Amplify Math
TENNESSEE

Teacher Edition

Grade 6 | Volume 2

FEATURING Amps POWERED BY desmos

Amplify Math

Grade 6

Volume 2: Units 5–8

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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55 Washington Street, Suite 800,
Brooklyn, NY 11201
www.amplify.com

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Program Advisors

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Phil Daro
Board member: Strategic Education Research Partnership (SERP)
Area of focus: Content strategy

Fawn Nguyen
Rio School District, California
Area of focus: Problem solving

Sunil Singh
Educator, author, storyteller
Area of focus: Narrative and storytelling

Paulo Tan, Ph.D.
Johns Hopkins University, School of Education
Area of focus: Meeting the needs of all students

Educator Advisory Board

Amplify gratefully acknowledges the outstanding contributions and work of esteemed members of our Educator Advisory Board. This product exhibits their respected observations and reflections.

Melvin Burnett
Alamance Burlington Schools, North Carolina

Jessica Childers
Putnam County Schools, Tennessee

Brent Christensen
Spokane Public Schools, Washington

Rhonda Creed-Harry
New York City Schools, New York

Jenny Croitoru
Chicago Public Schools, Illinois

Tara DeVaughn
Fairbanks Northstar Borough School District, Alaska

Elizabeth Hailey
Springfield R-XII School District, Missouri

Howie Hua
California State University at Fresno, California

Rachael Jones
Durham Public Schools, North Carolina

Rita Leskovec
Cleveland Metropolitan School District, Ohio

Corey Levin
New York City Schools, New York

Sandhya Raman
Berryessa Union School District, California

Jerry Schmidt
Brentwood School District, Missouri

Deloris Scott
Yazoo County School District, Mississippi

Noah Sharrow
Clarkston Community Schools, Michigan

Myla Simmons
Plainfield Public Schools, New Jersey

Michele Stassfurth
North Plainfield School District, New Jersey

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Young Women's Leadership School of Brooklyn, New York

Amplify Math Product Development

Product

Molly Batchik
Candy Bratton
Stephanie Cheng
Rebecca Copley
Marsheela Evans
Christina Lee
Brad Shank
Kathleen Sheehy
Jennifer Skelley
Rey Vargas
Allen von Pallandt
Steven Zavari

Curriculum and Editorial

Toni Brokaw
Anna Buchina
Nora Castiglione
Jaclyn Claiborne
Kristina Clayton
Drew Corley
Karen Douglass
Karen Everly
Chris Ignaciuk
Justine Jackson
Louise Jarvis
Brian Kam
Rachel King
Suzanne Magargee
Mark Marzen
Nana Nam
Kim Petersen
Molly Pooler
Elizabeth Re
Allison Shatzman
Kristen Shebek
Ben Simon
Evan Spellman
Amy Sroka
Shelby Strong
Rajan Vedula
Zach Wissner-Gross

Digital Curriculum

Ian Cross
Phil DeOrsey
Ryan de la Garza
Sheila Jaung
Nokware Knight
Michelle Palker
Vincent Panetta
Aaron Robson
Sam Rodriguez
Eileen Rutherford
Elliot Shields
Gabe Turow

Design and Illustration

Amanda Behm
Irene Chan
Tim Chi Ly
Cindy Chung
Caroline Hadilaksono
Justin Moore
Christina Ogbotiti
Renée Park
Eddie Peña
Todd Rawson
Jordan Stine
Veronica Tolentino
J Yang

Narrative Design

Bill Cheng
Gala Mukomolova
Raj Parameswaran

Marketing

Megan Hunter
Zach Slack
Heath Williams

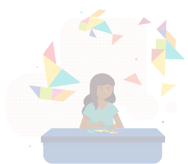
Engineering

Jessica Graham
Matt Hayes
Bardh Jahjaga
Eduard Korolchuk
Nick Maddalena
Syed Rizvi
Jon Tully

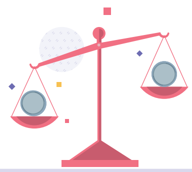
Digital Production

Andrew Avery
Ryan Cooper
Jessica Yin Gerena
Edward Johnson
Charvi Magdaong
Julie Palomba
Heather Ruiz
Ana Zapata

Program Scope and Sequence

	Unit 1	Unit 2	Unit 3	Unit 4
				
Grade 6 160 days total	Area and Surface Area 20 Instructional Days 3 Assessment Days 23 days total	Introducing Ratios 20 Instructional Days 2 Assessment Days 22 days total	Rates and Percentages 15 Instructional Days 2 Assessment Days 17 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 153 days total	Scale Drawings 13 Instructional Days 2 Assessment Days 15 days total	Introducing Proportional Relationships 17 Instructional Days 2 Assessment Days 19 days total	Measuring Circles 12 Instructional Days 2 Assessment Days 14 days total	Percentages 13 Instructional Days 2 Assessment Days 15 days total
Grade 8 145 days total	Rigid Transformation and Congruence 18 Instructional Days 3 Assessment Days 21 days total	Dilations and Similarity 12 Instructional Days 2 Assessment Days 14 days total	Linear Relationships 19 Instructional Days 2 Assessment Days 21 days total	Linear Equations and Systems of Linear Equations 17 Instructional Days 2 Assessment Days 19 days total
Algebra 1 157 days total	Linear Equations, Inequalities, and Systems 26 Instructional Days 3 Assessment Days 29 days total	Data Analysis and Statistics 22 Instructional Days 3 Assessment Days 25 days total	Functions and Their Graphs 22 Instructional Days 3 Assessment Days 25 days total	Introducing Exponential Functions 22 Instructional Days 3 Assessment Days 25 days total

Volume 2

Unit 5	Unit 6	Unit 7	Unit 8
			
Arithmetic in Base Ten	**Expressions and Equations**	**Rational Numbers**	**Data Sets and Distributions**
14 Instructional Days	19 Instructional Days	19 Instructional Days	17 Instructional Days
2 Assessment Days	2 Assessment Days	2 Assessment Days	3 Assessment Days
16 days total	21 days total	21 days total	20 days total
Rational Number Arithmetic	Expressions, Equations, and Inequalities	Angles, Triangles, and Prisms	Probability and Sampling
20 Instructional Days	23 Instructional Days	18 Instructional Days	17 Instructional Days
3 Assessment Days	3 Assessment Days	3 Assessment Days	3 Assessment Days
23 days total	26 days total	21 days total	20 days total
Functions and Volume	Exponents and Scientific Notation	Irrationals and the Pythagorean Theorem	Associations in Data
21 Instructional Days	15 Instructional Days	16 Instructional Days	9 Instructional Days
3 Assessment Days	2 Assessment Days	2 Assessment Days	2 Assessment Days
24 days total	17 days total	18 days total	11 days total
Introducing Quadratic Functions	Quadratic Equations		
23 Instructional Days	24 Instructional Days		
3 Assessment Days	3 Assessment Days		
26 days total	27 days total		

Volume 1

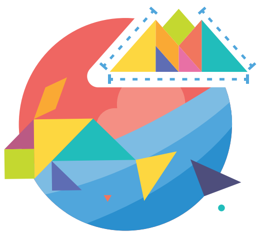
Unit 1 Area and Surface Area

Students extend their elementary understanding of area as compositions and decompositions for covering, shifting from limited experiences with rectangles and unit-square thinking to more general formulas for parallelograms and triangles. They leverage these in working with three-dimensional figures as well, recognizing surface area as a different measure than volume.

Unit Narrative:
A Place for Space

PRE-UNIT READINESS ASSESSMENT

LAUNCH



Sub-Unit 1 Area of Special Polygons 17

Sub-Unit Narrative:
Can a sum ever really be greater than its parts?
Polygons are shapes whose sides are all line segments, and they can be decomposed and rearranged without changing their area.

MID-UNIT ASSESSMENT

Sub-Unit 2 Nets and Surface Area 89

Sub-Unit Narrative:
How did a misplaced ruler change the way you shop?
Polyhedra are three-dimensional figures composed of polygon faces. Their surfaces can be decomposed.

CAPSTONE

END-OF-UNIT ASSESSMENT

Unit 2 Introducing Ratios

Students understand ratios using three of their five senses. They use written and visual representations to learn the language of ratios, and scale up (with multiplication) or down (with division) to calculate equivalent ratios. Ratios are also used for thinking about constant rates or occurrences happening at the same rate.

Unit Narrative:
Sensing a Ratio

PRE-UNIT READINESS ASSESSMENT

LAUNCH

Sub-Unit Narrative:
How does an eggplant become a plum?
Ratios represent comparisons between quantities by multiplication or division. First, you must first learn the language of ratios and how quantities "communicate."

Sub-Unit Narrative:
How do you put your music where your mouth is?
Equivalent ratios involve relationships between ratios themselves. They speak to each other through music and rhythm, beats and time.

Sub-Unit Narrative:
Who brought Italy to India and back again?
Now it is your turn to choose the information to represent and compare ratios.

CAPSTONE

END-OF-UNIT ASSESSMENT

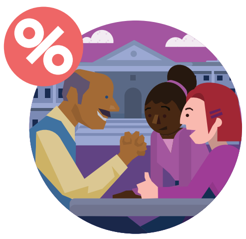
Unit 3 Rates and Percentages

Students understand the concept of unit rate in the contexts of constant price and speed, recognizing that equivalent ratios have the same unit rates. They use several visual and algebraic representations of percentages to determine missing percentages, parts, and wholes.

Unit Narrative:
Stand and Be Counted

LAUNCH

PRE-UNIT READINESS ASSESSMENT



Sub-Unit 1 Rates 281

Sub-Unit Narrative:
How did student governments come to be?
Rates describe relationships between quantities like price and speed. Unit rates reveal which is a better deal or who is faster.

Sub-Unit 2 Percentages 323

Sub-Unit Narrative:
What can a corpse teach us about governing?
Percentages are rates per 100. They can compare relationships between parts and wholes, even when two quantities have different total amounts.

CAPSTONE

END-OF-UNIT ASSESSMENT

Unit 4 Dividing Fractions

Students extend their understanding of partitive and quotitive division from whole numbers to fractions. They use this along with the relationship between multiplication and division to construct models and develop an algorithm for dividing fractions, and they apply it to problems involving lengths, areas, and volumes.

Unit Narrative:
Crossing the Fractional Divide

Sub-Unit Narrative:
Which item costs between 100 and 1,000 spök-bucks?
Multiplication and division are related, and the relationship between fractions and division can be used to estimate quotients.

Sub-Unit Narrative:
How long is the bolt Samira needs?
To divide fractions, you can use multiplication, common denominators, or an algorithm. Apply these to determine the length of an oddly labeled bolt.

Sub-Unit Narrative:
How can Maya fit Penny in the box?
When you know an area or volume, but not every side length, you will often divide fractions.

Volume 2

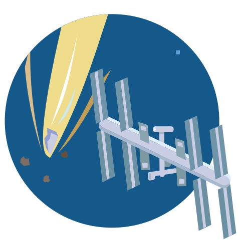
Unit 5 Arithmetic in Base Ten

Students synthesize previous learning of place value, properties of operations,and relationships between operations to complete their understanding of both the “whys” and “hows” of the four operations with positive rational numbers. They develop general algorithms for working with whole numbers and decimals, containing any arbitrary number of digits.

Unit Narrative:
Making Moves With Decimals

Sub-Unit Narrative:
How did a decimal decide an Olympic race?
Determine the results of high stakes competitions and identify record-setting moments by adding and subtracting decimals, as precisely as you need.

Sub-Unit Narrative:
What happens when you make a small change to a big bridge?
To reproduce something at large or small scales so it looks the same, you need decimals and multiplication.

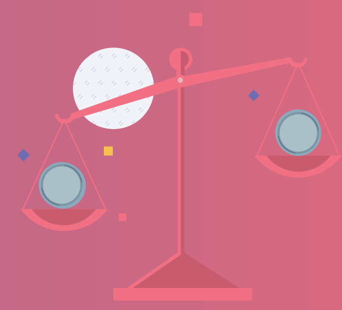


Sub-Unit Narrative:
How do you dodge a piece of space junk?
Dividing whole numbers and decimals with many digits is the final set of operations you need to complete your trophy case.

Unit 6 Expressions and Equations

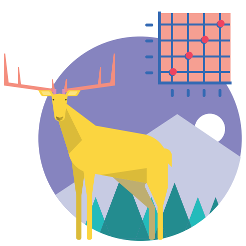
Students discover that the equal sign is more than a prompt, it's also a way to indicate balance — a critical understanding that allows them to move beyond the strictly numeric world and into the realm of algebra.

Unit Narrative:
The Power of Balance



Sub-Unit Narrative:
What's a bag of chips worth in Timbuktu?
Learn about the 14th century African salt trade, as you explore expressions and equations with tape diagrams and hanger diagrams.

Sub-Unit Narrative:
How did a Welshman equalize England's upper crust with its common folk?
Extend the concept of equality as you investigate equivalent expressions, the all-important Distributive Property, and exponents.



Sub-Unit Narrative:
What's more dangerous: a pack of wolves or a gang of elk?
Balance is everywhere, especially in ecosystems. You'll look at systems that are in and out of balance.

Unit 7 Rational Numbers

Students recognize a need to expand their concept of number to represent both magnitude and direction, extending the number line and coordinate plane to include negative rational numbers. They compare these numbers, as well as their absolute values, and write inequality statements using variables.

Unit Narrative:
Getting Where We're Going

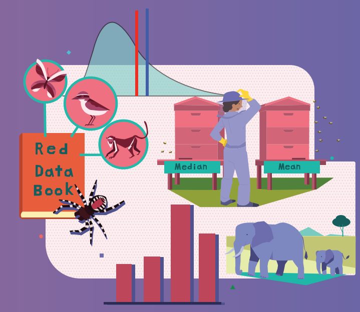
Sub-Unit Narrative:
What's the tallest mountain in the world?
Consider the most extreme locations on Earth as you discover negative numbers, which lend new meaning to positive numbers and zero.

Sub-Unit Narrative:
How do you keep a quantity from wandering off?
A variable represents an unknown quantity. And sometimes it represents many possible values, which can be expressed as an inequality.

Sub-Unit Narrative:
How did Greenland get so big?
Armed with the opposites of positive rational numbers, it's time you expanded your coordinate plane. Welcome to the four quadrants!

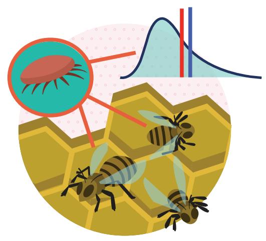
Unit 8 Data Sets and Distributions

Unit Narrative:
Walk on the Wild Side With Data



In this unit, students learn about populations and study variables associated with a population, focusing on populations of animal species and their respective endangerment classifications. They distinguish numerical and categorical data, relative to survey and statistical questions, and represent and describe the distributions of response data. Students first interpret frequency tables, dot plots, and histograms, before calculating measures of center — mean and median — and measures of variability — mean absolute deviation (MAD), range, and interquartile range (IQR). They then construct box plots in addition to interpreting these measures in context, and relating the shape and features of a distribution to the best choice of measures.

Sub-Unit Narrative:
How do you keep track of a disappearing animal?
When questions have more than one answer, it is helpful to visualize and describe a typical answer. For numbers, you can also identify the center and describe the spread of the numbers.



Sub-Unit Narrative:
What's the buzz on honey bees?
For numerical data, you can summarize an entire data set by a single value representing the center of the distribution. The mean and the median represent two ways you can do this.

Sub-Unit Narrative:
Where have the giant sea cows gone?
For numerical data, you can summarize an entire data set by a single value representing the variability of the distribution. The MAD, range, and IQR represent three ways you can do this.

UNIT 5

Arithmetic in Base Ten

Students synthesize previous learning of place value, properties of operations, and relationships between operations to complete their understanding of both the "whys" and "hows" of the four operations with positive rational numbers. They develop general algorithms for working with whole numbers and decimals, containing any arbitrary number of digits.

Essential Questions

- How can base ten units be composed and decomposed?
- How is the total number of decimal places in two decimal factors related to the total number of decimal places in their product?
- What is similar and different about using related whole-number expressions when multiplying versus dividing decimals?
- *(By the way, what place value position could also be called the "oneths" place?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students build on the understanding of place value and the properties of operations to extend their use of base ten algorithms to decimals of arbitrary length and also learn an efficient algorithm for division. The first Sub-Unit focuses on addition and subtraction, while the second Sub-Unit focuses on multiplication, and the third Sub-Unit focuses on division. Students learn long division, first evaluating whole number quotients of whole numbers, followed by quotients of whole numbers that result in decimals, quotients of decimals and whole numbers, and finally quotients of decimals.

Coherence

Previously . . .

Students saw how to use efficient algorithms to fluently calculate sums and differences (Grade 4), and products (Grade 5) of multi-digit whole numbers. In Grade 5, students calculated quotients of multi-digit whole numbers with up to four-digit dividends and two-digit divisors, and also sums, differences, products, and quotients of decimals to hundredths. They saw how all of these calculations could be represented using concrete representations, and they calculated them by applying strategies based on place value, the properties of operations, and the relationships between operations.

Coming soon . . .

In Unit 6, students will use these operations to evaluate expressions and solve equations with one variable involving whole numbers, decimals, and fractions. In Unit 7, students will extend the number system to include negative rational numbers. In Grade 8, students will express numbers using scientific notation and perform calculations on them, as well as extending the number system even further, to include irrational numbers.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students revisit concrete representations grounded in place value and properties of operations for each of the four operations throughout the unit: addition and subtraction (Lessons 2 and 3), multiplication (Lessons 5–7), and division (Lessons 9 and 11–13).

Procedural Fluency

Students formalize algorithms for each of the four operations with decimal numbers, as well as division with whole numbers. Depending on the context or numbers involved, they shift flexibly between strategies and representations, building fluency with operations involving multi-digit numbers (Lessons 2–8 and 10–14).

Application

Two applications of student understanding of the four operations with multi-digit numbers are using relationships between operations to solve problems, and solving one- and two-step problems in a variety of contexts that warrant precision, such as world records (Lessons 1, 4, 8, 13, and 14).

Making Moves With Decimals

SUB-UNIT

1

Lessons 2–4

Adding and Subtracting Decimals

Students revisit previously seen concrete models and strategies based on place value to formalize a standard algorithm for adding and subtracting multi-digit decimals, including those with decimal places beyond hundredths. They apply properties of operations and the relationship between addition and subtraction, along with an algorithm, to compare results from X Games events and solve problems in other contexts.

Narrative: Determine the results of high-stakes competitions and identify record-setting moments by adding and subtracting decimals.

SUB-UNIT

2

Lessons 5–8

Multiplying Decimals

Students revisit previously seen concrete models and strategies based on place value to formalize a standard algorithm for multiplying multi-digit decimals, including those with decimal places beyond hundredths. They solve problems in the context of world records, such as the smallest and fastest objects made by humans.

Narrative: From tremendous bridges to tiny chess sets, multiplying decimals can help you make sense of engineering marvels in the world around you.

Launch

Lesson 1

Precision and World Records

Students are introduced to two continuous, connected themes of this unit about performing operations with multi-digit base ten whole numbers and decimals — the need for precision when establishing world records. They will measure and tape together strips of paper to construct replicas of the world's longest fingernails.

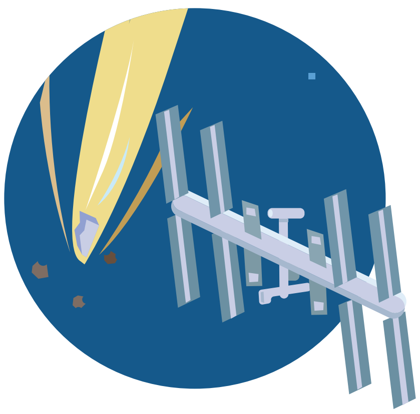
SUB-UNIT

3

Lessons 9–13

Dividing Decimals

Students revisit previously seen concrete models and strategies based on place value to formalize a standard algorithm for dividing both multi-digit whole numbers (including beyond thousands) and multi-digit decimals (including beyond hundredths). They apply long division, along with the properties of operations and the relationship between multiplication and division to solve problems in the contexts of Earth and space science.



Narrative: Dividing decimals can tell you precisely how fast to go to dodge some space junk, and a whole lot more.

Capstone

Lesson 14

The So-Called World's "Littlest Skyscraper"

The oil boom and a real-estate tycoon — what could possibly go wrong? Students recognize the importance of attending to precision in two different ways. They apply the appropriate operations with multi-digit whole numbers and decimal numbers to determine the extent of a legendary investment deal gone wrong, in the context of the past and current times.

Unit at a Glance

Spoiler Alert: You can add, subtract, multiply, and divide decimals by ignoring the decimal points and calculating with whole numbers first. Then you use place value to determine the location of the decimal point in the result.

Assessment

A **Pre-Unit Readiness Assessment**

Launch Lesson

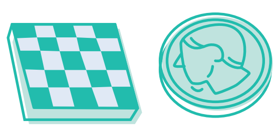
1 **Precision and World Records**

Review decimal place value in the context of world records.

Sub-Unit 1:

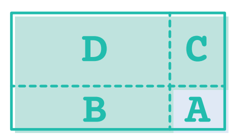
2 **Speaking of Decimals . . .**

Use the language of decimals represented in addition models and vertical calculations.



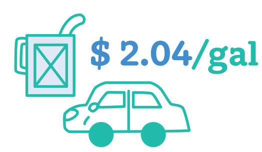
6 **Methods for Multiplying Decimals**

Multiply decimals by interpreting each factor as a product of a whole number and a unit fraction whose denominator is a power of 10.



7 **Representing Decimal Multiplication With Diagrams**

Use area diagrams to determine the product of decimals, while also beginning to generalize the process.



8 **Calculating Products of Decimals**

Choose a strategy to represent and reason about problems in real-world contexts that involve multiplication of decimals.

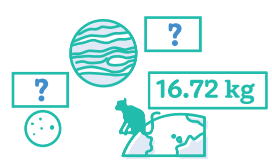
$46.368 \div 7.2$

$463.68 \div 72$

$46{,}368 \div 7{,}200$

12 **Using Related Expressions to Divide With Decimals**

Use related expressions to divide with whole number and decimal dividends.



13 **Dividing Multi-digit Decimals**

Formalize the steps to divide with decimals.

Capstone Lesson

14 **The So-called World's Littlest Skyscraper**

Apply multiple operations with decimals to solve multi-step real-world problems.

Key Concepts

Lesson 4: To add or subtract decimals, line up the decimal points. Compose or decompose units as needed.
Lesson 8: To multiply decimal numbers, convert them to whole numbers. Then convert the product back.
Lesson 13: Long division with decimals works similarly to division with whole numbers. In fact, you convert them to whole numbers.

Pacing

14 Lessons: 45–50 min each
2 Assessments: 45 min each
Full Unit: 16 days
● **Modified Unit:** 13 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Adding and Subtracting Decimals

$$\begin{array}{r} 3.21 \\ +\,4.5 \\ \hline \boxed{?} \end{array}$$

3 Adding and Subtracting Decimals

Add and subtract decimals of different lengths.

$$\begin{array}{r} 45.37 \\ -\,45.081 \\ \hline \boxed{?} \end{array}$$

4 X Games Medal Results

Add and subtract decimals with multiple digits efficiently using vertical calculations.

Sub-Unit 2: Multiplying Decimals

$$(0.4) \cdot (0.15)$$

5 Decimal Points in Products ●

Multiply decimals by multiplying equivalent fractions that have a power of 10 in the denominator.

Sub-Unit 3: Dividing Decimals

$365 \div 5$ or $5\overline{)365}$

9 Exploring Division

Compare and contrast division methods: base ten diagrams, partial quotients, and long division.

$$\begin{array}{r} 4 \\ 36\overline{)1476} \\ -144 \\ \hline \end{array}$$

10 Using Long Division ●

Use long division to divide whole numbers that result in a whole number quotient.

11 Dividing Numbers That Result in Decimals

Use base ten diagrams and long division to divide two whole numbers that result in a decimal quotient.

Assessment

A End-of-Unit Assessment

● Modifications to Pacing

Lesson 2: This lesson may be omitted, but be sure to introduce and reinforce the language of base ten place value and decimal numbers early and often in other lessons.

Lessons 5 and 6: These two lessons may be combined. One option is to use Lesson 6 with Activity 1 from Lesson 5, instead of Activity 1 from Lesson 6. Other options may require reducing the number of problems in some places or conducting activities as a whole class. However, be sure students understand the connections between decimals and both fractions and area models.

Lesson 10: This lesson may be omitted, or some of the problems from Lesson 10 could replace Problems 2 and 3 of Activity 2 in Lesson 9.

Unit Supports

Math Language Development

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
3, 6, 13	**MLR1:** Stronger and Clearer Each Time
3, 5, 7, 9, 13	**MLR2:** Collect and Display
5, 8, 10, 12	**MLR3:** Critique, Correct, Clarify
4, 9, 10	**MLR5:** Co-craft Questions
1, 4, 11	**MLR6:** Three Reads
2, 7–9, 11, 12	**MLR7:** Compare and Connect
1, 4–8, 10	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket   Additional Practice

Additional required materials include:

Lesson(s)	Materials
11	six 1-liter bottles
2–3, 9, 11	base ten blocks or cut-outs
1	blindfold
3, 9	colored pencils
1	decimal circle
2–4, 8–11, 14	graph paper
11	measuring cup
11	measuring spoons (tbsp and tsp)
1–3, 7–11, 13–14	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
1	rulers or tape measures
11	salt (approx. 7 cups)
11	spoons
1	strips of paper
1	tape

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
13	Gallery Tour
4, 8	Notice and Wonder
8–12	Number Talk
3, 9, 12	Think-Pair-Share
5	Turn and Talk

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 14

Amps
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Social & Collaborative Digital Moments

Featured Activity

Pin the Decimal on the Record

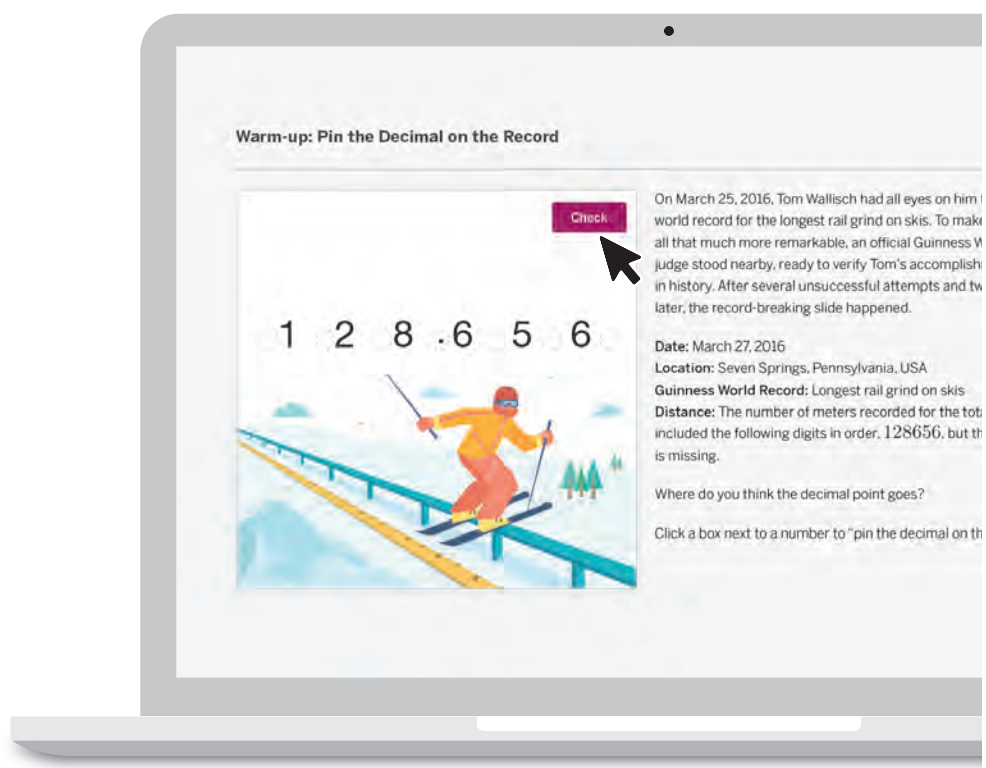
Put on your student hat and work through the **Lesson 1, Warm-up:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- The World's Smallest Newspaper **(Lesson 7)**
- Using Long Division **(Lesson 9)**
- "The Most Vertical Woman in the World" **(Lesson 10)**



Unit Study
Professional Learning

This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces students to multiplying decimals. Students previously worked with addition and subtraction of decimals. They will tackle division of decimals in Sub-Unit 3. Students learn to understand and rewrite each decimal number as a product of a whole number and a unit fraction. For example, $5.84 = 584\left(\frac{1}{100}\right)$. They examine how the decimal point "moves" based on the unit fraction and its related place value. Students are provided with many examples of decimals from real-world contexts, including quantities and prices of items, traveled distances, and elapsed times. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 8, Activity 1:

Activity 1 How Far Can One Tank of Gas Take You?

In 2011, Marko Tomac and Ivan Cvetkovic set the Guinness World Record for the "Greatest distance driven on a single tank of diesel fuel." They drove 1,581.88 miles through Croatia in a Volkswagen Passat 1.6 TDI BlueMotion, averaging 76.37 mpg.

1. In 2011, the average price, in U.S. dollars, for diesel fuel in Croatia was about \$6.038 per gallon. At the same time, in the United States, the average price for diesel was 64% of what it cost in Croatia.
   a. Estimate the cost of one gallon of diesel fuel in the United States in 2011. Explain your thinking.
   b. Determine the actual price of a gallon of diesel in the United States in 2011, rounded to the nearest thousandth. Show your thinking.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

- What was it like to engage in this problem as a learner?
- Students are often asked to estimate prior to solving for the actual answer. What range of estimates would you consider reasonable for Problem 1?
- Problem 2 asks for the price per gallon of diesel rounded to the thousandths place. Students may not notice that gasoline prices in the United States end in $\frac{9}{10}$ of a cent or 0.9 cent. Consider having them look into the history of this and/or determine how much this [small] amount can add up over someone's lifetime.
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Number Talk

Rehearse . . .

How you'll facilitate the ***Number Talk*** instructional routine in Lesson 9, Warm-up:

Warm-up Number Talk

Mentally evaluate the quotient of $657 \div 3$. Be prepared to explain your thinking.

Points to Ponder . . .

- How do you expect your students to leverage their experience with this problem when doing the same thing with larger numbers in Lesson 10? How will you support them in making connections?

This routine . . .

- Builds computational fluency by encouraging students to think about the numbers in a computation problem and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem.
- Allows students to share their thinking verbally, requiring them to be precise in their word choice and use of language.
- Allows students to see and hear the thinking and strategies of others.
- Sometimes shows how numbers and operations are related, through purposefully chosen and sequenced problems.

Anticipate . . .

- Some students will equate mental evaluation with estimation, stopping after evaluating a different problem, such as $660 \div 3$.
- Some students may struggle to proceed more than one or two steps without being able to write, while others may determine a correct response but struggle to recount all of their steps. What classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Build procedural fluency from conceptual understanding.

This effective teaching practice . . .

- Begins with a foundation of deep understanding so that students develop sense-making skills, before procedural skills are introduced.
- Provides students with the opportunity to connect procedural skills with contextual or mathematical problems, strengthening their problem solving abilities.

Points to Ponder . . .

- Before introducing a formula or procedure, how will you ensure that your students have a solid understanding of the mathematical concepts?
- Do your students connect procedures to concepts, or are they reliant on memorization of formulas or procedural steps? How can you be sure they understand the "why behind the what"?

Math Language Development

MLR3: Critique, Correct, Clarify

MLR3 appears in Lessons 5, 8, 10, and 12.

- In Lesson 5, students analyze an incorrect statement about the placement of zeros in decimals as they analyze the product of two decimal numbers.
- In Lesson 10, students may have misconceptions about where to place digits above the dividend when performing long division calculations. This is a good opportunity to present the misconception as a statement and have students critique it.
- **English Learners:** Allow students to speak or draft a response in their primary language first, and then have them generate a response in English.

Point to Ponder . . .

- In this routine, students analyze incorrect statements and work to correct them. How can you model an effective and respectful critique?

Differentiated Support

Accessibility: *Optimize Access to Tools, Optimize Access to Technology*

Opportunities to provide physical manipulatives, the use of technology (through the Amps slides), or other tools appear in Lessons 2–12 and 14.

- Throughout the unit, opportunities are provided for students to use the Amps slides to manipulate digital base ten blocks to deepen their conceptual understanding of the calculations.
- You may also choose to provide continued access throughout the unit to physical base ten blocks or distribute copies of the *Decimal Base Ten Blocks* PDF for students to use.
- In Lesson 6, students can view an animation comparing area models to related expressions. By zooming in and out, they see how the models reflect place value concepts, which deepens their understanding.

Point to Ponder . . .

- How will you decide when students are ready to move toward vertical calculations more often than using base ten blocks or diagrams? What clues will you gather from your students?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with understanding and executing each of the four operations? Do you think your students will generally:
  - Only be challenged by decimals, but be able to calculate accurately with whole numbers?
  - Be able to add and subtract, but struggle to multiply and divide?
  - Seem to make errors largely attributed to difficulties in remembering and properly following the steps of the algorithms?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-awareness and responsible decision-making skills.

Points to Ponder . . .

- Are students able to identify their strengths and work in such a way that it highlights their strengths? How do students accommodate for their personal weaknesses? Do they approach tasks with an optimistic spirit? How do students show a growth mindset?
- Are students able to make constructive choices about their behavior? Do their social interactions fall within the acceptable standards? Do they think ahead and consider the consequences of their choices prior to making a decision?

Precision and World Records

Let's look at the use of decimals in world records.

Focus

Goals

1. **Language Goal:** Analyze a number beyond the hundredths place based on place value. **(Speaking and Listening, Writing)**
2. **Language Goal:** Calculate sums and products of decimals in the context of world records, and explain the calculation strategy. **(Speaking and Listening, Writing)**

Coherence

Today

Students recall and build on the language of decimal numbers from earlier grades and experiences. Students connect the precision of their language to place value and the reasonableness of values in a context. Groups of students collaborate to use operations with decimals in exploring a unique world record that some students will be able to live out.

Previously

In previous grades, students learned how to read decimals to the thousandths place and used several strategies for making sense of and performing the operations of addition, subtraction, multiplication, and division with both whole numbers and decimals to the hundredths place.

Coming Soon

In this unit, students will build fluency, applying the four operations to base ten numbers of any place value. In Lessons 2–4, students focus on the operations of addition and subtraction with decimals.

Rigor

- Students **apply** a variety of strategies to perform decimal operations in a real-world context.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Whole Class	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- *Place Value Chart* PDF (for display)
- decimal circle, cut from black paper, with a diameter of approximately 5 in.
- 70–75 strips of paper, approximately $\frac{1}{2}$ in. wide by 11 in. long, one set per group
- blindfold (optional)
- rulers or tape measures
- tape

Amps | **Featured Activity**

Warm-up

Formative Feedback for Students

Students decide where to place a decimal, and are then shown an animation that demonstrates the resulting quantity.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may respond harshly if another person in their group measures incorrectly in Activity 1. Explain that there are standards for behavior and expectations for the way they speak to each other. Ask students to identify ways that they can handle the situation respectfully. Ask them to treat the other team the way they would want to be treated so that they can focus on attending to precision in Activity 1.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, instead of having students pin the decimal on the number, ask students what a reasonable distance would be for the world record. Have the whole class repeat the correct number.
- In **Activity 1**, groups could complete the table and class results can be compared. You may also skip the creation of the nail models or only make one for each record holder as a class.

Whole Class | 10 min

Warm-up Pin the Decimal on the Record

Students place a decimal point in different places within the same digits to recall place values and how to read decimal numbers.

Amps Featured Activity Formative Feedback for Students

Unit 5 | Lesson 1 – Launch

Precision and World Records

Let's look at the use of decimals in world records.

Warm-up Pin the Decimal on the Record

On March 25, 2016, Tom Wallisch had all eyes on him to break the world record for the longest rail grind on skis. To make this moment all that much more remarkable, an official Guinness World Records™ judge stood nearby, ready to verify Tom's accomplishment and place in history. After several unsuccessful attempts and two cold days later, the record-breaking slide happened.

Date: March 27, 2016
Location: Seven Springs, Pennsylvania, USA
Guinness World Record: Longest rail grind on skis
Distance: The number of meters recorded for the total distance included the following digits in order, 128656, but the decimal point is missing.

Where do you think the decimal point goes? As the decimal is "pinned," say the number aloud. Then complete the sentence.

Tom Wallish's record run was ....128.656.... meters.

"one hundred twenty-eight and six hundred fifty-six thousandths"
Sample responses:
- **1.28656 "one and twenty-eight thousand six hundred fifty-six hundred thousandths" (too low to be the record)**
- **12.8656 "twelve and eight thousand six hundred fifty-six ten thousandths" (too low to be the record)**
- **1,286.56 "one thousand two hundred eighty-six and fifty-six hundredths" (too high to be the record)**
- **12,865.6 "twelve thousand eight hundred sixty-five and six tenths" (too high to be the record)**

Log in to Amplify Math to complete this lesson online.

498 Unit 5 Arithmetic in Base Ten

1 Launch

Write or project the number 128656 on the board, leaving enough space between each digit for the pre-cut decimal point to be "pinned." Select students to blindfold (or close their eyes), spin three times, and then place the decimal point on the number. Consider "guiding" students so that a variety of numbers can be made. After each placement, ask, "Could that number make sense as the record?" Then show the video to see whether the number is correct.

2 Monitor

Help students get started by reminding how to read a decimal by using the place value chart on the *Place Value Chart* PDF. Consider also having students first saying the number to the right of the decimal point without the decimal point, or as if it is not there, to help transition to the decimal reading.

Look for points of confusion:

- **Using words such as *point* or *dot*, instead of *and* when referring to the decimal point.** Repeat what students said, but use and emphasize the word *and* for the decimal point.

Look for productive strategies:

- Being mindful of correctly saying the numbers using *and* and the ending *-ths*.
- Recognizing a reasonable number for the record, and writing and reading the number appropriately.

3 Connect

Display the animation of the record run in the digital version of the lesson, revealing the correct answer.

Highlight the use of the word *and* to verbally signify the decimal point and that language matters when reading a decimal number because it is describing the places.

Ask:
- "How would you write 168 in words?" one hundred sixty-eight
- "What about this number?" [Write out 100.68.] one hundred and sixty-eight hundredths

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

As students pin the decimal point, encourage them to explain their reasoning by asking:
- "Is it reasonable for 1.28656 to be the record?"
- "How do you know that the decimal point placement isn't too high or too low?"

Revoice student language to demonstrate the appropriate use of the word "and" and the ending "-ths" when reading decimal values.

English Learners

The term *record* might be new to many students. Be sure to define the term *record* in this context.

Small groups | 25 min

Activity 1 World's Longest Fingernails Challenge

Students recall how to perform operations with decimals in order to recreate fingernails of world record lengths.

Name: Date: Period:

Activity 1 World's Longest Fingernails Challenge

At some point in 1979, Lee Redmond decided to let all of her fingernails continue to grow. Fast forward 30 years and they would measure, in total, a staggering 8.65 m (28 ft 4.5 in.). Unfortunately, in early 2009, Lee's nails all broke in an automobile accident, but she still holds the Guiness World Record for the "Longest fingernails on a pair of hands ever (female)."

But wait! There's more.

In 2014, the Guiness World Record for the "Longest fingernails on a single hand ever" was recorded by Shridhar Chillal of India. They measured 909.6 cm (29 ft 10.1 in.) in total. He still held the record as of 2020, but, after 66 years of not doing so, he did cut his nails on July 11, 2018.

Part 1

1. Whose fingernails are longer and by how much?
   Shridhar Chillal's single hand of fingernails are longer than Lee Redmond's two hands of fingernails by 44.6 cm.
   $909.6 - 865 = 44.6$ cm

2. If Shridhar had let his nails grow to the exact same length on *both* hands, what would the total length of both hands have been? 1819.2 cm; Sample responses:
   - $909.6 \cdot 2 = 1819.2$
   - $909.6 + 909.6 = 1819.2$ cm

Part 2

Your group will be assigned to recreate either Shridhar's fingernails or Lee's fingernails using strips of paper. Decide who in your group will wear the model. Tape strips of paper together and then tape the long strips to this person's nails for a truly visual experience of a world record.

Record the measurements your group uses and creates for each fingernail in the table.

	Thumb	Pointer	Middle	Ring	Pinky	Total
Shridhar						
Lee						

Sample responses:
Possible measurement for Shridhar Chillal is 181.92 cm for each nail, if the length is divided by 5.
Possible measurement for Lee Redmond is 86.5 cm for each nail, if the length is divided by 10.

STOP

Lesson 1 Precision and World Records 499

1 Launch

Read the information and instructions aloud. Give students 3 minutes to complete Part 1, and then assign half the groups to Lee's nails and the other groups to Shridhar's nails.

2 Monitor

Help students get started by asking "If you know the total length of all five nails together, what can you do to find the length of one nail?"

Look for points of confusion:

- **Subtracting the meters from centimeters in Problem 1.** Ask "Can you subtract different units of measurements from each other?"
- **Measuring inaccurately.** Have students in the group check on each other's measurements.

Look for productive strategies:

- Converting Lee's measurement from in. to cm.
- Using operations and tools correctly for precision.
- Making one strip for the entire length and randomly cutting it into five pieces. Encourage students to then add the five lengths to show an operation, or ask, "How would the record checker have measured their nails?"
- Being creative with different lengths for each fingernail, while maintaining the correct sum. If a photo is shown, students may want to mimic each actual nail's length as they can be discerned.

3 Connect

Display each group's set of paper nails (on or off the student).

Have groups of students share how they determined the measurement for each fingernail and then share their answers to Problems 1 and 2.

Highlight that to be identified as a Guinness of World Records™ achievement, the measurements must be both accurate and precise.

Ask, "What work with decimals did you use?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to round the actual measurements to whole numbers of centimeters for Part 1 (Lee: 865 cm, Shridhar: 910 cm). If students need more processing time, assign them to work with Lee's nails during Part 2.

Accessibility: *Activate Prior Knowledge*

Remind students that they previously learned how to convert measurements within the metric system or U.S. customary system. Ask:

- "How many centimeters are in 1 m?"
- "How many inches are in 1 ft?"

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand there are two individuals who grew their fingernails out to earn world records.
- **Read 2:** Ask students to name given quantities, such as the length of each person's fingernails.
- **Read 3:** Ask students to brainstorm strategies for how they will respond to Problem 1.

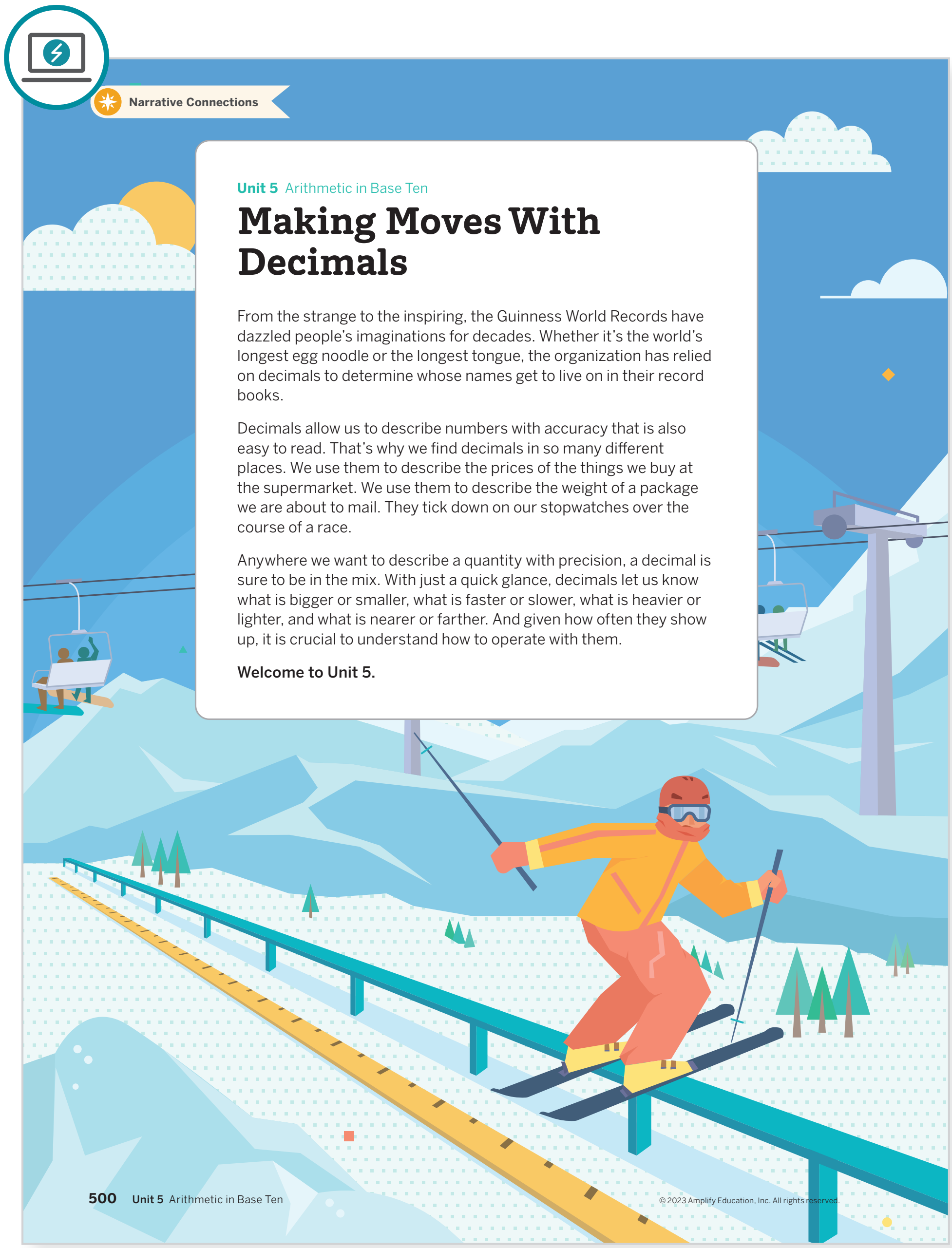
English Learners

Draw quick illustrations of two hands with the lengths of the fingernails labeled.

Whole Class | 5 min

Summary Making Moves With Decimals

Review and synthesize the language and precision of decimals.



Narrative Connections

Unit 5 Arithmetic in Base Ten

Making Moves With Decimals

From the strange to the inspiring, the Guinness World Records have dazzled people's imaginations for decades. Whether it's the world's longest egg noodle or the longest tongue, the organization has relied on decimals to determine whose names get to live on in their record books.

Decimals allow us to describe numbers with accuracy that is also easy to read. That's why we find decimals in so many different places. We use them to describe the prices of the things we buy at the supermarket. We use them to describe the weight of a package we are about to mail. They tick down on our stopwatches over the course of a race.

Anywhere we want to describe a quantity with precision, a decimal is sure to be in the mix. With just a quick glance, decimals let us know what is bigger or smaller, what is faster or slower, what is heavier or lighter, and what is nearer or farther. And given how often they show up, it is crucial to understand how to operate with them.

Welcome to Unit 5.

500 Unit 5 Arithmetic in Base Ten

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Ask:

- "Why do you think precision is important for world records?"
- "Why is language important when reading decimals?"

Highlight that the position of the word *and*, as well as the *-ths* suffix, is very important when speaking about precision and decimals.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "What did you already know and understand about decimals that was confirmed during this lesson?"
- "What is something new you learned or now understand about decimals after this lesson?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of place value and decimal language by naming the place of each digit of a number and writing it in words and by calculating with a decimal.

Printable

Name: ________ Date: ________ Period: ________

Exit Ticket 5.01

In 2019, the Guinness World Record for the "Fastest speed in a body-controlled jet engine powered suit (wind-guided)" was set at 136.891 kph. This was more than twice as fast as the previous record.

1. Name the place value of each digit in 136.891. Then write the entire number in words.
   1 hundred
   3 tens
   6 ones
   8 tenths
   9 hundredths
   1 thousandth
   One hundred thirty-six and eight hundred ninety-one thousandths

2. Estimate this record-setting speed in miles per hour. Use the approximate conversion of 1 km = 0.62 miles. Show your thinking.
   Student responses should be between 84 and 85 mph. Sample responses:
   - $137 \cdot 0.62 = 84.94$, or 85 mph
   - $136 \cdot \frac{62}{100} = 84.32$ mph
   - The ratio 1 : 0.62 is equivalent to the ratio 137 : ?
     The estimate is approximately 84.94 or 85 mph.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can identify the place values of decimals past the hundredths place. 1 2 3

b. I can use the language of decimals correctly. 1 2 3

Lesson 1 Precision and World Records

Success looks like . . .

- **Language Goal:** Analyzing a number beyond the hundredths place based on place value. **(Speaking and Listening, Writing)**
  - » Identifying each place value of the number 136.891.
- **Language Goal:** Calculating sums and products of decimals in the context of world records, and explaining the calculation strategy. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly identify a digit's place, consider:

- Giving students a smaller version of the place value chart. They can use this as a reference to correct their Exit Ticket and for future activities, if necessary, throughout the unit.

If students do not convert kilometers to miles correctly, consider:

- Setting up a ratio table so that students can identify the relationship between kilometers and miles.

Kilometers	Miles
1	0.62
2	1.24
136 or 137	?

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did collaborating to make the fingernails reveal about your students as learners?
- What expectations for communication enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ____________ Date: ____________ Period: ____________

Practice

1. The Guinness World Record for the “Longest horns on a yak” belongs to Jericho the Yak from Welch, Minnesota, measuring at a total of 346.4 cm. A yak has two horns. If both horns were of equal length, what is the length of one horn?
   173.2 cm; $346.4 \div 2 = 173.2$

2. Tickets to a show cost \$5.50 for adults and \$4.25 for students. A family is purchasing 2 adult tickets and 3 student tickets.
   a. Estimate the total cost.
      Sample responses: \$22
   b. What is the exact cost?
      $\$23.75; 2 \cdot 5.50 + 3 \cdot 4.25 = 23.75$
   c. If the family gives the ticket attendant \$25, what is the exact amount of change they should receive?
      $\$1.25; 25.00 - 23.75 = 1.25$

3. Complete each expression with one of the following values so that the corresponding quotient fits correctly in each category. A number may be used more than once. **Sample responses shown.**

0.001	0.1	10	1,000

Close to $\frac{1}{100}$	Close to 1	Greater than 100
0.1 $\div 9$	**0.1** $\div 0.12$	**1,000** $\div \frac{1}{3}$
$12 \div$ **1,000**	$\frac{1}{8} \div$ **0.1**	$700.7 \div$ **0.001**

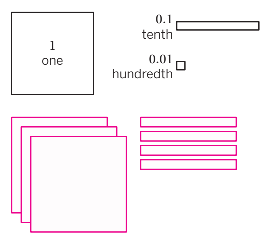
Name: ____________ Date: ____________ Period: ____________

Practice

4. Determine each quotient. Show your thinking.
   a. $\frac{5}{6} \div \frac{1}{6} = \frac{5}{6} \cdot \frac{6}{1} = 5$
   b. $1\frac{1}{6} \div \frac{1}{12} = \frac{7}{6} \cdot \frac{12}{1} = 14$
   c. $\frac{10}{6} \div \frac{1}{24} = \frac{10}{6} \cdot \frac{24}{1} = 40$

5. One bottle of water holds $1\frac{1}{2}$ liters.
   a. How many $\frac{1}{5}$-liter glasses can be filled with one bottle of water?
      $7\frac{1}{2}$ glasses; Sample response: $1\frac{1}{2} \div \frac{1}{5} = \frac{3}{2} \cdot \frac{5}{1} = \frac{15}{2} = 7\frac{1}{2}$
   b. How many bottles of water are needed to fill a 16-liter jug?
      $10\frac{2}{3}$ bottles; Sample response: $16 \div 1\frac{1}{2} = 16 \cdot \frac{2}{3} = \frac{32}{3} = 10\frac{2}{3}$

6. How would you represent the number 3.4:
   a. In words? **Three and four tenths**
   b. As a model using these blocks?



Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 10	1
	5	Unit 4 Lesson 16	2
Formative	6	Unit 5 Lesson 2	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Whole Class

Sub-Unit 1
Adding and Subtracting Decimals

In this Sub-Unit, students transition from concrete models and place-value strategies to formalize a standard algorithm for adding and subtracting multi-digit decimals, including those with decimal places beyond hundredths.

SUB-UNIT
1 Adding and Subtracting Decimals

Narrative Connections

How did a decimal decide an Olympic race?

Most of the time, whole numbers do a good job in telling us the outcome of any game or sport. With just a glance at a scoreboard, it should be clear which team is the champion. But while whole numbers are great for something like basketball, tennis, or competitive tuk tuk polo, there are some sports where you will want to be a bit more precise.

Take, for instance, the 1964 road cycling championship at the Tokyo Olympics. One hundred thirty-nine cyclists from 37 countries took to the race course, pelted by heavy rain. For much of the race, the peloton — the main group of cyclists — stayed clustered in a tight pack. Despite their best efforts, none of the cyclists could break out into a significant lead.

The race ended up having the tightest finish in history, with 51 cyclists all crossing the finish line in 4 hours 39 minutes and 51 seconds. But the race was not a tie. The winner, Italy's Mario Zanin finished first at 4:39:51.63, followed by Denmark's Kjell Rodian at 4:39:51.65 in second place, and Walter Godefroot from Belgium at 4:39:51.74 in third. The difference between the gold medalist Zanin and the cyclist who took 99th place was only 0.2 seconds!

As history has shown, a difference of a few hundredths of a second can make or break an Olympic championship. Understanding how to add and subtract these decimals can help you figure out who's in the lead and by how much.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to see the importance of precision when adding or subtracting decimals in sporting events in the following places:

- **Lesson 4, Activity 1:** Skateboard Big Air
- **Lesson 4, Activity 2:** Snowboard Cross

Speaking of Decimals . . .

Let's use the language of decimals.

Focus

Goals

1. **Language Goal:** Connect place value to describing decimals. **(Speaking and Listening, Writing)**
2. **Language Goal:** Compare and contrast vertical calculations and base ten diagrams that represent adding and subtracting decimals. **(Speaking and Listening, Writing)**
3. Interpret and create diagrams that represent 10, such as base ten units being composed into 1 unit of higher place value, e.g., 10 tenths as 1 one, and understand how the word *compose* is used in this context.

Coherence

Today

Students use and compare two methods — base ten diagrams and vertical calculations — to determine sums and differences of decimals. Central to both methods is an understanding of the meaning of each digit in the numbers and how the different digits are related. Students recall that digits are only added or subtracted if they represent the same base ten units. This idea is made explicit in both diagrams and vertical calculations, particularly when values need to be composed or decomposed. The use of precise mathematical language is critical in describing decimal numbers and attending to place value when adding and subtracting.

Previously

In Grades 4 and 5, students added and subtracted decimals to the hundredths by using a variety of methods, all of which emphasize an understanding of place value.

Coming Soon

In Lessons 3 and 4, students will continue to add and subtract decimals by composing and decomposing.

Rigor

- Students build their **conceptual understanding** of decimals through reading and writing decimals beyond hundredths.
- Students develop **fluency** in connecting place value and diagrams to addition and subtraction with decimals.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	12 min	12 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (display)
- *Decimal Base Ten Blocks* PDF (optional)
- *Place Value Chart* PDF (optional)
- base ten blocks
- graph paper

Math Language Development

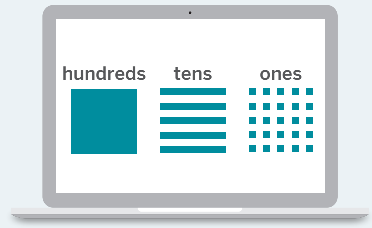
Review words

- *compose (base ten units)*
- *decimal*

Amps | Featured Activity

Activity 1
Multiple Representations

Students can toggle between multiple representations of decimal addition.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

When working through Activity 1, students may not understand that they need to use the tools. Remind students that the available tools will help them successfully complete the activity. Prior to the start of the activity, you might want to have them identify what tools could be helpful and discuss how to use them, if necessary.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be done as a whole class, focusing on the correct way to say *twenty-three hundredths* in Problem 1, and limiting the discussion to representations in Problem 2.
- In **Activity 1**, Part 1 may be done as a whole class. In Part 2, Problem 3 may be omitted by simply displaying the Activity 1 PDF, and students can choose to do one part of Problem 4.
- In **Activity 2**, Problem 2 may be omitted.

Pairs | 10 min

Warm-up What's the Number?

Students analyze a decimal number to review the language of decimals and of place value.

Unit 5 | Lesson 2

Speaking of Decimals . . .

Let's use the language of decimals.

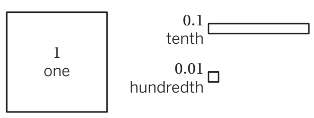
Warm-up What's the Number?

1. Think of as many different ways as possible that you could say the number 0.23 aloud.

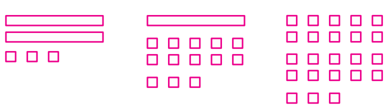
Sample responses:

- Zero point two three
- Zero point twenty-three
- Zero decimal twenty-three
- Zero dot two three
- Zero dot twenty-three
- No wholes and twenty-three hundredths
- Zero and twenty-three hundredths
- Zero and twenty tenths and three hundredths
- Twenty three hundredths
- Twenty tenths and three hundredths

2. Show one way to represent 0.23 using these base ten blocks. If you can think of other ways, show as many ways as possible.



Sample responses:



Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work on the Warm-up, monitoring to ensure that students have time for both problems.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "What is the place value of each digit? How does that help you read the number?"

Look for points of confusion:

- **Confusing the places and their values.** Have students reference the place value chart and ask, "What number would this be without the decimal point? What happens when you put it on the place value chart? At what place does the number end?"
- **Incorrectly representing the number.** Make decimal base ten blocks available to students.

Look for productive strategies:

- Using language correctly, including the word *and* to signify the decimal point (instead of *dot* or *point*).

3 Connect

Have pairs of students share their responses, focusing on correct decimal language (not using *dot* or *point* and articulating the *-th/-ths* suffixes).

Highlight that, when the value of a base ten unit is 10 or more, it can be expressed with a different unit that is 10 times less in value. For example, two tenths can be expressed as 20 hundredths. Therefore, each place represents a unit that is 10 times larger than the unit immediately to its right.

Ask:

- "You saw in Unit 3 that *cent* in the word *percent* refers to 100. What part of the word *decimal* could help with your understanding of decimals?"
- "Why do you think there is not a 'oneths' place?"

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students that they previously learned about place value for whole numbers and decimals in elementary school. Display or distribute copies of the *Place Value Chart* PDF for students to use as a reference.

Power-up

To power up students' ability to express decimals and understand place values, ask:

How would you represent the number 0.11 as a model using base ten blocks?

Sample responses:

Use: Before the Warm-up.

Informed by: Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 1 and 2.

Pairs | 12 min

Activity 1 Representing Addition With Decimals

Students use language, symbols, and diagrams to determine a sum that requires regrouping, or "composing," base ten units.

Amps Featured Activity Multiple Representations

Name: ____________ Date: ________ Period: ________

Activity 1 Representing Addition With Decimals

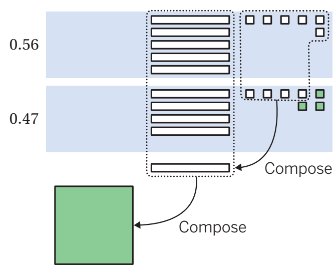
Part 1

Here are two methods you can use to calculate $0.56 + 0.47$. In the diagram, each large square represents 1, each rectangle represents 0.1, and each small square represents 0.01.

Vertical calculation:

$$\begin{array}{r} {}^{1}\ {}^{1}\ \ \\ 0.56 \\ +\,0.47 \\ \hline 1.03 \end{array}$$

Base ten diagram:



1. Write an addition equation to represent this sum using both numbers and words.
   $0.56 + 0.47 = 1.03$
   Fifty-six hundredths plus forty-seven hundredths equals one and three hundredths

2. Use what you know about base ten units and adding base ten numbers to respond to these questions.
   a. How can ten small squares be *composed* into a rectangle?
      Ten hundredths can be placed together to create one tenth.
   b. How can ten rectangles be *composed* into a large square?
      Ten tenths can be placed together to create one whole.
   c. How are the different types of *composition* represented in the vertical calculation?
      Sample response: The thirteen hundredths are partitioned to show ten hundredths moving to the tenths place and three staying in the hundredths place. One group of ten tenths are shown moving to the ones place, leaving zero in the tenths place.

1 Launch

Give groups 4–5 minutes to discuss and complete Part 1. Pause to have a whole-class discussion about how to label the key in Part 2. Then give 7–8 minutes to complete Part 2, Problems 4 and 5. Provide access to the *Place Value Chart* PDF, decimal base ten blocks, and graph paper.

2 Monitor

Help students get started by saying "Describe to your partner what you see happening in the diagram." Ask the partner to paraphrase what they heard.

Look for points of confusion:

- **Drawing diagrams that do not demonstrate composing.** Provide decimal base ten blocks for students to work with and exchange smaller units for the composed unit.
- **Not lining up decimal points.** Consider providing a graphic organizer, such as graph paper, to help students make sure the values in the same places are aligned.

Look for productive strategies:

- Articulating responses effectively using decimal language.
- Composing groups of ten in applicable place values (e.g., using ten thousandths to make one hundredth).
- Identifying when a tool (decimal base ten blocks, Place Value chart) or a strategy (base ten diagram or vertical calculation) is useful and then using it appropriately.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate digital base ten blocks to compose decimal numbers and expressions.

Accessibility: *Optimize Access to Tools*

If you choose not to use the Amps slides for this activity, provide continued access to physical base-ten blocks or distribute copies of the *Decimal Base Ten Blocks* PDF for students to use.

Math Language Development

MLR7: Compare and Connect

During the Connect, display two different representations for calculating the sum of 6.038 and 0.09:

- Base ten diagrams
- Vertical calculations

Ask students to identify where the *composing* occurred and how it is shown in each representation.

English Learners

Annotate the base ten diagrams to illustrate the composing by writing the term *composing*.

Pairs | 12 min

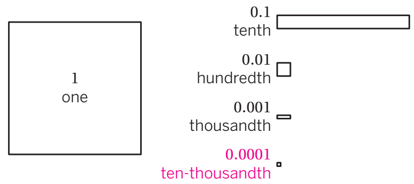
Activity 1 Representing Addition With Decimals (continued)

Students use language, symbols, and diagrams to determine a sum that requires regrouping, or “composing,” base ten units.

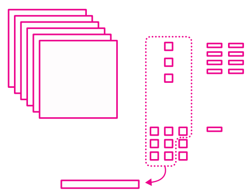
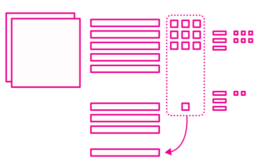
Activity 1 Representing Addition With Decimals (continued)

Part 2

3. Each larger shape can be composed of 10 of the next smaller shape. What value does this new smallest shape represent?



4. Decide who will be Partner A and who will be Partner B. Evaluate each sum as indicated, using either a base ten diagram or a vertical calculation.

Partner A	Partner B
Calculate $6.038 + 0.091$. $\begin{array}{r} \scriptstyle 1 \\ 6.038 \\ +\,0.091 \\ \hline 6.129 \end{array}$	Draw a diagram to represent $6.038 + 0.091$. 
Draw a diagram to represent $2.5936 + 0.3132$. 	Calculate $2.5936 + 0.3132$. $\begin{array}{r} \scriptstyle 1 \\ 2.5936 \\ +\,0.3132 \\ \hline 2.9068 \end{array}$

5. After you have both completed both of your problems, discuss the following questions with your partner. **Answers may vary.**
   - How did the vertical calculation and the diagram show composing units?
   - Is one method of calculating more efficient than the other? If so, why?

506 Unit 5 Arithmetic in Base Ten

3 Connect

Display the Activity 1 PDF for students to refer to as they share their thinking.

Have pairs of students share their responses, focusing on their responses to Part 2, Problem 5.

Define (if necessary) ***compose (base-ten units)*** as forming groups of ten of one place value to make groups of the next greater place value. **Note:** Students may have previously used other valid terms, such as *grouping, regrouping*, or *bundling*.

Ask:

- “Where would the decimal point go in the diagrams?” Between the large square and large rectangle.
- “Where have you already seen the term *composing* used this year? How is this similar?” In Unit 1, I put shapes together to form new shapes or to calculate area. The sizes of the base ten blocks in the diagrams represent composing areas in this same way.

Highlight that addition of decimals beyond hundredths works in the same way as addition of whole numbers and of decimals up to hundredths. It is still combining the values of like base ten units and composing as necessary.

Differentiated Support

Extension: *Math Around the World, Interdisciplinary Connections*

Ask students if they have ever wondered how the decimal system was developed. Provide them with the information shown:

- Many historians believe that the base ten place value system was developed because humans used 10 fingers for counting.
- The first known ruler is from the Indus Valley Civilization and was divided into both units and sub-units, revealing an understanding of fractional or decimal subdivisions.
- Ancient Egyptian mathematicians used a decimal system with hieroglyphics.
- Decimal fractions were used by Chinese mathematicians near the end of the 4th century BCE and later made their way to the Middle East and then to Europe.
- The Chinese number system used bamboo rods and different arrangements of the rods indicated the numbers from 1 to 9. They placed digits next to each other to indicate a place value system, similar to the one we use today.

Have students research one of these number systems or another one that they find through their own research, create a visual display, and present their findings to the class. **(History)**

Pairs | 12 min

Activity 2 Representing Subtraction With Decimals

Students use diagrams and vertical calculations to subtract, identifying the need to pair the digits of like base ten units, and explain why it is helpful to line up the decimal points.

Name: ______ Date: ______ Period: ______

Activity 2 Representing Subtraction With Decimals

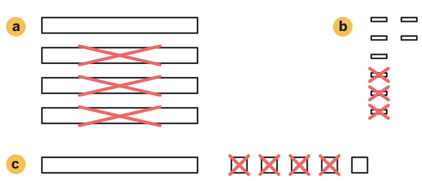
Common language and representations have always been important for sharing mathematical results.

Two symbols that are important in communicating decimal operations are the digit 0 and the decimal point. Believe it or not, both of these are only about a thousand years old. In ancient China, they were introduced around 1247 CE. Qin Jiushao used the ○ symbol as the first digit in a number with a value less than 1. So by writing 096644 with that symbol as the 0, he was really indicating the number 0.96644.

Plan ahead: After evaluating your use of base ten blocks in Activity 1, how will you change your interactions for Activity 2?

Complete these problems using the same key from Activity 1 as a common language to interpret the values represented by each base ten diagram.

1. Here are diagrams representing three subtraction expressions. The pieces marked with "X" have been removed. With your partner, say the expressions represented by each diagram aloud, then evaluate the difference. Write the subtraction equation it represents using numbers.



a. $0.4 - 0.3 = 0.1$
b. $0.008 - 0.003 = 0.005$
c. $0.15 - 0.04 = 0.11$

Featured Mathematician

Qin Jiushao

Qin Jiushao was a 13th century Chinese mathematician, inventor, and politician. In *Shùshū Jiǔzhāng* ("Mathematical Treatise in Nine Sections") he published a general form of the Chinese Remainder Theorem and several new discoveries in algebra and geometry. Qin is also attributed with the first written use of a symbol for 0 and introducing an understanding of decimal fractions in China.

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Keep the Activity 1 PDF displayed.

2 Monitor

Help students get started by asking "What does a rectangle represent? How many are there? What is happening in the diagram?"

Look for points of confusion:

- **Lining up the digits incorrectly**. Consider providing a graphic organizer, such as graph paper, to help students make sure the values in the same places are aligned. This will help students pay close attention to place value when calculating differences.

Look for productive strategies:

- Connecting their language to the diagrams represented.
- Presenting clear diagrams and vertical calculations.

Activity 2 continued ›

Differentiated Support

Accessibility: ***Optimize Access to Technology***

Have students use the Amps slides, in which they can manipulate digital base ten blocks to compose decimal numbers and expressions.

Accessibility: ***Guide Processing and Visualization***

In Problem 2, have students begin by using whole number expressions and then slowly moving the decimal point each time. For example, for Problem 2a, have them determine each of these differences, in the following order:

$5 - 2$ $\quad$ $0.5 - 0.2$ $\quad$ $0.05 - 0.02$

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, draw connections between the base ten diagrams and the vertical calculations. Ask students to respond to the questions posed in their Student Edition:

- "Where do you see 0.024 in the base ten diagram for Problem 2b?"
- "Where do you see 0.003 in the base ten diagram for Problem 2b?"

Pose similar questions for parts a and c.

Featured Mathematician

Qin Jiushao

Have students read about Chinese mathematician Qin Jiushao, who brought the concept of decimals to China and helped introduce notation involving a circle for 0. This was a novel idea when compared to most previous work in which the idea of absence of a digit for a place value was simply indicated by exaggerated spaces between numerals.

Pairs | 12 min

Activity 2 Representing Subtraction With Decimals (continued)

Students use diagrams and vertical calculations to subtract, identifying the need to pair the digits of like base ten units, and explain why it is helpful to line up the decimal points.

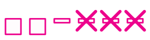
Activity 2 Representing Subtraction With Decimals (continued)

2. Determine each difference by drawing a base ten diagram *and* by using a vertical calculation. Compare your results with your partner. If you have different results, work together to discuss and resolve any differences until you both arrive at the same result.

a. 0.05 – 0.02

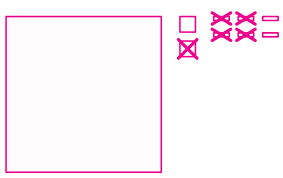
$$\begin{array}{r} 0.05 \\ -\ 0.02 \\ \hline 0.03 \end{array}$$

b. 0.024 – 0.003



$$\begin{array}{r} 0.024 \\ -\ 0.003 \\ \hline 0.021 \end{array}$$

c. 1.26 – 0.14



$$\begin{array}{r} 1.26 \\ -\ 0.14 \\ \hline 1.12 \end{array}$$

Compare and Connect: How do the base ten diagrams compare to the vertical calculations? Where do you see 0.024 in the base ten diagram for part b? Where do you see 0.003?

Are you ready for more?

A concession stand sells pretzels for $3.25 each, drinks for $1.85 each, and bags of popcorn for $0.99 each. Clare purchased at least one of each item and spent no more than $10. Could Clare have purchased each of the following combinations of items? Explain your thinking.

1. 2 pretzels, 2 drinks, and 2 bags of popcorn?

$$\begin{array}{r} \scriptstyle 1 \\ 3.25 \\ +\ 3.25 \\ \hline 6.50 \end{array} \quad \begin{array}{r} \scriptstyle 1\ 1 \\ 1.85 \\ +\ 1.85 \\ \hline 3.70 \end{array} \quad \$2.00 - 0.02 = 1.98 \quad \begin{array}{r} \scriptstyle 2 \\ 6.50 \\ 3.70 \\ +\ 1.98 \\ \hline 12.18 \end{array}$$

No, she would need $2.18 more.

2. 1 pretzel, 1 drink, and 5 bags of popcorn?

$$\begin{array}{r} \scriptstyle 4\ 4 \\ 0.99 \\ \times\quad 5 \\ \hline 4.95 \end{array} \quad \begin{array}{r} \scriptstyle 2\ 1 \\ 4.95 \\ 3.25 \\ +\ 1.85 \\ \hline 10.05 \end{array}$$

No, she would need 5 more cents.

STOP

508 Unit 5 Arithmetic in Base Ten

3 Connect

Display the given diagrams from Problem 1 for students' reference as they share their responses, and then display the student diagrams for Problem 2.

Have pairs of students share their responses and explain how the diagrams represent the given expressions, the process of subtraction and vertical calculations, and the differences.

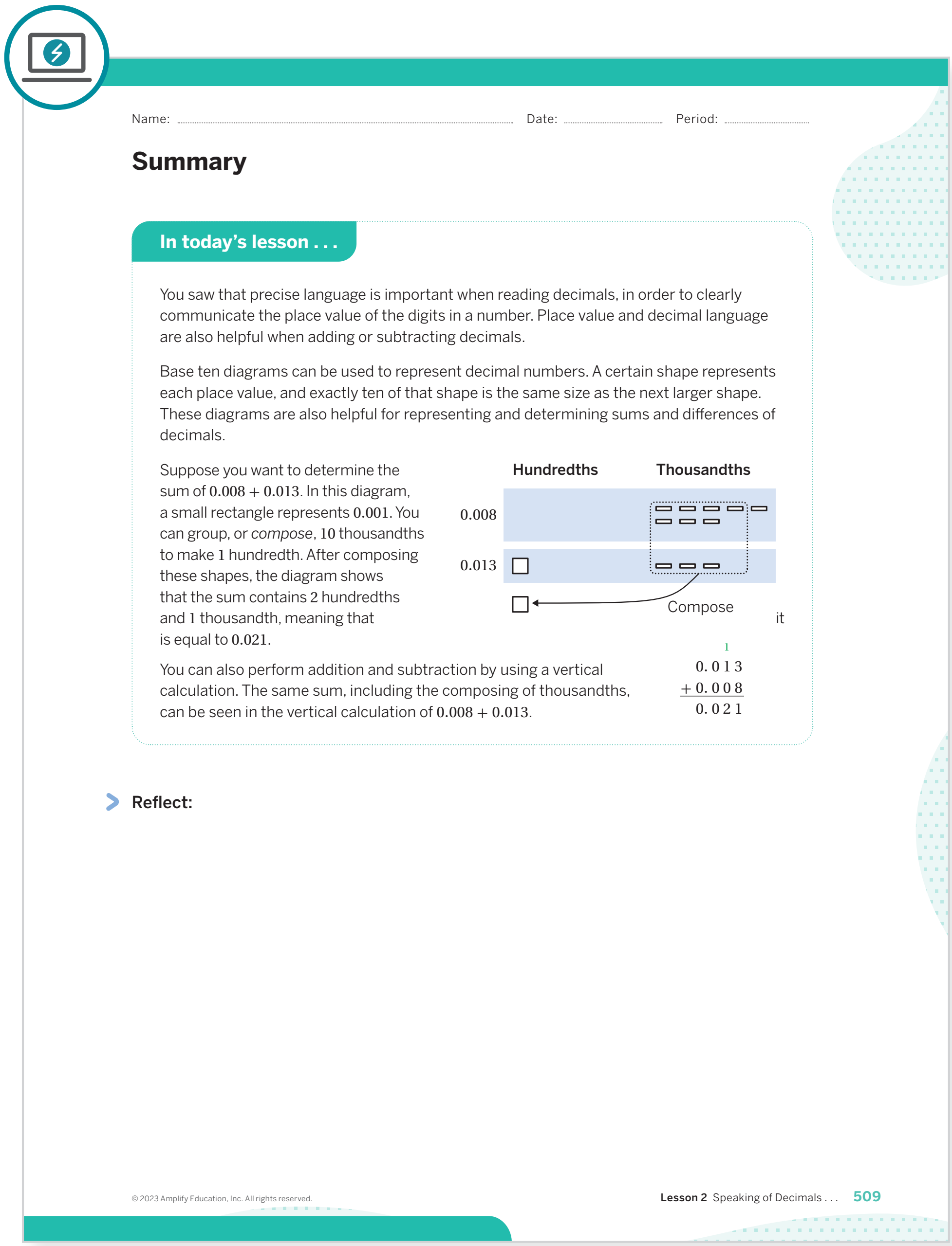
Ask:

- "Why is it helpful to line up the decimal points when calculating differences of decimals?" Aligning the points aligns the same place values.
- "Why was there no composing of units in these subtraction problems?" The values were not being added or put together and so, no place value could end up with a value greater than 9.

Highlight that, when students perform subtraction without diagrams, it is essential to pay close attention to the place value in the numbers.

Summary

Review and synthesize the connections between place value, decimal language, and calculations by using diagrams or vertically aligned numbers.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that precise language is important when reading decimals, in order to clearly communicate the place value of the digits in a number. Place value and decimal language are also helpful when adding or subtracting decimals.

Base ten diagrams can be used to represent decimal numbers. A certain shape represents each place value, and exactly ten of that shape is the same size as the next larger shape. These diagrams are also helpful for representing and determining sums and differences of decimals.

Suppose you want to determine the sum of $0.008 + 0.013$. In this diagram, a small rectangle represents 0.001. You can group, or *compose*, 10 thousandths to make 1 hundredth. After composing these shapes, the diagram shows that the sum contains 2 hundredths and 1 thousandth, meaning that it is equal to 0.021.

You can also perform addition and subtraction by using a vertical calculation. The same sum, including the composing of thousandths, can be seen in the vertical calculation of $0.008 + 0.013$.

$$\begin{array}{r} {\scriptstyle 1} \\ 0.013 \\ +\,0.008 \\ \hline 0.021 \end{array}$$

Reflect:

Synthesize

Highlight the idea of composing. Students can group 10 of any base ten unit into 1 of a base ten unit that is 10 times as large, and this is reflected in the language of decimals.

Ask:

- "How do the pieces representing ones, tenths, hundredths, etc., of a base ten diagram help you to add or to subtract two decimals?" They show clearly the values that should be added or subtracted in order to determine the value for each decimal place of the sum or difference.
- "How is adding and subtracting decimal numbers similar to adding and subtracting whole numbers?" It is important to attend to place value and to add or subtract numbers that represent the same base ten units.
- "How are the diagrams and calculations for adding and subtracting decimal numbers similar and different?" Each place value is represented by the same base ten shapes and aligned in calculations. For adding, you are joining them together, and for subtracting, you are removing one from the other.
- "Is one method of calculating more efficient? If so, when or why?" Choosing a method depends on the size of the numbers, but vertical calculations are more efficient for numbers with more decimal places and greater digits. Drawing could take a long time for some numbers, such as 2.315 or 9.999.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when adding or subtracting decimals? How were they helpful?"
- "How are addition and subtraction of decimals similar? How are they different?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of addition with decimals in the context of scores from the X Games.

Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 5.02

The top three winners of the BMX Vert competition from the X Games Minneapolis 2019 are shown here. To determine the final standings, the best scores for each of the riders from either of two runs are ranked.

Final					
Rank	Name		Run 1	Run 2	Best
1	Vince Byron	Gold	35.33	90.66 ✓	90.66
2	Jamie Bestwick	Silver	89.33 ✓	87.66	89.33
3	Mykel Larrin	Bronze	87.00	88.66 ✓	88.66

Suppose instead that the final standings were based on the total points from both runs. Would these three riders have been ranked in the same order? Explain your thinking.

No, their rankings would not have been in the same order; Sample response: Jamie Bestwick would have been ranked the highest, Mykel Larrin as the second highest, and Vince Byron as the third highest of these three riders.

Vince Byron: 125.99

$$\begin{array}{r} 35.33 \\ +90.66 \\ \hline 125.99 \end{array}$$

Jamie Bestwick: 176.99

$$\begin{array}{r} {}^{1} \\ 89.33 \\ +87.66 \\ \hline 176.99 \end{array}$$

Mykel Larrin: 175.66

$$\begin{array}{r} {}^{1} \\ 87.00 \\ +88.66 \\ \hline 175.66 \end{array}$$

Self-Assess

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it

a. I can use diagrams to represent and reason about addition and subtraction of decimals.

1 2 3

b. I know how to solve addition problems with decimals that require composing numbers.

1 2 3

c. I can use place value to explain addition and subtraction of decimals.

1 2 3

Success looks like . . .

- **Language Goal:** Connecting place value to describing decimals. **(Speaking and Listening, Writing)**
- **Language Goal:** Comparing and contrasting vertical calculations and base ten diagrams that represent adding and subtracting decimals. **(Speaking and Listening, Writing)**
  - » Performing vertical addition and comparing the sums.
- **Goal:** Interpreting and creating diagrams that represent 10, such as base ten units being composed into 1 unit of higher place value, e.g., 10 tenths as 1 one, and understand how the word *compose* is used in this context.

Suggested next steps

If students line up the decimals incorrectly, consider:

- Providing graph paper and showing them how to line up digits that have the same place value. Then have them perform the vertical calculation.

If students add incorrectly, consider:

- Providing base ten blocks to help them perform the addition. Then have them write the vertical calculation to match their process with the blocks.

If students spend too much time drawing diagrams, instead of using the vertical calculation method, consider:

- Asking, "Is there a more efficient way to calculate this sum?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

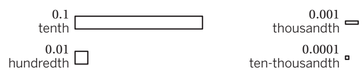
- What worked and didn't work today? A goal of this lesson was to make students mindful of their decimal language. How did that go?
- Which teacher actions made defining *composing (base ten units)* impactful? What might you change for the next time you teach this lesson?

Practice

Independent

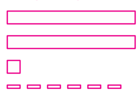
Name: ____________ Date: ________ Period: ________

1. Use this key to complete these problems.



a. What number does this diagram represent?
0.025

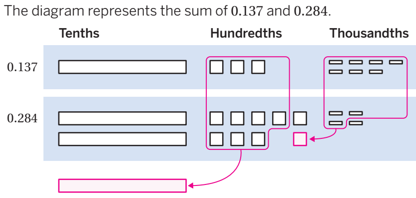
b. Draw a diagram that represents 0.216.
Sample response:



c. Draw a diagram that represents 0.304.

Sample response:

2. The diagram represents the sum of 0.137 and 0.284.



a. Use the diagram to determine $0.137 + 0.284$. Show your thinking.

b. Show how to determine the sum using a vertical calculation.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 0.137 \\ +\ 0.284 \\ \hline \boxed{0.421} \end{array}$$

c. How are the methods similar in representing addition? How are they different?
Sample response: Both show composing groups of 10 of a smaller place value to make 1 of the next larger place value. However, the diagrams use shapes to represent different place values while the vertical calculation only uses numbers.

Name: ____________ Date: ________ Period: ________

3. Circle the vertical calculation in which the digits of the same place value are lined up correctly. Then complete that corresponding calculation.

a. $3.25 - 1$

$$\begin{array}{r} 3.2\ \ 5 \\ -\ 1.0\ \ \\ \hline \end{array} \qquad \begin{array}{r} 3.25 \\ -1.0\ \\ \hline 2.25 \end{array} \qquad \begin{array}{r} 3.25 \\ -\ \ \ \ 1 \\ \hline \end{array}$$

b. $0.5 + 1.15$

$$\begin{array}{r} 0.5\ \ \\ +1.15 \\ \hline 1.65 \end{array} \qquad \begin{array}{r} 0.5 \\ +1.1\ \ 5 \\ \hline \end{array} \qquad \begin{array}{r} 0.50 \\ +1.1\ \ 50 \\ \hline \end{array}$$

4. Calculate each sum or difference using a vertical calculation. Show your thinking.

a. $10.6 + 1.7 = \mathbf{12.3}$

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \ \ \\ 10.6 \\ +\ \ 1.7 \\ \hline 12.3 \end{array}$$

b. $123.5 - 1.2 = \mathbf{122.3}$

$$\begin{array}{r} 123.5 \\ -\ \ 1.2 \\ \hline 122.3 \end{array}$$

5. Tyler's school is having a stair climbing challenge for "Better Health Week." He can now climb 135 stairs in 90 seconds.

a. If Tyler climbs stairs at a constant rate, how many stairs can he climb per second? **1.5 stairs per second; $135 \div 90 = 1.5$**

b. Shawn also participates in the challenge. Shawn can climb 75 stairs in 1 minute. Who climbs at a faster rate? Explain or show your thinking.
Tyler is faster; Sample response: Shawn can climb $75 \div 60 = 1.25$ stairs per second. 1.5 is greater than 1.25 so Tyler is faster than Shawn.

6. Show or explain how you would calculate the sum of $1.091 + 0.009$.
Sample responses shown.
I can add 91 and 9 which equals 100, and then I add the 1 to the zero in the ones place.

$$\begin{array}{r} {\scriptstyle 1\ 1}\ \ \\ 1.091 \\ +0.009 \\ \hline 1.100 \end{array}$$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activitits 1 and 2	2
	4	Activity 2	1
Spiral	5	Unit 3 Lesson 3	2
Formative	6	Unit 5 Lesson 3	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 5 | **LESSON 3**

Adding and Subtracting Decimals

Let's add and subtract decimals.

Focus

Goals

1. **Language Goal:** Add or subtract decimals, and explain the reasoning using multiple representations. **(Speaking and Listening, Writing)**
2. Comprehend that the term *decompose* means to take apart a larger base ten unit into 10 units of a lower place value (e.g., 1 tenth as 10 hundredths).
3. **Language Goal:** Recognize and explain that writing additional zeros or removing zeros after the last non-zero digit in a decimal does not change its value. **(Speaking)**

Coherence

Today

Students continue to add and subtract decimals by using base ten diagrams and vertical calculations, focusing on problems where the decimals are of different lengths. They recall that a base ten unit can be expressed as another unit that is $\frac{1}{10}$ of its size. For example, 1 tenth can be decomposed into 10 hundredths or into 100 thousandths. This reminds students that the same decimal number can be written in several equivalent ways, such as writing four tenths as 0.40, 0.400, or 0.4000, where the additional "trailing" zeros at the end of the decimal do not change its value. They use this idea to subtract a number with more decimal places from one with fewer decimal places, making use of the structure of base ten numbers to decompose when necessary.

Previously

In earlier grades, students decomposed whole numbers in order to subtract. In Lesson 2 of this unit, students used base ten diagrams and vertical calculations to add and to subtract decimals beyond hundredths, composing values as needed in determining and writing sums.

Coming Soon

In Lesson 4, students will add and subtract decimals to solve problems in context, before moving on to multiplication.

Rigor

- Students build on their **conceptual understanding** of place value while adding decimals with one or more decimal digits of 0 and subtracting decimals by decomposing.
- Students continue to develop **procedural skills** and fluency for adding and subtracting decimals.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- *Decimal Base Ten Blocks* PDF (optional)
- *Place Value Chart* PDF (optional)
- base ten blocks
- colored pencils
- graph paper

Math Language Development

New word

- ***decompose (base ten units)***

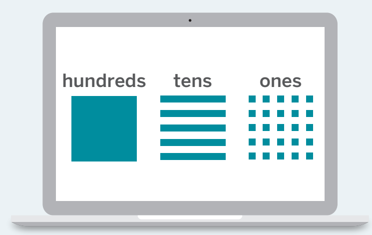
Review words

- *decimal*
- *compose*

Amps | **Featured Activity**

Activity 1
Multiple Representations

Students can toggle between multiple representations of decimal subtraction.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might be frustrated that they are not able to quickly relate the representations and different ways of writing subtraction for decimals. Remind students to have a growth mindset. While they might not be able to synthesize it all at this moment, they will be able to after putting in good effort.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- **Activity 1** may be omitted to give more time to the subtraction work in Activity 2.

Pairs | 5 min

Warm-up Do the Zeros Matter?

Students reason about regrouping and about when the zeros in a decimal affect the number that it represents.

Unit 5 | Lesson 3

Adding and Subtracting Decimals

Let's add and subtract decimals.

Warm-up Do the Zeros Matter?

1. Mentally evaluate the expression $1.009 + 0.391$. Be prepared to explain your thinking.
   1.4

2. Decide whether each equation is *true* or *false*. Be prepared to explain your thinking.
   a. $34.56000 = 34.56$
      True
   b. $25 = 25.0$
      True
   c. $2.405 = 2.45$
      False

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use the ***Think-Pair-Share*** routine. Give them 1 minute of individual think time, and then have them share with a partner.

2 Monitor

Help students get started by asking, "What would happen when you add the 9 and the 1 in the thousandths places?"

Look for points of confusion:

- **Saying that $2.405 = 2.45$ is true because any 0 to the right of the decimal point can be ignored.** Have students also write down the numbers 2.450 and 2.045, and then discuss the value of each digit in each number.

Look for productive strategies:

- Using mathematical language appropriately, such as *composing* for combining bundles, or number names, such as *one and nine thousandths*.
- Recognizing that 0s at the end of a number and to the right of the decimal point add precision, but do not change the value of the number.

3 Connect

Have pairs of students share their responses, focusing on how they wrote their answer to Problem 1 and their reasoning for Problem 2.

Ask:

- "Can zeros be added on at the end of a decimal without changing the number that it represents?"
- "Can zeros be eliminated from the end of a decimal without changing the value?"
- "Can zeros be written or taken away in the middle of a decimal without changing the value?"

Highlight that the number 0 is an important part of a number, indicating an absence of value for any place value and also serving as a placeholder for aligning place values when performing operations.

Math Language Development

MLR2: Collect and Display

During the Connect, as students respond to the Ask questions, begin a class display for this unit if you have not already done so. Add a chart similar to the following to the class display. Invite students to add to the class display during the remainder of this unit and to borrow phrases and mathematical language from the display during discussions.

Placement of zeros

End of a decimal	Middle of a decimal
Does not change the value.	Does change the value.
$0.37 = 0.370$	$0.37 \neq 0.307$

Power-up

To power up students' ability to determine the sum of two decimal values that have the same ending place value, have students complete:

1.
$$\begin{array}{r} 4.23 \\ +0.66 \\ \hline 4.89 \end{array}$$

2.
$$\begin{array}{r} {\scriptstyle 1} \\ 2.789 \\ +4.004 \\ \hline 6.793 \end{array}$$

Use: Before the Warm-up.

Informed by: Performance on Lesson 2, Practice Problem 6.

Activity 1 Adding Decimals of Different Lengths

Students analyze how two addends are presented in a vertical calculation to help ensure that correct values are combined.

Name: ______ Date: ______ Period: ______

Activity 1 Adding Decimals of Different Lengths

You have seen that the placement of the decimal point is important in communicating the value of a number. The Hindu-Arabic numerals we use today have been around since the 900s. During that time, it was Arab mathematician al-Uqlidisi who first introduced a unique symbol (′) for the decimal point.

Complete these problems, paying attention to the placement of the decimal point in numbers with decimals of different lengths.

1. Here are two calculations of $0.2 + 0.05$. One is correct and the other is incorrect.

$$\begin{array}{r} 0.2 \\ +\,0.05 \\ \hline 0.25 \end{array} \qquad \begin{array}{r} 0.2 \\ +\,0.05 \\ \hline 0.07 \end{array}$$

   a. Which calculation is correct?
   The first calculation is correct.

   b. Why is the other calculation incorrect?
   Sample response: The second calculation is incorrect because the decimal points are not lined up, and so the place values are not lined up either; 2 tenths is being added to 5 hundredths.

2. Calculate each sum. Consider setting up a vertical calculation or drawing base ten diagrams to help with your thinking.

   a. $0.11 + 0.005$
   $$\begin{array}{r} 0.11 \\ +\,0.005 \\ \hline 0.115 \end{array}$$

   b. $0.209 + 0.01$
   $$\begin{array}{r} 0.209 \\ +\,0.01 \\ \hline 0.219 \end{array}$$

   c. $10.2 + 1.8456$
   $$\begin{array}{r} {}^{1} \\ 10.2 \\ +\,1.8456 \\ \hline 12.0456 \end{array}$$

Featured Mathematician

Abu'l Hasan Ahmad ibn Ibrahim al-Uqlidisi

Abu'l Hasan Ahmad ibn Ibrahim al-Uqlidisi was a mathematician who lived in the mid-to-late 10th century, most likely in Syria. In one of his two major works, the *Kitab al-fusul fi al-hisab al-Hindi*, Al-Uqlidisi introduced the Hindu-Arabic system of numerals, and in addressing decimal fractions, he suggested a ′ symbol for the decimal point. Al-Uqlidisi went on to show how previously known methods of arithmetic translated into this new number system, which turned out to be more useful for basic operations.

1 Launch

Activate background knowledge by making connections to whole numbers. Ask, "Can you add 200 plus 50, even if they do not have the same number of digits? Could you give 50 another digit without changing its value so that it has the same number of digits as 200?"

2 Monitor

Help students get started by asking "What is different about the two calculation setups that may have caused different results?"

Look for points of confusion:

- **Not lining the decimal and places up correctly, or arbitrarily or incorrectly adding 0s.** Have students first line up the decimal points on graph paper. Next, have them line up the digits in each place for both numbers (possibly color coding them) and then add 0s as needed.

Look for productive strategies:

- Recognizing that a blank space at the end of a decimal is the same as a zero in its place and does not affect the addition of numbers of different lengths.

3 Connect

Have pairs of students share their responses, focusing on their strategies and thinking behind their actions.

Ask:

- "When adding 0.008 and 0.07, why do you not combine the 8 and the 7 to make 15?"
- "Can you look at these vertical calculations and imagine how they would look using base ten diagrams? Would you get the same result?"

Highlight that vertical calculations are an efficient way to determine the sums of decimals, especially for longer decimals or those requiring composing.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can manipulate digital base ten blocks to help them visualize the addition.

Accessibility: *Optimize Access to Tools*

Provide continued access to physical base ten blocks or distribute copies of the *Decimal Base Ten Blocks* PDF for students to validate their responses to Problem 1. In Problem 2c, consider altering the problem to be $10.2 + 1.845$.

Extension: *Math Around the World*

Tell students that in the U.S., a period is used to indicate the decimal point. In other countries, different notations are used. For example, most countries in Europe use a decimal comma. Have students research other countries to learn which countries use the decimal point or the decimal comma.

Decimal point	2.45
Decimal comma	2,45

Featured Mathematician

Al-Uqlidisi

Have students read about Al-Uqlidisi, who introduced the system of writing numerals we use today – known as the Hindu-Arabic system – and also authored one of the first mathematical texts containing an example of a decimal point being used.

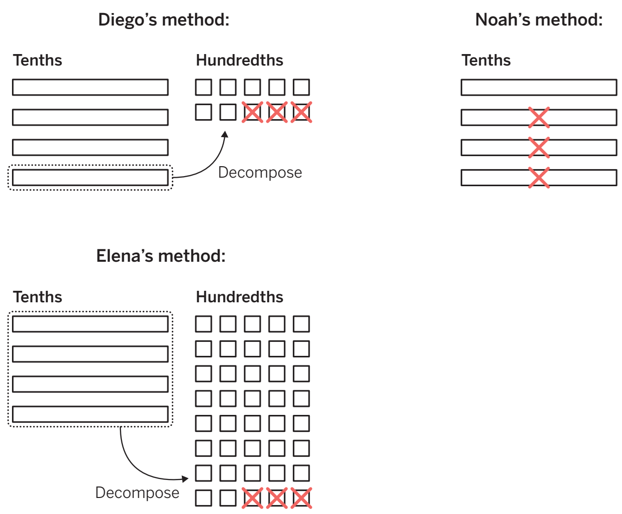
Activity 2 Subtracting Decimals of Different Lengths

Students analyze decimal subtraction problems represented by base ten diagrams to understand when decomposing numbers is necessary and then use vertical calculations.

Amps Featured Activity | Multiple Representations

Activity 2 Subtracting Decimals of Different Lengths

To represent the expression $0.4 - 0.03$, Diego, Noah, and Elena each drew a different base ten diagram. In each diagram, one rectangle represents 0.1 and one square represents 0.01.



1. With your partner, discuss how subtraction is shown in each student's diagram.

   Sample discussion response: Diego drew 4 rectangles to represent 0.4. He then replaced 1 rectangle with 10 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 3 rectangles and 7 squares.

   Noah drew 4 rectangles to represent 0.4. He then crossed out 3 of the rectangles to represent the subtraction, leaving 1 rectangle.

   Elena also drew 4 rectangles. She then replaced all 4 rectangles with 40 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 37 squares.

2. Does any diagram correctly represent $0.4 - 0.03$? If so, which one(s)? Explain your thinking.

   Diego's and Elena's diagrams represent $0.4 - 0.03$. Sample response: Diego decomposed one of the four tenths, whereas Elena decomposed all four tenths to make four hundredths, but they still showed the subtraction of three hundredths.

514 Unit 5 Arithmetic in Base Ten

1 Launch

Give partners 4–5 minutes to complete Problems 1 and 2 and then 4–5 minutes of individual work time to complete Problems 3 and 4. Provide base ten blocks, decimal base ten cut outs, and graph paper, as needed or requested.

2 Monitor

Help students get started by having pairs discuss whether and where they see the values of 0.4 and 0.03 represented in each diagram.

Look for points of confusion:

- **Shifting numbers to make the lengths match.** Have students use base ten diagrams (paper cut-outs or drawing), while recording each step in a vertical calculation next to it. Also consider having students use graph paper to line up the decimal points, digits, and place values.
- **Bringing down digits in smaller place values when there are no digits in the other given number to subtract them from.** If students start, for example, Problem 3a by "bringing down" the 5 as the value in the hundredths place of the difference, have them use base ten diagrams and ask, "How many thousandths are in 0.3? How can you show that in the written vertical calculation?"
- **Incorrectly calculating a difference.** Remind students that they can perform the inverse operation to check their answer.

Look for productive strategies:

- Clearly articulating what is happening in each diagram for Problems 1 and 2.
- Aligning the decimal points and all values in the same place values to the left and right of the decimal point and adding 0 as necessary.
- Efficiently solving Problems 3 and 4 by setting up vertical calculations and decomposing in order to subtract greater digits from lesser digits in the necessary places.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can manipulate digital base ten blocks to help them visualize the subtraction.

Accessibility: *Optimize Access to Tools*

Provide continued access to physical base-ten blocks or distribute copies of the *Decimal Base Ten Blocks* PDF for students to validate their responses to Problems 3 and 4. In Problem 3, consider having them focus only on determining the difference for Problem 3b first using models, and then using a vertical calculation. If time permits, have them describe *how* they could evaluate the expressions in Problems 3a and 3c.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 1, have pairs meet with $1 - 2$ other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "What is the same and what is different between each method?"
- "What is the relationship between Diego's rectangles and squares? Elena's rectangles and squares?"
- "Why doesn't Noah use squares?"

Have students revise their responses, as needed.

English Learners

Use intentional grouping so that students with different English language proficiency levels can interact and have an opportunity to listen to peers with more advanced proficiency.

Activity 2 Subtracting Decimals of Different Lengths (continued)

Students analyze decimal subtraction problems represented by base ten diagrams to understand when decomposing numbers is necessary and then use vertical calculations.

Name: ______________________ Date: __________ Period: __________

Activity 2 Subtracting Decimals of Different Lengths (continued)

Show or explain your thinking for each of the following problems.

3. Evaluate each expression. Then compare your results with a partner. If you and your partner have different results, consider drawing a base ten diagram or using a vertical calculation to determine the correct solution.

a. $0.3 - 0.05$

$$\begin{array}{r} {\scriptstyle 2\ 10} \\ 0.\cancel{3}\cancel{0} \\ -\,0.05 \\ \hline 0.25 \end{array}$$

b. $1.03 - 0.016$

$$\begin{array}{r} {\scriptstyle 2\ 10} \\ 1.0\cancel{3}\cancel{0} \\ -\,0.016 \\ \hline 1.014 \end{array}$$

c. $0.025 - 0.00735$

$$\begin{array}{r} {\scriptstyle 14\ 9} \\ {\scriptstyle 1\ 1\ \cancel{5}\ 1\cancel{0}\ 10} \\ 0.02\cancel{5}\cancel{0}\cancel{0} \\ -\,0.00735 \\ \hline 0.01765 \end{array}$$

4. Refer to the table which shows the three medalists of the Skateboard Big Air event at the X Games in 2013.

X Games Barcelona 2013 — Skateboard Big Air

Final		
Rank	**Name**	**Score**
1	Bob Burnquist	93.165
2	Mitchie Brusco	90.995
3	Elliot Sloan	90.83

a. Circle the pair of medal-winning scores that were the closest. Explain your thinking.

Gold and Silver Silver and Bronze

Sample response: Gold and silver were about 2 points apart, but silver and bronze both scored ninety-something and had a smaller difference.

b. What is the difference in the scores of the two closest medal winners?

0.165 points; Silver minus bronze: $90.995 - 90.830 = 0.165$

STOP

3 Connect

Define ***decompose (base ten units)*** as taking apart a value of one from one place value to make a group of ten of the next lesser place value. **Note:** Students may have previously used other valid terms, such as *borrowing*, *regrouping*, or *unbundling*. Consider also making connections to the use of the term in Unit 1, as you may have also done for *composing* in Lesson 2.

Ask:

- "What conclusion did you come to in Problem 2?" Diego's method and Elena's method both represented $0.4 - 0.03$.
- "What is the difference between Diego's method and Elena's method?" Diego only breaks up 1 tenth into 10 hundredths, whereas Elena breaks up all 4 tenths into hundredths. Consider displaying a table like this to compare the two methods:
- "What are some advantages to Diego's method?" Diego's method is quicker to draw. It shows the 3 tenths and 7 hundredths. Elena would need to count how many hundredths she has.
- "What are some advantages to Elena's method?" Elena's diagram shows a difference of 37 hundredths, which matches how we say 0.37 in words.

Have individual students share their responses for Problems 3 and 4.

Highlight that, when subtracting a lesser digit from a greater digit, in any place value, a group of 10 can be decomposed from the next greater place value in order to make the subtraction possible.

Whole Class | 5 min

Summary

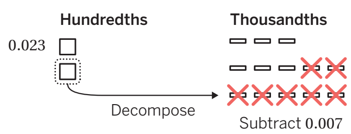
Review and synthesize the subtraction of decimals and the use of zeros in calculations.

Summary

In today's lesson . . .

You continued exploring adding and subtracting decimals, but in cases where the two numbers in the given expressions had decimals of "different lengths" — meaning two different place values. It is important to correctly line up all place values for adding and subtracting decimals, including those any digits that are 0. Whenever a subtraction problem requires subtracting a greater digit from a lesser digit in one place value, then you need to *decompose* a value of 1 from the next greater place value to make 10 of the place value that you need.

Suppose you want to determine the difference of $0.023 - 0.007$. You need to subtract 7 thousandths (7 small rectangles in a base ten diagram) from 3 thousandths. While you might not think this is possible, you can actually decompose 1 hundredth into 10 thousandths, and then you can subtract 7 thousandths from a total of 13 thousandths (10 thousandths + 3 thousandths).



Subtracting 7 thousandths from 13 thousandths leaves you with 6 thousandths. To complete the subtraction, you then have 1 hundredth left from 0.023, and you can subtract 0 hundredths. The difference is equal to 1 hundredth and 6 thousandths, which is written numerically as $0.023 - 0.007 = 0.016$.

$$
\begin{array}{r}
{\scriptstyle 1\ 13} \\
0.0\cancel{2}\cancel{3} \\
-\ 0.007 \\
\hline
0.016
\end{array}
$$

This difference can also be shown using a vertical calculation, with the same decomposing.

Reflect:

Synthesize

Formalize vocabulary:
decompose (base ten units)

Ask:

- "When using base ten blocks to represent subtraction of decimals, how do you take away a greater value from a lesser value in the same place value? For example, to determine $4.5 - 2.7$, how could you take away 7 tenths from 5 tenths?" I can unbundle a larger unit into 10 of a smaller unit. For the example, I can exchange a 1 from the ones place for 10 tenths, making 15 tenths which allows me to subtract 7 tenths.
- "When calculating differences of decimals, why should you line up the decimal points and digits in the same decimal places?" The value of any digit in a base ten number depends on its place. Lining up the decimal points and like units helps you subtract correctly.
- "How do you subtract a number with more decimal places from one with fewer decimal places (e.g., $4.1 - 1.0935$)?" I can write zeros at the end of the shorter decimal to help make them the same length.
- "Which might be more efficient to use for determining differences: base ten diagrams or vertical calculations?" It depends on the length of the number and the size of the digits. Base ten diagrams may take a while to draw.

Highlight that, in this lesson, students saw that decimal numbers of different lengths can still be added or subtracted. Decimal subtraction problems can be done with base ten diagrams or with vertical calculations, paying attention to when decomposing is necessary (as was done with composing for addition). In both cases, it is important to subtract the values that have the same place value. Zeros can be added to or removed from the end of a decimal and to the right of the decimal point without changing the value of the number.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can base ten units be composed and decomposed?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started. Ask them to review and reflect on any terms and phrases related to the term *decompose (base ten units)* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving addition and subtraction problems involving decimals of different lengths.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 5.03

Show or explain your thinking for each of these problems.

1. Evaluate the sum of $1.56 + 0.083$.

   1.643

$$\begin{array}{r} \scriptstyle 1 \\ 1.560 \\ +\,0.083 \\ \hline 1.643 \end{array}$$

2. Evaluate the difference of $0.2 - 0.05$.

   0.15

$$\begin{array}{r} \scriptstyle 1\,10 \\ 0.\cancel{2}\cancel{0} \\ -\,0.05 \\ \hline 0.15 \end{array}$$

3. An amusement park requires a height of at least 39.37 in. to ride in a bumper car. Diego is 35.75 in. tall. How much taller, in inches, will Diego need to grow before he can ride in a bumper car?

   3.62 in.

$$\begin{array}{r} \scriptstyle 8\,13 \\ 3\cancel{9}.\cancel{3}7 \\ -\,35.75 \\ \hline 3.62 \end{array}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can tell whether writing or removing a zero in a decimal will change its value.
1 2 3

b. I know how to solve subtraction problems with decimals that require decomposing numbers.
1 2 3

c. I can use vertical calculations to represent and reason about addition and subtraction of decimals.
1 2 3

Success looks like . . .

- **Language Goal:** Adding or subtracting decimals, and explaining the reasoning using multiple representations. **(Speaking and Listening, Writing)**
  - » Evaluating a sum and a difference of decimals in Problems 1 and 2.
- **Goal:** Comprehending that the term *decompose* means to take apart a larger base ten unit into 10 units of a lower place value (e.g., 1 tenth as 10 hundredths).
- **Language Goal:** Recognizing and explaining that writing additional zeros or removing zeros after the last non-zero digit in a decimal does not change its value. **(Speaking)**

Suggested next steps

If students do not solve Problem 1 correctly, consider:

- Having students use base ten diagrams (paper cut-outs or drawing) and record each step in a vertical calculation next to their diagrams.

If students struggle to solve Problem 2 correctly, due to the different lengths of the decimals being subtracted or due to needing to decompose, consider:

- Referring back to Problem 3a in Activity 2 and having students describe the procedure for subtraction and connect it to the problem.
- Having students use base ten diagrams, recording each step in a vertical calculation next to their diagrams.

If students have trouble understanding the scenario in Problem 3, consider:

- Having students draw a picture to represent what is happening.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did base ten diagrams set students up to develop their understanding of place value and the importance of zero?
- During the discussion about Diego's, Noah's, and Elena's diagrams, how did you encourage each student to listen to one another's strategies? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ____________ Date: ________ Period: ________

Practice

1. Determine each of the following sums. Consider drawing base ten diagrams to help with your thinking.
   a. $0.027 + 0.004 = 0.031$
   b. $0.203 + 0.01 = 0.213$
   c. $1.2 + 0.145 = 1.345$

2. Mai claims that 1.97 cannot be subtracted from 20 because 1.97 has two decimal places and 20 has none. Do you agree? Show or explain your thinking.
   I disagree; Sample response: 20 can be written with any number of zeros after the decimal point, such as 20.00.

3. Complete each calculation to determine the correct difference.

   a. $\begin{array}{r} 142.6 \\ -\ \ \ 1.4 \\ \hline \boxed{1}\boxed{4}\boxed{1}\boxed{.}\boxed{2} \end{array}$

   b. $\begin{array}{r} {\scriptstyle 7\ \ \ \ \overset{15}{\cancel{5}}\ 10} \\ 3\cancel{8}.\cancel{6}\cancel{0} \\ -\ \ 6.75 \\ \hline \boxed{3}\boxed{1}\boxed{.}\boxed{8}\boxed{5} \end{array}$

   c. $\begin{array}{r} {\scriptstyle 3\ 11\ \ \ 6\ 16} \\ 2\cancel{4}\cancel{1}.\cancel{7}\cancel{6} \\ -\ \ \ \ 2.18 \\ \hline \boxed{2}\boxed{3}\boxed{9}\boxed{.}\boxed{5}\boxed{8} \end{array}$

Name: ____________ Date: ________ Period: ________

Practice

4. A rectangular prism measures $7\frac{1}{2}$ cm by 12 cm by $15\frac{1}{2}$ cm.
   a. Calculate the number of cubes with an edge length of $\frac{1}{2}$ cm that will fit into this prism.
      11,160 cubes; Sample response:
      $7\frac{1}{2} \div \frac{1}{2} = 15$ $\quad 12 \div \frac{1}{2} = 24$ $\quad 15\frac{1}{2} \div \frac{1}{2} = 31$
      $15 \cdot 24 \cdot 31 = 11{,}160$
   b. What is the volume of the prism, in cubic centimeters? Show your thinking. **Hint:** Think about how many cubes with $\frac{1}{2}$-cm edge lengths fit into a cube with a volume of 1 cm^3.
      1,395 cm^3; Sample responses:
      - $7\frac{1}{2} \cdot 12 \cdot 15\frac{1}{2} = 1{,}395$
      - $11160 \div 8 = 1{,}395$

5. A car travels 75 miles in 60 minutes at a constant speed. How far does the car travel in 18 minutes? Consider using the table to help with your thinking.
   22.5 miles

Minutes	Distance (miles)
60	75
6	**7.5**
18	**22.5**

6. Which calculation shows the correct way to determine the difference of $0.3 - 0.006$? Explain your thinking.

   A. $\begin{array}{r} 0.3\ \ \ \\ -0.006 \\ \hline 0.306 \end{array}$

   B. $\begin{array}{r} 0.3\ \ \ \\ -0.006 \\ \hline 0.097 \end{array}$

   C. $\begin{array}{r} 0.30\ \\ -0.006 \\ \hline 0.024 \end{array}$

   D. $\begin{array}{r} 0.300 \\ -0.006 \\ \hline 0.294 \end{array}$

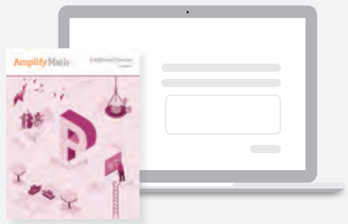
   Sample response: The two extra zeros added to the right of the 0.3 help line up the decimal places with the same place value. So, your subtraction shows thousandths from thousandths and hundredths from hundredths, and so on.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 16	2
	5	Unit 2 Lesson 11	2
Formative	6	Unit 5 Lesson 4	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 5 | **LESSON 4**

X Games Medal Results

Let's practice adding and subtracting decimals.

Focus

Goals

1. **Language Goal:** Add or subtract decimals with multiple non-zero digits, and explain the solution method. **(Speaking)**
2. **Language Goal:** Interpret a description of a real-world situation involving decimals, and write an addition or subtraction problem to represent it. **(Writing)**
3. **Language Goal:** Recognize and explain that vertical calculation is an efficient strategy for adding and subtracting decimals, especially for decimals with multiple non-zero digits. **(Speaking)**

Coherence

Today

Students work with decimals to the thousandths and beyond, mostly in the context of scoring at X Games competitions. They have to decide which operation to perform and which strategy to use when calculating sums and differences. The ideas that decimals can be expressed in different but equivalent ways, and that writing additional zeros after the last non-zero digit in a decimal does not change its value, are both necessary. Students use these and other understandings of base ten numbers, particularly when subtracting numbers with more decimal places from those with fewer decimal places. They should begin to see patterns in the calculations, enabling them to become increasingly fluent in finding sums and differences. As an opportunity for additional fluency practice, students determine missing addends and subtrahends in an optional activity.

Previously

Lessons 2 and 3 reviewed and introduced representations and strategies for adding and subtracting decimal numbers with increasing digits and more precise place values.

Coming Soon

In the next lesson, students will shift to multiplying decimals.

Rigor

- Students practice **procedural skills** and **fluency** to add and subtract decimals involving more digits.
- Students **apply** what they have learned about adding and subtracting decimals to calculating and comparing scores in a variety of X Games competitions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice
- graph paper

Math Language Development

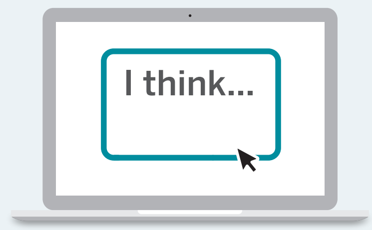
Review words

- *compose (base ten units)*
- *decompose (base ten units)*

Amps | Featured Activity

Activities 1 and 2
See Student Thinking

Students solve addition and subtraction problems with decimals that involve regrouping, and their work is available to you digitally, in real time.



Amps POWERED BY desmos

Building Math Identity and Community
Connecting to Mathematical Practices

Students might feel frustrated about determining the patterns when they first see the problems with missing numbers in Activity 3. Remind them that throughout mathematics, they have looked for patterns to make sense of new concepts. Encourage students to focus on the patterns that relate addition and subtraction. Because students are familiar with related facts, they should feel a boost in their self-confidence.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Optional **Activity 3** may be omitted, but you should consider making time to include it if your students would benefit from more fluency practice with decimals beyond thousandths; or, omit Activity 1 or Activity 2 to make time for it instead.

Independent | 5 min

Warm-up Notice and Wonder

Students use the ***Notice and Wonder*** routine to reason about the placement of digits in a decimal subtraction problem.

Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 4

X Games Medal Results

Let's practice adding and subtracting decimals.

Warm-up Notice and Wonder

Here are three ways to write a subtraction calculation. What do you notice? What do you wonder?

$\begin{array}{r} 5 \\ -0.17 \\ \hline \end{array}$ $\begin{array}{r} 5 \\ -0.17 \\ \hline \end{array}$ $\begin{array}{r} 5 \\ -0.17 \\ \hline \end{array}$

1. I notice . . .
   Sample responses:
   - The first calculation is the "most correct" because the 5 should be in the ones place.
   - All three could be determined if the decimal point was placed above the decimal in the second addend and zeros were placed in the other places.
   - The third calculation would result in a negative number if the 5 changed to 0.05.

2. I wonder . . .
   Sample responses:
   - Whether the first calculation was supposed to have a decimal point and two zeros.
   - What the context is, because then maybe I could understand if the decimal points and zeros are just missing.
   - Whether you need the zeros in the ones place.
   - What the results would be if I performed each calculation and how the digits would compare.

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use the *Notice and Wonder* routine to think about setting up a vertical calculation for a subtraction problem with different decimal places.

2 Monitor

Help students get started by asking, "What is the same about these three representations? What is different?"

Look for points of confusion:

- **Thinking in terms of "wrong" or "right."** Remind students that is not the focus here (and there are no solutions given). Ask, "What questions or thoughts come to mind as you look at these three calculation setups?"

Look for productive strategies:

- Noticing that the third representation, if corrected with a decimal point and a zero before the 5, would not be able to be subtracted because $17 > 5$.

3 Connect

Display the three representations for students to refer to as they show or explain their thinking.

Have individual students share what they notice or wonder, focusing on the points made about place value and placement of a decimal point to make the calculation possible using either 5.0 or 0.5.

Highlight that only the first representation would be set up properly if the top value is the whole number 5, and it is important to always show a decimal point for any number that is not a whole number.

Ask, "Identify the calculation that would match this scenario: Clare bought a photo for 17 cents and paid with a $5 bill. She wanted to know how much change she would get back." The first calculation. (This could be mentally calculated as non-decimals if it is seen as 500 minus 17.)

Math Language Development

MLR5: Co-craft Questions

After students independently complete Problems 1 and 2, have them share their responses with a partner and work together to write 2–3 mathematical questions they could ask about this situation. Sample questions shown.

- What value does number 5 have in each of the calculations?
- How does moving the number 5 affect each difference?
- Can I add zeros after the number 5 in the first two calculations?

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Power-up

To power up students' ability to align place values in subtraction problems involving decimals, ask:

Without solving, write how you would set up a vertical calculation to determine the difference of 2.4 and 0.004.

$\begin{array}{r} 2.400 \\ -0.004 \\ \hline \end{array}$

Use: Before the Warm-up

Informed by: Performance on Lesson 3, Practice Problem 6.

Pairs | 10 min

Activity 1 Skateboard Big Air

Students read a description of a skateboarding event and make sense of the given information to determine the calculations that help determine the gold medalist.

Amps Featured Activity See Student Thinking

Activity 1 Skateboard Big Air

In the fourth and final Summer X Games of 2013, in Los Angeles, California, the Skateboard Big Air Final came down to three athletes:

Athlete	Description
Bob Burnquist	Won gold at all three previous Summer X Games that same year. His event ended with a best score of 88.665 and a broken nose.
Elliot Sloan	Placed third in Barcelona. Placed second in the other two Summer X Games that year, in Munich and Foz do Iquaçu. On his fifth and final run, his total deductions were 9.835 from the base score of 100 points.
Tom Shaar	13-year old competitor who completed an amazing run and posted a leading score of 88.83.

Who won the gold medal and how much higher was his score compared to each of the other two competitors?

Elliot Sloan won the gold with a score of 90.165, which was 1.335 points more than Tom Schaar and 1.5 points more than Bob Burnquist.

$$\begin{array}{r} \cancel{1}\cancel{0}\cancel{0}.\cancel{0}\cancel{0}\cancel{0} \\ -9.835 \\ \hline 90.165 \end{array} \qquad \begin{array}{r} \cancel{9}\cancel{0}.\cancel{1}65 \\ -88.83 \\ \hline 1.335 \end{array} \qquad \begin{array}{r} \cancel{9}\cancel{0}.\cancel{1}65 \\ -88.665 \\ \hline 1.500 \end{array}$$

Are you ready for more?

In a *cryptarithmetic puzzle*, the digits 0–9 are represented using the first 10 letters of the alphabet, but they can be assigned in any order for a given puzzle. Determine the digits that correspond with the letters E, F, H, I, and J in this puzzle. Sample response:

$$\begin{array}{r} \text{I H F . I J} \\ +\ \text{J I I . F I} \\ \hline \text{E J I . I E} \end{array}$$

E = 5, F = 0, H = 1, I = 2, J = 3

$$\begin{array}{r} 210.23 \\ +\ 322.02 \\ \hline 532.25 \end{array}$$

520 Unit 5 Arithmetic in Base Ten

1 Launch

Read the information. Give students 5 minutes of quiet work time and then share with a partner.

2 Monitor

Help students get started by asking, "What information do you have? What information do you still need to know in order to determine who won the gold medal?"

Look for points of confusion:

- **Incorrecting calculating Sloan's score due to misalignment or decomposing mistakes.** Have students use base ten diagrams (paper cut-outs or drawing) while recording each step in a vertical calculation next to it.

Look for productive strategies:

- Recognizing that, before answering the questions, the score of Elliot Sloan is needed first. However, the first question could also be answered by estimating the difference between 100 and the deductions.
- Aligning the vertical calculation correctly, which shows understanding of how the place and digits line up.

3 Connect

Display the relevant numbers from the problem for all to see. Consider also showing an online video of the event to reveal the winner of the gold medal.

Ask:

- "What information did you need before you could determine the winner and by how much their score was over the competitors' scores?" Elliot Sloan's score
- "In what ways were your methods effective?"

Have pairs of students share which method or methods they used to calculate and why.

Highlight the ways the numbers have to line up in the vertical calculation when the lengths of the numbers are different.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Suggest that students round all numbers to the hundredths place, or you could suggest a combination of tenths and hundredths, to allow for more manageable calculations that will not impact the final results of the competition.

Accessibility: *Optimize Access to Tools*

Provide continued access to physical base-ten blocks or distribute copies of the *Decimal Base Ten Blocks* PDF for students to use if they choose to do so.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they will be comparing the scores of three Summer X Games skateboarders.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as Elliot Sloan's base score of 100 points.
- **Read 3:** Ask students to brainstorm strategies for how they will complete the problem.

English Learners

Annotate *total deductions* for Elliot Sloan's description by writing *subtraction* near the phrase to help students make sense of this terminology.

Pairs | 15 min

Activity 2 Snowboard Cross

Students calculate differences in times of races to practice adding and subtracting decimals in a real-world context.

Amps Featured Activity | See Student Thinking

Name: ________________ Date: ________ Period: ________

Activity 2 Snowboard Cross

Now, let's look at some events from the Winter X Games from 2015, held in Aspen, Colorado. In particularly, let's look at the Snowboarder X Adaptive Finals.

Avid snowboarder, Keith Gabel, had his leg amputated after a horrible accident at the age of just 21. But that did not stop him from becoming a gold medalist at the X Games in 2015. His winning time was officially recorded as 0:57.168.

1. The results of the event are shown in the table. Calculate the finishing times of the silver and bronze medal runs.

Snowboarder X Adaptive

Final		
Rank	Name	Time
1	Keith Gabel	0:57.168
2	Carl Murphy	−0.281
3	Alex Massie	−4.127

Murphy:
57.449 seconds

$$\begin{array}{r} 57.168 \\ +\ 0.281 \\ \hline 57.449 \end{array}$$

Massie:
61.295 seconds or 1:01.295 minutes

$$\begin{array}{r} 57.168 \\ +\ 4.127 \\ \hline 61.295 \end{array}$$

1 Launch

Consider showing an online video featuring Keith Gabel or another snowboard cross competition to introduce the activity and activate background knowledge. Explain, if necessary, that the notation for time is min:sec.sec, so the time 0:57.168 is read as 57 and 168 thousandths seconds.

2 Monitor

Help students get started by asking "What does −0.281 mean when compared to the first place time?" The time was slower than the first place time by 0.281 seconds. "How does knowing that help you decide what to do with these numbers?" I know that the second place time is equal to first place time plus that value.

Look for points of confusion:

- **Thinking that the operation must be subtraction for Problem 1 due to the minus signs in the table.** Ask, "Should the times of those competitors be more than or less than the first place time?"
- **Incorrectly calculating times due to misalignment of decimal point and place values.** Have students use graph paper to align the decimal points of each number and all places to the left and right of the decimal.

Look for productive strategies:

- Recognizing the operation needed to solve for each problem.
- Using vertical calculations and composing and decomposing efficiently. (Using diagrams can still produce correct responses, but students should be encouraged to move toward more efficient vertical calculations).
- Interpreting the results correctly and in each context.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, allow them to choose to complete either Problem 1 or Problem 2. Offering them the power of choice can result in greater engagement in the task.

Accessibility: *Optimize Access to Tools*

Provide continued access to physical base ten blocks or distribute copies of the *Decimal Base Ten Blocks* PDF for students to use if they choose to do so. Encourage them to move toward using vertical calculations by asking, "How might using a vertical calculation be more efficient than base ten blocks?"

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Connect, as students share their methods for solving each problem, revoice their ideas in the form of a question using appropriate mathematical language or language from the context. For example:

If a student says . . .	Revoice their ideas by asking . . .
"I used a vertical calculation to subtract the times in Problem 2."	"By subtracting, did you calculate the time difference between first and second place?"

English Learners

Model the language of comparison: *faster*, *fastest*, *slower*, and *slowest*.

Pairs | 15 min

Activity 2 Snowboard Cross (continued)

Students calculate differences in times of races to practice adding and subtracting decimals in a real-world context.

Activity 2 Snowboard Cross (continued)

Let's look at the Women's Snowboarder X Finals. Lindsey Jacobellis was not just the winner of the Women's snowboarder X race in 2015, but she also holds the Guinness World Record for "Most Winter X Games medals (11) for Snowboarder X by a female athlete."

2. The table shows the results of the 2015 Women's Snowboarder X finals, which Jacobellis won.

X Games Aspen 2015 — Women's Snowboarder X

Final		
Rank	**Name**	**Time**
1	Lindsey Jacobellis	0:53.085
2	Dominique Maltais	0:53.234
3	Nelly Moenne Loccoz	0:53.384

Fun fact! This race was the first time a live drone was used for an aerial camera in a sporting event.

Calculate the time difference between the first and second places. Then calculate the difference between the first and third places. Be prepared to explain what the differences in time mean within the context of the race.

Dominique Maltais was 0.149 of a second slower than Lindsey Jacobellis.
Nelly Moenne Loccoz was 0.299 of a second slower than Lindsey Jacobellis.

$$\begin{array}{r} \scriptstyle 12 \\ \scriptstyle 1\,\cancel{13}\,14 \\ 53.\cancel{2}\cancel{3}\cancel{4} \\ -53.085 \\ \hline 0.149 \end{array} \qquad \begin{array}{r} \scriptstyle 17 \\ \scriptstyle 2\,\cancel{7}\,14 \\ 53.\cancel{3}\cancel{8}\cancel{4} \\ -53.085 \\ \hline 0.299 \end{array}$$

3 Connect

Have pairs of students share how they determined what operation to use for each problem and the methods chosen to solve each.

Ask:

- "Why do you think times go to the thousandths of a second for races like these?" It is necessary to distinguish the times and finishes of athletes of very close races.
- "If you were to add the total times of all three medalists from the Women's Snowboard Cross, the tens place value in the number of seconds would sum to 15. Why would the total time not be 1 minute and 50-some seconds?" Because there are only 60 seconds in a minute and not 100, the actual time is 2 minutes and 30-some seconds.

Highlight that the numbers in these problems are good examples of when calculations with several decimal places using base ten diagrams may become cumbersome. A vertical calculation is favorable and is likely more efficient in these examples.

Optional

Pairs | 15 min

Activity 3 Missing Numbers

Students deepen their understanding of composing and decomposing in solving missing value problems, which require attention to and making use of structure.

Name: ______ Date: ______ Period: ______

Activity 3 Missing Numbers

Determine and write the missing digits in each calculation so that the value of each sum or difference is correct. Be prepared to show or explain your thinking.

1. $$\begin{array}{r} 0.404 \\ +\ \boxed{0.596} \\ \hline 1.000 \end{array}$$

2. $$\begin{array}{r} 9.8765 \\ +\ \boxed{0.1235} \\ \hline 10.0000 \end{array}$$

3. $$\begin{array}{r} 0.7 \\ -\ \boxed{0.688} \\ \hline 0.012 \end{array}$$

4. $$\begin{array}{r} 7 \\ -\ \boxed{3.5433} \\ \hline 3.4567 \end{array}$$

5. $$\begin{array}{r} 70 \\ -\ \boxed{69.9911} \\ \hline 0.0089 \end{array}$$

STOP

1 Launch

Give students 8–10 minutes of quiet work time and 2–3 minutes to discuss their solutions with a partner.

2 Monitor

Help students get started by asking, "What operation are you being asked to perform? What would you add to four to make a sum of zero?"

Look for points of confusion:

- **Beginning to fill in values from left to right.** Have students focus on how this strategy would not make sense using Problem 1 as an example, and asking, "What is $0.404 + 0.6$?"
- **Not continuing the composing or decomposing of values from place to place.** Suggest students use the inverse operations to check their calculations.

Look for productive strategies:

- Recognizing the pattern of determining sums to 10, and noticing the digits of the missing addend after the one that is farthest to the right is always one less than what it would be to make the 10.
- Eliminating the need to decompose by changing the addends in the subtraction problems to end in 9 then adding back in the 1.
- Using the inverse operation to solve, especially in the case of the subtraction problems.

3 Connect

Display the problems for students to refer to as they show or explain their thinking.

Have pairs of students share their responses, focusing on the variety of strategies used and how students applied repeated reasoning, both among addition or subtraction problems, but also between the two operations.

Highlight the flexibility of vertical calculations, the composing and decomposing place value units when necessary, and the use of the inverse operation to check solutions.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge, Guide Processing and Visualization***

If students need more processing time, have them focus on completing Problems 2 and 4. Consider using a think-aloud to demonstrate how to fill in the first missing value in Problem 1 by saying, "In vertical calculations, we start on the right because it's the smallest place value position. What number should I add to 4 to make 0? I can't, so I should add a number to 4 to make 10."

Extension: ***Math Enrichment***

Challenge students to create their own missing number problem. Tell them it needs to include composing or decomposing. Have them trade problems with a partner and have each partner determine the missing numbers.

Whole Class | 5 min

Summary

Review and synthesize how vertical calculations are efficient for determining the sums and differences of decimals, to any number of decimal digits.

Summary

In today's lesson . . .

You saw that for expressions having numbers with many non-zero digits, such as $0.25103 - 0.04671$, it would take a long time to draw a base ten diagram. With vertical calculations, you can determine the difference more efficiently. Even though you might not draw a base ten diagram, you can still think about base ten diagrams to help you make sense of the calculations, particularly with *composing* or *decomposing* units.

For example, drawing a base ten diagram to represent $0.25103 - 0.04671$ would require 29 base ten shapes and then 20 more to show the two sets of decomposed digits. Using a vertical calculation, as shown here, is more efficient.

$$\begin{array}{cccccccc}
 & & & & & 10 & & \\
 & & & & 4 & \cancel{0} & 10 & \\
 & 0 & . & 2 & \cancel{5} & \cancel{1} & \cancel{0} & 3 \\
- & 0 & . & 0 & 4 & 6 & 7 & 1 \\ \hline
 & 0 & . & 2 & 0 & 4 & 3 & 2
\end{array}$$

Reflect:

Synthesize

Ask, "What are some of the most important take-aways from what you learned today?"

Highlight that the biggest difference between adding and subtracting decimals versus adding and subtracting whole numbers is the need to line up the decimal points so that the place values are also lined up.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "In all of the activities today, did it matter whether the scores/times were given to the whole number? Tenths place? Hundredths place? Thousandths place? Did it change the way you went about solving the problems?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of subtracting decimals by determining the winner of an X Games surfing competition.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 5.04

The 2013 Summer X Games featured a Team Surfing competition between the East Coast and West Coast.

The total score for the West Coast team was 82.3 points and the total score for the East Coast team was 73.85 points.

Which team won and by how many points?

The West Coast team won by 8.45 points. Sample response:

$$\begin{array}{rrcrr} & 11 & & 12 & \\ 7 & \cancel{1}2 & & \cancel{1}3 & 10 \\ \cancel{8} & \cancel{2} & . & \cancel{3} & \cancel{0} \\ -7 & 3 & . & 8 & 5 \\ \hline & 8 & . & 4 & 5 \end{array}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can solve problems that involve addition or subtraction of decimals.

1 2 3

Success looks like . . .

- **Language Goal:** Adding or subtracting decimals with multiple non-zero digits, and explaining the solution method. **(Speaking)**
- **Language Goal:** Interpreting a description of a real-world situation involving decimals, and writing an addition or subtraction problem to represent it. **(Writing)**
  - » Determining the winning team by writing a subtraction problem.
- **Language Goal:** Recognizing and explaining that vertical calculation is an efficient strategy for adding and subtracting decimals, especially for decimals with multiple non-zero digits. **(Speaking)**

Suggested next steps

If students do not add the zero to the minuend, consider:

- Asking, "What is $1 - 0.85$?"
- Referring to or reviewing Activity 3, Problem 3.

If students have difficulty with all of the decomposing, consider:

- Having students check their calculation using the inverse operation.

If students have have difficulty with a conceptual or procedural issue, consider:

- Using base ten blocks to demonstrate decomposing.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find frustrating about subtracting decimals? What helped them work through this frustration?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

Practice

1. For each subtraction problem, circle the correct calculation.

   a. $7.2 - 3.67$

   $\begin{array}{r} 7.2 \\ -\,3.67 \\ \hline 3.05 \end{array}$ $\begin{array}{r} 07.2 \\ -\,3.67 \\ \hline 3.05 \end{array}$ $\begin{array}{r} 7.20 \\ -\,3.67 \\ \hline 3.53 \end{array}$

   b. $16 - 1.4$

   $\begin{array}{r} 16 \\ -\,1.4 \\ \hline 0.2 \end{array}$ $\begin{array}{r} 16.0 \\ -\,1.40 \\ \hline 0.20 \end{array}$ $\begin{array}{r} 16.0 \\ -\,1.4 \\ \hline 14.6 \end{array}$

2. Explain how you could determine the difference between 1 and 0.1978. You do not need to actually calculate the difference.

   Sample responses:
   - **I could subtract 0.1978 from 1.0000.**
   - **I could add on to 0.1978 to equal 1, or 1.0000, by making tens in each decimal place (by first adding 0.0002, then 0.002, and finally 0.8).**

3. The label on a bag of dried apples says it contains 0.384 lb of apples. The actual weight of one bag of apples is 0.3798 lb.

   a. Are the apples actually heavier or lighter than the weight stated on the label? Explain your thinking.

   They are lighter because 0.3798 is less than 0.384.

   b. How much heavier or lighter are the apples than stated on the label? Show your thinking.

   0.0042 lb lighter; Sample response:

   $\begin{array}{r} {\scriptstyle 7\ \overset{13}{\cancel{3}}\ 10} \\ 0.3\cancel{8}\cancel{4}\cancel{0} \\ -\,0.3798 \\ \hline 0.0042 \end{array}$

Name: Date: Period:

Practice

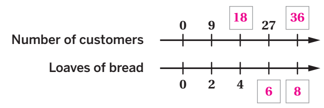
4. A shipping company is loading cube-shaped crates into a larger cube-shaped container. The smaller cubes have side lengths of $2\frac{1}{2}$ ft, and the larger shipping container has side lengths of 10 ft. How many crates will fit in the shipping container? Show your thinking.

   64 crates; Sample response:

   $10 \div 2\frac{1}{2} = 10 \cdot \frac{2}{5} = 4$ $\quad 4 \cdot 4 \cdot 4 = 64$

5. For every 9 customers, a chef prepares 2 loaves of bread.

   a. The double number line and the table show varying numbers of customers and the number of loaves prepared by the chef. Complete each representation with the missing values.



Number of customers	Number of loaves
9	2
18	4
27	**6**
63	14

   b. Use either representation to solve these problems.
   - How many loaves are needed for 63 customers?
     14 loaves
   - If the chef prepares 20 loaves, how many customers would receive loaves?
     90 customers
   - How much of a loaf is prepared for each customer?
     $\frac{2}{9}$ of a loaf

6. Which of the following expressions are equal to 7.3? Select *all* that apply.

   A. $73 \cdot \frac{1}{10}$
   B. $73 \div \frac{1}{10}$
   C. $73 \div 10$
   D. $73 \cdot 10$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 3	2
	3	Activity 2	2
	4	Unit 4 Lesson 14	2
	5	Unit 2 Lesson 12	2
Formative	6	Unit 5 Lesson 5	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 2
Multiplying Decimals

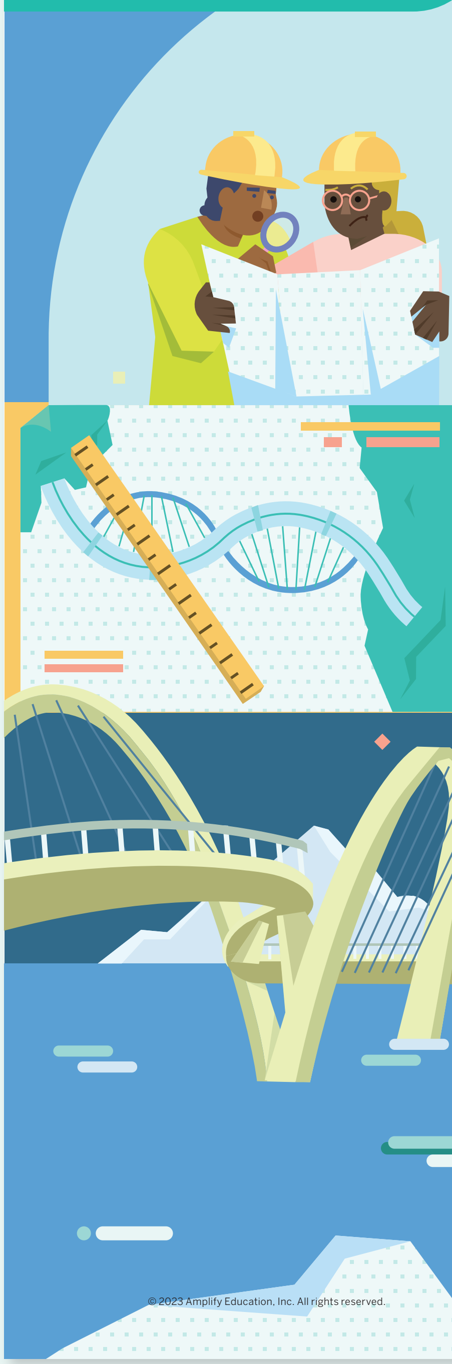
Whole Class

In this Sub-Unit, students transition from concrete models and place-value strategies to formalize a standard algorithm for multiplying multi-digit decimals, including those with decimal places beyond hundredths.

SUB-UNIT

2 Multiplying Decimals

Narrative Connections



What happens when you make a small change to a big bridge?

Since 2016, the Elizabeth Quay Bridge in Perth, Australia has been a landmark of the city. The bridge stands over the Swan River, suspended by two dramatic arches. Building it was a delicate job. Engineers had to install parts of the bridge, like the arch's stubs, even while the arches themselves were still being made.

To help with this, engineers needed to be accurate with their measurements. That way everything would match up once the finished arches were brought to the site. Bridges, buildings, and other structures often undergo changes as they move from design to the actual construction. Architects, engineers, and construction crews have to work together to accommodate any changes they might encounter.

This kind of flexibility and precision are what decimals are made for. A designer might dream of a building using nice whole numbers. But reality may require that a few tenths or hundredths of a meter be shaved off or added. Even seemingly small adjustments can have a major impact for things like square footage and costs of materials.

To compute these, you have to understand how to multiply decimals. Before the first construction crane makes it to the work site, it is math that does the heavy lifting.

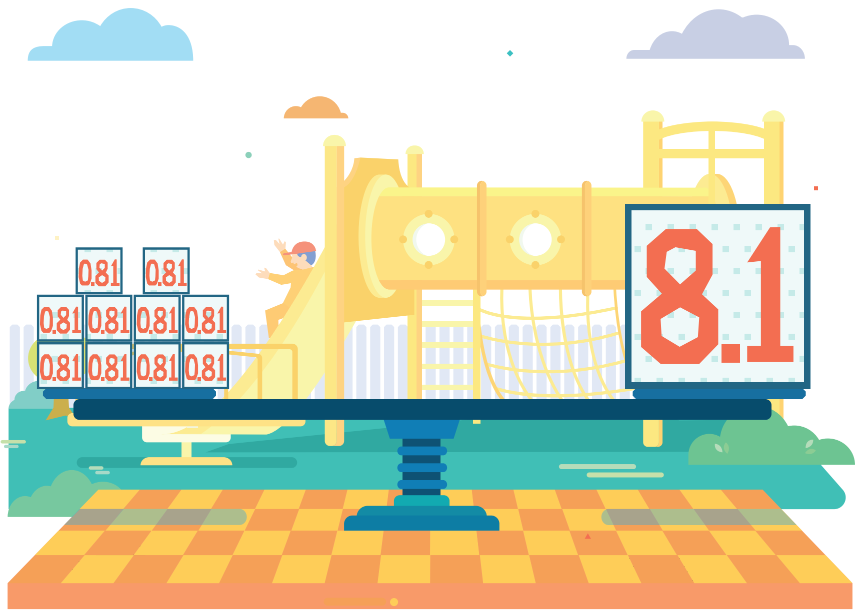
Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to multiply decimals in when studying engineering endeavors, large and small, in the following places:

- **Lesson 6, Activity 1:** The World's Smallest Chess Set
- **Lesson 7, Activity 2:** The World's Smallest Newspaper
- **Lesson 8, Activity 2:** Going Wheelie Fast

Decimal Points in Products

Let's look at products that are decimals.



Focus

Goals

1. **Language Goal:** Generalize that the number of decimal places in a product is related to the number of decimal places in the factors. **(Speaking and Listening, Writing)**

2. **Language Goal:** Justify the product of two decimals, by multiplying equivalent fractions that have a power of 10 in the denominator. **(Speaking and Listening)**

Rigor

- Students build their **conceptual understanding** of multiplying decimals beyond the hundredths by using what they know about fractions and place value.
- Students build **procedural fluency** with multiplying multi-digit numbers.

Coherence

Today

Students extend their Grade 5 work on decimal multiplication, using what they know about fractions and place value to calculate products of decimals beyond the hundredths. They express each decimal as a product of a whole number and a fraction, and then they use the commutative and associative properties to compute the product. As they look for and use the structure of the base ten system and equivalent expressions, they see how the number of decimal places in the factors can help them place the decimal point in the product.

Previously

In Grades 4 and 5, students represented fractions with denominators of 10 and 100 as decimals and explained patterns in the placement of the decimal point when multiplying or dividing by powers of 10. They also multiplied decimals to the hundredths using concrete models and place value strategies.

Coming Soon

In Lesson 6, students will continue developing methods for computing products of decimals, extending from fractions and powers of 10 to area diagrams.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials
- Exit Ticket
- Additional Practice

Math Language Development

Review words
- *associative property*
- *commutative property*

Amps | Featured Activity

Activity 1
See Student Thinking

Students solve multiplication problems that shift the decimal point, and their work is available to you digitally, in real time.

Building Math Identity and Community
Connecting to Mathematical Practices

Students might not exert self-discipline as they attempt to use the structure of decimals in Activities 1 and 2. Have students choose an accountability partner who will help them stay focused and in control of themselves throughout the lesson. You might want to discuss signs that a student is not using self-discipline and signals that partners can use to help bring each other back on track.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. Consider incorporating relevant key ideas in the Activity 1 synthesis.
- In **Activity 1**, have partners complete Problems 1 and 2. Complete Problems 3 and 4 as a class using the ***Turn and Talk*** routine.

Pairs | 5 min

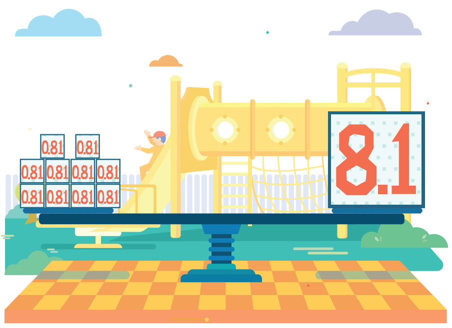
Warm-up Multiplying by 10

Students use a related set of multiplication equations to compare the positions of digits and the decimal point based on a factor that is a power of 10.

Unit 5 | Lesson 5

Decimal Points in Products

Let's look at products that are decimals.



Warm-up Multiplying by 10

1. Circle the equation for which the value of x is equal to 8.1. Be prepared to explain your thinking.

   $x \cdot 10 = 810$ $\quad$ $x \cdot 10 = 81$ $\quad$ $x \cdot 10 = 8.1$ $\quad$ $x \cdot 10 = 0.81$

   Sample response: Multiplying by 10 moves each digit one place value to the left. Multiplying 8.1 • 10 moves the 8 to the tens place and the 1 to the ones place to get 81.

2. Compare 0.81 and 810. Which number is greater? How many times greater?

   810 is greater; It is 1,000 times greater than 0.81.

Discussion Support: What math terms can you use to describe how the structure of the equations in Problem 1 compare? Share those terms with your partner.

Log in to Amplify Math to complete this lesson online.

528 Unit 5 Arithmetic in Base Ten

1 Launch

Have students use the ***Turn and Talk*** routine. Give them 2 minutes to discuss both problems with a partner before recording their responses.

2 Monitor

Help students get started by asking, "How are all of the equations in Problem 1 similar? How are they different?"

Look for points of confusion:

- **Struggling to multiply with decimals.** Have students solve $x \cdot 10 = 810$. Ask, "How is the value of x related to the product? How can you apply that to the other equations without actually calculating?"

Look for productive strategies:

- Recognizing that, when multiplying by 10, each digit is ten times greater than in the original factor.
- Connecting the relative placement of decimal points to powers of 10, and applying this to solve Problem 2.

3 Connect

Have students share their responses, focusing on how they used the structure of the equations to solve.

Ask, "When multiplying by 10, how are the digits in the other factor related to those in the product? What about the position of the decimal point?"

Highlight that, when multiplying by 10, each digit in the product is 10 times greater than it was; or, each digit moves to the left one place value and the decimal point moves to the right one place. When dividing by 10, each digit in the quotient is 10 times less than it was; or, each digit moves to the right one place value and the decimal point moves left one place.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses, have them respond to the question posed in their Student Edition, "What math terms can you use to describe how the structure of the equations in Problem 1 compare?" Listen for and amplify phrases such as the following:

- The left side of each equation is the same.
- The right side of each equation is divided by 10 each time.
- The decimal point in 810 moves to the left one place each time.

Power-up

To power up students' ability to understand the relationship between decimals, fractions, and multiplying or dividing by ten and one-tenth, ask:

Choose *all* the true equalities.

A. $155 \cdot 10 = 155 \cdot \frac{1}{10}$

B. $155 \div 10 = 155 \cdot \frac{1}{10}$

C. $155 \cdot 10 = 155 \div \frac{1}{10}$

D. $155 \div 10 = 155 \div \frac{1}{10}$

Use: Before Activity 1.

Informed by: Performance on Lesson 4, Practice Problems 6 and Pre-Unit Readiness Assessment, Problems 3 and 7.

Pairs | 20 min

Activity 1 Fractions and Powers of 10

Students multiply decimals by writing them in fraction form, and they recognize the relationship between the position of the decimal point in the factors and its position in the product.

Amps Featured Activity See Student Thinking

Name: ______ Date: ______ Period: ______

Activity 1 Fractions and Powers of 10

1. One of you will be Partner A and one of you will be Partner B. Determine the products and quotients for the problems in your assigned column. Then compare and discuss your responses, starting with part a.

Partner A	Partner B
a. $250 \cdot \frac{1}{10} = 25$	a. $250 \div 10 = 25$
b. $250 \cdot \frac{1}{100} = 2.5$	b. $250 \div 100 = 2.5$
c. $48 \div 10 = 4.8$	c. $48 \cdot \frac{1}{10} = 4.8$
d. $48 \div 100 = 0.48$	d. $48 \cdot \frac{1}{100} = 0.48$

2. Use your work from Problem 1 to determine the products of $720 \cdot (0.1)$ and $720 \cdot (0.01)$. Explain your thinking.

   72 and 7.2

   Sample response: Because 0.1 and 0.01 are equivalent to $\frac{1}{10}$ and $\frac{1}{100}$, respectively, they are also equivalent to dividing 720 by 10 and 100. When I divide by 10, I move each digit one place value to the right, and when I divide by 100, I move each digit two place values to the right.

1 Launch

Give pairs 5 minutes to complete Problems 1 and 2, and then pause for discussion, focusing on the relationship between multiplying by a power of $\frac{1}{10}$ and dividing by its reciprocal. Then give pairs another 5 minutes to complete Problems 3 and 4.

2 Monitor

Help students get started by activating prior knowledge. Ask, "How are multiplying a number by $\frac{1}{10}$ and dividing the same number by 10 related?"

Look for points of confusion:

- **Arbitrarily placing the decimal point in the product, particularly when multiplying two decimal factors.** Have students write both decimals as fractions, multiply them, and convert the product to a decimal. Ask, "What do you notice about the placement of the digits and decimal point in the factors when compared to the product?"

Look for productive strategies:

- Connecting the decimal and fraction forms, and recognizing that multiplying by 0.1, 0.01, or 0.001 is the same as dividing by 10, 100, or 1,000, respectively.
- Applying previous work with multiplying and dividing by powers of 10 to describe the movement of digits and the decimal point in the product. If students show their work solely by moving the decimal point, ask them why that strategy works.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can apply their work from previous slides to begin generalizing how to multiply decimals.

Accessibility: *Vary Demands to Optimize Challenge*

In Problems 1a and 1b, replace 250 with 200. In Problems 1c and 1d, replace 48 with 20. In Problem 2, replace 720 with 700. By doing so, students can still access the mathematical goal of the activity, using more manageable calculations.

Math Language Development

MLR2: Collect and Display

While students work, circulate and listen for the connections they make between Problems 1 and 2. Write the words and phrases they use on a visual display and update it throughout the remainder of the lesson. Refer to the language they used during the Connect. Listen for language, such as *the same*, *reciprocal*, and *inverse operation*. Remind students to use language from the display as needed.

Pairs | 20 min

Activity 1 Fractions and Powers of 10 (continued)

Students multiply decimals by writing them in fraction form, and they recognize the relationship between the position of the decimal point in the factors and its position in the product.

Activity 1 Fractions and Powers of 10 (continued)

3. Determine each product. Show your thinking.
   a. $36 \cdot 0.1 = 3.6$; Sample response: $36 \cdot (0.1) = 36 \cdot \frac{1}{10} = 36 \div 10 = 3.6$
   b. $24.5 \cdot 0.1 = 2.45$; Sample response: $24.5 \cdot (0.1) = 24.5 \cdot \frac{1}{10} = 24.5 \div 10 = 2.45$
   c. $1.8 \cdot 0.1 = 0.18$; Sample response: $1.8 \cdot (0.1) = 1.8 \cdot \frac{1}{10} = 1.8 \div 10 = 0.18$
   d. $54 \cdot 0.01 = 0.54$; Sample response: $54 \cdot (0.01) = 54 \cdot \frac{1}{100} = 54 \div 100 = 0.54$
   e. $9.2 \cdot 0.01 = 0.092$; Sample response: $9.2 \cdot (0.01) = 9.2 \cdot \frac{1}{100} = 9.2 \div 100 = 0.092$

4. Without calculating, determine how the values of the digits 7 and 5 change from the factor 750 to the product of $750 \cdot 0.001$. Explain your thinking.

   Each digit moves to the right 3 place values to make 0.750 or 0.75; Sample response: Multiplying by 0.001 is the same as dividing by 1,000, which means each digit will move to the right 3 place values.

Are you ready for more?

Ancient Romans wrote their numbers using symbols (or letters), now known as Roman numerals. They used the symbols I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, and M for 1,000. Typically, a number was expressed by listing the necessary values from greatest to least, and readers understood that meant to add. For example, XVI represents $10 + 5 + 1 = 16$. But, for efficiency, they never repeated the same letter more than three times in a row.

1. What numbers are represented by IX, XLIV, and CMXCIV?

   9, 44, and 994

2. The film of the game for Super Bowl LIII has a copyright of MMXIX. What is the number of the Super Bowl, and in what year was it played?

   Super Bowl 53 in 2019

3. What is the largest possible number you can write in Roman numerals using only these symbols and rules? Write it both as a number and in Roman numerals.

   3,999, which would be written as MMMCMXCIX.

530 Unit 5 Arithmetic in Base Ten

3 Connect

Have students share their responses to Problems 3 and 4, focusing on how they used the relationships between decimals and fractions, as well as multiplication and division, to know where to place the decimal point in each product.

Ask, "When a factor is multiplied by 0.1, 0.01, and 0.001, how does . . .

- The size of the product compare to the size of the factor?" The product is 10/100/1,000 times less than the factor.
- The placement of each digit change?" Each digit moves one/two/three place value(s) to the right.
- The placement of the decimal point change?" The decimal point moves to the left one/two/three place(s).

Highlight that the placement of each digit and the decimal point in the product is related to the placement of the decimal point in the factors, particularly when multiplying two decimals. For example, in Problem 3c, the product of $(1.8) \cdot (0.1)$ is 0.18 because multiplying by 0.1 is the same as dividing by 10. Therefore, each digit moves one place value to the right, and the decimal point moves one place to the left.

Pairs | 10 min

Activity 2 Fractions and Multiples of Powers of 10

Students extend their work from Activity 1, multiplying two decimals by writing and evaluating equivalent multiplication expressions involving fractions and powers of 10.

Name: ____________ Date: ________ Period: ________

Activity 2 Fractions and Multiples of Powers of 10

1. Select *all* the expressions that are equivalent to $(0.6) \cdot (0.5)$. Be prepared to explain your thinking.

   (A.) $6 \cdot 0.1 \cdot 5 \cdot 0.1$

   B. $6 \cdot 0.01 \cdot 5 \cdot 0.1$

   C. $6 \cdot 0.001 \cdot 5 \cdot 0.01$

   (D.) $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$

   E. $6 \cdot \frac{1}{1000} \cdot 5 \cdot \frac{1}{100}$

   (F.) $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$

   (G.) $\frac{6}{10} \cdot \frac{5}{10}$

   (H.) $6 \cdot 5 \cdot \frac{1}{100}$

2. Determine the value of $0.6 \cdot 0.5$. Show your thinking.

   0.30 or 0.3

   Sample response: $0.6 \cdot 0.5 = \frac{6}{10} \cdot \frac{5}{10} = \frac{30}{100} = 0.30$

3. Determine the value of each product by writing and evaluating an equivalent expression that contains fractions.

   a. $0.3 \cdot 0.02 = 0.006$

   Sample response: $0.3 \cdot 0.02 = 3 \cdot \frac{1}{10} \cdot 2 \cdot \frac{1}{100} = 3 \cdot 2 \cdot \frac{1}{10} \cdot \frac{1}{100} = 6 \cdot \frac{1}{1{,}000} = 0.006$

   b. $1.5 \cdot 0.007 = 0.0105$

   Sample response: $1.5 \cdot 0.007 = \frac{15}{10} \cdot \frac{7}{1{,}000} = \frac{105}{10{,}000} = 0.0105$

STOP

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having students rewrite $(0.6) \cdot (0.5)$ by using fractions.

Look for points of confusion:

- **Not recognizing all of the equivalent expressions.** Ask, "What are other ways you can write 0.6? 0.5? Where do you see that in the list of expressions?"
- **Incorrectly placing the decimal point in the product.** Ask, "What is your denominator? What is an equivalent division by a whole number? How does that help you decide where to place the decimal point?"

Look for productive strategies:

- Using the Associative and Commutative Properties of Multiplication to identify and write equivalent expressions that decompose decimals by place values and to rearrange factors in order to calculate more efficiently.
- Rewriting decimals as fractions and applying understandings from Activity 1 to determine the placement of the digits and the decimal point.

3 Connect

Have students share their responses for Problem 1, focusing on why expressions A, D, F, G, and H are all equivalent. Then have them share how they used these understandings to solve Problems 2 and 3.

Ask, "In Problem 2, why are both 0.3 and 0.30 correct products?"

Highlight how the expressions $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$ and $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$ use the Associative and Commutative Properties of Multiplication to express a product of decimals as a product of whole numbers and unit fractions involving powers of 10. This can be done for any product of decimals.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For Problem 1, display only the correct expressions (A, D, F, G, H). Ask, "Why are all of these expressions equivalent to $(0.6) \cdot (0.5)$?" Have students select one of these expressions to help them complete Problem 2. Have them use a similar structure to help them complete Problem 3.

Extension: *Math Enrichment*

Ask students to write an equivalent expression that uses fractions or whole number powers of 10 to determine the product of $(0.0015) \cdot 3{,}000$. $15 \cdot \frac{1}{10{,}000} \cdot 3 \cdot 1{,}000 = 4.5$

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement, such as "In Problem 1, Expressions B and E as equivalent to $(0.6) \cdot (0.5)$, because they both use the numbers 6 and 5 and the location of zeros in a decimal does not matter." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

Whole Class | 5 min

Summary

Review and synthesize how to determine the placement of the decimal point in the product when multiplying with decimals.

Summary

In today's lesson . . .

You used the relationship between place value and unit fractions with denominators that are powers of 10 to reason about the location of the decimal point in products involving decimals. For example, to evaluate $24 \cdot (0.1)$, you can think of the expression in multiple ways, all of which are equivalent and result in a product of 2.4.

Reason about place value.	Multiply by the unit fraction.	Divide.
24 groups of 1 tenth, or 24 tenths.	$24 \cdot \frac{1}{10}$, because 0.1 is equivalent to $\frac{1}{10}$.	$24 \div 10$, because multiplying by $\frac{1}{10}$ has the same result as dividing by 10.

When you divide a number by 10, the digits will remain the same, but they will each have a value that is 10 times less. This means the decimal point should be located one place to the left.

Similar reasoning can be applied when evaluating expressions such as $(1.5) \cdot (0.43)$, which is equal to $\frac{15}{10} \cdot \frac{43}{100}$, or $\left(15 \cdot \frac{1}{10}\right) \cdot \left(43 \cdot \frac{1}{100}\right)$. Using the associative and commutative properties, this can be rewritten as $(15 \cdot 43) \cdot \left(\frac{1}{10} \cdot \frac{1}{100}\right)$. That essentially makes the equivalent product of $645 \cdot \frac{1}{1{,}000}$, which is equal to $645 \div 1{,}000$. So, the product of $(1.5) \cdot (0.43)$ has the same digits as 645, but the decimal point is located 3 places to the left, giving a final result of 0.645.

> Reflect:

Synthesize

Highlight that rewriting decimals as fractions helps to reason about the placement of the decimal point in the product of two decimals. Each decimal place in a factor will correspond to a unit fraction with a denominator that is a power of 10. Then, by using the properties of multiplication, those unit fractions can be combined to get a new power of 10 that will correspond to the number of decimal places in the product (unless the product ends in trailing zeros, in which case they could be left off; e.g., $0.15 \cdot 0.04 = 0.0060$, or 0.006).

Ask, "When multiplying two decimals, what are some steps you could always follow to determine the product and the correct placement of the decimal point?"

Sample responses:

- I could change them to fractions and then multiply the numerators and multiply the denominators. The numerators will be two whole numbers, so their product is a whole number. The denominators will always be a power of 100, so I can just move the decimal point in the numerator.
- I can rewrite each factor as a whole number times a unit fraction with a denominator that is a power of 10 and then multiply the whole numbers and divide by the product of the fraction denominators. This tells me how many place values to move each digit to the right (or how many places to move the decimal point to the left).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is the total number of decimal places in two decimal factors related to the total number of decimal places in their product?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by explaining the placement of the digits and the decimal point in the product of two decimals.

Printable

Name: ________ Date: ________ Period: ________

Exit Ticket 5.05

Use what you know about decimals and fractions to explain why $0.2 \cdot 0.002 = 0.0004$.

Sample responses:

- I can rewrite the equation using fractions. For example:
  $$0.2 \cdot 0.002 = \frac{2}{10} \cdot \frac{2}{1{,}000} = \frac{4}{10{,}000} = 0.0004$$
- I can think of multiplication as division.
  For example, $0.2 \cdot 0.002 = \frac{2}{10} \cdot \frac{2}{1{,}000} = \frac{4}{10{,}000} = 4 \div 10{,}000$, which means I move the the digit 4 four place values to the right, which is the same as moving the decimal point four places to the left.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can use place value and fractions to reason about multiplication of decimals.

1 2 3

Success looks like . . .

- **Language Goal:** Generalizing that the number of decimal places in a product is related to the number of decimal places in the factors. **(Speaking and Listening, Writing)**
- **Language Goal:** Justifying the product of two decimals, by multiplying equivalent fractions that have a power of 10 in the denominator. **(Speaking and Listening)**
  - » Explaining why $0.2 \cdot 0.002 = 0.0004$ by using fractions.

Suggested next steps

If students are unable to explain why $(0.2) \cdot (0.002) = 0.0004$, consider:

- Reviewing Activity 2 and asking as many of the following questions as are needed:
  - » "How can you rewrite this expression by using fractions?"
  - » "What are some ways you can use the Associative and Commutative Properties of Multiplication to multiply the factors efficiently?"
  - » "How can you use the relationship between multiplication and division to determine where to place the decimal point in the product?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students used place value and fractions to multiply decimals. How will that support their work as they formalize an algorithm for multiplying decimals?
- In what ways have your students gotten better at identifying and using structure to make calculations more efficient? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

1. Determine the product. Use your work from parts a–d to complete part e.

a. $122.1 \cdot \frac{1}{100} = 1.221$

b. $11.8 \cdot \frac{1}{100} = 0.118$

c. $1350.1 \cdot \frac{1}{100} = 13.501$

d. $1.704 \cdot \frac{1}{100} = 0.01704$

e. What happens to the decimal point of the original number when you multiply it by $\frac{1}{100}$? Why do you think that is the case? Explain your thinking.

The decimal point moves two places to the left. Sample response: Because multiplying by $\frac{1}{100}$ is the same as dividing by 100, each digit will move two place values to the right, which is the same as the decimal point "moving" two places to the left.

2. Which expressions have the same value as $0.06 \cdot 0.154$? Select *all* that apply.

A. $6 \cdot \frac{1}{100} \cdot 154 \cdot \frac{1}{1{,}000}$

B. $6 \cdot 154 \cdot \frac{1}{100{,}000}$

C. $6 \cdot 0.1 \cdot 154 \cdot 0.01$

D. $6 \cdot 154 \cdot 0.00001$

E. 0.00924

3. Calculate the value of each expression by first writing the decimal factors as fractions, and then writing their product as a decimal. Show your thinking.

a. $0.1 \cdot 0.02 = 0.002$

Sample response:

$$\begin{aligned} 0.1 \cdot 0.02 &= 1 \cdot \frac{1}{10} \cdot 2 \cdot \frac{1}{100} \\ &= (1 \cdot 2) \cdot \left(\frac{1}{10} \cdot \frac{1}{100}\right) \\ &= 2 \cdot \frac{1}{1{,}000} \\ &= \frac{2}{1{,}000} \\ &= 0.002 \end{aligned}$$

b. $0.3 \cdot 0.2 = 0.06$

Sample response:

$$\begin{aligned} 0.3 \cdot 0.2 &= \frac{3}{10} \cdot \frac{2}{10} \\ &= \frac{6}{100} \\ &= 0.06 \end{aligned}$$

c. $1.2 \cdot 5 = 6$

Sample response:

$$\begin{aligned} 1.2 \cdot 5 &= \frac{12}{10} \cdot \frac{5}{1} \\ &= \frac{60}{10} \\ &= 6 \end{aligned}$$

d. $0.9 \cdot 1.1 = 0.99$

Sample response:

$$\begin{aligned} 0.9 \cdot 1.1 &= \frac{9}{10} \cdot \frac{11}{10} \\ &= \frac{99}{100} \\ &= 0.99 \end{aligned}$$

Name: ______ Date: ______ Period: ______

4. Evaluate each sum. Show your thinking.

a. $33.1 + 1.95 = 35.05$

$$\begin{array}{r} 33.10 \\ +\ \ 1.95 \\ \hline 35.05 \end{array}$$

b. $1.075 + 27.105 = 28.18$ **or 28.180**

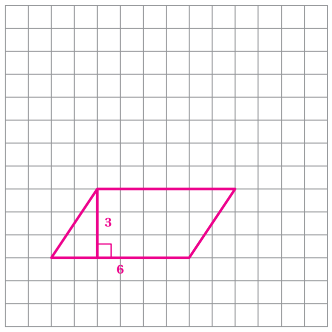
$$\begin{array}{r} 1.075 \\ +\ 27.105 \\ \hline 28.180 \end{array}$$

c. $0.401 + 9.28 = 9.681$

$$\begin{array}{r} 0.401 \\ +\ 9.280 \\ \hline 9.681 \end{array}$$

5. On the grid, draw a quadrilateral *that is not a rectangle* with an area of 18 square units. Show how you know the area is 18 square units.

Sample response shown; Area is 18 square units; $6 \cdot 3 = 18$



6. Write as many expressions using multiplication or division as you can think of that are equal to 6.

Answers may vary, but student responses may include factors, dividends, or divisors in whole number, fraction, or decimal form.

Sample responses:

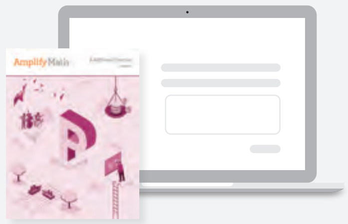
- $6 \cdot 1$
- $2 \cdot 3$
- $12 \div 2$
- $60 \cdot 0.1$
- $60 \cdot \frac{1}{10}$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 3	2
	5	Unit 1 Lesson 5	2
Formative	6	Unit 5 Lesson 6	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

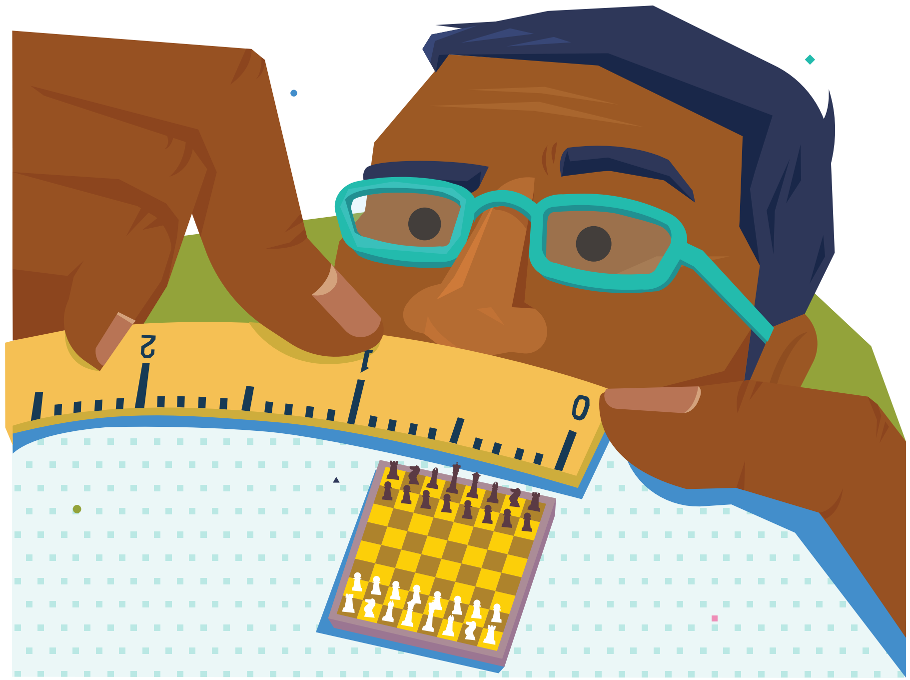


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 5 | **LESSON 6**

Methods for Multiplying Decimals

Let's look at some ways to represent multiplication of decimals.



Focus

Goals

1. **Language Goal:** Interpret different methods for computing the product of decimals, and evaluate their usefulness. **(Speaking and Listening)**
2. **Language Goal:** Justify where to place the decimal point in the product of two decimals with multiple non-zero digits. **(Speaking and Listening, Writing)**

Rigor

- Students build **conceptual understanding** of multiplying decimals by connecting to previous work with powers of ten, fractions, and area models.
- Students build **procedural fluency** with multiplying decimals.

Coherence

Today

Students continue developing methods for computing the products of decimals. Building on their work in Lesson 5, they multiply decimals by interpreting each factor as a product of a whole number and a unit fraction with a power of 10 in the denominator. This leads to extending multiplication area models to include decimal factors, and also to recognizing related expressions involving only whole number factors that can be used to determine products of decimals. For example, $0.4 \cdot 0.2$ can be represented by the same rectangle that represents $4 \cdot 2$, and the product of $4 \cdot 2$ can simply be divided by $10 \cdot 10 = 100$ to determine the product of $0.4 \cdot 0.2$.

Previously

In Lesson 5, students evaluated decimal multiplication expressions by rewriting the factors as fractions and then used the relationship between multiplication of unit fractions and division to place the decimal point.

Coming Soon

In Lesson 7, students will begin to generalize an algorithm for multiplying decimals. They will use another type of area diagram as a visual representation of vertical calculations and partial products.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Amps | Featured Activity

Activity 2
Dynamic Area Models

Students enter lengths of a base ten area model to visualize related decimal products.

Building Math Identity and Community

Connecting to Mathematical Practices

Students' behaviors may show frustration with determining structure in Activity 1. Allow students to take a break during which they reflect on their current emotions. Ask them to identify how those emotions are impacting their performance and what they can do to put their behavior back on track. If needed, you might need to help a student individually deal with the root cause of the emotion before sending them back to their task at hand.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Complete the table In **Activity 2** as a class. Have students use the ***Turn and Talk*** routine to discuss how the expressions, and therefore their products, are related.

Independent | 5 min

Warm-up Equivalent Expressions

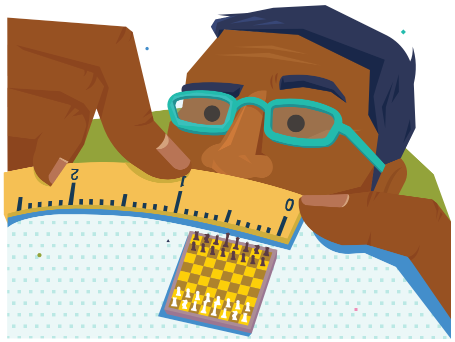
Students activate prior knowledge to represent a decimal with equivalent multiplication and division expressions, preparing them to multiply decimals.

Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 6

Methods for Multiplying Decimals

Let's look at some ways to represent multiplication of decimals.



Warm-up Equivalent Expressions

Write as many expressions using multiplication or division as you can think of that are equal to 0.6.

Answers may vary, but should demonstrate an understanding of place value.
Sample responses:

- $6 \cdot 0.1$
- $0.1 \cdot 6$
- $0.2 \cdot 3, 6 \cdot \frac{1}{10}$
- $60 \cdot \frac{1}{100}$
- $6 \div 10$
- $12 \div 20$
- $0.15 \cdot 4$

Log in to Amplify Math to complete this lesson online.

1 Launch

Give students 15 seconds to write as many expressions as they can. The purpose of attending to timing here is to keep the number of unique expressions manageable. Adjust the duration at your discretion.

2 Monitor

Help students get started if they do not understand how a single number can be represented by an expression by asking, "Can you represent 0.6 as a sum or difference of two numbers?"

Look for points of confusion:

- **Not knowing how to express 0.6 using multiplication or division.** Ask, "How is 0.6 related to the whole number 6?" or "Can you write 0.6 as a fraction?"

Look for productive strategies:

- Recognizing that 0.6 is 6 groups of one-tenth, which can be expressed as a fraction or decimal.
- Writing multiplication expressions with factors of 0.6 and division expressions with multiples of 0.6.
- Using previously determined expressions and place value to generate more equivalent expressions by multiplying and dividing powers of ten and one-tenth.

3 Connect

Display unique expressions as students share.

Have students share their expressions and explain why each is equivalent to 0.6. If not provided, be sure to include two expressions that illustrate the commutative property (e.g., $(0.1) \cdot 6$ and $6 \cdot (0.1)$) and at least one that can be generated from another using place value and powers of 10 (e.g., $60 \div 100$ from $6 \div 10$).

Highlight that place value and properties of operations can be helpful in generating equivalent expressions, which can be used to multiply decimals and to make sense of products.

Power-up

To power up students' ability to identify equivalent forms of a decimal value, ask:

Which of the following expressions are equal to 0.5? Select *all* that apply.

A. $5 \div 10$
B. $5 \div 0.1$
C. $\frac{1}{2}$
D. $\frac{20}{10}$
E. $50 \cdot 0.01$
F. $0.1 \cdot 5$

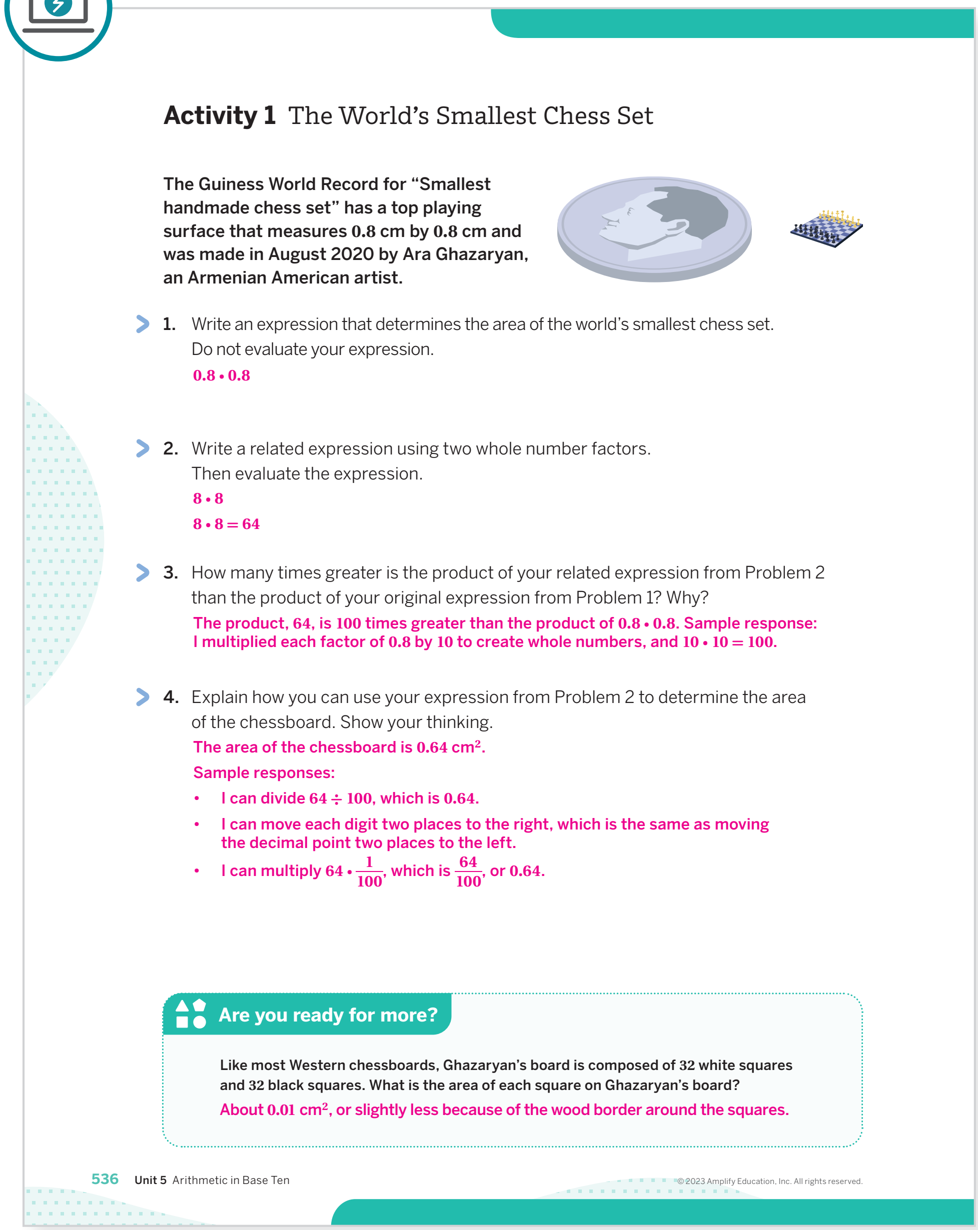
Use: Before the Warm-up.

Informed by: Performance on Lesson 5, Practice Problem 6.

Pairs | 15 min

Activity 1 The World's Smallest Chess Set

Students build upon their place value and fraction work, multiplying two decimals by writing and evaluating related expressions with two whole number factors.



Activity 1 The World's Smallest Chess Set

The Guiness World Record for "Smallest handmade chess set" has a top playing surface that measures 0.8 cm by 0.8 cm and was made in August 2020 by Ara Ghazaryan, an Armenian American artist.

1. Write an expression that determines the area of the world's smallest chess set. Do not evaluate your expression.
   $0.8 \cdot 0.8$
2. Write a related expression using two whole number factors. Then evaluate the expression.
   $8 \cdot 8$
   $8 \cdot 8 = 64$
3. How many times greater is the product of your related expression from Problem 2 than the product of your original expression from Problem 1? Why?
   The product, 64, is 100 times greater than the product of $0.8 \cdot 0.8$. Sample response: I multiplied each factor of 0.8 by 10 to create whole numbers, and $10 \cdot 10 = 100$.
4. Explain how you can use your expression from Problem 2 to determine the area of the chessboard. Show your thinking.
   The area of the chessboard is 0.64 cm².
   Sample responses:
   - I can divide $64 \div 100$, which is 0.64.
   - I can move each digit two places to the right, which is the same as moving the decimal point two places to the left.
   - I can multiply $64 \cdot \frac{1}{100}$, which is $\frac{64}{100}$, or 0.64.

Are you ready for more?

Like most Western chessboards, Ghazaryan's board is composed of 32 white squares and 32 black squares. What is the area of each square on Ghazaryan's board?
About 0.01 cm², or slightly less because of the wood border around the squares.

536 Unit 5 Arithmetic in Base Ten

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What information do you know? What information do you need to know?"

Look for points of confusion:

- **Writing an arbitrary or unrelated expression (e.g., $80 \cdot 8$).** Remind students that in a related expression, each value is multiplied or divided by the same value.
- **Struggling to compare the sizes of the products.** Ask, "What did you do to each factor to write your related expression? How much greater should the product be?"
- **Not using division to reverse the initial multiplication.** Ask, "How did you change the original expression? What operation can you use to "undo" that action?"

Look for productive strategies:

- Using prior knowledge of powers of 10 to compare the products of the two expressions, and extending this understanding to reason about using the related product to determine the original product.
- Using the structure of the base ten numbers and place value to reason about the placement of the decimal point in the final product.

3 Connect

Have students share their responses, focusing on how they used place value and powers of 10 to determine their solutions to Problems 3 and 4.

Display the equation $(0.8) \cdot (0.8) = \frac{8}{10} \cdot \frac{8}{10}$.

Ask, "How is the strategy you used today similar to the strategy from the last lesson? In both, I have a product of 64 (by multiplying whole numbers or numerators) divided by 100 (to undo the multiplication done to make whole number factors or because the denominator is 100).

Highlight that these strategies are equivalent and can be used to multiply any two decimals.

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

During the Launch, ask students if they are familiar with the game of chess. Consider showing images of a chessboard or bringing in a sample chessboard and displaying it. After students complete Problems 1 and 2, consider displaying the following to help students organize their thinking for Problem 3.

$0.8 \cdot 0.8 = 8 \cdot \frac{\square}{\square} \cdot 8 \cdot \frac{\square}{\square}$

$8 \cdot 8 = 0.8 \cdot \square \cdot 0.8 \cdot \square$

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problems 3 and 4, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "How do you know for sure that the product of $8 \cdot 8$ is 100 times greater than the product of $0.8 \cdot 0.8$?"
- "What math language did you use in your response?"

Have students revise their responses, as needed.

English Learners

Encourage students to draw a sketch of a chessboard and annotate the side lengths and refer to their sketches in their explanations.

Pairs | 15 min

Activity 2 Related Expressions and Area Diagrams

Students connect their work with related expressions and area models to determine the product of two decimals, preparing them for partial products in the next lesson.

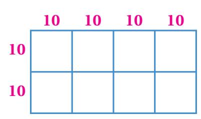
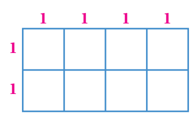
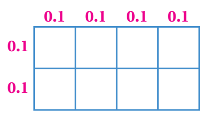
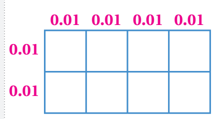
Amps Featured Activity Dynamic Area Models

Name: ________ Date: ________ Period: ________

Activity 2 Related Expressions and Area Diagrams

1. Decide who will be Partner A and who will be Partner B. Partner A will complete Rows 1 and 3 in the following table. Partner B will complete Rows 2 and 4.
   - Label the side length of each square in the diagram so that the area of the entire rectangle is represented by the expression.
   - Determine the area of each *inner square*. Show your thinking.
   - Determine the area of the *entire rectangle*. Show your thinking.

   Then share and compare your responses.

Expression	Diagram	Area of each square (square units)	Area of the entire rectangle (square units)
$40 \cdot 20$		100 $10 \cdot 10 = 100$	800 $40 \cdot 20 = 800$
$4 \cdot 2$		1 $1 \cdot 1 = 1$	8 $4 \cdot 2 = 8$
$0.4 \cdot 0.2$		0.01 $0.1 \cdot 0.1 = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100} = 0.01$	0.08 $\frac{4}{10} \cdot \frac{2}{10} = \frac{8}{100} = 0.08$
$0.04 \cdot 0.02$		0.0001 $0.01 \cdot 0.01 = \frac{1}{100} \cdot \frac{1}{100} = \frac{1}{10000} = 0.0001$	0.0008 $0.04 \cdot 100 = 4$ $0.02 \cdot 100 = 2$ $4 \cdot 2 = 8$ $100 \cdot 100 = 10000$ $8 \div 10000 = 0.0008$

1 Launch

Assign one student in each pair to be Partner A and the other to be Partner B. Explain that they will each complete two rows of the table and pause to share their work before completing Problems 3 and 4 together.

2 Monitor

Help students get started by pointing to $4 \cdot 2$ and asking, "How long is each side of the rectangle? What is the length of each square?"

Look for points of confusion:

- **Mislabeling the side length of each square.** Ask, "How many squares are there? What must the length of each square be if the total length is __?"
- **Incorrectly determining the area of each square (e.g., $0.1 \cdot 0.1 = 0.1$).** Have students rewrite the decimals as fractions and multiply.
- **Misplacing the decimal point in the product.** Depending on the strategy used, ask, "What is your denominator?" or "What did you multiply each factor by to make whole number factors? What is the product of that?" Then ask, "How does that help you determine the location of the decimal point and the value of each digit?"

Look for productive strategies:

- Using multiplicative or ratio thinking to determine the area of each rectangle (e.g., if one square has an area of 0.01 and there are 8 squares, then the area is 8 hundredths, or 0.08).
- Recognizing that each side length is ten times less than in the rectangle above, and therefore, the area (product) will be 100 times less.
- Using prior knowledge of powers of 10 to recognize that $21 \cdot 47 = 987$ is the result of multiplying $0.021 \cdot 1{,}000$ and $4.7 \cdot 10$, and, thus, 987 must be divided by 10,000 (Problem 3).

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can view an animation comparing area models to related expressions. By zooming in and out, they see how the models reflect place value concepts.

Extension: *Math Enrichment*

Tell students that the product of two decimals is 0.000006. Have each partner draw an area diagram that represents possible side lengths, and then trade diagrams with their partner. Each partner should determine the product of the side lengths and compare it to the given product. If any of the diagrams do not produce the correct product, have students determine the corrections needed.

Math Language Development

MLR8: Discussion Supports

While students work, display these sentence frames for the to use as they compare their responses to Problem 1.

- "Our expressions are similar because . . ."
- "Our expressions are different because . . ."
- "I agree with your diagram and area because . . ."
- "I disagree because . . ."
- "Our strategies are similar in that we both . . ."
- "Our strategies are different because I . . ., while you . . ."

Pairs | 15 min

Activity 2 Related Expressions and Area Diagrams (continued)

Students connect their work with related expressions and area models to determine the product of two decimals, preparing them for partial products in the next lesson.

Activity 2 Related Expressions and Area Diagrams (continued)

2. How does each of the products in the table relate to the equation $4 \cdot 2 = 8$? Show or explain your thinking.

   Sample responses:
   - **The product of $40 \cdot 20$ is 100 times greater than 8 because both factors are ten times greater than 4 and 2, and $10 \cdot 10 = 100$.**
   - **The product of $0.4 \cdot 0.2$ is 100 times less than 8 because both factors are ten times less than 4 and 2, and $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$.**
   - **The product of $0.04 \cdot 0.02$ is $10{,}000$ times less than 8 because both factors are one hundred times less than 4 and 2, and $\frac{1}{100} \cdot \frac{1}{100} = \frac{1}{10{,}000}$.**

3. Compute the product of $0.021 \cdot 4.7$ using the equation $21 \cdot 47 = 987$ and your thinking from Problem 2. Show or explain your thinking.

   0.0987

   Sample response: 0.021 is $1{,}000$ times less than 21, and 4.7 is 10 times less than 47. So, the product of $0.021 \cdot 4.7$ will be $10{,}000$ times less than 987 because $1{,}000 \cdot 10 = 10{,}000$. Therefore, each digit will move four place values to the right, which is the same as the decimal point moving four places to the left.

Reflect: In what ways did you show respect to your partner?

STOP

3 Connect

Have students share their responses to Problems 1 and 2, focusing on how the side lengths and products were related. Then have them share their responses to Problem 3, focusing on how they used powers of 10 to reason about the placement of the decimal in the product.

Highlight the following:

- Area diagrams are similar to the pieces in a base ten diagram in that they can represent different values. Just as a collection of base ten shapes can be said to represent either 103 or 0.103, the area of a rectangle composed of unit squares can represent many different products, depending on the side lengths of the squares.
- 100 of each successive unit square would fit into one of the squares from the diagram for the previous row's product.

Display a clean copy or a quick sketch of a 4 by 2 rectangle made up of 8 unit squares, labeling the side length of each square as 1 cm.

Ask, "Based on this diagram, what is the area of each small square and of the whole rectangle expressed as:

- Square centimeters?" 1 cm^2 and 8 cm^2
- Square millimeters?"100 mm^2 and 800 mm^2
- Square meters?" 0.0001 m^2 and 0.0008 m^2

Whole Class | 5 min

Summary

Review and synthesize the relationships among the different strategies to multiply decimals – using fractions, related expressions with whole numbers, and area models.

Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You saw at least two other ways to think about and calculate a product of two decimals based on place value and the fact that decimals can be written as equivalent fractions — using related expressions with whole numbers and area diagrams.

Consider the product of $(0.04) \cdot (0.07)$. This expression can be rewritten as each of the following equivalent expressions.

$\frac{4}{100} \cdot \frac{7}{100}$ $\left(4 \cdot \frac{1}{100}\right) \cdot \left(7 \cdot \frac{1}{100}\right)$ $(4 \cdot 7) \cdot \left(\frac{1}{100} \cdot \frac{1}{100}\right)$

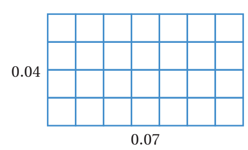
All of these expressions have a product of $\frac{28}{10{,}000}$, or 0.0028.

You can also multiply each factor by a power of 10 to create whole numbers, keeping track of how you changed the values, and then divide the product of those whole numbers by the product of both powers of 10.

- For $0.04 \cdot 0.07$, you could multiply both factors by 100.
- The expression $(0.04 \cdot 100) \cdot (0.07 \cdot 100)$ shows that is equal to $4 \cdot 7$.
- Because you multiplied by $100 \cdot 100$, you need to divide 28 by $10{,}000$ to get the actual product.

You can also use rectangular area models.

- For $0.04 \cdot 0.07$, you can represent $(4 \cdot 7) \cdot \left(\frac{1}{100} \cdot \frac{1}{100}\right)$ using unit squares that are $\frac{1}{100}$ by $\frac{1}{100}$, or 0.01 by 0.01.
- The rectangular area shown here looks the same as the rectangular area for multiplying $4 \cdot 7$.
- However, each square has an area of $\frac{1}{10{,}000}$ square units, instead of an area of 1 square unit.



Reflect:

Synthesize

Highlight the three strategies students have developed so far to multiply decimals: writing each decimal as a fraction and multiplying, multiplying by powers of ten to obtain whole number factors, and using area models.

Ask:

- "For any two of the strategies you have seen, explain how they are related."
- "When might one strategy be more efficient than another?" Answers may vary, but they should consider the number of place values in the decimal factors (and therefore the product), and they should acknowledge the tradeoffs between seeing the diagram (area model) and using fractional computation.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did today's lesson build upon your previous experience with area diagrams as a tool to multiply factors?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using a related expression with whole numbers to multiply two decimals.

Printable

Name: ______________________ Date: __________ Period: __________

Exit Ticket 5.06

1. Use the equation $135 \cdot 42 = 5{,}670$ and what you know about fractions, decimals, and place value to explain how to place the decimal point when you compute $(1.35) \cdot (4.2)$.

5.670 or 5.67

Sample response: 1.35 is 100 times less than 135, and 4.2 is 10 times less than 42. So, the product of $(1.35) \cdot (4.2)$ is 1,000 times less than 5,670 because $100 \cdot 10 = 1{,}000$. That means each digit in 5,670 will move three places to the right, which is the same as the decimal point moving three places to the left.

2. Which of the following is the correct value of $(0.22) \cdot (0.4)$? Show your thinking.

A. 8.8

B. 0.88

C. 0.088

D. 0.0088

Sample response: $(0.22) \cdot (0.4) = 22 \cdot 4 \cdot \frac{1}{100} \cdot \frac{1}{10}$

$= 22 \cdot 4 \cdot \frac{1}{1{,}000}$

$= 88 \cdot \frac{1}{1{,}000} = 0.088$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use area diagrams to represent and reason about multiplication of decimals.

1 2 3

b. I know how to use a product of whole numbers to determine a product of decimals.

1 2 3

c. I can explain in more than one way to multiply decimals.

1 2 3

Success looks like . . .

- **Language Goal:** Interpreting different methods for computing the product of decimals, and evaluating their usefulness. **(Speaking and Listening)**
- **Language Goal:** Justifying where to place the decimal point in the product of two decimals with multiple non-zero digits. **(Speaking and Listening, Writing)**
  - » Explaining where to place the decimal point when calculating $(1.35) \cdot (4.2)$.

Suggested next steps

If students incorrectly place the decimal point in the product of Problem 1, consider:

- Reviewing Activity 2, Problem 3, and asking, "How did you use powers of 10 to move from the decimal factors to the whole number factors? How did you use those powers of 10 to determine where the decimal point should be located in the product?"
- Asking, "How are 135 and 1.35 related? 42 and 4.2? How should 5,670 and the decimal product be related?"

If students select an incorrect answer choice in Problem 2, consider:

- Reviewing Activity 1. Ask, "How could you write a related expression with whole number factors? What do you need to do to the product of your related expression? Why?"
- Having students check their work by rewriting the expression using fractions.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In the previous lesson, students multiplied decimals by converting the factors to fractions. How did that support students' work with dividing by powers of 10 today?
- Which groups of students did and did not have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Interpreting different methods for computing the product of decimals, and evaluating their usefulness.

Reflect on students' language development toward this goal.

- How did using the sentence frames provided in the *Discussion Supports* routine in Activity 2 help students develop more precise language to compare their strategies?
- How have students progressed in their comfort using terms and phrases such as *factor, location of the decimal point, place value,* and *product?*

Practice

Independent

Name: ______ Date: ______ Period: ______

1. Priya determines the value of the expression $1.05 \cdot 2.8$ by calculating $105 \cdot 28 = 2{,}940$ and then moving the decimal point three places to the left. She determines a product of 2.94 for the original expression.

a. Why does Priya's method make sense?

Sample response: 1.05 is 100 times less than 105, and 2.8 is 10 times less than 28. Therefore, the product of $1.05 \cdot 2.8$ will be 1,000 times less than the product of $105 \cdot 28$. This means each digit will move three places to the right, which is the same as the decimal point moving three place values to the left.

b. Use Priya's method to calculate $0.0015 \cdot 0.024$.

0.000036;

Sample response: $15 \cdot 24 = 360$, and $\frac{1}{10{,}000} \cdot \frac{1}{1{,}000} = \frac{1}{10{,}000{,}000}$, and so, the product is 0000360.

2. Determine each product. Show your thinking.

a. $1.2 \cdot 0.11 = 0.132$;

Sample response: $1.2 \cdot 0.11 = 12 \cdot 11 \cdot \frac{1}{10} \cdot \frac{1}{100} = 132 \cdot \frac{1}{1{,}000} = 0.132$

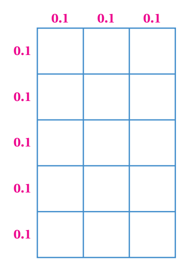
b. $0.34 \cdot 0.02 = 0.0068$;

Sample response: $0.34 \cdot 0.02 = 34 \cdot 2 \cdot \frac{1}{10000} = \frac{68}{10000} = 0.0068$

c. $120 \cdot 0.002 = 0.24$;

Sample response: $120 \cdot 0.002 = 120 \cdot 2 \cdot \frac{1}{1{,}000} = \frac{240}{1{,}000} = 0.240$

3. In this diagram, the area of the entire largest rectangle represents $0.3 \cdot 0.5$.



a. Label the side lengths of each square forming the rectangle.

b. What is the area represented by each square?

0.01 square units

c. What is the product of $0.3 \cdot 0.5$? Show or explain your thinking.

0.15; Sample response: If each square represents 0.01 square units, and there are 15 squares, the area is 15 hundredths, or 0.15 square units.

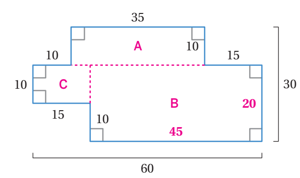
Name: ______ Date: ______ Period: ______

4. Evaluate $\frac{49}{50} \div \frac{7}{6}$. Show or explain your thinking.

$\frac{294}{350}$ or $\frac{84}{100}$ or 0.84

Sample response: $\frac{49}{50} \div \frac{7}{6} = \frac{49}{50} \cdot \frac{6}{7} = \frac{294}{350}$

5. Determine the area, in square units, of the figure. All angles are right angles. Show your thinking.



1,400 square units; Sample response:

Area of A: $35 \cdot 10 = 350$ square units

Area of B: $45 \cdot 20 = 900$ square units

Area of C: $10 \cdot 15 = 150$ square units

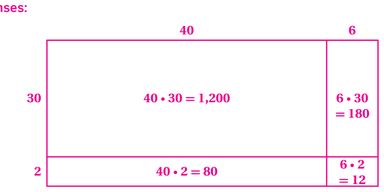
Total area:
$350 + 900 + 150 = 1{,}400$ square units

6. Evaluate $46 \cdot 32$. Show your thinking.

1,472

Sample responses:

$$\begin{array}{r} 46 \\ \times\ 32 \\ \hline 92 \\ +\ 1380 \\ \hline 1{,}472 \end{array}$$



$1{,}200 + 180 + 80 + 12 = 1{,}472$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 11	2
	5	Unit 1 Lesson 3	2
Formative	6	Unit 5 Lesson 7	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 5 | **LESSON 7**

Representing Decimal Multiplication With Diagrams

Let's use area diagrams to determine products.

Focus

Goals

1. **Language Goal:** Comprehend how the phrase *partial products* refers to decomposing a multiplication problem. **(Speaking and Listening, Writing)**
2. Coordinate area diagrams and vertical calculations that represent the same decimal multiplication problem.
3. **Language Goal:** Use an area diagram to represent and justify how to determine the product of two decimals. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue to use area diagrams to determine products of decimals, while also beginning to generalize the process. They revisit two methods used to determine products in earlier grades: decomposing a rectangle into sub-rectangles and determining the sum of their areas (partial products) and using a multiplication algorithm. Students extend this same reasoning to multiply two decimal values, connecting how these partial products correspond to the numbers in the multiplication algorithm. Students connect multiplication of decimals to that of whole numbers, look for correspondences between geometric diagrams and arithmetic calculations, and use these connections to calculate products of various decimals.

Previously

In Grades 4 and 5, students used area diagrams to multiply whole numbers and fractions. They connected their work to partial products and the related algorithms.

Coming Soon

In Lesson 8, students will formalize an algorithm to multiply decimals.

Rigor

- Students build **conceptual understanding** of multiplying decimals by using area diagrams and partial products.
- Students build **fluency** with multiplying decimals and whole numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials
- Exit Ticket
- Additional Practice
- Warm-up PDF, one per student
- Activity 1 PDF (for display)
- Activity 2 PDF (for display)

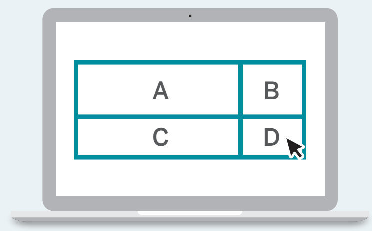
Math Language Development

Review word
- *partial products*

Amps | **Featured Activity**

Activity 2
Dynamic Area Diagrams

Students create digital area diagrams and connect their work to partial products. You can overlay them to see similarities and differences at a glance.



Amps POWERED BY desmos

Building Math Identity and Community
Connecting to Mathematical Practices

As students work with a partner, they might not make constructive choices about their social interactions impeding their work to determine the regularity in repeated reasoning. Prior to starting the activity, have students set some goals for how partners will work together. Have them examine the consequences of working against those goals and discuss how good choices can benefit both partners.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be done as a whole class, or students could use the ***Turn and Talk*** routine, with each pair assigned one strategy to consider before discussing as a class.
- In **Activity 1**, Problem 3 may be done as a whole class, using the Activity 1 PDF.

Independent | 10 min

Warm-up Representing Multiplication

Students activate prior knowledge of area diagrams representing multiplication of whole numbers and how those relate to calculations, preparing them to multiply decimals.

Unit 5 | Lesson 7

Representing Decimal Multiplication With Diagrams

Let's use area diagrams to determine products.

Warm-up Representing Multiplication

You will be given a table showing three strategies for representing and evaluating the expression $24 \cdot 13$.

1. How are the area models similar? How are they different?
   Sample response: The area models show one or both of the factors 24 and 13 decomposed by place value. The values in the vertical calculations are the areas of the sub-rectangles.

2. How are each of the area models related to their corresponding calculations?
   Sample responses:
   - The areas of the sub-rectangles in the diagrams are represented as partial products in the calculations.
   - Strategy A decomposes both factors into place value parts and multiplies each digit.
   - Strategy B decomposes 13 into place value parts and multiplies each digit by 24: $(3 \cdot 24) + (10 \cdot 24)$.
   - Strategy C decomposes 24 into place value parts and multiplies each digit by 13: $(4 \cdot 13) + (20 \cdot 13)$.

Log in to Amplify Math to complete this lesson online.

1 Launch

Give each student a copy of the Warm-up PDF. Set an expectation for the amount of time they will have to work independently on the activity.

2 Monitor

Help students get started by asking, for each area diagram, "Where do you see 24? 13?"

Look for points of confusion:
- **Not recognizing the partial product values as areas of the sub-rectangles.** Point to Strategy A's area model. Ask, "Where did 200 come from? Where do you see that same multiplication being done in the calculation?"

Look for productive strategies:
- Recognizing that each strategy decomposes one or both factors into place value parts.
- Connecting the areas of the sub-rectangles to the partial products in the vertical calculations.
- Recognizing why different factor decompositions lead to different partial products, but to the same total product.

3 Connect

Have students share their responses to each problem, focusing on why each strategy has different partial products, but result in the same total product.

Ask:
- "How is the calculation in Strategy B related to the multiplication algorithm?"
- "The numbers like 240 and 72 are called partial products. Why might that be?"
- "If I remove the zero from 240 in Strategy B's vertical calculation, why will the product be the same?"

Highlight that Strategies B and C use the Distributive Property, where one factor is decomposed by place value. Note the connection between the numbers of sub-rectangles and partial products, and also note their relative sizes and values.

Math Language Development

MLR2: Collect and Display

As students share what is similar and what is different between the three strategies, collect the language they use, such as *partial products*, *Distributive Property*, and *decompose*. Add this language to the class display.

English Learners

Display the area model diagrams and annotate them with appropriate mathematical language to help students make connections between the terms and the diagrams.

Power-up

To power up students' ability to multiply multi-digit whole numbers, have students complete:

a. Evaluate $17 \cdot 5$. Show your thinking.

$$\begin{array}{r} 17 \\ \times\ 5 \\ \hline 85 \end{array}$$

b. Evaluate $17 \cdot 20$. Show your thinking.

$$\begin{array}{r} 17 \\ \times 20 \\ \hline 340 \end{array}$$

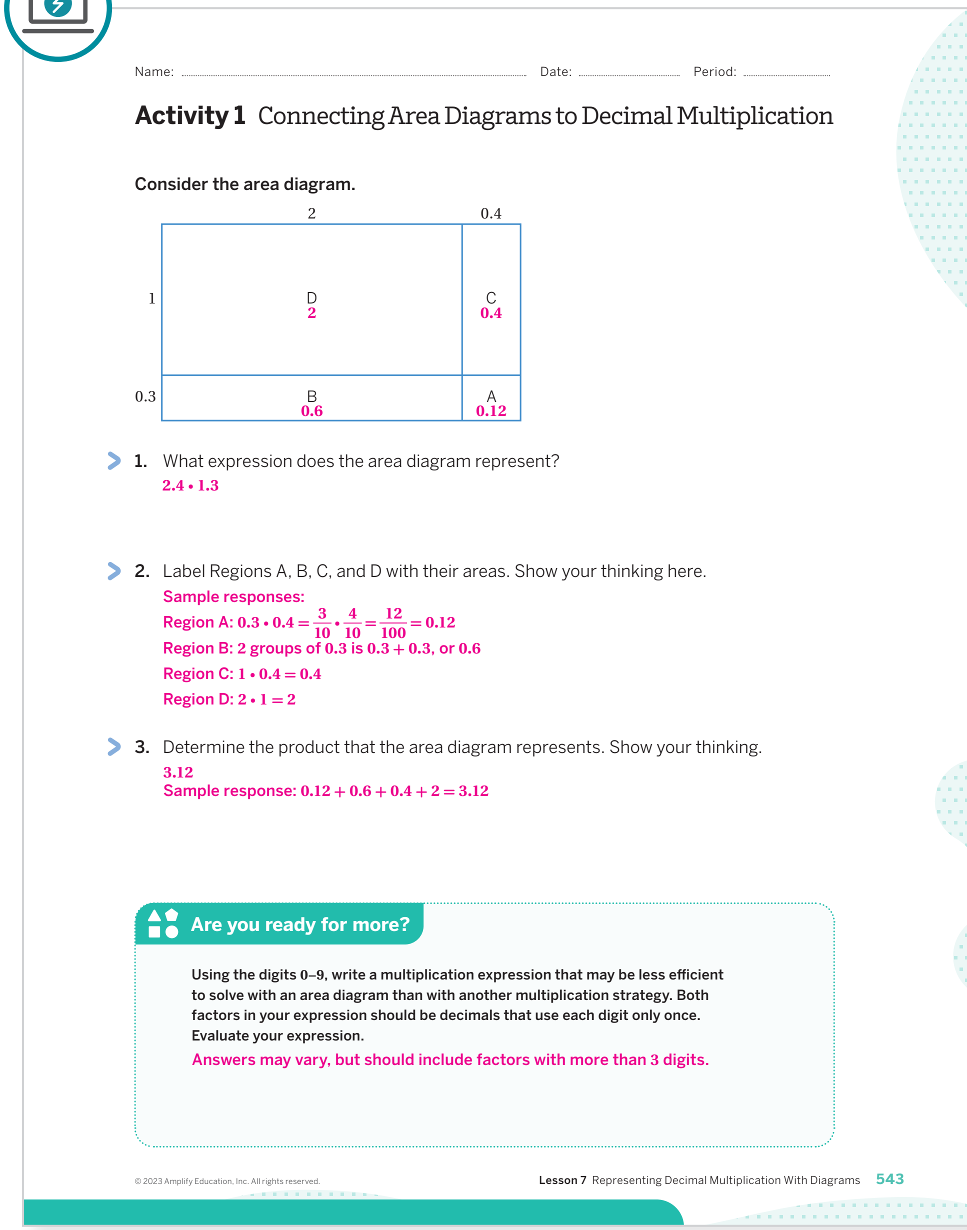
Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5.

Pairs | 10 min

Activity 1 Connecting Area Diagrams to Decimal Multiplication

Students extend their work with area models and partial products with whole numbers to multiply two decimal factors.



Name: Date: Period:

Activity 1 Connecting Area Diagrams to Decimal Multiplication

Consider the area diagram.

	2	0.4
1	D 2	C 0.4
0.3	B 0.6	A 0.12

1. What expression does the area diagram represent?
$2.4 \cdot 1.3$

2. Label Regions A, B, C, and D with their areas. Show your thinking here.
Sample responses:
Region A: $0.3 \cdot 0.4 = \frac{3}{10} \cdot \frac{4}{10} = \frac{12}{100} = 0.12$
Region B: 2 groups of 0.3 is $0.3 + 0.3$, or 0.6
Region C: $1 \cdot 0.4 = 0.4$
Region D: $2 \cdot 1 = 2$

3. Determine the product that the area diagram represents. Show your thinking.
3.12
Sample response: $0.12 + 0.6 + 0.4 + 2 = 3.12$

Are you ready for more?

Using the digits 0–9, write a multiplication expression that may be less efficient to solve with an area diagram than with another multiplication strategy. Both factors in your expression should be decimals that use each digit only once. Evaluate your expression.
Answers may vary, but should include factors with more than 3 digits.

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "How does this area model compare to the one in the Warm-up?"

Look for points of confusion:
- **Multiplying the wrong factors.** Have students outline each sub-rectangle. Ask, "What is the length? The width?"
- **Incorrectly placing the decimal point in the products.** Have students write the expression using fractions first, and then multiply. Ask, "What does your denominator tell you about the location of the decimal point?"

Look for productive strategies:
- Using fractions or whole numbers to multiply decimals.
- Recognizing the relationship between $2.4 \cdot 1.3$ and $24 \cdot 13$ (from the Warm-up), and using it to reason about the placement of the decimal point.
- Calculating the total area by adding the areas of the sub-rectangles.

3 Connect

Have students share their responses to Problems 1 and 2.

Display the Activity 1 PDF.

Ask:
- "Who solved similarly to Strategy A? B? C?"
- "Where do the partial products in each calculation come from?"
- "How are Strategies A and C and their products related? Why does that make sense?"
- "How could you have decomposed the factors in a fourth way?" Only decompose 2.4. This is similar to Strategy C in the Warm-up.

Highlight how the partial products are lined up by place value. Then note that Strategy B uses the Distributive Property and the multiplication algorithm for whole numbers.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the partial products and then color code the calculations that show their sum. Suggest students cover up the second column with an index card or scrap piece of paper as they work on the calculations for the first column.

Extension: *Math Enrichment*

Have students draw and decompose a rectangle to determine the product of $3.15 \cdot 0.26$. 0.819

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses and respond to the Ask questions, provide sentence frames to help students organize their thinking.

- "Strategy _____ reminds me of _____, because . . ."
- "I noticed _____, so I . . ."
- "_____ corresponds to . . ."
- "I already know _____, but I still need to know . . ."

English Learners

Use gestures such as pointing to the strategies from theActivity 1 PDF as you discuss different aspects of each strategy.

Pairs | 15 min

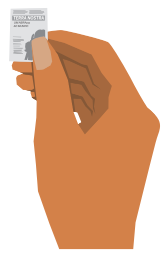
Activity 2 The World's Smallest Newspaper

Students apply their work with area models and vertical multiplication with decimals to determine the area of the front page of the world's smallest newspaper.

Amps Featured Activity Dynamic Area Diagrams

Activity 2 The World's Smallest Newspaper

The Guinness World Record for the "Smallest newspaper" was achieved in February 2012 by the Portuguese Nova Gráfica print shop. An exact replica of an issue of *Terra Nostra*, the miniature newspaper measured 0.72 in. by 0.99 in. and weighed 0.04 oz.



1. Determine the area of the newspaper's front page by drawing and labeling an area diagram. Show your thinking.

0.7128 in^2; Sample response:

	0.9	0.09
0.7	D 0.63	C 0.063
0.02	B 0.018	A 0.0018

A: $(0.02) \cdot (0.09) = 0.0018$
B: $(0.02) \cdot (0.9) = 0.018$
C: $(0.7) \cdot (0.09) = 0.063$
D: $(0.7) \cdot (0.9) = 0.63$
Total Area: $0.0018 + 0.018 + 0.063 + 0.63 = 0.7128$

2. Show how to calculate the area of the newspaper using vertical multiplication.

Sample responses:

$$\begin{array}{r} 0.99 \\ \times\ 0.72 \\ \hline 0.0018 \\ 0.018 \\ 0.063 \\ +\ 0.63 \\ \hline 0.7128 \end{array} \qquad \begin{array}{r} 0.99 \\ \times\ 0.72 \\ \hline 0.0198 \\ +\ 0.693 \\ \hline 0.7128 \end{array} \qquad \begin{array}{r} 0.99 \\ \times\ 0.72 \\ \hline 0.648 \\ +\ 0.0648 \\ \hline 0.7128 \end{array} \qquad \begin{array}{r} 99 \\ \times\ 72 \\ \hline 198 \\ +\ 6930 \\ \hline 7{,}128 \end{array}$$

$$7{,}128 \cdot \frac{1}{10{,}000} = 0.7128$$

STOP

544 Unit 5 Arithmetic in Base Ten

1 Launch

Set an expectation for the amount of time they will have to work independently on the activity.

2 Monitor

Help students get started by asking, "What information do you know? How can you draw an area diagram to represent the newspaper?"

Look for points of confusion:

- **Not attending to place value when decomposing factors and labeling side lengths (Problem 1).** Remind students to use place value when decomposing. Ask, "What is the value of each digit? How can you write that as a side length?"
- **Struggling to use vertical multiplication (Problem 2).** Ask, "How can you represent your computation in Problem 1 using vertical multiplication? Why does that work?"

Look for productive strategies:

- Drawing an area diagram that breaks at least one factor into place value parts.
- Calculating the area of each sub-rectangle by using whole numbers and adjusting the product, or using fractions; and then adding the areas.
- Using their understanding of place value and the multiplication algorithm for whole numbers to set up and use a vertical calculation to multiply.

3 Connect

Have pairs of students share their models and vertical calculations with another pair, such as those who used different strategies.

Display the Activity 2 PDF.

Ask, "How do Strategies B and C use the whole number multiplication algorithm? Why is there an extra step in Strategy C?"

Highlight that multiplying decimals can be done without a diagram, using the multiplication algorithm for whole numbers.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital area diagrams and connect their work to partial products. You can overlay them to see similarities and differences at a glance.

Accessibility: *Guide Processing and Visualization*

Have students first consider the values to be 99 in. and 72 in. and ask them to multiply these values. Consider also asking them to first determine $100 \cdot 72$ and then use that product to determine $99 \cdot 72$. Ask, "How can you use your calculations to help you think about multiplying $0.99 \cdot 0.72$?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their vertical calculations with another pair of students, ask them to look for similarities and differences among the strategies used. Ask partners to discuss these questions:

- "How is using partial products in a vertical calculation similar to drawing an area diagram?"
- "Did your partner use whole number multiplication? If so, what must be done at the end?"

English Learners

Encourage students to refer to the class display to use appropriate mathematical language as they discuss and interpret each other's work.

Whole Class | 5 min

Summary

Review and synthesize how to use area models and vertical calculations to multiply decimals.

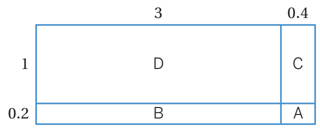
Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used what you know about base ten numbers and determining the area of a rectangle to help you multiply two decimals.

The rectangle shown has side lengths of 3.4 units and 1.2 units. Each side length has been partitioned by place value units, so $3.4 = 3 + 0.4$ and $1.2 = 1 + 0.2$. The area of the rectangle, in square units, is the product of $3.4 \cdot 1.2$.



The total area is equal to the sum of the areas of the four smaller rectangles, which can be calculated in several ways. One way is using partial products — determining the area of each smaller rectangle by multiplying its length and width. Another is using the Distributive Property — multiplying the common length or width of two smaller rectangles by the sum of their other dimensions. Both of these can also be seen in a vertical calculation of the product, as shown here.

Partial Products

$$
\begin{array}{rrl}
 & 3.4 & \\
\times & 1.2 & \\
\hline
 & 0.08 & \text{A} \\
 & 0.6 & \text{B} \\
 & 0.4 & \text{C} \\
+ & 3 & \text{D} \\
\hline
 & 4.08 &
\end{array}
$$

Distributive Property

$$
\begin{array}{rrl}
 & 3.4 & \\
\times & 1.2 & \\
\hline
 & 0.68 & \text{A + B} \\
+ & 3.4 & \text{C + D} \\
\hline
 & 4.08 &
\end{array}
$$

Reflect:

Synthesize

Display the Summary from the Student Edition.

Ask:

- "How are the two calculations similar? Different?" The numbers in each vertical calculation represent partial areas in the diagram. In Calculation A, each factor is decomposed into its place value parts and then multiplied by the other parts. Calculation B only decomposes the second factor into its place value parts, so the partial products are grouped.
- "How is the Distributive Property similar to the multiplication algorithm for whole numbers?" The first factor is multiplied by the second factor, one digit at a time, starting from the rightmost place value and moving left.
- "How could you use a related expression and what you know about whole number multiplication to also solve this problem?" Rewrite the expression as $34 \cdot 12 = 408$, and move the decimal point two places left, which is the same as dividing by 100 because each factor was multiplied by 10 to make whole number factors.

Highlight that students will formalize these strategies and determine an algorithm for multiplying decimals in the next lesson.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How were you able to use your previous experience with area diagrams and whole number multiplication to multiply decimals today?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by multiplying two decimals using both an area model and a vertical calculation.

Printable

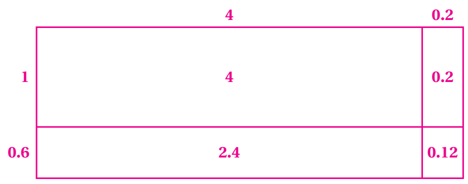
Name: ______ Date: ______ Period: ______

Exit Ticket 5.07

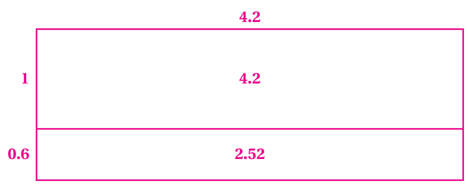
Draw an area diagram to represent the product of (4.2) • (1.6). Then use your diagram and a vertical calculation to determine the product. Show your thinking.

6.72

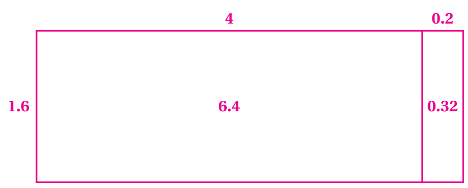
Sample responses:



$$\begin{array}{r} 4.2 \\ \times\ 1.6 \\ \hline 0.12 \\ 2.4 \\ 0.2 \\ +\ 4 \\ \hline 6.72 \end{array}$$



$$\begin{array}{r} 4.2 \\ \times\ 1.6 \\ \hline 2.52 \\ +\ 4.2 \\ \hline 6.72 \end{array}$$



$$\begin{array}{r} 4.2 \\ \times\ 1.6 \\ \hline 0.32 \\ +\ 6.4 \\ \hline 6.72 \end{array}$$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use area diagrams and partial products to represent and determine products of decimals.

1 2 3

Success looks like . . .

- **Language Goal:** Comprehending how the phrase *partial products* refers to decomposing a multiplication problem. **(Speaking and Listening, Writing)**
- **Goal:** Coordinating area diagrams and vertical calculations that represent the same decimal multiplication problem.
  - » Determining the product 4.2 • 1.6 using an area diagram and a vertical calculation.
- **Language Goal:** Using an area diagram to represent and justify how to determine the product of two decimals. **(Speaking and Listening, Writing)**

Suggested next steps

If students struggle to use place value to decompose the factors or draw the area diagram, consider:

- Asking, "How did you decompose the factors in Activity 2? How was that decomposition represented in your area diagram? How can that help you?"

If students use the incorrect side lengths for the sub-rectangle areas, consider:

- Reviewing Activity 2, and asking, "How did you determine what factors to multiply for each sub-rectangle?"

If students incorrectly place the decimal in any of the products, consider:

- Having them rewrite the problem using fractions and ask, "How can you use the denominator to place the decimal point?"

If students struggle to set up or use a vertical calculation, consider:

- Asking "What partial areas are represented in your diagram? How can you show those in a vertical calculation?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How were the area models used today similar to or different from those in Activity 2 of Lesson 6?
- What different ways did students approach Activity 2? What does that tell you about similarities and differences among your students?

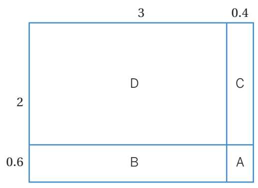
Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. The rectangle has an area in square units and has been partitioned into four smaller rectangles. For each expression, write the letter label of the smaller rectangle whose area matches each expression.



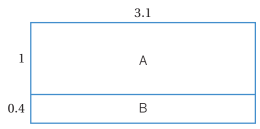
a. $3 \cdot 0.6$
Rectangle B

b. $0.4 \cdot 2$
Rectangle C

c. $0.4 \cdot 0.6$
Rectangle A

d. $3 \cdot 2$
Rectangle D

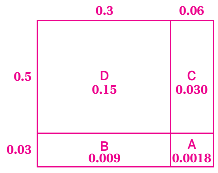
2. The area diagram represents $3.1 \cdot 1.4$.



a. Determine the areas of Rectangles A and B.
Rectangle A: $1 \cdot 3.1 = 3.1$
Rectangle B: $0.4 \cdot 3.1 = \frac{4}{10} \cdot \frac{31}{10} = \frac{124}{100} = 1.24$

b. What is the sum of the areas?
4.34; $3.1 + 1.24 = 4.34$

3. Draw and use an area diagram and a corresponding vertical calculation to determine the product of $0.36 \cdot 0.53$.
0.1908; Sample response:



$$\begin{array}{r} 0.36 \\ \times\ 0.53 \\ \hline 0.0018 \\ 0.009 \\ 0.030 \\ +\ 0.15 \\ \hline 0.1908 \end{array}$$

Name: ______ Date: ______ Period: ______

4. Complete each calculation so that it shows the correct sum.

a. $$\begin{array}{r} 2\,.\,3\,\boxed{1} \\ +\ \boxed{7}\,.\,6\,4 \\ \hline 9\,.\,\boxed{9}\,5 \end{array}$$

b. $$\begin{array}{r} 4\,.\,3\,\boxed{7} \\ +\ \boxed{2}\,.\,1\,5 \\ \hline 6\,.\,\boxed{5}\,2 \end{array}$$

c. $$\begin{array}{r} 2\,.\,3\,\boxed{8} \\ +\ \boxed{6}\,.\,6\,4 \\ \hline 9\,.\,\boxed{0}\,2 \end{array}$$

d. $$\begin{array}{r} 1\,.\,5\,\boxed{6} \\ +\ \boxed{0}\,.\,3\,8 \\ \hline 1\,.\,\boxed{9}\,4 \end{array}$$

5. Diego bought 12 mini muffins for \$4.20.

a. At this rate, how much would Diego pay for 4 mini muffins? Show your thinking.
\$1.40
Sample response: $12 \div 3 = 4$, and $4.20 \div 3 = 1.40$

b. How many mini muffins could Diego buy with \$3.00? Show or explain your thinking.
8 mini muffins
Sample response: Each mini muffin costs \$0.35, so 8 cost \$2.80, and 9 cost a little more than \$3. Therefore, Diego can buy 8 muffins.

Mini muffins	Price (\$)
12	4.20
1	0.35
8	2.80
9	3.15

6. Evaluate each expression.

a. $2 \cdot 50 = 100$

b. $2 \cdot 8 = 16$

c. $2 \cdot 0.4 = 0.8$

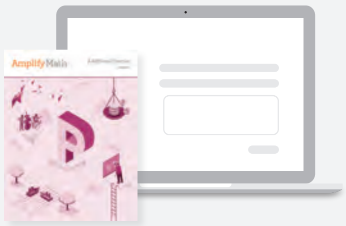
d. $2 \cdot 58.4 = 116.8$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 3	2
	5	Unit 2 Lesson 11	2
Formative	6	Unit 5 Lesson 8	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Calculating Products of Decimals

Let's multiply decimals.

Focus

Goals

1. **Language Goal:** Represent real-world contexts involving multiplication of decimals with an expression, and evaluate it using a chosen method. **(Writing)**
2. **Language Goal:** Use an algorithm to calculate the product of two decimals, and explain the solution method. **(Speaking and Listening)**

Coherence

Today

Students use strategies of their choosing to represent and reason about problems in real-world contexts that involve multiplication of decimals. They leverage this work to consolidate their understanding into a generalized process for multiplying any number by any decimal. Throughout the lesson, students should attend to precision as they use place value and powers of 10 to reason about and explain the location of the decimal point and the value of each digit in the products they determine.

Previously

In Lesson 7, students connected area models to vertical calculations with partial products. They saw how vertical calculations relate to the multiplication algorithm for whole numbers, which served as the foundation for this lesson.

Coming Soon

In Lesson 9, students will explore the relationships between base ten diagrams, partial quotients, and long division to divide multi-digit whole numbers.

Rigor

- Students build **fluency** with decimal multiplication, connecting previous strategies to a generalized algorithm.
- Students **apply** decimal multiplication strategies to represent and evaluate real-world contexts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- graph paper (optional)

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to show and explain their thinking behind their decimal calculations, and these explanations are available to you digitally, in real time.

Amps POWERED BY desmos

Building Math Identity and Community
Connecting to Mathematical Practices

While students might effectively think about how to complete Activity 2, they might not spend much time reflecting on their own abstract reasoning. As students build connections between previous and current concepts, they need to think about the connection between the concepts and the skills involved. Students should focus on creating a coherent representation of their own reasoning of the activity.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted or completed as a whole class.
- In **Activity 1**, you may omit Problem 1, but you will need to give students the U.S. price of diesel to use in Problem 2.
- **Activity 2** may be omitted. While this activity primarily provides additional practice, the context should be fun and engaging for students. Consider using it as a review when you have time.

Independent | 10 min

Warm-up Number Talk

Students use the structure of the base ten number system to mentally multiply a decimal that has been decomposed into place value parts.

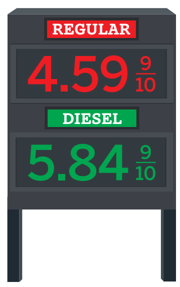
Unit 5 | Lesson 8

Calculating Products of Decimals

Let's multiply decimals.

Warm-up Number Talk

You need to buy 20 gallons of diesel gasoline. Mentally evaluate each expression. Be prepared to explain your thinking.



1. $20 \cdot 5 = 100$
   Sample response: I skip counted by 20 five times.
2. $20 \cdot 0.8 = 16$
   Sample response: I changed 0.8 into a whole number factor by multiplying by 10. Then I multiplied 20 by 8, and found a product of 160. I divided 160 by 10 to account for multiplying 0.8 by 10.
3. $20 \cdot 0.04 = 0.8$
   Sample response: I used my work from Problem 2. Half of 8 is 4, but the 4 in 0.04 is one place value to the right of the 8 in 0.8, so I divided 16 in half and then by 10.
4. $20 \cdot 5.84 = 116.8$
   Sample response: I added the products from the previous three problems, $100 + 16 + 0.8 = 116.8$, because 5.84 is the sum of $5 + 0.8 + 0.04$.

Log in to Amplify Math to complete this lesson online.

548 Unit 5 Arithmetic in Base Ten

1 Launch

Have students use the ***Notice and Wonder*** routine with the image of gas prices. Explain that, in the United States, gas prices are listed and calculated using this additional $\frac{9}{10}$ of a cent. Then conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by asking, "What do the 20 and 5 represent in context?"

Look for points of confusion:

- **Struggling to mentally evaluate with a decimal factor.** Ask, "How can you use whole number factors to help you evaluate mentally? How is this expression related to the expression(s) above it?"

Look for productive strategies:

- Using place value strategies to mentally multiply, such as using powers of 10 to multiply two whole numbers, and then dividing the result by the product of those powers of 10.
- Using Problems 1–3 to evaluate Problem 4.

3 Connect

Have students share their responses and strategies, focusing on how they used place value to mentally evaluate, and how they used the previous expression to evaluate the next expression.

Highlight that, as seen with area models, when one factor is broken into place value parts and multiplied by the other factor, the Distributive Property is applied and it will be covered more generally in the next unit.

Ask, "How is the Distributive Property related to the multiplication algorithm for whole numbers that you saw today?"

MLR Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses and how they mentally evaluated each expression, draw connections between the first three expressions and the fourth expression. Focus on the idea of "building up a number" by adding up its parts. Ask:

- "What do you notice about the second factors in each expression?"
- "Why do you think you can add the sums of the products for the first three expressions to obtain the product of the fourth expression?"

English Learners

Use gestures, such as pointing to each of the second factors in the first three expressions and then pointing to the second factor in the fourth expression, during the discussion.

Power-up

To power up students' ability to multiply decimals using the Distributive Property, have students complete:

a. Find the following products:
$2 \cdot 3 = 6$
$2 \cdot 0.3 = 0.6$

b. Recall that $2 \cdot 3.3 = 2 \cdot 3 + 2 \cdot 0.3$.
Use the previous results to find the product of 2 and 3.3.
$2 \cdot 3.3 = 6 + 0.6 = 6.6$

Use: Before the Warm-up.

Informed by: Performance on Lesson 7, Practice Problem 6.

Pairs | 15 min

Activity 1 How Far Can One Tank of Gas Take You?

Students apply decimal multiplication strategies to solve problems about the cost of a tank of diesel fuel, preparing them to generalize two multiplication algorithms.

Amps Featured Activity See Student Thinking

Name: ______________ Date: ________ Period: ________

Activity 1 How Far Can One Tank of Gas Take You?

In 2011, Marko Tomac and Ivan Cvetkovic set the Guinness World Record for the "Greatest distance driven on a single tank of diesel fuel." They drove 1,581.88 miles through Croatia in a Volkswagen Passat 1.6 TDI BlueMotion, averaging 76.37 mpg.

1. In 2011, the average price, in U.S. dollars, for diesel fuel in Croatia was about $6.038 per gallon. At the same time, in the United States, the average price for diesel was 64% of what it cost in Croatia.

   a. Estimate the cost of one gallon of diesel fuel in the United States in 2011. Explain your thinking.

   Answers may vary, but should be close to $3.84.

   Sample response: I rounded $6.038 to $6. I know that $\frac{1}{10}$ of $6 is $0.60 and $\frac{1}{100}$ of $6 is $0.06. The price in the United states is 6 groups of 0.60 plus 4 groups of 0.06, or $\left(6 \cdot \frac{6}{10}\right) + \left(4 \cdot \frac{6}{100}\right) = \frac{36}{10} + \frac{24}{100} = \frac{360}{100} + \frac{24}{100} = \frac{384}{100} = 3.84$.

   b. Determine the actual price of a gallon of diesel in the United States in 2011, rounded to the nearest thousandth. Show your thinking.

   $3.864

   Sample response:

   $$6.038 = 6038 \cdot \frac{1}{1{,}000}$$

   $$0.64 = 64 \cdot \frac{1}{100}$$

   $$6.038 \cdot 0.64 = 6{,}038 \cdot 64 \cdot \frac{1}{100{,}000}$$

   $$\begin{array}{r} 6038 \\ \times \quad 64 \\ \hline 24152 \\ +\,362280 \\ \hline 386{,}432 \end{array}$$

   $$386{,}432 \cdot \frac{1}{100{,}000} = \frac{386{,}432}{100{,}000} = 3.86432$$

Critique and Correct: Your teacher will present an incorrect response to Problem 1b. Be prepared to critique the response, correct it, and explain your thinking.

1 Launch

Consider activating prior knowledge by reviewing the multiplication algorithm for whole numbers. Provide graph paper to students who need help aligning place values.

2 Monitor

Help students get started by asking, "What information is given in Problem 1? How can that help you estimate?"

Look for points of confusion:

- **Struggling to estimate in Problem 1a.** Ask, "About how much is $6.038? What does 64% tell you about the price in the U.S.? How can you use benchmark percentages to help you think about this?
- **Not writing 64% as 0.64, and multiplying 6.038 • 64.** Ask, "What does *percent* mean? How can you represent 64% as a decimal factor?"
- **Incorrectly placing the decimal point in the products.** Depending on the strategy used, ask, "What is the denominator in your product?" or "How did you make whole number factors?" Then ask, "How does that help you locate the position of the decimal point in the product?"

Look for productive strategies:

- Using place value, benchmark percentages, fractions, or decimals to estimate (Problem 1).
- Using fractions, related expressions for whole numbers, or area models and partial products to multiply.
- Using the multiplication algorithm to vertically multiply either decimal factors or related whole numbers, and using powers of 10 to determine where the decimal point is located in the partial products (decimal factors) and the final product, as needed. If students line up the factors by the decimal points, ensure they add zeros so that each factor goes to the same place value. While this does not affect the product, it is more efficient to align both factors to the right.

Activity 1 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they are familiar with the phrase *miles per gallon*, and how many miles can typically be driven on one tank of gas for different types of vehicles. Help them visualize how 1,581.88 miles compares to the typical number of miles that can be driven on one tank of gas for other vehicles.

Accessibility: *Activate Prior Knowledge*

In Problem 1, suggest that students round the cost per gallon to the nearest dollar, if they have not done so already. Remind students that they previously learned to determine the percent of a number. Ask, "What is 64% of 6?"

Math Language Development

MLR3: Critique, Correct, Clarify

After students complete Problem 1b or during the Connect, present an incorrect response to Problem 1b such as, "If $6{,}038 \cdot 64 = 386{,}432$, then the product of 6.038 and 0.64 is 38.643." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "What mathematical language or reasoning can you use to verify that your statement is correct?"

English Learners

Have pairs offer a corrected statement that includes how to place the decimal point using powers of 10.

Activity 1 How Far Can One Tank of Gas Take You? (continued)

Students apply decimal multiplication strategies to solve problems about the cost of a tank of diesel fuel, preparing them to generalize two multiplication algorithms.

Activity 1 How Far Can One Tank of Gas Take You? (continued)

2. The Passat TDI's fuel tank can hold 18.5 gallons of diesel. How much less would it have cost Tomac and Cvetkovic to fill the tank in the United States compared to what they paid in Croatia?

   One partner should calculate the total cost in Croatia and the other partner should calculate the total cost in the United States. Then use both of your total costs to determine the price difference. Round your final response to the nearest cent. Show your thinking.

   $40.22; Sample responses:

   Croatia: $111.7030

   $6.038 = 6{,}038 \cdot \frac{1}{1{,}000}$

   $18.5 = 185 \cdot \frac{1}{10}$

   $6.038 \cdot 18.5 = 6{,}038 \cdot 185 \cdot \frac{1}{10{,}000}$

   $$\begin{array}{r} 6038 \\ \times \quad 185 \\ \hline 30190 \\ 483040 \\ +\ 603800 \\ \hline 1{,}117{,}030 \end{array}$$

   $1{,}117{,}030 \cdot \frac{1}{10{,}000} = 111.7030$

   United States: $71.4840

   $3.864 \cdot 1{,}000 = 3{,}864$

   $18.5 \cdot 10 = 185$

   $10 \cdot 1{,}000 = 10{,}000$

   $$\begin{array}{r} 3864 \\ \times \quad 185 \\ \hline 19320 \\ 309120 \\ +\ 386400 \\ \hline 714{,}840 \end{array}$$

   $714{,}840 \div 10{,}000 = 71.4840$

   Difference: $40.219

   $$\begin{array}{r} \cancel{1}\cancel{1}1.\cancel{7}\cancel{0}30 \\ -\ 71.4840 \\ \hline 40.2190 \end{array}$$

Are you ready for more?

1. Evaluate each of the following expressions. Write each result as a decimal.

   a. $1 - 0.1$ $= 0.9$

   b. $1 - 0.1 + 10 - 0.01$ $= 10.89$

   c. $1 - 0.1 + 10 - 0.01 + 100 - 0.001$ $= 110.889$

2. If the pattern in the expressions from Problem 1 repeated three more times so that the last number in the expression is 0.000001, what would be the value of the whole expression?

   111,110.888889

3. If all of the addition and subtraction symbols were multiplication symbols, how could you describe the final results for each expression in the pattern? Explain your thinking.

   They would be equal to 0.1, 0.01, 0.001, and so on, because each new whole number is equal to the reciprocal of the previous result (as a unit fraction).

3 Connect

Have students share their responses and strategies for Problem 1a, focusing on how they used place value, benchmark percentages, fractions, or decimals to estimate. Then have them share their responses and strategies for Problem 1b and Problem 2, focusing on how they used a vertical calculation and powers of 10 to multiply.

Display the Activity 1 PDF, noting that the calculations shown are for Problem 1b.

Ask:

- "What do the bold arrows represent in Calculation A?"
- "How would you know where to put the products in each partial product in Calculation B?"
- "Using these two calculations, what is an efficient process to multiply decimals?"

Highlight that the factors in Calculation B are not lined up by the decimal point because that requires writing an extra zero to make 0.64 appear as 0.640. Adding zeros is not wrong, but not necessary, because the product of 0, or essentially 0.000, and 6.038 is 0. Then note that the two algorithms shown can both be used to efficiently multiply decimals. Also, both use place value, powers of 10, and vertical multiplication, resulting in the same final product.

Pairs | 10 min

Activity 2 Going Wheelie Fast

Students practice using the decimal multiplication algorithm to determine which traveled faster – the world's fastest motorized toilet or the world's fastest motorized trash can.

Name: ________________ Date: ________ Period: ________

Activity 2 Going Wheelie Fast

What happens when you put wheels and a motor on a toilet or a trash can? Well, you have a unique new way to get around town!

In September 2018, four 18-year-old British engineering students — Robert English, Thomas Ellis, Joe Summer, and William Beaty — achieved the Guinness World Record for "Fastest motorized toilet." Complete with a toilet brush as a steering stick, the HAWC (Highly Advanced Water Closet) Mk1 reached a top speed of 70.545 mph. And at the end of the race, the driver flushed the toilet to show that it actually worked!

In October 2020, Andy Jennings created the "World's fastest motorized trash can" using a garbage bin, motor bike engine, and handle bars from a child's scooter. He reached a top speed of approximately 69.2 kph.

Plan ahead: How will you control your impulses while working with these fun scenarios?

Which traveled faster — the world's fastest motorized toilet or trash can? How much faster?
Note: Use 1 km = 0.6214 mile and 1 mile = 1.6093 km.

The toilet traveled 27.54412 mph (or 44.32591 kph) faster than the trash can.

Note: The sample response is shown only for a comparison of the speed in miles per hour.

km	miles
1	0.6214
69.2	?

$0.6214 \cdot 10{,}000 = 6{,}214$

$69.2 \cdot 10 = 692$

$10 \cdot 10{,}000 = 100{,}000$

Or

$0.6214 = 6{,}214 \cdot \frac{1}{10{,}000}$

$69.2 = 692 \cdot \frac{1}{10}$

$0.6214 \cdot 69.2 = 6{,}214 \cdot 692 \cdot \frac{1}{100{,}000}$

$$\begin{array}{r} 6214 \\ \times\quad 692 \\ \hline 12428 \\ 559260 \\ +\,3728400 \\ \hline 4{,}300{,}088 \end{array}$$

$$\begin{array}{r} \scriptstyle 9 \\ \scriptstyle 6\,10\quad 4\,\cancel{10}\,10 \\ \cancel{7}\cancel{0}.54\cancel{5}\cancel{0}\cancel{0} \\ -\,43.00088 \\ \hline 27.54412 \end{array}$$

$4{,}300{,}088 \div 100{,}000 = 4{,}3.00088$

Or

$4{,}300{,}088 \cdot \frac{1}{100{,}000} = 43.00088$

STOP

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What information do you know about the toilet? The trash can?"

Look for points of confusion:

- **Not converting to the same unit (km or mi).** Ask, "Are the units of distance the same? What information is given to you to help convert to the same units?"

Look for productive strategies:

- Using a ratio table and recognizing which conversion factor to use with a chosen given speed so multiplication and not division can be used.
- Using the multiplication algorithm and a vertical calculation to multiply either decimal factors or related whole numbers.
- Using powers of 10 to determine where the decimal point is located in the partial products (if multiplying decimal factors) and the final product.

3 Connect

Have students share their responses and strategies, including those who converted to mph and kph, focusing on how they knew where to place the decimal point at each step of their work and in the final product.

Display student work that shows the correct use of the multiplication algorithm with decimal factors and with whole number factors.

Ask, "What might be some pros and cons of each version of the algorithm?"

Highlight that, no matter which units are used or which version of the algorithm is used, it is important to use place value to identify where to place the decimal point in the final product.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see the immediate consequences of their solutions and determine how to fix any errors on their own.

Accessibility: *Vary Demands to Optimize Challenge*

Reduce the number of decimal places in the conversion factors and have students use the following conversion factors. This could be a good opportunity to assign different students different conversion factors and facilitate a discussion on how the final results compare to each other.

1 km = 0.62 miles 1 mile = 1.61 km

Math Language Development

MLR8: Discussion Supports— Press for Details

While students work, encourage them to document their steps and calculations clearly so that they can be understood by others. During the Connect, have students share their strategies and provide the following prompts for listeners to press for details:

- "Did you choose to convert miles to kilometers or vice versa? Was there a reason why you chose this conversion?"
- "Why did you do _____ first?"
- "Could you explain that in a different way?"

English Learners

Provide students time to rehearse and formulate their responses before sharing them with the class.

Whole Class | 5 min

Summary

Review and synthesize the strategies developed throughout this Sub-Unit for multiplying fractions.

Summary

In today's lesson . . .

You used the relationship between place value and unit fractions with base ten denominators to generate a general process for multiplying any two decimals. Equivalent decimals and fractions such as $0.1 = \frac{1}{10}$ and $0.01 = \frac{1}{100}$, and so on, allow you to determine the product of two decimals.

For example, you can follow these steps to determine the product of 3.02 and 4.1

Step	Example
1. Write each decimal as the product of a whole number and a fraction.	$3.02 = 302 \cdot \frac{1}{100}$ $4.1 = 41 \cdot \frac{1}{10}$
2. Multiply the whole numbers.	$302 \cdot 41 = 12382$
3. Multiply the fractions.	$\frac{1}{100} \cdot \frac{1}{10} = \frac{1}{1000}$
4. Determine the the final product by: • Multiplying the product of the whole numbers by the product of the fractions; **or** • Divide product of the whole numbers by the reciprocal of the product of the fractions; **or** • Use the product of the fractions to determine the place value of the last digit, then locate the decimal point relative to its position at the end of the product of the whole numbers.	• $12382 \cdot \frac{1}{1000} = 12.382$; **or** • $12382 \div 1000 = 12.382$; **or** • $\frac{1}{1000}$ is the ten thousandths place, meaning the last digit will be three places after the decimal point. So, the product is 12.382

Reflect:

Synthesize

Highlight the four main strategies developed throughout this Sub-Unit: using fraction equivalents of decimals, writing related expressions with whole numbers, decomposing decimal factors with area diagrams and partial products, and using vertical calculations.

Ask, "How do all of the strategies relate to each other?" Sample response: All of the strategies require you to use the structure of the base ten number system and place value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did the algorithm build upon what you already knew about fractions, decimals, powers of 10, place value, and multiplication?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by multiplying two decimal factors.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 5.08

Calculate $1.6 \cdot 0.215$. Show your thinking.

0.344 or 0.3440

Sample response:

$1.6 \cdot 10 = 16$

$0.215 \cdot 1{,}000 = 215$

$10 \cdot 1{,}000 = 10{,}000$

Or

$1.6 = 16 \cdot \frac{1}{10}$

$0.215 = 215 \cdot \frac{1}{1{,}000}$

$\frac{1}{10} \cdot \frac{1}{1{,}000} = \frac{1}{10{,}000}$

$$\begin{array}{r} 215 \\ \times\ 16 \\ \hline 1290 \\ +\ 2150 \\ \hline 3{,}440 \end{array}$$

$3{,}440 \div 10{,}000 = 0.3440$

Or

$3{,}440 \cdot \frac{1}{10{,}000} = 0.3440$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe and apply an algorithm for multiplying decimals. 1 2 3

Lesson 8 Calculating Products of Decimals

Success looks like . . .

- **Language Goal:** Representing real-world contexts involving multiplication of decimals with an expression, and evaluating it using a chosen method. **(Writing)**
- **Language Goal:** Using an algorithm to calculate the product of two decimals, and explaining the solution method. **(Speaking and Listening)**
  - » Determining the product of $1.6 \cdot 0.215$ using an algorithm.

Suggested next steps

If students incorrectly apply the algorithm, particularly when using vertical calculations, consider:

- Having them evaluate using fractions. Ask, "How does your fraction multiplication work help you identify the error in your original work?"

If students make calculation errors, consider:

- Having them draw an area diagram to decompose the factors into place value parts. Ask, "How can you use partial products as a way to solve this problem?"

If students incorrectly place the decimal point in the product, consider:

- Having them evaluate using fractions. Ask, "How does the denominator help you determine the location of the decimal point? Where do you see that same reasoning in your original strategy?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was using an algorithm to multiply decimals. How well did students accomplish this? What did you specifically do to help students accomplish this skill?
- Which teacher actions made the class discussion in Activity 1 strong? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

1. The Amazon rainforest covered 6.42 million km² in 1994. In 2014, it covered only $\frac{50}{59}$ as much. Which value is closest to the area of the Amazon forest in 2014? Explain how you know *without* calculating the exact area.

   A. 6.4 million km²
   B. 5.4 million km²
   C. 4.4 million km²
   D. 3.4 million km²
   E. 2.4 million km

   Sample response: I rounded $\frac{50}{59}$ to $\frac{50}{60}$ or $\frac{5}{6}$ and 6.42 million to 6 million. $\frac{5}{6}$ of 6 million is 5 million, so I looked for an answer choice that was closest to 5 million.

2. Use vertical calculation to determine each product. Show your thinking.

   a. $5.4 \cdot 2.4 = 12.96$

   Sample response:

   $$\begin{array}{r} 54 \\ \times\ 24 \\ \hline 216 \\ +\ 1080 \\ \hline 1{,}296 \end{array}$$

   $1{,}296 \cdot \frac{1}{100} = 12.96$

   b. $1.67 \cdot 3.5 = 5.845$

   Sample response:

   $$\begin{array}{r} 167 \\ \times\ 35 \\ \hline 835 \\ +\ 5010 \\ \hline 5{,}845 \end{array}$$

   $5{,}845 \div 1{,}000 = 5.845$

3. A pound of blueberries costs \$3.98 and a pound of clementines costs \$2.49. What is the combined cost of 0.6 lb of blueberries and 1.8 lb of clementines? Round your answer to the nearest cent. Show your thinking.

   \$6.87; Sample response:

   Blueberries:

   $3.98 \cdot 100 = 398$
   $0.6 \cdot 10 = 6$

   $$\begin{array}{r} {\scriptstyle 2\,5\,4} \\ 398 \\ \times\ 6 \\ \hline 2388 \end{array}$$

   $2{,}388 \cdot \frac{1}{1{,}000} = 2.388$

   Clementines:

   $2.49 \cdot 100 = 249$
   $1.8 \cdot 10 = 18$

   $$\begin{array}{r} {\scriptstyle 1\,3\,7} \\ 249 \\ \times\ 18 \\ \hline {\scriptstyle 1\,1} \\ 1992 \\ +\ 2490 \\ \hline 4482 \end{array}$$

   $4{,}482 \cdot \frac{1}{1{,}000} = 4.482$

   Total cost:

   $2.388 + 4.482 = 6.870$

   The total cost is \$6.87.

Name: Date: Period:

4. Which has a greater value: $7.4 - 0.0022$ or $7.39 - 0.0012$? Show your thinking.

   $7.4 - 0.0022$ has a greater value.

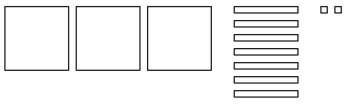
   Sample response:

   $$\begin{array}{r} {\scriptstyle 3\ 9\ 9\,10} \\ 7.\cancel{4}\cancel{0}\cancel{0}\cancel{0} \\ -\ 0.0022 \\ \hline 7.3978 \end{array} \qquad \begin{array}{r} {\scriptstyle 8\,9\,10} \\ 7.3\cancel{9}\cancel{0}\cancel{0} \\ -\ 0.0012 \\ \hline 7.3888 \end{array}$$

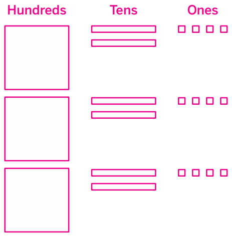
5. Andre is planting saplings (baby trees). It takes him 30 minutes to plant 3 saplings. If each sapling takes the same amount of time to plant, how long will it take Andre to plant 14 saplings?

   140 minutes or 2 hours and 20 minutes

6. Elena used base ten diagrams to find $372 \div 3$. She started by representing 372 as shown here.



   To show the division, she create 3 groups, each with 1 hundred. Then, she placed the tens and ones in each of the 3 groups. Draw what Elena's diagram might have looked like.

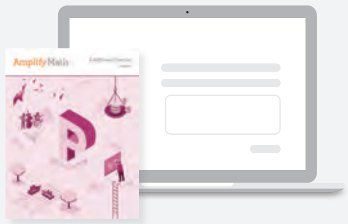


Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 4	2
	5	Unit 2 Lesson 11	2
Formative	6	Unit 5 Lesson 9	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

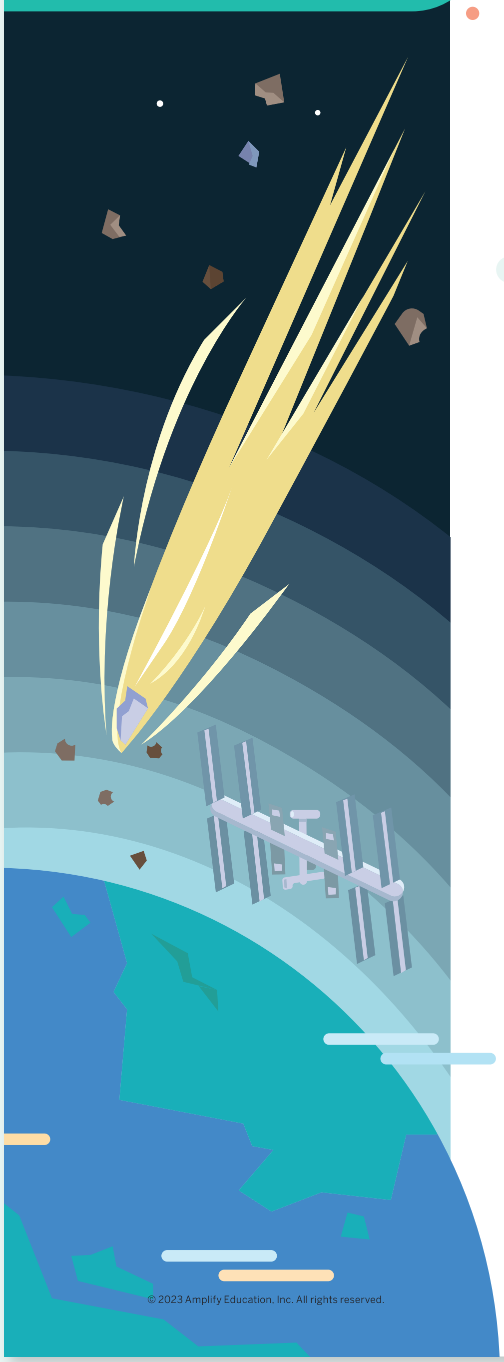


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 3
Dividing Decimals

In this Sub-Unit, students transition from concrete models and place-value strategies to formalize a standard algorithm for dividing both multi-digit whole numbers (including beyond thousands) and multi-digit decimals (including beyond hundredths).

SUB-UNIT
3 Dividing Decimals



Narrative Connections

How do you dodge a piece of space junk?

The International Space Station is the largest human-made structure in space. Astronauts have been living in it since 2000, conducting scientific research while the station races around the Earth at more than 17,000 mph.

But, in 2015, all that could have come to an end when a piece of space junk was detected moving toward the station. The debris was only a few inches long. However, the combined speeds of the object and the space station meant it could have caused catastrophic damage.

Fortunately, there was enough time for the Space Station to execute an avoidance maneuver. It fired thrusters for 2 minutes and 20 seconds, changing its velocity by 0.3 m per second. This tiny change actually raised the station's orbit by about half a kilometer! Disaster avoided.

To date, there have been nearly 10,000 satellites launched into space. As these satellites break down, they add to the space junk constantly threatening other spacecraft orbiting around the Earth.

To avert catastrophe, things such as distance and velocity must be computed as accurately as possible. An understanding of how to divide whole numbers and decimals is the starting point to making these calculations. Without it, you could end up lost in space.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore the importance of dividing fractions on and above Earth's surface in the following places:

- **Lesson 10, Activity 2:** "The Most Vertical Woman in the World"
- **Lesson 11, Activity 2:** Salinity Now
- **Lesson 12, Activity 1:** Ham the Astrochimp

Exploring Division

Let's explore different ways to divide whole numbers.

Focus

Goals

1. **Language Goal:** Divide whole numbers that result in a whole number quotient, and explain the reasoning using multiple representations. **(Speaking and Listening)**
2. **Language Goal:** Interpret base ten diagrams and partial quotients as different methods for computing the quotient of whole numbers, and evaluate their usefulness. **(Speaking and Listening)**
3. **Language Goal:** Interpret the long division method for computing the quotient of whole numbers, and compare and contrast it with other methods. **(Speaking and Listening)**

Coherence

Today

Students revisit two methods for determining a quotient of whole numbers without a remainder: base ten diagrams and partial quotients. Reviewing these strategies reinforces their understanding of the underlying principles and structure of base ten division, which are based on place value, the properties of operations, and the relationship between multiplication and division. This also paves the way for understanding the long division algorithm. Students also revisit the two interpretations of division — determining a number of equal-sized groups, or determining the size of each group.

Previously

In Grades 4 and 5, students reasoned about division of whole numbers and decimals to the hundredths using many of the same strategies being revisited in this lesson. In Unit 4 of this grade, students were already reminded of the two interpretations of division before dividing fractions.

Coming Soon

In Lesson 10, students use the long division method to build fluency in dividing whole numbers. In Lessons 11–13, these division strategies are applied to decimals.

Rigor

- Students build upon their **conceptual understanding** of division, and connect it to the standard algorithm.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Examples of Division Methods*
- base ten blocks
- graph paper

Math Language Development

New word

- *long division*

Review words

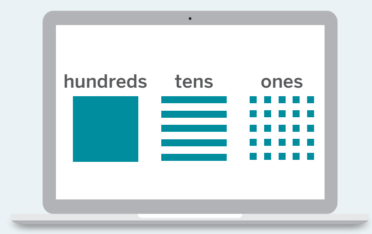
- *quotient*
- *divisor*
- *dividend*

Amps | Featured Activity

Activity 1

Interactive Base Ten Blocks

Students can hover over base ten blocks to help them understand division algorithms.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might fall into the routine of thoughtlessly completing an activity because it is easier than attempting to reason mathematically. Prior to the activity, have students set a goal about how they will focus on their own quantitative reasoning. Throughout the activity, take time-outs where students reflect on their progress toward the goal, and, after the activity, have students mentally score themselves. Students can use this score to motivate themselves to better direct their efforts in the future.

Modifications to Pacing

Note: this lesson is slightly longer and should take ~50 minutes. You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up** because the numbers used are continued into Activity 1. Then Activity 1, Problem 1 can be treated as the Warm-up.
- In **Activity 2**, the discussion surrounding long division can be conducted with the whole class.

Warm-up Number Talk

Students mentally evaluate or estimate a whole number quotient from division of two whole numbers to activate prior knowledge of division.

Unit 5 | Lesson 9

Exploring Division

Let's explore different ways to divide whole numbers.

Warm-up Number Talk

Mentally evaluate the quotient of $657 \div 3$.
Be prepared to explain your thinking.

219

Sample responses:

- I can use rounding:
  I know that $660 \div 3 = 220$, and I only need to subtract one group of 3 to get to 657. So, 219 groups of 3 go into 657.
- I can use expanded form:
  $600 \div 3$, $50 \div 3$, and $7 \div 3$
  $600 \div 3 = 200$
  $50 \div 3 = 16$, with a remainder of 2 that goes to the ones place and so,
  $7 \div 3$ becomes $9 \div 3 = 3$.
  $200 + 16 + 3 = 219$

Log in to Amplify Math to complete this lesson online.

556 Unit 5 Arithmetic in Base Ten

1 Launch

Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by asking, "Is there a "friendlier" number you can use to help you that is close to 657?"

Look for points of confusion:

- **Not knowing what to do with the remainders while dividing place by place, resulting in an answer of 212.** Ask, "What happens when you multiply the quotient (212) times the divisor (3)?"

Look for productive strategies:

- Thinking about the problem in terms of how many in each of 3 groups or how many groups of 3 (partitive and quotitive division, as seen in Unit 4).
- Rounding the divisor and recognizing that the difference is just one group of 3.
- Understanding what to do with a remainder when dividing place by place or with expanded form.

3 Connect

Have individual students share their strategy for mentally evaluating the quotient, focusing on those who used estimation or partial quotients.

Ask (after relevant responses), "Does that represent 'how many groups' or 'how many in each group' thinking?"

Highlight one way students can relate to the previous unit. Ask, "How many groups of three are in 657?" or "How many threes in each group?"

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share their strategies, ask for details in their reasoning by asking one or more of the following questions:

- "If you used estimation, how did you round? Why did you round that way?"
- "If you used partial quotients, how did you decompose the dividend? Why did you decompose it that way?"

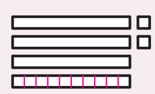
English Learners

Use wait time to allow students to formulate a response. Consider having students rehearse with a partner before sharing with the whole class.

Power-up

To power up students' ability to visualize division, ask:

The diagram shows 42 represented using base-ten blocks. How would you rearrange the blocks to represent $42 \div 3$? Explain or show your thinking.



Sample response:

Use: Before the Warm-up.

Informed by: Performance on Lesson 8, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6.

Pairs | 15 min

Activity 1 Diagrams and Partial Quotients

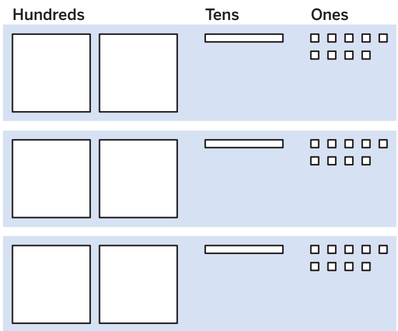
Students compare division using a base ten diagram and the partial quotient method to prepare them for using the long division algorithm.

Amps Featured Activity Interactive Base Ten Blocks

Name: ______________________ Date: __________ Period: __________

Activity 1 Diagrams and Partial Quotients

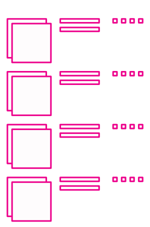
To calculate $657 \div 3$, Jada used a base ten diagram and Andre used partial quotients.

Jada's Method	Andre's Method
	$\begin{array}{r} \boxed{219} \\ 9 \\ 10 \\ 200 \\ 3\overline{)657} \\ -600 \\ \hline 57 \\ -30 \\ \hline 27 \\ -27 \\ \hline 0 \end{array}$

1. Discuss with your partner how Jada's and Andre's methods are similar and different.
   Answers may vary, but should include how one is represented with objects and the other with numbers, and how both represent dividing the value in each place into equal parts similarly.
   Discussion could also include references to partitive and quotitive division from the previous unit (e.g., Jada's method is more representative of how many are in each of the three groups, whereas Andre's method is more representative of how many 3s fit into 657).

2. Calculate $896 \div 4$ using one of these two methods. Show your thinking, including how you know your solution is correct.
   $896 \div 4 = 224$; Sample responses:

   Diagram:

   Partial Quotients:

   $$\begin{array}{r} \boxed{224} \\ 4 \\ 20 \\ 200 \\ 4\overline{)896} \\ -800 \\ \hline 96 \\ -80 \\ \hline 16 \\ -16 \\ \hline 0 \end{array}$$

   Check:

   $$\begin{array}{r} \scriptstyle 1 \\ 224 \\ \times\ 4 \\ \hline 896 \end{array}$$

1 Launch

Say, "You just calculated $657 \div 3$ mentally. Now you are going to see two ways of representing the calculation on paper."

2 Monitor

Help students get started by saying, "The number 657 has 5 tens, but Jada's diagram only shows 3 tens. Why?"

Look for points of confusion:

- **Making subtraction or multiplication errors because they did not align place values.** Prompt students to compare the structure of Andre's work with their own and check that like units are aligned in their vertical calculations.

Look for productive strategies:

- Making connections between division and subtraction, and the uses of decomposition in each method.
- Explaining the methods and results as "how many in each group" or "how many groups" of 3 in 657.

3 Connect

Display the two methods for students for reference as they share their thinking.

Have groups of students share their responses to Problem 1 first and then Problem 2, focusing how the two methods were used.

Highlight that, when using base ten diagrams to divide, especially when dividing by a relatively small whole number divisor, this method can be seen as showing "how many in each group." It may be more intuitive to look at partial quotients "as how many groups."

Ask:

- "Jada's diagram shows 3 groups of 2 hundreds. Where in Andre's method do you see the same value?"
- "Where in Jada's work do you see the 30 that Andre subtracts from 57?

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate digital base ten blocks to help them visualize the process of division.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code where they see 200, 10, and 9 in Jada's and Andre's methods. Then have them annotate where they see the division by 3 in each method.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display Jada's and Andre's methods without revealing any of the problems. Have students work with their partner to write 2–3 mathematical questions they could ask about one or both methods.
Sample questions shown.

- Why does Jada's method not show 5 tens? Why does her method show more than 7 ones?
- Why is Jada's method divided into 3 equal groups?
- How does Andre's method compare to Jada's method?

English Learners

Model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 15 min

Activity 2 Using Long Division

Students analyze a quotient being calculated using long division, compare long division to other methods, and then calculate other quotients using ***long division***.

Activity 2 Using Long Division

Lin calculated the quotient of 657 ÷ 3 using a method called *long division*. Her steps are shown and described here.

Lin arranged the numbers for vertical calculations. Her plan was to divide each digit of 657 into 3 groups, starting with the 6 hundreds.	There are 3 groups of 2 in 6, so Lin wrote 2 at the top and subtracted 6 from the 6, leaving 0. Then she brought down the 5 tens of 657.	There are 3 groups of 1 in 5, so she wrote 1 at the top and subtracted 3 from 5, which left a remainder of 2.	She brought down the 7 ones of 657 and wrote it next to the 2, which made 27. There are 3 groups of 9 in 27, so she wrote 9 at the top and subtracted 27, leaving 0.
$3\overline{)657}$	2 $3\overline{)657}$ -6 05	21 $3\overline{)657}$ -6 5 -3 2	219 $3\overline{)657}$ -6 5 -3 27 -27 0

1. Discuss the following questions with your partner.

   a. How is Lin's method similar to Jada's and Andre's methods from Activity 1? How is it different?

   Answers may vary.

   Discussion could include:

   - **How Jada's method looks more like determining how many are in each of three groups, but the other two methods look more like determining how many groups of 3 fit into 657 (or each digit and place value of 657).**
   - **Connections between Jada's and Lin's methods should be similar to previous connections made between base ten diagrams and partial quotients, noting how each digit in the quotient can be seen in the diagram because those values were determined similarly.**
   - **Connections between Andre's and Lin's methods should include how subtraction was used in both, but Lin's method subtracts one place at a time.**

   b. Explain why all three students were able to calculate the same solution.

   They all calculated the same answer because they all divided 657 into 3 equal groups, starting with the six hundreds and then continuing to either the five tens and seven ones, which can be thought of as three tens and twenty-seven ones.

1 Launch

Tell students that they will now consider a third method — called ***long division*** — for solving the same division problem as Jada and Andre. Use the ***Think-Pair-Share*** routine for Problem 1 before working on Problems 2 and 3.

2 Monitor

Help students get started by asking "How is this method *recorded* differently from the partial quotients method?"

Look for points of confusion:

- **Not aligning the digits of the quotient by place value.** Have students refer back to the partial quotients method and ask, "In the first step, Lin divided 6 by 3 to get 2. Did it matter where Lin wrote the 2? Why?" Yes, because the 6 represents 600, and she was dividing 600 by 3 in that step, which is 200. The 2 needs to be written in the hundreds place to show its correct place value of 200.
- **Not attending to place value throughout every step of the process: multiplying, subtracting, and bringing down digits.** Consider offering graph paper to help with aligning.

Look for productive strategies:

- Discussing that in the partial quotients method, the division is done in "chunks," resulting in a series of divisions and quotients. Each partial quotient is a multiple of the divisor and is written above the dividend according to place value; they are stacked so their sum is the overall quotient.
- Discussing that long division is performed digit by digit, from the greatest place value to the least, so the resulting quotient is also recorded one digit at a time.
- Recognizing that, although only one digit of the quotient is written down at a time, the full value of the digit is communicated by its place value in the entire quotient.

Activity 2 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

During the Launch, if you conduct the ***Notice and Wonder*** routine for Lin's method, ask these questions to help facilitate student thinking before having them continue with the activity.

- "Why did Lin write a 2 above the 6 in her first step?"
- "Why did Lin write a 1 above the 5 in her second step?"
- "Why did Lin determine a remainder of 2 in her second step?"
- "Why did Lin determine there were 9 groups of 3 in 27 in her third step? Where did the 27 come from?"

Math Language Development

MLR7: Compare and Connect

As students discuss Problem 1 with their partner, ask them to make connections between the three approaches used. Consider displaying these sample questions to help with their thinking.

- "Where do you see 3 equal groups used in each method?"
- "Both Andre's and Lin's methods use vertical calculations. How are they different?"
- "Both Andre's and Lin's methods show a final remainder of zero. Why is that important? What does that mean? Where can you see a similar concept in Jada's method?"

Pairs | 15 min

Activity 2 Using Long Division (continued)

Students analyze a quotient being calculated using long division, compare long division to other methods, and then calculate other quotients using ***long division***.

Name: ________ Date: ________ Period: ________

Activity 2 Using Long Division (continued)

2. Work with your partner to determine the following quotients using long division.

a. $1{,}332 \div 9 = 148$

Sample response:

$$\begin{array}{r} 148 \\ 9\overline{)1332} \\ -9 \\ \hline 43 \\ -36 \\ \hline 72 \\ -72 \\ \hline 0 \end{array}$$

b. $1{,}816 \div 4 = 454$

Sample response:

$$\begin{array}{r} 454 \\ 4\overline{)1816} \\ -16 \\ \hline 21 \\ -20 \\ \hline 16 \\ -16 \\ \hline 0 \end{array}$$

c. $4{,}352 \div 16 = 272$

Sample response:

$$\begin{array}{r} 272 \\ 16\overline{)4352} \\ -32 \\ \hline 115 \\ -112 \\ \hline 32 \\ -32 \\ \hline 0 \end{array}$$

3. Han used long division to divide $10{,}268 \div 4$. Using his work, determine the quotient.

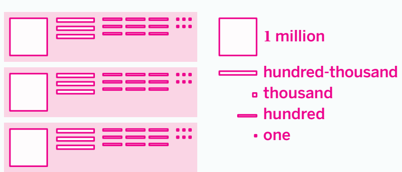
$10268 \div 4 = 2567$

$$\begin{array}{r} \square\boxed{2}\boxed{5}\boxed{6}\boxed{7} \\ 4\overline{)1\,0\,2\,6\,8} \\ -8 \\ \hline 2\,2 \\ -2\,0 \\ \hline 2\,6 \\ -2\,4 \\ \hline 2\,8 \\ -2\,8 \\ \hline 0 \end{array}$$

Are you ready for more?

Which method would you use to calculate $3{,}902{,}718 \div 3$? Why? Determine the quotient. If you use the base ten method, include a key for the values.

$3902718 \div 3 = 1300906$; Answers may vary. Students may find that the diagram or partial product methods are easier to use due to the "messiness" of the zeros present in long division.



$$\begin{array}{r} \boxed{1300906} \\ 6 \\ 900 \\ 300000 \\ 1000000 \\ 3\overline{)3902718} \\ -3000000 \\ \hline 902718 \\ -900000 \\ \hline 2718 \\ -2700 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1300906 \\ 3\overline{)3902718} \\ -3 \\ \hline 9 \\ -9 \\ \hline 00 \\ -0 \\ \hline 02 \\ -0 \\ \hline 27 \\ -27 \\ \hline 01 \\ -0 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$$

STOP

3 Connect

Display each of the division expressions from Problems 2 and 3 for students to reference. Consider having some students show their work after using the long division method for each.

Have pairs of students share what they discussed in Problem 1. Then have individual students share their solutions for Problems 2 and 3, focusing on those who chose other methods and why. Be sure to show long division for at least one of the methods.

Highlight these two big ideas related to long division:

- The method proceeds digit by digit, starting with the greatest base ten units, and continuing to the least units (or ones place).
- The placement of each digit in the quotient matters because, even alone, it conveys the full value of the digit and ensures the final quotient is correct.

Ask:

- "After writing down the 2, Lin subtracted 6. Why is this correct and 651 ($657 - 6 = 651$) not correct?" Though she wrote a subtraction of 6, she is actually subtracting 600, because 3 goes into 600 exactly 200 times and $3 \cdot 2 = 6$ or $3 \cdot 200 = 600$.
- "What are the advantages and disadvantages of each of the three methods you saw today?"
- "What might happen if you use a base ten diagram when the dividend or divisor are relatively large numbers? Does that affect whether you would choose that method or another?"

Define ***long division*** as a way to show the steps for dividing base ten numbers. The quotient is determined one digit at a time, from left to right.

Whole Class | 5 min

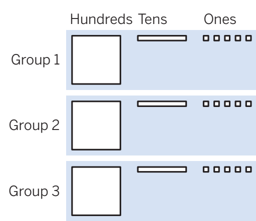
Summary

Review and synthesize the similarities and differences of the three division methods.

Summary

In today's lesson . . .

You saw three ways to divide whole numbers. Each method is shown for $345 \div 3$.

Base ten diagrams	Partial quotients		Long division
Different shapes represent the hundreds, tens, and ones in 345, and then the shapes are partitioned into 3 equal-sized groups, starting with the hundreds. When there is a remainder of one unit, decompose it into ten of the next smaller unit.	Determine how many times 3 goes into the full value of each digit in each place value, starting with the hundreds place (300), and then subtract from 345 to see how much is left. As you move from place to place, you may eventually recognize a multiple ($3 \cdot 15 = 45$).		Similar to partial quotients, divide place by place, from left to right, but you should always remove the greatest possible amount each time relative only to the current place value. The digits in the quotient are determined and recorded one at a time, all as part of a single number (rather than multiple numbers that are added).
	$\boxed{115}$ 5 10 100 $3\overline{)345}$ -300 45 -30 15 -15 0	$\boxed{115}$ 15 100 $3\overline{)345}$ -300 45 -45 0	115 $3\overline{)345}$ -3 4 -3 15 -15 0

Reflect:

Synthesize

Display the Anchor Chart PDF, *Examples of Division Methods*.

Ask:

- "How is dividing using long division similar to and different from drawing a base ten diagram?" Even though one method involves drawing and the other involves using only numbers, both methods rely on the same principle of dividing base ten units into equal-sized groups. In both methods, when there is not enough of a unit to divide equally into groups, you can unbundle the unit into the next smaller base ten units.
- "What about dividing using long division versus the partial quotients method?" They are similar in that you divide in "installments." In the partial quotients method, you can decide on the size of each installment, or group, being subtracted from the dividend. In long division, you follow a very specific order based on place value, you divide digit by digit—from left to right, and you subtract as large a group as possible at any step. In long division, you also do not write out all the numbers in the calculations; you use one digit at a time and rely on its place in the base ten system to convey its value.

Highlight that long division is another general method for determining quotients that will always work. It follows similar lines of reasoning for dividing by using a base ten diagram or the partial quotients method. All three methods rely on the structure of the base ten number system and place value. Also, highlight that students will have time to practice long division more in the next lesson.

Formalize vocabulary: ***long division***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What has changed about your understanding of division since fifth grade?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *long division* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving a division problem with a method of their choosing.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 5.09

Calculate $4{,}235 \div 11$ by using any method. Show your thinking.

385

Sample response:

$$\begin{array}{r} 385 \\ 11\overline{)4235} \\ -33 \\ \hline 93 \\ -88 \\ \hline 55 \\ -55 \\ \hline 0 \end{array}$$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use a method of my choosing to determine a quotient of two whole numbers. 1 2 3

b. I can analyze and explain how two different methods for division are related. 1 2 3

Success looks like . . .

- **Language Goal:** Dividing whole numbers that result in a whole number quotient, and explaining the reasoning using multiple representations. **(Speaking and Listening)**
  - » Calculating $4{,}235 \div 11$.
- **Language Goal:** Interpreting base ten diagrams and partial quotients as different methods for computing the quotient of whole numbers, and evaluating their usefulness. **(Speaking and Listening)**
- **Language Goal:** Interpreting the long division method for computing the quotient of whole numbers, and comparing and contrasting it with other methods. **(Speaking and Listening)**

Suggested next steps

If students do not complete the problem because they chose to solve using the diagram method, consider:

- Guiding students to understand that drawing the base ten blocks, while not incorrect, does take time and ask, "Is there another method that might take less time that you can try?"

If students have difficulty lining up their quotient, consider:

- Covering up the 35 in the dividend to view the problem as 42 divided by 11. Explain how the answer, 3, needs to be recorded over the 2. Then reveal the 3 in the dividend so students see that the next division will have the quotient over the 3.
- Referring back to Problem 2 in Activity 2, and reviewing how the partial quotients method is related to the long division method.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? This lesson asked students to connect and compare three division methods. Where in your students' work today did you see or hear evidence of them doing this?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

MLR Math Language Development

Language Goal: Interpreting the long division method for computing the quotient of whole numbers, and comparing and contrasting it with other methods.

Reflect on students' language development toward this goal.

- How did using the ***Compare and Connect*** routine in Activity 2 help students compare the long division method to methods involving base ten diagrams and partial quotients?
- During the Summary, as students responded to the Ask questions, did you see evidence of their developing math language, such as place value, digit, value of a digit, remainder, etc.

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Show how to determine $2{,}105 \div 5$ using the partial quotients method.

$$\begin{array}{r} \boxed{421} \\ 1 \\ 20 \\ 400 \\ 5\overline{)2105} \\ -2000 \\ \hline 105 \\ -100 \\ \hline 5 \\ -5 \\ \hline 0 \end{array}$$

2. Here is an incomplete calculation of $534 \div 6$. Complete the calculation.

$$\begin{array}{r} 8\,\boxed{9} \\ 6\overline{)5\;3\;4} \\ -\boxed{4}\boxed{8}\;\; \\ \hline \boxed{5}\boxed{4} \\ -\boxed{5}\boxed{4} \\ \hline 0 \end{array}$$

3. Why is the base ten diagram not an efficient way to evaluate $552 \div 46$? Explain your thinking by calculating the quotient using either the partial quotient or long division method.

Sample response: A base ten diagram would not be efficient because drawing out the base ten diagrams would take too long.

The quotient is 12.

$$\begin{array}{r} \boxed{12} \\ 2 \\ 10 \\ 46\overline{)552} \\ -460 \\ \hline 92 \\ -92 \\ \hline 0 \end{array} \qquad \begin{array}{r} 12 \\ 46\overline{)552} \\ -46\; \\ \hline 92 \\ -92 \\ \hline 0 \end{array}$$

Name: ______ Date: ______ Period: ______

Practice

4. Which of the polygons described has the greatest area? Explain your thinking

A. A rectangle that is 3.25 in. wide and 6.1 in. long.
19.825 square inches; $3.25 \bullet 6.1 = 19.825$

B. A square with side length of 4.6 in.
21.16 square inches; $4.6 \bullet 4.6 = 21.16$

C. A parallelogram with a base of 5.875 in. and a height of 3.5 in.
20.5625 square inches; $5.875 \bullet 3.5 = 20.5625$

D. A triangle with a base of 7.18 in. and a height of 5.4 in.
19.386 square inches; $\frac{1}{2} \bullet 7.18 \bullet 5.4 = 19.386$

5. A micrometer is equal to one millionth of a meter. A certain spider web is 4 micrometers thick. A fiber in a shirt is 1 hundred-thousandth of a meter thick.

a. Is the spider web or the fiber wider? Explain your thinking.
The fiber is wider. Sample response: Thickness of the spider web is 0.000004 m; thickness of a fiber is 0.00001 m. The fiber is wider because 4 millionths is less than one hundred-thousandths, or 10 millionths.

b. How many meters wider?
0.0000006 m;

$$\begin{array}{r} 0.000010 \\ -\,0.000004 \\ \hline 0.000006 \end{array}$$

6. Explain how you would estimate the quotient of $485 \div 8$. Determine an estimate.

Sample responses:

- **I could estimate the quotient by rounding the numbers: $490 \div 10 = 49$ or $500 \div 10 = 50$.**
- **I could estimate the quotient by using compatible numbers: $480 \div 8 = 60$.**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 8	2
	5	Unit 5 Lesson 4	2
Formative	6	Unit 5 Lesson 10	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Using Long Division

Let's use long division.

Focus

Goal

1. **Language Goal:** Recognize and explain that long division is an efficient strategy for dividing numbers, especially with multi-digit dividends. **(Speaking and Listening)**
2. Use long division to divide whole numbers that result in a whole number quotient, and multiply the quotient by the divisor to check the solution.

Coherence

Today

Students continue to practice dividing multi-digit whole numbers with a focus on long division. They see that, in long division, the meaning of each digit is indicated by its place value, and so instead of working with all the digits at once, the position of any digit – in the quotient, in the number being subtracted, or in a difference related to a partial quotient — conveys its meaning, which simplifies the calculation. This helps students recognize how this method is an efficient way to evaluate any division of whole numbers. Students conclude the lesson by solving division problems in a context that requires them to make sense of elevations above and below sea level, while using any method they choose.

Previously

In Lesson 9, students compared previous methods of dividing – base ten diagrams and partial quotients – to the new method of long division.

Coming Soon

In Lesson 11, students will divide two whole numbers with a remainder, and they will use long division to see how those remainders can be written as a decimal in the quotient. Then, in Lessons 12 and 13, students will continue working with division, but the dividends and divisors will be decimals, some of which will extend to the thousandths place or beyond.

Rigor

- Students reinforce the **procedural skills** of long division, including performing the inverse operation to check their solution.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (answers, for display)
- Activity 2 PDF (for display)
- *Decimal Base Ten Blocks* PDF (as needed)
- graph paper
- index cards

Math Language Development

Review word

- *long division*

Amps | Featured Activity

Activity 2
Interactive Map of Earth and Beyond

Students are able to zoom in and out using an interactive map of the far reaches of space to the trenches of the ocean.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to get the correct quotient because they are not able to keep their work organized effectively. Ask students to identify what tools they can use to help with organization of their work. Have graph paper ready in case they want to use it to help align their digits vertically.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 1**, have students only complete one of the two problems. Consider assigning Problems 1 and 2 to pairs so that both may be discussed in the whole-class discussion.

Independent | 5 min

Warm-up Number Talk

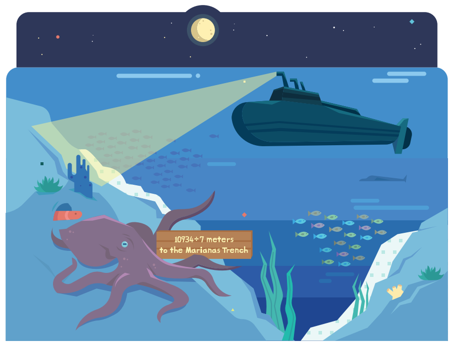
Students make reasonable estimates of a quotient of multi-digit whole numbers, activating their prior knowledge of base ten numbers, place value, and division.

Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 10

Using Long Division

Let's use long division.



Warm-up Number Talk

Mentally evaluate the quotient using estimation.

$328{,}624 \div 304$

Sample responses:

- $300{,}000 \div 300 = 1{,}000$
- $324{,}000 \div 300 = 1{,}080$
- $327{,}000 \div 300 = 1{,}090$
- $330{,}000 \div 300 = 1{,}100$
- $300 \cdot 1100 \approx 330{,}000$

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by acknowledging that the problem *looks* difficult, and then asking, "Could you round either or both numbers to make them more compatible and the division 'friendlier'?"

Look for points of confusion:

- **Trying to calculate an exact answer.** Encourage students to approximate the actual answer by rounding the dividend or divisor or friendlier "nearby" numbers.
- **Mixing up place values in mental calculations (e.g., $324{,}000 \div 300 = 1{,}800$ or $327{,}000 \div 300 = 1{,}900$).** Remind students to use the inverse operation (multiplication) to check their results, or have them consider $300{,}000 \div 300$ first, and ask whether their answer makes sense.

Look for productive strategies:

- Thinking in terms of high and low estimates to validate solutions or to hone in on a more precise answer (e.g., Because $330{,}000 \div 300 = 1{,}100$ and $300{,}000 \div 300 = 1{,}000$, the quotient should be between $1{,}000$ and $1{,}100$).

3 Connect

Have individual students share their solution and how they carried out the mental estimation.

Highlight the different ways estimation could be used: rounding, using "nearby, friendly" numbers, estimating to a specific place value, or using multiplication.

Ask, "How could you use multiplication to help you in this problem?" By thinking of 304 or 300 as one factor and trying to determine the other, or for checking the reasonableness of the quotient by multiplying.

Math Language Development

MLR8: Discussion Supports

During the Connect, display the following sentence frames to support students as they share their strategies for mental estimation.

- "I rounded 328,624 to ___ because . . ."
- "I rounded 304 to ___ because . . ."
- "I noticed ___, so I . . ."
- "I knew that ___, so I . . ."

English Learners

Provide students time to rehearse what they will say with a partner before they share with the whole class.

Power-up

To power up students' ability to estimate the quotient of two values, have students complete:

Recall that one way to estimate the value of a product or a quotient is to round one of the numbers.

Estimate the quotient for each of the following. Show your thinking.

a. $535 \div 9 \approx 60$; Sample response: $540 \div 9 = 60$

b. $798 \div 4 \approx 200$; Sample response: $800 \div 4 = 200$

c. $2823 \div 7 \approx 400$; Sample response: $2800 \div 7 = 400$

Use: Before the Warm-up

Informed by: Performance on Lesson 9, Practice Problem 6.

Pairs | 15 min

Activity 1 Long Division in Action

Students are given two opportunities to practice long division with multi-digit whole numbers.

Activity 1 Long Division in Action

Decide who will be Partner A and who will be Partner B. You will independently estimate the two quotients and then calculate them using long division. When you are both done, swap books and check your partner's results using multiplication.

1.

Partner A	Partner B
$7{,}680 \div 12 = 640$	$2{,}996 \div 14 = 214$
Estimate: Sample response: $12 \cdot 600 = 7200$ and $12 \cdot 700 = 8400$. So, the quotient must be between 600 and 700, but closer to 600.	**Estimate:** Sample response: $3000 \div 15 = 200$
Calculate: $$\begin{array}{r} 640 \\ 12\overline{)7680} \\ -72 \\ \hline 48 \\ -48 \\ \hline 00 \\ -0 \\ \hline 0 \end{array}$$	**Calculate:** $$\begin{array}{r} 214 \\ 14\overline{)2996} \\ -28 \\ \hline 19 \\ -14 \\ \hline 56 \\ -56 \\ \hline 0 \end{array}$$
SWAP!	
Multiply to check: $$\begin{array}{r} 640 \\ \times 12 \\ \hline 1280 \\ +6400 \\ \hline 7680 \end{array}$$	**Multiply to check:** $$\begin{array}{r} {}^{1} \\ 214 \\ \times 14 \\ \hline 856 \\ +2140 \\ \hline 2996 \end{array}$$

1 Launch

After reviewing the instructions, explain to students that, if there is a difference discovered at the "Multiply to check" step, pairs need to pause to identify and correct the issues. **Note:** This may mean that pairs will not get to work on Problem 2, which is fine, as these discussions are important and should be allowed to happen.

2 Monitor

Help students get started by reminding them that, for long division, the dividend should be larger than the divisor, and then asking, "Should 12 go into 7 or 12 go into 76?"

Look for points of confusion:

- **Not aligning the digits of the quotient by place values above the dividend.** Have students use index cards or other means to cover the digit(s) not being used at each step (e.g., so only the 12 and the 76 are showing first and then show the 12 and 768 in the dividend, aligning to the 48 below), to help them identify where each partial quotient should be placed.
- **Not attending to place value throughout every step of the process: multiplying, subtracting, and bringing down digits.** Consider offering graph paper to help with aligning.
- **Forgetting a step in the long division process.** Consider showing Lin's example from Lesson 9 for students to reference.

Look for productive strategies:

- Getting into the "rhythm" of the process of: divide, multiply, subtract, bring down, repeat.
- Attending to place value and aligning digits properly in the quotient and in work below the dividend.
- Clearly explaining the process of long division for each problem.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have pairs collaborate to set up the long division problem to calculate the quotient of $7{,}200 \div 12$ first. Provide access to physical base ten blocks or distribute copies of the *Decimal Base Ten Blocks* PDF for students to use to model the division and then capture the same steps using long division. Ask, "What happens when dividing 7 by 12? Dividing 72 by 12?" Have students work together on Problem 1.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display a calculation for Problem 1 that shows incorrect placement of the 6 in the hundreds digit of the quotient above the 7 in the thousands digit of the dividend. Ask:

- ***Critique:*** "Do you agree or disagree with this calculation? What error was made? Why does the placement of each digit matter?" Listen for students who address the placement of each digit by specifying the process they used, and invite these students to share with the class.
- ***Correct and Clarify:*** "Where should this digit be placed? Why should it be placed there?"

Pairs | 15 min

Activity 1 Long Division in Action (continued)

Students are given two opportunities to practice long division with multi-digit whole numbers.

Name: ______ Date: ______ Period: ______

Activity 1 Long Division in Action (continued)

2.

Partner A	Partner B
$21{,}835 \div 11 = 1{,}985$	$27{,}768 \div 12 = 2{,}314$
Estimate. $22{,}000 \div 11 = 2{,}000$	**Estimate.** $12 \cdot 2000 = 24{,}000$ $12 \cdot 3000 = 36{,}000$ So, the quotient must be between 2,000 and 3,000, but closer to 2,000.
Calculate. $11\overline{)21835}$ = 1985 -11 108 -99 93 -88 55 -55 0	**Calculate.** $12\overline{)27768}$ = 2314 -24 37 -36 16 -12 48 -48 0
SWAP!	
Multiply to check. 1985 $\times$ 11 1985 + 19850 21,835	**Multiply to check.** 2314 $\times$ 12 4628 + 23140 27,768

3 Connect

Display the Activity 1 PDF showing the correct solutions for Problems 1 and 2 for Partner A. Note that the solutions shown for "Estimate." represent only one way. Consider asking for other methods. Problem 2 for Partner B is blank to allow students to demonstrate how they solved it.

Have pairs of students share how each individual step – estimating, dividing, and multiplying to check – worked together in the calculation process. Use the displayed Problem 2B as a reference.

Highlight that, when possible, using inverse operations to check work and solutions is a good practice, but it can only indicate a mistake was made. It does not tell where the mistake is or what exactly the mistake was. It also only works in the case of division if the multiplication is performed correctly.

Ask, one or more of the following:

- For Problem 1:
  - » "What happened when you got to the 0 in $7{,}680 \div 12$?" I thought of the multiplication fact that would make it true, which was $12 \cdot 0 = 0$.
  - » 'Why are the partial products different in the long division (72, 48, and 0) and in the check (1,280 and 6,400)?" Because, in the long division, I was multiplying the 12 by each digit in 640, but, in the check, I was multiplying 640 by each digit in the 12.
- For Problem 2, "Did anyone notice anything different that happened when carrying out the long division process for $21{,}835 \div 11$?" It was the first time that after subtracting (21 – 11), there was still a two-digit number difference. "How did you know that it was still correct?" It was still less than the divisor.
- In general, "How are subtraction and division similar? Different?" Similar: With both operations, the solution is less than the starting value. Subtraction is used during long division. Different: With subtraction, you are taking away, which is why the solution is less than the starting value. With division, you are not really taking anything away, you are just separating a number into groups.

Pairs | 15 min

Activity 2 “The Most Vertical Woman in the World”

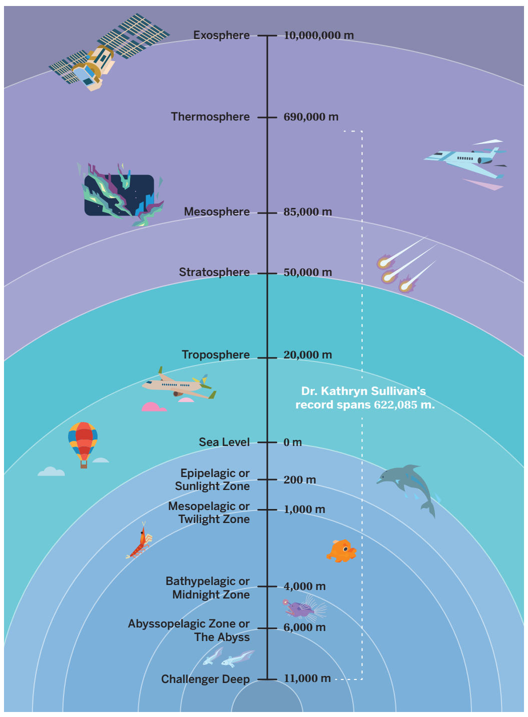
This lesson gives students the opportunity to practice division in another world-record context, while also supplying some background knowledge for Unit 7.

Amps Featured Activity Interactive Map of Earth and Beyond

Activity 2 “The Most Vertical Woman in the World”

The current Guinness World Records holder for “Greatest vertical extent travelled by an individual (within Earth’s exosphere)” is Dr. Kathryn Sullivan. She was the first person to visit both space and the deepest point on Earth. That’s right — Dr. Sullivan has traveled to the thermosphere layer in outer space all the way down to the Challenger Deep in the Mariana Trench, for a grand total of 622,085 vertical meters.

This image shows all of the different layers of Earth’s atmosphere and oceanic zones, and also the extent of Dr. Sullivan’s full vertical travels.



Co-craft Questions: Before you begin Problem 1, work with your partner to write 2–3 mathematical questions you have about this image.

566 Unit 5 Arithmetic in Base Ten

1 Launch

Read aloud the paragraph about Dr. Kathryn Sullivan. Use the ***Co-craft Questions*** routine as described in the Math Language Development section. Explain to students how to read the zones, making it clear that they are read above and below Sea Level, while Sea Level is not a zone on its own. For example, the dolphin is between the markings for Sea Level and Sunlight Zone. That region is all referred to as the Sunlight Zone.

2 Monitor

Help students get started by asking “What is the distance from sea level to the Challenger Deep? Where can you find that information?”

Look for points of confusion:

- **Still having trouble aligning digits, especially with six-digit dividends.** Have students use graph paper to organize their work. Consider also having students cover any digits not being used to visually reinforce the process.

Look for productive strategies:

- Remembering that multiplying by a unit fraction is the same as dividing by its denominator.
- Coordinating the correctly calculated quotients with the information about the location relative to sea level to identify the correct zones.

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive map to zoom in and out from the far reaches of space to the trenches of the ocean.

Extension: *Math Enrichment*

Have students complete the following problem:

A Dumbo octopus swims at a depth of 4,500 m below sea level at the same time an airplane flies at an altitude of 10,500 m above sea level. The distance between the octopus and a bird flying at the same time is one third the distance between the octopus and the plane. What is the bird’s altitude? 500 m

Math Language Development

MLR5: Co-craft Questions

During the Launch, have students work with their partner to write 2–3 questions they could ask about the image. Sample questions shown.

- How far is Earth’s exosphere from the Challenger Deep?
- Which is farther from sea level, Earth’s exosphere or the Challenger Deep?
- If someone travels from the Challenger Deep to Earth’s stratosphere, how far would they travel?

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 15 min

Activity 2 "The Most Vertical Woman in the World" (continued)

This lesson gives students the opportunity to practice division in another world-record context, while also supplying some background knowledge for Unit 7.

Name: ______________ Date: ________ Period: ________

Activity 2 "The Most Vertical Woman in the World" (continued)

Use the image on the previous page to help you complete Problems 1–3. Note the precise height Kathryn Sullivan reached in space was 611,151 m above sea level, and the precise depth she reached at the Mariana Trench was 10,934 m below sea level.

1. Dr. Sullivan is below sea level at a depth that is equal to her total distance traveled from sea level to the Mariana Trench divided by 14. Which zone is she in?

   She would be at 781 m in the Twilight or Mesopelagic Zone.
   Sample response: 10,934 ÷ 14 = 781

$$\begin{array}{r} 781 \\ 14\overline{)10934} \\ -98 \\ \hline 113 \\ -112 \\ \hline 14 \\ -14 \\ \hline 0 \end{array}$$

2. Dr. Sullivan is in the atmosphere, one third of the way to the farthest point into outer space that she reached. Which zone is she in?

   She would be at 203,717 m in Thermosphere;
   Sample response: 611,151 ÷ 3 = 203,717

$$\begin{array}{r} 203717 \\ 3\overline{)611151} \\ -6 \\ \hline 01 \\ -0 \\ \hline 11 \\ -9 \\ \hline 21 \\ -21 \\ \hline 05 \\ -3 \\ \hline 21 \\ -21 \\ \hline 0 \end{array}$$

3. Dr. Sullivan is above sea level, at an altitude that is equal to her total vertical distance traveled divided by 5. What object might Dr. Sullivan see when she looks around outside? She would be at 124,417 m and see a spacecraft;
   Sample response: 622,085 ÷ 5 = 124,417

$$\begin{array}{r} 124417 \\ 5\overline{)622085} \\ -5 \\ \hline 12 \\ -10 \\ \hline 22 \\ -20 \\ \hline 20 \\ -20 \\ \hline 08 \\ -5 \\ \hline 35 \\ -35 \\ \hline 0 \end{array}$$

STOP

Lesson 10 Using Long Division 567

3 Connect

Display the Activity 2 PDF for reference as students explain their solutions.

Have pairs of students share their responses.

Highlight that no matter how many digits a number has, the process of long division is the same, and produces a valid quotient, but in more steps.

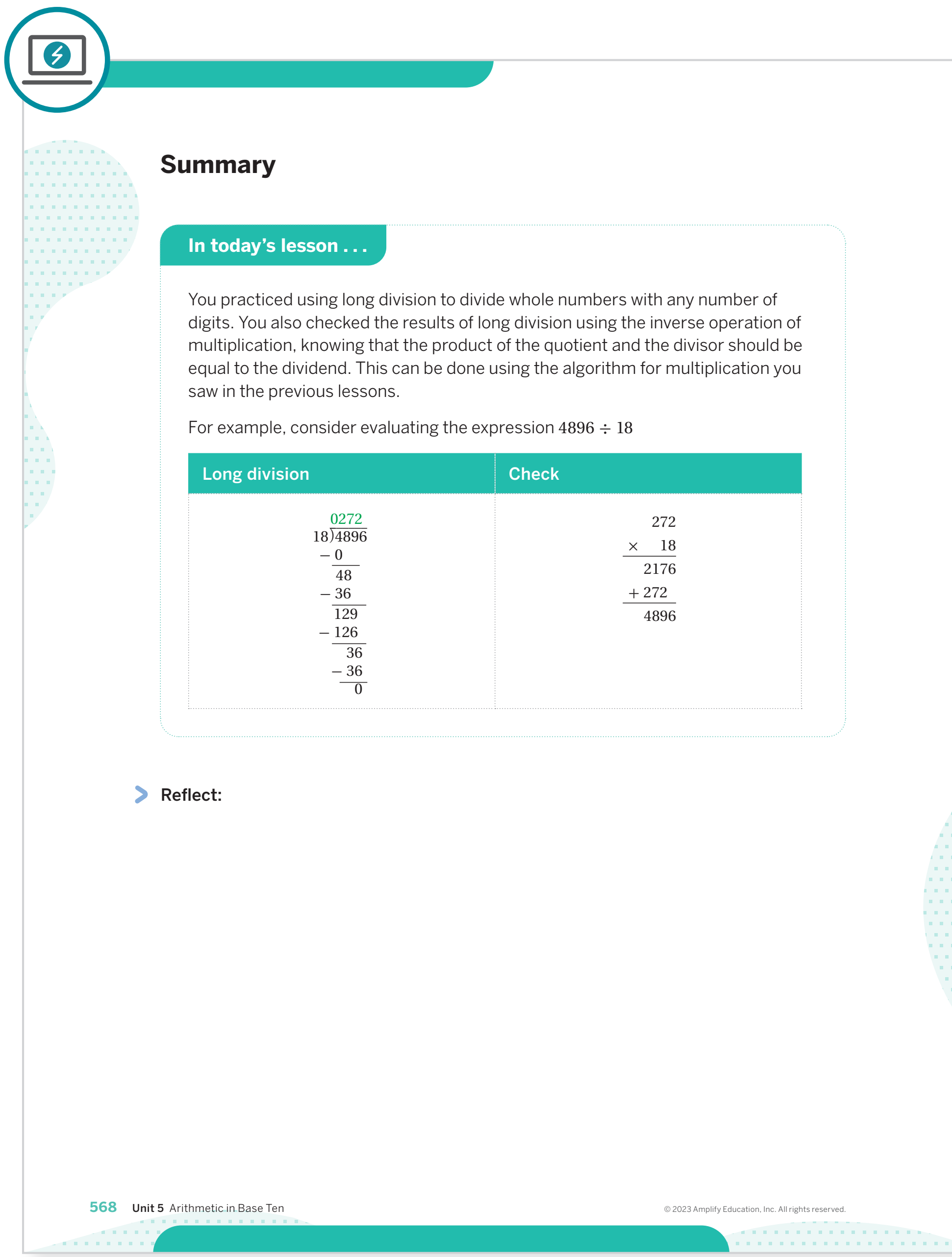
Ask:

- "What are some important things to remember when you are using long division?"
- "When going to outer space or down to the bottom of the ocean, is it important to know your precise location? Why or why not?"

Whole Class | 5 min

Summary

Review and synthesize the process of long division and how it can be used for any multi-digit whole numbers.



Summary

In today's lesson . . .

You practiced using long division to divide whole numbers with any number of digits. You also checked the results of long division using the inverse operation of multiplication, knowing that the product of the quotient and the divisor should be equal to the dividend. This can be done using the algorithm for multiplication you saw in the previous lessons.

For example, consider evaluating the expression $4896 \div 18$

Long division	Check
$\begin{array}{r} 0272 \\ 18\overline{)4896} \\ -0 \\ \hline 48 \\ -36 \\ \hline 129 \\ -126 \\ \hline 36 \\ -36 \\ \hline 0 \end{array}$	$\begin{array}{r} 272 \\ \times\ 18 \\ \hline 2176 \\ +\ 272 \\ \hline 4896 \end{array}$

Reflect:

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Synthesize

Highlight again that the process of long division requires working with one digit at a time from the dividend and proceeding from left to right. Each step should remove the largest group possible for that place value, and the placement of the resulting digit indicates the size of each group in the quotient.

Ask:

- "Which method for dividing relatively large whole numbers do you think is the most efficient?"
  It depends how large the numbers are, but drawing could take a long time and might be more likely to have unnoticed mistakes. Partial quotients could take up a lot of space if there are several steps. Long division might be the most efficient because you reason with one digit and one place value unit at a time, and it is compact to write.
- "How can you check that a quotient is correct?"
  Multiply the quotient by the divisor.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did you find challenging today? What did you do, or could you do, to overcome that challenge?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of using long division to determine quotients of multi-digit whole numbers.

Printable

Name: ____________________ Date: __________ Period: __________

Exit Ticket 5.10

Use long division to calculate $1{,}875 \div 15$. Use multiplication to check your response. Show your thinking.

$1{,}875 \div 12 = 125;$

$$\begin{array}{r} 125 \\ 15\overline{)1875} \\ -\,15 \\ \hline 37 \\ -\,30 \\ \hline 75 \\ -\,75 \\ \hline 0 \end{array} \qquad \begin{array}{r} 125 \\ \times\ \ 15 \\ \hline 625 \\ +\,1250 \\ \hline 1{,}875 \end{array}$$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use long division to determine a quotient of two whole numbers.

1 2 3

Success looks like . . .

- **Language Goal:** Recognizing and explaining that long division is an efficient strategy for dividing numbers, especially with multi-digit dividends. **(Speaking and Listening)**
- **Goal:** Using long division to divide whole numbers that result in a whole number quotient, and multiplying the quotient by the divisor to check the answer.
  - » Determining $1{,}875 \div 15$ by long division and checking the answer.

Suggested next steps

If students place the 1 in the quotient over the 1 of the dividend, consider:

- Asking "Did you divide the 1 by 15, or the 18 by 15, to get that 1 in the quotient?"

If students continue to misalign the digits, especially when subtracting and bringing down, consider:

- Having students use graph paper, or also have them describe why they placed each digit in its place, making the connection between the placement and the place value.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- What did students find frustrating about dividing with larger numbers? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

Practice

1. Determine each quotient. Use multiplication to check your response. Show your thinking.

   a. $3\overline{)1107}$
   369; Sample response:
   $3\overline{)1107}$ = 369; -9, 20, -18, 27, -27, 0
   $369 \times 3 = 1{,}107$

   b. $7\overline{)16198}$
   2,314; Sample response:
   $7\overline{)16198}$ = 2314; -14, 21, -21, 09, -7, 28, -28, 0
   $2314 \times 7 = 16{,}198$

2. Use the long division calculation of $917 \div 7$ to complete these problems.
   Sample responses shown.

   $7\overline{)917}$ = 131; -7, 21, -21, 07, -7, 0

   a. There is a 7 under the 9 of 917. What does this 7 represent?
   It represents 700. It is one group of 100, multiplied by 7.

   b. What does the subtraction of 7 from 9 mean?
   It means that the 700 is being subtracted from the 900.

   c. Why is 1 written next to the 2 from $9 - 7$?
   The one in the tens place of the dividend was brought down.

3. Han's calculation of $972 \div 9$ is shown here. His work is not correct.

   $9\overline{)972}$ = 180; -9, 72, -72, 0, -0, 0

   a. Identify and explain Han's mistake.
   Han should have divided 7 by 9 first before he brought down the 2 from the ones place.

   b. Determine the correct quotient. Show your thinking.
   108; Sample response:
   $9\overline{)972}$ = 108; -9, 07, -0, 72, -72, 0

Name: Date: Period:

Practice

4. One oz of yogurt contains 1.2 g of sugar. How many grams of sugar are in 14.25 oz of yogurt? Show your thinking.

   A. 0.171 g
   B. 1.71 g
   C. 17.1 g
   D. 171 g

   Sample response:
   14.25×1.2: 2850 + 1425 = 17,100

5. The mass of one coin is 16.718 g. The mass of a second coin is 27.22 g. How much greater is the mass of the second coin than the first? Show your thinking.
   10.502 g
   Sample response:
   $27.220 - 16.718 = 10.502$

6. Explain how to mentally calculate the quotient of $273 \div 3$, and then determine the quotient.
   Sample response: I can think of 273 as $270 + 3$, so $273 = (270 \div 3) + (3 \div 3)$. If $27 \div 3 = 9$, then $270 \div 3 = 90$ and $3 \div 3 = 1$. So, $273 \div 3 = 91$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 8	2
	5	Unit 5 Lesson 3	2
Formative	6	Unit 5 Lesson 11	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 5 | **LESSON 11**

Dividing Numbers That Result in Decimals

Let's determine quotients that are not whole numbers.

Focus

Goals

1. **Language Goal:** Interpret different methods for computing a quotient that is not a whole number, and express it in terms of "decomposing." **(Speaking and Listening, Writing)**

2. **Language Goal:** Use long division to divide whole numbers that result in a quotient with a decimal, and explain the solution method. **(Speaking and Listening)**

Rigor

- Students build **conceptual understanding** of how to express "remainders" as decimal quotients.
- Students build **fluency** using long division to divide two whole numbers, extending prior use cases to include cases that result in a decimal quotient.

Coherence

Today

Students extend their work with division to include cases in which dividing two whole numbers results in a decimal. They use and connect base ten diagrams and long division, relating the decomposition of ones into tenths with adding a decimal point and a zero to the whole-number dividend. Students apply this reasoning to calculate equivalent decimal forms of fractions, translating values between calculations and context, as they explore the salinity of different bodies of water on Earth.
Note: The expectation in this grade limits decimal quotients to terminating decimals only.

Previously

In Lessons 9 and 10, students reviewed division with base ten diagrams and partial quotients, and then they formalized the long division algorithm in order to fluently divide two multi-digit whole numbers without a remainder.

Coming Soon

In Lesson 12, students will use related expressions and long division to divide whole numbers by decimals and decimals by whole numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, two per pair
- Activity 2 PDF (instructions)
- base ten blocks, or copies of the *Decimal Base Ten Blocks* PDF (as needed)
- graph paper (as needed)
- Optional: 6 1-liter bottles, 7 cups of table salt, measuring cup, teaspoon, tablespoon, spoons (one per student)

Amps | **Featured Activity**

Activity 1
Digital Partners

Students work in pairs to solve a division problem using different strategies..

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not use effective communication methods when discussing division with their partners in Activity 2. Compare the need for precision in division to the need for precision in communication. Encourage students to use specific vocabulary words in their descriptions, as well as an appropriate speed and volume of speech.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, complete Problem 1 as a class. Then, have pairs of students complete Problem 2 together.
- In **Activity 2**, have pairs of students determine the salinity of only one location. Though a fun and engaging application of student computations, the taste test may also be omitted.

Independent | 5 min

Warm-up Number Talk

Students activate prior knowledge of division with whole numbers, using the structure and patterns of place value and partial quotients to evaluate a string of division expressions.

Name: ______________ Date: ________ Period: ________

Unit 5 | Lesson 11

Dividing Numbers That Result in Decimals

Let's determine quotients that are not whole numbers.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $400 \div 8 = 50$
   Sample response: I know that $40 \div 8 = 5$, and 400 is ten times 40, so the quotient will be ten times 5, or 50.

2. $80 \div 8 = 10$
   Sample response: I know that $10 \cdot 8 = 80$, so $80 \div 8 = 10$.

3. $16 \div 8 = 2$
   Sample response: I know that $8 \cdot 2 = 16$, so $16 \div 8 = 2$.

4. $496 \div 8 = 62$
   Sample response: Because $496 = 400 + 90 + 6$, or $400 + 80 + 16$, I can add the previous three quotients, $60 + 2 = 62$.

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use the ***Number Talk*** routine

2 Monitor

Help students get started by asking, "How are the digits 4 and 8 related? How can that help you?"

Look for points of confusion:

- **Not knowing how to evaluate $496 \div 8$ (Problem 4).** Ask, "How is 8 related to the divisors in Problems 1–3? How is 496 related to those dividends?"

Look for productive strategies:

- Using place value and powers of 10 to evaluate Problems 1 and 2 (e.g., because $40 \div 8 = 5$, then $400 \div 8 = 50$).
- Recognizing Problems 1–3 are the partial quotients of Problem 4, and using the structure and patterns of place value to evaluate.

3 Connect

Have students share their responses and strategies, allotting the most time to discuss Problem 4 and how it can be thought about using partial products.

Display a variety of student strategies for all to see for the expression in each problem.

Highlight that, when two or more quotients share the same divisor, relationships between the dividends can be used to help evaluate a new expression when the quotient of another related expression is known.

Ask:

- "If 8 is the divisor, what is another dividend for which you could determine the quotient?" Sample responses: 512, 576, 896
- "If 8 is the divisor and the dividend is a whole number, will the quotient always be a whole number? Why or why not?" No, because not all whole numbers are multiples of 8 and there can be a remainder.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses and how they mentally evaluated each expression, draw connections between the first three expressions and the fourth expression. Ask:

- "What do you notice about the dividends in each expression?"
- "Why do you think you can add the sums of the quotients for the first three expressions to obtain the quotient of the fourth expression?"

English Learners

Use gestures, such as pointing to each of the dividends in the first three expressions and then pointing to the dividend of the fourth expression, during the discussion.

Power-up

To power up students' ability to mentally evaluate the quotient of two whole number values, ask:

1. Choose a way to decompose 372 to make the mental division $372 \div 3$ more convenient.
   A. $372 = 300 + 70 + 2$
   B. $372 = 300 + 72$
   C. $372 = 300 + 60 + 12$
   D. $372 = 200 + 170 + 2$
2. Mentally evaluate $372 \div 3$. Show your thinking.
   $372 = 300 + 60 + 12$; $300 \div 3 = 100$, $60 \div 3 = 20$, and $12 \div 3 = 4$, therefore $372 \div 3 = 124$

Use: Before the Warm-up.
Informed by: Performance on Lesson 10, Practice Problem 6

Pairs | 15 min

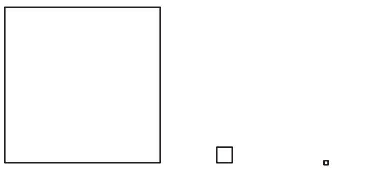
Activity 1 Keep Calm and Divide On

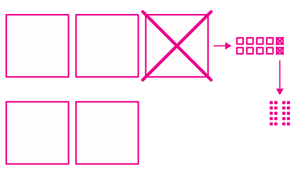
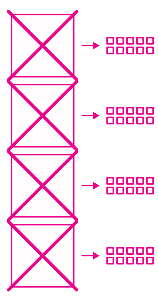
Students apply base ten and long division strategies from previous lessons to divide whole numbers that result in terminating decimal quotients.

Amps Featured Activity Digital Partners

Activity 1 Keep Calm and Divide On

1. Decide who will be Partner A and who will be Partner B. Show how you know that each equations in your assigned column is true. For part a, draw base ten diagrams. For part b, use long division. Then share and discuss your work with your partner.



Partner A	Partner B
a $5 \div 4 = 1.25$ 	a $4 \div 5 = 0.8$ 
b $4 \div 5 = 0.8$ $\begin{array}{r} 0.8 \\ 5\overline{)4.0} \\ -0 \\ \hline 4\,0 \\ -4\,0 \\ \hline 0 \end{array}$	b $5 \div 4 = 1.25$ $\begin{array}{r} 1.25 \\ 4\overline{)5.00} \\ -4 \\ \hline 1\,0 \\ -8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$

1 Launch

Activate prior knowledge by asking,

- "In previous lessons, what did you do with the remaining units when dividing?" I decomposed each into ten of the next smaller place value.
- "How do you think that would work for a remainder in the ones place?" Each one is decomposed into ten tenths.

Provide access to graph paper, as needed. Have pairs of students complete Problem 1, and then pause for a brief class discussion of how they represented the decomposition of ones into tenths by using both base ten diagrams and long division. Then have pairs complete Problem 2 together.

2 Monitor

Help students get started by asking, "How could you state your expression as a fair sharing problem? What kind of number would you expect the quotient to be?"

Look for points of confusion:

- **Struggling to divide when the divisor is greater than the dividend (e.g., $4 \div 5$ in Problem 1).** Ask, "Can you make 1 group of 5 out of 4? How can you use decomposition in this model to represent 5 with more than 5 parts that can be used to make equal groups?"
- **Not knowing how to represent a remainder as a decimal when using long division.** Ask, "What did you do with the remaining one(s) in your base ten diagram? How can you show that same decomposition in your long division?"

Look for productive strategies:

- Recognizing that, when decomposing ones into tenths, the quotient becomes a decimal, and applying their previous work with long division to represent this.
- Using the structure of the place value system to recognize that writing a decimal point and zeros to the right in the dividend does not change its value.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to physical base ten blocks or distribute copies of the *Decimal Base Ten Blocks* PDF for students to use if they choose to do so.

Accessibility: *Math Enrichment*

Ask students to use long division to evaluate each of the following to the thousandths place: $1 \div 9$, $2 \div 9$, $3 \div 9$, $4 \div 9$, $5 \div 9$, $6 \div 9$, $7 \div 9$, and $8 \div 9$. Have them describe what they notice. Sample response: Each quotient consists of the same digit as the dividend. For example, $7 \div 9 = 0.777$, to the nearest thousandth.

Math Language Development

MLR7: Compare and Connect

As partners share their work for Problem 1, ask them to compare the similarities and differences for their strategies. Display these questions for them to respond to before moving on to Problem 2.

- "For Problem 1a, why were all 4 ones blocks decomposed in the equation $4 \div 5 = 0.8$?"
- "For Problem 1a, how do you know that the quotient is a decimal for the equation $4 \div 5 = 0.8$ and not 8 ones?"

English Learners

Circle the five groups of 8 tenths in Problem 1a for the equation $4 \div 5 = 0.8$ and annotate each group as 0.8 or 8 tenths.

Pairs | 15 min

Activity 1 Keep Calm and Divide On (continued)

Students apply base ten and long division strategies from previous lessons to divide whole numbers that result in terminating decimal quotients.

Name: ______________________ Date: __________ Period: __________

Activity 1 Keep Calm and Divide On (continued)

2. Use long division to evaluate each expression. Show your thinking and express each quotient as a decimal.

a. $1 \div 8 = 0.125$

$$\begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ -0 \\ \hline 1\,0 \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

b. $1 \div 25 = 0.04$

$$\begin{array}{r} 0.04 \\ 25\overline{)1.00} \\ -0 \\ \hline 1\,0 \\ -0 \\ \hline 1\,00 \\ -1\,00 \\ \hline 0 \end{array}$$

Reflect: How well did you and your partner communicate? What did you do to build your relationship?

Are you ready for more?

Use long division to evaluate $1 \div 3$. What do you notice?

$1.3333\ldots$

Sample responses:

- No matter how many zeroes I add, such as writing 1.000, 1.00000, and so on, each step is a repeat of dividing 10 by 3 because there is always a remainder of 1.
- The equivalent fraction of $\frac{1}{3}$ does not have any other equivalent fractions with a denominator that is a power of 10, such as 100 or $1{,}000$.

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3 Connect

Display student work that correctly uses long division for Problem 2.

Have pairs of students share how their work compares with the displayed work, focusing on how they determined where to place zeroes in the dividends and divisors.

Ask:

- "How was long division in this activity similar to and different from your previous work with long division?" The steps were the same. I divided one digit at a time, from left to right, and then multiplied, subtracted, and brought the next digit down; I repeated this process. In this activity, there was still a remainder after subtracting from the ones place, so a decimal point and more place values (tenths, hundredths, etc.) had to be added as 0s. Then the process was similar again.
- "How was your work today related to equivalent fractions you learned in previous grades?" I could have thought about the division expressions as fractions (e.g., $4 \div 5 = \frac{4}{5}$).
- "How could you use equivalent fractions to help make sense of the decimals? You can make equivalent fractions with a denominator that is a power of 10. Then you can rewrite the equivalent fraction as a decimal.

Highlight that the steps in the long division of two whole numbers are the same, regardless of whether the quotient is a whole number or decimal, which may not be known ahead of time. The value of the dividend does not change when writing a decimal point and zeros to the right of the point.

Pairs | 15 min

Activity 2 Salinity Now

Students use long division to determine the salinity of different bodies of water, dividing larger whole number dividends and divisors that result in decimal quotients.

Activity 2 Salinity Now

Over 96% of the water on Earth is saltwater. The amount of salt, measured as the *salinity* of the water, changes some properties of the water — and it varies quite a lot. Two bodies of water known to have extremely high salinity are the Dead Sea in Asia and Don Juan Pond in Antarctica. It is easy to float in the Dead Sea, and yet it is nearly impossible to swim because of the high salt content. Due to its high salt content, Don Juan Pond rarely freezes — even at temperatures of 50°C below the normal freezing point of water!

Many factors affect the exact salinity of a body of water, so scientists often take samples to measure the number of grams of salt per liter of water at different locations and at different times or under different conditions. For example, the salinity of the Great Salt Lake in Utah can fluctuate from 50 g per liter to 250 g per liter depending on the season, recent weather, and the exact location or depth of where the sample was taken.

You will be given two cards that each shows an amount of salt per water measured in samples taken from various sources. Use long division to determine the salinity of your two water sources and record this information in the table. Show your thinking and express any remainders as decimals.

Water source	Salt (g)	Water (liters)	Salinity (g/liter)

Water source	Salt (g)	Water (liters)	Salinity (g/liter)
Human tears	18	3,000	0.006
Atlantic Ocean	1,593	45	35.4
Great Salt Lake	57,013	355	160.6
Dead Sea	2,532	8	316.5
Don Juan Pond	146,783	364	403.25
Gaet'ale Pond	2,186,751	5,050	433.02

STOP

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1 Launch

Give each pair of students two pre-cut cards from the Activity 2 PDF. Provide graph paper as needed. If time permits, consider concluding the activity with a taste test as outlined in the instructions of the Activity 2 PDF.

2 Monitor

Help students get started by asking, "What information do you know? What information do you need?"

Look for points of confusion:

- **Struggling to divide with 3-, 4-, and 5-digit divisors.** Moving digit by digit in the dividend, ask, "Does the divisor fit into this number at least one time? If so, how many times?" Have them check using multiplication.
- **Reversing the dividend and divisor.** Ask, "In what units is salinity expressed? Does your division expression match that?"

Look for productive strategies:

- Determining the salinity by dividing grams of salt by liters of water, and using the relative size of the divisor to determine at which place value in the dividend to start dividing.
- Using long division until there is no remainder, adding a decimal point and zeros as needed.

3 Connect

Have students share their responses and strategies, encouraging the use of precise mathematical language, such as *decompose*, to explain the procedural actions in long division.

Highlight how to check work by writing the remainder as a fraction and then an equivalent fraction, over a power of 10.

Ask, "Will using equivalent fractions to represent the remainder work for every problem?" It will not work when the denominator is not a factor of a power of 10.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see the immediate consequences of their solutions and determine how to fix any errors on their own. They can also visually observe how the salinity of their bodies of water compare to those of other groups.

Accessibility: *Vary Demands to Optimize Challenge*

For students who need more processing time, distribute the cards for the Dead Sea and the Atlantic Ocean. Suggest they begin by listing the first 5 multiples of the divisor prior to dividing.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they will be determining the salinity of various bodies of water. Clarify the meaning of the term *salinity* before moving on to the second read.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as "the salinity of the Great Salt Lake in Utah can fluctuate from 50 g per liter to 250 g per liter, depending on the season." Tell students that the word *fluctuate* means to *change*.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete the table.

Summary

Review and synthesize how to use long division to divide two multi-digit whole numbers when the quotient is a decimal.

Name: ____________ Date: ________ Period: ________

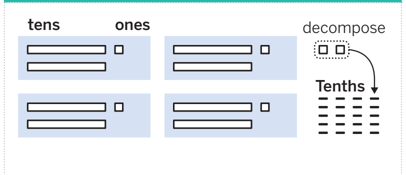
Summary

In today's lesson . . .

You used base ten diagrams and long division to divide two whole numbers whose quotient is not a whole number.

For example, to evaluate $86 \div 4$, you can think of dividing 86 into 4 equal groups. Whether you are using a base ten diagram or long division, the thought process is the same: there are 4 groups of 21 with 2 ones left over. The ones can be decomposed into 20 tenths and then distributed evenly to each of the 4 groups. Each group will have 2 tens, 1 one, and 5 tenths, so $86 \div 4 = 21.5$.

Base ten diagram



Long division

$$\begin{array}{r} 21.5 \\ 4\overline{)86.0} \\ -8 \\ \hline 6 \\ -4 \\ \hline 2\,0 \\ -2\,0 \\ \hline 0 \end{array}$$

In the long division strategy, the remaining 2 ones are decomposed into 20 tenths by extending the dividend of 86 to an equivalent form of 86.0, which allows you to bring a 0 down to the right of the remainder of 2 (ones). That also means a decimal point is added to the right of the 1 (which is in the ones place) in the quotient, which shows that the resulting 5 is in the tenths place.

Reflect:

Synthesize

Highlight how the two representations — base ten diagrams and long division — show the same mathematical thinking in different ways. When dividing two whole numbers, it will generally not be known whether the quotient will be a decimal or not, but that does not change the process in any of the general methods. Consider also mentioning that, in certain contexts where a particular level of precision is needed (or is all that is needed), students may stop the long division process once the quotient has reached the designated place value (or the one beyond in order to round), even if there is still a remainder.

Ask, "How is long division different when the dividend is less than the divisor (e.g., $4 \div 5$)?" The first digit in the quotient will be 0 because there are not enough ones to equally divide. Then, you decompose the dividend into ten of the next smaller unit. This will be represented by writing a decimal point in the dividend and quotient, writing a 0 on the end of the quotient, and continuing to divide.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did today's lesson build upon your previous understanding of long division? Of remainders?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using long division to divide two whole numbers, resulting in a decimal quotient.

Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 5.11

Use long division to determine each quotient. Show your thinking, and write each quotient as a decimal.

1. $22 \div 5 = 4.4$

$$\begin{array}{r} 4.4 \\ 5\overline{)22.0} \\ -20 \\ \hline 2\,0 \\ -2\,0 \\ \hline 0 \end{array}$$

2. $7 \div 8 = 0.875$

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ -0 \\ \hline 7\,0 \\ -6\,4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use long division to determine the quotient of two whole numbers when the quotient is not a whole number.

1 2 3

Success looks like . . .

- **Language Goal:** Interpreting different methods for computing a quotient that is not a whole number, and expressing it in terms of "decomposing." **(Speaking and Listening, Writing)**
- **Language Goal:** Using long division to divide whole numbers that result in a quotient with a decimal, and explaining the solution method. **(Speaking and Listening)**
  - » Dividing whole numbers that result in a decimal quotient in Problems 1 and 2.

Suggested next steps

If students are struggling to write the remainder as a decimal (Problem 1) or to divide when the divisor is greater than the dividend (Problem 2), consider:

- Reviewing their work from Problem 1 in Activity 1. If they are struggling with Problem 1 on the Exit Ticket, have them review their work for $5 \div 4$. If they are struggling with Problem 2, have them review their work for $4 \div 5$.
- Having students draw base ten diagrams first, and then represent their work with long division. For Problem 1, ask, "What did you do with the remaining ones in the diagram? How can you show that same decomposition in the long division?" For Problem 2, ask, "Can you make a full group of 8? How did you represent this in your diagram? How can you represent that in the long division?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did today's work with long division and decimal quotients build on student work in Lessons 9 and 10?
- In what ways did Activity 1 go as planned? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Use long division to show that the fraction and decimal in each pair are equal.

a. $\frac{3}{4}$ and 0.75

$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ -0 \\ \hline 3\,0 \\ -2\,8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

b. $\frac{3}{50}$ and 0.06

$$\begin{array}{r} 0.06 \\ 50\overline{)3.00} \\ -0 \\ \hline 3\,0 \\ -0\,0 \\ \hline 3\,00 \\ -3\,00 \\ \hline 0 \end{array}$$

c. $\frac{7}{25}$ and 0.28

$$\begin{array}{r} 0.28 \\ 25\overline{)7.00} \\ -0 \\ \hline 7\,0 \\ -5\,0 \\ \hline 2\,00 \\ -2\,00 \\ \hline 0 \end{array}$$

2. Use long division to determine each quotient. Write the quotient as a decimal.

a. $99 \div 12 = 8.25$

$$\begin{array}{r} 8.25 \\ 12\overline{)99.00} \\ -96 \\ \hline 3\,0 \\ -2\,4 \\ \hline 60 \\ -60 \\ \hline 0 \end{array}$$

b. $216 \div 5 = 43.2$

$$\begin{array}{r} 43.2 \\ 5\overline{)216.0} \\ -20 \\ \hline 16 \\ -15 \\ \hline 1\,0 \\ -1\,0 \\ \hline 0 \end{array}$$

c. $1988 \div 8 = 248.5$

$$\begin{array}{r} 248.5 \\ 8\overline{)1988.0} \\ -16 \\ \hline 38 \\ -32 \\ \hline 68 \\ -64 \\ \hline 4\,0 \\ -4\,0 \\ \hline 0 \end{array}$$

Name: ______ Date: ______ Period: ______

Practice

3. Tyler reasoned that $\frac{9}{25}$ is equivalent to $\frac{18}{50}$ and to $\frac{36}{100}$, so the decimal of $\frac{9}{25}$ is 0.36.

a. Use long division to show that Tyler is correct.

$$\begin{array}{r} 0.36 \\ 25\overline{)9.00} \\ -0 \\ \hline 9\,0 \\ -7\,5 \\ \hline 1\,50 \\ -1\,50 \\ \hline 0 \end{array}$$

b. Use long division to determine whether the decimal of $\frac{18}{50}$ is also 0.36.

$$\begin{array}{r} 0.36 \\ 50\overline{)18.00} \\ -0 \\ \hline 18\,0 \\ -15\,0 \\ \hline 3\,00 \\ -3\,00 \\ \hline 0 \end{array}$$

4. Complete each calculation so that the correct difference is shown.

a.
$$\begin{array}{r} 5 \\ -\,\boxed{0.6}\boxed{7}\boxed{1} \\ \hline 4.3\;2\;9 \end{array}$$

b.
$$\begin{array}{r} 1 \\ -\,\boxed{0.9}\boxed{8}\boxed{5} \\ \hline 0.0\;1\;5 \end{array}$$

c.
$$\begin{array}{r} 1 \\ -\,\boxed{0.1}\boxed{3}\boxed{7} \\ \hline 0.8\;6\;3 \end{array}$$

5. Use the equation $124 \cdot 15 = 1{,}860$ and what you know about fractions, decimals, and place value to explain where to place the decimal point as you compute $1.24 \cdot 0.15$.

0.186; Sample response: Because 1.24 is 100 times less than 124, and 0.15 is 100 times less than 15, the product of $1.24 \cdot 0.15$ will be 10,000 times less than 1,860. I can divide 1,860 by 10,000 by moving the decimal point 4 places to the left, which is the same as moving each digit 4 places to the right.

6. Explain how to mentally calculate $0.8 \div 4$.

0.2

Sample responses:

- **I know that $8 \div 4 = 2$, and 0.8 is ten times less than 8, so the quotient of $0.8 \div 4$ will be ten times less than 2.**
- **Because $0.8 \div 4$ means 0.8 divided into 4 equal groups, I can put 0.2 in each of the four groups.**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 5 Lesson 4	2
	5	Unit 5 Lesson 6	2
Formative	6	Unit 5 Lesson 12	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 5 | **LESSON 12**

Using Related Expressions to Divide With Decimals

Let's use related expressions to divide with decimals.

Focus

Goals

1. **Language Goal:** Compare and contrast a process for dividing with whole number versus decimal dividends. **(Speaking and Listening)**
2. **Language Goal:** Use related expressions and long division to divide decimals and whole numbers, and explain the strategy. **(Speaking and Listening, Writing)**
3. **Language Goal:** Generalize that multiplying both the dividend and the divisor by the same factor does not change the quotient. **(Speaking and Listening, Writing)**

Coherence

Today

Students use related expressions to divide whole numbers and decimals. They discover that, just as with multiplication (from Lesson 5), they can use the associative and commutative properties to write and evaluate a related expression involving only whole numbers. Students then build upon their work with equivalent ratios from Units 2 and 3 to recognize that the quotient of two division expressions is the same if both the dividend and divisor of one expression are multiplied (or divided) by the same number, which will likely be a power of 10. They use this reasoning to analyze errors in long division with decimal divisors and then correct those errors to evaluate the quotient. Throughout this lesson, students recognize and reiterate that the steps in long division are the same, whether they are dividing two whole numbers or a whole number and a decimal.

Previously

In Lesson 11, students used long division to divide two whole numbers whose quotient was a decimal.

Coming Soon

In Lesson 13, students will generalize a rule for using long division to divide any two decimals.

Rigor

- Students extend previous work with related expressions, equivalent ratios, and equivalent fractions to build **conceptual understanding** of how to divide with decimals.
- Students build **fluency** with the long division algorithm.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

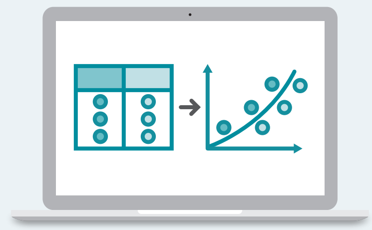
Materials

- Exit Ticket
- Additional Practice

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to show and explain their thinking behind their decimal division, and these explanations are available to you digitally, in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not see the benefits of working to determine structure with a partner in Activity 1 or Activity 2. Prior to the start of the lesson, have each student identify three of their mathematical strengths and three places where they can use help. As pairs align, have them compare their strengths and weaknesses to specify each person's unique ability to contribute to problem solving.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, Problems 1 and 2 may be omitted. You may also choose to model the long division for Problem 3 and discuss how it is similar to and different from performing long division with whole numbers.

Independent | 5 min

Warm-up Number Talk

Students use the structure of base ten numbers and the Distributive Property to mentally evaluate division problems involving decimals.

Unit 5 | Lesson 12

Using Related Expressions to Divide With Decimals

Let's use related expressions to divide with decimals.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $80 \div 4 = 20$
   Sample response: If $8 \div 4 = 2$, then $80 \div 4 = 20$.

2. $12 \div 4 = 3$
   Sample response: I know that $3 \cdot 4 = 12$, so $12 \div 4 = 3$.

3. $1.2 \div 4 = 0.3$
   Sample responses:
   - If $12 \div 4 = 3$, then $1.2 \div 4 = 0.3$.
   - I thought of $1.2 \div 4$ as $12 \cdot \frac{1}{10} \div 4$, which is equal to $12 \cdot \frac{1}{10} \cdot \frac{1}{4}$. Then, because of the commutative property, I can see that is the same as $12 \cdot \frac{1}{4} \cdot \frac{1}{10}$ which is equal to $12 \div 4 \cdot \frac{1}{10}$. I evaluate the division first to get $3 \cdot \frac{1}{10} = 0.3$.

4. $81.2 \div 4 = 20.3$
   Sample response: Because $81.2 = 80 + 1.2$, I thought of the expression as $(80 \div 4) + (1.2 \div 4)$. I evaluated these expressions in Problems 1 and 3, so I added their quotients together.

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use the ***Number Talk*** routine.

2 Monitor

Help students get started by asking, "How are the digits 4 and 8 related? How can that help you?"

Look for points of confusion:

- **Struggling to mentally divide $1.2 \div 4$.** Ask, "How are the expressions in Problems 2 and 3 related? How can you use your thinking and result from Problem 2 to help here?"
- **Not knowing how to evaluate $18.2 \div 4$ (Problem 4).** Ask, "How is the divisor related to the divisors in the other problems? How is the dividend related to the dividends in the other problems?"

Look for productive strategies:

- Using place value to evaluate Problems 2 and 3 (e.g., because 1.2 is one-tenth of 12, the product of $1.2 \div 4$ will be one-tenth the product of $12 \div 4$).
- Recognizing that Problems 1 and 3 are the partial quotients of Problem 4. If students start from scratch when evaluating Problem 4, ask them how they could use what they have already done to help them evaluate.

3 Connect

Have students share their responses and strategies, focusing on how they used partial products to evaluate the fourth expression.

Display student strategies for all to see.

Ask, "How does the expression $81.2 \cdot \frac{1}{4}$ also represent Problem 4? How could you use the Distributive Property to evaluate that expression?"

Highlight how student responses used the relationship between dividing by a whole number and multiplying by a unit fraction, as well as the Distributive Property, when thinking of $81.2 \div 4$ as $(80 \div 4) + (1.2 \div 4)$.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw connections between the first three expressions and the fourth expression. Ask:

- "What do you notice about the dividends in each expression?"
- "How did determining the quotient in the second expression help you determine the quotient in the third expression?"
- "Why can you add the quotients of the first and third expressions to determine the quotient of the fourth expression?"

English Learners

Provide sentence frames to support students as they explain their strategies. For example, "I noticed that ______, so I . . ." or "First, I ______ because . . ."

Power-up

To power up students' ability to determine the quotient of decimals, ask:

1. Find the quotient: $54 \div 9 = 6$
2. Use the previous result to evaluate the following expressions:
   - $5.4 \div 9 = 0.6$
   - $0.54 \div 9 = 0.06$
   - $5400 \div 9 = 600$

Use: Before the Warm-up.

Informed by: Performance on Lesson 11, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 15 min

Activity 1 Using Properties to Divide Decimals

Students recall the associative and commutative properties from Lessons 5 and 6, connecting their work to long division when dividing decimals by whole numbers.

Amps Featured Activity See Student Thinking

Name: ____________ Date: ________ Period: ________

Activity 1 Using Properties to Divide Decimals

Peggy Whitson, an American astronaut from Iowa, has held as many as five space-related Guinness World Records, four of which still stood as of 2020: "First woman to command the International Space Station," "Most spacewalks by a female," "Longest accumulated time on spacewalks by a female," and "Oldest female astronaut in space." Until December 2019, she also held the record for the "Longest continuous time in space by a female," spending about 289.2 straight days outside of Earth's atmosphere.

While many would say not a single minute in space feels like work, and, technically, there are regulations on official working hours, you can imagine that astronauts spend most of their time in space working. To put Whitson's accomplishment in perspective, consider the typical work week on Earth of 5 days, and think about how many "never-ending" work weeks she spent in space.

1. Explain how the expression $\left(2{,}892 \cdot \frac{1}{10}\right) \div 5$ represents the number of work weeks Whitson spent in space.

   Sample response: To determine how many work weeks Whitson spent in space, you need to divide $289.2 \div 5$. This can be written as the expression $\left(2{,}892 \cdot \frac{1}{10}\right) \div 5$ because $289.2 = 2{,}892 \cdot \frac{1}{10}$.

2. Show or explain how to evaluate $\left(2{,}892 \cdot \frac{1}{10}\right) \div 5$.

   Sample response: Based on the relationship between dividing by a whole number and multiplying by a unit fraction and the commutative property, I can divide the whole numbers first and then multiply the quotient by $\frac{1}{10}$. Multiplying by one tenth is the same as dividing by 10 or moving the decimal point in the quotient one place to the left.

$$\begin{array}{r} 578.4 \\ 5\overline{)2892.0} \\ -25 \\ \hline 39 \\ -35 \\ \hline 42 \\ -40 \\ \hline 2\,0 \\ -2\,0 \\ \hline 0 \end{array}$$

$$578.4 \cdot \frac{1}{10} = 57.84$$

3. Use what you know about long division to evaluate $289.2 \div 5$ and determine how many work weeks Whitson was in space.

   57.84 work weeks

   Sample response:

$$\begin{array}{r} 57.84 \\ 5\overline{)289.20} \\ -25 \\ \hline 39 \\ -35 \\ \hline 4\,2 \\ -4\,0 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

1 Launch

Read the introduction together as a class.

2 Monitor

Help students get started by asking, "How is the expression in Problem 1 similar to what you saw when multiplying decimals?"

Look for points of confusion:

- **Using order of operations and struggling to evaluate.** Ask, "How can you use the commutative property to rearrange the values and make this easier to evaluate?"
- **Leaving a remainder in the quotient.** Ask, "How did you handle remainders in Lesson 11?"
- **Ignoring or misplacing the decimal point in the quotient.** Ask, "What value does the 2 represent? What does that tell you about the quotient?"

Look for productive strategies:

- Determining and evaluating related expressions using properties of operations and relationships between specific numbers and operations.
- Using long division to divide until there is no remainder.
- Coordinating multiplication and division by powers of 10 to locate the decimal point in the quotient.

3 Connect

Have students share their responses and strategies for Problems 1 and 2, focusing on those who reasoned using the associative and commutative properties.

Display the solutions for Problems 2 and 3.

Ask:

- "How is the long division in Problems 2 and 3 similar? How is it different?"
- "How did you know where to locate the decimal point?"

Highlight that, whether using a related expression or a decimal dividend, the steps to divide by long division are the same as when dividing two whole numbers.

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Display the number of straight days Peggy Whitson spent outside of Earth's atmosphere, 289.2 next to the expression $\left(2{,}892 \cdot \frac{1}{10}\right) \div 5$ in Problem 1. Ask:

- "Where does the number 5 come from?" There are 5 days in a typical work week.
- "How is 289.2 related to $\left(2{,}892 \cdot \frac{1}{10}\right)$?" Sample response: 2,892 divided by 10 is 289.2. Dividing by 10 is the same as multiplying by $\frac{1}{10}$.

Pairs | 15 min

Activity 2 Related Expressions and Decimal Division

Students reason about related division expressions involving decimal divisors, and they use this reasoning to evaluate errors, preparing them to divide a decimal by a decimal.

Activity 2 Related Expressions and Decimal Division

1. Without evaluating, explain why $100 \div 5$, $10 \div 0.5$, and $1 \div 0.05$ have the same quotient.

   Sample response: The dividends and divisors have been multiplied or divided by the same power of 10. For example, both the dividend and divisor in $100 \div 5$ are ten times those in $10 \div 0.5$ and 100 times those in $1 \div 0.05$.

2. Calculations A–D all show incorrect attempts of using related expressions and long division to determine the quotient of $64 \div 1.6$. What mistake do you see in each calculation? Explain your thinking.

Calculation A	Calculation B	Calculation C	Calculation D
$\begin{array}{r} 0.4 \\ 1.6\overline{)64} \\ -\ 0 \\ \hline 64 \\ -\ 64 \\ \hline 0 \end{array}$	$\begin{array}{r} 0.4 \\ 16\overline{)64.0} \\ -\ 64 \\ \hline 0\ 0 \\ -\ 0 \\ \hline 0 \end{array}$	$\begin{array}{r} 0.4 \\ 16\overline{)640} \\ -\ 64 \\ \hline 00 \\ -\ 0 \\ \hline 0 \end{array}$	$\begin{array}{r} 0.4 \\ 16\overline{)6.4} \\ -\ 0 \\ \hline 6\ 4 \\ -\ 6\ 4 \\ \hline 0 \end{array}$

Sample response:

- Calculation A ignores the decimal point in 1.6 and divides 64 by 16 to get 4. The decimal point in 4 is moved one place to the left to account for the decimal point in 1.6.
- Calculation B divides $64 \div 16$, which is not a related expression because only the divisor was multiplied by 10.
- Calculation C shows a correct related expression where both the dividend and divisor are multiplied by the same power of 10, but the decimal is incorrectly placed. It should be placed where the decimal point *was* in 64, not where it is shown in 640.
- Calculation D divides $6.4 \div 16$, which is not a related expression because the dividend was multiplied by 10, but the divisor was divided by 10. Both values should have been multiplied by 10 or divided by 10 to make a related expression.

1 Launch

Have students use the ***Think-Pair-Share*** routine for Problem 1. Give them 2 minutes to work independently before sharing with a partner. Then pause for a brief class discussion on how the expressions are related because the dividends and divisors were both multiplied or divided by the same factor. Have pairs complete Problems 2 and 3 together.

2 Monitor

Help students get started by asking, "What do you notice about the dividends in all of the expressions? The divisors?"

Look for points of confusion:

- **Not recognizing the mistakes in Problem 2.** Ask, "What expression does each calculation represent? Is each expression related to $64 \div 1.6$?" For Calculations A and C, ask, "Does the quotient make sense? What mistake must have been made?"
- **Writing an incorrect related expression (Problem 3).** Refer to Problem 1. Ask, "Why are these expressions and their quotients related? How can you use that same thinking to write a related expression here?"
- **Inserting a decimal point in the quotient (Problem 3).** Ask, "What should be true about the quotients of two related expressions? So what should be the quotient of $64 \div 1.6$?"

Look for productive strategies:

- Applying reasoning from work in previous units with equivalent ratios and related expressions to reason that quotients are the same when the dividends and divisors are multiplied or divided by the same factor (e.g., the same power of 10).
- Connecting place value errors in Calculations A and C to the misplacement of the decimal point in the quotients.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use their work from previous slides to build on their analysis to use long division to divide with a decimal dividend.

Extension: *Math Enrichment*

Challenge students to use what they learned in this activity to determine the quotient of $5.2 \div 0.00013$. Sample response: Multiply both 5.2 and 0.00013 by 100,000, which results in $520{,}000 \div 13$. Because $52 \div 13 = 4$, the quotient is 40,000.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present students with incorrect reasoning, such as "Calculation C is correct because $\frac{640}{16}$ and $\frac{64}{1.6}$ are related expressions." Ask:

- ***Critique and Correct:*** "Do you agree or disagree with this statement? How would you correct this statement?" Listen for explanations that use place value to correct the placement of the decimal point.
- ***Clarify:*** "What mathematical language or reasoning can you use to explain why and how you corrected the statement?"

English Learners

Encourage students to use language from the class display to support developing mathematical language in this unit.

Pairs | 15 min

Activity 2 Related Expressions and Decimal Division (continued)

Students reason about related division expressions involving decimal divisors, and they use this reasoning to evaluate errors, preparing them to divide a decimal by a decimal.

Name: ______ Date: ______ Period: ______

Activity 2 Related Expressions and Decimal Division (continued)

3. Use what you know about related expressions and long division to determine the quotient of $64 \div 1.6$.

   40

   $64 \div 1.6 = 640 \div 16$

$$\begin{array}{r} 40 \\ 16\overline{)640} \\ -64\downarrow \\ \hline 00 \\ -0 \\ \hline 0 \end{array}$$

Are you ready for more?

Using the digits 0–9, write a division expression that uses each digit once and has both numbers as decimals. Then evaluate it.

Answers may vary, but responses that represent higher-order thinking, number sense, and a strong grasp of decimal division are: $2,345,678.9 \div 0.1 = 23,456,789$ or $1,345,679.8 \div 2.0 = 6,728,399$.

STOP

3 Connect

Have students share their responses and strategies for Problems 2 and 3, focusing on how they reasoned about related expressions, place value, and the location of the decimal point in the divisor, dividend, and quotients.

Ask:

- "Besides $640 \div 16$, what are some other related expressions for $64 \div 1.6$?" Sample responses: $6.4 \div 0.16$, $0.64 \div 0.016$, and $6400 \div 160$.
- "Which of those related expressions might help you evaluate $64 \div 1.6$? Any of the expressions where the divisor is a whole number. The dividend does not need to be a whole number.

Display a collected list of related expressions for $64 \div 1.6$, and leave the list displayed for the Summary.

Highlight that, when dividing with a decimal divisor, it is efficient to write a related expression in which the divisor is a whole number. As long as the dividend and divisor are multiplied by the same power of 10, the value of the quotient will not change. From there, the steps for long division are the same as with dividing two whole numbers.

Whole Class | 5 min

Summary

Review and synthesize how related expressions can help evaluate division expressions involving decimals.

Summary

In today's lesson . . .

You used related expressions and long division to reason about division with decimals. You know that fractions such as $\frac{6}{4}$ and $\frac{60}{40}$ are equivalent because of the following.

- The numerator and denominator of $\frac{60}{40}$ are each 10 times those of $\frac{6}{4}$.
- Both fractions can be simplified to $\frac{3}{2}$.
- 60 divided by 40 is 1.5, and 6 divided by 4 is also 1.5.

That means that, as with fractions, division expressions can be equivalent, which can also be called related expressions. For example, the expressions $540 \div 90$ and $5.4 \div 0.9$ are both equivalent to $54 \div 9$ because of the following.

- They all have a quotient of 6.
- The dividend and the divisor in $540 \div 90$ are each 10 times the dividend and divisor in $54 \div 9$. Those in $5.4 \div 0.9$ are each $\frac{1}{10}$ of the dividend and divisor in $54 \div 9$. In both cases, the quotient does not change.

In general, multiplying a dividend and a divisor by the same number does not change the quotient. Multiplying by powers of 10 (e.g., 10, 100, $1{,}000$, etc.) can be particularly useful for dividing decimals, as you will see in an upcoming lesson.

Reflect:

Synthesize

Display the list of related expressions for $64 \div 1.6$ that was generated by students during the discussion at the end of Activity 2.

Ask, "How was your thinking in coming up with these related expressions similar to previous work with equivalent ratios? Equivalent fractions?" Sample response: In a related expression, the dividend and divisor are multiplied or divided by the same factor. For two ratios to be equivalent, both quantities in the relationship must be multiplied or divided by the same factor. In equivalent fractions, the numerator and denominator of one fraction can be multiplied or divided by the same factor to get the other fraction.

Highlight that, as with equivalent ratios and fractions, related expressions have the same quotient because the dividend and divisor are multiplied or divided by the same factor. In fact, the dividend and divisor in every related expression can be used to write a set of equivalent ratios, or equivalent fractions (for those that are both whole numbers). Also, multiplying or dividing by the same powers of 10 can be particularly useful for dividing decimals, which are base ten numbers.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What is similar about using related whole number expressions when multiplying versus dividing decimals? What is different?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using long division to divide a decimal by a whole number.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 5.12

Use long division to determine the quotient of $43.5 \div 3$. Show your thinking.

14.5

Sample responses:

$$
\begin{array}{r}
14.5 \\
3\overline{)43.5} \\
-3 \\
\hline
13 \\
-12 \\
\hline
1\,5 \\
-1\,5 \\
\hline
0
\end{array}
$$

$$43.5 \div 3 = 435 \cdot \frac{1}{10} \div 3 = 435 \div 3 \cdot \frac{1}{10}$$

$$
\begin{array}{r}
145 \\
3\overline{)435} \\
-3 \\
\hline
13 \\
-12 \\
\hline
15 \\
-15 \\
\hline
0
\end{array}
$$

$$145 \cdot \frac{1}{10} = 145 \div 10 = 14.5$$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use related expressions and long division to divide a decimal by a whole number and a whole number by a decimal.
1 2 3

b. I understand how multiplying both the dividend and the divisor by the same power of 10 affects the quotient.
1 2 3

Success looks like . . .

- **Language Goal:** Comparing and contrasting a process for dividing with whole number versus decimal dividends. **(Speaking and Listening)**
- **Language Goal:** Using related expressions and long division to divide decimals and whole numbers, and explaining the strategy. **(Speaking and Listening, Writing)**
  - » Dividing $43.5 \div 3$ by calculating $435 \div 3 \cdot \frac{1}{10}$.
- **Language Goal:** Generalizing that multiplying both the dividend and the divisor by the same power of 10 does not change the quotient. **(Speaking and Listening, Writing)**

Suggested next steps

If students forget or misplace the decimal point in the quotient, consider:

- Reviewing Activity 1, Problems 2 and 3, and then asking:
  - » "How was a related expression used in Problem 2? How could you use that here?"
  - » "What was done with the decimal point in Problem 3? How could you use that here?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to divide decimals and whole numbers using related expressions and long division. How well did students accomplish this?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Consider these four expressions:

$4.5 \div 0.09$ $45 \div 0.9$ $450 \div 9$ $4{,}500 \div 90$

a. Without evaluating, explain how you know all four expressions have the same value.

Sample response: All of the expressions are related because the dividends and divisors were multiplied or divided by the same power of 10. Moving from left to right, the dividends and divisors are each multiplied by 10 to get the dividends and divisors in the next expression. Moving right to left, the dividends and divisors are divided by ten or multiplied by one tenth.

b. What is the quotient of $4.5 \div 0.09$?

50

2. Use long division to determine each quotient. Show your thinking.

a. $7.89 \div 2 = \mathbf{3.945}$

$$\begin{array}{r} 3.945 \\ 2\overline{)7.890} \\ -6 \\ \hline 1\,8 \\ -1\,8 \\ \hline 09 \\ -8 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

b. $39.54 \div 3 = \mathbf{13.18}$

$$\begin{array}{r} 13.18 \\ 3\overline{)39.54} \\ -3 \\ \hline 09 \\ -9 \\ \hline 0\,5 \\ -3 \\ \hline 24 \\ -24 \\ \hline 0 \end{array}$$

c. $176 \div 0.5 = \mathbf{352}$

$\mathbf{176 \div 0.5 = 1760 \div 5}$

$$\begin{array}{r} 352 \\ 5\overline{)1760} \\ -15 \\ \hline 26 \\ -25 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

3. Kiran, Mai, Noah, and Clare set up a lemonade stand. At the end of one day they had made a profit of $17.52. If the profit is divided equally, how much money would each student receive? Show or explain your thinking.

$4.38

Sample response:

$$\begin{array}{r} 4.38 \\ 4\overline{)17.52} \\ -16 \\ \hline 1\,5 \\ -1\,2 \\ \hline 32 \\ -32 \\ \hline 0 \end{array}$$

Name: ______ Date: ______ Period: ______

Practice

4. At Bard's school, 460 of the students walk to school.

a. The number of students who take public transit is 20% of the number of students who walk. How many students take public transit?

92 students; $460 \bullet \frac{1}{5} = 92$

b. The number of students who bike to school is 5% of the number of students who walk. How many students bike to school?

23 students; $460 \bullet \frac{5}{100} = 23$

c. The number of students who ride the school bus is 110% of the number of students who walk. How many students ride the school bus?

506 students; $460 \bullet \frac{110}{100} = 506$

5. Calculate each difference. Show your thinking.

a. $13.2 - 1.78 = \mathbf{11.42}$

$$\begin{array}{r} 13.20 \\ -\ 1.78 \\ \hline 11.42 \end{array}$$

b. $23.11 - 0.376 = \mathbf{22.734}$

$$\begin{array}{r} 23.110 \\ -\ 0.376 \\ \hline 22.734 \end{array}$$

c. $0.9 - 0.245 = \mathbf{0.655}$

$$\begin{array}{r} 0.900 \\ -\ 0.245 \\ \hline 0.655 \end{array}$$

6. Explain how to mentally calculate $720{,}000 \div 9{,}000$.

Sample response: I can think of a related expression that has fewer zeros. To do this, I can divide the dividend and divisor by the same power of 10. If I divide the dividend and divisor each by 1,000, I can then evaluate $720 \div 9$, which has the same quotient, and that is 80.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1 Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 13	2
	5	Unit 5 Lesson 4	2
Formative	6	Unit 5 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 5 | **LESSON 13**

Dividing Multi-digit Decimals

Let's divide any decimal by any other decimal.

Focus

Goals

1. **Language Goal:** Compare and contrast division problems with whole number divisors and problems with decimal divisors. **(Speaking and Listening, Writing)**
2. **Language Goal:** Generate another division expression that has the same value as a given expression, and justify they are equal. **(Speaking and Listening, Writing)**
3. **Language Goal:** Divide whole numbers or decimals by decimals, and explain and generalize the reasoning, including choosing to divide a different expression that has the same quotient. **(Speaking and Listening, Writing)**

Coherence

Today

Students generalize that to divide a number by a decimal, they can multiply both the dividend and divisor by the same power of 10 so that the divisor is a whole number and then use a known algorithm. Using the ***Stronger and Clearer Each Time*** routine, they generalize this relationship and process as a rule for dividing with decimals. Students then practice applying it by dividing decimals in both abstract and contextual problems.

Previously

In Lesson 12, students used related expressions to divide with whole numbers and decimals. They saw that multiplying both the dividend and the divisor by the same power of 10 does not change the quotient.

Coming Soon

In Lesson 14, students will apply their work with decimal operations throughout the unit to explore a real-world legend. In Unit 6, students will work extensively with algebraic equations in which variables can take on decimal values.

Rigor

- Students co-construct a rule for long division to further their **conceptual understanding**.
- Students divide decimals to develop **procedural fluency**.
- Students **apply** work with decimal division to determine how the gravitational pull of other planets compares to that of Earth.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one per pair
- graph paper (optional)

Math Language Development

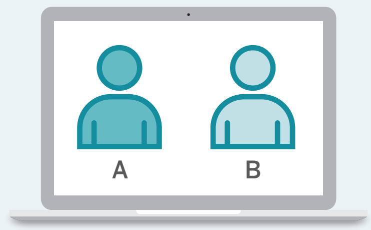
Review words

- *divisor*
- *dividend*
- *quotient*

Amps | Featured Activity

Activity 1
Digital Collaboration

Students collaborate to compose a rule for dividing decimals, and division in general.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might find it challenging to collaborate with others as they look for repeated reasoning. Throughout the activity, students should demonstrate appropriate and effective social engagement. As a team, they should speak when it is useful and demonstrate active listening skills when others are speaking.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be completed as a class or omitted entirely.
- **Activity 2** may be omitted. While this activity primarily provides additional practice, the context should be fun and engaging for students. Consider using it as a review when you have time.

Independent | 5 min

Warm-up Same Quotients

Students identify equivalent division expressions, activating prior knowledge that quotients are the same when the expressions differ by the same power of 10 factor.

Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 13

Dividing Multi-digit Decimals

Let's divide any decimal by any other decimal.

Warm-up Same Quotients

Select *all* of the expressions that have the same quotient as $5.04 \div 7$. Be prepared to share your thinking.

A. $0.504 \div 700$

B. $5.04 \div 70$

C. $50.4 \div 70$

D. $504 \div 700$

E. $504{,}000 \div 700$

F. $504{,}000 \div 700{,}000$

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking "What is the connection between the 7 in the given expression and the 70 in Expression B? What needs to be true about the dividends for the quotients to be the same?"

Look for points of confusion:

- **Thinking the dividend and divisor do not need to change by the same power of 10.** Ask, "How did the divisor change? Did the dividend change in the same way?" And if not, "How would that impact the quotient?"

Look for productive strategies:

- Connecting their prior work with related expressions, and equivalent ratios and fractions, to recognize that, if both the dividend and divisor are multiplied by the same power or 10, the quotient will be the same.

3 Connect

Display the expressions, indicating that C, D, and F are the only three that are equivalent.

Have students share why Expressions C, D, and F are equivalent and Expressions A, B, and E are not.

Highlight that C, D, and F are equivalent expressions because the dividend and divisor from $5.04 \div 7$ have each been multiplied by the same power of 10 to make those expressions, but that is not true for A, B, or E. Reveal that, if each of the related expressions were to actually be divided, they would all result in the same quotient of 0.72.

Ask, "How would the divisor have to change in Expression A for it to have the same quotient as $5.04 \div 7$?"

Power-up

To power up students' ability to use equivalent expressions to mentally calculate the quotient of two large whole number values, have students complete:

Determine if each equation is true or false.

1. $640000 \div 8000 = 640 \div 8$ True
2. $550000 \div 1100 = 550 \div 11$ False
3. $360000 \div 400 = 36 \div 4$ False
4. $100000 \div 500 = 1000 \div 5$ True

Use: Before the Warm-up

Informed by: Performance on Lesson 12, Practice Problem 6.

Small Groups | 15 min

Activity 1 Generalizing a Decimal Division Rule

Students collaborate to write a rule for division with decimals that can be generalized to every division problem involving any decimal dividend and any decimal divisor.

Amps Featured Activity Digital Collaboration

Activity 1 Generalizing a Decimal Division Rule

Your small group, and then your whole class, will collectively write a general rule for division with decimals. You will test and revise your rules along the way.

Who	What
You	Evaluate the expression $3 \div 0.12$ in Problem 1.
Trios	**Discuss how you solved the problem.**
	Write a rule about how to divide decimals. This is your first attempt, and you will be able to revise it later. **First write:**
	Evaluate the expressions $7.5 \div 1.25$ and $1.8 \div 0.004$ in Problem 2.
	Discuss whether your rule holds true, and why or why not?
	Rewrite or revise your rule based on this discussion and record it here. This will be your group's final general rule. **Second write:**
Class	**Share group rules. Then, as a class, you will discuss and agree on a final rule.**
	Record your class rule here. **Third write (Class Rule):** **Sample rule:** **To divide decimals, the dividend and divisor can be multiplied by the same power of 10 so that the divisor is a whole number. Then the resulting numbers can be divided using long division or another algorithm.**
	Evaluate the expression $7.89 \div 2$ in Problem 3.
	Does the rule still hold true? Yes No

586 Unit 5 Arithmetic in Base Ten

1 Launch

Arrange students in groups of three. Review the steps for the activity and note the second page of the activity as space for calculations. As trios collaborate, listen for and encourage the use of division language, such as *divisor*, *dividend*, and *quotient*.

2 Monitor

Help students get started by asking "How would you set up this problem? Can you calculate the expression as written?"

Look for points of confusion:

- **Writing a step-by-step procedure or only thinking of one specific problem, instead of a general rule.** If students write down everything they did for a problem step-by-step, explain that this should be a general rule that can be applied to more than just a specific problem.
- **Multiplying a divisor and dividend by different powers of 10, such as to make them both the smallest possible whole numbers.** Refer to the Warm-up and ask, "Why was Expression F equivalent to $5.04 \div 7$, but Expression E was not?"

Look for productive strategies:

- Making connections to the Warm-up and how what is done for the divisor must be done for the dividend and vice versa.
- Using prior knowledge to state that, for long division, the divisor should always be a whole number, but the dividend can be a decimal or a whole number.
- Connecting how the decimal point shifts relative to the power of 10 being multiplied by the dividend and the divisor.
- Using the base ten structure while carrying out an algorithm to divide with decimal divisors, and using the structure of each successive problem to refine their rules.

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Before students evaluate $3 \div 0.12$ in Problem 1, display the expression $300 \div 12$ and have students determine the quotient, documenting the steps they use. Have them apply this thinking as they complete Problem 1.

Math Language Development

MLR1: Stronger and Clearer Each Time

The entirety of this activity is designed similarly to the routine *Stronger and Clearer Each Time*. Ensure that students receive peer feedback prior to refining and revising their responses. Encourage reviewers to ask clarifying questions such as:

- "Did the first write of your rule work as you evaluated the expressions in Problem 2? What worked? What did not work?"
- "How can you use what did not work to help revise your rule?"

English Learners

Allow students to write their first write in their primary language and then translate it to English during the second write.

Small Groups | 15 min

Activity 1 Generalizing a Decimal Division Rule (continued)

Students collaborate to write a rule for division with decimals that can be generalized to every division problem involving any decimal dividend and any decimal divisor.

Name: ______ Date: ______ Period: ______

Activity 1 Generalizing a Decimal Division Rule (continued)

Use this space for your calculations as you test your division rule for decimals.

1. $3 \div 0.12 = 25$

$$\begin{array}{r} 25. \\ 0.12\overline{)3.00} \\ -2\,4 \\ \hline 60 \\ -60 \\ \hline 0 \end{array}$$

2. a $7.5 \div 1.25 = 6$

$$\begin{array}{r} 6. \\ 1.25\overline{)7.50} \\ -7\,50 \\ \hline 0 \end{array}$$

b $1.8 \div 0.004 = 450$

$$\begin{array}{r} 450. \\ 0.004\overline{)1.800} \\ -1\,6 \\ \hline 20 \\ -20 \\ \hline 00 \\ \hline 0 \end{array}$$

3. $7.89 \div 2 = 3.945$

$$\begin{array}{r} 3.945 \\ 2\overline{)7.890} \\ -6 \\ \hline 1\,8 \\ -1\,8 \\ \hline 09 \\ -8 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

3 Connect

Have students share their small group rules. One group proposes the initial wording of the rule, and successive groups can add, clarify, or pass until the class agrees the rule is complete.

Display Problem 3 and determine the quotient as a class, using the class's final rule.

Ask:

- "Does the rule hold true even if the dividend and divisor are whole numbers, such as for $184 \div 4$?"
- "How would you use the rule to create a related expression for $0.004 \div 1.8$?"

Highlight that the rule can be applied to *any* division situation: a whole number divided by a whole number, a decimal divided by a whole number, a whole number divided by a decimal, and a decimal divided by a decimal.

Pairs | 15 min

Activity 2 Ham the Astrochimp

Students apply their generalized rule from Activity 1 to compare the gravitational pull of another planet to that of Earth.

Activity 2 Ham the Astrochimp

The first astronauts to travel into space were not actually human — they were primates! While not the first primate in space, Ham the astrochimp was the first to successfully pull levers while in space. His mission proved that it was physically possible for a human to pilot a spacecraft, paving the way for successful and safe human space exploration.

In 1961, when Ham was launched into space and successfully landed back on Earth, he weighed 37 lb, or about 16.7829 kg, on Earth. Weight is a measure of the pull of gravity between an object and the planet it is on. Because each planet has a different gravitational pull, objects — including Ham — do not weigh the same on every planet.

NASA

You will be given a card that shows Ham's weight, in kilograms, on a different planet. Write a division expression to determine how many times as much the gravitational pull of that planet is, as a factor relative to the gravitational pull of Earth. To prepare for the class *Gallery Tour*, record your results in the table and show any additional thinking that would help others understand.

Planet	Division expression	Gravitational pull relative to Earth (times as much)
Mercury	$6.3271533 \div 16.7829$	0.377
Venus	$15.15831528 \div 16.7829$	0.9032
Mars	$6.53693955 \div 16.7829$	0.3895
Jupiter	$44.306856 \div 16.7829$	2.64
Saturn	$19.1157231 \div 16.7829$	1.139
Uranus	$15.3899193 \div 16.7829$	0.917
Neptune	$19.2667692 \div 16.7829$	1.148

STOP

1 Launch

Distribute one pre-cut card from the Activity 2 PDF to each pair, and provide graph paper, as needed.

2 Monitor

Help students get started by activating prior knowledge. Ask, "What operation will help you answer a question that is asking 'how many times as much'?"

Look for points of confusion:

- **Reversing the dividend and divisor (i.e., not comparing their planet to Earth).** Ask, "Do you expect your quotient to be greater than or less than 1? Is that what your expression will produce?"
- **Multiplying the divisor and the dividend by different powers of 10.** Refer students to the class rule.

Look for productive strategies:

- Using prior knowledge of multiplicative comparison to understand they need to divide Ham's weight on their planet by his weight on Earth.
- Applying the class rule generated in Activity 1, and recognizing that it is most efficient to make the divisor a whole number, but leave the dividend a decimal.

3 Connect

Display student work using the ***Gallery Tour*** routine.

Have students share the similarities and differences they notice in how each group used the class rule.

Highlight that powers of 10 can be used to generate related expressions (with the same quotient) in which the divisor, and sometimes also the dividend, is a whole number.

Ask, "How did the numbers in these problems make it more efficient to change only the divisor into a whole number and not the dividend?" The number of decimal places in each are different and to make both whole numbers, the divisor would have several zeros on the end.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Have students round the dividend to the same number of decimal places as the divisor. Display the class rule for dividing and ask them to think about how the class rule can help them complete the activity.

Extension: *Math Enrichment*

Have students determine the weight of the heaviest animal on Earth, the blue whale, if the whale could exist on each of the other planets in our solar system. Tell students the average weight of an adult blue whale is about 350,000 lb.

Math Language Development

MLR2: Collect and Display

During the ***Gallery Tour***, as students examine the work of their classmates, listen for and amplify the mathematical language used, such as *divisor*, *dividend*, *quotient*, *power of 10*, *whole number*, and *related expressions*. Annotate the class rule with these terms and phrases.

Summary

Review and synthesize how to use a generalized rule to divide with decimals.

Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You developed a general rule and a process for dividing any two decimals, based on your work in the previous lessons with base ten numbers. Place value and related expressions can be used to transform any division problem into one where long division can be used.

When the divisor is a decimal, you can multiply it by a power of 10 to make it a whole number. But, remember, you must multiply the dividend by the same power of 10 for the expressions to be equivalent. You can also choose to multiply by a power of 10 that makes *both* the divisor and the dividend whole numbers. For example, $7.65 \div 1.2$ can be transformed into either of these related expressions.

```
        6.375
1.2.)7.6.500
    -7 2
      4 5
     -3 6
        90
       -84
        60
       -60
         0
```

```
           6.375
1.20.)7.65.000
     -7 20
       45 0
      -36 0
        9 00
       -8 40
         600
        -600
           0
```

Reflect:

Synthesize

Highlight that, as with equivalent fractions and ratios, related expressions used for division require the dividend and divisor to be multiplied by the same factor. Then it does not matter whether values start as decimals or remain decimals, the quotients will be the same; so, number sense and reasoning can help make such work even more efficient.

Ask:

- "In the two examples shown in the Summary, what were the divisors and dividend multiplied by and why?" In the first example, they are both multiplied by 10 to make the divisor a whole number. In the second example, they are both multiplied by 100 to make both the divisor and the dividend into whole numbers.
- "Why were zeros added to the dividend?" You still need to add zeros to extend the place value to more decimal points until there is no remainder, such as using 76.500 or 765.000.
- "As you are dividing with decimals, why is it helpful to multiply by a power of 10, instead of another factor that is not a power of 10?" These are base ten numbers, so multiplying by a power of 10 does not change any of the digits but simply adjusts the place values of the digits and the location of the decimal point, which makes it easier to still see the original values.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking students to mark up their work using the following key:

! Something that surprised you.

? Something you still have a question about.

☆ Something you want to remember.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by dividing two decimals using powers of 10 to create a related expression and using long division to evaluate.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 5.13

1. Write two different division expressions that have the same quotient as $36.8 \div 2.3$.

   $368 \div 23$

   $3{,}680 \div 230$

2. Evaluate $36.8 \div 2.3$. Show your thinking.

   $36.8 \div 2.3 = 16$;

$$\begin{array}{r} 16 \\ 2.3\overline{)36.8} \\ -23 \\ \hline 13\ 8 \\ -13\ 8 \\ \hline 0 \end{array}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can generalize a rule and a process for dividing decimals. 1 2 3

b I can determine the quotient of two decimals. 1 2 3

Success looks like . . .

- **Language Goal:** Comparing and contrasting division problems with whole number and decimal divisors. **(Speaking and Listening, Writing)**
- **Language Goal:** Generating another division expression that has the same value as a given expression, and justify they are equal. **(Speaking and Listening, Writing)**
  - » Writing two different division expressions that have the same quotient as $36.8 \div 2.3$ in Problem 1.
- **Language Goal:** Dividing whole numbers or decimals by decimals, and explaining and generalizing the reasoning, including choosing to divide a different expression that has the same quotient. **(Speaking and Listening, Writing)**
  - » Dividing $36.8 \div 2.3$.

Suggested next steps

If students write unrelated expressions in Problem 1, or do not use the related expression to evaluate Problem 2, consider:

- Referring them to the class rule in Activity 1, and asking:
  - » "Why did you have to multiply the dividend and divisor by the same power of 10?"
  - » "How did you use long division to divide when the divisor was a decimal?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did the use of MLR1 in Activity 1 reveal about your students as learners?
- Which teacher actions made the generation of a class rule particularly strong? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Mai claims, "To determine the value of $109.2 \div 6$, I can divide $1{,}092$ by 60."

   a. Do you agree with Mai? Explain your reasoning.

   I agree; Sample response: Both the divisor and dividend were multiplied by the same power of 10, or the number 10.

   b. Calculate the quotient of $109.2 \div 6$. Show your thinking.

   18.2

$$\begin{array}{r} 18.2 \\ 6\overline{)109.2} \\ -6 \\ \hline 49 \\ -48 \\ \hline 1\,2 \\ -1\,2 \\ \hline 0 \end{array}$$

2. Write two different division expressions that have the same quotient as $61.12 \div 3.2$. Then evaluate $61.12 \div 3.2$. Show your thinking.

   $61.12 \div 3.2 = 19.1$; Sample responses: $611.2 \div 32$ or $6{,}112 \div 320$

$$\begin{array}{r} 19.1 \\ 3.2\overline{)61.12} \\ -32 \\ \hline 29\,1 \\ -28\,8 \\ \hline 32 \\ -32 \\ \hline 0 \end{array}$$

3. A bag of pennies weighs 5.1 kg. Each penny weighs 2.5 g. Which is the best estimate for the number of pennies in the bag? Show your thinking.

   A. 20 B. 200 **C.** 2,000 D. 20,000

   Sample response: 5.1 kg $= 5{,}100$ g

   $5{,}100 \div 2.5$ is about $5{,}000 \div 2.5$, which is $2{,}000$.

Name: ______ Date: ______ Period: ______

Practice

4. Determine each difference. Show your thinking.

   a. $2.5 - 1.6$ **$= 0.9$**

$$\begin{array}{r} \scriptstyle 1\,15 \\ \cancel{2}.\cancel{5} \\ -1.6 \\ \hline 0.9 \end{array}$$

   b. $0.72 - 0.4$ **$= 0.32$**

$$\begin{array}{r} 0.72 \\ -0.40 \\ \hline 0.32 \end{array}$$

   c. $11.3 - 1.75$ **$= 9.55$**

$$\begin{array}{r} \scriptstyle 0\,10\,12\,10 \\ \cancel{1}\cancel{1}.\cancel{3}\cancel{0} \\ -1.75 \\ \hline 9.55 \end{array}$$

   d. $73 - 1.3$ **$= 71.7$**

$$\begin{array}{r} \scriptstyle 2\,10 \\ 7\cancel{3}.\cancel{0} \\ -1.3 \\ \hline 71.7 \end{array}$$

5. Plant B is $6\frac{2}{3}$ in. tall. Plant C is $4\frac{4}{15}$ in. tall. Complete the sentences and show your thinking.

   a. Plant C is **$\frac{16}{25}$** times as tall as Plant B.

   Sample response:

$$4\frac{4}{15} \div 6\frac{2}{3} = \frac{64}{15} \div \frac{20}{3} = \frac{64}{15} \cdot \frac{3}{20} = \frac{16}{5} \cdot \frac{1}{5} = \frac{16}{25} \text{ (or equivalent)}$$

   b. Plant C is **$2\frac{2}{5}$** in. **shorter** (taller or shorter) than Plant B.

   Sample response:

$$\begin{aligned} 6\frac{2}{3} - 4\frac{4}{15} &= \frac{20}{3} - \frac{64}{15} \\ &= \frac{100}{15} - \frac{64}{15} \\ &= \frac{36}{15} \\ &= 2\frac{6}{15} \\ &= 2\frac{2}{5} \end{aligned}$$

6. One floor of a rectangular building measures 25 ft long, 50 ft wide, and 10 ft high.

   a. What is the square footage of the ground surface of the floor, the part on which people can stand?

   $1{,}250$ ft^2; $25 \bullet 50 = 1{,}250$

   b. What would the total square footage be if the building contained 10 identical floors?

   $12{,}500$ ft^2; $1{,}250 \bullet 10 = 12{,}500$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 3	1
	5	Unit 4 Lesson 13	2
Formative	6	Unit 5 Lesson 14	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

The So-called World's "Littlest Skyscraper"

Let's use what we know about decimals to explore a skyscraper of a scam.

Focus

Goals

1. Apply multiple operations with decimals to solve multi-step, real-world problems.
2. **Language Goal:** Describe sources of differences in precision, and justify an appropriate level of precision for reporting an answer in context. **(Speaking and Listening)**

Coherence

Today

Students conclude this unit and their year-long journey of investigating, understanding, and developing fluency with the four operations as applied to base ten numbers and decimals. They apply all of this to determine how much an infamously sly real estate developer conned out of some investors in a real-life deal gone wrong. Students must work with both relatively large and relatively small numbers, allowing them to determine whether and when to round, such as in using different conversion factors, impacts successive calculations. They must make sense of the problems, determine which information is important, and choose and justify a strategy and an appropriate level of precision for solving in the given context. To focus student thinking on process and precision, consider providing calculators to aid with the time involved in calculations.

Previously

In this unit, students generalized algorithms to add, subtract, multiply, and divide with multi-digit whole numbers and multi-digit decimals beyond the hundredths.

Coming Soon

In Unit 6, students will extend their work with related expressions and properties of operations to algebraic expressions. In Grade 7, students will build upon their learning from this unit as they use operations with rational numbers.

Rigor

- Students build **fluency** with subtracting, multiplying, and dividing with decimals beyond the hundredths.
- Students **apply** their work with decimal operations to determine how much a legendary swindler stole from investors.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
5 min	30 min	5 min	5 min
Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

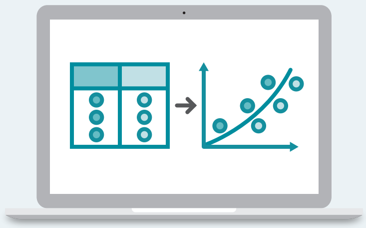
Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one per pair
- calculators, one per student
- graph paper (optional)

Amps | Featured Activity

Warm-up
See Student Thinking

Students are asked to describe the importance of units in measurement, and these explanations are available to you digitally, in real time.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In this activity, students might struggle to critique the arguments of their peers. Students will make choices about their calculations based on their analysis of the provided situation. While McMahon ventured outside of ethical norms, the students will evaluate the mathematical consequences of their own decisions.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 1**, have students determine the different meter conversions. Then complete the rest of the activity as a class.

Independent | 5 min

Warm-up The So-called World's "Littlest Skyscraper"

Students read about the Newby-McMahon building scam, a real-world example of the importance of precision, and establish the context for Activity 1.

Amps Featured Activity See Student Thinking

Unit 5 | Lesson 14 – Capstone

The So-called World's "Littlest Skyscraper"

Let's use what we know about decimals to explore a skyscraper of a scam.

Warm-up The So-called World's "Littlest Skyscraper"

There is no official definition, but It is generally agreed that a skyscraper is a building containing 10–20, or more, floors. In 1919, when J.D. McMahon drafted the blueprints for what is now known as the Newby-McMahon Building in the then-booming oil town of Wichita Falls, TX, investors were eager to contribute $200,000 for a 480-foot skyscraper.

But what happened? How did the "world's littlest skyscraper" end up containing only 4 floors (without any stairs!) and standing a mere 40 ft tall?

It turns out, according to local legend, that McMahon was a bit of a swindler. The investors had not examined the plans carefully when they approved the blueprints for a building with a listed height of 480″ (not 480′). In a lawsuit following the building's completion, the courts ultimately ruled in favor of McMahon, who, most likely with a large sum of the unspent investment still in his pockets, was never seen or heard from in the area again.

Newby-McMahon Building, c. 1919, also known as the "Worlds Littlest Skyscraper". Wichita Falls, Texas, by Travis K Witt, courtesy of Wikimedia Commons, is licensed under the Creative Commons Attribution-Share Alike 4.0 International license: https://creativecommons.org/licenses/by-sa/4.0/deed.en

What is the mathematical moral of this story?

Sample responses:

- It is important to pay attention to details.
- Precision matters.

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students read the story. Activate background knowledge by displaying the values 480″ and 480′. Ask, "What do these values and symbols mean?" 480 in.; 480 ft

2 Monitor

Help students get started by asking, "What do you know about skyscrapers? Does the image look like a skyscraper?"

Look for productive strategies:

- Connecting the investors' lack of attention to detail to the importance of precision, not only when performing calculations but also when writing (and reading) mathematical symbols and statements.

3 Connect

Have students share the mathematical morals they took away from the story, focusing on responses, such as "precision" and "reading mathematical symbols carefully."

Highlight that precision refers to the quality of being exact and accurate. Although the plans were misleading, McMahon did build a 480-in.-tall building as the agreed-upon plans showed.

Ask, "How might this story and precision be connected to the work you have done with decimals in this unit?" Sample response: As more place values are used in the calculations, your answer becomes more precise. For example, the Newby-McMahon building is actually 480.27 in. tall, which is more precise than saying it is 480 in. tall.

Power-up

To power up students' ability to determine the necessary information needed to calculate the square footage of rectangle, ask:

A classroom in an elementary school building measures 12 feet long, 8 feet wide, and 10 feet high.

a. Determine the square footage of the floor.
96 ft^2

b. If there are 6 classrooms in each wing of the building, what is the total square footage of each wing?
576 ft^2

Use: Before the Warm-up.

Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 30 min

Activity 1 Then and Now: How Much Did It Cost?

Students apply their work with all four decimal operations to determine how much McMahon “stole” from his investors, in both the legend and an imagined sequel.

Name: ______ Date: ______ Period: ______

Activity 1 Then and Now: How Much Did It Cost?

Part 1

Refer to the Warm-up about the so-called world's "littlest skyscraper." Use the following facts, as needed, to evaluate each problem.

- **Investors thought the Newby-McMahon building would be 480 ft tall.**
- **The completed building measured 480 in., or 40 ft tall.**
- **Each of the 4 floors measured approximately 12 ft by 9 ft.**
- **$200,000 was invested to construct a 480-ft building in 1919.**
- **Inflation causes the prices of goods and services to rise over time. An item costing $1 in 1919 is estimated to have cost $14.78 in 2019.**

1. Assuming all relative costs have been adjusted at the same inflation rate, how much more would a 480-ft Newby-McMahon building cost to construct in 2019 than in 1919? Show your thinking.

   $2,756,000; Sample response:

   $200{,}000 \cdot 14.78 = 2 \cdot 14.78 \cdot 100{,}000$

   $14.78 \times 2 = 29.56$

   $29.56 \cdot 100{,}000 = 2{,}956{,}000$

   $2{,}956{,}000 - 20{,}000 = 2{,}756{,}000$

2. You will be given a calculator to help with your computations. In 1919, how much more did investors pay, per square foot, than they thought they would be paying? Round to the nearest penny. Show each step of your work, using the calculator.

   $424.38 per square foot

Actual: $462.96 ft²	Anticipated: $38.58 ft²	Difference: $424.38 ft²
$4 \cdot 9 \cdot 12 = 432$ ft²	1 ft = 12 in. $4 \cdot 12 \cdot 9 \cdot 12 = 5{,}184$ or $432 \cdot 12 = 5{,}184$	
$200000.000 \div 432 = 462.962$ − 1728 2720 − 2592 1280 − 864 416 0 − 388 8 27 20 − 25 92 1 280 − 864 416	$200000.000 \div 5184 = 38.580$ − 15552 44480 − 41472 3008 0 − 2592 0 416 00 − 414 72 1 280 − 0 1 280	462.96 − 38.58 424.38

1 Launch

Give pairs 5–7 minutes to complete Part 1, providing calculators after they complete Problem 1. Pause for a brief discussion of their responses and strategies and the impact of rounding to the nearest cent in Problem 2. Then give each pair one pre-cut card from the Activity 1 PDF, and have them complete Part 2 together.

2 Monitor

Help students get started by having them read the problems and then asking, "What given information will be important?" Repeat this for Part 2 as necessary.

Look for points of confusion:

- **Struggling to operate with 200,000.** Ask, "How can you use what you know about powers of 10 to make this more efficient?" (Problem 1) or "What decimal place does your quotient need to go to?" (Problem 2)
- **Not knowing how to determine the area in square meters (Part 2).** Have students sketch one floor, and label the dimensions in feet first, and then use their given conversion to rewrite each in meters. Ask, "How can you use this to determine the area of all the floors, in square meters?"
- **Dividing by 1.298 instead of 1.298 million (Part 2).** Have students reread the conversion and ask, "How would you write the full value of 1.298 million?"

Look for productive strategies:

- Using the structure of base ten numbers to multiply $14.78 \cdot 2$ by 100,000 (Part 1, Problem 1).
- Recognizing they only need to divide to the hundredths or thousandths (rounding with more precision) place to determine the nearest cent (Part 1, Problem 2).
- Reasoning about which digits are meaningful in context to help them decide when and how to round calculator results (Part 2).

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can build upon their work from previous slides after they determine how much the swindler stole from his Swiss investors.

Extension: *Math Enrichment, Interdisciplinary Connections*

Ask students to research other moments in history when not paying careful attention to units resulted in some significant problems.

For example, in 1999, NASA lost signals from a Mars orbiter that was sent to study Mars from an orbital view. The investigation determined that the failure resulted in a navigational error because the commands that were sent from Earth were in pounds-seconds, but needed to be converted to metric units, Newton-seconds. **(History, Science)**

Pairs | 30 min

Activity 1 Then and Now: How Much Did It Cost? (continued)

Students apply their work with all four decimal operations to determine how much McMahon "stole" from his investors, in both the legend and an imagined sequel.

Activity 1 Then and Now: How Much Did It Cost? (continued)

Part 2

As the legend goes, McMahon was never seen or heard from again in Wichita Falls. Supposedly, he secretly fled to Switzerland in 1920 to run the exact same scam.

As a class, you will determine how much more investors paid in 1920, per square *meter*, than they thought they would be paying. But first, you and your partner will be assigned one of the following preliminary questions to answer. Circle your assigned question.

How much did the investors *think* they would be paying, per square meter?

How much did the investors *actually* pay, per square meter?

You will also be given a card showing conversion rates between feet and meters and U.S. dollars and Swiss francs, and you will have access to a calculator. Use these tools and your work from Activity 1 to answer your assigned question.

Convert square feet to square meters	Card 1: 0.09 Card 2: 0.092416 Card 3: 0.9290304	Card 1: $0.3 \cdot 0.3 = 0.09$ Card 2: $0.304 \cdot 0.304 = 0.092416$ Card 3: $0.3048 \cdot 0.3048 = 0.09290304$
Area of anticipated building (m^2)	Card 1: 466.56 Card 2: 479.084544 Card 3: 481.60935936	Card 1: $5{,}184 \cdot 0.09 = 466.56$ Card 2: $5{,}184 \cdot 0.092416 = 479.084544$ Card 3: $5{,}184 \cdot 0.09290304 = 481.60935936$
Cost/square meter of anticipated building (in francs/m^2)	Card 1: 2,782.0644718793 Card 2: 2,709.3339082966 Card 3: 2,695.1303473938	Card 1: $1{,}298{,}000 \div 466.56 = 2{,}782.0644718793$ Card 2: $1{,}298{,}000 \div 479.084544 = 2{,}709.3339082966$ Card 3: $1{,}298{,}000 \div 481.60935936 = 2{,}695.1303473938$
Area of actual building (m^2)	Card 1: 38.88 Card 2: 39.923712 Card 3: 40.13411328	Card 1: $432 \cdot 0.09 = 38.88$ Card 2: $432 \cdot 0.092416 = 39.923712$ Card 3: $432 \cdot 0.09290304 = 40.13411328$
Cost/square meter of actual building (francs/m^2)	Card 1: 33,384.773662551 Card 2: 32,512.006899559 Card 3: 32,341.564168725	Card 1: $1{,}298{,}000 \div 38.88 = 33{,}384.773662551$ Card 2: $1{,}298{,}000 \div 39.923712 = 32{,}512.006899559$ Card 3: $1{,}298{,}000 \div 40.13411328 = 32{,}341.564168725$

STOP

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3 Connect

Display unique responses as pairs share.

Have pairs of students share their responses and strategies for Part 2, calling on groups who used 0.3 to convert first, and then on groups whose given conversion factors were more precise.

Ask:

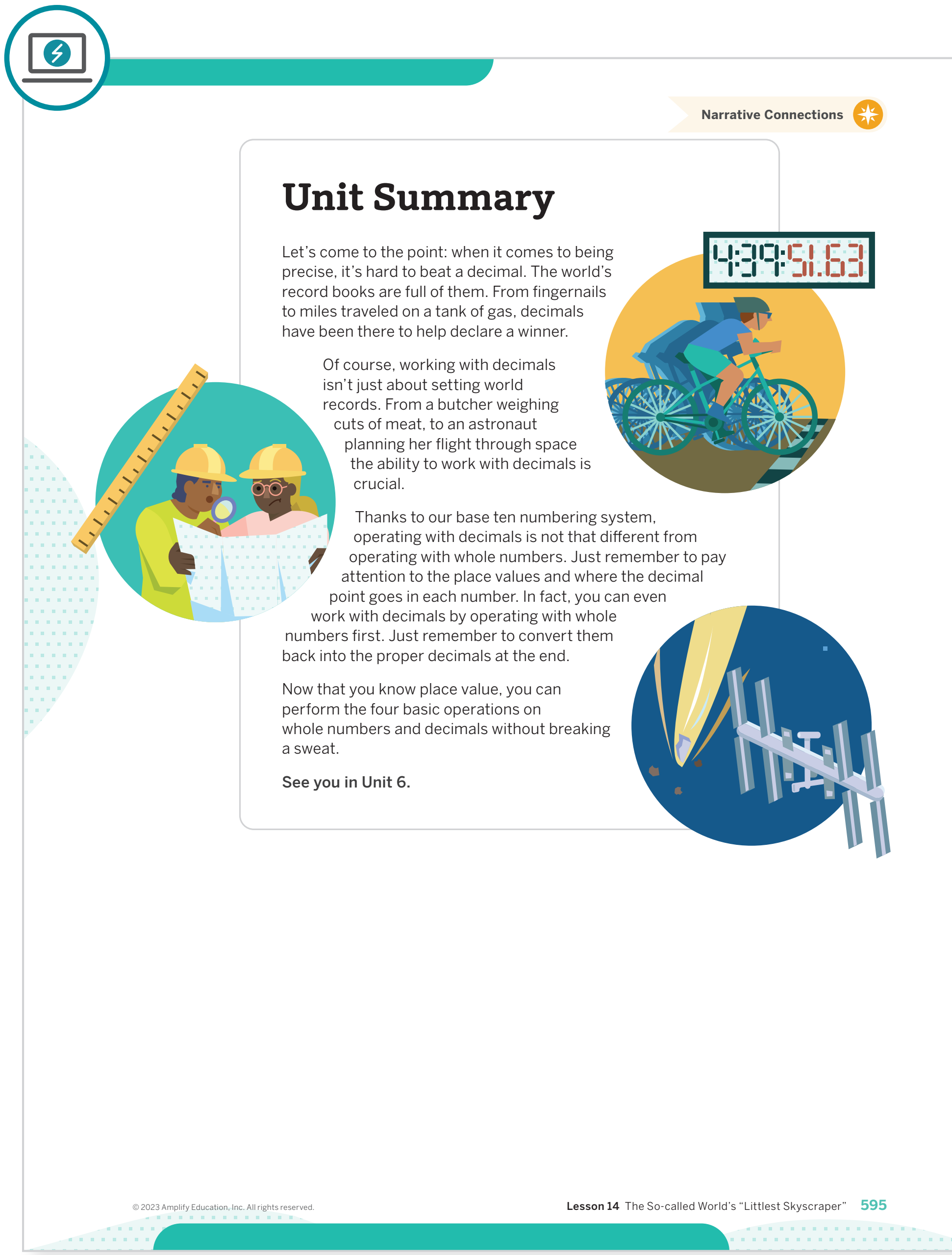
- "What was the impact of your conversion factor on the rest of your calculations?"
- "Did anyone round any values? Which ones and how did you choose the place value? How did that impact the rest of the calculations?"
- "Because 0.3 goes to the tenths and 0.3048 goes to the ten-thousandths, a difference in precision of 10^3, why would you not expect the results to be off by that same power of 10 (e.g., something in the hundreds would become something in the tenths)?"
- "In this context, explain some advantages or disadvantages of using 0.3048 m versus 0.3 m to convert." Sample responses: 0.3048 provides the most precise and accurate final answer, but the calculations may take more time and the additional decimal places don't add much. Using 0.3 describes the general relationship clearly enough, and the values were easier to work with and understand in context.

Highlight that, when determining how much McMahon stole from his investors, values beyond hundredths may be too precise to make sense within the context. Have students round the actual and anticipated costs per square meter for Card 3 to the hundredths and then determine the difference. 29,646.43 francs/m^2 Consider also showing students a current average of dollars per square foot for property values in the local area, so they have a point of reference for comparison (dollars/m^2 can be very roughly calculated as dollars/$ft^2 \cdot 10$).

Whole Class | 5 min

Unit Summary

Review and synthesize the connection between decimals, precision, and context.


Narrative Connections

Unit Summary

Let's come to the point: when it comes to being precise, it's hard to beat a decimal. The world's record books are full of them. From fingernails to miles traveled on a tank of gas, decimals have been there to help declare a winner.

Of course, working with decimals isn't just about setting world records. From a butcher weighing cuts of meat, to an astronaut planning her flight through space the ability to work with decimals is crucial.

Thanks to our base ten numbering system, operating with decimals is not that different from operating with whole numbers. Just remember to pay attention to the place values and where the decimal point goes in each number. In fact, you can even work with decimals by operating with whole numbers first. Just remember to convert them back into the proper decimals at the end.

Now that you know place value, you can perform the four basic operations on whole numbers and decimals without breaking a sweat.

See you in Unit 6.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Highlight that students have been working with decimals and operations involving decimals for a few years, but they now know algorithms for all four operations that work regardless of the number of decimals places involved. Different levels of precision may be acceptable and useful, depending on the context in which the information is being considered.

Ask:

- "When might precision or precise values, such as those with more decimal places, be more important?" Less important?" Sample responses: Precision is important when there are multiple steps and small value changes can result in large differences after the final calculation, or when the relative sizes of the numbers are extremely small and precise. It is less important when the additional place values do not change the final numbers in a meaningful way, such as thinking about large whole numbers with decimal parts.
- "Do you think rounding decimals has a different impact depending on which operations are being used? If so, which ones and why?" Sample response: Yes. When you add or subtract with decimals, the digits in each place value are only combined with each other, and maybe one more if you need to compose or decompose, so the results will be relatively close. But when you multiply or divide, the digits in each place value interact with those in every place value of the other number, which could mean many more compositions or decompositions, and so the result can differ across more units.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "Did anything surprise you while reading the narratives of this unit?"
- "Would you like to learn more about these topics? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by considering how decimals and precision are connected.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 5.14

Consider the role of precision in a multi-step process, such as in the problems you solved in this lesson. What caused different groups to arrive at different answers?

Sample responses: When the initial calculations are less precise or rounded early in the process, then the successive calculations will become less precise. When each successive calculation has as many digits as possible, then the final calculation will be more precise. Precise answers may be more challenging to interpret within context.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use the four operations with decimals to reason about and solve real-world problems.

1 2 3

Success looks like . . .

- **Goal:** Applying multiple operations with decimals to solve multi-step, real-world problems.
- **Language Goal:** Describing sources of differences in precision, and justifying an appropriate level of precision for reporting an answer in context. **(Speaking and Listening)**
  - » Explaining how precision can affect the final calculation of a multi-step process.

Suggested next steps

If students are unable to name a source of differences in precision, consider:

- Reviewing the list of different solutions generated during Activity 1, Part 2. Ask, "Why were there so many different solutions for the anticipated cost per square meter, and then again for the actual cost per square meter?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Reflect upon the unit as a whole. How did it build on and connect to the previous three units? Think about the mathematics and your students as learners.
- Have you changed any ideas you used to have about decimal operations, particularly multiplication and division?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

The Willis Tower in Chicago, IL, has the highest observation deck in the United States. The Skydeck is located on the 103rd floor at an elevation of 1,353 ft.

1. The elevators reach the Skydeck in about 60 seconds. How many *meters* per second do the elevators travel? Use 1 ft = 0.3 m to evaluate. Show your thinking.

6.765 m per second

Sample response:

$1{,}353 \cdot 0.3 = 1{,}353 \cdot 3 \cdot \frac{1}{10}$

$$\begin{array}{r} 1353 \\ \times\ \ \ 3 \\ \hline 4{,}059 \end{array}$$

$4{,}059 \cdot \frac{1}{10} = 405.9$

$$\begin{array}{r} 6.765 \\ 60\overline{)405.900} \\ -360 \\ \hline 45\,9 \\ -42\,0 \\ \hline 3\,90 \\ -3\,60 \\ \hline 300 \\ -300 \\ \hline 0 \end{array}$$

2. By how much would your answer change if you used 1 ft = 0.3048 m to evaluate Problem 1? Show your thinking.

0.10824 meters per second; Sample response:

$1{,}353 \cdot 0.3048 = 1{,}353 \cdot 3{,}048 \cdot \frac{1}{10{,}000}$

$$\begin{array}{r} 1353 \\ \times 3048 \\ \hline 10824 \\ 54120 \\ 00000 \\ +4059000 \\ \hline 4{,}123{,}944 \end{array}$$

$4{,}123{,}944 \cdot \frac{1}{10{,}000} = 412.3944$

$$\begin{array}{r} 6.87324 \\ 60\overline{)412.39440} \\ -360 \\ \hline 52\,3 \\ -48\,0 \\ \hline 4\,39 \\ -4\,20 \\ \hline 194 \\ -180 \\ \hline 144 \\ -120 \\ \hline 240 \\ -240 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 6.87324 \\ -6.76500 \\ \hline 0.10824 \end{array}$$

3. Does the extra precision in Problem 2 make a difference, in the context of the problem? Explain your thinking.

No; Sample response: If I am riding in the elevator, I do not know what 0.10824 m per second feels like. It is not a significant difference.

Practice

Name: ______ Date: ______ Period: ______

4. Determine each difference. Show your thinking.

a $3.572 - 2.6014$
$= 0.9706$

$$\begin{array}{r} 3.5720 \\ -2.6014 \\ \hline 0.9706 \end{array}$$

b $0.106 - 0.0315$
$= 0.0745$

$$\begin{array}{r} 0.1060 \\ -0.0315 \\ \hline 0.0745 \end{array}$$

c $0.151 - 0.028$
$= 0.123$

$$\begin{array}{r} 0.151 \\ -0.028 \\ \hline 0.123 \end{array}$$

5. Write three numerical expressions that are equivalent to $0.0004 \cdot 0.005$.

Sample responses:

$4 \cdot \frac{1}{10{,}000} \cdot 5 \cdot \frac{1}{1{,}000}$, $4 \cdot (0.0001) \cdot 5 \cdot (0.001)$,

$4 \cdot 5 \cdot \frac{1}{10{,}000} \cdot \frac{1}{1{,}000}$, $4 \cdot 5 \cdot \frac{1}{10{,}000{,}000}$,

$\frac{4}{10{,}000} \cdot \frac{5}{1{,}000}$, $(4 \cdot 5) \div 10{,}000{,}000$

6. Determine the following quotients. Show your thinking.

a $24.2 \div 1.1$
$= 22$

$24.2 \div 1.1 = 242 \div 11$

$$\begin{array}{r} 22 \\ 11\overline{)242} \\ -22 \\ \hline 22 \\ -22 \\ \hline 0 \end{array}$$

b $13.25 \div 0.4$
$= 33.125$

$13.25 \div 0.4 = 132.5 \div 4$

$$\begin{array}{r} 33.125 \\ 4\overline{)132.500} \\ -12 \\ \hline 12 \\ -12 \\ \hline 0\,5 \\ -4 \\ \hline 10 \\ -8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

c $170.28 \div 0.08$
$= 2{,}128.5$

$170.28 \div 0.08 = 17{,}028 \div 8$

$$\begin{array}{r} 2128.5 \\ 8\overline{)17028.0} \\ -16 \\ \hline 10 \\ -8 \\ \hline 22 \\ -16 \\ \hline 68 \\ -64 \\ \hline 4\,0 \\ -4\,0 \\ \hline 0 \end{array}$$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 5 Lesson 4	2
	5	Unit 5 Lesson 5	2
	6	Unit 5 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

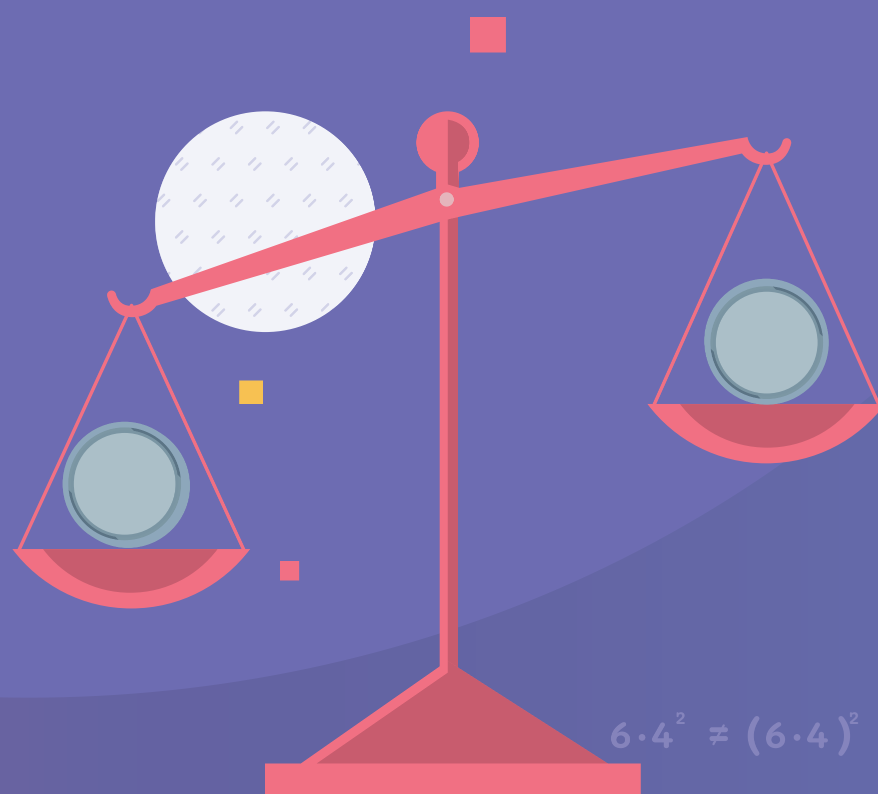
UNIT 6

Expressions and Equations

Students discover that the equal sign is more than a prompt, it's also a way to indicate balance — a critical understanding that allows them to move beyond the strictly numeric world and into the realm of algebra.

Essential Questions

- What does it mean for an equation to be true? Can an equation be false?
- How can two quantities be equal when one is partially or totally unknown?
- What does it mean for two expressions to be equal? Equivalent? Is there a difference?
- *(By the way, what do a good yogi and a good accountant have in common?)*



Key Shifts in Mathematics

Focus

In this unit . . .

Students build on their understanding of the equal sign and use it as a representation of equality (or balance), at which point they can perform balanced operations to solve for unknown values, and relate equivalent expressions, including those involving two variables or exponents.

Coherence

Previously . . .

In prior grades, students have encountered the equal sign as a prompt, asking them to perform an evaluation. Relevant prior work In Grade 6 includes working closely with ratio relationships, dividing fractions, and operating with base ten arithmetic.

Coming soon . . .

With the ability to solve single-step equations from Unit 6, students will be empowered to grapple with proportional relationships and two-step equations in Grade 7. Students' exposure to exponents in this unit will be extended and formalized in Grade 8.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Expressions and equations with unknowns are introduced using visual models, such as tape diagrams and hanger diagrams, so that students can recognize what it means to maintain equality or balance (Lessons 3–6). Along with area models, these visual representations are also leveraged for equivalent expressions (Lessons 10–13).

Procedural Fluency

Solving numerous equations involving positive rational numbers of all forms helps students develop fluency with operations as well as solving equations, including revisiting fractions as representing division and working with percentages (Lessons 7–9, 16).

Application

Combining prior understandings of ratio relationships with developing skills for writing and solving equations, students begin to represent and solve equations relating real-world quantities — independent and dependent variables — that are proportional (Lessons 17–19).

The Power of Balance

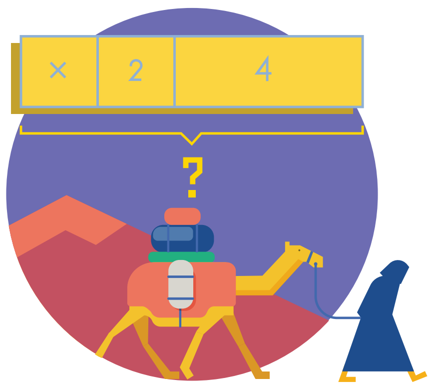
SUB-UNIT

1

Lessons 2–9

Expressions and Equations in One Variable

Students explore expressions and equations with one variable, often in the context of the 14th century African salt trade. Throughout, they connect verbal scenarios, symbolic mathematical expressions, and visual models: tape diagrams and hanger diagrams. Students first explore what it means for an equation to be true or false, followed by what it means to preserve equality. Then, they hone in on the properties of equality as strategies for determining a solution to a one-step equation, including those involving positive rational numbers and percentages.



Narrative: Learn about the 14th century African salt trade, as you explore expressions and equations.

SUB-UNIT

2

Lessons 10–16

Equivalent Expressions

Students extend the concept of equality to investigate ***equivalent expressions***, which doubles down on the utility of the equal sign and leverages the meanings and properties of operations building up to the all-important ***Distributive Property*** and working with ***exponents***. Algebraically, two expressions are equivalent if they always represent the same value, no matter what values for the variables are substituted into them. Once again, students will use tape diagrams to explore and verify these equivalent expressions, and will also revisit the area model for multiplication.

Narrative: Extend the concept to equality to investigate equivalent expressions and exponents.

Launch

Lesson 1

Launch Lesson Title

In this classic low-floor high-ceiling puzzle, students begin to explore balance literally, while applying logic and repeated reasoning. You're given a handful of coins, all of which are identical except for one — a counterfeit — which weighs ever-so-slightly more (or less) than the others. Using as few weighings as possible, how can you identify the counterfeit?

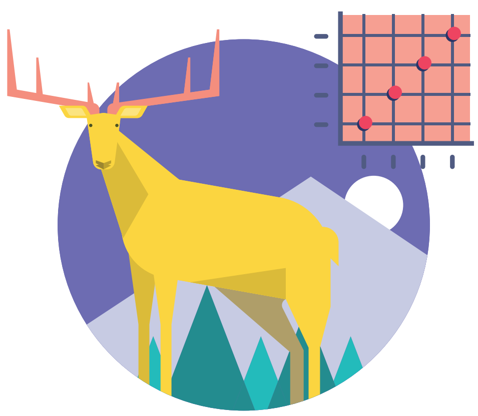
SUB-UNIT

3

Lessons 17–18

Relationships Between Quantities

Students compare and contrast different representations of mathematical relationships with ***independent*** and ***dependent variables***, often in the context of balanced (or unbalanced) ecosystems. They will look at written scenarios, visual models, tables, graphs, and equations with respect to ratios and unit rates. To close out the unit, students will examine a multi-tiered mobile, thereby ending where they began — with balance — but now empowered by their algebraic thinking.



Narrative: Balance is everywhere, from mathematical equations to ecosystems.

Capstone

Lesson 19

Capstone Lesson Title

Students create personal artifacts to hang on a class mobile. They apply their understanding of balance and equations to position one artifact after another, considering their weights and some intuitive trial and error with physics, to achieve a final state of balance.

Unit at a Glance

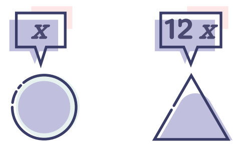
Spoiler Alert: To solve algebraic equations, such as $x + 1 = 2$ or $2x = 6$, perform the same operation with the same value on each side.

Assessment | **Launch Lesson** | **Sub-Unit 1: Expressions and Equations in One Variable**

A Pre-Unit Readiness Assessment

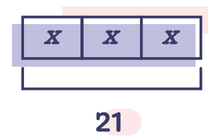
1 Detecting Counterfeit Coins

Explore balance with concrete objects and apply logic to solve puzzles that plant some early seeds for mathematical equality and working with unknowns.



2 Write Expressions Where Letters Stand for Numbers

Represent relationships between quantities using mathematical operations, where one quantity is unknown.



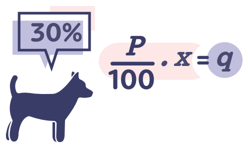
3 Tape Diagrams and Equations

Re-introduce tape diagrams, but now with variables/unknowns. It's just about getting the right algebraic equation, not solving!

$$\frac{\frac{5}{2}}{\frac{3}{4}}$$

8 A New Way to Interpret a Over b

Practice focusing on the connection between multiplicative equations and the division interpretation of fractions.



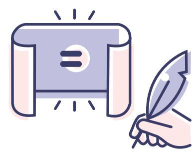
9 Revisiting Percentages

Now practice focusing on the connection between multiplicative equations and percentages.

Assessment

A Mid-Unit Assessment

Sub-Unit 2: Equivalent Expressions



10 Equal and Equivalent (Part 1)

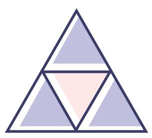
Introduce equations with a variable on both sides, leading to the possibility of more than one value that makes the equation true.

Sub-Unit 3: Relationships Between Quantities

$$6 \cdot 4^2 \neq (6 \cdot 4)^2$$

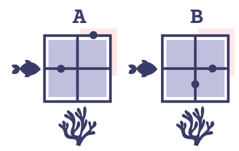
15 Evaluating Expressions With Exponents •

Use the meaning of exponents and order of operations to determine whether two expressions are equivalent.



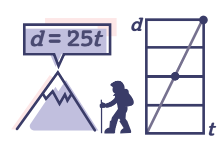
16 Analyzing Exponential Expressions and Equations •

Evaluate equivalent expressions involving exponents and a variable as either the base or exponent to solve equations, again by guess and check.



17 Two Related Quantities (Part 1)

Explore relationships between two variables, where one value depends on the other but either can be unknown, using tables, graphs, and now equations.



18 Two Related Quantities (Part 2)

Continue exploring relationships between two quantities with unknowns, focusing on unit rates and the relationship between a unit rate and a solution to an equation.

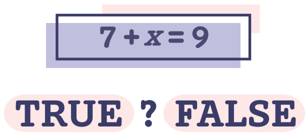
Key Concepts

Lesson 6: The properties of equality are formally introduced for solving equations in one variable.
Lesson 10: Two expressions are equivalent when they represent the same value, no matter what number is substituted for the variable into them.

Pacing

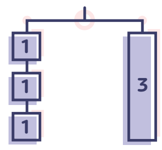
19 Lessons: 45 min each
3 Assessments: 45 min each
Full Unit: 22 days
● **Modified Unit:** 20 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



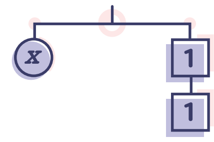
4 Truth and Equations

Understand that a solution is a value that can be substituted for the variable to make the equation true — by guess and check.



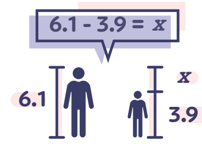
5 Staying in Balance

Introduce hanger diagrams to show balance and equality, but only using numerical values and equations, not variables. Don't skip this!



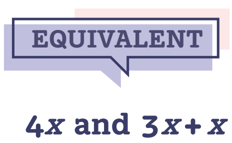
6 Staying in Balance With Variables

This is it! Solve equations for a variable using properties of operations based on maintaining balance with hanger diagrams.



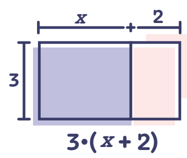
7 Practice Solving Equations

Plenty of practice solving equations to develop fluency, which often relates to familiar real-world scenarios from prior lessons.



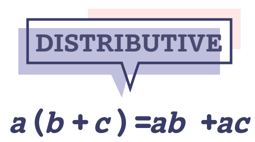
11 Equal and Equivalent (Part 2)

Apply the associative and commutative properties, along with the meanings of operations, to generate and verify equivalent expressions with variables.



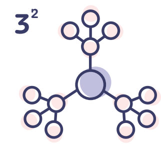
12 The Distributive Property (Part 1) ●

Relate partitioned rectangles with unknown measures to two equivalent expressions containing variables.



13 The Distributive Property (Part 2) ●

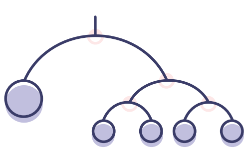
Generalize the Distributive Property as relating two equivalent expressions with common factors among terms.



14 Meaning of Exponents ●

Notice patterns in the value of an expression with an exponent and relate them to repeated multiplication.

Capstone Lesson



19 Creating a Class Mobile ●

Bring the critical mathematical properties of equality back full circle to the practical concept of balance and apply them in creating and discussing a personal, collective, and creative model.

Assessment

A End-of-Unit Assessment

● Modifications to Pacing

Lessons 12–13: It is strongly recommended to give the Distributive Property its intended pacing, due to its importance and generalizability across many areas in higher mathematics. However, if truly pressed for time, both Warm-ups may be omitted, and, in Lesson 13, students can complete Activities 1 and 2 using a shorter list of expressions.

Lessons 14–16: In the event your students have previously been well exposed to exponents and are comfortable with them, these three lessons taught together can be shortened to 90 minutes by omitting the Warm-ups, concatenating Exit Tickets or Summaries, and possibly omitting Lesson 15, Activity 2.

Lesson 19: The Capstone lesson for Unit 6 is a creative application of the concepts developed in the unit, and while some students may be sad to see this hands-on lesson omitted, we will all understand.

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	coefficient variable
4	solution to an equation
6	Addition Property of Equality Division Property of Equality Multiplication Property of Equality Subtraction Property of Equality
10	equivalent expressions
17	dependent variable independent variable

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
1, 2, 8, 14	**MLR1:** Stronger and Clearer Each Time
2, 4–6, 10, 11, 17	**MLR2**: Collect and Display
15, 16	**MLR3:** Critique, Correct, Clarify
5, 15	**MLR5:** Co-craft Questions
1, 7, 14, 15	**MLR6:** Three Reads
3–7, 9–12, 14, 16–18	**MLR7:** Compare and Connect
2, 4, 7, 8, 10, 13, 17	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket

Additional Practice

Additional required materials include:

Lesson(s)	Materials
4	poster paper
1, 10, 16–18	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
18	colored pencils
19	optional building materials

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routine
4	Gallery Tour
11	I Have, You Have
5, 13, 14, 19	Notice and Wonder
9, 12	Number Talk
1, 3, 8	Think-Pair-Share
4	Two Lies and a Truth
1	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 9
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 18 or 19

Amps
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Social & Collaborative Digital Moments

Featured Activity

Storytime With Tape Diagrams

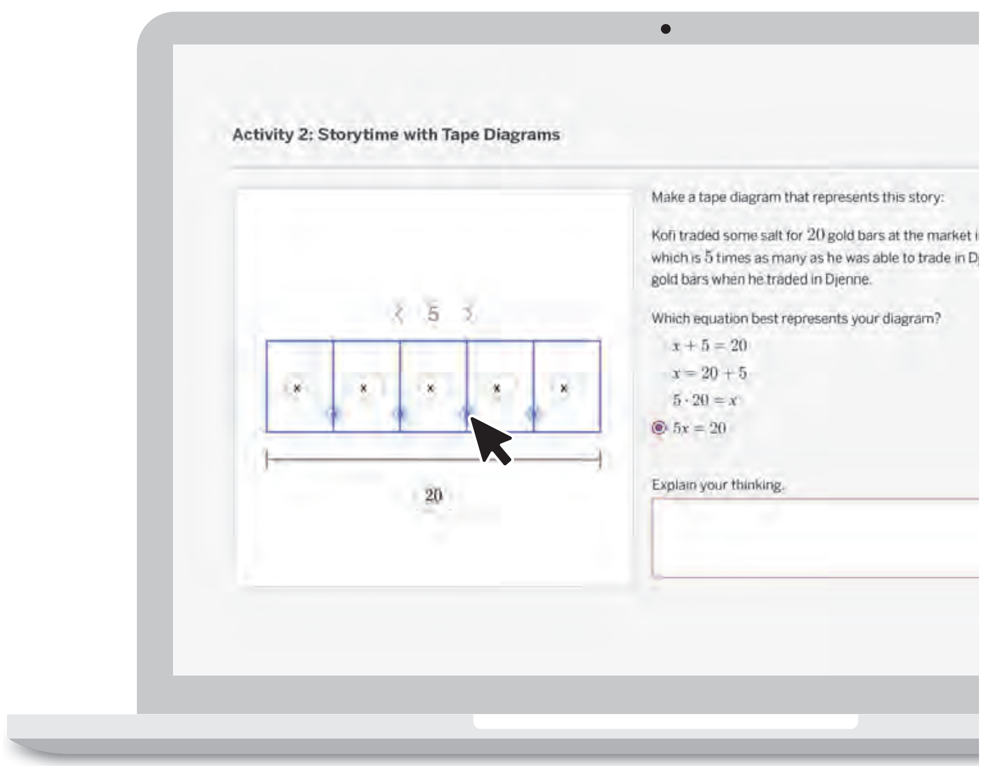
Put on your student hat and work through **Lesson 3, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities

- Three Coins **(Lesson 1)**
- Using Hangers to Write and Solve Equations With Variables **(Lesson 6)**
- Puppies Grow Up, Revisited **(Lesson 9)**
- Bear Populations In Yellowstone National Park **(Lesson 17)**



Unit Study
Professional Learning

This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces students to equivalent expressions. They have worked between drawing tape diagrams and writing equations. Students learn that a scenario can be represented by more than one equation and solving these equations yields the same solution. They also create story problems given an equation. Using problems in real-world contexts, students also learn to represent and solve percentage problems with equations. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 9, Activity 1:

Activity 1 Representing and Solving Percentage Problems With Equations

You arrive at the market in Niani with 16 kg of cloth to trade.

1. Your first stop is the spice seller. You trade all of your cloth for rare and expensive spices. The spices weigh 25% of the weight of your cloth. How many kilograms of spices did you buy?
   a. Adaeze represents the scenario with the equation $\frac{25}{100} \cdot 16 = h$. Explain how her equation represents the scenario.
   b. Solve the equation to determine the number of kilograms of spices you bought.
2. Your next stop is the jeweler. You trade all your rare spices for diamonds. The diamonds weigh 150% of the weight of your spices. How many kilograms of diamonds did you buy?
   a. Write an equation to represent the scenario.
   b. Solve the equation to determine the number of kilograms of diamonds you bought.
3. Your final stop is the salt seller, where you buy 12 kg of salt. What percent is this of the original weight of cloth that you had when you arrived at the market?
   a. Write an equation to represent the scenario.
   b. Solve the equation to determine the percent.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

- What was it like to engage in this problem as a learner?
- This lesson has students revisit percentages by adding them to the story problems. Since students had a lot of practice with tape diagrams, it is possible that they continue to use them to solve the problems instead of writing out the equations. How might you help them connect the tape diagrams to the equations?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Think-Pair-Share

Rehearse . . .

How you'll facilitate the *Think-Pair-Share* instructional routine in Lesson 8, Activity 1:

Activity 1 Interpreting $\frac{a}{b}$

Solve each equation for x.

1. $35 = 7x$
2. $35 = 11x$
3. $7x = 7.7$
4. $0.3x = 2.1$
5. $\frac{2}{5} = \frac{1}{2}x$

Points to Ponder . . .

- Consider each part of the *Think-Pair-Share* routine separately for each activity in which it is used:
  - Should pairs be assigned at the onset, or based on strategies or relative completion? Would they benefit more from homogeneous or heterogeneous partners?
  - Is the sharing primarily to answer-check, to practice articulating one's thinking, or to foster open discussion and make decisions that guide further collaboration on the presented task(s)?

This routine . . .

- Allows students to practice and apply newly introduced ideas, immediately check their work, practice sharing their thinking, and listening to the ideas of others.
- Gives students a low-stakes environment to refine and rehearse how to share their thinking.
- Allows you to focus on students who need the most support, while others can continue to actively participate and receive feedback from peers.
- Provides an opportunity for students to work at their own initial pace, without being pressured or influenced by faster or more vocal peers.

Anticipate . . .

- Students may be more comfortable discussing and comparing just their answers, rather than their strategies and how and why they were the same or different across problems. What further prompts or guidance could you give these students to encourage better collaboration?
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening your Effective Teaching Practices

Use and connect mathematical representations.

This effective teaching practice . . .

- Deepens student understanding of mathematical concepts and procedures and builds a toolkit for problem solving.
- Facilitates meaningful mathematical discourse by drawing connections between the different representations used and how each one illustrates the same mathematical ideas.

Points to Ponder . . .

- What representations will be presented in this unit?
- Where do you see opportunities to make connections among the different representations used, and when should students be allowed choice of the representation used?

Math Language Development

MLR6: Three Reads

MLR6 appears in Lessons 1, 7, 14, and 15.

- Encourage students to read introductory text multiple times before jumping into a task. By doing so, they will have more opportunities to understand the task and the quantities and relationships presented. The ***Three Reads*** routine asks students to focus on the following for each read:
  - **Read 1:** Make sense of the overall information or scenario, without focusing on specific quantities.
  - **Read 2:** Look for specific quantities and relationships and make note of them.
  - **Read 3:** Brainstorm strategies for how to approach the task.
- **English Learners:** Annotate or highlight key words and phrases in the introductory text to help students understand the relationships between quantities, such as *each*, *twice*, etc.

Point to Ponder . . .

- Some students may resist reading information multiple times. How will you help them see the benefits to doing so before jumping into the actual task?

Differentiated Support

Accessibility: *Optimize Access to Tools, Optimize Access to Technology*

Opportunities to provide physical manipulatives, the use of technology (through the Amps slides), or other tools appear in Lessons 1, 3, 5–10, 17, and 18.

- Throughout the unit, provide students with counters, pennies, or other objects that they can use to model and make sense of relationships between quantities.
- Throughout the unit, provide students with copies of the *Tape Diagrams* PDF and *Double Number Lines*: *Percentage Problems* PDF that they can use to partition pre-made blank tape diagrams or partition and label pre-made blank double number lines.
- In Lessons 5 and 6, students can use a digital tool to enter a weight for a variable to animate the hanger diagram. By doing so, they will receive real-time feedback that shows whether the hanger is balanced.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology, physical manipulatives, or other tools to deepen student understanding?

Unit Assessments

- Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students in solving one-step equations with a variable throughout the unit?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-awareness and self-management.

Points to Ponder . . .

- How did students demonstrate their ability to recognize their strengths and weaknesses? Did they use their strengths to work through challenges? Did they remain positive and optimistic throughout the tasks? Did they persevere, seeking understanding that they had not achieved yet?
- Can students regulate their emotions, preventing their emotions from hindering them in tasks? Are they able to effectively manage their stress levels? How do they motivate themselves to stay on task instead of impulsively being distracted? Can they set goals? Are they organized? Do they have the self-discipline required to complete tasks?

Detecting Counterfeit Coins

How can a balance scale help you detect a counterfeit coin?

Focus

Goal

Understand how the concept of balance relates to math, both concretely and abstractly.

Coherence

Today

Students explore the physical concept of balance as they model weighing different numbers of coins to identify a counterfeit among them. All students will be able to engage with these tasks by applying some repeated reasoning, taking the form of a combination of guess-and-check, brute force, or higher-order thinking and logic. This lesson sets the stage for the important concept of balance as it relates to algebraic expressions and equations, which will be carried into all the lessons that follow.

Previously

In Grades 3–5, students wrote and evaluated numerical expressions involving positive rational numbers, and used the equal sign to indicate when two expressions, or an expression and a number, represented the same value.

Coming Soon

In Lessons 2–9, students will expand their understanding of balance as an important mathematical concept as it relates to numerical and algebraic expressions, and also to solving equations with variables representing unknown values. In particular, the balances seen here will connect with the hanger diagrams in Lessons 5–6.

Rigor

- Students build **conceptual understanding** of balance as it relates to math.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	10 min	20 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice | Independent

Materials

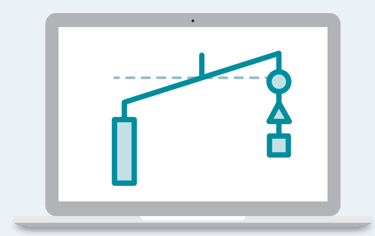
- Exit Ticket
- Additional Practice
- Activity 2 PDF (answers)

Amps | Featured Activity

Activity 1

Dynamic Balance Scale

When students weigh coins on a balance scale, the balance will animate, giving real-time feedback. Students see the consequences of their answers, and can work out for themselves any errors that need fixing.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel their stress level increase as they try to determine a mathematical strategy for finding the counterfeit coin in Activity 1. Rather than either being overwhelmed by stress or reacting impulsively, students should develop a plan and persevere with it. By thinking metacognitively, their strategic approach provides a structure in which they can work successfully.

Modifications to Pacing

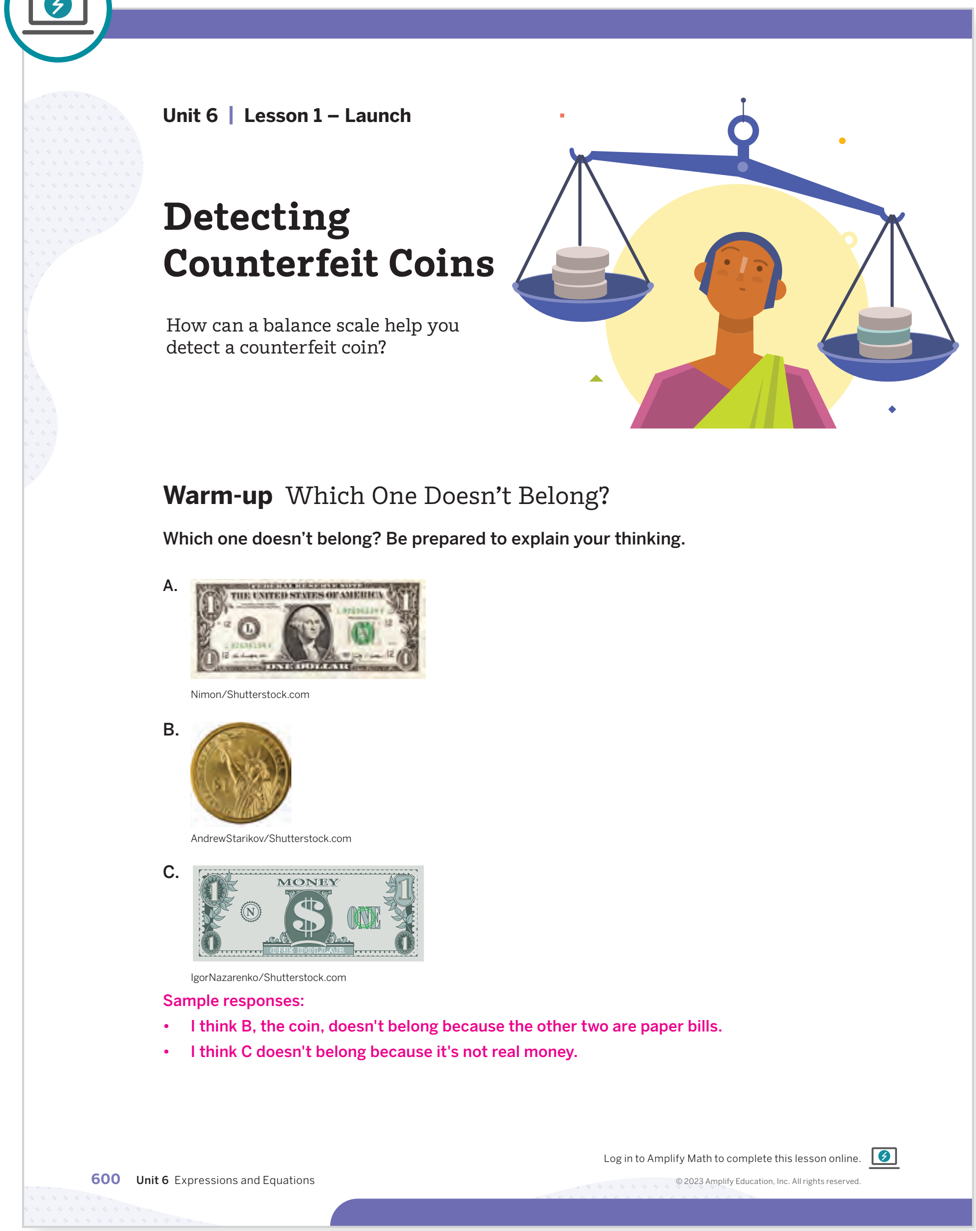
You may want to consider these additional modifications if you are short on time.

- The **Warm-Up** may be omitted.
- **Activity 1** may be completed as a class.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students look at three images of currency and identify one they believe does not belong, in order to begin thinking about the context of money and the word *counterfeit*.


Unit 6 | Lesson 1 – Launch

Detecting Counterfeit Coins

How can a balance scale help you detect a counterfeit coin?

Warm-up Which One Doesn't Belong?

Which one doesn't belong? Be prepared to explain your thinking.

A.

Nimon/Shutterstock.com

B.

AndrewStarikov/Shutterstock.com

C.

IgorNazarenko/Shutterstock.com

Sample responses:

- I think B, the coin, doesn't belong because the other two are paper bills.
- I think C doesn't belong because it's not real money.

Log in to Amplify Math to complete this lesson online.

600 Unit 6 Expressions and Equations

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1 Launch

Display the images and have students use the *Which One Doesn't Belong* routine.

2 Monitor

Help students get started by leveraging part of another familiar routine and asking them, "What do you notice?" Consider asking additional probing questions such as, "How are the images the same? How are any of the images different?"

3 Connect

Have students share their selection of which image does not belong and the rationale for their choice.

Highlight that the denomination of a piece of currency indicates its value, but only real money is actually worth its stated value. Money that has been created for the purpose of a board game, for example, has no real value, but also was not intended to. However, pieces of currency created to imitate the real object identically and pass off as real are called *counterfeit*.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Students who are familiar with U.S. currency may be able to identify the image in Choice C as "play money," or not real currency. Students from other countries may benefit from the showing of examples of real U.S. currency and examples of "play money" from a board game.

Accessibility: *Clarify Vocabulary and Symbols*

Students may not be familiar with the terms *currency* or *counterfeit*. The term *counterfeit* will be used in the upcoming activities.

- Explain that *currency* refers to the system of money that a country uses. Ask students from other countries to describe their country's currency or show examples of currency from other countries.
- Mention that the term *counterfeit* means more than just something that is "fake" or "not real." When using the term *counterfeit*, it is implied that the intention behind creating or using the fake object was to pass it off as real. Counterfeit currency often looks identical to real currency.

Pairs | 10 min

Activity 1 Three Coins

In this low-floor, high-ceiling activity, students investigate how many weighings are needed to determine which of three coins is counterfeit.

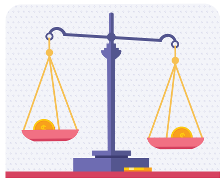
Amps Featured Activity Dynamic Balance Scale

Name: ______ Date: ______ Period: ______

Activity 1 Three Coins

Plan ahead: How will you organize your plan in order to manage your stress level?

There are three coins that look completely identical, but exactly one of the coins is counterfeit, and is slightly heavier than the two real coins. You cannot feel the difference in weight when you hold them. But you have a balance scale that is sensitive enough to detect it.



1. If you weigh two of the coins, you might get lucky and determine that one of them is heavier than the other. But you might not be so lucky. Will you always know which coin is the counterfeit coin after just one weighing? Why or why not?
   Yes, because if I pick any two coins (call them A and B) and place one on each side of the balance, then if one is heavier, I know it is the counterfeit coin. But if they weigh the same, then the third coin (call it C) that was not weighed must be counterfeit.

2. Now imagine you again have three coins and one of them is counterfeit. But this time, you don't know if the counterfeit coin is slightly heavier or slightly lighter than the other two coins.
   a. Is it possible to know which coin is counterfeit after just one weighing?
   Yes, but only if I weigh the two real coins (call them A and B). Then the third coin (call it C) must be counterfeit.
   b. Will you *always* know which coin is the counterfeit after just one weighing?
   No, because if A and B do not balance, I know one of them is counterfeit, but not which one.
   c. What is the smallest number of weighings you would need to perform so you could always say which coin is counterfeit?
   Two weighings, because after one weighing I would either know C is counterfeit *or* that A *or* B is counterfeit, but not which one. So if A and B do not balance, I need to take one of them, say A, and compare it with C (which is now known to be a real coin). If A and C balance, then B is counterfeit; but if A and C do not balance, then A is counterfeit.

Lesson 1 Detecting Counterfeit Coins 601

1 Launch

Activate background knowledge by asking students if they have ever seen or used a balance scale before. Make sure all students are familiar with how a balance scale works. Then, arrange students in pairs and have them use the *Think-Pair Share* routine.

2 Monitor

Help students get started by asking, "What would you put on the scale first? What will the result of that weighing tell you?"

Look for points of confusion:

- **Not distinguishing a single trial from a generalization.** Have students recount and record their weighings that lead to an identification. Ask, "What if one of those weighings went the other way? Could you have still identified the counterfeit coin after the same number of weighings?"
- **Double counting different possible weighings.** Each weighing has two possible results (balanced or unbalanced), each of which leads to a different next weighing. But only one result will happen at any given time, so only one next weighing should be counted.

3 Connect

Display a model of a balance scale and three coins labeled A, B, and C.

Have students share their strategies for determining the counterfeit coin. Start with students who used more weighings, and work toward students with more efficient strategies. Ask, "Was anyone able to identify the counterfeit coin in fewer weighings?"

Ask, "What does a balanced weighing tell you? An unbalanced weighing? How does knowing whether the counterfeit coin is heavier or lighter affect your weighings?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can interact with a digital, dynamic balance scale to deepen their conceptual understanding of how the three coins compare in weight to each other.

Accessibility: *Clarify Vocabulary and Symbols, Guide Processing and Visualization*

Clarify the meaning of the term *weighing* as it is used in this activity. Tell students that one weighing is the actual act of weighing the coins. If they need to weigh two coins one time, and then two different coins another time, this means there are two weighings.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should summarize the situation to be sure everyone understands. Clarify the term *counterfeit*, as needed.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the counterfeit coin is heavier than the two real coins.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problem 1.

Small Groups | 20 min

Activity 2 More Coins

Students move beyond three coins, again identifying strategies for determining the counterfeit coin.

Activity 2 More Coins

Many mathematicians LOVE good puzzles like these! Some solve them for fun. Some create them — often taking a known puzzle and tweaking it to make a new puzzle. And others, like Tanya Khovanova, study and research the deep structure and theory in recreational mathematics.

Try another. For each number of coins, assume one coin is counterfeit, and that the counterfeit coin is either slightly heavier or slightly lighter than the other coins. Determine the least number of weighings needed so that you could *always* say which coin is counterfeit.

1. Four coins
   Two weighings
   See Activity 2 PDF for sample responses.

2. Eight coins
   Three weighings
   See Activity 2 PDF for sample responses.

3. 12 coins
   Three weighings. In fact, it can be determined with certainty with only three weighings for every total number of coins from four up to 12!

Stronger and Clearer: You will meet with another group to give and receive feedback on your response to Problem 1. Use this feedback to refine and improve your response.

Featured Mathematician

Tanya Khovanova

Born in Russia, Tanya Khovanova is an American mathematician. In high school she won silver and gold medals at the International Mathematical Olympiad (becoming only the second female gold medalist at the time), and then went on to earn a PhD in Mathematics from Moscow State University. Khovanova has found success teaching at the university level, coaching mathletes, contracting as a systems engineer and applied mathematician, and conducting research on quantum group theory and superstring theory. Her recent work explores combinatorics and recreational mathematics, creating and solving mathematical puzzles, including some related to the counterfeit coin problems in this lesson.

Rebecca Frankel

1 Launch

Set an expectation for the amount of time students have to work collectively on the activity. See the Activity 2 PDF for additional explanations for each problem.

2 Monitor

Help students get started by asking, "What would you put on the scale first? What will the result of that weighing tell you?"

Look for points of confusion:

- **Determining the counterfeit coin in more than three weighings.** Students might only weigh two coins at a time. Ask, "What if I told you it could be done in fewer weighings? Can you think of a different way to eliminate more coins at the same time?"

Look for productive strategies:

- Systematically and simultaneously considering the results and implications of each weighing.

3 Connect

Display a model of a balance scale and each total number of coins, labeled using the letters A, B, C, etc.

Have students share the number of weighings it took for each number of coins and their strategies for determining the counterfeit. Start with students who used more weighings, and work toward students with more efficient strategies.

Ask, "What was the same about your strategy for each different number of coins? Do you think that strategy would still work for, say, 24 coins? How would you change your strategy for an odd number of coins?"

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide actual coins, counters, or concrete objects that students can physically hold to help them simulate the weighings.

Extension: *Math Enrichment*

Have students complete the following problem:

In this activity, the counterfeit could be determined with certainty in just three weighings. How many coins would there have to be for the number of weighings needed to be four? 13 coins

Math Language Development

MLR1: Stronger and Clearer Each Time

After groups respond to Problem 1, have them meet with another group to share and receive feedback, and then revise their explanation. Display these prompts that reviewers can use as they discuss their responses.

- "What if the first two coins weigh the same? What is your next step?"
- "What if the first two coins do not weigh the same?"

Featured Mathematician

Tanya Khovanova

Have students read about Tanya Khovanova who creates and solves mathematical puzzles, including some related to the counterfeit coin problems in this lesson.

Optional

Small Groups | 15 min

Activity 3 A Cruel Twist

Students grapple with 13 coins, figuring out how many weighings are needed to determine which coin, if any, is counterfeit.

Name: ______ Date: ______ Period: ______

Activity 3 A Cruel Twist

Consider the same setup as the previous activities, with 13 total coins, except now you only know that either one coin is counterfeit (and could be slightly heavier or slightly lighter than the other coins) *or* all the coins are real.

1. What is the smallest number of weighings it would take so you could *always* say:

   a. Whether any of the coins is actually counterfeit?

   Two weighings

   Compare A + B + C + D to E + F + G + H. If one is heavier, there is a counterfeit.

   - **Take five coins from the first weighing, say A + B + C + D + E, and compare them to I + J + K + L + M. If one is heavier, there is a counterfeit. Otherwise, there is not.**

   b. Which coin is counterfeit (if one of them is)?

   Three weighings

   Compare A + B + C + D to E + F + G + H.

   - **If they are balanced, take one, say A, and three coins not previously weighed, say I, J, and K, and compare A + I and J + K.**
   - **If they are not balanced, then use one coin now known to be real and five of the coins from the first weighing, switching sides with a couple to compare, say A + B + E and C + F + I.**
     - **At this point, there are too many possible scenarios to list, but in all of them, the possible counterfeit coin has been narrowed to two or three possible coins. So, the last weighing should be one of the following: (a) If narrowed to two, compare one of those two to any of the known real coins, or (b) if narrowed to three, you will also know whether the counterfeit is in fact lighter *or* heavier, so just compare two of the three.**

STOP

1 Launch

Set an expectation for the amount of time students have to work collectively on the activity.

2 Monitor

Help students get started by asking, "To determine if there is even a counterfeit coin at all, what would you need to know? Would you weigh the coins any differently than before?"

Look for points of confusion:

- **Adding 1 to the number of weighings it took in previous examples without trying.** Remind them that they will need to justify their answers.

3 Connect

Display a model of a balance scale and each total number of coins, labeled using the letters A, B, C, etc.

Have students share the number of weighings needed to determine whether there was a counterfeit, and how those looked. Then have them share the number of weighings needed to determine which coin was counterfeit when there was one. Again, start with strategies that used more weighings, and move toward more efficient strategies.

Differentiated Support

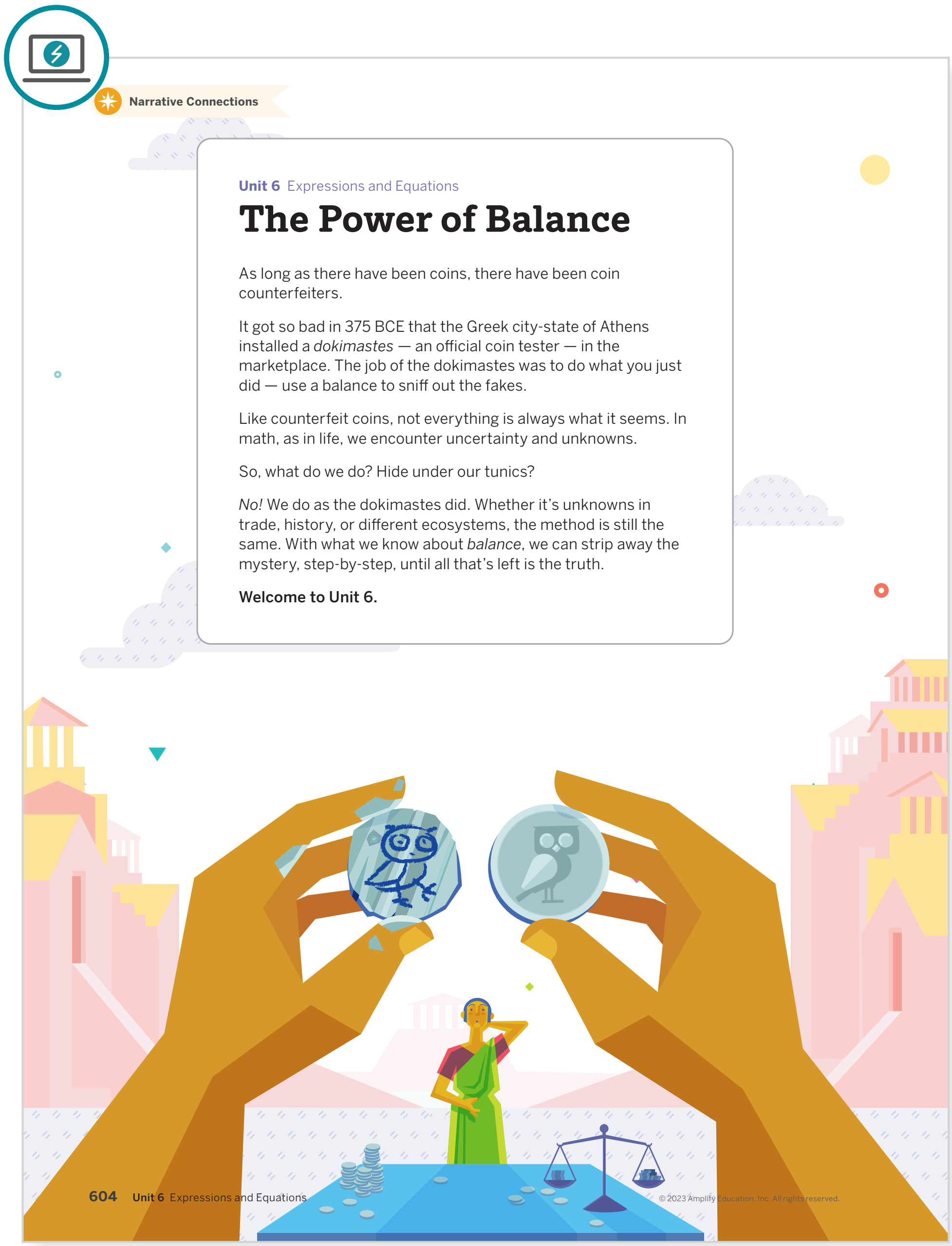
Accessibility: *Optimize Access to Tools, Vary Demands to Optimize Challenge*

Provide actual coins, counters, or concrete objects that students can physically hold to help them simulate the weighings. Consider reducing the number of coins to 7, but keep the rest of the scenario and task the same.

Whole Class | 5 min

Summary The Power of Balance

Synthesize the critical aspect of these puzzles as the concept of balance, which will play a significant role in the unit.


Narrative Connections

Unit 6 Expressions and Equations

The Power of Balance

As long as there have been coins, there have been coin counterfeiters.

It got so bad in 375 BCE that the Greek city-state of Athens installed a *dokimastes* — an official coin tester — in the marketplace. The job of the dokimastes was to do what you just did — use a balance to sniff out the fakes.

Like counterfeit coins, not everything is always what it seems. In math, as in life, we encounter uncertainty and unknowns.

So, what do we do? Hide under our tunics?

No! We do as the dokimastes did. Whether it's unknowns in trade, history, or different ecosystems, the method is still the same. With what we know about *balance*, we can strip away the mystery, step-by-step, until all that's left is the truth.

Welcome to Unit 6.

604 Unit 6 Expressions and Equations

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Ask students how the different variations of the counterfeit coin problem changed their thinking and strategies, using questions such as:

- "What changes to the scenario had the greatest impact on the number of weighings needed?"
- "Were there any aspects of your strategies that were used consistently, regardless of the number of coins or given information?"

Highlight that *balance* is an important mathematical concept, which was very obvious with weighing the coins, but also as a more abstract representation of the concept of equality. In either case, there are many possible moves, as well as some natural laws guiding what can be done or what makes sense.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "How does the idea of balance relate to math?"

Independent | 5 min

Exit Ticket

Students express their initial understandings of balance, representative of how math is not just about numbers and calculations, but is also a general way of thinking.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 6.01

Reflect on how you thought mathematically (not just with numbers) as you tried to find the smallest number of weighings needed to detect counterfeit coins. What did you do the same, and what did you do differently, as you attempted new scenarios?

Answers may vary.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can see how the idea of balance relates to math.

1 2 3

Success looks like . . .

- **Goal:** Understanding how the concept of balance relates to math, both concretely and abstractly.
  - » Explaining how to use balance when trying to determine the smallest number of weighings needed to detect the counterfeit coins.

Suggested next steps

There are no right or wrong responses, but students should recognize the partitioning of coins into equal groups as relating to math. Exemplary responses would be those that reference repeated reasoning and logic, such as "if, then" thinking, and using words, such as equal or even.

Draw attention back to the general notion of balance, as both a physical representation and a mathematical principle, as it relates to the concepts presented in subsequent lessons. Hanger diagrams will be presented in Lessons 5–6, offering a direct connection to the balances seen here.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to understand how the concept of balance relates to math. How well did students accomplish this? What did you specifically do to help students accomplish it?
- How did students make sense of problems and persevere in solving them today? How are you helping students become aware of how they are progressing in this area?

Practice

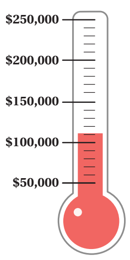
Independent

Name: ______________ Date: ________ Period: ________

Practice

1. Write at least two different mathematical expressions or equations about the thermometer shown. Include either a fraction, a decimal number, or a percentage in each.

FUNDRAISER
OUR GOAL: $250,000



Sample responses:
$110{,}000 \div 250{,}000 = 0.44$
44% of $250,000 is $110,000
$\frac{44}{100} \cdot \$250{,}000 = \$110{,}000$
$0.44 \cdot \$250{,}000 = \$110{,}000$

2. Determine each product.

a. $21.2 \cdot 0.02 = 0.424$

b. $2.05 \cdot 0.004 = 0.0082$

Name: ______________ Date: ________ Period: ________

Practice

3. Calculate $141.75 \div 2.5$ using a method of your choice. Explain your thinking.
56.7
Sample response: I multiplied both numbers by 100 to determine an equivalent expression with whole numbers, and then I used long division to calculate $14{,}175 \div 250 = 56.7$.

4. There are 90 students in the band. Most band members rent their instruments, but the remaining 20% of the band members own their instruments.

a. How many students in the band own their instruments?
18 students; $90 \cdot \frac{1}{5} = 18$

b. How many students in the band rent their instruments?
72 students; $90 - 18 = 72$

c. What percent of students rent their instruments?
80%; $100 - 20 = 80$

5. Mai brought 12 apples to school in the morning and then shared 6 apples with friends at lunch. Write an expression to represent the number of apples Mai had left after lunch.
$12 - 6$

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 3 Lesson 10	2
	2	Unit 5 Lesson 8	2
	3	Unit 5 Lesson 13	2
	4	Unit 3 Lesson 12	2
Formative	5	Unit 6 Lesson 2	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

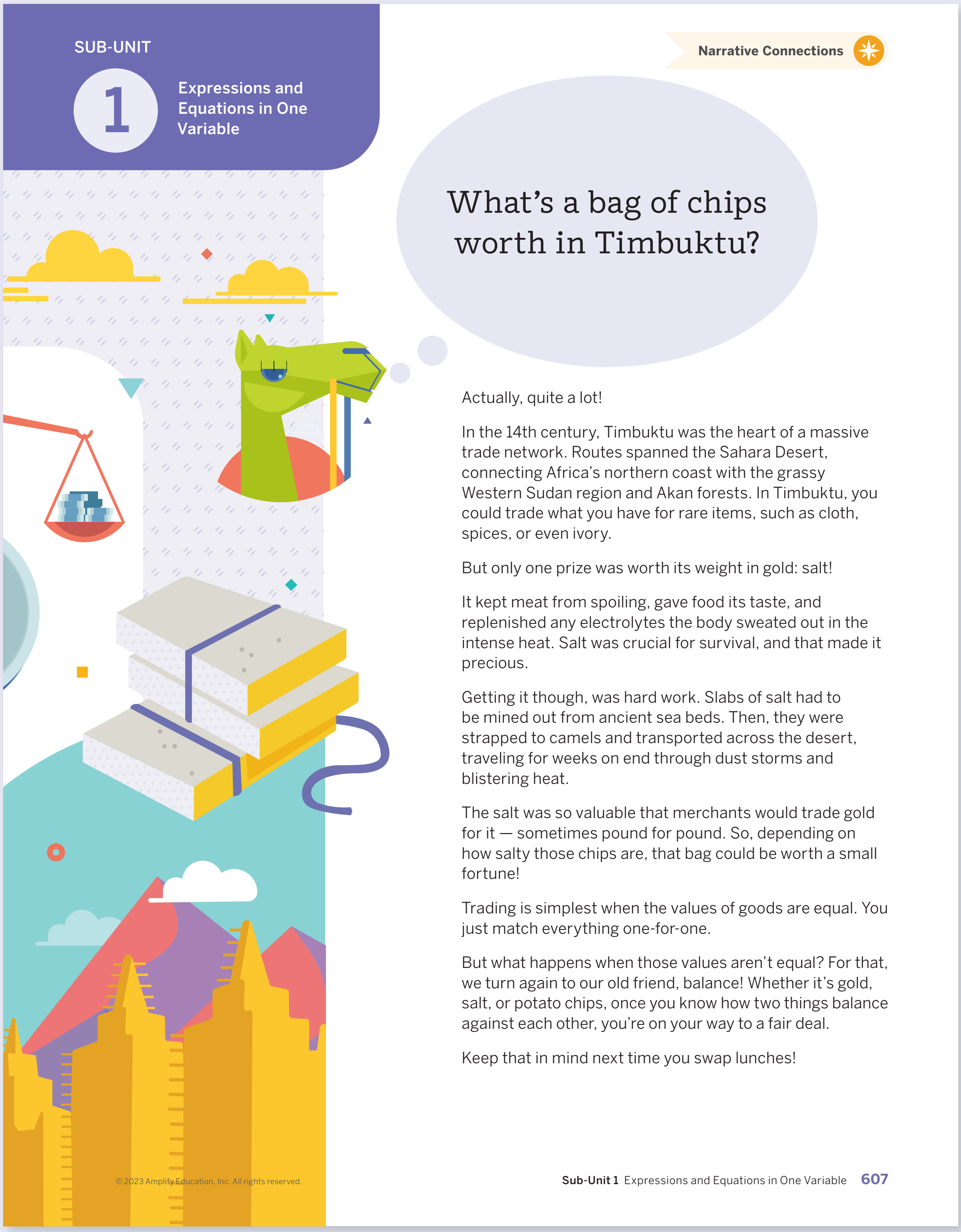
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Whole Class

Sub-Unit 1
Expressions and Equations in One Variable

In this Sub-Unit, students model relationships between quantities, and in story problems, with expressions and equations involving variables. They also represent these using tape diagrams and hanger diagrams, developing an understanding of the properties of equality that preserve balance and lead to solutions.



SUB-UNIT 1 Expressions and Equations in One Variable

Narrative Connections

What's a bag of chips worth in Timbuktu?

Actually, quite a lot!

In the 14th century, Timbuktu was the heart of a massive trade network. Routes spanned the Sahara Desert, connecting Africa's northern coast with the grassy Western Sudan region and Akan forests. In Timbuktu, you could trade what you have for rare items, such as cloth, spices, or even ivory.

But only one prize was worth its weight in gold: salt!

It kept meat from spoiling, gave food its taste, and replenished any electrolytes the body sweated out in the intense heat. Salt was crucial for survival, and that made it precious.

Getting it though, was hard work. Slabs of salt had to be mined out from ancient sea beds. Then, they were strapped to camels and transported across the desert, traveling for weeks on end through dust storms and blistering heat.

The salt was so valuable that merchants would trade gold for it — sometimes pound for pound. So, depending on how salty those chips are, that bag could be worth a small fortune!

Trading is simplest when the values of goods are equal. You just match everything one-for-one.

But what happens when those values aren't equal? For that, we turn again to our old friend, balance! Whether it's gold, salt, or potato chips, once you know how two things balance against each other, you're on your way to a fair deal.

Keep that in mind next time you swap lunches!

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to encounter expressions and equations related to the African salt trade in the following places:

- **Lesson 2, Warm-up:** The African Salt Trade
- **Lesson 4, Activity 1:** Revisiting the Market
- **Lesson 5, Activity 1:** Akan Weights

Write Expressions Where Letters Stand for Numbers

Let's use expressions with variables to describe scenarios.

Focus

Goals

1. Write an expression with a variable to generalize the relationship between quantities in a situation.
2. **Language Goal:** Comprehend that a variable is a letter standing in for a number, and recognize that a coefficient next to a variable indicates multiplication. **(Speaking and Listening, Writing)**
3. **Language Goal:** Describe a situation that could be represented by an expression of the form $x + p$, or px, for a rational number p and unknown x. **(Speaking and Listening)**

Rigor

- Students build **conceptual understanding** of how to use expressions with variables to describe real-world scenarios.

Today

Students begin to formally cross from arithmetic to algebra. They write expressions that record operations with numbers and letters (now defined as variables), which stand in for unknown numbers and can take on different values. They connect stories relating two quantities using expressions, and identify which quantity the variable represents.

Previously

In Grades 3–5, students worked strictly with numerical expressions and equations. They informally used symbols and letters for missing values, and informally determined these values using properties of operations and/or relationships between operations.

Coming Soon

In Lessons 3–6, students will work with algebraic equations, as they progress toward understanding and applying properties of operations to more efficiently solve for variables.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	Independent or Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New words

- *variable*
- *coefficient*

Review word

- *expression*

Amps | Featured Activity

Exit Ticket

Real-Time Exit Ticket

Check in real time whether your students can turn situations into expressions with variables using a digital Exit Ticket.

Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students begin to abstractly connect between mathematical symbols and their real-world meanings, they might feel overwhelmed, especially as they write the scenarios in Activity 2. Each part of the activity is a small step towards connecting the abstract and the quantitative qualities of the scenarios. Each acts as a small goal to help keep them motivated to continue to completion.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, complete Problems 1 and 2 as a class. Then, have pairs complete problem 3.
- In **Activity 2**, have each pair work on either Problem 1 or 2, and then share their responses with the class.

Independent | 10 min

Warm-up The African Salt Trade

Students review translating verbal statements to numerical expressions, using letters for unknown values.

Unit 6 | Lesson 2

Write Expressions Where Letters Stand for Numbers

Let's use expressions with variables to describe scenarios.

Warm-up The African Salt Trade

Write an expression to represent the underlined quantity from each statement. Use a letter to represent any unknown numbers.

Statement	Expression
The <u>total distance a caravan traveled</u>, if it traveled 5 km before sunrise, and 3 more km after sunrise.	$5 + 3$
The <u>total kilograms of salt</u> a small caravan can carry, if each camel can carry a load of 9 kg, and there are 3 camels.	$3 \cdot 9$
The <u>high temperature of the day</u>, if it is 37 degrees warmer than the overnight low temperature.	$a + 37$
The <u>total number of blocks of salt traded</u> for bars of gold, if the number of blocks of salt is equal to one-half the number of bars of gold.	$\frac{1}{2} \cdot b$

Collect and Display: As you explain how you wrote your expressions, your teacher will add any math language you use to a class display. Continue to add to this display throughout this unit.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate prior knowledge by reviewing what an expression is. Then, set an expectation for the amount of time students have to work individually on the activity.

2 Monitor

Help students get started by asking, "What number words do you see? What operation words do you see? How would you write an unknown number?"

Look for points of confusion:

- **Trying to evaluate expressions.** Remind students that they need to write expressions, not determine the solution.
- **Writing > for the phrase *more than*.** Focus on the phrases *more than* (meaning addition) versus *is more than* (an inequality).
- **Not recalling *one-half of* means multiplication.** Ask, "What is $\frac{1}{2}$ of 6? What operation did you use to come up with 3?"
- **Selecting an actual number instead of a letter.** Ask, "Is that the only number you could use?" Tell students that the phrase *a number* indicates an unknown, and should be represented using a letter.

Look for productive strategies:

- Recognizing the underlined text represents the "answer," but expressions should not be evaluated here.
- Knowing which operation to use for each scenario.

3 Connect

Display correct expressions for all four statements.

Have students share the different ways they wrote their expressions and which are equivalent (or not).

Highlight that the phrase *a number* indicates an *unknown* (the term *variable* will be introduced in the next activity). Any letter could be used, and they could be the same or different, even if they represent the same or different values, across scenarios.

MLR Math Language Development

MLR2: Collect and Display

During the Connect, as students share how they wrote their expressions, collect and display the language they use, such as *total, each, warmer, number, sum, product, letter,* etc. Continue adding to this display throughout the unit and encourage students to refer to it during their discussions.

English Learners

Highlight how the term *total* doesn't always indicate addition. In the second and third statements, multiplication is used. In the second statement, "each camel" can carry a load, indicating repeated addition (multiplication) for the 3 camels.

Power-up

To power up students' ability to write an expression to represent a problem, ask:

Read each scenario. Then match it to the expression that can be used to solve the problem.

a. Han has 10 granola bars. He shares 5 with his cousins. How many granola bars does he have left?

b. Han has 10 granola bars. He shares them between 5 of his cousins. How many granola bars does each cousin get?

__b__ $10 \div 5$

__a__ $10 - 5$

Use: Before the Warm-up.

Informed by: Performance on Lesson 1, Practice Problem 5.

Independent or Pairs | 10 min

Activity 1 Known, Known, Unknown

Students write expressions describing situations with an unknown quantity, and recognize how variables can take on multiple values.

Name: ______ Date: ______ Period: ______

Activity 1 Known, Known, Unknown

1. There are 43 shops in the city market of Niani. The cities of Gao, Timbuktu, and Djenne each have more shops than Niani. Complete the table to show how many shops each city has.

Number of gold bars	Gao	Timbuktu	Djenne
How many more shops than Niani?	7	2	d
Number of shops	50	45	$43 + d$

2. A gold bar is worth the same as 12.5 kg of salt. Complete the table to show how much salt, in kilograms, is worth the same as each number of gold bars.

Number of gold bars	2	12	b
Salt (kg)	25	150	$12.5b$

3. Enoch had 31.25 g of salt on Monday. On Tuesday, he received x more grams of salt than he had on Monday. Write an expression to show how much salt Enoch had on Tuesday.
$31.25 + x$

4. Two-fifths of the goods Enyonam is selling at her market stand are bolts of cloth. Write an expression to show the number of bolts of cloth if Enyonam's stand has:
   a. 20 goods.
   $\frac{2}{5} \cdot 20$
   b. x goods.
   $\frac{2}{5} \cdot x$ or $\frac{2}{5}x$

1 Launch

Set an expectation for the amount of time students have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "For each scenario, what are the two quantities involved, and what operation relates them?"

Look for points of confusion:

- **Using incorrect operations.** Have students identify key words, and draw a diagram to check the reasonableness of solutions.
- **Writing multiplication as repeated addition.** Acknowledge this as correct, but inefficient, and ask them to write it another way.

Look for productive strategies:

- Applying formulas consistently by continuing the pattern throughout each example given.
- Testing values to check if the formula is accurate.

3 Connect

Have students share each of their expressions, identifying the information that helped them determine the operation.

Highlight that the "next to" notation represents multiplication by a variable without an operation symbol. There is a repeated structure of common terms and operations among all the expressions within each scenario.

Define a ***variable*** as a letter that represents an unknown number in expressions and equations, and a ***coefficient*** as a number that is multiplied by a variable, typically written in front of and "next to" the variable without an operation symbol.

Ask, "For the tables, if you knew the expressions first, how could they have been used to determine specific values?"

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Extend the tables for Problems 1 and 2. Have students add the values of 1, 2, 3, 4, and 10 in each scenario (gold bars, "how many more shops than Niani)." Ask them to describe what changes and what stays the same for each set of calculations.

Extension: ***Math Enrichment***

Have students complete the following problem:

How would the table from Problem 1 change if the top row was labeled, "How many fewer shops than Niani"?

Gao: 36, Timbuktu: 41, Djenne: $43 - d$

Math Language Development

MLR8: Discussion Supports — Press for details

During the Connect, as students share how they determined what operation(s) to use, provide sentence frames to support their thought organization, such as:

- "To determine the amount of bolts of cloth in Enyonam's market, I would _____ because . . ."
- "To determine the amount of salt Enoch had on Tuesday, I would use _____ because . . ."

Encourage students to press for more detail by asking:

- "So, what I think I heard you say is . . ."
- "Can you tell me more about . . .?"

Pairs | 15 min

Activity 2 Storytime

Students invent scenarios that could correspond to expressions with unknown amounts.

Activity 2 Storytime

Think of a story that might be represented by each expression. For each, state what quantity x represents and what quantity the expression represents.

1. $30 + x$
   a. My story:
   Sample response: My mother and I have the same birth date, but she is 30 years older than me.
   b. In my story, x represents:
   Sample response: my age
   c. In my story, the expression, $30 + x$, represents:
   Sample response: my mother's age

2. $12x$
   a. My story:
   Sample response: I have several cartons of eggs that each contain a dozen eggs.
   b. In my story, x represents:
   Sample response: the number of dozens of eggs
   c. In my story, the expression $12x$ represents:
   Sample response: the number of total eggs

STOP

610 Unit 6 Expressions and Equations

1 Launch

Keep students in the same pairs. Clarify that for every equation, each pair should come up with one story to be recorded.

2 Monitor

Help students get started by asking, "How many quantities should be in your story? Is one always greater than the other? How?"

- **Describing different operations; confusing the variable and the expression.** Have students substitute values for the variable to assess the expression's meaning and reasonableness.
- **Choosing unrealistic quantities.** While there are no incorrect scenarios, discuss the appropriateness of chosen values in the context of their stories.

Look for productive strategies:

- Writing stories that correctly represent each term in the equation.

3 Connect

Have students share their group's story for each expression, what quantity x represents, and what quantity the expression represents.

Highlight that the same variable x was used in each expression, but does not represent the same quantity, or value, across expressions.

Ask, "How do the values 30 and 12 fit into some of the stories shared? What do they represent?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider changing the expressions to $1 + x$ and $2x$ if students need more support thinking of realistic or reasonable meanings for the values of 30 and 12 in the original expressions.

Extension: *Math Enrichment*

Challenge students to create a new scenario that involves three related quantities, where x represents the same quantity throughout, and the other two quantities make sense as $30 + x$ and $12x$. Sample response: x represents the number of dozens of eggs, $12x$ represents the number of total eggs, and $30 + x$ represents the total number of dozens of eggs in the inventory.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, have pairs of groups meet to share and receive feedback on their stories and what the variables represent. Provide these prompts for feedback to help strengthen ideas and clarify language.

- "How are the parts of the expression represented in your story?"
- "How does the operation fit into your story?"

Allow time to complete a final draft based on feedback.

English Learners

Provide time for students to brainstorm contexts for their stories. Consider offering a "menu" of story contexts to help students get started.

Whole Class | 5 min

Summary

Review and synthesize how variables are used to represent unknowns in expressions and stories.

Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You wrote expressions where letters epresented numbers. We often use a letter, such as x or a, as a placeholder for an unknown number in expressions. This letter is called a ***variable***. For example, in the expression $u + 1$, the variable is u.

When a number is written "next to" a variable without an operation symbol, the number and the variable are multiplied. A number written next to a variable in this way is called a ***coefficient***. For example, $7x$ means the same as $7 \cdot x$. The variable is x, and the coefficient is 7.

If no coefficient is written next to the variable, the coefficient is 1. For example, in the expression $p + 3$, the coefficient of p is 1. You could also write $1p$, but it is not necessary because $1 \cdot p = p$.

Consider the following expressions:

Expression	Variable	Coefficient
$2x + 8$	x	2
$y - 4$	y	1

Reflect:

Synthesize

Display the expressions $p + 3$ and $7x$ to support the discussion.

Highlight that when two quantities are related mathematically, a variable can be used as a placeholder when writing an expression for that relationship. You can choose different numbers for the quantity represented by the variable, and then evaluate the expression to determine the corresponding value for the other quantity.

Ask, "What is the coefficient of p in the expression $p + 3$? What is the value of $7x$ when x is 5? What operation did you use to determine this?"

Formalize vocabulary:

- *variable*
- *coefficient*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did you use variables today?"
- "How did your work today build upon your previous experiences with expressions?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on any terms and phrases related to the terms *variable* and *coefficient* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by matching an algebraic expression to a given scenario, and describing a scenario for a given expression.

Amps Featured Activity | Real-Time Exit Ticket | Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.02

A plant was 8 in. tall last week, and grew x in. this week.

1. Which expression represents how tall, in inches, the plant is now?

A. $x + 8$

B. $8x$

Explain your thinking.

Answers may vary, but should focus on the plant growing additively or not growing multiplicatively.

Sample response: The plant is x inches taller than 8, which would be written as $8 + x$, but this is the same as $x + 8$.

2. For the expression not chosen, describe a scenario the expression might represent.

Answers may vary, but should focus on two quantities where one is multiplicatively greater, or a certain number of times greater, than the other quantity.

Sample response: A second plant is always 8 times as tall as the first plant. When the first plant is x inches tall, the second plant will be $8x$ inches tall.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write an expression with a variable to represent a calculation in which I do not know one of the numbers.
1 2 3

b. I can match an expression to a real-world scenario that it could represent.
1 2 3

c. When I see an expression, I can make up a story that it might represent and explain what the variable represents in the story.
1 2 3

Lesson 2 Write Expressions Where Letters Stand for Numbers

Success looks like . . .

- **Goal:** Writing an expression with a variable to generalize the relationship between quantities in a situation.
  - » Selecting the expression that represents the present height of the plant.
- **Language Goal:** Comprehending that a variable is a letter standing in for a number, and recognizing that a coefficient next to a variable indicates multiplication. **(Speaking and Listening, Writing)**
- **Language Goal:** Describing a situation that could be represented by an expression of the form $x + p$, or px, for rational number p and unknown x. **(Speaking and Listening)**
  - » Describing a scenario that represents a quantity as 8 times greater than the quantity x.

Suggested next steps

If students select the incorrect expression for Problem 1, consider:

- Reviewing critical mathematical cue words in both statements from Activity 1, Problem 1.
- Assigning Practice Problem 3.

If students use additive (not multiplicative) language in their scenario for Problem 2, consider:

- Reviewing critical mathematical cue words in Activity 1, Problem 2.
- Asking, "In the term $8x$, 8 is the coefficient of x. What operation does that represent? How can your story show a multiplicative action?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did Activity 2 reveal about your students as learners?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

1. Instructions for a craft project indicate that the length of a piece of red ribbon should be 7 in. less than the length of a piece of blue ribbon. Determine the length of the blue ribbon if the length of the red ribbon is:

   a. 10 in.
   17 in., or $(10 + 7)$ in.

   b. 27 in.
   34 in., or $(27 + 7)$ in.

   c. x in.
   $(x + 7)$ in.

2. Gadisa has 3 times as many books as Folami. Determine the number of books that Gadisa has if Folami has:

   a. 15 books.
   45 books, or $3 \cdot 15$ books

   b. 21 books.
   63 books, or $3 \cdot 21$ books

   c. x books.
   $3x$ books, or $3 \cdot x$ books

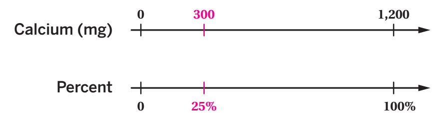
3. A bottle can hold 24 oz of water. It currently holds x oz of water.

   a. What does $24 - x$ represent in this situation?
   Sample response: The number of ounces needed to fill the bottle completely

   b. Write a question about this situation that has $24 - x$ as the answer.
   Sample response: How many more ounces of water can the bottle hold?

Name: Date: Period:

4. The daily recommended allowance of calcium for a sixth grader is 1,200 mg. One cup of milk has 25% of the recommended daily allowance of calcium. How many milligrams of calcium are in a cup of milk? Consider using the double number line to help with your thinking.



300 mg

5. A trash bin has a capacity of 50 gallons. What percent of its capacity is each amount? Show your thinking.

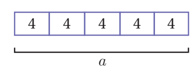
   a. 5 gallons
   10%; $\frac{5}{50} \cdot 100 = 10$

   b. 30 gallons
   60%; $\frac{30}{50} \cdot 100 = 60$

   c. 45 gallons
   90%; $\frac{45}{50} \cdot 100 = 90$

   d. 100 gallons
   200%; $\frac{100}{50} \cdot 100 = 60$

6. Lin, Shawn, and Tyler each write an equation to represent the given diagram.



Shawn's equation: $4 + 4 + 4 + 4 + 4 = a$
Lin's equation: $4 \cdot 5 = a$
Tyler's equation: $5 + 4 = a$
Which student is correct? Explain your thinking.
Lin and Shawn; Sample response: There are five segments each labeled with 4 and together they make a. Lin showed that the five 4's could be added, while Shawn showed the same relationship through multiplication.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 11	2
	5	Unit 3 Lesson 13	2
Formative	6	Unit 6 Lesson 3	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 6 | **LESSON 3**

Tape Diagrams and Equations

Let's see how tape diagrams and equations can show relationships between amounts.

Focus

Goals

1. Draw tape diagrams to represent equations of the forms $x + p = q$ and $px = q$.
2. **Language Goal:** Interpret tape diagrams that represent equations of the form $p + x = q$ or $px = q$. **(Speaking and Listening, Writing)**

Coherence

Today

Students build on previous work with tape diagrams to represent operations with letters standing in for numbers. These tape diagrams serve as a tool to help students visualize the relationships between quantities represented by expressions and equations. Students interpret the structure of tape diagrams and create their own, based on the relationships in given story problems.

Previously

Since Grade 3, students have used tape diagrams to represent operations with numbers. In Lesson 2 of this unit, students wrote numerical and algebraic expressions with letters standing in for numbers, to represent scenarios and to relate operations with numbers.

Coming Soon

In Lesson 4, students will continue to leverage tape diagrams for determining and recognizing solutions to equations with unknown values. Later, in Lessons 5–9, students will solve a variety of real-world and mathematical problems by writing and solving equations in which tape diagrams may still prove to be a useful strategy.

Rigor

- Students develop **conceptual understanding** connecting expressions to equations using tape diagrams.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Pairs	Independent or Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- *Tape Diagrams* PDF

Math Language Development

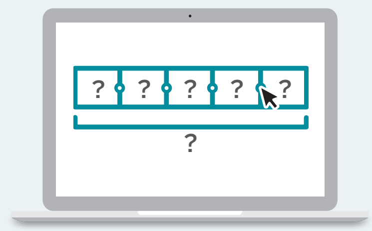
Review words

- *tape diagram*
- *expression*
- *equation*
- *variable*
- *coefficient*

Amps | Featured Activity

Activity 2
Dynamic Tape Diagrams

Students can create digital tape diagrams, and you can overlay them all to see similarities and differences at a glance.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may not immediately connect the tape diagram and an equation in Activity 1. As they practice writing an equation, or even more than one equation, for each diagram, their confidence will increase in their ability to use the mathematical structure of each. By accessing their own strengths, they can begin to discern the patterns that they will apply in all of their algebraic reasoning.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 2**, Part B may be omitted. Part A could also be modified by having each partner work on either Problem 1 or 2, and then share.

Independent | 5 min

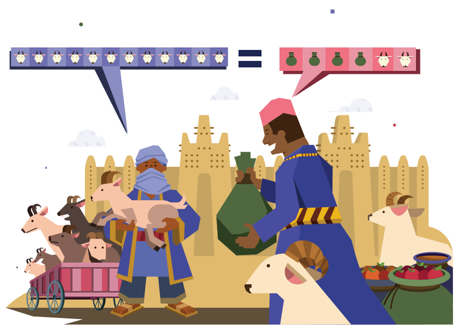
Warm-up Moving From Expressions to Equations

Students recall how tape diagrams can be used to represent addition and multiplication relationships.

Unit 6 | Lesson 3

Tape Diagrams and Equations

Let's see how tape diagrams and equations can show relationships between amounts.



Warm-up Moving From Expressions to Equations

1. Write an expression that could be represented by each tape diagram.

Tape diagram	Expression
a. 2 | 2 | 2 | 2 | 2	$5 \cdot 2$ $2 \cdot 5$ $2+2+2+2+2$
a. 2 | 5	$2+5$

2. Label the length of each tape diagram. Write as many equations as you can that could be represented by each tape diagram.

Tape diagram	Equations
a. 2 | 2 | 2 | 2 | 2	$5 \cdot 2 = 10$ $2 \cdot 5 = 10$ $2+2+2+2+2=10$
b. 2 | 5	$2+5=7$ $5+2=7$ $7-2=5$ $7-5=2$

Log in to Amplify Math to complete this lesson online.

614 Unit 6 Expressions and Equations

1 Launch

Set an expectation for the amount of time students will have to work individually on the Warm-up.

2 Monitor

Help students get started by asking, "How is the first pair of tape diagrams different from the second pair? How are they similar?"

Look for points of confusion:

- **Not identifying the 5 in the first diagram.** Ask, "How many 2s do you see?"
- **Not making the jump from expressions to equations.** Ask, "What changed from the first diagram to the third?

Look for productive strategies:

- Using repeated addition. Ask, "Is there a more efficient way you could represent this sum?"
- Representing repeated addition as multiplication.

3 Connect

Have students share expressions that represent each diagram, then equations that represent each diagram.

Ask:

- "How do you see the 5 represented in the first diagram?"
- "How did you determine the total length for the first diagram in Problem 2? Where does that total length appear in the equation?"
- "How do the expressions from Problem 1 relate to the equations from Problem 2? How are they alike? How are they different?"

Highlight that boxes within a tape diagram can always be interpreted as addends whose sum is equal to the total length, and that same-sized boxes can also be represented by multiplication.

Differentiated Support

Accessibility: ***Optimize Access to Tools***

Provide counters or other objects that students could use to create physical models related to the tape diagrams. For example, for Problem 1, students could use 10 counters and place 2 counters in each group to help visualize the types of expressions that can be written.

Power-up

To power up students' ability to create an equation to represent a diagram, ask:

Which of the following expressions represents the diagram? Select *all* that apply.

A. $3+7=21$

B. $7+7+7=21$

C. $7 \div 21 = 3$

D. $3 \times 7 = 21$

E. $21 \div 7 = 3$

7 | 7 | 7

21

Use: Before the Warm-up.

Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6.

Pairs | 10 min

Activity 1 Tape Diagrams With Variables

Students use relationships between operations to construct multiple equations represented by the same tape diagram.

Name: ______ Date: ______ Period: ______

Activity 1 Tape Diagrams With Variables

Write as many different equations as you can that could be represented by each tape diagram.

Tape diagram	Equations
x | x | x | x — 12	$x + x + x + x = 12$ $4x = 12$ or $12 = 4x$ $4 \cdot x = 12$ or $12 = 4 \cdot x$ $12 \div 4 = x$ or $12 \div x = 4$
4 | x — 12	$4 + x = 12$ or $12 = 4 + x$ $12 - 4 = x$ or $x = 12 - 4$ $12 - x = 4$ or $4 = 12 - x$

Reflect: How did you apply your strengths to write the equations?

Historical Moment

The history of variables . . . varies

Why letters? Why x? Depending who you ask, or what you read, you may hear quite different stories. And to some extent the only fact is, no one *really* knows.

The Ancient Greeks, such as Aristotle and Diophantus, used letters to represent numbers in the third and fourth centuries BCE. So did a smattering of Europeans from the 1200s CE all the way up to one name you might often hear as an answer to those questions – René Descartes (1596–1650 CE). He may have been the first to publish a widely read work, *La géometrie*, that clearly used variables consistently, to represent both knowns and unknowns, and yes x was one of them.

But symbolic algebra has also been traced to ancient India (c. 800–1000 CE), however much of that work has not been preserved. Whoever it was, or all of the whoevers, definitely contributed to the development of algebra as it is still studied today. And if you know a number, call it x, can be added to $1{,}928.37$, call it y, to make $9{,}182.73$, call it z, it is sure a relief to just simply write $x + y = z$. Of course, that doesn't get you the answer of what the value of x is. What is the value of x? **7254.36**

1 Launch

Activate prior knowledge and remind students that x is a *variable*, standing in for an unknown number. Have students use the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by asking, "What does the 12 represent? Where does 12 fit into an equation representing the diagram?"

Look for points of confusion:

- **Not identifying the 4 in the first diagram.** Ask, "How many xs do you see?"
- **Not yet using "next to" notation.** Remind students that $4 \cdot x$ can be written as a variable with a coefficient, $4x$.
- **Thinking only one equation is possible.** Remind students to think about inverse operations.
- **Thinking x must represent the same value in all of the equations.** Students are not expected to "solve" these equations yet, but note that "the same variable can be used to represent different values in different scenarios."

Look for productive strategies:

- Using repeated addition. Ask, "Is there a more efficient way you could represent the sum $x + x + x + x$?"

3 Connect

Display the tape diagrams.

Have students share one equation at a time, until all possible valid equations have been recorded for each diagram.

Highlight that a multiplicative relationship can also be expressed using division, and an additive relationship can also be expressed using subtraction.

Ask:

- "How are all of the equations for each tape diagram related?"
- "How are the two diagrams the same? Different?"

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Have students refer to the tape diagrams and expressions from the Warm-up. Ask them how they can use what they did in the Warm-up to help them write equations in this activity. Provide counters or other objects that students could use to create physical models related to the tape diagrams.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their equations, relate the different equations and how they represent the same tape diagram. Ask:

- "Where do you see addition in each tape diagram? Multiplication? Subtraction? Division?"
- "How are the addition and subtraction equations related? Multiplication and division?"
- "How is addition related to multiplication?"

Historical Moment

The history of variables . . . varies

Have students read about the history of variables.

Independent or Pairs | 20 min

Activity 2 Storytime With Tape Diagrams

Students connect scenarios with an unknown amount to tape diagrams and equations.

Amps Featured Activity Dynamic Tape Diagrams

Activity 2 Storytime With Tape Diagrams

Part A

Draw a tape diagram to represent each story. Then use your drawings to determine which of the four equations best represents each story.

$x + 5 = 20$ $\quad$ $x = 20 + 5$ $\quad$ $5 \cdot 20 = x$ $\quad$ $5x = 20$

1. After Amara sold 5 kg of salt at the market in Gao on Friday, she had sold a total of 20 kg of salt for the week. She sold x kg before Friday.

Tape diagram	Equation
$\underbrace{\boxed{\quad x \quad}\boxed{5}}_{20}$	$x + 5 = 20$

2. Kofi traded some salt for 20 gold bars at the market in Timbuktu, which is 5 times as many as he was able to trade in Djenne. He received x gold bars when he traded in Djenne.

Tape diagram	Equation
$\underbrace{\boxed{x}\boxed{x}\boxed{x}\boxed{x}\boxed{x}}_{20}$	$5x = 20$

Part B

3. Choose one of the remaining equations *not* used in Part A and create your own story that represents the equation. Then draw a tape diagram for the equation.

Sample response: $5 \cdot 20 = x$. Noah's sister has 5 times as many pieces of gold as he does. Noah has 20 pieces of gold. How many pieces of gold does his sister have?

$\underbrace{\boxed{20}\boxed{20}\boxed{20}\boxed{20}\boxed{20}}_{x}$

STOP

616 Unit 6 Expressions and Equations

1 Launch

Give students 5 minutes of quiet work time for Part A. Then, have students compare responses with a partner and then work together to complete Part B.

2 Monitor

Help students get started by asking, "What quantity is not known in the first story?"

Look for points of confusion:

- **Drawing unreasonably sized parts.** Refer to previous diagrams as examples. Ask, "Which part should be larger here, the x or the 5?"
- **Being misled by key words.** If students interpret *total* to mean they need to add 5 and 20, or 5 *times as many* to mean they need to multiply 20 by 5, suggest they act out the scenarios or draw literal pictures to represent each scenario.
- **Not knowing how to interpret $5x$.** Explain that $5x$ is the same as $5 \cdot x$, and is read as "5 times x."
- **Not knowing what the x represents.** Say, "The variable x represents the unknown value." Ask, " In this case, is it a part or the total?"

Look for productive strategies:

- Drawing tape diagrams that represent the equations. Students may draw all four tape diagrams first, and then match them to the scenarios before writing equations.

3 Connect

Have students share their tape diagrams, equations, and thinking for each story in Part A, then their story problem for Part B, and by which equation each can be represented.

Ask, "All of the equations and scenarios involve the same three numbers. Did you rule out any equations right from the start? Which ones? Why?"

Highlight the connections among each scenario, a corresponding tape diagram, and an equation.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital tape diagrams. You can overlay them to see similarities and differences at a glance.

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide students with copies of the *Tape Diagrams* PDF that they can use to partition pre-made blank tape diagrams. For Problem 3, display a sentence frame to represent $x = 20 + 5$ that mirrors Problem 1, but has the values and variables replaced with blanks that students can fill in.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their tape diagrams and equations, draw their attention to the connections among each scenario, and their corresponding tape diagrams and equations. Ask these questions to help strengthen students' mathematical language use and reasoning with multiple representations.

- "What is similar about the equations used for Part A?"
- "How are these different from the equation used in Part B?"

English Learners

Annotate key words and phrases in the text for Problems 1 and 2, such as "sold a total of" and "5 times as many."

Whole Class | 5 min

Summary

Review and synthesize the connections between the visual appearance of tape diagrams and the equations they represent.

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You used tape diagrams to help you visualize relationships between quantities. Here are two examples:

Tape diagram	Interpretations	Equations
x | x | x, total 21	3 equal parts whose sum is 21	$x + x + x = 21$
	A number multiplied by 3, for which the product is 21	$3 \cdot x = 21$ $3x = 21$
y | 3, total 21	Two unequal parts, 3 and another number, whose sum is 21	$y + 3 = 21$
	3 taken away from the total of 21	$21 - 3 = y$
	Two unequal parts, 21 and another number, whose difference is 3	$21 - y = 3$

Reflect:

Synthesize

Display the summary table showing the two tape diagrams.

Ask:

- "How are tape diagrams useful in visualizing a relationship between two quantities?"
- "How can you tell whether a tape diagram represents an addition or multiplication relationship?"
- "What element of a tape diagram relates to the equal sign that would be in the equation the diagram represents?"
- "Why can a diagram be represented by more than one equation?"
- "The top diagram can be seen as either addition or multiplication. Why might multiplication be the better operation to use here?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did you use tape diagrams today?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how tape diagrams and equations can show relationships between values.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 6.03

Complete the diagrams so that the first diagram represents the equation $5 \cdot x = 15$ and the second diagram represents the equation $5 + y = 15$.

a $5 \cdot x = 15$

x	x	x	x	x

15

b $5 + y = 15$

5	y

15

Self-Assess

1 I don't really get it — 2 I'm starting to get it — 3 I got it

a I can tell whether a tape diagram could represent an equation. 1 2 3

b I can use a tape diagram to represent a story. 1 2 3

Success looks like . . .

- **Goal:** Drawing tape diagrams to represent equations of the forms $x + p = q$ and $px = q$.
- **Language Goal:** Interpreting tape diagrams that represent equations of the form $p + x = q$ or $px = q$. **(Speaking and Listening, Writing)**
  - » Completing each tape diagram to represent the given equation.

Suggested next steps

If students are unclear about how to partition the tape diagrams based on the operations in each equation, consider:

- Referring to Activity 1 and how the operation was identified based on how the diagrams were partitioned.
- Asking, "How did you determine which tape diagram represented addition and which represented multiplication? How can that help you partition these tape diagrams?"

If students are having difficulty with the variable, consider:

- Asking "What times 5 gives 15? Where would you place the 3 in the tape diagram?" Then have them take the 3 out and replace it with the variable x, showing that the x is representing the 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What routines enabled all students to do math in today's lesson?
- The focus of this lesson was connecting tape diagram representations to equations. How did students make those connections? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ________ Date: ________ Period: ________

1. Consider the equation: $x + 4 = 17$.

   a. Draw a tape diagram to represent the equation. All three parts of the equation should be identified.

x	4

   17

   b. How does your diagram show that $x + 4$ has the same value as 17?
   Sample response: The diagram shows that $x + 4$ has the same value as 17 because the total length of the tape diagram is labeled.

2. Diego is trying to determine the value of x in the equation $5 \cdot x = 35$. He draws this diagram, but is not certain how to proceed.

x	x	x	x	x

   35

   a. Complete the tape diagram so it represents the equation $5 \cdot x = 35$.

   b. Determine the value of x.
   $x = 7$

Practice

Name: ________ Date: ________ Period: ________

3. For each tape diagram, write which of the following equation(s) could represent it. Write as many equations as you can.

$x + 3 = 9$	$3 \cdot x = 9$	$9 = 3 \cdot x$	$x + x + x = 9$
$3 + x = 9$	$x = 9 - 3$	$x = 9 \div 3$	

Tape diagram	Equation(s)
a. x | x | x ; 9	$3 \cdot x = 9$ $9 = 3 \cdot x$ $x + x + x = 9$ $x = 9 \div 3$
b. x | 3 ; 9	$x + 3 = 9$ $3 + x = 9$ $x = 9 - 3$

4. A shopper paid \$2.52 for 4.5 lb of potatoes, \$7.75 for 2.5 lb of broccoli, and \$2.45 for 2.5 lb of pears. What is the unit price of each item she bought? Explain your thinking.
   Potatoes: \$0.56 per pound
   Broccoli: \$3.10 per pound
   Pears: \$0.98 per pound
   I need to determine how much one lb of each item costs. I can do this by dividing the total dollar amount by the total weight.

5. A sports drink bottle contains 16.9 fluid ounces. Andre drank 80% of the bottle. How many fluid ounces did Andre drink? Explain your thinking.
   Andre drank 13.52 fluid ounces.
   I need to determine 80% of the total fluid ounces, and I can do this by multiplying $0.80 \cdot 16.9$.

6. Is the following equation true or false? Explain your thinking.
   $a + 6 = 11$, when $a = 5$
   The equation is true because when I substitute 5 for a, $5 + 6 = 11$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 5 Lesson 13	3
	5	Unit 3 Lesson 13	3
Formative	6	Unit 6 Lesson 4	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Truth and Equations

Let's represent stories with equations, and see what makes them true or false.

Focus

Goals

1. **Language Goal:** Generate values that make an equation true or false and justify whether they are *solutions to the equation*. **(Speaking and Listening, Writing)**
2. Use substitution to determine whether a given number makes an equation true.

Coherence

Today

Students explore what it means for an equation with a variable to be true or false. They use systematic substitution to determine whether a number is a *solution to an equation*, making the equation true, by evaluating to check for equality. Students further demonstrate their understanding by explaining how they determine values that make equations true or false, defending their results, and also exploring and critiquing the work of others during a ***Gallery Tour*** routine.

Previously

In Lesson 3, students used variables to stand in for unknown numbers in both tape diagrams and equations.

Coming Soon

In Lesson 6, students will begin to discover and develop systematic ways to determine solutions to equations without having to guess and check.

Rigor

- Students continue to build **conceptual understanding**, writing equations to represent scenarios.
- Students build **procedural fluency** determining the solution for equations.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- poster paper

Math Language Development

New word

- ***solution to an equation***

Review words

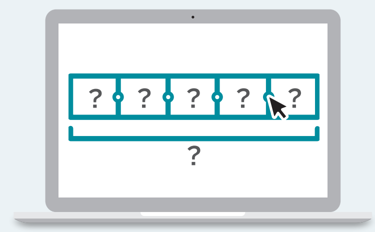
- *variable*
- *coefficient*

Amps | Featured Activity

Warm-up

Dynamic Tape Diagrams

Students can create digital tape diagrams, and you can overlay them all to see similarities and differences at a glance.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might attempt to evaluate each equation for all of the values posted in Activity 2. Point out that, once the truth, or the true value, is known, the rest will be considered false, or lies. Discuss that truth-telling is an ethical responsibility of all citizens. Continue to have students evaluate why the truth is so important. Ask students to consider what would happen if they all made decisions on lies, rather than the truth.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In the **Warm-Up**, one problem may be omitted.
- In **Activity 2**, small groups can determine and share the solution that makes their equation true, omitting the ***Gallery Tour*** routine and "two lies" components.

Independent | 10 min

Warm-up Making Equations True or False

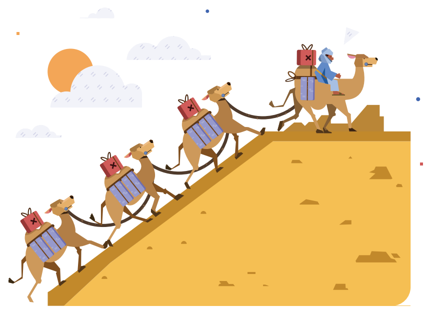
Students substitute different values for variables in an equation in order to determine what it means for an equation to be true or false.

Amps Featured Activity Dynamic Tape Diagrams

Unit 6 | Lesson 4

Truth and Equations

Let's represent stories with equations, and see what makes them true or false.



Warm-up Making Equations True or False

1. The equation $a + b = c$ could be true or false.

   a. Determine values to replace a, b, and c that make the equation true. Draw a tape diagram that represents your equation to help you with your thinking.
   Sample response: $12 + 9 = 21$

12	9
21	

   b. Determine values to replace a, b, and c that make the equation false.
   Sample response: $15 + 4 = 20$

2. The equation $x \cdot y = z$ could be true or false.

   a. Determine values to replace x, y, and z that make the equation true. Draw a tape diagram that represents your equation to help you with your thinking.
   Sample response: $3 \cdot 5 = 15$

3	3	3	3	3
15				

   b. Determine values to replace x, y, and z that make the equation false.
   Sample response: $3 \cdot 4 = 16$

Log in to Amplify Math to complete this lesson online.

620 Unit 6 Expressions and Equations

1 Launch

Set an expectation for the amount of time students will have to work individually on the Warm-up.

2 Monitor

Help students get started by saying, "You don't know the values, so choose some and substitute them into the equations." Offer some values, such as, $a = 3$, $b = 4$, $c = 5$ for Problem 1 and $x = 3$, $y = 4$, $z = 12$ for Problem 2.

Look for points of confusion:

- **Making a calculation error.** Have students check their computation to make sure the equation is identified correctly.
- **Drawing an incorrect diagram or placing variables and values incorrectly.** Remind students how to draw a tape diagram for addition.

Look for productive strategies:

- Substituting several numbers, in logical or sequential ways, into the equations, demonstrating repeated reasoning.

3 Connect

Have students share their values for both true and false equations. Ensure students use appropriate vocabulary (*sum*, *addend*, *product*, *factor*, *groups*, *size of the group*) as they share.

Highlight that equations can be true or false, and the truth of an equation with variables depends on the value(s) of the variable(s).

Ask, "What makes an equation true? What makes it false?"

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their values, display examples of true equations and false equations using numerical values. Annotate the equations as *true* or *false*. Ask students to identify parts of the equations and relationships among values, such as *sum, addend, product, factor, groups*, and *size of the group*. Add this language to the class display.

Power-up

To power up students' ability to use substitution to evaluate whether an equation is true for a given value, ask:

Recall that to evaluate an expression for a given value, you must first replace the variable in the expression with the given value, then evaluate the resulting expression.

Evaluate the following expressions for $b = 4$.

a. $b + 7 = (4) + 7 = 11$

b. $19 - b = 19 - (4) = 15$

c. $7 \cdot b = 7 \cdot (4) = 28$

d. $20 \div b = 20 \div (4) = 5$

Use: Before the Warm-up.

Informed by: Performance on Lesson 3, Practice Problem 6.

Activity 1 Revisiting the Market

Students revisit expressions from Lesson 2, but are now given additional information that allows them to write equations and determine solutions to the equations.

Name: ______________ Date: __________ Period: __________

Activity 1 Revisiting the Market

Complete the following problems about the salt and gold trade in Mali.

1. Priya brought back bars of gold from the market, and each bar weighs 12.5 kg. The total weight of the gold she brought back is 75 kg. You want to determine how many bars of gold Priya brought back from the market.

   a. Write an equation for this scenario using g as the variable.

   $12.5g = 75$

   b. If you substitute 5 or 6 as the value for g, does either value make the equation true? Be prepared to explain your thinking.

   The value 6 makes the equation true.

2. The average family in Timbuktu needs a minimum of 14.8 kg of salt to preserve a month's worth of food. Noah brought back 15.25 kg of salt from the market at the beginning of the month. You want to determine how much more he brought back than the minimum needed.

   a. Write an equation for this scenario, using s as the variable.

   $14.8 + s = 15.25$

   b. If you substitute 0.45 or 0.5 as the value for s, does either value make the equation true? Be prepared to explain your thinking.

   The value 0.45 makes the equation true.

1 Launch

Remind students of the markets in the ancient Mali scenarios from Lesson 2. Say, "You will now be using some additional information for each scenario to answer some new questions."

2 Monitor

Help students get started by presenting the first table from Lesson 2 and asking, "How would 75 fit into the table? What do you not know in this scenario?"

Look for points of confusion:

- **Thinking the word *total* in Problem 1 means they need to add 12.5 and 75.** Suggest students draw a picture of the scenario.

Look for productive strategies:

- Writing and performing multiplication and addition appropriately, as each is represented by the scenarios.

3 Connect

Have students share an equation for each scenario, and how they came up with that equation. Then, for each equation, discuss how they determined whether each of the two possible values was a solution to the equation. Note students' use of the terms *solution to an equation*, *true*, and *false*.

Define a ***solution to an equation*** as a number that can be substituted in place of the variable to make the equation true.

Ask, "What happened when you substituted the given values into each equation?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students begin with a smaller multiple than 75, or a whole number multiple, such as 9 or 45.

Extension: *Math Enrichment*

Have students determine values for each variable that make the equation $a + b = x \cdot y$ true. Sample response: $a = 5$, $b = 3$, $x = 2$, and $y = 4$.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their equations and whether the given values are solutions to their equations, draw their attention to the connections between the story problems and the parts of the equations.

Ask:

- "What operation did you use in your equation for this problem?"
- "Why did you choose that operation? What clues did you look for?"

English Learners

Students may be unfamiliar with the term *minimum* in Problem 2. Annotate this word by writing *at least* next to it.

Small Groups | 15 min

Activity 2 Two Lies and a Truth

Students work in groups to determine values that make equations true or false, and are exposed to additional equations via a ***Gallery Tour*** routine.

Activity 2 Two Lies and a Truth

Your group will be assigned one of the following equations containing a variable. Circle the equation you are assigned.

$18 = 3 + z$	$1000 - a = 400$	$h + \frac{3}{7} = 1$	$10c = 1$
$\frac{2}{3}d = \frac{10}{9}$	$10 = 0.5f$	$12.6 = b + 4.1$	$18 = 3q$

Determine a *solution to the equation* — a value that makes the equation true. Then determine two values that make the equation false. Record your group's equation and true/false values.

Equation	True	False
$18 = 3 + z$	15; $z = 15$ or $18 = 3 + 15$	5; $18 = 3 + 5$ 14; $18 = 3 + 14$

Complete the table as you visit each group's poster.

Equation	Solution to the equation	False values
$18 = 3 + z$	15	All false answers are sample responses. 5, 10
$1{,}000 - a = 400$	$a = 600$	1400, 60
$h + \frac{3}{7} = 1$	$\frac{4}{7}$	$\frac{13}{7}$, $\frac{10}{7}$
$10c = 1$	$\frac{1}{10}$	$\frac{10}{10}$, 10
$\frac{2}{3}d = \frac{10}{9}$	$\frac{5}{3}$	$\frac{5}{2}$, $\frac{8}{6}$
$10 = 0.5f$	20	50, 2
$12.6 = b + 4.1$	8.5	16.7, 18.5
$18 = 3q$	6	54, 2

STOP

622 Unit 6 Expressions and Equations

1 Launch

Post the eight equations around the room. Arrange students in eight small groups and assign each group an equation. Explain that each group's first job is to post the value that makes their equation true and two values that make it false — next to their equation — without labeling them as true or false. Then, a ***Gallery Tour*** will allow other students to determine which of the values posted for each equation is a solution to the equation, and complete the table.

2 Monitor

Help students get started by suggesting that they substitute a number, such as 3, for their variable. Then consider asking, "Does that number make the equation true? If not, do you think you should try a greater or lesser number next?"

Look for points of confusion:

- **Incorrectly applying the operation in an equation to both given numbers rather than to the variable term.** Encourage students to express the relationship of the equation in words and/or draw a diagram to represent the equation.

Look for productive strategies:

- Recognizing that once a solution is found, no other value will make the equation true because the same operations would be applied.

3 Connect

Have students share, for an equation that was *not* theirs, which values are or are not solutions, showing or explaining their work. Encourage students' precise use of the terms ***solution to the equation***, *true*, and *false*.

Highlight that, in this activity, students generated different values to substitute for a variable. For each equation, they found the one solution that made it true.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to select values to test into the first equation, $18 = 3 + z$. Test a value that makes the equation true, such as $z = 15$, and a value that makes the equation false, such as $z = 4$.

Extension: *Math Enrichment*

Have each student pair up with one other group member to come up with "two truths and a lie" for the equation $a - b = 6$. Then ask them to challenge their other group members to find the lie. Sample response: Truth, $a = 10$, $b = 4$. Lies, $a = 3$, $b = 3$ and $a = 12$, $b = 2$.

Math Language Development

MLR8: Discussion Supports — Revoicing

During the Connect, as students share values that are not solutions to some of the equations, revoice their statements and reasoning to demonstrate the use of precise mathematical language. For example, if a student says, "500 doesn't work for the equation $1{,}000 - a = 400$,

Ask:

- "What do you mean by *doesn't work*? What math terms can you use to describe this?"
- "Do you mean the value 500 *is not a solution to the equation*?"
- "Do you mean the value 500 *makes the equation false*?"

Whole Class | 5 min

Summary

Review and synthesize the idea that equations can be true or false, and values can be substituted for the variable to determine the solution to an equation.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that an equation can be true or false. An example of a true equation is $7 + 1 = 4 \cdot 2$ because both sides are equivalent to 8. An example of a false equation is $7 + 1 = 9$. Normally you would not write an equation that is false, especially when it only includes numbers. False equations more often appear when you substitute a value for a variable, giving you an equation involving only numbers.

An equation can also have variables in it, like $u + 1 = 8$. This equation is neither true nor false on its own. If u is equal to 3, the equation is false because $3 + 1$ does not equal 8. The equation is true if u is equal to 7 because $7 + 1 = 8$.

A ***solution to an equation*** is a number that can be substituted in place of the variable so that it makes the equation true. In $u + 1 = 8$, there is only one solution, which is 7.

Reflect:

Synthesize

Display the equations $3 + 15 = 3 \cdot 6$ and $18 = 3 + 5$.

Highlight that these equations might look familiar because they are similar to ones from Activity 2: $18 = 3q$ and $18 = 3 + z$.

Formalize vocabulary: ***solution to an equation***

Ask:

- "Using these equations, explain what it means for an equation to be true? What does it mean for an equation to be false?"
- "How can you determine whether an equation is true or false?"
- "Is an equation with a variable *always* true?"
- "What do you call a number that makes an equation with a variable true?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean for an equation to be true? Can an equation be false?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the term *solution to an equation* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students will demonstrate their understanding of what a solution to an equation means both in written and mathematical forms.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.04

Explain how you know that 88 is a solution to the equation $\frac{1}{8}x = 11$ by completing these sentences.

1. I know 88 is a solution because . . .
   Sample response: When you multiply $\frac{1}{8}$ and 88, the product is $\frac{88}{8}$, which equals 11.

2. The phrase *solution to an equation* means . . .
   Sample response: A number that can be used in place of a variable that makes the equation true.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write an equation to represent a real-world scenario.
1 2 3

b. I can replace a variable in an equation with a number that makes the equation true, and I know that this number is called a *solution to the equation.*
1 2 3

Lesson 4 Truth and Equations

Success looks like . . .

- **Language Goal:** Generating values that make an equation true or false and justify whether they are *solutions to the equation*. **(Speaking and Listening, Writing)**
  - » Explaining why 88 is a solution to the equation in Problem 1.
- **Goal:** Using substitution to determine whether a given number makes an equation true.

Suggested next steps

If students are unable to write anything, consider:

- Giving them the opportunity to respond verbally, or suggest that they do not need to use words for the second prompt, but can just show their work for solving the equation.

If students cannot write a correct definition or description of the word *solution*, consider:

- Referring them to Activity 2 and having them determine an example of a solution. Ask, "How can you describe what that number means for the equation?"

If students do not substitute 88 into the equation, consider:

- Referring them to Activity 1 and saying, "Explain what you would do if either Problem 1b or Problem 2b asked you to substitute 88 as the value for the variable."

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about the term *solution to an equation*, how did you encourage each student to share their understandings?
- How well was the context supported? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ______________________ Date: __________ Period: __________

1. Select *all* the true equations.
   - A. $5 + 0 = 0$
   - B. $15 \cdot 0 = 0$
   - C. $1.4 + 2.7 = 4.1$
   - D. $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$
   - E. $4\frac{2}{3} = 5 - \frac{1}{3}$

2. Match each equation with its solution.

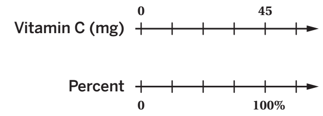
Equation	Solution
a $k + \frac{3}{8} = 2$	a $1\frac{5}{8}$
b $4.6 - m = 0.6$	c 2.3
c $2p = 4.6$	d $\frac{8}{5}$
d $r \div \frac{8}{5} = 1$	b 4

3. Mai's water bottle had 24 oz in it. After she drank x oz of water, there were 10 oz left. Select *all* the equations that represent this scenario.
   - A. $24 \div 10 = x$
   - B. $24 + 10 = x$
   - C. $24 - 10 = x$
   - D. $x + 10 = 24$
   - E. $10x = 24$

Name: ______________________ Date: __________ Period: __________

4. The daily recommended allowance of vitamin C for a sixth grader is 45 mg. One orange has about 75% of the recommended daily allowance of vitamin C. How many milligrams of vitamin C are in one orange? Consider using the double number line to help with your thinking.

   33.75 mg



5. Calculate each product.
   - a $0.25 \cdot 1.4 = 0.35$
   - b $0.061 \cdot 0.43 = 0.02623$
   - c $1.017 \cdot 0.072 = 0.073224$
   - d $5.226 \cdot 0.037 = 0.193362$

6. What value would make the equation $5 + ? = 9$ true? Show or explain your thinking.

   4; Sample responses:
   - I subtracted 5 from 9 to get 4
   - I thought about what number added to 5 gets to 9. I know that $4 + 5 = 9$, so the missing number must be 4.

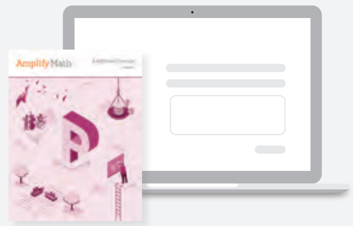
Practice

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 11	2
	5	Unit 5 Lesson 8	1
Formative	6	Unit 6 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 6 | **LESSON 5**

Staying in Balance

Let's use balanced hangers to help us write equations.

Focus

Goals

1. **Language Goal:** Interpret hanger diagrams that display numerical and pictorial relationships. **(Speaking and Listening, Writing)**
2. Write equations that represent relationships between the weights on a balanced hanger diagram.
3. **Language Goal:** Explain how to balance a hanger diagram by adding or subtracting the same amount from each side. **(Speaking and Listening, Writing)**
4. Represent the actions used to balance a hanger diagram in an equation.

Coherence

Today

Students recognize the relationship between a balanced hanger diagram and a true equation, and they discover that maintaining balance requires the same action to be performed on both sides of the hanger and equation. While the expectation for Grade 6 is limited to the Subtraction and Division Properties of Equality, this lesson challenges students to consider the repetitive nature of balancing hangers and equations using all four operations to build their understanding of solving equations in a more general sense.

Previously

Since Grade 1, students have worked with the concept of balance in terms of equivalent numerical expressions and the meaning of the equal sign (e.g., $6 = 3 + 3$ or $3 + 2 = 4 + 1$). In Lesson 2, students used variables to represent missing values, and informally determined missing values using the relationships between operations and properties of operations.

Coming Soon

In Lesson 6, students will use equations with variables to represent balanced hangers with unknown values. They will also solve for an unknown value using the properties of equality introduced in this lesson.

Rigor

- Students develop **conceptual understanding** connecting balanced hangars to true equations.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
5 min	15 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

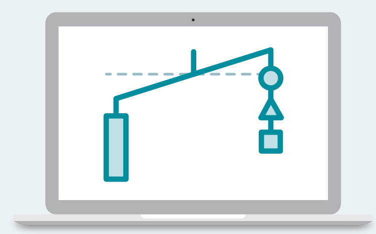
- *equation*
- *expression*

Amps | Featured Activity

Activity 2

Dynamic Hanger Diagrams

When students enter a weight for a variable in a hanger diagram, the hanger will animate, giving real-time feedback that shows whether the hanger is balanced.



Building Math Identity and Community

Connecting to Mathematical Practices

Students consider how they make decisions throughout Activity 3 and learn through repeated reasoning how to keep an equation true. The regularity of making the same changes on the left and right sides of an equation is what keeps the equation true. Similarly, students should recognize that working with truth in everyday decisions is a methodical process.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have each partner consider either Hanger A or B. Then have them share and compare responses. This activity could be modified more by having pairs consider A or B and comparing responses during the class discussion.
- Optional **Activity 2** may be omitted. Consider launching Activity 2 by explaining that each number on the hanger represents a weight, and the hanger remains balanced when the weight on both sides is equal.

Independent | 5 min

Warm-up Notice and Wonder

Students reason with a concrete representation of scales to develop an understanding of the terms *balanced* and *unbalanced*.

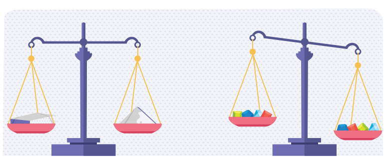
Unit 6 | Lesson 5

Staying in Balance

Let's use balanced hangers to help us write equations.

Warm-up Notice & Wonder

The people from Taghaza are ready to trade their salt for jewels. Refer to the picture. What do you notice? What do you wonder?



1. I notice . . .

   Sample responses:
   - The scale with the salt is level. The salt slab and ground-up salt weigh the same.
   - The scale with jewels is not level. The jewels on the right are heavier than the jewels on the left.

2. I wonder . . .

   Sample responses:
   - Which is heavier — the jewels or the salt?
   - How much salt would the people of Taghaza need to buy the jewels?

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by activating prior knowledge about balance scales. Ask, "Do both scales look the same? Why do you think that might be?"

Look for points of confusion:

- **Noticing only low-floor features (e.g., color).** "Do you see anything that can be represented as numbers?"
- **Saying the jewelry sets have the same weight because they look the same.** "What do you notice about the scale? What does that tell you about the weight of the jewelry?"
- **Making comparisons between salt and jewelry.** "You only know enough to compare the weights of items on a single scale."

Look for productive strategies:

- Noticing that the scale with salt is level, while the scale with gems is not, and relating this to weight.

3 Connect

Display the image of the two balance scales.

Have students share low-floor observations first, followed by those that incorporate some math — specifically numbers and/or the concept of equality.

Highlight that, because the scale on the left is balanced, the salt block and mound of salt must have an equal weight. Because the scale on the right is unbalanced, one set of jewelry (the right one) must be heavier than the other (the left one).

Ask, "How is this image related to math?"

Math Language Development

MLR5: Co-craft Questions

Before the Connect, have students share what they noticed and wondered with a partner, and work together to write 2–3 mathematical questions they have about the image of the two balance scales. Ask volunteers to share their questions with the class and amplify questions that compare the weights of the items measured to the balance scales, such as *balanced, level, heavier, weigh the same*, or *not balanced*.

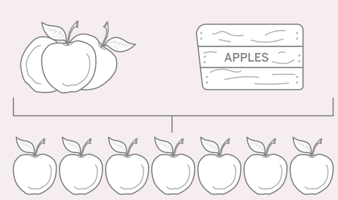
English Learners

Annotate the balance scale in which the objects do not weigh the same by writing *heavier* and *weighs more* for the items lower to the ground.

Power-up

To power up students' ability to explain the process for determining the missing value in an equation, ask:

How many apples are in the box? Explain your thinking.



4 apples. If there are 7 apples in total and 3 apples are outside the box, then 4 apples are inside the box because $7 - 3 = 4$.

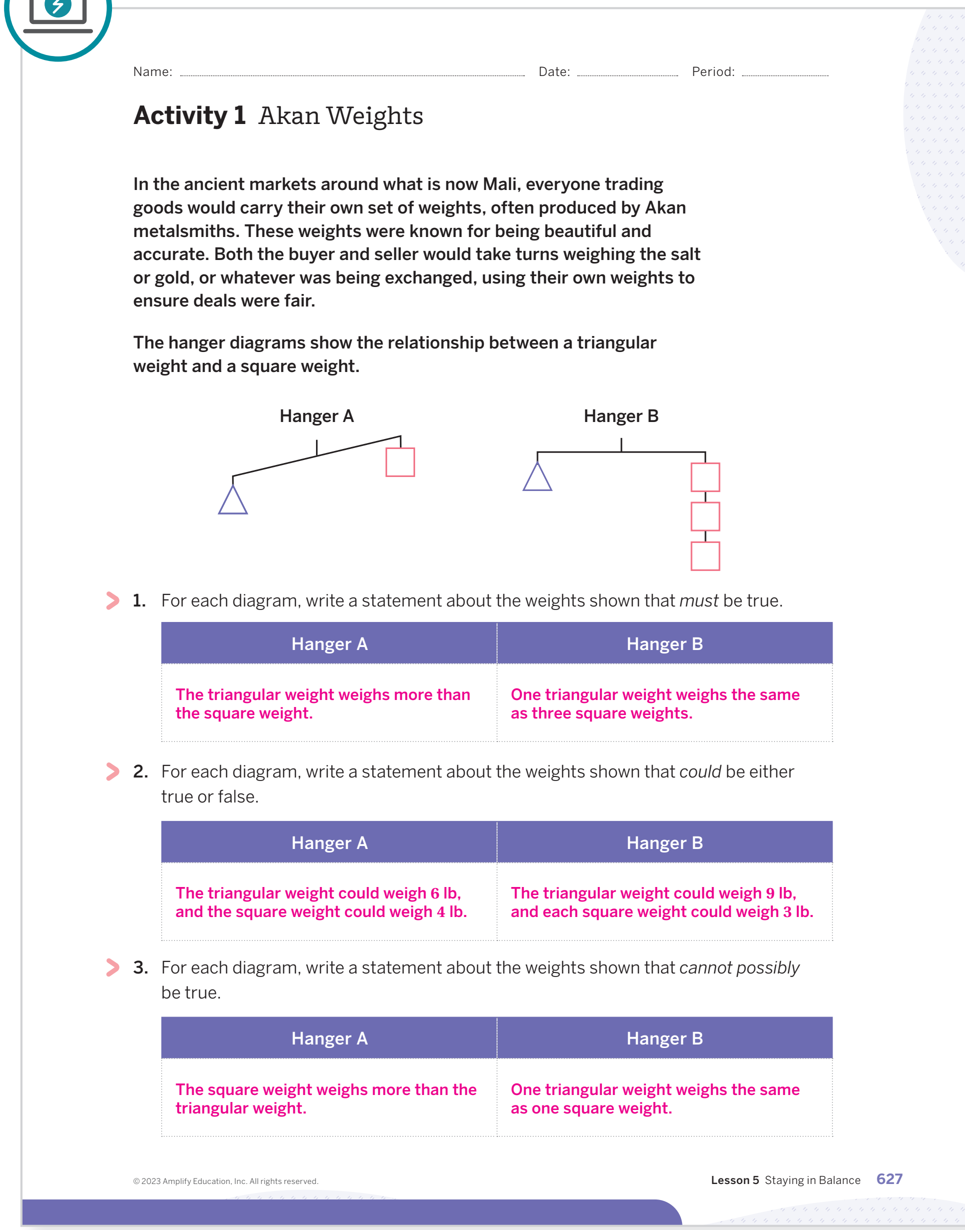
Use: Before Activity 1.

Informed by: Performance on Lesson 4, Practice Problems 6 and Pre-Unit Readiness Assessment, Problem 3.

Pairs | 15 min

Activity 1 Akan Weights

Students use what they know about balanced and unbalanced scales to reason about hanger diagrams with abstract shapes.



Name: ________ Date: ________ Period: ________

Activity 1 Akan Weights

In the ancient markets around what is now Mali, everyone trading goods would carry their own set of weights, often produced by Akan metalsmiths. These weights were known for being beautiful and accurate. Both the buyer and seller would take turns weighing the salt or gold, or whatever was being exchanged, using their own weights to ensure deals were fair.

The hanger diagrams show the relationship between a triangular weight and a square weight.

1. For each diagram, write a statement about the weights shown that *must* be true.

Hanger A	Hanger B
The triangular weight weighs more than the square weight.	One triangular weight weighs the same as three square weights.

2. For each diagram, write a statement about the weights shown that *could* be either true or false.

Hanger A	Hanger B
The triangular weight could weigh 6 lb, and the square weight could weigh 4 lb.	The triangular weight could weigh 9 lb, and each square weight could weigh 3 lb.

3. For each diagram, write a statement about the weights shown that *cannot possibly* be true.

Hanger A	Hanger B
The square weight weighs more than the triangular weight.	One triangular weight weighs the same as one square weight.

1 Launch

Ask students how the hanger diagrams are similar to the scales from the Warm-up. Focus the discussion on how both show balance. Then explain the meaning of "one thing that could either be true or false." Have students complete the activity in pairs.

2 Monitor

Help students get started by asking, "Do both hangers look the same? What looks different? Why do you think that might be?"

Look for points of confusion:

- **Using inconsistent values across the diagrams.** Ask, "If the triangle is the same in both hangers, what does that mean about its weight?"
- **Substituting numbers that work for Hanger A but not Hanger B.** Ask, "If Hanger B is balanced, what does that mean? Does your value for the triangle equal the value of three squares?"

Look for productive strategies:

- Recognizing that one triangle weighs the same as three squares, and using this to reason about the weight of each shape.

3 Connect

Have students share general statements first, followed by those involving numerical values. Students share why some values work for Hanger A, but not for Hanger B.

Highlight that Hanger A is unbalanced, so one triangle must be heavier than one square. Because Hanger B is balanced, one triangle must weigh exactly the same as three squares. Students don't know how much either shape weighs, but there are many possible values that would represent either true or false statements.

Ask, "Because Hanger B is balanced, why does it make sense for Hanger A to be unbalanced?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problem 1 (*must be true*) and Problem 3 (*cannot possibly be true*). Or have them focus only on the balanced hanger. Students will need these understandings in Activity 2.

Extension: *Math Enrichment*

Have students draw two hangers with 2 triangles on the left side of each hanger. Have them place squares on the right side of each hanger so that one hanger is balanced and the other hanger is unbalanced. Ask them how many squares they placed on each hanger. Sample response: Balanced, 6 squares. Unbalanced, 2 squares.

Extension: *Math Around the World*

Tell students that gold was believed by many of the Akan peoples to be the earthly counterpart to the Sun. Many of the earlier gold weights they created from mined gold dust were cast into geometric shapes. Later, the weights were cast into beautiful images, such as elephant weights and bird weights. These were inspired by Akan proverbs and carried special meaning. One such bird weight, on display at the Metropolitan Museum of Art, in New York City, has a height of $1\frac{5}{8}$ in. and a width of $\frac{1}{2}$ in.

Optional

Pairs | 15 min

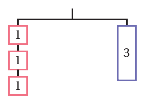
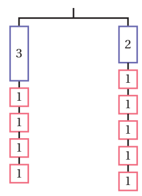
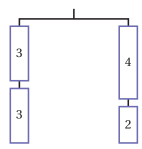
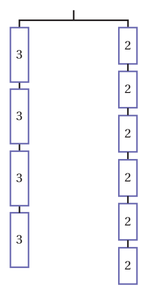
Activity 2 Maintaining Balance

Students work in partners to unbalance and rebalance hangers by adding or removing the same total weight on each side of the hanger.

Amps Featured Activity Dynamic Hanger Diagrams

Activity 2 Maintaining Balance

Hangers A, B, C, and D are balanced. Take turns using the given superpower to unbalance and then rebalance each hanger. Each superpower may be applied more than once. Be ready to explain how you knew what action would rebalance each hanger.

Balanced hanger	Superpower
Hanger A 	Add a weight of 1. I rebalanced the hanger by adding *or* removing 1s from either side so that the same number of 1s are on the left and right sides of the hanger.
Hanger B 	Remove a weight of 1. I rebalanced the hanger by adding or removing 1s from either side so that the same number of 1s are on the left and right sides of the hanger.
Hanger C 	Make a copy of the total weight on a side. I rebalanced the hanger by replicating the total weight on each side. Weight should be added in groups of 6: $3 + 3$ on the left and $4 + 2$ on the right.
Hanger D 	You have two superpowers! Remove a weight of 3 from the left. Remove a weight of 2 from the right. I rebalanced the hanger by removing groups of equal weight. To remove $3 + 3 = 6$ on the left, I removed $2 + 2 + 2 = 6$ on the right.

628 Unit 6 Expressions and Equations

1 Launch

Explain that each number on the hanger represents a weight, and the hanger remains balanced when the weight on both sides is equal. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What happens if you add 1 to the left side?"

Look for points of confusion:

- **Rebalancing by removing all of the weight.** Ask, "Is there another point of balance before removing all the weight?"
- **Getting Hanger B to a $3 = 2$ state. Have students restart.** Remind them that balanced hangers need have the same amount of weight on each side, not the same number of weights.
- **Being unable to rebalance Hanger D.** Ask, "How could you remove equal weight from each side?"

Look for productive strategies:

- Changing Hanger D's weights into unitized values. Challenge students to rebalance by keeping the weight in its original form.
- Recognizing that the same amount of weight must be added or removed from both sides of the hanger.

3 Connect

Display one hanger at a time.

Have students share how they rebalanced the hangers by adding or removing the same total weight from each side.

Highlight that a hanger will remain balanced when performing the same action on each side. If students add to or remove from one side, they must add to or remove the same amount from the other side.

Ask, "How do Hangers C and D show doubling and halving?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter a weight for a variable to animate the hanger diagram. By doing so, they will receive real-time feedback that shows whether the hanger is balanced.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Hangers A and B.

Extension: *Math Enrichment*

Tell students that they can represent weights on a hanger as equations. Each time the hanger is rebalanced, have them write an equation to represent the weights.

Sample response for Hanger A:

$1 + 1 + 1 = 3$; $1 + 1 + 1 + 1 = 3 + 1$

Pairs | 15 min

Activity 3 Keeping Equations True

Students write equations to represent their actions rebalancing a hanger, and relate a balanced hanger to a true equation.

Name: ______ Date: ______ Period: ______

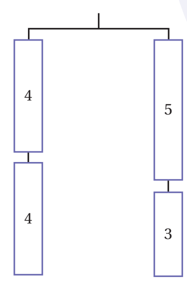
Activity 3 Keeping Equations True

Plan ahead: How does truth impact your decision making?

Hanger A is balanced and represents the equation $4 + 4 = 5 + 3$.

1. Each scenario in the table represents a change to Hanger A. Complete the table by describing how to rebalance the hanger. Then write an equation to represent the rebalanced hanger.

Hanger A



Change to left	Change to right	Equation
Add 2.	**Add 2.**	$4 + 4 + 2 = 5 + 3 + 2$
Subtract 6.	Subtract 6.	$4 + 4 - 6 = 5 + 3 - 6$
Double the weight.	**Double the weight.**	$4 + 4 + 4 + 4 = 5 + 3 + 5 + 3$ $2(4 + 4) = 2(5 + 3)$
Subtract half of the weight.	Subtract half of the weight.	$4 + 4 - 4 = 5 + 3 - 4$ $(4 + 4) \div 2 = (5 + 3) \div 2$ $\frac{4+4}{2} = \frac{5+3}{2}$ $\frac{1}{2} \cdot (4 + 4) = \frac{1}{2} \cdot (5 + 3)$

2. Choose one equation from the table above. Explain how you know the equation represents a balanced hanger.

Answers may vary, but should explain that the equation represents a balanced hanger because it is true — the expression on the left side of the equal sign is equivalent to the expression on the right side. That means the total weight on the left side of the hanger is equal to the total weight on the right side.

STOP

1 Launch

Explain that the weights on the balanced hangers can be represented using an equation, and the actions used to change the hanger can be represented using operations. Ensure students know how the equation represents the hanger before completing the activity in pairs.

2 Monitor

Help students get started by asking, "In the last activity, if your partner added 2, how would you rebalance the hanger?"

Look for points of confusion:

- **Incorrectly doubling the weight.** Remind students that doubling means adding a copy of the existing total weight on each side.

Look for productive strategies:

- Writing equations with totals (i.e., $10 = 10$). Have students use an operation to show how they rebalanced.
- Using addition/subtraction to represent doubling/halving.
- Writing equations that show all rebalancing actions, and using multiplication and division for doubling and subtracting half.

3 Connect

Have students share their equations, starting with those who used addition or subtraction, followed by those who used multiplication and division. Then have them share how the operations in an equation and actions used to rebalance a hanger are related.

Ask, "When subtracting half of the weight, why were you able to represent your actions using subtraction, multiplication, and division?"

Highlight that a true equation can be used to represent a balanced hanger. Just like adjusting weights on a hanger, operations can be used to change both sides of an equation. To keep the equation true, the same operation must be done on both sides of the equal sign.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with counters, pennies, or other objects that they can use to model the weights on Hanger A and keep the hanger balanced with each change described in the table.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on describing the changes for all four scenarios, and then writing equations for only the first two scenarios.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their equations, prompt them to compare and contrast their actions and equations. Ask:

- "When you doubled the weight on the left side, what mathematical operation did you use to represent this action in your equation? Why did you choose this operation?"
- "When you subtracted half of the weight from the left side, how did you represent this action in your equation?"

English Learners

Annotate the phrase *double the weight* with the phrases *multiply by* 2 and *twice the weight*. Annotate the phrase *half of the weight* with the phrases *halving the weight* and *divide the weight by* 2.

Whole Class | 5 min

Summary

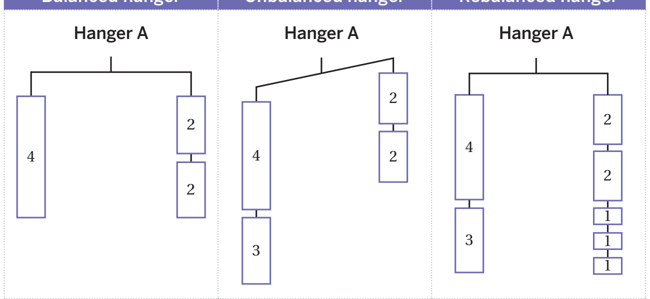
Formalize the properties of equality, and connect them to balancing hanger diagrams and writing equations.

Summary

In today's lesson . . .

You balanced hanger diagrams. A hanger is balanced when the total weights on both sides are equivalent. If you start with a balanced hanger and then change the weight on only one side, the hanger will no longer be balanced. But, if you change the weights on both sides in exactly the same way, the hanger will stay balanced.

Balanced hanger	Unbalanced hanger	Rebalanced hanger



An equation can be used to represent a balanced hanger. When an equation is true, the expressions on both sides of the equal sign represent the same value. Just like changing the weights on a hanger, you can use operations to change both sides of an equation. The ***properties of equality*** tell you the equation will still be true if you perform the exact same operations (addition, subtraction, multiplication, or division) on both sides of the equal sign.

In the example above, the original balanced hanger can be represented using the equation $4 = 2 + 2$. The rebalanced hanger can be represented using the equation $4 + 3 = 2 + 2 + 1 + 1 + 1$. The hanger is rebalanced because the total weight on both sides is equal to 7.

Reflect:

630 **Unit 6** Expressions and Equations

Synthesize

Formalize vocabulary: ***properties of equality***

Ask, "How did your work today demonstrate these properties of equality?"

Highlight:

- The properties of equality tell that an equation will remain true if the same exact operations (addition, subtraction, multiplication, or division) are performed on both sides of the equal sign.
- **Note:** While the expectation for Grade 6 is limited to the Subtraction and Division Properties of Equality, this lesson challenges students to take note of, and apply repeated reasoning in balancing hangers and equations, using all four operations. They will continue this in Grade 7, to build their understanding of solving equations in a more general sense.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did you use hangar diagrams today?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the term *properties of equality* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by rebalancing a hanger diagram and writing an equation to represent their actions.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.05

Hanger A is balanced.

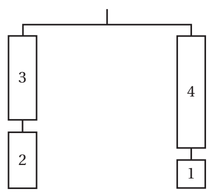
1. Write an equation to represent Hanger A.

$3 + 2 = 4 + 1$

2. A weight of 5 is added to the right side of Hanger A. Describe how to rebalance Hanger A. Then write an equation to represent the rebalanced hanger.

Add a weight of 5 to the left side.
$3 + 2 + 5 = 4 + 1 + 5$

Hanger A



3. Explain how you know your equation in Problem 2 represents a balanced hanger.

The equation represents a balanced hanger because it is true — the expression on the left side of the equal sign is equivalent to the expression on the right. That means the total weight on the left side of the hanger is equal to the total weight on the right.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can relate a balanced hanger to a true equation. 1 2 3

b. I can describe and show how a balanced hanger can be changed, while maintaining balance. 1 2 3

c. I can relate the properties of equality to true equations and balanced hangers. 1 2 3

 Lesson 5 Staying in Balance

Success looks like . . .

- **Language Goal:** Interpreting hanger diagrams that display numerical and pictorial relationships. **(Speaking and Listening, Writing)**
  - » Writing an equation to represent Hanger A in Problem 1.
- **Goal:** Writing equations that represent relationships between the weights on a balanced hanger diagram.
- **Language Goal:** Explaining how to balance a hanger diagram by adding or subtracting the same amount from each side. **(Speaking and Listening, Writing)**
- **Goal:** Representing the actions used to balance a hanger diagram in an equation.
  - » Explaining how to rebalance Hanger A after the weight is added to the right side in Problem 2.

Suggested next steps

If students write an inaccurate equation to represent Hanger A in Problem 1, consider:

- Reviewing the connection between Hanger A and its equation from Activity 3.
- Assigning Practice Problem 1.

If students do not add a weight of 5 to the left side in Problem 2, consider:

- Reviewing Scenario 1 from Activity 2.
- Assigning Practice Problem 2.

If students write an equation that does not show adding 5 to both sides in Problem 2, consider:

- Reviewing how equations reflect the properties of equality from Activity 3.
- Assigning Practice Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students connected balanced hangar diagrams to equations. How did that build on the earlier work students did in the Launch with the counterfeit coins?
- What did you see in the way some students approached Activity 2 that you would like other students to try? What might you change for the next time you teach this lesson?

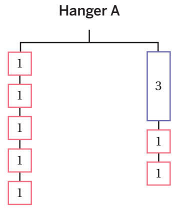
Practice

Independent

Name: Date: Period:

Use Hanger A to complete Problems 1 and 2.

Hanger A



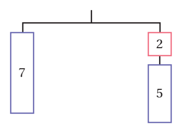
1. Hanger A is balanced. Write an equation that represents Hanger A.
$1+1+1+1+1=3+1+1$

2. Each scenario below presents a change to Hanger A. Describe how to rebalance the hanger. Then write an equation to represent the rebalanced hanger.

Change to left	Change to right	Equation
Add 3	Add 3	$1+1+1+1+1+1+1+1=3+1+1+1+1+1$ or $1+1+1+1+1+3=3+1+1+3$
Double the weight	Double the weight	$1+1+1+1+1+1+1+1+1+1=3+1+1+3+1+1$ or $2(1+1+1+1+1)=2(3+1+1)$

3. Hanger B is balanced. Izem added weights of 3 and 4 to the left side of Hanger B. Hodan then said it could be rebalanced by adding weights of 2 and 5 to the right side.

Hanger B



But Izem disagreed because the two weights added to each side would not be exactly the same. Do you agree with either Izem or Hodan? Explain your thinking.
Izem is incorrect. Sample Response: A balanced hanger has the same total weight on each side. When Izem adds weights of 3 and 4 to the left side, the total weight on the left side increases by 7 to 14. Hodan can rebalance the hanger by adding a total weight of 7 in any combination of weights, such as $2+5$, to the right side.

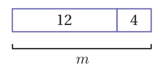
Practice

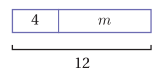
Name: Date: Period:

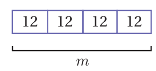
Practice

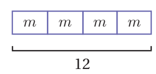
4. Match each equation with the diagram it represents.

Diagram

a 

b 

c 

d 

Equation

b $12-m=4$

d $12=4\cdot m$

a $m-4=12$

c $\frac{m}{4}=12$

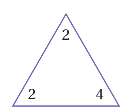
5. The area of a rectangle is 14 square units. It has side lengths x and y. Given each value for x, determine y.

a. $x=2\frac{1}{3}$ $y=6$

b. $x=4\frac{1}{5}$ $y=3\frac{1}{3}$

c. $x=\frac{7}{6}$ $y=12$

6. Write *all* the equations you can think of, for the fact family below.



Sample responses:
$2+2=4$ $2\cdot 2=4$
$4-2=2$ $4\div 2=2$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activities 1 and 2	2
Spiral	4	Unit 6 Lesson 3	3
	5	Unit 4 Lesson 14	2
Formative	6	Unit 6 Lesson 6	1

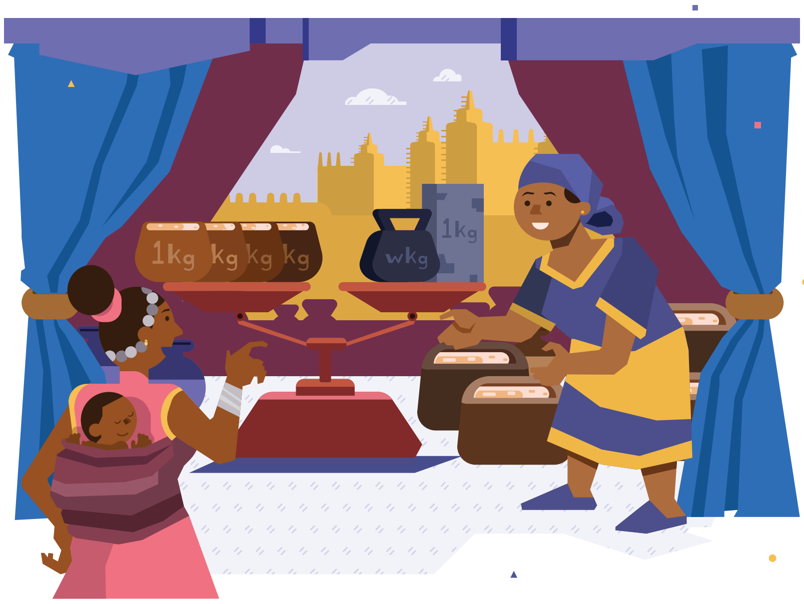
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Staying in Balance With Variables

Let's use balanced hangers to help us write and solve equations with variables.



Focus

Goals

1. **Language Goal:** Interpret hanger diagrams and write equations that represent relationships between the weights on a balanced hanger diagram. **(Speaking and Listening, Writing)**
2. **Language Goal:** Solve equations of the form $x + p = q$ or $px = q$ and explain the solution method. **(Speaking and Listening, Writing)**

Coherence

Today

Students write equations to represent balanced hangers with variables. They solve for an unknown value by using the properties of equality, and they recognize the similarities between balancing a hanger and solving an equation in context. While the expectation for Grade 6 is limited to the Subtraction and Division Properties of Equality, this lesson challenges students to consider the repetitive nature of balancing hangers and equations using all four operations to build their understanding of solving equations in a more general sense.

Previously

In Lesson 3, students used tape diagrams to visually represent expressions and equations, representing unknown values with variables. In Lesson 5, they worked with hanger diagrams to represent true equations through the concept of balance.

Coming Soon

In Lessons 7–9, students will build fluency by using the properties of equality to solve many equations of the form $x + p = q$ or $px = q$.

Rigor

- Students continue developing **conceptual understanding** that balanced hangars represent true equations, including equations with variables.
- Students build **procedural fluency** writing and solving equations with variables.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Whole Class	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New words

- *Addition Property of Equality*
- *Subtraction Property of Equality*
- *Multiplication Property of Equality*
- *Division Property of Equality*

Review words

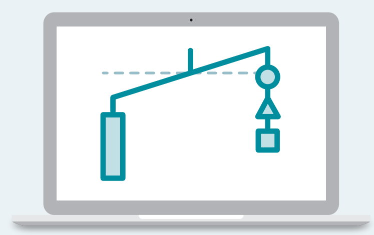
- *coefficient*
- *properties of equality*
- *solution to an equation*
- *variable*

Amps | Featured Activity

Activity 1

Dynamic Hanger Diagrams

When students enter a weight for a variable in a hanger diagram, the hanger will animate, giving real-time feedback that shows whether the hanger is balanced.



Building Math Identity and Community

Connecting to Mathematical Practices

While students have worked with hanger diagrams before, the introduction of the abstract nature of variables might cause them to express some confusion. They may lose sight of how the manipulative process of keeping equations balanced is exactly the same whether with numbers or variables. Their ability to reflect on and apply the different properties of operations can help them stay organized in the abstract variable manipulation.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be completed as a class. Give students one minute of think time before having them share their responses.
- In **Activities 1 and 2**, Problems 1–3 may be completed as a class. Have pairs complete Problems 4–6 together.

Whole Class | 5 min

Warm-up Understanding an Ancient Sales Record

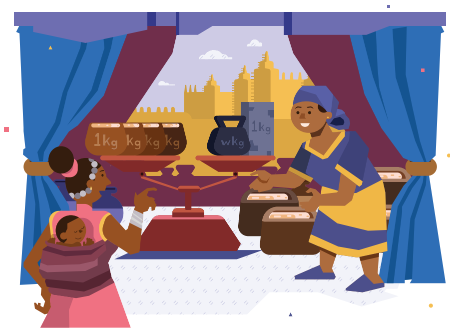
Students activate prior knowledge of fact families to write equivalent equations.

Name: ______________ Date: ________ Period: ________

Unit 6 | Lesson 6

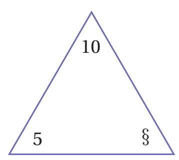
Staying in Balance With Variables

Let's use balanced hangers to help us write and solve equations with variables.



Warm-up Understanding an Ancient Sales Record

The diagram shown is a sales record from the most successful spice seller in Timbuktu. The seller has used numbers and a symbol to track the sales. Write *all* the equations you can think of to represent the relationship between 5, 10 and §. Leave the symbol in your equation.



There are six possible families in which the § has the same value. All possible equations for two of the families are shown.

$§ = 10 + 5$	$§ = 10 - 5$	$§ = 5 - 10$	$§ = 5 \cdot 10$	$§ = 10 \div 5$	$§ = 5 \div 10$
$§ = 5 + 10$			$§ = 10 \cdot 5$		
$10 = § - 5$			$10 = § \div 5$		
$5 = § - 10$			$5 = § \div 10$		

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate prior knowledge by asking, "What is a fact family?" Consider providing a numeric example. Remind them to keep the symbol § in their equations.

2 Monitor

Help students get started by asking, "How can you represent a mathematical relationship that involves the numbers 5 and 10 in an equation? The symbol § represents the third, unknown value, and should be left as a symbol in your equations."

Look for points of confusion:

- **Only writing equations for one value of** §. Ask, "How else are 5 and 10 related?"

Look for productive strategies:

- Writing equations that result in a fraction or negative number. A brief discussion of the fraction and decimal numbers may be beneficial, but a discussion of negative numbers is not appropriate here.
- Writing correct equations and organizing them in families where § has the same value.

3 Connect

Have students share one equation from their lists at a time, using correct vocabulary when describing operations and naming parts of equations. Organize their responses by families.

Highlight that symbols, such as §, are similar to variables. They represent unknown values, which can be a term in an expression or the result of an operation. Even when a value is unknown, you can show the relationship among the values using several different equations.

Ask, "Why are $10 = § + 5$ and $§ + 5 = 10$ equivalent?

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Students may think they cannot proceed because they have not previously seen the symbol used in the diagram. Help them understand this is merely a symbol and change it to another symbol, if their discomfort persists.

Accessibility: *Activate Prior Knowledge*

Remind students they have worked with fact families in prior grades. A *fact family* consists of three numbers that are used together to create a set of math facts. Those math facts can be made for addition, subtraction, multiplication, or division.

$3 \cdot 4 = 12$
$4 \cdot 3 = 12$
$12 \div 4 = 3$
$12 \div 3 = 4$

Power-up

To power up students' ability to represent fact families with equations, ask:

Write all of the equations you can think of using the numbers 2, 3, and 6.

$2 \cdot 3 = 6$ $\quad$ $6 \div 2 = 3$

$3 \cdot 2 = 6$ $\quad$ $6 \div 3 = 2$

Use: Before the Warm-up.

Informed by: Performance on Lesson 5, Practice Problem 6.

Activity 1 Using Hanger Diagrams to Write and Solve Equations With Variables

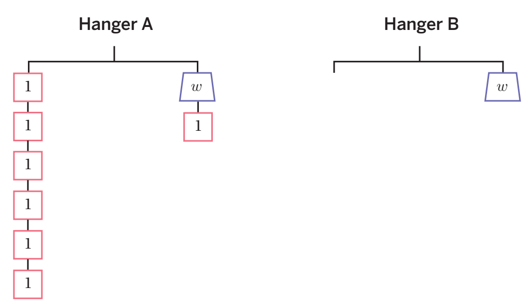
Pairs | 15 min

Students write equations to represent hangers with unit and variable weights, and manipulate the hangers to solve for an unknown value.

Amps Featured Activity Dynamic Hanger Diagrams

Activity 1 Using Hangers to Write and Solve Equations With Variables

You are buying spices at the market in Timbuktu. The seller has placed several 1-kg bags of spices on the left side of a balance. The right side of the balance has a 1-kg weight and a weight labeled as w kg.



1. Hanger A represents how the seller balanced the spices and weights. Write an equation that represents Hanger A.
   $1 + 1 + 1 + 1 + 1 + 1 = w + 1$
   $6 = w + 1$

2. You want to know how much the unknown weight w weighs, to ensure a fair deal.
   a. Complete the left side of Hanger B so it is balanced.
      Hanger B should show a total weight of 5 on the left side, and the one w should remain alone on the right side.
   b. What is the weight of w, in kilograms?
      5 kg or $1 + 1 + 1 + 1 + 1$ kg

3. Take the weight of w, in kg, from Hanger B, and substitute it for the value of w in Hanger A's equation. Is Hanger A's equation true? Explain your thinking.
   This equation is true because the expressions on either side of the equal sign are equivalent:
   $1 + 1 + 1 + 1 + 1 + 1 = 5 + 1$

634 **Unit 6** Expressions and Equations

1 Launch

Review vocabulary: *term*, *variable*, and *coefficient*. Explain that each shape labeled with a letter has an unknown weight. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What does it mean if the hanger is balanced? What is the total weight on each side? Can you write an expression for each side?"

Look for points of confusion:

- **Guessing and checking values for variables.** Ask, "What is different about the right side of Hangers A and B? How could that have happened? What needs to happen to the left side?"
- **Determining $x = 4$ for Hanger D.** Students subtracting in a $1:1$ ratio here means they didn't recognize the equal groups. Ask, "Since Hanger C is balanced, what does that tell you about the three xs and six 1s? And how could you then add or remove the same total weight, which is not going to be the same number of weights, from each side?"
- **Incorrectly substituting the value of w or x.** Say, "You said the variable equals __. Wherever you see the variable in Hanger A/C's equation, replace it with __."

Look for productive strategies:

- Using repeated addition for Hanger C's equation.
- Determining that $x = 2$, and showing this relationship by drawing equal groups on Hanger C.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter a weight for a variable to animate the hanger diagram. By doing so, they will receive real-time feedback that shows whether the hanger is balanced.

Accessibility: *Optimize Access to Tools*

Provide students with counters, pennies, or other objects that they can use to model the weights on Hangers A–D to help them visualize the relationships. Have them designate one object, such as a counter, to represent x with each penny representing the value of 1.

Extension: *Math Enrichment*

Have students complete the following problem:

Examine Hanger C. How would your equation and value of x change if the balanced hanger only had four 1s on the right side? The equation would become $3x = 4$ (or equivalent) and the value of x would be $\frac{4}{3}$ (or equivalent).

Activity 1 Using Hanger Diagrams to Write and Solve Equations With Variables (continued)

Pairs | 15 min

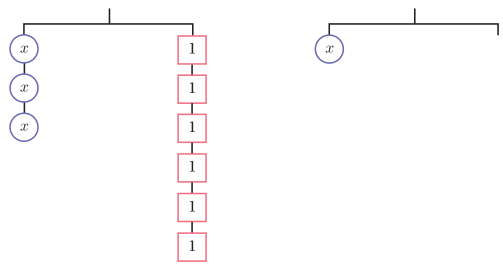
Students write equations to represent hangers with unit and variable weights, and manipulate the hangers to solve for an unknown value.

Name: ______________________ Date: __________ Period: __________

Activity 1 Using Hangers to Write and Solve Equations With Variables (continued)

Happy with your first purchase, you decide to buy some salt. The seller has placed several 1-kg blocks of salt on the right side of a balance. Three identical weights, each labeled as x kg, are on the left side.

Hanger C Hanger D



4. Hanger C represents how the seller balanced the weights and salt. Write an equation that represents Hanger C.

$3x = 1 + 1 + 1 + 1 + 1 + 1$
$3x = 6$
$x + x + x = 1 + 1 + 1 + 1 + 1 + 1$
$x + x + x = 6$

5. You want to know how much the unknown weight x weighs, to ensure a fair deal.

a Complete the right side of Hanger D so that it is balanced.

Hanger D should still have one x alone on the left side, and should show a total weight of 2 on the right side.

b What is the weight of each x, in kilograms?

2 kg or 1 + 1 kg

6. Take the weight of x, in kilograms, from Hanger D, and substitute it for the value of x in Hanger C's equation. Is the equation true? Explain your thinking.

This equation is true because the expressions on either side of the equal sign are equivalent: $2 + 2 + 2 = 1 + 1 + 1 + 1 + 1 + 1$ or $2 + 2 + 2 = 6$

3 Connect

Display, in pairs, Hangers A and B, and then Hangers C and D.

Have students share how they determined from the scenarios whether the expressions were addition or multiplication, then which side changed and how, and, lastly, how they manipulated both the hangers and equations to solve for the variable. Remind students to use appropriate vocabulary, such as *variable, term*, and *coefficient*.

Ask:

- "How do you know from looking at a hanger whether it can be represented by an equation involving addition? Multiplication?"
- "In each scenario, you removed weight from both sides. What operations can be used to represent these actions?"

Highlight that an unknown weight on a hanger or a solution to an equation can be found by isolating the variable, or in other words, determining the weight of just one variable. This can be done by performing the same action or operation on each side.

Define ***Subtraction Property of Equality*** and ***Division Property of Equality***.

Ask:

- "How is the Subtraction Property of Equality used to isolate the variable in the first hanger?" 1 was removed (subtracted) from both sides of the hanger so it was still balanced.
- "How is the Division Property of Equality used to isolate the variable in the first hanger?" There were three xs and six 1s in the original hanger and the new hanger has one x and two 1s. Both sides are ⅓ of the original so it is still balanced.

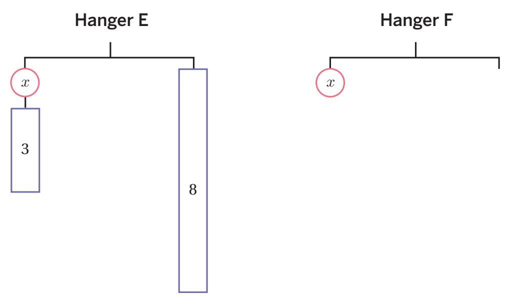
Pairs | 15 min

Activity 2 Writing and Solving Equations With Variables

Students write equations to represent hangers with non-unit and variable weights, and manipulate the hangers to solve for an unknown value.

Activity 2 Writing and Solving Equations With Variables

Hanger E is balanced. Use it to complete the following problems.



1. Write an equation that represents Hanger E.
   $x + 3 = 8$

2. Complete the right side of Hanger F so that it is balanced. Then write an equation to represent the balanced hanger.
   Hanger F should still have one x alone on the left side, and should show a total weight of 5 on the right side.
   $x = 5$

3. Explain how you know the value representing the weight of x is a solution to Hanger E's equation.
   I know that 5 is a solution because when I substitute 5 for x in Hanger E's equation, the equation is true — the expressions on both sides of the equal sign are equivalent ($5 + 3 = 8$).

636 **Unit 6** Expressions and Equations

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "How can you redraw Hanger E using weights of 1? How can you represent that using an equation?"

Look for points of confusion:

- **Not knowing how to model removal or equal grouping with non-unitized values.** Have students unitize the values on Hanger E and balance Hanger F. Ask, "How could you show the same actions using the original weights?"
- **Solving for y by removing the same number of weights from each side, rather than the same total weight.** Remind students that hangers remain balanced when equal weight, not necessarily an equal number of weights, is added to or removed from each side.

Look for productive strategies:

- Using repeated addition for Hanger G's equation. Challenge students to represent this using a related operation (multiplication).
- Determining that $y = 6$, and showing this relationship by dividing the block of 12 into two equal-sized parts of 6.

Activity 2 continued ›

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on Problems 1, 2, 4, and 5.

Extension: ***Math Enrichment***

Have students complete the following problems:

Examine Hanger G. How would your equation and solution change if you replaced:

- $2y$ with $4y$ on the right side of the hanger? $12 = 4y$; $3 = y$
- 12 with 36 on the left side of the hanger? $36 = 2y$; $18 = y$

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they wrote their equations and how they solved them, draw their attention to the connections between manipulating weights on a hanger diagram and determining the solution to a corresponding equation. Ask:

- "In Hanger G, how would you manipulate these weights to determine the weight of y? How does this connect to determining the solution to the equation you wrote?"
- "Look at Hanger E. If a classmate said that you can determine the weight of x by adding 3 to each side, how would you respond?"

Activity 2 Writing and Solving Equations With Variables (continued)

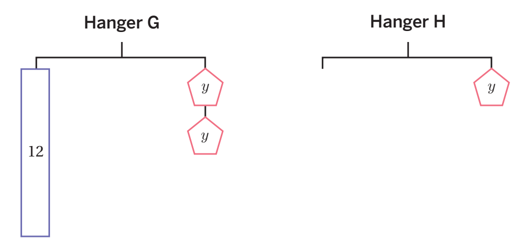
Pairs | 15 min

Students write equations to represent hangers with non-unit and variable weights, and manipulate the hangers to solve for an unknown value.

Name: ______________________ Date: __________ Period: __________

Activity 2 Writing and Solving Equations With Variables (continued)

Hanger G is balanced. Use it to complete the following problems.



4. Write an equation that represents Hanger G.

   Sample responses:
   - $12 = 2y$
   - $12 = 2 \cdot y$
   - $12 = 2 \times y$
   - $12 = y + y$

5. Complete the left side of Hanger H so that it is balanced. Then write an equation to represent the balanced hanger.

   Hanger H should show a total weight of 6 on the left side, and one y should remain alone on the right side.
   $6 = y$

6. Explain how you know the value representing the weight of y is a solution to Hanger G's equation.

   I know that 6 is a solution because, when I substitute 6 for y in Hanger G's equation, the equation is true — the expressions on both sides of the equal sign are equivalent ($12 = 6 + 6$ or $12 = 2 \cdot 6$).

STOP

3 Connect

Display, in pairs, Hangers E and F and then Hangers G and H.

Have students share how they determined whether the expressions in their equations used addition or multiplication, which side changed and how it changed. Then have them explain how they manipulated both the hangers and the equations to solve for the unknown value. Have them share how their work connects to working with unitized values in the previous activity.

Ask:

- "What common actions did you use for solving an addition equation and a multiplication equation?"
- "How can you use subtraction and division to represent your actions for Hanger H?"
- "Which property of equality did you use to solve each equation?"

Highlight that the process to determine the unknown weight on a hanger or to solve an equation remains the same whether you have many weights — as was the case in Activity 1 — or just one weight, as was the case here. The variable is isolated on one side by performing the same action, or operation, to both sides.

Whole Class | 5 min

Summary

Review and synthesize how the properties of equality are related to the process of solving equations

Summary

In today's lesson . . .

You used balanced hangers to write and solve equations. The table shows the four *properties of equality*, using the variables a, b, and c to represent any numbers.

Property of Equality	Formula	Equation
Addition Property of Equality	If $a = b$, then $a + 2 = b + 2$.	As $3 + 3 = 4 + 2$, then $3 + 3 + 2 = 4 + 2 + 2$
Subtraction Property of Equality	If $a = b$, then $a - 2 = b - 2$.	As $6 = 4 + 2$, then $6 - 2 = 4 + 2 - 2$
Multiplication Property of Equality	If $a = b$, then $a \cdot 2 = b \cdot 2$.	As $3 + 3 = 4 + 2$, then $2(3 + 3) = 2(4 + 2)$
Division Property of Equality	If $a = b$, then $a \div 2 = b \div 2$.	As $3 + 3 = 6$, then $(3 + 3) \div 2 = 6 \div 2$

Reflect:

638 Unit 6 Expressions and Equations

Synthesize

Formalize vocabulary:

- ***Addition Property of Equality*** — For rational numbers a, b, and c, if $a = b$, then $a + c = b + c$.
- ***Subtraction Property of Equality*** — For rational numbers a, b, and c, if $a = b$, then $a - c = b - c$.
- ***Multiplication Property of Equality*** — For rational numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$.
- ***Division Property of Equality*** — For rational numbers a, b, and c, if $a = b$, then $a \div c = b \div c$.

Ask:

- "How did your work today demonstrate these properties of equality?"
- "Why did you use some properties and not others?"

Highlight:

- All of the properties of equality state that a true equation remains true if you perform the exact same operation to the expressions on both sides of the equal sign.
- **Note:** While the expectation for Grade 6 is limited to the Subtraction and Division Properties of Equality, this lesson challenges students to consider the repetitive nature of balancing hangers and equations, using all four operations. This will build their understanding of solving equations in a more general sense.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can two quantities be equal when one is partially or totally unknown?"

MLR Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the terms *Addition Property of Equality, Subtraction Property of Equality, Multiplication Property of Equality,* and *Division Property of Equality* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

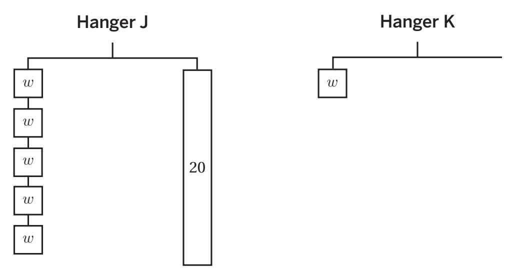
Students demonstrate their understanding by manipulating a hanger diagram and an equation to solve for an unknown value.

Printable

Name: ________________ Date: ________ Period: ________

Exit Ticket 6.06

Hanger J is balanced.



1. Write an equation to represent Hanger J.
   $5w = 20$, or $5 \cdot w = 20$, or $5 \times w = 20$, or $w + w + w + w + w = 20$

2. Balance Hanger K by completing the right side of the hanger. Then write an equation to represent the balanced hanger.
   Hanger K should show one w on the left side and a block of 4 on the right side.
   $w = 4$

3. Explain how you know the value for w is a solution to Hanger J's equation.
   I know that 4 is a solution because when I substitute 4 for w in Hanger J's equation, the equation is true ($5 \times 4 = 20$ or $5 \cdot 4 = 20$ or $5 + 5 + 5 + 5 = 20$).

Self-Assess

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it

a. I can write an equation to represent a balanced hanger with variables.
   1 2 3

b. I can solve for a variable using the properties of equality.
   1 2 3

c. I can explain what a balanced hanger and a true equation have in common.
   1 2 3

d. I can compare the actions needed to balance a hanger and to solve an equation.
   1 2 3

Success looks like . . .

- **Language Goal:** Interpreting hanger diagrams and writing equations that represent relationships between the weights on a balanced hanger diagram. **(Speaking and Listening, Writing)**
  » Writing a equation to represent Hanger J.
- **Language Goal:** Solving equations of the form $x + p = q$ or $px = q$ and explaining the solution method. **(Speaking and Listening, Writing)**
  » Explaining why $w = 4$ is a solution to the equation for Hanger J in Problem 3.

Suggested next steps

If students write an inaccurate equation to represent Hanger J in Problem 1, consider:

- Reviewing the equation for Hanger G in Activity 2, Problem 4.
- Assigning Practice Problem 1.

If students do not show a balanced Hanger K as representing the equation $w = 4$ in Problem 2, consider:

- Reviewing Problem 5 from Activity 2, focusing on how students were able to manipulate the hanger to solve for one weight.
- Assigning Practice Problem 2.
- Asking, "How can you use the properties of equality to rebalance the hanger?"

If students do not write an explanation that connects the value of w to the equation's solution, consider:

- Reviewing the connection between the weight of one variable and the solution to an equation in Activity 2.
- Assigning Practice Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was today's work similar to or different from the previous lesson's work with hanger diagrams?
- In what ways did Activity 2 go as planned? What might you change for the next time you teach this lesson?

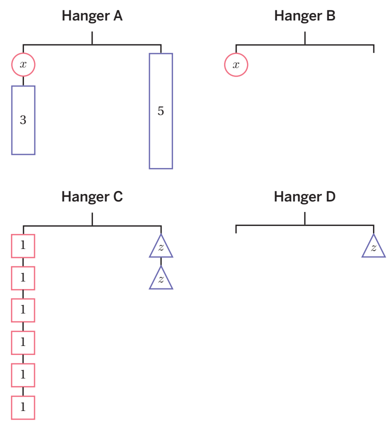
Practice

Independent

Name: ________ Date: ________ Period: ________

Practice

Hangers A and C are balanced. Use them to complete Problems 1–3.



1. Write equations that represent Hanger A and Hanger C.

Hanger A: $x+3=5$

Hanger C: $1+1+1+1+1+1=2z$ or
$1+1+1+1+1+1=2 \cdot z$ or
$1+1+1+1+1+1=z+z$

2. Balance Hangers B and D by completing the empty sides of the hangers. Then, write equations that represent Hanger B and Hanger D.

Hanger B: The hanger should show one x on the left side and 2 on the right side.
$x=2$

Hanger D: The hanger should show 3 ones on the left side and one z on the right side.
$1+1+1=z$ or $3=z$

3. Explain how you know the value for x is a solution to the equation for Hanger A, and the value for z is a solution to the equation for Hanger C.

I know that 2 is a solution for Hanger A's equation because when I substitute 2 for x, the equation is true ($2+3=5$).

I know that 3 is a solution for Hanger C's equation because when I substitute 3 for z, the equation is true ($1+1+1+1+1+1=2 \cdot 3$ or $1+1+1+1+1+1=3+3$).

Name: ________ Date: ________ Period: ________

Practice

4. Lin needs to save up $20 for a new game. How much money has she saved at each percentage of her goal?

a. 25%
$\$5;\ 20 \cdot \frac{1}{4}=5$

b. 75%
$\$15;\ 20 \cdot \frac{3}{4}=15$

c. 125%
$\$25;\ 20 \cdot \frac{125}{100}=25$

5. Calculate each quotient.

a. $0.009 \div 0.001 = 9$

b. $0.009 \div 0.002 = 4.5$

c. $0.0045 \div 0.001 = 4.5$

d. $0.0045 \div 0.002 = 2.25$

6. Evaluate each expression. Then, explain how you can use the first expression to evaluate the second expression.

a. $2-1=1$

b. $2-1.1=0.9$

In the second expression, I subtract 0.1 more than in the first expression, so the value will be 0.1 less. The number 1 is really 1.0 or $\frac{10}{10}$, so if I subtract an extra $\frac{1}{10}$, I get $\frac{9}{10}$, or 0.9.

Practice Problem Analysis

Type	Problems	Refer to	DOK
On-lesson	1	Activities 1–2	2
	2	Activities 1–2	2
	3	Activities 1–2	2
Spiral	4	Unit 3 Lesson 11	2
	5	Unit 5 Lesson 13	2
Formative	6	Unit 6 Lesson 7	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Practice Solving Equations

Let's solve equations by doing the same to each side.

Focus

Goals

1. Interpret and coordinate sentences, equations, and diagrams that represent the same addition or multiplication situation.
2. **Language Goal:** Solve equations of the form $x + p = q$ or $px = q$, and explain the solution method. **(Writing)**

Rigor

- Students practice **procedural fluency** representing and solving equations.
- Students **apply** solving equations to real-world scenarios.

Coherence

Today

Students solve more equations of the forms $x + p = q$ and $px = q$, which now involve whole number, fraction, and decimal values. Students move from using diagrams to reasoning about unknown quantities by looking at the structure in equations. Through their experiences with a variety of representations and strategies, students develop flexibility and fluency in writing and solving equations with a variable, including those in real-world contexts.

Previously

In Lessons 5 and 6, students took an in-depth look at balance and related hanger diagrams to balancing equations. They saw that, if one side of a hanger or true equation is changed, the other side must be changed in the exact same way to keep it balanced or equal.

Coming Soon

In Lesson 8, students will further explore solving equations of the form $px = q$ with fractional values, extending their understanding of fractions as division.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Whole Class	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

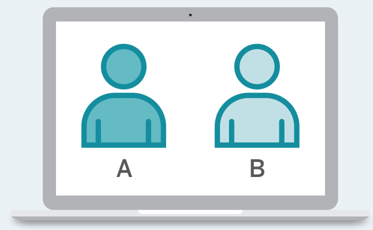
Review words

- *coefficient*
- *equation*
- *variable*

Amps | **Featured Activity**

Activity 1
Digital Tables

Students can enter values into a table as an organizer for solving equations.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students work together to solve equations in Activity 1, either person might, at any point, feel lost within the structure of the solution process. The partners need to recognize that they might have different strengths and that their partner can help them if they find themselves at their limits. By honing in on each other's strengths, they can confidently approach and complete the task.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1** have students complete the fourth and fifth row of the table. Consider assigning the remaining rows as additional practice.
- In **Activity 2** have students complete Problems 1 and 2. Consider assigning the remaining problems as additional practice

Whole Class | 5 min

Warm-up Subtracting From Five

Students recall how to subtract with fractions and decimals, which often need regrouping, in preparation for solving equations for variables involving such operations.

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 7

Practice Solving Equations

Let's solve equations by doing the same to each side.

Warm-up Subtracting From Five

Using mental math, determine the value of each expression.

1. $5 - 2 = 3$
2. $5 - 2.1 = 2.9$
3. $5 - 2.17 = 2.83$
4. $5 - 2\frac{7}{8} = 2\frac{1}{8}$

Compare and Connect: How are these expressions similar and different? Can you use your approach to Problem 1 to help you complete the other problems?

Log in to Amplify Math to complete this lesson online.

1 Launch

Display one expression at a time, and give students 30 seconds of quiet think time. Discuss each expression as a whole class.

2 Monitor

Help students get started by saying, "Think about how Problems 2–4 are related to the first expression, $5 - 2$."

Look for points of confusion:

- **Not recognizing when regrouping is needed**. Refer to the strategies discussed in previous examples.

Look for productive strategies:

- Visualizing a number line. Students can mentally subtract an extra 0.1 and then 0.07 from the previous problem, if the problems are all seen on a number line.

3 Connect

Display one expression at a time, but keep each visible throughout the remaining discussion. Record responses for all to see.

Have students share the strategies they used for each expression.

Ask, "Did anyone solve the problem in a different way?"

Highlight the common strategy of regrouping across each of the last three expressions, regardless of number types. Also, note and discuss how each difference relates to the first result, $5 - 2 = 3$, with respect to how the subtrahend of each expression also relates to 2.

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to respond to the question posed in the Student Edition, "How are these expressions similar and different? Can you use your approach to Problem 1 to help you complete the other problems?" Listen for students who recognize the first number remains the same and the second number increases each time, resulting in a difference that decreases each time. For example, illustrate how 2.1 in Problem 2 is 0.1 more than 2 which means the difference is 0.1 less than the difference in Problem 1.

Power-up

To power up students' ability to relate subtraction expressions in order to calculate the difference, ask:

Evaluate each expression. Then explain how you can use the first problem to solve the second problem.

a. $7 - 3 = 4$

b. $7 - 3\frac{1}{4} = 3\frac{3}{4}$

Sample response: In the second problem, I subtract $\frac{1}{4}$ more than in the first problem, so the answer is $\frac{1}{4}$ less than the answer in the first. $\frac{1}{4}$ less than 4 is $3\frac{3}{4}$.

Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 6.

Pairs | 15 min

Activity 1 Solving Equations With a Partner

Students solve equations by performing the same operation on each side of the equal sign. They recognize that isolating a variable corresponds to determining a solution.

Amps Featured Activity Digital Tables

Activity 1 Solving Equations With a Partner

Work together to solve the following equations.

Equation	What I do to the variable side . . .		. . . I do to the other side.	Solve and check
$18 = 2x$	$\frac{2}{2}x$ Because this is multiplying x by 2, I need to divide by 2 to make it 1 x, or x.	=	$\frac{18}{2}$ I need to divide my side by 2, which equals 9.	Solution: $x = 9$ Check: $18 = 2 \cdot 9$
$36 = 4x$	$\frac{4}{4}x$ Sample response: Divide by 4, which leaves $1x$, or just x.	=	$\frac{36}{4}$ Sample response: Divide 36 by 4, which equals 9.	Solution: $x = 9$ Check: $36 = 4 \cdot 9$
$17 = x + 9$	$x + (9 - 9)$ Subtract 9 from the right side to leave just the x.	=	$17 - 9$ Subtract 9 from 17, which equals 8.	Solution: $x = 8$ Check: $17 = 8 + 9$
$8x = 56$	$\frac{8}{8}x$ Divide by 8, which leaves $1x$ or just x.	=	$\frac{56}{8}$ Divide 56 by 8, which equals 7.	Solution: $x = 7$ Check: $8 \cdot 7 = 56$
$x + 3\frac{5}{6} = 8$	$x + \left(3\frac{5}{6} - 3\frac{5}{6}\right)$ Subtract $3\frac{5}{6}$ to leave just the x.	=	$8 - 3\frac{5}{6}$ Subtract $3\frac{5}{6}$ from 8, which equals $4\frac{1}{6}$.	Solution: $x = 4\frac{1}{6}$ Check: $4\frac{1}{6} + 3\frac{5}{6} = 8$

1 Launch

Arrange students in pairs, Partner A and Partner B. Explain that Partner A always "owns" the left side of the equation, and Partner B always "owns" the right side of the equation. Say, "For each equation, the person who has the x on their side will go first." Model using the completed example in the Student Edition, noting how Partner B would start because the variable is on the right side of the equation.

2 Monitor

Help get students started by asking, "What can you do to isolate the x? Think back to the hanger diagrams. What did you do to balance them?"

Look for points of confusion:

- **Having difficulty solving the equations where a variable is multiplied by a fraction**. Walk through the process step by step, reminding students that dividing by a fraction is equivalent to multiplying by its reciprocal.

Look for productive strategies:

- Using precise language. Encourage students' discussions to accurately describe the process.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter values into an interactive table and solve equations simultaneously.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing only the first six rows of the table.

Extension: *Math Enrichment*

Have students choose one equation and create a word problem that could be represented by that equation. For an added challenge, ask them to choose an equation that includes fractional or decimal values.

Math Language Development

MLR8: Discussion Supports

During the Launch, display these prompts that partners could ask each other as they progress through the activity.

- "Can you tell me why you ______ by ______?" (use for multiplication/division)
- "Can you tell me why you ______ to/from each side?" (use for addition/subtraction)

English Learners

Annotate the first equation $18 = 2x$ by writing "variable side" next to the side that contains the variable so that students can connect this phrase to the algebraic representation. Draw an arrow that points to the variable x.

Pairs | 15 min

Activity 1 Solving Equations With a Partner (continued)

Students solve equations by performing the same operation on each side of the equal sign. They recognize that isolating a variable corresponds to determining a solution.

Name: ______ Date: ______ Period: ______

Activity 1 Solving Equations With a Partner (continued)

Equation	What I do to the variable side . . .		. . . I do to the other side.	Solve and check
$21 = \frac{1}{4}x$	$\left(\frac{1}{4} \div \frac{1}{4}\right)x$ or $\left(\frac{1}{4} \cdot \frac{4}{1}\right)x$ Divide by $\frac{1}{4}$, which is the same as multiplying by $\frac{4}{1}$ or 4, leaving just the x.	= =	$21 \div \frac{1}{4} = 21 \cdot \frac{4}{1}$ or $21 \cdot 4 = x$ $21 \cdot \frac{4}{1}$ Divide 21 by $\frac{1}{4}$ (multiply by the reciprocal $\frac{4}{1}$ or 4,) which equals 84.	Solution: $x = 84$ Check: $21 = \frac{1}{4}(84)$
$2.17 + x = 5$	$(2.17 - 2.17) + x$ Subtract 2.17 to leave just the x.	=	$5 - 2.17$ Subtract 2.17 from 5, which equals 2.83.	Solution: $x = 2.83$ Check: $2.17 + 2.83 = 5$
$10x = \frac{1}{3}$	$\frac{10}{10}x$ Divide by 10, which leaves $1x$ or just x.	=	$\frac{1}{3} \div \frac{10}{1}$ then $\frac{1}{3} \cdot \frac{1}{10}$ Divide $\frac{1}{3}$ by 10 or $\frac{10}{1}$, which then becomes $\frac{1}{3} \cdot \frac{1}{10}$, which equals $\frac{1}{30}$.	Solution: $x = \frac{1}{30}$ Check: $10 \cdot \frac{1}{30} = \frac{1}{3}$
$17.05 = 14.88 + x$	$(14.88 - 14.88) + x$ Subtract 14.88 to leave just the x.	=	$17.05 - 14.88$ Subtract 14.88 from 17.05, which equals 2.17.	Solution: $x = 2.17$ Check: $17.05 = 14.88 + 2.17$

Lesson 7 Practice Solving Equations 643

3 Connect

Have students share their strategies and solutions for each equation, and how the check shows the relationships between multiplication and division, and addition and subtraction.

Highlight that, while the equations and operations changed for every row, whatever the one partner did to the variable side, the other partner did the exact same to the other side. Also, for an equation, such as $21 = \frac{1}{4}x$, there are two possible strategies for solving: divide each side by $\frac{1}{4}$, or multiply both sides by 4.

Ask, "Could all these equations have been solved using mental math? How can the check help you?"

Independent | 15 min

Activity 2 Representing Scenarios With Equations

Students match equations to scenarios and then solve those equations by performing the same exact operation on each side.

Activity 2 Representing Scenarios With Equations

Circle *all* the equations that represent each scenario. Then calculate a solution for each scenario. Consider drawing a diagram to help with your thinking.

1. Kofi has 8 fewer slabs of salt than Ime. If Ime has 26 slabs of salt, how many slabs of salt does Kofi have?

A. $26 - x = 8$

B. $x = 26 + 8$

C. $x + 8 = 26$

D. $26 - 8 = x$

$x =$ 18 slabs

2. A market in Djenne has shops with 8 goats in each. There are 14 shops. How many total goats are there in the market?

A. $y = 14 \div 8$

B. $\frac{y}{8} = 14$

C. $\frac{1}{8}y = 14$

D. $y = 14 \cdot 8$

$y =$ 112 goats

3. A caravan bringing salt from Taghaza traveled 489 miles to the market in Timbuktu. If a second caravan from the gold mines of Lobi traveled 292 more miles to get to Timbuktu, how many miles did the second caravan travel?

A. $292 = 489 - z$

B. $z - 292 = 489$

C. $489 + 292 = z$

D. $292 = 489 + z$

$z =$ 781 miles

4. Amara traveled 27 miles last week from Gao to the Niger River, which was three times as far as Neela traveled. How far did Neela travel?

A. $3w = 27$

B. $w = \frac{1}{3} \cdot 27$

C. $w = 27 \div 3$

D. $w = 3 \cdot 27$

$w =$ 9 miles

STOP

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1 Launch

Say, "You now have strategies for solving equations with variables, but let's revisit how those equations represent a scenario, which may actually be able to be represented by more than one equation." Allow 10 minutes of quiet work time.

2 Monitor

Help students get started by asking, "What values do you know? What value do you not know? How are those related?"

Look for points of confusion:

- **Not identifying all equations that match the scenario**. Encourage them to express the relationships in their own words or draw a diagram representing the scenario, and remind them about inverse operations.

Look for productive strategies:

- After determining the solution, students analyze each remaining equation to determine additional possible equations that represent the scenario. Encourage students to draw tape diagrams to help visualize the scenarios instead, making the connection between the equations and the scenario.

3 Connect

Have students share their strategies for matching equations to the stories and for solving the equations, including those who drew diagrams to help them understand and reason about the relationships.

Highlight the similarities in the structure of additive scenarios, such as Problems 1 and 3, and multiplicative scenarios, such as Problems 2 and 4.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2. These problems represent one additive scenario and one multiplicative scenario, which are the two different structures examined in this activity.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of each scenario.

- **Read 1: Ask,** "What is this scenario about? Describe it in your own words, without using the numbers."
- **Read 2: Ask,** "What are the quantities or relationships in the scenario? Tell me one of them."
- **Read 3:** Ask students to brainstorm possible strategies to connect the scenario with the appropriate equation.

English Learners

Annotate key words and phrases in the text, such as *fewer*, *in each*, 292 more *miles than*, and *three times as far*.

Whole Class | 5 min

Summary

Review and synthesize all the different ways to represent a scenario mathematically.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that writing and solving equations can help you answer questions about mathematical scenarios, which may involve whole numbers, fractions, or decimals. These equations can also be represented by tape diagrams or hanger diagrams, which may be helpful in solving them.

In some scenarios, the two quantities can be related by addition.	$x + 1.5 = 3.25$
You can keep the equality by subtracting the same value, 1.5, from both sides.	$x + 1.5 - 1.5 = 3.25 - 1.5$
This isolates the variable x on one side and reveals the solution.	$x = 1.75$
In other scenarios, the two quantities can be related by multiplication	$\frac{3}{4}x = 54$
You can keep the equality by dividing both sides by the same value $\frac{3}{4}$.	$\frac{3}{4}x \div \frac{3}{4} = 54 \div \frac{3}{4}$
This isolates the variable x on one side and reveals the solution.	$x = 72$

Reflect:

Synthesize

Ask:

- "What are some ways to understand how a scenario can be represented mathematically?"
- "What have you learned about equations that surprised you?"
- "As you think about using diagrams to help you understand relationships, where else did you see diagrams used earlier this year? Where were they most helpful to you? Least helpful?"
- "What are some connections you see between the types of diagrams used in the last four lessons?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did you use inverse operations today? How did that connection help you solve equations?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by matching equations to a scenario and then solving each equation.

Printable

Name: ______________________ Date: __________ Period: __________

Exit Ticket 6.07

1. Diego's sister is 3.91 ft tall. Their dad is 6.08 ft tall. To determine how many feet taller Diego's dad is than his sister, which equation(s) can you use? Select *all* that apply.

   A. $6.08 + 3.91 = d$

   B. $6.08 - 3.91 = d$

   C. $6.08 - d = 3.91$

   D. $3.91 + d = 6.08$

2. Choose one of the equations that represent the scenario from Problem 1. Determine the solution. Show how you solved the equation.

   $d =$ 2.17 ft

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain why different equations can describe the same scenario. 1 2 3

b. I can solve an equation that has whole numbers, fractions, and decimals. 1 2 3

Lesson 7 Practice Solving Equations

Success looks like . . .

- **Goal:** Interpreting and coordinating sentences, equations, and diagrams that represent the same addition or multiplication situation.
  - » Selecting all equations that can be used to determine how much taller Diego's dad is than his sister in Problem 1.
- **Language Goal:** Solving equations of the form $x + p = q$ or $px = q$, and explaining the solution method. **(Writing)**
  - » Solving one of the correct equations in Problem 2.

Suggested next steps

If students are unable to determine an equation, consider:

- Suggesting they draw a picture or diagram to represent the scenario.

If students are having trouble determining more than one equation, consider:

- Reminding them to think of the inverse operation.

If students have difficulty solving the equation, consider:

- Referring back to Activity 2, Problem 1, which has a similar structure that can be connected to this problem.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students gotten better at representing scenarios with equations?
- What challenges did students encounter as they worked on understanding balance in equations? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

For Problems 1 and 2, circle *all* the equations that represent each scenario. Then calculate the solution for each scenario.

1. Kiran's backpack weighs 3 lb less than Clare's backpack. Clare's backpack weighs 14 lb. How much does Kiran's backpack weigh?
   - A. $x + 3 = 14$
   - B. $3x = 14$
   - C. $x = 14 - 3$
   - D. $x = 14 \div 3$

   $x =$ 11 lb

2. Each notebook contains 60 sheets of paper. Andre has 5 notebooks. How many sheets of paper do Andre's notebooks contain in total?
   - A. $y = 60 \div 5$
   - B. $y = 5 \cdot 60$
   - C. $\frac{y}{5} = 60$
   - D. $5y = 60$

   $y =$ 300 sheets

3. Solve each equation. Show your work. Be sure to check your solution by substituting it back into the original equation.

   a. $y + 1.8 = 14.7$
   $y = 12.9$
   Solve:
   $y + 1.8 - 1.8 = 14.7 - 1.8$
   $y = 12.9$
   Check:
   $(12.9) + 1.8 = 14.7$
   $14.7 = 14.7$

   b. $6 = \frac{1}{2}z$
   $z = 12$
   Solve:
   $6 \div \frac{1}{2} = \frac{1}{2}z \div \frac{1}{2}$
   $12 = z$
   Check:
   $6 = \frac{1}{2}(12)$
   $6 = 6$

   c. $3\frac{1}{4} = \frac{1}{2} + w$
   $w = 2\frac{3}{4}$
   Solve:
   $3\frac{1}{4} - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} + w$
   $2\frac{3}{4} = w$
   Check:
   $3\frac{1}{4} = \frac{1}{2} + \left(2\frac{3}{4}\right)$
   $3\frac{1}{4} = 3\frac{1}{4}$

   d. $2.5t = 10$
   $t = 4$
   Solve:
   $2.5t \div 2.5 = 10 \div 2.5$
   $t = 4$
   Check:
   $2.5(4) = 10$
   $10 = 10$

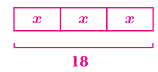
   e. $6x = 45$
   $x = 7.5$
   Solve:
   $6x \div 6 = 45 \div 6$
   $x = 7.5$
   Check:
   $6(7.5) = 45$
   $45 = 45$

Name: ______ Date: ______ Period: ______

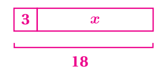
Practice

4. Draw a tape diagram that represents each equation.

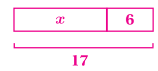
   a. $3 \cdot x = 18$



   b. $3 + x = 18$



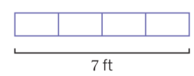
   c. $17 - 6 = x$



5. For a science experiment, students need to determine 25% of 60 grams.
   - Jada says, "I can determine this by calculating $\frac{1}{4}$ of 60."
   - Andre says, "25% of 60 means $\frac{25}{100} \cdot 60$."

   Do you agree with either of them? Explain your thinking.
   I agree with both of them; Sample response: 25% is the same as $\frac{1}{4}$ and $\frac{25}{100}$.

6. Lin is making a square planter for her garden. She has a 7 ft long piece of wood she is using for the sides. The marks where she is planning on cutting the wood are shown.



   a. Write an expression that can be used to determine the length of each section of wood:
   $7 \div 4$

   b. What is the length of each section of wood?
   $\frac{7}{4}$ ft or $1\frac{3}{4}$ ft

Practice Problem Analysis

Type	Problems	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 1	1
Spiral	4	Unit 6 Lesson 3	2
	5	Unit 3 Lesson 10	2
Formative	6	Unit 6 Lesson 8	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

A New Way to Interpret a Over b

Let's investigate what a fraction means when the numerator and denominator are not whole numbers.

Focus

Goals

1. Comprehend that the notation $\frac{a}{b}$ can be used to represent division generally, and the numerator and denominator can include fractions, decimals, or variables.
2. **Language Goal:** Describe a situation that could be represented by a given equation of the form $x + p = q$ or $px = q$. **(Speaking and Listening)**
3. Express division as a fraction when solving equations of the form $px = q$.

Coherence

Today

Students apply the general procedure they just learned for solving $px = q$ in order to define what $\frac{a}{b}$ means when a and b are not whole numbers. Up to this point, students have likely only seen a fraction bar separating two whole numbers, but now an expression like $\frac{2.5}{8.9}$ or $\frac{\frac{1}{2}}{\frac{3}{5}}$ can also be well-defined as division. They apply this to writing and interpreting fraction solutions in real-world contexts.

Previously

In Grade 3, students learned about fractions as being the result of partitioning wholes into equal parts and then selecting some of the parts. This worked well for whole-number numerators and denominators, and it allowed students to locate fractions on the number line. In Grade 5, students expanded their understanding to also see fractions as representing division. Then, in Lessons 5–6 of this unit, students discovered the Division Property of Equality as a way to maintain balance and equality, and as a means to solve an equation of the form $px = q$.

Coming Soon

In Lesson 9, students will conclude the first section of this unit by writing equations in one variable and applying their understanding of properties of equality and solving equations to revisit percentages, now with missing values.

Rigor

- Students develop **conceptual understanding** of how to represent and interpret fractions when the numerator and denominator are not whole numbers.
- Students **build procedural fluency** representing division as a fraction.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *coefficient*
- *solution to an equation*

Amps | Featured Activity

Exit Ticket

Real-Time Exit Ticket

Check in real time whether your students can identify multiple expressions that represent a solution to an equation using a digital Exit Ticket.

Building Math Identity and Community

Connecting to Mathematical Practices

Some students may immediately give up when asked to work with numbers that are not whole numbers or when they find a solution that is not a whole number. But, finding the pattern to the solution strategy for equations of the form $px = q$ brings more meaning to the process. By thinking of a fraction as division, students are able to see a complicated expression as a familiar division expression.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-Up** may be omitted.
- in **Activity 1,** solve Problem 1 as a class. Then assign each pair one remaining problem to complete.
- In **Activity 2,** have each pair work on either Problem 1 or 2, and then share their responses with the class.

Independent | 5 min

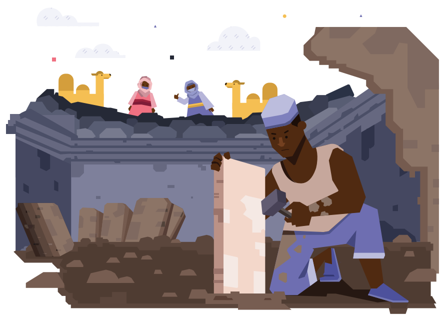
Warm-up Division and Fractions

Students solve for unknown values in word problems involving a missing factor or division, and recall that division expressions can be written as fractions.

Unit 6 | Lesson 8

A New Way to Interpret a Over b

Let's investigate what a fraction means when the numerator and denominator are not whole numbers.



Warm-Up Division and Fractions

Determine a fraction that represents a solution for each scenario.

1. A buyer and seller agree that 4 blocks of salt weigh the same as 1 gold bar. If this is true, how many gold bars would weigh the same as 21 blocks of salt?
   $\frac{21}{4}$

2. Kofi dug up 7 kg of salt in a mine in Taoudenni. He wants to transport equal amounts to 10 different markets. How many kilograms of salt should he transport to each market?
   $\frac{7}{10}$

Log in to Amplify Math to complete this lesson online.

648 Unit 6 Expressions and Equations

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by activating prior knowledge about ratios. For Problem 1, ask "How could you write this as a ratio? How can you rewrite the ratio as a fraction?"

Look for points of confusion:

- **Dividing 4 by 21, rather than dividing 21 by 4.** Have them create a table for gold bars and blocks of salt, similar to those used in Lesson 2.
- **Only writing equivalent whole-number ratios** Ask, "How many gold bars for 1 block of salt?"

Look for productive strategies:

- Using ratio or division thinking to solve Problem 1. If students write the solution as $\frac{5}{(1)(4)}$, acknowledge this is correct, but ask them to write it as an improper fraction.
- Recognizing Problem 2 as a fair-share problem, and using a fraction to represent the division.

3 Connect

Have students share how they determined a solution for each scenario, focusing on the fraction solutions and how they determined the numerator and denominator.

Highlight that the relationship between multiplication and division can be used to rewrite $4 \cdot t = 21$ as $t = 21 \div 4$. Also, the numerators and denominators of the fractions relate to both the original equations and the coefficients.

Ask, "Can you always rewrite a multiplication equation like that in Problem 1 as a corresponding division equation? Will the solution to division equations, such as these, always be a fraction?"

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with counters, pennies, or other objects that they can use to model and make sense of the relationships between quantities. For example, in Problem 1, they could designate pennies to represent blocks of salt and an index card to represent one gold bar.

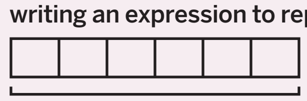
Extension: *Math Enrichment*

Challenge students to write as many equations as they can that represent each scenario. Sample equations shown for Problem 1, where t represents the number of gold bars.

$4 \cdot t = 21, \frac{21}{4} = t, 21 \div 4 = t$

Power-up

To power up students' ability to use a diagram to assist them in writing an expression to represent a real-world scenario, ask:



a. Write an expression that can be used to determine the length of each section in the diagram shown. $10 \div 6$

b. What is the length of each section? $\frac{10}{6}$ or $\frac{5}{3}$

Use: Before the Warm-up.

Informed by: Performance on Lesson 7, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 15 min

Activity 1 Interpreting $\frac{a}{b}$

Students apply the general procedure for solving $px = q$ to define what $\frac{a}{b}$ means when a and b are not whole numbers.

Name: ______ Date: ______ Period: ______

Activity 1 Interpreting $\frac{a}{b}$

Solve each equation for x.

1. $35 = 7x$

   $x = \frac{35}{7}$ or $x = 5$

2. $35 = 11x$

   $x = \frac{35}{11}$ or $x = 3\frac{2}{11}$

3. $7x = 7.7$

   $x = \frac{7.7}{7}$ or $x = 1.1$

4. $0.3x = 2.1$

   $x = \frac{2.1}{0.3}$ or $x = 7$

5. $\frac{2}{5} = \frac{1}{2}x$

   $x = \frac{\frac{2}{5}}{\frac{1}{2}}$ or $x = \frac{4}{5}$

Are you ready for more?

Solve the equation for x.

$\frac{1}{6} \cdot \frac{3}{20} \cdot \frac{5}{42} \cdot \frac{7}{72} \cdot x = \frac{1}{384}$

$x = 9$

1 Launch

Have students use the ***Think-Pair-Share*** routine. Give them 5 minutes to work independently and 5 minutes to compare responses with their partner.

2 Monitor

Help students get started by reminding them that, in the expression $7x$, 7 is the coefficient and x is the variable, and it is equivalent to $7 \cdot x$.

Look for points of confusion:

- **Trying to convert $\frac{35}{11}$ into a decimal.** Remind them that $\frac{35}{11}$ is a number, and there is no need to determine a decimal equivalent.
- **Multiplying $\frac{2}{5} \cdot \frac{1}{2}$.** Ask, "How would you solve for x if the equation was $5 = 2x$? $\frac{2}{5} = 2x$?"

Look for productive strategies:

- Simplifying, or not writing solutions as fractions. Acknowledge this is correct, but challenge students to rewrite as fractions.
- Writing equivalent equations. By multiplying both sides by the same number, students can recognize that, solving the equation $7x = 7.7$ is the same as solving the equation $70x = 77$ for example.

3 Connect

Have students share their solutions and strategies for solving each equation.

Highlight that all these equations are of the form $px = q$ and both sides can be divided by p, the coefficient of x, to determine the solution, which can always be written as a fraction.

Ask, "How do the numerators and denominators of each solution relate to the original equations?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Students may feel successful with Problem 1 and be unsure how to approach Problem 2. Ask, "What is the only difference between the equations in Problems 1 and 2? How did you solve Problem 1? Can you solve the equation in Problem 2 a similar way?"

Have students write the solutions as expressions that show $x = a$ number $\div$ another number.

Accessibility: *Clarify Vocabulary and Symbols*

During the Connect, be sure students understand that these equations are all of the general form $px = q$. Emphasize that p and q are numbers, while x remains the unknown (variable). Any number — including whole numbers, fractions, and decimals — can be substituted for p and q. Ask:

- "How is the equation in Problem 5 of the form $px = q$? What are the values of p and q?"
- "What strategies can you use to solve the equation in Problem 5?"

Pairs | 15 min

Activity 2 Storytime Again

Students continue to practice representing particular, concrete relationships with equations to solve for missing values, which now also involve multiple rational numbers.

Activity 2 Storytime Again

Think of a story that could be represented by each equation. For each, state what quantity x represents, and the value for x that represents a solution.

1. $\frac{7}{10} + x = 1$

   a. In my story, x represents:

   Sample response: the amount of pizza already eaten

   b. A solution to the equation is: $x = \frac{3}{10}$

2. $\frac{1}{4}x = \frac{3}{2}$

   a. In my story, x represents:

   The total length of a ribbon; Sample response: When divided into 4 equal parts, each part has a length of $\frac{3}{2}$.

   b. A solution to the equation is: $x = \frac{\frac{3}{2}}{\frac{1}{4}}$ or $\frac{12}{2}$ or 6

STOP

650 Unit 6 Expressions and Equations

1 Launch

Keep students in their same pairs, and remind them of previous work with scenarios and equations, noting the key difference here will be working with fractions and decimals.

2 Monitor

Help students get started by asking, "Where might you see a value of $\frac{7}{10}$ in real life? What about a value of $\frac{3}{2}$? What is something that you can calculate $\frac{1}{4}$ of?"

Look for points of confusion:

- **Creating scenarios where fractions and decimals don't make sense.** Have them explain what each quantity represents first, and then determine a solution.
- **Not writing a scenario for the multiplication equation.** "What does $\frac{1}{4}x$ mean? Can you rewrite it as a division expression?"

Look for productive strategies:

- Arriving at the complex fraction $\frac{\frac{3}{2}}{\frac{1}{4}}$. Acknowledge this as correct, but not easy to interpret in the context of the story. Have them divide the fractions.

Note: Students are not expected to know the term *complex fraction* in Grade 6.

- Writing scenarios that use each term as intended by the provided expressions, and solving each expression.

3 Connect

Have students share their stories for each equation. Ask them to interpret the solutions in terms of their scenarios.

Highlight that not all quantities make sense to have fractional values, but that does not impact how to solve the equations or their mathematical solutions.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

If students feel intimidated by the fractions in each equation, ask, "Can you imagine stories with similar structures that involve whole numbers? How could you tweak or alter those stories to use the numbers given in these equations?"

Extension: *Math Enrichment*

Direct students to Problem 2. Tell them that because this equation is of the form $px = q$, they can divide both sides of the equation by p, resulting in the solution shown. Ask them how they can interpret the structure of this complicated fraction.

$$x = \frac{\frac{3}{2}}{\frac{1}{4}}$$

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, have students meet with another pair of students to share and receive feedback on their stories, what the variables represent, and their solutions. Have reviewers ask these questions:

- "How are the parts of the expression represented in your story?"
- "How does your solution fit into your story?"

Allow time to complete a final draft based on feedback.

English Learners

Suggest that students draw diagrams or pictures to help represent their stories. For example, they could draw a pizza in Problem 1 that is divided into 10 equal-sized slices.

Whole Class | 5 min

Summary

Review and synthesize how the relationship between division and fractions is connected to numerators and denominators of all number types.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You investigated fractions in which the numerator and denominator were *not* whole numbers. In prior grades, you learned that a fraction, such as $\frac{4}{5}$, can be thought of in a few ways:

- $\frac{4}{5}$ is a number that you can locate on the number line by dividing the section between 0 and 1 into five equal parts and then counting four of those parts to the right of 0.
- $\frac{4}{5}$ is how much each person would receive if four wholes were shared equally among five people.
- $\frac{4}{5}$ is the quotient resulting from dividing $4 \div 5$.

You can extend this quotient meaning of a fraction to include numerators and denominators that are *not* whole numbers. For example, $\frac{1.5}{4.5} = 1.5 \div 4.5 = \frac{1}{3}$.

When you solve any equation using division, the solution can be written as a fraction. These fractions can also involve non-whole numbers in the numerator and denominator.

Reflect:

Synthesize

Highlight:

- Any fraction can be written as division, and also any division can be written as a fraction. So, now students can see how it makes sense for numerators and denominators to be numbers that aren't just whole numbers.
- When solving an equation where the variable is multiplied by a coefficient, the solution can always be written as a fraction, where the coefficient will be the denominator.

Ask, "In thinking about how fractions also represent parts and wholes, how could you interpret a fraction, such as $\frac{1.5}{4.5}$, in terms of parts and wholes?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are division and fractions related? How did your work today build upon your previous work with division as fractions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by relating the solution of an equation of the form $px = q$ to both a division expression and a fraction.

Amps Featured Activity | Real-Time Exit Ticket | Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.08

1. Select *all* the expressions that represent a solution to the equation $5 = \frac{2}{3}x$.

   A. $5 \cdot \frac{2}{3}$

   B. $\frac{5}{\frac{2}{3}}$

   C. $5 \div \frac{2}{3}$

   D. $\frac{15}{2}$

   E. $\frac{10}{3}$

2. Write a story that can be represented by the equation $5 = \frac{2}{3}x$. Explain what x, $\frac{2}{3}$, and 5 each represent in the context of your story.

   Sample response: I ate 5 slices of veggie pizza, which was $\frac{2}{3}$ of the total slices in the box.
   - **x represents the total number of slices of veggie pizza in the box.**
   - **$\frac{2}{3}$ represents the fraction of all the veggie pizza's slices that I ate. I divided all the slices into 3 equal groups and ate 2 of those groups.**
   - **5 represents the number of slices of veggie pizza that I ate.**

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I understand the meaning of a fraction made up of fractions or decimals, such as $\frac{2.1}{0.07}$ or $\frac{\frac{4}{5}}{\frac{3}{2}}$. 1 2 3

b. When I see an equation, I can make up a story that the equation might represent, explain what the variable represents in the story, and solve the equation. 1 2 3

Success looks like . . .

- **Goal:** Comprehending that the notation $\frac{a}{b}$ can be used to represent division generally, and the numerator and denominator can include fractions, decimals, or variables.
- **Language Goal:** Describing a situation that could be represented by a given equation of the form $x + p = q$ or $px = q$. **(Speaking and Listening)**
- **Goal:** Expressing division as a fraction when solving equations of the form $px = q$.
  - » Selecting equivalent division expressions for the solution in Problem 1.

Suggested next steps

If students are unable to write anything, consider:

- Giving them the opportunity to respond verbally.
- Suggesting that they do not need to use words for the second prompt and can instead show the steps for solving the equation.

If students did not select Expression B, $\frac{5}{\frac{2}{3}}$, consider:

- Reminding them that fractions can have non-whole numbers as numerators, as well as denominators.

If students did not select Expression D, $\frac{15}{2}$, consider:

- Reviewing Problem 5 from Activity 1, noting that the fractions could have been divided to get $\frac{4}{5}$, and asking, "What would you get if you actually divided 5 by $\frac{2}{3}$?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was Activity 2 similar to or different from Activity 2 in Lesson 2?
- What challenges did students encounter as they worked on Activity 1? How did they work through them? What might you change for the next time you teach this lesson?

MLR Math Language Development

Language Goal: Describing a situation that could be represented by a given equation of the form $x + p = q$ or $px = q$.

Reflect on students' language development toward this goal.

- How have students progressed in writing story contexts that can be represented by division equations? Are their descriptions of what each quantity or term represents in the story accurate and precise?
- How can you help them be more precise in their descriptions?

Sample descriptions for Problem 2 from the Exit Ticket:

Emerging	Expanding
x is the whole pizza.	x is the total number of slices of pizza.

Practice

Independent

Name: ______ Date: ______ Period: ______

1. Select *all* the expressions that are equal to $\frac{3.15}{0.45}$.

A. $3.15 \cdot 0.45$

B. $3.15 \div 0.45$

C. $3.15 \cdot \frac{1}{0.45}$

D. $3.15 \div \frac{45}{100}$

E. $3.15 \cdot \frac{100}{45}$

F. $\frac{0.45}{3.15}$

2. Solve each equation. Show your thinking.

a $4a = 32$

$4a \div 4 = 32 \div 4$

$a = \frac{32}{4}$ or $a = 8$

b $4 = 32b$

$4 \div 32 = 32b \div 32$

$b = \frac{4}{32}$ or 0.125 or $\frac{1}{8}$

c $10c = 26$

$10c \div 10 = 26 \div 10$

$c = \frac{26}{10}$ or $2\frac{6}{10}$ or 2.6

d $26 = 100d$

$26 \div 100 = 100d \div 100$

$d = \frac{26}{100}$ or 0.26

Name: ______ Date: ______ Period: ______

3. For each equation, write a story problem represented by the equation and state what quantity x represents. Consider drawing a diagram to help with your thinking.

a $\frac{3}{4} + x = 2$

Sample response: x represents the amount I ate of the 2 muffins, if my brother ate $\frac{3}{4}$ of the muffins; $x = 1\frac{1}{4}$

b $1.5x = 6$

Sample response: x represents the number of pages I read, if my sister read 6 pages and read 1.5 times as many pages as I read; $x = 4$

4. In a lilac paint mixture, 40% of the mixture is white paint, 20% is blue, and the rest is red. There are 4 cups of blue paint used in a batch of lilac paint. Consider drawing a diagram to help with your thinking.

a How many cups of white paint are used?

8 cups

b How many cups of red paint are used?

8 cups

c How many cups of lilac paint will this batch yield?

20 cups

5. Triangle P has a base of 12 in. and a corresponding height of 8 in. Triangle Q has a base of 15 in. and a corresponding height of 6.5 in. Which triangle has a greater area? Explain your thinking.

Triangle Q has a greater area; Sample response: $\frac{1}{2} \cdot 15 \cdot 6.5 > \frac{1}{2} \cdot 12 \cdot 8$.

6. 20% of 50 is equivalent to which of the following?

A. 5 **C.** 2

B. 10 **D.** 20

Practice Problem Analysis

Type	Problems	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	1
	3	Activity 2	3
Spiral	4	Unit 3 Lesson 12	2
	5	Unit 1 Lesson 11	2
Formative	6	Unit 6 Lesson 9	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Revisiting Percentages

Let's use equations to determine percentages.

Focus

Goals

1. **Language Goal:** State explicitly what the chosen variable represents when creating an equation. **(Speaking and Listening, Writing)**
2. **Language Goal:** Use equations to solve problems involving percentages and explain the solution method. **(Speaking and Listening)**
3. Write equations of the form $px = q$, or equivalent, to represent situations where the amount that corresponds to 100% is unknown.

Coherence

Today

Students connect their previous understanding of percentages to their recent work with equations. They write, solve, and explain how equations of the form $nw = p$ represent scenarios where the part (p), whole (w), or percent (n) is unknown. Using the structure of the equation, students explain how the placement of the variable in the equation affects their solution method.

Previously

In Unit 3, students learned what percentages are and how to use double number lines, tape diagrams, and multiplication and division to solve three types of percentage problems (corresponding to determining n, w, or p respectively when n% of w is p.). However, if w was unknown, they had to rely on ratio reasoning and guess-and-check strategies.

Coming Soon

In Lessons 10 and 11, students will discover the difference between equal and equivalent expressions. They will determine whether expressions are equal or equivalent by substituting a given set of values for variables and evaluating.

Rigor

- Students develop **procedural skills** for using equations to represent and solve for the missing whole in percentage problems.
- Students **apply** their work with expressions and equations to represent and solve percentage problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Whole Class	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF
- *Tape Diagrams* PDF (as needed)
- *Double Number Lines: Percentage Problems* PDF (as needed)

Math Language Development

Review word

- *percent*

Amps | Featured Activity

Activity 2
Dynamic Growing Dogs

When students enter a solution for their equations, the dog's growth will animate, giving real-time feedback that shows whether the solution is correct.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be able to determine a solution strategy that does not require them to write an equation as they are asked to do in Activity 1. They might think that their quantitative reasoning is enough. Encourage students to find the self-discipline to use equations as they can be applied in many ways. While they may not use the equation to find the correct solution, it is a stepping stone that they will use throughout their mathematical studies.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, complete Problem 1 as a class, ensuring students identify what each value represents in the equation. Have pairs complete Problems 2 and 3 together.
- In **Activity 2**, have pairs complete either Problem 1 or 2, and then the related part for Problem 3.

Whole Class | 5 min

Warm-up Number Talk

Students activate prior knowledge of percentages to reason when the whole, part, or percentage is unknown.

Unit 6 | Lesson 9

Revisiting Percentages

Let's use equations to determine percentages.

Warm-up Number Talk

Mentally solve each problem. Be prepared to explain your thinking.

1. 50% of 10 equals what number?
   5
2. 10 is 50% of what number?
   20
3. 8 is what percent of 10?
   80%

Log in to Amplify Math to complete this lesson online.

654 Unit 6 Expressions and Equations

1 Launch

Remind students that percent means $\frac{\text{part}}{\text{whole}} \cdot 100$. Have students use the *Number Talk* routine.

2 Monitor

Help students get started by saying, "50% means one half." Ask, "What is half of 10?"

Look for points of confusion:

- **Not knowing the meaning of given information.** Remind students that percentage problems have a percentage, part, and whole. Ask what each number represents.
- **Calculating 50% of 10 for Problem 2**. Ask, "How is this problem different from the first one?"
- **Being unable to determine the missing percentage.** Ask, "Would 8 be more or less than 50% of 10?" Have students use the guess-and-check strategy.

Look for productive strategies:

- Recognizing 50% as "half," and using this information to help reason about Problem 3.
- Recognizing each value as a percentage, part, or whole, and using the relationships among them to solve.

3 Connect

Display one problem at a time, and keep all previous problems displayed throughout the talk.

Have students share different representations and ways of reasoning, starting with those who mentally pictured tape diagrams and double number lines, followed by those who reasoned using multiplication and division.

Highlight that percentage problems include a percentage, part, and whole.

Ask, "What are the percentage, part, and whole in each scenario?

Math Language Development

MLR8: Discussion Supports — Press for Reasoning

During the Connect, as students share how they solved the problems mentally, press for details in their reasoning. Ask:

- "How are Problems 1 and 2 similar? How are they different?"
- "How do these differences affect the solution?"

Ask students to identify the *part, whole,* and *percentage* in each problem. Annotate these terms in each problem.

English Learners

Circle the phrases that include the word "what" ("equals what number," "of what number," and "what percent"). Show how these phrases help identify the part, whole, and percentage.

Power-up

To power up students' ability to solve percentage problems, have students complete:

Recall that to find 10% of a number, you multiply the number by $\frac{10}{100}$; to find 25% of a number, you multiply the number by $\frac{25}{100}$; to find n% of a number, you multiply the number by $\frac{n}{100}$.

Complete these sentences:

1. 10% of 20 is 2
2. 25% of 20 is 5
3. 35% of 20 is 7

Use: Before the Warm-up.
Informed by: Performance on Lesson 8, Practice Problem 6.

Activity 1 Representing and Solving Percentage Problems With Equations

Pairs | 15 min

Students write and solve equations of the form $nw = p$ to represent scenarios where the whole, percentage, or part is unknown.

Name: ______ Date: ______ Period: ______

Activity 1 Representing and Solving Percentage Problems With Equations

You arrive at the market in Niani with 16 kg of cloth to trade.

1. Your first stop is the spice seller. You trade all of your cloth for rare and expensive spices. The spices weigh 25% of the weight of your cloth. How many kilograms of spices did you buy?

   a. Adaeze represents the scenario with the equation $\frac{25}{100} \cdot 16 = b$. Explain how her equation represents the scenario.

   This means 25% of 16 equals the weight of the spices. $\frac{25}{100}$ is the percentage, 16 is the whole, and b, or the weight of the spices, is the part.

   b. Solve the equation to determine the number of kilograms of spices you bought.

   4 kg

2. Your next stop is the jeweler. You trade all your rare spices for diamonds. The diamonds weigh 150% of the weight of your spices. How many kilograms of diamonds did you buy?

   a. Write an equation to represent the scenario.

   Sample response: $\frac{150}{100} \cdot 4 = c$

   b. Solve the equation to determine the number of kilograms of diamonds you bought.

   6 kg

3. Your final stop is the salt seller, where you buy 12 kg of salt. What percent is this of the original weight of cloth that you had when you arrived at the market?

   a. Write an equation to represent the scenario.

   Sample response: $\frac{d}{100} \cdot 16 = 12$

   b. Solve the equation to determine the percent.

   75%

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What is the percentage, part, and whole in Adaeze's equation?"

Look for points of confusion:

- **Misidentifying the part and whole.** Say, "You determine a percentage of a whole, and the part is the result."
- **Incorrectly solving their equations for Problems 2 and 3.** Ask, "How would you solve if the equation was $5c = 15$? Use those same steps to solve."

Look for productive strategies:

- Using Adaeze's equation, but replacing the known and unknown values to match each scenario.
- Solving correctly by using division to isolate the variable.

3 Connect

Display each problem, one at a time.

Have students share their equations, how they decided where to use a variable, and what the variable represented in the story. Then have students share their solutions and how their methods changed as the location of the variable changed.

Highlight that an efficient way to solve problems about percentages is to use the equation $nw = p$, where n is the percentage of the whole, w is the whole, and p is the part of the whole. The solution strategy changes depending on what values are missing.

Ask, "When do you use multiplication to solve? Division?"

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide copies of the *Tape Diagrams* PDF and the *Double Number Lines: Percentage Problems* PDF to help students visualize the scenarios.

Extension: *Clarify Vocabulary and Symbols*

Consistent with the prior unit on percentages, the equation $nw = p$ is used in this lesson. Students have also been working with the equation $px = q$. Help them see that these equations represent the same concept and it does not matter which variables are used.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their equation and where they decided to place their variables, have them compare how the placement of the variables varied in the structure of the equations, based on the unknown described in each context. Display the general form $nw = p$ and ask:

- "In your equation in Problem 1, what does n represent: the part, the percentage, or the whole?" The percentage
- "Did n represent the percentage in your equations for Problems 2 and 3?" Yes, and, in Problem 3, the percentage was the unknown.

English Learners

Annotate the general form of the equation $nw = p$, where n represents the percentage, q represents the part, and w represents the whole.

Pairs | 15 min

Activity 2 Puppies Grow Up, Revisited

Students represent percentage scenarios where the whole is unknown using equations of the form $nw = p$, and justify the meaning and placement of the variable.

Amps Featured Activity Dynamic Growing Dogs

Activity 2 Puppies Grow Up, Revisited

Aadan and Chinyelu visit the market in Gao to buy an Azawakh, a breed of dog that is popular for guarding ancient Mali villages.

1. When Aadan's dog was a puppy, it weighed 8 kg, which is 30% of its current adult weight. What is the current weight of Aadan's dog?
   a. Write an equation to represent the situation.
      Sample response: $8 = \frac{30}{100}w$
   b. Solve the equation to determine the adult weight of Aadan's dog.
      $26\frac{2}{3}$ kg
2. When Chinyelu's dog was a puppy, it weighed 8 kg, which is 86% of its current adult weight. What is the current weight of Chinyelu's dog?
   a. Write an equation to represent the situation.
      Sample response: $8 = \frac{86}{100}w$
   b. Solve the equation to determine the adult weight of Chinyelu's dog.
      $9\frac{26}{86}$ kg or $9\frac{13}{43}$ kg
3. How would your equations and solutions change if:
   a. Aadan's dog weighed 5.8 kg as a puppy, which is 30% of its current adult weight.
      $5.8 = \frac{30}{100}w$ or $19\frac{1}{3}$ kg
   b. 8 kg is 43% of the adult weight of Chinyelu's dog.
      $8 = \frac{43}{100}w$
      $18\frac{26}{43}$ kg

Are you ready for more?

Kofi wants to paint his room purple. He buys one gallon of a purple paint mixture that is 70% blue paint. Kofi wants to add more blue so that the mixture is 80% blue.

1. How much blue paint should Kofi add? Test these possibilities: 0.2 gallons, 0.3 gallons, 0.4 gallons, and 0.5 gallons.
   Kofi should add 0.5 gallons of blue paint.
2. Write an equation in which x represents the amount of paint Kofi should add.
   $\frac{80}{100} \cdot (1 + x) = 0.7 + x$
3. Check that the amount of paint Kofi should add is a solution to your equation.
   $\frac{80}{100} \cdot (1 + 0.5) = 0.7 + 0.5$; $\frac{80}{100} \cdot (1.5) = 1.2$; $1.2 = 1.2$

STOP

1 Launch

Explain that responses should be written as mixed numbers. Set an expectation for the amount of time students will have to work individually before sharing their responses with a partner.

2 Monitor

Help students get started by having them identify the percentage, part, and whole in the scenario, and then writing an equation.

Look for points of confusion:

- **Making 8 the whole.** Ask, "Do you expect the dog to weigh more as a puppy or an adult? So, which weight represents the whole?"
- **Incorrectly solving for the unknown.** "How would you solve if the equation was $5c = 15$? Use those same steps to solve." If necessary, remind students how to divide with fractions.

Look for productive strategies:

- Recognizing that the whole is missing in each scenario, and writing equations of the form $nw = p$, where w is unknown.
- Solving using division to isolate the variable.

3 Connect

Display each problem, one at a time, to the entire class.

Have students share their equations and solutions, focusing on how they used division in each scenario because the total weight was missing.

Ask, "How would the meaning and value of your variable have changed if you had written the equation $\frac{25}{100} \cdot 8 = w$?

Highlight that the equation $nw = p$ is particularly helpful when you solve equations that have difficult values. To solve for a missing whole, divide each side by the percentage.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter a solution for their equations and watch an animation of the dog's growth. By doing so, they will receive real-time feedback that shows whether the solution is correct.

Accessibility: *Optimize Access to Tools*

Provide students with copies of the *Tape Diagrams* PDF and the *Double Number Lines: Percentage Problems* PDF that they can use to help visualize the scenarios prior to writing their equations.

Math Language Development

MLR8: Discussion Supports — Press for Details

During the Connect, display these sentence frames to help students explain their reasoning.

- "I knew I needed to use a variable to represent _____ because . . ." (Listen for students who recognize the variable represents the unknown quantity, the quantity that is asked for.)
- "The variable I chose to represent _____ is _____, because . . ."

Point out that the variable in each scenario represents the missing whole.

English Learners

Circle the phrases that indicate the missing whole, such as "30% of its current adult weight" and "what is the current adult weight" in Problem 1.

Whole Class | 5 min

Summary

Review the structure of a percentage equation, and generalize how the unknown in a percentage equation of the form $nw = p$ affects the solution method.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw how equations can help you represent and solve problems in which one amount is a percentage of another amount. The phrase "n percent of w equals p," where n is the percent, w is the whole, and p is the part, can be represented by the equation $\frac{n}{100} \cdot w = p$.

You can substitute the known values into the equation $\frac{n}{100} \cdot w = p$, and use a variable to represent the unknown value. Your solution method will depend on which value is unknown — the percentage, the whole, or the part.

Reflect:

Synthesize

Display the equations $nw = p$ and $\frac{n}{100} \cdot w = p$

Ask:

- "How are these equations related?"
- "How can you use these equations to solve percentage problems?"
- "Can you use the same solution method no matter which variable is missing?"

Highlight that students can always represent problems involving percentages by using the equation $nw = p$, where n is the percentage of the whole, w is the whole, and p is the part of the whole. Then, depending on which quantity is unknown, they can always multiply or divide to efficiently solve the equation to determine the value of the missing quantity.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did your work today build upon your previous work with percentages in Unit 3?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing an equation of the form $nw = p$ to represent and solve a percentage problem.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.09

Efe raised $54 to support the animal shelter, which is 60% of his fundraising goal. What is Efe's fundraising goal?

1. Write an equation to represent the scenario.

   Sample response: $54 = \frac{60}{100}x$

2. Determine Efe's fundraising goal.

   $x = 90$
   Efe's fundraising goal is $90.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can solve percent problems by writing and solving an equation.

1 2 3

Success looks like . . .

- **Language Goal:** Stating explicitly what the chosen var iable represents when creating an equation. **(Speaking and Listening, Writing)**
- **Language Goal:** Using equations to solve problems involving percentages and explain the solution method. **(Speaking and Listening)**
  - » Determining Efe's fundraising goal by solving the equation in Problem 2.
- **Goal:** Writing equations of the form $nw = p$, or equivalent, to represent situations where the amount that corresponds to 100% is unknown.

Suggested next steps

If students write an inaccurate equation for Problem 1, consider:

- Reviewing the connections between the scenarios and equations in Activity 1.
- Asking, "What information is known and unknown in the scenario, and how can you write an equation to represent the relationship between those values?"

If students are unable to solve the equation for Problem 2, consider:

- Reviewing solution strategies from Activity 2.
- Asking, "How can you use the properties of equality to isolate the variable on one side of the equation?"

If students make a computation error when solving their equation in Problem 2, consider:

- Reviewing solution strategies from Activity 2, and having them compare their steps in Activity 2 to those taken in the Exit Ticket.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students wrote and solved equations to solve percentage problems. How did that build on the earlier work students did with percentages in Unit 3?
- During the discussion about Activity 2, how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ________ Date: ________ Period: ________

1. A crew has paved $\frac{3}{4}$ of a mile of road. If they have completed 50% of the work, how long is the road they are paving?

   $\frac{6}{4}$ miles or $1\frac{1}{2}$ miles or 1.5 miles

2. Consider the statement: 40% of x is 35.

   a. Write an equation that shows the relationship of 40%, x, and 35.

   Sample response: $\frac{40}{100}x = 35$

   b. Use your equation to determine x. Show your thinking.

   $x = 87.5$ or $x = 87\frac{1}{2}$

   $$\frac{40}{100} \cdot x = 35$$
   $$0.40x = 35$$
   $$x = \frac{35}{0.40}$$
   $$x = 87.5$$

3. Ime has completed 9 exam questions. This is 60% of the questions on the exam.

   a. Write an equation representing this situation. Explain the meaning of any variables you use.

   Sample response: $9 = \frac{60}{100}x$

   x represents the total number of questions on the exam.

   b. How many questions are on the exam? Show your thinking.

   There are 15 questions on the exam.

   $$9 = \frac{60}{100} \cdot x$$
   $$9 = 0.60x$$
   $$\frac{9}{0.60} = x$$
   $$15 = x$$

Name: ________ Date: ________ Period: ________

4. Consider the equation $2n - 3 = 7$.

   a. What is the variable?

   n

   b. What is the coefficient of the variable?

   2

   c. Which of these is the solution to the equation? $2, 3, 5, 7, n$

   5

5. Which of these is a solution to the equation $\frac{1}{8} = \frac{2}{5} \cdot x$?

   A. $\frac{2}{40}$ B. $\frac{5}{16}$ C. $\frac{11}{40}$ D. $\frac{17}{40}$

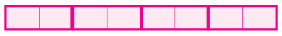
6. Draw tape diagrams to represent the following expressions:

   a. $4 + 2$

   b. $2 + 2 + 2 + 2$

   c. $2 \cdot 4$

   d. $4 \cdot 2$

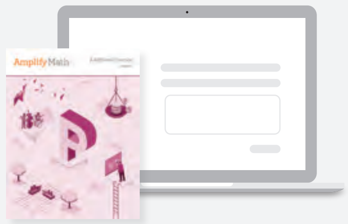


Practice Problem Analysis

Type	Problems	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	1
	3	Activity 2	1
Spiral	4	Unit 6 Lesson 4	1
	5	Unit 6 Lesson 4	1
Formative	6	Unit 6 Lesson 10	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

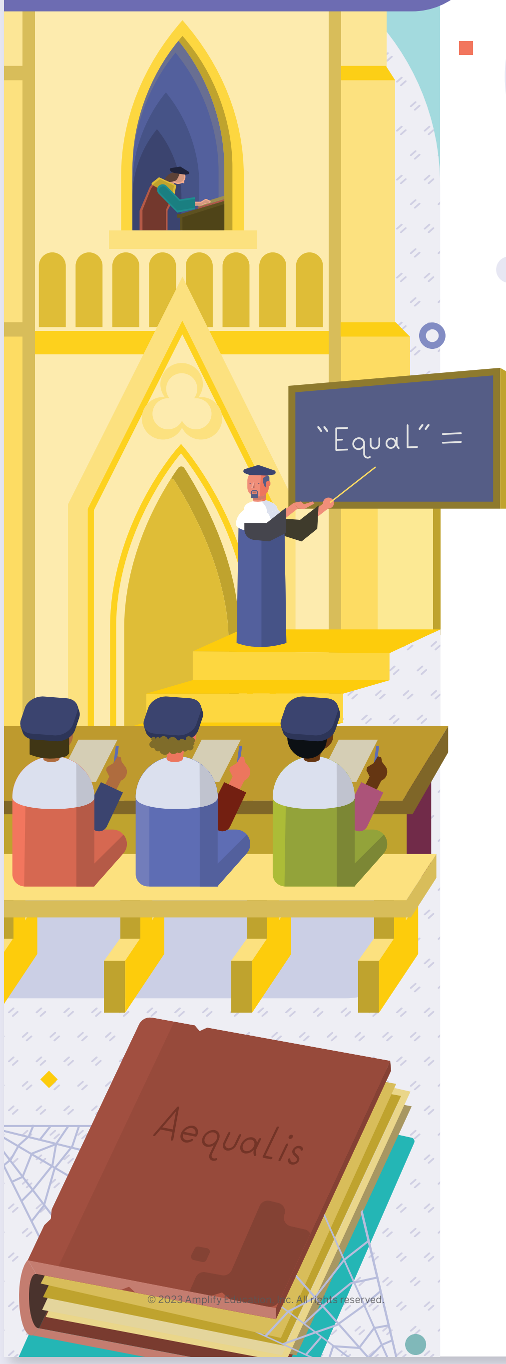
Sub-Unit 2
Equivalent Expressions

Whole Class

In this Sub-Unit, students distinguish between algebraic expressions that are equal for one value and those that are equal for any value, which makes them equivalent, such as the expressions that represent the Distributive Property. Students also work with expressions involving exponents and variables.

SUB-UNIT

2 Equivalent Expressions



Narrative Connections

How did a Welshman equalize England's upper crust with its common folk?

Quite literally, with an equal sign.

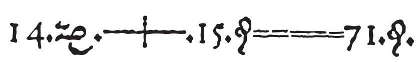
For this, let's go way back to the middle ages. Back then, in England, all math was written out in full sentences, and in Latin. To express that things were equal, English mathematicians used the word *aequalis*, which means "is equal to." But learning Latin was a luxury, and was only for the elite. The lower classes, who spoke only English, were effectively cut off, not just from math, but also from science, rhetoric, music, and philosophy.

Such was life until the mid-1550s, when a Welsh physician and teacher, Robert Recorde, set out to make learning math possible for everybody. Unlike mathematicians before him, Recorde wrote and taught in English, which gave commoners access to the mathematical understanding they were previously kept from.

In his 1557 book, *The Whetstone of Witte*, he simplified the process by inventing what we now call the equal sign:

"And to avoid the tedious repetition of these words, *is equal to*, I will set as I do often in work use, a pair of parallels . . . of one length, thus: =, because no two things can be more equal."

The two lines in his original equal sign (along with his plus sign) were much longer, as you can see in the following equation he wrote:



And the rest, as they say, is history.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to work with equivalent expressions and use language to describe this equivalence in the following places:

- **Lesson 10, Warm-up:** Representing and Visualizing Mathematical Equality
- **Lesson 13, Activity 2:** Writing Equivalent Expressions Using the Distributive Property
- **Lesson 14, Activity 1:** Fundraising Outreach for the Animal Shelter

Equal and Equivalent (Part 1)

Let's use diagrams to determine which expressions are equivalent and which are just sometimes equal.

Focus

Goals

1. Draw a diagram to represent the value of an expression for a given value of its variable.
2. **Language Goal:** Explain that some pairs of expressions are equal for one value of their variable, but not for other values. **(Writing)**

Coherence

Today

Students are introduced to the concept of equivalent expressions through tape diagrams. They create multiple diagrams, adjusting the represented length for several different values of the variable to decide when the expressions are equal. They also notice when this is always true, and thus when the expressions are equivalent. They then recognize that equivalent expressions can also be identified using familiar facts and properties of operations.

Previously

In Lessons 3 through 9, students developed an understanding of algebraic expressions and equations, and how different values substituted for variables can make equations true (equal) or false (unequal).

Coming Soon

In Lesson 11, students will deepen their understanding of equivalent expressions by relating them to the associative and commutative properties. Then, in Lessons 12–13, students will identify and generate equivalent expressions using the Distributive Property.

Rigor

- Students build **conceptual understanding** of equal and equivalent using tape diagrams.
- Students develop **procedural skills** for solving equations, recognizing fact families when determining solutions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF

Math Language Development

New word

- ***equivalent expressions***

Review words

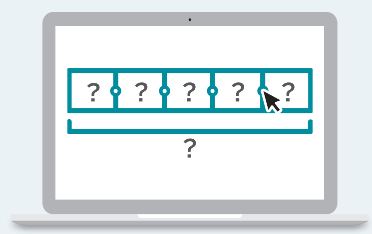
- *solution an equation*
- *variable*

Amps | **Featured Activity**

Activity 1

Sketchable Tape Diagrams

Students can sketch tape diagrams, and you can overlay them all to see similarities and differences at a glance.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might not understand the difference between *equal* and *equivalent* is. Encourage them to use the visuals as well as any class discussion to analyze the situation and reflect on how they were different. By deciding to approach the problem through this lens, students will see that the differences are what help distinguish the meaning of the two terms.

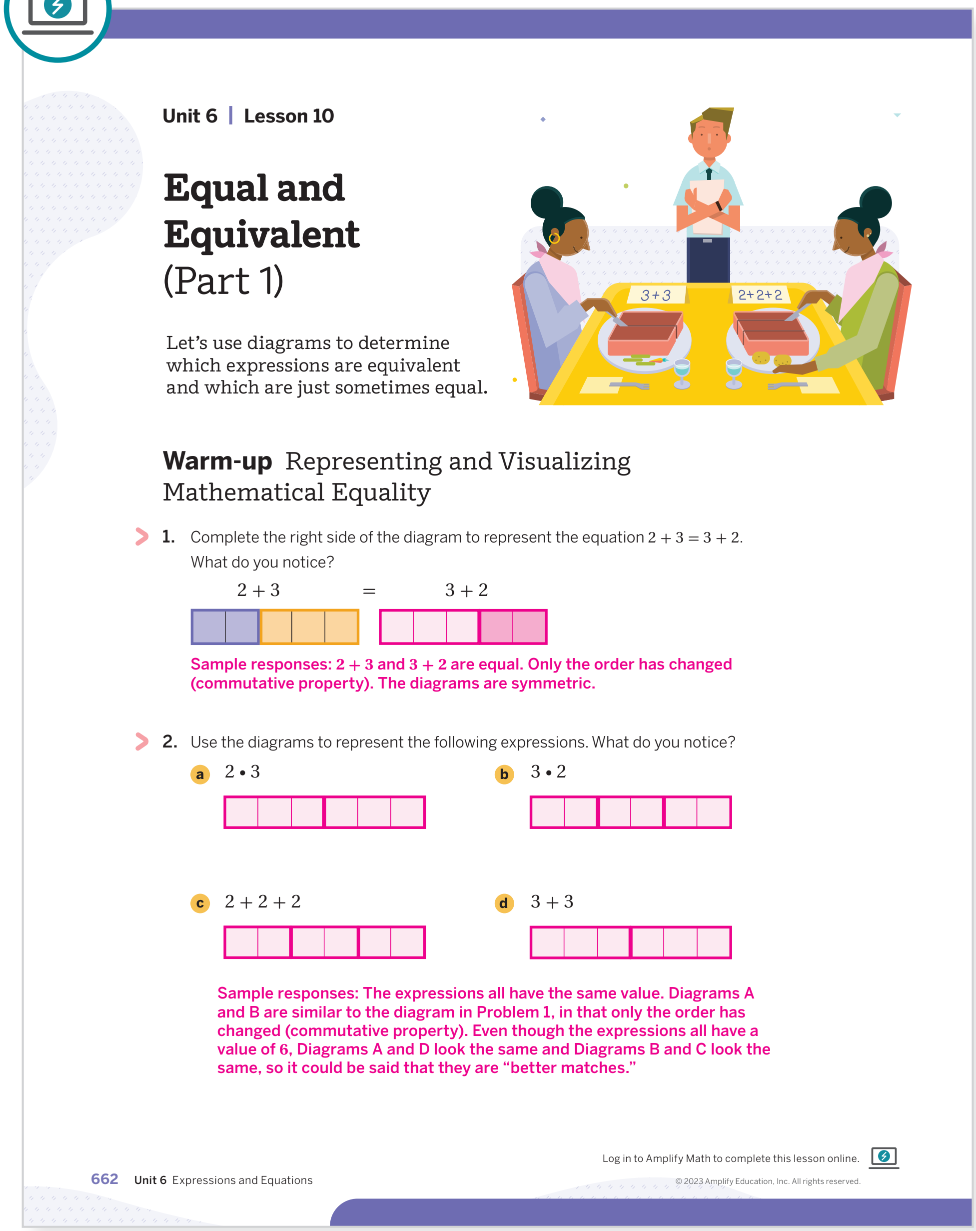
Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- For **Activity 2**, small groups can solve for one of the five problems. Then answers can be shared with the whole class.

Warm-up Representing and Visualizing Mathematical Equality

Students use diagrams to show that expressions can be equal, and then make generalizations about equal expressions.


Unit 6 | Lesson 10

Equal and Equivalent (Part 1)

Let's use diagrams to determine which expressions are equivalent and which are just sometimes equal.

Warm-up Representing and Visualizing Mathematical Equality

1. Complete the right side of the diagram to represent the equation $2 + 3 = 3 + 2$. What do you notice?

$2 + 3 \quad = \quad 3 + 2$

Sample responses: $2 + 3$ and $3 + 2$ are equal. Only the order has changed (commutative property). The diagrams are symmetric.

2. Use the diagrams to represent the following expressions. What do you notice?

a. $2 \cdot 3$

b. $3 \cdot 2$

c. $2 + 2 + 2$

d. $3 + 3$

Sample responses: The expressions all have the same value. Diagrams A and B are similar to the diagram in Problem 1, in that only the order has changed (commutative property). Even though the expressions all have a value of 6, Diagrams A and D look the same and Diagrams B and C look the same, so it could be said that they are "better matches."

Log in to Amplify Math to complete this lesson online.

662 Unit 6 Expressions and Equations

1 Launch

Activate background knowledge by saying, "Make sure the left-hand sides of all your diagrams line up with the given diagram for $2 + 3$."

2 Monitor

Help students get started by asking, "How can you use the diagram for $2 + 3$ to help you draw a diagram for $3 + 2$?"

Look for points of confusion:

- **Having difficulty drawing diagrams for multiplication expressions.** Ask, "How could you read that expression as a number of groups? How can you draw that?"

Look for productive strategies:

- Color-coding the grid boxes. Draw the total boxes and then color in each group or color in each addend.

3 Connect

Display the full set of correct diagrams to the class for students to check their work and to aid discussion.

Have students share what they notice about the diagrams.

Highlight that students can determine that $2 + 3$ and $3 + 2$ are equal because the lengths of both diagrams are the same. They can also determine that the next three expressions are equal as well because the lengths of all their diagrams are the same, yet different from the first two expressions.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share what they notice about the diagrams, listen for students who recognize the diagrams are the same length which means the expressions have equal values. If students do not mention this, ask these questions:

- "Are the individual sections of your diagrams the same size/length? Why would it be important to draw them to be the same size?"
- "What do you notice about the total length of your diagrams?"
- "How many individual sections do you have in each diagram? What does this tell you about the expressions?"

Power-up

To power up students' ability to create diagrams to represent expressions, ask:

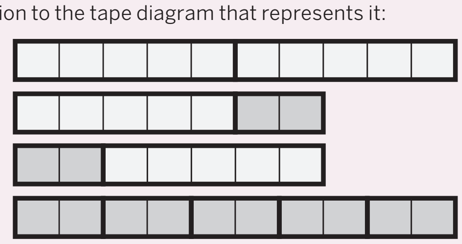
Match each expression to the tape diagram that represents it:

a. $5 + 2$ ...d...

b. $2 + 5$ ...a...

c. $5 \cdot 2$ ...b...

d. $2 \cdot 5$ ...c...



Use: Before the Warm-up

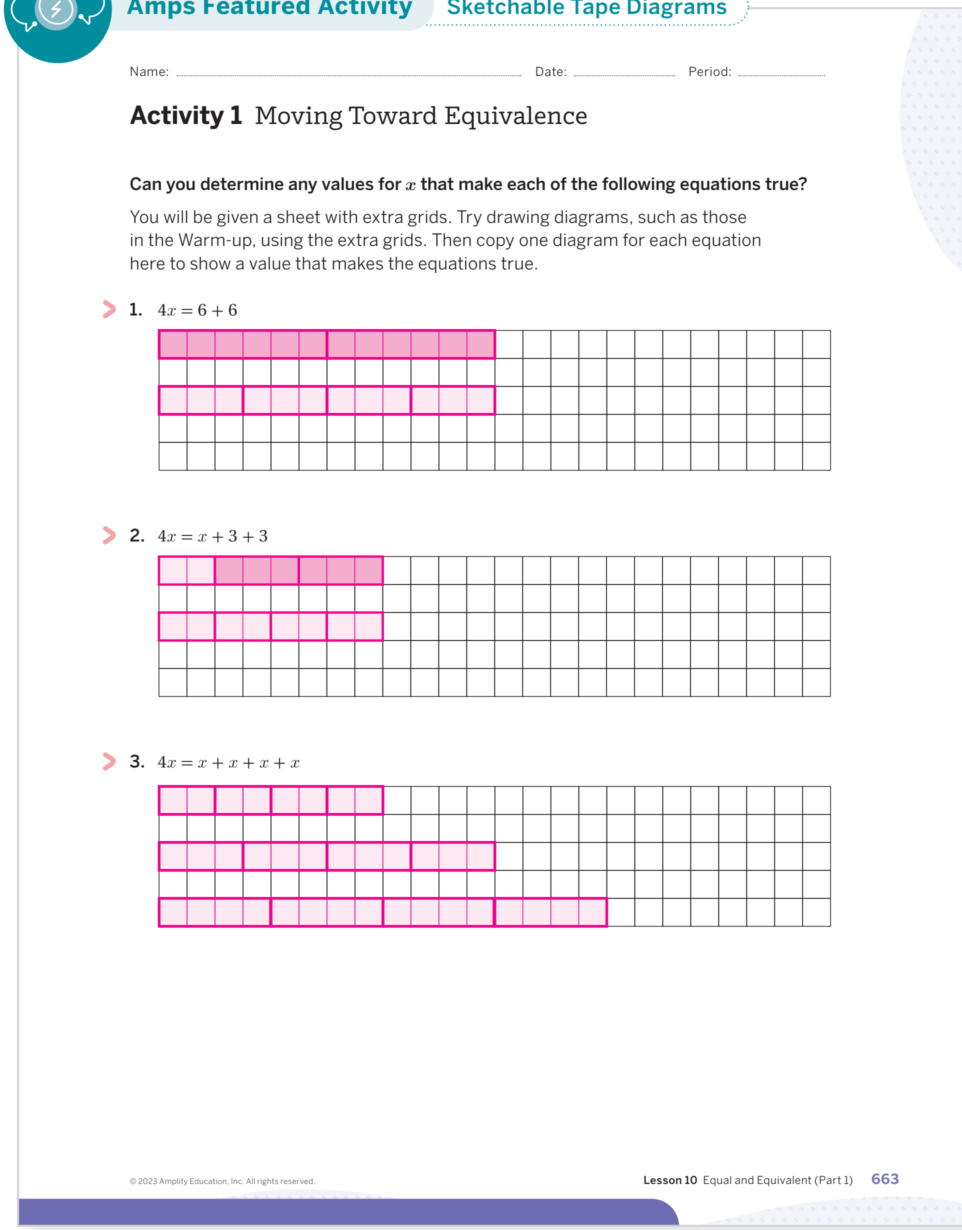
Informed by: Performance on Lesson 9, Practice Problem 6.

Independent | 15 min

Activity 1 Moving Toward Equivalence

Students diagram two equations that are each equal for one value of x, and one that is equal for all values of x. The difference between *equal* and *equivalent* is introduced.

Amps Featured Activity Sketchable Tape Diagrams


Name: ______ Date: ______ Period: ______

Activity 1 Moving Toward Equivalence

Can you determine any values for x that make each of the following equations true?

You will be given a sheet with extra grids. Try drawing diagrams, such as those in the Warm-up, using the extra grids. Then copy one diagram for each equation here to show a value that makes the equations true.

1. $4x = 6 + 6$

2. $4x = x + 3 + 3$

3. $4x = x + x + x + x$

1 Launch

Have students work independently on each problem and compare responses with their partner.

2 Monitor

Help students get started by asking, "What value does the right side represent? How would you draw that? What should the diagram for the left side look like?"

Look for points of confusion:

- **Representing x as an arbitrary length.** Say, "As a variable, the length of x is unknown, but here you want to determine values that make the equations true, so choose a value and substitute it first before drawing your diagrams."

Look for productive strategies:

- Trying additional values to test for equivalence if the first value chosen makes the expressions equal, or continuing to try new values until they find one that makes them equal.

3 Connect

Have students share their solutions and how they relate to the diagrams. Move through Problems 1 and 2 quickly, and leave more time for discussing and sharing solutions to Problem 3.

Highlight that everyone determined the same solution, the only possible one, for each of the first two equations. So, they are equal when $x =$ some specific number. Several different solutions could be found for the third equation. When this happens, then the expressions on each side of the equal sign are equivalent, and therefore equal all the time.

Define ***equivalent expressions*** as two expressions whose values are equal regardless of the value of the variable.

Ask, "Do you think there's a way to tell whether the two expressions in an equation are just *equal sometimes* or *equivalent all the time*, without drawing diagrams?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital tape diagrams. You can overlay them all to see similarities and differences at a glance.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 3.

Extension: *Math Enrichment*

After the Connect, challenge students to write an equation in which the two expressions on either side of the equal sign are equivalent expressions. Answers will vary.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their solutions and how they relate to the diagrams, draw connections between the number of solutions for each problem. Ask:

- "How many diagrams did you draw for Problem 1? Problem 3?"
- "Why is it that you could draw several different diagrams for Problem 3?"

Amplify language students use to make sense of what it means for two expressions to be equivalent, such as "It does not matter what the value of x is because the left and right sides of the equation (the two expressions) will always be equal to each other."

Pairs | 10 min

Activity 2 Detecting Equal and Equivalent Expressions

Students substitute given values for the variables in pairs of expressions, to determine whether the expressions are equal, equivalent, or neither.

Activity 2 Detecting Equal and Equivalent Expressions

For each pair of expressions, determine which of the values 0, 1, and 3 make them equal. What does it mean if *all* values make them equal? Hint: You may want to draw diagrams to help with your thinking.

1. $a + 3$ and $9 - a$
   They are equal when $a = 3$.
2. $x + 2$ and $x + 3$
   They are not equal for any values.
3. $b \div 3$ and $b \cdot \frac{1}{3}$
   They are equal for *all* values, meaning they are equivalent.
4. $a + a + a + a + a$ and $5a$
   They are equal for *all* values, meaning they are equivalent.
5. $2x$ and $3x$
   They are equal when $x = 0$.

Are you ready for more?

Are there any values that would make the following two expressions equal? Are they equivalent?

$3x + 1$ and $0x + 1$

They are equal when $x = 0$. They cannot be equivalent because there is only one value that makes them equal.

STOP

1 Launch

Keep students in their same pairs. Say, "After you substitute all the given values into each pair of expressions, discuss your results and label each pair as *equal*, *equivalent*, or *not equal* (for any values)."

2 Monitor

Help students get started by saying, "Let's start with the value of 1. Where would you place this value in the first pair of expressions?"

Look for points of confusion:

- **Not checking other values after finding one that makes the expressions equal.** Remind students to substitute *all* the given values for every pair of expressions.
- **Thinking $x + 2$ and $x + 3$ may be equal for some other value.** This is a good instinct, but focus their attention on the structure of the expressions.

Look for productive strategies:

- Recognizing equivalence based on operations. Make sure these students still substitute the values to verify their claims.
- Recognizing or remembering that the product of any number and 0 is equal to 0 (Problem 5).

3 Connect

Have students share for one pair of expressions at a time, how they determined whether the expressions were equal, equivalent, or neither.

Highlight that, if all values make the expressions equal, then they are ***equivalent expressions***. The expressions are *equal* when there is only one value that makes them equal. If no values make them equal, then they are *neither equal nor equivalent*.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1—3.

Accessibility: *Activate Prior Knowledge*

If you have students complete Problem 5 and students are unsure of an approach they could use, ask, "What do you know about the product of any number and 0? How could you use this knowledge to make sense of when these two expressions might be equal?"

Math Language Development

MLR2: Collect and Display

While students work, and during the Connect, listen to their conversations about the concepts of *equal* and *equivalent*. Add any appropriate mathematical language they use to the class display. During the Connect, as you highlight these terms, consider displaying or providing students with a graphic organizer to help them distinguish between *equal* and *equivalent*.

Two expressions are . . .

Equal	Equivalent	Neither equal nor equivalent
When *only one value* of the variable makes the expressions equal.	When *any and all* values of the variable make the expressions equal.	When *no value* of the variable makes the expressions equal.

Whole Class | 5 min

Summary

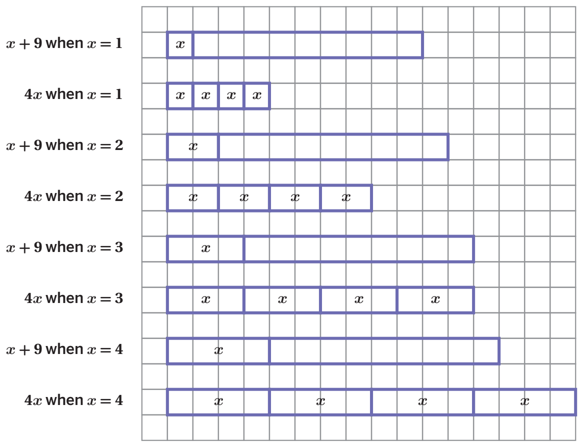
Review and synthesize how expressions can be equal or equivalent (or not equal), comparing and contrasting equivalence with equality.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used diagrams showing lengths of rectangles to see whether two expressions are equal for a particular value of the variable. For example, the expressions $x + 9$ and $4x$ are *equal* when x is 3, but they are not equal for any other values of x.



Other times, two expressions seem to be equal no matter what the value of the variable is, such as $3x + 4x$ and $5x + 2x$. When x is 3, both expressions equal 21. When x is 10, both expressions equal 70. In fact, these expressions will be equal for any value of x, because they are ***equivalent expressions***.

Reflect:

Synthesize

Display the following values for a variable: $x = 3$ and $x = 10$. Then, display the following two pairs of expressions: $x + 9$ and $4x$ together on one side, and $3x + 4x$ and $5x + 2x$ together on the other side.

Formalize vocabulary: ***equivalent expressions***

Have students share with a partner how they can determine whether two expressions are equal or equivalent. Use the expressions displayed and the given values of x to help guide the discussion around using properties of operations to recognize equivalent expressions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean for two expressions to be equal? Equivalent? Or is there a difference?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the term *equivalent expressions* that were added to the display during the lesson.

Consider displaying or providing students with a graphic organizer, such as the one shown in the Math Language Development section for Activity 2 to help them distinguish between *equal* and *equivalent*.

Two expressions are . . .

Equal	Equivalent	Neither equal nor equivalent
When *only one value* of the variable makes the expressions equal.	When *any and all* values of the variable make the expressions equal.	When *no value* of the variable makes the expressions equal.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using substitution to determine whether expressions are equal or equivalent.

Printable

Name: ______________________ Date: __________ Period: __________

Exit Ticket 6.10

1. Substitute the values 1, 3, and 4 for a into each of the following expressions. Then identify one pair of expressions that is equal and one pair of expressions that is equivalent. For the expressions that are equal, include the value that makes them equal.

$a + 12$	$7 \cdot a$	$a \cdot 7$	$5a$

Equal	Equivalent
$5a = a + 12$ when $a = 3$	$a \cdot 7 = 7 \cdot a$

2. Explain your thinking.

Sample response: The expressions $5a$ and $a + 12$ are only equal when $a = 3$, so they are not equivalent. The expressions $a \cdot 7$ and $7 \cdot a$ are equal for all values of a, so they are equivalent, which I also know because of the Commutative Property of Multiplication.

Self-Assess

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it

a I can use a tape diagram to determine when two expressions are equal. 1 2 3

b I can explain what it means for two expressions to be equivalent. 1 2 3

Success looks like . . .

- **Goal:** Drawing a diagram to represent the value of an expression for a given value of its variable.
- **Language Goal:** Explaining that some pairs of expressions are equal for one value of their variable, but not for other values. **(Writing)**
  - » Explaining which pair of expressions is equal for one value of a and which pair of expressions is equal for all values of a in Problem 2.

Suggested next steps

If students have difficulty identifying equivalent expressions, consider:

- Referring back to Activity 2 and guiding students to connect expressions to the similar structures $a \cdot b = b \cdot a$ and $a \cdot 3 = a \div \frac{1}{3}$.

If students have difficulty pairing expressions, consider:

- Creating a table for each expression, and include each value of a and the value of the expression when a is replaced by each value. Look for any expressions whose tables look the same, which would indicate they are equivalent.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students used tape diagrams to understand *equal* vs. *equivalent*. How did that build on the earlier work students did with hanger diagrams?
- What did students find challenging about distinguishing equal and equivalent expressions? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Explaining that some pairs of expressions are equal for one value of their variable, but not for other values.

Reflect on students' language development toward this goal.

- How have students progressed in their understanding of the terms *equal* and *equivalent*, as they relate to algebraic expressions?
- What is an example of a developing explanation for why the expressions $a \cdot 7$ and $7 \cdot a$ are equivalent?

Sample descriptions:

Emerging	Expanding
$a \cdot 7$ and $7 \cdot a$ are equivalent because they look the same, just switched.	$a \cdot 7$ and $7 \cdot a$ are equivalent because they are equal for all values of a.

Practice

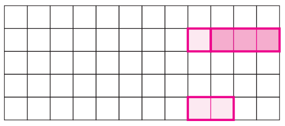
Independent

Name: ______ Date: ______ Period: ______

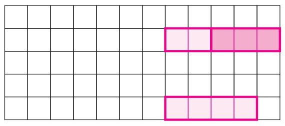
Practice

1. Use the grids below to draw a diagram of $x + 3$ and a diagram of $2x$ for each given value of x. Remember to line up the diagrams on one side.

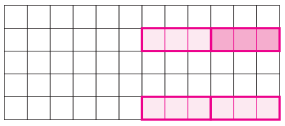
a x is 1



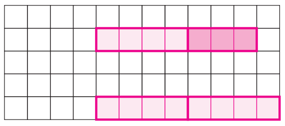
b x is 2



c x is 3



d x is 4



e When are $x + 3$ and $2x$ equal? *Not* equal? Use your diagrams to explain.

They are equal when $x = 3$. I can tell because the diagrams are the same length in part c, but not in the others.

Name: ______ Date: ______ Period: ______

Practice

2. Select *all* the expressions that are *equivalent* to $3b$.

- (A.) $b + b + b$
- (B.) $b \div \frac{1}{3}$
- C. $b \cdot b \cdot b$
- D. $b + 3$
- (E.) $2b + b$

3. Are the two expressions in the equation $2b + 5 + 1b = 3b + 5$ *equal* or *equivalent*? How do you know?

Equivalent; Sample responses: Because when I substitute values for the variable, they are always *equal*. I get the same expression on both sides when I combine the bs.

4. For each story problem, write an equation to represent the problem and then solve the equation. Be sure to explain the meaning of any variable you use.

Sample equations shown.

a Jada's dog was $5\frac{1}{2}$ in. tall when it was a puppy. Now her dog is $14\frac{1}{2}$ in. taller. How tall is Jada's dog now?

$5\frac{1}{2} + 14\frac{1}{2} = j$ **$j = 20$ in., where j is the height of Jada's dog now**

b Lin picked $9\frac{3}{4}$ lb of apples, which was 3 times the weight of the apples Andre picked. How many pounds of apples did Andre pick?

$3a = 9\frac{3}{4}$ **$a = 3\frac{1}{4}$ lb, where a is the weight of the apples Andre picked**

5. Calculate each product.

a $2.3 \cdot 1.4 = 3.22$

b $1.72 \cdot 2.6 = 4.472$

c $18.2 \cdot 0.2 = 3.64$

d $15 \cdot 1.2 = 18$

6. Identify the property demonstrated by the equation $x + (3 + 5) = (x + 3) + 5$.

Associative Property of Addition

Practice Problem Analysis

Type	Problems	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 8	3
	5	Unit 5 Lesson 8	1
Formative	6	Unit 6 Lesson 11	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Equal and Equivalent (Part 2)

Let's use what we know about operations to decide whether two expressions are equivalent.

Focus

Goals

1. Use the associative and commutative properties of operation to describe equivalent expressions.
2. **Language Goal:** Justify whether two expressions are *equivalent*, i.e., equal to each other for every value of their variable. **(Speaking and Listening, Writing)**

Coherence

Today

Students further explore the concept of equivalent expressions, and leverage the associative and commutative properties to identify and generate equivalent expressions using the structure of the expressions themselves. They substitute a given set of values for variables into pairs of expressions to determine whether they are equal, equivalent, or neither. By the end of the lesson, students generate their own equivalent expressions.

Previously

In Lesson 10, students recognized the difference between the terms *equal* and *equivalent* with respect to algebraic expressions, using guess-and-check strategies.

Coming Soon

Lessons 12 and 13 focus on identifying and generating equivalent expressions using the Distributive Property. Then, in Lessons 14 and 15, students will explore more equivalent expressions involving exponents, as well as variables.

Rigor

- Students continue to develop **conceptual understanding** of equivalent expressions.
- Students strengthen **procedural fluency** substituting values for variables.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (Optional)	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

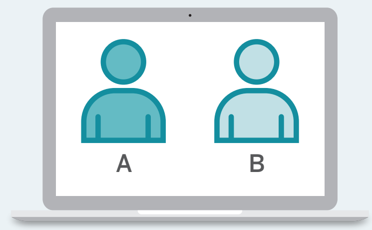
- *Associative Property of Multiplication*
- *Commutative Property of Multiplication*
- *equivalent expressions*

Amps | Featured Activity

Activity 2

Collaborative Equivalent Expressions

Students can take turns completing in a table to create equivalent expressions.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may doubt that they have the skills to determine equivalent expressions. Encourage students to overcome their pessimism with a growth mindset. The idea that it is just not something they know yet, along with a dose of perseverance, will create a more optimistic outlook.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Optional **Activity 2** may be omitted.

Independent | 5 min

Warm-up Associative or Commutative Properties With Variables

Students identify the property used in each equation to activate their prior knowledge of the commutative and associative properties.

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 11

Equal and Equivalent (Part 2)

Let's use what we know about operations to decide whether two expressions are equivalent.

Warm-up Associative or Commutative Properties With Variables

Identify the property or properties shown in each equation.

a. $12 + x = x + 12$
commutative

b. $(3 \cdot 5) \cdot x = 3 \cdot (5 \cdot x)$
associative

c. $3 + (x + 5) = x + 8$
commutative and associative

d. $12 \cdot x = x \cdot 12$
commutative

e. $2b + 5 + 3b = 5 + 5b$
commutative and associative

Log in to Amplify Math to complete this lesson online.

668 Unit 6 Expressions and Equations

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by reminding them that *associative* relates to grouping, while *commutative* relates to order.

Look for points of confusion:

- **Wanting to try to solve the equations.** Redirect them to the instructions.

Look for productive strategies:

- Correctly identifying the property. Students notice either the change in ordering or grouping.

3 Connect

Display each equation, possibly one at a time.

Ask, "How do you know whether the equation illustrates the commutative or associative property? Did any equations illustrate both properties?"

Have students share how they identified which property or properties were illustrated by the equations, and which did not.

Highlight the difference between the associative and commutative properties. Refer to the Math Language Development section for a suggested way to illustrate the difference between grouping and order.

Math Language Development

MLR2: Collect and Display

During the Connect, have students help you create a visual display that shows the equations from the Warm-up and how they illustrate the commutative and/or associative properties. The display should connect the idea of grouping to the associative property and the idea of order to the commutative property. Continue adding to the display in Activity 1 to help reinforce the concepts of order and grouping.

English Learners

Annotate the equations in the visual display to illustrate the difference between *grouping* and *order*. For example, in Problem a, the addends merely switched order. In Problem b, the factors grouped by the parentheses changed.

Power-up

To power up students' ability to identify the Commutative and Associative Properties of Addition, ask:

Match each property with an equation that demonstrates it. You may use each property more than once.

a. Commutative Property of Addition

b. Associative Property of Addition

__b__ $(5 + 2) + 1 = 5 + (2 + 1)$

__a__ $3 + 6 = 6 + 3$

__a__ $a + 7 = 7 + a$

__b__ $4 + (b + 9) = (4 + b) + 9$

Use: Before the Warm-up.

Informed by: Performance on Lesson 10, Practice Problem 6

Small Groups | 15 min

Activity 1 Experimenting With Expressions and Variables

Students work in groups to determine whether two expressions are equal, given specified values for the variable.

Name: ______ Date: ______ Period: ______

Activity 1 Experimenting With Expressions and Values

1. Han was given the two expressions $5 + x$ and $x + 5$. He tried substituting 1, 2, 3, and 1,273,673 for x into both expressions. He was shocked by what he saw. What did he see?

   He saw:

$5 + 1 = 6$	$1 + 5 = 6$
$5 + 2 = 7$	$2 + 5 = 7$
$5 + 3 = 8$	$3 + 5 = 8$
$5 + 1{,}273{,}673 = 1{,}273{,}678$	$1{,}273{,}673 + 5 = 1{,}273{,}678$

   He was shocked because it did not matter what value was substituted; the sums were equal regardless of the order.

2. Han was then given another pair of expressions, $2y$ and $y + 3$. Again, he substituted 1, 2, and 3, and 1,273,673 for y into both expressions. Did Han have the same reaction that he had in Problem 1?

$2(1) = 2$	$1 + 3 = 4$
$2(2) = 4$	$2 + 3 = 5$
$2(3) = 6$	$3 + 3 = 6$
$2(1{,}273{,}673) = 2{,}547{,}346$	$1{,}273{,}673 + 3 = 1{,}273{,}676$

   Han did not have the same reaction. Only one pair was equal when the values were substituted.

3. You are now given the expressions $2z + 4z$ and $5z + z$. Choose some values to substitute into the expressions and record your findings. Is your reaction like Han's in Problems 1 or 2?

   Sample responses:

$z = 2$	$2(2) + 4(2) = 12$	$5(2) + 2 = 12$
$z = 3$	$2(3) + 4(3) = 18$	$5(3) + 3 = 18$
$z = 5$	$2(5) + 4(5) = 30$	$5(5) + 5 = 30$

   My reaction is like Han's from Problem 1. It does not matter what value is substituted for z — the sums are always equal.

1 Launch

Arrange students in groups of four. Have each student evaluate the first pair of expressions with a different given value. Have them share their solutions in their groups and discuss Han's reaction. Then have them repeat these steps for the next two problems.

2 Monitor

Help students get started by asking, "What happens if you substitute 1 for x?"

Look for points of confusion:

- **Stopping before substituting each value because students already found one that "works."** Remind students that to get the full understanding of the activity, they need to see what happens when all values are substituted into the expressions.

Look for productive strategies:

- Solving by substituting the given set of numbers. Some students may be able to solve these mentally.

3 Connect

Display each pair of expressions, possibly one at a time.

Have students share what they believe Han noticed in each problem, and how they evaluated the expressions for each value of the variable, as well as how they think his reactions relate to those mathematical discoveries.

Ask, "What does it mean when all the results are equal, or only one value makes a pair equal?"

Highlight that the first and third examples are different from the second in that the expressions are equal for *all* values, which means they are equivalent.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them use the values of 1 and 2 for Problems 1 and 2. They can use these same values in Problem 3.

Math Language Development

MLR2: Collect and Display

If you started a visual display in the Warm-up, add these equations to the display to reinforce the concepts or order and grouping. Ask:

- "Does changing the order of the addends in Problem 1 change the value of the sums? What property illustrates this?" The sums are the same because of the commutative property.
- "Does it matter which values are substituted for x? What does this tell you about the expressions $5 + x$ and $x + 5$?" They are equivalent expressions.
- "Are these expressions in Problem 2 equivalent? Equal? Neither? Explain." These expressions are not equivalent, but they are equal when the value 3 is substituted for y.
- "Do you see the commutative or associative properties in Problem 2?"

Ask similar questions for Problem 3.

Optional

Pairs | 15 min

Activity 2 Making Equivalent Expressions

Students, working in pairs, take turns or work together, to make equivalent expressions.

Amps Featured Activity Collaborative Equivalent Expressions

Activity 2 Making Equivalent Expressions

Write an equivalent expression for each given expression.

If I have	. . . you have
$2x \cdot 5$	$5 \cdot 2x$ or $10x$
$(a \cdot 5) \cdot 2$	$a \cdot (5 \cdot 2)$ or $a \cdot 10$
$3b + 5b$	$2b + 6b$ or $b + 7b$ or $4b + 4b$ or $8b$
$2q + 7 + q$	$3q + 7$
$5y$	$y + y + y + y + y$
$9 + (c + 3)$	$c + 12$
$5m + 4n + 3m$	$8m + 4n$

Reflect: Were you able to be optimistic when using the guess-and-check strategy to see if expressions are equivalent? Why or why not?

Are you ready for more?

For each question, decide whether you think the expressions are equivalent. Test your guess by choosing values for x and/or y.

1. $\frac{x \cdot x \cdot x \cdot x}{x}$ and $x \cdot x \cdot x$
   Yes. (Technically no, because the first expression is undefined when $x = 0$.)
2. $\frac{x + x + x + x}{x}$ and $x + x + x$
   No
3. $2(x + y)$ and $2x + 2y$
   Yes
4. $2xy$ and $2x \cdot 2y$
   No

STOP

670 Unit 6 Expressions and Equations

1 Launch

Have students work in pairs. They can take turns or work together writing equivalent expressions. The focus should be on the discussion about how to make the equivalent expression(s).

2 Monitor

Help students get started by asking, "Think about the associative or commutative properties. Is there something you can do with the order or the grouping of the addends?"

Look for points of confusion:

- **Thinking of only one equivalent expression.** For Rows 2, 3, and 5 challenge students to determine more than one equivalent expression.
- **Not adding the** q**s in Row 4.** Students may just want to reorder or group this expression. If they group $2q$ and q together, ask them what that could also represent.

Look for productive strategies:

- Using associative and/or commutative properties. Students may recognize the connection to the Warm-up.
- Combining like terms. For example, in Row 4, realizing that $2q + 7 + q = 3q + 7$.

3 Connect

Display the "If I have . . . you have" table to the class.

Have students share the expressions they added to the "you have" side of the table until all possible expressions are recorded.

Highlight that Rows 3, 4, and 7 have variables that can be grouped.

Ask, "How is Row 6 similar to Rows 3, 4, and 7?" The variable terms are added, just like the numbers without variables are added.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on completing the first five rows of the table.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask question, have students compare their expressions for Rows 3, 4, 6, and 7 to make sense of the similarities between them. Ask:

- "How is the expression in Row 3 similar to Row 4? How is it different?"
- "How is the expression in Row 7 similar to Rows 3 and 4? How is it different?"
- "What do you notice about the expression in Row 6?"

English Learners

To support students' metalinguistic awareness, provide an exemplar sentence after students have had a chance to think and respond. For example, "If I have $9 + (c + 3)$, then you have $c + 12$ because you can group 9 *and* 3 together. This is similar to Row 7, where you can group $5m + 4n + 3m$ to get $8m + 4n$."

Whole Class | 5 min

Summary

Review and synthesize how properties of operations can help determine when two expressions are equivalent.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how to determine whether two expressions were equivalent. It would be impossible to test every possible value for a variable to know whether two expressions are always equal. So, how can you know for sure if two expressions are equivalent? One way is to use the meaning of operations and the properties of operations to show how the expressions are equivalent, because they can be rewritten or evaluated to look identical.

Property	Example
Associative Property of Addition	$(x + 4) + 6 = x + (4 + 6)$ $= x + 10$
Associative Property of Multiplication	$(x \cdot 4) \cdot 6 = x \cdot (4 \cdot 6) = 24x$
Commutative Property of Addition	$y + 4 = 4 + y$
Commutative Property of Multiplication	$y \cdot 4 = 4 \cdot y$

> Reflect:

Synthesize

Display the question, "How can you know for sure that two expressions are equivalent?"

Highlight the associative and commutative properties being exemplified, and that they are simple cases of recognizing equivalent expressions.

Ask, "Can you identify when two expressions are equivalent without evaluating them for specific values of the variables?" Yes, by looking at the structure of the expressions, I can tell:
$(a + b) + c = a + (b + c)$
$a(bc) = (ab)c$
$a + b = b + a$
$a \cdot b = b \cdot a$

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can two quantities be equal when one is partially or totally unknown?"
- "When did you feel like a balance being balanced with your partner today? How does that relate to the math?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying equivalent expressions, with suggested next steps.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.11

1. Are the expressions $4 \cdot (a \cdot 3)$ and $12a$ equivalent? Explain or show your thinking.
   Yes, they are equivalent.
   Sample responses:

$a = 2$	$4 \cdot (2 \cdot 3) = 24$	$12(2) = 24$
$a = 3$	$4 \cdot (3 \cdot 3) = 36$	$12(3) = 36$

   Sample responses: I know they are equivalent because I substituted different values for a and the expressions were always equal. I know they are equivalent because I determined $4 \cdot 3 = 12$ and then multiplied by a. This shows the associative property, because each expression involved grouping the factors differently.

2. Complete the equation so that it shows two equivalent expressions.

   $5c + 3b + 2c = b + \underline{2}\, b + 4c + \underline{3}\, c$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use what I know about the properties of operations to decide whether two expressions are equivalent. 1 2 3

b. I can use what I know about the properties of operations to generate two expressions that are equivalent. 1 2 3

Success looks like . . .

- **Goal:** Using the associative and commutative properties of operation to describe equivalent expressions.
  - » Using the associate property to show $4 \cdot (a \cdot 3)$ and $12a$ are equivalent in Problem 1.
- **Language Goal:** Justifying whether two expressions are *equivalent*, i.e., equal to each other for every value of their variable. **(Speaking and Listening, Writing)**
  - » Explaining why $4 \cdot (a \cdot 3)$ and $12a$ are equivalent in Problem 1.

Suggested next steps

If students do not identify the expressions as equivalent in Problem 1, consider:

- Having them substitute values into the expressions to see if more than one will make the expressions equal.

If students have trouble explaining their thinking in Problem 1, consider:

- Demonstrating, using two values and substitution, how the expressions are equal, and then having students explain how these examples explain equivalency.

If students are unable to complete the equation for Problem 2, consider:

- Grouping the like terms together. Ask, "How many bs are there? How many cs?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did experimenting with expressions and values reveal about your students as learners?
- During the discussion about Activity 2, how did you encourage each student to listen to one another's strategies? What might you change for the next time you teach this lesson?

MLR Math Language Development

Language Goal: Justifying whether two expressions are equivalent, i.e., equal to each other for every value of their variable.

- Reflect on students' language development toward this goal.
- How have students progressed in justifying whether two expressions are equivalent, using precise mathematical language, since Lesson 10? Do they use properties to defend their responses?
- How have the language routines used in this lesson helped students develop their mathematical language, particularly as it relates to using properties to demonstrate equivalence?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. a. Do $4x$ and $15 + x$ have the same value when x is 5?
   Yes.

   b. Are $4x$ and $15 + x$ equivalent expressions? Explain your thinking.
   No, because if you substitute other values for x, they are not equal.

2. a. Check that $2b + b$ and $3b$ have the same value when b is 1, 2, and 3.
   $2 \cdot 1 + 1 = 3$ and $3 \cdot 1 = 3$
   $2 \cdot 2 + 2 = 6$ and $3 \cdot 2 = 6$
   $2 \cdot 3 + 3 = 9$ and $3 \cdot 3 = 9$

   b. Do $2b + b$ and $3b$ have the same value for all the given values of b?
   Yes.

   c. Are $2b + b$ and $3b$ equivalent expressions? Explain your thinking.
   Yes, I can substitute any value for b, and they will always be equal.

3. Create your own equivalent expressions. What property helps you know that they are equivalent?

   a. **$x + 14$ and $14 + x$**

   Property: **commutative**

   b. **$12 + (3a + 4)$ and $3a + 16$**

   Property: **associative and commutative**

Name: ______ Date: ______ Period: ______

4. Clare said the value of g is the same in the following statements because they both include 30%, 90, and g. Do you agree or disagree? Explain your thinking.

   30% of 90 is equal to g. 30% of g is equal to 90.

   I disagree; Sample response: In the first statement, g represents part of 90, so it will be less than 90. In the second statement, g represents the whole, or 100%, so g will be greater than 90. In the first statement, $g = 27$, and in the second statement, $g = 300$.

5. A television has a length of ℓ in., a width of 28 in., and an area of 1,358 in^2. Select *all* the equations that represent the relationship between the side lengths and area of the television.

   A. $\ell \cdot 1{,}358 = 28$
   B. $28 \cdot \ell = 1{,}358$
   C. $1{,}358 \div 28 = \ell$
   D. $28 \cdot 1{,}358 = \ell$
   E. $1{,}358 \div \ell = 28$

6. Explain how evaluating the expression $4(200 + 90 + 2)$ can help you to mentally calculate the product of $4 \cdot 292$.

   The expression $4(200 + 90 + 2)$ allows me to calculate the answer mentally using a "break-apart" strategy (the Distributive Property), by multiplying 4 by the hundreds, then the tens, and then the ones, and then adding them all together.

Practice

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 9	3
	5	Unit 5 Lesson 6	2
Formative	6	Unit 6 Lesson 12	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice.**

The Distributive Property (Part 1)

Let's use rectangles to understand the Distributive Property with variables.

Focus

Goals

1. Generate algebraic expressions that represent the area of a rectangle with an unknown length.
2. **Language Goal:** Justify (using multiple representations) that algebraic expressions that are related by the Distributive Property are equivalent. **(Speaking and Listening)**

Coherence

Today

Students recall the use of area models for representing the Distributive Property, and extend this work to include situations where one of the quantities is represented by a variable. They recognize and explain (orally and in writing) that the Distributive Property can be used to represent the area of a partitioned rectangle in two different ways — as a sum of products, or as the product of a number and a sum — which emphasizes the concept of using equivalent expressions to write the same quantity multiple ways.

Previously

In Grade 4, students used the area model to multiply and divide using partial products and quotients. In Lessons 10–11 of this unit, students identified and generated equivalent expressions.

Coming Soon

In Lesson 13, students will generalize the use of the Distributive Property for identifying and generating equivalent expressions, including those with more than one variable or more than two terms.

Rigor

- Students build **conceptual understanding** of the distributive property by using expressions to represent the area of joined rectangles, including cases with variables.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *area*
- *coefficient*
- *Distributive Property*
- *equivalent expressions*
- *term*
- *variable*

Amps | Featured Activity

Exit Ticket
Real-Time Exit Ticket

Check in real time if your students can identify multiple expressions representing the total area of a partitioned rectangle using a digital Exit Ticket.

Building Math Identity and Community
Connecting to Mathematical Practices

Students might see Activity 2 as something that can be quickly completed because it is short. Before the activity begins, have students set goals about their responses. They need to predict what will be needed for a complete explanation that includes both their quantitative and qualitative reasoning. Rather than acting impulsively, students need to focus on doing what it takes to reach their goals.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- **Activitities 1 and 2** may be combined. The problem in Activity 1 can be done as a whole-class demonstration, with students working alongside you. Then have students complete the problem in Activity 2 before having the full class discussion.

Independent | 5 min

Warm-up Number Talk

Students perform mental calculations by applying strategies involving the Distributive Property.

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 12

The Distributive Property (Part 1)

Let's use rectangles to understand the Distributive Property with variables.

Warm-up Number Talk

Calculate the product mentally. Be prepared to explain your thinking.

$5 \cdot 98 = 490$

Log in to Amplify Math to complete this lesson online.

674 Unit 6 Expressions and Equations

1 Launch

Have students use the *Number Talk* routine.

2 Monitor

Help students get started by asking, "How can you make 98 friendlier?"

Look for points of confusion:

- **Changing 98 to friendlier numbers (e.g. $90 + 8$ or $100 - 2$), but not finishing the process.** "You have turned the problem into 5 groups of 90 or 100 and 5 groups of 8 or 2. What is your next step?"

Look for productive strategies:

- Using the standard algorithm in their heads. Challenge students to consider how to solve by making 98 "friendlier."
- Using the Distributive Property to rewrite 98 as $90 + 8$ or $100 - 2$, multiplying both terms by 5, and then adding or subtracting.

3 Connect

Have students share their different ways of thinking about the product, focusing on explanations using the Distributive Property, such as $5 \cdot 98 = 5 \cdot (90 + 8)$ or $5 \cdot (100 - 2)$. Write out the two products resulting from these distributions.

Highlight that, when writing one factor as a sum of friendlier numbers, this demonstrates the Distributive Property. Students can use "next to" notation to write $5 \cdot (90 + 8)$ or $5 \cdot (100 - 2)$ as $5(90 + 8)$ and $5(100 - 2)$.

Ask, "How are the expressions $5(90 + 8)$ and $5 \cdot 90 + 5 \cdot 8$ similar and different, and how does this relate to equivalent expressions?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they used mental math to determine the product, display the different strategies used and connect the ones that illustrate the Distributive Property. For example, a student may say, "I know that 5 times 100 is 500, and 98 is two less than 100. This means that 5 times 98 is 5 times "two less" than 500, which is 10 less than 500.

English Learners

Illustrate each strategy by writing it as an equation, such as $5 \cdot 98 = (5 \cdot 100) - (5 \cdot 2)$, which is $5\,(100-2)$

Power-up

To power up students' ability to recognize and use properties to identify equivalent expressions:

1. Mentally evaluate each of the following expressions:

   a. $5 \cdot 200 = 1000$ **b.** $5 \cdot 30 = 150$ **c.** $5 \cdot 8 = 40$

2. Explain how evaluating the above expressions can help you calculate $5 \cdot 238$.
   Sample response: The expressions show 238 broken down into the hundreds, tens, and ones place, so I can find the product of 5 and 238 by adding their products together, which is 1190.

Use: Before Activity 1.

Informed by: Performance on Lesson 11, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

Pairs | 15 min

Activity 1 Representing the Area of Joined Rectangles

Students write expressions to represent lengths of sides and areas of rectangles, connecting the Distributive Property to equivalent expressions.

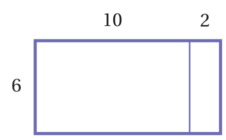
Name: ______ Date: ______ Period: ______

Activity 1 Representing the Area of Joined Rectangles

Part 1

Write an expression that represents the area of the largest, outlined rectangle in Figure A. Explain your thinking.

Figure A



Expression: $6(10+2)$, or $6 \cdot 10 + 6 \cdot 2$, or $6 \cdot 12$

Answers may vary, depending on the expression written. Student responses should include how their expression uses the formula area = length • width.

Compare and Connect: You will compare your expressions and approaches with another pair of students. Be ready to explain how you wrote your expression.

1 Launch

Activate prior knowledge by reviewing the area formula for a rectangle and the terms: *variable*, *term*, and *coefficient*. Explain that the values in the area models represent a decomposed value, not place value. Have pairs of students complete Part 1 before sharing their expressions and reasoning with the class. As students share, record their expressions in two unlabeled groups: area as a sum of the smaller areas, and area as a product of the width times the length. Have students identify how the expressions in each group are related, and then label the groups. Explain that both strategies use the Distributive Property to determine the total area, and, because they yield the same total area, the expressions are equivalent. Then have pairs of students complete Part 2.

2 Monitor

Help students get started by asking, "How can you determine the area of each small rectangle?"

Looking for points of confusion:

- **Multiplying or adding all three terms**. Ask, "What is the length? The width? The area?"
- **Writing the expression $6 \cdot 10 + 2$ or $5 \cdot x + 8$.** Ask, "What does your expression tell you to do first? Next? What area does that yield? How can your expression represent the exact steps you are taking?"
- **Struggling to use x in the expression (Part 2).** Ask, "If x was a number, how would you write the length?"

Look for productive strategies:

- Considering the rectangle in two ways — as the large, bold rectangle or as two smaller rectangles — and using the Distributive Property to write expressions that represent the total area as a sum or product.
- Recognizing that multiple expressions represent the total area, and because they will yield the same result, they are equivalent expressions.

Activity 1 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

Remind students they have previously determined the area of a rectangle using the area formula. Display the formula $A = \ell \cdot w$ and clarify the meaning of each variable. Students may also benefit from a review of rectangle diagrams that were used in a previous unit to represent multiplication.

Extension: *Math Enrichment*

In Part 2, have students re-label the diagram so that x and 5 trade places. Ask them to rewrite their expression that represents the area of the largest, outlined rectangle. $13x$ (or equivalent)

Math Language Development

MLR7: Compare and Connect

Identify pairs of students who wrote the area as a sum of the smaller products, $6 \cdot 10 + 6 \cdot 2$, and those who wrote the area as a product of the width times the sum of the lengths, $6 \cdot (10 + 2)$. During the Connect, display these expressions as you highlight the Distributive Property.

English Learners

Display a table, similar to the following, to help students connect the language used.

Distributive Property	
Sum of two products	Product of a number and a sum
$6 \cdot 10 + 6 \cdot 2$	$6 \cdot (10 + 2)$

Pairs | 15 min

Activity 1 Representing the Area of Joined Rectangles (continued)

Students write expressions to represent lengths of sides and areas of rectangles, connecting the Distributive Property to equivalent expressions.

Activity 1 Representing the Area of Joined Rectangles (continued)

Part 2

Write as many expressions as you can for the total area of the largest, outlined rectangle in figure B. Explain your thinking.

Figure B

x 8

5

Expression: $5x + 5 \cdot 8$ or $5 \cdot x + 5 \cdot 8$ or $5x + 40$ or $5(x + 8)$

Answers may vary, depending on the expression written. Student responses should include how their expression uses the formula area = length • width.

Are you ready for more?

Write an expression to represent the total area of each rectangle.

a. x 3 y, 2

$2(x + 3 + y)$

b. 3 y, x

$x(3 + y)$

c. x y, x

$x(x + y)$

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3 Connect

Have students share their expressions and reasoning for Part 2, focusing on how they used the Distributive Property to write equivalent expressions. Encourage students to use the terms coefficient and term to connect expressions such, as $5 \cdot x + 5 \cdot 8$, $5x + 5 \cdot 8$ and $5x + 40$.

Ask, "Are the expressions $5 \cdot x + 8$ and $5(x + 8)$ equivalent? Could they both represent the total area of the joined rectangle in Part 2? Why or why not?"

Highlight how the Distributive Property represents the total area as a sum or product in both Parts 1 and 2. Explain that when a rectangle has an unknown side length the total area can be determined in the same way.

Pairs | 15 min

Activity 2 Determining Areas of Partitioned Rectangles

Students represent the total area of partitioned rectangles as a product and a sum, and reason why their expressions are equivalent.

Name: ______ Date: ______ Period: ______

Activity 2 Determining Areas of Partitioned Rectangles

For each rectangle, write expressions for the length and width, and two expressions for the total area of the largest, outlined rectangle. Record them in the table.

Rectangle A: a, 5; 3

Rectangle B: 6, x; $\frac{1}{3}$

Rectangle C: 6, 8; m

Rectangle D: $3x$, 8; 5

Rectangle	Length	Width	Area as a product of width and sum of the lengths	Area as the sum of the areas of the smaller rectangles
A	$a+5$	3	$3(a+5)$	$3a+15$ or $3a+3\cdot5$
B	$6+x$	$\frac{1}{3}$	$\frac{1}{3}(6+x)$	$2+\frac{1}{3}x$ or $\frac{1}{3}\cdot6+\frac{1}{3}x$
C	$6+8$	m	$m(6+8)$	$6m+8m$
D	$3x+8$	5	$5(3x+8)$	$15x+40$ or $5\cdot3x+5\cdot8$ or $15x+5\cdot8$

Choose one rectangle. Assign a value to the variable, and evaluate both expressions to determine the total area. Repeat with a new value for the variable. What do you notice?

I notice that the expressions for area are equal no matter what value I substitute for the variable. This means the expressions are equivalent.

STOP

1 Launch

Remind students that *partitioned* means split into smaller parts. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "How is Rectangle A similar to the rectangle in Activity 2?"

Look for points of confusion:

- **Confusing which expressions represent area as a product and area as a sum.** Review the terms *product* and *sum*. Ask, "Which expression shows two areas being added together?"
- **Not substituting consistent variables in both expressions.** Say, "The expressions represent the total area for the same rectangle, so the variables represent the same value in both expressions."

Look for productive strategies:

- Writing expressions correctly in $a(b+c)$ and $ab+ac$ form. If students do not use, or incorrectly use, coefficients (e.g., $3\cdot(a+5)$, $m6$, or $5\cdot3x$), remind them of "next to" notation.

3 Connect

Display each rectangle, one at a time.

Have students share their expressions and a few variables to substitute for each expression, focusing on how this illustrates the expressions are equivalent.

Highlight that each pair of expressions is equivalent because they represent the total area, and no matter what value is chosen for the variable, the expressions will yield the same value.

Ask, "Why is one expression considered a *sum* and the other a *product*, even though they both have sums and products?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to complete the table for Rectangle A. Highlight how the expressions in the *Area as a product* column show a product, where the two factors are the width (the quantity outside the parentheses) and the sum of the lengths (the quantities inside the parentheses).

Extension: *Math Enrichment*

Have students draw a diagram in which the expressions $a(4+3x)$ and $4a+3ax$ both represent the area of the largest, outlined rectangle. Students' diagrams may vary, as long as the Distributive Property is satisfied.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask question, display the expressions for one of the rectangles and have students examine their structures. For example, display the expressions $3(a+5)$ and $3a+15$ for Rectangle A. Ask:

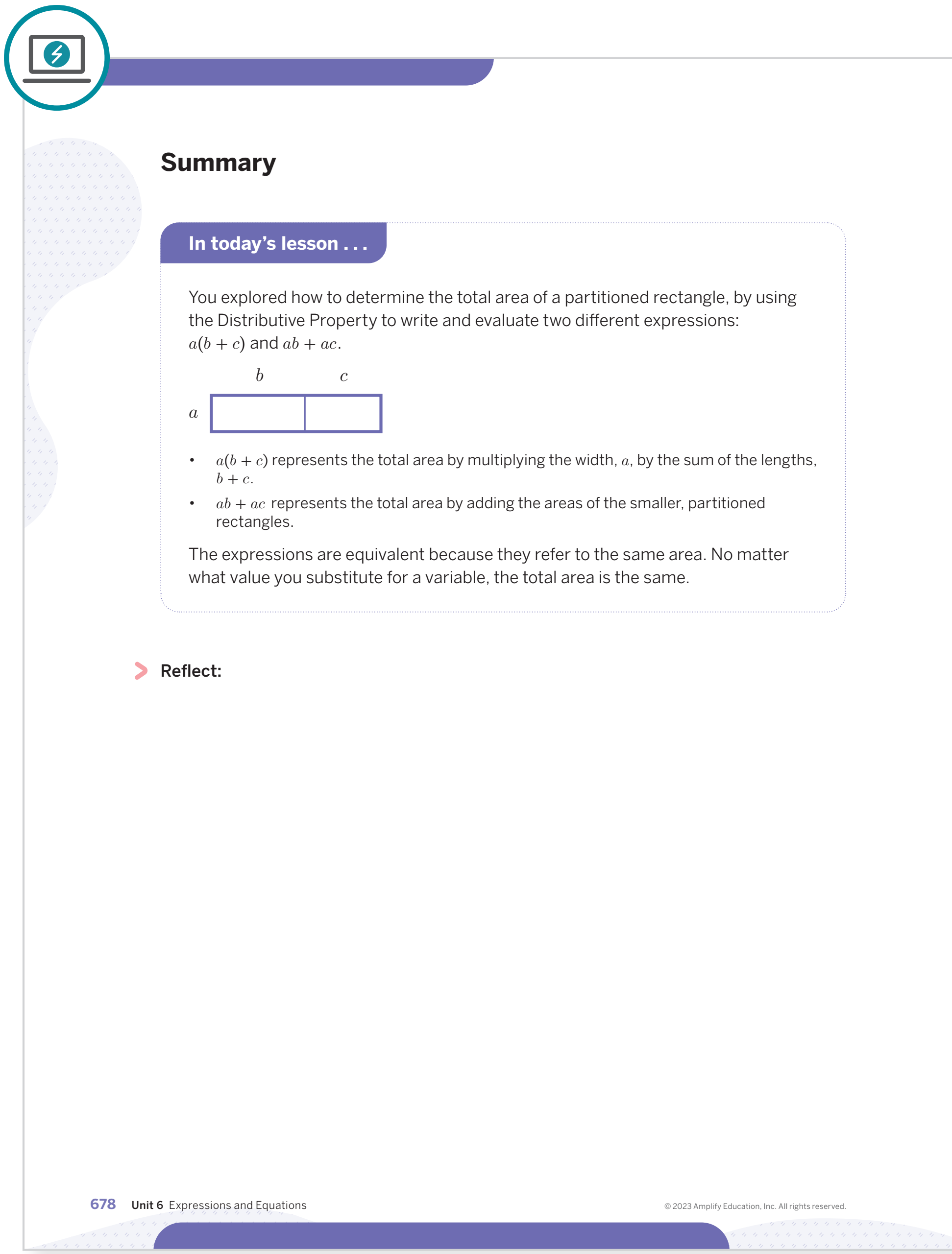
- "Would you consider the expression $3(a+5)$ as a *sum of two products, or the product of a number and a sum*? Why?"
- "What about the expression $3a+15$?"

Listen for students who recognize how the structure of the expressions relate to the order of operations.

Whole Class | 5 min

Summary

Review and synthesize how the Distributive Property can help determine the total area of a partitioned rectangle.



Summary

In today's lesson . . .

You explored how to determine the total area of a partitioned rectangle, by using the Distributive Property to write and evaluate two different expressions: $a(b + c)$ and $ab + ac$.

- $a(b + c)$ represents the total area by multiplying the width, a, by the sum of the lengths, $b + c$.
- $ab + ac$ represents the total area by adding the areas of the smaller, partitioned rectangles.

The expressions are equivalent because they refer to the same area. No matter what value you substitute for a variable, the total area is the same.

Reflect:

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Synthesize

Display the expressions $a(b + c)$ and $ab + ac$.

Ask:

- "How do these expressions represent the total area of a partitioned rectangle?"
- "How can you prove that these expressions are equivalent?"

Highlight that $a(b + c)$ represents the total area of a partitioned rectangle as the product of the width, a, times the sum of the lengths, $b + c$, and the expression $ab + ac$ represents the total area as the sum of the areas of the smaller rectangles. Both expressions are equivalent because, no matter what is substituted for the variable, they yield the same value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did your work today build upon your experience with the Distributive Property and area models in previous grades? In Unit 5?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying expressions with variables that represent the total area of a partitioned rectangle.

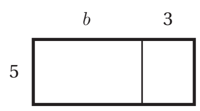
Amps Featured Activity Real-Time Exit Ticket

Printable

Name: ________ Date: ________ Period: ________

Exit Ticket 6.12

Select *all* the expressions that represent the larger, bold rectangle's total area.



A. $3(5 + b)$

B. $5(b + 3)$

C. $5b + 15$

D. $15 + 5b$

E. $3 \cdot 5 + 3b$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use a diagram of a rectangle partitioned into two smaller rectangles to write different expressions representing its area.
1 2 3

b. I can use a diagram of a partitioned rectangle to write different expressions, with variables, representing its area.
1 2 3

c. I can use the Distributive Property to write equivalent expressions with variables.
1 2 3

Success looks like . . .

- **Goal:** Generating algebraic expressions that represent the area of a rectangle with an unknown length.
  - » Selecting all expressions that represent the total area of the larger rectangle.
- **Language Goal:** Justifying (using multiple representations) that algebraic expressions that are related by the Distributive Property are equivalent. **(Speaking and Listening)**

Suggested next steps

If students select Choice A or E, consider:

- Reviewing how to identify the length and width in a partitioned rectangle from Activity 1.
- Asking, "How do you determine the area of any rectangle? What is the width and length in our example?"

If students do not select Choice B, consider:

- Reviewing the two ways to determine the total area of a partitioned rectangle from Activity 2.
- Asking, "Which answer choice represents the area as the width times the sum of the lengths?"

If students do not select Choice C or D, consider:

- Reviewing the two ways to determine the total area of a partitioned rectangle from Activity 2.
- Asking, "Which answer choice(s) represent(s) the area as the sum of the areas of the smaller rectangles?"

If students select Choice C or D, but not both, consider:

- Reviewing the commutative property.
- Asking, "How are the expressions similar? Are they equivalent? How do you know?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- When you compare and contrast today's work with work students did earlier this year on using area models to multiply decimals, what similarities and differences do you see?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

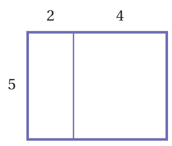
Independent

Name: ________ Date: ________ Period: ________

Practice

1. Select *all* of the expressions that represent the area of the largest, outlined rectangle in Figure A.

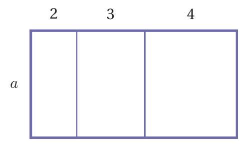
Figure A



- **A.** $5(2 + 4)$
- **B.** $5 \cdot 2 + 4$
- **C.** $5 \cdot 2 + 5 \cdot 4$
- **D.** $5 \cdot 2 \cdot 4$
- **E.** $5 + 2 + 4$
- **F.** $5 \cdot 6$

2. Refer to Figure B.

Figure B



a Explain why the area of the larger, bold rectangle is $2a + 3a + 4a$.

The area of the largest rectangle is the sum of the areas of the three smaller rectangles.

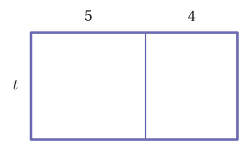
The area of the left rectangle is $a \cdot 2$ or $2a$, the middle is $a \cdot 3$ or $3a$, and the right is $a \cdot 4$ or $4a$. When you add these areas, the expression is $2a + 3a + 4a$.

b Explain why the area of the larger, bold rectangle is $(2 + 3 + 4)a$.

To determine the area of a rectangle, you multiply length and width. The length of the largest, outlined rectangle is broken into three parts, so the total length is $2 + 3 + 4$. To determine the area, you multiply the length $(2 + 3 + 4)$ by the width (a), and the expression is $(2 + 3 + 4)a$.

3. Choose the expressions that do *not* represent the total area of the larger, bold rectangle in Figure C. Select *all* that apply.

Figure C



- **A.** $5t + 4t$
- **B.** $t + 5 + 4$
- **C.** $9t$
- **D.** $4 \cdot 5 \cdot t$
- **E.** $t(5 + 4)$

Name: ________ Date: ________ Period: ________

Practice

4. Consider the statement: 120% of x is equal to 78.

a Write an equation that shows the relationships between 120%, x, and 78.

$\frac{120}{100}x = 78$

b Use your equation to determine x.

$x = 65$

5. Kiran's aunt is 17 years older than Kiran.

a How old will Kiran's aunt be when Kiran is 15 years old?

32 years old

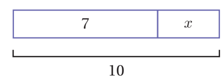
b How old will Kiran's aunt be when Kiran is x years old?

$x + 17$

c How old will Kiran be when his aunt is 60 years old?

43 years old

6. What is the value of x in the rectangle shown? Explain your thinking.



$x = 3$; Sample response: The entire length is 10, and $10 - 7 = 3$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 9	1
	5	Unit 6 Lesson 2	1
Formative	6	Unit 6 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

The Distributive Property (Part 2)

Let's practice writing equivalent expressions by using the Distributive Property.

Focus

Goals

1. Use the Distributive Property to write equivalent algebraic expressions, including where the common factor is a variable.
2. Draw a diagram to justify that two expressions that are related by the Distributive Property are equivalent.
3. **Language Goal:** Justify (using multiple representations) that algebraic expressions that are related by the Distributive Property are equivalent. **(Speaking and Listening)**

Coherence

Today

Students draw partitioned rectangles to represent expressions of the form $a(b + c)$, and use their drawings to write equivalent expressions of the form $ab + ac$. By examining the structure of their equivalent expressions, students discover the connection between common factors and the coefficient, a. They explain how to use common factors to efficiently move from expressions of $ab + ac$ form to $a(b + c)$ form.

Previously

In Lesson 12, students used rectangle diagrams to represent the Distributive Property, including situations where one of the quantities was represented by a variable. They saw that the Distributive Property can arise out of writing areas of rectangles in two different ways, which emphasizes the idea of equivalent expressions as being two different ways of writing the same quantity.

Coming Soon

In Lessons 14–16, students will extend their understanding of equivalent expressions to include expressions with exponents.

Rigor

- Students develop **conceptual understanding** using area models to connect equivalent expressions and the Distributive Property.
- Students develop **procedural fluency** writing expressions to represent the Distributive Property with variables.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

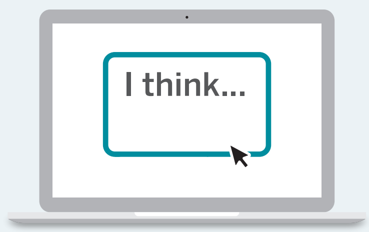
- *coefficient*
- *Distributive Property*
- *equivalent expressions*
- *term*
- *variable*

Amps | Featured Activity

Activity 1

See Student Thinking

Students are asked to draw partitioned rectangles to represent expressions, and these drawings are passed to you.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students learn to represent equivalent expressions geometrically and algebraically. Because one way might seem more natural than the other, students might lack motivation to complete both ways. Remind students that having more than one way to do something oftens lessens their stress because, if the first way does not work, they have another approach to take. By understanding the benefits of additional tools in their mathematical tool chests, students should feel more motivated.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1,** have pairs work on corresponding rows (e.g., rows 1 and 2, rows 3 and 4, rows 5 and 6, or rows 7 and 8).
- In **Activity 2,** the first two rows may be done as a whole-class demonstration, with students working alongside you. Then have pairs choose two rows to complete together, ideally one addition and one subtraction expression.

Independent | 5 min

Warm-up Notice and Wonder

Students reason with two partitioned rectangles to develop an understanding of using the Distributive Property in addition and subtraction scenarios.

Name: ________ Date: ________ Period: ________

Unit 6 | Lesson 13

The Distributive Property (Part 2)

Let's practice writing equivalent expressions using the Distributive Property.

Warm-up Notice and Wonder

Refer to these rectangles. What do you notice? What do you wonder?

1. I notice . . .

   Sample responses:
   - That the length of Rectangle A's shaded rectangle is w, and Rectangle B's is $w - 4$.
   - That Rectangle A's total length is $4 + w$ and Rectangle B's is w.
   - That Rectangle A's total area is $6(4 + w)$ or $24 + 6w$.
   - That Rectangle B's total area is $6w$.

2. I wonder . . .

   Sample responses:
   - How I can determine the area of just the shaded rectangles.
   - Whether the areas of Rectangle A and Rectangle B are the same.

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking, "How are the rectangles similar? Different?"

Look for points of confusion:

- **Not recognizing the shaded length of Rectangle B as $w - 4$.** Replace w with 10. Ask, "Now what is the length of the shaded rectangle? How did you get that? Since you don't know the value of w, how can you represent the length of the shaded rectangle?"
- **Using addition to represent the total area of Rectangle B:** "What is the length of the shaded rectangle?"

Look for productive strategies:

- Noticing that both rectangles are partitioned, but one is missing the length of the shaded rectangle, requiring subtraction.
- Writing expressions for the total area that show addition for Rectangle A and subtraction for Rectangle B.

3 Connect

Have students share low-floor observations first, followed by those that incorporate math — specifically the length of Rectangle B's shaded rectangle and expressions to determine the total area of Rectangle A and Rectangle B.

Ask, "Why does one expression for the total area use addition, and the other subtraction?"

Highlight that there are two ways to determine the total area of a partitioned rectangle, no matter what values are unknown. If the entire side length is known, use subtraction to represent the length of the smaller rectangle.

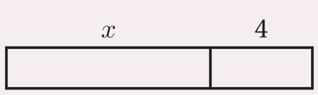
Differentiated Support

Accessibility: *Guide Processing and Visualization*

To support students in making sense of what they notice and wonder, encourage them to try different values they can use to substitute for w in Rectangle B. Have them record their calculations without simplifying them first, which will help them to see the pattern. After trying a few values for w, students should begin to see that the missing side is represented by the expression $w - 4$.

Power-up

To power up students' ability to determine the unknown value in a tape diagram.



a. What expression represents the diagram?
Sample response: $x + 4$

b. What value would you add to 4 to get 5? 6? 10? 12?
1, 2, 6, 8

Use: Before the Warm-up

Informed by: Performance on Lesson 12, Practice Problem 6.

Activity 1 Drawing Partitioned Rectangles and Writing Equivalent Expressions

Pairs | 15 min

Students represent the total area of partitioned rectangles by drawing models and writing equivalent expressions of the forms $a(b + c)$ and $(ab + ac)$.

Amps Featured Activity | See Student Thinking

Activity 1 Drawing Partitioned Rectangles and Writing Equivalent Expressions

Complete the table. Each row should have:

- An expression that shows the length and width of its partitioned rectangle.
- A partitioned rectangle whose area and labels match the expression. If the rectangle has been started, label any missing dimensions.
- An equivalent expression for the rectangle's total area.

Expression	Rectangle	Equivalent expression
$2(12 + 4)$	12, 4; 2	$24 + 8$
$8(3 + 1)$	8; 3, 1	$24 + 8$
$2(2a + 10)$	$2a$, 10; 2	$4a + 20$
$4(a + 5)$	a, 5; 4	$4a + 20$
$a(4 + 8)$	4, 8; a	$4a + 8a$
$4a + 8a$	$4a$; 2, 1	$4a(1 + 2)$
$3(2a + 4b)$	$2a$, $4b$; 3	$6a + 12b$
$6(a + 2b)$	a, $2b$; 6	$6a + 12b$

682 Unit 6 Expressions and Equations

1 Launch

Review how the length and width of a rectangle are terms, and terms can be a number (4), variable (a), or a variable with a coefficient ($4a$). Have pairs of students complete the activity together.

2 Monitor

Help students get started by asking, "In the expression $2(12 + 4)$, which term is the width? the length? Draw a rectangle to show this."

Look for points of confusion:

- **Struggling to complete a row when the first column is given.** Ask, "What is the width? Length? What is the area of each smaller rectangle?"
- **Struggling to complete a row when the third column is given.** Have student use the expression to label the areas of the smaller rectangles. Ask, "What can you multiply the known term by to get the area of the small rectangle?"

Look for productive strategies:

- Drawing rectangles with a as the width and $b + c$ as the length, and writing equivalent expressions in $ab + ac$ form.
- Using multiplicative reasoning to factor the expressions in the third column and determine the missing terms in the rectangles.

3 Connect

Have students share responses and reasoning for each row.

Display the table with correct responses.

Ask, "How are the coefficients in the first column related to the terms in the third column? How could you use common factors to write an equivalent expression in $a(b + c)$ form without drawing a rectangle?"

Highlight that, in $ab + ac$ and $a(b + c)$, a is a common factor of b and c. Dividing ab and ac by a results in the expression $a(b + c)$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Draw a partitioned rectangle with a length of 2 and a width of $3 + 4$. Have students write expressions that represent the total area of the largest rectangle. Then ask them to identify the length and width in the expression. Keep this rectangle displayed and annotated for students to refer to as they complete the activity.

Total area: $2(3 + 4)$
Length: 2
Width: $(3 + 4)$

Extension: *Math Enrichment*

Have students draw a partitioned rectangle whose total area can be represented by the expression $a(2a + 3b + 4c)$. Have them assign values for the variables to ensure that their diagram matches the expression. Students' diagrams may vary, as long as the Distributive Property is satisfied.

Activity 2 Writing Equivalent Expressions Using the Distributive Property

Pairs | 15 min

Students use the Distributive Property and common factors to write equivalent expressions of the forms $a(b + c)$ and $ab + ac$.

Name: ______ Date: ______ Period: ______

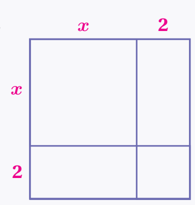
Activity 2 Writing Equivalent Expressions Using the Distributive Property

With a partner, use the Distributive Property to write an equivalent expression for each row, and explain your thinking to your partner. Take turns selecting which row to work on next. Consider drawing a diagram to help with your thinking.

Sum or difference	Product
$5b + 6b$	$(5 + 6)b$
$10x - 5$	$5(2x - 1)$
$4x + 7x$	$x(4 + 7)$
$\frac{1}{2}x - 3$	$\frac{1}{2}(x - 6)$
$6c + 2c$	$2c(3 + 1)$, or $2(3c + c)$, or $c(6 + 2)$
$20p - 10z$	$10(2p - z)$, or $5(4p - 2z)$, or $2(10p - 5z)$
$x + 2x + 3x$	$x(1 + 2 + 3)$
$3xy + 4zy$	$y(3x + 4z)$

Are you ready for more?

The total area of the largest, outlined rectangle shown can be written as $(x + 2)^2$, where the length is $x + 2$, and the width is also $x + 2$.



1. Label the length and width of the largest, outlined rectangle.
2. Determine the area of each smaller rectangle.
   Top left: $x \cdot x$ or x^2, Top right: $2x$
   Bottom left: $2x$, Bottom right: 4
3. Using the areas of the smaller rectangles, write another expression to represent the total area of the largest, outlined rectangle.
   $x^2 + 4x + 4$ or
   $x^2 + 2x + 2x + 4$ or
   $x^2 + 2x + 2x + 2 \cdot 2$

STOP

1 Launch

Activate prior knowledge, reminding students of the Commutative Property of Multiplication. Consider using the expression in the first row of the table as an example. Then have pairs complete the activity.

2 Monitor

Help students get started by asking, "In the expression $(5 + 6)b$, what is the width? How can you draw a rectangle to help?"

Look for points of confusion:

- **Not using a common factor (e.g., when it is a variable).** "What do the two terms have in common?"
- **Not dividing by a common factor.** "In the expression $5b - 6b$, the common factor is b. How did you move it to make $b(5 + 6)$?"

Look for productive strategies:

- Writing an equivalent expression by dividing by a common factor, or multiplying the coefficient by each term.

3 Connect

Have students share their expressions, focusing on how they found a common factor and used it to write their expressions.

Ask, "Why do some expressions have multiple equivalent expressions, and others only have one?"

Highlight that, in the expression $ab - ac$, a is a common factor of both terms, and a may represent a number, such as 2, or another expression, such as $2d$. If there are multiple common factors, then there will be multiple possible equivalent expressions.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Clarify Vocabulary and Symbols*

Display the first expression given in the Product column, $(5 + 6)b$, and write the expression $b(5 + 6)$ next to it. Ask:

- "How are these expressions similar? Different? What do you notice?"
- "Are these expressions equivalent?" Yes; If students say *no*, display the expressions $b(11)$ and $11b$ and ask them if they are equivalent.
- "What property illustrates that $(5 + 6)b$ and $b(5 + 6)$ are equivalent? Did the order or the grouping change?" The commutative property. The order of the products changed.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share how they determined a common factor, use a think-aloud approach to highlight the connections between common factors and the coefficient. Use the expression $ab - ac$ and say something similar to the following:

"What factor do the terms ab and ac have in common? They are both multiplied by a, so I can divide each term by a to see what's left. I can show this by writing the equivalent expression $a(b - c)$."

English Learners

Annotate the number outside the parentheses in these expressions as the *common factor.*

Whole Class | 5 min

Summary

Review and synthesize the connection between common factors and equivalent expressions written using the Distributive Property.

Summary

In today's lesson . . .

You saw how the Distributive Property can be used to write a sum or difference as a product: $a(b + c)$ or $a(b - c)$. It can also be used to write a product as a sum or difference: $ab + ac$ or $ab - ac$.

When you are given an expression in one form, you can write an equivalent expression in the other form by drawing a partitioned rectangle, where a is the width, and b and c are the lengths.

To write an equivalent expression for $a(b + c)$, multiply a by both b and c to get $ab + ac$. This is known as distributing a to both b and c. For example, to write an equivalent expression for $2(3 + 4)$, multiply 2 by both 3 and 4 to get $(2 \cdot 3) + (2 \cdot 4)$, or $6 + 8$.

To write an equivalent expression for $ab + ac$, divide ab and ac by their common factor, a. The equivalent expression is $a(b + c)$. For example, to write an equivalent expression for $(3 \cdot 4) + (3 \cdot 5)$, which is $12 + 15$, divide 12 and 15 by the common factor 3. The equivalent expression is $3(4 + 5)$.

Reflect:

Synthesize

Display the expressions $a(b + c)$ and $ab + ac$.

Ask:

- "How are common factors related to these expressions?" The coefficient, a, is a common factor of ab and ac.
- "How do you know whether the coefficient is the greatest common factor?" The terms b and c will no longer have any factors in common.
- "How can you use common factors to write an equivalent expression for $ab + ac$?" Divide ab and ac by the common factor of a. The expression becomes $a(b + c)$.

Highlight that common factors can be used to write equivalent expressions because, in both expressions, a is a common factor of ab and ac. To write an equivalent expression for $a(b + c)$ multiply a by both b and c to get $ab + ac$. To write an equivalent expression for $ab + ac$, divide ab and ac by the common factor a. The equivalent expression is $a(b + c)$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you use area models and the Distributive Property to generate equivalent expressions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by dividing by common factors to write an equivalent expression of the form $a(b + c)$.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.13

1. Use the Distributive Property to write an expression that is equivalent to the expression $12 + 4x$.
   Sample responses: $4(3 + x)$ or $2(6 + 2x)$

2. Show or explain how you know they are equivalent expressions.
   Answers may vary, but should include drawings of rectangles or an explanation that the total areas are the same.
   Sample response: $12 + 4x$ is equivalent to $4(3 + x)$ because when you draw a partitioned rectangle with a width of 4, and a length of $3 + x$, the areas of the smaller rectangles are $4 \cdot 3 = 12$ and $4 \cdot x = 4x$. When you add those areas together, you get $12 + 4x$. Because the sum of the smaller areas is the same as the area of the larger rectangle, the expressions are equivalent.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use the Distributive Property to write equivalent expressions with variables. 1 2 3

b. I can explain that algebraic expressions that are related by the Distributive Property are equivalent. 1 2 3

Success looks like . . .

- **Goal:** Using the Distributive Property to write equivalent algebraic expressions, including where the common factor is a variable.
- **Goal:** Drawing a diagram to justify that two expressions that are related by the Distributive Property are equivalent.
- **Language Goal:** Justifying (using multiple representations) that algebraic expressions that are related by the Distributive Property are equivalent. **(Speaking and Listening)**
  - » Explaining why the expression is equivalent to $12 + 4x$ in Problem 2.

Suggested next steps

If students do not identify a common factor in Problem 1, consider:

- Reviewing how to identify common factors from Activity 1.
- Asking, "For the second expression in Activity 2, how did 5 become the coefficient in $5(2x - 1)$?"

If students do not or incorrectly divide both terms by the common factor in Problem 1, consider:

- Reviewing how to use common factors from Activity 1.
- Asking, "For the second example In Activity 2, where did $(2x - 1)$ come from?"

If students do not explain that the total areas are the same in Problem 2, consider:

- Reviewing how to prove equivalence by drawing partitioned rectangles for each expression from Activity 1.
- Asking, "How did you use your diagrams in Activity 1 to help you prove that $2(12 + 4)$ and $24 + 8$ are equivalent?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did Activity 1 reveal about your students as learners?
- What did students find frustrating about Activity 2? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______________ Date: ________ Period: ________

Practice

1. For each expression, use the Distributive Property to write an equivalent expression.
   a. $4(x+2) = 4x + 8$
   b. $(6-8)\cdot x = 6x - 8x$
   c. $8x + 12 = 4(2x+3)$ or $2(4x+6)$
   d. $6x - 9y = 3(2x - 3y)$
   e. $6x - 4y - 2z = 2(3x - 2y - z)$

2. Select *all* the expressions that are equivalent to $16x + 36$.
   A. $16(x+20)$
   B. $x(16+36)$
   C. $4(4x+9)$
   D. $2(8x+18)$
   E. $2(8x+36)$

3. Priya rewrites the expression $8y - 24$ as $8(y-3)$. Han rewrites $8y-24$ as $2(4y-12)$. Are Priya and Han's expressions both equivalent to $8y - 24$? Explain your thinking.
   They are both correct. Priya divided $8y$ and 24 by 8, which is the greatest common factor. Han divided $8y$ and 24 by 2, which is another common factor. Since both are common factors, both expressions are equivalent to $8y - 24$.

Name: ______________ Date: ________ Period: ________

Practice

4. Solve each equation. Show your thinking.
   a. $10 = 4a$
      $10 \div 4 = 4a \div 4$
      $a = 2.5$ or $2\frac{1}{2}$
   b. $5b = 17.5$
      $5b \div 5 = 17.5 \div 5$
      $b = 3.5$ or $3\frac{1}{2}$
   c. $1.036 = 10c$
      $1.036 \div 10 = 10c \div 10$
      $c = 0.1036$
   d. $0.6d = 1.8$
      $0.6d \div 0.6 = 1.8 \div 0.6$
      $d = 3$
   e. $15 = 0.1e$
      $15 \div 0.1 = 0.1e \div 0.1$
      $e = 150$

5. Select *all* the expressions that are equivalent to the expression $\frac{1}{2}z$.
   A. $z + z$
   B. $z \div 2$
   C. $z \cdot z$
   D. $\frac{1}{4}z + \frac{1}{4}z$
   E. $2z$

6. Clare said that both of the following expressions can be represented by writing 10^3. Do you agree or disagree? Explain your thinking.

   $10 + 10 + 10$ $\qquad$ $10 \cdot 10 \cdot 10$

   I disagree. Sample response: The first expression is repeated addition, which can be represented using multiplication ($3 \cdot 10$) for three groups of 10, or 30. The second expression is repeated multiplication, which can be represented using an exponent (10^3), and it equals 1,000.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1–2	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 8	1
	5	Unit 6 Lesson 11	1
Formative	6	Unit 6 Lesson 14	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 6 | LESSON 14

Meaning of Exponents

Let's see how exponents show repeated multiplication.

Focus

Goals

1. **Language Goal:** Describe a pattern that could be expressed using repeated multiplication.
2. Generate and evaluate numerical expressions involving whole number exponents. **(Speaking and Listening, Writing)**
3. **Language Goal:** Interpret expressions with exponents larger than 3, and comprehend the phrase *to the power* or *to the*. **(Speaking and Listening)**

Coherence

Today

Students explore a real-world scenario in which exponent notation is naturally useful, and they are motivated to represent and evaluate repeated calculations using exponent notation as a shorthand. Utilizing the structure and meaning of exponents, students discover how to evaluate terms with large exponents.

Previously

In Grade 5, students learned that exponents represent repeated multiplication when multiplying and dividing by powers of 10.
In Unit 1, students worked with exponents in the context of labeling area and surface area as *squared* and *cubed*.

Coming Soon

In Lessons 15–16, students will use the order of operations to evaluate expressions with two terms, one of which is a whole number exponent.

Rigor

- Students develop **conceptual understanding** of exponents and exponential form within the context of fundraising for an animal shelter.
- Students build **fluency** solving expressions, including those with exponents.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *base*
- *cube of a number*
- *equivalent expressions*
- *exponent*
- *power*
- *square of a number*

Amps | Featured Activity

Exit Ticket

Real Time Exit Ticket

Check in real time whether your students can evaluate expressions with exponents using a digital Exit Ticket.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack organizational skills that they need to express repeated addition or repeated multiplication. Explain the benefits of exponents and what they communicate. Guide students to conclude that using exponents eliminates some of the opportunity for error in either recording the expression or entering it into a calculator.

Modifications to Pacing

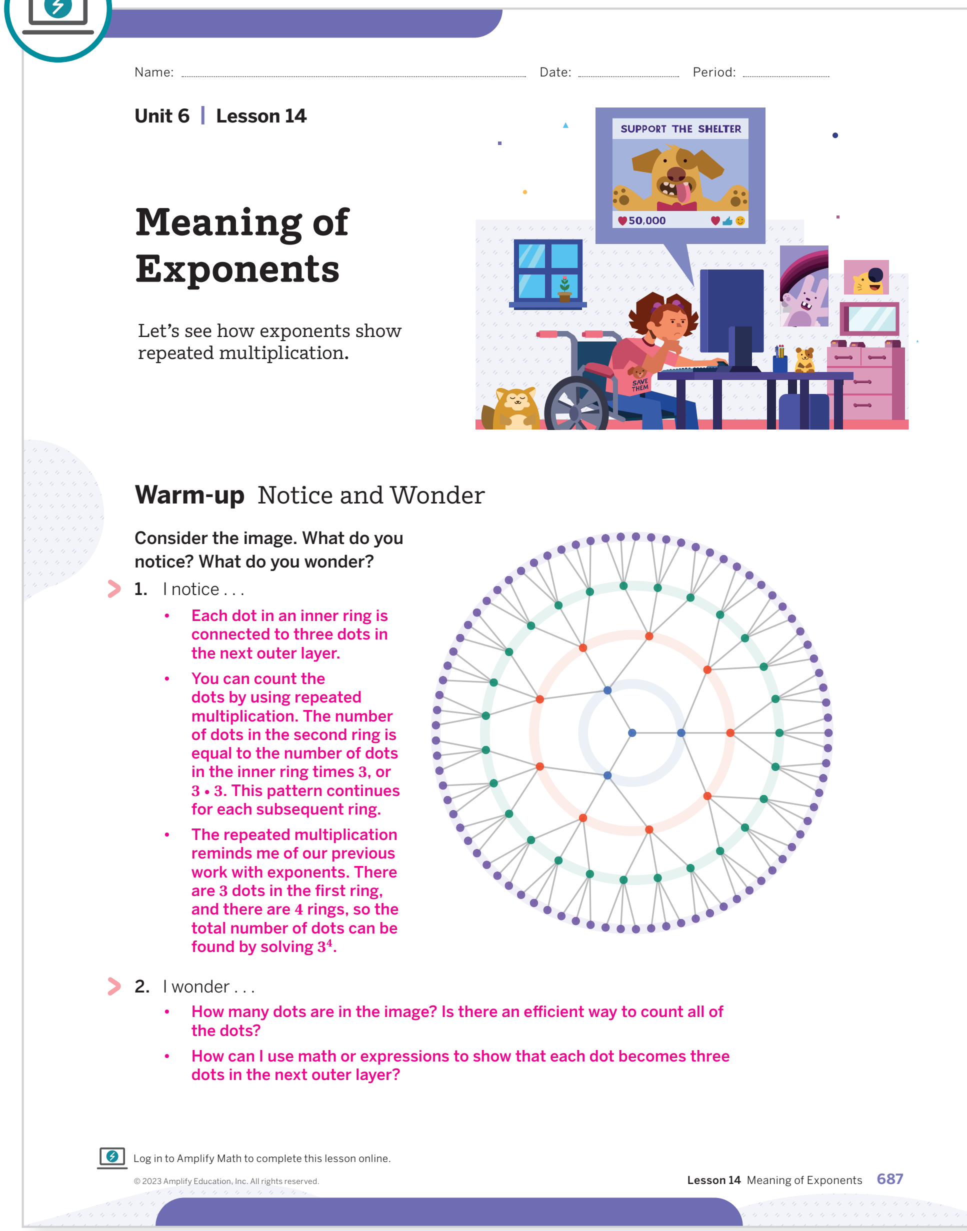
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be done using the ***Notice and Wonder*** routine. After students share their responses, model how to determine the number of dots in the image without counting.
- In **Activity 1,** have pairs complete Problem 1 and share their thinking with the class. Then complete Problems 2 and 3 as a class.

Independent | 5 min

Warm-up Notice and Wonder

Students reason about an image, preparing them to relate repeated multiplication and exponents.


Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 14

Meaning of Exponents

Let's see how exponents show repeated multiplication.

Warm-up Notice and Wonder

Consider the image. What do you notice? What do you wonder?

1. I notice . . .
   - Each dot in an inner ring is connected to three dots in the next outer layer.
   - You can count the dots by using repeated multiplication. The number of dots in the second ring is equal to the number of dots in the inner ring times 3, or $3 \cdot 3$. This pattern continues for each subsequent ring.
   - The repeated multiplication reminds me of our previous work with exponents. There are 3 dots in the first ring, and there are 4 rings, so the total number of dots can be found by solving 3^4.
2. I wonder . . .
   - How many dots are in the image? Is there an efficient way to count all of the dots?
   - How can I use math or expressions to show that each dot becomes three dots in the next outer layer?

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking, "How are the dots arranged in the image? Does each ring have the same number of dots?"

Look for productive strategies:

- Noticing that each dot becomes 3 dots in the next ring, so they can determine the number of dots in each ring by multiplying each previous ring count by 3.
- Recognizing that because each layer is three times greater than the previous ring, they can use repeated multiplication (e.g., the outer ring has $3 \cdot 3 \cdot 3 \cdot 3$ total dots).
- Activating prior knowledge of exponents from Unit 1 to represent the repeated multiplication using exponents.

3 Connect

Have students share their responses, starting with those who recognized the pattern of 1 dot becoming 3 dots, followed by those who reasoned about the total number of dots.

Highlight that every dot will become 3 new dots in the next ring, so the growing groups can be represented using repeated multiplication. The outer ring has $3 \cdot 3 \cdot 3 \cdot 3$ total dots. Remind students how they represented the volume of a cube in Unit 1 using repeated multiplication ($s \cdot s \cdot s$) and exponents (s^3).

Ask:

- "How could you represent the total number of dots in the outer ring using exponents?"
- "In the expressions $3 \cdot 3 \cdot 3 \cdot 3$ and 3^4, how can you tell what number ring from the center each expressions represents?"
- "How could you use repeated multiplication or exponents to represent the next ring (the fifth)? the 100th ring? the nth ring?"

MLR Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies for determining the number of dots without counting, consider displaying something similar to the following that highlights the connections between two strategies.

One strategy:	Another strategy:
Each dot becomes 3 dots in the next ring. First ring: 3 Second ring: $3 \cdot 3$ dots Third ring: $3 \cdot 3 \cdot 3$ dots Outer ring: $3 \cdot 3 \cdot 3 \cdot 3$ dots	Each next ring has 3 times as many dots as the previous ring. Because there are 4 rings, I can write this pattern: 3, 9, 27, 81.

Power-up

To power up students' ability to relate powers of 10, exponents, and multiplication. ask:

Evaluate:

a. $10 + 10 = 20$ **b.** $10 \cdot 10 = 100$

c. $10 + 10 + 10 = 30$ **d.** $10 \cdot 10 \cdot 10 = 1000$

e. $10 + 10 + 10 + 10 = 40$ **f.** $10 \cdot 10 \cdot 10 \cdot 10 = 10000$

Use: Before Activity 1.

Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1.

Pairs | 15 min

Activity 1 Fundraising Outreach for the Animal Shelter

Students differentiate between repeated addition and repeated multiplication scenarios, and use repeated multiplication to represent patterns of repeated growth.

Activity 1 Fundraising Outreach for the Animal Shelter

You want to raise money for the local animal shelter. Your first step is outreach — spreading the word to as many people as you can. You create three outreach options:

- **Option 1:** Post one message on social media that will instantly reach 50,000 people.
- **Option 2:** Email 2 different people each day for 28 days.
- **Option 3:** Start a chain email, in which each person forwards your email to 2 new people. On the first day, you email 2 people. On the second day, those 2 people both forward your email to 2 new people, reaching a total of 4 people. This will continue for 28 days.

Plan ahead: How does conducting a fundraiser demonstrate empathy for others?

1. Which outreach option would you choose? Explain your thinking.

   Answers may vary, but should include:
   - Option 3 (the chain email) is the best offer, followed by Option 1 (the social media post), and then Option 2 (2 people per day).
   - 2 people per day for 28 days is repeated addition, or $2 \cdot 28 = 56$.
   - The chain email doubles the number of people reached each day and is repeated multiplication ($2 \cdot 2 \cdot 2 \cdot 2 \ldots$). The chain email reaches more than 50,000 new people by day 16.

2. Priya claims you can use the expression $2 + 2 + 2 + 2 + 2$ to determine the total number of people who have received the chain email on the fifth day. Do you agree or disagree? Explain your thinking.

   Answers may vary, but should include that students disagree because:
   - Repeated addition represents the second outreach option—emailing 2 people each day.
   - The chain email doubles the number of new people who receive the email each day.
   - For the chain email, you can represent the number of new people using repeated multiplication: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

3. Write an expression to represent the number of new people the chain email will reach on Day 28. Do not evaluate. Explain your thinking.

   2^{28}; Sample response: 2^{28} represents the number of new people who receive the email on Day 28. This means you repeatedly multiply 2 by itself a total of 28 times.

688 Unit 6 Expressions and Equations

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them draw diagrams representing the first few days for each option. Ask, "How are they different? Why?"

Look for points of confusion:

- **Choosing Option 1 because it is such a large number.** Have students compute the first 15 days for each option.
- **Representing the chain email with repeated addition or $28 \cdot 2$.** Say, "This means you email 2 new people every day. How is the chain email different, and how can your expression show this?

Look for productive strategies:

- Representing the chain email using repeated multiplication, and computing enough days to determine that it is the best option. Some students may represent the chain email using exponents.

3 Connect

Display one problem at a time.

Have students share how they chose the best option, focusing on how expressions represent each option. Have them describe the difference between scenarios represented by repeated addition versus repeated multiplication.

Highlight that exponents are a shorthand notation for repeated multiplication. On Day 28, the chain email will reach 2^{28} people, read as "2 to the 28th power" or "2 to the 28th." The *base*, 2, represents the repeated factor. The *exponent*, 28, is how many times the base is used as a factor.

Ask, "How can you use exponents to represent Day 150? Day 1,000? Day n?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students create and extend a table, similar to the following, to help them organize their thinking. Days 1 and 2 are shown.

Day	Option 1	Option 2	Option 3
1	50,000	2	2
2	n/a	$2 + 2$	$2 \cdot 2$

Extension: *Math Enrichment*

Have students determine on what day Option 3 becomes the best option. Ask them how many people will receive the chain email on Day 28. On Day 16, 65,536 people will receive the chain email, which surpasses 50,000 people with Option 1.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that there are three options for spreading the word about the fundraising event.
- **Read 2:** Ask students to name or highlight given quantities and relationships, such as "Option 1 will reach 50,000 people instantly."
- **Read 3:** Ask students to brainstorm strategies for how they will choose the best option.

English Learners

Highlight and clarify key phrases, such as "instantly reach," "each day," and "forward(s)" (in the context of email).

Pairs | 15 min

Activity 2 Fundraising Success?

Students represent repeated multiplication using exponents and informally use the quotient rule for exponents to evaluate expressions with large exponents.

Name: ____________ Date: ________ Period: ________

Activity 2 Fundraising Success?

The animal shelter wants to raise $50,000 in 10 days. Having seen how powerful a chain email can be, you decide to send an email to three friends, asking them each to donate $1, and then forward the email and donation request to three more friends, and so on. Assume that each friend donates the requested amount.

1. The number of dollars raised on the third day will be $3 \cdot 3 \cdot 3$.
   a. Write an equivalent expression using exponents.
   3^3
   b. Evaluate the expression. What does this value mean?
   27; It means a total donation of $27.

2. You will have raised $81 on the fourth day. Show or explain how you can use this information to calculate the amount of money raised on the fifth day.
   I can multiply $81 \cdot 3 = \$243$. I know that 3^4 means 3 multiplied repeatedly 4 times, and 3^5 means 3 multiplied repeatedly 5 times. So, 3^5 means $3^4 \cdot 3$.

3. If you have raised $2,187 on one day, what day was it? Write an expression with an exponent to help explain your thinking.
   It is day 7. I know that $3^5 = 243$, and $3^7 = 3^5 \cdot 3 \cdot 3$, or $3^7 = 3^5 \cdot 9$. So I can multiply $243 \cdot 9 = \$2{,}187$.

4. Will you reach the $50,000 fundraising goal by the tenth day? Show or explain your thinking.
   Yes; Sample response: The fundraising goal will be met by the tenth day. On the tenth day alone, I will have raised 3^{10}, or $3^7 \cdot 3^3$. Because $3^7 = 2{,}187$, I can multiply that by 3^3, or 27, to get $3^{10} = \$59{,}049$.

STOP

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "How is this scenario different from the first activity?" Have students draw diagrams representing the first few days of the fundraiser.

Look for points of confusion:

- **Evaluating 3^3 as $3 \cdot 3$.** Remind students of the connection between multiplication and repeated addition, and between exponents and repeated multiplication.
- **Not seeing 3^5 as $3^4 \cdot 3$.** Have students list the multiplication strings for 3^3, and 3^4. Ask, "Where do you see 3^3 in the string for 3^4? How could you write 3^4 using 3^3?

Look for productive strategies:

- Writing and comparing repeated multiplication strings to evaluate large exponents.
- Using the structure of exponents to decompose exponents into smaller, known parts (e.g., $3^5 = 3^4 \cdot 3$).

3 Connect

Display each problem, one at a time.

Have students share their answers, focusing on their strategies to evaluate large exponents. Begin with students who used repeated multiplication strings, followed by those who used the structure of exponents to decompose terms.

Highlight that lesser exponents can be used to evaluate expressions with greater exponents (that have the same base). For example, $3^5 = 3^4 \cdot 3^1$.

Ask, "How can you use 3^{10} to evaluate 3^{14}?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Allow students to create a table, similar to the one suggested in Activity 1, to help them visualize how the amount raised is growing. Ask them how they can use an exponent to represent the repeated multiplication in a shorter way.

Extension: *Math Enrichment*

Have students complete the following as an extension to Problem 4. If you will have raised more than 50,000 on the tenth day alone, how much money will you have raised altogether by the tenth day? 88,572; The sum of the amounts raised from the first day to the tenth day.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 4, have them meet with 2–3 partners to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "How did you use your response from Problem 3 to help you with Problem 4?"
- "Did you use exponents in your response? If not, how can you revise your response to include exponents?"

Have students write a final response, based on the feedback they received.

English Learners

Allow pairs of students who speak the same primary language to provide feedback to each other.

Whole Class | 5 min

Summary

Review and synthesize how exponents and multiplication are related, highlighting the mathematical structure of terms with exponents.

Summary

In today's lesson . . .

You saw how exponents represent repeated multiplication. When you write an expression, such as 2^n, 2 is the base, and n is the exponent. If n is a positive whole number, it tells you how many factors of 2 you should multiply to determine the value of the expression. For example, $2^1 = 2$, and $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

There are different ways to say 2^5. You can say "two raised to the power of five," "two to the fifth power," or just "two to the fifth."

Reflect:

Synthesize

Ask:

- "How can you use the value of expressions with lesser exponents to evaluate expressions with greater exponents (that have the same base)?"
- "How would an exponent affect a fraction? For example, what does this expression mean: $\left(\frac{1}{5}\right)^2$?"

Highlight that for an expression, such as 2^n, 2 is the *base* and n is the *exponent*. If n is a positive whole number, it indicates how many factors of 2 should be multiplied to determine the value of the expression.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are exponents and multiplication related?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of exponents as repeated multiplication to evaluate expressions containing exponents.

Amps Featured Activity | Real Time Exit Ticke | Printable

Name: ________________ Date: ________ Period: ________

Exit Ticket 6.14

1. Which of the following is *not* equal to the others? Show or explain your thinking.

$2^3 \cdot 2$	4^2	8^2	16^1

8^2 is not equal to the others because $8^2 = 64$, while the other expressions are all equal to 16.

2. Given the equation $5^3 = 125$, explain how to use the value of 5^3 to evaluate the expression 5^4.

$5^4 = 5^3 \cdot 5^1$, so I can multiply $125 \cdot 5$ to obtain 625. So, $5^4 = 625$.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can evaluate expressions with exponents and write expressions with exponents that are equal to a given number. 1 2 3

b. I understand the meaning of an expression with an exponent, such as 3^5. 1 2 3

Success looks like . . .

- **Language Goal:** Describing a pattern that could be expressed using repeated multiplication.
- **Goal:** Generating and evaluating numerical expressions involving whole number exponents. **(Speaking and Listening, Writing)**
  - » Evaluating all four expressions to determine which one is not equal to the others in Problem 1.
- **Language Goal:** Interpreting expressions with exponents larger than 3, and comprehend the phrase *to the power* or *to the*. **(Speaking and Listening)**
  - » Explaining how to evaluate 5^4 in Problem 2.

Suggested next steps

If students evaluate exponents as base times exponent in Problem 1, consider:

- Reviewing how to evaluate exponents from Activity 2, Problem 1b.
- Assigning Practice Problem 2.

If students claim all expressions are equal in Problem 1, consider:

- Reviewing how to evaluate exponents from Activity 2, Problem 1b.
- Assigning Practice Problem 1.

If students are unable to explain how to use 5^3 to evaluate 5^4 in Problem 2, consider:

- Reviewing how to use lesser exponents to evaluate greater ones from Activity 2.
- Asking, "How can you rewrite each of these exponents as a multiplication string to help?"
- Assigning Practice Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was to interpret, evaluate and write expressions with exponents. How did this focus on exponents go?
- How did students make use of structure and use repeated reasoning today? How are you helping students become aware of how they are progressing in this area? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ________ Date: ________ Period: ________

Practice

1. Select *all* the expressions that are equivalent to 64.
   - A. 2^6
   - B. 2^8
   - C. 4^3
   - D. 8^2
   - E. 16^4
   - F. 32^2

2. Select *all* the expressions that are equivalent to 3^4.
   - A. 7
   - B. 4^3
   - C. 12
   - D. 81
   - E. 64
   - F. 9^2
   - G. $3 + 3 + 3 + 3$
   - H. $3 \cdot 3 \cdot 3 \cdot 3$

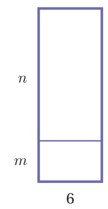
3. 4^5 is equal to $1{,}024$. Use this to evaluate each expression.
   - a $4^6 = 4{,}096$
   - b $4^4 = 256$
   - c $4^3 \cdot 4^2 = 1{,}024$

Name: ________ Date: ________ Period: ________

Practice

4. Solve each equation. Show your thinking.
   - a $a - 2.01 = 5.5$
     $a - 2.01 + 2.01 = 5.5 + 2.01$
     $a = 7.51$
   - b $b + 2.01 = 5.5$
     $b + 2.01 - 2.01 = 5.5 - 2.01$
     $b = 3.49$
   - c $10c = 13.71$
     $10c \div 10 = 13.71 \div 10$
     $c = 1.371$
   - d $100d = 13.71$
     $100d \div 100 = 13.71 \div 100$
     $d = 0.1371$

5. Which expressions represent the total area of the largest, outlined rectangle? Select *all* that apply.
   - A. $6(m + n)$
   - B. $6n + m$
   - C. $6n + 6m$
   - D. $6mn$
   - E. $(n + m) \cdot 6$



6. Using what you know about the properties of operations, rewrite each expression using a different operation. Do not evaluate.
   - a $3 + 3$
     Sample response: $2 \cdot 3$
   - b $3 \cdot 2$
     Sample response: $2 + 2 + 2$
   - c 3^2
     Sample response: $3 \cdot 3$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	3
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 8	1
	5	Unit 6 Lesson 13	2
Formative	6	Unit 6 Lesson 15	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 6 | **LESSON 15**

Evaluating Expressions With Exponents

Let's determine the value of expressions with exponents.

Focus

Goals

1. **Language Goal:** Evaluate numerical expressions that have an exponent and one other operation, and justify the process. **(Speaking and Listening)**
2. **Language Goal:** Explain that the convention is to evaluate the exponent before the other operations in an expression with no grouping symbols. **(Speaking and Listening, Writing)**
3. **Language Goal:** Justify whether numerical expressions involving whole number exponents are equal. **(Speaking and Listening, Writing)**
4. **Language Goal:** Critique arguments that claim two different numerical expressions are equal. **(Speaking and Listening, Writing)**

Coherence

Today

Students represent scenarios using expressions that contain two terms, one of which has an exponent, and they evaluate using the order of operations. Using the structure of exponents and the order of operations, students determine whether two expressions are equivalent, justifying their thinking, and critiquing the reasoning of others.

Previously

In Grade 5, students used the order of operations to evaluate expressions with parentheses, brackets, and braces. In Lesson 14, students used exponents to represent scenarios featuring repeated multiplication, and they evaluated expressions with exponents.

Coming Soon

In Lesson 16, students will evaluate expressions and equations, where the variable may be the base or exponent, by substituting given values for the variable. They will connect this process to making true equations.

Rigor

- Students build **fluency** representing real-world scenarios with expressions that include exponents.
- Students **apply** their understanding of exponents and the order of operations to solve expressions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

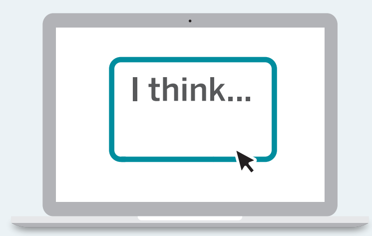
Review words

- *base*
- *coefficient*
- *equivalent*
- *exponent*
- *order of operations*
- *term*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to explain their thinking behind choosing a fundraising strategy, and these explanations are passed on to you.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

If students feel confident in their own responses, they might forget to respectfully listen to the reasoning and critiques of their partners. Have students brainstorm ways to actively listen and then guide them to see the benefits of working with a partner. Besides developing or strengthening their own mathematical arguments, students might hear something they had not considered before, and they can work through the disagreement with each other and both come out with better mathematical knowledge.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be done as a whole class.
- In **Activity 1,** determine the expressions that represent each option as a class. Then have pairs use the expressions to determine which option is best.
- In **Activity 2,** have pairs choose two rows to complete. Ensure that each row is solved by at least one pair of students.

Independent | 5 min

Warm-up Is the Equation True?

Students use the structure of exponents to determine whether equations are true.

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 15

Evaluating Expressions With Exponents

Let's determine the values of expressions with exponents.

Warm-up Is the Equation True?

Without evaluating, decide whether each equation is true or false. Be prepared to explain your thinking.

a. $2^4 = 2 \cdot 4$
False

b. $3 + 3 + 3 + 3 + 3 = 3^5$
False

c. $5^3 = 5 \cdot 5 \cdot 5$
True

d. $2^3 = 3^2$
False

e. $14^1 = 7^2$
False

Log in to Amplify Math to complete this lesson online.

Lesson 15 Evaluating Expressions With Exponents 693

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking, "What does an exponent represent? How is this different from multiplication?"

Look for points of confusion:

- **Evaluating each expression.** Have students rewrite the expression in a different form, rather than evaluate it.
- **Missing the connection between exponents and repeated multiplication.** Remind students that exponents represent repeated multiplication.

Look for productive strategies:

- Comparing by rewriting the expressions in a different form.
- Comparing the expressions using their structure, and interpreting exponents as representing repeated multiplication and multiplication as representing repeated addition.

3 Connect

Have students share their responses and reasoning, starting with those who rewrote each expression in a different form, followed by those who used the structure or meaning of exponents and operations.

Ask:

- "What change can you make to the equation $3 + 3 + 3 + 3 + 3 = 3^5$ to make it true?"
- "If I said $14^1 = 7^2$, what mistake did I make?"
- "How is using the meaning of exponents often more efficient than evaluating each expression?"

Highlight that exponents represent repeated multiplication, and multiplication represents repeated addition.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the following incorrect statement and reasoning, "The equation $2^4 = 2 \cdot 4$ is true because $2 \cdot 4 = 8$." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? If you agree, what do you think the exponent notation means? If you disagree, what was the likely misunderstanding here?"
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "How do you know that the statement is now true?"

English Learners

Allow students time to rehearse what they will say with a partner before sharing with the whole class.

Power-up

To power up students' ability to relate addition, multiplication, and exponents to create equivalent expressions, ask:

Without evaluating, use what you know about the properties of operations to rewrite each expression using a different operation.

1. $4 + 4 + 4 = 3 \cdot 4$
2. $4 \cdot 3 = 3 + 3 + 3 + 3$
3. $4^3 = 4 \cdot 4 \cdot 4$

Use: Before the Warm-up.

Informed by: Performance on Lesson 14, Practice Problem 6.

Pairs | 15 min

Activity 1 Revisiting Your Fundraising Strategy

Students reason about scenarios and write expressions that contain two terms, one of which has an exponent.

Amps Featured Activity See Student Thinking

Activity 1 Revisiting Your Fundraising Strategy

A local radio station was so impressed with your efforts to raise money for the local animal shelter, they offered to help in one of four possible ways:

- **Offer 1:** Donate $2 on the first day and then double the amount every day for 6 days. On the seventh day, they will give you an extra $100.
- **Offer 2:** Donate $2 on the first day and then double the amount every day for 9 days. On the tenth day, they will give $100 from the money they raised to a listener in a raffle.
- **Offer 3:** Donate $2 on the first day and then double the amount every day for 7 days. On the eighth day, they will triple the total amount up to that point.
- **Offer 4:** Donate $50 on the first day and and then donate $2 every day, for 20 days.

Which offer would you choose? Use expressions or equations to explain your thinking.

The best offer is the second offer.

Sample response: I can represent each offer with an expression and evaluate to show that the second offer is best:

- Offer 1: $2^6 + 100 = \$164$
- Offer 2: $2^9 - 100 = \$412$
- Offer 3: $2^7 \cdot 3 = \$384$
- Offer 4: $50 + 2 \cdot 20 = \$90$

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "How are the first three offers different from the fourth? How can you write an expression to represent each?"

Look for points of confusion:

- **Choosing an option based on the operation and second term.** Have students evaluate each offer. Ask, "Why are the outcomes so different?"
- **Representing doubling as repeated addition.** Say, "You have $3. I double it. How much do you have? Now I double that. How much do you have? What operation are you really performing?"
- **Not representing the final option as multiplication.** Say, "Is getting $2 each day the same as doubling each day?"

Look for productive strategies:

- Writing appropriate expressions with multiplication, repeated multiplication, or exponents, and evaluating. Students may use the structure of exponents to evaluate 2^7 and 2^9 efficiently.

3 Connect

Have students share their expressions, focusing on the differences in wording between the first three offers and the fourth. Then have students share their strategies for deciding the best offer, focusing on how they evaluated each expression.

Highlight that when an expression has an exponent and a second term, the order of operations says to evaluate the term with the exponent first.

Ask, "Would you evaluate $2^3 \cdot 5$ and $5 \cdot 2^3$ in the same order? Why or why not?" Yes, because you evaluate the exponent first.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students create and extend a table, similar to the following, to help them organize their thinking. Days 1 and 2 are shown. Have students annotate the days in which the growing pattern changes. For example, for Offer 1, the doubling pattern only occurs up through Day 6.

Day	Offer 1 ($)	Offer 2 ($)	Offer 3 ($)	Offer 4 ($)
1	2	2	2	50
2	$2 \cdot 2$	$2 \cdot 2$	$2 \cdot 2$	$50 + 2$

Math Language Development

MLR6: Three Reads

- **Read 1:** Students should understand that there are four different offers to choose from to help raise money for the animal shelter.
- **Read 2:** Ask students to identify quantities and relationships, such as recognizing that Offers 1–3 include doubling for different numbers of days.
- **Read 3:** Ask students to brainstorm strategies for how they will choose the best offer.

English Learners

Highlight and clarify key phrases, such as "double the amount" "every day," and "triple the total amount."

Pairs | 15 min

Activity 2 Are They Equal?

Students evaluate expressions with two terms, one of which has an exponent, to determine whether they are equivalent. They explore how parentheses affect the order of operations.

Name: ______ Date: ______ Period: ______

Activity 2 Are They Equal?

Determine whether the expressions in each row are equivalent. Be prepared to share your thinking with your partner.

Column A	Column B	Equivalent?
$5^2 + 4$	$2^2 + 25$	Yes
$2^4 \cdot 5$	$2^3 \cdot 10$	Yes
$3 \cdot 4^2$	$12 \cdot 2^1$	No
$5 + 4^3$	9^3	No
$9 \cdot 2^1$	$3 \cdot 6^1$	Yes
$(6 + 2)^2$	$6^2 + 2^2$	No

STOP

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity before sharing and comparing their answers with their partners.

2 Monitor

Help students get started by reminding them of the order of operations, and asking, "What is the first step to evaluate the expression $5^2 + 4$?"

Look for unproductive strategies:

- **Multiplying the base times the exponent.** Ask, "How is 5^2 different than $5 \cdot 2$?"
- **Multiplying or adding before evaluating the term with the exponent.** Ask, "Using the order of operations, what is your first step?"
- **Not evaluating $(6 + 2)$ first.** Ask, "What do parentheses tell you?"

Look for productive strategies:

- Evaluating expressions using the order of operations, including examples with parentheses, and recognizing they are equivalent when they have the same value.

3 Connect

Have students share their claims about whether each pair of expressions was equivalent, focusing on those for which some partners disagreed at first, and how they explained and critiqued one another to come to agreement, such as by using the order of operations or the meaning of exponents.

Highlight that exponents are evaluated first, unless there are parentheses. Parentheses indicate to perform the operation(s) inside the parentheses first and then evaluate using the exponent.

Ask, "How could you tell that the first two pairs of expressions are equivalent, without evaluating?"

Differentiated Support

Extension: *Math Enrichment*

Challenge students to write two more pairs of expressions, one in which the expressions are equivalent, and one in which the expressions are not equivalent. Have them trade their sets of expressions with another pair of students and have each pair determine whether the expressions are equivalent. Students' responses will vary.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the table. Have students work with their partner to write 2–3 mathematical questions they could ask about the expressions in the table. Sample questions shown.

- In the fourth row, I wonder if I can add 5 and 4 to get 9? Would that mean those expressions are equivalent?
- In the last row, can I distribute the exponent to each term inside the parentheses?

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question based on the table. Consider displaying one of the sample questions.

Whole Class | 5 min

Summary

Review and synthesize how parentheses affect the order of operations, noting the order when parentheses and exponents are in the same expression.

Summary

In today's lesson . . .

You explored evaluating expressions with exponents. To ensure everyone evaluates expressions, such as $6 \cdot 4^2$, in the same way, and gets the same answer, the convention is to evaluate the part of the expression with the exponent first.

$$\begin{aligned} 6 \cdot 4^2 &= 6(4 \cdot 4) \\ &= 6 \cdot 16 \\ &= 96 \end{aligned}$$

If you want to communicate that 6 and 4 should be multiplied first, and then squared, you should use parentheses to group parts together: $(6 \cdot 4)^2$.

$$\begin{aligned} (6 \cdot 4)^2 &= 24^2 \\ &= 24 \cdot 24 \\ &= 576 \end{aligned}$$

Reflect:

Synthesize

Display the expressions $6 \cdot 4^2$ and $(6 \cdot 4)^2$.

Ask:

- "How are these expressions similar?"
- "How are these expressions different?" The first means multiply 6 by 4 squared, and the second means square $6 \cdot 4$ or 24.
- "How do parentheses affect the order of operations when evaluating expressions with exponents?"

Highlight that the expressions may look very similar, but there is a big difference in how they are evaluated. In the first expression, the order of operations tells to evaluate the part of the expression with the exponent first. The value is 96. But in the second expression, the parentheses tell to perform the operations inside them first. The value here is 576.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did you use the order of operations today?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using the order of operations to evaluate expressions with two terms, one of which has an exponent. They explain how parentheses affects the order of operations.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.15

1. Evaluate the following expressions.

   a. $6 + 6^2$ **42**

   b. $3 \cdot 2^3$ **24**

2. Priya says that the expressions on both sides of the equal sign equal 64, and this equation is true. Do you agree or disagree? Explain your thinking.

$$3 + 5^2 = (3 + 5)^2$$

I disagree because $3 + 5^2 = 3 + 25 = 28$, and $(3 + 5)^2 = 8^2$. On the left side of the equation, Priya needs to evaluate 5^2 first and then add 3. On the right side of the equation, the parentheses tell Priya to add $5 + 3$ first and then square the sum.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I know how to evaluate expressions that have both an exponent and addition or subtraction. 1 2 3

b. I know how to evaluate expressions that have both an exponent and multiplication or division. 1 2 3

c. I can decide whether expressions with exponents are equal by evaluating the expressions or by understanding what exponents mean. 1 2 3

Success looks like . . .

- **Language Goal:** Evaluating numerical expressions that have an exponent and one other operation, and justify the process. **(Speaking and Listening)**
- **Language Goal:** Explaining that the convention is to evaluate the exponent before the other operations in an expression with no grouping symbols. **(Speaking and Listening, Writing)**
- **Language Goal:** Justifying whether numerical expressions involving whole number exponents are equal. **(Speaking and Listening, Writing)**
- **Language Goal:** Critiquing arguments that claim two different numerical expressions are equal. **(Speaking and Listening, Writing)**
  - » Explaining why Priya's evaluation of one of the expressions is incorrect in Problem 2.

Suggested next steps

If students do not evaluate the exponents first in Problem 1, consider:

- Reviewing the order of operations from Activity 2.
- Assigning Practice Problem 1.

If students evaluate exponents by multiplying the base by the exponent in Problem 1, consider:

- Reviewing how to evaluate exponents from Activity 1 or Lesson 14, Activities 1 and 2.
- Asking, "How is 6^2 different than $6 \cdot 2$?"

If students agree with Priya for Problem 2, consider:

- Reviewing the order of operations from Activity 2, focusing on how the parentheses affect the order.
- Assigning Practice Problems 2 and 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- One instructional goal for this lesson was to evaluate numerical expressions that have an exponent and one other operation. How well did students accomplish this? What did you specifically do to help students accomplish it?
- What different ways did students approach Activity 1? What does that tell you about similarities and differences among your students?

Practice

Independent

Name: ______________________ Date: __________ Period: __________

Practice

1. Evaluate each expression. Show your thinking.

   a. $7 + 2^3 = 15$

   b. $9 \cdot 3^1 = 27$

   c. $20 - 2^4 = 4$

   d. $2 \cdot 6^2 = 72$

   e. $8 \cdot \left(\frac{1}{2}\right)^2 = 2$

   f. $\left(\frac{1}{5} \cdot 5\right)^5 = 1$

2. Lin says, "I took the number 8, and then multiplied it by the square of 3." Select *all* the expressions that are equal to Lin's result.

   A. $8 \cdot 3^2$
   B. $(8 \cdot 3)^2$
   C. $8 \cdot 2^3$
   D. $3^2 \cdot 8$
   E. 24^2
   F. 72

Name: ______________________ Date: __________ Period: __________

Practice

3. Andre says, "I multiplied 4 by 5 and then cubed the result." Select *all* the expressions that are equal to Andre's result.

   A. $4 \cdot 5^3$
   B. $(4 \cdot 5)^3$
   C. $(4 \cdot 5)^2$
   D. $5^3 \cdot 4$
   E. 20^3
   F. 500
   G. $8{,}000$

4. Solve each problem. Show or explain your thinking.

   a. 125% of e is 30. What is e?

      24; $30 \cdot \frac{100}{125} = 24$

   b. 35% of f is 14. What is f?

      40; $14 \cdot \frac{100}{35} = 40$

5. Which expressions have values that are solutions to the equation $2.4y = 13.75$? Select *all* that apply.

   A. $13.75 - 2.4$
   B. $13.75 \cdot 2.4$
   C. $13.75 \div 2.4$
   D. $\frac{13.75}{2.4}$
   E. $2.4 \div 13.75$

6. Evaluate each expression when $x = 2$.

   a. $3 + x$

      $3 + x = 3 + (2) = 5$

   b. $3 \cdot x$

      $3 \cdot x = 3 \cdot (2) = 6$

   c. x^3

      $x^3 = (2)^3 = 8$

   d. 3^x

      $3^x = 3^{(2)} = 9$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 9	1
	5	Unit 6 Lesson 8	2
Formative	6	Unit 6 Lesson 15	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 6 | LESSON 16

Analyzing Exponential Expressions and Equations

Let's investigate expressions and equations with variables and exponents.

Focus

Goals

1. **Language Goal:** Describe the values that result from evaluating expressions in which a fraction is raised to a power. **(Speaking and Listening)**
2. Determine whether a given value is a solution to an equation that includes an exponent.
3. Evaluate expressions that have a variable, an exponent, and one other operation for a given value of the variable, carrying out the operations in the conventional order.

Coherence

Today

Students continue to work with expressions with exponents and a variable, which can be either the base or the exponent. They practice using precise language related to exponents and order of operations in trying to identify mystery expressions. Students also connect the concept of equality from this unit to equations with exponent terms, by applying their understanding of exponents and operations to identify a solution from a given list of values.

Previously

In Lessons 14–15, students saw how exponents can represent repeated multiplication. They evaluated expressions with exponents and created equivalent expressions with exponents.

Coming Soon

Students will continue their work with exponents in Grade 8 when they apply the properties of integer exponents to generate equivalent numerical expressions.

Rigor

- Students build **conceptual understanding** of expressions with exponents.
- Students build **fluency** evaluating expressions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Whole Class	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- Activity 1 PDF (answers)

Math Language Development

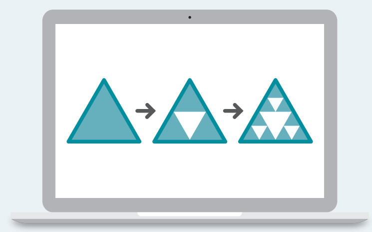
Review words

- *base*
- *coefficient*
- *exponent*
- *solution to an equation*
- *variable*

Amps | Featured Activity

Warm-up
Animated Sierpiński Triangle

An animation of the Sierpiński Triangle is presented. Students follow along by completing a table showing what is happening in the animation.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may not treat each other kindly as they ask each other questions during Activity 1. Beyond the need to use precise mathematical language as they discuss expressions with exponents, students must also consider the social aspect of the activity. Remind students that they must treat each other with respect. They can check their language and their tone if they consider how they would feel if someone else talked to them in that way.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- For **Activity 1,** reduce the number of expression cards and give students 3^x and $\frac{x^3}{4}$.
- In **Activity 2,** problems can be assigned to small groups or pairs and then answers can be shared as a whole class.

Whole Class | 10 min

Warm-up The Sierpiński Triangle

Using an animation of the Sierpiński triangle, students look for patterns in expressions using variables as the whole number and exponent.

Amps Featured Activity Animated Sierpiński Triangle

Name: ______________ Date: ________ Period: ________

Unit 6 | Lesson 16

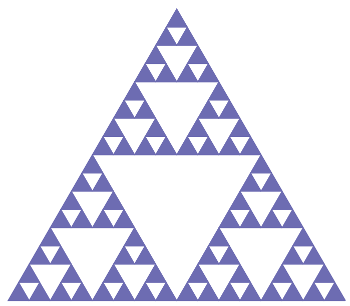
Analyzing Exponential Expressions and Equations

Let's investigate expressions and equations with variables and exponents.

Warm-up The Sierpiński Triangle

Complete the column in the table for x^3 using the different values of x. Then, as you watch the animation, follow along to complete the column for 3^x, which represents the number of triangles in the animation. What do you notice about the two columns?

Sample response: They do not result in the same answers, with the exception of 3^3. The first column's answers are not related, but the second column's answers are multiples of 3.



x	x^3	3^x
1	$1^3 = 1 \cdot 1 \cdot 1 = 1$	$3^1 = 3$
2	$2^3 = 2 \cdot 2 \cdot 2 = 8$	$3^2 = 3 \cdot 3 = 9$
3	$3^3 = 3 \cdot 3 \cdot 3 = 27$	$3^3 = 3 \cdot 3 \cdot 3 = 27$
4	$4^3 = 4 \cdot 4 \cdot 4 = 64$	$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

Log in to Amplify Math to complete this lesson online.

Lesson 16 Analyzing Exponential Expressions and Equations 699

1 Launch

Give students 2 minutes to complete the first column, then show the animation. Have students complete the second column as they watch the animation.

2 Monitor

Help students get started by asking "What would you write if you substituted 1 in for the x? What does that mean?"

Look for points of confusion:

- **Not focusing on only the shaded triangles.** Pause the animation as needed and ask, "How many shaded triangles are there now? Where did the new ones come from?"

Look for productive strategies:

- Recording all three parts to the solution of the problem: the expression, the work, and the solution. Students can return later to complete any part that they did not record to show the thinking behind the procedure.

3 Connect

Display the table and the solutions.

Have students share what they noticed about the expressions they wrote.

Highlight how x and 3 changed positions — exponent and base — and the effect the change has on the solutions.

Ask, "What patterns do you notice in the table and why do you think they happened?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share what they noticed about the expressions they wrote in the table, highlight connections between the value of the base and exponent, and the value of each factor and number of factors in the expressions. Show that, for example, the expression 4^3 repeatedly multiplies 4 by itself 3 times. The expression 3^4 repeatedly multiplies 3 by itself 4 times.

English Learners

Annotate the expressions with the terms *base, exponent,* and *factor.*

Power-up

To power up students' ability to evaluate algebraic expressions by substituting a given value, ask:

Recall that to evaluate an expression for a given value, you must first replace the variable in the expression with the given value, then evaluate the resulting expression.

Evaluate each expression when $x = 3$

1. $x \cdot 2 = (3) \cdot 2 = 6$
2. $x^2 = (3)^2 = 9$
3. $2^x = 2^3 = 8$

Use: Before the Warm-up.

Informed by: Performance on Lesson 15, Practice Problem 6.

Activity 1 Twenty Questions: Evaluating Expressions With Variables and Exponents

Pairs | 15 min

Students work in pairs to evaluate expressions with variables and exponents, focusing on the order of operations.

Activity 1 Twenty Questions: Evaluating Expressions With Variables and Exponents

You will given two sets of cards – Expression Cards and Value Cards. With your partner, determine who will be Partner A and Partner B. Partner A starts with an expression card, and Partner B starts with a value card. Do not show or read your card to your partner. The partner with the value card will ask questions to determine the expression. Then, if possible, evaluate the expression together by substituting the information from the value card. Responses may vary.

Information gathered	Expression	Solution		
		Value card:		
		Value card:		
		Value card:		
		Value card:		
		Value card:		
		Value card:		
		Value card:		

Sample Questions Bank:
What is the coefficient of the variable term?
Is something added to the variable?
Is the variable an exponent, a base, or neither?

1 Launch

Distribute the **Activity 1 PDF** pre-cut cards to each pair in two piles; Expression Cards and Value Cards. Have one student (Partner A) in each pair chose an Expression Card and the other student (Partner B) chose a Value Card. Explain that Partner B should ask Partner A questions to determine what expression is on their card. Then they should work together to substitute and evaluate for the value on Partner B's card, when possible. Once they have finished one pair of cards, they should shuffle the cards, swap roles, and repeat the process.

2 Monitor

Help students get started by suggesting they choose a question from the Sample Questions Bank at the bottom of the page in the Student Edition.

Look for points of confusion:

- **Thinking that the questions in the Sample Questions Bank are the only questions they can ask.** Ask, "If I had the expression $6x + 1$ what features does it have? What questions could you ask to discover similar features in another expression?"

Look for productive strategies:

- Asking questions using explicit and precise mathematical language related to exponents and operations.

3 Connect

Display the table from Activity 1.

Have pairs of students share combinations of expressions and values they worked with, and how they evaluated the expressions.

Highlight that, if the variable is "by itself," for example $9 + x^2$, its coefficient is 1.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students do one trial round, where both partners can see the Expression card and have them discuss what series of questions they could ask to identify the expression using as few questions as possible.

Refer them to the Sample Questions Bank at the bottom of the activity. As students progress through the activity, they may want to revise their questions as they think of more efficient approaches, or add more questions to their list.

Independent | 10 min

Activity 2 Experimenting With Exponents in Equations

Students recall what is meant by a solution to an equation as they replace variables with values to make two expressions equal.

Name: ______ Date: ______ Period: ______

Activity 2 Experimenting With Exponents in Equations

Determine a solution to each equation from the list of numbers. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

2	4	5	$\frac{4}{3}$	6	8	$\frac{8}{125}$	3	$\frac{8}{9}$

1. $64 = x^2$
   $x = 8$
2. $64 = 4^x$
   $x = 3$
3. $2^x = 32$
   $x = 5$
4. $x = \left(\frac{2}{5}\right)^3$
   $x = \frac{8}{125}$
5. $\frac{16}{9} = x^2$
   $x = \frac{4}{3}$
6. $2 \cdot 2^5 = 2^x$
   $x = 6$
7. $2x = 2^4$
   $x = 8$
8. $4^3 = 8^x$
   $x = 2$

Critique and Correct: Describe and correct any errors in this statement: "The solution to the equation $x = \left(\frac{2}{5}\right)^3$ is $\frac{6}{15}$ because $\left(\frac{2}{5}\right)^3$ is equal to $\frac{2}{5} \cdot 3$."

STOP

1 Launch

Say, "Now you will solve equations with variables and exponents independently." Use $x^2 = 100$ as a model to activate background knowledge. Discuss why 10, not 50, would be the solution. Give students 10 minutes of work time.

2 Monitor

Help students get started by asking, "Which number from the choices can you try first?"

Look for points of confusion:

- **Evaluating exponents by multiplying the base by the value of the exponent.** Remind students of what the exponent indicates. Refer back to the equation modeled in the Launch section.
- **Incorrectly solving an equation due to an incorrect computation.** Have students check their work.

Look for productive strategies:

- Showing how they solved each problem. Students can show their thinking by analyzing the structure of the equations and not just guessing the answers.

3 Connect

Display the equations, all at once, or one at a time.

Have students share their solutions and how they solved each equation.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge, Guide Processing and Visualization***

Chunk this task into smaller, more manageable parts by providing students with a subset of equations to solve first, such as the first four equations. In doing so, provide them with a subset of possible solutions.

Extension: ***Math Enrichment***

For any number in the list that students did not determine was a solution to the given equations, challenge students to write an equation for which that number is a solution.

Sample responses: $2^x = 16$; $x = 4$ and $x^2 = \frac{64}{81}$; $x = \frac{8}{9}$

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, ask students to critique and correct the statement shown in their Student Edition, "The solution to the equation $x = \left(\frac{2}{5}\right)^3$ is $\frac{6}{15}$ because $\left(\frac{2}{5}\right)^3$ is equal to $\frac{2}{5} \cdot 3$." Ask:

- **Critique:** "Do you agree with this reasoning? Why or why not?" Listen for students who recognize the difference between multiplication by 3 and an exponent of 3.
- **Correct:** "Write a corrected statement, including correct reasoning."
- **Clarify:** "Could you write a correct statement in which the solution to the equation is $\frac{6}{15}$? How would you alter the equation?" Remove the exponent and multiply the fraction $\frac{2}{5}$ by 3.

Summary

Review and synthesize how the order of operations is used when solving equations with exponents.

Summary

In today's lesson . . .

You saw expressions with exponents that had a variable as either the base or the exponent. You evaluated these expressions using the *order of operations,* which meant evaluating the exponent term before any other operations outside of any parentheses. You also used this order of operations when evaluating the expressions on one or both sides of an equation with a variable, such as x. This allowed you to determine a *solution* — what value of x made the equation true. For example, to evaluate the expression $2x^3$ when x is 5, you replace the letter x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$ or 250.

Reflect:

Synthesize

Ask:

- "How did you use the order of operations when evaluating expressions with exponents?"
- "What did you notice about a number less than 1 raised to a power, compared to a number greater than 1 raised to a power?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Where does an exponent fall in the order of operations"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving equations with exponents.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.16

Match each equation with its solution.

a. $9 + x^4 = 90$

b. $\frac{x^3}{4} = \frac{1}{32}$

c. $3 \cdot (3^4) = 3^x$

d. $\frac{16}{25} = x^2$

b $\frac{1}{2}$

d $\frac{4}{5}$

c 5

a 3

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can replace a variable with a number in an expression with exponents and operations and use the correct order to evaluate the expression.
1 2 3

b. I can find solutions to equations with exponents using a list of numbers.
1 2 3

Lesson 16 Analyzing Exponential Expressions and Equations

Success looks like . . .

- **Language Goal:** Describing the values that result from evaluating expressions in which a fraction is raised to a power. **(Speaking and Listening)**
- **Goal:** Determining whether a given value is a solution to an equation that includes an exponent.
- **Goal:** Evaluating expressions that have a variable, an exponent, and one other operation for a given value of the variable, carrying out the operations in the conventional order.
  - » Evaluating the equations in order to determine their solutions.

Suggested next steps

If students make no attempt at solving part a, consider:

- Asking, "Would it make sense that x is a fraction?"

If students answer incorrectly due to an error in procedure (order of operations), consider:

- Referring back to Activity 1 and the expression $9 + x^7$. Ask, "How did you evaluate that expression?"

If students answer incorrectly due to the miscalculation of a fraction in part b and/or d, consider:

- Reviewing how to multiply and divide fractions.

If students make no attempt at solving part c and/or d, consider:

- Asking, "Would it make sense that x is a fraction?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did the Sirepinski animation set students up to see patterns in expressions?
- Have you changed any ideas you used to have about evaluating expressions as a result of today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Evaluate each expression for the given value of each variable.

   a. $2 + x^3$, when x is 3
   $2 + x^3 = 2 + (3)^3 = 2 + 27 = 29$

   b. x^2, when x is $\frac{1}{2}$
   $x^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

   c. $3x^2 + y$, when x is 5 and y is 3
   $3x^2 + y = 3(5)^2 + 3 = 75 + 3 = 78$

   d. $10y + x^2$, when x is 6 and y is 4
   $10y + x^2 = 10(4) + (6)^2 = 40 + 36 = 76$

2. Decide whether the expressions have the same value. If not, determine which expression has the greater value.

   a. 2^3 and 3^2
   8 and 9, so 3^2 is greater

   b. 1^{31} and 31^1
   1 and 31, so 31^1 is greater

   c. 4^2 and 2^4
   16 and 16, so they have the same value

   d. $\left(\frac{1}{2}\right)^3$ and $\left(\frac{1}{3}\right)^2$
   $\frac{1}{8}$ and $\frac{1}{9}$, so $\left(\frac{1}{2}\right)^3$ is greater

3. Match each equation with its solution.

   a. $7 + x^2 = 16$ ...... d ...... $x = 1$

   b. $5 - x^2 = 1$ ...... b ...... $x = 2$

   c. $2 \cdot 2^3 = 2^x$ ...... a ...... $x = 3$

   d. $\frac{3^4}{3^x} = 27$ ...... c ...... $x = 4$

Name: ______ Date: ______ Period: ______

Practice

4. An adult pass at the amusement park costs 1.6 times as much as a child's pass.

   a. Complete the table.

Child's pass ($)	5	10	w
Adult pass ($)	8	16	$1.6w$

   b. If a child's pass costs $15, how much does an adult pass cost?
   $24

5. Jada reads 5 pages every 20 minutes. At this rate, how many pages can she read in 1 hour? Show your work using the double number line and the table.
   60

   Double number line:

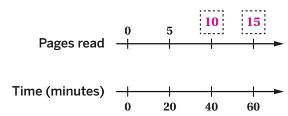


   Table:

Pages read	Time (minutes)
5	20
10	40
15	60

   Which strategy do you think is better, and why?
   Sample response: I think the double number line is easier because I can better see the relationship between pages read and minutes when they are stacked on top of one another.

6. Han works at a grocery store. Last week, he worked 8 hours and earned 84.

   a. Han always earns the same amount per hour worked. Determine the missing values in the table.

Hours	8	2	10
Pay ($)	84	21	105

   b. What is the relationship between the number of hours worked and Han's pay?
   Sample response: Han earns 10.50 for every hour he works.

Practice Problem Analysis

Type	Problems	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	1
Spiral	3	Unit 6 Lesson 2	2
	5	Unit 2 Lesson 14	2
Formative	4	Unit 6 Lesson 17	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

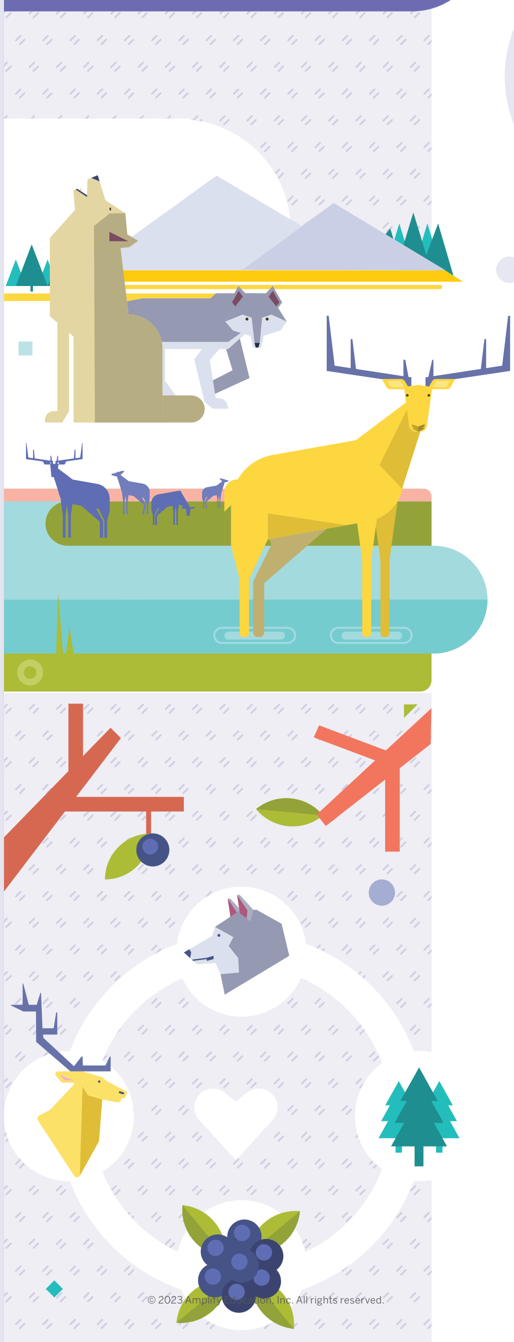
Sub-Unit 3
Relationships Between Quantities

Whole Class

In this Sub-Unit, students combine some ideas introduced earlier in the year with new ideas from earlier in this unit – ratio relationships and their graphs from Unit 2, and equations involving variables from this unit. They identify independent and dependent variables as they write equations, graph them, and solve problems.

SUB-UNIT

3 Relationships Between Quantities



Narrative Connections

What's more dangerous: a pack of wolves or a gang of elk?

Let's go back to the early 20th century, when the U.S. government allowed hunting and killing of "undesirable predators," namely wolves, in its national parks. So many wolves were killed, that, by 1926, there wasn't a single one left in Yellowstone National Park.

So, where do elk come in?

As it turns out, the elk's greatest predator was the gray wolf. With the wolves gone, the elk were free to multiply. And multiply. And multiply. Soon, Yellowstone had more elk than you could shake an antler at.

The elk ate everything in sight — grass, shrubs, and saplings. This meant no grazeland for bison, no berries for grizzlies, no trees for beavers or birds. The whole ecosystem was out of whack.

Finally, in 1995, the National Parks Service reintroduced wolves back into the park. Since then, the park has seen remarkable changes. Willow, aspen, and cottonwood trees have grown back, and animal populations are rebounding.

A healthy ecosystem is a balanced ecosystem — its different pieces pushing and pulling on each other: the predators, the prey, the fruiting plants, the shade-giving trees . . . When we understand how these pieces work together, we can make meaningful choices that support the system — deciding what to plant, what to nourish, and what species to protect.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to analyze balance and equivalence in ecological settings in the following places:

- **Lesson 17, Activity 2:** Bear Populations in Yellowstone National Park
- **Lesson 18, Activity 1:** Reintroduction of the Grey Wolf

Two Related Quantities (Part 1)

Let's use equations and graphs to describe relationships with ratios.

Focus

Goals

1. **Language Goal:** Compare and contrast graphs and equations that represent a relationship between the same quantities, but have the independent and dependent variables switched. **(Speaking and Listening)**
2. **Language Goal:** Comprehend the terms *independent variable* and *dependent variable*. **(Speaking and Listening, Writing)**
3. Create a table, graph, and equation to represent the relationship between quantities in a set of equivalent ratios.

Coherence

Today

Students revisit and combine their understanding of both equivalent ratios and algebraic equations. The new vocabulary terms of *independent* and *dependent variables* are introduced. A scenario involving bear populations in Yellowstone National Park provides the context for writing equations that represent ratio relationships between two quantities. Students relate these equations to tables of values and graphs, all showing how changes in one quantity affect changes in the other.

Previously

In Grade 5, students identified relationships between two quantities, formed ordered pairs and graphed the ordered pairs on a coordinate plane. In Units 2 and 3 of this grade, students represented a set of equivalent ratios, including unit rates, for a relationship between two quantities using tables and graphs of points. Throughout the earlier lessons in this unit, students reasoned and solved one-variable equations.

Coming Soon

In Lesson 18, students will continue to explore ratio relationships using tables, graphs, and equations. Students will study proportional relationships in more depth in Grade 7.

Rigor

- Students build **conceptual understanding**, connecting equivalent ratios to equivalent equations.
- Students **apply** their understanding of equivalent ratios and algebraic expressions to bear populations in Yellowstone National Park.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- list of resource websites

Math Language Development

New words

- ***independent variable***
- ***dependent variable***

Review words

- *equivalent ratios*
- *ratio*

Amps | **Featured Activity**

Activity 2
Interactive Graphs

Given a table, students create equations to represent the table and then plot the points on a coordinate plane. You can overlay student responses to give immediate feedback.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students will build three models that capture exactly the same information. If they struggle to identify the critical information for each scenario, ask them to highlight or underline what they need to complete a model. As they work through the different models, they might have to make assumptions and/or adjustments. Help them to understand that this is normal, as long as the changes are rooted in the mathematics of the problem.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 2,** Problem 5 may be omitted.

Independent | 5 min

Warm-up Increases and Decreases

Students think about pairs of real-world quantities that might be related, where one increases or decreases as a result of changes in the other.

Unit 6 | Lesson 17

Two Related Quantities (Part 1)

Let's use equations and graphs to describe relationships with ratios.

Warm-up Increases and Decreases

1. What are two real-world quantities you think might increase together or decrease together? In other words, as the amount of one goes up, the amount of the other also goes up; or both amounts go down.
   Sample response: Flower and honey bee populations might increase together.

2. What are two real-world quantities you think might increase and decrease opposite of one another? In other words, as the amount of one goes up, the amount of the other goes down.
   Sample response: As the number of people who get flu shots increases, the number of people who get the flu may decrease.

Log in to Amplify Math to complete this lesson online.

706 Unit 6 Expressions and Equations

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Look for points of confusion:

- **Not understanding what the questions are asking.** Suggest that students try to think of quantities for which there is an actual cause-and-effect relationship.

Look for productive strategies:

- Identifying one quantity as directly affecting the other. Do not introduce the terms *independent variable* and *dependent variable* here, as they will be defined in Activity 1.

3 Connect

Have students share their responses, focusing first on relationships that represent a correlation between quantities, and then those that more likely represent causation.

Note: Students are not expected to know the terms *correlation* and *causation* in this grade.

Highlight that the way in which a relationship is presented or described can provide important mathematical information, as will be explored in the rest of the lesson; but sometimes it simply represents a choice.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses, provide time for classmates to decide whether they agree or disagree with the real-world quantities described. Display the following sentence frames to support their thought organization.

- "I agree because . . ."
- "I disagree because . . ."
- "I think that ___ might increase, while ___ decreases because . . ."
- "I think that ___ might increase, while ___ increases because . . ."

Power-up

To power up students' ability to describe a relationship between two quantities, have students complete:

Jada's father put 12 gallons of gas in the car on Monday and paid $36.

a. The price per gallon of gas remains the same all day. Determine the missing values in the table.

Gallons of Gas	12	3	4
Total Price ($)	36	9	12

b. What is the relationship between the amount of gas purchased, in gallons, on Monday and the total price.
Sample response: The total price of gas is $3 per gallon.

Use: Before the Warm-up.
Informed by: Performance on Lesson 16, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2.

Independent | 10 min

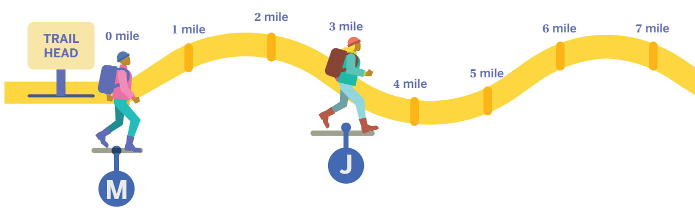
Activity 1 Introducing Equations With Two Variables

Students are introduced to independent and dependent variables. They identify the independent and dependent variables in an equation.

Name: ______ Date: ______ Period: ______

Activity 1 Introducing Equations With Two Variables

Jada and Mai hiked the same trail in Yellowstone National Park. They walked at the same speed, but started at different locations; Jada started at mile marker 3, while Mai started at the beginning of the trail (mile marker 0).



1. If Jada's location on the trail is represented by J, and Mai's location on the trail is represented by M, what are two equations that can represent this relationship?

   Equation 1: $J =$ $M + 3$

   Equation 2: $M =$ $J - 3$

2. Complete these sentences. Be prepared to explain your thinking.

   a. In Equation 1, you need to know the value of M to determine the value of J, so J is called the dependent variable. M is called the independent variable because it is used to calculate the value of J.

   b. In Equation 2, which is the ***independent variable*** — the variable you use to perform the calculation? J

      Which is the ***dependent variable*** — the variable you determine after performing the calculation? M

Are you ready for more?

If you did not know the mile marker where Jada started hiking, the equation could be written as $M + s = J$, where s represents where Jada started. Solve the equation to determine where Jada started, if she was at mile marker 15 when Mai was at mile marker 7.

$7 + s = 15$, which means $s = 8$

1 Launch

Lead a brief discussion on the meaning of the words *dependent* and *independent* in general. (Something that is *dependent* needs help from something else. Something that is *independent* can stand on its own.) Then define the mathematical terms ***independent variable*** and ***dependent variable***. Give students 5 minutes of quiet work time.

2 Monitor

Help students get started by asking, "Which variable needs the other?"

Look for points of confusion:

- **Confusing *dependent* and *independent*.** Ask, "Which variable needs the other? Meaning, it is dependent upon the other variable before it can be found."

Look for productive strategies:

- Using precise language to correctly identify and reason which variable is dependent, and which is independent.

3 Connect

Define the ***independent variable*** in an equation as the variable used to perform the calculation, and the ***dependent variable*** as the variable whose value is found after performing the calculation.

Display the scenario to the whole class.

Have students share the two equations representing the scenario first, and then identify which variable is the independent variable and which is the dependent variable in each equation.

Highlight that $J = M + 3$ and $M = J - 3$ are equivalent equations.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Use the map to orient students more concretely to the relationship between Jada's and Mai's locations. For example, annotate the map to show how starting with Mai's location and adding 3 more miles shows where Jada is located.

Math Language Development

MLR8: Discussion Supports

During the Connect, draw students' attention to the terms *independent variable* and *dependent variable*. They may be familiar with the terms *independent* and *dependent* from experiences outside of math class. Ask:

- "Where have you heard the term *independent* or *independence* before? What does it mean in that context?"
- "Where have you heard the term *dependent* or *dependence* before? What does it mean in that context?"
- "How can these everyday meanings of these terms help you understand what they mean in mathematics?

Pairs | 20 min

Activity 2 Bear Populations in Yellowstone National Park

Students will solve a table involving ratios and then extend their thinking to representing the ratios in equations and graphs.

Amps Featured Activity Interactive Graphs

Activity 2 Bear Populations In Yellowstone National Park

Did you know that the Greater Yellowstone Ecosystem is one of only a few areas in North America where both black bears and grizzly bears coexist? Their current populations are approximately 600 black bears and 700 grizzly bears.

1. Assuming the ratio of black bears to grizzly bears is always the same, complete the table.

Black bears, (b)	Grizzly bears, (g)	Total bears, (t)
6	7	13
18	21	39
24	28	52
42	49	91
48	56	104
60	70	130
600	700	1,300
900	1,050	1,950

2. Refer to the table.
   a. Write a fraction that represents the ratio of black bears to total bears.
      $\frac{6}{13}$
   b. Write an equation that represents the relationship between the number of black bears b as the dependent variable and the total number of bears t as the independent variable.
      $b = \frac{6}{13}t$
3. Write an equation that will always describe the relationship between b and g, where g is the independent variable.
   $b = \frac{6}{7} \cdot g$
4. Write an equation that will always describe the relationship between b and g, where b is the independent variable.
   $g = \frac{7}{6} \cdot b$

708 Unit 6 Expressions and Equations

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by activating prior knowledge. Ask, "What do you notice happened from the first row to the second row of the b column?" Can that relationship be carried through to the g and t columns?" This is assuming t in the first row is completed (if not, ask students to do so).

Look for points of confusion:

- **Identifying the relationship as additive instead of multiplicative.** Have students complete the first column and ask, "Does addition result in 48? What other operation could you have used to go from 6 to 18?"
- **Struggling to identify the fraction in Problem 2.** "Ask, If you think of the fraction bar as "out of," how many black bears are there out of the total bears? Where in the table can you find this information?"
- **Writing an equation (e.g. $13 - b = g$), but not the equivalent equation that always shows the relationship (Problems 3 and 4).** Ask, "Would this equation work for the second row in the table?"

Look for productive strategies:

- Recognizing that all the values in a row are the same multiples of all the values in another row.
- Understanding that the equations for Problems 3 and 4 are equivalent.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can plot the points from an equation on a coordinate plane. You can overlay them to see similarities and differences and provide immediate feedback.

Extension: *Math Enrichment*

After students complete Problem 5, provide them with a blank graph so that the horizontal axis is labeled Black bears, b and the vertical axis is labeled Grizzly bears, g. Have them graph both equations $b = \frac{6}{7}g$ and $g = \frac{7}{6}b$ on the same graph, using different colors for each equation, and describe what they notice.

Math Language Development

MLR7: Compare and Connect

Use this routine to help students understand the connections between the equations in Problems 3 and 4. Ask, "What is similar about these two equations? What is different?" Listen for and amplify student responses that recognize:

- These two equations are of the form: independent variable = [fraction] • dependent variable
- The fractions are reciprocals of each other. Remind students of the meaning of the term reciprocal.

English Learners

Annotate each equation to illustrate which variable is considered the independent variable and which variable is considered the dependent variable, based on the structure of the equation.

Pairs | 20 min

Activity 2 Bear Populations in Yellowstone National Park (continued)

Students will solve a table involving ratios and then extend their thinking to representing the ratios in equations and graphs.

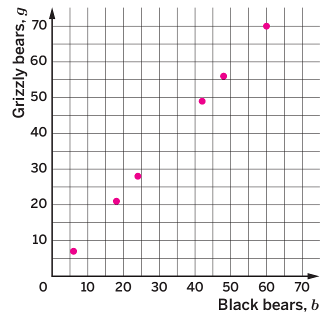
Name: ____________ Date: ________ Period: ________

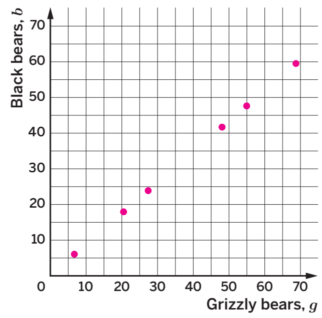
Activity 2 Bear Populations In Yellowstone National Park (continued)

Visual mathematical representations, such as graphs, that model the relationship between two quantities are important tools. They can aid in understanding relationships more deeply but also make it easier to show and explain them to a wider audience. Not all relationships are simple, or happy. For instance Rae Wynn-Grant has studied and modeled many aspects of the Greater Yellowstone Ecosystem, including the mortality risk of bears living in areas near human populations.

Even with the same values, a graph can show different things, depending on choices a researcher makes. You will see a small example of this as you complete Problem 5.

5. Use the points in the table to create two graphs that show the relationship between b and g. Match each relationship to one of the equations you wrote.





Featured Mathematician

Rae Wynn-Grant

Wildlife ecologist Rae Wynn-Grant was born in California and holds bachelor's and master's degrees in Environmental Studies and a PhD in Ecology and Evolution. She is a Research Faculty member at the Bren School of Environmental Science and Management where she conducts research on carnivore behavior and ecology. She has also studied and published work focused on the Greater Yellowstone Ecosystem and carnivores there, such as wolves and bears.

STOP

3 Connect

Display the incomplete table to the class, which will be completed as students share their responses.

Have students share their responses and explain their thinking, focusing on how their equations show the relationship between black bears and grizzly bears.

Highlight that all three representations, — tables, equations, and graphs, — capture the same information but in a different way.

Ask:

- "Why is it possible to write two different equations to describe the same situation?"
- "How do you know the equations in Problems 3 and 4 are correct?"
- "What do you notice about the numbers that are multiplied by the independent variable in each equation?"
- "What similarities and differences are there between the two graphs?"

Featured Mathematician

Rae Wynn-Grant

Have students read about Rae Wynn-Grant who is a wildlife ecologist conducting research on carnivore behavior and ecology.

Whole Class | 5 min

Summary

Review and synthesize how various ways of representing ratio relationships – tables, equations, and graphs – are connected to independent and dependent variables.

Summary

In today's lesson . . .

You revisited representing equivalent ratios with tables, and connected those tables to writing equations and graphing the points. You did this with the populations of black bears and grizzly bears in Yellowstone National Park.

In an equation, the ***dependent variable*** is the one resulting from a calculation, and the ***independent variable*** is the one used to calculate the value of the dependent variable.

When you previously studied equivalent ratios, you used double number lines and tables. Graphs and equations provide other tools for working with equivalent ratios.

Reflect:

710 **Unit 6** Expressions and Equations

Synthesize

Display the table, graphs, and equations during the discussion.

Highlight that this lesson revisited equivalent ratios, but, now, students can write equations to represent sets of equivalent ratios and their graphs.

Formalize vocabulary:

- *dependent variable*
- *independent variable*

Ask:

- "How do you know which quantity to choose as the independent variable when you write an equation to describe a scenario?"
- "How can you be sure an equation shows the correct relationship?"
- "When you first worked with equivalent ratios, you used double number lines and tables to represent them. How do graphs and equations add to your understanding of equivalent ratios? Do they help in solving problems? If so, how?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Think about the terms *dependent variable* and *independent variable*? What does it mean to be dependent on something? Independent of something?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the terms *dependent variable* and *independent variable* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining which graph represents the given scenario.

Printable

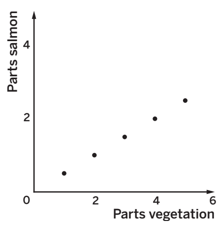
Name: ______________ Date: ________ Period: ________

Exit Ticket 6.17

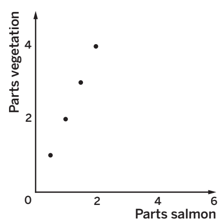
Grizzly bears are omnivores, meaning they eat both plants and meat. For each 1 part vegetation, they eat $\frac{1}{2}$ part salmon, even during the peak of the salmon season. The relationships between salmon and vegetation is shown by the equation $s = \frac{1}{2}v$, where s represents the amount of salmon and v represents the amount of vegetation.

Which graph also represents the relationship? Select *all* that apply. Explain your thinking.

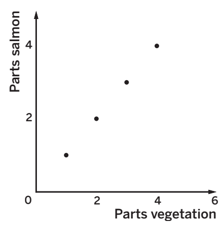
Graph A



Graph B



Graph C



Graph A and Graph B;

Sample explanations shown.

- Graph A shows a ratio of 1 part vegetation to $\frac{1}{2}$ part salmon relationship. The ratio continues: 2 parts vegetation to 1 part salmon, 3 parts vegetation to $1\frac{1}{2}$ part salmon, and 4 parts vegetation to 2 parts salmon. The amount of salmon is always half the vegetation.
- Graph B shows the opposite ratio of $\frac{1}{2}$ salmon: 1 part vegetation, which is the same relationship as 2 parts salmon: 1 vegetation, but the independent and dependent variables are switched.
- Graph C shows that the ratio is 1:1.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can create tables and graphs that show the relationship between two amounts in a given ratio. 1 2 3

b. I can write an equation with variables that shows the relationship between two amounts in a given ratio. 1 2 3

Success looks like . . .

- **Language Goal:** Comparing and contrasting graphs and equations that represent a relationship between the same quantities, but have the independent and dependent variables switched. **(Speaking and Listening)**
  - » Comparing and contrasting Graphs A and B.
- **Language Goal:** Comprehending the terms *independent variable* and *dependent variable*. **(Speaking and Listening, Writing)**
- **Goal:** Creating a table, graph, and equation to represent the relationship between quantities in a set of equivalent ratios.

Suggested next steps

If students select C, consider setting up the ratio given in the problem: $1 : \frac{1}{2}$ (vegetation : salmon) and asking:

- "Where is that first dot on the graph for the vegetation?" 1
- "What about the salmon?" 1
- "Is that the correct ratio?" No.

If students do not select Graph B, consider asking:

- "What is the first point on the graph?" $(\frac{1}{2}, 1)$
- "What does it represent in context?" $\frac{1}{2}$ parts salmon for 1 part vegetation
- "Is that the same relationship as 1 part vegetation for $\frac{1}{2}$ part salmon?" Yes

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students explored relationships between the same quantities. How did that build on the earlier work students did with ratios?
- In what ways did Activity 2 go as planned? What might you change for the next time you teach this lesson?

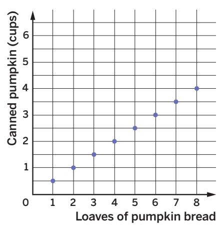
Practice

Independent

Name: ______________________ Date: __________ Period: __________

Practice

1. This graph shows some values for the number of cups of canned pumpkin p needed to make b loaves of pumpkin bread.



a. What does the point $(8, 4)$ mean in terms of the amount of canned pumpkin and number of loaves of bread?

To make 8 loaves of pumpkin bread, 4 cups of canned pumpkin are needed.

b. Complete the table so that the numbers in each column represent the coordinates of a point on the graph.

b	1	2	3	4	5	6	7
p	0.5	1	1.5	2	2.5	3	3.5

c. Write an equation that gives the number of cups of canned pumpkin in terms of the number of loaves of bread.

$p = \frac{1}{2}b$

2. Each serving of a certain fruit snack contains 90 calories.

a. Han wants to know how many calories he gets from the fruit snacks. Write an equation that shows the number of calories c in terms of the number of servings n.

$c = 90n$

b. Tyler needs some extra calories each day during his sports season. He wants to know how many servings he can have each day if all the extra calories come from the fruit snack. Write an equation that shows the number of servings n in terms of the number of calories c.

$n = \frac{c}{90}$

Name: ______________________ Date: __________ Period: __________

Practice

3. Kiran shops for books during a 20% off sale.

a. What percent of the original price of a book does Kiran pay during the sale?

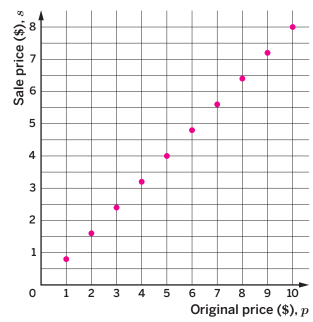
80%

b. Complete the table to show how much Kiran pays for books during the sale. Write an equation that relates the sale price s to the original price p.

$s = 0.8p$

Original price, p ($)	Sale price, s ($)
1	0.80
2	1.60
3	2.40
4	3.20
5	4.00
6	4.80
7	5.60
8	6.40
9	7.20
10	8.00

c. Plot the points on the graph to show the relationship between the sale price and the original price.



4. Calculate $12.34 \cdot 0.7$. Show your thinking.

8.638

5. Han is planning to ride his bike 24 miles. How long will it take him if he rides at a constant speed of 3 mph? Write an equation to represent this scenario, and then solve the equation.

$3x = 24$, $x = 8$ hours

Practice Problem Analysis

Type	Problems	Refer to	DOK
On-lesson	1	Activity 2	3
	2	Activity 2	3
	3	Activity 2	3
Spiral	4	Unit 5 Lesson 8	1
Formative	5	Unit 6 Lesson 18	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Two Related Quantities (Part 2)

Let's use equations and graphs to describe stories with constant rates.

Focus

Goals

1. Create a table, graph, and equations to represent the relationship between two quantities.
2. **Language Goal:** Identify the independent and dependent variable in an equation. **(Writing)**
3. **Language Goal:** Interpret an equation that represents the relationship between two quantities. **(Speaking and Listening, Writing)**

Coherence

Today

In this second lesson on representing relationships between two quantities, students shift their thinking from more discrete ratios to more continuous rates, using new contexts for writing equations that represent the relationships. Students use and make connections between tables, graphs, and now equations, all representing the same relationship. They use their representations to compare rates and consider how each of the representations would change if the independent and dependent variables were switched.

Previously

In Lesson 17, students learned about dependent and independent variables and saw how equations could also be used to represent a ratio relationship between two quantities.

Coming Soon

Students will study proportional relationships in more depth in Grade 7.

Rigor

- Students continue to build **conceptual understanding** of ratio relationships between two quantities in tables, graphs, and equations.
- Students **apply** their understanding to the effects of the reintroduction of the grey wolf in Yellowstone National Park.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Independent	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *independent variable*
- *dependent variable*

Amps | Featured Activity

Activity 1
Interactive Graphs

Given a table, students create equations to represent the table and then plot the points on a coordinate plane. You can overlay student responses to give immediate feedback.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Throughout Activity 1, students are asked to analyze and model the population of the grey wolf in Yellowstone National Park. Students might be frustrated that they are not able to identify the values that they will use in the analysis. Encourage students to mark up the text, identifying key information and the relationships between the variables. By working from the quantitative to the qualitative information, students will be able to pause throughout their manipulations and draw connections between values and symbols involved.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 1**, focus on Problems 1–3. Problem 5 may be discussed as a whole class.

Independent | 10 min

Warm-up Hiking Around Old Faithful

Students reason about the relationship between distance, rate, and time to solve a problem to reactivate what they know about constant speeds.

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 18

Two Related Quantities (Part 2)

Let's use equations and graphs to describe stories with constant rates.

Warm-up Hiking Around Old Faithful

Lin and Priya each start their hike at 9:00 a.m. They each hike at a steady rate. About what time do they finish their hike?

1. Do you have enough information to solve this problem? If not, what other information would be useful?
   No, I do not have enough information. I need to know what their rate is and how far they hike to know when they arrive.

2. Lin and Priya are hiking the Upper Geyser Basin and Observation Point Loop, which is a 4.5-mile trail. Lin can hike 10 miles in 5 hours, and Priya can hike 23 miles in 10 hours. If they each leave the trailhead at 9:00 a.m. and hike at a steady rate, at about what times do they finish the trail?
   Lin will finish at about 11:15 a.m.
   Priya will finish a little before 11:00 a.m., or at 10:57 a.m.

Log in to Amplify Math to complete this lesson online.

1 Launch

Give students 5 minutes of quiet work time. Alternatively, consider displaying the first scenario and pausing for discussion before students work with the second, full scenario.

2 Monitor

Help students get started by activating prior knowledge and reminding them of tools they can use including double number lines or tables of equivalent ratios.

Look for points of confusion:

- **Thinking they have enough information to solve.** Have students try to solve. Ask, "Why was it not possible?"
- **Working with numbers and operations incorrectly.** Guide students to determine how many miles each person can walk in 1 hour.

Look for productive strategies:

- Using double number lines or tables of equivalent ratios where the columns represent distance and time.

3 Connect

Display the second, full scenario.

Have students share different representations and their reasoning, starting with those who used visual representations such as double number lines, followed by those who used tables of equivalent ratios.

Highlight that whoever walks farther in 10 hours will be the one walking at a faster rate, and therefore would arrive sooner.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students how familiar they are with Yellowstone National Park or what Old Faithful is. Consider showing an image of the Old Faithful geyser, and some of the hiking trails around Yellowstone National Park. This will help students visualize the context as they consider whether they have enough information. Mention that the Upper Geyser Basin and Observation Point Loop represents one of the trails. Ask students if they have ever gone hiking and how many miles they have hiked along a single trial.

Power-up

To power up students' ability to write an equation to represent a ratio relationship:

A certain phone app charges $4 a month for its subscription service. Complete the table to show how much it would cost to subscribe to this app.

Number of months subscribed to app	1	2	4	x
Total cost of subscription ($)	4	8	19	$4x$

Use: Before the Warm-up.

Informed by: Performance on Lesson 17, Practice Problem 6.

Independent | 25 min

Activity 1 Reintroduction of the Grey Wolf

Students complete a table representing a real-world scenario. They write equations and use graphs to represent and analyze the relationships between two quantities.

Amps Featured Activity | Interactive Graphs

Activity 1 Reintroduction of the Grey Wolf

In 1995, the grey wolf was reintroduced into Yellowstone National Park. The effects of the reintroduction amazed scientists, who recorded increases in the populations of everything from vegetation and migrating birds, to other animals, such as beavers, otters, and bears. As a result, the whole landscape of Yellowstone has dramatically changed.

On January 12, 1995, the first of 31 grey wolves were released into the park. That same year, there was only one colony of beavers left (a colony has, on average, 6 beavers), and most willow trees were nothing more than shrubs.

The following table represents how the reintroduction of the grey wolf could result in positive changes for beaver populations and willow tree growth.

1. Complete the table representing the different animal populations and the average height of willow trees at the end of each year, assuming that all the rates of increase are constant each year.

Year	Wolves	Beavers	Average Willow Tree Height (ft)
1	**31**	6	**3.5 or $3\frac{1}{2}$ or $\frac{7}{2}$**
2	62	**12**	7
3	**93**	**18**	**10.5 or $10\frac{1}{2}$ or $\frac{21}{2}$**
6	186	36	21
8	**248**	**48**	**28**

2. How much did each population and the average height of willow trees increase per year?

   Wolves: **31**

   Beavers: **6**

   Average height of willow trees: **3.5 or $3\frac{1}{2}$ or $\frac{7}{2}$ ft**

3. How long does it take for each animal population to increase by 1 animal and for the willow tree height to increase by 1 ft?

   Wolves: **$\frac{1}{31}$ of a year, or $365 \div 31 \approx 12$ days**

   Beavers: **$\frac{1}{6}$ of a year, or $365 \div 6 \approx 61$ days or about 2 months**

   Willow trees: **$\frac{2}{7}$ of a year, or $365 \div 3.5 \approx 104$ days**

714 Unit 6 Expressions and Equations

1 Launch

Give students access to colored pencils.
Have students work for 5–8 minutes (partners optional).

2 Monitor

Help students get started by asking, "Where is there a relationship between time and growth? How can that help get us started?"

Look for points of confusion:

- **Working only in the first row.** Ask, "Do you have enough information to figure all those out yet?"
- **Plotting the incorrect pairs.** Remind students to use their table and make sure they are maintaining the relationships between the time and each participant.
- **Struggling with the average willow tree height.** Students may need to be reminded of the different ways to show this either as a decimal, mixed number, or improper fraction, before choosing the one they want to use.
- **Struggling with the fractional values for the willow tree in Problem 3.** It may be easier for students to divide 365 by the decimal 3.5, instead of a fraction or mixed number.

Look for productive strategies:

- Talking about the relationship between the year and the growth (either in population or height). Students should interpret the unit ratios: for the wolf, $1 : 31$; for the beaver, $1 : 6$; and, for the willow tree, $1 : 3\frac{1}{2}$.
- Using the fraction $\frac{7}{2}$ for the willow tree. This will make calculations easier in later problems.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can plot the points from an equation on a coordinate plane. You can overlay them all to see similarities and differences at a glance and provide immediate feedback.

Extension: *Math Enrichment*

Ask students what the point (0, 0) would represent in this scenario. The point (0, 0) represents the population of the grey wolf as 0 in Year 0.

Math Language Development

MLR7: Compare and Connect

Have students create a visual display of their table, graph, and equations that relate population or height and time for the wolf, beaver, and willow trees. Display student work and conduct a ***Gallery Tour*** so that other students can compare and analyze each other's displays. As students they do so, ask them to share what is especially clear in a particular representation. Listen for and amplify the language students use to describe how the distance traveled increases by a constant amount per hour and how this pattern can be seen on the table and graph.

Independent | 25 min

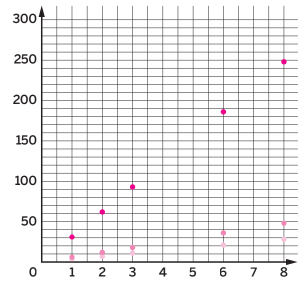
Activity 1 Reintroduction the Grey Wolf (continued)

Students complete a table representing a real-world scenario. They write equations and use graphs to represent and analyze the relationships between two quantities.

Name: ______ Date: ______ Period: ______

Activity 1 Reintroduction of the Grey Wolf (continued)

4. Graph the populations or average heights for the wolf, beaver, and willow trees, through 2002, where 1995 corresponds to Year 1 of the program. Use a different color for each of the three quantities.



5. Diego says that the equation $w = 31y$ represents the relationship between the years of the program and the wolf population, where w represents the number of wolves and y represents the number of years.

   a. Explain why the equation $w = 31y$ relates the wolf population to the number of years.
   Sample response: Because there are 31 wolves for every year.

   b. Write equations that relate the beaver population b and the growth of the willow tree t to years of the program y.
   $b = 6y$ $t = 3.5y$

   c. Is the independent variable the same in all three equations?
   Yes, the number of years, y, is the independent variable in all three.

6. Use the equations you wrote to predict the animal and plant growth, if they continued to increase at the same rates, 12 years into the program.

   Wolves: 372

   Beavers: 72

   Willow trees: 42 ft

STOP

Lesson 18 Two Related Quantities (Part 2) 715

3 Connect

Display the table, graph, and equations.

Highlight how the table, graph, and equations represent the scenario and how they are connected to each other.

Ask:

- "How can you determine from the table who had the greatest growth, in population or height, each year?"
- "How can you determine from the graph who had the greatest growth each year?"
- "How can you determine from the equation who had the greatest growth?"
- "If growth was the independent variable, how would the equations and graphs be different?"

Whole Class | 5 min

Summary

Review and synthesize the different representations used for describing and analyzing a scenario involving time, distance, and a constant rate.

Summary

In today's lesson . . .

You used equations and graphs to represent stories that have constant rates. Equations can be used to represent the relationship between two quantities more generally, such as time and distance traveled.

Equations can also help you answer questions like: "How far can a boat travel in 3.25 hours if it is traveling at a constant speed of 25 mph?" or, "How long does it take for the boat to travel 60 miles?" Once you have an equation, if you know the value of one quantity, you can substitute and evaluate to solve for the value of the other quantity.

For example, you can use t to represent the time in hours and d to represent the distance in miles that the boat travels.

When you know the time and want to determine the distance:	When you know the distance and want to determine the time:
You can write $d = 25t$. In this equation, if t changes, d is affected by the change, so t is the independent variable and d is the dependent variable. This equation can help you determine d when you have any value of t. In 3.25 hours, the boat can travel 25(3.25) or 81.25 miles.	You can write $t = \frac{d}{25}$. In this equation, if d changes, t is affected by the change so d is the independent variable and t is the dependent variable. This equation can help you determine t for any value of d. To travel 60 miles, it will take $\frac{60}{25}$ or $2\frac{2}{5}$ hours.

Reflect:

716 Unit 6 Expressions and Equations

Synthesize

Display the equations and graphs representing the scenario, and these questions:

- "How far can a boat travel in 3.25 hours if it is traveling at a constant speed of 25 ph?"
- "How long does it take for the boat to travel 60 miles?"

Ask:

- "Which representation would be most helpful in determining unknown quantities in different situations?"
- "What factors would you consider in deciding which quantity to set as the independent variable when writing an equation to describe a situation?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did you learn about balance in nature? What connections can you make with the math you did today"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding identifying the independent variable for a scenario and interpreting independent and dependent variables in context.

Printable

Name: ________ Date: ________ Period: ________

Exit Ticket 6.18

During the Yellowstone Marathon, Noah's time in hours t and the distance d, in miles, he has walked are related by the equation $\frac{1}{3}d = t$. A graph of the equation includes the point $(12, 4)$.

1. Identify the independent variable.
   The independent variable is d, the number of miles walked.

2. What does the point $(12, 4)$ represent in this situation?
   12 represents the distance in miles and 4 represents the time in hours. Noah walked 12 miles in 4 hours (assuming a steady rate).

3. What point would correspond to walking $7\frac{1}{2}$ miles?
   $\left(7\frac{1}{2}, 2\frac{1}{2}\right)$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can create tables and graphs to represent the relationship between two quantities.
1 2 3

b. I can write an equation with variables to represent the relationship between two quantities.
1 2 3

Success looks like . . .

- **Goal:** Creating a table, graph, and equations to represent the relationship between two quantities.
- **Language Goal:** Identifying the independent and dependent variable in an equation. **(Writing)**
  - » Identifying the independent variable in Problem 1.
- **Language Goal:** Interpreting an equation that represents the relationship between two quantities. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of the point $(12, 4)$ in relation to the equation.

Suggested next steps

If students cannot identify the independent variable, consider:

- Having them identify the dependent variable. Ask, "Which variable is the result after the calculation? That is the *dependent variable*. So, which is the *independent variable*?"

If students do not know what the point represents, consider:

- Asking, "Which would make sense in this calculation? Can you substitute the numbers so it is true?"

If students cannot identify the corresponding point, consider:

- Asking, "Does $7\frac{1}{2}$ represent the time or the distance? What should you do with the number $7\frac{1}{2}$ then?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- How did the context set students up to develop equations involving constant rates? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Interpreting an equation that represents the relationship between two quantities.

Reflect on students' language development toward this goal.

- How did using the ***Compare and Connect*** routine in Activity 1 help students make connections between multiple representations of the three relationships presented? Did using this routine help them be able to interpret the equations that they wrote in the activity?
- Would you change anything the next time you use this routine?

Practice

Independent

Name: Date: Period:

Practice

1. A car is traveling down a road at a constant speed of 50 mph.

a Complete the table with the amounts of time it takes the car to travel certain distances, or the distances traveled for certain amounts of time.

Time (hours)	Distance (miles)
2	**100**
1.5	**75**
t	**$50 \cdot t$**
1	50
6	300
$\frac{d}{50}$	d

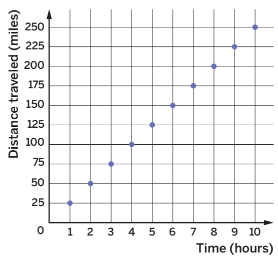
b Write an equation that represents the distance d, traveled by the car in miles, given the time t, in hours.

$d = 50 \cdot t$

c In your equation, which is the dependent variable and which is the independent variable?

The time t is the independent variable, and the distance d is the dependent variable.

2. The graph represents the amount of time, in hours, it takes a ship to travel various distances in miles.



a Write the coordinates for one point on the graph. What does the point represent?

Sample response: (1, 25). The point shows that it takes 1 hour for the ship to travel 25 miles.

b What is the speed of the ship in miles per hour?

25 mph

c Write an equation that relates the time t it takes to travel a given distance d.

$d = 25 \cdot t$

3. Han is planning to ride his bike 24 miles.

a How long will it take if he rides at a rate of 4 mph? 6 mph?

6 hours; 4 hours

b Write an equation that Han can use to determine the time t will take to ride 24 miles, if his rate in miles per hour is represented by r.

$t = \frac{24}{r}$

Name: Date: Period:

Practice

4. Determine a solution to each equation. Show or explain your thinking.

a $2^x = 8$

$x = 3$

b $2^x \cdot 2^3 = 2^7$

$x = 4$

c $\frac{2^x}{2^3} = 2^5$

$x = 8$

5. Select *all* the expressions that are equivalent to $5x + 30x - 15x$.

A. $5(x + 6x - 3x)$

B. $(5 + 30 - 15) \cdot x$

C. $x(5 + 30x - 15x)$

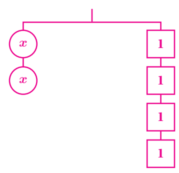
D. $5x(1 + 6 - 3)$

E. $5(x + 30x - 15x)$

6. Create an example of a balanced hangar diagram using the given figures. You may use each figure more than once. Then write and solve an equation that matches your diagram.

$\boxed{1}$ x

Sample response:



$2x = 4$

$x = 2$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	3
Spiral	4	Unit 6 Lesson 16	1
	5	Unit 6 Lesson 13	1
Formative	6	Unit 6 Lesson 19	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Creating a Class Mobile

Let's make a class mobile that represents our class and our unit on expressions and equations.

Focus

Goals

1. Create a visual model of balance.
2. Explain how balancing a model is similar to equality.
3. Make connections to the algebraic procedures of solving an equation and the actions of keeping a model in balance.

Coherence

Today

In this culminating lesson, Alexander Calder's *Double Gong*, exemplifies the concept of balance in the form of a complex mobile. Each student creates an object to be attached to a hanger in order to create a mobile as a class. Before they build their model, students discuss the possible effects of the size and weight of the objects being placed on the hanger in mathematical terms. As the mobile is being assembled, balance is repeatedly created, lost, and then corrected. Students connect discussions to the math of the unit by referencing expressions and equations as being *balanced*, *true*, or *equal*, and their objects as representing variables.

Previously

This unit has focused on how equations, including those with variables, represent two expressions as balanced. Students have represented equations using tape diagrams and hanger diagrams, to help them visualize applying properties of operations to maintain balance, and to determine solutions.

Coming Soon

In Unit 7, students plot points with signed rational number coordinates on the number line, and recognize and use the connection between the relative position of two points on the number line and inequalities involving the coordinates of the points.

Rigor

- Students **apply** algebraic expressions and equations to a class mobile project.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	20 min	5 min	5 min
Independent	Independent	Whole Class	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (instructions)

Optional building materials:

- wire hanger or wire
- construction paper
- paper clips
- clay
- colored pencils
- paint
- stickers

Math Language Development

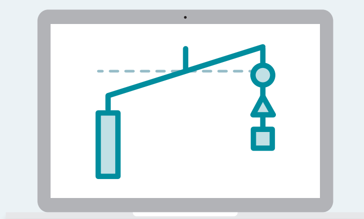
Review words

- *expression*
- *equation*
- *coefficient*
- *variable*
- *term*
- *equal*
- *equivalent*

Amps | Featured Activity

Activity 2
Hanger Puzzle

Students design their own ornament that has special meaning for them, and then solve a detailed hanger puzzle that includes their ornament.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students work together to build a class mobile, the conversation and directives might become uncontrolled. Remind students that effective communication requires clarity and precision of language. Clear communication in Activity 3 includes precise physical and algebraic descriptions.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm up** may be omitted.
- The **Activity 1** portion of the lesson can be skipped. Have students create something directly to the chosen medium**.**

Warm-up Notice and Wonder

Students observe a mobile by Alexander Calder, titled *Double Gong*, and discuss how it relates to the mathematical concept of balance present throughout this unit.

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 19 – Capstone

Creating a Class Mobile

Let's make a class mobile that represents our class and our unit on expressions and equations.

Warm-up Notice & Wonder

What do you notice about this mobile? What do you wonder?

1. I notice . . .
   Sample response:
   The mobile looks balanced, even though there are more items on the left.

2. I wonder . . .
   Sample responses:
   - How can the mobile be balanced?
   - How heavy the pieces are to make it balanced?

Log in to Amplify Math to complete this lesson online.

Lesson 19 Creating a Class Mobile 719

1 Launch

Project Alexander Calder's *Double Gong*. Give students 2 minutes to look at and think about what they notice and wonder. If necessary, clarify that a mobile (pronounced moh-beel) is this type of hanging structure, not a cell phone.

2 Monitor

Help students get started by asking, "What jumps out at you?"

Look for points of confusion:

- **Noticing only low-floor criteria (color, shape).** Ask, "Do you see anything that relates to our unit? How does this relate to math?"
- **Sharing opinions on whether they like the piece or not.** Students should be focusing on the structure of the mobile.

Look for productive strategies:

- Questioning the possibility that this could even be in balance. Ask, "What could that mean about the objects on the right side?"
- Counting the objects on each side and determining values that could make them equal (e.g. $13 = 6.5 + 6.5$).

3 Connect

Have students share what they noticed or still wonder, starting with low-floor observations, followed by those connecting the mobile to the mathematical ideas explored in this unit (balance, quantities, etc.).

Ask:

- "How does this relate to the unit?"
- "How can you relate this to equations, or keeping things equal and in balance?"

Differentiated Support

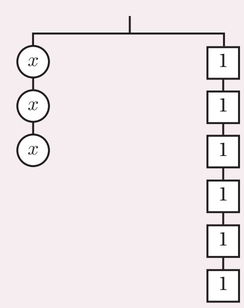
Accessibility: *Guide Processing and Visualization*

If students are unfamiliar with a mobile, consider displaying other images of mobiles, or bring in a mobile. Students with siblings may recognize mobiles that are commonly used to engage infants and placed hanging over cribs.

Power-up

To power up students' ability to make sense of a hanger diagram, ask:

Write an equation to match the hanger diagram. Then solve the equation you wrote.



$3x = 6$

$x = 2$

Use: Before Activity 1.

Informed by: Performance on Lesson 18, Practice Problem 6.

Independent | 20 min

Activity 1 Making a Mobile Ornament

Students will make an object or a drawing of an object of something that represents themselves. These will be hung to create a class mobile.

Activity 1 Making a Mobile Ornament

Write down a word or brief description of something that represents your identity. Using that as inspiration, sketch an ornament for the class mobile in the space provided. Various materials will be made available to you so that you can create a model of your ornament.

Something that represents me is:

Answers may vary.

Once you have completed creating your ornament, it will be assigned a variable. Record your variable here.

720 **Unit 6** Expressions and Equations

1 Launch

Offer students a variety of materials. Give them 20 minutes to create something that represents themselves in some way. You may want to have something already done that represents you as a model.

2 Monitor

Help students get started by asking them about their favorite animals, sports, food, places to visit, etc., to activate background knowledge and generate an idea for their object.

Look for points of confusion:

- **Asking whether it has to be related to math class.** Tell students the object they are making can be anything. It does not have to be related to math class.

Look for productive strategies:

- Creating something that represents them, using the materials provided.

3 Connect

Have students share about their object. Go around in a circle so every student has an opportunity to share. If all students do not feel comfortable sharing with the whole class, consider small group sharing. Personal sharing like this is meant to build classroom community, and therefore all voices should be treated equally.

Ask, "How are your objects similar? different?"

Assign variables to each student's object. This can be used to create or build expressions and equations in Activity 2.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students do not feel comfortable creating objects out of clay or construction paper, allow them to bring in objects or pictures from home. Consider having them do so prior to this activity.

Whole Class | 20 min

Activity 2 Assembling the Class Mobile

As a class, students and the teacher will build a balanced class mobile, representing the math learning over the course of the unit.

Amps Featured Activity Hanger Puzzle

Name: ______ Date: ______ Period: ______

Activity 2 Assembling the Class Mobile

As we build the mobile, make mental notes of when you apply the thinking you learned about expressions and equations in this unit.

Sketch the finished mobile that your class created in the space provided.
Answers may vary.

STOP

1 Launch

Make sure there is enough space for all students to gather around the mobile. Forming a circle ensures all students can see, be seen, and be active participants in the activity. Mobile assembly options can be found in the Activity 2 PDF.

2 Monitor

Help students get started by putting two pieces on one side of the hanger representing an expression, $1 + 1$. Find one object to balance them out on the other side of the hanger to form the "balanced" equation, or 2; hence $1 + 1 = 2$.

Look for points of confusion:

- **Students may want to exaggerate imbalance, which could distract from the ultimate goal of the activity.** Remind students of mathematical reasoning and why it's important that the mobile is balanced.

Look for productive strategies:

- Representing addition/subtracting by adding horizontally to the hanger, and representing multiplication/division by adding vertically, object to object. Be sure to encourage the use of precise mathematical language as students describe their actions both physically and algebraically.
- Highlight the operations using variables assigned to student objects. Build expressions and equations as balance is achieved.

3 Connect

Display the class mobile(s).

Highlight the various operations and actions of balancing.

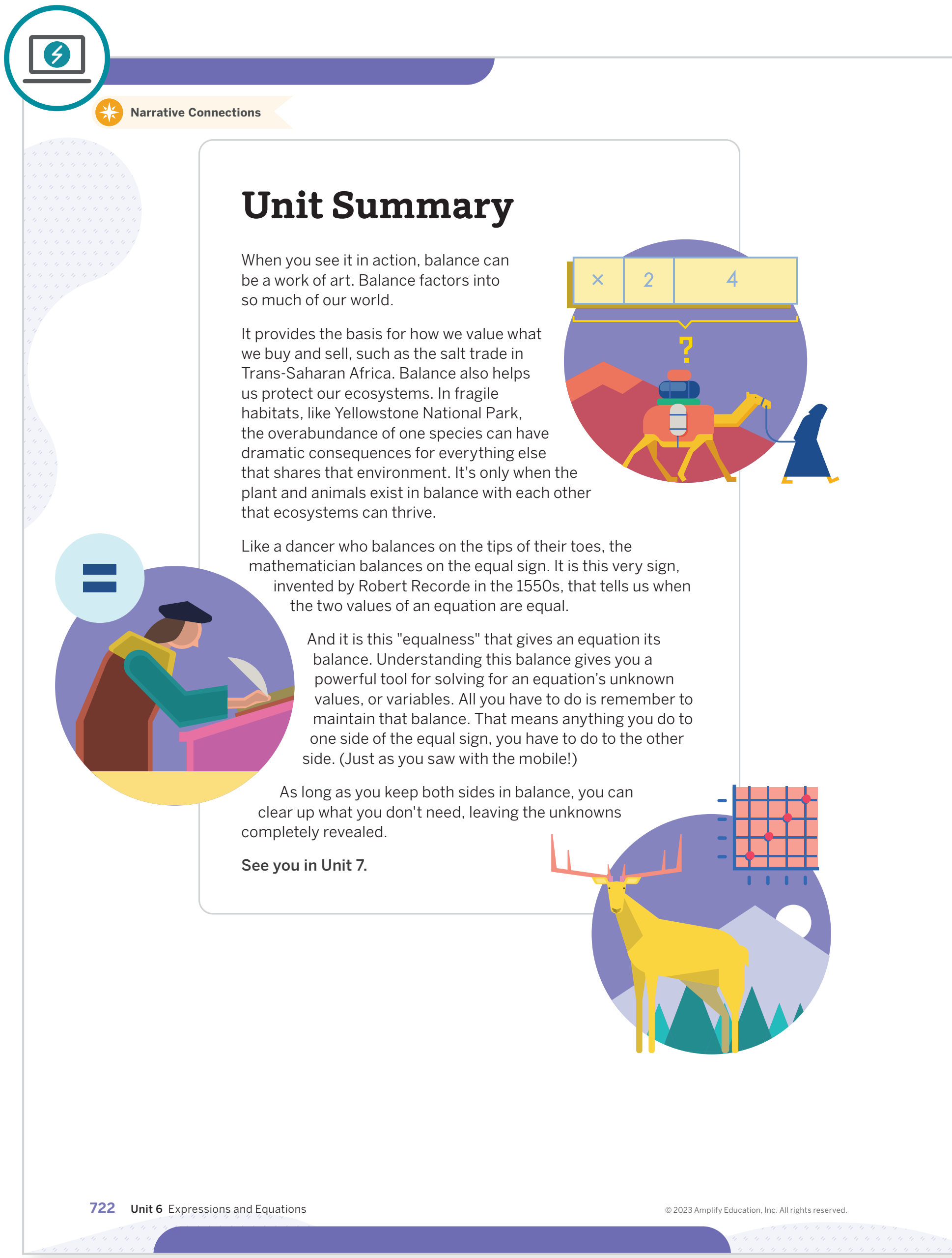
Have groups share how they created their mobiles.

- **Option 1:** Create one mobile on one hanger by balancing using all objects.
- **Option 2:** Balance a few pieces at a time representing different expressions with addition and multiplication. Put matching expressions and solutions on different hangers, that can be placed around the room.
- **Option 3:** Make groups of 6–8 students and have them balance their own mobile. Monitor the use of mathematical language.

Whole Class | 5 min

Unit Summary

Review and synthesize all the big ideas of the unit as students reflect on their learning across the lessons and how it is symbolized in the class mobile.


Narrative Connections

Unit Summary

When you see it in action, balance can be a work of art. Balance factors into so much of our world.

It provides the basis for how we value what we buy and sell, such as the salt trade in Trans-Saharan Africa. Balance also helps us protect our ecosystems. In fragile habitats, like Yellowstone National Park, the overabundance of one species can have dramatic consequences for everything else that shares that environment. It's only when the plant and animals exist in balance with each other that ecosystems can thrive.

Like a dancer who balances on the tips of their toes, the mathematician balances on the equal sign. It is this very sign, invented by Robert Recorde in the 1550s, that tells us when the two values of an equation are equal.

And it is this "equalness" that gives an equation its balance. Understanding this balance gives you a powerful tool for solving for an equation's unknown values, or variables. All you have to do is remember to maintain that balance. That means anything you do to one side of the equal sign, you have to do to the other side. (Just as you saw with the mobile!)

As long as you keep both sides in balance, you can clear up what you don't need, leaving the unknowns completely revealed.

See you in Unit 7.

722 Unit 6 Expressions and Equations

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Ask, "Reflect upon how this lesson connects to the learning done over the course of this unit."

Have students share their reflections on their learning.

Highlight the connections made from building the mobile to the mathematics of the unit, focusing on balance and equations and their solutions.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "How does the mobile represent your balance in your classroom community?"

Independent | 5 min

Exit Ticket

Students reflect on the creation process of the mobile(s) and identify at least one mathematical action that demonstrates the concept of balance.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 6.19

Draw or explain one mathematical action demonstrated while balancing the class mobile.
Answers may vary.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain how to balance a model. 1 2 3

b. I can make connections between solving an equation and keeping a model in balance. 1 2 3

Success looks like . . .

- **Goal:** Creating a visual model of balance.
- **Goal:** Explaining how balancing a model is similar to equality.
- **Goal:** Making connections to the algebraic procedures of solving an equation and the actions of keeping a model in balance.
  - » Explaining one mathematical action while balancing the class mobile.

Suggested next steps

If students cannot describe any balancing action that relates to the content learning over the course of the unit, consider:

- Referring back to a particular balancing action that can be related to a simple equation, such as $1 + 1 = 2$.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What was especially satisfying about creating a model that connects math to your classroom community?
- In what ways did creating the mobile go as planned? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

Practice

1. Select *all* the expressions that are equivalent to $4b$.
   - A. $b + b + b + b$
   - B. $b + 4$
   - C. $2b + 2b$
   - D. $b \cdot b \cdot b \cdot b$
   - E. $b \cdot \frac{1}{4}$

2. Complete each problem. Show or explain your thinking.
   - a. 20% of a is 11. What is a?
     55
   - b. 75% of b is 12. What is b?
     16
   - c. 80% of c is 20. What is c?
     25
   - d. 200% of d is 18. What is d?
     9

3. Solve each equation. Show your thinking.
   - a. $111 = 14a$
     $a = \frac{111}{14}$ or $a = 7\frac{13}{14}$
   - b. $13.65 = b + 4.88$
     $b = 8.77$
   - c. $c + \frac{1}{3} = 5\frac{1}{8}$
     $c = \frac{115}{24}$ or $c = 4\frac{19}{24}$
   - d. $\frac{2}{5}d = \frac{17}{4}$
     $d = \frac{\frac{17}{4}}{\frac{2}{5}}$ or $d = \frac{85}{8}$ or $d = 10\frac{5}{8}$
   - e. $5.16 = 4e$
     $e = \frac{5.16}{4}$ or $e = 1.29$ or $e = 1\frac{29}{100}$

Name: Date: Period:

Practice

4. Andre ran $5\frac{1}{2}$ laps around a track in 8 minutes, at a constant speed. It took Andre x minutes to run each lap. Select *all* the equations that represent this situation.
   - A. $\left(5\frac{1}{2}\right)x = 8$
   - B. $5\frac{1}{2} + x = 8$
   - C. $5\frac{1}{2} - x = 8$
   - D. $5\frac{1}{2} \div x = 8$
   - E. $x = 8 \div \left(5\frac{1}{2}\right)$
   - F. $x = \left(5\frac{1}{2}\right) \div 8$

5. Select *all* the expressions that are equal to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.
   - A. $3 \cdot 5$
   - B. 3^5
   - C. $3^4 \cdot 3$
   - D. $5 \cdot 3$
   - E. 5^3

6. Evaluate each expression. Show your thinking.
   - a. $5 + 3^2 = 14$
   - b. $4 \cdot 5^1 = 20$
   - c. $15 - 2^3 = 7$
   - d. $5 \cdot \left(\frac{1}{2}\right)^3 = \frac{5}{8}$
   - e. $\left(\frac{1}{4} \cdot 4\right)^4 = 1$

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 6 Lesson 11	1
	2	Unit 6 Lesson 9	1
	3	Unit 6 Lesson 7	1
	4	Unit 6 Lesson 4	2
	5	Unit 6 Lesson 14	1
	6	Unit 6 Lesson 15	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 7

Rational Numbers

Students recognize a need to expand their concept of number to represent both magnitude and direction, extending the number line and coordinate plane to include negative rational numbers. They compare these numbers, as well as their absolute values, and write inequality statements using variables.

Essential Questions

- What does it mean for a value to be less than zero?
- How can a number be closer to zero and have a greater value?
- How can two objects move the same distance, but end up in different places?
- *(By the way, how many lefts make a right?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students build on their understanding of numbers for representing quantities or values on the number line, now considering contexts in which there is meaning associated with both values less than zero and a number's distance from zero. They also extend these understandings to two related quantities using ordered pairs and a coordinate plane.

Coherence

Previously . . .

In prior grades, students used the number line to compare positive rational numbers and plotted ordered pairs of those numbers in the first quadrant of a coordinate plane. Relevant prior work in Grade 6 includes graphing ratio relationships and writing expressions and solving equations with one variable.

Coming soon . . .

With an understanding of negative rational numbers and their opposites from this unit, students will be prepared to extend the four operations to these numbers in Grade 7. The introduction of the four-quadrant coordinate plane will be leveraged in graphing more complex relationships between quantities — linear equations in Grades 7 and 8, and other functions in Grade 8 and high school courses.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each aspect is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Negative numbers are introduced using vertical and horizontal number lines and related to a variety of contexts, so students leverage familiar understandings to make sense of signed numbers and their relative order (Lessons 2–4, 9, 11). Relating two number lines to the axes of a coordinate plane, students plot and interpret relative position in two dimensions in the plane (Lessons 13–14).

Procedural Fluency

Ample practice comparing and ordering rational numbers and their absolute values using the number line prepares students for further fluency with graphing inequalities and plotting points in all four quadrants of the coordinate plane (Lessons 5, 8, 10, 15).

Application

Understanding the meaning of positive and negative values, and zero, within a context, students consider perspective in exchanges of money (Lesson 6) and explore extreme environments and their inhabitants (Lessons 7–8, 12). They also relate distances between positive and negative coordinates on a coordinate plane to maps, geometric shapes, puzzles, and artistic drawings (Lessons 16–19).

Getting Where We're Going

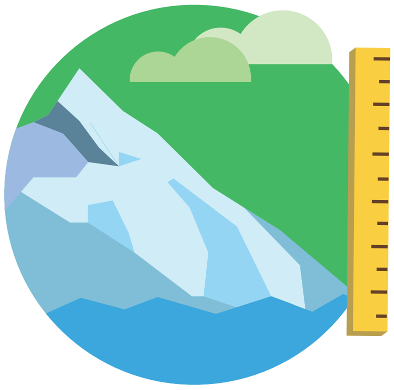
SUB-UNIT

1

Lessons 2–8

Negative Numbers and Absolute Value

Students explore negative rational numbers and make sense of them with respect to the meaning of zero. They plot these values on vertical and horizontal number lines, and explore the concept of absolute value.



Narrative: Knowing the tallest mountain on Earth depends on whether its height is measured *from sea level* or *from Earth's center.*

SUB-UNIT

2

Lessons 9–12

Inequalities

Students write inequality statements to represent upper or lower bounds. They graph the solutions on a number line and consider the real-world implications when interpreting whether negative values make sense, or if there is an implied lower bound to represent the range of possible values.

Narrative: Inequalities, such as <, ≤, >, and ≥, can describe real-world scenarios with precision.

Launch

Lesson 1

How Far? Which Way?

Students begin to explore the necessity of articulating both magnitude and direction precisely as they provide directions for one another to physically move about the classroom. They then apply these same principles more abstractly using pencil and paper to navigate one another through mazes.

SUB-UNIT

3

Lessons 13–18

The Coordinate Plane

Students extend their understanding of the coordinate plane to all four quadrants. They recognize patterns in the signs of coordinates and use absolute value to determine the distance between two vertically- or horizontally-aligned points, including those on opposite sides of either axis.

Narrative: Gerardus Mercator introduced the world to using parallel and perpendicular lines plot locations on a map grid.

Capstone

Lesson 19

Drawing on the Coordinate Plane

Students test out their technical and creative drawing skills on the coordinate plane, connecting points with segments to form a personal masterpiece. Aside from identifying and labeling positive and negative coordinates, they also consider absolute value and distance between points by sizing geometric shapes and creating symmetry.

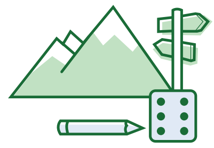
Unit at a Glance

Spoiler Alert: A number's sign tells you whether the value is less than 0 or greater than 0. On the number line or the coordinate plane, that means moving left or right, or up or down. You can ignore the sign when determining how far to go from 0.

Assessment

A **Pre-Unit Readiness Assessment**

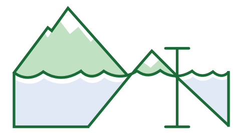
Launch Lesson



1 **How Far? Which Way?**

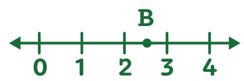
Close your eyes and listen to how words and numbers together can get you from A to B.

Sub-Unit 1: Negative Numbers and Absolute Value



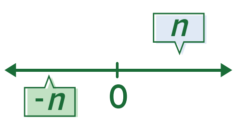
2 **Positive and Negative Numbers**

Use positive and negative integers to describe differences in elevation above and below sea level.



3 **Points on a Number Line**

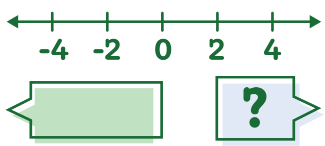
Recognize and interpret opposites and other positive and negative rational numbers representing degrees Celsius that are plotted on an extended number line.



8 **Comparing Numbers and Distances From Zero**

Distinguish a number from its absolute value and compare combinations of these by writing simple equations and inequalities.

Sub-Unit 2: Inequalities



9 **Writing Inequalities**

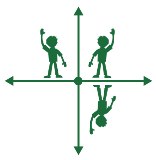
Use the symbols $<$, $>$, $\leq$, and $\geq$ to write inequalities representing verbal statements and scenarios in which a variable represents an unknown quantity.

10 **Graphing Inequalities**

Represent possible values for a quantity with a given bound as an inequality and as a collection of points on the number line.

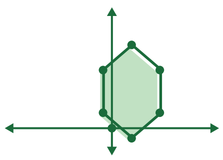
11 **Solutions to One or More Inequalities**

Understand a solution to an inequality as any value that makes it true, and graph all possible solutions to one or two inequalities on the number line.



16 **Distances on the Coordinate Plane**

Leverage understanding of opposites and the concept of absolute value to determine the distance between any two vertical or horizontal points on a coordinate plane or a map.



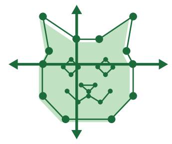
17 **Shapes on the Coordinate Plane**

Draw polygons on a coordinate plane and solve problems related to their attributes by determining distances between points.

18 **Lost and Found Puzzles**

Key word: puzzles! Practice plotting points and determining distances between them on a coordinate plane while navigating and creating mazes and solving a logic puzzle.

Capstone Lesson



19 **Drawing on the Coordinate Plane**

Bring it all together to create artistic images that rely on understanding negative numbers and the coordinate plane.

Key Concepts

Lesson 3: Negative rational numbers are the opposites of positive rational numbers, located on the other side of 0 on a number line.
Lesson 7: The absolute value of a number represents its distance from 0.
Lesson 10: Solutions to inequalities comparing positive or negative numbers to an unknown quantity can be graphed.
Lesson 14: The x- and y-axes of the coordinate plane can be extended to form four quadrants with ordered pairs of positive and negative numbers.

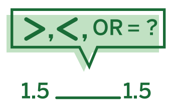
Pacing

19 Lessons: 45 min each
2 Assessments: 45 min each
Full Unit: 21 days
● **Modified Unit:** 18 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

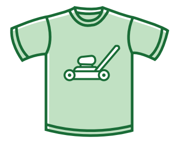
4 Comparing Integers

Compare integers on the number line and in the context of elevation, understanding the sign of a number indicates something about its position relative to zero.



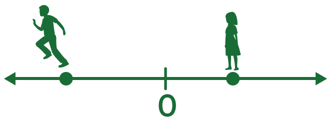
5 Comparing and Ordering Rational Numbers

Compare and order positive and negative rational numbers in all forms using the number line.



6 Using Negative Numbers to Make Sense of Contexts

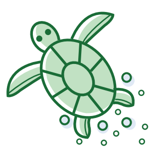
Hello, financial literacy! Consider the perspectives of buyers and sellers while using signed numbers to represent credits, debits, and changes in inventory.



7 Absolute Value of Numbers

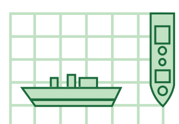
Recognize the absolute value of a number as its distance from 0, regardless of sign or direction. Use and interpret absolute value notation in revisiting the contexts of elevation and temperature.

Sub-Unit 3: The Coordinate Plane



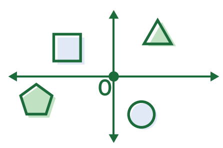
12 Interpreting Inequalities ●

Consider the reasonableness of mathematical solutions to one or more inequalities in the contexts of elevation and temperature.



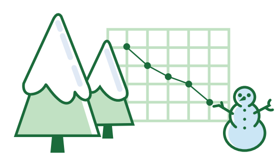
13 Extending the Coordinate Plane

Extend the coordinate plane to include negative values and form four quadrants, plotting and interpreting the locations of ordered pairs.



14 Points on the Coordinate Plane

Practice makes perfect, both in archery and in plotting ordered pairs of rational numbers on the coordinate plane.



15 Interpreting Points on the Coordinate Plane

Consider the values of two quantities, such as time and money or elevation and temperature, to identify an appropriate scale for the coordinate axes in order to plot and interpret ordered pairs in context.

Assessment

A End-of-Unit Assessment

● Modifications to Pacing

Lesson 8: Students are given opportunities to see and use absolute value in Lesson 7. If they have a sufficient grasp of the concept to apply it to distance on the coordinate plane in Lesson 16, then you may omit this lesson or simply use its Practice.

Lessons 11–12: These lessons both focus heavily on scenarios with both upper and lower bounds (examples of compound inequalities). Lesson 12 may be omitted, or students could complete Lesson 11, Activity 1 and Lesson 12, Activity 2 in one class period.

Lessons 18–19: These lessons are intended as fun applications of plotting points on the coordinate plane and using the distances between them to solve problems. Omitting them will likely reduce the fun factor, but may be a fair consideration.

Unit Supports

Math Language Development

Lesson	New vocabulary
1	magnitude
2	negative number positive number
3	opposite rational number
4	sign
7	absolute value
9	solution to an inequality
13	quadrant

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
3, 5, 7, 10, 16, 17	**MLR1:** Stronger and Clearer Each Time
1–4, 6, 7, 8, 19	**MLR2:** Collect and Display
3, 8, 12, 15	**MLR3:** Critique, Correct, Clarify
2, 8, 9, 15, 16, 18	**MLR5:** Co-craft Questions
7, 17, 18	**MLR6:** Three Reads
4–11, 14, 16	**MLR7:** Compare and Connect
1, 3, 7, 10–13	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Materials
1	blindfolds
16, 17	colored pencils
1	six-sided dice
3, 15, 17, 18, 19	graph paper
1, 2, 4, 5, 7, 9–13, 16, 18	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
3, 13	rulers

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routine
17, 19	Gallery Tour
2, 6	Notice and Wonder
7	Number Talk
9	Take Turns
5, 7, 9, 11, 12	Think-Pair-Share
4	True or False

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 19

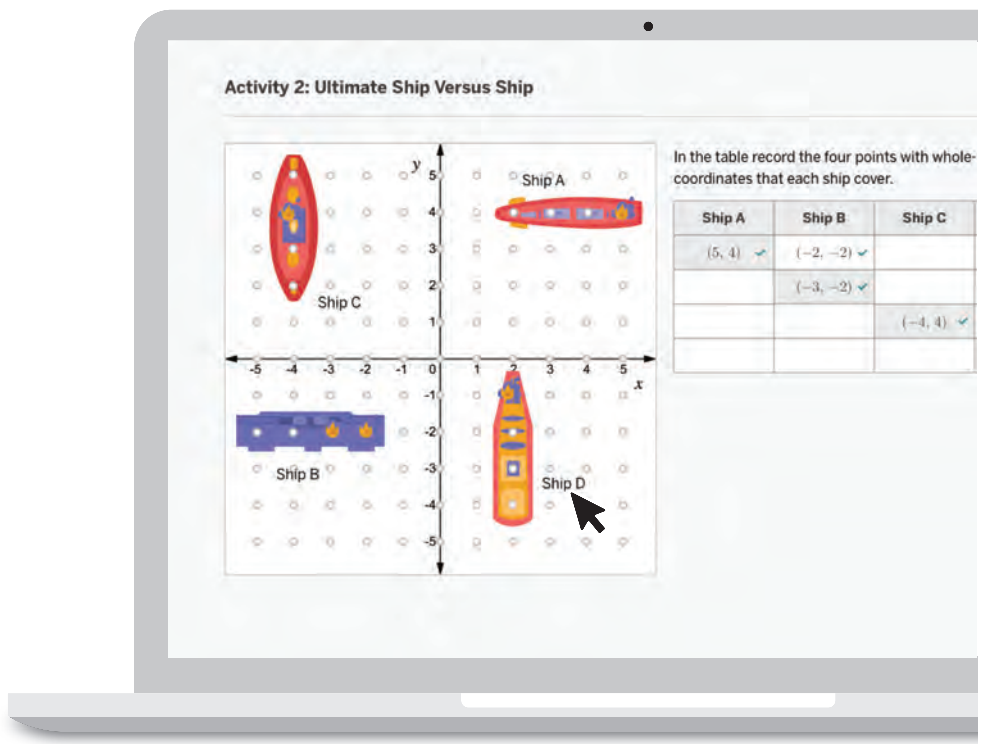
Amps POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

Ultimate Ship Versus Ship

Put on your student hat and work through **Lesson 13, Activity 2:**



Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities

- Jumping Fleas **(Lesson 7)**
- Extreme Temperatures **(Lesson 12)**
- Coordinated Archery **(Lesson 14)**
- Mystery Maze Design Challenge **(Lesson 18)**
- Image Race **(Lesson 19)**

Unit Study
Professional Learning

This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 1 introduces the idea of negative integers and negative rational numbers by having students move beyond zero on the number line. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

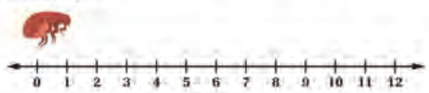
Do the Math

Put on your student hat and tackle Problems 1–3 from Lesson 7, Activity 1:

Activity 1 Jumping Fleas

The current world record for long jump is held by American Mike Powell, who jumped a distance of 8.95 m at the 1991 World Championships in Athletics. But the greatest jumper in the animal kingdom, relative to its size, is actually the flea! Fleas can jump over 200 times their own body length. Imagine a little 1.5-mm flea in a long jump competition jumping an impressive 300 mm (that's 0.3 m, or about 12 in.). Well, impressive for a flea — it's all relative!

A flea is jumping around on a number line, where each tick mark represents 1 in. You will be given the cut-out of a flea and a number line to record the flea's jumps. Use your flea to help you complete this activity. Each jump should be 12 in.



Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

- What was your strategy? Did you use more than one approach?
- What approaches might your students take?
- Do any approaches reveal a misconception that might arise for students?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

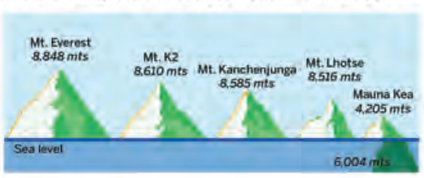
Notice and Wonder

Rehearse . . .

How you'll facilitate the ***Notice and Wonder*** instructional routine in the Lesson 2, Warm-up:

Warm-up Notice and Wonder

Consider the image showing some of the tallest mountains in the world and their heights. What do you notice? What do you wonder?



1. I notice . . .
2. I wonder . . .

Points to Ponder . . .

- What are low-floor observations that will allow all students to contribute? How will you track and sequence student ideas and questions to be shared?
- What are students wondering about the mathematical ideas that they will explore? How could you steer the conversation that direction if no students take it there?

This routine . . .

- Makes a mathematical task accessible to all students by asking two approachable, completely open-ended questions.
- Allows students to gain entry into the context, and ideally piques their curiosity.
- Is a first step to becoming familiar with a context and the mathematics that might be involved, so they can then make sense of related problems.

Anticipate . . .

- A wide variety of responses, both relevant and not. Some students may only notice or wonder one thing, while others may generate multiple responses for each.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Elicit and use evidence of student thinking.

This effective teaching practice . . .

- Helps you assess student progress toward the mathematical goals and objectives of the lessons and the unit. By knowing where your students are at, you can help them get to where they need to be!
- Allows you to adjust your instruction, based upon student responses, so that you can support your students and extend their learning.

Points to Ponder . . .

- How and when will I use student responses from this unit's assessments to help adjust my instruction?
- Where are there moments during the lessons' activities in which I can informally elicit evidence of my students' thinking? How will I approach these moments?

Math Language Development

MLR5: Co-craft Questions

MLR5 appears in Lessons 2, 6, 8, 9, 15, 16, and 18.

- In Lesson 6, after you conduct the ***Notice and Wonder*** routine, ask students to work with their partner to co-craft questions they have about the words and phrases indicating positive or negative numbers.
- In Lesson 18, ask students to study the maze before revealing the activity's task. Generating their own questions about the maze will help them produce mathematical language related to the coordinate plane.
- **English Learners:** DIsplay 1 or 2 sample questions and allow students to say their questions aloud as opposed to writing them, or vice versa.

Point to Ponder . . .

- As students work with each other to create mathematical questions they have about graphs or scenarios, how can you model for them how to use their developing mathematical vocabulary?

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Opportunities to provide visual support and guidance to help students process new information appear in Lessons 1–3, 9–16, 18, 19.

- In Lesson 3, demonstrate how to use the symmetry of a number line to label negative values once the positive values have been labeled.
- In Lessons 9 and 10, suggest students color code key words and phrases in the text that indicate certain inequality symbols.
- In Lesson 16, display a table that shows the coordinates of points that are directly across either axis from each other. Color code the coordinates to show how the signs change.
- In Lesson 17, have students cut out polygons using construction paper to overlay them on the coordinate plane to help them visualize the space as they design the wildlife refuge.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when your students may benefit from visual support or suggested guidance? What clues will you gather from your students?

Unit Assessments

- Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Read and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students in writing, interpreting, comparing, and locating negative rational numbers throughout this unit? Do you think your students will generally:
  - Wrestle with internalizing an understanding of negative values?
  - Have difficulty connecting their thinking and verbal descriptions of ideas to concrete representations?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-management skills.

Point to Ponder . . .

- Do your students step back and shift their perspective so that they are able to look for and make use of the structure of the number system — both as they previously understood it and now as it expands, particularly as represented by the number line and coordinate plane?

How Far? Which Way?

Let's think about magnitude and direction as we move around a flat surface.

Focus

Goals

1. **Language Goal:** Describe magnitude in the context of distance from one point to another. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Generate vocabulary associated with magnitude and direction. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Interpret magnitude and direction instructions to reach a given end point. **(Speaking and Listening)**

Coherence

Today

Students explore the concepts of magnitude and direction as they guide partners in a scavenger hunt and then a maze activity. In both activities, precise verbal language associated with magnitude and direction is necessary to guide a blindfolded partner. This lesson sets the stage for the important mathematical concepts of magnitude and direction as they relate to negative numbers, absolute value, and the four quadrants of the coordinate plane, which are explored in later lessons in this unit.

Previously

In Grade 5, students used ordered pairs to show how far and in what direction to travel from the origin of a single quadrant coordinate plane with a horizontal and vertical axis.

Coming Soon

In Lessons 2–8, students will be working with negative numbers on horizontal and vertical number lines. Relating the positions of opposite integers on number lines will be used to introduce the concept of magnitude for numbers (absolute value). Later, in Lessons 13–19, a pair of horizontal and vertical number lines come together to form the four-quadrant coordinate plane.

Rigor

- Students build **conceptual understanding** of magnitude and direction.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Small Groups	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (cards), pre-cut, one set per group
- Activity 1 PDF (grid), one per group
- Activity 2 PDF (as needed)
- blindfolds
- six-sided dice

Math Language Development

New words

- *magnitude**

*Students may confuse the mathematical meaning of magnitude with its everyday meaning related to the size of an object. Be ready to address the differences between these meanings.

Amps | **Featured Activity**

Exit Ticket
Real-Time Exit Ticket

Check in real time to see whether your students are correctly using magnitude and direction language with the digital Exit Ticket.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to trust their partner to lead them through the maze in Activity 2. Ask students to identify how they can build that relationship and trust so that they can be successful at getting through the maze. Guide students to see that precise and accurate communication lays a foundation for a healthy relationship.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 1**, limit the number of rounds to 1 or 2.

Small Groups | 5 min

Warm-up Hot 100

Students collectively generate words and phrases related to magnitude and direction to establish baseline language that will be referenced throughout the unit.

Unit 7 | Lesson 1 – Launch

How Far? Which Way?

Let's think about magnitude and direction as we move around a flat surface.

Warm-up Hot 100

In small groups, take turns writing down as many words or phrases as you can think of that are related to *magnitude* and *direction*.

- While one person writes, the person to their right will continue to roll a die until a 6 is rolled.
- At that time, the person to their left begins to write. The previous writer will roll the die.
- Continue until your group has 100 words or phrases, or time is called.

Magnitude	Direction
miles, nautical miles, kilometers, yards, feet, near, far, number of blocks/steps/etc., one/two/three/etc. "magnitude of an earthquake"	up, down, left, right, turn around, u turn, one way, vertical, horizontal, north, south, east, west, above, below, between, after, over, under, on top, under, behind, in front, next to, beside, upside down, right side up, outside, inside, through, around, middle, home, forward, backward

Log in to Amplify Math to complete this lesson online.

1 Launch

Distribute a die to each group and review the instructions. Briefly explain that the term *magnitude* informally means "distance or size."

2 Monitor

Help students get started by activating their prior knowledge. Ask, "What do helpful directions sound like if you are lost?"

Look for points of confusion:

- **Confusing magnitude with direction and writing words related to one column in the other column.** Help students understand that direction does not include the distance or size.

Look for productive strategies:

- Thinking of words or phrases related to different modes of transportation.

3 Connect

Display a two-column table with *Magnitude* and *Direction* at the top. Record words or phrases in the appropriate columns.

Define the term ***magnitude*** as the size of something or the distance of a number from 0.

Have groups of students share at least one word or phrase for each column, until every group has shared (and until all lists have been exhausted, if time permits). If students bring up the magnitude of an earthquake, explain that this number describes size as motion recorded by a seismograph.

Highlight words or phrases that *clearly* describe or relate to magnitude and distance (e.g., *to your left*, which is clear vs. *over there*, which is not clear).

Ask, "Would these words or phrases be enough to find your way around a place or town in which you are not familiar? What more information would you need?"

Math Language Development

MLR2: Collect and Display

Create a class display for the mathematical language students will develop in this unit. Begin by creating a two-column table for *Magnitude* and *Direction* as shown in the Warm-up. Ask students to share the words or phrases they came up with during the Warm-up and add these to the class display. Keep this class display posted throughout the unit for students to refer to during future class discussion.

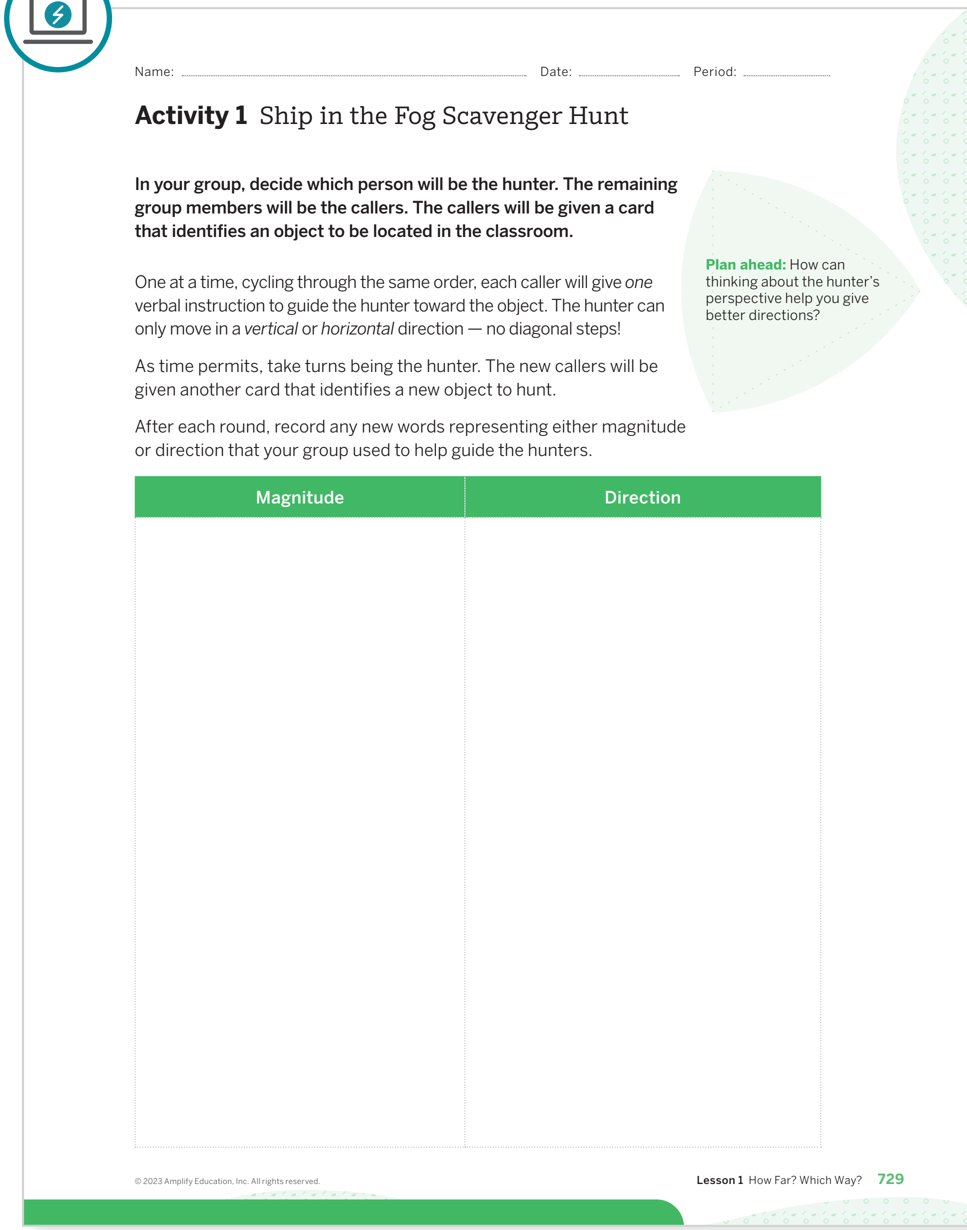
English Learners

Consider adding visual examples of words and phrases, such as the up/down and left/right arrows.

Small Groups | 15 min

Activity 1 Ship in the Fog Scavenger Hunt

Students work in groups to give directions for finding different objects around the classroom, focusing on early magnitude and direction language.


Name: Date: Period:

Activity 1 Ship in the Fog Scavenger Hunt

In your group, decide which person will be the hunter. The remaining group members will be the callers. The callers will be given a card that identifies an object to be located in the classroom.

One at a time, cycling through the same order, each caller will give *one* verbal instruction to guide the hunter toward the object. The hunter can only move in a *vertical* or *horizontal* direction — no diagonal steps!

As time permits, take turns being the hunter. The new callers will be given another card that identifies a new object to hunt.

After each round, record any new words representing either magnitude or direction that your group used to help guide the hunters.

Plan ahead: How can thinking about the hunter's perspective help you give better directions?

Magnitude	Direction

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Lesson 1 How Far? Which Way? 729

1 Launch

Review the directions at the top of the student page. Consider having groups start at different locations. Tell students to give clear and precise instructions related to magnitude and direction. Make sure the hunter's eyes are blindfolded or closed. Distribute the graph and one card at a time from the Activity 1 PDF to each group.

2 Monitor

Help students get started by having them review their lists of words from the Warm-up to identify ones that might be useful.

Look for points of confusion:

- **Not knowing what to do if an incorrect estimation of steps is given.** Additional instructions can be given such as, "Take X more steps forward," or "Take X steps backward."

Look for productive strategies:

- Using clear language that quickly navigates the hunter around the room.
- Accurately representing the classroom and its obstacles, and showing how the movements help guide the partner to the intended object.

3 Connect

Display the *Magnitude* and *Direction* table from the Warm-up and add to it any other words or phrases students found useful during the activity.

Ask, "What makes for an effective and clear instruction in this kind of activity?"

Highlight that, generally speaking, effective instructions will include the use of language describing both magnitude and direction.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students whether they have ever been lost or asked someone for directions and what kind of directions they received. Consider asking these questions:

- "What were some helpful words or phrases that you have received before when asking for directions? How were they helpful?"
- "What might be a sample instruction if someone only gave you the magnitude?" Sample response: Go 3 blocks.
- "What might be a sample instruction if someone only gave you the direction?" Sample response: Head East.

Math Language Development

MLR2: Collect and Display

While students work, continue to add words, phrases, and visual diagrams to the class display you started in the Warm-up as students complete this scavenger hunt activity. During the Connect, call students' attention to the words and phrases they used and ask them which words and phrases were the most helpful. Highlight those on the class display and ask students to record and circle those in the table in their Student Edition.

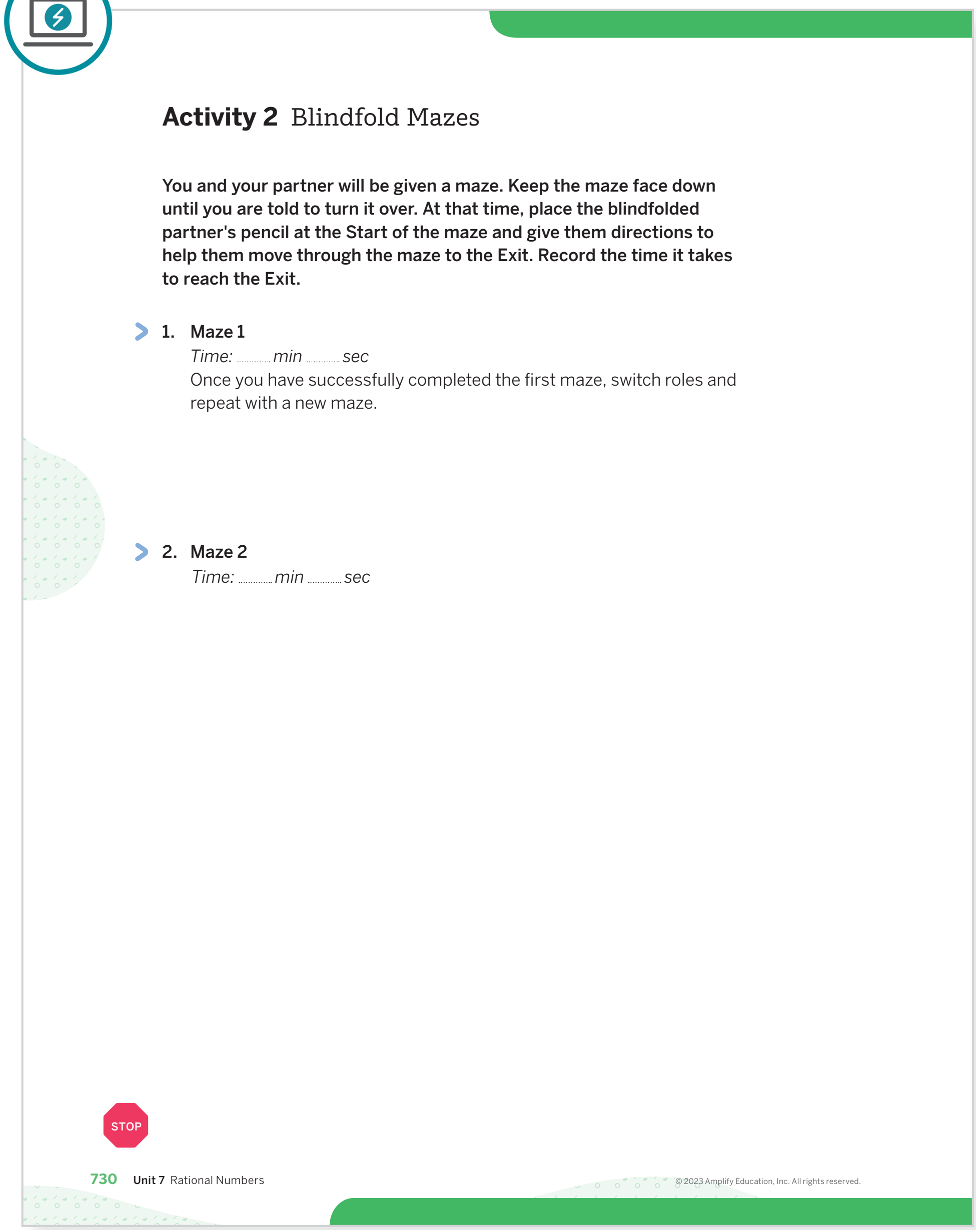
English Learners

Include visuals, such as arrow diagrams, to support students' visual understanding.

Pairs | 15 min

Activity 2 Blindfold Mazes

Partners take turns guiding each other through a paper maze puzzle, again using the language of magnitude and direction.



Activity 2 Blindfold Mazes

You and your partner will be given a maze. Keep the maze face down until you are told to turn it over. At that time, place the blindfolded partner's pencil at the Start of the maze and give them directions to help them move through the maze to the Exit. Record the time it takes to reach the Exit.

1. **Maze 1**

   Time: .... *min* .... *sec*

   Once you have successfully completed the first maze, switch roles and repeat with a new maze.

2. **Maze 2**

   Time: .... *min* .... *sec*

STOP

730 **Unit 7** Rational Numbers

1 Launch

Review the directions and say, "You will need to use precise magnitude and direction language to guide your partner through the maze. The challenge will be to describe magnitude so that your partner knows how far to travel each time."

2 Monitor

Help students get started by saying, "What magnitude and direction instructions can you give to your partner that will guide them to the first turning point?"

Look for points of confusion:

- **Saying "Move X units."** Explain that their partner cannot see the grid lines and therefore does not know what a unit is. Ask, "How can you describe what one unit is?"

Look for productive strategies:

- Using language that conveys the length of one unit.
- Interpreting what one unit looks like and "feels like."
- Confirming or communicating understanding of magnitude and direction.

3 Connect

Display the *Magnitude* and *Direction* table from the Warm-up and add to it any other words or phrases students found useful in the activity.

Have pairs of students share examples of clear and precise language, focusing on the way partners described magnitude.

Ask:

- "Why was it so important to establish what one unit represented?"
- "What have you come to understand about the preciseness of the language used for magnitude and direction?"

Highlight that magnitude and direction instructions need to be specific and clear in order to accurately convey instructions.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Differentiated mazes are provided in the Activity 2 PDF. If students need more processing time, have them use a maze that requires less instructing, such as Maze 3.

Accessibility: *Guide Processing and Visualization*

If students seem to spiral into unproductive work or discussion, allow them to give and receive a few instructions without the blindfold, and then have them complete the rest of the activity as intended.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share how they described the magnitude or the length of one unit, look for and highlight any similarities or differences among these descriptions. Students may have used approximate measurement units, such as inches or centimeters, or may have chosen to describe the length of one unit in different ways.

English Learners

Display the mazes and illustrate the different ways that students described the length of one unit.

Whole Class | 5 min

Summary Getting Where We're Going

Review and synthesize the use of magnitude and direction language across the various contexts and activities, within this lesson, involving moving around a plane.

Narrative Connections

Unit 7 Rational Numbers

Getting Where We're Going

Believe it or not, there are people to this day who still play chess by mail. There are lots of reasons you might want to play by mail. Maybe you prefer having time to think about your move. Maybe you do not live near others who like to play chess. Maybe your opponent happens to have breath like pickled gym socks.

Whatever the reason, these players need a way to coordinate with each other. Thankfully, chess boards are arranged in grids, using letters and numbers. So, to talk about any location on the board, players just use the right letters and numbers.

Grids are fantastic for helping us communicate about where things are. But to get there, you're going to need two things: magnitude and direction.

Magnitude tells us *how far* to travel, while direction tells us *which way* to travel.

By now, you noticed numbers are great for showing magnitude (in miles, in blocks, in captured pawns, in postage stamps, etc.), but is there a way to use numbers to talk about direction as well?

This is the question you will look at in these next lessons. Get on your hiking shoes! From the highest peaks to far off lands, we'll look at how we can use magnitude and direction to get us anywhere.

Welcome to Unit 7.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the *Magnitude* and *Direction* table from the Warm-up.

Highlight that the most effective language in regard to moving around on a flat plane combines numbers to describe magnitude and clear words and phrases to describe direction.

Formalize vocabulary: ***magnitude***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "What does *magnitude* mean?"
- "How did numbers help you talk about magnitude and direction?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on any terms and phrases related to the term *magnitude* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of movement on a flat plane by using precise language related to magnitude and direction.

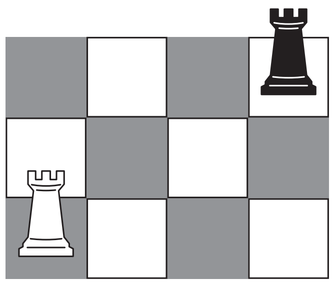
Amps Featured Activity Real-Time Exit Ticket

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.01

In chess, the rook can move one or more spaces in either the horizontal or vertical direction for each turn. Refer to this snapshot of part of a chess board.



1. Describe a set of moves to get from one rook to the square occupied by the opposing rook (assuming it is not moved).
   Sample response: Move up 2 spaces and then to the right 3 spaces.
2. What is the fewest number of moves it could take?
   2 moves
3. What is the most number of moves it could take, without crossing the same space more than once?
   Sample responses:
   - 11 moves (moving all by 1s; e.g., first to the right, then up and to the left, and then up and to the right, and so on)
   - 8 moves (if thinking moves of 1 space are only allowed when absolutely necessary)

Self-Assess

1 I don't really get it	2 I'm starting to get it	3 I got it

a. I can use words and phrases describing magnitude. 1 2 3

b. I can use words and phrases describing direction. 1 2 3

Lesson 1 How Far? Which Way?

Success looks like . . .

- **Language Goal:** Describing magnitude in the context of distance from one point to another. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Generating vocabulary associated with magnitude and direction. **(Speaking and Listening, Reading and Writing)**
  » Explaining moves of the rook in terms of direction and number of spaces.
- **Language Goal:** Interpreting magnitude and direction instructions to reach a given end point. **(Speaking and Listening)**

Suggested next steps

If students have trouble responding to the problems, consider having them draw different paths that could be taken from one rook to the other and then asking:

- For Problem 1, "Describe one of the routes you took to get the white/black rook to the white/black rook?"
- For Problem 2, "Which is the shortest route taken?"
- For Problem 3, "Which is the longest route taken?"

If students do not use a combination of magnitude and direction (i.e. they just say "move up" or "go one space"), consider:

- Referring them to Activity 1. Ask, "What were the best directions you gave to your partner/given to you by your partner? What made them effective?"

If students use diagonal routes from one rook to the other, consider:

- Reminding students that *horizontal* means left or right and *vertical* means up or down.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students used precise language associated with magnitude and direction to guide a blindfolded partner. How will this language support future work of magnitude and direction as they relate to negative numbers, absolute value, and the four quadrants of the coordinate plane?
- What surprised you as your students gave their partners directions in Activities 1–2? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Generating vocabulary associated with magnitude and direction.

Reflect on students' language development toward this goal.

- How did using the *Collect and Display* routine in Activity 1 help students make connections between the words and phrases they used to describe *magnitude* and *direction*?
- Did using this routine help them distinguish between *magnitude* and *direction*? Would you change anything the next time you use this routine?

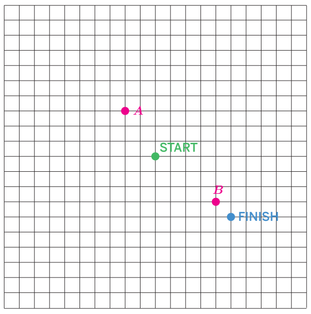
Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Use the grid and points labeled START and FINISH to answer parts a–c.



a. From the point labeled START, move two spaces to the left and three spaces up. Plot a point at this location and label it A.

b. From the point labeled START, move three spaces down and four spaces to the right. Plot a point at this location and label it B.

c. Describe how to move from point A to the point labeled FINISH.

Sample response: Move seven spaces to the right and then seven spaces down.

2. Use the symbols >, <, or = to compare each pair of numbers.

a. 0.4 **>** $\frac{10}{40}$

b. 12.030 **=** 12.03

c. $\frac{16}{17}$ **>** $\frac{11}{12}$

d. $\frac{13}{44}$ **<** $\frac{13}{43}$

e. 21 **>** 2.1

f. 31.2 **>** 31.02

3. The total force of a tug-of-war team is equal to 4.5 Newtons per team member. Complete the table to show the total force for each team size.

Team size (number of members)	8	23	n
Total force (Newtons)	**36**	**103.5**	**$4.5n$**

Name: ______ Date: ______ Period: ______

Practice

4. Which expressions are solutions to the equation $\frac{3}{4}x = 15$? Select *all* that apply.

(A.) $\frac{15}{\frac{3}{4}}$

B. $\frac{15}{\frac{4}{3}}$

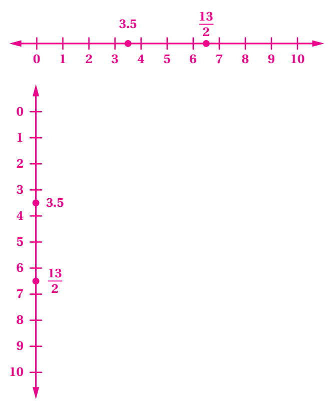
(C.) $\frac{4}{3} \cdot 15$

D. $\frac{3}{4} \cdot 15$

(E.) $15 \div \frac{3}{4}$

5. Andre and Clare's teacher asked them to draw a number line from 0 to 10. Andre drew his number line horizontally, and Clare drew hers vertically.

a. Draw both of their number lines. Then plot and label the locations of 3.5 and $\frac{13}{2}$.



b. Describe a real-world context for which Andre's number line might be the best representation.

Answers may vary.

c. Describe a real-world context for which Clare's number line might be the best representation.

Answers may vary.

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Grade 5	1
	2	Grade 5	1
	3	Unit 6 Lesson 2	2
	4	Unit 6 Lesson 8	2
Formative	5	Unit 7 Lesson 2	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 1

Negative Numbers and Absolute Value

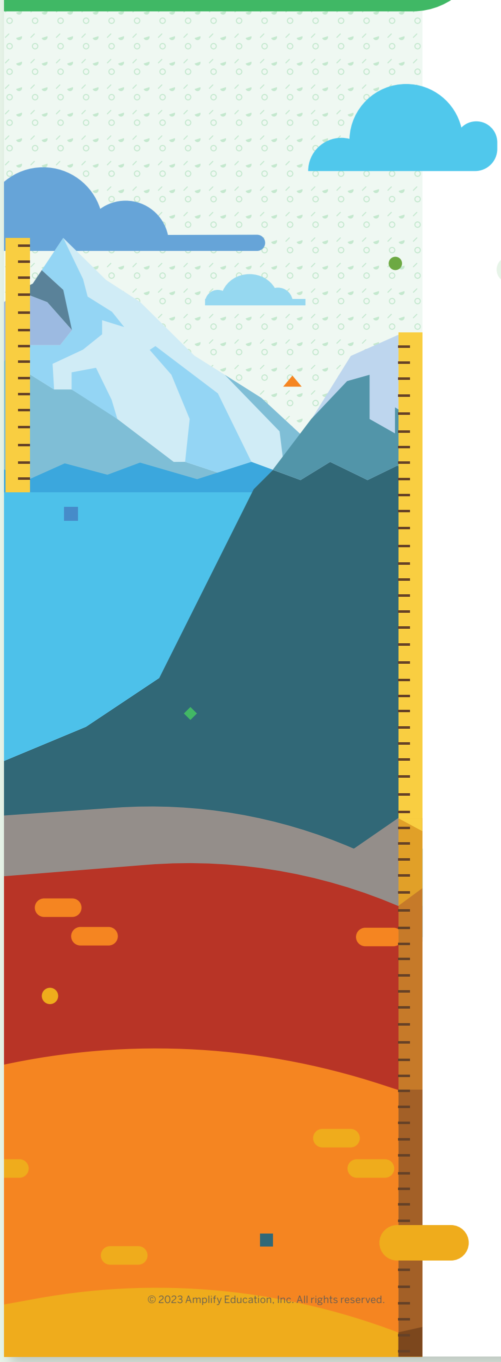
Whole Class

In this Sub-Unit, students explore the concept of negative numbers and absolute value, often in the familiar contexts of elevation and temperature.

SUB-UNIT

1 Negative Numbers and Absolute Value

Narrative Connections



What's the tallest mountain in the world?

If you said "Mount Everest," you're only *kind* of right.

Everest is certainly the tallest mountain *above sea level*. From its peak to where the ocean meets the land, Everest is roughly 29,035 ft tall.

As a surface-dwelling human, it figures you'd start at sea level. But if you were a devil worm nematode – a creature that spends its life more than a mile underground – you might suggest a different contender.

Meet Chimborazo of Ecuador, the tallest mountain measured *from the center of the Earth*.

You might think that title would go to Everest again. After all, if you're the farthest from sea level, you'd also have to be the farthest from the Earth's center...right?

While that would be true if the Earth were a perfect sphere, in reality, the Earth is more like a *slightly* squashed tomato (technically called an *oblate spheroid*). The Earth is a little flatter at its poles, and it bulges out at the equator. And since Chimborazo sits on that bulge, it's about 1.3 miles taller than Everest, putting it at a whopping 3,967.1 miles from the Earth's center.

All this goes to show that we should be careful when we talk about measuring distances. It's not always as simple as breaking out a really long measuring tape. Sometimes the first thing we have to do is decide *where we're measuring from*.

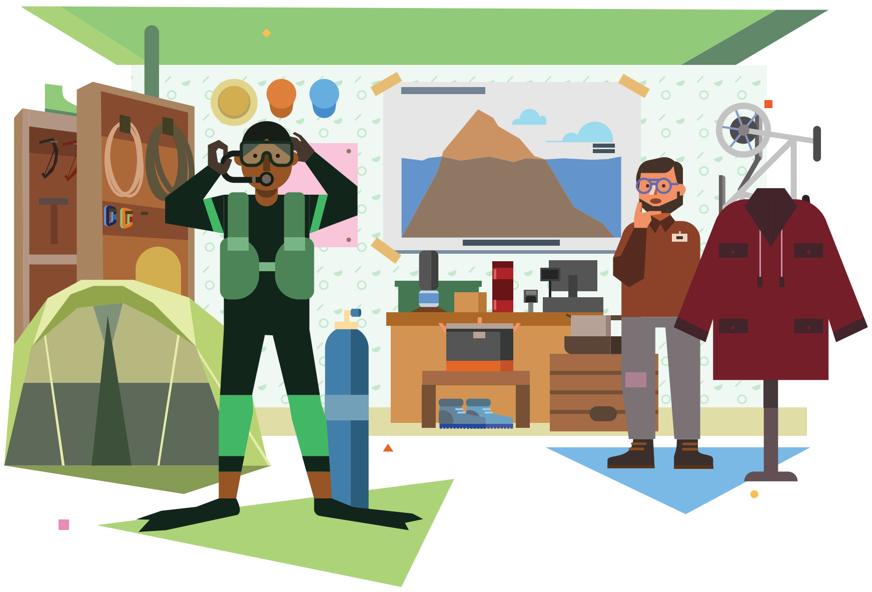
Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how numbers can be used to measure distance and direction in the following places:

- **Lesson 2, Activities 1–2:** The Apartment Building, High Places, Low Places
- **Lesson 4, Activity 2:** Comparing Elevations
- **Lesson 7, Activities 1–3:** Jumping Fleas, Absolute Value With Jumping Fleas, Absolute Value WIth Elevation and Temperature
- **Lesson 8, Activity 1:** Comparing Elevations and Distances From Sea Level

Positive and Negative Numbers

Let's explore how we represent elevations.



Focus

Goals

1. **Language Goal:** Comprehend the words *positive* and *negative* and the symbol "–". Say *negative* when reading numbers written with the "–" symbol. **(Speaking and Listening, Reading)**
2. **Language Goal:** Interpret positive and negative numbers that represent elevation, and understand the convention of what *below zero* typically means in this context. **(Speaking and Listening, Reading and Writing)**
3. Recognize that a number line can be extended to represent negative numbers.

Coherence

Today

Students extend their understanding of numbers to now include negative numbers by exploring elevation. In this context, they associate every value with a physical state in the real world. Zero represents sea level, numbers greater than zero are higher than sea level, and numbers less than zero are lower than sea level. Students then abstract elevations to positive and negative numbers on a vertical number line, which is used to compare the relative location of each elevation.

Previously

By Grade 6, students have spent considerable time developing their understanding of, and fluency operating with, positive numbers. They have also used both horizontal and vertical number lines to represent and compare the values of positive numbers.

Coming Soon

In Lesson 3, students will apply their knowledge from working with positive and negative numbers in the context of elevation to a new context of temperature. They will also investigate opposites, and be able to identify and generate examples.

Rigor

- Students build **conceptual understanding** of positive and negative integers in the context of elevation.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, *Are you ready for more?* (as needed)

Math Language Development

New words

- ***negative number***
- ***positive number***

Amps | **Featured Activity**

Activity 2
Interactive Graphs

Students plot elevations on a number line and are able to easily manipulate values to see highest and lowest elevation points.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not understand how to read a vertical number line, especially when it extends to negative numbers. Have students consider their understanding of a horizontal number line and then compare its structure to that of a vertical number line to build their confidence in Activity 2.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 2**, Problems 1 and 2 could be done as a whole class.

Independent | 5 min

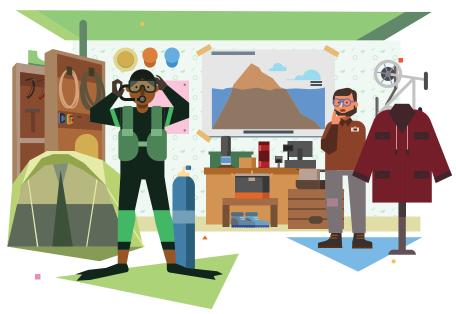
Warm-up Notice and Wonder

Students are introduced to the concept of zero elevation (sea level), and begin to recognize a need for different numbers to represent values above zero and values below zero.

Unit 7 | Lesson 2

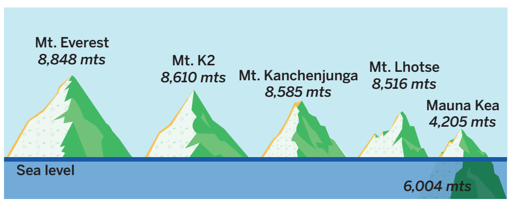
Positive and Negative Numbers

Let's explore how we represent elevations.



Warm-up Notice and Wonder

Consider the image showing some of the tallest mountains in the world and their heights. What do you notice? What do you wonder?



1. I notice . . .
   Sample responses:
   - One mountain, Mauna Kea, goes below sea level, while the others are all completely above sea level.
   - Mount Everest is the tallest.

2. I wonder . . .
   Sample responses:
   - If Mount Everest really is the tallest mountain in the world.
   - Why Mauna Kea goes below sea level while the others do not.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate students' background knowledge by asking, "What are some tall mountains you have heard of?" Conduct the ***Notice and Wonder*** routine with the image of the mountains.

2 Monitor

Help students get started by asking, "How are the mountains different? How are they similar?"

Look for points of confusion:

- **Thinking that these mountains are actually next to each other in real life.** Explain that this is just an image showing them next to each other, for purposes of comparison.
- **Only focusing on the peaks above sea level.** Remind students to look at the bases of the mountains as well.

Look for productive strategies:

- Recognizing sea level corresponds to an elevation of 0 m and that part of Mauna Kea is below sea level.

3 Connect

Display the image of the five mountains, as well as a two-column anchor chart with the headings *Positive* and *Negative*, to capture key words introduced in Lessons 2–8.

Have students share their observations, focusing on those who considered elevations above and below sea level, and those who claimed that Mauna Kea is the tallest mountain.

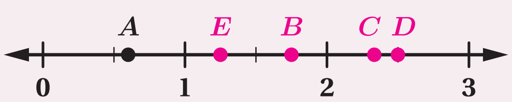
Ask, "How does this image add to *or* change what you know about distance measurement?"

Highlight that Mount Everest has the highest peak (above sea level), but Mauna Kea is the tallest mountain, from base to peak. *Elevation* is used to represent the vertical distance from sea level to a location. Have students discuss where to place terms such as *above/below sea level*, *trench*, *peak*, etc., on the anchor chart.

Power-up

To power up students' ability to plot values between tick marks on horizontal and vertical number lines, have students complete:

Plot each value on the number line. The first value is done for you.



A 0.6 B $1\frac{3}{4}$ C $2\frac{1}{3}$ D 2.5 E $1\frac{1}{4}$

Use: Before the Warm-up.

Informed by: Performance on Lesson 1, Practice Problem 5.

Pairs | 10 min

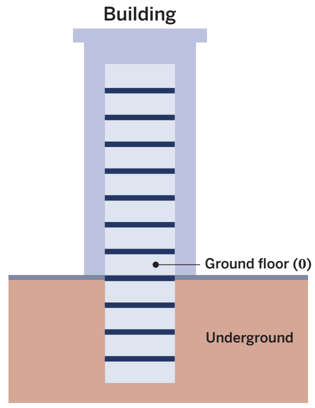
Activity 1 The Apartment Building

Students further discover the need for negative numbers, now in the context of labeling above-ground and below-ground floors in an apartment building.

Name: ______________________ Date: __________ Period: __________

Activity 1 The Apartment Building

An apartment building has a total of 12 floors. The ground floor is Floor 0, and the floor above that is Floor 1.



1. Priya lives on Floor 2 and wants to visit her friend Mai, who lives on Floor 7. How many floors up does Priya need to go?
   5 floors

2. Priya and Mai are now both on Floor 7 and decide to race down 9 floors. On what floor will they end up?
   2 floors below the ground floor or Floor −2

3. Priya and Mai now want to exit the building from the ground floor. How many floors away is the ground floor? In which direction do they need to go?
   2 floors up

4. How did you name the resulting floor in each situation?
   Answers may vary.

5. What does it mean when a floor is above Floor 0? Below Floor 0?
   Sample response: When a floor is above 0, it is positive. When a floor is below 0, it is negative.

6. Do numbers below 0 make sense outside of the context of an apartment building? Give at least one example to support your thinking.
   Sample response: Yes, in negative temperatures or the depths at which fish swim in the ocean.

1 Launch

Activate students' background knowledge by asking, "Have you ever been in or seen a building with multiple floors? What about a building that has underground floors?"

2 Monitor

Help students get started by modeling pointing at the starting floor and counting up or down one.

Look for points of confusion:

- **Counting the starting floor as a move.** Remind students that a change of even 1 floor will place them in a new location.
- **Labeling a below-ground and an above-ground floor the same.** Ask, "Is there a way to name the floors differently so these don't get confused?"

Look for productive strategies:

- Labeling below-ground floors as whole numbers with a negative sign.
- Using mental math in place of, or in addition to, using the apartment building visual while working through the problems.

3 Connect

Have students share what they discovered when moving down 9 floors from Floor 7, and how they named the floors below the ground floor.

Highlight how the building is like a vertical number line, with floors above ground level (Floor 0) labeled by positive numbers and floors below ground level labeled by negative numbers.

Define:

- ***positive numbers*** as numbers greater than zero.
- ***negative numbers*** as numbers less than zero.

Ask, "Are there any similarities or differences between Floor 1 and the floor directly below ground level?"

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they have been in a building with underground floors. Have them think of different ways they can describe a floor that is below the ground. Record and display these phrases. Sample responses: underground, below the ground, basement, sub-ground, cellar, lower level, etc.

Extension: *Math Enrichment*

Have students complete the following problem:
Suppose Priya is 3 floors below the ground floor and Mai is on Floor 6. What is the distance, in the number of floors, between Priya and Mai? They are 9 floors apart.

Math Language Development

MLR2: Collect and Display

During the Connect, as students respond to the Ask question, collect and display any key words and phrases they use to describe the similarities and differences between Floor 1 and the floor directly below ground level. For example, look for students who use language such as, "same distance from the ground floor," "same distance from Floor 0," "Floor 1 is above ground," "Floor −1 is below ground," etc.

English Learners

Annotate the image of the building by writing *positive numbers* to indicate the floors above the ground floor and *negative numbers* to indicate the floors below the ground floor.

Pairs | 20 min

Activity 2 High Places, Low Places

Students revisit the concept of elevation and use the structure of a vertical number line that includes negative numbers to compare relative locations.

Amps Featured Activity Interactive Graphs

Activity 2 High Places, Low Places

1. The table shows the elevations of several U.S. cities.

City	Elevation (ft)
Harrisburg, PA	320
Bethel, IN	1,211
Denver, CO	5,280
Coachella, CA	−22
Death Valley, CA	−282
New York City, NY	33
Miami, FL	0

a. Of the cities in the table, which has the second highest elevation?
Bethel, IN

b. How would you describe the elevation of Coachella, CA, in relation to sea level?
It is 22 ft below sea level.

c. How would you describe the elevation of Death Valley, CA, in relation to sea level?
It is 282 ft below sea level.

d. How would you describe the elevation of Miami, FL?
It is right at sea level.

2. A city not listed in the table has a higher elevation than Coachella, CA. Select *all* numbers that could represent the city's elevation. Be prepared to explain your thinking.

A. −11 ft
B. −35 ft
C. 4 ft
D. −8 ft
E. 0 ft

738 **Unit 7** Rational Numbers

1 Launch

Remind students that the term *sea level* is used to mean an elevation of zero. Make sure that students understand the meaning of *trench* and that all trenches are below sea level, with elevations represented with a "—."

2 Monitor

Help students get started by asking what an elevation with a negative sign would mean.

Look for points of confusion:

- **Only providing a number in their response.** Remind students to include 'below/at/above sea level' when appropriate (Problem 1).
- **Ignoring the signs when comparing the numbers in Problem 1.** Have students create a vertical number line and plot each point.
- **Choosing only positive numbers for Problem 2.** Refer back to the number line and remind students that negative numbers can still be greater than other negative numbers.
- **Plotting negative numbers incorrectly on the number line.** Model for students how to look above and below the number plot.

Look for productive strategies:

- Knowing that on a vertical number line, negative elevations are marked below zero and positive elevations are marked above zero.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Distribute copies of pre-made vertical number lines and demonstrate how to plot a positive elevation, such as the elevation of New York City at 33 ft. Annotate the number line showing sea level at 0 ft. Then ask students how the elevation of Coachella, CA, would relate to the elevation of New York City and where it should be plotted on the number line.

Extension: *Math Enrichment*

Have students complete the extension activity using the Activity 2 PDF, Are you ready for more?, in which they will study the elevation of a spider as it spins a web in a certain way. The spider's elevation will be −30 in. after an hour has passed.

Math Language Development

MLR5: Co-craft Questions

To help students use language related to positive and negative numbers within the context of elevation, show them the table in Problem 1 and ask pairs to work together to write 2–3 mathematical questions they have about the data they see in the table. Ask volunteers to share their questions with the class.

English Learners

Students might have heard the word *elevator* and know it is something that "takes you up and down". Tell them that *elevation* is similar in that it describes how high (up) or how low (down) a place is located, relative to a reference location, such as ground floor or sea level.

Pairs | 20 min

Activity 2 High Places, Low Places (continued)

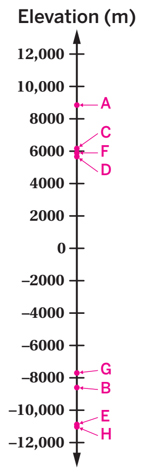
Students revisit the concept of elevation and use the structure of a vertical number line that includes negative numbers to compare relative locations.

Name: ________ Date: ________ Period: ________

Activity 2 High Places, Low Places (continued)

The table shows the elevations of several geological landmarks, representing some of the highest points on land and lowest points in the oceans.

Label	Landmark	Location	Elevation (m)
A	Mount Everest	Nepal, Asia	8,848
B	Puerto Rico Trench	Atlantic Ocean	−8,600
C	Denali	United States, North America	6,168
D	Pichu Pichu	Peru, South America	5,664
E	Tonga Trench	Pacific Ocean	−10,882
F	Mount Kilimanjaro	Tanzania, Africa	5,895
G	Sunda Trench	Indian Ocean	−7,725
H	Mariana Trench	Pacific Ocean	−11,033



3. Refer to the table.

   a. Plot a point on the vertical number line for each location, and label it using the corresponding capital letter from the table.

   b. Which landmark is the lowest? What is its elevation?
   Mariana Trench at −11,033 m

   c. Which landmark is the highest? What is its elevation?
   Mt. Everest at 8,848 m

   d. What would a point at 0 represent on your vertical number line? What do points above 0 represent? Points below 0?
   Sample response: A point at 0 represents sea level. Points below 0 represent elevations below sea level, like the trenches. Points above sea level represent elevations above sea level, like the mountains.

   e. Which is farther from sea level: the bottom of the Mariana Trench or the top of Mount Everest? Explain.
   Sample response: The bottom of the Mariana Trench is farther away from sea level (more than 11,000 m away from sea level), while the top of Mount Everest is not even 9,000 m away.

STOP

3 Connect

Display the table from Problem 1, followed by the table and vertical number line from Problem 3.

Have students share how they knew a city was below sea level (Problem 1) and how they determined where to plot the landmarks on their vertical number line in regard to the highest and lowest points (Problem 3).

Highlight that elevation of sea level is always zero. Elevations measured greater than zero are positive and elevations measured less than zero are negative, indicated by a "−" sign. Zero is neither less than nor greater than itself, so it is neither positive nor negative. Discuss with students where to add new words to the anchor chart, e.g., *trench*.

Ask:

- "If you are standing next to the water at the beach, will you always be at sea level?"
- "Looking at the table, what is the highest elevation? (Point out this is actually the highest point on Earth!) What is the lowest elevation? (Point out this is actually the lowest point on Earth!)"

Differentiated Support

Extension: *Math Around the World*

Where did the concept of negative numbers first appear? Tell students that around 200 BCE, Chinese mathematicians represented positive and negative numbers using a number rod system. Positive numbers were red and negative numbers were black. These rods were typically used in the context of receiving money (positive) and spending money (negative). Consider showing students a visual example of these number rods.

Around 300 CE, Alexandrian mathematician Diophantus created an equation in which the solution was negative. He called the negative solution "absurd." The ancient Greeks did not work with negative numbers because they mostly dealt with lengths, areas, and volumes.

In India, negative numbers appeared around 620 CE in the work of Indian mathematician Brahmagupta who discussed the concepts of "fortunes" represented by positive numbers and "debts" represented by negative numbers. He also documented the first set of "rules" for operations with positive and negative numbers.

Negative numbers did not begin to appear in Europe until the 15th century when mathematicians began to study ancient texts from non-Western mathematicians. Even then, it wasn't until the 19th century that negative numbers were more fully accepted and documented among European mathematicians. Let students know that mathematicians have wrestled with the concept of negative numbers, so they shouldn't feel bad if they wrestle with the concept as well.

Whole Class | 5 min

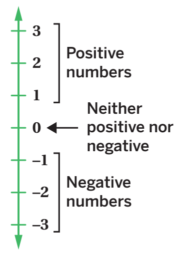
Summary

Review the fact that numbers less than (or below) zero are called *negative numbers* and synthesize their connections to elevation.

Summary

In today's lesson . . .

You have seen that numbers can be less than zero, such as numbers representing elevations below sea level. Until now, you have only been working with positive numbers and zero, which include whole numbers, fractions, and decimals. ***Positive numbers*** are numbers that are greater than zero. ***Negative numbers*** are numbers that are less than zero. Zero is *neither* positive *nor* negative. The meaning of a negative number in a context depends on the meaning of zero in that context.



For example, elevation describes the height or depth of a location or object relative to sea level. An elevation of 0 represents *sea level*. So, a *positive* elevation is *above* sea level, and a *negative* elevation is *below* sea level.

Reflect:

Synthesize

Display the number line from Activity 2.

Highlight that *positive numbers* are numbers that are greater than zero. *Negative numbers* are numbers that are less than zero, written with a "–" sign in front. Review that sea level is measured at 0, and that *elevation* refers to heights above or below sea level.

Formalize vocabulary:

- *positive number*
- *negative number*

Ask:

- "Can anyone think of other real-world examples of how to use negative numbers aside from elevation?" Sample responses: temperature, account balances, gains/losses in football
- "Can you think of any other places that have elevations above or below sea level?" Sample responses: lakes, deserts, cities or towns, any location on Earth
- "Can you think of any examples of animals that might swim or fly above or below sea level? Give an approximate elevation using the appropriate signs." Answers may vary. The focus here is not on the approximate elevations, but on using the appropriate signs.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean for a value to be less than zero?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *positive number* and *negative number* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying positive and negative numbers as greater than or less than another given number.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.02

State whether you agree or disagree with each statement. Explain your thinking.

1. An elevation of 35 ft is the same as an elevation of -35 ft.
   I disagree; Sample response: One is above sea level, and the other is below sea level.

2. A city that has an elevation of -17 m is closer to sea level than a city that has an elevation of -40 m.
   I agree; Sample response: -17 is closer to 0 than -40.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain what 0, positive numbers, and negative numbers mean in the context of elevation. 1 2 3

b. I can use positive and negative numbers to describe elevation. 1 2 3

c. I know what positive and negative numbers are. 1 2 3

Success looks like . . .

- **Language Goal:** Comprehending the words *positive* and *negative* and the symbol "–". Saying *negative* when reading numbers written with the "–" symbol. **(Speaking and Listening, Reading)**
- **Language Goal:** Interpreting positive and negative numbers that represent elevation, and understanding the convention of what *below zero* typically means in this context. **(Speaking and Listening, Reading and Writing)**
  - » Explaining the difference between 35 ft and –35 ft in Problem 1.
- **Goal:** Recognizing that a number line can be extended to represent negative numbers.

Suggested next steps

If students agree with the statement in Problem 1, consider:

- Asking, "Do the two numbers "look" identical?"
- Using a number line or having students draw one to show where the two elevations would be represented.

If students are confused with how to properly order the negative numbers in Problem 2, consider:

- Using a number line, or having students draw one, to show where the two elevations would be represented.
- Reviewing the locations of negative elevations plotted on the number line from Activity 2, Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

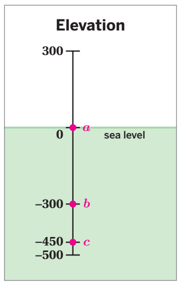
- In this lesson, students used negative numbers to describe elevations below sea or ground level. How did that build on students' work in previous grades with positive numbers?
- Students moved from recognizing a need for negative numbers (Warm-up and Activity 1) to using a vertical number line to compare relative locations of positive and negative values (Activity 2). What challenges did students encounter as they progressed through the lesson? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

1. The statements in parts a–d describe the movements of a whale swimming in the ocean. Each statement starts from the whale's elevation in the previous statement.



a. A whale is at the surface of the ocean to breathe. What is the whale's elevation?
0 ft

b. The whale swims down 300 ft to feed. What is the whale's elevation now?
–300 ft

c. The whale swims down 150 ft more to rest. What is the whale's elevation now?
–450 ft

d. Plot and label the three elevations as points on the vertical number line.

2. Using what you know about positive and negative numbers, answer these questions about sea level and elevation.

a. Is an elevation of –10 ft closer to sea level or farther from sea level than an elevation of –8 ft?
Farther

b. Is an elevation of 3 m closer to sea level or farther from sea level than an elevation of –4 m?
Closer

3. Complete the table for each elevation with the correct sign and its relation to sea level. If you have time, add in extra rows to think of your own examples.

Elevation	Corresponding sign (+/–/no sign)	Relation to sea level (above/below/at)
0 miles	no sign	at sea level
–10 ft	–	below sea level
12 m	+	above sea level

Name: Date: Period:

4. Write an equation to represent each situation. Then solve the equation.

a. Andre drinks 15 oz of water, which is $\frac{3}{5}$ of a bottle. How much does the bottle hold? Use x to represent the number of ounces of water the bottle holds.
25 oz; $\frac{3}{5}x = 15$;
$x = 25$

b. A bottle holds 15 oz of water. Jada drank 8.5 oz of water. How many ounces of water are left in the bottle? Use y to represent the number of ounces of water left in the bottle.
6.5 oz; $15 - 8.5 = y$;
$y = 6.5$

c. A bottle holds z ounces of water. A second bottle holds 16 oz, which is $\frac{8}{5}$ times as much water as the first bottle. How much does the first bottle hold?
10 oz; $\frac{8}{5}z = 16$;
$z = 10$

5. A rectangle has an area of 24 square units, and one side has a length of $2\frac{3}{4}$ units. Determine the length of the other side of the rectangle. Show your thinking.

$8\frac{8}{11}$ units;

$$24 = 2\frac{3}{4}s$$
$$24 \div 2\frac{3}{4} = 2\frac{3}{4}s \div 2\frac{3}{4}$$
$$24 \div \frac{11}{4} = s$$
$$24 \cdot \frac{4}{11} = s$$
$$s = \frac{96}{11} \text{ or } 8\frac{8}{11}$$

6. Estimate the value of the point plotted on the number line as a fraction and as a decimal.

Sample responses shown.

a. Fraction: $\frac{1}{2}$

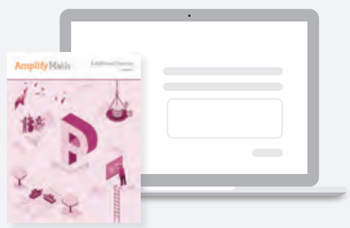
b. Decimal: 0.5

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activities 1 and 2	2
	3	Activities 1 and 2	2
Spiral	4	Unit 6 Lesson 7	2
	5	Unit 4 Lesson 13	3
Formative	6	Unit 7 Lesson 3	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

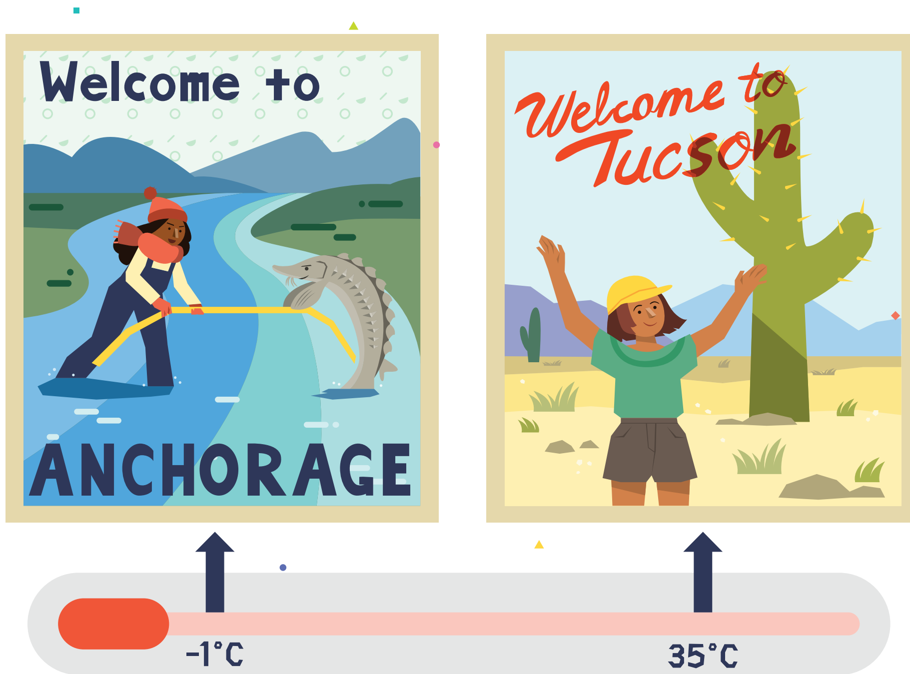


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 7 | **LESSON 3**

Points on the Number Line

Let's plot positive and negative numbers on the number line.



Focus

Goals

1. **Language Goal:** Comprehend that two numbers are called *opposites* when they are the same distance from zero, but on different sides of a number line. **(Speaking and Listening, Reading and Writing)**

2. **Language Goal:** Interpret a point on a number line that represents a positive or negative rational number. **(Speaking and Listening, Reading and Writing)**

3. Represent a positive or negative rational number as a point on a number line.

Coherence

Today

Students extend previous work with interpreting equally-spaced partitions on the negative side of the number line, but now in the context of temperature (in degrees Celsius), where $0°C$ is a meaningful pivot — the freezing point. They then create folded number lines to reason about opposites, which are pairs of numbers that are the same distance from 0 on a number line. They re-interpret distance on the number line in the context of negative numbers.

Previously

In Lesson 2, students first learned about negative numbers in the context of elevation. They also interpreted interval spaces on the negative side of a vertical number line.

Coming Soon

In Lessons 4–5, students will compare and order rational numbers, applying and reinforcing their understanding of the number line. Then, in Lessons 6–7, they will build on their knowledge of opposites and explore the concept of absolute value.

Rigor

- Students solidify **conceptual understanding** of rational numbers using number lines.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- graph paper
- rulers or straightedges

Math Language Development

New words

- ***integer***
- ***rational number***
- ***opposite***

Review words

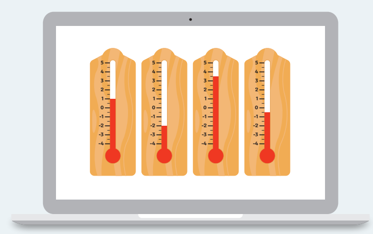
- *positive number*
- *negative number*

Amps | Featured Activity

Activity 1

Interactive Thermometers

Students apply their knowledge of the number line and plot values onto a thermometer.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might not immediately recognize the number line's symmetry across 0. They might become stressed because the numbers on either side of 0 have opposite signs. Ask students how they can regulate their thoughts to control their stress levels as they explore new concepts on the number line and its structure.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Problems 1–3 can be done as a whole class and Problem 4 may be omitted.

Pairs | 5 min

Warm-up A Point on the Number Line

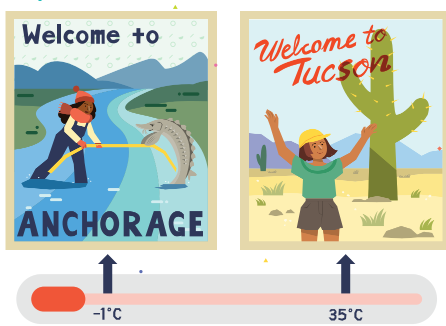
Students identify possible positive rational numbers that accurately represent a point on the number line. This prepares them to locate negative fractions on the number line later.

Name: ____________ Date: ________ Period: ________

Unit 7 | Lesson 3

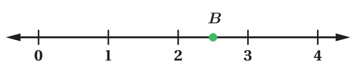
Points on the Number Line

Let's plot positive and negative numbers on the number line.



Warm-up A Point on the Number Line

Select *all* of the following numbers that *could* be the value of B.



A. 2.5

B. $\frac{2}{5}$

C. $\frac{5}{2}$

D. 2.01

E. $\frac{25}{10}$

F. 2.49

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by activating their prior knowledge. Ask them how a number line is structured.

Look for points of confusion:

- **Selecting $\frac{2}{5}$.** Have students write the fraction as an equivalent decimal.
- **Forgetting that there is more than one way to write a number.** Offer similar examples, such as "If $2 = \frac{10}{5}$, what might 2.5 be equal to?"
- **Thinking the point cannot possibly represent both 2.5 and 2.49.** Remind students that a precise value cannot be determined by visual inspection.

Look for productive strategies:

- Knowing that a fraction greater than 2 will be an improper fraction.
- Realizing that 2.49 is a plausible response, based on estimation or rounding strategies.

3 Connect

Highlight that benchmarks on the number line (in this case, 2 and 3) and can be used to identify equivalent expressions of a number on the number line.

Ask:

- "Were there any answer choices you knew right away were incorrect? How did you know?"
- "Were there any answer choices you initially dismissed, but then came back to as possibilities?"
- "What are some other numbers not listed that could be the value of Point B?"

MLR Math Language Development

MLR3: Critique, Correct, Clarify

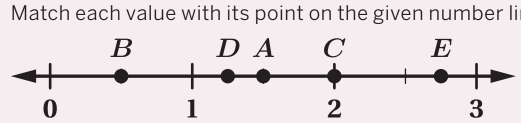
Before students begin the Warm-up or during the Connect, consider displaying an incorrect statement, such as "Point B could have a value of $\frac{7}{2}$." Ask these questions:

- ***Critique:*** "How do you know this statement is incorrect?"
- ***Correct:*** "If Point B had a value of $\frac{7}{2}$, where should it be located on the number line?"
- ***Clarify:*** "How can you use this incorrect statement to help you think about the numbers listed in the answer choices for the Warm-up? What strategies can you use to determine which numbers could be the value of B?

Power-up

To power up students' ability to identify fraction and decimal values plotted on a number line, have students complete:

Match each value with its point on the given number line.



A 1.5 D $1\frac{1}{4}$ E $2\frac{3}{4}$ C 2 B $\frac{1}{2}$

Use: Before the Warm-up.

Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3.

Pairs | 15 min

Activity 1 What is the Temperature?

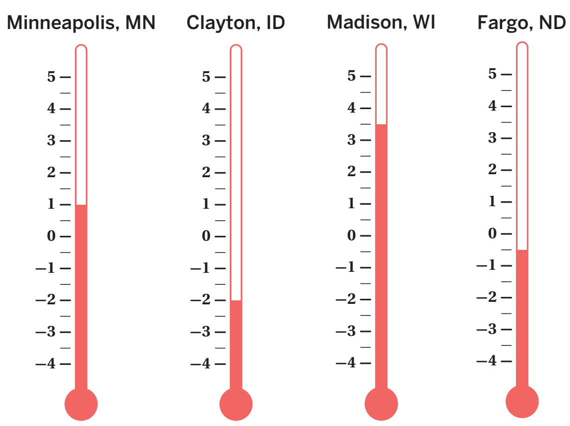
Students continue to build understanding of the negative side of the number line using a thermometer, including non-integer negative values.

Amps Featured Activity Interactive Thermometers

Activity 1 What is the Temperature?

1. Each of the four thermometers shows the temperature, in degrees Celsius, in a different city at the same time of day on March 2, 2020. Complete the table by writing the temperature in each city.

City	Temperature (°C)
Minneapolis	1
Clayton	-2
Madison	3.5
Fargo	-0.5



1 Launch

Activate students' background knowledge by asking them what they know about thermometers.

2 Monitor

Help students get started by asking what the scale of the ticks on the thermometer is. Then have them identify the thermometers showing positive temperatures.

Look for points of confusion:

- **Attempting to count or add the totals above and below 0.** If students claim, for example, the temperature in Minneapolis is 5°C, have them draw a vertical number line that is not a thermometer and plot a point to represent the temperature.
- **Estimating labels for the thermometer in Problem 2 without establishing a pattern.** Have students test out their pattern to make sure it applies throughout the thermometer.
- **Identifying the temperature in Duluth as −1.5°C or in Problem 3 as 2.5°C.** Have students identify the integers above and below the liquid mercury mark. Ask, "What would the middle tick mark represent if it was between 0 and 1?"
- **Incorrectly determining "cooler than" or "warmer than" temperatures.** Have students plot the temperatures on a number line or thermometer and count spaces to the left or right.

Look for productive strategies:

- Understanding that the positive and negative sides of a number line appear to be ordered differently. For example, 5 is above 4 on the positive side, but −5 is below −4 on the negative side.
- Using the "in between" marks correctly on both the positive and negative sides of 0, by looking at the integers above and below each mark.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can plot values on an interactive thermometer.

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

If students need more processing time, have them focus on Problems 1–2. For Problem 2, demonstrate how to complete the labels for the tick marks on the positive side of the thermometer, and then use the symmetry of values across 0 to complete the labels on the negative side of the thermometer.

Math Language Development

MLR8: Discussion Supports

During the Connect, as you highlight how Problem 3 is related to the Warm-up, provide these sentence frames for students to share their thinking as to whether they agree with either Elena or Jada's reasoning.

- "I agree with _____ because . . ."
- "I disagree with _____ because . . ."

Encourage students to explain how they are reading the thermometer.

English Learners

Annotate the diagram of the thermometer in Problem 3 to illustrate how −1.5°C is halfway between −1°C and −2°C.

Activity 1 What is the Temperature? (continued)

Students continue to build understanding of the negative side of the number line using a thermometer, including non-integer negative values.

Name: ______________________ Date: __________ Period: __________

Activity 1 What is the Temperature? (continued)

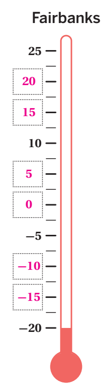
2. One thermometer shows the temperature, in degrees Celsius, in Fairbanks, Alaska on March 3, 2020.

a. What was the temperature?

-20°C

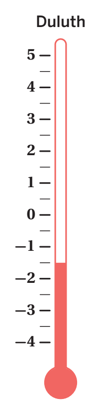
b. The thermometer is missing some labels. Write the missing numbers in the boxes on the thermometer.

$-15, -10, 0, 5, 15, 20$



3. Elena says that the thermometer for Duluth, Minnesota reads -2.5°C because the top of the liquid rises above -2°C. Jada disagrees and says that it reads -1.5°C because the top of the liquid is below -1°C. Do you agree with either of them? Explain your thinking.

Jada is correct; Sample response: The level of the liquid is between -1 and -2, and appears to be at the halfway mark, which would be -1.5°C.



4. The temperatures in Phoenix, Arizona, and Portland, Maine, are rarely the same.

a. One morning, the temperature in Phoenix was 8°C. That same morning in Portland, it was 12°C cooler than it was in Phoenix. What was the temperature in Portland?

-4°C

b. At noon on another day, Portland and Phoenix each had temperatures that measured the same distance from zero on the thermometer. Portland had a negative temperature and Phoenix had a positive temperature. If the temperature in Portland was 18°C colder than the temperature in Phoenix, what was the temperature, in Celsius, in each city? Explain your thinking.

It was -9°C in Portland, ME and 9°C in Phoenix, AZ. They are both 9°C away from 0 and 18°C away from each other.

3 Connect

Display the different thermometers as needed, and capture new language related to temperature (e.g., "freezing point") on the anchor chart.

Have students share how they determined the temperatures for Phoenix and Portland, and if they can think of any other city examples in which the cities' temperatures might be exactly the same distance from zero (one positive, one negative).

Highlight how Problem 3 is related to the Warm-up, because, as 2.5 is halfway between 2 and 3, -1.5°C is halfway between -1°C and -2°C.

Differentiated Support

Extension: *Interdisciplinary Connections*

Have students research the hottest and coldest temperatures ever recorded on Earth. The current Guinness World Record holder for Highest Recorded Temperature on Earth is Death Valley, California, which had a temperature of 56.7°C, or 134°F, measured on July 10, 1913. The current world record holder for Lowest Recorded Temperature on Earth is Vostok Station, located in Antarctica. The temperature was recorded as -89.2°C, or -128.6°F, measured on July 21, 1983.

Have students explore the online thermometer image, Solar System Temperatures, from NASA comparing the average temperatures of each planet in the solar system. Have students describe what they notice. **(Science)**

Sample response: Other than Venus, the farther a planet or object (in the case of Pluto) is from the Sun, the colder its average temperature.

Independent | 15 min

Activity 2 Folded Number Lines

Students build an understanding of symmetry across 0 on the number line, recognize opposites, and begin exploring distance from 0 (later defined as absolute value).

Activity 2 Folded Number Lines

You will be given a sheet of graph paper and a straightedge.

1. Follow the steps to create your own number line.

- Use the straightedge to draw a horizontal line with arrows on each end. Mark a point in the middle of the line and label it 0.
- To the right of 0, draw tick marks at every vertical grid line on your graph paper. Label the tick marks $1, 2, 3, \dots, 10$. This is the positive side of your number line.
- Fold your paper so that a vertical crease goes across the number line through 0 and the two sides of the number line match up perfectly.
- Use the fold to help you trace the tick marks that you already made onto the other side of the number line. Unfold and label the tick marks $-1, -2, -3, \dots, -10$. This is the negative side of your number line.

2. Use your number line to respond to the following.

a. Which number is the same distance from 0 as the number 4?

-4

b. Which number is the same distance from 0 as the number -7?

7

c. Two numbers that are the same distance from 0 on the number line are called ***opposites***. Both of the pairs 7 and -7, and -4 and 4 are opposites. What is another pair of opposites?

Sample response: 3 and -3

d. Determine how far the number 5 is from 0 on the number line. Then determine both a positive number and a negative number that are each farther from 0 on the number line than the number 5.

The number 5 is 5 units from 0; Sample response: 6 and -6 are both farther from zero than 5.

STOP

1 Launch

Provide students with straightedges and graph paper.

2 Monitor

Help students get started by modeling how to mark and fold the graph paper to create the positive and negative number line.

Look for points of confusion:

- **Marking the space between tick marks as opposed to the actual marks.** Model how to label a tick mark.

Look for productive strategies:

- Correctly representing the positive and negative sides of the number line by showing proper numbering and use of signs.
- Drawing equally-spaced tick marks.

3 Connect

Display a number line with integer tick marks labeled.

Have students share how they located and could plot the points in Problem 2.

Highlight that two points can be the same distance from 0, but on opposite sides, which means they represent different numbers — one positive and one negative. Discuss and add new vocabulary to the anchor chart.

Define:

- ***Opposites*** are two numbers that are the same distance from 0, but on different sides of the number line.
- ***Integers*** are whole numbers and their opposites.
- ***Rational numbers*** are integers, and also positive and negative fractions/decimals.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider one of these alternative approaches to this activity:

- Have students create a shorter number line that includes only -1, 0, and 1. Have them extend each side one number at a time.
- Provide students with partially-completed number lines, blank number lines, or blank number lines that have tick marks.
- Provide students with completed number lines and instruct them to fold their number lines so that they can see the symmetry across 0.

Extension: *Math Enrichment*

Have students complete the following problem:

Which two numbers are located 3 units from -4? Which number is farther from 0? Each of the numbers -7 and -1 is 3 units from -4, and -7 is farther from 0.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, after you have defined the vocabulary terms, display the question, "How do you know if two numbers are opposites?" After providing students time to respond in writing to the question, have them meet with 1–2 partners to give and receive feedback on their responses. Display these prompts that reviewers can use to press for details as they discuss their responses.

- "How does the number line support your response?"
- "Could you explain how you know that _____?"

English Learners

Allow students who speak the same primary language to share their responses with each other.

Whole Class | 5 min

Summary

Review and synthesize how the structure of the number line can be extended to include negative values, which are the opposites of positive rational numbers.

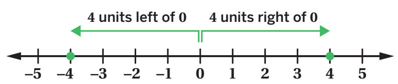
Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

Just as you can extend a number line to the right of 0 to show positive numbers, the number line can also be extended to the left of 0 to show negative numbers.

The number 4 is positive, so its location is 4 units to the *right* of 0 on the number line. The number -4 is negative, so its location is 4 units to the *left* of 0 on the number line.



Two numbers are ***opposites*** if they are the same distance from zero, but on different sides, on the number line. Therefore, -4 and 4 are opposites. Every number has an *opposite*, including fractions and decimals. In the case of 0, 0 is its own opposite.

All of the positive numbers you have ever seen — whole numbers, fractions, and decimals — can all be written as fractions, and can all be located precisely on the number line.

- All of the whole numbers and their *opposites*, including 0, are ***integers***.
- All of the numbers that can be written as fractions, including whole numbers and decimals, and their opposites are called ***rational numbers***.

The numbers 2 and -2 are both integers, and they are both rational numbers. The numbers 8.3, -8.3, $\frac{3}{2}$, and $-\frac{3}{2}$ are all rational numbers, but none are integers.

Reflect:

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *integers*, *rational numbers*, and *opposites* that were added to the display during the lesson.

Synthesize

Display a number line with tick marks and labels.

Highlight that the number line can be extended, either left or down, to include both positive and negative numbers. Every number on the number line has an opposite, a number that is the same distance from 0.

Have students share how to find the *opposite* of a number using a number line.

Formalize vocabulary:

- *integers*
- *rational numbers*
- *opposites*

Ask:

- "What is an example of an integer that is also a rational number? What is an example of a rational number that is not an integer?" Sample response: An example of an integer that is also a rational number is -3. An example of a rational number that is not an integer is $-\frac{3}{5}$.
- "What is the opposite of zero? Explain your thinking." (If this is not previously discussed in Activity 2) Zero is its own opposite, because 0 is located 0 units from 0 on the number line.
- "What are some differences in the way positive and negative numbers are ordered on a number line?" Sample response: Positive numbers are to the right of 0, increasing in value the farther they are to the right. Negative numbers are to the left of 0, decreasing in value the farther they are to the left.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does the number line to the left of 0 show you?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of positive and negative numbers, including opposites, using a thermometer to represent a vertical number line.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.03

1. Refer to the thermometer showing degrees Celsius.

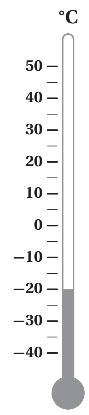
   a. What is the temperature shown?
   −20°C

   b. What would the temperature be if it was 5° colder?
   −25°C

   c. What would the temperature be if it was 5° warmer?
   −15°C

   d. What would the temperature be if it was one-half of a degree warmer?
   −19.5°C

   e. What would the temperature be if it was one-half of a degree colder?
   −20.5°C



2. Write two numbers that are opposites and that are each more than 6 units from 0 on a number line.
Sample response: −7 and 7

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine or approximate the value of any point on a number line. 1 2 3

b. I can represent negative numbers on a number line. 1 2 3

c. I can identify the opposite of a number. 1 2 3

Lesson 3 Points on the Number Line

Success looks like . . .

- **Language Goal:** Comprehending that two numbers are called opposites when they are the same distance from zero, but on different sides of the number line. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Interpreting a point on the number line that represents a positive or negative rational number. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Representing a positive or negative rational number as a point on a number line.
  - » Determining the value of each temperature from the thermometer in Problem 1.

Suggested next steps

If students have trouble calculating the positive and negative changes on the thermometer, consider:

- Pointing to tick marks and showing students how to move backward and forward on the number line as they count off the change.
- Reviewing Problem 4 of Activity 1.

If students are confused by how to increase or decrease by $0.5°$ in Problem 1e, consider:

- Showing a smaller distance number line (i.e., $0°$ and $1°$) to show where the half mark is and then extend it to other numbers.
- Asking, "What is 0.5 more than 20? What is 0.5 less than 20?"

If students cannot write two numbers that are opposites, consider:

- Showing them a number line and pointing to an example pair of opposites, and then having them try again to come up with their own example.
- Referencing their resulting number line creations from Activity 2, as well as the questions and answers from Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students created their own folded number lines to discover the symmetry across zero and how opposites are the same distance from zero (Activity 2). How did that build on the earlier work students did with distance and magnitude in Lesson 1? How will that support their work with absolute value in upcoming lessons?
- What did you see in the way some students approached identifying non-integer values in Activity 1 that you would like other students to try? Where can you introduce or highlight these approaches into the next few lessons? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. For each number, name its opposite.
   - a. -5 $\mathbf{5}$
   - b. 0.875 $\mathbf{-0.875}$
   - c. $\frac{4}{5}$ $\mathbf{-\frac{4}{5}}$
   - d. 0 $\mathbf{0}$
   - e. -10.4 $\mathbf{10.4}$
   - f. $-8{,}003$ $\mathbf{8{,}003}$

2. Plot and label each point at the correct location on the number line.

   - a. Point A is located at the opposite of -2.
   - b. Point B is located at the opposite of 0.5.
   - c. Point C is located at -2.
   - d. Point D is located at $\frac{3}{2}$.
   - e. Point E is located at -1.5.
   - f. Point F is located at $-\frac{4}{3}$.

3. Where would the temperature $-5.5°F$ be located on a thermometer? Select *all* that apply.
   - A. Between $-5°F$ and $-4°F$
   - B. Between $-5°F$ and $-6°F$
   - C. Between $5°F$ and $10°F$
   - D. Between $-10°F$ and $-5°F$
   - E. Between $0°F$ and $-5°F$

Name: ______ Date: ______ Period: ______

Practice

4. Solve each equation for x.
   - a. $8x = \frac{2}{3}$ $\quad x = \frac{1}{12}$
   - b. $1\frac{1}{2} = 2x$ $\quad x = \frac{3}{4}$
   - c. $5x = \frac{2}{7}$ $\quad x = \frac{2}{35}$
   - d. $\frac{1}{4}x = 5$ $\quad x = 20$
   - e. $\frac{1}{5} = \frac{2}{3}x$ $\quad x = \frac{3}{10}$

5. There are 15.24 cm in 6 in. How many centimeters are in each of the following?
   - a. 1 ft

     30.48 cm; Sample response: 1 ft is 12 in. $6 \bullet 2 = 12$, and $15.24 \bullet 2 = 30.48$.
   - b. 1 yd

     91.44 cm; Sample response: 1 yd is equal to 3 ft. $1 \bullet 3 = 3$ and $30.48 \bullet 3 = 91.44$.

6. The elevation of Memphis, TN, is 338 ft. The elevation of Nashville, TN, is 597 ft.
   - a. Which city has a higher elevation?

     Nashville, TN
   - b. Use the correct equality symbol ($<$, $>$, $=$) to complete the comparison statement.

     $338 \;\underline{\;<\;}\; 597$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activities 1 and 2	1
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 8	1
	5	Unit 2 Lesson 13	2
Formative	6	Unit 7 Lesson 4	2

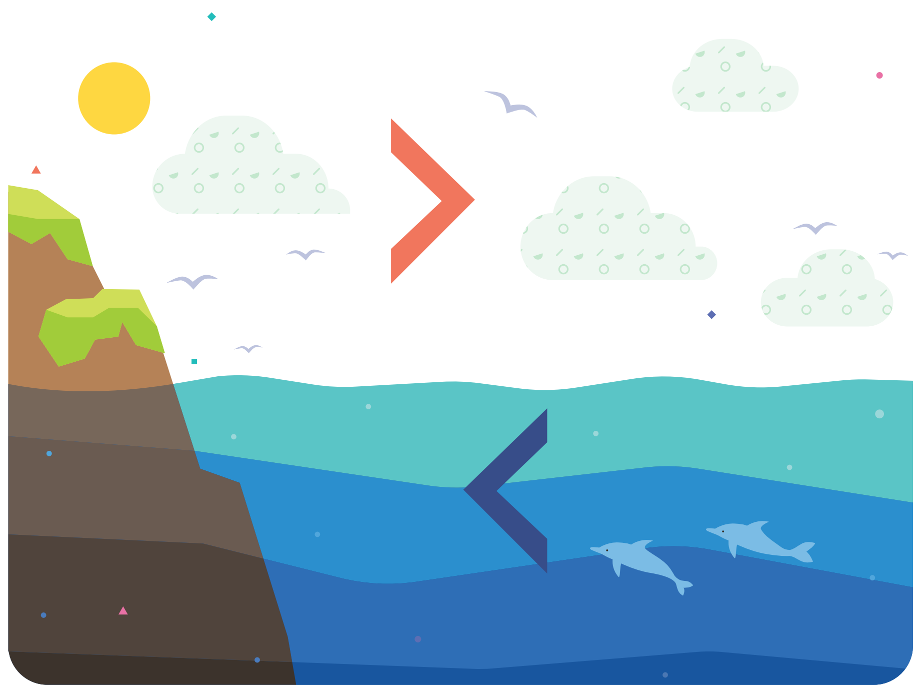
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Comparing Integers

Let's compare integers on the number line.



Focus

Goals

1. **Language Goal:** Compare integers, and express the comparisons using the symbols > and <. **(Reading and Writing)**
2. **Language Goal:** Comprehend the word *sign* to refer to whether a number is positive or negative. **(Speaking and Listening)**
3. **Language Goal:** Critique statements comparing integers, including claims about relative position and claims about distance from zero. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students develop strategies for comparing integers by creating a human number line and reasoning about their relative positions. Then, they use a number line to plot and compare positive and negative integers in the context of elevation, representing their comparisons using inequality statements. Students translate contextual words and phrases such as *above sea level, below sea level,* and *depth*, into mathematical language such as *greater than* and *less than*. They use the structure of the number line to reason about relationships between numbers, and evaluate and critique the reasoning of others, recognizing that if a is to the right of b, then $a > b$ and $b < a$.

Previously

In Lesson 3, students used vertical and horizontal number lines to reason about distance from 0 on the number line in the context of negative numbers and opposites.

Coming Soon

In Lesson 5, students will extend their work of comparing integers to comparing and ordering rational numbers.

Rigor

- Students **apply** their understanding of elevation and rational numbers to compare positive and negative numbers using a number line.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Whole Class	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one card per student

Math Language Development

New words

- ***sign*** (of a number)*

Review words

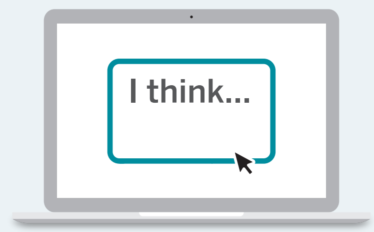
- *integer*
- *opposite*

*Students may confuse the mathematical meaning of *sign* with its everyday meaning related to a signature (signing a name) or a sign posted in a room or on the road. Be ready to address the differences between these meanings.

Amps | Featured Activity

Activity 2
See Student Thinking

Students are asked to explain their thinking when evaluating and critiquing arguments comparing integers, and these explanations are passed to you.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not be able to analyze the situation in Activity 2 without the aid of a number line. Ask students to identify what tools they have that can help them solve each problem that focuses on quantitative reasoning with abstract symbolic representations.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Independent | 5 min

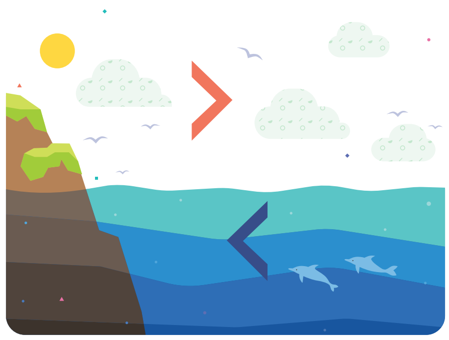
Warm-up True or False

Students determine whether an inequality is true or false, preparing them to write and evaluate inequality statements involving signed numbers in later activities.

Unit 7 | Lesson 4

Comparing Integers

Let's compare integers on the number line.



Warm-up True or False

Determine whether each inequality is true or false. Be prepared to explain your thinking.

1. $\frac{5}{4} < 2$
   True
2. $8.5 > 0.95$
   True
3. $8.5 < 7$
   False
4. $10.00 < 100$
   True

Log in to Amplify Math to complete this lesson online.

750 Unit 7 Rational Numbers

1 Launch

Conduct the ***True or False*** routine, displaying one inequality at a time. Give students about 15 seconds to quietly think and have them give a signal when they have an answer. Select students to share different ways of reasoning for each statement.

2 Monitor

Help students get started by activating their prior knowledge, asking them what the < and > symbols mean. Then, prior to evaluating, have them translate each expression either verbally, or in writing.

Look for points of confusion:

- **Switching the meanings of < and >.** Display $1 < 2$ and $2 > 1$. Say, "Both of these inequalities are true, but are read differently. What does each tell you?"
- **Struggling to compare $\frac{5}{4}$ to 2.** "What is another way to write $\frac{5}{4}$? Where is it on the number line?"

Look for productive strategies:

- Translating each symbolic expression to words, aloud or in writing, prior to evaluating.

3 Connect

Display the list of inequalities.

Have students share how each statement can be translated and read aloud using words. Encourage students to use terms, such as *greater than* or *less than*, in their explanations.

Highlight that the inequality symbols < and > are used to write inequalities that compare numbers. Every inequality can be interpreted in two ways. For example, $a < b$ means a is *less than* b, and also b is *greater than* a.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Display the inequality symbols < and > and be sure students understand what they mean. Suggest students write each inequality in words, such as "$\frac{5}{4}$ is less than 2" for Problem 1, before they determine whether they think the inequality is true or false.

Accessibility: *Optimize Access to Tools*

Provide copies of blank number lines for students to use, or suggest they draw a number line to help them visualize each inequality.

Power-up

To power up students' ability to use inequality symbols to compare positive integer values, have students complete:

Recall that > represents *greater than* and < represents *less than*.

Use the appropriate symbol, < or > to compare the given values.

a. $6 > 2$ b. $16 < 22$ c. $256 > 193$

Use: Before the Warm-up.

Informed by: Performance on Lesson 3, Practice Problem 6.

Whole Class | 15 min

Activity 1 Human Number Line

Students develop strategies to compare integers by constructing a human number line and reasoning about their relative positions.

Name: ______ Date: ______ Period: ______

Activity 1 Human Number Line

You will be given a number card. Pair up with the classmate whose number card is the opposite of your number. Then discuss where your numbers would each be located on the number line.

1. Prepare for the human number line. Complete the sentences.
   Sample responses shown.
   a. My number is **4**
   b. The opposite of my number is **–4**

2. Reflect on the activity and the human number line that was created. Write one statement about comparing positive and negative integers that is always true.
   Sample responses:
   - **Integers farther to the right are greater, and integers farther to the left are less.**
   - **In a negative integer, the greater the digit, the less the magnitude of the number.**
   - **Positive integers are always greater than negative integers.**
   - **Opposites are located an equal distance from 0, but a positive integer is always greater than its negative opposite.**

Compare and Connect:
As you respond to Problem 2, think about how the number line shows both magnitude and direction. Can you use the term *magnitude* in your response?

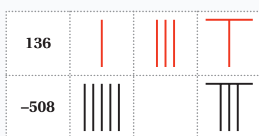
Historical Moment

Chinese Number Rods

One of the first known records of negative numbers being used comes from the Chinese number rod system seen here, which dates to around 200 BCE. The need for positive and negative numbers came from taxes — receiving and paying currency.

Use the image to decipher and determine how to write each of the following numbers using Chinese number rods.

1. 57,193
2. –60,428



Lesson 4 Comparing Integers 751

1 Launch

Create a large number line on the floor or on wall of your classroom with only 0 labeled at the center. Give each student a number card, and have them determine partner by locating their opposite. Once all students are in pairs, have students find their location, one pair at a time, relative to 0 and the numbers already standing on the number line. Once all values are placed, have students work with their opposite on Problem 2.

2 Monitor

Help students get started by saying, "Starting from 0, should you move left or right? Use the other integers already on our number line to help you find your place."

Look for points of confusion:

- **Plotting negative numbers incorrectly.** Ask, "What happens to the integers as you move from a positive number toward 0?"

Look for productive strategies:

- Recognizing that, for negative integers, the greater the digit (without its negative sign), the farther left it will be placed on the number line.
- Recognizing that their place will be the same number of ticks away from 0 as their opposite, but in the other direction.

3 Connect

Have students share their responses to Problem 2, focusing on those who generalized that integers to the left are less than those to the right, despite the size of the digit in negative integers.

Highlight that for two integers, you can determine which is greater by seeing which is further to right on the number line.

Ask, "Can a negative integer ever be greater than a positive integer?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses to Problem 2, highlight key words and phrases students use to compare positive and negative numbers, such as *farther to the right (greater magnitude), farther to the left (less magnitude),* etc. Ask these questions:

- "Which has the greater magnitude, 4 or 3?" 4; It is 4 units from 0.
- "Which has the greater magnitude, –4 or –3?" –4; It is 4 units from 0.
- "How would you complete this phrase: For a negative integer, the greater the digit, the ___ the magnitude of the number?" Less; –3 > –4, but –3 is closer to 0, so it has a lesser magnitude.

Historical Moment

Chinese Number Rod System

Have students complete the Historical Moment activity to see how the ancient Chinese mathematicians represented positive and negative numbers for credits and debts.

Pairs | 15 min

Activity 2 Comparing Elevations

Students use the context of elevation to compare integers using number lines and inequalities.

Amps Featured Activity See Student Thinking

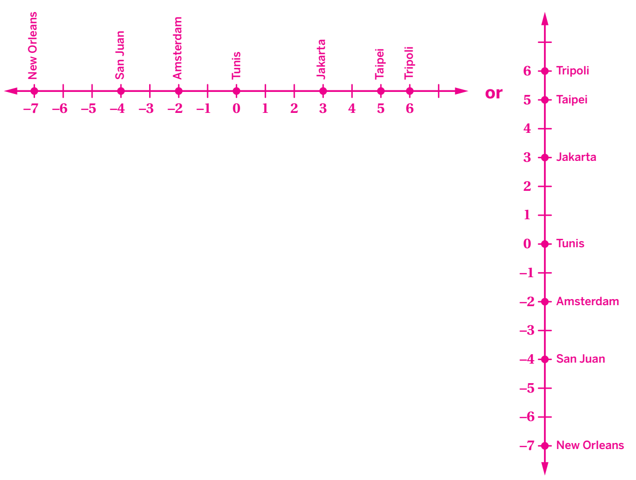
Activity 2 Comparing Elevations

The elevations of several cities around the world are shown in the table.

Location	Tripoli, Libya	San Juan, Puerto Rico	New Orleans, Louisiana	Amsterdam, Netherlands	Jakarta, Indonesia	Taipei, Taiwan	Tunis, Tunisia
Elevation (ft)	6	-4	-7	-2	3	5	0

1. What do the following numbers mean in this context?
   a. 0
   The elevation is at sea level.
   b. -7
   The elevation is 7 ft below sea level.

2. Plot all of the elevations on a number line.



752 Unit 7 Rational Numbers

1 Launch

Activate students' background knowledge by asking, "When might you measure the depth of something? What could *depth* mean when talking about elevation?"

2 Monitor

Help students get started by asking, "What does elevation mean? Where might you see a positive elevation? A negative elevation?"

Look for points of confusion:

- **Making 0 the left-most or bottom-most point shown.** Say, "The human number line and thermometers from the previous lesson did not start with 0 on the left or bottom. Why not?"
- **Not labeling equal-interval tick marks.** Remind students that tick marks should represent a constant interval or scale in both directions.
- **Reversing the order of negative numbers on the number line or comparing integers incorrectly.** Ask, "Where was -1 on our human number line? -2? -3? What is true about integers as you move to the left or down? To the right or up?"
- **Interpreting or writing inequality symbols incorrectly.** Have students translate the symbolic inequality statement into words. Refer them to the Warm-up to review the meaning of $<$ and $>$.

Look for productive strategies:

- Connecting terms *like above sea level, below sea level*, and *depth/deeper* to mathematical terms, such as *less* and *greater*.
- Translating the symbolic inequalities into words before determining whether they are true or false.
- Evaluating their responses in context.
- Using the structure of the number line to write inequalities, and evaluating and critiquing arguments about the connection between distance from 0 and direction on the number line.

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge, Clarify Vocabulary and Symbols*

Remind students what the term *elevation* means, and what *sea level* represents within the context of elevation. Ask students if they know the elevation of their city; consider providing this information to students so they can visualize how their city compares to the cities shown in the table.

Accessibility: *Optimize Access to Tools*

Provide students with partially-completed number lines, blank number lines, or blank number lines that have tick marks. Have both horizontal and vertical number lines available from which students can choose as they complete Problem 2.

Math Language Development

MLR7: Compare and Connect

To help students connect the language of an elevation context, a verbal inequality, and a symbolic inequality, ask students to compare the following statements.

- 6 ft above sea level is higher than 50 ft below sea level.
- 6 is greater than -50.
- $6 > -50$

English Learners

Display these statements next to a vertical number line and annotate 0 as *sea level*.

Pairs | 15 min

Activity 2 Comparing Elevations (continued)

Students use the context of elevation to compare integers using number lines and inequalities.

Name: ______ Date: ______ Period: ______

Activity 2 Comparing Elevations (continued)

3. Decide whether each inequality statement is true or false. Be prepared to explain what each statement means in the context of the elevation of the cities.
   - a. $-2 < 6$ True. Amsterdam has a lower elevation than Tripoli.
   - b. $-4 < -7$ False. The inequality says that San Juan has a lower elevation than New Orleans, but San Juan has a higher elevation.
   - c. $3 > -4$ True. Jakarta has a higher elevation than San Juan.
   - d. $-7 > 5$ False. The inequality says that New Orleans has a higher elevation than Taiwan, but New Orleans has a lower elevation.
4. Jada said, "I know that 2 is less than 4, so -2 must be less than -4. This means Amsterdam has a lower elevation than San Juan." Do you agree? Explain your thinking.
   I disagree; Sample response: -4 is located to the left of (or below) -2 on the number line.
5. Andre said, "3 is less than -7 because 3 is closer to 0 on the number line. This means Jakarta has a lower elevation than New Orleans." Do you agree? Explain your thinking.
   I disagree; Sample response: 3 is to the right of (or above) -7 on the number line. Positive numbers are always greater than negative numbers, no matter how close each is to 0.
6. The shore of the Dead Sea has an elevation of 1,419 ft below sea level. The Challenger Deep, part of the Mariana Trench in the western Pacific Ocean, has a depth of 36,201 ft.
   - a. Write the elevations of the Dead Sea and Challenger Deep.
     Dead Sea: $-1{,}419$ ft Challenger Deep: $-36{,}201$ ft
   - b. Which one has the lower elevation? Explain your thinking.
     Challenger Deep has the lower elevation; Sample response: $-36{,}201$ is to the left of (or below) $-1{,}419$ on the number line.
   - c. Write an inequality that compares the elevations.
     $-1{,}419 > -36{,}201$ or $-36{,}201 < -1{,}419$

STOP

Lesson 4 Comparing Integers 753

3 Connect

Display each problem, one at a time.

Have students share their responses to each problem one at a time, focusing on:

- How they represented and compared elevations using number lines and inequalities.
- How the elevation, and corresponding language, changed as they moved left or right on the horizontal number line or up and down on the vertical number line.
- How they used the structure of the number line to compare integers and critique Jada's and Andre's reasoning.

Highlight:

- The connection among contextual phrases, such as *above sea level*, *below sea level*, *depth/deeper*, and *higher/lower*, with mathematical terms, such as *greater* and *less*. Have students identify where to place these phrases on the anchor chart.
- If you write $6 > -7$, then you could say, "Tripoli has a higher elevation than New Orleans because 6 is farther to the right/up on the number line." Equivalently, you could write $-7 < 6$ and say, "New Orleans has a lower elevation because -7 is farther to the left/down on the number line."

Ask:

- "Does the integer farther from 0 always have the greater value?" Sample response: No, -20 is farther away from 0 than 4, but 4 has a greater value than -20.
- "Does the integer closer to 0 always have the lesser value?" Sample response: No, 3 is closer to from 0 than -9, but -9 has a lesser value than 3.

Whole Class | 5 min

Summary

Review and synthesize how to compare any two numbers using a number line and introduce the term *sign* (as in "the sign of a number").

Summary

In today's lesson . . .

You compared positive and negative integers on the number line. The ***sign*** of a number tells you whether it is positive or negative. For example you read the number -3 as "negative three". The symbol '–' tells you that the sign of -3 is negative. Although 0 is not written with a negative sign and looks to be written the same way we write positive numbers, it has *no sign* because it is *neither* positive *nor* negative.

You use the words *greater than* and *less than* to compare numbers when you refer to their values or corresponding positions on the number line. This is true for negative integers as well.

- The integer -3 is to the *left* of -1, so -3 is *less than* -1. You can write this as the inequality statement $-3 < -1$.
- The integer -1 is to the *right* of -3, so -1 is *greater than* -3. You can write this as the inequality statement $-1 > -3$.
- You can also write $1 > -1$ and $1 > -3$ because 1 is to the right of both of the other integers.

Any number that is located to the *left* of a number n is *less than* n. And any number that is located to the *right* of a number n is *greater than* n. This means that any positive number is *greater than* any negative number, and any negative number is *less than* any positive number.

Reflect:

Synthesize

Formalize vocabulary: ***sign*** (of a number).

Highlight:

- The sign of -3 is negative, and the sign of 5 is positive. We typically do not write $+5$.
- 0 has no sign and is neither positive nor negative.

Ask:

- "What is the sign of any number to the right of 0?" positive (+) "To the left of 0?" negative (–)
- "What is the sign of any elevation at sea level?" no sign (0) "Above sea level?" positive (+) "Below sea level?" negative (–)
- "If you plot two numbers on the number line, how can you tell which one has a greater value?" The number to the right has a greater value than the number to the left.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can a number be closer to zero than another number and have a greater value than the other number?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *sign* (of a number) that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing symbolic inequality statements to compare elevations, and evaluating comparison statements using a number line.

Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 7.04

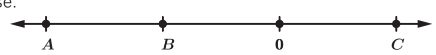
1. The elevations of three U.S. cities are shown in the table.

Location	Elevation (ft)
Death Valley, CA	-282
Tallahassee, FL	203
Westmorland, CA	-157

a. Write an inequality that compares the elevations of Death Valley and Tallahassee.
$-282 < 203$ or $203 > -282$

b. Write an inequality that compares the elevations of Death Valley and Westmorland.
$-282 < -157$ or $-157 > -282$

2. The points A, B, C, and 0 are plotted on a number line. The points B and C are opposites. Determine whether each statement about the values of these points is true or false.



a. A is greater than B. False

b. A is less than C. True

c. B and C are equally far from 0. True

d. B and C are equal. False

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain how to use the positions of numbers on a number line to compare them.
1 2 3

b. I can use inequalities to compare positive and negative numbers.
1 2 3

c. I can use phrases like *greater than*, *less than*, and *opposite* to compare integers.
1 2 3

 Lesson 4 Comparing Integers

Success looks like . . .

- **Language Goal:** Comparing integers, and expressing the comparisons using the symbols > and <. **(Reading and Writing)**
  - » Writing inequalities to compare the elevations of the three U.S. Cities in Problem 1.
- **Language Goal:** Comprehending the word *sign* to refer to whether a number is positive or negative. **(Speaking and Listening)**
- **Language Goal:** Critiquing statements comparing integers, including claims about relative position and claims about distance from zero. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students write incorrect inequalities for Problem 1a or 1b, consider having them verbally translate their expressions. If they translate them incorrectly, consider:

- Asking, for Problem 1a, "What is always true when you compare a positive number and a negative number?"
- Reminding them what the symbols > and < mean.

If students translate their answers to Problems 1a and 1b correctly, but write an incorrect inequality, consider:

- Referring them to the number line in Activity 2, and asking, "What happens to the integers when you move to the left/right on the number line?"

If students incorrectly evaluate the truth of any of the statements in Problem 2, consider:

- Referring them to the number line in Activity 2 and asking, "What happens to the integers when you move to the left and to the right on the number line?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- One instructional goal for this lesson was that students compare integers and express these comparisons using the greater than and less than symbols. How well did students accomplish this? What did you specifically do to help students accomplish it?
- What did the human number line in Activity 1 reveal about your students as learners? How can you use this understanding of your students to adjust future lessons? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Write four inequality statements using the integers -2, 4, -7, and 10.

Sample responses:

$-2 > -7$	$4 > -2$	$-7 < -2$	$10 > -2$
$-2 < 4$	$4 > -7$	$-7 < 4$	$10 > 4$
$-2 < 10$	$4 < 10$	$-7 < 10$	$10 > -7$

2. Determine whether each inequality statement is true or false. Explain your thinking.

a. $-5 < 2$

True; Sample response: -5 is to the left of 2 on the number line. Negative numbers are always less than positive numbers.

b. $3 > -8$

True; Sample response: 3 is to the right of -8 on the number line. Positive numbers are always greater than negative numbers.

c. $-12 > -15$

True; Sample response: -12 is to the right of -15 on the number line.

d. $-13 > -12$

False; Sample response: -13 is to the left of -12 on the number line.

3. Here is a true statement: $-8 < -7$. Select *all* the statements that are equivalent to $-8 < -7$.

A. -8 is farther to the right on the number line than -7.

B. -8 is farther to the left on the number line than -7.

C. -8 is less than -7.

D. -8 is greater than -7.

E. -7 is less than -8.

F. -7 is greater than -8.

Name: ______ Date: ______ Period: ______

Practice

4. Each lap around a track is 400 m.

a. How many meters did Kiran run if he ran:

2 laps?

800 m; $400 \bullet 2 = 800$

5 laps?

2,000 m; $400 \bullet 5 = 2{,}000$

x laps?

$400x$ m; $400 \bullet x = 400x$

b. If Noah ran 14 laps, how many meters did he run?

5,600 m; $400 \bullet 14 = 5{,}600$

c. If Jada ran $7{,}600$ m, how many laps did she run?

19 laps; $400x = 7{,}600$ so $x = 19$

5. A stadium has a total capacity of $16{,}000$ people.

a. If there are $13{,}920$ people in the stadium, what percent of its total capacity is filled? Show your thinking.

87%, $\frac{13{,}920}{16{,}000} = 0.87$.

b. What percent of its total capacity is *not* filled?

13%; $100 - 87 = 13$.

6. Complete the table.

Inequality	Description in words	True or false
$\frac{3}{2} > 2$	**$\frac{3}{2}$ is greater than 2 or three halves is greater than two**	**False**
$2.03 < 2.3$ or $2.3 > 2.03$	Two and three hundredths is less than two and three tenths.	**True**
$\frac{2}{3} < 0.5$	**$\frac{2}{3}$ is less than 0.5 or two thirds is less than five tenths**	**False**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activities 1 and 2	2
Spiral	4	Unit 6 Lesson 2	2
	5	Unit 3 Lesson 9	2
Formative	6	Unit 7 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 7 | **LESSON 5**

Comparing and Ordering Rational Numbers

Let's compare and order rational numbers.

Focus

Goals

1. **Language Goal:** Comprehend that all negative numbers are less than all positive numbers. **(Speaking and Listening, Reading)**
2. **Language Goal:** Compare rational numbers without a context and express the comparisons using the terms *greater than*, *less than*, and *opposite*. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Order rational numbers from least to greatest, or greatest to least, and justify a stated order using words or other representations. **(Speaking and Listening, Writing)**

Coherence

Today

Students extend their work with comparing integers to now include negative rational numbers that are not integers. Using the structure of the number line, students create another human number line, developing strategies to locate other rational numbers *between* integers, and using the terms *greater than*, *less than*, and *opposite* to compare them. Students evaluate and critique different strategies for locating and comparing positive and negative rational numbers.

Previously

In Lesson 4, students first created a human number line to compare integers using the structure of the number line and reasoning about relationships between numbers. They related contextual language for elevation to mathematical language and symbolic inequality statements using $<$ and $>$.

Coming Soon

In Lesson 6, students will apply their understanding of negative numbers to contexts involving other quantities, such as money and inventory. Then in Lessons 7–8, they will connect their previous number line work to the new concept of absolute value.

Rigor

- Students compare and order rational numbers to strengthen their **procedural fluency** using a number line.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Whole Class	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one card per student
- Activity 2 PDF, one per student (as needed)

Math Language Development

Review words

- *integer*
- *negative number*
- *opposite*
- *positive number*
- *rational number*
- *sign*

Amps | **Featured Activity**

Activity 2
Interactive Number Lines

Given a number line and one known point, students label other points. You can overlay student responses to see similarities and differences at a glance and give immediate feedback.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not use the structure of number lines to help build their thoughts about where they belong on the human number line in Activity 1. Have students monitor their own thinking and be willing to change the thought process if it is not getting the desired results.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Independent | 5 min

Warm-up How Do They Compare?

Students use inequality symbols to compare two positive rational numbers, or one positive and one negative integer, preparing them to compare and order any rational numbers.

Name: ____________ Date: ________ Period: ________

Unit 7 | Lesson 5

Comparing and Ordering Rational Numbers

Let's compare and order rational numbers.

Warm-up How Do They Compare?

Use the symbols >, <, = to compare each pair of numbers. Be prepared to explain your thinking.

1. $15 > 1.5$
2. $6.050 = 6.05$
3. $\frac{19}{24} < \frac{19}{21}$
4. $15 > -15$
5. $9.02 < 9.2$
6. $0.4 < \frac{3}{5}$
7. $-9 > -10$
8. $\frac{24}{20} = 1.2$

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students have to work individually on the activity.

2 Monitor

Help students get started by activating their prior knowledge. Ask them to compare 1 and 0.2 using the terms *greater than* and *less than*. Represent their words using the symbols < and >.

Look for points of confusion:

- **Writing incorrect comparisons**. Have them read their statements aloud using *greater than* or *less than*. If they:
  - » read greater than or less than for the wrong symbol, remind them what the < and > symbols mean.
  - » read the symbols correctly, have them draw a number line and consider the numbers in context (e.g., 15 apples versus 1.5 apples).
- **Not converting to, or considering common forms**. Ask, "How else can you think about the number ___? Is there another way to write it?"

Look for productive strategies:

- Using place value, the structure of fractions, or the structure of the number line to compare numbers.

3 Connect

Display the correct inequality symbols for each pair of numbers.

Have students share their thinking for each pair of numbers, focusing on productive and generalizable strategies.

Ask, "How can you compare rational numbers in different forms?"

Highlight that it is helpful to compare rational numbers when they are in the same form and written to the same place value.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Display the symbols <, >, and = along with their meanings for students to reference as they complete the Warm-up.

Symbol	Meaning
<	Less than
>	Greater than
=	Equal to

Accessibility: *Optimize Access to Tools*

Provide access to blank number lines that students can choose to use as they complete the Warm-up, or suggest they draw number lines in their Student Edition.

Power-up

To power up students' ability to write, translate, or evaluate inequalities with positive rational values, have students complete:

Recall that you can compare the size of fractions and decimals by rewriting them both as fractions with the same denominator or by rewriting each value as a decimal.

Compare each pair of numbers using <, >, or =.

a. $0.75 > \frac{2}{3}$ b. $1.8 > \frac{5}{4}$ c. $\frac{12}{5} = 2.4$ d. $\frac{6}{15} < \frac{8}{10}$

Use: Before the Warm-up.

Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

Whole Class | 15 min

Activity 1 Human Number Line, Revisited

Students develop strategies to compare and order positive and negative rational numbers by constructing another human number line, reasoning again about their positions.

Activity 1 Human Number Line, Revisited

You will be given a number card. Pair up with the classmate whose number card is opposite of your number. Then discuss where your numbers would each be located on the number line.

1. Prepare for the human number line. Complete the sentences.
   Sample responses shown.
   a. My number is $\frac{1}{4}$.
   b. The opposite of my number is $-\frac{1}{4}$.

2. Reflect on the activity and the human number line that was created. Write one statement about ordering rational numbers that is always true.
   Sample responses:
   - **To make ordering easier, I can think of rational numbers in any common form — decimals, fractions, or mixed numbers.**
   - **Numbers farther to the right are greater, and numbers farther to the left are less.**
   - **Positive rational numbers are always greater than negative rational numbers.**
   - **Opposites are located an equal distance from 0, but the positive rational number is always greater than its negative opposite.**

Stronger and Clearer: Share the reflection you wrote about rational numbers in Problem 2 with a partner to give and receive feedback. Revise your response based on the feedback you receive.

758 Unit 7 Rational Numbers

1 Launch

Create a large number line with only 0 labeled at the center. Give each student a number card from the Activity 1 PDF and have them partner up using opposites. Each pair takes a turn to go to their locations. Then have partners complete Problem 2.

2 Monitor

Help students get started by helping them convert their numbers to common forms.

Look for points of confusion:
- **Not recognizing opposites in different forms**. Ask, "What is another form of your number?"
- **Positioning themselves incorrectly**. Ask, "What happens as you move to the left/right? Is your number less/greater than ___?"

Look for productive strategies:
- Using 0 as a reference to plot positive numbers to the right and negative numbers to the left.
- Flexibly comparing rational numbers in all their forms — fractions, decimals, whole numbers.

3 Connect

Display the number line on the board.

Have students share strategies for thinking flexibly about rational number forms, using points already on the number line and using the structure of the number line. Have students use precise language, such as *greater than*, *less than*, and *opposite*.

Highlight that, to order numbers from *least* to *greatest*, list them as they appear on the number line from *left* to *right*. To order numbers from *greatest* to *least*, list them as they appear on the number line from *right* to *left*.

Ask, "How would you interpret $a = b = c$? What about $a < b < c$?"

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they have previously worked with positive numbers expressed as whole numbers, fractions, or decimals.

- Display two of the cards, such as $\frac{1}{4}$ and $\frac{8}{3}$. Ask students which fraction is greater and have them explain their thinking. $\frac{8}{3}$; Sample response: $\frac{8}{3}$ is between 2 and 3 and $\frac{1}{4}$ is less than 1.
- Then display a card with a negative value, such as -0.25, and ask them how this value compares with the other two cards you displayed. -0.25 is the least value because it is the only negative value and all positive numbers are greater than negative numbers.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 2, have them meet with 2–3 partners to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "How do you know that your statement is *always* true, or whether it is only true for certain rational numbers?"
- "What mathematical language did you use in your response?"

Have students write a final response, based on the feedback they received.

English Learners

Allow pairs of students who speak the same primary language to provide feedback to each other.

Pairs | 15 min

Activity 2 Comparing and Ordering Points on a Line

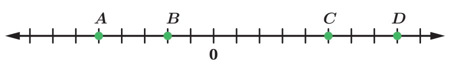
Students compare and order positive and negative rational numbers by considering their relative positions on a number line.

Amps Featured Activity Interactive Number Lines

Name: ____ Date: ____ Period: ____

Activity 2 Comparing and Ordering Points on a Line

The number line shows the points A, B, C, D, and 0. Copies of the number line can be provided if you need them.



1. If the tick marks on the number line increase by 1s, what is the position of each letter?

   A: -5 B: -2 C: 5 D: 8

2. If the tick marks on the number line do *not* increase by 1s, and D represents the location of 2, what do A, B, and C represent?

   A: -1.25, $-\frac{5}{4}$, or $-1\frac{1}{4}$ B: -0.5 or $-\frac{1}{2}$ C: 1.25, $\frac{5}{4}$, or $1\frac{1}{4}$

3. Suppose A represents $-2\frac{1}{2}$. Clare says that means D represents $2\frac{1}{2}$. Is she correct? Explain your thinking.

   Clare is incorrect; Sample response: The intervals on the number line are $\frac{1}{2}$, so D represents 4. Because C is the opposite of A, C represents $2\frac{1}{2}$, meaning D must be greater than $2\frac{1}{2}$.

4. Suppose the letters represent the elevations of four cities in Florida. Use the letters to list the cities in order of their elevations, from:

   a. Least to greatest.
   A, B, C, D

   b. Greatest to least.
   D, C, B, A

STOP

Lesson 5 Comparing and Ordering Rational Numbers 759

1 Launch

Distribute copies of the Activity 2 PDF as needed. Have students use the ***Think-Pair-Share*** routine. Provide them 5 minutes of individual work time. Then have them compare their answers with a partner.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "What happens as you move to the left or right on a number line?"

Look for points of confusion:

- **Struggling to determine intervals or label points.** Have students guess and check friendly intervals (e.g. 5, 10, 25) by starting at 0 and counting to the given point. Ask, "Is the interval greater than or less than ___? How do you know?"
- **Not understanding which numbers to use for Problem 4.** "Which point was the least in Problem 1? Problem 2? Does it matter?"

Look for productive strategies:

- Reasoning about the interval by using the structure of the number line to consider the number of equally-partitioned tick marks from 0.
- Using the concept of opposites to reason about the number line.

3 Connect

Display the correct solutions for each problem.

Have pairs of students share any places where they disagreed at first, and how they came to agreement, such as by using the structure of the number line or the meaning of opposites.

Highlight that, no matter the values of the points A, B, C, and D, students can order them in two ways — least to greatest, or greatest to least: $A < B < C < D$ or $D < C < B < A$.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive number line to plot and label points. You can overlay student responses to see similarities and differences, and to provide immediate feedback to students.

Accessibility: *Optimize Access to Tools*

Provide copies of the number lines from the Activity 2 PDF for students to use during this activity.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, draw their attention to the fact that no matter what the letters represent, the number line shows that $A < B < C < D$. Ask students what other information the number line shows. Sample responses: C and D are positive numbers, while A and B are negative numbers. D is the farthest from 0.

English Learners

Display the following sentence frames to help students organize their thinking:

- "___ is greater than ___ because . . ."
- "___ is the opposite of ___ because . . ."
- "I know that ___ is a negative number because . . ."

Whole Class | 5 min

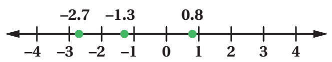
Summary

Review and synthesize how to compare and order rational numbers, using the structure of the number line, and how to represent comparisons with inequality statements.

Summary

In today's lesson . . .

You used a number line to compare and order positive and negative rational numbers, just as you previously did for integers. Any number to the *left* on the number line is *less than* a number to its right, and any number to the *right* on the number line is *greater than* a number to its left.



When ordering three or more rational numbers from *least to greatest*, list them in the order they appear on the number line going from *left to right*.

For example, the numbers -2.7, -1.3, and 0.8 are listed from least to greatest because of the order they appear on the number line. You can also represent this using a *compound inequality* statement $-2.7 < -1.3 < 0.8$.

When ordering rational numbers from *greatest to least*, list them in the order they appear on the number line going from *right to left*.

For example, 0.8, -1.3, -2.7 are listed from greatest to least, and you can write the compound inequality statement $0.8 > -1.3 > -2.7$.

Reflect:

Synthesize

Ask students to summarize the ideas they have developed in the last few lessons about plotting and comparing rational numbers. Consider asking one or both of these questions:

- "How can you tell whether one number is greater than or less than another number? How could you write that comparison?" Sample response: Any number to the right on the number line is greater than a number to its left. Any number to the left on the number line is less than a number to its right. I can use the inequality symbols > and < to show the comparisons.
- "How can you use the number line to help order three or more rational numbers from *least to greatest*? From *greatest to least*?" Sample response: List them in the order they appear on the number line from left to right (least to greatest) or from right to left (greatest to least).

Highlight:

- When ordering three or more rational numbers from *least to greatest*, list them in the order they appear on the number line, moving from *left to right*.
- When ordering three or more rational numbers from *greatest to least*, list them in the order they appear on the number line, moving from *right to left*.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How has comparing and ordering numbers developed or changed since fifth grade math?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by ordering rational numbers from least to greatest.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.05

Order the numbers from least to greatest.

$\frac{16}{5}$ -3 6 3.1 -2.5 $\frac{1}{4}$ $-\frac{3}{4}$ $-\frac{3}{8}$

$-3, -2.5, -\frac{3}{4}, -\frac{3}{8}, \frac{1}{4}, 3.1, \frac{16}{5}, 6$

or

$-3 < -2.5 < -\frac{3}{4} < -\frac{3}{8} < \frac{1}{4} < 3.1 < \frac{16}{5} < 6$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain what a rational number is. 1 2 3

b. I can compare and order rational numbers. 1 2 3

c. I can use phrases, such as *greater than*, *less than*, and *opposite*, to compare rational numbers. 1 2 3

Lesson 5 Comparing and Ordering Rational Numbers

Success looks like . . .

- **Language Goal:** Comprehending that all negative numbers are less than all positive numbers. **(Speaking and Listening, Reading)**
- **Language Goal:** Comparing rational numbers without a context and expressing the comparisons using the terms *greater than*, *less than*, and *opposite*. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Ordering rational numbers from least to greatest, or greatest to least, and justifying a stated order using words or other representations. **(Speaking and Listening, Writing)**
  - » Ordering the numbers from least to greatest.

Suggested next steps

If students list numbers out of order and do not show evidence of using a common form, consider:

- Asking, "In Activity 1, your number was in a different form than your partner's. How did you know you had a match?"
- Assigning Practice Problems 1 and 2.

If students list numbers out of order, but show evidence of using a common form, consider:

- Referring to Activity 1, and asking, "How did the human number line help you order from least to greatest? What happened to the numbers as you moved to the left or right on the number line?"
- Assigning Practice Problems 1 and 2.

If students list the numbers in a correct order, but from greatest to least, consider:

- Having students read their list, using the words *less than* or *greater than* between each number. Ask, "Does your answer match what was asked for?"
- Assigning Practice Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How was today's human number line (Activity 1) similar to or different from the human number line in Lesson 4? How did students use and build upon what they learned in Lesson 4 to compare and order rational numbers today?
- What did students find frustrating about Activity 2? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

Practice

1. Select *all* the numbers that are greater than -5.
   - A. 1.3
   - B. -6
   - C. -12
   - D. $\frac{1}{7}$
   - E. -1
   - F. -4

2. Order these numbers from least to greatest: $\frac{1}{2}, 0, 1 -1\frac{1}{2}, -\frac{1}{2}, -1$.
   $-1\frac{1}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$
   or
   $-1\frac{1}{2} < -1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1$

3. The table shows the boiling points of certain elements in degrees Celsius. List the elements in the order of their boiling points, from *least* to *greatest*.

Element	Boiling point (°C)
Argon	-185.8
Chlorine	-34
Fluorine	-188.1
Hydrogen	-252.87
Krypton	-153.2

Hydrogen, Fluorine, Argon, Krypton, Chlorine

4. Explain why zero is considered its own opposite.
   Zero is its own opposite because it is 0 units to the right of 0, so its opposite has to be the same distance to the left of 0, which is still 0.

Name: Date: Period:

Practice

5. Evaluate each quotient.
   - a. $\frac{1}{2} \div 2 = \frac{1}{4}$ or 0.25
   - b. $2 \div 2 = 1$
   - c. $\frac{1}{2} \div \frac{1}{2} = 1$
   - d. $\frac{38}{79} \div \frac{38}{79} = 1$

6. Each student starts with \$10. Write an expression to represent how much money each student has after the event described. *Do not evaluate the expressions.*
   - a. Elena buys \$3 worth of stickers.
     $10 - 3$
   - b. Tyler earns \$5 walking dogs.
     $10 + 5$
   - c. Diego is paid \$4 for fixing a computer.
     $10 + 4$
   - d. Lin pays \$8 for a new app.
     $10 - 8$
   - e. Priya sells a t-shirt for \$9.
     $10 + 9$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 3	2
	5	Unit 4 Lesson 11	2
Formative	6	Unit 7 Lesson 6	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Using Negative Numbers to Make Sense of Contexts

Let's make sense of negative amounts of money.

Focus

Goals

1. **Language Goal:** Interpret a table of signed numbers that represent how a quantity changed. **(Speaking and Listening, Reading and Writing)**
2. Recognize that signed numbers can be useful to represent changes in a quantity in opposite directions, e.g., money received and money paid, inventory bought and inventory sold, etc.

Coherence

Today

Students use signed numbers in business contexts to represent money spent and received, as well as inventory gained and lost. This lesson is an introduction to the conventions of "changes" in value, as modeled by mathematics in financial situations; however, students are not expected to perform operations with negative numbers in this grade.

Previously

In Lessons 2–5, students used rational numbers to represent elevation and temperature. They also compared and ordered rational numbers using number lines and distance from zero.

Coming Soon

In Grade 7, students will see how all of the previously learned operations on positive numbers — addition, subtraction, multiplication, and division — extend to signed rational numbers, including applying those concepts when they revisit financial situations.

Rigor

- Students **apply** their understanding of positive and negative numbers to represent cost and inventory in a business context.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

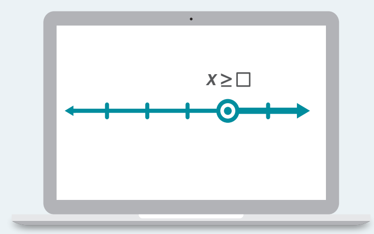
Review words

- *negative number*
- *opposite*
- *positive number*
- *rational number*
- *sign (of a number)*

Amps | Featured Activity

Activity 1
Interactive Number Lines

Students create digital number lines, and you can overlay them all to see similarities and differences at a glance.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might become anxious about relating mathematics to a real-world business in Activity 2. Ask students to identify the purpose of the mathematical model for the business and how they will regulate their stress level as they complete the activity.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- **Activity 1** may be done as a whole class.

Independent | 5 min

Warm-up Notice and Wonder

Students reason about positive and negative amounts of money to establish conventions for using signed numbers to represent money spent and received.

Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 6

Using Negative Numbers to Make Sense of Contexts

Let's make sense of negative amounts of money.

Warm-up Notice and Wonder

Consider the following table. What do you notice? What do you wonder?

Activity	Amount ($)
Mow my neighbor's lawn	30.00
Babysit my cousin	45.00
Buy my lunch	−10.80
Sell newspapers	15.00
Purchase a shirt	−18.69
Brush my teeth	0.00
Pay a library fine	−1.25

1. I notice . . .
   Sample response: The negative values correspond to spending money, and the positive values correspond to earning or gaining money.

2. I wonder . . .
   Sample response: How to make the most money in one week.

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the ***Notice and Wonder*** routine using the table.

2 Monitor

Help students get started by asking, "How could you group some of the amounts together? How are your groups different?"

Look for points of confusion:

- **Not connecting receiving/spending to the signed amounts.** Ask, "When looking at all of the negative amounts, what do the activities have in common?"

Look for productive strategies:

- Connecting receiving money to positive amounts and spending or owing money to negative amounts.

3 Connect

Display the table and anchor chart, with a new section labeled *Change* separated into *Positive* and *Negative*.

Have students share their responses, focusing on the connection between receiving money and positive amounts, and spending money to negative amounts. As students share, have them identify where their words belong on the anchor chart (e.g., *owe*, *fine*, *purchase*, *buy*, *sell*, etc.).

Ask, "What does $0 represent in this context?"

Highlight that what you label as a positive or negative change can depend on your perspective. For example, if you sell a newspaper for $1, you earn money and could think of this as a positive change. However, the person who buys the newspaper spends $1, so they could think of this as −$1, a negative change.

Power-up

To power up students' ability to write expressions to represent gaining or losing money, have students complete:

Determine the operation that would be used to describe each scenario, addition or subtraction.

a. Earning $10. addition

b. Being paid $32. addition

c. Spending $6.50. subtraction

d. Buying an item that costs $42. subtraction

e. Receiving a gift of $100. addition

f. Owing a debt of $12. subtraction

Use: Before the Warm-up.

Informed by: Performance on Lesson 6, Practice Problem 6.

Pairs | 15 min

Activity 1 Managing the School Store

Students use and interpret signed numbers that represent corresponding changes in money and inventory.

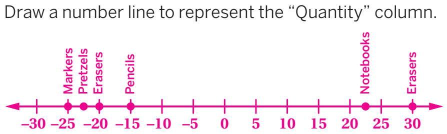
Amps Featured Activity Interactive Number Lines

Activity 1 Managing the School Store

As manager of the school store, Elena keeps records of all the items purchased to stock the store, and all the items sold to students. The table shows her records for Tuesday.

Item	Quantity	Amount ($)
Pack of pencils	−15	18.75
Erasers	30	−22.50
Erasers	−20	15.00
Notebooks	22	−29.70
Packs of markers	−25	62.50
Bags of pretzels	−22	16.50

1. For each column, identify what the following represents:
   a. A positive number.
   the number of items bought to stock the store and the amount of money the store made from selling an item
   b. A negative number.
   the number of items sold and the amount of money spent to restock an item
   c. 0
   No items were bought or sold, and no money was earned or spent.
2. Which item did Elena sell the most? Explain your thinking.
   Elena sold the most packs of markers because of the four items sold; −25 represents the greatest change in the number of items.
3. Mai said there was a greater change in the number of notebooks than in the number of bags of pretzels because 22 > −22. Do you agree or disagree? Explain your thinking.
   Sample response: I disagree because 22 and −22 represent the same amount of change, just in opposite directions.
4. Priya pays $1.25 for a pack of pencils. Represent the value of the pencils from both Priya's perspective and Elena's perspective.
   Priya: −$1.25; Elena: $1.25
5. Draw a number line to represent the "Quantity" column.



1 Launch

Give students 5 minutes of quiet work time, then 5 minutes to work with their partners.

2 Monitor

Help students get started by asking, "Why are the signs different for the two erasers entries?"

Look for points of confusion:

- **Misunderstanding what positive and negative values represent in each column.** Ask, "In the Warm-up, did selling newspapers result in a positive or negative change in money? What about in the quantity of newspapers you had? How is this similar?"
- **Saying Elena sold the most erasers because 30 is the greatest value.** Ask, "What do positive and negative values represent? How many erasers were sold?"

Look for productive strategies:

- Recognizing that all values represent change. Connecting a negative "quantity" to items sold, which corresponds to a positive "value."
- Comparing change as the distance (or difference) from 0. **Note:** The term *absolute value* is introduced in the next lesson.

3 Connect

Display the school store table and anchor chart.

Have students share their responses, one problem at a time, focusing on what the signed numbers represent relative to types of changes — increases or decreases — in quantity and money, and how perspective may change the sign. Add appropriate words to the anchor chart.

Highlight that a positive "quantity" results in a negative "value" because money is spent to increase the inventory. A negative "quantity" results in a positive "value" because money is received when someone buys an item, and this decreases the inventory.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital number lines. You can overlay student responses to see similarities and differences, and to provide immediate feedback to students.

Extension: *Math Enrichment*

Have students complete the following problems:

1. What is the net change in quantity for the erasers? 10 erasers are left in the store.
2. What is the unit price for each item? Sample response: One pack of pencils costs $1.25.

Math Language Development

MLR7: Compare and Connect

During the Connect, highlight how perspective may change the sign of a value by having students discuss their responses to Problem 4. Ask:

- "From Elena's perspective, what decreased? What increased?" The number of packs of pencils in the store decreased. The dollar amount the store has made increased.
- "From Priya's perspective, what decreased? What increased?" Priya's amount of money decreased. Priya now has a pack of pencils, so her quantity of this item increased.

English Learners

Display the term *perspective* along with its translation in students' primary languages to support their understanding.

Pairs | 15 min

Activity 2 Owning Your Own Business

Students create their own business and continue to represent changes in amounts of money and inventory using positive and negative values.

Name: ______ Date: ______ Period: ______

Activity 2 Owning Your Own Business

You and your partner co-own a business.

1. What is the name of your business?
   Sample response: Books-O-Fun

2. Identify three products that your business sells, and set their unit prices.
   Sample responses shown.

Product	Product Name	Unit Price ($)
A	Single books	3.50
B	3-pack book sets	12
C	Complete comic book series	45

3. Determine whether each scenario represents a *credit* or a *debit* for your business. Complete the table with the amount of money your business has gained or lost.
   Sample responses shown.

Scenario	Amount ($)
You buy $250.46 worth of supplies.	−250.46
A customer buys 100 of Product *A*.	350
A flood destroys $350 worth of products.	−350
You pay your $124.79 electricity bill.	−124.79
You sell 50 of Product *B*, and 30 of product *C*.	1,950
You gain 5 new followers on social media.	0

4. Has your business made a profit, or is it operating at a loss? Explain your thinking.
   Sample response: Our business has made a profit because we made more money than we spent. When we sold fifty 3-pack book sets and thirty complete comic book series, we made $1,950, which is more than we spent on supplies, the electricity bill, and on destroyed products.

Reflect: How can goal-setting help prepare you for success?

STOP

1 Launch

Introduce the words *credit*, *debit*, *profit*, and *loss*, and add them to the anchor chart.

2 Monitor

Help students get started by activating their background knowledge. Ask them to name real businesses or companies they know that sell at least three different products.

Look for points of confusion:

- **Listing unit price as an amount gained**. Remind students what unit price means. Ask, "How many did you sell in this scenario? How can you determine how much money you earned in total?"
- **Representing every *buy* and *gain* scenario as positive change**. Remind students of previous discussions about perspective. Ask, "Are you, the store owners, gaining or losing money?"

Look for productive strategies:

- Evaluating each scenario by asking, "Are we, the store owners, gaining or losing money?"
- Multiplying the unit price by the number sold.

3 Connect

Have students share which scenarios resulted in a credit, debit, or no change, and how they determined whether their businesses made a profit. As students share, add new words or phrases to the anchor chart (e.g., *customer buys*).

Ask, "Can you always rely on key words to determine whether a change is positive or negative?"

Highlight that it is important to consider the full scenario *and* perspective, not just key words, when determining whether a change should be represented as positive or negative. For example, consider *who* is buying or selling and *what* is being gained or lost.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students generate their own lists of products, have them choose three products from a list you generate, or from the following list by having them choose a store first, and then three products from that store. Consider allowing them to create prices that use whole numbers.

Book store
Single books
3-pack book sets
Complete comic book series

Clothing store
Pair of jeans
Sweatshirt
Pair of shoes

Bakery
Single bagel
Box of 12 bagels
Loaf of bread

Math Language Development

MLR2: Collect and Display

While students work, circulate and collect informal language they use to talk about change in quantity of product or change in dollar amount. For example, listen for words such as *get, lose, more, less, gain, earn, credit, debit, profit, loss,* etc. During the Connect, add these words to the class display or anchor chart.

English Learners

There are many different words that can be used to represent positive and negative values, and it depends also on the perspective considered. Consider creating copies of the class display that students can keep with them.

Whole Class | 5 min

Summary

Review and synthesize the different meanings of positive and negative numbers when describing changes in inventory or money.

Summary

In today's lesson . . .

You explored the meaning of negative numbers, given real-world contexts. Sometimes you use positive and negative numbers to represent *changes* in quantity. If the quantity *increases*, the change is *positive*. If the quantity *decreases*, the change is *negative*. This is especially common when representing money received (positive numbers) and money spent (negative numbers).

Whether a number is considered positive or negative depends on a person's perspective. For example, suppose Clare's grandmother gives her \$20 for her birthday. Clare could view this as positive 20, which might be written as +20, because her amount of money increased. But her grandmother could view this as negative 20, which would be written as −20, because her amount of money decreased.

When using positive and negative numbers to represent changes, you have to be very clear about what it means when the change is positive and what it means when the change is negative. You also need to consider whose perspective you want to represent.

Reflect:

Synthesize

Ask:

- "What did the positive and negative numbers represent in this lesson?" Sample response: positive numbers: earning or gaining money, increasing inventory by restocking items; negative numbers: owing or losing money, spending money, decreasing inventory by selling product.
- "How were the numbers that represented a change in inventory related to the numbers that represented a change in money?" Sample response: When restocking an item, the store manager needs to buy the item — which means from the store manager's perspective, he or she loses money (negative). When selling an item, the store manager gains money (positive).
- "How does perspective impact the way you represent changes in money and inventory?" Sample response: Based on the perspective of the person or company, the sign of the number might be positive or negative.
- "What other situations can you think of where you might gain or lose an amount of something? How could you use positive and negative numbers to talk about changes in those situations? Are there multiple perspectives?" Answers may vary.

Highlight that students can use positive and negative numbers to represent changes in inventory and money, but the sign depends on the perspective of who is keeping the record.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did a negative number represent in this lesson?"
- "If you owned your own business, would you want to see negative numbers or positive numbers?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of positive and negative numbers in context by interpreting a bakery owner's records.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.06

The table shows records of money coming into and going out of the bank account of an owner of a catering company.

Date	Item	Amount ($)
May 1	rent	850
May 2	catering order: 1 sandwich platter and 2 dozen salads	106.75
May 3	electricity	294.50
May 5	catering order: 1 barbeque dinner and 60 assorted side dishes	240.55
May 5	baking supplies	147.95
May 6	catering order: 350 dinners	428.30
May 7	equipment sales	415.65

1. For which items did the owner receive money?
   The catering orders and equipment sales.

2. What does the number $-\$147.95$ represent in this context?
   The owner spent $147.95.

3. Did the owner receive more money or spend more money on May 5? Explain your thinking.
   The owner received more money because $240.55 represents a greater change than $-\$147.95$.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain and use negative numbers in situations involving money. 1 2 3

b. I can interpret and use negative numbers in different contexts. 1 2 3

Lesson 6 Using Negative Numbers to Make Sense of Contexts

Success looks like . . .

- **Language Goal:** Interpreting a table of signed numbers that represent how a quantity changed. **(Speaking and Listening, Reading and Writing)**
  - » Interpreting the values of the table to determine when the owner received or spent money in Problems 1 and 3.
- **Goal:** Recognizing that signed numbers can be useful to represent changes in a quantity in opposite directions, e.g., money received and money paid, inventory bought and inventory sold, etc.
  - » Explaining the value of – $147.95 in the context of the table in Problem 2.

Suggested next steps

If students identify the negative amounts as money earned for Problems 1 and 2, consider:

- Reviewing the table in Activity 1 and asking, "When did Elena record positive dollar values? What did a negative dollar amount mean for Elena?"
- Asking, "If you own a business, when will you earn money? How will you represent this? When will you spend money? How will you represent thiat?"
- Assigning Practice Problems 1c, 1d, 2c, and 2d.

If students say the bakery owner spent more money in Problem 3, consider:

- Asking, "Which value represents the greater change?" or, "How much does the amount of the owner's money change for the catering order? How much does it change for purchasing baking supplies?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was using signed numbers to represent "changes" in value in money or inventory contexts. How well did students accomplish this? What did you specifically do to help students accomplish it?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

Practice

1. Write a positive or negative number to represent the change for each scenario.
   - a. Tuesday's high temperature was 4 degrees colder than Monday's high temperature.
     -4
   - b. Wednesday's high temperature was 3.5 degrees warmer than Tuesday's high temperature.
     3.5
   - c. I earned $6.50 for babysitting my brother.
     6.5 or 6.50
   - d. I spent $2 on a gift for a friend.
     -2

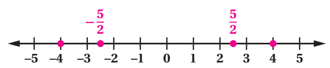
2. Decide whether the quantity in each scenario is positive or negative, from the perspective of the person or object indicated. Then describe a scenario, in the same context, that would have the opposite sign.
   - a. Tyler's puppy gained 5 lb.
     Positive; Tyler's puppy lost 5 lb.
   - b. The aquarium leaked 2 gallons of water.
     Negative; The manager added 2 gallons of water.
   - c. Andre received a gift of $10.
     Positive; Andre gave a gift of $10.
   - d. Kiran gave a gift of $10.
     Negative; Kiran received a gift of $10.
   - e. A climber descended 550 ft.
     Negative; A climber ascended 550 ft.

3. For the integers -15, 28, and 0, complete the following tasks.
   - a. Describe a real-world scenario where each integer represents a quantity.
     Sample responses: It is −15°C outside. My city is 28 ft above sea level. A city is at sea level.
   - b. Describe a real-world scenario where each integer represents a *change* in a quantity. State whose perspective matches the sign of the number.
     Sample responses:
     - **I paid $15 for my lunch. From my perspective, I have −$15.**
     - **My team gained 28 yd in the football game. From my team's perspective, we have positive 28 yd.**
     - **I earned no money for watching television. From my perspective, I have neither gained nor lost any money.**

Name: Date: Period:

Practice

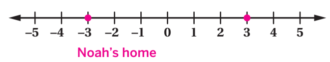
4. Refer to the number line.



   - a. Label the points that are 4 units from 0. What would happen to these points if you folded the number line so that a vertical crease goes through 0?
     The points 4 and −4 would match up (align).
   - b. Label the points that are $\frac{5}{2}$ units from 0. What is the distance between these points?
     The distance is 5.

5. Evaluate each expression.
   - a. $2^3 \cdot 3 = \mathbf{24}$
   - b. $\frac{4^2}{2} = \mathbf{8}$
   - c. $3^1 = \mathbf{3}$
   - d. $6^2 \div 4 = \mathbf{9}$
   - e. $2^3 - 2 = \mathbf{6}$
   - f. $10^2 + 5^2 = \mathbf{125}$

6. Han and Noah both leave the park and walk in opposite directions towards their homes. They both live the same distance away from the park. Let 0 represent the location of the park. If the location of Han's house can be represented by 3 on the number line, mark the location of Noah's home.



Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 7 Lesson 3	2
	5	Unit 6 Lesson 15	2
Formative	6	Unit 7 Lesson 7	2

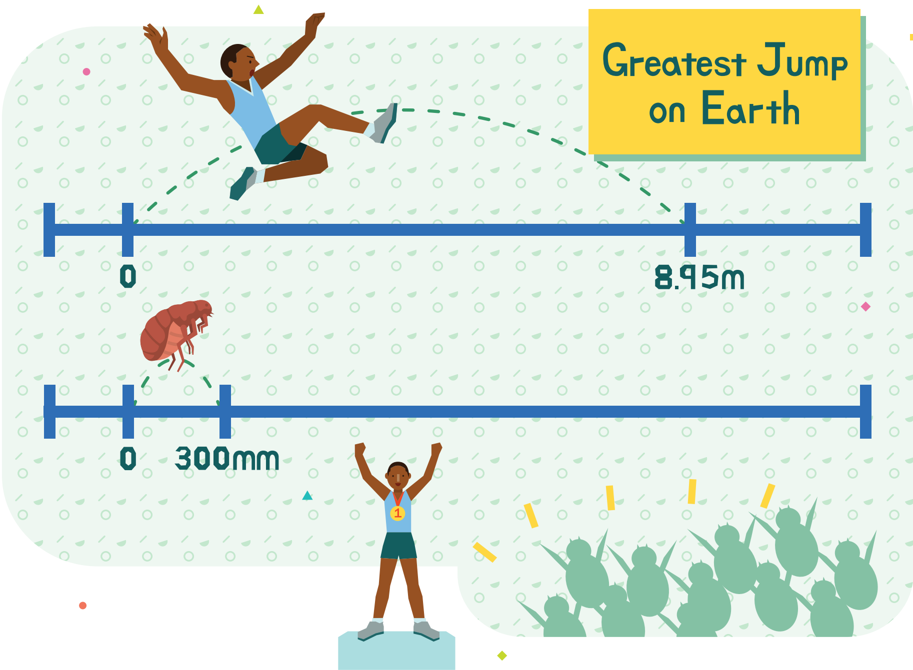
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Absolute Value of Numbers

Let's explore distances from zero more closely.



Focus

Goals

1. **Language Goal:** Compare rational numbers and their absolute values, and justify comparisons. **(Speaking and Listening, Writing)**
2. **Language Goal:** Comprehend the term *absolute value* and the symbol | | to refer to a number's distance from zero on the number line. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Interpret rational numbers and their absolute values in the contexts of elevation and temperature. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students formalize the concept of a number's magnitude using the term *absolute value* to mean a number's distance from zero. They recognize that opposite numbers have the same absolute value, interpret absolute value in the familiar contexts of temperature and elevation, and compare and contrast rational numbers and their absolute values. Students recognize the similarities in structure of the vertical number lines represented in elevation and temperature scenarios.

Previously

In Lessons 2–6, students reasoned about the structure of rational numbers by plotting them on a number line and noting their relative positions and distances from zero. They also learned that opposite numbers are located the same distance from zero.

Coming Soon

In Lesson 8, students will continue to write, interpret, and explain comparisons of rational numbers in real-world contexts. They will use absolute value to discuss distances on the coordinate plane in Lesson 16. In Grade 7, students will add and subtract positive and negative rational numbers, and represent these operations on horizontal or vertical number lines.

Rigor

- Students develop **conceptual understanding** of absolute value as the distance from 0 on a number line.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut, one flea per student
- Activity 1 PDF (number line for display)

Math Language Development

New words

- ***absolute value***

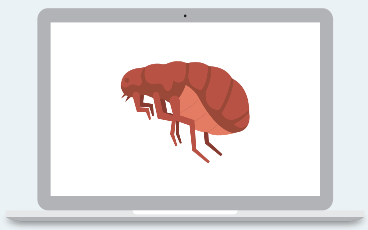
Review words

- *negative number*
- *positive number*
- *opposite*
- *rational number*
- *sign*

Amps | Featured Activity

Activity 1
Jumping Fleas

Students can use the interactive flea number line to simulate absolute values as distances from 0.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel frustrated in their attempts to reason abstractly with the introduction of the new symbol for absolute value in Activity 3. Explore with students how they can manage their frustration and move towards the goal of understanding of the symbol and being able to use it to solve problems.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 3**, assign or have students choose either Problem 1 or 2.

Independent | 5 min

Warm-up Number Talk

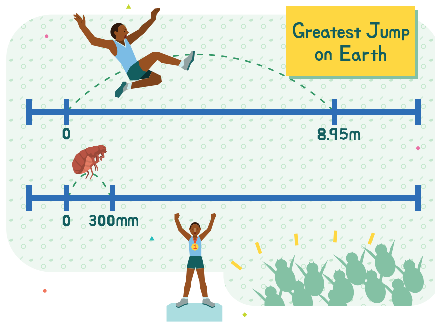
Students will determine which number in a given pair is closer to 0, developing fluency in their thinking about distances from 0.

Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 7

Absolute Value of Numbers

Let's explore distances from zero more closely.



Warm-up Number Talk

For each pair of numbers, mentally decide which one has a value that is closer to 0.

1. $\frac{9}{11}$ or $\frac{15}{11}$
2. $\frac{1}{5}$ or $\frac{1}{9}$
3. 1.25 or $\frac{5}{4}$
   Same distance from 0
4. 5 or -2
5. -0.33 or -0.25

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the *Number Talk* routine, displaying one pair of numbers at a time.

2 Monitor

Help students get started by asking "What do you notice about the denominators? the numerators? Can you imagine a number line with these points on it?"

Look for points of confusion:

- **Choosing the value that is greater or less.** Remind students that this is not simply a greater than or less than comparison.
- **Saying $\frac{1}{5}$, in part b, because the denominator is less than the other fraction's denominator.** Review comparing fractions.

Look for productive strategies:

- Imagining a number line in their head to visualize which is closer.

3 Connect

Display all the pairs of expressions. Record students' reasoning for which number is closer to 0 for each pair.

Have individual students share their reasoning for each pair of expressions. If there isn't time to discuss strategies for all five pairs, focus on Problems 2, 4, and 5.

Ask:

- "Is the number with the greater value always farther from 0?"
- "Is the number with the lesser value always closer to 0?"

Highlight strategies and understandings students have about the distances from 0 on the number line.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

During the Connect, as students share their reasoning for each pair of expressions, draw their attention as to how they compared two positive values (e.g., Problem 1), two negative values (e.g., Problem 5), and two values with different signs (e.g., Problem 4). Press them for details explaining how they knew which number was closer to 0. Display these sentence frames to support students as they explain their strategies.

- "First, I _____ because . . ."
- "I noticed _____, so I . . ."

English Learners

Provide students time to rehearse what they will say with a partner, before sharing with the whole class.

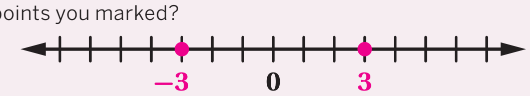
Power-up

To power up students' ability to locate opposite values on a number line, have students complete:

Recall that *opposite numbers* are two values that are the same distance from 0, but on different sides of the number line.

Mark a pair of opposite values on the given number line. What are the values of the two points you marked?

Sample response shown:



Use: Before Activity 1.

Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 10 min

Activity 1 Jumping Fleas

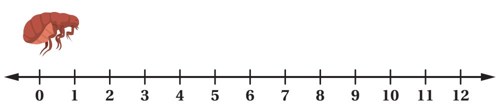
Students move a flea along a number line, noting its distance from 0, to foster initial discussions about absolute value, which will be further developed in the next activity.

Amps Featured Activity Jumping Fleas

Activity 1 Jumping Fleas

The current world record for long jump is held by American Mike Powell, who jumped a distance of 8.95 m at the 1991 World Championships in Athletics. But the greatest jumper in the animal kingdom, relative to its size, is actually the flea! Fleas can jump over 200 times their own body length. Imagine a little 1.5-mm flea in a long jump competition jumping an impressive 300 mm (that's 0.3 m, or about 12 in.). Well, impressive for a flea — it's all relative!

A flea is jumping around on a number line, where each tick mark represents 1 in. You will be given the cut-out of a flea and a number line to record the flea's jumps. Use your flea to help you complete this activity. Each jump should be 12 in.



1. The flea starts at 0 and jumps once.
   - a. Where might it land?
     12 or −12
   - b. Could it land somewhere else? If so, where? If not, why not?
     12 or −12, whichever is the opposite of the students' response to part a.

2. Suppose you do not know where the flea starts, but it jumps twice.
   - a. If the flea lands at 0, where could it have started?
     24 (if the flea jumped left twice), −24 (if the flea jumped right twice), or 0 (if the flea jumped left then right, or right then left)
   - b. Could the flea have started from a different point and still ended up at 0?
     24, −24, or 0 (whichever are missing from the student's response to part a)

3. Now there are two fleas, both starting at 1.
   - a. The first flea jumps once to the right, and the second flea jumps once to the left. Where does each flea land?
     First flea: **13** Second flea: **−11**
   - b. How far away from 0 does each flea land?
     First flea: **13 in.** Second flea: **11 in.**

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1 Launch

Distribute one pre-cut flea to each student from the Activity 1 PDF. Give students 5 minutes to complete Part 1 individually before sharing with a partner, and follow with a whole-class discussion. Repeat for Part 2. Students can "play" with moving the flea around more, if time allows.

2 Monitor

Help students get started by asking, "How many different ways could your flea jump?"

Look for points of confusion:

- **Jumping 1 space instead of 12.** Remind students that one jump equals 12 in. or 12 units on the number line.
- **Questioning the symmetry of ending up at −11 on the number line in Problem 3a.** Suggest students redo their jumps to verify the result.

Look for productive strategies:

- Fluidly moving along the number line to solve.
- Noticing that there could be two solutions for Problem 1a.

3 Connect

Display the number line from Activity 1 PDF.

Have individuals or pairs of students share their solutions and reasoning, focusing on opposites from Problems 1 and 2.

Ask, "How do you know the pairs of numbers from Problems 1 and 2 are opposites? Why are the pairs of numbers in Problem 3 *not* opposites?"

Highlight that opposites have the same absolute value because they are the same distance from zero.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive number line to simulate absolute values as distances from 0.

Accessibility: *Vary Demands to Optimize Challenge*

Have students use 2 in., instead of 12 in., to represent each jump of their fleas. They can still use the number line from the Activity 1 PDF.

Extension: *Math Enrichment*

As a follow-up to Problem 3, ask students how far away the first flea is from the second flea after they each land, for each of parts a and b. 24 in.; 2 in.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand a flea can jump a great distance and that they will use the cut-out of a flea to simulate a flea jumping on the number line.
- **Read 2:** Ask students what each tick mark represents on the number line. 1 in.
- **Read 3:** Ask students to preview Problems 1–2 to brainstorm how they can use the flea cut-out to simulate the flea jumping once or twice.

Pairs | 10 min

Activity 2 Absolute Value With Jumping Fleas

Students continue their exploration of the number line, relative to 0. The term *absolute value* is introduced.

Name: ______ Date: ______ Period: ______

Activity 2 Absolute Value With Jumping Fleas

The *absolute value* of a number represents its distance from 0. To represent the absolute value of a number n, use the notation $|n|$, which can also be interpreted as the distance from n to 0 on the number line.

Let's think about some more fleas jumping around on a number line.

1. If a flea is 6 units to the left of 0 on the number line, how do you write its distance from 0 as an absolute value expression?
   $|-6|$
2. If a flea is to the right of 0 and the absolute value of its location is equal to 12.5, where on the number line is the flea located?
   12.5
3. What does $|-7|$ represent? What is its value?
   $|-7|$ represents the distance from -7 to 0. Its value is 7.
4. What does $|1.8|$ represent? What is its value?
   $|1.8|$ represents the distance from 1.8 to 0. Its value is 1.8.

Are you ready for more?

Consider a flea that does not jump the maximum possible distance of 12 in. It can start anywhere on the number line and must jump at least 1 in., but always toward 0. List three possible combinations of starting points and distances jumped, in inches. Then determine the flea's ending point for each, and how far this final distance is from 0.

Sample responses shown.

Starting point	Distance jumped (in.)	Ending point	Final distance from 0
3	4	-1	1
-3	4	1	1
2	11	-9	9

1 Launch

Discuss the definition of *absolute value*. Have students use the ***Think-Pair-Share*** routine. Provide them 5 minutes of individual work time. Then have them compare responses with a partner.

2 Monitor

Help students get started by discussing the meaning of *absolute value*, and its notation.

Look for points of confusion:

- **Writing $|6|$ instead of $|-6|$.** Remind students that the sign stays in the notation and is read as *the absolute value of* -6.
- **Confusing absolute value with opposites.** Remind students that absolute value represents the distance of the number from 0, without worrying about the sign or direction.

Look for productive strategies:

- Using symbols and signs appropriately. Students can explain when to use the absolute value symbols and negative sign.

3 Connect

Display a number line to use, if necessary.

Have individuals or pairs of students share their responses using precise language representing absolute value and location.

Define the ***absolute value*** of a number as its distance from 0.

Highlight that absolute value is always positive, so there should never be a negative sign after evaluating an absolute value.

Ask, "Does finding a number's absolute value always mean changing the sign?"

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Before students begin Activity 2, have them use their flea cut-out and number line from Activity 1 to model the flea jumping once and landing on -12. Ask students how far away the flea is from 0 and tell them this distance is the *absolute value* of -12. Then display the notation $|-12|$ and clarify what this notation means.

Accessibility: *Optimize Access to Tools*

Provide access to blank number lines or the number line from Activity 1 for students to use or refer to during this activity.

Math Language Development

MLR1: Stronger and Clearer Each Time

To help students refine their understanding of absolute value, have pairs meet with 1–2 other pairs of students for feedback on their responses to Problems 1–4, using the following prompts:

- "Can you show on a number line where . . ."
- "Why are the values the same for this number?"
- "Why are the absolute values always positive?"

Students can use ideas and language to strengthen their final responses.

English Learners

Use intentional grouping so that students with different levels of English language proficiency have an opportunity to interact together.

Pairs | 10 min

Activity 3 Absolute Value With Elevation and Temperature

Students develop their understanding that the $|x|$ notation represents the distance x is from zero and that the expressions $|-x|$ and $|x|$ are equivalent.

Activity 3 Absolute Value With Elevation and Temperature

1. Part of the city of New Orleans is 6 ft below sea level. You can use -6 ft to describe its elevation, and $|-6|$ ft to describe its distance from sea level. In the context of elevation, what would each of the following describe?
   a. 25 ft **This represents an elevation of 25 ft, which is above sea level.**
   b. $|25|$ ft **This represents the distance of 25 ft from sea level.**
   c. -8 ft **This represents an elevation of -8 ft, which is below sea level.**
   d. $|-8|$ ft **This represents the distance that -8 ft is from sea level, which is a distance of 8 ft.**

2. You can use -5°C to describe a temperature that is 5 degrees below the freezing point, 0°C, and you can use 5°C to describe a temperature that is 5 degrees above the freezing point. In the context of temperature, in degrees Celsius, what do each of the following describe?
   a. 1 **This represents a temperature of 1 degree Celsius, which is 1 degree above the freezing point, or 1 degree warmer than 0°C.**
   b. -4 **This represents a temperature of -4 degrees Celsius, which is 4 degrees below the freezing point, or 4 degrees colder than 0°C.**
   c. $|12|$ **This represents the number of degrees that the temperature 12°C is above the freezing point, which is 12°C above the freezing point.**
   d. $|-7|$ **This represents the number of degrees that the temperature -7°C is below the freezing point, which is 7°C below the freezing point.**

Are you ready for more?

At a certain time, the difference between the temperatures in New York City and Boston was 7°C. At that same time, the difference between the temperatures in Boston and Chicago was also 7°C. Could the temperatures in New York City and Chicago be the same? Is it possible for one to be above freezing and the other below? Explain your thinking.

Yes, they could have been the same temperature. It is also possible for one to be above freezing and the other below. For example, if the temperature in Boston was 0°C and the temperature in New York City was 7°C, then the temperature in Chicago could have been -7°C (or vice versa).

STOP

772 Unit 7 Rational Numbers

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by focusing on the second sentence in Problem 1 and asking, "How are these descriptions different?"

Look for points of confusion:

- **Having difficulty translating the absolute value into words.** Connect a sentence frame to the language, such as "The distance from an elevation of ___ to sea level is ___."

Look for productive strategies:

- Using a vertical number line to visualize the scenarios.

3 Connect

Display a vertical number line to aid discussion.

Have individual students share their solutions and reasoning, noting the language used to describe each scenario with and without the absolute value notation.

Highlight the similarities in the structures of the vertical number line for both elevation and temperature.

Ask, "Consider the temperatures -6°C and 3°C. Which is colder? Closer to freezing? Has a greater absolute value?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1a, 1b, 2a, and 2d. This will allow them to access the goal of the activity and interpret a variety of expressions involving absolute value.

Math Language Development

MLR7: Compare and Connect

Before students begin, provide them with phrases that they could use to connect each scenario to the numerical values.

For Problem 1, display these phrases:

- _____ represents an elevation of _____, which is [above/below] sea level.
- _____ represents the distance that _____ is from sea level.

For Problem 2, display these phrases:

- _____ represents a temperature of _____, which is [above/below] the freezing point.
- _____ represents the distance that _____ is from the freezing point.

English Learners

Connect the phrase "below the freezing point" with "colder than 0°C" and the phrase "above the freezing point" with "warmer than 0°C."

Whole Class | 5 min

Summary

Review and synthesize how to relate the meaning of 0 to the absolute value of rational numbers, within a context.

Name: ______ Date: ______ Period: ______

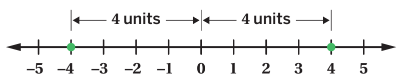
Summary

In today's lesson . . .

You explored how numbers are related to zero. In contexts such as elevation and temperature, it is important to understand what 0 represents. For elevation, 0 ft represents sea level. For temperature, $0°$ Celsius is the freezing point of water. However, this is *not* the freezing point for degrees Fahrenheit.

The ***absolute value*** of a number represents its distance from 0.

- For example, the absolute value of -4 is 4, because -4 is 4 units to the left of 0.
- The absolute value of 4 is also 4, because 4 is 4 units to the right of 0.
- Opposites always have the *same* absolute value because they are both located the same distance from 0, just in opposite directions.



The absolute value of any number is always a non-negative value because it represents distance, and distance is always non-negative.

- To represent the absolute value of 4, use the notation $|4|$, which equals 4.
- To represent the absolute value of -8, use the notation $|-8|$, which equals 8.
- Opposite values have the same absolute value since they are the same distance from 0: $|-6| = |6| = 6$

Reflect:

Synthesize

Display the number line from the Summary.

Highlight the difference between opposites and absolute value.

Formalize vocabulary: ***absolute value***

Ask:

- "How would you write the absolute value of -9?"
- "How would you read it?"
- "How would you evaluate it?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can two objects move the same distance, but end up in different places?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *absolute value* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by explaining how the absolute value of -12.9 is different from the number 12.9.

Printable

Name: ______________________ Date: __________ Period: __________

Exit Ticket 7.07

How are the expressions $|-12.9|$ and 12.9 different? How are they similar? Explain your thinking, and be sure to include any necessary mathematical symbols.

Sample response: The expressions are different in appearance. In the first expression, 12.9 is represented with a negative sign. In the second expression, 12.9 is positive (no negative sign). The first expression also includes the absolute value symbol | |.

The expressions are similar because they have the same value. The absolute value of -12.9 is 12.9, so it has the same value as 12.9.

Self-Assess

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it

a. I can explain what the absolute value of a number means. 1 2 3

b. I can determine the absolute value of a rational number. 1 2 3

c. I can recognize and use the notation for absolute value. 1 2 3

Lesson 7 Absolute Value of Numbers

Success looks like . . .

- **Language Goal:** Comparing rational numbers and their absolute values, and justifying comparisons. **(Speaking and Listening, Writing)**
  - » Comparing and contrasting the appearance and values of $|-12.9|$ and 12.9.
- **Language Goal:** Comprehending the term *absolute value* and the symbol | | to refer to a number's distance from zero on the number line. **(Speaking and Listening, Reading and Writing)**
  - » Explaining the meaning of $|-12.9|$.
- **Language Goal:** Interpreting rational numbers and their absolute values in the contexts of elevation and temperature. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students do not identify how the expressions are different, consider:

- Reviewing the reasoning behind the solutions from Activity 1, Problems 1–2.

If students do not identify how they are similar beyond that the numbers share the same digits, consider:

- Reminding students of the meaning and value of the absolute value expression $|-12.9|$.
- Referring back to the final paragraph of the Summary.

If students cannot articulate their reasoning, consider:

- Referring back to Activity 1 and asking, "How did you talk about absolute value in the first flea activity?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did students use the structure of the number line to formalize the concept of absolute value? How did that build on earlier work with magnitude, distance, and opposites?
- In Activity 2, you used intentional grouping with MLR1 to group students with different levels of English proficiency. What effect did this grouping strategy have on student discussions or revisions? Would you have anything the next time you use this routine?

Math Language Development

Language Goal: Comprehending the phrase absolute value and the symbol | | to refer to a number's distance from zero on the number line.

Reflect on students' language development toward this goal.

- How have students progressed in this unit in their understanding of *absolute value* representing the *magnitude* of a number, where the magnitude can be thought of as the number's distance from zero on the number line?
- Reflect on the language routines used in this lesson? Were there any that were more helpful than others? Why? Would you change anything the next time you use these routines?

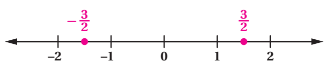
Practice

Independent

Name: ______________________ Date: __________ Period: __________

Practice

> **1.** Plot and label *all* the numbers that have an absolute value of $\frac{3}{2}$ on the number line.



> **2.** The temperature at dawn was 6°C away from 0°C. Select *all* the temperatures that could have been the temperature at dawn.

A. −12°C

B. −6°C

C. 0°C

D. 6°C

E. 12°C

> **3.** Order the values from least to greatest.

$|2.7|$ 0   1.3 $|-1|$ 2

$0,\ |-1|,\ 1.3,\ 2,\ |2.7|$

> **4.** Complete each sentence with a number (or an expression with an exponent) that makes it true.

a. 3^4 is ....3.... times greater than 3^3.

b. 5^3 is ....5.... times greater than 5^2.

c. 7^{10} is ....7^2 or 49.... times greater than 7^8.

d. 17^6 is ....17^2 or 289.... times greater than 17^4.

e. 5^{10} is ....5^6 or 15,625.... times greater than 5^4.

Name: ______________________ Date: __________ Period: __________

Practice

> **5.** Elena donates some money to charity each time she earns money as a babysitter. The table shows how much money in dollars d she donates for different amounts of money m that she earns.

d (\$)	4.44	1.80	3.12	3.69	2.16
m	37	15	26	30	18

a. What percent of her money earned does Elena donate to charity? Show your thinking.

She donates 12%; $\frac{4.44}{37} = 0.12$

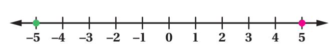
b. Which quantity, m or d, would be the better choice for the dependent variable in an equation describing the relationship between m and d? Explain your thinking.

d; Sample response: Because it is the result of multiplying m by 0.12.

c. Use your choice from part b to write an equation that relates m and d.

$d = 0.12 \cdot m$

> **6.** Plot a point at the opposite of −5 on the number line.



Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	1
Spiral	4	Unit 6 Lesson 15	2
	5	Unit 6 Lesson 17	2
Formative	6	Unit 7 Lesson 8	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 7 | **LESSON 8**

Comparing Numbers and Distances From Zero

Let's use absolute value and negative numbers to think about elevation.

Focus

Goals

1. **Language Goal:** Critique comparisons (expressed using words or symbols) of rational numbers and their absolute values. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Generate values that meet given conditions for their relative position and absolute value, and justify the comparisons (using words and symbols). **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Recognize that the value of $-n$ can be positive or negative, depending on the value of n. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students use precise language to distinguish between the order of rational numbers and their absolute values. They apply their understanding to visualize the possible elevations of several animals and compare absolute locations with relative distances. Students then reason with rational numbers and absolute values to write and explain mathematical comparison statements.

Previously

Since Grade 2, students have worked only with non-negative numbers, for which magnitude and order were indistinguishable. Most recently, in Lesson 7, they began to write and reason with absolute values and reasoning with distances from 0 with positive and negative numbers.

Coming Soon

In Lesson 16, students will apply absolute value to determine distances between points with positive and negative coordinates on the coordinate plane.

Rigor

- Students build **fluency** with determining and comparing rational numbers and their absolute values.
- Students **apply** absolute value to elevations and relative distance from sea level.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials
- Exit Ticket
- Additional Practice

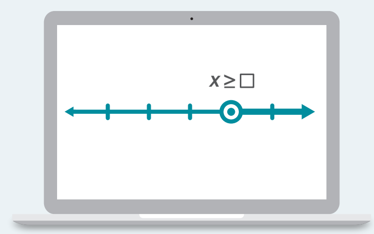
Math Language Development

Review words
- *absolute value*
- *negative number*
- *positive number*
- *opposite*
- *rational number*
- *sign*

Amps | Featured Activity

Activity 1
Interactive Number Line

Students move virtual animals along an interactive vertical number line. You can overlay student responses to provide immediate feedback to students.



Amps POWERED BY desmos

Building Math Identity and Community
Connecting to Mathematical Practices

Students might feel that they are ill-equipped to compare rational numbers in Activity 2. Have students work with their partners and their strengths to gain confidence in their ability to complete the activity. Have them identify both skills and tools that they can use to be successful.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 1**, assign one animal to a pair. Time is reduced by having students focus on one animal instead of all four.

Independent | 5 min

Warm-up Opposites

Students use the structure of the number line to plot a number and its opposite, leading to an initial understanding of the opposite of a negative number.

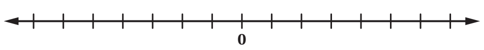
Unit 7 | Lesson 8

Comparing Numbers and Distances From Zero

Let's use absolute value and negative numbers to think about elevation.

Warm-up Opposites

1. Suppose n is a rational number. Choose a value for n and plot it on the number line.
   Answers will vary.



2. Refer to your value for n.
   a. Plot $-n$ on the number line.
   Answers may vary, but n and $-n$ should be opposites, located the same distance from 0 on opposite sides of the number line.
   b. What is the value of $-n$?
   Answers may vary, but if a positive value was chosen for n, the value of $-n$ should be negative; likewise, if a negative value was chosen for n, the value of $-n$ should be positive.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by having them first number the ticks on the number line, and choose one tick to label n.

Look for points of confusion:
- **Mislabeling a chosen point.** Have students label all of the ticks on the number line first.
- **Thinking that all opposites are negative numbers.** Remind students that "Opposites are on different sides of 0, so for a point located to the left of zero, its opposite is located to the right of zero." Demonstrate equal distances, if necessary.

Look for productive strategies:
- Saying the opposite of a negative is a positive. This could be shown as: $-(-3)$, and read as "the opposite of negative 3" or "the opposite of the opposite of 3".

3 Connect

Display a blank number line with only 0 labeled in the center.

Ask, "Who chose a value for n that was a point to the right of 0 on the number line? Who choose a value for n that was a point to the left of 0?"

Have students share the numbers they chose and the numbers they determined to be the opposites.

Highlight that $-n$ can be seen as the opposite of a number, even if that number itself is negative. The opposite of a positive number is negative, and the opposite of a negative number is positive because they must be on opposite sides of zero.

In general, the opposite of $-n$ is n, so $-(-n) = n$.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the following incorrect statement, "The value of $-n$ is always negative, for any value of n." Ask:

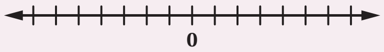
- ***Critique:*** "Do you agree or disagree with this statement? Can you think of a value for n, in which the value of $-n$ is positive?" Sample response: I disagree with this statement. If $n = -3$, then the $-n$ means the opposite of -3, which is positive 3.
- ***Correct:*** "Write a corrected statement that is now true." Sample response: The value of $-n$ is always the opposite of the value of n, for any value of n.
- ***Clarify:*** "How did you correct the statement? How do you know that the statement is now true?"

Power-up

To power up students' ability to locate the opposite of a negative value on a number line, have students complete:

Recall that *opposite numbers* are two values that are the same distance from 0, but on different sides of the number line.

Determine the opposite of each value. Use the number line to help you if needed.



a. 5 -5 **b.** 7 -7 **c.** -7 7 **d.** -3 3

Use: Before the Warm-up.
Informed by: Performance on Lesson 7, Practice Problem 6.

Small Groups | 20 min

Activity 1 Comparing Elevations and Distances From Sea Level

Students distinguish between absolute value and order in the context of elevation and then transition to writing inequality statements involving animals above and below sea level.

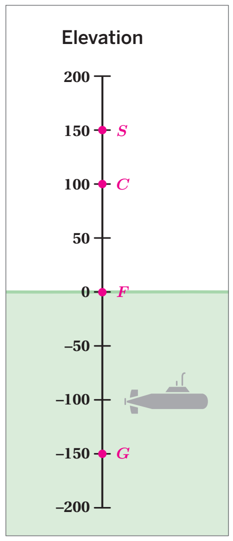
Amps Featured Activity Interactive Number Line

Name: ______ Date: ______ Period: ______

Activity 1 Comparing Elevations and Distances From Sea Level

The connection between absolute value and distance will return in more advanced mathematics. Absolute values and similar ideas are commonly used, even in recent mathematical research, such as the work of Mary Deconge-Watson.

Take a look at these examples of distance. A submarine is at an elevation of -100 ft, as shown on the vertical number line. Compare the elevations of the seagull, giant tube worm, flying fish, and coral reef to the elevation of the submarine.



1. Use the following information to plot and label a possible location where the seagull, giant tube worm, flying fish, and coral reef could each be found. Sample responses shown.
   a. A seagull is located at an elevation, S, that is greater than the elevation of the submarine. It is farther away from sea level than the submarine.
   b. A giant tube worm is located at an elevation, G, that is less than the elevation of the submarine.
   c. A flying fish is located at an elevation, F, that is greater than the elevation of the submarine. It is closer to sea level than the submarine.
   d. A coral reef is located at an elevation, C, that is the same distance from sea level as the submarine, but not at the same location.

1 Launch

Set an expectation for the amount of time students will have to work collaboratively on the activity.

2 Monitor

Help students get started by asking, "Should the seagull be above or below the submarine? Where could it be relative to sea level?"

Look for points of confusion:

- **Thinking *farther away from sea level* always means *greater than* 0.** Remind students this is true for positive numbers, but is not true for negative numbers.
- **Thinking the coral reef can only be below sea level based on prior knowledge.** Explain that many mountains and volcanoes above sea level today were below sea level hundreds of thousands of years ago, and coral reefs have been found at the top mountains and volcanoes that are now above sea level.
- **Thinking elevations that are *less than* always means moving toward sea level.** Ask, "What values could be less than -100, where the submarine is located?" This will help when making comparisons like $-150 < -100$.
- **Thinking the giant tube worm's elevation can be less than the submarine and closer to sea level.** Have students draw the giant tube worm where they think it could be found. Ask, "Does that elevation hold true with what you were thinking?"
- **Overlooking the negative sign when comparing a negative and a positive number.** Ask, "What does the negative sign tell you about the location of this number on the number line?"

Look for productive strategies:

- Using precise language in their reasoning. For example, saying "The ___'s elevation is greater than the submarine's, which means it must be greater than -100 ft."
- Translating reasoning to inequalities. "The ___'s elevation is greater than the submarine's, which means it must be greater than -100. I can write the inequality as I say it: ___ > -100."

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can move virtual animals along an interactive vertical number line. You can overlay student responses to provide immediate feedback to students.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to comparisons about elevation (greater elevation or less elevation) versus comparisons about distance from sea level. Highlight that an object, such as the flying fish, can be closer to *sea level* (which is a *smaller distance from sea level*) and yet have a *greater elevation* than another object, such as the submarine.

Featured Mathematician

Mary Deconge-Watson

Have students read about featured mathematician Mary Deconge-Watson, who received a Ph.D. for advanced work related to norming.

Small Groups | 20 min

Activity 1 Comparing Elevations and Distances From Sea Level (continued)

Students distinguish between absolute value and order in the context of elevation and then transition to writing inequality statements involving animals above and below sea level.

Activity 1 Comparing Elevations and Distances From Sea Level (continued)

2. Complete the table as follows:
   a. Write a possible elevation for each animal.
   b. Use <, >, or = to compare the elevation of each animal to the elevation of the submarine.
   c. Use absolute value to represent the distance from sea level to each animal.

	Possible elevation	Compare to submarine	Distance from sea level
Submarine	-100 ft	$-100 = -100$	$\lvert -100 \rvert$ or 100 ft
Seagull	120 ft	$120 > -100$	$\lvert 120 \rvert$ or 120 ft
Giant tube worm	-150 ft	$-150 < -100$	$\lvert -150 \rvert$ or 150 ft
Flying fish	0 ft	$0 > -100$	$\lvert 0 \rvert$ or 0 ft or at sea level
Coral reef	100 ft	$100 > -100$	$\lvert 100 \rvert$ or 100 ft

Sample answers are shown. The elevation of the flying fish could span above and below sea level.

3. Priya says the elevation of a sea turtle is less than the submarine's and closer to sea level. Is this possible? Explain your thinking.
   No, it would have to be below the submarine to have an elevation that is less than the submarine's, but that means it would be farther from sea level, not closer to it.

Featured Mathematician

Mary Deconge-Watson

Born in Louisiana in 1933, Deconge-Watson became the 15th African-American woman ever to earn a Ph.D. in mathematics. Her doctoral thesis is entitled *2-Normed Lattices and 2-Metric Spaces*. Mathematically, "norming" something is very much like taking its absolute value or determining its distance from zero. She is also known for her work related to the Cauchy problem for parabolic equations, which goes well beyond absolute values.

3 Connect

Display the elevation diagram and a blank version of the table.

Have groups of students share different possible elevations for each of the animals, focusing on the overall order from closest to sea level (top) to farthest from sea level (bottom).

Ask:

- "How is it possible that the distance from sea level to the submarine and the distance from sea level to the coral reef look different but represent the same value?" Sample response: The absolute value of -100 and 100 have the same value because they are the same distance from 0.
- "Which animal could have been at a positive or negative elevation, or at sea level?" Inform students that a flying fish can propel themselves up to 4 ft above the water.
- "What would an inequality look like that compares the elevation of one of the animals to sea level? Which ones would look similar? Which ones would look different? How would comparing the elevation of the submarine to sea level fit in?" Sample response: $120 > 0$ represents the seagull's elevation compared to sea level. All of the animals or objects that are above sea level will use the > symbol. All of the animals or objects that are at sea level will use the equal sign. All of the animals or objects that are below sea level will use the < symbol.

Highlight that the absolute values of opposite numbers look different when written symbolically, but have the same value. So, two different elevations can be the same distance from sea level, where one is below (a negative) and one is above (a positive).

Pairs | 10 min

Activity 2 Inequality Mix and Match

Students compare rational numbers — in different forms — and their absolute values.

Name: ______________ Date: ________ Period: ________

Activity 2 Inequality Mix and Match

Here are some numbers, expressions, and symbols. Work with your partner to write three true comparison statements and corresponding sentences in the table.

-0.7	$-\frac{3}{5}$	1	4	$\lvert -8 \rvert$	$<$
$-\frac{6}{3}$	-2.5	2.5	8	$\lvert 0.7 \rvert$	$=$
-4	0	$\frac{7}{2}$	$\lvert 3 \rvert$	$\left\lvert -\frac{5}{2} \right\rvert$	$>$

1. Each partner selects a number or expression. *At least* one should be a decimal, fraction, negative number, or absolute value expression. Decide which equality or inequality symbol would make a *true* comparison statement.
2. Once you agree upon and record your comparison statement, write a sentence (in words) to match the statement, using one or more of the following phrases:
   - is equal to
   - is the absolute value of
   - is greater than
   - is less than
3. Across your three comparisons, there must be *at least*:
   - two negative numbers
   - two absolute value expressions
   - one decimal
   - one fraction

Sample responses are shown.

Comparison statement	Sentence
$\lvert -\frac{5}{2} \rvert > \lvert -2 \rvert$	**The absolute value of $-\frac{5}{2}$ is greater than the absolute value of -2.**
$-\frac{3}{5} < \lvert 3 \rvert$	**$-\frac{3}{5}$ is less than the absolute value of 3.**
$2.5 = \lvert -\frac{5}{2} \rvert$	**2.5 is equal to the absolute value of $-\frac{5}{2}$, which is $\frac{5}{2}$, which is equal to 2.5.**

STOP

1 Launch

Have students read the directions silently, or read them aloud as a class. Answer any questions and make sure everyone understands how to complete the activity.

2 Monitor

Help students get started by activating their prior knowledge. Have them choose a symbol to work with first, instead of the numbers or expressions. Ask, "What needs to be true about your two numbers to use that symbol?"

Look for points of confusion:

- **Thinking that numbers, such as -2.5 and 2.5, are equal.** Ask, "What does that sign in front of the first number mean?"
- **Confusing negative numbers with their absolute values**. Ask, "Is -4 a number or an absolute value? How do you know?"

Look for productive strategies:

- Comparing any positive number to any negative number, including opposites.
- Recognizing absolute values are always positive.
- Recognizing that only one value listed is equal to 0.
- Converting between fractions and decimals.

3 Connect

Have pairs of students share their statements.

Highlight how the absolute value of a number can have a different value than the number, so inequalities for these comparisons can use different symbols.

Ask, "How would you read this expression: $\lvert -8 \rvert < 5$? Is it true?" Have students critique each other's responses and having them explain why they agree or disagree.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Reduce the list of possible values for students to work with, such as by suggesting they start writing comparison statements using only these values: $-4, 0, 4, 8, \lvert -8 \rvert$, and $-\frac{6}{3}$.

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing each number or symbol on a card and allow students to physically interact with the cards to select and sequence them as they create true comparison statements.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display only the list of numbers. Ask students to work with their partner to choose 2 numbers from the list and write 2–3 questions they have about those numbers. Ask volunteers to share their questions with the class. Sample responses:

- "Which number is greater? Less? Are the numbers equal?"
- "Which number is closer to 0? Farther from 0? Are they the same distance from 0?"
- "Are these numbers opposites?"
- "Are these numbers on the same side of 0? Opposite sides?"

Whole Class | 5 min

Summary

Review and synthesize the importance of using precise language when writing inequalities to compare rational numbers and their absolute values, across multiple contexts.

Summary

In today's lesson . . .

You used absolute value and negative numbers to think about elevation. The context of elevation can help you compare two rational numbers or two absolute values, and understand the difference between these comparisons.

For example, suppose an anchor has an elevation of -10 m and a house has an elevation of 12 m.

- To compare their *elevations* and describe the anchor as having a lower elevation than the house, you can write $-10 < 12$, which means -10 is less than 12.
- To compare their *distances from sea level* and describe the anchor as being closer to sea level than the house, you can write $|-10| < |12|$, which means the distance from -10 to 0 is less than the distance from 12 to 0.

You can compare any rational numbers and their absolute values outside of the context of elevation. For example:

- To compare the distances that -47.5 and 5.2 are each from 0, you can say $|-47.5|$ is 47.5 units from 0 and $|5.2|$ is 5.2 units from 0, so $|-47.5| > |5.2|$.
- The expression $|-18| > 4$ means that the absolute value of -18 is greater than 4, which means that -18 units is more than 4 units from 0 on the number line.

Reflect:

780 Unit 7 Rational Numbers

Synthesize

Ask, "What was one thing you learned in these last two lessons that really stood out to you?"

Have students share their responses after reflecting on the last two lessons.

Highlight any vocabulary words used in students' responses and refer to the anchor chart, or word list, as they are used. Emphasize the importance of keeping track of both the mathematical and contextual vocabulary and using them with precision.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can a number be closer to zero than another number and have a greater value than the other number?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining whether given inequalities involving absolute values are true or false, and by finding solutions to make them true.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.08

1. Determine whether each of the following inequality statements is true or false. Explain your thinking. Sample explanations shown.

   a. $-5 < 3$
   True; Sample response: -5 is on the left side of 0 on a number line.

   b. $|-5| < 3$
   False; Sample response: $|-5|$ is equal to 5, which is not less than 3.

   c. $|-5| > 3$
   True; Sample response: $|-5|$ is equal to 5, which is greater than 3.

   d. $|-5| < |3|$
   False; Sample response: $|-5|$ is equal to 5, and $|3|$ is equal to 3, so $|-5|$ is not less than $|3|$ because 5 is not less than 3.

2. Complete the statement that would make each inequality true.

   a. $4 > |\_\_\_|$
   Responses should be between -4 and 4, not inclusive.
   Sample responses: -3, -1, 2

   b. $\_\_\_ < |-4|$
   Responses should be strictly less than 4.
   Sample responses: 3, -2, -7

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use inequalities to compare rational numbers and the absolute values of rational numbers.
1 2 3

b. I can determine values to make equations involving absolute value true.
1 2 3

c. I can explain what the absolute value of a number means in different contexts.
1 2 3

Lesson 8 Comparing Numbers and Distances From Zero

Success looks like . . .

- **Language Goal:** Critiquing comparisons (expressed using words or symbols) of rational numbers and their absolute values. **(Speaking and Listening, Reading and Writing)**
  - » Determining whether each inequality statement is true or false in Problem 1.
- **Language Goal:** Generating values that meet given conditions for their relative position and absolute value, and justifying the comparisons (using words and symbols). **(Speaking and Listening, Reading and Writing)**
  - » Completing each inequality with a value that makes the inequality true in Problem 2.
- **Language Goal:** Recognizing that the value of $-n$ can be positive or negative, depending on the value of n. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students have difficulty determining whether an inequality involving an absolute value is true or false in Problem 1, consider:

- Highlighting -5 and $|-5|$, and have them explain the value of each. Then return to the problems and ask, "What did you say $|-5|$ was equal to? Now, substitute this value into the inequalities, and try comparing again."

If students cannot determine a value that makes the inequality in Problem 2b true, consider:

- Having them draw a number line and plotting both 4 and $|-4|$ to determine values that are less than each.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In Lesson 7 students formalized the concept of absolute value. How did that support their ability to distinguish between absolute locations and relative distances today?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What made it so effective? What questions might you change or add the next time you teach this lesson?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. In the context of elevation, what would |–7| ft mean?
   An elevation of –7 ft is a distance of 7 ft from sea level.

2. Match each verbal statement with an equivalent mathematical statement.

Verbal statement	Mathematical statement
a. The number -4 is a distance of 4 units from 0 on the number line.	b $\;\|-63\| > 4$
b. The number -63 is a distance of more than 4 units from 0 on the number line.	e $\;-63 < 4$
c. The number 4 is greater than the number -4.	f $\;\|-63\| > \|4\|$
d. The numbers 4 and -4 are the same distance from 0 on the number line.	a $\;\|-4\| = 4$
e. The number -63 is less than the number 4.	c $\;4 > -4$
f. The number -63 is farther away from 0 than the number 4 on the number line.	d $\;\|4\| = \|-4\|$

3. Use the symbols >, <, or = to make a true comparison statement for each pair of values.
   a. -32 __<__ $|15|$
   b. $|-32|$ __>__ $|15|$
   c. 5 __>__ -5
   d. $|5|$ __=__ $|-5|$
   e. 2 __>__ -17
   f. 2 __<__ $|-17|$
   g. $|-27|$ __<__ $|-45|$
   h. $|-27|$ __>__ -45

Practice

Name: ______ Date: ______ Period: ______

4. Mai received and spent money in the following ways last month. For each of the following, write either a positive or negative number to represent the change in money from her perspective.
   a. Her grandmother gave her $25 in a birthday card.
      +25 or 25
   b. She received $14 for babysitting.
      +14 or 14
   c. She spent $10 on a ticket to the concert.
      –10
   d. She donated $3 to a local charity.
      –3
   e. She earned $2 interest on money that was in her savings account.
      +2 or 2

5. The following list shows the record low temperatures in five U.S. cities.
   - Death Valley, CA: –45°F (January, 1937)
   - Danbury, CT: –37°F (February, 1943)
   - Monticello, FL: –2°F (February, 1899)
   - East Saint Louis, IL: –36°F (January, 1999)
   - Greenville, GA: –17°F (January, 1940)

   a. Which of these cities had the lowest record temperature?
      Death Valley, CA
   b. Which city had a lower record temperature: Monticello, FL, or Greenville, GA?
      Greenville, GA
   c. Which city had a lower record temperature: Danbury, CT, or East Saint Louis, IL?
      Danbury, CT
   d. How many more degrees colder is the record low temperature in Greenville, GA than the record low temperature in Monticello, FL?
      15°F

6. Write an expression that matches each phrase.
   a. 3 more than g
      $g + 3$ or $3 + g$
   b. Twice of h
      $2h$ or $2 \bullet h$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	1
Spiral	4	Unit 7 Lesson 6	2
	5	Unit 7 Lesson 2	2
Formative	6	Unit 7 Lesson 9	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 2

Inequalities

Whole Class

In this Sub-Unit, students build on their prior work with expressions and equations to explore inequalities, grappling with real-world contexts in which their solutions have an implied bound.

SUB-UNIT

2 Inequalities

Narrative Connections

How do you keep a quantity from wandering off?

Place it in bounds!

Most of the time, we use inequalities to compare values — such as when we write $5 > 4$. But inequalities can also be used as constraints, or *bounds,* for variables that can take on different values. If there are x people in a movie theater, and the fire marshal will shut the theater down if it exceeds a maximum capacity of 500 people, then you had better hope that $x < 500$. Otherwise, you can forget about catching the movie!

But what if there were *exactly* 500 people in the theater? That's the maximum capacity, which means it's still technically safe, and the fire marshal shouldn't be crashing the movie. So, saying $x < 500$ doesn't quite cut it, because it doesn't tell you that it's okay if x happens to equal 500 on the nose.

To express this mathematically, we'll need to add two more symbols to your arsenal: $\geq$ and $\leq$.

These are the "greater than or equal to" and the "less than or equal to" symbols. They're a mouthful to say, but they're fantastic for *including* a value within a bound.

In these next few lessons, you'll see there are *many* ways to describe inequalities: *less than*, *no more than*, *at least*, *at most*, *up to*, *between*, *betwixt* — well, maybe no one says *betwixt* anymore. But with the $<$, $>$, $\leq$, and $\geq$ symbols, you'll have everything you need to keep those unruly quantities in check.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how inequalities can be used to represent real-world scenarios in the following places:

- **Lesson 9, Activities 1–2:** Stories About 9, How High, How Low?
- **Lesson 10, Activity 1:** Stories About 9, Revisited
- **Lesson 11, Activity 1:** Amusement Park Rides
- **Lesson 12, Activities 1–2:** Extreme Elevations, Extreme Temperatures

Writing Inequalities

Let's write some inequalities.

Focus

Goals

1. **Language Goal:** Interpret and coordinate between verbal and algebraic representations of inequalities for a quantity with a given constraint, e.g., *at least*, *at most*, *up to*, *more than*, *less than*, etc. **(Speaking and Listening, Reading and Writing)**

2. **Language Goal:** Use substitution to justify or critique whether a value is a solution to an inequality representing a scenario with a constraint, including determining whether the boundary value should be included. **(Speaking and Listening, Reading and Writing)**

3. **Language Goal:** Recognize and explain that an inequality has infinitely many solutions. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students extend their work with inequalities to represent comparisons using a variable. These quantities often describe real-world situations, and their possible values are often constrained by a given minimum or maximum. Students represent these situations with inequality statements and identify values that make them true, which is often an infinite number. They also consider whether the constraint itself is included or excluded.

Previously

Lessons 2–6 introduced students to rational numbers, extending their concept of number. They wrote inequality statements to compare rational numbers, and in Lessons 7–8, they distinguished between the order of rational numbers and their absolute values.

Coming Soon

In Lesson 10, students will build on their understanding of inequalities with variables, see how to represent them on number lines, and further consider possible solutions involving continuous and discrete quantities.

Rigor

- Students interpret real-world scenarios to build **conceptual understanding** of strict and non-strict solutions to inequalities.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Whole Class	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair

Math Language Development

New words

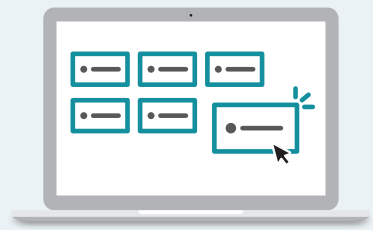
- ***solution to an inequality***

Amps | Featured Activity

Activity 1

Digital Card Sort

Students match inequalities to examples and descriptions.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be uncomfortable with making a guess without any boundary values to start in Activity 2. Ask students how they can evaluate the situation, making their guess more reasonable to start. Then have students identify how the symbolic representation of the inequalities and variables help narrow the possible solutions.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- **Activity 2** may be omitted.

Whole Class | 5 min

Warm-up Guess My Number

Students identify a mystery number based on a sequence of verbal clues, stating if it is less than or greater than specified positive and negative values.

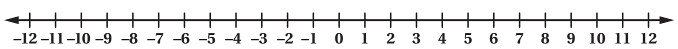
Unit 7 | Lesson 9

Writing Inequalities

Let's write some inequalities.

Warm-up Guess My Number

Each new guess should take into account all of the clues you have heard up to that point. Consider plotting your guesses on the number line to help your thinking.



Answers will vary. Answers will vary.

Clue	Guess

My *final* mystery number guess is: ____________.

Log in to Amplify Math to complete this lesson online.

1 Launch

Read each clue aloud and give students time to record a guess after each. Reiterate that each clue builds on the one before it. Tell students you are thinking of a number that is:

1. Greater than –7. **2.** Less than 5. **3.** Greater than –1.
4. Less than 3. **5.** Greater than 1.

2 Monitor

Help students get started by plotting the first given number, –7, on the number line. Ask, "On which side of that point would a number greater than –7 be?"

Look for points of confusion:

- **Not using previous clues.** Ask,, "Is your guess also still less/greater than ___?"
- **Picking invalid numbers for the clues.** Have students reference the number line to help them better understand where numbers *greater than* or *less than* a given number are located.

Look for productive strategies:

- Refining each guess with every subsequent clue.

3 Connect

Display a new two-column anchor chart, showing < and >, to capture key words related to algebraic inequalities introduced in Lessons 9–12.

Have students share how they determined the mystery number. Ask where *greater than* and *less than* belong on the anchor chart.

Highlight that the mystery number is 2. But, there are also possible non-whole numbers that could have been the mystery number — anything between 1 and 3 (e.g., 1.7 or 2.8).

Power-up

To power up students' ability to write variable expressions to represent written phrases, have students complete:

Match each statement with the expression that represents it.

a. 8 more than a number   __a__ $n + 8$

b. 8 times a number   __e__ $n - 8$

c. The quotient of a number and 8   __d__ $8 - n$

d. 8 minus a number.   __c__ $\frac{n}{8}$

e. 8 less than a number   __b__ $8n$

Use: Before Activity 1.

Informed by: Performance on Lesson 8, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1.

Pairs | 15 min

Activity 1 Stories About 9

Students extend their understanding of inequality statements by considering unknown quantities with one constraint in real-world situations.

Amps Featured Activity Digital Card Sort

Name: ______ Date: ______ Period: ______

Activity 1 Stories About 9

You will be given a set of cards.

1. For each scenario, match the appropriate description of all possible solutions with the corresponding inequality statement.
2. Write down one number that is a possible solution to each inequality.

Scenario	Description	Inequality	Possible solution to the inequality
Priya's mother lives less than 9 miles from her work.	Any value less than 9.	$x < 9$	Sample response: 8
A two-year-old sleeps more than 9 hours a day.	Any value greater than 9.	$x > 9$	Sample response: 10
Tyler can carry no more than 9 lb of apples in his bag.	Any value that is less than or equal to 9.	$x \leq 9$	Sample response: 9
Kiran wants to drink at least 9 cups of water a day.	Any value greater than or equal to 9.	$x \geq 9$	Sample response: 9.5

Compare and Connect: What do each of the phrases *more than, no more than, at least,* and *less than* mean? How do they relate to the inequality symbols? Discuss with your partner.

1 Launch

Give each pair a set of pre-cut cards from the Activity 1 PDF. Have students use the ***Take Turns*** routine to match cards.

2 Monitor

Help students get started by having them read all the scenarios first, and then starting with the one that makes the most sense to them.

Look for points of confusion:

- **Thinking that *less than* 9 and *no more than* 9 mean the same thing.** Have students start listing possible values for each example and explain that *no more than* includes 9, but *less than* does not.
- **Using some key words and not others.** Have students read the descriptions aloud and underline what they believe to be the key words.

Look for productive strategies:

- Distinguishing between *strict* inequalities (<, >) and *non-strict* inequalities (≤, ≥). **Note:** Students have not seen the ≤ and ≥ symbols before, and are not expected to know their meaning yet.
- Checking possible solutions against the inequality, as well as the scenario and description.

3 Connect

Have individual students share their strategies for matching, focusing on key words and the different inequality symbols, and discuss where to add the terms/symbols to the anchor chart.

Define a ***solution to an inequality*** as a value that makes the inequality statement true.

Highlight that the symbols ≤ and ≥ mean *less than or equal to* and *greater than or equal to*, respectively. These are called *non-strict* inequalities and the boundary values are possible solutions. Note that, unlike equations, there are *infinitely many* possible solutions to inequalities.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils or highlighters and have students annotate the key words and phrases they see on each card, such as *less than*, *more than*, *no more than*, and *at least*.

Accessibility: *Clarify Vocabulary and Symbols*

During the Connect, as you highlight the symbols ≤ and ≥ and their meanings, emphasize how these symbols look like they contain part of the = sign underneath the < and > signs. Tell students this indicates to them that this means the quantity could also be equal to a certain number.

Math Language Development

MLR7: Compare and Connect

During the Connect, have students respond to the Compare and Connect questions posed to them in their Student Edition. Sample responses shown for the phrase "no more than."

- "What do each of the phrases more than, no more than, at least, and less than mean?" A quantity cannot be greater than a certain number. It must be less than or equal to that certain number.
- "How do they relate to the inequality symbols?" This phrase would use the *less than or equal to symbol*, ≤.

English Learners

Have students refer to the class display that shows these inequality symbols and phrases. Add any new phrases to the display.

Pairs | 15 min

Activity 2 How High, How Low?

Students apply their developing understanding of inequalities to describe constraints of maximum and minimum possible values for an unknown quantity symbolically.

Activity 2 How High, How Low?

The picture shows an adult giraffe and a twelve-year old girl. Refer to this picture to help you complete the activity to estimate the height of the giraffe.

1. Complete the sentences.
   a. What do you think is a good estimate for the *minimum* height of the giraffe? Does this mean the giraffe cannot be *shorter* or taller than this height?
   15 ft; shorter
   b. What do you think is a good estimate for the *maximum* height of the giraffe? Does this mean the giraffe cannot be shorter or *taller* than this height?
   20 ft; taller
2. Write one inequality statement to represent your estimate for the *minimum* height, in feet, of the giraffe. Then write another inequality statement to represent your estimate for its *maximum* height, in feet. Use the variable h to represent the unknown height.
   Minimum: $h \geq 15$
   Maximum: $h \leq 20$
3. Determine an estimate of h. Use your inequalities for maximum and minimum heights to explain how you know your estimate is reasonable.
   Sample response: h could be 17 ft. This is greater than or equal to 15, but less than or equal to 20.

STOP

1 Launch

Have students use the ***Think-Pair-Share*** routine. Provide them 5 minutes of individual work time to complete Problems 1–2. Then have them compare answers with a partner and complete Problem 3 together. Ensure students know the meanings of minimum and maximum as boundary values, which should be included as possible solutions.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "What is an estimate? When are estimates useful?" Then have them estimate the girl's height.

Look for points of confusion:
- **Giving heights using the scale of the image.** Ask, "Can you use the relationship between your smaller measurements to still estimate the actual heights?"
- **Thinking they must have an exact answer.** Suggest ways to estimate using informal units, such as their thumb or pencil.

Look for productive strategies:
- Determining reasonable estimates for height.
- Approximating the giraffe's height to be 3 times that of the girl, or in a ratio of $3:1$.
- Correctly reading and evaluating their inequality statements with a value for h representing the unknown height.

3 Connect

Have students share how they estimated the heights and wrote corresponding inequalities. Collect all final estimates from Problem 3, including explanations of reasonableness.

Highlight that a *minimum* is a constraint giving the least possible value, and a *maximum* is a constraint giving the greatest possible value. These are represented using the symbols $\geq$ and $\leq$. Add new terms to the anchor chart.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing students a list of possible minimum and maximum heights for the giraffe and have them choose values they think would be good estimates for Problem 1. For example, consider providing them with this list of possible heights.

4 ft 10 ft 15 ft 20 ft 30 ft

Accessibility: *Clarify Vocabulary and Symbols*

Continue to use the class display or anchor chart to display the inequality symbols students have learned and key phrases that represent them.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display only the picture and introductory text. Have students work with their partner to write 2–3 mathematical questions they could ask about the picture. This will help them interpret the situation in their own words before beginning the activity.

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question based on the picture. For example, two sample questions are: "How tall is the girl? If I know her height, can I determine or estimate the height of the giraffe?"

Whole Class | 5 min

Summary

Review and synthesize how to relate a given scenario to either a strict or a non-strict inequality statement, and how to determine a possible solution to an inequality.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You wrote inequalities to represent real-world contexts. When two numbers are not equal, you can use an inequality to compare the numbers. For example, you can write $3 < 4$ to show that the value of 3 is less than the value of 4.

A ***solution to an inequality*** is any value that makes the inequality true. For example, you could say that 5 is a solution to the inequality $c < 10$ because $5 < 10$ is a true statement. You could also say that 12 is *not* a solution because $12 < 10$ is not a true statement. These types of inequalities with a variable have an infinite number of possible solutions — also called the *solution set* — because there is always another number that is greater/less than the last.

Inequalities can be categorized into two categories, base on whether or not their solution set includes the *boundary value* or not:

Strict inequalities		Non-strict inequalities	
$<$	$>$	$\leq$	$\geq$
less than	greater than	less than or equal to	greater than or equal to
Solution set does *not* include the boundary value.		Solution set *does* include the boundary value.	

Reflect:

Synthesize

Ask, "Can you think of an inequality that might represent the possible numbers of students that could fit in our classroom and have a seat?"

Have students share some inequalities in response to the question, and how they decided whether to include the boundary values.

Formalize vocabulary: ***solution to an inequality***.

Highlight that inequalities have infinite possible solutions, so inequality statements using variables and the symbols $<$, $>$, $\leq$, or $\geq$ can be used to represent all the solutions without having to list them.

Note: Lessons 10–12 will provide further opportunities to explore other representations of inequalities (e.g., number line graphs). Students will discuss constraints in a given scenario, such as when some values may be solutions to an inequality mathematically, but not be reasonable solutions in the context of the scenario.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What were some of the symbols you used today and what do they mean?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *solution to an inequality* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of inequalities with unknowns by writing inequality statements to match verbal scenarios and find possible solutions.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.09

Andre looks at a box of paper clips and says, "I think there are less than 1,000 paper clips in the box."

Lin looks at the same box of paper clips and says, "I think there are more than 500 paper clips in the box."

1. Write an inequality to show Andre's statement, using p to represent the number of paper clips.
   $p < 1{,}000$

2. Write another inequality to show Lin's statement, also using p to represent the number of paper clips.
   $p > 500$

3. Can Andre and Lin both be correct? Explain your thinking and give a possible number of paper clips that supports your argument.
   Sample response: Yes, Andre and Lin can both be right because there are many numbers that are greater than 500, but less than 1,000. Some examples are 650, 802, or 999.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can write an inequality to represent a scenario. 1 2 3

b I can describe a scenario that represents an inequality. 1 2 3

c I can identify a solution to an inequality. 1 2 3

Success looks like . . .

- **Language Goal:** Interpreting and coordinating between verbal and algebraic representations of inequalities for a quantity with a given constraint, e.g., *at least*, *at most*, *up to*, *more than*, *less than*, etc. **(Speaking and Listening, Reading and Writing)**
  » Writing inequalities for Andre's and Lin's statements in Problems 1 and 2.
- **Language Goal:** Using substitution to justify or critique whether a value is a solution to an inequality representing a scenario with a constraint, including determining whether the boundary value should be included. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Recognizing and explaining that an inequality has infinitely many solutions. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students use ≤ or ≥, consider:

- Reviewing the phrases *less than* and *more than*.
- Giving examples of different values that would make Andre's and Lin's statements true, and giving examples that would make the statements false.
- Reviewing the matching scenarios and inequalities from Activity 1.

If students do not think both Andre and Lin can be right in Problem 3, consider:

- Having students use a possible number of paperclips like 700 or 800 to substitute for the variable in their inequality statements from Problems 1 and 2. Ask, "Are these statements true?"
- Reviewing Problem 3 from Activity 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In Unit 6, students wrote and solved expressions with variables to represent real-world contexts in which a value was unknown. Earlier in this unit, students wrote inequality statements to compare known rational numbers. How did this previous work support students today as they wrote inequality statements with variables to describe possible solutions for real-world contexts?
- What trends do you see in participation? How might you encourage those who did not participate to do so in the upcoming lessons? What might you change for the next time you teach this lesson to ensure more voices are heard?

Practice

Independent

Practice

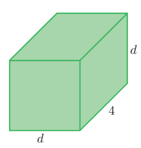
Name: ________ Date: ________ Period: ________

1. At a book sale, all books cost less than $5.
   a. What is the most expensive price a book could be?
   Sample response: $4.99
   b. Write an inequality to represent the possible costs of books, in dollars, at the sale.
   Sample response: $x < 5$

2. Kiran started his homework before 7:00 p.m. and finished his homework after 8:00 p.m. Let h represent the number of hours Kiran spent on his homework. Decide whether each statement is *definitely true*, *definitely not true*, or *possibly true*.
   a. $h > 1$ **Definitely true**
   b. $h > 2$ **Possibly true**
   c. $h < 1$ **Definitely not true**
   d. $h < 2$ **Possibly true**

3. Clare tries to drink 13 cups of water every day, but many days she drinks less. Write an inequality to represent the possible cups of water c that Clare drinks on any given day.
   $c \leq 13$

4. Consider a rectangular prism with length 4 units, width d units, and height d units.
   a. Write an expression for the volume of the prism, v, in terms of d.
   $v = 4d^2$
   b. Determine the volume of the prism when $d = 1$, when $d = 2$, and when $d = \frac{1}{2}$.
   4 cubic units, 16 cubic units, and 1 cubic unit



Name: ________ Date: ________ Period: ________

Practice

5. Match each statement with its corresponding inequality. All of the statements are true.

Statement	Inequality
a. The number -15 is farther from 0 than the number -12 on the number line.	c $\lvert -12\rvert > -15$
b. The number -12 is a distance of 12 units from 0 on the number line.	e $-15 < -12$
c. The distance between -12 and 0 on the number line is greater than -15.	a $\lvert -15\rvert > \lvert -12\rvert$
d. The numbers 12 and -12 are the same distance from 0 on the number line.	b $\lvert -12\rvert = 12$
e. The number -15 is less than the number -12.	f $12 > -12$
f. The number 12 is greater than the number -12.	d $\lvert 12\rvert = \lvert -12\rvert$

6. What is the difference between the inequality statements $n \leq 8$ and $n < 8$? Write a real-world scenario that could be represented by each inequality.
   Sample responses: The inequality $n \leq 8$ means any number that is 8 or less; The inequality $n < 8$ means any number less than 8, but not including 8. For the first inequality, a speed limit on a side road is 8 mph. For the second example, Mai wants to drive less than 8 miles to work.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	2
	2	Activities 1 and 2	2
	3	Activity 1	1
Spiral	4	Unit 6 Lesson 15	2
	5	Unit 7 Lesson 8	2
Formative	6	Unit 7 Lesson 10	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Graphing Inequalities

Let's graph some inequalities.

Focus

Goals

1. **Language Goal:** Interpret and coordinate among verbal, algebraic, and number line representations of inequalities for a quantity with a given constraint, e.g. *at least, at most, up to, more than, less than,* etc. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Critique or justify possible solutions to an inequality or number line graph representing a quantity with a given constraint, including determining whether the boundary value should be included. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students build on their understanding of inequalities with variables by representing all possible solutions on a number line. Having seen situations that can be represented by inequalities that have many values that make them true, and often even an infinite number, the number line representation of possible values is very useful. Students think through the constraints of various contexts. They decide whether boundary values are possible solutions in those contexts, in which open and closed circles are used on the number line to represent when to include the boundary value.

Previously

In Lesson 9, students represented situations with an unknown quantity, writing inequality statements and reasoning about possible values that make them true.

Coming Soon

In Lessons 11–12, students will use inequalities and number lines to reason through scenarios with two constraints. They will also consider the reasonableness of solutions in context, based on inherent minimums or maximums, or whether a quantity is continuous or discrete.

Rigor

- Students interpret and graph inequality scenarios to strengthen their **procedural skills** using number lines.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair

Math Language Development

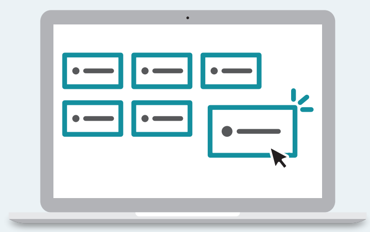
Review words

- *solution to an inequality*

Amps | Featured Activity

Activity 2
Digital Card Sort

Students match inequalities to open and closed number lines.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel lost when starting to make the connection between inequalities, number lines, and scenarios in Activity 2. Have students identify ways they can use their organizational skills to make it easier to match up corresponding representations.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, only distribute cards corresponding to three complete matches, instead of all six.
- The optional scenario cards for **Activity 2** may be omitted.

Pairs | 5 min

Warm-up Thinking About Limits

Students interpret two scenarios involving limits, distinguishing between inclusive and non-inclusive boundary values to see that they can represent these using inequalities.

Unit 7 | Lesson 10

Graphing Inequalities

Let's graph some inequalities.

Warm-up Thinking About Limits

1. You are riding in the car on a highway and pass a speed limit sign exactly like the one shown.
   a. What is the fastest, in mph, you could go without breaking the speed limit?
   65 mph
   b. Write an inequality to match your response that represents all the acceptable speeds, using s as the variable to represent speed, in miles per hour.
   $s \leq 65$
2. Water freezes and becomes ice when it reaches a temperature of 32°F.
   a. What are some temperatures at which water will *not* freeze?
   Sample responses: 35°F, 40°F, 51°F
   b. Write an inequality to represent all of the possible temperatures, in degrees Fahrenheit, at which water does not freeze. Use the variable d to represent temperature, in degrees Fahrenheit.
   $d > 32$

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate students' background knowledge by asking what they know about road signs, paying particular attention to speed limit signs.

2 Monitor

Help students get started by clarifying that a speed limit represents a maximum. Ask, "Should the boundary value be included? What about the boundary value for water's freezing point?"

Look for points of confusion:

- **Thinking the fastest speed is 64 mph.** Ask, "What about 64.1 mph? 64.2 mph?"
- **Thinking that "reaches . . . 32°F" means increasing temperatures.** Ask, "Would water be liquid or solid/ice at a temperature of 1°F? What about 100°F?"
- **Using inequality symbols incorrectly.** Remind students that the ≥ and ≤ symbols include the boundary value, while the > and < symbols do not.

Look for productive strategies:

- Determining reasonable solutions and distinguishing the differences between strict and non-strict inequalities in both words and symbols.

3 Connect

Have students share their responses to Problem 1, followed by Problem 2, or to both Part A's first, followed by both Part B's.

Highlight that the *boundary values* in these scenarios involve a *maximum* (Problem 1) and a *minimum* (Problem 2). Note that 0 and negative numbers are possible solutions to Problem 1 and that speed can take on fractional values, but these points are a focus of Lesson 12.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses, revoice their ideas to model precise mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students. For example, if students say "The fastest I could go without breaking the speed limit is 65 mph," ask the question "Does this speed limit represent a *maximum* or a *minimum*?"

Power-up

To power up students' ability to distinguish between strict (≥, ≤) and non-strict (>, <) inequalities, have students complete:

Recall that strict inequalities (≤, ≥) include boundary values while non-strict inequalities (<, >) do not.

Match each statement with the inequality that represents it.

a. Shawn spent no more than \$8	d $x > 8$
b. Tyler spent at least \$8.	a $x \leq 8$
c. Bard spent under \$8.	c $x < 8$
d. Priya spent more than \$8	b $x \geq 8$

Use: Before the Warm-up
Informed by: Performance on Lesson 9, Practice Problem 6 and Exit Ticket.

Activity 1 Stories About 9, Revisited

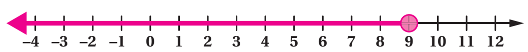
Students revisit familiar scenarios involving inequalities and explore representing solutions using number lines.

Name: ______ Date: ______ Period: ______

Activity 1 Stories About 9, Revisited

Refer to these scenarios from Lesson 9, Activity 1.

1. **Scenario 1:** Priya's mother wants to live less than 9 miles from her work.
   a. Write an inequality to represent this scenario using m as the variable.
   $m < 9$
   b. Use the number line to plot all the possible solutions to your inequality.
   

   c. Write three possible values for m.
   Sample responses: 2, 3.5, 7
2. **Scenario 2:** Tyler can carry no more than 9 lb of apples.
   a. Write an inequality to represent this scenario using p as the variable.
   $p \le 9$
   b. Use the number line to plot all the possible solutions to your inequality.
   

   c. Write three possible values for p.
   Sample responses: 3, 5.5, 9
3. How are the number lines from Scenario 1 and Scenario 2 similar? How are they different?
   Sample response: The number lines are similar because they both use the number 9 and involve values less than 9. They are different because Scenario 1 *does not* include 9 as a possible solution, whereas Scenario 2 *does* include it as a possible solution.

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What quantities do the variables m and p represent?"

Look for points of confusion:

- **Thinking the two scenarios have the same solution.** Have students identify key words.
- **Only plotting whole numbers on the number line.** Ask, "Could Priya's mother live 8.5 miles from her work? Could Tyler carry 8.3 pounds of apples?"

Look for productive strategies:

- Including non-integer solutions on the number line.
- Only shading positive numbers because these quantities cannot realistically be negative. Acknowledge this as a good instinct, but the number lines should represent the mathematical inequality statements.

3 Connect

Display two number lines that can be used to graph the correct inequalities.

Highlight that for a *strict* inequality (e.g., $m < 9$), the boundary point is an *open circle* because it is *not* a solution. For a *non-strict* inequality (e.g., $p \le 9$), the boundary point is a *closed circle* because it *is* a solution. The number line is shaded and the arrow on the end is bold, to show that every value, continuing forever, is a solution.

Ask, "How would you plot solutions representing *no more than 9* and *at least 9* on number lines?

Have students share their responses and where to add similar examples with number lines and new terms to the anchor chart such as *strict/non-strict inequalities* and *open/closed circles*.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils or highlighters and suggest students color code the key words or phrases in each scenario that indicate the inequality symbol.

Accessibility: *Clarify Vocabulary and Symbols*

Continue to use the class display or anchor chart to display the inequality symbols students have learned and key phrases that represent them.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, provide them time to meet with 2–3 partners to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "Does the response indicate which number line includes the boundary value and why it includes this value?"
- "Does the response describe how the graphs are similar?"

Have students use the feedback to improve their responses.

English Learners

Annotate the number lines with the words *open/closed circle*, *boundary value*, and *strict/non-strict*. Circle or highlight the key phrases in the text, such as *less than* and *no more than*.

Pairs | 20 min

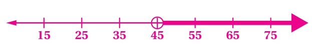
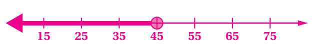
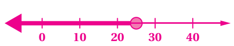
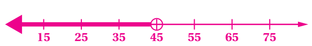
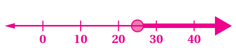
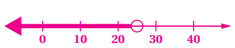
Activity 2 Match It Up

Students match inequalities to number lines and determine a possible solution for each. Time permitting, they also match a scenario to each inequality.

Amps Featured Activity | Digital Card Sort

Activity 2 Match It Up

You will be given a set of cards showing inequalities, number lines, and scenarios. Match each inequality with the corresponding number line showing all possible solutions. Then list one possible solution. If you have time, try matching each inequality with a scenario.

Inequality	Number line	Possible solution	Scenario (optional)
	$t > 45$	Sample response: 50	In the first week of April, the temperature was warmer than 45°F every day.
	$w \leq 45$	Sample response: 40	The wingspan of a Macaw parrot can be up to 45 in.
	$s \leq 25$	Sample response: 25	In a given class, there can be no more than 25 students.
	$m < 45$	Sample response: 40	It takes Mai less than 45 minutes to do her homework.
	$c \geq 25$	Sample response: 30	A video game costs at least \$25.
	$p < 25$	Sample response: 20	A doctor always sees less than 25 patients in one day.

STOP

792 Unit 7 Rational Numbers

1 Launch

Give each pair a complete set of pre-cut inequality and number line cards from the Activity 2 PDF. Leave out the scenario cards at first, and distribute those to pairs who finish within 10 minutes.

2 Monitor

Help students get started by encouraging them to look at all the cards before attempting to match them, making sure they can interpret each number line and inequality using words.

Look for points of confusion:

- **Mismatching strict and non-strict inequalities with closed and open circles.** Review the conventions for representing both types of inequality symbols on a number line graph.
- **Thinking that the boundary values are not solutions for any scenarios.** Use the anchor chart to remind students of keywords that might indicate strict versus non-strict inequalities.

Look for productive strategies:

- Checking their work by thinking of different numbers that would or would not fit each example.

3 Connect

Ask, "How did/would you match the scenarios?"

Highlight that when the variable is written on the left, the symbols $<$ or $\leq$ correspond to number line graphs that are shaded to the left, while the symbols $>$ or $\geq$ correspond to number line graphs that are shaded to the right. Similarly, the symbols $<$ or $>$ correspond to number line graphs with an open circle end point, while the symbols $\leq$ or $\geq$ correspond to number line graphs with a closed circle end point. This means there are four basic number line graphs for representing inequalities.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students first sort the cards into the following categories:

- Number lines or inequalities that include the boundary value
- Number lines or inequalities that do not include the boundary value

Have students match number lines with inequalities that include the boundary value, and number lines with inequalities that do not include the boundary value.

Extension: *Math Enrichment*

Using the same two boundary values of 25 and 45, have students generate two inequalities that are not represented in the table, and draw a number line and describe a scenario each could represent.

Math Language Development

MLR7: Compare and Connect

To foster students' language development, display the scenarios involving the Macaw parrot and Mai's homework. Ask:

- "What is the same and what is different about these two scenarios?"
- "What do you think the phrase 'can be up to' means? Does this mean the boundary value can be included? Why or why not?"
- "If it takes Mai 'less than 45 minutes' to do her homework, does this mean the boundary value can be included? Why or why not?"

English Learners

Provide pairs of students time to rehearse and formulate a response before sharing with the class.

Whole Class | 5 min

Summary

Review and synthesize how strict and non-strict inequality statements are related to corresponding number line graphs, verbal descriptions, and real-world scenarios.

Name: ______ Date: ______ Period: ______

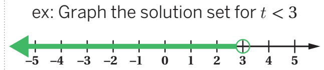
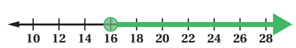
Summary

In today's lesson . . .

You explored how a scenario involving an unknown quantity and a given boundary value, or *constraint,* can be represented by an inequality which has infinite possible solutions.

To represent this on a number line, shade part of the number line to indicate that every possible point you could plot in the shaded region is a solution. This means every rational number in the shaded region is a solution. Then bold the arrow on one end of the number line to show that the possible solutions continue on forever in that direction.

To represent the boundary value, you first must consider which type of inequality symbol is used:

Strict inequalities		Non-strict inequalities	
$<$	$>$	$\leq$	$\geq$
The boundary value is *not* a solution, so an open (empty) circle is used on the boundary value.		The boundary value *is* a solution, so a closed (filled in) circle is used on the boundary value.	
ex: Graph the solution set for $t < 3$ 		ex: Graph the solution set for $p \geq 16$ 	

Reflect:

Synthesize

Display the two number lines from the Summary.

Ask:

- "How do you know in which direction to shade on a number line graph?" Sample response: I need to shade in the direction of all possible solutions.
- "How do you know whether to use an open circle or a closed circle on a number line graph?" Sample response: If the set of all possible solutions includes the boundary value, use a closed circle. If it does not include the boundary value, use an open circle.

Have students share their responses to these questions.

Highlight that number line graphs represent all possible solutions to an inequality statement, which could represent a real-world scenario. These solutions include non-integer values, which are shaded, and even those points beyond the values shown on the number line. The boundary values are not always included as possible solutions, so open and closed circles are used to indicate whether the end point on a number line graph is a solution.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "There were times today that you used an open circle or a closed circle on the number line. What did the open circle and the closed circle mean?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of graphing inequalities by representing scenarios using number line graphs and interpreting features of those types of graphs.

Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 7.10

Andre looks at a box of paper clips and says, "I think the number of paper clips in the box is less than 1,000."

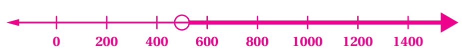
Lin also looks at the box and says, "I think the number of paper clips in the box is more than 500."

1. Graph an inequality to represent Andre's statement.

Andre:

2. Graph an inequality to represent Lin's statement.

Lin:



3. Describe in your own words the difference between a closed circle and an open circle on the graph of a number line for an inequality.

Sample response: A closed circle marking a number on a number line includes the value of that number as well as the values either less than or greater than, depending upon the direction of the inequality. An open circle marking a number on a number line does not include the value of that number; it only includes the values either less than or greater than, depending upon the direction of the inequality.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can graph an inequality on a number line. 1 2 3

b. I can use a number line to identify solutions to an inequality. 1 2 3

Lesson 10 Graphing Inequalities

Success looks like . . .

- **Language Goal:** Interpreting and coordinating among verbal, algebraic, and number line representations of inequalities for a quantity with a given constraint, e.g. *at least*, *at most*, *up to*, *more than*, *less than*, etc. **(Speaking and Listening, Reading and Writing)**
  - » Graphing the Andre's and Lin's statements on separate number lines in Problems 1 and 2.
- **Language Goal:** Critiquing or justifying possible *solutions to an inequality* or number line graph representing a quantity with a given constraint, including determining whether the boundary value should be included. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students shade the number lines in the wrong direction, consider:

- Asking them to come up with a possible solution to each inequality. Ask**,** "Where would that solution be located on the number line?"
- Reviewing that any number *greater than* a given number is to its right, and any number *less than* a given number is to its left.

If students incorrectly use open and closed circles, consider:

- Reviewing Problems 1 and 2 from Activity 1.
- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was using number lines to represent possible solutions to inequalities with constraints. How did this focus go?
- During the discussion about Activity 2, how did you encourage each student to share their understandings? How did you encourage each student to listen to one another's strategies? What might you change for the next time you teach this lesson?

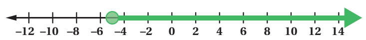
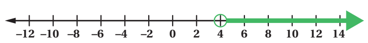
Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Complete the table to show the inequality statements paired with their corresponding number line graphs.

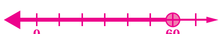
Inequality statement	Number line graph
$s < 7$	
$s \geq -5$	
$s < 10$	
$s > 4$	

2. A sign on the road reads: "Speed limit, 60 mph."

   a. Let s represent the speed, in miles per hour, of a car. Write an inequality that represents the information on the sign.

   $s \leq 60$

   b. Draw a number line graph to represent the solutions to the inequality.



   c. Could 60 be a value of s? Explain your thinking.

   Yes; Sample response: 60 is the maximum speed, and $s \leq 60$ means 60 or less.

3. Consider the inequality $k > 5$.

   a. Which of these numbers are solutions to the inequality: 4, 5, 6, 5.2, 5.01? List *all* that apply.

   6, 5.2, 5.01

   b. Draw a number line graph to represent the inequality.

Name: ______ Date: ______ Period: ______

Practice

4. Suppose the price of a cell phone at one store is $250.

   a. Elena's mom buys one of these cell phones on sale for $150. What percent of the price did she pay?

   $60\%; \frac{150}{250} = 0.6$

   b. Elena's dad also buys one of these cell phones from a different store, and he pays 75% of the price. How much did he pay?

   $\$187.50; 250 \cdot \frac{75}{100} = 187.5$

5. Here are five expressions that each show a sum or a difference. Use the Distributive Property to write an equivalent expression for each that is a product with two factors.

   a. $2a + 7a = a(2 + 7)$

   b. $5z - 10 = 5(z - 2)$

   c. $c - 2cd = c(1 - 2d)$

   d. $r + r + r + r + r = r(1 + 1 + 1 + 1 + 1)$

   e. $2x - \frac{1}{2} = 2\left(x - \frac{1}{4}\right)$ or $\frac{1}{2}(4x - 1)$

6. Han says he is thinking of a number that is greater than –3. Could –3 be the number? Explain your thinking.

   No; Sample response: The number has to be *greater than*, not equal to –3. An example could be –1 or 5.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	1
	2	Activities 1 and 2	2
	3	Activities 1 and 2	1
Spiral	4	Unit 3 Lesson 13	2
	5	Unit 6 Lesson 13	2
Formative	6	Unit 7 Lesson 11	1

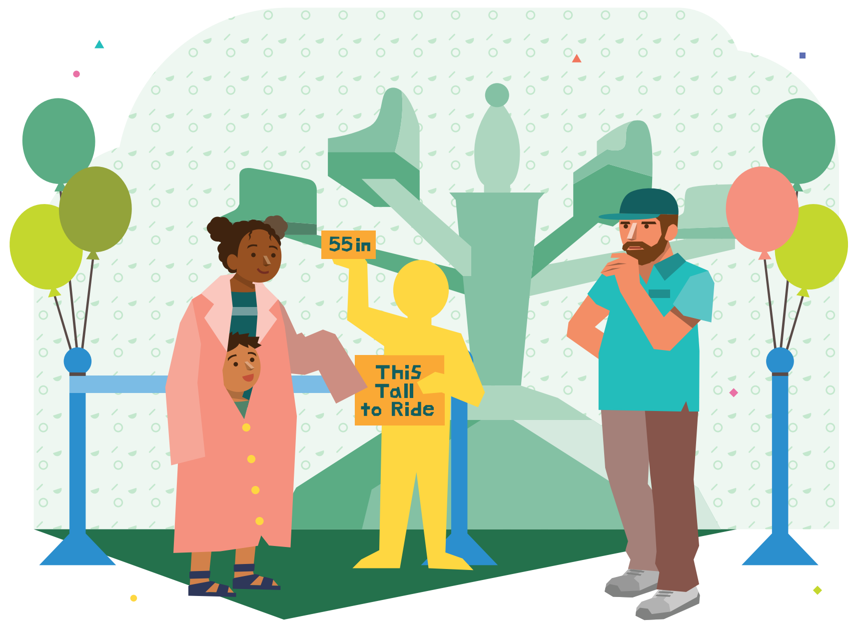
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Solutions to One or More Inequalities

Let's think about the solutions to inequalities.



Focus

Goals

1. **Language Goal:** Interpret and coordinate among verbal, algebraic, and number line representations of inequalities for a quantity with two given constraints, e.g. *between*. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Critique or justify possible solutions to inequalities or number line graphs representing a quantity with two given constraints, including determining whether upper and/or lower bounds should be included. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students use inequalities and number lines to reason about height restrictions at an amusement park, including situations with both upper and lower bounds. They consider whether or not the boundary values should be included as solutions to the inequalities in order to precisely describe allowed heights. Students then reason about whether given values make an inequality true as they play a game, justifying their responses using inequality statements and number lines.

Previously

In Lesson 9, students used inequality statements with a variable to represent unknown quantities, considering different types of constraints and reasoning about possible solutions. Then, in Lesson 10, students graphed inequalities, using open and closed circles to distinguish between included and excluded boundary values, and also using a shaded number line to represent infinite solutions.

Coming Soon

In Lesson 12, students will continue interpreting inequalities, revisiting the contexts of temperature and elevation, and thinking about how a context may limit the possible solutions that are reasonable.

Rigor

- Students continue to strengthen their **procedural fluency** with graphing inequalities.
- Students **apply** their understanding of inequalities to height requirements for amusement park rides.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, three sets of cards per group

Math Language Development

Review word

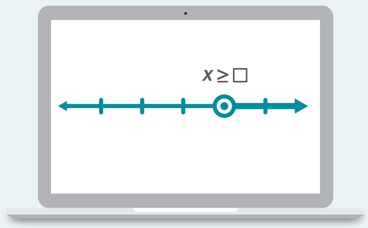
- *solution to an inequality*

Amps | Featured Activity

Activity 1

Interactive Number Lines

Students create digital number lines to support their thinking, and you can overlay them to see similarities and differences at a glance.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to determine a strategy together for minimizing the number of clue cards used in Activity 2. Prior to starting the activity, have groups engage in conversation and determine a strategy that they will follow for choosing clues. Remind them that they are to work as a team and that cooperation will be a key to success.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 1**, assign one amusement ride to each pair, distributing all four across the class.

Independent | 5 min

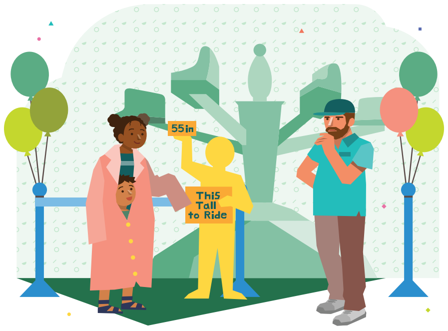
Warm-up Is Five a Solution?

Students interpret symbolic and graphed inequalities to determine whether 5 is a solution.

Unit 7 | Lesson 11

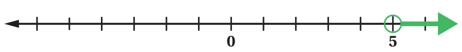
Solutions to One or More Inequalities

Let's think about the solutions to inequalities.

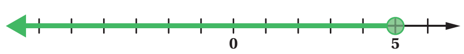


Warm-up Is Five a Solution?

Determine whether 5 is a solution to each inequality. If 5 is *not* a solution, provide at least one example of a solution. Be prepared to explain your thinking.

1. 

Sample response: No, because this represents $n > 5$, or *n is greater than 5*, so 5 is not a solution. A sample solution is 6.

2. 

Sample response: Yes, because this represents *n is less than or equal to 5*, so 5 is a solution.

3. $n \geq 5$

Sample response: Yes, because this represents *n is greater than or equal to 5*, so 5 is a solution.

4. $n < 5$

Sample response: No, because this represents *n is less than 5*, so 5 is not a solution. A sample solution is -1.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by having them translate each given inequality statement into a verbal or written form.

Look for points of confusion:

- **Incorrectly distinguishing between open and closed circles on number lines.** Ask, "Why do the points look different on the graphs in Problems 1 and 2?"
- **Incorrectly distinguishing when 5 should be included or excluded as a solution.** Ask, "What is the difference between the symbols < and >, and the symbols ≤ and ≥? How do they relate to the graphs on the number line and possible solutions?"

Look for productive strategies:

- Interpreting the representations as verbal inequality statements and substituting 5 to determine whether the statement is true.
- Recognizing that closed circles and the symbol ≥ includes 5, while open circles and the symbol < excludes 5.

3 Connect

Have students share, for each problem, a strategy to determine whether 5 is a solution. Focus on the use of verbal statements, and 5 as the boundary value, which is included sometimes and excluded other times.

Ask, "Are there any numbers that would be a solution for all four of the inequalities represented?"

Highlight that an open circle represents < or > on number line graphs where the boundary value is excluded, and a closed circle represents ≤ or ≥ where the boundary value is included.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Continue to use the class display or anchor chart to display the inequality symbols students have learned and key phrases that represent them. If the ≤ or ≥ symbols have not been added to the class display yet, add them to the display during this Warm-up.

Power-up

To power up students' ability to determine whether a value is a solution to an inequality, have students complete:

Recall that strict inequalities (≤, ≥) include boundary values while non-strict inequalities (<, >) do not.

Determine whether 2 is a solution to each inequality.

a. $x < 2$ No

b. $x > 2$ No

c. $x \leq 2$ Yes

d. $x \geq 2$ Yes

Use: Before the Warm-up

Informed by: Performance on Lesson 10, Practice Problem 6.

Pairs | 15 min

Activity 1 Amusement Park Rides

Students represent height restrictions for amusement park rides with inequalities, and investigate whether a given height is a solution to multiple inequalities at the same time.

Amps Featured Activity Interactive Number Lines

Name: ____________ Date: ________ Period: ________

Activity 1 Amusement Park Rides

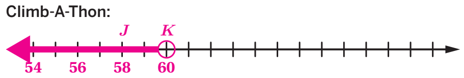
The brochure for an amusement park includes this table showing the height requirements for some of the rides.

To ride the . . .	You must be . . .
High Bounce	between 55 and 72 in. tall
Climb-A-Thon	under 60 in. tall
Twirl-O-Coaster	58 in. minimum
Tilt-A-Round	58 in. maximum

1. Jada is 58 in. tall. Kiran is 60 in. tall. What rides can they ride together? Use the number lines to support your response.

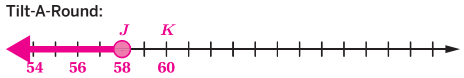
High Bounce:

Climb-A-Thon:



Twirl-O-Coaster:

Tilt-A-Round:



Sample response: They can go on both the High Bounce and the Twirl-O-Coaster together. Jada (represented by J on the number line) can go on every ride, but Kiran (represented by K on the number line) is too tall for the Climb-A-Thon and Tilt-A-Round.

1 Launch

Activate students' background knowledge by asking, "What do you know about amusement rides? Can everyone go on each ride?" Have students use the *Think-Pair-Share* routine. Provide them 5 minutes of individual work time. Then have them compare their answers with a partner and complete the activity together.

2 Monitor

Help students get started by listing one height that *is* a solution and one height that *is not* a solution for each ride.

Look for points of confusion:

- **Incorrectly interpreting words, such as *under*, *minimum*, and *maximum*.** Remind students of how each word relates to inequalities.
- **Including boundary values for statements that use *between*.** Draw a number line, labeling 0 and 10. Ask, "What values are located between 0 and 10? Why do you not include 0 and 10?"
- **Providing solutions that only satisfy one, but not both, inequalities.** Have students draw two number lines, one for each inequality, and then identify a solution as a number that falls within the shaded regions of both number lines.

Look for productive strategies:

- Interpreting and properly representing each restriction as inclusive or exclusive of boundary values.
- Writing a compound inequality for the High Bounce ride, such as $55 < h < 72$, but this is not expected in this grade.
- Using precise language such as "Priya must be less than 60 in. and she must be less than 55 in. because that is the minimum height for the High Bounce."

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital number lines to help support their thinking.

Extension: *Math Enrichment*

Have students complete the following as a follow-up to Problem 5: Draw a number line to represent all of the possible heights of riders who could ride the Roller Ride. Which other roller coaster has a similar number line? What do you notice about their phrases or inequalities? Students' number lines should have open circles at 64 and 75 and shaded between these values, similar to the High Bounce roller coaster.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the words and number line representing the High Bounce roller coaster. Ask these questions:

- "What do you think the word *between* means? Does it include the boundary values?"
- "How many boundary values are represented on the High Bounce number line?"
- "What are some differences between the High Bounce number line and the other number lines?"

English Learners

Act out the phrase *between* by having three students stand in a row and have the student in the middle hold up a piece of paper with the word *between* written on it.

Pairs | 15 min

Activity 1 Amusement Park Rides (continued)

Students represent height restrictions for amusement park rides with inequalities, and investigate whether a given height is a solution to multiple inequalities at the same time.

Activity 1 Amusement Park Rides (continued)

2. Han's cousin is 55 in. tall. Han says she is not tall enough to ride the High Bounce, but Tyler says that she is tall enough. Do you agree with either Han or Tyler? Explain your thinking.
Sample response: I agree with Han, because "between 55 and 72" does not include either 55 or 72. That means Han's cousin has to be at least taller than 55 in. but shorter than 72 in.

3. Priya can ride the Climb-A-Thon, but she cannot ride either the High Bounce or the Twirl-O-Coaster. Select *all* the heights that could represent Priya's height. Be prepared to explain your thinking.
   - A. 59 in.
   - B. 53 in.
   - C. 56 in.
   - D. 54.9 in.

4. Write at least one inequality for each of the height requirements. Use h for the unknown height, in inches.
   - a. High Bounce: $h > 55$ **and** $h < 72$
   - b. Climb-A-Thon: $h < 60$ **or** $60 > h$
   - c. Twirl-O-Coaster: $h \geq 58$ **or** $58 \leq h$
   - d. Tilt-A-Whirl: $h \leq 58$ **or** $58 \geq h$

5. The inequalities $h < 75$ and $h > 64$ represent the height restrictions, in inches, for the Roller Ride. If Diego is tall enough to ride the Roller Ride, how tall could he be? List at least 3 possible heights, in inches. Be prepared to explain your thinking.
Sample responses: 64.1, 67, $74\frac{3}{4}$. Diego can be any height taller than 64 in., but less than 75 in.

798 Unit 7 Rational Numbers

3 Connect

Have students share how they represented the height restrictions using number lines and inequality statements, and how they reasoned about possible solutions. As students share, encourage them to use both mathematical and contextual language, such as *solutions to the inequality* and *heights allowed*, and add these terms to the anchor chart.

Highlight:

- Students need to use two inequality statements to represent situations that have both an upper and a lower bound, such as the High Bounce ride.
- Students can substitute values for the variable to determine if they are solutions to an inequality.
- There are an infinite number of solutions to an inequality, and this is also true for a pair of inequalities, because values such as 58.1 in., 58.01 in., and 58.001 in. are also possible heights between 55 in. and 72 in.

Ask, as a precursor to Lesson 12, "Are there any solutions to the inequalities that might not make sense in a context about height?"

Small Groups | 15 min

Activity 2 What Number Am I?

Students identify and interpret symbolic and graphed inequalities — for which a given mystery value is a solution — in order to provide clues about the number, or to guess it.

Name: ______ Date: ______ Period: ______

Activity 2 What Number Am I?

Your group will be given three sets of cards — one set shows inequalities, another set shows number lines, and the third set shows rational numbers. Place the inequality and number line cards face up where everyone can see them. Shuffle the number cards and stack them face down.

Plan ahead: How can being prepared to justify your conclusions help your group be successful in the activity?

To play:

- One person in your group is the *detective*. The others will give clues.
- Select one number card from the stack — the *mystery number* — and show it to everyone except the detective.
- The people giving clues each choose either an inequality card or a number line card that will help the detective identify the mystery number.
- The detective studies the selected cards — inequalities and/or number lines — and then makes a guess of what the mystery number might be.
  - If the detective does not guess the number, everyone else chooses another card, which is again either an inequality card or number line card. All the previously selected cards should remain face up.
  - The game ends when the detective has either identified the mystery number (wins), or made three incorrect guesses (loses).
- Repeat the game until everyone has a turn being the detective. Be sure to select a new mystery number for each game.

STOP

Lesson 11 Solutions to One or More Inequalities 799

1 Launch

Place students in groups of 3–4. Give each group three sets of cards from the Activity 2 PDF. Review the directions as a class. Consider assigning 1 point for each clue used, and celebrating groups with the lowest scores. After one round, ask, "Were some clues more helpful than others? Why? How did you decide this?" Play more rounds as time allows.

2 Monitor

Help students get started by having groups verbally translate the meanings of one of each type of clue card before playing.

Look for points of confusion:

- **Choosing clues that only eliminate a few possible solutions.** Ask, "Is there another clue that would help the detective eliminate more solutions?"
- **Not using the clues together.** "Does your solution work for *all* the clues? How can you check?"

Look for productive strategies:

- Choosing accurate clues that, when possible, provide an upper and lower limit for solutions.
- Translating each clue, and considering which values satisfy all clues given.

3 Connect

Have groups share their successful strategies for choosing the most helpful clues, and how detectives used those clues to guess the mystery number. Ask for any instances of disagreement while playing, and focus on how groups resolved differences.

Highlight that students can check whether a value is a solution to an inequality by substituting it for the unknown and checking that the resulting statement is true. Any solution should also fall within the shaded region of the number line.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

During the Launch, conduct a small group or whole class demonstration of how to play the game. Check for understanding by asking students to rephrase the directions in their own words.

Extension: *Math Enrichment*

Display the two number lines that each represent an inequality *between* two boundary values. Ask students to compare and contrast these inequalities and write a verbal statement that describes each. Sample response: One inequality represents all values between –2 and 2 that also includes the boundary values. The other inequality represents all values between –5 and –1 and does *not* include the boundary values.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their strategies, display the following sentence frames to help them organize their thoughts:

- "In choosing a clue card, I chose the card ___ because . . ."
- "As the detective, I used the card ___ to help me guess the mystery number because . . ."
- "One way that I checked that a value was a solution to the inequality (number line) was to ___."

English Learners

Provide students time to rehearse what they will say with a partner, before sharing with the whole class.

Whole Class | 5 min

Summary

Review and synthesize how to determine whether a given maximum and minimum are included as possible solutions to corresponding inequalities.

Summary

In today's lesson . . .

Sometimes a scenario involves both upper and lower bounds, or both a maximum and minimum possible value. To represent these types of situations, you can write two inequalities — one for the upper bound and one for the lower bound.

- For example, if you know that it rained for more than 10 minutes, but less than 30 minutes, you write two separate inequalities and graph them on two separate number lines.

Inequality	Number line
$r > 10$	
$r < 30$	

Any number greater than 10 is a solution to $r > 10$, and any number less than 30 is a solution to $r < 30$. But to meet *both* conditions — more than 10 *and* less than 30 — a solution must be a number that makes both inequalities true. By substituting the same value for r into each inequality, you can determine whether the value is a solution to both inequalities.

You can also represent all the possible solutions that satisfy both inequalities by graphing the two inequalities on the same number line, but only shading the values that overlap. The values that overlap are solutions to both inequalities — as shown.

Reflect:

Synthesize

Have students share real-world contexts where either or both upper and lower bounds are given or useful. As students share, add any relevant new terms they use to describe situations to the anchor chart (e.g., *at most*, *limit*, *at least*, *no more than*, *not less than*).

Ask:

- "Are upper and lower bounds always solutions to the inequalities that represent them?" Sample response: No; for strict inequalities, the upper and lower bounds are not solutions (<, >). For non-strict inequalities, the upper and lower bounds are solutions (≤, ≥).
- "Can you think of any scenarios where an upper or lower bound might not be stated, but could be implied?" Answers may vary.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you use to help you with representing maximum and minimum values?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying possible solutions to a pair of inequalities, and graphing them on the same number line.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.11

1. Select *all* the numbers that are solutions to *both* $h < 10$ and $h > -5$.

   A. 0
   B. 10
   C. -1
   D. -5
   E. 4
   F. -5.01
   G. 9.999

2. Draw a number line to represent *all* the possible solutions to *both* $h < 10$ and $h > -5$.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain what it means for a number to be a solution to an inequality. 1 2 3

b. I can write two inequalities to represent a scenario with upper and lower bounds. 1 2 3

c. I can identify a solution to two inequalities. 1 2 3

d. I can use a number line to identify solutions to two inequalities. 1 2 3

Success looks like . . .

- **Language Goal:** Interpreting and coordinating among verbal, algebraic, and number line representations of inequalities for a quantity with two given constraints, e.g. *between*. **(Speaking and Listening, Reading and Writing)**
  - » Representing the solutions to both $h < 10$ and $h > -5$ on a number line.
- **Language Goal:** Critiquing or justifying possible solutions to inequalities or number line graphs representing a quantity with two given constraints, including determining whether upper and/or lower bounds should be included. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

For Problem 1:

- If students select Choice B or D, consider asking, "What is the difference between $h < 10$ and $h \leq 10$? How about $h > -5$ and $h \geq -5$?"
- If students select Choice F, consider having students plot -5 and -5.01 on a number line, and asking, "Which is greater, -5 or -5.01?"
- If students do not select Choice A, C, E, or G, consider having students substitute each answer choice for h in $h < 10$ and $h > -5$.

If students draw a number line with closed circles for Problem 2, consider:

- Referring them to Activity 1, Problem 1.

If students draw two numbers lines, consider:

- Reviewing how two bounds can be represented on one number line.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How was Activity 1 similar or different from Stories About 9 (Lesson 10, Activity 1)? How were student approaches to boundary values similar or different in both activities?
- What surprised you as your students worked on Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ____________ Date: ________ Period: ________

Practice

1. An amusement park ride has the following restrictions on height h, in inches: $h > 48$ and $h < 65$.

   a. Draw one number line to represent all the possible values that satisfy both height restrictions.

   Sample response:

   b. Mai is 48 in. tall, and Clare is 65 in. tall. Can either of them get on this ride? Explain your thinking.

   Neither Mai nor Clare can get on this ride because the inequalities do not include the boundary points. A person needs to be taller than 48 in. and shorter than 65 in.

2. Select *all* the numbers that are solutions to both $c < 5$ and $c > -1$.

   A. -1

   B. -1.5

   C. 0

   D. 5

   E. 5.05

   F. 4

   G. -0.5

3. One day in Boston, MA, the high temperature was 60°F and the low temperature was 52°F.

   a. Write two inequalities to describe all the possible temperatures t in Boston, MA on that day.

   $t \le 60$ and $t \ge 52$

   b. Represent all the possible temperatures on one number line.

Name: ____________ Date: ________ Period: ________

Practice

4. Select *all* the true statements.

   A. $-5 < |-5|$

   B. $|-6| < -5$

   C. $|-6| < 3$

   D. $4 < |-7|$

   E. $|-7| < |-8|$

5. Match each equation with its solution.

Equation	Solution
a. $x^4 = 81$	d 2
b. $x^2 = 100$	a 3
c. $x^3 = 64$	c 4
d. $x^5 = 32$	b 10

6. A number n is greater than -5, but less than 5.

   a. Represent *all* the solutions that are negative numbers on a number line.

   b. Represent *all* the solutions that are non-negative numbers on a number line.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 7 Lesson 8	2
	5	Unit 6 Lesson 16	2
Formative	6	Unit 7 Lesson 12	1

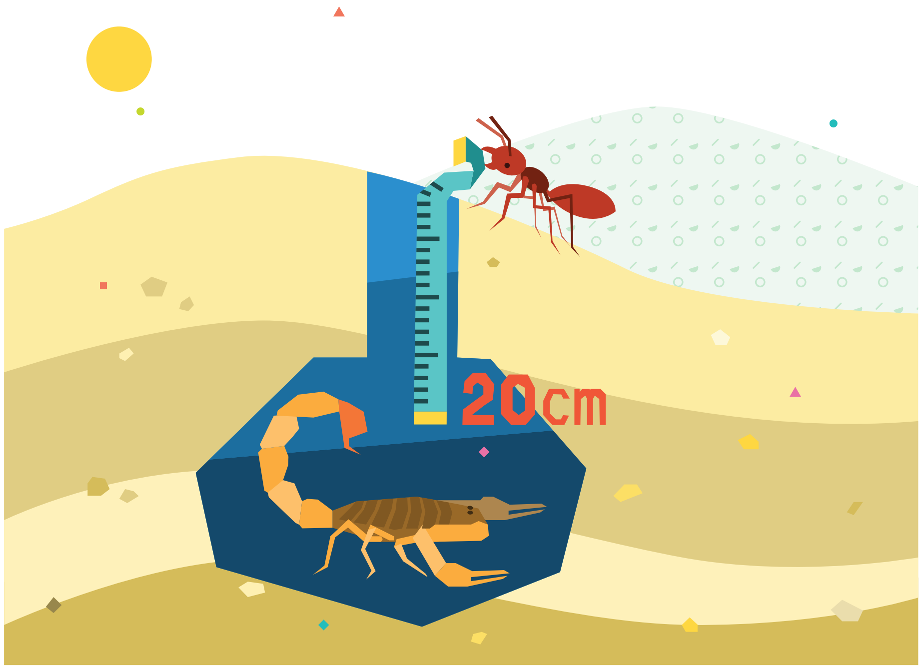
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Interpreting Inequalities

Let's examine what inequalities can tell us.



Focus

Goals

1. **Language Goal:** Interpret and connect verbal, algebraic, and number-line representations of inequalities representing a real-world scenario, e.g., elevation. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Critique or justify possible solutions to inequalities or number lines representing a quantity with one or two given constraints, including determining whether rational-number values are reasonable. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Write and interpret algebraic inequality statements that include more than one variable. **(Reading and Writing)**

Coherence

Today

Students describe scenarios using inequalities and interpret inequalities that represent given constraints in real-world problems. They also reason about limitations inherent to a context, such as continuous and discrete quantities, that have implications for possible solutions to an inequality. Students interpret solutions in the familiar contexts of elevation and temperature, reflecting on whether the solutions make sense. They make claims about valid solutions and consider their reasonableness in context.

Previously

In Lessons 9–11, students wrote inequality statements and graphed possible solutions on a number line. They began to explore the idea of constraints in considering the inclusion or exclusion of boundary values.

Coming Soon

In Grade 7, students will solve inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. They will graph the possible solutions to these inequalities and interpret them in context.

Rigor

- Students strengthen their **conceptual understanding** by determining reasonable solutions to inequalities.
- Students **apply** rational number and absolute value understanding in the contexts of real-world elevation and temperature models.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, *Extreme Temperatures Map,* one per student

Math Language Development

Review word

- *solution to an inequality*

Amps | Featured Activity

Activity 2
Interactive Map

Students click on various animals or people on the map to determine temperatures at the locations these animals or people can be found.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel uncomfortable with any disagreements that arise in Activity 1. Ask students to identify strategies that they can employ to show respect even when there is conflict. Encourage them to remain open to learning from others.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- **Activity 2** may be done as a whole class or omitted.

Independent | 5 min

Warm-up Turtle Clutch

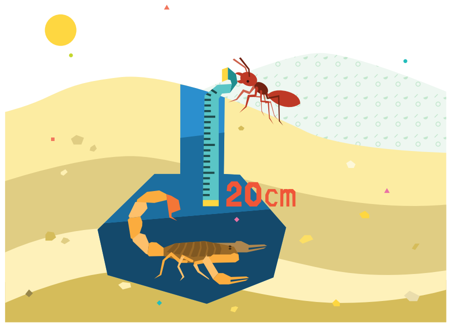
Students interpret two inequality statements with the same variable representing a single scenario, and determine possible solutions to both inequalities.

Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 12

Interpreting Inequalities

Let's examine what inequalities can tell us.



Warm-up Turtle Clutch

On average, sea turtles lay n eggs in a *clutch* (a group of eggs).

1. What does $50 \le n$ mean in this scenario?
   A sea turtle laid 50 or more eggs.
2. What does $n \le 110$ mean in this scenario?
   A sea turtle laid 110 or fewer eggs.
3. Draw two number lines to represent the solutions to each of the two inequalities.

4. Name a possible value for n that is a solution to both inequalities.
   Answers will vary, but should be between 50 and 110, inclusive. Sample responses: 50, 51, 86, 102, 109, 110

Log in to Amplify Math to complete this lesson online.

1 Launch

Explain that a *clutch* is a group of eggs.

2 Monitor

Help students get started by asking, "Do you know how many eggs any particular turtle laid in this scenario?"

Look for points of confusion:

- **Misinterpreting an inequality with the variable on the right side.** Have students rewrite the inequality with the variable on the left side (e.g., rewrite $50 < n$ as $n > 50$). Then have them name one solution and substitute that value into the original inequality with the variable on the right side.
- **Not including 50 and 110 as possible solutions.** Ask, "Could 50 and 110 be amounts that a turtle could lay in a clutch?"

Look for productive strategies:

- Thinking that a solution is a possible number of eggs one turtle could lay, which must be between 49 and 111.

3 Connect

Have individual students share their interpretations of each inequality in context, and how their thinking in finding a solution relates to their number line graphs.

Ask, "Is it possible to have a fraction or decimal solution to the inequality? What about in this context?"

Highlight that a solution to both inequalities must be a solution to each inequality individually, so any value that is shaded on both number line graphs is a solution. Note that only *whole numbers* are reasonable solutions (i.e., fractions or decimals are not possible solutions) in the context of turtle eggs, which is a *discrete* quantity. Add any relevant new words and phrases to the anchor chart.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the following incorrect statement for Problem 4, "The value of 47 is a possible value because it is both less than 50 and less than 110." Ask:

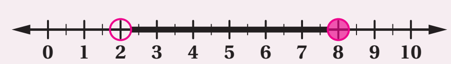
- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "How did you correct the statement? How do you know that the statement is now true?"

Power-up

To power up students' ability to set up constraints involving two inequalities, have students complete:

Recall that when graphing strict inequalities (≤, ≥) you use a closed circle and when graphing non-strict inequalities (<, >) you use an open circle.

A certain number is greater than 2 but no more than 8. Add the appropriate boundary circles, open or closed, to the diagram.



Use: Before the Warm-up
Informed by: Performance on Lesson 11, Practice Problem 6.

Pairs | 20 min

Activity 1 Extreme Elevations

Students use diagrams showing the elevations at which various animals live to write inequality statements and determine possible values that are solutions.

Activity 1 Extreme Elevations

You will compare the elevations where some animals, including humans, live on Earth. Throughout this activity, be prepared to explain your thinking.

Part 1

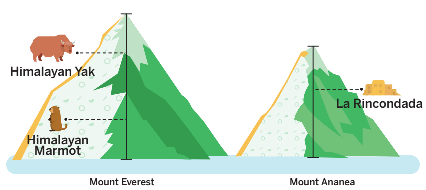
The deathstalker scorpion and Sahara desert ant live at the same elevation in the Sahara desert.

1. Write an equation to represent this scenario. Use d to represent the elevation of the deathstalker scorpion and a to represent the elevation of the Sahara desert ant.
   $d = a$

2. To escape the scorching heat of the desert, the deathstalker scorpion spends most of the day buried 20 cm below the ground. Write an inequality to compare the elevations of a buried deathstalker scorpion and a Sahara desert ant.
   Sample responses: $-20 \leq a$, $d \leq a$, $a \geq -20$, $a \geq d$

Part 2

The image shows Mount Everest, shared by Nepal and China, and Mount Ananea in Peru.



3. Write an inequality to compare the Himalayan yak's elevation y to the Himalayan marmot's elevation m.
   $y > m$

4. The town of La Rinconada is home to 50,000 people living at the extreme elevation of 16,700 ft on Mount Ananea.
   a. Write an inequality to compare the marmot's elevation m to the elevation of La Rinconada.
      $m < 16{,}700$
   b. What is one possible elevation for the Himalayan yak?
      Sample response: 17,000 ft
      Sample unreasonable response: 40,000 ft
   c. What is one possible elevation for the Himalayan marmot?
      Answers will vary, but should be less than 16,700 ft.
      Sample response: 12,000 ft

1 Launch

Activate students' background knowledge by asking, "What are some real-world examples of things that are extreme? What might *extreme* mean in the context of elevation?" Have students use the ***Think-Pair-Share*** routine. Provide them 5 minutes of individual work time. Then have them compare responses with a partner.

2 Monitor

Help students get started by having them visualize the elevations of the deathstalker scorpion and Sahara desert ant described before and after the scorpion goes underground.

Look for points of confusion:

- **Forgetting the negative sign for below sea level values.** Remind students that *below sea level* is represented by locations *below* 0 on a vertical number line (e.g., 26,716 ft below sea level is $-26{,}716$ ft).
- **Writing an inequality using an incorrect symbol.** Remind students that the symbols $\geq$ and $\leq$ include the boundary value as a possible solution, and ask where that makes sense based on the given information.
- **Choosing unreasonable values for solutions.** Ask, "Would that elevation be reasonable in this scenario?"

Look for productive strategies:

- Understanding that a negative number that is closer to 0 is *greater than* a negative number that is farther from 0.
- Using precise language in their thinking. For example, saying "The Goblin shark is closer to sea level, which means its elevation must be greater than $-11{,}000$ ft."

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Students may feel as if they need to know the numerical values of the elevations in order to write an equation for Problem 1. Ask, "Suppose the elevation was 15 cm. What equation would you write?"

Accessibility: *Clarify Vocabulary and Symbols*

In Part 3, clarify that the *depth* of an object or animal underwater is typically represented as a positive value when the term *depth* is included. Point out that the *depth* of the Mariana Trench is given as a positive value, but this does not mean the Mariana Trench is above sea level. Because the term *depth* was included, this refers to the *distance below sea level*.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share how they resolved any disagreements about writing the inequalities, have them use these sentence frames to help organize their thoughts:

- I knew the inequality ______ was correct because . . .
- I knew the inequality ______ was not correct because . . .
- A reasonable elevation for the ______ is ______ because . . . *(Use for Problem 4)*
- An unreasonable elevation for the ______ is ______ because . . . *(Use for Problem 4)*
- While the inequality ______ was correct, I knew that ______ could not be a reasonable value because ______. *(Use for Problem 6c)*

Pairs | 20 min

Activity 1 Extreme Elevations (continued)

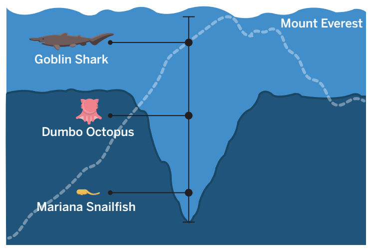
Students use diagrams showing the elevations at which various animals live to write inequality statements and determine possible values that are solutions.

Name: ______ Date: ______ Period: ______

Activity 1 Extreme Elevations (continued)

Part 3

The maximum depth of the Mariana Trench is 36,201 ft, which is more than 7,100 ft farther below sea level than the distance that Mount Everest rises above sea level! The image shows some creatures that live at different depths in and around the Mariana trench.



5. Mariana snailfish have been found as deep as 26,716 ft below sea level. Write an inequality comparing this location to either of the locations of the Dumbo octopus o or the Goblin shark g.
   Sample responses: $-26,716 < o$, $-26,716 < g$, $o > -26,716$, $g > -26,716$

6. The Dumbo octopus is the deepest dwelling octopus, living around 11,000 ft below sea level.
   a. Write an inequality comparing the locations of the goblin shark and the Dumbo octopus.
   Sample responses: $g > -11,000$, $-11,000 < g$
   b. List two possible locations where the Goblin shark could be found swimming, if it does not swim more than 3,200 ft below sea level.
   Sample response: $-2,000$ ft and -150 ft
   c. Tyler says that the Goblin shark could be found at an elevation of 2,000 ft because $2,000 > -11,000$? Do you agree or disagree with Tyler? Explain your thinking.
   I disagree with Tyler because while the inequality is true, the goblin shark could not be at an elevation of 2,000 ft because it would be out of the water.

7. Another sea creature living in the Mariana trench is the fangtooth fish. It can be found swimming between 1,640 and 16,400 ft, inclusive, below sea level.
   a. Write two inequalities that represent the possible locations where the fangtooth fish could be found swimming.
   $f \geq -16,400$ and $f \leq -1,640$
   b. Complete the following inequality statements, about the locations where the Dumbo octopus and fangtooth fish live, to make them true. Use *each* $<$, $>$, $\leq$, $\geq$ *symbol only once.*

   $-1,640 \boxed{>} o$ $\quad -16,400 \boxed{<} o$ $\quad -11,000 \boxed{\geq} f$ $\quad -11,000 \boxed{\leq} f$

Lesson 12 Interpreting Inequalities 805

3 Connect

Display the elevation diagrams for Part 2 and then Part 3.

Have individuals share their thinking about writing inequalities when there was more than one unknown, and then have pairs share any disagreements that arose — and how they were resolved — as a result of critiquing each others' thinking.

Ask:

- "Are the boundary values in the inequalities that use the symbols $\geq$ and $\leq$ also solutions to the inequalities?" Sample response: Yes, the fangtooth fish can be found swimming at the boundary value depths.
- "Is it possible to have a fraction or decimal solution to any of these inequalities? What about in this context?" Sample response: Yes, any of these animals or people could be found at fractional or decimal elevations or depths. It does not make sense to restrict their locations to whole-number values.

Highlight the reasonableness of solutions based on the information given about the locations and each animal's elevation. Note that while elevation is a *continuous* quantity (meaning fractions and decimals are possible solutions), when dealing with heights and depths that have large absolute values — such as many of these — the nearest foot (or even nearest 10 ft or 100 ft) is probably sufficient. Add any new, relevant terms to the anchor chart.

Independent | 10 min

Activity 2 Extreme Temperatures

Students interpret and find solutions to inequalities in real-world scenarios involving temperature.

Amps Featured Activity Interactive Map

Activity 2 Extreme Temperatures

Usually extreme elevations mean extreme temperatures. You will be given a map showing the extreme temperatures in which some animals live.

1. The Himalayan yak and the people of La Rinconada live at extreme elevations above sea level. Let's look at the temperatures in which they can survive.
   a. Write *two* inequalities that represent the temperatures, in degrees Fahrenheit, at which the Himalayan yak can survive.
      Sample response: $y \leq 60$ and $y \geq -40$
   b. What is an example of a negative temperature in which the Himalayan yak could survive?
      Sample responses: -30°F, -39°F, -16°F, -40°F
   c. Write *two* inequalities that represent the temperatures, in degrees Fahrenheit, seen in La Rinconada.
      Sample response: $L \leq 48$ and $L \geq 9$
   d. What is an example of a temperature that might be observed during the summer months in La Rinconada?
      Sample responses: 48°F, 47.3°F, 45°F

2. The animals living in the Mariana Trench live at extreme elevations below sea level, but the water temperature around the habitat of the Dumbo octopus does not fluctuate much. What are all the possible whole number temperatures of the water where it lives?
   33°F, 34°F, 35°F, 36°F, 37°F, 38°F, 39°F

3. What is an example of a temperature in which the Sahara desert ant could survive, but the deathstalker scorpion could not?
   Sample responses: 115.5°F, 112°F, 119°F, 139.99°F

STOP

806 Unit 7 Rational Numbers

1 Launch

Distribute the Activity 2 PDF, *Extreme Temperatures Map*.

2 Monitor

Help students get started by letting them explore the map and verbalize what they notice.

Look for points of confusion:

- **Writing an inequality that is not inclusive.** Ask, "If the highest the temperature rises is 48°F, can it be equal to 48°F? How would you represent this using an inequality statement?"

Look for productive strategies:

- Recognizing that for the given temperature information, the inequalities should be inclusive.
- Understanding that two given constraints can also mean a value *between* two numbers.

3 Connect

Display the map from the Activity 2 PDF.

Have individuals share their thinking, focusing on how they used the information on the map to write inequalities and find solutions. Then have pairs share any disagreements — and how they were resolved — as a result of critiquing each others' thinking.

Highlight the constraints students needed to determine in order to identify solutions. Note that temperature is a *continuous* quantity, but similar to elevation, it is usually only presented to the nearest degree. Add any relevant terms to the anchor chart.

Ask:

- "Who agrees/disagrees with ___'s thinking? Explain why you agree or disagree."
- "Is it possible to have a fraction or decimal solution to any of these inequalities? What about in this context?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can explore an interactive map. Students can click on various animals or people represented on the map to determine the temperatures for those locations.

Accessibility: *Guide Visualization and Processing*

During the Launch, as you distribute the Activity 2 PDF, *Extreme Temperatures Map,* conduct the ***Notice and Wonder*** routine about the information presented on the map. Ask them in particular what they notice or wonder about the temperature shown for the Himalayan Yak, the Dumbo Octopus, or La Rinconada.

Math Language Development

MLR8: Discussion Supports

During the Connect, be sure students understand that while an inequality, such as $y \leq 60$, can have an infinite number of solutions when the inequality does not represent a context, sometimes the context will limit the number of solutions that are reasonable. Consider asking these follow-up questions:

- "Temperature is a context in which fractional or decimal values are reasonable. What are some other contexts in which fractional or decimal values are reasonable?"
- "What are some contexts in which fractional or decimal values are *not* reasonable?"

Whole Class | 5 min

Summary

Review and synthesize how inequalities with two limits are used to define a range of possible values, or solutions, in real-world scenarios.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used inequalities to represent possible elevations and temperatures in which animals on Earth can survive. When you are determining the ***solutions to an inequality*** that represent a real-world scenario, you need to consider what values are reasonable in the context. Some numbers that are solutions to an inequality outside of a context may not make sense when you consider the context.

Scenario	Noah scored less than 25 points in a basketball game.	The temperature in the summer is between 65°F and 103°F.
Reasonable values	24, 22, 8	65.8, 93, 103
Inequality or Inequalities	$p < 25$ and $p \geq 0$	$t \geq 65$ and $t \leq 103$
Discrete or Continuous?	Discrete Only whole-number values of points can be scored in basketball, so not *all* values between 0 and 25 points are solutions.	Continuous Temperature can be measured in decimal and fraction values.

Reflect:

Synthesize

Highlight the reasonableness of a solution to an inequality in the context of the given scenario about Noah's points scored in a basketball game.

Ask, "What are the values that could be a solution in this scenario?" Answers may vary, but students should notice that negative values (negative points) do not make sense in this context. Whole-number values ranging from 0 to 24 make sense in this context.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean for a value to be less than zero?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by interpreting two inequalities that both represent the same scenario. They identify reasonable solutions considering the context.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.12

1. Lin says that the inequalities $h > 150$ and $h < 160$ describe her height in centimeters. What do these inequalities tell you about her height?
   Lin is taller than 150 cm, but shorter than 160 cm. Her height could be as short as 150.1 cm or as tall as 159.9 cm, or any other rational number between 150 and 160. She could not be exactly 150 cm tall or 160 cm tall.

2. Select *all* the heights that could be Lin's height.
   A. 150 cm
   B. 154 cm
   C. 160 cm
   D. 159.75 cm
   E. 164 cm

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine whether a given number is a solution to two inequalities that represent a scenario. 1 2 3

b. I can determine whether a solution to one or more inequalities is a reasonable solution for a given scenario. 1 2 3

c. I can explain what solutions to an inequality mean for a given scenario. 1 2 3

Lesson 12 Interpreting Inequalities

Success looks like . . .

- **Language Goal:** Interpreting and connecting verbal, algebraic, and number-line representations of inequalities representing a real-world scenario, e.g., elevation. **(Speaking and Listening, Reading and Writing)**
  - » Explaining the meanings of the two inequalities in the context of Lin's height in Problem 1.
- **Language Goal:** Critiquing or justifying possible solutions to inequalities or number lines representing a quantity with one or two given constraints, including determining whether rational-number values are reasonable. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Writing and interpreting algebraic inequality statements that include more than one variable. **(Reading and Writing)**

Suggested next steps

If students cannot describe what is happening in the scenario, consider:

- Having them draw a sketch of Lin next to a vertical number line.

If students have trouble identifying the possible solutions in the scenario, consider:

- Having them draw a number line to help with their thinking.

If students identify Choices A and C as possible solutions, consider:

- Asking, "What is the difference between $h > 150$ and $h \geq 150$?"

If students do not identify the decimal solution, consider:

- Asking, "Where would 159.75 be located on a number line?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students represented and interpreted inequalities that describe real-world contexts with inherent limitations. How did that build on the earlier work with constraints that students did in Lessons 10 and 11? How did their work today advance their understanding of solutions to an inequality?
- What challenges did students encounter as they worked on Activity 2? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent

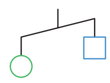
Practice

Name: ______ Date: ______ Period: ______

1. There is a closed carton of eggs in Mai's refrigerator. The carton contains e eggs, and it can hold up to 12 eggs.
   a. What does the inequality $e < 12$ mean in this context?
      Sample responses: There are less than 12 eggs in the carton; The carton is not full.
   b. What does the inequality $e > 0$ mean in this context?
      Sample responses: The number of eggs in the carton is greater than zero; the carton is not empty.
   c. What are some possible realistic values of e that will make both $e < 12$ and $e > 0$ true?
      Sample responses: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

2. Tyler has more than \$10. Elena has more money than Tyler. Mai has more money than Elena. Let t be the amount of money that Tyler has, e be the amount of money that Elena has, and m be the amount of money that Mai has. Select *all* the statements that must be true.
   A. $m = e$
   B. $m > 10$
   C. $e > 10$
   D. $t > 10$
   E. $e > m$
   F. $t < e$

3. Here is a diagram of an unbalanced hanger.



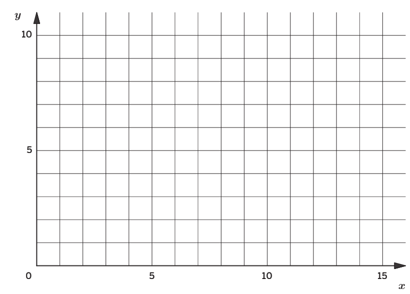
   a. Write an inequality to represent the relationship between the two weights. Use s to represent the weight, in grams, of the square, and use c to represent the weight, in grams, of the circle.
      $c > s$
   b. One circle weighs 12 grams. Write an inequality to represent the weight of one square.
      $s < 12$
   c. Could 0 be a value of s? Explain your thinking.
      In terms of the inequality, yes, 0 can be a value for s because 0 is less than 12. However, in this context, a weight has to weigh something, so 0 is not a realistic or reasonable answer.

Name: ______ Date: ______ Period: ______

4. Write an inequality statement to represent each scenario.
   a. Jada is taller than Diego. Diego is 54 in. tall. Write an inequality that compares Jada's height j, in inches, to Diego's height.
      $j > 54$
   b. Jada is shorter than Elena. Elena is 5 ft tall. Write an inequality that compares Jada's height j, in inches, to Elena's height.
      $j < 60$

5. Select *all* the expressions that are equivalent to $\left(\frac{1}{2}\right)^3$.
   A. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
   B. $\left(\frac{1}{3}\right)^2$
   C. $\frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2}$
   D. $\frac{1}{6}$
   E. $\frac{1}{8}$

6. Select four points and plot them on the coordinate plane. Label them as ordered pairs.



Answers will vary. Students' responses should have the correct format for ordered pairs of (x, y).

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 9	2
	5	Unit 6 Lesson 15	1
Formative	6	Unit 7 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 3
The Coordinate Plane

Whole Class

In this Sub-Unit, students extend the coordinate plane to include negative values, applying their understanding to locations on a map, geometric shapes, mazes, and artistic drawings.

SUB-UNIT 3 The Coordinate Plane

Narrative Connections

How did Greenland get so big?

Maybe it's not as big as you think . . .

If you remember your geography, Greenland is that big, chunky landmass, just northeast of Canada. It's almost as big as Africa! Except . . . it isn't.

Greenland is actually *14 times smaller* than Africa. So why does it look so distorted?

For that, we can thank the mapmaker Gerardus Mercator. In 1569, he invented a novel way of visualizing maps that we still use today, called the "Mercator Projection."

You see, longitude and latitude (the horizontal and vertical lines on maps) aren't really straight lines at all. They're arcs that trace a path along the curved surface of the Earth.

These circles are perfectly sensible on a 3D globe. But for 2D maps, they're a nightmare, especially for sailors. Figuring out longitude, in particular, required precise measurements of time, speed, and bearing, just to even *approximate* where you were.

Before Mercator, mapmakers (sensibly) drew latitude and longitude as curves. But that rebel Mercator, depicted them as straight, perpendicular lines. This allowed ships to follow one bearing, without having to make too many little adjustments.

However, straightening these lines meant having to stretch and distort the continents, especially closer to the poles. And that's why places like Greenland and Antarctica look humongous — a tradeoff sailors were happy to make. By using parallel and perpendicular lines, Mercator effectively placed a grid over the world, making it simpler to locate things and plot an effective course to any spot on the map.

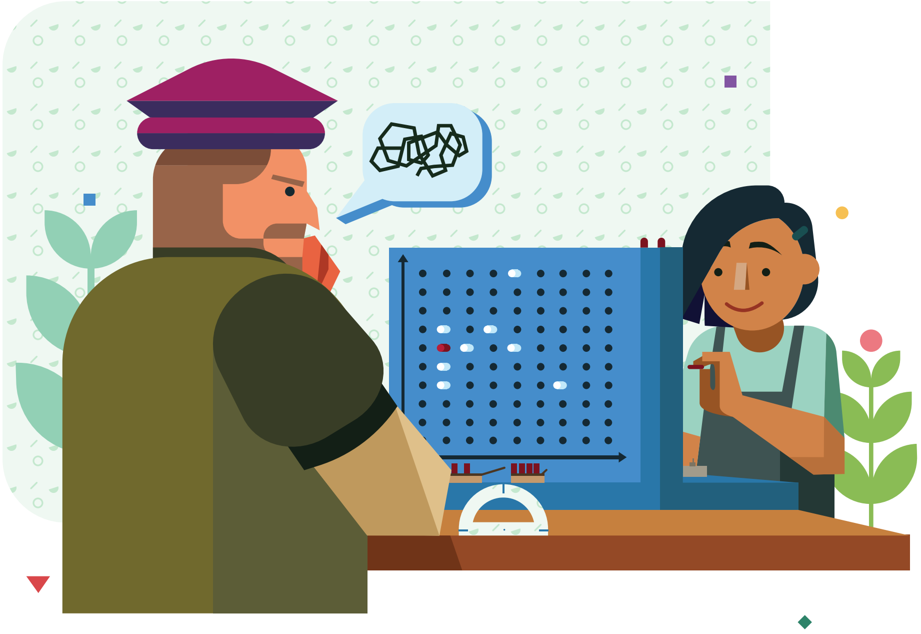
Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how the four quadrants of the coordinate plane can be used to represent real-world locations in the following places:

- **Lesson 14, Activities 1–2:** Coordinate Archery, A Coordinate Maze
- **Lesson 15, Activity 1:** Elevation and Temperature on Mauna Kea
- **Lesson 16, Activity 1:** Determining Distances on a Map
- **Lesson 17, Activity 2:** Fencing for a Wildlife Refuge
- **Lesson 18, Activities 1–2:** Found in the Maize Maze?, Lost Treasures

Extending the Coordinate Plane

Let's explore and extend the coordinate plane.



Focus

Goals

1. **Language Goal:** Recognize that the axes of the coordinate plane can be extended to form four quadrants that represent ordered pairs of positive and negative numbers. **(Speaking and Listening, Reading)**
2. Plot a point given integer coordinates, or identify the integer coordinates of a given point on the coordinate plane.
3. **Language Goal:** Generalize about the signs of coordinates that represent locations in each quadrant of the coordinate plane. **(Speaking and Listening, Reading)**

Rigor

- Students extend their **conceptual understanding** of rational numbers as related to a four-quadrant coordinate plane.
- Students develop **procedural skills** for locating and plotting ordered pairs on a four-quadrant coordinate plane.

Coherence

Today

Students construct an expanded coordinate plane where negative numbers appear on both the vertical and horizontal axes. These crossing axes create the four regions of the coordinate plane, called *quadrants*. Students then plot points with integer coordinates, and identify the integer coordinates of plotted points, in all four quadrants of the coordinate plane.

Previously

In Lesson 3, students extended the number line to include negative numbers. In Grade 5, students plotted points and wrote ordered pairs for points with positive coordinates, located in the first quadrant of a coordinate plane.

Coming Soon

In Lesson 14, students will explore how to appropriately select and interpret different end points and scales on the axes of the coordinate plane to plot real-world data where coordinates are made up of rational numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Warm-up PDF, one per student
- Activity 1 PDF, one per student
- Activity 2 PDF, one per student
- Warm-up/Activity 2 PDF, *Ships*, pre-cut, four ships per student
- rulers or straightedges

Math Language Development

New word

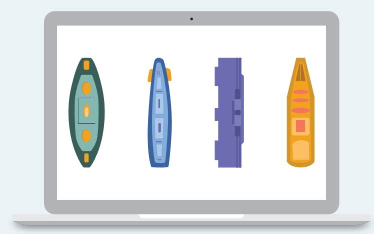
- *quadrant*

Amps | Featured Activity

Warm-up and Activity 2

Digital Game of Ship vs. Ship

Students play an interactive game with partners, practicing labeling and plotting ordered pairs with one another.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might want to impulsively and randomly guess while playing the game, instead of using the structure of the grid in Activity 2. Ask students to identify ways to control their impulses and motivate themselves to focus on the goal of the game. They can describe how organizational skills will make better guesses and be more likely to be successful when playing the game.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

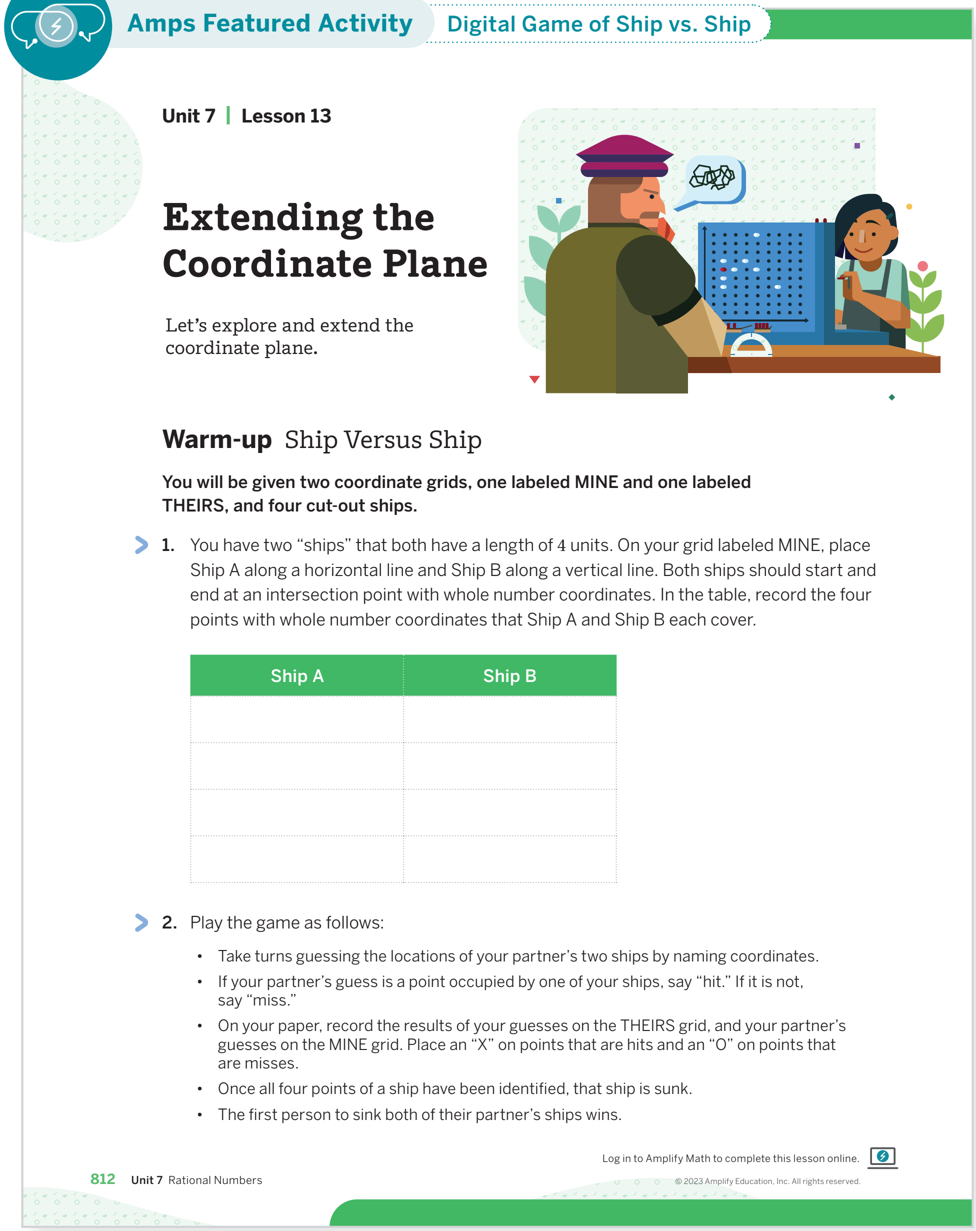
- **Activity 1** can be done as a whole-class demonstration, with students working alongside you.

Pairs | 10 min

Warm-up Ship Versus Ship

Students review how to plot and identify the coordinates of points in the first quadrant of the coordinate plane by playing a game with a partner.

Amps Featured Activity Digital Game of Ship vs. Ship


Unit 7 | Lesson 13

Extending the Coordinate Plane

Let's explore and extend the coordinate plane.

Warm-up Ship Versus Ship

You will be given two coordinate grids, one labeled MINE and one labeled THEIRS, and four cut-out ships.

1. You have two "ships" that both have a length of 4 units. On your grid labeled MINE, place Ship A along a horizontal line and Ship B along a vertical line. Both ships should start and end at an intersection point with whole number coordinates. In the table, record the four points with whole number coordinates that Ship A and Ship B each cover.

Ship A	Ship B

2. Play the game as follows:
   - Take turns guessing the locations of your partner's two ships by naming coordinates.
   - If your partner's guess is a point occupied by one of your ships, say "hit." If it is not, say "miss."
   - On your paper, record the results of your guesses on the THEIRS grid, and your partner's guesses on the MINE grid. Place an "X" on points that are hits and an "O" on points that are misses.
   - Once all four points of a ship have been identified, that ship is sunk.
   - The first person to sink both of their partner's ships wins.

Log in to Amplify Math to complete this lesson online.

812 Unit 7 Rational Numbers

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1 Launch

Distribute the Warm-up PDF. Make sure students understand how to play the game, and particularly how to record their guesses. Tell them to label one ship as Ship A and the other as Ship B. Give them 8 minutes to play (they might not finish the game, which is okay).

2 Monitor

Help students get started by activating prior knowledge. Ask them to define or use their arms to show the difference between horizontal and vertical. Have students place Ship A on a horizontal line and Ship B on a vertical line.

Look for points of confusion:

- **Reversing the coordinates in ordered pairs.** Remind students of the convention for writing ordered pairs as (x, y), or (horizontal, vertical).

Look for productive strategies:

- Carefully listening to their partner's coordinates and marking Xs and Os at the correct points.
- Recognizing that because ships must be vertical or horizontal, then all of the points covered by a ship must also share one coordinate, either the same values of x or the same value of y.

3 Connect

Display a blank version of the Warm-up PDF, with an X plotted at $(3, 3)$ representing a 'hit.'

Have students share the coordinates of a next guess, and why they believe it could also result in a 'hit.'

Highlight that points in the coordinate plane are represented by ordered pairs, (x, y), showing horizontal and then vertical position. Points on the same horizontal line share the same x-coordinate, and points on the same vertical line share the same y-coordinate.

Differentiated Support

Accessibility: Optimize Access to Technology

Have students use the Amps slides for this activity, in which they can play an interactive game of Ship vs. Ship with their partners.

Accessibility: *Guide Processing and Visualization*

Demonstrate, or ask a student volunteer to demonstrate, how to place one ship along a horizontal line and the other ship along a vertical line so that each ship starts and ends at an intersection point with whole number coordinates. Consider demonstrating both a correct sample placement and an incorrect sample placement.

Power-up

To power up students' ability to identify and write coordinate pairs that represent points on a coordinate plane:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up.

Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6.

Independent | 10 min

Activity 1 Double-Folded Coordinate Planes

Students extend vertical and horizontal number lines to form axes and construct a four-quadrant coordinate plane. They then plot ordered pairs with negative coordinates.

Name: ______ Date: ______ Period: ______

Activity 1 Double-Folded Coordinate Planes

You will be given a straightedge and a coordinate plane, labeled with horizontal axis x and vertical axis y.

Part 1

1. Follow the steps to your coordinate plane in one direction.
   - Carefully fold your paper along the y-axis and mark a crease there.
   - Use the straightedge to draw a vertical line along the crease that extends the y-axis downward. Draw an arrow at the end of the line.
   - Use what you know about number lines to label the new tick marks on the y-axis.

2. Plot a point in the space that is below the x-axis and to the right of the y-axis. How can you identify the location of this point using coordinates?
   Answers will vary, but a point in the lower right quadrant should have a positive x-coordinate and a negative y-coordinate.

Part 2

3. Follow the steps to extend your coordinate plane in the other direction.
   - Carefully fold your paper along the x-axis and mark a crease there.
   - Use the straightedge to draw a horizontal line along the crease that extends the x-axis to the left. Draw an arrow at the end of the line.
   - Use what you know about number lines to label the new tick marks on the x-axis.

4. Plot a point in each of the spaces that are to the left of the y-axis: one above the x-axis, and one below it. How can you identify the locations of these points using coordinates?
   Answers will vary, but a point in the upper left quadrant should have a negative x-coordinate and a positive y-coordinate, while a point in the lower left quadrant should have both a negative x-coordinate and a negative y-coordinate.

5. Study your coordinate plane.
   a. How many distinct areas are there on your coordinate plane now?
   Four
   b. What do you think a good name for each of these areas might be?
   Answers will vary.

1 Launch

Distribute the Activity 1 PDF and rulers or straightedges. Pause after students complete Part 1 and show how the extended y-axis should look.

2 Monitor

Help students get started by demonstrating how to create the first fold along the y-axis.

Look for points of confusion:
- **Only extending the axis by a unit or two.** Model extending the line to the edge of the page.
- **Creating an inaccurate scale for the negative sides of the axes.** Remind students about number lines with positive and negative sides as well as opposites to ensure equal distances from zero.
- **Reversing the coordinates in an ordered pair.** Refer to the Warm-up and explain that the (x, y) order remains the same.

Look for productive strategies:
- Accurately plotting and naming an ordered pair in each of the four regions of the coordinate plane, as well as accurately labeling negatives on the axes.

3 Connect

Display a blank, four-quadrant coordinate plane.

Have students share sample points from Problems 2 and 4, and the terms they used to name the four areas.

Define the term ***quadrant*** as each of the four regions of the coordinate plane, formed by the vertical and horizontal axes.

Highlight that the four quadrants are named using Roman numerals, moving counterclockwise from the upper right, as: I, II, III, IV. Points in all quadrants are still labeled using (x, y) ordered pairs, which include negative coordinates. The origin and points located on either axis are not in any quadrant.

Differentiated Support

Accessibility: *Activate Prior Knowledge*
Remind students they have previously plotted points in the first quadrant of the coordinate plane, although they may be familiar with the term quadrant. As you distribute the Activity 1 PDF, ask students to explain how they would plot the point $(2, 3)$ on the coordinate plane. Do not actually have them plot this point.

Accessibility: *Vary Demands to Optimize Challenge*

Consider using one or more of these alternative approaches:
- Have students work with a partner to fold the coordinate planes.
- Demonstrate how to fold the coordinate planes as students follow along and fold their own coordinate planes at the same time.
- Demonstrate how to number the negative axes and have students continue the pattern.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses to Problems 4 and 5, provide these sentence frames to help organize their thinking:
- "Points in the [top/bottom] [left/right] area have [positive/negative] x-coordinates and [positive/negative] y-coordinates."
- "I would name each of these four areas _____ because . . ."

Connect the term *quadrant* to other words students may know that use the same prefix *quad-*, such as *quadrilateral* or *quadriceps*. Emphasize the prefix *quad-* means "four."

English Learners

Ask students what "four" translates to in their primary language, such as cuatro (Spanish), quatre (French), etc.

Pairs | 15 min

Activity 2 Ultimate Ship Versus Ship

Students revisit the game from the Warm-up, but now playing with all four quadrants of the coordinate plane.

Amps Featured Activity Digital Game of Ship vs. Ship

Activity 2 Ultimate Ship Versus Ship

You will be given two more coordinate grids, one labeled MINE and the other labeled THEIRS. Now, there will be four *quadrants* in each grid. You will need the same four cut-out ships you used in the Warm-up.

Plan ahead: How will you apply your previous knowledge of the game structure to better your chances of winning?

1. You have four "ships" that each have a length of 4 units. On the grid labeled MINE, place each ship along either a horizontal or vertical line. You can only place one ship in each quadrant. Each ship should start and end at an intersection point with whole number coordinates. In the table, record the four points with integer coordinates that each of your ships cover.

Ship A	Ship B	Ship C	Ship D

2. Play the game just like in the Warm-up:
   - Take turns guessing the locations of your partner's ships by naming coordinates.
   - If your partner's guess is a point occupied by one of your ships, say "hit." If it is not, say "miss."
   - On your paper, record the results of your guesses on the THEIRS grid, and your partner's guesses on the MINE grid. Place an "X" on the points that are hits and an "O" on points that are misses.
   - Once all four points of a ship have been identified, that ship is sunk.
   - The first person to sink all four of their partner's ships wins.

STOP

1 Launch

Distribute the Activity 2 PDF and give students time to play the game with a partner (again, it is okay if they do not finish).

2 Monitor

Help students get started by pointing out that there are now four quadrants in each grid.

Look for points of confusion:

- **Reversing the coordinates in ordered pairs.** Ask students how to write an ordered pair for the first quadrant (x, y) and remind them that ordered pairs are written in the same way for the other quadrants. If needed, refer back to the Warm-up or Activity 1.
- **Naming the coordinates of a ship using only one point.** Remind students that each "ship" is four units long and therefore must have four different sets of coordinates.
- **Ignoring +/– symbols when locating or naming coordinates.** Ask students to locate the positive and negative sections of the x- and y-axes.

Look for productive strategies:

- Recognizing that points on the same horizontal or vertical line will share one coordinate, but the other coordinate can include either positive or negative numbers.

3 Connect

Have students share an ordered pair of one of their ships.

Ask, for each ordered pair that is shared, "In which quadrant is that ship located? How do you know?"

Highlight that both axes can be extended to include negative numbers. This creates a four-quadrant coordinate plane where ordered pairs can have positive and negative coordinates.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can play an interactive game of Ultimate Ship vs. Ship with their partners.

Extension: *Math Enrichment*

Challenge students to play the game with only two ships, where one ship must be placed in quadrant I and the other ship must be placed in quadrant III.

Whole Class | 5 min

Summary

Review and synthesize how the x- and y-axes were extended to construct a four-quadrant coordinate plane, in which ordered pairs can have positive and negative values.

Name: ______________________ Date: __________ Period: __________

Summary

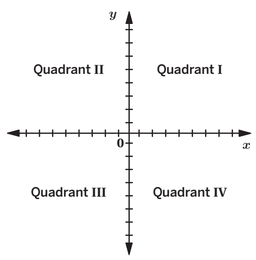
In today's lesson . . .

You extended the coordinate plane to include negative numbers. Just as a number line can be extended to the left (horizontally) or down (vertically) to include negative numbers, the x- and y-axes of the coordinate plane can also be extended to include negative numbers. In doing so, the axes cross at the origin, $(0, 0)$, to create four regions, called ***quadrants***. The names of these quadrants use Roman numerals, so that we can reference them easily.

- For example, "a point in quadrant III" indicates a point that is located in the lower left quadrant, left of the y-axis and below the x-axis.

Points represented by ordered pairs can have negative x- and y-coordinates.

This coordinate plane shows the locations of quadrants I, II, III, and IV. The table shows the signs of the x- and y-coordinates of any point located in each quadrant.



Quadrant	x-coordinate	y-coordinate
I	positive	positive
II	negative	positive
III	negative	negative
IV	positive	negative

Reflect:

Synthesize

Display the four-quadrant coordinate plane from Activity 1.

Formalize vocabulary: ***quadrant***

Ask:

- "What are the names of the quadrants and where is each one located on the coordinate plane?"
- "In which quadrant is the point $(-4, 5)$ located? How do you know?"
  Quadrant 2; The x-coordinate is negative and the y-coordinate is positive.
- "In which quadrant is the point $(4, -5)$ located? How do you know?"
  Quadrant 4; The x-coordinate is positive and the y-coordinate is negative.

Have students share how to locate and label a point on the coordinate plane.

Highlight that the phrases *ordered pairs* and *coordinates of a point* are often used interchangeably. Every point on the coordinate plane has x- and y-coordinates, written as the ordered pair (x, y). Points on the axes are not considered to be located in any quadrant, which means the origin at $(0, 0)$ is also not located in any quadrant. The signs of the coordinates for every point in a given quadrant are the same, because they are all located in the same direction — right or left, and up or down — from the origin. This concept will be explored further in Lesson 14.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you use to help you when you played Ship vs. Ship?"
- "How did your understanding of the coordinate plane change from Grade 5?"

Independent | 5 min

Exit Ticket

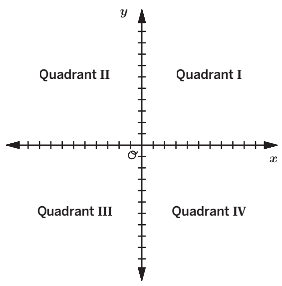
Students demonstrate their understanding of the quadrants in the coordinate plane, and how to plot and name points.

Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 7.13

Refer to the coordinate plane.



Point	Coordinates	Quadrant
G	$(5, 2)$	I
H	$(-1, -5)$	III
I	$(7, -4)$	IV
J	Sample response: $(-3, 4)$	II

1. In the table, write the names of the quadrants where points G, H, and I are located.
2. Point J is not located in the same quadrant as any of points G, H, or I. In the table, write the name of the quadrant where point J is located.
3. In the table, write an ordered pair that could be the coordinates of point J.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe a coordinate plane that has four quadrants.
1 2 3

b. I can identify the quadrant in which an ordered pair with positive and/or negative integer coordinates is located.
1 2 3

c. I can plot points with integer coordinates in the coordinate plane.
1 2 3

d. I can write an ordered pair representing the location of a given point with integer coordinates.
1 2 3

Lesson 13 Extending the Coordinate Plane

Success looks like . . .

- **Language Goal:** Recognizing that the axes of the coordinate plane can be extended to form four quadrants that represent ordered pairs of positive and negative numbers. **(Speaking and Listening, Reading)**
- **Goal:** Plotting a point given integer coordinates, or identifying the integer coordinates of a given point on the coordinate plane.
- **Language Goal:** Generalizing about the signs of coordinates that represent locations in each quadrant of the coordinate plane. **(Speaking and Listening, Reading)**
  - » Completing the table to identify the quadrant of each point in Problem 1.

Suggested next steps

If students have trouble identifying the quadrant for the given points, consider:

- Having students label the tick marks with the appropriate number(s) to help track where the points will be located.
- Using a straightedge and pencil to extend tick marks — up, down, or across — by drawing lines to help find the coordinates of each point.
- Having students reflect back on Activity 2 and say, "If this were a point on your partner's ship, in which quadrant would you start your search?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In Sub-unit 1, students used number lines to compare and order rational numbers. How did that work support students today as they created their own four-quadrant coordinate planes and played Ship vs. Ship?
- What different ways did students approach the two games of Ship vs. Ship (Warm-up and Activity 2)? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

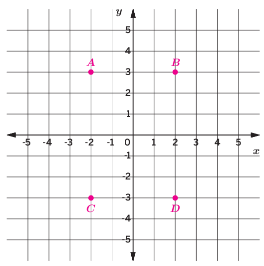
Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

Use the coordinate plane to plot points for Problems 1 and 2.



1. Plot and label the following four points: $A(-2, 3)$, $B(2, 3)$, $C(-2, -3)$, and $D(2, -3)$.

2. Plot a point on one of the axes and label it E. What are the coordinates of point E?
Answers may vary.

3. These three coordinates form a line: $(-3, 4)$, $(0, 4)$, and $(6, 4)$.

a. Is the line vertical or horizontal? Explain how you know.
Horizontal. Sample response: They all have the same y-coordinate.

b. Write the coordinates for two other points that are also on this same line.
Sample response: (1, 4), (10, 4)

4. Lin ran 29 m in 10 seconds. She ran at a constant speed.

a. How far did Lin run every second?
2.9 m

b. At this rate, how far can she run in 1 minute?
174 m

Name: ______ Date: ______ Period: ______

Practice

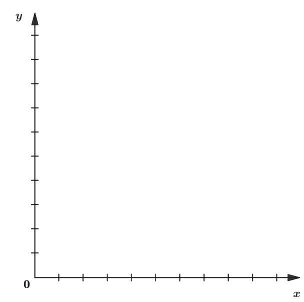
5. One night, it was 24°C degrees warmer in Tucson, AZ, than it was in Minneapolis, MN. If the temperatures in Tucson and Minneapolis were opposites, what was the temperature in Tucson?

A. −24°C
B. −12°C
C. 12°C
D. 24°C

6. Refer to the coordinate plane showing the first quadrant. If the intervals of the ticks on the x- and y-axes increased by 1s, then a point that is plotted exactly halfway between 0 and the first tick on either axis would have a coordinate value of 0.5.

What would be the coordinate value of a point that is plotted exactly halfway between 0 and the first tick on either axis if the intervals increased by:

a. 2s **1**
b. 10s **5**
c. 0.5s **0.25**



Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	1
	3	Activities 1 and 2	2
Spiral	4	Unit 3 Lesson 3	2
	5	Unit 7 Lesson 3	3
Formative	6	Unit 7 Lesson 14	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

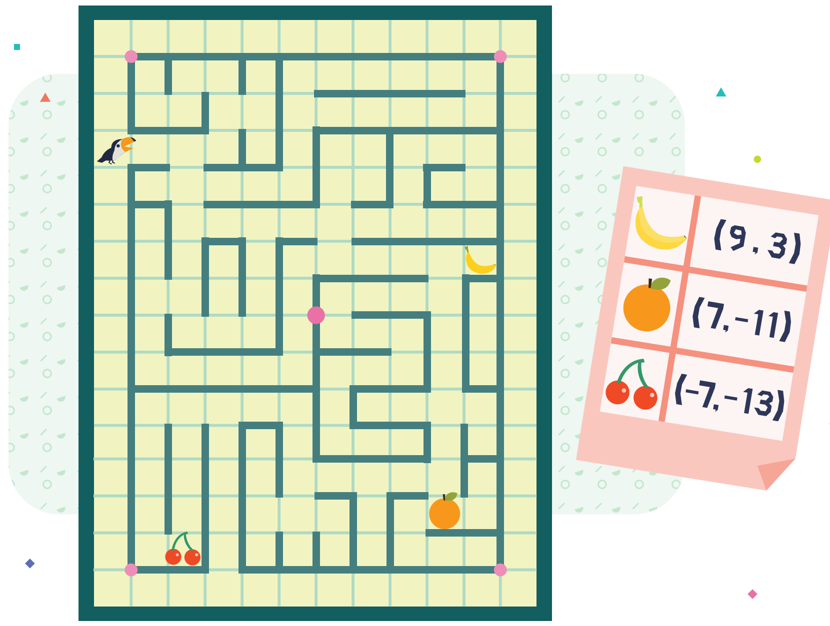
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 7 | **LESSON 14**

Points on the Coordinate Plane

Let's plot points between the tick marks on a coordinate plane.



Focus

Goals

1. Plot a point given its rational-number coordinates or identify the rational-number coordinates of a given point on the coordinate plane.
2. **Language Goal:** Compare and contrast different scales for the axes of a coordinate plane. **(Speaking and Listening, Reading)**

Coherence

Today

Students use the structure of the four-quadrant coordinate plane and attend to precision when plotting and labeling rational-number coordinates. Students not only plot and identify points between the tick marks on a coordinate plane, but also begin to consider the locations of ordered pairs with rational-number coordinates using axes with different scales — particularly those with scales where the tick marks represent values other than 1 unit. They also begin to consider how points in the coordinate plane can be used to show movement between locations as they navigate through a maze.

Previously

Students learned how to plot and read negative numbers on a number line. In Lesson 13, they extended this knowledge to explore negative numbers in a four-quadrant coordinate plane, learning how to plot and label points, as well as identify the different quadrants.

Coming Soon

In Lesson 15, students will continue to see differently-scaled axes and think about the most appropriate scale to create to plot and label coordinates on their individually created planes. They will also think about ways the coordinate plane is used and applied in real-world contexts.

Rigor

- Students strengthen their **fluency** with plotting rational-number coordinates efficiently and accurately on a four-quadrant coordinate plane.
- Students **apply** their understanding of rational-number coordinates to models of contexts using a four-quadrant coordinate plane.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (as needed)

Math Language Development

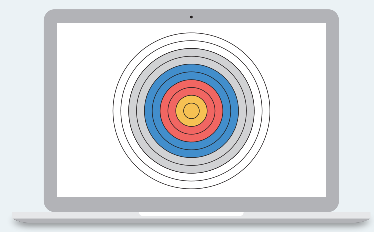
Review word

- *quadrant*

Amps | **Featured Activity**

Activity 1
Dynamic Archery

Students can input coordinates and watch their arrows fly to receive real-time feedback.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not fully make sense of the problem of deciding which points to mark in Activity 2. Encourage students to make sure that they study and analyze the maze before starting the path. Then challenge them to guide their decision making by thinking about not only how to solve the problem, but also how to do so efficiently.

Modifications to Pacing

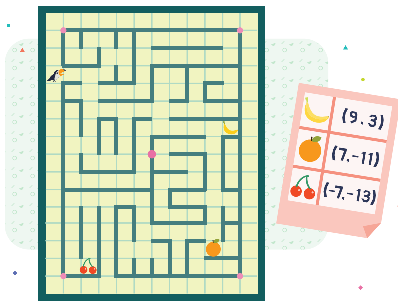
You may want to consider this additional modification if you are short on time.

- **Activity 2** can be done as a whole class, having students identify aloud the physical locations of turns, and then determining coordinates independently before you confirm the correct values.

Independent | 5 min

Warm-up A Map of the Town

Students use a four-quadrant coordinate plane to find and plot locations of a town.



Unit 7 | Lesson 14

Points on the Coordinate Plane

Let's plot points between the tick marks on a coordinate plane.

Warm-up A Map of the Town

Use the map to complete the table indicating the place, quadrant, and coordinates of each location in the town.

1.

Place	Quadrant	Coordinates
School	I	(3, 4)
Library	IV	(1, –5)
Supermarket	III	(–5, –2)
Park	II	(–3, 2)

2. Think of another place to include in the town. Plot its location on the town map, and then write down the quadrant in which it is located, and its coordinates.

Sample responses shown.
Place: Post Office

Quadrant: IV

Coordinates: (2, –2)

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work independently on the activity.

2 Monitor

Help students get started by having them match the coordinates of the school to the point showing its location on the grid.

Look for points of confusion:

- **Confusing the (x, y) order for writing coordinates.** Remind them that, just as x comes before y in the alphabet, the ordered pair is written (x, y).
- **Forgetting to include negative signs.** Have students trace the grid lines from a point to each axis, x and then y, and note the label for the corresponding tick mark.
- **Mislabeling quadrants.** Remind students that quadrant I is located in the top right corner, and then numbering continues counterclockwise, ending with quadrant IV in the bottom right corner.

Look for productive strategies:

- Accurately identifying coordinates and corresponding quadrants.

3 Connect

Display the coordinate plane containing the map of the town and the table.

Have students share their responses for completing the table, and then their examples of other locations with their identified quadrant and coordinates, focusing on a variety across all four quadrants.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Add the following information to the class display or anchor chart and keep it posted for students to refer to as they complete this activity and the upcoming activities.

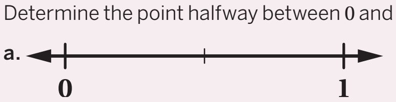
Quadrant	x-coordinate	y-coordinate
I	positive	positive
II	negative	positive
III	negative	negative
IV	positive	negative

Power-up

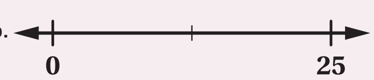
To power up students' ability to identify values between 0 and a given value on the axis of a coordinate plane have students complete:

Determine the point halfway between 0 and the given value on each number line.

a.



b.



Use: Before Activity 2.
Informed by: Performance on Lesson 13, Practice Problem 6.

Pairs | 20 min

Activity 1 Coordinated Archery

Students locate and name specific coordinates in order to hit different parts of an archery target on the coordinate plane.

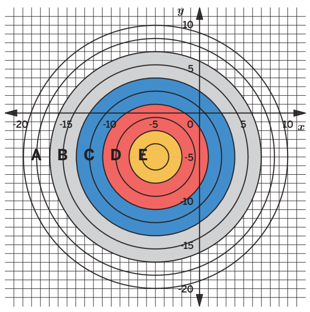
Amps Featured Activity Dynamic Archery

Name: ______ Date: ______ Period: ______

Activity 1 Coordinated Archery

Part 1

A single-spot target like this one is commonly used in many archery competitions, and the scoring system is shown in the table.



	Outer ring (points)	Inner ring (points)
White ring (A)	1	2
Grey ring (B)	3	4
Blue ring (C)	5	6
Red ring (D)	7	8
Yellow ring (E)	9	10

1. How many points would you score if you hit the target at each of these coordinates?
   - a. $(-5, -2)$ 8 points
   - b. $(-9.5, 1)$ 5 points

2. Write an ordered pair for a hit that would score 2 points.
   Sample response: $(7, 0)$

3. Clare hit the target four times, but each arrow hit the target in a different quadrant. Write possible coordinates for each hit in the table and determine her total score.

   Clare's total score would be 10 points.

	Coordinates	Point(s)
Quadrant I	$(7, 2)$	1
Quadrant II	$(-2, 7)$	2
Quadrant III	$(-15, -5)$	3
Quadrant IV	$(5, -4)$	4

1 Launch

Activate students' background knowledge by asking what they know about the rules of archery or darts. Briefly explain that the closer to the center of a target — the "bullseye" — an arrow lands, the greater the score, and ensure students can interpret the scoring tables for both Parts 1 and 2.

2 Monitor

Help students get started by asking one or both of these questions: "In which ring is the origin, $(0, 0)$, located? What are the integer coordinates of one of the intersection points that is in the "bullseye" — the innermost ring?"

Look for points of confusion:

- **Ignoring the inner and outer rings for each color.** Reference the table and ensure students understand the terms *inner* and *outer*.
- **Thinking all points/hits must have two integer coordinates.** Ask, "Do you think an arrow could hit the target in between the grid lines? How would you determine those coordinates?"

Look for productive strategies:

- Using the scales of the axes to correctly identify and label points with accurate and precise integer coordinates.
- Recognizing that when an entire unit square on the grid lies within the same scoring ring, any non-integer coordinates representing values between those grid lines would work.
- Noticing that, for the five-spot target, there are many possible solutions where all of the ordered pairs have either the same or opposite x- and/or y-coordinates.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital archery target to input coordinates and watch their arrows fly, receiving real-time feedback on their accuracy and progress.

Accessibility: *Guide Processing and Visualization*

Provide access to the single-rings target and scoring system from the Activity 1 PDF. Sample responses: 1a. 8 points; 1b. 6 points; 2. $(7, 0)$; 3. 2, 2, 4, 4, and 12 points (based on double-ring sample coordinates)

Pairs | 20 min

Activity 1 Coordinated Archery (continued)

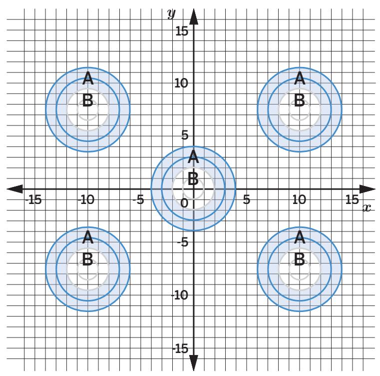
Students locate and name specific coordinates in order to hit different parts of an archery target on the coordinate plane.

Activity 1 Coordinated Archery (continued)

Part 2

A five-spot target like the one shown is used in some other archery competitions, and has the following scoring system:

- **4 points for any blue ring (A)**
- **5 points for any white ring (B)**



4. Choose either 16 or 20 points as your goal. You have four arrows to hit the target and score *exactly* the chosen point total. Each arrow must hit the target in a different quadrant. Write the coordinates of four possible hits that allow you to achieve your goal.

My Goal: 20 points or 16 points (circle one)

	Coordinates
Quadrant I	Answers may vary.
Quadrant II	Answers may vary.
Quadrant III	Answers may vary.
Quadrant IV	Answers may vary.

3 Connect

Display the archery targets from Part 1 first, and then from Part 2.

Have students share their responses to each of the problems for Part 1 first, and then for Part 2.

Ask, "How were your strategies similar for the one-spot and five-spot targets? How were they different?"

Independent | 10 min

Activity 2 A Coordinate Maze

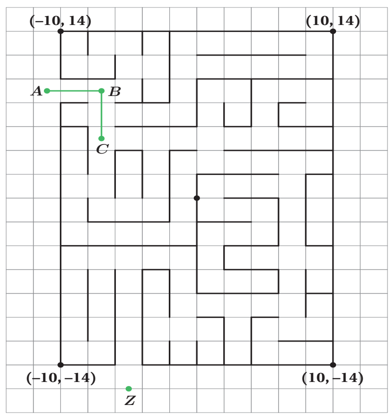
Students identify a route through a maze on a coordinate plane by strategically plotting points and labeling their coordinates.

Name: ______ Date: ______ Period: ______

Activity 2 A Coordinate Maze

Refer to the maze. The point in the center is located at $(0, 0)$, and the side of each grid square is 2 units long.

1. You enter the maze by traveling from point A to point B. You turn at point B and travel to point C. Continue drawing line segments to show your way through the maze, all the way to the exit to reach point Z. As you work, label each turning point with a letter and write its coordinates in the table.



Sample responses shown.

Point	Coordinates
A	$(-11, 9)$
B	$(-7, 9)$
C	$(-7, 5)$
D	$(1, 5)$
E	$(1, 3)$
F	$(7, 3)$
G	$(7, -9)$
H	$(-1, -9)$
I	$(-1, -5)$
J	$(-5, -5)$
Z	$(-5, -16)$

2. Choose any two turning points from your path that share the same line segment. What is the same about their coordinates? Explain why they share that feature.

Sample response: Points A and B share a line segment and they both have the same y-coordinate, because the segment represents moving horizontally in the coordinate plane. Segments that represent moving horizontally will share the same y-coordinate. Segments that represent moving vertically will share the same x-coordinate.

STOP

1 Launch

Read the directions together.

2 Monitor

Help students get started by having them trace the route from A to C, matching the points with the coordinates in the table.

Look for points of confusion:

- **Trying to list every point along the path.** Use one of their unnecessary points and ask, "As you follow the path from ___ to ___, does the path then turn to go in a different direction next?"
- **Misinterpreting the scale of the maze.** Have students place their pencil at $(0, 0)$ and move horizontally to the right edge of the maze (x-coordinate of 10), counting the grid lines. Ask, "What is the scale of the x-axis?" Have them repeat this for the y-axis.
- **Choosing random turning points for Problem 2.** Demonstrate what two points "sharing the same line segment" looks like, and have students physically identify two valid turning points.

Look for productive strategies:

- Plotting and labeling turning points accurately along an efficient route through the maze, refining moves as necessary.

3 Connect

Display an uncompleted version of the maze.

Have students share turning points to navigate through the maze, allowing multiple students to contribute one "next" move (and also share when they did it differently), until the class has reached the exit. Then have students share their responses to Problem 2.

Highlight that points on a horizontal line share the same x-coordinate and points on a vertical line share the same y-coordinate.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students use a pencil to draw their route to allow for any mistakes to be corrected. Have them first focus on drawing the route from beginning to end (marking turning points as they go) and then return later to label the points.

Accessibility: *Vary Demands to Optimize Challenge*

Consider making copies of the grid maze and re-labeling the points so that the side length of each square grid is 1 unit, instead of 2 units. This will allow students to access the activity goal without the added step of interpreting the scale.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their turning points and "next" moves, ask, "How did you determine the coordinates for each turning point?" As pairs of students discuss their strategies, their exchanges strengthen their use of mathematical language and thinking about the structure of the grid.

English Learners

Provide students time to formulate and rehearse a response with their partner before sharing with the class.

Whole Class | 5 min

Summary

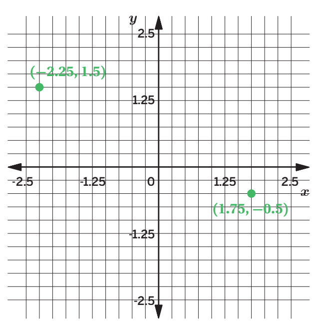
Review and synthesize how plotting positive and negative rational numbers on number lines — from earlier in the unit — is extended in this lesson to the coordinate plane.

Summary

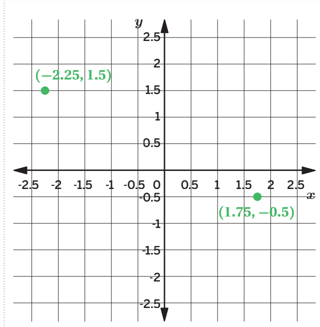
In today's lesson . . .

You worked with points that are plotted between the tick marks on a coordinate plane. The coordinate plane can be used to represent ordered pairs of numbers. When plotting or interpreting points on the coordinate plane, you need to pay attention to the scales of the axes — the intervals between the tick marks. A scale of 1 is often used, but sometimes different scales are more advantageous to show a set of really large or really small values. No matter the scale, every possible combination of values can be plotted, but some scales help you to more efficiently read and interpret coordinates.

For example, consider the coordinates of the points $(1.75, -0.5)$ and $(-2.25, 1.5)$.



- A scale of 0.25 is used on both axes.
- Both sets of coordinates are divisible by 0.25.
- Both points are plotted at intersection points on the grid.



- A scale of 0.5 is used on both axes.
- -2.25 and 1.75 are not multiples of 0.5, so neither point is plotted at an intersection point on the grid.
- 1.75 is plotted halfway between 1.5 and 2.
- -2.25 is plotted halfway between -2.5 and -2.

Reflect:

Synthesize

Ask one or both of the following questions:

- "How many points do you think there could be on a given coordinate plane?"
- "What types of points might be difficult to identify the coordinates of based on just looking at them? Why? What could make it easier to identify them?"

Have students share their thoughts in response to the question(s), and focus on their uses of relevant mathematical vocabulary.

Highlight that coordinates will not always be integers, but can consist of any combination of positive and negative numbers — whole numbers, integers, fractions, decimals, and rational numbers. Different scales on the axes can be used to more precisely plot and interpret non-integer coordinates.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does changing the intervals represented by the tick marks change how a coordinate plane looks and what it represents?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of naming coordinates by analyzing the placement of four ordered pairs on a coordinate plane.

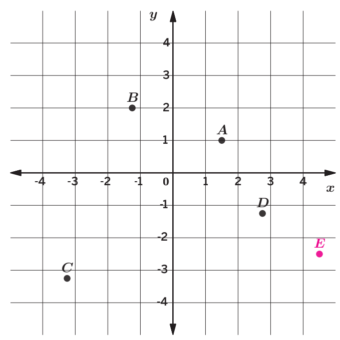
Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 7.14

Tyler plotted four points on the coordinate plane and recorded their coordinates as follows:

$A(1, 1.5)$ $B(-1.25, 2)$
$C(-3.25, 3.5)$ $D(2.75, 1.25)$



1. Some of the coordinates are incorrect. If the coordinates of the point are correct, write *correct*. For each incorrect ordered pair, write *incorrect*. Then write the correct ordered pair and explain what Tyler's mistake might have been.

   a. Point A: **Incorrect. The correct answer is (1.5, 1). Tyler might have confused the (x, y) order.**

   b. Point B: **Correct**

   c. Point C: **Incorrect. The correct answer is (−3.25, −3.25). Tyler might have overlooked that the point is *between* −3 and −3.5 on the y-axis.**

   d. Point D: **Incorrect. The correct answer is (2.75, −1.25). Tyler might have forgotten to include the negative sign on the y-coordinate.**

2. Plot a fifth point labeled E on the graph that has x- and y-coordinates that are both non-integers, and write its coordinates.
   Answers will vary, but the point should be plotted so that it is not on any horizontal or vertical grid line.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can label points with negative rational number coordinates on the coordinate plane.
1 2 3

b. I can plot points with negative rational number coordinates on the coordinate plane.
1 2 3

Lesson 14 Points on the Coordinate Plane

Success looks like . . .

- **Goal:** Plotting a point given its rational-number coordinates, or identifying the rational-number coordinates of a given point on the coordinate plane.
  - » Determining whether Tyler's coordinates are correct and writing the correct coordinates for the incorrect ordered pairs in Problem 1.
- **Language Goal:** Comparing and contrasting different scales for the axes of a coordinate plane. **(Speaking and Listening, Reading)**

Suggested next steps

If students have trouble explaining or identifying why some of the coordinates are incorrect, consider:

- Reviewing Activity 1 to help with thinking about rational-number values for coordinates.
- Asking:
  - » "What should the signs be for the coordinates of points in each quadrant?"
  - » "What are the scales on the axes?" 1
  - » "What is half of 1?" 0.5

If students have trouble plotting point E using non-integers on the grid, consider:

- Reviewing some of the turning points from Activity 2 that were not on the grid lines.
- Asking, "Do points exist between the grid lines? How would you determine the coordinates of points like these?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Knowing where students need to be by the end of this unit, how did the synthesis of ideas during the lesson summary influence that future goal?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

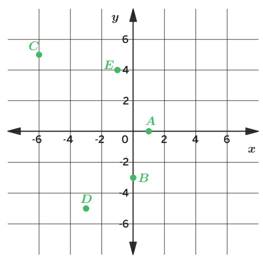
Practice

Independent

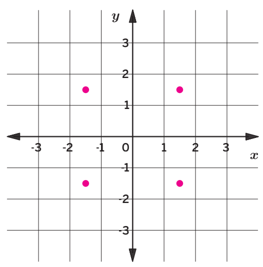
Name: ________ Date: ________ Period: ________

1. Write the coordinates of each point.

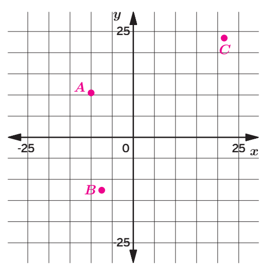
Point	Coordinates
A	$(1, 0)$
B	$(0, -3)$
C	$(-6, 5)$
D	$(-3, -5)$
E	$(-1, 4)$



2. Refer to the coordinate plane.
   a. Name four points with non-integer coordinates that would form a square with the origin at its center.
   Sample responses: $(1.5, 1.5)$, $(1.5, -1.5)$, $(-1.5, -1.5)$, $(-1.5, 1.5)$
   b. Plot these points on the coordinate plane to verify that they form a square.



3. Refer to the coordinate plane.
   a. What is the scale for this coordinate plane?
   5
   b. Plot and label points A, B and C on the coordinate plane, approximating as needed.
   $A(-10, 11)$
   $B(-7.5, -12.5)$
   $C(21, 24)$



Practice

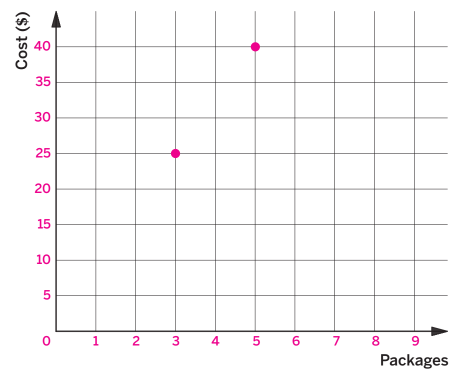
Name: ________ Date: ________ Period: ________

4. Which of the following changes would you represent using a negative number? Explain what a positive number would represent in that same context.
   A. a loss of 4 points Sample response: A gain of 4 points
   B. a gain of 50 yards
   C. a loss of \$10 Sample response: A gain of \$10
   D. an elevation above sea level

5. A corn field has an area of 28.6 acres. It requires about 15,000,000 gal of water. Approximately how many gallons of water does it require per acre?
   A. 5,000 gallons
   B. 50,000 gallons
   C. 500,000 gallons
   D. 5,000,000 gallons

6. Shawn went to the post office twice in the last week. The first time, Shawn sent 3 packages for a total of 25. The second time, Shawn sent 5 packages for a total of 40.

   Label the tick marks on the x- and y-axis, and then plot two points to represent the relationship between packages sent and money spent.
   Sample response:



Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 6	2
	5	Unit 5 Lesson 13	2
Formative	6	Unit 7 Lesson 15	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 7 | **LESSON 15**

Interpreting Points on the Coordinate Plane

Let's examine what points on the coordinate plane can tell us.

Focus

Goals

1. **Language Goal:** Choose and label appropriate scales for the axes of a coordinate plane, based on the coordinates to be plotted, and explain the impact of different scales. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Identify and interpret points on a graph to solve problems about situations involving temperature, elevation, or money. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Describe (using words and inequality symbols) and interpret the range of coordinates on a graph, including the meaning of negative values. **(Speaking and Listening, Reading and Writing)**

Rigor

- Students continue to build **conceptual understanding** of rational-number coordinates on four-quadrant coordinate planes by interpreting the meaning of a point in context.
- Students **apply** the representation of data on a four-quadrant coordinate plane to real-world scenarios involving temperature or account balances.

Coherence

Today

Students create and interpret graphs that represent relationships in the contexts of temperature, elevation, and money. Students attend to precision as they consider where to place the axes on a plane and how to scale them appropriately to accommodate coordinates that are large and small rational numbers. They then identify and interpret points, including those with negative x- and y-values.

Previously

In Lesson 12, students used the structure of number lines extended to negative values in order to construct a coordinate plane with four quadrants. They also recognized patterns in the signs of coordinates in each quadrant. In Lesson 13, students fluently labeled and plotted points in all four quadrants.

Coming Soon

In Lesson 16, students will explore the relationship between points with opposite coordinates, and they will develop strategies for determining the vertical or horizontal distance between two points.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- graph paper

Math Language Development

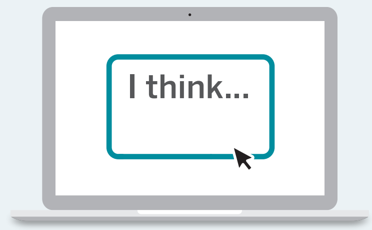
Review word

- *quadrant*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to interpret plotted points in the context of bank account balances, and those explanations are passed on to you.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might become frustrated as they try to organize the data in Activity 2. Have students discuss how to decrease their frustration levels as they begin to reason quantitatively and abstractly by pausing as needed to reconsider their choices, making sure that they are sensical for the given situation.

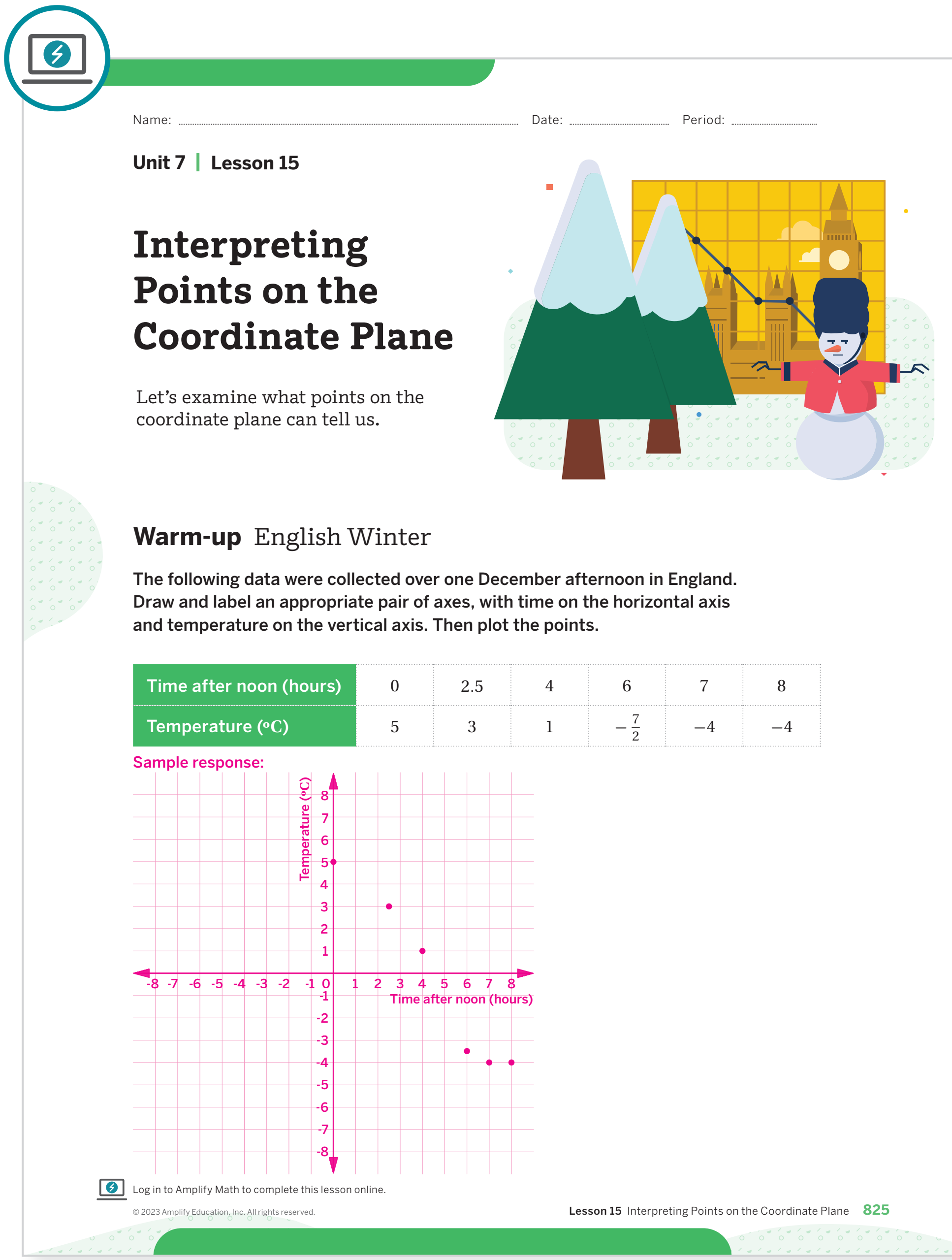
Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be done as a whole class with a coordinate plane displayed for all to see and complete together.

Warm-up English Winter

Students draw and scale their own axes, and then plot coordinates to represent a relationship between temperature and time.


Name: Date: Period:

Unit 7 | Lesson 15

Interpreting Points on the Coordinate Plane

Let's examine what points on the coordinate plane can tell us.

Warm-up English Winter

The following data were collected over one December afternoon in England. Draw and label an appropriate pair of axes, with time on the horizontal axis and temperature on the vertical axis. Then plot the points.

Time after noon (hours)	0	2.5	4	6	7	8
Temperature (°C)	5	3	1	$-\frac{7}{2}$	-4	-4

Sample response:

Log in to Amplify Math to complete this lesson online.

1 Launch

Read the scenario and directions together.

2 Monitor

Help students get started by having them consider the maximum and minimum values to create appropriate scales on each axis.

Look for points of confusion:

- **Creating inconsistent scales.** Remind students that, similar to a number line, each interval must represent the same distance.
- **Plotting rational numbers incorrectly.** Suggest students convert $-\frac{7}{2}$ to -3.5. For integer-only scales, ask, "Between which two integers is 2.5? -3.5? How can you show these on the graph?"

Look for productive strategies:

- Identifying the maximum and minimum values to define an appropriate scale and placement of the axes and origin.

3 Connect

Display students' graphs that use different scales.

Ask:

- "Which intervals best represent the data?"
- "How would a scale that is too big (e.g., 10) or too small (e.g., 0.25) impact your ability to create and interpret the graph?"
- "How did you plot rational numbers that were not integers?"
- "What do the points $(0, 5)$ and $(7, -4)$ mean in context? What would the point $(-2, -4)$ mean in context?"

Highlight that graphs help to visualize relationships between quantities, such as time and temperature. When creating a graph, consider the *range* of the data, or the difference between the greatest and least values, for each axis. This helps ensure the placement and scale of the axes are appropriate.

Differentiated Support

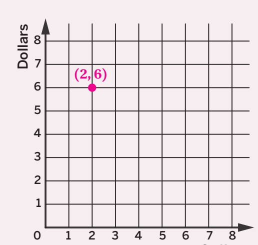
Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Consider providing students with coordinate planes in which the axes are drawn and labeled, but the numerical scales are missing. Have students determine an appropriate numerical scale for each axis and plot the points. Suggest they focus on the horizontal axis first by thinking of it as a horizontal number line.

Power-up

To power up students' ability to plot and interpret points representing a situation on a coordinate plane have students complete:

Plot the point that represents the cost of $6 for 2 coffees.



Use: Before the Warm-up

Informed by: Performance on Lesson 14, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7.

Pairs | 10 min

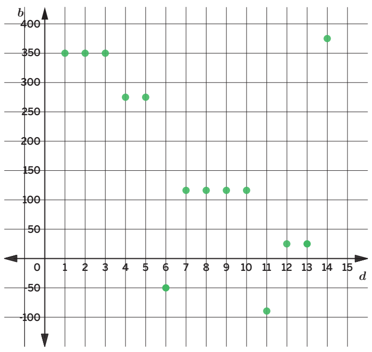
Activity 1 Interpreting Account Balances

Students interpret points in the coordinate plane that represent a bank account balance over time.

Amps Featured Activity | See Student Thinking

Activity 1 Interpreting Account Balances

The graph shows the balance in a bank account over a period of 14 days. The axis labeled b represents the account balance in dollars, and the axis labeled d represents the day.



1. Estimate the greatest account balance. On which day did it occur?
   Sample response: $375 on Day 14

2. Estimate the least account balance. On which day did it occur?
   Sample response: −$90 on Day 11

3. What does the point $(6, -50)$ tell you about the account balance?
   On Day 6, the account balance was −$50.

4. How can you interpret $|-50|$ within this context?
   Sample response: The account has a debt of $50.

5. Write two inequalities to describe the account balance b in dollars over the entire 14-day period.
   Sample responses: $b > -\$100$ and $b < \$400$, or $b \geq -\$90$ and $b \leq \$375$

826 Unit 7 Rational Numbers

1 Launch

Activate students' background knowledge by asking them what they know about bank accounts. Explain that an account balance is the amount of money in the account. A positive balance means there is money in the account. A negative balance means you have borrowed money from the bank, and you are in debt. For example, if you borrow $30 from the bank, you are $30 in debt and have an account balance of $30.

2 Monitor

Help students get started by having them read a few values on each axis using the labels (i.e., on the x-axis, read "Day 1, Day 2, Day 3," etc.; on the y-axis, read "balance of $50, $100," etc.).

Look for points of confusion:

- **Reversing the meaning of the x- and y-coordinates.** Ask, "How are the axes labeled? What do the variables represent?"
- **Not connecting negative values of y to money borrowed.** Ask, "When would you have a negative bank account balance?"

Look for productive strategies:

- Using the axes' labels to interpret ordered pairs as "On Day _____, the account balance was y."

3 Connect

Have students share how they interpreted the contextual meaning of the x- and y-coordinates, and how their inequalities represent the range of data on the graph.

Ask, "In this context, what do negative y-coordinates represent? Would negative x-coordinates make sense? Why or why not?"

Highlight that the axes labels can be used to interpret ordered pairs within a context. Also, different scales can be used for the x- and y-axes, as long as the intervals on each axis are consistent.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students add the labels for each axis to help them complete the activity. For example, suggest they label the horizontal axis "Day" and the vertical axis "Account balance ($)."

Accessibility: *Clarify Vocabulary and Symbols, Activate Prior Knowledge*

Preview Problem 4 before students begin and remind them they learned about this notation in prior lessons in this unit. Ask them what this notation means and have them write a note next to this problem to support them when they get to it during the activity. For example, have them write the term *absolute value* next to Problem 4.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introduction and graph without revealing any of the problems. Have students work with their partner to write 2–3 questions they have about the graph and this scenario. Sample questions could be:

- "What happened on Day 13?"
- "Why are some points below the horizontal axis?"
- "What does it mean when two or more points are in a row?"

English Learners

Display 1 or 2 sample questions and allow students to say their questions aloud as opposed to writing them, or vice versa.

Pairs | 15 min

Activity 2 Elevation and Temperature on Mauna Kea

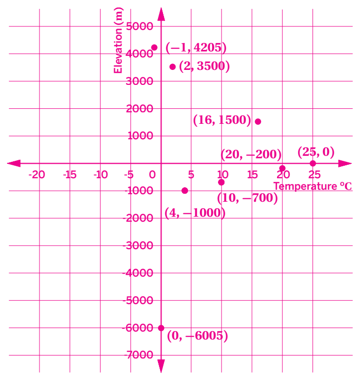
Students create a graph and write inequalities to represent and interpret the relationship between elevation and temperature on Mauna Kea, Hawai'i.

Name: ________ Date: ________ Period: ________

Activity 2 Elevation and Temperature on Mauna Kea

Mauna Kea, the tallest mountain in the world, has a total height of 10,210 m. Its height extends from its base that is 6,005 m below sea level to its peak that is 4,205 m above sea level. The following data were collected one afternoon at different elevations on Mauna Kea.

Temperature (°C)	Elevation (m)
-1	4,205
2	3,500
16	1,500
25	0
20	-200
10	-700
4	$-1{,}000$
0	$-6{,}005$



1. On a piece of graph paper, draw and label an appropriate pair of axes, with temperature on the x-axis and the elevation on the y-axis. Then plot the data.

2. What does a point in quadrant III represent in this context?
   A point in quadrant III represents an elevation below sea level and a temperature below freezing.

3. Write two inequalities to describe all the possible elevations e from the base of Mauna Kea to its peak.
   $e \geq -6{,}005$ and $e \leq 4{,}205$

4. Write two inequalities to describe the temperatures t recorded on Mauna Kea during that afternoon.
   $t \leq 25$ and $t \geq -1$

Critique and Correct: Your teacher will display two incorrect inequalities. Work with your partner to critique the inequalities, correct them, and clarify why and how you corrected them.

STOP

Lesson 15 Interpreting Points on the Coordinate Plane 827

1 Launch

Provide each student with a piece of graph paper.

2 Monitor

Help students get started by having them identify the maximum and minimum x- and y-values, and asking, "Does it make sense to have the same scale on each axis?"

Look for points of confusion:

- **Not using a reasonable scale for the y-axis.** Ask, "What is a reasonable scale if all of the values of y are in the hundreds or thousands?" Have students count the spaces needed for different scales to determine which best fits the paper and data.
- **Writing strict inequalities.** Ask, "If the point ___ is in the table, should it be included or excluded as a solution to the inequality?"

Look for productive strategies:

- Choosing an appropriate interval for the data on each axis, as well as plotting points with temperature on the x-axis, and elevation on the y-axis.

3 Connect

Display student graphs with different scales.

Ask:

- "Which graphs best represent the data?"
- "How does the scale affect how you plot and interpret the data?"
- "Does the origin always have to be in the center of the graph?"
- "What would a point in each quadrant represent in this context?"
- "Why are two inequalities needed to represent all possible elevations? All possible temperatures?"

Highlight that inequalities can be written to represent all possible values for x- and y-coordinates by considering the maximum and minimum values shown in the data set or on the graph.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with graph paper they can use for Problem 1. Consider providing them with a coordinate plane that has the horizontal and vertical axes already drawn and labeled as "Elevation (m)" and "Temperature (°C)" and ask students to decide on appropriate scales for the axes.

Extension: *Math Enrichment*

Ask students to write a few sentences describing how the temperature compares to the elevation. Sample response: The closer the elevation is to 0 m, the warmer the temperature. The farther from 0 m (whether positive or negative elevations), the colder the temperature.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display two incorrect inequalities for the temperature of Mauna Kea. Consider displaying an error that confuses the horizontal and vertical axes labels, such as $t \geq -6{,}005$ and $t \leq 4{,}205$ for Problem 3 and $e \leq 25$ and $e \geq -1$ for Problem 4. Ask:

- **Critique:** "Why are these inequalities incorrect?"
- **Correct:** "Write two corrected inequalities for each problem."
- **Clarify:** "How do you know that these inequalities are now true?"

English Learners

If students have not already done so, be sure they label their horizontal and vertical axes with the variables e and t, along with the text labels.

Whole Class | 5 min

Summary

Review and synthesize how to clearly represent and interpret the relationship between two quantities, using a graph with appropriate scales on the axes.

Summary

In today's lesson . . .

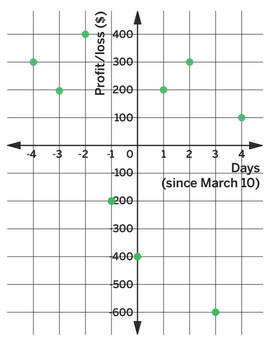
You interpreted points plotted on the coordinate plane. The coordinate plane can be used to show relationships between two quantities that are not just horizontal and vertical locations. Each axis can represent a different quantity, and in order to plot all the values, you can change the scales along the axes of a coordinate plane by selecting appropriate:

- maximum and minimum coordinates, which may need to be different on each axis, and
- intervals between tick marks or grid lines — the scale of the axis — which also may need to be different on each axis.

The origin is always the point on the coordinate plane where the x- and y-axes cross, but it does not always have to be shown as the center of a graph.

Points on a coordinate plane can also be used to represent and interpret information about a given scenario that involves two related quantities.

- For example, this graph shows a company's daily profits or losses recorded on different days around March 10 of one year.



Knowing what quantities the x- and y-axes each represent, you can interpret both the meaning of individual coordinates as well as the meaning of the ordered pair as a whole.

- For example, on the graph shown, an x-coordinate of -4 represents the day 4 days *before* March 10 (or March 6), and a y-coordinate of 300 represents a profit of \$300. Together, the point $(-4, 300)$ represents that on March 6, the company made a profit of \$300.

Reflect:

828 Unit 7 Rational Numbers

Synthesize

Ask:

- "When creating a graph to represent the relationship between two real-world quantities, why is it important to consider the placement of the axes and the scale used on each axis?"
- "When might you want to move the origin from the 'center' of a grid?"
- "How do axes labels help you interpret data?"
- "How does the context of the quantities being graphed impact how you interpret a coordinate of -50?"

Highlight that graphs can be used to visually display information about the relationship between two quantities. In order to make the information clear, students must ensure that the origin's location and the scales used on each axis are appropriate for the range of data being plotted.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do the labels on the x- and y-axes help you interpret data?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by creating and interpreting a graph that shows a relationship between temperature and time.

Printable

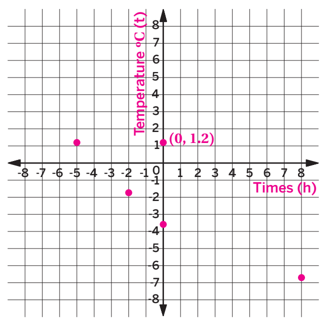
Name: ______________________ Date: __________ Period: __________

Exit Ticket 7.15

The temperature in Berlin, Germany, was recorded at various times one day. The times, relative to midnight, and corresponding temperatures are shown in the table.

Time (hours relative to midnight)	Temperature (°C)
-5	1.2
-2	-1.6
0	-3.5
8	-6.7

1. Plot the points from the table. Be sure to label each of the axes.
2. In Munich, Germany, the temperature at midnight on that same day was 1.2°C.
   a. Plot and label this point with its coordinates.
   b. Which city was warmer at midnight that day, Berlin or Munich?
   Munich was warmer.
3. If the point $(3, -2.5)$ was also plotted on the graph, what would it represent?
   Sample response: 3 hours after midnight, the temperature was 2.5 degrees (Celsius) below zero.



Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can construct a four-quadrant coordinate plane with appropriate axes and scales to represent a given set of points.
1 2 3

b. I can plot points in a four-quadrant coordinate plane to represent situations and solve problems.
1 2 3

c. I can explain what points on the coordinate plane represent in a situation, including account balances in a money context.
1 2 3

Lesson 15 Interpreting Points on the Coordinate Plane

Success looks like . . .

- **Language Goal:** Choosing and labeling appropriate scales for the axes of a coordinate plane, based on the coordinates to be plotted, and explaining the impact of different scales. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Identifying and interpreting points on a graph to answer questions about situations involving temperature, elevation, or money. **(Speaking and Listening, Reading and Writing)**
  - » Interpreting the point $(3, -2.5)$ from the graph in terms of temperature.
- **Language Goal:** Describing (using words and inequality symbols) and interpreting the range of coordinates on a graph, including the meaning of negative values. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students plot time on the y-axis and temperature on the x-axis, consider:

- Reviewing the Warm-up, and asking, "On which axis did you plot temperature? Time?"
- Asking, "What do you notice about the maximum and minimum on each axis? For which data set — time or temperature — do those work best?"

If students do not recognize midnight as an x-value of 0 in Problem 2, consider:

- Reviewing the Warm-up and asking, "What did 0 mean in this context?"
- Referring to the table and asking, "What would a value of -2 represent in the time column? 2? 0?"

If students misinterpret the point $(3, -2.5)$ in Problem 3, or do not interpret it in context, consider:

- Ensuring their axes are appropriately labeled.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students created and interpreted graphs that represent the contexts of temperature, elevation, and money. How did that build on the earlier work students did with rational numbers, absolute value, and determining solutions to inequalities with constraints?
- In what ways have students gotten better at interpreting points or values within the context of the problem? What might you change for the next time you teach this lesson?

Practice

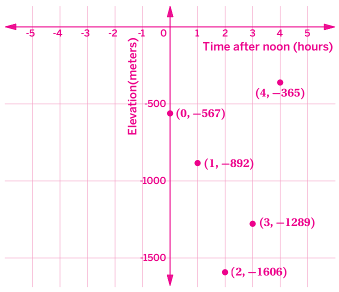
Independent

Practice

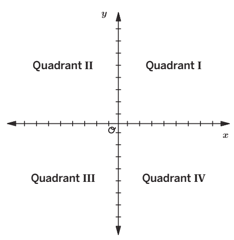
Name: ______ Date: ______ Period: ______

1. The elevation of a submarine at different times is shown in the table. Draw and label an appropriate pair of axes. Then plot the points.

Time after noon (hours)	Elevation (m)
0	−567
1	−892
2	−1,606
3	−1,289
4	−365



2. The x-axis represents the number of hours from noon (12 p.m.), and the y-axis represents the temperature in degrees Celsius.



a. At 9 a.m., it was below freezing. In which quadrant would this point be plotted?
Quadrant III

b. At 11 a.m., it was 10°C. In which quadrant would this point be plotted?
Quadrant II

c. Choose another time and temperature. Then determine the quadrant in which your point would be plotted.
Sample response: At 1 p.m., it was 10°C. This point would be plotted in quadrant I.

d. What does the point $(0, 0)$ represent in this context?
A temperature of 0°C at 12 p.m. (noon)

Practice

Name: ______ Date: ______ Period: ______

3. Diego will plot these points: $(-50, 0)$, $(150, 100)$, $(200, -100)$, $(350, 50)$, $(-250, 0)$. What interval could he use for the tick marks on each axis? Explain your thinking.
Sample response: He could use an interval of 50 on both axes, because every x- and y-coordinate is a multiple of 50.

4. The inequalities $h > 42$ and $h < 60$ represent the height requirements for an amusement park ride, where h represents a person's height in inches. Write a sentence or draw a sign that describes these rules as clearly as possible.
Sample response: A person needs to be taller than 42 in. but shorter than 60 in. to go on the ride.

5. Solve each equation.

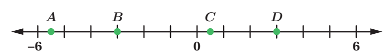
a. $3a = 12$
$3a \div 3 = 12 \div 3$
$a = 4$

b. $b + 3.3 = 8.9$
$b + 3.3 - 3.3 = 8.9 - 3.3$
$b = 5.6$

c. $1 = \frac{1}{4}c$
$1 \div \frac{1}{4} = \frac{1}{4}c \div \frac{1}{4}$
$c = 4$

d. $5\frac{1}{4} = d + \frac{1}{4}$
$5\frac{1}{4} - \frac{1}{4} = d + \frac{1}{4} - \frac{1}{4}$
$d = 5$

6. Refer to the number line.



a. For each point, determine the distance from zero.
A: 5.5 units, B: 3 units, C: $\frac{1}{2}$ units, D: 3 units

b. What is the distance from B to D?
6 units

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Warm-up, Activity 2	2
	3	Warm-up, Activity 2	2
Spiral	4	Unit 7 Lesson 11	2
	5	Unit 6 Lesson 7	2
Formative	6	Unit 7 Lesson 16	1

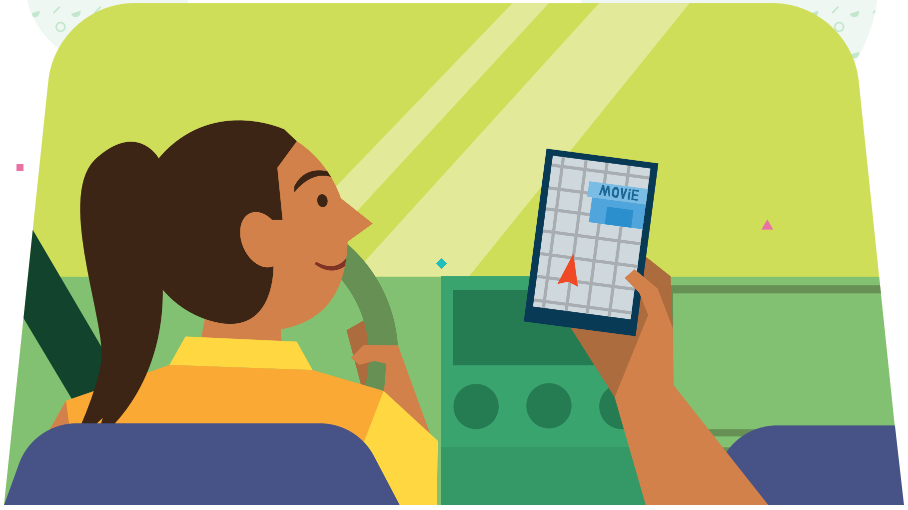
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Distances on the Coordinate Plane

Let's explore distance on the coordinate plane.



Focus

Goals

1. **Language Goal:** Compare and contrast the coordinates for points in different locations on the coordinate plane. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Generalize about the coordinates of points that are reflected across the x- or y-axis. **(Speaking and Listening)**
3. Determine the vertical or horizontal distance between two points on the coordinate plane that share the same x- or y-coordinate.

Coherence

Today

Students explore ways to find vertical and horizontal distances on the coordinate plane. They apply repeated reasoning to determine the relationship between points and opposite coordinates. Then, they develop strategies for finding the distance between any two points located vertically or horizontally across an axis, including with coordinates that are not integers. Using absolute value to relate distance from the x- or y-axis is a strategy presented to students.

Previously

In Lessons 13–15, students constructed and explored coordinate planes containing four quadrants, making connections between number lines and the x- and y-axes. They interpreted points on the coordinate plane where each axis represented a particular quantity in a real-world context.

Coming Soon

In Lessons 17–19, students will continue working with distances and polygons on the coordinate plane, ultimately applying their understanding to puzzles and drawings. They will revisit distances on the coordinate plane when operations with rational numbers are introduced in Grade 7.

Rigor

- Students build **conceptual understanding** of vertical and horizontal distances on the four-quadrant coordinate plane by connecting coordinates to absolute value.
- Students **apply** their understanding of absolute value to determine distances on the coordinate plane.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per pair
- Activity 2 PDF, one per pair
- colored pencils

Math Language Development

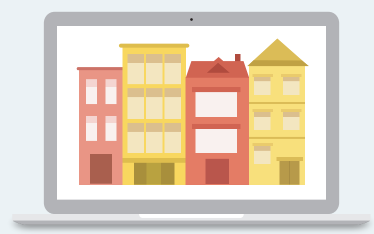
Review words

- *quadrant(s)*
- *coordinate plane*
- *ordered pairs*

Amps | Featured Activity

Activity 2
Interactive Map

Students simulate Priya and Diego walking around their town, helping them connect the coordinate plane to an interactive map.



Amps POWERED BY desmos

Building Math Identity and Community
Connecting to Mathematical Practices

Students might choose not to implement the self-discipline required for generalizing patterns in Activity 1. Ask students to set goals for the activity and include what they will need to do both mentally and behaviorally to achieve them. If a student seems lost in the process, refocus them on the goal of determining patterns.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be done as a whole class.

Pairs | 10 min

Warm-up Reflections on the Coordinate Plane

Students develop strategies to find the distance from a point to an axis, and relate reflections across the axes to changes in the signs of coordinates of points.

Name: ______ Date: ______ Period: ______

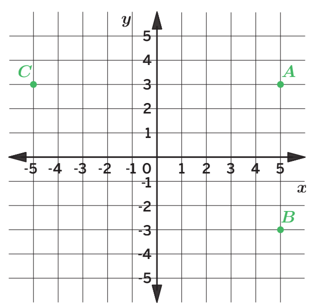
Unit 7 | Lesson 16

Distances on the Coordinate Plane

Let's explore distance on the coordinate plane.

Warm-up Reflections on the Coordinate Plane

Refer to the coordinate plane.



1. Label points A, B, and C with their coordinates.
   $A(5, 3)$, $B(5, -3)$, $C(-5, 3)$
2. Which point would line up with point A if you folded the graph along the y-axis? How far away are each of these points from the y-axis?
   point C, 5 units
3. Which other point would line up with point A if you folded the graph along the x-axis? How far away are each of these points from the x-axis?
   point B, 3 units
4. What should the coordinates be for a point D in quadrant III, that would complete this set of "similar" points? Explain how you determined those coordinates.
   $(-5, -3)$; Sample response: I used the coordinates of point C and kept the x-coordinate the same, but determined the opposite of the y-coordinate (*or* I used the coordinates of point B and kept the y-coordinate the same, but determined the opposite of the x-coordinate).

Log in to Amplify Math to complete this lesson online.

Lesson 16 Distances on the Coordinate Plane 831

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What do you notice about the three points plotted here?"

Look for points of confusion:

- **Not understanding how to imagine the folding action.** Have students isolate the axis they are working with and relate it to a horizontal or vertical number line. This can be related to the work done with opposites in Lesson 5.
- **Plotting a point at (–3, 5).** Ask, "Can you use an example, like points A and B to help you? Which coordinate of point A changes in point B? Could you use point C to do something similar for point D?"

Look for productive strategies:

- Using absolute value to describe the distance to the x-axis or y-axis.
- Noticing a polygon is formed once all four points are plotted and connecting equal distances to the properties of the rectangle.

3 Connect

Display the coordinate grid with points A, B, and C. Record students' thinking about absolute value on the plane using different colored lines to connect each point to the axes.

Have pairs of students share their responses and strategies for finding the distance from a point to an axis.

Highlight that, for every point on the coordinate plane, there is a point in each quadrant that is the same distance from the x-axis and the same distance from the y-axis.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students respond to Problem 4, have them meet with 1–2 other pairs of students to give and receive feedback on their responses. Display these prompts that reviewers can use to press for details as they discuss their responses.

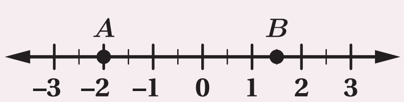
- "How did you use the horizontal/vertical distance to help you?"
- "How can you verify that your coordinates of point D are correct?"

English Learners

Suggest students annotate the graph with the distance from each point to each axis to show how the distances are the same.

Power-up

To power up students ability to determine the distance between two values on a number line, have students complete:



1. What is the distance from A to 0? 2 units
2. What is the distance from 0 to B? 1.5 units
3. What is the distance from A to B? 3.5 units

Use: Before the Warm-up.

Informed by: Performance on Lesson 15, Practice Problem 6b.

Pairs | 10 min

Activity 1 Crossing an Axis

Students plot and label points that share a coordinate and an opposite to generalize patterns in the coordinates for each quadrant.

Activity 1 Crossing an Axis

You will be given a blank coordinate plane.

1. Point E is 2 units from the x-axis and 6 units from the y-axis. Plot point E on your graph and write its coordinates here.
   Sample responses: (6, 2), (6, −2), (−6, −2), *or* (−6, 2)

2. Point F has the same coordinates as point E, except its y-coordinate has the opposite sign.
   a. Plot point F on your graph and write its coordinates here.
   Sample response: (6, −2)
   b. In which quadrant is point F? To travel from point E to point F, which axis do you cross?
   Sample response: quadrant IV; x-axis
   c. How many units apart are points E and F?
   4 units

3. Point G has the same coordinates as point E, except its x-coordinate has the opposite sign.
   a. Plot point G on your graph and write its coordinates here.
   Sample response: (−6, 2)
   b. In which quadrant is point G? To travel from point E to point G, which axis do you cross?
   Sample response: quadrant II; y-axis
   c. How many units apart are points E and G?
   12 units

4. Think about the signs of the coordinates of points in different quadrants of the coordinate plane.
   a. When two points are directly across the x-axis from each other, what is true about the y-coordinates of these points?
   The y-coordinates are opposites.
   b. When two points are directly across the y-axis from each other, what is true about the y-coordinates of these points?
   The x-coordinates are opposites.

Reflect: How well do you understand the material? How did repetition help your self-confidence?

832 Unit 7 Rational Numbers

1 Launch

Distribute the Activity 1 PDF (blank coordinate plane) to each pair of students.

2 Monitor

Help students get started by activating their prior knowledge. Have them plot a point in the first quadrant and ask, "In which direction would you move away from the x-axis?"

Look for points of confusion:

- **Plotting a combination of (2, 6) instead of (6, 2).** Ask, "If you move away from the x-axis, in which direction(s) should you go?"
- **Not changing the sign of the coordinate specified in the problem.** Ask, "Which is the x-coordinate in the ordered pair? The y-coordinate?"
- **Not knowing the quadrants' names.** Refer back to Lesson 13 and label the quadrants.

Look for productive strategies:

- Using absolute value to determine distances across the axes.

3 Connect

Display the blank coordinate plane.

Have pairs of students share their responses and their thinking, focusing on how they plotted the first point based on the information given in Problem 1, and how they determined each point's coordinates after that.

Ask, "What strategy did you use to find the total distance between any of the two points?"

Highlight that, when a point crosses over the y-axis, the x-coordinate changes sign, and when a point crosses over the x-axis, the y-coordinate changes sign.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display a table, such as the one shown and have students complete the table using the points they plotted in the activity. Suggest they use colored pencils to color code the coordinates that are the same in one color.

Point E	Point F, crossing the x-axis	Point G, crossing the y-axis
(6, 2)	(6, −2)	(−6, 2)

Math Language Development

MLR7: Compare and Connect

During the Connect, have partners compare traveling across the x-axis and traveling across the y-axis. Ask:

- "How does traveling across the x-axis, in either direction, change the coordinates of a point? Traveling across the y-axis?"
- "Without plotting the points (3, −2) and (3, 2), what must be true about them?" Sample response: They are directly across the x-axis from each other.
- "Name two ordered pairs that are directly across the y-axis from each other." Sample response: (5, 1) and (−5, 1).

English Learners

Provide students time to formulate and rehearse a response with their partner before sharing with the class.

Pairs | 15 min

Activity 2 Determining Distances on a Map

Students apply strategies for finding the distance between points, including those with non-integer coordinates, on a coordinate plane representing a map of a town.

Amps Featured Activity Interactive Map

Name: ______ Date: ______ Period: ______

Activity 2 Determining Distances on a Map

You will be given a map of a town represented on a coordinate plane. Each unit represents one block.

Place	Coordinates
Priya's house	$(-1.5, 2)$
Supermarket	$(3.5, 2)$
Post Office	$(3.5, 0)$
School	$(3.5, -3)$
Library	$(3.5, -4.5)$
Town Hall	$(0.5, -4.5)$
Diego's house	$(-1.5, -4.5)$

1. Label each place in town with its coordinates.
2. Determine the number of blocks Priya and Diego walk as they walk around the town.
   a. Priya walks from her house to the supermarket. **5 blocks**
   b. Priya then walks from the supermarket to school. **5 blocks**
   c. Diego walks from his house to the library. **5 blocks**
   d. Diego then walks to the supermarket. **6.5 blocks**
   e. How far did Priya walk all together? Diego? **Priya: 10 blocks; Diego: 11.5 blocks**
3. If Priya stops at the post office after the supermarket, how could you determine the distance between the post office and the supermarket *without* counting? **Hint:** Study the coordinates of the supermarket and the post office.
   I subtracted the y-coordinates, $2 - 0$ and did nothing with the x-coordinates because they are the same.
4. The movie theater is located outside of town at $(32, 2)$. What is the distance between Priya's house and the movie theater? Explain your thinking.
   33.5 units; The movie theater is 32 units from the x-axis. Priya's house is 1.5 units from the x-axis. Add these distances.
5. Which place in town is located 2 blocks from Town Hall?
   Diego's house
6. Noah's house is located in quadrant III. One coordinate of this point is the opposite of the coordinate of the point representing the school.
   a. What are the coordinates of Noah's house? Plot and label Noah's house on the map.
   Sample response: $(-3.5, -3)$
   b. How many blocks does Noah walk to school?
   7 blocks

STOP

Lesson 16 Distances on the Coordinate Plane 833

1 Launch

Activate students' background knowledge by asking, "How can you determine distance between two locations on a map?" Distribute the Activity 2 PDF.

2 Monitor

Help students get started by having them start with the post office. Ask, "Where is it along Rock Road? Main Street? How would this help you find the coordinates of the supermarket?"

Look for points of confusion:

- **Not counting the halves.** Ask, "What could the coordinates be for a point that is between 3 and 4?"
- **Finding distance only by counting units.** Say, "If the x-coordinates are the same, you can find the distance between the two points as the difference between the y-coordinates."

Look for productive strategies:

- Noticing the relationship between coordinates and distances.
- Using subtraction to find distances with coordinates.
- Using absolute value to find distances.

3 Connect

Display the map of the town.

Have pairs of students share their responses, using the map to support their thinking and focusing on different strategies for finding distance.

Ask, "How did you determine the number of blocks Noah walks to school?"

Highlight the different strategies for finding distance: totaling absolute values, subtracting coordinates (in specific cases), and counting units.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can explore an interactive map to simulate Priya and Diego walking around their town.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1–4.

Extension: *Math Enrichment*

Ask students to determine the distance between each of the following pairs of points *without* plotting them on a coordinate plane.

$(-3, 4)$ and $(5, 4)$ 8 units

$(2.5, 6)$ and $(2.5, -7)$ 13 units

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the Activity 2 PDF that shows the map of the town represented on a coordinate plane along with the introductory text of the activity. Ask students to work with their partner to write 2–3 mathematical questions they have about the grid or the scenario. Invite pairs to share their questions with the class. This routine will help students begin the conversation of horizontal and vertical distance.

English Learners

Consider displaying images of a house, supermarket, post office, town hall, and school to help students visualize these terms on the map.

Whole Class | 5 min

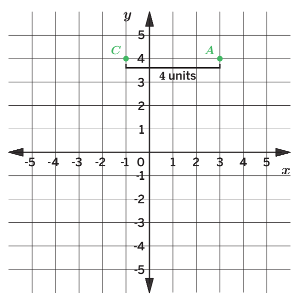
Summary

Review and synthesize strategies for plotting and determining the distance between two points on a coordinate plane.

Summary

In today's lesson . . .

You discovered that for any two points that lie on the same horizontal or vertical line, you can determine the distance between them. Specifically you explored the following strategies for determining the distance between two points:

Strategy	Example
Count the units between them.	
If they are in the same quadrant, subtract the coordinates that are different.	For (6, −2) and (3, −2), the distance between them is: $6 - 3 = 3$ 3 units
If they are in different quadrants, use the absolute value to determine the distance each point is from the axes between them.	For (−6, −2) and (12, −2), first determine the distance that −6 and 12 are from the y-axis. $\lvert(-6)\rvert = 6$ $\lvert 12\rvert = 12$ The distance is the sum of their respective distances. $6 + 12 = 18$ 18 units

Reflect:

834 Unit 7 Rational Numbers

Synthesize

Display a copy of the map from Activity 2, to be used when asking students the following questions.

Ask:

- "How does knowing the coordinates of one point help you find the reflection of that point across one of the axes?" Sample response: The coordinates of the reflected point will differ by their signs, depending upon which axis they are reflected across.
- "What difference did it make when points were plotted between grid lines?" Sample response: It did not make a difference, except I have to be careful to calculate the correct distance from the point to one of the axes.
- "How does knowing the distances from each of two points (that are reflections across one axis) to that axis help you find the distance between the two points?" Sample response: If the two points are reflections across one axis and you know the distance each point is to that axis, then the distance between the two points is the sum of the individual distances.

Have students share a strategy for finding the distance between two points, such as two locations on the map.

Highlight that counting units may not always be the most efficient strategy for finding the distance between two points that are farther away, and other strategies can help in those scenarios.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can two objects move the same distance, but end up in different places?"

Independent | 5 min

Exit Ticket

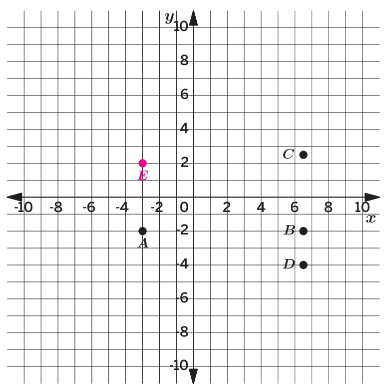
Students demonstrate their understanding by finding the distances between points and reflections of points on the coordinate plane.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.16

Points A, B, C, and D have been plotted and labeled on the coordinate plane.



1. What is the distance between points A and B?
   9.5 units
2. What is the distance between points C and D?
   6.5 units
3. Plot the point $(-3, 2)$. Label it E.
4. Which point, located in a different quadrant, could be the result of reflecting point E across an axis?
   Point A

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine the coordinates of points that are reflected across the x-axis or the y-axis.
1 2 3

b. I can find distances from a point to the x-axis and to the y-axis.
1 2 3

c. I can calculate horizontal or vertical distances between two points on the coordinate plane.
1 2 3

Lesson 16 Distances on the Coordinate Plane

Success looks like . . .

- **Language Goal:** Comparing and contrasting the coordinates for points in different locations on the coordinate plane. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Generalizing about the coordinates of points that are reflected across the x- or y-axis. **(Speaking and Listening)**
- **Goal:** Determining the vertical or horizontal distance between two points on the coordinate plane that share the same x- or y-coordinate.
  - » Determining the distance between points A and B in Problem 1 and between points C and D in Problem 2.

Suggested next steps

If students cannot determine the distance between the points in Problems 1 and 2, consider:

- Having them use colored pencils to connect the points to an axis, note the distances, and then add the distances.

If students cannot identify the reflected point as point E, consider:

- Referring back to the Warm-up and ask, "What helped you determine point D?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

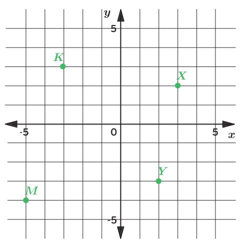
- What did student responses to Activity 1, Problem 4 reveal about your students as learners? How are you helping students become aware of how they are progressing in their ability to recognize structure and make generalizations?
- Which students' ideas were you able to highlight during the class discussion of Activity 2? What might you change for the next time you teach this lesson to ensure even more voices are heard?

Practice

Independent

Name: Date: Period:

1. Points K, M, X and Y are plotted and labeled on the coordinate plane.

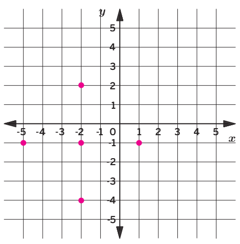


   a. Write the coordinates of each point.
   $K(-3, 3)$, $M(-5, -4)$, $X(3, 2)$, $Y(2, -3)$

   b. Plot a point that is located a distance of 3 units from point K. Label it P. What are its coordinates?
   Sample responses: $(0, 3)$, $(-3, 0)$, $(-6, 3)$, $(-3, 6)$

   c. Plot a point that is located a distance of 2 units from point M. Label it W. What are its coordinates?
   Sample responses: $(-5, -2)$, $(-3, -4)$, $(-5, -6)$, $(-7, -4)$

2. Plot and label four points on the coordinate plane that are each located a distance of 3 units from point P at $(-2, -1)$. Write the coordinates of each of your four points.
$(1, -1)$, $(-2, -4)$, $(-5, -1)$, $(-2, 2)$



3. Each pair of points is connected to form a line segment in the coordinate plane. What is the length of each line segment?

   a. $A(3, 5)$ and $B(3, 6)$
   1 unit

   b. $C(-2, -3)$ and $D(-2, -6)$
   3 units

   c. $E(-3, 1)$ and $F(-3, -1)$
   2 units

Practice

Name: Date: Period:

4. Noah's recipe for sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

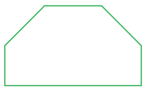
   a. Noah prepares large batches of sparkling orange juice for school parties. He usually knows the total number of liters t that he needs to prepare. Write an equation that shows how Noah can determine the number of liters s of soda water he needs if he knows t.
   $s = \frac{5}{9} \cdot t$

   b. Sometimes the school purchases a certain number of liters j of orange juice and Noah needs to determine how much sparkling orange juice he can make. Write an equation that Noah can use to determine t if he knows j.
   $t = 2.25 \cdot j$

5. For a suitcase to be checked on a flight (instead of carried by hand), it can weigh at most 50 lb. Andre's suitcase weighs 23 kg. Can Andre check his suitcase? Explain or show your thinking. **Hint:** 10 kg ≈ 22 lb.
Andre cannot take his suitcase because it weighs about 50.6 lb.

Weight (kg)	Weight (lb)
10	22
1	2.2
23	50.6

6. Describe the properties of the figure shown. Be sure to use the following words to describe the figure: *vertices, parallel lines, angles.* Is it a polygon?
Sample response: The figure has 6 vertices, 2 sets of parallel lines, 2 right angles. It is a polygon because it is a closed figure with all straight lines and no lines crossing each other.

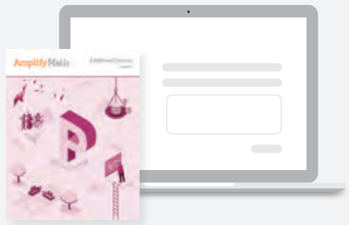


Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	2
	3	Activity 2	1
Spiral	4	Unit 6 Lesson 17	2
	5	Unit 2 Lesson 19	2
Formative	6	Unit 6 Lesson 17	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

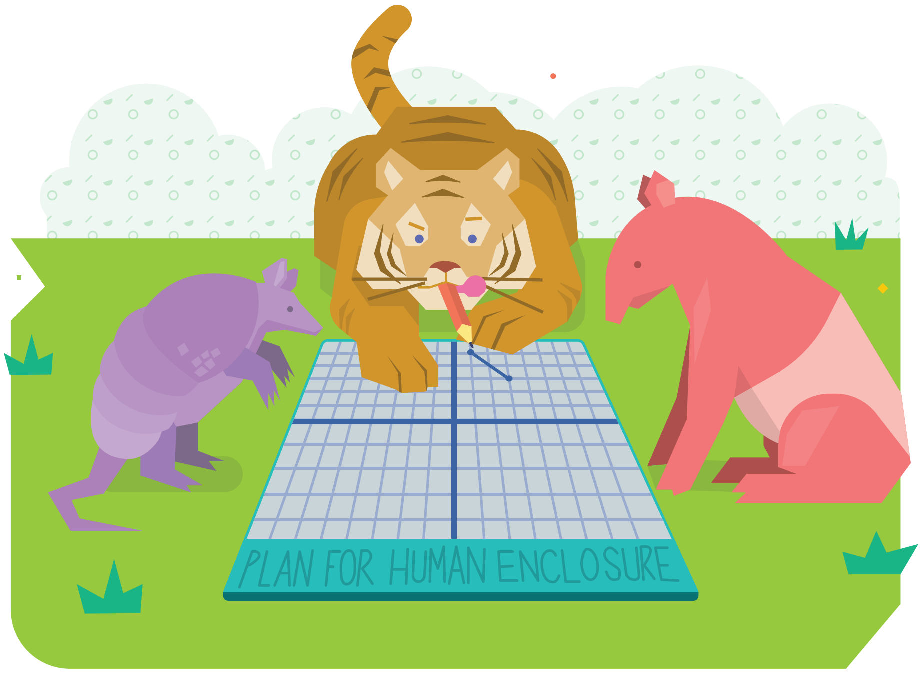
Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Shapes on the Coordinate Plane

Let's use the coordinate plane to solve problems and determine perimeters.



Focus

Goals

1. Determine the total length of multiple horizontal and vertical segments in the coordinate plane that are connected end-to-end.
2. Draw a polygon on the coordinate plane, given the coordinates for its vertices.
3. **Language Goal:** Explain that coordinates can be a useful way of describing geometric figures or modeling real-world locations. **(Speaking and Listening)**

Coherence

Today

Students plot polygons and determine their perimeters on the coordinate plane, in the context of fencing for a wildlife refuge, by relating side lengths to distances between points. Their work is presented using the ***Gallery Tour*** routine to observe and critique others' thinking about shapes and perimeter in the coordinate plane.

Previously

In Lessons 13–16, students plotted points with rational-number coordinates in all four quadrants of the coordinate plane, determining distances between those points, and interpreting them in context.

Coming Soon

In Lesson 18, students will apply their understanding of plotting, interpreting, and calculating distances on a coordinate plane to a series of puzzles. Then in Grade 7, they will solve problems involving scale drawings of geometric figures, including finding the lengths and areas of the figures.

Rigor

- Students build **conceptual understanding**, connecting geometrical figures to the coordinate plane.
- Students strengthen their **procedural skills** to plot rational-number coordinates on a four-quadrant coordinate plane by plotting polygons.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils
- graph paper

Math Language Development

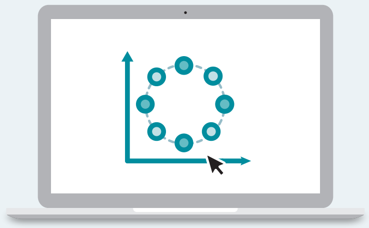
Review words

- *quadrant(s)*
- *coordinate plane*
- *ordered pairs*

Amps | Featured Activity

Activity 2
Designing a Wildlife Refuge

You can overlay student responses to compare designs.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not actively engage in the evaluation of other groups' plans in Activity 2. Review the importance of honest evaluation, as well as respect for others. Explain that through this process, everyone learns from both the successes and mistakes of others.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 2**, students may determine the fencing needed for one or two animals, instead of all three, in Problem 2.

Independent | 5 min

Warm-up Plotting Polygons

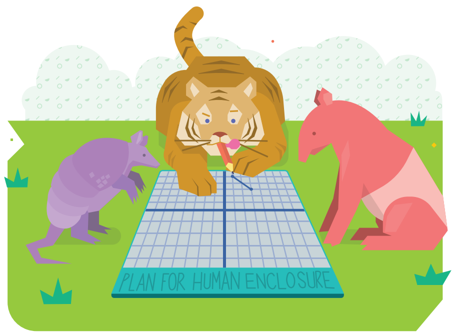
Students plot a figure on the coordinate plane that has multiple specific properties and review the properties of a polygon.

Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 17

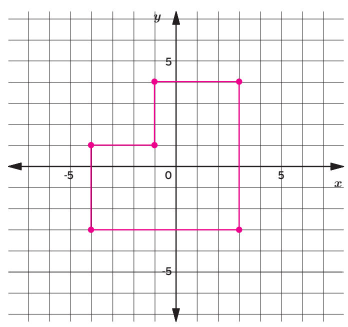
Shapes on the Coordinate Plane

Let's use the coordinate plane to solve problems and determine perimeters.



Warm-up Plotting Polygons

1. Draw a figure on the coordinate plane with at least three of the following properties:
   - 6 vertices
   - 1 pair of parallel sides
   - 1 or more right angles
   - 2 sides of the same length



2. Is your figure a polygon? Explain your thinking.
   Sample response: Yes, my figure is a polygon because it has sides that are straight-line segments connecting at least three vertices.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students have to work independently on the activity.

2 Monitor

Help students get started by activating their prior knowledge. Have them identify one property with which to start, and ask them to define the key words (e.g., vertices, parallel, and right angle).

Look for points of confusion:

- **Thinking that they can use only the number of properties given in each bullet.** Say, "1 is the minimum number of right angles you should have, but there could be more than 1."

Look for productive strategies:

- Experimenting with different combinations of the required properties.

3 Connect

Have individual students share the figure that they plotted, starting with simpler figures, such as rectangles. Have them share their thinking or the process they followed in creating their figure using the given criteria.

Ask, "How do you know whether a figure is a polygon?"

Highlight the characteristics of a polygon:

- Composed only of line segments.
- Each line segment intersects one and only one other line segment at each end.
- No line segments intersect each other at any points other than their endpoints.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, as you highlight the characteristics of polygons, ask students to return to their responses from Problem 2 and write an improved response that includes these characteristics.

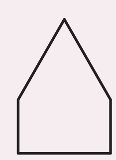
English Learners

Use gestures when describing parallel lines and describe characteristics of polygons. For example, point to each line segment and its endpoints to illustrate how each line segment intersects one and only one other line segment.

Power-up

To power up students' ability to describe properties of polygons using appropriate mathematical vocabulary have students complete:

Complete each statement about the given polygon:



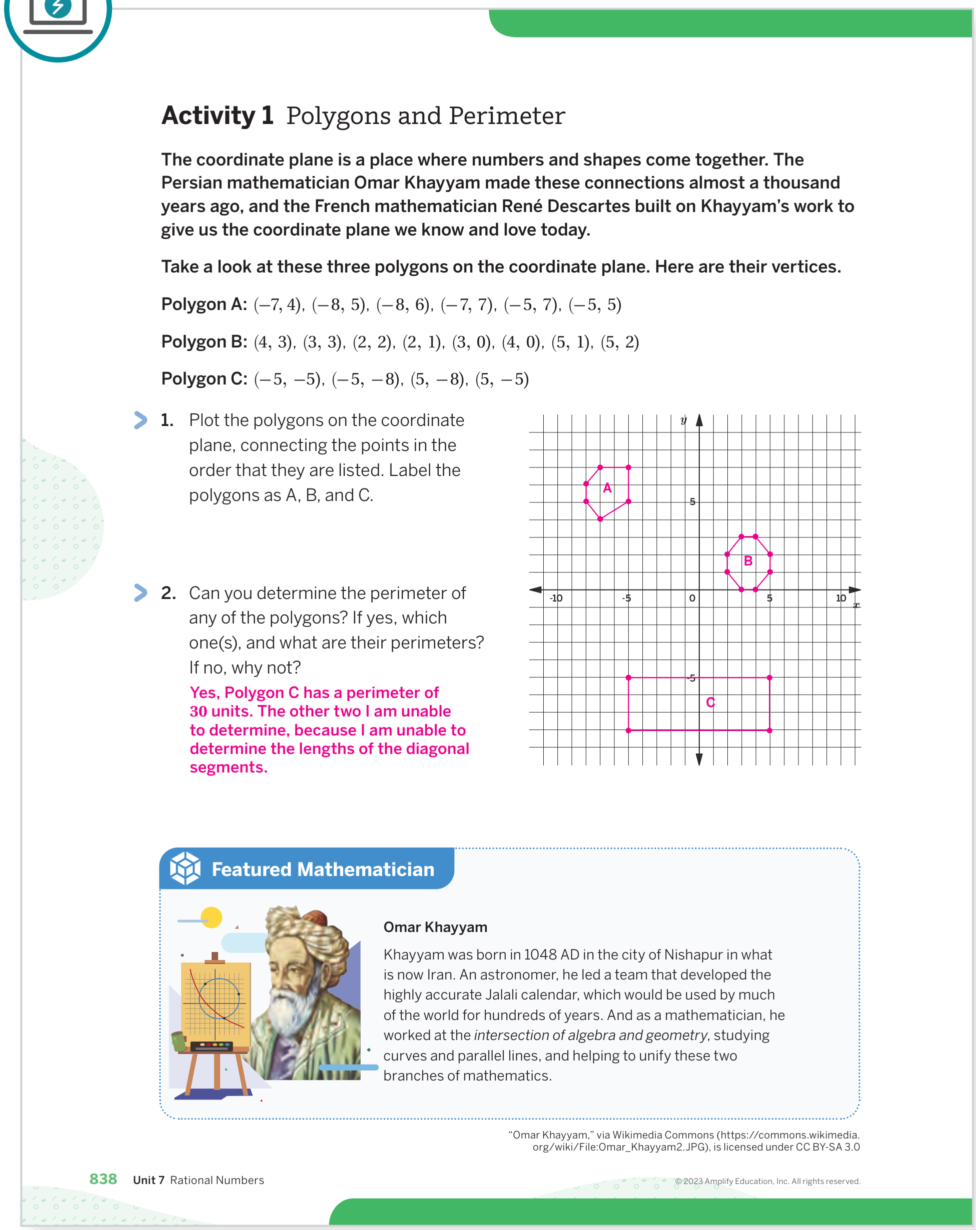
1. There are <u>5</u> angles.
2. There are <u>2</u> two right angles.
3. There is/are <u>1</u> pair(s) of parallel sides.
4. There is/are <u>2</u> pair(s) of perpendicular sides.

Use: Before the Warm-up.
Informed by: Performance on Lesson 16, Practice Problem 6.

Independent | 10 min

Activity 1 Polygons and Perimeter

Students plot coordinates to create different polygons on the coordinate plane. They then consider whether any of the three perimeters can be calculated.



Activity 1 Polygons and Perimeter

The coordinate plane is a place where numbers and shapes come together. The Persian mathematician Omar Khayyam made these connections almost a thousand years ago, and the French mathematician René Descartes built on Khayyam's work to give us the coordinate plane we know and love today.

Take a look at these three polygons on the coordinate plane. Here are their vertices.

Polygon A: $(-7, 4)$, $(-8, 5)$, $(-8, 6)$, $(-7, 7)$, $(-5, 7)$, $(-5, 5)$

Polygon B: $(4, 3)$, $(3, 3)$, $(2, 2)$, $(2, 1)$, $(3, 0)$, $(4, 0)$, $(5, 1)$, $(5, 2)$

Polygon C: $(-5, -5)$, $(-5, -8)$, $(5, -8)$, $(5, -5)$

1. Plot the polygons on the coordinate plane, connecting the points in the order that they are listed. Label the polygons as A, B, and C.

2. Can you determine the perimeter of any of the polygons? If yes, which one(s), and what are their perimeters? If no, why not?
Yes, Polygon C has a perimeter of 30 units. The other two I am unable to determine, because I am unable to determine the lengths of the diagonal segments.

Featured Mathematician

Omar Khayyam

Khayyam was born in 1048 AD in the city of Nishapur in what is now Iran. An astronomer, he led a team that developed the highly accurate Jalali calendar, which would be used by much of the world for hundreds of years. And as a mathematician, he worked at the *intersection of algebra and geometry*, studying curves and parallel lines, and helping to unify these two branches of mathematics.

1 Launch

Activate students' prior knowledge by asking, "How do you determine the perimeter of a polygon?"

2 Monitor

Help students get started by saying, "Let's label the quadrants with the signs of their coordinates." Ask, "In which quadrant would this first point of Polygon A be placed?"

Look for points of confusion:

- **Confusing the x- and y-coordinates.** Remind students that the coordinates of points are written as ordered pairs in the form (x, y).
- **Not closing the figure by connecting the final ordered pair to the first.** Remind students of the properties of polygons.

Look for productive strategies:

- Plotting points in all four quadrants of the coordinate plane with accuracy.
- Using the structure of ordered pairs and absolute value to find distances between points in order to calculate perimeter.

3 Connect

Display the coordinate plane with the three polygons plotted and labeled.

Ask:

- "How do the coordinate plane and number line relate?"
- "How can you make polygons in the coordinate plane?"
- "Why were the perimeters of the other two polygons not able to be determined?"

Have individual students share their strategies for finding the perimeter of Polygon C.

Highlight determining the length of the top and bottom lines by finding the distances from the x- coordinates to the y-axis.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students plot Polygon C and one other polygon of their choice on the coordinate plane and respond to Problem 2.

Extension: *Math Enrichment*

Have students estimate the perimeters of Polygons A and B by determining a possible range of values for the length of a slanted line segment. For example, in Polygon A, they can estimate the length of each of the two shorter slanted line segments as somewhere between 1 and 2 units, and could estimate it at about 1.5 units. Accept reasonable estimates. Sample response: Polygon A's perimeter is about 10 units. Polygon B's perimeter is about 10 units.

Featured Mathematician

Omar Khayyam

Have students read about featured mathematician Omar Khayyam, who was one of the first to study connections between algebra and geometry.

Small Groups | 20 min

Activity 2 Fencing for a Wildlife Refuge

Students plot three polygons, representing enclosures for animals in a wildlife refuge. They then observe and critique each other's work during a *Gallery Tour*.

Amps Featured Activity Designing a Wildlife Refuge

Name: ____________ Date: ________ Period: ________

Activity 2 Fencing for a Wildlife Refuge

The coordinate plane on the next page represents all the land on a wildlife refuge. The locations of an existing observation tower and an elevated walkway are shown. You have been tasked with designing the size and locations of the enclosures for three endangered animals from the Amazon Rainforest that will live at the refuge: South American tapirs, giant armadillos, and tiger cats.

Your enclosures must meet the following criteria:

- The tiger cats need the most space to run, followed by the tapirs. The armadillos need the least space.
- None of the animal enclosures can share any fence.
- There should be at least one foot between the observation tower and any enclosure. *Note: this does not include the walkway.*

1. Plot your three enclosures on the coordinate plane on the next page. Record the coordinates of each "corner" in the table.

2. Determine how much fencing, in feet, will you need for each enclosure, and record those values in the table. **Sample responses shown.**

Animal	Enclosure coordinates	Fencing needed (ft)
Tiger cats	(–1, 6), (–1, 3), (3, 3), (3, –7), (7, –7), (7, 6)	42 ft
Tapirs	(–3, 0), (–8, 0), (–8, –7), (2, –7), (2, –3), (–3, –3)	34 ft
Armadillos	(–3, 1), (–8, 1), (–8, 6), (–2, 6), (–2, 3), (–3, 3)	22 ft

1 Launch

Provide students with 10 minutes of collaborative work time, and conduct the *Gallery Tour* routine to display student work.

2 Monitor

Help students get started by asking, "With which animal would you like to start? What information do you have about that animal and its enclosure?"

Look for points of confusion:

- **Thinking the enclosures can only be rectangles.** Refer back to examples from the Warm-up that were not rectangles.
- **Finding distance only by counting units.** Ask, "What do you know about two points that share a coordinate? How can you find the distance between them?"

Look for productive strategies:

- Calculating perimeters using distances from 0 (i.e., absolute values).
- Calculating distances by subtracting the x- or y-coordinates, depending upon which coordinates are the same.
- Plotting points with non-integer coordinates.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital technology to design the space for the wildlife refuge.

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Provide access to three different colors of construction paper and suggest that students cut out polygons using the paper. Students can overlay these polygons on the coordinate plane to help visualize the space.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that the coordinate grids represent all the land on a wildlife refuge.
- **Read 2:** Ask students to name or highlight the given criteria, such as "None of the animal enclosures can share any fence."
- **Read 3:** Ask students to plan out possible layouts for the enclosures.

Small Groups | 20 min

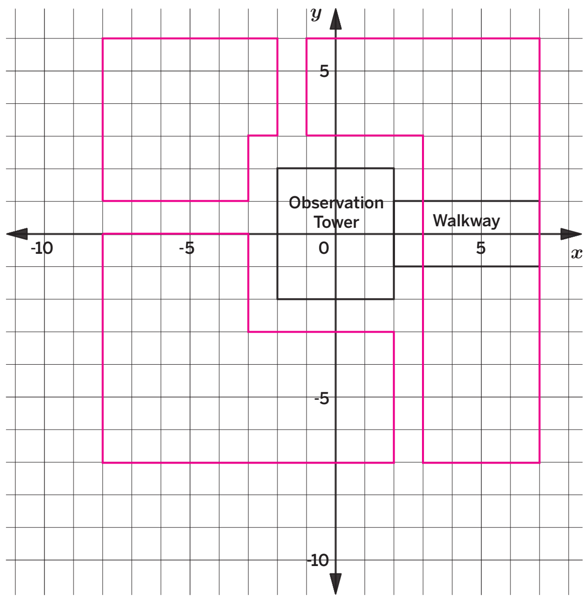
Activity 2 Fencing for a Wildlife Refuge (continued)

Students plot three polygons, representing enclosures for animals in a wildlife refuge. They then observe and critique each other's work during a *Gallery Tour*.

Activity 2 Fencing for a Wildlife Refuge (continued)

Sample response shown.

Hint: One unit = 1 ft.



Are you ready for more?

Determine the area of each of your enclosures. Sample responses shown.

Animal	Area (ft^2)
Tiger cat	64 ft^2
Tapir	55 ft^2
Armadillo	28 ft^2

STOP

3 Connect

Display group work around the classroom for the *Gallery Tour* routine.

Have groups of students share how they designed their enclosures and their strategies for calculating the amount of fencing needed.

As students participate in the *Gallery Tour*, **ask**, "Did every group's plans follow all of the given constraints?"

Highlight the strategies for finding how much fencing was needed, which represents the perimeters of the figures.

Whole Class | 5 min

Summary

Review and synthesize how plotting coordinates and finding lengths of line segments on the coordinate plane can be related to determining measurements of shapes, e.g., perimeter.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You can construct a polygon in the coordinate plane by plotting points that represent the vertices, and then connecting those points with line segments to form the sides. You can use the coordinate plane to identify certain properties of polygons. Suppose a polygon is graphed in the coordinate plane. You can determine:

- The number of sides, and in some cases, the lengths of those sides.
- Whether two sides are parallel or perpendicular.

This information can help you identify and describe shapes based on the points that form their vertices. One way you can use information like this is to determine the lengths of the line segments that form the sides of a polygon, and add them together to calculate its perimeter.

Reflect:

Synthesize

Ask, "In what other real-world situations could you apply the strategies you used today?"

Highlight:

- If two points have the same x-coordinate, they will be on the same vertical line, and students can determine the distance between them by either subtracting the y-coordinates or adding the absolute values of each point's distance to the x-axis.
- If two points have the same y-coordinate, they will be on the same horizontal line, and students can determine the distance between them by either subtracting the x-coordinates or adding the absolute values of each point's distance to the y-axis.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful when determining the length of a line segment, especially when it crossed over an axis?"
- "Were there any strategies that were not helpful? Why?"

Independent | 5 min

Exit Ticket

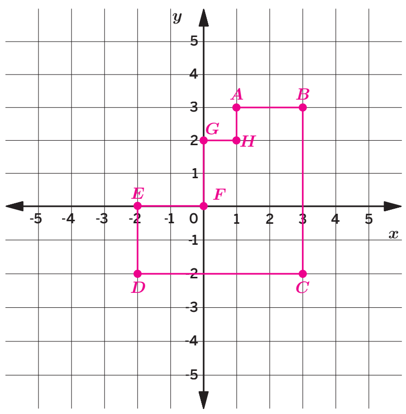
Students demonstrate their understanding by plotting points to make a polygon and then calculating its perimeter.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 7.17

1. Plot these points on the coordinate plane and connect them to create a polygon: (1, 3), (3, 3), (3, −2), (−2, −2), (−2, 0), (0, 0), (0, 2), (1, 2)



2. Determine the perimeter of the polygon.
   20 units

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can find the lengths of horizontal and vertical segments on the coordinate plane.
1 2 3

b. I can plot polygons on the coordinate plane when I know the coordinates for the vertices.
1 2 3

c. I can find the length of a side joining points with the same x-coordinate or the same y-coordinate.
1 2 3

Lesson 17 Shapes on the Coordinate Plane

Success looks like . . .

- **Goal:** Determining the total length of multiple horizontal and vertical segments in the coordinate plane that are connected end-to-end.
  - » Determining the perimeter of the polygon in Problem 2.
- **Goal:** Drawing a polygon on the coordinate plane, given the coordinates for its vertices.
  - » Plotting the points on the coordinate plane to make a polygon in Problem 1.
- **Language Goal:** Explaining that coordinates can be a useful way of describing geometric figures or modeling real-world locations. **(Speaking and Listening)**

Suggested next steps

If students have trouble plotting the points, consider:

- Referring back to Activity 1 and asking, "How did you plot Polygon B?"
- Ask, "With which number do you start in a ordered pair?"

If students have trouble calculating the perimeter, consider:

- Having students determine and record the distance of each side separately and then add the distances.
- Referring back to Activity 1 and asking, "How did you determine the perimeter of Polygon C?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Students moved from plotting polygons on the coordinate plane (Warm-up) to using distance between points to calculate the perimeter of polygons (Activity 1). They then applied these skills to create fencing for a wildlife sanctuary in Activity 2. What challenges did students encounter as they progressed through the lesson? How did they work through them?
- Thinking about the questions you asked students today and what the students said or did as a result of those questions, which question was the most effective? What made it so effective? What questions might you change or add the next time you teach this lesson?

Practice

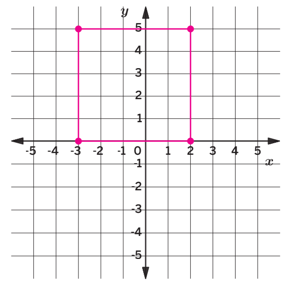
Independent

Name: ______ Date: ______ Period: ______

Practice

1. Refer to the coordinate plane.

a Draw a square with a perimeter of 20 units and one vertex at the point $(-3, 5)$.

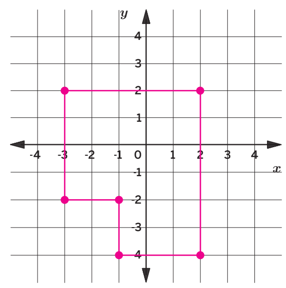


b Write the coordinates of the other vertices.

Sample responses: $(-3, 0)$, $(2, 0)$, $(2, 5)$

2. Refer to the coordinate plane.

a Plot and connect the following points in the order they are listed to form a polygon: $(-3, 2)$, $(2, 2)$, $(2, -4)$, $(-1, -4)$, $(-1, -2)$, $(-3, -2)$.



b What is the perimeter of the polygon?

22 units

Name: ______ Date: ______ Period: ______

Practice

3. The coordinates of a rectangle are $(3, 0)$, $(3, -5)$, $(-4, 0)$ and $(-4, -5)$.

a What are the length and width of the rectangle?

length: 7 units, width: 5 units or
length: 5 units, width: 7 units

b What is the perimeter of the rectangle?

24 units

c What is the area of the rectangle?

35 square units

4. Circle *all* the equations that could represent each scenario. Then determine the solution for each scenario using one of the selected equations.

a Jada's cat weighs 3.45 kg. Andre's cat weighs 1.2 kg more than Jada's cat. How much does Andre's cat weigh?

$x = 3.45 + 1.2$ $\quad x = 3.45 - 1.2$ $\quad x + 1.2 = 3.45$ $\quad x = 1.2 + 3.45$

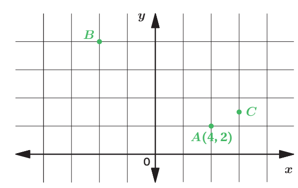
Solution: $x = 4.65$

b Apples cost \$1.60 per pound at the farmer's market. They cost 1.5 times as much at the grocery store. How much do apples cost per pound at the grocery store?

$y = 1.5 \cdot 1.60$ $\quad y = 1.60 \div 5$ $\quad 1.5y = 1.60$ $\quad \frac{y}{1.5} = 1.60$

Solution: $y = \$2.40$

5. Use the coordinates of point A to help you determine the coordinates of points B and C. Explain your thinking.



$B(-4, 8)$ and $C(6, 3)$; Sample response: Both axes count by 2s because point A is located 2 ticks to the right of the origin, but the x-coordinate is 4. The point is also located 1 tick above the origin, but the y-coordinate is 2.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	1
Spiral	4	Unit 6 Lesson 7	2
Formative	5	Unit 7 Lesson 18	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 7 | **LESSON 18**

Lost and Found Puzzles

Let's use the coordinate plane to solve problems and puzzles.

Focus

Goals

1. Determine the total length of multiple horizontal and vertical segments in the coordinate plane that are connected end-to-end.
2. **Language Goal:** Explain that coordinates can be a useful way of describing geometric figures or modeling real-world locations. **(Listening and Speaking)**

Coherence

Today

Students continue to work with the coordinate plane, revisiting the context of mazes from Lesson 14. They determine the scale of the axes and determine the lengths of line segments in order to calculate the total distances along a solution path through a maze. Then, they solve a logic puzzle based on lost objects on a coordinate plane, identifying where lost objects are based on several clues, choosing to use a coordinate plane, a table, or both. This helps students visualize and organize their thinking. As an optional part of the lesson, students can work with a partner to create their own mazes on the coordinate plane, and then swap with another pair to solve each other's maze.

Previously

Lessons 13–17 have all focused on the four-quadrant coordinate plane, from constructing the quadrants and understanding relationships among the signs of coordinates, to finding distances between two vertical or horizontal points, to determining appropriate scales for the x- and y-axes when graphing points to represent a relationship between two real-world quantities.

Coming Soon

In Lesson 19, the final lesson of this unit, students will apply all of the ideas about rational numbers and the four-quadrant coordinate plane in order to create pictures that are not strictly geometric shapes.

Rigor

- Students **apply** rational number and coordinate graphing concepts to explore and solve puzzles.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	10 min	20 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, (answers)
- Activity 3 PDF (optional), one per pair
- graph paper

Math Language Development

Review words

- *quadrants*
- *coordinate plane*
- *ordered pairs*

Amps | Featured Activity

Activity 1

Interactive Maize Maze

Students are able to plot an exit route and show how they compute the distance using ordered pairs. You can view student work to provide immediate feedback.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might impatiently jump into the solution without thinking through each clue in Activity 2. Ask students to make a plan for how they will use the clues. By setting goals and organizing their approach to the problem, students will present a plan that encourages the perseverance required to complete the activity.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Optional **Activity 3** may be omitted.

Pairs | 5 min

Warm-up Lost in the Maize Maze

Students practice plotting and labeling points on a coordinate plane representing a maze.

Unit 7 | Lesson 18

Lost and Found Puzzles

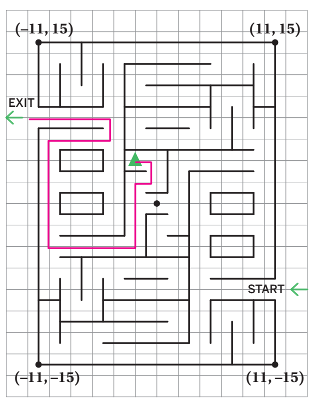
Let's use the coordinate plane to solve problems and puzzles.

Warm-up Lost in the Maize Maze

Noah got lost in the maize maze. The triangle represents his stopping point. Help him find his way out.

1. Write the coordinates that represent the point at which Noah stopped.
   $(-2, 4)$

2. Plot and label the points where Noah needs to turn to make his way out of the maze, starting from the point where he realized he was lost. **Sample response:** $(-2, 4)$, $(0, 4)$, $(0, 2)$, $(-2, 2)$, $(-2, -4)$, $(-10, -4)$, $(-10, 6)$, $(-4, 6)$, $(-4, 8)$, $(-12, 8)$



Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by suggesting they draw in the axes, and then asking, "What is the scale of each axis?"

Look for points of confusion:

- **Incorrectly labeling the point where Noah stopped or the turning points to make his way out of the maze.** Have students draw the x- and y-axes and number them to help them to see where the point lies on the coordinate plane.
- **Counting by 1s between grid lines.** Demonstrate with a number line that is counting by 2s. Ask, "If each tick represents 2 units, what would be the the first coordinate of a point located halfway between the horizontal tick marks representing 6 and 8?"

Look for productive strategies:

- Drawing the x- and y-axes.
- Plotting the points leading to the exit of the maze.
- Recognizing that there is more than one way to exit the maze.

3 Connect

Display the Maize Maze for students to refer to as they explain their thinking.

Ask, "What were some strategies you used to help you get started?"

Have pairs of students share their ordered pairs representing turns in the maze to get to the Exit. Both possible routes should be shown.

Highlight that it is possible to describe situations involving movement using ordered pairs on a coordinate plane.

Differentiated Support

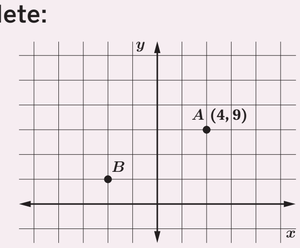
Accessibility: *Guide Processing and Visualization*

Suggest students use colored pencils to track and count the units along the path, if seeing the grid lines is challenging. They could also use colored pencils to draw the x-axis and y-axis to make them more visible.

Power-up

To power up students' ability to understand the relationship between a coordinate pair and the scale used on each axis in a coordinate plane, have students complete:

Use the given coordinate for point A to respond to each question.



a. What is the scale on the x-axis? By 2's

b. What is the scale on the x-axis? By 3's

c. What are the coordinates of point B? $(-4, 3)$

Use: Before the Warm-up.
Informed by: Performance on Lesson 17, Practice Problem 5.

Pairs | 10 min

Activity 1 Found in the Maize Maze?

Students continue to work with the same maze from the Warm-up, plotting and labeling more points and finding horizontal and vertical distances.

Amps Featured Activity Interactive Maize Maze

Name: ______ Date: ______ Period: ______

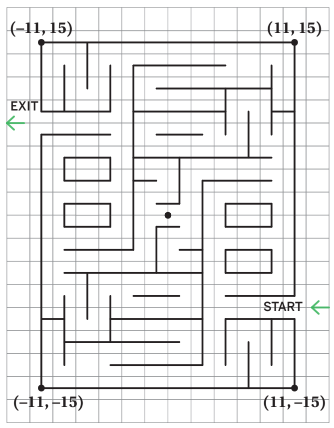
Activity 1 Found in the Maize Maze?

Han entered the maze at the Exit to help Noah. He did not know that Noah was already making his way out (following the path you mapped out in the Warm-up).

Co-craft Questions: Before you begin the activity, study this maze. Work with your partner to write 2–3 mathematical questions you have about this maze or scenario.

1. Han stopped at (–7, –2). Plot where Han stopped.

2. Do you think Han was able to find Noah? Explain your thinking.
   Responses will depend on the paths students chose to help Noah exit the maze.



3. What is the shortest distance Han could have walked before stopping? Record the ordered pairs of each line segment representing Han's path, and the distance between each pair of coordinates. The first part of his route has been completed in the table.

Line segment	Ordered pairs	Distance (units)
1	(–12, 8) and (–4, 8)	8
2	(–4, 8) and (–4, –2)	10
3	(–4, –2) and (–7, –2)	3
	Total Distance:	21

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them draw the x- and y-axes to make the quadrants more visible.

Look for points of confusion:

- **Counting by 1s between grid lines.** Demonstrate with a number line that is counting by 2s. Ask, "If each tick is 2 units, what would be the the first coordinate of a point located halfway between the horizontal tick marks representing 6 and 8?"
- **Not understanding where Noah is located.** Have students refer back to the Warm-up and copy how they plotted Noah's route out of the maize maze.

Look for productive strategies:

- Finding the lengths of each vertical or horizontal line segment along Han's path using the coordinates of turning points.
- Drawing Noah's route from the Warm-up.
- Using another strategy, such as counting by 2s, to check their answers.

3 Connect

Display the maze for students to refer to as they explain their thinking.

Have pairs of students share their responses and their thinking, focusing on how they calculated the lengths of the line segments and how this represents distance.

Highlight that it is possible to further describe situations involving movement using both ordered pairs and distances on the coordinate plane.

Ask, "How did the table help you calculate the distance Han walked before he stopped?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can plan and plot an exit route, showing how they computed the distances using ordered pairs.

Accessibility: *Vary Demands to Optimize Challenge*

Have students draw and label intermediate grid lines to represent the axes scales increasing by 1s.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the maze and the introductory text. Ask pairs of students to write 2–3 questions they have about this maze or scenario. A sample question could be, "If Han stopped at (–7, –2), where is he in relation to Noah?" Invite pairs to share their questions with another pair of students or the class. This routine will help students produce mathematical language related to the coordinate plane.

English Learners

Allow students to orally communicate their questions, instead of writing them down, or vice versa.

Pairs | 20 min

Activity 2 Lost Treasures

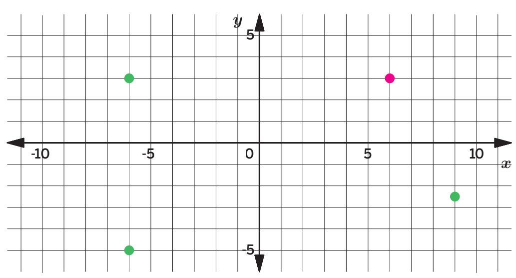
Students solve a logic puzzle using clues referencing the locations of places and objects on a coordinate plane.

Activity 2 Lost Treasures

Can you determine the mystery locations of all the lost treasures? The location of three of the treasures are marked on the coordinate grid. Use the clues and the given points to determine which treasure is at each location as well as the coordinates of the final treasure.

- The places where the necklace and puppy can be found are points that can be seen as a reflection across one axis on the coordinate plane.
- The puppy ran out from under a bed and ran 5.5 blocks north and 3 blocks west to a friend's house.
- The sports gear can be found at the point farthest away from the y-axis.
- The lost treasure under the bed is located at the point closest to the x-axis.
- The person who lost the granola bar also lost a different treasure. They found the treasure that was in the locker first. Then they walked 8 blocks to find the granola bar.
- That same person was given a look of disgust by someone who works where one of the treasures was found, but thought "Well, at least I only walked 8 blocks to come get it."

You can use the coordinate plane, the table, or both, to help track the given information and solve the mystery.



846 Unit 7 Rational Numbers

1 Launch

Activate students' background knowledge by asking, "Have you ever read a mystery book or watched a mystery movie? How did clues help the characters?" Say, "You will be finding the location and coordinates of four lost treasures. There are three parts to what you are finding: the location, the coordinates of the location, and which lost treasure is at each location. Each clue might not immediately lead to a solution on its own, but the information from multiple clues can be put together to reach conclusions".

2 Monitor

Help students get started by reading through the clues and focusing on those that provide known information right away (Clues 3 and 4).

Look for points of confusion:

- **Not recognizing that there is a fourth location not plotted on the coordinate plane.** Suggest that students look for some other information that might be shown on the table.
- **Having difficulty knowing how to use the table.** Demonstrate how the table can be used to keep track of what is known to be true, or not true, about a treasure and/or a location, using Xs and check marks.
- **Making incorrect assumptions without thinking through the clues.** Say, "Read through the clues and see whether that holds true."

Look for productive strategies:

- Discussing possible combinations as partners work through the clues.
- Using the coordinate grid and/or table effectively.

Activity 2 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Before students begin, ask them to study the coordinate plane and see what they notice. If they do not notice that there are only 3 points plotted, ask them, "How many lost treasures are there? How many points do you see on the coordinate plane?" Direct them to study the table to see that they will need to determine the coordinates of this missing point as well.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text and the clues.

Read 1: Students should understand that there are four lost treasures (necklace, puppy, sports gear, granola bar) and that there are several clues given to help find these items.

Read 2: Ask students to name given quantities and relationships, such as the sports gear is located the farthest from the y-axis.

Read 3: Ask students to brainstorm whether they think they will use the graph, the table, or both to help determine the locations of the lost treasures.

Pairs | 20 min

Activity 2 Lost Treasures (continued)

Students solve a logic puzzle using clues referencing the locations of places and objects on a coordinate plane.

Name: ______ Date: ______ Period: ______

Activity 2 Lost Treasures (continued)

		Lost treasures				Ordered pairs			
		Gold necklace	Granola bar	Sports gear	Puppy	(9, −2.5)	?	(−6, −5)	(−6, 3)
Locations	Locker at school								
	Under the bed								
	Friend's house								
	Gym								
Ordered pairs	(9, −2.5)								
	?								
	(−6, −5)								
	(−6, 3)								

Lost treasure	Location	Ordered pairs
Gold necklace	Locker at school	(−6, 3)
Granola bar	Gym	(−6, −5)
Sports gear	Under the bed	(9, −2.5)
Puppy	Friend's house	(6, 3)

3 Connect

Display the blank coordinate plane and table. The map and table can be used to refer to as students share their thinking.

Have pairs of students share their mystery solutions, focusing on what clues helped them the most as well as how each tool was used to help solve the mystery.

Highlight the connections to the math learned throughout the unit, such as reflected points on the coordinate plane, direction and magnitude from a given point, and distances from the axes.

Optional

Pairs | 15 min

Activity 3 Mystery Maze Design Challenge

Students design mazes using a scale of their choice. They then have to guide someone through the maze without giving the scale.

Activity 3 Mystery Maze Design Challenge

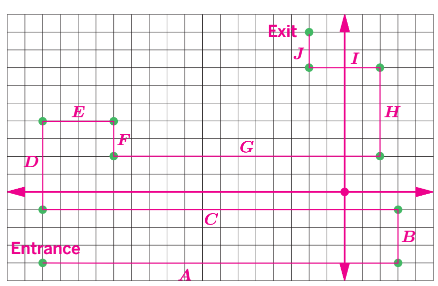
You will be given a blank grid.

1. Work with a partner to create a path that goes through all the given points on the coordinate plane, connected in any order, but the path *cannot* cross itself. This will be a *solution path* to your maze. Then design a maze around that path.

   You must:

   - Plot and label the location of the origin. It can be anywhere on the grid, but there has to be at least one turning point in each of the four quadrants.
   - Identify the scale of your grid, as going by either 1s, 2s, or 3s.
   - Add two more turning points, but they cannot be in the same quadrant.
   - Label one point as the *Entrance* and label one point as the *Exit*, but they cannot be in the same quadrant.
   - Label each line segment using capital letters starting with A representing the segment from the *Entrance* to the next point on the solution path. Even if two line segments are on the same line, they should be labeled separately.

   Sample maze: Scale = 1



1 Launch

Challenge students to create a maze. They can use a scale of 1, 2, or 3, and will need to record the coordinates for each turn and the length of each segment. After 10 minutes of collaborative work time, form new pairs of students, one with a finished maze and one without. The student without (The Maze Walker) should be given a blank grid from the Activity 3 PDF, and needs to find their way out of the maze using only verbal information given by the student with the maze (The Caller). The challenge is finished when the scale and path have been found. This activity is similar to the *Blindfold Mazes* activity in Lesson 1.

2 Monitor

Help students get started by saying "Decide on the scale you want to use and then plan out the path. You can place the walls later."

Look for points of confusion:

- **Miscalculating distances, or lengths of line segments.** Review strategies used to find lengths, or distance, on the coordinate plane.
- **Thinking that the maze has to start at one edge and end at the opposite edge.** Remind students that they have full creative range and can design their maze however they want. There are no strict rules for a starting or ending point.
- **Not knowing what to write in the "Line segment" column.** Refer back to the table in Activity 1 to remind them to number the line segments.

Look for productive strategies:

- Determining distances by finding differences of coordinates when x- or y-coordinates are the same.
- Determining distances based on absolute values.
- Flexibly working within the coordinate plane to create the path. First ideas may not work, so perseverance and creativity to plan a path are necessary.

Activity 3 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider showing students a sample maze, created either by yourself or by another student (perhaps from a prior class). Turn the directions given in the Student Edition into a checklist and have students check off each bulleted list item as they complete their maze to ensure it meets all the criteria.

Extension: *Math Enrichment*

Have students go to the Guinness World Record online site to explore the Yancheng Dafeng Dream Maze, located in Yancheng, Jiangsu, China, which currently is the Guinness World Record holder for Largest maze (permanent) and Largest Hedge Maze (permanent). The maze covers an astounding area of 35,596.74 m^2. The maze also holds the title for the Largest Pathway Network in a Hedge Maze (permanent) with a path measuring 9.45 km.

Pairs | 15 min

Activity 3 Mystery Maze Design Challenge (continued)

Students design mazes using a scale of their choice. They then have to guide someone through the maze without giving the scale.

Name: ______ Date: ______ Period: ______

Activity 3 Mystery Maze Design Challenge (continued)

2. Record the coordinates of each line segment and its distance.

Line segment	Ordered pairs	Distance
A	(−17, −4) and (3, −4)	20
B	(3, −4) and (3, −1)	3
C	(3, −1) and (−17, −1)	20
D	(−17, −1) and (−17, 4)	5
E	(−17, 4) and (−13, 4)	4
F	(−13, 4) and (−13, 2)	2
G	(−13, 2) and (2, 2)	15
H	(2, 2) and (2, 7)	5
I	(2, 7) and (−2, 7)	4
J	(−2, 7) and (−2, 9)	2

3. When your maze is complete, you will switch partners and challenge each other to solve your mazes, using the information in the table.

STOP

3 Connect

Display students' mazes, noting the variety of configurations for those pairs that used the same scale.

Have pairs of students share what information helped the Maze Walkers the most. To help elicit responses, consider also asking, "What was your *Ah-ha!* moment?"

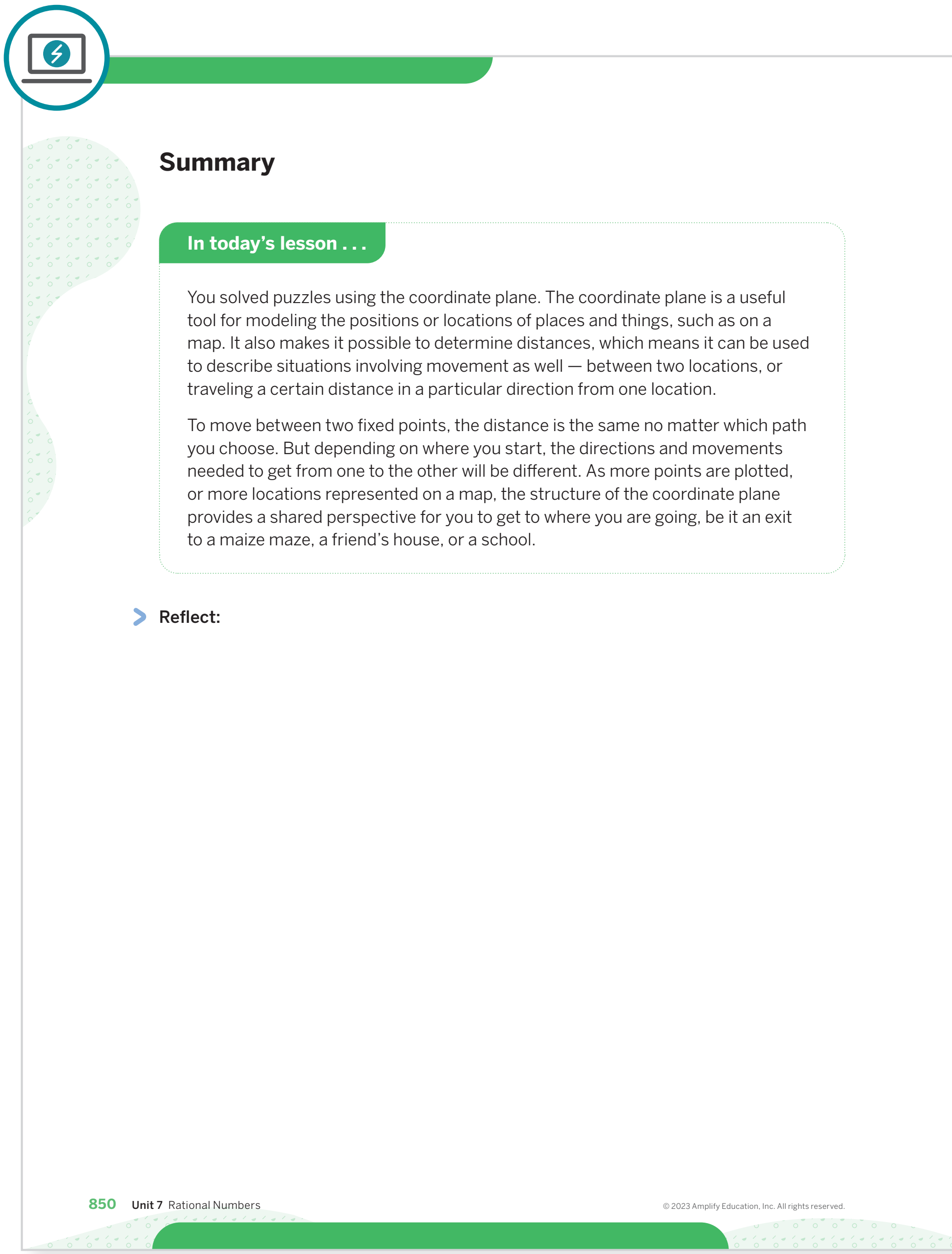
Ask, "Why are coordinates useful for communicating information about flat space?"

Highlight the importance of making relevant inferences from the information that is presented (e.g., deriving the scale based on given coordinates).

Whole Class | 5 min

Summary

Review and synthesize how coordinates and distance can be used to represent movement on the coordinate plane.



Synthesize

Highlight the information needed for movement around the coordinate plane: scale, coordinates, distance, and direction.

Ask:

- "How were you or your partner able to find the coordinates in the maze? Did you come up with any strategies or shortcuts?"
- "Was there any movement shown when finding the lost treasures? Why was it necessary?"
- "What other situations involving movement could be represented on a coordinate plane?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "By the way, how many lefts make a right?"
- "How did you use numbers to talk about direction?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by calculating the distance between two points.

Printable

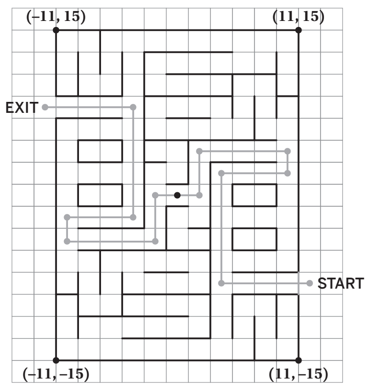
Name: ______ Date: ______ Period: ______

Exit Ticket 7.18

Explain how you would find the distance from START to the second turn located at $(4, 2)$.

Sample response:

- I would find the coordinates of START first, $(12, -8)$, and the first turn, $(4, -8)$, to find the length of the first line segment, 8 (finding the difference of 12 and 4).
- Then I would continue the same process for the second line segment. The points are $(4, -8)$ and $(4, 2)$. The length of this segment is 10. (I can add the absolute values of the y-coordinates because the segment crosses the x-axis.)
- Finally, I would find the sum of 8 and 10, which is 18.



Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine the lengths of multiple horizontal and vertical segments on the coordinate plane.
1 2 3

b. I can explain how coordinates can be used to model real-world locations.
1 2 3

c. I can use given information to determine solutions to puzzles on the coordinate plane.
1 2 3

Lesson 18 Lost and Found Puzzles

Success looks like . . .

- **Goal:** Determining the total length of multiple horizontal and vertical segments in the coordinate plane that are connected end-to-end.
  - » Explaining how to determine the total distance from START to the second turn in the maze.
- **Language Goal:** Explaining that coordinates can be a useful way of describing geometric figures or modeling real-world locations. **(Speaking and Listening)**

Suggested next steps

If students only count by 2s to find the distance, consider:

- Reviewing the strategies for finding the length of line segments using ordered pairs in Activity 1.
- Having students write the coordinates of the first two points and then saying, "Because the y-coordinates are the same, you can subtract the x-coordinates."
- Reviewing how to use absolute values to calculate the length of a vertical line segment crossing the x-axis.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

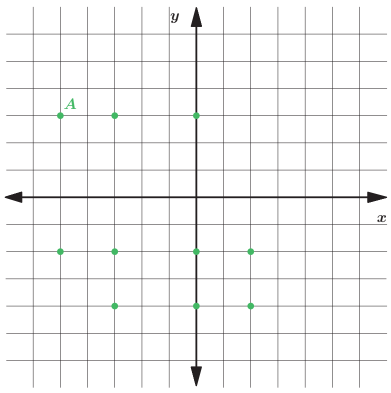
- How was student work with mazes in Activity 2 similar to or different from their work with mazes in Lesson 14?
- In what ways did Activity 2 go as planned? In what ways did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

Use the points plotted in this coordinate plane for Problems 1–3. The coordinates of point A are $(-15, 9)$.



1. What is the scale of the axes? Explain your thinking.
   The scale increases by 3s on both axes. I know because -15 along the x-axis is 5 ticks to the left of the y-axis, and 9 along the y-axis is 3 ticks up from the x-axis.

2. Connect the points to create a maze-like path starting from point A. The line segments should not intersect each other.
   Answers will vary, but should show a path connecting all of the points without intersecting.

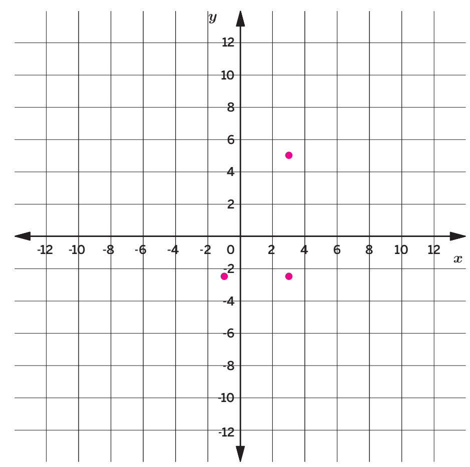
3. What is the total distance along your path from point A to the end of the path?
   Answers will vary, but should be accurate based on the scale of the grid being 3 units.

4. Explain how to calculate a number that is equivalent to $\frac{2.1}{1.5}$.
   Sample response: Divide 2.1 by 1.5, or 21 by 15, to obtain 1.4. Students could also divide both 2.1 and 1.5 by 0.3 to obtain $\frac{7}{5}$.

Name: Date: Period:

5. Lin's family needs to travel 325 miles to reach her grandmother's house.
   a. After they have traveled 26 miles, what percent of the distance to her grandmother's house have they completed?
      8%; $\frac{26}{325} = 0.08$
   b. How far will they have they traveled when they have completed 72% of the distance to her grandmother's house?
      234 miles; $325 \cdot \frac{72}{100} = 234$
   c. Suppose they traveled 377 miles. What percent of the distance to her grandmother's house have they completed?
      116%; They are past her grandmother's house by 16%, or 52 miles beyond her house (assuming they traveled in the same direction the entire time), $\frac{377}{325} = 1.16$.

6. Plot the following points on the coordinate plane: $(3, 5)$, $(3, -2.5)$, $(-1, -2.5)$.



Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 8	1
	5	Unit 3 Lesson 13	2
Formative	6	Unit 7 Lesson 19	1–3

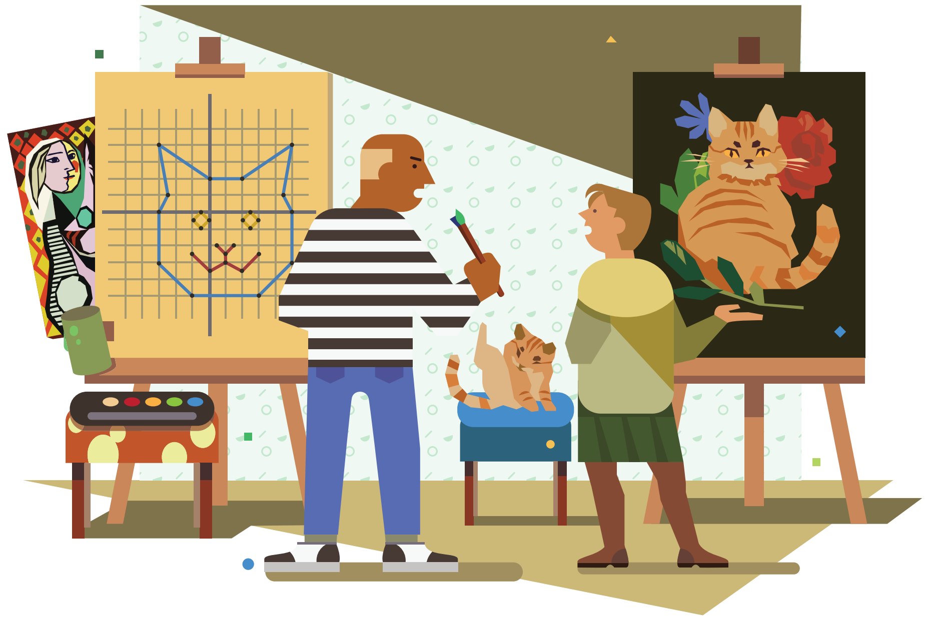
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Drawing on the Coordinate Plane

Let's draw on the coordinate plane.



Focus

Goal

1. **Language Goal:** Generate a list of ordered pairs to create an image in the coordinate plane, and explain the reasoning. **(Speaking and Listening)**

Coherence

Today

In the culminating lesson of this unit, students plot ordered pairs to create more artistic images. Students determine the ordered pairs needed to make their images look precisely as they intended. This allows them to apply their accumulated knowledge from across the unit – thinking about the axes as number lines, using absolute values to determine distances, working with signs in ordered pairs of coordinates, and leveraging reflections across an axis and distance from zero to create symmetries. When possible, using graphing technology or the digital version of this lesson is highly recommended.

Previously

Throughout this unit, students extended previous understandings of the number system to now include signed rational numbers. They also used mathematical tools such as inequality statements, number lines, and the four-quadrant coordinate plane, in order to describe and represent the relationship between rational numbers, identify solutions to inequalities, and interpret quantities in real-world contexts.

Coming Soon

Students will continue to work with the four-quadrant coordinate plane as a mathematical tool for representing and solving real-world and mathematical problems. In Grade 7, they will extend their work with discrete graphing of ratio relationships to continuous graphs of proportional relationships, driving toward graphing all linear functions, and other relationships, in Grade 8.

Rigor

- Students **apply** rational number and coordinate graphing concepts to creative drawings on a coordinate plane.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Independent	Pairs, Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- graph paper (if completed offline only)

Math Language Development

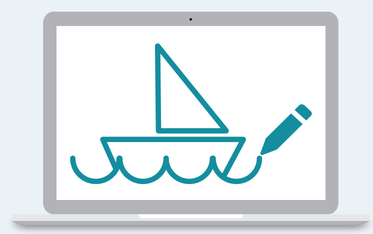
Review word

- *quadrant*

Amps | Featured Activity

Activity 1
Drawing on the Coordinate Plane

Students create a digital image of their own design, easily adjusting their ordered pairs until the image looks just the way they want it.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might get frustrated if the results of their drawing or their calls are not as precise as desired in Activity 1. Remind students to think about their partner and how they feel about the failed results. Have students find ways to encourage each other, providing constructive feedback without being critical of the other person.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 1**, the number of points can be reduced to a maximum of twelve (three points in each quadrant), or Part 2 may be omitted.

Independent | 10 min

Warm-up Cat Pictures

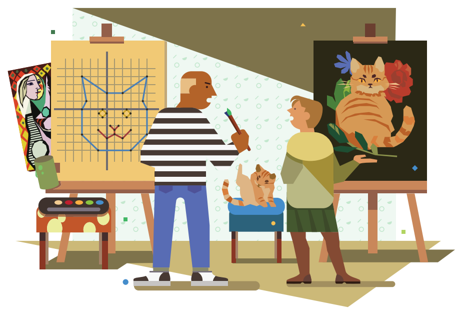
Students use the structure of the coordinate plane to identify and plot ordered pairs in creating a detailed image of a cat's face.

Name: ____________ Date: ________ Period: ________

Unit 7 | Lesson 19 – Capstone

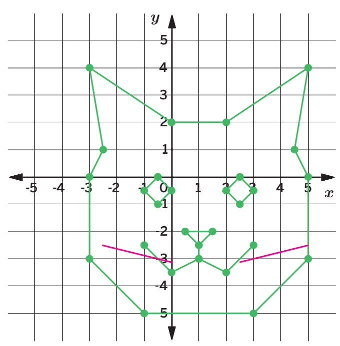
Drawing on the Coordinate Plane

Let's draw on the coordinate plane.



Warm-up Cat Pictures

Refer to the image of the cat drawn on the coordinate plane.



1. List the coordinates of the points plotted that form the cat's eyes.

   Left eye: $(-0.5, 0)$, $(-1, -0.5)$, $(-0.5, -1)$, $(0, -0.5)$
   Right eye: $(2.5, 0)$, $(3, -0.5)$, $(2.5, -1)$, $(2, -0.5)$

2. Add more detail to the image, such as whiskers, the inside of the ears, a bow, or whatever you would like — but you must add at least three new points. Then add the coordinates to your list.

   Sample answers:
   Whiskers on the right: $(2, -2.5)$, $(4, -2)$
   Whiskers on the left: $(0, -2.5)$, $(-2, -2)$

Log in to Amplify Math to complete this lesson online.

Lesson 19 Drawing on the Coordinate Plane 853

1 Launch

Display the cat face and ask students what they notice.

2 Monitor

Help students get started by asking, "Is this picture centered on the coordinate plane? How do you know?"

Look for points of confusion:

- **Labeling points as if the origin were the center of the picture.** Ask, "What is the scale on the x- and y-axes? Where is the origin, $(0, 0)$? How does that change the ordered pairs you wrote?"
- **Identifying and/or plotting non-integer numbers incorrectly.** Have students label each grid line along the axis and ask, "What is a value between the tick marks for ___ and ___?"

Look for productive strategies:

- Using the scale to accurately identify the origin and the signs and coordinates of each point.

3 Connect

Ask, "Because the image has a vertical line of symmetry that is not the y-axis, how does that affect the relationship between the coordinates of symmetric points on the image (e.g., the tops of the eyes)?"

Have students share their additions to the image, any challenges they experienced, and how they overcame them.

Highlight that, similar to when students created polygons and mapped paths through mazes, their points should be an ordered list of ordered pairs. When drawing on the coordinate plane, this helps them to know which points to connect with a straight line. If there is more than one polygon, such as the two cat eyes, they should list their ordered pairs on separate lines.

Power-up

To power up students' ability to plot points with rational number coordinates:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up.

Informed by: Performance on Lesson 18, Practice Problem 6.

Pairs | Small Groups | 25 min

Activity 1 Image Race

Students design an image on the coordinate plane, and then compete against classmates to recreate and guess what each other's image is.

Amps Featured Activity Drawing on the Coordinate Plane

Activity 1 Image Race

Part 1

With your partner, draw a recognizable image on a piece of graph paper. Your picture must include the following

- A vertical and a horizontal axis
- At least four points in each quadrant of your coordinate plane
- A maximum of 20 points

Part 2

Pair up with another group to compete in a drawing game using each other's images. Play the game as follows:

- With your partner, choose who will be the artist, and who will be the caller.
- The callers on each team exchange images. Do not let the artists see the drawing.
- The callers direct the artists, but are only allowed to give the following types of directions:
  - » Where to draw a vertical or horizontal axis
  - » The scale to label the ticks on an axis
  - » The coordinates of a point to be plotted
  - » Whether to connect two points
- As they draw, the artists try to guess what the image is.
- The first team to guess correctly wins.

STOP

854 Unit 7 Rational Numbers

1 Launch

Provide each student with a piece of graph paper. Review the directions as a class, and reiterate that their image should be recognizable, like the cat in the Warm-up. Explain that, in Part 2, the caller can tell the artist only three things: the coordinates of a point, directions to connect the points, or directions to start a new shape. The first team to guess correctly wins. Give students 10 minutes to draw their pictures, and 10 minutes to play the game. Use the ***Gallery Tour*** routine to display student work.

2 Monitor

Help students get started by offering ideas for simple images such as still life objects – a flower, a vehicle, a building, or an animal other than a cat.

Look for points of confusion:

- **Plotting points haphazardly.** Have students designate an x- and y-axis and sketch a more freeform image that extends to all four quadrants before identifying points.
- **Identifying or plotting coordinates incorrectly.** Ask, "In the quadrant ____, are the ____-coordinates positive or negative? What happens to the numbers as you move left/right/up/down along the ____-axis?"

Look for productive strategies:

- Using the distance from the axes and between points to adjust ordered pairs until the image looks just the way they want.

3 Connect

Display student images around the room, and conduct the ***Gallery Tour*** routine.

Have pairs of students share their strategies for plotting their original images, if time permits.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing students with a pre-labeled coordinate plane using a scale of 1 on each axis.

Accessibility: *Guide Processing and Visualization*

Provide more detailed guidance around selecting and constructing images. Consider providing a set of images from which students could use to construct a similar image on the coordinate plane, such as flowers, animals, buildings, cars, or landscapes. Suggest students begin by drawing larger sections with less detail and add more detail later, as desired.

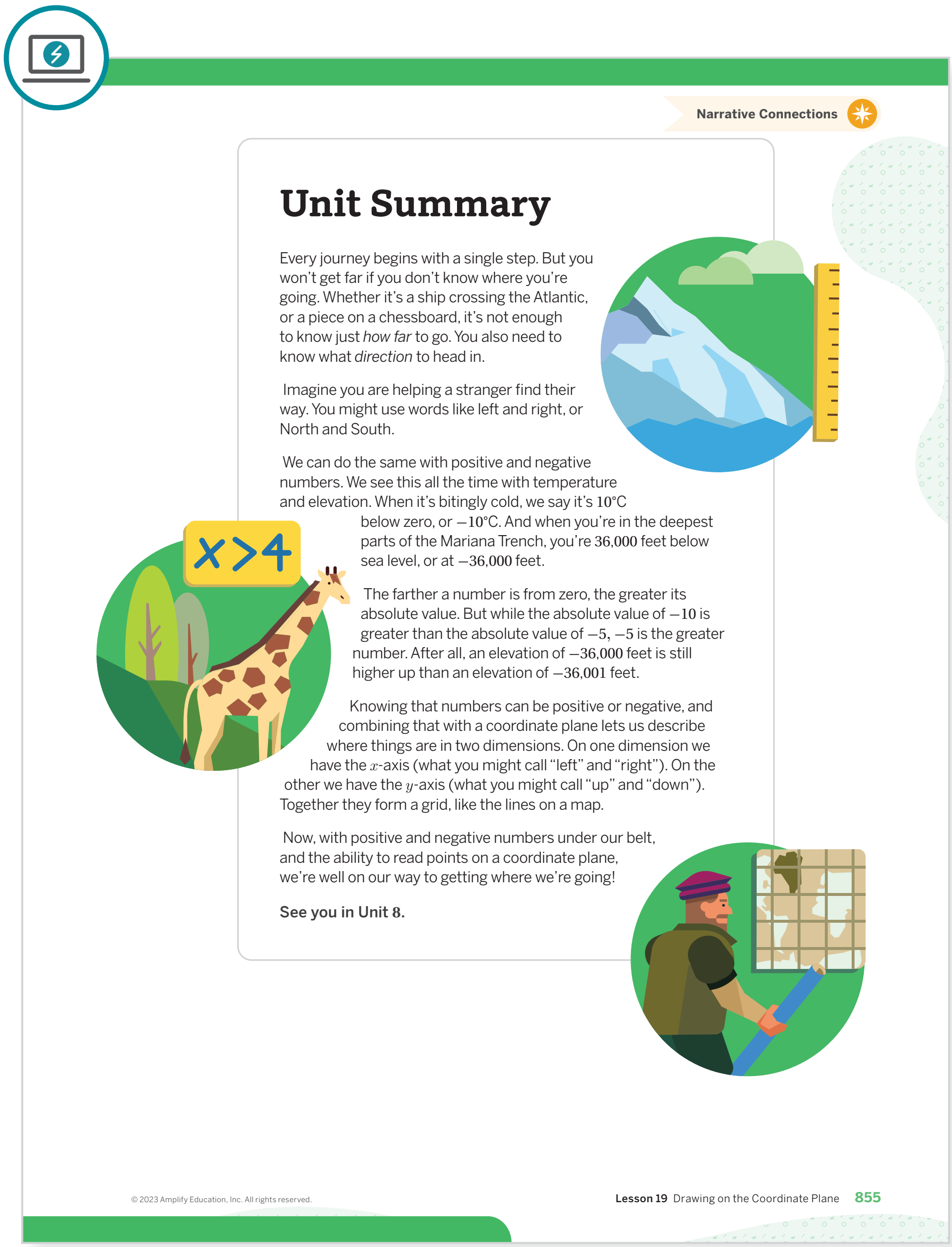
Math Language Development

MLR2: Collect and Display

As students participate in the ***Gallery Tour***, ask them to highlight and reflect upon the key words and phrases they learned during this unit, such as *magnitude, direction, quadrants, positive, negative, integers, rational numbers*, etc. Keep the class display posted and suggest students refer to it as they share their strategies for plotting their images, as time permits.

Unit Summary

Review and synthesize how this lesson connects to student learning across this unit.



Narrative Connections

Unit Summary

Every journey begins with a single step. But you won't get far if you don't know where you're going. Whether it's a ship crossing the Atlantic, or a piece on a chessboard, it's not enough to know just *how far* to go. You also need to know what *direction* to head in.

Imagine you are helping a stranger find their way. You might use words like left and right, or North and South.

We can do the same with positive and negative numbers. We see this all the time with temperature and elevation. When it's bitingly cold, we say it's 10°C below zero, or -10°C. And when you're in the deepest parts of the Mariana Trench, you're $36{,}000$ feet below sea level, or at $-36{,}000$ feet.

The farther a number is from zero, the greater its absolute value. But while the absolute value of -10 is greater than the absolute value of -5, -5 is the greater number. After all, an elevation of $-36{,}000$ feet is still higher up than an elevation of $-36{,}001$ feet.

Knowing that numbers can be positive or negative, and combining that with a coordinate plane lets us describe where things are in two dimensions. On one dimension we have the x-axis (what you might call "left" and "right"). On the other we have the y-axis (what you might call "up" and "down"). Together they form a grid, like the lines on a map.

Now, with positive and negative numbers under our belt, and the ability to read points on a coordinate plane, we're well on our way to getting where we're going!

See you in Unit 8.

Lesson 19 Drawing on the Coordinate Plane 855

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Ask students to quietly reflect, mentally or in writing, on how this lesson connects to their learning throughout this unit.

Have students share their reflections on their learning.

Highlight the connections students make from drawing and plotting their images to the mathematics of the unit, focusing on those who mention signed rational numbers, absolute value and distance, and location and movement in the four quadrants of the coordinate plane.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the unit narratives. To help them engage in meaningful reflection, consider asking:

- Where exactly would you measure from in order to find a person's true height?
- What other kinds of signs can you think of that describe relationships that are "greater than or equal to" or "less than or equal to" a number?
- What's more useful on a map: having accurately-sized geographical features, or having accurate latitude and longitude?

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying at least one mathematical action demonstrated in their image creation process.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 7.19

Explain one mathematical action demonstrated while creating your image in the lesson.

Sample response: When I drew my image, I wanted it to be symmetric across the y-axis, so I had to keep thinking about distance each point was from 0. If I placed one point at $(5, 1)$, I needed to place the symmetrical point at $(-5, 1)$.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use ordered pairs to draw a picture.

1 2 3

Lesson 19 Drawing on the Coordinate Plane

Success looks like . . .

- **Language Goal:** Generating a list of ordered pairs to create an image in the coordinate plane, and explain the reasoning. **(Speaking and Listening)**
  - » Presenting their reasoning for choosing points for their image.

Suggested next steps

If a student cannot describe any mathematical action that relates to the content learned in this unit, consider:

- Asking one or all of the following:
  - » "When you created your own image, did you plot points in only the first quadrant?"
  - » "When you made the ordered pair list for your image, how did you identify the points? What if the points were between the grid lines?"
  - » "When you were drawing your opponent's image, how did you know where to plot the points? How did that help you guess the image before it was fully created?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this Capstone lesson, students plotted ordered pairs to create more artistic images. How did this apply their accumulated knowledge from across this unit?
- From this lesson and the unit in its entirety, what trends do you see in participation? What might you change — for the next time you teach this lesson, or more immediately, for the next unit — to increase the variety of voices heard and methods of participation?

Independent

Practice

Practice

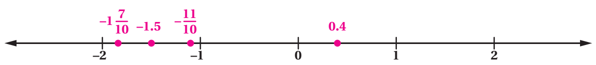
Name: ______ Date: ______ Period: ______

1. Order the following temperatures, in degrees Fahrenheit, from coldest to warmest:
   - 5 degrees above zero
   - 3 degrees below zero
   - 6 degrees above zero
   - $2\frac{3}{4}$ degrees below zero

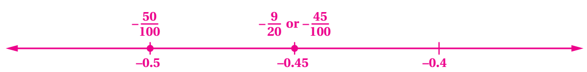
   3 degrees below zero, $2\frac{3}{4}$ degrees below zero, 5 degrees above zero, 6 degrees above zero

2. Plot each of the following numbers on the number line. Label each point with its numeric value.

0.4	-1.5	$-1\frac{7}{10}$	$-\frac{11}{10}$



3. Which value is greater: $-\frac{9}{20}$ or -0.5? Show your thinking. Consider plotting the numbers on a number line to help with your thinking.



$-\frac{9}{20}$ is greater than -0.5. Sample response: I know this because $-\frac{9}{20}$ is equivalent to $-\frac{45}{100}$, and -0.5 is equivalent to $-\frac{50}{100}$. The fraction $-\frac{45}{100}$ is greater than $-\frac{50}{100}$.

Name: ______ Date: ______ Period: ______

Practice

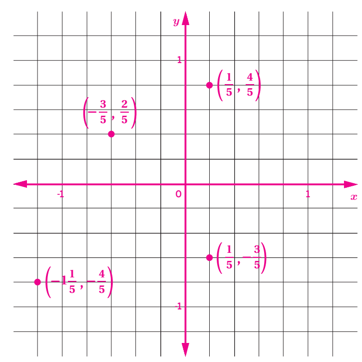
4. Write a number that matches each description.
   a. A number whose value is equal to the absolute value of -12.9.
      12.9
   b. A number whose absolute value is equal to 5.
      Sample response: -5
   c. A positive number whose value is less than $|4.7|$.
      Sample response: 4
   d. A negative number whose absolute value is greater than $|-2.6|$.
      Sample response: -3

5. Noah said, "If a is a rational number, $-a$ will always be a negative number." Do you agree with Noah? Explain your thinking.

   Sample response: I disagree with Noah because $-a$ means "the opposite of a." If a is -2, the opposite is 2, which is not a negative number.

6. Draw and label an appropriate pair of axes and plot the points.

$\left(\frac{1}{5}, \frac{4}{5}\right)$	$\left(-\frac{3}{5}, \frac{2}{5}\right)$	$\left(-1\frac{1}{5}, -\frac{4}{5}\right)$	$\left(\frac{1}{5}, -\frac{3}{5}\right)$



Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 7 Lesson 2	2
	2	Unit 7 Lesson 3	2
	3	Unit 7 Lesson 4	2
	4	Unit 7 Lesson 7	2
	5	Unit 7 Lesson 8	2
	6	Unit 7 Lesson 14	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

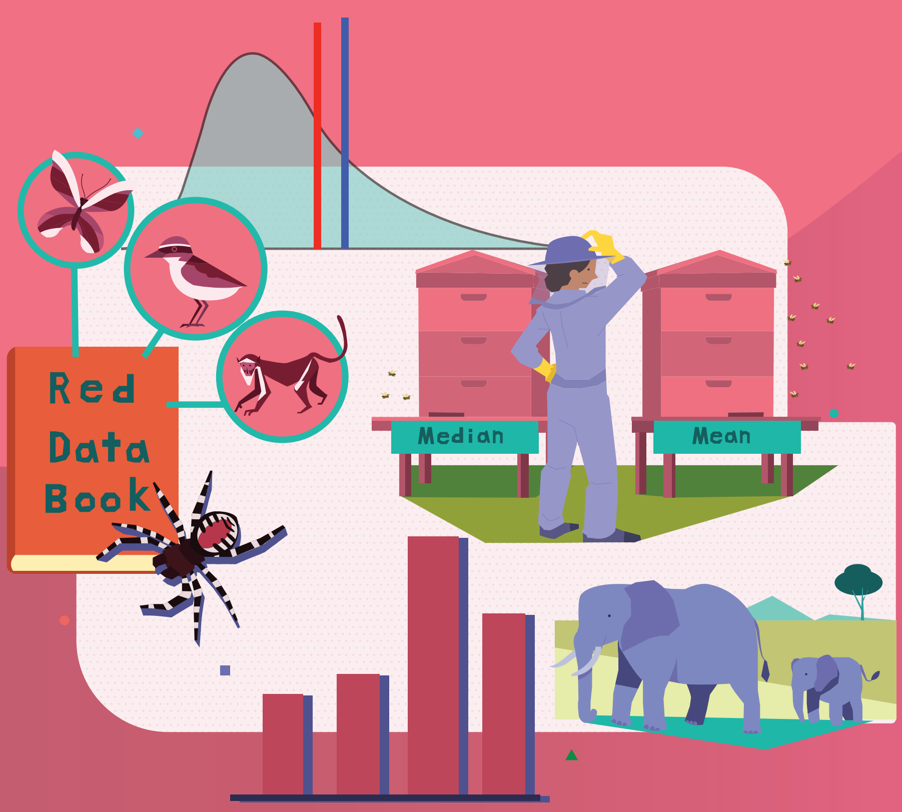
UNIT 8

Data Sets and Distributions

In this unit, students learn about populations and study variables associated with a population, focusing on populations of animal species and their respective endangerment classifications. They distinguish numerical and categorical data, relative to survey and statistical questions, and represent and describe the distributions of response data. Students first interpret frequency tables, dot plots, and histograms, before calculating measures of center — mean and median — and measures of variability — mean absolute deviation (MAD), range, and interquartile range (IQR). They then construct box plots in addition to interpreting these measures in context, relating the shape and features of a distribution to the best choice of measures.

Essential Questions

- What makes a question statistical?
- What does a measure of center tell you about a distribution?
- What does a measure of variability tell you about a distribution?
- *(By the way, if mean people get MAD, do median people get IQR?)*



Key Shifts in Mathematics

Focus

In this unit . . .

Students analyze data to answer statistical questions, particularly those involving numerical data, accounting for variability and interpreting their answers in context. They shift from line plots and bar graphs to dot plots and histograms, as well as box plots to represent and visualize the distribution of a data set. Initially, they identify typical values, focusing qualitatively on general features and characteristics related to the center and spread of a distribution. Then students move to summarizing data sets quantitatively by calculating measures of center and variability. Lastly, they consider which measures of center and variability are most appropriate for a given distribution and context, and justify their choices.

Coherence

Previously . . .

In Grades 2–5, students represented data using bar graphs and line plots. The analysis was limited to frequency comparisons and some minor calculations involving frequencies or data values.

Coming soon . . .

In Grade 7, students will extend their work with statistics to include sampling and probability.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students recognize and interpret the overall shape of a distribution and its meaningful features (Lessons 2–7). They also make sense of how to summarize a set of numerical data with a single number to describe its center (Lessons 8–10) and its variability (Lessons 12–15).

Procedural Fluency

Students understand and follow the processes for creating different visual representations of numerical data — dot plots, histograms, and box plots (Lessons 4, 6, and 14). They also become proficient at calculating the mean and median of a data set (Lessons 9 and 11), as well as MAD, range, and IQR (Lessons 13 and 14).

Application

In addition to many applications of statistics in context, students also apply their understanding of measures of center and variability to compare measures for a data set, and then choose the appropriate measures, based on the distribution and context (Lessons 11, 16, and 17).

Walk on the Wild Side With Data

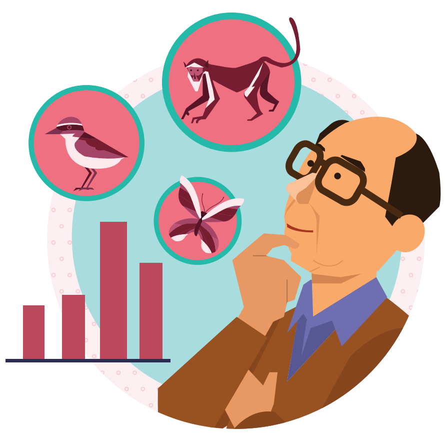
SUB-UNIT

1

Lessons 2–7

Statistical Questions and Representing Data

Students recognize and answer ***statistical questions*** involving numerical data by analyzing ***dot plots*** and ***histograms*** and by describing the overall shape, ***center***, and ***spread*** of these distributions.



Narrative: Collecting data on the populations of animals can help us understand if they are at risk for extinction.

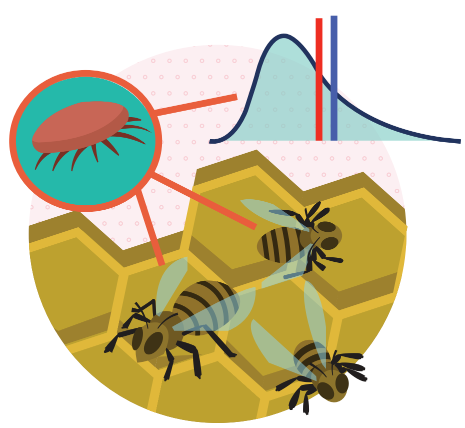
SUB-UNIT

2

Lessons 8–11

Measures of Center

Students formalize their understanding of the center of a distribution by determining different ***measures of center***, the ***mean*** and ***median***. They recognize how each measure may be affected by ***outliers*** and based on the shape of the distribution, choose an appropriate measure.



Narrative: To understand the issues facing honey bees, we need precise numbers to describe them.

Launch

Lesson 1

Plausible Variation or New Species?

Students informally work with data and statistical questions involving variability in order to determine whether a spider is a different-looking member of an existing species or a newly discovered species altogether. The scientific names will not disappoint pop culture fanatics!

SUB-UNIT

3

Lessons 12–16

Measures of Variability

Students formalize their understanding of the ***variability*** of a distribution by determining different ***measures of variability***, the ***mean absolute deviation***, ***range***, and interquartile range. Based on the shape of the distribution, they choose an appropriate measure. Students also construct and analyze ***box plots***.

Narrative: Measures of variability in data help us understand even more about the manatee population.

Capstone

Lesson 17

Asian Elephant Populations

Students apply the analytical and procedural math skills learned in this unit to a real-world scenario involving Asian elephant populations. They consider the IUCN Red List classification of the Asian elephant and its implications from different perspectives. In their final Exit Ticket, students reflect on their growth as mathematicians and collaborators over the school year.

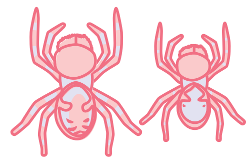
Unit at a Glance

Spoiler Alert: You can determine which measures of center and variability are most appropriate by considering the distribution of the data, the context, and the questions being asked.

Assessment

A Pre-Unit Readiness Assessment

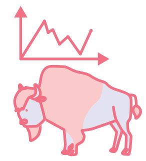
Launch Lesson



1 Plausible Variation or New Species?

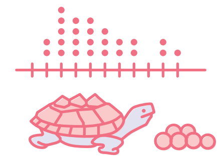
Determine whether an unspecified spider is a variation or new species.

Sub-Unit 1: Statistical Questions and Representing Data



2 Statistical Questions

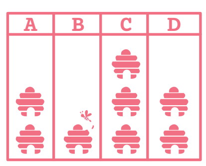
Analyze questions, the variables being investigated, appropriate units, and expected responses to define statistical questions.



3 Interpreting Dot Plots

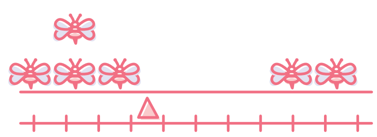
Represent and interpret distributions, including typical values, by using ordered lists, frequency tables, and dot plots.

Sub-Unit 2: Measures of Center



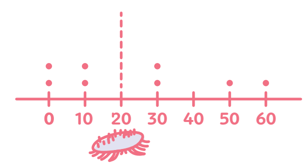
8 Mean as a Fair Share

Determine and interpret the mean of a numerical distribution as a fair share or equal redistribution.



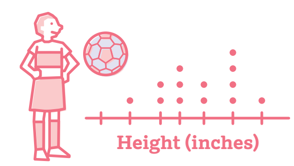
9 Mean as the Balance Point

Interpret the mean as a balance point of a numerical distribution.



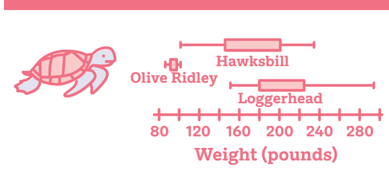
10 Median

Determine and interpret the median for a numerical data set.



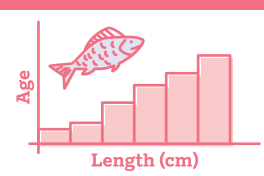
11 Comparing Mean and Median

Justify whether the mean or median is a more appropriate measure of center for a distribution in context.



15 Box Plots

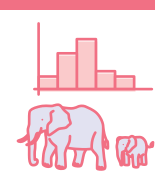
Use the five-number summary to create a new type of visual representation of data.



16 Comparing MAD and IQR

Determine what the different measures (mean and MAD or median and IQR) represent in context and select an appropriate representation for distributions.

Capstone Lesson



17 Asian Elephant Populations

Select and justify an appropriate representation for distributions in a real-world context.

Assessment

A End-of-Unit Assessment

Key Concepts

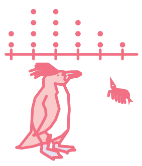
Lesson 2: Define a statistical question as one expecting variability.
Lesson 11: Determine the best measure of center for a given distribution and context.
Lesson 16: Determine the best measure of variability for a given distribution and context.

Pacing

17 Lessons: ~45–50 min each
3 Assessments: 45 min each

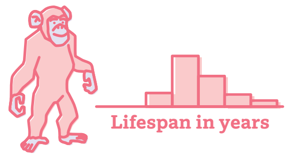
Full Unit: 20 days
● **Modified Unit:** 16 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



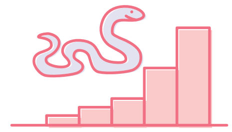
4 Using Dot Plots to Answer Statistical Questions ●

Use informal observations of the center and spread to describe distributions and identify typical values.



5 Interpreting Histograms

Compare and contrast information representation in dot plots and histograms.



6 Using Histograms to Answer Statistical Questions

Read and interpret histograms based on the shape, spread, and center.

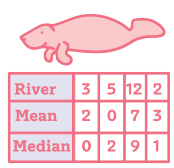
7 Describing Distributions With Histograms

Use the structure of histograms to look for symmetry, peaks, clusters, gaps, and any unusual values.

Assessment

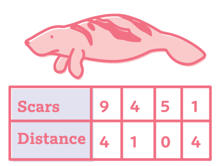
A Mid-Unit Assessment

Sub-Unit 3: Measures of Variability



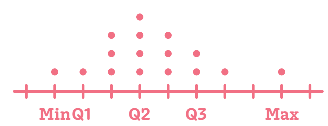
12 Describing Variability ●

Explore variability of a distribution and ways to describe it.



13 Variability and MAD ●

Determine and interpret the MAD as a single number description of variability.



14 Variability and IQR

Determine the five-number summary of a distribution, and the range and IQR. Use these measures as another way to describe variability.

● Modifications to Pacing

Lessons 3 and 4: These two lessons may be combined. After completing either Warm-up, Lesson 3 could become one activity, for students to consider the same data presented multiple ways: an unorganized list, frequency table, and dot plot. Introduce the terms frequency, mode, and distribution, and then have students complete Activity 2 from Lesson 4 to distinguish between typical values, center, and spread.

Lessons 8 and 9: These two lessons may be combined, as long as students see both interpretations of the mean (as a fair share and equal redistribution), how to calculate the mean using division, and how to identify a missing data value given the mean. One option is to do Lesson 8, Activity 1, Part 1 as a whole class, and then have students complete Lesson 9, Activity 1. If there is time, they could also work on Lesson 8, Activity 2 (perhaps just Problem 3).

Lessons 12 and 13: These two lessons may be combined, although, if possible, students should still be given the opportunity to explore and "invent" ways of summarizing variability with a single number, such as by extending and using the Lesson 12, Warm-up and asking them to also to suggest one number to summarize the variability in each dot plot, before then proceeding through Lesson 13.

Lesson 17: The Capstone lesson may be omitted, but students miss out on the opportunity to apply their analytical and mathematical skills to a real-world scenario with messy data. It is highly recommended you still present the opportunity for them to reflect on their year.

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	categorical data numerical data statistical question variability*
3	distribution dot plot frequency mode
4	center spread
5	histogram
6	maximum minimum
8	average mean measure of center
10	median
12	variability
13	mean absolute deviation (MAD)
14	five-number summary interquartile range (IQR) quartile range
15	box plot

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
1, 6, 9	**MLR1:** Stronger and Clearer Each Time
1–6, 8, 10–15	**MLR2:** Collect and Display
1, 4, 7	**MLR5:** Co-craft Questions
4	**MLR6:** Three Reads
3, 5–8, 10, 13–16	**MLR7:** Compare and Connect
4, 5, 8, 11, 13	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket

Additional Practice

Additional required materials include:

Lesson(s)	Materials
11–13	calculators
10, 15	index cards
1, 2, 4, 5, 7, 9–17	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
8	snap cubes
5–7, 15	straightedges
17	tools to create a visual display

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
14, 15	Notice and Wonder
13	Number Talk
5	Take Turns
8, 12	Think-Pair-Share
6, 7	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 11
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 17

Amps
POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

Balancing Bumble Bees

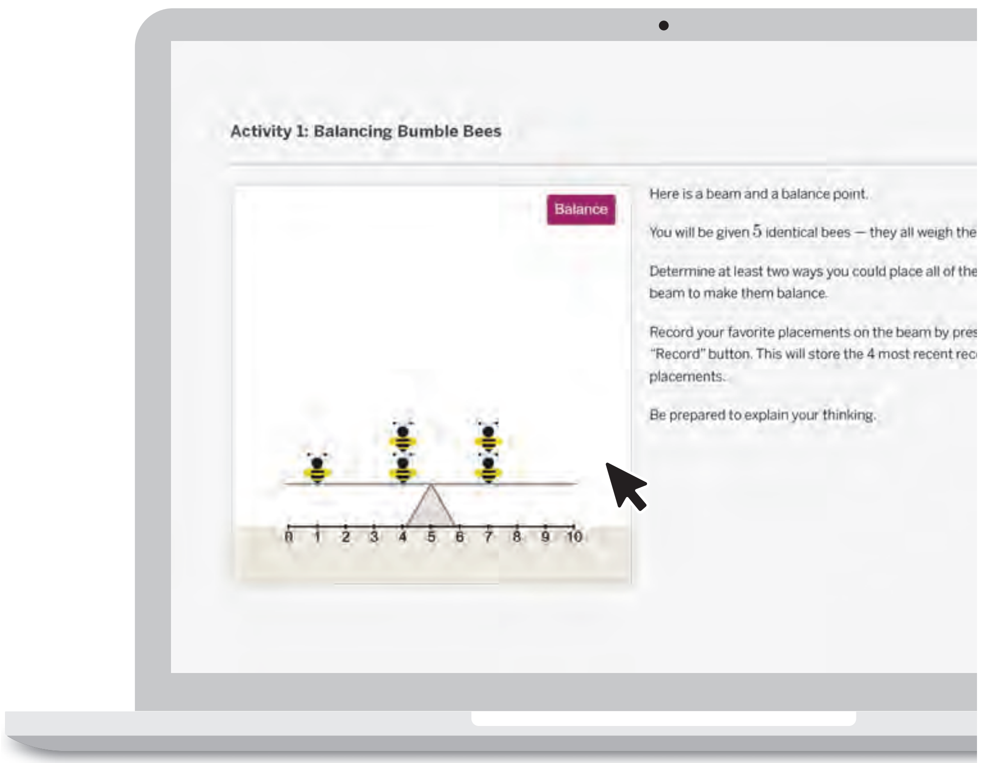
Put on your student hat and work through **Lesson 9, Activity 1:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- The Hunt for Red Krill **(Lesson 4)**
- Monarch Butterfly Migration **(Lesson 7)**
- Moving the Middle **(Lesson 11)**
- Living Box Plot **(Lesson 15)**



Unit Study
Professional Learning

This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

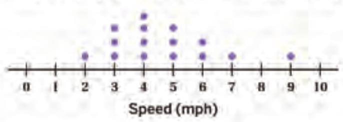
Sub-Unit 3 introduces students to two measures of variability — the mean absolute deviation (MAD) and the interquartile range (IQR) — and how these measures respectively tie to the mean and the median. Students learn the relevant statistics while analyzing data of populations of endangered species. Students understand that data are messy and can be misinterpreted (or data can be used to make misleading claims). Therefore, learning statistics is paramount in helping them interpret data appropriately and hopefully find a worthy cause that they may champion.

Do the Math

Put on your student hat and tackle these problems from Lesson 14, Activity 2:

Activity 2 Range and Interquartile Range

Here is a dot plot that shows 15 recorded speeds of a manatee, in miles per hour.



1. Write the five-number summary for this data set. Show your thinking.

   Minimum: **Q1:** **Q2:** **Q3:** **Maximum:**

2. One way to describe the *spread* of values in a data set is to look at the difference between the maximum and minimum values. This is called the ***range***. What is the range of the speeds of the manatee?

3. Another way to describe the spread of values in a data set is to look at the difference between the upper quartile (Q3) and the lower quartile (Q1). This is called the ***interquartile range (IQR)***.
   a. What is the interquartile range (IQR) of this manatee's speeds?
   b. How does the IQR relate to typical values?

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

- What was it like to engage in this problem as a learner?
- A student wants to know why they need to learn about the interquartile range (IQR) when they can just use the range. How might you address this?
- How might you help students differentiate when it's more appropriate to use MAD versus IQR in discussing the variability of a data set?

Focus on Instructional Routines

Which One Doesn't Belong?

Rehearse . . .

How you'll facilitate the ***Which One Doesn't Belong?*** instructional routine in Lesson 6, Warm-up:

Warm-up Which One Doesn't Belong?

Four questions about the population of Alaska are shown. Which question does not belong? Be prepared to explain your thinking.

a. At what age do Alaska residents generally retire?
b. At what age can Alaskans vote?
c. What is the age difference between the youngest and oldest Alaska residents with a full-time job?
d. Which age group makes up the largest percent of the population: under 18 years, 18–24 years, 25–34 years, 35–44 years, 45–54 years, 55–64 years, or 65 years or older?

Point to Ponder . . .

- How will you support students in the routine differently here because the mathematical information presented in each possibility is a text-based question?

This routine . . .

- Fosters a need to define terms carefully and use words precisely in order to compare and contrast a set of mathematical information presented in a common format.
- Offers a "low floor" point of entry for all students, in that typically each of the four options "doesn't belong" for a different reason, and the similarities and differences are mathematically significant.
- Prompts students to explain their rationale for deciding that one option doesn't belong, and then possibly refine their rationale.

Anticipate . . .

- Students may focus on the words and questions without thinking about the corresponding answers and data, and they may need a nudge in that direction if they struggle to choose one that does not belong.
- Students may think there is only one possible correct response, but should be reminded that reasoning is more important than choice.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Pose purposeful questions.

This effective teaching practice . . .

- Helps you assess the reasoning behind student responses. They may arrive at a correct response using flawed reasoning; probing for their reasoning helps you know if they truly understand the concept.
- Helps you advance student reasoning and sense making by asking deeper questions about mathematical ideas and relationships.

Points to Ponder . . .

- How can you probe for student reasoning to ensure they understand a mathematical concept, beyond just providing a correct response?
- What kinds of questions can you ask that will help advance their understanding to a deeper level?

Math Language Development

MLR7: Compare and Connect

MLR7 appears in Lessons 3, 5–8, 10, and 13–16.

- In Lesson 5, students make connections between dot plots and histograms to see how each display shows the center and spread. Rotate the displays 90 degrees clockwise to illustrate how the bar heights of the histogram compare to the heights of the dots on the dot plot.
- In Lesson 16, students compare and contrast dot plots, box plots, and histograms and discuss the similarities and differences in the information that each display provides.
- **English Learners:** Multiple strategies are provided to support students' understanding of mathematical language, including providing sentence frames, providing sufficient wait time for students to formulate a response, annotating the class display with mathematical terms and phrases, and allowing students to speak first in their primary language.

Point to Ponder . . .

- How can you help students draw connections between the multiple statistical representations in this unit, instead of only presenting each type of display separately?

Differentiated Support

Accessibility: *Optimize Access to Tools, Optimize Access to Technology*

Opportunities to provide physical manipulatives, the use of technology (through the Amps slides), or other tools appear in Lessons 1, 3–12, and 15–17.

- In Lesson 5, students can virtually compare shapes of distributions by overlaying dot plots and histograms.
- In Lesson 8, students can experiment with, and discover, the mean as an equal redistribution — or "fair share" — by relocating honey bee hives among different fields.
- In Lesson 8, consider providing students with physical objects that they can use to experiment with equal redistribution to visualize the mean.
- In Lesson 11, students can use technology to see, in real time, how adding values to a data set affects the mean and median.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology, physical manipulatives, or other tools to deepen student understanding?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with analyzing and interpreting data, in visual representations and through calculations? Do you think your students will generally:
  - Grapple with the notion that, in statistics, there is inherent variability and uncertainty, meaning there is not always a single numerical answer and there is some subjectivity to responses?
  - Have difficulty constructing or interpreting visual representations of numerical data?
  - Understand how to perform the necessary calculations, but struggle to determine which is more appropriate to apply in a given scenario?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their relationship skills and self-management.

Points to Ponder . . .

- Can students act in a way that maintains healthy relationships as they discuss and debate statistical questions? Are they able to use the language of statistics to clearly communicate their arguments?
- Do students display the self-discipline required for a thorough and accurate analysis of data? Are they motivated to understand what the data tell them? Do they have the organizational skills required to create graphs, if needed?

Plausible Variation or New Species?

Let's determine whether a spider is just a rare variation or a new species.

Focus

Goals

1. **Language Goal:** Use observational and numerical data as evidence to construct an argument or counterargument. **(Speaking and Listening, Writing)**
2. **Language Goal:** Describe examples of expected variation and of variation that would not be expected or typical. **(Speaking and Listening, Writing)**

Coherence

Today

Students build an informal understanding of variability – a critical foundation of statistical thinking. They make observations and analyze data to draw conclusions about spiders in the *Loureedia* genus. Students take on the role of zoologists classifying a specimen as an existing species or a new species. They first focus on describing the physical characteristics, recognizing that variations exist within the genus. Then they collaborate to review and analyze information about distribution (location), markings, and measurements to form initial claims or provide skeptical counterarguments to rule out possibilities. Throughout the lesson, students ask and answer questions involving data with variability (priming them to work with statistical questions), consider what might be typical or reasonable variation (a focus throughout the unit), and use data analysis as evidence.

Previously

In Grades 2–5, students represented and interpreted data using bar graphs and line plots.

Coming Soon

In Lessons 2–7, students will begin to identify and answer statistical questions by representing, analyzing, and interpreting data in context, starting by informally describing typical values and the shape of a distribution.

Rigor

- Students build a **conceptual understanding** of variability as they use observable characteristics and ask questions to determine whether a spider is a rare variation or new species.

Note: This lesson could be used as supplementary material to the Amplify Science *Traits and Reproduction* unit.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

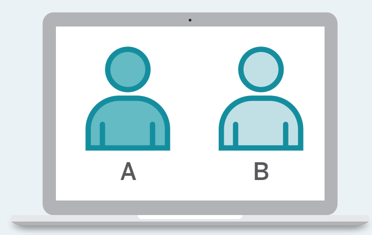
Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per group

Amps | **Featured Activity**

Activity 1
Digital Collaboration

Students claim whether an unclassified spider is a variation or new species. They jigsaw with another group, using evidence to provide counterarguments.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might disagree over an acceptable amount of variation and turn the disagreement from a mathematical discussion to something personal. Before students begin the activity, discuss the rules for engagement. Have students set boundaries for the discussions with the goal of showing respect to others at all times. Ask the groups to hold each other accountable to following those rules.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be completed as a class.
- In **Activity 1**, Part 2 may be omitted. After groups complete Part 1, have them share their unique claims and evidence with the class, one at a time. If time permits, have students determine one piece of evidence that could provide a counterargument for each claim.

Independent | 5 min

Warm-up Genus Variation

Students describe the physical characteristics of six spiders from three different known species in the *Loureedia* genus, recognizing that physical variations exist.

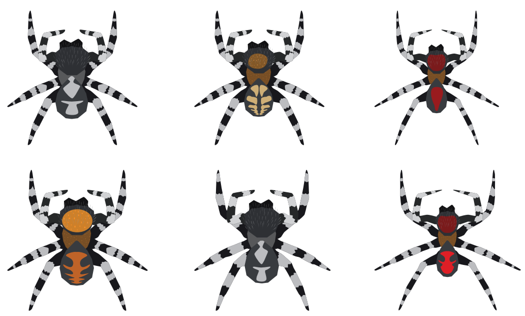
Unit 8 | Lesson 1 – Launch

Plausible Variation or New Species?

Let's determine whether a spider is just a rare variation or a new species.

Warm-up Genus Variation

Analyze the images of 6 specimens of male spiders belonging to the *Loureedia* genus. Briefly describe what you notice and think might be true about the sizes, shapes, colors, markings, and any other features of this genus of spiders. Use words such as: *mostly, generally, typically, normally, sometimes, rarely, never*, etc.



Sample responses:

- Generally, they have 8 striped legs.
- Most have a rounded head and an oval body.
- Typically, they have patterns or markings on their bodies.
- The spiders are normally small, about the size of a pebble.
- Some spiders are black and white, and some have bright colors like orange or red.
- They rarely have the exact same pattern on their bodies.
- None of the spiders are one solid color.

Log in to Amplify Math to complete this lesson online.

1 Launch

Supply background knowledge by reviewing the taxonomic ranks with students (*domain, kingdom, phylum, class, order, family, genus, species*). Explain that, in this lesson, they will be examining spiders in the *Loureedia* genus.

2 Monitor

Help students get started by asking, "Do all six spiders look exactly alike? What is different about them?"

Look for points of confusion:

- **Not thinking about quantities when using adverbs.** Consider providing a familiar example such as, "In this class, students typically . . ." Then ask, "Does typically refer to all students? Most students? Very few students?"

Look for productive strategies:

- Using mathematical benchmarks (e.g., all, half, fewer than half, etc.) to state conclusions.
- Using the provided words about relative frequency accurately and appropriately.

3 Connect

Have students share their responses, asking whether others agree or disagree and why.

Ask, "How would you describe any spider belonging to the *Loureedia* genus?" Sample responses: 8 black and white striped legs and distinctive markings on the bottom section that are usually white or red.

Highlight that members of a group are not always identical and often display variations that may be physical or behavioral. Explain that it can be expected that a group that is higher in the taxonomic ranks has more variation among its members. (Consider also noting that there can even be unique features or anomalies at the lowest level, where some may just be individual differences and others could be genetic mutations — such as in humans, and how even identical twins have differences).

Math Language Development

MLR2: Collect and Display

During the Connect, collect the phrases students use to describe the spiders and add these phrases to a class display, such as, "Typically, they have patterns" and "The spiders are normally small." Highlight that such phrases are used to describe the relative frequency of these traits in this genus. In addition to frequency, note the language that students may use to describe variation.

English Learners

Clarify the meaning of the terms *specimen* and *genus* for students as these terms may be unfamiliar to many students.

Small Groups | 15 min

Activity 1 Beyond a Reasonable Doubt?

Students use observational and numerical data to claim whether an unclassified specimen is a member of an existing species, or a new species, in the *Loureedia* genus.

Amps Featured Activity Digital Collaboration

Name: ____________ Date: ________ Period: ________

Activity 1 Beyond a Reasonable Doubt?

You will be given fact sheets for 3 known species of spiders belonging to the *Loureedia* genus and one unclassified specimen, as seen here.

Part 1

With your group, decide whether you believe the unclassified specimen belongs to any of the three known species or is a new, previously unidentified species. Record your observations and the scientific or mathematical evidence you would use to support your claim.

Answers may vary. Sample responses should include comparisons of corresponding measurements of the specimen and all of the other species, and could also mention a need for additional specimens similar to the unclassified one, or suggest other tools, such as observation of behaviors or DNA collection, to rule out the possibility of, for example, *Loureedia colleni* ever having red markings.

Part 2

1. You will now jigsaw with other groups and take turns sharing your claims and the related evidence, one at a time. For each claim that does not match yours, take turns providing skeptical counterarguments. For example, you could say, "As a skeptic, I would say the specimen is not as long as the average *Loureedia annulipes*, but it may not be fully grown."

2. With your new group, write two to three questions that would help you come to a consensus about the classification. These can be questions you already answered in making your claims, new questions that could also be answered using the data available to you, or even questions that would require additional information. If a question requires more information, be sure to state that extra information, as well.

   Answers may vary. Sample responses could include using the ranges of average measurements and considering extreme cases not captured, or also referencing the numbers of specimens documented and used in determining the values for each species versus only the one unclassified spider.

Co-craft Questions: You will share your questions with another group. Work together to decide which questions might be the most helpful. You may decide to write new questions.

1 Launch

Arrange students into groups of 3–4 and have them complete Part 1 together. You can decide whether each group should come to a consensus first or allow individuals to form their own opinions. Then have one student from each group join other students representing different claims to complete Part 2 together.

2 Monitor

Help students get started by asking, "What information is on each card?"

Look for points of confusion:

- **Using only one data value or one possible data value to make a claim.** Have students compare each data value. Ask, "Has your claim changed?"
- **Thinking the ranges of values exhaust all possibilities.** Note that the average human is 5–6 ft tall, and ask, "Are there any humans taller or shorter than that?"

Look for productive strategies:

- Systematically comparing the unknown species' data to the data of the other species (e.g., one data value at a time across all cards, or one entire species at a time).
- Having a clear argument for how much variation is reasonable within a species and which data values may matter more or less than others.

3 Connect

Have students share the pieces of evidence that best supported their claims and counterarguments, followed by the questions that would bring consensus in Part 2, Problem 2.

Ask, "How might the number of specimens impact the data and your claims?"

Highlight that appearance is a good indicator that a new species may exist, but it is not evidence enough on its own. Rather, it is a cause or call for further investigation. Scientists draw conclusions by asking and investigating questions that yield data that can be compared to existing species.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can claim whether an unclassified spider is a variation or a new species. They jigsaw with another group, using evidence to provide counterarguments.

Math Language Development

MLR5: Co-craft Questions

Have pairs of students within each group work together to co-craft mathematical and/or scientific questions for Part 2, Problem 2. Then have pairs share their questions with another pair of students. Provide students with time to discuss their questions and any additional information they would need to answer their questions. Sample questions shown.

- Are there any extreme cases not represented here?
- How many specimens were documented and used to determine the values for each species?

English Learners

Circulate as students share their questions with one another and amplify exemplar questions by displaying them. This will give students an opportunity to build metalinguistic awareness as they compare their own questions to exemplar questions.

Small Groups | 15 min

Activity 2 Further Microscopic Evidence

Students use new evidence to determine more definitively whether the unclassified spider is a variation or new species in the *Loureedia* genus.

Activity 2 Further Microscopic Evidence

You will be given another data sheet that provides some measurements of the *pedipalps* – the antenna-like sensory organ on a spider's head – for the unclassified specimen and the three known species.

Analyze the measurements. Using that analysis and previously gathered information, investigate and make one final claim: Which species does the unclassified specimen belong to? Or, do you believe it is a new, previously unidentified species?

Be prepared to share all of the questions you asked and the evidence you used to answer them in support of your claim.

Answers may vary. Sample responses should include comparisons of the lengths of both the prolateral arms and the retrolaternal arms of the unclassified specimen to each of the other species, and may also reference the general shape of the conductor and include the real differentiating factor which is the ratio of those lengths (with the unclassified specimen be closer to a 1 : 1 ratio and the others being closer to 1 : 2).

STOP

1 Launch

Arrange students in groups of 3–4 (consider utilizing groups from Activity 1). Set an expectation for the amount of time that groups will have to work on the activity.

2 Monitor

Help students get started by asking, "Which values appear to be similar? Different?"

Look for points of confusion:

- **Making a claim simply based on the shape and appearance of the conductors.** Remind students that the images represent "typical" shapes but there could be some variation.

Look for productive strategies:

- Making a claim based on average values and expected variations of length measurements.
- Using a ratio comparing the lengths of Pa and Ra to justify the difference in shape of the unclassified specimen as being beyond expected variation.

3 Connect

Have groups of students share their final claims and evidence. As they share, ask whether others agree or disagree, and encourage them to use evidence to support their skeptical counterarguments.

Ask, "What information did you, or would you, consider to change your claim from Activity 1?"

Highlight that the unclassified specimen is, in fact, a new species, *Loureedia phoenixi!* The species was named after an American actor whose facial makeup in a movie resembles the male abdominal pattern of the species. The new species differs from the others in that its prolateral arm of the conductor is pointed and about the same length as the retrolateral arm. (Zamani and Marusik, 2020) Interested students can research more how the new species got its name.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students organize their work in a systematic way, such as recording each measurement for the unclassified specimen and then determining how it relates to the same measurement for the other species. Suggest they form "mini-claims" that can then be combined to form a final claim.

Math Language Development

MLR1: Stronger and Clearer Each Time

After groups construct their final claim, have them share their claim with another group of students. Encourage groups to ask clarifying questions and press for mathematical and scientific evidence to support their claims. Provide clarifying prompts, such as:

- "How do you know that . . .?"
- "What evidence do you have to support your claim?"

Have groups revise their claims, as needed.

Whole Class | 5 min

Summary Walk on the Wild Side With Data

Review and synthesize the scientific, mathematical, and statistical processes students utilized during the lesson. Consider noting language that will be useful in this unit.

Narrative Connections

Unit 8 Data Sets and Distributions

Walk on the Wild Side With Data

In 2012, a team of international scientists journeyed to Israel. It was part of a broad study on a family of *Eresidae* spiders, or velvet spiders. It was there, in Israel's sandy terrain, that the team identified a new type of a velvet spider. These spiders are small, with velvety hairs, and live in underground burrows. Inspired, the researchers named them *Loureedia* after the 1960s rock icon Lou Reed and his band the Velvet Underground.

Much of what we know about the world's diverse species is thanks to the scientists that study and catalogue them. But what exactly is a *species*? Put simply, a species is a category of living things that share common characteristics and can breed together.

To date, there are only four species of *Loureedia* spiders in existence. Since their discovery, many of them have been sold in an illegal pet trade, putting them at risk of becoming endangered.

Deciding what counts as being part of one species rather than another can get complicated. But it is important for understanding when a species is at risk of extinction. That is why specialists monitor the populations of different species, sometimes enlisting the help of local observers and animal enthusiasts.

With reliable data, researchers can encourage humans to be more responsible, and protect all wildlife, no matter the number of legs.

Welcome to Unit 8.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Highlight that, in this unit, students will study data and variables associated with populations – much like the population of *Loureedia* spiders explored in this lesson. They will represent and interpret population data visually, and also numerically by doing some calculations, largely trying to identify and explain or describe occurrences of variation that seem reasonable and those that are not, as well as considering what all of that may mean in the given context.

When making decisions, new data may override previous conclusions. In most real-world problems, even mathematical ones, there is not one clear-cut answer. Sometimes a range of values makes sense, because variation is expected. Numbers and calculations can be used to identify and quantify variation, and help support claims about what is typical and what is not.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "How have you represented or interpreted data in previous grades? What did you enjoy the most? What was most challenging?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by summarizing how they determined whether a specimen was a variation or new species by using and asking questions to elicit data.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.01

Your work today was very similar to how scientists, specifically zoologists, often begin investigations of unclassified animals (and plants!). How did you and your group use data and evidence to determine whether the unclassified specimen was part of the *Loureedia* genus?

We considered similarities and differences in characteristics (data) that we could see or that were measured and recorded. We asked questions that we could answer using data or observed characteristics.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use both observational and numerical data to construct an argument or a counterargument.
1 2 3

b. I can describe at least one example of expected variation and at least one example of variation that would not be expected or typical.
1 2 3

Success looks like . . .

- **Language Goal:** Using observational and numerical data as evidence to construct an argument or counterargument. **(Speaking and Listening, Writing)**
  - » Determining whether the specimen was part of the *Loureedia* genus using data.
- **Language Goal:** Describing examples of expected variation and of variation that would not be expected or typical. **(Speaking and Listening, Writing)**

Suggested next steps

If students are unable to recall how they determined whether the unclassified specimen was part of the *Loureedia* genus, consider:

- Asking, "What did you do in the Warm-up? How did you build on that in Activity 1? What did you do with the new information given in Activity 2?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did Activity 1 set students up to develop an understanding of statistical questions in the next lesson?
- During the discussion about Activity 2, how did you encourage each student to share their claims? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. You see a herd of deer and want to know whether they are mule deer.

a Write two questions you could ask that would result in data you can observe.

Sample responses:
Generally, what color tail do the deer have?
Overall, what shape are the antlers?

b Write two questions you could ask that would result in data you need to measure.

Sample responses:
How much do the deer weigh?
How long are mule deer's antlers?

2. Darwin's bark spiders display an extreme example of dimorphism, with females being much larger than males. The *average* total body lengths can be represented as 0.8 +/– 0.1 in. for females, and 0.2 +/– 0.05 in. for males. The lengths listed here each represent a spider. For each length, state whether you believe it could reasonably be a female Darwin's bark spider (F), a male Darwin's bark spider (M), either a female or a male Darwin's bark spider (E), or is most likely not a Darwin's bark spider (N).

Sample responses based on ranges within 3 times the indicated variability for average lengths (i.e., females from 0.5 to 1.1 in. and males from 0.05 to 0.35 in.).

a 0.88 in. **F**

b 0.16 in. **M**

c 0.70 in. **F**

d 0.11 in. **M**

e 0.39 in. **N (or M)**

f 0.62 in. **F (or N)**

3. The largest known species of jumping spider is *Hyllus giganteus*. The average lengths of these spiders are 2.15 +/– 0.35 cm. Consider a *Hyllus giganteus* whose length matches the shortest possible average length based on this. If that spider recorded a jump covering a horizontal distance of 14.4 cm, then:

a What would be the horizontal distance for a similar jump by a *Hyllus giganteus* whose length matches the longest possible average length?

20 cm

b For fun, how far of a horizontal distance could you jump similarly, if you were a *Hyllus giganteus*? **Hint:** Use your height.

Answers may vary, but responses should reflect a horizontal distance of 8 times the student's height. (For a 67.5-in person, the distance would be 45 ft – approximately the length of a school bus.)

Name: ______ Date: ______ Period: ______

Practice

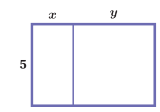
4. Order the following numbers from least to greatest.

$-4, \frac{1}{4}, 0, 4, -3\frac{1}{2}, \frac{7}{4}, -\frac{5}{4}$

-4	$-3\frac{1}{2}$	$-\frac{5}{4}$	0	$\frac{1}{4}$	$\frac{7}{4}$	4

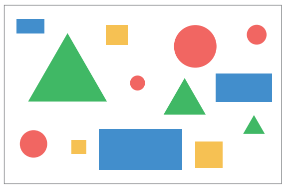
Least — Greatest

5. Select *all* the expressions that represent the total area of the largest, outlined rectangle.



A. $5(x + y)$
B. $5 + xy$
C. $5x + 5y$
D. $2(5 + x + y)$
E. $5xy$

6. Describe how you would sort the items in the group into two or more categories. Then write how many items would belong in each category.



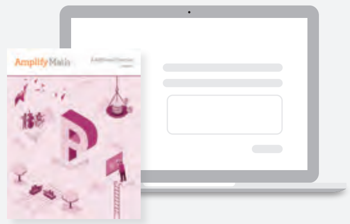
Answers may vary, but responses should describe what characteristic students used to sort (e.g., size, polygon name, number of sides, or a combination of those characteristics).

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 3	1
	5	Unit 6 Lesson 13	2
Formative	6	Unit 8 Lesson 2	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 1

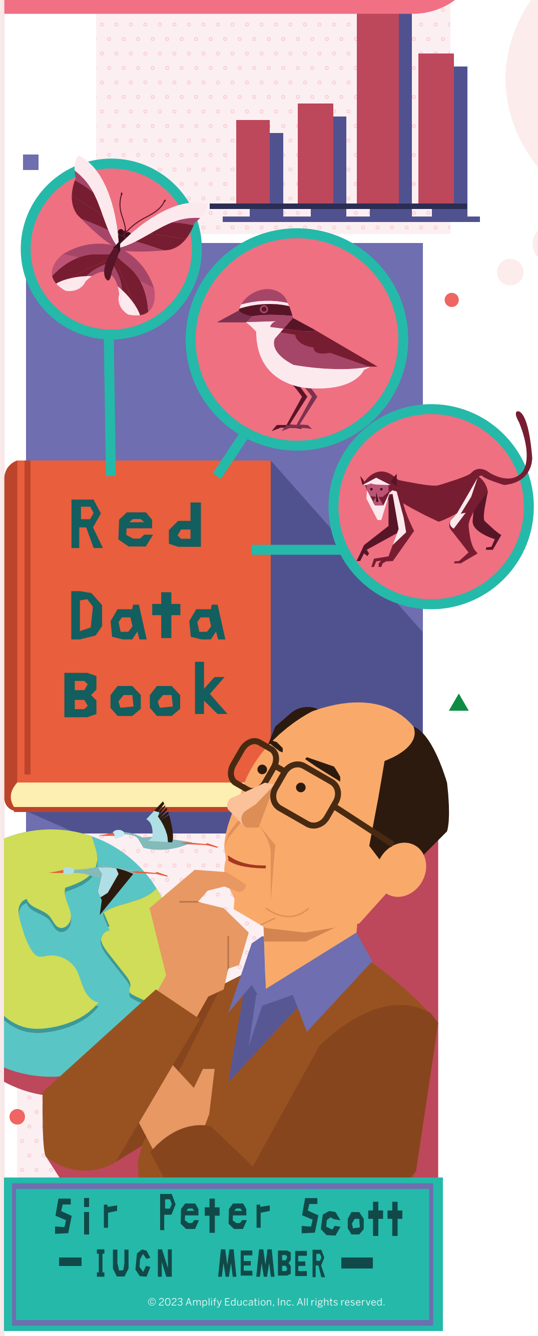
Statistical Questions and Representing Data

Whole Class

In this Sub-Unit, students answer statistical questions involving five different animal species and analyze dot plots and histograms to describe distributions and identify typical values.

SUB-UNIT

1 Statistical Questions and Representing Data



Narrative Connections

How do you keep track of a disappearing animal?

Imagine if a species of bird that normally visits your neighborhood suddenly disappeared. How would you know if they went south for the winter, or went extinct?

This was the problem Sir Peter Scott pondered in the 1950s and 60s. Scott was an avid ornithologist and a member of the International Union for Conservation of Nature (IUCN). Under the IUCN, he worked on a system that tracked different species in danger of extinction. Using reliable data, this system would document the populations of different species across the globe.

Finally, in 1966, Scott debuted the "Red Data Books." These books drew on data from observers and scientists around the world. The Red Data Books would eventually become the IUCN Red List still used today. These lists provide information on different species' population distributions. They categorize a species' extinction risk and provide a baseline against which a species' population can be measured.

With more and more species under threat, it is important to use every resource available. Data and statistics can be a powerful tool for understanding conservation issues. With these skills, conservationists can persuade people to act and ultimately help to save the natural world from going extinct.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how data and statistics can help understand conservation issues in the following places:

- **Lesson 2, Activity 1:** Matching Questions to Their Data
- **Lesson 3, Activities 1–2:** Organizing Data With Frequency Tables, Using Dot Plots to Represent and Describe Data
- **Lesson 4, Activities 1–2:** The Hunt for Red Krill, Seasonal Hunting Patterns
- **Lesson 5, Activity 1:** Chimpanzee Lifespans
- **Lesson 6, Activities 1–2:** Measuring Anitguan Racers, Match the Histogram
- **Lesson 7, Activity 2:** Monarch Butterfly Migration

Statistical Questions

Let's explore different kinds of data and the questions they can help answer.

Focus

Goals

1. **Language Goal:** Comprehend and use the terms *numerical* and *categorical* to describe data sets. **(Speaking and Listening, Writing)**
2. **Language Goal:** Justify whether a question is statistical based on whether variability is expected in the data that could be collected. **(Speaking and Listening)**
3. **Language Goal:** Match survey questions to data sets representing possible responses and justify why they match. Include units of measurement when reporting numerical data. **(Speaking and Listening)**

Coherence

Today

Students analyze different questions and the kinds of responses they can expect from those questions. They begin by sorting questions about the American bison, leading them to consider whether a question elicits numerical and categorical data. Students then focus on questions with numerical responses, and appropriate units of measure. They match questions to data sets and identify statistical questions. Students apply their understanding to generate their own statistical and non-statistical questions, explaining their thinking and critiquing the reasoning of others.

Previously

In Grades 1–5, students organized, represented, and interpreted categorical and numerical data in picture graphs, bar graphs, and line plots.

Coming Soon

In Lesson 3, students will represent and interpret numerical data in frequency tables and dot plots, and they will identify *typical values* for a given data set.

Rigor

- Students build **conceptual understanding** of variability in data that can be collected, helping them to distinguish between statistical and non-statistical questions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, pre-cut cards, one set per student
- Activity 1 PDF, pre-cut cards, one set per pair

Math Language Development

New words

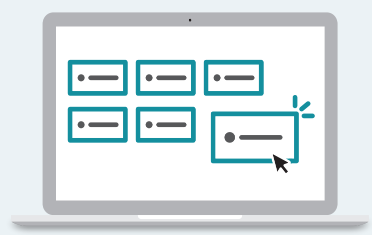
- ***categorical data***
- ***numerical data***
- ***statistical question***
- ***variability****

Note: Students will only be given a brief working definition of *variability* in this lesson, as a reference to being a feature of statistical questions. *Variability* in a distribution of data, as a measurable quantity, will be formally defined in Lesson 12.

Amps | **Featured Activity**

Activity 1
Digital Card Sort

Students match data sets to the questions they answer by dragging and connecting them on screen. Then they sort the pairs of cards into categories and name their new categories.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting Mathematical Practices

As students discuss the placement of the questions, they might forget to include their partner in the decision-making process. Remind students that the purpose of the activity is for both students to learn to construct and critique arguments as they match the cards. Remind students to listen to their partner and evaluate what is said.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, show students the questions already sorted into two categories. Have students use the *Think-Pair-Share* routine to name each category.
- In **Activity 1**, complete Part 3 as a class.

Independent | 10 min

Warm-up Sorting Questions

Students sort questions about the American bison, leading them to distinguish between numerical and categorical data.

Unit 8 | Lesson 2

Statistical Questions

Let's explore different kinds of data and the questions they can help answer.

Warm-up Sorting Questions

In 2016, the American bison (*Bison bison*) was named the national mammal of the United States. Despite this honor, the International Union for Conservation of Nature (IUCN) lists it as Near Threatened, because its survival is heavily dependent upon conservation efforts.

O.S. Fisher/Shutterstock.com

You will be given a set of cards with different questions about the bison on each card. Sort the questions into two or more categories, and name your categories. Be prepared to explain your thinking.

Answers may vary. Sample responses show how students may sort by the type of expected responses, the question words used, or different characteristics being investigated, such as weight.

Sorted by	Category names	Question
Type of expected responses	Numbers	2, 5, 6, 8
	Words	1, 3, 4, 7
Question words used	What	1, 4, 7
	How much/many	2, 5, 6, 8
	Other	3
Characteristic or topic being investigated	Appearance (color, horns)	1, 5
	Weight	2, 6
	Endangered status and threats	3, 7
	Food	4, 8

Log in to Amplify Math to complete this lesson online.

1 Launch

Distribute one set of cards from the Warm-up PDF to each student. Give students 2 minutes to sort independently before sharing with a partner.

2 Monitor

Help students get started by asking, "Do the question words help you?"

Look for productive strategies:

- Sorting based on the characteristic being investigated.
- Sorting by considering the question words and phrases. If students focus solely on sorting by the question words, ask, "Would you expect the same types of responses for every question in each group?"
- Using expected responses to sort into two groups — categorical and numerical.

3 Connect

Have students share how they sorted the questions, focusing on how they considered the type of responses each would yield.

Display the cards in two groups — categorical and numerical. Leave the groups unlabeled.

Ask, for each card, "What is being investigated? What data would you collect? How would you collect it? Are units of measurement appropriate here? What type of unit?"

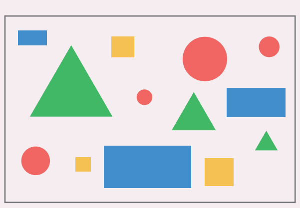
Define *numerical data* as responses that are measurements or quantities that can be meaningfully compared, and ***categorical data*** as responses that are words or labels and can be sorted into categories.

Highlight how to name the groups. Explain that data, such as phone numbers, are categorical data because the numbers are labels rather than meaningfully comparable quantities.

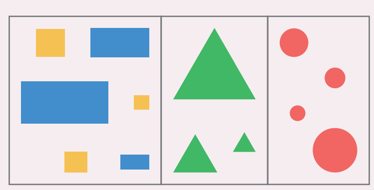
Power-up

To power up students' ability to sort a set of shapes in a meaningful way, ask:

Elena sorted the items shown into the following categories. Explain how she sorted them.



Elena's work:



Sample response: I think that Elena sorted the items by the number of sides. The first category of items has four sides, the second has three sides, and the third has zero sides.

Use: Before the Warm-up.
Informed by: Performance on Lesson 1, Practice Problem 6.

Pairs | 25 min

Activity 1 Matching Questions to Their Data

Students match numerical questions to their data sets, leading them to recognize variability and distinguish between statistical and non-statistical questions.

Amps Featured Activity Digital Card Sort

Name: ______ Date: ______ Period: ______

Activity 1 Matching Questions to Their Data

Part 1

Ten conservationists each answered all of the *numerical* questions from the Warm-up. You will be given another set of cards that each contain the responses of the ten conservationists to one of the questions. Match each data set with the one question to which it most likely corresponds.

Data Set A and Question 5
Data Set B and Question 2
Data Set C and Question 8
Data Set D and Question 6

Part 2

Sort the four pairs of cards (questions and corresponding data) from Part 1 into two or more categories based on the data sets. Be prepared to explain your thinking.

Answers may vary, but students should conclude that the data in Data sets A and B do not vary, while the data in Data sets C and D do vary.

1 Launch

Each pair should keep one set of cards from the Warm-up and be given one set of cards from the Activity 1 PDF. Pause after students complete Part 2 and have students share how they matched and sorted the cards. Provide a working definition of ***variability*** (which will be formally defined in Lesson 12) as a way to describe a data set with different values. Model how to label the table for Part 3 (consider creating this in a large format), and explain that categorical data can also have variability. Use the categorical cards from the Warm-up as examples, if needed. Then give pairs 2–3 minutes to complete Part 3.

2 Monitor

Help students get started by asking, "What are some reasonable answers for Question 5?"

Look for points of confusion:

- **Using unrealistic units to force matches, or not considering multiple viable data sets to help narrow and coordinate options.** Ask, "What units do those numbers represent? Does that make sense? Is there another unit and data set that could also work?"
- **Miscategorizing questions in Part 3.** Ask, "What type of answers do you expect? Are those measurements considered numbers or labels? Is there more than one possible and reasonable response?"

Look for productive strategies:

- Recognizing that some questions elicit different answers (variability), and using this to sort (Part 2).
- Generating questions by considering both the type of expected answers (categorical or numerical) and then also whether all answers will be the same (Part 3).

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing the matches for Part 1 and ask, "*Why* do these data sets match the questions being asked?" Have students complete Parts 2 and 3.

Extension: *Math Enrichment*

Have students choose one of the questions they wrote in Part 3 of the activity that did not anticipate variability and determine whether they can rewrite it so that it does anticipate variability.

Math Language Development

MLR2: Collect and Display

During the Connect, ask pairs of students to share how they wrote new questions for each category in Part 3. Collect the language they use and add it to a class display that students can refer to throughout the unit. Consider providing these sentence frames to help them organize their thinking.

- "_____ is/is not a statistical question because . . ."
- "_____ is a categorical/numerical question because . . ."
- "I am unsure about this question because . . . ."

Encourage students to respectfully challenge ideas and reasoning when they disagree.

English Learners

Provide students wait time to formulate their questions before sharing with a partner.

Pairs | 25 min

Activity 1 Matching Questions to Their Data (continued)

Students match numerical questions to their data sets, leading them to recognize variability and distinguish between statistical and non-statistical questions.

Activity 1 Matching Questions to Their Data (continued)

Part 3

With your partner, write one new question about the American bison for each category.
Sample responses shown.

	Categorical	Numerical
Data *without* variability	What are adult female bison called?	How old is that bison?
Data *with* variability	Where do bison live?	How long do bison typically live?

Collect and Display: Be prepared to share how you wrote new questions for each category. Your teacher will add the language you use to a class display that you can refer to during this unit.

Are you ready for more?

Tyler and Han want to collect data for the question, "Which sixth grader lives the farthest from school?"

- Tyler says, "The data that we collect will not have variability because only one person lives the farthest from school."
- Han says, "There will be variability in the data we collect. We would not actually be asking everyone, 'Which sixth grader lives the farthest from school?' Instead, we can ask, "How far do you live from school?' Responses to that question are expected to have variability."

Do you agree with either one of them? Explain your reasoning.

I agree with Han; Sample response: There will be variability if you answer the original question by actually asking a different, but related question: "How far away from school do you live?"

STOP

870 Unit 8 Data Sets and Distributions

3 Connect

Display the table you modeled labeling for Part 3 to record students' questions as they share.

Have pairs of students share one question at a time, either verbally stating the question and where it belongs in the table or by copying it onto a notecard, sticky note, or strip of paper and taping it onto the display. For each question shared, have the pair also share why it belongs there. Give others an opportunity to explain why they agree or disagree with the placement, and adjust the placements as necessary. Repeat this process until all cells have at least one valid question, or more as time allows.

Define a ***statistical question*** as a question that can be answered by collecting data that has variability.

Highlight that recognizing whether a question is statistical or not is important because data will need to be collected in order to answer a statistic question. Similarly, recognizing whether the responses to a question are categorical or numerical is important because that could impact how you collect the data and the tools needed. A data set alone does not usually answer a statistical question because it usually requires analysis of the data.

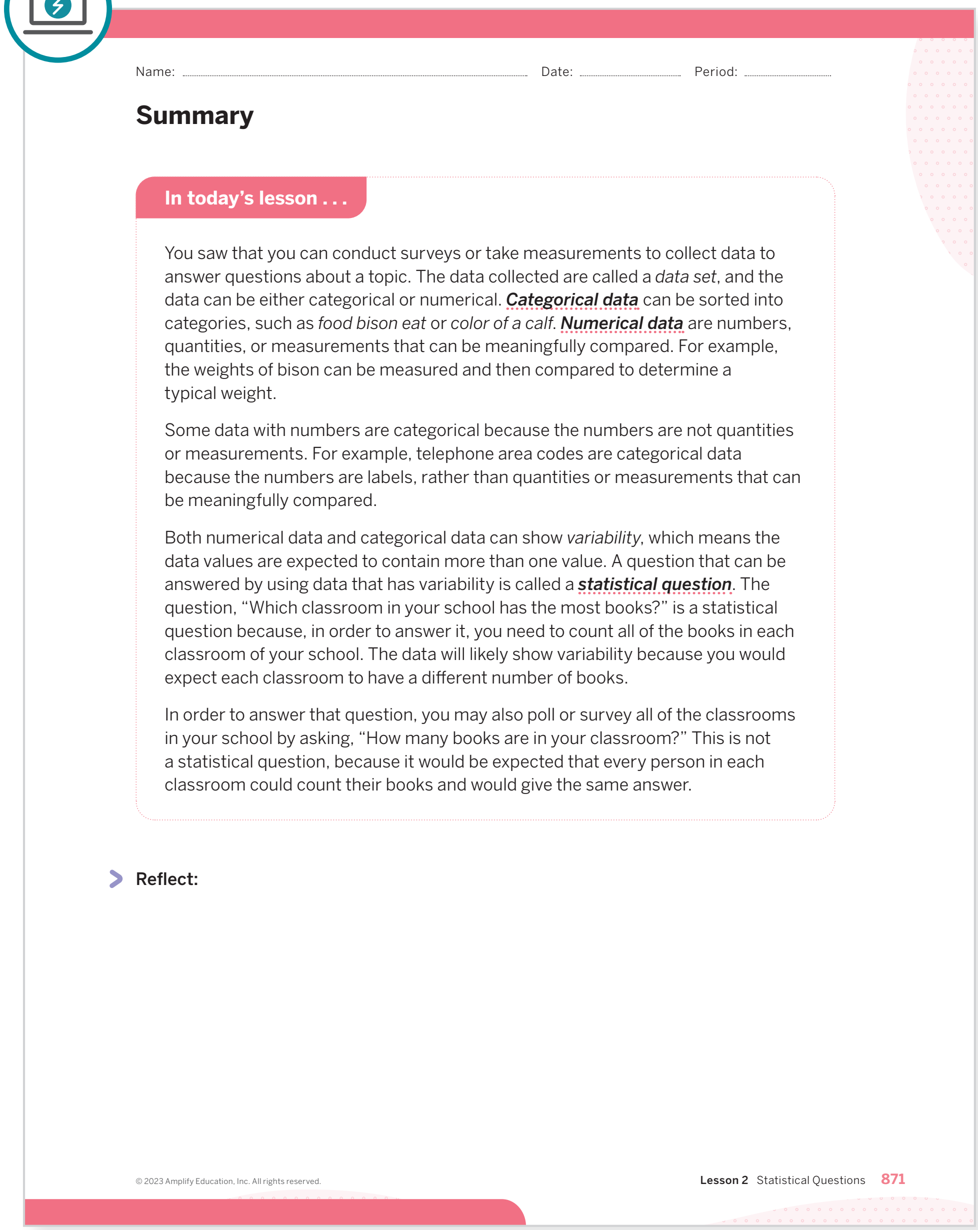
Ask:

- "Which entire rows or columns represent statistical questions?" Entire bottom row only.
- "Select one of your statistical questions and explain how you might collect the data, including required tools and units, to answer it? What units would be involved?" Answers may vary, but should indicate either asking survey questions for categorical data or numerical data. If asking for numerical data, measurement units should be appropriate for the question.

Whole Class | 5 min

Summary

Review and synthesize the relationship between variability and statistical questions.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that you can conduct surveys or take measurements to collect data to answer questions about a topic. The data collected are called a *data set*, and the data can be either categorical or numerical. ***Categorical data*** can be sorted into categories, such as *food bison eat* or *color of a calf*. ***Numerical data*** are numbers, quantities, or measurements that can be meaningfully compared. For example, the weights of bison can be measured and then compared to determine a typical weight.

Some data with numbers are categorical because the numbers are not quantities or measurements. For example, telephone area codes are categorical data because the numbers are labels, rather than quantities or measurements that can be meaningfully compared.

Both numerical data and categorical data can show *variability*, which means the data values are expected to contain more than one value. A question that can be answered by using data that has variability is called a ***statistical question***. The question, "Which classroom in your school has the most books?" is a statistical question because, in order to answer it, you need to count all of the books in each classroom of your school. The data will likely show variability because you would expect each classroom to have a different number of books.

In order to answer that question, you may also poll or survey all of the classrooms in your school by asking, "How many books are in your classroom?" This is not a statistical question, because it would be expected that every person in each classroom could count their books and would give the same answer.

Reflect:

Synthesize

- "How would you have approached your work in Lesson 1 differently if you had known about categorical and numerical variables, and statistical questions?"
- "When might you expect data to have no variability?"
- "Are the following questions statistical? Why or why not?"
  - » Who is the tallest student in the class?
    Statistical; Even if it is obvious who is the tallest, and everyone in the class would give the same answer (so there is no variability), that is not always the case. You need to collect data to answer the question by measuring the heights of each student in the class.
  - » How long is the longest river in the United States?
    Non-statistical; There is only one correct answer and you can readily look up the answer.

Highlight that characteristics, such as the length of the longest river in the United States are non-statistical because they are facts (assuming there are no new rivers discovered or no major changes to the lengths of relevant rivers occur). However, in order to be considered a fact, many questions like this were once treated as statistical. The lengths of all of the rivers in the U.S. had to be measured, collected, and compared in order to determine the answer. P.S., the answer is the Missouri River, 2,341 mi.

Formalize vocabulary:

- ***numerical data***
- ***categorical data***
- ***statistical question***
- ***variability***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- What makes a question statistical?

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on any terms and phrases related to the terms *numerical data*, *categorical data*, *statistical question*, and *variability* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by distinguishing statistical from non-statistical questions and identifying what data to collect and units to use.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.02

1. Determine whether each question is a statistical question. Explain your thinking.
   a. In general, students in your class generally live in which zip code?
      Statistical; Sample response: It is likely that students will live in different zip codes, so there will be variability in the data set.
   b. In which zip code is your school located?
      Non-statistical; Sample response: The question is asking about one school and there is one right answer. There will be no variability in the data set.
   c. How many hours did you spend outdoors today?
      Not statistical; Sample response: The question asks for only one person's response and there is one answer. There will be no variability in the data.
   d. How many hours did a typical sixth grader spend outdoors today?
      Statistical; Sample response: It is unlikely that every sixth-grader spent the exact same amount of time outdoors, so there will be variability in the data.

2. For each question you identified as statistical, describe how you would answer the question. What data would you collect? For numerical data, include the unit of measurement that you would use.
   Sample responses:
   - **Problem 1a: I would ask every student to state the zip code in which they live.**
   - **Problem 1d: I would ask each student the amount of time, in minutes, they spent outdoors today. Or, I would time how long each student is outside, in seconds.**

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain the difference between categorical and numerical data. 1 2 3

b. I can distinguish between non-statistical and statistical questions, based on the expected variability of the data. 1 2 3

c. I can identify the correct data to answer a question and use the correct units. 1 2 3

Success looks like . . .

- **Language Goal:** Comprehending and using the terms *numerical* and *categorical* to describe data sets. **(Speaking and Listening, Writing)**
- **Language Goal:** Justifying whether a question is *statistical* based on whether variability is expected in the data that could be collected. **(Speaking and Listening)**
  - » Determining whether each question is statistical and explaining why in Problem 1.
- **Language Goal:** Matching survey questions to data sets representing possible responses and justifying why they match. Include units of measurement when reporting numerical data. **(Speaking and Listening)**

Suggested next steps

If students misidentify a question as statistical or non-statistical, consider:

- Reviewing the table in Activity 3, and asking, "What makes a question statistical? Non-statistical?" Then ask, "Do you expect variability in the data for this question? Do you need to analyze the data to answer the question?"

If students do not mention using survey questions or taking measurements in Problem 2, consider:

- Asking, "What is a reasonable answer you would expect for this question? What would you do to get that answer?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did the sorting and matching exercises in the Warm-up and Activity 1 reveal about your students as learners?
- In what ways have your students improved at recognizing and making use of structure? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Sixth graders were asked, "What grade are you in?" Explain why this is *not* a statistical question.
   All of the students asked were in the sixth grade, so there will be no variability in their responses and data analysis is not required to answer the question.

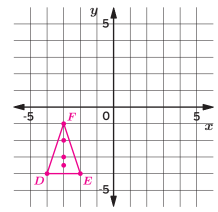
2. Lin and her friends went out for muffins after school. The following questions came up during their trip. Select *all* the questions that are statistical questions.
   A. How far is the bakery?
   B. What is the most popular muffin this week?
   C. What does a group of 4 people typically spend on muffins at this shop?
   D. Do kids usually prefer to get one large muffin or two mini muffins?
   E. How many flavors are there to choose from?

3. Here is a list of statistical questions. What data would you collect and analyze to answer each question? For numerical data, include the unit of measurement that you would use.
   a. What is a typical height of female athletes on the U.S. Olympic Gymnastics team?
   Collect the height, in inches, of female athletes on the team.
   b. Are most adults who work at the school football fans?
   Sample responses:
   Ask the adults who work at the school whether they are football fans.
   c. How long do drivers generally need to wait at a red light in Washington, D.C.?
   Sample responses:
   Time how long drivers wait in seconds at a red light in multiple intersections throughout Washington, D.C.

Practice

Name: ______ Date: ______ Period: ______

4. Triangle DEF has vertices $D(-4, -4)$, $E(-2, -4)$, and $F(-3, -1)$.
   a. Plot and connect the vertices of the triangle in the coordinate plane. Label the vertices.
   b. Name the coordinates of 3 points that lie inside the triangle.
   Sample responses: $(-3, -2)$, $(-3, -3)$, $(-3, -3.5)$



   c. What is the area of the triangle? Show or explain your thinking.
   3 square units
   Sample response:
   $\frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 2 \cdot 3 = 3$

5. Order these numbers from least to greatest:

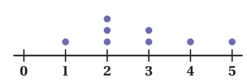
$\lvert 17 \rvert$	$\lvert -18 \rvert$	-18	$\lvert 19 \rvert$	20

-18	**$\lvert 17 \rvert$**	**$\lvert -18 \rvert$**	**$\lvert 19 \rvert$**	**20**

Least **Greatest**

6. Are the set of data and the line plot showing the same information? Explain your thinking.

2	1	5	2
3	4	2	4



No; Sample response: The set of data lists 3 once and 4 twice, while the dot plot lists 3 twice and 4 once.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1 Activity 2	2
Spiral	4	Unit 7 Lesson 17	2
	5	Unit 7 Lesson 8	2
Formative	6	Unit 8 Lesson 3	1

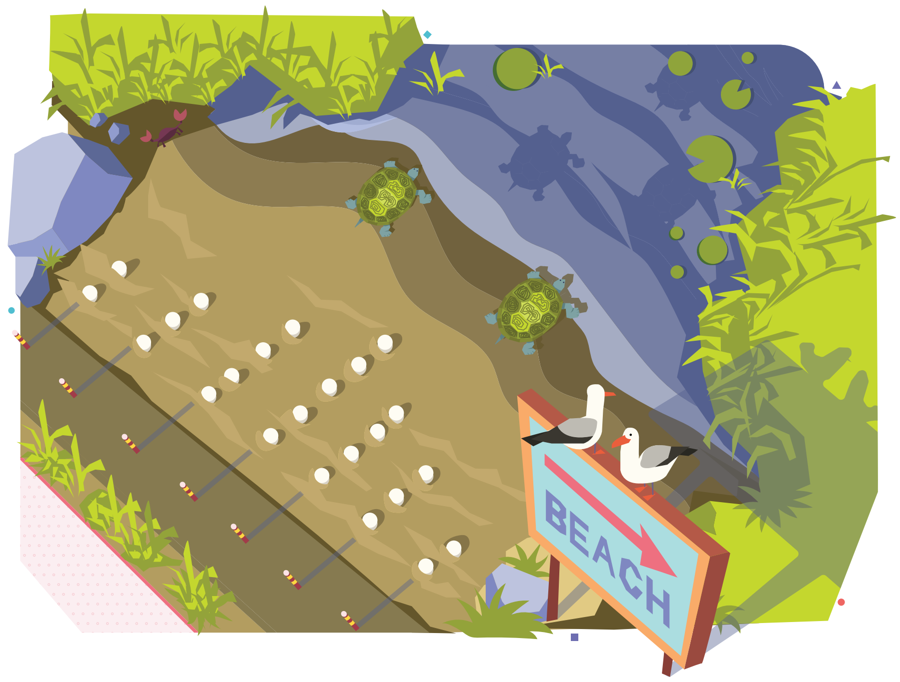
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Interpreting Dot Plots

Let's represent and interpret data with dot plots.



Focus

Goals

1. **Language Goal:** Comprehend the word *frequency* to refer to the number of times a particular value occurs in a data set. **(Speaking and Listening)**

2. Create and interpret a dot plot to answer statistical questions about a numerical data set.

3. **Language Goal:** Describe a distribution represented by a dot plot, including identifying what types of values are *typical* for the distribution. **(Speaking and Listening, Writing)**

Coherence

Today

Students represent distributions of numerical data by organizing them into ordered lists, frequency tables, and dot plots. They use the everyday meaning of the word *typical* to describe a characteristic of a group and justify their reasoning. This begins the idea that the values near the center of the distribution can be considered *typical* in some sense. Using dot plots, they develop a spatial understanding of distributions in preparation for a more mathematical understanding of the concepts *center* and *spread*. Students use dot plots to answer statistical questions in context.

Previously

In Grades 4 and 5, students created and interpreted line plots. In Lesson 2, students distinguished between categorical and numerical data, as well as statistical and non-statistical questions.

Coming Soon

In Lesson 4, students will describe distributions represented by dot plots and begin to describe center and spread.

Rigor

- Students build **conceptual understanding** of the distribution of a data set and what *typical values* mean in context.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New words

- ***distribution***
- ***dot plot***
- ***frequency***
- ***mode***

Review words

- *categorical data*
- *numerical data*
- *statistical question*
- *variability*

Amps | Featured Activity

Activity 2
Interactive Dot Plots

Students create dot plots to represent the data from their frequency tables. You can overlay student responses to provide immediate feedback.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively draw a conclusion from a dot plot without conducting a thorough quantitative investigation. Have students set goals for the process they will use to draw conclusions. Ask them to describe the steps they will take to make sure that their response is reasonable.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted, but you may want to consider having students begin **Activity 1** by ordering the list from the Warm-up first. Additional modifications for Activity 1 could include answering Problem 2 as a class, or having students use the ***Think-Pair-Share*** routine for Problem 3.
- In **Activity 2**, complete Problems 1 and 2 as a class before reviewing the definition of *distribution*. Have pairs of students complete the rest of the activity.

Independent | 5 min

Warm-up Counting Diamondback Terrapin Eggs

Students determine the most common value in a data set, recognizing the benefit of organizing data in order to interpret it.

Unit 8 | Lesson 3

Interpreting Dot Plots

Let's represent and interpret data with dot plots.

Warm-up Counting Diamondback Terrapin Eggs

The IUCN lists the Diamondback Terrapin (*Malaclemys terrapin*) as Vulnerable, meaning it faces a high risk of extinction in the wild. The decline in the terrapin population is attributed to two threats: crabbing and the loss of nesting lands caused by human development.

Jay Ondreicka/Shutterstock.com

A team of conservationists are studying the breeding patterns of terrapins in the Chesapeake Bay. Here are the data representing the number of eggs found in 25 different clutches (nests).

13, 11, 7, 9, 6, 6, 5, 8, 10, 7, 14, 11, 9, 6, 7, 8, 10, 6, 9, 8, 7, 6, 13, 5, 8

What was the most common number of eggs found in the clutches? Show or explain your thinking.

6 eggs

Sample response: I ordered the list of data from least to greatest, which was more efficient than counting the number of times that each value appeared in the list. 6 appeared five times, which means the most common number of eggs found in a clutch was 6.

$$\underbrace{5, 5}_{2}, \underbrace{6, 6, 6, 6, 6}_{5}, \underbrace{7, 7, 7, 7}_{4}, \underbrace{8, 8, 8, 8}_{4}, \underbrace{9, 9, 9}_{3}, \underbrace{10, 10}_{2}, \underbrace{11, 11}_{2}, \underbrace{13, 13}_{2}, \underbrace{14}_{1}$$

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work independently on the activity.

2 Monitor

Help students get started by asking, "What do you notice about the data set? Can you tell how many 13s there are? What might make that easier?"

Look for points of confusion:

- **Identifying the largest values as the most common.** Explain that *most common* means the value that appears the most often in the data set.
- **Confusing frequency with value (e.g., saying 5 because there are five 6s).** Ask, "What does the 5 in your response mean? What does the 6 mean? Which number – 5 or 6 — answers the question asked?"

Look for productive strategies:

- Counting or tallying the frequency of each value as they appear in the original, unordered list.
- Ordering the data from least to greatest and counting the frequency of each value.

3 Connect

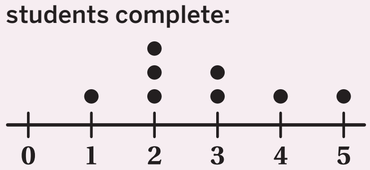
Have students share their answers and strategies, focusing on how they organized the data to efficiently identify the most common value.

Define *frequency* as a count of how often a value shows up in a data set. Usually, "most common" means "most frequent." The corresponding value is then called the ***mode*** of the data set.

Highlight that organizing the data by grouping common values together makes identifying and interpreting frequencies more efficient.

Power-up

To power up students' ability to construct and interpret line plots, have students complete:



Use the line plot to determine the number of times each of the following numbers appear in the data set:

a. 0 : 0

b. 1 : 1

c. 2 : 3

d. 3 : 2

e. 4 : 1

f. 5 : 1

Use: Before Activity 2.

Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 4 and 5.

Pairs | 10 min

Activity 1 Organizing Data With Frequency Tables

Students organize the data from the Warm-up in a frequency table, and use this representation to identify "typical" or expected values.

Name: ____________ Date: ________ Period: ________

Activity 1 Organizing Data With Frequency Tables

Data are often collected and analyzed to identify what is "typical," or expected, of that data. As a class, you will organize the data of turtle eggs from the Warm-up in a frequency table, and then analyze it to identify typical values.

Number of eggs	Frequency
5	2
6	5
7	4
8	4
9	3
10	2
11	2
12	0
13	2
14	1

1. Revisit your work and response from the Warm-up question. How might a frequency table make it more efficient to determine the most common number of eggs in a clutch? Explain your thinking.

   Sample response: The frequency table is more efficient because the counts for each value have already been done, so I can just scan the frequency column to identify the greatest value, which corresponds to the value in the number of eggs column that occurred the most.

2. The lead conservationist discovers another clutch in the marsh.

   a. Would you expect 10 or more eggs or less than 10 eggs in this clutch? Explain your thinking.

   I expect less than 10 eggs because more than half (72%) of the clutches had less than 10 eggs in them.

   b. How many eggs would you typically expect there to be in any other new clutch that is discovered? Explain your thinking.

   Sample responses:
   - **6 to 8 eggs; Because these represent the numbers of eggs found in 52% of the clutches.**
   - **6 eggs; This was the most common amount (found in 20% of the clutches).**
   - **8 eggs; 8 is in the "middle" of all the other data values, when the data are written in numerical order.**

1 Launch

Guide the class in completing the frequency table. Then have pairs to complete the activity.

2 Monitor

Help students get started by asking, "Where do you see the mode in the table?

Look for points of confusion:

- **Using the number of eggs rather than the frequency to determine an expected value (Problem 2a).** Ask, "How many clutches had 10 or more eggs? Fewer than 10?"
- **Struggling to identify a typical number of eggs (Problem 2b). Ask,** "How might your response to Problem 2a relate to a typical number of eggs?"

Look for productive strategies:

- Recognizing that the table displays the frequencies as singular values, making it an efficient representation to determine the mode (Problem 1).
- Comparing the total frequencies of clutches with fewer than 10 eggs to those with 10 or more eggs (Problem 2a).
- Determining a typical single value or a range of values by considering the mode, middle, or location of "most" of the data (Problem 2b). If students use the term *center*, acknowledge it, but note that it will be a focus in Lesson 4.

3 Connect

Display a completed frequency table.

Have students share their responses for Problems 1 and 2a, focusing on how they used the frequency table. Then have several students share responses to Problem 2b.

Ask, "Do you agree or disagree with any other responses for Problem 2b? How can you use the data to support your argument?"

Highlight that there is often a "range" of typical values, where most of the data lie.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Allow students to respond to Problem 1 verbally with their partners and record notes that will prepare them to share their thoughts during the Connect. For Problem 2a, suggest that students draw a horizontal line in the frequency table that divides it into two sections — 10 or more eggs or less than 10 eggs. Ask, "What do you notice about the frequency values in the second column?"

Extension: *Math Enrichment*

Have students add up to three more data values to the frequency table that would change their response to Problem 2b. Have them explain how and why their responses would change.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the different responses and reasoning students may have for Problem 2b. For example, display something similar to the following. Draw students' attention to the similarities and differences between the values — or ranges of values — and the reasoning behind a "typical" number of eggs in a clutch.

6 to 8 eggs (or 5 to 9 eggs)	6 eggs	8 eggs
More than 50% of clutches were in this range	Most common value	The "middle" when the values are written in numerical order

Pairs | 20 min

Activity 2 Using Dot Plots to Represent and Describe Data

Students create a dot plot to represent data, and then use the structure of the dot plot to describe the distribution of the data in context.

Amps Featured Activity Interactive Dot Plots

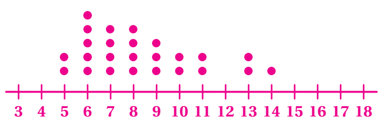
Activity 2 Using Dot Plots to Represent and Describe Data

The IUCN is made up of over 1,400 organizations and the collective 17,000 experts working in them. The employees of IUCN, such as Programme Officers, oversee the work. In their role, Programme Officers contribute to, edit, and publish reports, which involve reviewing data and information gathered, and then interpreting it to share out. They also make recommendations for future actions and efforts.

Laura Máiz-Tomé served several years as the Programme Officer for the Freshwater Biodiversity Unit at IUCN, focusing on the protection and conservation of wetland-dependent species, which would include the Diamondback terrapin.

Represent the terrapin egg data from Activity 1 and think about how a dot plot and a frequency table could each be used to share the information the data represents.

1. Construct a dot plot representing the number of eggs in each clutch.



2. How are the frequency table from Activity 1 and your dot plot similar? How are they different?
They both show the different values of the data and the frequency of each value. The table shows the frequency as a single quantity (already counted). The dot plot shows the frequency using dots above each value (which can be counted).

Featured Mathematician

Laura Máiz-Tomé

Laura Máiz-Tomé is a Spanish political-ecologist who has a bachelor's degree in Environmental Policy and two master's degrees in Natural Protected Areas, and Environmental Assessment and Management. As the Programme Officer for the Freshwater Biodiversity Unit at IUCN, she led large-scale biodiversity assessments, including species extinction risk assessments for the IUCN Red List and the identification of Freshwater Key Biodiversity Areas. She served a similar role in the Ecosystem Assessment and Policy Support Unit at the Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services (IPBES), and continues this work as a consultant.

876 Unit 8 Data Sets and Distributions

1 Launch

Activate prior knowledge by asking students what a line plot shows and how it is constructed. Consider showing an example to support student descriptions.

Define a ***dot plot*** as a representation of numerical data that uses a number line and stacked dots to represent the frequencies of each value in the data set.

2 Monitor

Help students get started by asking, "What is the greatest number of eggs in a clutch? The least?" Have students use these values to draw a number line.

Look for points of confusion:

- **Struggling to describe the distribution.** Have students trace the shape of the data and describe the motion. Say, "Now try to describe that shape in context."
- **Mistaking reasonable for the *possible* number of eggs (Problem 5).** Ask, "While it is possible, do you expect there to be 20 eggs in a clutch? How can you use the distribution of the data to help you?"

Look for productive strategies:

- Recognizing that the dot plot allows them to make observations about the distribution that are difficult to make looking at a frequency table or a list (Problem 2).
- Describing the distribution of the data (Problem 3) in terms of individual values (e.g., no clutches had 12 eggs) or in broader terms (e.g., almost all clutches had 11 or fewer eggs).
- Using the structure of the dot plot to draw conclusions about the data (Problem 4) and visually identify typical and non-typical values (Problems 5 and 6).

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create dot plots to represent the data from their frequency tables. You can overlay student responses to provide immediate feedback.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their responses to Problems 2 and 3, collect the words and phrases they use to describe the distribution of the data and add them to the class display. Group them by similar features they describe, such as *highest point, most common, mode*, and *peak*.

Featured Mathematician

Laura Máiz-Tomé

Have students read about Laura Máiz-Tomé, a former Programme Officer for the Freshwater Biodiversity Unit at IUCN who led large-scale biodiversity assessments, including species extinction risk assessments for the IUCN Red List and the identification of Freshwater Key Biodiversity Areas.

Pairs | 20 min

Activity 2 Using Dot Plots to Represent and Describe Data (continued)

Students create a dot plot to represent data, and then use the structure of the dot plot to describe the distribution of the data in context.

Name: ____________ Date: ____________ Period: ____________

Activity 2 Using Dot Plots to Represent and Describe Data (continued)

The *distribution* of a data set refers to the combination of all of the values in the data set and their frequencies, which can be described by features of the overall shape of the data when represented visually, such as being arranged in a dot plot.

3. Use your dot plot of the turtle egg data to describe the distribution of the data set.
   Sample responses: There were between 5 and 14 eggs in the clutches. The most frequent number of clutches (mode) is 6, and then the frequencies generally taper off to the right with fewer and fewer clutches containing more and more eggs. No clutches in the study had exactly 12 eggs, so there is a gap in the distribution there. Almost all clutches had 11 or fewer eggs in them, and the data are clustered around 6 to 8 because most clutches had that amount of eggs in them.

4. What percent of clutches had:
   a. More than 6 eggs in them?
      72%
   b. 8 or fewer eggs in them?
      60%

5. The lead conservationist learns that a 26th clutch has been discovered in the marsh. She says, "There are probably 20 eggs in that clutch!" Is her statement reasonable? Explain your thinking.
   Sample response: No, because most of the clutches had between 6 and 8 eggs, and no clutch had more than 14 eggs. While a clutch *could* have 20 eggs in it, I would not *expect* it because 20 is not a typical number of eggs, based on the other 25 clutches.

6. Revisit your work in Problem 2b from Activity 1. How might a dot plot be more efficient for determining a typical number of eggs in a clutch? Explain your thinking.
   Sample response: It is more efficient to identify a typical number of eggs in a clutch using the dot plot because frequencies are shown as dots arranged in a vertical line. Just by looking at the heights of the dots above each value, I can see where there is a cluster of higher frequencies. The dot plot seems to indicate that there are typically between 6 and 8 (or 6 and 9) eggs in a clutch.

STOP

3 Connect

Display a completed dot plot of the data.

Define the ***distribution*** of a data set as the combination of all of the values in the data set and their frequencies, which can be described by features of the overall shape of the data when represented visually.

Have students share their responses to Problems 2 and 3 focusing on descriptions that refer to individual categories or values and those that characterize the distribution in broader terms. Allow students to share as many observations about the distribution as time permits, and record the words and phrases they use on the board. Then have students share their responses and strategies for Problems 4–6, focusing on how they used the structure of the dot plot to identify typical and non-typical values.

Highlight students' words and phrases that can be used to describe distributions: *skewed, symmetric, cluster, peak, gap, extreme value* or *outlier, maximum, minimum, mode, typical, range, spread*, or *variability*. Each of these words for describing a distribution will be formalized in later lessons, and some words will be formally defined. Consider supplying these words to students and encouraging their use, once introduced as appropriate.

Whole Class | 5 min

Summary

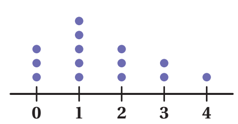
Review and synthesize how dot plots represent the distribution of a data set and how they help to identify typical values.

Summary

In today's lesson . . .

You saw that the term ***frequency*** refers to the number of times a "value" occurs in a data set. The most frequently occurring value is called the ***mode***. You can also represent and analyze the ***distribution*** of a data set – that is, information that describes all the data values and their frequencies. One way to describe a distribution is by identifying *typical values* – those that would be expected, based on the other values observed.

Organizing data is helpful for describing a data set and answering both non-statistical and statistical questions about it. For example, consider the data about number of siblings a group of sixth graders have:

Data list	0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 4,	Allows you to see all of the data
Frequency Table	Siblings: 0, 1, 2, 3, 4 Frequency: 3, 5, 3, 2, 1	Allows you to compare the frequency of each response (for categorical or numerical data)
Dot Plot		Allows you to compare frequencies in numerical data as well as provides a visual of the data.

Reflect:

Synthesize

Highlight that being able to see and describe the distribution of a data set helps determine typical or common values, as well as those that are not typical or would not be expected.

Ask:

- "Which representation of data that you have worked with so far most clearly displays the distribution of a data set? Why?" Dot plot; Because you can see the actual shape of the data.
- "How does the distribution of a data set help you identify typical values?" You can see how the data are arranged, so you can look at where most of the data are located, where it might be clustered, or where the middle of the data values might be. The corresponding values are typical of the data set.

Formalize vocabulary:

- ***frequency***
- ***mode***
- ***dot plot***
- ***distribution***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did your work today with dot plots build on your previous work with line plots? What can you do now that you could not do previously?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started. Ask them to review and reflect on any terms and phrases related to the terms *frequency, mode, dot plot,* and *distribution* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

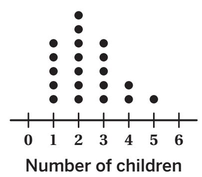
Students demonstrate their understanding by using a dot plot to describe the distribution of a data set and drawing conclusions in context.

Printable

Name: ______________________ Date: __________ Period: __________

Exit Ticket 8.03

A group of students was asked, "How many children, including yourself, live with you?" Their responses are displayed in the dot plot.



1. How many students responded to the question?
   20

2. What percent of the students live with at least one other child?
   75%

3. Write a sentence that describes the distribution of the data shown on the dot plot.
   Sample responses:
   - Typically, there are between 1 and 3 children in the family. The most frequently occurring number of children in the family (mode) is 2.
   - Overall, the data peaks at 2 children per family and then tapers out. This means that in the data there are fewer families with a greater amount of children.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe the information presented in tables and dot plots. 1 2 3

b. I can use tables and dot plots to represent distributions and frequencies of data. 1 2 3

c. I can describe a typical data point for a given distribution. 1 2 3

Success looks like . . .

- **Language Goal:** Comprehending the word *frequency* to refer to the number of times a particular value occurs in a data set. **(Speaking and Listening)**
- **Goal:** Creating and interpreting a dot plot to answer statistical questions about a numerical data set.
  » Answering statistical question about the dot plot in Problems 1 and 2.
- **Language Goal:** Describing a distribution represented by a dot plot, including identifying what types of values are *typical* for the distribution. **(Speaking and Listening, Writing)**
  » Describing typical values of the data set of number of children in Problem 3.

Suggested next steps

If students misinterpret the dots as each representing the numerical value which they are above (rather than as representing one response), consider:

- Reviewing Activity 2, Problem 1. Ask, "What did each dot represent in the dot plot of the data of turtle eggs?"

If students include 1 (i.e., say 100%) for Problem 2, consider asking:

- "For the students who responded 1 child, what does that mean?" They are the only child in the house.
- "Should those responses be included for this particular question? Why or why not?" No, because the question asks about children *other than* the student surveyed.

If students struggle to describe the distribution of the data, consider:

- Reviewing Activity 2, Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- One instructional goal for this lesson was to describe the distribution and typical values for a data set. How well did students accomplish this? What did you specifically do to help students accomplish it?
- What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

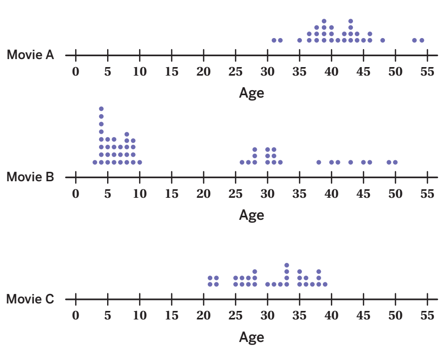
Independent

Name: ______________________ Date: __________ Period: __________

Practice

1. Here are descriptions of data sets. Select *all* the descriptions that could be graphed as dot plots.
   - **A.** Class size for the classes at an elementary school.
   - **B.** Colors of cars in a parking lot.
   - **C.** Favorite sport of each student in a sixth grade class.
   - **D.** Birth weights for the babies born during October at a hospital.
   - **E.** Number of goals scored in each of 20 games played by a soccer team.

2. A movie theater is showing three different movies. These dot plots represent the ages of the people who were at the Saturday afternoon matinee of each of these movies.



   a. One of these movies was an animated movie rated G for general audiences. Do you think it was movie A, B, or C? Explain your thinking.
   Movie B, because the typical age is between 4 and 8 years old, which makes the audience generally young kids.

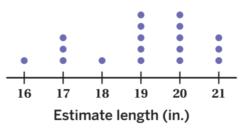
   b. For which movie was the typical audience member about 30 years old?
   Movie C

   c. What was a typical age for an audience member seeing Movie A?
   About 40, or between 38 and 43

Name: ______________________ Date: __________ Period: __________

Practice

3. A teacher drew a line segment that was 20 in. long on the blackboard. She asked each of her students to estimate the length of the segment and used all of their estimates to construct this dot plot.

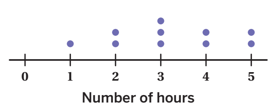


   a. How many students are in the class?
   18

   b. Were students generally accurate in their estimates of the length of the line segment the teacher drew? Explain your thinking.
   Sample response: Yes, because a typical guess was between 19 in. and 21 in., so 13 out of 18 estimations were within 1 in. of the exact length.

4. Determine the value of each expression.
   a. $3.727 + 1.384$ $= 5.111$
   b. $3.727 - 1.384$ $= 2.343$
   c. $5.01 \cdot 4.8$ $= 24.048$
   d. $5.01 \div 4.8$ $= 1.04375$

5. The dot plot shows the numbers of hours spent last night on homework by a group of students.



   a. What does each value on the number line represent?
   The number of hours a student spent on homework last night.

   b. What does each dot represent?
   1 student in the group

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 14	2
Formative	5	Unit 8 Lesson 4	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 8 | **LESSON 4**

Using Dot Plots to Answer Statistical Questions

Let's use dot plots to answer statistical questions.

Focus

Goals

1. **Language Goal:** Describe a distribution represented by a dot plot, including observations about its center and spread. **(Speaking and Listening, Writing)**
2. **Language Goal:** Compare and contrast dot plots that represent two different data sets measuring the same quantity, paying attention to the *center* and *spread* of each distribution. **(Speaking and Listening, Writing)**
3. **Language Goal:** Critique or justify claims about the center of a distribution represented on a dot plot. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue using dot plots to develop a spatial understanding of distributions. They are introduced to the idea of using the center and spread to describe distributions generally, and they see that typical values are a range of values around the center of the data, which can be identified by a single number. Students make use of the structure of distributions to identify values of center, describe the spread of the data, and compare centers and spreads of different distributions related to the hunting habits of the Macaroni penguin. Throughout the lesson, they must justify their claims using evidence from the dot plots.

Previously

In Lesson 3, students created and used dot plots to describe the distribution of a data set, including typical values.

Coming Soon

In Lesson 5, students will be introduced to histograms, and they will compare and contrast dot plots and histograms.

Rigor

- Students build **conceptual understanding** of how a data set can be described by its distribution including its center and spread.
- Students continue to develop **procedural skills** for using the distribution of a data set to identify typical values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Activity 1 PDF, pre-cut cards, one set per pair
- Activity 2 PDF, one per student

Math Language Development

New words

- *center*
- *spread**

Review words

- *distribution*
- *dot plot*
- *frequency*
- *numerical data*
- *statistical question*

*Students may confuse the statistical term *spread* with the various everyday uses of the term. Be ready to address the similarities and differences between them.

Amps | Featured Activity

Activity 1

Animated Penguins

Students see dot plots generated in real time as penguins dive into water.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might doubt that they are capable of the thorough analysis required within the structure of the dot plot. Encourage them to lean on their partner, relying on each other's strengths and supporting each other with their limitations. While they have previously been focused on the details of getting the data into the plot, now students need to step back and look at the overview of the graph in order to get an accurate interpretation.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. Consider reading the introductory text in the Warm-up prior to Activity 1 to supply background knowledge of the recurring context for the lesson.
- In **Activity 1**, complete Part 1 together as a class. Distribute one card to each pair, ensuring each card is equally distributed among the pairs. Have partners complete Part 2 together. Then complete Part 3 as a class.

Warm-up The Macaroni Penguin

Students use a dot plot to determine how many krill are typically caught by a Macaroni penguin, preparing them to relate the terms *typical* and *center* in the next activity.

Name: ______ Date: ______ Period: ______

Unit 8 | Lesson 4

Using Dot Plots to Answer Statistical Questions

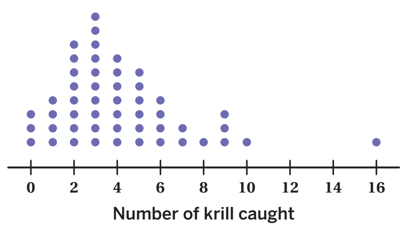
Let's use dot plots to answer statistical questions.

Warm-up The Macaroni Penguin

The IUCN classifies the Macaroni penguin (*Eudyptes chrysolophus*) as Vulnerable. The overall global population of Macaroni penguins has experienced a 47% decline over three generations, and they still face ongoing threats from climate change, commercial fishing, and food competition with the increasing fur seal population.

james_stone76/Shutterstock.com

A scientist is studying the hunting and feeding habits of 50 Macaroni penguins on Bird Island, South Georgia, Antarctica. The dot plot shows the number of krill that each of several penguins caught on their first dive one day.



What is a typical number of krill that one of these 50 penguins caught on their first dive? Be prepared to explain your thinking.

Answers may vary, but should reflect a range of about 2 to 6 krill because that is where the dot plot shows the largest cluster of data and it also contains a peak at 3.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work independently.

2 Monitor

Help students get started by asking, "What does each dot represent in the dot plot?"

Look for points of confusion:

- **Thinking a typical value is in the middle of the number line, not the data.** Point to 8 on the number line, and ask, "Is this the middle of the data values, with about half to the left and half to the right?"
- **Struggling to name a typical value.** Ask, "Where are most of the data located in the dot plot?"

Look for productive strategies:

- Using the distribution on the dot plot to identify values located near the "middle" of the data.
  If students:
  » Only identify the mode, ask, "Could you reasonably expect penguins to catch more or less krill than that? Why?"
  » Identify the range of values from 0 to 16, ask, "Can you make a more precise observation?"

3 Connect

Have students share their responses and evidence from the dot plot to support their thinking. Ensure a variety of responses are shared.

Ask, "Could all of your responses be correct? Why?"

Highlight that there is not usually one clear value that is typical for a data set, but it is often a "range" of values that are characteristic, or could be expected. Formalize terms for describing distributions by explaining that a characteristic range of values often describes where the largest *cluster* of data is, which is also often around a *peak*, especially when the overall shape is *symmetric*. However, if there are *gaps* or *outliers* that *skew* the shape, the center could shift.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they are examining the decline and feeding habits of Macaroni penguins.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the overall population of Macaroni penguins has declined 47% over three generations.
- **Read 3:** Encourage students to ask questions about the dot plot before revealing the prompt.

English Learners

Clarify the meaning of terms, such as *vulnerable species*.

Power-up

To power up students' ability to interpret dot plots, Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up.

Informed by: Performance on Lesson 3, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 6.

Pairs | 15 min

Activity 1 The Hunt for Red Krill

Students are introduced to *center* and *spread* more formally, as conventional ways to describe the generally distribution of a data set.

Amps Featured Activity Animated Penguins

Activity 1 The Hunt for Red Krill

The scientist knows that, during their winter migration, Macaroni penguins spend most of their day foraging for food. She notices that the males tend to stay underwater longer than the females, so she poses the question: "Do male and female penguins typically hunt at the same depth?" To help answer this question, she put trackers on 25 male penguins and 25 female penguins to determine how deep each penguin dives to hunt.

You and your partner will each be given a dot plot. One partner will examine the data for the male penguins and the other will examine the data for the female penguins.

Part 1
Work together with your partner to complete these problems.

1. Is the scientist's question a statistical question? Explain your thinking.
   Yes, the scientist collected data by tracking how deep the penguins dive, and the dot plots show variability because not every penguin went to the same depth.
2. How would you interpret one dot above 0 on the dot plot in this context?
   Sample responses:
   - **It represents a penguin who hunts at 0 m.**
   - **It means one penguin hunts around the surface of the water.**

Part 2
Use your dot plot to complete these problems.

3. How would you describe the *center* of the data? What might that tell you in context?
   Answers may vary, but should be a single value between 30 m and 40 m for male penguins and between 10 m and 20 m for female penguins. The center value means the general depth that a male or female penguin dives to hunt for food.
4. How could you use the center to describe a typical hunting depth for this group of penguins? Explain your thinking.
   Answers may vary, but should reflect an understanding of typical values as those around the center (i.e., close to and including the center value).
   Sample responses:
   - **Male penguins: A typical depth is between 10 m and 40 m, which includes a center value of 30 m.**
   - **Female penguins: A typical depth is between 0 m and 20 m, which includes a center value of 10 m.**

882 Unit 8 Data Sets and Distributions

1 Launch

Distribute one set of cards of the Activity 1 PDF to each pair of students, ensuring each partner gets one card in the set. Pause after Part 1 to discuss students' responses. Encourage them to respond in context (e.g., "Not all penguins dive to the same depth" rather than "The dots are not all in the same place." Then have students complete Part 2 independently and Part 3 with their partner.

2 Monitor

Help students get started by asking, "What is a statistical question?"

Look for points of confusion:

- **Misinterpreting 0 (Part 1)**. Ask, "What does each dot represent? What do the number line values represent?"
- **Thinking the center is the middle value on the number line (Part 2)**. Ask, "Is that where most of the data are located?"
- **Thinking males and females hunt at the same depth because 10 m is in the typical range for both (Part 3)**. Have them review Part 2, Problem 3. Ask, "How does the center value help you compare the penguins?"

Look for productive strategies:

- Using the structure of the dot plot and distribution of the data to recognize the presence of variability (Part 1), determine a center value and typical values (Part 2), and compare the spreads of the data (Part 3) in the context of the problem.
- Determining the center value by identifying a point at which about half of the data are on either side, and recognizing that a typical value is either the center or located near it (Part 2).
- Determining which data set is more alike by either considering the entire range of the data or an interval around the center (Part 3).

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use virtual tools to identify the center and spread of a data set. You can overlay student responses to compare them.

Accessibility: *Vary Demands to Optimize Challenge*

Allow pairs to complete Part 2 together. If a pair struggles to formulate ideas for Parts 2 or 3, have them join another pair to respond to one of the questions, after which they should return to complete the activity as an independent pair.

Math Language Development

MLR5: Co-craft Questions

After students share their responses from Part 2 with a partner and before they respond to the questions in Part 3, have them work together to write 2–3 questions that could be answered by the data displayed in their dot plots. Ask volunteers to share their questions with the class. Note questions that vary in complexity, making sure to have students share examples that ask about *percent, center,* or *spread,* if applicable. Allow students to ask clarifying questions of their peers regarding how the dot plot could be used to answer the questions.

English Learners

Consider structured pairing to pair students together who speak the same primary language. They can support each other as they ask and respond to clarifying questions.

Pairs | 15 min

Activity 1 The Hunt for Red Krill (continued)

Students are introduced to *center* and *spread* more formally, as conventional ways to describe the generally distribution of a data set.

Name: ______ Date: ______ Period: ______

Activity 1 The Hunt for Red Krill (continued)

Part 3

Share your work from Part 2 with your partner. Then work together to complete these problems. Be prepared to explain your thinking.

5. Do male and female penguins typically hunt at the same depth?

No, males typically hunt at greater depths than females; Sample response: A typical depth for males is between 10 m and 40 m, but the center of the data shows they tend to hunt on the upper end of that range. Females typically hunt between 0 m and 20 m, but the center of the data shows they tend to hunt right in the middle of that range at 10 m or 15 m.

6. Overall, which group – male penguins or female penguins – are more alike in the depth at which they hunt? Explain your thinking.

Sample responses:

- **The males are more alike because more of the data are in the typical range.**
- **The females are more alike because the data are more clustered, but the males are more spread out.**

Are you ready for more?

Consider this data set: 20, 20, 21, 23, 23, 26, 26, 26, 26, 27, 27, 27, 28, 28, 29, 29, 29, 30, 32, 34.

1. Describe the center and spread of the data set.

Sample response: The center is 27, and the data are widely spread from 20 to 34, but clustered between 26 and 29.

2. Describe how the center and spread would change (or not) if:

a. Each value is increased by 5.

Sample response: The center would be 32, and the data would remain widely spread, but from 25 to 39 and clustered between 31 and 34.

b. Each value is decreased by 5.

Sample response: The center would be 22, and the data would remain widely spread, but from 15 to 29 and clustered between 21 and 24.

c. All values equal to the center you named in Problem 1 remains the same. Each value less than the center is increased by 5, and each value greater than the center is decreased by 5.

Sample response: The center would remain 27, but the data would have a narrower spread from 23 to 31, clustering between 24 and 27.

3 Connect

Display the Activity 1 PDF.

Have students share their answers and strategies to Parts 1 and 2, focusing on how their center and typical values are related. Then have pairs share their answers to Part 3, focusing on how they determined which set of data was more alike by considering the entire range of the data or an interval around the center. Nudge students to share their thinking in the context of the problem, rather than discussing decontextualized dots on a number line.

Define:

- The ***center*** of a distribution as a value in the middle of a data set that represents a typical value
- The ***spread*** of a distribution as a description of how alike or different the values in the data set are, often in relationship to the center.

Highlight that distributions are generally described using the center and spread. The center of a distribution helps describe the data set using one typical value. Note that the range of typical values is located near the center. If time permits, enrich the context of the problem by explaining that male Macaroni penguins tend to dive deeper because, on average, they weigh more than females. By hunting at different depths, males and females can dive in the same areas without competing for food.

Pairs | 15 min

Activity 2 Seasonal Hunting Patterns

Students determine whether the hunting behaviors of Macaroni penguins are consistent throughout the year by comparing the centers and spreads of two distributions.

Activity 2 Seasonal Hunting Patterns

The scientist then became curious whether the hunting behaviors of the Macaroni Penguin is consistent throughout the year. She posed the question, "Do the penguins' dives last the same amount of time during the summer (breeding season) and the winter (migration)?" She recorded the average duration of 25 penguins' dives in both the summer and the winter.

You will be given two dot plots of these two data sets. Based on the dot plots, state whether you agree or disagree with each of the following statements about this group of penguins. Be prepared to explain your thinking.

1. 72 seconds is a good description of the center of the data for how long the penguins spend per dive in the winter.

   Disagree
   Sample response: 72 seconds is the minimum number of seconds a penguin spent diving in the winter. The center of the data is approximately 86 seconds.

2. In general, 72 seconds is a good estimate for how long the penguins typically spend per dive in the summer.

   Agree
   Sample response: There are approximately the same number of data points to the left and right of 72 seconds.

3. Overall, the penguins' dives typically lasted the same amount of time in both the summer and the winter.

   Disagree
   Sample response: The dives lasted longer in the winter because the typical dive time was 86 seconds in winter as compared to 72 seconds in the summer.

4. The penguins' dive times were more alike in the winter.

   Agree
   Sample response: The dives in the summer are more spread out from the typical range than in the winter. In the summer, dives typically last about 72 seconds, but the dives recorded go all the way from 48 to 92 seconds. In the winter, dives typically last 86 seconds, and the data values are more tightly clustered around the typical range. The dives recorded show a 28 seconds span from 72 to 100 seconds.

STOP

884 **Unit 8** Data Sets and Distributions

1 Launch

Distribute one copy of the Activity 2 PDF to each pair of students. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What does each dot represent? How is that different than in Activity 1?"

Look for points of confusion:

- **Struggling to identify the center by looking at one single dot plot**. Ask, "How are the distributions in the two dot plots different? Is there a cluster of data in any?"
- **Focusing only on the existence of data values anywhere in the range**. Ask, "Where are most of the data located?"

Look for productive strategies:

- Evaluating each claim by using the structure of the distributions to:
  - » Identify the center of the data as the location about which most of the data can be found.
  - » Conclude that dives typically last longer in the winter than summer because the winter data is "shifted," or clustered, to the right.

3 Connect

Have students share their responses and strategies using the structure of the distributions to justify their thinking.

Ask, "How do the distributions differ from summer to winter?"

Highlight that meaningful comparisons between two data sets can be made by comparing the center and spread of their distributions. (Students are not expected to connect spread and variability here). If time permits, explain that in the winter, krill and other prey are located deeper in the water column.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students draw a vertical line through 72 on each dot plot to help them respond to Problems 1 and 2. As students complete Problem 4, clarify the meaning of the phrase *more alike*, as needed. Ask, "If the data values were a lot alike, what do you think the distribution would look like compared to one in which the data values were very different?"

Extension: *Math Enrichment, Interdisciplinary Connections*

Tell students that a Macaroni Penguin can dive at speeds up to 24 kph. Have students determine the depth of a penguin's dive at this speed if they dive for 100 seconds in the same direction. **(Science)** $\frac{2}{3}$ km

Math Language Development

MLR8: Discussion Supports

Consider chunking this activity by revealing each problem one at a time, giving pairs time to think independently and share their reasoning together, before sharing their responses with the class. Once the class comes to a consensus, reveal the next problem and continue this routine.

English Learners

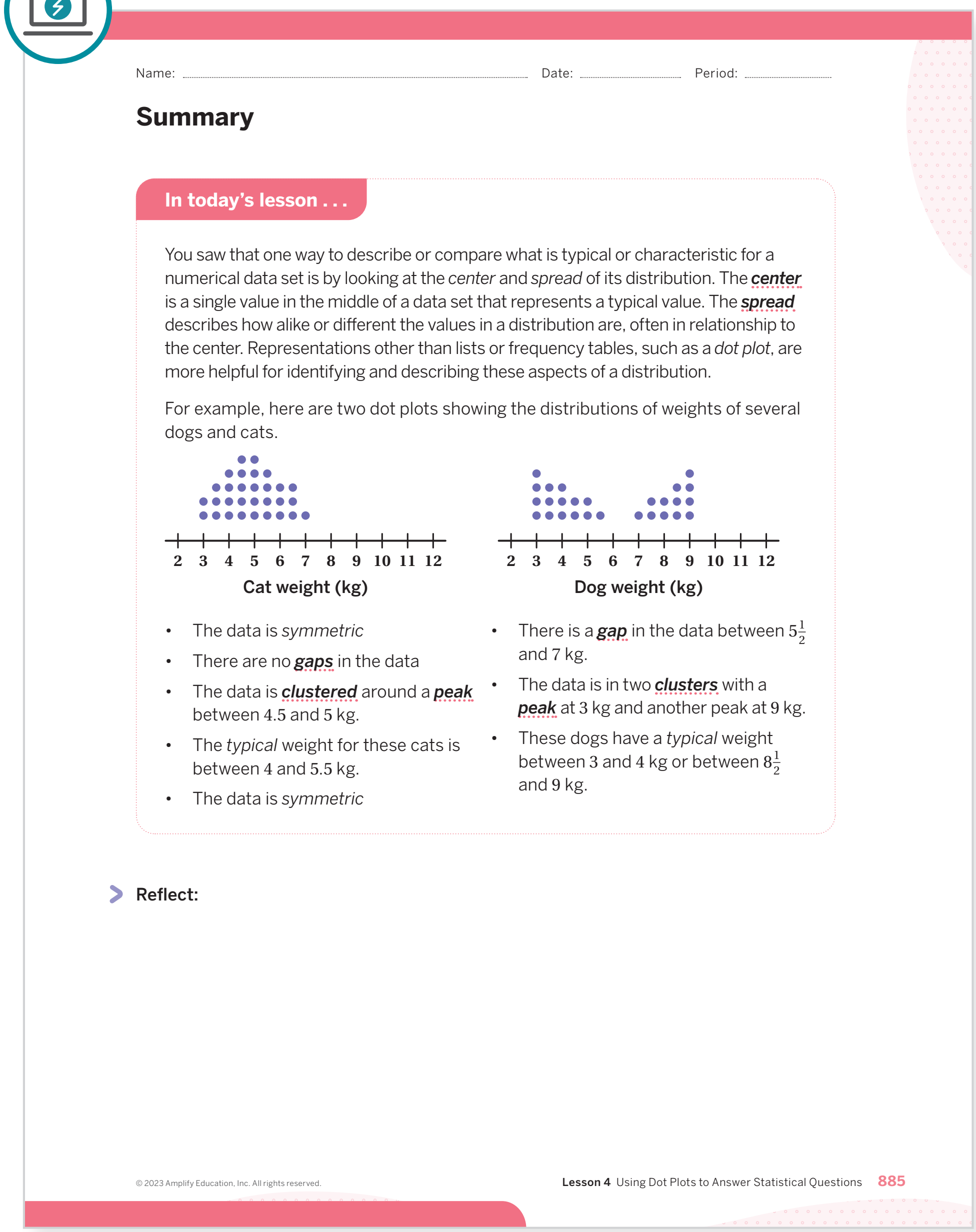
Display sentence frames for students to use as they evaluate each statement. For example:

- "I agree because . . ."
- "I disagree because . . ."
- "That could be true because . . ."
- "That could not be true because . . ."

Whole Class | 5 min

Summary

Review and synthesize how to use the center and spread of a distribution to describe a data set.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that one way to describe or compare what is typical or characteristic for a numerical data set is by looking at the *center* and *spread* of its distribution. The ***center*** is a single value in the middle of a data set that represents a typical value. The ***spread*** describes how alike or different the values in a distribution are, often in relationship to the center. Representations other than lists or frequency tables, such as a *dot plot*, are more helpful for identifying and describing these aspects of a distribution.

For example, here are two dot plots showing the distributions of weights of several dogs and cats.

- The data is *symmetric*
- There are no ***gaps*** in the data
- The data is ***clustered*** around a ***peak*** between 4.5 and 5 kg.
- The *typical* weight for these cats is between 4 and 5.5 kg.
- The data is *symmetric*

- There is a ***gap*** in the data between $5\frac{1}{2}$ and 7 kg.
- The data is in two ***clusters*** with a ***peak*** at 3 kg and another peak at 9 kg.
- These dogs have a *typical* weight between 3 and 4 kg or between $8\frac{1}{2}$ and 9 kg.

Reflect:

Synthesize

Ask, "How do the center and spread help describe the distribution of a data set, and how do they relate to typical values?" The center allows you to describe the data set using one, typical value. The spread allows you to describe how alike or different the data points are from each other, often relative to how spread out the data are from the center. A narrow spread means many data values are close to the center, so many items in the group are considered "typical." A wide spread means either the range of typical is wider or fewer items in the group are considered "typical."

Highlight that, in future lessons, students will learn how to calculate measures of center and spread precisely, using all of the values in a data set, rather than just visual inspection of a representation.

Formalize vocabulary:

- *center*
- *spread*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do you determine the center of a distribution?"
- "How are the center and typical values related?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *center* and *spread* that were added to the display during the lesson.

Exit Ticket

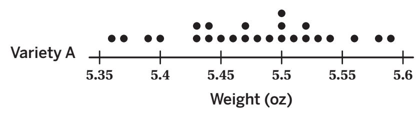
Students demonstrate their understanding of comparing distributions by analyzing typical values for two varieties of tomatoes.

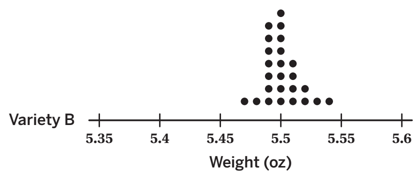
Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.04

A farmer would like to buy tomatoes that are about the same size and close to 5.5 oz in weight. She is considering two different tomato varieties: Variety A and Variety B. She weighs 25 tomatoes of each variety. The dot plots show her results.





1. What would be a typical weight of a:
   a. Variety A tomato?
   Answers may vary but should be between 5.45 and 5.5 oz.
   b. Variety B tomato?
   Answers may vary, but should be between 5.49 and 5.51 oz.

2. The farmer is going to buy a bushel (approximately 53 lb) of one variety of tomatoes without inspecting each one. Based on the size and consistency she wanted, which tomato variety should the farmer choose? Explain your thinking.
   Variety B; Sample response: The data are tightly clustered around 5.5 oz and has less spread than Variety A. This means that most tomatoes in Variety B weight close to or exactly 5.5 oz.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use a dot plot to represent the distribution of a data set and answer questions about the real-world context.
1 2 3

b. I can use center and spread to describe data sets, including what is typical in a data set.
1 2 3

Lesson 4 Using Dot Plots to Answer Statistical Questions

Success looks like . . .

- **Language Goal:** Describing a distribution represented by a dot plot, including informal observations about its center and spread. **(Speaking and Listening, Writing)**
  - » Describing the typical weight of each variety of tomatoes in Problem 1.
- **Language Goal:** Comparing and contrasting dot plots that represent two different data sets measuring the same quantity, paying attention to the center and spread of each distribution. **(Speaking and Listening, Writing)**
  - » Comparing the centers and spreads of the distributions of weights in Problem 2.
- **Language Goal:** Critiquing or justifying claims about the center of a distribution represented on a dot plot. **(Speaking and Listening, Writing)**

Suggested next steps

If students struggle to identify a typical weight for either variety of tomato, consider:

- Reviewing the Part 2 of Activity 1, and asking, "How is the center of a distribution related to a typical value? How did you determine the center or a typical value?"

If students say the farmer should choose Variety A or either variety for Problem 2, consider:

- Reviewing Activity 1 and asking, "What was the mode in each dot plot? Was the mode always the center value? Why not? How can that help you here?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students used center and spread to describe the distribution of a data set. How did that build on the work students did with typical values in Lesson 3?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent

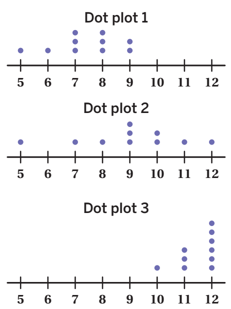
Name: Date: Period:

1. Three sets of data about ten sixth graders were used to make three unlabeled dot plots. Match each dot plot with the most appropriate label.

   a. Ages in years
   Dot plot 3

   b. Hours of sleep on school nights
   Dot plot 1

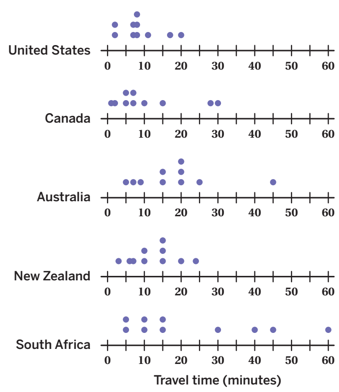
   c. Hours spent doing chores each week
   Dot plot 2



2. These dot plots represent the time it takes for ten sixth graders from the United States, Canada, Australia, New Zealand, and South Africa to get to school.

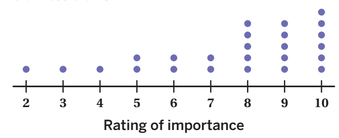
   a. List the countries in order of typical travel times, from least to greatest.
   United States, Canada, New Zealand, Australia, South Africa

   b. Which country has the least spread in travel times? The greatest?
   United States; South Africa

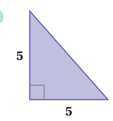


Name: Date: Period:

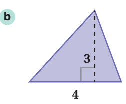
3. Twenty-five students were asked to rate, on a scale of 0 to 10, how important it is to reduce pollution. A rating of 0 means "not at all important" and a rating of 10 means "very important." The dot plot shows their responses. Explain why a rating of 6 is *not* a good description of the center of this data set.
   Sample response: There are a lot more values that are greater than 6 than less than 6.



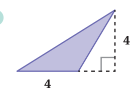
4. Determine the area, in square units, of each triangle.

   a.


   12.5 square units

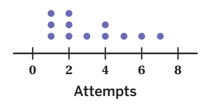
   b.


   6 square units

   c.


   8 square units

5. Priya created a dot plot of the number of attempts it took each of 12 of her classmates to successfully throw a ball into a basket. Write a question for which the answer would be:



   Sample responses shown.

   a. More than half the classmates.
      - **How many students needed 2 attempts or more to make the basket?**
      - **How many students needed less than 4 attempts to make the basket?**

   b. 3.
      - **What is a typical number of attempts students needed to make the basket?**
      - **How many students needed more than 4 attempts to make the basket?**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 11	2
Formative	5	Unit 8 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Interpreting Histograms

Let's explore how histograms represent data sets.

Focus

Goals

1. **Language Goal:** Compare and contrast dot plots and histograms in terms of how useful they are for answering different statistical questions. **(Speaking and Listening)**
2. Create a histogram to represent a numerical data set.
3. **Language Goal:** Interpret a histogram to answer statistical questions about a numerical data set. **(Writing)**

Coherence

Today

Students are introduced to histograms as another representation of the distribution of a numerical data set. They compare and contrast the information shown by dot plots and histograms, noticing that histograms only show the frequencies of groups of values, rather than individual values. Students analyze dot plots and histograms representing the same data sets to determine what information each representation makes easier to interpret, including in context. They also construct histograms for given sets of data.

Previously

In Lesson 4, students represented and interpreted numerical data corresponding to statistical questions using dot plots.

Coming Soon

In Lessons 6 and 7, students will continue to use histograms to answer statistical questions and describe features of the distributions of numerical data sets, driving toward identifying typical values.

Rigor

- Students build **conceptual understanding** of interpreting distributions of data by relating dot plots and histograms.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 1 PDF, *Dot Plot and Histogram* (for display)
- Activity 2 PDF, one per group
- straightedges

Math Language Development

New word

- *histogram*

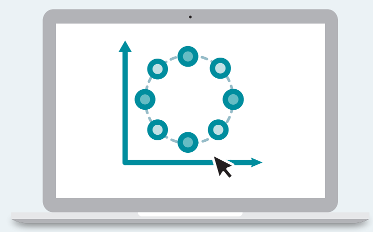
Review words

- *center*
- *distribution*
- *frequency*
- *spread*

Amps | Featured Activity

Activity 1
Multiple Representations

Students are able to compare shapes of distributions by switching between dot plots and histograms.



Amps POWERED BY desmos

Building Math Identity and Community
Connecting to Mathematical Practices

When comparing a dot plot and a histogram, students might be confused as to why they might want to change their interpretations of the data. The different presentations of data have different purposes, and choosing the correct graph requires abstract reasoning. At the end of the activity, have students come together and create a note card that lists when it is better to use a dot plot and when a histogram is the better choice. This card can be used in the future when analyzing statistical situations.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 1 could be completed as a whole class.
- In **Activity 2**, the frequency table for Problem 2 could be completed first and then given to the students to use.

Pairs | 5 min

Warm-up Chimpanzee Lifespans (Part 1)

Students activate prior knowledge of writing and responding to statistical questions, as well as identifying typical values based on a dot plot, preparing them for histograms.

Unit 8 | **Lesson 5**

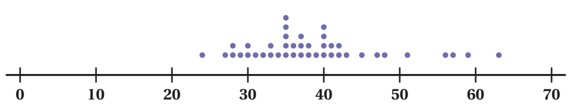
Interpreting Histograms

Let's explore how histograms represent data sets.

Warm-up Chimpanzee Lifespans (Part 1)

The chimpanzee (*Pan troglodytes*) is listed as Endangered by IUCN and its population is decreasing. Here is a dot plot showing the life spans of 40 chimpanzees that lived in the wild.

Helen J Davies/Shutterstock.com



1. Write two statistical questions that can be answered using the dot plot.
   Sample responses:
   Is it typical for chimpanzees to live more than 40 years in the wild?
   What is the longest expected lifespan of a chimpanzee living in the wild?

2. What would you consider to be a typical lifespan for a chimpanzee in the wild? Explain your thinking.
   About 37 or 38 years; Sample response: 37 or 38 seems to be the center of the distribution and the largest cluster of data goes from about 35–42.

Log in to Amplify Math to complete this lesson online.

1 Launch

Arrange students in pairs. Give students 1 minute of quiet work time, followed by 2 minutes to share their responses with a partner. For Problem 1, have students use the ***Take Turns*** routine to share one question at a time. Then their partner must agree or disagree as to whether it is a statistical question that can be answered using the dot plot.

2 Monitor

Help students get started by asking "What two things must be true about statistical questions?"

Look for points of confusion:

- **Writing questions that are statistical but cannot be answered with the dot plot, or vice versa.** Ask, "What does each dot represent? What can you ask about that?"
- **Thinking that 35 must be the typical lifespan because it is the mode.** Ask, "What if there were 10 total dots above 63, would you then say that is the typical lifespan? Why or why not?"

Look for productive strategies:

- Asking questions about the lifespans of more than one specific chimpanzee.
- Identifying a small range of values between 35–40 or 35–42, where the largest cluster of data is.
- Identifying one value at or near the physical center of the data by counting (e.g., more or less determining the median, approximately 20 dots from the left end or right end of the dot plot).

3 Connect

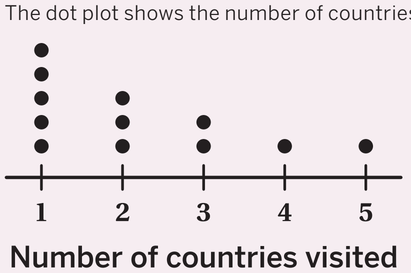
Have pairs of students share their statistical questions that could be answered using the dot plot from Problem 1, followed by sharing and explaining their responses for Problem 2.

Highlight that these skills will be important to remember as they look at a new way to represent data during the day's activities.

Power-up

To power up students' ability to form statistical questions, have students complete:

The dot plot shows the number of countries visited by members of a travel club.



a. How many members of the travel club visited more than 3 countries? 2

b. How many members of the travel club visited more than 1 country? 7

c. What is a typical number of countries visited by a member of the travel club? 2

Use: Before the Warm-up.

Informed by: Performance on Lesson 4, Practice Problem 5.

Pairs | 10 min

Activity 1 Chimpanzee Lifespans (Part 2)

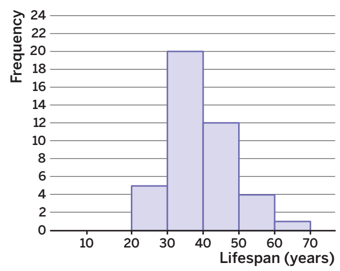
Students are introduced to histograms as another representation of the distribution of the numerical data set from the Warm-up, and they begin to interpret its features.

Amps Featured Activity Multiple Representations

Name: ______ Date: ______ Period: ______

Activity 1 Chimpanzee Lifespans (Part 2)

In a histogram, each bar includes the left boundary value but not the right boundary value. For example, in this histogram showing the lifespans of the same 40 chimpanzees from the Warm-up, the first bar includes those that lived from 20 to 29.99999 . . . years, but not 30 years.



1. Refer to the histogram.
   - **a** How many chimpanzees lived at least 40 years?
     17 chimpanzees
   - **b** How many chimpanzees lived exactly 30 years?
     It cannot be determined.
   - **c** How many chimpanzees lived at least 50 years and less than 70 years?
     5 chimpanzees
   - **d** What was the longest a chimpanzee lived?
     Sample response:
     The exact number of years cannot be determined, but it was at least 60 years and not 70 years.
   - **e** Refer back to Problem 2 from the Warm-up. Would your answer be different based on this histogram of the data? Explain your thinking.
     Sample response:
     A little bit. Because the bars are for every 10 years, I would say about 40 years is the center, and typical lifespans are between 30–50 years.

2. Discuss these questions with a partner and record your responses:
   - **a** If you used the dot plot from the Warm-up to answer the questions in Problems 1a–1d, how might your answers be different?
     Sample response:
     You would be able to answer Problem 1b and you could give the exact lifespan for Problem 1d.
   - **b** How are the histogram and the dot plot alike? How are they different?
     Sample responses:
     They are alike because they represent the same values, and are close to the same shape. They are different because the dot plot shows every data value. Dots are placed by increments of 1 and there are no more than 5 data values for any number, but bars increase by 10s and contain up to 20 data values.

1 Launch

Have students complete Problem 1 individually, and then compare their responses with a partner before completing Problem 2 together.

2 Monitor

Help students get started by asking them to clarify the meaning of "at least 40 years."

Look for points of confusion:

- **Having difficulty interpreting the histogram.** Have students label the vertical axis as "Number of chimpanzees" and provide the sentence frame: "There are ___ chimpanzees that are between ___ and ___ years old."

Look for productive strategies:

- Distinguishing questions with one answer (Problems 1a and 1c) from those where there could be more than one feasible answer.
- Making connections between the overall shape of distribution is represented in the dot plot and in the histogram, and stating how it could affect interpreting the data.

3 Connect

Display the histogram using the Activity 1 PDF.

Have pairs of students share their responses, focusing on Problem 2.

Highlight how dot plots and histograms are structurally similar and structurally different. You do not need to focus on the merits or limits of either, but should discuss the implications for analyzing data and answering questions.

Define a ***histogram*** as a way to represent frequencies of data values that have been grouped into intervals, called *bins*, along a number line. Bars are drawn above the bins where data exist, and the height of each bar represents a frequency. **Note:** The left boundary value of each bar is included in the frequency, but the right boundary value is excluded.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can compare shapes of distributions by overlaying dot plots and histograms.

Extension: *Math Enrichment*

Tell students, "Suppose two more data values of 40 were added to the data set. Would you be able to identify that the most frequently occurring value is now 40 on either display?" The dot plot will show this, but the histogram is arranged by intervals and will not show this.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight the structure of the displays, display the histogram from this activity and the dot plot from the Warm-up, using the Activity 1 PDF. Draw students' attention to the connections between the displays and how they each represent the same information, yet in different ways. Ask:

- "Where do you see the most frequently occurring value or interval of values in each display?"
- "How does each display show the center of the data? The spread?"
- "How do the heights of the bars of the histogram compare to the heights of the dot plots for the same intervals?" Rotate the displays 90 degrees clockwise to illustrate how the heights compare.

English Learners

Annotate the displays where these features are shown.

Small Groups | 20 min

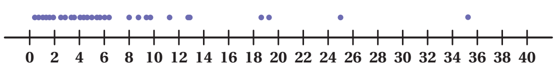
Activity 2 Populations of U.S. States and D.C.

Students compare a dot plot and a histogram, evaluating how each represents a data set and answers statistical questions.

Activity 2 Populations of U.S. States and D.C.

Part 1

Every ten years, the United States conducts a census, which is an effort to count its entire human (*Homo sapiens*) population. The dot plot shows the population data from the 2010 census for each of the fifty states and the District of Columbia (D.C.).



1. Some statistical questions are shown about the populations of these 50 states and D.C. Decide whether it would be *possible* to answer each question from the dot plot. For those that would be possible, also decide whether the answer, for this data, is *clearly shown on the dot plot*. Be prepared to explain your thinking.

   In the middle column of this table (Dot plot), mark your decision for each question by writing: C (clearly shown), P (possible to answer, but not clearly shown), or NP (not possible to answer).

Statistical question	Dot plot	Histogram
a How many states have populations greater than 15 million?	C	C
b Which states have populations greater than 15 million?	NP	NP
c How many states have populations less than 5 million?	NP or P	C
d What is a typical state population?	P	P or C
e Are there more states with fewer than 5 million people or more states with between 5 and 10 million people?	P	C
f How would you describe the distribution of the state populations?	P or C	C

1 Launch

In reference to the data used in this activity, see **Note** after the Connect section on the next page.

Activate background knowledge by providing a brief overview of census and population data. Consider also facilitating a brief class discussion about the dot plot, using orienting questions, such as:

- "How many total dots are there?" 51.
- "What is the population of the state with the largest population?" Between 36 and 38 million.

Arrange students in groups of 3 or 4. Explain that once groups have completed Problem 1, they will use data in the Activity 2 PDF and straightedges to represent the population data in a histogram. Give groups 10–12 minutes to complete the activity.

2 Monitor

Help students get started by asking, "What is the first question asking? The second question? Can either be answered using only the dot plot?"

Look for points of confusion:

- **Not knowing where to place 5.03**. Students may need more clarification about the boundaries of the bins. Ask, "If you imagine 5.03 on a number line, would it come before the 5, at the 5, or after the 5?"

Look for productive strategies:

- Articulating why some decisions changed when they transitioned from using dot plots to using histograms to answer statistical questions.
- Constructing a histogram with precision, considering values along both the horizontal and vertical axes.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to choose three parts to respond to for Part 1, Problem 1. Demonstrate how to complete one row of the table in Part 2, e.g., 15–20, and draw the corresponding bar height of the histogram.

Extension: *Math Enrichment*

Have students imagine reconstructing the histograms using different bin sizes (intervals). Have them respond to these questions:

- "What would the histogram look like if the bin sizes were increased by 1? 10? 20?
- "Do you think the bin size of 5 is the *best* or *most appropriate* bin size for this data? Why or why not?"

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask question, display these sentence frames to help them organize their thinking.

- "Dot plots show _____ clearly, because . . ."
- "Histograms show _____ clearly, because . . ."
- "It is not possible to see _____ on a _____ because . . ."

This will help students make decisions about the type of representation to use for different data sets and questions throughout the unit.

English Learners

Provide students time to rehearse and formulate a response before sharing with the class.

Small Groups | 20 min

Activity 2 Populations of U.S. States and D.C. (continued)

Students compare a dot plot and a histogram, evaluating how each represents a data set and answers statistical questions.

Name: ______________________ Date: __________ Period: __________

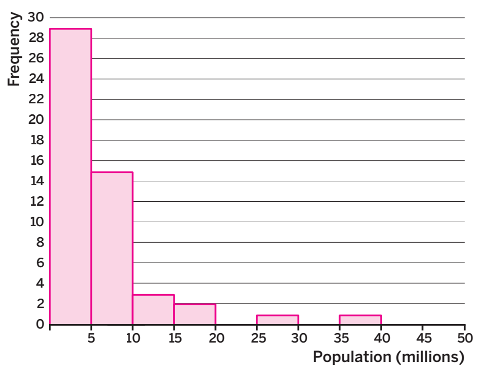
Activity 2 Populations of U.S. States and D.C. (continued)

Part 2

You will now be given the population data for each of the 50 states and the District of Columbia from the 2010 census.

2. Using the population data for each country, complete the table. Then use the grid and the information in your table to construct a histogram.

Population (millions)	0 to less than 5	5 to less than 10	10 to less than 15	15 to less than 20	20 to less than 25	25 to less than 30	30 to less than 35	35 to less than 40
Frequency	29	15	3	2	0	1	0	1



3. Revisit the questions in the table in Problem 1, and complete the same exercise as you did with the dot plot. This time, make your decisions for each question about whether it is possible to answer and if the answer is clearly shown *in the histogram*. In the last column of the table on the previous page, mark these decisions by again writing: C (clearly shown), P (possible to answer, but not clearly shown), or NP (not possible to answer). Be prepared to explain your thinking.

STOP

3 Connect

Have groups of students share their decisions that changed when they transitioned from using dot plots to histograms, focusing on comparing the features and effectiveness of each relative to answering specific statistical questions.

Ask, "What are some similarities that you noticed between the dot plot and the histogram of this same data? What are some differences?"

Highlight that a dot plot may not be best for representing a data set when there is a lot of variability (or when few values are repeated), or when a data set has a large number of values. Histograms may help visualize the distribution of the data more clearly in these situations; however individual values are not clearly shown on histograms.

Note: At the time of production of this version of the curriculum, 2020 census data were not available, nor was it known whether that data contained anomalies due to the conditions under which it was administered. You may consider using more recent data if available and considered accurate (or if not, and engage your students in an extension conversation about data collection). If you have additional time, you could also consider allowing your students to research the data for themselves.

Whole Class | 5 min

Summary

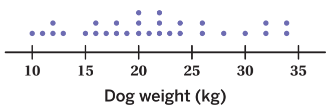
Review and synthesize how a histogram displays numerical data, how it shows the overall shape of a distribution, and how it is different from a dot plot.

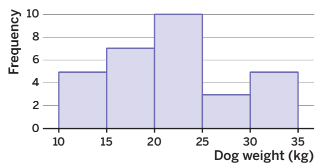
Summary

In today's lesson . . .

You saw that in addition to using dot plots, distributions of numerical data can be represented using ***histograms***.

The dot plot and histogram represent the same data set of the weights, in kilograms, of 30 dogs.





In a histogram, data values are grouped into *bins* that cover a range of values, and each *bin* has the same width. The total frequency of all values in that range is represented by a bar, including the left boundary (least value) but excluding the right boundary (greatest value). For example, the height of the tallest bar, between 20 and 25, represents weights of 20 kg up to (but not including) 25 kg.

Notice that the histogram and the dot plot have similar shapes – roughly symmetric around the center between 20 kg and 25 kg, with a spread going from about 10 kg less than that to about 10 kg more. One advantage of a dot plot is that it always shows all of the data values. A histogram generally never shows individual values, and its shape can change based on the chosen bin size. However, histograms may be more efficient to construct and interpret for large data sets, when there is a wider range of values, or when very few data values are the same.

> Reflect:

892 Unit 8 Data Sets and Distributions

Synthesize

Highlight that students saw histograms as another way to represent the distribution of a numerical data set. Histograms and dot plots can both be useful for answering statistical questions, but depending on the nature of the data or the context and the question being asked, or both, one representation may not be as useful as the other, or one may be slightly preferable.

Formalize vocabulary: *histogram*

Ask:

- "In general, what information does a histogram allow you to see? How is it different from a dot plot?" The histogram provides a "bigger picture" of the distribution that is "simpler" because it shows less detail. But that means some information cannot be seen as with a dot plot, such as individual data values.
- "When might it be more useful to use a histogram than a dot plot?" When the data are very spread out, when there are not very many data values with the same value, or when an overall idea of the shape and features of the distribution are more important than a detailed view.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What do you think was the most important thing that you learned about today?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *histogram* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to construct a histogram and interpret the information in the context of daily rainfall for one month.

Printable

Name: ________________ Date: ________ Period: ________

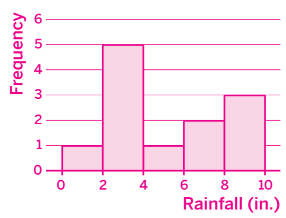
Exit Ticket 8.05

The tables show the average amount of rainfall, in inches, for each month in Miami, Florida.

Month	Rainfall (in.)	Month	Rainfall (in.)	Month	Rainfall (in.)
January	1.61	May	5.35	September	9.84
February	2.24	June	9.69	October	6.34
March	2.99	July	6.5	November	3.27
April	3.14	August	8.9	December	2.05

1. Complete the frequency table. Then make a histogram of the Miami, FL, rainfall data using the information from the frequency table.

Rainfall (in.)	Frequency
0–2	1
2–4	5
4–6	1
6–8	2
8–10	3



2. What is a typical amount of rainfall in one month in Miami?
About 5 in.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can recognize when a histogram is an appropriate graphical display of a data set.
1 2 3

b. I can use a histogram to get information about the distribution of data and explain what it means in a real-world context.
1 2 3

Success looks like . . .

- **Language Goal:** Comparing and contrasting dot plots and histograms in terms of how useful they are for answering different statistical questions. **(Speaking and listening)**
- **Goal:** Creating a histogram to represent a numerical data set.
  » Creating a histogram for the average rainfall in Miami, FL in Problem 1.
- **Language Goal:** Interpreting a histogram to answer statistical questions about a numerical data set. **(Writing)**
  » Determining the typical monthly amount of rainfall in Problem 2.

Suggested next steps

If students have difficulty setting up the histogram, consider asking:

- "Which column in the frequency table would go on the horizontal axis? Which would go on the vertical axis?"

If students incorrectly identify the typical amount of rainfall, consider:

- Refer them back to Lesson 3 to review what "typical" means. Ask, "What feature of a histogram could help you identify a typical value?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- In earlier lessons, students learned several vocabulary terms. How did knowing these terms support student descriptions and understanding of histograms? What might you change for the next time you teach this lesson?

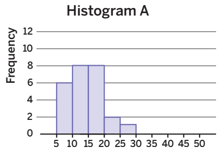
Practice

Independent

Name: Date: Period:

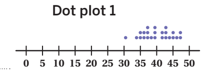
1. Match each histogram with its dot plot.

Histogram A

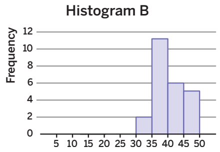


Dot plot 1

B

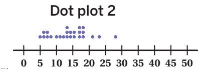


Histogram B

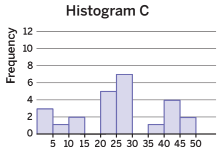


Dot plot 2

A

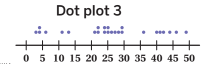


Histogram C



Dot plot 3

C



2. Here is a histogram that summarizes the lengths, in feet, of a group of adult female Great White Shark (*Carcharodon carcharias*). Select *all* the statements that are true about the histogram.

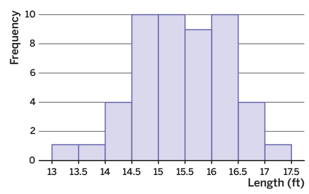
A. A total of 9 sharks were measured.

B. A total of 50 sharks were measured.

C. The longest shark was 10 ft long.

D. Most of the sharks were over 17 ft long.

E. Two sharks were less than 14 ft long.

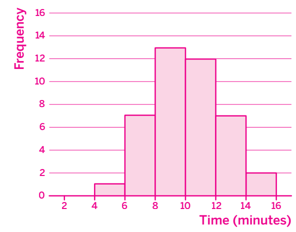


Practice

Name: Date: Period:

3. This frequency table shows the times, in minutes, it took 40 sixth grade students to run 1 mile. Use the frequency table to draw a histogram of the data.

Time (minutes)	Frequency
4 to less than 6	1
6 to less than 8	7
8 to less than 10	13
10 to less than 12	12
12 to less than 14	7
14 to less than 16	2



4. The point (−2, 3) is one vertex of a square on a coordinate plane. Name three points that could be the other vertices.
Sample response: (2, 3), (2, −1), (−2, −1)

5. How are these questions similar or different? How would you expect the answers to these questions to be similar or different?
   - What is the height of the tallest and lowest loop of the 5-loop roller coaster?
   - What is the minimum and maximum speed of the roller coaster?

Sample response:
The answers to both questions will have two numbers, set up as low to high.
They are both statistical.
They will both have heights or speeds in the middle.
The second question will have more data values (variability).

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 14	2
Formative	5	Unit 8 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 8 | **LESSON 6**

Using Histograms to Interpret Statistical Data

Let's draw histograms and use them to answer questions.

Focus

Goals

1. **Language Goal:** Compare and contrast histograms that represent two different data sets measuring the same quantity. **(Writing)**
2. **Language Goal:** Critique a description of a distribution, recognizing that there are multiple valid ways to describe its center and spread. **(Speaking and Listening)**
3. **Language Goal:** Describe the distribution shown on a histogram, including making claims about the center and spread. **(Speaking and Listening, Writing)**

Coherence

Today

Students create, read, and interpret histograms. They characterize the distribution displayed in a histogram in terms of its shape and spread, and identify a measurement that is typical for the data set by looking for the center in a histogram. Students also use histograms to make comparisons and interpret the meanings of different spreads and centers in a given context.

Previously

In Lesson 5, students were introduced to histograms — how to draw them, how they differ from dot plots, and what information they can display.

Coming Soon

In Lesson 7, students will use data to create histograms and bar graphs in order to compare numerical data representations to categorical data representations.

Rigor

- Students continue to build **conceptual understanding** in this lesson by creating histograms from frequency tables and interpreting the data they represent.
- Students build **fluency** with constructing histograms.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- straightedges

Math Language Development

New words

- ***maximum***
- ***minimum***

Note: The term *range* appears in this lesson in reference to the span of values, such as from minimum to maximum, or for a particular bin on a histogram. However, these are being treated as common usage. The formal definition of the term as a measure of variability will be introduced in Lesson 14.

Review words

- *histogram*
- *center*
- *distribution*
- *frequency*
- *spread*

Amps | **Featured Activity**

Activity 1
Interactive Histogram

Given a table, students can digitally move bars on a histogram. You can overlay student responses to provide immediate feedback.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Because histograms do not show individual pieces of data, students might think that they are not a very useful graphical representation in Activity 2. However, through the analysis of the structure of the histogram, students learn that histograms are a good way to visualize the shape and spread of the data, along with the location of the center. Encourage students to take notes on the histograms to build their descriptions, to stay organized, and to remind them how they drew their conclusions.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, a completed frequency table could be given to students.
- **Activity 2** can be done as a whole class. Also, the Connect part of Activity 2 could be used in place of the Summary, but it would be recommended to include a few of the Ask questions from the Summary.

Warm-up Which One Doesn't Belong?

Students consider whether questions are statistical and the types of data they will show, as well as their possible distributions.

Name: ________ Date: ________ Period: ________

Unit 8 | Lesson 6

Using Histograms to Interpret Statistical Data

Let's draw histograms and use them to answer questions.

Warm-up Which One Doesn't Belong?

Four questions about the population of Alaska are shown. Which question does not belong? Be prepared to explain your thinking.

Sample responses shown.

a. At what age do Alaska residents generally retire?
This is the only question that focuses on the center/typical value.

b. At what age can Alaskans vote?
This is not a statistical question.

c. What is the age difference between the youngest and oldest Alaska residents with a full-time job?
This is the only question that focuses on the distribution or spread of the ages.

d. Which age group makes up the largest percent of the population: under 18 years, 18–24 years, 25–34 years, 35–44 years, 45–54 years, 55–64 years, or 65 years or older?
This is the only question that has categories.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by reminding them there is not one correct answer, and having them think about the first choice that stands out to them as different in order to explain how or why it is different.

Look for points of confusion:

- **Not thinking in statistical terms.** Remind students to think about how they might go about answering the questions and what type of data they expect.

Look for productive strategies:

- Using statistical language in their justifications, such as *center*, *spread*, *typical*, *statistical*, *non-statistical*, *numerical data*, or *categorical data*.

3 Connect

Have individual students share a reason why a particular question does not belong. After each response, ask the class who agrees. Because there is no single correct answer for why a question does not belong, allow for a variety of reasons to be given for each question and attend to students' explanations to ensure the reasons given are accurate.

Highlight that question b is the only non-statistical question because there would not be any variability in the answers given. Question d is categorical because the age groups represent categories, even though they consist of numbers.

Math Language Development

MLR2: Collect and Display

During the Connect, as students describe which questions do not belong, collect and display the statistical language they use to justify their reasoning, such as, *center*, *spread*, *typical*, *statistical*, *non-statistical*, *numerical data*, or *categorical data*.

English Learners

Consider providing students with a Frayer model diagram labeled with the term *statistical question*. Have students write their definition of a statistical question and include questions a, c, and d as examples and question b as an example of a non-statistical question.

Power-up

To power up students' ability to relate statistical questions to the data that can be collected from them, ask:

Three questions are shown about a certain middle school's student population. Match each question to the type of data its answer tells you.

a. Are students required to wear uniforms? — __b__ center

b. What is the average daily attendance? — __c__ spread

c. What is the difference in size between the smallest class and the largest class? — __a__ non-statistical

Use: Before the Warm-up.

Informed by: Performance on Lesson 5, Practice Problem 5.

Pairs | 20 min

Activity 1 Measuring Antiguan Racers

Students practice drawing a histogram for a given data set and using the histogram to answer statistical questions.

Amps Featured Activity Interactive Histogram

Activity 1 Measuring Antiguan Racers

The Antiguan racer (*Alsophis antiguae*) is one of the rarest snakes in the world, listed as Critically Endangered by IUCN, only living only on islands near Antigua and Barbuda. It is so rare, in fact, that it has been thought to have been extinct two separate times – in 1936 and 2009. Most of the Antiguan racers alive today have been microchipped to monitor their growth and survival.


Public Domain

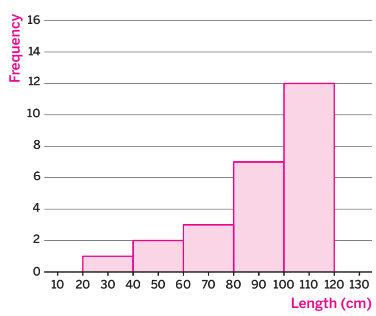
The table shows the lengths, in centimeters, of 25 Antiguan racers.

39	41.3	59.9	67	72.5	79.8	89	96.1	97.8	98	98.6	99.8	99.9
100	100	100.3	100.7	102.8	103.9	104	104	104.1	104.7	106.7	115.2	

1. Complete the frequency table representing the lengths of these 25 Antiguan racers.

Length (cm)	Frequency
0 to less than 20	0
20 to less than 40	1
40 to less than 60	2
60 to less than 80	3
80 to less than 100	7
100 to less than 120	12

2. Use the grid and the information in the table to draw a histogram for the data of snake lengths. Be sure to label the axes of your histogram.



1 Launch

Activate students' background knowledge by explaining that the lengths of the Antiguan racers in the bins provide information about their ages, which can be useful for monitoring its population. Give students 8–10 minutes of quiet work time and then 3–4 minutes to discuss their work and complete the activity with a partner.

2 Monitor

Help students get started by asking "What does *frequency* refer to?" The number of racers in the data set whose lengths are anywhere in the corresponding range.

Look for points of confusion:

- **Losing track of their counting when determining frequencies.** Suggest students reorganize the data in an ordered list and use tally marks to help keep track of the number of values for each bin.
- **Confusing histograms with bar graphs by leaving spaces between the bars.** Have them look back at the bars in other histograms they have seen so far and ask, "What would a gap mean, considering that histograms are built on a number line?"

Look for productive strategies:

- Accurately capturing the data into the frequency table and then in a histogram with correctly labeled axes.
- Using their histogram to explain what a typical length would be by noting a peak or cluster of data in the bars.
- Relating the distribution to the length and recognizing there are increasingly more racers with longer lengths.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally move bars on a histogram, given a table of values. You can overlay student responses to provide immediate feedback.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, provide a completed frequency table for them in Problem 1 and have them begin the activity in Problem 2.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you select students to share their responses to Problems 3 and 4, ask them to compare and contrast their responses with those of their classmates. This will help students better understand there are multiple, valid ways for describing a "typical" value.

English Learners

Display and annotate the completed histogram with where different students think the display shows a "typical" value, along with the minimum/maximum values, clusters, center and spread.

Pairs | 20 min

Activity 1 Measuring Antiguan Racers (continued)

Students practice drawing a histogram for a given data set and using the histogram to answer statistical questions.

Name: ______ Date: ______ Period: ______

Activity 1 Measuring Antiguan Racers (continued)

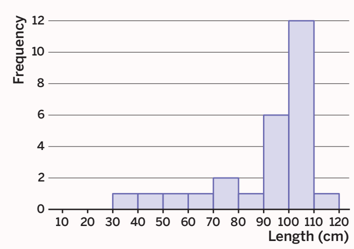
3. Based on your histogram, describe a typical length for an Antiguan racer? Explain your thinking.
   A typical length would be between 80 cm and 120 cm because most of the data are represented in those two bars and most likely around 100 cm because that is in the middle of 80 cm and 120 cm.

4. Based on the data, your histogram and your description of a typical length in Problem 3, describe the distribution of the data.
   The shortest Antiguan racer was 39 cm and the longest was 115.2 cm, so the snakes can differ in length as much as 76.2 cm. Most are more than 80 cm long, and are typically close to 100 cm long.

Are you ready for more?

Here is a histogram representing the Antiguan racer measurement data. In this histogram, the measurements are given in different bins.

1. Based on this histogram, what would you estimate is a typical length for the 25 Antiguan racers?
   100–110 cm
2. Compare this histogram with the one you created in Problem 2 of the activity. How are the distributions of data summarized in the two histograms the same? How are they different?
   The cluster is still between 90 and 110 cm. This one is much more spread out because the data are shown in groups of 10 cm, instead of 20 cm.



3 Connect

Have pairs of students share their completed histograms for all to see and briefly describe the overall distribution. Then select a few other previously identified students to share their responses and explanations for Problems 3 and 4.

Ask: (as time permits; the first three questions are specific to this activity, while the last two are about histograms in general)

- (If this has not already been addressed) "How did you determine what a typical length was?" Some students might describe it in terms of the size of the bins (e.g., "a typical length is between 100 and 120 cm"); others might choose a value within a bin or a boundary between bins (e.g., 100).
- "What does this tell you about the typical age of the Antiguan racers?"
- "Why would this information be important for animal scientists to know?"
- "How is identifying the center and spread using a histogram different from doing so using a dot plot?"
- "Explain whether each of these is visible on a histogram, and, if so, where or how you can see them:
  - » Maximum and minimum values.
  - » Clusters of values.
  - » Center of the distribution.
  - » Spread of the distribution."

Define:

- The ***maximum*** for a data set is the data value that is the greatest.
- The ***minimum*** for a data set is the data value that is the least.

Highlight that students should begin to see that identifying a typical value for a distribution is not a straightforward or precise process, yet. Explain that, in upcoming lessons, they will explore how to describe a typical value and characterize a distribution more systematically.

Pairs | 10 min

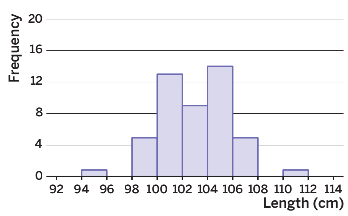
Activity 2 Match the Histogram

Students analyze two histograms using their overall shapes, as well as their centers and spreads, to compare the distributions of the lengths of male and female Antiguan racers.

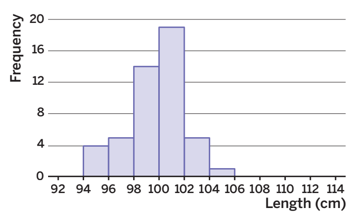
Activity 2 Match the Histogram

Here are two histograms representing the lengths of 50 adult male and 50 adult female Antiguan racers.

Histogram A



Histogram B



1. Describe the distribution of the lengths of the Antiguan racers represented in Histogram A in terms of the center and spread of the data.
   The distribution is centered around 103 cm.
   Sample response:
   The distribution is approximately symmetric around the center, with most of the racers' lengths being clustered between 98–108 cm. The overall spread goes from 94–112 cm, so these racers can have a difference in length of up to 18 cm. There are two gaps because there are no racers with lengths between 96–98 cm or 108–110 cm. The longest and shortest racers' lengths are each about 9 cm away from the center.

2. Describe the distribution of the lengths of the Antiguan racers represented in Histogram B, in terms of the center and spread of the data.
   The distribution of data is centered around 100 cm.
   Sample response:
   The data are spread out from 94 cm to 106 cm, so these racers can have a difference in length of up to 12 cm. The distribution is also symmetric, but there is a peak from 100–102 cm, with less and less of each length out from the center to around 4 cm longer or 6 cm shorter.

3. If the female Antiguan racer is typically longer than the male, which histogram do you think represents the lengths of female racers? About how much longer would you say a typical adult female is compared to a typical adult male?
   Histogram A represents the lengths of female racers.
   A typical female racer is about 103 cm long. A typical male is about 100 cm long. So, the female is about 3 cm longer than the male.

STOP

1 Launch

Use the *Think-Pair-Share* routine for this activity.

2 Monitor

Help students get started by having them first verbally describe the shapes of the distributions. Consider having students loosely outline the shapes of the distribution.

Look for points of confusion:

- **Misinterpreting the histograms.**
  - » **Thinking that the center of Histogram A is 105 because it is the tallest bar.** Ask, "Where would you fold the histogram in half?"
  - » **Thinking that since Histogram B has the highest peak, it must represent the female (typically longer).** Ask, "What is a typical length represented in Histogram A? in Histogram B? What does that mean about the general lengths of the racers that are represented?"

Look for productive strategies:

- Recognizing that the distributions of the two groups of Antiguan racers are quite different based on features or a description of either center or spread, or both.
- Using clear and precise mathematical language when describing and comparing the distributions.

3 Connect

Have pairs of students share their descriptions about the distributions presented in the two histograms. After each student shares, ask others whether they agree with the descriptions and, if not, how they might revise or elaborate on them.

Highlight that students are using approximations of center and different adjectives to characterize a distribution or a typical length, which leads to slight variations in the descriptions. In some situations, these variations might make it challenging to compare groups more precisely.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2. Problem 3 can be discussed as a whole class during the Connect.

Extension: *Math Enrichment*

Have students merge the data to represent all 100 adult Antiguan racers, or they can use the histograms to construct a new histogram representing the merged data set. Ask them to describe the center and spread of the new distribution and the length of a typical Antiguan racer.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 1, have pairs meet with 2–3 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "How do you know the distribution is approximately symmetric?"
- "What do the gaps in the histogram represent?"
- "What math language can you use in your response?"

Have students revise their responses, as needed.

English Learners

Use intentional grouping so that students with different English language proficiency levels can interact and have an opportunity to listen to peers with more advanced proficiency.

Whole Class | 5 min

Summary

Review and synthesize how to construct a histogram and the information about the distribution of a data set that can be interpreted from a histogram.

Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You learned how to construct a histogram, using a list or table of data.

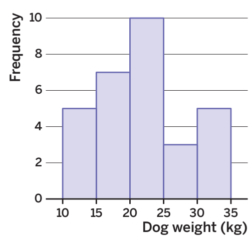
- First, you construct a number line that includes both the ***minimum*** (least value) and ***maximum*** (greatest value) from the data set.
- Next, you use that *range* of values to determine a reasonable bin size, remembering that each bin should have the same width. The number of bins impacts the shape of the distribution that is represented, so too many or too few could also impact how the data are interpreted.
- Finally, you need to draw the bars so that the heights represent the total number of data values corresponding to the range for each bin. Remember that each bin includes the left boundary value, but not the right boundary value.

For example, consider this ordered list of the weights, in kilograms, of 30 dogs:

10	11	12	12	13	15	16	16	17	18	18	19	20	20	20
21	22	22	22	23	24	24	26	26	28	30	32	32	34	34

Using the data, you can see that the range of values goes from 10 to 34 kg, so a reasonable bin size could be 5 kg, which means your number line should have values from 10 to 35, as shown here.

The histogram shows that a typical weight for this group of dogs is between 20 and 25 kg.



Reflect:

Synthesize

Highlight the importance of understanding bin sizes and what values are included in each bin.

Ask:

- "What are some questions you should ask yourself, or decisions you need to make, as you construct a histogram?"
- "What does the horizontal axis of a histogram tell you? What about the vertical axis?"
- "How do you know how tall to make each bar?"
- "Once a histogram is drawn, how can you use it to identify typical values?"
- "What else can you say about the weights of the group of dogs represented by the histogram?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Does it matter that one histogram was by 20s and another was by 2s?"
- "What are some generalizations you can make about the population of Antiguan racers?"
- "Is there anything you would like to learn more about the Antiguan racer?"

Independent | 5 min

Exit Ticket

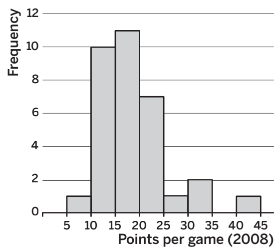
Students demonstrate their understanding of how histograms represent distributions of data by describing their centers and spreads.

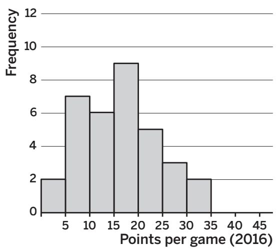
Printable

Name: _______________ Date: _______ Period: _______

Exit Ticket 8.06

The histograms show the points scored per game by a college basketball player in 2008 and in 2016.





1. Write 2 to 3 sentences that describe the centers of the two distributions, including what the center could tell you, in this context.
   2008: 20 is at the center of the 2008 histogram. This means that a typical number of points per game was around 20.
   2016: 17 is at the center of the 2016 histogram. This means that a typical number of points per game was around 17.

2. Write 2 to 3 sentences that describe the spreads of the two distributions, including what the spread could tell you, in this context.
   2008: 5 to 44 is a spread of 39 points.
   2016: 0 to 34 is a spread of 34 points.
   This tells me that, while the 2008 team typically scored more points per game, the 2016 team was more consistent. Or, the 2008 team had more "up and down" games.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can draw a histogram from a table of data.
1 2 3

b. I can use a histogram to describe the distribution of data and determine a typical value for the data.
1 2 3

Lesson 6 Using Histograms to Interpret Statistical Data

Success looks like . . .

- **Language Goal:** Comparing and contrasting histograms that represent two different data sets measuring the same quantity. **(Writing)**
  » Comparing the centers and spreads of the two histograms in Problems 1 and 2.
- **Language Goal:** Critiquing a description of a distribution, recognizing that there are multiple valid ways to describe its center and spread. **(Speaking and Listening)**
- **Language Goal:** Describing the distribution shown on a histogram, including making claims about the center and spread. **(Speaking and Listening, Writing)**
- Explaining what each measure of center or spread means in the context of points scored in Problems 1 and 2.

Suggested next steps

If students confuse the two terms *center* and *spread*, consider:

- Reviewing the vocabulary, or directing them to the class display. If these terms have not been added to the class display, add them along with visual examples.

If students give unreasonable answers when identifying the centers, consider:

- Suggesting students fold the page slightly to divide each histogram in half to help them identify the middle.

If students cannot interpret the center or spread in this context, consider:

- Rephrasing the questions using other words from the unit with similar meanings. For example, *center* can be another way of describing a typical number; spread can be another way to describe whether the data overall are consistent or has a lot of variability.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did understanding the bins or intervals set students up to interpret histograms?
- What challenges did students encounter as they worked on interpreting data in a histogram? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

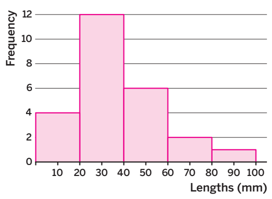
Name: Date: Period:

1. The table shows the lengths, in millimeters, of 25 earthworms.

6	11	18	19	20	23	23	25	25	26	27	27	28
29	32	33	41	42	48	52	54	59	60	77	93	

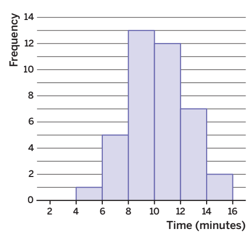
a Complete the frequency table for the lengths of the earthworms. Then use the grid to draw a histogram. Be sure to label the horizontal axis of your histogram.

Length (mm)	Frequency
0 to less than 20	4
20 to less than 40	12
40 to less than 60	6
60 to less than 80	2
80 to less than 100	1

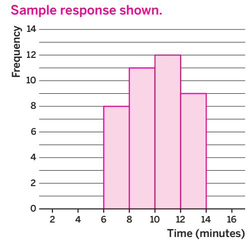


2. Forty sixth graders ran 1 mile. The histogram summarizes their times, in minutes. The center of the distribution is approximately 10 minutes. On the blank axes, draw a second histogram that has:

- A distribution of times for a different group of 40 sixth graders.
- A center at 10 minutes.
- Less variability than the distribution shown in the first histogram.

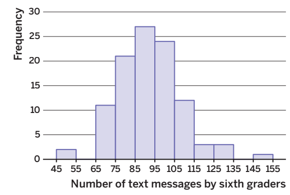


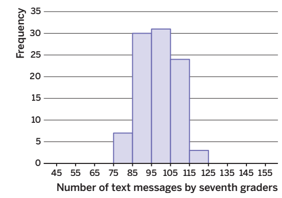
Sample response shown.



Name: Date: Period:

3. These two histograms show the number of text messages sent in one week by 2 groups of 100 students. The first histogram summarizes the data for sixth graders. The second histogram summarizes the data for seventh graders.





a Do the two data sets have approximately the same center and spread? Explain your thinking.

Sample response:
- **The sixth grade data's center is 90. The seventh grade data's center is 100. So, the seventh grade center is greater than the sixth grade center.**
- **No. The sixth grade data set has a greater spread from 45 to 154. The seventh grade spread is 75–124.**

b Overall, which group of students — sixth or seventh grader — typically sends more text messages? Explain your thinking.

Seventh graders typically send more text messages. The typical value for sixth graders is 90 and the typical value for seventh graders is 100.

4. Jada has d dimes, and no other money. She has more than 30 cents but less than a dollar.

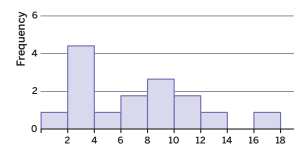
a Write two inequalities that represent how many dimes Jada has.

Let d represent the number of dimes. $d > 3$ and $d < 10$

b How many possible solutions make both inequalities true? If possible, describe or list the solutions.

There are six possible solutions: 4, 5, 6, 7, 8, 9.

5. How would you describe this histogram? Be sure to use all of the relevant language from this unit, such as: *peak, gap, cluster, spread, center,* and *typical*.



Sample response:
There is a peak at 2–4. There is a gap between 14 and 16. The spread is from 0 to 18. The center is at 9. A typical value would be between 6 and 10.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 11	2
Formative	5	Unit 8 Lesson 7	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

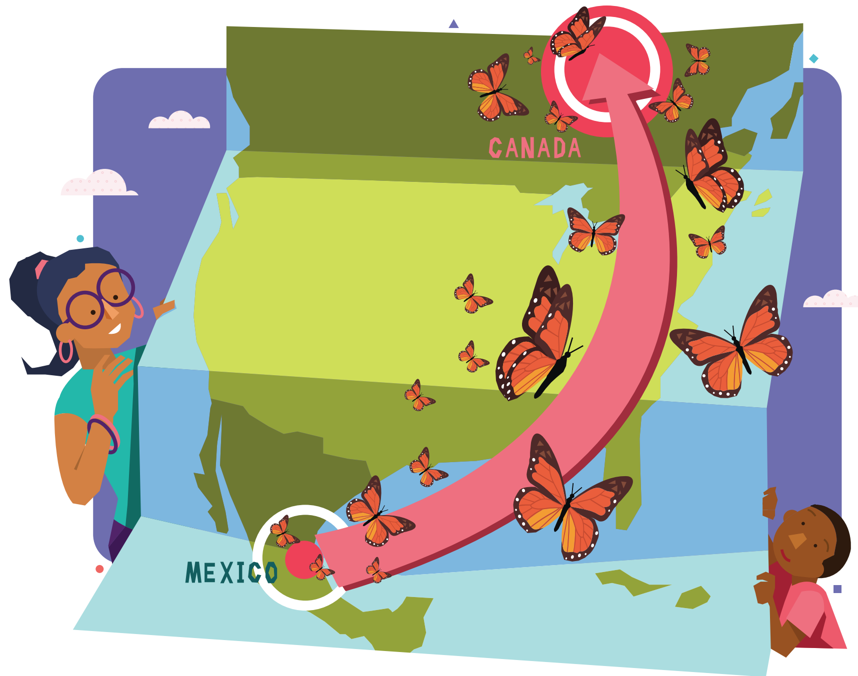
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 8 | **LESSON 7**

Describing Distributions With Histograms

Let's describe distributions by shapes and features displayed on histograms.



Focus

Goals

1. **Language Goal:** Compare and contrast bar graphs and histograms, recognizing that descriptions of shape, center, and spread do not pertain to bar graphs. **(Speaking and Listening)**
2. **Language Goal:** Describe the overall shape and features of a distribution represented on a histogram, including peaks, clusters, gaps, and symmetry. **(Speaking and Listening, Writing)**
3. Identify histograms that display distributions with specific features.

Coherence

Today

Students explore various shapes and features of a distribution displayed in a histogram. They use the structure to look for symmetry, peaks, clusters, gaps, and any unusual values in histograms. Students also begin to consider how these features might affect how data can be described. For example, students might wonder how they can describe what is typical in a distribution that shows symmetry or in a distribution that has one peak, but is not symmetric. This work is informal, but helps to prepare students to better understand measures of center and spread later in the unit. Students also distinguish between the uses and construction of bar graphs and histograms in this lesson.

Previously

In Lessons 5 and 6, students learned about histograms and how they represent data.

Coming Soon

In Lessons 9–14, students focus on determining measures of center.

Rigor

- Students solidify their **conceptual understanding** of histograms by relating them to frequency tables and interpreting them.
- Students **apply** their understanding of histograms in a real-world scenario about butterfly migration.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Symmetry* (for display)
- Activity 1 PDF, pre-cut cards, one set per group
- Activity 2 PDF, one per pair
- straightedges

Math Language Development

Review words

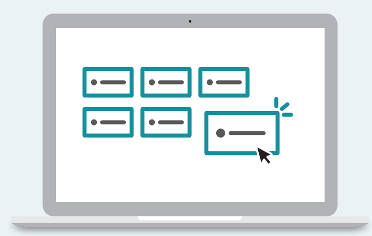
- *histogram*
- *center*
- *distribution*
- *frequency*
- *spread*
- *bar graph*
- *symmetric*

Amps | **Featured Activity**

Activity 1

Sorting Histograms

Students can use virtual tools to sort histograms, based on similarities of symmetry.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Throughout Activity 1, students should be debating how to sort the histograms, and poor communication might hinder the group's progress. When precise language is not used, students might waste time arguing about an issue that is really a point of agreement. Have each group keep a tally of the number of times they used precise mathematical language to clarify an argument or resolve a disagreement.

Modifications to Pacing

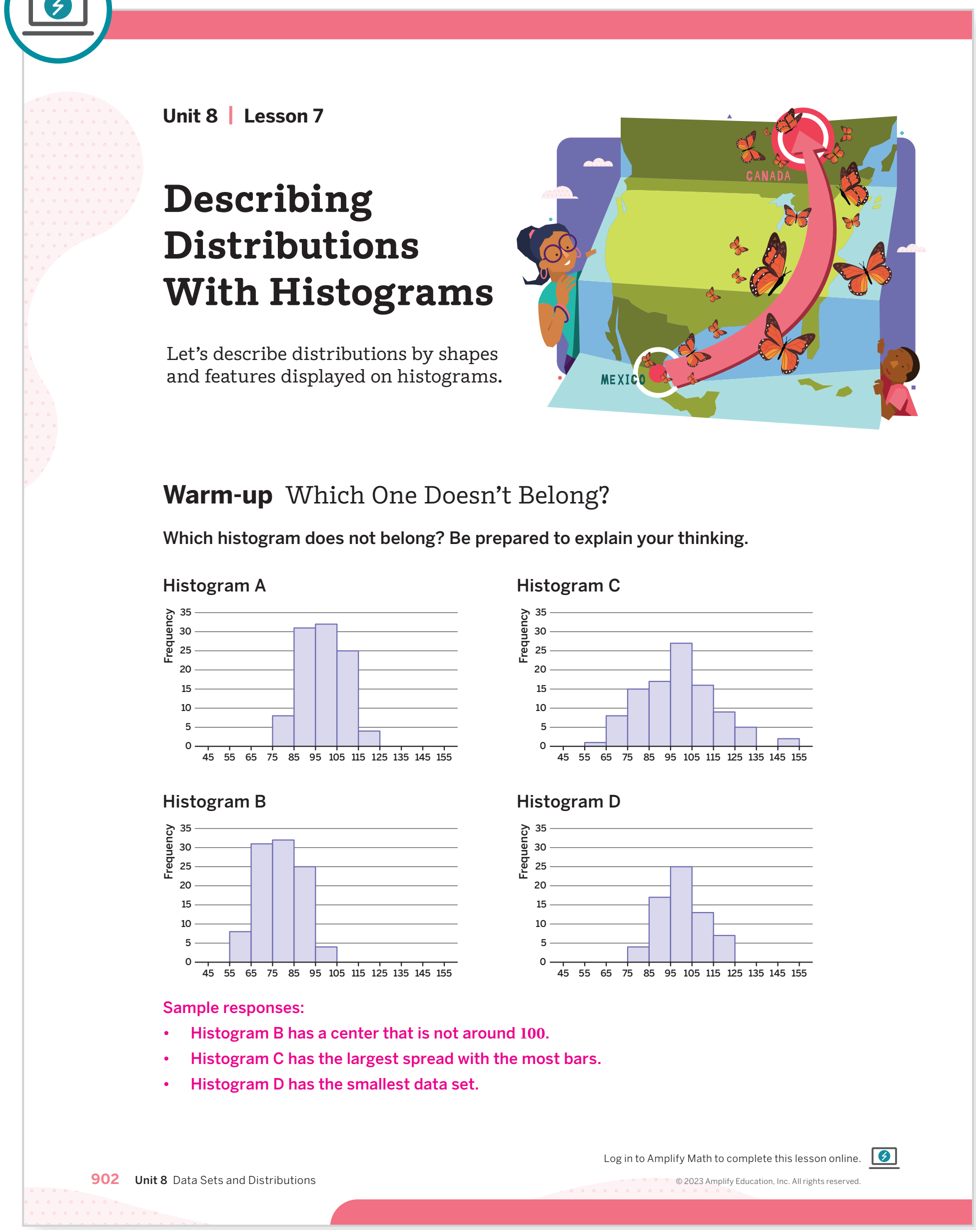
You may want to consider these additional modifications if you are short on time.

- In **Activity 1,** the number of cards could be reduced using only one of the pages of the Activity 1 PDF (cards).
- In **Activity 2,** complete the tables on the Activity 2 PDF ahead of time and display them for students to reference, or complete the tables together as a class.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students make sense of histograms in terms of center and spread and use mathematical language to explain their reasoning.



Unit 8 | Lesson 7

Describing Distributions With Histograms

Let's describe distributions by shapes and features displayed on histograms.

CANADA

MEXICO

Warm-up Which One Doesn't Belong?

Which histogram does not belong? Be prepared to explain your thinking.

Histogram A

Histogram C

Histogram B

Histogram D

Frequency

45 55 65 75 85 95 105 115 125 135 145 155

Sample responses:

- **Histogram B has a center that is not around 100.**
- **Histogram C has the largest spread with the most bars.**
- **Histogram D has the smallest data set.**

Log in to Amplify Math to complete this lesson online.

902 Unit 8 Data Sets and Distributions

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1 Launch

Conduct the ***Which One Doesn't Belong?*** routine.

2 Monitor

Help students get started by saying "Histograms A and B look exactly the same in terms of shape, but how are they different?"

Look for productive strategies:

- Recognizing that three of the four histograms have a center around 100 (choosing Histogram B).
- Comparing the greatest and least values in the data sets (choosing Histogram C).
- Calculating the heights to determine the total numbers of data values in each distribution (choosing Histogram D).

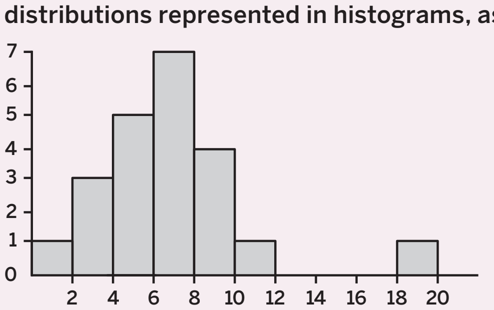
3 Connect

Have individual students share which histogram they thought did not belong and why, focusing on their use of mathematical language from this unit (such as *center, spread, distribution, frequency,* etc.). After each response, ask who agrees or disagrees, and if anyone chose the same histogram but for a different reason.

Highlight that, because there is no single correct answer to the question of which histogram does not belong, what is most important is that the explanations are reasonable.

Power-up

To power up students' ability to use mathematical language to describe distributions represented in histograms, ask:



Which of the following statements are true about the histogram shown. Select *all* that apply.

A. The spread is from 0 to 20.

B. There are 7 gaps in the data.

C. The distribution is not symmetrical.

D. There is a peak at 6–8.

E. A typical value would fall between 12 and 18.

Use: Before the Warm-up.

Informed by: Performance on Lesson 6, Practice Problem 5.

Small Groups | 15 min

Activity 1 Sorting Histograms

Students sort histograms based on features, such as symmetry, gaps, clusters, and unusual or extreme values, expanding their exposure to various features of distributions.

Amps Featured Activity Sorting Histograms

Name: ____________ Date: ________ Period: ________

Activity 1 Sorting Histograms

Any diagram of a distribution — dot plot or histogram — is described as *symmetric* if you can draw a line on the diagram so that the parts on one side of the line mirror the parts on the other side. It is actually quite rare for distributions of real-world data to be perfectly symmetric, but many are close enough, and the distributions are called *approximately symmetric*.

1. Your group will be given a set of histogram cards. Sort them into two piles — one for histograms that are approximately symmetric, and another for those that are not.

2. *Discuss* your sorting decisions *with another group*. Resolve any disagreements and record your final decisions.

   Approximately symmetric: C, E, G, I, J
   Not symmetric: A, B, D, F, H, K, L

3. Histograms are also described by how many peaks they have. Histogram A is an example of a distribution with at least one peak. Which other histograms have this feature?
   B, D, E, F, G, H, J, K, L

4. Histogram A is also an example of a histogram with a *gap*, which is an interval between two bars where there are no data values. Which other histograms have this feature?
   D, F, E, K, L

5. Sometimes there are a few data values in a data set that are *far* from the center. Histogram A is an example of a distribution with this feature, in the 80 to 85 interval. Which other histograms have this feature?
   A, K, L

Are you ready for more?

Create a scenario that Histogram E could represent.
Answers will vary.

1 Launch

Display the Activity 1 PDF, *Sorting Histograms*. Activate students' prior knowledge of symmetry in distributions from their work with dot plots and point out that symmetry and other features of distributions in dot plots can also be seen in histograms. Then give groups 10–12 minutes to complete the activity.

2 Monitor

Help students get started by asking students what they notice (or what they wonder) about Histogram A.

Look for points of confusion:
- **Not knowing whether a uniform or near uniform distribution is symmetric (Histograms G and I).** Ask, "If you were to fold them in the middle, would there be about the same amount of data on each side, and would each side be close to the same shape?"

Look for productive strategies:
- Using the mathematical language of the unit to accurately classify and sort the histograms.

3 Connect

Ask
- "When a distribution is approximately symmetric, where might its center be? What about those that are not approximately symmetric (such as Histogram B)?"
- "How might a gap (such as the ones in Histograms D or K) affect your description of what is typical in a group?"
- "Do unusual or extreme values (such as those in Histograms A or L) affect your descriptions of center and spread? If those values were not there, would your descriptions of center and spread change, and if so, how?"

Highlight that students have been using the center of a distribution to talk about what is typical in a group, but the shape and features of distributions also have an affect on how to interpret the data.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use virtual tools to sort histograms, based on similarities of symmetry.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them analyze two cards, such as Histograms C and F. They can discuss the features, similarities, and differences of these histograms and still participate in the class discussion.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the Activity 1 PDF, *Sorting Histograms*. Have students work with their group to write 2–3 mathematical questions they could ask about the histogram. Invite students to share their questions with the class and compare the language of the questions before having students begin the activity. Sample questions shown.

- Does this histogram show symmetry? Is it symmetric or approximately symmetric?
- Does the left and right side of the center need to look exactly the same in order for it to be symmetric or approximately symmetric?

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 15 min

Activity 2 Monarch Butterfly Migration

Students use both numerical and categorical data related to the same context to construct both a histogram and a bar graph, and then compare both representations.

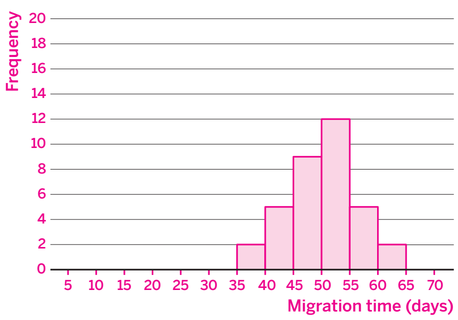
Activity 2 Monarch Butterfly Migration

In the 1980s, the IUCN created a new threat level of Not Evaluated as a result of the decrease in monarch butterfly (*Danaus plexippus*) populations counted during their migration, calling this a "threatened phenomenon." The Not Evaluated status acknowledges that the species needs to be assessed because there are concerns about its population, but not enough data to make another classification. The IUCN's Special Survival Commission Butterfly Specialist Group is currently assessing three butterfly groups, one of which is the monarch.

Sue Bishop/Shutterstock.com

You will be given information related to the migration of monarch butterflies from as far north as Canada down to their wintering place in Mexico. One partner will create the histogram for Problem 1 and the other will create the bar graph for Problem 3; then you will work on Problems 2, 4, and 5 together.

1. Use the data to create a histogram that shows how many days it took 75 tagged monarch butterflies to migrate from as far north as Canada down to Mexico.



2. Describe the distribution of the migration times. Comment on the center and spread of the data, as well as the shape and features.
   The center is around 50.
   The spread is 30, from 35 to 65.
   It is approximately symmetric.
   There is a peak in the 50–55 bar.

904 Unit 8 Data Sets and Distributions

1 Launch

Distribute the Activity 2 PDF. Have one partner create a histogram for Problem 1 and the other create a bar graph for Problem 2. They can then complete the remaining problems together.

2 Monitor

Help students get started by asking, "What would be an appropriate bin size?" If students do not know how to answer or give an off hyphen base answer, suggest sizes of 5, 10, or 20 and ask which would be best.

Look for points of confusion:

- **Having bins of different sizes.** Remind students that the bins have to be of equal size.
- **Losing track of their counting when determining frequencies.** Suggest students circle or cross out data values as they assign them to bins or bars.

Look for productive strategies:

- Fluently calculating the different percentages to determine the frequencies for Problem 3.
- Noticing similarities and differences in how to characterize distributions in histograms and bar graphs, including how to describe and interpret typical values and frequencies.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, provide them with either a pre-completed histogram or a pre-completed bar graph. This will allow them to spend more time analyzing, comparing, and contrasting the displays.

Extension: *Interdisciplinary Connections*

Students may wonder how the monarch butterflies know how to travel to their southern destination. Tell them that researchers are still investigating this phenomenon, but some researchers think that both the magnetic pull of Earth and the position of the Sun help the butterflies reach their destination. **(Science)**

Math Language Development

MLR7: Compare and Connect

During the Connect, have students share their responses to the question posed in their Student Edition, "As you respond to Problem 5, what math language can you use to compare histograms and bar graphs? What kind of data does each graph show?" Draw their attention to the similarities and differences by referencing the following types of features:

- Horizontal axis labels, numerical or categorical data
- Center and spread and whether they make sense for each type of display

English Learners

Annotate the histogram and bar graph with their similarities and differences.

Pairs | 15 min

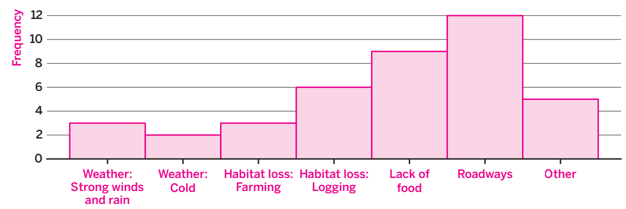
Activity 2 Monarch Butterfly Migration (continued)

Students use both numerical and categorical data related to the same context to construct both a histogram and a bar graph, and then compare both representations.

Name: ____ Date: ____ Period: ____

Activity 2 Monarch Butterfly Migration (continued)

3. Use the data to create a *bar graph* on the dangers monarch butterflies face during migration that prevent them from reaching their destination. Include labels for the vertical and horizontal axes.



4. Describe what you learned about the dangers to monarch butterflies during their migration. Comment on any patterns you noticed.

   Roadways are the biggest danger for monarch butterflies. Weather combined is less of a danger than the habitat loss combined, so habitat loss in general is more of a danger to monarch butterflies during their migration than the weather.

5. Think about histograms and the bar graphs. How are they the same? How are they different?

   Numerical data are presented on a histogram, but a bar graph shows categorical data. When a histogram has gaps, there is no data. With a bar graph, the gaps have no meaning. Both use the y-axis to show frequency or the vertical bars show frequency in both. The widths of the bars in a histogram are important, but they are not in a bar graph.

Compare and Connect: As you respond to Problem 5, what math language can you use to compare histograms and bar graphs? What kind of data does each graph show?

Are you ready for more?

Research one of these data sets (or choose your own) to create a histogram. Then describe the distribution.

- Heights of 30 athletes from multiple sports or the same sport.
- High temperatures for each day of the last month in a city you would like to visit.
- Prices for all the menu items at a local restaurant.

STOP

3 Connect

Have students share their representations and descriptions of what each shows, being sure to also include histograms constructed using different bin sizes. Then have students share their responses to Problem 5.

Ask:

- "How were your descriptions for the histogram (Problem 2) different from those for the bar graph (Problem 4)?"
- "Could you have used a bar graph to display the data on travel times? Why or why not?"
- "Could you have used a histogram to display the data on methods of travel? Why or why not?"

Highlight the fact that the notions of shape, center, and spread do not apply to bar graphs, because order does not matter, and those critical factors for the meaning and interpretation of those features. If time permits, connect the frequencies between the data sets — how many butterflies made it and how many succumbed to one of the dangers relative to the total of 75 tagged butterflies.

Whole Class | 5 min

Summary

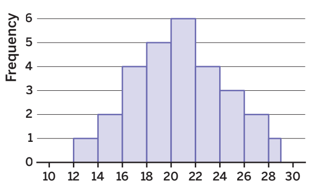
Review and synthesize the features of distributions represented by a histogram and their interpretations, and then compare a histogram to a bar graph.

Summary

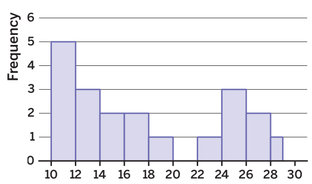
In today's lesson . . .

You continued to explore ways to describe the shape and features of a distribution represented by a histogram. Here are two distributions with very different shapes and features.

Histogram A



Histogram B



- Histogram A is symmetric and has a peak near 21. Histogram B is not symmetric and has two peaks, one near 11 and one near 25.
- Histogram B has two clusters.
- Histogram B also has a gap between 20 and 22.

Bar graphs and histograms may seem alike, but they are very different.

- Bar graphs represent categorical data. Histograms represent numerical data.
- Bar graphs have spaces between the bars. Histograms show a space between bars *only* when there is a gap and no data values fall between the bars.
- Bars in a bar graph can be in any order, and the number of bars depends on the number of categories. Bars in a histogram always represent ranges of values in numerical order, and you can choose how many bars to use.

Reflect:

Synthesize

Have students share words they have been using to describe different distributions, in this lesson and previous ones.

Highlight that the words used to describe distributions help in analyzing and interpreting data, and should be used precisely and consistently when communicating findings to someone else.

Ask:

- "What does it mean for a histogram to have a symmetric distribution?"
- "Think about some features of distributions — peaks, clusters, gaps. How might each of those features be interpreted differently on a histogram than a dot plot?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Think about other times you have discussed symmetry. Maybe you discussed it in art class in previous grades?"

Exit Ticket

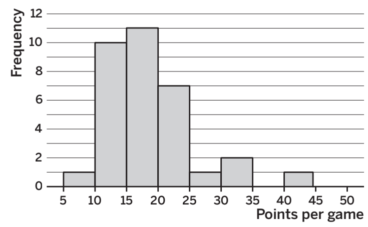
Students demonstrate their understanding by describing the shape and features of a distribution represented by a histogram.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 8.07

The histogram that shows the number of points scored by a college basketball player during the 2008 season. Describe the shape and features of the data.



The data are not symmetric.

There is a gap between 35 and 39 which means that the college basketball player did not score 35–39 points during the season.

A typical amount of points scored by the college basketball player is between 10 and 19 points.

There is a peak at 15 to 19.

The college basketball player had one really high scoring game and one really low scoring game.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe the shape and features of a histogram and explain what they mean in the context of the data.
1 2 3

b. I can distinguish histograms and bar graphs.
1 2 3

Success looks like . . .

- **Language Goal:** Comparing and contrasting bar graphs and histograms, recognizing that descriptions of shape, center, and spread do not pertain to bar graphs. **(Speaking and Listening)**
- **Language Goal:** Describing the overall shape and features of a distribution represented on a histogram, including peaks, clusters, gaps, and symmetry. **(Speaking and Listening, Writing)**
  - » Describing the shape and features of the histogram for the number of points scored by a basketball player.
- **Goal:** Identifying histograms that display distributions with specific features.

Suggested next steps

If students have trouble describing the overall shape or recognizing and describing other features of the distribution, consider:

- Having students reference a class anchor chart or word wall, or construct a list of terms they could use to describe the histogram. Then have them choose at least two that match the histogram of number of points scored by the basketball player and interpret them in context.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which teacher actions were especially helpful to students today as they created the histogram in Activity 2?
- In earlier lessons, students were introduced to a lot of vocabulary and language to describe data. How did you model using the language today to support students? What might you change for the next time you teach this lesson?

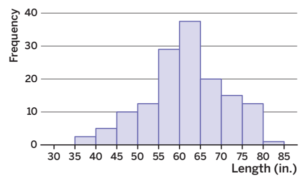
Practice

Independent

Name: ______________ Date: ________ Period: ________

Practice

1. The histogram summarizes the data on the body lengths of 143 black bears (*Ursus americanus*). Describe the distribution of body lengths. Be sure to comment on the overall shape, center, and spread.



Symmetric shape. A typical body length (the center) for wild bears is between 60–65 in. There is a peak between 60–65. The spread is 50 in.

2. Which data set is more likely to produce a histogram with a symmetric distribution? Explain your thinking.
   - a. Data on the length of songs, in seconds, on a pop album.
   - b. Data on the time, in seconds, spent talking on the phone yesterday by every person in the school.

   The data on songs would be more symmetric because they are all about the same length of time. The data representing everyone in the school talking on the phone would have a lot of variability, so it is less likely to be symmetric.

3. Select *all* of the data sets that would likely have one or more gaps when represented by a histogram. For each data set that may have a gap, briefly describe or give an example of why.
   - A. The ages of students enrolled in a sixth grade class.
   - B. The ages of people in an elementary school building on any given day.
     There would be both students and adults who are working there, but it is unlikely there would be any high school or college-aged people there.
   - C. The ages of people eating in a family restaurant.
     The people eating there would likely include children of all ages and younger adults, but the ages of say grandparents could vary more and it might be less likely every age group of older adults would be represented.
   - D. The ages of runners in a marathon.
     A wide range of people run marathons, but it could be likely that only a few younger or older people are running, which means one or more groups in the ends of the age range might not be represented.

Name: ______________ Date: ________ Period: ________

Practice

4. Evaluate the expression $4x^3$ for each value of x.
   - a. 1
     $4; 4 \cdot (1 \cdot 1 \cdot 1) = 4$
   - b. 2
     $32; 4 \cdot (2 \cdot 2 \cdot 2) = 32$
   - c. $\frac{1}{2}$
     $4 \cdot \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right); 4 \cdot \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

5. Jada drank 12 oz of water from her bottle. This is 60% of the water the bottle holds.
   - a. Write an equation to represent this situation. Explain the meaning of any variables you use.
     Let B represent the number of ounces the bottle can hold. $\frac{60}{100} \cdot B = 12$
   - b. How much water does the bottle hold?
     20 oz

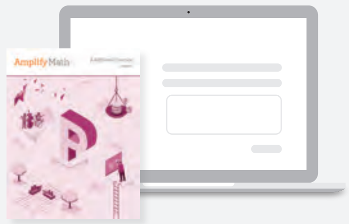
6. Determine each quotient.
   - a. $128 \div 8 = 16$
   - b. $228 \div 6 = 38$
   - c. $12.4 \div 8 = 1.55$
   - d. $0.99 \div 0.3 = 3.3$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 15	1
	5	Unit 3 Lesson 12	2
Formative	6	Unit 8 Lesson 8	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 2

Measures of Center

Whole Class

In this Sub-Unit, students calculate and compare means and medians of data sets, and determine when and why one of the measures of center may be more appropriate.

SUB-UNIT

2 Measures of Center

Narrative Connections

What's the buzz on honey bees?

You've seen bees before. They buzz around our gardens, busy as — well, a bee! Studying one bee can tell you a lot about it, but it is only a small part of the bigger picture. Each bee is part of a collective called a hive. This hive includes workers, drones, and, most notably, the queen. Beekeepers and scientists the world over are working to make sure that honey bee hives increase in health and number.

While honey bees pollinate about 75% of the world's food production, their numbers have been declining. Colony Collapse Disorder (CCD) is the largest cause of this decline. CCD occurs when worker bees disappear, leaving their queen behind. There are many theories behind what causes CCD. Some possible explanations include climate change, mite infestation, and pesticide use.

As biologists continue to study the causes of CCD, the Environmental Protection Agency has banned pesticides harmful to bee habitats in the U.S. Even tech companies have gotten involved. They have developed devices that alert beekeepers to problems in their hives.

While these are good first steps, there is still more work to be done. After all, honey bees are just one species among millions. Prioritizing this one species can create imbalance and do harm to another. To get a precise picture of the issues bees face, we need a precise way to describe their numbers. By studying the mean and median values in available data sets, we not only learn about the honey bees, but also their role in our ecosystem.

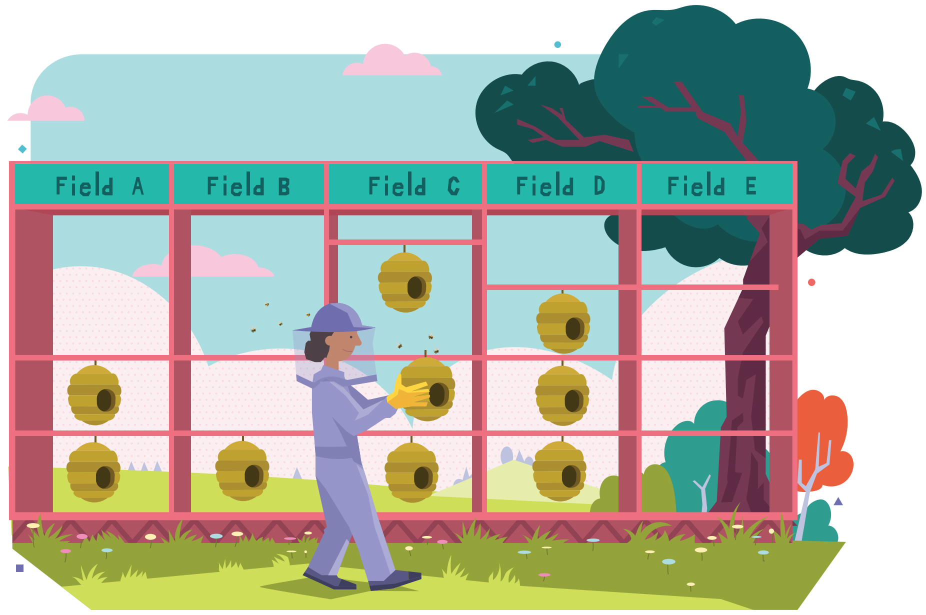
Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how measures of center can be used to understand the honey bee population in the following places:

- **Lesson 8, Activity 1:** Relocating Honey Bee Hives
- **Lesson 9, Activities 1-2:** Balancing Bumble Bees, Threats to the Honey Bees
- **Lesson 10, Activity 2:** Varroa Mites, Bee Gone!
- **Lesson 11, Activity 2:** Mean or Median?

Mean as a Fair Share

Let's explore the mean of a data set and what it tells us.



Focus

Goals

1. **Language Goal:** Comprehend the words *mean* and *average* as a measure of center that summarizes the data using a single number. **(Speaking and Listening)**
2. **Language Goal:** Explain how to calculate the mean for a numerical data set. **(Speaking and Listening, Writing)**
3. **Language Goal:** Interpret diagrams that represent determining the mean as a process of leveling out the data to determine a "fair share." **(Speaking and Listening, Writing)**

Coherence

Today

Students determine and interpret the mean of a distribution as the amount each member of the group would get if everything is distributed equally. This is sometimes called the "leveling out" or the "fair share" interpretation of the mean. For an already measured quantity that cannot actually be redistributed, students can imagine redistribution but should also begin relating the mean to the idea of an (arithmetic) average. This allows them to better understand how a single number can be used to summarize a data set when there is variability, recognizing some values will be less and some will be greater, but how much more and less overall relates both the average and fair share interpretations. Students also make an explicit connection between the mean and the idea of "typical," or the center of a distribution, and thus explain why the mean is called a measure of center for a distribution.

Previously

In Lessons 3 and 4, students used the shape of a distribution to describe the center and identify typical values in a data set.

Coming Soon

In Lesson 9, students will determine and interpret the mean of a distribution as the balance point for a data set.

Rigor

- Students build **conceptual understanding** of the mean as an equal redistribution so that each value has the same frequency.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- snap cubes (optional)

Math Language Development

New words

- ***average***
- ***mean***
- ***measure of center***

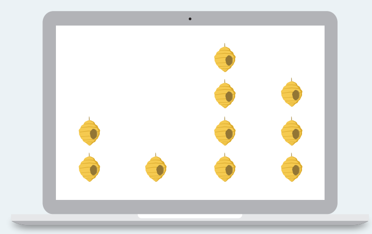
Review words

- *center*
- *distribution*
- *typical*

Amps | **Featured Activity**

Activity 1
Interactive Honey Bee Hive Relocation

Students experiment with, and discover, mean as an equal redistribution — or "fair share" — by relocating honey bee hives among different fields. You can overlay student work to provide immediate feedback.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might feel stressed at having to discover the missing data. Prior to the activity, ask students to identify ways that they will control their stress levels. Have them share with their partner which method they chose and at any time when they are feeling overwhelmed, their partner can remind them to use it. Also, reinforce that they have all of the skills necessary to reason quantitatively within the structure of the dot plot to complete the task.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1,** Part 1 may be completed as a class. Consider showing students the sample answers for Problem 1 in Part 2 and asking, "Do these arrangements work? Why or why not?" If time permits, have pairs use the *Turn and Talk* routine to come up with a third arrangement. Then have pairs complete Problem 2 before sharing with the class.

Independent | 5 min

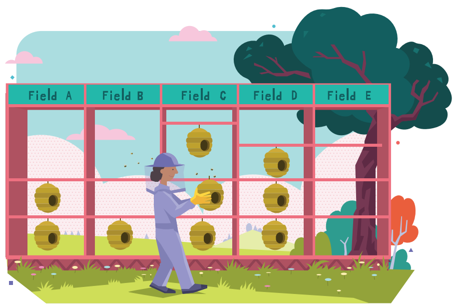
Warm-up Four for Four

Students create an equation with a value of 4, preparing them to calculate the mean of a data set by determining the average value in Activity 1.

Unit 8 | Lesson 8

Mean as a Fair Share

Let's explore the mean of a data set and what it tells us.



Warm-up Four for Four

Complete the equation using four *different* single digit addends from 0–9. Show your thinking

$(\square + \square + \square + \square) \div 4 = 4$

Answers may vary, but the sum of the digits should equal 16.
Sample response: $1 + 2 + 4 + 9$

$(a + b + c + d) \div 4 = 4$
$(a + b + c + d) = 4 \cdot 4$
$a + b + c + d = 16$
$1 + 2 + 4 + 9 = 16$

or

$x \div 4 = 4$
$x = 4 \cdot 4 = 16$
$1 + 2 + 4 + 9 = 16$

Log in to Amplify Math to complete this lesson online.

910 Unit 8 Data Sets and Distributions

1 Launch

Have students use the *Think-Pair-Share* routine. Give them 1 minute of quiet work time and then 2 minutes to share with a partner.

2 Monitor

Help students get started by asking, "What part of the expression on the left side of the equal sign would need to be calculated first?"

Look for points of confusion:

- **Struggling to identify addends that are different.** Ask, "If the addends could all be the same, what would they be? What would need to happen if one of those increased by 1 (and became a 5)? If one of those decreased by 2?"

Look for productive strategies:

- Using the structure of the equation and understanding of division to determine that the addends should total 16 because $16 \div 4 = 4$.
- Using their understanding of fractions as division to determine the addends.
- Starting with $4 + 4 + 4 + 4$ and adjusting the addends to maintain a sum of 16.

3 Connect

Have students share their equations and strategies, making sure that at least two different sets of addends are shown.

Highlight how the 4 on the right side of the equal sign represents an average of the four addends. Then show how writing division as a fraction can be used to rewrite the equation as $\frac{a+b+c+d}{4} = 4$. Explain that this could also be written as a sum of fractions (dividing each addend in the numerator by 4), but this single fraction format or the given whole-number division format will be most efficient for determining averages.

Ask, "How might your strategy change if the denominator was 6? 10?"

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share their equations and strategies, press for more detail in their reasoning by asking:

- "Why does the sum of the digits need to be equal to 16?"
- "What does the 4 on the right side of the equal sign represent?"
- "In this equation, the divisor and quotient were both 4. Do they represent the same concept? Why or why not? What does the divisor of 4 represent?"

English Learners

Provide students time to rehearse and formulate a response with a partner before sharing with the class.

Power-up

To power up students' ability to divide values whose quotients are positive rational values, have students complete:

Hint: Use the result from the first problem in each column to help you evaluate the second problem in each column.

a. $108 \div 9$
$= 12$

b. $8.4 \div 4$
$= 2.1$

c. $1.08 \div 9$
$= 0.12$

d. $8.4 \div 0.4$
$= 21$

Use: Before Activity 2.

Informed by: Performance on Lesson 7, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1.

Pairs | 15 min

Activity 1 Relocating Honey Bee Hives

Students activate prior knowledge of division as they are introduced to the concept of mean as an average, in terms of the equal distribution of honey bee hives.

Amps Featured Activity Interactive Honey Bee Hive Relocation

Name: ______ Date: ______ Period: ______

Activity 1 Relocating Honey Bee Hives

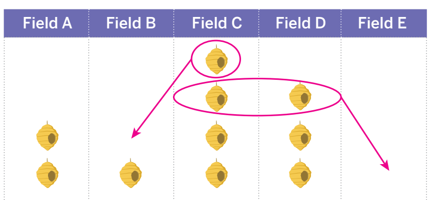
Pollinators like bees, birds, bats, beetles, and butterflies are crucial to life on Earth. Without them, many species of flora (flowers, plants, and trees) would die, limiting the diversity of food available for animals and humans.

Daniel Prudek/Shutterstock.com

The honey bee (*Apis mellifera*) is one of the most prolific pollinators in the world. Farmers use these bees to increase crop production, moving hives to support different crops that grow at different times of the year. However, moving bees can be disruptive to them and, thus, to their ability to pollinate. Because bees map efficient routes between their colonies and the surrounding flora, farmers use the "rule of three" when relocating a hive. Either move the hive no more than 3 ft, placing them close enough to be familiar with the surroundings, or move it at least 3 miles, which is far enough for them to know the hive is at a completely different location.

Part 1

1. Consider a farmer with 5 fields and 10 honey bee hives. This table shows where the hives are currently located. For the upcoming growing season, the farmer wants the hives distributed equally among all 5 fields. How many hives will end up in each field? Use the table to show how it could be done, and explain your thinking.



There will be 2 hives in each field. Sample response: Move 1 hive from Field C to Field B. Move 1 hive each from Fields C and D to Field E.

The number of hives in each field after they are equally distributed represents the *mean* or the *average* number of hives per field given the total, regardless of where they are located.

2. Write an expression for determining the *mean* number of hives the farmer has for each of the 5 fields. Show or explain your thinking.

Sample response: The farmer had a total of 10 hives to be equally distributed in 5 locations. This can be represented using the division expression $10 \div 5$ because $(2 + 1 + 4 + 3) \div 5$ or $(2 + 2 + 2 + 2 + 2) \div 5$.

Lesson 8 Mean as a Fair Share 911

1 Launch

Have students pause after completing Part 1 to explain how their representations and expressions are related, ensuring they see a variety of responses, including the sample responses given. Then have pairs complete Part 2 together.

2 Monitor

Help students get started by asking, "What does the table show?"

Look for points of confusion:

- **Adding new hives (Part 1, Problem 1).** Clarify that there still must be only 10 hives. Ask, "When have you created equal groups before?"
- **Thinking that there cannot be 3 hives because 3 is the mean (Part 2, Problem 1).** Explain that the mean can be a value in the set, but it is not necessary.

Look for productive strategies:

- Relating prior work with equal groups and division to write a division expression to calculate the mean as an average.
- Using the relationship between multiplication and division to determine the total as equal groups before redistributing into unequal parts (Part 2, Problem 1).
- Identifying data sets that could have a mean of 11 by considering whether redistributing could make all values equal to 11.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they experiment with, and discover, the mean as an equal redistribution — or "fair share" — by relocating honey bee hives among different fields.

Accessibility: *Clarify Vocabulary and Symbols*

Demonstrate what the phrase "redistribute equally" means by giving three students a different number of pencils, or other objects. For example, give one student 5 pencils, one student 1 pencil, and one student 3 pencils. Ask them how many pencils need to move from one student to another so that they all have the same number of pencils. Tell them this is what the phrase "redistribute equally" means.

Math Language Development

MLR7: Compare and Connect

After students complete Part 2, Problem 1, have them share their responses and strategies with another pair of students. Have them consider what is the same and what is different about the different approaches used. Provide clarifying prompts for partners to press each other to prove their arrangements create the given average, such as:

- "How do you know the average is 3 hives per field?"
- "Does every field need to have a hive?"

English Learners

Point out the phrase ". . . where hives *could* be located" means that not every field needs to have a hive.

Pairs | 15 min

Activity 1 Relocating Honey Bee Hives (continued)

Students activate prior knowledge of division as they are introduced to the concept of mean as an average, in terms of the equal distribution of honey bee hives.

Activity 1 Relocating Honey Bee Hives (continued)

Part 2

3. Another farmer has 6 fields where hives could be located. Every field has a different number of hives, and there is an *average* of 3 hives per field. Complete the table to show two different possible arrangements of hives that match this description.
Answers may vary, but should show that 18 hives are divided across the 6 fields, and no two fields in the same arrangement should have the same number of hives.

	Field A	Field B	Field C	Field D	Field E	Field F
Arrangement 1	0	1	2	4	5	6
Arrangement 2	0	1	2	3	5	7

4. A third farmer has 5 fields where hives are located, and the *mean* number of hives per field is 11.

a. By simply looking at the data sets, and without calculating, describe how you could identify which of the following data sets could possibly represent the hives in each of the farmer's fields, and which could not. Explain your thinking.

Data set A: 11, 8, 7, 9, 8

Data set B: 12, 7, 13, 9, 14

Data set C: 11, 20, 6, 9, 10

Data set D: 8, 10, 9, 11, 11

Sample response: Data sets B and C could be the farmer's because, while there are some values less than 11, there are also some values greater than 11 that could be redistributed to make the lesser ones equal to 11. Data sets A and D could not be the farmer's because I can tell the mean number of hives will be less than 11. All values that are not equal to 11 are all less than 11, and, if they were redistributed to make equal amounts, it would result in less than 11 per field.

b. Determine which data set is the farmer's. Explain how you know.
Data set B; Sample response: I know that the 5 values need to add up to 55 in order for the mean to be 5, because $55 \div 5 = 11$. Data set B is the only one that adds up to 55: $12 + 7 + 13 + 9 + 14 = 55$.

912 Unit 8 Data Sets and Distributions

3 Connect

Have pairs of students share their responses and strategies for Part 2, focusing on how they can prove that their arrangements make an average of 3 hives per field (Problem 1). Then have them share their reasoning for Problem 2.

Define:

- A ***measure of center*** uses a single number to summarize a data set in terms of its typical values.
- The ***average*** of a set of numbers is equal to the sum of all their values divided by how many numbers are in the set.
- The ***mean*** is a measure of center for a numerical data set that is equal to the average of all its values.

Ask:

- "How does the mean of a data set relate to your previous work with division and fair sharing?"
- "Could the expression $18 \div 3$ help create arrangements for Part 2, Problem 1? Why or why not?" No, because the quotient of 6 represents the 6 fields, and those are not being redistributed, the hives are being redistributed.

Highlight that the mean can be thought of as an equal redistribution or as "leveling out" the distribution, so that each value has the same frequency, which can also be thought of as the "fair share." This would create a uniform data set in which the values of some data values changed, but the sum of all the values in the data set has not changed. Also, note that it is possible for some values in a data set to be equal to the mean, but it is also possible that none are. Sometimes, the mean is a decimal value even when all the values are whole numbers.

Pairs | 15 min

Activity 2 Creating a Consistent Schedule

Students formally connect the mean to the "typical" value and the center of a distribution by determining the mean and identifying missing values from two data sets.

Name: ______ Date: ______ Period: ______

Activity 2 Creating a Consistent Schedule

An apple orchard owner has 50 honey bee hives. He hires a beekeeper (also called an *apiarist*) to tend the bees and collect the honey they produce to sell alongside his apples. This table shows the number of hours that the apiarist worked last week.

Monday	Tuesday	Wednesday	Thursday	Friday
3	6	11	7	4

1. How many hours did the apiarist typically work each day?

   6 hours, around 6 hours, or about 6–7 hours

2. The apiarist knows the amount of work will be the same next week, but wants to evenly distribute his hours throughout the week – Monday and Friday were nice, but he was exhausted after Wednesday! How long would he typically work each day next week if the hours were evenly distributed? Show or explain your thinking.

   $6\frac{1}{5}$ hours, 6.2 hours, or 6 hours and 12 minutes;

   $3 + 6 + 11 + 7 + 4 = 31$

   $\frac{31}{5} = 6\frac{1}{5}$

3. The following list shows the amounts of honey, in pounds, the apiarist collected Monday through Thursday last week.
   amount of honey (in pounds): 15, 9, 15, 12

   After finishing work on Friday, the apiarist claimed that he had collected an average of 15 lb of honey per day that week. If he is correct, then how many pounds of honey did he collect on Friday? Show or explain your thinking.

   24 lb

   Sample responses:

   $15 \cdot 5 = 75$

   $15 + 9 + 15 + 12 + x = 75$

   $75 - 51 = x$

   $x = 24$

   Because 9 is 6 less than 15, and 12 is 3 less than 15, he must have collected $15 + 3 + 6$, or 24 lb, on Friday.

Are you ready for more?

Starting next month, the apiarist is going to need to work 7 more hours each week. If he decides to work the same number of hours each day from Monday through Saturday, should he expect the typical number of hours worked each day to increase or decrease from the typical value you found in Problem 2? Explain your thinking.

The typical number of hours worked will slightly increase because 7 hours is slightly more than the original mean of 6.2 hours.

STOP

Lesson 8 Mean as a Fair Share 913

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them make a dot plot of the data and use it to identify the center of the distribution, or the typical number of hours worked.

Look for points of confusion:

- **Thinking the mean must be a whole number (Problem 2).** Ask, "If the apiarist had worked that number of hours every day, would the total hours worked be the same?"
- **Struggling to use the mean to determine a missing data value (Problem 3).** Ask, "What is the average number of pounds of honey collected Monday to Thursday? What does that tell you about Friday, relative to 15 lb?"

Look for productive strategies:

- Creating a dot plot or listing the values from least to greatest to determine the center (Problem 1).
- Redistributing the hours incrementally from one day to another until the hours level out, or adding the hours and dividing by the days (Problem 2).
- Using the structure of a data set to reason about a missing value (Problem 3).

3 Connect

Have students share their responses and strategies for each problem. For Problem 2, start with students who reallocated the hours incrementally, followed by those who divided the sum by the number of days.

Ask:

- "How are your answers to Problems 1 and 2 related?"
- "Which strategy for Problem 2 was the most efficient?"

Highlight that the pounds of honey have already been measured, so the original data set cannot be changed. Therefore, it does not really make sense to interpret the mean as an equal redistribution, but the *average* pounds of honey collected each day does make sense.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students whether they have seen a beekeeper tend bee hives. If they struggle with the term *apiarist*, have them replace it with the term *beekeeper.*

Accessibility: *Optimize Access to Tools*

Provide students with objects, such as paper clips or counters, that they can use to represent the number of hours the apiarist worked. They can physically redistribute them to help them visualize the process.

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students explain their responses and strategies, press them for detail in their reasoning and strategies used. For example:

If a student says . . .	Ask . . .
"I moved the hours around to make them all the same."	"Which hours did you move around and to where did you move them? Did you need to make any adjustments along the way? How did you know when you were done?"

English Learners

Provide sentence frames to help students organize their thinking.

- "I noticed that ___, so . . ."
- "I agree/disagree with ___ because . . ."

Whole Class | 5 min

Summary

Review and synthesize how to determine and interpret the mean as an equal redistribution, or the average, of the data.

Summary

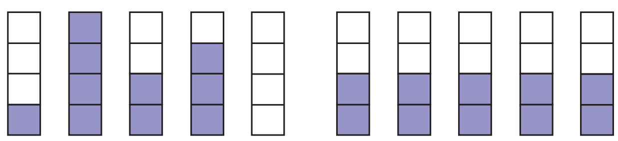
In today's lesson . . .

You saw that you can summarize a data set in terms of typical values using a single number, called a ***measure of center***. One way to *measure the center* of a data set is to think about it as "determining a fair share," or "leveling out the distribution," so that each value would have the same frequency. And one way to calculate that value is to determine the ***average*** value of all the data values.

For example, suppose this data set represents the numbers of liters of water in 5 bottles: $1, 4, 2, 3, 0$.

To calculate an *average*, you first add up all of the values to determine the total (10 liters). Then you divide that sum by the number of values (5 bottles), which in this example can be represented by the expression $(1 + 4 + 2 + 3 + 0) \div 5$, or $10 \div 5$. So, the average liters of water in the 5 bottles is 2 liters (per bottle).

This average value also represents the "fair share," which can be determined by redistributing equal amounts to the original number of "groups" (the 5 bottles), as shown in this diagram.



As a *measure of center* for a data set, the average of all the values in the data set is called the ***mean***.

Reflect:

914 Unit 8 Data Sets and Distributions

Synthesize

Ask:

- "Suppose a data set contains the amounts of money in 5 piggy banks. What does the mean of this data set represent in context?" The amount of money in each piggy bank, if the money was equally redistributed among them.
- "Describe a general process that will always work for calculating the mean of a data set?" Divide the sum of all the values in the data set by the number of values in the set.

Highlight that, in previous lessons, students described the center of a distribution or typical values of a data set by only considering the shape of the distribution — more qualitatively. The *mean* is a way to calculate (quantitatively) a single value that summarizes a distribution by its center, which is why it is called a *measure of center*. However, it is not the only measure of center that can be calculated (students will also calculate the median, beginning in Lesson 10).

Formalize vocabulary:

- *average*
- *mean*
- *measure of center*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does the mean, or the average, of a data set relate to division?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *average*, *mean*, and *measure of center* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the average or a missing value in a data set.

Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 8.08

1. The data set represents the daily low temperatures for Anchorage, Alaska, in degrees Celsius.

   5, 8, 6, 5, 10, 7, 1

   What was the average low temperature? Show your thinking.

   6°C; Sample response: $5 + 8 + 6 + 5 + 10 + 7 + 1 = 42$

   $\frac{42}{7} = 6$

2. The mean of four numbers is 7. Three of the numbers are 5, 7, and 7. What is the fourth number? Show or explain your thinking.

   9; Sample responses:

   Because 5 is 2 less than 7, the fourth number must be 7 + 2 or 9.

   $(5 + 7 + 7 + x) \div 4 = 7$

   $4 \cdot \frac{5 + 7 + 7 + x}{4} = 7 \cdot 4$

   $5 + 7 + 7 + x = 4$

   $19 + x = 28$

   $x = 9$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain how the mean for a data set represents a "fair share." 1 2 3

b. I can determine the mean for a numerical data set. 1 2 3

Success looks like . . .

- **Language Goal:** Comprehend the words *mean* and *average* as a measure of center that summarizes the data using a single number. **(Speaking and Listening)**
- **Language Goal:** Explain how to calculate the mean for a numerical data set. **(Speaking and Listening, Writing)**
  - » Calculating the mean for the daily low temperatures in Problem 1.
- **Language Goal:** Interpret diagrams that represent determining the mean as a process of leveling out the data to determine a "fair share." **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly calculate the average low temperature in Problem 1 (e.g., they do not divide by 7), consider:

- Reviewing Activity 1, Problem 2, and asking, "How is determining the mean or average like equal redistribution?"

If students struggle to determine the missing value in the data set in Problem 2, consider:

- Reviewing Activity 2, Problem 3.
- Asking:
  - » "What does a mean of 7 indicate in this problem?" When redistributed, every value in the data set would be 7.
  - » "Is every value equal to 7? How can that help you think about the missing value?" No, one value is 5, which is 2 less than 7, so the missing value must be 2 more than 7.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students determined and interpreted the mean as equal redistribution. How did that build on the earlier work students did with division in a prior unit?
- How did students interpret the mean in context today? How are you helping students become aware of how they are progressing in this area?

Math Language Development

Language Goal: Interpret diagrams that represent determining the mean as a process of leveling out the data to determine a "fair share."

Reflect on students' language development toward this goal.

- How did using the *Compare and Connect* routine in Activity 1 and the *Discussion Supports — Press for Details* routine in Activity 2 help students interpret the mean as a fair share? Would you change anything the next time you use these routines?
- Are students able to clearly explain how they would rebalance a data set by using equal redistribution? Are they connecting this idea to the concept of the mean as a measure of center?

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. A preschool teacher is rearranging four boxes of playing blocks so that each box contains an equal number of blocks. Currently Box A has 32 blocks, Box B has 18, Box C has 41, and Box D has 9 blocks. Select *all* the ways he could make each box have the same number of blocks.

   A. Remove all the blocks and make four equal piles, which will contain 25 blocks. Then put each pile in one of the boxes.

   B. Remove 7 blocks from Box A and place them in Box B.

   C. Remove 23 blocks from Box A, 9 blocks from Box B, and 32 blocks from Box C. Redistribute the 64 blocks that were removed, putting 16 in each box.

   D. Remove 7 blocks from Box A and place them in Box B. Remove 21 blocks from Box C and place them in Box D.

   E. Remove 7 blocks from Box A and place them in Box B. Remove 16 blocks from Box C and place them in Box D.

2. In a round of mini-golf, Clare records the number of putts she takes on each hole, as shown in the table. She claims that her mean number of putts per hole is 3. Explain why Clare is correct.

2	3	1	4	5	2	3	4	3

She can redistribute the numbers by calculating the sum of her putts and dividing the sum by 9, or $\left(\frac{2+3+1+4+5+2+3+4+3}{9}\right) = \frac{27}{9} = 3$, or an average of 3 putts per hole.

3. In her English class, Mai's teacher gives 4 quizzes, each worth 5 points. After 3 quizzes, Mai has scores of 4, 3, and 4. How many points on the last quiz are needed for Mai to have an average score of 4? Show or explain your thinking.

5 points

Sample responses:

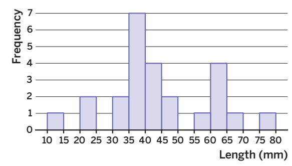
Because 3 is one less than 4, the final quiz has to be $4 + 1$, or 5 points.

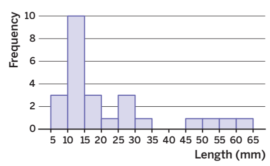
$$\frac{4+3+4+x}{4} = 4$$
$$4 \cdot \left(\frac{4+3+4+x}{4}\right) = 4 \cdot 4$$
$$4+3+4+x = 16$$
$$11 + x = 16$$
$$x = 5$$

Name: ______ Date: ______ Period: ______

Practice

4. An earthworm farmer examined two containers of a certain species of earthworms so that he could learn about their lengths. He measured 25 earthworms in each container and recorded their lengths in millimeters. The histograms of the lengths for each container are shown.





   a. Which container typically has longer worms than the other container?
   Container A

   b. For which container would 15 mm be a reasonable description of a typical length of the worms in the container?
   Container B

   c. If length is related to age, which container had the younger worms?
   Container B

5. Diego thinks that $x = 3$ is a solution to the equation $x^2 = 16$. Do you agree? Show or explain your thinking.

No; Sample response: If $x = 3$, then $3^2 = 3 \cdot 3 = 9$, and 9 is not equal to 16.

6. Identify at least two ways in which the expressions are alike.

$\frac{3+3+3}{3}$ $\qquad$ $\frac{3+3+3+3}{4}$

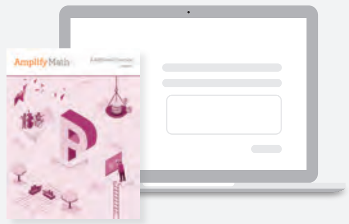
Sample responses: Both expressions have the value 3 as the repeated number in the numerator. They both have a denominator that is equal to the number of threes in the numerator. Both are expressions that can determine a mean.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 8 Lesson 6	2
	5	Unit 6 Lesson 15	2
Formative	6	Unit 8 Lesson 12	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Mean as the Balance Point

Let's look at another way to understand the mean of a data set.

Focus

Goals

1. **Language Goal:** Calculate and interpret distances between data values and the mean of the data set. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret diagrams that represent the mean as a "balance point" for both symmetric and non-symmetric distributions. **(Speaking and Listening, Writing)**
3. **Language Goal:** Represent the mean of a data set on a dot plot and interpret it in the context of the situation. **(Speaking and Listening, Writing)**

Coherence

Today

Students use the structure of the data to interpret the mean as the balance point of a numerical distribution. They begin by physically balancing objects (bees). And they discover that balance is achieved, or maintained, when the sums of the distances from the mean to all of the points on the left and to all of the points to the right are equal. This leads to another interpretation of the mean – as a balancing point for a data set. Students then calculate and compare the means of two data sets in order to make a claim about whether the varroa mites or the Colony Collapse Disorder is the greater threat to honey bees.

Previously

In Lesson 8, students interpreted the mean as equal redistribution or a fair share value — the amount in each group if the values were redistributed such that all groups have the same value.

Coming Soon

In Lesson 10, students will determine and interpret a second measure of center – the median of a data set.

Rigor

- Students build **conceptual understanding** of mean as a balance point.
- Students practice **procedural skills** for calculating the mean of a data set.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Problem 1,* pre-cut cards, two rows per pair
- Activity 1 PDF, *Problem 4,* pre-cut cards, one set per pair
- Activity 2 PDF, one per student

Math Language Development

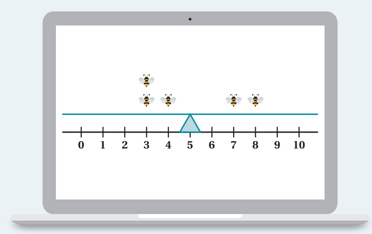
Review words

- *average*
- *mean*
- *measure of center*

Amps | Featured Activity

Activity 1
Interactive Bee Balances

Students experiment with, and discover, the mean as a balance point by balancing bumble bees on a beam. You can overlay student work to provide immediate feedback.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students begin working with the visual representation in Activity 1, they might forget the goal of the task. The distraction of manipulating bees can distract them and draw them farther off task. Remind them that the visual representation is a structure that is to help them better understand the mathematics. Prior to starting the activity, have each student write their goal for the activity in a place that will be visible the whole time. Have them use it as a visual cue to focus on the purpose of the activity.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1,** complete Problems 1–3 as a class. Then have pairs of students complete Problem 5.
- **Activity 2** may be completed as a class. Using the ***Think-Pair-Share*** routine, have students consider how they would answer the question. Then have half of the class calculate the mean for varroa mites and the other half calculate the mean for CCD. Compare the means as a class.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students analyze the structure of four expressions, determining which does not belong and connecting to the process of calculating a mean.

Name: ________ Date: ________ Period: ________

Unit 8 | Lesson 9

Mean as the Balance Point

Let's look at another way to understand the mean of a data set.

Warm-up Which One Doesn't Belong?

Which expression does not belong? Be prepared to explain your thinking.

Expression A	Expression B	Expression C	Expression D
$\frac{8+8+4+4}{4}$	$\frac{10+10+4}{4}$	$\frac{9+9+5+5}{4}$	$\frac{6+6+6+6+6}{5}$

Answers may vary, but students should be able to make a reasonable argument that any one of the four expressions does not belong.

Sample responses:

- Expression A does not belong because it is the only expression in which the denominator divides evenly into each addend in the numerator.
- Expression B does not belong because it is the only expression in which the denominator is not the same as the number of addends in the numerator. It is the only expression that would not calculate the mean of the data set.
- Expression C does not belong because it is the only one with odd numbers in the numerator; or because when evaluated, it is the only expression that does not equal 6.
- Expression D does not belong because it is the only one where all of the addends in the numerator are the same; or because it is the only expression with a denominator of 5.

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use the ***Which One Doesn't Belong?*** routine.

2 Monitor

Help students get started by asking, "Which expression looks different from the others? Why?"

Look for productive strategies:

- Using the structure of the expressions, as well as considering the specific values, to justify a claim about which expression does not belong.
- Connecting the structure of Expressions A, C, and D to the process of calculating a mean as an average.

3 Connect

Have students share at least one reason why each expression could be the one that does not belong. After each response, ask the class whether they agree or disagree. Because there is no single correct answer for why a question does not belong, attend to students' explanations and ensure the reasons given are correct.

Ask, if not already suggested, "How do any of these expressions relate to the process of calculating an average or a mean of a data set?" All expressions, except for Expression B, could represent how to determine the mean of a data set because the number of terms in the numerator is the same as the value of the denominator.

Highlight how the process of calculating the mean relative to Expressions A, C, and D is also related to the concept of equal redistribution, or "fair share," from Lesson 8.

Power-up

To power up students' ability to describe similarities between two expressions, ask:

Which of the following statements are true about the expressions shown? Select *all* that apply.

$\frac{5+5+5}{3}$ $\qquad$ $\frac{5+5+6+6}{4}$

A. Both have the same value when evaluated.

B. Both have the same number of addends in the numerator.

C. Both can be used to determine a mean.

D. Both have the same denominator.

E. Both have a denominator that is the same as the number of addends in the numerator.

Use: Before the Warm-up.

Informed by: Performance on Lesson 8, Practice Problem 6.

Pairs | 20 min

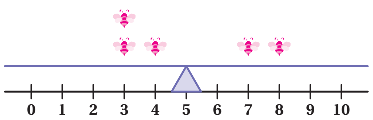
Activity 1 Balancing Bumble Bees

Students explore and manipulate a visual representation (balancing bees on a beam) that mimics a dot plot to develop an understanding of mean as a balance point.

Amps Featured Activity Interactive Bee Balances

Activity 1 Balancing Bumble Bees

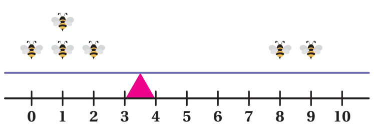
1. Here is a beam and a balance point. You will be given 5 identical bees — they all weigh the same. Determine at least two ways you could place all of the bees on the beam to make them balance. Record your favorite placement on the beam below. Be prepared to explain your thinking.



Answers may vary, but should show the five bees placed so that the sum of the distances of the bees on the left and right of the balance is the same.

Sample response shown on beam.

2. Place the triangle under the beam to show you you could balance the bees. Explain your thinking.



Sample response: When the triangle is at 3.5, the bees on the left will be the same distance from the triangle as the bees on the right, and the bees will be balanced.

1 Launch

Distribute one row from the Activity 1 PDF, *Problem 1* to each student. Have them complete Problems 1–3 before then giving each pair one set of pre-cut cards from the Activity 1 PDF, *Problem 4* to use in completing Problems 4 and 5 together.

2 Monitor

Help students get started by activating background knowledge. Ask, "How is the beam like a see-saw? How do you balance a see-saw?"

Look for points of confusion:

- **Thinking balance requires the same number of bees on each side of the balance (Problem 1).** Ask, "Can a see-saw balance even with different numbers of kids on each side? How?" Reiterate that because all bees weigh the same, distance is the only thing that can be adjusted here.
- **Struggling to balance the bees in Problem 2.** Ask, "How is the beam like a number line? How are the locations of the bees related to your work with hanger diagrams? How can that help you to balance the bees?"
- **Confusing or mixing distances from the balance point and actual values on the number line.** Consider sketching an example of a balance point with one bee on each that are both 1 unit away. Ask, "Does it matter what value each of these bees represents?" Show sample numbers as needed.
- **Not knowing what to do with the bees located above the triangle balance point (Problem 4).** Ask, "If that bee was the only one on the beam, would it be balanced? Why?"

Look for productive strategies:

- Using prior knowledge of hanger diagrams and absolute value to recognize that when the sums on each side are equal, the model is balanced.
- Recognizing when the sum on one side is less, the triangle moves to the right (e.g., the balance point must be greater than 3 in Problem 2).
- Recognizing the balance point represents the mean of the data sets.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they experiment with, and discover, the mean as a balance point by balancing bumble bees on a beam. You can overlay student work to provide immediate feedback.

Extension: *Math Enrichment*

Have students place a "tower" of all five cut-out bees placed above the value 5 and ask them to determine the balance point. Then have them move the "top" bee to the value 6 and determine whether and how the balance point changes. Repeat this process until all of the bees are in a "tower" above the value 6.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "How does the distances of the bees from the triangle relate to the bees being balanced?"
- "Can you describe Mai's strategy in your own words?"

Have students revise their responses, as needed.

English Learners

Group students together with different levels of language proficiencies. This will allow them to interact with a diverse group of peers as they speak about, write, and refine their responses.

Pairs | 20 min

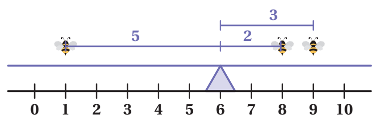
Activity 1 Balancing Bumble Bees (continued)

Students explore and manipulate a visual representation (balancing bees on a beam) that mimics a dot plot to develop an understanding of mean as a balance point.

Name: ______ Date: ______ Period: ______

Activity 1 Balancing Bumble Bees (continued)

3. Here is Mai's strategy to balance three bees.



   a. Are the bees balanced? Explain your thinking.
   Yes; Sample response: There is one bee that is 5 units away, on the left; then there are two bees that are 2 units away and 3 units away, on the right, which makes a total of 5. So the total distances away for all the bees on the left and right are equal.

   b. If the bees are balanced, do you think Mai's strategy will always work? If not, explain how Mai might not have been thinking about it correctly.
   Mai's strategy will always work because the total distance from the balance is the same on the left and right. The distances balance each other.

4. You will be given four cards that show bees on beams. Determine which sets of bees, if any, are balanced. If they are balanced, explain why. If they are not balanced, explain why and which direction — left or right — the balance (triangle) would need to move to balance the bees.
   Cards A and D are balanced, but Cards B and C are not balanced.
   Sample responses:
   - **Cards A and D are balanced because the total distance of the bees to the left of the balance is equal to the distance of the bee(s) to the right of the balance.**
   - **Card B is not balanced because the distance to the left of the balance is 6, but the distance to the right is 5, so the triangle would need to move to the left.**
   - **Card C is not balanced because the distance to the left of the balance is 2, and the total distance to the right is 6. The triangle will need to move to the right.**

5. Determine the means for each set of bees in Problem 4. How do the means relate to your responses from Problem 4?
   - **Card A: 3**
   - **Card B: 5.8**
   - **Card C: 8.8**
   - **Card D: 4**

   Sample response: The means for Cards A and D are where the balance (triangle) is placed in Problem 4. The means for Cards B and C are different than where the balance is placed in Problem 4. So, the mean represents the place where the triangle should be placed to balance the beam.

Lesson 9 Mean as the Balance Point 919

3 Connect

Have pairs of students share their answers to each problem, focusing most of the discussion on the connection between the mean and the balancing point. Consider showing a table for Mai's strategy in Problem 3 or any of the others where each row captures: the value of a data value, its distance from the mean (or proposed mean), and whether the data value is to the "left" or "right" of the mean — this strategy and organizational tool will be revisited in later lessons on variability.

Display Cards A and D from Problem 4.

Highlight how Card D represents a perfectly symmetric distribution and each point to the left balances an equidistant point to the right; but Card A represents a non-symmetric distribution and it is more important to focus on the *sums* of the distances of all of the points from the mean on each side.

Ask:

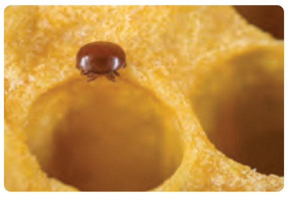
- "How does the interpretation of mean as a balance point support the idea that mean is a measure of center?" The mean is the center value of each value in the data set. Any values less than the mean are balanced by one or more values greater than the mean.
- "How does a dot plot help you make sense of the interpretation that the mean is a balance point of a data set?" You can see and, therefore, count the distance each point is from the mean (the bees were essentially the dots of a dot plot in this activity).

Activity 2 Threats to the Honey Bees

Students determine which threat — varroa mites or Colony Collapse Disorder — was more dangerous to bees in one apiary by calculating and comparing means.

Activity 2 Threats to the Honey Bees

The IUCN has not yet classified honey bees on their Red List. However, many are worried about decades of steep declines in the insect's population. The National Agricultural Statistics Service (NASS) is one of the groups within the U.S. Department of Agriculture that monitors honey bee populations. Cynthia Clark, an American statistician, has been recognized for her work improving the quality of data that the NASS and other organizations, such as the United States Census Bureau, collect and use.



Mirko Graul/Shutterstock.com

Two of the biggest threats to honey bees are varroa mite infestations and Colony Collapse Disorder (CCD). Varroa mites transmit varroosis and other deadly diseases that spread from hive to hive. CCD occurs when the majority of adult worker bees disappear from the hives at the same time, leaving too few adult bees to care for all of the young bees. This causes entire colonies to die.

You will be given two dot plots that show the number of colonies lost to varroa mites and CCD in ten months at a Mississippi apiary. On average, which threat — varroa mites or CCD — killed more honey bee hives? Show or explain your thinking.

Sample responses:

- **Varroa mites killed more honey bee hives. On average, varroa mites destroyed 6.9 hives, and CCD destroyed 6.4 hives.**
- **On average, they killed *about* the same number of hives because the mean was within half (0.5) of a hive.**

Varroa mites: $\frac{(0+2+4+7+7+8+8+9+9+15)}{10} = 6.9$

CCD: $\frac{(4+4+5+5+6+7+7+8+9+9)}{10} = 6.4$

Featured Mathematician

Cynthia Clark

Cynthia Clark is a statistician who was born in Colorado. She holds both a BS and MS in Mathematics, and went on to pursue a PhD in Mathematics at Iowa State University. However, she graduated with a PhD in Statistics instead. Throughout her career, Clark was known for her work improving the quality of data, and she served in positions for the Office of Federal Statistical Policy, the Office of Management and Budget, the National Agricultural Statistics Service, and the United States Census Bureau. In 2011, she was given the Presidential Meritorious Rank Award.

STOP

920 Unit 8 Data Sets and Distributions

1 Launch

Give each pair of students one copy of the Activity 2 PDF. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What data does each dot represent? What does *on average* mean here?"

Look for points of confusion:

- **Misapplying previously seen strategies before determining the mean (measuring distances between points, or moving points around an arbitrary value).** Remind students that the mean is not known, and ask, "How can you calculate it?"

Look for productive strategies:

- Determining means by choosing starting values and calculating sums of distances, then adjusting for balance.
- Calculating means by dividing the sums by 10.
- Recognizing that because both data sets have 10 values, comparing sums is equivalent to comparing means.
- Claiming varroa mites killed more hives, on average, because 6.9 is greater than 6.4.

3 Connect

Have students share their responses.

Ask:

- "Do you agree that overall varroa mites and CCD pose roughly equal threats to honey bees? Why or why not?"
- "How could the means be so close with such different distributions?"

Highlight that thinking of the mean as a balance point preserves the original values and is generally more applicable because there is not always a meaningful uniform distribution, such as relocating hives in Lesson 8.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Read aloud the introduction together with the class, or provide a summary. Display the two dot plots from the Activity 2 PDF and clarify what each dot plot displays. Ask these questions to help students make sense of the information:

- "Which threat — varroa mites or CCD — resulted in the greatest number of colonies lost? The least?" Varroa mites resulted in both.
- "Which threat — varroa mites or CCD — resulted in more variability among the number of colonies lost" Varroa mites.
- "Which dot plot — varroa mites or CCD — do you think displays a more consistent typical value? Why do you think so?" Sample response: I think the CCD dot plot displays a more consistent typical value because there is less variability. The data values are closer together.

Featured Mathematician

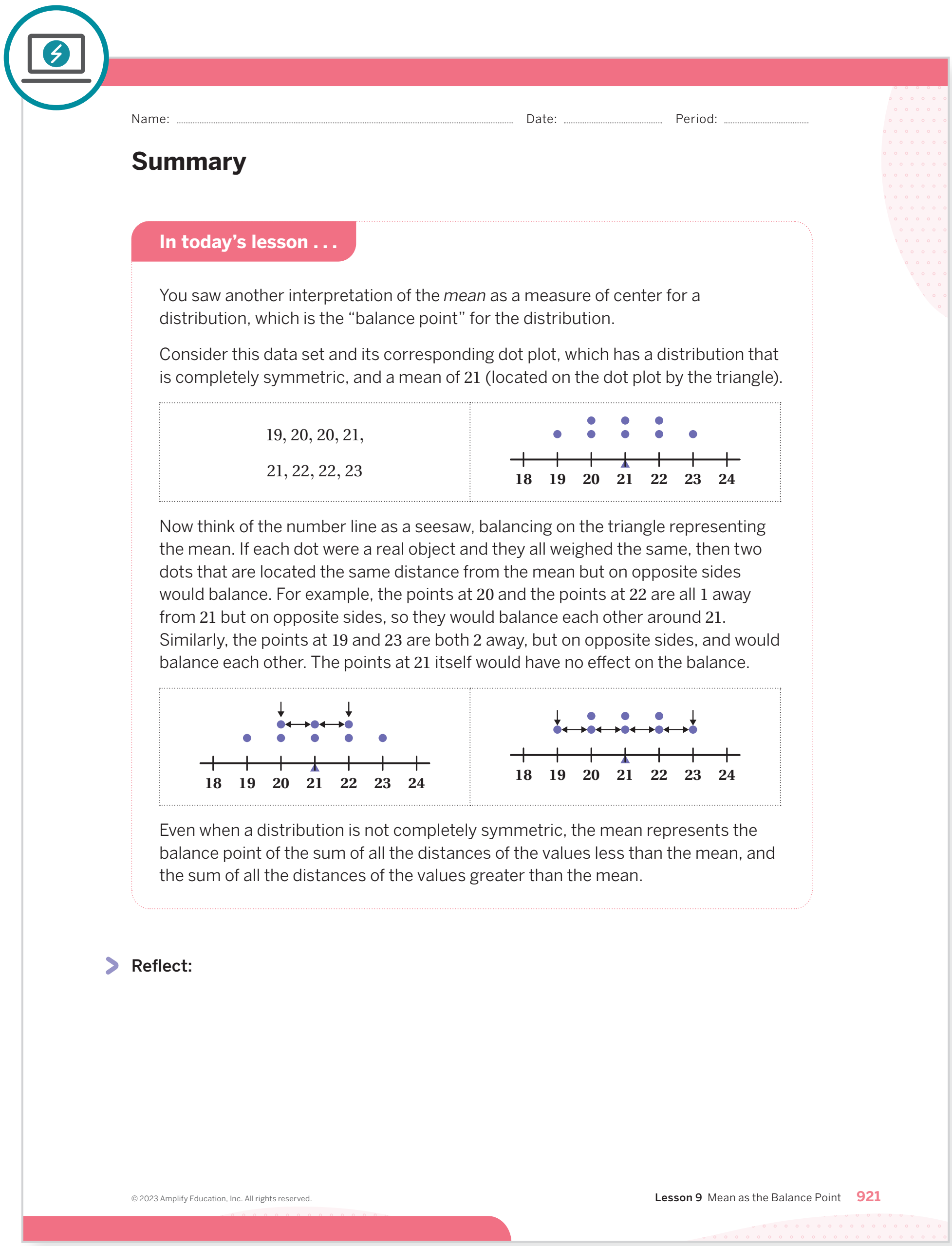
Cynthia Clark

Have students read about statistician Cynthia Clark, who spent much of her career improving the quality of data, at the Office of Federal Statistical Policy, the Office of Management and Budget, the National Agricultural Statistics Service, and the United States Census Bureau.

Whole Class | 5 min

Summary

Review and synthesize how the mean can be interpreted as a balance point for a data set.


Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw another interpretation of the *mean* as a measure of center for a distribution, which is the "balance point" for the distribution.

Consider this data set and its corresponding dot plot, which has a distribution that is completely symmetric, and a mean of 21 (located on the dot plot by the triangle).

Now think of the number line as a seesaw, balancing on the triangle representing the mean. If each dot were a real object and they all weighed the same, then two dots that are located the same distance from the mean but on opposite sides would balance. For example, the points at 20 and the points at 22 are all 1 away from 21 but on opposite sides, so they would balance each other around 21. Similarly, the points at 19 and 23 are both 2 away, but on opposite sides, and would balance each other. The points at 21 itself would have no effect on the balance.

Even when a distribution is not completely symmetric, the mean represents the balance point of the sum of all the distances of the values less than the mean, and the sum of all the distances of the values greater than the mean.

Reflect:

Synthesize

Ask:

- "Could another value, beside the mean, balance a data distribution? Why or why not?" No; If you are considering the relative values of each data point (the distances) — there is only one value that balances the data that way, which is the mean. However, if you only consider the number of data points and not their values, then, if there is an odd number of data values, you could find the one in the middle of the data set where the counts of data points on each side are equal.
- "In earlier lessons, you estimated the center of a distribution to describe a typical value. Why does it make sense for the mean to describe a typical feature or characteristic of a group?" The mean considers the value of each point and it is one value that is directly in the center of those values, although it may not be in the center of the dot plot. It can be considered typical of that specific data set.

Highlight that students have worked with two interpretations of the mean of a data set — as equal redistribution and as a balancing point. While the same process to calculate the mean can be used, it may not always make sense to explain the mean in context using either interpretation. That would not make the interpretations inaccurate — just less meaningful.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are the two interpretations of the mean of a data set — fair share and balance point — similar? How are they different?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the mean of three data sets and interpreting a comparison of the means within context.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.09

The table shows the number of text messages sent by Jada, Diego, and Lin over 6 days. One of the data sets has a mean of 4, one has a mean of 5, and one has a mean of 6.

Jada	4	4	4	6	6	6
Diego	4	5	5	6	8	8
Lin	1	1	2	2	9	9

1. Which mean corresponds to each person's data set?
   Lin has a mean of 4 text messages, Jada has a mean of 5 text messages, and Diego has a mean of 6 text messages.

2. What do these values tell you about the text message habits of these three students?
   Sample responses:
   - On average, Lin sends the least text messages per day and Diego sends the most text messages per day.
   - They all typically send about the same number of text messages per day.

Self-Assess

1 I don't really get it — 2 I'm starting to get it — 3 I got it

a I can describe what the mean explains in the context of the data. 1 2 3

b I can explain how the mean represents a balance point for the data on a dot plot. 1 2 3

Success looks like . . .

- **Language Goal:** Calculating and interpreting distances between data values and the mean of the data set. **(Speaking and Listening, Writing)**
- **Language Goal:** Interpreting diagrams that represent the mean as a "balance point" for both symmetric and non-symmetric distributions. **(Speaking and Listening, Writing)**
- **Language Goal:** Representing the mean of a data set on a dot plot and interpreting it in the context of the situation. **(Speaking and Listening, Writing)**
  » Interpreting the means in the context of text message habits of the three students.

Suggested next steps

If students incorrectly match the means to the data sets in Problem 1, consider:

- Reviewing Activity 1, Problems 2–4. Then have students make a dot plot for each data set and test each mean as a balance point, one at a time.

If students struggle to meaningfully compare the students' texting habits in Problem 2, consider:

- Reviewing Activity 2, specifically focusing on how they used the mean of each dot plot to draw conclusions about the data.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- When you compare and contrast today's work with prior students' work with hanger diagrams and balancing equations, what similarities and differences do you see?
- What did you see in the way some students approached Activity 1 that you would like other students to try? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Interpreting diagrams that represent the mean as a "balance point" for both symmetric and non-symmetric distributions.

Reflect on students' language development toward this goal.

- How did using the *Stronger and Clearer Each Time* routine in Activity 1 help students interpret the mean as a balance point? Would you change anything the next time you use this routine?
- Are students able to clearly explain the similarities and differences in the two interpretations of the mean — as a fair share and as a balance point? What strategies can you use to help them clarify their thinking?

Practice

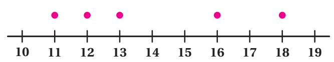
Independent

Name: Date: Period:

1. The data set shows 5 times, in minutes, for Kiran's walks on 5 school days.

16	11	18	12	13

a. Create a dot plot for Kiran's data.



Time (in minutes)

b. Without calculating, decide whether 15 minutes would be a good estimate of the mean. If you think it is a good estimate, explain your thinking. If not, give a better estimate and explain your thinking.

No; Sample response: 12 and 18 balance, but then even 13 alone would not balance 16, and there is still 11 on the left as well. So, the mean would need to be less than 15, and a better estimate would be 14.

c. Calculate the mean for Kiran's data. Show your thinking.

14 minutes

Sample response: $\frac{(11+12+13+16+18)}{5} = \frac{70}{5} = 14$

d. In the table, record the distance of each data value from the mean and its location relative to the mean.

Time (minutes)	Distance from the mean	Greater or less than the mean?
16	2	Greater than
11	3	Less than
18	4	Greater than
12	2	Less than
13	1	Less than

e. Calculate the sum of all distances to the left of the mean, and then calculate the sum of distances to the right of the mean. Explain how these sums show that the mean is a balance point for the values in the data set.

Distance to the left: 6;

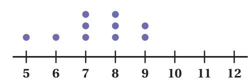
Distance to the right: 6;

The mean is a balance point because the sum of the distances to the left and right of the mean are the same.

Practice

Name: Date: Period:

2. This dot plot shows the hours of sleep received by ten sixth grade students last night.



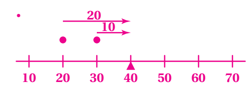
Sleep (hours)

Elena estimated the mean number of hours of sleep to be 8.5 hours, Andre estimated it to be 7.5 hours, and Noah estimated it to be 6.5 hours. Which estimate do you think is best? Explain your thinking.

Andre's estimate is the best ; Sample response: It is in the center of the data with 5 points on either side, and the total distance of the points to the left and right of 7.5 is about equal. Neither 6.5 hours nor 8.5 hours are good estimates because the distances of the points to the left and right of 6.5 and 8.5 are not equal or close to being equal.

3. In a basketball game, Shawn scored 20 points and Bard scored 30 points. The mean number of points scored by Shawn, Bard, and Tyler for that game was 40 points. How many points did Tyler score in that game? Show or explain your thinking.

70 points; Sample responses:

- 

  Height (in.)

  The total distance of the points to the left of 40 is 30, so Tyler's score must be 30 points greater than 40, or 70.

- **Because 3 players averaged 40 points, they scored a total of 120 points.**

  $3 \cdot 40 = 120$

  $20 + 30 + x = 120$

  $x = 70$

4. Evaluate each quotient.

a. $\frac{2}{5} \div 2$

$\frac{2}{10}$ (or equivalent)

b. $\frac{2}{5} \div 5$

$\frac{2}{25}$ (or equivalent)

c. $2 \div \frac{2}{5}$

5 (or equivalent)

d. $5 \div \frac{2}{5}$

$12\frac{1}{2}$ (or equivalent)

5. The data set shows how many hours 10 students spent playing outside last week.

5	3	4	2	2	4	5	2	1	3

What is the middle of the data?

Answers may vary but should be close to 3 hours.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 11	2
Formative	5	Unit 8 Lesson 10	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Median

Let's explore the median of a data set and what it tells us.

Focus

Goals

1. **Language Goal:** Comprehend that the median is another measure of center, which uses the middle of all the values in an ordered list to summarize the data. **(Speaking and Listening)**
2. **Language Goal:** Identify and interpret the median, given a data set or a dot plot. **(Speaking and Listening, Writing)**
3. **Language Goal:** Informally estimate the center of a data set and then compare both the mean and the median with this estimate. **(Speaking and Listening, Writing)**

Coherence

Today

Students consider another measure of center for a numerical data set — the median. They begin by analyzing a non-symmetric distribution in which the mean is different from expected typical values. Students are then introduced to the median as a measure of center that represents the *middle* value in a data set when the values are listed in order from least to greatest (or greatest to least). They recognize that the median partitions the data into two equal groups, with half of the data greater than and half of the data less than the median. They determine the median when there are even or odd numbers of data values. Throughout the lesson, students interpret the median in context, as a way to describe what is typical in a distribution, but is interpreted differently than the mean.

Previously

In Lessons 8 and 9, students determined and interpreted the mean for a data set.

Coming Soon

In Lesson 11, students will make arguments for which measure of center is the "best fit" based on attributes of the data sets and contexts.

Rigor

- Students build **conceptual understanding** of the median as a measure of center that represents the *middle* value in a data set, and is, therefore, interpreted differently than the mean.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Independent	Whole Class	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Instructions*
- index cards, one per student

Math Language Development

New word

- *median*

Review words

- *mean*
- *measure of center*

Amps | Featured Activity

Activity 2
Interactive Dot Plots

When it is time to determine whether varroa mite interventions worked, students can see their thinking visually represented in interactive dot plots.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not feel motivated to learn another measure of center. Ask them to speculate why they might need another measure of center. While they might rely on quantitative reasoning to find the median, the abstract reasoning about the median will lead students to understanding its different interpretations. At the end of the activity, have students compare and contrast the mean and the median, explaining why it was worthwhile to learn about the median.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students complete Problem 1 mentally and share verbally. Then display the mean on the dot plot and discuss Problem 3 as a class.
- **Activity 1** may be completed without the kinesthetic component. List an odd number of values on the board and have students order them from least to greatest. Ask them how they might identify the median, or middle number. Modify the data and repeat the process two more times so students see all three possible cases noted in the Activity 1 PDF, *Instructions*.

Independent | 10 min

Warm-up Monitoring Infestation Rates

Students analyze the distribution of varroa mite infestation rates, recognizing that the mean may not always be the best representation of a typical value for a data set.

Unit 8 | Lesson 10

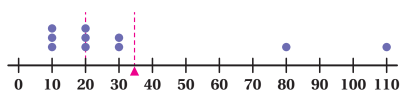
Median

Let's explore the median of a data set and what it tells us.

Warm-up Monitoring Infestation Rates

You have seen that varroa mites pose a serious threat to honey bees' survival. However, the mere presence of these parasites does not guarantee a death sentence for a bee or its colony. If the parasite population is kept in check, entire colonies can remain healthy. The general consensus is that an infestation rate of less than 3% is safe. This is calculated using a sugar roll test.

This dot plot shows the number of colonies with infestation rates greater than 3% at ten North Dakota apiaries.



Number of colonies

1. Use the dot plot to think about what a typical number of colonies with infestation rates greater than 3% is for these ten apiaries. Then without making any calculations, estimate the *center* of the data, and mark its location on the dot plot.
   20 colonies
2. Determine the *mean* for the data, and mark its location on the dot plot with a triangle.
   34 colonies
3. How does the mean compare to the value that you identified as the center of the data? Why might that be?
   Sample response: A typical number of infested colonies would be between 10 and 30, so for the center I was not really considering the ones with 80 and 110 colonies because they were not typical. But the mean is greater than what I said would be typical. To calculate the mean, I have to consider the ones with 80 and 110 colonies, which is what "pulls" the mean up from, or to the right of, the typical values.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work independently on the Warm-up.

2 Monitor

Help students get started by asking, "How would you describe the distribution of this data?"

Look for points of confusion:

- **Estimating the mean for Problem 1.** Acknowledge that the mean does provide a measure of center but may be difficult to estimate, and then ask, "Is there another way you could interpret the word center here, for just looking at the dot plot?"

Look for productive strategies:

- Identifying typical values by considering where the data are clustered (Problem 1).
- Recognizing that the mean is greater than expected typical values because the 80 and 110 colonies values "pull" the mean up or to the right (Problem 3).

3 Connect

Display the dot plot for reference and marking as students share their responses.

Have students share their responses, gathering class consensus, or the majority, for marking a center and mean. Then focus most of the discussion on Problem 3. Prompt students to respond in context.

Ask, "Do you think the mean summarizes the distribution well? Why or why not?"

Highlight that the mean may not always represent a typical value for a data set because it is sensitive to extreme values. Therefore, another measure of center might better describe the typical value.

MLR Math Language Development

MLR7: Compare and Connect

As students share their responses to Problem 3, highlight how the mean, is greater than what students may describe as the typical value. Ask:

- "If the data values 80 and 110 were removed from this data set, what would the new mean be?"
- "Is the new mean closer to the center you estimated in Problem 1?"
- "How can you describe, in your own words, how the mean compares to typical values when extreme values are or are not included in the data set?"

English Learners

Annotate the dot plot with where the mean is located and where students estimated the *center*, or a *typical value*.

Power-up

To power up students' ability to determine the middle, or typical, value from a set of data, ask:

The data set shows how many hours 6 students spent on social media last week: 18, 9, 21, 5, 12, 7

a. Order the data values from least to greatest.
5, 7, 9, 12, 18, 21

b. What is the typical value? Explain your thinking.
12 because that is the average number of hours that the students spent on social media.

Use: Before the Warm-up.
Informed by: Performance on Lesson 9, Practice Problem 5.

Whole Class | 15 min

Activity 1 Determining the Middle

Students are introduced to the median as another measure of center by physically representing a data set and finding the "middle."

Name: ______________ Date: ________ Period: ________

Activity 1 Determining the Middle

Part 1

Your teacher will give you an index card. Write your first and last name on the card and record the total number of letters in your name. As a class, you will determine the middle of the data set, which is another measure of center called the ***median***.

Part 2

1. How did you determine the median?
   Sample response: I ordered the data from least to greatest, and eliminated the farthest data point on either end (left and right) until the middle value remained.

2. How does the process for determining the median change depending on the number of data values?
   Sample response: If there was an odd number of data values, the remaining value was the median. If there was an even number of data values, I found the average of the two remaining values.

3. In the context of what the values in the data set represent, what does the median tell you?
   Sample responses:
   - **It tells me a typical number of letters that all of the students in my class have in their names.**
   - **It is the middle, or center, of the number of letters in our names.**

Stronger and Clearer: Share your responses to Problems 1–3 with another pair. Ask clarifying questions, such as "If you have a data set with 10 values, is the median the 5th value? Why or why not?" Revise your responses, as needed.

1 Launch

Explain that another measure of center is the ***median***, or the "middle" value in an ordered list of data. Give each student an index card. Use the Activity 1 PDF, *Instructions* to guide students through Part 1 of the activity. Then have pairs of students complete Part 2.

2 Monitor

Help students get started by providing an example of how to complete the index card.

Look for points of confusion:

- **Thinking the median could be two values (Part 1).** Explain that the median summarizes the data set using one value. Ask, "If the median is the middle, how could you determine the middle of two values?"
- **Not distinguishing how the process to determine the median differs for odd and even numbers of values (Part 2).** Ask, "Was the median determined the exact same way in each round? Why not?"

Look for productive strategies:

- Reasoning that because the median is the middle value, there should be the same number of students sitting to the left and right of the median.
- Determining the median whether there is an odd or even number of values.
- Interpreting the median as a typical number of letters in the names of students in the class.

3 Connect

Have students share their responses.

Define *median* as a measure of center for a distribution that represents the middle value in a data set when the values are listed in order. Half of the values are less than or equal to the median, and half of the values are greater than or equal to the median.

Highlight that the median may or may not be a value in the data set.

Ask, "Can you predict when the median will *not* be a value in the data set?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

An alternate approach that does not require students to stand: Display the class data in numerical order. Have student volunteers tell you to cross off values from the beginning and end of the data set until you reach the middle. Have pairs brainstorm how to determine the middle number, as a single value, if there are two middle numbers.

Extension: *Math Enrichment*

Ask students how might the typical number of letters in the names of our class change if two new students, one with 8 letters and the other with 16 letters in their first and last names, join the class. Responses will vary, based on the original data set.

Math Language Development

MLR7: Compare and Connect

After pairs complete Problems 1–3, have them share their responses with another pair of students. Encourage reviewers to ask clarifying questions, such as "If you have a data set with 10 values, is the median the 5th value? Why or why not?" Have pairs of students revise their responses, as needed.

English Learners

For Problem 2, provide sentence frames for students to complete as they write their original drafts or as they revise their responses.

- When there is an odd number of data values, . . .
- When there is an even number of data values, . . .

Pairs | 10 min

Activity 2 Varroa Mites, Bee Gone!

Students determine whether a varroa mite intervention was successful by identifying and comparing the medians of both the pre- and post-intervention data sets.

Amps Featured Activity Interactive Dot Plots

Activity 2 Varroa Mites, Bee Gone!

1. Here is the data set from the Warm-up representing the number of colonies with varroa mite infestation rates greater than 3% in 10 North Dakota apiaries.

20	10	30	20	80	10	30	10	20	110

a. Determine the median of the data. Show your thinking.

20 colonies; Sample response:
10, 10, 10, 20, 20, 20, 30, 30, 80, 110

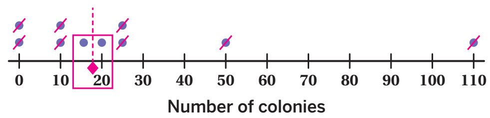
b. In the context of what the data set represents, what does the median tell you?

In general, there are typically 20 colonies with varroa mite infestation rates greater than 3%.

c. How does the median represent the center of the data differently than the mean?

Sample response: The median treats all data values as the same, and does not really use their magnitudes, to just locate the actual point in the middle. So, unlike the mean, which does consider magnitudes, the median shows that the number of colonies with infestation rates greater than 3% is within the range of typical values that I had estimated by looking at the distribution.

2. All 10 apiaries tried to control the varroa mite populations by using standard intervention techniques. This dot plot shows the number of colonies with infestation rates still greater than 3% *after* the interventions.



a. Determine the median number of colonies with infestation rates still greater than 3%. Mark the location of the median on your dot plot with a rhombus. Show your thinking.

17.5 colonies

$(15 + 20) \div 2 = 35 \div 2 = 17.5$

b. In general, were the interventions successful? Use your work to explain your thinking.

Yes, because the median number of colonies with infestation rates greater than 3% decreased from 20 colonies to 17.5 colonies.

Are you ready for more?

Create a data set with a mean that is much less than what you would consider to be a typical value for the data set you invented.

Answers may vary. Sample response: 0, 0, 0, 100, 100, 100, 100, 100, 100, 100. A typical value is 100, but the mean is 70.

STOP

926 Unit 8 Data Sets and Distributions

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "How might organizing the data help?"

Look for points of confusion:

- **Treating data values that have the same value as a single point.** Ask, "In Activity 1, how did we count off when students shared a value?"
- **Not knowing how to determine the median when data are on a dot plot.** Explain that data in a dot plot is also ordered least to greatest and points sharing the same value can be counted in any order.

Look for productive strategies:

- Ordering the data from least to greatest and crossing out one on each end until one or two values remain — averaging the values if two remain.
- Recognizing that the median represents both a measure of center *and* a typical number of hives with a dangerous infestation rate.
- Claiming that the interventions worked because the median number of hives with dangerous infestation rates was less post-intervention.

3 Connect

Have students share their responses.

Ask, "Why are you able to say the interventions generally worked, when the apiary with 110 infested hives did not improve at all?"

Highlight that while the mean and median are both measures of center, they may not identify the same typical value. The mean is more affected by outliers.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can use interactive dot plots.

Extension: *Math Enrichment*

Have students determine the fewest number of data values and their locations on the dot plot that if changed, would show that the interventions *did not* work. Have them show or explain their thinking. Sample response: If the data value 0 was moved to 30, the median would be 22.5, which is greater than the original 20.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their responses to Problems 1b and 1c, add the language they use that compares the mean and median to the class display. For example:

Measure of Center	Description	2 3 3 4 15	2 3 3 4 4
Mean	• Can describe a typical value • The average of the values • Affected by extreme values	**Mean:** 5.4 (not typical)	**Mean:** 3.2 (typical)
Median	• Can describe a typical value • The middle of the values • Not affected by extreme values	**Median:** 3 (typical)	**Median:** 3 (typical)

Summary

Review and synthesize what the median of a data set is, how it is useful, and how it is similar to and different from the mean.

Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You determined another measure of center for a distribution called the ***median***. The *median* is literally the "middle" value in a data set when the values are listed in order from least to greatest (or greatest to least). Half of the data values have values less than or equal to the median, and half of the data values have values greater than or equal to the median.

To determine the median from an ordered representation of the data, such as a list or a dot plot, you repeat a process of eliminating the pairs of least and greatest values.

Odd number of values	$\cancel{0}$ $\cancel{1}$ $\cancel{1}$ 2 $\cancel{2}$ $\cancel{4}$ $\cancel{5}$ Once all pairs have been eliminated, only one value remains in the middle, making it the *mean*. Mean: 2
Even number of values	$\cancel{0}$ $\cancel{1}$ $\cancel{1}$ 1 2 $\cancel{2}$ $\cancel{4}$ $\cancel{5}$ Once all pairs have been eliminated, two values remain. Their average is the mean. $(1 + 2) \div 2 = 1.5$ Mean: 1.5

Reflect:

Synthesize

Ask:

- "What does the median tell you about a data set? Why is it used as a measure of the center of a distribution?" It tells you where to divide a data set in the middle, so that half of the data are less than or equal to the median and the other half are greater than or equal to the median.
- "Why do you need another measure of center other than the mean?" Sometimes the mean is not a good indication of what is typical for the data set, such as when there are extreme values that "pull" the mean away (up or down) from other typical values.

Highlight that both mean and median are measures of center, but they could report the center differently. When choosing which to use as the one value to summarize the center of a data set, the overall shape of the distribution as well as the context and the question being asked are all factors to be considered. In particular, extreme values — those that are relatively different from typical values by a lot (much greater or much lesser) — affect the mean more so than the median. These types of data values are often called *outliers*.

Note: The term *outliers* will be used throughout the remainder of this unit in these materials, but it will not be further defined. It is not an expectation of students in grades 6–8 to understand or use a quantitative definition of *outliers*.

Formalize vocabulary: ***median***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are the mean and median of a data set similar? How are they different?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *median* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the median of two data sets and interpreting them in context.

Printable

Name: ______________ Date: __________ Period: __________

Exit Ticket 8.10

Jada and Andre are practicing the piano for an upcoming recital. The number of minutes each of them practiced on several days over the past few weeks are shown.

Jada:	10	10	20	15	25	25	8	15	20	20	35	25	40
Andre:	25	10	15	30	15	20	20	25	30	45			

1. Determine the median of each data set.
   Jada: 20 minutes Andre: 22.5 minutes

2. Explain what the medians tell you about Jada's and Diego's practice times on the piano.
   Sample responses: Jada typically practiced 20 minutes, and Andre typically practiced 22.5 minutes. Andre typically practiced 2.5 minutes longer than Jada.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can determine the median for a data set. 1 2 3

b I can say what the median represents in a given context. 1 2 3

Lesson 10 Median

Success looks like . . .

- **Language Goal:** Comprehending that the median is another measure of center, which uses the middle of all the values in an ordered list to summarize the data. **(Speaking and Listening)**
- **Language Goal:** Identifying and interpreting the median given a data set or a dot plot. **(Speaking and Listening, Writing)**
  - » Determining and interpreting the medians for Jada's and Andre's practice times in Problems 1 and 2.
- **Language Goal:** Informally estimating the center of a data set and then comparing both the mean and the median with this estimate. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly determine the median (e.g., they do not order from least to greatest or greatest to least or they do not know how to determine the median with an even number of values), consider:

- Reviewing Activity 1, Problem 2.

If students are unable to interpret the median as a typical value in context, consider:

- Reviewing Activity 2, Problem 1b, and asking, "What does the median tell you about Jada's and Diego's practice times?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was for students to identify and interpret the median of a data set. How did it go?
- In what ways did Activity 1 go as planned? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

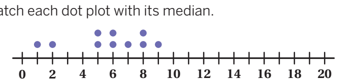
1. Priya attempted the first level of a video game 10 times. Her scores are shown. What is her median score?

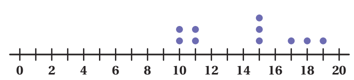
130	150	120	170	130
120	160	160	190	140

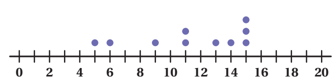
A. 125
B. 145
C. 147
D. 150

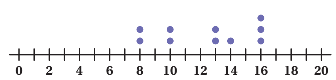
2. A teacher sorted a class's scores on their last test and noticed that 12 students scored higher than Clare, 12 students scored lower than Clare, and no one scored the same as Clare. Does this mean that Clare's score on the test is the median? Explain your thinking.
Yes; Sample response: There are 25 total students in the class, and about 50% of the students scored lower than Clare and about 50% scored higher than Clare.

3. Match each dot plot with its median.

a


b


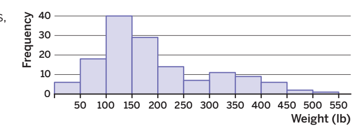
c


d


b 15
c 12
d 13
a 6

Name: ______ Date: ______ Period: ______

4. This histogram shows the weights, in pounds, of 143 wild bears. The ages of the bears ranged from newborn to 15 years old.



a. What can you say about the heaviest bear in this group?
It weighed between 500 and 550 lb.

b. What is a typical weight for the bears in this group?
200 lb; Sample response: A typical weight is between 150–200 lb.

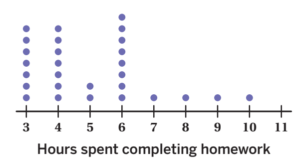
c. Do more than half of the bears in this group weigh less than 250 lb?
Yes

d. If weight is related to age, with older bears tending to have greater body weights, would you say that there were more old bears or more young bears in the group? Explain your thinking.
More young bears; Sample response: The data are clustered to the left of the histogram, so more of the bears weigh less.

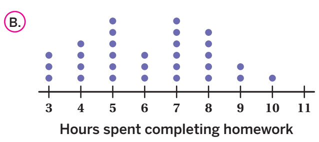
5. Noah's water bottle contains more than 1 qt of water but less than $1\frac{1}{2}$ qt. Let w represent the number of quarts that the bottle can hold. Select *all* the true statements.

A. w could be $\frac{3}{4}$.
B. w could be 1.
C. $w > 1$
D. w could be $\frac{4}{3}$.
E. w could be $\frac{5}{4}$.
F. w could be $\frac{5}{3}$.
G. $w > 1.5$

6. In one class, students typically spent 6 hours per week completing homework. Without calculating, which dot plot best represents this class?

A.


Hours spent completing homework

B.


Hours spent completing homework

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 8 Lesson 7	2
	5	Unit 7 Lesson 11	2
Formative	6	Unit 8 Lesson 11	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Comparing Mean and Median

Let's compare the mean and median of data sets.

Focus

Goals

1. **Language Goal:** Choose the measure of center to describe a given data set and justify the choice. **(Listening and Speaking, Writing)**
2. **Language Goal:** Explain that the median is a better estimate of a typical value than the mean for distributions that are not symmetric or contain values far from the center. **(Listening and Speaking, Writing)**
3. **Language Goal:** Generalize how the shape of the distribution affects the mean and median of a data set. **(Listening and Speaking)**

Rigor

- Students build **fluency** with determining the median and calculating the mean of a data set.
- Students **apply** their understanding of measures of center and the shapes of distributions to determine whether the mean or median is a more appropriate measure of center.

Coherence

Today

Students justify whether the mean or the median is a more appropriate measure of the center for a distribution in a given context. They see that a symmetric distribution has similar values for the mean and median. Likewise, for a distribution that is not symmetric, the mean is more affected by values that are far from the majority of the data values (even if there is only one outlier). In that case, the median could be a better measure of center. Students recognize that the choice of measure of center can be subjective, and the choice should always consider the context of the problem and the desired insights or questions being asked.

Previously

In Lesson 10, students determined and interpreted the median as a measure of center that, unlike the mean, is not impacted by outliers.

Coming Soon

In Lessons 12, students will describe variability in a data set, preparing them to measure the variability using the MAD and IQR in Lessons 13 and 14.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- calculators (optional)

Math Language Development

Review words

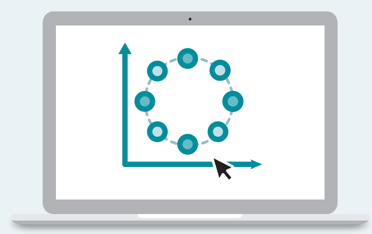
- *mean*
- *measure of center*
- *median*

Amps | Featured Activity

Activity 1

Interactive Dot Plots

Students see, in real time, how adding values to a data set affects the mean and median.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might react negatively to a difference of opinion while discussing which measure of center to use and when to use it. Ask them to take on the perspective of the person to whom they spoke rudely and describe how that would make them feel. Then ask them to rephrase their response in a way that focuses on the mathematics and treats the other person with the respect that they deserve.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, Problems 1 and 2 may be completed as a class. Have pairs complete Problem 3–5 before sharing their responses with the class.
- In **Activity 2**, Part 1 may be completed as a class. Then have students complete Part 2, Problem 2. Consider including the pros and cons of each measure of center as outlined in Part 2, Problem 1 in the class discussion.

Independent | 5 min

Warm-up Heights of the Presidents

Students review ways of describing typical values in data sets in order to compare the heights of the earliest and most recent presidents of the United States.

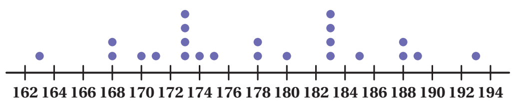
Unit 8 | Lesson 11

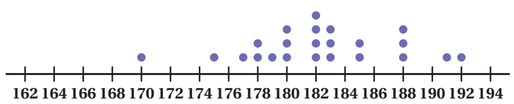
Comparing Mean and Median

Let's compare the mean and median of data sets.

Warm-up Heights of the Presidents

Two dot plots are shown. The first dot plot shows the heights of the first 23 U.S. presidents who served from 1789–1893. The second dot plot shows the heights of the next 23 presidents who served from 1893–2021. All heights are rounded to the nearest centimeter.





Without calculating, use the dot plots to compare the heights of the first 23 presidents to the heights of the second 23 presidents. Explain your thinking.

The second set of 23 presidents were typically taller than the first set of 23 presidents. Sample responses:

- **The median height of the first set of presidents is 178 cm, and the median of the second set of presidents is 182 cm.**
- **A typical value for the first set of presidents is between 173 cm and 183 cm, with 178 as the average height in that range. A typical value for the second set of presidents is between 180 cm and 188 cm, but likely closer to 182 because the data clusters between 180 and 183. Because the mean is the balance point, the sum of the distances to the left and right of 178 in the first data set and 182 in the second data set are approximately equal. Therefore, the second 23 presidents were typically taller.**

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work on the activity.

2 Monitor

Help students get started by asking, "What does each dot represent? How would you describe the distribution in each dot plot?"

Look for points of confusion:

- **Comparing the shortest or tallest presidents.** Ask, "Is that height typical of all the presidents?"
- **Struggling to identify a typical height.** Ask, "Where is the data clustered in each dot plot?"

Look for productive strategies:

- Determining and comparing the median heights.
- Identifying clustering and outliers to estimate mean heights.

3 Connect

Have students share their responses, focusing on how they compared the heights, including those looking at spread, typical values or clusters, and calculated or estimated measures of center.

Ask, if no students have mentioned it, "How did you consider any outliers? Why?"

Sample responses:

- I included them; they do not affect the median.
- I excluded them because they may shift the mean.
- They balanced out, so they don't affect the mean.

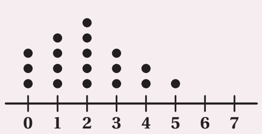
Highlight how to use outliers to reason about the mean without calculating. If they mostly balance each other, the mean is relatively unaffected; otherwise the mean will be shifted toward more extreme outliers.

Power-up

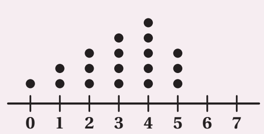
To power up students' ability to identify the typical value for a data set represented in a dot plot, have students complete:

Each of the dot plots show the number of siblings each student has in two different classes.

Class A



Class B



1. Which class has a greater number of students?
   They have the same number of students.
2. Estimate the center for each class.
   Class A: 2; Class B: 3
3. Without calculating, determine the class in which students typically have more siblings.
   Class B

Use: Before the Warm-up.

Informed by: Performance on Lesson 10, Practice Problem 6.

Pairs | 15 min

Activity 1 Moving the Middle

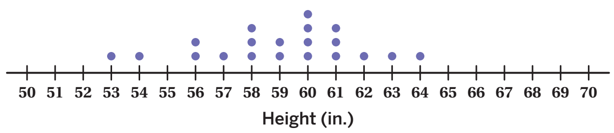
Students examine how adding a new data value to a distribution could affect the mean and median.

Amps Featured Activity Interactive Dot Plots

Name: ______ Date: ______ Period: ______

Activity 1 Moving the Middle

The dot plot shows the heights, in inches, of 20 players on Shawn's soccer team.



1. Determine one number to describe a typical height, in inches, of a player on the team by calculating:
   - a The mean height.
     59 in.
   - b The median height.
     59.5 in.

2. A new player joins Shawn's soccer team. For each description of how that player's height impacts the mean of the whole team when the new player is included, determine:
   - How tall the new player might be, without calculating.
   - What happens to the median.
   - a The mean height remains the same.
     59 in.; The median decreases to 59 in., so the mean and median are equal.
   - b The mean height increases.
     Answers may vary, but should be greater than 59 in. The median decreases slightly if the height is less than 59.5 in., remains the same if the height is exactly 59.5 in., or slightly increases if the height is greater than 59.5 in.

3. Would it be possible for one new player to join Shawn's team and for both the mean and median to remain the same as for the original data? Explain your thinking.
   No; Sample response: If the new player is 59 in. tall, the mean remains the same, but the median changes to 59 in. If the new player is 59.5 in., the median remains the same, but the mean increases slightly to a value between 59 in. and 59.5 in.

Lesson 11 Comparing Mean and Median 931

1 Launch

Consider providing calculators for this activity. If possible, have students complete the digital version of this activity as it affords real-time visual feedback of the impacts of their choices of measures. Alternatively, you could consider projecting the digital activity during the class discussion as they share.

2 Monitor

Help students get started by having them write the values in an ordered list first.

Look for points of confusion:

- **Struggling to think about both measures of center simultaneously.** Have students focus on one measure at a time, consider what happens to it, and then analyze the other measure.
- **Assuming that the mean and median will always move together.** Have students test and check values equal to, close to, and far from each.
- **Not knowing how to keep the mean the same.** Provide a data set such as 2, 2, 2. Ask, "What value can you add to keep the mean 2?"
- **Not knowing how to keep the median the same.** Have students test values close to, far from, and equal to the median. (one at a time for Problem 3; two at a time, and a variety of cases, for Problems 4b and 5a).

Look for productive strategies:

- Recognizing that the mean moves toward a newly added value, to maintain balance.
- Recognizing the number of data points added affects how the median is determined and only 59 in., 59.5 in., or 60 in. can be added to this data set.
- Using previous results and calculations to consider new data values.
- Using a systematic approach to test values and analyze outcomes, one measure of center at a time.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see, in real time, how adding values to a data set affects the mean and median.

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide students with sample heights to test for each description in Problem 2. For example, provide them with the following heights: 59 in., 59.5 in., 70 in., and 50 in.

Math Language Development

MLR8: Discussion Supports

While students work, provide sentence frames to help them organize and explain their thinking as they complete Problems 2–5. For example:

- "I tested the value _____, and the mean/median . . ." (Problem 2)
- "The heights of the players should be _____ and _____." (Problem 4)
- "This scenario is/is not possible if . . ." (Problems 3 and 5)
- "I predict _____ because . . ."

Pairs | 15 min

Activity 1 Moving the Middle (continued)

Students examine how adding a new data value to a distribution could affect the mean and median.

Activity 1 Moving the Middle (continued)

4. Two players join the soccer team. For each description of how those two players' heights impact the mean and median of the whole team when they are included, describe what you would know about the heights of the two players. Do not calculate. Be prepared to explain your thinking.

   a. The mean remains the same but the median decreases slightly.
   Both players are exactly 59 in. tall.

   b. The mean might change but the median does not change.
   Answers may vary, but one player should be shorter than 59.5 in., and the other player should be taller than 59.5 in.

5. Determine whether each scenario is possible when two new players join Shawn's team. Explain your thinking.

   a. Both the mean and median remain the same as in the original data.
   Yes; Sample response: It is possible, if one player is shorter and the other is taller than both the original mean and the original median, and the differences in their heights from the mean is the same amount (but in different directions). For example, if one player is 58 in. tall and the other is 60 in. tall.

   b. The mean and median end up a lot farther apart than in the original data.
   Yes; Sample response: It is possible, if both heights are much greater, or if both heights are much less than the mean in the original data.

Are you ready for more?

Create a data set with five numbers that has a mean of 10 and a median of 12. Show or explain your thinking.

Sample response: 5, 6, 12, 13, 14

Any five numbers including at least one 12 such that $\frac{a+b+12+c+d}{5} = 10$, and where *a* and *b* are less than 12, and *c* and *d* are greater than 12, will work.

$a+b+12+c+d=50$

$a+b+c+d=38$

$5+6+13+14=38$

932 Unit 8 Data Sets and Distributions

3 Connect

Have students share their responses for each problem, focusing most of the discussion on the number of data values involved in each scenario and what causes changes to the mean and median, and what keeps them the same. As students share, consider representing their thinking on the dot plot for all to see.

Ask:

- "Which measure of center – the mean or median – is more susceptible (more affected) to *large* changes? Why?" Both values can change when new values are added to the data set, but the mean changes more because it is more affected by outliers. The mean is pulled in the direction of the new value, or the average of the new values, to balance the distribution. The median only considers the number of values in the data set, so it will only change at most to the next data value in one direction when one value is added.
- "Is it even possible to add just one value and see a large change in the median? If so, can you give an example?" Yes, but only if there are few values around the original median or there are two clusters that are already relatively far apart. For example, the median of 1, 1, 2, 2, 3, 17, 17, 18, 18 is 3, but if you add another 17 (or anything greater) to the set, then the median will be 10.

Highlight how the difference between the mean and median as measures of center is what causes the mean to be more susceptible (more affected) to large changes.

Pairs | 15 min

Activity 2 Mean or Median?

Students analyze the shape of a distribution to choose the better measure of center for summarizing the data, and they justify their choices.

Name: ______ Date: ______ Period: ______

Activity 2 Mean or Median?

Despite the serious threats that honey bees face, there is some evidence that their situation is improving, in part because apiarists are working to renovate existing colonies by introducing a new queen or other new bees.

Part 1

You will be given six cards. Each card contains information about the number of honey bee colonies renovated across several apiaries in the state listed. Each card also includes either a dot plot or a histogram, as well as both the mean and the median.

1. Sort the cards into *two* groups based on the distributions and measures of center shown. Be prepared to explain your thinking.
   Sample responses:
   - **Group 1: West Virginia, Massachusetts, Florida; The distributions are generally symmetric, and the mean and median are the same or very close to each other.**
   - **Group 2: Oklahoma, Indiana, Texas; The distributions are not symmetric, and the mean and median are not close to each other.**
2. Discuss your sorting decisions with another group. Resolve any disagreements, and record your final decisions.

Part 2

You will be assigned one state. For your state:

3. Explain the pros and cons of using each measure of center to summarize the data.
   Answers may vary, but should consider:
   - **The mean is calculated using all the data values, so it includes the apiaries that renovated many more or many less colonies than was typical in the state. By including these outliers, the average number of renovated colonies may appear more or less than what is actually typical for that state.**
   - **The median considers the number of data values to the left and right of center all the same, so even if one or two apiaries did way more or less than most, it would not skew the interpretation of a typical value. However, this will discount the efforts of those same colonies.**
4. Make a case for which measure of center *best* summarizes the data. Explain your thinking.
   Answers may vary, but should consider:
   - **When the distribution is symmetric, such as Group 1 from Part 1, a typical number of colonies renovated in each state can be summarized using either the mean or median because the values are close.**
   - **When the distribution is not symmetric, such as Group 2 from Part 1, the mean number of colonies renovated has been skewed by those apiairies that did far more or far less than what is typical in their state. Therefore, the mean may be misleading. The median number of colonies renovated is the better choice.**

STOP

1 Launch

Distribute one set of cards from the Activity 2 PDF to each pair of students. Pause after Part 1, and have students share their categories and how they sorted. Ensure students connect roughly symmetric distributions with similar means and medians, and vice versa.

2 Monitor

Help students get started by displaying the West Virginia card and asking, "Describe the distribution. Which other states look similar?"

Look for points of confusion:

- **Sorting based on the range rather than distribution of the data (Part 1).** Ask, "Describe the shape of the distribution. Do any other states look similar?"
- **Not using the context of the data to consider the pros and cons of each measure of center (Part 2).** Ask, "Why might including or excluding outliers be misleading for the mean? median?"

Look for productive strategies:

- Relating symmetric distributions to similar means and medians, and non-symmetric distributions to the measures differing more.
- Considering the pros and cons of including or excluding outliers in context when choosing the best measure of center.

3 Connect

Display each card, one at a time.

Have students share their responses and justifications for Part 2, one state at a time. Have others agree or disagree and justify their thinking.

Ask, "Is the median *always* a good measure of center? Why or why not?"

Highlight that the shape of the distribution, the question being asked, and the context all impact which measure of center to use.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Pre-sort the cards into the two groups shown in the sample response for Part 1. Ask students to describe the similarities and differences *between the cards in each group* and the differences *between groups*.

Extension: *Math Enrichment*

Ask students to examine the distribution for West Virginia and explain why either the mean or the median is an appropriate measure of center, even though they are *not* the same value, as they are for Massachusetts or Florida. While they are not the same value, they are very close to each other, so either measure is appropriate to describe a typical value.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their responses, display the two categories of distributions: symmetric and non-symmetric. Ask students to complete the following statements and then add these statements to the class display.

- "When the distribution is symmetric or approximately symmetric, use _____ because . . ."
- "When the distribution is not symmetric, use _____ because . . ."

English Learners

Include at least one visual example of a symmetric and non-symmetric distribution to help make connections between the shape of the data distributions and the statements.

Whole Class | 5 min

Summary

Review and synthesize how the mean and median summarize a typical value for a data set differently, and how both can be misleading.

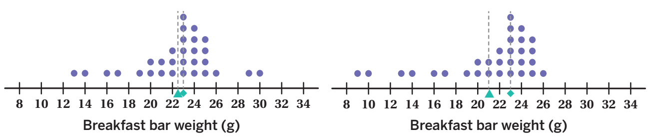
Summary

In today's lesson . . .

You compared how well the mean and median each describe or summarize the center of a distribution. Each measure of center tells you slightly different things, and one may be more appropriate depending on the distribution of the data and the context.

In general, when a distribution is symmetric, or approximately symmetric, the mean and median values are close. Less symmetric distributions, such as those with *outliers* (values much less or greater than typical values in the distribution), tend to have the mean and median farther apart. The mean is calculated using the values of every data value, so it shifts away from other typical values when there are outliers. However, the median counts all values the same (values of 1 less or 100 less than typical are both just one point), so it does not generally shift away from other typical values when there are outliers.

Consider these two dot plots showing the weights, in grams, of two different batches of 30 breakfast bars. The means (triangles) and medians (rhombuses) have been marked.



The mean is a good measure of center for the first batch because it represents a balance point and the data are symmetric, but it could be misleading for the second batch because the data are not symmetric and most values are greater.

The median is a good measure of center for both the first batch and the second batch, but it could be misleading to someone who does not know what the distribution looks like if they assume the data are symmetric.

> Reflect:

Synthesize

Ask, "A student reports that 7 is a typical number of pets that students in her class have. Do you think they used the mean or median number of pets? How do you know?" The mean. That number seems too high to be a typical number of pets. The data may have included one or more students who have a tank of fish or other small animals that get counted individually, which would make the mean greater.

Highlight that, so far, students have described distributions, including using mean and median to identify typical values. In the next few lessons, they will paint a more complete picture of a data set by considering ways to describe the spread or variability using numbers, and the relationship between those numbers, typical values, and the center.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- What does a measure of center tell you about a distribution?

Exit Ticket

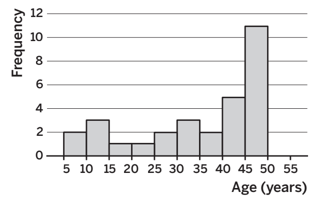
Students demonstrate their understanding of how to determine and justify whether the mean or median best describes a typical value.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 8.11

The histogram shows the ages of 30 people at a family dinner party.



1. In the context, what does each measure of center tell you about the data?

   Median:

   A typical age of a guest at the dinner party. There are the same number of guests that are younger than the median age as those that are older than the median age.

   Mean:

   The average age of everyone at the dinner party.

2. Without calculating, determine which has the greater value – the mean or the median. Explain your thinking.

   The median is greater than the mean because the distribution is not symmetric, and there are many values that pull the mean to the left of the center of the data.

3. Which is the *better* measure of center for this data? Explain your thinking.

   Answers may vary, and students may choose either the mean or the median, but the shape of the distribution and the context of the data should be used to support their thinking.

   Sample responses:

   - The median better describes the center because, overall, there are more adults and the typical age represented by the mean would be pulled down by the five children at the party that are 5–15 years old.
   - The mean better describes the center because the spread of ages goes all the way from 5–50 and a typical age needs to include the ages of the 9 people who are younger than 30, even though there are fewer of them.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine whether the mean or the median is more appropriate to describe the center of data.
1 2 3

b. I can explain how the distribution of data affects the mean and the median.
1 2 3

Success looks like . . .

- **Language Goal:** Choosing the measure of center to describe a given data set and justifying the choice. **(Listening and Speaking, Writing)**
  » Selecting the better measure of center for the ages of the guests and explaining why in Problem 3.
- **Language Goal:** Explaining that the median is a better estimate of a typical value than the mean for distributions that are not symmetric or contain values far from the center. **(Listening and Speaking, Writing)**
- **Language Goal:** Generalizing how the shape of the distribution affects the mean and median of a data set. **(Listening and Speaking)**

Suggested next steps

If students are unable to distinguish the differences between what mean and median tell them about the data in Problem 1, consider:

- Reviewing Activity 2, Part 2, Problem 1.

If students say the mean is greater or are unable to justify their thinking in Problem 2, consider:

- Asking, "Where is most of the data clustered in the histogram? What does that tell you about a typical age at the party? How do the people younger than the typical age impact the mean? The median?"
- Reviewing the Texas card from Activity 2, and asking, "Why is the mean greater than the median? How is this distribution generally the *opposite* of the distribution on the Exit Ticket? What does that tell you about the mean and median ages at the party?"

If students inadequately justify their thinking in Problem 3, consider:

- Reviewing Activity 2, particularly the reasoning used for Indiana, Oklahoma, and Texas.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

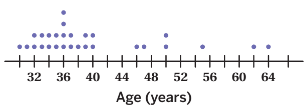
- In this lesson, students were asked to relate their choice of measure of center to the shape of the distribution and context of the data. How well did your students do this today?
- What did students find frustrating about Activity 1? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______________________ Date: __________ Period: __________

1. Here is a dot plot that shows the ages of teachers at a school. Which of these statements is true of the data set shown in the dot plot?



A. The mean is less than the median.
B. The mean is approximately equal to the median.
C. The mean is greater than the median.
D. The mean cannot be determined.

2. Bard asked each of five friends to attempt to throw a ball in a trash can until they succeeded. Bard recorded the number of unsuccessful attempts made by each friend as: 1, 8, 6, 2, 4. However, Bard made an error in recording. The 8 in the data set should have been 18. How would changing the 8 to 18 affect the mean and median of the data set?

A. The mean would decrease and the median would not change.
B. The mean would increase and the median would not change.
C. The mean would decrease and the median would increase.
D. The mean would increase and the median would increase.

3. For history class, Han's homework scores are shown below.

100	100	100	100	95	100	90	100	0

The history teacher uses the mean to calculate an overall grade for homework. Write an argument for Han to explain why median would be a better measure to use for his homework grades.

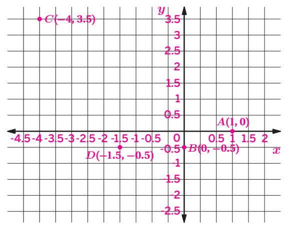
Sample response: The teacher should use the median because it more accurately reflects the center of Han's data. The one score of 0 pulls the mean to the left (decreases his mean) when the rest of his grades are between 90 and 100.

Name: ______________________ Date: __________ Period: __________

4. Zookeepers recorded the age, weight, sex, and height of the 10 pandas at their zoo. Write two statistical questions that could be answered using these data sets.

Sample responses:
What is a typical weight of a panda at this zoo?
Do female pandas eat the same amount of bamboo as male pandas?

5. Here is a set of coordinates. Draw and label an appropriate pair of axes and plot the points.
$A(1, 0)$, $B(0, -0.5)$, $C(-4, 3.5)$, $D(-1.5, -0.5)$



6. Elena and Kiran have been practicing their basketball free throws during recess. They record the number of baskets they make out of 10 attempts. Here are their data sets for 12 school days.

Elena:

4	5	1	6	9	7	2	8	3	3	5	7

Kiran:

2	4	5	4	6	6	4	7	3	4	8	7

a. Calculate the mean number of baskets each player made, and then compare the means.

Elena and Kiran have the same mean of 5; Sample response: Both sets have a sum of 60 and $60 \div 12 = 5$.

b. What do the means tell you about each player's free-throw shooting ability in this context?

Sample response:

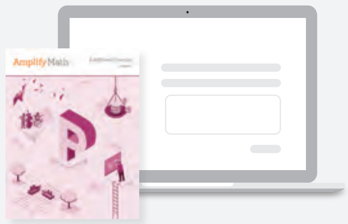
The means show that both students make, on average, half of the 10 attempts to get the basketball in the hoop.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 8 Lesson 2	2
	5	Unit 7 Lesson 14	2
Formative	6	Unit 8 Lesson 11	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 3
Measures of Variability

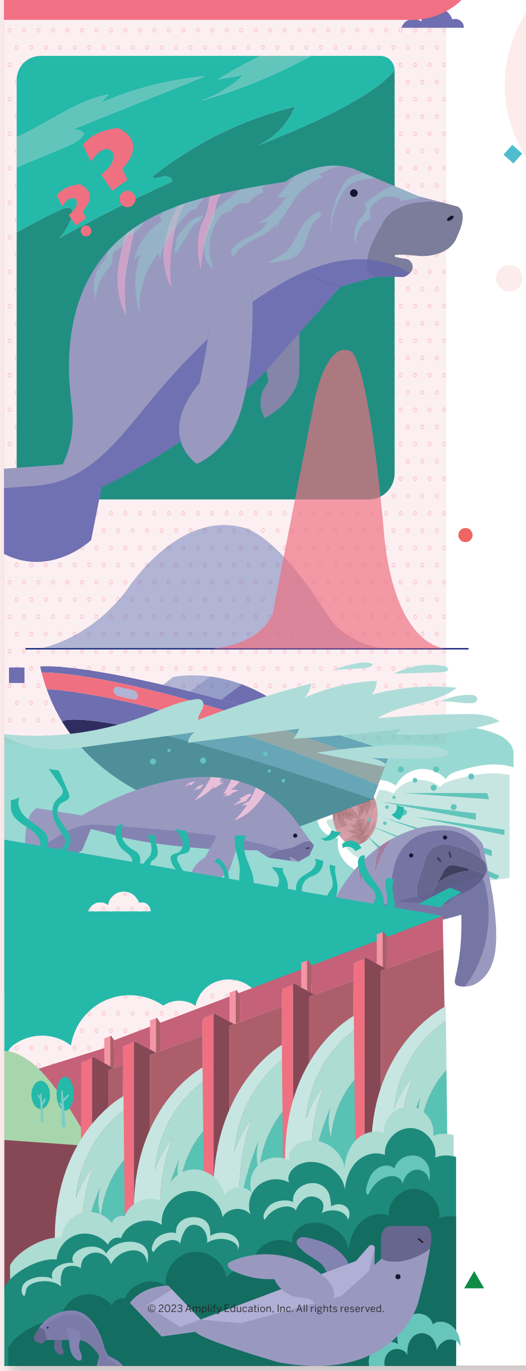
Whole Class

In this Sub-Unit, students calculate and compare mean absolute deviations and interquartile ranges of data sets, construct and interpret box plots, and determine when and why one of the measures of variability may be more appropriate.

SUB-UNIT

3 Measures of Variability

Narrative Connections



Where have the giant sea cows gone?

Look off the Florida coast and in the shallows, you might see a rush of bubbles followed by a whoosh of water and air. These are the tell-tale signs of a manatee. Also known as "sea cows," manatees are the gentle giants of the ocean. These aquatic mammals are huge. They average about ten ft in length and weigh around 1,200 lb.

To date there are only three known species of manatees in existence — the West Indian manatee, the Amazonian manatee, and the African manatee. And, yet, despite having only a few natural predators, manatees worldwide are in danger of extinction.

The reason? Humans.

Every year, hundreds of manatees are injured or killed by boaters. In the winter season, manatees congregate in warm shallow waters, like those off the Florida coast. Boat drivers, unable to see them, often run them over. Others have been trapped and killed in manmade structures like floodgates and canal locks. Others still get tangled up in fishing lines. Meanwhile climate change has changed ocean temperatures, spurring on habitat loss. Manatees suffer from overcrowding and exposure to cold stress that their sensitive bodies can't withstand.

The manatees' survival depends on us making smart choices to protect their environment. To do that, we have to be well-informed about where they live, their migration patterns, and the habits in their lifecycle. Through statistics, scientists can account for variabilities in the data and deepen their understanding of how to help manatee populations thrive.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how measures of variability can be used to understand the manatee population in the following places:

- **Lesson 12, Activities 1-2:** Counting Manatees, Variability at Chassahowitzka River
- **Lesson 13, Activity 1:** Manatee Scars
- **Lesson 14, Activities 1-2:** The Five-Number Summary, Range and Interquartile Range
- **Lesson 15, Activities 1-2:** Living Box Plot

Describing Variability

Let's study distances between data values and the mean or median.

Focus

Goals

1. **Language Goal:** Explain how the variability of a distribution relates to the mean and median. **(Writing)**
2. **Language Goal:** Compare and contrast distributions that have the same mean, but different amounts of variability. **(Writing)**
3. **Language Goal:** Describe the variability of a data set in their own words with a logical justification. **(Speaking and Listening)**

Coherence

Today

Students begin to think about how the variability of a distribution could be measured and summarized. They first compare distributions with the same mean but different medians, and recognize that the relationship between these two measures of center can provide some information about the shape of the distribution and its spread. They also use these measures and the dot plots of the data to try to quantify variability in some way that is related to what typical values are. As a bridge to the next lessons and the introduction of specific measures of variability, students are presented with the idea of calculating the distances between the data values and the mean or median to as a way to quantify the *variability*, or spread, of a distribution.

Previously

In Lessons 8–11, students determined a single number to summarize the center of a distribution using two possible measures of center – mean and median.

Coming Soon

In Lessons 13 and 14, students will similarly determine a single number to summarize the spread of a distribution using two possible measures of variability related to those measures of center: mean absolute deviation (MAD) and interquartile range (IQR).

Rigor

- Students build **conceptual understanding** of measures of variability to prepare them to calculate MAD and IQR.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Activity 1 PDF, one per student
- calculators

Math Language Development

New word

- ***variability***

Note: *Variability* was introduced informally in Lesson 2, relative to the type of data associated with statistical questions.

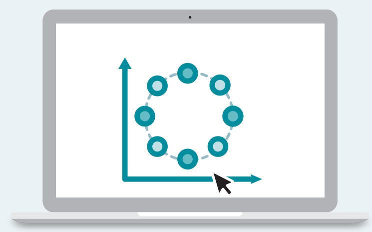
Review words

- *mean*
- *median*

Amps | **Featured Activity**

Activity 1
See Student Thinking

As students explain their thinking about dot plots, see their responses in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might either try to dominate the activity or let others in the group do all of the work as they compare distributions. While working in a small group, it is important that all students have the opportunity to seek help and provide help whenever needed. Have the group develop a plan, prior to starting the activity, that guarantees active participation by all.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted, but students should read the introductory sentences to supply background knowledge about the context for the rest of the lesson.
- In **Activity 2,** display a completed table for Problem 1 and have students focus on Problem 3.

Pairs | 5 min

Warm-up Spotting Manatees

Students compare the means, medians, and spreads of three sets of data to activate prior knowledge and also supply background knowledge about the context for this lesson.

Unit 8 | Lesson 12

Describing Variability

Let's study distances between data values and the mean or median.

Warm-up Spotting Manatees

Florida manatees (*Trichechus manatus latirostris*) are listed as Endangered by the IUCN, but, interestingly, the US Fish and Wildlife Service (USFWS) took Florida manatees off their endangered species list in 2017.

Alex Couto/Shutterstock.com

Crystal River, Homosassa River, and Chassahowitzka River are three locations in Citrus County, Florida, where aerial surveys are conducted to count and monitor Florida manatee populations. Given the competing labeling between the IUCN and USFWS, keeping track of their populations is very important.

Your teacher will show you data sets from these 3 locations showing the number of manatees sighted on 12 different days.

Without calculating, predict which location has the greatest mean, median, and spread.

Sample responses:

- I think Crystal River has the greatest median because I looked at the middle numbers.
- I think Homosassa has the greatest mean because it has the most values of 10 or greater.
- I think Chassahowitzka has the greatest spread because it has values from 0 to 15.

Log in to Amplify Math to complete this lesson online.

1 Launch

Use the *Think-Pair-Share* routine for this Warm-up. Display the Warm-up PDF. Explain to students that an *aerial survey* is a way to collect data from a helicopter or airplane flying above an area. People literally count the number of objects of interest that they see — in this case, manatees.

2 Monitor

Help students get started by asking, "Where can you look to determine an estimate of the median?"

Look for points of confusion:

- **Thinking that Crystal River has the lowest mean because it has no higher values.** Ask, "How many values greater than 9 were there at each site? Less than 3?"
- **Thinking that Homosassa and Chassahowitzka Rivers have close means because they each have double-digit values.** Ask students about the frequency of the lower values of each location and how that affects their means.

Look for productive strategies:

- Identifying medians precisely by quickly scanning the ordered lists of data.
- Using mental arithmetic and the equal redistribution interpretation of the mean to estimate means.
- Visualizing the data sets to help them think about spread and clusters of data to estimate a typical value.

3 Connect

Record and display their predictions.

Have pairs of students share their responses, focusing on their reasoning for the mean and spread, based on the frequencies of low and high values.

Highlight that students will use this same data in Activity 1.

Power-up

To power up students' ability to describe what the mean tells them in context, ask:

The math test scores of 12 students are shown:

55, 65, 65, 70, 75, 80, 85, 85, 90, 95, 95, 100

1. Calculate the mean test score. 80
2. Which of the following statements best describe what the mean tells you about students' performance on the math test.

A. 80% of the students earned a passing grade.
B. On average, students earned a score of 80.
C. Most of the students in the class scored 80.
D. Half the class earned scores above 80 and half earned scores below 80.

Use: Before the Warm-up.

Informed by: Performance on Lesson 11, Practice Problem 6.

Small groups | 15 min

Activity 1 Counting Manatees

Students compare distributions with the same mean and different medians, seeing how the relative values for those measures of center relate to the spread of a distribution.

Amps Featured Activity See Student Thinking

Name: ______ Date: ______ Period: ______

Activity 1 Counting Manatees

Refer to the dot plots of the same sightings data from the Warm-up, showing the mean and median.

1. Does either the mean or the median describe the variability in the data for each location?
   No, both the mean and the median reflect the center, not the shape or spread.
2. Describe the variability of the three locations based on their mean and median, thinking about how they are the same or different.
   Crystal River has the least variability and you can see that because all the data are clustered right around the mean and median, which are the same as well.
   Homosassa has more variability than Crystal River, but not as much as Chassahowitzka.
   Chassahowitzka has the most variability and this is visible by how spread apart the mean and median are, which is true about its data as well.
3. Use the dot plots, the data sets, and their measures of center (if you think they are useful), to compare the variability of the three data sets.
   Crystal River has the least variability because most of the data are within 1 unit of both the mean and the median (from 4 to 6 manatees).
   Chassahowitzka River probably has the second most variability because most of the data are within 3 or 4 units of the median. But it could be the most because the data are the most spread out from least to greatest (0 to 15).
   Homosassa River probably has the most variability, by a little bit, because almost all of the data are more than 3 units away from the median. It could also be second most because there are two clusters that are far apart, but the data around each are close.
4. Which location would you choose to go to if you wanted to see at least 4 manatees? Explain your thinking.
   Sample response: I would not go to Homosassa because most days only 1 or 2 are sighted. I would probably choose Crystal River because, even though 4 or 5 could be typical and larger numbers can be seen at Chassahowitzka River, it is typical to see 5 or 6 at Crystal River with more consistent sightings.

Are you ready for more?

Think about some issues that scientists and volunteers who monitor and count manatees may face in trying to gather accurate data.

1. How could each of those issues impact the counts and data available?
   Sample response: Not getting accurate counts could produce lower or higher data than the actual numbers.
2. How could that also impact how the data is interpreted by different groups, such as IUCN and USFWS?
   Sample response: It could make them think that populations are better off than they actually are or worse off than they actually are, which then could impact their classification.

1 Launch

Distribute a copy of the Activity 1 PDF to each student. Set an expectation for the amount of time students will have to work collectively on the activity.

2 Monitor

Help students get started by having them interpret what the mean and median would tell them in this context.

Look for points of confusion:

- **Thinking the mean or median (alone) can describe variability.** Present students with a data set, such as 11, 11, 11, 11, 15, 15, 15, 19, 19, 19, 19, where the mean and median are both 15. Ask, "Does 15 seem like it has anything to do with spread or variability here?"

Look for productive strategies:

- Connecting features of distributions to numerical descriptions of spread, such as a how spread relates to the differences between the mean and median.
- Discussing variability in terms of consistency, or typical or expected values (e.g., Crystal values are most consistent around the mean; Chassahowitzka values are least consistent – some very low, some very high values, none around the mean).

3 Connect

Have groups of students share their responses, focusing on Problems 2 and 3, connecting the qualitative descriptions of shape to the numbers.

Highlight that the center and spread or variability of a distribution describe different aspects of the data.

Define the ***variability*** of a data set as a description of how far away, or how spread out, data values generally are from the center.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can overlay the dot plots to obtain a more comprehensive visual of the data in order to make connections and comparisons.

Accessibility: *Clarify Vocabulary and Symbols, Activate Prior Knowledge*

Display the three dot plots from the Activity 1 PDF and remind students that the triangle symbol represents the mean and the rhombus symbol represents the median. Ask, "What does the mean describe about a set of data? The median?" This will prepare them for understanding why neither measure by itself can summarize the variability.

Math Language Development

MLR2: Collect and Display

During the Connect, collect and record ideas and language students use about spotting manatees "consistently" as the class discusses the data represented in the three dot plots. This will help students connect the visual representations to statistical terms such as *mean*, *median*, *center*, and *variability (spread)*. Continue adding to the class display in Activity 2 as students describe variability.

English Learners

Use gestures as students discuss spotting the manatees to emphasize the different degrees of variability.

Small groups | 15 min

Activity 2 Variability at Chassahowitzka River

Students quantify the variability in one data set based on how each value relates to either its mean or median, which is a preview for understanding the mean absolute deviation.

Activity 2 Variability at Chassahowitzka River

Refer to the aerial survey data from Chassahowitzka River to answer the problems.

1. Complete the table by determining the difference between each value and the:
   - Mean.
   - Median.

Chassahowitzka River	3	5	12	2	1	2	3	0	3	15	5	9
Distance from the mean, 5	2	0	7	3	4	3	2	5	2	10	0	4
Distance from the median, 3	0	2	9	1	2	1	0	3	0	12	2	6

2. Describe what the values in each row of your completed table might tell you about the *variability* of the data.
   Sample responses:
   - There are more values closer to the median, but the farthest two are farther away.
   - Not counting the ones that are equal to the mean or median, there are no values as close to the mean as to the median, but they are all about the same distance from the mean.
   - There are seven values 3 or more away from the mean and only four values 3 or more away from the median. Of those, the ones relative to the median are farther away.

3. Select the measure of center — mean or median — that you think could best be used to help summarize the *variability* of the data set with one number. Use the values in the corresponding row in the table to help you determine one number to describe the variability of the original data set. Explain your thinking.
   Sample responses shown.

Measure of center	One number for variability	Explanation
Mean	3.5 (mean of differences) or 3 (median of differences)	I determined the mean/median of the distances from center.
Median	2 (median of differences) or 3.2 (mean of differences)	

STOP

1 Launch

Have students work independently and then share their responses with their group.

2 Monitor

Help students get started by suggesting they use the dot plot (on the Activity 1 PDF) to help them calculate distances from the mean and median.

Look for points of confusion:

- **Having trouble calculating the differences.** Remind students that this is similar to absolute value, and direction does not matter. It is the distance from the mean (similar to distance from 0 for absolute value) to that value.

Look for productive strategies:

- Thinking about the distances from the mean or median as being similar to computing absolute value.
- Applying mathematical reasoning and number sense from this unit and previous units (e.g. average, median, ratios, or percents) to determine an informal measure of variability and justifying how it represents the spread of the distribution.

3 Connect

Display a completed table of differences for students to check their work.

Have individuals or groups of students share their responses to Problems 2 and 3, starting with students who ultimately worked with median and then those who ultimately worked with mean.

Ask, "Is there a way to describe variability and consistency in seeing manatees more precisely and in an objective way?"

Highlight that the spread or variability of a data set can also be described more formally and precisely by one number. The upcoming lessons will explore two common methods that are related to the mean and median.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider demonstrating how to complete a few cells of the table in Problem 1, and then provide students with a pre-completed table so that they can focus on thinking about and responding to Problems 2 and 3. To help them begin to think about Problem 2, ask:

- "What is the greatest distance from the mean? The least distance?"
- "What is the greatest distance from the median? The least distance?"
- "Do any distances from the mean seem to be grouped closer together than others? From the median?"
- "Do the values look closer to the mean or the median? Why do you think so?"

Extension: *Math Enrichment*

Have students re-examine the dot plot for the Chassahowitzka River. Ask:

- "Are most of the data values closer to the mean or the median?" Sample response: Most of the values are closer to the median.
- "How close do you think the data values are to this measure?" Sample response: Most values are within 2 (either side) of the median.
- "Refer to the number you determined in Problem 3. Does this value make sense when you visually examine the dot plot?" Sample response: Yes, it is close to the same number.

Whole Class | 5 min

Summary

Review and synthesize how variability or spread and center are different, but related, descriptions of a distribution.

Name: Date: Period:

Summary

In today's lesson . . .

You shifted away from focusing on how the values in a data set can be used to determine one or more numbers that describe the center of a distribution. You began to explore these same ideas of using the values in a data set to determine one or more numbers that instead describe the *spread* of a distribution. With spread, this can be a little more involved and can be interpreted a few different ways.

One way to think about it would be to consider how far apart the values are overall. Another way to think about it would be to consider how far apart the values are relative to one other number, such as the center. Yet another way to think about it would be to consider how far apart each number is from each and every one of the other numbers. And there are probably even more possibilities.

All of these possibilities have one thing in common — they are attempting to describe the amount of *variability* in a data set. The ***variability*** of a data set is a description of how far away, or how spread out, data values generally are from the center.

In the next lessons, you will see some commonly used *measures of variability* that relate to the measures of center you already know — mean and median.

Reflect:

Synthesize

Ask:

- "How might a distribution having a wide spread affect its measures of center?" It could pull the center in one direction if it is not symmetric.
- "How might the location of a distribution's center affect a numerical description (or measure) of its spread or variability?" If the center is in the middle, then all values will be relatively closer to it, or roughly equal distances from it, but if it is not in the center, some values will be really close and others might be really far and you need to account for both.

Highlight that similar to measures of center, a *measure of variability* considers all of the values in the data set and may not actually be equal to any of them. The next lessons will explore further this idea of measuring variability.

Formalize vocabulary: *variability*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Can you draw variability? What would it look like?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *variability* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of variability in a data set by determining which of three data sets has the most variability and explaining their reasoning.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.12

The data sets show the number of text messages sent by Jada, Diego, and Lin over 6 days as well as the mean and median numbers of text messages sent by each student over those days. Which data set do you think has the most variability? Show or explain your thinking.

Jade's data set	Diego's data set	Lin's data set
4 4 4 6 6 6	4 5 5 6 8 8	1 1 2 2 9 9
Mean: 5	Mean: 6	Mean: 4
Median: 5	Median: 5.5	Median: 2

Lin's has the most; Sample responses:

- I know this because the mean and median are the farthest apart, which means there must be numbers pulling the data farther apart.
- I know this because the difference between the maximum and minimum values for Lin's data set is the greatest.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain how the variability of distributions relates to the mean and the median. 1 2 3

b. I can summarize the variability of a data set with one number. 1 2 3

c. I can compare distributions that have the same mean, but different variability. 1 2 3

Lesson 12 Describing Variability

Success looks like . . .

- **Language Goal:** Explaining how the variability of a distribution relates to the mean and median. **(Writing)**
  - » Explaining which data set of text messages has the most variability.
- **Language Goal:** Comparing and contrasting distributions that have the same mean, but different amounts of variability. **(Writing)**
- **Language Goal:** Describing the variability of a data set in their own words with a logical justification. **(Speaking and Listening)**

Suggested next steps

If students have trouble identifying which data set has the most variability, consider:

- Having them order the data to see it more clearly.
- Having them create a dot plot for another visual representation of the data.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which students' ideas were you able to highlight during Activity 1?
- The goal of this lesson was just to explore the idea of measuring variability. In what ways was this evident? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Describing the variability of a data set in their own words with a logical justification.

Reflect on students' language development toward this goal.

- Are students' descriptions of variability logical and do they include mathematical language they have learned up to this point?
- What is an example of a developing description of variability for the Exit Ticket problem?

Sample descriptions:

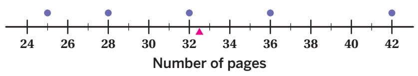
Emerging	Expanding
Lin's has the most variability because her values are farther apart.	Lin's has the most variability because the difference between the minimum and maximum values are the greatest.

Practice

Independent

Name: ____________ Date: ________ Period: ________

1. Han recorded the number of pages that he read each day for five days. This dot plot shows his data.



a. Is 30 pages a good estimate for the mean number of pages that Han read each day? Explain your thinking.

No; Sample response: 30 is in the center of the left four values, but 42 would increase its value.

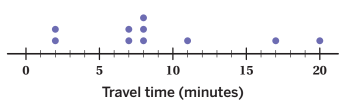
b. Determine the mean number of pages that Han read during the five days. Draw a triangle to mark the mean on the dot plot.

32.6 pages

c. Use the dot plot and the mean to complete the table.

Number of pages	Distance from mean	Left or right of mean
25	**7.6**	left
28	**4.6**	**left**
32	**0.6**	**left**
36	**3.4**	**right**
42	**9.4**	**right**

2. Ten sixth graders recorded the amounts of time each of them took to travel to school. The dot plot shows their travel times.



a. What is a typical amount of travel time for the 10 sixth graders? Explain your thinking.

8–10 minutes; Sample response: The mean is 9 and most of the travel times are around 9, and there are an equal number of data values that are farther from the mean.

b. Jada believes that travel times between 5 and 13 minutes are common for this group. Do you agree? Explain your thinking.

I agree; Sample response: More than half the data (6 of the 10 dots) are between 5 and 13.

Name: ____________ Date: ________ Period: ________

3. In an archery competition, scores for each round are calculated by averaging the distances of 3 arrows from the center of the target.

An archer's three arrows were 1.6 in., 0.5 in., and 2.9 in. from the center in the first round. In the second round, the archer's arrows were farther from the center *on average*, but more consistent. What values for the distances in the second round could fit this description? Explain your thinking.

Sample response: The average (mean) distance from the first round is 1.6 in.

The average (mean) distance for the second round could be 2 in., so the distances for those three arrows could be 2 in., 1.75 in., and 2.25 in.

4. Draw a number line.

Sample responses:

$-7 < -6$; $-3 > -6$; $-3 > -7$

a. Plot and label three numbers between -2 and -8 (not including -2 or -8).

b. Use the numbers you plotted and the symbols $<$ and $>$ to write three inequality statements.

5. Jada earns money from babysitting, walking her neighbor's dogs, and running errands for her aunt. Every four weeks, she combines her earnings and divides them into three equal parts — one for spending, one for saving, and one for donating to a charity. Jada donated $26.00 of her earnings from the past four weeks to charity.

Make two different lists showing possible amounts she could have earned from each job in the last four weeks, if she did not earn the same amount from any two jobs.

In the 4 weeks, Jada earned a total of $78; 26 • 3 = 78;

Sample response:

Amount from babysitting ($)	Amount from walking dogs ($)	Amount from errands ($)
25	**27**	**26**
30	**22**	**26**

6. Determine each sum or difference:

a. $3.3 + 2.1$ **$= 5.4$**

b. $33 + 2.1$ **$= 35.1$**

c. $3.3 - 2.1$ **$= 1.2$**

d. $33 - 2.1$ **$= 30.9$**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 4	2
	5	Unit 8 Lesson 8	2
Formative	6	Unit 8 Lesson 13	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Variability and MAD

Let's use mean and mean absolute deviation (MAD) to describe variability.

Focus

Goals

1. **Language Goal:** Compare the means and mean absolute deviations of different distributions, specifically those with the same mean but different MADs. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret the mean and mean absolute deviation (MAD) in the context of the data. **(Speaking and Listening, Writing)**

Coherence

Today

Students see one way to summarize the spread of a distribution with a single number by calculating a measure of variability. They learn that the mean absolute deviation (MAD) is the average distance of all the data values from the mean. Students also apply their knowledge of how to calculate averages and interpret the mean to interpret the MAD in the context of the data. They compare distributions with the same mean but different MADs, which helps them recognize that the greater the MAD, the greater the variability or spread in a data set.

Previously

In Lessons 8 and 9, students looked at mean in two ways: fair share and balance point. In Lesson 12, they computed and interpreted distances of data values from the mean and understood this as a form of describing variability.

Coming Soon

In Lesson 14, students will see another measure of variability — the interquartile range (IQR), which is related to the median.

Rigor

- Students continue to develop **conceptual understanding** of measures of variability.
- Students begin to develop **procedural skills** for calculating MAD, as well as building **fluency** in calculating averages in general.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs Small Groups	Pairs Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (answers, for display)
- *Percentage Algorithms* PDF (as needed)
- calculators

Math Language Development

New words

- ***measure of variability***
- ***mean absolute deviation (MAD)***

Review words

- *variability*
- *mean*

Amps | Featured Activity

Activity 2

Animated Dot Plots

Students see what happens to a dot plot when two outliers are taken out of the distribution.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel overwhelmed by all of the calculations involved with finding the mean absolute deviation (MAD). To simplify the process, have students use their own words to describe what they are doing in each step of the process. Students should note that in each step there is a lot of repetition. In order to attend to the details and not get lost in the process, students must stay very organized. Ask them to identify ways to organize their work on their paper. Remind them that it will take self-discipline to follow through with it.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- For **Activities 1 and 2**, consider making calculators available to reduce the time students spend on calculations.
- In **Activity 1**, display the Activity 1 PDF after students complete Problems 1–3, and then discuss Problems 4–7 as a class.

Independent | 5 min

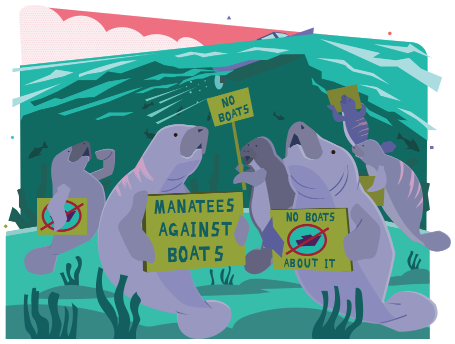
Warm-up Number Talk

Students review strategies for dividing a decimal number by a whole number to build fluency and to prepare them to calculate mean and MAD more efficiently.

Unit 8 | Lesson 13

Variability and MAD

Let's use the mean and mean absolute deviation (MAD) to describe variability.



Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

$42 \div 12 = 3.5$
I know $12 \cdot 3 = 36$, and then I need another 6 for 42, which is half of 12.

$2.4 \div 12 = 0.2$
I know that $12 \cdot 2 = 24$, but I have to move the decimal place once to the left.

$44.4 \div 12 = 3.7$
The dividend is the sum of the dividends in first two problems, so I can add the quotients.

$46.8 \div 12 = 3.9$
I added another 0.2 to the quotient of the third problem.

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the *Number Talk* routine. Reveal one problem at a time and give students 30 seconds of quiet think time for each. Ask them to give a signal when they have a response and a strategy. Keep all problems displayed throughout the activity.

2 Monitor

Help students get started by asking after the second or third expression, as needed, "Do you see any ways the expressions are related that might help you?"

Look for points of confusion:

- **Thinking quotients involving decimals cannot be evaluated mentally.** Ask, "Can you think of a related division expression you could evaluate mentally, or use multiplication, to help you?"

Look for productive strategies:

- Using related expressions, such as thinking of $2.4 \div 12$ as $24 \div 12$ and then understanding how to place the decimal, or recognizing that $42 \div 6$ would be twice as much as $42 \div 12$.
- Applying an understanding of the Distributive Property to separate dividends into two addends, such as how to relate $42 \div 12$ and $2.4 \div 12$ to $44.4 \div 12$.

3 Connect

Display all of the previous expressions for students to refer to as they share their responses and thinking.

Have individual students share their strategies for each expression, one at a time, and before revealing the next expression. Consider gathering only two or three different strategies for the first three expressions, saving most of the discussion time for the last one.

Highlight that students will likely see opportunities to apply some of these same strategies in this lesson.

Power-up

To power up students' ability to determine the sum or difference of decimal values, have students complete:

1. Which of the following is the correct vertical set up to determine the difference of $23 - 1.5$?

   A. $\begin{array}{r} 23 \\ -\ 1.5 \\ \hline \end{array}$ B. $\begin{array}{r} 23.0 \\ -\ 1.5 \\ \hline \end{array}$

2. Calculate the difference of $23 - 1.5$. Show your thinking.

   $\begin{array}{r} 23.0 \\ -\ 1.5 \\ \hline 21.5 \end{array}$

   21.5; 21.5

Use: Before Activity 2.

Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2

Pairs | Small groups | 20 min

Activity 1 Manatee Scars

Students are introduced to the mean absolute deviation (MAD) as one measure of variability, and they use it to compare three distributions with the same center, but different spreads.

Amps Featured Activity Animated Dot Plots

Name: ____________ Date: ________ Period: ________

Activity 1 Manatee Scars

Many manatees bear scars from having narrowly escaped encounters with one of their biggest threats – boats in shallow water. The tables show the number of manatees (out of every 10) spotted with scars over 12 days.

gary powell/Shutterstock.com

1. Record the distance between each data value and the mean at Crystal River.

Manatees with scars (out of every 10)	9	4	5	1	6	7	2	8	3	3	5	7
Distance from mean (5)	4	1	0	4	1	2	3	3	2	2	0	2

Calculate the *average of the distances from the mean* in the table. This value is called the ***mean absolute deviation (MAD)*** of the data set. Show your thinking and round to the nearest tenth.

Crystal River's MAD: 2

$4+1+0+4+1+2+3+3+2+2+0+2=24 \quad 24 \div 12 = 2$

2. Determine the mean absolute deviation of the data from Homosassa River. Show your thinking and round to the nearest tenth.

Number with scars (out of every 10)	2	4	5	4	6	6	4	7	3	4	8	7
Distance from mean (5)	3	1	0	1	1	1	1	2	2	1	3	2

Homosassa River's MAD: 1.5

$3+1+0+1+1+1+1+2+2+1+3+2=18 \quad 18 \div 12 = 1.5$

3. Determine the mean absolute deviation of the data from Chassahowitzka River. Show your thinking and round to the nearest tenth.

Number with scars (out of every 10)	7	8	9	0	1	2	1	3	2	10	9	8
Distance from mean (5)	2	3	4	5	4	3	4	2	3	5	4	3

Chassahowitzka River's MAD: 3.5

$2+3+4+5+4+3+4+2+3+5+4+3=42 \quad 42 \div 12 = 3.5$

1 Launch

Activate prior knowledge by reminding students of their work in Lessons 9 and 12 where they calculated the distance between each data value and the mean. Arrange students in groups of four, consisting of two pairs. Give them 4–5 minutes to complete Problems 1–3 with their partner, and then 8–10 minutes to complete Problems 4–7 as a group. Consider providing calculators if time is an issue.

2 Monitor

Help students get started by asking "How do you calculate an average? Will the average of the distances be equal to the mean of the data set?"

Look for points of confusion:

- **Thinking the MAD should always be equal to 0 because the left and right distances should be equal.** Remind students that distances are always positive, and point out that the table shows all of the values, both left and right, which should all be used together to calculate the MAD.

Look for productive strategies:

- Understanding that the MAD is the average of the distances from the mean and not the average of the data values.
- Relating a greater MAD to greater variability and spread around the mean; and vice versa for a lesser MAD.
- Using the mathematical language of the unit accurately, such as *mean* and *variability* (Problems 5 and 7).
- Using the MAD and the mean together to determine a range of typical values (Problem 6).
- Associating a greater MAD with greater variability and also a less consistency. So, the likelihood of observing any one value, even within the typical range, decreases (Problem 7).

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider using all or some of these supports for this activity:

- Provide students with pre-completed tables for Problems 1–3, so that they can focus on calculating the mean absolute deviations.
- Provide students with a checklist to help them organize their calculations for the mean absolute deviation. For example:
  - » Determine the mean.
  - » Determine the distance from each data value to the mean.
  - » Determine the average of these distances. This is the MAD.
- Clarify how to interpret the line segments above the triangles in the dot plots. Consider annotating parts of the segment below/above the mean as "1 MAD below" and "1 MAD above."

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, display the three dot plots along with the marks for the mean and MAD. Draw students' attention to how the means for all three dot plots were the same, yet the MADs were different. Ask:

- "Which dot plot had the greatest MAD? The least MAD? What does the distribution look like, compared to the others?"
- "For which river are the data values more consistent with the mean? How does the distribution and the measure of variability tell you this information?" The Homosassa River; The distribution is closer to the mean (less spread out). The MAD is the least, compared to the other data sets.

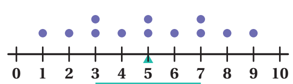
Pairs | Small groups | 20 min

Activity 1 Manatee Scars (continued)

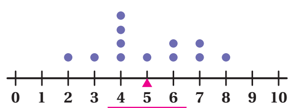
Students are introduced to the mean absolute deviation (MAD) as one measure of variability, and they use it to compare three distributions with the same center, but different spreads.

Activity 1 Manatee Scars (continued)

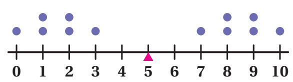
Here are dot plots representing the same data for the scarred manatees. For Crystal River, the mean is represented by a triangle, and the segment below the triangle represents the span of values between one mean absolute deviation less than the mean and one mean absolute deviation more than the mean.



Crystal River



Homosassa River



Chassahowitzka River

4. Add similar marks to the dot plots for Homosassa River and Chassahowitzka River to represent the mean (triangle) and the MAD (segment).

5. Use the mean and MAD to describe and compare the distributions at the three locations.
   Sample response: The mean is the same for all three, but the MAD shows that the data at Homosassa is closer to the mean because it has a lesser MAD than the other two locations. Chassahowitzka River's data are more spread out and so the MAD is greater.

6. Use the mean and the MAD to complete the statements describing typical values for each location:
   - a. At Crystal River, there were typically between **3** and **7** manatees with scars sighted each day.
   - b. At Homosassa River, there were typically between **3.5** and **6.5** manatees with scars sighted each day.
   - c. At Chassahowitzka River, there were typically between **1.5** and **8.5** manatees with scars sighted each day.

7. If you were to observe and count manatees with scars at one sight for one day, at which location do you think you would typically see 7 out of the first 10 manatees with scars? Explain your thinking.
   Answers may vary, but could include either Crystal River or Chassahowitzka River and should include using the MAD to describe what is typical at either location.

3 Connect

Display the Activity 1 PDF for students to check their work for Problems 1–4.

Have groups of students share their responses to Problems 5–7. For Problem 7, expect most students will choose Crystal River or Chassahowitzka, but regardless of their choice, be sure they support their choice with a reasonable, mathematical explanation. Students should understand that, in this context, a greater MAD indicates more variability and less consistency in the number of manatees seen with scars at the corresponding location.

Ask:

- "How are your responses to Problem 6 shown on the dot plots?" They are represented by the segments.
- "What can you say about the MAD for a data set whose data values are all very close to the mean? All very far from the mean?" Close values would have a lesser MAD; Far values would have a greater MAD.
- "What do the MAD values of 2, 1.5, and 3.5 mean in this context?" Numbers of manatees with scars that could be added to or subtracted from the mean to determine a range of typical numbers of manatees with scars seen out of 10.

Highlight that the center of the distribution is not always the only consideration when discussing data. The variability or spread can also influence how to interpret the data. Determining how far away, on average, the data values are from the mean is one way to summarize the amount of variability of a distribution with a single number.

Define:

- The ***measure of variability*** is a way to summarize how the values in a data set vary with a single number.
- The ***mean absolute deviation (MAD)*** is a measure of variability calculated by determining the average of the distances between each data value and the mean.

Pairs | Small groups | 10 min

Activity 2 Human Swimmers

In this activity, students continue to practice interpreting the mean and the MAD and using those values to answer statistical questions.

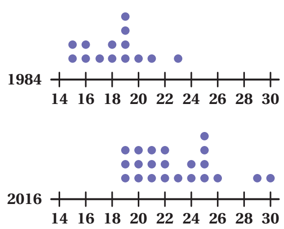
Name: ______________________ Date: __________ Period: __________

Activity 2 Human Swimmers

In 1984, the mean age of swimmers on the U.S. women's swimming team was 18.2 years and the MAD was 2.2 years. In 2016, the mean age of the swimmers was 22.8 years, and the MAD was 3 years.

1. Are the swimmers on the 1984 team closer in age to one another than the swimmers on the 2016 team are to one another? Explain your thinking.
   The 1984 team was closer in age to one another than in 2016, because the MAD for 1984 is less than the MAD for 2016.

2. Here are dot plots showing the ages of the women on the U.S. swimming team in 1984 and in 2016. For both parts, round to the nearest tenth.



Age of swimmers (years)

   a. What percent of the data is within the MAD on both sides of the mean in 1984?
   $18.2 - 2.2 = 16$ and $18.2 + 2.2 = 20.4$; between 16 and 20.4 is $\frac{10}{14} = 71.4\%$

   b. What percent of the data is within the MAD on both sides of the mean in 2016?
   $22.8 - 3 = 19.8$ and $22.8 + 3 = 25.8$; between 19.8 and 25.8. $\frac{16}{22} = 72.7\%$

3. If the 29- and 30-year old swimmers were not on the 2016 team, the mean age of swimmers for that year would be 22.2 years old and the MAD would be 1.9 years. What impact would that have on your answer to Problem 1?
   Sample response:
   The 2016 team has a lesser MAD than the 1984 team, so the remaining members of the 2016 team would be closer in age to one another than in 1984.

STOP

Lesson 13 Variability and MAD 947

1 Launch

Activate background knowledge by asking "What can you say about the average ages of the 1984 team compared to the and 2016 team?" It increased (by 4.6 years).

2 Monitor

Help students get started by asking "What would you expect a dot plot to look like if the ages are close, such as with you and your classmates?"

Look for points of confusion:

- **Having trouble calculating the percentages.** Have students write out an algorithm for calculating percentages. If they cannot remember one, consider referring them to the *Percentage Algorithms* PDF from Unit 3.

Look for productive strategies:

- Recognizing that the greater mean and MAD for the 2016 swimming team means that the team, as a whole, has both become older and more diverse in ages.

3 Connect

Have groups of students share their responses.

Highlight that students should consider both the center and variability of the distribution as ways of thinking about what is typical for a set of data and how consistent the data tends to be.

Ask, "What would you expect the MAD to be for a swimming team with very diverse ages represented? Very little diversity in ages?" A team with very diverse ages would have a higher MAD. A low MAD would mean there would be little diversity in ages.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Prior Knowledge*

Read the introduction and discuss Problem 1 together to help students make sense of the information before beginning the rest of the activity. Display or provide copies of the *Percentage Algorithms* PDF to help them review how to calculate percentages as they complete Problem 2.

Extension: *Math Enrichment*

Have students determine the percent of the data that is within two MADs on both sides of the mean for each year.
1984: 13 out of 14 data values, about 93%.
2016: 20 out of 22 data values, about 91%

Math Language Development

MLR8: Discussion Supports

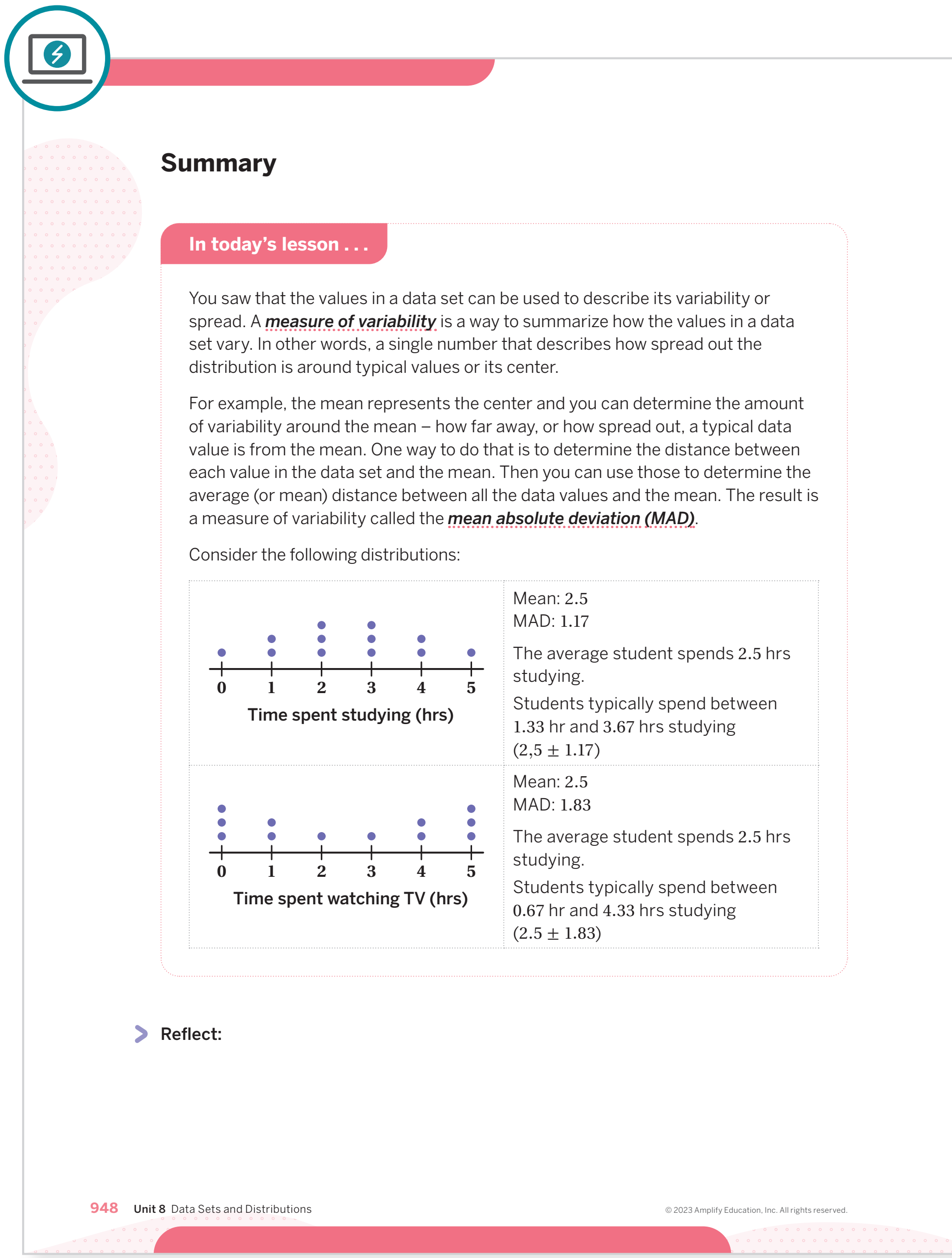
During the Connect, display the following sentence frames to support students as they produce statements about interpreting the mean and the MAD.

- "_____% of the data is within the MAD on both sides of the mean in _____."
- "Over the three decades, the _____ of the swimming team has changed by . . ."
- "The swimmers' ages in the year _____ are closer to one another because . . ."

Whole Class | 5 min

Summary

Review and synthesize how to determine mean absolute deviation (MAD) as a measure of variability around the mean, and what it means relative to a distribution or context.



Summary

In today's lesson . . .

You saw that the values in a data set can be used to describe its variability or spread. A ***measure of variability*** is a way to summarize how the values in a data set vary. In other words, a single number that describes how spread out the distribution is around typical values or its center.

For example, the mean represents the center and you can determine the amount of variability around the mean – how far away, or how spread out, a typical data value is from the mean. One way to do that is to determine the distance between each value in the data set and the mean. Then you can use those to determine the average (or mean) distance between all the data values and the mean. The result is a measure of variability called the ***mean absolute deviation (MAD)***.

Consider the following distributions:

0 1 2 3 4 5 Time spent studying (hrs)	Mean: 2.5 MAD: 1.17 The average student spends 2.5 hrs studying. Students typically spend between 1.33 hr and 3.67 hrs studying (2.5 ± 1.17)
0 1 2 3 4 5 Time spent watching TV (hrs)	Mean: 2.5 MAD: 1.83 The average student spends 2.5 hrs studying. Students typically spend between 0.67 hr and 4.33 hrs studying (2.5 ± 1.83)

Reflect:

Synthesize

Display the Summary from the Student Edition.

Formalize vocabulary:

- ***measure of variability***
- ***mean absolute deviation (MAD)***

Ask:

- "What does the mean absolute deviation (or MAD) tell you about a data set and its distribution?" The average distance between all data values and the mean, which relates to how spread out the data values are.
- "How do you calculate the MAD?"
- "Can the MAD be used to describe the center of a data set?" No, just as mean does not describe variability, you really need both to describe the overall spread of a distribution.

Highlight that both distributions have the same minimum and maximum values and the same mean (2.5 hr), but the MAD for time spent studying is lower than the time watching TV since most of the data is clustered around the center. In general, the MAD tells you how much the values in a data set vary around the center (the mean) of the distribution.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is the mean related to the MAD?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *measure of variability* and *mean absolute deviation (MAD)* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing and comparing the means and MADs of multiple data sets in the context of student travel times.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.13

Ten sixth graders from each of five different countries were asked about their daily travel times to school. The responses were organized by country and the mean and MAD of each data set are shown in the table.

	Mean (minutes)	MAD (minutes)
United States	9	4.2
Australia	18.1	7.9
South Africa	23.5	16.2
Canada	11	8
New Zealand	12.3	5.5

1. Which group of students has the greatest variability in their travel times? Explain your thinking.
   South Africa; Because the travel times for those students had the greatest MAD.

2. Consider the data for New Zealand and Canada.
   a. The mean for New Zealand is close to the mean for Canada. What does that tell you about the travel times of students in those countries?
      The average time it takes students to travel to school in those countries was similar.
   b. The MAD for New Zealand is quite different from the MAD for Canada. What does this tell you about the travel times of students in those two countries?
      The time it takes students in Canada to travel to school varies more, while the time it takes students in New Zealand to travel to school is more consistent. In Canada, times are typically anywhere from 3 to 19 minutes, but, in New Zealand, times are typically between 6.8 and 17.8 minutes.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine the mean absolute deviation (MAD) for a set of data. 1 2 3

b. I understand what the MAD measures and what information it provides. 1 2 3

c. I can use the mean and MAD to compare groups. 1 2 3

Success looks like . . .

- **Language Goal:** Comparing the means and mean absolute deviations of different distributions, specifically those with the same MAD but different means. **(Speaking and Listening, Writing)**
  - » Comparing the MADs of New Zealand and Canada in Problem 2.
- **Language Goal:** Interpreting the mean and mean absolute deviation (MAD) in the context of the data. **(Speaking and Listening, Writing)**
  - » Determining the country with the greatest variability in travel times in Problem 1.

Suggested next steps

If students are confused about what the MAD means in context, consider:

- Having them draw a number line with the mean in the middle and then identifying the range of data values within the MAD on both sides to help them visualize the spread and what the measure describes.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

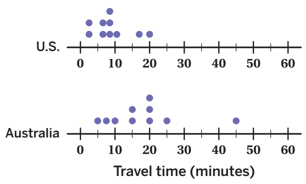
- What worked and didn't work today? The focus of this lesson was determining the MAD. Were students able to understand MAD in the context given?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

1. The dot plots show the amounts of time that 10 U.S. students and 10 Australian students took to get to school. Which statement is true about the MAD of the Australian data set?



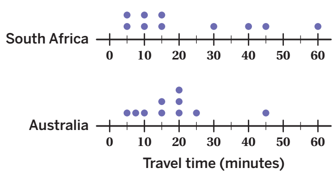
A. It is significantly less than the MAD of the U.S. data set.

B. It is exactly equal to the MAD of the U.S. data set.

C. It is approximately equal to the MAD of the U.S. data set.

D. It is significantly greater than the MAD of the U.S. data set.

2. The dot plots show the amounts of time that 10 South African students and 10 Australian students took to get to school. Without calculating, solve the problems.



a. Which data set has the lesser mean? Explain your thinking.

Australia; Sample response: South Africa's data have more greater numbers which means it will have a greater mean.

b. Which data set has the lesser MAD? Explain your thinking.

Australia; Sample response: South Africa has a value of 60, so the MAD would probably be greater.

c. What does a lesser mean tell you in this context?

The average time it takes for Australian students to get to school is less than the time it takes South African students to get to school.

d. What does a lesser MAD tell you in this context?

Most students in Australia have closer travel times to get to school.

Practice

Name: Date: Period:

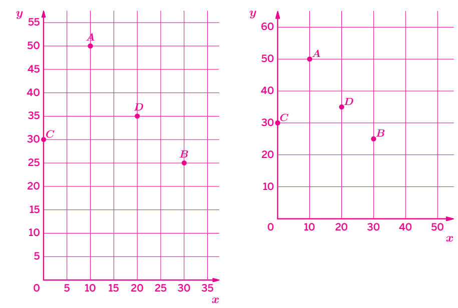
3. Two high school basketball teams have identical records of 15 wins and 2 losses. Sunnyside High School's mean number of points per game is 50 points, with a MAD of 4 points. Shadyside High School's mean number of points per game is 60 points, with a MAD of 15 points.

Which team's scores in all 17 games were the most consistent? Explain your thinking.

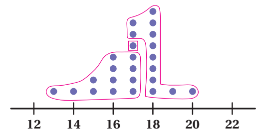
Sunnyside High School; Sample response: The team's points were mostly between 46 and 54.

4. Draw and label an appropriate pair of axes. Then plot and label the points $A(10, 50)$, $B(30, 25)$, $C(0, 30)$, $D(20, 35)$.

Sample responses:



5. The dot plot shows the birth weights, in ounces, of all the puppies born at a kennel in the past month. Determine the median using only the dot plot. Show or explain your thinking.



17; Sample response: There are 25 data values, so I thought about 12 on each side, and the median had to be the 13th number in the dot plot, which is 17.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 7 Lesson 14	1
Formative	5	Unit 8 Lesson 14	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

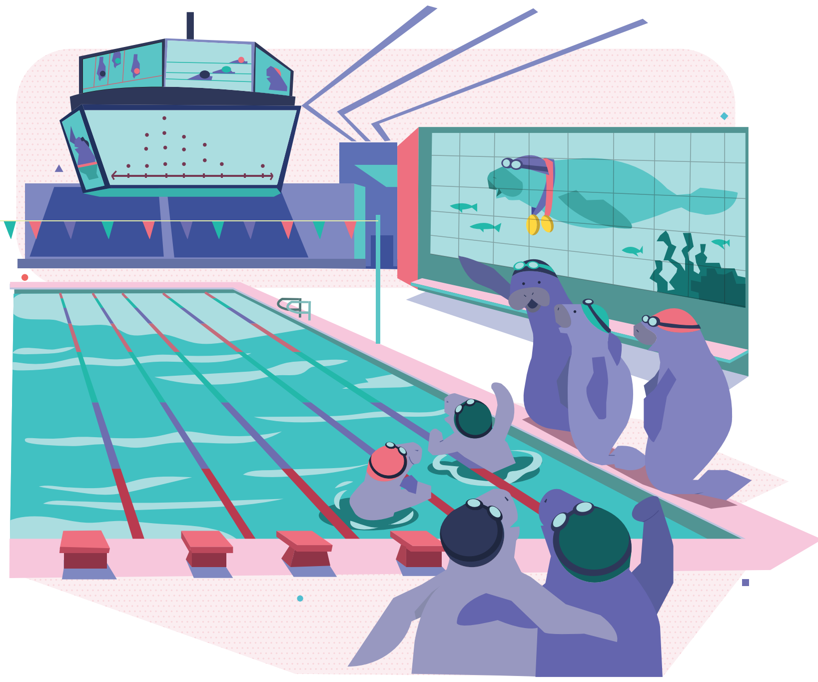
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 8 | LESSON 14

Variability and IQR

Let's use the median and the interquartile range (IQR) to describe variability.



Focus

Goals

1. **Language Goal:** Calculate the range and interquartile range (IQR) of a data set and interpret what they tell about a scenario. **(Speaking and Listening, Writing)**
2. **Language Goal:** Comprehend the interquartile range (IQR) as another measure of variability, which describes the span of the middle half of the data. **(Writing)**
3. **Language Goal:** Identify and interpret the numbers in the five-number summary for a data set: the minimum, first quartile (Q1), median (Q2), third quartile (Q3), and the maximum. **(Writing)**

Coherence

Today

Students expand on their understanding of median to split data into quarters by determining three values called *quartiles*. They relate the three quartiles to the 25th, 50th, and 75th percentiles, which are useful in describing a distribution. Students also identify the maximum and minimum values of the data set, and combining those with the quartiles, they can identify the five-number summary. Students also explore two more measures of variability for a distribution — the range and the interquartile range (IQR) — as two other ways to describe its spread and summarize its variability with a single number. Students interpret what the IQR, as the middle half of the data, tells them about a scenario in context.

Previously

In Lesson 10, students decomposed a data set into two halves by identifying the median. In Lesson 13, they also determined the MAD, as one measure of variability.

Coming Soon

In Lesson 15, students will use the five-number summary of a data set to construct another representation of the distribution — a box plot.

Rigor

- Students further their **conceptual understanding** of measures of variability.
- Students build **procedural skills** for constructing box plots, as well as calculating and representing IQR.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (answers, for display)

Math Language Development

New words

- *quartile*
- *five-number summary*
- *range**
- *interquartile range (IQR)*

Review words

- *variability*
- *median*

*Students may confuse the statistical term *range* with the various everyday uses of the term. Be ready to address the similarities and differences between them.

Amps | Featured Activity

Activity 1

Five-Number Formative Feedback

Students receive immediate feedback about their calculations so that they can move forward with correct values.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might not adequately analyze the situation in order to generate a five-number summary. The process requires both quantitative and abstract reasoning. Remind students that generating the summary is a matter of calculating the quartiles. Encourage them to identify the problem by listing what they need to find for the summary. Then the process of interpreting those values requires processing of a more abstract nature. By beginning with the end in mind, students can discern the information they need to identify and interpret the five-number summary of a data set.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- **Activity 1** can be done as a whole class, and Problem 4 may also be omitted.
- In **Activity 2**, discuss Problems 4–6 as a whole class, providing the ordered data sets (or even the five-number summary) for students in Problem 6.

Pairs | 5 min

Warm-up Notice and Wonder

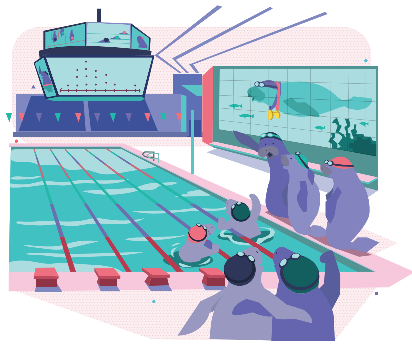
Students study two distributions that look very different but have the same MAD, demonstrating the need for another way to quantify and compare variability.

Name: ______ Date: ______ Period: ______

Unit 8 | Lesson 14

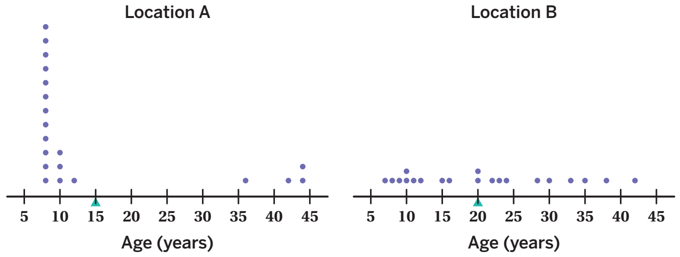
Variability and IQR

Let's use the median and the interquartile range (IQR) to describe variability.



Warm-up Notice and Wonder

The two dot plots show the ages of 20 manatees from 2 different locations. The mean of each data set is marked with a triangle.



1. I notice . . .
   Sample responses:
   - There are no values equal to the mean in A but there are in B.
   - The data are more consistent in data set B
   - The least and greatest values are basically the same for both data sets.

2. I wonder . . .
   Sample responses:
   - Why are there so many 8s?
   - Do those data values in the 40s of set A affect the data?
   - Why is there such a huge gap in set A? What context could make the data look like that?

Log in to Amplify Math to complete this lesson online.

1 Launch

Use the *Notice and Wonder* routine.

2 Monitor

Help students get started by asking "How would you describe the distribution for Location A? Location B?"

Look for points of confusion:

- **Thinking that one of the means must be incorrect.** Remind students how to calculate the mean.

Look for productive strategies:

- Using the mathematical language of the unit in their observations, such as *peak*, *gap*, *cluster*, *typical*, *center*, *spread*, *variability*, *mean*, and *MAD*.
- Noticing that both sets of data have similar minimum and maximum values, but the data points in between are distributed very differently.
- Connecting or comparing the two locations in their observations and questions.

3 Connect

Display the two dot plots.

Have students share their responses, recording them around the plots if possible. Have others share agreement or disagreement, and alternative ways of thinking.

Ask:

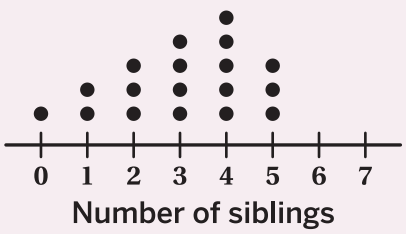
- What does a MAD of 10.5 years tell you in this context?"
- "Is the MAD a useful description of variability for Location A? What about Location B?"

Highlight that the MAD is a way to summarize the variability relative to the mean, but depending on the distribution, it may not be the best measure of variability.

Power-up

To power up students' ability to determine the median for a data set represented by a dot plot, ask:

Recall that the median is the middle value in a data set when the values are listed in order.



The dot plot shows the number of siblings each student has in a class. What is the median of this data set? 3

Use: Before Activity 1.

Informed by: Performance on Lesson 13, Practice Problem 5.

Pairs | 15 min

Activity 1 The Five-Number Summary

Students are introduced to quartiles and the five-number summary for a data set, identifying and interpreting those values, and preparing for the IQR in the next activity.

Amps Featured Activity Five-Number Formative Feedback

Activity 1 The Five-Number Summary

You have seen data sets that are not symmetric, have a wide spread, or have outliers, and, for those, the median is a good choice of center. But because the MAD corresponds to mean, what about describing and summarizing variability for those types of distributions? And what about the different reasons behind why the data may look like that?

Statisticians deal with these questions and issues all the time, because the reality is, reality is messy! Statistician Mary C. Christman, who has served as an advisor to the Florida Fish and Wildlife Commission Research Institute, has spent part of her career addressing exactly that. Collecting environmental data, such as about manatees, is not easy and has many challenges.

Here are the ages of twenty manatees from Location B, ordered from least to greatest. Use the data set to complete the problems, and think about how your work is dealing with the relatively wide spread of values in this data set.

7 8 9 10 10 | 11 12 15 16 20 | 20 22 23 24 28 | 30 33 35 38 42

Minimum Q1 Q2 Q3 Maximum

1. Circle the least data value and label it *Minimum*. Then circle the greatest data value and label it *Maximum*.

2. Determine the following values. Mark the position of each value in the data and label each as indicated.

	Value	Mark	Label
Median	20	♦	Q2
Middle value of the *lower half* of the data	10.5	↑	Q1
Middle value of the *upper half* of the data	29	↑	Q3

Featured Mathematician

Mary C. Christman

Mary C. Christman holds a BS in Biology from the University of Pennsylvania, an MS in Marine Biology and Physical Oceanography from the University of Delaware, and a PhD in Mathematical Statistics from George Washington University. She is currently the owner of MCC Statistical Consulting, which specializes in collecting and representing environmental and ecological data. She has advised the Florida Fish and Wildlife Commission Research Institute on the coastal ecosystems of Florida and the effects of the "red tide" phenomenon on both humans and sea life, including manatees.

1 Launch

Say, "You will work with the data from Location B in the Warm-up to determine alternative ways of summarizing the spread with numbers. Be sure to think about what each value represents both in the data and in context." Give pairs 8–10 minutes to complete the activity.

2 Monitor

Help students get started by asking "How can you determine the median of this set of data?"

Look for points of confusion:

- **Not knowing whether to include the 20s when calculating Q1 and Q3 (Activity 1).** Explain to students that when the median is not one of the data values, i.e., when it is the average of the middle two values, the two values are included with their lower and upper halves.
- **Not knowing what to do when there is an even number of values to the left or right of the mean.** Remind students that Q1 and Q3 are the medians of that half of the data. Consider covering one of the halves so students can see only the half of the data they are working with.

Look for productive strategies:

- Recognizing that the minimum and maximum are included in the lower and upper sections of data points.
- Understanding that Q1 and Q3 are determined as if they are the medians of the two halves of data around Q2.
- Analyzing and interpreting what the data between each of the quartiles represents, and, specifically understanding that, while 29 is not a data value, it is included in statements about both the lower 75% and upper 25% of data (and, possibly, noting the same is true for 20 being part of both the upper and lower halves).

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

In Problem 2, annotate the value marking Q1 with "one quarter of the manatees are 10.5 years old or younger" to help students visualize the data set divided into fourths.

Extension: *Math Enrichment*

Have students compare the five-number summaries for the three dot plots from Lesson 13, Activity 1. They have the same medians, but different quartiles and extreme values. Q1 and Q3 are closer together for the Crystal and Homosassa Rivers and farther apart for the Chassahowitzka River.

	Min	Q1	Q2	Q3	Max
Crystal River	1	3	5	7	9
Homosassa River	2	4.5	5	6.5	8
Chassahowitzka River	0	1.5	5	8.5	10

Featured Mathematician

Mary C. Christman

Have students read about Mary C. Christman, who uses statistics to collect and represent environmental and ecological data.

Pairs | 15 min

Activity 1 The Five-Number Summary (continued)

Students are introduced to quartiles and the five-number summary for a data set, identifying and interpreting those values, and preparing for the IQR in the next activity.

Name: ______ Date: ______ Period: ______

Activity 1 The Five-Number Summary (continued)

Look back at the ordered list of data on the previous page, now with the marks and labels. The data set has been split into four equal parts from the minimum to the maximum. The three values labeled Q1, Q2, and Q3 that divide the data are called *quartiles*.

- The *first quartile* represents an upper bound for the lowest 25% of the data, which is always contained between the minimum and Q1. It is also referred to as the *25th percentile*. Q1 is also a lower bound for the highest 75% of the data.
- The *second quartile* corresponds to the *median*, and it represents an upper bound for the lowest 50% of the data, which is always contained between the minimum and Q2. It is also referred to as the *50th percentile*. Q2 is also a lower bound for the highest 50% of the data.
- The *third quartile* represents an upper bound for the lowest 75% of the data, which is always contained between the minimum and Q3. It is also referred to as the *75th percentile*. Q3 is also a lower bound for the highest 25% of the data.

Together, these five numbers — minimum, Q1, Q2, Q3, maximum — make up what is called the *five-number summary* for a data set.

3. Record the *five-number summary* for data representing the ages of the manatees.

   Minimum: 7 **Q1:** 10.5 **Q2:** 20 **Q3:** 29 **Maximum:** 42

4. What does the value of the third quartile (Q3) tell you about the ages of the manatees at this location?

   Sample response: The youngest 75% of the manatees are 29 years old or younger. This also means that the oldest 25% of the manatees are 29 years old or older.

3 Connect

Display the Activity 1 PDF for students to check their responses from Problems 1–3.

Ask:

- "How did you determine where to mark Q1, Q2, and Q3 for this data set?" Note: This discussion should focus on how to work with an even number of data values and particularly what to do with the two numbers used to determine the median when determining Q1 and Q3. Activity 2 will present problems that are the opposite of this (i.e., the median is the middle value because it is an odd number of data values).
- "How do these five numbers help you understand the distribution and spread of the data? Because each section of the data contains (at least) 25% of the values, the closer a pair of numbers in the summary is, the more clustered the data values in that range; and vice versa.

Define:

- A ***quartile*** as one of three numbers (Q1, Q2, Q3) that divide a data set into 4 sections so that each contains the same number of data values.
- The ***five-number summary*** for a data set summarizes a distribution by five specific values: its minimum, first quartile, median, third quartile, and maximum.

Have individual students share their responses to Problem 4.

Highlight that the five-number summary helps describe a data set without listing or showing every value. It summarizes the data by dividing it into four equal parts, or quartiles, with the median determining the middle point of the data. The closer the values that bound a section of the data, the more of a cluster the data points in that range represent.

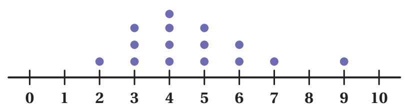
Pairs | 15 min

Activity 2 Range and Interquartile Range

Students determine two other measures of variability for data sets – range and interquartile range (IQR) – and they use IQR to describe variability in context.

Activity 2 Range and Interquartile Range

Here is a dot plot that shows 15 recorded speeds of a manatee, in miles per hour.



1. Write the five-number summary for this data set. Show your thinking.

   Minimum: 2 Q1: 3 Q2: 4 Q3: 6 Maximum: 9

2	3	3	3	4	4	4	4	5	5	5	6	6	7	9
Min			Q1				Q2				Q3			Max

2. One way to describe the *spread* of values in a data set is to look at the difference between the maximum and minimum values. This is called the ***range***. What is the range of the speeds of the manatee?

   7 mph, because $9 - 2 = 7$.

3. Another way to describe the spread of values in a data set is to look at the difference between the upper quartile (Q3) and the lower quartile (Q1). This is called the ***interquartile range (IQR)***.

   a. What is the interquartile range (IQR) of this manatee's speeds?

   3 mph, because $6 - 3 = 3$.

   b. How does the IQR relate to typical values?

   Sample response:

   Because 50% of the data is between Q1 and Q3, which are also the boundaries of the IQR, it would make sense to say typical values are clustered in a "range" that is 3 units wide, and, more specifically, between 3 mph (Q1) and 6 mph (Q3).

954 Unit 8 Data Sets and Distributions

1 Launch

Activate background knowledge by asking, "How is this data set different?" There is an odd number of data values.

2 Monitor

Help students get started by asking "How would you identify the quartiles?" List the values of all the data values in order and then count off to determine the median, and then do the same for Q1 and Q3.

Look for points of confusion:

- **Not knowing when to include certain values when determining quartiles.** Have students review the data set from Activity 1 and ask, "For Q1 and Q3, did you include the two values used to determine the median? Why?" Then have them look at the current data set and ask, "Is the median an average of two points or is it the data value?" Consider having students mark the median in the data set, whether it is a value or is between two values, in order to visually separate it from the lower and upper halves.

Look for productive strategies:

- Accurately identifying the five-number summaries, and using those to calculate ranges and IQRs.
- Associating the range of values around the median between Q1 and Q3 with typical values and with 50% of the data. Some students may also relate this range of values to the range of values within one MAD of the mean.
- Recognizing that a dot plot with less spread and more points clustered around the center will have a lesser IQR, and vice versa.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider displaying or providing a checklist similar to the one shown to help students organize their thinking around determining the five-number summary.

- Determine the minimum and maximum.
- Determine the median (Q2) — the middle of the data set.
- Determine Q1 — halfway between the minimum and Q2.
- Determine Q3 — halfway between the maximum and Q2.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the dot plots from Problem 4 and annotate which dot plot shows the lesser range and which dot plot shows the lesser IQR. Draw students attention to the connections between the visual distribution of the data values on the dot plots and the comparisons of their ranges and IQRs. Have students complete these statements.

- "A distribution with a _____ range means the entire data set will be more spread out than a data set with a _____ range."
- "A distribution with a _____ IQR means the middle half of the data values will be closer together than a data set with a _____ IQR."
- "The IQR is/is not affected by extreme values or outliers because . . ."

Pairs | 15 min

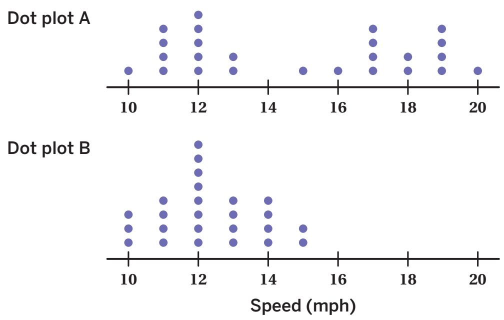
Activity 2 Range and Interquartile Range (continued)

Students determine two other measures of variability for data sets – range and interquartile range (IQR) – and they use IQR to describe variability in context.

Name: ________ Date: ________ Period: ________

Activity 2 Range and Interquartile Range (continued)

4. Here are two more dot plots showing the speeds of two other manatees.



Without doing any calculations, predict:

a. Which data set has the lesser range? **Dot plot B**

b. Which data set has the lesser IQR? **Dot plot B**

5. Check your predictions by calculating the ranges and IQRs for the data in each dot plot.

Data set A:

10 11 11 11 11 12 12 12 12 12 13 13 15 16 17 17 17 17 18 18
19 19 19 19 20

Range: 10 **IQR:** 6

Data set B:

10 10 10 11 11 11 11 12 12 12 12 12 12 12 12 13 13 13 13 14
14 14 14 15 15

Range: 5 **IQR:** 2.5

STOP

Lesson 14 Variability and IQR 955

3 Connect

Have pairs of students share their responses, briefly ensuring that students know how to determine range and IQR. Then focus the discussion on how these measures relate to the shape of a distribution, such as on a dot plot, and interpreting the information they provide in context.

Define:

- The ***range*** of a data set as a measure of variability that is calculated as the difference between the maximum and minimum values in the data set.
- The ***interquartile range (IQR)*** of a data set as a measure of variability that is calculated as the difference between the third quartile (Q3) and the first quartile (Q1).

Ask, as many as time permits:

- "What does a range of 7 mph tell you about the speeds of these manatees?" The slowest and fastest manatees' speeds differed by 7 mph. The greatest difference in speeds was 7 mph.
- "What does an IQR of 3 mph tell you about the speeds of these manatees?" The speeds of the most typical half of manatees are all within 3 mph.
- "In general, what does a greater range tell you? A greater IQR?" A wider overall spread in the data. More variability in the data set.
- "Can a data set have a large range and a small IQR?" Yes, for a data set with most points in one big cluster but a few points very far away from those.
- "Which is more informative about the variability of a single data set, the range or the IQR? Would you say the same or different for comparing data sets?" For both one data set or comparing two, the IQR is generally more informative, because outliers affect range. But for symmetric, single peak/cluster distributions, the range could be more informative.

Highlight how the range and IQR are represented in a dot plot and the five-number summary, and that the range encompasses 100% of the data and the IQR encompasses 50% of the data.

Whole Class | 5 min

Summary

Review and synthesize that range and IQR are both measures of variability, such as the MAD, but are centered around the median instead of the mean.

Summary

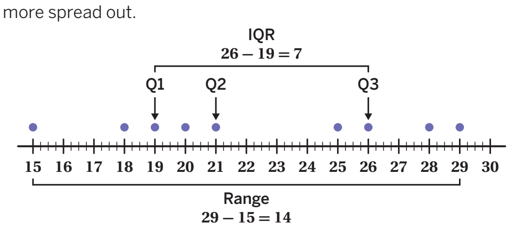
In today's lesson . . .

You saw how to calculate the ***five-number summary*** for a data set, which can be used to summarize its distribution. The *five-number summary* consists of the minimum, maximum, and the three quartiles, Q1, Q2, and Q3.

The ***first quartile (Q1)*** is the median of the lower half of the data.	The ***second quartile (Q2)*** is the median of the entire data set.	The ***third quartile (Q3)*** is the median of the upper half of the data.

You used the *five-number summary* to calculate two measures of ***variability*** that can be used to describe the distribution of a data set in terms of its spread.

- One measure, the ***range***, represents the difference between the maximum and minimum values of a data set.
  - The range gives you a basic overall sense of how spread out the data is, but it does not tell you about variability and how it is distributed between the minimum and maximum values.
- The other measure is called the ***interquartile range (IQR)***, which represents the range of the middle 50% of the data.
  - A greater IQR indicates more variability because the middle 50% of the data is more spread out.



Reflect:

Synthesize

Ask:

- "What are the quartiles for a numerical data set?" Numbers that show where you can divide the data set, so that the data are in quarters or fourths.
- "What is the relationship between the quartiles and the median?" The second quartile is also the median.
- "What is the interquartile range (IQR)? What does it mean?" The IQR is the difference between the third and first quartile. It is a measure of the variability or spread of the data. It tells the range of the middle half of the data.
- "Compare the MAD and IQR. How are they alike? How are they different?" They both provide information on the variability of a set of data. The MAD is based on the mean, while the IQR is based on the median. The MAD considers all the data values and tells the average distance between each data value and the mean, while the IQR focuses on the middle half of the data and tells how close or spread out the middle half of the data is.
- "When might the IQR be a better measure of variability than the MAD?" Whenever the median is the more appropriate measure of center, meaning the distribution is not symmetric or contains a small number of outliers.

Highlight that the range and the IQR are measures of variability, similar to the MAD, but all three provide different information. The MAD and the range are both sensitive to outliers, but the IQR is not. The MAD and the IQR both indicate the spread, but around different centers.

Formalize vocabulary:

- *quartile*
- *five-number summary*
- *range*
- *interquartile range (IQR)*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is the thinking you did today different from the thinking you did in the previous lesson?"
- "How is the IQR different from the MAD?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *quartile, five-number summary, range,* and *interquartile range* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the median and IQR and what it means in context.

Printable

Name: ______________ Date: ________ Period: ________

Exit Ticket 8.14

Diego wanted to know how far sixth graders could throw a dodgeball, so he asked 10 sixth graders to throw a dodgeball as far as they could and he measured the distances, in feet, from where the ball was thrown to where it first landed. The data he recorded is shown.

40	76	40	63	47	57	49	55	50	53

1. Determine the median and IQR of the data set.

40 40 47 49 50 53 55 57 63 76

Q1 (47), 51.5, Q3 (57)

Median: 51.5 ft

IQR: 10ft; 57 − 47 = 10

2. On another day, he asked the same group of students to throw a basketball and collected data the same way as he did for the dodgeball. The median distance of the basketball throws was 49 ft, with an IQR of 6 ft.

   a. Overall, did this group of sixth graders throw the basketball farther than the dodgeball? Explain how you know.

   No, they did not throw the basketball farther. I know this because the median is less, which means there were more shorter throws pulling the typical distance down.

   b. Overall, for which type of ball were the distances thrown by this group of sixth graders more consistent? Explain how you know.

   The distances for the basketball were less varied because the IQR is less, which means the distances were less spread out or more consistent.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I know what the quartiles and the interquartile range (IQR) measure and what each tells me about the data.
1 2 3

b. When given a list of data values or a dot plot, I can determine the quartiles and the interquartile range (IQR) for the data.
1 2 3

Success looks like . . .

- **Language Goal:** Calculating the range and interquartile range (IQR) of a data set and interpreting what they tell about a scenario. **(Speaking and Listening, Writing)**
  » Determining the range and IQR of the distances the dodgeball is thrown in Problem 1.
- **Language Goal:** Comprehending the interquartile range (IQR) as another measure of variability, which describes the span of the middle half of the data. **(Writing)**
- **Language Goal:** Identifying and interpreting the numbers in the five-number summary for a data set: the minimum, first quartile (Q1), median (Q2), third quartile (Q3), and the maximum. **(Writing)**

Suggested next steps

If students do not first place the data in numerical order, consider:

- Asking, "To determine the median, what must be true about the data set?"

If students have difficulty determining the median, consider asking:

- "How many data values are in this set?" 10
- "If you divide it in half, how many values should be on each side?" 5
- "What value is between the lower five values and the upper five values?" I need to find the average the fifth and sixth data values, which are 50 and 53.

If students think that Q1 and Q3 are averages of two numbers because they think that the values 50 and 53 are not used to calculate them, consider:

- Color coding the data sets in Activity 1 and Activity 2, Problem 1 to compare the two procedures.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

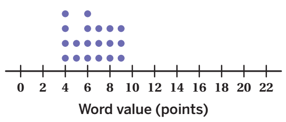
- What worked and didn't work today? What did determining the five-number summary and IQR reveal about your students as procedural learners?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent

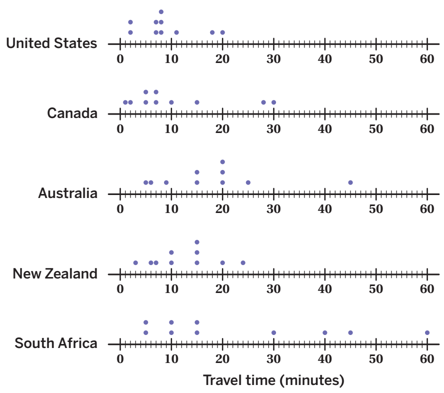
Name: ______________ Date: ________ Period: ________

1. In a word game, 1 letter is worth 1 point. This dot plot shows the scores for 20 words.



   a. What is the median score? 6 points
   b. What is the first quartile (Q1)? 5 points
   c. What is the third quartile (Q3)? 8 points
   d. What is the interquartile range (IQR)? 3 points

2. The 5 dot plots show the travel times to school of 10 sixth graders in 5 countries. Match each dot plot with the correct median and IQR by writing the country's name next to each set of statistics.



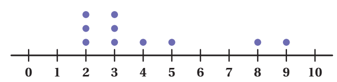
   a. Median: 17.5, IQR: 11 Australia
   b. Median: 15, IQR: 30 South Africa
   c. Median: 8, IQR: 4 United States
   d. Median: 7, IQR: 10 Canada
   e. Median: 12.5, IQR: 8 New Zealand

Practice

Name: ______________ Date: ________ Period: ________

3. Mai and Priya each bowled 10 games and recorded their scores. Mai's median score was 120, with an IQR of 5. Priya's median score was 118, with an IQR of 15. Whose scores had less variability? Explain how you know.
   Mai's scores probably had less variability; Sample response: Mai's IQR is 5 and Priya's IQR is 15 and 5 < 15.

4. Here is a dot plot that represents a data set. Explain why the mean of the data set is greater than its median.



   Because the distribution is not symmetric and the majority of the data values are clustered around 2 to 3, the median will be closer to that range. There are fewer points greater than 3, but two outliers, 8 and 9, will increase the mean to a value greater than the median.

5. Two TV shows each asked 100 viewers for their ages. For Show X, the mean age of the viewers was 25 years old, with a MAD of 15 years. For Show Y, the mean age of the viewers was 20 years old, with a MAD of 5 years. A sixth grader watches one of the two shows. Which show do you think the student most likely watches? Explain your thinking.
   Show X; Sample response: A sixth grader is most likely 11 or 12 years old, and typical viewers within one MAD of the mean for Show X would be 10 to 40 years old, while, for Show Y, they would be 15 to 25 years old.

6. Write a scenario that could be represented by this number summary.
   Minimum: 5 **Median:** 7.5 **Maximum:** 9
   Sample responses:
   - **The length, in inches, of the feet of a family.**
   - **The travel time, in minutes, for students to go to school.**
   - **The number of hours spent on watching TV during one week**

Practice

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 8 Lesson 11	2
	5	Unit 8 Lesson 13	2
Formative	6	Unit 8 Lesson 15	2

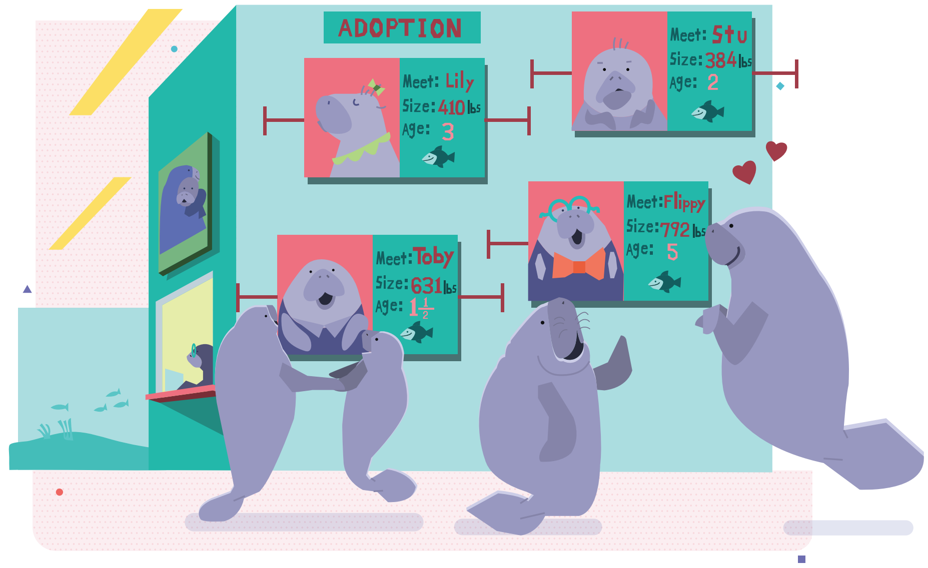
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Box Plots

Let's explore how box plots can summarize distributions.



Focus

Goals

1. Create a box plot to represent a data set.
2. **Language Goal:** Describe the parts of a box plot that correspond with each number in the five-number summary, the range, and the IQR of a data set. **(Speaking and Listening)**
3. **Language Goal:** Compare and contrast box plots that represent different data sets. **(Speaking and Listening, Writing)**
4. **Language Goal:** Interpret a box plot to answer statistical questions about a data set. **(Speaking and Listening)**

Coherence

Today

Students use the five-number summary to construct a new type of statistical data display – a box plot. They are first shown a box plot, without knowing what it is, but given a context, are asked to try and make sense of what it may be representing. Then the whole class creates a box plot through a kinesthetic activity in which each student represents a data value. Students then reproduce the class box plot on paper and use it to interpret the structure of the representation. Through these exercises, they develop an understanding of how a box plot represents and summarizes the distribution of a data set in terms of both center and spread. Students compare both typical values and variability in data sets using their box plots, and also explain the meaning of all types of interpretations in context.

Previously

In Lesson 14, students identified the five-number summary of a data set, and then calculated two measures of variability — range and interquartile range.

Coming Soon

In Lesson 16, students will compare the usefulness of the representations of MAD and IQR relative to different types of distributions of data.

Rigor

- Students continue to build **conceptual understanding** for interpreting data, now using box plots.
- Students **apply** their understanding of the five number summary to construct box plots.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Whole Class	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Activity 1 PDF, one per student
- Activity 1 PDF, *Instructions*
- Activity 1 PDF, *Data,* one per student
- Activity 2 PDF (answers, for display)
- index cards
- markers
- tape (thin tape and wide tape for the floor)
- scissors
- straightedges

Math Language Development

New word

- ***box plot***

Review words

- *quartile*
- *range*
- *interquartile range*
- *median*
- *variability*

Amps | Featured Activity

Activity 1

Interactive Box Plots

Students are able to create and manipulate digital box plots, seeing the immediate effect on the data values, and vice versa.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Some students may make decisions based on their individual thoughts and preferences rather than thinking about what is good for the whole class in Activity 1, but as they use the structure of the five-number summary to create a living box plot, it is critical that they make decisions based on what is good and safe for the entire class. If students are not acting responsibly, pause and ask them to reflect on their choices before returning to the activity.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- **Activity 1** can be done without students physically making the dot plot. As an alternative, the cards could still be used, but taped on the board for the box plot to be drawn around them.
- In **Activity 2,** display the data and have small groups complete Problem 2. Then display the Activity 2 PDF and have small groups complete Problems 4 and 5.
- In **Activity 2,** if you have *extra time,* consider having students make all three box plots instead of just one each, for practice.

Pairs | 5 min

Warm-up Notice and Wonder

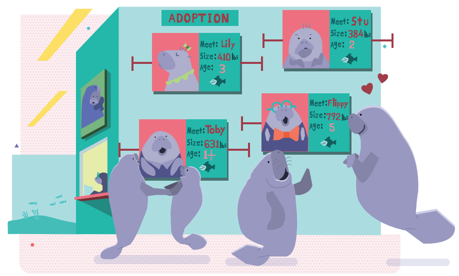
Students explore a box plot (likely) for the first time, with no other information besides the number line labeled "hours of sleep".

Name: ____________ Date: ________ Period: ________

Unit 8 | Lesson 15

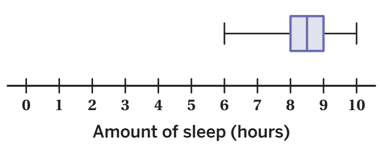
Box Plots

Let's explore how box plots can summarize distributions.



Warm-up Notice and Wonder

Several students were asked how much sleep, in hours, they usually receive on a school night. Here is a representation of the data collected. What do you notice? What do you wonder?



1. I notice . . .
   Sample responses:
   - The graph is drawn between 6 and 10.
   - The box or two boxes is/are between 8 and 9.
   - It is not a number line.

2. I wonder . . .
   Sample responses:
   - Whether the rectangle is one box cut in half or two boxes?
   - Does the graph represent the five-number summary?
   - Should there be a vertical axis?

Log in to Amplify Math to complete this lesson online.

1 Launch

Display the Warm-Up PDF. Students may reference it as they work and during the class discussion. Conduct the *Notice and Wonder* routine.

2 Monitor

Help students get started by asking "What is the first thing that you think of when you look at the graph?"

Look for productive strategies:

- Relating the structure of the box plot to the number line and to the context of hours of sleep.
- Using the mathematical language of the unit to relate parts or features of the box plot to prior work in the unit (e.g., the box from 8 – 9 hours is in the center, or this represents data that vary from 6 – 10 hours of sleep).

3 Connect

Have individual students share their responses, focusing on those who make connections to prior work in the unit, and especially anyone who notices there are five critical values.

Ask, if insufficient related responses have been shared, "How might this relate to any (other parts) of the work you have been focusing on during this unit?"

Highlight that this is another representation that students can use to interpret information about data sets and distributions. It is called a ***box plot***. Box plots are the focus of this lesson, and will be defined in the next activity after students physically participate in seeing how one is constructed.

Power-up

To power up students' ability to connect a data summary to a real-world scenario, have students complete, ask:

The number summaries of three different data sets are given. Match each number summary to the scenario that it could represent.

a. Minimum: 3.5
Median: 7.8
Maximum: 11

__b__ The height, in feet, of the players on an NBA team.

b. Minimum: 6
Median: 6.5
Maximum: 7.75

__c__ The height, in feet, of the trees in a national park.

c. Minimum: 165
Median: 220
Maximum: 380

__a__ The weight, in pounds, of the babies in a hospital nursery.

Use: Before Activity 2.

Informed by: Performance on Lesson 14, Practice Problem 6.

Whole Class | 15 min

Activity 1 Living Box Plot

Students explore a new representation of data kinesthetically: by creating a human box plot to represent data on the lengths of manatee names.

Amps Featured Activity Interactive Box Plots

Activity 1 Living Box Plot

Conservation groups have been monitoring manatees that return to the same locations year after year, noting migration habits, births, and deaths. Many of these returning visitors have been given names, and they can be identified by their markings as well as their personalities (one named Howie apparently likes to tip over researchers' canoes).

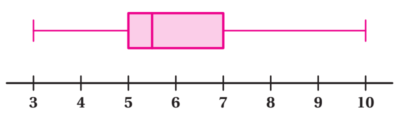
You will be given the names of 24 manatees from Blue Spring State Park that are up for "adoption," and an ordered list of values representing the lengths of their names.

1. Determine the five-number summary for the data set representing the lengths of the manatees' names.

   Minimum: 3 **Q1:** 5 **Q2:** 5.5 **Q3:** 7 **Maximum:** 10

Your class will make a ***box plot*** (sometimes called a *box-and-whisker plot*) to represent the distribution of the lengths of the manatees' names.

2. Record the completed box plot here.



3. Using your box plot to represent the length of manatee names data, determine the percentages of the data represented by each of these elements of the box plot.

   a. The left whisker: 25%

   b. The box: 50%

   c. The right whisker: 25%

960 **Unit 8** Data Sets and Distributions

1 Launch

Provide each student a copy of the Activity 1 PDF, *Data*. Give students 3–4 minutes to complete Problem 1 independently, and then pause for a discussion, being sure the class comes to a consensus for the five-number summary. Then use the Activity 1 PDF, *Instructions* to guide the class through making a physical box plot representation of the data. Once the box plot is complete, display the answers of the Activity 1 PDF for students to reference for the remainder of the Activity.

2 Monitor

Help students get started by asking "How can you determine the median (Q2) for this set of data?"

Look for points of confusion:

- **Incorrectly determining the quartiles (Problem 1).** Remind students that median is a quartile and the process for determining any of the quartiles is similar.
- **Not knowing how to complete Problem 3 using percentages.** Ask, "What percentage does the IQR represent? The whiskers?"

Look for productive strategies:

- Interpreting the sections of the box plot.

3 Connect

Display a ***box plot*** as a representation of the five-number summary for a numerical data set, positioned above a number line.

Have individual students share (optionally) their responses to Problem 1 again, quickly, and label the names of each value in the five-number summary on the box plot. Then have students share their responses to Problem 3.

Highlight how a box plot represents an entire data set using the five-number summary, and the other information noted on the Activity 1 PDF.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create and manipulate digital box plots, seeing the immediate effect on the data values, and vice versa.

Accessibility: *Guide Processing and Visualization*

Annotate the completed box plot in Problem 2 by recording the data values above each section. This will help students visualize how the length of each section indicates how close the data values are to each other, not the number of data values.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the completed box plot and annotate where the five-number summary values are located. Then annotate the percentages from Problem 3 on their respective locations of the box plot. Draw students' attention to the length of each whisker or box and what the length tells them about the data. Ask:

- "The right whisker is longer than the left whisker. Does this mean there are more data values from 7 to 10 than there are from 3 to 5? Explain your thinking."
- "The left side of the box is much smaller than the right side. What does this tell you?"

Small Groups | 15 min

Activity 2 Sea Turtles

Students practice creating box plots to represent data sets, and then interpret and compare the information shown for three distributions, particularly the variability (IQRs).

Name: ______________ Date: ________ Period: ________

Activity 2 Sea Turtles

Here are the data collected from weighing, in pounds, 11 different specimens for each of 3 different species of sea turtles that nest at the Outer Banks of North Carolina.

Chatchantharangsee/Shutterstock.com

Hawksbill (*Eretmochelys imbricata*; IUCN Critically Endangered)

210.75	155.5	100	120	235	180	190.25	198	200	145	165

Olive Ridley (*Lepidochelys olivacea*; IUCN Vulnerable)

98	87.5	85	90	90	90	100	92.75	95	90.25	91

Loggerhead (*Caretta caretta*; IUCN Vulnerable)

150	295	180	185	200	185.5	215	220.25	225	245	160

1. As a group, decide which turtle each of you will focus on. Write the five-number summary for the data for that turtle and then calculate the interquartile range (IQR).

	Minimum	Q1	Q2	Q3	Maximum	IQR
Hawksbill	100	145	180	200	235	55
Olive Ridley	85	90	90.25	95	100	5
Loggerhead	150	180	200	225	295	45

2. Share your data with your group and write 3 sentences explaining what the IQRs tell you about the weights of these three species of sea turtles.

   Sample responses:
   - The weights of the Olive Ridley turtles are mostly close to the median weight of 90.25 lb, so the median is a typical weight.
   - Hawksbill turtles have the most variation in their weights.
   - The Loggerhead turtles' weights also vary a lot more than the Olive Ridley, but are still less than the Hawksbill.

Lesson 15 Box Plots 961

1 Launch

Provide access to straightedges. Each student should determine the five-number summary for one type of turtle before discussing Problem 2 as a group. Then each student draws the box plot for their turtle and shares with the group. Have groups check their work for Problems 1–3 before moving on, either by displaying the Activity 2 PDF, or by having groups rotate to check with each other.

2 Monitor

Help students get started by asking, "How could you rewrite or represent the data differently to make it more efficient for determining the five-number summary?"

Look for points of confusion:

- **Having trouble making comparisons.** Prompt students to study the medians, IQRs, and ranges of the data sets.
- **Comparing only in abstract terms (e.g., "The medians are the same.").** Have students specify what their comparisons mean in the context (e.g., "What would equal medians tell you about the weights of those two types of turtles?").

Look for productive strategies:

- Connecting features of the box plots to the IQR, range, median, and minimum and maximum values of the data sets.
- Understanding and discussing the data in terms of typical weights for the three types of turtles.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, provide the five-number summaries already completed for them and have them begin the activity with Problem 2.

Accessibility: *Guide Processing and Visualization*

As students complete Problem 2, suggest they circle the Q2 column and the IQR column. Remind them that the IQR describes how the data vary about the center, in this case, Q2. Ask, "What does a low IQR tell you? A high IQR?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as you display the Activity 2 PDF and students share their responses, draw their attention to the connections between the box plots and the five-number summaries. Ask:

- "What do you notice about the box plot for Olive Ridley sea turtles? How do you see this same information in the five-number summary?"
- "Which dot plot has the greatest box length? Where do you see this in the five-number summary?"
- "Which representation — the table or the box plot — gives you a quicker sense of how the data sets compare to one another, including their centers and spread?"

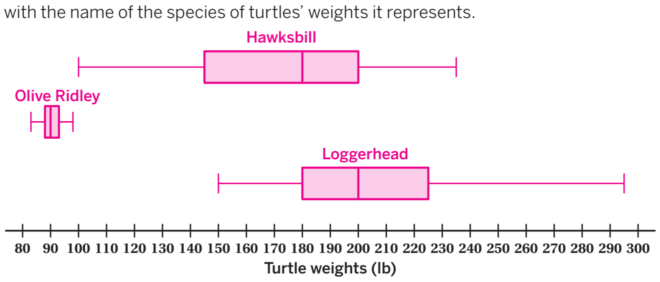
Small Groups | 15 min

Activity 2 Sea Turtles (continued)

Students practice creating box plots to represent data sets, and then interpret and compare the information shown for three distributions, particularly the variability (IQRs).

Activity 2 Sea Turtles (continued)

3. Draw the dot plot for your turtle, and then share with your group. Draw the box plot for each sea turtle, at different heights along this one number line. Label each box plot with the name of the species of turtles' weights it represents.



4. How are the weights of the Hawksbill and the Loggerhead the similar? How are they different?
   Sample response: The Q2 to Q3 range of the Hawksbill is the same as the Q1 to Q2 range of the Loggerhead, which means it would be typical to find one of each species that weighs the same, around that range. However, these would tend to be slightly heavier Hawksbills and slightly lighter Loggerheads.

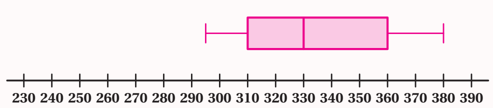
5. How are the weights of the Loggerhead and the Olive Ridley similar? How are they different?
   Sample response: The weights of both the Loggerhead and the Olive Ridley are not exactly symmetric around their medians, with more specimens weighing close to the median but less (from Q1 to Q2) compared to those weighing close to the median but more (from Q2 to Q3).

Are you ready for more?

Weights of a set of Green Sea turtles were also recorded.

- The minimum weight was the same as the maximum weight of the Loggerheads.
- The maximum weight was 3.8 times greater than the maximum weight of the Olive Ridleys.
- The IQR is equal to the sum of the IQRs of the Olive Ridley and Loggerhead.

1. Draw a box plot that could represent the Green Sea Turtle's data.



2. Can you estimate the median weight for the Green Sea Turtles? If so, how? If not, why not?
   Sample response: It is not possible other than knowing the median is probably between 300–375 lb, because I don't know how symmetric the data are. If the data are approximately symmetric, then the median is probably around 335–340 lb.

STOP

962 Unit 8 Data Sets and Distributions

3 Connect

Display the Activity 2 PDF.

Have groups of students share their analyses, and also interpretations of and connections between the box plots, focusing on Problems 4 and 5.

Ask:

- "Where can the IQR be seen in the box plot?" It is the length of the entire box.
- "Why might it be helpful to summarize a data set with a box plot?" It could help you see how close together or spread out the values are, and where they are clustered.

Highlight the usefulness of box plots in comparing distributions and characteristics of different populations. Notice that, unlike dot plots and histograms, box plots lend themselves well to being able to show multiple distributions on the same number line.

Whole Class | 5 min

Summary

Review and synthesize how box plots represent data, showing the center and variability in the corresponding distribution.

Name: ______ Date: ______ Period: ______

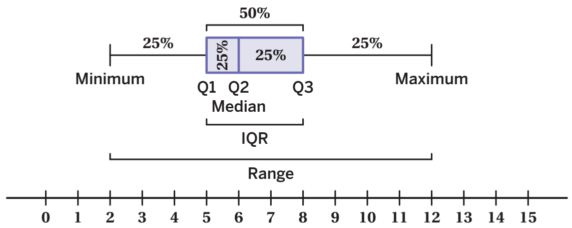
Summary

In today's lesson . . .

You saw how a ***box plot*** can be used to represent the five-number summary of a data set.

For example:

Minimum	Q1	Q2 or median	Q3	Maximum
2	5	6	8	12



Box plots on their own, however, do not show you how many data points were in each set, or the values of any individual data points, except the minimum and maximum. They do, however, make it possible to compare minimums, maximums, medians, quartiles, ranges, and IQRs for two or more sets of data at a glance.

Reflect:

Synthesize

Ask:

- "How can box plots be helpful in comparing two data sets?" You could compare the minimum and maximum values, where the median falls, and how the data are distributed among the four "quarters."
- "What are some questions you can ask about a data set, or comparing two data sets, that could be answered by looking only at a box plot?" Questions about the quartiles, maximum, minimum, center or spread can all be answered using box plots, whether for one data set or comparing multiple data sets.

Highlight that students saw another way to graphically represent a numerical data set — the *box plot*. The median is the line in the middle, which identifies the center. The IQR is the width of the box in the middle, which provides information about the spread, as does the range, shown by the full length between the ends of the whiskers. The relative lengths of each "quarter" gives an indication of clustering, and relatively equal spacing of those would indicate that the distribution is roughly symmetric.

Formalize vocabulary: *box plot*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why might it be useful to use a dot plot and a box plot together?"
- "What does a measure of variability tell you about a distribution?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *box plot* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

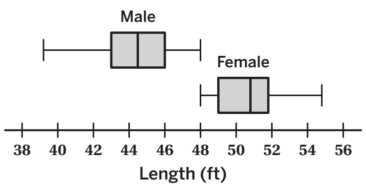
Students demonstrate their understanding by comparing two box plots and the data they represent.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.15

Researchers measured the lengths of 20 male humpback whales and 20 female humpback whales in feet. The box plots summarize their data.



For each statement about the whales that were measured, state whether you agree or disagree. Explain your thinking.

1. More than half of the female humpback whales measured were longer than 52 ft.
   Disagree: Sample response: 52 ft is greater than Q3, so no more than 25% would be longer than 52 ft.

2. The male humpback whales were typically longer than the females.
   Disagree; Sample response: The median length of males is only about 44.5 ft or between 43 and 46 ft. For females, the median is nearly 51 ft or between 49 and almost 52 ft. Also, the shortest female was as long as the longest male.

3. The lengths of the male humpback whales vary more than the lengths of the females.
   Sample responses:
   - Agree; The range for males is about 9 ft, while the range for females is a little less than 7 ft.
   - Disagree; The IQRs for both males and females are about 3 ft.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use the five-number summary to draw a box plot. 1 2 3

b. I know what information a box plot shows and how it is constructed. 1 2 3

c. I can use a box plot to answer questions about a data set. 1 2 3

d. I can use medians and IQRs to compare groups. 1 2 3

Success looks like . . .

- **Goal:** Creating a box plot to represent a data set.
- **Language Goal:** Describing the parts of a box plot that correspond with each number in the five-number summary, the range, and the IQR of a data set. **(Speaking and Listening)**
- **Language Goal:** Comparing and contrasting box plots that represent different data sets. **(Speaking and Listening, Writing)**
  - » Explaining whether they agree or disagree with the statements about the lengths of the whales in Problems 1–3.
- **Language Goal:** Interpreting a box plot to answer statistical questions about a data set. **(Speaking and Listening)**

Suggested next steps

If students agree with Problem 1 because they think the median is located at the far right edge of the box plot, consider:

- Having students label a box plot with the locations of the minimum, Q1, Q2, Q3, and maximum. Have them identify which corresponds to the half mark, or the median, and highlight this as the middle line in the box.

If students have difficulty with Problem 2, consider:

- Asking them where information regarding a typical value can be found on a box plot, and then how to interpret that in context.

If students have trouble explaining their thinking in writing for Problem 3, consider:

- Allowing them to verbally explain what they are thinking. Alternatively, they could color code the box plots to illustrate their thinking.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did the living box plot go as planned?
- In Lesson 14, students learned about the five-number summary. How did that support their understanding of box plots? What might you change for the next time you teach this lesson?

Practice

Independent

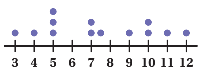
Name: Date: Period:

1. The data show the number of hours per week sixth and seventh graders spent doing homework. Create two box plots above the same number line to represent both sets of data.

Sixth graders:

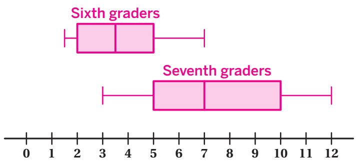
Minimum: 1.5 **Q1:** 2 **Median:** 3.5 **Q3:** 5 **Maximum:** 7

Seventh graders:



The data in numerical order are:

3	4	5
5	5	7
7	7.5	9
10	10	11
12		



a. How are the distributions of the time spent doing homework by the students in the two grades most alike? Explain your thinking.

Sample responses:

- About 50% of the students in each grade spent between 3 and 7 hours doing homework per week.
- The students in each grade who spent the most hours doing homework per week only spent up to 2 hours more time than typical students in their grade.
- A typical seventh grader spent as much time doing homework per week as the sixth graders who spent the most time doing homework per week.

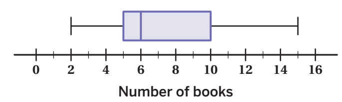
b. Which grade level of students corresponds to the data with the greatest variability in time spent on homework? Explain your thinking with measures of data.

The seventh graders' data have the greatest variability; Sample response: For the seventh grade, the range is 9 and the IQR is equal to 5. For the sixth grade, the range is 5.5 and the IQR is 3. The range and IQR are greater for seventh graders.

Practice

Name: Date: Period:

2. Each student in a class recorded how many books they read during the summer. The box plot summarizes their data.



a. What is the greatest number of books read by a student in this group?
15 books

b. What is the median number of books read by the students?
6 books

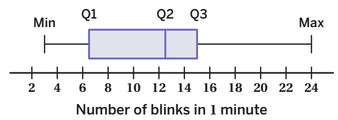
c. What is the interquartile range (IQR) for the number of books read by the students?
5 books

3. There are 20 pennies in a jar. If 16% of the coins in the jar are pennies, how many coins are there in the jar?
125 coins

4. Here is a list of questions about the students and teachers at a school. Select *all* the questions that are statistical questions.

A. What is the most popular lunch choice?
B. What school do these students attend?
C. How many math teachers are in the school?
D. What is a common age for the teachers at the school?
E. About how many hours of sleep do students generally get on a school night?
F. How do students usually travel from home to school?

5. Scientists believe people blink their eyes to keep the surface of the eye moist and also to give the brain a brief rest. Write two statistical questions that could be answered using the box plot.



Minimum: 3 **Q1:** 6.5 **Q2:** 12.25 **Q3:** 15 **Maximum:** 24

Sample responses:
What is the typical number of time someone blinks in a minute?
How many times did 50% of the people blink?

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
Spiral	3	Unit 3 Lesson 12	2
	4	Unit 8 Lesson 2	2
Formative	5	Unit 8 Lesson 16	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Comparing MAD and IQR

Let's compare data sets using visual displays.

Focus

Goals

1. **Language Goal:** Recognize that different graphical displays offer different insights into a distribution. **(Speaking and Listening, Writing)**
2. **Language Goal:** Recognize that different measures of center and variability offer different insights into a data set. **(Speaking and Listening, Writing)**
3. **Language Goal:** Choose an appropriate graphical display to represent a data set, and justify the choice. **(Speaking and Listening, Writing)**

Coherence

Today

Students review statistical questions by comparing the center and spread of different distributions related to the Grizzly bear. In the second activity, determining a typical number of yellow perch using a histogram may not be obvious, prompting students to interpret measures of center and spread more carefully. They determine what these different measures (mean and MAD, or median and IQR) represent in context. They select an appropriate representation for the distribution based on the structure of the data, an appropriate set of measures of center and spread, and interpret their meaning in the context.

Previously

In Lessons 12–15, students have focused on measures of variability. The particular measures were the MAD and IQR.

Coming Soon

In Lesson 17, students will apply their work with statistics and data throughout the unit to explore conservation through a variety of perspectives.

Rigor

- Students **apply** what they have learned in this Sub-Unit to a real-world population scenario.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- straightedges

Math Language Development

Review words

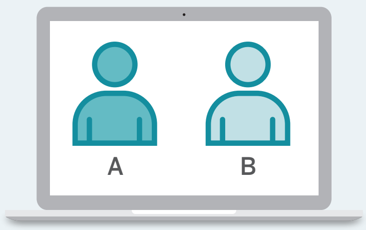
- *box plot*
- *quartile*
- *range*
- *interquartile range*
- *median*
- *mean*
- *MAD*
- *variability*

Amps | Featured Activity

Activity 1

Interactive Histogram

Students can digitally move bars on a histogram representing the yellow perch population. You can overlay student responses to provide immediate feedback.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Some students might question themselves and wonder whether they can use the data to answer the questions in Activity 1. Ask students to identify their emotions and change their thinking to be more optimistic. The mathematical models provided by the graphs contain all of the information needed. Beyond that, students must feel empowered to make decisions and interpret what they see with an understanding that it is okay to make a mistake.

Modifications to Pacing

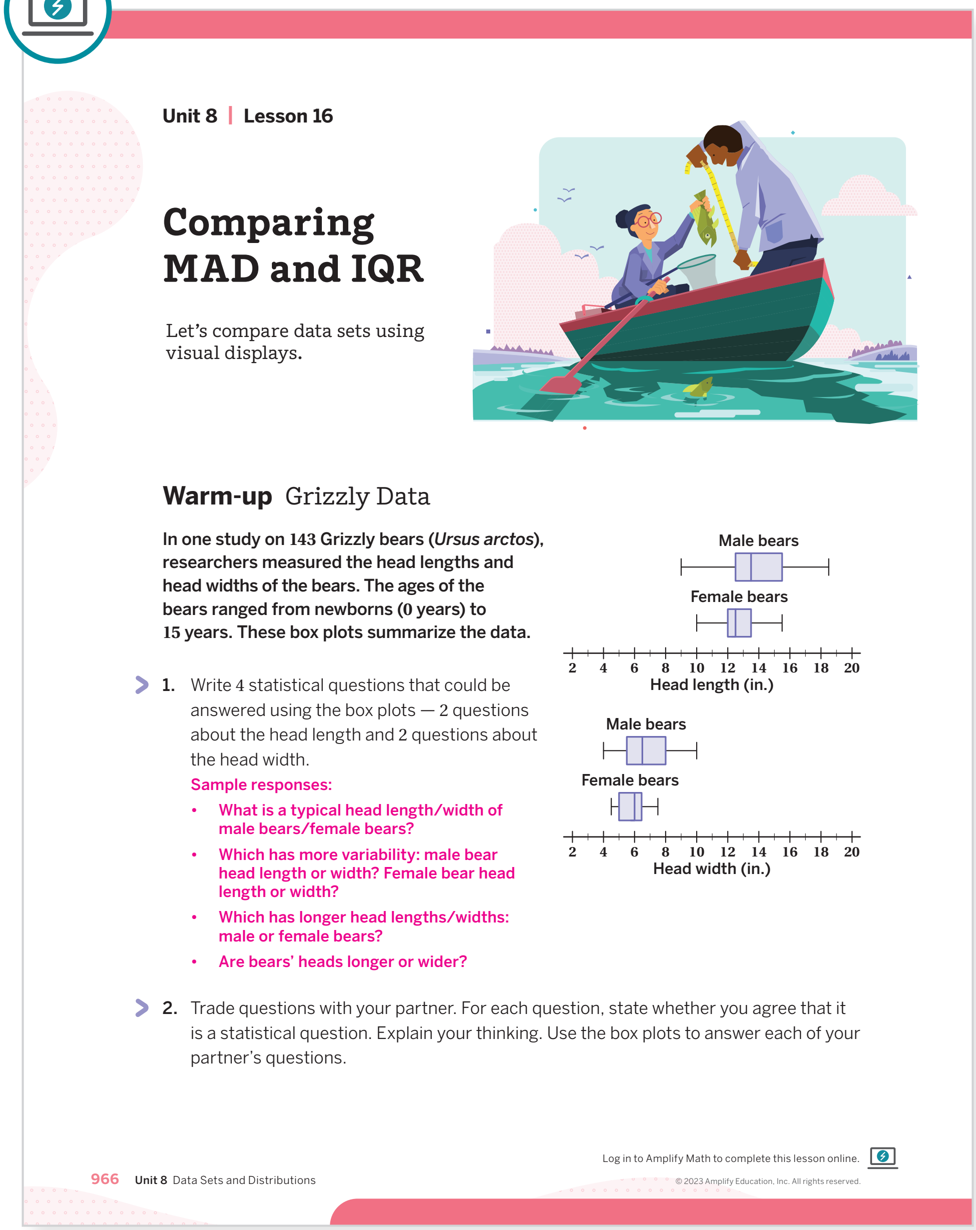
You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted and Part 1 of **Activity 1** could be used to remind students about the nature of statistical questions and the tools they have for describing or measuring variability in a data set.

Pairs | 10 min

Warm-up Grizzly Data

Students review writing statistical questions and interpreting box plots, preparing them to consider which measure of variability is better for comparing distributions.



Comparing MAD and IQR

Let's compare data sets using visual displays.

Warm-up Grizzly Data

In one study on 143 Grizzly bears (*Ursus arctos*), researchers measured the head lengths and head widths of the bears. The ages of the bears ranged from newborns (0 years) to 15 years. These box plots summarize the data.

1. Write 4 statistical questions that could be answered using the box plots — 2 questions about the head length and 2 questions about the head width.
   Sample responses:
   - What is a typical head length/width of male bears/female bears?
   - Which has more variability: male bear head length or width? Female bear head length or width?
   - Which has longer head lengths/widths: male or female bears?
   - Are bears' heads longer or wider?
2. Trade questions with your partner. For each question, state whether you agree that it is a statistical question. Explain your thinking. Use the box plots to answer each of your partner's questions.

Log in to Amplify Math to complete this lesson online.

1 Launch

Tell students that, for the first question, one partner should write two questions about the head lengths and the other partner should write two questions about the head widths. For the second question, they should exchange and review each other's questions. Remind students to consider units of measurement.

2 Monitor

Help students get started by asking "What is a value that could have been in the data set for [choose box plot]? What does that represent in context?"

Look for points of confusion:

- **Writing a non-statistical question.** Ask the partner whether they agree or disagree that their question is a statistical question. If they disagree, ask how they could revise the question so it is a statistical question.

Look for productive strategies:

- Writing statistical questions based on the given box plots that involve comparisons, such as "How much greater is the range of female head lengths compared to male head lengths?"
- Determining whether the question can or cannot be answered using the box plot, and then answering the question when possible.

3 Connect

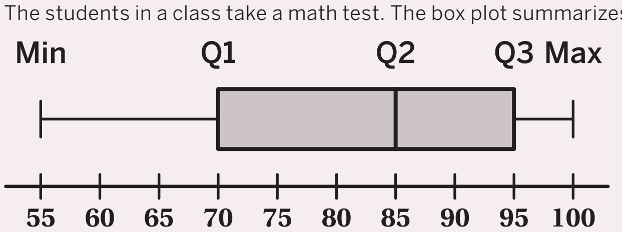
Have individual students share one of their questions, focusing on how the answer was determined using the box plots.

Highlight, as a review, what makes a question a statistical question, and how box plots show variability in data sets by representing both the range and IQR.

Power-up

To power up students' ability to write statistical questions that relate to a data set given as a box plot, ask:

The students in a class take a math test. The box plot summarizes their test scores.



Determine which of the following statistical questions can be answered using the box plot.

A. What is the typical score on the math test?
B. What percentage of students scored 70 or above?
C. How many students scored less than 70?
D. How many hours did students generally spend studying?

Use: Before the Warm-up.

Informed by: Performance on Lesson 15, Practice Problem 5.

Activity 1 Will the Yellow Perch Survive?

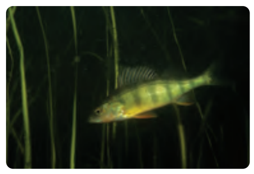
Students answer statistical questions by representing a data set, deciding on appropriate measures of center and variability, and then using their analyses to draw conclusions.

Amps Featured Activity Interactive Histogram

Name: ____________ Date: ________ Period: ________

Activity 1 Will the Yellow Perch Survive?

You are a scientist studying the yellow perch (*Perca flavescens*), a species of fish. You and your fellow scientists know that, in general, the length of a fish is related to its age. This means that the longer the fish, the older it is. Adult yellow perch vary in size, but they typically measure between 10 and 25 cm long.

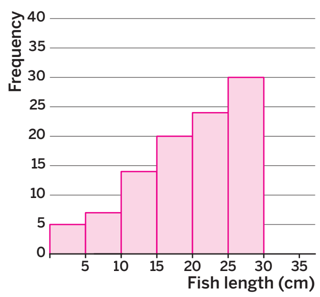


RLS Photo/Shutterstock.com

The Great Lakes Water Institute caught, measured, and released yellow perch at several locations in Lake Michigan. You and your colleagues have been asked to evaluate their data.

Part 1

1. Use the data to construct a **histogram** that shows the lengths of the captured yellow perch. Each bar should contain the lengths shown in each row in the table.



2. How many fish were measured? How do you know?
   100; I added the heights of each bar or the number of fish for each bar.

3. How would you describe the shape of the distribution?
   Non-symmetric

4. Estimate the median length for this sample. Describe how you made this estimate.
   Sample responses:
   - **Between 15–19 cm because that is the center of the histogram.**
   - **22 cm; half of the data values would be 50, so that would include all of the "25 to less than 30" group and most of the "20 to less than 25 group" because that totals 54, so it would be in the lower 20s (the last 30 and 20 of the data values, leaving 4 from the "20 to less than 25 group").**

5. Predict whether the mean length of this sample is *greater than*, *less than*, or *nearly equal* to the median length for this sample of fish? Explain your prediction.
   It would be less than the median because there are many lengths less than the median in the data.

1 Launch

Supply background knowledge by telling students that they will look at an example of how data analysis could be used to help conservation efforts. Provide access to straightedges.

Arrange students into groups of 3 or 4. Give individual students 7–8 minutes of quiet work time to complete Part 1. Then give them 10–12 minutes to compare responses with their group and complete Parts 2 and 3.

2 Monitor

Help students get started by asking "How can you use the information in the table to construct a histogram?"

Look for points of confusion:

- **Thinking they have to do some calculations in Part 2.** Ask, "What is it you are trying to calculate? Is that already provided or able to be interpreted from your data sheet?"
- **Having trouble interpreting the data analysis.** Ask students to support their claims using evidence to pinpoint where the confusion is, and then ask questions, such as "How does that support your thinking?" or "Is there another piece of data that would go with/counter that idea?" to have them reconsider their rationales.

Look for productive strategies:

- Choosing and justifying the pair of measures for center and variability that summarizes the distribution of the population better in Part 2, Problem 3
- Supporting their conclusions with specific pieces of evidence, such as their histogram, their analysis of the distribution, the measures of center and spread, etc.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can collaborate virtually to determine the status of the yellow perch population.

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to respond verbally to Part 3, as opposed to writing all of their responses. Tell them to record notes for each problem so that they can share their thinking with the class during the Connect.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the Activity 1 PDF that shows the dot plot and box plot. If possible, display the histogram nearby or have students refer to it in their Student Edition. Draw students' attention to how each display represents the same data set, and the similarities and differences in the information that each display provides. Ask:

- "How is the longer whisker on the box plot represented on the dot plot? The shorter whisker? Where do you see this information represented on the histogram?"
- "Which display easily shows the five-number summary? From which display can you calculate the five-number summary?"

Small groups | 25 min

Activity 1 Will the Yellow Perch Survive? (continued)

Students answer statistical questions by representing a data set, deciding on appropriate measures of center and variability, and then using their analyses to draw conclusions.

Activity 1 Will the Yellow Perch Survive? (continued)

Part 2

6. Using the data and other representations provided on the data sheet, interpret the mean and MAD. What do they tell you about the lengths and ages of the yellow perch?
   There were 49 fish with lengths between 13.2 cm and 24.8 cm, which is more than half of the fish. That tells me that there are more adult fish than younger fish.

7. Interpret the median and IQR. What do they tell you about the lengths and ages of the yellow perch?
   The lengths of half of the fish were between 14 and 25.5 cm. That tells that the yellow perch might typically be longer and older than what I thought from using the mean and MAD.

8. Which pair of measures of center and variability — mean and MAD, or median and IQR — do you think summarize the distribution of the lengths of the yellow perch better? Explain your thinking.
   I think the median and IQR summarize the data better because the distribution is not symmetric and the biggest cluster of data values is around 20 cm, with a peak at 22 cm.

Part 3

9. Based on your data analysis of this sample and the following classification: young (< 10 cm), adult (≥ 10 and < 25), and old (≥ 25), which graph best represents a typical age for the yellow perch — the histogram, dot plot, or box plot? Explain your thinking.
   Sample responses:
   - **Histogram: Because the typical length is around 15 cm, I would describe them as adult, but on the younger side of adulthood.**
   - **Histogram: I would describe it as adult because there are 58 fish in the 10–25 bins.**
   - **Dot plot: I would describe it as adult because the mean is 19 and the spread is from about 13 to 24 years, which are ages of adult fish.**
   - **Box plot: A typical age is adult, but more on the older side of adult, because Q3 is at 25.5 years.**

10. Some researchers are concerned about the survival of the yellow perch. Do you think the lengths (or the ages) of the fish in this sample are something to worry about? Explain your thinking.
    Sample responses:
    - **Yes, because there seems to be more adult and older fish than younger fish.**
    - **No, because the bigger fish may have been easier to catch and there could be a lot more younger and baby fish out there that weren't measured.**

STOP

3 Connect

Have groups of students share their responses to as many problems as possible that you choose. Consider a variety of ways for them to share and check their work, but focus the majority of the discussion on Part 3, Problem 1, being sure they provide the information they used to determine their response, and hearing from as many students as possible to highlight as many of the key ideas built up throughout the unit as possible.

Ask, "What do you think the IUCN classification for yellow perch is (or should be)?" Reveal to students that the most recent research showed more favorable length-age distributions: more of the fish were smaller or younger. Its IUCN classification, at last designation, was *Least Concern* (2013).

Highlight that sometimes the data needed to directly answer a question are not available or easily attained or measured, so researchers often need to consider different, but related, data, or consider analyzing the data differently, in order to make informed conclusions and decisions. It would be important in these cases to provide as much information as possible about the assumptions made and the process, so that the readers of the results understand what is known and what is calculated as a "best guess."

Summary

Review and synthesize how the mean and MAD, and the median and IQR, each answer different questions and give different insights about a distribution or data set.

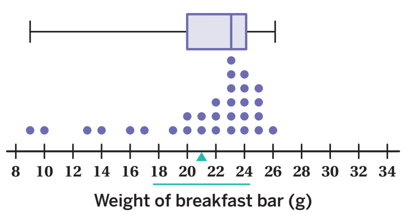
Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You compared how well the MAD and IQR each describe or summarize the variability in a distribution. Similar to the differences between mean and median, each measure of variability tells you slightly different things, and one may be more appropriate. It is still true that when a distribution is not symmetric, the median and IQR are often more representative measures of center and spread than the mean and MAD. However, the decision on which pair of measures to use depends on what you want to know about the data or the question you are answering.

Here is the dot plot from the example you saw in Lesson 11, and also a box plot for the same data representing the weights of 30 different breakfast bars.



The mean is 21 g and the MAD is 3.4 g. This suggests that a breakfast bar typically weighs between 17.6 g and 24.4 g, as represented by the triangle and horizontal segment around it. The median bar weight is 23 g and the IQR is 4 g. This suggests that a breakfast bar typically weighs between 20 g and 24 g, as represented by the box.

These two pairs of measures paint very different pictures of the variability of the weights of the breakfast bars. But this is not because the MAD and IQR vary a lot. Rather it is the combination of differences in both the measures of center and the measures of variability together that make up the difference. Because the distribution is not symmetric, the median is closer to the middle of the largest cluster of values. If you were to ignore the lighter weights, the median and IQR would give a more accurate picture of how much a breakfast bar typically weighs.

Reflect:

Synthesize

Ask:

- "What do the mean and MAD tell you?" The mean represents an average value. The MAD represents the average distance between a value and the mean.
- "How would you interpret this statement? Noah's mean number of homework problems completed per day is 10 and the MAD is 6." If Noah did the same number of problems each day, he would do 10 per day. Based on what he actually did, Noah typically completes 4–16 problems per day.
- "What do the median and IQR tell you?" The median represents the value where half of the data set is greater, and half of the data set is less than it. The IQR represents the range covered by the middle half of the data set.
- "How would you interpret this statement: Lin's median number of homework problems completed per day is 10 and the IQR is 6?" On half of the days Lin completed 10 or fewer problems, and the other half of the days she completed 10 or more problems. Also, she typically completes around 7–13 problems per day, if the distribution is approximately symmetric or that's how many problems she completed on the most typical half of the days.

Highlight that students practiced selecting and then calculating measures of center and variability (mean and MAD, or median and IQR) and made sense of them in the context of a given situation. Each pair (and sometimes all four pieces) of information provides different insights about the distribution of a data set. Often questions require more than one piece of information to tell a more complete story.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What are the ways in which you can represent a data set?"
- "Which representations are helpful for summarizing a distribution?"

Independent | 5 min

Exit Ticket

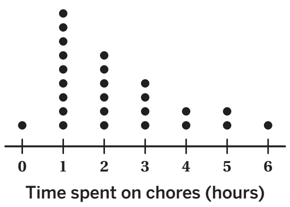
Students demonstrate their understanding by using either the mean and MAD, or the median and IQR, to summarize the center and spread of the data.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.16

Lin surveyed her classmates on the number of hours they spend doing chores each week. She represented her data with a dot plot and a histogram.



1. Lin thinks that she could identify the median, the minimum, and the maximum of the data set using either the dot plot or the histogram. Do you agree? Explain your thinking.
   I disagree; Sample response: She can determine those three values using the dot plot, but not the histogram. The histogram does not show any of these values because the data are shown in ranges. You cannot identify an exact number for any of them.
2. Should Lin use the mean and MAD, or the median and IQR, as the most appropriate measures to summarize the center and spread of the data? Explain your thinking.
   The median and IQR would be most appropriate because the data are not symmetric. Most of the data values are between 1 and 3, but the data value at 6 would increase the mean.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can draw an appropriate representation for a set of data.
1 2 3

b. I can explain what the mean and MAD or the median and IQR tell me in the context of a situation and use either pair of measures to answer questions.
1 2 3

c. I can decide which graphical display would be more appropriate as a representation of the center and spread of a data set.
1 2 3

Lesson 16 Comparing MAD and IQR

Success looks like . . .

- **Language Goal:** Recognizing that different graphical displays offer different insights into a distribution. **(Speaking and Listening, Writing)**.
  - » Explaining which measures Lin could identify from the dot plot and the histogram in Problem 1.
- **Language Goal:** Recognizing that different measures of center and variability offer different insights into a data set. **(Speaking and Listening, Writing)**
- **Language Goal:** Choosing an appropriate measure of center and variability to describe a data set, and justifying the choice. **(Speaking and Listening, Writing)**
  - » Explaining which set of measure is more appropriate to summarize the center and spread of the data in Problem 2.

Suggested next steps

If students agree with Problem 1, consider:

- Asking them to show how the data supports their claim.
  - » Having students review the Summary before reevaluating any responses.

If students choose the mean and MAD, consider:

- Asking, "Would you describe this distribution as symmetric or not symmetric? What does that mean for the measure of center?" Not symmetric; The median is often a better measure.
  - » Having students review the Summary before reevaluating any responses.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

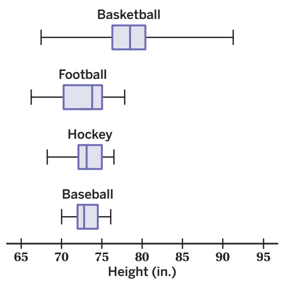
- What worked and didn't work today? During Parts 2 and 3 in Activity 1, how did you encourage each student to listen to one another's ideas?
- In what ways have your students gotten better at mathematical collaboration? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

1. The box plots summarize the heights of 20 professional male athletes in basketball, football, hockey, and baseball.



a. In which two sports are the players' height distributions most alike? Explain your thinking.

Hockey and baseball;

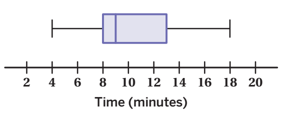
Sample response:

The center of the data for basketball players is greater, while it is about the same for athletes who play the other three sports. There is also about the same amount of hockey and baseball players, while there is more variability in the heights of football players.

b. Which sport shows the greatest variability in players' heights? Which sport shows the least variability?

Greatest variability is basketball; Least variability is baseball.

2. Here is a box plot that summarizes data for the time, in minutes, that a fire department took to respond to 100 emergency calls. Select *all* the statements that must be true based on the box plot.



A. About 75% of the response times were 8 minutes or less.

B. At least half of the response times were 11 minutes or longer.

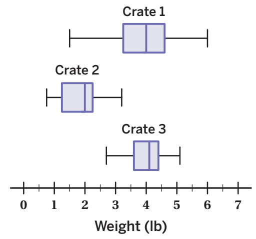
C. Most of the response times were under 13 minutes.

D. Fewer than 30 of the response times were over 13 minutes.

E. There were more response times that were greater than 13 minutes than those that were less than 9 minutes.

Name: ______ Date: ______ Period: ______

3. Pineapples were packed in 3 large crates. For each crate, the weight of every pineapple in the crate was recorded. Here are three box plots that summarize the weights of the pineapples in each crate. Select *all* of the statements that are true, according to the box plots.



A. The weights of the pineapples in Crate 1 were the most varied.

B. The heaviest pineapple was in Crate 1.

C. The lightest pineapple was in Crate 1.

D. Crate 3 had the greatest median weight and the greatest IQR.

E. More than half the pineapples in Crate 1 and Crate 3 were heavier than the heaviest pineapple in Crate 2.

4. Solve each equation.

a. $9v = 1$ $\quad v = \frac{1}{9}$

b. $1.37w = 0$ $\quad w = 0$

c. $1 = \frac{7}{10}x$ $\quad x = \frac{10}{7}$

d. $12.1 = 12.1 + y$ $\quad y = 0$

e. $\frac{3}{5} + z = 1$ $\quad z = \frac{2}{5}$

5. When might it be acceptable to ignore a data value and leave it out of calculations for measures of center and variability? Would it ever be acceptable? Explain your thinking.

Answers will vary, but responses should include some mention of outliers and depending on the measure of center or variability being used and how outliers affect those. The responses should also have considerations of the context and question being asked. It could be noted that it is sometimes acceptable, or justifiable (but it is okay if all students do not state this).

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 7	1
Formative	5	Unit 8 Lesson 17	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Asian Elephant Populations

Let's look at a real-world scenario involving messy data and competing interests.

Focus

Goals

1. **Language Goal:** Choose an appropriate measure of center and variability to describe a data set, and justify the choice. **(Speaking and Listening, Writing)**
2. **Language Goal:** Reflect upon the year of mathematics as an individual learner and as a peer collaborator. **(Writing)**

Coherence

Today

Students apply the skills learned throughout this unit to consider the endangered status of the Asian elephant. They begin by analyzing the population data for the elephants, recognizing that there are many challenges when collecting data in the real world. Students then take on a real-world persona to interpret the population data. They consider which values and measures of center and variability best support their perspective, and use what they know about those measures to justify their choices. The lesson concludes with students presenting their perspectives and supporting evidence. The Exit Ticket allows students to reflect on their goals and growth as mathematicians and collaborators.

Previously

In this unit, students developed a foundation of statistical thinking. They recognized that statistical questions have answers that display variability. Using measures of center and variability, they described distributions of data sets and answered such questions. They also considered which pair of measures is best for a given distribution and context.

Coming Soon

In Grade 7, students will extend their work with statistics to include sampling and probability.

Rigor

- Students **apply** the statistics skills learned in this unit to a real-world scenario involving Asian elephant populations as considered from different perspectives.

Note: Part 3 of Activity 1 offers multiple options for presentation. See the Connect section for Activity 1.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	20 min	5 min	10 min
Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, one per pair
- Warm-up PDF (teacher instructions)
- Activity 1 PDF, *Data*, one per group
- Activity 1 PDF, *Example Persona Card* (for display)
- Activity 1 PDF, *Persona Cards*, pre-cut cards, one card per group
- Activity 1 PDF, *Resource* (optional)
- tools for creating a visual display (optional)

Math Language Development

Review words

- *mean*
- *mean absolute deviation*
- *median*
- *interquartile range*

Amps | Featured Activity

Activity 1
Data Presentation

Students can view multiple representations of wildlife data and choose which representations to include in a presentation slide. Then, they can add their sketches to the slide.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to work from a persona with whom they do not agree or have anything in common. Point out that the class knows that they were assigned a persona and that what you are giving is not necessarily your opinion. Challenge students to fully take on that person's perspective and create a mathematical argument that represents the given persona. Tell them to use their mathematical knowledge to show empathy and respect for another.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In Part 3 of **Activity 3**, have groups of students share their claims and evidence with another group. Then have each group share one item that they learned from the other group with the class.

Pairs | 10 min

Warm-up Looking at Messy Data

Students look at Asian elephant population data, which is presented as rounded values, precise values, and ranges of values, supplying background knowledge for Activity 1.

Unit 8 | Lesson 17 – Capstone

Asian Elephant Populations

Let's look at a real-world scenario involving messy data and competing interests.

Warm-up Looking at Messy Data

The IUCN lists the Asian elephant (*Elephas maximus*) as Endangered, citing a general decline in population that has most likely been occurring for centuries. You will be given the population data used by IUCN in their 2018 survey assessment to complete these problems.

Mogens Trolle/Shutterstock.com

1. What questions do you have about how this data is represented by country or region?
   Answers may vary. Responses may include questions about the form of numbers (exact values, estimations, ranges), the countries listed, and who is collecting the data.
2. How are the data different from the data you have seen throughout the unit?
   Answers may vary. Responses may highlight the mix of ranges and exact values, and how the ranges are inconsistent and differ from the bins used with histograms.
3. Why do you think the data are different in these ways?
   Answers may vary. Responses may highlight the difficulty of counting elephants, and differences in counting methods including guessing or estimating.
4. Can you trust the data?
   Answers may vary, including *yes*, *no* or *maybe*. Students may support their opinions by considering the form of numbers (exact values, estimations, ranges), the countries listed, and who is collecting the data and how they are collecting it.

Log in to Amplify Math to complete this lesson online.

972 Unit 8 Data Sets and Distributions

1 Launch

Arrange students in pairs and give each a copy of the Warm-up PDF, *Student*. Read aloud the introductory paragraph in the Student Edition. Give students 5–6 minutes to complete the problems with their partner.

2 Monitor

Help students get started by asking "Do you think all the data was collected the same way?"

Look for points of confusion:

- **Focusing on the differences in populations rather than the different ways values are presented.** Have students look at the population values for Bangladesh, China, and India. Ask, "What do you notice about how these values are listed?"
- **Struggling to identify why the data are different (Problem 3).** Explain that elephants live in dense brush. Ask, "How might that impact data collection?"

Look for productive strategies:

- Recognizing the different ways the data are presented, and using this to formulate questions about data collection and reporting.
- Generalizing that each way the data is presented could raise skepticism about its accuracy.

3 Connect

Have pairs of students share their most interesting or important question or idea.

Ask:

- "Who do you think is collecting the data and what motivations or interests do they have in the data?"
- "How might saying 'completely extinct' in Pakistan create issues?"
- "How could you make the population values reported as a range easier to work with?"

Highlight how all of this reflects the challenges of collecting accurate data, which may lead to different ways of counting and reporting.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use the interactive data map to analyze the population of Asian elephants.

Power-up

To power up students' ability to determine which values should be included, or excluded, in statistical calculations, ask:

When calculating measures of center or variability, when might it be appropriate to exclude a value from a data set?

A. When the data includes extreme values.
B. When the distribution of the data is symmetric.
C. When the range of values in the data set is large.
D. When there are large gaps in the data.

Use: Before Activity 1.
Informed by: Performance on Lesson 16, Practice Problem 5,

Small Groups | 20 min

Activity 1 Population Perspectives

Students look at the same data from the Warm-up, but through the lens of a specific persona and perspective that guides decisions about how to use the data.

Amps Featured Activity Data Presentation

Name: ________ Date: ________ Period: ________

Activity 1 Population Perspectives

Part 1: Be the Person

Your group will be given a persona. Use that persona to respond *as that person would* to the following prompts.

1. What is your goal and how does the population of Asian elephants impact you and your goals?

Answers may vary. Sample responses:

Ecotourism Developer	**My goal is to increase tourism so that my country can make more money and give more back to the people. I want the elephant to remain on the Endangered list.**
Non-profit Worker	**My goal is to report accurate data so that the Asian elephant can be protected if it is endangered. Or, if the elephant population is stable or increasing, the funding can go toward protecting other species.**
Community Hunter	**My goal is to keep my rights to hunt big game, such as the Asian elephant, so that I can feed the people who are important to me. I want the elephant to be removed from the Endangered list.**

2. Consider how the data on elephant populations from the Warm-up can support your goals. How will you choose the population value to use for each country? Explain your thinking.

Answers may vary. Students may include or exclude Pakistan, claiming either will give a more accurate picture. Sample responses:

Ecotourism Developer	**I will use the lowest available population numbers, including Pakistan, because this will help the elephants stay on the Endangered list. This means they will be rare, and people will pay to come see them. If I use the largest numbers, they may no longer be considered endangered, and as a result, they will be less of a draw to tourists.**
Non-profit Worker	**I will use values near the center of the available population numbers to paint the most accurate picture of the Asian elephant. This will help the IUCN decide if it should remain on the Endangered list and be protected or be removed from the list, allowing the funding to help another endangered species.**
Community Hunter	**I will use the greatest population numbers available because this will help make the case that the elephants can be removed from the Endangered list. I will exclude Pakistan because it skews the data. That means I can hunt them as needed to feed my family and neighbors. If I use the lowest numbers, they will remain on the Endangered list, and I will have to find other ways to help feed everyone.**

1 Launch

Display the Activity 1 PDF, *Example Persona Card*, and discuss the 4 questions. Then arrange students in groups of 4 and give each group 1 one persona card from Activity 1 PDF, *Persona Cards*. Give them 5 minutes to discuss Part 1, and then give each group one copy of the Activity 1 PDF, *Data*. Have students continue to work on Part 2. For Part 3, there are several options for students to share and discuss their work, as listed on the next page.

2 Monitor

Help students get started by giving them the Activity 1 PDF, *Resource*.

Look for points of confusion:

- **Reasoning using personal opinions.** Remind students that they are taking on the worries, opinions, and goals of the persona on their card.
- **Not connecting goals to how they will choose population values.** Ask, "Does your persona want elephants to remain on the Endangered list? What type of population values — low, middle, or high — would help you achieve that goal?"
- **Choosing measures of center and variability based on the distribution and not their persona's goals.** Ask, "What type of values would best support your persona's goals? Would the mean or median be the best measure of center?"

Look for productive strategies:

- Using their persona's goals and concerns, as indicated on the card, to determine whether they want the elephants to remain Endangered and considering which values — low, middle, or high — would achieve those goals.
- Reasoning about how including or excluding Pakistan would impact their goals.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use the interactive data map to analyze the population of Asian elephants.

Accessibility: *Clarify Vocabulary and Symbols*

Ask students if they have heard of the term *persona*. Tell them that a *persona* is the description of an individual (fictional) that is related to the topic of your research. Personas represent the different aspects of individuals affected by your research and creating or using personas can help you understand those needs and experiences.

Small Groups | 20 min

Activity 1 Population Perspectives (continued)

Students look at the same data from the Warm-up, but through the lens of a specific persona and perspective that guides decisions about how to use the data.

Activity 1 Population Perspectives (continued)

Part 2: Be the Statistician

You will be given measures of center and variability that were each calculated for the data using different values. Choose the measure of center and variability calculated with the data values that best support your perspective. Explain your thinking.

Answers may vary.

Ecotourism Developer	There are typically 605 Asian elephants per country. I used the median and IQR because the data are not symmetric. I included Pakistan because although it is an outlier, it provides a clearer picture about the state of the Asian elephants.
Non-profit Worker	The typical population of Asian elephants is 1,067. I used the median and IQR because the data are not symmetric, and I wanted to determine the middle value that is not skewed by outliers, such as 0 elephants in Pakistan.
Community Hunter	The typical population of Asian elephants is 3,691. I used the mean and MAD. While the data are not symmetric, I wanted to ensure the typical value considers those countries that have significantly more elephants than is typical in other countries. I chose to exclude Pakistan because it is such an outlier that would skew the average population values far to the left of what is typical and representative of the region.

Part 3: Be the Voice

You now need to convince IUCN officials of your position to either keep the Asian Elephant as *Endangered*, downgrade it to *Vulnerable*, or reclassify it as *Critically Endangered*. Include any data analysis and visuals that support your claims.

STOP

- Choosing the measure of center and variability based on the persona's goal, and using the advantages of either the mean and MAD or the median and IQR to justify their choices and anticipate counterarguments.

3 Connect

Here are some different options for students to share their responses to Part 3:

- ***Gallery Tour* routine:** Have students make a poster.
- **Convince a Skeptic:** Using the *Jigsaw* or *Mix and Mingle* routine, have students present their arguments to groups with differing perspectives.
- **Debate:** Have students present their positions to the IUCN (consider asking a colleague to be the IUCN representative). Groups could combine (those who want to downgrade the Asian elephant to *Vulnerable*, for example) to form debate "teams."

Display student artifacts based on the option chosen for Part 3.

Have pairs of students share their perspective and argument, depending on the option chosen for Part 3.

Ask:

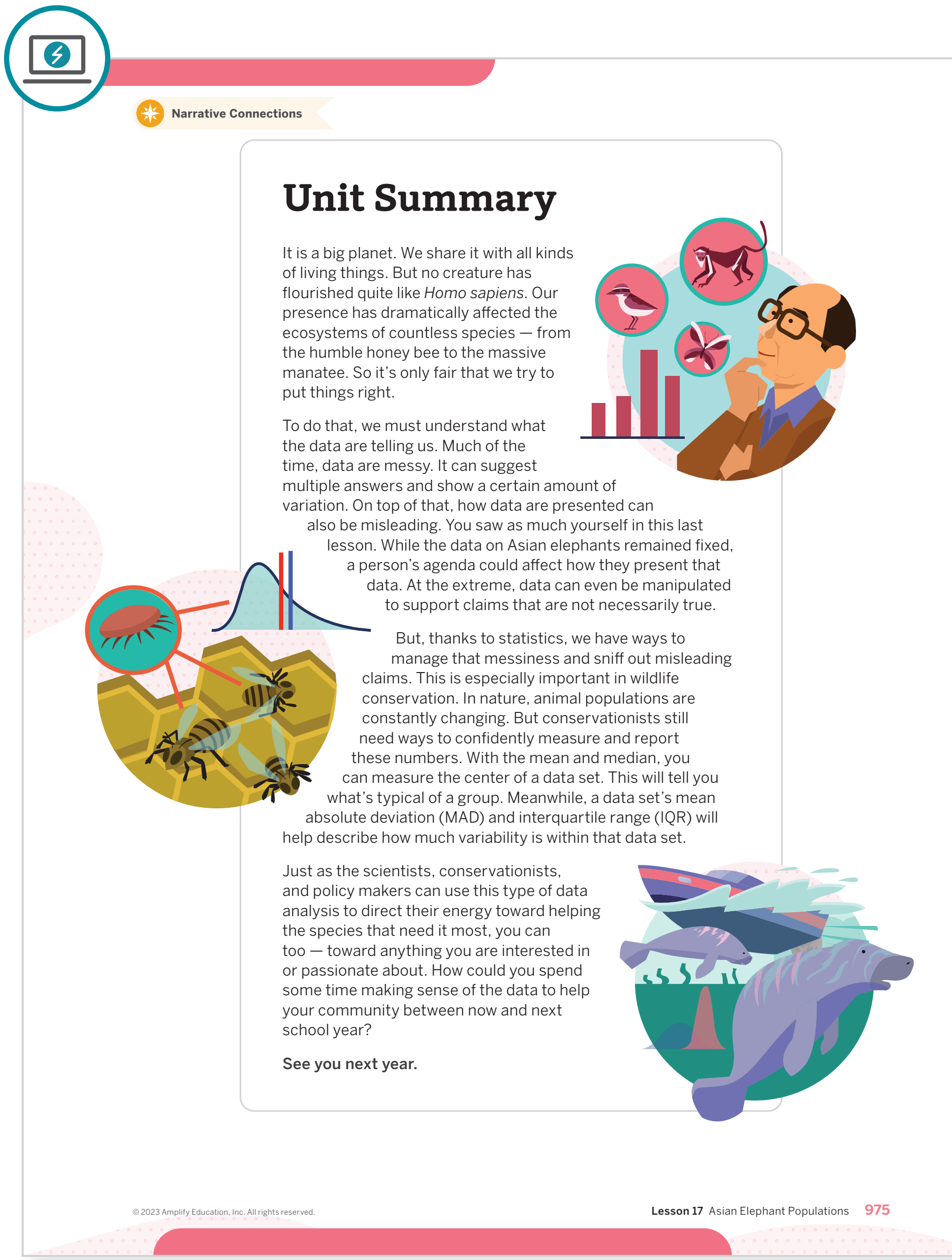
- "Did you include every country from the population table? Why or why not?"
- "Which measure of center and variability did you use to form your argument? Why?"

Highlight that no matter how accurate the data collected may be, the interpretation of that data can also be subjective and used to support preconceived opinions. When the data are not symmetric and there are outliers, one measure of center may better support a point of view than another. Additionally, outliers can be included or excluded. It is important to be clear about assumptions and choices made when analyzing data, and, especially, being transparent and objective when excluding data.

Whole Class | 5 min

Unit Summary

Review and synthesize the importance of accurate data and how to be a critical consumer of statistical information.



Narrative Connections

Unit Summary

It is a big planet. We share it with all kinds of living things. But no creature has flourished quite like *Homo sapiens*. Our presence has dramatically affected the ecosystems of countless species — from the humble honey bee to the massive manatee. So it's only fair that we try to put things right.

To do that, we must understand what the data are telling us. Much of the time, data are messy. It can suggest multiple answers and show a certain amount of variation. On top of that, how data are presented can also be misleading. You saw as much yourself in this last lesson. While the data on Asian elephants remained fixed, a person's agenda could affect how they present that data. At the extreme, data can even be manipulated to support claims that are not necessarily true.

But, thanks to statistics, we have ways to manage that messiness and sniff out misleading claims. This is especially important in wildlife conservation. In nature, animal populations are constantly changing. But conservationists still need ways to confidently measure and report these numbers. With the mean and median, you can measure the center of a data set. This will tell you what's typical of a group. Meanwhile, a data set's mean absolute deviation (MAD) and interquartile range (IQR) will help describe how much variability is within that data set.

Just as the scientists, conservationists, and policy makers can use this type of data analysis to direct their energy toward helping the species that need it most, you can too — toward anything you are interested in or passionate about. How could you spend some time making sense of the data to help your community between now and next school year?

See you next year.

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Highlight that another important part of data is how data are collected and how much you can trust it. If the data are not accurate, analyzing it may lead to poor decisions.

Ask:

- "Can you think of other times you have seen data and how that data might have sculpted or changed your thinking? Did you ever wonder how accurate the data were? Will you now be aware of that?"
- "How can you be a critical consumer of statistical data? What questions can you ask yourself when you hear statistics, such as 50% of people believe . . ."

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Differentiated Support

Extension: ***Math Enrichment, Interdisciplinary Connections***

The table shows the percentage of the U.S. population for each state and the District of Columbia that are under 18 years of age or over 65 years of age. Provide students with the data and ask them to create a statistical display for each data set and describe their centers and spreads, taking into account the shape of the data distribution. Have students prepare a visual display and share their displays and descriptions with the class. **(Social Studies)**

Under 18 years	23	25	24	24	24	23	22	22	18	20	25	22	26	23	24	23	25
	22	24	24	23	22	25	23	20	23	24	21	23	23	23	25	22	21
	31	19	22	23	21	23	24	23	24	20	23	21	20	22	24	23	26
Over 65 years	15	10	16	16	13	13	15	16	11	19	12	16	14	14	16	14	15
	15	14	14	15	17	14	14	16	15	15	15	15	14	16	15	16	17
	12	10	17	14	14	18	15	14	18	14	15	14	16	16	16	15	15

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of their growth as an individual and as a peer by responding to three reflection questions.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.17

1. In Units 1 and 4, you identified areas of strengths and areas for growth. Reflect upon what you have done to build and grow as a:

   a. Mathematician.
   Answers may vary.

   b. Peer and collaborator.
   Answers may vary.

2. What have been some of your favorite experiences and new understandings from this year in math? Why?
   Answers may vary.

3. How might you be able to apply what you learned this year in the future, both inside and outside of school and of math class?
   Answers may vary.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can decide whether the mean and MAD or the median and IQR would be more appropriate for describing the center and spread of a data set in a real-world scenario.
1 2 3

b. I can justify why the mean and MAD or median and IQR would be more appropriate to support my perspective on a real-world scenario.
1 2 3

c. I can reflect upon the year as a growing math learner and collaborator.
1 2 3

Success looks like . . .

- **Language Goal:** Choosing an appropriate measure of center and variability to describe a data set, and justify the choice. **(Speaking and Listening, Writing)**
- **Language Goal:** Reflecting upon the year of mathematics as an individual learner and as a peer collaborator. **(Writing)**
  - » Reflecting on growth as a mathematician and a peer and collaborator in Problem 1.

Suggested next steps

If students feel overwhelmed with the questions or are feeling rushed to finish responding in the time given, consider:

- Letting them finish at home and return the responses the following day. Reminding students that this is an opportunity for them to reflect on the learning they have done this year, both as a mathematician and as a collaborator. They should be honest with themselves because this is not something that will be "graded."

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Have you changed any ideas you used to have about data collection as a result of today's lesson?
- What was especially satisfying about what students created in Activity 1, Part 3? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

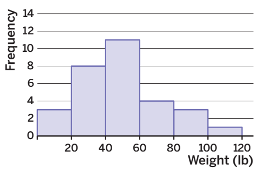
Practice

1. In this final unit, you answered statistical questions by representing and interpreting data. A common saying is "Data doesn't lie." Do you agree with this statement? Explain your thinking.

Sample response: I agree that data itself do not lie, but data interpretation can be used to mislead or draw conclusions that support your opinion.

2. Use the data and histogram of the weights of dogs at an adoption shelter. Is the histogram a good representation of the data? Why or why not?

17	17	18	20	25	26	27	29	32	37	39	41	45	46	47
48	52	52	53	57	58	59	62	70	73	78	82	89	95	101

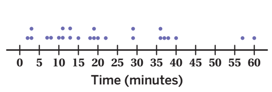


Sample response:

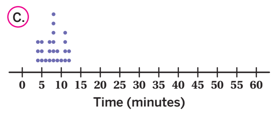
This histogram shows that the typical weight of the dogs is in the 50–60 range. It is not a good representation of the lowest and highest weights though.

3. The dot plots show how much time, in minutes, students in a class took to complete each of five different tasks. Select *all* the dot plots of tasks for which the mean time is approximately equal to the median time.

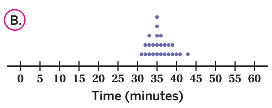
A.



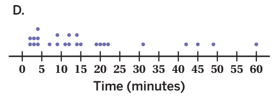
C.



B.



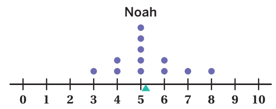
D.



Name: ______ Date: ______ Period: ______

Practice

4. Noah and Lin recorded their basketball scores the same way: the number of baskets made out of 10 attempts. Each collected 12 data values. Noah's mean number of baskets was 5.25, and his MAD was 1. Lin's mean number of baskets was 6.



Lin	6	5	4	7	6	5	7	8	5	6	5	8
Distance from 6	**0**	**1**	**2**	**1**	**0**	**1**	**1**	**2**	**1**	**0**	**1**	**2**

a. Calculate Lin's MAD and compare it to Noah's. What do you notice?

They have the same MAD of 1.

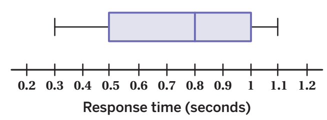
b. What do you notice about the distributions? Draw a dot plot if necessary.

Noah's distribution in symmetric, while Lin's is not.

c. What can you say about the two players' shooting accuracy and consistency?

Noah is more consistent, but Lin is more accurate.

5. The box plot displays the data on the response times of 100 mice to seeing a flash of light. How many mice are represented by the rectangle between 0.5 and 1 second?



50 mice

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
Spiral	2	Unit 8 Lesson 5	2
	3	Unit 8 Lesson 11	2
	4	Unit 8 Lesson 13	2
	5	Unit 8 Lesson 15	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Glossary/Glosario

English | **Español**

A

absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $|-3| = 3$.

Addition Property of Equality A property stating that if $a = b$, then $a + c = b + c$.

area The number of unit squares needed to fill a two-dimensional shape without gaps or overlaps.

average The average of a set of values is their sum divided by the number of values in the set. The average represents a fair share, or a leveling out of the distribution, so that each value in the set has the same frequency.

valor absoluto Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $|-3| = 3$.

Propiedad de igualdad en la suma Propiedad que establece que si $a = b$, entonces $a + c = b + c$.

área Número de unidades cuadradadas necesario para llenar una forma bidimensional sin dejar espacios vacíos ni superposiciones.

promedio El promedio de una serie de valores es su suma dividida por la cantidad de valores en el conjunto. El promedio representa una repartición justa, o igualada, de la distribución, de manera que cada valor del conjunto tenga la misma frecuencia.

B

base (of an exponential expression) The number that is raised to a power. The power indicates the number of times the base is multiplied by itself.

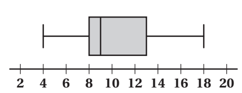
base (of a parallelogram) Any chosen side of the parallelogram.

base (of a prism) Either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.

base (of a pyramid) The face of a pyramid that is opposite from the vertex, where all the triangular faces connect.

base (of a triangle) Any chosen side of the triangle.

box plot A visual representation of the five-number summary for a numerical data set.



base (de una expresión exponencial) Número que se eleva a una potencia. La potencia indica el número de veces que la base debe multiplicarse por sí mismo.

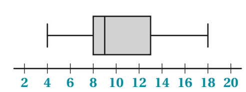
base (de un paralelogramo) Cualquier lado escogido del paralelogramo.

base (de un prisma) Cualquiera de las dos caras idénticas y paralelas de un prisma que están conectadas por un conjunto de caras rectangulares.

base (de una pirámide) La cara de una pirámide que se opone al vértice, donde todas las caras triangulares se conectan.

base (de un triángulo) Cualquier lado escogido del triángulo.

diagrama de cajas Representación visual del resumen de cinco números de un conjunto de datos numéricos.



C

categorical data Data that can be sorted into categories rather than counted, such as the different types of food bison eat or the colors of the rainbow.

center A value that represents the typical value of a data set.

datos categóricos Datos que pueden ser clasificados en categorías en vez de ser contados, como por ejemplo los diferentes tipos de comida que come un bisonte o los colores del arcoíris.

centro Valor que representa el valor típico de un conjunto de datos.

Glossary/Glosario

English

coefficient A number that is multiplied by a variable, typically written in front of or "next to" the variable, often without a multiplication symbol.

common factor A number that divides evenly into each of two or more given numbers.

common multiple A number that is a multiple of two or more given numbers.

compose To place together shapes or numbers, or to combine them.

coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both take on values that are positive, negative, or zero.

cubed The raising of a number to the third power (with an exponent of 3). This is read as that number, "cubed."

D

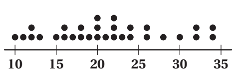
decompose To take apart a shape or number.

dependent variable In a relationship between two variables, the dependent variable represents the output values. The output values are unknown until the indicated calculations are performed on the independent variable.

distribution A collection of all of the data values and their frequencies. A distribution can be described by its features when represented visually, such as in a dot plot.

Division Property of Equality A property stating that if $a = b$ and c does not equal 0, then $a \div c = b \div c$

dot plot A representation of data in which the frequency of each value is shown by the number of dots drawn above that value on a horizontal number line. A dot plot can only be used to represent numerical data.



Español

coeficiente Número por el cual una variable es multiplicada, escrito comúnmente frente o junto a la variable sin un símbolo de multiplicación.

factor común Número que divide en partes iguales cada número de entre dos o más números dados.

múltiplo común Número que es múltiplo de dos o más números dados.

componer Unir formas o números, o combinarlos.

plano de coordenadas Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.

al cubo Un número elevado a la tercera potencia (con un exponente de 3) se lee como ese número "al cubo".

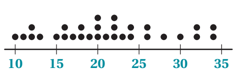
descomponer Desmontar una forma o un número.

variable dependiente En una relación entre dos variables, la variable dependiente representa los valores de salida. Los valores de salida son desconocidos hasta que se realizan los cálculos indicados sobre la variable independiente.

distribución Una colección de todos los valores de datos y sus frecuencias. Una distribución puede ser descrita según sus características cuando es representada en forma visual, como por ejemplo en un diagrama de puntos.

Propiedad de igualdad en la división Propiedad que establece que si $a = b$ y c no equivale a 0, entonces $a \div c = b \div c$.

diagrama de puntos Representación de datos en la cual la frecuencia de cada valor es equivalente al número de puntos que aparecen sobre dicho valor en una línea numérica horizontal. Un diagrama de puntos solo se puede usar para representar datos numéricos.



English	Español
E	**E**
edge A line segment where two faces of a three-dimensional figure meet. The term *edge* can also refer to the side of a two-dimensional shape.	**arista** Segmento de una línea donde se encuentran dos caras de una figura tridimensional. *Arista* puede también referirse al lado de una forma bidimensional.
equation Two expressions with an equal sign between them. When the two expressions are equal, the equation is true. An equation can also be false when the values of the two expressions are not equal.	**ecuación** Dos expresiones con un signo de igual entre ellas. Cuando las dos expresiones son iguales, la ecuación es verdadera. Una ecuación también puede ser falsa, cuando los valores de las dos expresiones no son iguales.
equivalent If two mathematical quantities (especially fractions, ratios, or expressions) are equal in any form, then they are *equivalent*.	**equivalente** Si dos objetos matemáticos (especialmente fracciones, razones o expresiones) son iguales de cualquier manera, entonces son *equivalentes*.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
equivalent fractions Two fractions that represent the same value or location on the number line.	**fracciones equivalentes** Dos fracciones que representan el mismo valor o la misma ubicación en la línea numérica.
equivalent ratios Any two ratios where the values for one quantity in each ratio can be multiplied or divided by the same number to get the values for the other quantity in each ratio.	**razones equivalentes** Dos razones para las cuales los valores de una cantidad en cada razón pueden ser multiplicados o divididos por el mismo número para obtener así los valores de la otra cantidad en cada razón.
exponent The number of times a factor is multiplied by itself.	**exponente** Número de veces que un factor es multiplicado por sí mismo.
expression A set of numbers, letters, operations, and grouping symbols that represent a quantity that can be calculated.	**expresión** Conjunto de números, letras, operaciones y símbolos de agrupamiento que representa una cantidad que puede ser calculada.
F	**F**
face One of many two-dimensional shapes that form the outer surface of a three-dimensional figure.	**cara** Una de muchas formas bidimensionales que forman la superficie externa de una figura tridimensional.
factor A number that divides evenly into a given whole number. For example, the factors of 15 are 1, 3, 5, and 15.	**factor** Número que divide de manera exacta a otro número dado. Por ejemplo, los factores de 15 son 1, 3, 5 y 15.
five-number summary The minimum, first quartile, median, third quartile, and maximum values of a data distribution.	**resumen de cinco números** El mínimo, el primer cuartil, la mediana, el tercer cuartil y los valores máximos de una distribución de datos.
frequency The number of times a value occurs in a data set.	**frecuencia** Número de veces que un valor está presente en un conjunto de datos.

Glossary/Glosario

English | Español

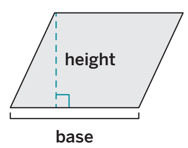
G

greatest common factor The common factor of two or more given whole numbers whose value is the greatest (often abbreviated as "GCF").

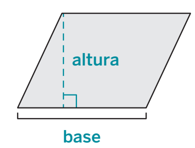
máximo factor común Factor común de dos o más números enteros dados, cuyo valor es el mayor (comúnmente abreviado como "MFC").

H

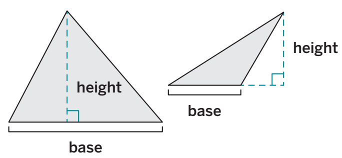
height (of a parallelogram) A segment measuring the shortest distance from the chosen base to the opposite side.



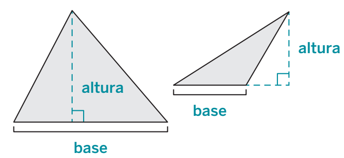
altura (de un paralelogramo) Segmento que mide la distancia más corta desde la base escogida hasta el lado opuesto.



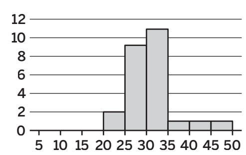
height (of a triangle) A segment representing the distance between the base and the opposite vertex.



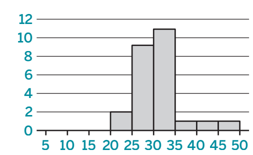
altura (de un triángulo) Segmento que representa la distancia entre la base y el vértice opuesto.



histogram A visual way to represent frequencies of numerical data values that have been grouped into intervals, called bins, along a number line. Bars are drawn above the bins where data exists, and the height of each bar reflects the frequency of the data values in that interval.



histograma Forma visual de representar frecuencias de valores de datos que han sido agrupados en intervalos, llamados contenedores, a lo largo de una línea numérica. Se dibujan barras sobre los contenedores donde existen los datos, y la altura de cada barra refleja la frecuencia de los valores de datos en ese intervalo.



I

independent variable In a relationship between two variables, the independent variable represents the input values. Calculations are performed on the input values to determine the values of the dependent variable.

variable independiente En una relación entre dos variables, la variable independiente representa los valores de entrada. Se realizan cálculos con los valores de entrada para determinar los valores de la variable dependiente.

integers Whole numbers and their opposites.

enteros Números completos y sus opuestos.

interquartile range (IQR) A measure of spread (or variability) that is calculated as the difference between the third quartile (Q3) and the first quartile (Q1).

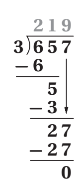
rango intercuartil (RIC) Medida de dispersión (es decir, de variabilidad) que es calculada mediante la diferencia entre el tercer cuartil (C3) y el primer cuartil (C1).

L

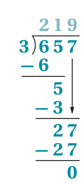
least common multiple The common multiple of two or more given whole numbers whose value is the least (often abbreviated as "LCM").

mínimo común múltiplo Múltiplo común de dos o más números enteros dados, cuyo valor es el menor (comúnmente abreviado como "MCM").

long division A method that shows the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right.



división larga Método que muestra los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha.



English

M

magnitude (of a number) The absolute value of a number, or the distance of a number from 0 on the number line.

maximum The value in a data set that is the greatest.

mean A measure of center that represents the average of all values in a data set. The mean represents a fair share distribution or a balancing point of all of the values in the data set.

mean absolute deviation (MAD) A measure of spread (or variability) calculated by determining the average of the distances between each data value and the mean.

measure of center A single number used to summarize the typical value of a data set.

measure of variability A single number used to summarize how the values in a data set vary.

median The middle value in the data set when the values are listed in order from least to greatest. When there is an even number of data points, the median is the average of the two middle values.

minimum The value in a data set that is the least.

mode The most frequently occurring value in a data set. A data set may have no mode, one mode, or more than one mode.

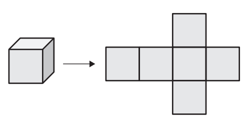
multiple A number that is the product of a given number and a whole number. For example, multiples of 7 include 7, 14, and 21.

Multiplication Property of Equality A property stating that, if $a = b$, then $a \cdot c = b \cdot c$.

N

negative number A number whose value is less than zero.

net A two-dimensional representation, or "flattening," of a three-dimensional solid's surface that shows all of its faces.



numerical data Numbers, quantities, or measurements that can be meaningfully compared.

Español

M

magnitud (de un número) Valor absoluto de un número, o la distancia de un número con respecto al 0 en la línea numérica.

máximo El valor más grande en un conjunto de datos.

media Medida del centro que representa el promedio de todos los valores de un conjunto de datos. La media representa una distribución equitativa o un punto de equilibrio entre todos los puntos del conjunto de datos.

desviación absoluta media (DAM) Medida de dispersión (o variabilidad) que se calcula mediante la obtención del promedio de la distancia entre cada valor de datos y la media.

medida de centro Número individual que se utiliza para resumir el valor típico en un conjunto de datos.

medida de variabilidad Número individual que se utiliza para resumir cómo varían los valores en un conjunto de datos.

mediana Valor medio de un conjunto de datos cuando sus valores están ordenados de menor a mayor. Cuando la cantidad de puntos de datos es par la mediana es el promedio de los dos valores medios.

mínimo Valor que es el menor de un conjunto de datos.

modo Valor que aparece con mayor frecuencia en un conjunto de datos. Un conjunto de datos puede tener un modo, más de un modo o ningún modo.

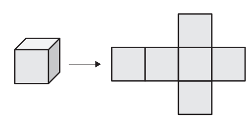
múltiplo Número que es el producto de un número dado y un número entero. Por ejemplo, entre los múltiplos de 7 se incluyen 7, 14 y 21.

Propiedad de igualdad en la multiplicación Propiedad que establece que si $a = b$, entonces $a \cdot c = b \cdot c$.

N

número negativo Número cuyo valor es menor que cero.

red Representación bidimensional, o "aplanamiento", de la superficie de un sólido tridimensional, para mostrar todas sus caras.



datos numéricos Números, cantidades o medidas que pueden ser comparadas de manera significativa.

Glossary/Glosario

O

English

opposite numbers Two numbers that are the same distance from 0, but are on different sides of the number line.

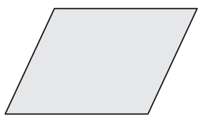
Español

números opuestos Dos números que están a la misma distancia de 0, pero que están en lados diferentes de la línea numérica.

P

English

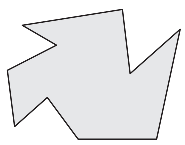
parallelogram A type of quadrilateral with two pairs of parallel sides.



per For each.

percentage A rate per 100. (A specific *percentage* is also called a *percent*, such as "70 percent.")

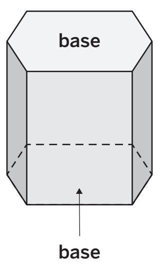
polygon A closed, two-dimensional shape with straight sides that do not cross each other.



polyhedron A closed, three-dimensional shape with flat sides. (The plural of *polyhedron* is *polyhedra*.)

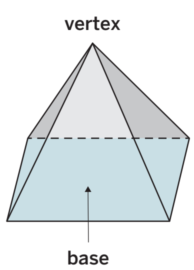
positive number A number whose value is greater than zero.

prism A three-dimensional figure with two parallel, identical faces (called *bases*) that are connected by a set of rectangular faces.



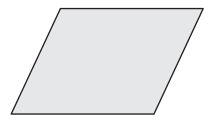
properties of equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that if an equation is true, then performing the same operation to both sides will result in an equivalent equation.

pyramid A three-dimensional figure with one base and a set of triangular faces that meet at a singular vertex.



Español

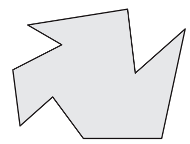
paralelogramo Tipo de cuadrilátero con dos pares de lados paralelos.



por Por cada uno de los elementos.

porcentaje Tasa por cada 100. (Un *porcentaje* específico también es llamado *por ciento*, como por ejemplo "70 por ciento".)

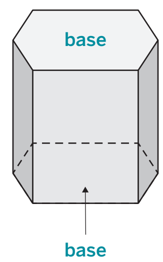
polígono Forma cerrada y bidimensional de lados rectos que no se entrecruzan.



poliedro Forma cerrada y tridimensional de lados planos.

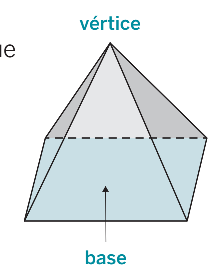
número positivo Número cuyo valor es mayor que cero.

prisma Figura tridimensional con dos caras iguales y paralelas (llamadas *bases*) que se conectan entre sí a través de un conjunto de caras rectangulares.



propiedades de igualdad Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al realizar la misma operación en ambos lados se obtendrá una ecuación equivalente.

pirámide Figura tridimensional con una base y un conjunto de caras triangulares que se conectan en un solo vértice.

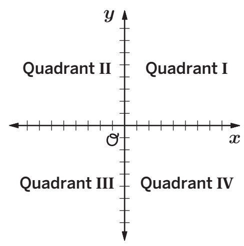


English

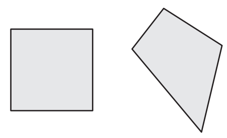
Español

Q

quadrant Each of the four regions of the coordinate plane formed by the vertical and horizontal axes. The quadrants are labeled counterclockwise from top right to bottom right as I, II, III, IV.

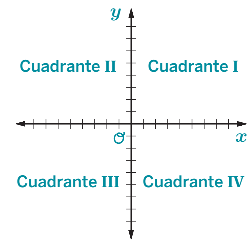


quadrilateral A polygon with exactly four sides.

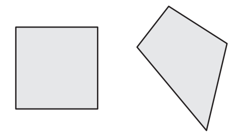


quartile One of three numbers (Q1, Q2, Q3) that divide an ordered data set into four sections so that each section contains 25% of data points.

cuadrante Cada una de las cuatro regiones del plano de coordenadas formado por los ejes vertical y horizontal. Los cuadrantes se identifican en sentido contrario a las agujas del reloj, desde la parte superior derecha a la parte inferior derecha, como I, II, III y IV.



cuadrilátero Polígono de exactamente cuatro lados.



cuartil Uno de los tres números (C1, C2, C3) que dividen un conjunto ordenado de datos en cuatro secciones, de manera que cada sección contenga el 25% de los puntos de datos.

R

range A measure of spread (or variability) that is calculated as the difference between the maximum and minimum values in the data set.

rate A comparison of how two quantities change together.

ratio A comparison of two quantities, such that for every a units of one quantity, there are b units of another quantity.

rational numbers The set of all the numbers that can be written as positive or negative fractions.

ratio relationship A relationship between quantities that establishes that the values for each quantity will always change together in the same way.

ratio table A table of values organized in columns and rows that contains equivalent ratios.

reciprocal Two numbers whose product is 1 are *reciprocals* of each other. (When written in simplest fraction form, the numerator of each number corresponds to the denominator of the other number. For example, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals.)

region The space inside a shape or figure.

rango Medida de dispersión (o variabilidad) que es calculada mediante la diferencia entre los valores máximos y mínimos de un conjunto de datos.

tasa Comparación de cuánto cambian dos cantidades en conjunto.

razón Una comparación entre dos cantidades, de modo tal que por cada a unidades de una cantidad, hay b unidades de la otra cantidad

números racionales Conjunto que consta de todos los números que pueden ser escritos como fracciones positivas o negativas.

relación de razón Relación entre cantidades que establece que los valores para cada cantidad siempre cambiarán en conjunto de la misma manera.

tabla de razones Tabla de valores organizada en columnas y filas que contiene razones equivalentes.

recíproco/a Dos números cuyo producto es 1 son *recíprocos* entre sí. (Al escribirlo en la forma de fracción más simple, el numerador de cada número corresponde al denominador del otro número. Por ejemplo, $\frac{3}{5}$ y $\frac{5}{3}$ son recíprocos.)

región Espacio al interior de una forma o figura.

Glossary/Glosario

S

sign (of a number) Indication of whether a number is positive or negative.

solution to an equation A number that can be substituted in place of a variable to make an equation true.

solution to an inequality Any number that can be substituted in place of a variable to make an inequality true.

spread The variability of a distribution. A description of how the data values in the distribution vary from the center of the distribution.

squared The raising of a number to the second power (with an exponent of 2). This is read as that number, "squared."

statistical question A question that anticipates variability and can be answered by collecting data.

Subtraction Property of Equality For rational numbers a, b, and c, if $a = b$, then $a - c = b - c$.

surface area The number of unit squares needed to cover all of the faces of a three-dimensional figure without gaps or overlaps.

signo (de un número) Indicación de si un número es positivo o negativo.

solución a una ecuación Número que puede sustituir una variable para volver verdadera una ecuación.

solución a una desigualdad Cualquier número que puede sustituir una variable para volver verdadera una desigualdad.

dispersión Variabilidad de una distribución. Descripción de cuánto varían los valores en comparación con el centro de una distribución de datos.

al cuadrado Un número elevado a la segunda potencia (con un exponente de 2) se lee como ese número "al cuadrado".

pregunta estadística Pregunta que anticipa variabilidad y que se puede responder mediante la recolección de datos.

Propiedad de igualdad en la resta Para los números racionales a, b y c, si $a = b$, entonces $a - c = b - c$.

área de superficie Número de unidades cuadradas necesarias para cubrir todas las caras de una figura tridimensional sin dejar espacios vacíos ni superposiciones.

T

tape diagram A model in which quantities are represented as lengths (of tape) placed end-to-end, and which can be used to show addition, subtraction, multiplication, or division.

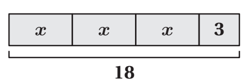
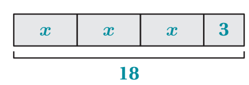


diagrama de cinta Modelo en el cual las cantidades están representadas como longitudes (de una cinta) colocadas de forma continua, y que pueden ser usadas para mostrar suma, resta, multiplicación y división.



U

unit rate How much one quantity changes when the other changes by 1.

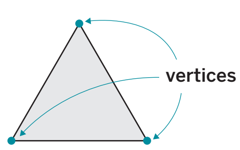
tasa unitaria Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.

V

variability The spread of a distribution. A description of how the data values in the distribution vary from the center of the distribution.

variable A letter that represents an unknown number in an expression or equation.

vertex A point where two sides of a two-dimensional shape or two or more edges of a three-dimensional figure intersect. (The plural of *vertex* is *vertices*.)

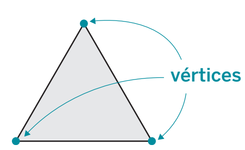


volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.

variabilidad La dispersión de una distribución. Descripción de cuánto varían los valores en comparación con el centro de una distribución de datos.

variable Letra que representa un número desconocido en una expresión o ecuación.

vértice Punto donde se intersecan dos lados de una forma bidimensional, o dos o más aristas de una figura tridimensional.



volumen Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

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