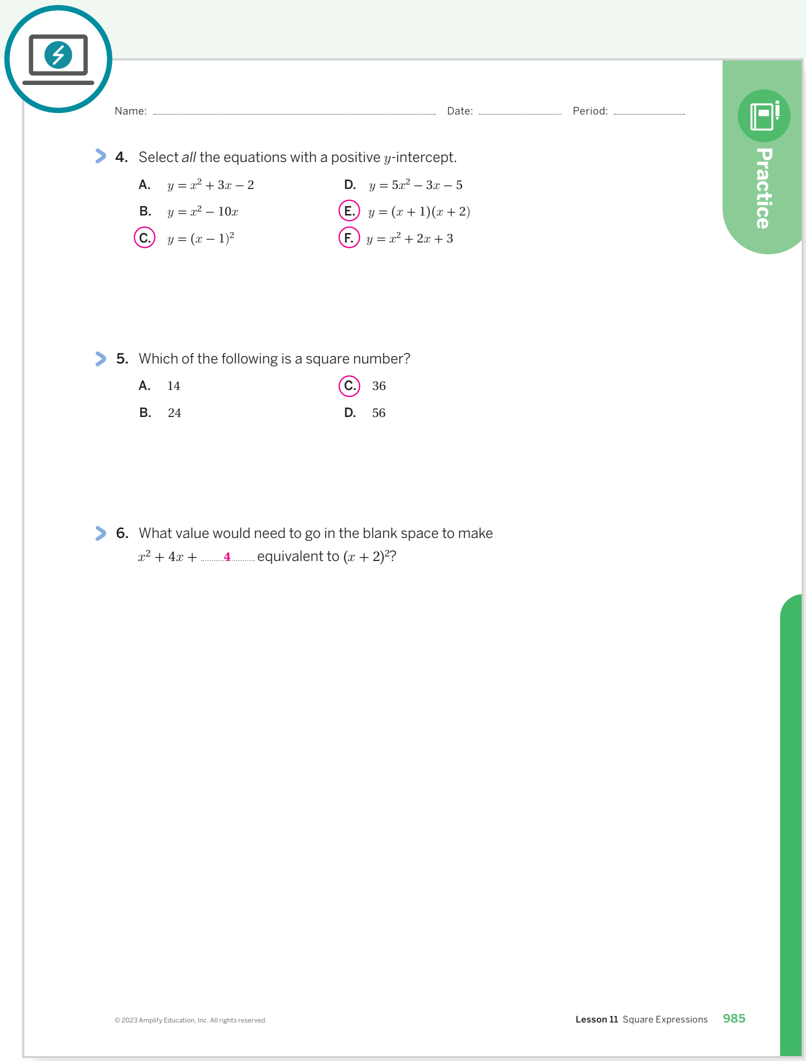
3. The equations $y=(x+2)(x+3)$ and $y=x^2+5x+6$ are equivalent.

a. Which equation would you use to determine the x-intercepts? Explain your thinking.

Sample response: The equation written in factored form, $y=(x+2)(x+3)$, because the factors in the expression allow me to see when the expression equals 0.

b. Which equation would you use to determine the y-intercept? Explain your thinking.

Sample response: The equation written in standard form, $y=x^2+5x+6$, because the constant term helps me determine the y-intercept.



Name: ________ Date: ________ Period: ________

4. Select *all* the equations with a positive y-intercept.

A. $y=x^2+3x-2$
B. $y=x^2-10x$
[✓ C.] $y=(x-1)^2$
D. $y=5x^2-3x-5$
[✓ E.] $y=(x+1)(x+2)$
[✓ F.] $y=x^2+2x+3$

5. Which of the following is a square number?

A. 14
B. 24
[✓ C.] 36
D. 56

6. What value would need to go in the blank space to make x^2+4x+ ....4.... equivalent to $(x+2)^2$?

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	3
	3	Activity 2	3
Spiral	4	Unit 5 Lesson 13	3
	5	Unit 5 Lesson 4	2
Formative ⚡	6	Unit 6 Lesson 12	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 12**

Completing the Square

Let's learn a new strategy for solving quadratic equations.

Focus

Goals

1. **Language Goal:** Explain that to "complete the square" is to determine the value of c that will make the expression $x^2 + bx + c$ a square expression. **(Speaking and Listening, Writing)**
2. **Language Goal:** Describe how to complete the square. **(Speaking and Listening, Writing)**

Coherence

Today

Students derive the formula for completing the square using manipulatives (algebra tiles). Then they complete the square for monic quadratic expressions. They model rewriting factorable standard form expressions in factored form using algebra tiles and area diagrams. Students use their understanding to complete the square algebraically, making use of structure and patterns.

Previously

In Lesson 11, students examined square quadratic expressions and equations. Students expanded expressions in factored form to write equivalent expressions in standard form.

Coming Soon

In Lesson 13, students will solve monic quadratic equations by completing the square.

Rigor

- Students build **conceptual understanding** of completing the square.
- Students strengthen their **procedural skills** of square expressions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	12 min	12 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- Anchor Chart PDF, *Square Expressions*
- algebra tiles

Math Language Development

New words

- ***completing the square***

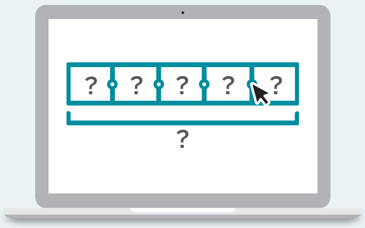
Review words

- *coefficient*
- *linear*
- *monic quadratic*
- *square expressions*

Amps | Featured Activity

Activity 1
Digital Algebra Tiles

Students create models with digital algebra tiles to visualize completing the square.



Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may feel lost if they do not notice the structure of an expression of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ and how it relates to the structure of an expression of the form $\left(x + \frac{b}{2}\right)^2$. Encourage them to persist as they look for structure. For example, ask them to shift their perspective by relating the parts of the expression to the algebra tiles and area diagrams that represent them.

Modifications to Pacing

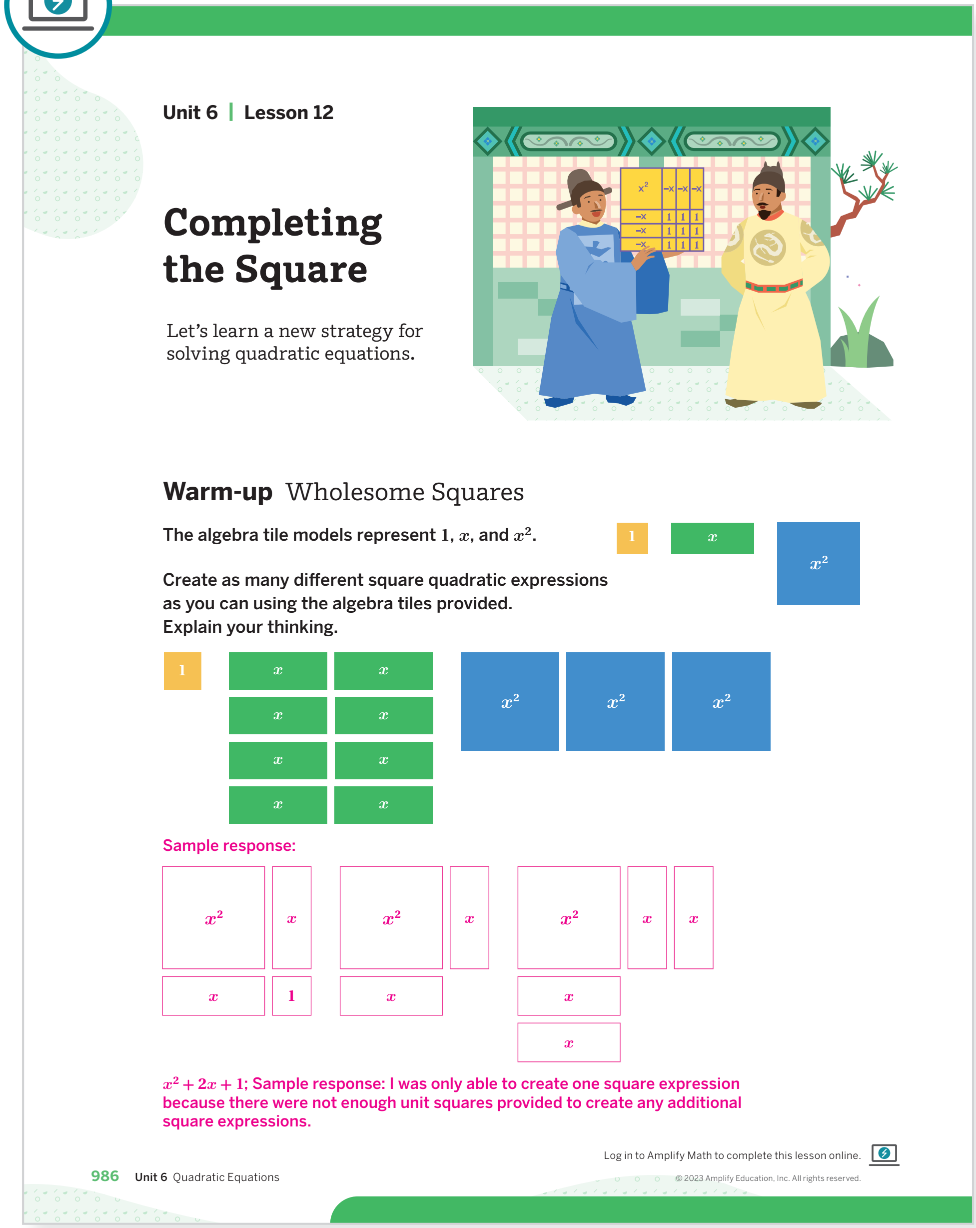
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, have students focus on the expressions in 1a, 1b, 1c, or 1f.

Pairs | 5 min

Warm-up Wholesome Squares

Students attempt to create square expressions with a limited number of algebra tiles, which allows them to visualize incomplete squares.



Unit 6 | Lesson 12

Completing the Square

Let's learn a new strategy for solving quadratic equations.

Warm-up Wholesome Squares

The algebra tile models represent 1, x, and x^2.

Create as many different square quadratic expressions as you can using the algebra tiles provided. Explain your thinking.

Sample response:

$x^2 + 2x + 1$; Sample response: I was only able to create one square expression because there were not enough unit squares provided to create any additional square expressions.

Log in to Amplify Math to complete this lesson online.

986 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide a set of algebra tiles to each pair of students and remind students of what each tile represents. First, have students work independently to attempt to create three square expressions with the listed tiles, then share with a partner. **Note:** It is only possible to create one square expression.

2 Monitor

Help students get started by having them write a few square quadratic expressions first and then try to build them.

Look for points of confusion:

- **"Needing" more tiles.** Have students build almost complete squares, leaving the missing tiles out.

Look for productive strategies:

- Noticing there are not enough unit tiles to build more than one square quadratic expression.
- Attempting to create small complete squares, due to the limited number of tiles.

3 Connect

Have pairs of students share strategies and any incomplete squares they built.

Ask, "How many additional tiles would you need to complete the square?" Sample responses: One unit square, four unit squares.

Display student solutions and incomplete squares with what students stated they needed to complete the square.

Highlight that when given an incomplete square, students can create a complete square by adding, or possibly subtracting, tiles.

Math Language Development

MLR8: Discussion Supports — Press for Reasoning

Before the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Explaining My Steps* to support students as they explain their strategy. Allow students to rehearse what they will say before sharing with the whole class during the Connect.

Power-up

To power up students' ability to determine the missing value in square expressions, have students complete:

Determine the missing value in the equation. Consider expanding the expression on the left side of the equation to support your answer.

$(x + 3)^2 = x^2 + 6x + \boxed{9}$

Use: Before Activity 1

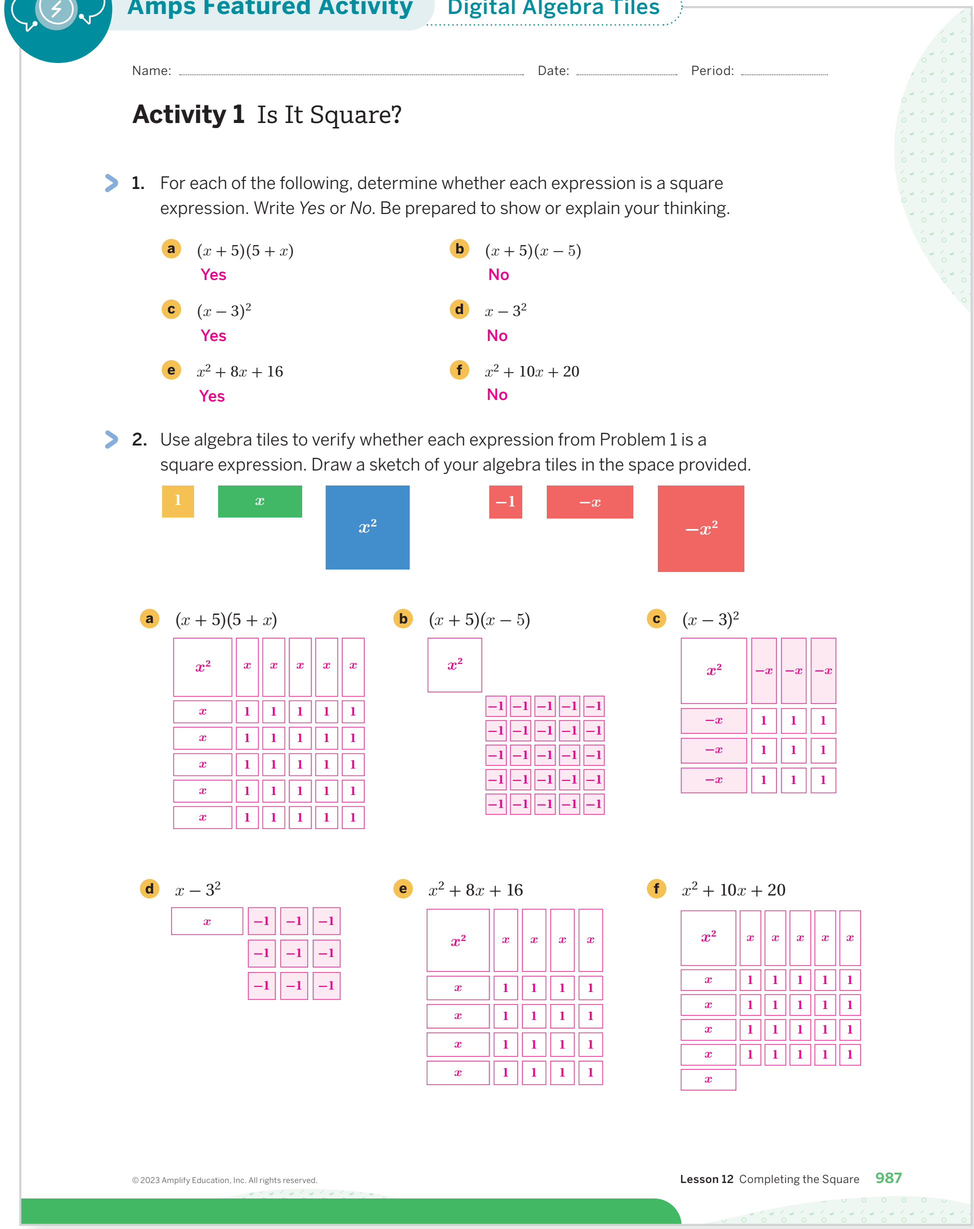
Informed by: Performance on Lesson 11, Practice Problem 6

Pairs | 12 min

Activity 1 Is It Square?

Students reinforce their understanding of square expressions by comparing square and non-square expressions algebraically and visually.

Amps Featured Activity Digital Algebra Tiles



Name: ______ Date: ______ Period: ______

Activity 1 Is It Square?

1. For each of the following, determine whether each expression is a square expression. Write *Yes* or *No*. Be prepared to show or explain your thinking.

a $(x + 5)(5 + x)$ Yes	b $(x + 5)(x - 5)$ No
c $(x - 3)^2$ Yes	d $x - 3^2$ No
e $x^2 + 8x + 16$ Yes	f $x^2 + 10x + 20$ No

2. Use algebra tiles to verify whether each expression from Problem 1 is a square expression. Draw a sketch of your algebra tiles in the space provided.

a $(x + 5)(5 + x)$

b $(x + 5)(x - 5)$

c $(x - 3)^2$

d $x - 3^2$

e $x^2 + 8x + 16$

f $x^2 + 10x + 20$

Lesson 12 Completing the Square 987

1 Launch

Provide a set of algebra tiles to each pair of students. Use the *Poll the Class* routine for each expression in Problem 1. Have students explain their thinking before they continue with the activity.

2 Monitor

Help students get started by asking them what strategies they can use to determine whether an expression is a square expression.

Look for points of confusion:

- **Thinking the expression in Problem 1b is a square expression.** Ask, "Can you rewrite the factors as $(x + n)^2$ or $(x - n)^2$?" No.

Look for productive strategies:

- Using patterns from factored form or standard form to determine if an expression is square.
- Building all expressions in a square frame, to note the missing tiles for a complete square.

3 Connect

Ask:

- "What does a square expression look like?" Sample response: The product of a linear expression and itself.
- "What does a square expression modeled with algebra tiles look like?" Sample response: A complete square, with no missing tiles.

Have pairs of students share strategies for determining whether an expression is square.

Display each expression modeled with algebra tiles.

Highlight that when trying to build square expressions, students can identify what tiles would be needed to build a complete square and how that relates to what terms might be missing from the expression.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital algebra tiles to help them visualize completing the square.

Accessibility: *Guide Processing and Visualization*

Display or provide access to the Anchor Chart PDF, *Square Expressions*, for students to use as a reference during this activity.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you display each expression modeled with algebra tiles, draw students' attention to the connections between the expressions that are square expressions and what their arrangement of algebra tiles looks like. Highlight that the expressions that are *not* square expressions look "uncompleted." Ask:

- "How does the arrangement in Problem 2b show a difference of squares? How does it show that there is no linear term, when written in standard form?"
- "What terms are missing from this arrangement that would 'complete the square'? How would the corresponding expression be altered?"

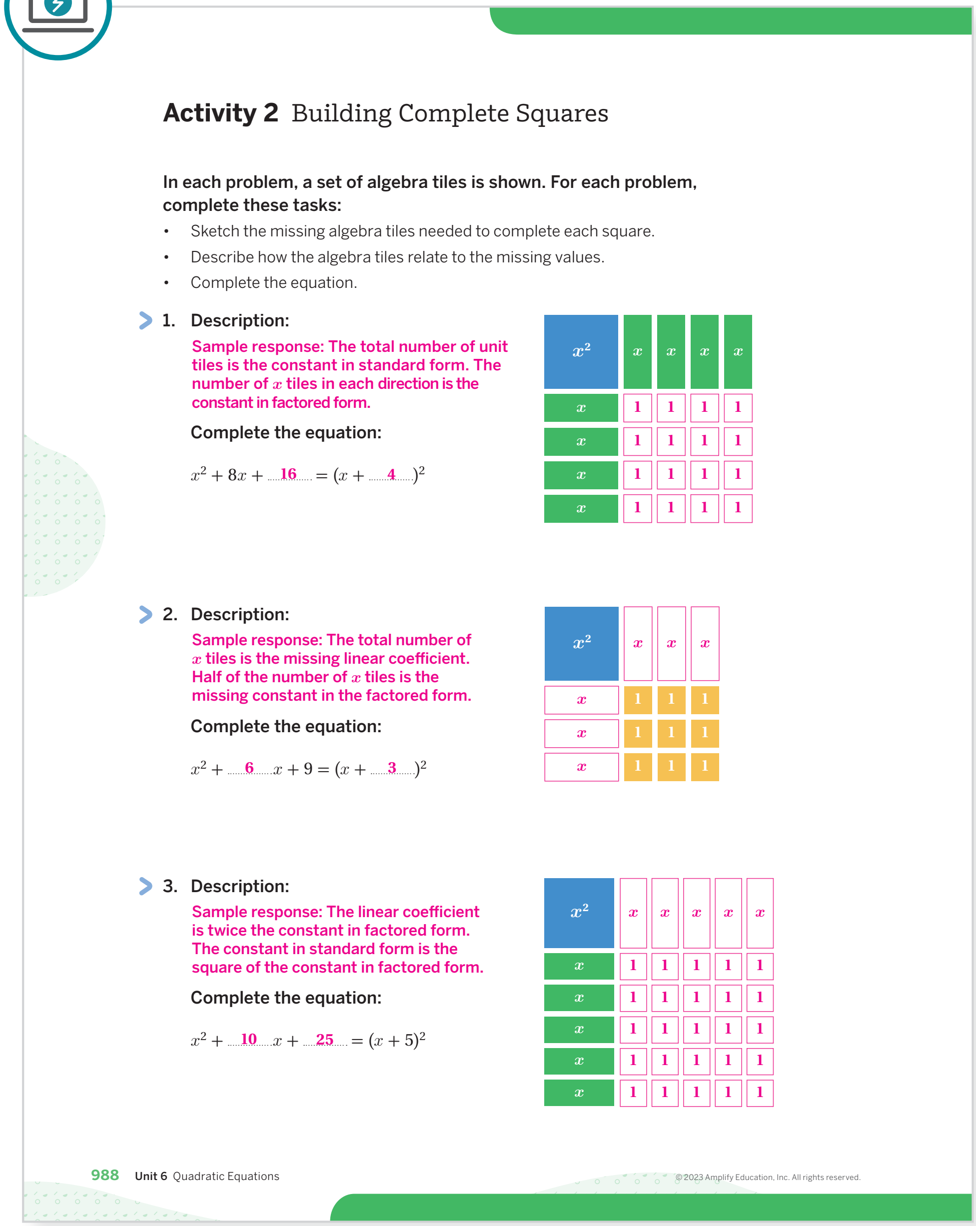
English Learners

Annotate the algebra tile arrangements with the phrases *square expression* or *not a square expression*.

Pairs | 12 min

Activity 2 Building Complete Squares

Students visually complete the square with algebra tiles to prepare them to algebraically complete the square.



Activity 2 Building Complete Squares

In each problem, a set of algebra tiles is shown. For each problem, complete these tasks:

- Sketch the missing algebra tiles needed to complete each square.
- Describe how the algebra tiles relate to the missing values.
- Complete the equation.

1. **Description:**

   Sample response: The total number of unit tiles is the constant in standard form. The number of x tiles in each direction is the constant in factored form.

   Complete the equation:

   $x^2 + 8x + \underline{16} = (x + \underline{4})^2$

2. **Description:**

   Sample response: The total number of x tiles is the missing linear coefficient. Half of the number of x tiles is the missing constant in the factored form.

   Complete the equation:

   $x^2 + \underline{6}x + 9 = (x + \underline{3})^2$

3. **Description:**

   Sample response: The linear coefficient is twice the constant in factored form. The constant in standard form is the square of the constant in factored form.

   Complete the equation:

   $x^2 + \underline{10}x + \underline{25} = (x + 5)^2$

988 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide a set of algebra tiles to each pair of students. Have student pairs discuss each problem before completing the problems individually. Then, compare solutions and patterns.

2 Monitor

Help students get started by having them label the side lengths of the algebra tiles to determine the factored form expression, then multiply to determine the standard form expression.

Look for points of confusion:

- **Having difficulty recognizing that b is twice the square root of the value of c.** Have students relate the side lengths to the area, then to the expressions.

Look for productive strategies:

- Annotating each expression to draw connections between representations.
- Using precise language when describing each term and expression type.
- Noticing and applying the structure between the standard form and factored form.

3 Connect

Have pairs of students share strategies and patterns noticed in the tiles and expressions.

Ask, "What patterns did you notice between the standard form square expressions and factored form square expressions?" Sample response: The coefficient of the linear term in the standard form is 2 times the constant term in the factored form. Both the coefficient of the squared variable term and the constant term are squares.

Highlight that the squared variable term and the constant term in each expression are squares. The coefficient of the linear term is twice the product of the constant term and the coefficient of the squared variable term.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital algebra tiles to help them visualize completing the square.

Accessibility: *Guide Processing and Visualization*

Suggest that students draw the outline of a square around the algebra tile arrangement to help them visualize the missing tiles that would be needed to complete the square.

Extension: *Math Enrichment*

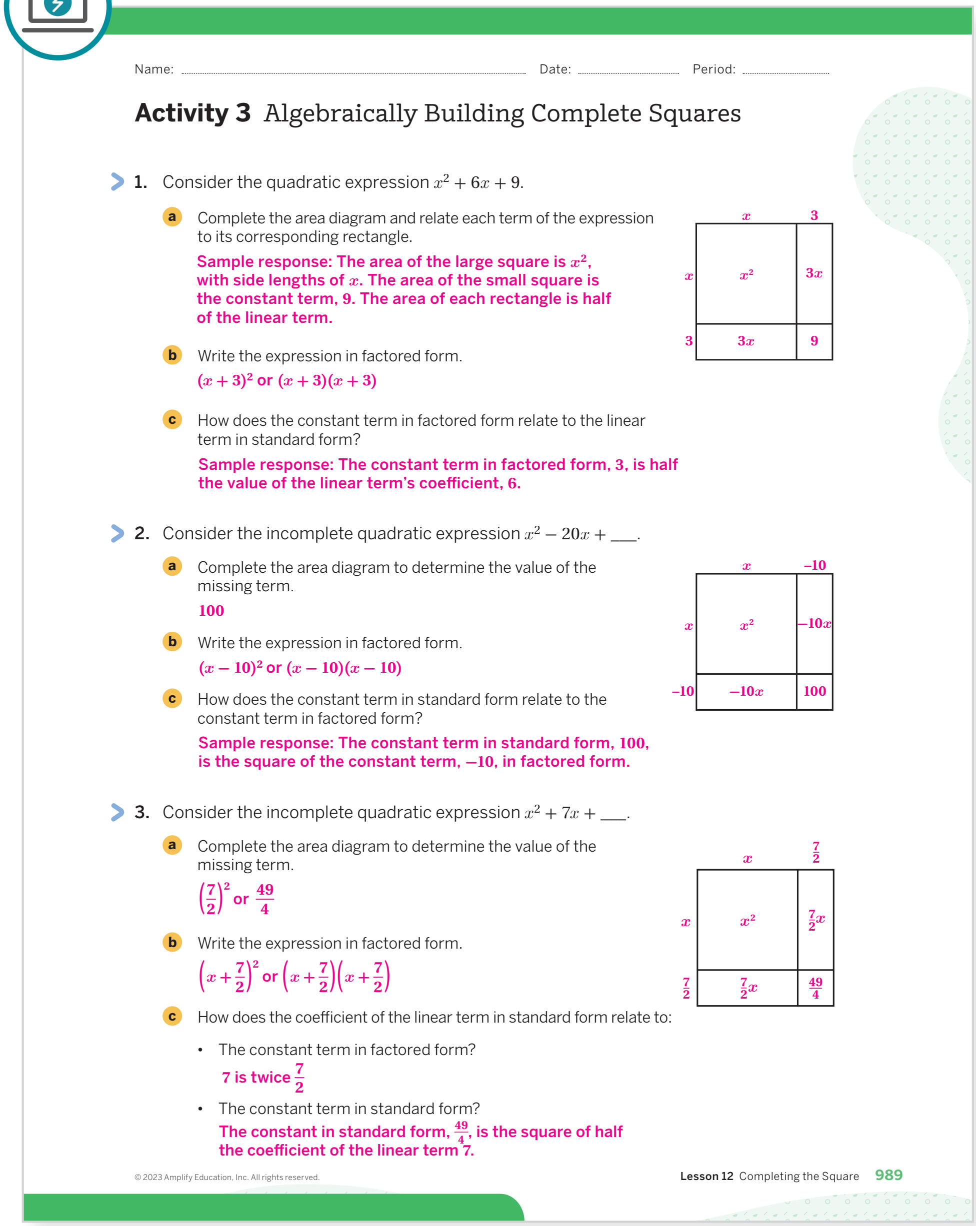
Have students use the algebra tile arrangements to explain why the following statements are true for square expressions. Sample responses shown.

- The constant of the standard form is the square of the constant in factored form. The total number of 1-tiles is the square of the side length, e.g., $4^2 = 16$, $3^2 = 9$, and $5^2 = 25$.
- The coefficient of the linear term in standard form is twice the constant in factored form. The total number of x-tiles represents half the perimeter of the square of 1-tiles. This means there are twice as many x-tiles as the side length of the square, in 1-tiles.

Pairs | 12 min

Activity 3 Algebraically Building Complete Squares

Students complete area diagrams by completing the square to informally prove the general formula.



Name: ____ Date: ____ Period: ____

Activity 3 Algebraically Building Complete Squares

1. Consider the quadratic expression $x^2 + 6x + 9$.

   a. Complete the area diagram and relate each term of the expression to its corresponding rectangle.

   Sample response: The area of the large square is x^2, with side lengths of x. The area of the small square is the constant term, 9. The area of each rectangle is half of the linear term.

	x	3
x	x^2	$3x$
3	$3x$	9

   b. Write the expression in factored form.

   $(x + 3)^2$ **or** $(x + 3)(x + 3)$

   c. How does the constant term in factored form relate to the linear term in standard form?

   Sample response: The constant term in factored form, 3, is half the value of the linear term's coefficient, 6.

2. Consider the incomplete quadratic expression $x^2 - 20x + \_\_\_$.

   a. Complete the area diagram to determine the value of the missing term.

   100

	x	-10
x	x^2	$-10x$
-10	$-10x$	100

   b. Write the expression in factored form.

   $(x - 10)^2$ **or** $(x - 10)(x - 10)$

   c. How does the constant term in standard form relate to the constant term in factored form?

   Sample response: The constant term in standard form, 100, is the square of the constant term, –10, in factored form.

3. Consider the incomplete quadratic expression $x^2 + 7x + \_\_\_$.

   a. Complete the area diagram to determine the value of the missing term.

   $\left(\frac{7}{2}\right)^2$ **or** $\frac{49}{4}$

	x	$\frac{7}{2}$
x	x^2	$\frac{7}{2}x$
$\frac{7}{2}$	$\frac{7}{2}x$	$\frac{49}{4}$

   b. Write the expression in factored form.

   $\left(x + \frac{7}{2}\right)^2$ **or** $\left(x + \frac{7}{2}\right)\left(x + \frac{7}{2}\right)$

   c. How does the coefficient of the linear term in standard form relate to:

   - The constant term in factored form?

     7 is twice $\frac{7}{2}$

   - The constant term in standard form?

     The constant in standard form, $\frac{49}{4}$, is the square of half the coefficient of the linear term 7.

1 Launch

Display the first problem. Provide one minute of think-time, then have student pairs discuss solutions and strategies before discussing as a whole class. Afterwards, have student pairs complete the remaining problems.

2 Monitor

Help students get started by referring back to patterns noticed in Activity 2.

Look for points of confusion:

- **Having difficulty determining the constant term in Problem 2.** Prompt students to work backward in the area diagram to determine the factors along the sides.
- **Not writing a difference in Problem 2b.** Have students rewrite their factored form to standard form to notice their error.
- **Struggling to generalize the process in Problem 4.** Prompt students to describe the steps they took to write each equivalent expression in Problems 1–3 and to apply similar steps.

Look for productive strategies:

- Annotating each expression to draw connections between representations.
- Using precise language when describing each term and expression type.
- Noticing and applying the structure between the standard form and factored form.

Activity 3 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the linear coefficients of each expression in one color, the constant term of the standard form (whether provided or missing) in another color, and the constant of the factored expression (once determined) in a third color. Ask them to look for connections between the values they color coded.
For example: $x^2 + 6x + 9 = (x + 3)(x + 3)$

- The coefficient of the linear term, $+ 6$, is twice the constant of the factored term, $+ 3$.
- The constant of the standard form, $+ 9$, is the square of the constant of the factor term, $+ 3$.

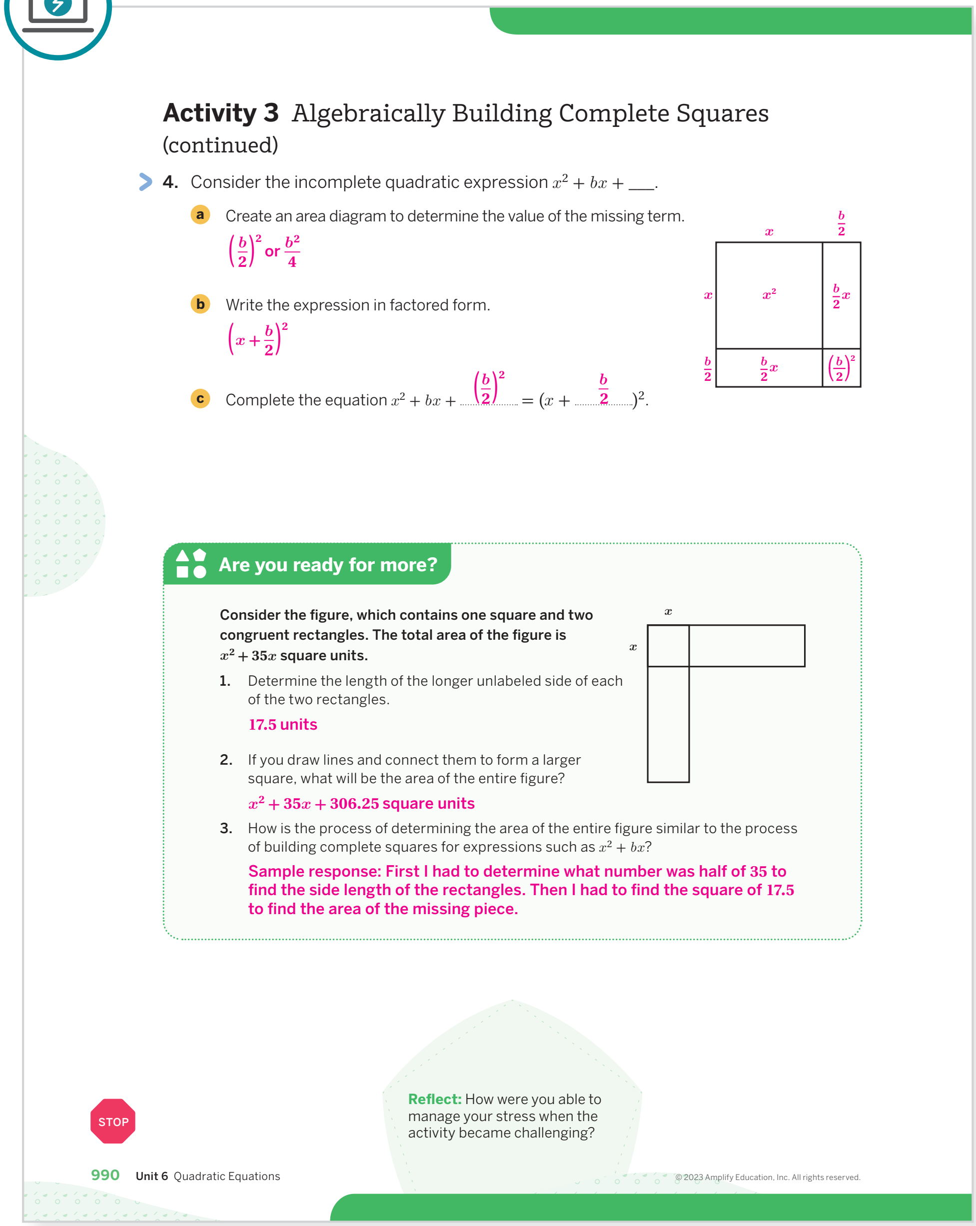
Accessibility: *Optimize Access to Tools*

Allow the continued use of algebra tiles should students choose to use them during this activity.

Pairs | 12 min

Activity 3 Algebraically Building Complete Squares (continued)

Students complete area diagrams by completing the square to informally prove the general formula.



Activity 3 Algebraically Building Complete Squares (continued)

4. Consider the incomplete quadratic expression $x^2 + bx +$ ___.

a Create an area diagram to determine the value of the missing term.

$\left(\frac{b}{2}\right)^2$ or $\frac{b^2}{4}$

b Write the expression in factored form.

$\left(x + \frac{b}{2}\right)^2$

c Complete the equation $x^2 + bx + \underline{\left(\frac{b}{2}\right)^2} = (x + \underline{\frac{b}{2}})^2$.

Are you ready for more?

Consider the figure, which contains one square and two congruent rectangles. The total area of the figure is $x^2 + 35x$ square units.

1. Determine the length of the longer unlabeled side of each of the two rectangles.

   17.5 units

2. If you draw lines and connect them to form a larger square, what will be the area of the entire figure?

   $x^2 + 35x + 306.25$ square units

3. How is the process of determining the area of the entire figure similar to the process of building complete squares for expressions such as $x^2 + bx$?

   Sample response: First I had to determine what number was half of 35 to find the side length of the rectangles. Then I had to find the square of 17.5 to find the area of the missing piece.

Reflect: How were you able to manage your stress when the activity became challenging?

3 Connect

Have pairs of students share their thinking and strategies for determining the missing value in the standard form expression, factored form expression, and strategies for completing Problem 4.

Display the correct responses to Problem 4.

Highlight the connection between the visual and algebraic general square expressions. Then, connect the general square expression with specific problems in the activity.

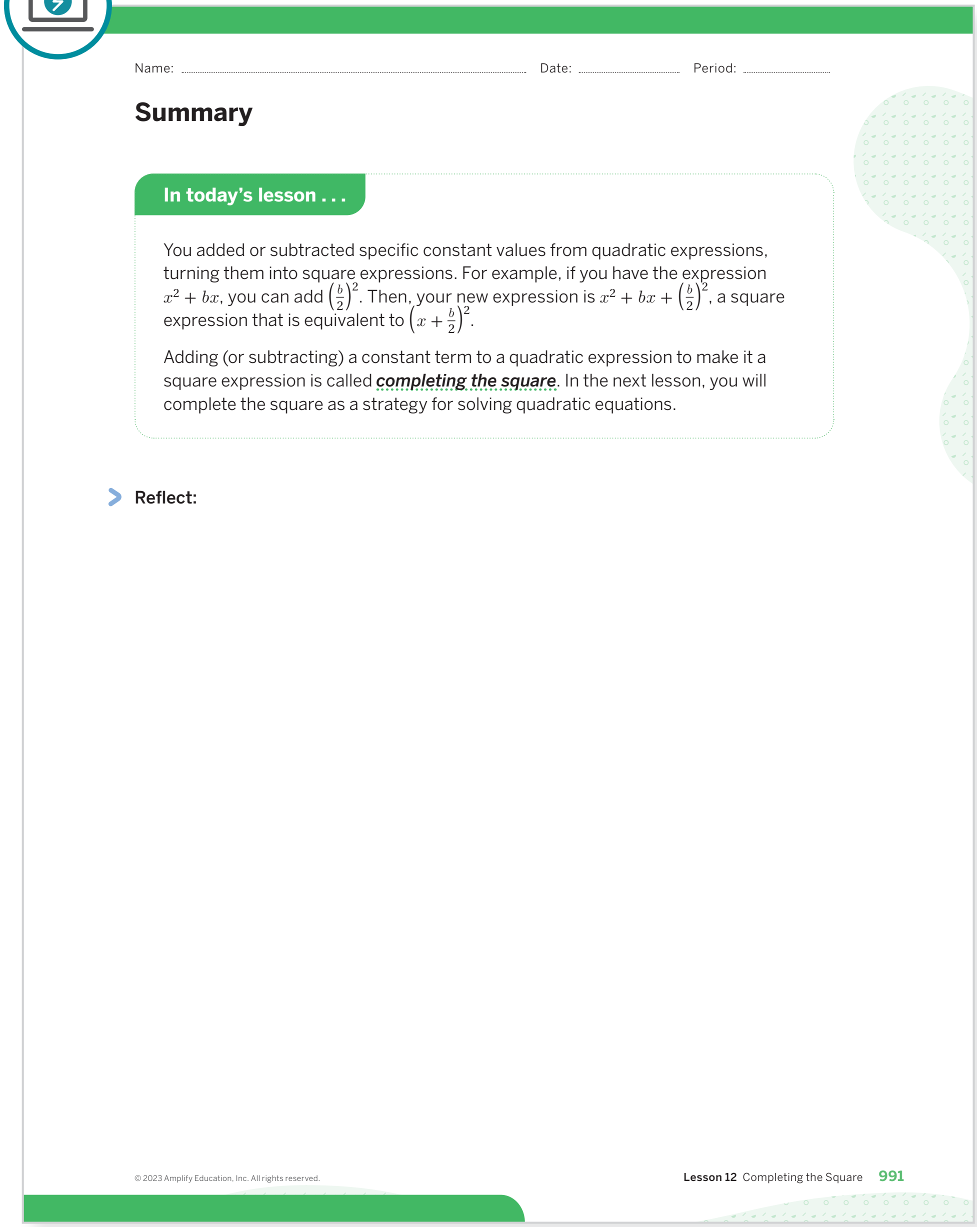
Define ***completing the square*** as determining the constant term to add or subtract in order to create a square expression.

Ask, "What are the general steps for completing the square?" Sample response: Determine the missing constant term when the expression is written in standard form, such that the expression becomes a square expression. This can be done by determining half the value of the coefficient of the linear term and then squaring it.

Whole Class | 5 min

Summary

Review and synthesize completing the square, visually and algebraically.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You added or subtracted specific constant values from quadratic expressions, turning them into square expressions. For example, if you have the expression $x^2 + bx$, you can add $\left(\frac{b}{2}\right)^2$. Then, your new expression is $x^2 + bx + \left(\frac{b}{2}\right)^2$, a square expression that is equivalent to $\left(x + \frac{b}{2}\right)^2$.

Adding (or subtracting) a constant term to a quadratic expression to make it a square expression is called ***completing the square***. In the next lesson, you will complete the square as a strategy for solving quadratic equations.

Reflect:

Synthesize

Display an example of an incomplete square modeled with algebra tiles and written in standard form.

Have students share the process of completing the square and strategies to complete the square algebraically.

Highlight the benefits of completing the square. The goal is to create a square expression in order to help solve a related quadratic equation.

Formalize vocabulary: ***completing the square***

Ask, "When might completing the square be useful?" Sample responses: Solving quadratic equations that cannot be factored, writing quadratic equations in vertex form.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did using algebra tiles and area diagrams help you understand how to complete the square in quadratic expressions?"
- "What is the value you see in completing the square?"

Math Language Development

MLR2: Collect and Display

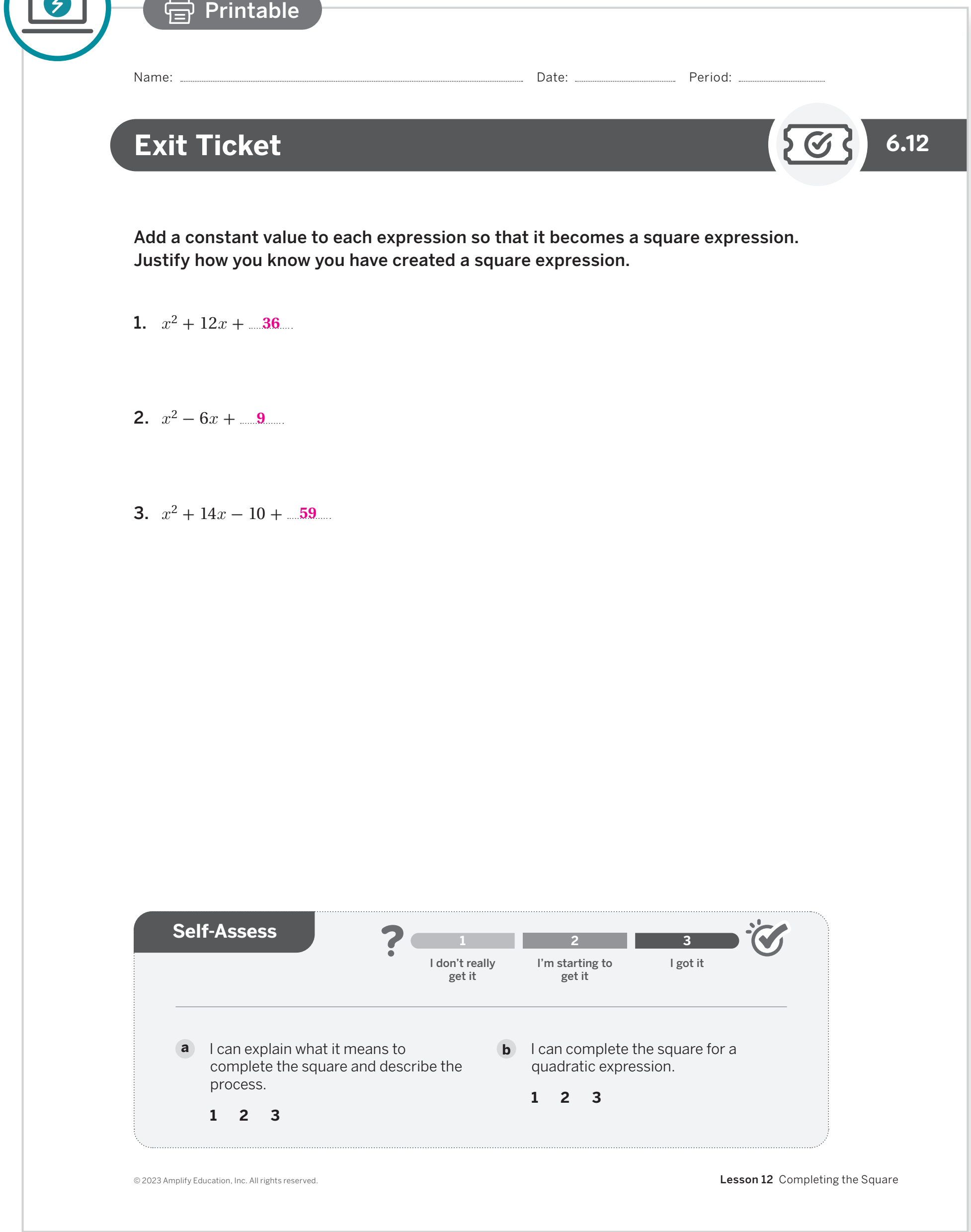
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *completing the square* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of completing the square by adding different values to standard form expressions to make square expressions.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 6.12

Add a constant value to each expression so that it becomes a square expression. Justify how you know you have created a square expression.

1. $x^2 + 12x +$ 36
2. $x^2 - 6x +$ 9
3. $x^2 + 14x - 10 +$ 59

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain what it means to complete the square and describe the process.

1 2 3

b I can complete the square for a quadratic expression.

1 2 3

Lesson 12 Completing the Square

Success looks like . . .

- **Language Goal:** Explaining that to "complete the square" is to determine the value of c that will make the expression $x^2 + bx + c$ a square expression. **(Speaking and Listening, Writing)**
- **Language Goal:** Describing how to complete the square. **(Speaking and Listening, Writing)**
  - » Completing the square in Problems 1–3.

Suggested next steps

If students do not correctly complete the square in Problems 1 and 2, consider:

- Reviewing visual and algebraic strategies from Activities 1 and 2.
- Assigning Practice Problem 2.
- Asking, "What is the relationship between the linear and constant term when completing the square?" Sample response: The coefficient of the linear term in standard form is twice that of the constant term in factored form.

Note: Problem 3 is a formative question for Lesson 14.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did using algebra tiles reveal about your students as learners?
- How did using algebra tiles and area diagrams set students up to develop their conceptual understanding of completing the square? What might you change for the next time you teach this lesson?

Math Language Development

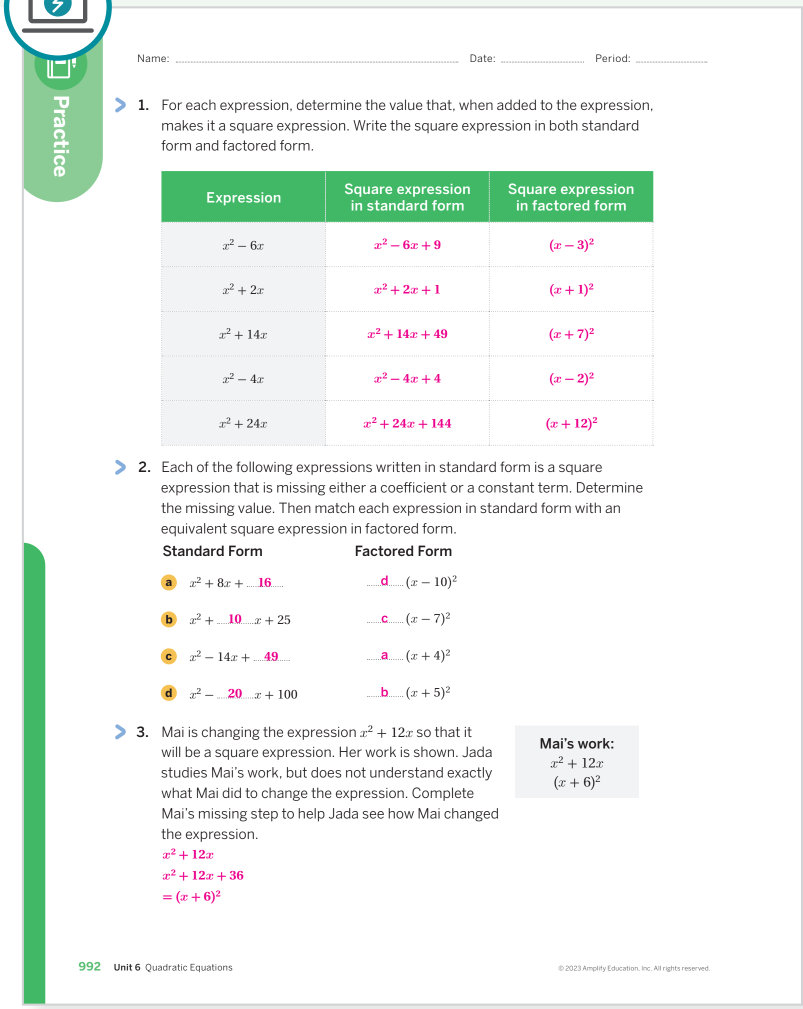
Language Goal: Explaining that to "complete the square" is to determine the value of c that will make the expression $x^2 + bx + c$ a square expression.

Reflect on students' language development toward this goal.

- Do students' explanations or work shown in their responses to the Exit Ticket problems demonstrate they understand what it means to "complete the square"?
- How have the language routines used in this lesson helped students develop their mathematical language to describe what it means to "complete the square"? Do they use terms, such as *square expression* or *two equal factors*?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. For each expression, determine the value that, when added to the expression, makes it a square expression. Write the square expression in both standard form and factored form.

Expression	Square expression in standard form	Square expression in factored form
$x^2 - 6x$	$x^2 - 6x + 9$	$(x - 3)^2$
$x^2 + 2x$	$x^2 + 2x + 1$	$(x + 1)^2$
$x^2 + 14x$	$x^2 + 14x + 49$	$(x + 7)^2$
$x^2 - 4x$	$x^2 - 4x + 4$	$(x - 2)^2$
$x^2 + 24x$	$x^2 + 24x + 144$	$(x + 12)^2$

2. Each of the following expressions written in standard form is a square expression that is missing either a coefficient or a constant term. Determine the missing value. Then match each expression in standard form with an equivalent square expression in factored form.

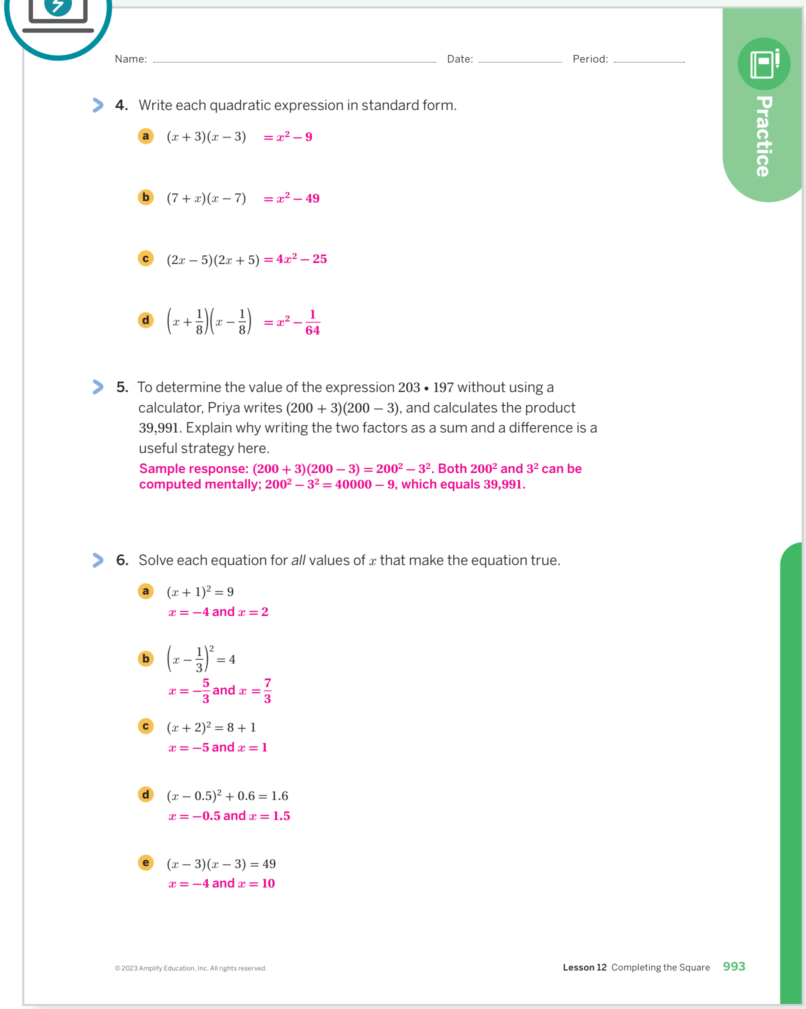
Standard Form	Factored Form
a $x^2 + 8x +$ 16	d $(x - 10)^2$
b $x^2 +$ 10 $x + 25$	c $(x - 7)^2$
c $x^2 - 14x +$ 49	a $(x + 4)^2$
d $x^2 -$ 20 $x + 100$	b $(x + 5)^2$

3. Mai is changing the expression $x^2 + 12x$ so that it will be a square expression. Her work is shown. Jada studies Mai's work, but does not understand exactly what Mai did to change the expression. Complete Mai's missing step to help Jada see how Mai changed the expression.

Mai's work:
$x^2 + 12x$
$(x + 6)^2$

$x^2 + 12x$
$x^2 + 12x + 36$
$= (x + 6)^2$

992 **Unit 6** Quadratic Equations



Practice

Name: ______ Date: ______ Period: ______

4. Write each quadratic expression in standard form.
   - a $(x + 3)(x - 3)$ $= x^2 - 9$
   - b $(7 + x)(x - 7)$ $= x^2 - 49$
   - c $(2x - 5)(2x + 5) = 4x^2 - 25$
   - d $\left(x + \frac{1}{8}\right)\left(x - \frac{1}{8}\right)$ $= x^2 - \frac{1}{64}$

5. To determine the value of the expression 203 • 197 without using a calculator, Priya writes $(200 + 3)(200 - 3)$, and calculates the product 39,991. Explain why writing the two factors as a sum and a difference is a useful strategy here.
   Sample response: $(200 + 3)(200 - 3) = 200^2 - 3^2$. Both 200^2 and 3^2 can be computed mentally; $200^2 - 3^2 = 40000 - 9$, which equals 39,991.

6. Solve each equation for *all* values of x that make the equation true.
   - a $(x + 1)^2 = 9$
     $x = -4$ and $x = 2$
   - b $\left(x - \frac{1}{3}\right)^2 = 4$
     $x = -\frac{5}{3}$ and $x = \frac{7}{3}$
   - c $(x + 2)^2 = 8 + 1$
     $x = -5$ and $x = 1$
   - d $(x - 0.5)^2 + 0.6 = 1.6$
     $x = -0.5$ and $x = 1.5$
   - e $(x - 3)(x - 3) = 49$
     $x = -4$ and $x = 10$

Lesson 12 Completing the Square 993

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 3	2
	3	Activity 3	2
Spiral	4	Unit 6 Lesson 8	2
	5	Unit 6 Lesson 8	3
Formative ⚡	6	Unit 6 Lesson 13	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 13**

Solving Quadratic Equations by Completing the Square

Let's see if completing the square can help us solve equations.

Focus

Goals

1. **Language Goal:** Describe a process for completing the square to express any monic quadratic equation in the form $(x + p)^2 = q$. **(Speaking and Listening, Writing)**
2. Express any monic quadratic equation in the form $(x + p)^2 = q$ and solve the equation by calculating square roots.
3. Solve quadratic equations of the form $x^2 + bx + c$ by rearranging terms and completing the square.

Coherence

Today

Students continue to build on their understanding of completing the square and use it to determine solutions of a monic quadratic equation. They learn that completing the square can be used to solve any quadratic equation, including equations that have non-integer rational number coefficients. Students make use of the same structure that helped them with less complicated expressions.

Previously

In Lesson 12, students derived the formula for completing the square and used concrete models to build their conceptual understanding of the process.

Coming Soon

In Lesson 14, students will complete the square to write quadratic functions in vertex form.

Rigor

- Students solve equations by completing the square to develop **procedural fluency** for completing the square.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
5 min	20 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, Sentence *Stems, Comparing and Contrasting*
- Anchor Chart PDF, Sentence *Stems, Math Talk*
- Anchor Chart PDF, *Completing the Square*
- algebra tiles (as needed)

Math Language Development

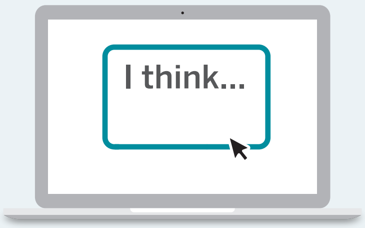
Review words

- *completing the square*
- *monic quadratic*
- *quadratic equation*
- *square expression*

Amps | **Featured Activity**

Warm-up
See Student Thinking

Students are asked to explain their thinking behind solving equations, and these explanations are available to you digitally, in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may lack attention toward, be dismissive of, or be defensive of other students' perspectives and thinking that do not align with theirs or lack clarity as students attempt to communicate their thinking precisely with one another in Activity 3. Have students repeat their group members' expressed thinking back to them to help their peer more precisely clarify their thinking, and also to encourage active listening and consideration of the perspective of others.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

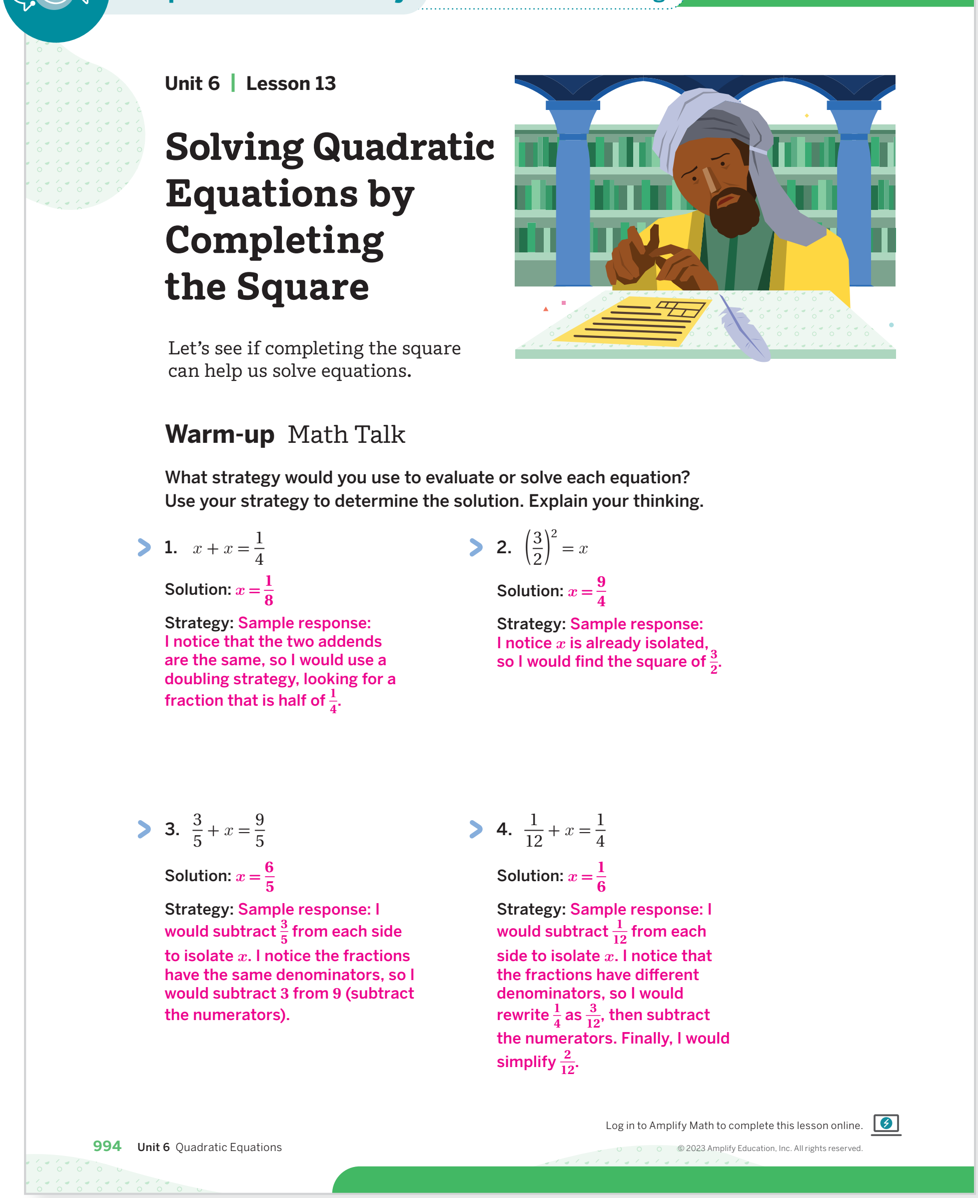
- In **Activity 1**, have students only complete 2 problems for each strategy.
- Optional **Activity 2** may be omitted.
- In **Activity 3**, assign between 1–3 equations per group.

Independent | 5 min

Warm-up Math Talk

Students discuss strategies for solving simple equations with fractions to develop fluency for similar computations used to solve quadratic equations.

Amps Featured Activity See Student Thinking



Unit 6 | **Lesson 13**

Solving Quadratic Equations by Completing the Square

Let's see if completing the square can help us solve equations.

Warm-up Math Talk

What strategy would you use to evaluate or solve each equation? Use your strategy to determine the solution. Explain your thinking.

1. $x + x = \frac{1}{4}$

Solution: $x = \frac{1}{8}$

Strategy: Sample response: I notice that the two addends are the same, so I would use a doubling strategy, looking for a fraction that is half of $\frac{1}{4}$.

2. $\left(\frac{3}{2}\right)^2 = x$

Solution: $x = \frac{9}{4}$

Strategy: Sample response: I notice x is already isolated, so I would find the square of $\frac{3}{2}$.

3. $\frac{3}{5} + x = \frac{9}{5}$

Solution: $x = \frac{6}{5}$

Strategy: Sample response: I would subtract $\frac{3}{5}$ from each side to isolate x. I notice the fractions have the same denominators, so I would subtract 3 from 9 (subtract the numerators).

4. $\frac{1}{12} + x = \frac{1}{4}$

Solution: $x = \frac{1}{6}$

Strategy: Sample response: I would subtract $\frac{1}{12}$ from each side to isolate x. I notice that the fractions have different denominators, so I would rewrite $\frac{1}{4}$ as $\frac{3}{12}$, then subtract the numerators. Finally, I would simplify $\frac{2}{12}$.

Log in to Amplify Math to complete this lesson online.

994 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the *Math Talk* routine. Display one problem at a time. Give students think-time for each problem and provide a signal to use when they have a solution and a strategy.

2 Monitor

Help students get started by first asking, "What value(s) of y make the equation $y + y = 10$ true? What strategies did you use to solve it?" 5; Sample response: I added the two y terms together to get $2y$ and then I divided each side of the equation by 2 to get $y = 5$.

Look for points of confusion:

- **Determining a solution, but unable to explain a strategy.** Prompt students to solve the problem using a two-way chart, explaining each step in their strategy.

Look for productive strategies:

- Using precise language when explaining their strategies.
- Using the structure of the equations to solve them mentally.

3 Connect

Have individual students share their strategies for solving each equation.

Ask:

- "Who can restate _____'s reasoning in a different way?"
- "Did anyone use the same strategy, but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Can anyone add on to _____'s strategy?"

Display the equations and solutions.

Highlight that to square a fraction, square both the numerator and denominator. Dividing by 2 is equivalent to multiplying by $\frac{1}{2}$.

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Connect, as students share their strategies for solving each equation, revoice their statements by demonstrating precise mathematical language. For example:

If a student says . . .	Revoice by asking . . .
"There are two xs in the equation in Problem 1. So, I added them."	"Why is it a valid strategy to add $x + x$? Are these like terms? How do you know?"

English Learners

Display or provide students the Anchor Chart PDF, *Sentence Stems, Math Talk* to support students as they explain their strategies.

Power-up

To power up students' ability to solve equations of the form $(x + m)^2 = p$, have students complete:

1. What are the solutions to the equation $x^2 = 9$? 3 and −3
2. What must $(x + 1)$ be equal to in the equation $(x + 1)^2 = 9$? $x + 1 = 3$ or $x + 1 = -3$
3. What must x be equal to in the equation $(x + 1)^2 = 9$? How do you know this? $x = 2$ or $x = -4$; Subtract 1 from each side of the equations $x + 1 = 3$ and $x + 1 = -3$.

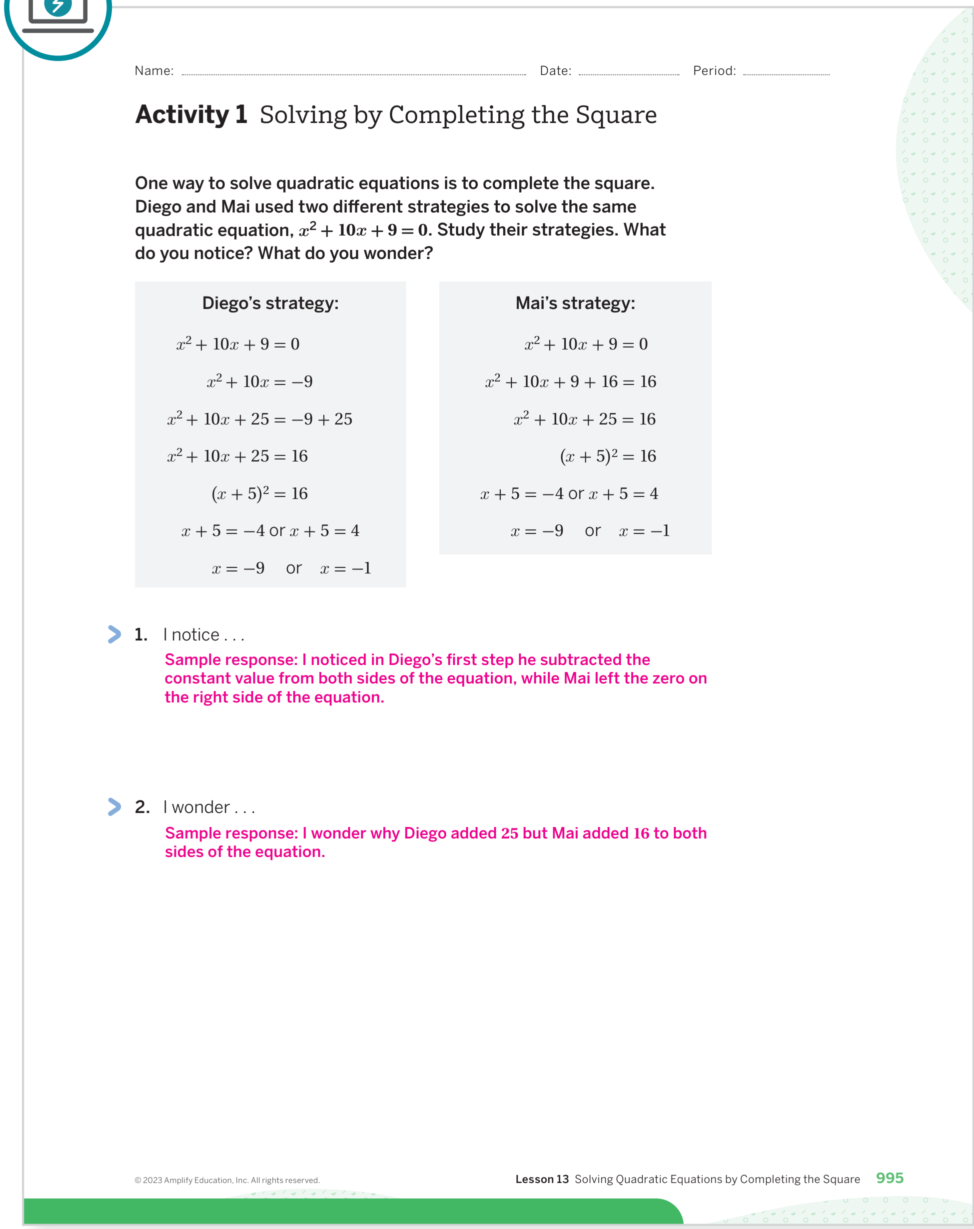
Use: Before Activity 1

Informed by: Performance on Lesson 12, Practice Problem 6

Small Groups | 20 min

Activity 1 Solving by Completing the Square

Students use completing the square to solve quadratic equations.



Name: ____________ Date: ________ Period: ________

Activity 1 Solving by Completing the Square

One way to solve quadratic equations is to complete the square. Diego and Mai used two different strategies to solve the same quadratic equation, $x^2 + 10x + 9 = 0$. Study their strategies. What do you notice? What do you wonder?

Diego's strategy:	Mai's strategy:
$x^2 + 10x + 9 = 0$	$x^2 + 10x + 9 = 0$
$x^2 + 10x = -9$	$x^2 + 10x + 9 + 16 = 16$
$x^2 + 10x + 25 = -9 + 25$	$x^2 + 10x + 25 = 16$
$x^2 + 10x + 25 = 16$	$(x + 5)^2 = 16$
$(x + 5)^2 = 16$	$x + 5 = -4$ or $x + 5 = 4$
$x + 5 = -4$ or $x + 5 = 4$	$x = -9$ or $x = -1$
$x = -9$ or $x = -1$	

1. I notice . . .

Sample response: I noticed in Diego's first step he subtracted the constant value from both sides of the equation, while Mai left the zero on the right side of the equation.

2. I wonder . . .

Sample response: I wonder why Diego added 25 but Mai added 16 to both sides of the equation.

1 Launch

Conduct the ***Notice and Wonder*** routine.

Ask:

- "Is the original quadratic a square expression?" No.
- "How are the two solution strategies alike? Different?" Sample response: Both strategies complete the square. Mai's strategy requires less steps, but Diego's strategy allows me to more readily see the square number that needs to be added to both sides of the equation.

2 Monitor

Help students get started by prompting them to describe the first step in Diego's example, mimic the step in Diego's strategy column in the table in Problem 3, and repeat for each step.

Look for points of confusion:

- **Forgetting subtraction in their factored square expressions.** Have students expand their expression to notice it is not equivalent to the expression in their previous step.
- **Having difficulty using Mai's strategy of completing the square in the problems in her strategy column.** Highlight the constant values in Steps 2 and 3.
- **Having difficulty using Diego's strategy of completing the square in the problems in his strategy column.** Highlight the constant values in Steps 1, 2, and 3.
- **Not using algebraic techniques.** Prompt them to try the algebraic strategy first, and then make sense of their work with algebra tiles or area diagrams.

Look for productive strategies:

- Annotating or highlighting the important steps in the example problems.
- Attempting the new problems using both strategies to determine their preferred strategy.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

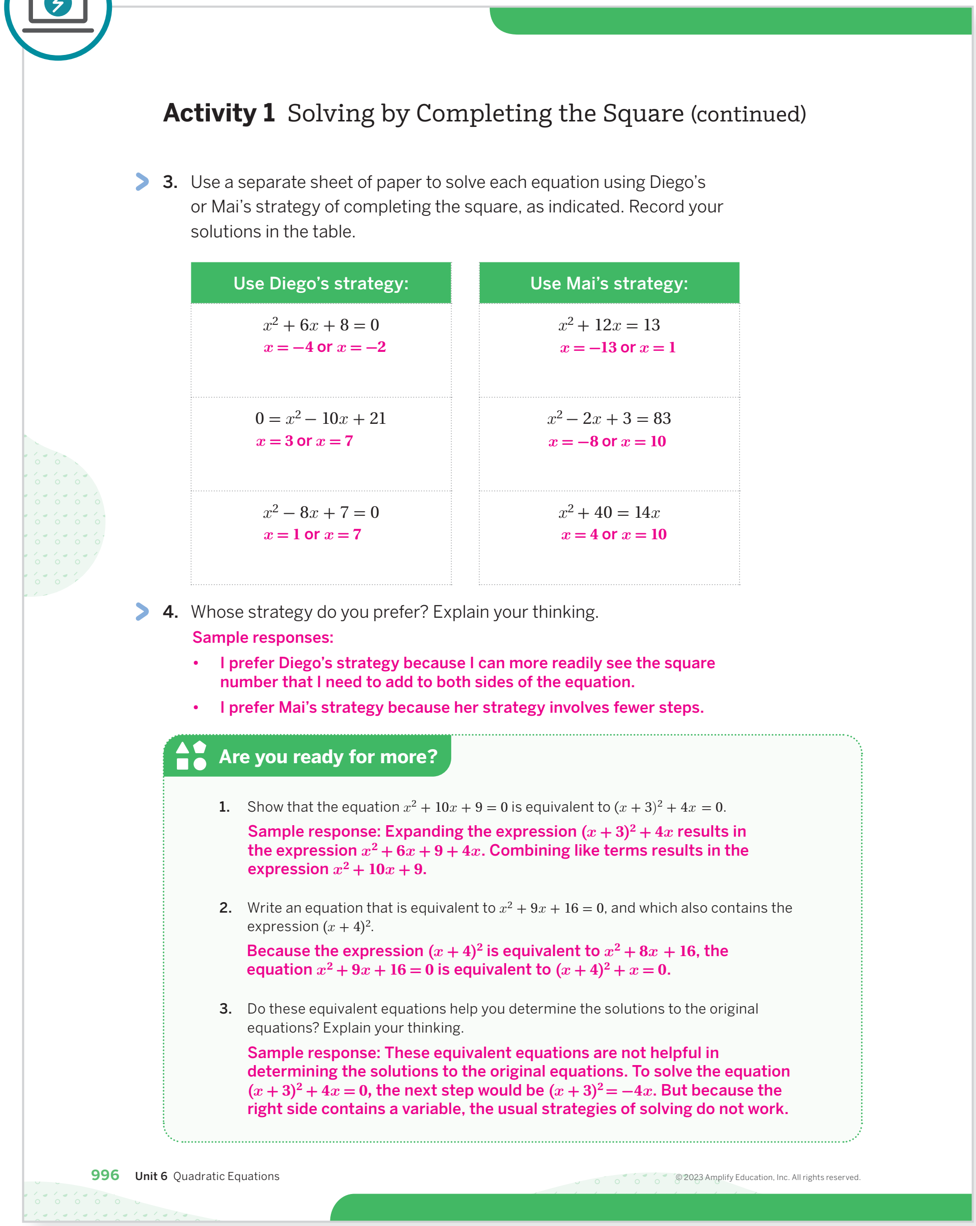
After students describe what they notice and wonder about Diego's and Mai's strategies, ask them to annotate what each person did for each step and the mathematical justification behind what they did. For example, they could annotate Diego's strategy with the following:

- Subtract 9 from each side.
- Complete the square for the left side by adding 25. Add 25 to the right side to maintain equality.
- Simplify the right side.
- Write the expression on the left side in factored form.
- Take the square root of each side.
- Solve the two equations to isolate x.

Small Groups | 20 min

Activity 1 Solving by Completing the Square (continued)

Students use completing the square to solve quadratic equations.



Activity 1 Solving by Completing the Square (continued)

3. Use a separate sheet of paper to solve each equation using Diego's or Mai's strategy of completing the square, as indicated. Record your solutions in the table.

Use Diego's strategy:	Use Mai's strategy:
$x^2 + 6x + 8 = 0$ $x = -4$ or $x = -2$	$x^2 + 12x = 13$ $x = -13$ or $x = 1$
$0 = x^2 - 10x + 21$ $x = 3$ or $x = 7$	$x^2 - 2x + 3 = 83$ $x = -8$ or $x = 10$
$x^2 - 8x + 7 = 0$ $x = 1$ or $x = 7$	$x^2 + 40 = 14x$ $x = 4$ or $x = 10$

4. Whose strategy do you prefer? Explain your thinking.

Sample responses:

- **I prefer Diego's strategy because I can more readily see the square number that I need to add to both sides of the equation.**
- **I prefer Mai's strategy because her strategy involves fewer steps.**

Are you ready for more?

1. Show that the equation $x^2 + 10x + 9 = 0$ is equivalent to $(x + 3)^2 + 4x = 0$.

   Sample response: Expanding the expression $(x + 3)^2 + 4x$ results in the expression $x^2 + 6x + 9 + 4x$. Combining like terms results in the expression $x^2 + 10x + 9$.

2. Write an equation that is equivalent to $x^2 + 9x + 16 = 0$, and which also contains the expression $(x + 4)^2$.

   Because the expression $(x + 4)^2$ is equivalent to $x^2 + 8x + 16$, the equation $x^2 + 9x + 16 = 0$ is equivalent to $(x + 4)^2 + x = 0$.

3. Do these equivalent equations help you determine the solutions to the original equations? Explain your thinking.

   Sample response: These equivalent equations are not helpful in determining the solutions to the original equations. To solve the equation $(x + 3)^2 + 4x = 0$, the next step would be $(x + 3)^2 = -4x$. But because the right side contains a variable, the usual strategies of solving do not work.

3 Connect

Have groups of students share their preferred strategy. Select students who have different preferences to share.

Display student work and solutions.

Highlight that balance must be maintained on both sides of the equation, similar to solving linear equations, in order for completing the square to work with either strategy.

Ask:

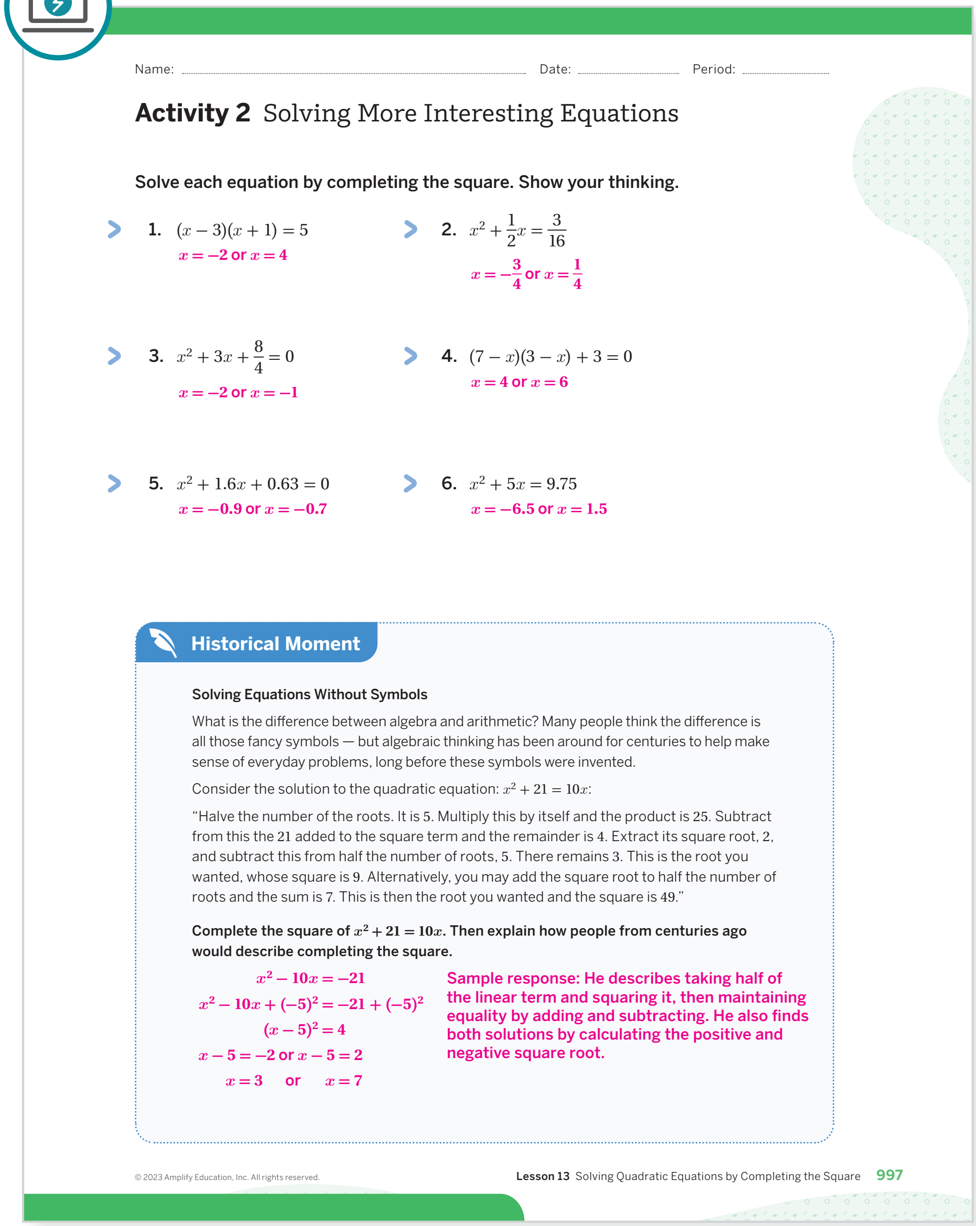
- "Which strategy do you prefer? Why?" Answers may vary.
- "How can you check your solutions?" By using substitution.
- "Is there another strategy to solve these quadratic equations?" I can also solve these quadratic equations by factoring.
- "When would completing the square be a better strategy for solving quadratic equations?" When the quadratic is not easily factorable.

Optional

Small Groups | 15 min

Activity 2 Solving More Interesting Equations

Students solve quadratic equations with rational numbers to improve fluency in solving quadratic equations by completing the square.



Name: ______ Date: ______ Period: ______

Activity 2 Solving More Interesting Equations

Solve each equation by completing the square. Show your thinking.

1. $(x-3)(x+1)=5$
   $x=-2$ or $x=4$
2. $x^2+\frac{1}{2}x=\frac{3}{16}$
   $x=-\frac{3}{4}$ or $x=\frac{1}{4}$
3. $x^2+3x+\frac{8}{4}=0$
   $x=-2$ or $x=-1$
4. $(7-x)(3-x)+3=0$
   $x=4$ or $x=6$
5. $x^2+1.6x+0.63=0$
   $x=-0.9$ or $x=-0.7$
6. $x^2+5x=9.75$
   $x=-6.5$ or $x=1.5$

Historical Moment

Solving Equations Without Symbols

What is the difference between algebra and arithmetic? Many people think the difference is all those fancy symbols — but algebraic thinking has been around for centuries to help make sense of everyday problems, long before these symbols were invented.

Consider the solution to the quadratic equation: $x^2+21=10x$:

"Halve the number of the roots. It is 5. Multiply this by itself and the product is 25. Subtract from this the 21 added to the square term and the remainder is 4. Extract its square root, 2, and subtract this from half the number of roots, 5. There remains 3. This is the root you wanted, whose square is 9. Alternatively, you may add the square root to half the number of roots and the sum is 7. This is then the root you wanted and the square is 49."

Complete the square of $x^2+21=10x$. Then explain how people from centuries ago would describe completing the square.

$$x^2-10x=-21$$
$$x^2-10x+(-5)^2=-21+(-5)^2$$
$$(x-5)^2=4$$
$$x-5=-2 \text{ or } x-5=2$$
$$x=3 \quad \text{or} \quad x=7$$

Sample response: He describes taking half of the linear term and squaring it, then maintaining equality by adding and subtracting. He also finds both solutions by calculating the positive and negative square root.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Solving Quadratic Equations by Completing the Square 997

1 Launch

Display the problems to the class. Give students think-time to notice how each equation is alike or unlike other equations they have seen before.

2 Monitor

Help students get started by prompting them to refer back to the strategies used by Diego and Mai in Activity 1.

Look for points of confusion:

- **Struggling to complete the square with equations involving fractions and decimals in Problems 2, 5, and 6.** Allow students to use calculators.
- **Not understanding the process when the equation is written in factored form in Problem 4.** Prompt students to review problems in Activity 1, then ask, "What form are the problems in? How can you rewrite this equation to be in that form?" Standard form. I can use the Distributive Property to rewrite the problem in standard form.

Look for productive strategies:

- Applying the steps for completing the square with non-integer coefficients.

3 Connect

Have groups of students share strategies, solutions, and any errors or challenges they experienced.

Highlight that the process of solving by completing the square is the same whether the equation includes integers or other rational numbers.

Ask, "What is the general process of solving by completing the square?" Answers may vary.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students solve the equations in Problems 3 and 5 first because those equations are written in standard form. Then have them solve the equations in Problems 2 and 6, and ask, "How are these equations different?" Finally, have them solve the equations in Problems 1 and 4, and ask, "How can you transform these equations to be in standard form?"

Accessibility: *Guide Processing and Visualization*

Display or provide access to the Anchor Chart PDF, *Completing the Square*, for students to reference as they complete the activity.

Historical Moment

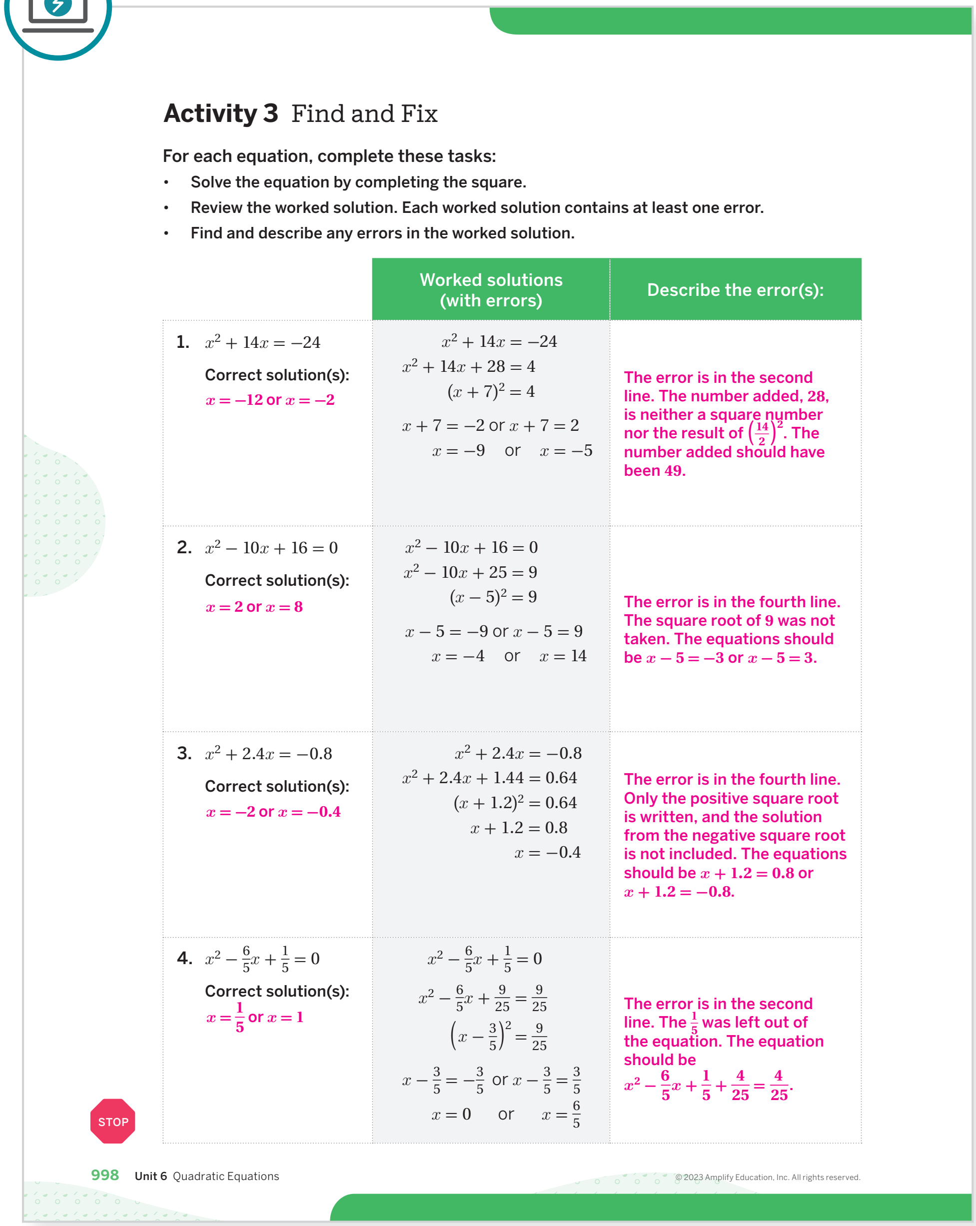
The Father of Algebra

Have students complete the *Historical Moment* activity to see how the Father of Algebra, Al-Khwārizmī, studied algebra without the use of symbols.

Small Groups | 15 min

Activity 3 Find and Fix

Students analyze worked examples of equations solved by completing the square that contain errors, to further develop their understanding of the strategy.



Activity 3 Find and Fix

For each equation, complete these tasks:

- Solve the equation by completing the square.
- Review the worked solution. Each worked solution contains at least one error.
- Find and describe any errors in the worked solution.

	Worked solutions (with errors)	Describe the error(s):
1. $x^2 + 14x = -24$ Correct solution(s): $x = -12$ or $x = -2$	$x^2 + 14x = -24$ $x^2 + 14x + 28 = 4$ $(x + 7)^2 = 4$ $x + 7 = -2$ or $x + 7 = 2$ $x = -9$ or $x = -5$	The error is in the second line. The number added, 28, is neither a square number nor the result of $\left(\frac{14}{2}\right)^2$. The number added should have been 49.
2. $x^2 - 10x + 16 = 0$ Correct solution(s): $x = 2$ or $x = 8$	$x^2 - 10x + 16 = 0$ $x^2 - 10x + 25 = 9$ $(x - 5)^2 = 9$ $x - 5 = -9$ or $x - 5 = 9$ $x = -4$ or $x = 14$	The error is in the fourth line. The square root of 9 was not taken. The equations should be $x - 5 = -3$ or $x - 5 = 3$.
3. $x^2 + 2.4x = -0.8$ Correct solution(s): $x = -2$ or $x = -0.4$	$x^2 + 2.4x = -0.8$ $x^2 + 2.4x + 1.44 = 0.64$ $(x + 1.2)^2 = 0.64$ $x + 1.2 = 0.8$ $x = -0.4$	The error is in the fourth line. Only the positive square root is written, and the solution from the negative square root is not included. The equations should be $x + 1.2 = 0.8$ or $x + 1.2 = -0.8$.
4. $x^2 - \frac{6}{5}x + \frac{1}{5} = 0$ Correct solution(s): $x = \frac{1}{5}$ or $x = 1$	$x^2 - \frac{6}{5}x + \frac{1}{5} = 0$ $x^2 - \frac{6}{5}x + \frac{9}{25} = \frac{9}{25}$ $\left(x - \frac{3}{5}\right)^2 = \frac{9}{25}$ $x - \frac{3}{5} = -\frac{3}{5}$ or $x - \frac{3}{5} = \frac{3}{5}$ $x = 0$ or $x = \frac{6}{5}$	The error is in the second line. The $\frac{1}{5}$ was left out of the equation. The equation should be $x^2 - \frac{6}{5}x + \frac{1}{5} + \frac{4}{25} = \frac{4}{25}$.

STOP

998 Unit 6 Quadratic Equations

1 Launch

Display the four equations. Conduct the *Find and Fix* routine. Students should solve independently before consulting with group members to agree on the solution and identify the errors.

2 Monitor

Help students get started by prompting them to list and explain the steps for completing the square.

Look for points of confusion:

- **Having difficulty discerning the miscalculations.** Ask, "What values changed from the last step? Adding or subtracting what value to both sides of the equation would result in the new step?"

Look for productive strategies:

- Creating a checklist to compare their work against each worked problem.
- Using their developing math language when identifying and explaining the error(s).
- Marking the inconsistencies between their own work and the work provided.

3 Connect

Have groups of students share the errors they identified and their proposed corrections.

Highlight different examples of common errors and the ways to avoid them when solving quadratic equations by completing the square.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display or provide access to the Anchor Chart PDF, *Completing the Square,* for students to reference as they complete the activity. Provide access to colored pencils and have students annotate the error in each worked solution.

Accessibility: *Vary Demands to Optimize Challenge*

Consider telling students in what line of the worked solution the error is in, but not identifying the error. For example, for Problem 1, tell them the error is in the second line.

Math Language Development

MLR3: Critique, Correct, Clarify

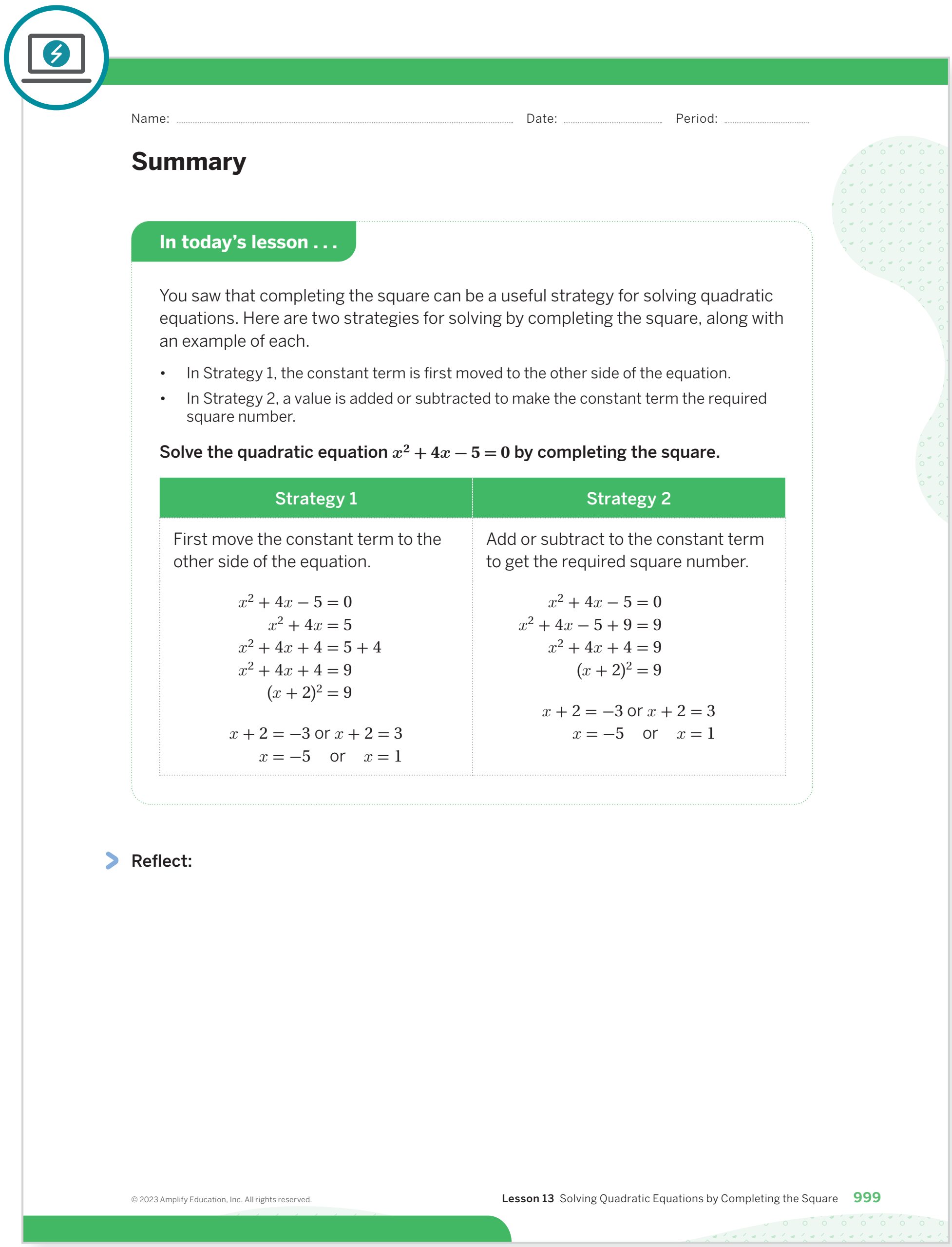
The entirety of this activity is structured similarly to the *MLR3: Critique, Correct, and Clarify* routine. While students work, consider displaying these questions that group members can ask themselves.

- ***Critique:*** "What mistake or error was made? Why do you think the person who attempted this solution made this mistake? What might they have been thinking?"
- ***Correct:*** "What should have been the correct step?"
- ***Clarify:*** "How could you convince the person who attempted the solution that the way you solved the equation is correct?"

Whole Class | 5 min

Summary

Review and synthesize the process of solving quadratic equations by completing the square.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that completing the square can be a useful strategy for solving quadratic equations. Here are two strategies for solving by completing the square, along with an example of each.

- In Strategy 1, the constant term is first moved to the other side of the equation.
- In Strategy 2, a value is added or subtracted to make the constant term the required square number.

Solve the quadratic equation $x^2 + 4x - 5 = 0$ by completing the square.

Strategy 1	Strategy 2
First move the constant term to the other side of the equation.	Add or subtract to the constant term to get the required square number.
$x^2 + 4x - 5 = 0$ $x^2 + 4x = 5$ $x^2 + 4x + 4 = 5 + 4$ $x^2 + 4x + 4 = 9$ $(x + 2)^2 = 9$ $x + 2 = -3$ or $x + 2 = 3$ $x = -5$ or $x = 1$	$x^2 + 4x - 5 = 0$ $x^2 + 4x - 5 + 9 = 9$ $x^2 + 4x + 4 = 9$ $(x + 2)^2 = 9$ $x + 2 = -3$ or $x + 2 = 3$ $x = -5$ or $x = 1$

Reflect:

Synthesize

Display the two quadratic equations:

Equation 1: $x^2 + 10x + 9 = 0$
Equation 2: $x^2 + 5x - \frac{75}{4} = 0$

Have students share all the possible strategies to solve each equation. Sample responses: Factor, graph, complete the square, guess-and-check.

Ask, "When would you prefer to solve quadratic equations by factoring instead of completing the square? When would you prefer to solve by completing the square?" Sample response: I would prefer to solve quadratic equations by factoring if the quadratic is easily factorable. I would prefer to solve by completing the square if the quadratic cannot be factored easily.

Highlight that completing the square can be a useful strategy for solving quadratic equations in cases in which it is not straightforward to rewrite an expression in factored form.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

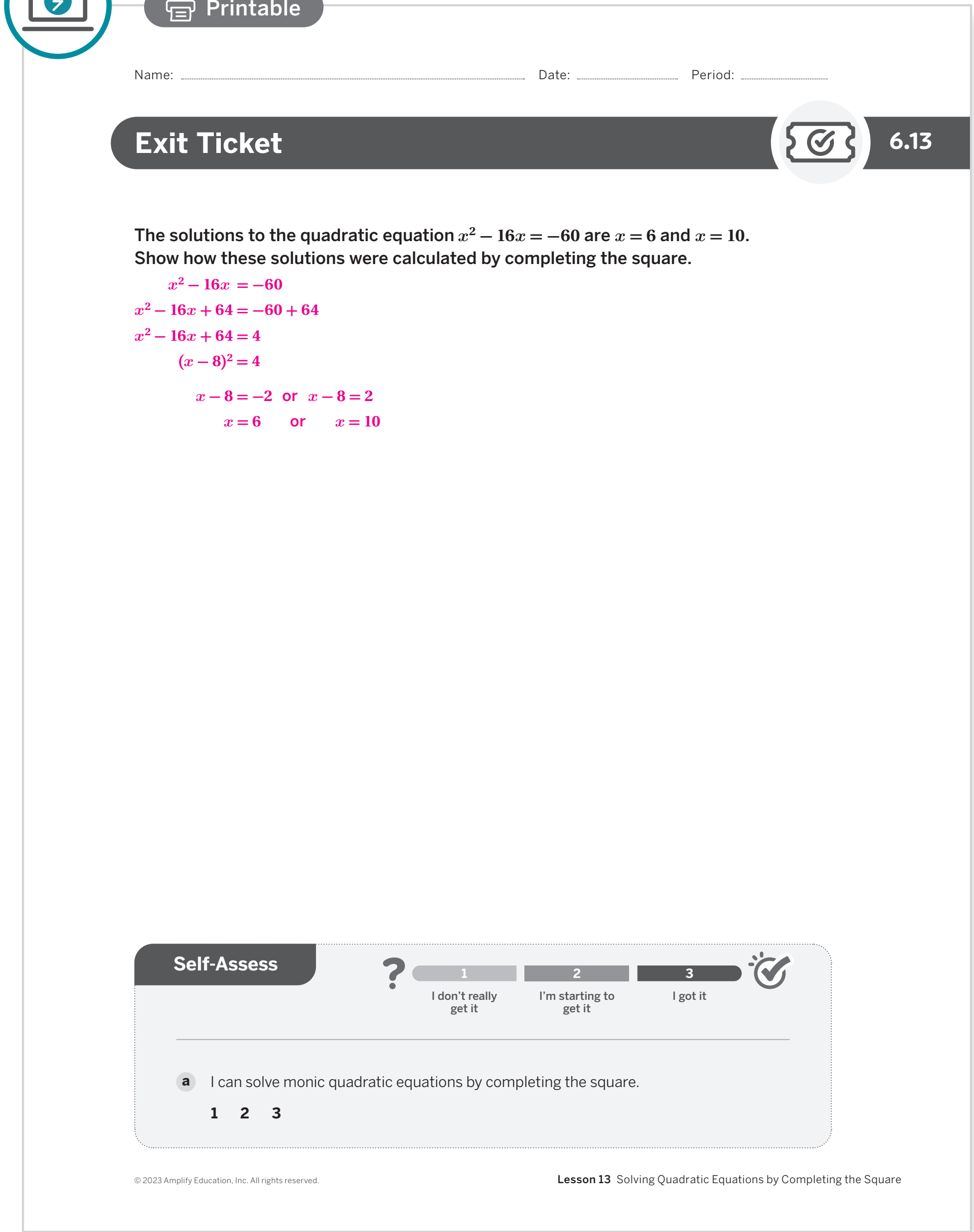
- "When do you think it is useful to solve a quadratic equation by completing the square?"
- "What do you like or dislike about the two strategies you learned about for solving by completing the square?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of completing the square by solving a quadratic equation by completing the square.

Printable



Name: Date: Period:

Exit Ticket 6.13

The solutions to the quadratic equation $x^2 - 16x = -60$ are $x = 6$ and $x = 10$. Show how these solutions were calculated by completing the square.

$$x^2 - 16x = -60$$
$$x^2 - 16x + 64 = -60 + 64$$
$$x^2 - 16x + 64 = 4$$
$$(x - 8)^2 = 4$$
$$x - 8 = -2 \text{ or } x - 8 = 2$$
$$x = 6 \quad \text{or} \quad x = 10$$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can solve monic quadratic equations by completing the square.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Solving Quadratic Equations by Completing the Square

Success looks like . . .

- **Language Goal:** Describing a process for completing the square to express any monic quadratic equation in the form $(x + p)^2 = q$. **(Speaking and Listening, Writing)**
- **Goal:** Expressing any monic quadratic equation in the form $(x + p)^2 = q$ and solving the equation by calculating square roots.
- **Goal:** Solving quadratic equations of the form $x^2 + bx + c$ by rearranging terms and completing the square.
    - » Showing how to solve the quadratic equation by completing the square.

Suggested next steps

If students do not change the expression to form a square expression, consider:

- Reviewing strategies from Activity 1.
- Assigning Practice Problem 3.
- Asking students to model the equation with algebra tiles.

If students do not include both the positive and negative solution, consider:

- Reviewing Lesson 4.

Professional Learning

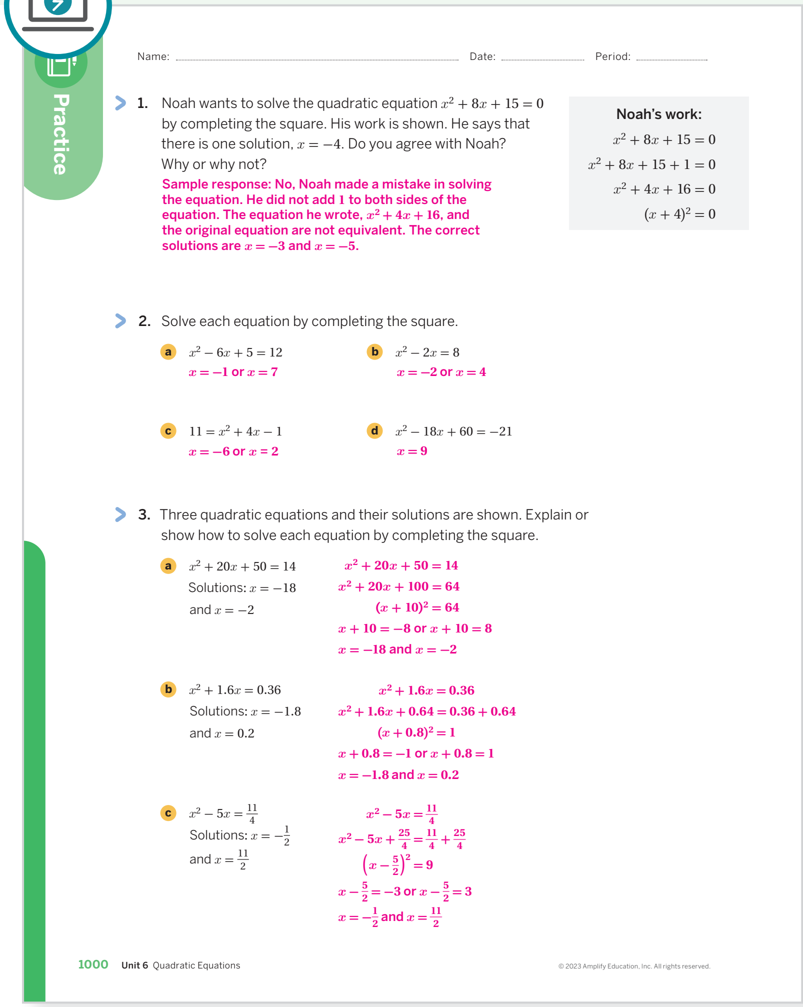
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on Activity 1?
- What resources did students use as they worked on Activity 3? Which resources were especially helpful? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ____ Date: ____ Period: ____

1. Noah wants to solve the quadratic equation $x^2 + 8x + 15 = 0$ by completing the square. His work is shown. He says that there is one solution, $x = -4$. Do you agree with Noah? Why or why not?
Sample response: No, Noah made a mistake in solving the equation. He did not add 1 to both sides of the equation. The equation he wrote, $x^2 + 4x + 16$, and the original equation are not equivalent. The correct solutions are $x = -3$ and $x = -5$.

Noah's work:

$$x^2 + 8x + 15 = 0$$
$$x^2 + 8x + 15 + 1 = 0$$
$$x^2 + 4x + 16 = 0$$
$$(x + 4)^2 = 0$$

2. Solve each equation by completing the square.
   - a $x^2 - 6x + 5 = 12$ **$x = -1$ or $x = 7$**
   - b $x^2 - 2x = 8$ **$x = -2$ or $x = 4$**
   - c $11 = x^2 + 4x - 1$ **$x = -6$ or $x = 2$**
   - d $x^2 - 18x + 60 = -21$ **$x = 9$**

3. Three quadratic equations and their solutions are shown. Explain or show how to solve each equation by completing the square.

a $x^2 + 20x + 50 = 14$
Solutions: $x = -18$
and $x = -2$

$$x^2 + 20x + 50 = 14$$
$$x^2 + 20x + 100 = 64$$
$$(x + 10)^2 = 64$$
$$x + 10 = -8 \text{ or } x + 10 = 8$$
$$x = -18 \text{ and } x = -2$$

b $x^2 + 1.6x = 0.36$
Solutions: $x = -1.8$
and $x = 0.2$

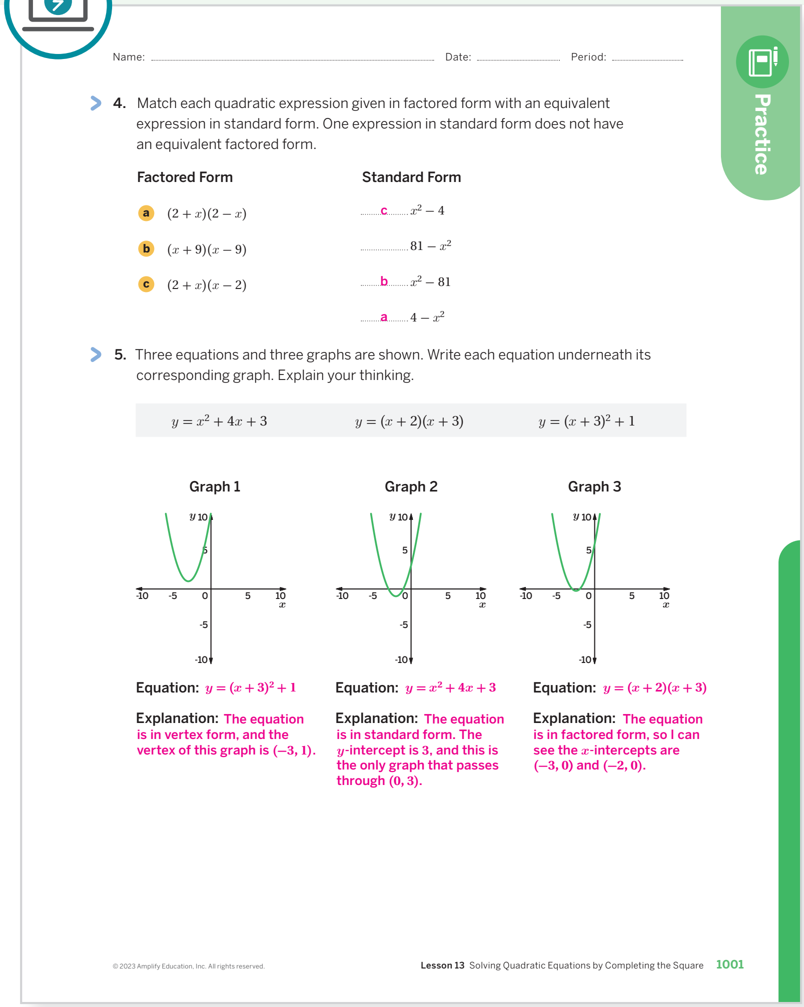
$$x^2 + 1.6x = 0.36$$
$$x^2 + 1.6x + 0.64 = 0.36 + 0.64$$
$$(x + 0.8)^2 = 1$$
$$x + 0.8 = -1 \text{ or } x + 0.8 = 1$$
$$x = -1.8 \text{ and } x = 0.2$$

c $x^2 - 5x = \frac{11}{4}$
Solutions: $x = -\frac{1}{2}$
and $x = \frac{11}{2}$

$$x^2 - 5x = \frac{11}{4}$$
$$x^2 - 5x + \frac{25}{4} = \frac{11}{4} + \frac{25}{4}$$
$$\left(x - \frac{5}{2}\right)^2 = 9$$
$$x - \frac{5}{2} = -3 \text{ or } x - \frac{5}{2} = 3$$
$$x = -\frac{1}{2} \text{ and } x = \frac{11}{2}$$

1000 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ____ Date: ____ Period: ____

4. Match each quadratic expression given in factored form with an equivalent expression in standard form. One expression in standard form does not have an equivalent factored form.

Factored Form	Standard Form
a $(2 + x)(2 - x)$	**c** $x^2 - 4$
b $(x + 9)(x - 9)$	____ $81 - x^2$
c $(2 + x)(x - 2)$	**b** $x^2 - 81$
	a $4 - x^2$

5. Three equations and three graphs are shown. Write each equation underneath its corresponding graph. Explain your thinking.

$y = x^2 + 4x + 3$ $y = (x + 2)(x + 3)$ $y = (x + 3)^2 + 1$

Graph 1	Graph 2	Graph 3
Equation: **$y = (x + 3)^2 + 1$**	Equation: **$y = x^2 + 4x + 3$**	Equation: **$y = (x + 2)(x + 3)$**
Explanation: **The equation is in vertex form, and the vertex of this graph is $(-3, 1)$.**	Explanation: **The equation is in standard form. The y-intercept is 3, and this is the only graph that passes through $(0, 3)$.**	Explanation: **The equation is in factored form, so I can see the x-intercepts are $(-3, 0)$ and $(-2, 0)$.**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Solving Quadratic Equations by Completing the Square 1001

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 3	2
Spiral	4	Unit 6 Lesson 8	1
Formative ⚡	5	Unit 6 Lesson 14	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 14**

Writing Quadratic Expressions in Vertex Form

Let's find other uses for completing the square.

Focus

Goals

1. **Language Goal:** Analyze and explain the steps for completing the square and how they transform a quadratic expression from standard to vertex form. **(Speaking and Listening, Writing)**
2. Identify the vertex of a graph of a quadratic function in vertex form.
3. Write equivalent quadratic expressions in vertex form by completing the square.

Coherence

Today

Students complete the square to write monic quadratic expressions in standard form into a new form, called vertex form. They review the different forms of quadratic functions and make connections to their graphs. Students examine different strategies for rewriting standard form expressions to vertex form by completing the square.

Previously

In Lesson 13, students completed the square to determine the solutions to a quadratic equation.

Coming Soon

In Lesson 15, students will complete the square for non-monic quadratic expressions.

Rigor

- Students strengthen their **conceptual understanding** of rewriting quadratic expressions in different forms to determine information about the graph of the function.
- Students strengthen their **procedural fluency** for rewriting expressions by completing the square.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	15 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Different Forms of Quadratic Expressions*
- Instructional Routine PDF, *Info Gap: Instructions*
- Instructional Routine PDF, *Info Gap: Types of Questioning*
- algebra tiles (as needed)

Math Language Development

Review words

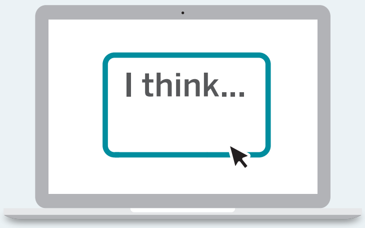
- *addend*
- *completing the square*
- *minuend*
- *quadratic function*
- *square numbers*
- *vertex form*
- *zeros*

Amps | **Featured Activity**

Activity 1

See Student Thinking

Students are asked to explain their thinking behind rewriting expressions to vertex form, and these explanations are available to you digitally, in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel irritated as they learn another form of quadratic equations. Help students organize all that they have learned by making a chart of the different forms with examples of the process as well as an explanation of the purpose. While the chart might not be completely filled out yet, knowing that the reason for this skill is coming might provide hope and allow students to see purpose in their work.

Modifications to Pacing

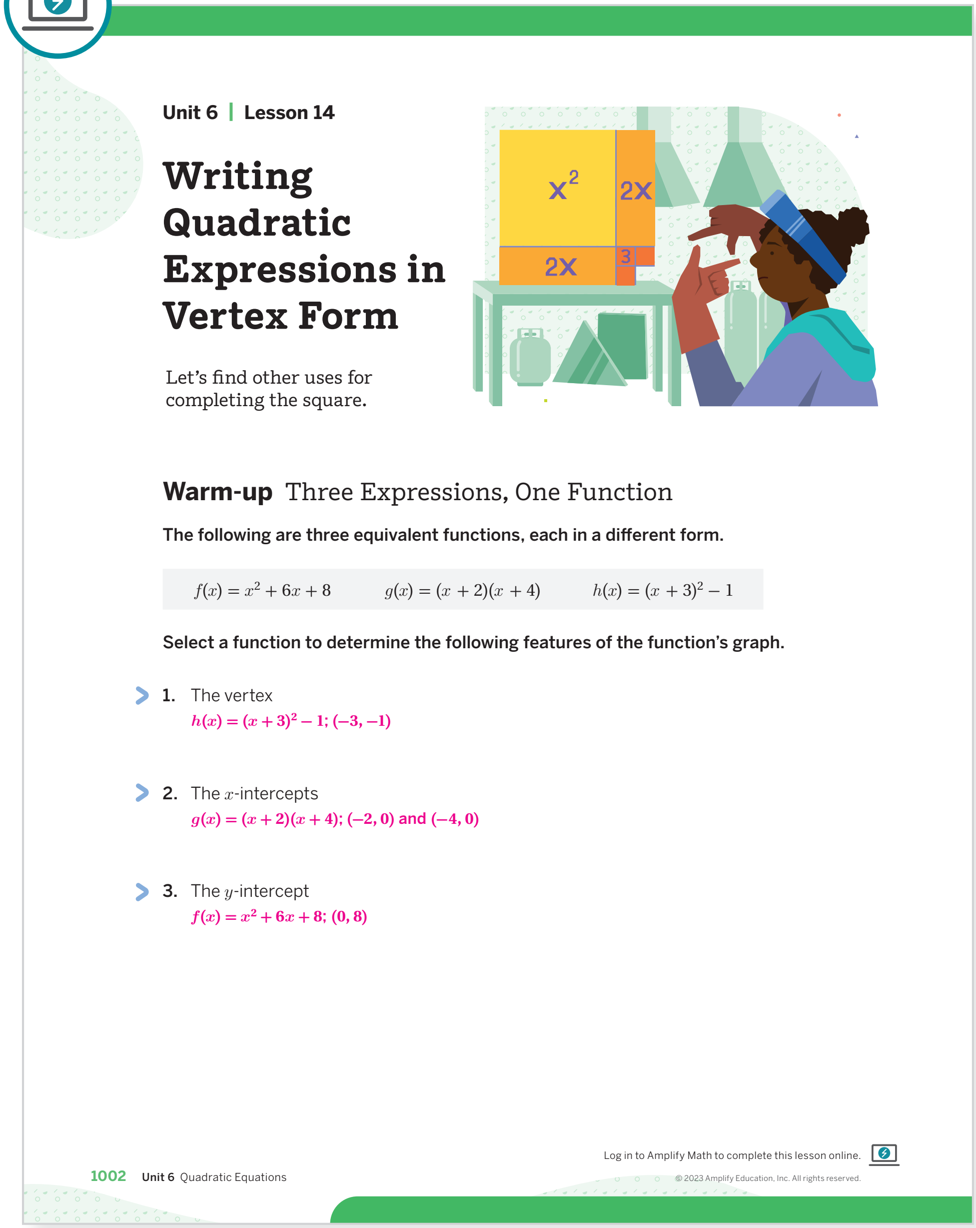
You may want to consider this additional modification if you are short on time.

- **Activity 1**, Problem 3 may be omitted.

Independent | 10 min

Warm-up Three Expressions, One Function

Students examine different forms of a quadratic function to recall the features of the graph recognizable in each form.



Unit 6 | Lesson 14

Writing Quadratic Expressions in Vertex Form

Let's find other uses for completing the square.

Warm-up Three Expressions, One Function

The following are three equivalent functions, each in a different form.

$f(x) = x^2 + 6x + 8$ $\quad g(x) = (x + 2)(x + 4)$ $\quad h(x) = (x + 3)^2 - 1$

Select a function to determine the following features of the function's graph.

1. The vertex
   $h(x) = (x + 3)^2 - 1; (-3, -1)$
2. The x-intercepts
   $g(x) = (x + 2)(x + 4); (-2, 0)$ and $(-4, 0)$
3. The y-intercept
   $f(x) = x^2 + 6x + 8; (0, 8)$

1002 Unit 6 Quadratic Equations

Log in to Amplify Math to complete this lesson online.

1 Launch

Display the three equivalent functions, each in a different form. Have students work on the problems individually, and then compare their solutions and strategies with a partner.

2 Monitor

Help students get started by prompting them to label the form of each function. Ask, "What information can be obtained from each different form?" Standard form identifies the y-intercept, factored form identifies the x-intercepts, and vertex form identifies the vertex.

Look for points of confusion:

- **Having difficulty determining which form provides certain information.** Display the Anchor Chart PDF, *Different Forms of Quadratic Expressions*. Have students annotate each function with its form and the information it provides.

Look for productive strategies:

- Setting each factor in $g(x)$ equal to zero to determine the x-intercepts.
- Annotating, labeling, or underlining the expressions to highlight key information.

3 Connect

Have students share which expression they used to determine each graph feature and why.

Display the graph of the function and label the vertex and intercepts.

Highlight that the constant in standard form identifies the y-intercept, factored form identifies the x-intercepts, and vertex form identifies the vertex.

Ask, "How can you rewrite a factorable quadratic function in standard form to factored form? Vertex form?" Sample response: I can rewrite a function in standard form to factored form by factoring the quadratic. I can rewrite a function in standard form to vertex form by completing the square.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the three equivalent functions (without revealing Problems 1–3) and have students work with a partner to write 2–3 questions they could ask about the functions. Sample questions shown.

- How can I verify these functions are equivalent?
- In what form are each of these functions written: standard, factored, or vertex form?
- What information does each form provide?
- Do the graphs of these functions look the same? Why or why not?

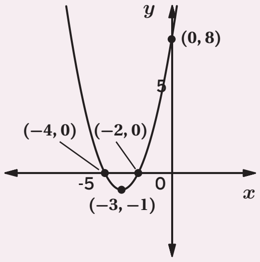
English Learners

Display one of the sample questions that students could use as a model for how to craft a question.

Power-up

To power up students' ability to identify key features of a graph of a quadratic function, have students complete:

1. What are the coordinates of the x-intercepts of this function? $(-4, 0)$ and $(-2, 0)$
2. What are the coordinates of the y-intercept of this function? $(0, 8)$
3. What are the coordinates of the vertex of this function? $(-3, -1)$



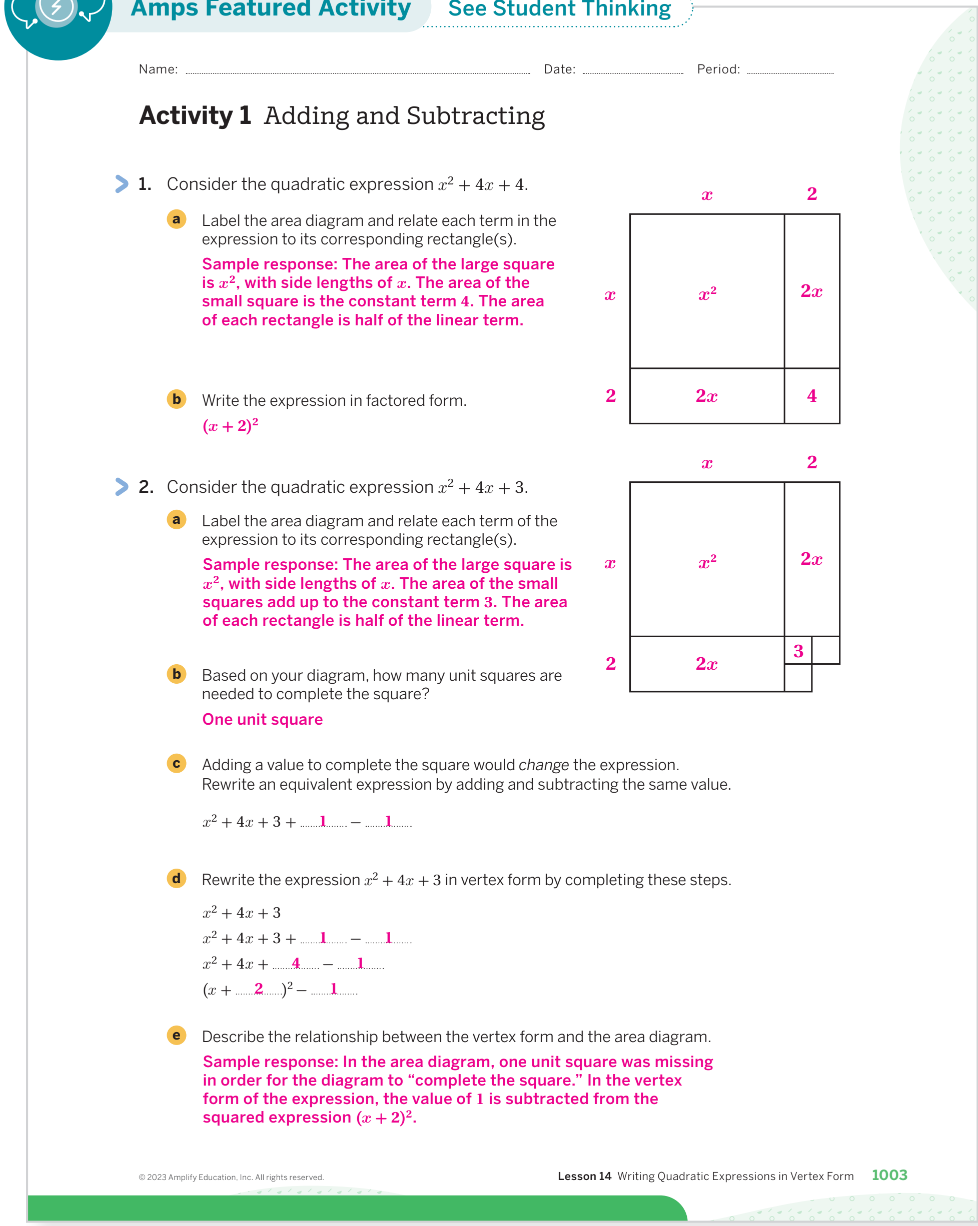
Use: Before the Warm-up
Informed by: Performance on Lesson 13, Practice Problem 5

Pairs | 15 min

Activity 1 Adding and Subtracting

Students use area diagrams to discover a different strategy to translate quadratic expressions in standard form to vertex form by completing the square.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 1 Adding and Subtracting

1. Consider the quadratic expression $x^2 + 4x + 4$.

   a. Label the area diagram and relate each term in the expression to its corresponding rectangle(s).

   Sample response: The area of the large square is x^2, with side lengths of x. The area of the small square is the constant term 4. The area of each rectangle is half of the linear term.

   b. Write the expression in factored form.

   $(x + 2)^2$

2. Consider the quadratic expression $x^2 + 4x + 3$.

   a. Label the area diagram and relate each term of the expression to its corresponding rectangle(s).

   Sample response: The area of the large square is x^2, with side lengths of x. The area of the small squares add up to the constant term 3. The area of each rectangle is half of the linear term.

   b. Based on your diagram, how many unit squares are needed to complete the square?

   One unit square

   c. Adding a value to complete the square would *change* the expression. Rewrite an equivalent expression by adding and subtracting the same value.

   $x^2 + 4x + 3 + \underline{1} - \underline{1}$

   d. Rewrite the expression $x^2 + 4x + 3$ in vertex form by completing these steps.

   $x^2 + 4x + 3$
   $x^2 + 4x + 3 + \underline{1} - \underline{1}$
   $x^2 + 4x + \underline{4} - \underline{1}$
   $(x + \underline{2})^2 - \underline{1}$

   e. Describe the relationship between the vertex form and the area diagram.

   Sample response: In the area diagram, one unit square was missing in order for the diagram to "complete the square." In the vertex form of the expression, the value of 1 is subtracted from the squared expression $(x + 2)^2$.

Lesson 14 Writing Quadratic Expressions in Vertex Form 1003

1 Launch

Have students examine and discuss how to approach each problem together with their partner, complete individually, and then compare solutions and strategies.

2 Monitor

Help students get started by prompting them to label the area of the largest and smallest squares first.

Look for points of confusion:

- **Not adding and subtracting the same value in Problem 2.** Discuss the difference between the expression, $x^2 + 4x + 3$, and the equation, $x^2 + 4x + 3 = 0$. Have students experiment with adding the missing value to the expression. Ask, "Are the expressions still equivalent?" No
- **Having difficulty explaining their process in Problem 3e.** Prompt students to describe the steps they took to write the equivalent expression. Have them note the values added and subtracted, then explain their thinking.

Look for productive strategies:

- Noticing the relationship between the constant value in vertex form and the incomplete square in the area diagram.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to algebra tiles and blank area diagram templates for students to use should they choose to do so.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the terms in each expression written in standard form with their corresponding terms represented in the area diagram. For example, in Problem 1, they could color x^2 in each representation in one color, $4x$ ($2x$ and $2x$) in a second color, and 4 in a third color.

Math Language Development

MLR7: Compare and Connect

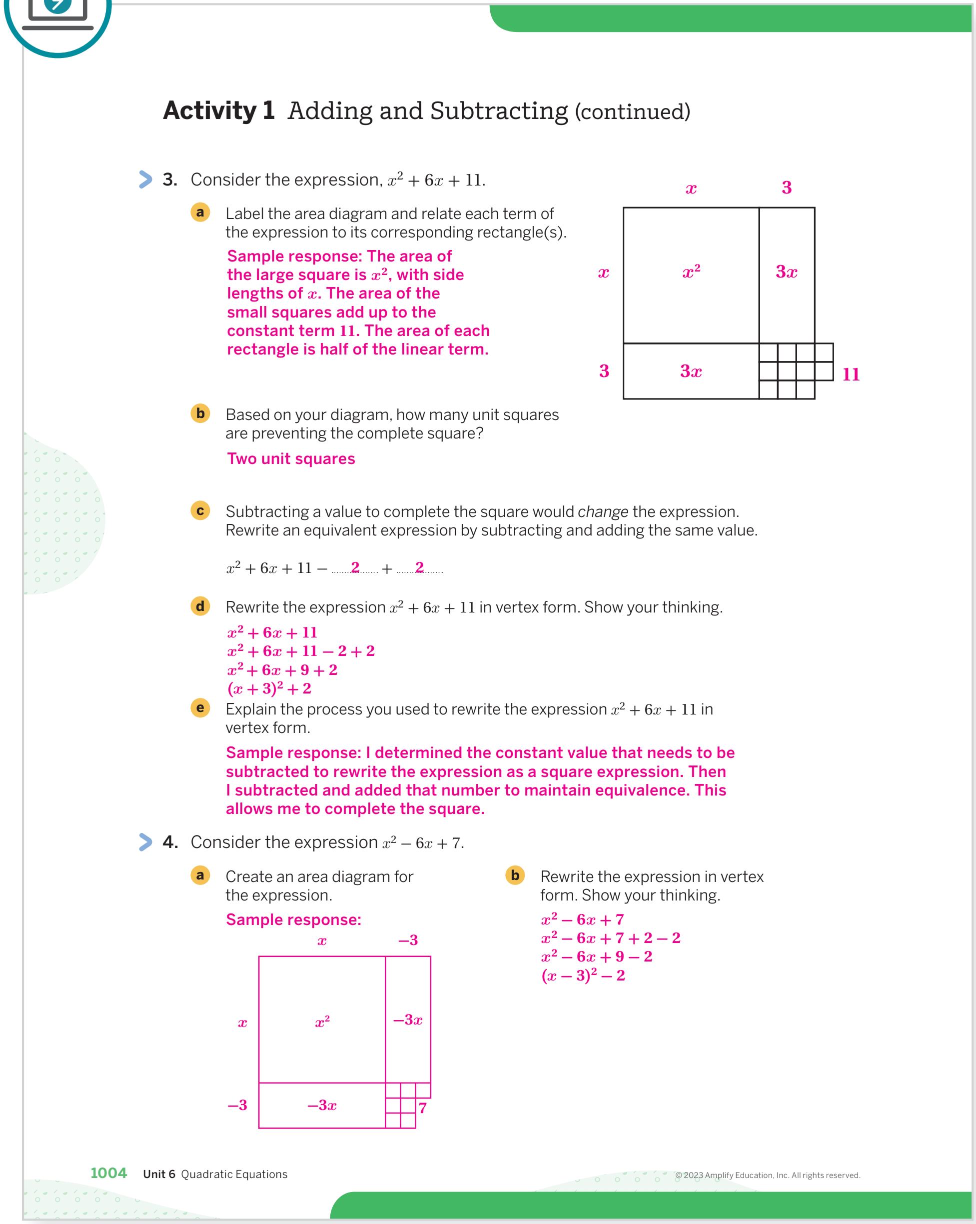
During the Connect, draw students' attention to the ways in which the area diagrams are similar and different, based on the values of the constant terms in the standard form expressions. Ask:

- "Which expressions are square expressions? How do you know?"
- "Which expressions are not square expressions? How do you know?"
- "For the expressions that are not square expressions, what value can you add to make it a square expression?"
- "If you add this value, what do you also need to do in order to maintain an equivalent expression?"
- "How does completing the square help you write the expression in vertex form?"

Pairs | 15 min

Activity 1 Adding and Subtracting (continued)

Students use area diagrams to discover a different strategy to translate quadratic expressions in standard form to vertex form by completing the square.



Activity 1 Adding and Subtracting (continued)

3. Consider the expression, $x^2 + 6x + 11$.

a Label the area diagram and relate each term of the expression to its corresponding rectangle(s).

Sample response: The area of the large square is x^2, with side lengths of x. The area of the small squares add up to the constant term 11. The area of each rectangle is half of the linear term.

b Based on your diagram, how many unit squares are preventing the complete square?

Two unit squares

c Subtracting a value to complete the square would *change* the expression. Rewrite an equivalent expression by subtracting and adding the same value.

$x^2 + 6x + 11 -$ ....**2**.... $+$ ....**2**....

d Rewrite the expression $x^2 + 6x + 11$ in vertex form. Show your thinking.

$x^2 + 6x + 11$
$x^2 + 6x + 11 - 2 + 2$
$x^2 + 6x + 9 + 2$
$(x + 3)^2 + 2$

e Explain the process you used to rewrite the expression $x^2 + 6x + 11$ in vertex form.

Sample response: I determined the constant value that needs to be subtracted to rewrite the expression as a square expression. Then I subtracted and added that number to maintain equivalence. This allows me to complete the square.

4. Consider the expression $x^2 - 6x + 7$.

a Create an area diagram for the expression.

Sample response:

b Rewrite the expression in vertex form. Show your thinking.

$x^2 - 6x + 7$
$x^2 - 6x + 7 + 2 - 2$
$x^2 - 6x + 9 - 2$
$(x - 3)^2 - 2$

1004 **Unit 6** Quadratic Equations

3 Connect

Have pairs of students share what they noticed in the relationship between the area diagrams and expressions in vertex form.

Highlight that to rewrite standard form expressions in vertex form, students need to add and subtract the same value to result in the required square number in order to maintain equality in the expression.

Ask:

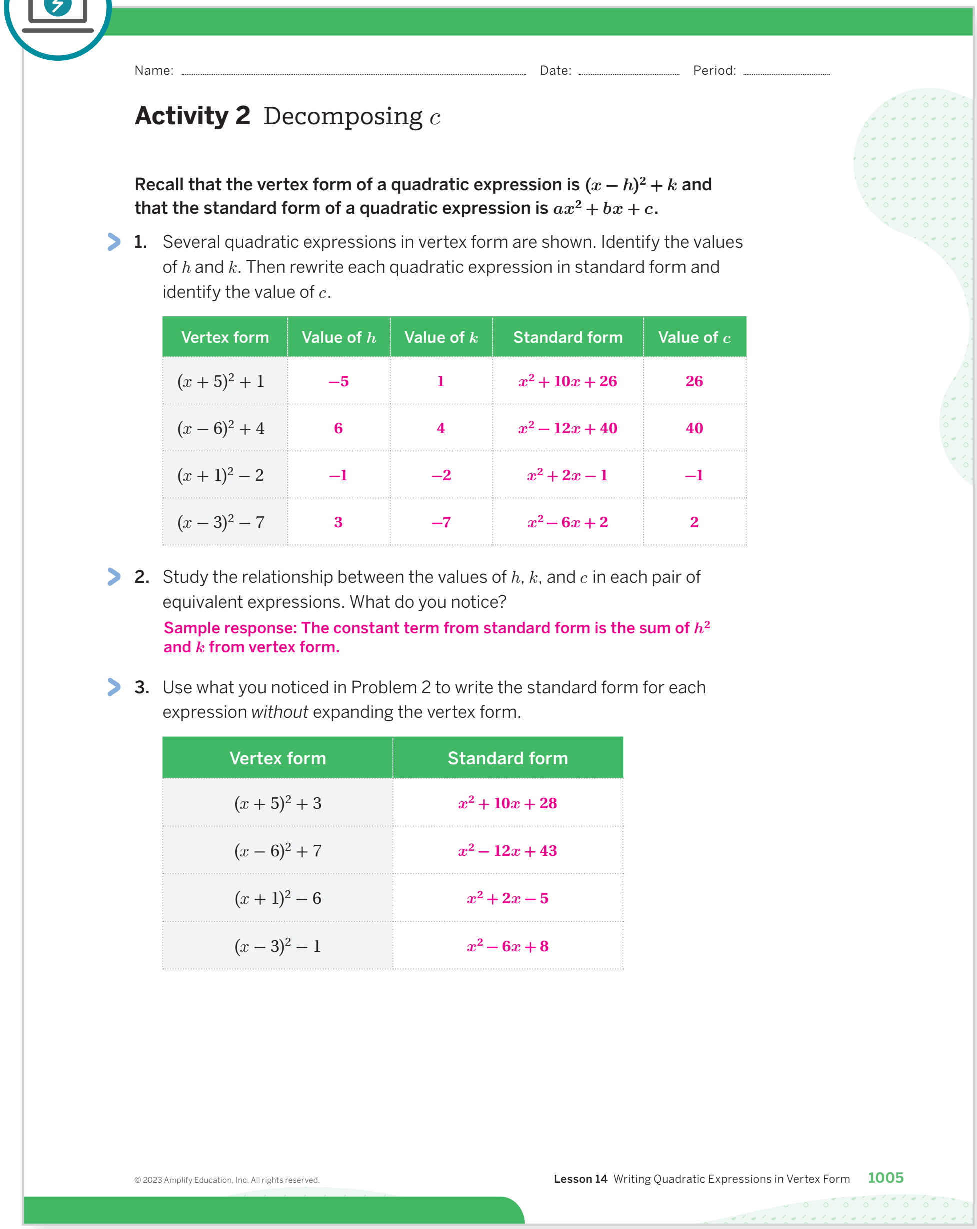
- "Can you think of a different method to translate a quadratic expression in standard form to vertex form?" Add and subtract the square number itself.
- "How can you check whether an expression in vertex form is equivalent to the original expression in standard form?" Use the Distributive Property to rewrite the expression in standard form.

Display Problem 4. Write equivalent expressions by adding and subtracting 2 and also by adding and subtracting 9. Highlight that the resulting expressions in vertex form are equivalent.

Pairs | 15 min

Activity 2 Decomposing c

Students rewrite quadratic expressions from standard form to vertex form to reveal the steps for completing the square.



Name: ______ Date: ______ Period: ______

Activity 2 Decomposing c

Recall that the vertex form of a quadratic expression is $(x - h)^2 + k$ and that the standard form of a quadratic expression is $ax^2 + bx + c$.

1. Several quadratic expressions in vertex form are shown. Identify the values of h and k. Then rewrite each quadratic expression in standard form and identify the value of c.

Vertex form	Value of h	Value of k	Standard form	Value of c
$(x + 5)^2 + 1$	**–5**	**1**	$x^2 + 10x + 26$	**26**
$(x - 6)^2 + 4$	**6**	**4**	$x^2 - 12x + 40$	**40**
$(x + 1)^2 - 2$	**–1**	**–2**	$x^2 + 2x - 1$	**–1**
$(x - 3)^2 - 7$	**3**	**–7**	$x^2 - 6x + 2$	**2**

2. Study the relationship between the values of h, k, and c in each pair of equivalent expressions. What do you notice?

Sample response: The constant term from standard form is the sum of h^2 and k from vertex form.

3. Use what you noticed in Problem 2 to write the standard form for each expression *without* expanding the vertex form.

Vertex form	Standard form
$(x + 5)^2 + 3$	$x^2 + 10x + 28$
$(x - 6)^2 + 7$	$x^2 - 12x + 43$
$(x + 1)^2 - 6$	$x^2 + 2x - 5$
$(x - 3)^2 - 1$	$x^2 - 6x + 8$

Lesson 14 Writing Quadratic Expressions in Vertex Form 1005

1 Launch

Review the terms *addend, minuend* and *square numbers*. Have students individually complete Problems 1 and 2 and then facilitate a whole-class discussion. Have student pairs complete the remaining problems together.

2 Monitor

Help students get started by reviewing vertex form, $(x - h)^2 + k$, and standard form, $ax^2 + bx + c$, of quadratic expressions.

Look for points of confusion:

- **Not recognizing the relationship between h, k, and c in Problem 2.** Ask, "How can you decompose c into a sum of two numbers, using h and k?" $c = h^2 + k$
- **Not writing a difference in Problems 5b and 5c.** Have students rewrite their vertex form into standard form to notice their error.

Look for productive strategies:

- Determining the square number before rewriting the constant value.
- Noticing and applying the relationship between the constant term of the standard form and vertex form.

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Have students annotate the vertex and standard form expressions given in the introductory text with the following information.

- **Coordinates of the vertex:** (h, k)
- **Constant of standard form:** c

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students color code the values of h and k in vertex form with the value of c in standard form for the table in Problem 1.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the relationship between the constant term c from the standard form expression and the values of h and k in the vertex form expression. Add the following to the class display, emphasizing the use of precise language, such as *constant, sum, standard form,* and *vertex form.*

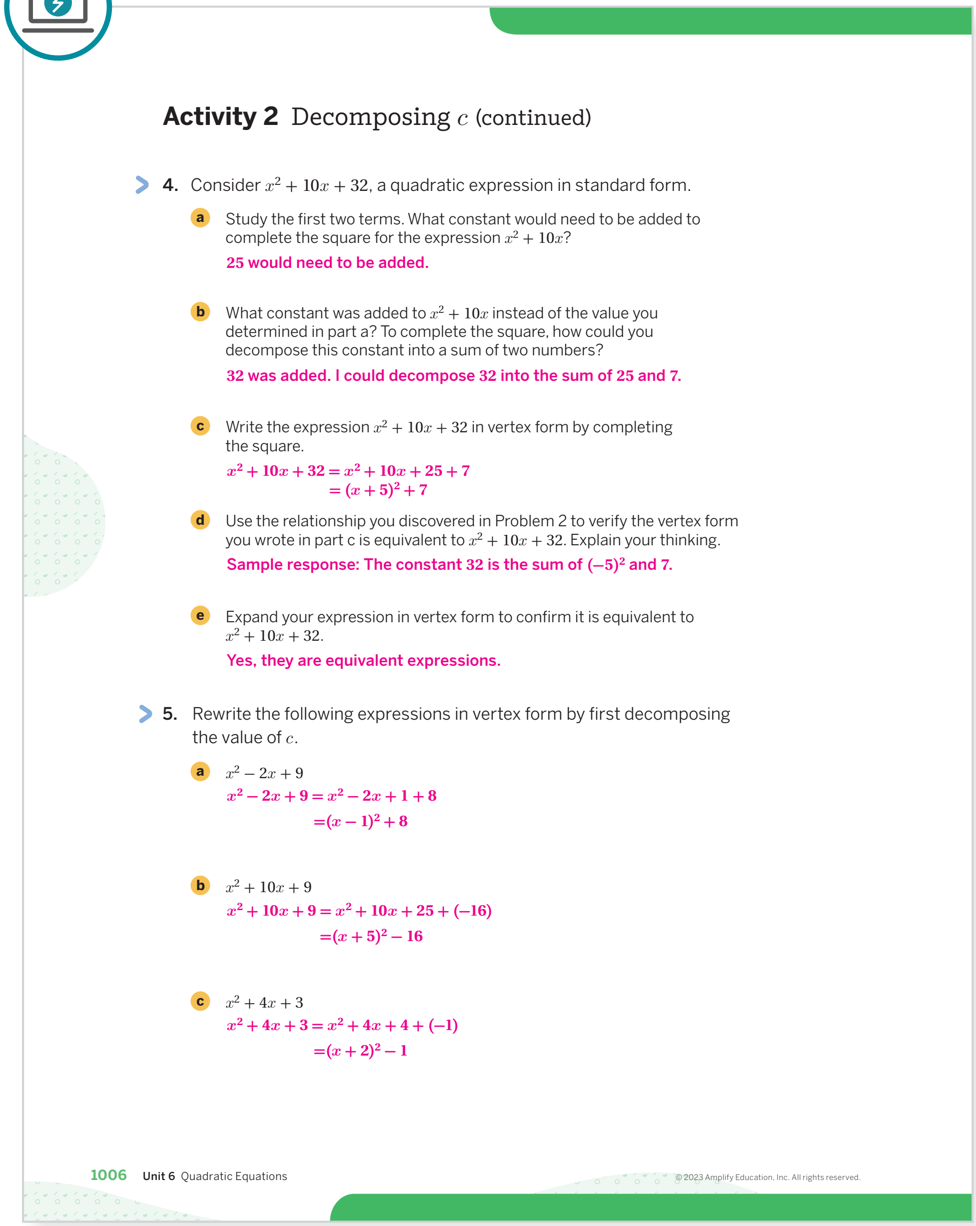
Words	Symbols
The constant c in standard form is equal to the sum of h^2 and k in vertex form.	$c = h^2 + k$

Display the three expressions from Problem 5 and ask, "How can you decompose c so that it is the sum of a perfect square (h^2) and another number?"

Pairs | 15 min

Activity 2 Decomposing c (continued)

Students rewrite quadratic expressions from standard form to vertex form to reveal the steps for completing the square.



Activity 2 Decomposing c (continued)

4. Consider $x^2 + 10x + 32$, a quadratic expression in standard form.
   - a Study the first two terms. What constant would need to be added to complete the square for the expression $x^2 + 10x$?

     25 would need to be added.
   - b What constant was added to $x^2 + 10x$ instead of the value you determined in part a? To complete the square, how could you decompose this constant into a sum of two numbers?

     32 was added. I could decompose 32 into the sum of 25 and 7.
   - c Write the expression $x^2 + 10x + 32$ in vertex form by completing the square.

     $$\begin{aligned} x^2 + 10x + 32 &= x^2 + 10x + 25 + 7 \\ &= (x + 5)^2 + 7 \end{aligned}$$
   - d Use the relationship you discovered in Problem 2 to verify the vertex form you wrote in part c is equivalent to $x^2 + 10x + 32$. Explain your thinking.

     Sample response: The constant 32 is the sum of $(-5)^2$ and 7.
   - e Expand your expression in vertex form to confirm it is equivalent to $x^2 + 10x + 32$.

     Yes, they are equivalent expressions.

5. Rewrite the following expressions in vertex form by first decomposing the value of c.
   - a $x^2 - 2x + 9$

     $$\begin{aligned} x^2 - 2x + 9 &= x^2 - 2x + 1 + 8 \\ &= (x - 1)^2 + 8 \end{aligned}$$
   - b $x^2 + 10x + 9$

     $$\begin{aligned} x^2 + 10x + 9 &= x^2 + 10x + 25 + (-16) \\ &= (x + 5)^2 - 16 \end{aligned}$$
   - c $x^2 + 4x + 3$

     $$\begin{aligned} x^2 + 4x + 3 &= x^2 + 4x + 4 + (-1) \\ &= (x + 2)^2 - 1 \end{aligned}$$

1006 **Unit 6** Quadratic Equations

3 Connect

Have pairs of students share the patterns noticed and strategies used to rewrite the standard form expressions into vertex form in Problem 5.

Highlight that to complete the square, students need to decompose the constant from standard form into the sum of a perfect square and another number. In this sum, the perfect square is the value of h^2 and the other number is the value of k.

Ask:

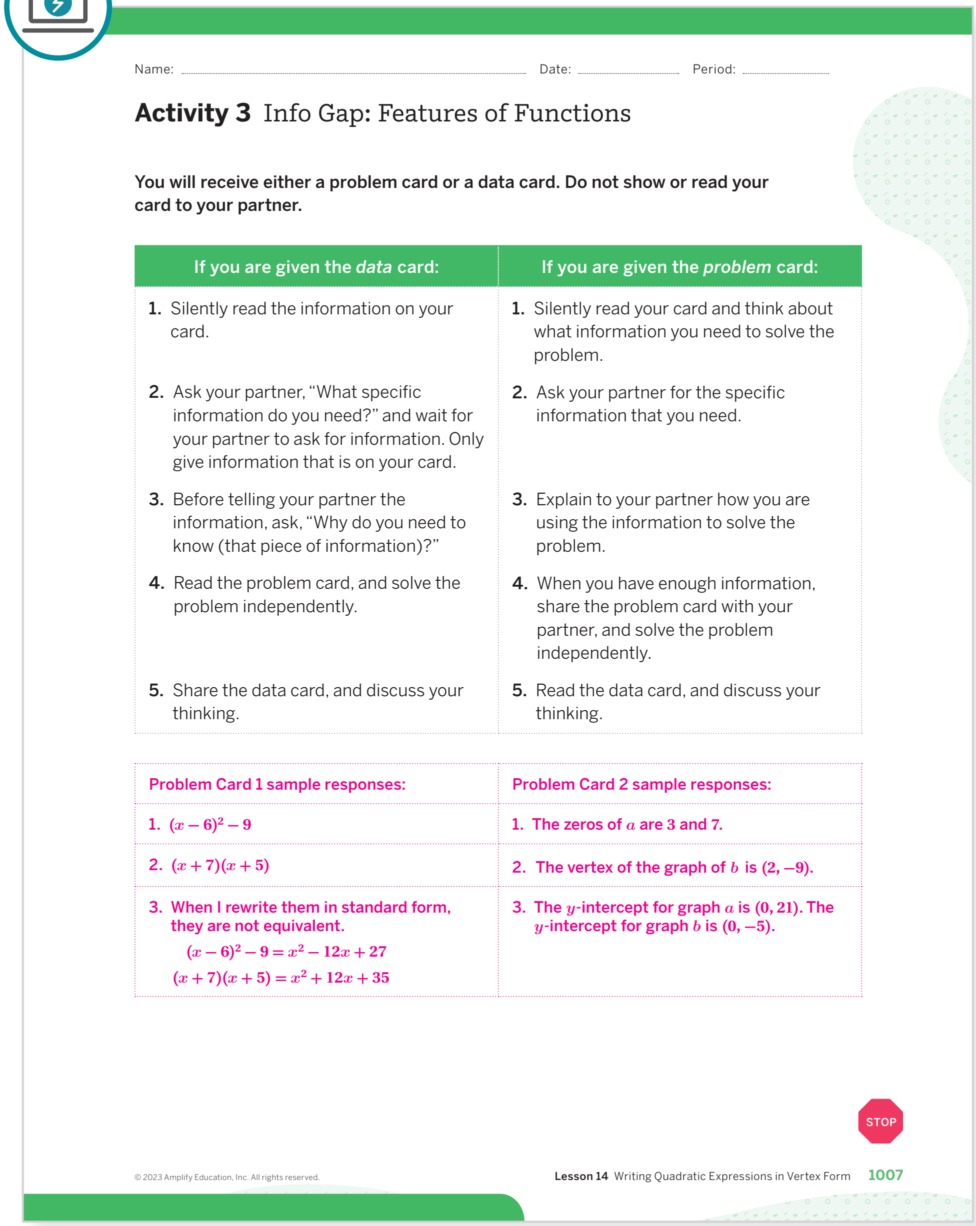
- "Describe the different patterns and relationships you saw today between standard form and vertex form." Sample response: I saw that there is a relationship between the constant in standard form and the values of h and k in vertex form.
- "How is this different from how you completed the square in prior lessons? How is this similar?" Sample response: In prior lessons, I completed the square by adding a value to c so that it becomes a perfect square. Then I subtracted that value from the end of the expression to maintain equivalence. In this lesson, I decomposed c so that it was the sum of a perfect square and another number. This was helpful so that I could write the expression in vertex form.

Optional

Pairs | 15 min

Activity 3 Info Gap: Features of Functions

Students determine and request the information needed to write expressions that define quadratic functions with certain graphical features.



Name: ______ Date: ______ Period: ______

Activity 3 Info Gap: Features of Functions

You will receive either a problem card or a data card. Do not show or read your card to your partner.

If you are given the *data* card:	If you are given the *problem* card:
1. Silently read the information on your card.	**1.** Silently read your card and think about what information you need to solve the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	**2.** Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	**3.** Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	**4.** When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	**5.** Read the data card, and discuss your thinking.

Problem Card 1 sample responses:	Problem Card 2 sample responses:
1. $(x-6)^2-9$	1. The zeros of a are 3 and 7.
2. $(x+7)(x+5)$	2. The vertex of the graph of b is $(2, -9)$.
3. When I rewrite them in standard form, they are not equivalent. $(x-6)^2-9=x^2-12x+27$ $(x+7)(x+5)=x^2+12x+35$	3. The y-intercept for graph a is $(0, 21)$. The y-intercept for graph b is $(0, -5)$.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Writing Quadratic Expressions in Vertex Form 1007

1 Launch

Display the Instructional Routine PDF, *Info Gap: Instructions*, and consider demonstrating the ***Info Gap*** routine if students are unfamiliar with it. Provide pre-cut cards to each pair of students from the Activity 3 PDF.

2 Monitor

Help students get started by modeling the questioning process.

Look for points of confusion:

- **Struggling to show the two functions are not the same in Problem Card 1, Problem 3.** Prompt students to discuss the different forms. Ask, "How could you rewrite these expressions so they are in the same form to compare?" Use the Distributive Property to rewrite the expressions in standard form.

Look for productive strategies:

- Rewriting the expressions in different forms to determine the solutions.

3 Connect

Have pairs of students share the strategies they used and any challenges they faced.

Highlight that different forms of quadratic equations are useful in different ways. Factored form identifies the zeros and x-intercepts, vertex form identifies the coordinates of the vertex, and standard form identifies the y-intercept.

Ask:

- "Which form do you think provides the most information? Why?" Answers may vary.
- "What form do you prefer to use? Why?" Answers may vary.

Differentiated Support

Extension: ***Math Enrichment***

For Problem Card 1 and Data Card 1, ask students to show that functions f and g do not define the same function, *without* writing an expression for either function. Sample response: The x-coordinate of the vertex is the average of the x-intercepts. The x-coordinate of the vertex of function g is -6, which is not the same as the x-coordinate of the vertex of function f.

Math Language Development

MLR4: Information Gap

During the Launch, display Problem Card 1, without revealing any of the information on Data Card 1. Ask students to work with their partner to write questions they could ask that might help them determine the solution to the first problem, "Write an expression in vertex form that defines a quadratic function f." Sample questions are shown.

- "Can you tell me the coordinates of the vertex of the function f?"
- "Can you tell me any other features of the graph, such as the x-intercepts?"

Ask students why the x-intercepts might be helpful if the coordinates of the vertex are not known.

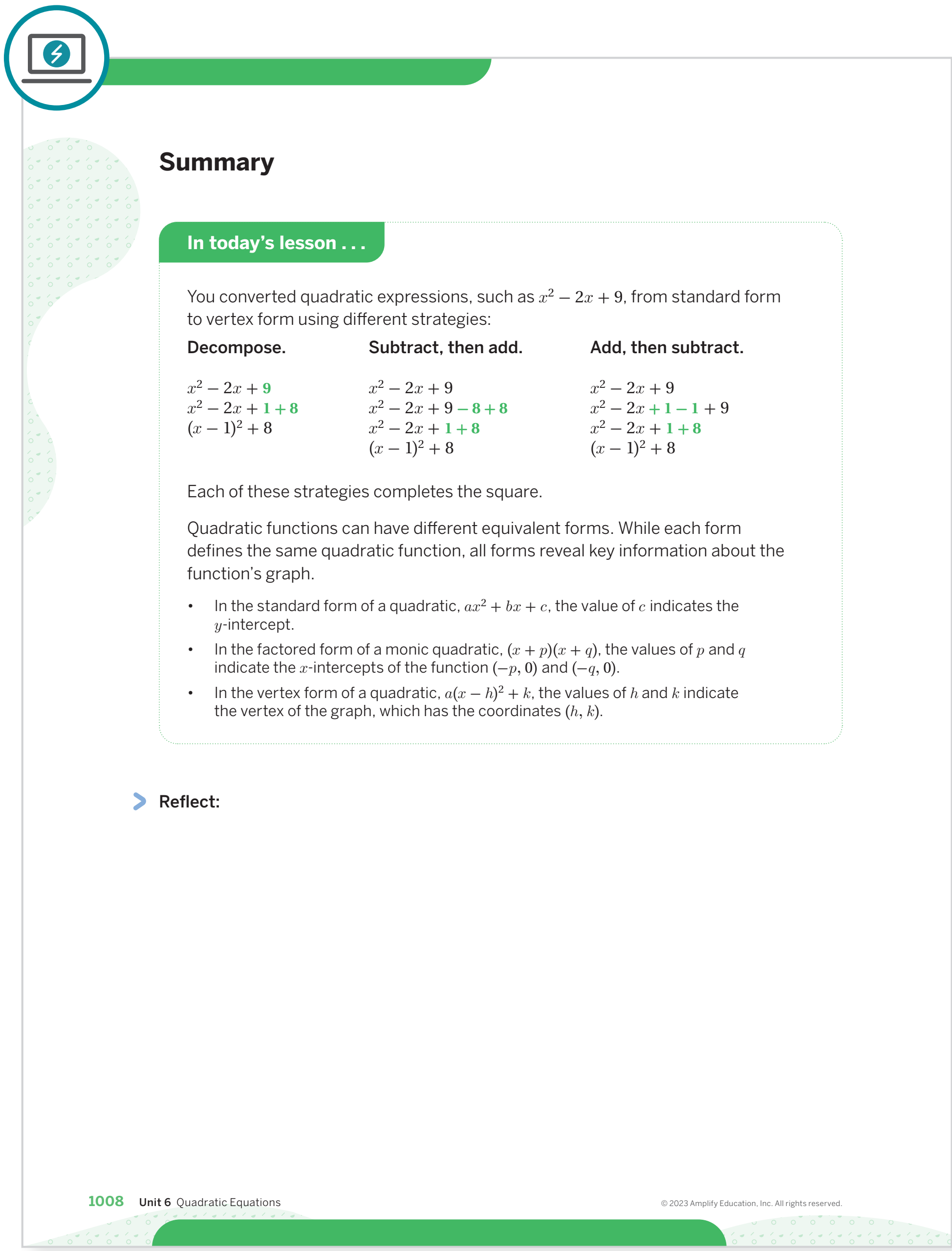
English Learners

Display or provide the Instructional Routine PDF, *Info Gap: Types of Questioning,* for students who would benefit from having a starting point to form questions.

Whole Class | 5 min

Summary

Review and synthesize rewriting standard form quadratic expressions in vertex form by completing the square.



Summary

In today's lesson . . .

You converted quadratic expressions, such as $x^2 - 2x + 9$, from standard form to vertex form using different strategies:

Decompose.	**Subtract, then add.**	**Add, then subtract.**
$x^2 - 2x + \mathbf{9}$	$x^2 - 2x + 9$	$x^2 - 2x + 9$
$x^2 - 2x + \mathbf{1 + 8}$	$x^2 - 2x + 9 \mathbf{- 8 + 8}$	$x^2 - 2x \mathbf{+ 1 - 1} + 9$
$(x - 1)^2 + 8$	$x^2 - 2x + \mathbf{1 + 8}$	$x^2 - 2x + \mathbf{1 + 8}$
	$(x - 1)^2 + 8$	$(x - 1)^2 + 8$

Each of these strategies completes the square.

Quadratic functions can have different equivalent forms. While each form defines the same quadratic function, all forms reveal key information about the function's graph.

- In the standard form of a quadratic, $ax^2 + bx + c$, the value of c indicates the y-intercept.
- In the factored form of a monic quadratic, $(x + p)(x + q)$, the values of p and q indicate the x-intercepts of the function $(-p, 0)$ and $(-q, 0)$.
- In the vertex form of a quadratic, $a(x - h)^2 + k$, the values of h and k indicate the vertex of the graph, which has the coordinates (h, k).

Reflect:

1008 **Unit 6** Quadratic Equations

Synthesize

Display the function $f(x) = x^2 - 2x + 9$.

Ask, "What features of the graph of f can be determined without sketching the graph?" The y-intercept can be identified by the constant term 9. The y-intercept is $(0, 9)$. The x-intercepts can be determined by rewriting the function in factored form. However, this function cannot be written in factored form, which means there are no rational x-intercepts. The vertex can be identified by rewriting the function in vertex form $f(x) = (x - 1)^2 + 8$. The vertex is $(1, 8)$.

Have students share how they determined each feature of the graph of f.

Highlight that the x-intercepts and coordinates of the vertex aren't readily identified by standard form, but students can rewrite the expression in other forms to determine them. Illustrate the different strategies that can be used to complete the square to rewrite a quadratic expression in vertex form. Demonstrate how factoring can be used to rewrite the expression in factored form. If time permits, have students sketch a graph by hand.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

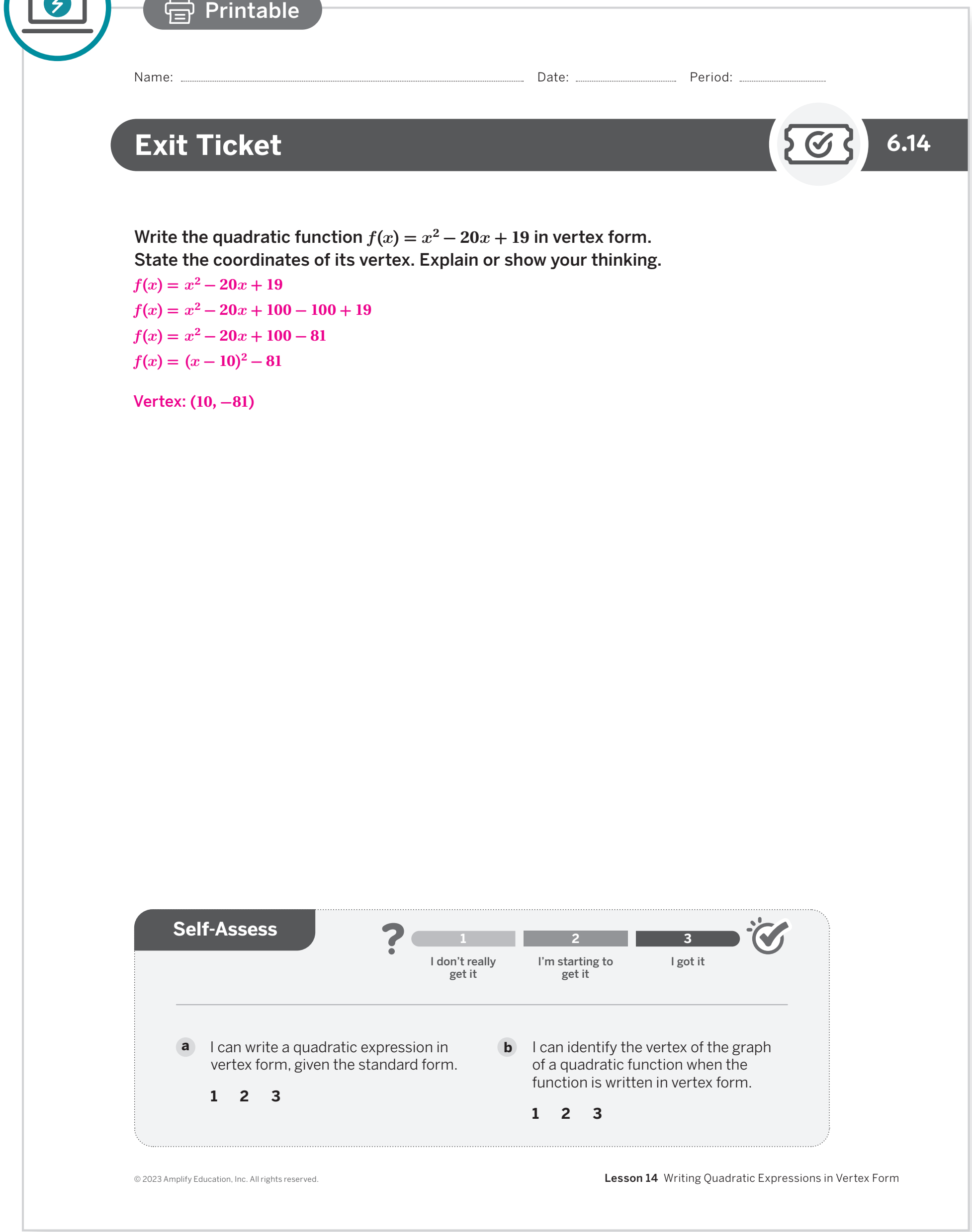
- "Why is it valuable to rewrite a quadratic expression in standard form to vertex form?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of rewriting quadratic expressions in vertex form by completing the square.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 6.14

Write the quadratic function $f(x) = x^2 - 20x + 19$ in vertex form. State the coordinates of its vertex. Explain or show your thinking.

$f(x) = x^2 - 20x + 19$

$f(x) = x^2 - 20x + 100 - 100 + 19$

$f(x) = x^2 - 20x + 100 - 81$

$f(x) = (x - 10)^2 - 81$

Vertex: $(10, -81)$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can write a quadratic expression in vertex form, given the standard form.

1 2 3

b I can identify the vertex of the graph of a quadratic function when the function is written in vertex form.

1 2 3

Lesson 14 Writing Quadratic Expressions in Vertex Form

Success looks like . . .

- **Language Goal:** Analyzing and explaining the steps for completing the square and how they transform a quadratic expression from standard to vertex form. **(Speaking and Listening, Writing)**
- **Goal:** Identifying the vertex of a graph of a quadratic function in vertex form.
- **Goal:** Writing equivalent quadratic expressions in vertex form by completing the square.
    - » Completing the square to write the quadratic expression in the left-hand side of the equation in vertex form.

Suggested next steps

If students incorrectly complete the square, consider:

- Reviewing strategies from Activities 1 and 2.
- Assigning Practice Problem 3.
- Asking, "How would you change the expression to be a square expression? How could you decompose the constant into a sum of a perfect square and another number?"

If students identify the vertex as $(-10, -81)$, consider:

- Reviewing the general form of the vertex form for quadratics as $f(x) = (x - h)^2 + k$ and ask, "What is the value of h for this function? If the function was $f(x) = (x + 10)^2 - 81$, what would be the value of h?"

Professional Learning

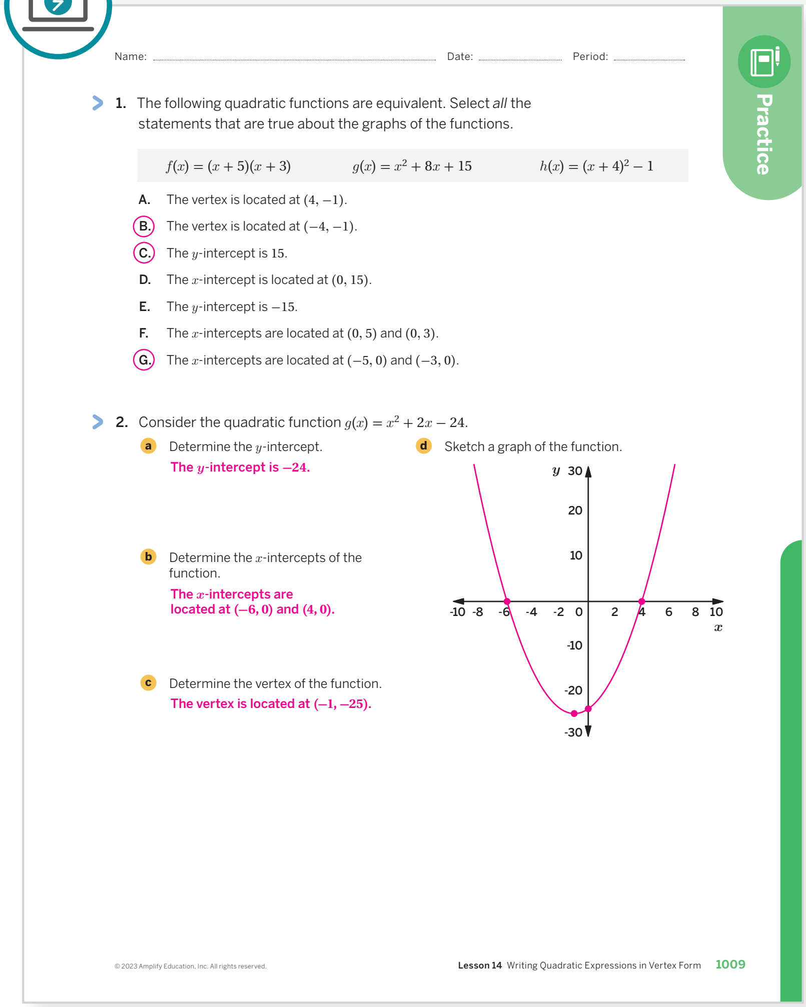
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students gotten better at factoring and completing the square?
- What different ways did students approach rewriting quadratic expressions from standard to vertex form? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. The following quadratic functions are equivalent. Select *all* the statements that are true about the graphs of the functions.

$f(x) = (x+5)(x+3)$ $g(x) = x^2 + 8x + 15$ $h(x) = (x+4)^2 - 1$

- A. The vertex is located at $(4, -1)$.
- [✓ B.] The vertex is located at $(-4, -1)$.
- [✓ C.] The y-intercept is 15.
- D. The x-intercept is located at $(0, 15)$.
- E. The y-intercept is -15.
- F. The x-intercepts are located at $(0, 5)$ and $(0, 3)$.
- [✓ G.] The x-intercepts are located at $(-5, 0)$ and $(-3, 0)$.

2. Consider the quadratic function $g(x) = x^2 + 2x - 24$.

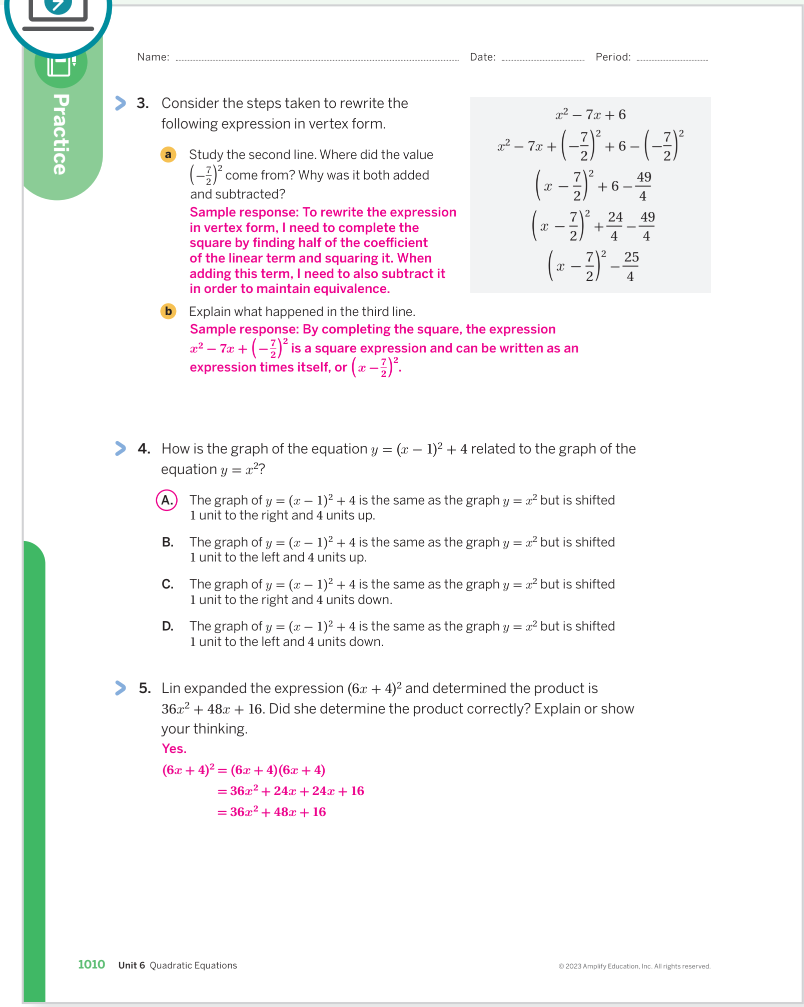
a. Determine the y-intercept.
The y-intercept is –24.

b. Determine the x-intercepts of the function.
The x-intercepts are located at $(-6, 0)$ and $(4, 0)$.

c. Determine the vertex of the function.
The vertex is located at $(-1, -25)$.

d. Sketch a graph of the function.

Practice



Name: Date: Period:

Practice

3. Consider the steps taken to rewrite the following expression in vertex form.

$$x^2 - 7x + 6$$
$$x^2 - 7x + \left(-\frac{7}{2}\right)^2 + 6 - \left(-\frac{7}{2}\right)^2$$
$$\left(x - \frac{7}{2}\right)^2 + 6 - \frac{49}{4}$$
$$\left(x - \frac{7}{2}\right)^2 + \frac{24}{4} - \frac{49}{4}$$
$$\left(x - \frac{7}{2}\right)^2 - \frac{25}{4}$$

a. Study the second line. Where did the value $\left(-\frac{7}{2}\right)^2$ come from? Why was it both added and subtracted?
Sample response: To rewrite the expression in vertex form, I need to complete the square by finding half of the coefficient of the linear term and squaring it. When adding this term, I need to also subtract it in order to maintain equivalence.

b. Explain what happened in the third line.
Sample response: By completing the square, the expression $x^2 - 7x + \left(-\frac{7}{2}\right)^2$ is a square expression and can be written as an expression times itself, or $\left(x - \frac{7}{2}\right)^2$.

4. How is the graph of the equation $y = (x-1)^2 + 4$ related to the graph of the equation $y = x^2$?

- [✓ A.] The graph of $y = (x-1)^2 + 4$ is the same as the graph $y = x^2$ but is shifted 1 unit to the right and 4 units up.
- B. The graph of $y = (x-1)^2 + 4$ is the same as the graph $y = x^2$ but is shifted 1 unit to the left and 4 units up.
- C. The graph of $y = (x-1)^2 + 4$ is the same as the graph $y = x^2$ but is shifted 1 unit to the right and 4 units down.
- D. The graph of $y = (x-1)^2 + 4$ is the same as the graph $y = x^2$ but is shifted 1 unit to the left and 4 units down.

5. Lin expanded the expression $(6x+4)^2$ and determined the product is $36x^2 + 48x + 16$. Did she determine the product correctly? Explain or show your thinking.
Yes.

$$\begin{aligned}(6x+4)^2 &= (6x+4)(6x+4)\\ &= 36x^2 + 24x + 24x + 16\\ &= 36x^2 + 48x + 16\end{aligned}$$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 3	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 21	2
Formative	5	Unit 6 Lesson 15	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

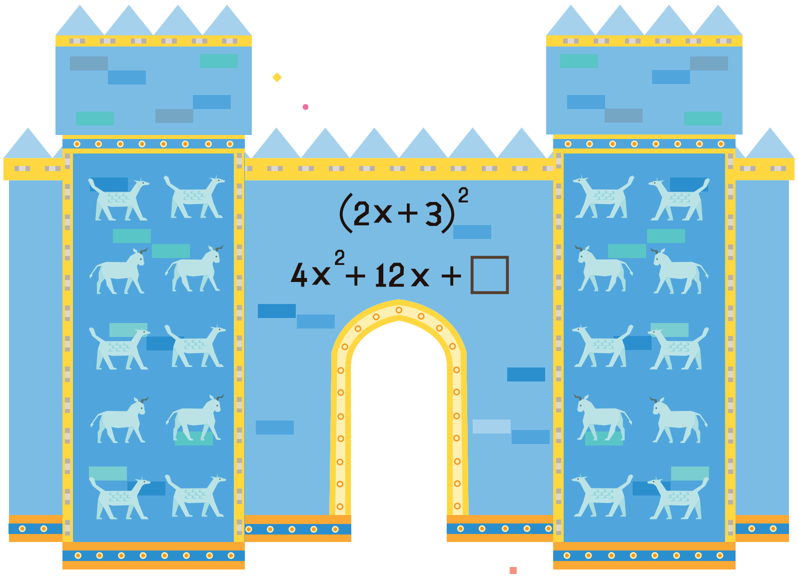
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 15**

Solving Non-Monic Quadratic Equations by Completing the Square

Let's solve other quadratic equations by completing the square.



Focus

Goals

1. **Language Goal:** Generalize a process for completing the square to express any non-monic quadratic equation in the form $(mx+p)^2=q$. **(Speaking and Listening)**
2. Solve non-monic quadratic equations by completing the square.

Coherence

Today

Students complete the square to solve non-monic quadratic equations. They examine the structure for expanding and factoring expressions that are square expressions. Then they use different strategies to solve quadratic equations, exposing the efficiency and limitations of certain strategies.

Previously

In Lesson 14, students rewrote standard form monic quadratic expressions in vertex form by completing the square.

Coming Soon

In Lesson 16, students will further their understanding of solving quadratic equations by investigating irrational solutions to quadratic equations.

Rigor

- Students build a **conceptual understanding** of how completing the square can be applied to non-monic quadratic expressions.
- Students develop **procedural fluency** of solving non-monic quadratic equations by completing the square.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- Anchor Chart PDF, *Sentence Stems, Generalizing*
- Anchor Chart PDF, *Solving Non-Monic Quadratic Equations by Factoring*
- algebra tiles

Math Language Development

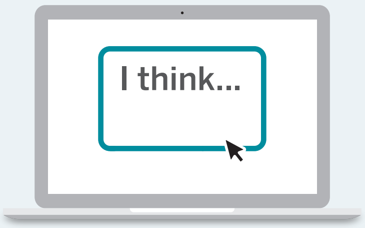
Review words

- *completing the square*
- *leading coefficient*
- *monic quadratic expression*
- *non-monic quadratic expression*
- *square expression*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to explain and show their thinking behind solving equations by completing the square, and these explanations are available to you digitally, in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overwhelmed making sense of and applying the strategies in Activity 3. Ask students how they are feeling and listen deeply and reflect what you heard about their feelings. For example, “It sounds like you are feeling very frustrated right now . . .” Then have students describe other challenging lessons or concepts they have preserved and succeeded in.

Modifications to Pacing

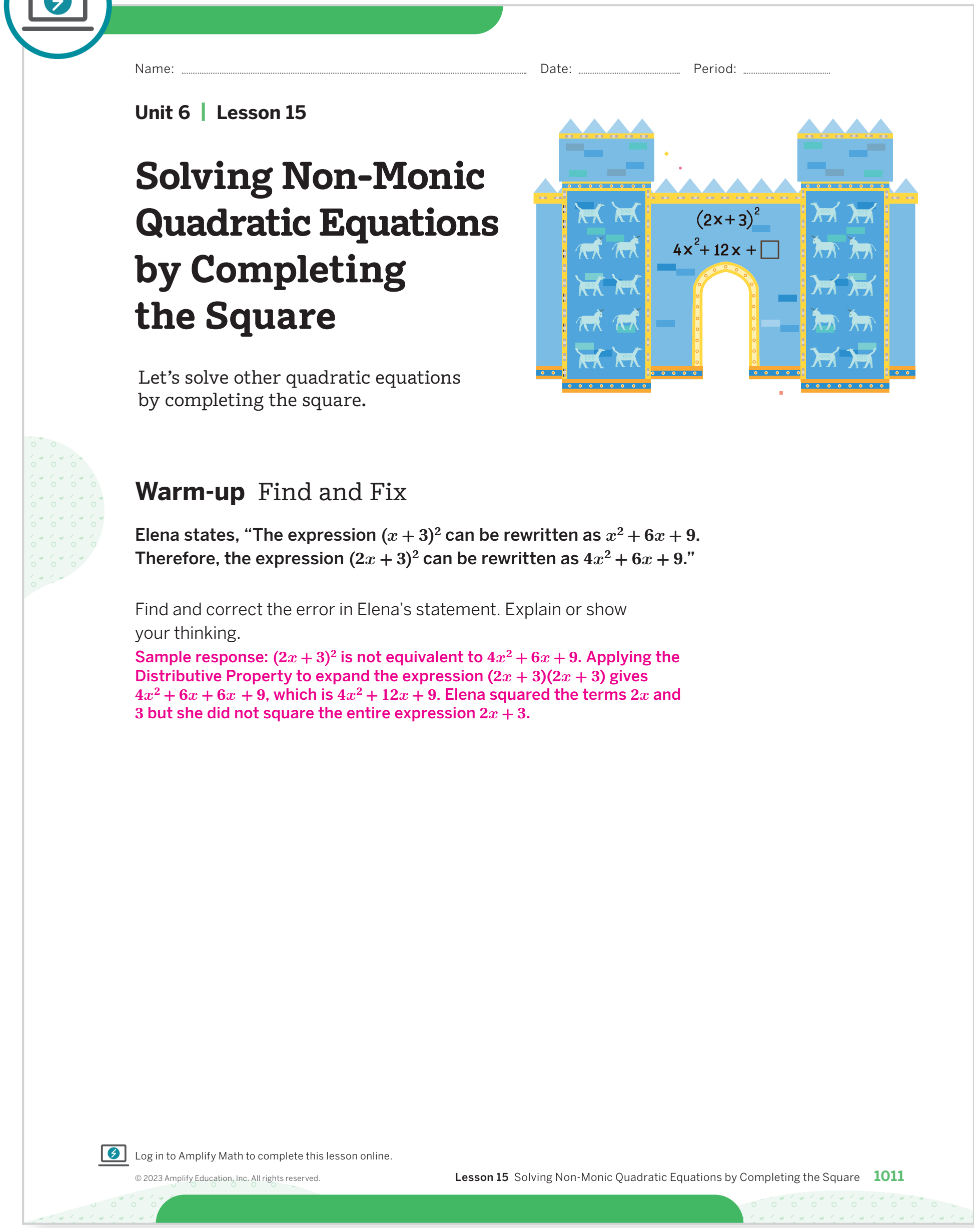
You may want to consider this additional modification if you are short on time.

- In **Activity 3**, have students only complete three problems or arrange students in groups and have each student complete a different equation using a different strategy, then compare together.

Independent | 5 min

Warm-up Find and Fix

Students analyze the expansion of an expression of the form $(mx+p)^2$ to compare with the expansion of the form $(x+p)^2$.



Name: ____________ Date: ________ Period: ________

Unit 6 | Lesson 15

Solving Non-Monic Quadratic Equations by Completing the Square

Let's solve other quadratic equations by completing the square.

Warm-up Find and Fix

Elena states, "The expression $(x+3)^2$ can be rewritten as x^2+6x+9. Therefore, the expression $(2x+3)^2$ can be rewritten as $4x^2+6x+9$."

Find and correct the error in Elena's statement. Explain or show your thinking.

Sample response: $(2x+3)^2$ is not equivalent to $4x^2+6x+9$. Applying the Distributive Property to expand the expression $(2x+3)(2x+3)$ gives $4x^2+6x+6x+9$, which is $4x^2+12x+9$. Elena squared the terms $2x$ and 3 but she did not square the entire expression $2x+3$.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Solving Non-Monic Quadratic Equations by Completing the Square 1011

1 Launch

Display the prompt. Conduct the ***Find and Fix*** routine. Provide 1 minute of think-time, then have students complete the problem individually.

2 Monitor

Help students get started by suggesting they use an area diagram to expand the factored expressions.

Look for points of confusion:

- **Having difficulty describing Elena's error.** Prompt students to explain the shortcut for expanding square expressions like $(x+3)^2$, then have them describe the difference between $(x+3)^2$ and $(2x+3)^2$.

Look for productive strategies:

- Using area diagrams to expand the expressions.
- Writing each factor twice, then using the Distributive Property to expand the expression.
- Using the generalized form $x^2+2px+p^2$ to spot the error.

3 Connect

Have students share their strategies and thinking for determining Elena's error. Sequence and select presenters by order of the productive strategies.

Display the correct standard form expression.

Ask, "How do the terms in $(2x+3)^2$ relate to the terms in $4x^2+12x+9$?" Sample response: The squared variable term in standard form is the square of the linear term in the factored form. The linear term in standard form is twice the product of $2x$ and 3. The constant term in standard form is the square of the constant term in the factored form.

Highlight the similar structure in expanding $(x+p)^2$ and $(mx+p)^2$.

Math Language Development

MLR3: Critique, Correct, Clarify

This Warm-up is structured similarly to the *MLR3: Critique, Correct, and Clarify* routine. While students work, display these questions that they can ask themselves.

- ***Critique:*** "Why do you think Elena made this mistake? What might she have been thinking?"
- ***Correct and Clarify:*** "How could you convince Elena of the correct standard form expression?"

English Learners

Before the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Explaining My Steps* to support students in explaining their thinking.

Power-up

To power up students' ability to expand expressions of the form $(mx+p)^2$, have students complete:

Complete the area diagram to rewrite the expression $(2x+5)^2$ in standard form.

$4x^2+20x+25$

	$2x$	5
$2x$	$4x^2$	$10x$
5	$10x$	25

Use: Before the Warm-up

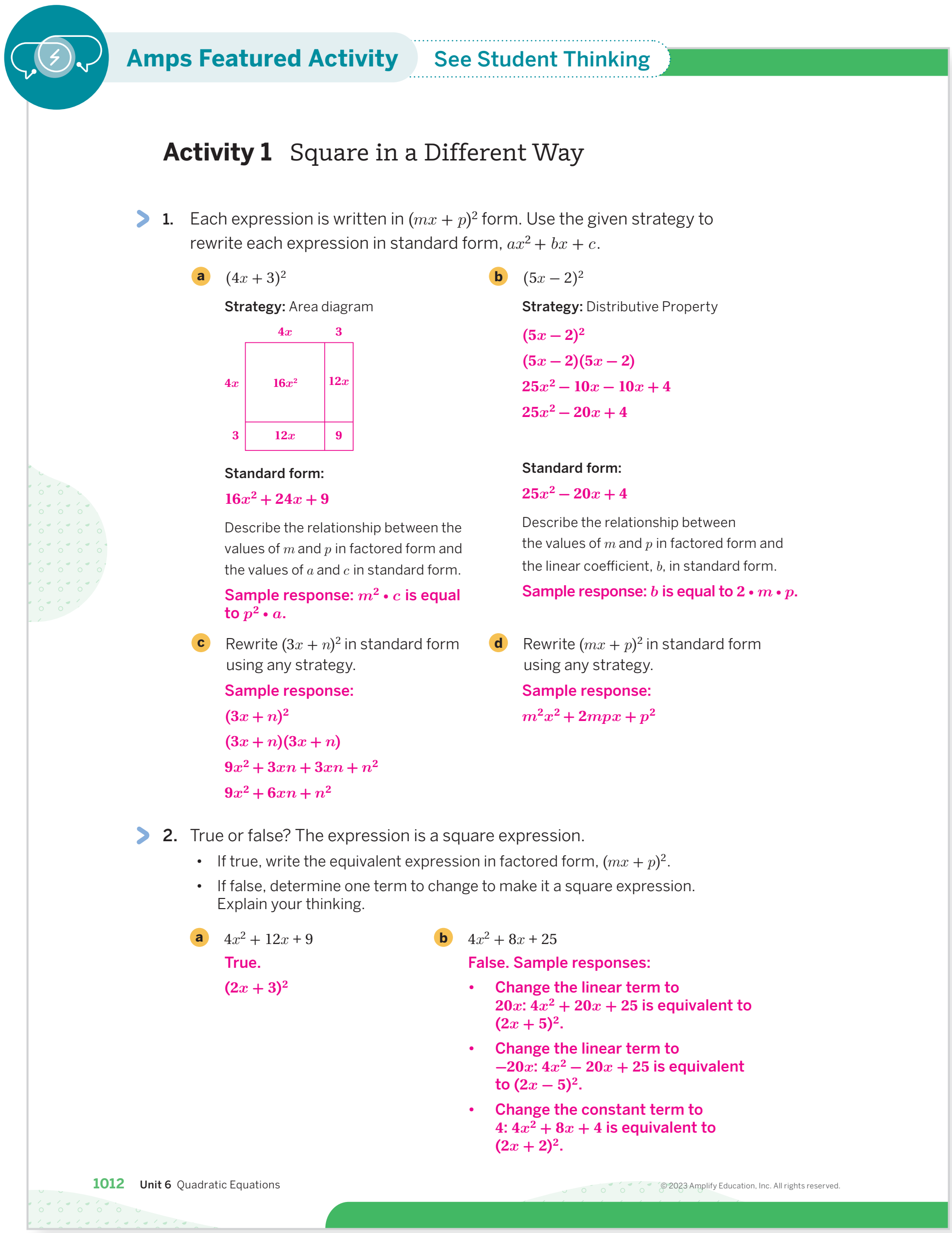
Informed by: Performance on Lesson 14, Practice Problem 5

Pairs | 10 min

Activity 1 Square in a Different Way

Students rewrite factored expressions $(mx + p)^2$, where m is not 1, into standard form to generalize the pattern for expanding non-monic expressions.

Amps Featured Activity See Student Thinking



Activity 1 Square in a Different Way

1. Each expression is written in $(mx + p)^2$ form. Use the given strategy to rewrite each expression in standard form, $ax^2 + bx + c$.

a $(4x + 3)^2$

Strategy: Area diagram

	$4x$	3
$4x$	$16x^2$	$12x$
3	$12x$	9

Standard form:

$16x^2 + 24x + 9$

Describe the relationship between the values of m and p in factored form and the values of a and c in standard form.

Sample response: $m^2 \cdot c$ is equal to $p^2 \cdot a$.

b $(5x - 2)^2$

Strategy: Distributive Property

$(5x - 2)^2$

$(5x - 2)(5x - 2)$

$25x^2 - 10x - 10x + 4$

$25x^2 - 20x + 4$

Standard form:

$25x^2 - 20x + 4$

Describe the relationship between the values of m and p in factored form and the linear coefficient, b, in standard form.

Sample response: b is equal to $2 \cdot m \cdot p$.

c Rewrite $(3x + n)^2$ in standard form using any strategy.

Sample response:

$(3x + n)^2$

$(3x + n)(3x + n)$

$9x^2 + 3xn + 3xn + n^2$

$9x^2 + 6xn + n^2$

d Rewrite $(mx + p)^2$ in standard form using any strategy.

Sample response:

$m^2x^2 + 2mpx + p^2$

2. True or false? The expression is a square expression.
   - If true, write the equivalent expression in factored form, $(mx + p)^2$.
   - If false, determine one term to change to make it a square expression. Explain your thinking.

a $4x^2 + 12x + 9$

True.

$(2x + 3)^2$

b $4x^2 + 8x + 25$

False. Sample responses:

- Change the linear term to $20x$: $4x^2 + 20x + 25$ is equivalent to $(2x + 5)^2$.
- Change the linear term to $-20x$: $4x^2 - 20x + 25$ is equivalent to $(2x - 5)^2$.
- Change the constant term to 4: $4x^2 + 8x + 4$ is equivalent to $(2x + 2)^2$.

1012 Unit 6 Quadratic Equations

1 Launch

Ask students to discuss each problem with their partner before completing them individually. Then have them compare solutions and describe any patterns they notice.

2 Monitor

Help students get started by providing blank area diagrams for students to complete.

Look for points of confusion:

- **Having difficulty writing the expression as a square expression in Problem 2b.** Prompt students to use an area diagram to model the standard form expression. Then have them explain how they could complete the square.

Look for productive strategies:

- Determining and applying a general pattern.

3 Connect

Have pairs of students share their strategies for determining the relationship between m and p in factored form and a, b, and c in standard form.

Display student solutions for each strategy and each expression rewritten in standard form.

Highlight the relationship between each term in factored form, $(mx + p)^2$, and standard form, $ax^2 + bx + c$.

Ask, "What term would you change to make the expression, $9x^2 + 24x + 25$, a square expression? By just changing that one term, would the new expression be equivalent?" I would change the linear term to $30x$ or $-30x$ or I would change the constant term to 16. The new expression would not be equivalent.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to algebra tiles or blank area diagram templates for students to choose to use in this activity.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the values of m, p, a, b, and c for each problem to help them notice the relationship.

Math Language Development

MLR3: Critique, Correct, Clarify

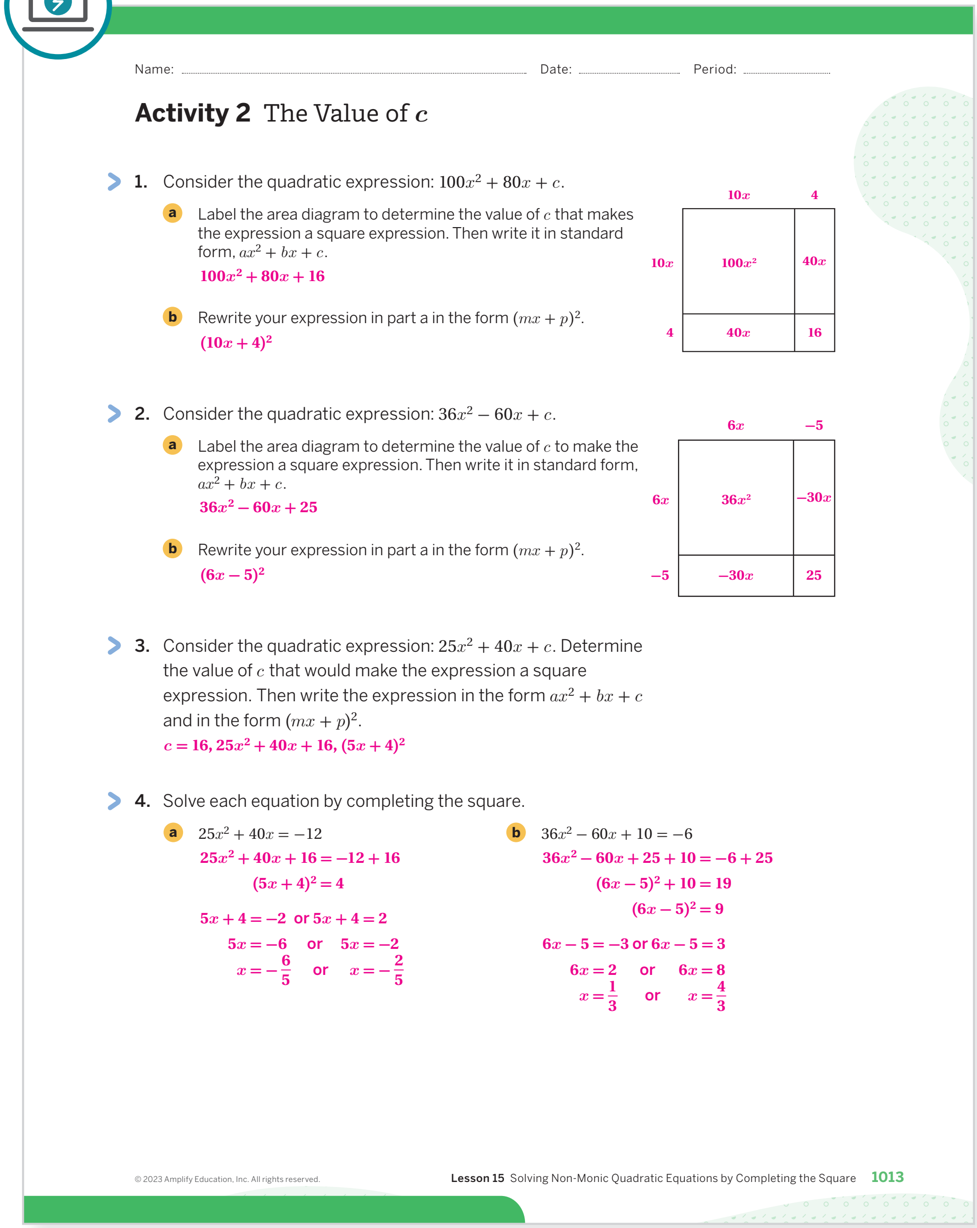
During the Connect, present students with an incorrect statement that reflects a common misunderstanding, such as "$(5x - 2)^2$ written in standard form is $25x^2 - 4$." Ask:

- ***Critique and Correct:*** "Do you agree or disagree with this statement? How would you correct this statement?" Listen for students who reason that the exponent cannot be distributed to each term inside the parentheses.
- ***Clarify:*** "What mathematical language or reasoning can you use to explain why and how you corrected the statement?"

Pairs | 10 min

Activity 2 The Value of c

Students complete the square for non-monic quadratic expressions and rewrite them in factored form to realize this strategy can be used to help solve quadratic equations.



Name: ________ Date: ________ Period: ________

Activity 2 The Value of c

1. Consider the quadratic expression: $100x^2 + 80x + c$.

   a. Label the area diagram to determine the value of c that makes the expression a square expression. Then write it in standard form, $ax^2 + bx + c$.

   $\mathbf{100x^2 + 80x + 16}$

	$10x$	4
$10x$	$100x^2$	$40x$
4	$40x$	16

   b. Rewrite your expression in part a in the form $(mx + p)^2$.

   $\mathbf{(10x + 4)^2}$

2. Consider the quadratic expression: $36x^2 - 60x + c$.

   a. Label the area diagram to determine the value of c to make the expression a square expression. Then write it in standard form, $ax^2 + bx + c$.

   $\mathbf{36x^2 - 60x + 25}$

	$6x$	-5
$6x$	$36x^2$	$-30x$
-5	$-30x$	25

   b. Rewrite your expression in part a in the form $(mx + p)^2$.

   $\mathbf{(6x - 5)^2}$

3. Consider the quadratic expression: $25x^2 + 40x + c$. Determine the value of c that would make the expression a square expression. Then write the expression in the form $ax^2 + bx + c$ and in the form $(mx + p)^2$.

   $\mathbf{c = 16, 25x^2 + 40x + 16, (5x + 4)^2}$

4. Solve each equation by completing the square.

   a. $25x^2 + 40x = -12$

   $25x^2 + 40x + 16 = -12 + 16$

   $(5x + 4)^2 = 4$

   $5x + 4 = -2$ or $5x + 4 = 2$

   $5x = -6$ or $5x = -2$

   $x = -\frac{6}{5}$ or $x = -\frac{2}{5}$

   b. $36x^2 - 60x + 10 = -6$

   $36x^2 - 60x + 25 + 10 = -6 + 25$

   $(6x - 5)^2 + 10 = 19$

   $(6x - 5)^2 = 9$

   $6x - 5 = -3$ or $6x - 5 = 3$

   $6x = 2$ or $6x = 8$

   $x = \frac{1}{3}$ or $x = \frac{4}{3}$

1 Launch

Ask students to discuss each problem with their partner before completing them individually. Then have them compare solutions and describe any patterns they notice.

2 Monitor

Help students get started by reviewing the terms of $(mx + p)^2$ and $ax^2 + bx + c$ and relating the terms to each partial area of the rectangle.

Look for points of confusion:

- **Struggling to complete the square to solve the equations in Problem 4.** Prompt students to refer back to the previous problems to complete the square. Then remind them to maintain equality when adding or subtracting values.

Look for productive strategies:

- When solving, adding the value of the square number.
- Rewriting the constant value using a square number.
- Determining the value needed to add to or subtract from the square number.

3 Connect

Have pairs of students share their solutions and strategies for Problems 3 and 4.

Highlight how completing the square on non-monic quadratic expressions can be used to help solve some quadratic equations.

Display the expressions $100x^2 + 80x + 16$, $36x^2 - 60x + 25$, and $25x^2 + 40x + 16$. Highlight how the coefficient of the squared variable term is a square number.

Ask, "How could you complete the square if the leading coefficient is not a square number?" I could factor the leading coefficient out of the expression and complete the square on the remaining expression. The result would be multiplied by the leading coefficient.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Help students make connections after they complete Problem 1 by displaying the equation $100x^2 + 80x + c = (mx + p)^2$ and asking:

- "What must be the value of m? Why?" 10, because the square of $10x$ is $100x^2$.
- "If the linear term in standard form is $80x$, how does this help you know what the value of c should be?" The linear term is equal to $2pmx$, so $80 = 2pm$. I know $m = 10$, so $p = 4$.
- "What must be the value of c? Why?" 16, because $p^2 = 16$.

Extension: *Math Enrichment*

Challenge students to determine the solutions to the equation $3x^2 - 6x + \frac{9}{4} = 0$ and explain their thinking. $x = \frac{3}{2}$, $x = \frac{1}{2}$.

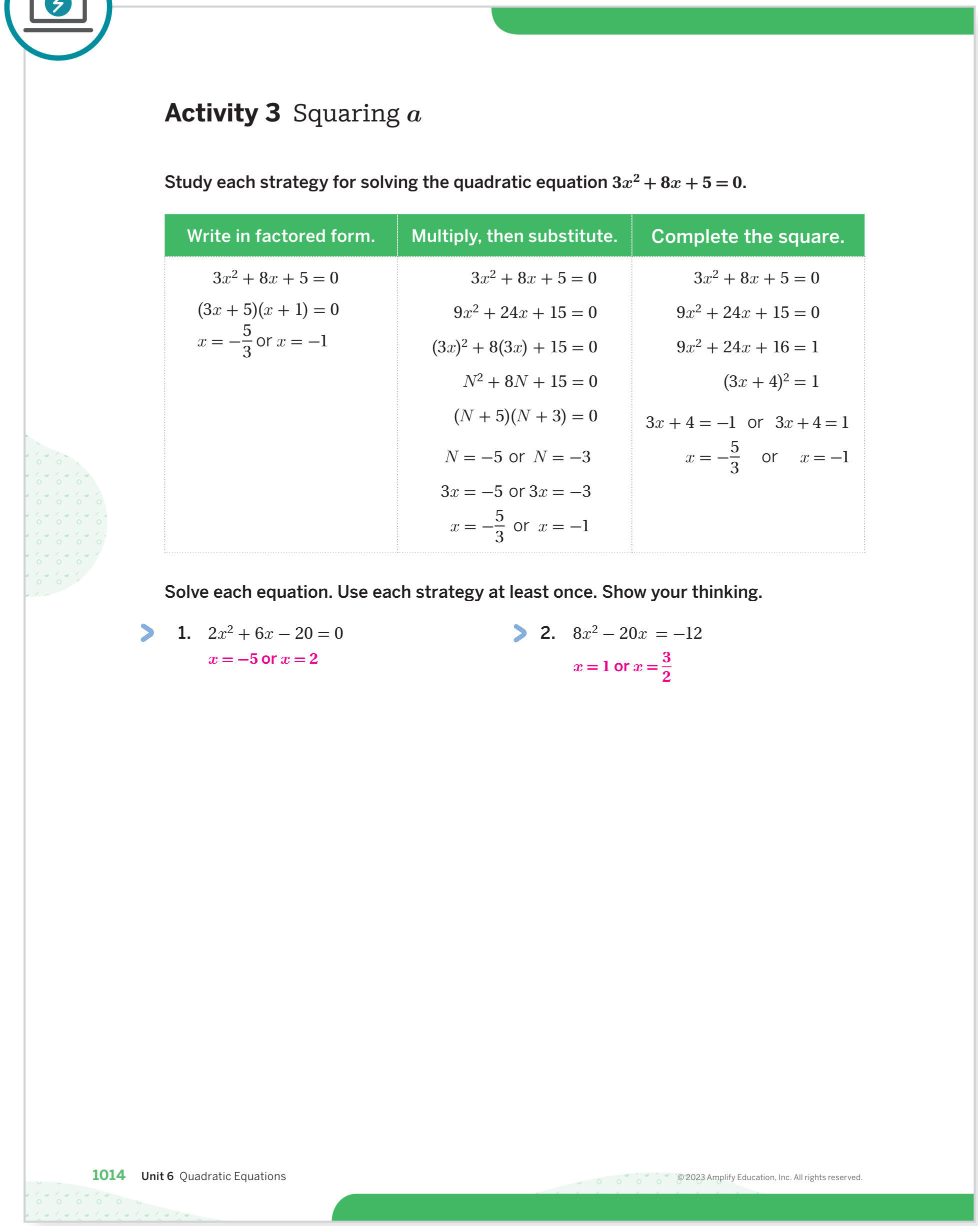
Sample response:

- Factor 3 out of the expression, $3(x^2 - 2x + \frac{3}{4}) = 0$.
- Write the expression in factored form by completing the square, $3((x - 1)^2 - \frac{1}{4}) = 0$.
- Divide both sides by 3, and then add $\frac{1}{4}$ to each side, $(x - 1)^2 = \frac{1}{4}$.
- Take the square root of each side, $x - 1 = \frac{1}{2}$ and $x - 1 = -\frac{1}{2}$.
- Add 1 to each side, $x = \frac{3}{2}$ or $x = \frac{1}{2}$.

Pairs | 15 min

Activity 3 Squaring a

Students analyze and apply three strategies to solve non-monic quadratic equations to determine the efficiency of certain strategies.



Activity 3 Squaring a

Study each strategy for solving the quadratic equation $3x^2 + 8x + 5 = 0$.

Write in factored form.	Multiply, then substitute.	Complete the square.
$3x^2 + 8x + 5 = 0$ $(3x + 5)(x + 1) = 0$ $x = -\frac{5}{3}$ or $x = -1$	$3x^2 + 8x + 5 = 0$ $9x^2 + 24x + 15 = 0$ $(3x)^2 + 8(3x) + 15 = 0$ $N^2 + 8N + 15 = 0$ $(N + 5)(N + 3) = 0$ $N = -5$ or $N = -3$ $3x = -5$ or $3x = -3$ $x = -\frac{5}{3}$ or $x = -1$	$3x^2 + 8x + 5 = 0$ $9x^2 + 24x + 15 = 0$ $9x^2 + 24x + 16 = 1$ $(3x + 4)^2 = 1$ $3x + 4 = -1$ or $3x + 4 = 1$ $x = -\frac{5}{3}$ or $x = -1$

Solve each equation. Use each strategy at least once. Show your thinking.

1. $2x^2 + 6x - 20 = 0$
   $x = -5$ or $x = 2$

2. $8x^2 - 20x = -12$
   $x = 1$ or $x = \frac{3}{2}$

1 Launch

Have students analyze the three strategies, then take turns explaining each to their partner. Have students complete the problems independently, and then compare strategies and solutions with their partner.

2 Monitor

Help students get started by asking, "What could you multiply the leading coefficient by to result in a square number?" I could multiply it by itself to get a square number.

Look for points of confusion:

- **Struggling to determine which strategy to use.** Prompt students to try any strategy to start and to explain why they had to switch strategies, if necessary.
- **Having difficulty applying the *multiply, then substitute* strategy.** Provide annotations and fill in the blank support to help students make connections between representations.
- **Forgetting how to factor non-monic quadratics.** Display the Anchor Chart PDF, *Solving Non-Monic Quadratic Equations by Factoring.*

Look for productive strategies:

- Using area diagrams.
- Changing to another strategy when the chosen strategy proves ineffective.
- Noticing and applying the structure to determine the best strategy to use for each equation.

Activity 3 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students choose four of the six problems to complete. Allowing them the power of choice can result in greater engagement and ownership of the task.

Extension: *Math Enrichment*

Have students create a flowchart from the three strategies in this activity to help determine an efficient strategy for solving a quadratic equation. For example, the first part of the flowchart could look like the one shown here.

Determining an efficient strategy:

1. Is the equation set equal to 0?
   YES: Move to Step 2.
   NO: Set it equal to 0.
2. Can you factor the equation?
   YES: Factor to solve the equation.
   NO: Move to Step 3.

Math Language Development

MLR8: Discussion Supports—Press for Details

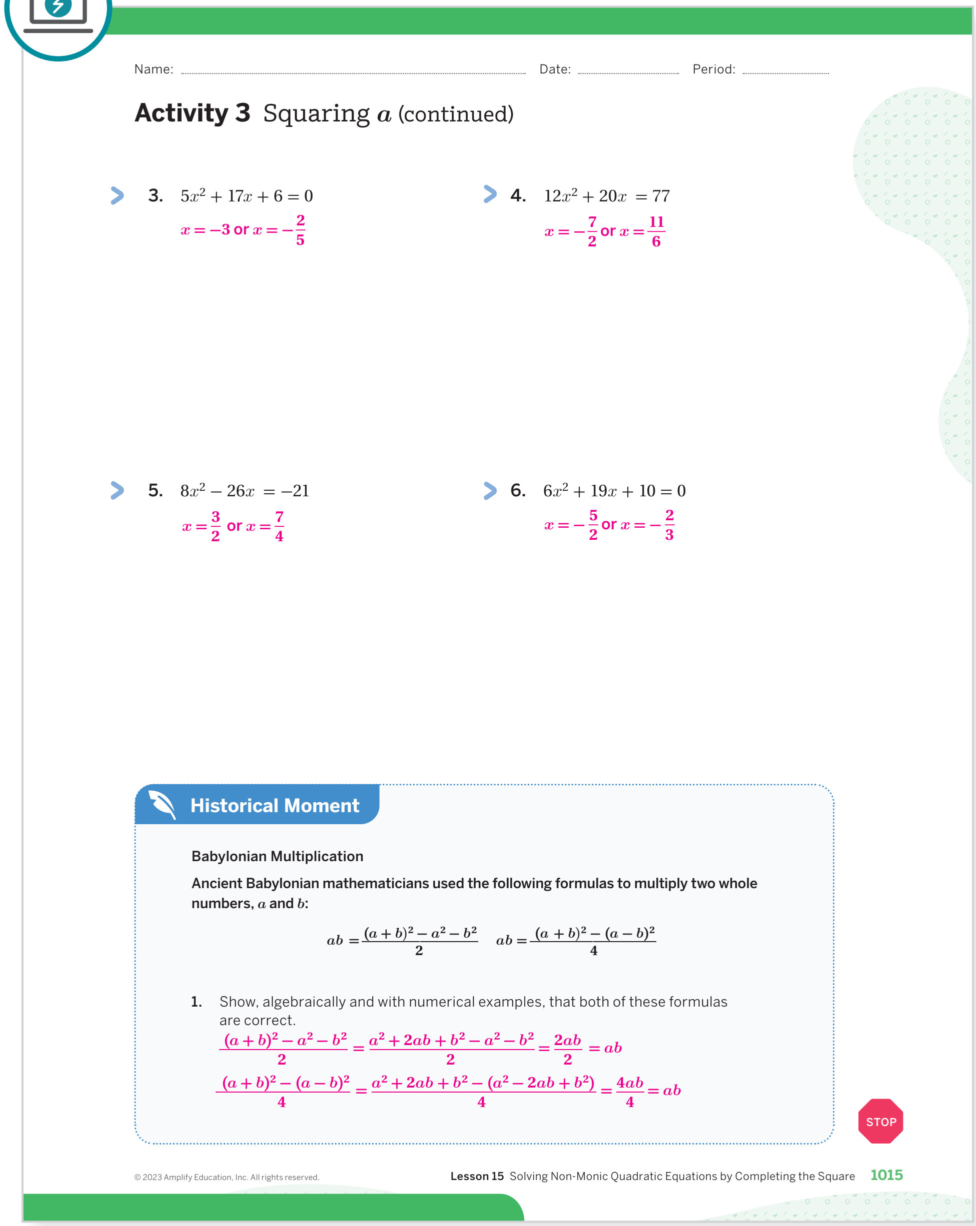
During the Connect, as students respond to the Ask questions, press for details in their reasoning as to how the structure of each equation helped them determine a strategy. For example:

If a student says . . .	Press for details by asking . . .
"I completed the square for Problem 6."	"What did you notice about the structure of the equation that helped you choose this strategy?"

Pairs | 15 min

Activity 3 Squaring a (continued)

Students analyze and apply three strategies to solve non-monic quadratic equations to determine the efficiency of certain strategies.



Name: ________ Date: ________ Period: ________

Activity 3 Squaring a (continued)

3. $5x^2 + 17x + 6 = 0$

$x = -3$ or $x = -\frac{2}{5}$

4. $12x^2 + 20x = 77$

$x = -\frac{7}{2}$ or $x = \frac{11}{6}$

5. $8x^2 - 26x = -21$

$x = \frac{3}{2}$ or $x = \frac{7}{4}$

6. $6x^2 + 19x + 10 = 0$

$x = -\frac{5}{2}$ or $x = -\frac{2}{3}$

Historical Moment

Babylonian Multiplication

Ancient Babylonian mathematicians used the following formulas to multiply two whole numbers, a and b:

$$ab = \frac{(a+b)^2 - a^2 - b^2}{2} \qquad ab = \frac{(a+b)^2 - (a-b)^2}{4}$$

1. Show, algebraically and with numerical examples, that both of these formulas are correct.

$$\frac{(a+b)^2 - a^2 - b^2}{2} = \frac{a^2 + 2ab + b^2 - a^2 - b^2}{2} = \frac{2ab}{2} = ab$$

$$\frac{(a+b)^2 - (a-b)^2}{4} = \frac{a^2 + 2ab + b^2 - (a^2 - 2ab + b^2)}{4} = \frac{4ab}{4} = ab$$

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Solving Non-Monic Quadratic Equations by Completing the Square 1015

3 Connect

Have pairs of students share the strategies used to solve the equation in Problem 1 and their thinking.

Display the equation in Problem 1 and student strategies and thinking.

Ask:

- "Did you solve the equation in Problem 2 using the same strategy or a different strategy? Explain your thinking." Answers may vary.
- "Did the structure of some equations help you know when it might be more efficient to solve by one strategy, rather than the others?" Sample response: There are many factors of 12 in Problem 4, so I decided to multiply by 3 to create a perfect square variable term instead and then I used the *multiply, then substitute* strategy.
- "What are the advantages of each strategy? Disadvantages?" Answers may vary.
- "Is there a strategy you prefer or find reliable? Which one and why?" Answers may vary.

Highlight there are many different strategies to solve quadratic equations. Some are more efficient than others, depending on the structure of the equation.

Historical Moment

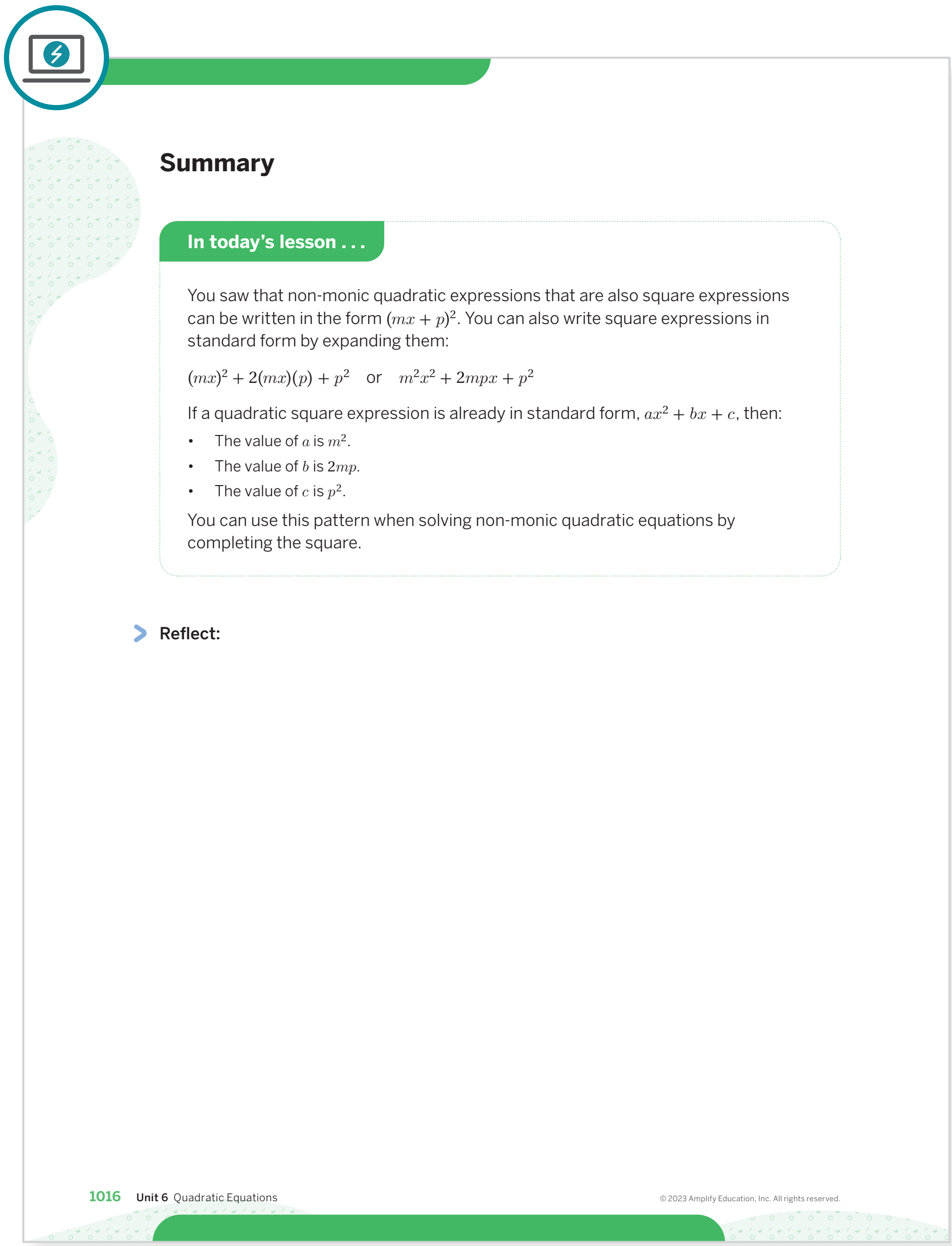
Babylonian Multiplication

Have students complete the *Historical Moment* activity to see how ancient Babylonian mathematicians multiplied whole numbers using two different formulas.

Whole Class | 5 min

Summary

Review and synthesize the process of solving non-monic quadratic equations by completing the square for different types of quadratic equations.



Summary

In today's lesson . . .

You saw that non-monic quadratic expressions that are also square expressions can be written in the form $(mx + p)^2$. You can also write square expressions in standard form by expanding them:

$(mx)^2 + 2(mx)(p) + p^2$ or $m^2x^2 + 2mpx + p^2$

If a quadratic square expression is already in standard form, $ax^2 + bx + c$, then:

- The value of a is m^2.
- The value of b is $2mp$.
- The value of c is p^2.

You can use this pattern when solving non-monic quadratic equations by completing the square.

Reflect:

1016 Unit 6 Quadratic Equations

Synthesize

Display the following equations: $x^2 + 8x = -12$, $16x^2 + 8x = -12$, $4x^2 + 32x = -7$, $5x^2 + 32x = -7$, and $9x^2 + 18x = 6$.

Ask:

- "Which equations would you solve by completing the square? Why?"
- "What number would you add to both sides to solve the equations $4x^2 + 32x = -7$ and $9x^2 + 18x = 6$ by completing the square? Why?" Add 16; add 9.
- "Do certain features or numbers in an equation make it more or less challenging to solve by completing the square?" Solving by completing the square can be less challenging when the coefficient of the squared variable term is 1 or a different square number. It is more challenging when the coefficients are fractions.

Highlight that there are different strategies to solve quadratic equations. So far, students have learned the following strategies:

- Solving by square roots
- Factoring
- Completing the square

Later in this unit, students will learn a new strategy.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

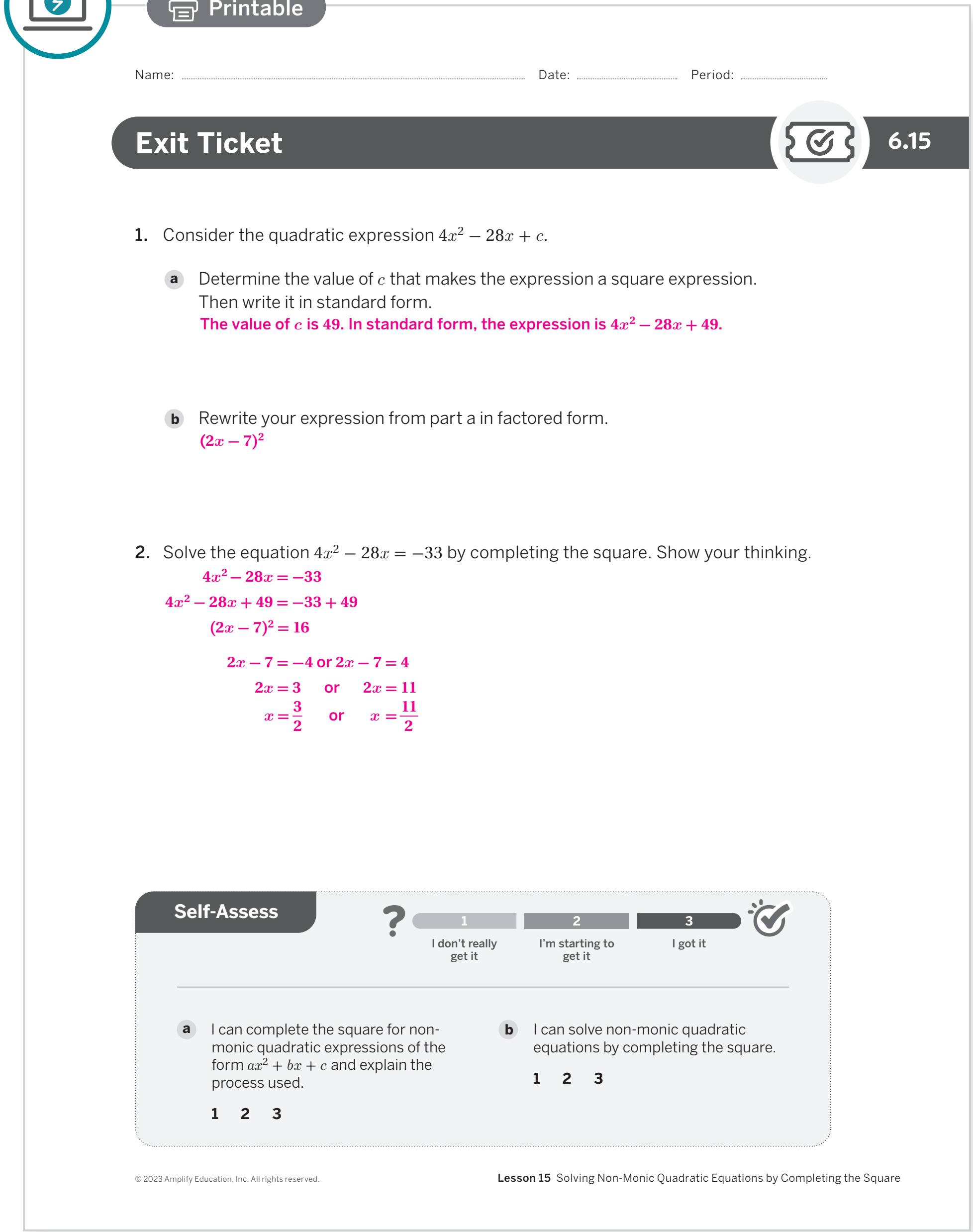
- "When is completing the square an efficient strategy for solving a non-monic quadratic equation?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of completing the square with non-monic quadratics by solving a quadratic equation.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.15

1. Consider the quadratic expression $4x^2 - 28x + c$.

   a Determine the value of c that makes the expression a square expression. Then write it in standard form.
   The value of c is 49. In standard form, the expression is $4x^2 - 28x + 49$.

   b Rewrite your expression from part a in factored form.
   $(2x - 7)^2$

2. Solve the equation $4x^2 - 28x = -33$ by completing the square. Show your thinking.

$$4x^2 - 28x = -33$$
$$4x^2 - 28x + 49 = -33 + 49$$
$$(2x - 7)^2 = 16$$
$$2x - 7 = -4 \text{ or } 2x - 7 = 4$$
$$2x = 3 \quad \text{or} \quad 2x = 11$$
$$x = \frac{3}{2} \quad \text{or} \quad x = \frac{11}{2}$$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can complete the square for non-monic quadratic expressions of the form $ax^2 + bx + c$ and explain the process used.

1 2 3

b I can solve non-monic quadratic equations by completing the square.

1 2 3

Success looks like . . .

- **Language Goal:** Generalizing a process for completing the square to express any non-monic quadratic equation in the form $(mx + p)^2 = q$. **(Speaking and Listening)**
- **Goal:** Solving non-monic quadratic equations by completing the square.
  - » Solving the quadratic equation in Problem 2.

Suggested next steps

If students incorrectly determine the value of c or incorrectly rewrite the expression in Problem 1a, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 2.
- Asking students to use an area diagram to model the problem.

If students incorrectly solve the equation in Problem 1c, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 3.
- Providing students with a monic quadratic to solve by completing the square to determine whether they understand the general strategy. Ask them to explain the strategy in their own words.

Professional Learning

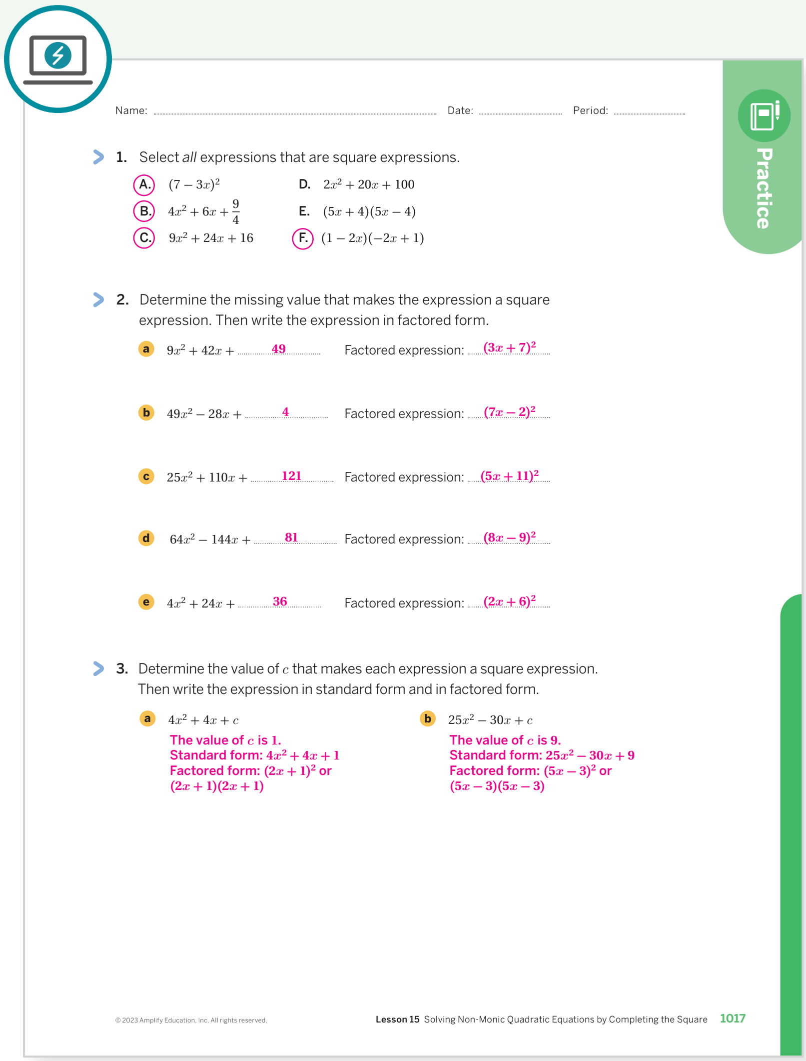
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students solve non-monic quadratic equations by completing the square. How did that build on the earlier work that students did with completing a square?
- What different ways did students approach solving non-monic quadratic equations in Activity 3? What does that tell you about similarities and differences among your students?

Practice

Independent



Name: .... Date: .... Period: ....

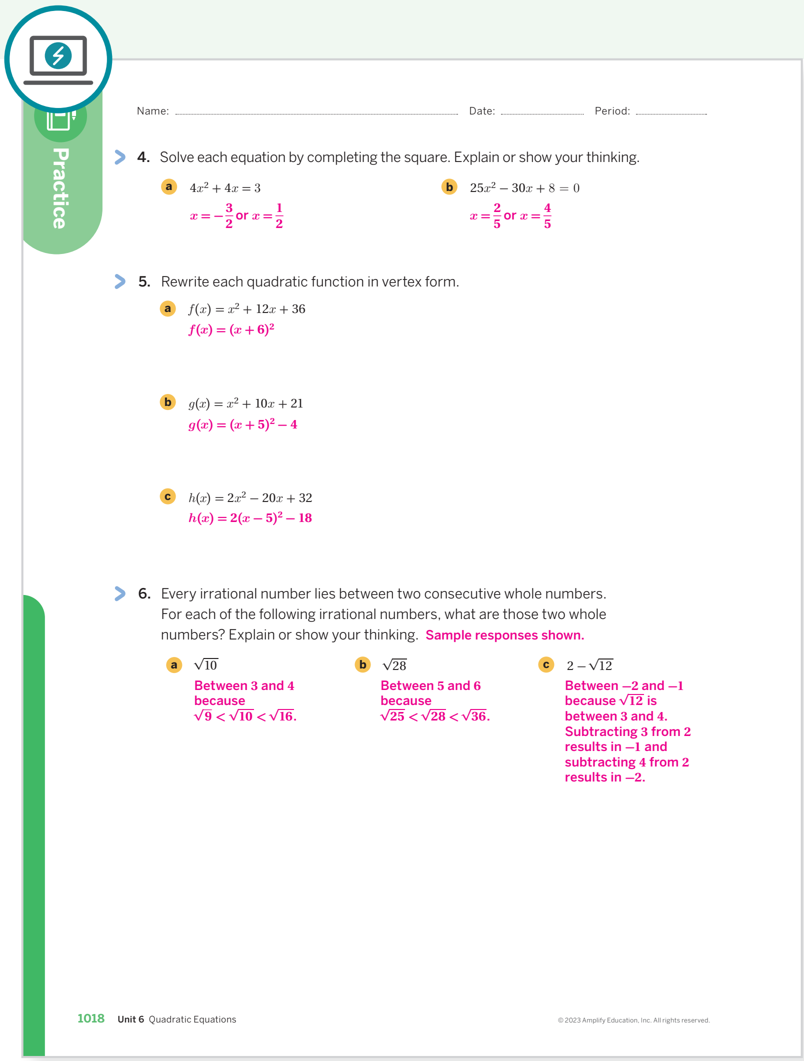
1. Select *all* expressions that are square expressions.
   - [✓ A.] $(7-3x)^2$
   - [✓ B.] $4x^2+6x+\frac{9}{4}$
   - [✓ C.] $9x^2+24x+16$
   - D. $2x^2+20x+100$
   - E. $(5x+4)(5x-4)$
   - [✓ F.] $(1-2x)(-2x+1)$

2. Determine the missing value that makes the expression a square expression. Then write the expression in factored form.
   - a $9x^2+42x+$ **49** Factored expression: $(3x+7)^2$
   - b $49x^2-28x+$ **4** Factored expression: $(7x-2)^2$
   - c $25x^2+110x+$ **121** Factored expression: $(5x+11)^2$
   - d $64x^2-144x+$ **81** Factored expression: $(8x-9)^2$
   - e $4x^2+24x+$ **36** Factored expression: $(2x+6)^2$

3. Determine the value of c that makes each expression a square expression. Then write the expression in standard form and in factored form.
   - a $4x^2+4x+c$
     The value of c is 1.
     Standard form: $4x^2+4x+1$
     Factored form: $(2x+1)^2$ or $(2x+1)(2x+1)$
   - b $25x^2-30x+c$
     The value of c is 9.
     Standard form: $25x^2-30x+9$
     Factored form: $(5x-3)^2$ or $(5x-3)(5x-3)$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Solving Non-Monic Quadratic Equations by Completing the Square 1017



Name: .... Date: .... Period: ....

4. Solve each equation by completing the square. Explain or show your thinking.
   - a $4x^2+4x=3$
     $x=-\frac{3}{2}$ or $x=\frac{1}{2}$
   - b $25x^2-30x+8=0$
     $x=\frac{2}{5}$ or $x=\frac{4}{5}$

5. Rewrite each quadratic function in vertex form.
   - a $f(x)=x^2+12x+36$
     $f(x)=(x+6)^2$
   - b $g(x)=x^2+10x+21$
     $g(x)=(x+5)^2-4$
   - c $h(x)=2x^2-20x+32$
     $h(x)=2(x-5)^2-18$

6. Every irrational number lies between two consecutive whole numbers. For each of the following irrational numbers, what are those two whole numbers? Explain or show your thinking. **Sample responses shown.**
   - a $\sqrt{10}$
     Between 3 and 4 because $\sqrt{9}<\sqrt{10}<\sqrt{16}$.
   - b $\sqrt{28}$
     Between 5 and 6 because $\sqrt{25}<\sqrt{28}<\sqrt{36}$.
   - c $2-\sqrt{12}$
     Between −2 and −1 because $\sqrt{12}$ is between 3 and 4. Subtracting 3 from 2 results in −1 and subtracting 4 from 2 results in −2.

1018 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 13	2
	5	Unit 6 Lesson 14	2
Formative ⚡	6	Unit 6 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 4

Roots and Irrationals

Whole Class

In this Sub-Unit, students apply their understanding of solving quadratic equations to explore quadratic equations with irrational solutions.

SUB-UNIT

4 Roots and Irrationals

Narrative Connections

Where does a number call its home?

Whenever you add two whole numbers, such as 3 and 4, you get another whole number (in this case, 7). This works for *any two* whole numbers you add. Try as you might, it is impossible to get anything else. You can imagine whole numbers as a town, and addition as a bus that runs through it. It's fine for a while — you can see a lot this way — but eventually you realize you have never seen what lies beyond. For all you know, the town is enclosed by a fence, a great wall, or an abyss. If you never try to leave, you will never know.

But then, along comes a new operation: subtraction. If you subtract 5 from 10, you stay within the town's limits. But what happens if you try to subtract 10 from 5? Suddenly, you have broken through into someplace different! Here, there are negatives and opposites. Along with positive integers, negative integers and zero come together to make up a bustling metropolis. But over time — with just adding, subtracting, and multiplying — you eventually realize you have never seen what is outside this metropolis.

Ah, but what about division? Try splitting 5 into 3 equal parts, and you will see this metropolis of integers is just part of a larger country of rational numbers. But what is beyond this country? Take the square root of many rational numbers, such as 2, and you are out in a wider world that includes irrational numbers. Will this always be the case? And can you ever put these new numbers together to find your way back to where you started, with whole numbers, integers, and rationals?

Narrative Connections

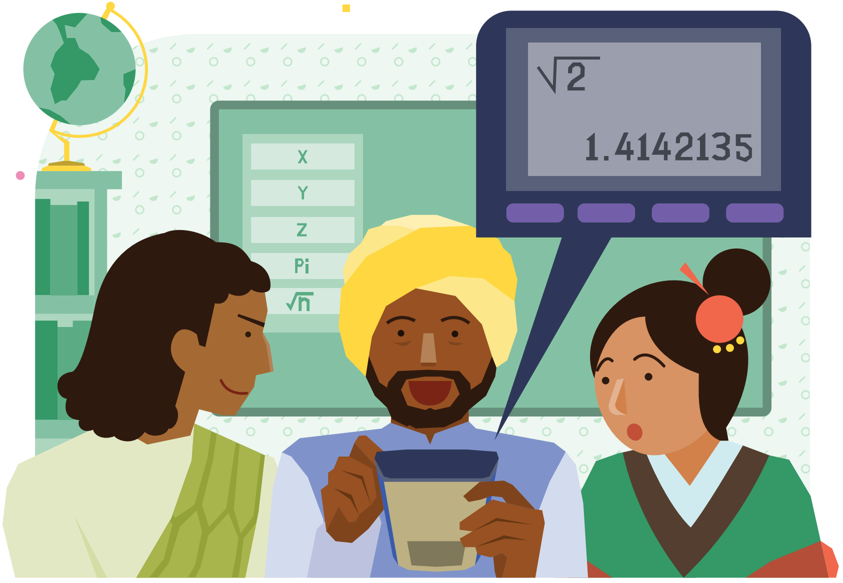
Read the narrative aloud as a class or have students read it individually. Students continue to explore the world of numbers that lie beyond rational numbers, including how they relate to quadratic equations, in the following places:

- **Lesson 16, Activities 1–2:** Square Root Solutions, Irrational Solutions
- **Lesson 17, Activities 1–3:** Exploring With Products and Sums, Sums and Products of Rational Numbers, Sums and Productions of Rational and Irrational Numbers
- **Lesson 18, Activities 1-3:** Exploring Irrational Denominators, Rational or Irrational Solutions?, Equations With Irrational Solutions

UNIT 6 | **LESSON 16**

Quadratic Equations With Irrational Solutions

Let's examine exact solutions to quadratic equations.



Focus

Goals

1. **Language Goal:** Compare solutions to quadratic equations solved by completing the square and graphing. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain that the plus-or-minus symbol is used to represent both square roots of a number and that the square root notation without a sign is understood to represent only the positive square root. **(Speaking and Listening, Writing)**
3. Use radical and plus-or-minus symbols to express solutions to quadratic equations.

Coherence

Today

Students solve quadratic equations with irrational solutions. They use the plus-or-minus symbol as a way to express both positive and negative exact solutions. Students determine and make sense of the decimal approximations by solving the equations by graphing.

Previously

In Lesson 15, students completed the square to solve non-monic quadratic equations.

Coming Soon

In Lesson 19, students will derive the quadratic formula by using the difference of squares and completing the square.

Rigor

- Students develop **conceptual understanding** of approximate and exact solutions of quadratic equations.
- Students strengthen their **procedural skills** of solving quadratic equations by taking the square root.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- *Anchor Chart PDF, Rational and Irrational Numbers*
- *Anchor Chart PDF, Sentence Stems, Explaining My Steps*
- *Anchor Chart PDF, Sentence Stems, Partner and Group Questioning*
- colored pencils
- graphing technology
- scientific calculators
- scissors

Math Language Development

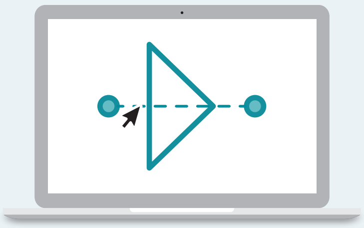
Review words

- *completing the square*
- *irrational number*
- *plus-or-minus* (±)
- *rational number*
- *square expression*

Amps | **Featured Activity**

Activity 2
Step-by-Step Solving

As students solve equations with irrational solutions, see their algebraic manipulations step by step.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel dismissive of or lack attention to communicating precisely about exact and approximate solutions in Activity 2. Have students brainstorm ways to organize their work to self-assess their progress towards learning goals.

● Modifications to Pacing

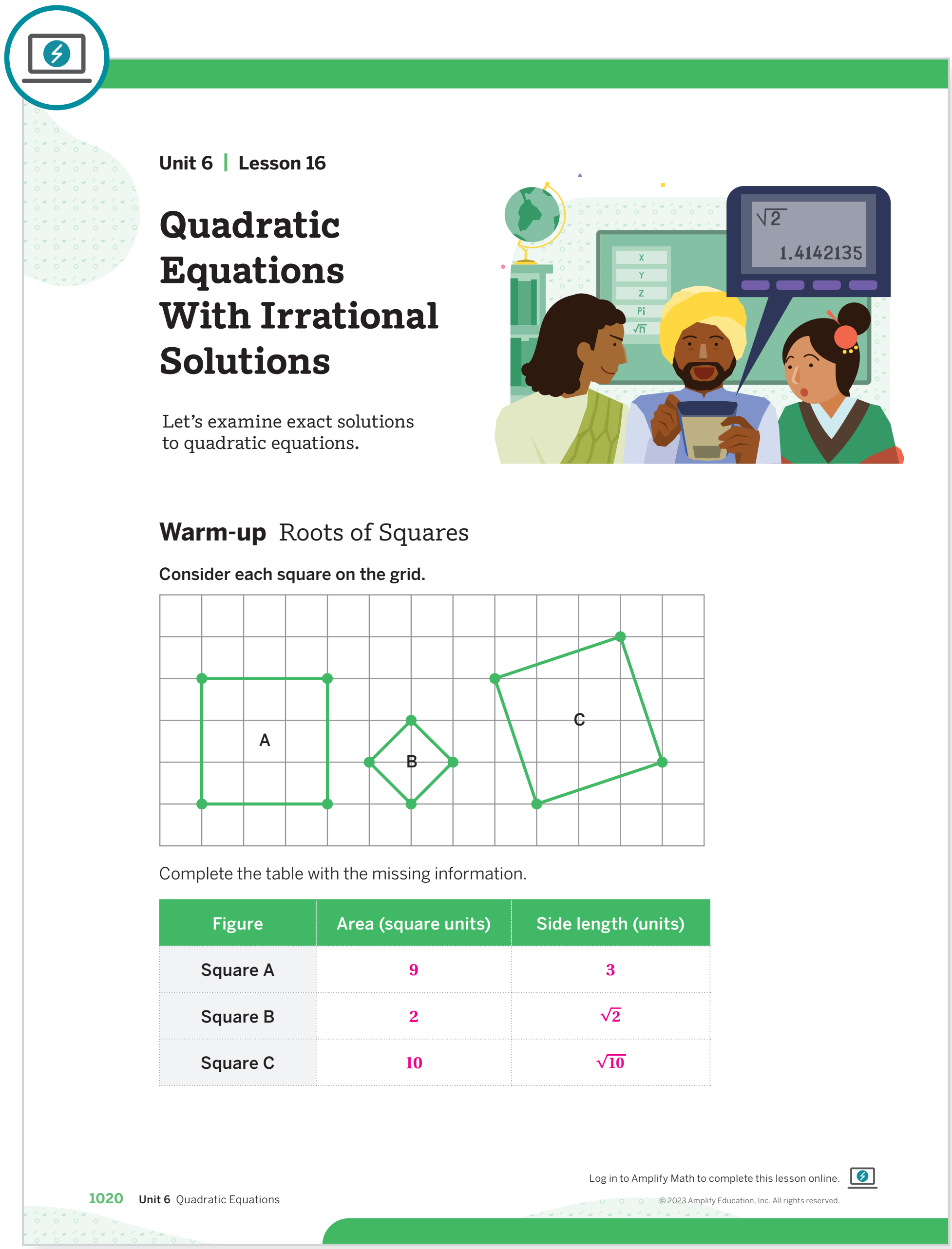
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problems 1 and 2 may be omitted.
- In **Activity 2**, have students only complete 2–3 problems.

Independent | 10 min

Warm-up Roots of Squares

Students express the side lengths of squares using the square root symbol to recall irrational values.



Unit 6 | Lesson 16

Quadratic Equations With Irrational Solutions

Let's examine exact solutions to quadratic equations.

Warm-up Roots of Squares

Consider each square on the grid.

Complete the table with the missing information.

Figure	Area (square units)	Side length (units)
Square A	9	3
Square B	2	$\sqrt{2}$
Square C	10	$\sqrt{10}$

Log in to Amplify Math to complete this lesson online.

1020 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Consider providing an additional copy of the Warm-up, colored pencils, and/or scissors for students who choose a strategy of decomposition.

2 Monitor

Help students get started by having them describe strategies for determining the area of a square using a grid.

Look for points of confusion:

- **Having difficulty determining the side length of Square C.** Have students use color to help visualize the pieces of the unit square.

Look for productive strategies:

- Decomposing the square and rearranging into rectangles.
- Composing Square B into another square using 4 triangles, then subtracting the areas of the extra triangles.
- Using the Pythagorean Theorem.

3 Connect

Have students share their strategies for determining the side lengths of each square.

Highlight that if the value of the square root is not a whole number, it can be written as a square root. Any positive number has two square roots.

Display the Anchor Chart PDF, *Rational and Irrational Numbers*.

Ask:

- "Is $\sqrt{10}$ or 3.16 the *exact* side length of the square?" $\sqrt{10}$
- "Because every positive number has two square roots, why is the other side length of the square not $-\sqrt{10}$?" Sample response: $-\sqrt{10}$ would represent a negative side length which is not possible.

Power-up

To power up students' ability to relate square roots to their rational approximations, have students complete:

1. A square has an area of 10 in^2. Noah claims that the *exact* side length of the square is $\sqrt{10}$ in. Lin claims the *exact* side length is 3.16 in. Who is correct? Explain your thinking. Noah; The exact side length is $\sqrt{10}$. Lin approximated the side length.
2. Is $\sqrt{10}$ rational or irrational? Explain your thinking. Irrational, because it cannot be written as a fraction representing the ratio of two integers.
3. Which is greater, 3.16 or $\sqrt{10}$? Explain your thinking. $\sqrt{10}$ is greater; $(3.16)^2 = 9.9856$, which is less than 10.

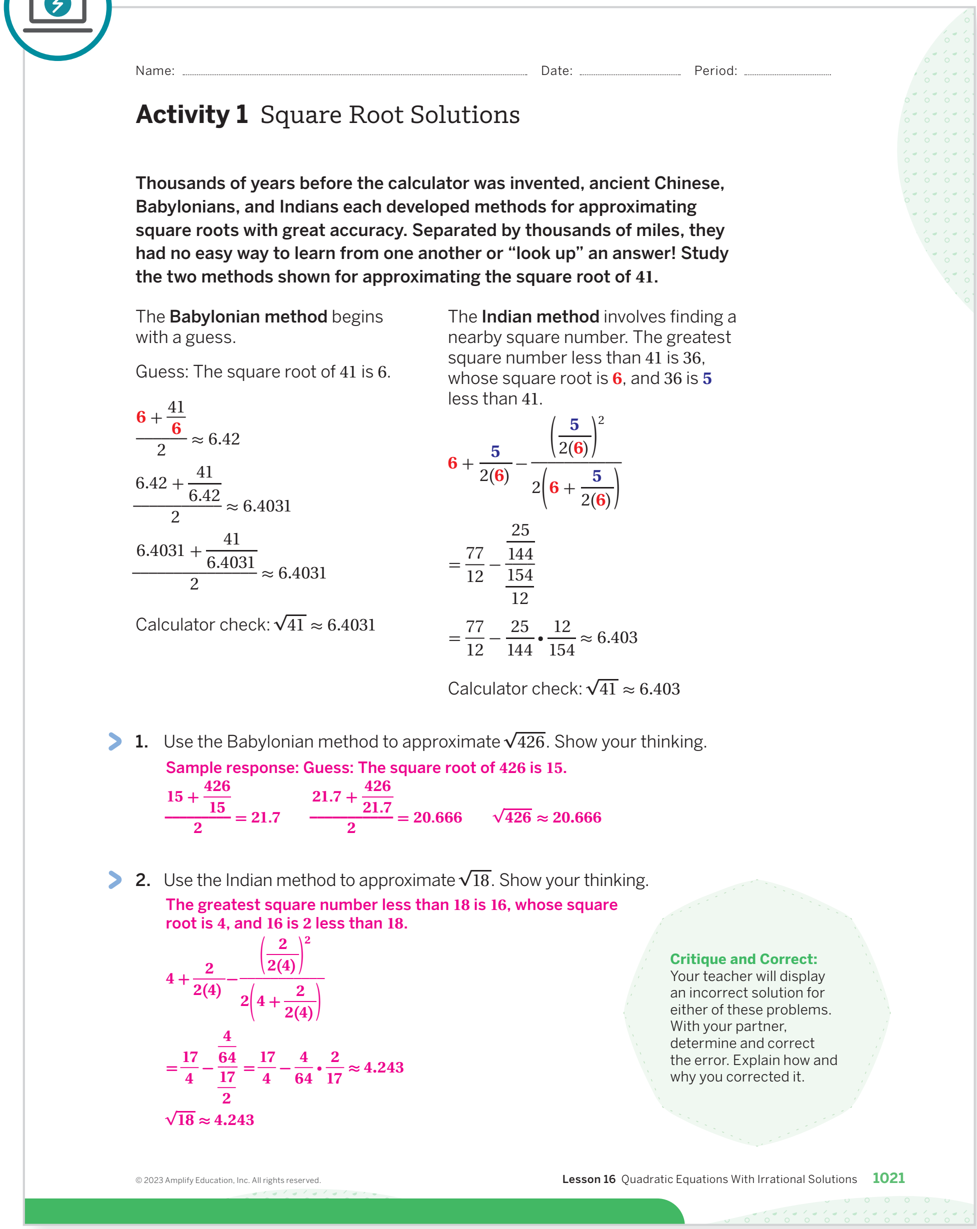
Use: Before Activity 1

Informed by: Performance on Lesson 15, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 4, 6, and 7

Pairs | 15 min

Activity 1 Square Root Solutions

Students solve quadratic equations by taking the square root of each side to practice using the plus-or-minus notation in solutions.



Name: ______ Date: ______ Period: ______

Activity 1 Square Root Solutions

Thousands of years before the calculator was invented, ancient Chinese, Babylonians, and Indians each developed methods for approximating square roots with great accuracy. Separated by thousands of miles, they had no easy way to learn from one another or "look up" an answer! Study the two methods shown for approximating the square root of 41.

The **Babylonian method** begins with a guess.

Guess: The square root of 41 is 6.

$$\frac{6+\frac{41}{6}}{2} \approx 6.42$$

$$\frac{6.42+\frac{41}{6.42}}{2} \approx 6.4031$$

$$\frac{6.4031+\frac{41}{6.4031}}{2} \approx 6.4031$$

Calculator check: $\sqrt{41} \approx 6.4031$

The **Indian method** involves finding a nearby square number. The greatest square number less than 41 is 36, whose square root is 6, and 36 is 5 less than 41.

$$6+\frac{5}{2(6)}-\frac{\left(\frac{5}{2(6)}\right)^2}{2\left(6+\frac{5}{2(6)}\right)}$$

$$=\frac{77}{12}-\frac{\frac{25}{144}}{\frac{154}{12}}$$

$$=\frac{77}{12}-\frac{25}{144}\bullet\frac{12}{154} \approx 6.403$$

Calculator check: $\sqrt{41} \approx 6.403$

1. Use the Babylonian method to approximate $\sqrt{426}$. Show your thinking.

Sample response: Guess: The square root of 426 is 15.

$$\frac{15+\frac{426}{15}}{2} = 21.7 \qquad \frac{21.7+\frac{426}{21.7}}{2} = 20.666 \qquad \sqrt{426} \approx 20.666$$

2. Use the Indian method to approximate $\sqrt{18}$. Show your thinking.

The greatest square number less than 18 is 16, whose square root is 4, and 16 is 2 less than 18.

$$4+\frac{2}{2(4)}-\frac{\left(\frac{2}{2(4)}\right)^2}{2\left(4+\frac{2}{2(4)}\right)}$$

$$=\frac{17}{4}-\frac{\frac{4}{64}}{\frac{17}{2}}=\frac{17}{4}-\frac{4}{64}\bullet\frac{2}{17} \approx 4.243$$

$$\sqrt{18} \approx 4.243$$

Critique and Correct: Your teacher will display an incorrect solution for either of these problems. With your partner, determine and correct the error. Explain how and why you corrected it.

1 Launch

Read the passage aloud and have a brief whole-class discussion about life before calculators. Have student pairs discuss each problem first, before working independently, and then comparing strategies and solutions. Provide access to scientific calculators.

2 Monitor

Help students get started by modeling the Babylonian and Indian methods for approximating square roots.

Look for points of confusion:

- **Evaluating square roots by dividing by 2.** Demonstrate the value of a square root by referring back to the Warm-up. Relate the square root to the area and side length of a square.
- **Not using the ± notation.** Prompt students to write their answer with and without notation to reinforce the meaning. Use language cues "a number and its opposite" to help in the reinforcement.
- **Attempting to take the square root of irrational numbers.** Remind students that numbers that are not square do not have an exact square root value.

Look for productive strategies:

- Noticing and applying the shortcut of writing the ± immediately after taking the square root.
- Including the exact and approximate solutions.
- Writing their solutions in different, equivalent ways.

Activity 1 continued ›

Math Language Development

MLR3: Critique, Correct, Clarify

After students complete Problems 1 and 2, display an incorrect solution to one of these problems, such as the following:

Guess: The square root of 426 is 20.

$$\frac{20+\frac{426}{20}}{20} \approx 2.065 \qquad \frac{2.065+\frac{426}{2.065}}{20} \approx 10.418 \qquad \frac{10.418+\frac{426}{10.418}}{20} \approx 2.565$$

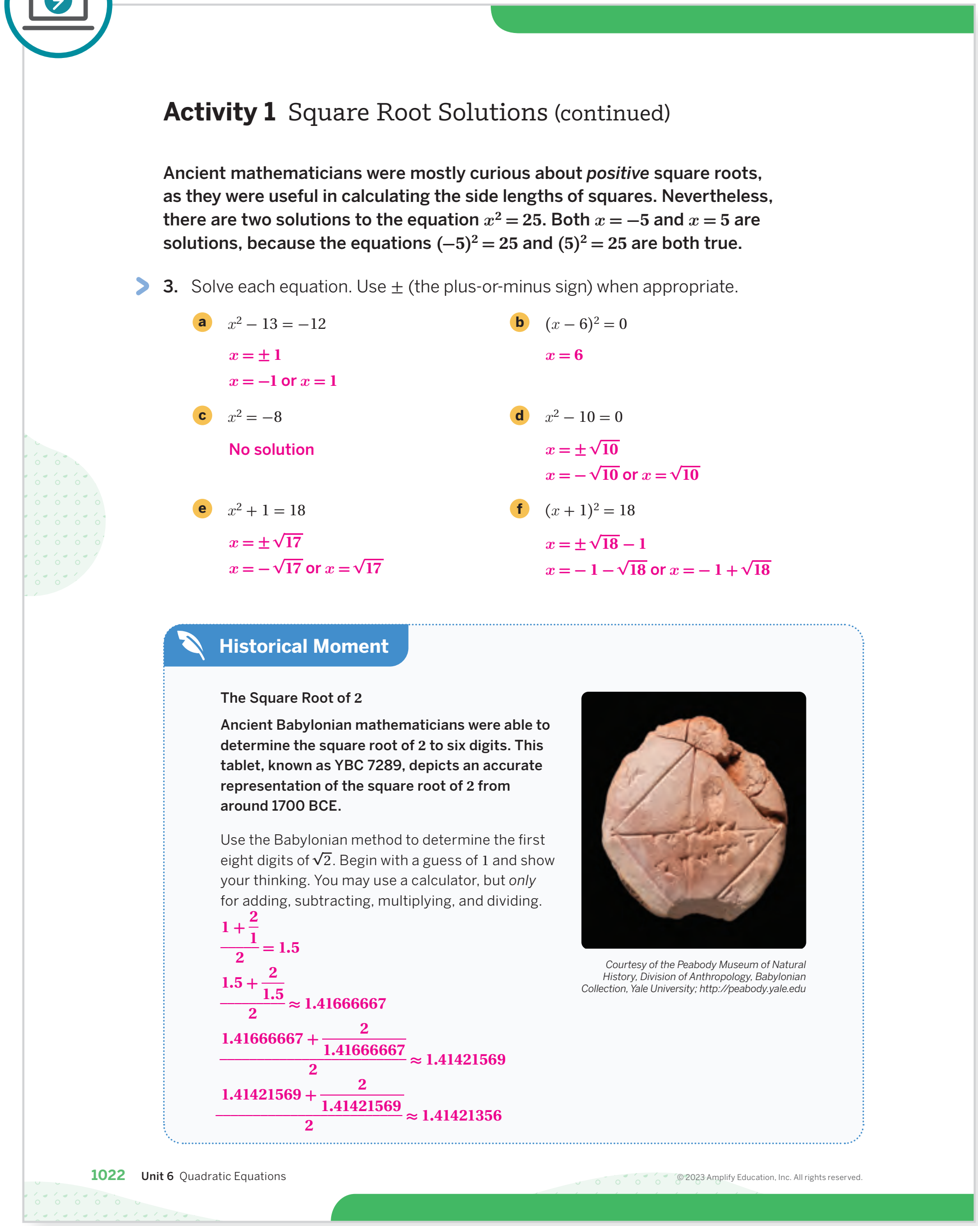
Ask:

- ***Critique and Correct:*** "Where was the error made in this solution attempt? Why do you think the person who attempted this solution may have made this mistake? What should they have done?"
- ***Clarify:*** "Study the approximations for each step. How might you know, based on these approximations, that you may be making a mistake?" Listen for students who reason that the decimal values are not getting closer to one decimal approximation.

Pairs | 15 min

Activity 1 Square Root Solutions (continued)

Students solve quadratic equations by taking the square root of each side to practice using the plus-or-minus notation in solutions.



Activity 1 Square Root Solutions (continued)

Ancient mathematicians were mostly curious about *positive* square roots, as they were useful in calculating the side lengths of squares. Nevertheless, there are two solutions to the equation $x^2 = 25$. Both $x = -5$ and $x = 5$ are solutions, because the equations $(-5)^2 = 25$ and $(5)^2 = 25$ are both true.

3. Solve each equation. Use ± (the plus-or-minus sign) when appropriate.

a. $x^2 - 13 = -12$

$x = \pm 1$

$x = -1$ or $x = 1$

b. $(x - 6)^2 = 0$

$x = 6$

c. $x^2 = -8$

No solution

d. $x^2 - 10 = 0$

$x = \pm\sqrt{10}$

$x = -\sqrt{10}$ or $x = \sqrt{10}$

e. $x^2 + 1 = 18$

$x = \pm\sqrt{17}$

$x = -\sqrt{17}$ or $x = \sqrt{17}$

f. $(x + 1)^2 = 18$

$x = \pm\sqrt{18} - 1$

$x = -1 - \sqrt{18}$ or $x = -1 + \sqrt{18}$

Historical Moment

The Square Root of 2

Ancient Babylonian mathematicians were able to determine the square root of 2 to six digits. This tablet, known as YBC 7289, depicts an accurate representation of the square root of 2 from around 1700 BCE.

Use the Babylonian method to determine the first eight digits of $\sqrt{2}$. Begin with a guess of 1 and show your thinking. You may use a calculator, but *only* for adding, subtracting, multiplying, and dividing.

$$\frac{1 + \frac{2}{1}}{2} = 1.5$$

$$\frac{1.5 + \frac{2}{1.5}}{2} \approx 1.41666667$$

$$\frac{1.41666667 + \frac{2}{1.41666667}}{2} \approx 1.41421569$$

$$\frac{1.41421569 + \frac{2}{1.41421569}}{2} \approx 1.41421356$$

Courtesy of the Peabody Museum of Natural History, Division of Anthropology, Babylonian Collection, Yale University; http://peabody.yale.edu

1022 **Unit 6** Quadratic Equations

3 Connect

Have pairs of students share their solutions for Problem 3.

Display and record student solutions.

Highlight that when a solution is irrational, the exact solution is written with a square root and the decimal solution is an approximation.

Ask:

- "What are the different ways to write the *exact* solutions to the equation $(x + 3)^2 = 15$?" $x = \pm\sqrt{15} - 3$, $x = -\sqrt{15} - 3$ or $x = \sqrt{15} - 3$, $x = -3 \pm \sqrt{15}$, and $x = -3 - \sqrt{15}$ or $x = -3 + \sqrt{15}$
- "What are the *approximate* solutions to the equation $(x + 3)^2 = 15$?" $x \approx -6.873$ or $x \approx 0.873$

Historical Moment

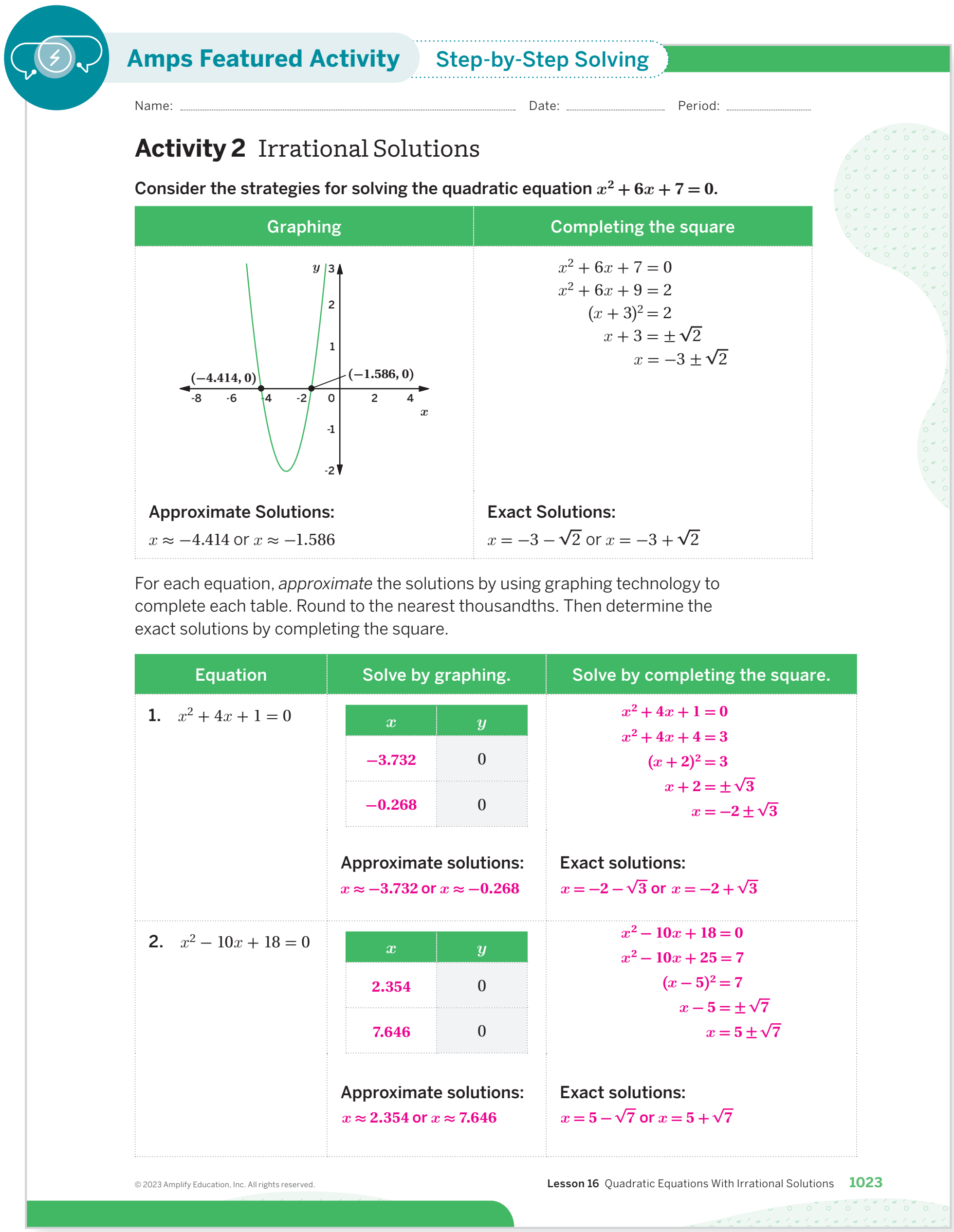
The Square Root of 2

Have students complete the *Historical Moment* activity to see how ancient Babylonian mathematicians were able to determine the square root of 2 to six digits.

Pairs | 15 min

Activity 2 Irrational Solutions

Students solve quadratic equations by graphing and completing the square to compare the two strategies and weigh their advantages and disadvantages.



Amps Featured Activity Step-by-Step Solving

Name: ______ Date: ______ Period: ______

Activity 2 Irrational Solutions

Consider the strategies for solving the quadratic equation $x^2 + 6x + 7 = 0$.

Graphing	Completing the square
Approximate Solutions: $x \approx -4.414$ or $x \approx -1.586$	$x^2 + 6x + 7 = 0$ $x^2 + 6x + 9 = 2$ $(x + 3)^2 = 2$ $x + 3 = \pm\sqrt{2}$ $x = -3 \pm \sqrt{2}$ **Exact Solutions:** $x = -3 - \sqrt{2}$ or $x = -3 + \sqrt{2}$

For each equation, *approximate* the solutions by using graphing technology to complete each table. Round to the nearest thousandths. Then determine the exact solutions by completing the square.

Equation	Solve by graphing.	Solve by completing the square.
1. $x^2 + 4x + 1 = 0$	x: −3.732, y: 0 x: −0.268, y: 0 **Approximate solutions:** $x \approx -3.732$ or $x \approx -0.268$	$x^2 + 4x + 1 = 0$ $x^2 + 4x + 4 = 3$ $(x + 2)^2 = 3$ $x + 2 = \pm\sqrt{3}$ $x = -2 \pm \sqrt{3}$ **Exact solutions:** $x = -2 - \sqrt{3}$ or $x = -2 + \sqrt{3}$
2. $x^2 - 10x + 18 = 0$	x: 2.354, y: 0 x: 7.646, y: 0 **Approximate solutions:** $x \approx 2.354$ or $x \approx 7.646$	$x^2 - 10x + 18 = 0$ $x^2 - 10x + 25 = 7$ $(x - 5)^2 = 7$ $x - 5 = \pm\sqrt{7}$ $x = 5 \pm \sqrt{7}$ **Exact solutions:** $x = 5 - \sqrt{7}$ or $x = 5 + \sqrt{7}$

1 Launch

Have one partner solve by completing the square and the other solve by graphing. Have them compare their solutions and describe what they notice. If the solutions are not approximately the same, have them discuss and resolve any differences. Partners should switch roles for each equation. Provide access to graphing technology.

2 Monitor

Help students get started by reminding them the x-intercepts of the graph of the equation represent the solutions to the equation.

Look for points of confusion:

- **Having difficulty completing the square with fractions in Problem 3.** Have students create a checklist of steps from the previous problems to apply to the new problem.
- **Struggling to determine the approximate solutions from the graph.** Ask, "Where are the solutions to a quadratic equation located on the graph?" The x-intercepts.

Look for productive strategies:

- Estimating the value of the square root to compare to the value of the x-intercepts.
- Using ± instead of writing out the two exact solutions.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on the equations in Problems 1–3, and as time permits, have them complete Problem 4.

Math Language Development

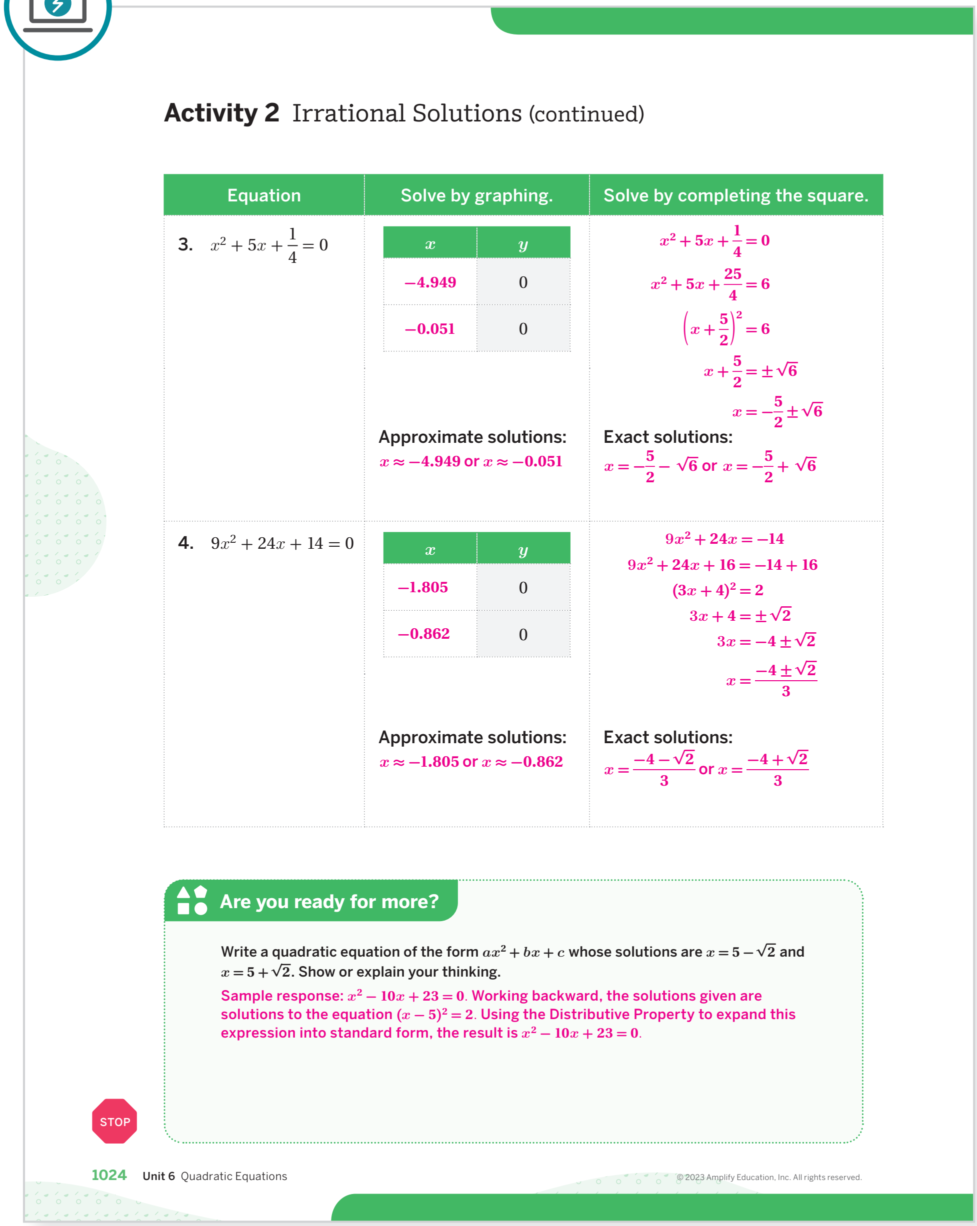
MLR8: Discussion Supports—Press for Reasoning

While students work, display or provide the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*. Encourage students to respectfully challenge each other's work and reasoning if they disagree on the solutions to each equation before moving on to the next equation.

Pairs | 15 min

Activity 2 Irrational Solutions (continued)

Students solve quadratic equations by graphing and completing the square to compare the two strategies and weigh their advantages and disadvantages.



Activity 2 Irrational Solutions (continued)

<table>
<tr><th>Equation</th><th>Solve by graphing.</th><th>Solve by completing the square.</th></tr>
<tr><td>3. $x^2 + 5x + \frac{1}{4} = 0$</td><td><table><tr><th>$x$</th><th>$y$</th></tr><tr><td>$-4.949$</td><td>0</td></tr><tr><td>-0.051</td><td>0</td></tr></table>Approximate solutions:
$x \approx -4.949$ or $x \approx -0.051$</td><td>$x^2 + 5x + \frac{1}{4} = 0$
$x^2 + 5x + \frac{25}{4} = 6$
$\left(x + \frac{5}{2}\right)^2 = 6$
$x + \frac{5}{2} = \pm\sqrt{6}$
$x = -\frac{5}{2} \pm \sqrt{6}$
Exact solutions:
$x = -\frac{5}{2} - \sqrt{6}$ or $x = -\frac{5}{2} + \sqrt{6}$</td></tr>
<tr><td>4. $9x^2 + 24x + 14 = 0$</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>-1.805</td><td>0</td></tr><tr><td>-0.862</td><td>0</td></tr></table>Approximate solutions:
$x \approx -1.805$ or $x \approx -0.862$</td><td>$9x^2 + 24x = -14$
$9x^2 + 24x + 16 = -14 + 16$
$(3x + 4)^2 = 2$
$3x + 4 = \pm\sqrt{2}$
$3x = -4 \pm \sqrt{2}$
$x = \frac{-4 \pm \sqrt{2}}{3}$
Exact solutions:
$x = \frac{-4 - \sqrt{2}}{3}$ or $x = \frac{-4 + \sqrt{2}}{3}$</td></tr>
</table>

Are you ready for more?

Write a quadratic equation of the form $ax^2 + bx + c$ whose solutions are $x = 5 - \sqrt{2}$ and $x = 5 + \sqrt{2}$. Show or explain your thinking.

Sample response: $x^2 - 10x + 23 = 0$. Working backward, the solutions given are solutions to the equation $(x - 5)^2 = 2$. Using the Distributive Property to expand this expression into standard form, the result is $x^2 - 10x + 23 = 0$.

STOP

1024 **Unit 6** Quadratic Equations

3 Connect

Have pairs of students share any challenges that arose from solving by either strategy.

Highlight these three strategies students can use to solve quadratic equations — graphing, factoring, and completing the square.

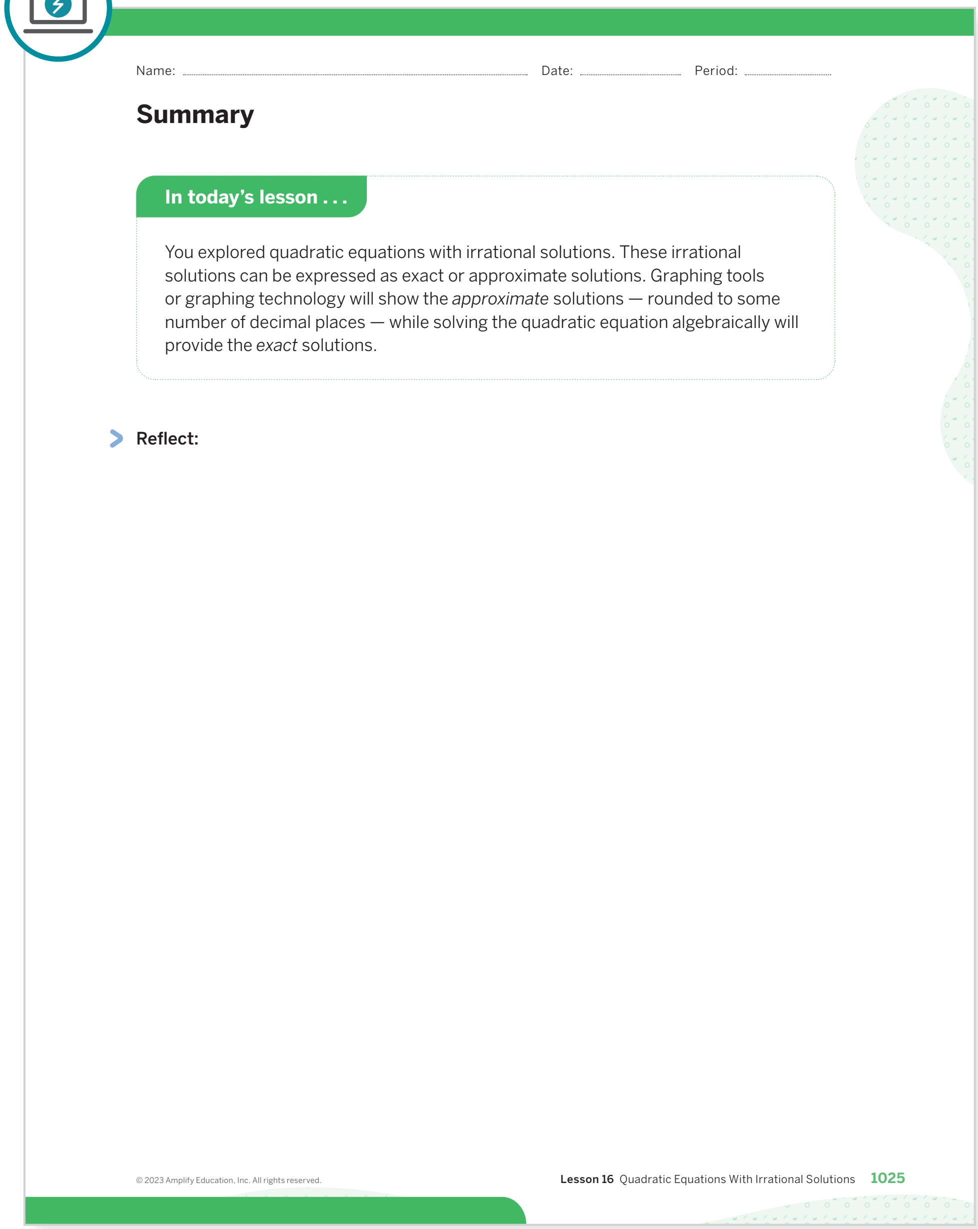
Ask:

- "Would you be able to solve any of these equations by factoring?" No.
- "What are some benefits of solving by graphing? Drawbacks?" Sample response:
- Benefit: It is quick and straightforward, even when the equations involve fractions or very large numbers.
- Drawback: It does not always give exact solutions.
- "What are some benefits of solving by completing the square? Drawbacks?" Sample response:
- Benefit: It can be used to determine exact solutions to any equation.
- Drawbacks: It can be time consuming. When the equations have fractions or very large or small numbers, the calculations can become complex and may be prone to error.

Whole Class | 5 min

Summary

Review and synthesize that some solutions to quadratic equations are irrational and can be expressed exactly or approximately.



Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored quadratic equations with irrational solutions. These irrational solutions can be expressed as exact or approximate solutions. Graphing tools or graphing technology will show the *approximate* solutions — rounded to some number of decimal places — while solving the quadratic equation algebraically will provide the *exact* solutions.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Quadratic Equations With Irrational Solutions 1025

Synthesize

Display the equation $(x + 4)^2 = 11$.

Have students share the different ways to express the solution.

Ask:

- "How is the square root symbol useful when solving quadratic equations?" It is used to express exact solutions.
- "Does the expression $\sqrt{11}$ represent both the positive and negative solutions?" No, just the positive solution.
- "What are some benefits and drawbacks of expressing solutions exactly and approximately?"

Sample responses:

- Exact solutions: The benefit is that the solutions are exact. The drawback is that they are challenging to determine the value of.
- Approximate solutions: The benefit is that they are more straightforward to understand the value. The drawback is that it is not an exact answer and is not as precise as the exact solutions.

Highlight that some solutions to quadratic equations are irrational and can be expressed exactly with square root symbols or approximately as decimals.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

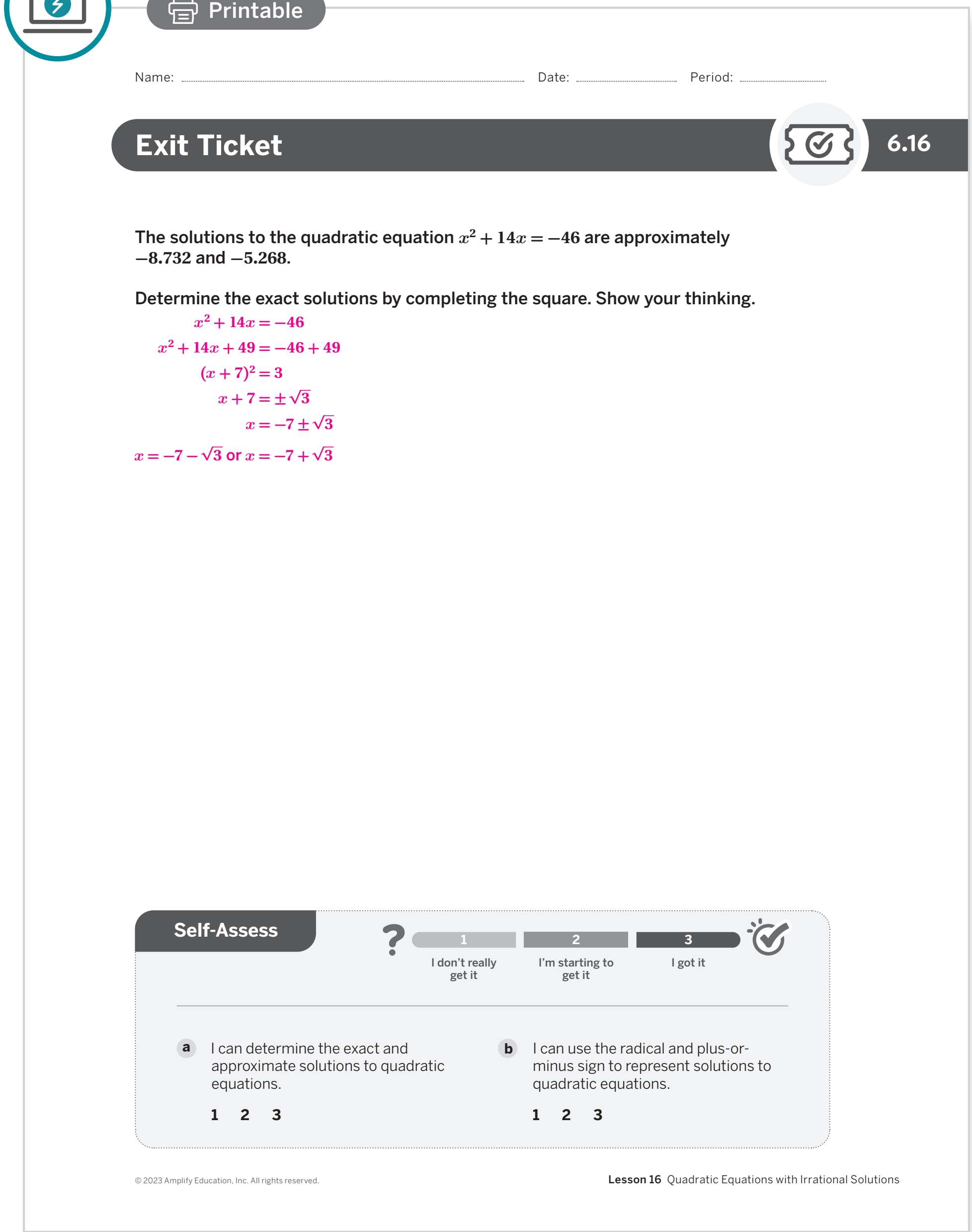
- "Which of the strategies for solving quadratic equations that you have learned so far will always yield exact solutions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of exact solutions by solving a quadratic equation and expressing the solution using the plus-or-minus symbol.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.16

The solutions to the quadratic equation $x^2 + 14x = -46$ are approximately -8.732 and -5.268.

Determine the exact solutions by completing the square. Show your thinking.

$$x^2 + 14x = -46$$
$$x^2 + 14x + 49 = -46 + 49$$
$$(x + 7)^2 = 3$$
$$x + 7 = \pm\sqrt{3}$$
$$x = -7 \pm \sqrt{3}$$
$$x = -7 - \sqrt{3} \text{ or } x = -7 + \sqrt{3}$$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can determine the exact and approximate solutions to quadratic equations.
1 2 3

b I can use the radical and plus-or-minus sign to represent solutions to quadratic equations.
1 2 3

Lesson 16 Quadratic Equations with Irrational Solutions

Success looks like . . .

- **Language Goal:** Comparing solutions to quadratic equations solved by completing the square and graphing. **(Speaking and Listening, Writing)**
- **Language Goal:** Explaining that the plus-or-minus symbol is used to represent both square roots of a number and that the square root notation without a sign is understood to represent only the positive square root. **(Speaking and Listening, Writing)**
- **Goal:** Using radical and plus-or-minus symbols to express solutions to quadratic equations.
  - » Expressing the exact solutions of the equation using the radical and plus-or-minus symbols.

Suggested next steps

If students do not write the exact solution, consider:

- Reviewing types of solutions in Activity 2.
- Assigning Practice Problem 3.

If students do not use the plus-or-minus symbol, consider:

- Reviewing notation from Activity 1.
- Assigning Practice Problem 1.

Professional Learning

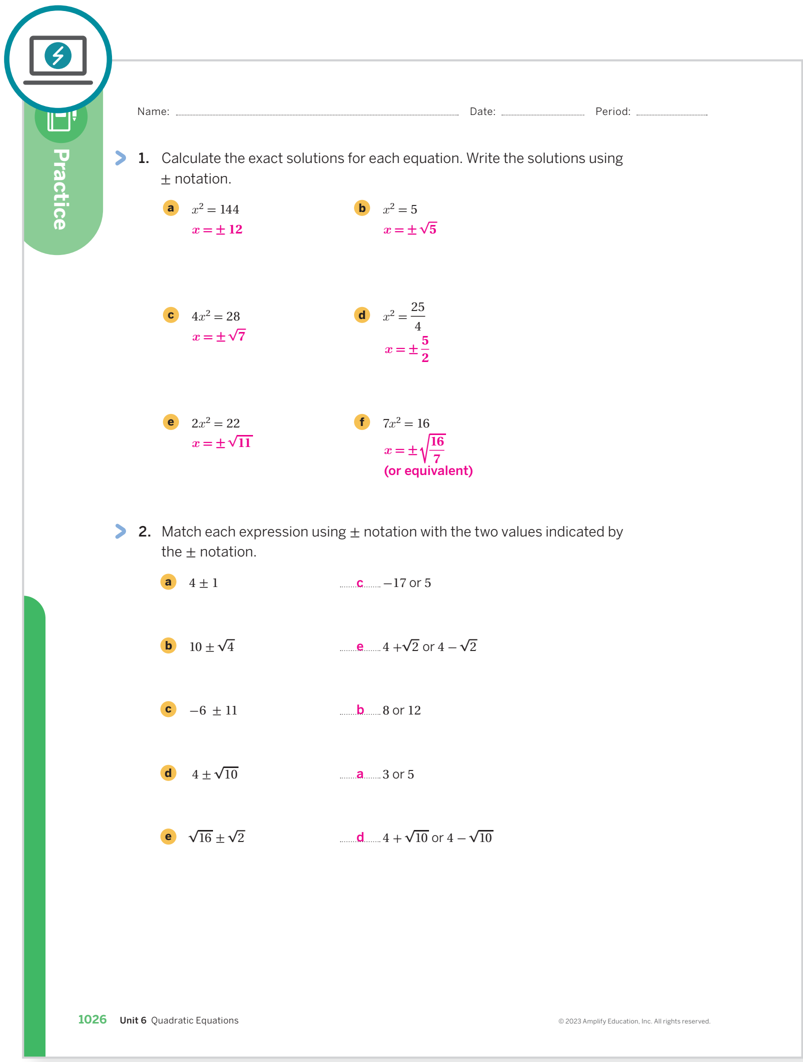
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students give the exact solutions of quadratic equations. How will this knowledge prepare them for their upcoming work understanding and using the quadratic formula?
- How did students attend to precision today? How are you helping students become aware of how they are progressing in this area?

Practice

Independent



Name: .... Date: .... Period: ....

Practice

1. Calculate the exact solutions for each equation. Write the solutions using ± notation.

   a. $x^2 = 144$
   $x = \pm 12$

   b. $x^2 = 5$
   $x = \pm\sqrt{5}$

   c. $4x^2 = 28$
   $x = \pm\sqrt{7}$

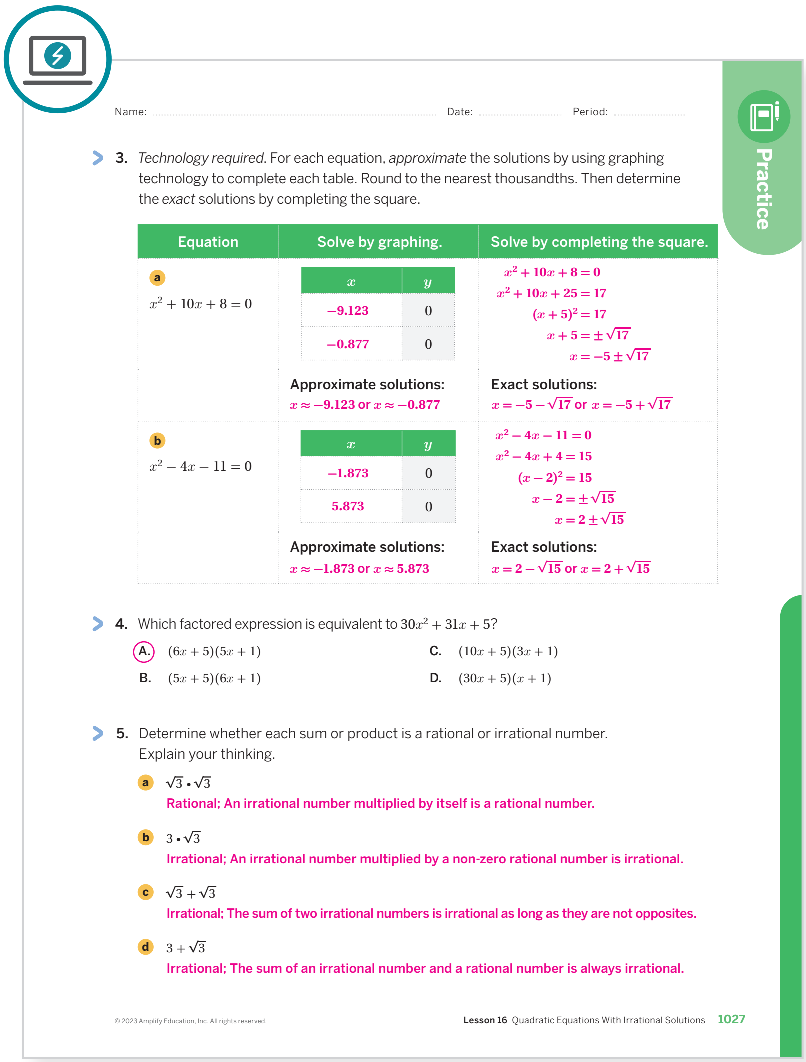
   d. $x^2 = \frac{25}{4}$
   $x = \pm\frac{5}{2}$

   e. $2x^2 = 22$
   $x = \pm\sqrt{11}$

   f. $7x^2 = 16$
   $x = \pm\sqrt{\frac{16}{7}}$
   (or equivalent)

2. Match each expression using ± notation with the two values indicated by the ± notation.

a. 4 ± 1	c -17 or 5
b. $10 \pm \sqrt{4}$	e $4 + \sqrt{2}$ or $4 - \sqrt{2}$
c. -6 ± 11	b 8 or 12
d. $4 \pm \sqrt{10}$	a 3 or 5
e. $\sqrt{16} \pm \sqrt{2}$	d $4 + \sqrt{10}$ or $4 - \sqrt{10}$



Name: .... Date: .... Period: ....

Practice

3. *Technology required*. For each equation, *approximate* the solutions by using graphing technology to complete each table. Round to the nearest thousandths. Then determine the *exact* solutions by completing the square.

<table>
<tr><th>Equation</th><th>Solve by graphing.</th><th>Solve by completing the square.</th></tr>
<tr><td>a
$x^2 + 10x + 8 = 0$</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>-9.123</td><td>0</td></tr><tr><td>-0.877</td><td>0</td></tr></table>Approximate solutions:
$x \approx -9.123$ or $x \approx -0.877$</td><td>$x^2 + 10x + 8 = 0$
$x^2 + 10x + 25 = 17$
$(x+5)^2 = 17$
$x + 5 = \pm\sqrt{17}$
$x = -5 \pm \sqrt{17}$
Exact solutions:
$x = -5 - \sqrt{17}$ or $x = -5 + \sqrt{17}$</td></tr>
<tr><td>b
$x^2 - 4x - 11 = 0$</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>-1.873</td><td>0</td></tr><tr><td>5.873</td><td>0</td></tr></table>Approximate solutions:
$x \approx -1.873$ or $x \approx 5.873$</td><td>$x^2 - 4x - 11 = 0$
$x^2 - 4x + 4 = 15$
$(x-2)^2 = 15$
$x - 2 = \pm\sqrt{15}$
$x = 2 \pm \sqrt{15}$
Exact solutions:
$x = 2 - \sqrt{15}$ or $x = 2 + \sqrt{15}$</td></tr>
</table>

4. Which factored expression is equivalent to $30x^2 + 31x + 5$?

   [✓ A.] $(6x + 5)(5x + 1)$
   B. $(5x + 5)(6x + 1)$
   C. $(10x + 5)(3x + 1)$
   D. $(30x + 5)(x + 1)$

5. Determine whether each sum or product is a rational or irrational number. Explain your thinking.

   a. $\sqrt{3} \cdot \sqrt{3}$
   Rational; An irrational number multiplied by itself is a rational number.

   b. $3 \cdot \sqrt{3}$
   Irrational; An irrational number multiplied by a non-zero rational number is irrational.

   c. $\sqrt{3} + \sqrt{3}$
   Irrational; The sum of two irrational numbers is irrational as long as they are not opposites.

   d. $3 + \sqrt{3}$
   Irrational; The sum of an irrational number and a rational number is always irrational.

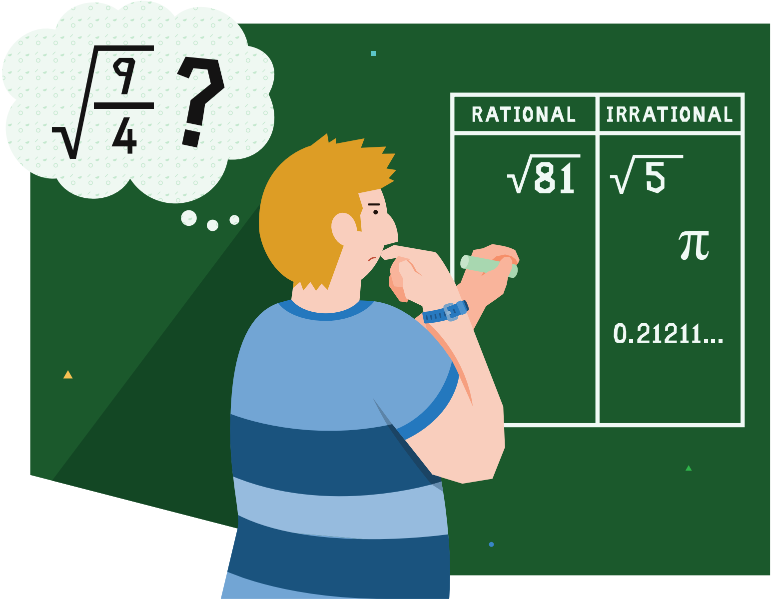
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 10	2
Formative ⚡	5	Unit 6 Lesson 17	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.



Rational and Irrational Numbers

Let's explore irrational numbers.

Focus

Goals

1. **Language Goal:** Explain why the product of a nonzero rational number and irrational number is irrational. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain why the sum of a rational and irrational number is irrational. **(Speaking and Listening, Writing)**
3. **Language Goal:** Explain why the sum or product of two rational numbers is rational. **(Speaking and Listening, Writing)**

Coherence

Today

Students build on their Grade 8 understanding of rational and irrational numbers by determining and classifying the sums and products of rational or irrational numbers. Calculators will be used to address multiplication of irrational numbers. They construct arguments to categorize specific results as rational or irrational numbers.

Previously

Students determined exact solutions to quadratic equations by completing the square.

Coming Soon

In the next lesson, students will classify solutions to quadratic equations as rational or irrational.

Rigor

- Students solidify their **conceptual understanding** of the real number system and operations that can be performed with real numbers.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	15 min	12 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket PDF
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- scientific calculator

Math Language Development

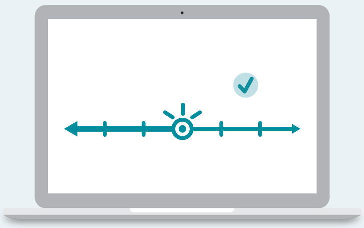
Review words

- *irrational number*
- *rational number*

Amps | **Featured Activity**

Activity 1
View Work From Previous Slides

Students explore when different operations can result in rational or irrational numbers. As they choose pairs of values on which to operate, their results are stored in a table on a later slide to help organize their thinking.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, as students share their thinking for a part of Problem 4, they may not know or exhibit collaborative or productive ways to share any disagreements they have with their classmates' thinking. Consider posting sentence frames for students to use when they disagree or want to challenge another student's reasoning, such as "I disagree because . . ." Model how to actively listen to another student's reasoning before stating whether you agree or disagree.

Modifications to Pacing

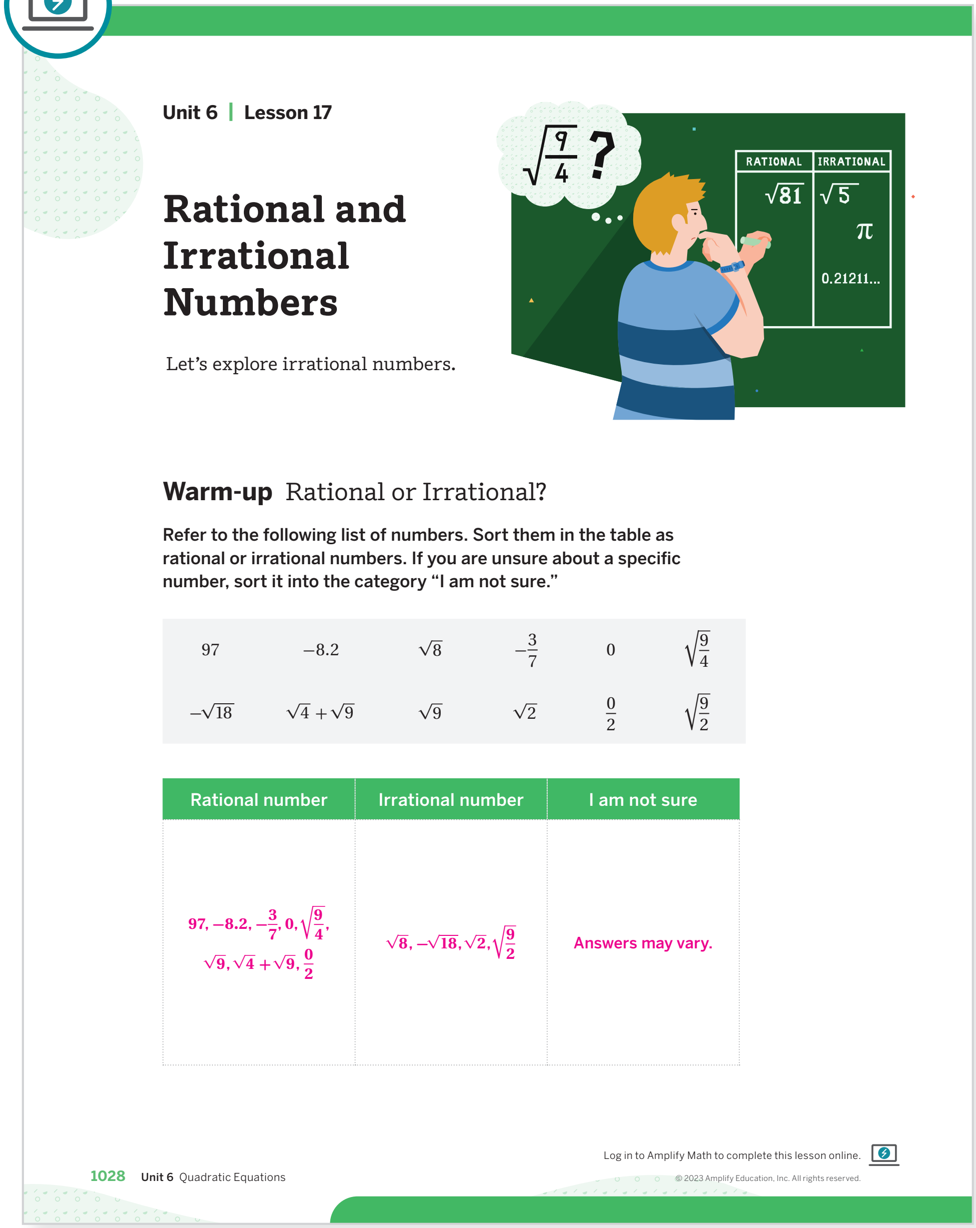
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, reduce the time to 10 minutes.
- In **Activity 3**, reduce the time to 10 minutes.

Independent | 5 min

Warm-up Rational or Irrational?

Students classify rational and irrational numbers to activate prior knowledge, preparing to perform operations with rational and irrational numbers.



Unit 6 | Lesson 17

Rational and Irrational Numbers

Let's explore irrational numbers.

Warm-up Rational or Irrational?

Refer to the following list of numbers. Sort them in the table as rational or irrational numbers. If you are unsure about a specific number, sort it into the category "I am not sure."

97	-8.2	$\sqrt{8}$	$-\frac{3}{7}$	0	$\sqrt{\frac{9}{4}}$
$-\sqrt{18}$	$\sqrt{4}+\sqrt{9}$	$\sqrt{9}$	$\sqrt{2}$	$\frac{0}{2}$	$\sqrt{\frac{9}{2}}$

Rational number	Irrational number	I am not sure
$97, -8.2, -\frac{3}{7}, 0, \sqrt{\frac{9}{4}}, \sqrt{9}, \sqrt{4}+\sqrt{9}, \frac{0}{2}$	$\sqrt{8}, -\sqrt{18}, \sqrt{2}, \sqrt{\frac{9}{2}}$	Answers may vary.

Log in to Amplify Math to complete this lesson online.

1028 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students time to work independently. As they finish, have them discuss their thinking with a partner.

2 Monitor

Help students get started by displaying examples of rational $\left(2, -2.8, \frac{2}{5}\right)$ and irrational numbers $\left(\sqrt{12}, \sqrt{\frac{5}{9}}\right)$.

Look for points of confusion:

- **Classifying all radical values as irrational.** Ask them how to evaluate $\sqrt{36}$.
- **Classifying all fractional values as rational.** Display $\sqrt{\frac{9}{2}}$. Ask them if they can write this number as the ratio of two integers.

Look for productive strategies:

- Recognizing perfect squares that can be written as rational numbers.
- Sorting by whole, rational, decimal, or radical numbers to make sense of each group.

3 Connect

Have individual students share their responses. Record and display the responses. Ask students to critique each other's responses and discuss and resolve any disagreements.

Ask, "What is different about the radical expressions of rational and irrational numbers?" Radical expressions of rational numbers can be rewritten as exact values without the radical sign.

Highlight that irrational numbers cannot be written as a fraction representing the ratio of two integers. They often contain a radical sign.

Power-up

To power up students' ability to classify square roots as rational or irrational, have students complete:

Recall that a number is *rational* if it can be simplified to a ratio of two integers. Determine whether each value is *rational* or *irrational*. Be prepared to explain your thinking.

1. $\sqrt{4}$ Rational, because $\sqrt{4}=2$, which is an integer and all integers are rational numbers.
2. -5 Rational, because -5 is an integer and all integers are rational numbers.
3. $\sqrt{\frac{9}{16}}$ Rational, because $\sqrt{\frac{9}{16}}=\frac{3}{4}$, which can be written as a fraction representing the ratio of two integers.
4. $\sqrt{2}$ Irrational, because $\sqrt{2}$ cannot be written as a fraction representing the ratio of two integers.

Use: Before the Warm-up

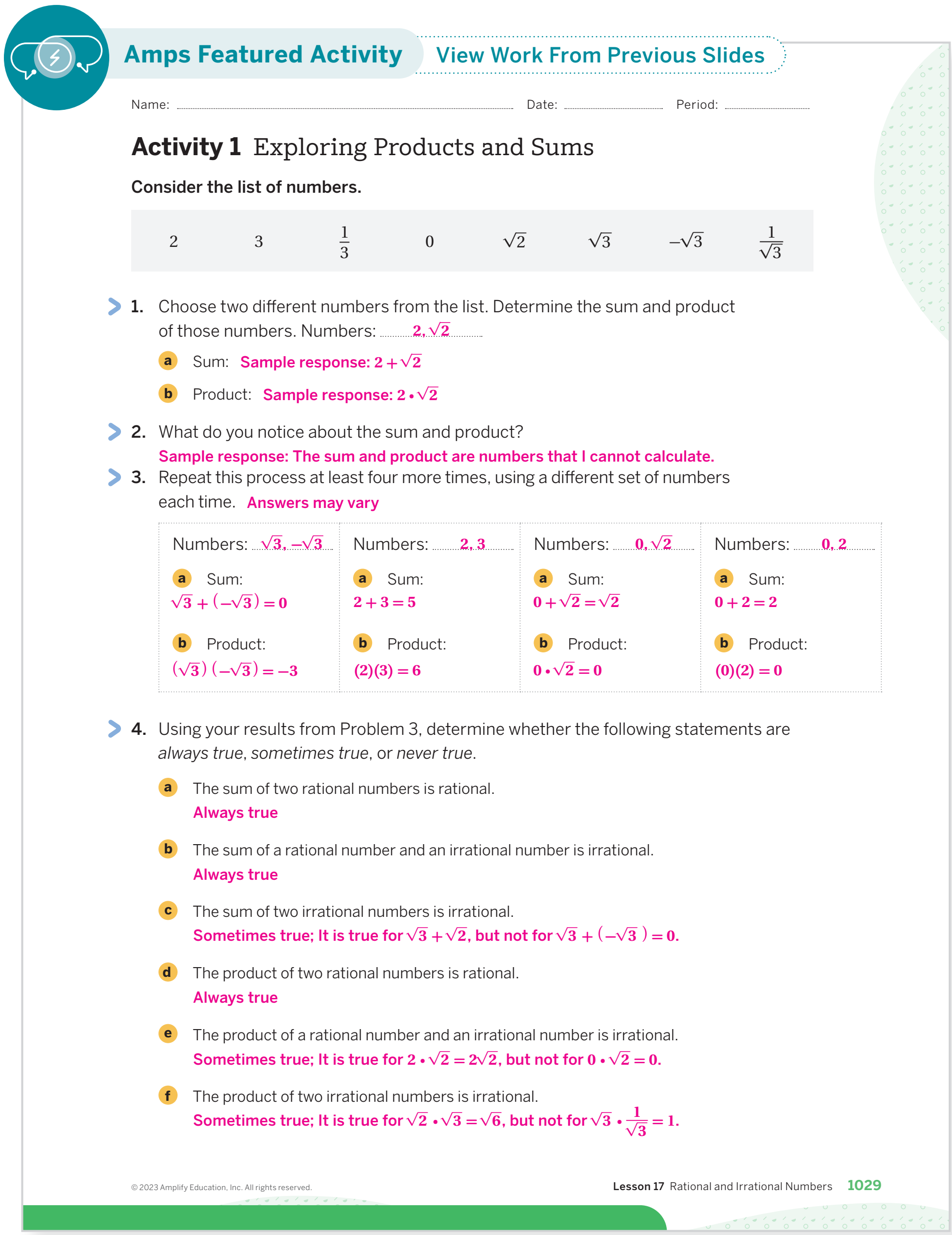
Informed by: Performance on Lesson 16, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 6

Pairs | 12 min

Activity 1 Exploring With Products and Sums

Students make conjectures using specific numbers to explore the relationship of sums and products with rational and irrational numbers.

Amps Featured Activity View Work From Previous Slides



Name: Date: Period:

Activity 1 Exploring Products and Sums

Consider the list of numbers.

2	3	$\frac{1}{3}$	0	$\sqrt{2}$	$\sqrt{3}$	$-\sqrt{3}$	$\frac{1}{\sqrt{3}}$

1. Choose two different numbers from the list. Determine the sum and product of those numbers. Numbers: $2, \sqrt{2}$
   - a Sum: **Sample response: $2 + \sqrt{2}$**
   - b Product: **Sample response: $2 \bullet \sqrt{2}$**
2. What do you notice about the sum and product?
   Sample response: The sum and product are numbers that I cannot calculate.
3. Repeat this process at least four more times, using a different set of numbers each time. **Answers may vary**

Numbers: $\sqrt{3}, -\sqrt{3}$	Numbers: $2, 3$	Numbers: $0, \sqrt{2}$	Numbers: $0, 2$
a Sum: $\sqrt{3} + (-\sqrt{3}) = 0$	a Sum: $2 + 3 = 5$	a Sum: $0 + \sqrt{2} = \sqrt{2}$	a Sum: $0 + 2 = 2$
b Product: $(\sqrt{3})(-\sqrt{3}) = -3$	b Product: $(2)(3) = 6$	b Product: $0 \bullet \sqrt{2} = 0$	b Product: $(0)(2) = 0$

4. Using your results from Problem 3, determine whether the following statements are *always true*, *sometimes true*, or *never true*.
   - a The sum of two rational numbers is rational.
     Always true
   - b The sum of a rational number and an irrational number is irrational.
     Always true
   - c The sum of two irrational numbers is irrational.
     Sometimes true; It is true for $\sqrt{3} + \sqrt{2}$, but not for $\sqrt{3} + (-\sqrt{3}) = 0$.
   - d The product of two rational numbers is rational.
     Always true
   - e The product of a rational number and an irrational number is irrational.
     Sometimes true; It is true for $2 \bullet \sqrt{2} = 2\sqrt{2}$, but not for $0 \bullet \sqrt{2} = 0$.
   - f The product of two irrational numbers is irrational.
     Sometimes true; It is true for $\sqrt{2} \bullet \sqrt{3} = \sqrt{6}$, but not for $\sqrt{3} \bullet \frac{1}{\sqrt{3}} = 1$.

1 Launch

Give students time to individually experiment with sums and products of two numbers from the list. When students have at least five sums and products, have them discuss their thinking with a partner for Problem 4. Provide access to scientific calculators.

2 Monitor

Help students get started by helping them choose two numbers for Problems 1 and 2.

Look for points of confusion:

- **Having difficulty multiplying radical numbers.** Students may not have seen multiplication of radical numbers before. Encourage students to use their calculator to compute products.

Look for productive strategies:

- Choosing numbers that have similar properties to add or multiply, such as $\sqrt{3}$ and $-\sqrt{3}$. These numbers help students determine whether the statements in Problem 4 are sometimes true.

3 Connect

Display the six statements from Problem 4.

Have pairs of students share their thinking for each statement. If there is disagreement from the class, have another pair provide evidence for why they disagree. Allow for a productive debate until a consensus is reached.

Highlight that it is important to be precise in definitions and explorations.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide pairs of numbers for students to use in Problems 1–3, such as ones shown in the sample responses in the Student Edition.

Extension: *Math Enrichment*

Tell students that a set of numbers is "closed" under a given mathematical operation if performing that operation on any numbers in that set *always* produces another number within that same set. For example, the set of integers is closed under addition because the sum of any two integers is always another integer. Ask students to determine whether the set of irrational numbers is closed under addition or multiplication and to explain their thinking.

Addition: Not closed. For example, $\sqrt{2} + (-\sqrt{2}) = 0$, which is a rational number.

Multiplication: Not closed. For example, $\sqrt{2} \bullet \sqrt{2} = 2$, which is a rational number.

Math Language Development

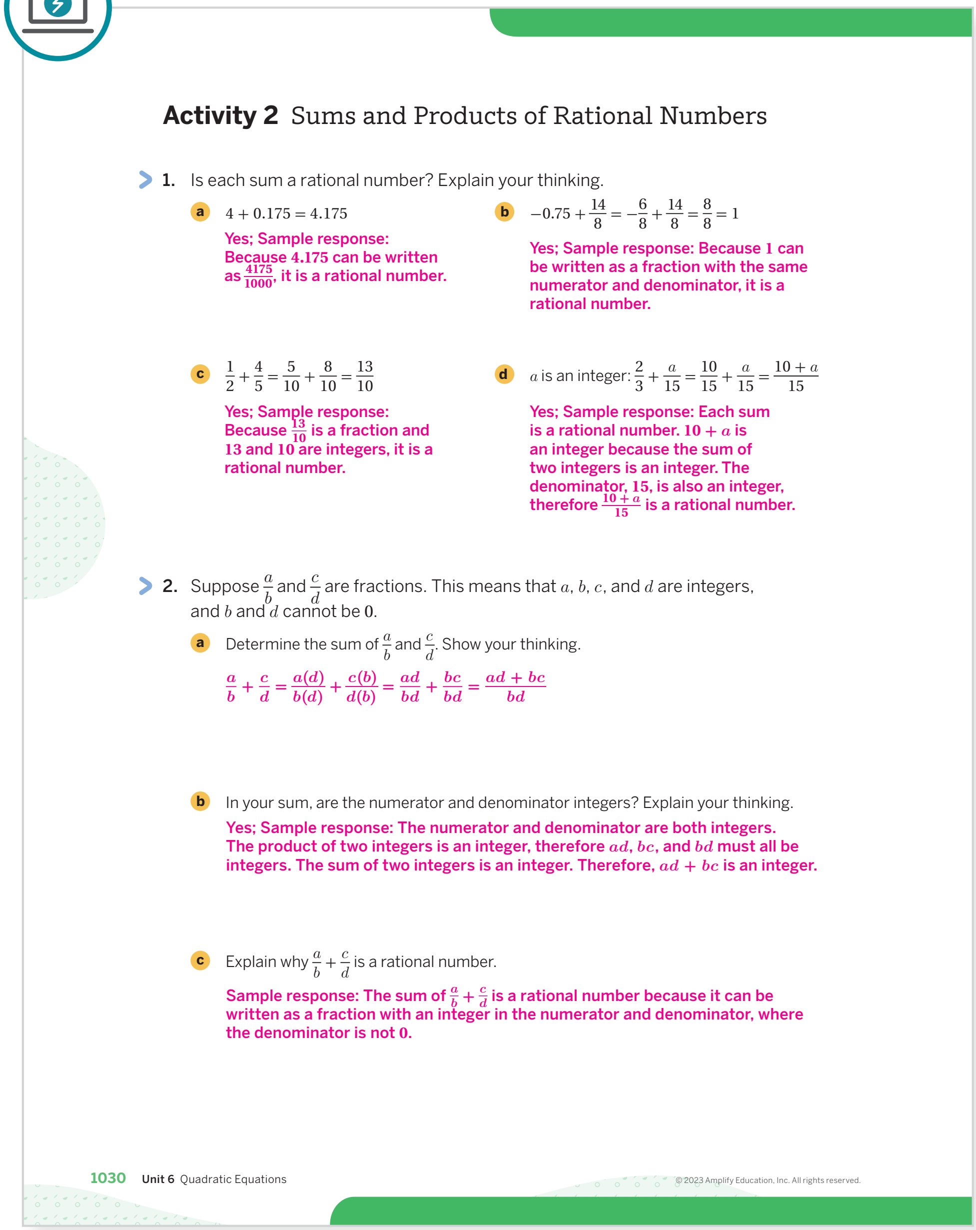
MLR8: Discussion Supports—Press for Reasoning

While partners complete Problem 4, and also during the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*. Encourage students to respectfully challenge each other's reasoning if they disagree as to whether the statements in Problem 4 are *always true*, *sometimes true*, or *never true*.

Pairs | 15 min

Activity 2 Sums and Products of Rational Numbers

Students determine sums and products of rational numbers to prove they will always result in a rational sum or product.



Activity 2 Sums and Products of Rational Numbers

1. Is each sum a rational number? Explain your thinking.

a $4 + 0.175 = 4.175$

Yes; Sample response: Because 4.175 can be written as $\frac{4175}{1000}$, it is a rational number.

b $-0.75 + \frac{14}{8} = -\frac{6}{8} + \frac{14}{8} = \frac{8}{8} = 1$

Yes; Sample response: Because 1 can be written as a fraction with the same numerator and denominator, it is a rational number.

c $\frac{1}{2} + \frac{4}{5} = \frac{5}{10} + \frac{8}{10} = \frac{13}{10}$

Yes; Sample response: Because $\frac{13}{10}$ is a fraction and 13 and 10 are integers, it is a rational number.

d a is an integer: $\frac{2}{3} + \frac{a}{15} = \frac{10}{15} + \frac{a}{15} = \frac{10+a}{15}$

Yes; Sample response: Each sum is a rational number. $10 + a$ is an integer because the sum of two integers is an integer. The denominator, 15, is also an integer, therefore $\frac{10+a}{15}$ is a rational number.

2. Suppose $\frac{a}{b}$ and $\frac{c}{d}$ are fractions. This means that a, b, c, and d are integers, and b and d cannot be 0.

a Determine the sum of $\frac{a}{b}$ and $\frac{c}{d}$. Show your thinking.

$$\frac{a}{b} + \frac{c}{d} = \frac{a(d)}{b(d)} + \frac{c(b)}{d(b)} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

b In your sum, are the numerator and denominator integers? Explain your thinking.

Yes; Sample response: The numerator and denominator are both integers. The product of two integers is an integer, therefore ad, bc, and bd must all be integers. The sum of two integers is an integer. Therefore, $ad + bc$ is an integer.

c Explain why $\frac{a}{b} + \frac{c}{d}$ is a rational number.

Sample response: The sum of $\frac{a}{b} + \frac{c}{d}$ is a rational number because it can be written as a fraction with an integer in the numerator and denominator, where the denominator is not 0.

1030 Unit 6 Quadratic Equations

1 Launch

Tell students they will further examine the sums and products of rational numbers.

2 Monitor

Help students get started by having them circle each sum in Problem 1 and think about whether it is rational.

Look for points of confusion:

- **Struggling to determine a common denominator in Problem 2a.** Refer students back to addition problems from Problem 1. Ask them how common denominators were determined in those problems.
- **Being unsure of the definition of a rational number.** Remind students a rational number is of the form $\frac{p}{q}$, where p and q are both integers. Ask, "Can the number be represented as a fraction, where the numerator and denominator are both integers?"

Look for productive strategies:

- Asking themselves the same question for each part of Problem 1, e.g., "Can the sum be written as a fraction where the numerator and denominator are both integers?"

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the following questions students can ask themselves as they think about how to approach Problem 2.

- "How do you add two fractions with unlike denominators?"
- "What would be a common denominator for the fractions $\frac{a}{b}$ and $\frac{c}{d}$?"
- "How can you rewrite each fraction using that common denominator?"
- "Now that the fractions are written with a common denominator, how can you add them?"

Extension: *Math Enrichment*

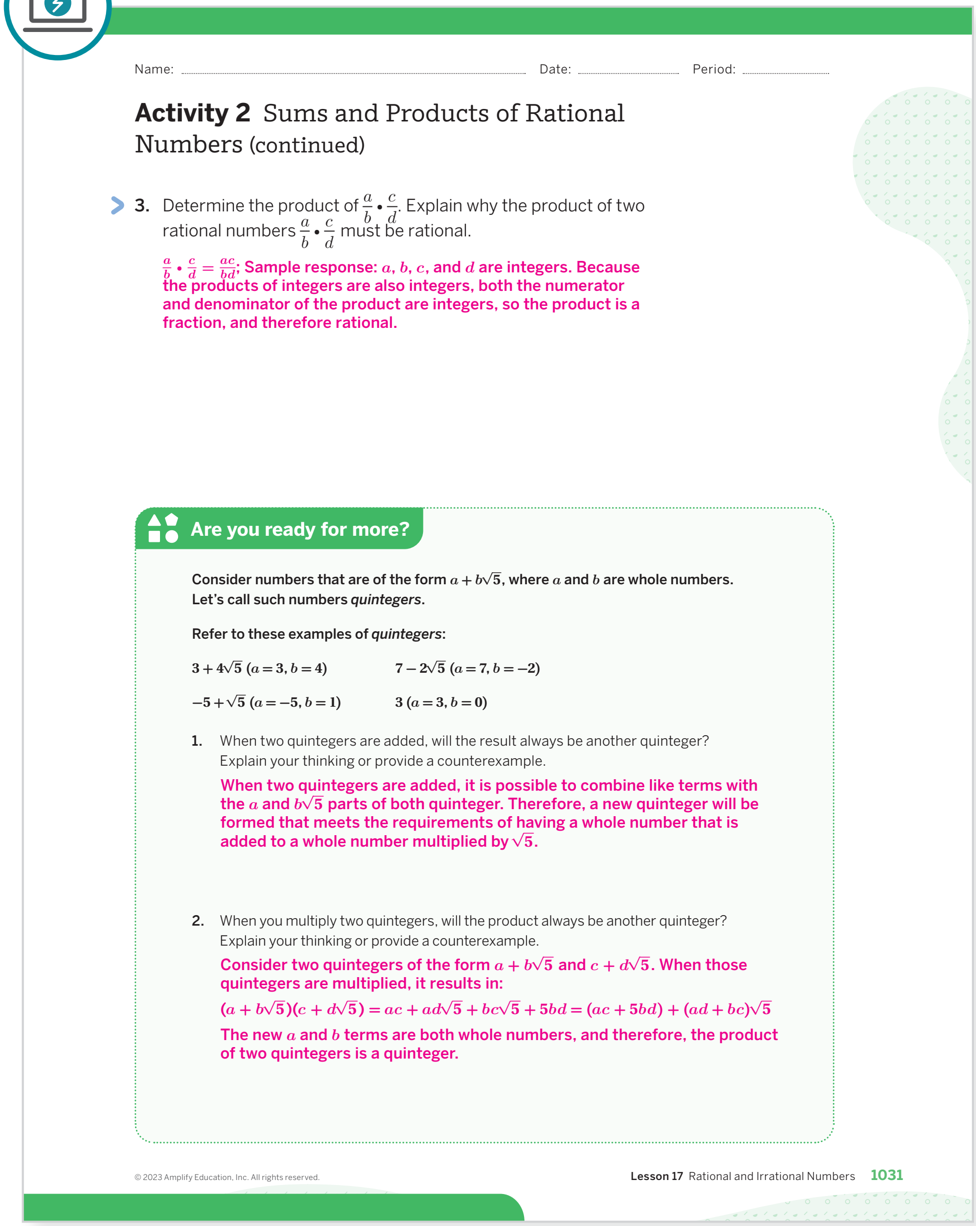
Tell students that a set of numbers is "closed" under a given mathematical operation if performing that operation on any numbers in that set *always* produces another number within that same set. Have them determine whether each of the following statements is true. If the statement is not true, they should provide a counterexample.

- Odd numbers are closed under addition. False; $3 + 3 = 6$, which is an even number.
- Even numbers are closed under multiplication. True.
- Whole numbers are closed under addition. True.
- Whole numbers are closed under subtraction. False; $4 - 9 = -5$, which is not a whole number.

Pairs | 15 min

Activity 2 Sums and Products of Rational Numbers (continued)

Students determine sums and products of rational numbers to prove they will always result in a rational sum or product.



Name: Date: Period:

Activity 2 Sums and Products of Rational Numbers (continued)

3. Determine the product of $\frac{a}{b} \bullet \frac{c}{d}$. Explain why the product of two rational numbers $\frac{a}{b} \bullet \frac{c}{d}$ must be rational.

   $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$; Sample response: a, b, c, and d are integers. Because the products of integers are also integers, both the numerator and denominator of the product are integers, so the product is a fraction, and therefore rational.

Are you ready for more?

Consider numbers that are of the form $a + b\sqrt{5}$, where a and b are whole numbers. Let's call such numbers *quintegers*.

Refer to these examples of *quintegers*:

$3 + 4\sqrt{5}\ (a = 3, b = 4)$

$7 - 2\sqrt{5}\ (a = 7, b = -2)$

$-5 + \sqrt{5}\ (a = -5, b = 1)$

$3\ (a = 3, b = 0)$

1. When two quintegers are added, will the result always be another quinteger? Explain your thinking or provide a counterexample.

   When two quintegers are added, it is possible to combine like terms with the a and $b\sqrt{5}$ parts of both quinteger. Therefore, a new quinteger will be formed that meets the requirements of having a whole number that is added to a whole number multiplied by $\sqrt{5}$.

2. When you multiply two quintegers, will the product always be another quinteger? Explain your thinking or provide a counterexample.

   Consider two quintegers of the form $a + b\sqrt{5}$ and $c + d\sqrt{5}$. When those quintegers are multiplied, it results in:

   $$(a + b\sqrt{5})(c + d\sqrt{5}) = ac + ad\sqrt{5} + bc\sqrt{5} + 5bd = (ac + 5bd) + (ad + bc)\sqrt{5}$$

   The new a and b terms are both whole numbers, and therefore, the product of two quintegers is a quinteger.

3 Connect

Have a pair of students share how they determined the product of two rational numbers must be rational.

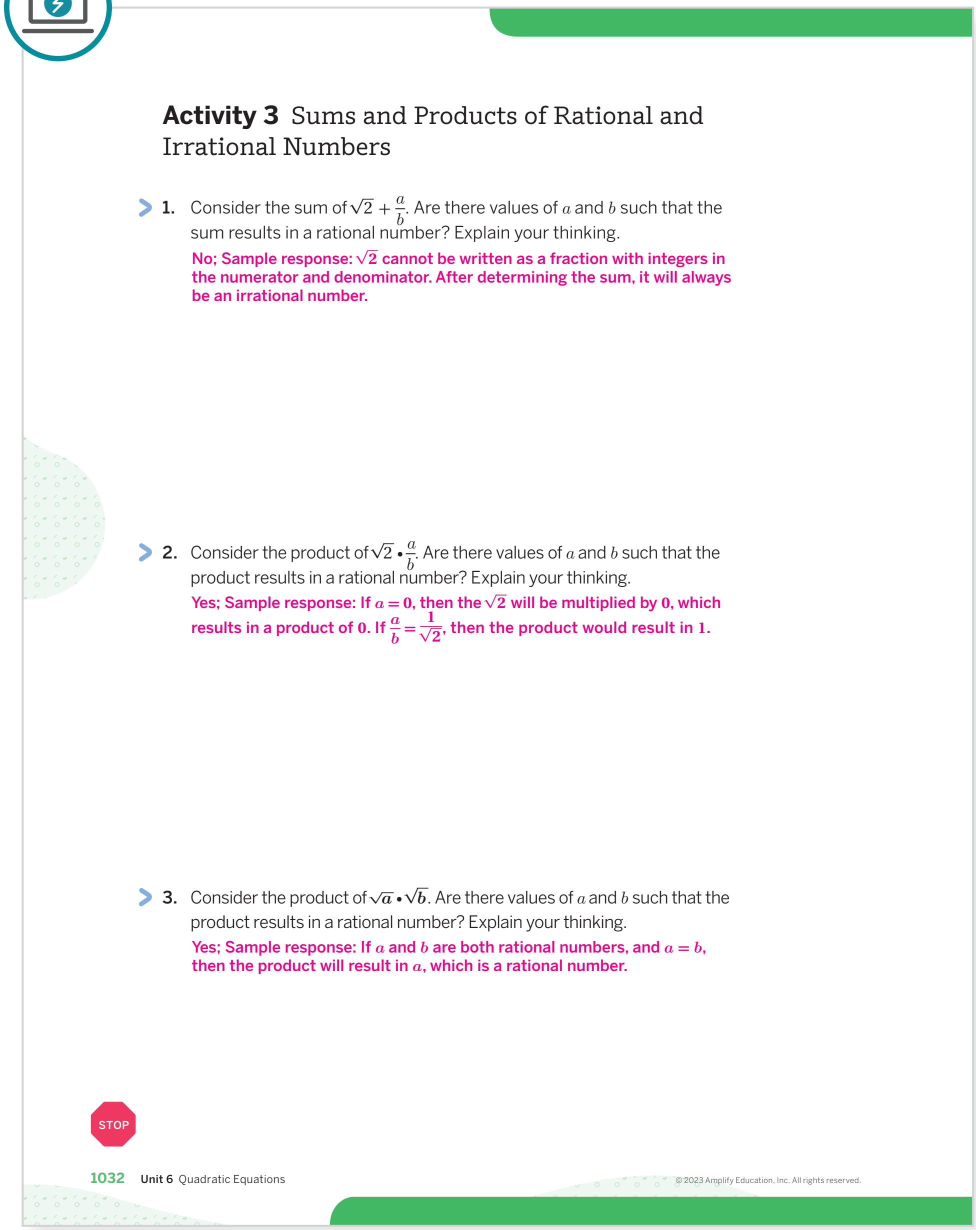
Ask, "How do you know whether a sum or product is a rational number?" When the sum or product can be written as a fraction with integer values in the numerator and denominator.

Highlight that when adding or multiplying rational numbers, the result will always be a rational number.

Activity 3 Sums and Products of Rational and Irrational Numbers

Pairs | 12 min

Students explore sums and products to determine which specific cases result in rational or irrational numbers.



Activity 3 Sums and Products of Rational and Irrational Numbers

1. Consider the sum of $\sqrt{2} + \frac{a}{b}$. Are there values of a and b such that the sum results in a rational number? Explain your thinking.
   No; Sample response: $\sqrt{2}$ cannot be written as a fraction with integers in the numerator and denominator. After determining the sum, it will always be an irrational number.

2. Consider the product of $\sqrt{2} \cdot \frac{a}{b}$. Are there values of a and b such that the product results in a rational number? Explain your thinking.
   Yes; Sample response: If $a = 0$, then the $\sqrt{2}$ will be multiplied by 0, which results in a product of 0. If $\frac{a}{b} = \frac{1}{\sqrt{2}}$, then the product would result in 1.

3. Consider the product of $\sqrt{a} \cdot \sqrt{b}$. Are there values of a and b such that the product results in a rational number? Explain your thinking.
   Yes; Sample response: If a and b are both rational numbers, and $a = b$, then the product will result in a, which is a rational number.

STOP

1032 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask students to try values of a or b to come to conclusions about the sums and products. Encourage students to use their calculators.

2 Monitor

Help students get started by helping them choose values of a or b.

Look for points of confusion:

- **Being unsure of the definition of irrational numbers.** Remind students that irrational numbers cannot be written as fractions, where the numerator and denominator are both integers.

Look for productive strategies:

- Organizing their explorations in a table or chart.
- Experimenting with several different values for a and b to determine their responses.

3 Connect

Have pairs of students share their thinking and approaches to each problem.

Highlight the specific cases that result in a rational number for Problems 2 and 3. For example, multiplying by 0 or the multiplicative inverse of an irrational number will result in either 0 or 1, which are both rational. Multiplying an irrational number by itself or multiplying two irrational numbers where the number inside the radicand is a perfect square will result in the square root being a rational number.

Differentiated Support

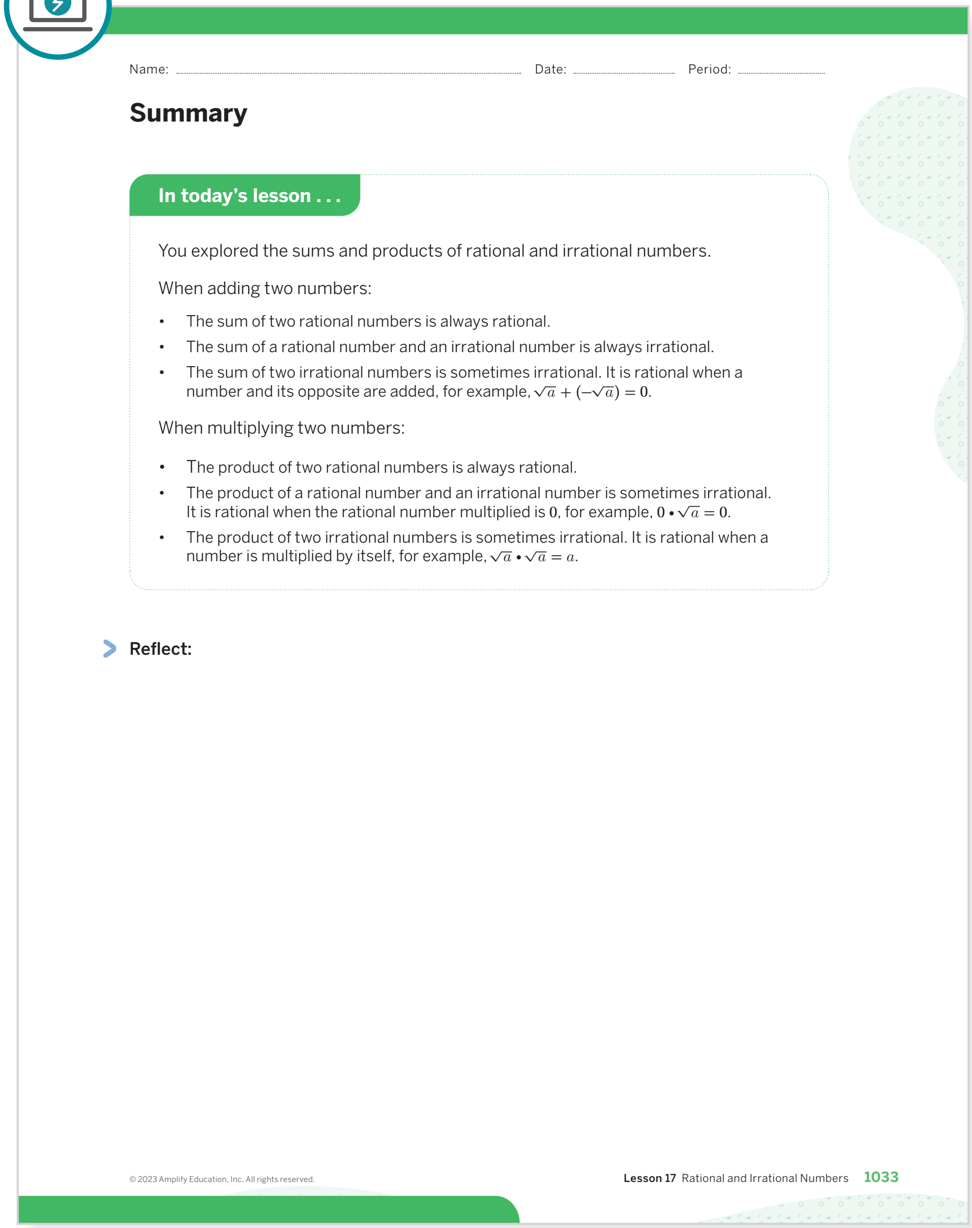
Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can explore and try values of a and b that result in rational or irrational expressions. A digital number line will represent these values to help students visualize and make sense of them.

Whole Class | 5 min

Summary

Review and synthesize how sums and products of rational and irrational numbers can be classified.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You explored the sums and products of rational and irrational numbers.

When adding two numbers:

- The sum of two rational numbers is always rational.
- The sum of a rational number and an irrational number is always irrational.
- The sum of two irrational numbers is sometimes irrational. It is rational when a number and its opposite are added, for example, $\sqrt{a} + (-\sqrt{a}) = 0$.

When multiplying two numbers:

- The product of two rational numbers is always rational.
- The product of a rational number and an irrational number is sometimes irrational. It is rational when the rational number multiplied is 0, for example, $0 \cdot \sqrt{a} = 0$.
- The product of two irrational numbers is sometimes irrational. It is rational when a number is multiplied by itself, for example, $\sqrt{a} \cdot \sqrt{a} = a$.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Rational and Irrational Numbers 1033

Synthesize

Display the following expressions:

$\frac{1}{2} + \sqrt{2}$

$\sqrt{12} + (-\sqrt{12})$

$\sqrt{2} \cdot \sqrt{10}$

Ask students to choose a sum or product from the displayed list and explain to a partner how they know whether the sum or product is rational or irrational. After both partners have given and received feedback, they should use this feedback to revise their explanation.

Highlight that in this lesson, students explored sums and products of rational and irrational numbers to make sense of the types of numbers that can result from performing these operations. In the next lesson, they will explore more properties of rational and irrational numbers and how they relate to solutions of quadratic equations.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

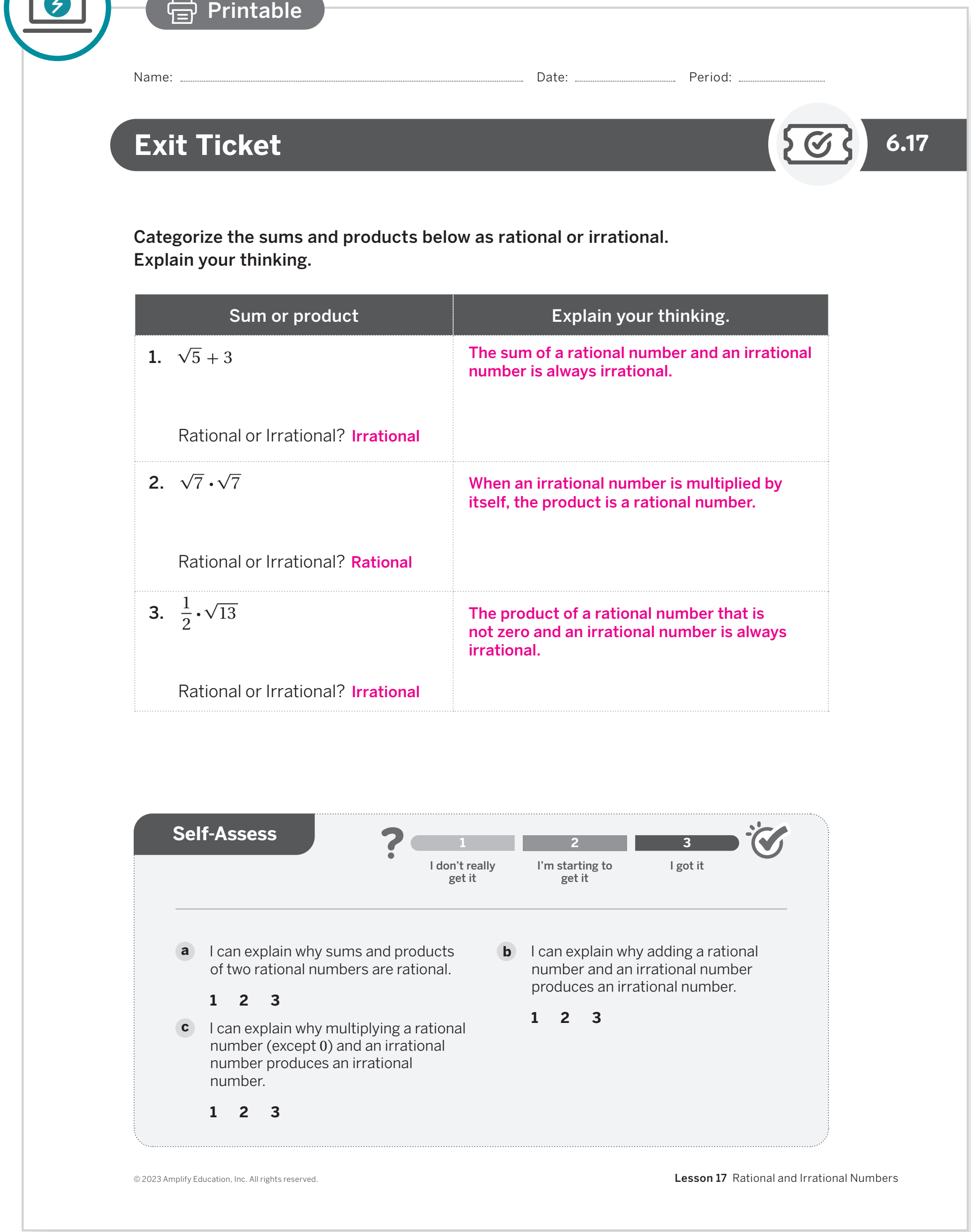
- "How can a sum or product of rational or irrational numbers be classified?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by classifying a sum or product as rational or irrational and explaining their thinking.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.17

Categorize the sums and products below as rational or irrational. Explain your thinking.

Sum or product	Explain your thinking.
1. $\sqrt{5}+3$ Rational or Irrational? Irrational	The sum of a rational number and an irrational number is always irrational.
2. $\sqrt{7}\cdot\sqrt{7}$ Rational or Irrational? Rational	When an irrational number is multiplied by itself, the product is a rational number.
3. $\frac{1}{2}\cdot\sqrt{13}$ Rational or Irrational? Irrational	The product of a rational number that is not zero and an irrational number is always irrational.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain why sums and products of two rational numbers are rational.
1 2 3

b. I can explain why adding a rational number and an irrational number produces an irrational number.
1 2 3

c. I can explain why multiplying a rational number (except 0) and an irrational number produces an irrational number.
1 2 3

Lesson 17 Rational and Irrational Numbers

Success looks like . . .

- **Language Goal:** Explaining why the product of a nonzero rational number and irrational number is irrational. **(Speaking and Listening, Writing)**
  - » Explaining why in Problem 3.
- **Language Goal:** Explaining why the sum of a rational and irrational number is irrational. **(Speaking and Listening, Writing)**
  - » Explaining why in Problem 1.
- **Language Goal:** Explaining why the sum or product of two rational numbers is rational. **(Speaking and Listening, Writing)**

Suggested next steps

If students classify Problem 1 as rational, consider:

- Reviewing Activity 3, Problem 1.
- Assigning Practice Problem 1.
- Asking, "What fraction could you use to represent the sum of those two numbers?"

If students classify Problem 2 as irrational, consider:

- Reviewing Activity 3, Problem 3.
- Assigning Practice Problem 2.
- Asking, "Is $\sqrt{100}$ irrational?" If students respond, "Yes," then review perfect squares.

If students classify Problem 3 as rational, consider:

- Reviewing Activity 3, Problem 2.
- Assigning Practice Problem 2.
- Asking, "What fraction could you use to represent the sum of those two numbers?"

Professional Learning

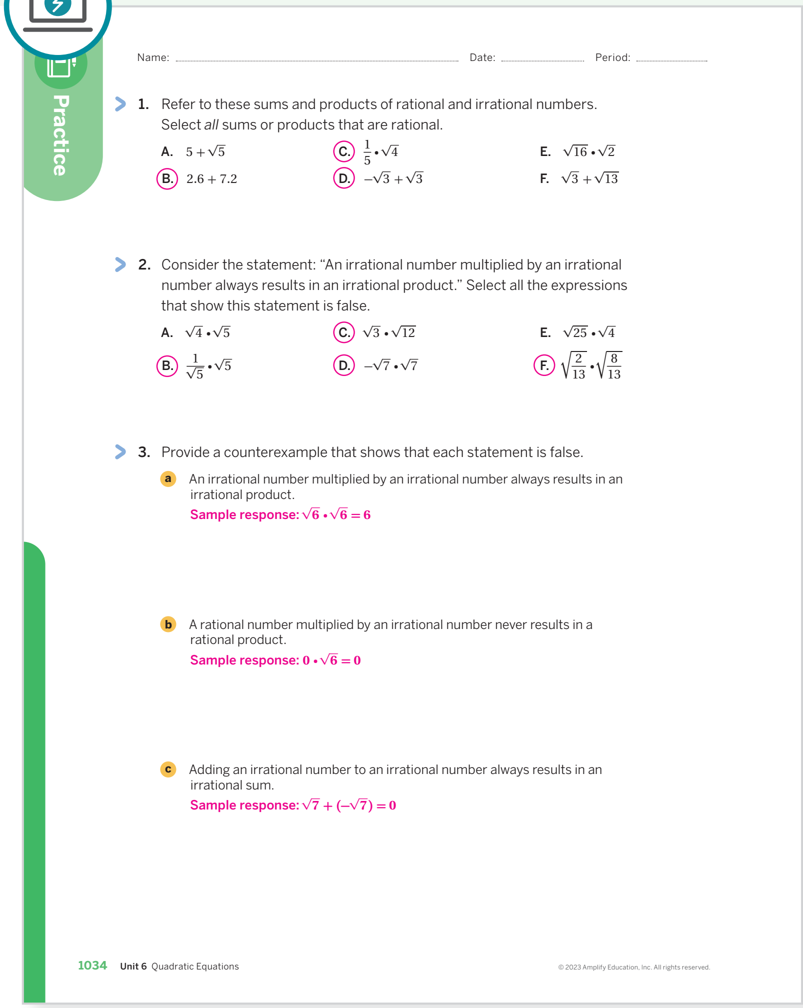
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you look at how students performed operations with irrational numbers, how did it compare with their ability to perform operations with rational numbers?
- In this lesson, students explored irrational sums and products. How will this support students' thinking about the approximate value of irrational numbers? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Refer to these sums and products of rational and irrational numbers. Select *all* sums or products that are rational.

A. $5 + \sqrt{5}$
[✓ B.] $2.6 + 7.2$
[✓ C.] $\frac{1}{5} \cdot \sqrt{4}$
[✓ D.] $-\sqrt{3} + \sqrt{3}$
E. $\sqrt{16} \cdot \sqrt{2}$
F. $\sqrt{3} + \sqrt{13}$

2. Consider the statement: "An irrational number multiplied by an irrational number always results in an irrational product." Select all the expressions that show this statement is false.

A. $\sqrt{4} \cdot \sqrt{5}$
[✓ B.] $\frac{1}{\sqrt{5}} \cdot \sqrt{5}$
[✓ C.] $\sqrt{3} \cdot \sqrt{12}$
[✓ D.] $-\sqrt{7} \cdot \sqrt{7}$
E. $\sqrt{25} \cdot \sqrt{4}$
[✓ F.] $\sqrt{\frac{2}{13}} \cdot \sqrt{\frac{8}{13}}$

3. Provide a counterexample that shows that each statement is false.

a. An irrational number multiplied by an irrational number always results in an irrational product.

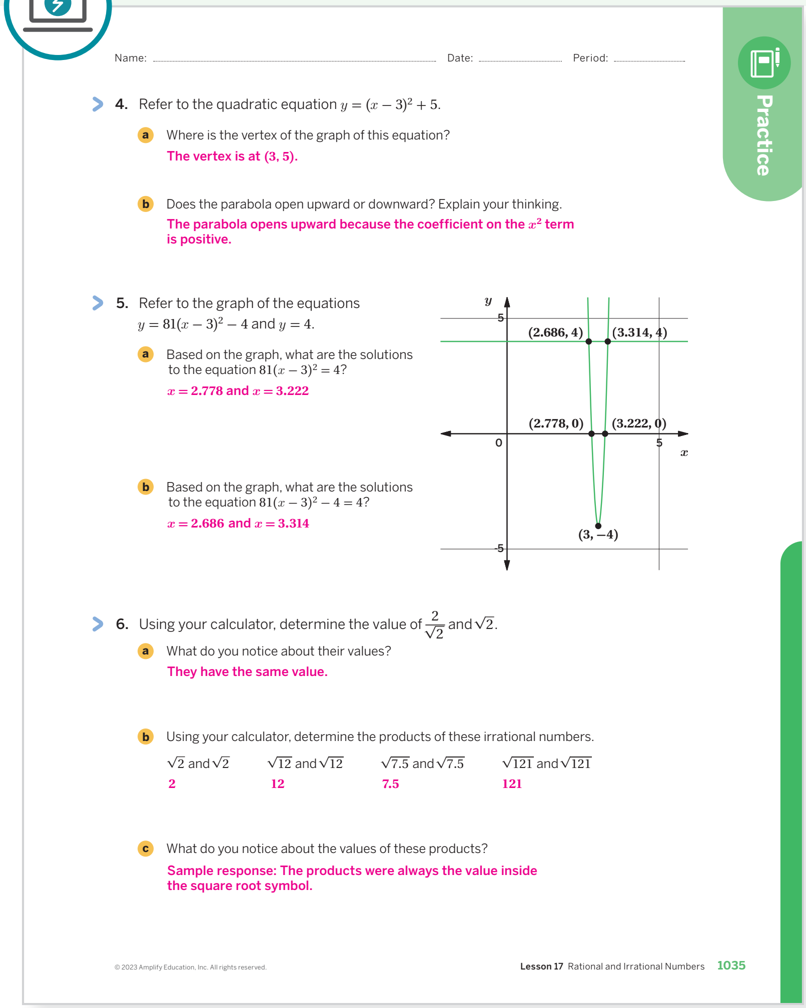
Sample response: $\sqrt{6} \cdot \sqrt{6} = 6$

b. A rational number multiplied by an irrational number never results in a rational product.

Sample response: $0 \cdot \sqrt{6} = 0$

c. Adding an irrational number to an irrational number always results in an irrational sum.

Sample response: $\sqrt{7} + (-\sqrt{7}) = 0$



Practice

Name: ______ Date: ______ Period: ______

4. Refer to the quadratic equation $y = (x - 3)^2 + 5$.

a. Where is the vertex of the graph of this equation?

The vertex is at (3, 5).

b. Does the parabola open upward or downward? Explain your thinking.

The parabola opens upward because the coefficient on the x^2 term is positive.

5. Refer to the graph of the equations $y = 81(x - 3)^2 - 4$ and $y = 4$.

a. Based on the graph, what are the solutions to the equation $81(x - 3)^2 = 4$?

$x = 2.778$ and $x = 3.222$

b. Based on the graph, what are the solutions to the equation $81(x - 3)^2 - 4 = 4$?

$x = 2.686$ and $x = 3.314$

6. Using your calculator, determine the value of $\frac{2}{\sqrt{2}}$ and $\sqrt{2}$.

a. What do you notice about their values?

They have the same value.

b. Using your calculator, determine the products of these irrational numbers.

$\sqrt{2}$ and $\sqrt{2}$	$\sqrt{12}$ and $\sqrt{12}$	$\sqrt{7.5}$ and $\sqrt{7.5}$	$\sqrt{121}$ and $\sqrt{121}$
2	**12**	**7.5**	**121**

c. What do you notice about the values of these products?

Sample response: The products were always the value inside the square root symbol.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 3	2
	3	Activity 1	3
Spiral	4	Unit 5 Lesson 19	1
	5	Unit 6 Lesson 5	1
Formative	6	Unit 6 Lesson 18	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 18**

Rational and Irrational Solutions

Let's explore irrational solutions.

Focus

Goals

1. Rationalize an irrational denominator to approximate the value of an irrational number located between two whole numbers.
2. **Language Goal:** Explain why the solution to a given quadratic equation is a rational or irrational number. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue making sense of irrational numbers through rationalizing denominators. They then make connections between different representations of quadratic equations to determine what specific structures result in irrational or rational solutions.

Previously

Students solved quadratic equations and explored sums and products of rational and irrational numbers.

Coming Soon

In the next lessons, students will solve quadratic equations that result in irrational solutions using the quadratic formula.

Rigor

- Students build their **conceptual understanding** that solutions to some quadratic equations can be rational or irrational.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	12 min	12 min	5 min	5 min
Pairs	Pairs	Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket PDF
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?*

Math Language Development

New words

- ***rationalize the denominator***

Review words

- *irrational number*
- *rational number*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students determine solutions to a quadratic equation and use an interactive graph to determine whether they are rational or irrational solutions.

Building Math Identity and Community

Connecting to Mathematical Practices

Some students may feel anxiety or struggle to persevere as they approach the problems in Activity 1 due to the radical sign in the denominator of the fraction. Encourage students to pause and make sense of each expression by describing the expression in their own words before thinking about how to rationalize the denominator. For example, they could describe the expression $\frac{8}{\sqrt{8}}$ by saying, "The number 8 is divided by the square root of 8."

Modifications to Pacing

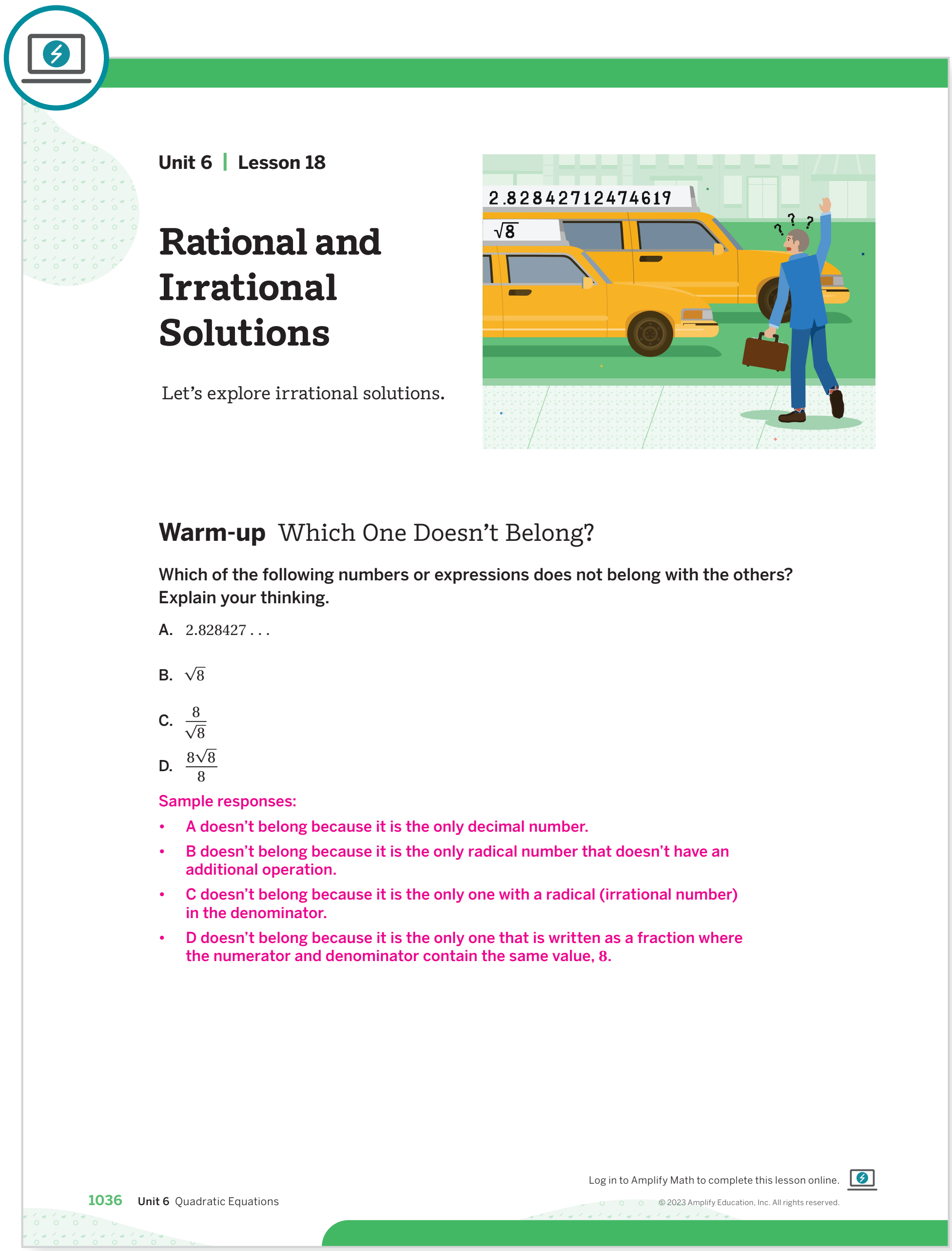
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problem 4 may be omitted.
- In **Activity 3**, reduce the time to 10 minutes.

Pairs | 5 min

Warm-up Which One Doesn't Belong?

Students compare four numbers to see how different number structures can evaluate to the same numerical value.



Unit 6 | Lesson 18

Rational and Irrational Solutions

Let's explore irrational solutions.

Warm-up Which One Doesn't Belong?

Which of the following numbers or expressions does not belong with the others? Explain your thinking.

A. 2.828427 . . .

B. $\sqrt{8}$

C. $\frac{8}{\sqrt{8}}$

D. $\frac{8\sqrt{8}}{8}$

Sample responses:

- A doesn't belong because it is the only decimal number.
- B doesn't belong because it is the only radical number that doesn't have an additional operation.
- C doesn't belong because it is the only one with a radical (irrational number) in the denominator.
- D doesn't belong because it is the only one that is written as a fraction where the numerator and denominator contain the same value, 8.

Log in to Amplify Math to complete this lesson online.

1036 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the *Which One Doesn't Belong?* routine. Provide students time to individually think through their responses before sharing their thinking with their partner.

2 Monitor

Help students get started by having them identify the location of the radical in Choices B, C, and D.

Look for points of confusion:

- **Forgetting the meaning of ". . ."** Remind them that it means the decimal values do not terminate.

Look for productive strategies:

- Using their calculator to determine decimal approximations of the irrational numbers.

3 Connect

Have students share their responses as to why each choice might not belong with the others.

Ask, "Do any of these numbers look different than what you have seen before?" Sample response: The number $\frac{8}{\sqrt{8}}$ looks different because it has an irrational number in the denominator.

Highlight that $\frac{8}{\sqrt{8}}$ has an irrational denominator. Many mathematicians choose to rewrite numbers that have an irrational denominator so that they do not have an irrational denominator. Students will explore this strategy in this lesson.

Math Language Development

MLR8: Discussion Supports

While students work, and also during the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?* Encourage students to use the sentence frames displayed on the Anchor Chart to help them organize their thinking and explanations.

Power-up

To power up students' ability to determine products with square roots, have students complete:

Determine each product. Use a calculator to verify your answers.

1. $\sqrt{3} \cdot \sqrt{3}$ 3
2. $\sqrt{27} \cdot \sqrt{27}$ 27
3. $\sqrt{4.5} \cdot \sqrt{\frac{9}{2}}$ $\frac{9}{2}$ or 4.5

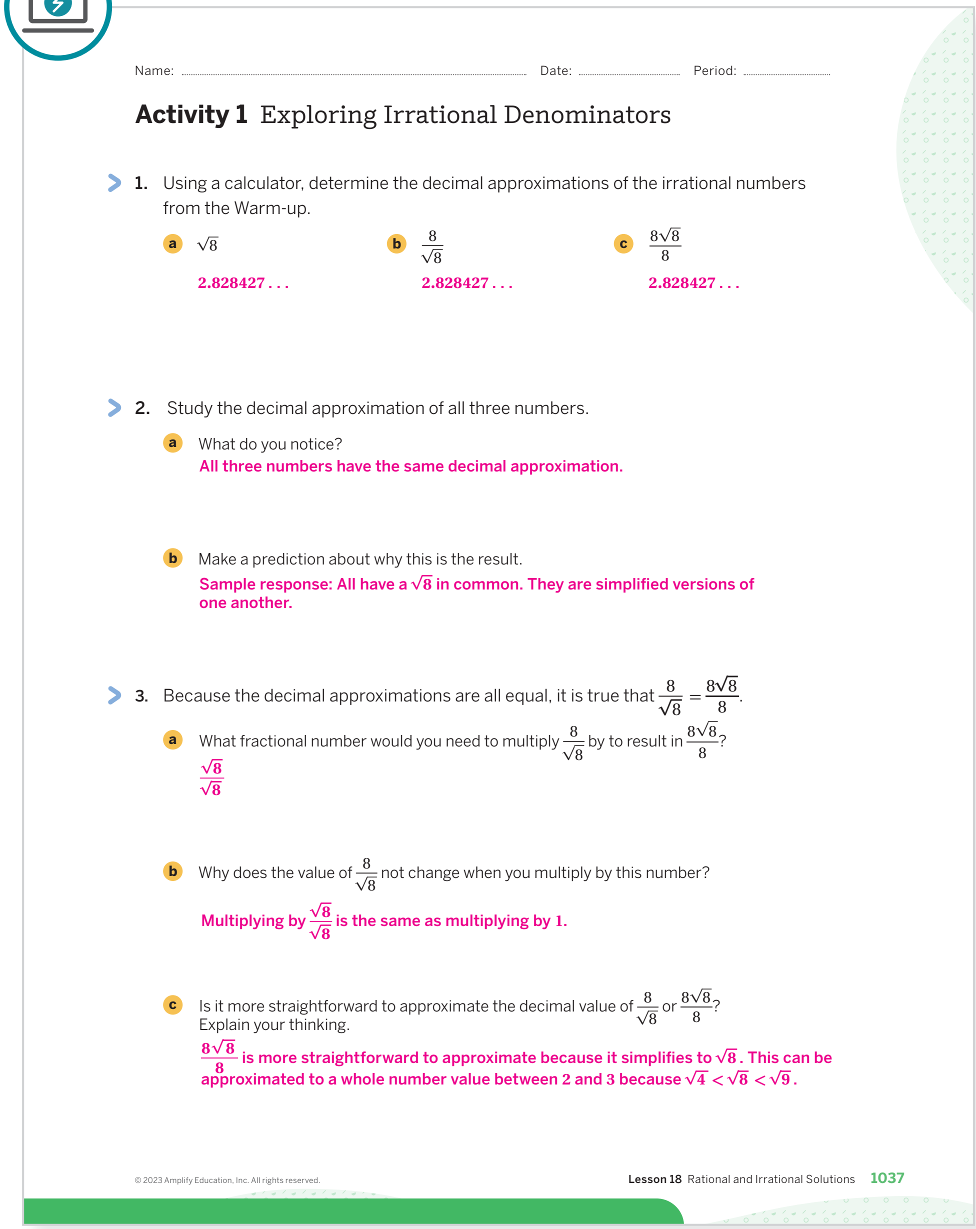
Use: Before Activity 1

Informed by: Performance on Lesson 17, Practice Problem 6

Pairs | 15 min

Activity 1 Exploring Irrational Denominators

Students explore how to rewrite an irrational number in which the denominator is irrational by rationalizing the denominator, which helps them to approximate the number's location on a number line.



Name: ______________ Date: ________ Period: ________

Activity 1 Exploring Irrational Denominators

1. Using a calculator, determine the decimal approximations of the irrational numbers from the Warm-up.
   - a $\sqrt{8}$ — **2.828427 . . .**
   - b $\frac{8}{\sqrt{8}}$ — **2.828427 . . .**
   - c $\frac{8\sqrt{8}}{8}$ — **2.828427 . . .**

2. Study the decimal approximation of all three numbers.
   - a What do you notice?
     All three numbers have the same decimal approximation.
   - b Make a prediction about why this is the result.
     Sample response: All have a $\sqrt{8}$ in common. They are simplified versions of one another.

3. Because the decimal approximations are all equal, it is true that $\frac{8}{\sqrt{8}} = \frac{8\sqrt{8}}{8}$.
   - a What fractional number would you need to multiply $\frac{8}{\sqrt{8}}$ by to result in $\frac{8\sqrt{8}}{8}$?
     $\frac{\sqrt{8}}{\sqrt{8}}$
   - b Why does the value of $\frac{8}{\sqrt{8}}$ not change when you multiply by this number?
     Multiplying by $\frac{\sqrt{8}}{\sqrt{8}}$ is the same as multiplying by 1.
   - c Is it more straightforward to approximate the decimal value of $\frac{8}{\sqrt{8}}$ or $\frac{8\sqrt{8}}{8}$? Explain your thinking.
     $\frac{8\sqrt{8}}{8}$ is more straightforward to approximate because it simplifies to $\sqrt{8}$. This can be approximated to a whole number value between 2 and 3 because $\sqrt{4} < \sqrt{8} < \sqrt{9}$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 Rational and Irrational Solutions 1037

1 Launch

Distribute or provide access to scientific calculators.

2 Monitor

Help students get started by having them consider the value of the expression $\sqrt{8} \cdot \sqrt{8}$, and whether it is rational or irrational.

Look for points of confusion:

- **Incorrectly multiplying irrational numbers.** Have them use a calculator and then try to determine a pattern.
- **Not realizing that multiplying any value by 1 does not change the value of a number.** Ask, "If you multiply the denominator by a certain value, what do you need to do to the numerator so that the value of the fraction remains the same?"
- **Being unsure determining the two integers between which each irrational number is located.** Have students estimate the locations of the irrational numbers and square roots on a number line.

Look for productive strategies:

- Multiplying the numerator and denominator by the same value so that the value of the fraction is not changed.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the three expressions in Problem 1, and ask:

- "Can you write the first expression as a fraction? What is the denominator of this fraction?"
- "Which of these three expressions have a rational number in the denominator? Which has an irrational number in the denominator?"

Extension: *Math Enrichment*

Have students determine whether multiplying $\frac{8}{\sqrt{8}}$ by $\frac{\sqrt{2}}{\sqrt{2}}$ would also rationalize the denominator, and explain their thinking.

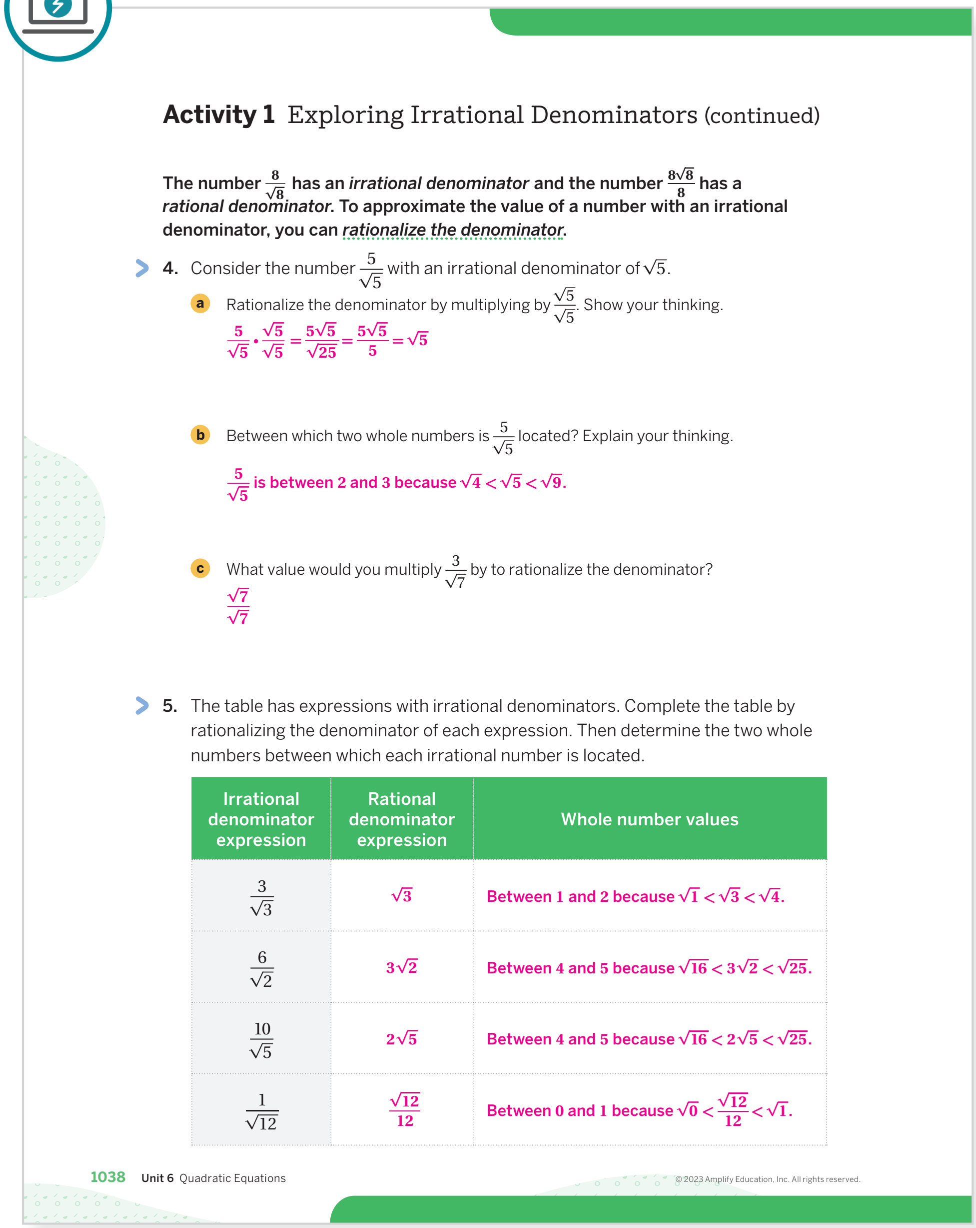
Yes; Sample response:

$$\begin{aligned}\frac{8}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} &= \frac{8\sqrt{2}}{\sqrt{16}}\\ &= \frac{8\sqrt{2}}{4}\\ &= 2\sqrt{2}\end{aligned}$$

Pairs | 15 min

Activity 1 Exploring Irrational Denominators (continued)

Students explore how to rewrite an irrational number in which the denominator is irrational by rationalizing the denominator, which helps them to approximate the number's location on a number line.



Activity 1 Exploring Irrational Denominators (continued)

The number $\frac{8}{\sqrt{8}}$ has an *irrational denominator* and the number $\frac{8\sqrt{8}}{8}$ has a *rational denominator*. To approximate the value of a number with an irrational denominator, you can *rationalize the denominator*.

4. Consider the number $\frac{5}{\sqrt{5}}$ with an irrational denominator of $\sqrt{5}$.

   a. Rationalize the denominator by multiplying by $\frac{\sqrt{5}}{\sqrt{5}}$. Show your thinking.

   $\frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{\sqrt{25}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

   b. Between which two whole numbers is $\frac{5}{\sqrt{5}}$ located? Explain your thinking.

   $\frac{5}{\sqrt{5}}$ is between 2 and 3 because $\sqrt{4} < \sqrt{5} < \sqrt{9}$.

   c. What value would you multiply $\frac{3}{\sqrt{7}}$ by to rationalize the denominator?

   $\frac{\sqrt{7}}{\sqrt{7}}$

5. The table has expressions with irrational denominators. Complete the table by rationalizing the denominator of each expression. Then determine the two whole numbers between which each irrational number is located.

Irrational denominator expression	Rational denominator expression	Whole number values
$\frac{3}{\sqrt{3}}$	$\sqrt{3}$	Between 1 and 2 because $\sqrt{1} < \sqrt{3} < \sqrt{4}$.
$\frac{6}{\sqrt{2}}$	$3\sqrt{2}$	Between 4 and 5 because $\sqrt{16} < 3\sqrt{2} < \sqrt{25}$.
$\frac{10}{\sqrt{5}}$	$2\sqrt{5}$	Between 4 and 5 because $\sqrt{16} < 2\sqrt{5} < \sqrt{25}$.
$\frac{1}{\sqrt{12}}$	$\frac{\sqrt{12}}{12}$	Between 0 and 1 because $\sqrt{0} < \frac{\sqrt{12}}{12} < \sqrt{1}$.

3 Connect

Display the expression $\frac{6}{\sqrt{2}}$.

Have students share how they determined the value by which to multiply to rationalize the denominator.

Ask:

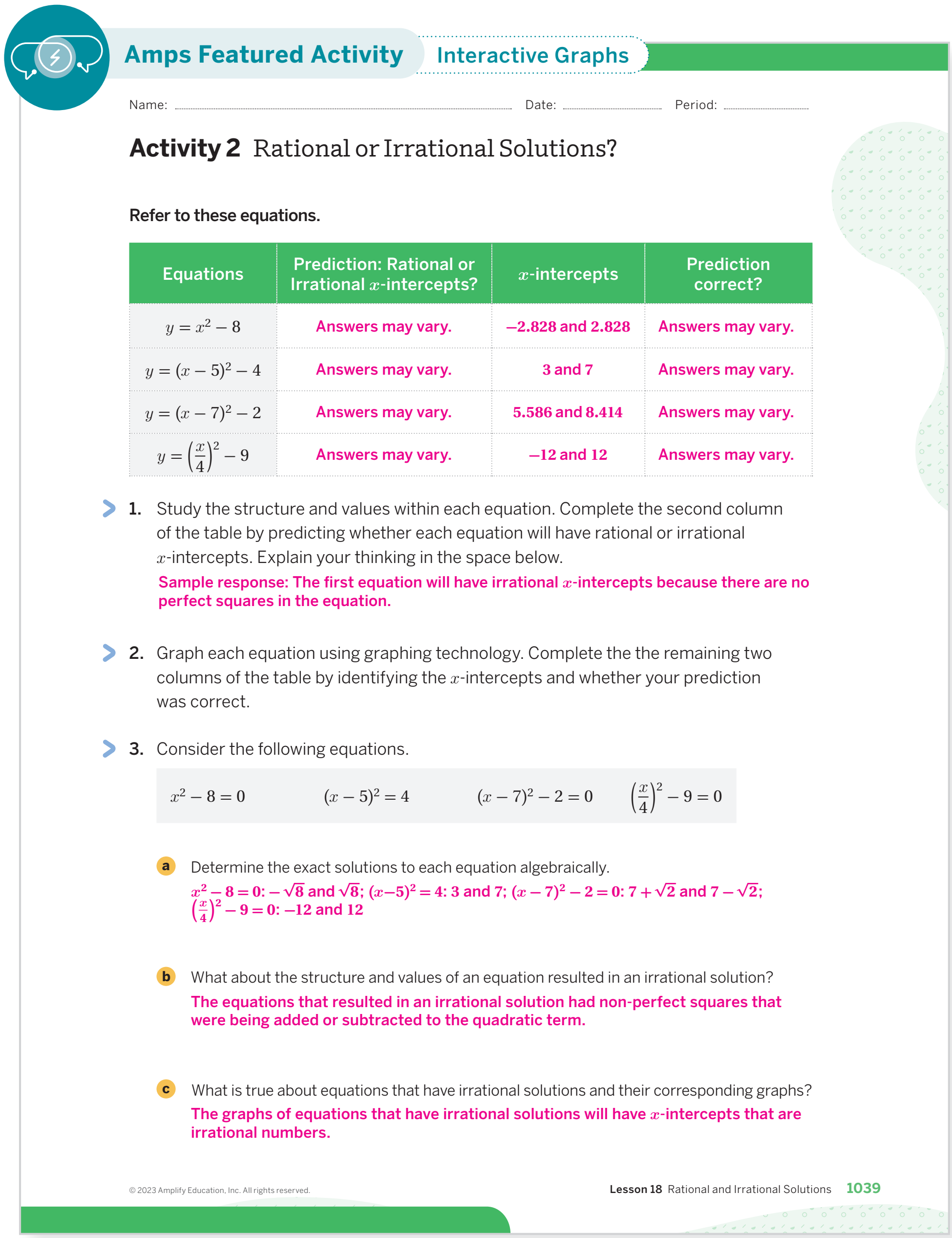
- "Given the number $\frac{2}{\sqrt{a}}$, what value do you multiply by to rationalize the denominator?"
  $\frac{\sqrt{a}}{\sqrt{a}}$
- "Why will this work to rationalize the denominator?"
  When a radical number is multiplied by itself, it will result in a rational number ($\sqrt{a} \cdot \sqrt{a} = \sqrt{a^2} = a$).

Highlight that when students ***rationalize the denominator***, they are multiplying the irrational number by 1, which does not change its value.

Independent | 12 min

Activity 2 Rational or Irrational Solutions?

Students explore solutions to quadratic equations to determine properties of quadratic equations that result in rational or irrational solutions.



Amps Featured Activity Interactive Graphs

Name: ______ Date: ______ Period: ______

Activity 2 Rational or Irrational Solutions?

Refer to these equations.

Equations	Prediction: Rational or Irrational x-intercepts?	x-intercepts	Prediction correct?
$y = x^2 - 8$	Answers may vary.	−2.828 and 2.828	Answers may vary.
$y = (x - 5)^2 - 4$	Answers may vary.	3 and 7	Answers may vary.
$y = (x - 7)^2 - 2$	Answers may vary.	5.586 and 8.414	Answers may vary.
$y = \left(\frac{x}{4}\right)^2 - 9$	Answers may vary.	−12 and 12	Answers may vary.

1. Study the structure and values within each equation. Complete the second column of the table by predicting whether each equation will have rational or irrational x-intercepts. Explain your thinking in the space below.
   Sample response: The first equation will have irrational x-intercepts because there are no perfect squares in the equation.

2. Graph each equation using graphing technology. Complete the the remaining two columns of the table by identifying the x-intercepts and whether your prediction was correct.

3. Consider the following equations.

$x^2 - 8 = 0$ $(x - 5)^2 = 4$ $(x - 7)^2 - 2 = 0$ $\left(\frac{x}{4}\right)^2 - 9 = 0$

a Determine the exact solutions to each equation algebraically.
$x^2 - 8 = 0$: $-\sqrt{8}$ and $\sqrt{8}$; $(x-5)^2 = 4$: 3 and 7; $(x - 7)^2 - 2 = 0$: $7 + \sqrt{2}$ and $7 - \sqrt{2}$; $\left(\frac{x}{4}\right)^2 - 9 = 0$: −12 and 12

b What about the structure and values of an equation resulted in an irrational solution?
The equations that resulted in an irrational solution had non-perfect squares that were being added or subtracted to the quadratic term.

c What is true about equations that have irrational solutions and their corresponding graphs?
The graphs of equations that have irrational solutions will have x-intercepts that are irrational numbers.

1 Launch

Display the four equations and ask students to make a prediction about the x-intercepts of the four functions. Provide access to graphing technology.

2 Monitor

Help students get started once they have made their predictions by asking them to graph the equations.

Look for points of confusion:

- **Forgetting the ± when taking square roots.** Display the equation $x^2 = 9$. Ask how many solutions the equation has.
- **Not remembering the relationship between x-intercepts and solutions to quadratic equations.** Display both a graph and a worked solution to a quadratic equation. Highlight the x-intercepts of the graph and the solutions to the equation. Ask students what they notice about the relationship.

Look for productive strategies:

- Recognizing perfect square k values will result in rational solutions.
- Connecting the x-intercepts of a graph and the related solutions to the related quadratic equation.

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can determine the solutions to quadratic equations and use interactive graphs to determine whether they are rational or irrational.

Math Language Development

MLR7: Compare and Connect

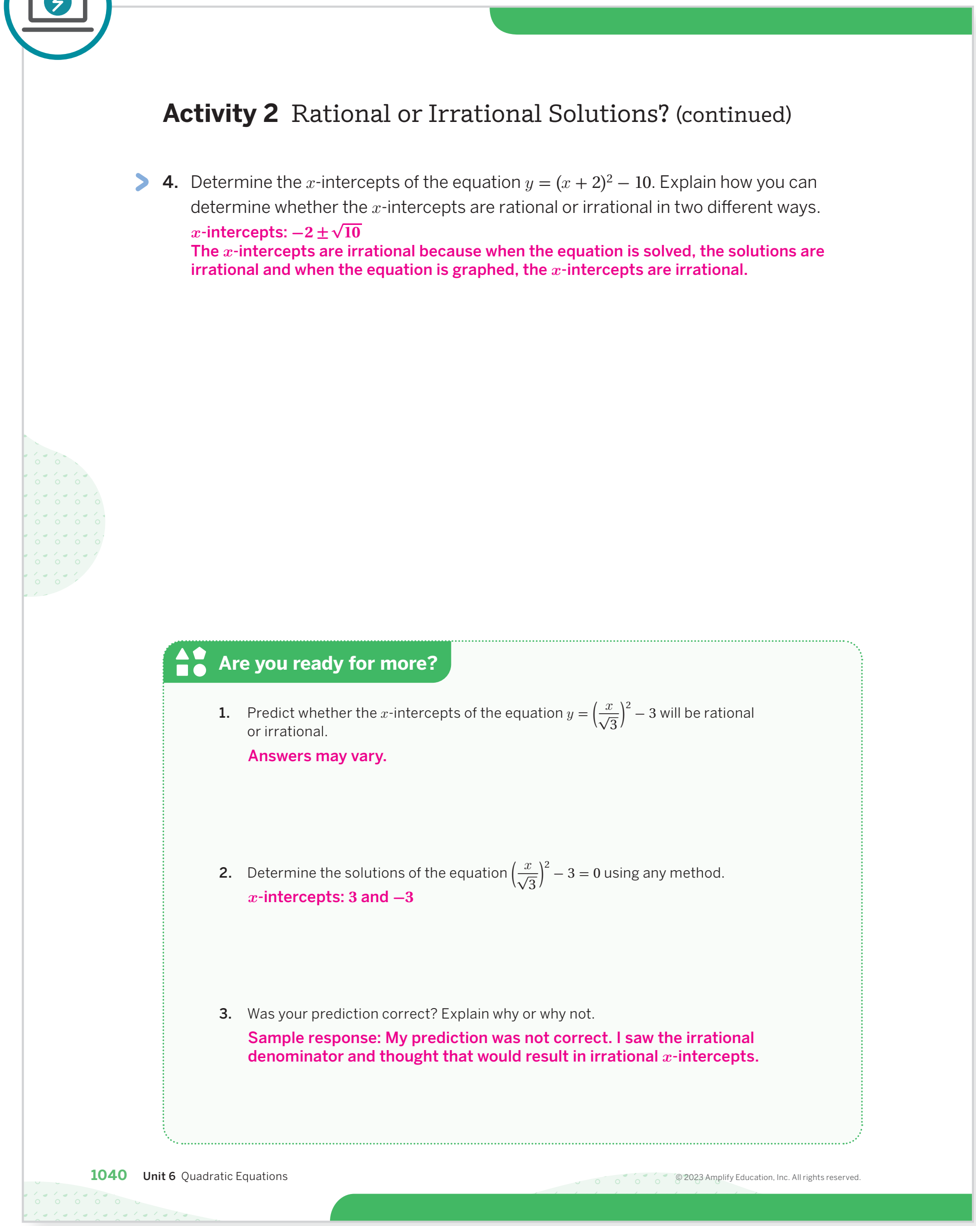
During the Connect, draw students' attention to the connections between the structure of a quadratic equation and whether the solutions to the equation were rational or irrational. Display the equation from Problem 4, $y = (x + 2)^2 - 10$. Ask:

- "To solve this equation algebraically, what is the first step?" Set it equal to 0, $(x + 2)^2 - 10 = 0$.
- "What is the next step?" Add 10 to each side, $(x + 2)^2 = 10$.
- "How do you know that the solutions will be irrational, just by studying the structure of this equation?" 10 is not a perfect square, so when taking the square root of each side and then subtracting 2, the result will be irrational.

Independent | 12 min

Activity 2 Rational or Irrational Solutions? (continued)

Students explore solutions to quadratic equations to determine properties of quadratic equations that result in rational or irrational solutions.



Activity 2 Rational or Irrational Solutions? (continued)

4. Determine the x-intercepts of the equation $y = (x + 2)^2 - 10$. Explain how you can determine whether the x-intercepts are rational or irrational in two different ways.

   x-intercepts: $-2 \pm \sqrt{10}$
   The x-intercepts are irrational because when the equation is solved, the solutions are irrational and when the equation is graphed, the x-intercepts are irrational.

Are you ready for more?

1. Predict whether the x-intercepts of the equation $y = \left(\frac{x}{\sqrt{3}}\right)^2 - 3$ will be rational or irrational.

   Answers may vary.

2. Determine the solutions of the equation $\left(\frac{x}{\sqrt{3}}\right)^2 - 3 = 0$ using any method.

   x-intercepts: 3 and −3

3. Was your prediction correct? Explain why or why not.

   Sample response: My prediction was not correct. I saw the irrational denominator and thought that would result in irrational x-intercepts.

1040 **Unit 6** Quadratic Equations

3 Connect

Display the equation $y = (x + 2)^2 - 10$.

Have students share their strategy for determining the x-intercepts of the equation.

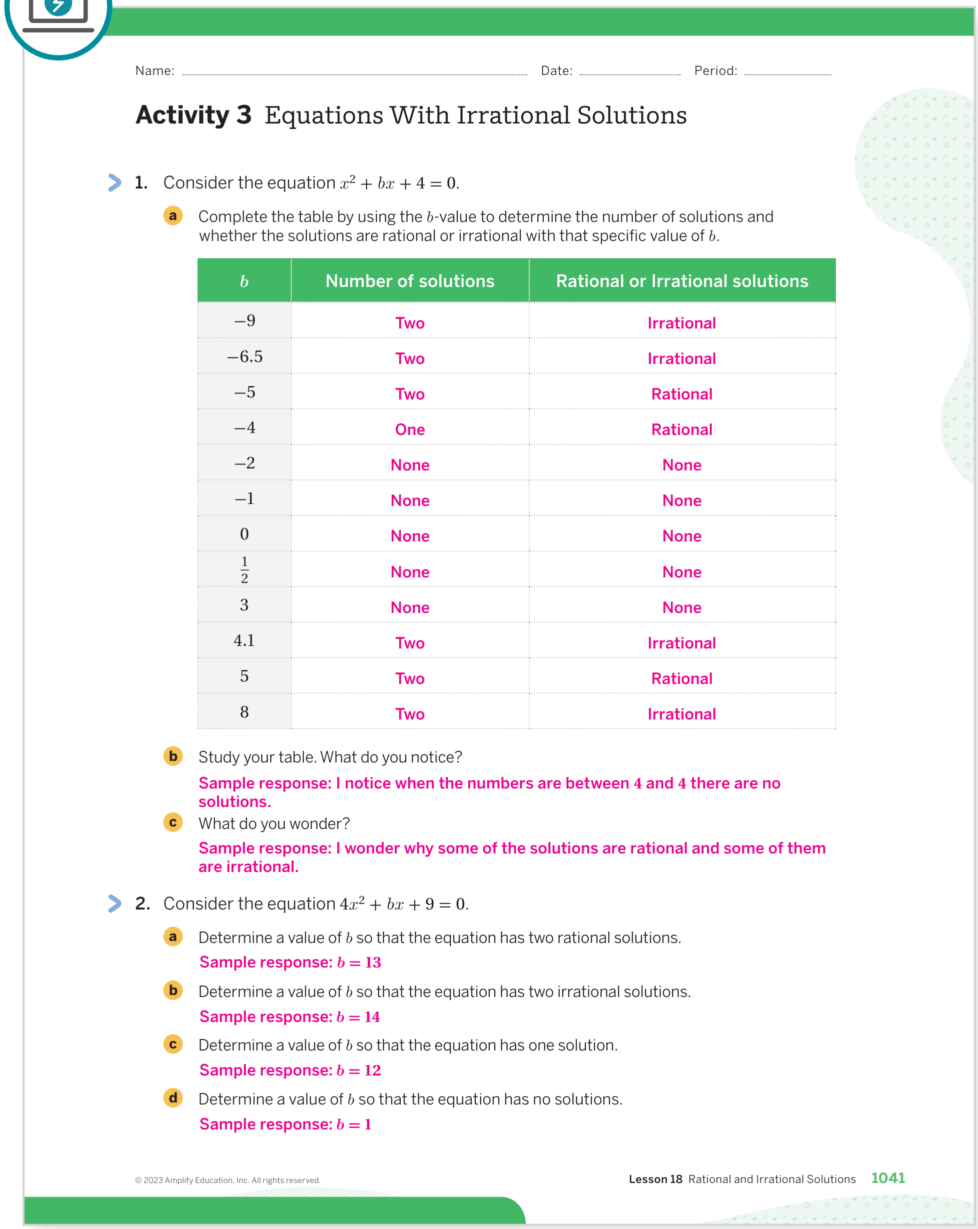
Ask, “How can you determine whether x-intercepts of quadratic equations are rational or irrational?” It is sometimes difficult to determine from a graph, but when we solve a quadratic equation, if the number inside the square root symbol is not a perfect square, then the x-intercepts will be irrational.

Highlight that solutions are irrational when they cannot be written as fractions where the numerator and denominator are integers. Graphing quadratic equations can help to estimate or visualize solutions, but are not reliable in providing exact values. Using algebraic methods to solve quadratic equations yields the exact solutions.

Small Groups | 12 min

Activity 3 Equations With Irrational Solutions

Students explore values of b to determine the number of rational or irrational solutions to a quadratic equation of the form $x^2 + bx + c = 0$.



Name: ______ Date: ______ Period: ______

Activity 3 Equations With Irrational Solutions

1. Consider the equation $x^2 + bx + 4 = 0$.

a Complete the table by using the b-value to determine the number of solutions and whether the solutions are rational or irrational with that specific value of b.

b	Number of solutions	Rational or Irrational solutions
-9	Two	Irrational
-6.5	Two	Irrational
-5	Two	Rational
-4	One	Rational
-2	None	None
-1	None	None
0	None	None
$\frac{1}{2}$	None	None
3	None	None
4.1	Two	Irrational
5	Two	Rational
8	Two	Irrational

b Study your table. What do you notice?

Sample response: I notice when the numbers are between 4 and 4 there are no solutions.

c What do you wonder?

Sample response: I wonder why some of the solutions are rational and some of them are irrational.

2. Consider the equation $4x^2 + bx + 9 = 0$.

a Determine a value of b so that the equation has two rational solutions.

Sample response: $b = 13$

b Determine a value of b so that the equation has two irrational solutions.

Sample response: $b = 14$

c Determine a value of b so that the equation has one solution.

Sample response: $b = 12$

d Determine a value of b so that the equation has no solutions.

Sample response: $b = 1$

Lesson 18 Rational and Irrational Solutions 1041

1 Launch

Tell students they will explore how the value of the linear term can help indicate the number of solutions to a quadratic equation. Provide access to graphing technology.

2 Monitor

Help students get started by encouraging them to use graphing technology to determine the number of solutions.

Look for points of confusion:

- **Struggling to classify the solutions as rational or irrational.** Encourage students to think about whether they can factor the quadratic equation with the specific value of b.
- **Focusing too much on graphical representations.** Prompt students to look for patterns in the algebraic representation. Suggest they focus on whether the equation can be factored.

Look for productive strategies:

- Categorizing quadratic equations into those that can be factored and those that cannot.
- Making connections between the graphical and algebraic representations of the quadratic equations.
- Making a table for Problem 2 to help organize their thinking. Look for those who use a wide range of numbers (both fractional and negative values).

Activity 3 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, provide them with a range of values they can use for Problems 2 and 4 to help narrow their focus.

Math Language Development

MLR3: Critique, Correct, Clarify

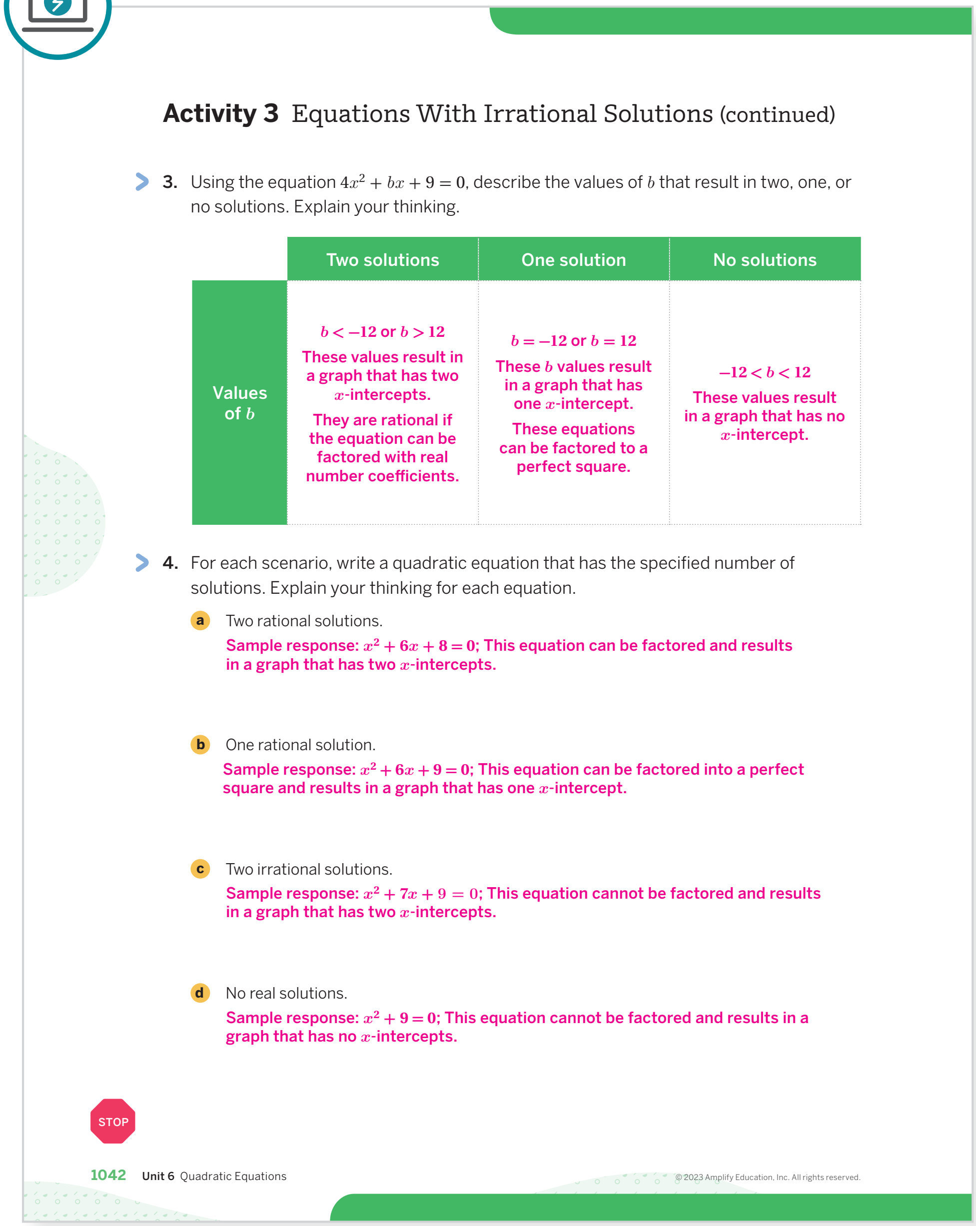
During the Connect, present an incorrect statement, such as "The equation $x^2 + 6x + 9 = 0$ has two rational solutions because the equation can be factored." Ask:

- ***Critique:*** "Do you agree with this statement? Explain your thinking." Listen for students who reason that while the solution will be rational, there will only be one solution because the expression is a square expression.
- ***Correct and Clarify:*** "Write a corrected statement. How would you convince someone that your statement is correct?"

Small Groups | 12 min

Activity 3 Equations With Irrational Solutions (continued)

Students explore values of b to determine the number of rational or irrational solutions to a quadratic equation of the form $x^2 + bx + c = 0$.



Activity 3 Equations With Irrational Solutions (continued)

3. Using the equation $4x^2 + bx + 9 = 0$, describe the values of b that result in two, one, or no solutions. Explain your thinking.

	Two solutions	One solution	No solutions
Values of b	$b < -12$ or $b > 12$ These values result in a graph that has two x-intercepts. They are rational if the equation can be factored with real number coefficients.	$b = -12$ or $b = 12$ These b values result in a graph that has one x-intercept. These equations can be factored to a perfect square.	$-12 < b < 12$ These values result in a graph that has no x-intercept.

4. For each scenario, write a quadratic equation that has the specified number of solutions. Explain your thinking for each equation.

a Two rational solutions.

Sample response: $x^2 + 6x + 8 = 0$; This equation can be factored and results in a graph that has two x-intercepts.

b One rational solution.

Sample response: $x^2 + 6x + 9 = 0$; This equation can be factored into a perfect square and results in a graph that has one x-intercept.

c Two irrational solutions.

Sample response: $x^2 + 7x + 9 = 0$; This equation cannot be factored and results in a graph that has two x-intercepts.

d No real solutions.

Sample response: $x^2 + 9 = 0$; This equation cannot be factored and results in a graph that has no x-intercepts.

STOP

1042 Unit 6 Quadratic Equations

3 Connect

Display the four statements from Problem 4

Have groups of students share a quadratic equation they created to meet the criteria for each statement. Have them share the strategies they used to determine the quadratic equation.

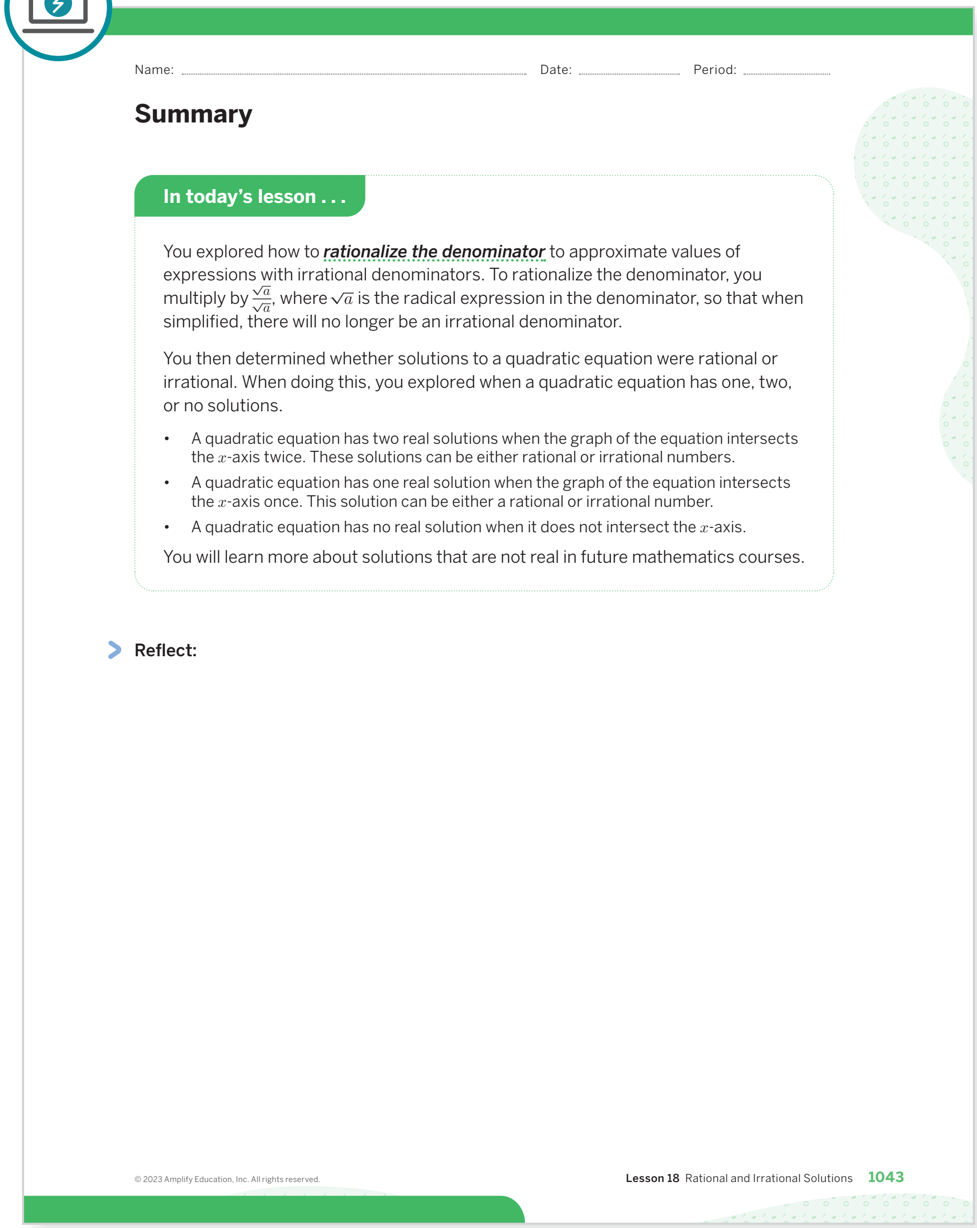
Ask, "What is the difference between when a quadratic equation has two rational solutions or two irrational solutions?" When the quadratic equation can be factored (with real number coefficients), then it has two rational solutions. When the equation cannot be factored (with real number coefficients), then it has two irrational solutions.

Highlight that it is often helpful to examine multiple representations to make sense of a relationship. A graphical representation can help determine the number of x-intercepts quickly. An algebraic representation can help determine whether a solution is a rational or irrational number.

Whole Class | 5 min

Summary

Review and synthesize how rationalizing the denominator of an irrational number can help approximate the irrational number's value and location on a number line.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how to ***rationalize the denominator*** to approximate values of expressions with irrational denominators. To rationalize the denominator, you multiply by $\frac{\sqrt{a}}{\sqrt{a}}$, where $\sqrt{a}$ is the radical expression in the denominator, so that when simplified, there will no longer be an irrational denominator.

You then determined whether solutions to a quadratic equation were rational or irrational. When doing this, you explored when a quadratic equation has one, two, or no solutions.

- A quadratic equation has two real solutions when the graph of the equation intersects the x-axis twice. These solutions can be either rational or irrational numbers.
- A quadratic equation has one real solution when the graph of the equation intersects the x-axis once. This solution can be either a rational or irrational number.
- A quadratic equation has no real solution when it does not intersect the x-axis.

You will learn more about solutions that are not real in future mathematics courses.

Reflect:

Synthesize

Ask, "What is one method you can use to determine what types of solutions a quadratic equation has?"

Have students share their thinking with a partner.

Ask, "What is your favorite way of solving a quadratic equation? What benefits does your strategy have over other strategies? What downsides does your strategy have?"

Have students share their thinking with a partner. Then have students share with the class about something their partner said that changed their thinking.

Highlight that in this lesson, students explored the types of solutions that occur when solving quadratic equations. There are many methods and strategies that can be used to highlight the types and number of solutions.

Formalize vocabulary: *rationalize the denominator*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean to rationalize a denominator? Describe this process in your own words."

Math Language Development

MLR2: Collect and Display

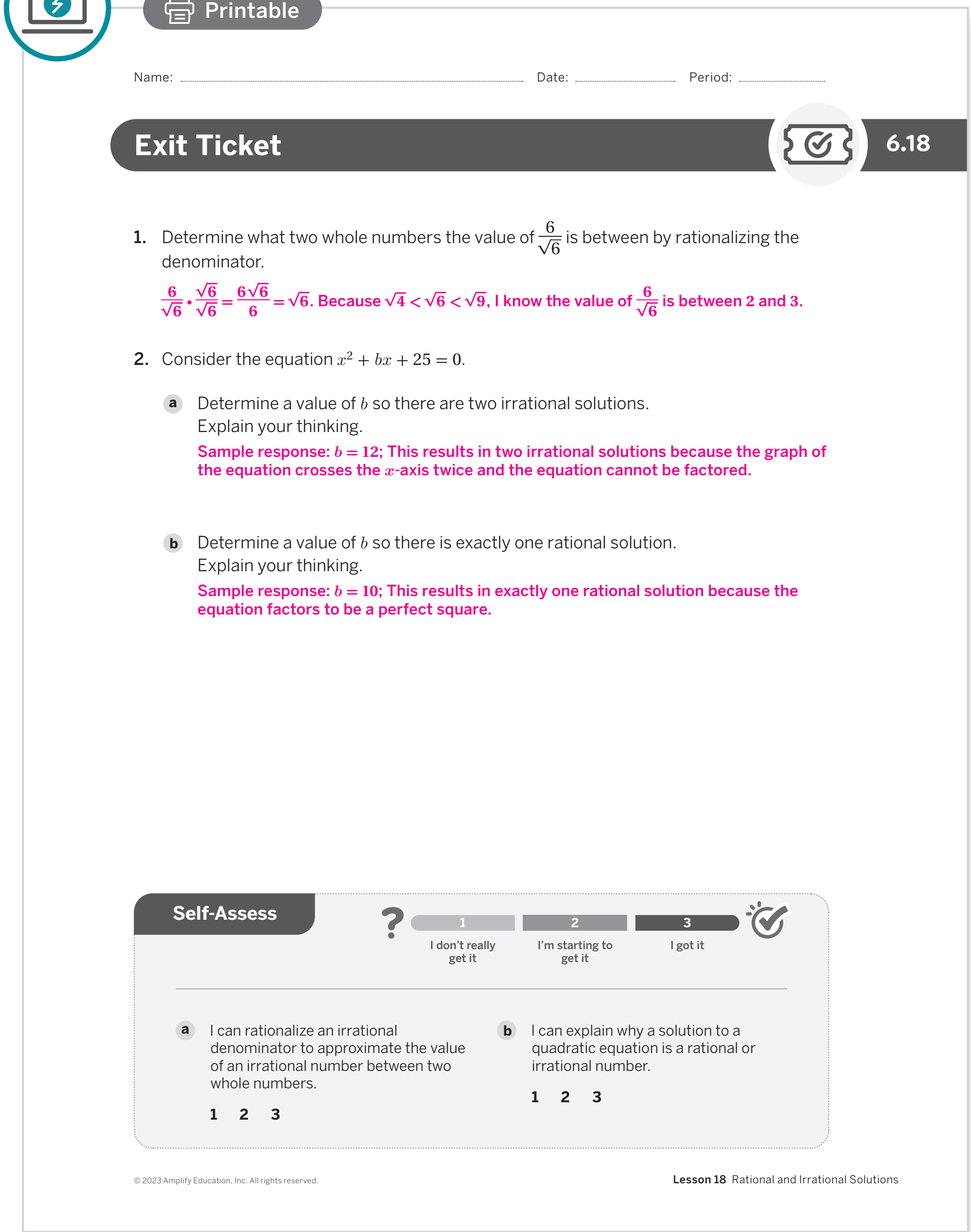
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *rationalize the denominator* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by classifying a sum or product as rational or irrational and explaining their thinking.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.18

1. Determine what two whole numbers the value of $\frac{6}{\sqrt{6}}$ is between by rationalizing the denominator.

   $\frac{6}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{6\sqrt{6}}{6} = \sqrt{6}$. Because $\sqrt{4} < \sqrt{6} < \sqrt{9}$, I know the value of $\frac{6}{\sqrt{6}}$ is between 2 and 3.

2. Consider the equation $x^2 + bx + 25 = 0$.

   a. Determine a value of b so there are two irrational solutions.
   Explain your thinking.
   Sample response: $b = 12$; This results in two irrational solutions because the graph of the equation crosses the x-axis twice and the equation cannot be factored.

   b. Determine a value of b so there is exactly one rational solution.
   Explain your thinking.
   Sample response: $b = 10$; This results in exactly one rational solution because the equation factors to be a perfect square.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can rationalize an irrational denominator to approximate the value of an irrational number between two whole numbers.

1 2 3

b. I can explain why a solution to a quadratic equation is a rational or irrational number.

1 2 3

Lesson 18 Rational and Irrational Solutions

Success looks like . . .

- **Goal:** Rationalizing an irrational denominator to approximate the value of an irrational number between two whole numbers.
  - » Approximating the two values by rationalizing $\frac{6}{\sqrt{6}}$ in Problem 1.
- **Language Goal:** Explaining why the solution to a quadratic equation is a rational or irrational number. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly rationalize the denominator in Problem 1, consider:

- Reviewing Activity 1, Problem 3.
- Assigning Practice Problem 1.
- Asking, "How could you multiply this number by 1 and rationalize the denominator at the same time?"

If students do not determine the two whole numbers between which the value is located in Problem 1, consider:

- Reviewing Activity 3, Problems 4 and 5.
- Assigning Practice Problem 1.
- Asking, "What is the value of $\sqrt{25}$?"

If students incorrectly respond to Problem 2, consider:

- Reviewing Activity 3, Problem 2.
- Assigning Practice Problem 3.
- Having students graph their equation and determine whether they should revise their response, based on the graph.

Professional Learning

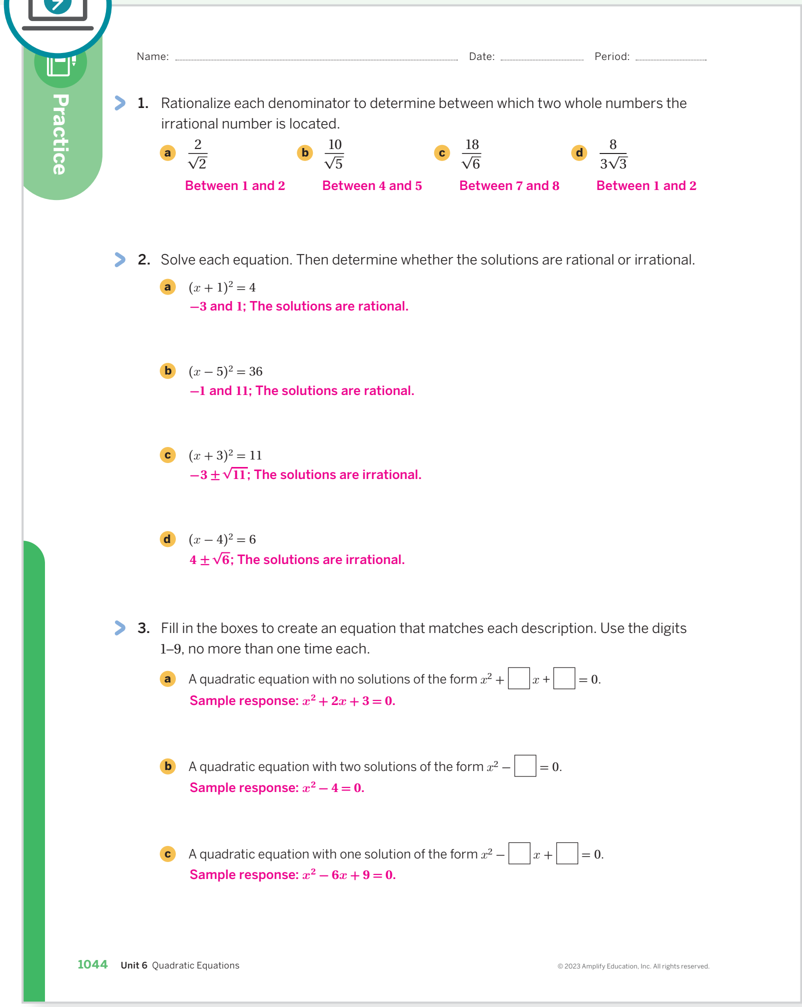
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students explored why rational and irrational solutions occur within specific quadratic equations. How did that build on the earlier work students did with solving quadratic equations?
- During the discussion on how different representations highlight specific properties about quadratic equations, how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Rationalize each denominator to determine between which two whole numbers the irrational number is located.

   a $\frac{2}{\sqrt{2}}$ Between 1 and 2
   b $\frac{10}{\sqrt{5}}$ Between 4 and 5
   c $\frac{18}{\sqrt{6}}$ Between 7 and 8
   d $\frac{8}{3\sqrt{3}}$ Between 1 and 2

2. Solve each equation. Then determine whether the solutions are rational or irrational.

   a $(x+1)^2 = 4$
   −3 and 1; The solutions are rational.

   b $(x-5)^2 = 36$
   −1 and 11; The solutions are rational.

   c $(x+3)^2 = 11$
   $-3 \pm \sqrt{11}$; The solutions are irrational.

   d $(x-4)^2 = 6$
   $4 \pm \sqrt{6}$; The solutions are irrational.

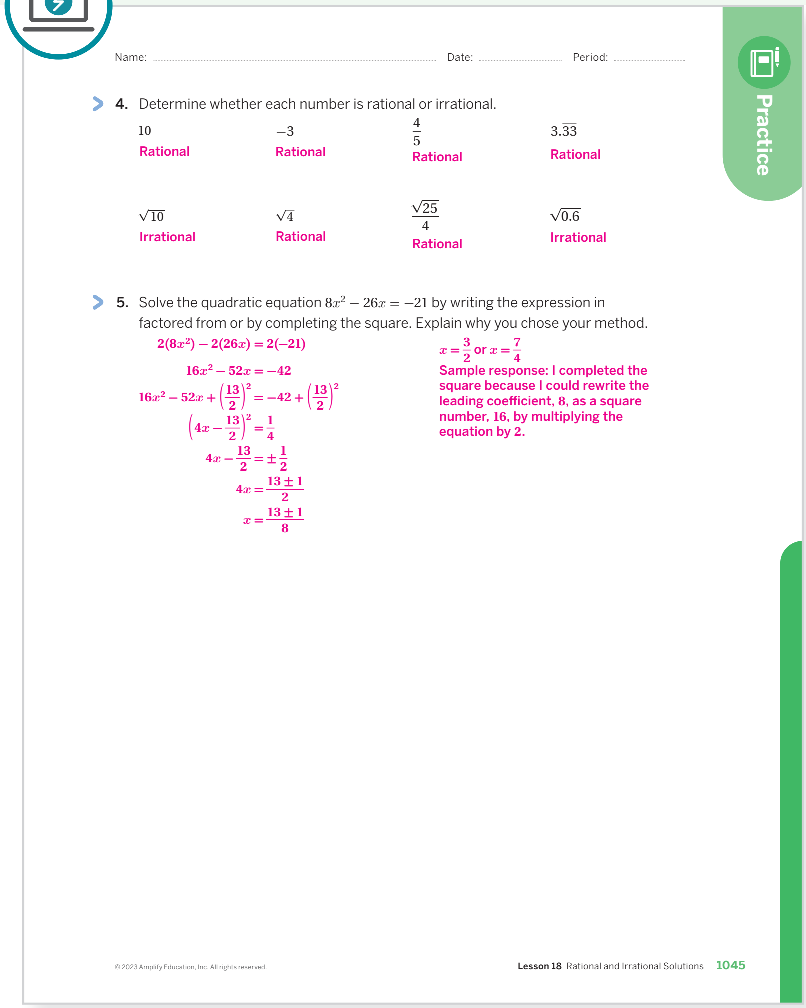
3. Fill in the boxes to create an equation that matches each description. Use the digits 1–9, no more than one time each.

   a A quadratic equation with no solutions of the form $x^2 + \square x + \square = 0$.
   Sample response: $x^2 + 2x + 3 = 0$.

   b A quadratic equation with two solutions of the form $x^2 - \square = 0$.
   Sample response: $x^2 - 4 = 0$.

   c A quadratic equation with one solution of the form $x^2 - \square x + \square = 0$.
   Sample response: $x^2 - 6x + 9 = 0$.

1044 Unit 6 Quadratic Equations © 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ______ Date: ______ Period: ______

4. Determine whether each number is rational or irrational.

10 Rational	-3 Rational	$\frac{4}{5}$ Rational	$3.\overline{33}$ Rational
$\sqrt{10}$ Irrational	$\sqrt{4}$ Rational	$\frac{\sqrt{25}}{4}$ Rational	$\sqrt{0.6}$ Irrational

5. Solve the quadratic equation $8x^2 - 26x = -21$ by writing the expression in factored from or by completing the square. Explain why you chose your method.

$$2(8x^2) - 2(26x) = 2(-21)$$
$$16x^2 - 52x = -42$$
$$16x^2 - 52x + \left(\frac{13}{2}\right)^2 = -42 + \left(\frac{13}{2}\right)^2$$
$$\left(4x - \frac{13}{2}\right)^2 = \frac{1}{4}$$
$$4x - \frac{13}{2} = \pm\frac{1}{2}$$
$$4x = \frac{13 \pm 1}{2}$$
$$x = \frac{13 \pm 1}{8}$$

$x = \frac{3}{2}$ or $x = \frac{7}{4}$

Sample response: I completed the square because I could rewrite the leading coefficient, 8, as a square number, 16, by multiplying the equation by 2.

© 2023 Amplify Education, Inc. All rights reserved. Lesson 18 Rational and Irrational Solutions 1045

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 3	3
Spiral	4	Unit 6 Lesson 17	2
Formative ⚡	5	Unit 6 Lesson 19	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 5

The Quadratic Formula

Whole Class

In this Sub-Unit, students examine an efficient strategy for solving quadratic equations. History comes full circle in the final lesson, when they discover recent contributions.

SUB-UNIT

5 The Quadratic Formula

Narrative Connections

What was the House of Wisdom?

In the late 8th century, while Europe was in the throes of its Dark Ages, the Middle East was a center of thought and knowledge. Under the rule of the Abbasid Caliphate, in what is now Iraq, a great library was built called the House of Wisdom.

Scholars accumulated knowledge from across the known world — from Babylonian, Greek, Jewish, Chinese, and Indian cultures and civilizations — translating texts of philosophy, astronomy, science, mathematics, and literature into Arabic. The House was a place where the ideas of Greek scholars such as Aristotle, Plato, and Euclid could crash together with Eastern thinkers such as Brahmagupta and Aryabhata. Overseeing it all was Muḥammad ibn Mūsā al-Khwārizmī.

Born around 780 CE, Al-Khwārizmī was one of House of Wisdom's first directors. Building on the works of Babylonian, Greek, Jewish, and Indian scholars, he wrote the first major treatise on algebra — a word that comes from the Arabic *al-jabr*, meaning "balance."

Unlike past mathematicians, Al-Khwārizmī presented math *systematically* — first providing rules for how the math behaved, then demonstrating how it worked. But Al-Khwārizmī's most influential achievement may have been his discovery of the quadratic formula — a skeleton key capable of unlocking *any* quadratic equation.

Narrative Connections

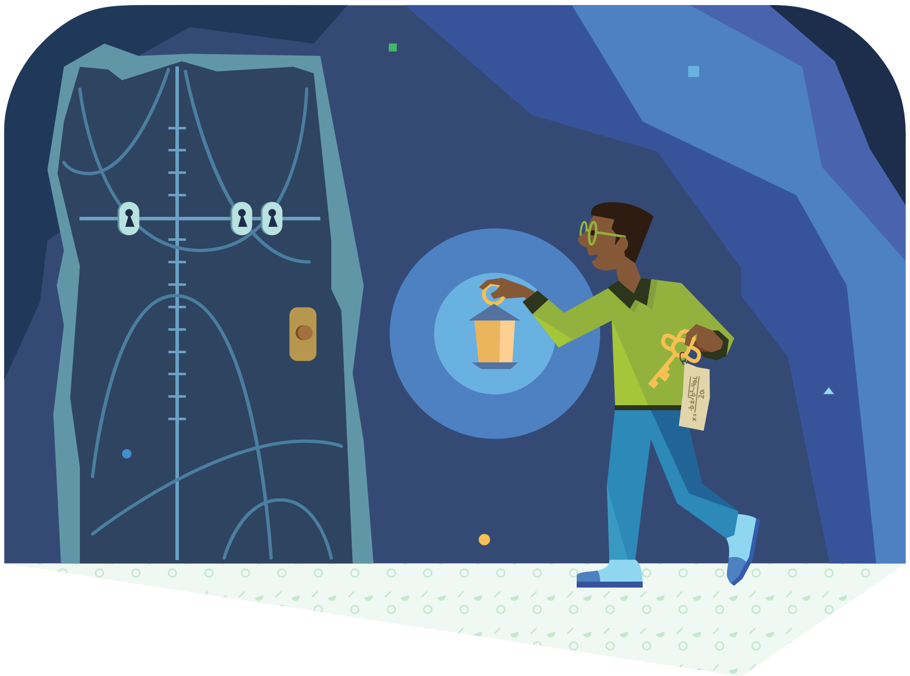
Read the narrative aloud as a class or have students read it individually. Students continue to explore different methods for solving quadratic equations discovered throughout history in the following places:

- **Lesson 19, Activities 1–2:** Deriving by Difference of Squares, Deriving by Completing the Square
- **Lesson 20, Activity 1:** Check It
- **Lesson 21, Activity 3:** Beyond the Quadratic Formula
- **Lesson 22, Activity 1:** A Babylonian Problem
- **Lesson 23, Activities 1–2:** Historical Origins, Making Math History in 2019

UNIT 6 | **LESSON 19**

A Formula For Any Quadratic

Let's examine a new strategy for solving quadratic equations.



Focus

Goals

1. **Language Goal:** Explain the steps used to derive the quadratic formula. **(Writing)**
2. **Language Goal:** Explain how the solutions obtained by completing the square are expressed by the quadratic formula. **(Speaking and Listening, Writing)**
3. Demonstrate that the quadratic formula can be derived by generalizing the process of completing the square.

Coherence

Today

Students derive the quadratic formula using the difference of squares and completing the square strategies. Starting with concrete examples, students model expressions with area diagrams and algebraically to make sense of the process and apply to the general standard form equation, $ax^2 + bx + c = 0$.

Previously

In Lessons 13 and 15, students solved quadratic equations with irrational solutions by completing the square.

Coming Soon

In Lesson 20, students will use the quadratic formula and verify that it produces the same solutions as those found using other strategies.

Rigor

- Students build **conceptual understanding** of the quadratic formula by deriving it by completing the square.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Anchor Chart PDFs:

  - *Difference of Squares*
  - *Sentence Stems, Partner and Group Questioning*
  - *Completing the Square*
  - *Solving Monic Quadratic Equations by Factoring*
  - *Solving Non-Monic Quadratic Equations by Factoring*
- algebra tiles (as needed)

Math Language Development

New words

- *quadratic formula*

Review words

- *completing the square*
- *difference of squares*

Amps | Featured Activity

Activity 1

Formative Feedback for Students

Instead of just being told if they are correct or incorrect, students see the consequences of their solutions and can determine any needed corrections on their own.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may have difficulty managing stress and self-motivating when deriving the quadratic formula in Activities 1 and 2. Lead a discussion on barriers students may encounter and have them think and discuss about ways they could overcome them. Have students consider who might be able to help, or what other resources might be available.

Modifications to Pacing

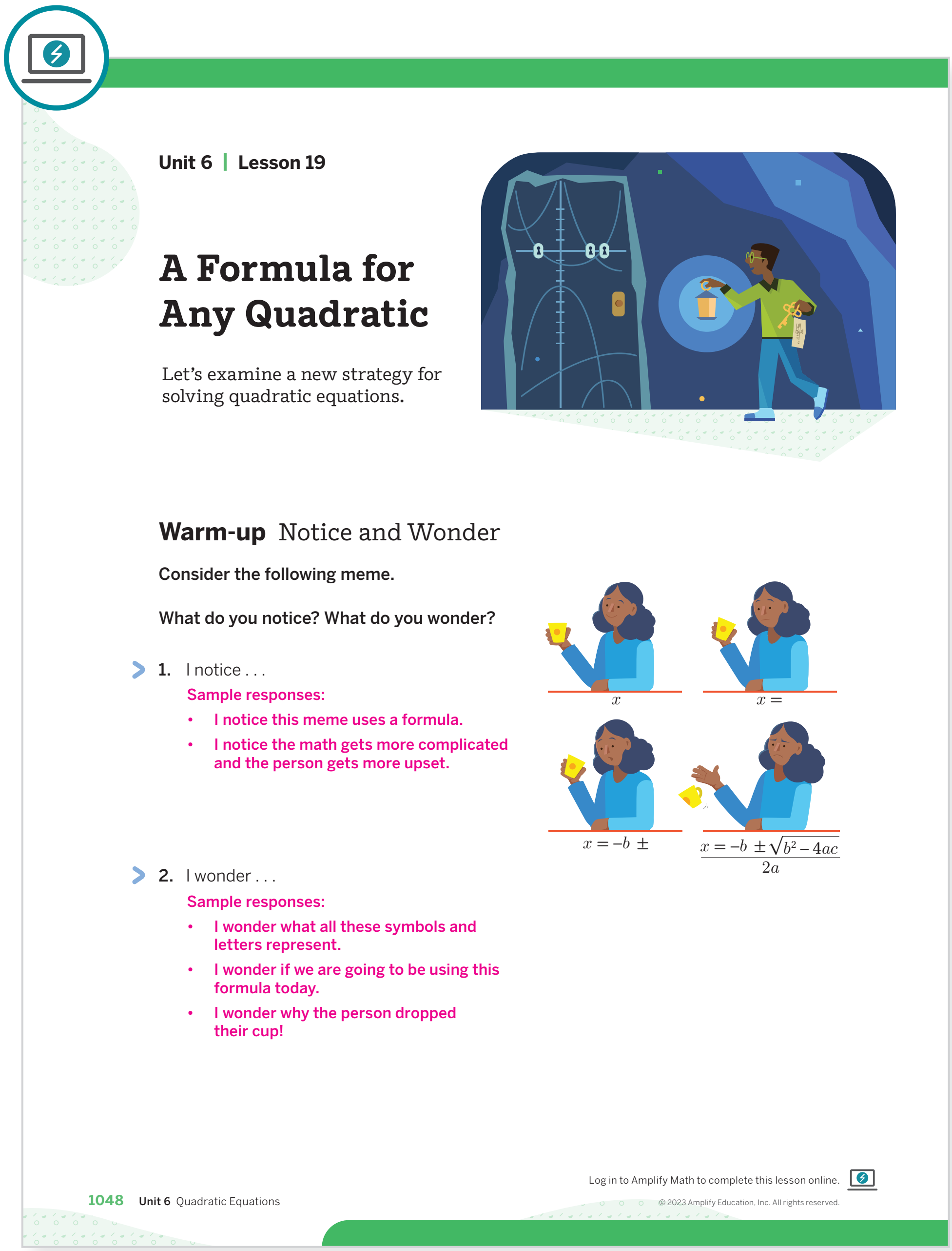
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students choose one equation in Problem 1 and Problem 3 may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students study a meme about the pop culture references of the quadratic formula to activate background knowledge before deriving the formula.



Unit 6 | **Lesson 19**

A Formula for Any Quadratic

Let's examine a new strategy for solving quadratic equations.

Warm-up Notice and Wonder

Consider the following meme.

What do you notice? What do you wonder?

1. I notice . . .

   Sample responses:
   - **I notice this meme uses a formula.**
   - **I notice the math gets more complicated and the person gets more upset.**

2. I wonder . . .

   Sample responses:
   - **I wonder what all these symbols and letters represent.**
   - **I wonder if we are going to be using this formula today.**
   - **I wonder why the person dropped their cup!**

1048 **Unit 6** Quadratic Equations

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the image. Have students study the meme independently before conducting the *Notice and Wonder* routine. Be prepared to record their responses during the Connect.

2 Monitor

Help students get started by prompting them to write what they find interesting about the meme.

Look for productive strategies:

- Circling the symbols and expressions.
- Noticing the person's facial expressions and body language.
- Noting their thoughts next to and within the image.

3 Connect

Have students share what they noticed and wondered.

Display the quadratic formula.

Highlight that the quadratic formula is a new strategy to solve quadratic equations of the form $ax^2 + bx + c = 0$.

Define the ***quadratic formula*** as a formula that provides the solution to a quadratic equation, or $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Ask:

- "Why do you think the quadratic formula is so well known?" Answers may vary.
- "Have you ever heard the quadratic formula set to a song?" Sing as a class, using call and response, to the *Pop Goes the Weasel* tune.

Power-up

To power up students' ability to solve quadratic equations by completing the square, have students complete:

Match each equation with its equivalent equation having a perfect square written on one side.

a $x^2 + 6.4x - 8.9 = 0$ b $(x - 2.5)^2 = 17.25$

b $x^2 - 5x = 11$ c $\left(x - \frac{9}{2}\right)^2 = \frac{83}{4}$

c $x^2 + 9x = \frac{1}{2}$ a $(x + 3.2)^2 = 19.14$

Use: Before Activity 1

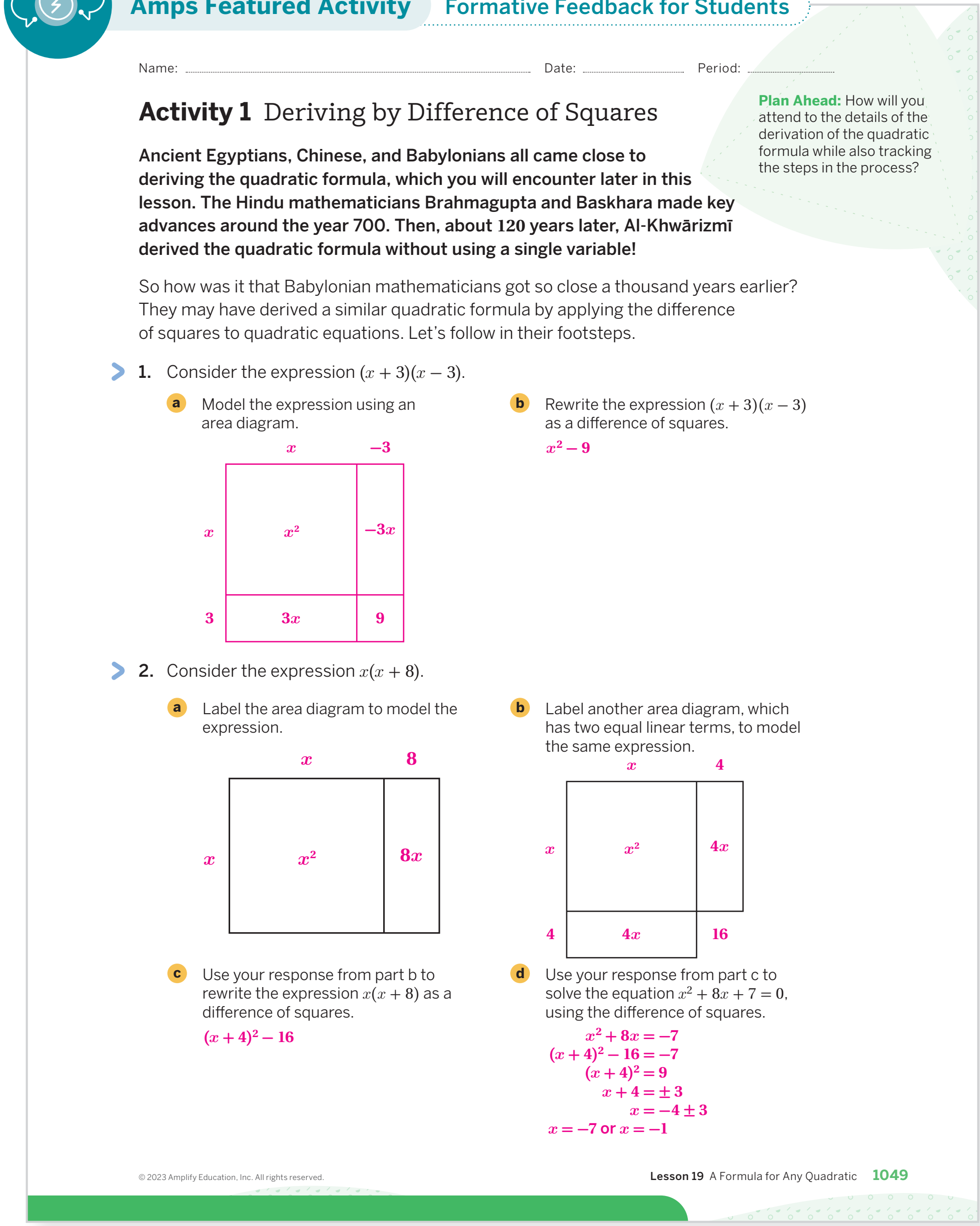
Informed by: Performance on Lesson 18, Practice Problem 5

Small Groups | 15 min

Activity 1 Deriving by Difference of Squares

Students derive the quadratic formula using the difference of squares to make sense of the formula.

Amps Featured Activity Formative Feedback for Students



Name: ______ Date: ______ Period: ______

Activity 1 Deriving by Difference of Squares

Plan Ahead: How will you attend to the details of the derivation of the quadratic formula while also tracking the steps in the process?

Ancient Egyptians, Chinese, and Babylonians all came close to deriving the quadratic formula, which you will encounter later in this lesson. The Hindu mathematicians Brahmagupta and Baskhara made key advances around the year 700. Then, about 120 years later, Al-Khwārizmī derived the quadratic formula without using a single variable!

So how was it that Babylonian mathematicians got so close a thousand years earlier? They may have derived a similar quadratic formula by applying the difference of squares to quadratic equations. Let's follow in their footsteps.

1. Consider the expression $(x+3)(x-3)$.

   a. Model the expression using an area diagram.

   b. Rewrite the expression $(x+3)(x-3)$ as a difference of squares.

   $x^2 - 9$

2. Consider the expression $x(x+8)$.

   a. Label the area diagram to model the expression.

   b. Label another area diagram, which has two equal linear terms, to model the same expression.

   c. Use your response from part b to rewrite the expression $x(x+8)$ as a difference of squares.

   $(x+4)^2 - 16$

   d. Use your response from part c to solve the equation $x^2 + 8x + 7 = 0$, using the difference of squares.

$$x^2 + 8x = -7$$
$$(x+4)^2 - 16 = -7$$
$$(x+4)^2 = 9$$
$$x + 4 = \pm 3$$
$$x = -4 \pm 3$$
$$x = -7 \text{ or } x = -1$$

1 Launch

Display the prompt and use the ***Co-craft Questions*** routine as described in the Math Language Development section.

For each problem, students should discuss strategies as a group, complete individually, and then compare solutions before moving on to the next problem.

2 Monitor

Help students get started by displaying the Anchor Chart PDF, *Difference of Squares*.

Look for points of confusion:

- **Having difficulty rewriting the expression as a difference of squares in Problems 2c and 3c.** Highlight the large square and the missing small square in each area diagram. Ask, "What is the area of the large square, including the small square? What is the area of the small square?"
- **Struggling to solve the equation using the difference of squares in Problem 3d.** Prompt students to explain or annotate the steps from Problem 2d to apply to Problem 3d.

Look for productive strategies:

- Using diagrams, color, or annotations to help make sense of the problems.
- Annotating, organizing, or highlighting the steps from previous examples to generalize with variables.
- Writing their solutions in different, equivalent ways.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see the consequences of their solutions and can determine any needed corrections on their own.

Accessibility: *Optimize Access to Tools*

Provide access to algebra tiles and blank area diagrams for students to choose to use during this activity.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory text and the expression in Problem 1. Ask students to work together with members of their small group to write 2–3 questions they could ask about the scenario or expression given in Problem 1. Sample questions shown.

- How did Al-Khwārizmī derive the quadratic formula without using variables?
- How is the difference of squares related to the quadratic formula?
- The quadratic formula uses the values of a, b, and c. What are those values for the expression given in Problem 1? Does the expression need to be written in standard form first?

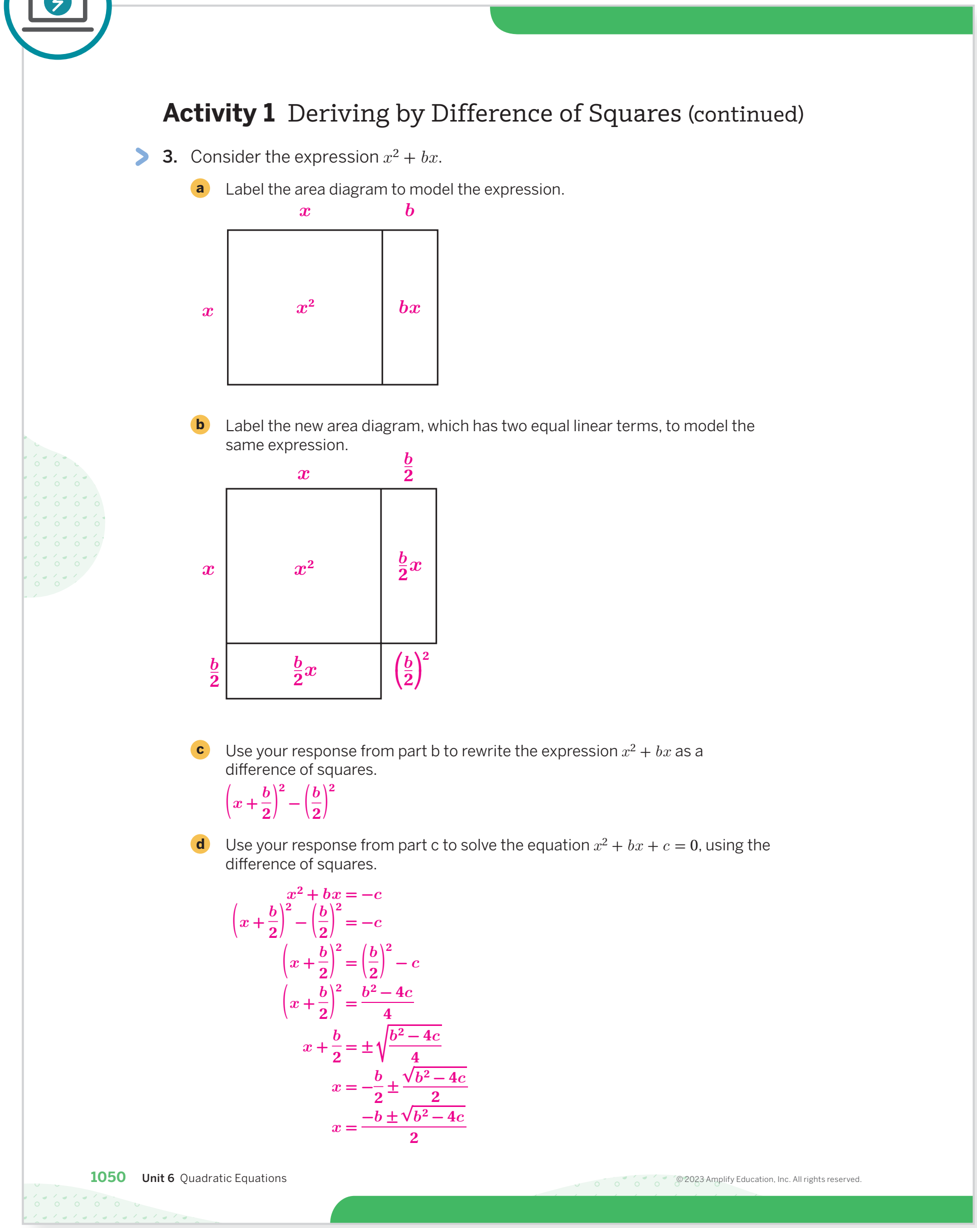
English Learners

Display one of the sample questions that students could use as a model for how to craft a question.

Small Groups | 15 min

Activity 1 Deriving By Difference of Squares (continued)

Students derive the quadratic formula using the difference of squares to make sense of the formula.



Activity 1 Deriving by Difference of Squares (continued)

3. Consider the expression $x^2 + bx$.

   a. Label the area diagram to model the expression.

   b. Label the new area diagram, which has two equal linear terms, to model the same expression.

   c. Use your response from part b to rewrite the expression $x^2 + bx$ as a difference of squares.

$$\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2$$

   d. Use your response from part c to solve the equation $x^2 + bx + c = 0$, using the difference of squares.

$$x^2+bx=-c$$
$$\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2=-c$$
$$\left(x+\frac{b}{2}\right)^2=\left(\frac{b}{2}\right)^2-c$$
$$\left(x+\frac{b}{2}\right)^2=\frac{b^2-4c}{4}$$
$$x+\frac{b}{2}=\pm\sqrt{\frac{b^2-4c}{4}}$$
$$x=-\frac{b}{2}\pm\frac{\sqrt{b^2-4c}}{2}$$
$$x=\frac{-b\pm\sqrt{b^2-4c}}{2}$$

3 Connect

Have groups of students share their work and solutions for Problem 3.

Highlight the connection between using the difference of squares and completing the square in Problem 3c. Emphasize that deriving the quadratic formula involves arithmetic calculations after applying the difference of squares.

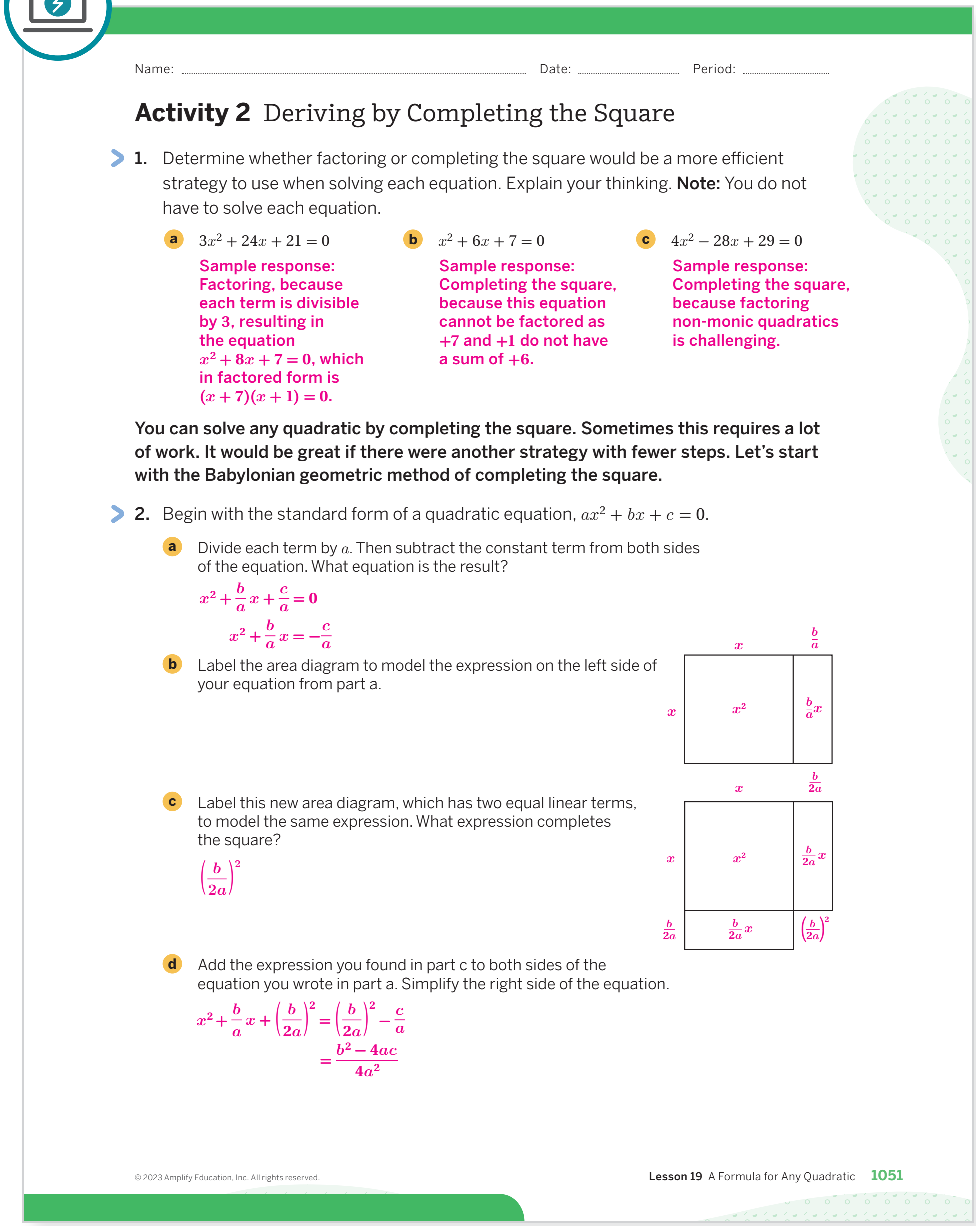
Ask:

- "What is different about the solution in Problem 3d and the quadratic formula? Why do you think that is?" The quadratic formula has an a term, but the solutions for Problem 3 do not have an a term. That is because the value of a in this equation is 1.
- "Describe what happened in each step in Problem 3d." First, c was subtracted from both sides of the equation and the left side was factored by completing the square. Then, $\left(\frac{b}{2}\right)^2$ was added to both sides of the equation. Next, $\left(\frac{b}{2}\right)^2$ was expanded and added to c using a common denominator. FInally, the square root was taken of both sides of the equation and simplified to get the final solution.

Small Groups | 20 min

Activity 2 Deriving by Completing the Square

Students derive the quadratic formula by completing the square to make sense of the formula.



Name: .... Date: .... Period: ....

Activity 2 Deriving by Completing the Square

1. Determine whether factoring or completing the square would be a more efficient strategy to use when solving each equation. Explain your thinking. **Note:** You do not have to solve each equation.

   a. $3x^2 + 24x + 21 = 0$

   Sample response: Factoring, because each term is divisible by 3, resulting in the equation $x^2 + 8x + 7 = 0$, which in factored form is $(x + 7)(x + 1) = 0$.

   b. $x^2 + 6x + 7 = 0$

   Sample response: Completing the square, because this equation cannot be factored as +7 and +1 do not have a sum of +6.

   c. $4x^2 - 28x + 29 = 0$

   Sample response: Completing the square, because factoring non-monic quadratics is challenging.

You can solve any quadratic by completing the square. Sometimes this requires a lot of work. It would be great if there were another strategy with fewer steps. Let's start with the Babylonian geometric method of completing the square.

2. Begin with the standard form of a quadratic equation, $ax^2 + bx + c = 0$.

   a. Divide each term by a. Then subtract the constant term from both sides of the equation. What equation is the result?

   $$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
   $$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

   b. Label the area diagram to model the expression on the left side of your equation from part a.

   c. Label this new area diagram, which has two equal linear terms, to model the same expression. What expression completes the square?

   $$\left(\frac{b}{2a}\right)^2$$

   d. Add the expression you found in part c to both sides of the equation you wrote in part a. Simplify the right side of the equation.

   $$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$
   $$= \frac{b^2 - 4ac}{4a^2}$$

1 Launch

Display Problem 1. First provide individual think-time, have groups discuss, and share as a whole class. Then allow students to work together on the remaining problems.

2 Monitor

Help students get started by displaying the Anchor Chart PDFs, *Solving Monic Quadratic Equations by Factoring*, *Solving Non-monic Quadratic Equations by Factoring*, and *Completing the Square*.

Look for points of confusion:

- **Struggling to label the area diagrams in Problems 2b and 2c.** Have students refer back to the area diagrams in Activity 2.
- **Making arithmetic errors.** Prompt students to complete the problem, then compare their work with group members to determine any error(s).
- **Having difficulty substituting the values into the quadratic formula in Problem 3.** Have students use color coding to distinguish the values for a, b, and c.

Look for productive strategies:

- Using the structure of the equation to determine the best strategy for solving.
- Using a table, annotations, or color to help make sense of the problems.
- Annotating, organizing, or highlighting the steps from the previous activity to generalize with variables.

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Provide students with their own copy of the following Anchor Chart PDFs that they can use as a reference and mark their own notes.

- *Solving Monic Quadratic Equations by Factoring*
- *Solving Non-monic Quadratic Equations by Factoring*
- *Completing the Square*

Accessibility: *Guide Processing and Visualization*

As students complete Problem 2e, consider providing a two-column organizer for them to record their steps and reasons for each step. Alternatively, provide them with a worked solution with several values missing and ask students to complete the missing values.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have groups meet with 1–2 other groups to share their responses. Encourage reviewers to ask clarifying questions such as:

- "Why is 28 substituted in the numerator before the square root and not −28?"
- "Why are there two 4s inside the square root?"

Have students revise their responses, as needed.

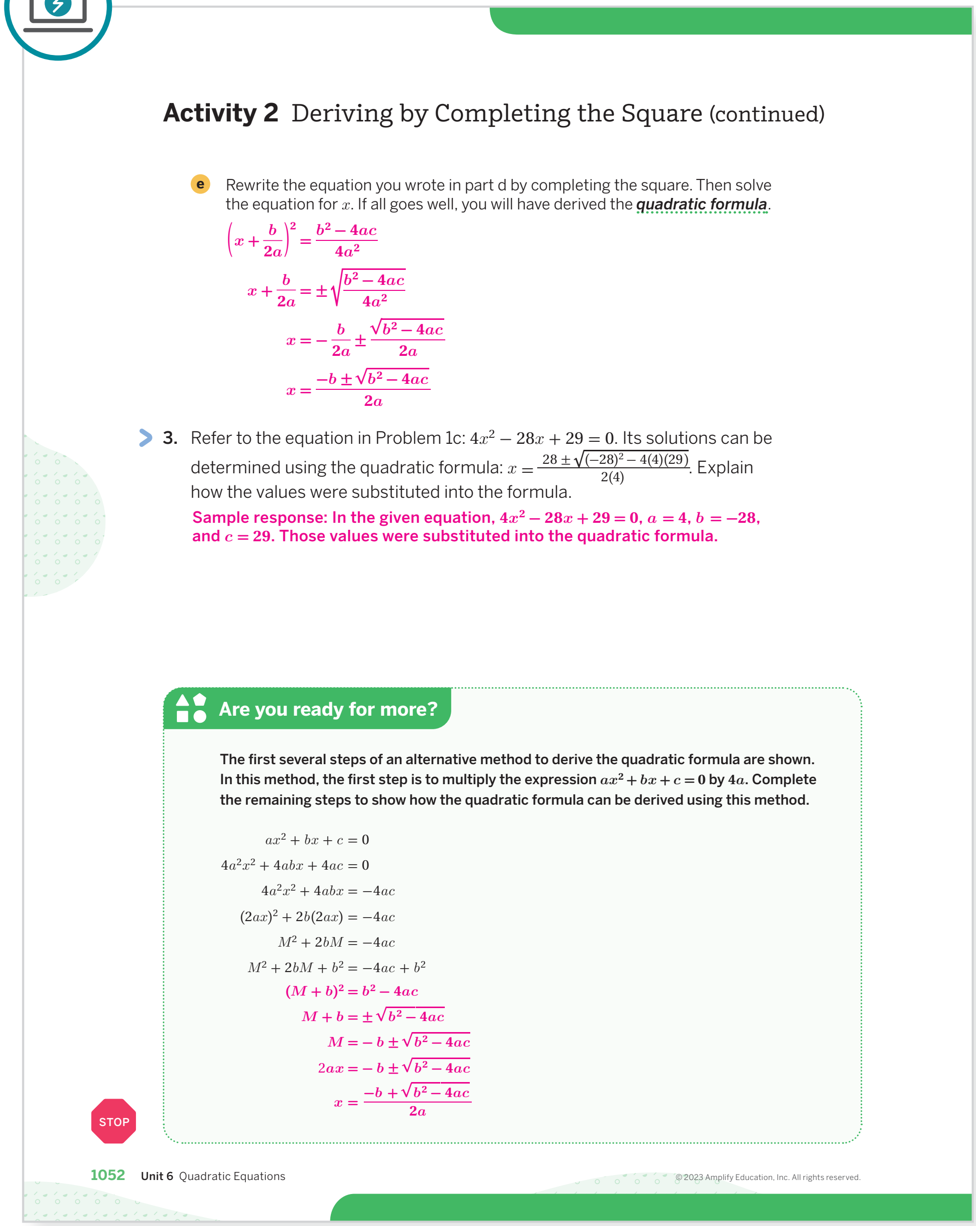
English Learners

Display the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*, to support students as they review each other's work.

Small Groups | 20 min

Activity 2 Deriving by Completing the Square (continued)

Students derive the quadratic formula by completing the square to make sense of the formula.



Activity 2 Deriving by Completing the Square (continued)

e Rewrite the equation you wrote in part d by completing the square. Then solve the equation for x. If all goes well, you will have derived the ***quadratic formula***.

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

3. Refer to the equation in Problem 1c: $4x^2-28x+29=0$. Its solutions can be determined using the quadratic formula: $x=\frac{28\pm\sqrt{(-28)^2-4(4)(29)}}{2(4)}$. Explain how the values were substituted into the formula.

Sample response: In the given equation, $4x^2-28x+29=0$, $a=4$, $b=-28$, and $c=29$. Those values were substituted into the quadratic formula.

Are you ready for more?

The first several steps of an alternative method to derive the quadratic formula are shown. In this method, the first step is to multiply the expression $ax^2+bx+c=0$ by $4a$. Complete the remaining steps to show how the quadratic formula can be derived using this method.

$$ax^2+bx+c=0$$

$$4a^2x^2+4abx+4ac=0$$

$$4a^2x^2+4abx=-4ac$$

$$(2ax)^2+2b(2ax)=-4ac$$

$$M^2+2bM=-4ac$$

$$M^2+2bM+b^2=-4ac+b^2$$

$$(M+b)^2=b^2-4ac$$

$$M+b=\pm\sqrt{b^2-4ac}$$

$$M=-b\pm\sqrt{b^2-4ac}$$

$$2ax=-b\pm\sqrt{b^2-4ac}$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

STOP

1052 **Unit 6** Quadratic Equations

3 Connect

Have groups of students share their work and solutions for Problems 2 and 3.

Highlight the connections between the values in the solution and the quadratic formula, $-b$, b^2, $4ac$, and $2a$. Emphasize that the quadratic formula essentially captures the steps for completing the square in one expression. Every time students solve a quadratic equation by completing the square, they are essentially using the quadratic formula, but in a less condensed way. State that because completing the square works for solving all quadratic equations, so does the quadratic formula.

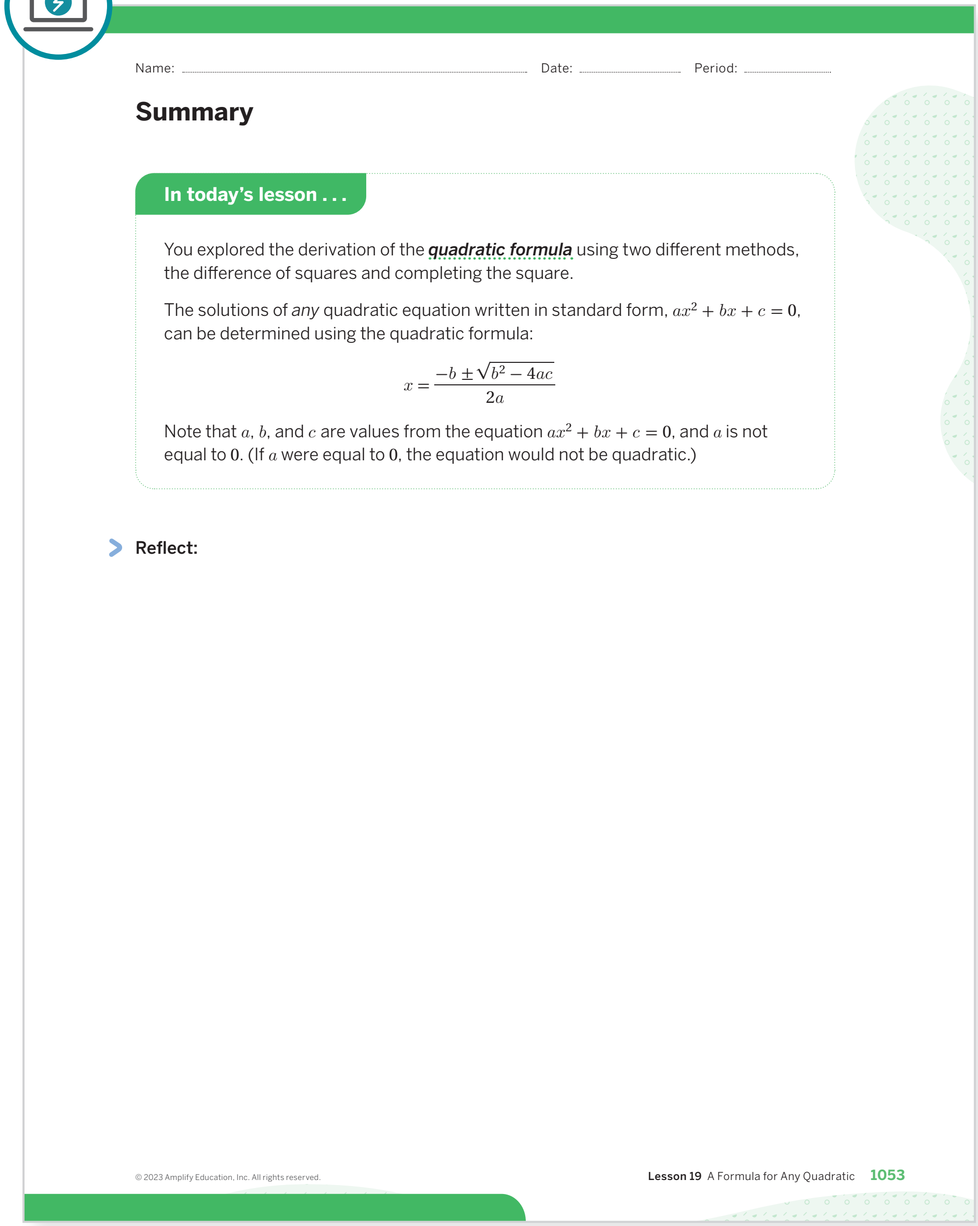
Ask:

- "What strategies to solve quadratic equations can you now use?" Factoring, graphing, completing the square, and the quadratic formula.
- "When do you think the quadratic formula would be better to use? Why?" When completing the square is too challenging, factoring is not possible, and graphing does not provide an exact solution.

Whole Class | 5 min

Summary

Review and synthesize how the quadratic formula is derived from using the difference of squares and completing the square strategies.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored the derivation of the ***quadratic formula*** using two different methods, the difference of squares and completing the square.

The solutions of *any* quadratic equation written in standard form, $ax^2 + bx + c = 0$, can be determined using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note that a, b, and c are values from the equation $ax^2 + bx + c = 0$, and a is not equal to 0. (If a were equal to 0, the equation would not be quadratic.)

Reflect:

Synthesize

Display the quadratic formula.

Highlight that the quadratic formula is not an isolated strategy that mysteriously produces solutions to quadratic equations. It derives from completing the square and the difference of squares.

Formalize vocabulary: *quadratic formula*

Ask:

- "A classmate who is absent today is not sure where the quadratic formula came from. What would you say to help them understand the quadratic formula and its use?" Sample response: The quadratic formula helps solve for quadratic equations. It can be derived using the difference of squares or by completing the square.
- "If the formula is connected to the steps of completing the square, why not just complete the square when you need to solve equations?" Sample response: Solving by completing the square can be challenging when the coefficient of the squared variable term is not 1, or when the coefficients are fractions.
- "Why do you think it is helpful to have a formula, even if it involves quite a few operations?" Sample response: The quadratic formula is helpful because it gives the exact solution every time.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why is the quadratic formula a useful strategy for solving quadratic equations?"

Math Language Development

MLR2: Collect and Display

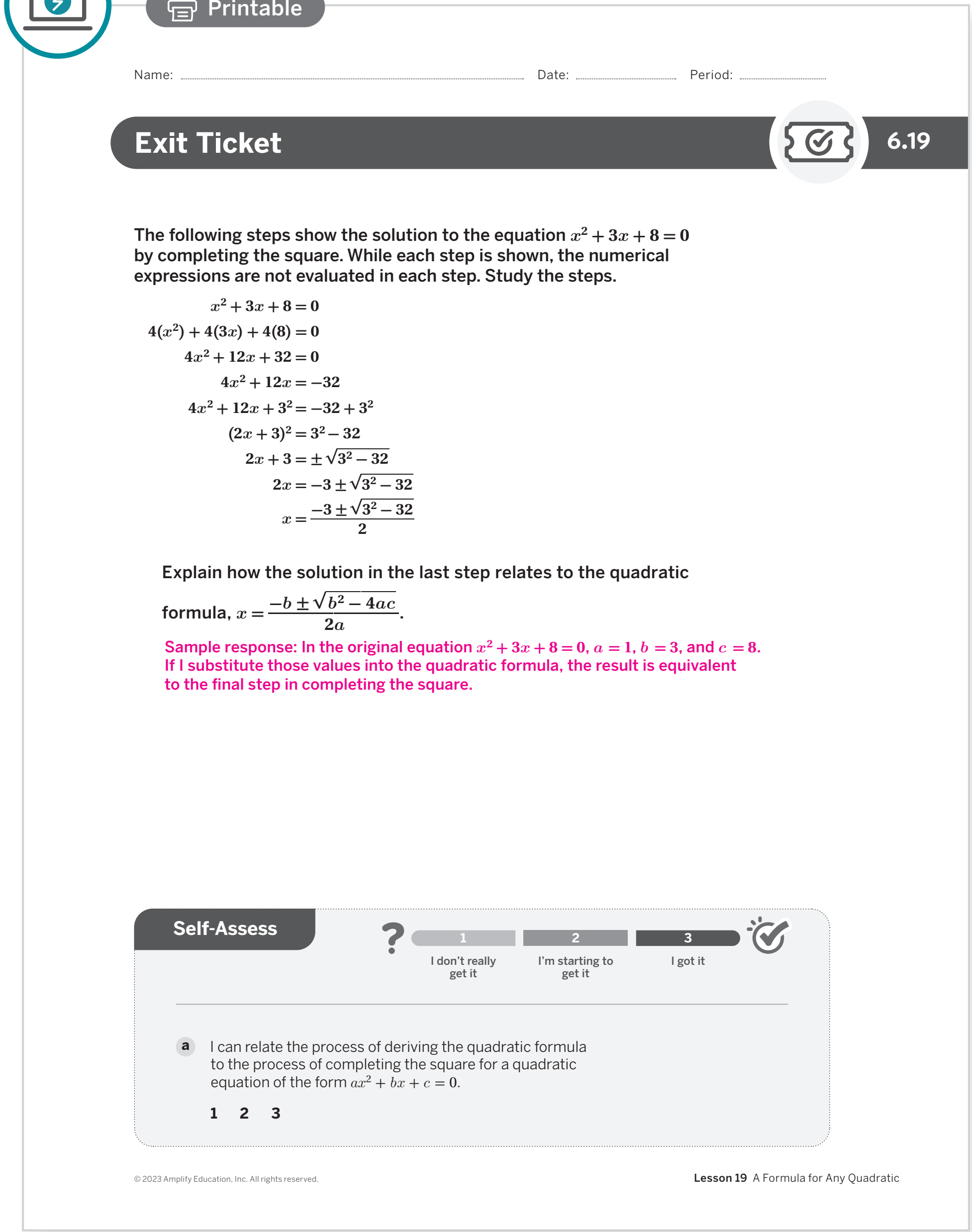
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *quadratic formula* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by relating the completing the square strategy to the quadratic formula.

Printable



Name: ________________ Date: __________ Period: __________

Exit Ticket

6.19

The following steps show the solution to the equation $x^2 + 3x + 8 = 0$ by completing the square. While each step is shown, the numerical expressions are not evaluated in each step. Study the steps.

$$\begin{aligned} x^2 + 3x + 8 &= 0 \\ 4(x^2) + 4(3x) + 4(8) &= 0 \\ 4x^2 + 12x + 32 &= 0 \\ 4x^2 + 12x &= -32 \\ 4x^2 + 12x + 3^2 &= -32 + 3^2 \\ (2x + 3)^2 &= 3^2 - 32 \\ 2x + 3 &= \pm\sqrt{3^2 - 32} \\ 2x &= -3 \pm \sqrt{3^2 - 32} \\ x &= \frac{-3 \pm \sqrt{3^2 - 32}}{2} \end{aligned}$$

Explain how the solution in the last step relates to the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Sample response: In the original equation $x^2 + 3x + 8 = 0$, $a = 1$, $b = 3$, and $c = 8$. If I substitute those values into the quadratic formula, the result is equivalent to the final step in completing the square.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can relate the process of deriving the quadratic formula to the process of completing the square for a quadratic equation of the form $ax^2 + bx + c = 0$.

1 2 3

Lesson 19 A Formula for Any Quadratic

Success looks like . . .

- **Language Goal:** Explaining the steps used to derive the quadratic formula. **(Writing)**
- **Language Goal:** Explaining how the solutions obtained by completing the square are expressed by the quadratic formula. **(Speaking and Listening, Writing)**
- **Goal:** Demonstrating that the quadratic formula can be derived by generalizing the process of completing the square.
  - » Demonstrating how completing the square leads to a solution in the form of the quadratic formula.

Suggested next steps

If students do not relate the values of a, b, and c, consider:

- Reviewing how the quadratic formula is derived from Activity 2.
- Assigning Practice Problem 1.
- Asking, "What are the values of a, b, and c for the equation $x^2 + 3x + 8 = 0$? How are these used in the quadratic formula?" $a = 1$, $b = 3$, and $c = 8$. These values are substituted into the quadratic formula to solve the quadratic equation.

Professional Learning

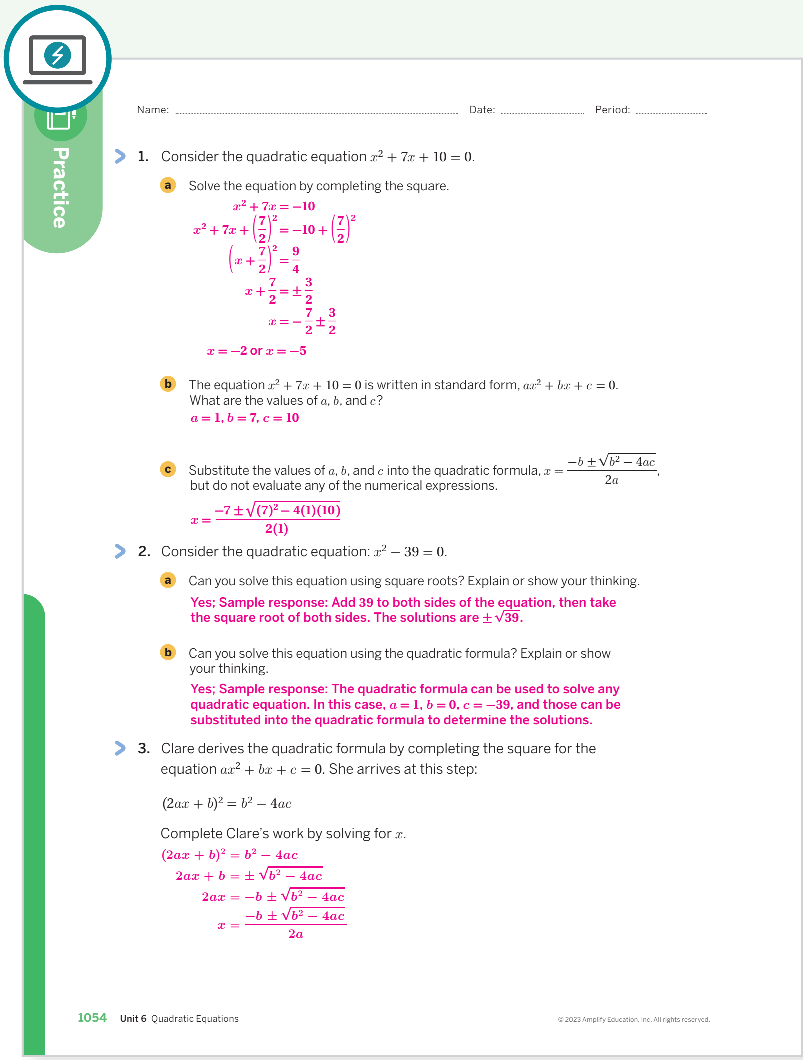
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What did students find frustrating about deriving the quadratic formula in Activity 1? What about in Activity 2? What helped them work through this frustration?
- The instructional goal for this lesson was to draw connections between the quadratic formula and the process used to complete the square. How well did students accomplish this? What did you specifically do to help students see this connection?

Practice

Independent



Practice

Name: ………… Date: ………… Period: …………

1. Consider the quadratic equation $x^2 + 7x + 10 = 0$.

   a Solve the equation by completing the square.

$$x^2 + 7x = -10$$
$$x^2 + 7x + \left(\frac{7}{2}\right)^2 = -10 + \left(\frac{7}{2}\right)^2$$
$$\left(x + \frac{7}{2}\right)^2 = \frac{9}{4}$$
$$x + \frac{7}{2} = \pm\frac{3}{2}$$
$$x = -\frac{7}{2} \pm \frac{3}{2}$$
$$x = -2 \text{ or } x = -5$$

   b The equation $x^2 + 7x + 10 = 0$ is written in standard form, $ax^2 + bx + c = 0$. What are the values of a, b, and c?

   $a = 1, b = 7, c = 10$

   c Substitute the values of a, b, and c into the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, but do not evaluate any of the numerical expressions.

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(10)}}{2(1)}$$

2. Consider the quadratic equation: $x^2 - 39 = 0$.

   a Can you solve this equation using square roots? Explain or show your thinking.

   Yes; Sample response: Add 39 to both sides of the equation, then take the square root of both sides. The solutions are $\pm\sqrt{39}$.

   b Can you solve this equation using the quadratic formula? Explain or show your thinking.

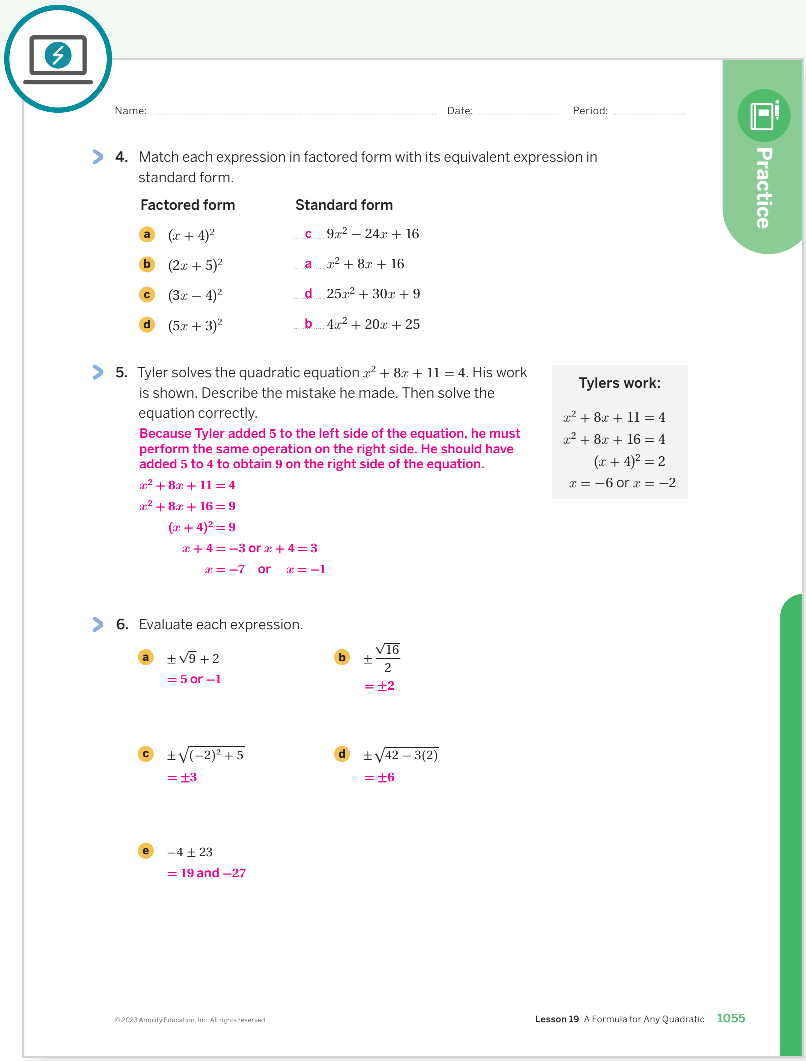
   Yes; Sample response: The quadratic formula can be used to solve any quadratic equation. In this case, $a = 1$, $b = 0$, $c = -39$, and those can be substituted into the quadratic formula to determine the solutions.

3. Clare derives the quadratic formula by completing the square for the equation $ax^2 + bx + c = 0$. She arrives at this step:

$$(2ax + b)^2 = b^2 - 4ac$$

   Complete Clare's work by solving for x.

$$(2ax + b)^2 = b^2 - 4ac$$
$$2ax + b = \pm\sqrt{b^2 - 4ac}$$
$$2ax = -b \pm \sqrt{b^2 - 4ac}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Practice

Name: ………… Date: ………… Period: …………

4. Match each expression in factored form with its equivalent expression in standard form.

Factored form	Standard form
a $(x + 4)^2$	c $9x^2 - 24x + 16$
b $(2x + 5)^2$	a $x^2 + 8x + 16$
c $(3x - 4)^2$	d $25x^2 + 30x + 9$
d $(5x + 3)^2$	b $4x^2 + 20x + 25$

5. Tyler solves the quadratic equation $x^2 + 8x + 11 = 4$. His work is shown. Describe the mistake he made. Then solve the equation correctly.

   Tylers work:

$$x^2 + 8x + 11 = 4$$
$$x^2 + 8x + 16 = 4$$
$$(x + 4)^2 = 2$$
$$x = -6 \text{ or } x = -2$$

   Because Tyler added 5 to the left side of the equation, he must perform the same operation on the right side. He should have added 5 to 4 to obtain 9 on the right side of the equation.

$$x^2 + 8x + 11 = 4$$
$$x^2 + 8x + 16 = 9$$
$$(x + 4)^2 = 9$$
$$x + 4 = -3 \text{ or } x + 4 = 3$$
$$x = -7 \quad \text{or} \quad x = -1$$

6. Evaluate each expression.

   a $\pm\sqrt{9} + 2$

   $= 5$ or -1

   b $\pm\frac{\sqrt{16}}{2}$

   $= \pm 2$

   c $\pm\sqrt{(-2)^2 + 5}$

   $= \pm 3$

   d $\pm\sqrt{42 - 3(2)}$

   $= \pm 6$

   e -4 ± 23

   $= 19$ and -27

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 11	2
	5	Unit 6 Lesson 12	3
Formative ⚡	6	Unit 6 Lesson 20	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 20**

The Quadratic Formula

Let’s put the quadratic formula to work.

Focus

Goal

1. Use the quadratic formula to solve quadratic equations of the form $ax^2 + bx + c = 0$.

Coherence

Today

Students use the quadratic formula and verify that it produces the same solutions as those found using other strategies. They solve quadratic equations using different methods and discuss the efficiency of certain strategies for certain equations.

Previously

In Lesson 19, students derived the quadratic formula by completing the square for the equation $ax^2 + bx + c = 0$.

Coming Soon

In Lesson 21, students will use the quadratic formula to solve problems that they did not previously have the algebraic tools to solve.

Rigor

- Students strengthen their **procedural fluency** of solving quadratic equations using the quadratic formula.
- Students develop **fluency** in determining the best method for solving a given quadratic equation.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	12 min	12 min	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one set per group

Anchor Chart PDFs:

  - *Sentence Stems, Which One Doesn't Belong?*
  - *Solving Monic Quadratic Equations by Factoring*
  - *Solving Non-Monic Quadratic Equations by Factoring*
  - *Completing the Square*
  - *The Order of Operations*
  - *The Quadratic Formula*
- scientific calculators

Math Language Development

Review words

- *completing the square*
- *factor*
- *quadratic formula*
- *radicand*

Amps | Featured Activity

Activity 1

See Student Thinking

Students show their work using the quadratic formula, and their steps are available to you digitally, in real time.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel intimidated or frustrated as they explain their thinking, defend their strategies, and reason with group members in Activity 3. Ask students to communicate calmly, repeat themselves clearly, listen actively, and seek help when they feel stuck.

Modifications to Pacing

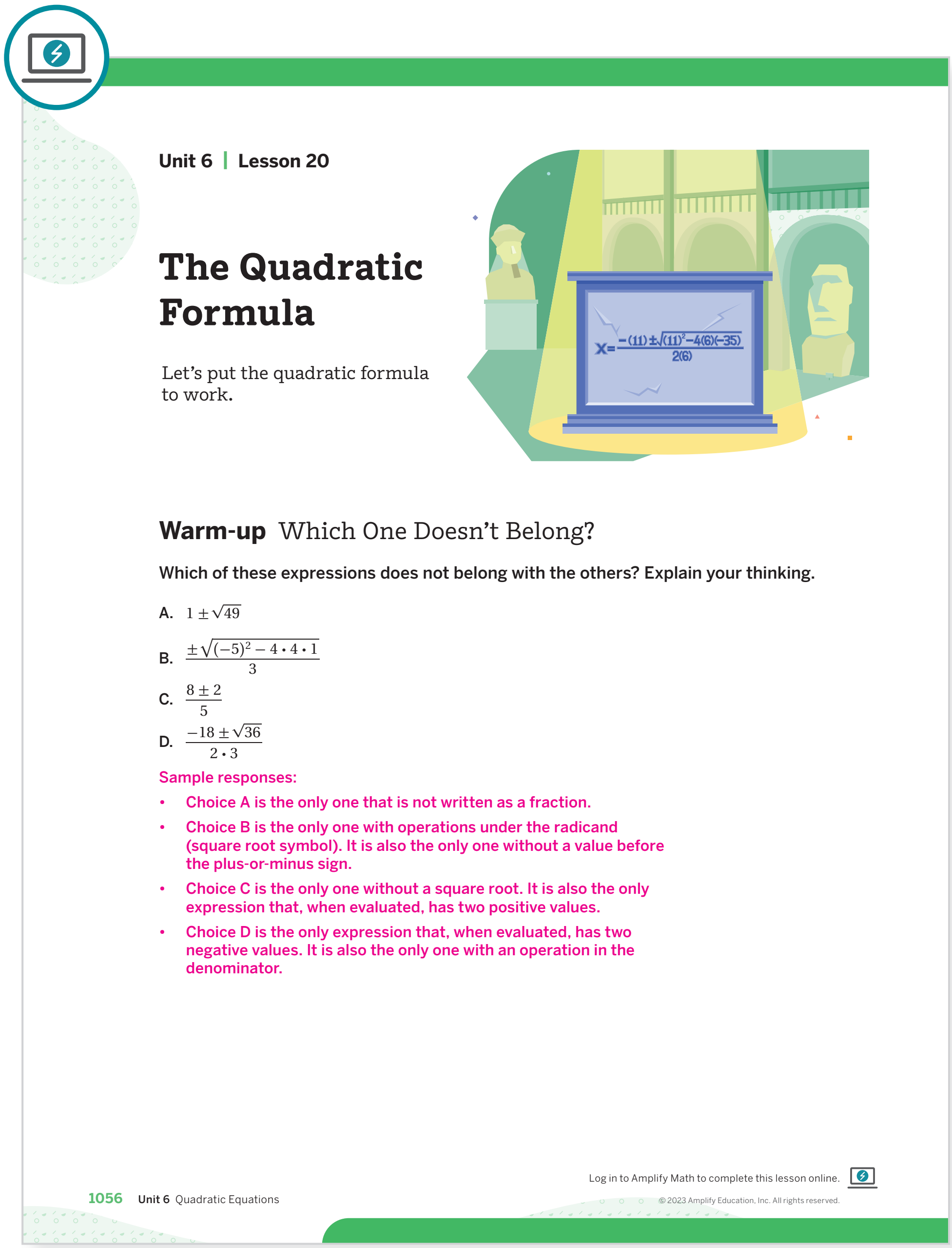
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have each student complete one part of Problem 2, then have students rotate work and check their group members' thinking.
- In **Activity 2**, Problem 2 may be omitted.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students consider different numerical expressions to practice evaluating expressions with rational square roots, fractions, and the plus-or-minus symbol.



Unit 6 | **Lesson 20**

The Quadratic Formula

Let's put the quadratic formula to work.

Warm-up Which One Doesn't Belong?

Which of these expressions does not belong with the others? Explain your thinking.

A. $1 \pm \sqrt{49}$

B. $\frac{\pm\sqrt{(-5)^2 - 4 \cdot 4 \cdot 1}}{3}$

C. $\frac{8 \pm 2}{5}$

D. $\frac{-18 \pm \sqrt{36}}{2 \cdot 3}$

Sample responses:

- **Choice A is the only one that is not written as a fraction.**
- **Choice B is the only one with operations under the radicand (square root symbol). It is also the only one without a value before the plus-or-minus sign.**
- **Choice C is the only one without a square root. It is also the only expression that, when evaluated, has two positive values.**
- **Choice D is the only expression that, when evaluated, has two negative values. It is also the only one with an operation in the denominator.**

Log in to Amplify Math to complete this lesson online.

1056 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the ***Which One Doesn't Belong?*** routine. Ask, "Which one doesn't belong? Why? Be prepared to share at least one reason why a choice doesn't belong." Provide think-time before students share their thinking with a partner. Provide access to a scientific calculator for numerical computations.

2 Monitor

Help students get started by having them compare two choices at a time.

Look for points of confusion:

- **Having difficulty evaluating Choice C.** Prompt students to decompose the fraction into a sum of two fractions.

Look for productive strategies:

- Annotating, highlighting, or color coding to note differences or similarities.
- Evaluating each expression to compare the solutions.

3 Connect

Have students share one reason why a choice doesn't belong. After each response, ask the class whether they agree or disagree.

Highlight that the plus-or-minus symbol indicates two solutions.

Ask, "In Choice C, why does the calculator show the incorrect answer if you do not put parentheses around the numerator when $8 + 2 \div 5$ is entered?" Sample response: Based on the order of operations, the calculator is dividing 2 by 5, then adding 8. I can use parentheses to enter this expression into the calculator as $(8 + 2) \div 5$.

Math Language Development

MLR8: Discussion Supports

Display or provide access to the Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?* To support students as they organize their thinking.

Power-up

To power up students' ability to evaluate expressions involving square roots and the plus-or-minus symbol, have students complete:

Recall that, when simplifying an expression, a square root acts as both a grouping symbol for the expression under the radical and as an exponent. Follow the steps to evaluate the expression $\pm\sqrt{(4^2 + 9)} + 1$.

a. Simplify the expression under the radical following the order of operations. $\pm\sqrt{(25)} + 1$

b. Evaluate the square root. $\pm 5 + 1$

c. Add 1 to the positive and negative values of the square root and write the values of the expression. $5 + 1 = 6$
$-5 + 1 = -4$

Use: Before Activity 1

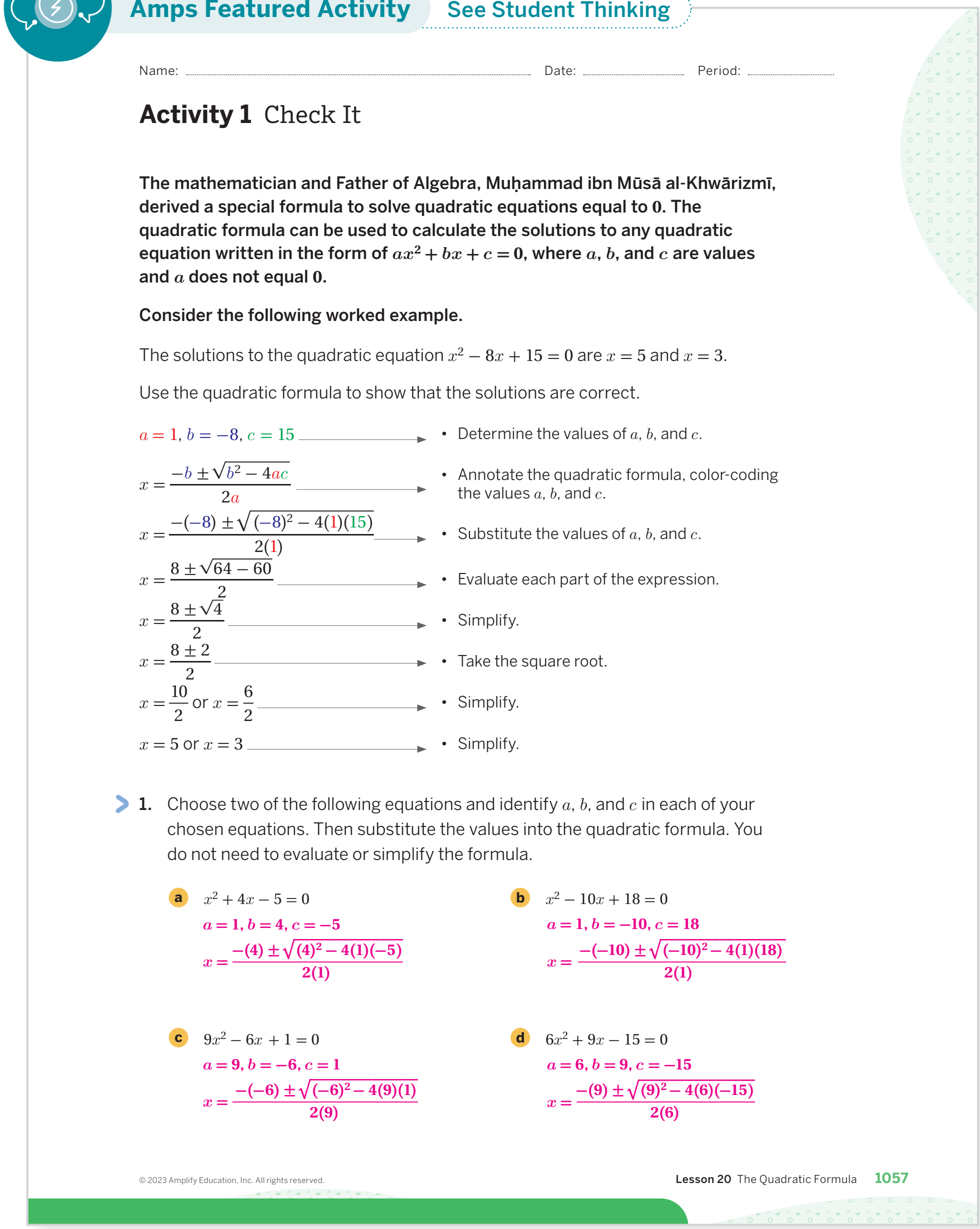
Informed by: Performance on Lesson 19, Practice Problem 6

Small Groups | 12 min

Activity 1 Check It

Students verify the solutions to quadratic equations to develop fluency using the quadratic formula.

Amps Featured Activity See Student Thinking



Name: ______ Date: ______ Period: ______

Activity 1 Check It

The mathematician and Father of Algebra, Muḥammad ibn Mūsā al-Khwārizmī, derived a special formula to solve quadratic equations equal to 0. The quadratic formula can be used to calculate the solutions to any quadratic equation written in the form of $ax^2 + bx + c = 0$, where a, b, and c are values and a does not equal 0.

Consider the following worked example.

The solutions to the quadratic equation $x^2 - 8x + 15 = 0$ are $x = 5$ and $x = 3$.

Use the quadratic formula to show that the solutions are correct.

$a = 1, b = -8, c = 15$ $\longrightarrow$	Determine the values of a, b, and c.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\longrightarrow$	Annotate the quadratic formula, color-coding the values a, b, and c.
$x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$ $\longrightarrow$	Substitute the values of a, b, and c.
$x = \dfrac{8 \pm \sqrt{64 - 60}}{2}$ $\longrightarrow$	Evaluate each part of the expression.
$x = \dfrac{8 \pm \sqrt{4}}{2}$ $\longrightarrow$	Simplify.
$x = \dfrac{8 \pm 2}{2}$ $\longrightarrow$	Take the square root.
$x = \dfrac{10}{2}$ or $x = \dfrac{6}{2}$ $\longrightarrow$	Simplify.
$x = 5$ or $x = 3$ $\longrightarrow$	Simplify.

1. Choose two of the following equations and identify a, b, and c in each of your chosen equations. Then substitute the values into the quadratic formula. You do not need to evaluate or simplify the formula.

a $x^2 + 4x - 5 = 0$

$a = 1, b = 4, c = -5$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-5)}}{2(1)}$$

b $x^2 - 10x + 18 = 0$

$a = 1, b = -10, c = 18$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(18)}}{2(1)}$$

c $9x^2 - 6x + 1 = 0$

$a = 9, b = -6, c = 1$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$

d $6x^2 + 9x - 15 = 0$

$a = 6, b = 9, c = -15$

$$x = \frac{-(9) \pm \sqrt{(9)^2 - 4(6)(-15)}}{2(6)}$$

1 Launch

Provide time for students to read and annotate the worked example. Have students explain the process of using the quadratic formula in their own words to a group member. As a class, highlight the steps and possible errors. Then, release students to complete Problems 1 and 2 individually, and compare their work and solutions in groups. Provide access to a scientific calculator for numerical computations.

2 Monitor

Help students get started by prompting them to explain what happened in each step.

Look for points of confusion:

- **Squaring negative numbers incorrectly.** Remind students that the product of two negative numbers results in a positive number. Demonstrate the importance of using parentheses in the calculator.
- **Having difficulty simplifying fractions.** Have students decompose each fraction into the sum and difference of two fractions as a step, even if it cannot be simplified.
- **Struggling to evaluate the radicand.** Prompt students to ignore the radical while they evaluate the radicand, applying the square root as one of the last steps.

Look for productive strategies:

- Annotating or highlighting to stay organized.
- Noticing and applying shortcuts and patterns to evaluate parts of the expression.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose two of the four problems to complete in Problem 1, and two of the four problems to complete in Problem 2. Different group members should choose different problems so that at least one group member is solving each of the problems. Allowing students the power of choice can increase their engagement and ownership of the task.

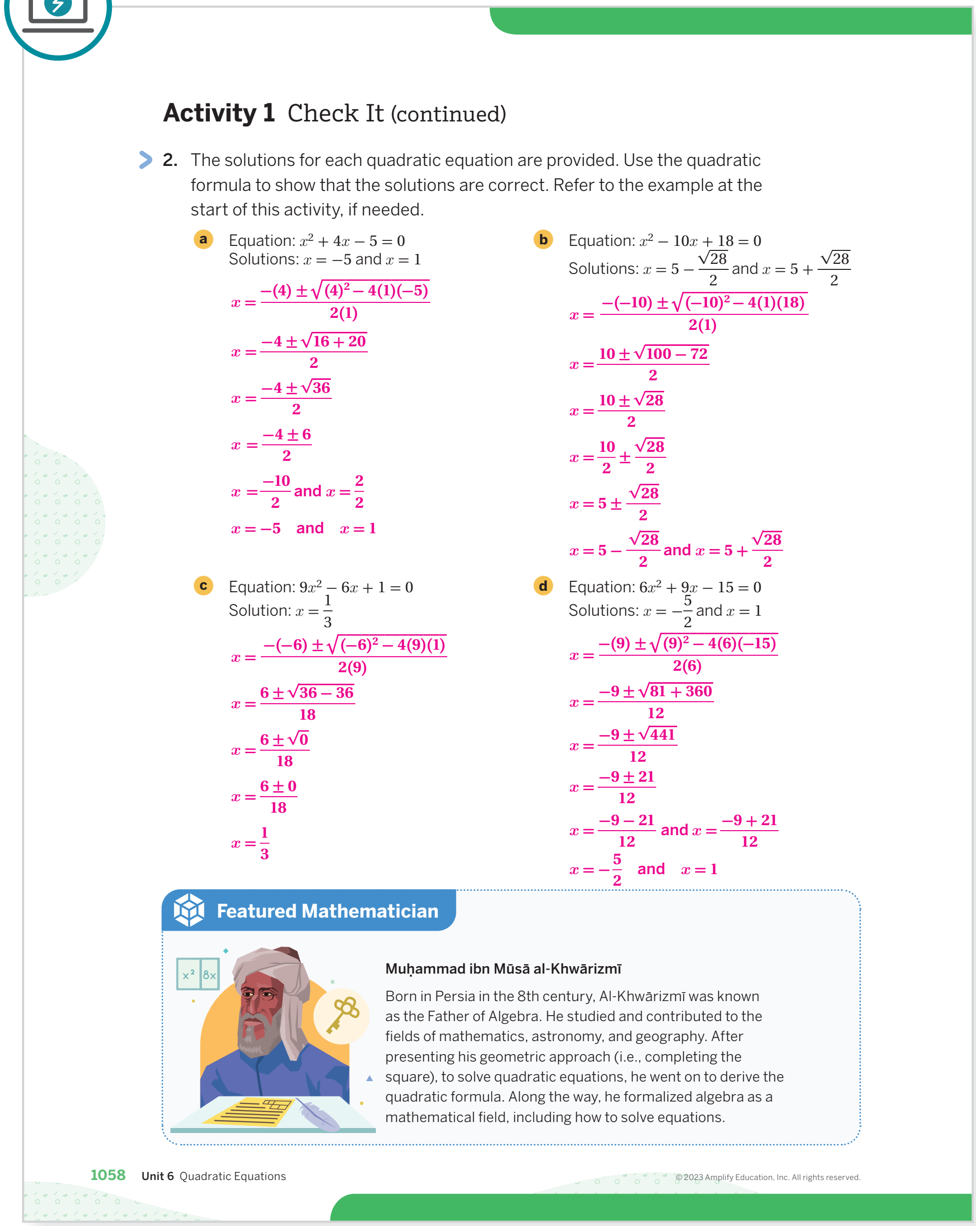
Accessibility: *Guide Processing and Visualization*

Provide students with the Anchor Chart PDF, *The Quadratic Formula*, for them to use as a reference. Provide access to colored pencils and suggest they color code the values of a, b, and c in each equation and as they substitute those values into the formula.

Small Groups | 12 min

Activity 1 Check It (continued)

Students verify the solutions to quadratic equations to develop fluency using the quadratic formula.



Activity 1 Check It (continued)

2. The solutions for each quadratic equation are provided. Use the quadratic formula to show that the solutions are correct. Refer to the example at the start of this activity, if needed.

a. Equation: $x^2 + 4x - 5 = 0$
Solutions: $x = -5$ and $x = 1$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-5)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{16 + 20}}{2}$$
$$x = \frac{-4 \pm \sqrt{36}}{2}$$
$$x = \frac{-4 \pm 6}{2}$$
$$x = \frac{-10}{2} \text{ and } x = \frac{2}{2}$$
$$x = -5 \quad \text{and} \quad x = 1$$

b. Equation: $x^2 - 10x + 18 = 0$
Solutions: $x = 5 - \frac{\sqrt{28}}{2}$ and $x = 5 + \frac{\sqrt{28}}{2}$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(18)}}{2(1)}$$
$$x = \frac{10 \pm \sqrt{100 - 72}}{2}$$
$$x = \frac{10 \pm \sqrt{28}}{2}$$
$$x = \frac{10}{2} \pm \frac{\sqrt{28}}{2}$$
$$x = 5 \pm \frac{\sqrt{28}}{2}$$
$$x = 5 - \frac{\sqrt{28}}{2} \text{ and } x = 5 + \frac{\sqrt{28}}{2}$$

c. Equation: $9x^2 - 6x + 1 = 0$
Solution: $x = \frac{1}{3}$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$
$$x = \frac{6 \pm \sqrt{36 - 36}}{18}$$
$$x = \frac{6 \pm \sqrt{0}}{18}$$
$$x = \frac{6 \pm 0}{18}$$
$$x = \frac{1}{3}$$

d. Equation: $6x^2 + 9x - 15 = 0$
Solutions: $x = -\frac{5}{2}$ and $x = 1$

$$x = \frac{-(9) \pm \sqrt{(9)^2 - 4(6)(-15)}}{2(6)}$$
$$x = \frac{-9 \pm \sqrt{81 + 360}}{12}$$
$$x = \frac{-9 \pm \sqrt{441}}{12}$$
$$x = \frac{-9 \pm 21}{12}$$
$$x = \frac{-9 - 21}{12} \text{ and } x = \frac{-9 + 21}{12}$$
$$x = -\frac{5}{2} \quad \text{and} \quad x = 1$$

Featured Mathematician

Muḥammad ibn Mūsā al-Khwārizmī

Born in Persia in the 8th century, Al-Khwārizmī was known as the Father of Algebra. He studied and contributed to the fields of mathematics, astronomy, and geography. After presenting his geometric approach (i.e., completing the square), to solve quadratic equations, he went on to derive the quadratic formula. Along the way, he formalized algebra as a mathematical field, including how to solve equations.

1058 Unit 6 Quadratic Equations

3 Connect

Have groups of students share their work and any challenges they experienced when using the quadratic formula.

Ask, "Because there are a lot of possible arithmetic errors, why is the quadratic formula still a worthwhile strategy?" Sample response: The quadratic formula is a useful strategy when the leading coefficient is not 1, and the linear term is not even. It also can be helpful to use if the expression is very complicated.

Highlight that the quadratic formula is a useful tool that always works, but is not always the quickest or most efficient due to the number of calculations involved which can lead to possible errors. When a problem cannot be factored, the quadratic formula may be an efficient strategy to use.

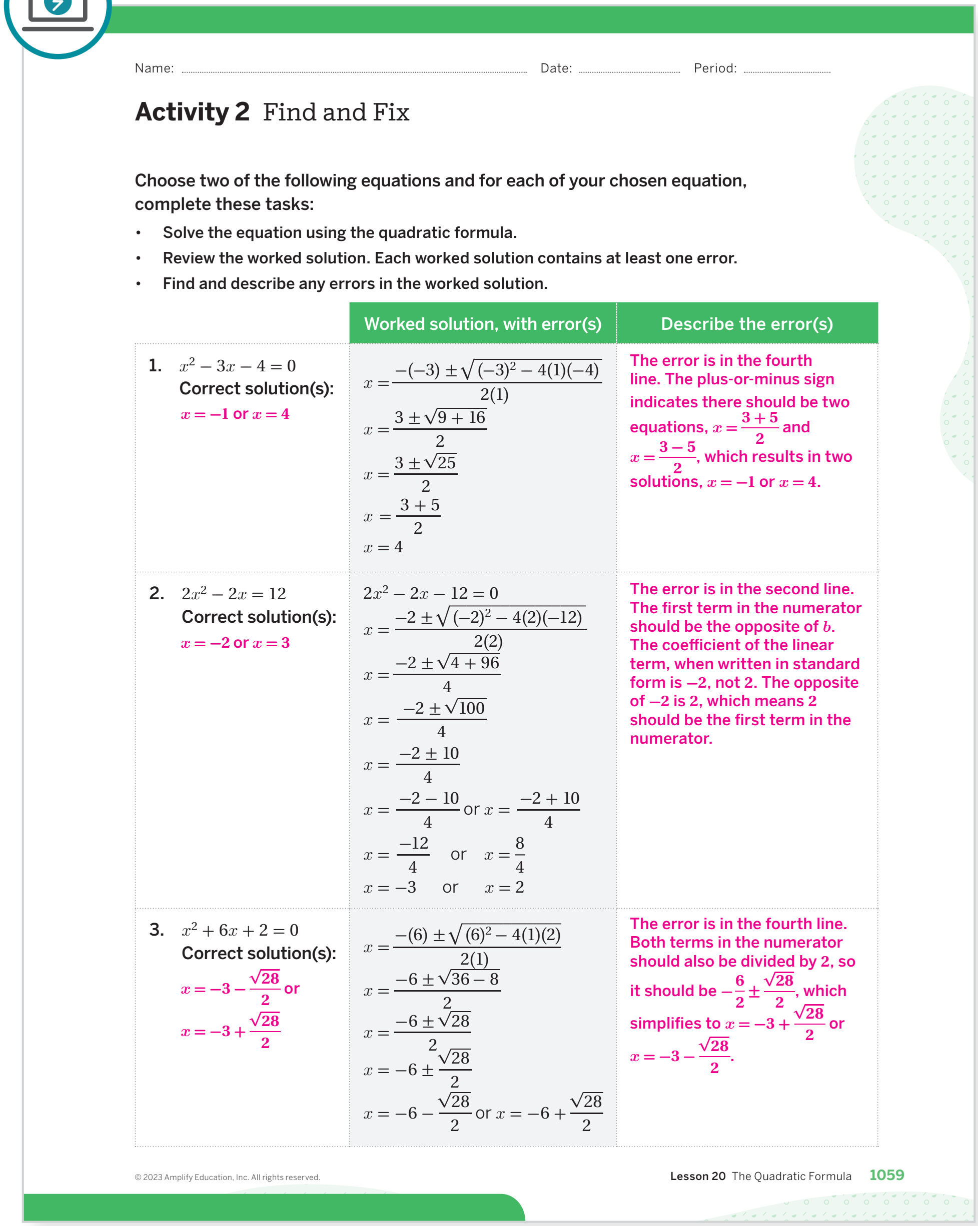
Featured Mathematician

Have students read about featured mathematician Muḥammad ibn Mūsā al-Khwārizmī who used a geometric approach (i.e., completing the square), to solve quadratic equations. He went on to derive the quadratic formula.

Small Groups | 12 min

Activity 2 Find and Fix

Students analyze worked examples of equations solved by the quadratic formula containing errors, to further develop their understanding of the strategy.



Name: ______ Date: ______ Period: ______

Activity 2 Find and Fix

Choose two of the following equations and for each of your chosen equation, complete these tasks:

- Solve the equation using the quadratic formula.
- Review the worked solution. Each worked solution contains at least one error.
- Find and describe any errors in the worked solution.

	Worked solution, with error(s)	Describe the error(s)
1. $x^2 - 3x - 4 = 0$ Correct solution(s): $x = -1$ or $x = 4$	$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2(1)}$ $x = \frac{3 \pm \sqrt{9 + 16}}{2}$ $x = \frac{3 \pm \sqrt{25}}{2}$ $x = \frac{3 + 5}{2}$ $x = 4$	The error is in the fourth line. The plus-or-minus sign indicates there should be two equations, $x = \frac{3+5}{2}$ and $x = \frac{3-5}{2}$, which results in two solutions, $x = -1$ or $x = 4$.
2. $2x^2 - 2x = 12$ Correct solution(s): $x = -2$ or $x = 3$	$2x^2 - 2x - 12 = 0$ $x = \frac{-2 \pm \sqrt{(-2)^2 - 4(2)(-12)}}{2(2)}$ $x = \frac{-2 \pm \sqrt{4 + 96}}{4}$ $x = \frac{-2 \pm \sqrt{100}}{4}$ $x = \frac{-2 \pm 10}{4}$ $x = \frac{-2 - 10}{4}$ or $x = \frac{-2 + 10}{4}$ $x = \frac{-12}{4}$ or $x = \frac{8}{4}$ $x = -3$ or $x = 2$	The error is in the second line. The first term in the numerator should be the opposite of b. The coefficient of the linear term, when written in standard form is −2, not 2. The opposite of −2 is 2, which means 2 should be the first term in the numerator.
3. $x^2 + 6x + 2 = 0$ Correct solution(s): $x = -3 - \frac{\sqrt{28}}{2}$ or $x = -3 + \frac{\sqrt{28}}{2}$	$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)}$ $x = \frac{-6 \pm \sqrt{36 - 8}}{2}$ $x = \frac{-6 \pm \sqrt{28}}{2}$ $x = -6 \pm \frac{\sqrt{28}}{2}$ $x = -6 - \frac{\sqrt{28}}{2}$ or $x = -6 + \frac{\sqrt{28}}{2}$	The error is in the fourth line. Both terms in the numerator should also be divided by 2, so it should be $-\frac{6}{2} \pm \frac{\sqrt{28}}{2}$, which simplifies to $x = -3 + \frac{\sqrt{28}}{2}$ or $x = -3 - \frac{\sqrt{28}}{2}$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 The Quadratic Formula 1059

1 Launch

Display the three equations. Conduct the ***Find and Fix*** routine. Have students solve their chosen equations independently, and then consult with group members to agree on the solution and examine the errors.

2 Monitor

Help students get started by prompting them to list and explain the steps for using the quadratic formula.

Look for points of confusion:

- **Struggling to identify the errors.** Have students compare their work, line by line, to the work provided, looking for differences.
- **Forgetting to write the opposite of b in Problem 2.** Have students compare their work to the example in Activity 1.

Look for productive strategies:

- Creating a checklist to compare their work against each worked problem.
- Using precise language when identifying and explaining the error(s).
- Marking the inconsistencies between their own work and the work provided.

3 Connect

Have groups of students share the errors they identified and proposed corrections.

Highlight the different types of errors and the ways to avoid them when solving quadratic equations using the quadratic formula.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the Anchor Chart PDF, *The Quadratic Formula*, for them to use as a reference. Provide access to colored pencils and suggest they color code the values of a, b, and c in each equation to show how those values were substituted into the formula in the worked solutions. This will help them identify the errors.

Math Language Development

MLR8: Discussion Supports

Before the Connect, provide students time to ensure that everyone in their group can explain the errors they identified and proposed corrections. Have groups rehearse what they will say before they share with the whole class.

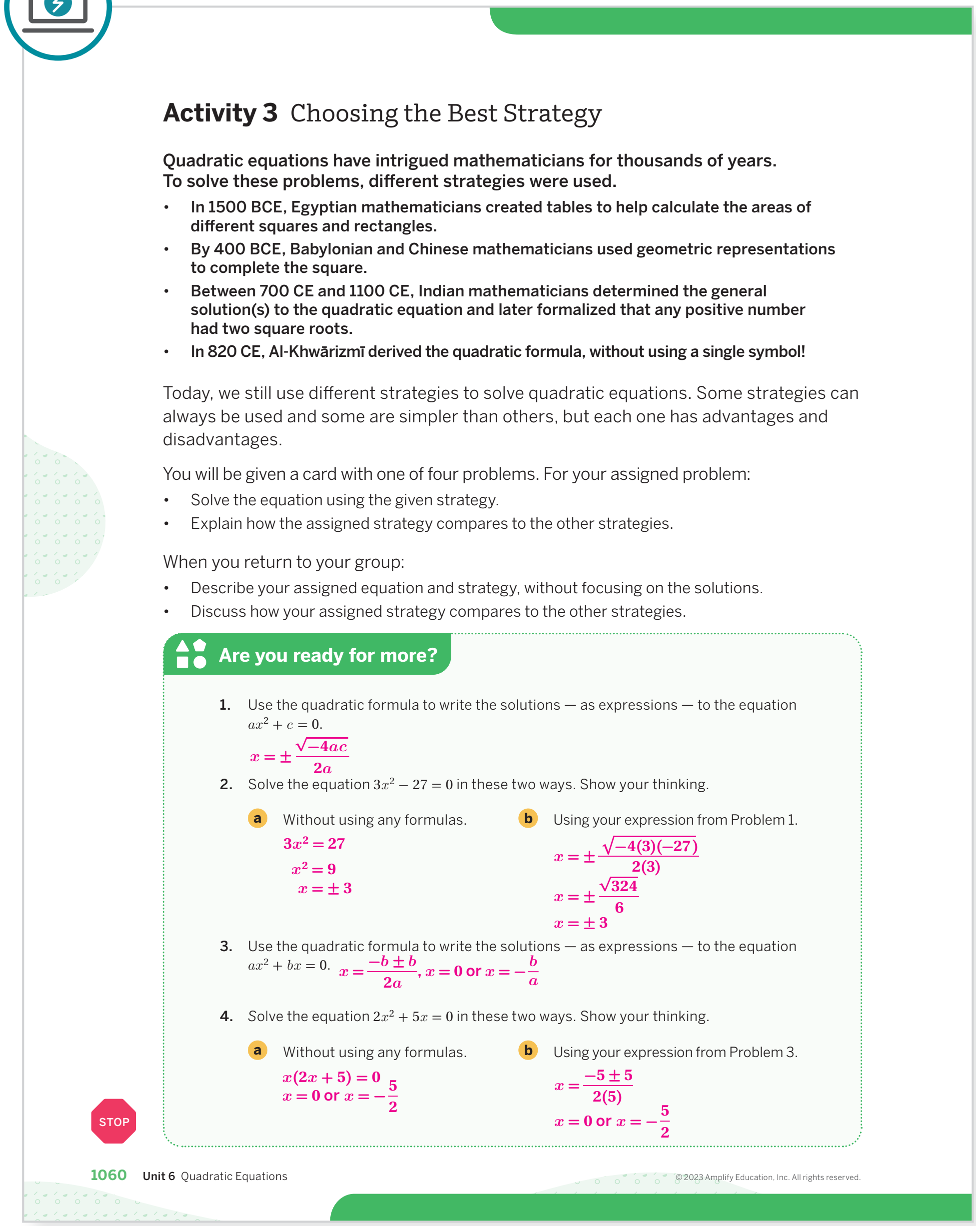
English Learners

As students critique the errors, display or provide access to the Anchor Chart PDF, *Sentence Stems, Critiquing*, to help them organize their thinking.

Small Groups | 12 min

Activity 3 Choosing the Best Strategy

Students compare strategies for solving quadratic equations to reason about when each strategy might be an efficient one to use.



Activity 3 Choosing the Best Strategy

Quadratic equations have intrigued mathematicians for thousands of years. To solve these problems, different strategies were used.

- **In 1500 BCE, Egyptian mathematicians created tables to help calculate the areas of different squares and rectangles.**
- **By 400 BCE, Babylonian and Chinese mathematicians used geometric representations to complete the square.**
- **Between 700 CE and 1100 CE, Indian mathematicians determined the general solution(s) to the quadratic equation and later formalized that any positive number had two square roots.**
- **In 820 CE, Al-Khwārizmī derived the quadratic formula, without using a single symbol!**

Today, we still use different strategies to solve quadratic equations. Some strategies can always be used and some are simpler than others, but each one has advantages and disadvantages.

You will be given a card with one of four problems. For your assigned problem:

- Solve the equation using the given strategy.
- Explain how the assigned strategy compares to the other strategies.

When you return to your group:

- Describe your assigned equation and strategy, without focusing on the solutions.
- Discuss how your assigned strategy compares to the other strategies.

Are you ready for more?

1. Use the quadratic formula to write the solutions — as expressions — to the equation $ax^2 + c = 0$.
   $x = \pm\frac{\sqrt{-4ac}}{2a}$
2. Solve the equation $3x^2 - 27 = 0$ in these two ways. Show your thinking.
   - a Without using any formulas.
     $3x^2 = 27$
     $x^2 = 9$
     $x = \pm 3$
   - b Using your expression from Problem 1.
     $x = \pm\frac{\sqrt{-4(3)(-27)}}{2(3)}$
     $x = \pm\frac{\sqrt{324}}{6}$
     $x = \pm 3$
3. Use the quadratic formula to write the solutions — as expressions — to the equation $ax^2 + bx = 0$. $x = \frac{-b \pm b}{2a}$, $x = 0$ or $x = -\frac{b}{a}$
4. Solve the equation $2x^2 + 5x = 0$ in these two ways. Show your thinking.
   - a Without using any formulas.
     $x(2x + 5) = 0$
     $x = 0$ or $x = -\frac{5}{2}$
   - b Using your expression from Problem 3.
     $x = \frac{-5 \pm 5}{2(5)}$
     $x = 0$ or $x = -\frac{5}{2}$

STOP

1060 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Use the ***Jigsaw*** routine. Remind students to focus on strategies rather than solutions during group discussions.

Provide each group with a set of pre-cut cards from the Activity 3 PDF. Assign each student in the group a different problem. Provide 5 minutes to complete individually, then have students come back together and discuss the equations and strategies for 5 minutes.

2 Monitor

Help students get started by asking them what they remember about their assigned strategy.

Look for points of confusion:

- **Using incorrect coefficients in the quadratic formula in Problem A.** Remind students the equation must be set equal to 0 before determining the values of a, b, and c.
- **Unable to complete the square in Problem D.** Provide fill in the blank support to help students recall the process of completing the square.

Look for productive strategies:

- Noticing the structure of the equation to determine the most efficient strategy.
- Using evidence and mathematical language to justify their thinking to group members.

3 Connect

Display the four equations and use the ***Poll the Class*** routine to determine which strategy would be most efficient to solve each equation.

Highlight that the quadratic formula is a useful tool that always works, but is not always the most efficient strategy. When a problem cannot be factored, the quadratic formula may be an efficient strategy to use.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

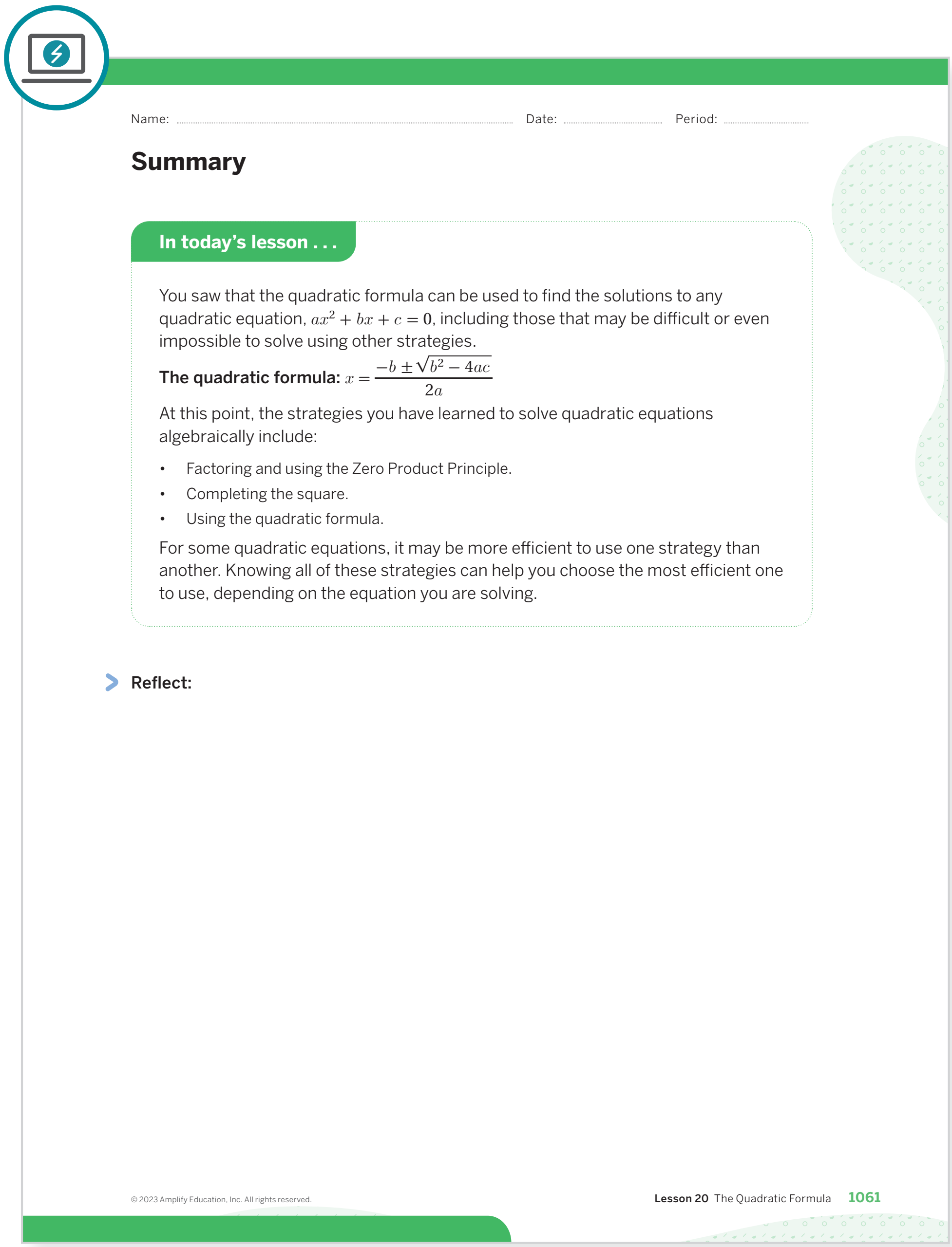
Provide students with their own copy of the following Anchor Chart PDFs that they can use as a reference and mark their own notes.

- *Solving Monic Quadratic Equations by Factoring*
- *Solving Non-Monic Quadratic Equations by Factoring*
- *Completing the Square*
- *The Quadratic Formula*

Whole Class | 5 min

Summary

Review and synthesize the strategies students have learned for solving quadratic equations, including the quadratic formula.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that the quadratic formula can be used to find the solutions to any quadratic equation, $ax^2 + bx + c = 0$, including those that may be difficult or even impossible to solve using other strategies.

The quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

At this point, the strategies you have learned to solve quadratic equations algebraically include:

- Factoring and using the Zero Product Principle.
- Completing the square.
- Using the quadratic formula.

For some quadratic equations, it may be more efficient to use one strategy than another. Knowing all of these strategies can help you choose the most efficient one to use, depending on the equation you are solving.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 The Quadratic Formula 1061

Synthesize

Display the equation $25x^2 - 50x + 16 = 0$.

Ask:

- "Which strategy would you prefer to use to solve the equation? Why?" Factoring, completing the square, or the quadratic formula.
- "Solve the equation using your preferred strategy." The solutions are $\frac{2}{5}$ and $\frac{8}{5}$.

Have students share their preferred strategy for solving and why.

Highlight that the quadratic formula can be used to solve any quadratic equation, but is not always the most efficient strategy.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

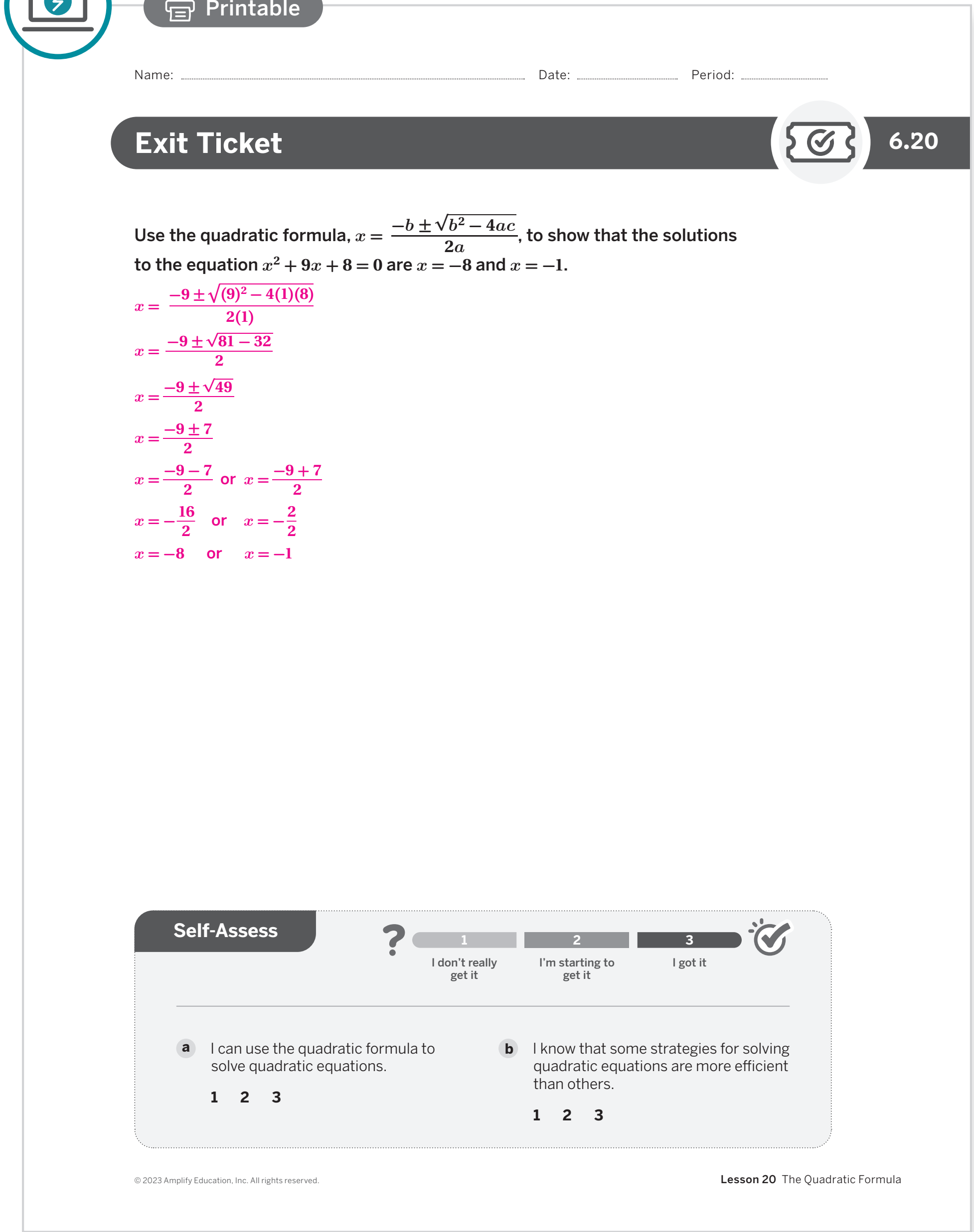
- "Even though it is possible to solve any quadratic equation using the quadratic formula, explain why it may not be the most efficient strategy for solving any given quadratic equation. When would you use the quadratic formula?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving a quadratic equation using the quadratic formula.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

6.20

Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to show that the solutions to the equation $x^2 + 9x + 8 = 0$ are $x = -8$ and $x = -1$.

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-9 \pm \sqrt{81 - 32}}{2}$$

$$x = \frac{-9 \pm \sqrt{49}}{2}$$

$$x = \frac{-9 \pm 7}{2}$$

$$x = \frac{-9 - 7}{2} \text{ or } x = \frac{-9 + 7}{2}$$

$$x = -\frac{16}{2} \quad \text{or} \quad x = -\frac{2}{2}$$

$$x = -8 \quad \text{or} \quad x = -1$$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use the quadratic formula to solve quadratic equations.

1 2 3

b I know that some strategies for solving quadratic equations are more efficient than others.

1 2 3

Lesson 20 The Quadratic Formula

Success looks like . . .

- **Goal:** Using the quadratic formula to solve quadratic equations of the form $ax^2 + bx + c = 0$.
  - » Using the quadratic formula to determine the solutions of the quadratic equation.

Suggested next steps

If students incorrectly substitute the values of a, b, and c into the quadratic formula, Consider:

- Reviewing Activity 1.

If students incorrectly evaluate the radicand, consider:

- Reviewing how to evaluate the expressions from the Warm-up.
- Assigning Practice Problem 6.
- Asking, "According to the order of operations, what must be evaluated first? Next?"

If students incorrectly apply the plus-or-minus symbol, Consider:

- Assigning Practice Problem 2.

Professional Learning

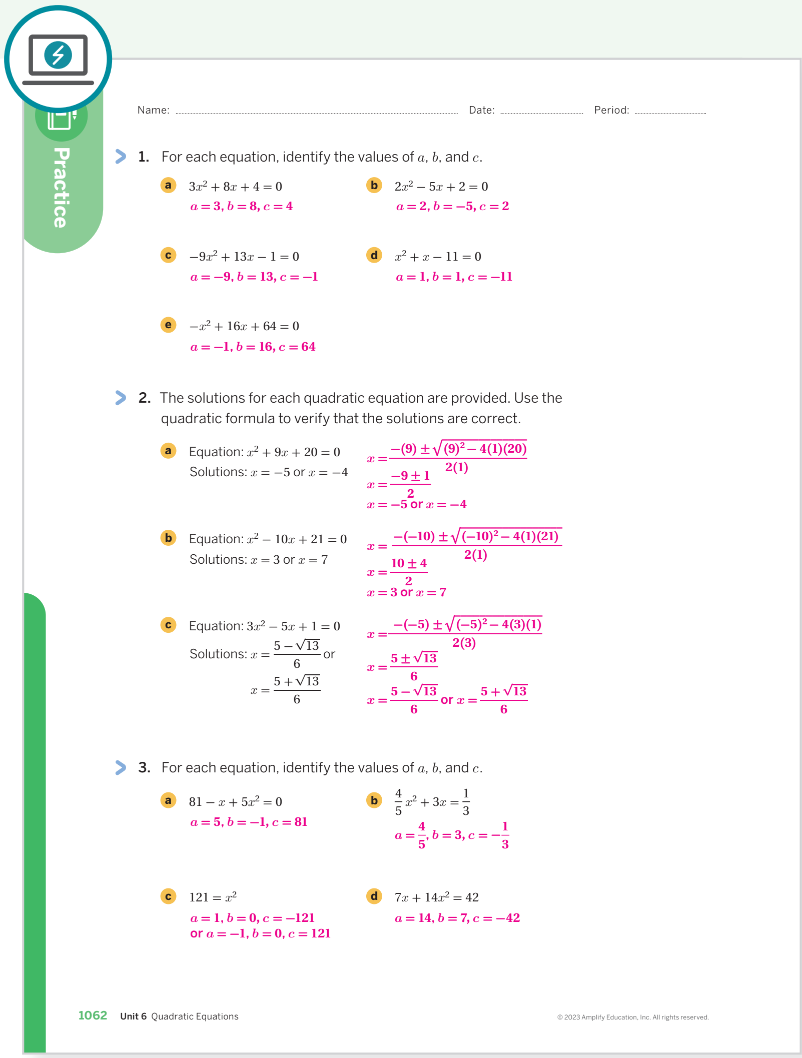
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What challenges did students encounter as they worked on Activity 1? How did they work through them?
- What did you see in the way some students approached solving quadratic equations using the strategy to which they were assigned in Activity 3 that you would like other students to try?

Practice

Independent



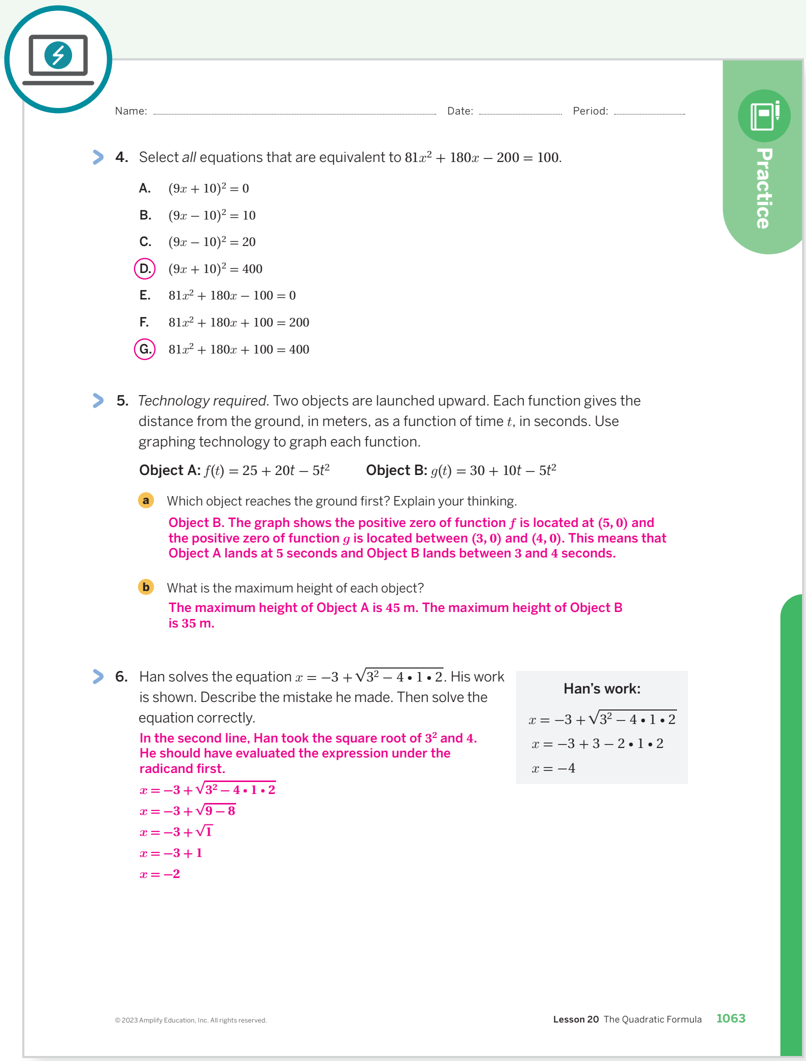
Name: .... Date: .... Period: ....

1. For each equation, identify the values of a, b, and c.
   - a $3x^2 + 8x + 4 = 0$
     $a = 3, b = 8, c = 4$
   - b $2x^2 - 5x + 2 = 0$
     $a = 2, b = -5, c = 2$
   - c $-9x^2 + 13x - 1 = 0$
     $a = -9, b = 13, c = -1$
   - d $x^2 + x - 11 = 0$
     $a = 1, b = 1, c = -11$
   - e $-x^2 + 16x + 64 = 0$
     $a = -1, b = 16, c = 64$

2. The solutions for each quadratic equation are provided. Use the quadratic formula to verify that the solutions are correct.
   - a Equation: $x^2 + 9x + 20 = 0$
     Solutions: $x = -5$ or $x = -4$
     $x = \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(20)}}{2(1)}$
     $x = \frac{-9 \pm 1}{2}$
     $x = -5$ or $x = -4$
   - b Equation: $x^2 - 10x + 21 = 0$
     Solutions: $x = 3$ or $x = 7$
     $x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$
     $x = \frac{10 \pm 4}{2}$
     $x = 3$ or $x = 7$
   - c Equation: $3x^2 - 5x + 1 = 0$
     Solutions: $x = \frac{5 - \sqrt{13}}{6}$ or $x = \frac{5 + \sqrt{13}}{6}$
     $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$
     $x = \frac{5 \pm \sqrt{13}}{6}$
     $x = \frac{5 - \sqrt{13}}{6}$ or $x = \frac{5 + \sqrt{13}}{6}$

3. For each equation, identify the values of a, b, and c.
   - a $81 - x + 5x^2 = 0$
     $a = 5, b = -1, c = 81$
   - b $\frac{4}{5}x^2 + 3x = \frac{1}{3}$
     $a = \frac{4}{5}, b = 3, c = -\frac{1}{3}$
   - c $121 = x^2$
     $a = 1, b = 0, c = -121$
     or $a = -1, b = 0, c = 121$
   - d $7x + 14x^2 = 42$
     $a = 14, b = 7, c = -42$

1062 **Unit 6** Quadratic Equations



Name: .... Date: .... Period: ....

4. Select *all* equations that are equivalent to $81x^2 + 180x - 200 = 100$.
   - A. $(9x + 10)^2 = 0$
   - B. $(9x - 10)^2 = 10$
   - C. $(9x - 10)^2 = 20$
   - [✓ D.] $(9x + 10)^2 = 400$
   - E. $81x^2 + 180x - 100 = 0$
   - F. $81x^2 + 180x + 100 = 200$
   - [✓ G.] $81x^2 + 180x + 100 = 400$

5. *Technology required.* Two objects are launched upward. Each function gives the distance from the ground, in meters, as a function of time t, in seconds. Use graphing technology to graph each function.

   Object A: $f(t) = 25 + 20t - 5t^2$ **Object B:** $g(t) = 30 + 10t - 5t^2$

   - a Which object reaches the ground first? Explain your thinking.
     Object B. The graph shows the positive zero of function f is located at $(5, 0)$ and the positive zero of function g is located between $(3, 0)$ and $(4, 0)$. This means that Object A lands at 5 seconds and Object B lands between 3 and 4 seconds.
   - b What is the maximum height of each object?
     The maximum height of Object A is 45 m. The maximum height of Object B is 35 m.

6. Han solves the equation $x = -3 + \sqrt{3^2 - 4 \bullet 1 \bullet 2}$. His work is shown. Describe the mistake he made. Then solve the equation correctly.

   Han's work:
   $x = -3 + \sqrt{3^2 - 4 \bullet 1 \bullet 2}$
   $x = -3 + 3 - 2 \bullet 1 \bullet 2$
   $x = -4$

   In the second line, Han took the square root of 3^2 and 4. He should have evaluated the expression under the radicand first.
   $x = -3 + \sqrt{3^2 - 4 \bullet 1 \bullet 2}$
   $x = -3 + \sqrt{9 - 8}$
   $x = -3 + \sqrt{1}$
   $x = -3 + 1$
   $x = -2$

Lesson 20 The Quadratic Formula 1063

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 15	2
	5	Unit 5 Lesson 8	2
Formative ⚡	6	Unit 6 Lesson 21	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

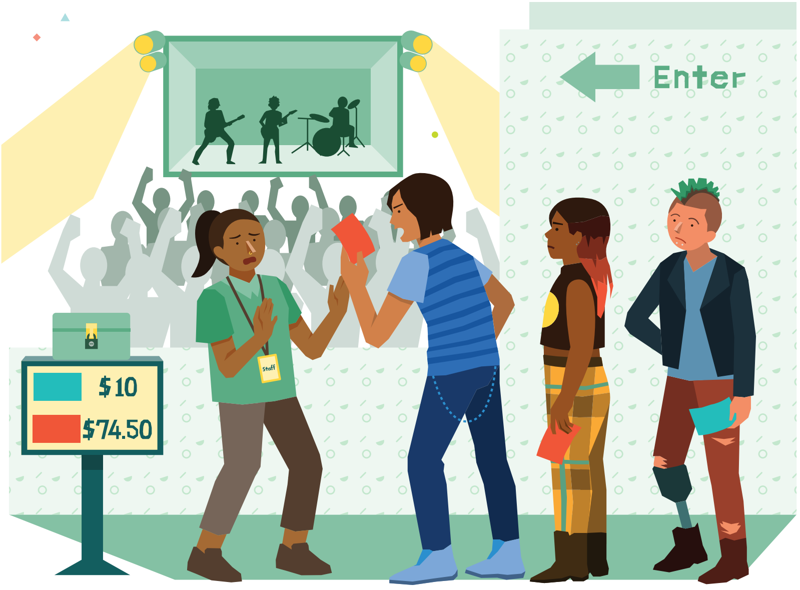
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 21**

Error Analysis: Quadratic Formula

Let’s analyze common mistakes made when using the quadratic formula.



Focus

Goals

1. **Language Goal:** Analyze and critique solutions to quadratic equations that are found using the quadratic formula. **(Reading and Writing, Speaking and Listening)**
2. Determine whether a given value is a solution to a quadratic equation.

Rigor

- Students develop **fluency** using the quadratic formula by classifying common mistakes.

Coherence

Today

Students continue to build on their understanding of using the quadratic formula by analyzing and classifying errors commonly made when applying the quadratic formula. They practice attending to precision and critiquing the reasoning of others while developing an awareness of the advantages and disadvantages of the formula.

Previously

In Lesson 20, students analyzed simple errors that arose from using the quadratic formula.

Coming Soon

In Lesson 22, students write quadratic equations to represent relationships and use the quadratic formula to solve problems that they previously could not solve.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- Anchor Chart PDF, *The Order of Operations*
- Anchor Chart PDF, *The Quadratic Formula*
- scientific calculators

Math Language Development

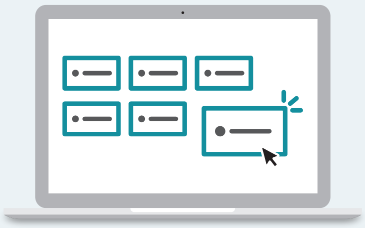
Review words

- *quadratic formula*

Amps | Featured Activity

Activity 1
Spot Those Errors!

Students will examine completed work samples to locate errors and correct them. They will receive real-time feedback regarding their own corrections.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may lack motivation, become upset, and lose focus before they have attempted spotting the errors in Activity 1. Have students brainstorm ways to motivate and calm themselves, focus on their learning goals, and use linear graphic organizers to self-access their progress toward learning goals.

Modifications to Pacing

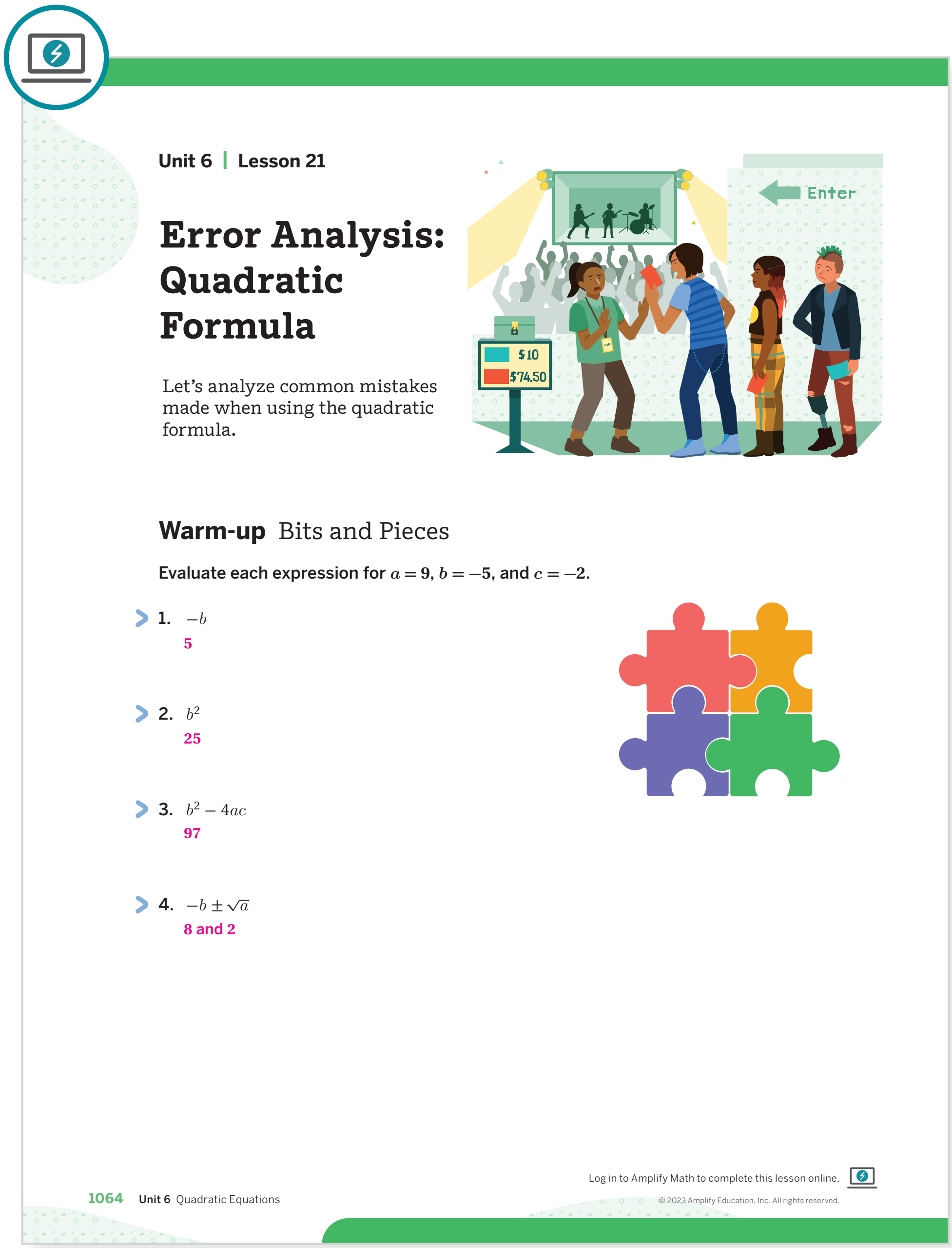
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have half the class complete Problem 1 and the other half complete Problem 2.

Independent | 5 min

Warm-up Bits and Pieces

Students evaluate expressions resembling those in the quadratic formula to preview calculations that are the source of some common errors while using the quadratic formula.



Unit 6 | Lesson 21

Error Analysis: Quadratic Formula

Let's analyze common mistakes made when using the quadratic formula.

Warm-up Bits and Pieces

Evaluate each expression for $a = 9$, $b = -5$, and $c = -2$.

1. $-b$
   5
2. b^2
   25
3. $b^2 - 4ac$
   97
4. $-b \pm \sqrt{a}$
   8 and 2

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1064 Unit 6 Quadratic Equations

1 Launch

Provide access to scientific calculators. Set an expectation for the amount of time students will have to complete the Warm-up independently.

2 Monitor

Help students get started by having them substitute the values into the expressions.

Look for points of confusion:

- **Evaluating b without the double negatives in Problems 1 and 4.** Have students write the given expression first, and then use parentheses to organize the negative inputs.
- **Providing one value for Problem 4.** Remind students that the plus-or-minus symbol implies that there are two possible values.

Look for productive strategies:

- Organizing the use of negatives with parentheses for b and c.
- Using appropriate tools to check their calculations.

3 Connect

Display the expressions one at a time.

Have individual students share or model their responses as they evaluate each expression. Select and sequence responses that include common errors alongside correct and productive strategies.

Ask, "What are some other possible errors that someone may make when evaluating these expressions?" Answers may vary. Record possible errors to display for the next activity.

Highlight that students must be careful with their computations for each part of the quadratic formula.

Power-up

To power up students' ability to identify mistakes when simplifying expression involving square roots, have students complete:

Andre is trying to simplify the expression $4 + \sqrt{3^2 - 4 \cdot 2 \cdot 1}$.

1. What operation should Andre complete first?
   - **A.** Take the square root of 3^2.
   - [✓ **C.**] Evaluate 3^2.
   - **B.** Evaluate $4 \cdot 2 \cdot 1$.
   - **D.** Square the expression.
2. Determine the value of the expression. 5

Use: Before Activity 1

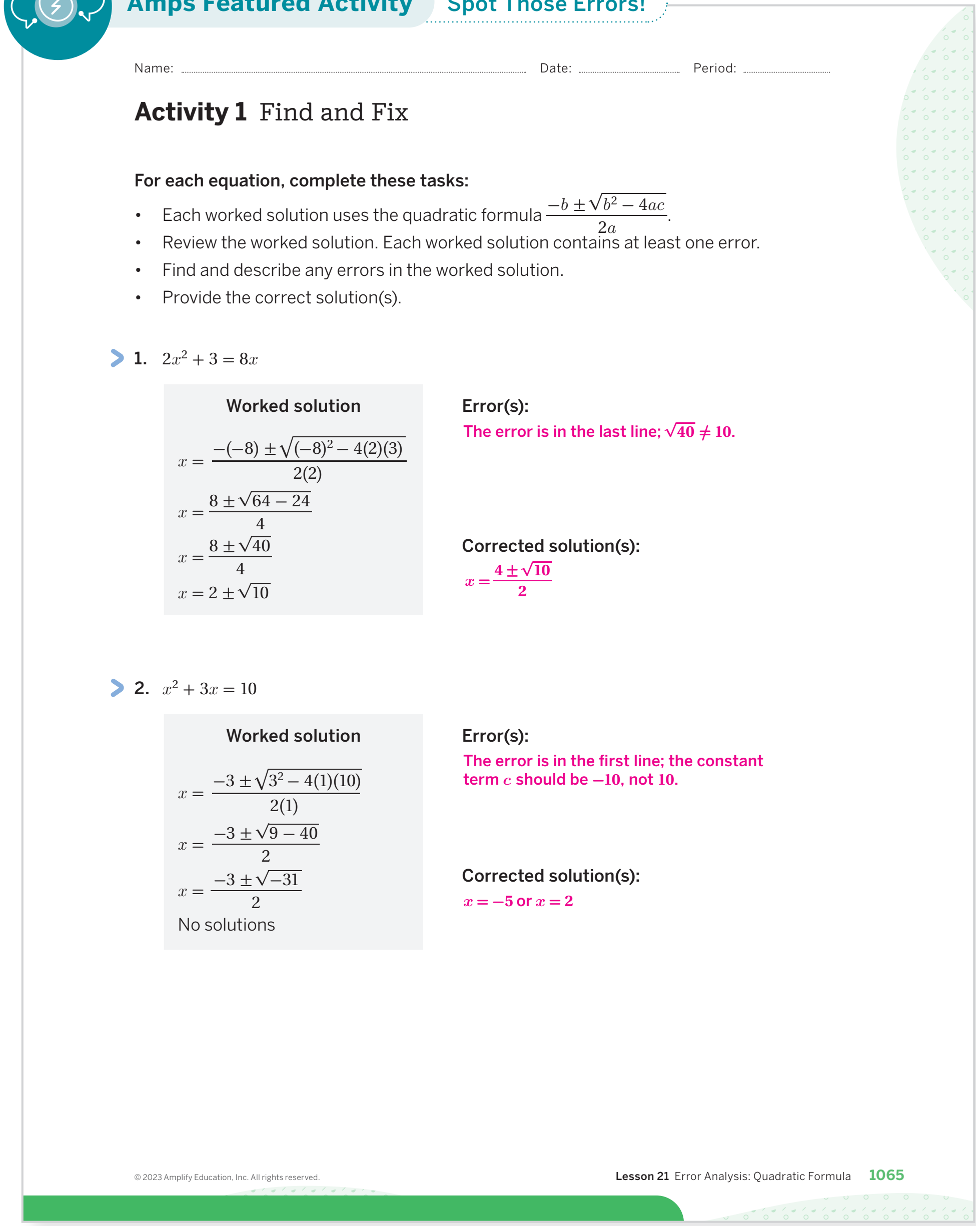
Informed by: Performance on Lesson 20, Practice Problem 6

Small Groups | 20 min

Activity 1 Find and Fix

Students analyze and correct worked examples of the quadratic formula to hone critical-thinking skills and address common misconceptions.

Amps Featured Activity Spot Those Errors!



Name: Date: Period:

Activity 1 Find and Fix

For each equation, complete these tasks:

- Each worked solution uses the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- Review the worked solution. Each worked solution contains at least one error.
- Find and describe any errors in the worked solution.
- Provide the correct solution(s).

1. $2x^2 + 3 = 8x$

Worked solution

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{64 - 24}}{4}$$

$$x = \frac{8 \pm \sqrt{40}}{4}$$

$$x = 2 \pm \sqrt{10}$$

Error(s):

The error is in the last line; $\sqrt{40} \neq 10$.

Corrected solution(s):

$$x = \frac{4 \pm \sqrt{10}}{2}$$

2. $x^2 + 3x = 10$

Worked solution

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 40}}{2}$$

$$x = \frac{-3 \pm \sqrt{-31}}{2}$$

No solutions

Error(s):

The error is in the first line; the constant term c should be -10, not 10.

Corrected solution(s):

$x = -5$ or $x = 2$

Lesson 21 Error Analysis: Quadratic Formula 1065

1 Launch

Arrange students in groups. Conduct the *Find and Fix* routine. Read the directions aloud, emphasizing that there is at least one error in each problem. Students should work independently, before consulting with group members to discuss errors and solutions.

2 Monitor

Help students get started by displaying the list of common errors your students helped you create during the Warm-up.

Look for points of confusion:

- **Having difficulty discerning the miscalculations.** Ask, "What values changed from the last step?"

Look for productive strategies:

- Creating a checklist to compare their work against each worked problem.
- Using precise language when identifying and explaining the error(s).
- Marking the inconsistencies between their own work and the work provided.

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the Anchor Chart PDF, *The Quadratic Formula* for them to use as a reference. Provide access to colored pencils and suggest they color code the values of a, b, and c in each equation to show how those values were substituted into the formula in the worked solutions to help them identify the errors.

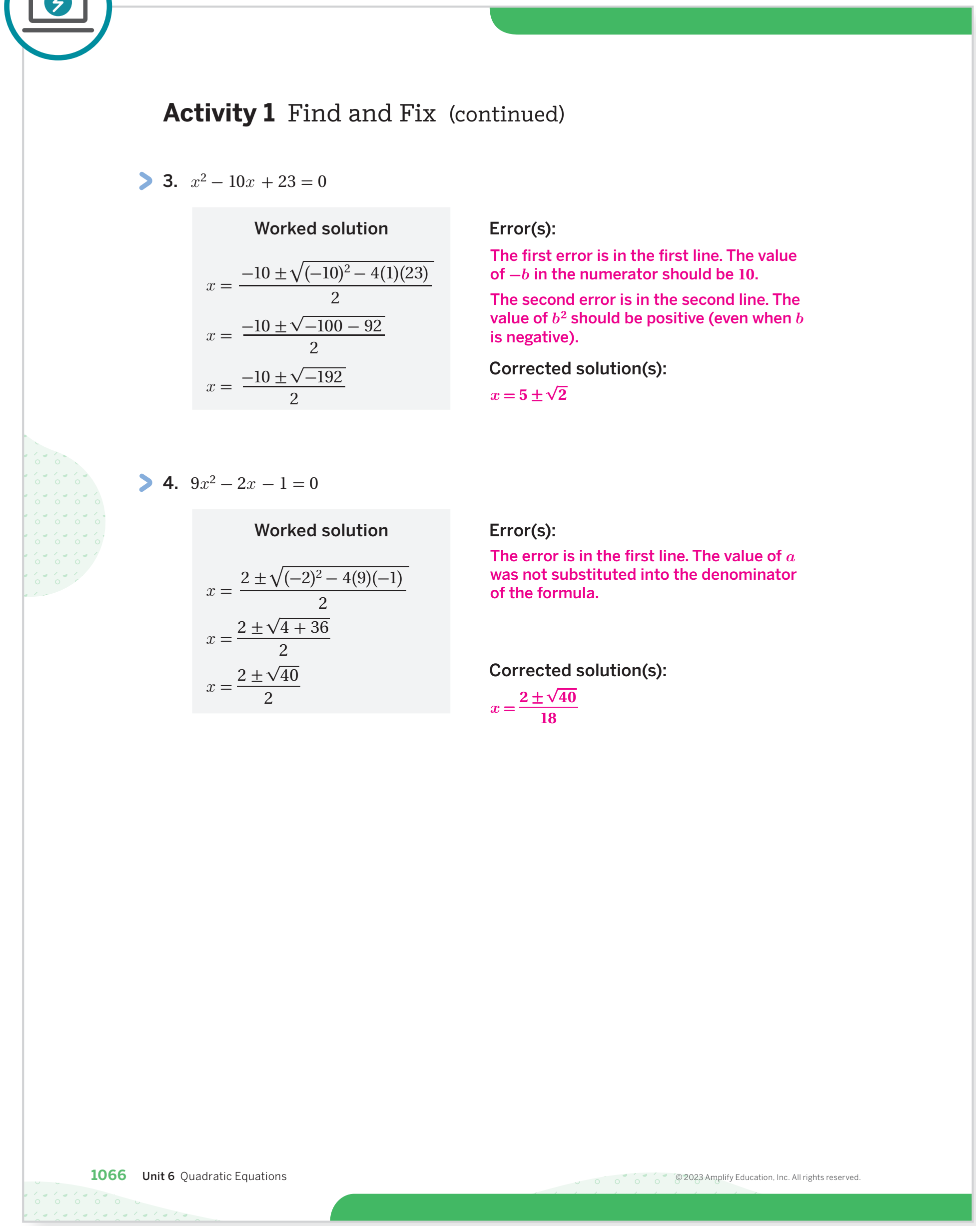
Extension: *Math Enrichment*

Have students use the quadratic formula to try to solve the equation $x^2 - 5x + 9 = 0$ and describe what they notice. There is no solution. The expression under the radical, when simplified, is negative. It is not possible to take the square root of a negative value. Note: Students will learn about imaginary and complex solutions in Algebra 2.

Small Groups | 20 min

Activity 1 Find and Fix (continued)

Students analyze and correct worked examples of the quadratic formula to hone critical-thinking skills and address common misconceptions.



Activity 1 Find and Fix (continued)

3. $x^2 - 10x + 23 = 0$

Worked solution

$$x = \frac{-10 \pm \sqrt{(-10)^2 - 4(1)(23)}}{2}$$

$$x = \frac{-10 \pm \sqrt{-100 - 92}}{2}$$

$$x = \frac{-10 \pm \sqrt{-192}}{2}$$

Error(s):

The first error is in the first line. The value of $-b$ in the numerator should be 10.

The second error is in the second line. The value of b^2 should be positive (even when b is negative).

Corrected solution(s):

$x = 5 \pm \sqrt{2}$

4. $9x^2 - 2x - 1 = 0$

Worked solution

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(9)(-1)}}{2}$$

$$x = \frac{2 \pm \sqrt{4 + 36}}{2}$$

$$x = \frac{2 \pm \sqrt{40}}{2}$$

Error(s):

The error is in the first line. The value of a was not substituted into the denominator of the formula.

Corrected solution(s):

$x = \frac{2 \pm \sqrt{40}}{18}$

1066 Unit 6 Quadratic Equations

©2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have groups of students share errors they identified and their proposed corrections.

Display the Anchor Chart PDF, *The Quadratic Formula.*

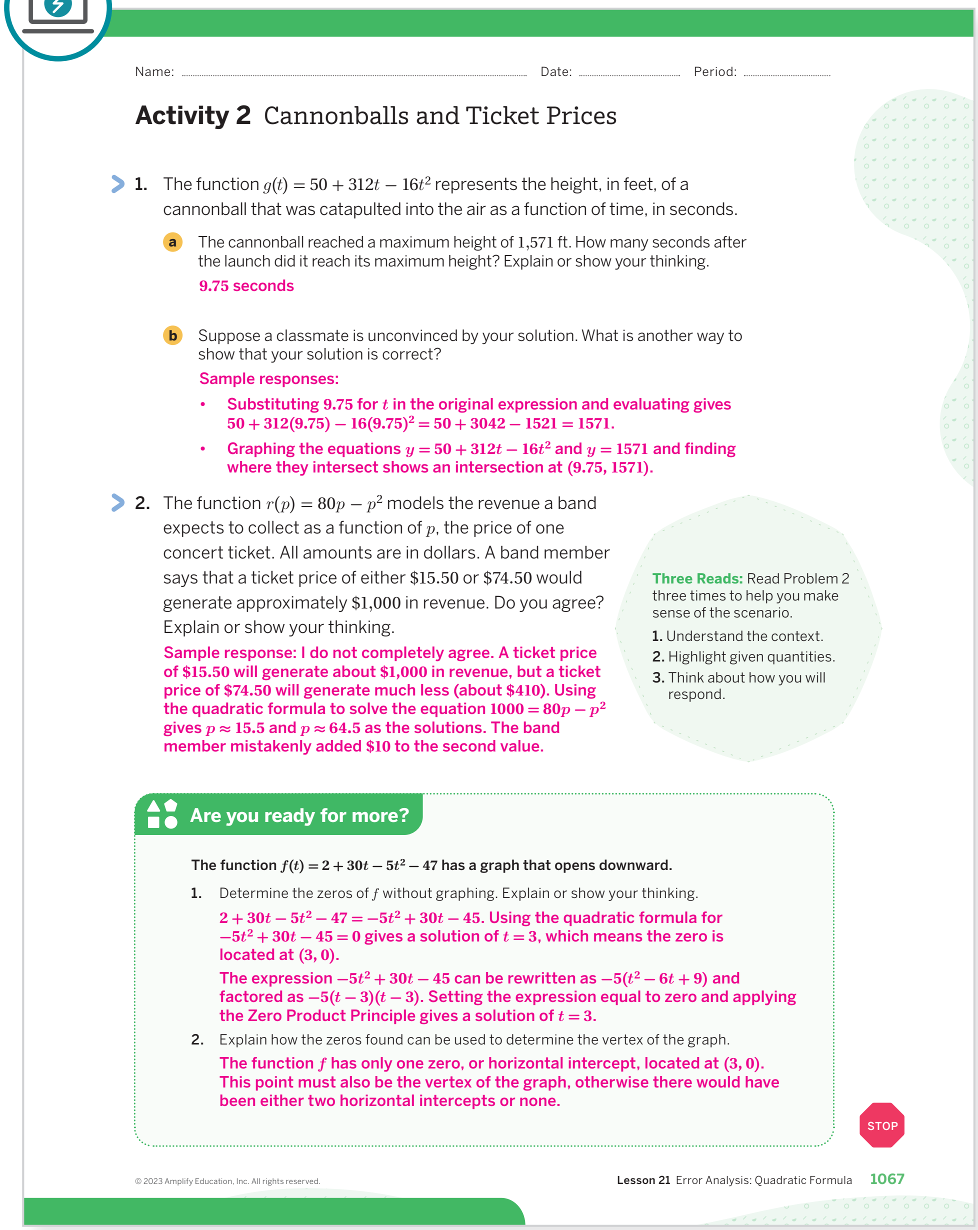
Highlight the common errors to avoid when using the quadratic formula to solve quadratic equations, including:

- Using the wrong values for a, b, or c in the formula.
- Not taking the opposite of b in the numerator outside the square root sign.
- Forgetting to multiply a by 2 in the formula's denominator.
- Forgetting that squaring a negative number produces a positive number.
- Forgetting that the product of a negative number and a positive number is negative.

Pairs | 15 min

Activity 2 Cannonballs and Ticket Prices

Students check the solutions of quadratic equations to recognize that there are several approaches to consider when solving.



Name: .... Date: .... Period: ....

Activity 2 Cannonballs and Ticket Prices

1. The function $g(t) = 50 + 312t - 16t^2$ represents the height, in feet, of a cannonball that was catapulted into the air as a function of time, in seconds.

   a. The cannonball reached a maximum height of 1,571 ft. How many seconds after the launch did it reach its maximum height? Explain or show your thinking.

   9.75 seconds

   b. Suppose a classmate is unconvinced by your solution. What is another way to show that your solution is correct?

   Sample responses:

   - **Substituting 9.75 for t in the original expression and evaluating gives $50 + 312(9.75) - 16(9.75)^2 = 50 + 3042 - 1521 = 1571$.**
   - **Graphing the equations $y = 50 + 312t - 16t^2$ and $y = 1571$ and finding where they intersect shows an intersection at $(9.75, 1571)$.**

2. The function $r(p) = 80p - p^2$ models the revenue a band expects to collect as a function of p, the price of one concert ticket. All amounts are in dollars. A band member says that a ticket price of either $15.50 or $74.50 would generate approximately $1,000 in revenue. Do you agree? Explain or show your thinking.

   Sample response: I do not completely agree. A ticket price of $15.50 will generate about $1,000 in revenue, but a ticket price of $74.50 will generate much less (about $410). Using the quadratic formula to solve the equation $1000 = 80p - p^2$ gives $p \approx 15.5$ and $p \approx 64.5$ as the solutions. The band member mistakenly added $10 to the second value.

Three Reads: Read Problem 2 three times to help you make sense of the scenario.

1. Understand the context.
2. Highlight given quantities.
3. Think about how you will respond.

Are you ready for more?

The function $f(t) = 2 + 30t - 5t^2 - 47$ has a graph that opens downward.

1. Determine the zeros of f without graphing. Explain or show your thinking.

   $2 + 30t - 5t^2 - 47 = -5t^2 + 30t - 45$. Using the quadratic formula for $-5t^2 + 30t - 45 = 0$ gives a solution of $t = 3$, which means the zero is located at $(3, 0)$.

   The expression $-5t^2 + 30t - 45$ can be rewritten as $-5(t^2 - 6t + 9)$ and factored as $-5(t - 3)(t - 3)$. Setting the expression equal to zero and applying the Zero Product Principle gives a solution of $t = 3$.

2. Explain how the zeros found can be used to determine the vertex of the graph.

   The function f has only one zero, or horizontal intercept, located at $(3, 0)$. This point must also be the vertex of the graph, otherwise there would have been either two horizontal intercepts or none.

STOP

Lesson 21 Error Analysis: Quadratic Formula 1067

1 Launch

Provide students with 1 minute of think-time before discussing their thinking with their partners.

Note: If time permits, consider using a ***Gallery Tour*** routine during the Connect for students to present strategies for Problem 1b.

2 Monitor

Help students get started by asking "What are you being asked to solve for?" The time that the cannonball reached its maximum height.

Look for points of confusion:

- **Choosing an insufficient or inefficient strategy to solve the equation.** Suggest students use the quadratic formula.
- **Struggling to determine what to set the equation equal to in Problem 1.** Remind students that $g(t)$ represents the height, so they should set the equation equal to the given height.

Look for productive strategies:

- Solving the equation using another algebraic method and checking that the solutions are the same.

3 Connect

Display the equation $6x^2 - 17x + 5 = 0$.

Ask, "What might be some ways to check whether $x = \frac{1}{3}$ and $x = \frac{5}{2}$ are the solutions to the equation?" Sample response: I can substitute the values for x into the original equation.

Highlight that there is more than one way to check the solutions to the equations, but some methods are more efficient than others, such as graphing or substitution.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students choose one of the problems to complete. Allowing them the power of choice can result in increased engagement and ownership of the task.

Accessibility: *Guide Processing and Visualization*

Provide students with the Anchor Chart PDF, *The Quadratic Formula* for them to use as a reference. Provide access to colored pencils and suggest they color code the values of a, b, and c in each equation to show how those values were substituted into the formula, if they choose to solve the equations using this strategy.

Math Language Development

MLR6: Three Reads

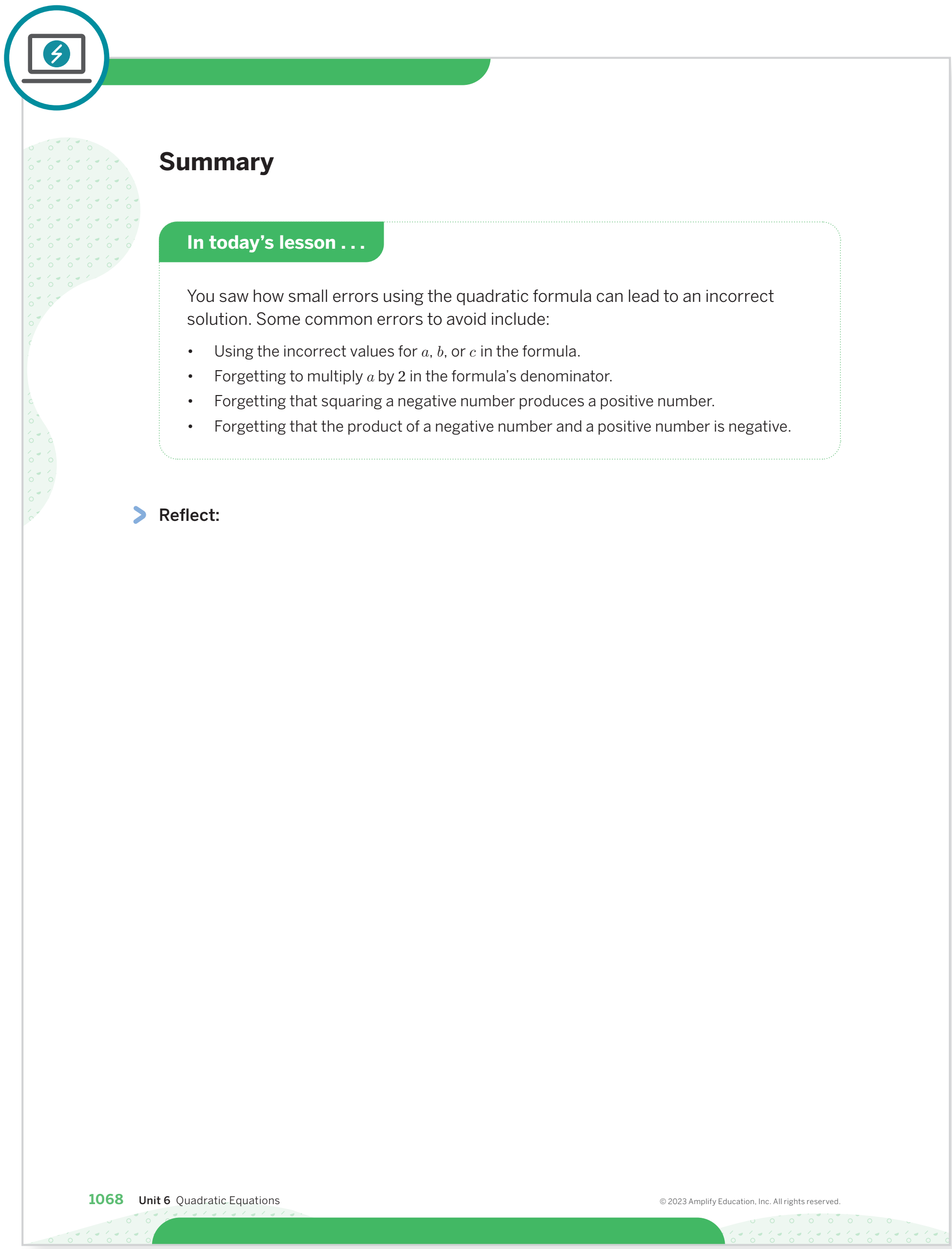
Use this routine to help students make sense of the introductory text for each scenario.

- **Read 1:** Students should understand the general scenario e.g., a cannonball is launched into the air for Problem 1.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as "the cannonball reached a maximum height of 1,571 ft" for Problem 1.
- **Read 3:** Ask students to plan their solution strategy as to how they will solve each problem.

Whole Class | 5 min

Summary

Review and synthesize the big ideas of the lesson by discussing the common errors that occur when using the quadratic formula and the advantages and disadvantages of the quadratic formula.



Summary

In today's lesson . . .

You saw how small errors using the quadratic formula can lead to an incorrect solution. Some common errors to avoid include:

- Using the incorrect values for a, b, or c in the formula.
- Forgetting to multiply a by 2 in the formula's denominator.
- Forgetting that squaring a negative number produces a positive number.
- Forgetting that the product of a negative number and a positive number is negative.

Reflect:

1068 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the list of errors recorded during the Warm-up.

Ask, "Think about some of the problems you encountered today. Did you find yourself making some of these mistakes? Or did you notice your partner making some of these errors?" Answers may vary.

Have students share some of the mistakes they noticed during Activities 1 and 2. Have them reflect on what those errors might imply and determine some proactive approaches for addressing each type of error.

Highlight that there are four types of errors typically made when using the quadratic formula:

- Using the wrong values for a, b, or c.
- Forgetting to multiply a by 2 in the denominator.
- Forgetting that squaring a negative number produces a positive number.
- Forgetting that the product of a negative number and a positive number is negative.

Discuss the importance of always checking solutions to verify the solution is correct.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

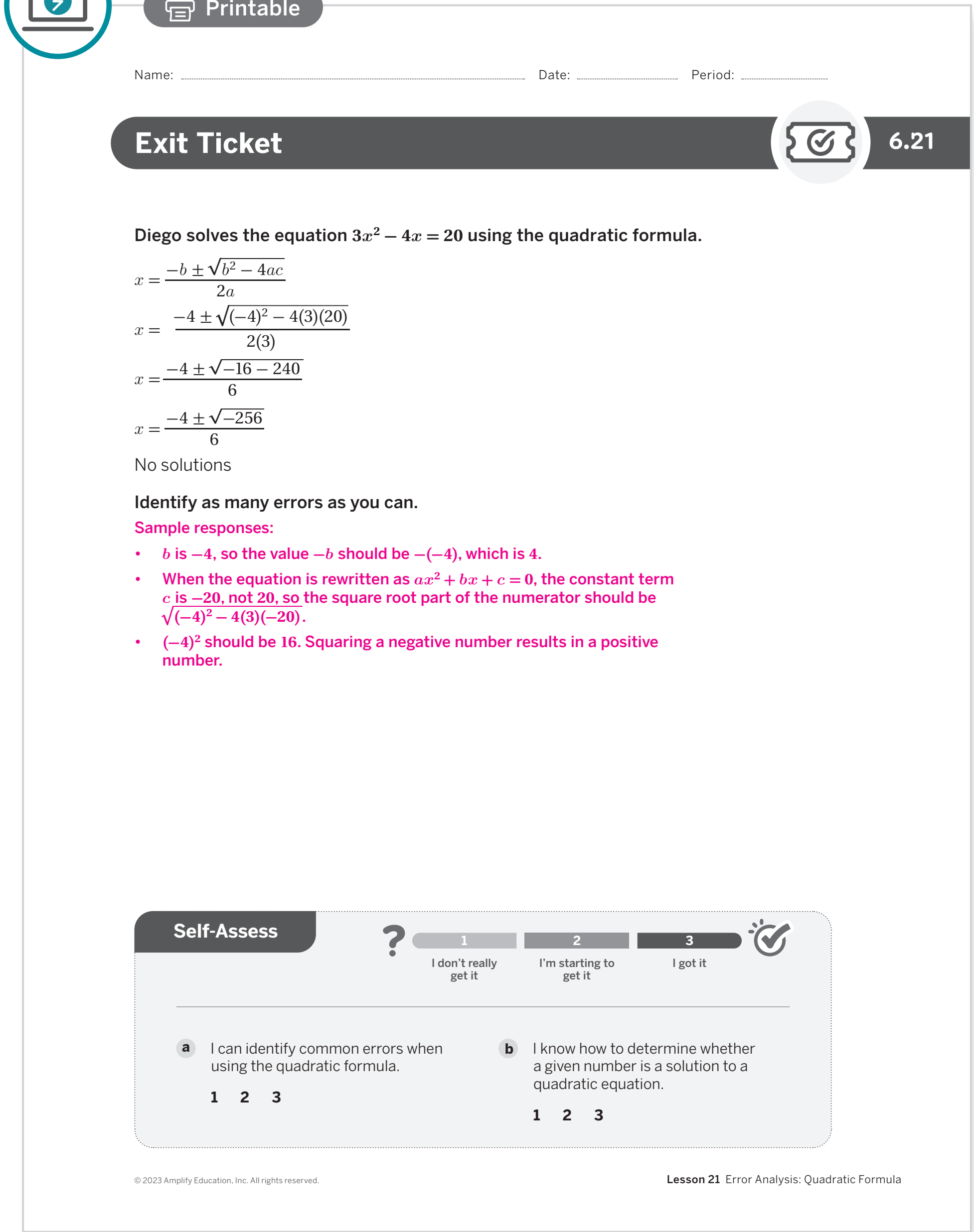
- "Why is considering common errors when using the quadratic formula beneficial? How will it help you to attend to precision when you use this strategy?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying errors in the use of the quadratic formula.

Printable



Name: __________ Date: __________ Period: __________

Exit Ticket 6.21

Diego solves the equation $3x^2 - 4x = 20$ using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(3)(20)}}{2(3)}$$

$$x = \frac{-4 \pm \sqrt{-16 - 240}}{6}$$

$$x = \frac{-4 \pm \sqrt{-256}}{6}$$

No solutions

Identify as many errors as you can.

Sample responses:

- b is -4, so the value $-b$ should be $-(-4)$, which is 4.
- When the equation is rewritten as $ax^2 + bx + c = 0$, the constant term c is -20, not 20, so the square root part of the numerator should be $\sqrt{(-4)^2 - 4(3)(-20)}$.
- $(-4)^2$ should be 16. Squaring a negative number results in a positive number.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can identify common errors when using the quadratic formula.

1 2 3

b I know how to determine whether a given number is a solution to a quadratic equation.

1 2 3

Success looks like . . .

- **Language Goal:** Analyzing and critiquing solutions to quadratic equations that are found using the quadratic formula. **(Reading and Writing, Speaking and Listening)**
  - » Identifying the errors in the steps of Diego's solution.
- **Goal:** Determining whether a given value is a solution to a quadratic equation.

Suggested next steps

If students do not recognize that Diego used a positive value for b, consider:

- Reviewing the Warm-up.
- Asking, "What is the value of $-(-4)$?" 4

If students do not identify that c is -20, consider:

- Reviewing the Warm-up.

If students do not recognize that $(-4)^2$ is 16, consider:

- Reviewing the properties of exponents for negative bases.

Professional Learning

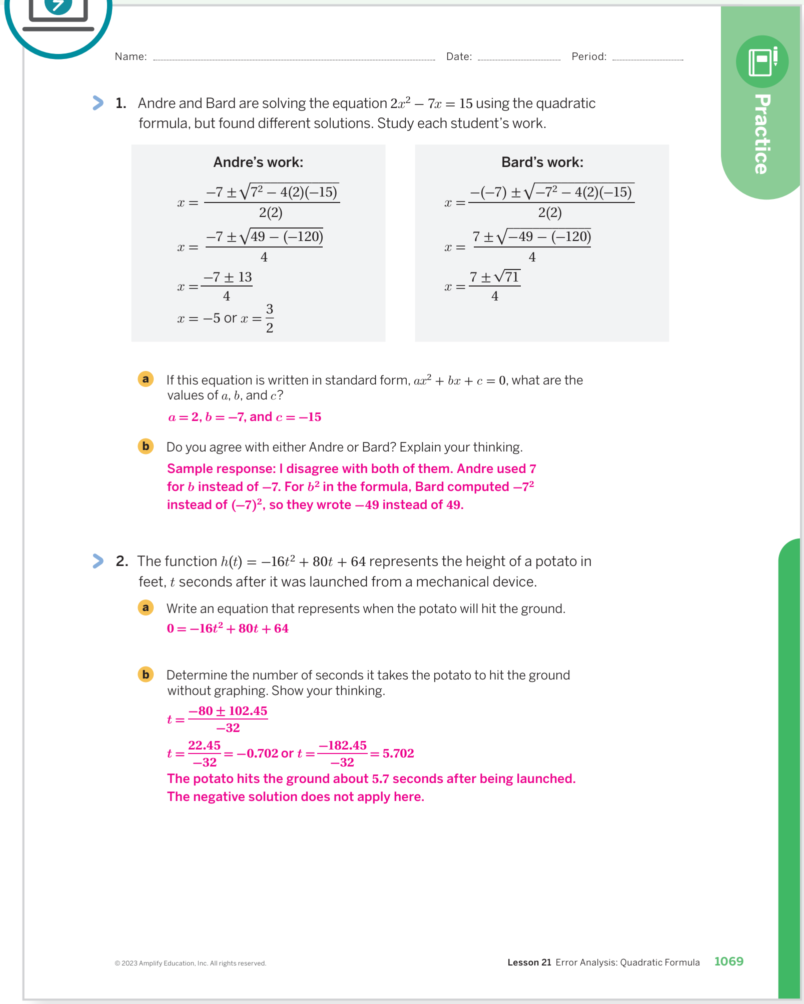
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Which students' ideas were you able to highlight during the Connect of Activity 1?
- Thinking about the questions you asked students today about common errors and what the students said or did as a result of the questions, which question was the most effective?

Practice

Independent



Name: Date: Period:

1. Andre and Bard are solving the equation $2x^2 - 7x = 15$ using the quadratic formula, but found different solutions. Study each student's work.

Andre's work:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$$
$$x = \frac{-7 \pm \sqrt{49 - (-120)}}{4}$$
$$x = \frac{-7 \pm 13}{4}$$
$$x = -5 \text{ or } x = \frac{3}{2}$$

Bard's work:

$$x = \frac{-(-7) \pm \sqrt{-7^2 - 4(2)(-15)}}{2(2)}$$
$$x = \frac{7 \pm \sqrt{-49 - (-120)}}{4}$$
$$x = \frac{7 \pm \sqrt{71}}{4}$$

a. If this equation is written in standard form, $ax^2 + bx + c = 0$, what are the values of a, b, and c?

$a = 2$, $b = -7$, and $c = -15$

b. Do you agree with either Andre or Bard? Explain your thinking.

Sample response: I disagree with both of them. Andre used 7 for b instead of –7. For b^2 in the formula, Bard computed -7^2 instead of $(-7)^2$, so they wrote –49 instead of 49.

2. The function $h(t) = -16t^2 + 80t + 64$ represents the height of a potato in feet, t seconds after it was launched from a mechanical device.

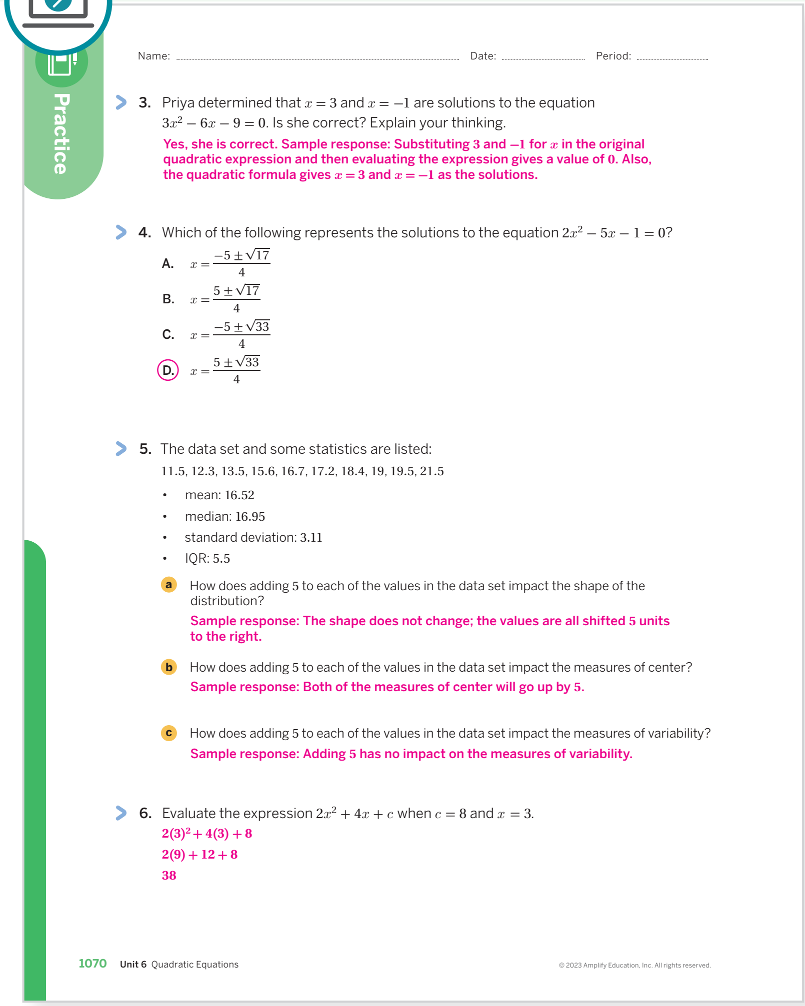
a. Write an equation that represents when the potato will hit the ground.

$0 = -16t^2 + 80t + 64$

b. Determine the number of seconds it takes the potato to hit the ground without graphing. Show your thinking.

$$t = \frac{-80 \pm 102.45}{-32}$$
$$t = \frac{22.45}{-32} = -0.702 \text{ or } t = \frac{-182.45}{-32} = 5.702$$

The potato hits the ground about 5.7 seconds after being launched. The negative solution does not apply here.



Name: Date: Period:

3. Priya determined that $x = 3$ and $x = -1$ are solutions to the equation $3x^2 - 6x - 9 = 0$. Is she correct? Explain your thinking.

Yes, she is correct. Sample response: Substituting 3 and –1 for x in the original quadratic expression and then evaluating the expression gives a value of 0. Also, the quadratic formula gives $x = 3$ and $x = -1$ as the solutions.

4. Which of the following represents the solutions to the equation $2x^2 - 5x - 1 = 0$?

A. $x = \frac{-5 \pm \sqrt{17}}{4}$

B. $x = \frac{5 \pm \sqrt{17}}{4}$

C. $x = \frac{-5 \pm \sqrt{33}}{4}$

[✓ D.] $x = \frac{5 \pm \sqrt{33}}{4}$

5. The data set and some statistics are listed:

11.5, 12.3, 13.5, 15.6, 16.7, 17.2, 18.4, 19, 19.5, 21.5

- mean: 16.52
- median: 16.95
- standard deviation: 3.11
- IQR: 5.5

a. How does adding 5 to each of the values in the data set impact the shape of the distribution?

Sample response: The shape does not change; the values are all shifted 5 units to the right.

b. How does adding 5 to each of the values in the data set impact the measures of center?

Sample response: Both of the measures of center will go up by 5.

c. How does adding 5 to each of the values in the data set impact the measures of variability?

Sample response: Adding 5 has no impact on the measures of variability.

6. Evaluate the expression $2x^2 + 4x + c$ when $c = 8$ and $x = 3$.

$2(3)^2 + 4(3) + 8$

$2(9) + 12 + 8$

38

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	3
	2	Activity 2	3
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 20	3
Formative ⚡	5	Unit 6 Lesson 22	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 22**

Applying the Quadratic Formula

Let's use the quadratic formula to solve problems within a real-world context.

Focus

Goals

1. **Language Goal:** Interpret the solutions to quadratic equations in context. **(Speaking and Listening, Writing)**
2. Practice using the quadratic formula to solve quadratic equations, rearranging the equations into $ax^2 + bx + c = 0$ if not already given in this form.

Coherence

Today

Students write quadratic equations to represent relationships and use the quadratic formula to solve problems that they previously could not solve. The work in this lesson encourages students to reason quantitatively and abstractly. Students notice that the quadratic formula is the most efficient and practical way to solve some equations.

Previously

In Lesson 21, students analyzed errors commonly made when applying the quadratic formula to solve quadratic equations.

Coming Soon

Students continue to build on their understanding of the quadratic formula and examine its advantages and disadvantages of usage.

Rigor

- Students **apply** the quadratic formula to solve real-world problems.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *The Quadratic Formula*
- scientific calculators

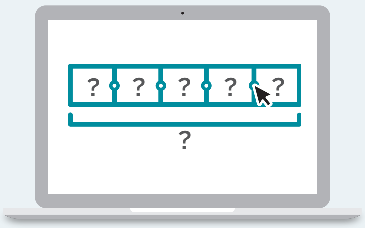
Math Language Development

Review words

- *quadratic formula*

Activity 2
See Student Thinking

Students revisit the framing problem from the beginning of the unit, now that they have more strategies to use to solve the quadratic equation. As they solve it, follow along with their thinking.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overwhelmed making sense of and applying the strategies in Activity 2. Ask students how they are feeling and listen deeply and reflect what you heard about their feelings. For example, “It sounds like you’re feeling very frustrated right now. . .” Then have students describe other challenging lessons or concepts they have preserved and succeeded in.

Modifications to Pacing

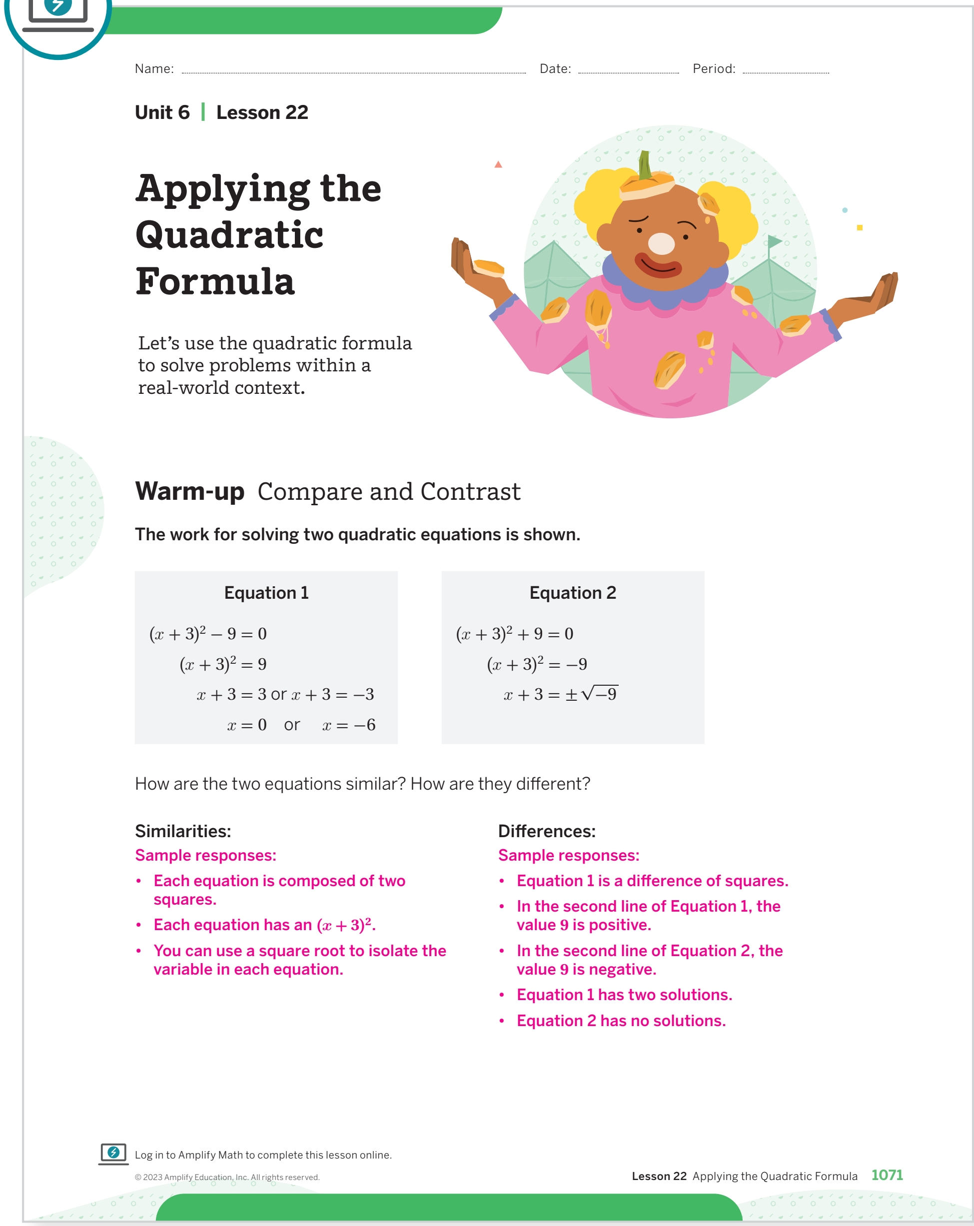
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have a student in each pair determine the solution to one equation.
- In **Activity 2**, have students only complete Problem 1.

Pairs | 5 min

Warm-up Compare and Contrast

Students recall there are no solutions when taking the square root of a negative number, by comparing and contrasting examples.



Name: ______________________ Date: __________ Period: __________

Unit 6 | Lesson 22

Applying the Quadratic Formula

Let's use the quadratic formula to solve problems within a real-world context.

Warm-up Compare and Contrast

The work for solving two quadratic equations is shown.

Equation 1	Equation 2
$(x+3)^2-9=0$	$(x+3)^2+9=0$
$(x+3)^2=9$	$(x+3)^2=-9$
$x+3=3$ or $x+3=-3$	$x+3=\pm\sqrt{-9}$
$x=0$ or $x=-6$	

How are the two equations similar? How are they different?

Similarities:
Sample responses:
- Each equation is composed of two squares.
- Each equation has an $(x+3)^2$.
- You can use a square root to isolate the variable in each equation.

Differences:
Sample responses:
- Equation 1 is a difference of squares.
- In the second line of Equation 1, the value 9 is positive.
- In the second line of Equation 2, the value 9 is negative.
- Equation 1 has two solutions.
- Equation 2 has no solutions.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Applying the Quadratic Formula 1071

1 Launch

Give students one minute of think-time and then two minutes to turn and talk with a partner before completing the activity.

2 Monitor

Help students get started by circling what is alike and underlining what is different for each worked solution.

Look for points of confusion:
- **Not noticing that the first equation consists of squares.** Remind students about the structure of a square equation.

Look for productive strategies:
- Setting up a chart for similarities and differences.

3 Connect

Have student pairs share the similarities and differences for each equation. Record the similarities and differences. Engage students by asking "Who can add on to ___'s thinking?"

Highlight that (in Algebra 1) they cannot take the square root of a negative value, so there are no solutions for Equation 2.

Ask, "How do you know $49+x^2=0$ has no solutions?" Sample response: Subtracting 49 from each side gives the equation $x^2=-49$ and you cannot take the square root of a negative number (in Algebra 1).

Power-up

To power up students' ability to evaluate quadratic expressions for given values, have students complete:

Jada wants to evaluate the expression $3x^2+bx+c$ for $x=-1$, $b=2$ and $c=-3$.

1. Which expression shows the correct substitution?

A. $3(-1)^2+2+(-3)$

[✓ **B.**] $3(-1)^2+2(-1)+(-3)$

C. $3(-1)+2(-1)+(-3)$

D. $3(-1)^2+2(-1)+(-3)(-1)$

2. Evaluate the expression you chose in Problem 1.

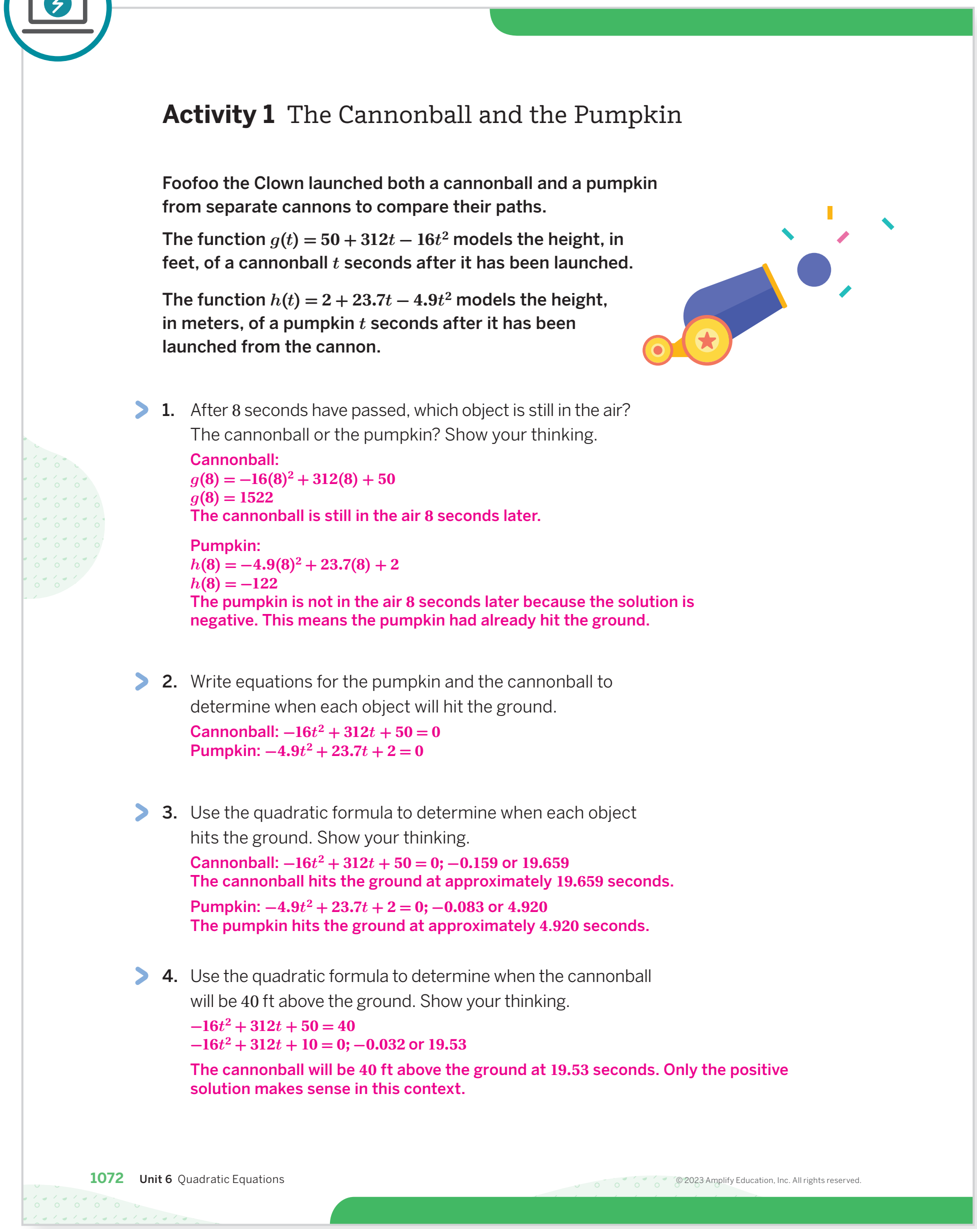
-2; $3(-1)^2+2(-1)+(-3)=3-2-3=-2$

Use: Before Activity 1
Informed by: Performance on Lesson 21, Practice Problem 5

Pairs | 15 min

Activity 1 The Cannonball and the Pumpkin

Students apply the quadratic formula to solve a problem previously encountered that they thought was only solvable by graphing.



Activity 1 The Cannonball and the Pumpkin

Foofoo the Clown launched both a cannonball and a pumpkin from separate cannons to compare their paths.

The function $g(t) = 50 + 312t - 16t^2$ models the height, in feet, of a cannonball t seconds after it has been launched.

The function $h(t) = 2 + 23.7t - 4.9t^2$ models the height, in meters, of a pumpkin t seconds after it has been launched from the cannon.

1. After 8 seconds have passed, which object is still in the air? The cannonball or the pumpkin? Show your thinking.

   Cannonball:
   $g(8) = -16(8)^2 + 312(8) + 50$
   $g(8) = 1522$
   The cannonball is still in the air 8 seconds later.

   Pumpkin:
   $h(8) = -4.9(8)^2 + 23.7(8) + 2$
   $h(8) = -122$
   The pumpkin is not in the air 8 seconds later because the solution is negative. This means the pumpkin had already hit the ground.

2. Write equations for the pumpkin and the cannonball to determine when each object will hit the ground.

   Cannonball: $-16t^2 + 312t + 50 = 0$
   Pumpkin: $-4.9t^2 + 23.7t + 2 = 0$

3. Use the quadratic formula to determine when each object hits the ground. Show your thinking.

   Cannonball: $-16t^2 + 312t + 50 = 0$; -0.159 or 19.659
   The cannonball hits the ground at approximately 19.659 seconds.

   Pumpkin: $-4.9t^2 + 23.7t + 2 = 0$; -0.083 or 4.920
   The pumpkin hits the ground at approximately 4.920 seconds.

4. Use the quadratic formula to determine when the cannonball will be 40 ft above the ground. Show your thinking.

   $-16t^2 + 312t + 50 = 40$
   $-16t^2 + 312t + 10 = 0$; -0.032 or 19.53

   The cannonball will be 40 ft above the ground at 19.53 seconds. Only the positive solution makes sense in this context.

1072 Unit 6 Quadratic Equations ©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide access to scientific calculators for numerical computations.

Note: Do not provide access to graphing technology.

2 Monitor

Help students get started by modeling the path of one of the objects. Ask, "What value represents the object hitting the ground?" 0

Look for points of confusion:

- **Setting the function equal to 8 instead of substituting in Problem 1.** Remind students that 8 is the number of seconds which should be substituted for t.
- **Thinking that they need to input 0 in for t in Problem 2.** Remind students that 0 represents the height of the pumpkin and cannonball when they hit the ground, not 0 seconds.

Look for productive strategies:

- Understanding positive solutions are needed, based on the context of the problem.

3 Connect

Have individual students share their solution and explanation to Problem 3.

Highlight that students should understand what equations to write and what it means to solve each equation in the given contexts. Then focus the discussion on how the solutions can be found, interpreted (why only positive solutions are viable), and verified (other strategies such as factoring or completing the square). Display the graphs of each object.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the Anchor Chart PDF, *The Quadratic Formula*, for them to use as a reference. Provide access to colored pencils and suggest they color code the values of a, b, and c in each equation to show how those values were substituted into the formula.

Extension: *Math Enrichment*

Have students use the equations to determine the maximum heights of the cannonball and pumpkin and at what time they reach those maximum heights.
Cannonball: 1,571 ft at 9.75 seconds
Pumpkin: about 30.66 m at about 2.42 seconds

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present an incorrect statement, such as "There are two times when the pumpkin will hit the ground because there are two solutions to the quadratic equation." Ask:

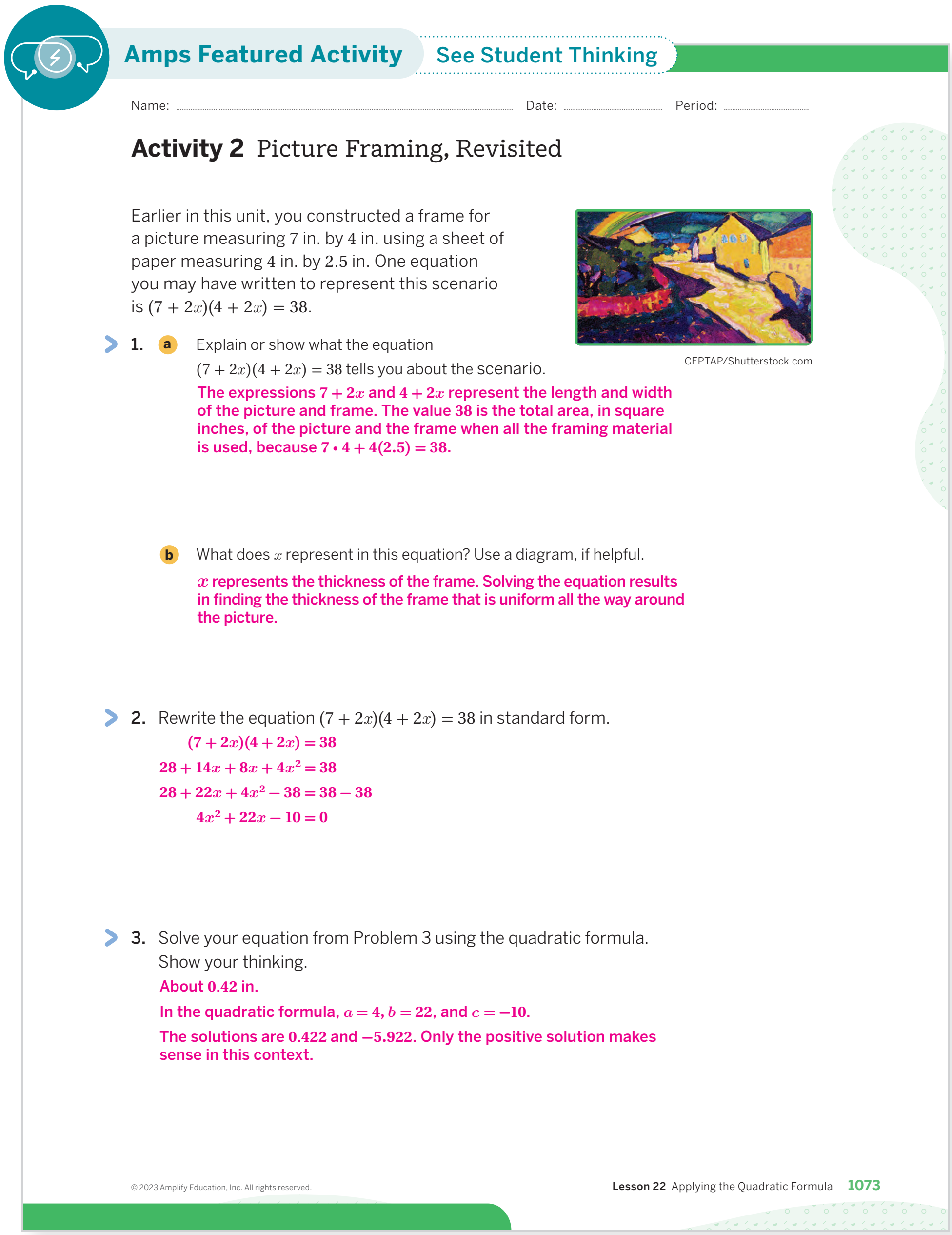
- ***Critique:*** "Do you agree with this statement? Explain your thinking." Listen for students who reason that while there are two solutions to the quadratic equation, only the positive solution makes sense in this context because x represents time and negative time does not make sense.
- ***Correct and Clarify:*** "Write a corrected statement. How would you convince someone that your statement is correct?"

Pairs | 20 min

Activity 2 Picture Framing, Revisited

Students identify the constraints in a situation, formulate a problem, construct a model, and interpret their solutions in context.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 2 Picture Framing, Revisited

Earlier in this unit, you constructed a frame for a picture measuring 7 in. by 4 in. using a sheet of paper measuring 4 in. by 2.5 in. One equation you may have written to represent this scenario is $(7 + 2x)(4 + 2x) = 38$.

CEPTAP/Shutterstock.com

1. a Explain or show what the equation $(7 + 2x)(4 + 2x) = 38$ tells you about the scenario.

   The expressions $7 + 2x$ and $4 + 2x$ represent the length and width of the picture and frame. The value 38 is the total area, in square inches, of the picture and the frame when all the framing material is used, because $7 \cdot 4 + 4(2.5) = 38$.

   b What does x represent in this equation? Use a diagram, if helpful.

   x represents the thickness of the frame. Solving the equation results in finding the thickness of the frame that is uniform all the way around the picture.

2. Rewrite the equation $(7 + 2x)(4 + 2x) = 38$ in standard form.

$$(7 + 2x)(4 + 2x) = 38$$
$$28 + 14x + 8x + 4x^2 = 38$$
$$28 + 22x + 4x^2 - 38 = 38 - 38$$
$$4x^2 + 22x - 10 = 0$$

3. Solve your equation from Problem 3 using the quadratic formula. Show your thinking.

   About 0.42 in.

   In the quadratic formula, $a = 4$, $b = 22$, and $c = -10$.

   The solutions are 0.422 and -5.922. Only the positive solution makes sense in this context.

Lesson 22 Applying the Quadratic Formula 1073

1 Launch

Display the framing problem encountered in Lesson 2. Discuss how students initially attempted to solve it and the challenges faced. Give students one minute of think-time to interpret the equation in Problem 1 before turning and talking with a partner. Discuss Problem 1 and then have students complete the activity.

2 Monitor

Help students get started by displaying sentence frames to get started such, as:

- "The two factors in the equation represent . . ."
- "The number 38 represents . . ."
- "The variable x represents . . ."

Look for points of confusion:

- **Thinking the equation can be set equal to 38 to solve using the quadratic formula.** Remind students that the equation needs to be set equal to 0.

Look for productive strategies:

- Correctly labeling the frame with the dimensions from the equation.
- Setting up the quadratic formula from the equation $4x^2 + 22x - 10 = 0$.

Activity 2 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students annotate the picture with its dimensions and include a frame around it. Ask, "What is the thickness of the frame?" x represents the thickness of the frame.

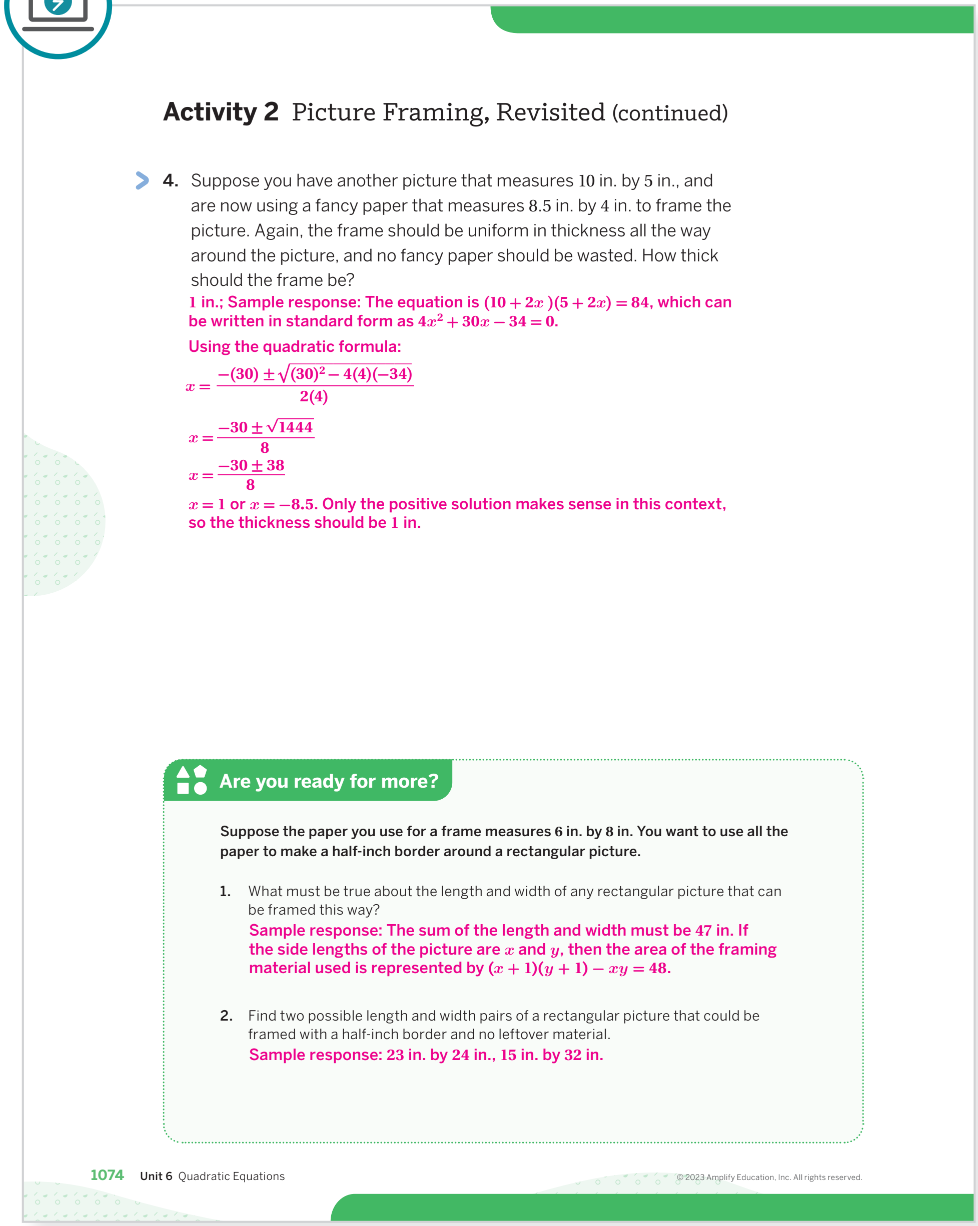
Accessibility: *Guide Processing and Visualization*

Provide students with the Anchor Chart PDF, *The Quadratic Formula*, for them to use as a reference. Provide access to colored pencils and suggest they color code the values of a, b, and c in each equation to show how those values were substituted into the formula.

Pairs | 20 min

Activity 2 Picture Framing, Revisited (continued)

Students identify the constraints in a situation, formulate a problem, construct a model, and interpret their solutions in context.



Activity 2 Picture Framing, Revisited (continued)

4. Suppose you have another picture that measures 10 in. by 5 in., and are now using a fancy paper that measures 8.5 in. by 4 in. to frame the picture. Again, the frame should be uniform in thickness all the way around the picture, and no fancy paper should be wasted. How thick should the frame be?
1 in.; Sample response: The equation is $(10+2x)(5+2x)=84$, which can be written in standard form as $4x^2+30x-34=0$.

Using the quadratic formula:

$$x=\frac{-(30)\pm\sqrt{(30)^2-4(4)(-34)}}{2(4)}$$

$$x=\frac{-30\pm\sqrt{1444}}{8}$$

$$x=\frac{-30\pm 38}{8}$$

$x=1$ or $x=-8.5$. Only the positive solution makes sense in this context, so the thickness should be 1 in.

Are you ready for more?

Suppose the paper you use for a frame measures 6 in. by 8 in. You want to use all the paper to make a half-inch border around a rectangular picture.

1. What must be true about the length and width of any rectangular picture that can be framed this way?
Sample response: The sum of the length and width must be 47 in. If the side lengths of the picture are x and y, then the area of the framing material used is represented by $(x+1)(y+1)-xy=48$.

2. Find two possible length and width pairs of a rectangular picture that could be framed with a half-inch border and no leftover material.
Sample response: 23 in. by 24 in., 15 in. by 32 in.

1074 **Unit 6** Quadratic Equations

3 Connect

Display the equation in standard form, $4x^2+22x+28=38$.

Have pairs of students share how they set up their quadratic equation to solve for the thickness of the frame.

Highlight that the quadratic equation cannot be solved until the original equation is set equal to 0.

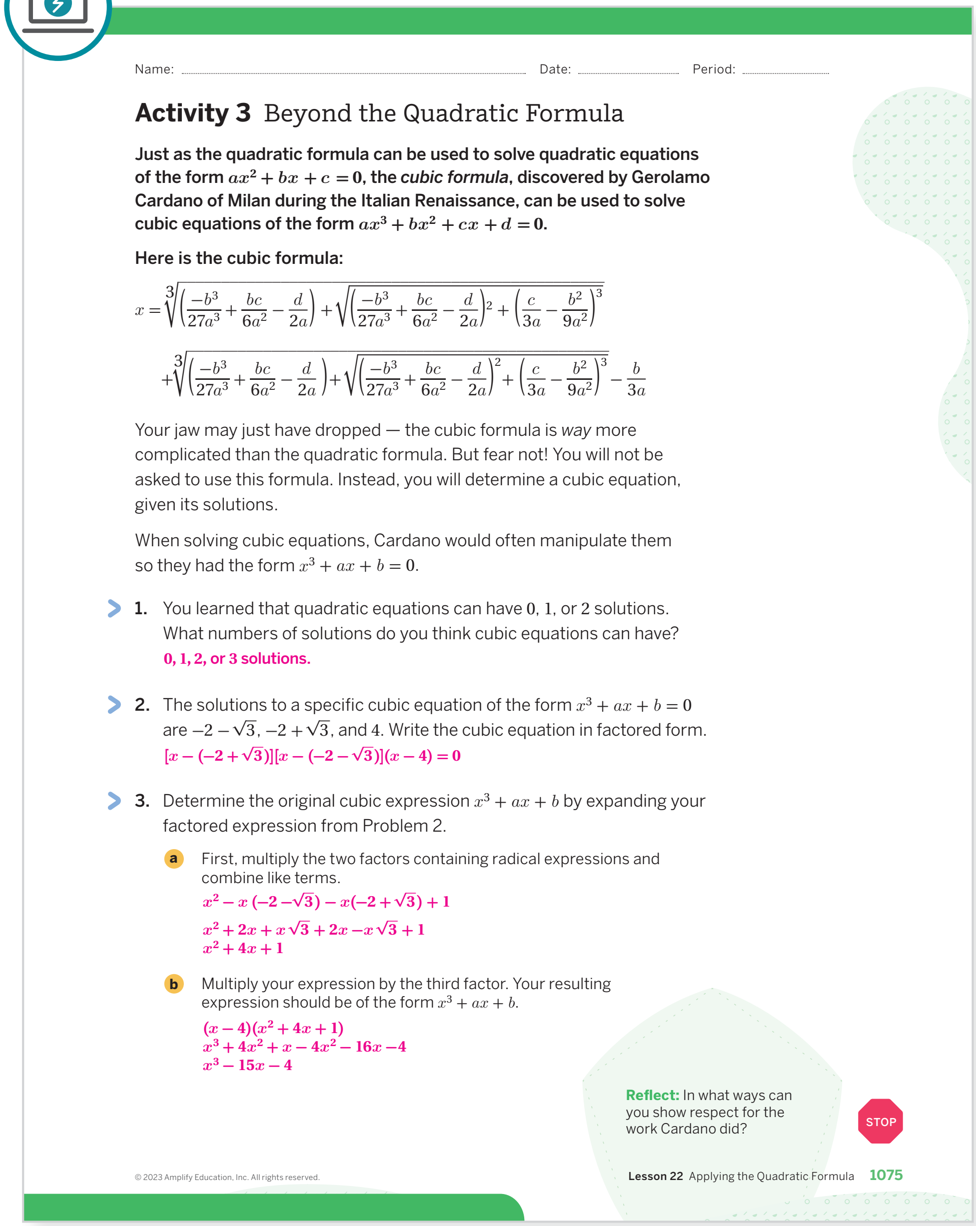
Ask, "Why would the negative solution not work in this context?" It does not make sense to have a negative thickness. Negative measurements are not possible.

Optional

Pairs | 10 min

Activity 3 Beyond the Quadratic Formula

Students use the solutions (derived from the cubic formula) to determine the original equation.



Name: .... Date: .... Period: ....

Activity 3 Beyond the Quadratic Formula

Just as the quadratic formula can be used to solve quadratic equations of the form $ax^2 + bx + c = 0$, the *cubic formula*, discovered by Gerolamo Cardano of Milan during the Italian Renaissance, can be used to solve cubic equations of the form $ax^3 + bx^2 + cx + d = 0$.

Here is the cubic formula:

$$x = \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}}$$

$$+\sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

Your jaw may just have dropped — the cubic formula is *way* more complicated than the quadratic formula. But fear not! You will not be asked to use this formula. Instead, you will determine a cubic equation, given its solutions.

When solving cubic equations, Cardano would often manipulate them so they had the form $x^3 + ax + b = 0$.

1. You learned that quadratic equations can have 0, 1, or 2 solutions. What numbers of solutions do you think cubic equations can have?
   0, 1, 2, or 3 solutions.

2. The solutions to a specific cubic equation of the form $x^3 + ax + b = 0$ are $-2 - \sqrt{3}$, $-2 + \sqrt{3}$, and 4. Write the cubic equation in factored form.
   $[x - (-2 + \sqrt{3})][x - (-2 - \sqrt{3})](x - 4) = 0$

3. Determine the original cubic expression $x^3 + ax + b$ by expanding your factored expression from Problem 2.
   - **a** First, multiply the two factors containing radical expressions and combine like terms.
     $x^2 - x(-2 - \sqrt{3}) - x(-2 + \sqrt{3}) + 1$
     $x^2 + 2x + x\sqrt{3} + 2x - x\sqrt{3} + 1$
     $x^2 + 4x + 1$
   - **b** Multiply your expression by the third factor. Your resulting expression should be of the form $x^3 + ax + b$.
     $(x - 4)(x^2 + 4x + 1)$
     $x^3 + 4x^2 + x - 4x^2 - 16x - 4$
     $x^3 - 15x - 4$

Reflect: In what ways can you show respect for the work Cardano did?

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Applying the Quadratic Formula 1075

1 Launch

Display the cubic formula. Have pairs of students work together on the problems before having a class discussion.

2 Monitor

Help students get started by comparing the equations $ax^2 + bx + c = 0$ and $ax^3 + ax + b = 0$.

Ask, "How do you know how many solutions there are in the equation?" In $ax^2 + bx + c = 0$, there are no more than two solutions.

Look for points of confusion:

- **Forgetting the signs should be opposite of their roots in the factored form.** Remind students that when factoring, the equation is equal to zero and will, in turn, makes the root the opposite of the constant in the factor.
- **Forgetting how to multiply with square roots in Problem 3.** Remind students that when multiplying with square roots, multiplying by the square root of the same value yields the value.

Look for productive strategies:

- Correctly setting up the cubic equation in factored form.
- Multiplying the expression in Problem 3 correctly.

3 Connect

Display the factored form $[x - (-2 + \sqrt{3})][x - (-2 - \sqrt{3})](x - 4) = 0$.

Have individual students share how they used the factored form to multiply the factors in Problem 3.

Highlight that the cubic formula was derived using the roots that students just determined.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

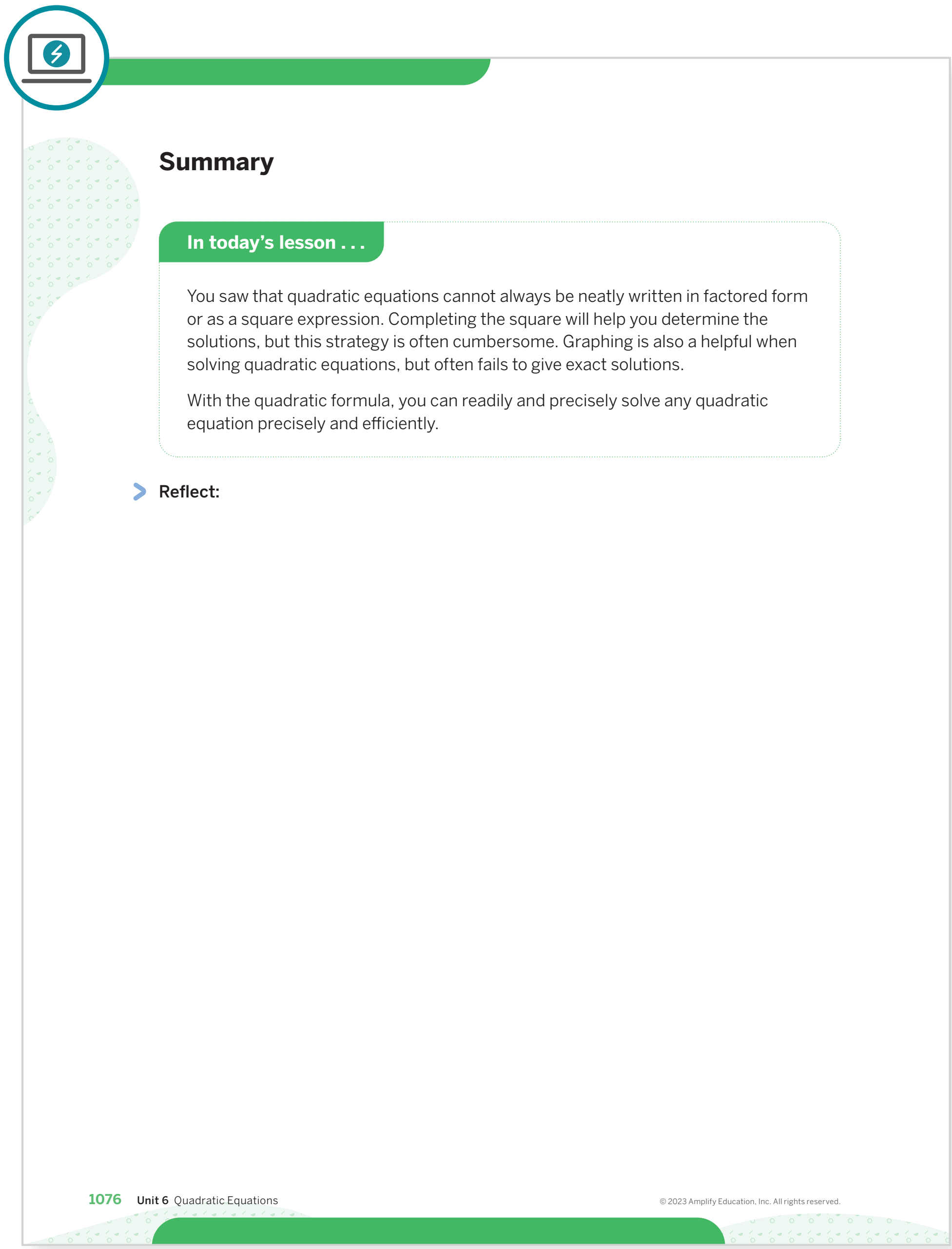
Display the quadratic formula nearby the cubic formula, or have students write the quadratic formula in the margin of their Student Edition. Ask students to compare and contrast the formulas, based on what they see. Consider asking:

- "Why do you think the quadratic formula only includes a square root, but the cubic formula includes cubic roots?" To undo a cube, you need to take the cube root.
- "What do you notice about the type of equation that can be solved by using the cubic formula, compared to the type of equation that can be solved by using the quadratic formula?" The cubic formula is used to solve cubed equations, where the greatest exponent on x is 3, not 2.

Whole Class | 5 min

Summary

Review and synthesize how the quadratic formula can be used to solve real-world problems, and how it compares to other strategies students have learned in this unit.



Summary

In today's lesson . . .

You saw that quadratic equations cannot always be neatly written in factored form or as a square expression. Completing the square will help you determine the solutions, but this strategy is often cumbersome. Graphing is also a helpful when solving quadratic equations, but often fails to give exact solutions.

With the quadratic formula, you can readily and precisely solve any quadratic equation precisely and efficiently.

> **Reflect:**

1076 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the equation $(2x + 30)(-4x + 12) = 75$.

Ask:

- "How could you identify what the values of a, b, and c are in this equation?" I could rewrite it in standard form and then identify those values.
- "What is the most efficient strategy to solve this equation?" Answers may vary.

Have students share their thinking for determining which strategy is most efficient to solve the equation. Select students using different strategies to solve from most inefficient (guess-and-check) to the most efficient (quadratic formula).

Highlight that the quadratic formula can be the most efficient and practical way to solve some equations.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

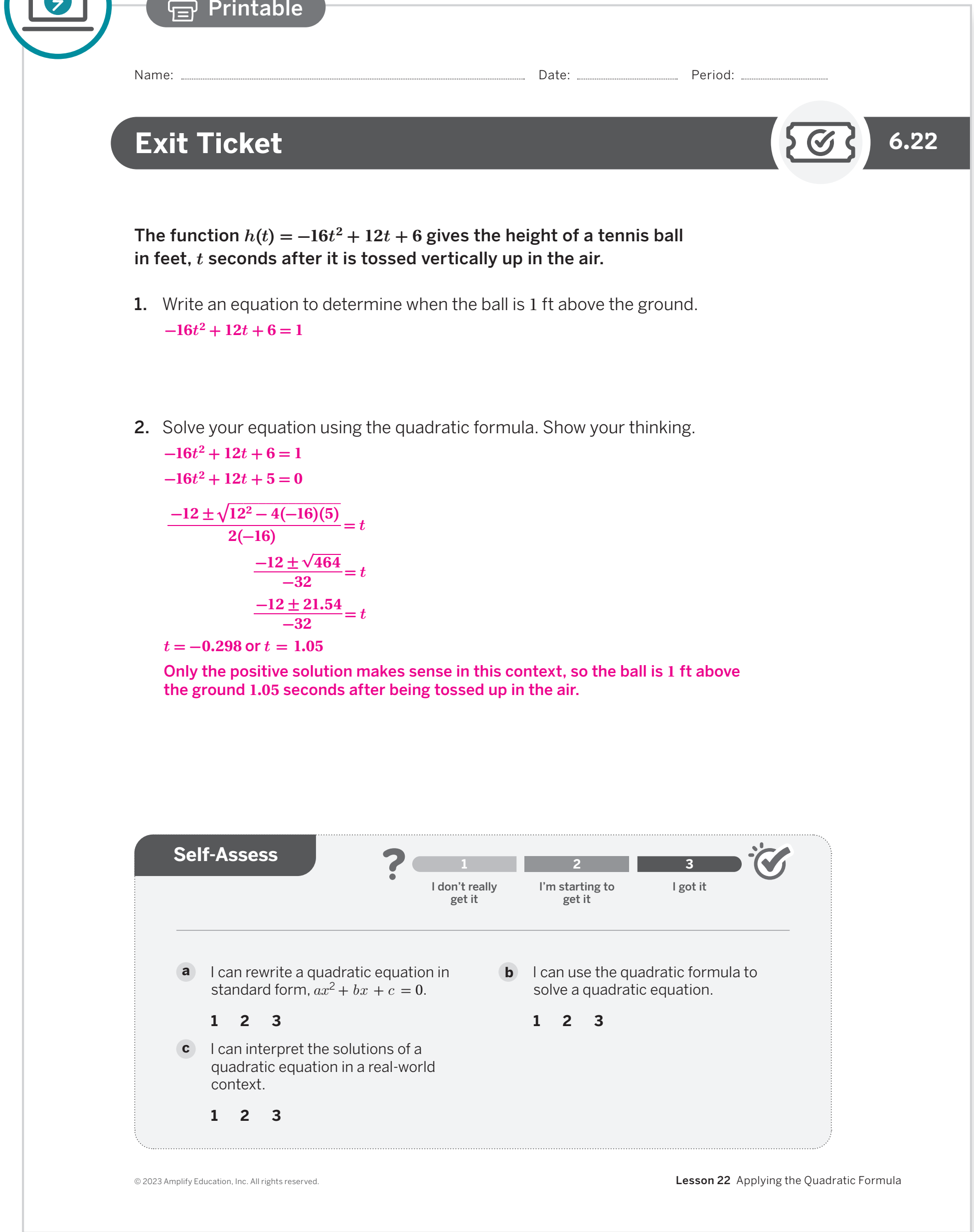
- "How did knowledge of the quadratic formula enable you to more efficiently solve problems that you encountered earlier in the unit?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by applying the quadratic formula in context.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 6.22

The function $h(t) = -16t^2 + 12t + 6$ gives the height of a tennis ball in feet, t seconds after it is tossed vertically up in the air.

1. Write an equation to determine when the ball is 1 ft above the ground.

$-16t^2 + 12t + 6 = 1$

2. Solve your equation using the quadratic formula. Show your thinking.

$-16t^2 + 12t + 6 = 1$

$-16t^2 + 12t + 5 = 0$

$$\frac{-12 \pm \sqrt{12^2 - 4(-16)(5)}}{2(-16)} = t$$

$$\frac{-12 \pm \sqrt{464}}{-32} = t$$

$$\frac{-12 \pm 21.54}{-32} = t$$

$t = -0.298$ or $t = 1.05$

Only the positive solution makes sense in this context, so the ball is 1 ft above the ground 1.05 seconds after being tossed up in the air.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can rewrite a quadratic equation in standard form, $ax^2 + bx + c = 0$.

1 2 3

b. I can use the quadratic formula to solve a quadratic equation.

1 2 3

c. I can interpret the solutions of a quadratic equation in a real-world context.

1 2 3

Lesson 22 Applying the Quadratic Formula

Success looks like . . .

- **Language Goal:** Interpreting the solutions to quadratic equations in context. **(Speaking and Listening, Writing)**
  - » Interpreting the solutions in the context of the ball toss in Problem 2.
- **Goal:** Practicing using the quadratic formula to solve quadratic equations, rearranging the equations into $ax^2 + bx + c = 0$ if not already given in this form.
  - » Solving the equation when the ball is 5 ft above the ground in Problem 2.

Suggested next steps

If students substitute 5 for t in solving the equation, consider:

- Reviewing Problem 1 from Activity 1.
- Assigning Practice Problem 2.
- Reminding students that t is the number of seconds, not the height of the ball.

If students do not rewrite the equation into standard form for solving the quadratic equation, consider:

- Reviewing Activity 1, Problem 1.

Professional Learning

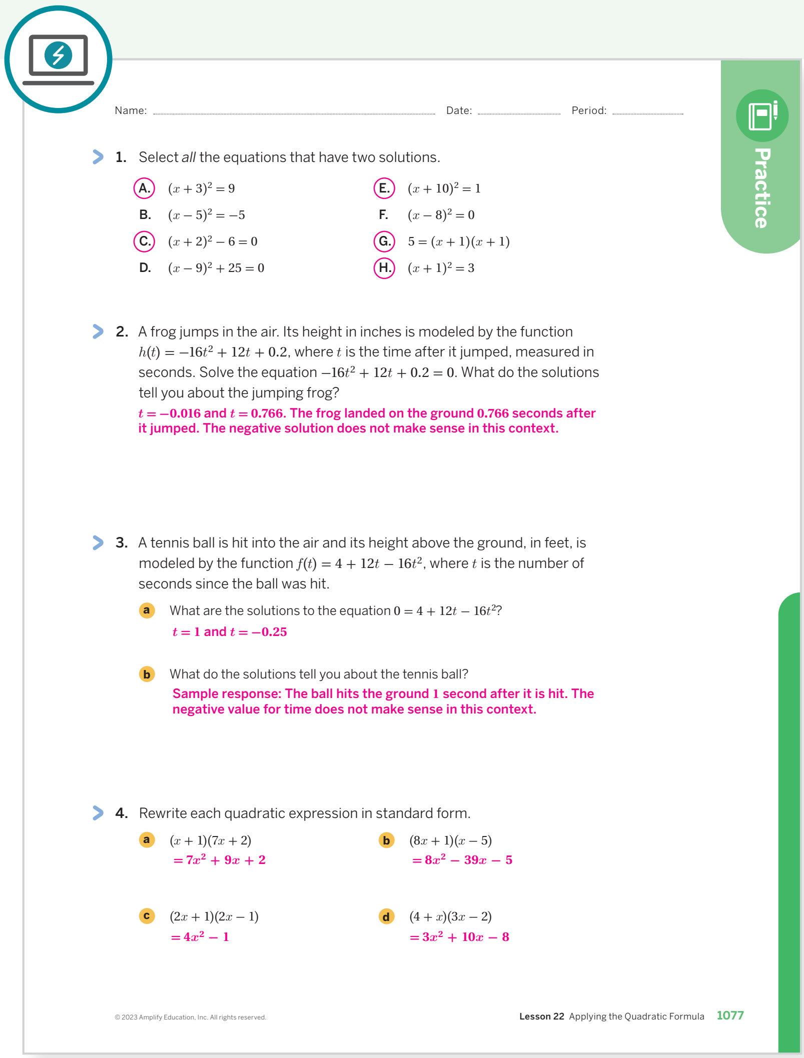
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What different ways did students approach solving the quadratic equations in Activity 1? How was this similar to or different from the ways they approached problems like these in earlier lessons?
- Students encountered the framing problem in Activity 2 in an earlier lesson. In what ways did revisiting this problem today go as planned?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Select *all* the equations that have two solutions.

[✓ A.] $(x+3)^2 = 9$

B. $(x-5)^2 = -5$

[✓ C.] $(x+2)^2 - 6 = 0$

D. $(x-9)^2 + 25 = 0$

[✓ E.] $(x+10)^2 = 1$

F. $(x-8)^2 = 0$

[✓ G.] $5 = (x+1)(x+1)$

[✓ H.] $(x+1)^2 = 3$

2. A frog jumps in the air. Its height in inches is modeled by the function $h(t) = -16t^2 + 12t + 0.2$, where t is the time after it jumped, measured in seconds. Solve the equation $-16t^2 + 12t + 0.2 = 0$. What do the solutions tell you about the jumping frog?

$t = -0.016$ and $t = 0.766$. The frog landed on the ground 0.766 seconds after it jumped. The negative solution does not make sense in this context.

3. A tennis ball is hit into the air and its height above the ground, in feet, is modeled by the function $f(t) = 4 + 12t - 16t^2$, where t is the number of seconds since the ball was hit.

a. What are the solutions to the equation $0 = 4 + 12t - 16t^2$?

$t = 1$ and $t = -0.25$

b. What do the solutions tell you about the tennis ball?

Sample response: The ball hits the ground 1 second after it is hit. The negative value for time does not make sense in this context.

4. Rewrite each quadratic expression in standard form.

a. $(x+1)(7x+2)$

$= 7x^2 + 9x + 2$

b. $(8x+1)(x-5)$

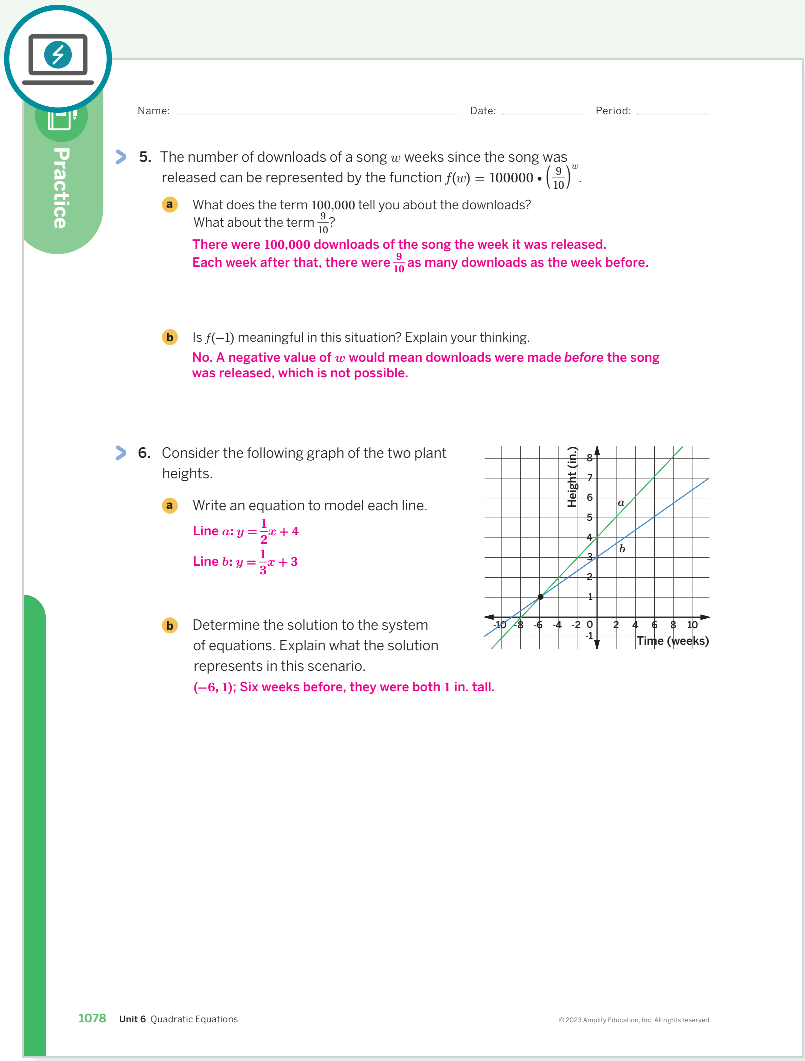
$= 8x^2 - 39x - 5$

c. $(2x+1)(2x-1)$

$= 4x^2 - 1$

d. $(4+x)(3x-2)$

$= 3x^2 + 10x - 8$



Name: ______ Date: ______ Period: ______

Practice

5. The number of downloads of a song w weeks since the song was released can be represented by the function $f(w) = 100000 \bullet \left(\frac{9}{10}\right)^w$.

a. What does the term $100,000$ tell you about the downloads? What about the term $\frac{9}{10}$?

There were 100,000 downloads of the song the week it was released. Each week after that, there were $\frac{9}{10}$ as many downloads as the week before.

b. Is $f(-1)$ meaningful in this situation? Explain your thinking.

No. A negative value of w would mean downloads were made *before* the song was released, which is not possible.

6. Consider the following graph of the two plant heights.

a. Write an equation to model each line.

Line a: $y = \frac{1}{2}x + 4$

Line b: $y = \frac{1}{3}x + 3$

b. Determine the solution to the system of equations. Explain what the solution represents in this scenario.

$(-6, 1)$; Six weeks before, they were both 1 in. tall.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	3
	3	Activity 1	3
Spiral	4	Unit 6 Lesson 6	2
	5	Unit 4 Lesson 9	3
Formative ⚡	6	Unit 6 Lesson 23	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

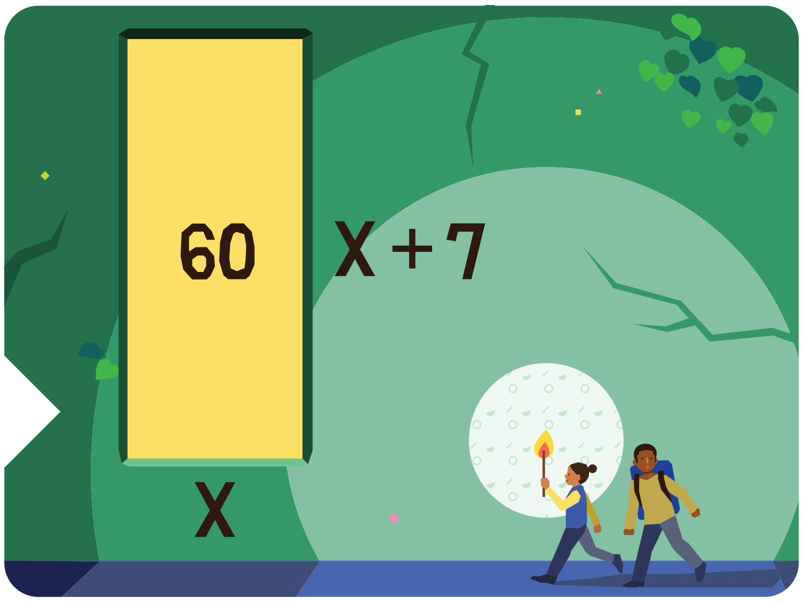
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 23**

Systems of Linear and Quadratic Equations

Let's find the intersections of the graphs of linear and quadratic functions.



Focus

Goals

1. Interpret the features of graphs and expressions that represent quadratic functions to gain information about the scenarios being modeled.
2. Write and solve systems of linear and quadratic equations to represent the constraints in a scenario.

Coherence

Today

Students synthesize strategies of solving quadratic equations and graphing quadratic functions within a context. They use tools learned throughout this unit to explore solving systems of linear and monic quadratic equations in unfamiliar scenarios.

Previously

In the previous lesson, students practiced solving monic and non-monic quadratic equations using the quadratic formula. They determined instances when it made sense or was more efficient to apply the quadratic formula rather than use other strategies.

Coming Soon

In the Capstone lesson, students culminate this unit by learning a modern strategy for solving quadratic equations and making connections between ancient and modern strategies.

Rigor

- Students **apply** their knowledge of quadratic equations to solve systems of quadratic and linear equations.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Anchor Chart PDFs:

  - *Sentence Stems, Math Talk*
  - *Solving Monic Quadratic Equations by Factoring*
  - *Solving Non-Monic Quadratic Equations by Factoring*
  - *Completing the Square*
  - *The Quadratic Formula*
- Graphic Organizer PDF, *Algebraic Connections*
- scientific calculators
- sticky notes

Math Language Development

Review words

- *linear equation*
- *slope*
- *system of equations*
- *zero product principle*

Amps | Featured Activity

Activity 3
See Student Thinking

View student explanations and sketches as they solve problems involving rectangles inscribed in parabolas.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may lack confidence as they approach Activity 2. Encourage students to use the Graphic Organizer PDF, *Algebraic Connections*, to identify what they know and make a list to determine what they want to know. Having them sort through their thinking and recognize that they do know a lot about the topic already will provide them the self-confidence to approach the task with optimism instead of doubt.

Modifications to Pacing

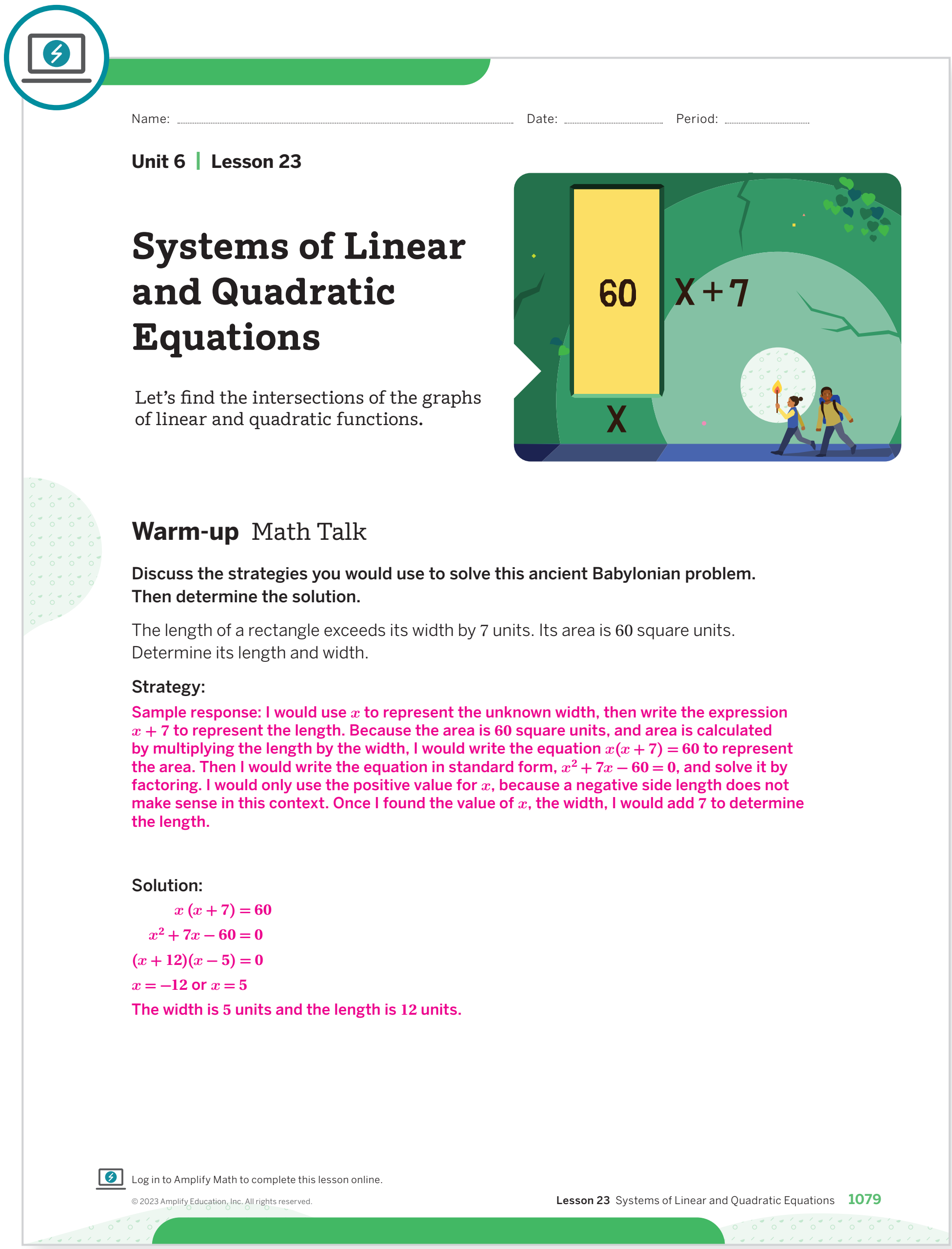
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, determining the solution may be omitted.
- In **Activity 3**, Problem 2 may be omitted.

Independent | 5 min

Warm-up Math Talk

Students discuss strategies for solving an ancient Babylonian area problem to practice writing and solving quadratic equations given certain constraints.



Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 23

Systems of Linear and Quadratic Equations

Let's find the intersections of the graphs of linear and quadratic functions.

Warm-up Math Talk

Discuss the strategies you would use to solve this ancient Babylonian problem. Then determine the solution.

The length of a rectangle exceeds its width by 7 units. Its area is 60 square units. Determine its length and width.

Strategy:

Sample response: I would use x to represent the unknown width, then write the expression $x + 7$ to represent the length. Because the area is 60 square units, and area is calculated by multiplying the length by the width, I would write the equation $x(x + 7) = 60$ to represent the area. Then I would write the equation in standard form, $x^2 + 7x - 60 = 0$, and solve it by factoring. I would only use the positive value for x, because a negative side length does not make sense in this context. Once I found the value of x, the width, I would add 7 to determine the length.

Solution:

$x\,(x + 7) = 60$

$x^2 + 7x - 60 = 0$

$(x + 12)(x - 5) = 0$

$x = -12$ or $x = 5$

The width is 5 units and the length is 12 units.

Log in to Amplify Math to complete this lesson online.

Lesson 23 Systems of Linear and Quadratic Equations 1079

1 Launch

Conduct the ***Math Talk*** routine. Give students think-time and provide a signal to use when they have a strategy and a different signal when they have a solution.

2 Monitor

Help students get started by prompting them to draw a sketch to model the scenario.

Look for points of confusion:

- **Forgetting to set the equation equal to 0.** Remind students that to apply the Zero Product Principle, the product must equal zero.

Look for productive strategies:

- Using a diagram or other visual to solve the problem.
- Using precise language when explaining their strategies.

3 Connect

Have individual students share their strategies.

Ask:

- "Who can restate ___'s thinking in a different way?"
- "Did anyone have the same strategy, but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Can anyone add on to ___'s strategy?"

Display the problem and student responses.

Highlight that the side lengths are linear and the area is quadratic. To determine the side lengths, students can model the area with a quadratic equation, which could be solved using any strategy learned in this unit.

Math Language Development

MLR8: Discussion Supports

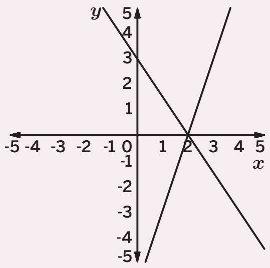
During the Connect, display or provide students the Anchor Chart PDF *Sentence Stems, Math Talk* to support students when they explain their strategy. Some students may benefit from rehearsing with a partner before sharing with the whole class.

Encourage students to draw a diagram to help them visualize the area problem.

Power-up

To power up students' ability to determine the solution to a system of linear equations from a graph, have students complete:

1. What is the solution to the system of equations shown in the graph?



   A. (0, 3)
   B. (0, 2)
   [✓ **C.**] (2, 0)
   D. (3, 3)

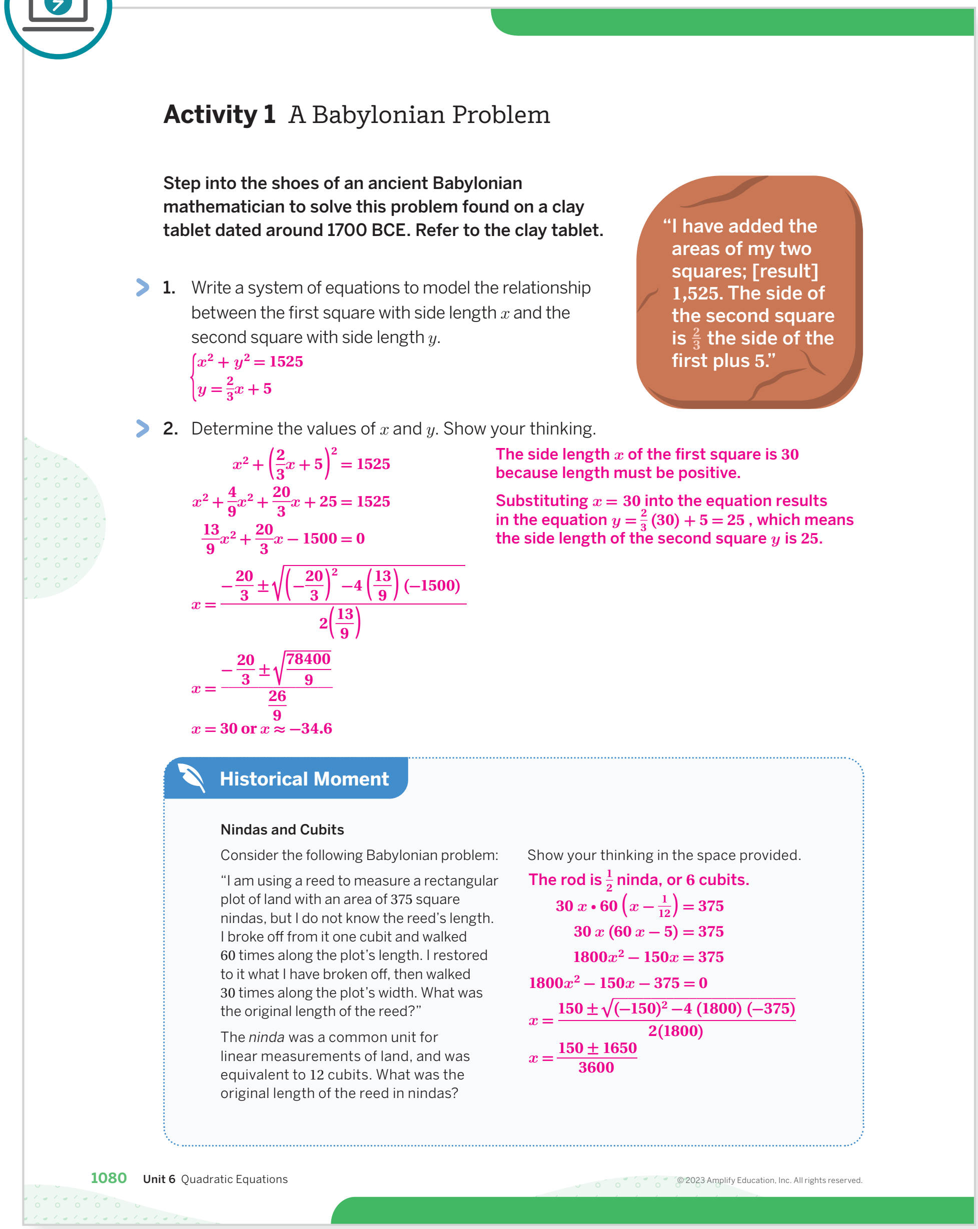
Use: Before the Warm-up

Informed by: Performance on Lesson 22, Practice Problem 6

Pairs | 15 min

Activity 1 A Babylonian Problem

Students solve an ancient Babylonian mixed system to apply learned skills for writing and solving quadratic and linear equations.



Activity 1 A Babylonian Problem

Step into the shoes of an ancient Babylonian mathematician to solve this problem found on a clay tablet dated around 1700 BCE. Refer to the clay tablet.

"I have added the areas of my two squares; [result] 1,525. The side of the second square is $\frac{2}{3}$ the side of the first plus 5."

1. Write a system of equations to model the relationship between the first square with side length x and the second square with side length y.

$$\begin{cases} x^2 + y^2 = 1525 \\ y = \frac{2}{3}x + 5 \end{cases}$$

2. Determine the values of x and y. Show your thinking.

$$x^2 + \left(\frac{2}{3}x + 5\right)^2 = 1525$$
$$x^2 + \frac{4}{9}x^2 + \frac{20}{3}x + 25 = 1525$$
$$\frac{13}{9}x^2 + \frac{20}{3}x - 1500 = 0$$
$$x = \frac{-\frac{20}{3} \pm \sqrt{\left(-\frac{20}{3}\right)^2 - 4\left(\frac{13}{9}\right)(-1500)}}{2\left(\frac{13}{9}\right)}$$
$$x = \frac{-\frac{20}{3} \pm \sqrt{\frac{78400}{9}}}{\frac{26}{9}}$$
$$x = 30 \text{ or } x \approx -34.6$$

The side length x of the first square is 30 because length must be positive.

Substituting $x = 30$ into the equation results in the equation $y = \frac{2}{3}(30) + 5 = 25$, which means the side length of the second square y is 25.

Historical Moment

Nindas and Cubits

Consider the following Babylonian problem:

"I am using a reed to measure a rectangular plot of land with an area of 375 square nindas, but I do not know the reed's length. I broke off from it one cubit and walked 60 times along the plot's length. I restored to it what I have broken off, then walked 30 times along the plot's width. What was the original length of the reed?"

The *ninda* was a common unit for linear measurements of land, and was equivalent to 12 cubits. What was the original length of the reed in nindas?

Show your thinking in the space provided.

The rod is $\frac{1}{2}$ ninda, or 6 cubits.

$$30\,x \bullet 60\left(x - \frac{1}{12}\right) = 375$$
$$30\,x\,(60\,x - 5) = 375$$
$$1800x^2 - 150x = 375$$
$$1800x^2 - 150x - 375 = 0$$
$$x = \frac{150 \pm \sqrt{(-150)^2 - 4\,(1800)\,(-375)}}{2(1800)}$$
$$x = \frac{150 \pm 1650}{3600}$$

1080 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Use the *Three Reads* routine to help students make sense of the introductory text

2 Monitor

Help students get started by prompting them to annotate each sentence written on the clay tablet using symbols.

Look for points of confusion:

- **Having difficulty working with two variables in Problem 2.** Prompt students to list the strategies learned for solving systems of linear equations with two variables (e.g., graphing, substitution, or elimination), and when each strategy is best to use.

Look for productive strategies:

- Using annotations, diagrams, or lists to analyze constraints and make sense of the problem.
- Analyzing the structure of the system to apply strategies for solving systems of linear equations.

3 Connect

Have pairs of students share their strategies and solutions or challenges they faced and how they attempted to work through them.

Highlight that a system of quadratic and linear equations can be solved using the same strategies as a system of linear equations.

Ask:

- "What other strategies could be used to solve the system? Would they be more efficient?" Answers may vary.
- "What other strategies could be used to solve the quadratic equation? Would they be more efficient?" Answers may vary.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that the scenario presented on the clay tablet involves the area of two squares.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as "The side length of the second square is $\frac{2}{3}$ the side length of the first square, plus 5."
- **Read 3:** Ask students to think about how they will write a system of equations to model the scenario.

English Learners

Sketch two squares and have students help you annotate how the side lengths compare.

Historical Moment

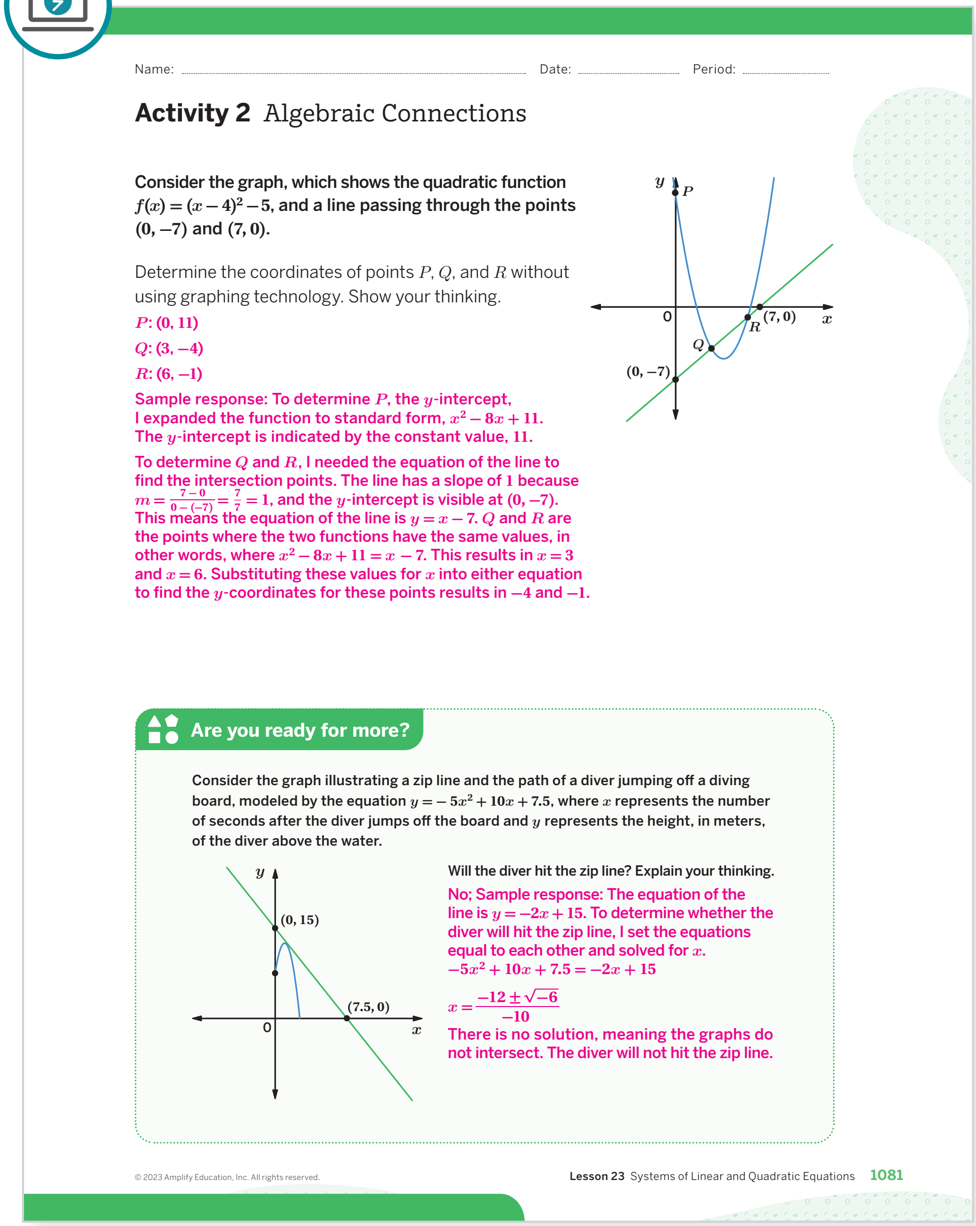
Nindas and Cubits

Have students complete the *Historical Moment* activity to explore an ancient Babylonian problem, solving for the length of a reed in nindas.

Pairs | 10 min

Activity 2 Algebraic Connections

Students apply their knowledge of linear and quadratic functions to write and solve a system of equations.



Name: Date: Period:

Activity 2 Algebraic Connections

Consider the graph, which shows the quadratic function $f(x) = (x-4)^2 - 5$, and a line passing through the points $(0, -7)$ and $(7, 0)$.

Determine the coordinates of points P, Q, and R without using graphing technology. Show your thinking.

P: $(0, 11)$

Q: $(3, -4)$

R: $(6, -1)$

Sample response: To determine P, the y-intercept, I expanded the function to standard form, $x^2 - 8x + 11$. The y-intercept is indicated by the constant value, 11.

To determine Q and R, I needed the equation of the line to find the intersection points. The line has a slope of 1 because $m = \frac{7-0}{0-(-7)} = \frac{7}{7} = 1$, and the y-intercept is visible at $(0, -7)$. This means the equation of the line is $y = x - 7$. Q and R are the points where the two functions have the same values, in other words, where $x^2 - 8x + 11 = x - 7$. This results in $x = 3$ and $x = 6$. Substituting these values for x into either equation to find the y-coordinates for these points results in -4 and -1.

Are you ready for more?

Consider the graph illustrating a zip line and the path of a diver jumping off a diving board, modeled by the equation $y = -5x^2 + 10x + 7.5$, where x represents the number of seconds after the diver jumps off the board and y represents the height, in meters, of the diver above the water.

Will the diver hit the zip line? Explain your thinking.

No; Sample response: The equation of the line is $y = -2x + 15$. To determine whether the diver will hit the zip line, I set the equations equal to each other and solved for x.

$-5x^2 + 10x + 7.5 = -2x + 15$

$x = \frac{-12 \pm \sqrt{-6}}{-10}$

There is no solution, meaning the graphs do not intersect. The diver will not hit the zip line.

Lesson 23 Systems of Linear and Quadratic Equations 1081

1 Launch

Provide students one minute of think-time, and then discuss possible strategies for solving the problem with a partner. Have students complete the activity independently and share responses with their partner.

2 Monitor

Help students get started by prompting them to list what they know about each function.

Look for points of confusion:

- **Having difficulty solving the quadratic equation $(x-4)^2 - 5 = 0$.** Have students label those solutions on the graph to see they are not the points Q and R.
- **Struggling to determine the y-coordinate of point P.** Ask, "Which graph contains this point? Does it matter which function you use to determine y?" P lies on the quadratic function, so I would use the quadratic to determine y for point P.

Look for productive strategies:

- Using annotations, diagrams, or lists to make sense of the problem.
- Rewriting the function in different forms to determine different information.

3 Connect

Have pairs of students share their strategies, solutions and any challenges the experienced.

Highlight that different information about functions can be determined through different strategies and rewriting the equations in different forms.

Ask:

- "How was vertex form useful in this problem? Factored form? Standard form?"
- "How was completing the square useful? The quadratic formula? Factoring?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display or provide students with the Graphic Organizer PDF, *Algebraic Connections*, to help them organize their thinking. The chart is similar to a KWL chart, using the categories *What I know* and *What I want to know.*

Extension: *Math Enrichment*

Have students experiment, using graphing technology, to determine the equations of a quadratic function and a linear function that intersect in only one point. Then have them determine the equations of a quadratic function and a linear function that do not intersect. Sample responses: $y = x^2 - 2$ and $y = -2$ intersect in one point; $y = x^2 - 2$ and $y = -4$ do not intersect

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete the task and share responses with their partner, have pairs meet with 1–2 other pairs of students to give and receive feedback on their strategies and solutions. Encourage reviewers to ask clarifying questions such as:

- "Which point did you decide to determine the coordinates for first? Why?"
- "How did you use the given points to determine the coordinates for points Q and R?"
- "How can you verify your solution is correct?"

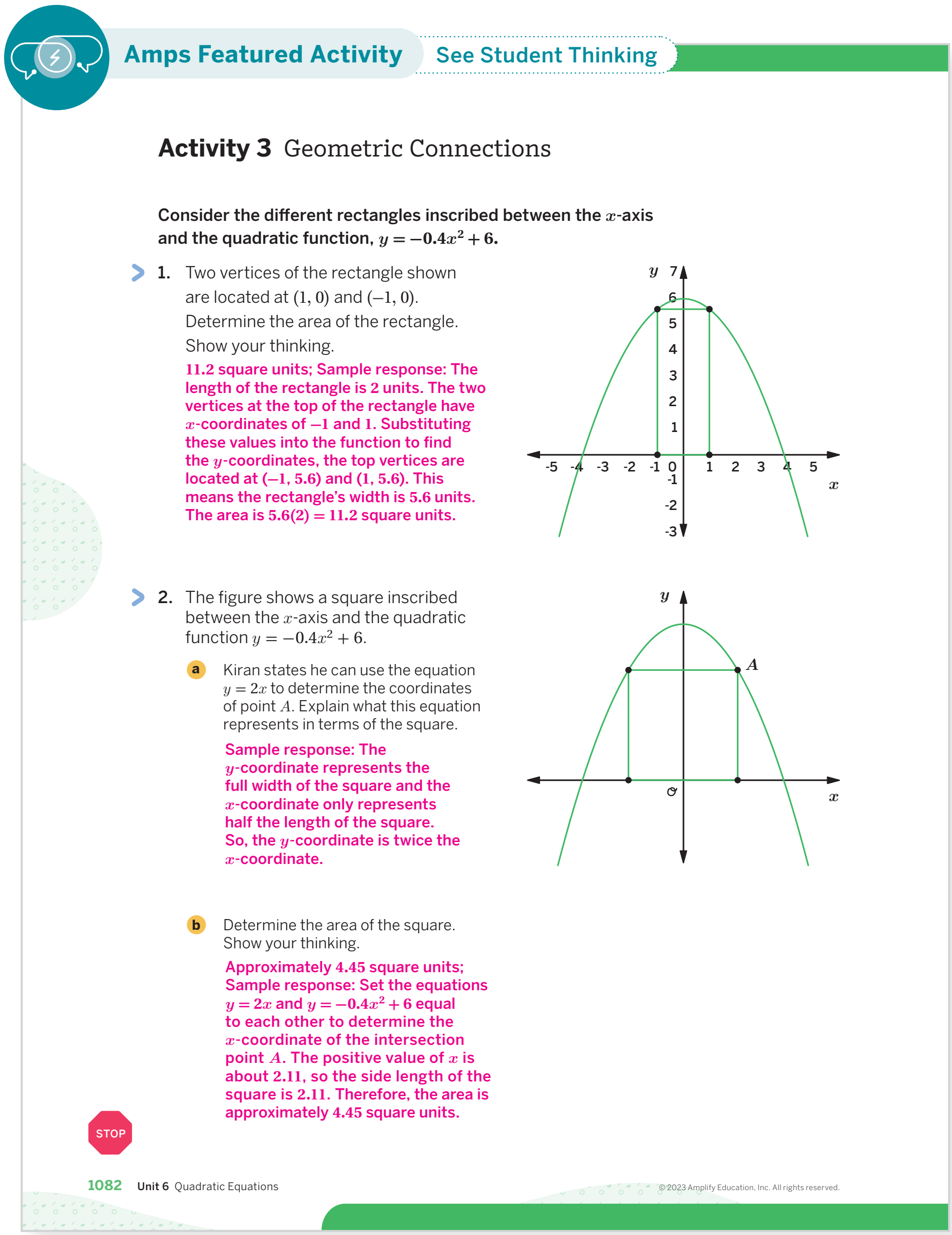
Have students revise their responses, as needed.

Pairs | 15 min

Activity 3 Geometric Connections

Students apply their knowledge of linear and quadratic functions to determine the area of rectangles inscribed in quadratic functions.

Amps Featured Activity See Student Thinking



Activity 3 Geometric Connections

Consider the different rectangles inscribed between the x-axis and the quadratic function, $y = -0.4x^2 + 6$.

1. Two vertices of the rectangle shown are located at $(1, 0)$ and $(-1, 0)$. Determine the area of the rectangle. Show your thinking.

   11.2 square units; Sample response: The length of the rectangle is 2 units. The two vertices at the top of the rectangle have x-coordinates of -1 and 1. Substituting these values into the function to find the y-coordinates, the top vertices are located at $(-1, 5.6)$ and $(1, 5.6)$. This means the rectangle's width is 5.6 units. The area is $5.6(2) = 11.2$ square units.

2. The figure shows a square inscribed between the x-axis and the quadratic function $y = -0.4x^2 + 6$.

   a Kiran states he can use the equation $y = 2x$ to determine the coordinates of point A. Explain what this equation represents in terms of the square.

   Sample response: The y-coordinate represents the full width of the square and the x-coordinate only represents half the length of the square. So, the y-coordinate is twice the x-coordinate.

   b Determine the area of the square. Show your thinking.

   Approximately 4.45 square units; Sample response: Set the equations $y = 2x$ and $y = -0.4x^2 + 6$ equal to each other to determine the x-coordinate of the intersection point A. The positive value of x is about 2.11, so the side length of the square is 2.11. Therefore, the area is approximately 4.45 square units.

STOP

1082 **Unit 6** Quadratic Equations

1 Launch

Use the *Co-craft Questions* routine. Then have student pairs discuss each problem, complete individually, and compare strategies and solutions before moving to the next problem. Provide access to scientific calculators.

2 Monitor

Help students get started by having them label the graph with everything they know and write a list of what they want to know.

Look for points of confusion:

- **Having difficulty interpreting the equation in Problem 2a.** Highlight the distance from the y-axis and x-axis to point A. Ask, "What is the relationship between these two distances and the side lengths of the square?" The distance from the y-axis to point A is half the distance from the x-axis to point A.

Look for productive strategies:

- Using annotations, color coding, or lists to make sense of the problem.
- Using the system to determine the intersection point.
- Determining the vertices to calculate the side lengths.

3 Connect

Have pairs of students share their strategies, solutions and any challenges they experienced.

Display student work and responses.

Highlight the relationship between geometry and algebra required to solve this problem.

Ask:

- "What skills and thinking were necessary to solve these problems?" Answers may vary.
- "What is another way you could have approached these problems?" Answers may vary.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide additional scaffolding by asking, "Can you determine from the graph either the length or the width of the rectangle? How can this help you determine the other dimension?"

Extension: *Math Enrichment*

Have students draw a different rectangle for Problem 1 whose vertices also touch the quadratic function and x-axis and determine the area of the rectangle they drew. Answers may vary.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the graph for Problem 1 and the introductory text, and have students work with their partner to write 2–3 mathematical questions they could ask about the graph. Have volunteers share their questions with the class. Sample questions shown.

- What is the area of this rectangle?
- Is this the largest rectangle whose vertices touch the quadratic function and the x-axis?
- How can I determine the vertices of this rectangle?

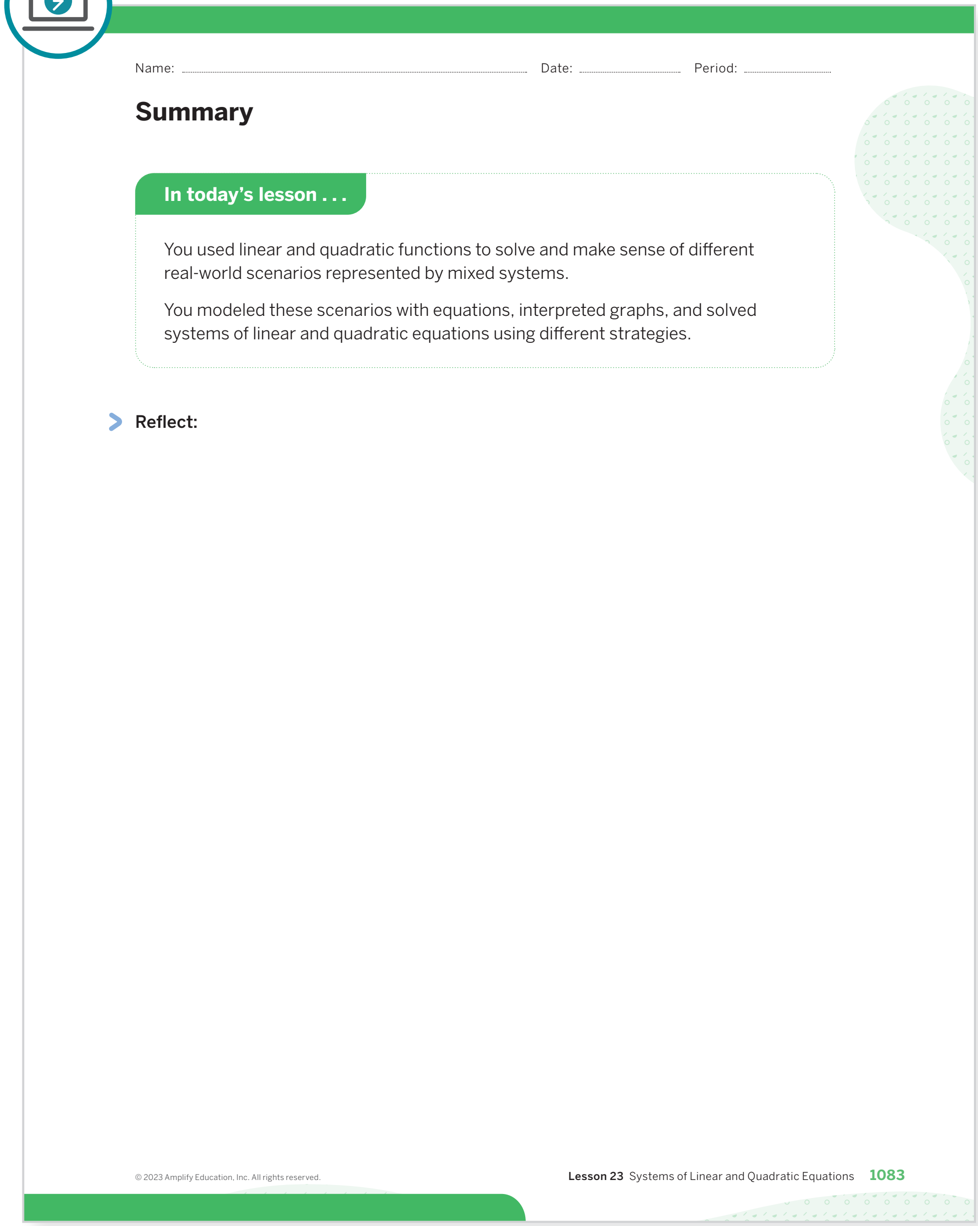
English Learners

Clarify the meaning of the word *inscribed* for this activity. Tell students it means that the vertices of the rectangle touch the quadratic function and the other two vertices of the rectangle touch the x-axis.

Whole Class | 5 min

Summary

Review and synthesize strategies for solving systems of linear and quadratic equations.



Name: Date: Period:

Summary

In today's lesson . . .

You used linear and quadratic functions to solve and make sense of different real-world scenarios represented by mixed systems.

You modeled these scenarios with equations, interpreted graphs, and solved systems of linear and quadratic equations using different strategies.

Reflect:

Synthesize

Display student work from the activities in this lesson and conduct a ***Gallery Tour***.

Have students share a question or observation on a sticky note to one or more of their classmate's solutions. Then, after reviewing their own feedback, share anything new they learned or questions they have after seeing some of their classmate's work.

Highlight that students have observed many skills and strategies to solve quadratic equations and there is no one right way to approach a problem.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

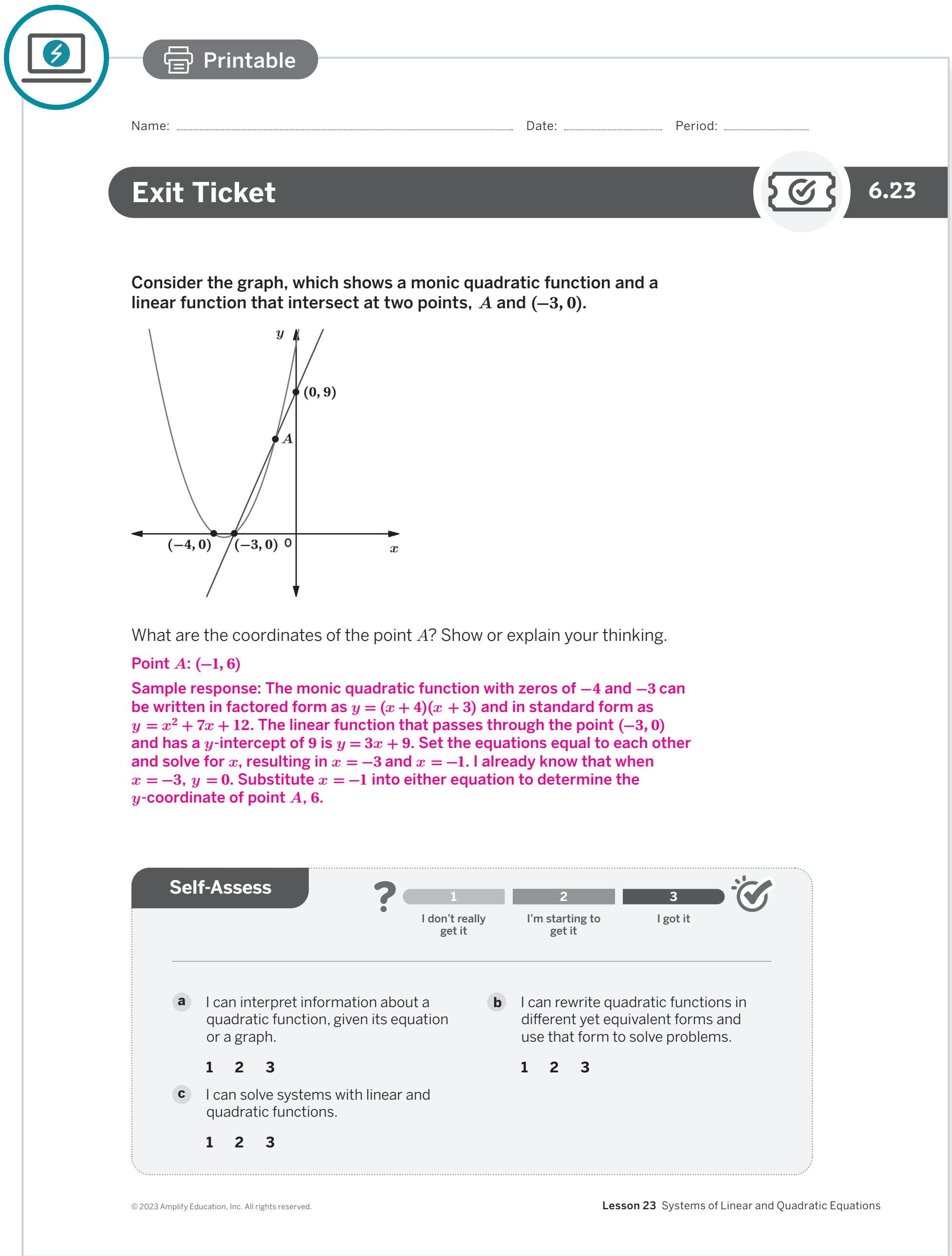
- "How is solving a quadratic-linear system similar to solving a system of linear equations? How is it different?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of quadratic equations by writing and solving a system of quadratic and linear equations.

Printable



Name: Date: Period:

Exit Ticket

6.23

Consider the graph, which shows a monic quadratic function and a linear function that intersect at two points, A and $(-3, 0)$.

What are the coordinates of the point A? Show or explain your thinking.

Point A: $(-1, 6)$

Sample response: The monic quadratic function with zeros of -4 and -3 can be written in factored form as $y = (x + 4)(x + 3)$ and in standard form as $y = x^2 + 7x + 12$. The linear function that passes through the point $(-3, 0)$ and has a y-intercept of 9 is $y = 3x + 9$. Set the equations equal to each other and solve for x, resulting in $x = -3$ and $x = -1$. I already know that when $x = -3$, $y = 0$. Substitute $x = -1$ into either equation to determine the y-coordinate of point A, 6.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can interpret information about a quadratic function, given its equation or a graph.
1 2 3

b I can rewrite quadratic functions in different yet equivalent forms and use that form to solve problems.
1 2 3

c I can solve systems with linear and quadratic functions.
1 2 3

Lesson 23 Systems of Linear and Quadratic Equations

Success looks like . . .

- **Goal:** Interpreting the features of graphs and expressions that represent quadratic functions to gain information about the scenarios being modeled.
- **Goal:** Writing and solving linear and quadratic systems to represent the constraints in a scenario.
  - » Writing a quadratic function and a linear function and then setting them equal to find the coordinates of Point A.

Suggested next steps

If students determine the incorrect quadratic or linear equation, consider:

- Assigning Practice Problem 2.

If students do not provide a coordinate point solution, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 1.
- Asking students to reread the directions and explain if their solution answers what is being asked.

Professional Learning

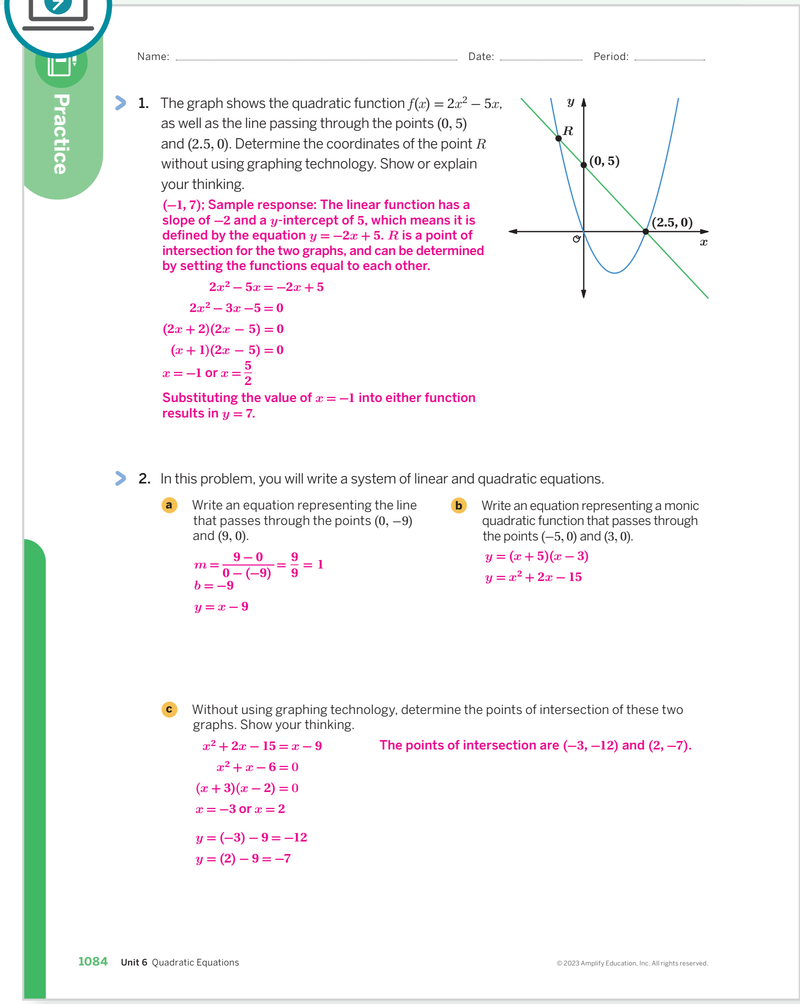
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- When you compare and contrast today's work with work students did earlier this year on solving systems, what similarites and differences do you see?
- How did the ***Gallery Tour*** during the Summary support students in determining appropriate strategies for solving systems of linear and quadratic equations?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. The graph shows the quadratic function $f(x) = 2x^2 - 5x$, as well as the line passing through the points $(0, 5)$ and $(2.5, 0)$. Determine the coordinates of the point R without using graphing technology. Show or explain your thinking.

(–1, 7); Sample response: The linear function has a slope of –2 and a y-intercept of 5, which means it is defined by the equation $y = -2x + 5$. R is a point of intersection for the two graphs, and can be determined by setting the functions equal to each other.

$$2x^2 - 5x = -2x + 5$$
$$2x^2 - 3x - 5 = 0$$
$$(2x + 2)(2x - 5) = 0$$
$$(x + 1)(2x - 5) = 0$$
$$x = -1 \text{ or } x = \frac{5}{2}$$

Substituting the value of $x = -1$ into either function results in $y = 7$.

2. In this problem, you will write a system of linear and quadratic equations.

a. Write an equation representing the line that passes through the points $(0, -9)$ and $(9, 0)$.

$$m = \frac{9 - 0}{0 - (-9)} = \frac{9}{9} = 1$$
$$b = -9$$
$$y = x - 9$$

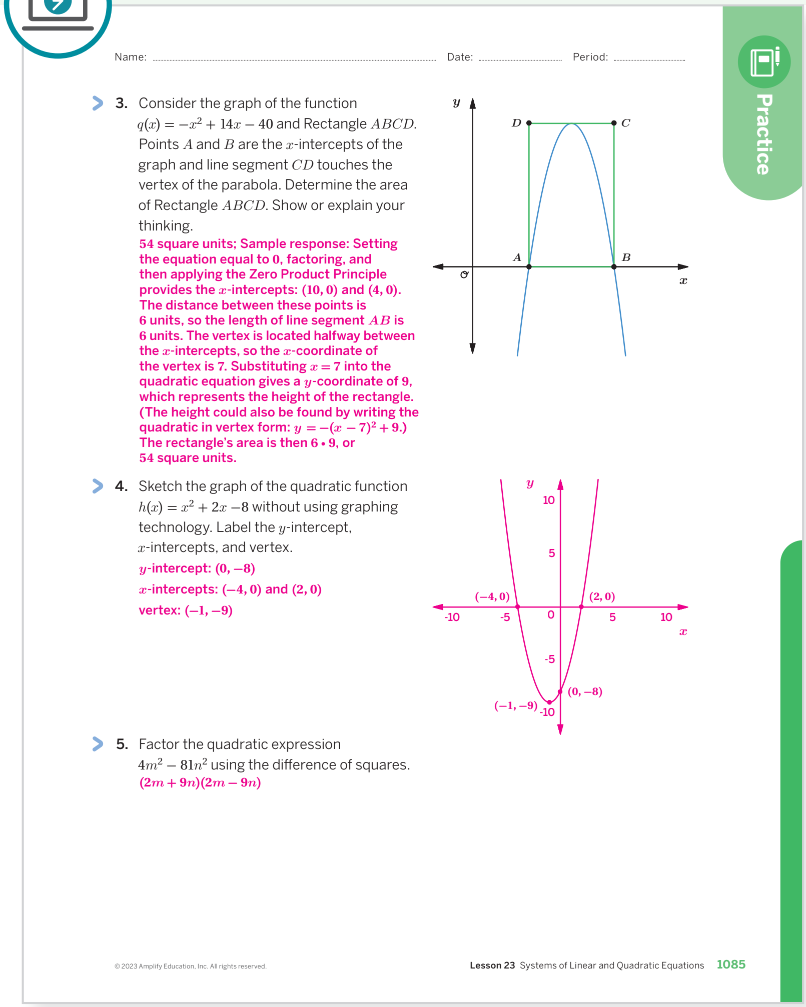
b. Write an equation representing a monic quadratic function that passes through the points $(-5, 0)$ and $(3, 0)$.

$$y = (x + 5)(x - 3)$$
$$y = x^2 + 2x - 15$$

c. Without using graphing technology, determine the points of intersection of these two graphs. Show your thinking.

$$x^2 + 2x - 15 = x - 9$$
$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x = -3 \text{ or } x = 2$$
$$y = (-3) - 9 = -12$$
$$y = (2) - 9 = -7$$

The points of intersection are (–3, –12) and (2, –7).



Practice

Name: .... Date: .... Period: ....

3. Consider the graph of the function $q(x) = -x^2 + 14x - 40$ and Rectangle $ABCD$. Points A and B are the x-intercepts of the graph and line segment CD touches the vertex of the parabola. Determine the area of Rectangle $ABCD$. Show or explain your thinking.

54 square units; Sample response: Setting the equation equal to 0, factoring, and then applying the Zero Product Principle provides the x-intercepts: (10, 0) and (4, 0). The distance between these points is 6 units, so the length of line segment AB is 6 units. The vertex is located halfway between the x-intercepts, so the x-coordinate of the vertex is 7. Substituting $x = 7$ into the quadratic equation gives a y-coordinate of 9, which represents the height of the rectangle. (The height could also be found by writing the quadratic in vertex form: $y = -(x - 7)^2 + 9$.) The rectangle's area is then 6 • 9, or 54 square units.

4. Sketch the graph of the quadratic function $h(x) = x^2 + 2x - 8$ without using graphing technology. Label the y-intercept, x-intercepts, and vertex.

y-intercept: (0, –8)

x-intercepts: (–4, 0) and (2, 0)

vertex: (–1, –9)

5. Factor the quadratic expression $4m^2 - 81n^2$ using the difference of squares.

$(2m + 9n)(2m - 9n)$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 3	3
Spiral	4	Unit 6 Lesson 14	2
Formative ⚡	5	Unit 6 Lesson 24	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 6 | **LESSON 24 – CAPSTONE**

The Latest Way to Solve Quadratic Equations

Let's explore (yet another) way to solve quadratic equations.

Focus

Goals

1. **Language Goal:** Explain how ancient approaches of solving quadratics connect to modern approaches. **(Speaking and Listening, Writing)**
2. Solve quadratic equations using Po-Shen Loh's strategy.

Coherence

Today

In today's lesson, students explore a new strategy for solving quadratic equations discovered by Dr. Po-Shen Loh of Carnegie Mellon University in 2019. Students examine the relationship between a quadratic equation and its solutions, using Viète's formula, and express the solutions in terms of their average, as ancient Babylonian mathematicians did, to determine their exact values.

Previously

In the previous lesson, students built upon their knowledge of systems of equations and solving quadratic equations as they explored quadratic-linear systems in context.

Coming Soon

In Algebra 2, students will revisit quadratic equations, among other polynomial equations, and will use some familiar strategies as well as some newer ones to graph and solve them.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Different Forms of Quadratic Expressions*

*Graphing technology should only be used for activities that specifically call for it.

Math Language Development

Review words

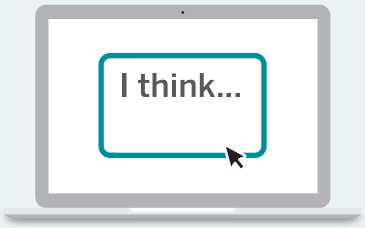
- *constant term*
- *difference of squares*
- *factored form*
- *leading coefficient*
- *linear term*
- *monic quadratic equation*
- *standard form*

Amps | Featured Activity

Activity 2

Step-by-Step Solving

Students participate in making math history as they practice solving quadratic equations using a recently discovered strategy. They can list out their steps in a dynamic table.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may initially struggle to see how or why a quadratic equation needs to be structured as described in each of the activities. Prompt students to self-assess their progress using strategies to solve quadratics that they already know. Offer authentic feedback to students when they persevere and point out how their feelings might have changed after having experienced success with this new strategy.

Modifications to Pacing

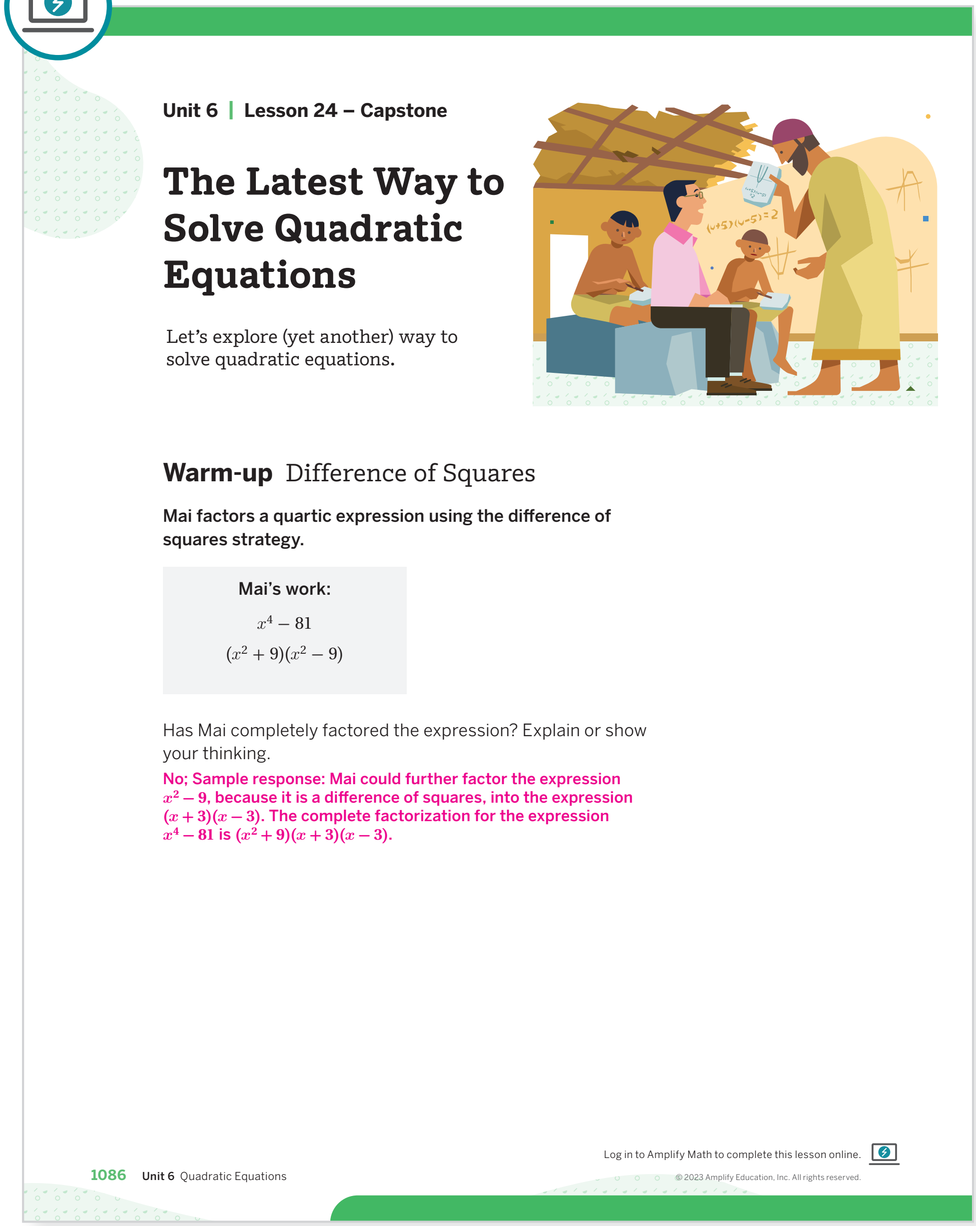
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students only complete Problems 1–4.
- In **Activity 2**, have students only complete Problems 1–4.

Independent | 5 min

Warm-up Difference of Squares

Students analyze and critique the work of a student to determine whether a quadratic expression is completely factored.



Unit 6 | **Lesson 24 – Capstone**

The Latest Way to Solve Quadratic Equations

Let's explore (yet another) way to solve quadratic equations.

Warm-up Difference of Squares

Mai factors a quartic expression using the difference of squares strategy.

Mai's work:

$$x^4 - 81$$

$$(x^2 + 9)(x^2 - 9)$$

Has Mai completely factored the expression? Explain or show your thinking.

No; Sample response: Mai could further factor the expression $x^2 - 9$, because it is a difference of squares, into the expression $(x + 3)(x - 3)$. The complete factorization for the expression $x^4 - 81$ is $(x^2 + 9)(x + 3)(x - 3)$.

Log in to Amplify Math to complete this lesson online.

1086 **Unit 6** Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide a minute of think-time and then a minute for students to ***Turn-and-Talk*** with a partner before completing the activity individually. Tell students that a *quartic expression* involves the variable term to the fourth power.

2 Monitor

Help students get started by having them take the square root of the square root of 81. Ask, "Why is it possible to take the square root twice?" It is always possible to take a square root of a number, as long as that number is positive.

Look for points of confusion:

- **Thinking $x^2 + 9$ is factorable.** Have students set $x^2 + 9$ equal to 0 and try solving for x. Ask, "What does its solution imply?" The quadratic cannot be factored.

Look for productive strategies:

- Expanding $(x + 3)(x - 3)(x^2 + 9)$ to check whether it produces the original expression.
- Identifying $x^2 - 9$ as a difference of squares.

3 Connect

Display Mai's work. ***Poll the Class*** and ask, "How many of you agree that Mai completely factored the expression? How many of you disagree?"

Have individual students share their thinking. Select and sequence responses by having students who agree with Mai share first.

Highlight that an expression is factored completely when it can no longer be factored. At times, an expression might be able to be factored more than once, using one or more of the factoring strategies learned. Note that the skill of factoring a difference of squares will be used in Activity 2.

Math Language Development

MLR3: Critique, Correct, Clarify

This Warm-up is structured similarly to the MLR3: *Critique, Correct,* and *Clarify routine*. During the Connect, as students share whether they agree or disagree, press them for details in their reasoning by asking:

- ***Critique:*** "Is Mai's work correct, incorrect, complete, or incomplete? Tell me as many of these words that you think apply to her work." Listen for students who reason that while Mai's work is correct, it is incomplete because $x^2 - 9$ is also a difference of squares, which can be factored as $(x + 3)(x - 3)$.
- ***Correct and Clarify:*** "How would you explain to Mai that her work is not complete? What math language can you use?"

Power-up

To power up students' ability to recognize and factor the difference of squares in non-monic quadratic expressions, have students complete:

Recall that $a^2 - b^2 = (a - b)(a + b)$. Rewrite each quadratic expression using the difference of squares.

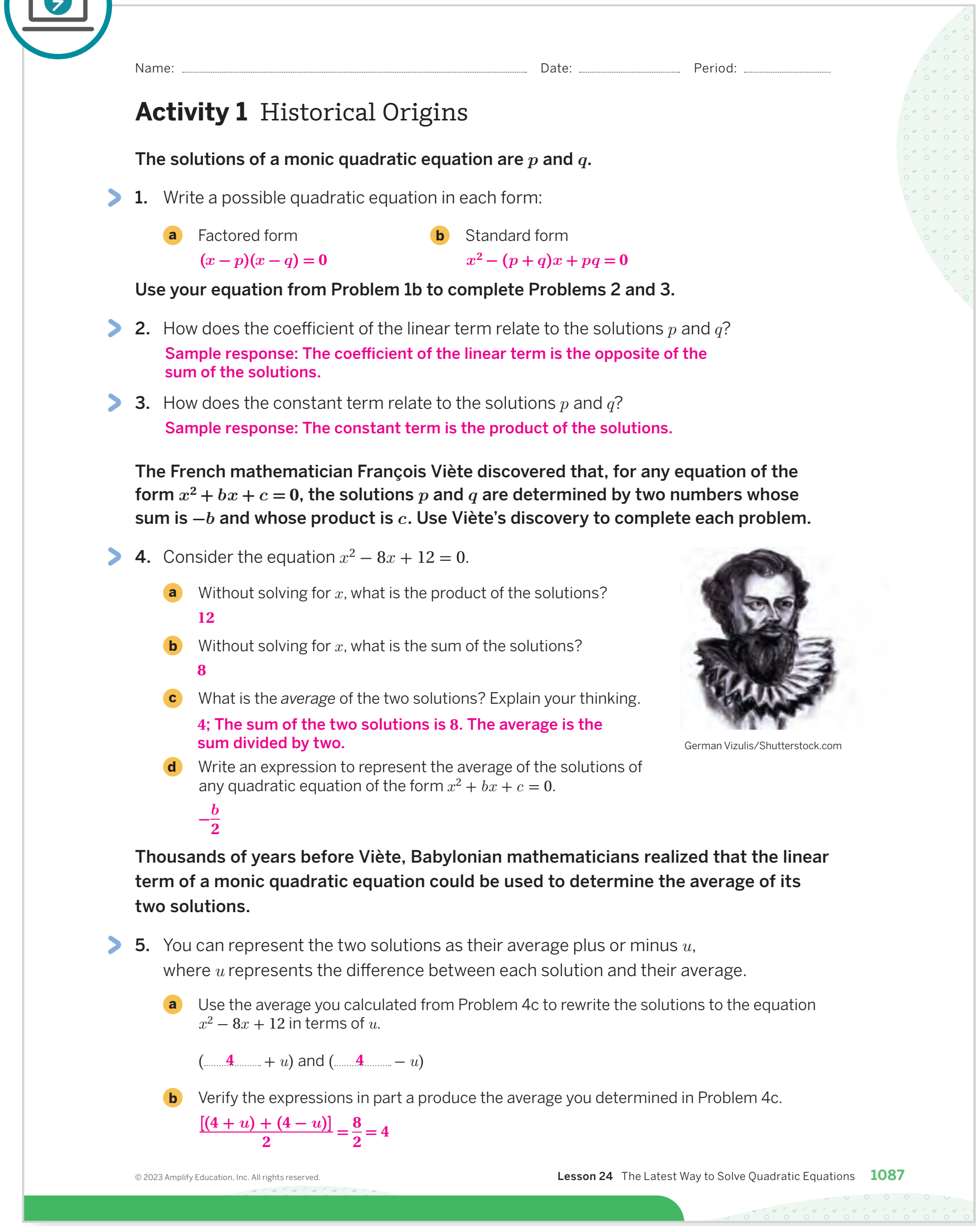
1. $x^2 - 9$ $(x + 3)(x - 3)$
2. $4m^2 - 25$ $(2m + 5)(2m - 5)$
3. $144 - 16p^2$ $(12 + 4p)(12 - 4p)$
4. $49a^2 - 81b^2$ $(7a + 9b)(7a - 9b)$

Use: Before the Warm-up
Informed by: Performance on Lesson 23, Practice Problem 5

Pairs | 15 min

Activity 1 Historical Origins

Students use the solutions of an equation to determine the relationship between the linear and constant terms of a quadratic equation.



Name: .... Date: .... Period: ....

Activity 1 Historical Origins

The solutions of a monic quadratic equation are p and q.

1. Write a possible quadratic equation in each form:

   a. Factored form

   $(x - p)(x - q) = 0$

   b. Standard form

   $x^2 - (p + q)x + pq = 0$

Use your equation from Problem 1b to complete Problems 2 and 3.

2. How does the coefficient of the linear term relate to the solutions p and q?

   Sample response: The coefficient of the linear term is the opposite of the sum of the solutions.

3. How does the constant term relate to the solutions p and q?

   Sample response: The constant term is the product of the solutions.

The French mathematician François Viète discovered that, for any equation of the form $x^2 + bx + c = 0$, the solutions p and q are determined by two numbers whose sum is $-b$ and whose product is c. Use Viète's discovery to complete each problem.

4. Consider the equation $x^2 - 8x + 12 = 0$.

   a. Without solving for x, what is the product of the solutions?

   12

   b. Without solving for x, what is the sum of the solutions?

   8

   c. What is the *average* of the two solutions? Explain your thinking.

   4; The sum of the two solutions is 8. The average is the sum divided by two.

   d. Write an expression to represent the average of the solutions of any quadratic equation of the form $x^2 + bx + c = 0$.

   $-\frac{b}{2}$

German Vizulis/Shutterstock.com

Thousands of years before Viète, Babylonian mathematicians realized that the linear term of a monic quadratic equation could be used to determine the average of its two solutions.

5. You can represent the two solutions as their average plus or minus u, where u represents the difference between each solution and their average.

   a. Use the average you calculated from Problem 4c to rewrite the solutions to the equation $x^2 - 8x + 12$ in terms of u.

   (....4.... $+ u$) and (....4.... $- u$)

   b. Verify the expressions in part a produce the average you determined in Problem 4c.

   $\frac{[(4 + u) + (4 - u)]}{2} = \frac{8}{2} = 4$

1 Launch

Have students write the factored form and standard form of a quadratic equation if -2 and 4 are the solutions. $(x + 2)(x - 4) = 0$

2 Monitor

Help students get started by prompting them to write the general form of a factored expression for Problem 1a, and to factor x from the linear terms in Problem 1b.

Look for points of confusion:

- **Thinking the linear term is the sum of the solutions in Problem 2.** Prompt students to note the sign in front of the linear term after they have factored out x.
- **Attempting to determine the factors of the constant.** Remind students Viète considered the solutions to the equations, not the factors.

Look for productive strategies:

- Factoring x from $-qx - px$ in $x^2 + -qx - px + pq$.
- Labeling -8 as b in the quadratic equation.

3 Connect

Have student pairs share their equations for Problem 1. Have students explain why the linear term is the opposite of the sum of the solutions.

Display the equation from Problem 4 and have students determine the solutions. Select students to verify that the product of the solutions is c, the sum of the solutions is $-b$, and the average of the solutions is $-\frac{b}{2}$.

Highlight that the solutions can be written in terms of the average, modeling how to do so by writing $4 + u$ and $4 - u$ and showing that their average is, in fact, 4.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the Anchor Chart PDF, *Different Forms of Quadratic Expressions*, for students to use as a reference in this activity. Ask these questions to help students complete Problem 1:

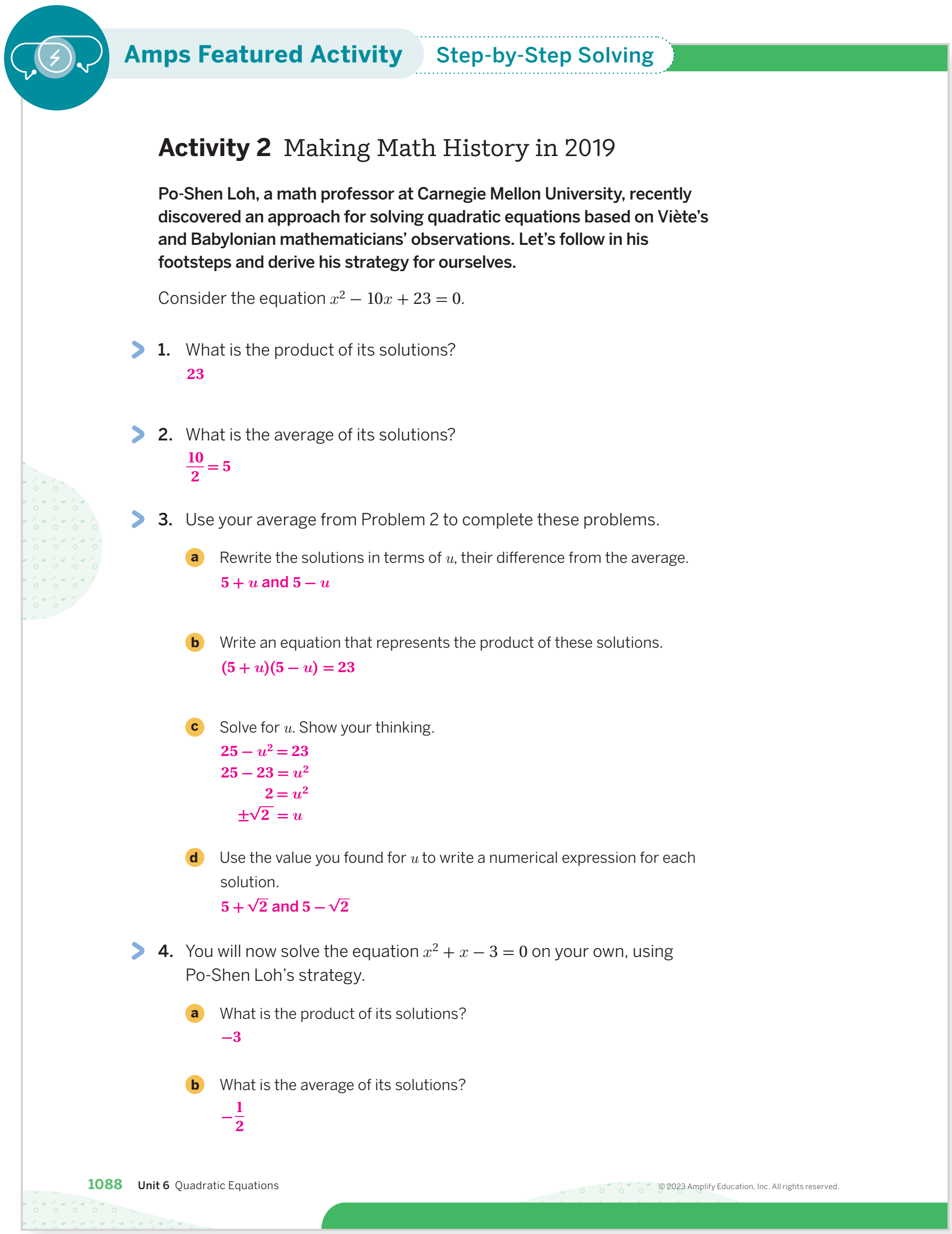
- "How does factored form show the solutions? What must the equation be set equal to?"
- "Once you know the factored form, how can you write the standard form?"

If students write $x^2 - px - qx + pq = 0$ for standard form, ask, "Look at the two linear terms. Can you combine these into one term by using the Distributive Property?"

Pairs | 20 min

Activity 2 Making Math History in 2019

Students derive Po-Shen Loh's strategy to learn a systematic strategy for solving any quadratic equation.



Amps Featured Activity Step-by-Step Solving

Activity 2 Making Math History in 2019

Po-Shen Loh, a math professor at Carnegie Mellon University, recently discovered an approach for solving quadratic equations based on Viète's and Babylonian mathematicians' observations. Let's follow in his footsteps and derive his strategy for ourselves.

Consider the equation $x^2 - 10x + 23 = 0$.

1. What is the product of its solutions?
   23
2. What is the average of its solutions?
   $\frac{10}{2} = 5$
3. Use your average from Problem 2 to complete these problems.
   - a. Rewrite the solutions in terms of u, their difference from the average.
     $5 + u$ and $5 - u$
   - b. Write an equation that represents the product of these solutions.
     $(5 + u)(5 - u) = 23$
   - c. Solve for u. Show your thinking.
     $$\begin{aligned} 25 - u^2 &= 23 \\ 25 - 23 &= u^2 \\ 2 &= u^2 \\ \pm\sqrt{2} &= u \end{aligned}$$
   - d. Use the value you found for u to write a numerical expression for each solution.
     $5 + \sqrt{2}$ and $5 - \sqrt{2}$
4. You will now solve the equation $x^2 + x - 3 = 0$ on your own, using Po-Shen Loh's strategy.
   - a. What is the product of its solutions?
     -3
   - b. What is the average of its solutions?
     $-\frac{1}{2}$

1088 Unit 6 Quadratic Equations

1 Launch

Read the Featured Mathematician aloud. Point out that Dr. Loh used Viète's formula and ancient Babylonian mathematicians' knowledge of averages to develop a new strategy. Ask, "What strategy might you discover?"

2 Monitor

Help students get started by reminding them that the linear term is the opposite of the sum of the solutions.

Look for points of confusion:

- **Having difficulty representing the solutions in terms of u.** Ask students what the average is. Then refer students to Problem 5 of Activity 1.
- **Struggling to write an equation to represent the product of the solutions.** Remind students that each expression written in terms of u represents a solution.
- **Not understanding the use of the positive and negative values of u.** Prompt students to use either value of u. They will notice both values produce the same responses.
- **Having difficulty eliminating the leading coefficient in Problem 5.** Ask students what they need to do to get $a = 1$.

Look for productive strategies:

- Applying the structure of difference of squares to determine the product of the expressions written in terms of u.
- Denoting the values of c and $-\frac{b}{2}$.
- Generalizing Po-Shen Loh's strategy in Problem 2.
- Noticing that $-\frac{b}{2}$, which represents the average, is part of the quadratic formula when $a = 1$.
- Noticing that $-\frac{b}{2}$, which represents the average, is the same formula to determine the axis of symmetry for a monic quadratic equation.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the following that students can use as a reference when applying Po-Shen Loh's strategy.

(average + u)(average − u) = c

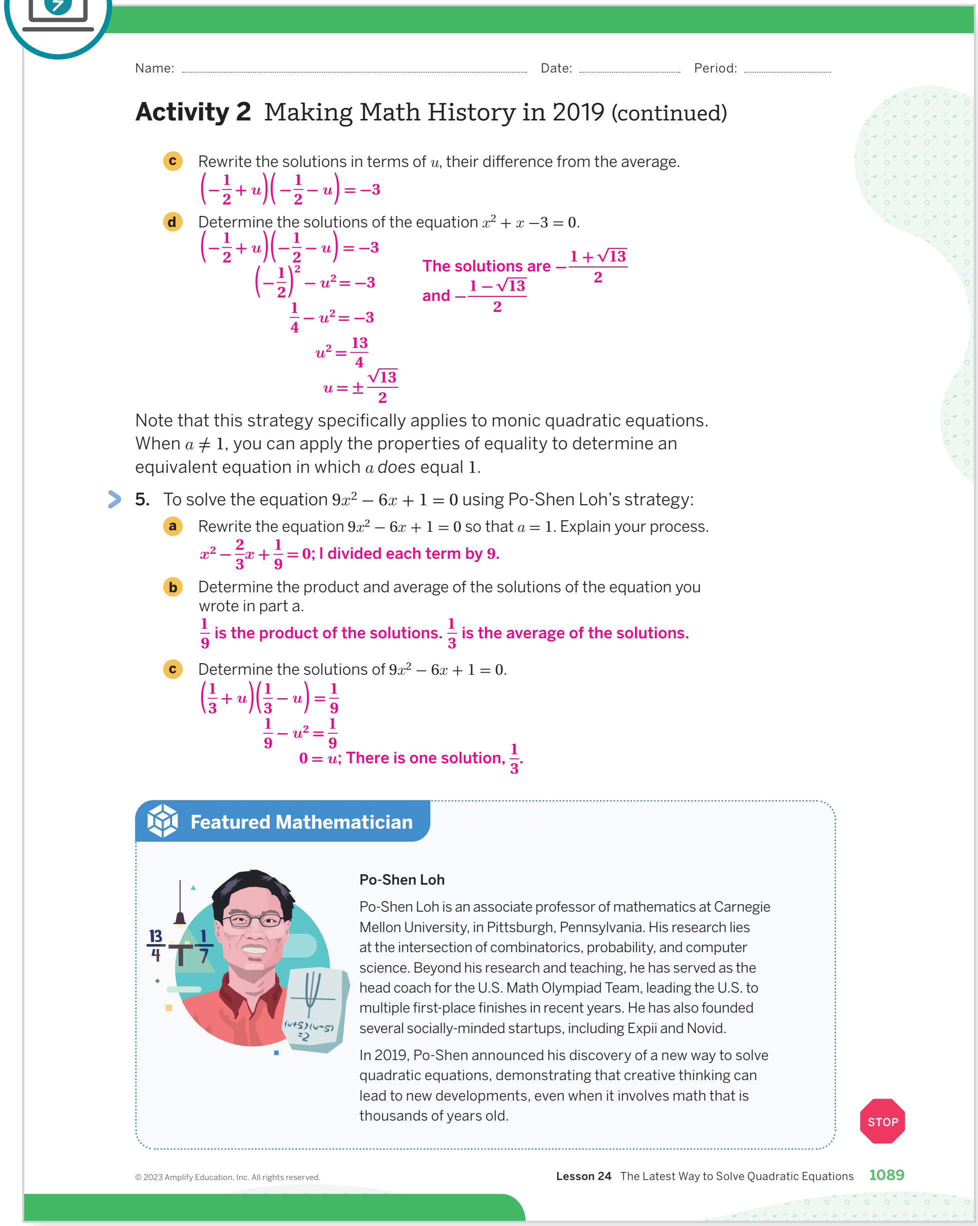
Extension: *Math Enrichment*

Have students verify the solutions they determined for the quadratic equations in this activity using one of the other strategies they have studied in this unit, such as the quadratic formula.

Pairs | 20 min

Activity 2 Making Math History in 2019 (continued)

Students derive Po-Shen Loh's strategy to learn a systematic strategy for solving any quadratic equation.



Name: ________ Date: ________ Period: ________

Activity 2 Making Math History in 2019 (continued)

c Rewrite the solutions in terms of u, their difference from the average.

$$\left(-\frac{1}{2}+u\right)\left(-\frac{1}{2}-u\right)=-3$$

d Determine the solutions of the equation $x^2 + x - 3 = 0$.

$$\left(-\frac{1}{2}+u\right)\left(-\frac{1}{2}-u\right)=-3$$
$$\left(-\frac{1}{2}\right)^2-u^2=-3$$
$$\frac{1}{4}-u^2=-3$$
$$u^2=\frac{13}{4}$$
$$u=\pm\frac{\sqrt{13}}{2}$$

The solutions are $-\frac{1+\sqrt{13}}{2}$ and $-\frac{1-\sqrt{13}}{2}$

Note that this strategy specifically applies to monic quadratic equations. When $a \neq 1$, you can apply the properties of equality to determine an equivalent equation in which a *does* equal 1.

5. To solve the equation $9x^2 - 6x + 1 = 0$ using Po-Shen Loh's strategy:

a Rewrite the equation $9x^2 - 6x + 1 = 0$ so that $a = 1$. Explain your process.

$x^2-\frac{2}{3}x+\frac{1}{9}=0$; I divided each term by 9.

b Determine the product and average of the solutions of the equation you wrote in part a.

$\frac{1}{9}$ is the product of the solutions. $\frac{1}{3}$ is the average of the solutions.

c Determine the solutions of $9x^2 - 6x + 1 = 0$.

$$\left(\frac{1}{3}+u\right)\left(\frac{1}{3}-u\right)=\frac{1}{9}$$
$$\frac{1}{9}-u^2=\frac{1}{9}$$

$0 = u$; There is one solution, $\frac{1}{3}$.

Featured Mathematician

Po-Shen Loh

Po-Shen Loh is an associate professor of mathematics at Carnegie Mellon University, in Pittsburgh, Pennsylvania. His research lies at the intersection of combinatorics, probability, and computer science. Beyond his research and teaching, he has served as the head coach for the U.S. Math Olympiad Team, leading the U.S. to multiple first-place finishes in recent years. He has also founded several socially-minded startups, including Expii and Novid.

In 2019, Po-Shen announced his discovery of a new way to solve quadratic equations, demonstrating that creative thinking can lead to new developments, even when it involves math that is thousands of years old.

STOP

Lesson 24 The Latest Way to Solve Quadratic Equations 1089

3 Connect

Display each equation from the activity.

Have student pairs share and model their steps for solving each equation. Encourage students to be as specific as possible about their values.

Ask:

- "What did you notice when you multiplied the solutions expressed in terms of u?" The product is always a difference of squares.
- "What happens if you used the negative root of u to write the solution? Does it change your solution?" Using the negative root of u yields the same solutions as using the positive root. It does not change the solution.
- "How could you check that your solutions are correct?" By applying the quadratic formula, or by substitution for Problem 3.

Display Problem 5.

Highlight that Po-Shen Loh's strategy focuses on the average of the solutions, which can be represented by $-\frac{b}{2}+u$ and $-\frac{b}{2}-u$. When these expressions are multiplied, the product will always yield a difference of squares, which enables them to find an exact value of u. Loh's strategy can be used to determine the solutions of any quadratic equation.

Featured Mathematician

Po-Shen Loh

Have students read about featured mathematician Po-Shen Loh, who developed a new method for solving quadratic equations.

Whole Class | 5 min

Unit Summary

Review and synthesize the historical origins of Po-Shen Loh's strategy for solving quadratic equations.

Narrative Connections

Unit Summary

You can solve most problems — both in math and out in the world — through different approaches. And, as you saw throughout this unit, quadratic equations are no different.

First, you journeyed back thousands of years to ancient Egypt, where the annual flooding of the Nile led to the development of a system for efficiently calculating the area of different plots of land.

Now some equations you can solve by factoring — but certainly not all of them. So we pick up our story almost a thousand years later in Mesopotamia and ancient China, who tackled quadratics through a method called "completing the square." It is a phrase that gets thrown around a lot in algebra courses. But it really is just that — finding the missing piece of a regular, bread-and-butter square. That way, the quadratic is changed into a simpler squared expression. Where there used to be two x's floating around in your equation, now there is just one!

But the greatest triumph was still to come. With a firm foundation in Indian mathematics, Arab thinkers developed the quadratic formula — a skeleton key that may be a pain to memorize, but solves any quadratic equation for you in a pinch.

Much of this might look like ancient history, but it is not. It is the start of a long dialogue between different people, not least of all yourselves. Just today, you saw a modern cousin of the quadratic formula — Po-Shen Loh's method — which takes advantage of the symmetry between a quadratic equation's roots.

Math is more than just numbers. It is a conversation, an exchange of ideas. Some of these ideas took centuries to develop, surviving through conflict, war, and superstition. But in spite of it all, people learned from each other, incorporated each other's ideas, and moved forward a conversation that we continue to this day.

See you next year.

1090 Unit 6 Quadratic Equations

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Ask:

- "What or who inspired Po-Shen Loh's strategy for solving quadratic equations?" Ancient Babylonian mathematicians and Viète, a French mathematician.
- "What information is needed to use Po-Shen Loh's strategy?" The product and the average of the solutions.
- "How is the average of the solutions determined?" The opposite of the sum of the solutions $(-b)$ divided by 2.
- "How do you represent the solutions algebraically using Po-Shen Loh's strategy?" $-\frac{b}{2} \pm u$

Have students share which strategy they prefer for solving quadratic equations. Have them explain their thinking.

Highlight that Po-Shen Loh's strategy can be used to determine the exact solutions of any quadratic equation, even if they are not factorable. It is more efficient than factoring and less complicated than using the quadratic formula.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

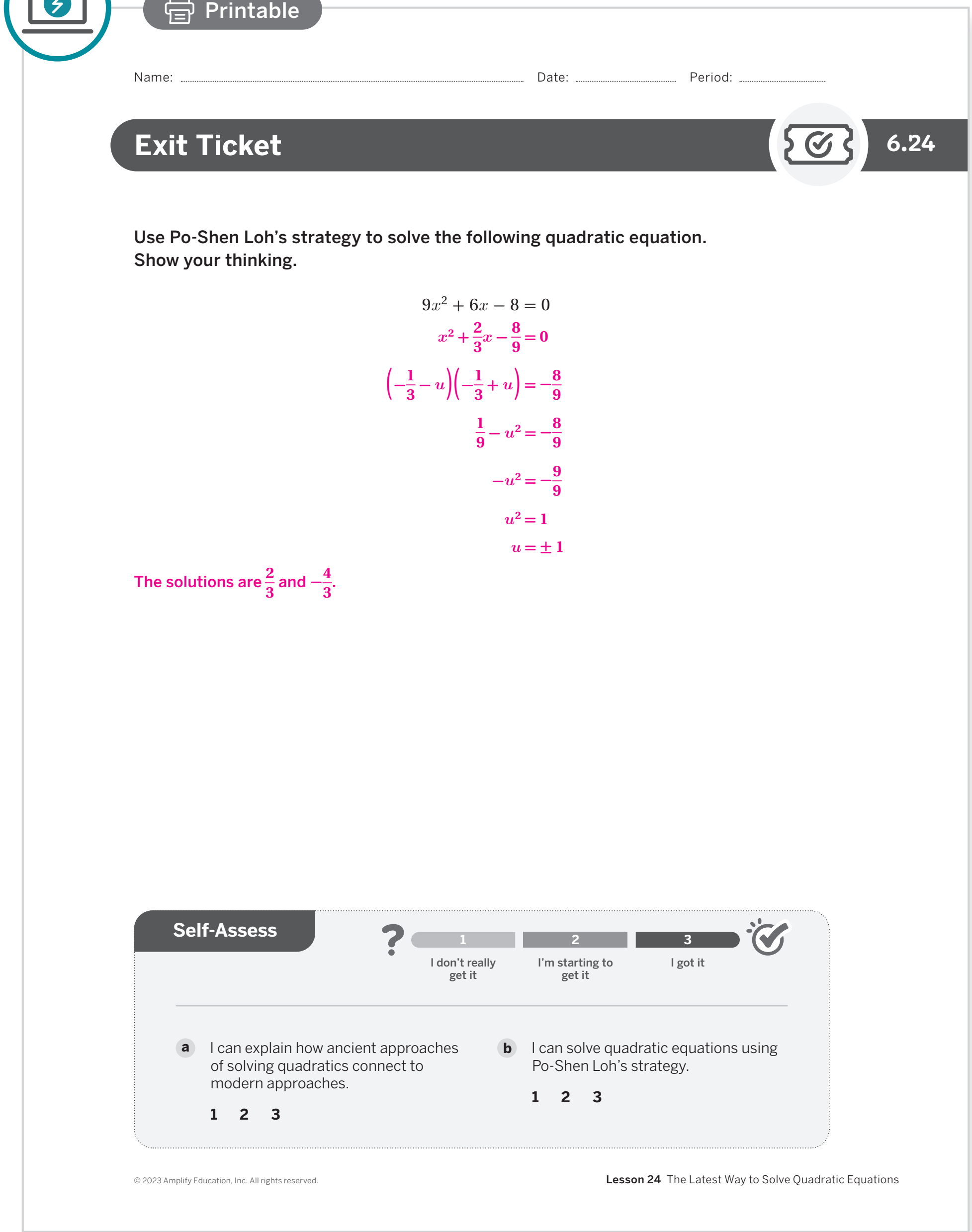
- "Describe, in your own words, how Po-Shen Loh's strategy compares to the other strategies you learned for solving quadratic equations."

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the Po-Shen Loh strategy by solving a quadratic equation.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 6.24

Use Po-Shen Loh's strategy to solve the following quadratic equation. Show your thinking.

$$9x^2 + 6x - 8 = 0$$

$$x^2 + \frac{2}{3}x - \frac{8}{9} = 0$$

$$\left(-\frac{1}{3} - u\right)\left(-\frac{1}{3} + u\right) = -\frac{8}{9}$$

$$\frac{1}{9} - u^2 = -\frac{8}{9}$$

$$-u^2 = -\frac{9}{9}$$

$$u^2 = 1$$

$$u = \pm 1$$

The solutions are $\frac{2}{3}$ and $-\frac{4}{3}$.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can explain how ancient approaches of solving quadratics connect to modern approaches.
1 2 3

b. I can solve quadratic equations using Po-Shen Loh's strategy.
1 2 3

Lesson 24 The Latest Way to Solve Quadratic Equations

Success looks like . . .

- **Language Goal:** Explaining how ancient approaches of solving quadratics connect to modern approaches. **(Speaking and Listening, Writing)**
- **Goal:** Solving quadratic equations using Po-Shen Loh's strategy.
  - » Using Po-Shen Loh's strategy to solve a quadratic equation.

Suggested next steps

If students are unable to solve the quadratic equation using Po-Shen Loh's strategy, consider:

- Reviewing each part of Problem 1 in Activity 2.
- Assigning Practice Problems 1 and 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Have you changed any ideas you used to have about solving quadratic equations as a result of today's lesson?
- What surprised you as your students worked on Po-Shen Loh's strategy? What might you change for the next time you teach this lesson?

Math Language Development

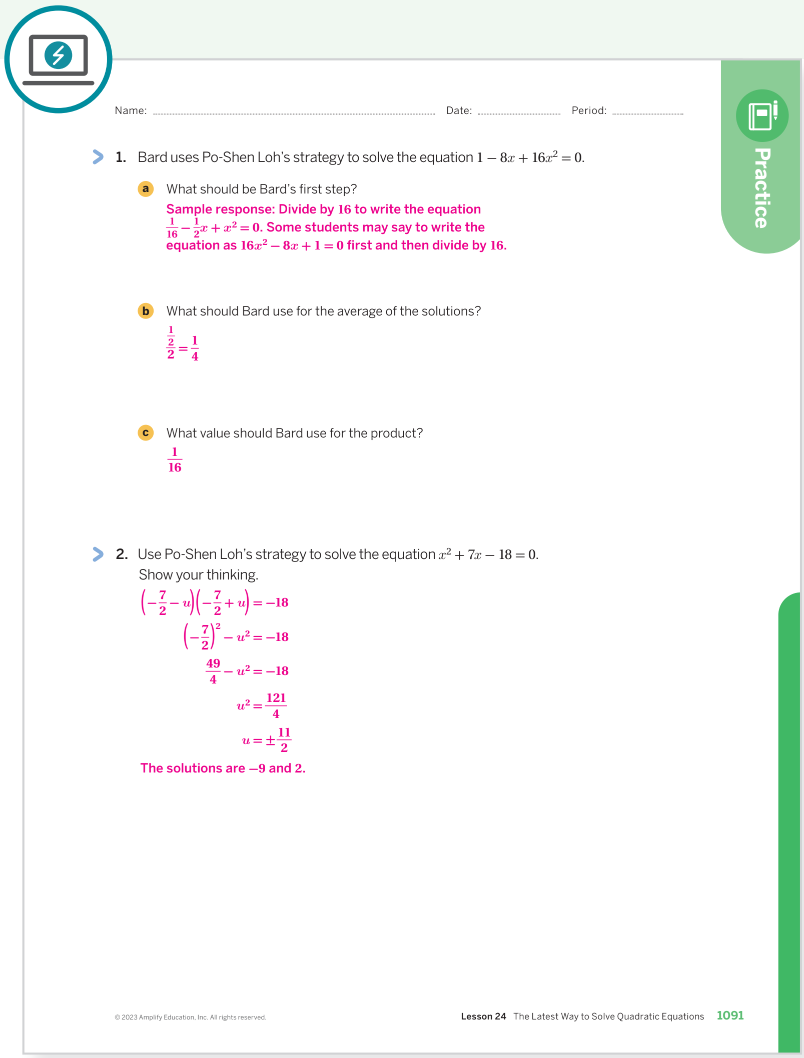
Language Goal: Explaining how ancient approaches of solving quadratics connect to modern approaches.

Reflect on students' language development toward this goal.

- How have students progressed in their explanations of the various strategies for solving quadratic equations presented in this unit? Do they generally understand the benefits of each and when one strategy might be more helpful to use than another?
- Are students able to explain the connections between the various approaches used to solve quadratic equations?

Practice

Independent



Name: ………… Date: ………… Period: …………

Practice

1. Bard uses Po-Shen Loh's strategy to solve the equation $1 - 8x + 16x^2 = 0$.

   a. What should be Bard's first step?

   Sample response: Divide by 16 to write the equation $\frac{1}{16} - \frac{1}{2}x + x^2 = 0$. Some students may say to write the equation as $16x^2 - 8x + 1 = 0$ first and then divide by 16.

   b. What should Bard use for the average of the solutions?

   $\frac{\frac{1}{2}}{2} = \frac{1}{4}$

   c. What value should Bard use for the product?

   $\frac{1}{16}$

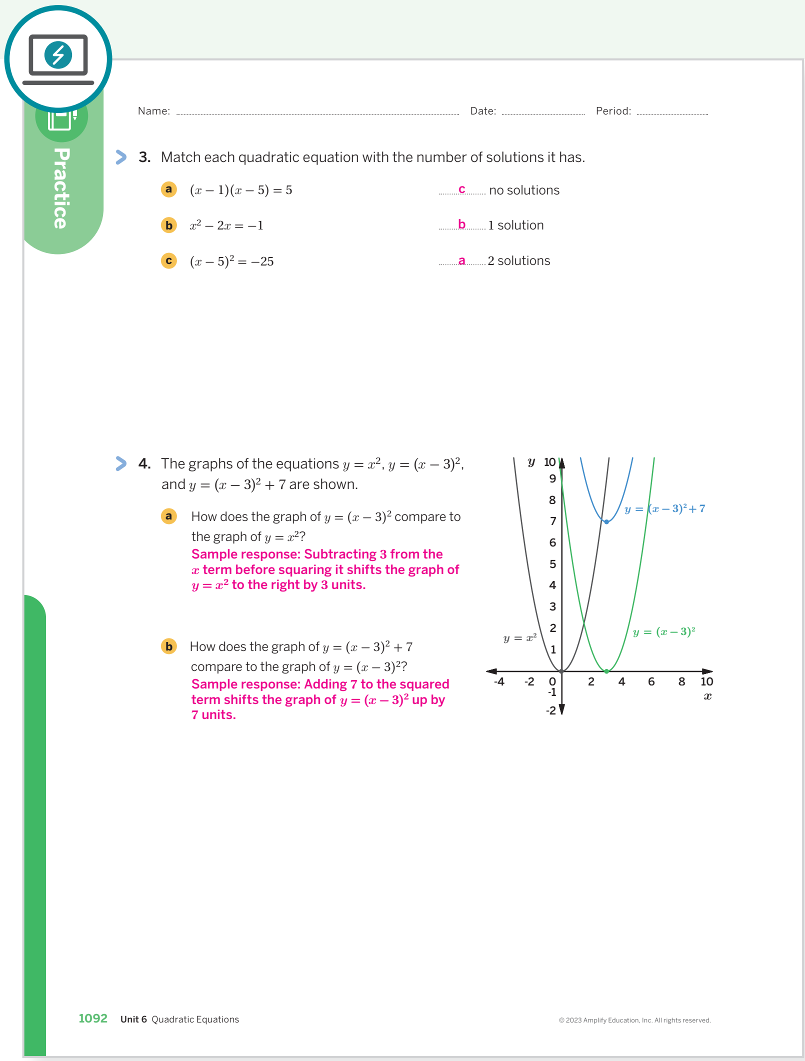
2. Use Po-Shen Loh's strategy to solve the equation $x^2 + 7x - 18 = 0$. Show your thinking.

$$\left(-\frac{7}{2} - u\right)\left(-\frac{7}{2} + u\right) = -18$$
$$\left(-\frac{7}{2}\right)^2 - u^2 = -18$$
$$\frac{49}{4} - u^2 = -18$$
$$u^2 = \frac{121}{4}$$
$$u = \pm\frac{11}{2}$$

The solutions are −9 and 2.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 24 The Latest Way to Solve Quadratic Equations 1091



Name: ………… Date: ………… Period: …………

Practice

3. Match each quadratic equation with the number of solutions it has.

a. $(x - 1)(x - 5) = 5$	c no solutions
b. $x^2 - 2x = -1$	b 1 solution
c. $(x - 5)^2 = -25$	a 2 solutions

4. The graphs of the equations $y = x^2$, $y = (x - 3)^2$, and $y = (x - 3)^2 + 7$ are shown.

   a. How does the graph of $y = (x - 3)^2$ compare to the graph of $y = x^2$?

   Sample response: Subtracting 3 from the x term before squaring it shifts the graph of $y = x^2$ to the right by 3 units.

   b. How does the graph of $y = (x - 3)^2 + 7$ compare to the graph of $y = (x - 3)^2$?

   Sample response: Adding 7 to the squared term shifts the graph of $y = (x - 3)^2$ up by 7 units.

1092 Unit 6 Quadratic Equations

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
Spiral	3	Unit 6 Lesson 20	2
	4	Unit 5 Lesson 22	2

Additional Practice Available

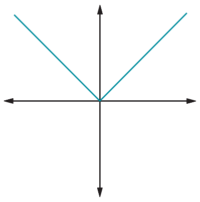
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Glossary/Glosario

English | Español

A

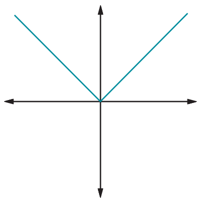
absolute value function A function whose output value is the distance of its input value from 0. In other words, the absolute value function is a piecewise function that takes negative input values and makes them positive.



association When a change in one variable suggests another may change as well, the variables have an *association* and are said to be *associated* with one another.

average rate of change The ratio of the change in the outputs to the change in the inputs, for a given interval of a function.

función de valor absoluto Función cuya salida es la distancia entre su entrada y 0. En otras palabras, la función de valor absoluto es una función definida a trozos que toma entradas negativas y las hace positivas.

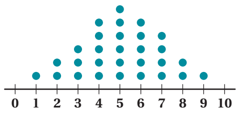


asociación Cuando un cambio en una variable sugiere que otra también podría cambiar, las variables tienen una *asociación* y están *asociadas* entre sí.

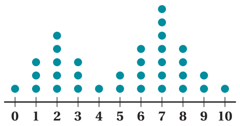
tasa de cambio promedio Razón entre el cambio de las salidas y el cambio de las entradas para un determinado intervalo de una función.

B

bell shaped A distribution that looks like a bell, with most of the data near the center and fewer points farther from the center, is called *bell shaped*.

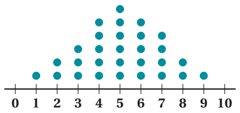


bimodal A distribution with two distinct peaks is called *bimodal*.

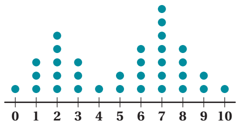


boundary line The line that represents the boundary between the region containing solutions and the region containing non-solutions for an inequality.

acampanada Una distribución que asemeja a una campana, con la mayoría de los datos cerca del centro y una menor cantidad de puntos más lejos del centro, es llamada *acampanada*.



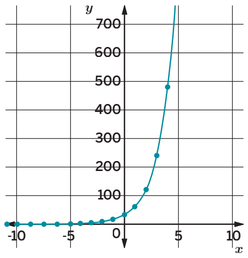
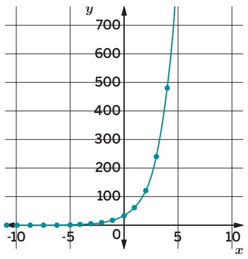
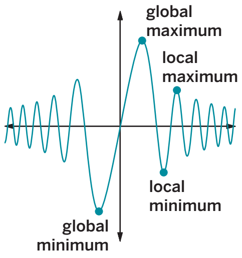
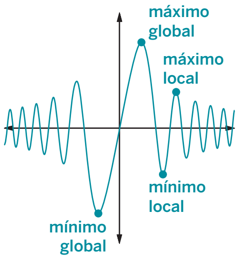
bimodal Una distribución con dos picos distintivos es llamada *bimodal*.



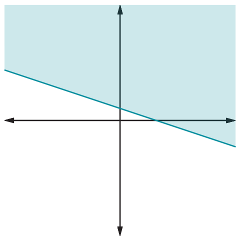
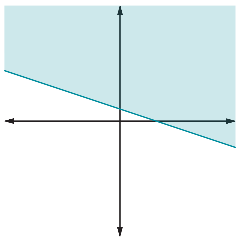
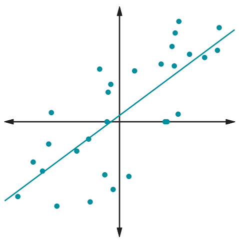
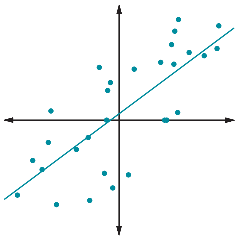
línea límite Línea que representa el límite entre la región que contiene soluciones a una desigualdad y la región que contiene no-soluciones.

Glossary/Glosario

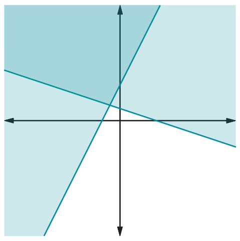
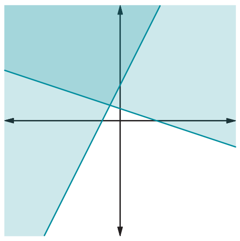
English	Español
C	
categorical variable A variable that can be partitioned into groups or categories.	**variable categórica** Variable que puede partirse en grupos o categorías.
causation When a change in one variable is shown, through careful experimentation, to cause a change in another variable.	**causalidad** Cuando se muestra que un cambio en una variable causa un cambio en otra variable, a través de cuidadosa experimentación.
common difference The difference between two consecutive terms in a linear pattern.	**diferencia común** Diferencia entre dos términos consecutivos de un patrón lineal.
common factor The factor by which each term is multiplied to generate an exponential pattern.	**factor común** Factor por el cual multiplicamos cada término para generar un patrón exponencial.
commutative property Changing the order in which numbers are either added or multiplied does not change the value of the sum or product.	**propiedad conmutativa** Cambiar el orden en que los números se suman o multiplican no cambia el valor de la suma o el producto.
completing the square Completing the square in a quadratic expression means transforming it into the form $a(x-h)^2+k$.	**completar el cuadrado** Completar el cuadrado en una expresión cuadrática significa transformarla en la forma $a(x-h)^2+k$.
compounding (interest) When interest itself earns further interest, it is said to be compounded, or applied to itself multiple times.	**(interés) compuesto** Cuando el interés genera más interés, se dice que es compuesto, o que se aplica a sí mismo múltiples veces.
constraint A limitation on the possible values of variables, often expressed by equations or inequalities. For example, distance above the ground d might be constrained to be non-negative: $d \geq 0$.	**limitación** Restricción de los posibles valores de las variables, usualmente expresada por ecuaciones o desigualdades. Por ejemplo, la distancia desde el suelo d puede ser limitada a ser no negativa: $d \geq 0$.
correlation coefficient A value that describes the strength and direction of a linear association between two variables. Strong positive associations have correlation coefficients close to 1, strong negative associations have correlation coefficients close to -1, and weak associations have correlation coefficients close to 0.	**coeficiente de correlación** Valor que describe la fuerza y dirección de una asociación lineal entre dos variables. Asociaciones positivas fuertes tienen coeficientes de correlación cercanos a 1, mientras que asociaciones negativas fuertes tienen coeficientes de correlación cercanos a -1, y asociaciones débiles tienen coeficientes de correlación cercanos a 0.
D	
decay factor A common factor in an exponential pattern that is between 0 and 1.	**factor de decaimiento** Factor común en un patrón exponencial que se encuentra entre 0 y 1.
difference of squares Two squared terms that are separated by a subtraction sign.	**diferencia de cuadrados** Dos términos al cuadrado que están separados por un signo de resta.
discrete Separate and distinct values or points.	**discreto** Valores o puntos separados y distintivos.
discriminant For a quadratic equation of the form $ax^2+bx+c=0$, the discriminant is b^2-4ac.	**discriminante** Para una ecuación cuadrática de la forma $ax^2+bx+c=0$, el discriminante es b^2-4ac.
domain The set of all of possible input values for a given function.	**dominio** Conjunto de todos los posibles valores de entrada para una determinada función.

English	Español
E	**E**
effective rate The actual interest amount earned over a year, taking into account the interest payment.	**tasa efectiva** Monto del interés real ganado en un año, después de tomar en cuenta el pago de intereses.
elimination The removal of a variable from a system of equations by adding or subtracting equations.	**eliminación** Anulación de una variable de un sistema de ecuaciones por medio de la suma o resta de ecuaciones.
equivalent equations Equations that have the same solution or solutions.	**ecuaciones equivalentes** Ecuaciones que tienen la misma solución o soluciones.
equivalent systems Systems of equations that have the exact same solution or solutions.	**sistemas equivalentes** Sistemas de ecuaciones que tienen exactamente la misma solución o soluciones.
exponential (growth) Describes a change characterized by the repeated multiplication of a common factor. 	**(crecimiento) exponencial** Describe un cambio caracterizado por la multiplicación repetida de un factor común. 
exponential function A one-to-one relationship in which a constant is raised to a variable power.	**función exponencial** Relación uno a uno en la cual una constante se eleva a una potencia variable.
F	**F**
factored form (of a quadratic expression) A quadratic expression that is written as the product of a constant and two linear factors is said to be in factored form.	**forma factorizada (de una expresión cuadrática)** Una expresión cuadrática escrita como el producto de una constante multiplicada por dos factores lineales se considera que está en forma factorizada.
first difference The difference between two consecutive dependent terms for a function.	**primera diferencia** Diferencia entre dos términos dependientes y consecutivos de una función.
function notation A way of writing the output of a named function. For example, if the function f has an input x, then $f(x)$ denotes the corresponding output.	**notación de función** Forma de escribir la salida de una determinada función. Por ejemplo, si la función f tiene una entrada x, entonces $f(x)$ denota la salida correspondiente.
G	**G**
global maximum The greatest value of a function over its entire domain. 	**máximo global** El mayor valor de una función por sobre la totalidad de su dominio. 
global minimum The least value of a function over its entire domain.	**mínimo global** El menor valor de una función por sobre la totalidad de su dominio.
growth factor The common factor that is multiplied over equal intervals in an exponential pattern. In exponential functions of the form $f(x) = a \bullet (1 + r)^x$, the growth factor is $1 + r$.	**factor de crecimiento** Factor común que es multiplicado en intervalos iguales como parte de un patrón exponencial. En funciones exponenciales de la forma $f(x) = a \bullet (1 + r)^x$, el factor de crecimiento es $1 + r$.
growth rate The percent change of an exponential function. In exponential functions of the form $f(x) = a \bullet (1 + r)^x$, the growth rate is r.	**tasa de crecimiento** El cambio porcentual de una función exponencial. En funciones exponenciales de la forma $f(x) = a \bullet (1 + r)^x$, la tasa de crecimiento es r.

Glossary/Glosario

English	Español
H	
half-plane The set of points in the coordinate plane on one side of a boundary line. 	**medio plano** Conjunto de puntos en el plano de coordenadas que está a un solo lado de una línea límite. 
I	
index fund An investment fund constructed to track segments of a financial market.	**fondo indexado** Fondo de inversiones elaborado para seguir segmentos de un mercado financiero.
infinity A boundless value, greater than that of any real number.	**infinito** Valor ilimitado, mayor que el valor de cualquier número real.
interest A percentage of the principal that is paid or owed over a specific amount of time.	**interés** Un porcentaje del principal que se paga o debe durante un periodo de tiempo específico.
interval notation A way to represent a set of numbers using parentheses and brackets. For example, the interval $(3, 5]$ represents all the values greater than 3 and less than or equal to 5.	**notación de intervalo** Forma de representar un conjunto de números por medio de paréntesis y corchetes. Por ejemplo, el intervalo $(3, 5]$ representa todos los valores mayores que 3 y menores o iguales que 5.
inverse of a function The inverse of a function is created by reversing all of the function's input-output pairs. It can be determined by reversing the process that defined the original function.	**inverso de una función** El inverso de una función es creado al revertir todos los pares entrada-salida de la función. Se le puede determinar revirtiendo el proceso que definió a la función original.
L	
line of best fit The linear model that has the smallest possible sum of the squares of the residuals. 	**línea de ajuste óptimo** Modelo lineal que tiene la menor suma posible de los cuadrados de los residuos. 
linear function A function with a constant rate of change.	**función lineal** Función con una tasa de cambio constante.
local maximum The value of a function that is greater than the nearby or surrounding values of the function.	**máximo local** Valor de una función que es mayor a los valores cercanos o circundantes de la función.
local minimum The value of a function that is less than the nearby or surrounding values of the function.	**mínimo local** Valor de una función que es menor a los valores cercanos o circundantes de la función.
M	
monic quadratic An expression of the form $x^2 + bx + c$, where the coefficient of the x^2 term is 1.	**ecuación cuadrática mónica** Expresión de la forma $x^2 + bx + c$, en la cual el coeficiente del término x^2 es 1.
N	
nominal rate The stated or published rate.	**tasa nominal** Tasa declarada o publicada.
non-monic quadratic An expression of the form $ax^2 + bx + c$, where a does not equal 1 or 0.	**ecuación cuadrática no mónica** Expresión de la forma $ax^2 + bx + c$, en la cual a no es igual a 1 o 0.
nonlinear relationship A relationship between two quantities in which there is no constant rate of change.	**relación no lineal** Una relación entre dos cantidades que no tiene una tasa de cambio constante.

O

English	Español
overlap of graphs of inequalities The set of points that satisfy two or more inequalties. 	**superposición de gráficas de desigualdades** Conjunto de puntos que satisfacen dos o más desigualdades. 

P

English	Español
piecewise function A function defined using different expressions for different intervals in its domain.	**función definida a trozos** Función definida por el uso de diferentes expresiones para diferentes intervalos de su dominio.
plus-or-minus symbol A symbol used to represent both the positive and negative of a number ($\pm$).	**símbolo de más menos** Usado para representar tanto el positivo como el negativo de un número ($\pm$).
principal Initial amount of a loan, investment, or deposit.	**principal** Monto inicial de un préstamo, inversión o depósito.

Q

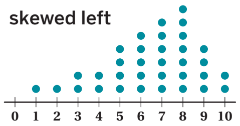
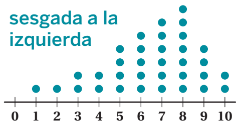
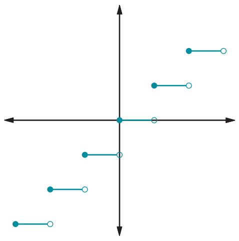
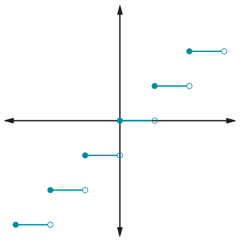
English	Español
quadratic equation An equation in which the highest power of the variable is 2. Also called an equation of the second degree.	**ecuación cuadrática** Ecuación en la cual la potencia más alta de la variable es 2. También se llama ecuación de segundo grado.
Quadratic Formula The formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions to the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.	**Fórmula cuadrática** Fórmula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ que provee las soluciones de la ecuación cuadrática $ax^2 + bx + c = 0$, en la cual $a \neq 0$.
quadratic function A function in which the output is given by a quadratic expression.	**función cuadrática** Función en la cual la salida está dada por una expresión cuadrática.

R

English	Español
range In algebra, a function's *range* is the set of all possible output values for the function. In statistics, the *range* of a data distribution is the difference between the maximum and minimum occurring values.	**rango** En algebra, el *rango* de una función es el conjunto de todos los posibles valores de salida de la función. En estadística, el *rango* de una distribución de datos es la diferencia entre los valores máximo y mínimo existentes.
relative frequency table A two-way table that shows the proportion of each value — expressed as fractions, decimals, or percentages — compared to the total in each row, column or in the entire table.	**tabla de frecuencia relativa** Tabla de doble entrada que muestra la proporción de cada valor (expresada como fracciones, decimales o porcentajes), en comparación con el total de cada fila, columna o con toda la tabla.
residual The difference between the actual y-coordinate and y-coordinate predicted by a model, given the x-coordinate.	**residuo** Diferencia entre la coordenada y real y la coordenada y pronosticada por un modelo, dada la coordenada x.
revenue The income generated from selling of a product or service.	**ingreso** Entrada de dinero generada por la venta de un producto o servicio.

Glossary/Glosario

S

English	Español
second difference The difference between two consecutive first differences.	**segunda diferencia** Diferencia entre dos primeras diferencias consecutivas.
skewed A distribution with a long tail, where data extends far away from the center, is called *skewed*. 	**sesgada** Una distribución de cola larga, en la cual los datos se extienden en dirección opuesta al centro, se conoce como *sesgada*. 
solution set The set of all values that satisfy an equation or inequality.	**conjunto de soluciones** Conjunto de todos los valores que satisfacen una ecuación o una desigualdad.
square expression An expression that represents the product of two identical expressions.	**expresión cuadrada** Expresión que representa el producto de dos expresiones idénticas.
standard deviation A commonly used measure of variability. It is the square root of the average of the squares of the distances between data values and the mean.	**desviación estándar** Medida de variabilidad de uso común. Se trata de la raíz cuadrada del promedio de las distancias elevadas al cuadrado entre los valores de los datos y la media.
standard form (of a quadratic expression) The standard form of a quadratic expression in x is $Ax^2 + Bx + C$, where A, B, and C are constants, and $A \neq 0$.	**forma estándar (de una expresión cuadrática)** La forma estándar de una expresión cuadrática en x es $Ax^2 + Bx + C$, en la cual A, B y C son constantes, y $A \neq 0$.
step function A piecewise function whose pieces are all constant values. 	**función escalonada** Función definida a trozos, cuyos trozos son todos valores constantes. 
system of linear inequalities Two or more inequalities that represent the constraints in the same situation.	**sistema de desigualdades lineales** Dos o más desigualdades que representan las limitaciones en la misma situación.

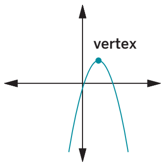
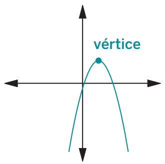
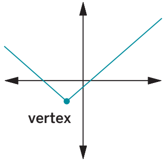
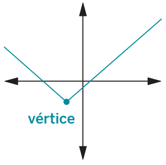
T

two-way table A table that organizes categorical data into cells. The categories do not overlap, so that each data value is recorded in exactly one cell.	**tabla de doble entrada** Tabla que organiza datos categóricos en celdas. Las categorías no se superponen, de manera que el valor de cada dato es registrado exactamente en una sola celda.

U

uniform A distribution in which data is evenly distributed throughout the range is called *uniform*.	**uniforme** Distribución en la cual los datos son distribuidos de manera regular a través del rango.

V

vertex (of a graph) The *vertex* of the graph of a quadratic function or of an absolute value function is the point where the graph changes from increasing to decreasing or vice versa. It is the highest or lowest point on the graph. 	**vértice (de una gráfica)** El *vértice* de la gráfica de una función cuadrática o de una función de valor absoluto es el punto en que la tendencia de la gráfica cambia de aumentar a disminuir o viceversa. Es el punto más alto o más bajo de la gráfica. 
vertex form An equation of the form $y = a(x - h)^2 + k$ where (h, k) represents the coordinates of the vertex of a quadratic function. 	**forma de vértice** Ecuación de la forma $y = a(x - h)^2 + k$, en la cual (h, k) representa las coordenadas del vértice de una función cuadrática. 

Z

Zero Product Principle This principle states that $a \cdot b = 0$, if and only if $a = 0$ or $b = 0$.	**Principio de producto cero** Este principio establece que $a \cdot b = 0$ si y solo si $a = 0$ o $b = 0$.
zeros (of a function) The values at which the function is zero.	**ceros (de una función)** Valores para los cuales la función es cero.

Index

Index

Index

Index

© 2023 Amplify Education, Inc. All rights reserved.

Index

Index