Amplify Math
TENNESSEE

Teacher Edition
Grade 7 | Volume 1

FEATURING Amps POWERED BY desmos

Amplify Math

Grade 7

Volume 1: Units 1–4

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Dear Teacher,

Thank you for choosing Amplify Math. Our team has worked hard to ensure you have strong, easy-to-follow instructional supports that get all students talking and thinking together about grade-level math.

You can learn more about the program design in the pages that follow, but we wanted to call out a few things that really drive our point home about the student experience:

Make math social

The student experience is **social and collaborative**. Your students will have plenty of opportunities to talk through their reasoning and work with their peers to come to new understandings. Thanks to our **partnership with Desmos**, you can kick off these social math experiences both offline and while logged in.

Power-ups

Your priority is teaching grade-level math to each and every student in your classroom, every day. To help with that, we've developed **Power-ups** to provide just-in-time support for your students.

Narrative

We kick off each sub-unit with a short, **engaging narrative** about historical or current-day events or phenomena. That way, your students can see the relevance of math in their everyday lives.

Featured Mathematicians

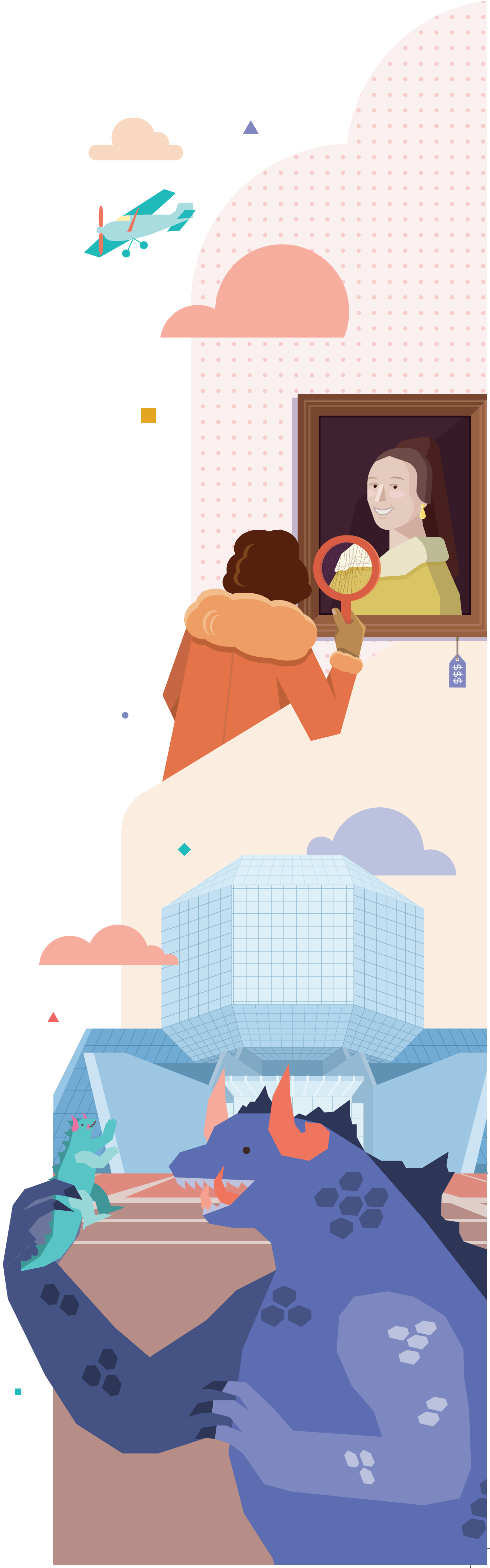
It's important to us that students see themselves in our materials. To that end, we've woven in **the work of innovative mathematical thinkers**. We've also included some of their personal stories, so that students can see themselves mirrored in the living history of mathematics.

Data

We provide plenty of **data to help you drive your instruction** and talk about student performance with your colleagues, as well as with caregivers who may not have had the best experiences with math when they were in school.

We hope you see the quality work of our team of editors and advisors in the program.

Sincerely,
The Amplify Math Team



Acknowledgments

Program Advisors

Amplify gratefully acknowledges the outstanding contributions and work of distinguished program advisors who have been integral to the development of Amplify Math. This product is testimony to their expertise, understanding of student learning needs, and dedication to rigorous and equitable mathematics instruction.

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Amplify gratefully acknowledges the outstanding contributions and work of esteemed members of our Educator Advisory Board. This product exhibits their respected observations and reflections.

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- San Juan Unified School District, California
- Santa Paula Unified School District, California
- Silver Summit Academy, Utah
- Streetsboro City Schools, Ohio
- West Contra Costa Unified School District, California
- Wyoming City Schools, Ohio
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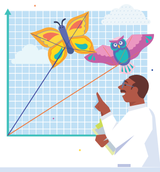
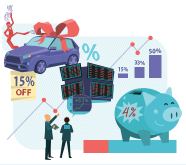
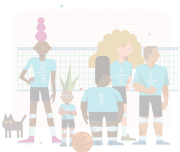
Program Scope and Sequence

Volume 1

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 160 days total	Area and Surface Area 20 Instructional Days 3 Assessment Days 23 days total	Introducing Ratios 20 Instructional Days 2 Assessment Days 22 days total	Rates and Percentages 15 Instructional Days 2 Assessment Days 17 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 153 days total	 **Scale Drawings** 13 Instructional Days 2 Assessment Days 15 days total	 **Introducing Proportional Relationships** 17 Instructional Days 2 Assessment Days 19 days total	 **Measuring Circles** 12 Instructional Days 2 Assessment Days 14 days total	 **Percentages** 13 Instructional Days 2 Assessment Days 15 days total
Grade 8 145 days total	Rigid Transformation and Congruence 18 Instructional Days 3 Assessment Days 21 days total	Dilations and Similarity 12 Instructional Days 2 Assessment Days 14 days total	Linear Relationships 19 Instructional Days 2 Assessment Days 21 days total	Linear Equations and Systems of Linear Equations 17 Instructional Days 2 Assessment Days 19 days total
Algebra 1 157 days total	Linear Equations, Inequalities, and Systems 26 Instructional Days 3 Assessment Days 29 days total	Data Analysis and Statistics 22 Instructional Days 3 Assessment Days 25 days total	Functions and Their Graphs 22 Instructional Days 3 Assessment Days 25 days total	Introducing Exponential Functions 22 Instructional Days 3 Assessment Days 25 days total

Unit 5	Unit 6	Unit 7	Unit 8
Arithmetic in Base Ten 14 Instructional Days 2 Assessment Days 16 days total	**Expressions and Equations** 19 Instructional Days 2 Assessment Days 21 days total	**Rational Numbers** 19 Instructional Days 2 Assessment Days 21 days total	**Data Sets and Distributions** 17 Instructional Days 3 Assessment Days 20 days total
 **Rational Number Arithmetic** 20 Instructional Days 3 Assessment Days 23 days total	 **Expressions, Equations, and Inequalities** 23 Instructional Days 3 Assessment Days 26 days total	 **Angles, Triangles, and Prisms** 18 Instructional Days 3 Assessment Days 21 days total	 **Probability and Sampling** 17 Instructional Days 3 Assessment Days 20 days total
Functions and Volume 21 Instructional Days 3 Assessment Days 24 days total	**Exponents and Scientific Notation** 15 Instructional Days 2 Assessment Days 17 days total	**Irrationals and the Pythagorean Theorem** 16 Instructional Days 2 Assessment Days 18 days total	**Associations in Data** 9 Instructional Days 2 Assessment Days 11 days total
Introducing Quadratic Functions 23 Instructional Days 3 Assessment Days 26 days total	**Quadratic Equations** 24 Instructional Days 3 Assessment Days 27 days total		

Volume 1

Unit 1 Scale Drawings

Certain objects in our universe exist at sizes and distances that are impossible for our eyes to see (such as a red blood cell, or Jupiter). In this unit, students harness the power of scaling — bringing large and small objects to a manageable size without distorting them.

Unit Narrative:
Life in the
Little Big City

Sub-Unit Narrative:
How do you get the perfect fit?

If we are making a larger or smaller copy of something, it needs to look right. The key is the scale factor.

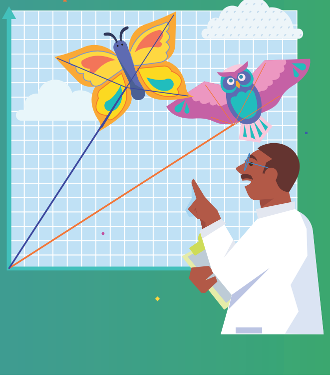
Sub-Unit Narrative:
Who was the King of Monsters?

We use maps and other scale drawings to help simplify large, complex places. Interpreting them is about knowing the scale and how to measure.

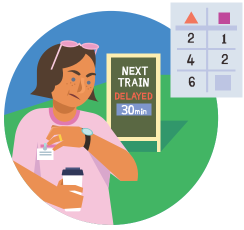
Unit 2 Introducing Proportional Relationships

When we exchange money from one currency to another, there is a rate that helps us find the amount of one currency equal in value to the other. Students see that a rate is at the heart of every proportional relationship as they encounter problems across cultures where two quantities are directly related.

Unit Narrative:
The World in Proportion



Sub-Unit Narrative:
Who was the original globetrotter?
Tables help keep us organized, but equations tell an entire story with just a few symbols. We'll use both of them to represent proportional relationships.



Sub-Unit Narrative:
Narrative: What good is a graph?
We turn to drawing, interpreting, and comparing proportional relationships in graphs, and notice what is particular to these types of graphs.

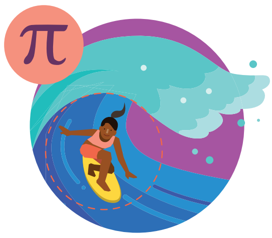
Unit 3 Measuring Circles

Identifying a circle may be straightforward, but measuring it is decidedly not. Students experience both the usefulness and challenges presented by this “perfect” shape.

Unit Narrative:
‘Round and
‘Round We Go

Sub-Unit Narrative:
Why do aliens love circles?

Circles are famously difficult to measure precisely, but that won’t stop us from trying. Let’s see how close we can get.



Sub-Unit Narrative:
What makes a circle so perfect?

Squares and circles may not have much in common, but we’ll need both to measure a circle’s area.

Unit 4 Percentages

From the supermarket to the stock market, percents are relied on to communicate quickly about how much something has changed. Students build on their experience with proportional relationships while using percentages to compare quantities within the friendly confines of the number 100.

Unit Narrative:
Keepin' it 100

Sub-Unit 1 Percent Increase and Decrease

Sub-Unit Narrative:
Is there truth in numbers?

Numbers never lie, but should we always believe them? Percentages can show how something changes – if we pay careful attention to the original amount.

Sub-Unit 2 Applying Percentages

Sub-Unit Narrative:
Did a quarantined U.S. keep a healthy economy?

See why percentages are used to calculate taxes, tips, interest, and other amounts when spending or saving money.

Volume 2

Unit 5 Rational Number Arithmetic

Students discover the need to work with both positive and negative values to describe the vastness of the world around them. With the entire set of rational numbers and all four operations now at their disposal, the sky (or the sea floor) is the limit.

Unit Narrative:
A World of Opposites

Sub-Unit Narrative:
What was Jeanne Baret's big secret?
Sure, you've probably been adding and subtracting for many years, but have you ever tried to take something away when you had less than zero to start with?

Sub-Unit Narrative:
Who was the toughest Grandma to ever hike the Appalachian Trail?
Travel forwards and backwards in time to help make sense of multiplication and division of negative numbers.

Sub-Unit Narrative:
How do you climb the world's most dangerous mountain?
Put it all together — adding, subtracting, multiplying, and dividing with rational numbers — while exercising your algebraic thinking muscles in a sneak preview of the next unit.

Unit 6 Expressions, Equations, and Inequalities

Unit Narrative:
Solving One Step
at a Time

Students return to the study of algebra and focus on how representation plays such a large role in communicating mathematical ideas. In this unit, the symbols, language, and drawings students use will help them tell the stories they see in the numbers.

Sub-Unit Narrative:
What are the first words you learn in "Caveman"?
Dog walking, tools of early civilization, and hangers all come together to help you explore new ways of solving equations.

Sub-Unit Narrative:
Who were the VIPs of ancient Egypt?
Solving word problems is about making meaning of the quantities, and tape diagrams return to help.

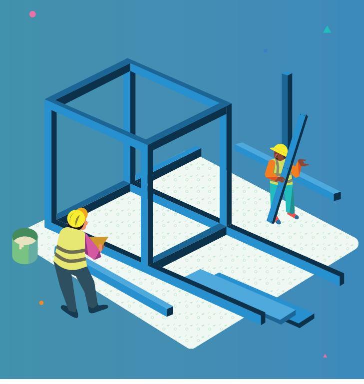
Sub-Unit Narrative:
Did a member of the School of Night infiltrate your math class?
Expressions are not always equal, so we must reckon with inequalities. Thankfully, finding their solutions will feel familiar.

Sub-Unit Narrative:
Which three blockheads did NASA send into space?
Find efficiencies for simplifying expressions like the Distributive Property and combining like terms.

Unit 7 Angles, Triangles, and Prisms

This unit is about the math of what can be seen and what can be held. Through constructing and drawing, students explore relationships among angles, lines, surfaces, and solids.

Unit Narrative:
Journey to the Third Dimension



Sub-Unit Narrative:
Did radio kill the aviation star?

As you'll see, some angles were just meant to go together. Here, you'll be introduced to complementary, supplementary, and vertical angles.

Sub-Unit Narrative:
How did triangles help win a war?

In this Sub-Unit, you will find that constructing polygons with specific lengths and angle measures can have dramatically different results.

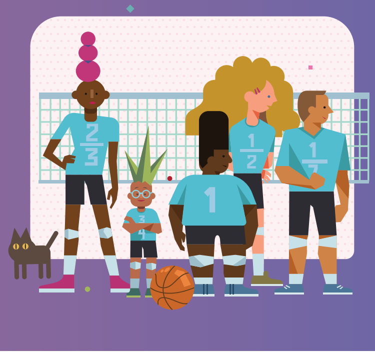
Sub-Unit Narrative:
This machine will slice, but will it dice?

You've studied the surfaces of three-dimensional figures and the spaces inside them. Now, let's see what happens when we slice them open.

Unit 8 Probability and Sampling

For the first time, students encounter how to quantify the chances of something happening. Though the future is unwritten, probability and statistics help us make better predictions and thus better decisions.

Unit Narrative:
Winning Chance



Sub-Unit Narrative:
How did the women of Bletchley Park save the free world?

Welcome to probability, the math of games and chance. Discover how probability can reveal hidden information, even secret codes.

Sub-Unit Narrative:
How did a blazing shoal bring the Philadelphia Convention Center to its feet?

When predicting the chances gets complicated, a simulation can help make predictions.

Sub-Unit Narrative:
What's on your mind?

Not all data is created equal. It is important to know how to identify when a sample is representative of a population.

Get all students talking and thinking about grade-level math.

Amplify Math was designed around the idea that core math needs to serve 100% of students in accessing grade-level math every day. To that end, the program delivers:

1 Productive discourse made easier to facilitate and more accessible for students

Clean and clear lesson design

The lessons all include straightforward "1, 2, 3 step" guidance for launching and facilitating discussions around the tasks. Thoughtful and specific differentiation supports are included for every activity. Every lesson ends with a summary and reflection moment, an Exit Ticket, and a practice problem set.

Narrative and storytelling

All students ask "Why do I need to know this? When am I ever going to use this in the real world?" Amplify Math helps students make the connections with math and their everyday lives to help them see and appreciate the relevance of the math they're figuring out in class. Throughout the units, students will be introduced to historical and current narratives that show the many places mathematics inhabits in our world and how the work they do in class connects to our history and their own reality.

2 Flexible, social problem-solving experiences online

Social learning experiences online

By partnering with Desmos, we've been able to deliver digital lessons, which we call **Amps**, that get students thinking, talking, revising, and celebrating their ideas. As students work in the interactive slides, new functionality may appear and they will often be asked to justify their actions and thinking. All of this is made visible to the teacher in real time.

Automatically differentiated activities

Our **Power-ups** automatically provide differentiated activities to students who need pre-requisite support, based on performance on past problems and assessments. They're available in this Teacher Edition, too. Phil Daro partnered with us on this feature to ensure we were giving all students, even the ones who might be three years behind in math, but only 15 minutes behind the day's lesson, the chance to experience success in math.

3 Real-time insights, data, and reporting that inform instruction

Teacher orchestration tools

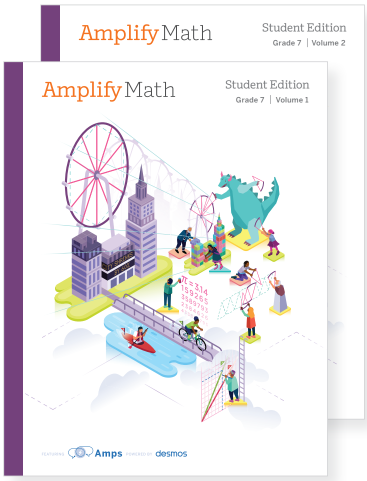
Once a teacher launches an Amp, students will be automatically moved to the lesson of the day and will see the interactive screens. Teachers will have the ability not only to pace the lesson the way they want to, but also to see student work in real time. The orchestration tools offer teachers ways to overlay student work to spot misconceptions and also the ability to spotlight student work anonymously to discuss with the class.

Embedded and standalone assessments

Amplify Math includes both a suite of standalone assessments and embedded assessments that allow teachers and leaders insights into where students are and how they might best be supported. The full reporting suite covers student and class performance based on work done in lessons, Exit Tickets, and practice sets, performance by standards, and performance on Interim assessments.

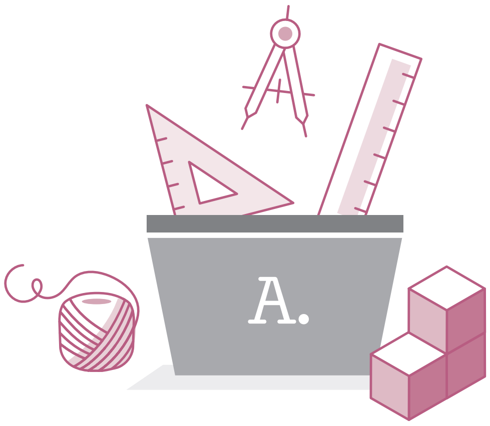
Amplify Math resources

Student Materials



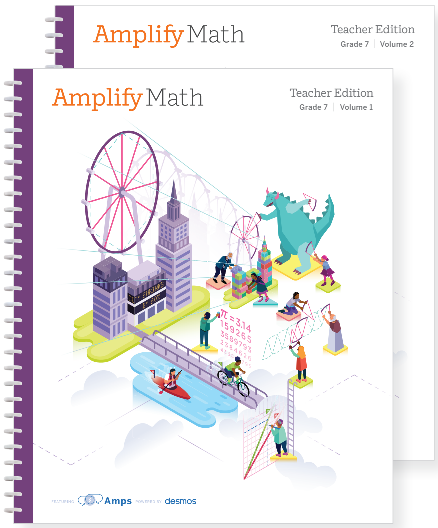
Student workbooks, 2 volumes

Amps, our exclusive collection of digital lessons powered by desmos



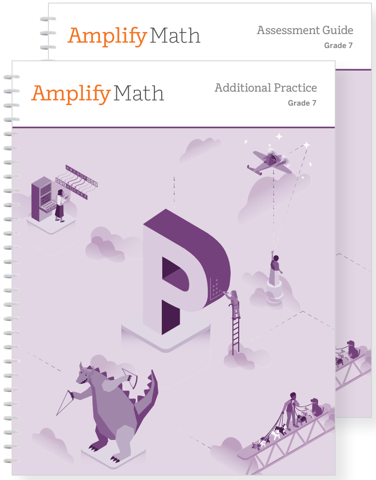
Hands-on manipulatives (optional)

Teacher Materials



Teacher Edition, 2 volumes

Digital Teacher Edition and classroom monitoring tools



Additional Practice and Assessment Guide blackline masters

Program architecture

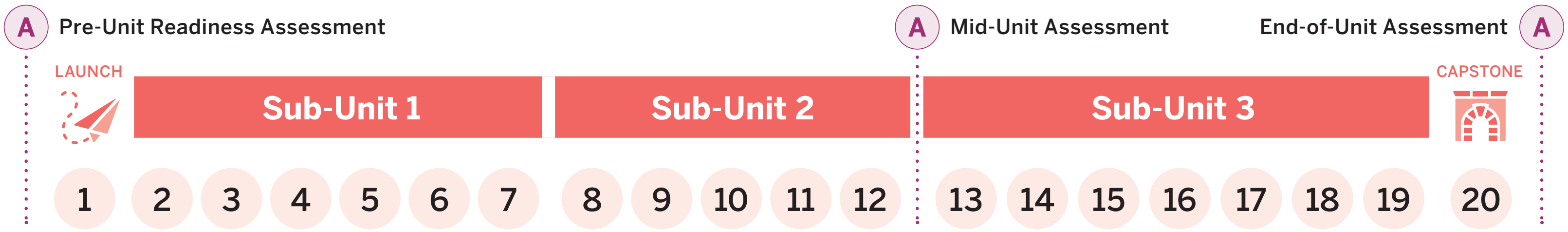
Course

I Interim Assesments

UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6	UNIT 7	UNIT 8
13 lessons	17 lessons	12 lessons	13 lessons	20 lessons	23 lessons	18 lessons	17 lessons

Note: Interim assessments may be administered according to your district/school's timeline; this depiction is just one of many possible administrations.

Unit



Note: The number of sub-units and lessons vary from unit to unit; this depiction shows the general structure of a unit.

Lesson

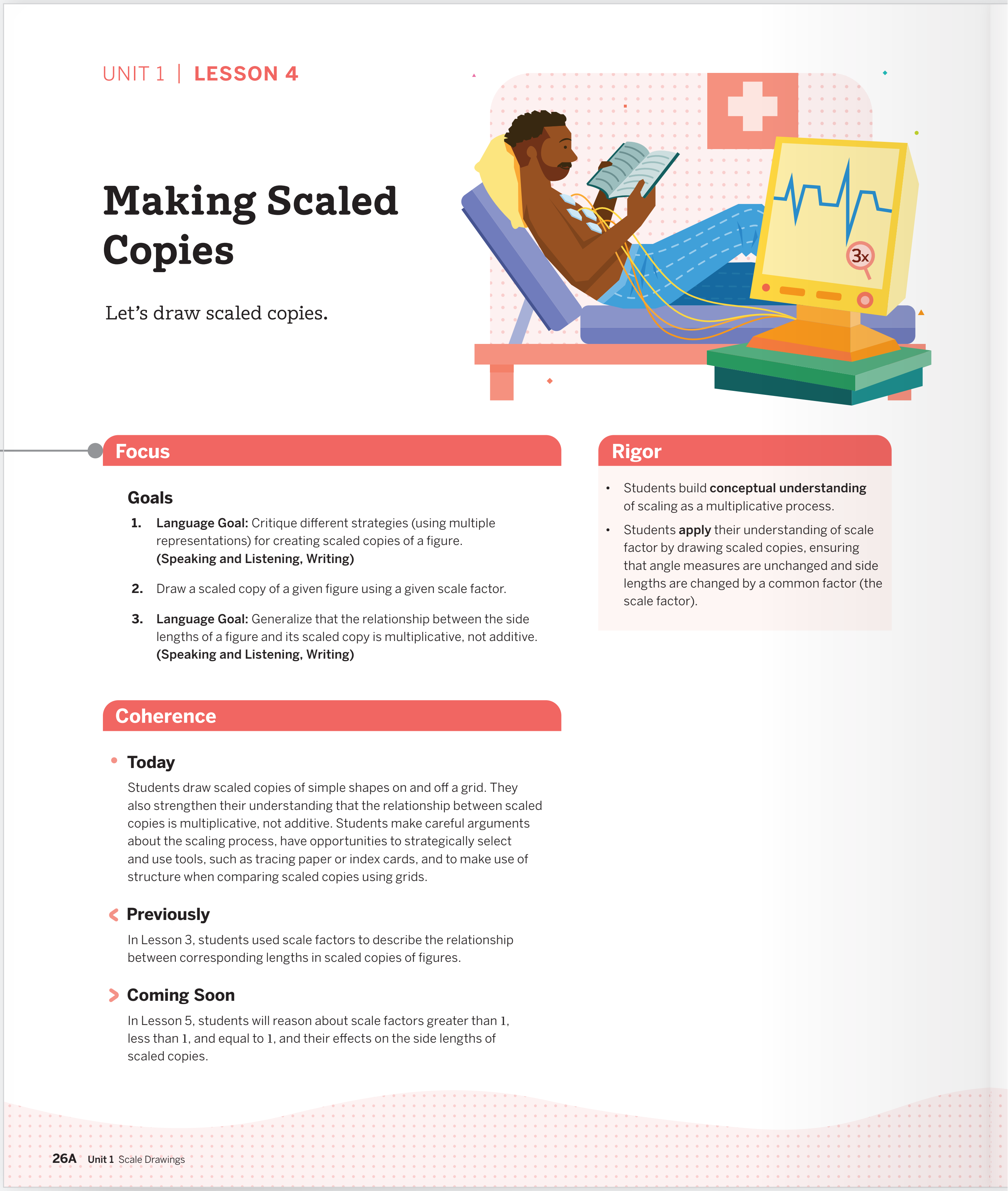
Note: The number of activities vary from lesson to lesson; this depiction shows the general structure of a lesson.

Key:
- Independent
- Pairs
- Small Groups
- Whole Class

Navigating This Program

Lesson Brief

Lesson goals, **coherence mapping**, and a breakdown for how **conceptual understanding**, **procedural fluency**, and **application** are addressed are included for each lesson.



UNIT 1 | **LESSON 4**

Making Scaled Copies

Let's draw scaled copies.

Focus

Goals

1. **Language Goal:** Critique different strategies (using multiple representations) for creating scaled copies of a figure. **(Speaking and Listening, Writing)**
2. Draw a scaled copy of a given figure using a given scale factor.
3. **Language Goal:** Generalize that the relationship between the side lengths of a figure and its scaled copy is multiplicative, not additive. **(Speaking and Listening, Writing)**

Coherence

Today

Students draw scaled copies of simple shapes on and off a grid. They also strengthen their understanding that the relationship between scaled copies is multiplicative, not additive. Students make careful arguments about the scaling process, have opportunities to strategically select and use tools, such as tracing paper or index cards, and to make use of structure when comparing scaled copies using grids.

Previously

In Lesson 3, students used scale factors to describe the relationship between corresponding lengths in scaled copies of figures.

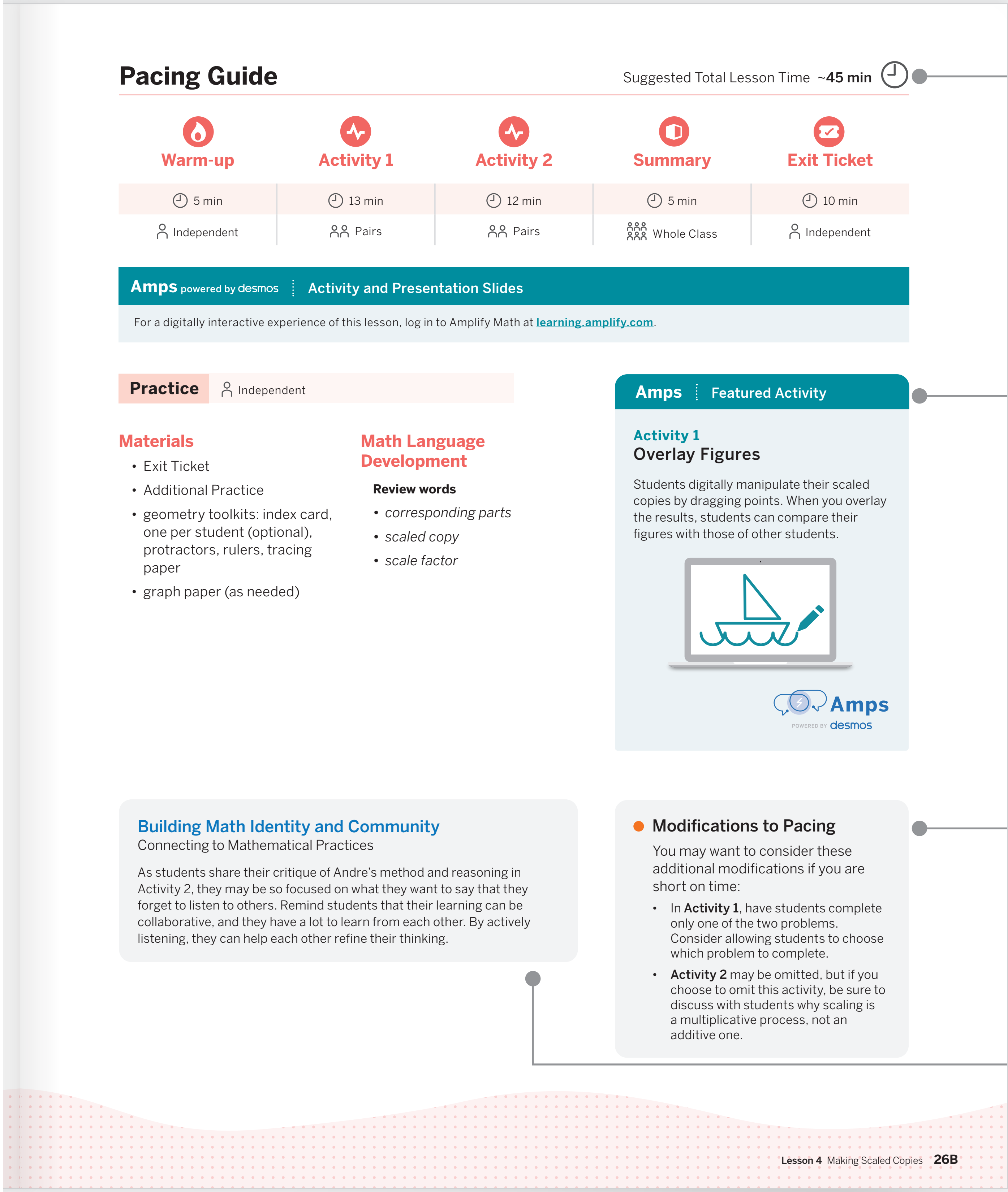
Coming Soon

In Lesson 5, students will reason about scale factors greater than 1, less than 1, and equal to 1, and their effects on the side lengths of scaled copies.

Rigor

- Students build **conceptual understanding** of scaling as a multiplicative process.
- Students **apply** their understanding of scale factor by drawing scaled copies, ensuring that angle measures are unchanged and side lengths are changed by a common factor (the scale factor).

26A Unit 1 Scale Drawings



Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	13 min	12 min	5 min	10 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: index card, one per student (optional), protractors, rulers, tracing paper
- graph paper (as needed)

Math Language Development

Review words

- *corresponding parts*
- *scaled copy*
- *scale factor*

Amps | Featured Activity

Activity 1
Overlay Figures

Students digitally manipulate their scaled copies by dragging points. When you overlay the results, students can compare their figures with those of other students.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students share their critique of Andre's method and reasoning in Activity 2, they may be so focused on what they want to say that they forget to listen to others. Remind students that their learning can be collaborative, and they have a lot to learn from each other. By actively listening, they can help each other refine their thinking.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, have students complete only one of the two problems. Consider allowing students to choose which problem to complete.
- **Activity 2** may be omitted, but if you choose to omit this activity, be sure to discuss with students why scaling is a multiplicative process, not an additive one.

Lesson 4 Making Scaled Copies 26B

Suggested timing for the lesson and each activity is included for quick reference.

The benefits of teaching one or more of the activities **online** are outlined for each lesson.

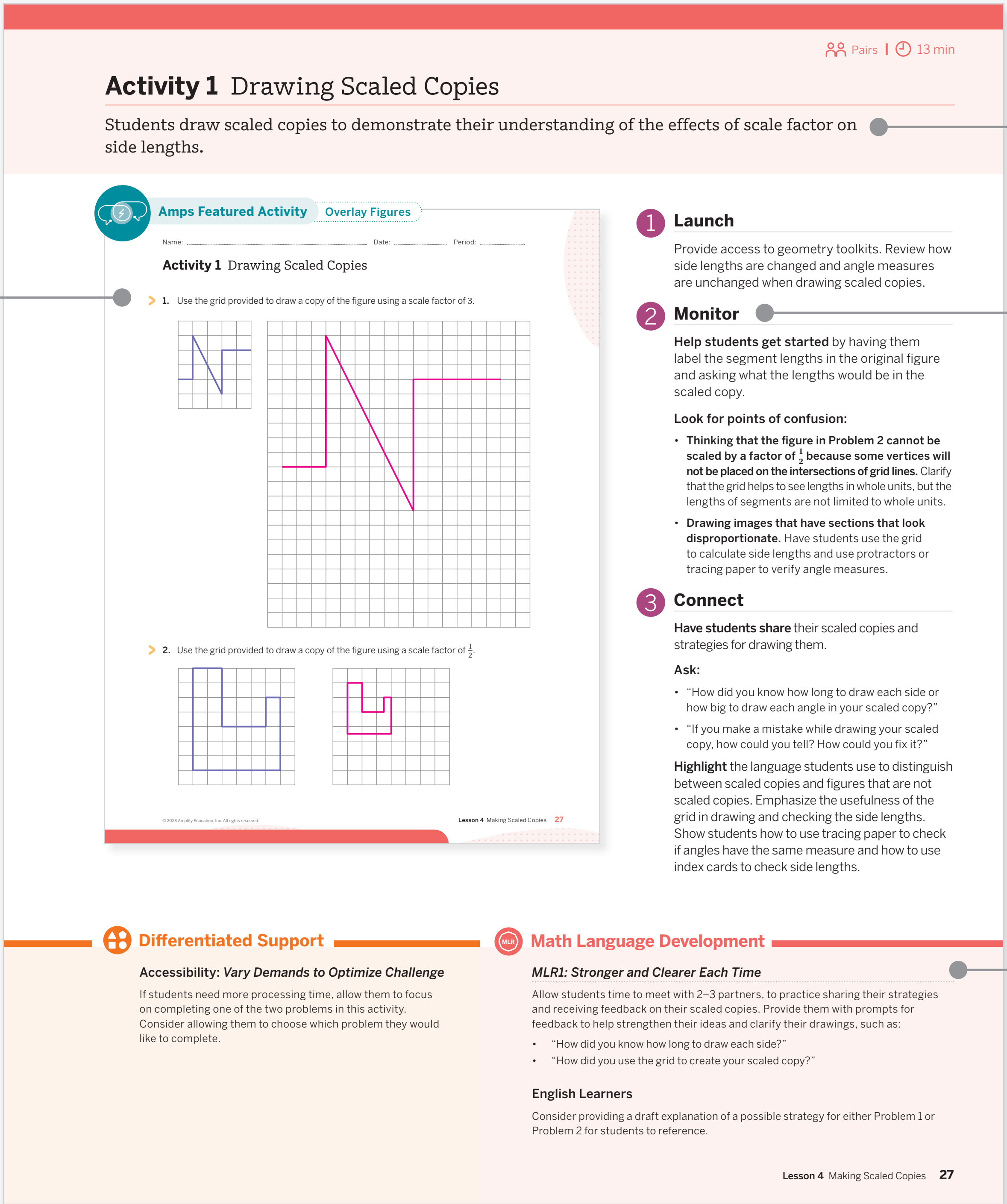
Every lesson pacing guide includes **modification** suggestions.

Building Math Identity and Community supports for teachers are included in the Lesson Brief. Student supports appear online and in the printed Student Edition.

Navigating This Program

Lesson

The **student-facing** content is presented to the left.



Pairs | 13 min

Activity 1 Drawing Scaled Copies

Students draw scaled copies to demonstrate their understanding of the effects of scale factor on side lengths.

Amps Featured Activity Overlay Figures

Name: Date: Period:

Activity 1 Drawing Scaled Copies

1. Use the grid provided to draw a copy of the figure using a scale factor of 3.

2. Use the grid provided to draw a copy of the figure using a scale factor of $\frac{1}{2}$.

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Lesson 4 Making Scaled Copies 27

1 Launch

Provide access to geometry toolkits. Review how side lengths are changed and angle measures are unchanged when drawing scaled copies.

2 Monitor

Help students get started by having them label the segment lengths in the original figure and asking what the lengths would be in the scaled copy.

Look for points of confusion:

- **Thinking that the figure in Problem 2 cannot be scaled by a factor of $\frac{1}{2}$ because some vertices will not be placed on the intersections of grid lines.** Clarify that the grid helps to see lengths in whole units, but the lengths of segments are not limited to whole units.
- **Drawing images that have sections that look disproportionate.** Have students use the grid to calculate side lengths and use protractors or tracing paper to verify angle measures.

3 Connect

Have students share their scaled copies and strategies for drawing them.

Ask:

- "How did you know how long to draw each side or how big to draw each angle in your scaled copy?"
- "If you make a mistake while drawing your scaled copy, how could you tell? How could you fix it?"

Highlight the language students use to distinguish between scaled copies and figures that are not scaled copies. Emphasize the usefulness of the grid in drawing and checking the side lengths. Show students how to use tracing paper to check if angles have the same measure and how to use index cards to check side lengths.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, allow them to focus on completing one of the two problems in this activity. Consider allowing them to choose which problem they would like to complete.

Math Language Development

MLR1: Stronger and Clearer Each Time

Allow students time to meet with 2–3 partners, to practice sharing their strategies and receiving feedback on their scaled copies. Provide them with prompts for feedback to help strengthen their ideas and clarify their drawings, such as:

- "How did you know how long to draw each side?"
- "How did you use the grid to create your scaled copy?"

English Learners

Consider providing a draft explanation of a possible strategy for either Problem 1 or Problem 2 for students to reference.

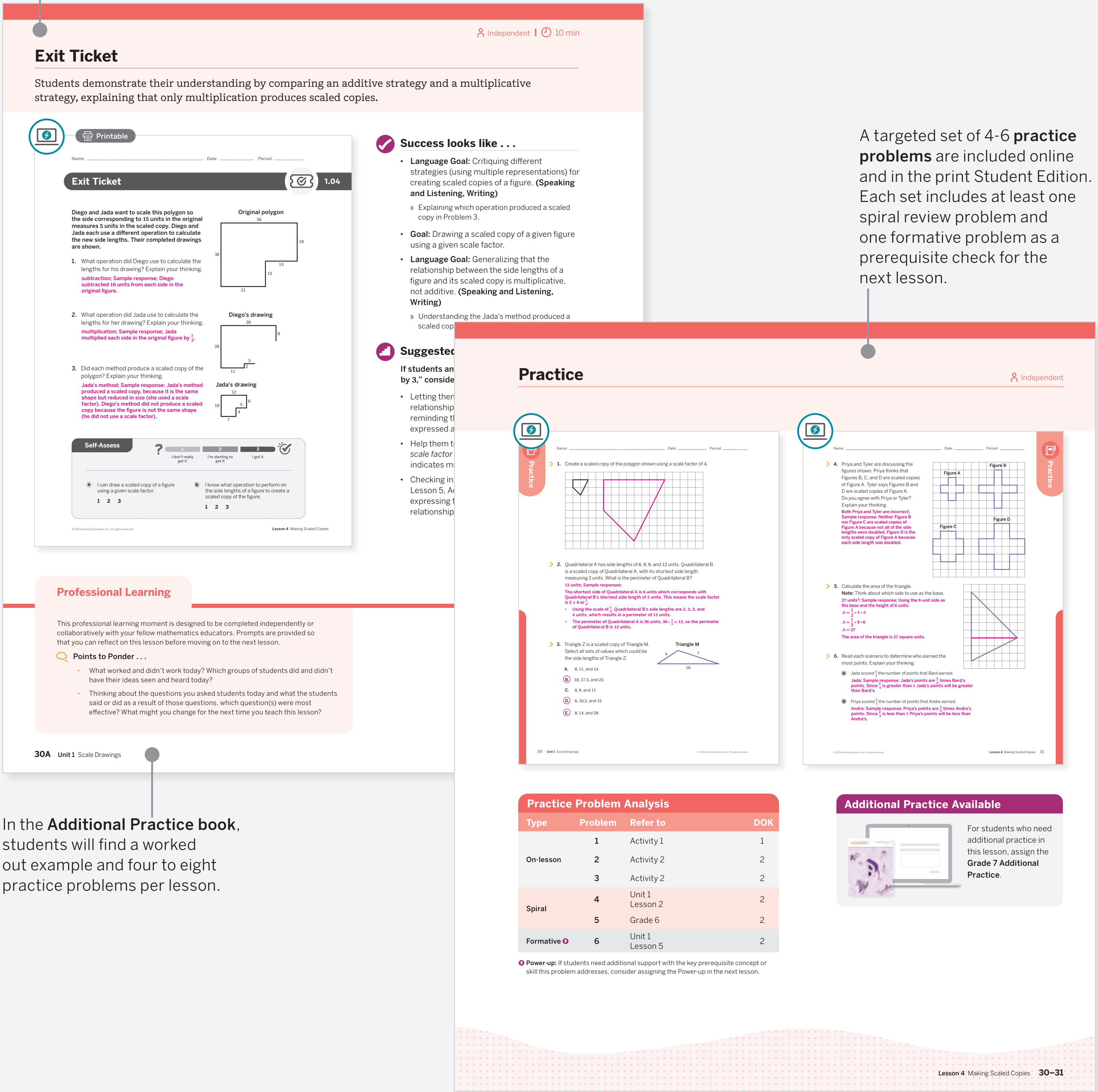
Lesson 4 Making Scaled Copies **27**

A short **description of the activity and its targeted goal** is outlined at the top.

Easy 1-2-3 guidance for teachers shortens the amount of time required to plan. The "look for" prompts are helpful to scan while teaching.

Differentiation supports, including our alternative warm-ups called Power-ups, provide practical guidance for scaffolding or extending the learning for all students. Differentiation supports, including our just-in-time supports called Power-ups, provide practical guidance for scaffolding or extending the learning for all students.

Each lesson ends with an **Exit Ticket** which includes a self assessment for students.



Independent | 10 min

Exit Ticket

Students demonstrate their understanding by comparing an additive strategy and a multiplicative strategy, explaining that only multiplication produces scaled copies.

Printable

Name: Date: Period:

Exit Ticket 1.04

Diego and Jada want to scale this polygon so the side corresponding to 15 units in the original measures 5 units in the scaled copy. Diego and Jada each use a different operation to calculate the new side lengths. Their completed drawings are shown.

1. What operation did Diego use to calculate the lengths for his drawing? Explain your thinking.
   subtraction; Sample response; Diego subtracted 10 units from each side in the original figure.
2. What operation did Jada use to calculate the lengths for her drawing? Explain your thinking.
   multiplication; Sample response; Jada multiplied each side in the original figure by $\frac{1}{3}$.
3. Did each method produce a scaled copy of the polygon? Explain your thinking.
   Jada's method; Sample response: Jada's method produced a scaled copy, because it is the same shape but reduced in size (she used a scale factor). Diego's method did not produce a scaled copy because the figure is not the same shape (he did not use a scale factor).

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can draw a scaled copy of a figure using a given scale factor. 1 2 3

b I know what operation to perform on the side lengths of a figure to create a scaled copy of the figure. 1 2 3

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Success looks like . . .

- **Language Goal:** Critiquing different strategies (using multiple representations) for creating scaled copies of a figure. **(Speaking and Listening, Writing)**
  - » Explaining which operation produced a scaled copy in Problem 3.
- **Goal:** Drawing a scaled copy of a given figure using a given scale factor.
- **Language Goal:** Generalizing that the relationship between the side lengths of a figure and its scaled copy is multiplicative, not additive. **(Speaking and Listening, Writing)**
  - » Understanding the Jada's method produced a scaled cop

Suggested

If students an by 3," conside

- Letting then relationship reminding t expressed a
- Help them t *scale factor* indicates m
- Checking in Lesson 5, A expressing t relationship

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- Thinking about the questions you asked students today and what the students said or did as a result of those questions, which question(s) were most effective? What might you change for the next time you teach this lesson?

30A **Unit 1** Scale Drawings

Practice

Independent

Name: Date: Period:

1. Create a scaled copy of the polygon shown using a scale factor of 4.
2. Quadrilateral A has side lengths of 6, 9, 9, and 12 units. Quadrilateral B is a scaled copy of Quadrilateral A, with its shortest side length measuring 2 units. What is the perimeter of Quadrilateral B?
   12 units; Sample responses:
   The shortest side of Quadrilateral A is 6 units which corresponds with Quadrilateral B's shortest side length of 2 units. This means the scale factor is $2 \div 6$ or $\frac{1}{3}$.
   - **Using the scale of $\frac{1}{3}$, Quadrilateral B's side lengths are 2, 3, 3, and 4 units, which results in a perimeter of 12 units.**
   - **The perimeter of Quadrilateral A is 36 units. $36 \cdot \frac{1}{3} = 12$, so the perimeter of Quadrilateral B is 12 units.**
3. Triangle Z is a scaled copy of Triangle M. Select *all* sets of values which could be the side lengths of Triangle Z.
   A. 8, 11, and 14
   [✓ B.] 10, 17.5, and 25
   C. 6, 9, and 11
   [✓ D.] 6, 10.5, and 15
   [✓ E.] 8, 14, and 20

30 **Unit 1** Scale Drawings © 2023 Amplify Education, Inc. All rights reserved.

4. Priya and Tyler are discussing the figures shown. Priya thinks that Figures B, C, and D are scaled copies of Figure A. Tyler says Figures B and D are scaled copies of Figure A. Do you agree with Priya or Tyler? Explain your thinking.
   Both Priya and Tyler are incorrect; Sample response: Neither Figure B nor Figure C are scaled copies of Figure A because not all of the side lengths were doubled. Figure D is the only scaled copy of Figure A because each side length was doubled.

5. Calculate the area of the triangle.
   Note: Think about which side to use as the base.
   27 units^2; Sample response: Using the 9-unit side as the base and the height of 6 units:
   $A = \frac{1}{2} \cdot b \cdot h$
   $A = \frac{1}{2} \cdot 9 \cdot 6$
   $A = 27$
   The area of the triangle is 27 square units.
6. Read each scenario to determine who earned the most points. Explain your thinking.
   a Jada scored $\frac{5}{4}$ the number of points that Bard earned.
   Jada; Sample response: Jada's points are $\frac{5}{4}$ times Bard's points. Since $\frac{5}{4}$ is greater than 1 Jada's points will be greater than Bard's.
   b Priya scored $\frac{2}{3}$ the number of points that Andre earned.
   Andre; Sample response: Priya's points are $\frac{2}{3}$ times Andre's points. Since $\frac{2}{3}$ is less than 1 Priya's points will be less than Andre's.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 2	2
	5	Grade 6	2
Formative	6	Unit 1 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Lesson 4 Making Scaled Copies **30–31**

A targeted set of 4-6 **practice problems** are included online and in the print Student Edition. Each set includes at least one spiral review problem and one formative problem as a prerequisite check for the next lesson.

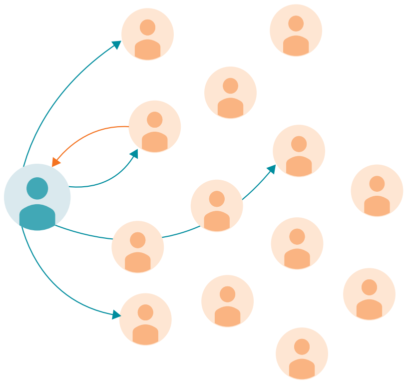
In the **Additional Practice book**, students will find a worked out example and four to eight practice problems per lesson.

Social, collaborative digital experiences

Digital lessons, when designed the right way, can be powerful in their ability to surface student thinking and spark interesting and productive discussions. To bring our vision of what digital lessons can and should be to life, we've partnered with Desmos to create our complete library of **Amps**—social, collaborative lessons powered by Desmos technology that recast technology from simply mirroring what can be done in a workbook to presenting captivating scenarios where students work together and interact with the mathematics in real time.

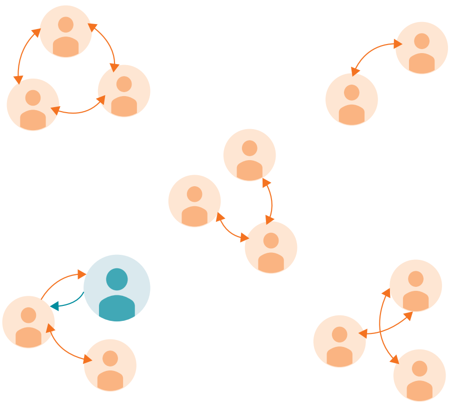
1 Launch

Teachers launch an activity and ensure students understand what's being asked.

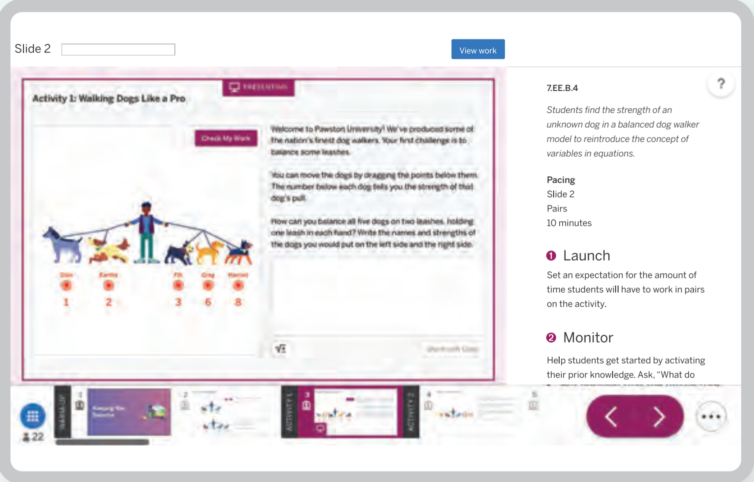


2 Monitor

Students interact with each other to discuss and work out strategies for solving a problem.



Teacher experience



Activity 1: Walking Dogs Like a Pro

Welcome to Pawston University! We've produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

You can move the dogs by dragging the points below them. The number below each dog tells you the strength of that dog's pull.

How can you balance all five dogs on two leashes, holding one leash in each hand? Write the names and strengths of the dogs you would put on the left side and the right side.

7.EE.B.4

Students find the strength of an unknown dog in a balanced dog walker model to reintroduce the concept of variables in equations.

Pacing
Slide 2
Pairs
10 minutes

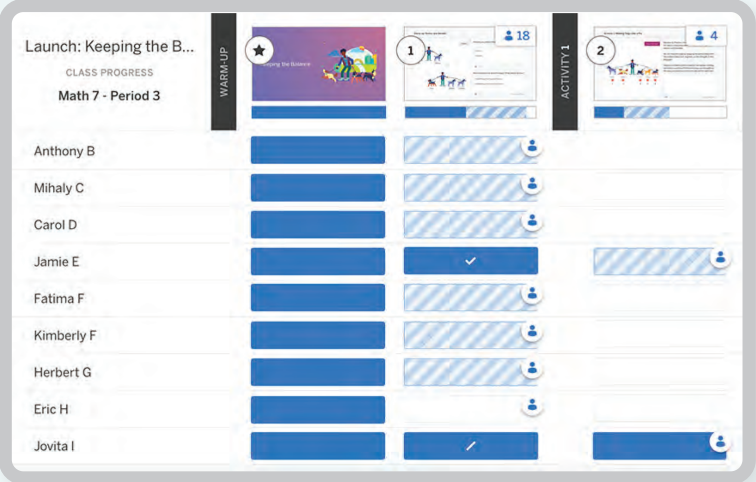
❶ Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

❷ Monitor

Help students get started by activating their prior knowledge. Ask, "What do

When you launch a lesson, you'll have access to **easy-to-skim teacher notes and all of the controls necessary** to manage the lesson.



Launch: Keeping the B...
CLASS PROGRESS
Math 7 - Period 3

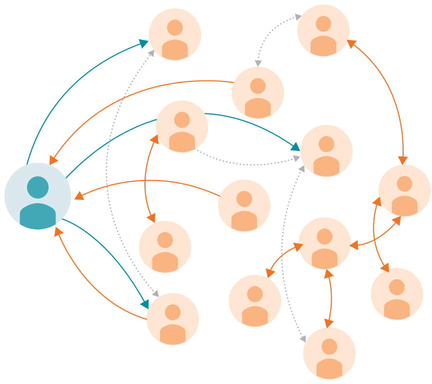
Anthony B
Mihaly C
Carol D
Jamie E
Fatima F
Kimberly F
Herbert G
Eric H
Jovita I

After students have started working you can access the Class Progress screen to **see where students are in the lesson and even control which problems they have access to.**

When you launch an **Amp**, you will be kickstarting small group and whole class discussions where students can see how their thinking can impact a situation and learn how their peers are justifying their actions and decisions.

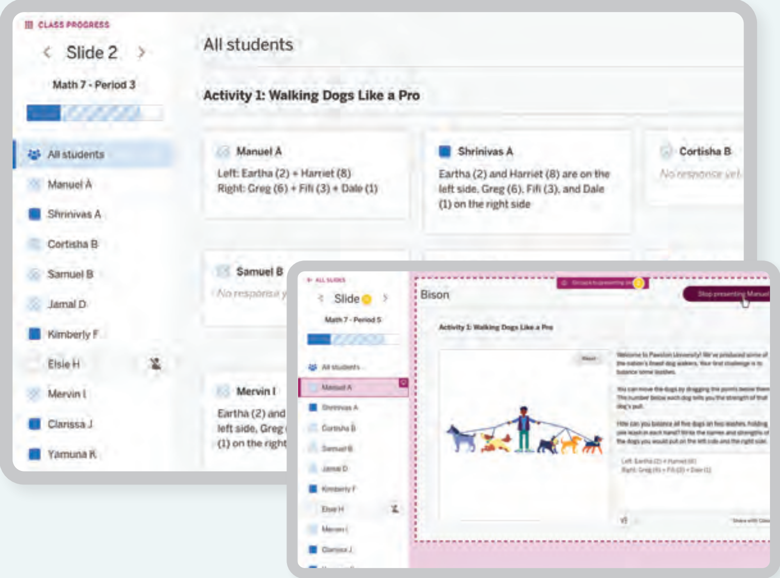
3 Connect

Students construct viable arguments and critique each other's reasoning, then synthesize with the teacher at the end.



4 Review

After class, teachers can provide feedback on submitted student work and run reports.



All students

Activity 1: Walking Dogs Like a Pro

Manuel A
Left: Eartha (2) + Harriet (8)
Right: Greg (6) + Fifi (3) + Dale (1)

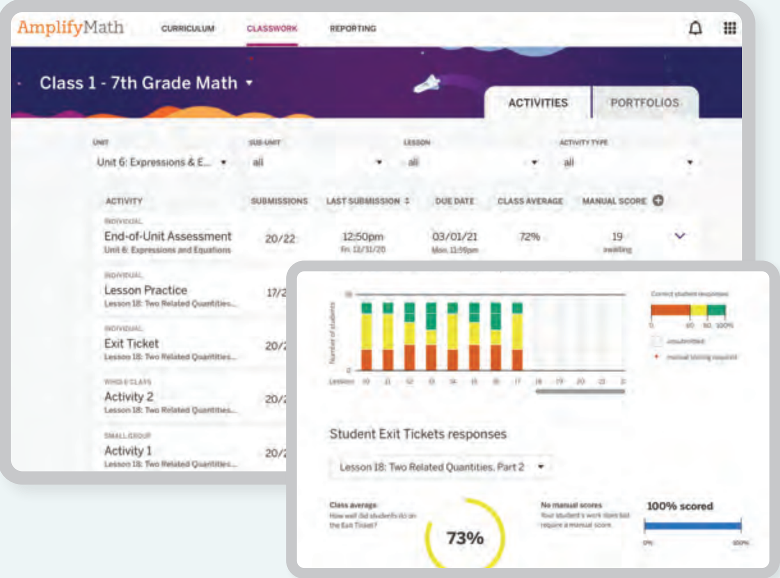
Shrinivas A
Eartha (2) and Harriet (8) are on the left side, Greg (6), Fifi (3), and Dale (1) on the right side

Cortisha B
No response yet

Samuel B
No response y

Mervin I
Eartha (2) and
left side, Greg
(1) on the right

All student responses can be viewed easily on the All Students screen. You can often view a composite view of responses and spotlight student work anonymously.



UNIT: Unit 6: Expressions & E...
SUB-UNIT: all
LESSON: all
ACTIVITY TYPE: all

ACTIVITY	SUBMISSIONS	LAST SUBMISSION	DUE DATE	CLASS AVERAGE	MANUAL SCORE
INDIVIDUAL End-of-Unit Assessment Unit 6: Expressions and Equations	20/22	12:50pm Fri. 12/31/20	03/01/21 Mon. 11:59pm	72%	19 awaiting
INDIVIDUAL Lesson Practice Lesson 18: Two Related Quantities…	17/2				
INDIVIDUAL Exit Ticket Lesson 18: Two Related Quantities…	20/2				
WHOLE CLASS Activity 2 Lesson 18: Two Related Quantities…	20/2				
SMALL GROUP Activity 1 Lesson 18: Two Related Quantities…	20/2				

Student Exit Tickets responses

Lesson 18: Two Related Quantities, Part 2

Class average: 73%

100% scored

After students complete work that's ready for grading, you can head to Classwork to **quickly provide feedback.**

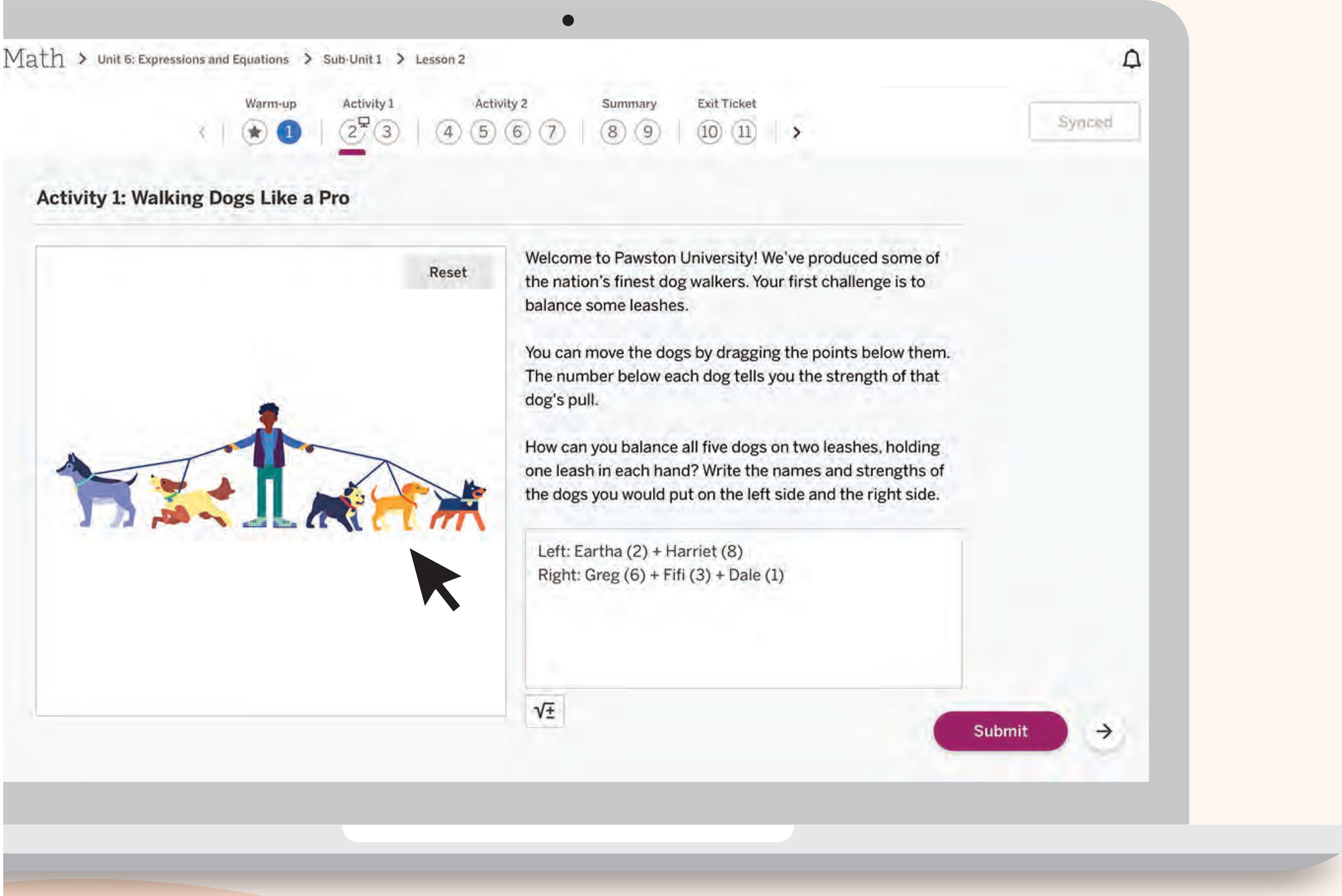
Once students have completed an Exit Ticket, a practice problem set, or an assessment, you can **run reports at the class, student, and standards levels to check in on student progress.**

Connecting everyone in the classroom

The student experience is interactive and responsive. As students manipulate the interactive elements of the Amps, they will be asked to justify their thinking, and often they will get to see how their peers are thinking, too. All along the way, teachers can monitor work in real time.

Student experience

The student experience is intuitive and engaging, offering students **low floors and high ceilings** as they engage with the lesson content.



Activity 1: Walking Dogs Like a Pro

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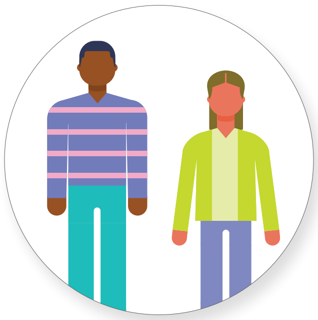
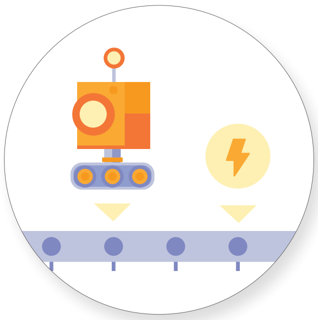
You can move the dogs by dragging the points below them. The number below each dog tells you the strength of that dog's pull.

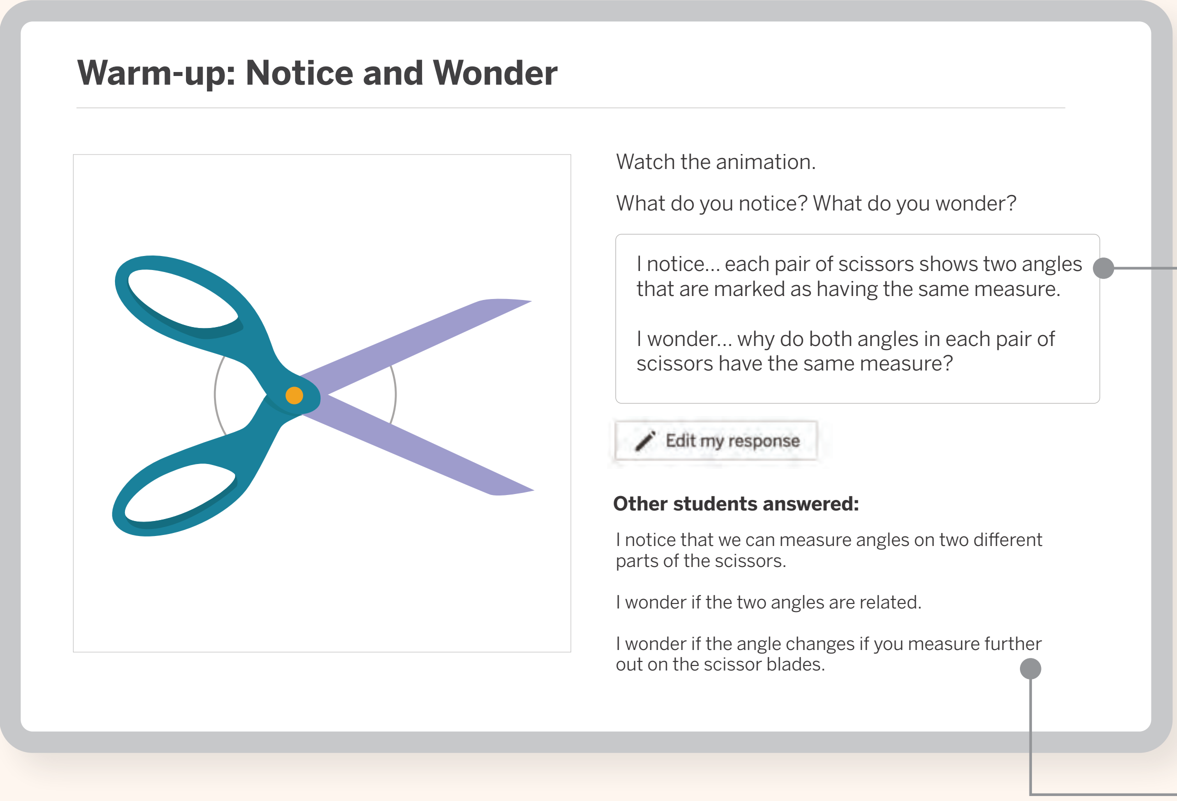
How can you balance all five dogs on two leashes, holding one leash in each hand? Write the names and strengths of the dogs you would put on the left side and the right side.

Left: Eartha (2) + Harriet (8)
Right: Greg (6) + Fifi (3) + Dale (1)

Many of the lessons feature rich, visual moments that students can interact with. Amps are meant to feel intuitive, immersive, and sometimes even playful.







Warm-up: Notice and Wonder

Watch the animation.

What do you notice? What do you wonder?

I notice... each pair of scissors shows two angles that are marked as having the same measure.

I wonder... why do both angles in each pair of scissors have the same measure?

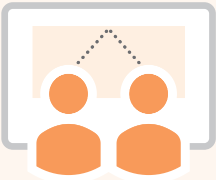
Other students answered:

I notice that we can measure angles on two different parts of the scissors.

I wonder if the two angles are related.

I wonder if the angle changes if you measure further out on the scissor blades.

As students work, the slides change, prompting students to **describe their strategies**. Teachers can see student work in real time and spotlight responses anonymously to support in-class discussion.



When working online, students will sometimes **see their peers' thinking** on their own screens. These connection points are great for sparking discussions ahead of the synthesis moment in the lesson.

Routines in Amplify Math

Routines help you and your students to maintain a sense of familiarity and structure throughout the school year. As the year progresses, routines free up time you would otherwise spend giving new directions for each activity.

Routine	What is it?	Where is it?
Turn and Talk	Turn and Talk can be done anytime, needs little preparation, and only takes a minute or two. Students turn and talk to their partner about another student's thinking, eventually deciding whether they agree or not and why. The students then switch roles. Eventually you will call on a few students to say what they think, or report what their partner said.	Use anytime students are working
Ask Three Before Me	Do you find yourself responding to all of your students' questions? This routine saves you time and empowers your students' agency and voice, while allowing them to view each other as knowledgeable resources. The routine is simple: When a student has a question, let them know they should ask three other students before they ask you.	
Go Find a Good Idea	When students are stuck and productive struggle has stalled, direct them to get up and walk around the room to find a good idea in the written work of other students. Encourage them to ask questions of other students and to explain their work. Then, they should bring a good idea back and continue working with it, citing from whom they got it.	
Notice and Wonder	Students are shown some media or a mathematical representation. They are prompted with "What do you notice? What do you wonder?," and are given a few minutes to think and share with a partner. Then you then ask several students to share what they noticed and wondered, recording responses for all to see. ***Note:*** Notice and Wonder and I Notice/I Wonder are trademarks of NCTM and the Math Forum and used in these materials with permission.	Warm-ups, Activity launches
Math Talks and Strings	Typically, one problem is displayed at a time. Problems are purposefully chosen to elicit different approaches, often in a way that builds from one problem to the next. These problems could be numerical or algebraic in nature. Give students a few moments to quietly think and give you a signal when they have an answer and a strategy, then select some to share different strategies for each problem. Ask questions like "Who thought about it a different way?"	Warm-ups
Which One Doesn't Belong?	Students are asked "Which one doesn't belong?" when presented with several figures, diagrams, graphs, or expressions. The best part is that every answer choice is correct! It's the similarities and differences among the options that are mathematically meaningful here. Prompt students to explain their rationale for deciding which option doesn't belong, and give them opportunities to make their rationale more precise.	Warm-ups
Card Sort	A card sort uses cards or slips of paper that can be manipulated and moved around. Individually, in pairs, or in small groups students put things into categories or groups based on shared characteristics or connections.	Activities
Find and Fix	Students are presented with the work of another hypothetical student whose work contains a flaw or error. They first identify any errors, then show or explain how to correct these errors, and explain why the person may have made the error.	Activities
Group Presentations and Gallery Tours	Instruct students—typically in groups—to create a visual display of their work, such as how they solved a problem with mathematical modeling, invented a new problem, designed a simulation or experiment, or organized and displayed data. In the Gallery Tour version of this routine, student work is captured on a piece of paper, a poster, or on an assigned portion of the board. Students then move around the room to observe, record notes or questions on their own paper, or write on each other's work (posing clarifying questions, giving kudos, or identifying portions they may disagree with). You lead a discussion, allowing students to respond to questions or critiques of their work.	Activities
Info Gap	One partner receives a problem card with a math question that doesn't have enough given information, and the other partner receives a data card with information relevant to the problem card. Students ask each other questions, such as "What information do you need?" and "Why do you need this information?" and are expected to explain what they will do with the information.	Activities

Math language development

All students, including English Learners, benefit from math lessons that are designed with strong math language development supports. Working in close partnership with the **English Learners Success Forum** (ELSF), the Amplify curriculum writers ensured the math language development strategies and supports in the program were clear, useful, and appropriate for all students. This was achieved by infusing the instruction with research-based Math Language Routines (MLRs) and by providing sentence frames where appropriate, both in the teacher language provided for each task and in the differentiation supports section. ELSF has helped review all student-facing content to ensure it's developmentally appropriate and reflects the fact that math is a new language students are mastering.

ENGLISH
LEARNERS
SUCCESS
FORUM

The math language development story starts at the unit level where teachers will see new vocabulary and a correlation of MLRs to lessons listed in the unit planning materials. In the Unit Study materials, one of the MLRs is highlighted under the Focus on Differentiated support section.

Embedded language development support

- **Course level:** The course design centers the development of communication skills.
- **Unit level:** Teachers will understand how language development progresses throughout the unit.
- **Lesson level:** Each lesson includes definitions of new vocabulary and language goals.
- **Activities:** Math Language Routines support the development of new vocabulary and activities often include additional supports based on language demands.
- **Assessments:** Suggested next steps offer guidance for teachers to support students' mathematical language development, based on their performance on assessment items.

Sentence frames

Sentence frames can give a student a sensible jumping off point for verbalizing or writing out an idea. Amplify Math leans on sentence frames that all serve one or more language functions, including describing a topic, explaining it, justifying thinking, generalizing, critiquing, representing, and interpreting ideas.

Math Language Routines

The Math Language Routines deployed throughout the Teacher Edition:

MLR1: Stronger and Clearer Each Time

MLR2: Collect and Display

MLR3: Critique, Correct, Clarify

MLR4: Information Gap

MLR5: Co-craft Questions

MLR6: Three Reads

MLR7: Compare and Connect

MLR8: Discussion Supports

Some routines adapted from Zwiers, J. (2014). Building academic language: Meeting Common Core Standards across disciplines, grades 5–12 (2nd ed.). San Francisco, CA: Jossey-Bass.

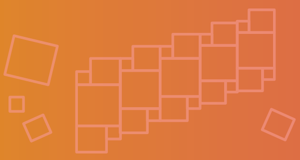
UNIT 1

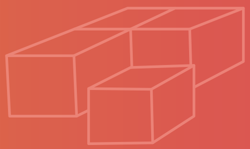
Scale Drawings

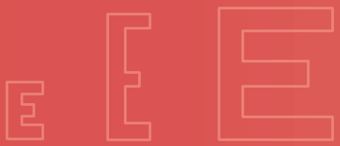
Certain objects in our universe exist at sizes and distances that are impossible for our eyes to see (such as a red blood cell, or Jupiter). In Unit 1, students harness the power of scaling — bringing large and small objects to a manageable size without distorting them.

Essential Questions

- Why is it important to be precise when making scaled copies?
- Why are lengths and areas affected in different ways when creating scaled copies?
- How do scale models help you make sense of the world around you?
- *(By the way, how do you make a guy in a lizard suit taller than a skyscraper?)*







Key Shifts in Mathematics

Focus

In this unit . . .

Students study scaled copies of pictures and plane figures, then apply what they have learned to scale drawings, such as maps and floor plans. They begin by looking at copies of a picture and describe what differentiates scaled and nonscaled copies. They go on to draw their own scaled copies, and notice how the size of a scale factors affects the lengths and area of the copy. In the second half of the unit, students see that the principles and strategies that they used to reason about scaled copies of figures can be used with scale drawings.

Coherence

Previously . . .

In Grade 4, students applied area and perimeter formulas for rectangles to solve real-world and mathematical problems, and they learned to use protractors. In Grade 5, students extended the formula for the area of a rectangle to include rectangles with fractional side lengths. In Grade 6, students built on their knowledge of geometry and geometric measurement to produce formulas for the areas of parallelograms and triangles, using these formulas to find surface areas of polyhedra.

Coming soon . . .

Students will build on their understanding of geometric scaling to reason about proportional relationships in Units 2, 3, and 4 in Grade 7. In Grade 8, students will perform dilations and examine similarity of plane figures.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students use visual models to build conceptual understanding of scale factor by examining the relationships among corresponding points, sides, and angles of scaled copies (Lesson 3).

Procedural Fluency

Students strengthen their procedural skills in measuring precisely using rulers or informal measuring tools (Lesson 9).

Application

Students apply their understanding of scale factor by drawing scaled copies, ensuring that angle measures are unchanged and side lengths are changed by a common factor (Lesson 4).

Life in the Little Big City

SUB-UNIT

1

Lessons 2–6

Scaled Copies

Students recognize that making two-dimensional figures larger or smaller requires precision. ***Scale factor*** emerges as an essential detail for identifying ***scaled copies***. Students discover that the area of a scaled copy changes in a different way than the length changes.

Narrative: In the story *Alice in Wonderland*, Alice increased and decreased her size to help her solve problems.

SUB-UNIT

2

Lessons 7–12

Scale Drawings

Students use actual distances to calculate scaled distances and create ***scale drawings*** at different ***scales***. They apply their understanding of scale drawings to solve problems and express scales without units to highlight the scale factor.

Narrative: Movies often use scale drawings and models to create the illusion of cities and objects, including monsters!

Launch

Lesson 1

Scale-y Shapes

Students are welcomed to Grade 7 work with an exploratory geometry lesson. They play with the arrangement of copies of the same shape, noticing the difference between shapes that simply have the same name and shapes that are proportionally enlarged. Additionally, they are introduced to the wonderful tradition of creating games from math, as featured mathematician Solomon Golomb famously did in his day.

Capstone

Lesson 13

Build Your Brand

In the graphic design world, all good logos are scalable. Students design a personal logo to share a piece of what makes their logo unique. Because a logo can be displayed just about anywhere, students must use scaling to fit their logo to appropriate spaces on a variety of promotional items.

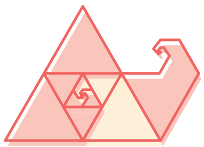
Unit at a Glance

Spoiler Alert: You can use perspective to help identify whether two shapes are scaled copies by holding them in your sight line, with one further away, and seeing if they match up perfectly.

Assessment

A **Pre-Unit Readiness Assessment**

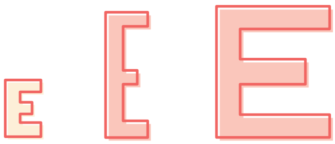
Launch Lesson



1 **Scale-y Shapes**

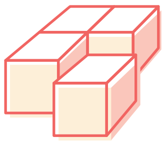
Investigate how certain shapes can be used to build larger versions of themselves — the cleverly-named rep-tile.

Sub-Unit 1: Scaled Copies



2 **What Are Scaled Copies?**

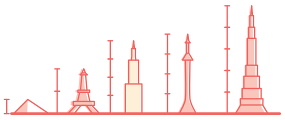
Distinguish scaled copies from those which are not — first informally, and later, with increasing precision.



6 **Scaling Area**

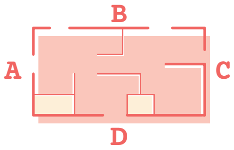
Recognize that the area of the scaled copy changes by the square of the scale factor.

Sub-Unit 2: Scale Drawings



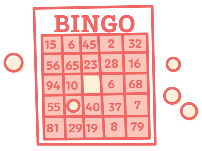
7 **Scale Drawings**

Begin the study of scale drawings, or scaled two-dimensional representations, of actual objects or places.



8 **Creating Scale Drawings**

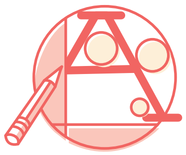
Use actual distances to calculate the scaled distance and create scale drawings.



12 **Units in Scale Drawings**

Explore how equivalent scales relate scaled and actual measurements by the same scale factor, even though the scales may be expressed differently.

Capstone Lesson



13 **Build Your Brand** ●

Create personal logos and scale them to fit on promotional items of one's choosing.

Assessment

A **End-of-Unit Assessment**

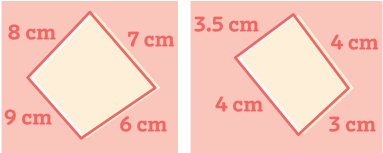
Key Concepts

Lesson 3: Scaled copies must be related by the same scale factor for all corresponding lengths.
Lesson 4: The relationship between scaled copies is multiplicative, not additive.
Lesson 11: You can convert scales to ratios without units to determine whether they are equivalent.

Pacing

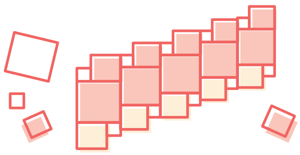
13 Lessons: 45 min each
2 Assessments: 45 min each
Full Unit: 15 days
● **Modified Unit:** 13 days

Assumes 45-minute class periods per day.
For block scheduling or other durations, adjust the number of days accordingly.



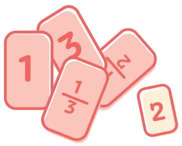
3 Corresponding Parts and Scale Factors

Develop the vocabulary for discussing scaling and scaled copies more precisely.



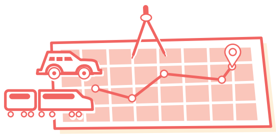
4 Making Scaled Copies

Draw scaled copies of simple shapes and strengthen understanding that the relationship between scaled copies is multiplicative, not additive.



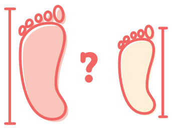
5 The Size of the Scale Factor

Notice how the size of the scale factor determines if the scaled copy is larger, smaller, or the same size.



9 Scale Drawings and Maps (optional) ●

Apply understanding of scale drawings to solve problems involving traveling at a constant speed (or average speed).



10 Changing Scales in Scale Drawings

Given a scale drawing, recreate it at a different scale.

11 Scales Without Units

Express scales without units to highlight the scale factor relating a scale drawing to an actual object.

● Modifications to Pacing

Lesson 9: This lesson focuses on interpreting speed using maps, but this context is revisited in Unit 2, and may be omitted.

Lesson 13: This lesson gives students a chance to apply the skills and knowledge from the unit, but does not introduce new understanding, and may be omitted.

Unit Supports

Math Language Development

Lesson	New vocabulary
2	scaled copy
3	corresponding parts scale factor
7	scale scale drawing
11	equivalent scales

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
2, 4, 10	**MLR1:** Stronger and Clearer Each Time
1, 2, 5, 7, 10, 12	**MLR2:** Collect and Display
1, 3–5, 7, 8	**MLR3:** Critique, Correct, Clarify
8, 9, 13	**MLR7:** Compare and Connect
2, 3, 6–9, 11, 12	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket Additional Practice

Lesson(s)	Additional required materials
2–5, 7, 9, 10, 13	geometry toolkits
4, 10	graph paper
5	markers or colored pencils
8	markers or highlighters
1	pattern blocks
1–3, 5–8, 10, 12, 13	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
9, 10, 13	rulers
12	small objects for marking bingo cards, e.g., linking cubes or paper clips
8	sheet protectors and dry erase markers
5	tape or glue

Instructional Routines

Activities throughout this unit include these routines:

Lesson(s)	Instructional Routines
2, 5	Card Sort
3	Find and Fix
13	Gallery Tour
4, 5, 8	Number Talk
6	Partner Problems
7, 9, 11, 12	Poll the Class
2, 3, 6, 10, 13	Think-Pair-Share
1, 11	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 13

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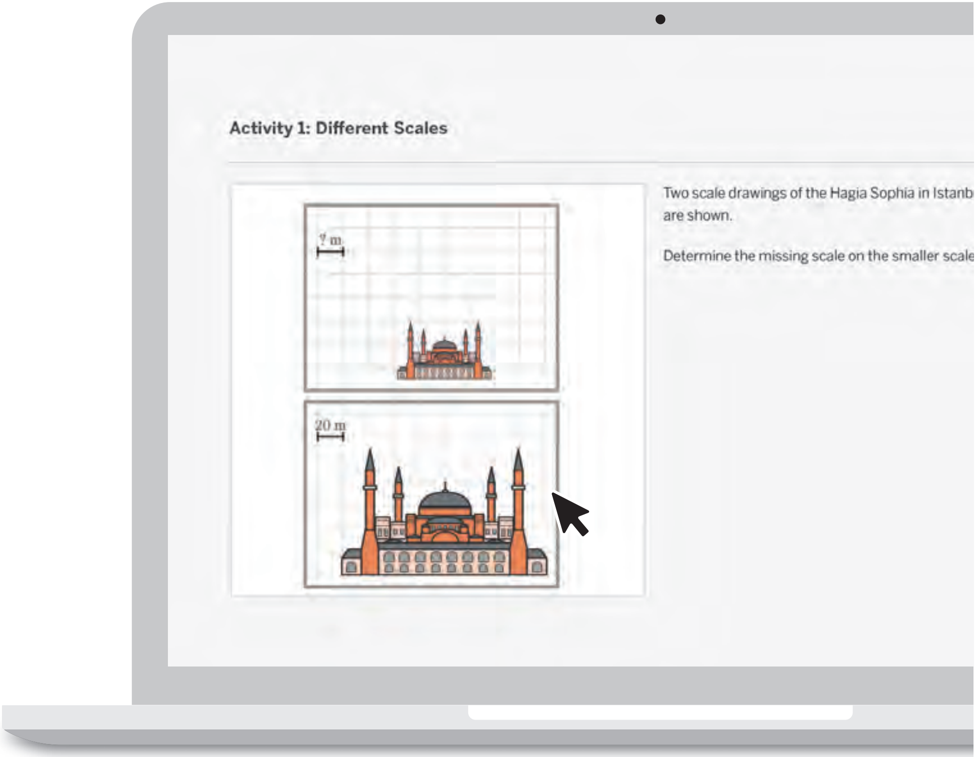
Social & Collaborative Digital Moments

Featured Activity

Different Scales

Put on your student hat and work through **Lesson 10, Activity 1:**



Activity 1: Different Scales

Two scale drawings of the Hagia Sophia in Istanb
are shown.

Determine the missing scale on the smaller scale

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Activity 1: Rep-tiles **(Lesson 1)**
- Activity 2: Scaled Triangles **(Lesson 3)**
- Activity 2: Same Drawing, Different Scales **(Lesson 11)**
- Activity 1: Large- and Small-Scale **(Lesson 13)**

Unit Study
Professional Learning

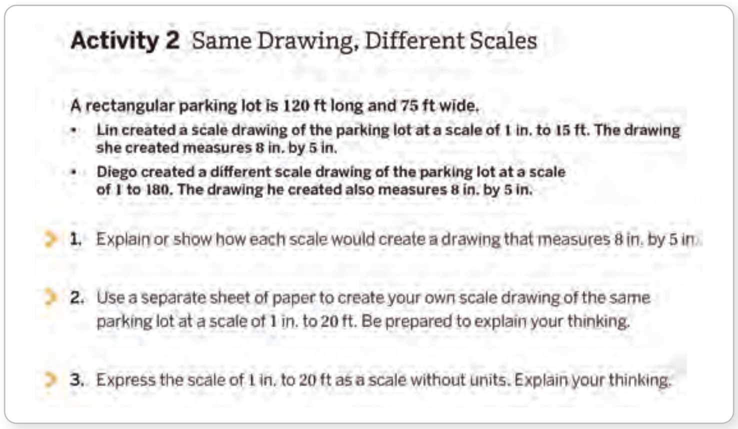
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces the idea of scale drawings and their applications in real life, including maps and models. Students learn to determine the scale factor between two drawings or between a real object and its scaled drawing. Given the scale factor, they are able to calculate the corresponding area and volume, or vice versa. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 11, Activity 2:**



Activity 2 Same Drawing, Different Scales

A rectangular parking lot is 120 ft long and 75 ft wide.

- **Lin created a scale drawing of the parking lot at a scale of 1 in. to 15 ft. The drawing she created measures 8 in. by 5 in.**
- **Diego created a different scale drawing of the parking lot at a scale of 1 to 180. The drawing he created also measures 8 in. by 5 in.**

1. Explain or show how each scale would create a drawing that measures 8 in. by 5 in.
2. Use a separate sheet of paper to create your own scale drawing of the same parking lot at a scale of 1 in. to 20 ft. Be prepared to explain your thinking.
3. Express the scale of 1 in. to 20 ft as a scale without units. Explain your thinking.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

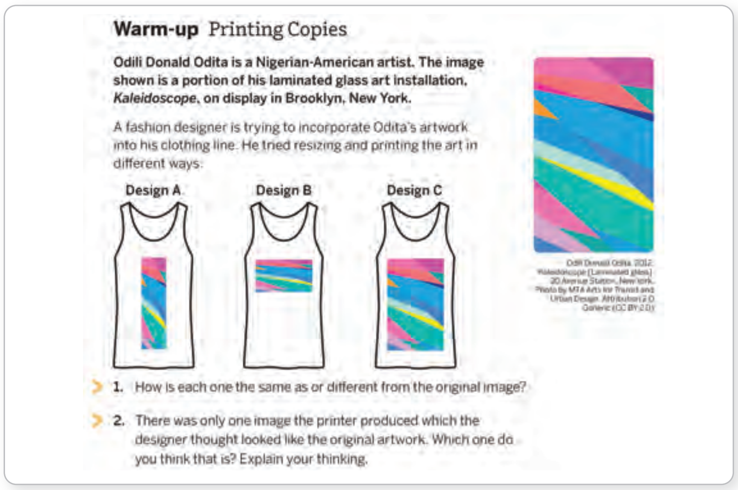
- In question 1, how might one scale (1 in. to 15 ft.) be considered more or less useful than the other (1 to 180)?
- Many students know that there are 12 inches in 1 foot. What do you think students would say if you asked them how many cubic inches are there in 1 cubic foot?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Think-Pair-Share

Rehearse . . .

How you'll facilitate the ***Think-Pair-Share*** instructional routine in **Lesson 2, Warm-up:**



Warm-up Printing Copies

Odili Donald Odita is a Nigerian-American artist. The image shown is a portion of his laminated glass art installation, *Kaleidoscope*, on display in Brooklyn, New York.

A fashion designer is trying to incorporate Odita's artwork into his clothing line. He tried resizing and printing the art in different ways.

Odili Donald Odita, 2012. Kaleidoscope [Laminated glass]. 20 Avenue Station, New York. Photo by MTA Arts for Transit and Urban Design. Attribution 2.0 Generic (CC BY 2.0)

1. How is each one the same as or different from the original image?
2. There was only one image the printer produced which the designer thought looked like the original artwork. Which one do you think that is? Explain your thinking.

Points to Ponder . . .

- *Think-Pair-Share* allows students to process their ideas first individually, then with a partner, then with the class.

This routine . . .

- Places students in the position of mathematical knowledge-sources.
- Reinforces the idea that learning math can be a social activity.
- Allows for students to test out their ideas with a single peer before sharing with the whole class.

Anticipate . . .

- Some students may not be comfortable sharing with a peer they have not met before. How can you help students break the ice?
- If you haven't used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you have used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Establish mathematics goals to focus learning.

This effective teaching practice . . .

- Establishes clear goals for both you and your students so that you both know what the lesson is targeting. If you know the target, you know whether or not you hit it.
- Provides a benchmark which will help you to make instructional decisions based on your students' performance.

Points to Ponder . . .

- How will understanding the target goals for each lesson or activity help you when planning how to spend your instructional time?
- How can you use the lesson goals to know whether you need to redirect instruction or provide additional support?

Math Language Development

MLR3: Critique, Correct, Clarify

MLR3 appears in Lessons 1, 3–5, 7, and 8.

- In Lesson 4, students critique a student's reasoning as to how to scale a copy of a drawing. Students can take a break from providing their own reasoning and look critically at another student's reasoning.
- In Lesson 5, students may have misconceptions about a scale factor of 0. This is a good opportunity to present the misconception as a statement and have students critique it.
- **English Learners:** Allow students to speak or draft a response in their primary language first, and then generate a response in English.

Point to Ponder . . .

- In this routine, students analyze incorrect statements and work to correct them. How can you model what an effective and respectful critique looks like?

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Opportunities to vary the demands of a task or activity appear in Lessons 1–7 and 9–13.

- Chunking a complex task into smaller, more manageable parts allows students to focus on one part at a time.
- Providing measurements, instead of having students measure, allows them to focus on the mathematical goal of the activity.
- Some students may benefit from more processing time. When restricting the number of tasks or problems students need to complete, consider allowing them to choose which problem(s) to complete. Students are often more engaged when they have choice.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to vary the demands of a particular task or activity? What clues will you gather from your students?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving one-step equations with a variable throughout the unit? Do you think your students will generally:
  - miss the underlying concept of balance and mathematical equality?
  - simply struggle with the concept of variables and unknowns?
  - be fully ready to solve procedurally and efficiently, but misapply the properties of equality?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their social awareness and self-awareness skills.

Points to Ponder . . .

- Are students able to understand the approaches of others? Can they see the benefits of different perspectives while completing a task? Do they appreciate the work of others?
- Are students able to identify a problem, analyze the situation, and develop a strategy for solving the problem? Can they evaluate how they did, reflecting on ways that they could improve in the future?

UNIT 1 | **LESSON 1 – LAUNCH**

Scale-y Shapes

Let's see which shapes can be used to build larger copies of themselves.

Focus

Goal

1. **Language Goal:** Understand and explain how some shapes can be tiled to form similar, but differently-sized versions of themselves. **(Writing, Speaking and Listening)**

Coherence

Today

Students investigate how certain shapes can be used to build larger versions of themselves — the cleverly-named *rep-tile*.

The concept of scaling emerges from this work, and students notice some of the attributes of scaled copies of shapes and copies of shapes that are not scaled.

Previously

In Grade 6, students found areas of polygons by decomposing the shapes.

Coming Soon

In Grade 8, students will explore the concept of similarity through the transformation of two-dimensional shapes.

Rigor

- Students build **conceptual understanding** of scaling by building larger copies of shapes and noticing relationships between them.
- Students use visual models to develop **conceptual understanding** of the effect of scaling on the area of scaled figures.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	20 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

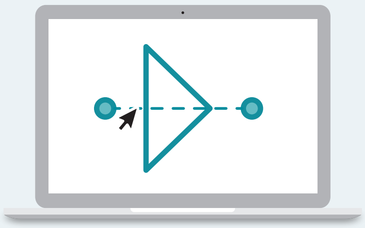
- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Pattern Blocks*, one set per pair: 6 copies with triangles, rhombuses (larger version), and trapezoids (optional, if not using physical pattern blocks)
- pattern blocks, one set per pair

Amps | **Featured Activity**

Activity 1

Interactive Geometry

Students can build digital rep-tiles by manipulating pattern blocks.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel that they are too old to be using pattern blocks in Activity 1. Point out that their maturity allows them to do more advanced activities, such as this one with the pattern blocks. Students can use repeated reasoning to think about the patterns of shapes they can create with pattern blocks, which allows them to engage on a higher cognitive level.

Modifications to Pacing

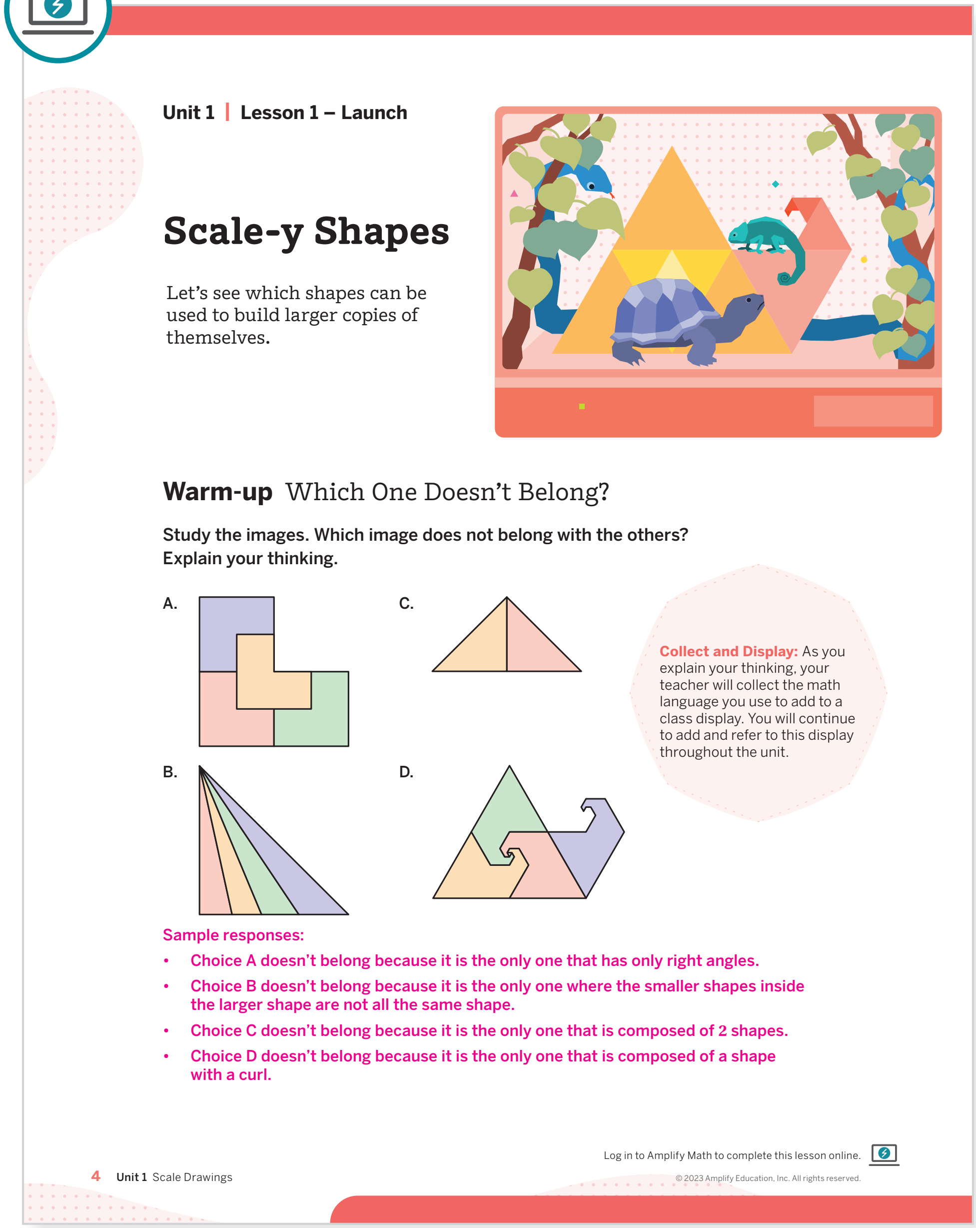
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, Problem 3 may be omitted. Students will revisit the relationship between scaling, side length, and area in Lesson 6.
- **Activity 2** may be omitted. It provides another example of rep-tiles, the *pentomino*, also created by mathematician Samuel Golomb.

Pairs | 7 min

Warm-up Which One Doesn't Belong?

Students explore a type of geometric tessellation called a *rep-tile* to introduce the idea of a scaled copy of a figure.



Unit 1 | **Lesson 1 – Launch**

Scale-y Shapes

Let's see which shapes can be used to build larger copies of themselves.

Warm-up Which One Doesn't Belong?

Study the images. Which image does not belong with the others? Explain your thinking.

Collect and Display: As you explain your thinking, your teacher will collect the math language you use to add to a class display. You will continue to add and refer to this display throughout the unit.

Sample responses:

- Choice A doesn't belong because it is the only one that has only right angles.
- Choice B doesn't belong because it is the only one where the smaller shapes inside the larger shape are not all the same shape.
- Choice C doesn't belong because it is the only one that is composed of 2 shapes.
- Choice D doesn't belong because it is the only one that is composed of a shape with a curl.

Log in to Amplify Math to complete this lesson online.

4 **Unit 1** Scale Drawings

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1 Launch

Conduct the *Which One Doesn't Belong?* routine, explaining to student that there will always be multiple correct responses to this type of problem. Encourage them to seek out a reason for why *each* image does not belong.

2 Monitor

Help students get started by saying, "Describe one attribute that all of the shapes have in common."

Look for points of confusion:

- **Citing a reason that excludes *more than one* of the shapes.** Have students refine their thinking so that only one shape does not belong.

Look for productive strategies:

- Noticing that the images in Choices A, C, and D are composed of smaller figures that are identical copies of one another. **Note:** The term *congruent* is not officially introduced until Grade 8.

3 Connect

Display the four images from the Student Edition.

Have students share their thinking for why each choice might not belong with the others. Record any informal vocabulary that emerges from students sharing. Display this language in a place where it can be referred to throughout the unit.

Highlight that the images in Choices A, C, and D are examples of a mathematical figure called a *rep-tile*, named by mathematician Samuel Golomb. A rep-tile is a figure that is created entirely from smaller copies of itself. The image in Choice B is not a rep-tile; it is composed of triangles, but each triangle is *not* a smaller copy of the larger triangle.

Ask, "Why do you think Solomon Golomb named this type of figure a *rep-tile*? Sample response: He may have named it a rep-tile because you are "rep"-eating or "rep"-licating a figure.

Math Language Development

MLR2: Collect and Display

Collect and display language that students use to describe their reasoning. Pay attention to informal language that students use to describe the attributes, such as the size, color, and shape of the images. Display this language so that it can be referred to throughout the unit.

English Learners

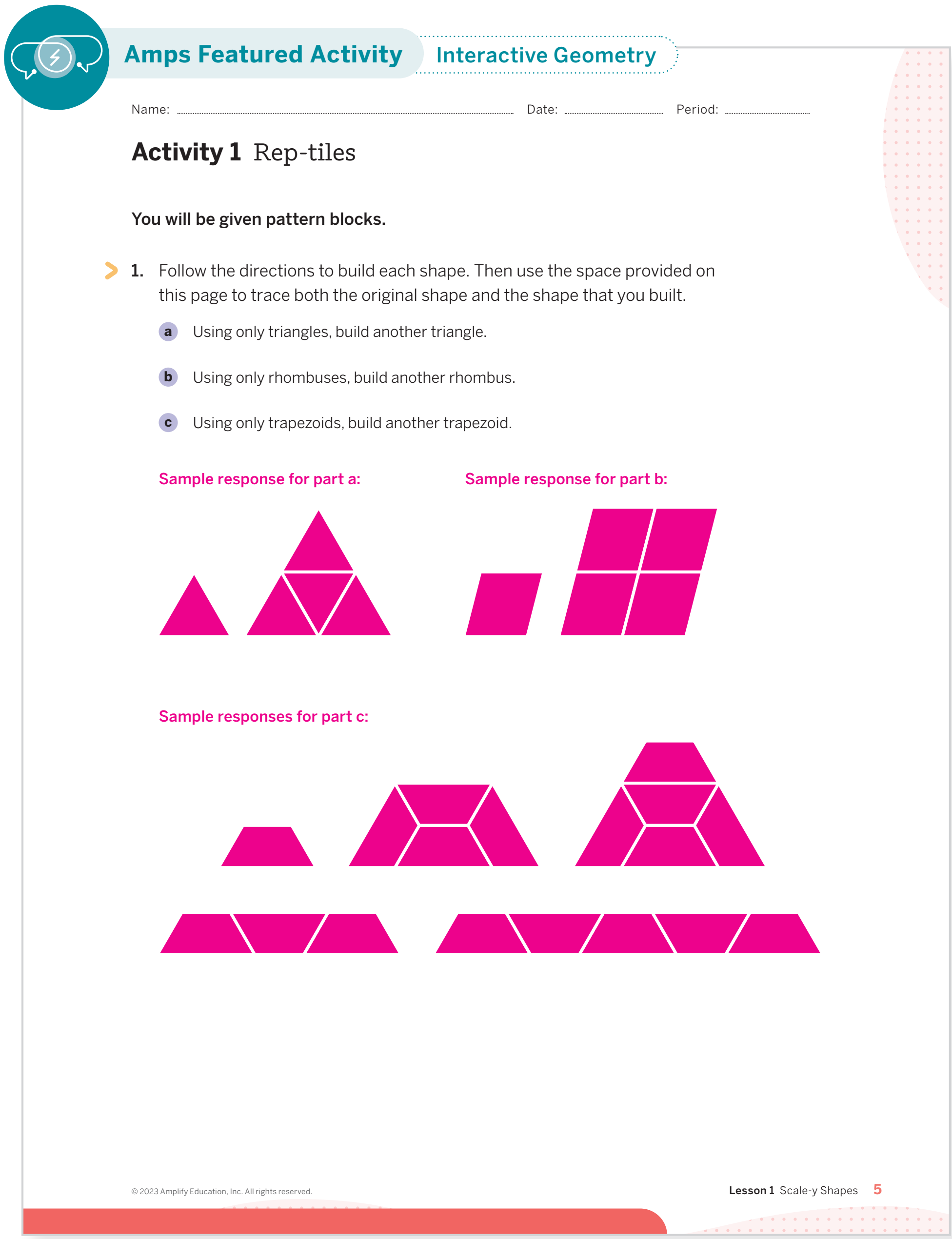
Display the following sentence frames for students to use as they begin to share their reasoning: "I think that _____ doesn't belong because . . ." "Something that makes _____ special is . . ."

Pairs | 20 min

Activity 1 Rep-tiles

Students explore a type of geometric tessellation called a *rep-tile* to introduce the idea of a scaled copy of a figure.

Amps Featured Activity Interactive Geometry



Name: ____________ Date: ________ Period: ________

Activity 1 Rep-tiles

You will be given pattern blocks.

1. Follow the directions to build each shape. Then use the space provided on this page to trace both the original shape and the shape that you built.
   - a Using only triangles, build another triangle.
   - b Using only rhombuses, build another rhombus.
   - c Using only trapezoids, build another trapezoid.

Sample response for part a:

Sample response for part b:

Sample responses for part c:

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Lesson 1 Scale-y Shapes 5

1 Launch

Display images of the pattern blocks students will use. Activate prior knowledge by inviting students to share what they already know about equilateral triangles, rhombuses, and trapezoids. Distribute pattern blocks to pairs of students. If not using physical pattern blocks, distribute pre-cut pattern blocks from the Activity 1 PDF.

2 Monitor

Help students get started by having them select a pattern block shape — such as a triangle — and setting one triangle aside for comparison. Suggest they use the remaining triangles to build a larger copy of the triangle they set aside.

Look for points of confusion:

- **Leaving a space in the middle of the larger shape.** Ask students if the smaller version of the shape also contains a space in the middle.
- **Using different shapes, such as triangles and rhombuses to build a larger triangle.** Ask students to re-read the requirements for building each shape.
- **Thinking a parallelogram is a rhombus.** Have students describe what attributes a rhombus has that a parallelogram does not have, i.e., all side lengths are the same.

Look for productive strategies:

- Comparing the side lengths of the figures by placing the smaller, original shape next to each side of the larger shape.
- Noticing that only one copy of the larger triangle and rhombus can be built, but that multiple copies of a larger trapezoid can be built.

Activity 1 continued ›

Differentiated Support

Accessibility: ***Optimize Access to Tools***

Provide students with pre-cut copies of each figure from the Activity 1 PDF. Demonstrate how to use the cutouts to check whether a figure is a larger or smaller copy of itself.

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on building a triangle and a rhombus in Problems 1a and 1b.

Math Language Development

MLR3: Critique, Correct, Clarify

Use this routine to address the point of confusion students might have thinking a parallelogram is a rhombus. Display the following statements. Ask students to critique each statement, determine whether each is correct or incorrect, and then craft a response clarifying their thinking.

- A parallelogram is always a rhombus.
- A parallelogram is sometimes a rhombus.
- A parallelogram is never a rhombus.

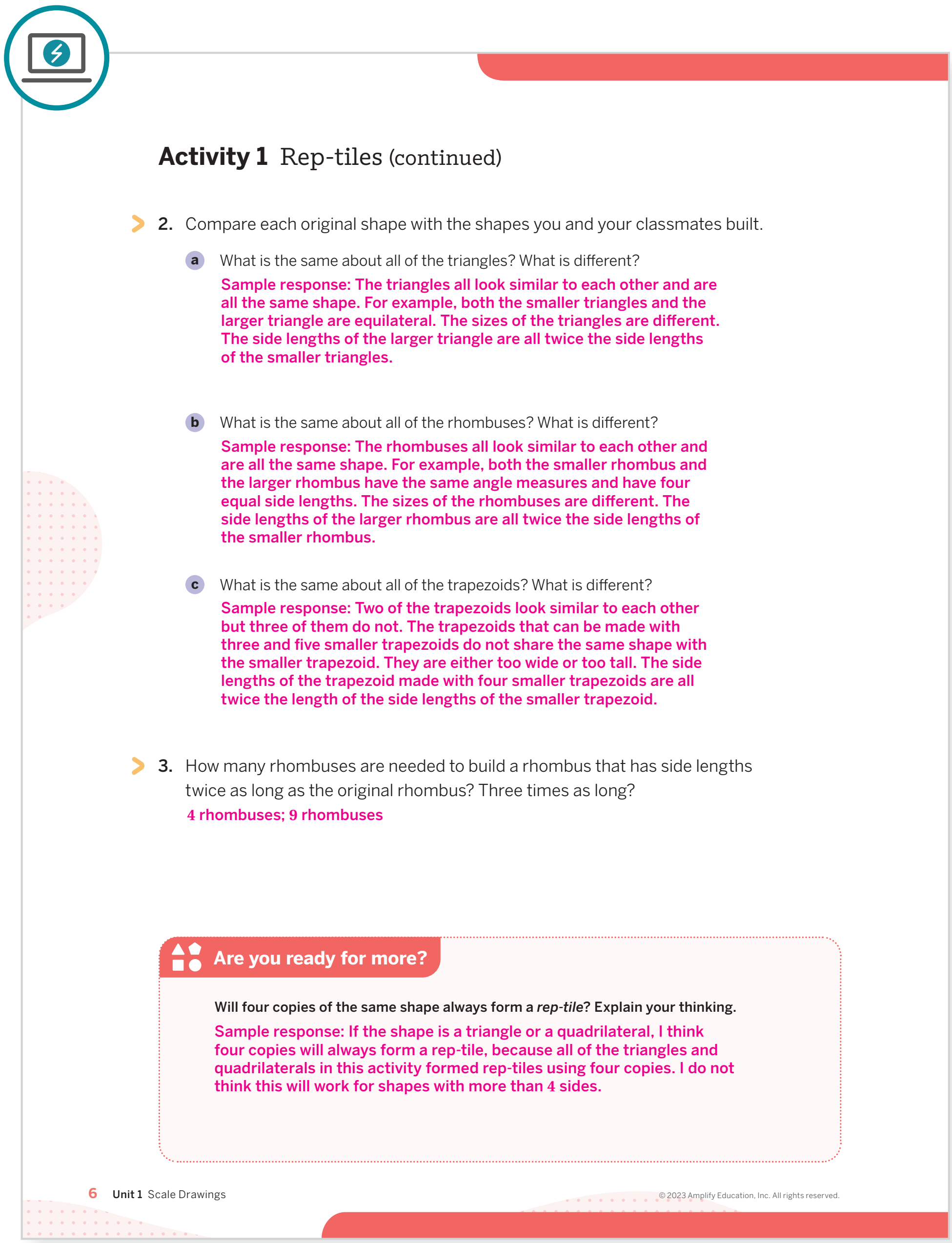
English Learners

Encourage students to draw various parallelograms and rhombuses to help them critique each statement.

Pairs | 20 min

Activity 1 Rep-tiles (continued)

Students explore a type of geometric tessellation called a *rep-tile* to introduce the idea of a scaled copy of a figure.



Activity 1 Rep-tiles (continued)

2. Compare each original shape with the shapes you and your classmates built.
   - a What is the same about all of the triangles? What is different?
     Sample response: The triangles all look similar to each other and are all the same shape. For example, both the smaller triangles and the larger triangle are equilateral. The sizes of the triangles are different. The side lengths of the larger triangle are all twice the side lengths of the smaller triangles.
   - b What is the same about all of the rhombuses? What is different?
     Sample response: The rhombuses all look similar to each other and are all the same shape. For example, both the smaller rhombus and the larger rhombus have the same angle measures and have four equal side lengths. The sizes of the rhombuses are different. The side lengths of the larger rhombus are all twice the side lengths of the smaller rhombus.
   - c What is the same about all of the trapezoids? What is different?
     Sample response: Two of the trapezoids look similar to each other but three of them do not. The trapezoids that can be made with three and five smaller trapezoids do not share the same shape with the smaller trapezoid. They are either too wide or too tall. The side lengths of the trapezoid made with four smaller trapezoids are all twice the length of the side lengths of the smaller trapezoid.

3. How many rhombuses are needed to build a rhombus that has side lengths twice as long as the original rhombus? Three times as long?
   4 rhombuses; 9 rhombuses

Are you ready for more?

Will four copies of the same shape always form a *rep-tile*? Explain your thinking.

Sample response: If the shape is a triangle or a quadrilateral, I think four copies will always form a rep-tile, because all of the triangles and quadrilaterals in this activity formed rep-tiles using four copies. I do not think this will work for shapes with more than 4 sides.

3 Connect

Display the different shapes students built for the whole class to see.

Have students share their observations from Problem 2.

Ask:

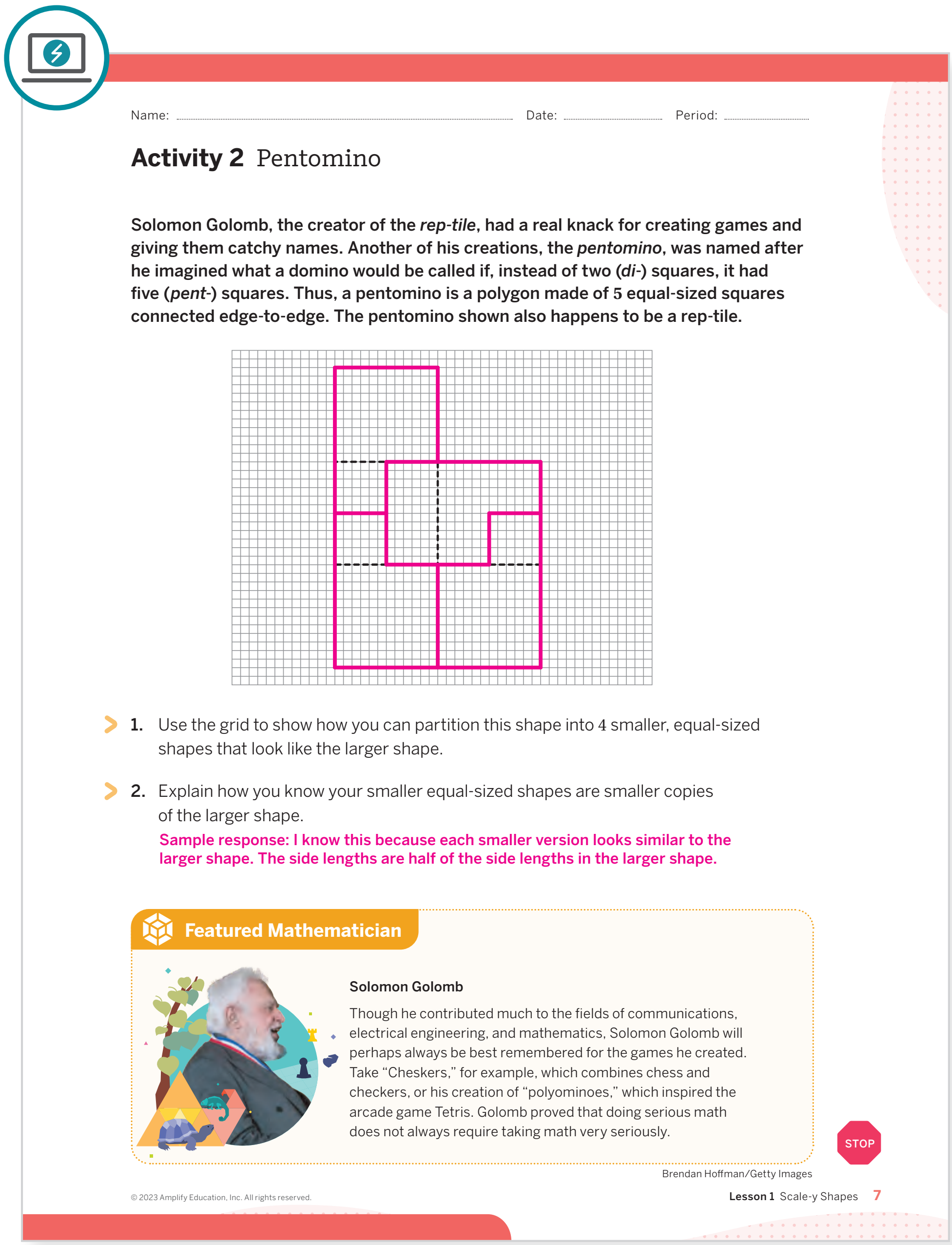
- "How can you tell if one figure is a larger or a smaller copy of another?" The figures will look similar, and I can check that the side lengths have been enlarged or reduced in a consistent manner. **Note:** The term *similar* here is informal. Allow students to use the informal language until *similarity* is formally defined in Grade 8.
- "How many rhombuses are needed to build a shape whose side lengths are four times longer than the original shape? Five times longer?" 16 rhombuses; 25 rhombuses
- How many rhombuses are needed to build a shape whose side lengths are n times longer than the original shape? Is this also true for the triangle and the trapezoid?" n^2; Yes, this is also true for the triangle and the trapezoid.

Highlight that only the shapes that look like a larger copy of the smaller original shapes are considered rep-tiles. This was true for all the rhombuses and triangles that were built, but not for all the trapezoids. Let students know that, in this unit, they will explore shapes and figures that look like larger and smaller copies of themselves.

Pairs | 10 min

Activity 2 Pentomino

Students are given a rep-tile and consider how to partition it into smaller copies of itself. This puzzle challenges students to reason about proportional scaling.



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Activity 2 Pentomino

Solomon Golomb, the creator of the *rep-tile*, had a real knack for creating games and giving them catchy names. Another of his creations, the *pentomino*, was named after he imagined what a domino would be called if, instead of two (*di-*) squares, it had five (*pent-*) squares. Thus, a pentomino is a polygon made of 5 equal-sized squares connected edge-to-edge. The pentomino shown also happens to be a rep-tile.

1. Use the grid to show how you can partition this shape into 4 smaller, equal-sized shapes that look like the larger shape.
2. Explain how you know your smaller equal-sized shapes are smaller copies of the larger shape.
   Sample response: I know this because each smaller version looks similar to the larger shape. The side lengths are half of the side lengths in the larger shape.

Featured Mathematician

Solomon Golomb

Though he contributed much to the fields of communications, electrical engineering, and mathematics, Solomon Golomb will perhaps always be best remembered for the games he created. Take "Cheskers," for example, which combines chess and checkers, or his creation of "polyominoes," which inspired the arcade game Tetris. Golomb proved that doing serious math does not always require taking math very seriously.

STOP

Brendan Hoffman/Getty Images

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Lesson 1 Scale-y Shapes 7

1 Launch

Let students know that this is a challenging puzzle and it is okay if they need to make multiple attempts in partitioning the shape.

2 Monitor

Help students get started by offering a hint. Hint: Because they need to partition the pentomino into 4 smaller copies of itself, each of the copies will have an area one-fourth of the larger shape.

Look for points of confusion:

- **Thinking there needs to be five smaller shapes because the pentomino is composed of five equal-sized squares.** Ask students to reread the directions in Problem 1.
- **Struggling to partition the shape on the first attempt and then giving up.** Encourage students to persevere, assuring them that great mathematicians also struggle with complex problems.

Look for productive strategies:

- Using the five equal-sized squares inside the larger shape to reason about how many smaller equal-sized squares there should be in each smaller shape.
- Drawing and cutting out a smaller copy of the shape, cutting it out, and manipulating it to see how it can fit multiple times inside the larger shape.

3 Connect

Display students' work to the whole class.

Have students share their moments where they felt like they made progress or "had a breakthrough" with the puzzle. Ask them to describe how they encouraged themselves to continue to persevere.

Highlight that students can compare the side lengths of the smaller shapes to the larger shape and notice a consistent relationship. Students will explore this relationship further in Lessons 2–6.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with copies of the figure and access to scissors so they can cut the shapes out and rearrange them. Alternatively, provide access to different colored pencils for students to sketch and shade where they think the smaller shapes will be placed inside the larger shape.

Math Language Development

MLR2: Collect and Display

Circulate and listen for students' reasoning for how they know the smaller equal-sized shapes are smaller copies of the larger shape. Add their reasoning to the class anchor chart for students to refer back to during discussions.

English Learners

Provide students with a smaller copy of the shape that they can cut out and manipulate to see how it can fit multiple times inside the larger shape.

Featured Mathematician

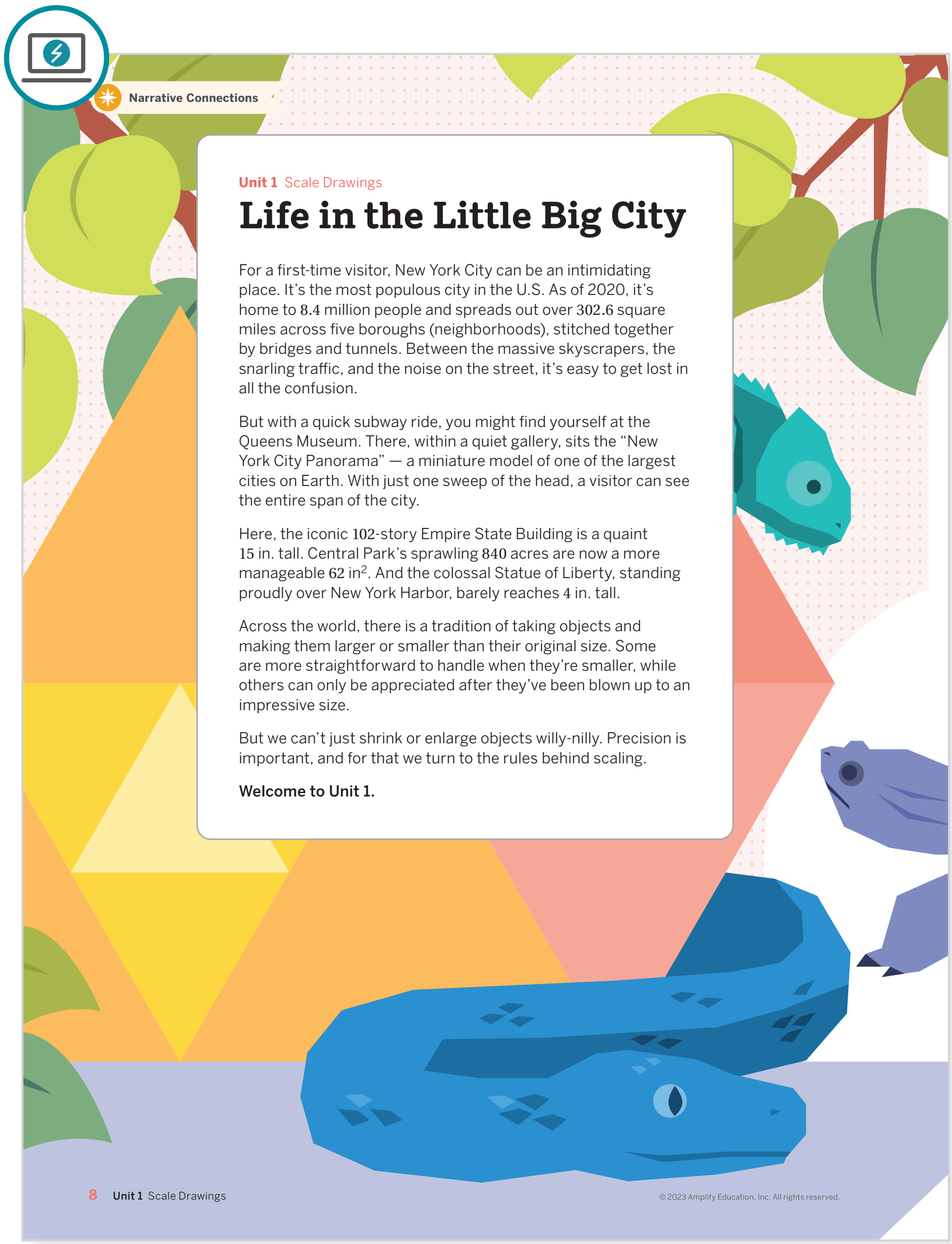
Solomon Golomb

Have students read about featured mathematician Solomon Golomb, a polymath who created several games and puzzles, and is credited as the "Godfather of Tetris." A *polymath* is someone who has a broad range of knowledge from multiple areas and can apply this knowledge to solve complex problems.

Whole Class | 5 min

Summary Life in the Little Big City

Review and synthesize how some shapes can be copied and rearranged to form larger copies of themselves.



Narrative Connections

Unit 1 Scale Drawings

Life in the Little Big City

For a first-time visitor, New York City can be an intimidating place. It's the most populous city in the U.S. As of 2020, it's home to 8.4 million people and spreads out over 302.6 square miles across five boroughs (neighborhoods), stitched together by bridges and tunnels. Between the massive skyscrapers, the snarling traffic, and the noise on the street, it's easy to get lost in all the confusion.

But with a quick subway ride, you might find yourself at the Queens Museum. There, within a quiet gallery, sits the "New York City Panorama" — a miniature model of one of the largest cities on Earth. With just one sweep of the head, a visitor can see the entire span of the city.

Here, the iconic 102-story Empire State Building is a quaint 15 in. tall. Central Park's sprawling 840 acres are now a more manageable 62 in^2. And the colossal Statue of Liberty, standing proudly over New York Harbor, barely reaches 4 in. tall.

Across the world, there is a tradition of taking objects and making them larger or smaller than their original size. Some are more straightforward to handle when they're smaller, while others can only be appreciated after they've been blown up to an impressive size.

But we can't just shrink or enlarge objects willy-nilly. Precision is important, and for that we turn to the rules behind scaling.

Welcome to Unit 1.

8 **Unit 1** Scale Drawings

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary silently or have a student volunteer read it aloud.

Highlight that during this unit, students will continue to work with polygons, focusing on how they can be resized and still maintain their shape.

Ask:

- "Have you ever built a model version of something? If so, what was it?" Sample responses: a model car, a model airplane, a model building
- "If you were to build a model of the Empire State Building that was only 15 in. tall, but looked exactly like the Empire State Building in every other way, what information might you need?" Sample responses: the actual height of the building, other dimensions of the building, number of floors or windows, what the building looks like

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

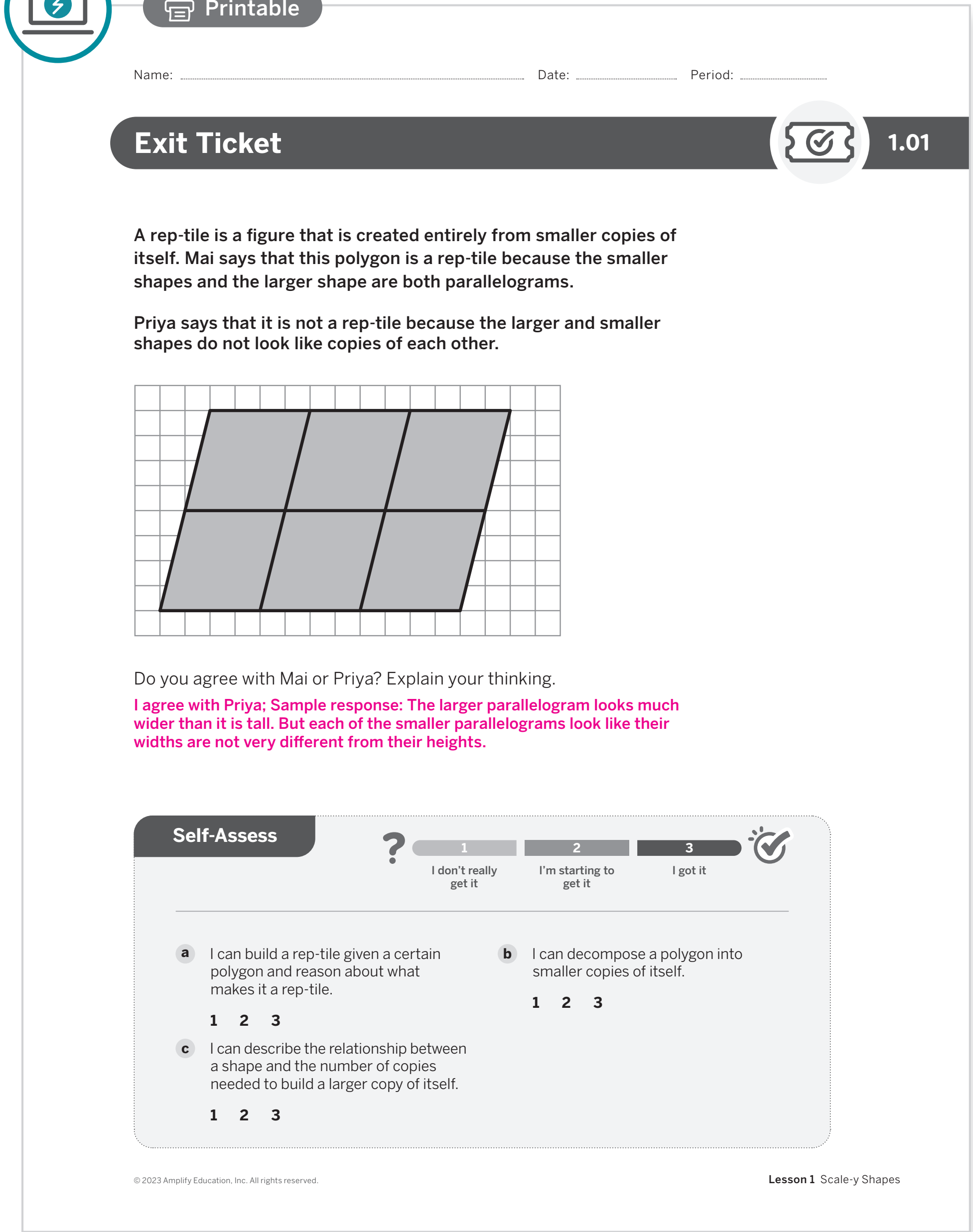
- "What did you find most challenging about creating rep-tiles? What strategies helped?"
- "What strategies or tools were helpful in creating larger copies of shapes? How were they helpful?"
- "What strategies or tools were helpful in partitioning large shapes into smaller copies of themselves?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of rep-tiles by determining whether the smaller shapes look similar to the larger shape.

Printable



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Exit Ticket 1.01

A rep-tile is a figure that is created entirely from smaller copies of itself. Mai says that this polygon is a rep-tile because the smaller shapes and the larger shape are both parallelograms.

Priya says that it is not a rep-tile because the larger and smaller shapes do not look like copies of each other.

Do you agree with Mai or Priya? Explain your thinking.

I agree with Priya; Sample response: The larger parallelogram looks much wider than it is tall. But each of the smaller parallelograms look like their widths are not very different from their heights.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can build a rep-tile given a certain polygon and reason about what makes it a rep-tile.

1 2 3

b I can decompose a polygon into smaller copies of itself.

1 2 3

c I can describe the relationship between a shape and the number of copies needed to build a larger copy of itself.

1 2 3

Lesson 1 Scale-y Shapes

Success looks like . . .

- **Language Goal:** Understanding and explaining how some figures can be tiled to form similar, but differently sized versions of themselves. **(Speaking and Listening, Writing)**
    - » Selecting that Priya is correct that the larger parallelogram is not a rep-tile.

Suggested next steps

If students think Mai is correct, consider:

- Asking them to explain how they know both polygons are parallelograms — which is actually correct. Follow up with a question about what makes a certain polygon a *rep-tile*.

Professional Learning

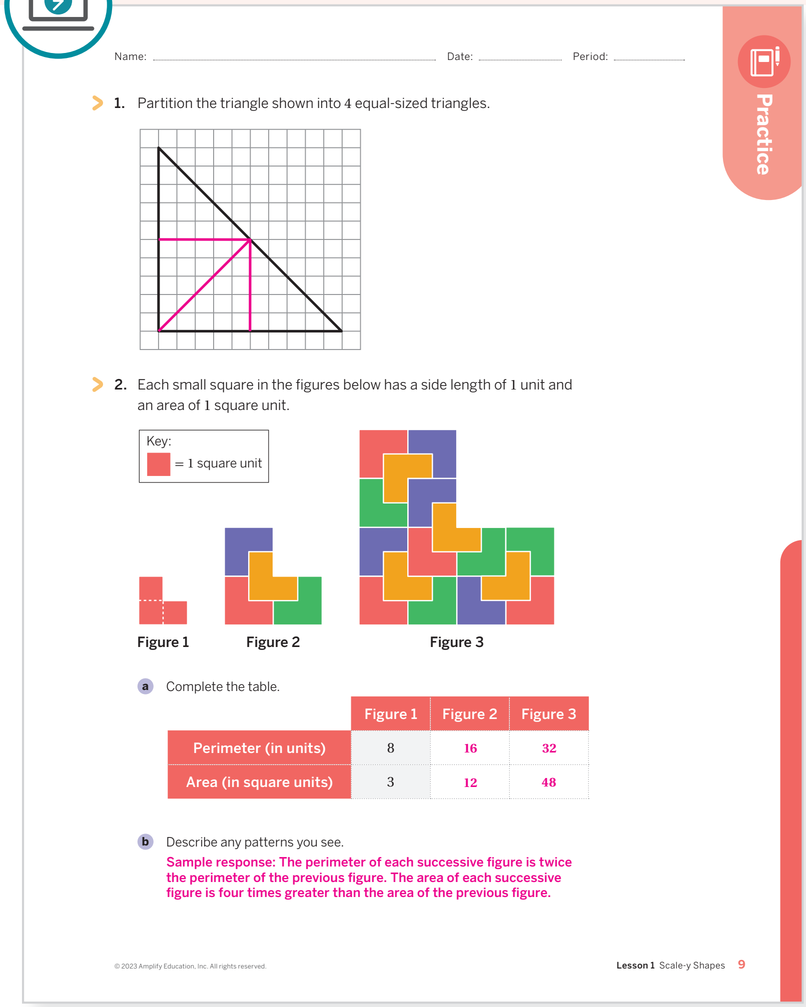
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did the process of building rep-tiles in Activity 1 reveal about your students as learners?
- In what ways did sharing student work go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Partition the triangle shown into 4 equal-sized triangles.

2. Each small square in the figures below has a side length of 1 unit and an area of 1 square unit.

Key:
🟥 = 1 square unit

Figure 1 Figure 2 Figure 3

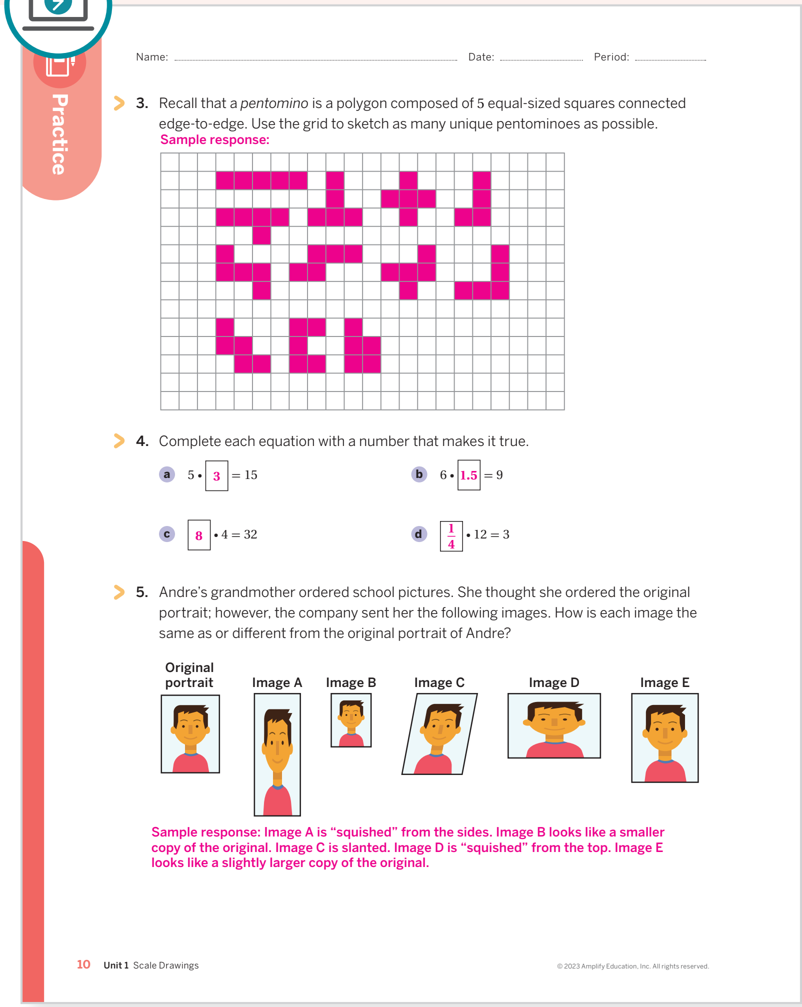
a Complete the table.

	Figure 1	Figure 2	Figure 3
Perimeter (in units)	8	16	32
Area (in square units)	3	12	48

b Describe any patterns you see.

Sample response: The perimeter of each successive figure is twice the perimeter of the previous figure. The area of each successive figure is four times greater than the area of the previous figure.

Lesson 1 Scale-y Shapes 9



Name: Date: Period:

3. Recall that a *pentomino* is a polygon composed of 5 equal-sized squares connected edge-to-edge. Use the grid to sketch as many unique pentominoes as possible.

Sample response:

4. Complete each equation with a number that makes it true.

a $5 \cdot \boxed{3} = 15$

b $6 \cdot \boxed{1.5} = 9$

c $\boxed{8} \cdot 4 = 32$

d $\boxed{\frac{1}{4}} \cdot 12 = 3$

5. Andre's grandmother ordered school pictures. She thought she ordered the original portrait; however, the company sent her the following images. How is each image the same as or different from the original portrait of Andre?

Original portrait Image A Image B Image C Image D Image E

Sample response: Image A is "squished" from the sides. Image B looks like a smaller copy of the original. Image C is slanted. Image D is "squished" from the top. Image E looks like a slightly larger copy of the original.

10 **Unit 1** Scale Drawings

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Grade 5	1
Formative ⚡	5	Unit 1 Lesson 2	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

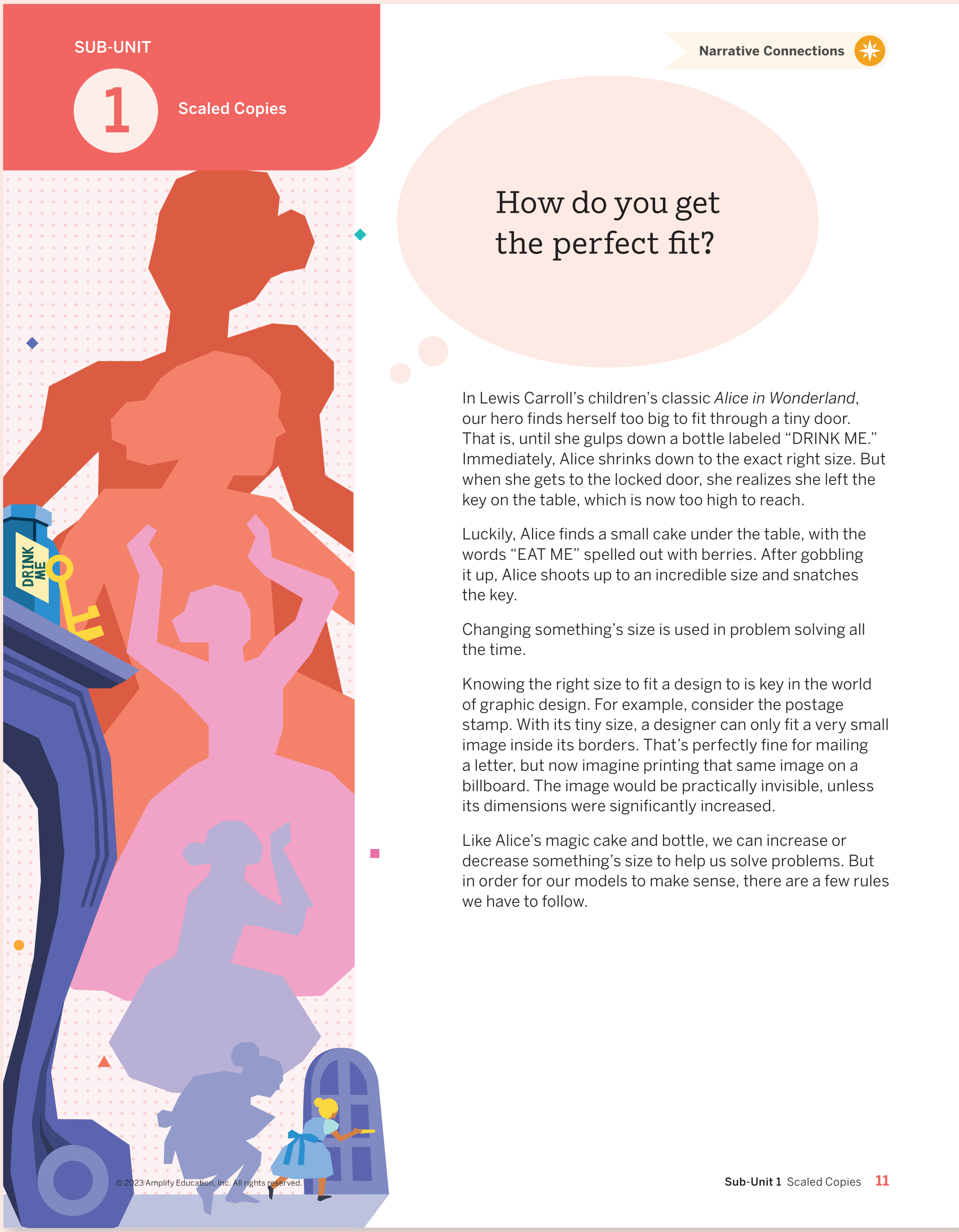
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 1
Scaled Copies

Whole Class

In this Sub-Unit, students notice that making two-dimensional figures larger and smaller involves paying careful attention to details, such as the scale factor.



SUB-UNIT

1 Scaled Copies

Narrative Connections

How do you get the perfect fit?

In Lewis Carroll's children's classic *Alice in Wonderland*, our hero finds herself too big to fit through a tiny door. That is, until she gulps down a bottle labeled "DRINK ME." Immediately, Alice shrinks down to the exact right size. But when she gets to the locked door, she realizes she left the key on the table, which is now too high to reach.

Luckily, Alice finds a small cake under the table, with the words "EAT ME" spelled out with berries. After gobbling it up, Alice shoots up to an incredible size and snatches the key.

Changing something's size is used in problem solving all the time.

Knowing the right size to fit a design to is key in the world of graphic design. For example, consider the postage stamp. With its tiny size, a designer can only fit a very small image inside its borders. That's perfectly fine for mailing a letter, but now imagine printing that same image on a billboard. The image would be practically invisible, unless its dimensions were significantly increased.

Like Alice's magic cake and bottle, we can increase or decrease something's size to help us solve problems. But in order for our models to make sense, there are a few rules we have to follow.

Sub-Unit 1 Scaled Copies 11

Narrative Connections

Read the narrative aloud as a class or have students read it individually. As fantastical as Lewis Carroll's writing is, students continue analyzing scaled copies of real-world shapes and figures in the following places:

- **Lesson 2, Warm-up:** Printing Copies
- **Lesson 5, Activity 3:** Scaling a Puzzle
- **Lesson 6, Warm-up:** Sharing a Vegetable Lasagna

UNIT 1 | **LESSON 2**

What Are Scaled Copies?

Let's explore scaled copies.

Focus

Goals

1. **Language Goal:** Describe the characteristics of scaled copies and copies that are not scaled. **(Speaking and Listening)**
2. **Language Goal:** Identify a scaled copy of a figure and justify that the copy is a scaled copy**. (Speaking and Listening, Writing)**

Coherence

Today

Students distinguish scaled copies from those which are not — first informally, and later, with increasing precision. They may start by saying scaled copies have the same shape as the original figure, or they do not appear to be distorted in any way, though they may have a different size. Next, students notice that the lengths of segments in a scaled copy vary from the lengths in the original figure in a consistent manner. Students work toward carefully articulating the characteristics of scaled copies quantitatively (e.g., "all the segments are twice as long in the copy as in the original").

Previously

In Grade 6, students determined common multiples and factors of multi-digit numbers and will continue that work as they explore scaling lengths and areas.

Coming Soon

In Lesson 3, students will recognize that scaled copies of figures have corresponding sides which are related by a scale factor. They will also notice that angle measures in a scaled copy are unchanged from the original figure.

Rigor

- Students use visual models to build **conceptual understanding** of scaling by analyzing side lengths and angle measures of copies to determine what criteria defines a scaled copy.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Activity 2 PDF, pre-cut cards, one set per group
- Activity 2 PDF, pre-cut cards (answers, optional)
- geometry toolkits: protractors, rulers, tracing paper

Math Language Development

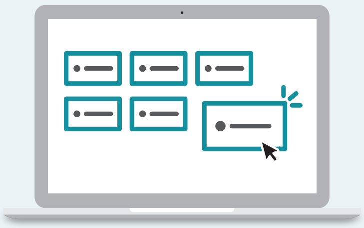
New word

- *scaled copy*

Amps | Featured Activity

Activity 2
Digital Card Sort

Students sort cards digitally to match scaled copies.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students discuss and resolve any disagreements during the Card Sort in Activity 2, they may not realize the impact their critique of another student's reasoning has on that student's feelings. Prior to the activity, have students plan for how they will show agreement as well as disagreement. Remind them that their choice of words and behaviors should show respect.

Modifications to Pacing

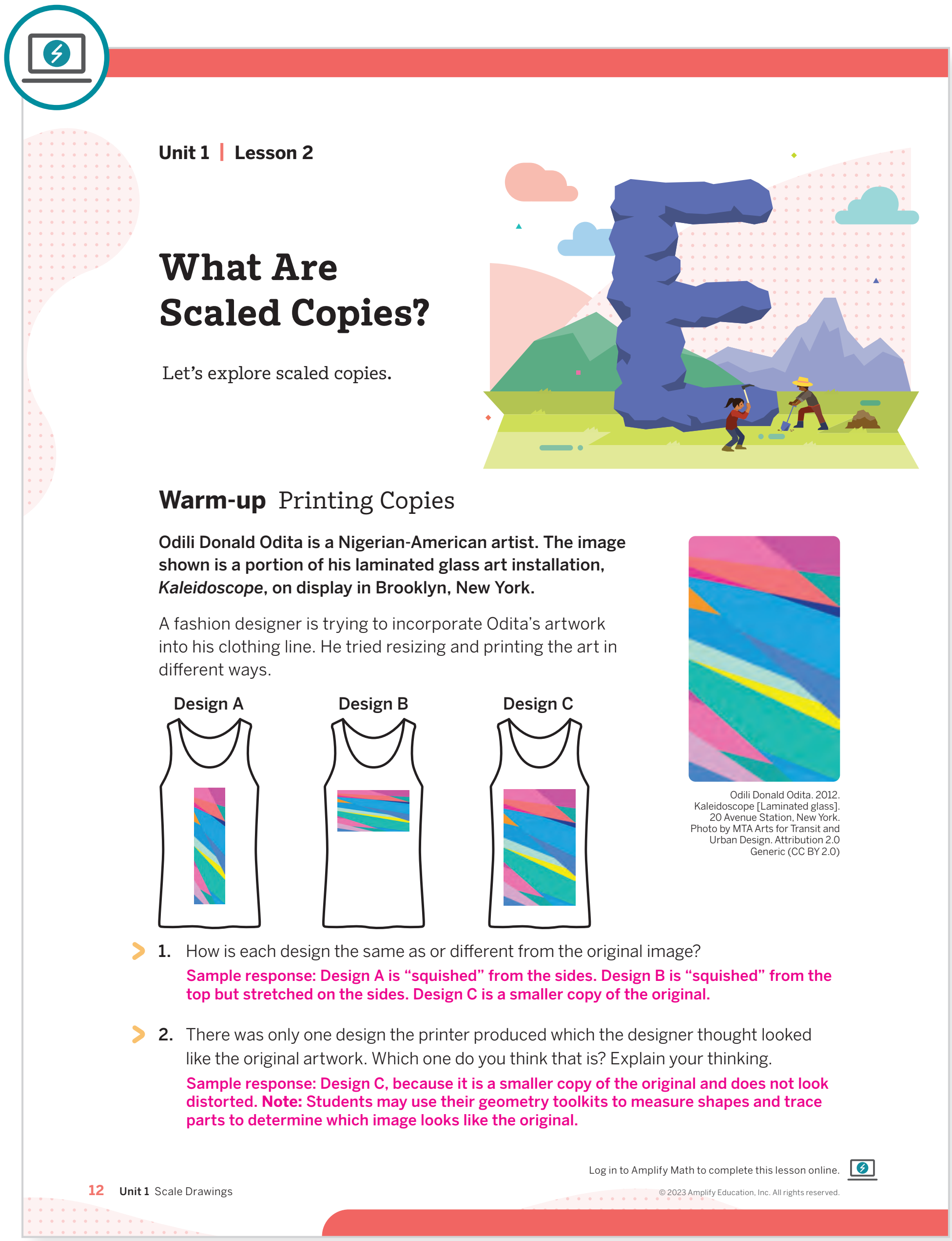
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, divide the figures among the pairs of students (e.g., have one pair determine if Figure 1 is a scaled copy, have another pair determine if Figure 2 is a scaled copy, etc.). Assign each pair a figure and have pairs share their findings.
- **Activity 2** may be omitted. Invest a little more time in Activity 1, Problem 3, discussing how to be sure a student's drawing of the letter "E" is a scaled copy or not.

Pairs | 10 min

Warm-up Printing Copies

Students analyze copies of a glass art installation to determine which are distorted, to begin recognizing what criteria defines a scaled copy.



Unit 1 | Lesson 2

What Are Scaled Copies?

Let's explore scaled copies.

Warm-up Printing Copies

Odili Donald Odita is a Nigerian-American artist. The image shown is a portion of his laminated glass art installation, *Kaleidoscope*, on display in Brooklyn, New York.

A fashion designer is trying to incorporate Odita's artwork into his clothing line. He tried resizing and printing the art in different ways.

Design A Design B Design C

Odili Donald Odita. 2012. Kaleidoscope [Laminated glass]. 20 Avenue Station, New York. Photo by MTA Arts for Transit and Urban Design. Attribution 2.0 Generic (CC BY 2.0)

1. How is each design the same as or different from the original image?
   Sample response: Design A is "squished" from the sides. Design B is "squished" from the top but stretched on the sides. Design C is a smaller copy of the original.

2. There was only one design the printer produced which the designer thought looked like the original artwork. Which one do you think that is? Explain your thinking.
   Sample response: Design C, because it is a smaller copy of the original and does not look distorted. Note: Students may use their geometry toolkits to measure shapes and trace parts to determine which image looks like the original.

Log in to Amplify Math to complete this lesson online.

12 Unit 1 Scale Drawings

1 Launch

Provide access to and explain the contents of their geometry toolkits. Conduct the ***Think-Pair-Share*** routine, providing students with 1 minute of individual think-time. Then have them complete the Warm-up with a partner.

2 Monitor

Help students get started by asking which painting looks the most similar to the original and have them share their reasoning. **Note:** The goal at this point is not to critique their reasoning, but to get a discussion started by having them share their thoughts.

Look for points of confusion:

- **Thinking there must be a right answer.** Encourage students to focus on the images and determine what is different or the same about the artwork.

Look for productive strategies:

- Using adjectives such as "stretched," "squished," "skewed," "reduced," etc., in imprecise ways. This is acceptable, as students' intuitive definition of scaled copies will be refined over the course of the lesson.
- Describing Design C with the idea that the length and width were changed by the same value.

3 Connect

Display the images of the designs.

Highlight the title of the lesson and ask students what the term *scaled copy* means. Let students know they may have the opportunity to make their own design in Lesson 13.

Have students share their working definitions of *scaled copy*. They will use their working definition in the next Activity.

Ask, "Do any of the designs appear to be scaled copies? Why or why not?"

Math Language Development

MLR2: Collect and Display

Write "scaled copy" and "non-scaled copy" on the board. Collect and display words students use to describe scaled copies (e.g., *not distorted*, *looks the same*, *but smaller*, etc.) and non-scaled copies (e.g., *distorted*, *squished from the top*, *squished from the sides*, etc.). Edit this class display as the class progresses through the lesson and this unit. Remind them to refer back to the class display during discussions.

English Learners

Use gestures to help students visualize the words that are used, such as moving hands together to indicate "squished from the top" or moving hands apart to indicate "stretched."

Power-up

To power up students' ability to identify differences in altered figures:

Provide students with a copy of the Power-up PDF.

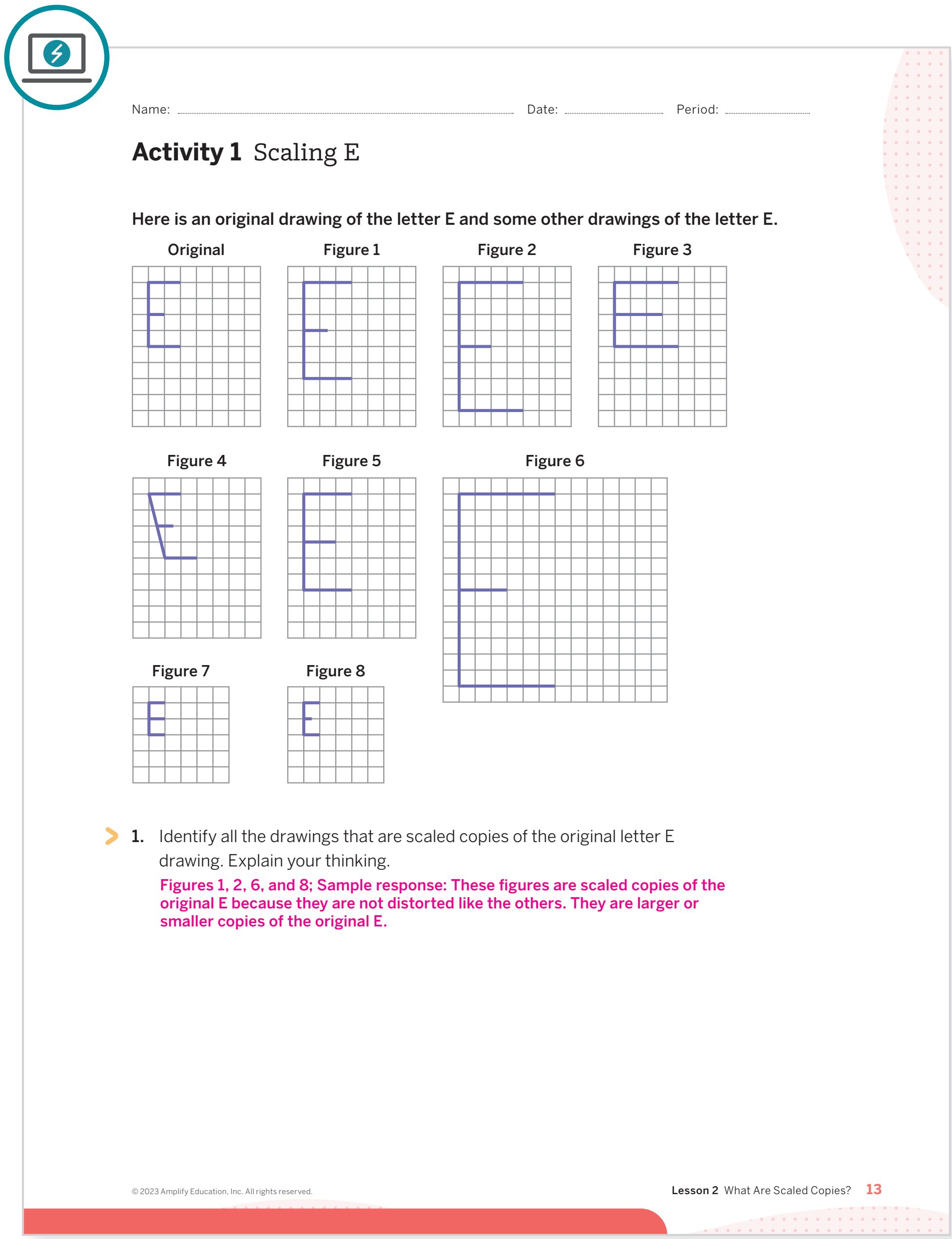
Use: Before Activity 1.

Informed by: Performance on Lesson 1, Practice Problem 5.

Pairs | 10 min

Activity 1 Scaling E

Students observe copies of a line drawing on a grid to describe more precisely the characteristics of scaled copies.



Name: ____________ Date: ________ Period: ________

Activity 1 Scaling E

Here is an original drawing of the letter E and some other drawings of the letter E.

Original

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

1. Identify all the drawings that are scaled copies of the original letter E drawing. Explain your thinking.
   Figures 1, 2, 6, and 8; Sample response: These figures are scaled copies of the original E because they are not distorted like the others. They are larger or smaller copies of the original E.

1 Launch

Explain the classroom procedure for partner work.

2 Monitor

Help students get started by asking them to quickly glance at the figures and circle the ones they think are scaled copies based solely on their quick observations. Then have students look closely at the ones they did not circle and explain why they think they are not scaled copies. Remind students they can change their responses as they analyze the figures more closely during the activity.

Look for points of confusion:

- **Analyzing only one segment of the letter E to determine whether the drawings are scaled copies.** Refer students to the designs from the Warm-up where only one dimension was changed. Changing one dimension did not create scaled copies.

Look for productive strategies:

- Measuring the lengths of the segments to compare each figure to the original.
- Noticing the original drawing and its scaled copies have equivalent width-to-height ratios by counting the grid units.
- Using a multiplier (or a scale factor) to compare the lengths of different figures to determine if they are scaled copies of the original.

Activity 1 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students compare Figure 1 to Figure 5 to determine which of those is a scaled copy of the original. Both figures are stretched vertically by the same amount, but the interior line segments are different lengths. Have students compare Figures 7 and 8 in a similar way.

Extension: *Math Enrichment*

Have students draw several scaled copies of the letter E for Problem 3. You may also choose to have them draw scaled copies of other letters of the alphabet that are composed of straight line segments, such as the letters F, H, K, L, or Z.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have students read Problem 1 and create a first draft response in writing. Have students meet with 2–3 partners to refine their writing by asking questions such as, "What did you mean by . . .?" and "Can you say that another way?" Have them revise their responses based on the feedback they were given and encourage them to borrow words and phrases they heard while working with others.

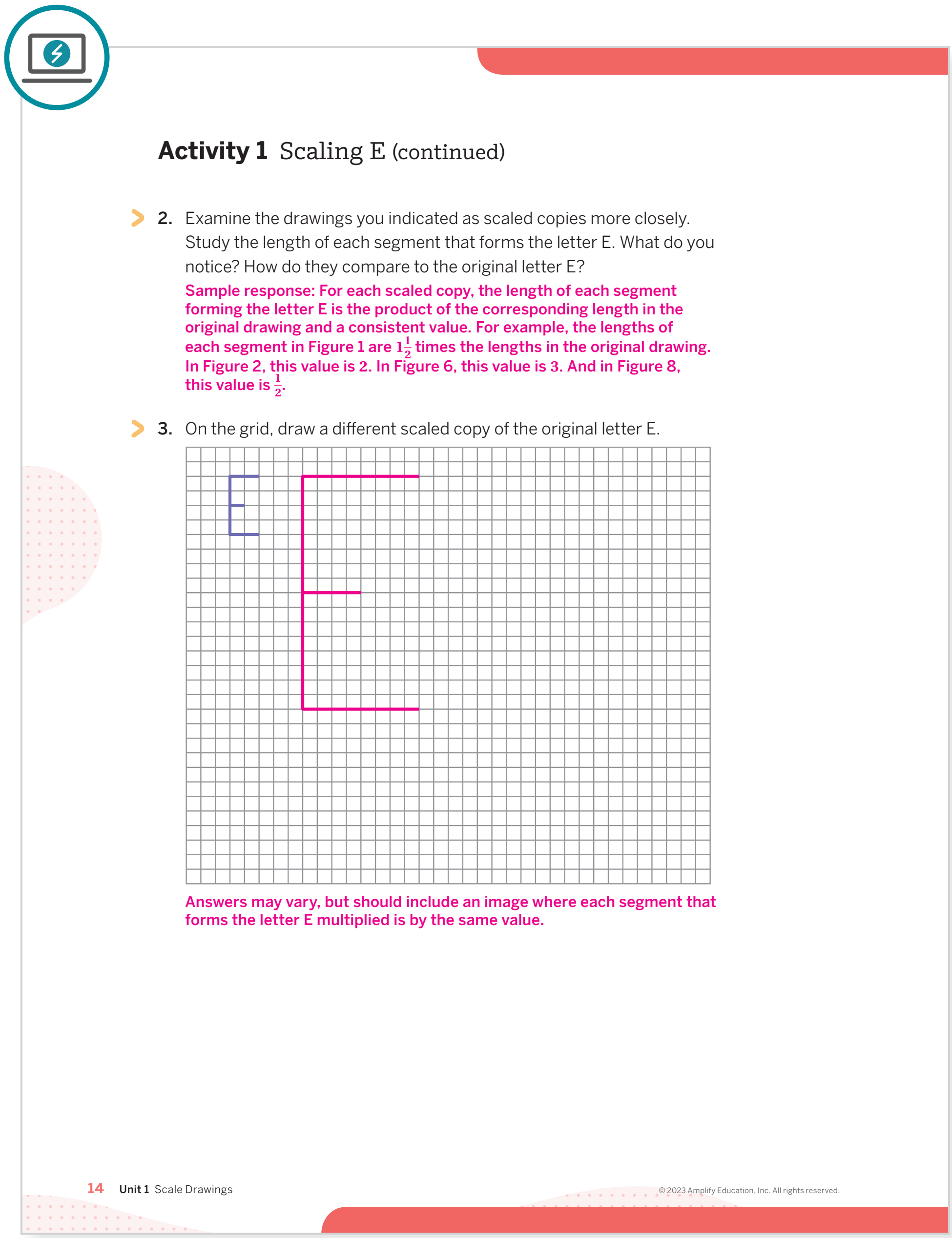
English Learners

Encourage students to write their first draft in their primary language. After meeting with 2–3 strategic partners, students should work to translate their revised draft to English.

Pairs | 10 min

Activity 1 Scaling E (continued)

Students observe copies of a line drawing on a grid to describe more precisely the characteristics of scaled copies.



Activity 1 Scaling E (continued)

2. Examine the drawings you indicated as scaled copies more closely. Study the length of each segment that forms the letter E. What do you notice? How do they compare to the original letter E?

   Sample response: For each scaled copy, the length of each segment forming the letter E is the product of the corresponding length in the original drawing and a consistent value. For example, the lengths of each segment in Figure 1 are 1$\frac{1}{2}$ times the lengths in the original drawing. In Figure 2, this value is 2. In Figure 6, this value is 3. And in Figure 8, this value is $\frac{1}{2}$.

3. On the grid, draw a different scaled copy of the original letter E.

   Answers may vary, but should include an image where each segment that forms the letter E multiplied is by the same value.

3 Connect

Display the figures from the activity.

Have students share whether they think each one is a *scaled copy* of the original letter E. Record and display the results. For contested drawings, ask 1–2 students to briefly say why they ruled these out. For Problem 1, expect students to explain their choices of scaled copies in intuitive, qualitative terms. For Problem 2, students should begin to distinguish scaled copies and copies that are not scaled in more specific and quantifiable ways. If it does not occur to students to observe the lengths of segments, suggest they do so.

Highlight students who identify distinguishing features of the scaled copies by determining similarities and differences in the shapes. They may see that corresponding parts increase or decrease by the same multiplier.
Note: It is not necessary for students to properly determine the scale factor as this concept will be addressed in the next lesson.

Ask:

- "What do the scaled copies have in common?" Be sure to invite students who were thinking along the lines of scale factors and angle measures to share.
- "How do the other copies fail to show these attributes?" Sample response: Sometimes, the lengths of the sides in the copy use different multipliers for different side lengths. So, they are not scaled copies. Other times, the angles in the copy do not have the same angle measures as the original. So, they are not scaled copies.

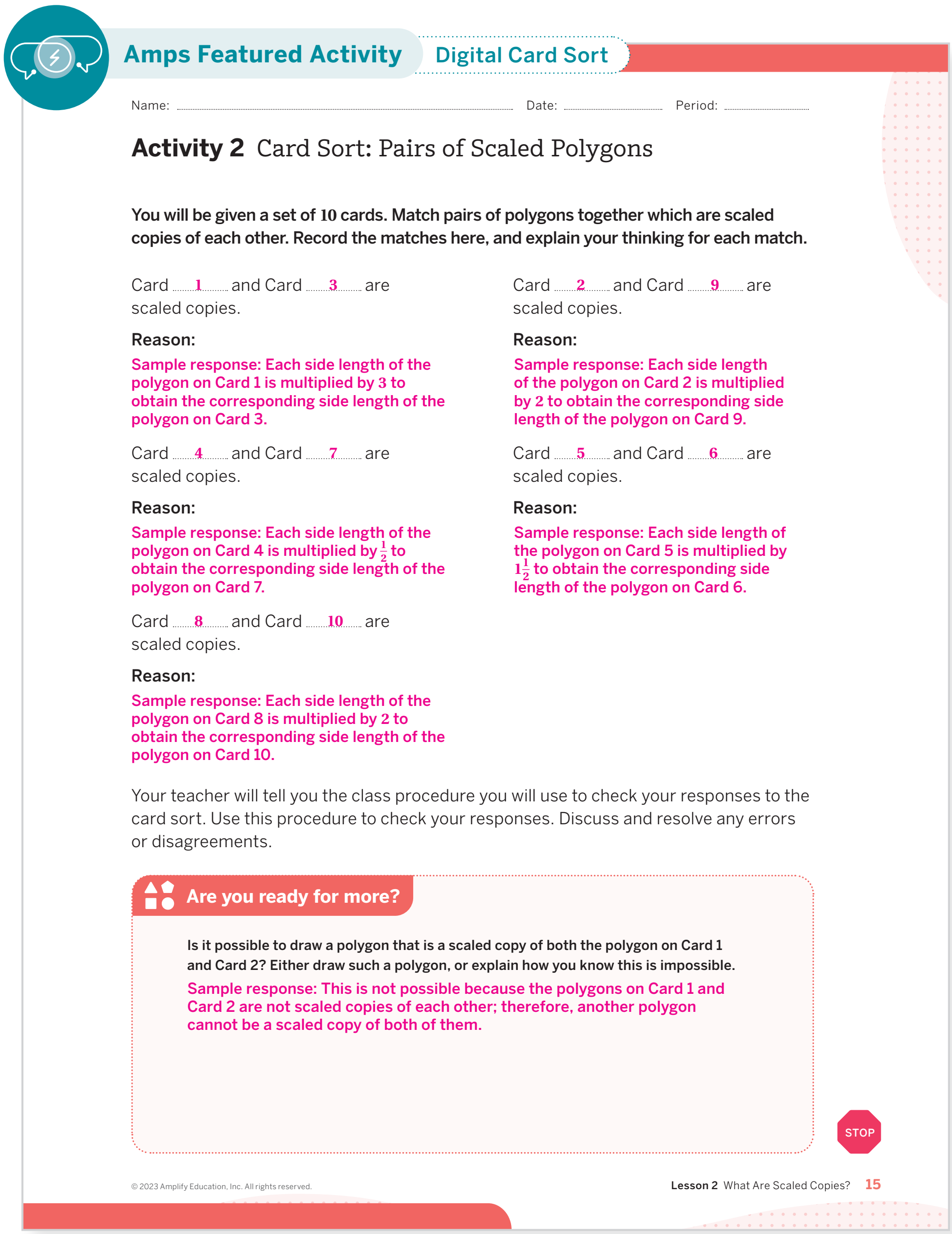
Define the term ***scaled copy*** as a copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy.

Pairs | 15 min

Activity 2 Card Sort: Pairs of Scaled Polygons

Students match pairs of polygons to refine their understanding of scaled copies.

Amps Featured Activity Digital Card Sort



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Activity 2 Card Sort: Pairs of Scaled Polygons

You will be given a set of 10 cards. Match pairs of polygons together which are scaled copies of each other. Record the matches here, and explain your thinking for each match.

Card 1 and Card 3 are scaled copies.

Reason:

Sample response: Each side length of the polygon on Card 1 is multiplied by 3 to obtain the corresponding side length of the polygon on Card 3.

Card 4 and Card 7 are scaled copies.

Reason:

Sample response: Each side length of the polygon on Card 4 is multiplied by $\frac{1}{2}$ to obtain the corresponding side length of the polygon on Card 7.

Card 8 and Card 10 are scaled copies.

Reason:

Sample response: Each side length of the polygon on Card 8 is multiplied by 2 to obtain the corresponding side length of the polygon on Card 10.

Card 2 and Card 9 are scaled copies.

Reason:

Sample response: Each side length of the polygon on Card 2 is multiplied by 2 to obtain the corresponding side length of the polygon on Card 9.

Card 5 and Card 6 are scaled copies.

Reason:

Sample response: Each side length of the polygon on Card 5 is multiplied by $1\frac{1}{2}$ to obtain the corresponding side length of the polygon on Card 6.

Your teacher will tell you the class procedure you will use to check your responses to the card sort. Use this procedure to check your responses. Discuss and resolve any errors or disagreements.

Are you ready for more?

Is it possible to draw a polygon that is a scaled copy of both the polygon on Card 1 and Card 2? Either draw such a polygon, or explain how you know this is impossible.

Sample response: This is not possible because the polygons on Card 1 and Card 2 are not scaled copies of each other; therefore, another polygon cannot be a scaled copy of both of them.

STOP

Lesson 2 What Are Scaled Copies? 15

1 Launch

Distribute one set of pre-cut cards from the Activity 2 PDF to each pair of students and conduct the ***Card Sort*** routine. Create a classroom procedure for pairs to check their work. If displaying answers around the room, display the Activity 2 PDF, (answers).

2 Monitor

Help students get started by selecting two cards and explaining why they think the cards do or do not match. Be sure to demonstrate productive ways to agree or disagree (e.g., by explaining your thinking, asking clarifying questions, etc.).

Look for points of confusion:

- **Thinking the vertices must land at intersections of grid lines and concluding that, e.g., card 5 cannot be a copy of card 6.** Ask them to consider how a 1-unit-long segment would change if scaled to be half its original side length. Where must one or both of its vertices land?

Look for productive strategies:

- Determining the multiplier to scale Card 1 to Card 3 and the multiplier to scale Card 3 to Card 1.

3 Connect

Display any cards needed for the discussion.

Have pairs of students share their responses.

Highlight concrete methods for deciding if two polygons are scaled copies of one another. Focus on using quantitative descriptors such as “half as long” or “three times as long”.

Ask:

- “How many sides did you compare before you decided that the polygon was or was not a scaled copy?” Checking two sides is sufficient to tell that polygons are not scaled copies, however, *all* sides need to be verified to ensure a polygon is a scaled copy.
- “Did anyone check the angles of the polygons? If so, what did you notice?“ They stayed the same.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with Cards 1–4, 7, and 9 first. Once they match these pairs, provide them with the remaining cards.

Accessibility: *Guide Processing and Visualization*

Provide a range of examples and counterexamples. During the demonstration of how to set up and complete the matching activity, select two cards that do not match. Invite students to suggest a shared justification.

Math Language Development

MLR8: Discussion Supports

Highlight the relationship between the two polygons on the cards. For example, ask, “How are the numbers related to each other? If I start with this number, what operation can I use to get the second number?” Emphasize that the relationship is multiplicative by using phrases, such as *half as long* and *three times as long* during the discussion.

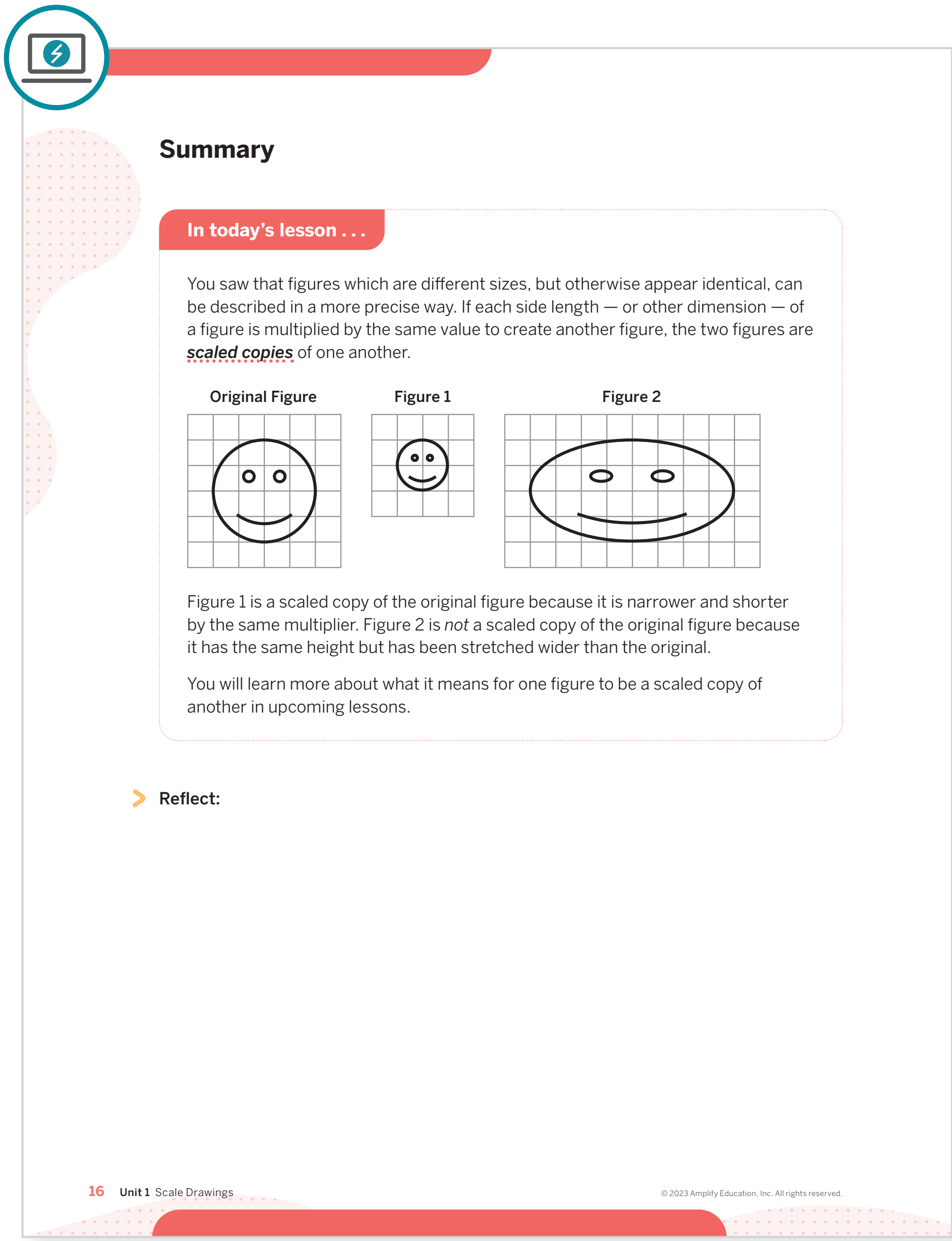
English Learners

Provide sentence frames, such as “Each side length of the polygon on Card _____ is multiplied by _____ to get the corresponding side length of the polygon on Card _____.”

Whole Class | 5 min

Summary

Review and synthesize what criteria determines whether two figures are scaled copies of each other.



Summary

In today's lesson . . .

You saw that figures which are different sizes, but otherwise appear identical, can be described in a more precise way. If each side length — or other dimension — of a figure is multiplied by the same value to create another figure, the two figures are *scaled copies* of one another.

Figure 1 is a scaled copy of the original figure because it is narrower and shorter by the same multiplier. Figure 2 is *not* a scaled copy of the original figure because it has the same height but has been stretched wider than the original.

You will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

Reflect:

16 Unit 1 Scale Drawings

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Synthesize

Formalize vocabulary: *scaled copy*

Have students share their strategies for determining whether figures are scaled copies.

Highlight that the lengths of segments in a scaled copy are related to the lengths in the original figure in a consistent way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their original counterparts. We might say, "All the segments of the original are twice as long as the scaled copy," or "All the segments of the scaled copy are one-half the length of the segments in the original." **Note:** While initial answers need not be particularly precise at this stage of the unit (for example, "scaled copies look the same, but are a different size"), guide the discussion toward making careful, precise statements that can be tested or measured.

Ask, "What are some characteristics of scaled copies? How are they different from figures that are not scaled copies?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What tools did you find helpful today in determining whether two figures were scaled copies? How were they helpful?
- "What specific information did you look for when determining whether something was a scaled copy of an original?"

Math Language Development

MLR2: Collect and Display

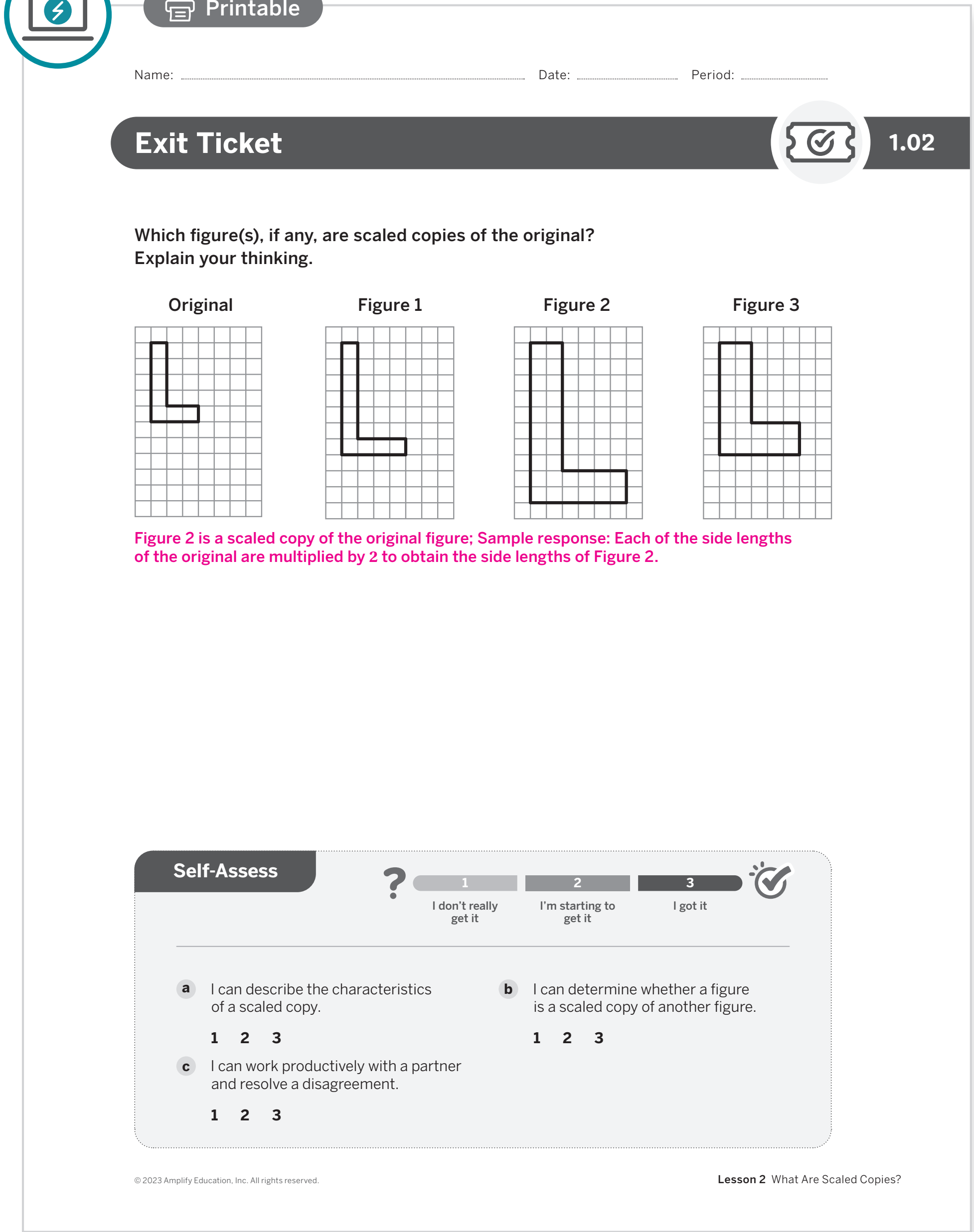
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on any terms and phrases related to the term *scaled copy* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of scaled copies by determining which figures, if any, are scaled copies of an original figure.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.02

Which figure(s), if any, are scaled copies of the original? Explain your thinking.

Figure 2 is a scaled copy of the original figure; Sample response: Each of the side lengths of the original are multiplied by 2 to obtain the side lengths of Figure 2.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe the characteristics of a scaled copy.
1 2 3

b. I can determine whether a figure is a scaled copy of another figure.
1 2 3

c. I can work productively with a partner and resolve a disagreement.
1 2 3

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Lesson 2 What Are Scaled Copies?

Success looks like . . .

- **Language Goal:** Describing the characteristics of scaled copies and copies that are not scaled. **(Speaking and Listening)**
- **Language Goal:** Identifying a scaled copy of a figure and justifying that the copy is a scaled copy. **(Speaking and Listening, Writing)**
    - » Selecting Figure 2 as the scaled copy and explaining their reasoning.

Suggested next steps

If students choose Figure 1 as the scaled copy, consider:

- Reviewing that all side lengths need to be multiplied by the same value. In Figure 1, the width of the vertical section remained unchanged, while the width of the horizontal section increased by a factor of $\frac{4}{3}$.

If students choose Figure 3 as the scaled copy, consider:

- Reviewing that all side lengths need to be multiplied by the same value. In Figure 3, it appears 1 unit was *added* to most of the sides. Because each side length is not multiplied by the same value, it is not a scaled copy.

If students choose Figure 2 as the scaled copy but are unable to provide an explanation, consider:

- Assigning Practice Problems 1 and 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find challenging about Identifying scaled copies? What helped them work through this challenge?
- In this lesson, students identified scaled copies. How will this understanding support drawing scaled copies? What might you change for the next time you teach this lesson?

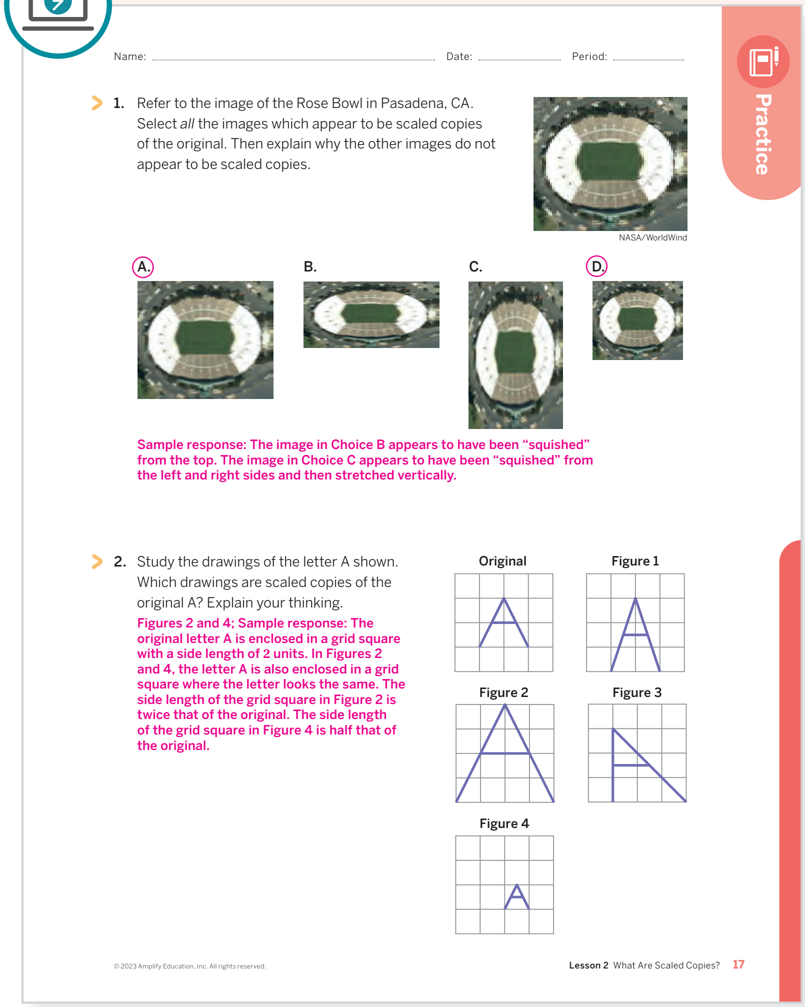
Math Language Development

ML7: Compare and Connect

For students who struggle with making sense of Figure 3, highlight the differences between additive and multiplicative properties. Use drawings to show the difference between multiplying by a scale of 2 compared to adding 2 units to each side.

Practice

Independent



Practice

Name: Date: Period:

1. Refer to the image of the Rose Bowl in Pasadena, CA. Select *all* the images which appear to be scaled copies of the original. Then explain why the other images do not appear to be scaled copies.

NASA/WorldWind

[✓ A.] B. C. [✓ D.]

Sample response: The image in Choice B appears to have been "squished" from the top. The image in Choice C appears to have been "squished" from the left and right sides and then stretched vertically.

2. Study the drawings of the letter A shown. Which drawings are scaled copies of the original A? Explain your thinking.

Figures 2 and 4; Sample response: The original letter A is enclosed in a grid square with a side length of 2 units. In Figures 2 and 4, the letter A is also enclosed in a grid square where the letter looks the same. The side length of the grid square in Figure 2 is twice that of the original. The side length of the grid square in Figure 4 is half that of the original.

Original

Figure 1

Figure 2

Figure 3

Figure 4

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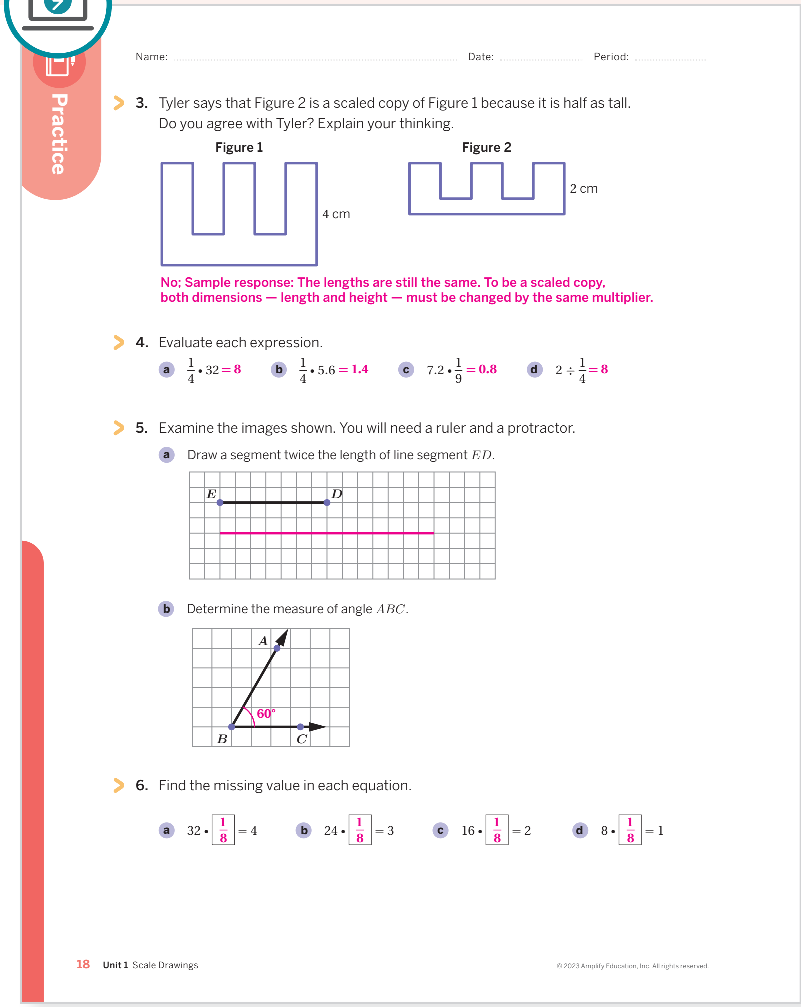
Lesson 2 What Are Scaled Copies? 17



Practice

Name: Date: Period:

3. Tyler says that Figure 2 is a scaled copy of Figure 1 because it is half as tall. Do you agree with Tyler? Explain your thinking.

Figure 1

4 cm

Figure 2

2 cm

No; Sample response: The lengths are still the same. To be a scaled copy, both dimensions — length and height — must be changed by the same multiplier.

4. Evaluate each expression.

a $\frac{1}{4} \cdot 32 = 8$ b $\frac{1}{4} \cdot 5.6 = 1.4$ c $7.2 \cdot \frac{1}{9} = 0.8$ d $2 \div \frac{1}{4} = 8$

5. Examine the images shown. You will need a ruler and a protractor.

a Draw a segment twice the length of line segment ED.

E D

b Determine the measure of angle ABC.

A $60°$ B C

6. Find the missing value in each equation.

a $32 \cdot \boxed{\frac{1}{8}} = 4$ b $24 \cdot \boxed{\frac{1}{8}} = 3$ c $16 \cdot \boxed{\frac{1}{8}} = 2$ d $8 \cdot \boxed{\frac{1}{8}} = 1$

18 **Unit 1** Scale Drawings

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Grade 5	1
	5	Grade 4	1
Formative ⚡	6	Unit 1 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 1 | **LESSON 3**

Corresponding Parts and Scale Factors

Let's describe the attributes of scaled copies.

Focus

Goals

1. **Language Goal:** Explain what it means to say one part in a figure corresponds to a part in another figure. **(Speaking and Listening)**
2. **Language Goal:** Identify and describe corresponding points, corresponding segments, or corresponding angles in a pair of figures. **(Speaking and Listening, Writing)**
3. **Language Goal:** Understand the term *scale factor* and explain how it relates the corresponding lengths of a figure to its scaled copy. **(Speaking and Listening)**
4. **Language Goal:** Explain that corresponding angles in a figure and its scaled copies have the same measure. **(Speaking and Listening, Writing)**

Coherence

Today

This lesson develops the vocabulary for discussing scaling and scaled copies more precisely and identifying the structures in common between two figures. Students use the term *corresponding* to refer to a pair of points, segments, or angles in two figures that are scaled copies. Students also begin to describe the numerical relationship between the side lengths of scaled copies using a scale factor.

Previously

Students were introduced to the idea of a scaled copy of a figure in Lesson 2 and learned to distinguish scaled copies from those that were not.

Coming Soon

Students will apply their knowledge of scale factor in Lesson 4 to draw scaled copies of simple shapes on and off a grid.

Rigor

- Students use visual models to build **conceptual understanding** of scale factor by examining the relationships among corresponding points, sides, and angles of scaled copies.
- Students **apply** the concept of scale factor to determine whether two figures are scaled copies.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	15 min	15 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- Anchor Chart PDF, *Scale Drawings (Part 1)*
- Anchor Chart PDF, *Scale Drawings (Part 1)* (answers)
- geometry toolkits: protractors, rulers, tracing paper

Math Language Development

New words

- ***corresponding parts***
- ***scale factor****

Review words

- *scaled copy*

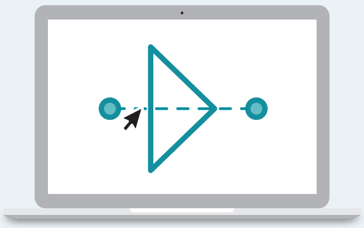
*Students may be familiar with the term *scale* as a household item used for weighing objects. Be ready to address how the term *scale* has a different meaning in this unit.

Amps | Featured Activity

Activity 1

Interactive Geometry

Students use a digital protractor to measure angles precisely.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might work through Activity 2 impulsively, not taking the time to analyze the structure of each triangle. Motivate students by referring to the Anchor Chart PDF or by telling them that they will learn about two attributes of scaled figures. Encourage them to challenge themselves to focus through their discovery of these facts.

Modifications to Pacing

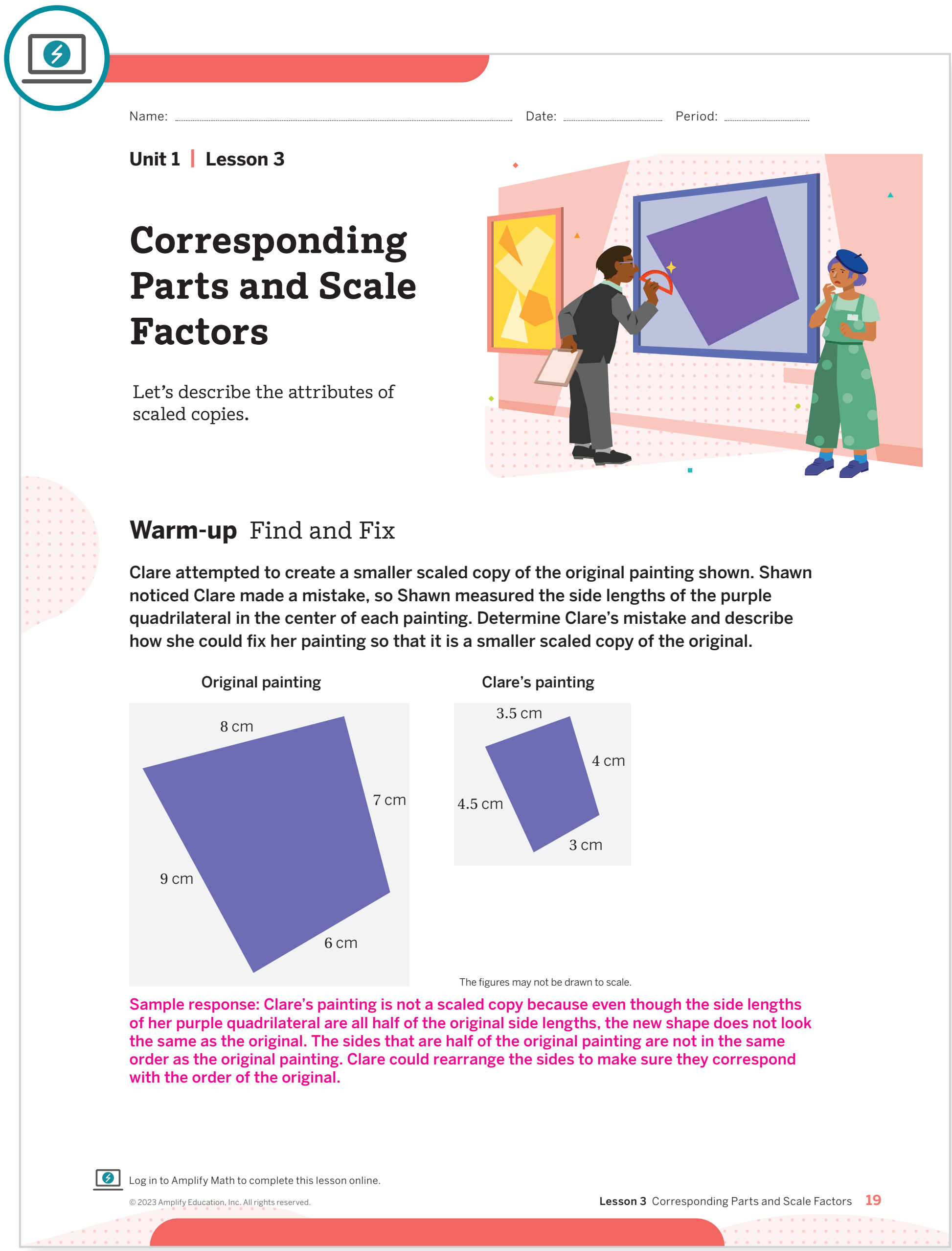
You may want to consider these additional modifications if you are short on time:

- Omit the **Warm-up**. Then, during Activity 1, highlight that scaled copies have both corresponding angles and corresponding side lengths.
- In **Activity 1**, have students measure only the first two or three angles of each quadrilateral.
- In **Activity 2**, have students focus on Triangles B, C, D, and E.

Pairs | 8 min

Warm-up Find and Fix

Students compare two quadrilaterals with scaled side lengths to recognize that lengths must correspond in order for the quadrilaterals to be scaled copies.



Name: ______________ Date: ________ Period: ________

Unit 1 | Lesson 3

Corresponding Parts and Scale Factors

Let's describe the attributes of scaled copies.

Warm-up Find and Fix

Clare attempted to create a smaller scaled copy of the original painting shown. Shawn noticed Clare made a mistake, so Shawn measured the side lengths of the purple quadrilateral in the center of each painting. Determine Clare's mistake and describe how she could fix her painting so that it is a smaller scaled copy of the original.

The figures may not be drawn to scale.

Sample response: Clare's painting is not a scaled copy because even though the side lengths of her purple quadrilateral are all half of the original side lengths, the new shape does not look the same as the original. The sides that are half of the original painting are not in the same order as the original painting. Clare could rearrange the sides to make sure they correspond with the order of the original.

Log in to Amplify Math to complete this lesson online.

Lesson 3 Corresponding Parts and Scale Factors 19

1 Launch

Activate students' background knowledge by asking, "Would you believe me if I told you a painting of a shape could sell for $85 million?" Introduce the painter Kazimir Malevich, a Russian avant-garde artist who used abstract geometric shapes in some of his art. Then conduct the *Think-Pair-Share* routine using the Warm-up.

2 Monitor

Help students get started by reminding them that in the previous lesson, they learned that scaled copies are different sizes, but otherwise appear identical.

Look for points of confusion:

- **Thinking that the figures are scaled copies because Clare's copy has side lengths that are half of the original's.** Ask, "Can two figures have the same side lengths, but look completely different?"

Look for productive strategies:

- Noticing the side lengths of Clare's copy are half of the original's, but in a different order.
- Turning the paper to try and match the side lengths with the corresponding sides of the original figure.

3 Connect

Display the original painting and a copy of Clare's painting on tracing paper. Show that turning or flipping Clare's copy does not represent a reduced copy of the original.

Have students share how they would fix Clare's copy.

Define the term ***corresponding parts*** as parts of two scaled copies that match up, or correspond with each other. These corresponding parts could be points, segments, or angles.

Highlight that when creating a scaled copy, the sides must correspond with each other.

Math Language Development

MLR3: Critique, Correct, Clarify

Use this routine during the Connect discussion. Use these questions:

- ***Critique:*** Why does turning or flipping Clare's copy not produce a reduced copy of the original?
- ***Correct:*** How would you correct Clare's copy? Would altering measurements work? Why or why not?
- ***Clarify:*** Describe in 1 or 2 sentences how Clare could correct her copy.

English Learners

Pair students together that have the same primary language to respond to your questions. Allow them to discuss in their primary language first and then clarify their thinking in English.

Power-up

To power up students' ability to multiply by fractions, ask:

Determine the missing value in each equation:

$8 \cdot \boxed{\frac{1}{2}} = 4$ $\quad 6 \cdot \boxed{\frac{1}{2}} = 3$

$9 \cdot \boxed{\frac{1}{2}} = 4.5$ $\quad 7 \cdot \boxed{\frac{1}{2}} = 3.5$

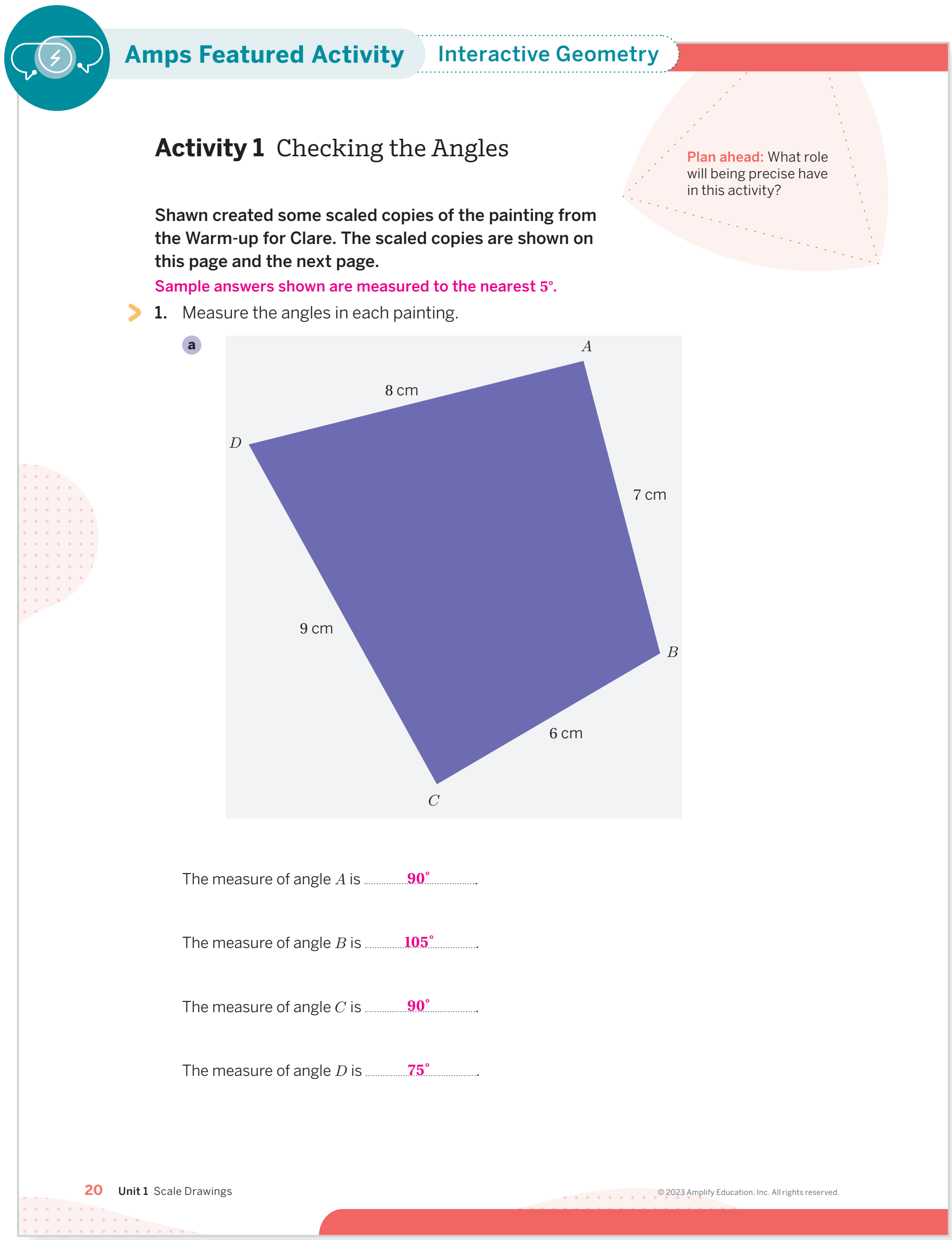
Use: Before the Warm-up.

Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1.

Small Groups | 15 min

Activity 1 Checking the Angles

Students measure the angles of scaled copies of a figure to recognize that all corresponding angles have the same measure.



Amps Featured Activity Interactive Geometry

Activity 1 Checking the Angles

Plan ahead: What role will being precise have in this activity?

Shawn created some scaled copies of the painting from the Warm-up for Clare. The scaled copies are shown on this page and the next page.

Sample answers shown are measured to the nearest 5°.

1. Measure the angles in each painting.

a

The measure of angle A is 90°.

The measure of angle B is 105°.

The measure of angle C is 90°.

The measure of angle D is 75°.

20 **Unit 1** Scale Drawings

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1 Launch

Provide access to geometry toolkits. Direct students to assign one of the copies of the painting to each group member. It may be helpful to review measuring angles with a protractor and class expectations for group work, as this may be the first time they have done either in a while.

2 Monitor

Help students get started by activating their prior knowledge. Have them predict if the angle they are measuring will be greater than or less than 90°.

Note: The actual measurements of the angles are 76°, 90°, 106°, and 88°. However, allowing students to measure to the nearest 5° is sufficient for this activity.

Look for points of confusion:

- **Measuring imprecisely, resulting in different angle measurements for equivalent angles.** Have students ask a member of their group to check their measurements and compare the results.
- **Not concluding that scaled copies have the same corresponding angle measures because the angles in part c are in a different order.** Have students check the corresponding sides, then compare if the angles are formed by the corresponding sides.

Look for productive strategies:

- Concluding that scaled copies will always have the same angle measures, because each copy has the same corresponding angle measures as the original.
- Noticing that copies of each painting will match each other when corresponding angles are overlaid on one another.
- Using tracing paper to check if corresponding angles have the same measure.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students are uncomfortable or have difficulty using their protractor, have them focus on completing part a. The larger size will assist them in determining the measures of the angles. Alternatively, provide the angle measures for students. The goal of this activity is to notice the pattern among the angle measurements, not to become proficient with using a protractor.

Math Language Development

MLR8: Discussion Supports—Revoicing

Amplify mathematical language used to communicate about corresponding points, segments, and angles. As students share what they notice about the figures, revoice their statements using the term *corresponding*, and encourage them to use the term in their discussion.

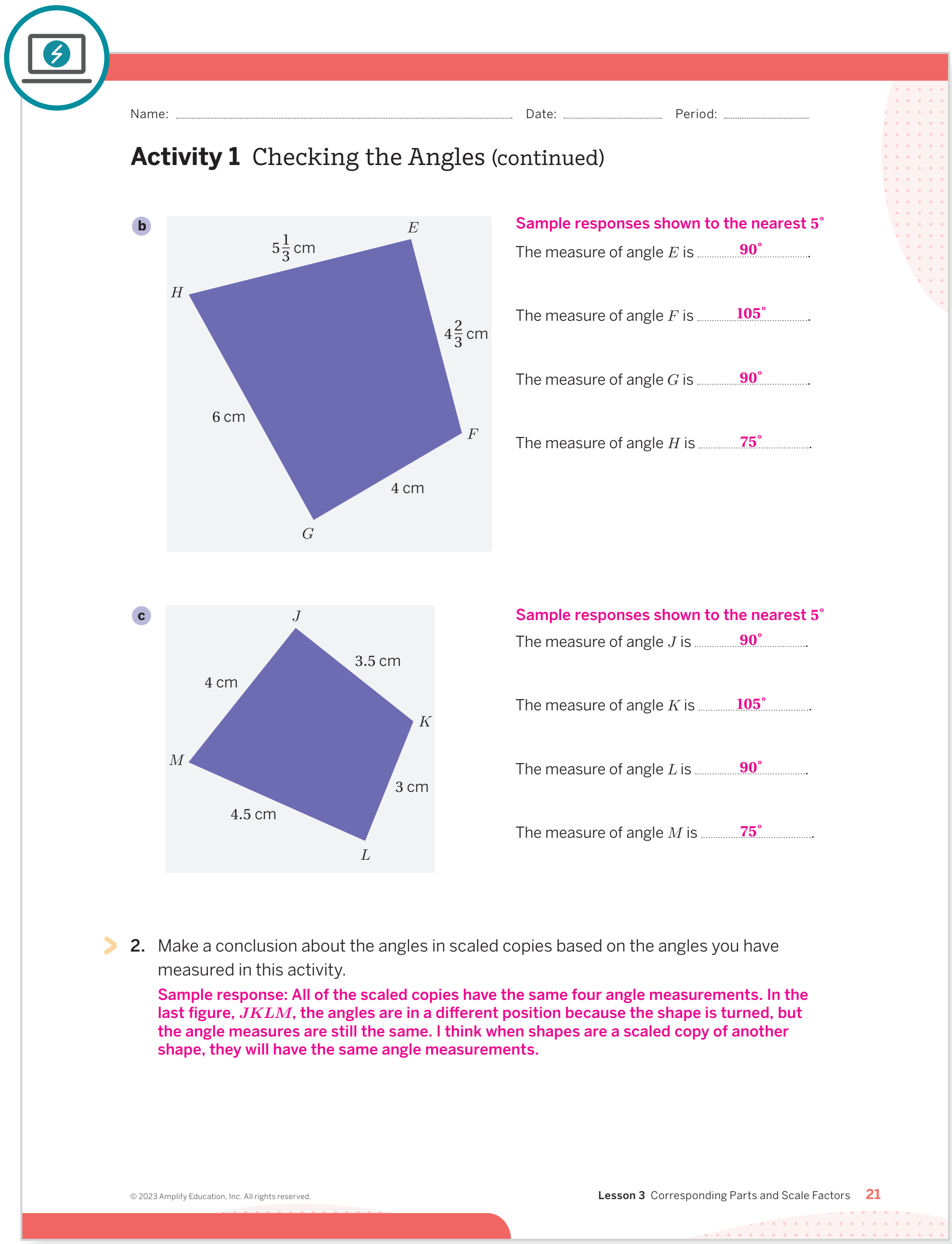
English Learners

Use diagrams or hand gestures to illustrate the term *corresponding*. Display such diagrams for students to reference.

Small Groups | 15 min

Activity 1 Checking the Angles (continued)

Students measure the angles of scaled copies of a figure to recognize that all corresponding angles have the same measure.



Name: ______ Date: ______ Period: ______

Activity 1 Checking the Angles (continued)

b

Sample responses shown to the nearest 5°

The measure of angle E is 90°.

The measure of angle F is 105°.

The measure of angle G is 90°.

The measure of angle H is 75°.

c

Sample responses shown to the nearest 5°

The measure of angle J is 90°.

The measure of angle K is 105°.

The measure of angle L is 90°.

The measure of angle M is 75°.

2. Make a conclusion about the angles in scaled copies based on the angles you have measured in this activity.
   Sample response: All of the scaled copies have the same four angle measurements. In the last figure, $JKLM$, the angles are in a different position because the shape is turned, but the angle measures are still the same. I think when shapes are a scaled copy of another shape, they will have the same angle measurements.

3 Connect

Have groups of students share conclusions they made for Problem 2.

Display the three paintings, with two copies traced on tracing paper. Demonstrate how overlaying the copies on the original show that the angle measures match exactly.

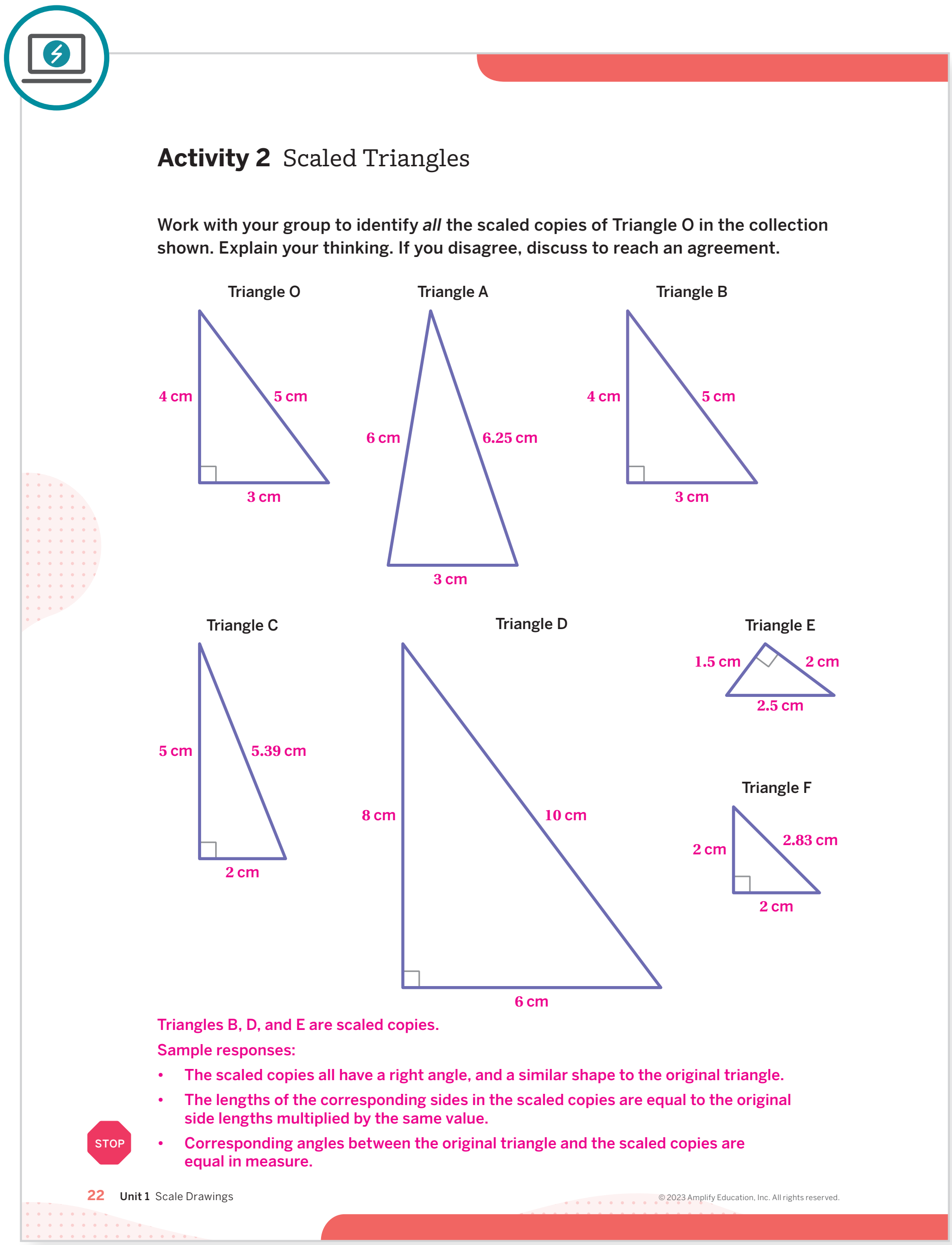
Highlight that unlike corresponding sides of scaled copies, corresponding angles of scaled copies will always have the same measure. Making a scaled copy preserves the measures of the angles from the original.

Ask, "What evidence do you have that these paintings are scaled copies of each other?" Sample response: I can see that the sides are all created using a consistent scale factor, and that their order corresponds with the original painting. I also now know that the measures of corresponding angles remain the same in scaled copies, and that is true of all these copies.

Small Groups | 15 min

Activity 2 Scaled Triangles

Students investigate triangles, identifying whether they are scaled copies to formalize the definition of scaled copy. They are introduced to the definition of *scale factor*.



Activity 2 Scaled Triangles

Work with your group to identify *all* the scaled copies of Triangle O in the collection shown. Explain your thinking. If you disagree, discuss to reach an agreement.

Triangles B, D, and E are scaled copies.

Sample responses:

- The scaled copies all have a right angle, and a similar shape to the original triangle.
- The lengths of the corresponding sides in the scaled copies are equal to the original side lengths multiplied by the same value.
- Corresponding angles between the original triangle and the scaled copies are equal in measure.

22 **Unit 1** Scale Drawings

1 Launch

Provide access to geometry toolkits. Remind students they have explored a few strategies to determine whether two figures are scaled copies. They can choose any of these strategies in this activity. **Note:** Direct students to use centimeters for any measurements.

2 Monitor

Help students get started by asking them to recall some shared attributes of scaled copies.

Look for points of confusion:

- **Thinking that because two triangles have a right angle in common, they are likely to be scaled copies.** Ask, "What did you learn in Activity 1 about corresponding angles in scaled copies?"

Look for productive strategies:

- Labeling the vertices on the triangles to check for corresponding side lengths and angle measures.
- Using tracing paper to check if corresponding angles have the same measure.
- Listening to the ideas of their group members and working productively and collaboratively.

3 Connect

Have groups of students share which triangles they determined were *scaled copies*, and their reasoning for their conclusion. Begin with students who reasoned informally about the sizes of the triangles, and conclude with students who measured corresponding angles and side lengths.

Display the Activity 2 PDF.

Ask, "What patterns do you notice in the table?"

Highlight that the value that is used to multiply the side lengths in scaled copies has a special name.

Define the term ***scale factor*** as the value that side lengths are multiplied by to produce a certain scaled copy.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, assign them to study Triangles A and B to determine whether either of these two triangles are scaled copies of Triangle O, instead of assigning all of the triangles. If students need support with fine motor skills, allow them to record and organize the measurements as opposed to measuring themselves.

Extension: *Math Enrichment*

Have students draw a new triangle with their geometry toolkits that is a scaled copy of Triangle O.

Math Language Development

MLR8: Discussion Supports

Encourage students to use their developing understanding of new vocabulary terms as they reason about the scaled triangles. To facilitate a productive discussion, ask questions that encourage them to compare their thinking. For example, ask, "What do they have in common? What is different?" As students discuss, encourage students to challenge each other when they disagree.

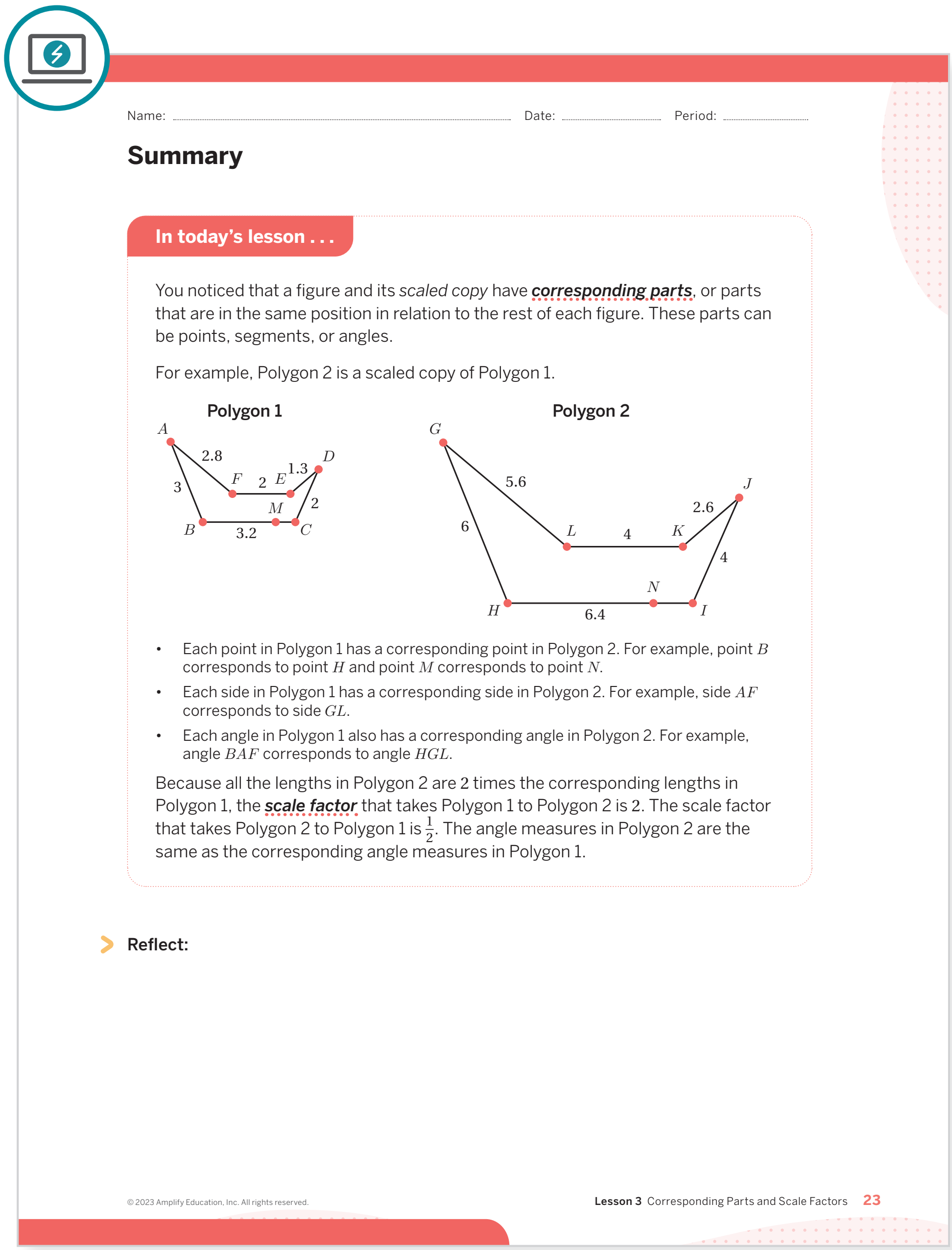
English Learners

Display the sentence frames, "___ corresponds with ___ because . . ." and "I noticed ___, so I realized . . ."

Whole Class | 5 min

Summary

Review and synthesize the important attributes of scaled copies: corresponding parts and a shared scale factor.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You noticed that a figure and its *scaled copy* have ***corresponding parts***, or parts that are in the same position in relation to the rest of each figure. These parts can be points, segments, or angles.

For example, Polygon 2 is a scaled copy of Polygon 1.

- Each point in Polygon 1 has a corresponding point in Polygon 2. For example, point B corresponds to point H and point M corresponds to point N.
- Each side in Polygon 1 has a corresponding side in Polygon 2. For example, side AF corresponds to side GL.
- Each angle in Polygon 1 also has a corresponding angle in Polygon 2. For example, angle BAF corresponds to angle HGL.

Because all the lengths in Polygon 2 are 2 times the corresponding lengths in Polygon 1, the ***scale factor*** that takes Polygon 1 to Polygon 2 is 2. The scale factor that takes Polygon 2 to Polygon 1 is $\frac{1}{2}$. The angle measures in Polygon 2 are the same as the corresponding angle measures in Polygon 1.

Reflect:

Lesson 3 Corresponding Parts and Scale Factors 23

Synthesize

Display the Anchor Chart PDF, *Scale Drawings (Part 1)* and complete it together as the class discusses the following questions.

Highlight that scaled copies of figures have some important attributes relating to their measurements.

- Corresponding angles have equal measures.
- Corresponding side lengths are related to each other by a common scale factor.

Formalize vocabulary:

- ***corresponding parts***
- ***scale factor***

Ask:

- "If you need to create a scaled copy of a figure, how would you begin?"
- "What are some different strategies you can use to determine whether two figures are scaled copies?"
- "How can you determine the scale factor when you are given two figures that are scaled copies?" Identify the corresponding sides. Then either use division or write and solve an equation to determine the scale factor.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What attributes did you compare to determine if two figures were scaled copies?"
- "How did you determine which sides or angles to compare when determining if two figures are scaled copies?"

Math Language Development

MLR2: Collect and Display

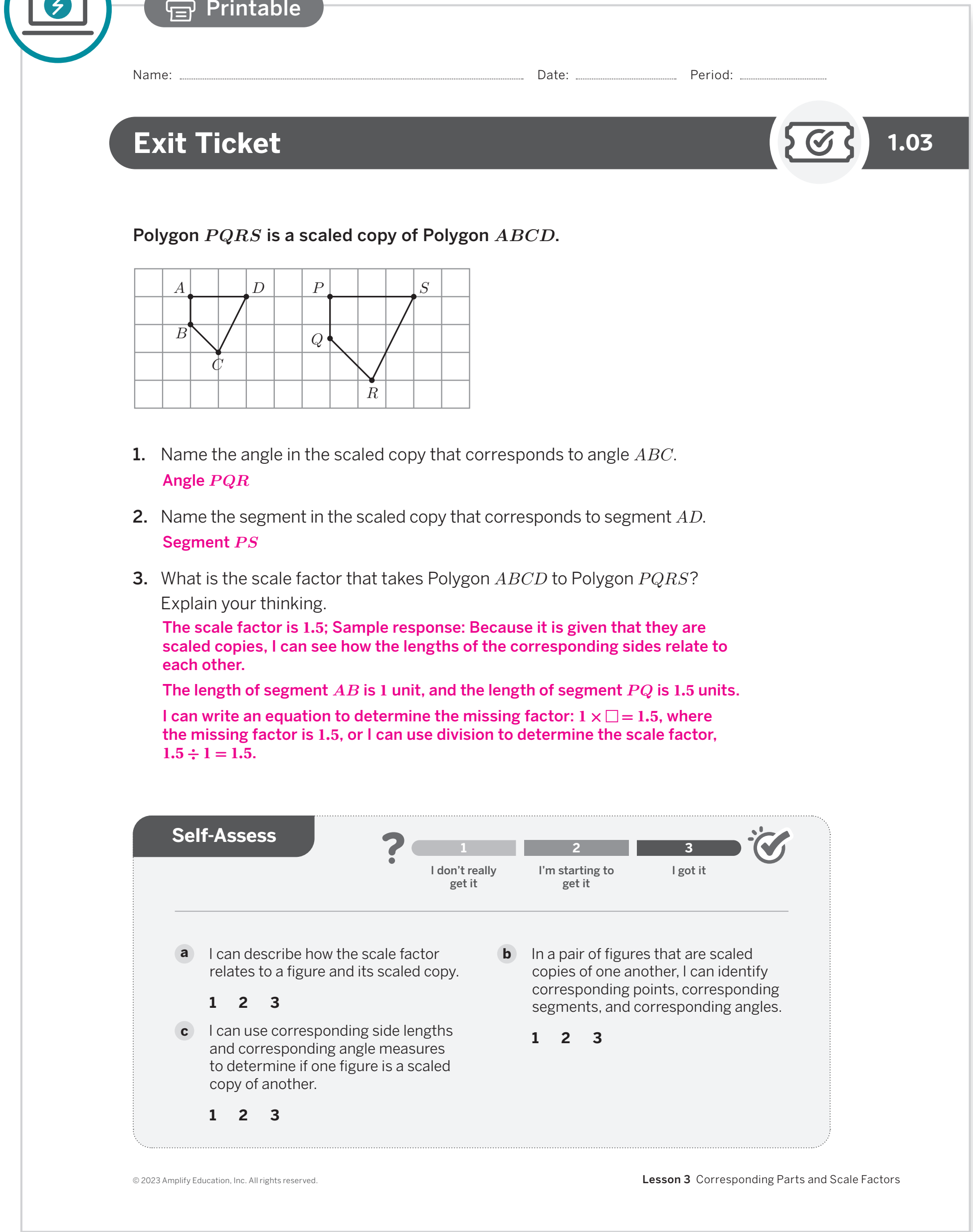
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *corresponding parts* and *scale factor* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of correspondence and scale factor by comparing a scaled copy to an original polygon.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.03

Polygon $PQRS$ is a scaled copy of Polygon $ABCD$.

1. Name the angle in the scaled copy that corresponds to angle ABC.
   Angle PQR
2. Name the segment in the scaled copy that corresponds to segment AD.
   Segment PS
3. What is the scale factor that takes Polygon $ABCD$ to Polygon $PQRS$? Explain your thinking.
   The scale factor is 1.5; Sample response: Because it is given that they are scaled copies, I can see how the lengths of the corresponding sides relate to each other.

   The length of segment AB is 1 unit, and the length of segment PQ is 1.5 units.

   I can write an equation to determine the missing factor: $1 \times \square = 1.5$, where the missing factor is 1.5, or I can use division to determine the scale factor, $1.5 \div 1 = 1.5$.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

(a) I can describe how the scale factor relates to a figure and its scaled copy.
1 2 3

(b) In a pair of figures that are scaled copies of one another, I can identify corresponding points, corresponding segments, and corresponding angles.
1 2 3

(c) I can use corresponding side lengths and corresponding angle measures to determine if one figure is a scaled copy of another.
1 2 3

Lesson 3 Corresponding Parts and Scale Factors

Success looks like . . .

- **Language Goal:** Explaining what it means to say one part in a figure corresponds to a part in another figure. **(Speaking and Listening)**
- **Language Goal:** Identifying and describing corresponding points, corresponding segments, or corresponding angles in a pair of figures. **(Speaking and Listening, Writing)**
  - » Identifying a corresponding angle and segment in Problems 1 and 2.
- **Language Goal:** Understanding the term *scale factor* and explaining how it relates the corresponding lengths of a figure to its scaled copy. **(Speaking and Listening)**
  - » Explaining how to determine the scale factor in Problem 3.
- **Language Goal:** Explaining that corresponding angles in a figure and its scaled copies have the same measure. **(Speaking and Listening, Writing)**

Suggested next steps

If students have difficulty identifying the corresponding parts of the scaled copy, consider:

- Having them use a colored pencil or marker to highlight the corresponding parts in Problem 1.

If students are unable to determine the scale factor, consider:

- Having them write and solve several equations to relate the corresponding side lengths. Practice Problem 3 offers an opportunity for further practice with scale factor.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- During the discussion about corresponding parts, how did you encourage students to listen to one another's strategies? What might you change the next time you teach this lesson?

Math Language Development

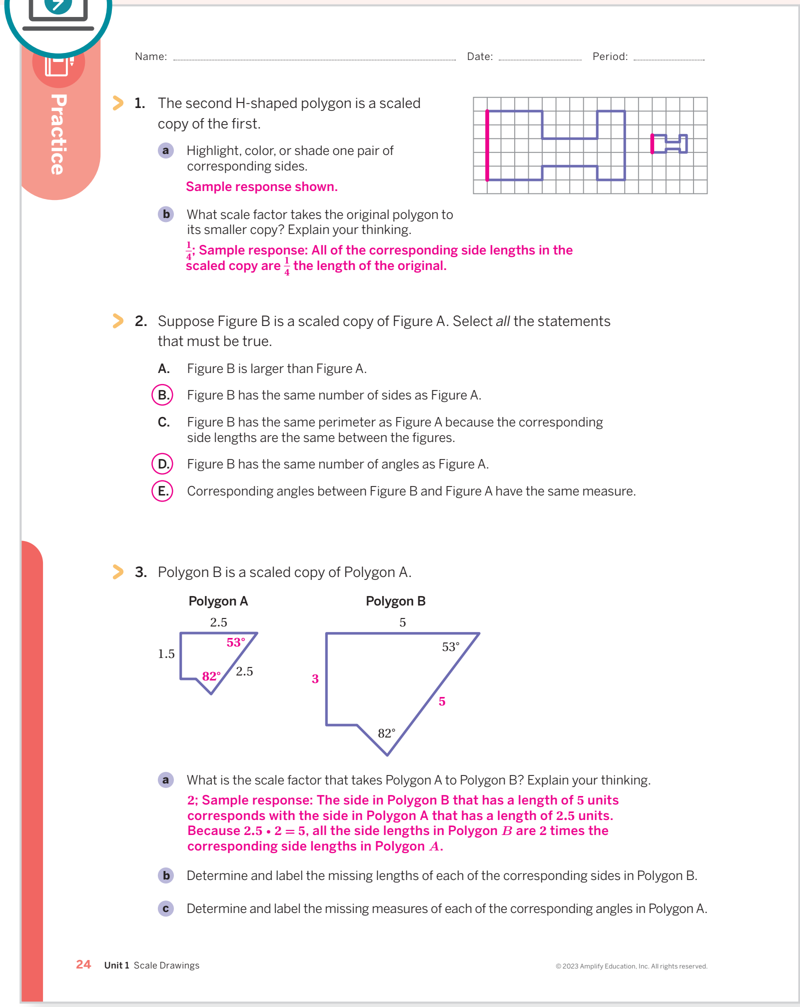
Language Goal: Understanding the term *scale factor* and explaining how it relates the corresponding lengths of a figure to its scaled copy.

Reflect on students' language development toward this goal.

- In Lesson 2, how did students begin to describe the characteristics of scaled copies and copies that are not scaled?
- How have they progressed toward using more precise language to describe scaled copies?

Practice

Independent



Name: ______ Date: ______ Period: ______

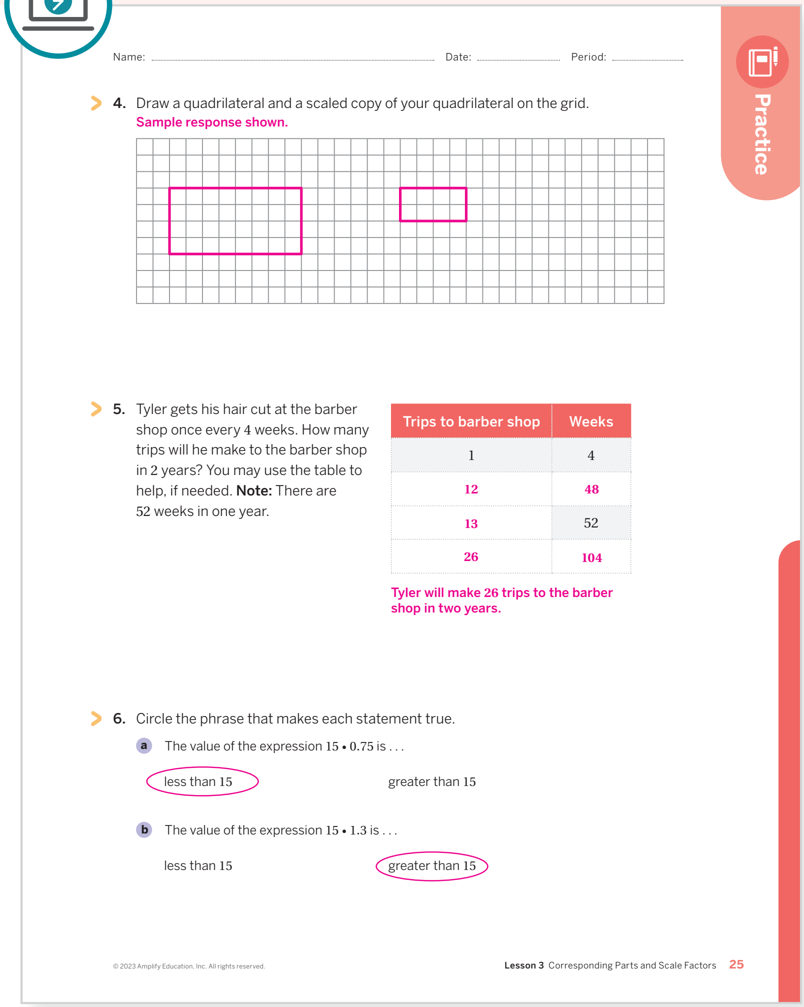
1. The second H-shaped polygon is a scaled copy of the first.
   - a Highlight, color, or shade one pair of corresponding sides.
     Sample response shown.
   - b What scale factor takes the original polygon to its smaller copy? Explain your thinking.
     $\frac{1}{4}$; Sample response: All of the corresponding side lengths in the scaled copy are $\frac{1}{4}$ the length of the original.

2. Suppose Figure B is a scaled copy of Figure A. Select *all* the statements that must be true.
   - A. Figure B is larger than Figure A.
   - [✓ B.] Figure B has the same number of sides as Figure A.
   - C. Figure B has the same perimeter as Figure A because the corresponding side lengths are the same between the figures.
   - [✓ D.] Figure B has the same number of angles as Figure A.
   - [✓ E.] Corresponding angles between Figure B and Figure A have the same measure.

3. Polygon B is a scaled copy of Polygon A.

   - a What is the scale factor that takes Polygon A to Polygon B? Explain your thinking.
     2; Sample response: The side in Polygon B that has a length of 5 units corresponds with the side in Polygon A that has a length of 2.5 units. Because $2.5 \cdot 2 = 5$, all the side lengths in Polygon B are 2 times the corresponding side lengths in Polygon A.
   - b Determine and label the missing lengths of each of the corresponding sides in Polygon B.
   - c Determine and label the missing measures of each of the corresponding angles in Polygon A.

24 **Unit 1** Scale Drawings



Name: ______ Date: ______ Period: ______

4. Draw a quadrilateral and a scaled copy of your quadrilateral on the grid.
   Sample response shown.

5. Tyler gets his hair cut at the barber shop once every 4 weeks. How many trips will he make to the barber shop in 2 years? You may use the table to help, if needed. **Note:** There are 52 weeks in one year.

Trips to barber shop	Weeks
1	4
12	48
13	52
26	104

Tyler will make 26 trips to the barber shop in two years.

6. Circle the phrase that makes each statement true.
   - a The value of the expression $15 \cdot 0.75$ is . . .
     [✓ less than 15] greater than 15
   - b The value of the expression $15 \cdot 1.3$ is . . .
     less than 15 [✓ greater than 15]

Lesson 3 Corresponding Parts and Scale Factors 25

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 2	2
	5	Grade 6	2
Formative ⚡	6	Unit 1 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 1 | **LESSON 4**

Making Scaled Copies

Let's draw scaled copies.

Focus

Goals

1. **Language Goal:** Critique different strategies (using multiple representations) for creating scaled copies of a figure. **(Speaking and Listening, Writing)**
2. Draw a scaled copy of a given figure using a given scale factor.
3. **Language Goal:** Generalize that the relationship between the side lengths of a figure and its scaled copy is multiplicative, not additive. **(Speaking and Listening, Writing)**

Coherence

Today

Students draw scaled copies of simple shapes on and off a grid. They also strengthen their understanding that the relationship between scaled copies is multiplicative, not additive. Students make careful arguments about the scaling process, have opportunities to strategically select and use tools, such as tracing paper or index cards, and to make use of structure when comparing scaled copies using grids.

Previously

In Lesson 3, students used scale factors to describe the relationship between corresponding lengths in scaled copies of figures.

Coming Soon

In Lesson 5, students will reason about scale factors greater than 1, less than 1, and equal to 1, and their effects on the side lengths of scaled copies.

Rigor

- Students build **conceptual understanding** of scaling as a multiplicative process.
- Students **apply** their understanding of scale factor by drawing scaled copies, ensuring that angle measures are unchanged and side lengths are changed by a common factor (the scale factor).

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	13 min	12 min	5 min	10 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: index card, one per student (optional), protractors, rulers, tracing paper
- graph paper (as needed)

Math Language Development

Review words

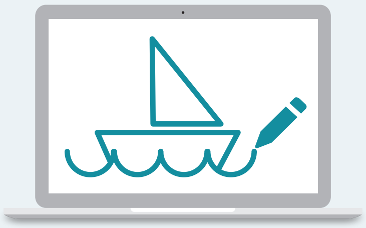
- *corresponding parts*
- *scaled copy*
- *scale factor*

Amps | Featured Activity

Activity 1

Overlay Figures

Students digitally manipulate their scaled copies by dragging points. When you overlay the results, students can compare their figures with those of other students.



Building Math Identity and Community

Connecting to Mathematical Practices

As students share their critique of Andre's method and reasoning in Activity 2, they may be so focused on what they want to say that they forget to listen to others. Remind students that their learning can be collaborative, and they have a lot to learn from each other. By actively listening, they can help each other refine their thinking.

Modifications to Pacing

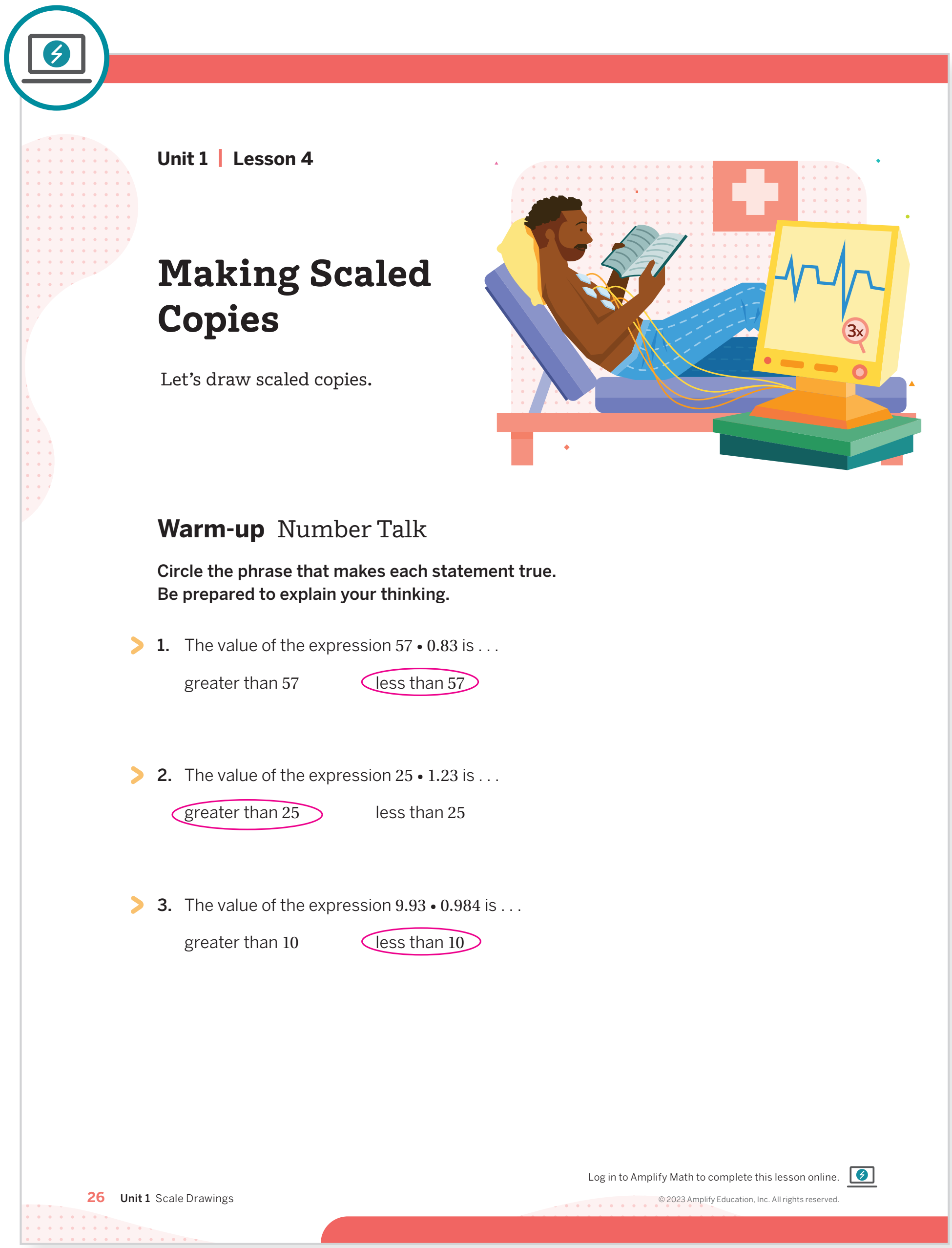
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, have students complete only one of the two problems. Consider allowing students to choose which problem to complete.
- **Activity 2** may be omitted, but if you choose to omit this activity, be sure to discuss with students why scaling is a multiplicative process, not an additive one.

Independent | 5 min

Warm-up Number Talk

Students activate prior knowledge to determine how multiplying by numbers less than or greater than 1 affects the value of the product.



Unit 1 | Lesson 4

Making Scaled Copies

Let's draw scaled copies.

Warm-up Number Talk

Circle the phrase that makes each statement true.
Be prepared to explain your thinking.

1. The value of the expression 57 • 0.83 is . . .

   greater than 57 [✓ less than 57]

2. The value of the expression 25 • 1.23 is . . .

   [✓ greater than 25] less than 25

3. The value of the expression 9.93 • 0.984 is . . .

   greater than 10 [✓ less than 10]

Log in to Amplify Math to complete this lesson online.

26 Unit 1 Scale Drawings

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1 Launch

Suggest to students that when they see an instruction line of "Be prepared to explain your thinking," they may want to write a few notes next to each problem, so that they will remember their thought process during the class discussion later. Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by asking them what they notice about the factors in each expression. Prompt them to notice the first factor is what needs to be compared to the product, and the second factor is either less than 1 or greater than 1.

Look for points of confusion:

- **Calculating each value instead of thinking about how the product compares to the first factor.** Remind students that they only need to compare the value to the first factor.

3 Connect

Display the expressions from the Student Edition.

Have students share their responses and explain their thinking. If there are any disagreements, facilitate a class discussion regarding how to have productive conversations regarding conflict resolution.

Ask, "Based on the instructions, is it necessary to calculate the value of each expression? Why or why not" No; Sample response: I only need to compare the size of the product to the size of the first factor.

Highlight the structure of the expressions and the values of the first and second factors. Multiplying the first factor by a number less than 1 results in a product less than the first factor. Multiplying the first factor by a number greater than 1 results in a product greater than the first factor.

Power-up

To power up students' ability to determine the size of the product of two values in relation to the size of the factors, have students complete:

Circle the correct response that completes each statement. Be prepared to explain your thinking.

1. The value of the expression 10 • 0.8 is. . .

   [✓ less than 10] greater than 10

2. The value of the expression 10 • 1.1 is. . .

   less than 10 [✓ greater than 10]

Use: Before the Warm-up.

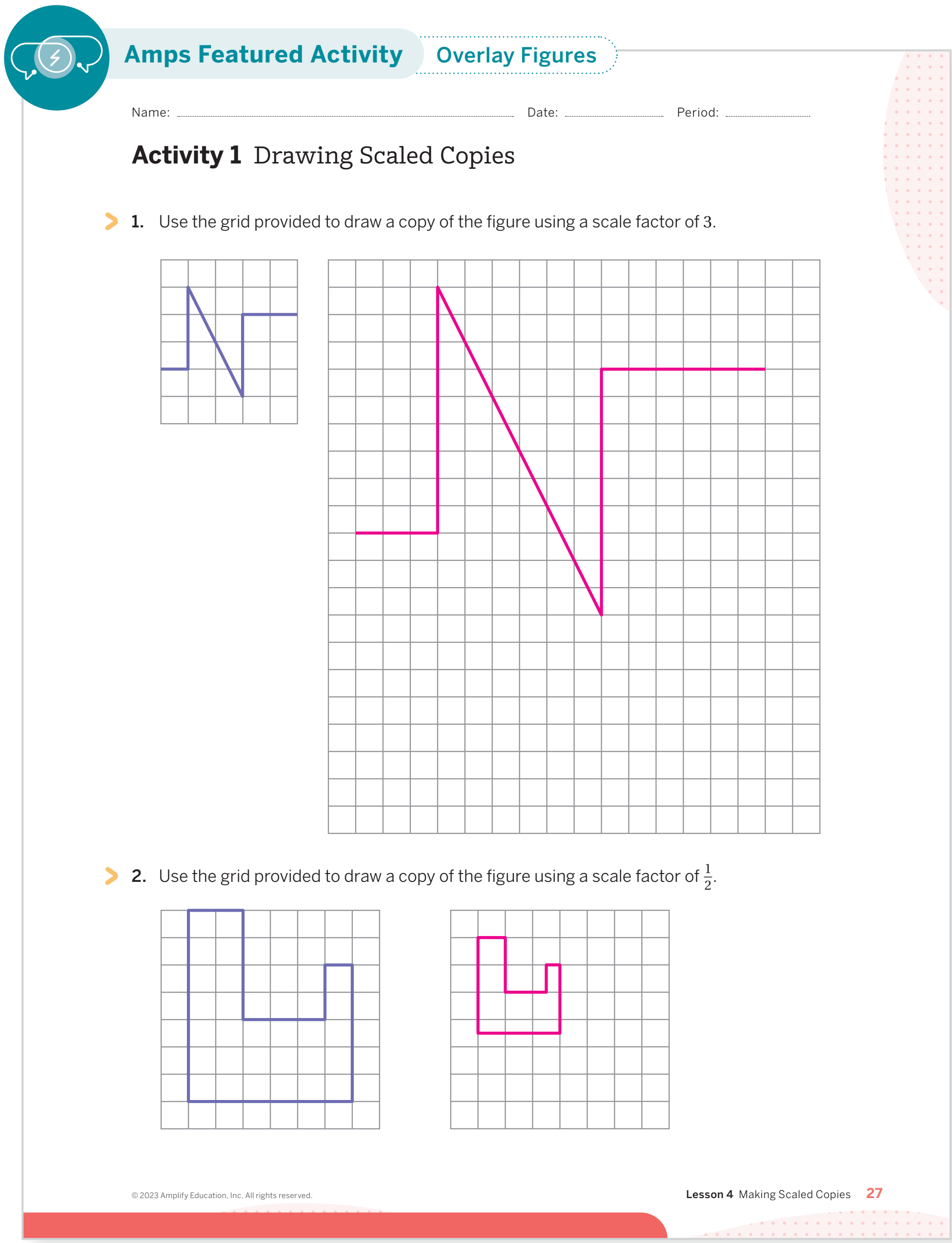
Informed by: Performance on Lesson 3, Practice Problem 6.

Pairs | 13 min

Activity 1 Drawing Scaled Copies

Students draw scaled copies to demonstrate their understanding of the effects of scale factor on side lengths.

Amps Featured Activity Overlay Figures



Name: Date: Period:

Activity 1 Drawing Scaled Copies

1. Use the grid provided to draw a copy of the figure using a scale factor of 3.

2. Use the grid provided to draw a copy of the figure using a scale factor of $\frac{1}{2}$.

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Lesson 4 Making Scaled Copies 27

1 Launch

Provide access to geometry toolkits. Review how side lengths are changed and angle measures are unchanged when drawing scaled copies.

2 Monitor

Help students get started by having them label the segment lengths in the original figure and asking what the lengths would be in the scaled copy.

Look for points of confusion:

- **Thinking that the figure in Problem 2 cannot be scaled by a factor of $\frac{1}{2}$ because some vertices will not be placed on the intersections of grid lines.** Clarify that the grid helps to see lengths in whole units, but the lengths of segments are not limited to whole units.
- **Drawing images that have sections that look disproportionate.** Have students use the grid to calculate side lengths and use protractors or tracing paper to verify angle measures.

3 Connect

Have students share their scaled copies and strategies for drawing them.

Ask:

- "How did you know how long to draw each side or how big to draw each angle in your scaled copy?"
- "If you make a mistake while drawing your scaled copy, how could you tell? How could you fix it?"

Highlight the language students use to distinguish between scaled copies and figures that are not scaled copies. Emphasize the usefulness of the grid in drawing and checking the side lengths. Show students how to use tracing paper to check if angles have the same measure and how to use index cards to check side lengths.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, allow them to focus on completing one of the two problems in this activity. Consider allowing them to choose which problem they would like to complete.

Math Language Development

MLR1: Stronger and Clearer Each Time

Allow students time to meet with 2–3 partners, to practice sharing their strategies and receiving feedback on their scaled copies. Provide them with prompts for feedback to help strengthen their ideas and clarify their drawings, such as:

- "How did you know how long to draw each side?"
- "How did you use the grid to create your scaled copy?"

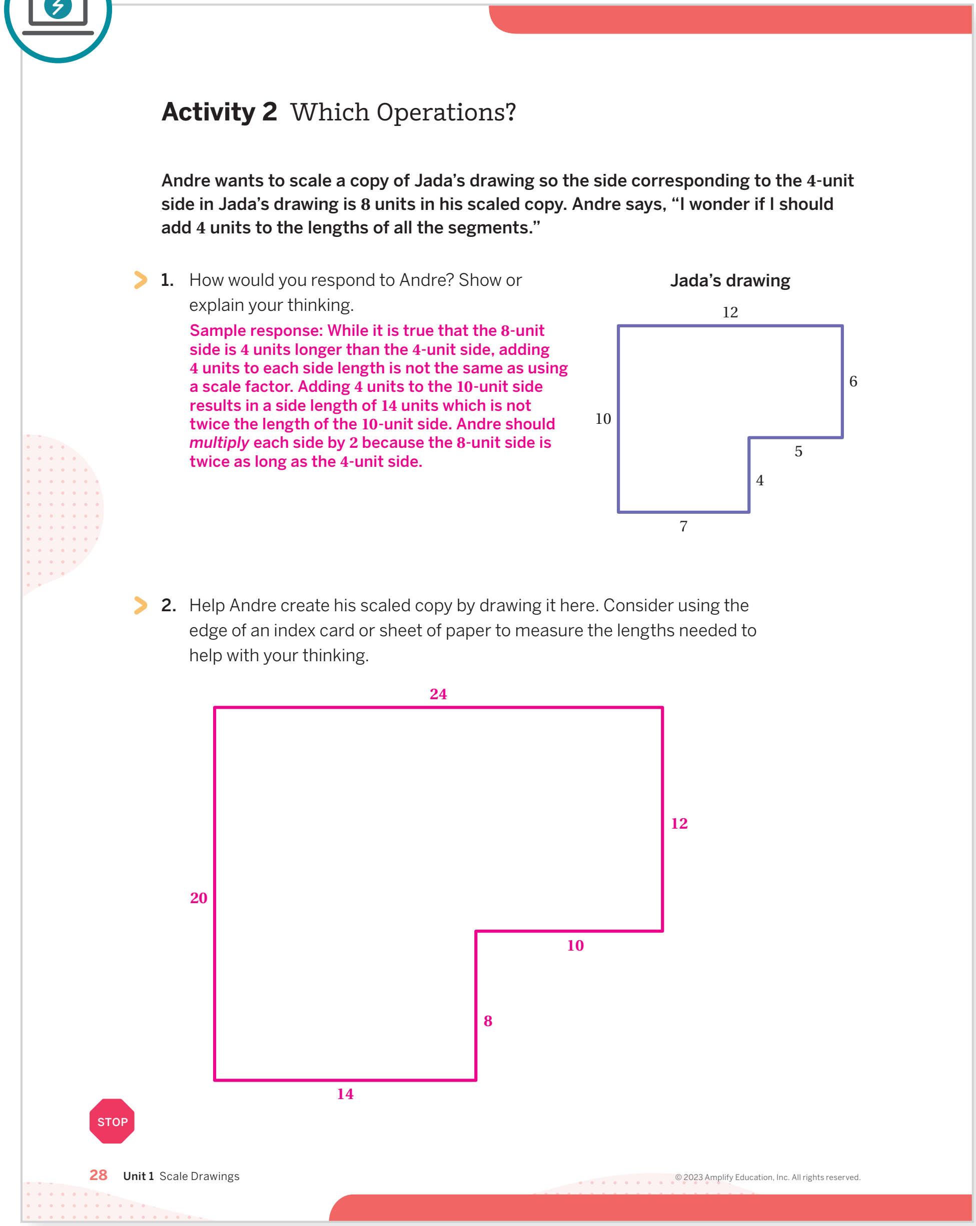
English Learners

Consider providing a draft explanation of a possible strategy for either Problem 1 or Problem 2 for students to reference.

Pairs | 12 min

Activity 2 Which Operations?

Students learn that adding the same value to all of the side lengths in a figure does not produce a (larger) scaled copy, reinforcing that scaling involves multiplication.



Activity 2 Which Operations?

Andre wants to scale a copy of Jada's drawing so the side corresponding to the 4-unit side in Jada's drawing is 8 units in his scaled copy. Andre says, "I wonder if I should add 4 units to the lengths of all the segments."

1. How would you respond to Andre? Show or explain your thinking.
   Sample response: While it is true that the 8-unit side is 4 units longer than the 4-unit side, adding 4 units to each side length is not the same as using a scale factor. Adding 4 units to the 10-unit side results in a side length of 14 units which is not twice the length of the 10-unit side. Andre should *multiply* each side by 2 because the 8-unit side is twice as long as the 4-unit side.

Jada's drawing

2. Help Andre create his scaled copy by drawing it here. Consider using the edge of an index card or sheet of paper to measure the lengths needed to help with your thinking.

STOP

1 Launch

Provide access to index cards for students to use as a measuring tool. Consider not explicitly directing them about how to use them, so that they have the opportunity to select and use tools strategically.

2 Monitor

Look for points of confusion:

- **Being unconvinced that drawing each segment 4 units longer will not produce a scaled copy.** Have them use graph paper to draw the original figure and a new figure with all side lengths 4 units longer. Ask them if the figures are scaled copies.
- **Adding 4 units to all side lengths and managing to create a polygon, but changing the angle measures along the way.** Remind students that the angles in a scaled copy have the same measures of their corresponding angles in the original figure.

3 Connect

Display Jada's drawing.

Have students share their explanations of how adding 4 units to the length of each segment does not work (e.g., the copy is no longer a polygon, or the copy has different angle measures from the original figure).

Ask:

- "What scale factor did you use to create your copy? Why?"
- "How can you use an index card (or a sheet of paper) to measure the side lengths needed for the scaled copy?" Mark the length of each original segment and transfer it twice onto the drawing surface.
- "How did you measure the angles for the copy?"

Highlight that scale factors are *factors*, not addends. Existing side lengths are multiplied by a common factor — the scale factor — rather than added to a common length.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the original figure on graph paper and allow them to create the scaled copy on graph paper.

Extension: *Math Enrichment*

Have students complete the following problem:
The side lengths of Triangle B are each 5 in. longer than the side lengths of Triangle A. Can Triangle B be a scaled copy of Triangle A? Explain your thinking. Yes; Sample response: This is only possible if Triangle A is an equilateral triangle. This is the only way to ensure that a scale factor has been used.

Math Language Development

MLR3: Critique, Correct, Clarify

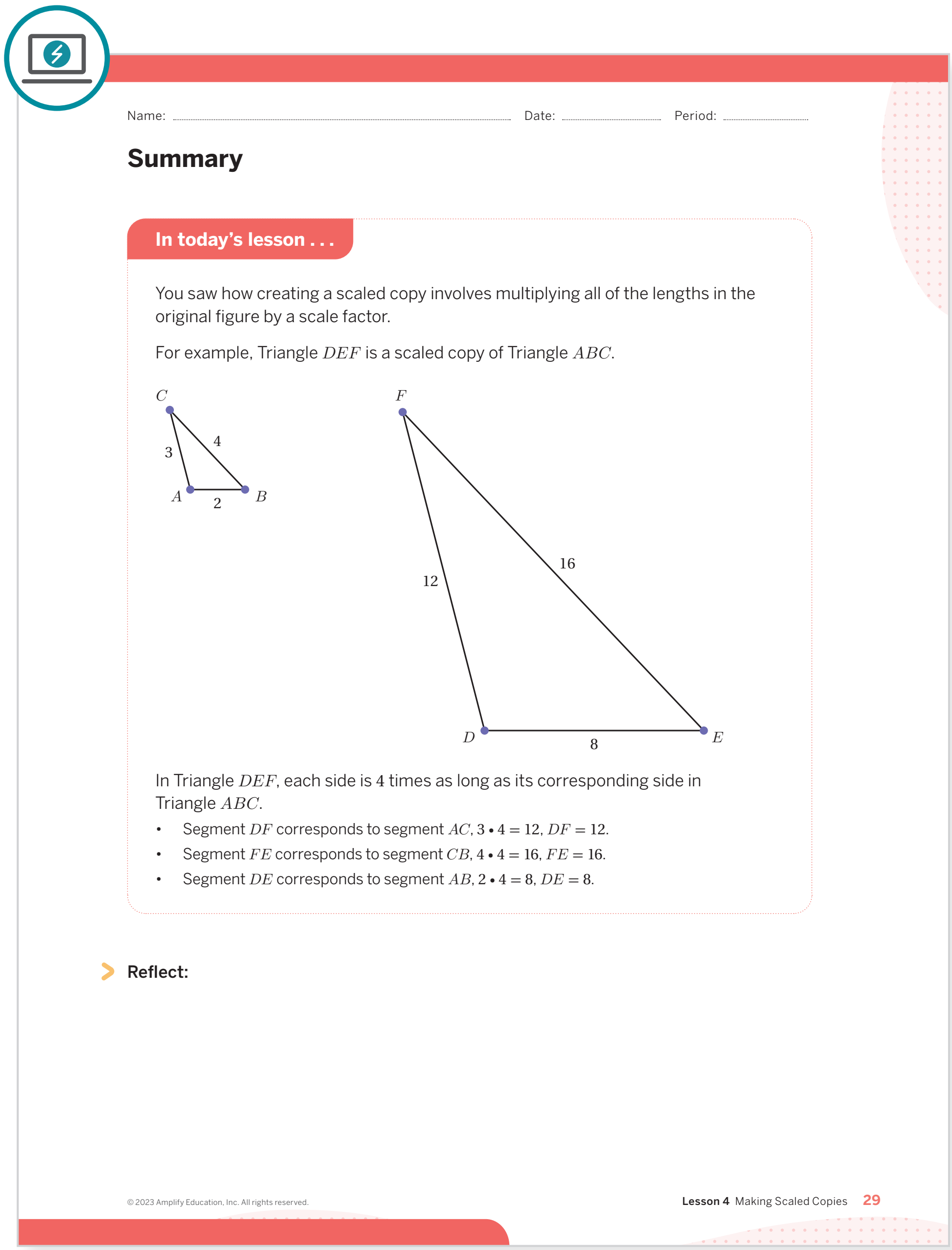
Use this routine as students work to make sense of Andre's reasoning in Problem 1.

- ***Critique:*** Have students critique why adding 4 units to each side length will not scale the drawing.
- ***Correct:*** Have them correct how they can accurately scale the drawing by multiplying each side by 2.
- ***Clarify:*** Have students clarify what it means to scale a copy. As students discuss, highlight language around the difference between an *additive* and *multiplicative* relationship.

Whole Class | 5 min

Summary

Review and synthesize how to use the scale factor to produce scaled copies of a figure.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw how creating a scaled copy involves multiplying all of the lengths in the original figure by a scale factor.

For example, Triangle DEF is a scaled copy of Triangle ABC.

In Triangle DEF, each side is 4 times as long as its corresponding side in Triangle ABC.

- Segment DF corresponds to segment AC, $3 \cdot 4 = 12$, $DF = 12$.
- Segment FE corresponds to segment CB, $4 \cdot 4 = 16$, $FE = 16$.
- Segment DE corresponds to segment AB, $2 \cdot 4 = 8$, $DE = 8$.

> **Reflect:**

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Lesson 4 Making Scaled Copies 29

Synthesize

Ask:

- "How do you draw a scaled copy of a figure?"
- "Can you create scaled copies by adding or subtracting the same value from all of the side lengths? Why or why not?"
- "If you know corresponding side lengths of both the original and the scaled copy, how can you determine the scale factor?" Divide one of the side lengths in the scaled copy by its corresponding side length in the original figure.

Highlight that scaling is a multiplicative process. To draw a scaled copy of a figure, students need to multiply all the side lengths of the original figure by the scale factor. Emphasize that students saw in the lesson that adding or subtracting the same value to all of the side lengths will *not* create scaled copies.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why is it important to be precise when creating scaled copies?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by comparing an additive strategy and a multiplicative strategy, explaining that only multiplication produces scaled copies.

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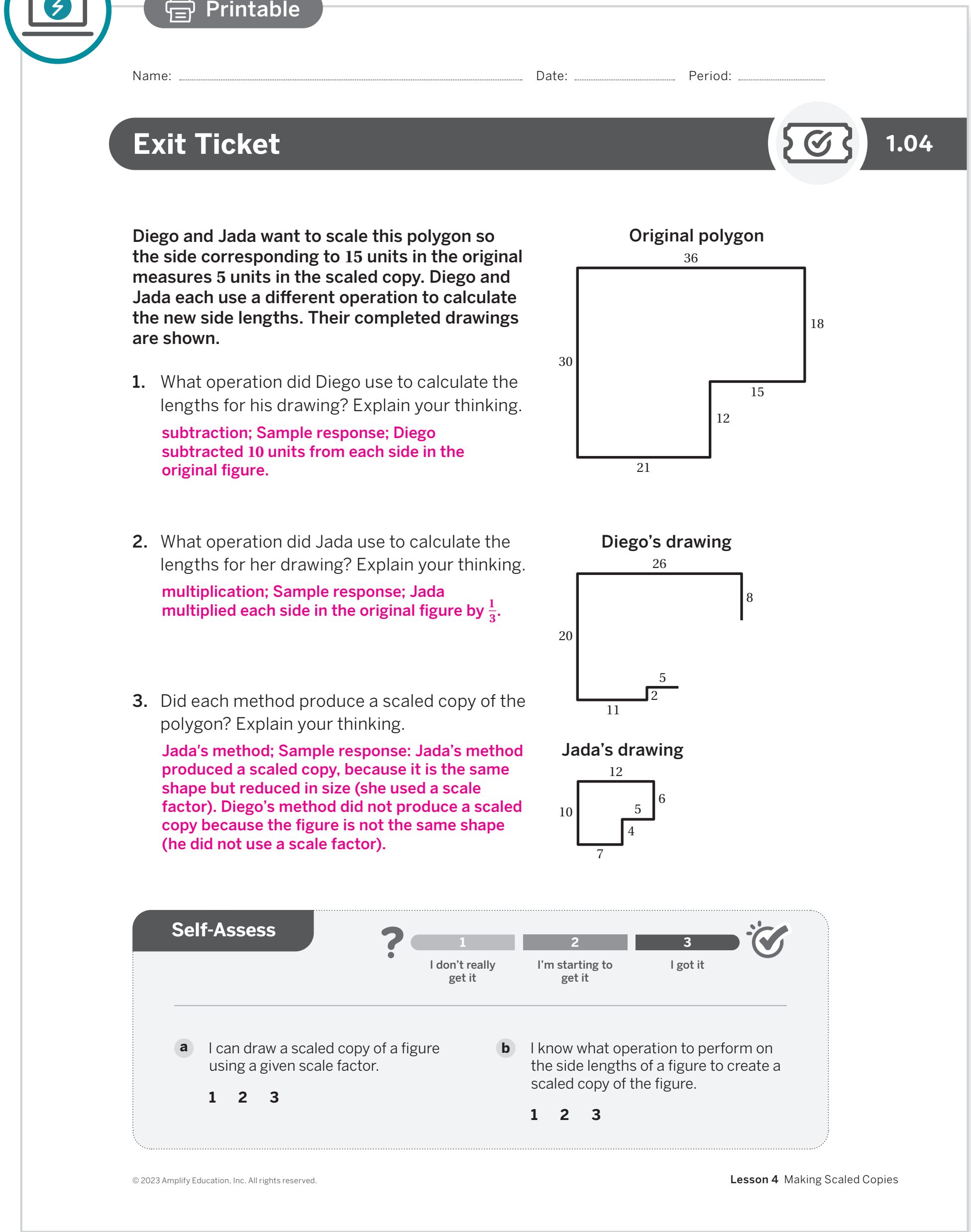


Name: ______________ Date: ________ Period: ________

Exit Ticket 1.04

Diego and Jada want to scale this polygon so the side corresponding to 15 units in the original measures 5 units in the scaled copy. Diego and Jada each use a different operation to calculate the new side lengths. Their completed drawings are shown.

1. What operation did Diego use to calculate the lengths for his drawing? Explain your thinking.

   subtraction; Sample response: Diego subtracted 10 units from each side in the original figure.

2. What operation did Jada use to calculate the lengths for her drawing? Explain your thinking.

   multiplication; Sample response; Jada multiplied each side in the original figure by $\frac{1}{3}$.

3. Did each method produce a scaled copy of the polygon? Explain your thinking.

   Jada's method; Sample response: Jada's method produced a scaled copy, because it is the same shape but reduced in size (she used a scale factor). Diego's method did not produce a scaled copy because the figure is not the same shape (he did not use a scale factor).

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can draw a scaled copy of a figure using a given scale factor.

   1 2 3

b. I know what operation to perform on the side lengths of a figure to create a scaled copy of the figure.

   1 2 3

Success looks like . . .

- **Language Goal:** Critiquing different strategies (using multiple representations) for creating scaled copies of a figure. **(Speaking and Listening, Writing)**
  - » Explaining which operation produced a scaled copy in Problem 3.
- **Goal:** Drawing a scaled copy of a given figure using a given scale factor.
- **Language Goal:** Generalizing that the relationship between the side lengths of a figure and its scaled copy is multiplicative, not additive. **(Speaking and Listening, Writing)**
  - » Understanding the Jada's method produced a scaled copy in Problem 3.

Suggested next steps

If students answered Problem 2 with "divide by 3," consider:

- Letting them know they understand the relationship between the side lengths, and reminding them that scale factors are expressed as multiplication.
- Help them to understand that the term *scale factor* has the word *factor* in it, which indicates multiplication.
- Checking in with those students during Lesson 5, Activity 2 to make sure they are expressing the scale factors as multiplicative relationships.

Professional Learning

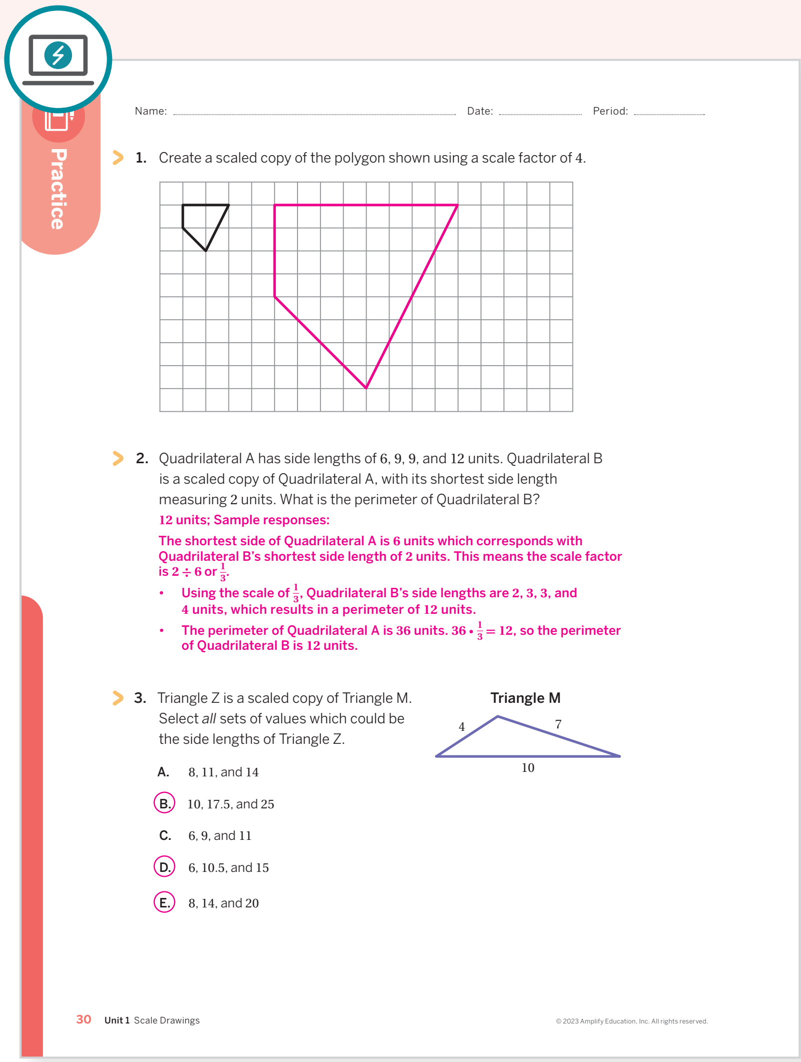
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- Thinking about the questions you asked students today and what the students said or did as a result of those questions, which question(s) were most effective? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Create a scaled copy of the polygon shown using a scale factor of 4.

2. Quadrilateral A has side lengths of 6, 9, 9, and 12 units. Quadrilateral B is a scaled copy of Quadrilateral A, with its shortest side length measuring 2 units. What is the perimeter of Quadrilateral B?

   12 units; Sample responses:

   The shortest side of Quadrilateral A is 6 units which corresponds with Quadrilateral B's shortest side length of 2 units. This means the scale factor is $2 \div 6$ or $\frac{1}{3}$.

   - **Using the scale of $\frac{1}{3}$, Quadrilateral B's side lengths are 2, 3, 3, and 4 units, which results in a perimeter of 12 units.**
   - **The perimeter of Quadrilateral A is 36 units. $36 \cdot \frac{1}{3} = 12$, so the perimeter of Quadrilateral B is 12 units.**

3. Triangle Z is a scaled copy of Triangle M. Select *all* sets of values which could be the side lengths of Triangle Z.

   Triangle M

   A. 8, 11, and 14

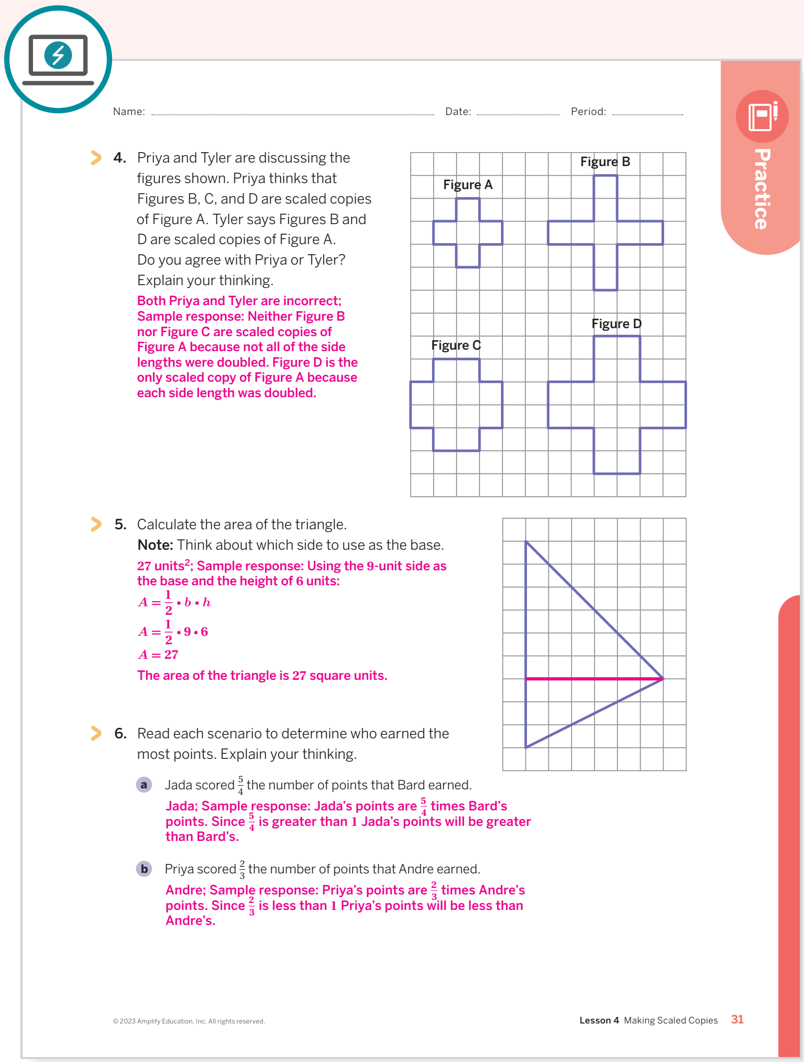
   [✓ B.] 10, 17.5, and 25

   C. 6, 9, and 11

   [✓ D.] 6, 10.5, and 15

   [✓ E.] 8, 14, and 20

30 **Unit 1** Scale Drawings



Practice

Name: Date: Period:

4. Priya and Tyler are discussing the figures shown. Priya thinks that Figures B, C, and D are scaled copies of Figure A. Tyler says Figures B and D are scaled copies of Figure A. Do you agree with Priya or Tyler? Explain your thinking.

   Both Priya and Tyler are incorrect; Sample response: Neither Figure B nor Figure C are scaled copies of Figure A because not all of the side lengths were doubled. Figure D is the only scaled copy of Figure A because each side length was doubled.

5. Calculate the area of the triangle.
   Note: Think about which side to use as the base.

   27 units^2; Sample response: Using the 9-unit side as the base and the height of 6 units:

   $A = \frac{1}{2} \cdot b \cdot h$

   $A = \frac{1}{2} \cdot 9 \cdot 6$

   $A = 27$

   The area of the triangle is 27 square units.

6. Read each scenario to determine who earned the most points. Explain your thinking.

   a. Jada scored $\frac{5}{4}$ the number of points that Bard earned.

   Jada; Sample response: Jada's points are $\frac{5}{4}$ times Bard's points. Since $\frac{5}{4}$ is greater than 1 Jada's points will be greater than Bard's.

   b. Priya scored $\frac{2}{3}$ the number of points that Andre earned.

   Andre; Sample response: Priya's points are $\frac{2}{3}$ times Andre's points. Since $\frac{2}{3}$ is less than 1 Priya's points will be less than Andre's.

Lesson 4 Making Scaled Copies 31

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 2	2
	5	Grade 6	2
Formative ⚡	6	Unit 1 Lesson 5	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 1 | **LESSON 5**

The Size of the Scale Factor

Let's observe the effects of different scale factors.

Focus

Goals

1. **Language Goal:** Describe how scale factors of 1, less than 1, and greater than 1 affect the size of scaled copies. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain and show how to recreate the original figure given a scaled copy and its scale factor. **(Speaking and Listening, Writing)**
3. **Language Goal:** Recognize that the scale factor that takes the scaled copy to its original figure is the reciprocal of the scale factor that takes the original figure to its scaled copy. **(Speaking and Listening, Writing)**

Coherence

Today

Students deepen their understanding of scale factor by classifying scale factors by size — less than 1, equal to 1, and greater than 1 — and noticing how each classification affects the scaled copies. They come to understand that the scale factor that takes the original figure to its scaled copy and the scale factor that takes the copy to the original are reciprocals.

Previously

In Lessons 2–4, students developed their understanding of scale factor and drew scaled copies.

Coming Soon

In Lesson 6, students will determine the effects of scale factor on the perimeter and the area of a figure.

Rigor

- Students build **conceptual understanding** of the effects of scale factors of different sizes by sorting scaled copies of images.
- Students gain **procedural fluency** by calculating the scale factors of many scaled copies.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	15 min	10 min	20 min	5 min	5 min
Pairs	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per group
- Activity 3 PDF, pre-cut squares, one set of puzzle pieces per group and one blank square per student
- Anchor Chart PDF, *Scale Drawings (Part 2)*
- Anchor Chart PDF, *Scale Drawings (Part 2)* (answers)
- geometry toolkits: index cards, protractors, rulers, tracing paper
- markers or colored pencils (optional)
- tape or glue (optional)

Math Language Development

Review words

- *corresponding parts*
- *reciprocal*
- *scaled copy*
- *scale factor*

Building Math Identity and Community

Connecting to Mathematical Practices

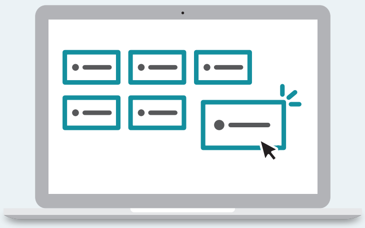
Students may attempt to try to do the work for every card in Activity 2, and thus miss the overall pattern among the scale factors that produced smaller or lager copies. Ask them to analyze the task and determine an efficient approach that the group can take. Based on those conclusions, have each student plan for how they can make good decisions that help them contribute effectively to the group.

Amps | Featured Activity

Activity 1

Digital Card Sort

Students sort cards in 3 to 5 categories of their choice. Then students use the cards to determine the scale factors for each card.



Modifications to Pacing

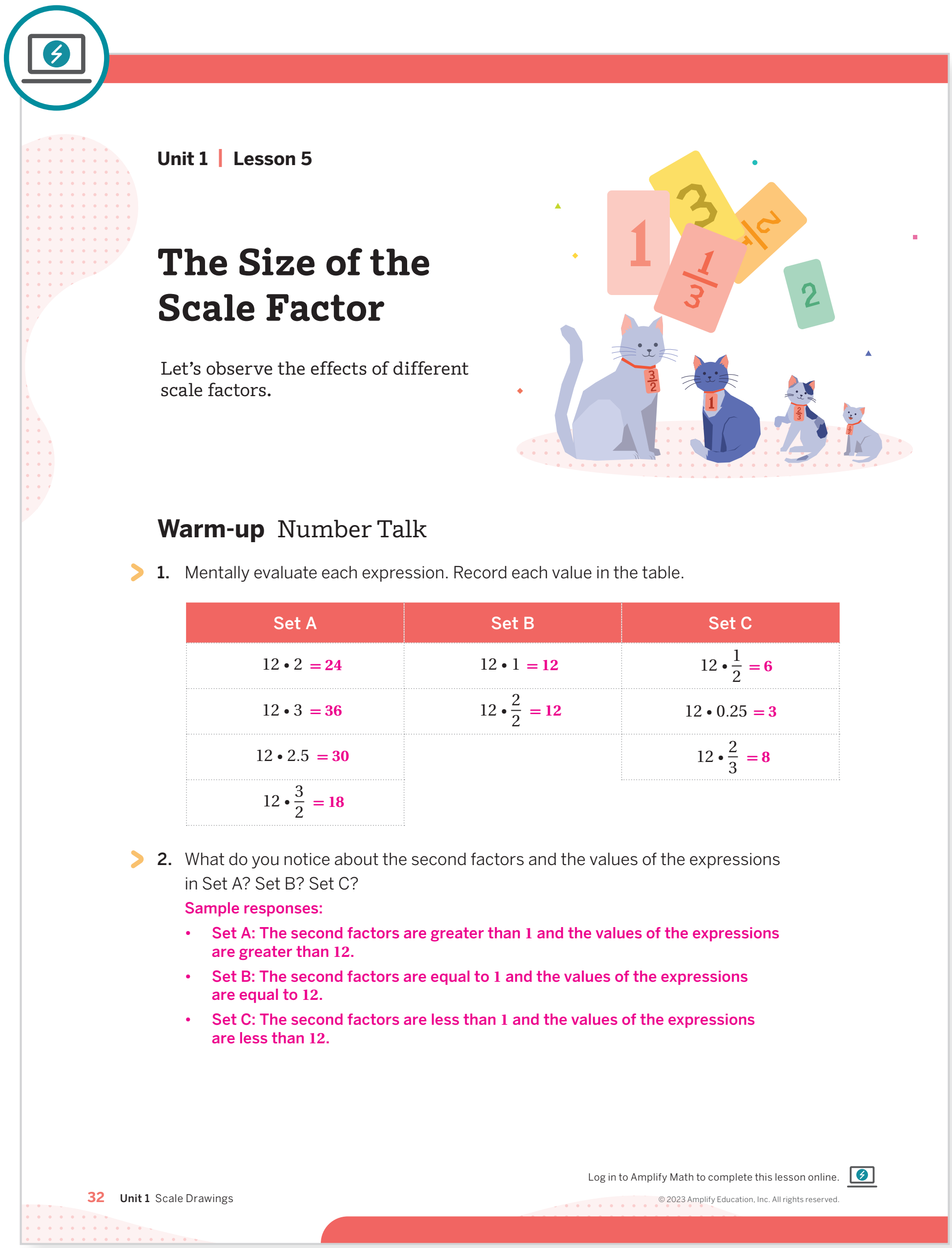
You may want to consider these additional modifications if you are short on time:

- Omit the **Warm-up**. Then, during Activity 2, write multiplication sentences to show how scale factors greater than and less than one affect the lengths of the scaled copy.
- In **Activity 2**, have groups determine the scale factors for cards 1–4, and then display the other scale factors for students to verify.
- Optional **Activity 3** may be omitted.

Pairs | 10 min

Warm-up Number Talk

Students evaluate and analyze expressions to notice how the size of the second factor affects the size of the product, in relationship to the first factor.



Unit 1 | Lesson 5

The Size of the Scale Factor

Let's observe the effects of different scale factors.

Warm-up Number Talk

1. Mentally evaluate each expression. Record each value in the table.

Set A	Set B	Set C
$12 \cdot 2$ **= 24**	$12 \cdot 1$ **= 12**	$12 \cdot \frac{1}{2}$ **= 6**
$12 \cdot 3$ **= 36**	$12 \cdot \frac{2}{2}$ **= 12**	$12 \cdot 0.25$ **= 3**
$12 \cdot 2.5$ **= 30**		$12 \cdot \frac{2}{3}$ **= 8**
$12 \cdot \frac{3}{2}$ **= 18**		

2. What do you notice about the second factors and the values of the expressions in Set A? Set B? Set C?

Sample responses:

- **Set A: The second factors are greater than 1 and the values of the expressions are greater than 12.**
- **Set B: The second factors are equal to 1 and the values of the expressions are equal to 12.**
- **Set C: The second factors are less than 1 and the values of the expressions are less than 12.**

Log in to Amplify Math to complete this lesson online.

32 **Unit 1** Scale Drawings

1 Launch

Emphasize to students that they should use mental math to evaluate each expression. Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by activating prior knowledge of multiplication and have them complete the problems with whole numbers first.

Look for points of confusion:

- **Thinking multiplying by a fraction always yields a smaller number.** Point out the last two expressions in Set A to show how multiplying by some fractions results in greater values. Point out the second expression in Set B to show how multiplying by $\frac{2}{2}$ does not change the value. For Set C, clarify that multiplying by fractions less than 1 results in smaller values.

Look for productive strategies:

- Using the first and second expressions in Set A to help evaluate the third expression.

3 Connect

Display the expressions and their values.

Have students share their observations.

Ask:

- "For Set A, what did you notice? How do the products compare with 12?" They are greater than 12.
- "For Set B, what did you notice? How do the products compare with 12?" They are equal to 12.
- "For Set C, what did you notice? How do the products compare with 12?" They are less than 12.

Highlight that when multiplying . . .

- by a number greater than 1 the product is greater than the other factor.
- by 1, the number is equal to the other factor.
- by a number less than 1, the product is less than the other factor.

Power-up

To power up students' ability to compare the size of a product to the size of one factor from a verbal statement, ask:

1. If 1 US dollar is $\frac{3}{4}$ the value of 1 British pound, which currency is worth more? Be prepared to explain your thinking.

   The British pound. Sample response: Because $\frac{3}{4}$ is less than 1, the value of the US dollar is less than that of the British pound.

2. If 1 US dollar is $\frac{7}{5}$ the value of 1 New Zealand dollar, which currency is worth more? Be prepared to explain your thinking.

   The US dollar. Sample response: Because $\frac{7}{5}$ is greater than 1, the value of the US dollar is greater than that of the New Zealand dollar.

Use: Before Activity 1.

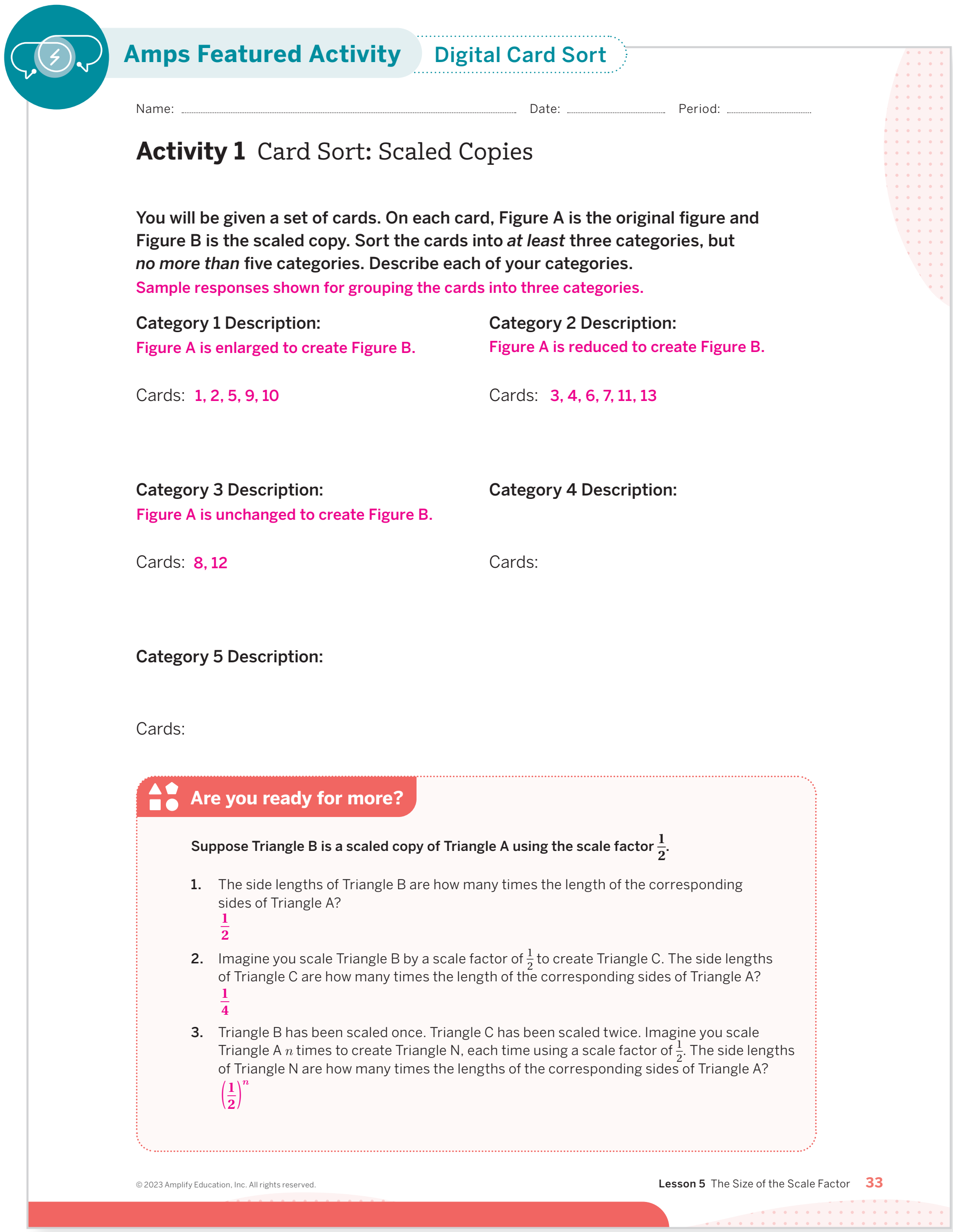
Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

Small Groups | 15 min

Activity 1 Card Sort: Scaled Copies

Students sort cards into groups to help them develop understanding of how the size of the scale factor affects whether a figure is enlarged or reduced.

Amps Featured Activity Digital Card Sort



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Activity 1 Card Sort: Scaled Copies

You will be given a set of cards. On each card, Figure A is the original figure and Figure B is the scaled copy. Sort the cards into *at least* three categories, but *no more than* five categories. Describe each of your categories.

Sample responses shown for grouping the cards into three categories.

Category 1 Description:
Figure A is enlarged to create Figure B.

Cards: 1, 2, 5, 9, 10

Category 2 Description:
Figure A is reduced to create Figure B.

Cards: 3, 4, 6, 7, 11, 13

Category 3 Description:
Figure A is unchanged to create Figure B.

Cards: 8, 12

Category 4 Description:

Cards:

Category 5 Description:

Cards:

Are you ready for more?

Suppose Triangle B is a scaled copy of Triangle A using the scale factor $\frac{1}{2}$.

1. The side lengths of Triangle B are how many times the length of the corresponding sides of Triangle A?
   $\frac{1}{2}$
2. Imagine you scale Triangle B by a scale factor of $\frac{1}{2}$ to create Triangle C. The side lengths of Triangle C are how many times the length of the corresponding sides of Triangle A?
   $\frac{1}{4}$
3. Triangle B has been scaled once. Triangle C has been scaled twice. Imagine you scale Triangle A n times to create Triangle N, each time using a scale factor of $\frac{1}{2}$. The side lengths of Triangle N are how many times the lengths of the corresponding sides of Triangle A?
   $\left(\frac{1}{2}\right)^n$

Lesson 5 The Size of the Scale Factor 33

1 Launch

Distribute a set of pre-cut cards from the Activity 1 PDF. Remind students to pay careful attention how Figure A is changed to create the scaled copy of Figure B. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by having them study Card 1 and Card 3 and asking, "Would you put these cards in the same group or different groups? Why?"

Look for points of confusion:

- **Sorting by broad characteristics of the figures, e.g., round, quadrilateral, polygon, etc.** Encourage students to think more deeply about their categories and remind them they are studying scaled copies.

Look for productive strategies:

- Sorting by scale factors of $1, 2, 3, \frac{1}{2}$, and $\frac{1}{3}$.

3 Connect

Display any necessary cards to help facilitate the discussion.

Have groups of students share their categories for how they sorted the cards. Begin with groups who sorted by whether the scaled copies were enlarged, reduced, or unchanged. Then progress to groups who sorted by scale factor. Try to steer the discussion away from specific scale factors, as students will determine those in Activity 2.

Ask, "What strategies can you use to determine the scale factors for each card?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Restrict the number of cards students sort to Cards 1–8. Then have students examine Cards 10 and 13 to complete Problem 4. Alternatively, provide the following categories students can use to sort their cards:

- Figure A is enlarged to create Figure B.
- Figure A is reduced to create Figure B.
- Figure A is unchanged to create Figure B.

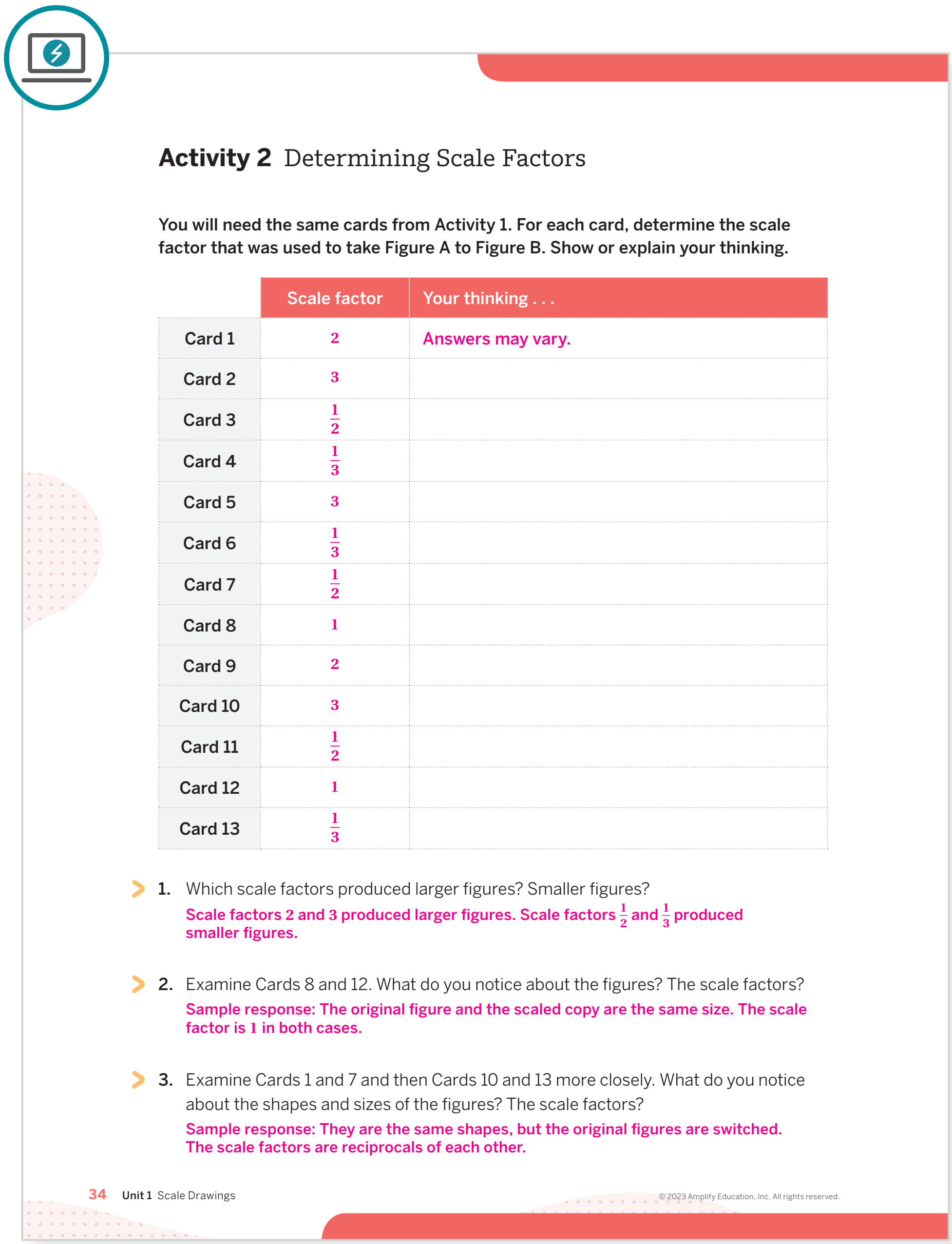
Accessibility: *Guide Processing and Visualization*

Begin the activity with a small group or whole class think-aloud demonstration using Card 1. Ask these questions:

- As I examine these figures, I notice they are not the same size. Which figure is smaller? Which figure is larger?
- I am told that Figure A is the original figure. What does this tell me about the relationship between Figure A and Figure B?
- What is one possible category I could create in which Card 1 would belong?

Activity 2 Determining Scale Factors

Students calculate scale factors to notice that factors greater than 1, less than 1, and equal to 1 create scale copies that are larger, smaller, and the same size, respectively.



Activity 2 Determining Scale Factors

You will need the same cards from Activity 1. For each card, determine the scale factor that was used to take Figure A to Figure B. Show or explain your thinking.

	Scale factor	Your thinking . . .
Card 1	2	Answers may vary.
Card 2	3	
Card 3	$\frac{1}{2}$	
Card 4	$\frac{1}{3}$	
Card 5	3	
Card 6	$\frac{1}{3}$	
Card 7	$\frac{1}{2}$	
Card 8	1	
Card 9	2	
Card 10	3	
Card 11	$\frac{1}{2}$	
Card 12	1	
Card 13	$\frac{1}{3}$	

1. Which scale factors produced larger figures? Smaller figures?
   Scale factors 2 and 3 produced larger figures. Scale factors $\frac{1}{2}$ and $\frac{1}{3}$ produced smaller figures.

2. Examine Cards 8 and 12. What do you notice about the figures? The scale factors?
   Sample response: The original figure and the scaled copy are the same size. The scale factor is 1 in both cases.

3. Examine Cards 1 and 7 and then Cards 10 and 13 more closely. What do you notice about the shapes and sizes of the figures? The scale factors?
   Sample response: They are the same shapes, but the original figures are switched. The scale factors are reciprocals of each other.

34 **Unit 1** Scale Drawings

1 Launch

Students will need the same cards they used in Activity 1. **Note:** Consider discussing group work procedures and having groups divide the cards among group members. Ask them to determine the scale factors individually, share their results, and resolve any disagreements.

2 Monitor

Help students get started by asking them how to determine a scale factor. If more support is needed, ask, "If the original side length is 5 units and the scaled copy's corresponding side length is 10 units, what is the scale factor? How did you determine it?"

Look for points of confusion:

- **Calculating all the scale factors to be greater than 1.** Remind students that Figure A is the original figure. The side lengths of Figure B must be able to be determined by multiplying corresponding side lengths of Figure A by the scale factor.
- **Writing scale factors that represent division, such as 3 for Cards 4, 6, and 13.** Remind students that scaling is a multiplicative process. Division can be expressed as multiplication by the reciprocal.

3 Connect

Display any cards needed for the discussion.

Have groups of students share their scale factors. Then have groups re-sort their cards based on these scale factors.

Ask, "What can you say about the scale factors that produced larger copies? Smaller copies? Same-size copies?"

Highlight the effects of scale factors greater than 1, less than 1, and equal to 1. Point out that scaling can be reversed; if Figure B is a scaled copy of Figure A, then Figure A is also a scaled copy of Figure B. Their scale factors are *reciprocals*.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Restrict the number of cards for which students need to determine a scale factor. Have them use Cards 1–8.

Accessibility: *Guide Processing and Visualization*

Demonstrate how to determine the scale factor for Card 1. Keep the worked-out calculations on display for students to reference as they work through the rest of the activity.

Math Language Development

MLR3: Critique, Correct, Clarify

Present an incorrect statement that reflects a possible point of confusion. For example, "The scale factor for Cards 8 and 12 is 0 because the shapes are the same size." Prompt students to identify the error, provide the correct scale factor, and clarify their thinking.

English Learners

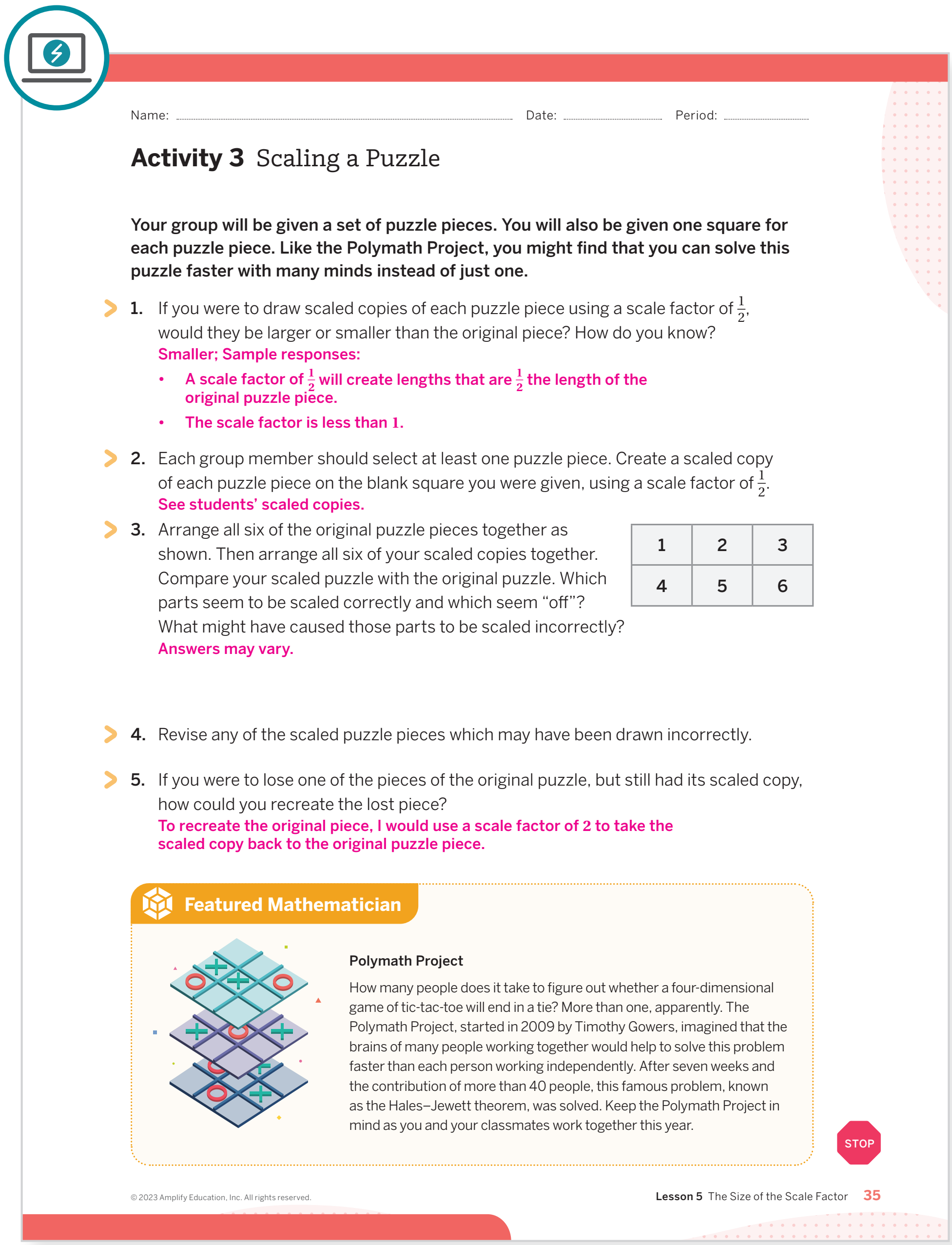
After the discussion, stress that students understand the statement you provided was an incorrect statement by clearly labeling it as *incorrect* or *false*.

Optional

Small Groups | 20 min

Activity 3 Scaling a Puzzle

Students apply what they know about scale factors, lengths, and angles to create scaled copies — without the support of a grid.



Name: ______________________ Date: __________ Period: __________

Activity 3 Scaling a Puzzle

Your group will be given a set of puzzle pieces. You will also be given one square for each puzzle piece. Like the Polymath Project, you might find that you can solve this puzzle faster with many minds instead of just one.

1. If you were to draw scaled copies of each puzzle piece using a scale factor of $\frac{1}{2}$, would they be larger or smaller than the original piece? How do you know?
   Smaller; Sample responses:
   - **A scale factor of $\frac{1}{2}$ will create lengths that are $\frac{1}{2}$ the length of the original puzzle piece.**
   - **The scale factor is less than 1.**
2. Each group member should select at least one puzzle piece. Create a scaled copy of each puzzle piece on the blank square you were given, using a scale factor of $\frac{1}{2}$.
   See students' scaled copies.
3. Arrange all six of the original puzzle pieces together as shown. Then arrange all six of your scaled copies together. Compare your scaled puzzle with the original puzzle. Which parts seem to be scaled correctly and which seem "off"? What might have caused those parts to be scaled incorrectly?
   Answers may vary.

1	2	3
4	5	6

4. Revise any of the scaled puzzle pieces which may have been drawn incorrectly.
5. If you were to lose one of the pieces of the original puzzle, but still had its scaled copy, how could you recreate the lost piece?
   To recreate the original piece, I would use a scale factor of 2 to take the scaled copy back to the original puzzle piece.

Featured Mathematician

Polymath Project

How many people does it take to figure out whether a four-dimensional game of tic-tac-toe will end in a tie? More than one, apparently. The Polymath Project, started in 2009 by Timothy Gowers, imagined that the brains of many people working together would help to solve this problem faster than each person working independently. After seven weeks and the contribution of more than 40 people, this famous problem, known as the Hales–Jewett theorem, was solved. Keep the Polymath Project in mind as you and your classmates work together this year.

STOP

Lesson 5 The Size of the Scale Factor 35

1 Launch

Distribute pre-cut copies of the puzzle pieces and one pre-cut square from the second page of the Activity 3 PDF for each puzzle piece. Provide access to geometry toolkits and markers or colored pencils.

2 Monitor

Help students get started by having them explain what a scale factor of $\frac{1}{2}$ means.

Look for points of confusion:

- **Not verifying that the angle measures in their copies are unchanged from the original.** Ask them to study the angles and recall how the angle measures of a scaled copy compare to the original figure.
- **Neglecting to incorporate the scale factor when scaling distances between two points that are not connected by a segment.** Remind them that *all* distances are scaled by the same factor.

Look for productive strategies:

- Measuring distances between points and not just drawn segments, e.g., between the corner of a square and where a segment begins.

3 Connect

Display students' completed puzzles.

Have groups of students share how they worked together, even though each person had their own assigned task.

Ask:

- "How is this task more challenging than creating scaled copies of polygons on a grid?"
- "Other than using distances or lengths, what helped you create an accurate scaled copy?"
- "How did you decide which distances to measure?"
- "Before your drawings were assembled, how did you check if they were correct?"

Highlight how students measured distances and whether they considered angle measures.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider assigning Piece 4 or Piece 6 of the puzzle to students who need more processing time.

Accessibility: *Optimize Access to Tools*

Before beginning the activity, ask students which tools they think they will need from their geometry toolkits, and have them describe how they might use them.

Extension: *Math Enrichment*

Consider assigning Piece 3 to students as a challenge activity. Have students color their puzzle pieces before arranging them to form the larger puzzle.

Featured Mathematician

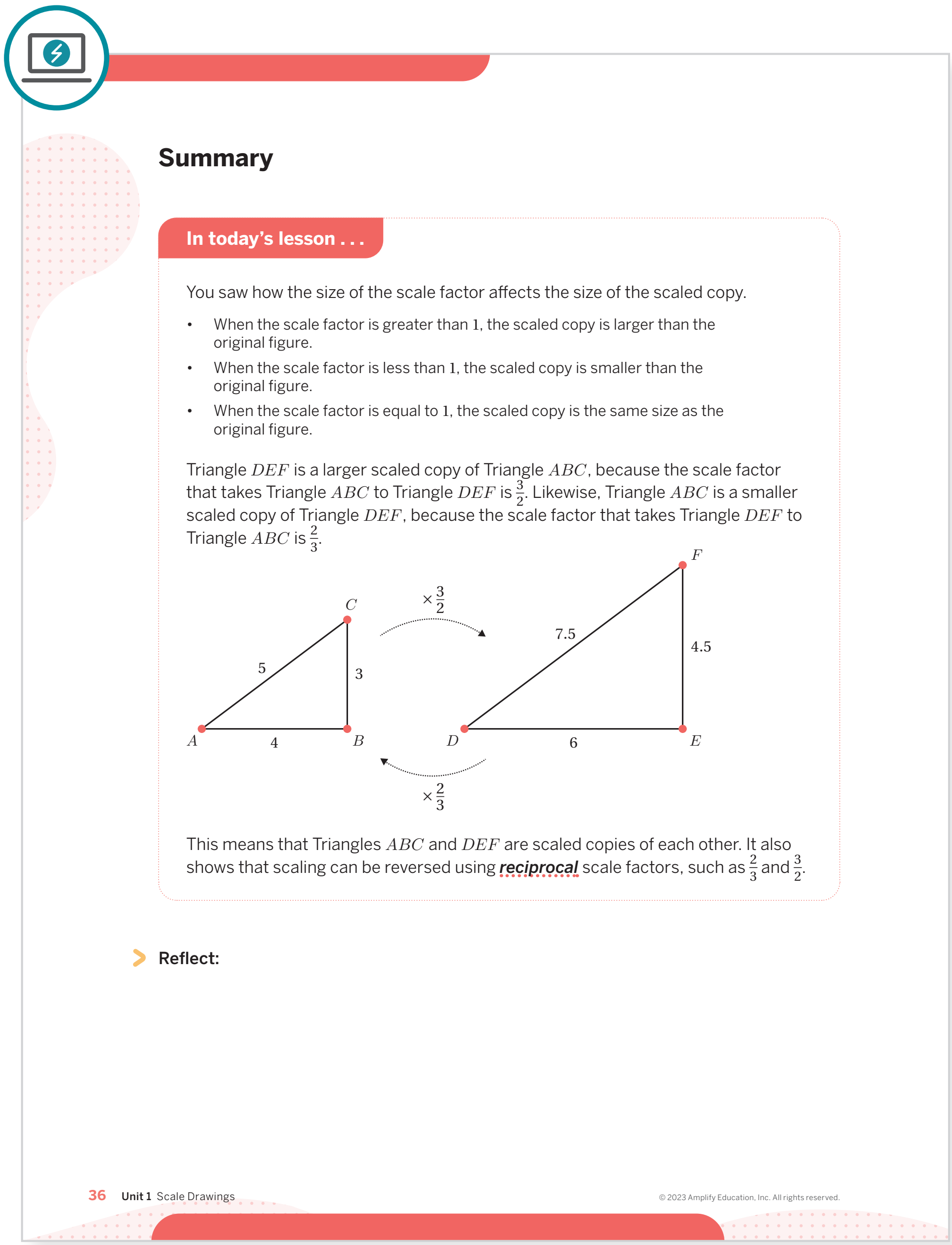
Polymath Project

Have students read about the Polymath Project which brings together mathematicians from all over the world to solve complex problems!

Whole Class | 5 min

Summary

Review and synthesize how the size of the scale factor affects the size of the scaled copy.



Summary

In today's lesson . . .

You saw how the size of the scale factor affects the size of the scaled copy.

- When the scale factor is greater than 1, the scaled copy is larger than the original figure.
- When the scale factor is less than 1, the scaled copy is smaller than the original figure.
- When the scale factor is equal to 1, the scaled copy is the same size as the original figure.

Triangle DEF is a larger scaled copy of Triangle ABC, because the scale factor that takes Triangle ABC to Triangle DEF is $\frac{3}{2}$. Likewise, Triangle ABC is a smaller scaled copy of Triangle DEF, because the scale factor that takes Triangle DEF to Triangle ABC is $\frac{2}{3}$.

This means that Triangles ABC and DEF are scaled copies of each other. It also shows that scaling can be reversed using ***reciprocal*** scale factors, such as $\frac{2}{3}$ and $\frac{3}{2}$.

Reflect:

36 **Unit 1** Scale Drawings

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Synthesize

Display the Anchor Chart PDF, *Scale Drawings (Part 2)* and complete it as you facilitate a class discussion using the following questions.

Ask:

- "What happens to the scaled copy when it is created using a scale factor greater than 1? Less than 1? Equal to 1?"
- "How can you reverse the scaling to return to the original figure after you have created a scaled copy?"

Highlight that when the scale factor is greater than 1, the scaled copy is larger than the original figure. When the scale factor is less than 1, the scaled copy is smaller than the original figure. A scale factor that is equal to 1 produces a scaled copy that is the same size as the original figure. Scaling can be *reversed* by using *reciprocal* factors. Referring to the figures shown in the Summary in the Student Edition, the scale factors $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What characteristics did you use to determine the categories for your card sort? Why did you choose those characteristics?"
- "How is scale factor related to the size of the scaled copy when compared to the original figure?"

Math Language Development

MLR2: Collect and Display

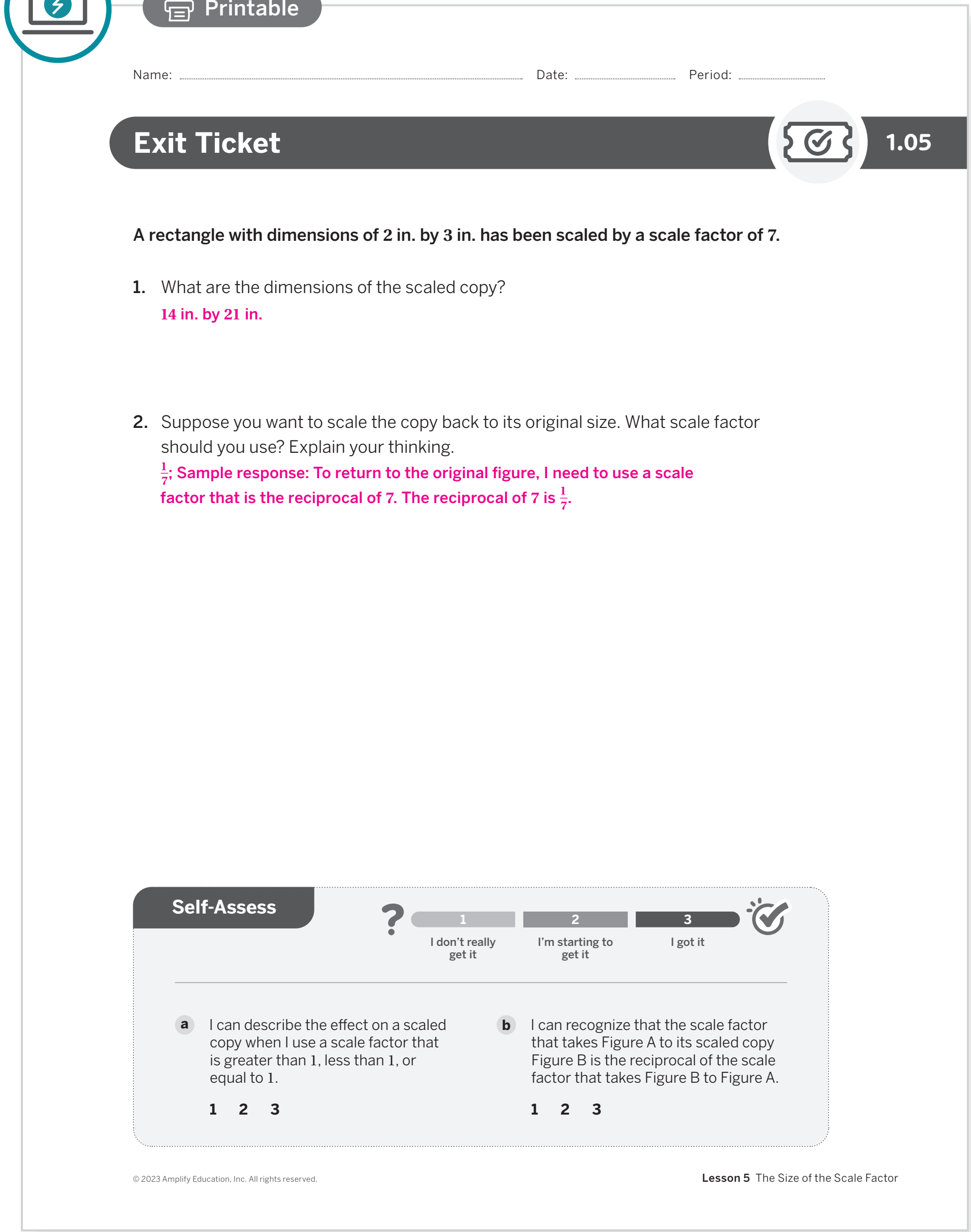
Add language highlighted in the Synthesize section to the class display. Pay particular attention to statements, such as, "A scale factor that is equal to 1 produces a scaled copy that is the same size as the original figure." Add this type of reasoning to the class display and remind students to refer back to it during class discussions.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the scale factor and its reciprocal to reverse the process.

Printable



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Exit Ticket 1.05

A rectangle with dimensions of 2 in. by 3 in. has been scaled by a scale factor of 7.

1. What are the dimensions of the scaled copy?

14 in. by 21 in.

2. Suppose you want to scale the copy back to its original size. What scale factor should you use? Explain your thinking.

$\frac{1}{7}$; Sample response: To return to the original figure, I need to use a scale factor that is the reciprocal of 7. The reciprocal of 7 is $\frac{1}{7}$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can describe the effect on a scaled copy when I use a scale factor that is greater than 1, less than 1, or equal to 1.

1 2 3

b I can recognize that the scale factor that takes Figure A to its scaled copy Figure B is the reciprocal of the scale factor that takes Figure B to Figure A.

1 2 3

Lesson 5 The Size of the Scale Factor

Success looks like . . .

- **Language Goal:** Describing how scale factors of 1, less than 1, and greater than 1 affect the size of scaled copies. **(Speaking and Listening, Writing)**
  - » Calculating that the a scale factor of 7 makes the scaled copy larger in Problem 1.
- **Language Goal:** Explaining and showing how to recreate the original figure given a scaled copy and its scale factor. **(Speaking and Listening, Writing)**
  - » Explaining the scale factor needed to scale the copy back to its original size in Problem 2.
- **Language Goal:** Recognizing that the scale factor that takes the scaled copy to its original figure is the reciprocal of the scale factor that takes the original figure to its scaled copy. **(Speaking and Listening, Writing)**
  - » Calculating and explaining that a scale factor of $\frac{1}{7}$ is needed to for the copy to scale back to its original size in Problem 2.

Suggested next steps

If students write the scale factor as dividing by 7 for Problem 2, consider:

- Reminding them that scale factor is represented as a multiplicative process.
- Asking, "When you make a smaller copy, what does the scale factor need to be?" Less than 1
- Assigning Practice Problems 1 and 2.

Professional Learning

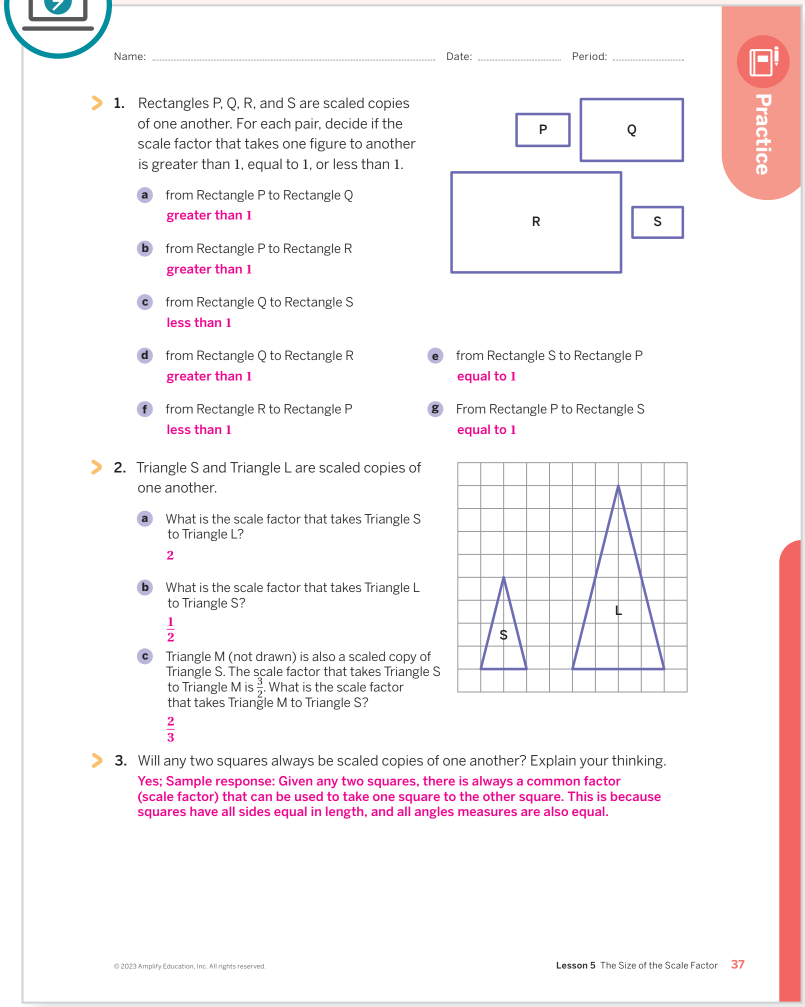
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did sorting the cards in Activity 1 set students up to develop the understanding of how the size of the scale factor affects the size of the copy?
- What surprised you as your students worked on computing the scale factors? What might you change for the next time you teach this lesson?

Practice

Independent



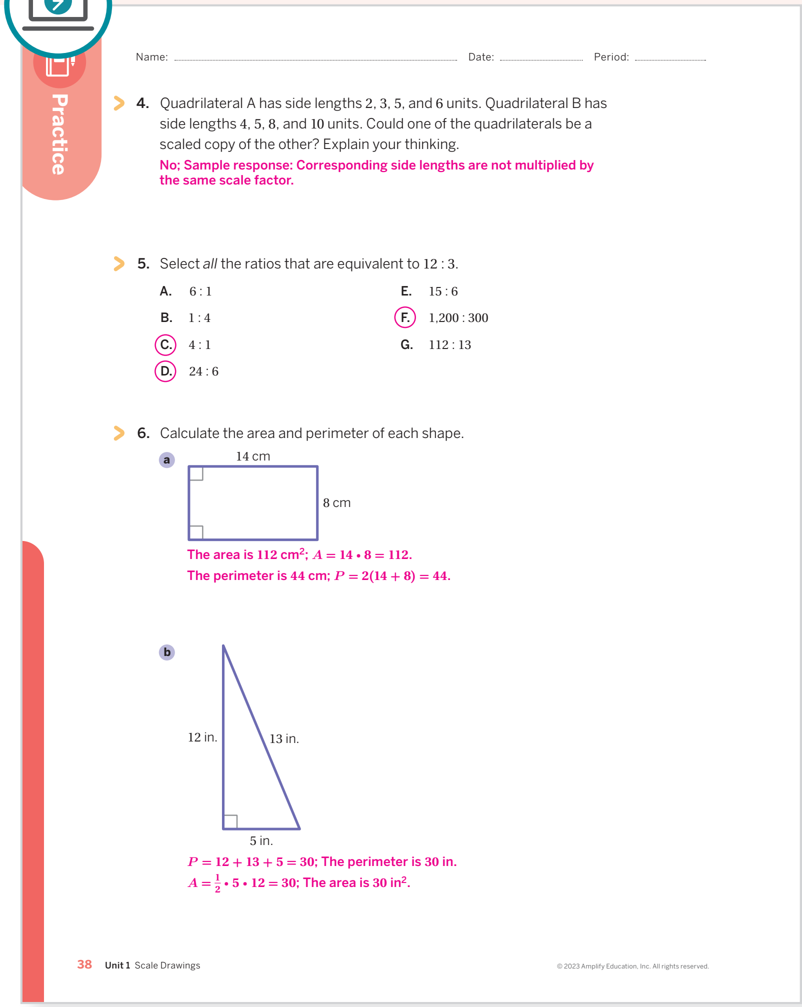
Name: Date: Period:

1. Rectangles P, Q, R, and S are scaled copies of one another. For each pair, decide if the scale factor that takes one figure to another is greater than 1, equal to 1, or less than 1.
   - a from Rectangle P to Rectangle Q
     greater than 1
   - b from Rectangle P to Rectangle R
     greater than 1
   - c from Rectangle Q to Rectangle S
     less than 1
   - d from Rectangle Q to Rectangle R
     greater than 1
   - e from Rectangle S to Rectangle P
     equal to 1
   - f from Rectangle R to Rectangle P
     less than 1
   - g From Rectangle P to Rectangle S
     equal to 1

2. Triangle S and Triangle L are scaled copies of one another.
   - a What is the scale factor that takes Triangle S to Triangle L?
     2
   - b What is the scale factor that takes Triangle L to Triangle S?
     $\mathbf{\frac{1}{2}}$
   - c Triangle M (not drawn) is also a scaled copy of Triangle S. The scale factor that takes Triangle S to Triangle M is $\frac{3}{2}$. What is the scale factor that takes Triangle M to Triangle S?
     $\mathbf{\frac{2}{3}}$

3. Will any two squares always be scaled copies of one another? Explain your thinking.
   Yes; Sample response: Given any two squares, there is always a common factor (scale factor) that can be used to take one square to the other square. This is because squares have all sides equal in length, and all angles measures are also equal.

Lesson 5 The Size of the Scale Factor 37



Name: Date: Period:

4. Quadrilateral A has side lengths 2, 3, 5, and 6 units. Quadrilateral B has side lengths 4, 5, 8, and 10 units. Could one of the quadrilaterals be a scaled copy of the other? Explain your thinking.
   No; Sample response: Corresponding side lengths are not multiplied by the same scale factor.

5. Select *all* the ratios that are equivalent to 12 : 3.
   - A. 6 : 1
   - B. 1 : 4
   - [✓ C.] 4 : 1
   - [✓ D.] 24 : 6
   - E. 15 : 6
   - [✓ F.] 1,200 : 300
   - G. 112 : 13

6. Calculate the area and perimeter of each shape.
   - a
     The area is 112 cm^2; $A = 14 \cdot 8 = 112$.
     The perimeter is 44 cm; $P = 2(14 + 8) = 44$.
   - b
     $P = 12 + 13 + 5 = 30$; The perimeter is 30 in.
     $A = \frac{1}{2} \cdot 5 \cdot 12 = 30$; The area is 30 in^2.

38 Unit 1 Scale Drawings

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 1	3
Spiral	4	Unit 1 Lesson 3	2
	5	Grade 6	1
Formative ⚡	6	Unit 1 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 1 | **LESSON 6**

Scaling Area

Let’s investigate the area of scaled copies.

Focus

Goals

1. **Language Goal:** Calculate and compare the areas of multiple scaled copies of the same figure. **(Speaking and Listening, Writing)**
2. **Language Goal:** Generalize that the area of a scaled copy is the product of the area of the original figure and the square of the scale factor. **(Speaking and Listening)**

Coherence

Today

Students are introduced to how the area of a scaled copy relates to the area of the original figure. Students build on their knowledge of exponents to recognize the patterns of the areas changing by the square of the scale factors used.

Previously

In Lesson 5, students used broader terms, such as *enlarge* or *reduce*, to describe how the size of the scale factor affects the size of the scaled copy.

Coming Soon

In Lesson 7, students will expand on their understanding of scaled copies to reason about scale drawings.

Rigor

- Students build **conceptual understanding** of how scale factor affects areas and lengths by noticing the patterns of how they change.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (answers, for display)

Math Language Development

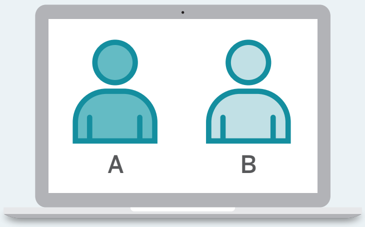
Review words

- *area*
- *corresponding parts*
- *perimeter*
- *reciprocal*
- *scale factor*
- *scaled copy*

Amps | Featured Activity

Activity 2
Digital Partner Problems

Students individually complete a series of problems digitally, and check responses with their partners before moving on to the next one.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel stressed during Activity 1 because the answer to the question is not explicitly given. Their anxiety might come from not knowing how they are supposed to have the knowledge to answer the question. Remind them that the goal is to make a conjecture based on their data. Their stress level can be reduced by being reassured that the activity will guide them to the information they need.

Modifications to Pacing

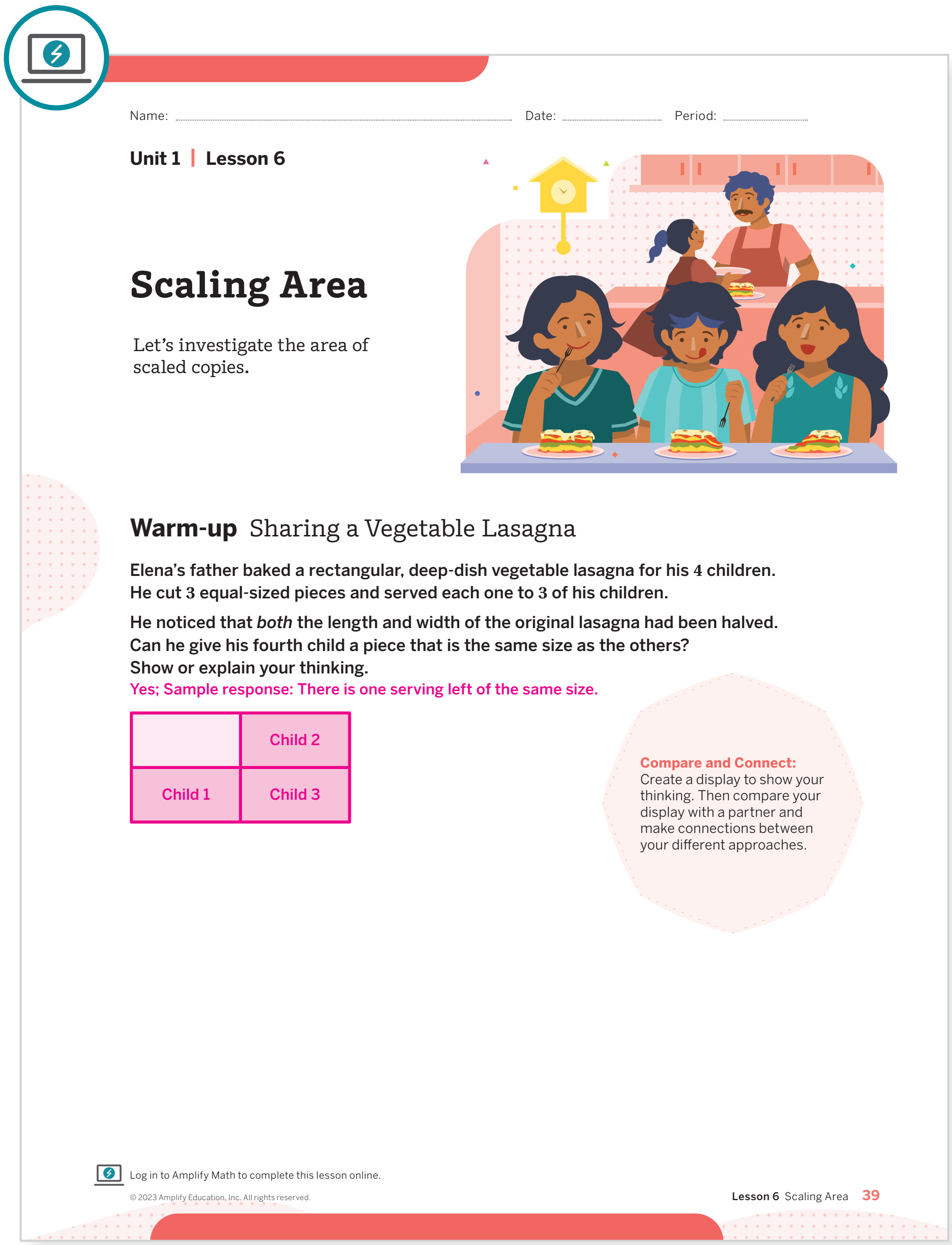
You may want to consider these additional modifications if you are short on time:

- Omit the **Warm-up** and have students complete the first row of the table in **Activity 1** as their Warm-up.
- Omit the last row in Problem 1 of **Activity 1**, or have students complete one row and share responses with the class.
- Postpone using **Activity 2** until the end of the unit, perhaps as part of a unit review.

Independent | 5 min

Warm-up Sharing a Vegetable Lasagna

Students activate prior knowledge of area and perimeter by analyzing the dimensions of a rectangular vegetable lasagna partitioned into equal-sized pieces.



Name: ________________ Date: __________ Period: __________

Unit 1 | Lesson 6

Scaling Area

Let's investigate the area of scaled copies.

Warm-up Sharing a Vegetable Lasagna

Elena's father baked a rectangular, deep-dish vegetable lasagna for his 4 children. He cut 3 equal-sized pieces and served each one to 3 of his children.

He noticed that *both* the length and width of the original lasagna had been halved. Can he give his fourth child a piece that is the same size as the others? Show or explain your thinking.

Yes; Sample response: There is one serving left of the same size.

	Child 2
Child 1	Child 3

Compare and Connect: Create a display to show your thinking. Then compare your display with a partner and make connections between your different approaches.

Log in to Amplify Math to complete this lesson online.

Lesson 6 Scaling Area 39

1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by having them draw a rectangle to visually present the situation. Ask them what it means for the length and width of the vegetable lasagna to be "halved."

Look for points of confusion:

- **Not realizing that they can draw a picture to help them understand the situation.** Remind students that representing a situation with a drawing or picture is a great problem-solving tool they can use often, and they do not need to be directed to do so.
- **Thinking there are 6 total pieces because Elena's father cut the lasagna into 3 pieces and the dimensions of the lasagna were halved.** Ask students to draw a rectangle partitioned into 6 equal-sized pieces to see if the dimensions were halved.

Look for productive strategies:

- Drawing pictures to represent the scenario. Note students who do this to share during the discussion.

3 Connect

Have students share their strategies for determining their responses. Include strategies of students who did not arrive at the correct response and strategies of students who assigned dimensions to the lasagna.

Ask:

- "How do the dimensions (length and width) of the individual servings relate to the dimensions (length and width) of the lasagna?" The dimensions of the individual servings are half the length of the lasagna.
- "How does the area of each serving compare to the area of the lasagna?" The area of each serving is one fourth the area of the lasagna.

Highlight that Activity 1 will help students determine the specific relationship between scale factor and area.

Power-up

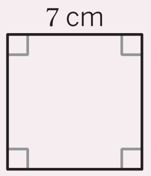
To power up students' ability to determine the area and perimeter of polygons, ask:

Recall that the *perimeter* is the sum of the lengths of the sides of the polygon.

The *area* is the number of square units that covers a polygon.

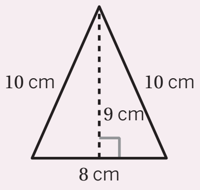
Calculate the perimeter and area of each shape.

1.



Perimeter: 28 cm
Area: 49 cm^2

2.



Perimeter: 28 cm
Area: 36 cm^2

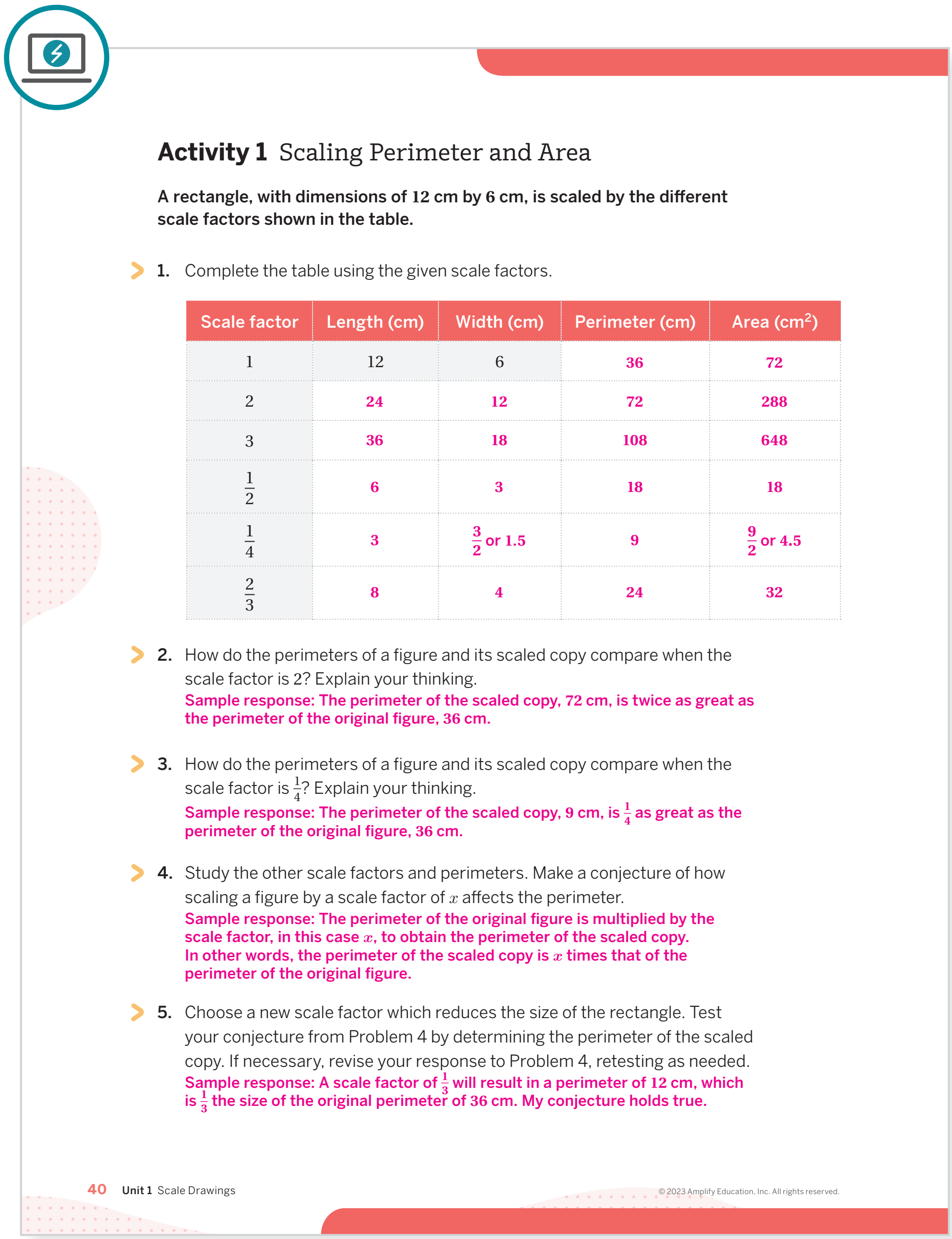
Use: Before Activity 1.

Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 5, 6, and 8.

Small Groups | 20 min

Activity 1 Scaling Perimeter and Area

Students study the effects of different scale factors on the dimensions of a rectangle to determine how the scale factor affects the perimeter and area of a figure.



Activity 1 Scaling Perimeter and Area

A rectangle, with dimensions of 12 cm by 6 cm, is scaled by the different scale factors shown in the table.

1. Complete the table using the given scale factors.

Scale factor	Length (cm)	Width (cm)	Perimeter (cm)	Area (cm^2)
1	12	6	**36**	**72**
2	**24**	**12**	**72**	**288**
3	**36**	**18**	**108**	**648**
$\frac{1}{2}$	**6**	**3**	**18**	**18**
$\frac{1}{4}$	**3**	**$\frac{3}{2}$ or 1.5**	**9**	**$\frac{9}{2}$ or 4.5**
$\frac{2}{3}$	**8**	**4**	**24**	**32**

2. How do the perimeters of a figure and its scaled copy compare when the scale factor is 2? Explain your thinking.
Sample response: The perimeter of the scaled copy, 72 cm, is twice as great as the perimeter of the original figure, 36 cm.

3. How do the perimeters of a figure and its scaled copy compare when the scale factor is $\frac{1}{4}$? Explain your thinking.
Sample response: The perimeter of the scaled copy, 9 cm, is $\frac{1}{4}$ as great as the perimeter of the original figure, 36 cm.

4. Study the other scale factors and perimeters. Make a conjecture of how scaling a figure by a scale factor of x affects the perimeter.
Sample response: The perimeter of the original figure is multiplied by the scale factor, in this case x, to obtain the perimeter of the scaled copy. In other words, the perimeter of the scaled copy is x times that of the perimeter of the original figure.

5. Choose a new scale factor which reduces the size of the rectangle. Test your conjecture from Problem 4 by determining the perimeter of the scaled copy. If necessary, revise your response to Problem 4, retesting as needed.
Sample response: A scale factor of $\frac{1}{3}$ will result in a perimeter of 12 cm, which is $\frac{1}{3}$ the size of the original perimeter of 36 cm. My conjecture holds true.

40 **Unit 1** Scale Drawings

1 Launch

Activate students' background knowledge by asking, "Why are the largest animals on Earth found in the water? You will find out after this activity." Review classroom expectations of group work.

2 Monitor

Help students get started by activating their prior knowledge of determining the perimeter and area of a rectangle. Let them know a *conjecture* is a conclusion based on information gathered.

Look for points of confusion:

- **Thinking the perimeter or area is changed by the addition or subtraction of a value.** Ask students to test their ideas using other scale factors in the table.
- **Choosing a scale factor greater than 1 for Problem 5 or Problem 9.** Although the results should still match the pattern, ask students what kind of scale factor creates a larger rectangle (greater than 1) and what kind of scale factor creates a smaller figure (less than 1).

Look for productive strategies:

- Articulating how the scale factor changes a figure's perimeter and area in a manner understood by their group members.
- Noticing an error in their conjecture and working to revise their conjecture by testing other values.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students complete the *Length, Width,* and *Perimeter* columns, followed by Problems 2–5. Then have students complete the *Area* column, followed by Problems 6–9. Alternatively, provide a partially-completed table with more of the cells pre-completed than what is currently shown in the Student Edition.

Math Language Development

MLR8: Discussion Supports

To foster productive mathematical discussions, encourage students to paraphrase their partners' conjectures for Problems 4 and 8. After paraphrasing their partners' ideas, students should highlight how their partners' conjectures compare to their own. Provide support for students to understand the language *twice as big, half as big, four times as big,* etc.

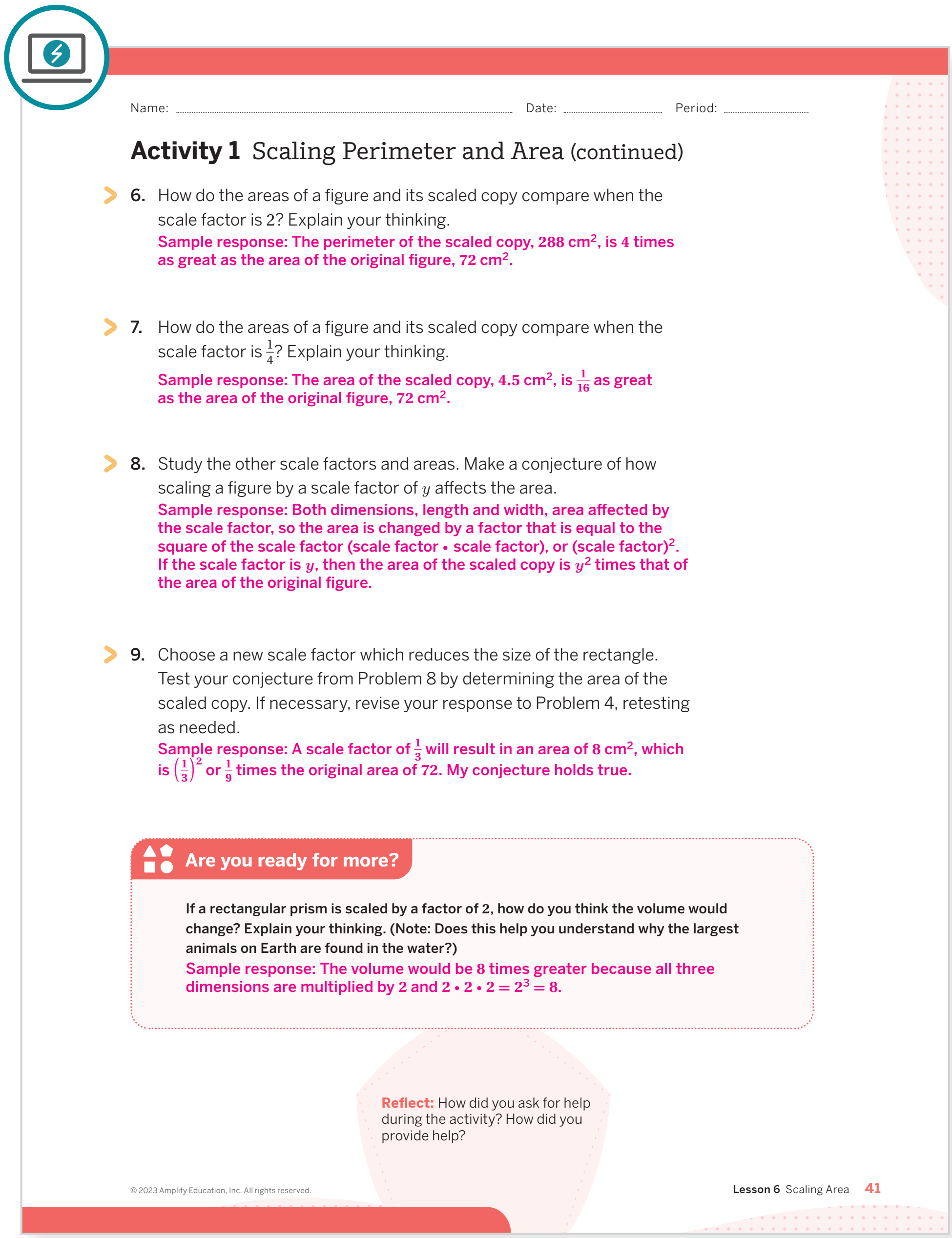
English Learners

To help students understand and explain how perimeter and area change as scale factors change, provide the following sentence starter, "When the side length of a rectangle is scaled by _____, the perimeter/area is scaled by _____."

Small Groups | 20 min

Activity 1 Scaling Perimeter and Area (continued)

Students study the effects of different scale factors on the dimensions of a rectangle to determine how the scale factor affects the perimeter and area of a figure.



Name: ______ Date: ______ Period: ______

Activity 1 Scaling Perimeter and Area (continued)

6. How do the areas of a figure and its scaled copy compare when the scale factor is 2? Explain your thinking.
   Sample response: The perimeter of the scaled copy, 288 cm^2, is 4 times as great as the area of the original figure, 72 cm^2.

7. How do the areas of a figure and its scaled copy compare when the scale factor is $\frac{1}{4}$? Explain your thinking.
   Sample response: The area of the scaled copy, 4.5 cm^2, is $\frac{1}{16}$ as great as the area of the original figure, 72 cm^2.

8. Study the other scale factors and areas. Make a conjecture of how scaling a figure by a scale factor of y affects the area.
   Sample response: Both dimensions, length and width, area affected by the scale factor, so the area is changed by a factor that is equal to the square of the scale factor (scale factor • scale factor), or (scale factor)^2. If the scale factor is y, then the area of the scaled copy is y^2 times that of the area of the original figure.

9. Choose a new scale factor which reduces the size of the rectangle. Test your conjecture from Problem 8 by determining the area of the scaled copy. If necessary, revise your response to Problem 4, retesting as needed.
   Sample response: A scale factor of $\frac{1}{3}$ will result in an area of 8 cm^2, which is $\left(\frac{1}{3}\right)^2$ or $\frac{1}{9}$ times the original area of 72. My conjecture holds true.

Are you ready for more?

If a rectangular prism is scaled by a factor of 2, how do you think the volume would change? Explain your thinking. (Note: Does this help you understand why the largest animals on Earth are found in the water?)

Sample response: The volume would be 8 times greater because all three dimensions are multiplied by 2 and $2 \cdot 2 \cdot 2 = 2^3 = 8$.

Reflect: How did you ask for help during the activity? How did you provide help?

3 Connect

Display the Activity 1 PDF (answers).

Have students share their conjectures and their reasoning for how they completed the table and their responses to Problems 4 and 8.

Ask:

- "Did anyone need to revise their conjecture after testing a scale factor?"
- "Why is the perimeter changed by a factor that is equal to the scale factor?" Perimeter is a one-dimensional measure.
- "Why is the area changed by a factor that is equal to the square of the scale factor?" Area is a two-dimensional measure and both dimensions are changed by the scale factor at the same time.
- "Based on your discoveries, why do you think the largest animals on Earth are found in the water?" Sample response: When the dimensions are scaled by a scale factor — for example, 2 — each dimension changes by a factor of 2, so the animal's volume (or mass) changes by a factor of 2^3 or 8. Life on land becomes almost impossible because the animals' legs would collapse due to the increased volume (or mass). Water buoyancy works against the gravitational pull on the body, so animals that live in water have the ability to grow larger than land animals.

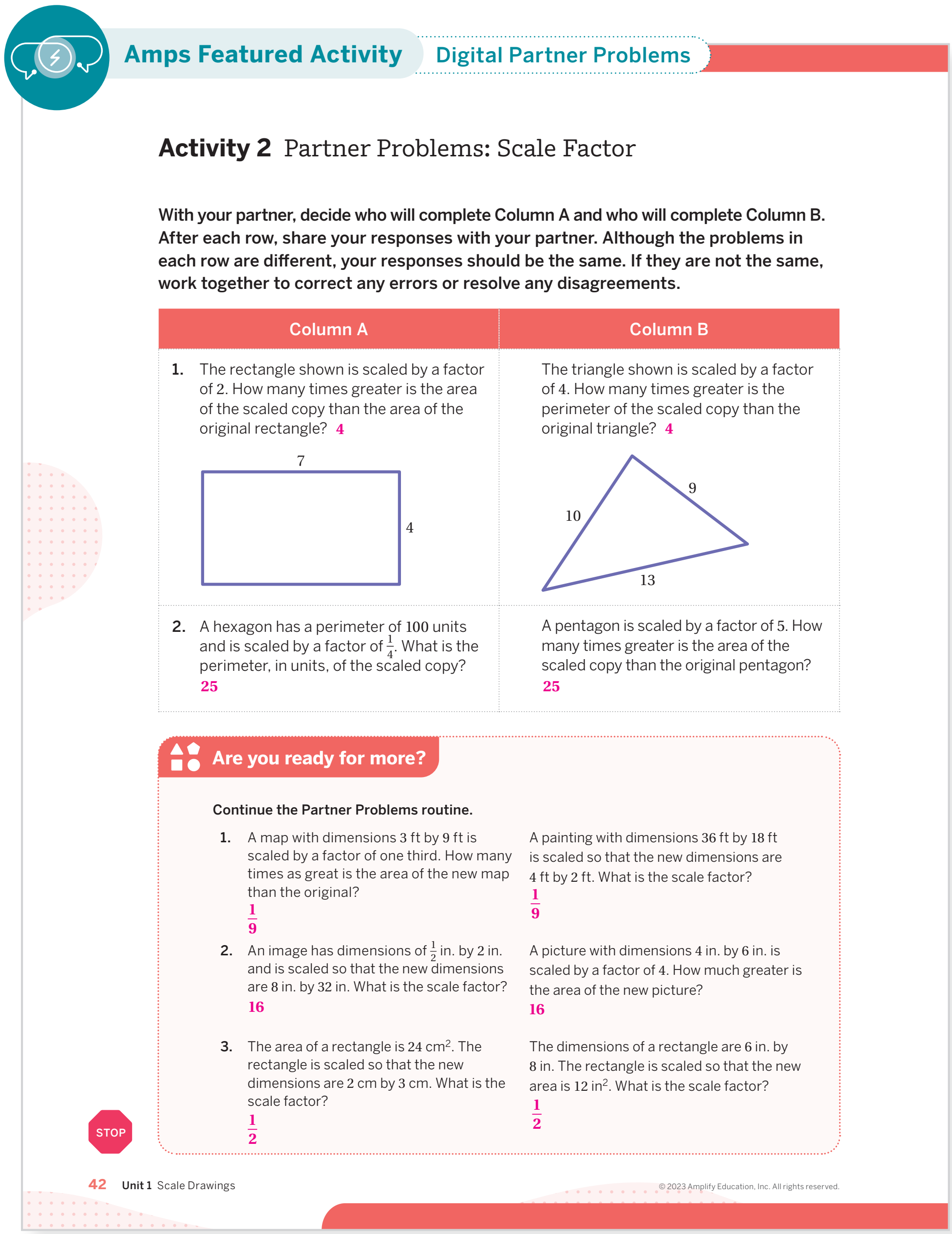
Highlight the connection between the perimeter of a rectangle and its length or width as both being one-dimensional attributes. This is why the perimeter of a scaled copy changes by a factor that is equal to the scale factor, just as with the length or width. Then emphasize that area is a two-dimensional attribute. This is why the area of a scaled copy changes by a factor that is equal to the square of the scale factor. This also works for shapes which are not rectangles. **Note:** Some students may be ready for an algebraic explanation. For a rectangle, $A = \ell \cdot w$. If both dimensions are multiplied by the scale factor s, then $A = (s \cdot \ell) \cdot (s \cdot w)$. This can be rewritten as $A = s \cdot s \cdot \ell \cdot w$ or $A = s^2 \cdot \ell \cdot w$.

Pairs | 10 min

Activity 2 Partner Problems: Scale Factor

Students strengthen their understanding of how scale factor affects the perimeter and area of figures by solving problems and receiving immediate feedback from their partners.

Amps Featured Activity Digital Partner Problems



Activity 2 Partner Problems: Scale Factor

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with your partner. Although the problems in each row are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

Column A	Column B
1. The rectangle shown is scaled by a factor of 2. How many times greater is the area of the scaled copy than the area of the original rectangle? 4	The triangle shown is scaled by a factor of 4. How many times greater is the perimeter of the scaled copy than the original triangle? 4
2. A hexagon has a perimeter of 100 units and is scaled by a factor of $\frac{1}{4}$. What is the perimeter, in units, of the scaled copy? 25	A pentagon is scaled by a factor of 5. How many times greater is the area of the scaled copy than the original pentagon? 25

Are you ready for more?

Continue the Partner Problems routine.

1. A map with dimensions 3 ft by 9 ft is scaled by a factor of one third. How many times as great is the area of the new map than the original? $\frac{1}{9}$	A painting with dimensions 36 ft by 18 ft is scaled so that the new dimensions are 4 ft by 2 ft. What is the scale factor? $\frac{1}{9}$
2. An image has dimensions of $\frac{1}{2}$ in. by 2 in. and is scaled so that the new dimensions are 8 in. by 32 in. What is the scale factor? 16	A picture with dimensions 4 in. by 6 in. is scaled by a factor of 4. How much greater is the area of the new picture? 16
3. The area of a rectangle is 24 cm^2. The rectangle is scaled so that the new dimensions are 2 cm by 3 cm. What is the scale factor? $\frac{1}{2}$	The dimensions of a rectangle are 6 in. by 8 in. The rectangle is scaled so that the new area is 12 in^2. What is the scale factor? $\frac{1}{2}$

STOP

42 **Unit 1** Scale Drawings

1 Launch

Explain and conduct the *Partner Problems* routine.

2 Monitor

Help students get started by having them calculate the area or perimeter of their figure, performing the scaling, and determining the new area or perimeter.

Look for points of confusion:

- **Solving all of the problems before checking in with their partner.** Remind students that the point is to receive immediate feedback on your response to help clear up any misunderstandings before moving to the next problem.

Look for productive strategies:

- Explaining their thinking or helping to identify their or their partner's mistake in a clear and productive manner.

3 Connect

Display any problems necessary for the discussion.

Have students share how the structure of their problem helped them determine which strategy to use.

Highlight any problems which were of particular interest to your class or ones with which most students struggled.

Ask, "Did anyone have any disagreements? How did you resolve them?"

Differentiated Support

Extension: *Math Enrichment*

Have students complete the following problem:
The side lengths of a cube are each scaled by a factor of 2. How many times greater is the surface area and volume of the scaled cube than the surface area and volume of the original cube? Explain your thinking.
Surface area: 4 times greater
Volume: 8 times greater
Sample response: Let the side length of a cube be 5 units.
Surface area of cube: 150 square units
Volume of cube: 125 cubic units
Surface area of scaled cube: 600 square units
Volume of scaled cube: 1,000 cubic units

Math Language Development

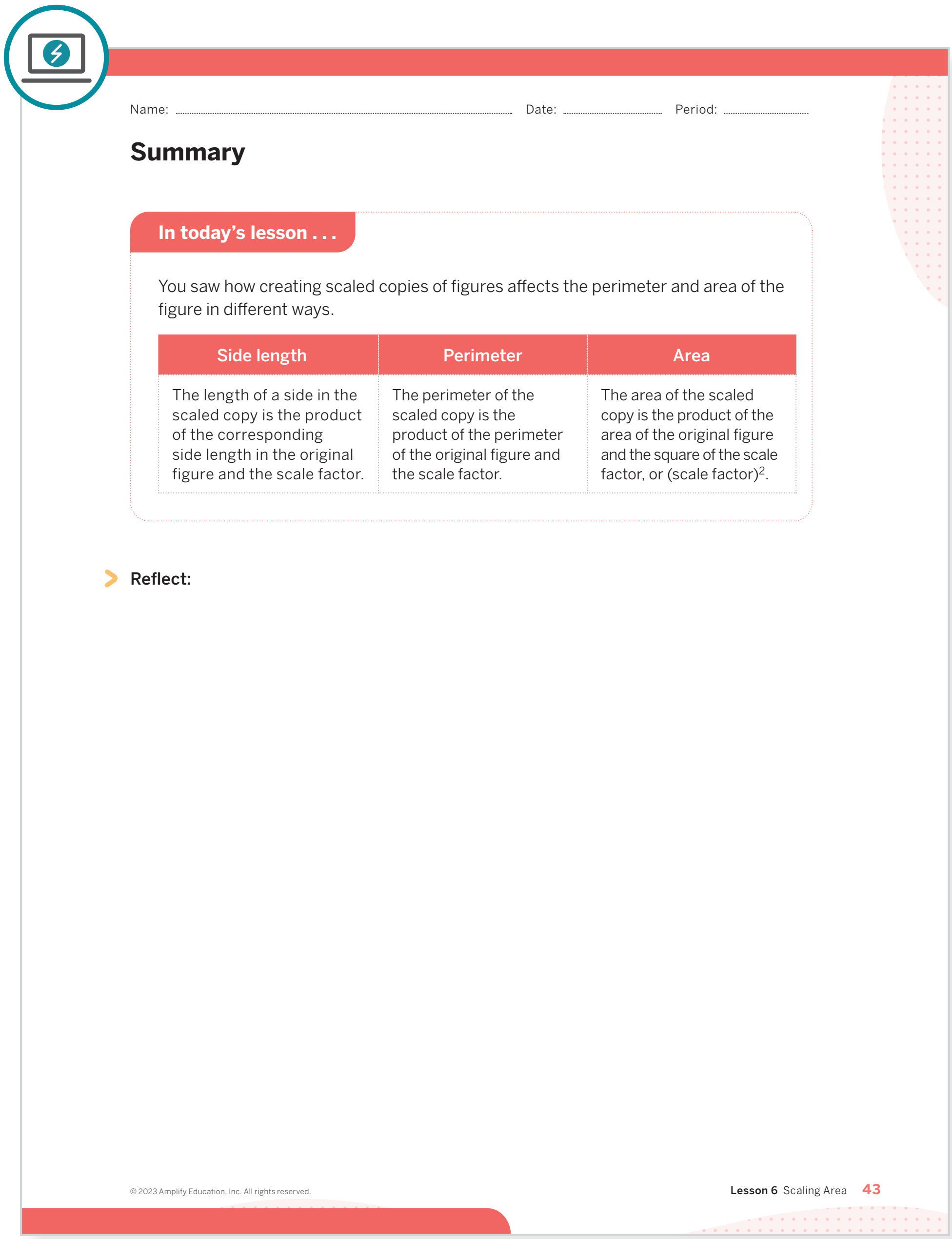
MLR8: Discussion Supports—Revoicing

To foster productive mathematical discussions as students work together to correct and resolve any disagreements, encourage them to revoice their partners' reasoning before correcting and resolving any errors. Use sentence starters, such as "I hear you saying that when the side length of a rectangle is scaled by ____, the perimeter/area is scaled by ____, however, based on my work, I found the perimeter/area to be scaled by ____, because . . ."

Whole Class | 5 min

Summary

Review and synthesize how scale factor affects the perimeter and area of figures.



Name: Date: Period:

Summary

In today's lesson . . .

You saw how creating scaled copies of figures affects the perimeter and area of the figure in different ways.

Side length	Perimeter	Area
The length of a side in the scaled copy is the product of the corresponding side length in the original figure and the scale factor.	The perimeter of the scaled copy is the product of the perimeter of the original figure and the scale factor.	The area of the scaled copy is the product of the area of the original figure and the square of the scale factor, or (scale factor)^2.

Reflect:

Synthesize

Have students share their explanations of how scale factor affects the perimeter and area of a figure.

Highlight that these effects work for all shapes and not just rectangles or triangles.

Ask:

- "If all of the dimensions of a scaled copy are twice as long as the original figure, will the area of the scaled copy also be twice as great? Why or why not?" No; Sample response: Both the length and the width are multiplied by the scale factor 2, so the area is actually multiplied by 4 (the square of the scale factor).

Reflect

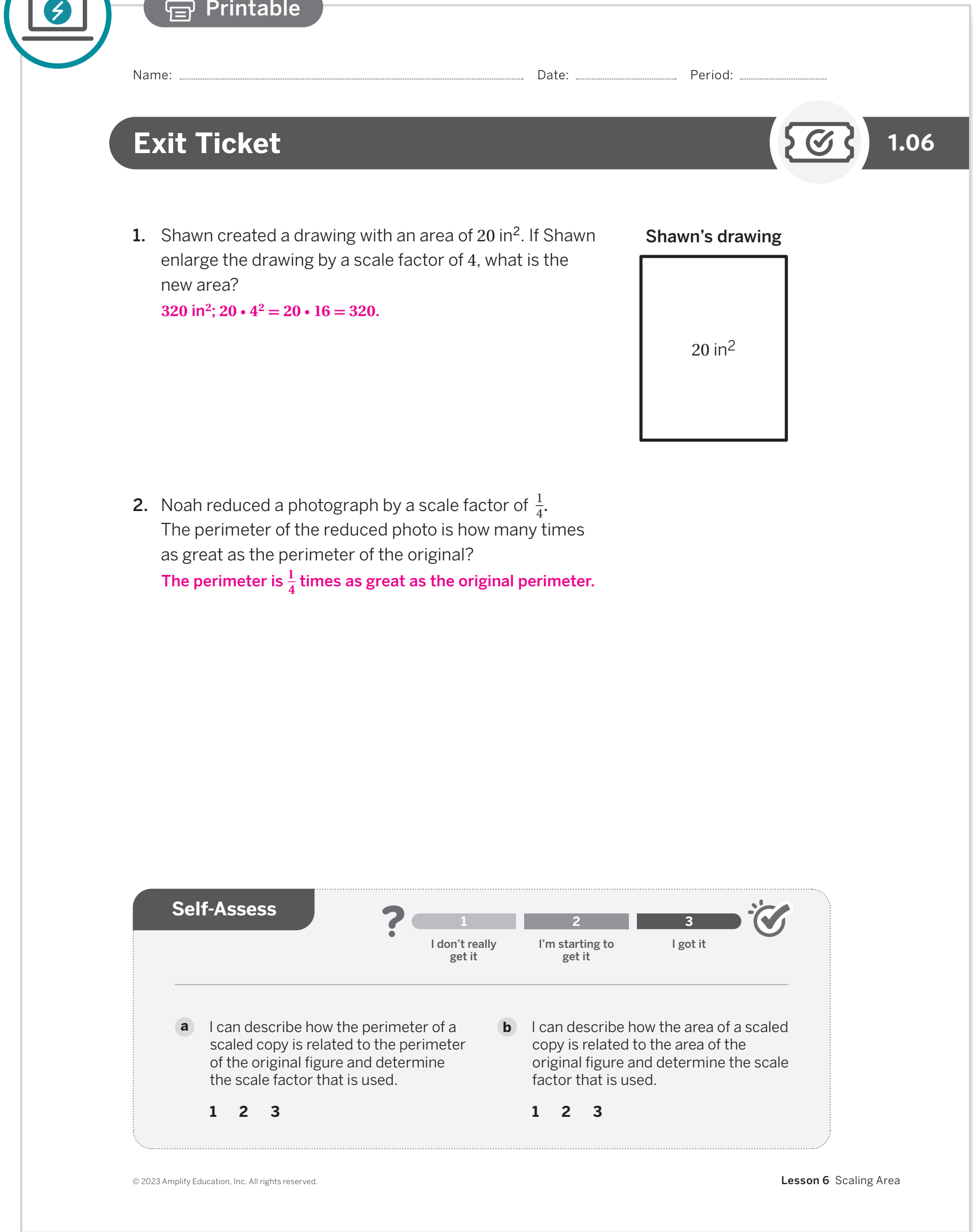
After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why are lengths and areas affected in different ways when creating scaled copies?"

Exit Ticket

Students demonstrate their understanding by calculating the area of a scaled copy and explaining the effects on the perimeter of a scaled copy.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.06

1. Shawn created a drawing with an area of 20 in^2. If Shawn enlarge the drawing by a scale factor of 4, what is the new area?
   320 in^2; $20 \cdot 4^2 = 20 \cdot 16 = 320$.

Shawn's drawing

20 in^2

2. Noah reduced a photograph by a scale factor of $\frac{1}{4}$. The perimeter of the reduced photo is how many times as great as the perimeter of the original?
   The perimeter is $\frac{1}{4}$ times as great as the original perimeter.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe how the perimeter of a scaled copy is related to the perimeter of the original figure and determine the scale factor that is used. 1 2 3

b. I can describe how the area of a scaled copy is related to the area of the original figure and determine the scale factor that is used. 1 2 3

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Lesson 6 Scaling Area

Success looks like . . .

- **Language Goal:** Calculating and comparing the areas of multiple scaled copies of the same shape. **(Speaking and Listening, Writing)**
  - » Calculating the area of the scaled copy of the rectangle in Problem 1.
- **Language Goal:** Generalizing that the area of a scaled copy is the product of the area of the original figure and the square of the scale factor. **(Speaking and Listening)**

Suggested next steps

If students multiply the area by 4 in Problem 1, consider:

- Reviewing Problems 5–9 from Activity 1.
- Assigning Practice Problems 1 and 2.

If students answer Problem 2 incorrectly, consider:

- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on Activity 1?
- What did the Partner Problems reveal about your students as learners? What might you change for the next time you teach this lesson?

Math Language Development

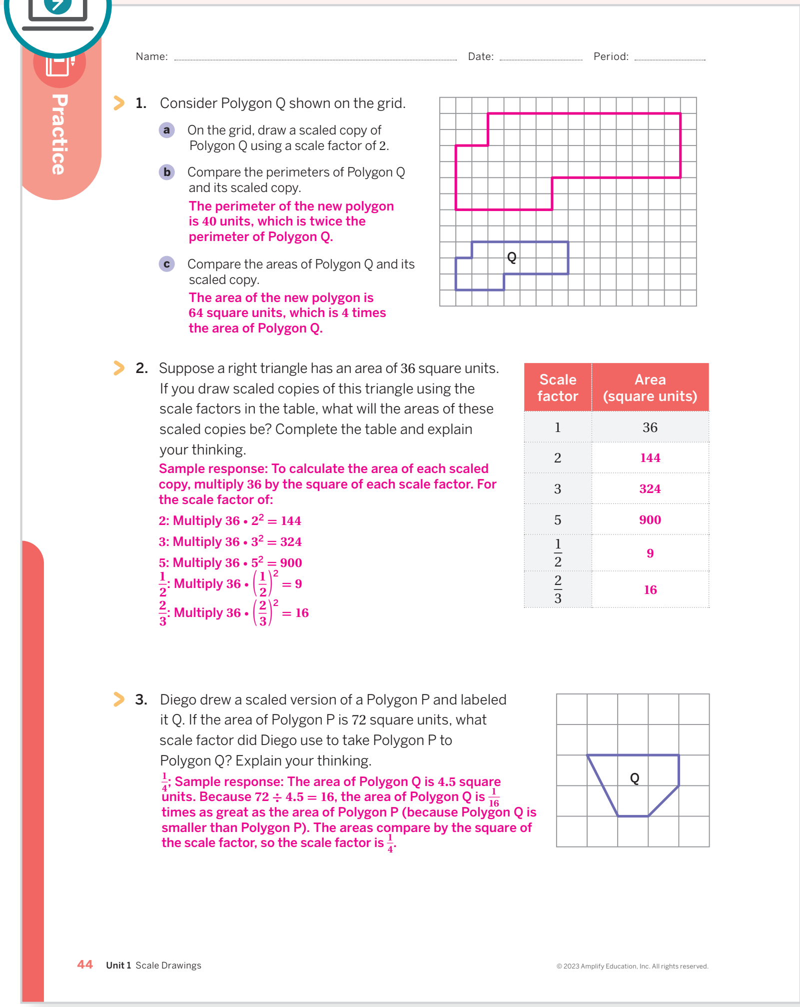
Language Goal: Generalizing that the area of a scaled copy is the product of the area of the original figure and the square of the scale factor.

Reflect on students' language development toward this goal.

- How did using the *Discussion Supports* routines in Activities 1 and 2 help them develop the math language describing how the area of a scaled copy relates to the area of the original figure?
- Did providing them with sentence frames help them develop this language? Would you change anything the next time you use this routine?

Practice

Independent


Practice

Name: Date: Period:

1. Consider Polygon Q shown on the grid.
   - a On the grid, draw a scaled copy of Polygon Q using a scale factor of 2.
   - b Compare the perimeters of Polygon Q and its scaled copy.
     The perimeter of the new polygon is 40 units, which is twice the perimeter of Polygon Q.
   - c Compare the areas of Polygon Q and its scaled copy.
     The area of the new polygon is 64 square units, which is 4 times the area of Polygon Q.

2. Suppose a right triangle has an area of 36 square units. If you draw scaled copies of this triangle using the scale factors in the table, what will the areas of these scaled copies be? Complete the table and explain your thinking.
   Sample response: To calculate the area of each scaled copy, multiply 36 by the square of each scale factor. For the scale factor of:
   2: Multiply $36 \cdot 2^2 = 144$
   3: Multiply $36 \cdot 3^2 = 324$
   5: Multiply $36 \cdot 5^2 = 900$
   $\frac{1}{2}$: Multiply $36 \cdot \left(\frac{1}{2}\right)^2 = 9$
   $\frac{2}{3}$: Multiply $36 \cdot \left(\frac{2}{3}\right)^2 = 16$

Scale factor	Area (square units)
1	36
2	**144**
3	**324**
5	**900**
$\frac{1}{2}$	**9**
$\frac{2}{3}$	**16**

3. Diego drew a scaled version of a Polygon P and labeled it Q. If the area of Polygon P is 72 square units, what scale factor did Diego use to take Polygon P to Polygon Q? Explain your thinking.
   $\frac{1}{4}$; Sample response: The area of Polygon Q is 4.5 square units. Because $72 \div 4.5 = 16$, the area of Polygon Q is $\frac{1}{16}$ times as great as the area of Polygon P (because Polygon Q is smaller than Polygon P). The areas compare by the square of the scale factor, so the scale factor is $\frac{1}{4}$.

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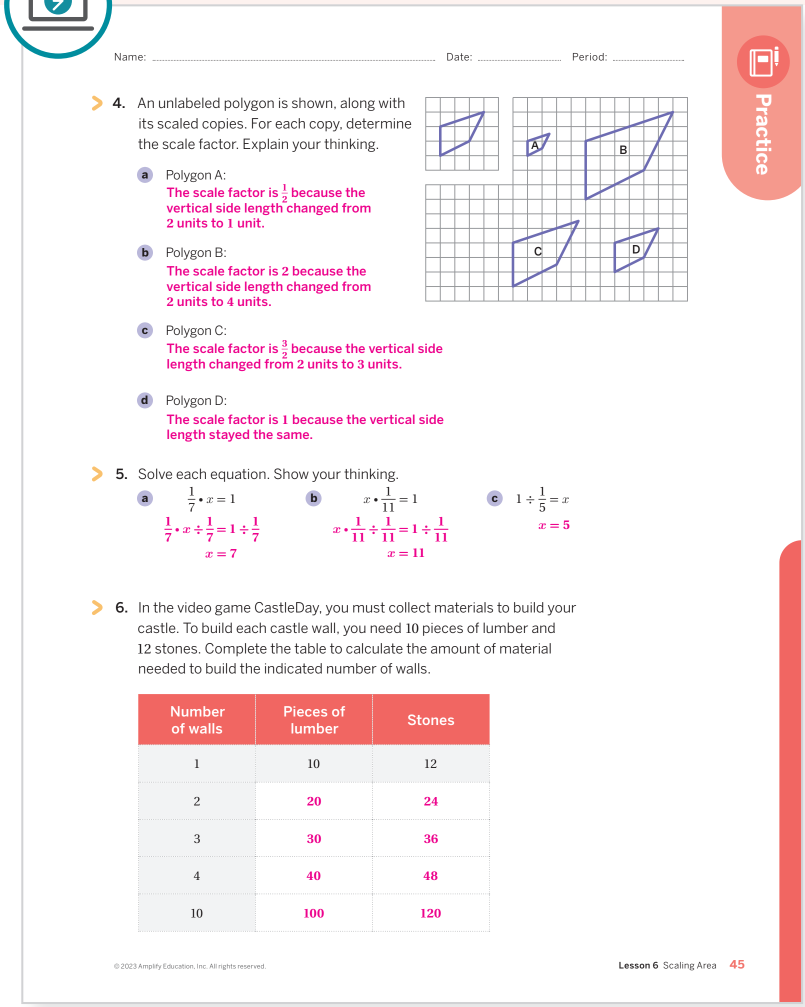

Practice

Name: Date: Period:

4. An unlabeled polygon is shown, along with its scaled copies. For each copy, determine the scale factor. Explain your thinking.
   - a Polygon A:
     The scale factor is $\frac{1}{2}$ because the vertical side length changed from 2 units to 1 unit.
   - b Polygon B:
     The scale factor is 2 because the vertical side length changed from 2 units to 4 units.
   - c Polygon C:
     The scale factor is $\frac{3}{2}$ because the vertical side length changed from 2 units to 3 units.
   - d Polygon D:
     The scale factor is 1 because the vertical side length stayed the same.

5. Solve each equation. Show your thinking.
   - a $\frac{1}{7} \cdot x = 1$
     $\frac{1}{7} \cdot x \div \frac{1}{7} = 1 \div \frac{1}{7}$
     $x = 7$
   - b $x \cdot \frac{1}{11} = 1$
     $x \cdot \frac{1}{11} \div \frac{1}{11} = 1 \div \frac{1}{11}$
     $x = 11$
   - c $1 \div \frac{1}{5} = x$
     $x = 5$

6. In the video game CastleDay, you must collect materials to build your castle. To build each castle wall, you need 10 pieces of lumber and 12 stones. Complete the table to calculate the amount of material needed to build the indicated number of walls.

Number of walls	Pieces of lumber	Stones
1	10	12
2	**20**	**24**
3	**30**	**36**
4	**40**	**48**
10	**100**	**120**

Lesson 6 Scaling Area 45

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	2
	2	Activity 1	2
	3	Activities 1 and 2	2
Spiral	4	Unit 1 Lesson 2	2
	5	Grade 6	1
Formative ⚡	6	Unit 1 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 2
Scale Drawings

Whole Class

In this Sub-Unit, students will learn that scale drawings have long served as a means for comprehending and communicating about the parts of our world that are too small or too large to be seen with our eyes.

SUB-UNIT

2 Scale Drawings

Narrative Connections

Who was the King of Monsters?

Before CGI, there was Haruo Nakajima.

You might not recognize his face, but for years Nakajima played the most recognizable monster in movie history: Godzilla!

Born in 1929 in Yamagata, Japan, Nakajima began his career as a stunt actor in samurai films. When a director noticed the energy Nakajima put into his stunt work, he was chosen to play the 15-story-tall monster.

It was hard work. The movie's special effects team put Nakajima in a suit made from mixed-concrete. Under the hot studio lights, Nakajima had to destroy a miniature scale model of Tokyo that the team had meticulously built.

A single set could take weeks for the team to build. The special effects director was Eiji Tsubaraya. Under his leadership, his team built their models at $\frac{1}{25}$th-to-$\frac{1}{50}$th the scale of their real life counterparts. At these scales, the team's attention to detail made the scenes look realistic. And once the sets were built, it was up to Nakajima to knock them down!

Rather than build a gigantic monster, which would have been costly and impractical, the special effects team used mathematically scaled models. These models helped create the illusion of a very large, very real Godzilla. With Nakajima's fearsome acting, as well as a few camera tricks, the monster would go on to terrify movie-goers across the globe.

Sub-Unit 2 Scale Drawings 47

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students create and analyze scaled drawings—most of which are not quite as massive as Godzilla—in the following places:

- **Lesson 8, Activity 2:** Two Maps of Utah
- **Lesson 11, Activity 1:** Godzilla
- **Lesson 13, Activity 1:** Large- and Small-Scale

UNIT 1 | **LESSON 7**

Scale Drawings

Let's explore scale drawings.

Focus

Goals

1. **Language Goal:** Describe what a "scale drawing" is. **(Speaking and Listening)**
2. **Language Goal:** Explain how to use scales and scale drawings to calculate actual and scaled distances. **(Speaking and Listening, Reading and Writing)**
3. Interpret the scale of a scale drawing.

Coherence

Today

Students begin to study scale drawings, or scaled two-dimensional representations of actual objects or places. Students learn that scale can be expressed in a number of ways, and they use scale, scale drawings, and a variety of geometric tools to calculate actual and scaled lengths. They see that the principles and strategies they used to reason about scaled copies are applicable to scale drawings.

Previously

In Lessons 2 through 6, students explored the idea of scaled copies and the concept of correspondence. They learned that scale factor is a value that describes how lengths in a figure correspond to lengths in a copy of the figure (and vice versa).

Coming Soon

In Lessons 8 and 9, students will create and reproduce scale drawings at specified scales, and will determine appropriate scales to use based on certain constraints.

Rigor

- Students build **conceptual understanding** of scale drawings by comparing examples and non-examples.
- Students **apply** their understanding of scale factor to scale drawings.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	15 min	5 min	10 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, one per student
- geometry toolkits: index cards, rulers

Math Language Development

New words

- *scale*
- *scale drawing*

Review words

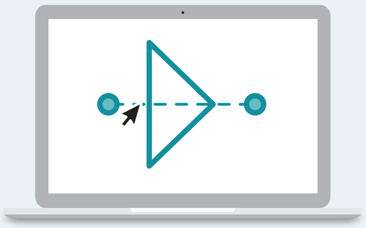
- *scale factor*
- *scaled copy*

Amps | **Featured Activity**

Activity 1

Digital Tape Measure

Students measure the structures from around the world using a scaled digital tape measure.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may get excited about being able to use tools for Activity 1 and just start using them without a plan or strategy. Remind them that they should be strategic about their tool section. Have students summarize rules for using the tools and explain how responsible decision making with respect to the tools can keep everyone safe.

Modifications to Pacing

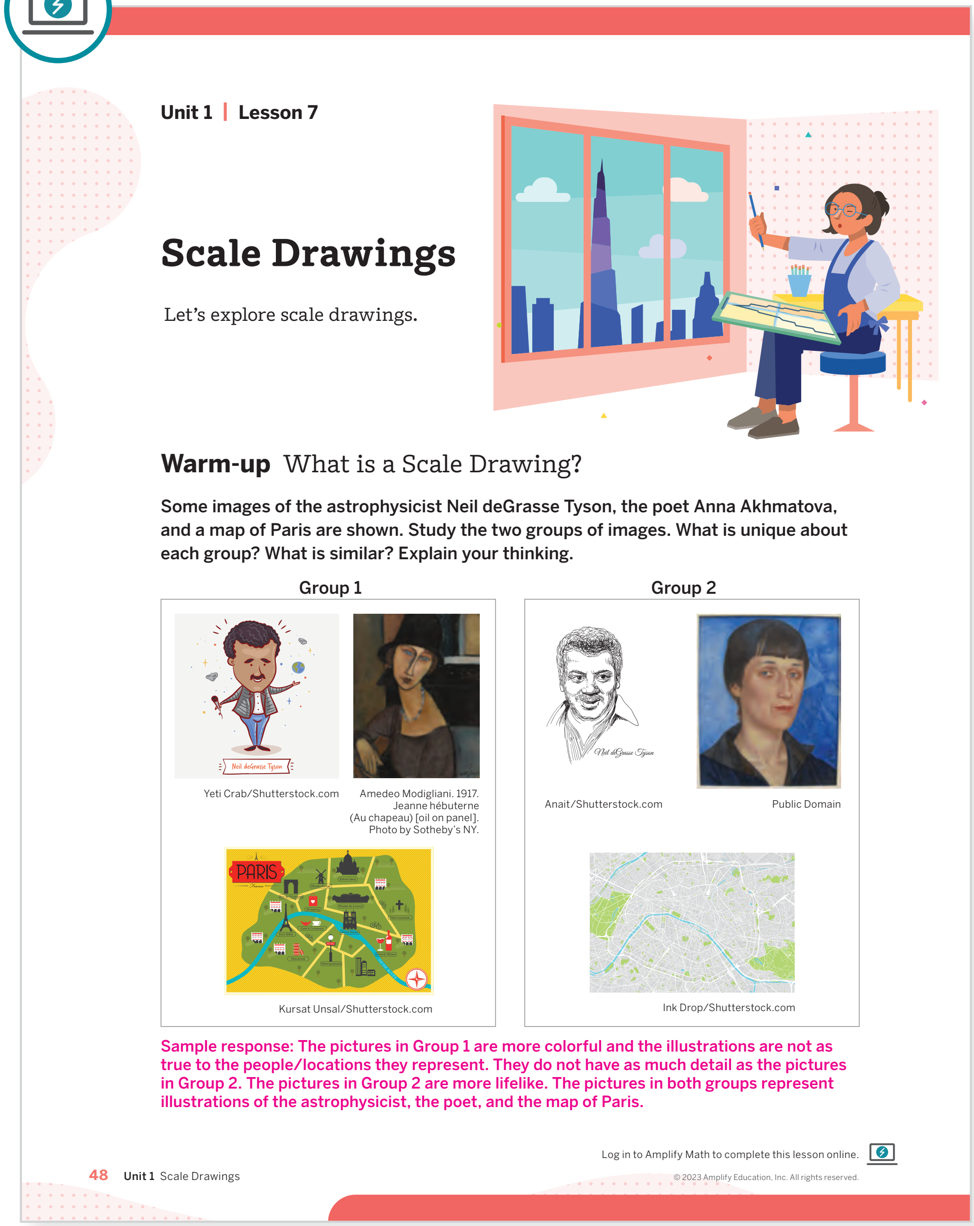
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, have students complete Problems 1 and 2.
- In **Activity 2**, have students complete the first two rows of the table.
- Alternatively, omit Activity 2. Instead, during Activity 1, discuss with students how the written scale "1 cm : 100 m" could be used to determine the actual heights of the buildings.

Pairs | 5 min

Warm-up What is a Scale Drawing?

Students compare examples and non-examples of scale drawings to learn about their characteristics and purpose.



Unit 1 | **Lesson 7**

Scale Drawings

Let's explore scale drawings.

Warm-up What is a Scale Drawing?

Some images of the astrophysicist Neil deGrasse Tyson, the poet Anna Akhmatova, and a map of Paris are shown. Study the two groups of images. What is unique about each group? What is similar? Explain your thinking.

Group 1

Yeti Crab/Shutterstock.com

Amedeo Modigliani. 1917. Jeanne hébuterne (Au chapeau) [oil on panel]. Photo by Sotheby's NY.

Kursat Unsal/Shutterstock.com

Group 2

Anait/Shutterstock.com

Public Domain

Ink Drop/Shutterstock.com

Sample response: The pictures in Group 1 are more colorful and the illustrations are not as true to the people/locations they represent. They do not have as much detail as the pictures in Group 2. The pictures in Group 2 are more lifelike. The pictures in both groups represent illustrations of the astrophysicist, the poet, and the map of Paris.

Log in to Amplify Math to complete this lesson online.

48 **Unit 1** Scale Drawings

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1 Launch

Conduct the *Poll the Class* routine to see if students are familiar with the astrophysicist Neil DeGrasse Tyson or the poet Anna Akhmatova. If time allows, provide a bit of background on each. Let them know they will be looking at and comparing two groups of related images.

2 Monitor

Help students get started by having them compare two matching representations (for example, just the images of Neil deGrasse Tyson) and covering the others.

Look for points of confusion:

- **Thinking only about the artistic merits of each group.** Ask, "Which group is a better representation of the *real* version? Why do you think that?"

3 Connect

Have pairs of students share what is unique and similar about the images in each group.

Highlight that the drawings in Group 2 are called *scale drawings*.

Ask:

- "What type of images would you consider to be *scale drawings*?"
- "How would you define what a *scale drawing* is, in your own words? Discuss with a partner."

Define a ***scale drawing*** as a drawing that represents an actual place, object, or person. All of the measurements in the scale drawing correspond to the measurements of the actual object by the same ***scale***.

Math Language Development

MLR8: Discussion Supports

Display the following sentence frames prior to the discussion:

- "Something that makes this group of pictures unique is . . ."
- "Something this group of pictures has in common is . . ."

English Learners

Allow students to record observations in their primary language first, before participating in the discussion.

Power-up

To power up students' ability to use a ratio table to scale quantities, have students complete:

Using the given ratio of 2:4, determine the missing values in the ratio table.

2	4
3	6
5	10
9.5	19

Use: Before Activity 2.

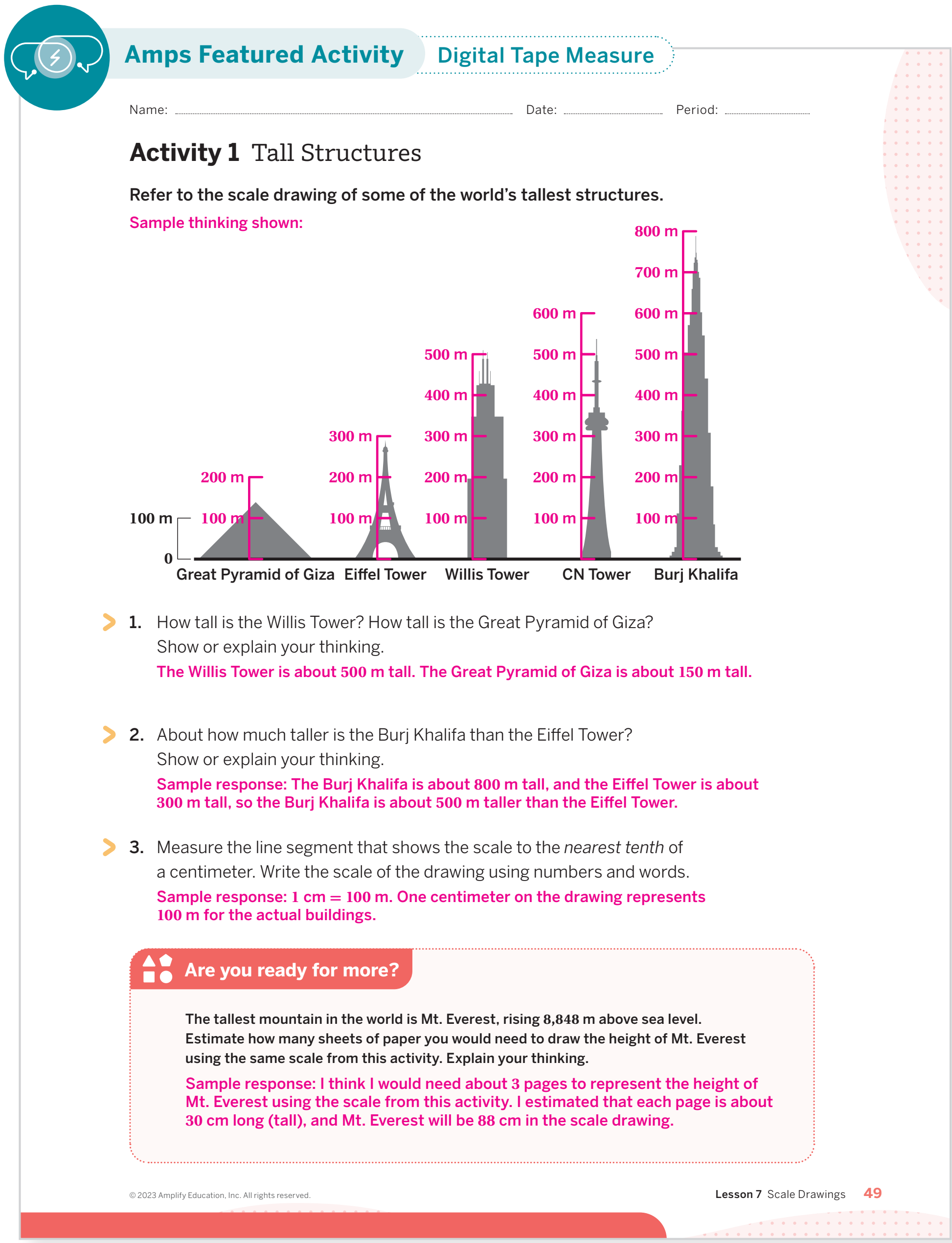
Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7.

Pairs | 10 min

Activity 1 Tall Structures

Students use a scale drawing and select measuring tools to calculate the actual heights of some famous buildings around the world.

Amps Featured Activity Digital Tape Measure



Name: Date: Period:

Activity 1 Tall Structures

Refer to the scale drawing of some of the world's tallest structures.

Sample thinking shown:

1. How tall is the Willis Tower? How tall is the Great Pyramid of Giza? Show or explain your thinking.
   The Willis Tower is about 500 m tall. The Great Pyramid of Giza is about 150 m tall.

2. About how much taller is the Burj Khalifa than the Eiffel Tower? Show or explain your thinking.
   Sample response: The Burj Khalifa is about 800 m tall, and the Eiffel Tower is about 300 m tall, so the Burj Khalifa is about 500 m taller than the Eiffel Tower.

3. Measure the line segment that shows the scale to the *nearest tenth* of a centimeter. Write the scale of the drawing using numbers and words.
   Sample response: 1 cm = 100 m. One centimeter on the drawing represents 100 m for the actual buildings.

Are you ready for more?

The tallest mountain in the world is Mt. Everest, rising 8,848 m above sea level. Estimate how many sheets of paper you would need to draw the height of Mt. Everest using the same scale from this activity. Explain your thinking.

Sample response: I think I would need about 3 pages to represent the height of Mt. Everest using the scale from this activity. I estimated that each page is about 30 cm long (tall), and Mt. Everest will be 88 cm in the scale drawing.

Lesson 7 Scale Drawings 49

1 Launch

Provide access to the geometry toolkits from which students can choose to measure the segments.

2 Monitor

Help students get started by asking them to use the scale to estimate the height of the Great Pyramid.

Look for points of confusion:

- **Not measuring the heights of the structures perpendicular to the base.** Remind students that height is always perpendicular to the base.

Look for productive strategies:

- Using the index card to mark off the length of the segment, then counting the number of lengths to equal the height of each structure.
- Using a ruler to measure the length of the scale segment and each structure, then multiplying or dividing to calculate the actual height.

3 Connect

Display the scale drawing of the structures.

Have pairs of students share their strategies for Problems 1 and 2.

Highlight the different approaches for comparing the heights of the Burj Khalifa and the Eiffel Tower. Prompt students to consider why the different approaches yield the same result.

Ask, "How does this scale segment provide information about the scale factor?" Sample response: It provides a length and indicates what it represents. If I know how many times that segment maps onto the height of one of the objects, I can determine the scale factor.

Define a ***scale*** as a ratio, sometimes drawn as a segment — as shown here — that indicates how the measurements in a scale drawing represent the actual measurements of the object.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2, and only work on Problem 3 as time allows.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they are able to select from several possible tools to assist in measuring the structures.

Accessibility: *Guide Processing and Visualization*

Draw a short, perpendicular segment connecting the top and base of each structure to allow for clearer organization of the measurement.

Math Language Development

MLR3: Critique, Correct, Clarify

Present groups of students with an incorrect solution to Problem 3, such as, "1 cm on the drawing represents 10 m for the actual buildings." As students measure to the *nearest tenth*, this incorrect statement can be used to show how the *nearest tenth* does not refer to 10 m. Students should critique the response, work with a partner to provide the correct response, and clarify their thinking.

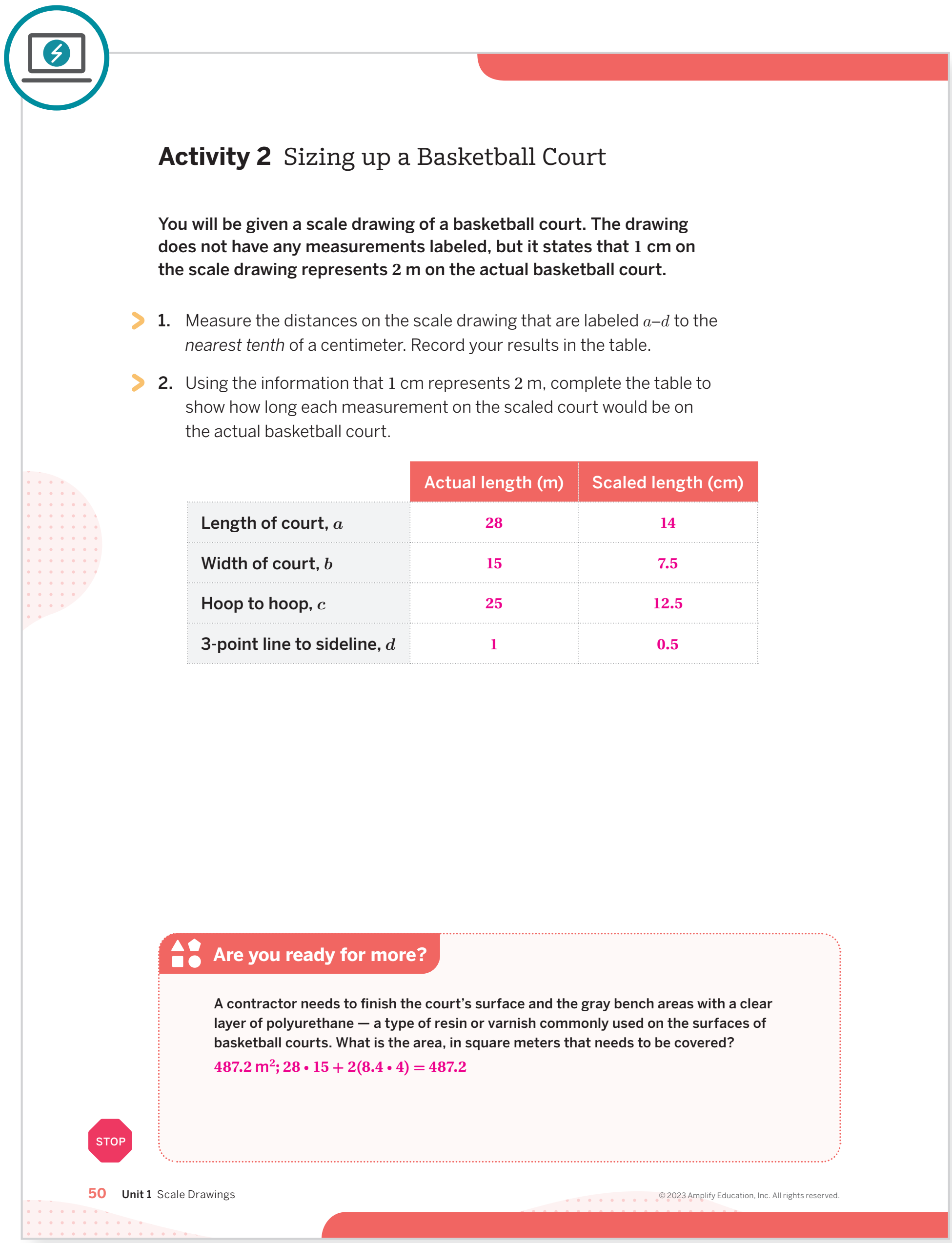
English Learners

Provide visuals and examples of what the *nearest tenth* means and display them for students to reference.

Pairs | 15 min

Activity 2 Sizing up a Basketball Court

Students notice how a written scale communicates the relationship between lengths on a drawing and corresponding lengths in the actual objects they represent.



Activity 2 Sizing up a Basketball Court

You will be given a scale drawing of a basketball court. The drawing does not have any measurements labeled, but it states that 1 cm on the scale drawing represents 2 m on the actual basketball court.

1. Measure the distances on the scale drawing that are labeled a–d to the *nearest tenth* of a centimeter. Record your results in the table.
2. Using the information that 1 cm represents 2 m, complete the table to show how long each measurement on the scaled court would be on the actual basketball court.

	Actual length (m)	Scaled length (cm)
Length of court, a	28	14
Width of court, b	15	7.5
Hoop to hoop, c	25	12.5
3-point line to sideline, d	1	0.5

Are you ready for more?

A contractor needs to finish the court's surface and the gray bench areas with a clear layer of polyurethane — a type of resin or varnish commonly used on the surfaces of basketball courts. What is the area, in square meters that needs to be covered?

$487.2\ \text{m}^2$; $28 \cdot 15 + 2(8.4 \cdot 4) = 487.2$

STOP

50 **Unit 1** Scale Drawings

1 Launch

Activate students' background knowledge by having them visualize a full-sized basketball court. Ask them to estimate how many of their classrooms would fit inside the size of a basketball court. Distribute copies of the Activity 2 PDF and the geometry toolkits.

2 Monitor

Help students get started by asking, "If you measure 1 cm on the drawing, what length does that represent on an actual basketball court?"

Look for points of confusion:

- **Ignoring the difference in units on the scale.** Show a visual representation of 1 cm and 1 m and ask students if their responses are reasonable.

Look for productive strategies:

- Using a ratio table to organize the scaling from the drawing to the actual dimensions.

3 Connect

Display the table, first showing only the scaled lengths, so students can check their measurements (they may have rounded differently).

Ask:

- "Does the scale 1 cm represents 2 m mean that the actual distance is twice that on the drawing?" No, each meter is 100 times the length of a centimeter. We need to take into account the units in the scale.
- "Which parts of the court should be drawn by using the scale 1 cm represents 2 m?" *Every* part, in order for it to be a scale drawing.
- "Can you reverse the order in which you list the scaled and actual distances? For example, can you say '2 m of actual distance to 1 cm on the drawing' or '2 m to 1 cm'?" The first one is clear what is meant, but the second could be considered too vague. It's not clear in the second one whether 2 m means 2 m on the scale drawing or the court.

Highlight that students can use a *scale* on a drawing to understand the scale factor, and vice versa. 2 m is 200 times larger than 1 cm, so the drawing was created using a $\frac{1}{200}$ scale factor.

Differentiated Support

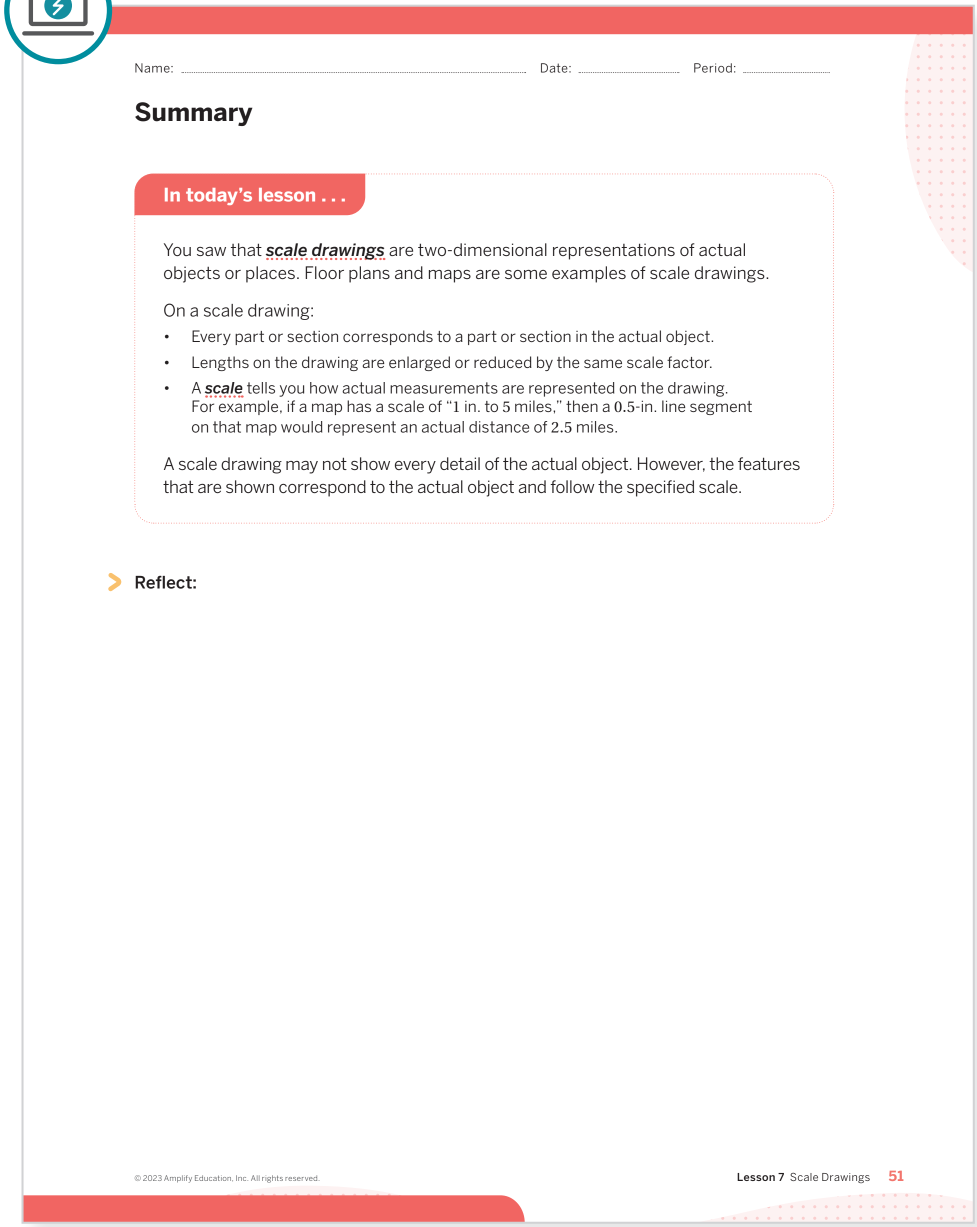
Accessibility: ***Vary Demands to Optimize Challenge***

Provide students with the measurement for the length of the court, a, so they can focus first on scaling. Allow them to measure the other dimensions to the nearest centimeter.

Whole Class | 5 min

Summary

Review and synthesize how to use a scale to determine distances on scale drawings.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that ***scale drawings*** are two-dimensional representations of actual objects or places. Floor plans and maps are some examples of scale drawings.

On a scale drawing:

- Every part or section corresponds to a part or section in the actual object.
- Lengths on the drawing are enlarged or reduced by the same scale factor.
- A ***scale*** tells you how actual measurements are represented on the drawing. For example, if a map has a scale of "1 in. to 5 miles," then a 0.5-in. line segment on that map would represent an actual distance of 2.5 miles.

A scale drawing may not show every detail of the actual object. However, the features that are shown correspond to the actual object and follow the specified scale.

> **Reflect:**

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Lesson 7 Scale Drawings 51

Synthesize

Formalize vocabulary:

- *scale drawing*
- *scale*

Have students share how to use a scale to calculate the actual size of objects represented in a scale drawing.

Highlight that a *scale drawing* is a scaled representation of an object. The *scale* tells students how lengths on the drawing relate to lengths on the actual object. For example, in the basketball court activity, students saw that 1 cm on the drawing represented 2 m of actual distance on the actual basketball court.

Ask:

- "When is it important or useful to use or to create a scale drawing?"
- "When might a scale drawing *not* be very useful or appropriate to use or create?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do scaled models help us make sense of the world around us?"

Math Language Development

MLR2: Collect and Display

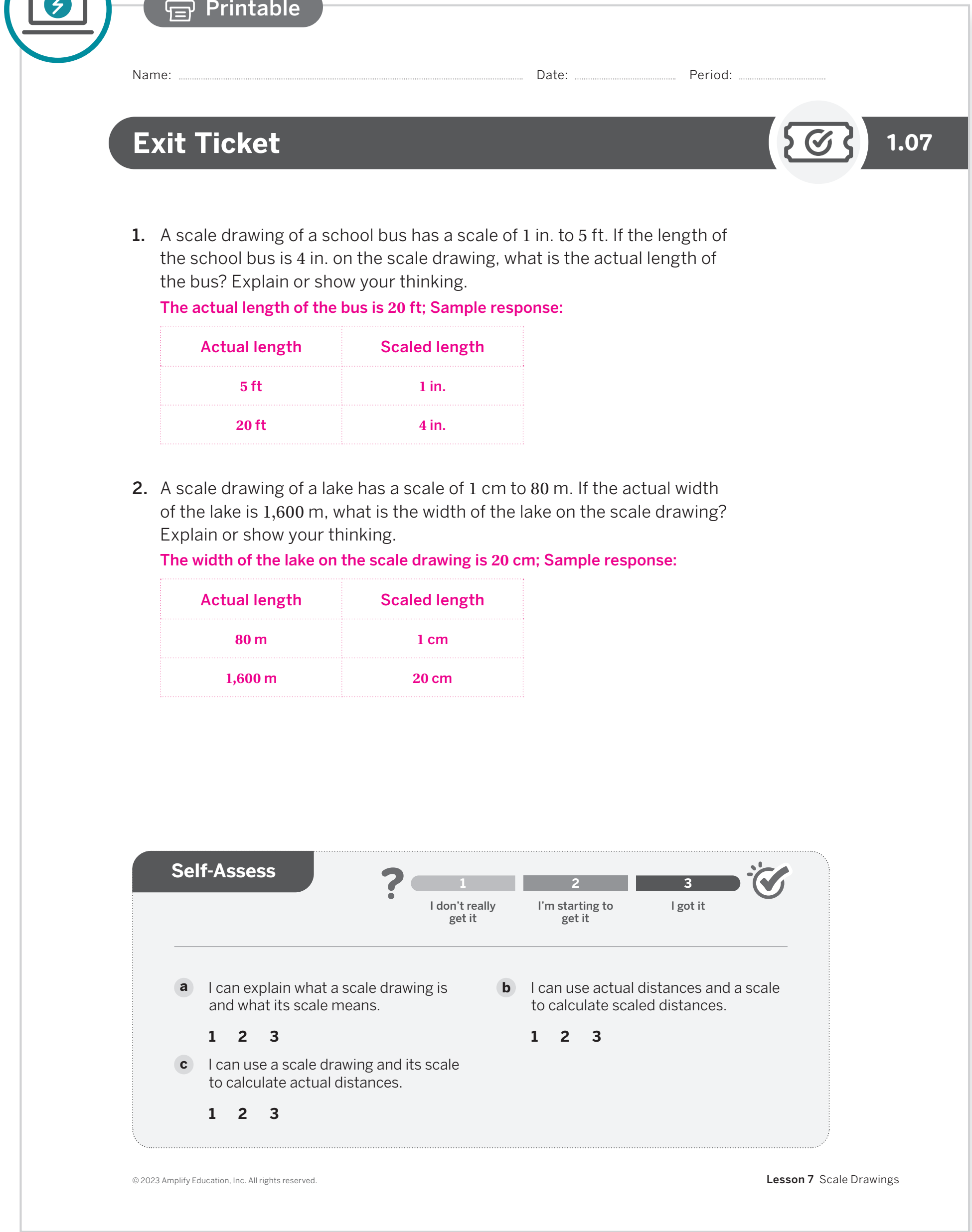
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *scale drawing* and *scale* that were added to the display during the lesson.

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of calculating actual and scaled distances using a scale.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.07

1. A scale drawing of a school bus has a scale of 1 in. to 5 ft. If the length of the school bus is 4 in. on the scale drawing, what is the actual length of the bus? Explain or show your thinking.

The actual length of the bus is 20 ft; Sample response:

Actual length	Scaled length
5 ft	1 in.
20 ft	4 in.

2. A scale drawing of a lake has a scale of 1 cm to 80 m. If the actual width of the lake is 1,600 m, what is the width of the lake on the scale drawing? Explain or show your thinking.

The width of the lake on the scale drawing is 20 cm; Sample response:

Actual length	Scaled length
80 m	1 cm
1,600 m	20 cm

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can explain what a scale drawing is and what its scale means.
1 2 3

b I can use actual distances and a scale to calculate scaled distances.
1 2 3

c I can use a scale drawing and its scale to calculate actual distances.
1 2 3

Lesson 7 Scale Drawings

Success looks like . . .

- **Language Goal:** Describing what a scale drawing is. **(Speaking and Listening)**
- **Language Goal:** Explaining how to use scales and scale drawings to calculate actual and scaled distances. **(Speaking and Listening, Reading and Writing)**
  - » Explaining how the actual length and scaled width were determined in Problems 1 and 2.
- **Goal:** Interpreting the scale of a scale drawing.

Suggested next steps

If students ignore the units in the scale, consider:

- Drawing a diagram comparing the different lengths of 1 in. compared to 1 ft.

If students use addition for scaling, rather than multiplication, consider:

- Referring them back to Activity 2 and asking how they scaled the values in that activity.

If students compare the wrong quantities to find the scale factor, consider:

- Suggesting they use a table and label both the columns and the rows.

Professional Learning

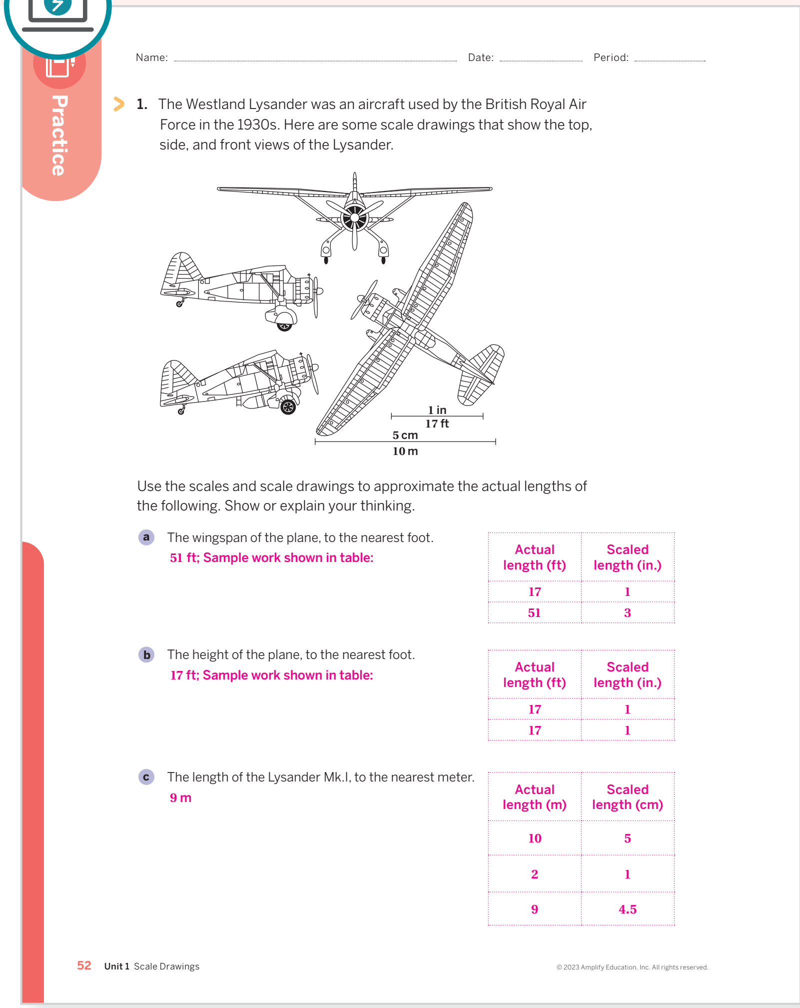
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What are some other ways you can determine which students' strategies to share during the Connect section of each activity?
- What did students find frustrating about determining the actual lengths of the basketball court? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. The Westland Lysander was an aircraft used by the British Royal Air Force in the 1930s. Here are some scale drawings that show the top, side, and front views of the Lysander.

Use the scales and scale drawings to approximate the actual lengths of the following. Show or explain your thinking.

a. The wingspan of the plane, to the nearest foot.
51 ft; Sample work shown in table:

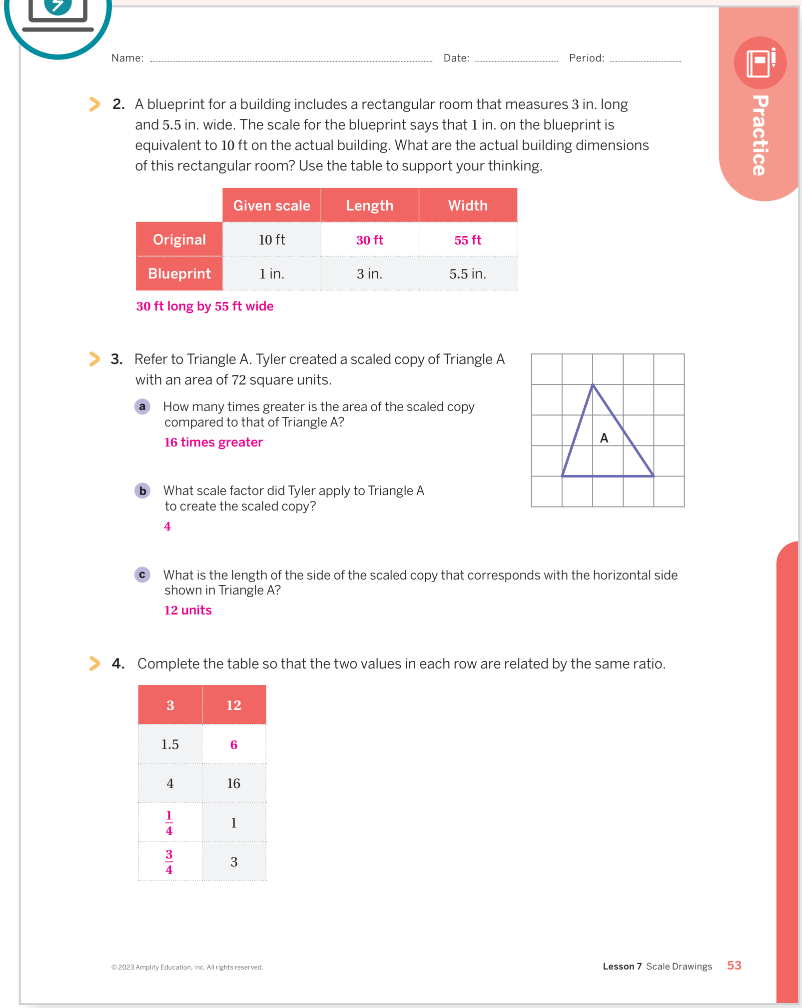
Actual length (ft)	Scaled length (in.)
17	1
51	3

b. The height of the plane, to the nearest foot.
17 ft; Sample work shown in table:

Actual length (ft)	Scaled length (in.)
17	1
17	1

c. The length of the Lysander Mk.I, to the nearest meter.
9 m

Actual length (m)	Scaled length (cm)
10	5
2	1
9	4.5



Name: Date: Period:

2. A blueprint for a building includes a rectangular room that measures 3 in. long and 5.5 in. wide. The scale for the blueprint says that 1 in. on the blueprint is equivalent to 10 ft on the actual building. What are the actual building dimensions of this rectangular room? Use the table to support your thinking.

	Given scale	Length	Width
Original	10 ft	30 ft	55 ft
Blueprint	1 in.	3 in.	5.5 in.

30 ft long by 55 ft wide

3. Refer to Triangle A. Tyler created a scaled copy of Triangle A with an area of 72 square units.

a. How many times greater is the area of the scaled copy compared to that of Triangle A?
16 times greater

b. What scale factor did Tyler apply to Triangle A to create the scaled copy?
4

c. What is the length of the side of the scaled copy that corresponds with the horizontal side shown in Triangle A?
12 units

4. Complete the table so that the two values in each row are related by the same ratio.

3	12
1.5	6
4	16
$\frac{1}{4}$	1
$\frac{3}{4}$	3

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 1 Lesson 6	2
Formative	4	Unit 1 Lesson 8	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

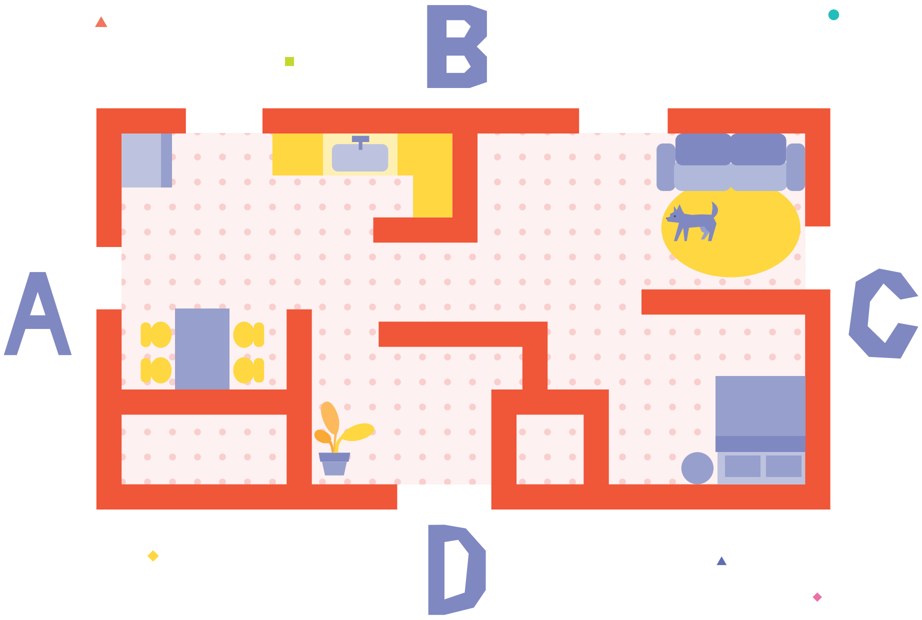
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 1 | **LESSON 8**

Creating Scale Drawings

Let's create our own scale drawings.



Focus

Goals

1. **Language Goal:** Compare and contrast different scale drawings of the same object, and describe how the scale affects the size of the drawing. **(Speaking and Listening)**
2. Create a scale drawing, given the actual dimensions of the object and the scale.
3. **Language Goal:** Determine the scale used to create a scale drawing and generate multiple ways to express it. **(Writing)**

Coherence

Today

This is the first lesson in which students use the actual distance to calculate the scaled distance and create their own scale drawings. They see how different scale drawings can be created of the same actual object, using different scales. Noticing how a scale drawing changes with the choice of scale develops important structural understanding of scale drawings.

Previously

In Lesson 7, students used scale drawings to calculate actual distances or lengths.

Coming Soon

In Lesson 9, students will use their understanding of scale drawings to solve real-world problems involving maps and speed.

Rigor

- Students build **conceptual understanding** of scaling by using both the scale and the ratio between actual lengths and scaled lengths.
- Students **apply** scaling by producing multiple scaled copies at different scales.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per pair (as needed)
- Activity 2 PDF (for display)
- markers or highlighters (optional)
- sheet protectors and dry erase markers (optional)

Math Language Development

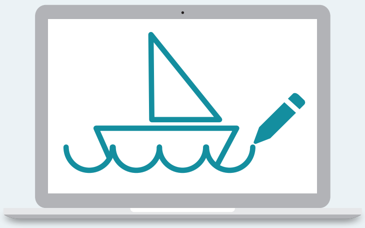
Review words

- *scale*
- *scale drawing*

Amps ⋮ **Featured Activity**

Activity 2
Overlay Scaled Maps

Students draw multiple scaled copies of maps. When you overlay the results, you can display student work.



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Building Math Identity and Community

Connecting to Mathematical Practices

As students begin Activity 2, they may be unsure how to approach a scale drawing set in this real-world context. Remind them that the concept of scale drawings is the same whether it is a purely mathematical context or set in a real-world context. They can still use the structure of the figures representing the map of Utah in the same way they analyzed the structure of figures without a real-world context.

Modifications to Pacing

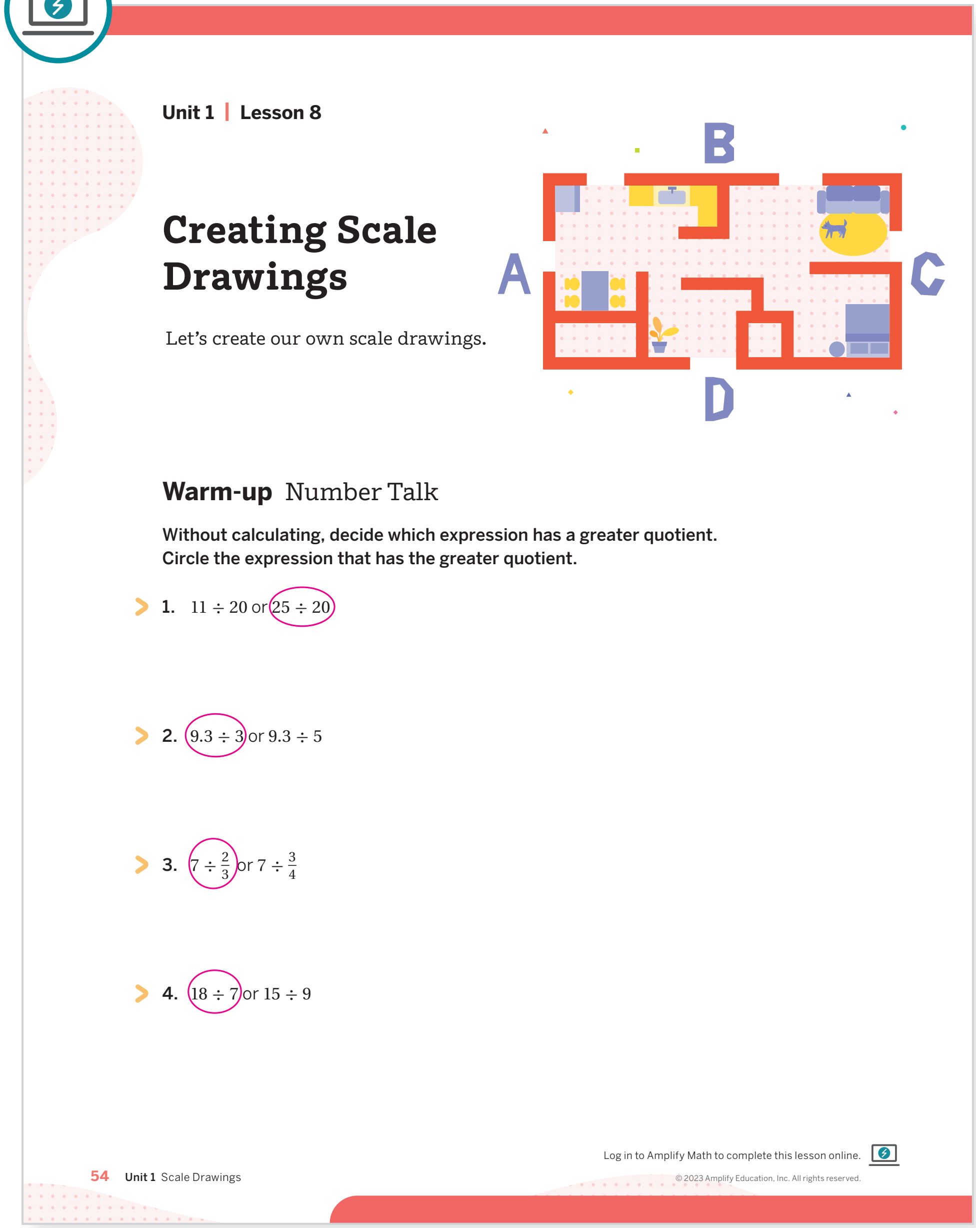
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, divide problems among students in the class and then have everyone share their responses.
- In **Activity 2**, omit Problems 3-5. Instead, prepare and display the scale drawing for Problem 4 for students to refer to during the discussion.

Independent | 5 min

Warm-up Number Talk

Students use the structure of division expressions to mentally compare the sizes of the quotients, preparing them to use similar reasoning when working with scales.



Unit 1 | Lesson 8

Creating Scale Drawings

Let's create our own scale drawings.

Warm-up Number Talk

Without calculating, decide which expression has a greater quotient. Circle the expression that has the greater quotient.

1. $11 \div 20$ or [✓ $25 \div 20$]
2. [✓ $9.3 \div 3$] or $9.3 \div 5$
3. [✓ $7 \div \frac{2}{3}$] or $7 \div \frac{3}{4}$
4. [✓ $18 \div 7$] or $15 \div 9$

Log in to Amplify Math to complete this lesson online.

54 **Unit 1** Scale Drawings

1 Launch

Activate prior knowledge of comparing dividends and emphasize to students that they should use mental math to decide which quotient is greater, without performing the calculations. Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by asking what they notice about the structure of each pair of expressions.

Look for points of confusion:

- **Thinking they need to find the precise quotient.** Remind students that they only need to determine which quotient is greater. The structure of the expressions are sufficient to determine this.
- **Getting stuck on Problem 4 because neither the dividends nor the divisors are the same.** Ask students if they can use estimation or other strategies to help them determine which quotient is greater.

Look for productive strategies:

- Making use of the structure of each pair of expressions by realizing that if the divisors are the same, the expression with the greater dividend will have a greater quotient and if the dividends are the same, the expression with the smaller divisor will have the greater quotient.

3 Connect

Display the expressions.

Have students share their thinking for how they determined which quotient is larger.

Highlight different approaches for comparing quantities that avoid direct calculations.

Math Language Development

MLR8: Discussion Supports

During the Connect discussion, display sentence frames to support students in explaining their strategies. For example, "First, I ___ because . . ." or "I noticed ___, so I . . ."

English Learners

Students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class. Refer students to the class displays that may help them.

Power-up

To power up students' ability to use a ratio table to scale non-integer quantities, have students complete:

Determine the unknown values in the table, such that each ratio is equivalent to 2 : 3.

2	3
$\frac{1}{2}$	$\frac{3}{4}$
$\frac{2}{9}$	$\frac{1}{3}$
1	1.5

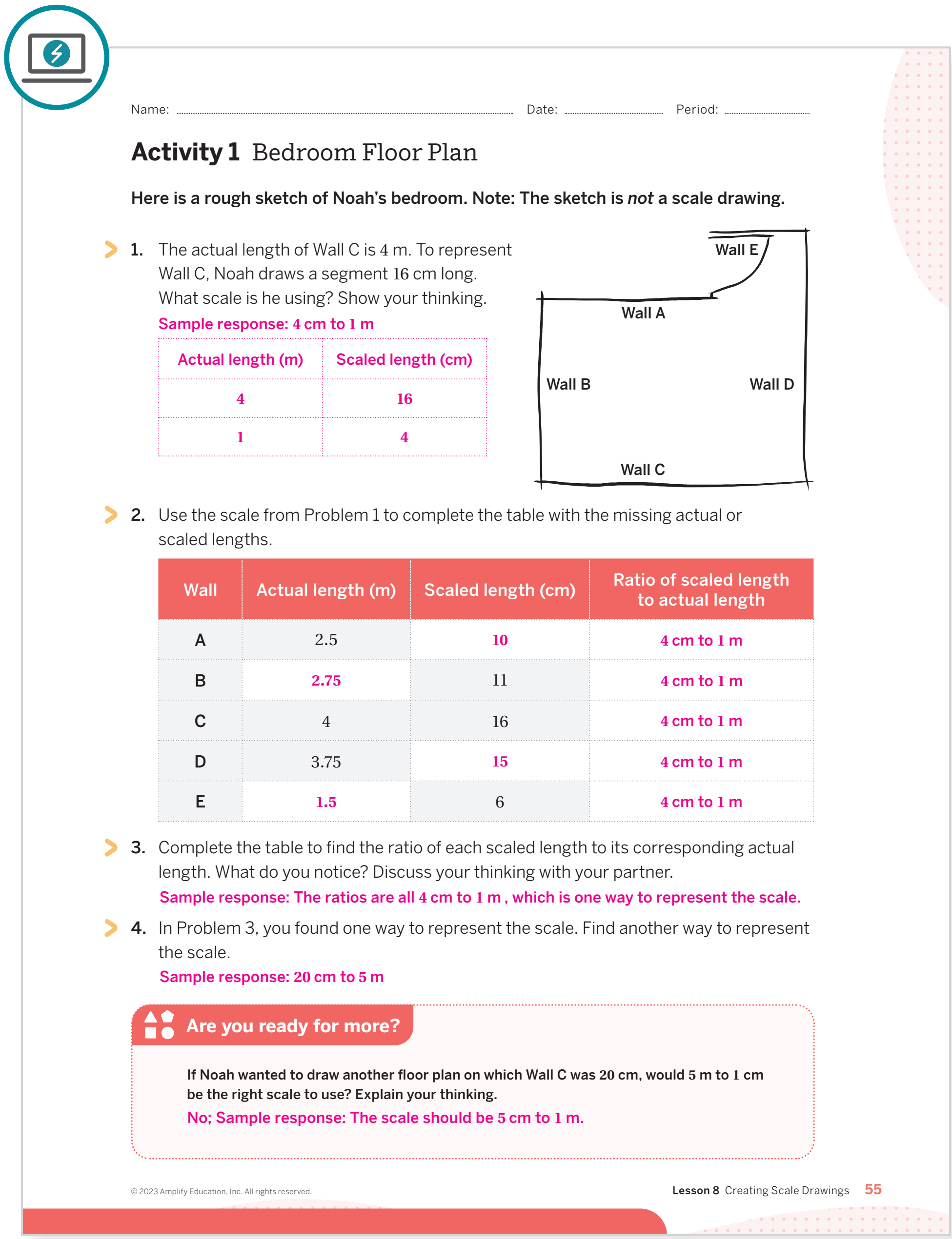
Use: Before Activity 1.

Informed by: Performance on Lesson 7, Practice Problem 4.

Pairs | 15 min

Activity 1 Bedroom Floor Plan

Students use a scale to calculate actual and scaled lengths, leading them to discover these quantities are all related by the same ratio — which is one way to represent the scale.



Name: ………… Date: ………… Period: …………

Activity 1 Bedroom Floor Plan

Here is a rough sketch of Noah's bedroom. Note: The sketch is *not* a scale drawing.

1. The actual length of Wall C is 4 m. To represent Wall C, Noah draws a segment 16 cm long. What scale is he using? Show your thinking.
   Sample response: 4 cm to 1 m

Actual length (m)	Scaled length (cm)
4	16
1	4

2. Use the scale from Problem 1 to complete the table with the missing actual or scaled lengths.

Wall	Actual length (m)	Scaled length (cm)	Ratio of scaled length to actual length
A	2.5	10	4 cm to 1 m
B	2.75	11	4 cm to 1 m
C	4	16	4 cm to 1 m
D	3.75	15	4 cm to 1 m
E	1.5	6	4 cm to 1 m

3. Complete the table to find the ratio of each scaled length to its corresponding actual length. What do you notice? Discuss your thinking with your partner.
   Sample response: The ratios are all 4 cm to 1 m , which is one way to represent the scale.

4. In Problem 3, you found one way to represent the scale. Find another way to represent the scale.
   Sample response: 20 cm to 5 m

Are you ready for more?

If Noah wanted to draw another floor plan on which Wall C was 20 cm, would 5 m to 1 cm be the right scale to use? Explain your thinking.

No; Sample response: The scale should be 5 cm to 1 m.

1 Launch

Activate background knowledge by asking students what they know about floor plans. Explain that floor plans are usually scale drawings.

2 Monitor

Help students get started by having them label the known lengths on the diagram using two colors: one color to represent the scaled lengths and another to represent the actual lengths.

Look for points of confusion:

- **Writing the scale as "1 cm to 4 m in Problem 1."** Prompt students to pay attention to the units and the meaning of each number in the scale.
- **Saying that the scaled and actual lengths are related by a scale factor of 4.** Ask: "Are the actual lengths 4 times the lengths on the drawing?" Point out that because the units for the two quantities are different, multiplying a scaled length in centimeters (e.g., 2.5 cm) by 4 will yield another length in centimeters (e.g., 10 cm), which is not the actual length. **Note:** It is not essential that students know that the scale factor here is actually 250 (1,000 cm to 4 cm).

3 Connect

Have students share how they determined the scale of the drawing, e.g. 4 cm to 1 m, 1 cm to 0.25 m, 16 cm to 4 m , etc. Discuss how all of these express the same relationship, and are therefore equivalent.

Highlight different ways to express the same scale. Although scales can be expressed in multiple, yet equivalent ways, they are often simplified to show 1 scaled unit for the corresponding actual distance. It is common to express the scaled distance as a whole number, a benchmark fraction (e.g., $\frac{1}{2}, \frac{1}{4}$) or a benchmark decimal (e.g., 0.25, 0.5, 0.75). The scaled and actual lengths are all related by the same ratio.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a larger copy of the room plan (Activity 1 PDF) where they can write the actual lengths on the copy. They can use the image in their Student Edition for the scaled version. If possible, place the larger copy in a sheet protector and provide students with dry erase markers so these can be reused across multiple classes.

Math Language Development

MLR7: Compare and Connect

Look for students who expressed the scale in different ways. Ask them to share what is especially clear about a particular approach. Then encourage them to explain why there are different, yet equivalent ways to express the scale, such as 1 m to 4 cm and 0.25 m to 1 cm.

English Learners

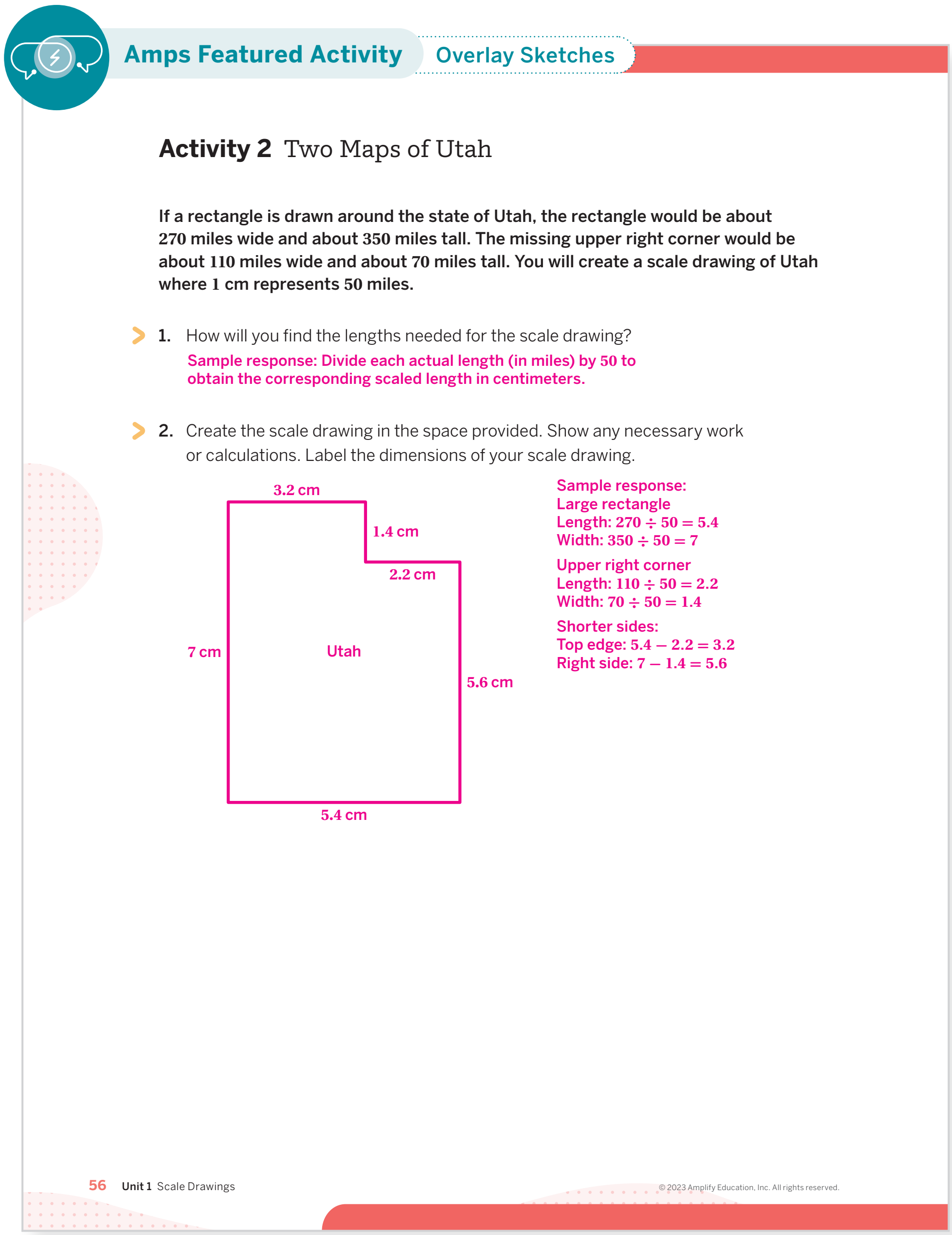
Have students record the different, equivalent scales used and annotate them as "equivalent." Students may benefit from a review of what this term means.

Pairs | 15 min

Activity 2 Two Maps of Utah

Students reproduce a scale drawing at a different scale to realize how the scale affects the size of the scale drawing.

Amps Featured Activity Overlay Sketches



Activity 2 Two Maps of Utah

If a rectangle is drawn around the state of Utah, the rectangle would be about 270 miles wide and about 350 miles tall. The missing upper right corner would be about 110 miles wide and about 70 miles tall. You will create a scale drawing of Utah where 1 cm represents 50 miles.

1. How will you find the lengths needed for the scale drawing?
   Sample response: Divide each actual length (in miles) by 50 to obtain the corresponding scaled length in centimeters.

2. Create the scale drawing in the space provided. Show any necessary work or calculations. Label the dimensions of your scale drawing.

Sample response:
Large rectangle
Length: 270 ÷ 50 = 5.4
Width: 350 ÷ 50 = 7

Upper right corner
Length: 110 ÷ 50 = 2.2
Width: 70 ÷ 50 = 1.4

Shorter sides:
Top edge: 5.4 − 2.2 = 3.2
Right side: 7 − 1.4 = 5.6

56 **Unit 1** Scale Drawings

1 Launch

Display the Activity 2 PDF while engaging students in a discussion regarding their background knowledge of Utah and why its borders are straight. The borders of Utah are intended to run parallel to the latitude and longitude lines.

2 Monitor

Help students get started by asking what operation can be performed on each actual distance given for the state of Utah to scale the state correctly.

Look for points of confusion:

- **Thinking a scale of 1 cm to 50 miles will produce a smaller scale drawing than a scale of 1 cm to 75 miles because 50 is less than 75 (Problem 5).** Have them compare the two drawings and ask how many centimeters it takes to represent 150 miles in each scale.

Look for productive strategies:

- Noticing the scale drawing at a scale of 1 cm to 75 miles is a scaled copy of the other drawing, with a scale factor of 1.5. Ask them to share their observations between scale drawings and scaled copies.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Activity 2 PDF in a sheet protector and access to dry erase markers. Have them mark the dimensions of Utah. The sheet protector allows you to reuse the maps for multiple classes. Alternatively, provide hard copies of the PDF and allow students to write directly on them with markers or highlighters.

Extension: *Math Enrichment*

Have students find the areas of all three maps of Utah and compare them.

Math Language Development

MLR3: Critique, Correct, Clarify

Present an incorrect drawing. Ask students to identify the error, critique the reasoning, and correct the statement so that the drawing is a correct scale drawing of Utah. Then have them clarify their thinking by describing why their drawing is a correct scale drawing.

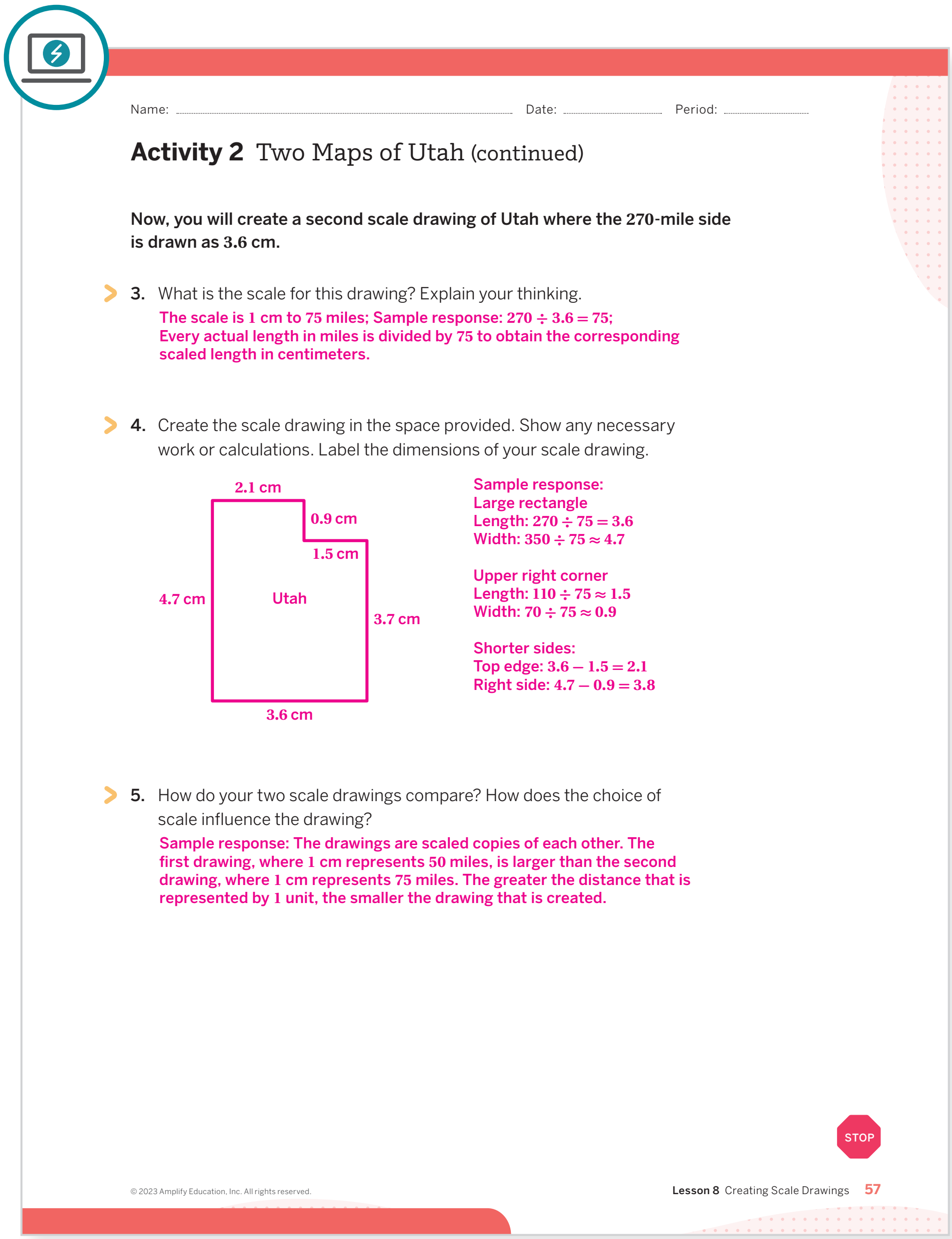
English Learners

Have students annotate the scale used in Problem 3 as “greater distance.” Then have them annotate the drawing in Problem 4 as “smaller drawing.” Help them make connections between the scale used and the size of the drawing.

Pairs | 15 min

Activity 2 Two Maps of Utah (continued)

Students reproduce a scale drawing at a different scale to realize how the scale affects the size of the scale drawing.



Name: ______ Date: ______ Period: ______

Activity 2 Two Maps of Utah (continued)

Now, you will create a second scale drawing of Utah where the 270-mile side is drawn as 3.6 cm.

3. What is the scale for this drawing? Explain your thinking.

The scale is 1 cm to 75 miles; Sample response: $270 \div 3.6 = 75$; Every actual length in miles is divided by 75 to obtain the corresponding scaled length in centimeters.

4. Create the scale drawing in the space provided. Show any necessary work or calculations. Label the dimensions of your scale drawing.

2.1 cm
0.9 cm
1.5 cm
4.7 cm
Utah
3.7 cm
3.6 cm

Sample response:
Large rectangle
Length: $270 \div 75 = 3.6$
Width: $350 \div 75 \approx 4.7$

Upper right corner
Length: $110 \div 75 \approx 1.5$
Width: $70 \div 75 \approx 0.9$

Shorter sides:
Top edge: $3.6 - 1.5 = 2.1$
Right side: $4.7 - 0.9 = 3.8$

5. How do your two scale drawings compare? How does the choice of scale influence the drawing?

Sample response: The drawings are scaled copies of each other. The first drawing, where 1 cm represents 50 miles, is larger than the second drawing, where 1 cm represents 75 miles. The greater the distance that is represented by 1 unit, the smaller the drawing that is created.

STOP

③ Connect

Display the scale drawings students created.

Have students share how they calculated the scaled distances for each scale drawing.

Highlight the differences in the scales and how the scale using 1 cm to represent a larger distance (75 miles) created a smaller drawing.

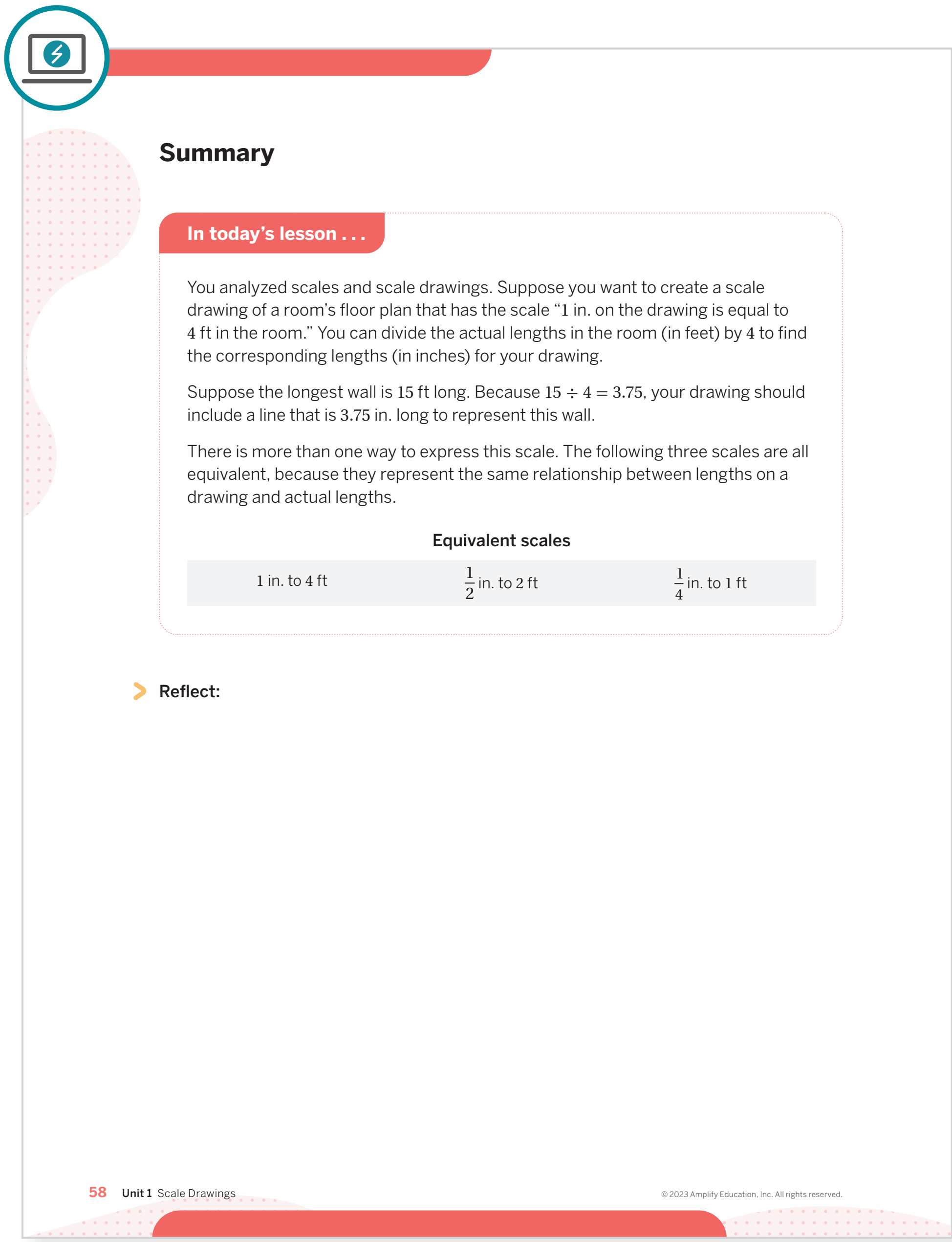
Ask:

- "What do the two scale drawings have in common?" Sample response: They both represent Utah, have the same shape, and can be used to measure actual distances for the state of Utah.
- "How do the two scale drawings differ?" The scale drawing that uses a scale of 1 cm to 50 miles is larger than the scale draw that uses a scale of 1 cm to 75 miles.
- "How can you tell if a scaled copy will be smaller or larger than another scaled copy?" The greater the distance being represented by 1 unit, the smaller the drawing.
- "When thinking about making maps, why is it important to choose a good scale?" Sample response: A scale that produces a large scale drawing (a large map) would be cumbersome for a person to use. A scale producing a smaller scale drawing (a smaller map) will be somewhat limited in the amount of detail shown.

Whole Class | 5 min

Summary

Review and synthesize how different scales affect the size of a scale drawing.



Summary

In today's lesson . . .

You analyzed scales and scale drawings. Suppose you want to create a scale drawing of a room's floor plan that has the scale "1 in. on the drawing is equal to 4 ft in the room." You can divide the actual lengths in the room (in feet) by 4 to find the corresponding lengths (in inches) for your drawing.

Suppose the longest wall is 15 ft long. Because $15 \div 4 = 3.75$, your drawing should include a line that is 3.75 in. long to represent this wall.

There is more than one way to express this scale. The following three scales are all equivalent, because they represent the same relationship between lengths on a drawing and actual lengths.

Equivalent scales

1 in. to 4 ft	$\frac{1}{2}$ in. to 2 ft	$\frac{1}{4}$ in. to 1 ft

Reflect:

58 **Unit 1** Scale Drawings

Synthesize

Ask:

- "Suppose there are two scale drawings of the same house. One uses the scale of 1 cm to 2 m, and the other uses the scale 1 cm to 4 m. Which scale drawing is larger? Why?" The one with the 1 cm to 2 m scale is larger, because it takes 2 cm on the drawing to represent 4 m of actual length.
- "Another scale drawing of the house uses the scale of 5 cm to 10 m. How does its size compare to the other two?" It is the same size as the drawing with the 1 cm to 2 m scale.

Highlight that different-sized scale drawings can represent the same actual object, but the size of the actual object does not change. Sometimes, two different scales are equivalent, such as 5 cm to 10 m and 1 cm to 2 m. It is common to write a scale so that it indicates what 1 unit on the scale drawing represents, for example, 1 cm to 2 m, instead of the scale 2 cm to 4 m.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

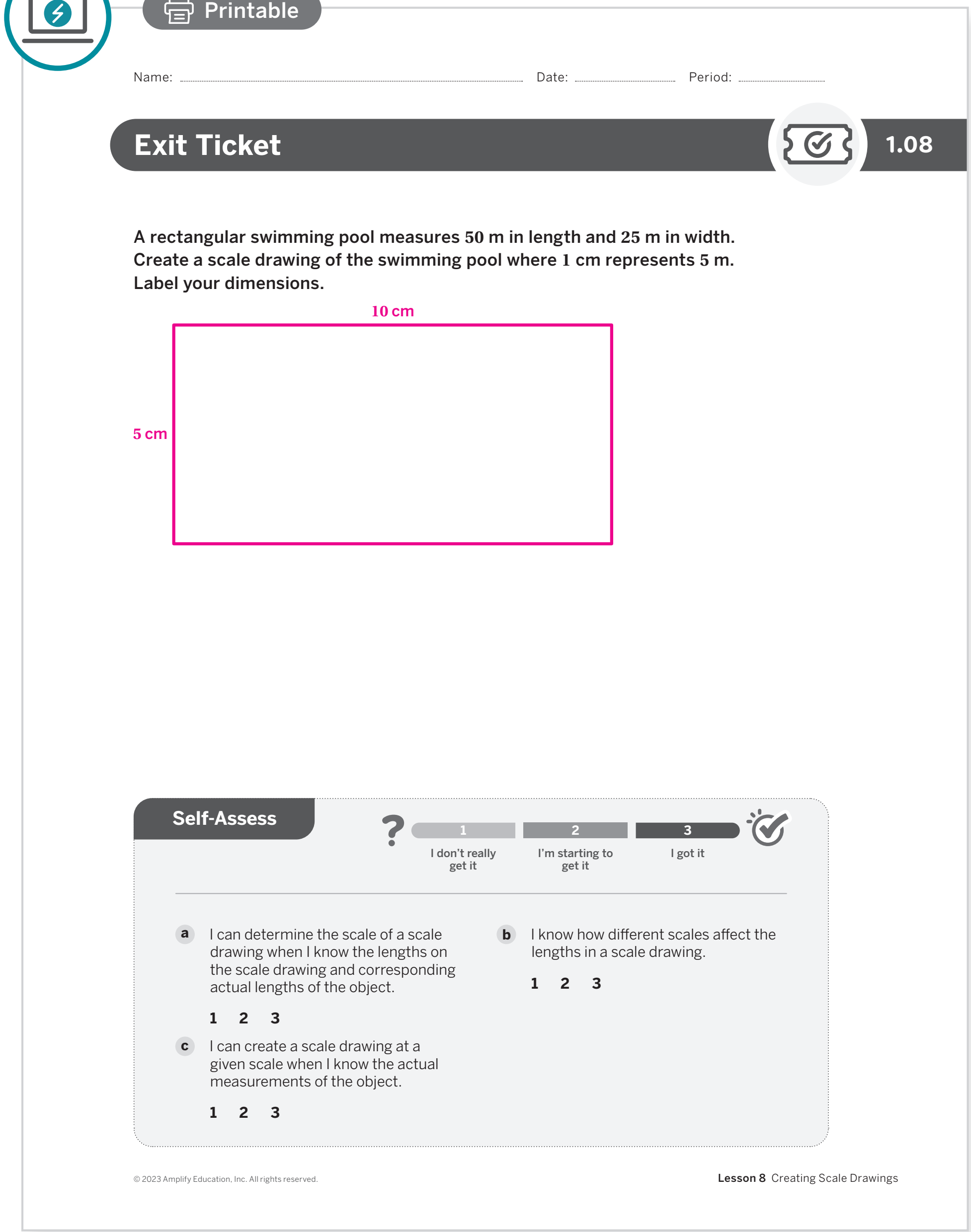
- "Why is it important to be precise when creating scale drawings?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by creating a scale drawing and identifying its dimensions.

Printable



Name: __________ Date: __________ Period: __________

Exit Ticket 1.08

A rectangular swimming pool measures 50 m in length and 25 m in width. Create a scale drawing of the swimming pool where 1 cm represents 5 m. Label your dimensions.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine the scale of a scale drawing when I know the lengths on the scale drawing and corresponding actual lengths of the object.

1 2 3

b. I know how different scales affect the lengths in a scale drawing.

1 2 3

c. I can create a scale drawing at a given scale when I know the actual measurements of the object.

1 2 3

Lesson 8 Creating Scale Drawings

Success looks like . . .

- **Language Goal:** Comparing and contrasting different scale drawings of the same object, and describing how the scale affects the size of the drawing. **(Speaking and Listening)**
- **Goal:** Creating a scale drawing, given the actual dimensions of the object and the scale.
  - » Creating a scale drawing of a swimming pool.
- **Language Goal:** Determining the scale used to create a scale drawing and generating multiple ways to express it. **(Writing)**

Suggested next steps

If students write the correct dimensions, but draw an incorrect scale drawing, consider:

- Clarifying that the expectation was for them to draw an actual scaled copy, not just a sketch. Have them attempt the problem again.
- Assigning Practice Problem 2.

Professional Learning

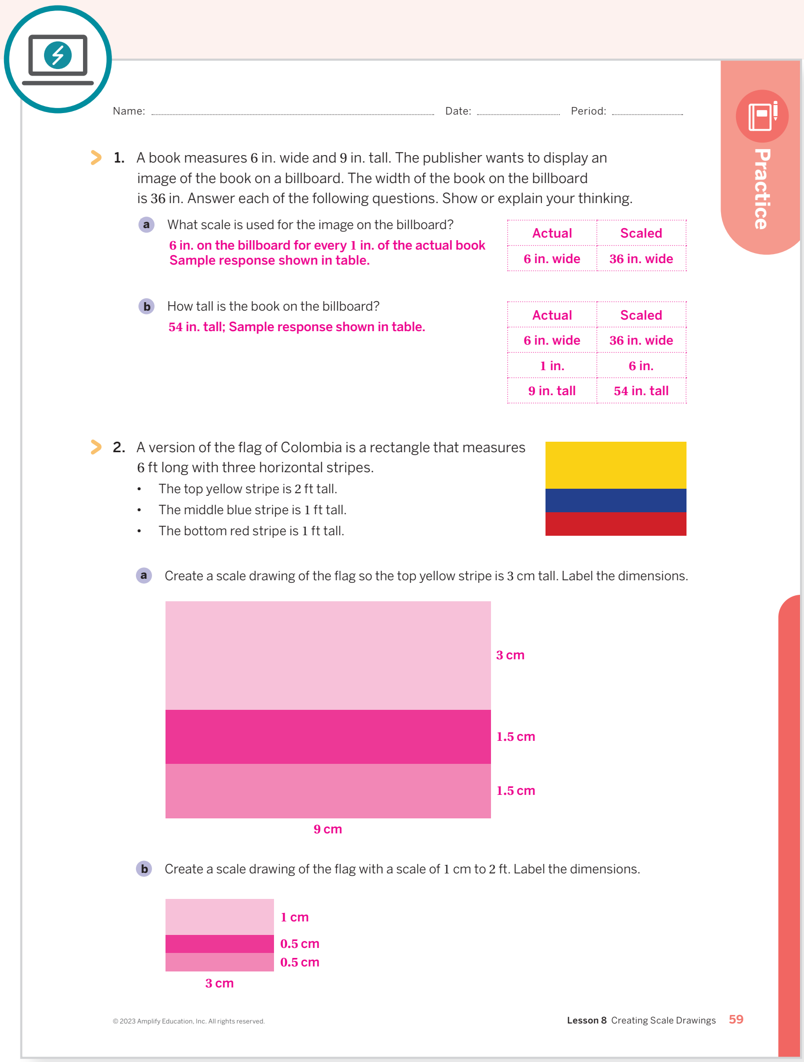
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did finding scaled or actual lengths in Activity 1 set students up to develop a process for drawing scaled copies in Activity 2?
- What challenges did students encounter as they progressed through Activity 2? How did they work through these challenges? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A book measures 6 in. wide and 9 in. tall. The publisher wants to display an image of the book on a billboard. The width of the book on the billboard is 36 in. Answer each of the following questions. Show or explain your thinking.

   a. What scale is used for the image on the billboard?

   6 in. on the billboard for every 1 in. of the actual book Sample response shown in table.

Actual	Scaled
6 in. wide	36 in. wide

   b. How tall is the book on the billboard?

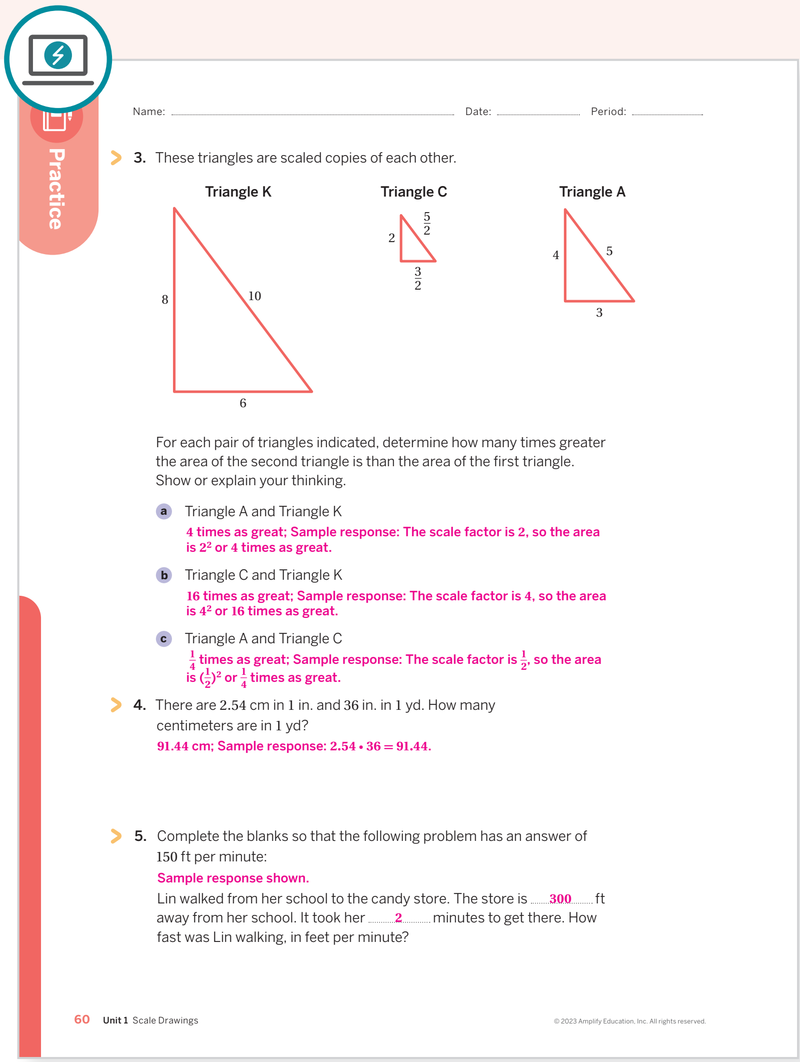
   54 in. tall; Sample response shown in table.

Actual	Scaled
6 in. wide	36 in. wide
1 in.	6 in.
9 in. tall	54 in. tall

2. A version of the flag of Colombia is a rectangle that measures 6 ft long with three horizontal stripes.
   - The top yellow stripe is 2 ft tall.
   - The middle blue stripe is 1 ft tall.
   - The bottom red stripe is 1 ft tall.

   a. Create a scale drawing of the flag so the top yellow stripe is 3 cm tall. Label the dimensions.

   b. Create a scale drawing of the flag with a scale of 1 cm to 2 ft. Label the dimensions.



Name: Date: Period:

3. These triangles are scaled copies of each other.

For each pair of triangles indicated, determine how many times greater the area of the second triangle is than the area of the first triangle. Show or explain your thinking.

   a. Triangle A and Triangle K

   4 times as great; Sample response: The scale factor is 2, so the area is 2^2 or 4 times as great.

   b. Triangle C and Triangle K

   16 times as great; Sample response: The scale factor is 4, so the area is 4^2 or 16 times as great.

   c. Triangle A and Triangle C

   $\frac{1}{4}$ times as great; Sample response: The scale factor is $\frac{1}{2}$, so the area is $(\frac{1}{2})^2$ or $\frac{1}{4}$ times as great.

4. There are 2.54 cm in 1 in. and 36 in. in 1 yd. How many centimeters are in 1 yd?

   91.44 cm; Sample response: $2.54 \cdot 36 = 91.44$.

5. Complete the blanks so that the following problem has an answer of 150 ft per minute:

   Sample response shown.

   Lin walked from her school to the candy store. The store is **300** ft away from her school. It took her **2** minutes to get there. How fast was Lin walking, in feet per minute?

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 1 Lesson 6	2
	4	Grade 6	1
Formative ⚡	5	Unit 1 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Optional

UNIT 1 | **LESSON 9**

Scale Drawings and Maps

Let's use scale drawings to solve problems.

Focus

Goals

1. **Language Goal:** Determine which of two objects is traveling at a faster rate and justify the reasoning. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Use a scale drawing to estimate the distance an object traveled, as well as its average speed or elapsed time, and explain the solution method. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students apply what they have learned about scale drawings to solve problems involving traveling at a constant speed. They make strategic use of maps, scales, and tools as they estimate distances. In some cases, the paths traveled are not straight; and students are encouraged to use their problem-solving skills and a strategic mathematical tool choice.

Previously

In Grade 6, students examined various contexts involving travel at a constant, or average, speed.

Coming Soon

In Unit 2, students will gain further familiarity with constant speed — through important contexts.

Rigor

- Students strengthen their **procedural skills** in measuring precisely using rulers or informal measuring tools.
- Students **apply** their knowledge of distance, speed, and time relationships to a new context involving maps with scales.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- rulers

Math Language Development

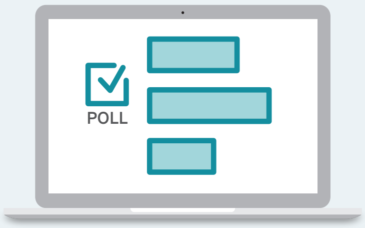
Review words

- *scale*
- *scale drawing*

Amps | **Featured Activity**

Activity 1
Poll the Class

Students leverage their intuition about speed, distance, and time based on a quick glance at a map. Get real-time insight into how students are thinking while the stakes are lower, before measuring and calculations take place.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overwhelmed as they work through these activities because of the amount of information given and that it is similar but not the same in both problems. Have students organize the information in the problem. They might want to use a simple graphic organizer with two columns, "Given" and "Need to find." By organizing the information in the problem, students set themselves up for success as a solution strategy becomes more apparent.

Modifications to Pacing

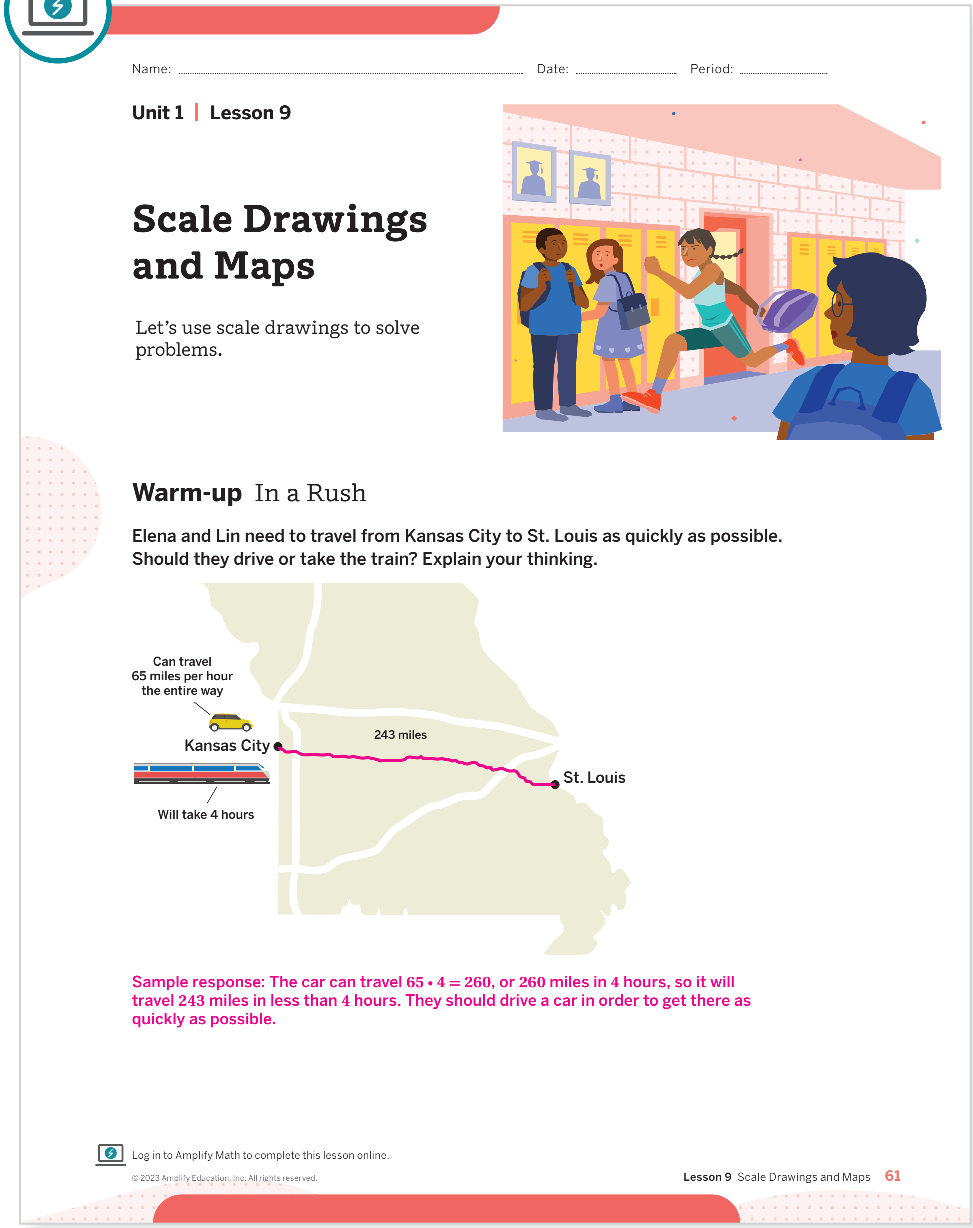
You may want to consider this additional modifications if you are short on time.

- In **Activities 1**, you may choose to omit having students write their responses to Problem 1.
- **Activity 2** may be omitted.

Pairs | 7 min

Warm-up In a Rush

Students revisit the relationship between distance, speed, and time to prepare for solving speed-related problems using scale drawings.



Name: ____________ Date: ________ Period: ________

Unit 1 | Lesson 9

Scale Drawings and Maps

Let's use scale drawings to solve problems.

Warm-up In a Rush

Elena and Lin need to travel from Kansas City to St. Louis as quickly as possible. Should they drive or take the train? Explain your thinking.

Sample response: The car can travel $65 \cdot 4 = 260$, or 260 miles in 4 hours, so it will travel 243 miles in less than 4 hours. They should drive a car in order to get there as quickly as possible.

Log in to Amplify Math to complete this lesson online.

Lesson 9 Scale Drawings and Maps 61

1 Launch

Activate background knowledge by asking students to think of a time when they needed to get somewhere quickly, and how they decided which route to take. Ask, "What things were important to consider when making the decision?"

2 Monitor

Help students get started by asking, "What are the important pieces of information in this problem? How should you use them?"

Look for points of confusion:

- **Multiplying 65 by 243.** Ask, "What does 65 miles per hour mean?"

Look for productive strategies:

- Noticing that traveling by car can be reasoned about by using total time or distance for the trip.
- Calculating the speed of the train.

3 Connect

Display the image from the Warm-up.

Have students share their reasoning for comparing the car and the train, making sure to hear from students who reasoned in different ways about the comparison.

Ask:

- "How could you find the speed of the train?"
- "In the real world, are there other factors you might want to consider when choosing which way to travel? What are they?"

Highlight that this problem involved comparing distance, speed, and time. Often, in similar problems, students will be given two of these pieces of information and must use those to find the third. Say, "In this lesson, you will continue to work with distance, speed, and time using information on maps with scales."

Math Language Development

MLR7: Compare and Connect

Have two students who used different approaches share their thinking and represent important parts visually. Ask them to compare the strategies, focusing on which approach they thought was clearer or which approach they better understood. As students compare and share their understanding of the two strategies, highlight developing mathematical language being used, particularly *distance*, *speed*, and *time*.

English Learners

Have students use annotations or gestures to illustrate their strategies.

Power-up

To power up students' ability to relate speed to distance and time, ask:

Determine which of the following vehicles is traveling at a speed of 60 miles per hour.

A. A ferry traveling a distance of 30 miles in 1 hour.

B. A car traveling a distance of 120 miles in 3 hours.

C. A train traveling a distance of 275 miles in 5 hours.

[✓ **D.**] A bus traveling a distance of 360 miles in 6 hours.

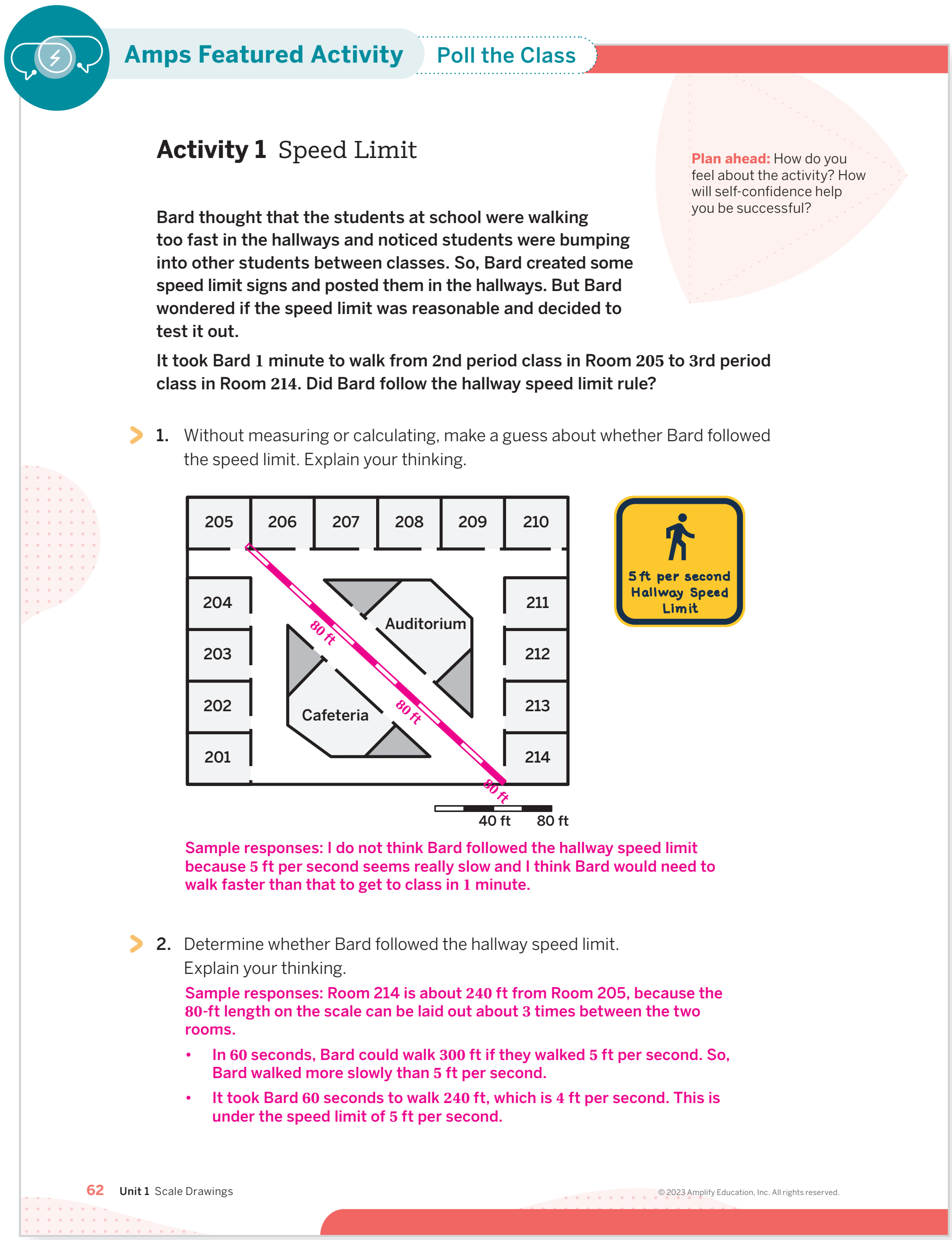
Use: Before the Warm-up.

Informed by: Performance on Lesson 8, Practice Problem 5.

Pairs | 15 min

Activity 1 Speed Limit

Students use a scale drawing and a new type of scale to determine average speed and solve a real-world problem.



Amps Featured Activity | Poll the Class

Activity 1 Speed Limit

Plan ahead: How do you feel about the activity? How will self-confidence help you be successful?

Bard thought that the students at school were walking too fast in the hallways and noticed students were bumping into other students between classes. So, Bard created some speed limit signs and posted them in the hallways. But Bard wondered if the speed limit was reasonable and decided to test it out.

It took Bard 1 minute to walk from 2nd period class in Room 205 to 3rd period class in Room 214. Did Bard follow the hallway speed limit rule?

1. Without measuring or calculating, make a guess about whether Bard followed the speed limit. Explain your thinking.

Sample responses: I do not think Bard followed the hallway speed limit because 5 ft per second seems really slow and I think Bard would need to walk faster than that to get to class in 1 minute.

2. Determine whether Bard followed the hallway speed limit. Explain your thinking.

Sample responses: Room 214 is about 240 ft from Room 205, because the 80-ft length on the scale can be laid out about 3 times between the two rooms.

- **In 60 seconds, Bard could walk 300 ft if they walked 5 ft per second. So, Bard walked more slowly than 5 ft per second.**
- **It took Bard 60 seconds to walk 240 ft, which is 4 ft per second. This is under the speed limit of 5 ft per second.**

62 **Unit 1** Scale Drawings

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1 Launch

Ask students what they notice about the new type of scale in this activity. Point out that the scale is divided into equal sections, yet still gives them information on how an actual distance compares to a distance on the map. Distribute rulers to students.

2 Monitor

Help students get started by asking them how they plan to complete Problem 1. Have them underline the important information they will use to solve Problem 2.

Look for points of confusion:

- **Finding the average speed per minute instead of per second.** Have students explain how they found the average speed and compare it to the speed limit on the sign.

Look for productive strategies:

- Measuring the length of the scale and the distance traveled on the drawing with a ruler, then using that information to find the distance Bard actually traveled.

3 Connect

Display the map of the school.

Have students share the steps they took to find whether Bard followed the speed limit. Sequence the responses to allow for different approaches to be heard.

Highlight that some students found how far Bard could have traveled by walking at the speed limit, and others found the average speed at which Bard actually walked.

Ask:

- "What actual length does each segment on the scale represent?" 20 ft
- "Could Bard have traveled along a different route? How would that affect the speed?" Sample response: Bard could have taken a longer route, not through the diagonal. To do so, Bard would have had to walk at a faster average speed.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Draw a direct line from Room 205 to Room 214 to provide students with a sense of what they need to measure, and then have them continue with the activity.

Extension: *Math Enrichment*

Ask students to determine whether Bard would still have walked under the speed limit by taking a path around the Cafeteria or Auditorium.

Math Language Development

MLR8: Discussion Supports

To help students unpack the concept of a walking speed limit and to foster mathematical discourse, ask "What does it mean to walk 5 ft per second? What if I walked 5 ft in 10 seconds? What if I walked 60 ft in 1 minute?"

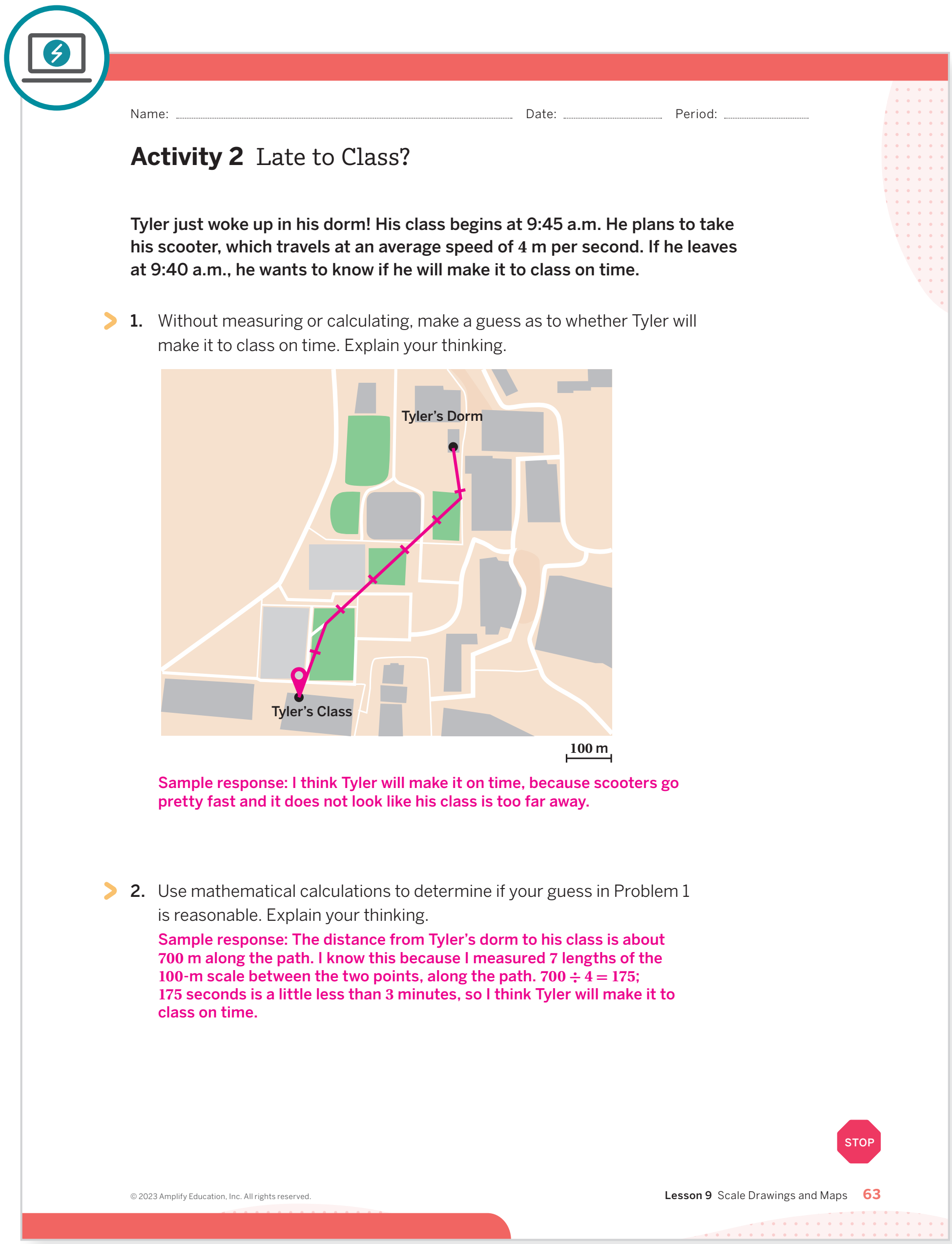
English Learners

As you present each of these three rates, display each one and ask a student volunteer to demonstrate what that walking speed might look like.

Pairs | 15 min

Activity 2 Late to Class?

Students use a map scale and average speed to solve a real-world problem, strengthening their understanding of and fluency with map scales and average speeds.



Name: Date: Period:

Activity 2 Late to Class?

Tyler just woke up in his dorm! His class begins at 9:45 a.m. He plans to take his scooter, which travels at an average speed of 4 m per second. If he leaves at 9:40 a.m., he wants to know if he will make it to class on time.

1. Without measuring or calculating, make a guess as to whether Tyler will make it to class on time. Explain your thinking.

Sample response: I think Tyler will make it on time, because scooters go pretty fast and it does not look like his class is too far away.

2. Use mathematical calculations to determine if your guess in Problem 1 is reasonable. Explain your thinking.

Sample response: The distance from Tyler's dorm to his class is about 700 m along the path. I know this because I measured 7 lengths of the 100-m scale between the two points, along the path. $700 \div 4 = 175$; 175 seconds is a little less than 3 minutes, so I think Tyler will make it to class on time.

STOP

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Lesson 9 Scale Drawings and Maps 63

1 Launch

Read the introduction and have students make a guess as to whether Tyler will make it to class on time. Conduct the ***Poll the Class*** routine about their guesses, then direct students to discuss their reasoning with a partner.

2 Monitor

Help students get started by having them use a ruler and the scale to find the actual distance.

Look for points of confusion:

- **Measuring in a straight line.** Ask, "What route would make the most sense for Tyler to take, given that he is riding a scooter?"
- **Forgetting to convert the units for the average speed to match the measurement for time.** Ask if the amount of time makes sense for traveling what looks like a few blocks on the map.

Look for productive strategies:

- First converting the times Tyler has to get to class and his speed to matching units.

3 Connect

Display the map from the Student Edition.

Ask:

- "In what ways were Activities 1 and 2 different?" Sample response: In Activity 1, we were given the time and needed to find the distance. Then we compared average speeds. In Activity 2, we were given the time and the average speed and needed to find the distance. Then we compared the lengths of time.
- "In what ways were Activities 1 and 2 similar?" Sample response: They both involved using a map with a scale and information about two of the following three values: distance, time, and average speed.

Highlight that the units used to measure speed will differ according to the context, but when comparing speeds, students should check if the units match or need to be converted in order to match.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to highlighters, markers, or colored pencils for students to use to mark the route they think Tyler should take. This will assist them in organizing their measurements.

Accessibility: *Vary Demands to Optimize Challenge*

Display the conversion from minutes to seconds for students to reference. Additionally, consider displaying a ratio table with the unit rate for minutes to seconds and have students complete the remainder of the table.

Math Language Development

MLR7: Compare and Connect

Look for students using different strategies to estimate the distance between the two buildings. Ask students to share what worked well in a particular approach. During this discussion, listen for and amplify any comments that make the estimation of the distance more precise.

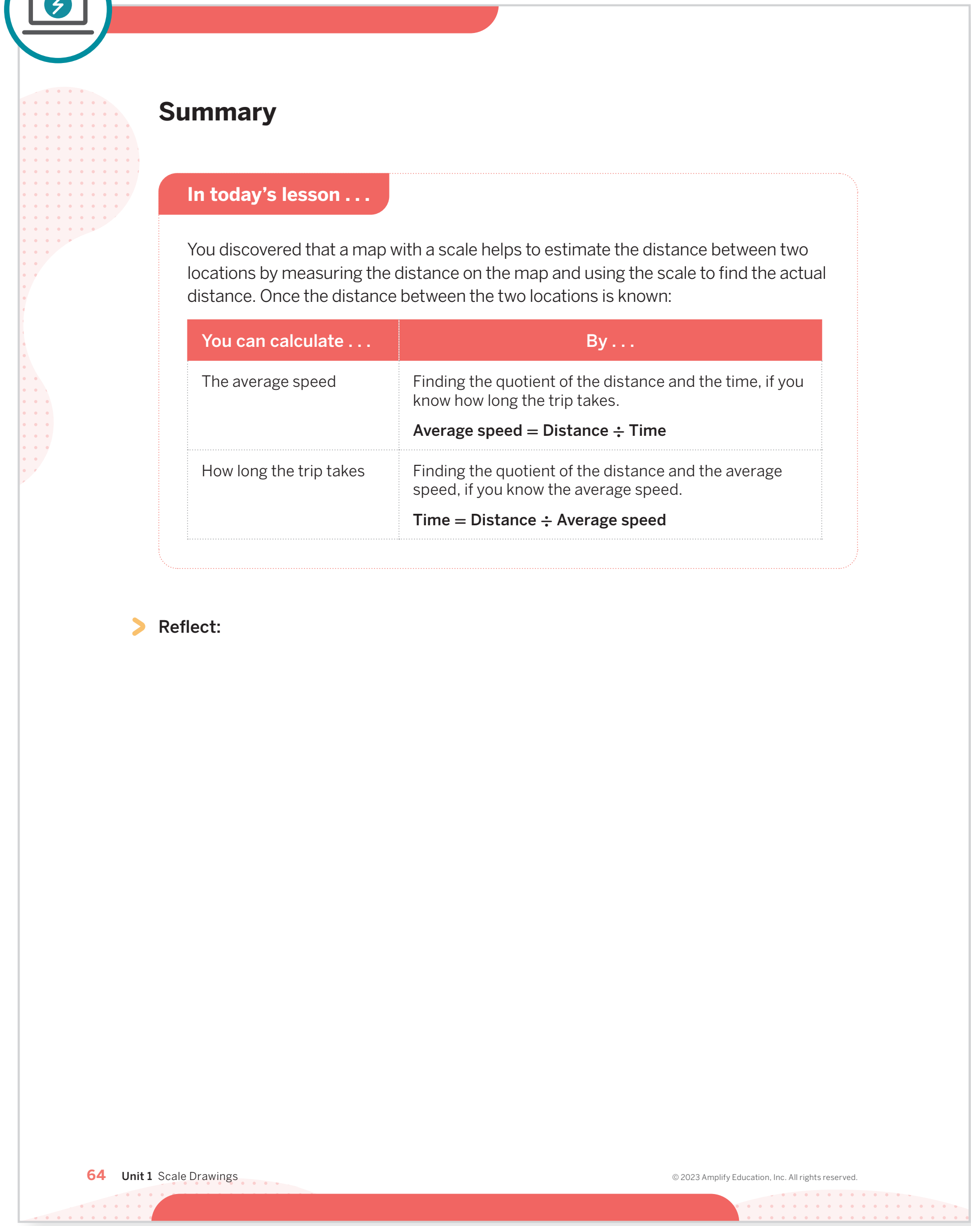
English Learners

Provide a graphic organizer with 3–4 different contexts labeled, such as car speed, bike speed, airplane speed, etc. Highlight for students how the units used to measure speed in these different contexts will differ.

Whole Class | 5 min

Summary

Review and synthesize how to use the scale on a map to find distances and make calculations involving distance, average speed, and time.



Summary

In today's lesson . . .

You discovered that a map with a scale helps to estimate the distance between two locations by measuring the distance on the map and using the scale to find the actual distance. Once the distance between the two locations is known:

You can calculate . . .	By . . .
The average speed	Finding the quotient of the distance and the time, if you know how long the trip takes. **Average speed = Distance ÷ Time**
How long the trip takes	Finding the quotient of the distance and the average speed, if you know the average speed. **Time = Distance ÷ Average speed**

Reflect:

64 **Unit 1** Scale Drawings

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Synthesize

Highlight that sometimes it makes sense to measure a straight-line distance between two points on a map and other times it makes sense to follow a path or a road. This decision should be based on the context of the problem.

Ask:

- "What information can be obtained from a map with a scale?" Sample response: We can estimate how far one place on the map is from another in real life. We can use it to help us find how long it might take to get from one place to another, if we know the average speed.
- "What information *cannot* be obtained from a map with a scale?" Sample response: We cannot tell if there are hills. It may be challenging to know exactly the length of some roads if they curve.
- "Why is it best to say that you are *estimating* the actual distance when using a scale on a map?" Sample response: Exact distances may not be known, if roads curve or routes are complex.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

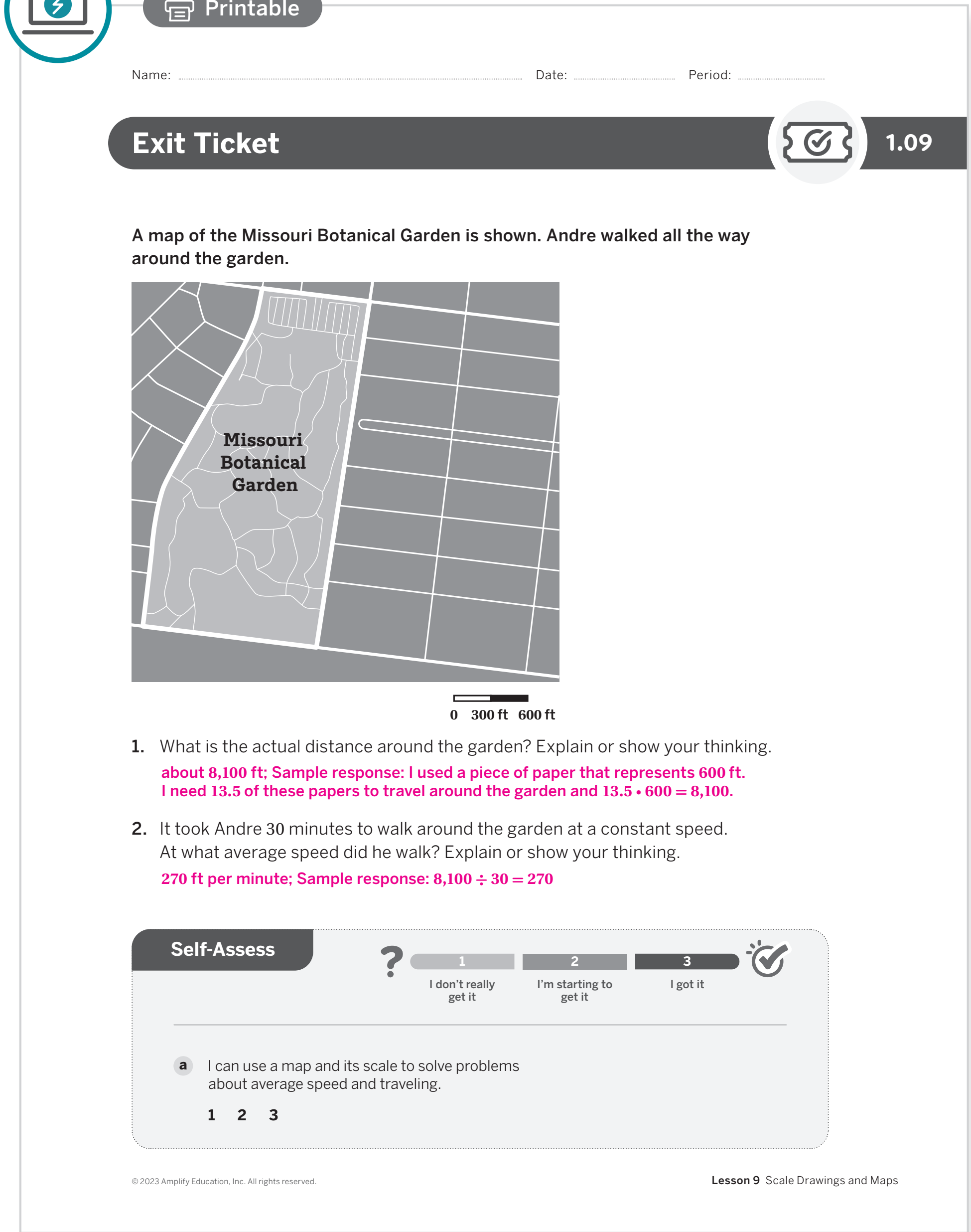
- "How do scale models help you make sense of the world around you?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to use a map scale by determining the actual distance on a map and calculating average speed.

Printable



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Exit Ticket 1.09

A map of the Missouri Botanical Garden is shown. Andre walked all the way around the garden.

1. What is the actual distance around the garden? Explain or show your thinking.
   about 8,100 ft; Sample response: I used a piece of paper that represents 600 ft. I need 13.5 of these papers to travel around the garden and $13.5 \cdot 600 = 8,100$.

2. It took Andre 30 minutes to walk around the garden at a constant speed. At what average speed did he walk? Explain or show your thinking.
   270 ft per minute; Sample response: $8,100 \div 30 = 270$

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a I can use a map and its scale to solve problems about average speed and traveling.
1 2 3

Lesson 9 Scale Drawings and Maps

Success looks like . . .

- **Language Goal:** Determining which of two objects is traveling at a faster rate and justifying the reasoning. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Using a scale drawing to estimate the distance an object traveled, as well as its speed or elapsed time, and explaining the solution method. **(Speaking and Listening, Reading and Writing)**
  - » Determining the average speed Andre traveled around the garden in Problem 2.

Suggested next steps

If students have difficulty measuring lengths on the map precisely, consider:

- Having them trace the path with their pencil first before measuring.

If students multiply in calculations where it is necessary to divide, or vice versa, consider:

- Asking them to think about the reasonableness of the numbers they get from their calculations.

If students improperly calculate the distance around the botanical garden after measuring with a ruler, consider:

- Having them measure informally using the scale length on an index card.

Professional Learning

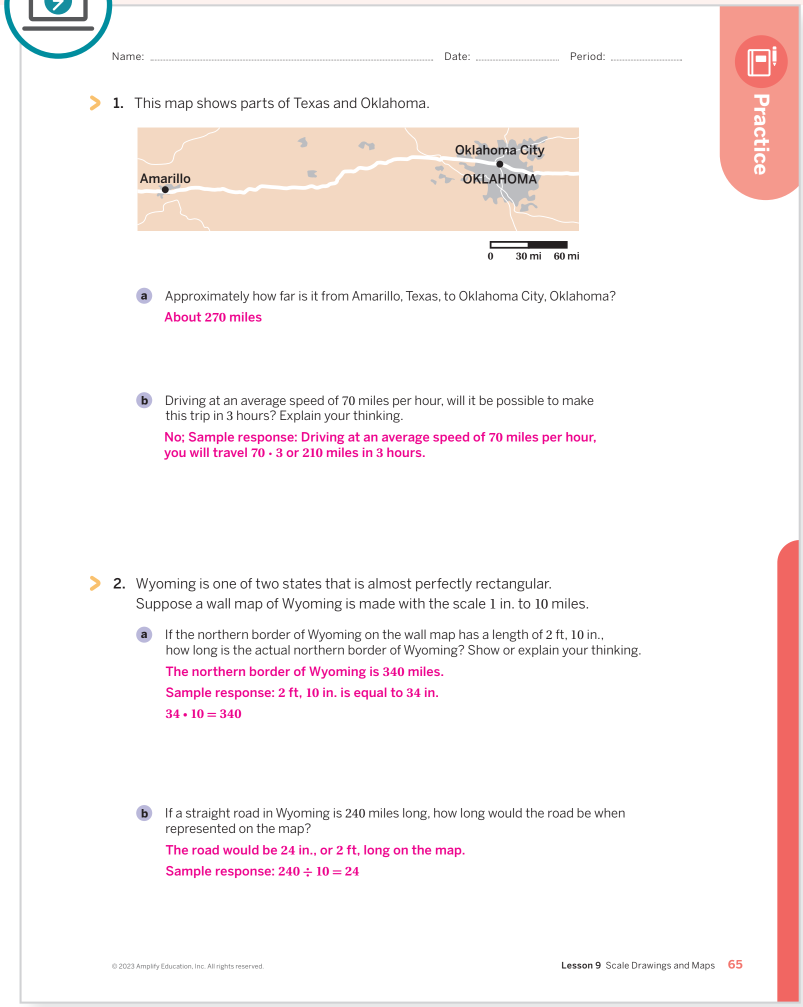
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In Activity 1, did anything surprise you or did anything happen that you did not expect?
- Thinking about the questions you asked students today and what the students said or did as a result of those questions, which question or questions were the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



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1. This map shows parts of Texas and Oklahoma.

a Approximately how far is it from Amarillo, Texas, to Oklahoma City, Oklahoma?

About 270 miles

b Driving at an average speed of 70 miles per hour, will it be possible to make this trip in 3 hours? Explain your thinking.

No; Sample response: Driving at an average speed of 70 miles per hour, you will travel 70 · 3 or 210 miles in 3 hours.

2. Wyoming is one of two states that is almost perfectly rectangular. Suppose a wall map of Wyoming is made with the scale 1 in. to 10 miles.

a If the northern border of Wyoming on the wall map has a length of 2 ft, 10 in., how long is the actual northern border of Wyoming? Show or explain your thinking.

The northern border of Wyoming is 340 miles.

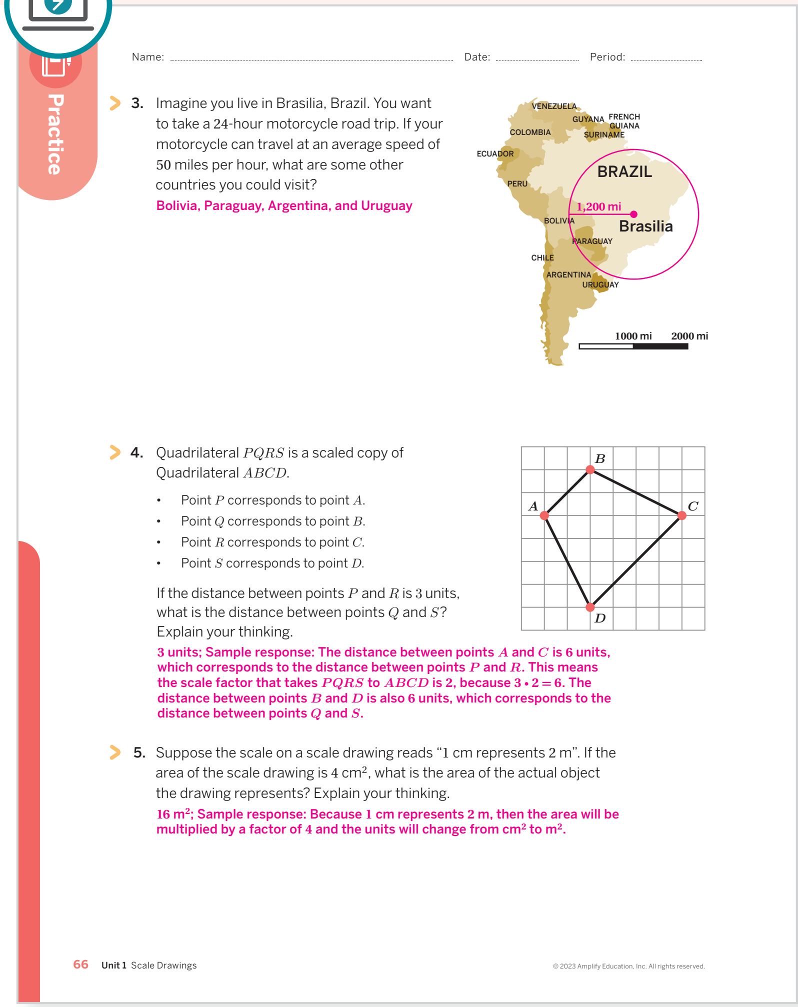
Sample response: 2 ft, 10 in. is equal to 34 in.

$34 \cdot 10 = 340$

b If a straight road in Wyoming is 240 miles long, how long would the road be when represented on the map?

The road would be 24 in., or 2 ft, long on the map.

Sample response: $240 \div 10 = 24$



Name: ______________________ Date: ____________ Period: ____________

3. Imagine you live in Brasilia, Brazil. You want to take a 24-hour motorcycle road trip. If your motorcycle can travel at an average speed of 50 miles per hour, what are some other countries you could visit?

Bolivia, Paraguay, Argentina, and Uruguay

4. Quadrilateral $PQRS$ is a scaled copy of Quadrilateral $ABCD$.

- Point P corresponds to point A.
- Point Q corresponds to point B.
- Point R corresponds to point C.
- Point S corresponds to point D.

If the distance between points P and R is 3 units, what is the distance between points Q and S? Explain your thinking.

3 units; Sample response: The distance between points A and C is 6 units, which corresponds to the distance between points P and R. This means the scale factor that takes $PQRS$ to $ABCD$ is 2, because $3 \cdot 2 = 6$. The distance between points B and D is also 6 units, which corresponds to the distance between points Q and S.

5. Suppose the scale on a scale drawing reads "1 cm represents 2 m". If the area of the scale drawing is 4 cm^2, what is the area of the actual object the drawing represents? Explain your thinking.

16 m^2; Sample response: Because 1 cm represents 2 m, then the area will be multiplied by a factor of 4 and the units will change from cm^2 to m^2.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 2	2
Formative ⚡	5	Unit 1 Lesson 10	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 1 | **LESSON 10**

Changing Scales in Scale Drawings

Let's explore different scale drawings of the same object or location.

Focus

Goals

1. **Language Goal:** Generalize that as the actual distance represented by one unit on the drawing increases, the size of the scale drawing decreases. **(Speaking and Listening)**
2. **Language Goal:** Reproduce a scale drawing at a different scale and explain the solution method. **(Speaking and Listening)**
3. Determine how much actual area is represented by one square unit in a scale drawing.

Coherence

Today

Students are given a scale drawing and asked to recreate it at a different scale. They extend their work with areas of scaled copies by comparing areas of scale drawings of the same object with different scales and examining how much area, on the actual object, is represented by 1 cm^2 on the scale drawing. Students observe and explain structure, both when they reproduce a scale drawing at a different scale and when they study how the area of a scale drawing depends on the scale.

Previously

In Lesson 6, students saw how the area of a scaled copy relates to the area of the original figure. In Lesson 9, students created multiple scale drawings using different scales.

Coming Soon

In Lesson 11, students will use scales without units and produce equivalent scales in preparation for the unit on ratios and proportional relationships.

Rigor

- Students further their **conceptual understanding** of scale drawings by resizing figures and comparing areas.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, *Map* (for display)
- Activity 2 PDF, *Scales*, pre-cut, one per student
- Activity 2 PDF (answers, for display)
- graph paper, one sheet per student
- rulers

Math Language Development

Review words

- *scale*
- *scale drawing*

Amps | Featured Activity

Activity 1

Formative Feedback for Students

Students test their rescaling and watch the building change size to receive immediate feedback on their accuracy.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may have difficulty making sense of the quantities they are given in the context of Activity 1. Guide students to pause to determine how they can apply the process that they have been using to this new situation. They need to pay close attention to the labels on the measurements, to better conceptualize what they are being asked to do.

Modifications to Pacing

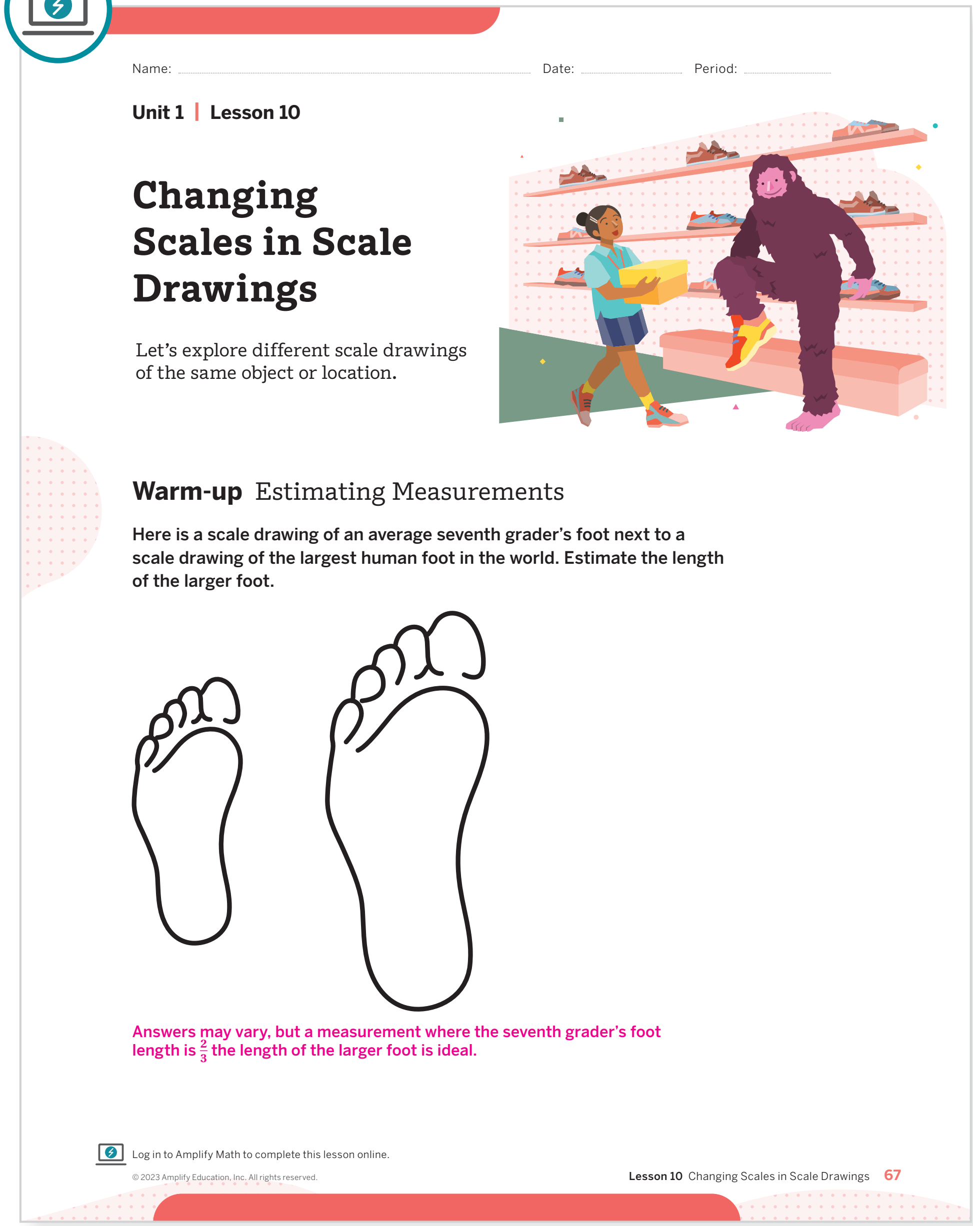
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted so that you can immediately proceed with Activity 1.
- Focus on Problem 1 in **Activity 1**. Consider using the other problems during later lessons if you have additional time.
- In **Activity 2**, limit the number of scales used to three or four. This may reduce the length of the discussion.

Independent | 5 min

Warm-up Estimating Measurements

Students estimate lengths using the relationship between two scale drawings, preparing them to work with scale drawings produced at different scales in the upcoming activities.



Name: Date: Period:

Unit 1 | Lesson 10

Changing Scales in Scale Drawings

Let's explore different scale drawings of the same object or location.

Warm-up Estimating Measurements

Here is a scale drawing of an average seventh grader's foot next to a scale drawing of the largest human foot in the world. Estimate the length of the larger foot.

Answers may vary, but a measurement where the seventh grader's foot length is $\frac{2}{3}$ the length of the larger foot is ideal.

Log in to Amplify Math to complete this lesson online.

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Lesson 10 Changing Scales in Scale Drawings 67

1 Launch

Conduct the Warm-up using the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by asking them to estimate the length of their feet, in inches.

Look for points of confusion:

- **Not knowing how to respond without a given scale.** Ask students what fraction of the larger foot is the smaller foot.
- **Estimating a length based on the actual length of the drawing.** Remind students both drawings are scale drawings of the actual feet.

Look for productive strategies:

- Not focusing on precise measurements, but paying attention to the relationship between the two scale drawings.
- Not needing the actual scale to estimate. Note students who compare sizes and create lengths using the scale factors of $\frac{2}{3}$ or $\frac{3}{2}$.

3 Connect

Display the images of the scaled feet.

Have students share their estimations.

Highlight that although students do not know the scale of the drawings, they can still estimate the length of the larger foot by using the relationship between the two feet. Point out that the smaller foot is about $\frac{2}{3}$ the size of the larger foot (or the larger foot is about $\frac{3}{2}$ the size of the smaller foot).

Ask, "If the size of the drawings changed, what would remain the same?" The relationship between the two feet would continue to have the same scale factor.

Differentiated Support

Extension: ***Interdisciplinary Connections***

Provide the heights and foot lengths of the Guinness World Records holders for Tallest Man Ever (Robert Wadlow) and Tallest Living Man (Sultan Kösen). Have students compare the heights and foot lengths of each person using mathematics of their choosing. **(History)**

	Robert Wadlow	Sultan Kösen
Height (cm)	272	251
Foot length (cm)	47 cm	Left: 36.5, Right: 35.5

Power-up

To power up students' ability to relate scales for lengths to areas, ask:

Suppose the scale on a rectangular scale drawing reads "1 cm represents 3 ft". The dimensions of an object in the scale drawing is 1 cm by 2 cm.

1. What are the dimensions of the actual object the drawing represents? 3 ft by 6 ft
2. What is the area of the actual object the drawing represents? 18 ft^2

Use: Before the Activity 2.

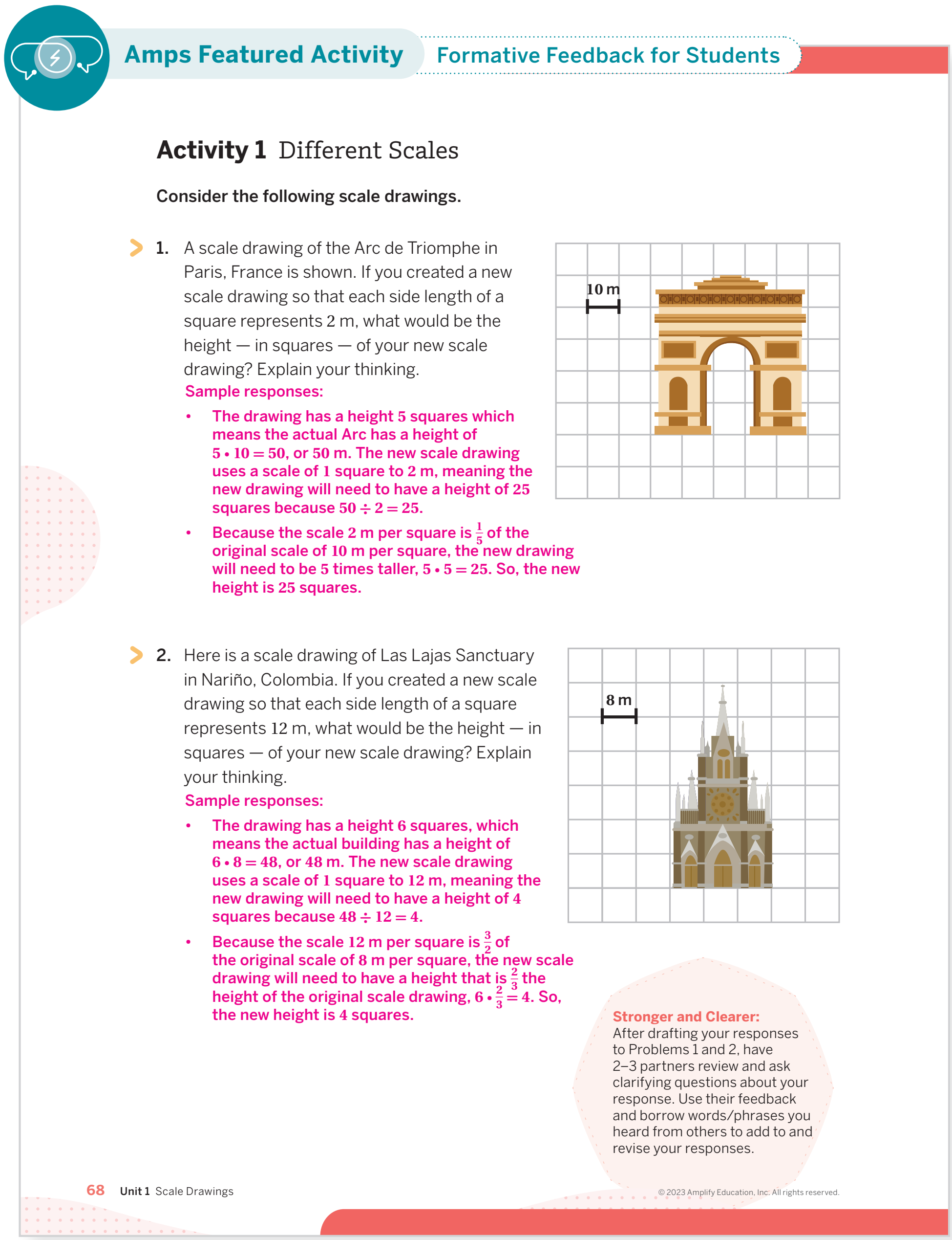
Informed by: Performance on Lesson 9, Practice Problem 5.

Small Groups | 15 min

Activity 1 Different Scales

Students reason about what a new scale drawing would look like, when produced at a different scale, based on an existing scale drawing.

Amps Featured Activity Formative Feedback for Students



Activity 1 Different Scales

Consider the following scale drawings.

1. A scale drawing of the Arc de Triomphe in Paris, France is shown. If you created a new scale drawing so that each side length of a square represents 2 m, what would be the height — in squares — of your new scale drawing? Explain your thinking.

Sample responses:

- **The drawing has a height 5 squares which means the actual Arc has a height of $5 \cdot 10 = 50$, or 50 m. The new scale drawing uses a scale of 1 square to 2 m, meaning the new drawing will need to have a height of 25 squares because $50 \div 2 = 25$.**
- **Because the scale 2 m per square is $\frac{1}{5}$ of the original scale of 10 m per square, the new drawing will need to be 5 times taller, $5 \cdot 5 = 25$. So, the new height is 25 squares.**

2. Here is a scale drawing of Las Lajas Sanctuary in Nariño, Colombia. If you created a new scale drawing so that each side length of a square represents 12 m, what would be the height — in squares — of your new scale drawing? Explain your thinking.

Sample responses:

- **The drawing has a height 6 squares, which means the actual building has a height of $6 \cdot 8 = 48$, or 48 m. The new scale drawing uses a scale of 1 square to 12 m, meaning the new drawing will need to have a height of 4 squares because $48 \div 12 = 4$.**
- **Because the scale 12 m per square is $\frac{3}{2}$ of the original scale of 8 m per square, the new scale drawing will need to have a height that is $\frac{2}{3}$ the height of the original scale drawing, $6 \cdot \frac{2}{3} = 4$. So, the new height is 4 squares.**

Stronger and Clearer: After drafting your responses to Problems 1 and 2, have 2–3 partners review and ask clarifying questions about your response. Use their feedback and borrow words/phrases you heard from others to add to and revise your responses.

68 **Unit 1** Scale Drawings

1 Launch

Activate background knowledge by asking students what they know about the metric system and the U.S. customary system of measurement. Let them know they will use scales involving both systems but the process will remain the same.

2 Monitor

Help students get started by asking them to calculate the actual height of the Arc de Triomphe.

Look for points of confusion:

- **Thinking they are finished when they find the height of the actual building.** Have them read the directions carefully and ask if they found the solution to the problem.
- **Confusing their calculations.** Encourage students to write down how they found the values and label what the numbers represent.

Look for productive strategies:

- Drawing a rectangle around the buildings and scaling the rectangle.
- Checking the scaling for the horizontal sections in Problems 1 and 2.
- Finding the actual height of the buildings or comparing scales. Note students who use either method and have them share during the class discussion.
- Creating a ratio table to find values and realizing they need two separate tables to show both scalings.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 3, and only work on Problem 2 as they have time available.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can test their rescaling and watch the building change size to receive immediate feedback on their accuracy.

Math Language Development

MLR1: Stronger and Clearer Each Time

Students draft a response, then allow partners to read and critique it. Encourage students to listen for words they can borrow and use to improve their own explanations. Students make edits and work to create responses using developing mathematical language.

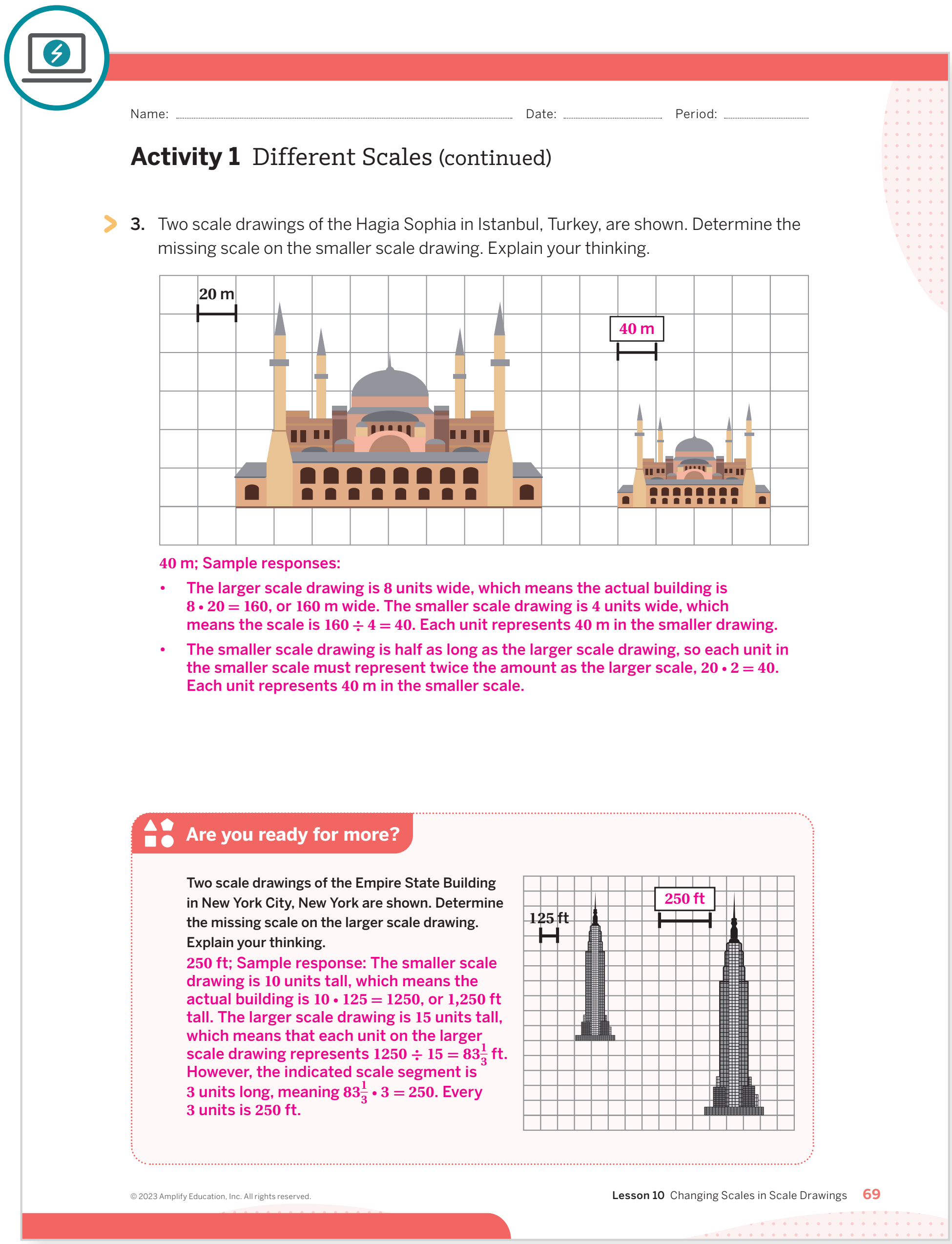
English Learners

Partner students with peers that speak the same primary language, and then allow the first draft response to be written in their primary language. Allow students time to rehearse what they will say before sharing out.

Small Groups | 15 min

Activity 1 Different Scales (continued)

Students reason about what a new scale drawing would look like, when produced at a different scale, based on an existing scale drawing.



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Activity 1 Different Scales (continued)

3. Two scale drawings of the Hagia Sophia in Istanbul, Turkey, are shown. Determine the missing scale on the smaller scale drawing. Explain your thinking.

40 m; Sample responses:

- The larger scale drawing is 8 units wide, which means the actual building is $8 \cdot 20 = 160$, or 160 m wide. The smaller scale drawing is 4 units wide, which means the scale is $160 \div 4 = 40$. Each unit represents 40 m in the smaller drawing.
- The smaller scale drawing is half as long as the larger scale drawing, so each unit in the smaller scale must represent twice the amount as the larger scale, $20 \cdot 2 = 40$. Each unit represents 40 m in the smaller scale.

Are you ready for more?

Two scale drawings of the Empire State Building in New York City, New York are shown. Determine the missing scale on the larger scale drawing. Explain your thinking.

250 ft; Sample response: The smaller scale drawing is 10 units tall, which means the actual building is $10 \cdot 125 = 1250$, or 1,250 ft tall. The larger scale drawing is 15 units tall, which means that each unit on the larger scale drawing represents $1250 \div 15 = 83\frac{1}{3}$ ft. However, the indicated scale segment is 3 units long, meaning $83\frac{1}{3} \cdot 3 = 250$. Every 3 units is 250 ft.

3 Connect

Display any needed images for the discussion.

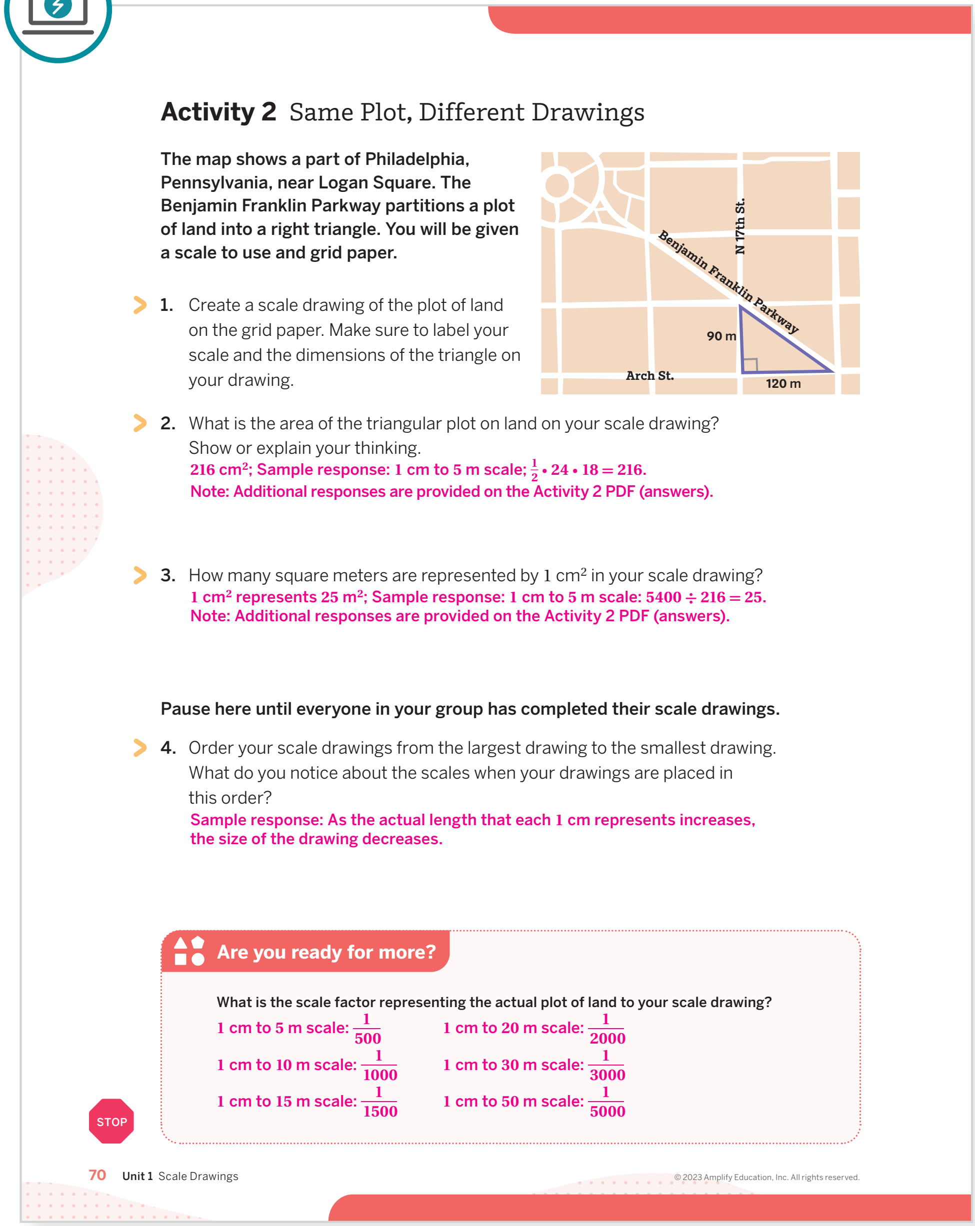
Have students share their responses and reasoning to the problems. Some students may use the structure of the scale to find the actual height and then rescale the building. Others may reason about the quantitative relationship between the two scales to find the lengths of the new drawing.

Highlight the methods used by students and mention any methods not discussed. Explain how to use the given scale to find the actual heights, either through multiplication or through the use of a ratio table. Show students how to find the relationship between the two scales, either with multiplication and division or with a ratio table. **Note:** Bridging these methods will be explored deeper in Unit 2.

Small Groups | 20 min

Activity 2 Same Plot, Different Drawings

Students create scale drawings to explore the relationship between the scaled area and actual area.



Activity 2 Same Plot, Different Drawings

The map shows a part of Philadelphia, Pennsylvania, near Logan Square. The Benjamin Franklin Parkway partitions a plot of land into a right triangle. You will be given a scale to use and grid paper.

1. Create a scale drawing of the plot of land on the grid paper. Make sure to label your scale and the dimensions of the triangle on your drawing.

2. What is the area of the triangular plot on land on your scale drawing? Show or explain your thinking.
   216 cm^2; Sample response: 1 cm to 5 m scale; $\frac{1}{2} \cdot 24 \cdot 18 = 216$.
   Note: Additional responses are provided on the Activity 2 PDF (answers).

3. How many square meters are represented by 1 cm^2 in your scale drawing?
   1 cm^2 represents 25 m^2; Sample response: 1 cm to 5 m scale: $5400 \div 216 = 25$.
   Note: Additional responses are provided on the Activity 2 PDF (answers).

Pause here until everyone in your group has completed their scale drawings.

4. Order your scale drawings from the largest drawing to the smallest drawing. What do you notice about the scales when your drawings are placed in this order?
   Sample response: As the actual length that each 1 cm represents increases, the size of the drawing decreases.

Are you ready for more?

What is the scale factor representing the actual plot of land to your scale drawing?

1 cm to 5 m scale: $\frac{1}{500}$　**1 cm to 20 m scale: $\frac{1}{2000}$**
1 cm to 10 m scale: $\frac{1}{1000}$　**1 cm to 30 m scale: $\frac{1}{3000}$**
1 cm to 15 m scale: $\frac{1}{1500}$　**1 cm to 50 m scale: $\frac{1}{5000}$**

STOP

70 **Unit 1** Scale Drawings

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1 Launch

Display the Activity 2 PDF, *Map* and find the area of the actual triangular plot of land as a class. Distribute a pre-cut slip with a scale factor from the Activity 2 PDF, *Scales*, rulers, and graph paper to each student.

2 Monitor

Help students get started by finding the side lengths of their scaled drawing using the scale and the actual side lengths of the plot of land.

Look for points of confusion:

- **Thinking everyone in their group should have the same responses to each problem.** Remind students that they each received different scales.
- **Thinking 1 cm^2 is equivalent to 5 m^2 in Problem 3.** Remind students both dimensions are changed, so the area will be $5 \cdot 5$ or 25 m^2. Drawing the square can help students visualize this change.

Look for productive strategies:

- Visualizing what a centimeter square represents at a given scale (e.g., at a scale of 1 cm to 5 m, each 1 cm^2 represents 25 m^2 in Problem 3).
- Dividing the actual area represented by the scale drawing by the area of their scale drawing.

3 Connect

Display the Activity 2 PDF (answers).

Ask:

- "How do the lengths of the scale drawing — where 1 cm represents 5 m — compare to the lengths of the scale drawing where 1 cm represents 15 m?"
- "How does the area of the scale drawing — where 1 cm represents 5 m — compare to the area of the scale drawing where 1 cm represents 15 m?"

Highlight these relationships:

- As the number of meters represented by 1 cm increases, the lengths in the scale drawing decrease.
- As the number of meters represented by 1 cm increases, the area of the scale drawing also decreases; but, it decreases by the square of the scale factor representing the lengths.

Math Language Development

MLR2: Collect and Display

As students talk, write down phrases they use. Capture a variety of students' words in a display that they can refer to, build on, or make connections with during future discussions, and to increase their awareness of language used in mathematics.

English Learners

Support students in developing the inverse relationship language around phrases, such as "As the number of meters represented by 1 cm increases, the lengths in the scale drawing decrease" and "As the number of meters represented by 1 cm increases, the area of the scale drawing also decreases; but it decreases by the square of the scale factor representing the lengths." This language should be added to the class display.

Differentiated Support

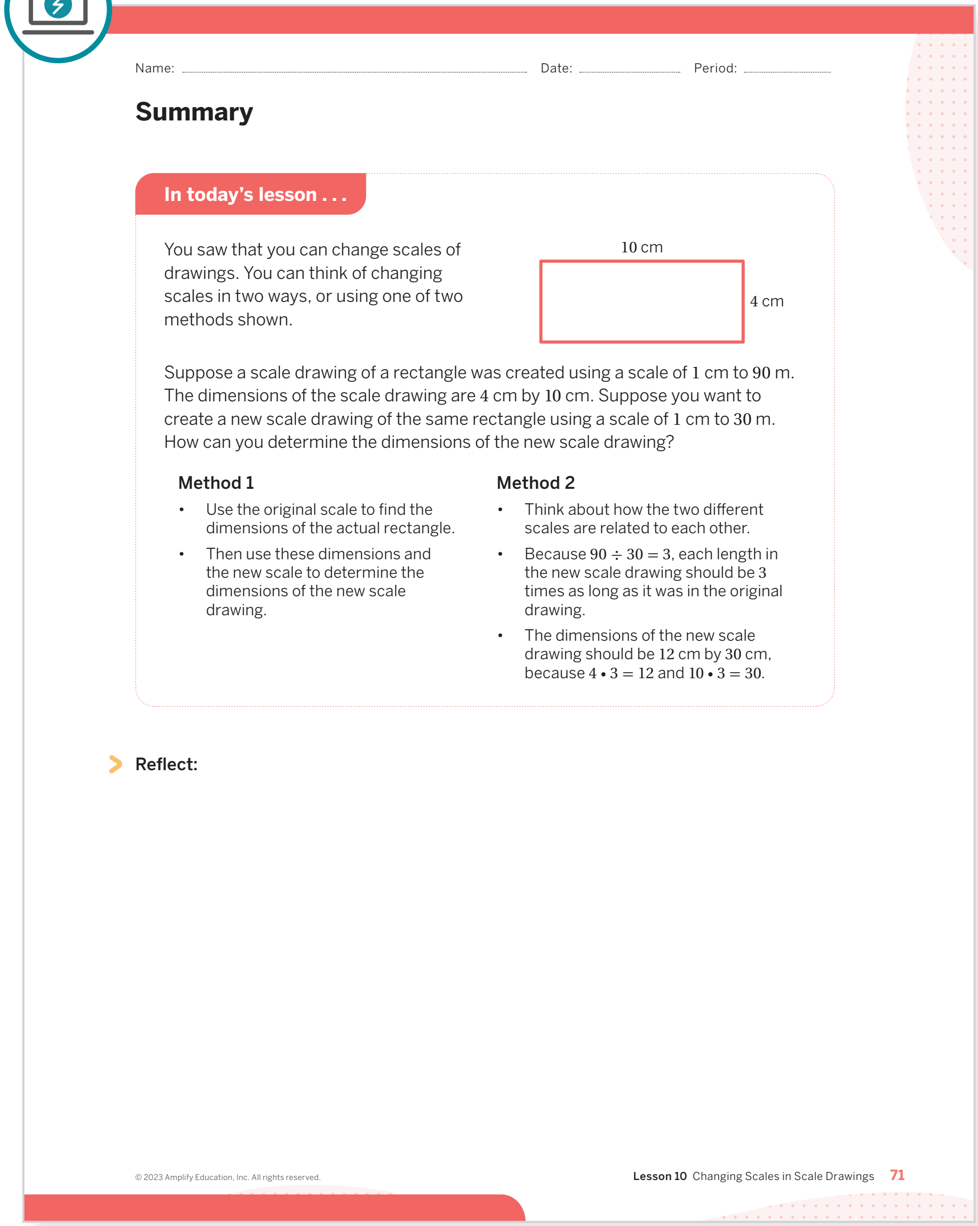
Accessibility: *Activate Prior Knowledge*

Before students complete Problem 2, activate or supply prior knowledge about various strategies that can be used to determine the area of a triangle, including:

- Using the formula for the area of a triangle.
- Finding the area of the related parallelogram or rectangle and decomposing into two equal-sized triangles.

Summary

Review and synthesize methods that can be used for scaling and re-scaling figures.



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Summary

In today's lesson . . .

You saw that you can change scales of drawings. You can think of changing scales in two ways, or using one of two methods shown.

10 cm

4 cm

Suppose a scale drawing of a rectangle was created using a scale of 1 cm to 90 m. The dimensions of the scale drawing are 4 cm by 10 cm. Suppose you want to create a new scale drawing of the same rectangle using a scale of 1 cm to 30 m. How can you determine the dimensions of the new scale drawing?

Method 1

- Use the original scale to find the dimensions of the actual rectangle.
- Then use these dimensions and the new scale to determine the dimensions of the new scale drawing.

Method 2

- Think about how the two different scales are related to each other.
- Because $90 \div 30 = 3$, each length in the new scale drawing should be 3 times as long as it was in the original drawing.
- The dimensions of the new scale drawing should be 12 cm by 30 cm, because $4 \cdot 3 = 12$ and $10 \cdot 3 = 30$.

Reflect:

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Lesson 10 Changing Scales in Scale Drawings 71

Synthesize

Display this context: "Suppose you have a map using the scale 1 cm to 200 m. You draw a new map of the same location using the scale 1 cm to 20 m."

Ask:

- "How does your new map compare to your original map?" The lengths are 10 times as long and the area is 100 times as great.
- "How much of the actual area does 1 cm^2 on your new map represent?" 400 m^2
- "How much of the actual area did 1 cm^2 on your original map represent?" 40,000 m^2

Highlight the different methods for thinking about these scales and moving between them:

- **Method 1:** Using the original scale drawing to calculate the actual lengths. Then use these actual lengths and the new scale to calculate the corresponding lengths on the new drawing.
- **Method 2:** Scaling lengths in the original scale drawing by a scale factor that related the two different scales. Let students know the discussions and concepts from this unit will be used in the next unit when they discuss ratios and proportional relationships.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

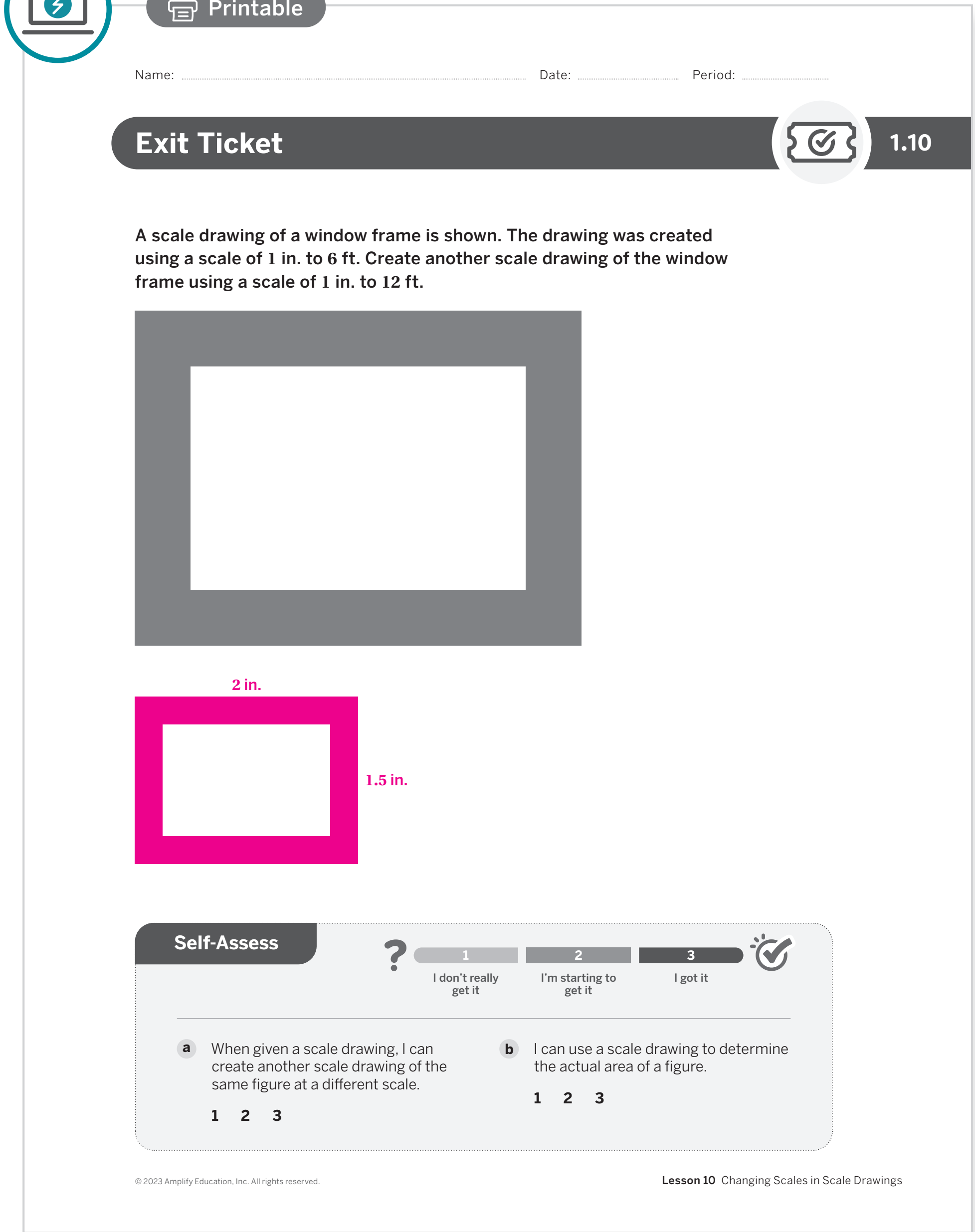
- "Why is it important to be precise when re-creating scaled copies?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by taking a scaled figure and drawing a new figure with a different scale.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket

1.10

A scale drawing of a window frame is shown. The drawing was created using a scale of 1 in. to 6 ft. Create another scale drawing of the window frame using a scale of 1 in. to 12 ft.

2 in.

1.5 in.

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a. When given a scale drawing, I can create another scale drawing of the same figure at a different scale.

1 2 3

b. I can use a scale drawing to determine the actual area of a figure.

1 2 3

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Lesson 10 Changing Scales in Scale Drawings

Success looks like . . .

- **Language Goal:** Generalizing that as the actual distance represented by one unit on the drawing increases, the size of the scale drawing decreases. **(Speaking and Listening)**
- **Language Goal:** Reproducing a scale drawing at a different scale and explaining the solution method. **(Speaking and Listening)**
  - » Creating a scale drawing of a window for a specific scale.
- **Goal:** Determining how much actual area is represented by one square unit in a scale drawing.

Suggested next steps

If students draw a frame without right angles, consider:

- Reminding them that the measures of angles remain unchanged after scaling.

If students do not draw the thickness of the frame to scale, consider:

- Reminding them all lengths, including the size of the frame, are changed by the scale factor.

If students draw a frame which is larger than the original frame, consider:

- Reviewing Activity 2, Problem 4.
- Assigning Practice Problems 1 and 2.

Professional Learning

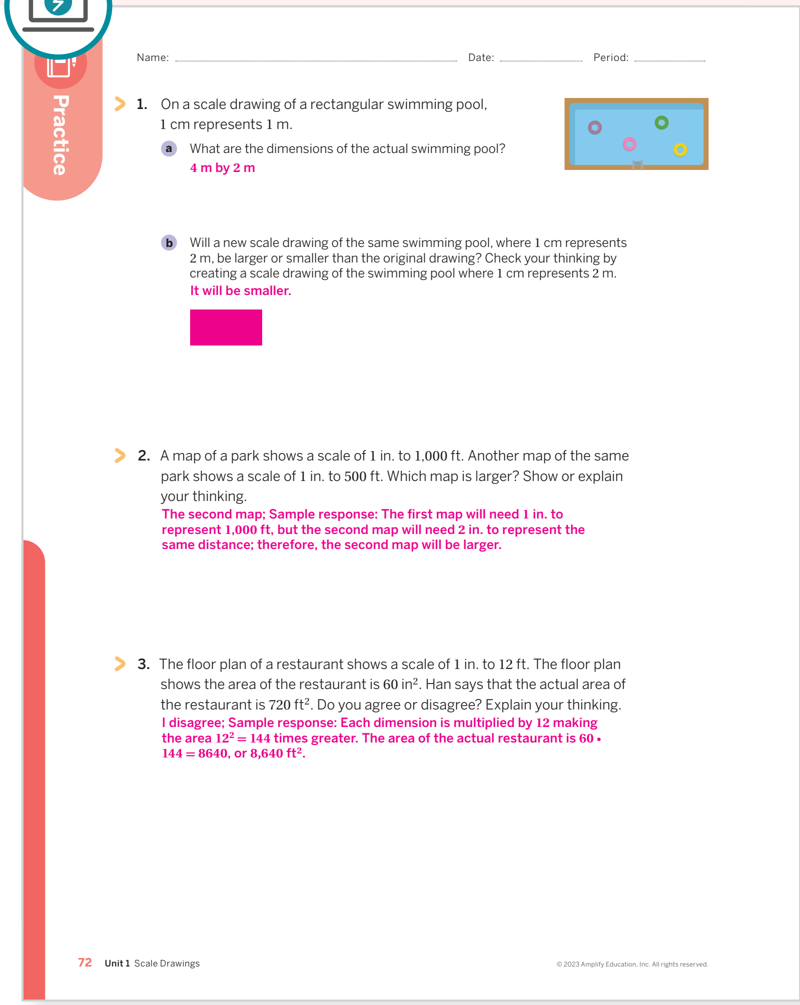
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach rescaling the buildings? What does that tell you about the similarities and differences among your students?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

Practice

Independent



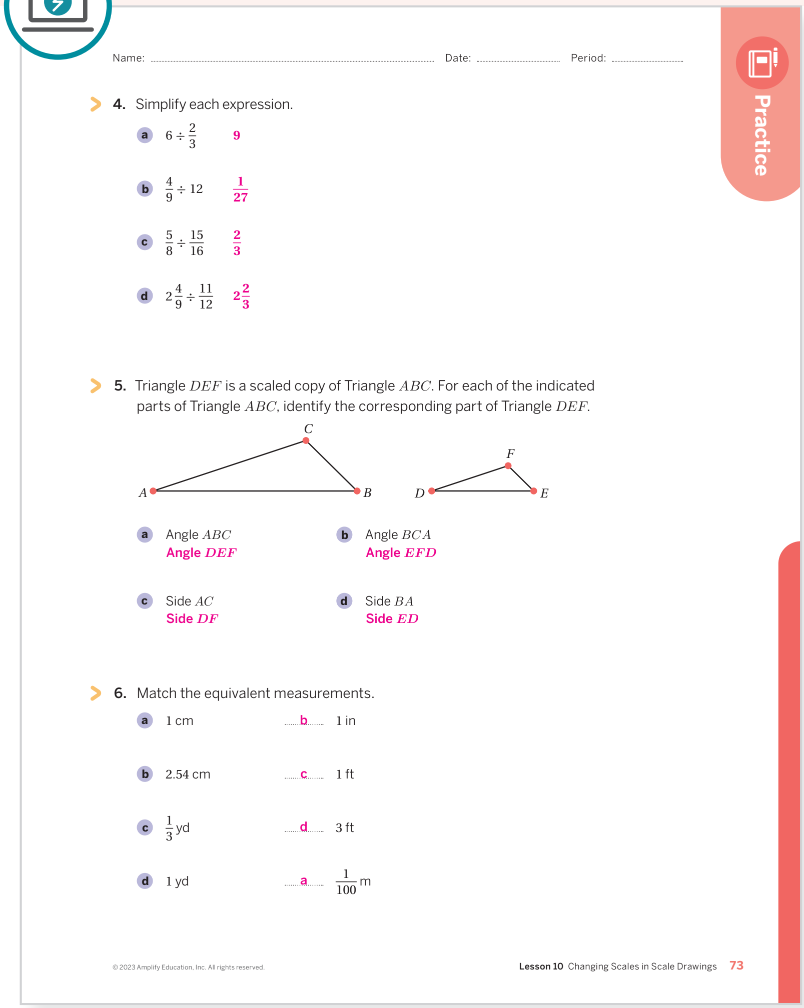
Name: ______ Date: ______ Period: ______

1. On a scale drawing of a rectangular swimming pool, 1 cm represents 1 m.
   - **a** What are the dimensions of the actual swimming pool?
     4 m by 2 m
   - **b** Will a new scale drawing of the same swimming pool, where 1 cm represents 2 m, be larger or smaller than the original drawing? Check your thinking by creating a scale drawing of the swimming pool where 1 cm represents 2 m.
     It will be smaller.

2. A map of a park shows a scale of 1 in. to 1,000 ft. Another map of the same park shows a scale of 1 in. to 500 ft. Which map is larger? Show or explain your thinking.
   The second map; Sample response: The first map will need 1 in. to represent 1,000 ft, but the second map will need 2 in. to represent the same distance; therefore, the second map will be larger.

3. The floor plan of a restaurant shows a scale of 1 in. to 12 ft. The floor plan shows the area of the restaurant is 60 in^2. Han says that the actual area of the restaurant is 720 ft^2. Do you agree or disagree? Explain your thinking.
   I disagree; Sample response: Each dimension is multiplied by 12 making the area $12^2 = 144$ times greater. The area of the actual restaurant is $60 \cdot 144 = 8640$, or 8,640 ft^2.

72 Unit 1 Scale Drawings



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4. Simplify each expression.
   - **a** $6 \div \frac{2}{3}$ **9**
   - **b** $\frac{4}{9} \div 12$ $\frac{1}{27}$
   - **c** $\frac{5}{8} \div \frac{15}{16}$ $\frac{2}{3}$
   - **d** $2\frac{4}{9} \div \frac{11}{12}$ $2\frac{2}{3}$

5. Triangle DEF is a scaled copy of Triangle ABC. For each of the indicated parts of Triangle ABC, identify the corresponding part of Triangle DEF.
   - **a** Angle ABC
     Angle DEF
   - **b** Angle BCA
     Angle EFD
   - **c** Side AC
     Side DF
   - **d** Side BA
     Side ED

6. Match the equivalent measurements.
   - **a** 1 cm ....**b**.... 1 in
   - **b** 2.54 cm ....**c**.... 1 ft
   - **c** $\frac{1}{3}$ yd ....**d**.... 3 ft
   - **d** 1 yd ....**a**.... $\frac{1}{100}$ m

Lesson 10 Changing Scales in Scale Drawings 73

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Grade 6	1
	5	Unit 1 Lesson 3	1
Formative ⚡	6	Unit 1 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 1 | **LESSON 11**

Scales Without Units

Let’s explore a different way to express scales.

Focus

Goals

1. **Language Goal:** Explain how to use scales without units to determine scaled or actual distances. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret scales expressed without units, e.g., “1 to 50.” **(Speaking and Listening, Writing)**

Coherence

Today

Students use scales expressed without units and attend to precision as they work between scales with units and those without units. Expressing the scale without units highlights the scale factor relating the scale drawing to the actual object. Students gain a better understanding of both scaled copies and scale drawings as they work to understand the common underlying structure.

Previously

In Lessons 7–10, students worked with scales associating two distinct measurements — one for the distance on a drawing and one for actual distance. In Lessons 1–6, students found scale factor between scaled copies.

Coming Soon

In Lesson 12, students will rewrite given scales as scales without units to make comparisons.

Rigor

- Students build **conceptual understanding** of how scales without units are related to scales with units.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- calculators

Math Language Development

New words

- ***equivalent scales***

  Note: Students will use an informal definition in this lesson, as this term will be formalized in Lesson 12.

Review words

- *scale*
- *scale drawing*

Amps | Featured Activity

Activity 2
Overlay Sketches

Students digitally draw their scaled copies using the sketch tool. When you overlay the results, students can compare their drawings with other students.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively try to give up because there is no scale given for part of Activity 2. Prepare them for the activity by discussing how self-discipline can help them stay focused on the task and pay close attention to details. This leads to a level of precision in their work and communication that bolsters success.

Modifications to Pacing

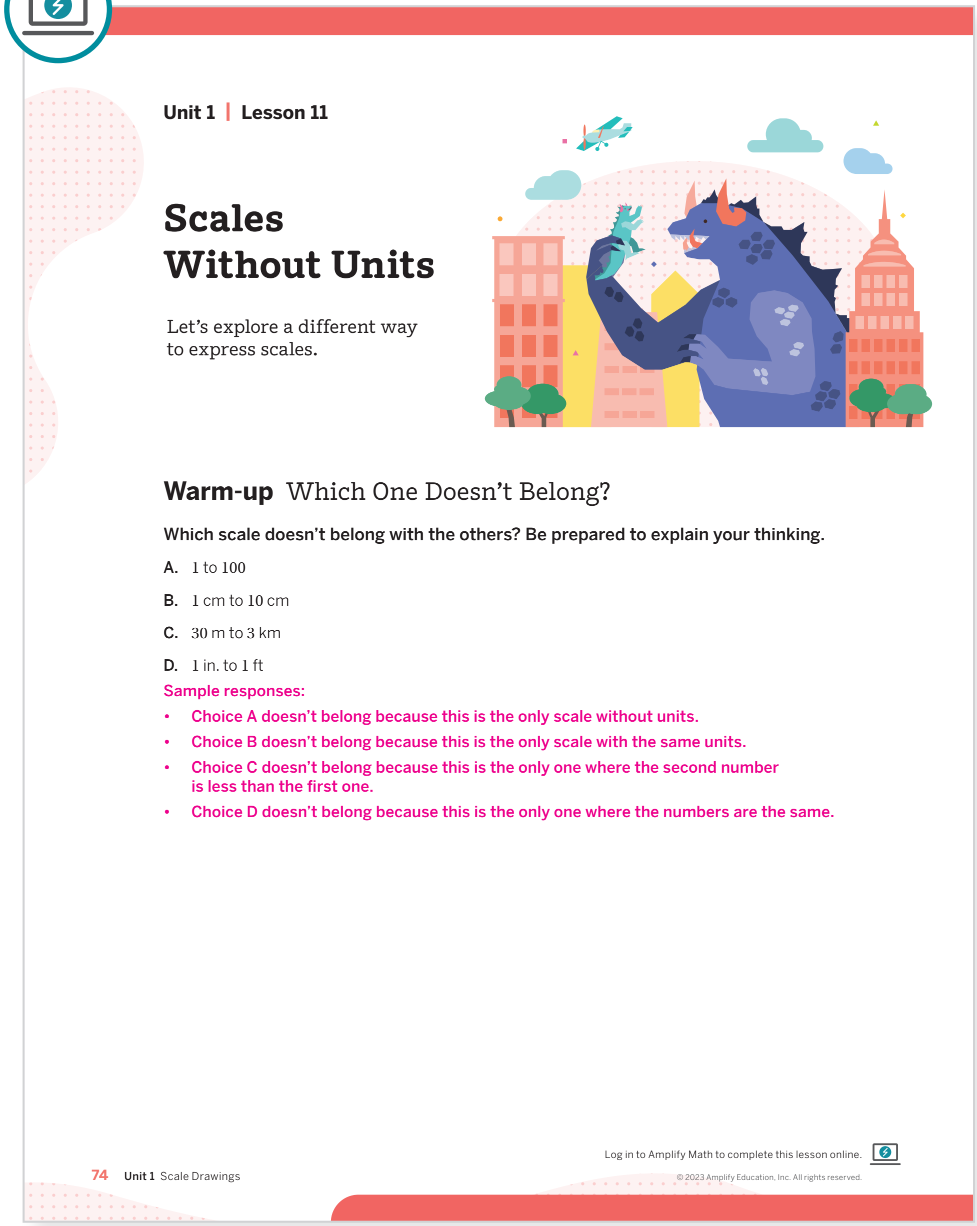
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problems 2 and 3 may be omitted.
- Complete **Activity 2** as a whole class to help guide students as they process the scales with and without units. Consider omitting Problem 2.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students decide which one of the scales does not belong in order to compare a scale without units.



Unit 1 | **Lesson 11**

Scales Without Units

Let's explore a different way to express scales.

Warm-up Which One Doesn't Belong?

Which scale doesn't belong with the others? Be prepared to explain your thinking.

A. 1 to 100

B. 1 cm to 10 cm

C. 30 m to 3 km

D. 1 in. to 1 ft

Sample responses:

- Choice A doesn't belong because this is the only scale without units.
- Choice B doesn't belong because this is the only scale with the same units.
- Choice C doesn't belong because this is the only one where the second number is less than the first one.
- Choice D doesn't belong because this is the only one where the numbers are the same.

Log in to Amplify Math to complete this lesson online.

74 **Unit 1** Scale Drawings

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1 Launch

Remind students to seek additional reasons for the other scales after they have discovered their first reason for one of the scales. Conduct the *"Which One Doesn't Belong?"* routine.

2 Monitor

Help students get started by asking, "Which scale stands out as being different? Why?"

Look for points of confusion:

- **Thinking there is only one right answer.** Ask students to think of reasons why each scale might not belong.

Look for productive strategies:

- Finding at least one reason for why each scale does not belong with the others.

3 Connect

Display the four scales. Conduct the *Poll the Class* routine to assess student thinking.

Have students share their reasoning for why each scale does not belong.

Highlight that the scales they have used so far have had units. Today, they will work with scales without units. If there is a scale without units, it is understood that both numerical values represent the same unit. 1 to 100 could mean 1 cm to 100 cm or 1 ft to 100 ft. But, it could not mean 1 cm to 1 m (if the scale is written without units).

Ask, "Could the scale 1 to 100 represent 1 in. to 100 ft?" No, because the units are not the same.

Math Language Development

MLR8: Discussion Supports

Choice A is the first time students will see scales without units. Facilitate a discussion that highlights how assumptions are made in mathematics when information is not provided. For example, with a variable, such as x, the coefficient is understood to be 1. To help students unpack the scale without units in Choice A, ask:

- "Can you have a scale without units?"
- "How can you make sense of a scale without units?"
- "Can you assume that the scale 1 to 100 represents 1 in. to 100 ft? Why or why not?"

Power-up

To power up students' ability to convert between units, have students complete:

Convert the following scales to scales with the same units.

1. 1 in. to 1 ft.
1 in. to 12 in., or $\frac{1}{12}$ ft to 1 ft

2. 1 in. to 2 ft.
1 in. to 24 in., or $\frac{1}{24}$ ft to 1 ft

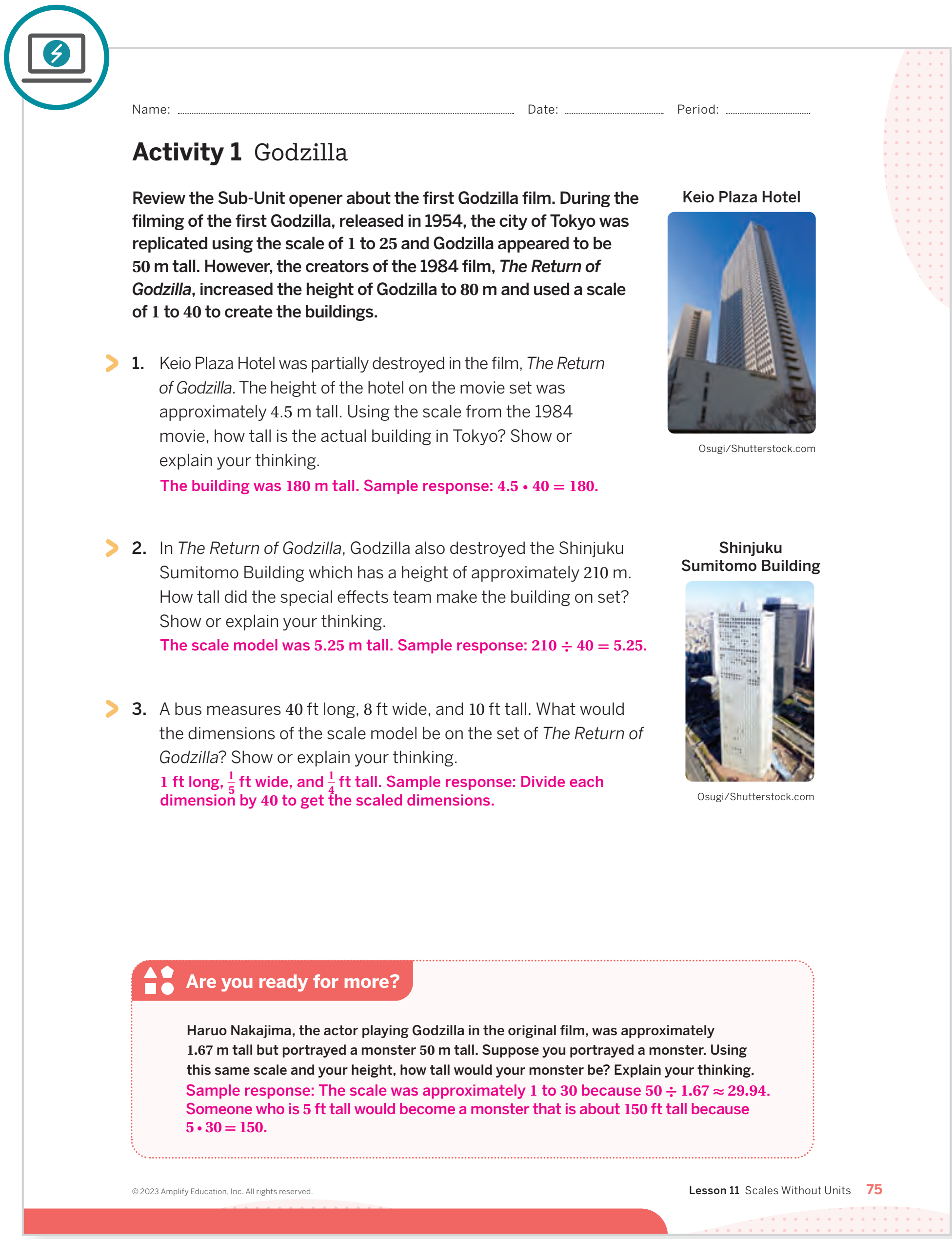
Use: Before the Warm-up.

Informed by: Performance on Lesson 10, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2.

Small Groups | 15 min

Activity 1 Godzilla

Students use the structure of the problem and a scale without units to calculate actual lengths or scaled lengths.



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Activity 1 Godzilla

Review the Sub-Unit opener about the first Godzilla film. During the filming of the first Godzilla, released in 1954, the city of Tokyo was replicated using the scale of 1 to 25 and Godzilla appeared to be 50 m tall. However, the creators of the 1984 film, *The Return of Godzilla*, increased the height of Godzilla to 80 m and used a scale of 1 to 40 to create the buildings.

Keio Plaza Hotel

Osugi/Shutterstock.com

1. Keio Plaza Hotel was partially destroyed in the film, *The Return of Godzilla*. The height of the hotel on the movie set was approximately 4.5 m tall. Using the scale from the 1984 movie, how tall is the actual building in Tokyo? Show or explain your thinking.
   The building was 180 m tall. Sample response: $4.5 \cdot 40 = 180$.

Shinjuku Sumitomo Building

Osugi/Shutterstock.com

2. In *The Return of Godzilla*, Godzilla also destroyed the Shinjuku Sumitomo Building which has a height of approximately 210 m. How tall did the special effects team make the building on set? Show or explain your thinking.
   The scale model was 5.25 m tall. Sample response: $210 \div 40 = 5.25$.

3. A bus measures 40 ft long, 8 ft wide, and 10 ft tall. What would the dimensions of the scale model be on the set of *The Return of Godzilla*? Show or explain your thinking.
   1 ft long, $\frac{1}{5}$ ft wide, and $\frac{1}{4}$ ft tall. Sample response: Divide each dimension by 40 to get the scaled dimensions.

Are you ready for more?

Haruo Nakajima, the actor playing Godzilla in the original film, was approximately 1.67 m tall but portrayed a monster 50 m tall. Suppose you portrayed a monster. Using this same scale and your height, how tall would your monster be? Explain your thinking.

Sample response: The scale was approximately 1 to 30 because $50 \div 1.67 \approx 29.94$. Someone who is 5 ft tall would become a monster that is about 150 ft tall because $5 \cdot 30 = 150$.

1 Launch

Activate students' background knowledge by asking them if they have seen the Godzilla films or how they think the films were made prior to CGI (computer-generated imagery). Consider displaying a picture of an early filming of Godzilla.

2 Monitor

Help students get started by asking how the scales given in this problem are different from the other scales they have seen.

Look for points of confusion:

- **Thinking they have to convert the units in Problem 3 to meters.** Point out that scale does not have units which means they do not need to convert.
- **Not understanding the phrase "to scale."** Explain it means "at the same scale" or "at the specified scale."

Look for productive strategies:

- Being able to easily find the requested measurements. Note students who do this and ask them to share how they know whether to multiply or divide by 40 in each problem.

3 Connect

Have students share their strategies and solutions.

Highlight the scaled measurements are multiplied by 40 to find the actual measurements. Finding the length on the scale models involves dividing the actual measurement by 40. Actual measurements translate to scaled measurements with a scale factor of $\frac{1}{40}$.

Ask:

- "Does it matter which units are used to measure the buildings?" No, because the scale is without units.
- "Why do you think the creators changed the scale from 1 to 25 in the original film to 1 to 40 in the 1984 film?" The creators did not want Godzilla to be dwarfed by the taller buildings. The new scale will make the buildings smaller than the 1 to 25 scale.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into more manageable parts by having students work on one problem at a time. After each problem, have a class discussion before moving on to the next problem. If students need more processing time, have them focus on completing Problems 1 and 2, and only work on Problems 3 and 4 as time allows.

Math Language Development

MLR8: Discussion Supports—Revoicing

For each response or observation that is shared, ask students to restate and/or revoice what they heard using their developing mathematical language.

English Learners

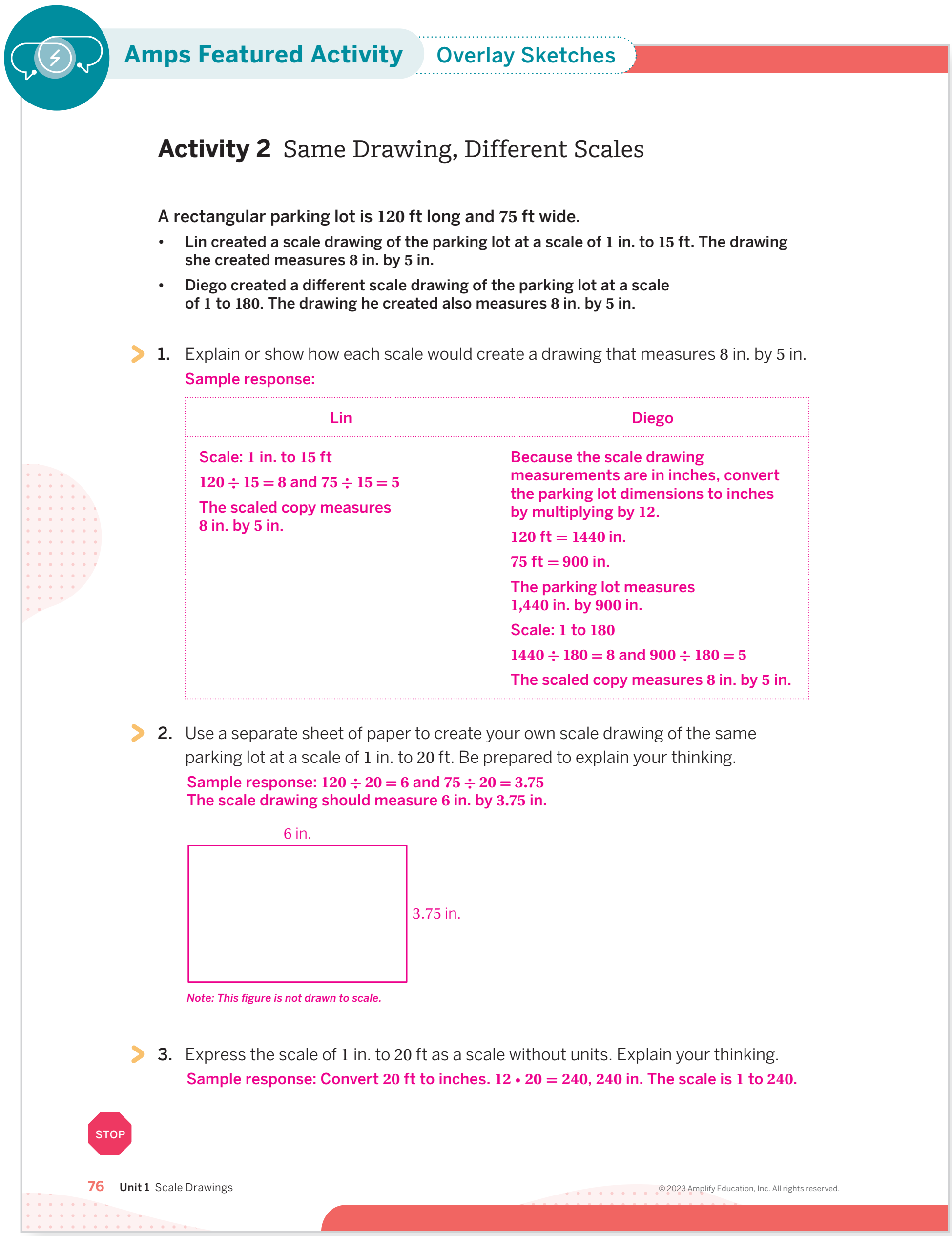
Spend time breaking down the phrase *to scale* as this may be unfamiliar. Explain that it means "at the same scale" or "at the specified scale." Add these phrases to the class display and encourage students to refer to these phrases during their discussions.

Small Groups | 15 min

Activity 2 Same Drawing, Different Scales

Students work with two equivalent scales (one with units and the other without) and make sense of how the two could yield the same scaled measurements of an actual object.

Amps Featured Activity Overlay Sketches



Activity 2 Same Drawing, Different Scales

A rectangular parking lot is 120 ft long and 75 ft wide.

- **Lin created a scale drawing of the parking lot at a scale of 1 in. to 15 ft. The drawing she created measures 8 in. by 5 in.**
- **Diego created a different scale drawing of the parking lot at a scale of 1 to 180. The drawing he created also measures 8 in. by 5 in.**

1. Explain or show how each scale would create a drawing that measures 8 in. by 5 in.
   Sample response:

Lin	Diego
Scale: 1 in. to 15 ft $120 \div 15 = 8$ and $75 \div 15 = 5$ The scaled copy measures 8 in. by 5 in.	Because the scale drawing measurements are in inches, convert the parking lot dimensions to inches by multiplying by 12. $120 \text{ ft} = 1440 \text{ in.}$ $75 \text{ ft} = 900 \text{ in.}$ The parking lot measures 1,440 in. by 900 in. Scale: 1 to 180 $1440 \div 180 = 8$ and $900 \div 180 = 5$ The scaled copy measures 8 in. by 5 in.

2. Use a separate sheet of paper to create your own scale drawing of the same parking lot at a scale of 1 in. to 20 ft. Be prepared to explain your thinking.
   Sample response: $120 \div 20 = 6$ and $75 \div 20 = 3.75$
   The scale drawing should measure 6 in. by 3.75 in.

6 in.

3.75 in.

Note: This figure is not drawn to scale.

3. Express the scale of 1 in. to 20 ft as a scale without units. Explain your thinking.
   Sample response: Convert 20 ft to inches. $12 \cdot 20 = 240$, 240 in. The scale is 1 to 240.

STOP

76 **Unit 1** Scale Drawings

1 Launch

Ask, "Is it possible to express the 1 to 50 scale from Activity 1 as a scale with units? If so, what units would you use?" Students are likely to say "1 in. to 50 in.," and "1 cm to 50 cm." Explain their next task is to explore how a scale without units and a scale with units could express the same relationship between scaled lengths and actual lengths.

2 Monitor

Help students get started by having them focus on one scale at a time. Ask, "How can the scale of 1 in. to 15 ft produce something which is 8 in. by 5 in.?"

Look for productive strategies:

- Noticing 15 ft is equal to 180 in. Thus, recognizing 1 in. to 15 ft is an equivalent scale to 1 in. to 180 in.

3 Connect

Have students share their responses and reasoning for Problem 1.

Highlight the need to attend to precision as they work with scales with units and without units. In the case of the scale 1 to 180, the actual lengths are 180 times as long as the scaled lengths (or the scaled lengths are $\frac{1}{180}$ times as long as the actual lengths). In the case of the scale 1 in. to 15 ft, converting the units helps identify the scale factor. Because 1 ft equals 12 in. and $15 \cdot 12 = 180$, the scale of 1 in. to 15 ft is equivalent to the scale of 1 in. to 180 in., or 1 to 180.

Ask, "How did you express the scale of 1 in. to 20 ft as a scale without units?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Represent the same given information from the beginning of the activity, but using a different modality: diagrams. Provide a diagram of the rectangular parking lot to students and ask them to label the dimensions. Then provide copies of the two scale drawings, each measuring 8 in. by 5 in. Ask students to annotate each drawing as they respond to Problem 1.

Extension: *Math Enrichment*

Have students write as many equivalent scales as they can for the rectangular parking lot scenario.

Math Language Development

MLR8: Discussion Supports

Allow students additional time to make sure that each group member can explain their responses for Problem 1. Invite groups to rehearse as rehearsing provides additional opportunities to speak and clarify thinking. Rehearsing will improve the quality of explanations shared during the whole class discussion. Provide feedback as students share to ensure they describe their steps and reasoning.

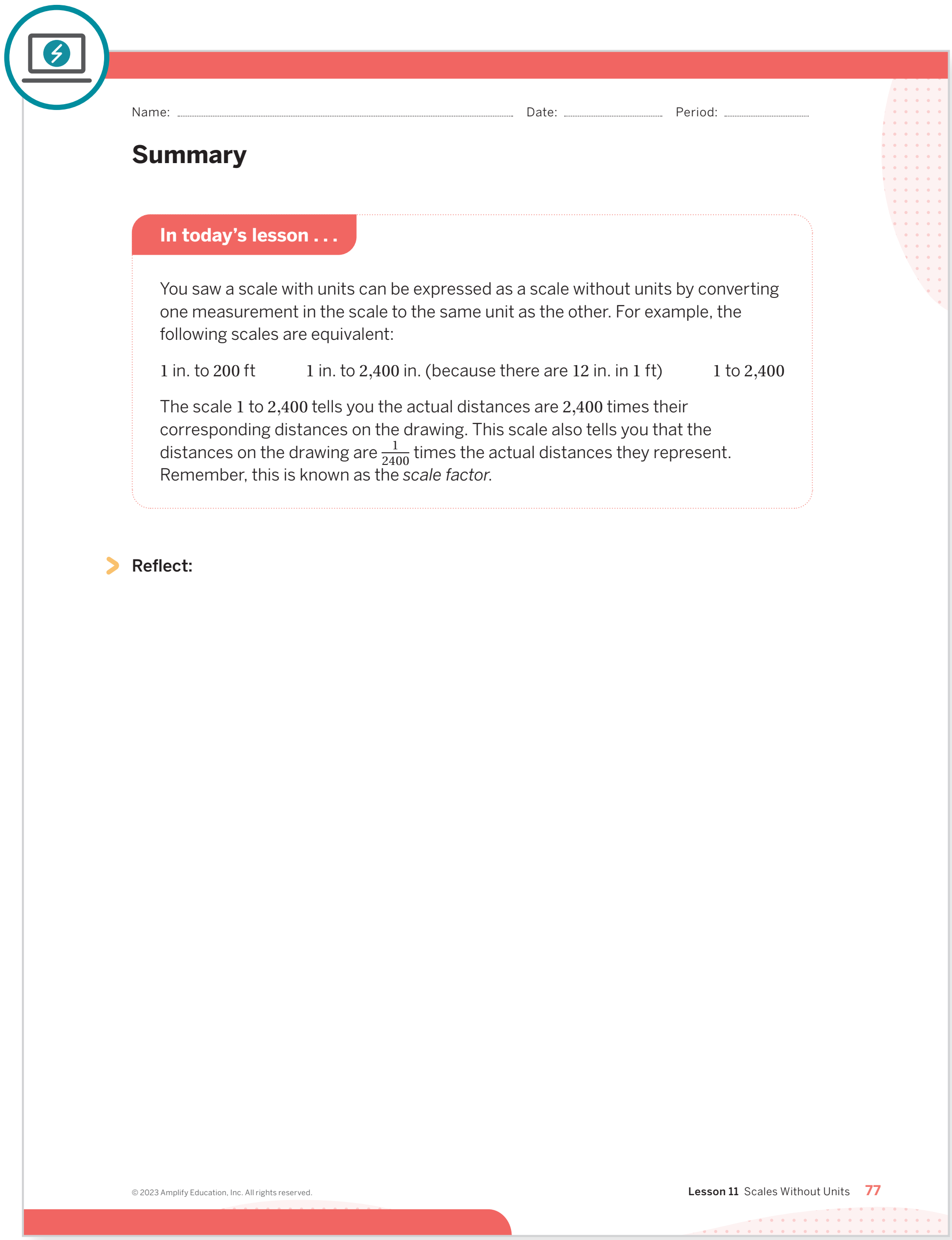
English Learners

Encourage the use of illustrations and annotations as students share their responses.

Whole Class | 5 min

Summary

Review and synthesize how to use and interpret scales without units.



Name: Date: Period:

Summary

In today's lesson . . .

You saw a scale with units can be expressed as a scale without units by converting one measurement in the scale to the same unit as the other. For example, the following scales are equivalent:

1 in. to 200 ft　　1 in. to 2,400 in. (because there are 12 in. in 1 ft)　　1 to 2,400

The scale 1 to 2,400 tells you the actual distances are 2,400 times their corresponding distances on the drawing. This scale also tells you that the distances on the drawing are $\frac{1}{2400}$ times the actual distances they represent. Remember, this is known as the *scale factor.*

Reflect:

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Lesson 11 Scales Without Units 77

Synthesize

Highlight that when a scale does not show units, it is understood that the same unit is used for both the scaled distance and the actual distance. For instance, a scale of 1 to 500 can mean many things:

- 1 in. on the drawing represents 500 in. of actual distance.
- 10 mm on the drawing represents 5,000 mm of actual distance.
- The actual distance is 500 times the distance on the drawing, and the scaled distance is $\frac{1}{500}$ of the actual distance.
- To calculate actual distances, students can multiply all distances on the drawing by the factor 500, regardless of the units given.
- To find scaled distances, students can multiply actual distances by $\frac{1}{500}$, regardless of the units given.
- 500 and $\frac{1}{500}$ are scale factors relating the actual and scaled measurements.

Note: Equivalent scales, or different scales relating scaled and actual measurements by the same scale factor, will be formalized in the next lesson.

Ask:

- "What does it mean when the scale on a scale drawing does not indicate any units?"
- "How can a scale without units be used to calculate scaled or actual distances?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is a scale without units the same as or different from a scale with units?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining actual lengths and reasoning about the sizes of scale drawings using scales without units.

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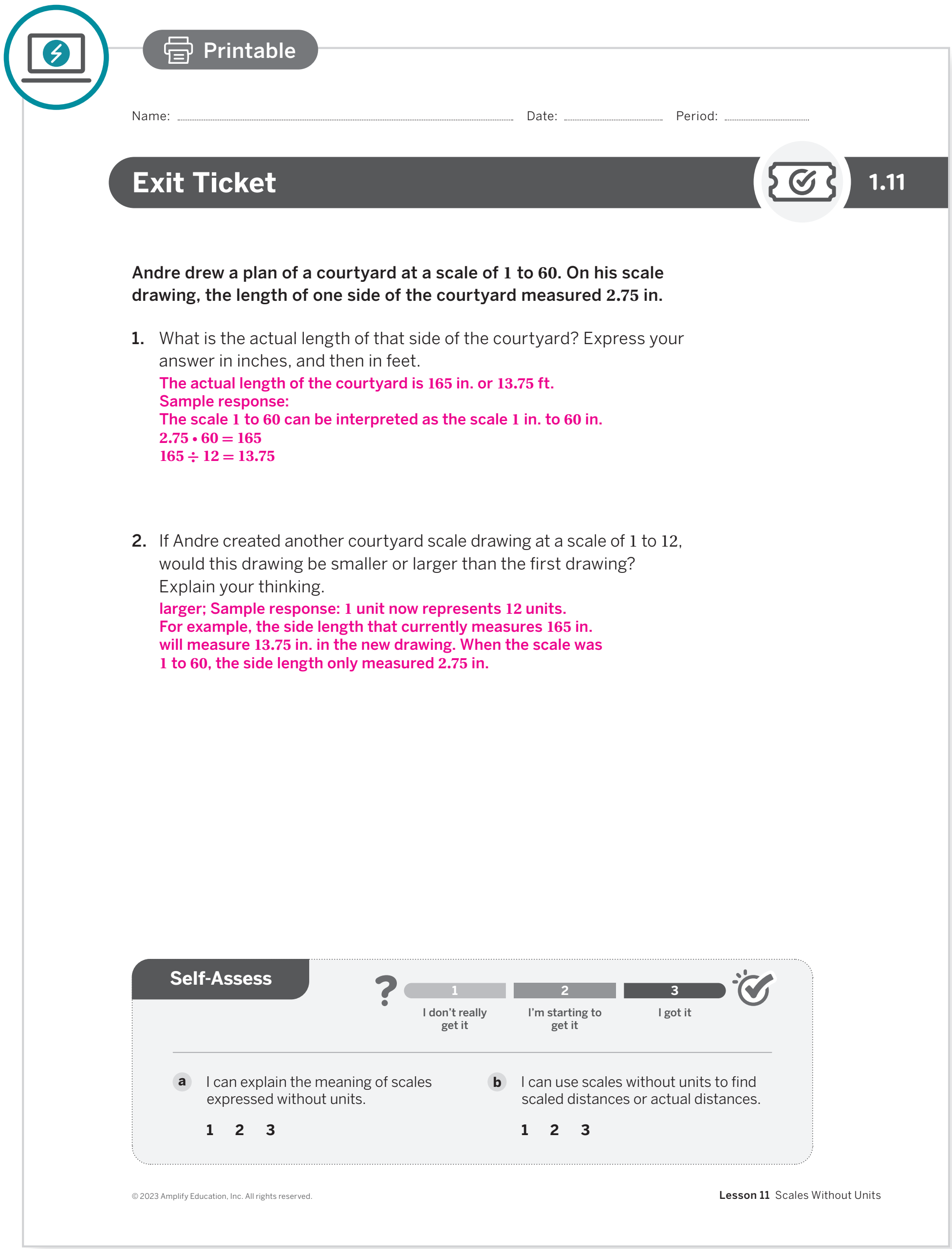


Name: ______ Date: ______ Period: ______

Exit Ticket 1.11

Andre drew a plan of a courtyard at a scale of 1 to 60. On his scale drawing, the length of one side of the courtyard measured 2.75 in.

1. What is the actual length of that side of the courtyard? Express your answer in inches, and then in feet.
   The actual length of the courtyard is 165 in. or 13.75 ft.
   Sample response:
   The scale 1 to 60 can be interpreted as the scale 1 in. to 60 in.
   $2.75 \cdot 60 = 165$
   $165 \div 12 = 13.75$

2. If Andre created another courtyard scale drawing at a scale of 1 to 12, would this drawing be smaller or larger than the first drawing? Explain your thinking.
   larger; Sample response: 1 unit now represents 12 units. For example, the side length that currently measures 165 in. will measure 13.75 in. in the new drawing. When the scale was 1 to 60, the side length only measured 2.75 in.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can explain the meaning of scales expressed without units.
1 2 3

b I can use scales without units to find scaled distances or actual distances.
1 2 3

Lesson 11 Scales Without Units

Success looks like . . .

- **Language Goal:** Explaining how to use scales without units to determine scaled or actual distances. **(Speaking and Listening, Writing)**
  - » Determining the actual length of the courtyard side in inches and in feet in Problem 1.
- **Language Goal:** Interpreting scales expressed without units, e.g., "1 to 50." **(Speaking and Listening, Writing)**
  - » Determining the actual length of the courtyard in inches and in feet in Problem 1.

Suggested next steps

If students respond to Problem 1 incorrectly, consider:

- Assigning Practice Problem 2.

If students respond to Problem 2 incorrectly, consider:

- Assigning Practice Problem 1.

Professional Learning

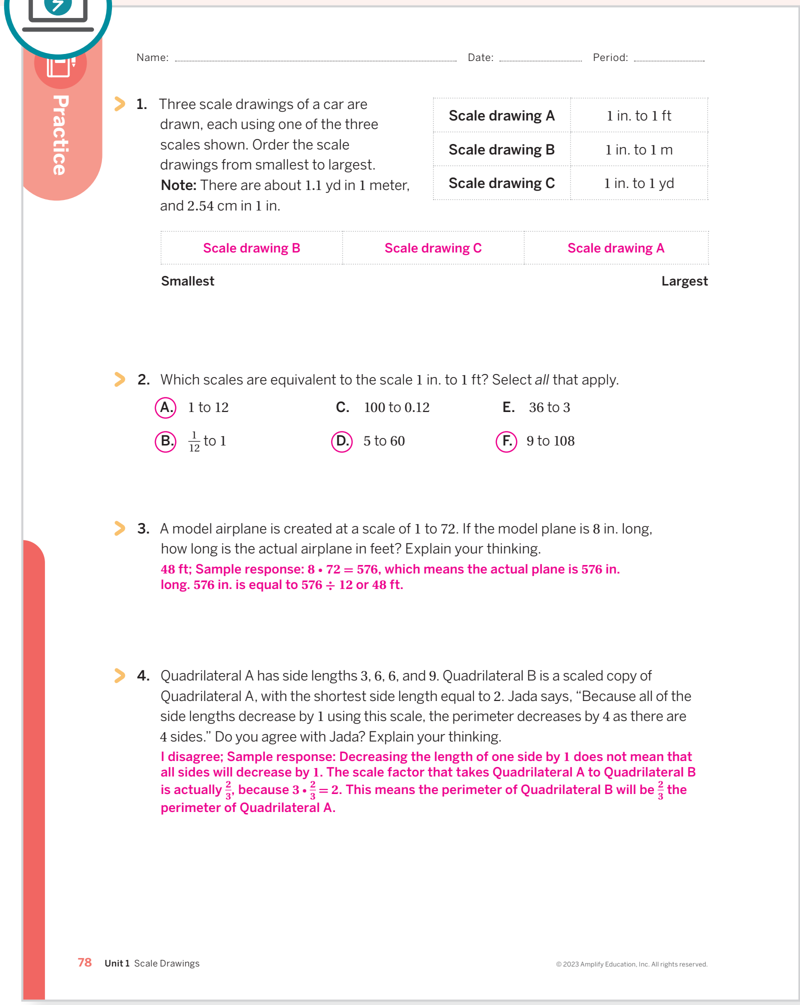
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on converting to scales without units in Activity 1?
- In earlier lessons, students drew scaled copies. How did that support writing scales without units? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Three scale drawings of a car are drawn, each using one of the three scales shown. Order the scale drawings from smallest to largest.
Note: There are about 1.1 yd in 1 meter, and 2.54 cm in 1 in.

Scale drawing A	1 in. to 1 ft
Scale drawing B	1 in. to 1 m
Scale drawing C	1 in. to 1 yd

Scale drawing B	**Scale drawing C**	**Scale drawing A**

Smallest **Largest**

2. Which scales are equivalent to the scale 1 in. to 1 ft? Select *all* that apply.

[✓ A.] 1 to 12
[✓ B.] $\frac{1}{12}$ to 1
C. 100 to 0.12
[✓ D.] 5 to 60
E. 36 to 3
[✓ F.] 9 to 108

3. A model airplane is created at a scale of 1 to 72. If the model plane is 8 in. long, how long is the actual airplane in feet? Explain your thinking.
48 ft; Sample response: $8 \cdot 72 = 576$, which means the actual plane is 576 in. long. 576 in. is equal to $576 \div 12$ or 48 ft.

4. Quadrilateral A has side lengths 3, 6, 6, and 9. Quadrilateral B is a scaled copy of Quadrilateral A, with the shortest side length equal to 2. Jada says, "Because all of the side lengths decrease by 1 using this scale, the perimeter decreases by 4 as there are 4 sides." Do you agree with Jada? Explain your thinking.
I disagree; Sample response: Decreasing the length of one side by 1 does not mean that all sides will decrease by 1. The scale factor that takes Quadrilateral A to Quadrilateral B is actually $\frac{2}{3}$, because $3 \cdot \frac{2}{3} = 2$. This means the perimeter of Quadrilateral B will be $\frac{2}{3}$ the perimeter of Quadrilateral A.

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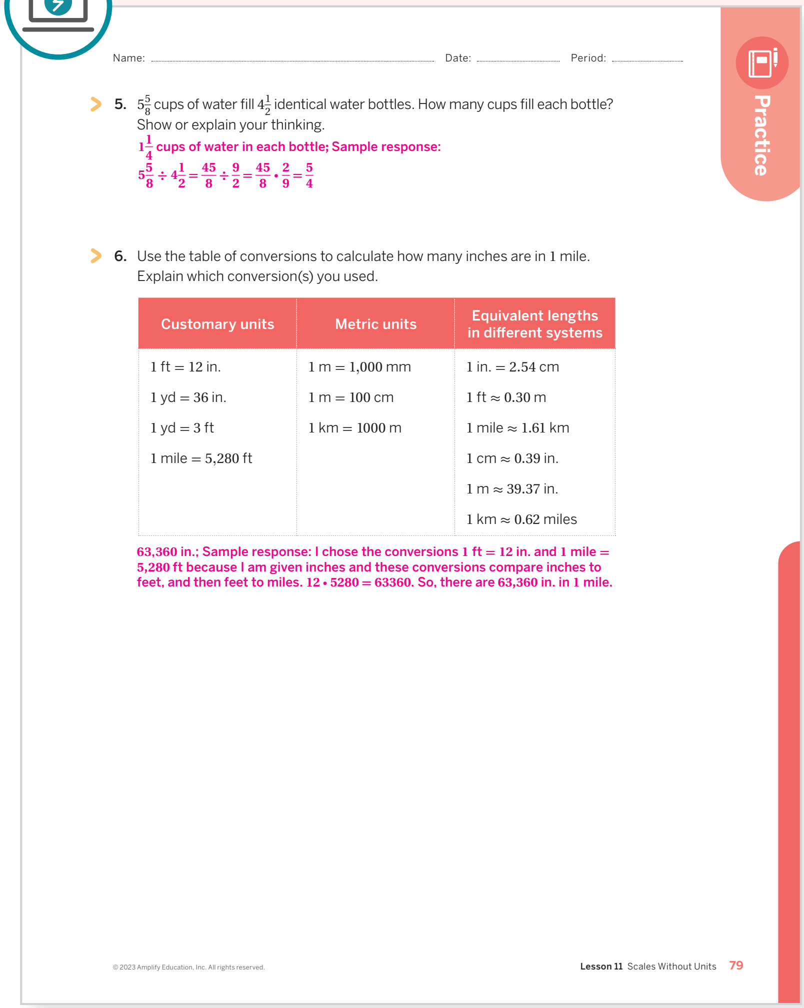

Practice

Name: Date: Period:

5. $5\frac{5}{8}$ cups of water fill $4\frac{1}{2}$ identical water bottles. How many cups fill each bottle? Show or explain your thinking.
$1\frac{1}{4}$ cups of water in each bottle; Sample response:
$5\frac{5}{8} \div 4\frac{1}{2} = \frac{45}{8} \div \frac{9}{2} = \frac{45}{8} \cdot \frac{2}{9} = \frac{5}{4}$

6. Use the table of conversions to calculate how many inches are in 1 mile. Explain which conversion(s) you used.

Customary units	Metric units	Equivalent lengths in different systems
1 ft = 12 in.	1 m = 1,000 mm	1 in. = 2.54 cm
1 yd = 36 in.	1 m = 100 cm	1 ft ≈ 0.30 m
1 yd = 3 ft	1 km = 1000 m	1 mile ≈ 1.61 km
1 mile = 5,280 ft		1 cm ≈ 0.39 in.
		1 m ≈ 39.37 in.
		1 km ≈ 0.62 miles

63,360 in.; Sample response: I chose the conversions 1 ft = 12 in. and 1 mile = 5,280 ft because I am given inches and these conversions compare inches to feet, and then feet to miles. $12 \cdot 5280 = 63360$. So, there are 63,360 in. in 1 mile.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 4	2
	5	Grade 6	2
Formative ⚡	6	Unit 1 Lesson 12	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Units in Scale Drawings

Let's see how different scales can describe the same relationship.

Focus

Goals

1. **Language Goal:** Comprehend that the phrase *equivalent scales* refers to different scales that relate scaled and actual measurements by the same scale factor. **(Speaking and Listening, Reading)**
2. Generate a scale without units that is equivalent to a given scale with units, or vice versa.
3. **Language Goal:** Justify that scales are equivalent, including scales with and without units. **(Speaking and Listening, Writing)**

Coherence

Today

Students analyze various scales and find that sometimes it is helpful to rewrite scales with units as scales without units in order to compare them. They see that equivalent scales relate scaled and actual measurements by the same scale factor, even though the scales may be expressed differently.

Previously

In Lesson 11, students saw that expressing a scale without units highlighted the scale factor that related the scale drawing to the actual object.

Coming Soon

In Lesson 13, students will conclude the unit with a design and scaling lesson, using many of the skills and understandings acquired throughout this unit.

Rigor

- Students build **conceptual understanding** of equivalent scales to see that they relate scaled and actual measurements by the same scale factor.
- Students convert units to create equivalent scales to build **procedural fluency.**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
7 min	25 min	5 min	8 min
Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one per student
- small objects for marking bingo cards, e.g., linking cubes or paper clips

Math Language Development

New word

- *equivalent scales*

Review word

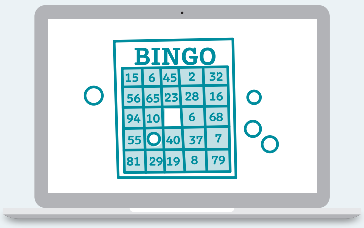
- *scale factor*

Amps | Featured Activity

Activity 1

Interactive Bingo Cards

Students digitally draw their scaled copies using the sketch tool. When you overlay the results, students can compare their drawings with other students.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students provide feedback and critique their classmates' responses, their manner in doing so may not yet be fully developed. Instead, they may inadvertently cause others to feel bad for any mistakes or errors they have made. Emphasize that learning can involve making mistakes, and that we have all made them. Have students discuss how they like to be treated when they make mistakes. Then explain that they should treat others that same way.

Modifications to Pacing

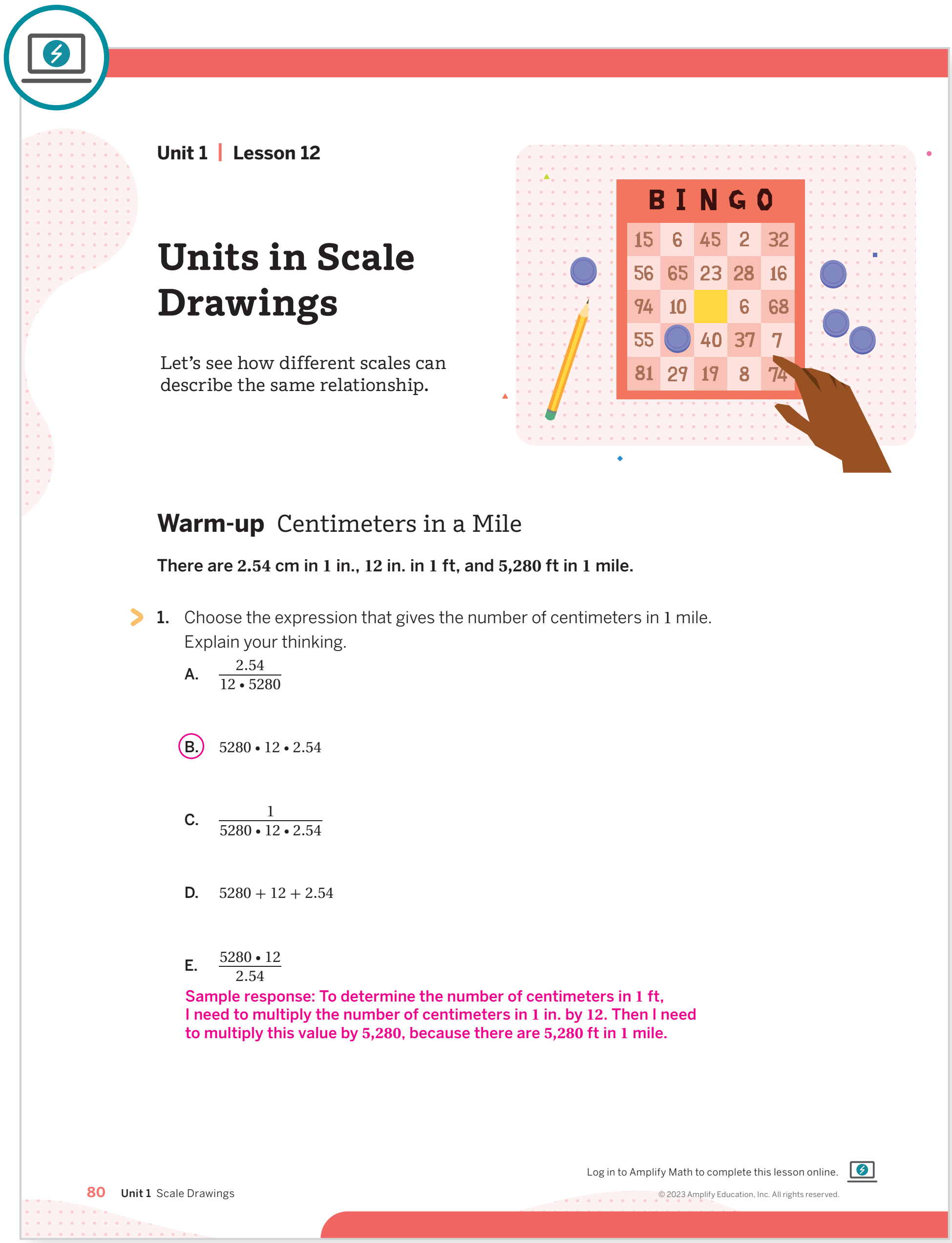
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, the bingo game can be timed. Set a timer at the beginning of the activity and have students count how many boxes they have marked at the end of the timer.

Pairs | 7 min

Warm-up Centimeters in a Mile

Students review how to convert units by analyzing expressions that could be used to accomplish the conversion.



Unit 1 | **Lesson 12**

Units in Scale Drawings

Let's see how different scales can describe the same relationship.

Warm-up Centimeters in a Mile

There are 2.54 cm in 1 in., 12 in. in 1 ft, and 5,280 ft in 1 mile.

1. Choose the expression that gives the number of centimeters in 1 mile. Explain your thinking.

   A. $\frac{2.54}{12 \cdot 5280}$

   [✓ B.] $5280 \cdot 12 \cdot 2.54$

   C. $\frac{1}{5280 \cdot 12 \cdot 2.54}$

   D. $5280 + 12 + 2.54$

   E. $\frac{5280 \cdot 12}{2.54}$

   Sample response: To determine the number of centimeters in 1 ft, I need to multiply the number of centimeters in 1 in. by 12. Then I need to multiply this value by 5,280, because there are 5,280 ft in 1 mile.

80 **Unit 1** Scale Drawings

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate background knowledge by asking if students have ever visited another country that used a different measuring system than they were used to. Read the introduction, and then conduct the ***Poll the Class*** routine by asking students to predict whether the number of centimeters in a mile is greater than or less than 5,280.

2 Monitor

Help students get started by focusing on the first step. Ask, "How can you find how many centimeters are in one foot?"

Look for points of confusion:

- **Thinking that division is needed to convert the units.** Show students a scale comparing centimeters to inches, then ask to find how many centimeters would be in 12 in. Ask, "What operation helps to make this calculation most efficiently?"

Look for productive strategies:

- Reasoning about the total number of centimeters in a mile, then noticing that only one of the expressions can reasonably produce the value.

3 Connect

Display the results of the poll from the Launch.

Ask, "Did anyone change their mind after they took the poll? If so, why?"

Have students share how the expression connects to the reasoning of how to find the number of centimeters in a mile.

Highlight that because 5,280 ft is equal to 1 mile the scale 1 cm to 1 mile represents the same as the scale 1 cm to 5,280 ft.

Define the term ***equivalent scales*** as different scales that relate scaled and actual measurements by the same scale factor.

Power-up

To power up students' ability to convert between multiple different sized standard units of measure within a given measurement system, have students complete:

1. There are 2.54 cm in 1 in., and 12 in. in 1 ft. Write an expression (do not complete the calculations) for the number of centimeters in 1 ft.
   $2.54 \cdot 12$
2. There are 12 in. in 1 ft, and 5,280 ft in 1 mile. Write an expression (do not complete the calculations) for the number of inches in 1 mile.
   $12 \cdot 5280$

Use: Before the Warm-up.

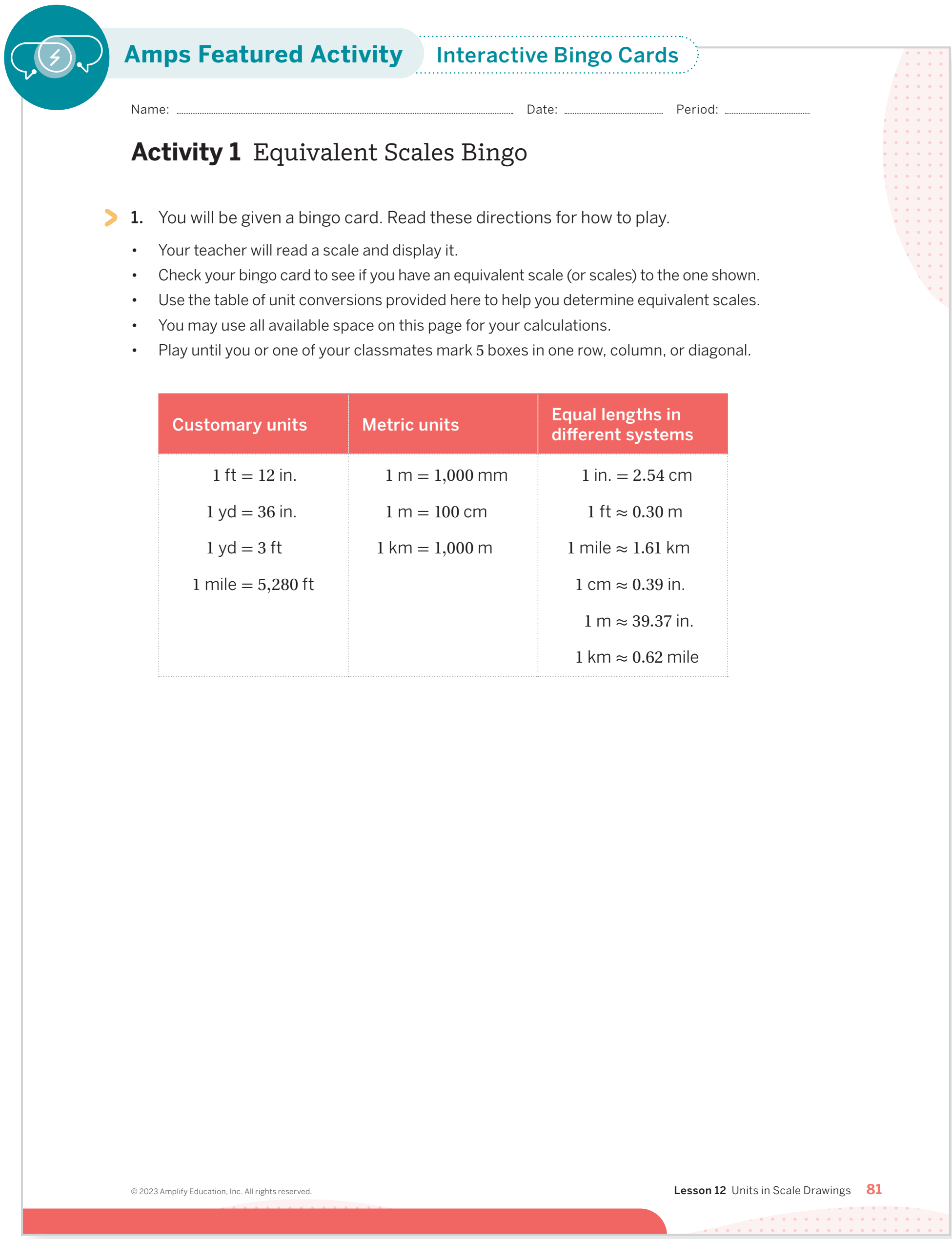
Informed by: Performance on Lesson 11, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3.

Pairs | 25 min

Activity 1 Equivalent Scales Bingo

Students play a game of bingo and notice how converting to scales without units helps to find equivalent scales.

Amps Featured Activity Interactive Bingo Cards



Name: ______________________ Date: __________ Period: __________

Activity 1 Equivalent Scales Bingo

1. You will be given a bingo card. Read these directions for how to play.
   - Your teacher will read a scale and display it.
   - Check your bingo card to see if you have an equivalent scale (or scales) to the one shown.
   - Use the table of unit conversions provided here to help you determine equivalent scales.
   - You may use all available space on this page for your calculations.
   - Play until you or one of your classmates mark 5 boxes in one row, column, or diagonal.

Customary units	Metric units	Equal lengths in different systems
1 ft = 12 in.	1 m = 1,000 mm	1 in. = 2.54 cm
1 yd = 36 in.	1 m = 100 cm	1 ft ≈ 0.30 m
1 yd = 3 ft	1 km = 1,000 m	1 mile ≈ 1.61 km
1 mile = 5,280 ft		1 cm ≈ 0.39 in.
		1 m ≈ 39.37 in.
		1 km ≈ 0.62 mile

1 Launch

Distribute the Activity 1 PDF and objects for marking cards to students. Model how the games will proceed with a sped-up practice round, and remind them that the scales will remain visible throughout the game.

2 Monitor

Help students get started by having them identify where to find different units in the conversion table. You might have them list the different units in each table at the top of the column, for easy reference.

Look for points of confusion:

- **Thinking that each scale shown will only have one equivalent scale on the bingo card.** Have students continue to look for additional equivalent scales even after they have found one. Let them know that some scales have 2 or 3 equivalent scales.

Look for productive strategies:

- Converting each scale to a scale without units first.
- Estimating the magnitude of the scale factor before computing the exact scale without units.
- Annotating Tyler's work, in Problem 2, to notice where the students would have performed a different action.

Activity 1 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Differentiate the degree of complexity by beginning with single-step conversions. Alternatively, have students concentrate on finding one equivalent scale for each scale you display. After all of the scales have been displayed, have students connect their unmarked boxes to other scales that are equivalent.

Extension: *Math Enrichment*

Allow students to play on more than one bingo card at a time.

Math Language Development

MLR8: Discussion Supports

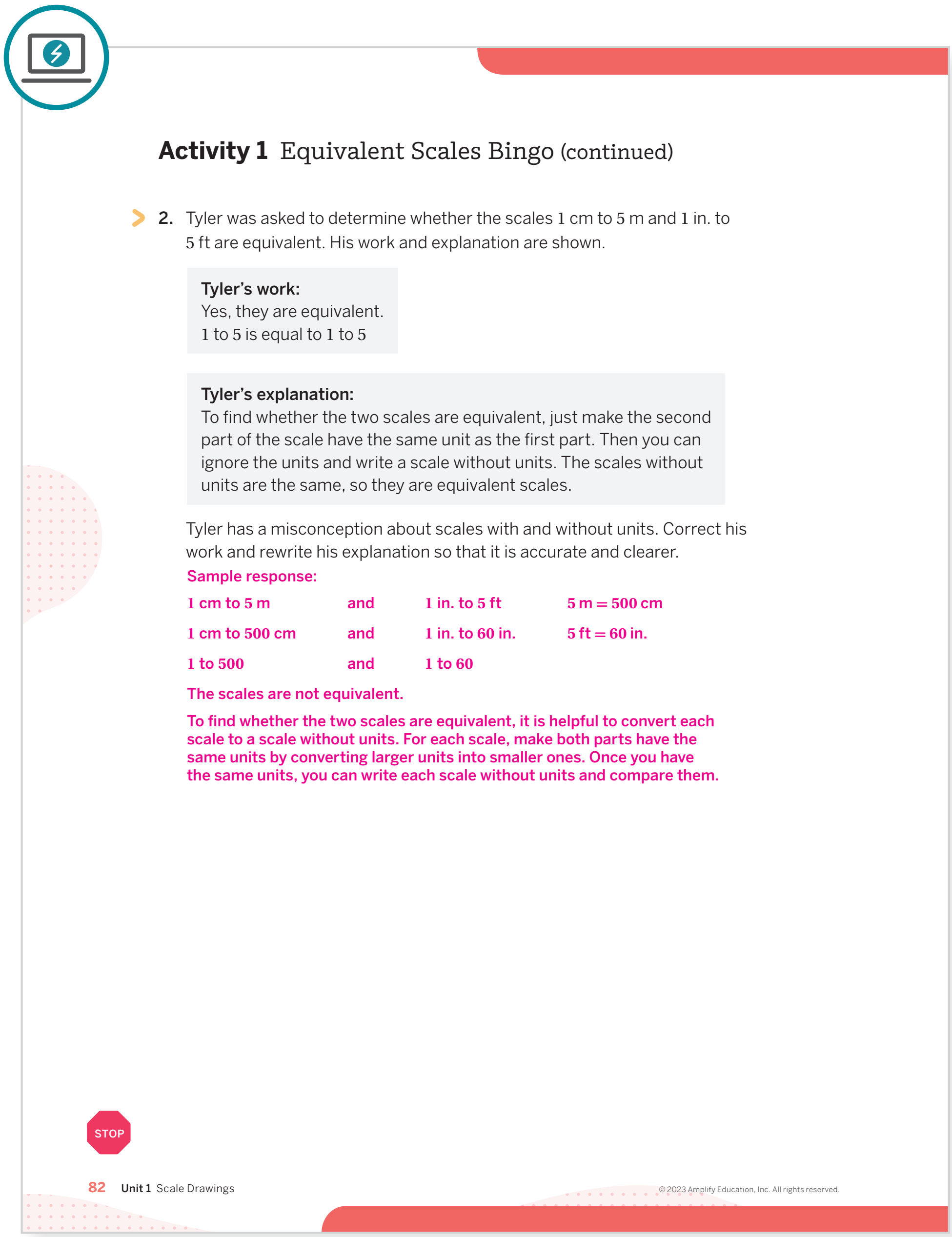
Conduct a meta-think-aloud analyzing one of Tyler's misconceptions in Problem 1. The meta-think-aloud should be collaborative in nature, modeling for students what productive struggle looks like and inviting them to participate in the struggle. For example, math thinkers might consider thinking aloud:

- "I noticed Tyler changed 1 cm to 5 m to 1 cm to 5 cm. Is this valid?"
- "I wonder if it matters if we convert the scale with larger units to smaller units, or vice versa?"
- "Once the units are the same, what can we do?"

Pairs | 25 min

Activity 1 Equivalent Scales Bingo (continued)

Students play a game of bingo and notice how converting to scales without units helps to find equivalent scales.



Activity 1 Equivalent Scales Bingo (continued)

2. Tyler was asked to determine whether the scales 1 cm to 5 m and 1 in. to 5 ft are equivalent. His work and explanation are shown.

Tyler's work:
Yes, they are equivalent.
1 to 5 is equal to 1 to 5

Tyler's explanation:
To find whether the two scales are equivalent, just make the second part of the scale have the same unit as the first part. Then you can ignore the units and write a scale without units. The scales without units are the same, so they are equivalent scales.

Tyler has a misconception about scales with and without units. Correct his work and rewrite his explanation so that it is accurate and clearer.

Sample response:

1 cm to 5 m	and	1 in. to 5 ft	5 m = 500 cm
1 cm to 500 cm	and	1 in. to 60 in.	5 ft = 60 in.
1 to 500	and	1 to 60	

The scales are not equivalent.

To find whether the two scales are equivalent, it is helpful to convert each scale to a scale without units. For each scale, make both parts have the same units by converting larger units into smaller ones. Once you have the same units, you can write each scale without units and compare them.

STOP

82 **Unit 1** Scale Drawings

3 Connect

Display Tyler's work and explanation from Problem 2.

Have pairs of students share how they corrected Tyler's work. Display their changes visually. Then have another pair of students explain those changes. This challenges students to be careful listeners and interpreters of others' thinking.

Highlight that to find if two scales are equivalent i.e., having the same scale factor, it makes sense to convert each to a scale without units first.

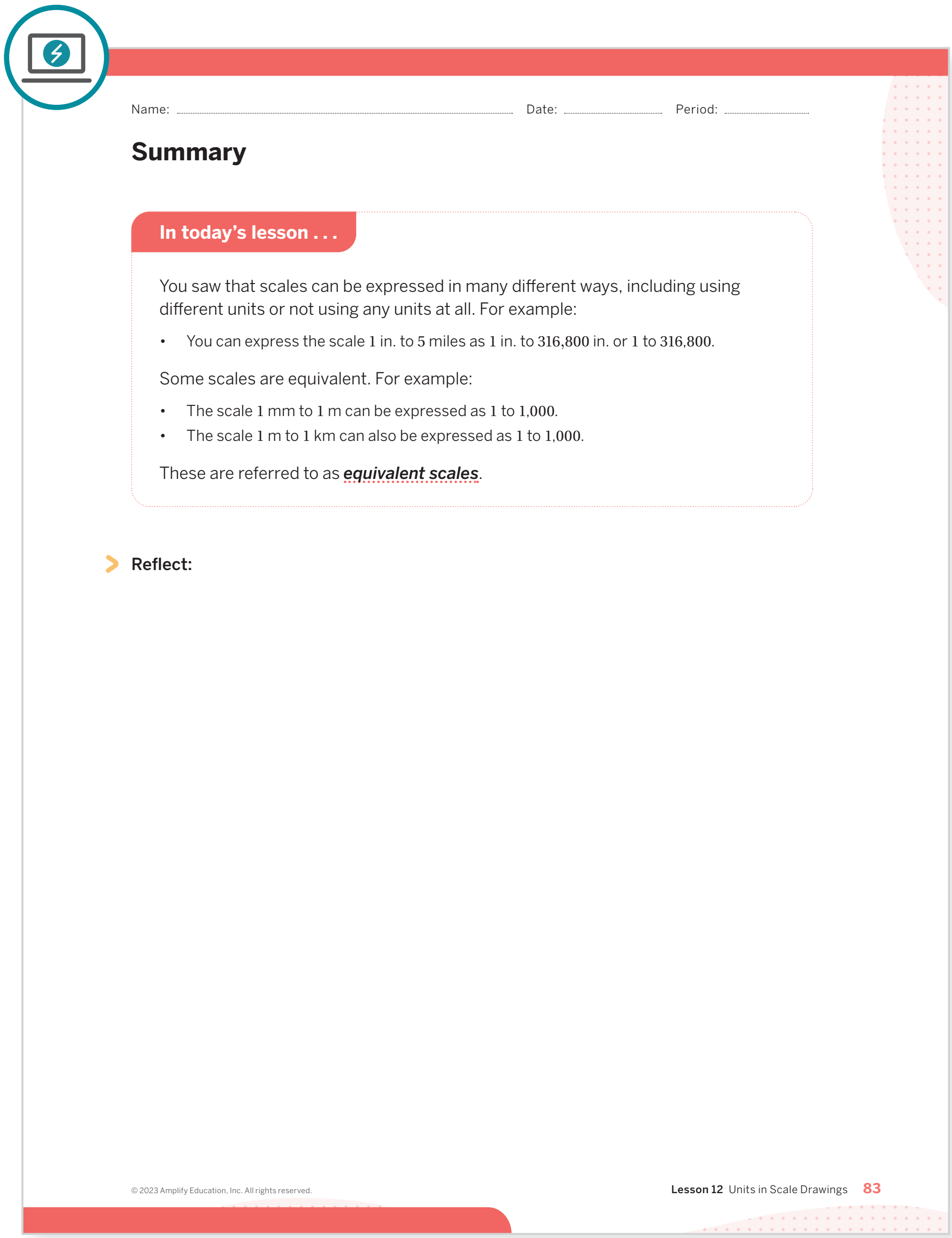
Ask:

- "Why might a conversion table, like the one in Activity 1, not include all possible conversions?" There are too many conversions to list all possible conversions. The table lists just the ones you might need to find.
- "Are there any conversion scales that you would add to a personal conversion chart, with a conversion you find particularly helpful? Why?"

Whole Class | 5 min

Summary

Review and synthesize how to find if two scales with different units are equivalent scales.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that scales can be expressed in many different ways, including using different units or not using any units at all. For example:

- You can express the scale 1 in. to 5 miles as 1 in. to 316,800 in. or 1 to 316,800.

Some scales are equivalent. For example:

- The scale 1 mm to 1 m can be expressed as 1 to 1,000.
- The scale 1 m to 1 km can also be expressed as 1 to 1,000.

These are referred to as ***equivalent scales***.

Reflect:

Synthesize

Highlight that when converting units, students need to pay attention to words and symbols. A scale is represented with words like "to" or "represents." An equality statement is represented with an equal sign. For example, if a scale is 1 in. to 2 ft, students know that 2 ft = 24 in., so they can rewrite this scale as 1 in. to 24 in.

Formalize vocabulary: *equivalent scales*

Ask, "How are scales without units like a scale factor?" Sample response: Scales without units are like a scale factor because I only need to find the factor that relates the first number to the second number. When the first number is 1, this is even easier, and the scale factor is simply the same as the second number.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do conversion tables help you to understand scales?"
- "How are conversion tables different or the same as scales?"

Math Language Development

MLR2: Collect and Display

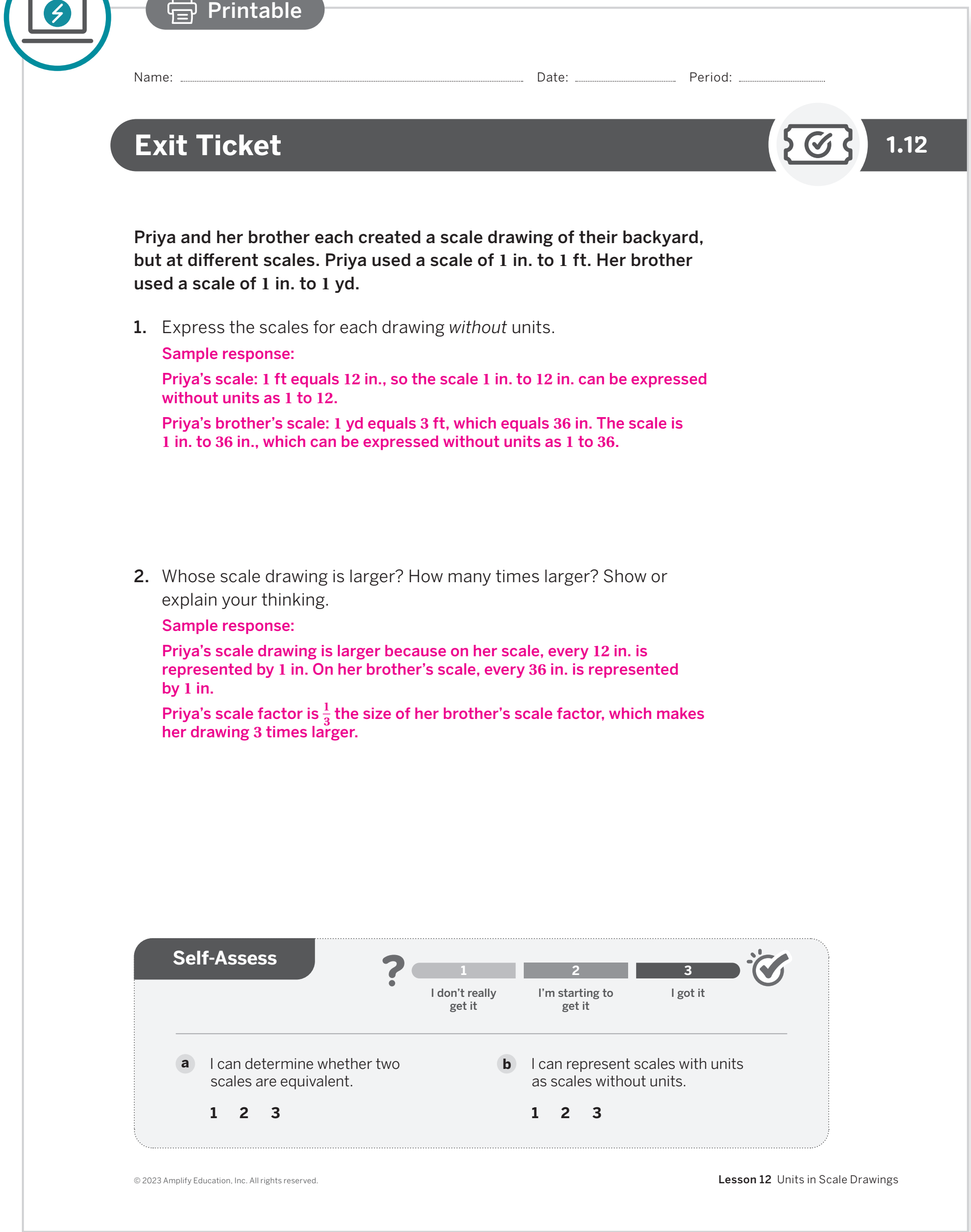
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the term *equivalent scales* that were added to the display during the lesson.

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of comparing scales by reasoning about which scale will produce a larger scaled copy.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.12

Priya and her brother each created a scale drawing of their backyard, but at different scales. Priya used a scale of 1 in. to 1 ft. Her brother used a scale of 1 in. to 1 yd.

1. Express the scales for each drawing *without* units.

   Sample response:

   Priya's scale: 1 ft equals 12 in., so the scale 1 in. to 12 in. can be expressed without units as 1 to 12.

   Priya's brother's scale: 1 yd equals 3 ft, which equals 36 in. The scale is 1 in. to 36 in., which can be expressed without units as 1 to 36.

2. Whose scale drawing is larger? How many times larger? Show or explain your thinking.

   Sample response:

   Priya's scale drawing is larger because on her scale, every 12 in. is represented by 1 in. On her brother's scale, every 36 in. is represented by 1 in.

   Priya's scale factor is $\frac{1}{3}$ the size of her brother's scale factor, which makes her drawing 3 times larger.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can determine whether two scales are equivalent.

1 2 3

b I can represent scales with units as scales without units.

1 2 3

Lesson 12 Units in Scale Drawings

Success looks like . . .

- **Language Goal:** Comprehending that the phrase *equivalent scales* refers to different scales that relate scaled and actual measurements by the same scale factor. **(Speaking and Listening, Reading)**
- **Goal:** Generating a scale without units that is equivalent to a given scale with units, or vice versa.
  - » Expressing the scales for the scale drawings of Priya and her brother in Problem 1.
- **Language Goal:** Justifying that scales are equivalent, including scales with and without units. **(Speaking and Listening, Writing)**

Suggested next steps

If students think that the scale with yards will produce a larger drawing, consider

- Asking them to think about drawing an object $\frac{1}{12}$ the size it actually is and also $\frac{1}{36}$ the size. Ask, "Which drawing will be larger?"

If students continue to have difficulty reasoning about the size of the drawings, consider:

- Having them compare two simpler scales with the same units and have them generalize about the scales in the problem.

If students misunderstand that the scale factor being $\frac{1}{3}$ the size makes the lengths 3 times longer, consider:

- Providing more examples of this reciprocal relationship by using the cards from Lesson 5, Activity 1. For example, on Card 1, the scale factor that takes Figure A to Figure B is 2, so the lengths of Figure A are $\frac{1}{2}$ the length of their corresponding side lengths of Figure B.

Professional Learning

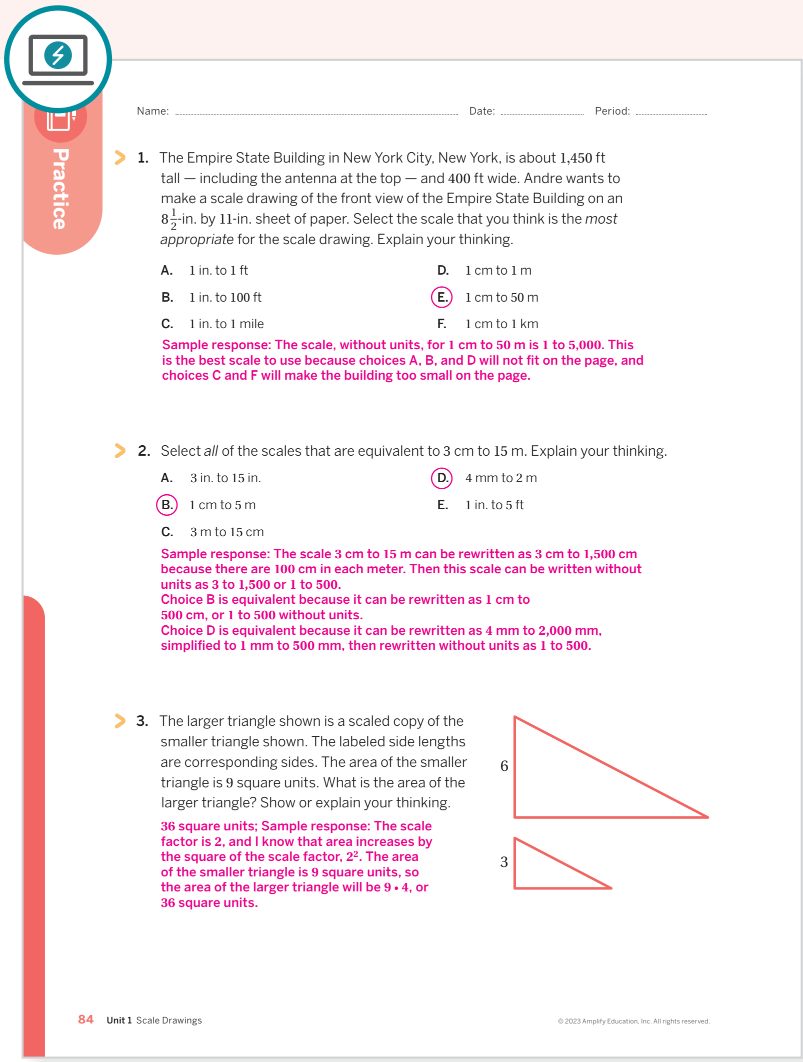
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What challenges did students encounter as they completed Activity 1? How did they work through them?
- In what ways did Activity 1 go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. The Empire State Building in New York City, New York, is about 1,450 ft tall — including the antenna at the top — and 400 ft wide. Andre wants to make a scale drawing of the front view of the Empire State Building on an $8\frac{1}{2}$-in. by 11-in. sheet of paper. Select the scale that you think is the *most appropriate* for the scale drawing. Explain your thinking.

   A. 1 in. to 1 ft
   B. 1 in. to 100 ft
   C. 1 in. to 1 mile
   D. 1 cm to 1 m
   [✓ E.] 1 cm to 50 m
   F. 1 cm to 1 km

   Sample response: The scale, without units, for 1 cm to 50 m is 1 to 5,000. This is the best scale to use because choices A, B, and D will not fit on the page, and choices C and F will make the building too small on the page.

2. Select *all* of the scales that are equivalent to 3 cm to 15 m. Explain your thinking.

   A. 3 in. to 15 in.
   [✓ B.] 1 cm to 5 m
   C. 3 m to 15 cm
   [✓ D.] 4 mm to 2 m
   E. 1 in. to 5 ft

   Sample response: The scale 3 cm to 15 m can be rewritten as 3 cm to 1,500 cm because there are 100 cm in each meter. Then this scale can be written without units as 3 to 1,500 or 1 to 500.
   Choice B is equivalent because it can be rewritten as 1 cm to 500 cm, or 1 to 500 without units.
   Choice D is equivalent because it can be rewritten as 4 mm to 2,000 mm, simplified to 1 mm to 500 mm, then rewritten without units as 1 to 500.

3. The larger triangle shown is a scaled copy of the smaller triangle shown. The labeled side lengths are corresponding sides. The area of the smaller triangle is 9 square units. What is the area of the larger triangle? Show or explain your thinking.

   36 square units; Sample response: The scale factor is 2, and I know that area increases by the square of the scale factor, 2^2. The area of the smaller triangle is 9 square units, so the area of the larger triangle will be 9 • 4, or 36 square units.

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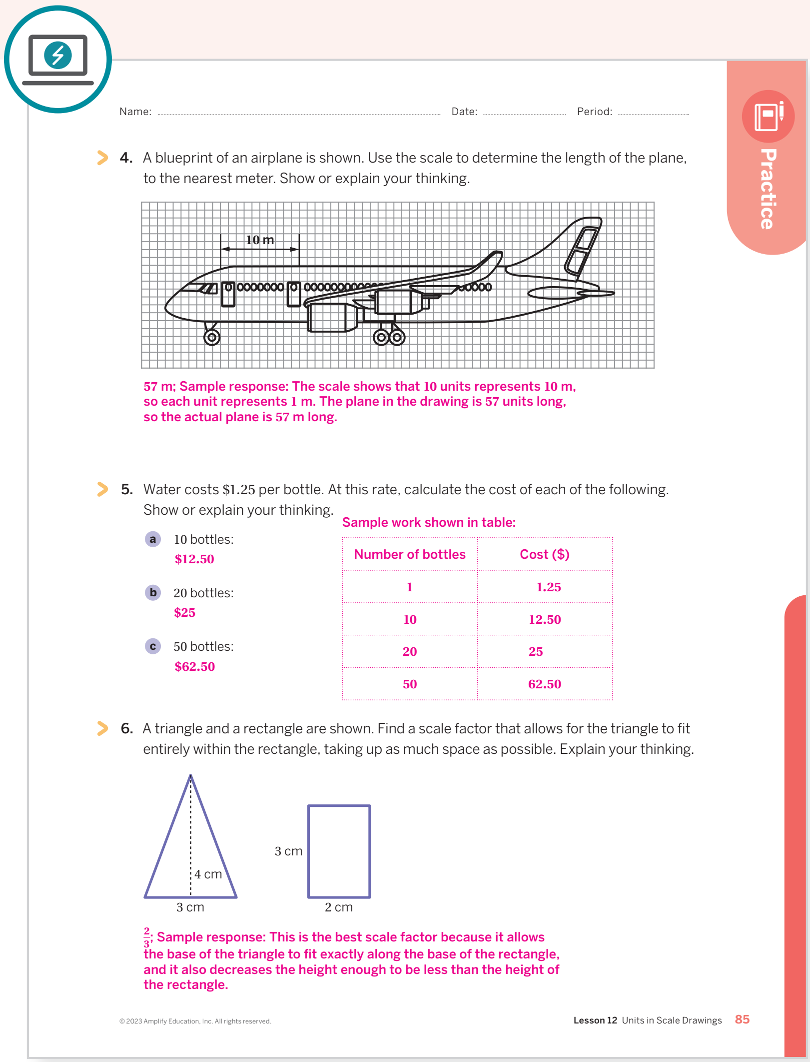

Name: Date: Period:

4. A blueprint of an airplane is shown. Use the scale to determine the length of the plane, to the nearest meter. Show or explain your thinking.

   57 m; Sample response: The scale shows that 10 units represents 10 m, so each unit represents 1 m. The plane in the drawing is 57 units long, so the actual plane is 57 m long.

5. Water costs $1.25 per bottle. At this rate, calculate the cost of each of the following. Show or explain your thinking.

   a. 10 bottles: **$12.50**
   b. 20 bottles: **$25**
   c. 50 bottles: **$62.50**

   Sample work shown in table:

Number of bottles	Cost ($)
1	1.25
10	12.50
20	25
50	62.50

6. A triangle and a rectangle are shown. Find a scale factor that allows for the triangle to fit entirely within the rectangle, taking up as much space as possible. Explain your thinking.

   $\frac{2}{3}$; Sample response: This is the best scale factor because it allows the base of the triangle to fit exactly along the base of the rectangle, and it also decreases the height enough to be less than the height of the rectangle.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
Spiral	3	Unit 1 Lesson 6	2
	4	Unit 1 Lesson 9	1
	5	Grade 6	1
Formative ⚡	6	Unit 1 Lesson 13	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 1 | **LESSON 13 – CAPSTONE**

Build Your Brand

Let's design a logo for your personal brand.

Focus

Goals

1. Create a logo that is simple and reflective of the creator's personality.
2. **Language Goal:** Generate an appropriate scale to represent an actual distance on a limited drawing size, and explain the reasoning. **(Speaking and Listening)**
3. Make assumptions and determine what information is needed to create a scaled copy of an image.

Coherence

Today

Students create personal logos and scale them to fit on blank canvases of their choosing. The task combines creative personal expression with the skills and understandings gained throughout the unit.

Previously

Students saw that converting scales with units to scales without units can help to find the scale factor and compare one scale to another.

Coming Soon

In Unit 2, students will see how making scaled copies is related to proportional relationships. They will make connections between how a proportional relationship grows and changes and how a figure that has been scaled grows or changes.

Rigor

- Students build **conceptual understanding** of how and why a scaled copy is made to fit a limited canvas size.
- Students build **fluency** when multiplying by non-whole number values when scaling lengths.
- Students **apply** their knowledge of scaling to help solve a real-world task.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Promotional Items*, one per student
- Activity 1 PDF, *Scaling Table* (optional)
- Activity 1 PDF, *Scaling Table* (answers, optional)
- rulers

Math Language Development

Review words

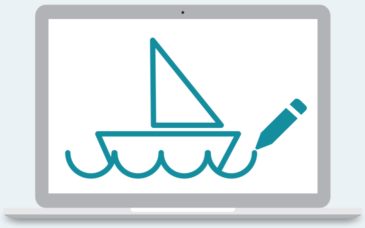
- *scaled copy*
- *scale factor*

Amps | Featured Activity

Activity 1

Designs Come to Life

Students see their scaled logos printed on promotional items of their choosing.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may struggle getting started because they are worried about what their classmates may think about their design. Guide them to find one or more possible entry points in their design process to help get them started. Have students think about their strengths and how these strengths will be put into action as they design their logos.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students write their responses for Problem 2 may be omitted.
- In **Activity 1**, you may choose to ask students to complete a scaled copy for only one promotional item.

Warm-up Sketch Your Logo

Students prepare to make scale drawings of an original design by brainstorming what a logo for their personal brand would look like.

Unit 1 | Lesson 13 – Capstone

Build Your Brand

Let's design a logo for your personal brand.

Warm-up Sketch Your Logo

An important step in building your personal brand is designing an iconic logo.

1. Sketch at least two ideas for your logo. Your logo must include your initials, but you may choose how to arrange them. You may also choose to include any design elements.
Sample responses shown.

2. Which design is your top choice? Explain your thinking.
Sample response: I chose the design on the left because I really like to talk and text with my friends. That's a big part of my personality and I want to reflect that in my logo design.

Log in to Amplify Math to complete this lesson online.

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86 Unit 1 Scale Drawings

1 Launch

Activate prior knowledge by asking students if they have ever thought about owning their own business. Ask about how brands help make their logos memorable. Have students use the *Think-Pair-Share* routine after they sketch a couple of designs. **Note:** You may want to display examples of logos with which your students may be familiar. Distribute rulers.

2 Monitor

Help students get started by prompting them to list some of their strengths or things they love about themselves. Suggest that they incorporate one of these ideas into their design.

Look for points of confusion:

- **Creating designs that are too complicated.** Ask, "How challenging will it be to make sure all parts of your design are appropriately scaled? What could reduce that challenge?"

Look for productive strategies:

- Creating designs that utilize mostly vertical and horizontal lines or use common shapes, such as rectangles or triangles.

3 Connect

Ask, "What feedback would you give to your classmate about their design?

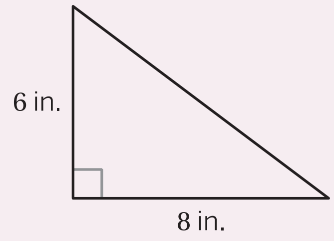
Have pairs of students share their designs with each other. Allow time for students to write their reasoning for selecting their design in Problem 2.

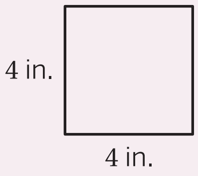
Highlight that marketing agencies will often build a brand stylebook, which includes a collection of different ways to display a business's logo. In the next activity, students will build a brand stylebook using their logo. As they build their stylebook, they will make strategic choices about how they measure and scale their design. Emphasize that precise measurements will be needed to ensure proper scaling.

Power-up

To power up students' ability to determine an appropriate scale factor between two figures, ask:

A triangle and a rectangle are shown.





Find a scale factor that would allow for the triangle to fit entirely within the square, while taking up as much space as possible. Be prepared to explain your thinking. $\frac{1}{2}$; Sample response: This is the best scale factor because it allows the base of the triangle to fit exactly along the base of the square, and it also decreases the height enough to be 1 in. less than the height of the rectangle.

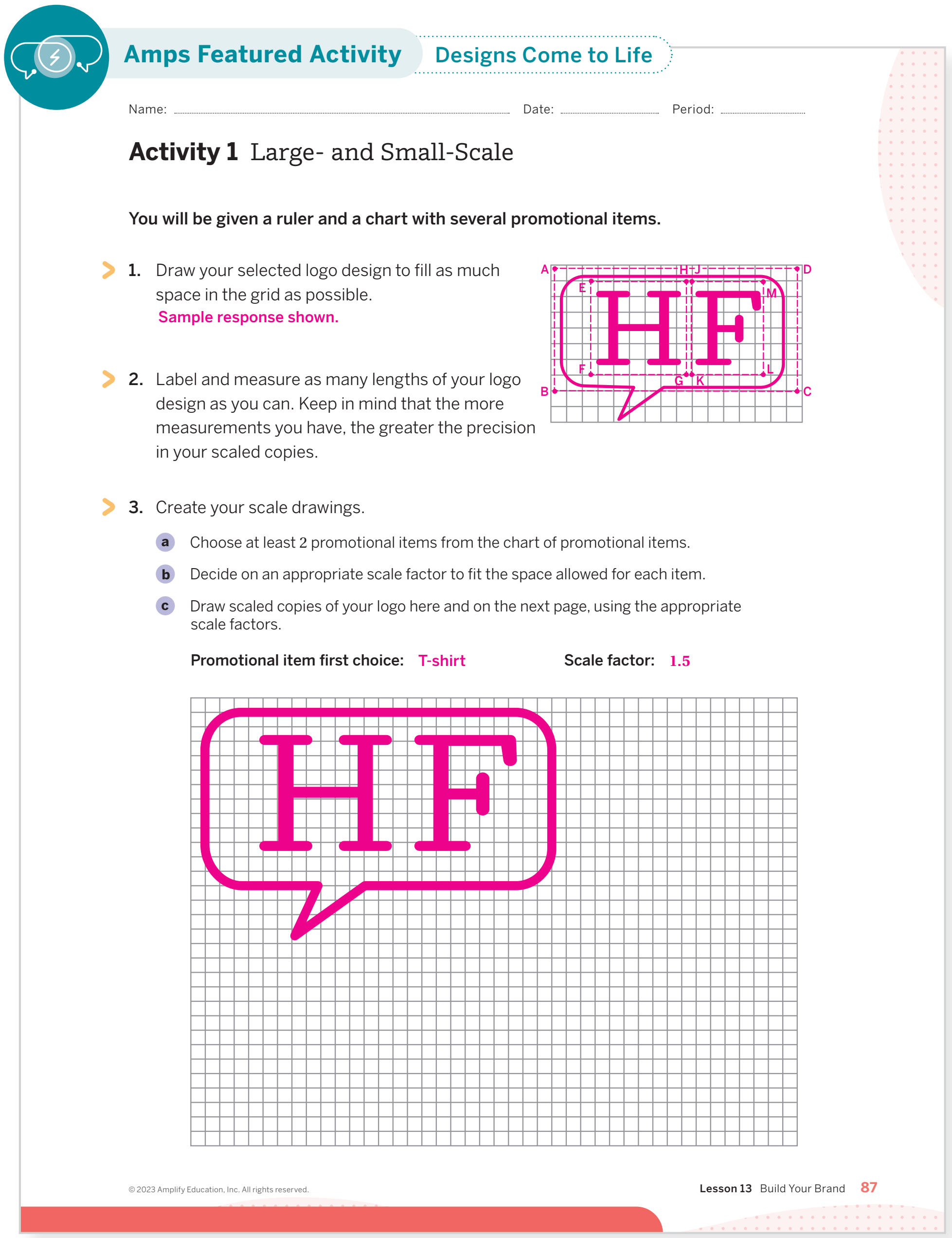
Use: Before Activity 1.

Informed by: Performance on Lesson 12, Practice Problem 6.

Independent | 25 min

Activity 1 Large- and Small-Scale

Students create an original design and two scaled copies of their brand's logo design, deciding how to measure strategically to ensure the copies fit on promotional items.



Amps Featured Activity Designs Come to Life

Name: Date: Period:

Activity 1 Large- and Small-Scale

You will be given a ruler and a chart with several promotional items.

1. Draw your selected logo design to fill as much space in the grid as possible.
   Sample response shown.
2. Label and measure as many lengths of your logo design as you can. Keep in mind that the more measurements you have, the greater the precision in your scaled copies.
3. Create your scale drawings.
   - **a** Choose at least 2 promotional items from the chart of promotional items.
   - **b** Decide on an appropriate scale factor to fit the space allowed for each item.
   - **c** Draw scaled copies of your logo here and on the next page, using the appropriate scale factors.

Promotional item first choice: T-shirt **Scale factor:** 1.5

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Lesson 13 Build Your Brand 87

1 Launch

Provide access to rulers and distribute the Activity 1 PDF, *Promotional Items*, to each student. Provide a brief walk-through of all of the steps students will need to take to complete the activity. Let students know it may not be possible to scale every single aspect of the design precisely; they should decide which elements are most important to scale precisely.

2 Monitor

Help students get started by asking if they need to enlarge or reduce their design to fit on the promotional item.

Look for points of confusion:

- **Creating a scaled copy that does not fit in the space given.** Have students find the dimensions of the given space and compare it with their planned scaled dimensions.
- **Labeling and measuring too many distances.** Have students select a maximum of 8 to 10 lengths to scale precisely.
- **Starting a scaled copy in a place that will not accommodate its eventual full size.** Ask, "Which part of your design might be best to start with to make sure you have enough room?"

Look for productive strategies:

- Strategically choosing which promotional items to use, based on the how the dimensions of the original design fill the item's space.
- Finding a balance between precision and working to completion.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to the Activity 1 PDF, *Scaling Table*. This will help students organize the lengths of their original and scaled copies.

Accessibility: *Vary Demands to Optimize Challenge*

Have students only use letters with vertical and horizontal lines or rectangles in their design.

Extension: *Math Enrichment*

Have students select a promotional item for which the dimensions are given in a different unit than their measurements.

Math Language Development

MLR7: Compare and Connect

Provide students time to consider what is the same and what is different about the scaled designs of different sizes. Then ask them to discuss what they noticed with a partner. Listen for and amplify reasoning about how the choice of scale affects the process of creating scaled drawings. Encourage students to refer to the class display of collected language to assist them in their reasoning.

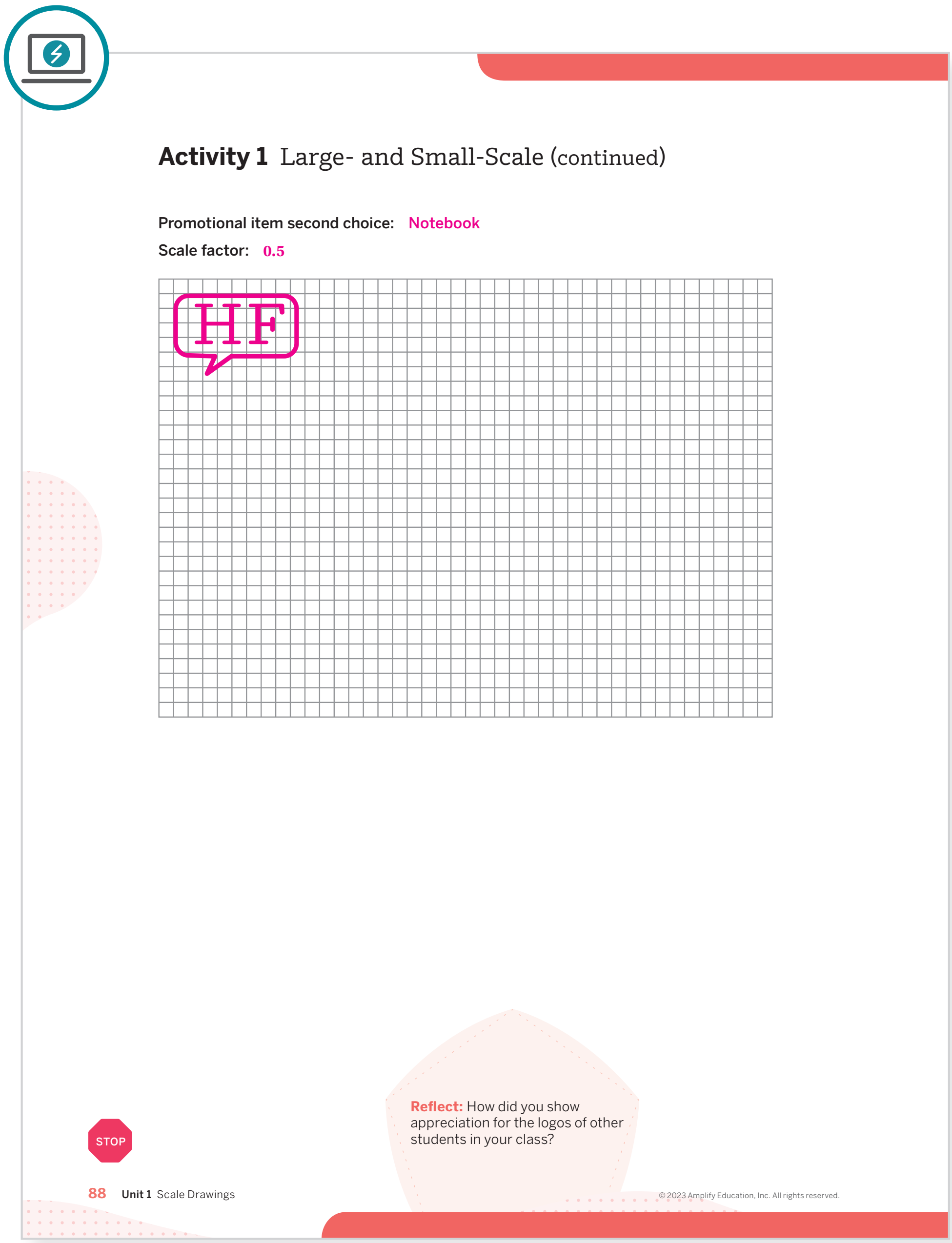
English Learners

Pair students together that have the same primary language. Allow them to discuss in their primary language first and then use language in English from the class display.

Independent | 25 min

Activity 1 Large- and Small-Scale (continued)

Students create an original design and two scaled copies of their brand's logo design, deciding how to measure strategically to ensure the copies fit on promotional items.



Activity 1 Large- and Small-Scale (continued)

Promotional item second choice: Notebook

Scale factor: 0.5

STOP

Reflect: How did you show appreciation for the logos of other students in your class?

3 Connect

Have individual students share their designs using the ***Gallery Tour*** routine. Ask half of the students to visit and ask questions while the other half remains at their desks and answers questions about their designs. Switch and repeat with the other half of the students.

Ask:

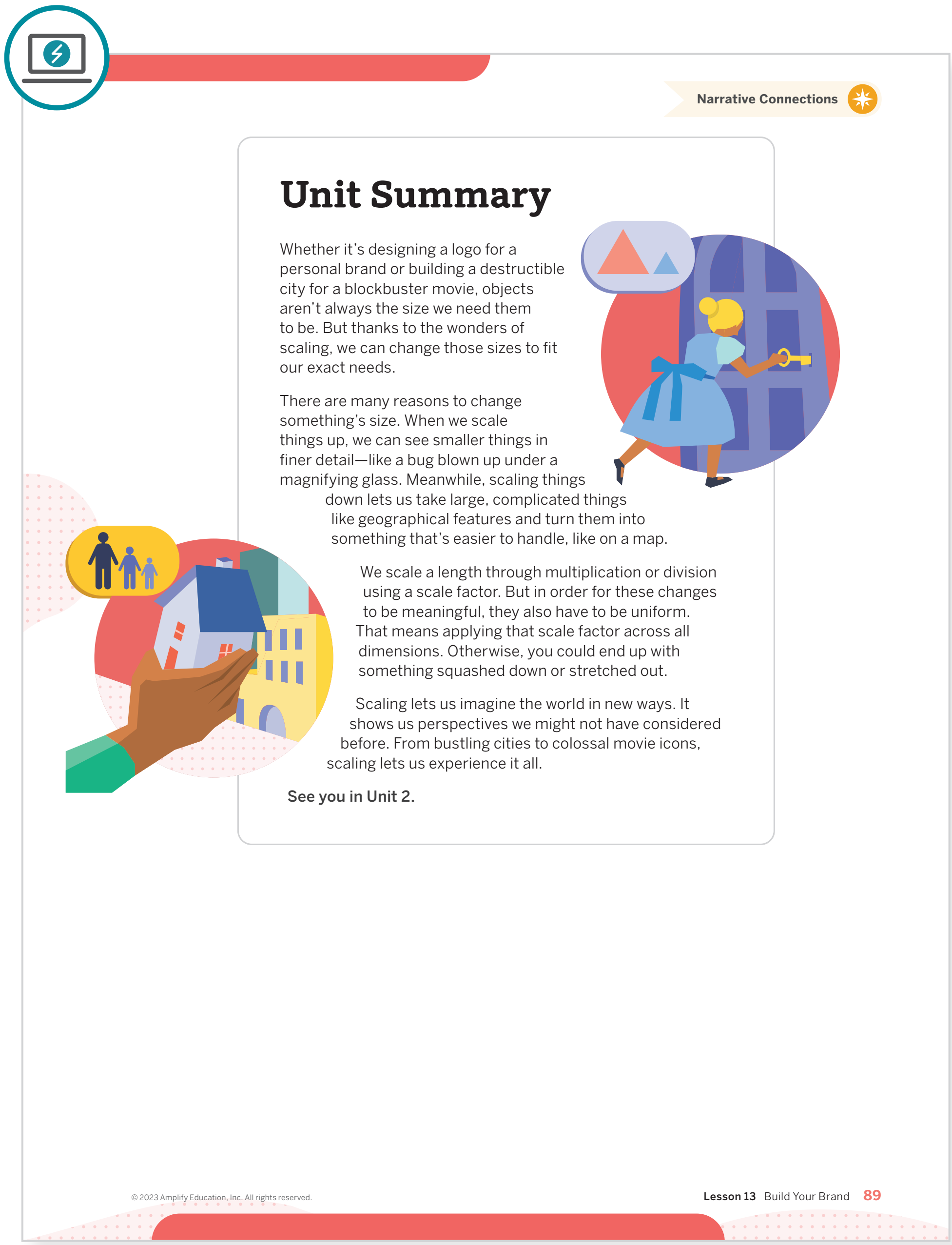
- "How did you decide what scale to use?"
- "If you had created a different design, would that affect your scale choices?"
- "How did you decide which parts of your design were most important to scale precisely?"
- "If you knew that your design would need to be scaled in different ways, would you have made a different design? How would you change it?"

Highlight that choosing a scale is a decision that impacts many aspects of your scaled copy. Sometimes the right scale factor is a non-whole number between 1 and 2, or between 1 and 0.5. Say, "As we prepare to move into the next unit, we will work more closely with fractional and decimal values."

Whole Class | 5 min

Unit Summary

Review and synthesize the major concepts of this unit.



Narrative Connections

Unit Summary

Whether it's designing a logo for a personal brand or building a destructible city for a blockbuster movie, objects aren't always the size we need them to be. But thanks to the wonders of scaling, we can change those sizes to fit our exact needs.

There are many reasons to change something's size. When we scale things up, we can see smaller things in finer detail—like a bug blown up under a magnifying glass. Meanwhile, scaling things down lets us take large, complicated things like geographical features and turn them into something that's easier to handle, like on a map.

We scale a length through multiplication or division using a scale factor. But in order for these changes to be meaningful, they also have to be uniform. That means applying that scale factor across all dimensions. Otherwise, you could end up with something squashed down or stretched out.

Scaling lets us imagine the world in new ways. It shows us perspectives we might not have considered before. From bustling cities to colossal movie icons, scaling lets us experience it all.

See you in Unit 2.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Highlight that it is very useful to spend some time reflecting on one's own learning at the conclusion of a unit. This helps to retain information and make connections that students may not have been able to do while focusing on each individual lesson.

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Ask students to take a few minutes to recall what they have learned about scale throughout this unit.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the Unit narratives. To help students engage in meaningful reflection, consider asking:

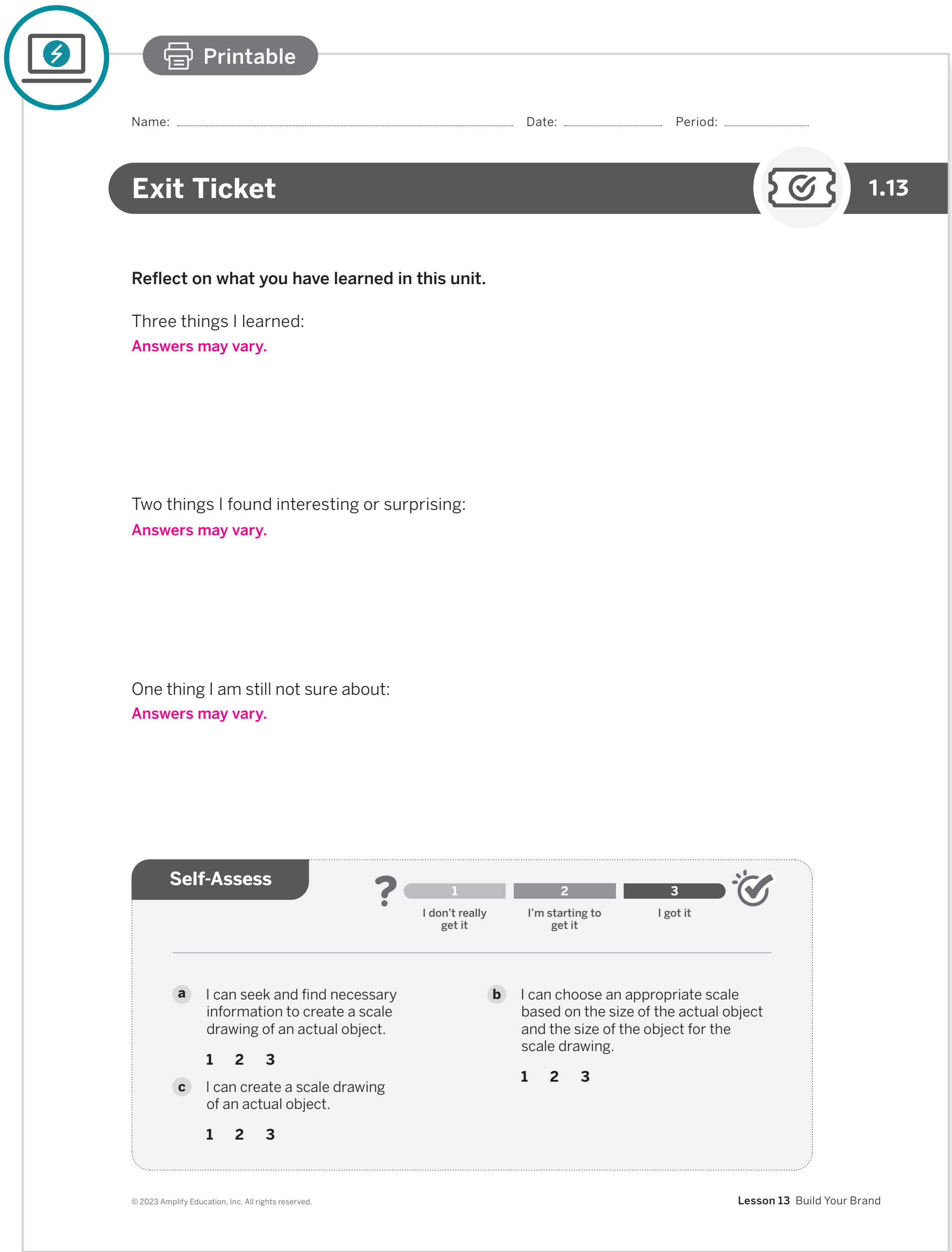
- "Can you think of examples of real world uses for scaled models?"
- "Can you think of a time that you have used a scale model (outside of math class)?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the unit by reflecting on what they learned and voicing any unresolved questions they may have.

Printable



Name: __________ Date: __________ Period: __________

Exit Ticket 1.13

Reflect on what you have learned in this unit.

Three things I learned:

Answers may vary.

Two things I found interesting or surprising:

Answers may vary.

One thing I am still not sure about:

Answers may vary.

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I can seek and find necessary information to create a scale drawing of an actual object.

1 2 3

b I can choose an appropriate scale based on the size of the actual object and the size of the object for the scale drawing.

1 2 3

c I can create a scale drawing of an actual object.

1 2 3

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Lesson 13 Build Your Brand

Success looks like . . .

- **Goal:** Creating a logo that is simple and reflective of the creator's personality.
- **Language Goal:** Generating an appropriate scale to represent an actual distance on a limited drawing size, and explaining the reasoning. **(Speaking and Listening)**
- **Goal:** Making assumptions and determining what information is needed to create a scaled copy of an image.

Suggested next steps

If students cannot think of three things they have learned, consider:

- Allowing them to review what they wrote in their Summary notes from each lesson.

If students cannot think of two things they found interesting or surprising, consider:

- Changing the prompt to "Two things that I enjoyed during this unit" and have them respond.

Professional Learning

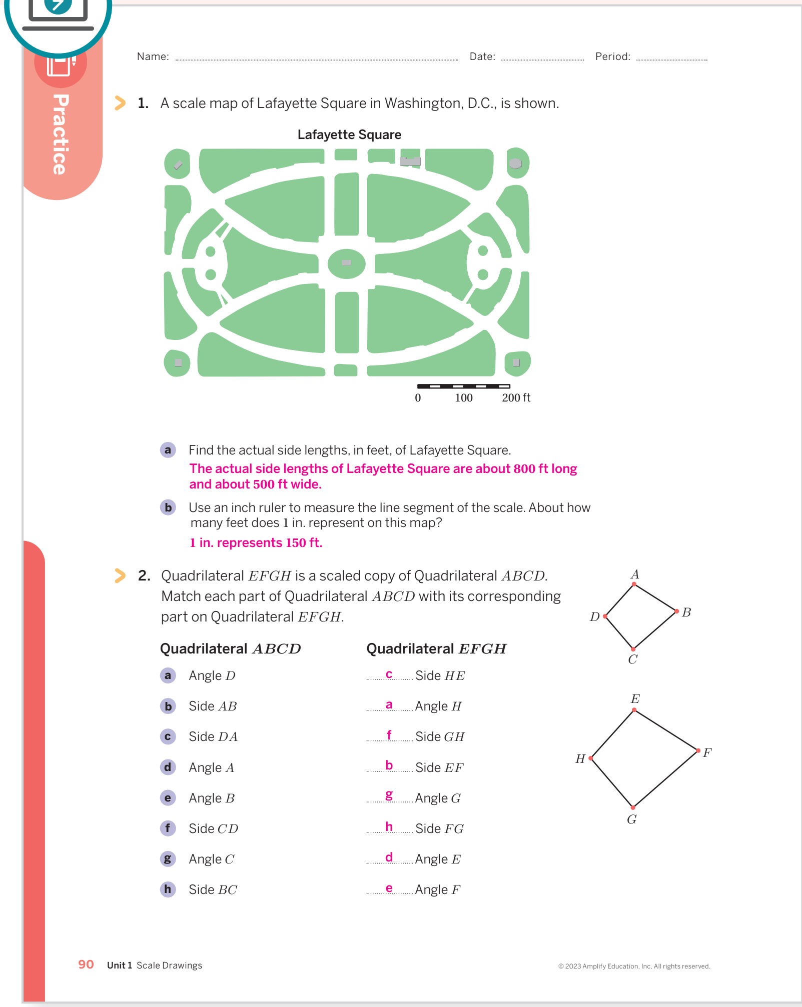
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which teacher actions encouraged students to give strong feedback to each other during the Warm-up?
- In Activity 1, in what ways did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. A scale map of Lafayette Square in Washington, D.C., is shown.

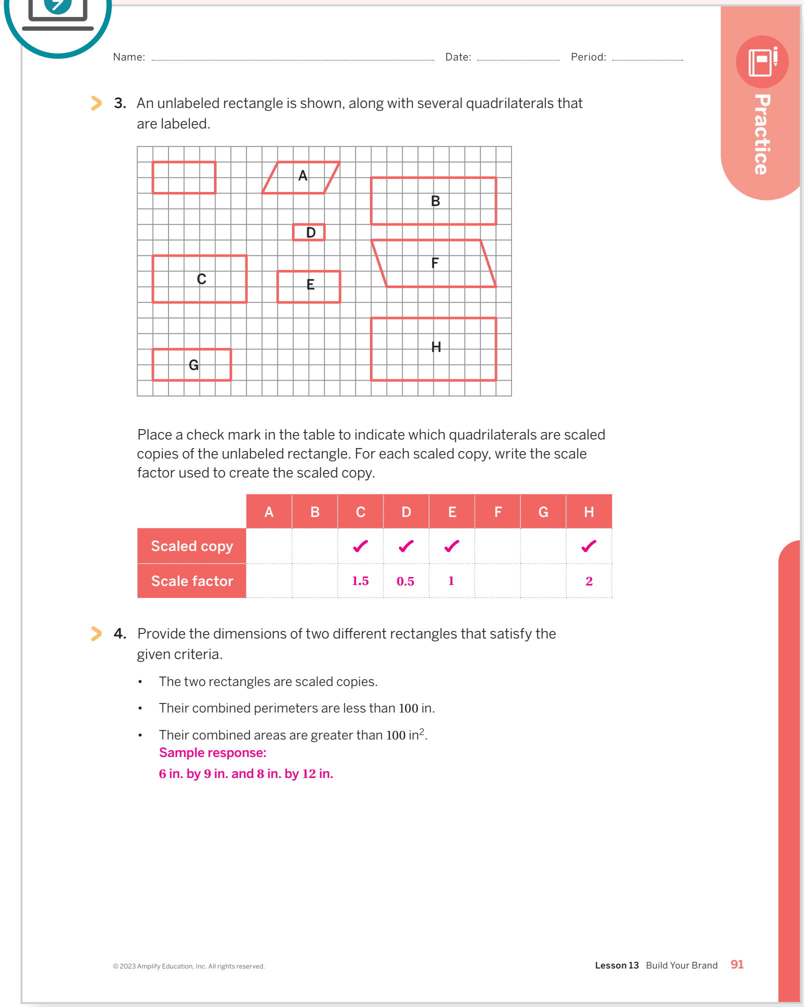
a. Find the actual side lengths, in feet, of Lafayette Square.
The actual side lengths of Lafayette Square are about 800 ft long and about 500 ft wide.

b. Use an inch ruler to measure the line segment of the scale. About how many feet does 1 in. represent on this map?
1 in. represents 150 ft.

2. Quadrilateral $EFGH$ is a scaled copy of Quadrilateral $ABCD$. Match each part of Quadrilateral $ABCD$ with its corresponding part on Quadrilateral $EFGH$.

Quadrilateral $ABCD$	Quadrilateral $EFGH$
a. Angle D	**c** Side HE
b. Side AB	**a** Angle H
c. Side DA	**f** Side GH
d. Angle A	**b** Side EF
e. Angle B	**g** Angle G
f. Side CD	**h** Side FG
g. Angle C	**d** Angle E
h. Side BC	**e** Angle F

90 Unit 1 Scale Drawings



Name: .... Date: .... Period: ....

3. An unlabeled rectangle is shown, along with several quadrilaterals that are labeled.

Place a check mark in the table to indicate which quadrilaterals are scaled copies of the unlabeled rectangle. For each scaled copy, write the scale factor used to create the scaled copy.

	A	B	C	D	E	F	G	H
Scaled copy			✓	✓	✓			✓
Scale factor			1.5	0.5	1			2

4. Provide the dimensions of two different rectangles that satisfy the given criteria.

- The two rectangles are scaled copies.
- Their combined perimeters are less than 100 in.
- Their combined areas are greater than 100 in^2.

Sample response:
6 in. by 9 in. and 8 in. by 12 in.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 1 Lesson 9	2
	2	Unit 1 Lesson 3	1
	3	Unit 1 Lesson 2	2
	4	Unit 1 Lesson 4	3

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2

Introducing Proportional Relationships

When we exchange money from one currency to another, there is a rate that helps us find the amount of one currency equal in value to the other. Students see that a rate is at the heart of every proportional relationship as they encounter problems across cultures where two quantities are directly related.

Essential Questions

- What does it mean for two things to be proportionally related? How can you tell?
- What are the different ways you can represent proportional relationships? How are the representations related?
- *(By the way, how can fractions impact the way you feel about something?)*

Key Shifts in Mathematics

Focus

This unit . . .

Students see that proportional relationships are a collection of equivalent ratios and can be represented in different ways: tables, equations, graphs, and verbal descriptions. They see that for each of these representations, there exists a constant of proportionality that relates every pair of quantities in the relationship.

Coherence

Previously . . .

In Grade 6, students learned two ways of looking at equivalent ratios: using a scale factor and using a unit rate. They produced tables of equivalent ratios and reasoned about contexts involving speed, recipes, food, and unit conversions.

Coming soon . . .

In Grade 8, students will use their understanding of proportionality to reason about linear relationships. The ratio of y to x is extended to the change in y and the change in x and produces a rate of change.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students notice that using an equation to solve a problem can be more efficient than other methods, especially when dealing with larger quantities (Lesson 8).

Procedural Fluency

Students repeatedly check the ratio $\frac{y}{x}$ to see whether it remains constant, therefore indicating that the relationship between x and y is proportional (Lesson 9).

Application

Students apply their knowledge of proportionality, nonproportionality, multiple representations, and constants of proportionality to make sense of scenarios in unfamiliar contexts (Lesson 16).

The World in Proportion

SUB-UNIT

1

Lessons 2–10

Representing Proportional Relationships With Tables and Graphs

Students revisit equivalent ratios to explore ***proportional relationships*** using tables as a way to provide structure and organization. They identify the ***constant of proportionality***, using it to write an equation that describes the entire collection of equivalent ratios.

Narrative: Explorer Ibn Battuta may have used proportionality to navigate different currencies and units.

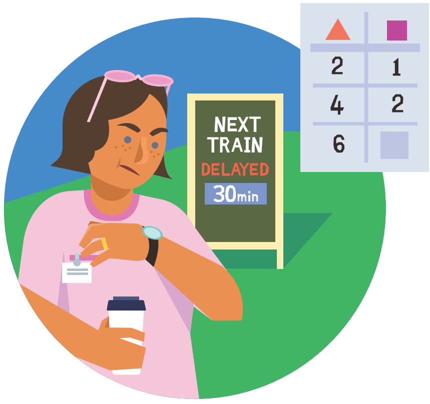
SUB-UNIT

2

Lessons 11–16

Representing Proportional Relationships With Tables and Graphs

Students notice that the graphs of proportional relationships have a certain look and reason about why this is. They move flexibly between the different representations of proportional relationships and note connections between them.



Narrative: Using a graph is an intuitive way to analyze proportionality and understand more about a relationship.

Launch

Lesson 1

Making Music

Students see and hear the role that ratios play in making different sounds on a stringed instrument. By associating those sounds with the emotions they feel when they hear them, students generalize patterns among the ratios in music similar to those made by some of history's greatest composers.

Capstone

Lesson 17

Welcoming Committee

We all know how nice it is to feel welcome. In this lesson, students make a plan for showing a new student around their community. By converting currencies and mapping a thoughtful route around town to various activities, students show the importance of being both a mathematician and a friend.

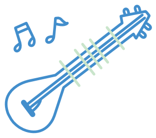
Unit at a Glance

The Spoiler: Finding the constant of proportionality, k, is the most important step in solving problems about any proportional relationship.

Assessment

A **Pre-Unit Readiness Assessment**

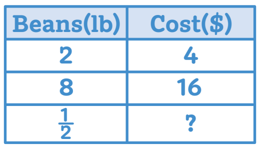
Launch Lesson



1 **Making Music**

Explore how to play a digital two-stringed instrument and notice how the ratio of the strings' length is important to making certain sounds.

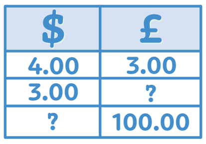
Sub-Unit 1: Representing Proportional Relationships With Tables



Beans(lb)	Cost($)
2	4
8	16
$\frac{1}{2}$	?

2 **Introducing Proportional Relationships With Tables**

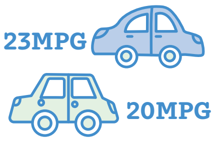
Introduce the concept of a proportional relationship by looking at tables of equivalent ratios.



$	£
4.00	3.00
3.00	?
?	100.00

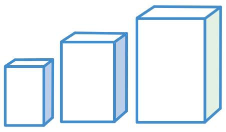
3 **More About the Constant of Proportionality**

Recognize that proportional relationships have two constants of proportionality that are reciprocals by generalizing repeated operations in tables of values.



8 **Using Equations to Solve Problems**

Notice situations where using the equation is a more efficient way of solving problems than other methods.



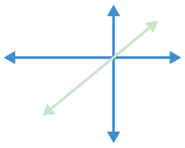
9 **Comparing Relationships With Equations**

Compare proportional and nonproportional relationships, focusing on the structure of the equation for each.

$y = kx$

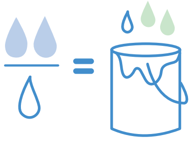
10 **Solving Problems About Proportional Relationships**

Learn to recognize proportional relationships from descriptions of the context.



11 **Introducing Graphs of Proportional Relationships**

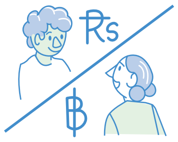
Determine that proportional relationships are straight lines passing through the origin and categorize graphs as proportional or nonproportional.



16 **Four Ways to Tell One Story** (Part 2) ●

Utilize the information from the entire unit to match representations of relationships, then determine which are proportional and which are not.

Capstone Lesson



17 **Welcoming Committee** ●

Help a new student from another country feel welcome and become acquainted with their new home.

Assessment

A **End-of-Unit Assessment**

Key Concepts

Lesson 2: Identify the constant of proportionality using a table of values.
Lesson 5: Write equations relating the quantities in a proportional relationship.
Lesson 11: Notice the graph of a proportional relationship always has certain specific characteristics.

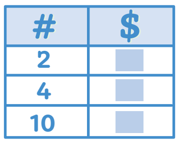
Pacing

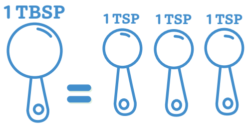
17 Lessons: 45 min each
2 Assessments: 45 min each

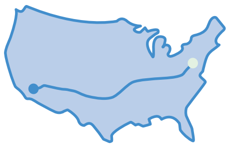
Full Unit: 19 days
● **Modified Unit:** 16 days

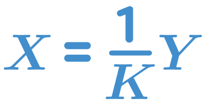
Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

and Equations









4 Comparing Relationships With Tables

Build on understanding of the constant of proportionality to determine if a table of values is representing a proportional or nonproportional relationship.

5 Proportional Relationships and Equations

For a proportional relationship, notice that the pairs of values in the table satisfy the equation $y = kx$ for the constant of proportionality k.

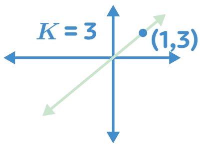
6 Speed and Equations ●

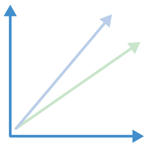
Represent proportional relationships using equations of the form $y = kx$ for contexts involving time, distance, and speed.

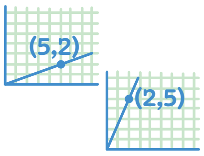
7 Two Equations for Each Relationship

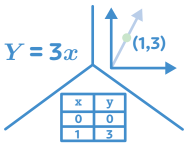
Write equations for the two ways a proportional relationship can be considered.

Sub-Unit 2: Representing Proportional Relationships With Graphs









12 Interpreting Graphs of Proportional Relationships

Make connections between the graph, the context modeled by the proportional relationship, and the equation.

13 Using Graphs to Compare Relationships

Interpret graphs of proportional relationships and reason that graphs can be used to compare constants of proportionality.

14 Two Graphs for Each Relationship

Notice the constant of proportionality in graphs at the special point $(1, y)$ and in any point (x, y) using $k = \frac{y}{x}$.

15 Four Ways to Tell One Story (Part 1) ●

Use all of the information from this unit to examine the connections between verbal descriptions, tables, equations, and graphs of proportional relationships.

● Modifications to Pacing

Lesson 6: This lesson focuses on the relationship between distance, time, and speed. Because this concept is revisited in Unit 4, you might choose to omit this lesson here.

Lessons 15–16: This pair of lessons brings all of the different representations of proportional relationships together in slightly different ways. You may choose to omit one of these lessons in favor of the other.

Lesson 17: This capstone lesson serves to highlight some practical uses of the work students completed in the unit, but does not introduce any new mathematical concepts. Though it serves as a summary lesson reinforcing how math can help build bridges between people with different backgrounds, it may be omitted.

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	**constant of proportionality** **proportional relationship**
4	**nonproportional relationship**

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
2, 4	**MLR1:** Stronger and Clearer Each Time
3, 4, 9, 14, 15	**MLR2:** Collect and Display
7, 9, 10, 11, 17	**MLR3:** Critique, Correct, Clarify
10	**MLR4:** Information Gap
4, 7	**MLR5:** Co-craft Questions
5, 6, 8	**MLR6:** Three Reads
2, 5, 6, 12, 13, 15, 17	**MLR7:** Compare and Connect
1, 2, 3, 7, 8, 11, 12, 16	**MLR8:** Discussion Supports

Materials

Every lesson includes . . .

Exit Ticket

Additional Practice

Additional required materials include:

Lesson(s)	Additional required materials
1*	**headphones (optional)**
1	**computer/digital instrument**
2, 3, 4, 7, 8, 9, 13, 14,17	**calculators**
9*	**snap cubes**
11*, 12, 13, 14, 15, 17	**rulers**
13*	**colored pencils**
15*	**poster-making supplies**
1*, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17	**PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.**

Note: An asterisk (*) indicates the material is optional for that lesson.

Instructional Routines

Activities throughout this unit include these routines:

Lesson(s)	Instructional Routines
4, 10, 11, 16	**Card Sort**
17	**Gallery Tour**
16	**I Have, Who Has?**
10	**Info Gap**
1, 13, 14	**Notice and Wonder**
6	**Number Talk**
5	**Partner Problems**
4, 5, 9	**Think-Pair-Share**
15	**True or False?**
11	**Which One Doesn't Belong?**

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 17

Amps POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

Playing With Ratios

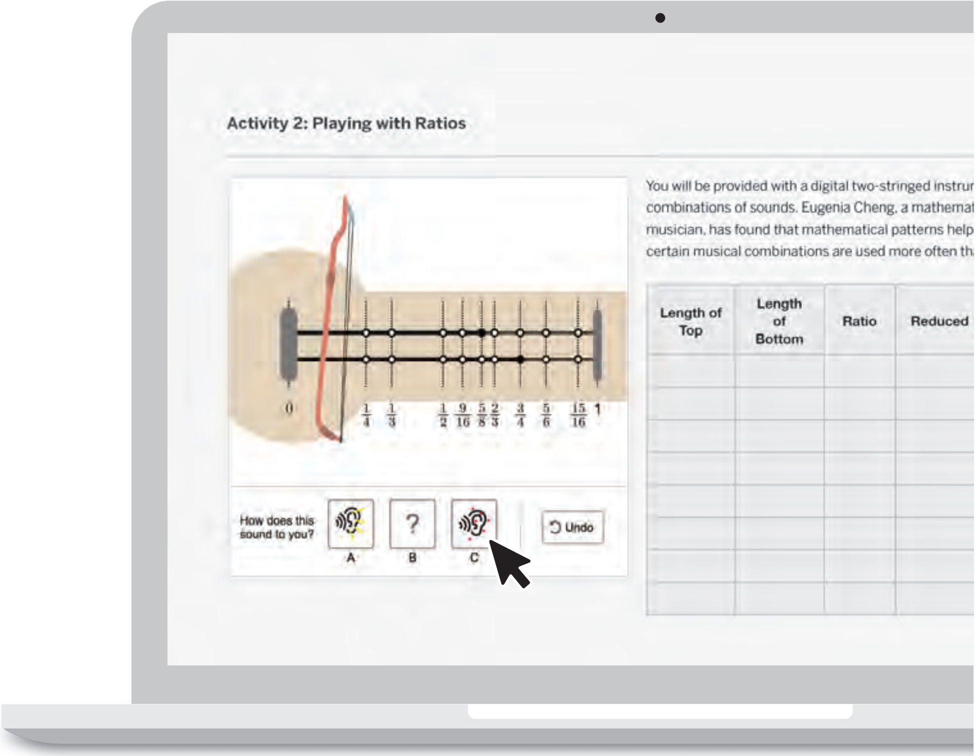
Put on your student hat and work through **Lesson 1, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Activity 1: Zipping Along **(Lesson 6)**
- Activity 2: Trading Gold for Salt **(Lesson 7)**
- Activity 1: Making Tea with a Chai Wallah **(Lesson 12)**
- Activity 2: Finding the Constant of Proportionality **(Lesson 15)**



Activity 2: Playing with Ratios

You will be provided with a digital two-stringed instru
combinations of sounds. Eugenia Cheng, a mathema
musician, has found that mathematical patterns help
certain musical combinations are used more often th

Length of Top	Length of Bottom	Ratio	Reduced

Unit Study
Professional Learning

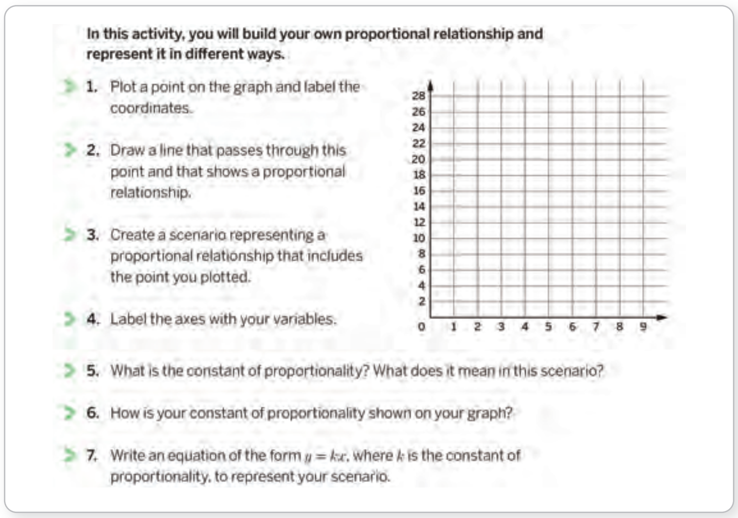
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 turns to graphs to represent proportional relationships. This work builds upon previous work of seeing proportional relationships in tables and equations. Students learn about these proportional relationships in real-world contexts, from ingredients in recipes to currency exchange. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 12, Activity 3:



In this activity, you will build your own proportional relationship and represent it in different ways.

1. Plot a point on the graph and label the coordinates.
2. Draw a line that passes through this point and that shows a proportional relationship.
3. Create a scenario representing a proportional relationship that includes the point you plotted.
4. Label the axes with your variables.
5. What is the constant of proportionality? What does it mean in this scenario?
6. How is your constant of proportionality shown on your graph?
7. Write an equation of the form $y = kx$, where k is the constant of proportionality, to represent your scenario.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

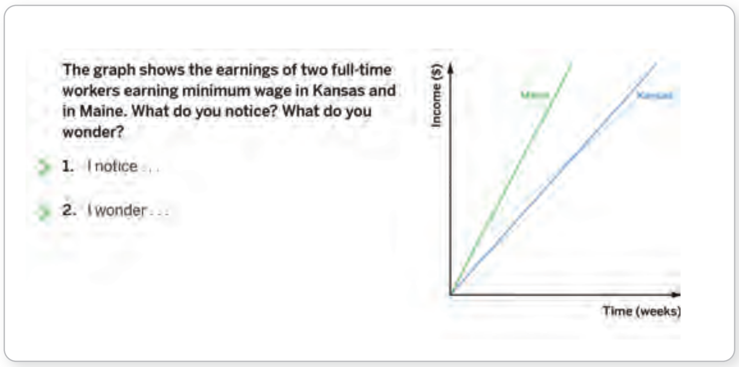
- What was it like to engage in this problem as a learner?
- Students get to create their own scenario for this activity. Which question(s) might your students have trouble with?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Notice and Wonder

Rehearse . . .

How you'll facilitate the *Notice and Wonder* instructional routine in Lesson 13, Warm-up:



The graph shows the earnings of two full-time workers earning minimum wage in Kansas and in Maine. What do you notice? What do you wonder?

1. I notice . . .
2. I wonder . . .

Points to Ponder . . .

- *Notice and Wonder* often has students grapple with a thought-provoking image or problem. Allow students time and space for their own ideas to emerge.

This routine . . .

- Allows students to let their intuition guide them through the problem, without focusing on arriving at a specific answer.
- Can sometimes have an unsettling effect on students who desire to be given a problem to solve.
- Reinforces that many students can make connections between the math they are learning and their experiences from their own lives.

Anticipate . . .

- Students noticing both mathematical and non-mathematical aspects of the problem. Acknowledge all observations that students may make, while guiding them to make more mathematical observations.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Implement tasks that promote reasoning and problem solving.

This effective teaching practice . . .

- Provides opportunities for students to engage in low floor, high ceiling tasks that allow for multiple entry points and a variety of solution strategies.
- Requires the use of reasoning and problem solving strategies as opposed to merely requiring the use of established procedures or skills.

Points to Ponder . . .

- Where do you see opportunities to highlight different strategies that students used to approach the tasks in this unit?
- How can you encourage students to consider the alternative approaches that other students may have used?

Math Language Development

MLR7: Compare and Connect

MLR7 appears in Lessons 2, 5, 6, 12, 13, 15, and 17.

- Throughout this unit, students compare and contrast multiple representations of proportional relationships and make connections as to how the constant of proportionality is represented in each.
- In Lesson 12, students compare the graphs of proportional and nonproportional relationships and use language to describe the characteristics of the graphs of proportional relationships.
- **English Learners:** Use visuals and annotations to show the constant of proportionality in each representation.

Point to Ponder . . .

- How will you help students connect how the constant of proportionality is represented in tables, graphs, equations, or verbal descriptions of proportional relationships?

Fostering Diverse Thinking

Use these opportunities for students to connect mathematics to the world around them:

- In the Sub-Unit 1 narrative, students learn about Ibn Battuta and discuss how they can learn more about explorers they might not have heard of previously.
- In Lesson 6, students compare the times their classmates take to travel to school each day and discuss if any steps can be taken to help all students get to school more easily.

Point to Ponder . . .

- How can I help raise my students' awareness of the contributions of mathematicians around the world, and connect the math they are learning in this unit to conversations about equity?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving problems involving proportional relationships throughout the unit? Do you think your students will generally:
  - » have difficulty determining the constant of proportionality for a given relationship?
  - » get stuck performing calculations involving non-whole numbers?
  - » struggle with solving one-step equations?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their social awareness and self-awareness skills.

Points to Ponder . . .

- Are students able to accurately recognize their emotions? Can they see how their thoughts influence their behaviors? Are they able to identify their strengths and use them to overcome their weaknesses?
- Are students able to keep themselves focused and on task? What do they do to prevent feeling overwhelmed? Can they divide a task into smaller steps to help them achieve their goals?

UNIT 2 | **LESSON 1 – LAUNCH**

Making Music

Let's explore ratios on a stringed instrument.

Focus

Goals

1. **Language Goal:** Describe the role ratios play in making music with a stringed instrument. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Understand that a ratio of two fractions can be simplified to a single ratio. **(Speaking and Listening)**

Coherence

Today

Students explore how to play a digital two-stringed bowed instrument. They notice how the length of the string is changed by pressing on frets, and that the ratio of the changed length to the original creates certain sounds. Further, by collecting data on how the strings sound, students generalize that certain ratios produce more pleasant sounds.

Previously

In Unit 1, students worked with scaling figures and quantities. This work prepared them for Unit 2 by building a basic conceptual understanding of proportional relationships.

Coming Soon

In Lessons 2 and 3, students will determine that groups of equivalent ratios, related by a quantity they will come to know as the constant of proportionality, are known as proportional relationships.

Rigor

- Students **apply** prior experience with ratios to understand how music is made on a stringed instrument.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Whole Class	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, *How to Use the Digital Instrument* (optional)
- computer/digital instrument
- headphones (optional)

Math Language Development

Review word

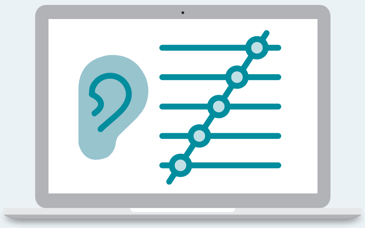
- *ratio*

Amps | **Featured Activity**

Warm-up and Activity 2

Digital Instrument

Students will explore the relationship between ratios and music by playing a digital two-stringed instrument.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle with paying attention to the video as it is shown in Activity 1. Before starting the video, explain that they will be sharing what they learn from it. Provide some specific questions, if needed, beforehand, to guide their listening. As students move toward exploring the digital instrument sounds in Activity 2, they will engage in identifying patterns, structures, and connections between the ratios of strings played.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

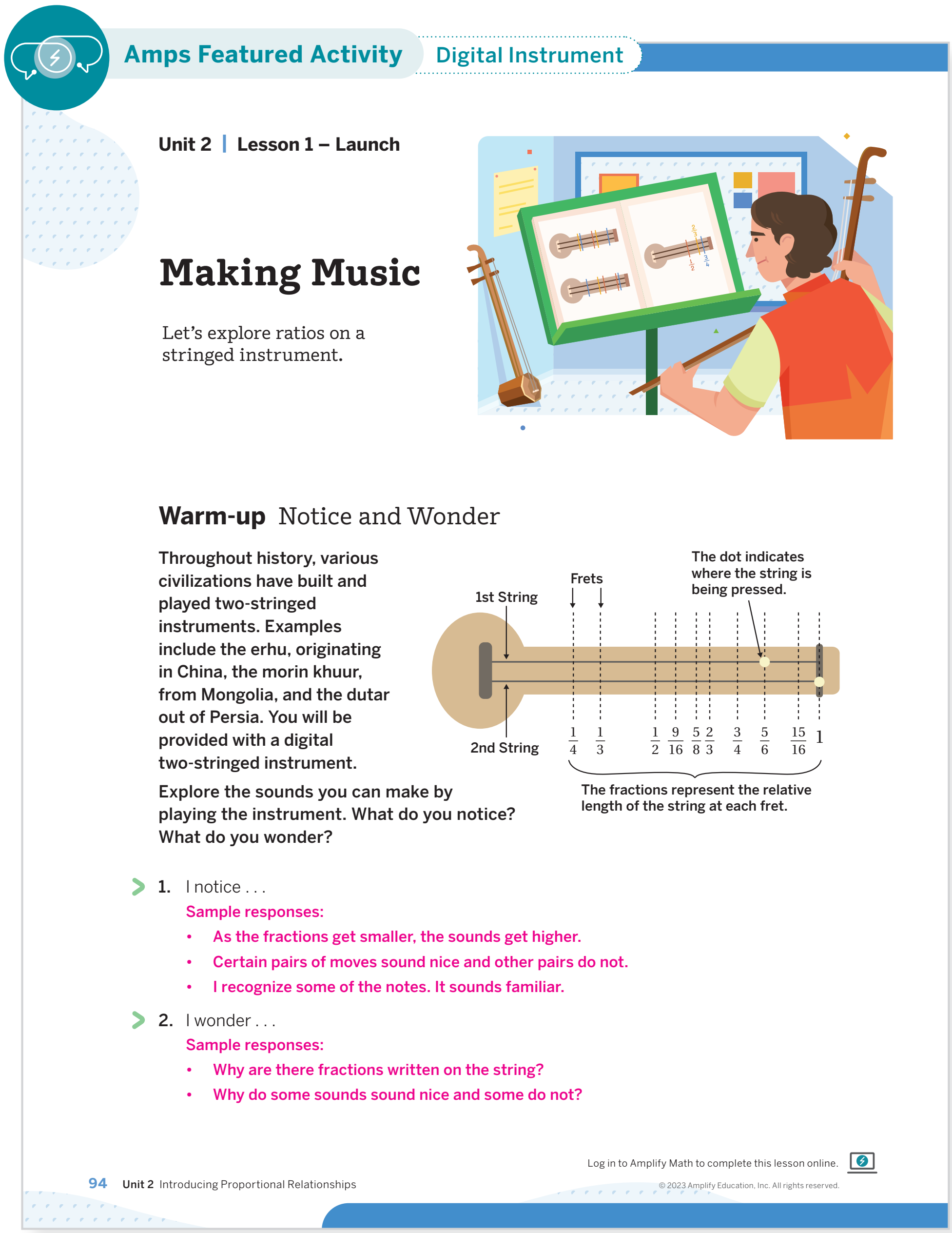
- In **Activity 1**, have students watch the video without completing the KWL chart.
- In **Activity 2**, have students complete only the first four rows of the table.

Independent | 5 min

Warm-up Notice and Wonder

Students explore playing a digital instrument to familiarize themselves with the parts of the instrument and how to make different sounds.

Amps Featured Activity Digital Instrument



Unit 2 | Lesson 1 – Launch

Making Music

Let's explore ratios on a stringed instrument.

Warm-up Notice and Wonder

Throughout history, various civilizations have built and played two-stringed instruments. Examples include the erhu, originating in China, the morin khuur, from Mongolia, and the dutar out of Persia. You will be provided with a digital two-stringed instrument.

Explore the sounds you can make by playing the instrument. What do you notice? What do you wonder?

Frets

1st String

2nd String

The dot indicates where the string is being pressed.

$\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{9}{16}$ $\frac{5}{8}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{15}{16}$ 1

The fractions represent the relative length of the string at each fret.

1. I notice . . .

   Sample responses:
   - As the fractions get smaller, the sounds get higher.
   - Certain pairs of moves sound nice and other pairs do not.
   - I recognize some of the notes. It sounds familiar.

2. I wonder . . .

   Sample responses:
   - Why are there fractions written on the string?
   - Why do some sounds sound nice and some do not?

Log in to Amplify Math to complete this lesson online.

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1 Launch

Activate background knowledge by asking if anyone plays an instrument. Display the digital two-stringed instrument to the class. Conduct the ***Notice and Wonder*** routine. You may want to have students wear headphones to better hear their instrument.

Note: The Warm-up PDF, *How to Use the Digital Instrument*, is intended to help teachers and students familiarize themselves with the operation of the digital instrument.

2 Monitor

Help students get started by asking them to determine what can be changed on the instrument, and what cannot.

Look for points of confusion:

- **Thinking they should always "press" both strings at the same ratio.** Let students know that musicians often play different notes together by pressing each string on a different fret.

Look for productive strategies:

- Keeping one string length constant, and listening for how "pressing" the other string sounds, one step at a time.

3 Connect

Display the digital instrument.

Have students share what they learned about the instrument.

Highlight that pressing strings at specific locations on the neck of a stringed instrument is exactly how a musician would play it.

Ask:

- "What do you notice about how the frets on the digital instrument are marked?"
- "What other fractions might you have expected to see on the frets?"

Differentiated Support

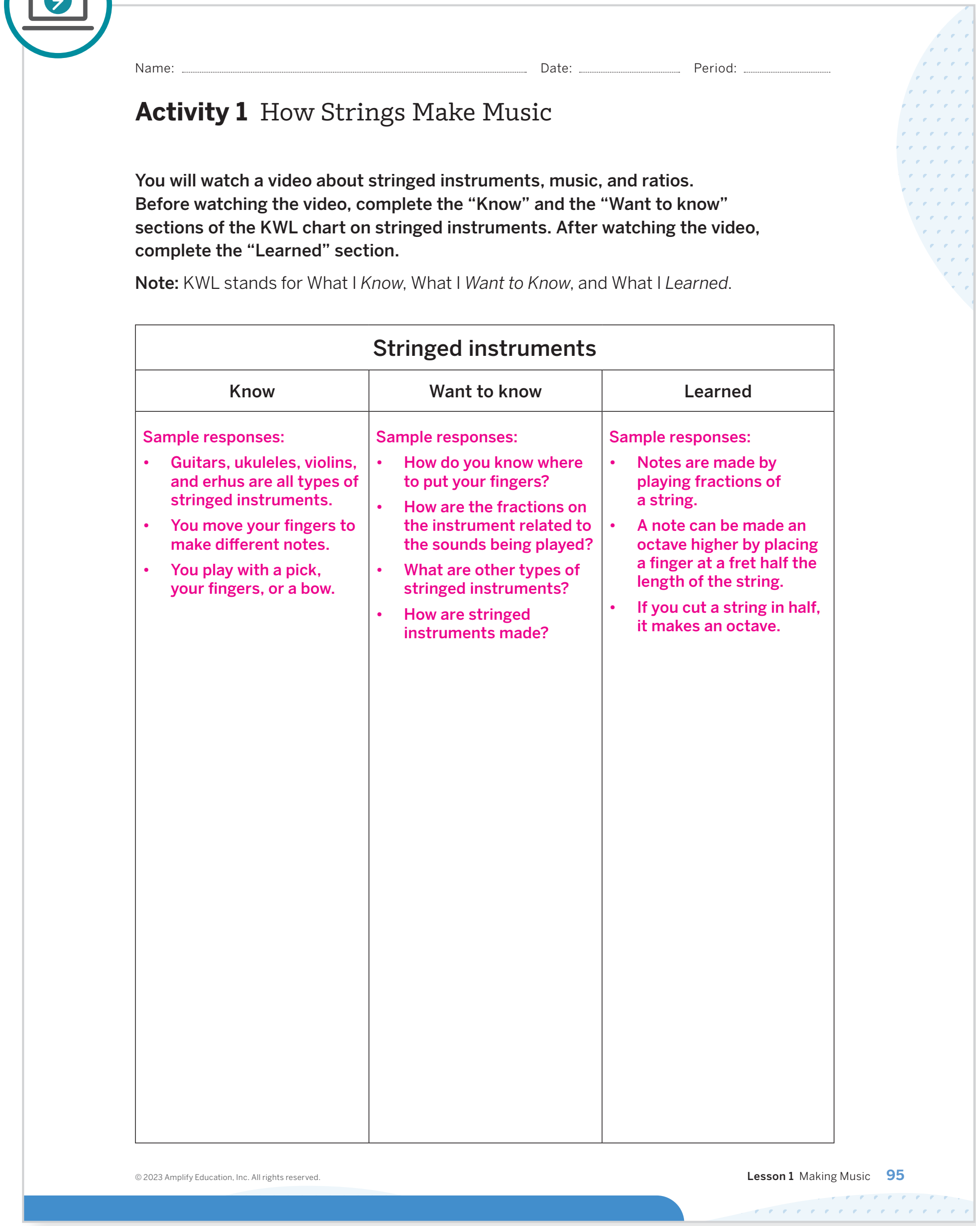
Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can play a digital two-stringed instrument to help make the connection between ratios and music. If time allows, consider playing a short segment from the online 3-minute video "The Erhu" from CBC News. This will allow students to hear a real erhu being played before they play the digital erhu in this activity.

Whole Class | 10 min

Activity 1 How Strings Make Music

Students complete a KWL chart and watch a video to discover the relationship between ratios and the sounds made by stringed instruments.



Name: ______ Date: ______ Period: ______

Activity 1 How Strings Make Music

You will watch a video about stringed instruments, music, and ratios. Before watching the video, complete the "Know" and the "Want to know" sections of the KWL chart on stringed instruments. After watching the video, complete the "Learned" section.

Note: KWL stands for What I *Know*, What I *Want to Know*, and What I *Learned*.

Stringed instruments		
Know	Want to know	Learned
Sample responses: • Guitars, ukuleles, violins, and erhus are all types of stringed instruments. • You move your fingers to make different notes. • You play with a pick, your fingers, or a bow.	Sample responses: • How do you know where to put your fingers? • How are the fractions on the instrument related to the sounds being played? • What are other types of stringed instruments? • How are stringed instruments made?	Sample responses: • Notes are made by playing fractions of a string. • A note can be made an octave higher by placing a finger at a fret half the length of the string. • If you cut a string in half, it makes an octave.

1 Launch

Allow students a few minutes to complete the "K" and "W" of their KWL chart prior to playing the video, *How Strings Make Music*.

2 Monitor

Help students get started by reminding them that there are no right or wrong answers for their KWL chart.

Look for points of confusion:

- **Thinking that they need a background in music to complete the KWL chart.** Encourage students to write down anything they know or want to know about stringed instruments even if it does not seem directly related to math class. Remind them that it is better to have more completed in the "Want" section than the "Know" section.

3 Connect

Display the video, *How Strings Make Music*, and then give students a few minutes to complete the "L" of their KWL chart.

Have students share something they learned from the video with their class.

Highlight that different sounds, or pitches, from stringed instruments are created from changing the length of the string. Certain ratios, such as $\frac{1}{2}$ and $\frac{2}{3}$, create the common notes that are the building blocks for making music.

Ask:

- "From watching the video and moving the finger around your digital instrument, what did you notice about the relationship between the length of a string and the notes it plays?" Sample response: As the string gets shorter, the sounds are higher pitched.
- "What questions do you still have about stringed instruments and ratios?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Prior to playing the video for your students, watch it on your own. Take notes of appropriate stopping points. When you play the video for your students, pause the video at these stopping points to allow students time to process what they watched and write down what they have learned, before continuing the video.

Math Language Development

MLR8: Discussion Supports

Prior to playing the video for your students, watch it on your own, taking notes of music-specific vocabulary that may be unfamiliar to your students. Provide students with a list of these terms for reference while watching the video. For example, the list may include words such as *octave*, *note*, and *harp*.

English Learners

Include a visual diagram or picture next to each word on the list.

Pairs | 20 min

Activity 2 Playing With Ratios

Students use the digital two-stringed instrument to collect data on how ratios of strings sound in pairs and generalize how certain ratio pairs elicit different reactions.

Amps Featured Activity Digital Instrument

Activity 2 Playing With Ratios

You will be provided with a digital two-stringed instrument to explore combinations of sounds. Eugenia Cheng, a mathematician and musician, has found that mathematical patterns help explain why certain musical combinations are used more often than others.

1. Move the fingers on the strings of the instrument. Complete the table for each pair of notes you play. **Sample responses shown.**

	Length (1st string)	Length (2nd string)	Ratio (2nd : 1st)	Simplified ratio	How would you describe the pair of notes?		
					A	B	C
1st pair	$\frac{9}{16}$	$\frac{5}{6}$	$\frac{5}{6}:\frac{9}{16}$	$\frac{40}{27}$	[✓ A]	B	C
2nd pair	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}:\frac{3}{4}$	$\frac{2}{3}$	A	B	[✓ C]
3rd pair	$\frac{9}{16}$	1	$1:\frac{9}{16}$	$\frac{16}{9}$	[✓ A]	B	C
4th pair	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{2}{3}:\frac{5}{8}$	$\frac{16}{15}$	[✓ A]	B	C
5th pair	1	$\frac{3}{4}$	$\frac{3}{4}:1$	$\frac{3}{4}$	A	B	[✓ C]
6th pair	1	$\frac{1}{2}$	$\frac{1}{2}:1$	$\frac{1}{2}$	A	B	[✓ C]
7th pair	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}:\frac{3}{4}$	$\frac{1}{3}$	A	[✓ B]	C
8th pair	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}:\frac{1}{2}$	$\frac{3}{2}$	A	B	[✓ C]

1 Launch

Instruct students that they will be playing pairs of notes on their digital two-stringed instrument. As students play each pair of notes, they will select an icon to indicate their initial reaction toward the sound produced by the note pair. As they click the icon, the table will automatically populate with the ratio of the lengths as well as the simplified ratio. You may choose to also have your students complete the table in the Student Edition.

2 Monitor

Help students get started by modeling how to collect the information from the instrument to complete their table.

Look for points of confusion:

- **Thinking that they can only move the dot on one string.** Explain to students that there is no right or wrong way to choose string pairs and that they are free to move the dots wherever they choose.

Look for productive strategies:

- Trying to recreate note pairs by using different ratios. For example, students observe from the video that 1 and $\frac{1}{2}$ are an octave apart, so they test to see whether pressing a string at a ratio played with 1 elicits the same reaction as when it is played with $\frac{1}{2}$.
- Looking for repeated reactions and determining whether there is a pattern in the sounds made by pairs of notes that results in reaction A, B, or C (unresolved, neutral, or resolved).

Activity 2 continued >

Differentiated Support

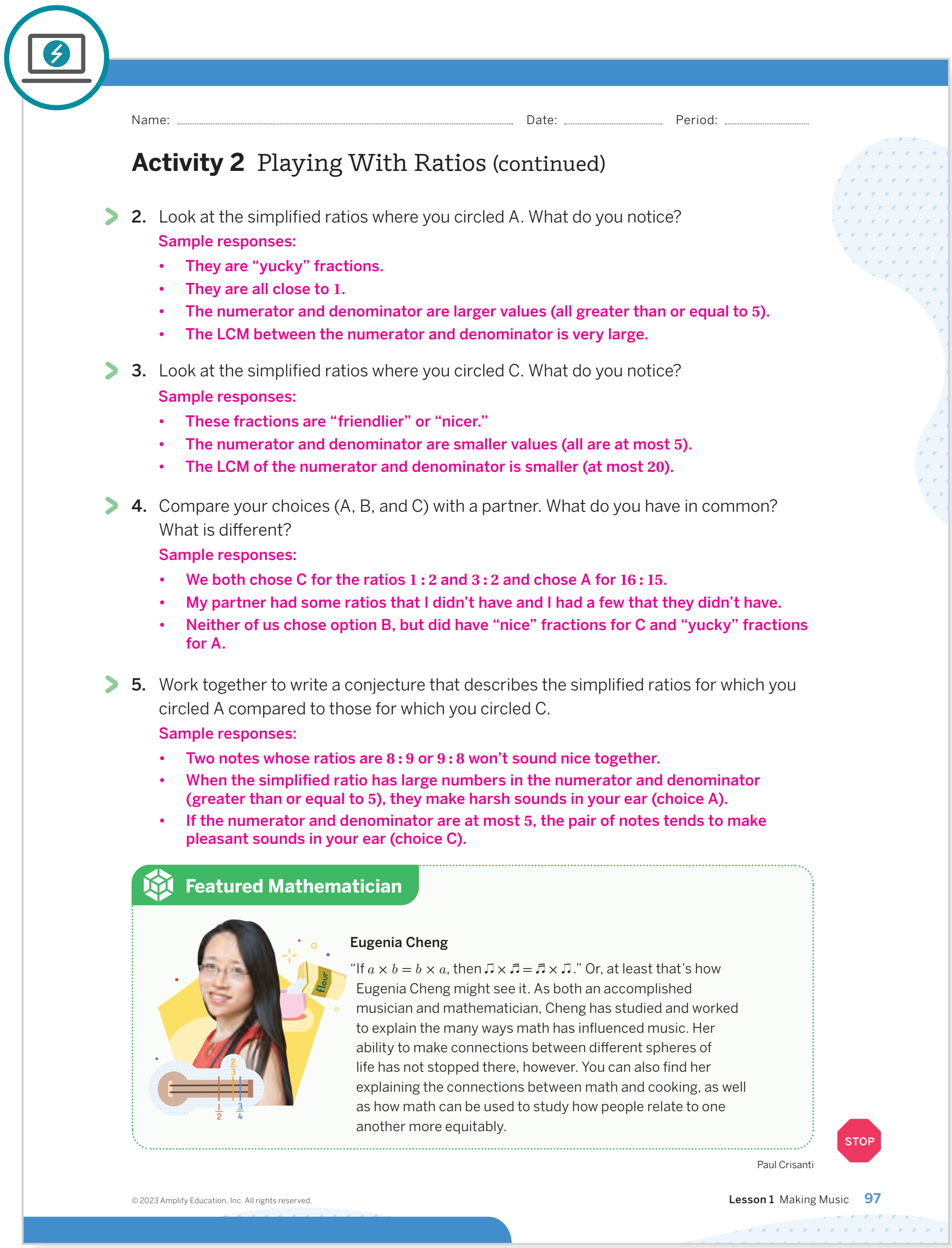
Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can play pairs of notes on a digital two-stringed instrument. As they click each icon, the table auto-populates with the ratios. Provide students with their own copy of the Warm-up PDF, *How to Use the Digital Instrument*, for reference. Consider allowing students to use headphones or to complete the activity in a quiet place, if they are sensitive to loud sounds.

Pairs | 20 min

Activity 2 Playing With Ratios (continued)

Students use the digital two-stringed instrument to collect data on how ratios of strings sound in pairs and generalize how certain ratio pairs elicit different reactions.



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Activity 2 Playing With Ratios (continued)

2. Look at the simplified ratios where you circled A. What do you notice?

   Sample responses:
   - They are "yucky" fractions.
   - They are all close to 1.
   - The numerator and denominator are larger values (all greater than or equal to 5).
   - The LCM between the numerator and denominator is very large.

3. Look at the simplified ratios where you circled C. What do you notice?

   Sample responses:
   - These fractions are "friendlier" or "nicer."
   - The numerator and denominator are smaller values (all are at most 5).
   - The LCM of the numerator and denominator is smaller (at most 20).

4. Compare your choices (A, B, and C) with a partner. What do you have in common? What is different?

   Sample responses:
   - We both chose C for the ratios 1 : 2 and 3 : 2 and chose A for 16 : 15.
   - My partner had some ratios that I didn't have and I had a few that they didn't have.
   - Neither of us chose option B, but did have "nice" fractions for C and "yucky" fractions for A.

5. Work together to write a conjecture that describes the simplified ratios for which you circled A compared to those for which you circled C.

   Sample responses:
   - Two notes whose ratios are 8 : 9 or 9 : 8 won't sound nice together.
   - When the simplified ratio has large numbers in the numerator and denominator (greater than or equal to 5), they make harsh sounds in your ear (choice A).
   - If the numerator and denominator are at most 5, the pair of notes tends to make pleasant sounds in your ear (choice C).

Featured Mathematician

Eugenia Cheng

"If $a \times b = b \times a$, then ♫ × ♬ = ♬ × ♫." Or, at least that's how Eugenia Cheng might see it. As both an accomplished musician and mathematician, Cheng has studied and worked to explain the many ways math has influenced music. Her ability to make connections between different spheres of life has not stopped there, however. You can also find her explaining the connections between math and cooking, as well as how math can be used to study how people relate to one another more equitably.

Paul Crisanti

STOP

3 Connect

Display the aggregated class data from the table.

Have students share any patterns they notice in the data.

Ask:

- "Study the ratios for the pairs that were labeled with C. What do they have in common?" I noticed that these ratios are more common fractions, such as $\frac{1}{2}, \frac{2}{3}, \frac{4}{5}$. The numerator and denominator are smaller; at most 5.
- "Study the ratios for the pairs that were labeled with A. What do they have in common?" These fractions are less frequently used. They are not as "friendly" as the fractions that have the positive emoji. Their numerator and denominator are larger, at least 5.

Highlight that patterns made by these ratios help make music. When humans discovered these patterns long ago, they started to use them to intentionally create music that makes people feel happy, sad, or any of the emotions that can be communicated through music. They used math to figure out how to make certain sounds, and that often included comparing ratios.

Note: You may want to connect the ratios to sound waves. The smaller the values in the simplified ratio, the more frequently the peaks of the sound waves from the notes meet (consider drawing sine waves on the board whose roots frequently match). The larger the values, the less frequently they overlap. The more frequently the peaks meet, the more resolved the pair of notes tends to feel.

Featured Mathematician

Eugenia Cheng

Have students read about Eugenia Cheng, an accomplished pianist and mathematician who has made several videos explaining the connections between these two fields of study.

Whole Class | 5 min

Summary The World in Proportion

Review and synthesize the relationship between ratios and making music.

Narrative Connections

Unit 2 Introducing Proportional Relationships

The World in Proportion

In the early 1900s, Wuxin was an up-and-coming city in China. It budded with smokestacks, textile factories — even a new railway. And walking through its streets you might hear the musician Abing, playing a mournful song.

Abing had a hard life. Born in 1893, he was orphaned and sent to live at the local daoist temple. As an adult, he lost the use of his eyes and wandered the city, playing the erhu for money.

The erhu is a two-stringed fiddle originating from the nomadic peoples of Central Asia. When a bow is drawn across the strings, it makes a distinctive, sad sound.

But his fortunes changed in 1950. Musicologists traveled to Wuxin, and there discovered Abing. They recorded three of his songs, including "The Moon's Reflection on the Erquan Spring." This song — as well as Abing's story — would become well known across China, influencing the country's distinctive musical character.

In this unit, you'll look at some different ways people have exchanged cultural ideas, such as the music of the erhu. This sort of communication can be difficult at times, due to differences in language, customs, and even units of measurements. Math, and ratios in particular, gives us a way to line those measurements up. Once lined up, we open ways to exchange information, from weights and measures to the precise tuning of an erhu.

Welcome to Unit 2.

98 Unit 2 Introducing Proportional Relationships

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Highlight that during this unit, students will continue working with ratios, focusing on proportional relationships and using ratios to model real-world relationships.

Ask:

- "Does anyone play a string instrument, and if so, which one? Have you ever noticed any ratios or markings on the instrument?" Sample response: I play the guitar and it has frets on it.
- "If you were to build your own stringed instrument, how would you decide where to place your fingers to create different notes?" Sample response: I would mark $\frac{1}{2}$ and $\frac{2}{3}$ because I know those create a nice sound when played together.
- "How does music help people communicate with each other?"
- "Can you think of any other ways that ratios may help people to communicate or exchange ideas?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

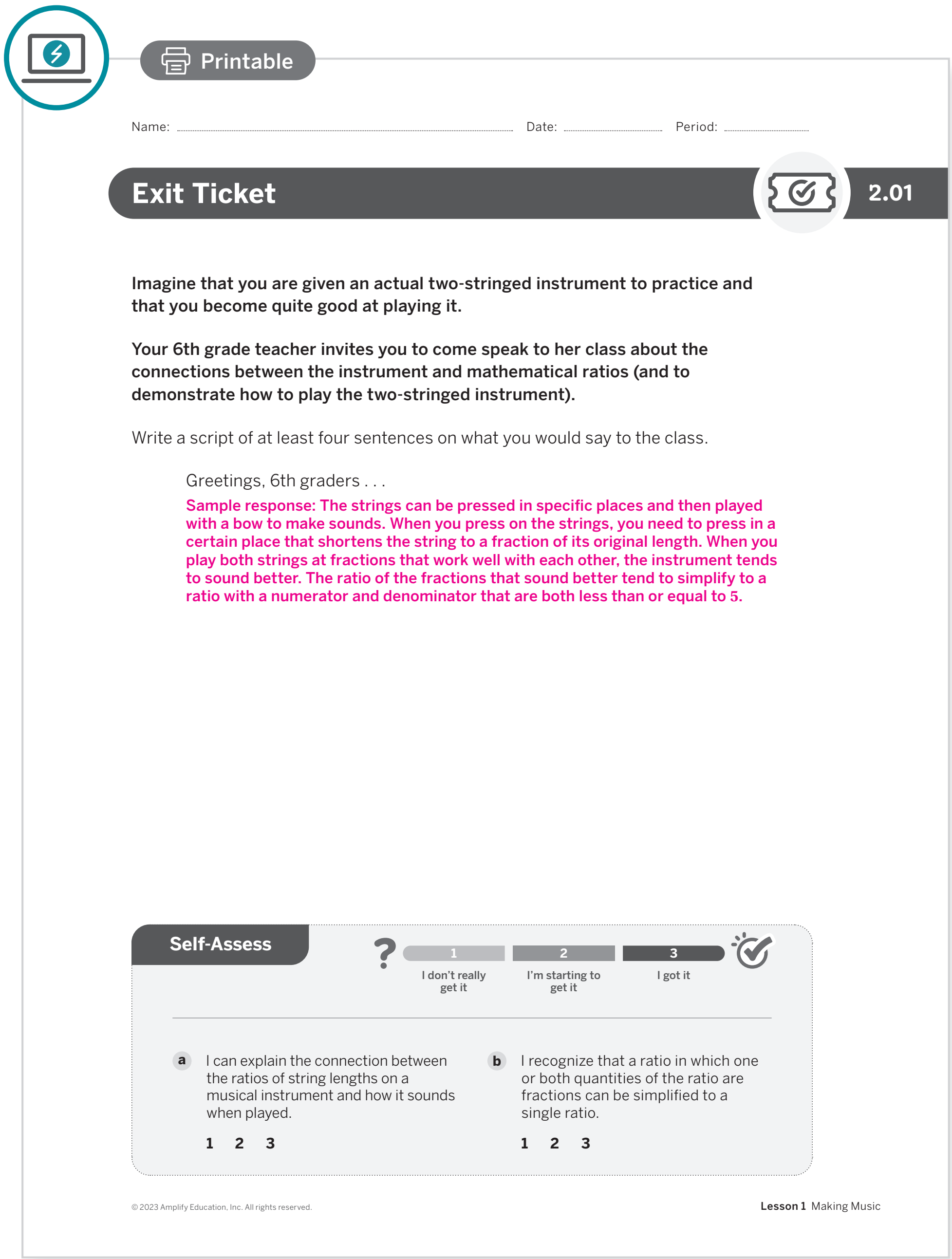
- "What was the most surprising thing when working with the digital instrument?"
- "What questions do you still have about the relationship between ratios and music?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the sounds made with a stringed instrument by teaching others about using ratios to change the string length.

Printable



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Exit Ticket 2.01

Imagine that you are given an actual two-stringed instrument to practice and that you become quite good at playing it.

Your 6th grade teacher invites you to come speak to her class about the connections between the instrument and mathematical ratios (and to demonstrate how to play the two-stringed instrument).

Write a script of at least four sentences on what you would say to the class.

Greetings, 6th graders . . .

Sample response: The strings can be pressed in specific places and then played with a bow to make sounds. When you press on the strings, you need to press in a certain place that shortens the string to a fraction of its original length. When you play both strings at fractions that work well with each other, the instrument tends to sound better. The ratio of the fractions that sound better tend to simplify to a ratio with a numerator and denominator that are both less than or equal to 5.

Self-Assess

?	1	2	3	✓
	I don't really get it	I'm starting to get it	I got it	

a. I can explain the connection between the ratios of string lengths on a musical instrument and how it sounds when played.

1 2 3

b. I recognize that a ratio in which one or both quantities of the ratio are fractions can be simplified to a single ratio.

1 2 3

Lesson 1 Making Music

Success looks like . . .

- **Language Goal:** Describing the role ratios play in making music with a stringed instrument. **(Speaking and Listening, Reading and Writing)**
  - » Writing a script about how the instrument and ratios are related.
- **Language Goal:** Understanding that a ratio of two fractions can be simplified to a single ratio. **(Speaking and Listening)**
  - » Including in the script how a ratio can be simplified to a value that relates to a better sound.

Suggested next steps

If students are unsure of what they would say in front of a class, consider:

- Changing the context to them writing a letter to a student in a lower grade.

If students do not mention fractions or ratios in their script, consider:

- Asking, "How can you revise your script to include fractions or ratios?"

Professional Learning

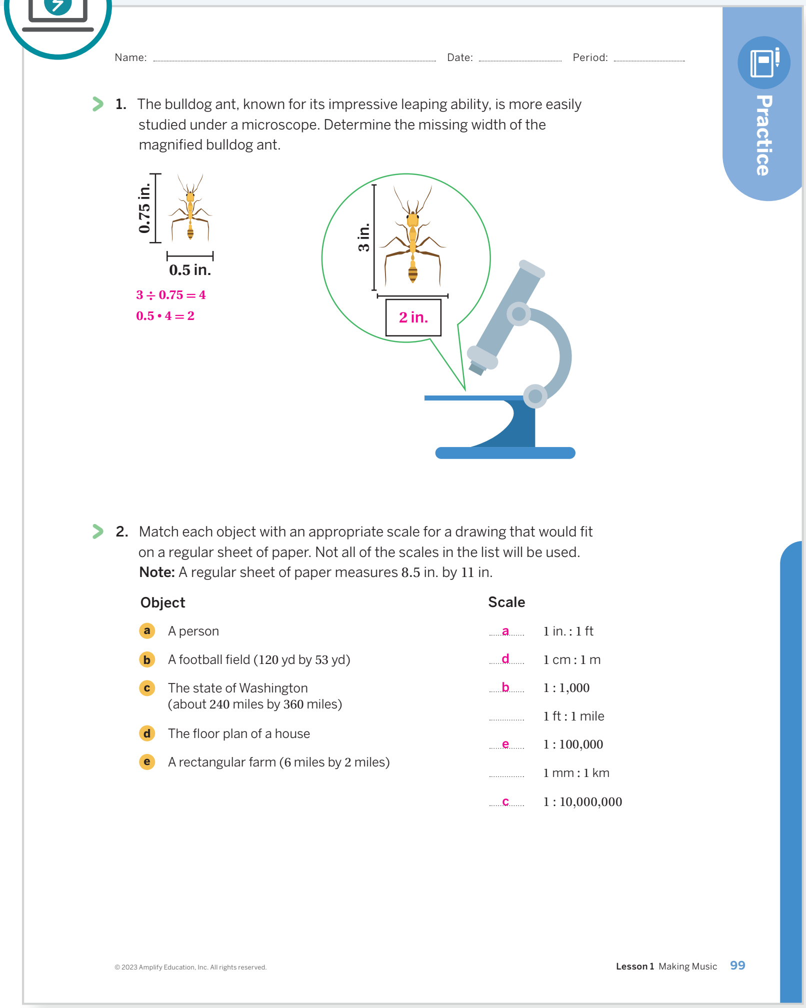
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did comparing fractions on the digital instrument set students up to develop their understanding of proportional relationships?
- Which teacher actions made analyzing the table in Activity 2 strong? What might you change the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

Practice

1. The bulldog ant, known for its impressive leaping ability, is more easily studied under a microscope. Determine the missing width of the magnified bulldog ant.

$3 \div 0.75 = 4$

$0.5 \cdot 4 = 2$

2. Match each object with an appropriate scale for a drawing that would fit on a regular sheet of paper. Not all of the scales in the list will be used.
Note: A regular sheet of paper measures 8.5 in. by 11 in.

Object

- a. A person
- b. A football field (120 yd by 53 yd)
- c. The state of Washington (about 240 miles by 360 miles)
- d. The floor plan of a house
- e. A rectangular farm (6 miles by 2 miles)

Scale

a 1 in. : 1 ft

d 1 cm : 1 m

b 1 : 1,000

.... 1 ft : 1 mile

e 1 : 100,000

.... 1 mm : 1 km

c 1 : 10,000,000

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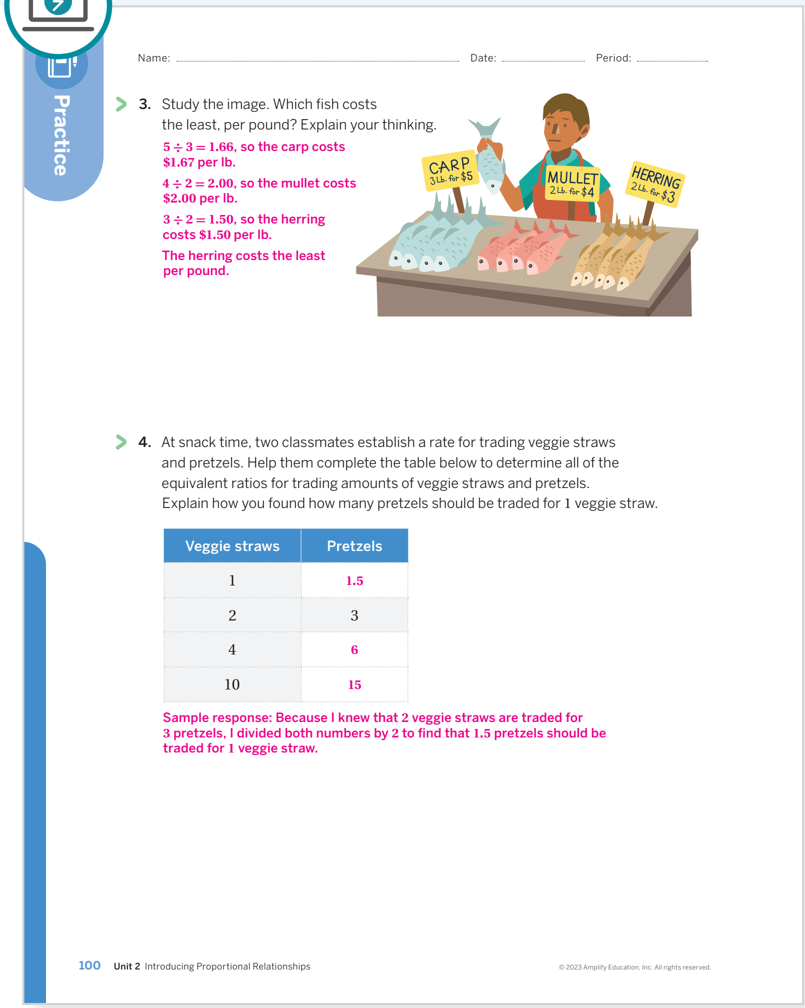
Lesson 1 Making Music 99



Practice

Name: .... Date: .... Period: ....

3. Study the image. Which fish costs the least, per pound? Explain your thinking.

$5 \div 3 = 1.66$, so the carp costs $1.67 per lb.

$4 \div 2 = 2.00$, so the mullet costs $2.00 per lb.

$3 \div 2 = 1.50$, so the herring costs $1.50 per lb.

The herring costs the least per pound.

4. At snack time, two classmates establish a rate for trading veggie straws and pretzels. Help them complete the table below to determine all of the equivalent ratios for trading amounts of veggie straws and pretzels. Explain how you found how many pretzels should be traded for 1 veggie straw.

Veggie straws	Pretzels
1	1.5
2	3
4	6
10	15

Sample response: Because I knew that 2 veggie straws are traded for 3 pretzels, I divided both numbers by 2 to find that 1.5 pretzels should be traded for 1 veggie straw.

100 **Unit 2** Introducing Proportional Relationships

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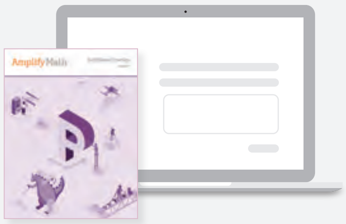
Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 1 Lesson 8	2
	2	Unit 1 Lesson 12	2
	3	Grade 6	2
Formative ⚡	4	Unit 2 Lesson 2	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



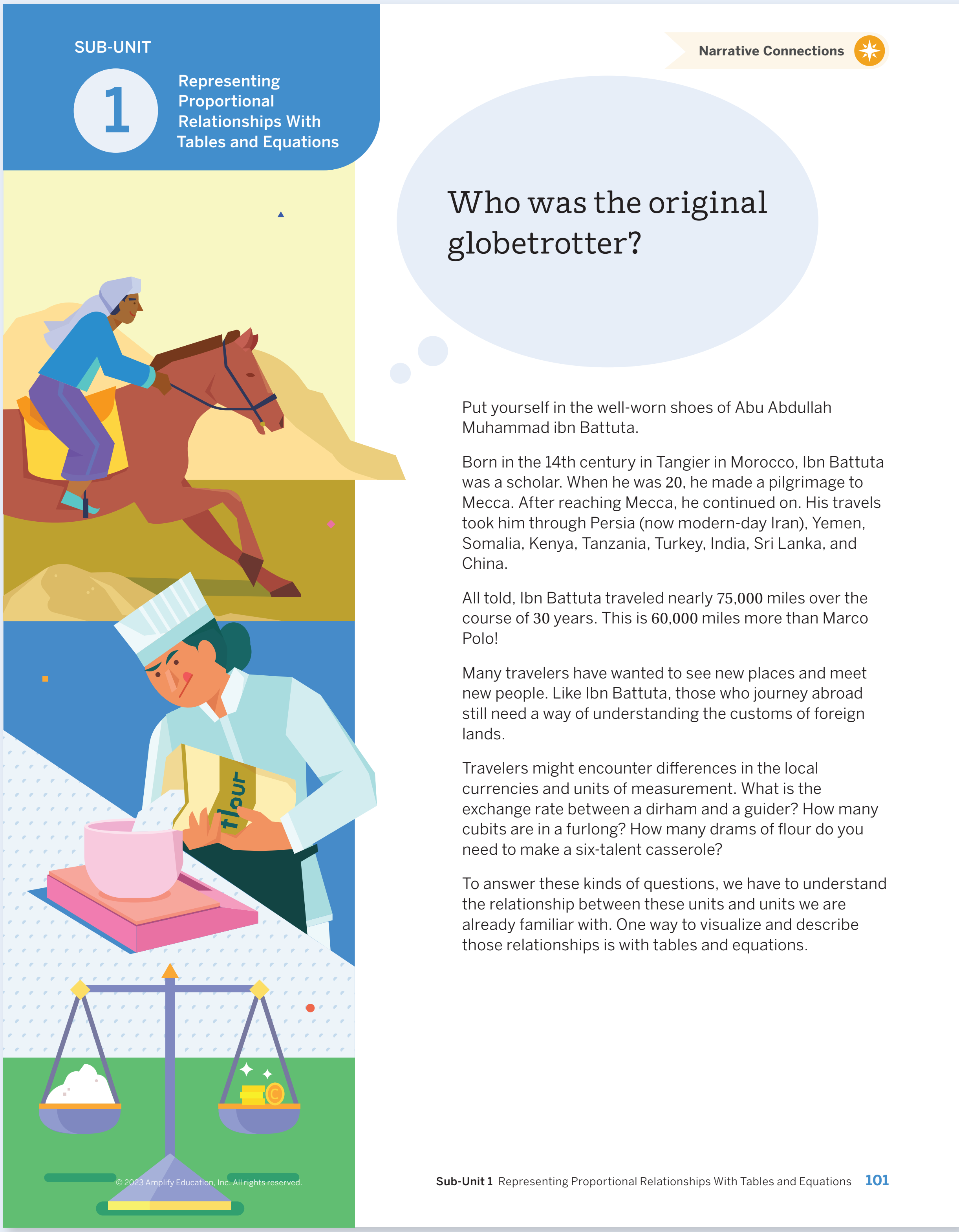
For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 1

Representing Proportional Relationships With Tables and Equations

Whole Class

In this Sub-Unit, using contexts from various cultures, students realize that the relationship between the numbers in a proportional relationship is the same no matter where — or when — you've lived.



SUB-UNIT

1 Representing Proportional Relationships With Tables and Equations

Narrative Connections

Who was the original globetrotter?

Put yourself in the well-worn shoes of Abu Abdullah Muhammad ibn Battuta.

Born in the 14th century in Tangier in Morocco, Ibn Battuta was a scholar. When he was 20, he made a pilgrimage to Mecca. After reaching Mecca, he continued on. His travels took him through Persia (now modern-day Iran), Yemen, Somalia, Kenya, Tanzania, Turkey, India, Sri Lanka, and China.

All told, Ibn Battuta traveled nearly 75,000 miles over the course of 30 years. This is 60,000 miles more than Marco Polo!

Many travelers have wanted to see new places and meet new people. Like Ibn Battuta, those who journey abroad still need a way of understanding the customs of foreign lands.

Travelers might encounter differences in the local currencies and units of measurement. What is the exchange rate between a dirham and a guider? How many cubits are in a furlong? How many drams of flour do you need to make a six-talent casserole?

To answer these kinds of questions, we have to understand the relationship between these units and units we are already familiar with. One way to visualize and describe those relationships is with tables and equations.

Sub-Unit 1 Representing Proportional Relationships With Tables and Equations 101

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how proportionality is used to compare currencies or units in the following places:

- **Lesson 2, Activity 1:** Pence and Wampum
- **Lesson 3, Activity 1:** Using Tables to Convert Currency
- **Lesson 5, Activities 1–2:** Teaspoons and Tablespoons, Baking Bread
- **Lesson 7, Activity 2:** Trading Gold for Salt

UNIT 2 | **LESSON 2**

Introducing Proportional Relationships With Tables

Let's solve problems involving proportional relationships by using tables.

Focus

Goals

1. **Language Goal:** Understand that the term *proportional relationship* refers to when one value is related to another by multiplying by a constant of proportionality. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Describe relationships between rows or between columns in a table that represent a proportional relationship. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Explain how to calculate missing values in a table that represents a proportional relationship. **(Speaking and Listening)**

Coherence

Today

This lesson introduces the concept of a proportional relationship by looking at tables of equivalent ratios. Students notice that all entries in the right column of the table can be obtained by multiplying entries in the left column by the same number, and that this number is called the *constant of proportionality.* Students identify contexts that make using the constant of proportionality a more convenient approach than thinking about equivalent ratios.

Previously

In Lesson 1, students explored the connections between music and mathematical ratios.

Coming Soon

In Lesson 3, students continue to explore proportional relationships in the contexts of currency and geometry.

Rigor

- Students build **conceptual understanding** of the characteristics of proportional relationships in tables.
- Students develop the **procedural skills** of determining unit rates and scaling from one ratio to another.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	13 min	12 min	5 min	10 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, *Proportional Relationship in a Table* (for display)
- calculators

Math Language Development

New words

- ***constant of proportionality***
- ***proportional relationship***

Review words

- *equivalent ratios*
- *scale factor*
- *unit rate*

Amps | Featured Activity

Activity 2
Instant Feedback

Students can check their tables and get instant feedback on whether they need to continue working.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

While working with unfamiliar currency, students may doubt their ability to determine the constant of proportionality. Have students replace the names of the currency with something more familiar so that the unknown words do not limit their ability to discern a pattern or structure. Ask them to relate their answers back to the original question.

Modifications to Pacing

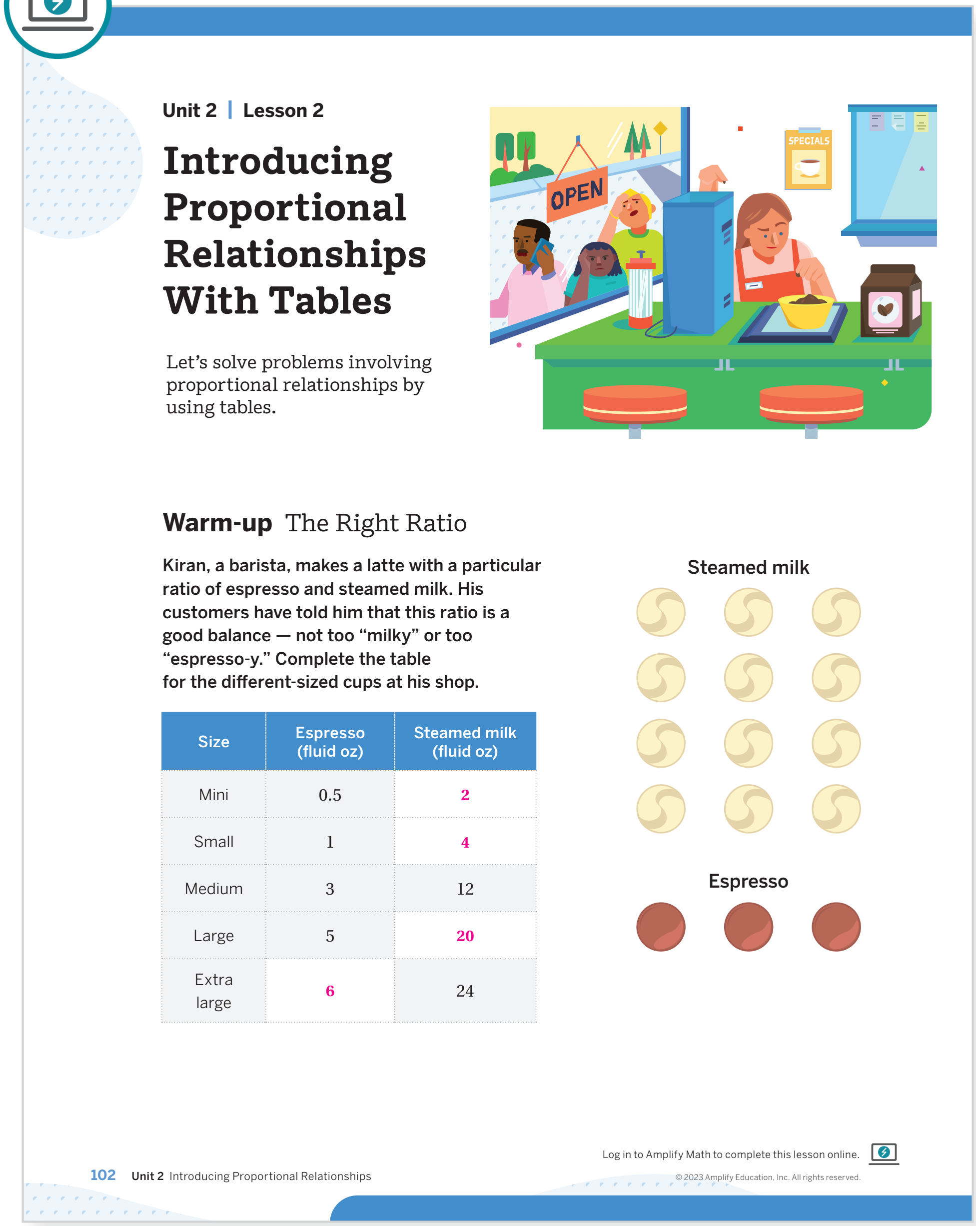
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2 may be used as a discussion question only.
- In **Activity 1**, have students only analyze Tuesday through Thursday in the table.
- In **Activity 2**, Problem 1 may be omitted.

Pairs | 5 min

Warm-up The Right Ratio

Students complete a table of equivalent ratios to review the characteristics of proportionality in a table.



Unit 2 | Lesson 2

Introducing Proportional Relationships With Tables

Let's solve problems involving proportional relationships by using tables.

Warm-up The Right Ratio

Kiran, a barista, makes a latte with a particular ratio of espresso and steamed milk. His customers have told him that this ratio is a good balance — not too "milky" or too "espresso-y." Complete the table for the different-sized cups at his shop.

Size	Espresso (fluid oz)	Steamed milk (fluid oz)
Mini	0.5	2
Small	1	4
Medium	3	12
Large	5	20
Extra large	6	24

Log in to Amplify Math to complete this lesson online.

102 **Unit 2** Introducing Proportional Relationships

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① Launch

Ask students whether they have ever heard anyone ask to add milk or sugar when ordering a coffee. Describe how a latte, a particular style of coffee, can be ordered in different-sized cups but still taste the same.

② Monitor

Help students get started by asking what the information for the medium-sized cup tells them about how much espresso and steamed milk are used to make a latte.

Look for points of confusion:

- **Calculating the small-cup values by subtracting 2 from 3 to get 1, and then subtracting 2 from 12 to get 10.** Have students study the diagram of the medium latte. Ask, "How much steamed milk is needed if you only use 1 ounce of espresso?"

Look for productive strategies:

- Using the relationship of $3 \cdot 4 = 12$, to calculate that each steamed milk is 4 times the amount of espresso.
- Determining the unit rate for the small cup first, using scale factors to calculate the others.

③ Connect

Display the Warm-up PDF, *Proportional Relationship in a Table.*

Have students share the different ways they found the missing values in the table. Look for someone who used the unit rate and also for someone who reasoned from one row to another.

Highlight that there is more than one way to calculate the ratios in a table of missing values. In this unit, the focus will be on the constant factor that relates one value to the other. Write this factor, 4, across each row of the table.

Define a ***proportional relationship*** as one where the values for one quantity are each multiplied by the same number to get the values for the other quantity.

Math Language Development

MLR7: Compare and Connect

During the Connect discussion, highlight the language used in the various strategies. For example, encourage students who used the unit rate to actually use that mathematical language, *unit rate*. Highlight other language or phrases used, such as *multiply by the unit rate.*

English Learners

Be sure students understand what is meant by the phrases "milky" and "espresso-y." Consider providing some examples, using drawings of the circles representing steamed milk and espresso of ratios that might be too "milky" or too "espresso-y."

Power-up

To power up students' ability to reason about ratio tables and unit rates have students complete:

Recall that you can use multiplication and division to determine an unknown value in a ratio table.

1. Determine the missing values in the provided ratio table.
2. What is the unit rate of steamed milk to espresso? 4 oz of milk per 1 oz of espresso.

Espresso (oz)	Steamed milk (oz)
1	4
3	12
6	24

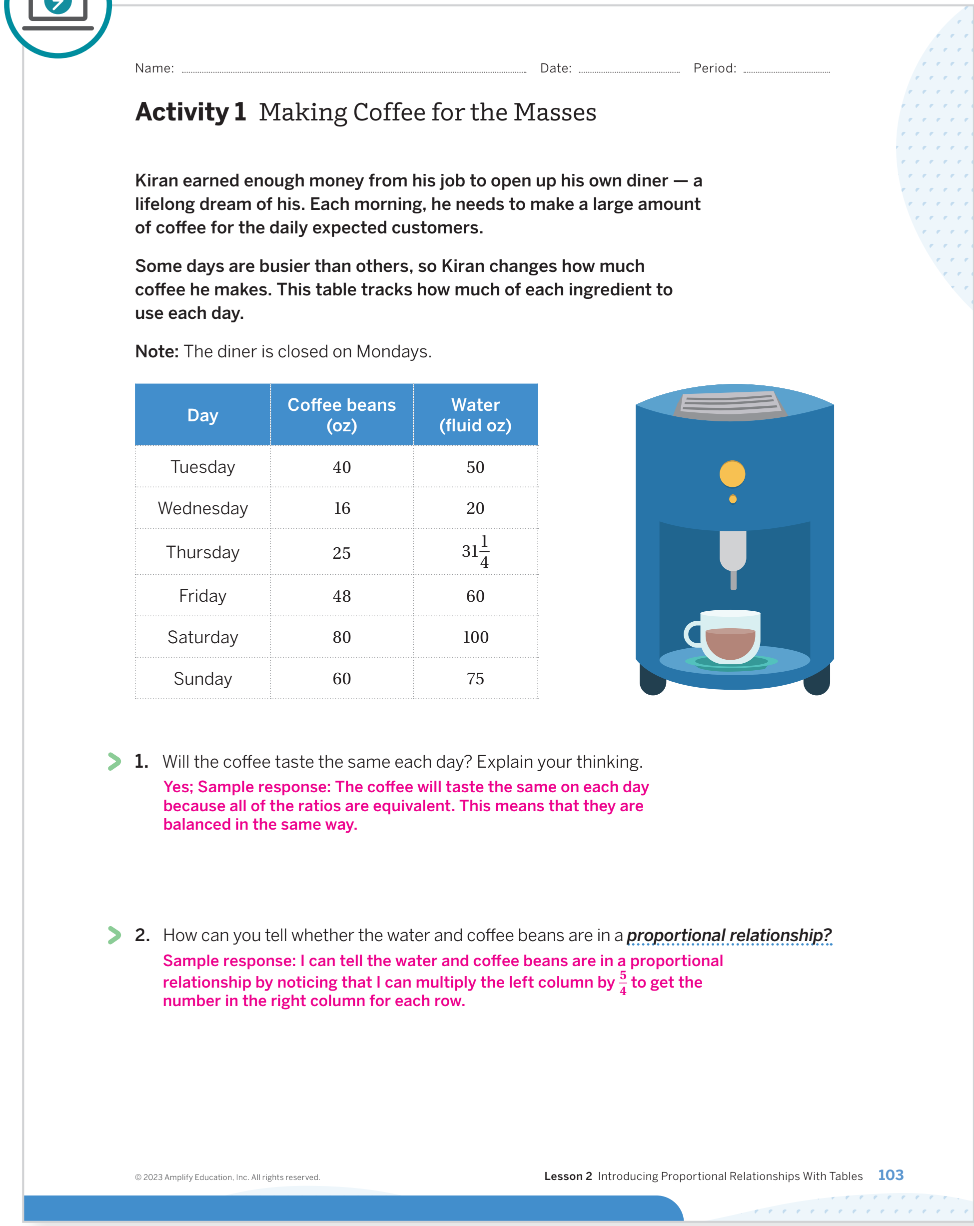
Use: Before the Warm-up.

Informed by: Performance on Lesson 1, Practice Problem 4 and Pre-Unit Readiness Assessment, Problems 1, 2, and 4.

Pairs | 13 min

Activity 1 Making Coffee for the Masses

Students examine a table of ratios to determine whether the values are proportional and notice that all the ratios are related by the same factor.



Name: ____________ Date: ________ Period: ________

Activity 1 Making Coffee for the Masses

Kiran earned enough money from his job to open up his own diner — a lifelong dream of his. Each morning, he needs to make a large amount of coffee for the daily expected customers.

Some days are busier than others, so Kiran changes how much coffee he makes. This table tracks how much of each ingredient to use each day.

Note: The diner is closed on Mondays.

Day	Coffee beans (oz)	Water (fluid oz)
Tuesday	40	50
Wednesday	16	20
Thursday	25	$31\frac{1}{4}$
Friday	48	60
Saturday	80	100
Sunday	60	75

1. Will the coffee taste the same each day? Explain your thinking.
   Yes; Sample response: The coffee will taste the same on each day because all of the ratios are equivalent. This means that they are balanced in the same way.

2. How can you tell whether the water and coffee beans are in a ***proportional relationship?***
   Sample response: I can tell the water and coffee beans are in a proportional relationship by noticing that I can multiply the left column by $\frac{5}{4}$ to get the number in the right column for each row.

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Lesson 2 Introducing Proportional Relationships With Tables 103

1 Launch

Activate background knowledge by asking students whether they have ever cooked, or helped cook, for a large group. Ask, "How did you adjust the recipes?" Provide access to calculators.

2 Monitor

Help students get started by suggesting they calculate the amount of water needed for 1 oz of coffee beans.

Look for points of confusion:

- **Thinking the coffee will not taste the same because the difference in the numbers is not the same.** Have students refer to the Warm-up to determine whether this was true for the latte.

Look for productive strategies:

- Dividing the number in the second column by the number in the first column to determine the missing factor.

3 Connect

Have students share how they found the factor that relates the amount of coffee beans to the amount of water. Look for a strategy that determines the unit rate and also one that divides a value in the second column by the first.

Highlight that this table shows a proportional relationship, even though it is not obvious without performing some calculations. Students may be more familiar with tables of equivalent ratios from Grade 6, where the relationship was more evident.

Define the ***constant of proportionality*** as the number in a proportional relationship, by which the value for one quantity is multiplied to get the value for the other quantity.

Ask, "Where in the table can you see the constant of proportionality? I can see it as a factor in each row or as the unit rate for ounces of coffee.

Note: If students have not yet done this, annotate the table in each row to show the constant of proportionality.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students first compare only Tuesday and Wednesday to determine whether the ratios of coffee beans to water are equivalent. Then have them compare to each next day, pausing after each one to discuss.

Extension: *Math Enrichment*

Have students complete the following problem:
How much does Kiran use each day, on average, of each ingredient?
About 44.8 ounces of coffee beans and about 56 fluid ounces of water.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have students share their responses to Problem 2 with 2 other partners, asking questions for clarity and reasoning. Have them write a second draft that reflects shared ideas and refinement of their initial thoughts.

English Learners

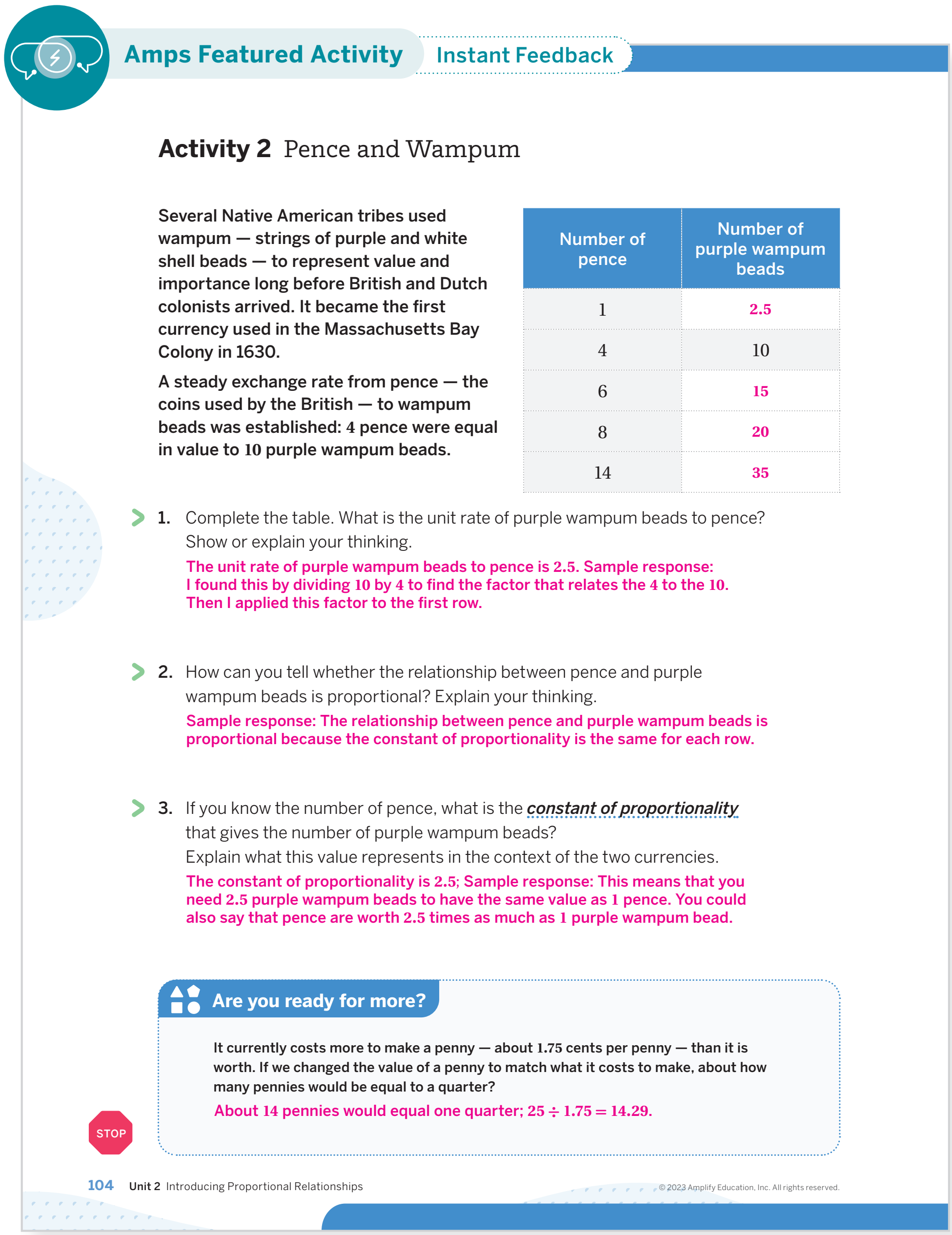
Allow students to write their first draft in their primary language.

Pairs | 12 min

Activity 2 Pence and Wampum

Students explore the historical exchange rate between pence and wampum to accurately use the vocabulary terms *proportional relationship* and *constant of proportionality*.

Amps Featured Activity Instant Feedback



Activity 2 Pence and Wampum

Several Native American tribes used wampum — strings of purple and white shell beads — to represent value and importance long before British and Dutch colonists arrived. It became the first currency used in the Massachusetts Bay Colony in 1630.

A steady exchange rate from pence — the coins used by the British — to wampum beads was established: 4 pence were equal in value to 10 purple wampum beads.

Number of pence	Number of purple wampum beads
1	**2.5**
4	10
6	**15**
8	**20**
14	**35**

1. Complete the table. What is the unit rate of purple wampum beads to pence? Show or explain your thinking.
   The unit rate of purple wampum beads to pence is 2.5. Sample response: I found this by dividing 10 by 4 to find the factor that relates the 4 to the 10. Then I applied this factor to the first row.

2. How can you tell whether the relationship between pence and purple wampum beads is proportional? Explain your thinking.
   Sample response: The relationship between pence and purple wampum beads is proportional because the constant of proportionality is the same for each row.

3. If you know the number of pence, what is the ***constant of proportionality*** that gives the number of purple wampum beads?
   Explain what this value represents in the context of the two currencies.
   The constant of proportionality is 2.5; Sample response: This means that you need 2.5 purple wampum beads to have the same value as 1 pence. You could also say that pence are worth 2.5 times as much as 1 purple wampum bead.

Are you ready for more?

It currently costs more to make a penny — about 1.75 cents per penny — than it is worth. If we changed the value of a penny to match what it costs to make, about how many pennies would be equal to a quarter?

About 14 pennies would equal one quarter; $25 \div 1.75 = 14.29$.

STOP

104 Unit 2 Introducing Proportional Relationships

1 Launch

Ask whether students are aware of the Native American tribes that once lived on or near the land where they live. If possible, research this and have a short history ready to share.

2 Monitor

Help students get started by referring them to the strategy from Activity 1 where students divided to determine the constant of proportionality.

Look for points of confusion:

- **Dividing the first column by the second to get the constant of proportionality.** Ask students to multiply the value in the first column by their number to see if they get the value in the second column.

Look for productive strategies:

- Using the unit rate to determine the constant of proportionality.

3 Connect

Display the completed table.

Have students share how they found the unit rate. Look for strategies that involved determining the constant of proportionality first and strategies that scaled down from the given ratio.

Ask:

- "How are the unit rate and the constant of proportionality related?"
- "How else can you test that a relationship is proportional?" Check whether the other ratios can be simplified to the same ratio, like equivalent fractions.

Highlight that the constant of proportionality lets students know the amount of the second column per one unit of the first column. In this way, it is very much like the unit rate, only without units attached to the value.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students draw a picture to represent the exchange rate of 4 pence to 10 purple beads. Ask them what they would have if each quantity was cut in half, and then cut in half again, to determine the unit rate.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

To support student conversation during the Connect, have them also share their responses to Problem 3, and ask students why the constant of proportionality asked for in this problem is 2.5 and not 0.4. The problem says I know the number of pence and I need to find the number of purple wampum beads.

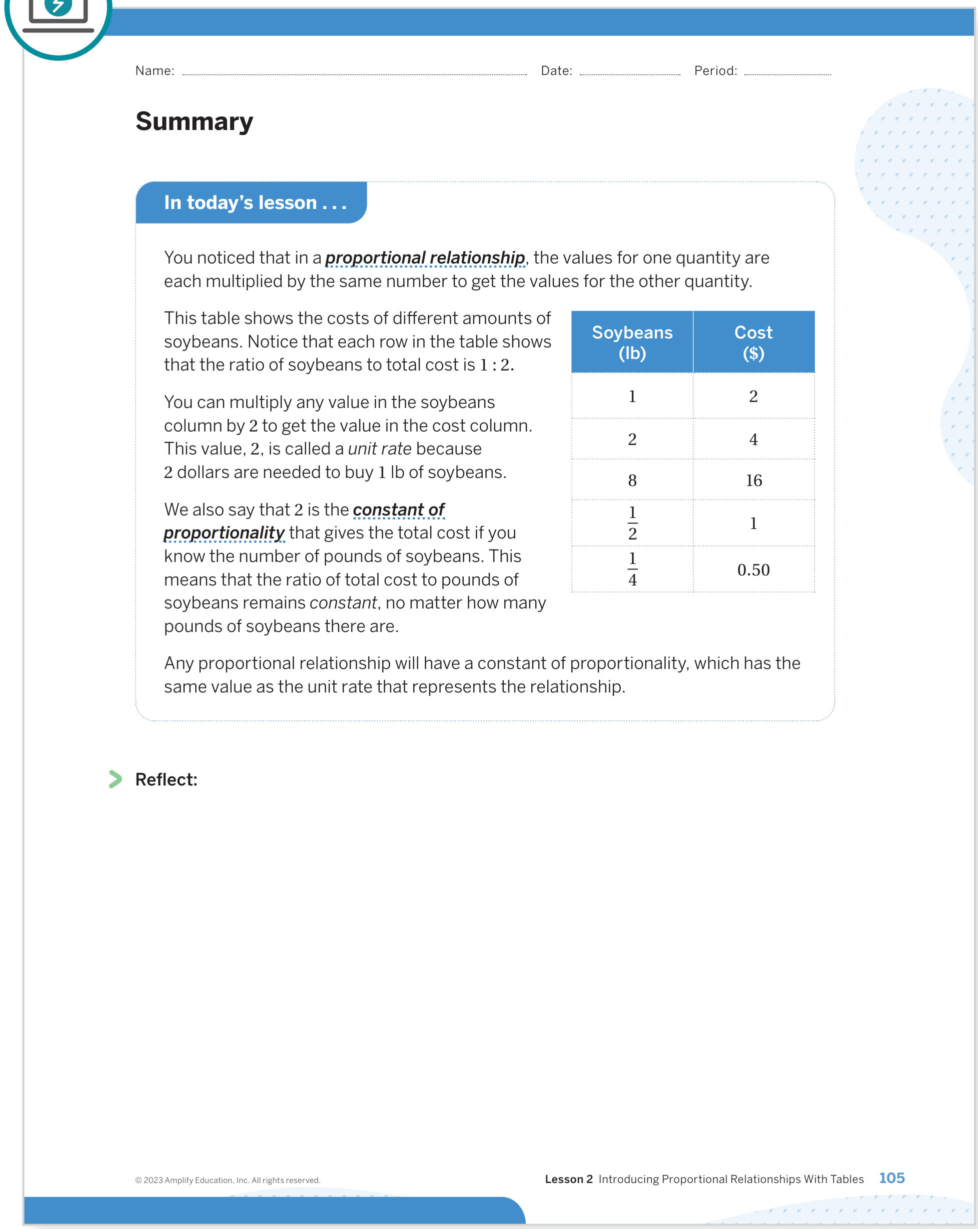
English Learners

Show the calculations for each row in the table that illustrates the constant of proportionality, such as $\frac{10}{4} = \frac{5}{2} = 2.5$, $\frac{15}{6} = \frac{5}{2} = 2.5$, and $\frac{20}{8} = \frac{5}{2} = 2.5$. Circle 2.5 in each and say "constant of proportionality."

Whole Class | 5 min

Summary

Review and synthesize how all proportional relationships have a constant of proportionality, which has the same value as the relationship's unit rate.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You noticed that in a ***proportional relationship***, the values for one quantity are each multiplied by the same number to get the values for the other quantity.

This table shows the costs of different amounts of soybeans. Notice that each row in the table shows that the ratio of soybeans to total cost is 1 : 2.

Soybeans (lb)	Cost ($)
1	2
2	4
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

You can multiply any value in the soybeans column by 2 to get the value in the cost column. This value, 2, is called a *unit rate* because 2 dollars are needed to buy 1 lb of soybeans.

We also say that 2 is the ***constant of proportionality*** that gives the total cost if you know the number of pounds of soybeans. This means that the ratio of total cost to pounds of soybeans remains *constant*, no matter how many pounds of soybeans there are.

Any proportional relationship will have a constant of proportionality, which has the same value as the unit rate that represents the relationship.

Reflect:

Synthesize

Display the table from the Summary.

Highlight that in a table of ratios, if students determine a number by which values for one quantity are each multiplied to get the values for the other quantity, then the relationship is proportional. The word *constant* in the term *constant of proportionality* is a reminder that the value used for multiplication stays the same throughout the relationship.

Formalize vocabulary:

- *proportional relationship*
- *constant of proportionality*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

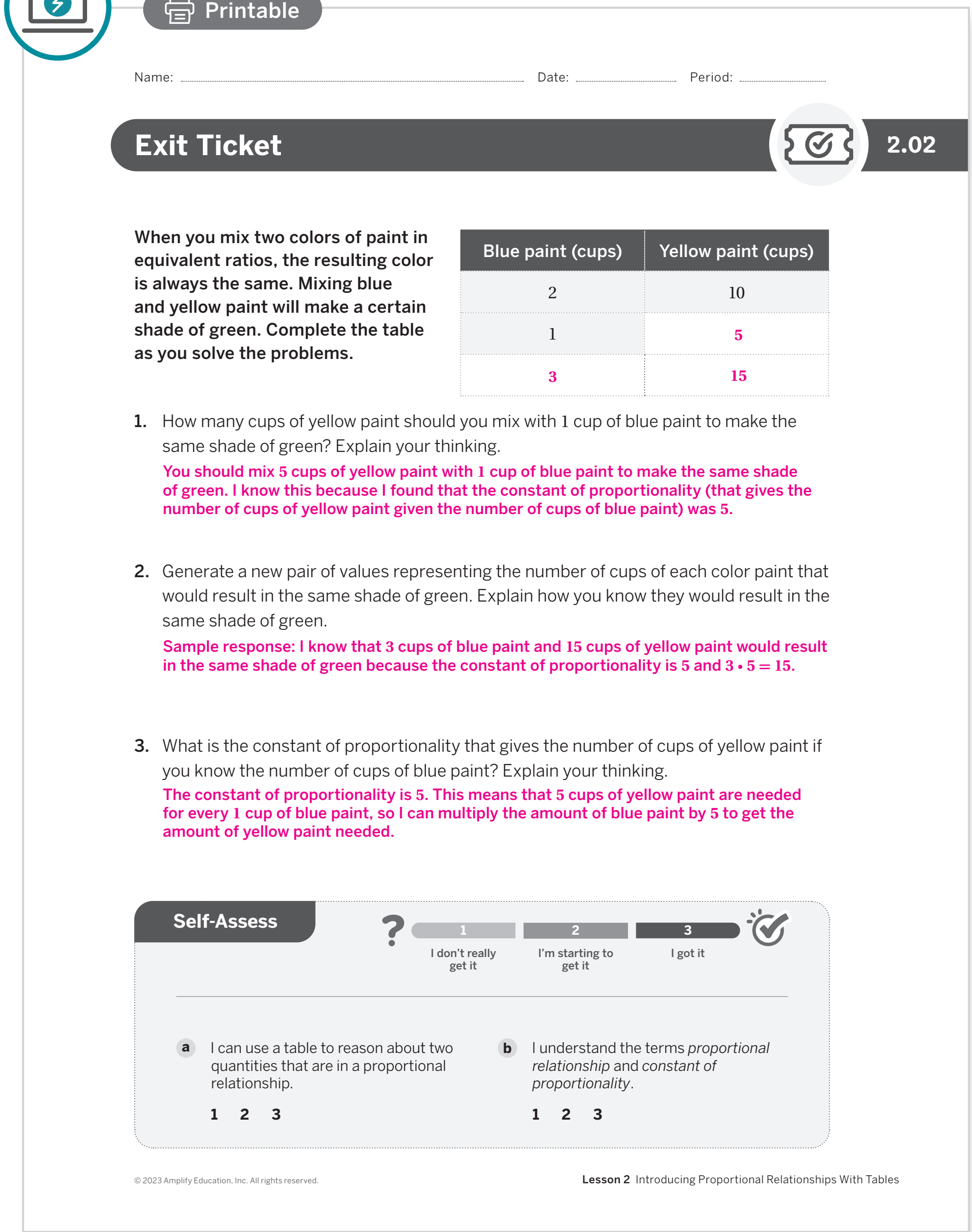
- "What does it mean for two things to be proportionally related? How can you tell?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of proportional relationships by identifying the constant of proportionality and using it to complete a table of equivalent ratios.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.02

When you mix two colors of paint in equivalent ratios, the resulting color is always the same. Mixing blue and yellow paint will make a certain shade of green. Complete the table as you solve the problems.

Blue paint (cups)	Yellow paint (cups)
2	10
1	5
3	15

1. How many cups of yellow paint should you mix with 1 cup of blue paint to make the same shade of green? Explain your thinking.
   You should mix 5 cups of yellow paint with 1 cup of blue paint to make the same shade of green. I know this because I found that the constant of proportionality (that gives the number of cups of yellow paint given the number of cups of blue paint) was 5.

2. Generate a new pair of values representing the number of cups of each color paint that would result in the same shade of green. Explain how you know they would result in the same shade of green.
   Sample response: I know that 3 cups of blue paint and 15 cups of yellow paint would result in the same shade of green because the constant of proportionality is 5 and $3 \cdot 5 = 15$.

3. What is the constant of proportionality that gives the number of cups of yellow paint if you know the number of cups of blue paint? Explain your thinking.
   The constant of proportionality is 5. This means that 5 cups of yellow paint are needed for every 1 cup of blue paint, so I can multiply the amount of blue paint by 5 to get the amount of yellow paint needed.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can use a table to reason about two quantities that are in a proportional relationship.
1 2 3

b. I understand the terms *proportional relationship* and *constant of proportionality*.
1 2 3

Lesson 2 Introducing Proportional Relationships With Tables

Success looks like . . .

- **Language Goal:** Understanding that the term *proportional relationship* refers to when one value is related to another by multiplying by a constant of proportionality. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Describing relationships between rows or between columns in a table that represent a proportional relationship. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Explaining how to calculate missing values in a table that represents a proportional relationship. **(Speaking and Listening)**
  - » Explaining how to generate a new pair of values that gives the same shade of green in Problem 2.

Suggested next steps

If students use addition or subtraction to solve Problem 2, consider:

- Reviewing multiplicative strategies from Activity 1.
- Assigning Practice Problem 2.

If students reverse the relationship in Problem 3, this will be addressed further in Lessons 3–5.

Professional Learning

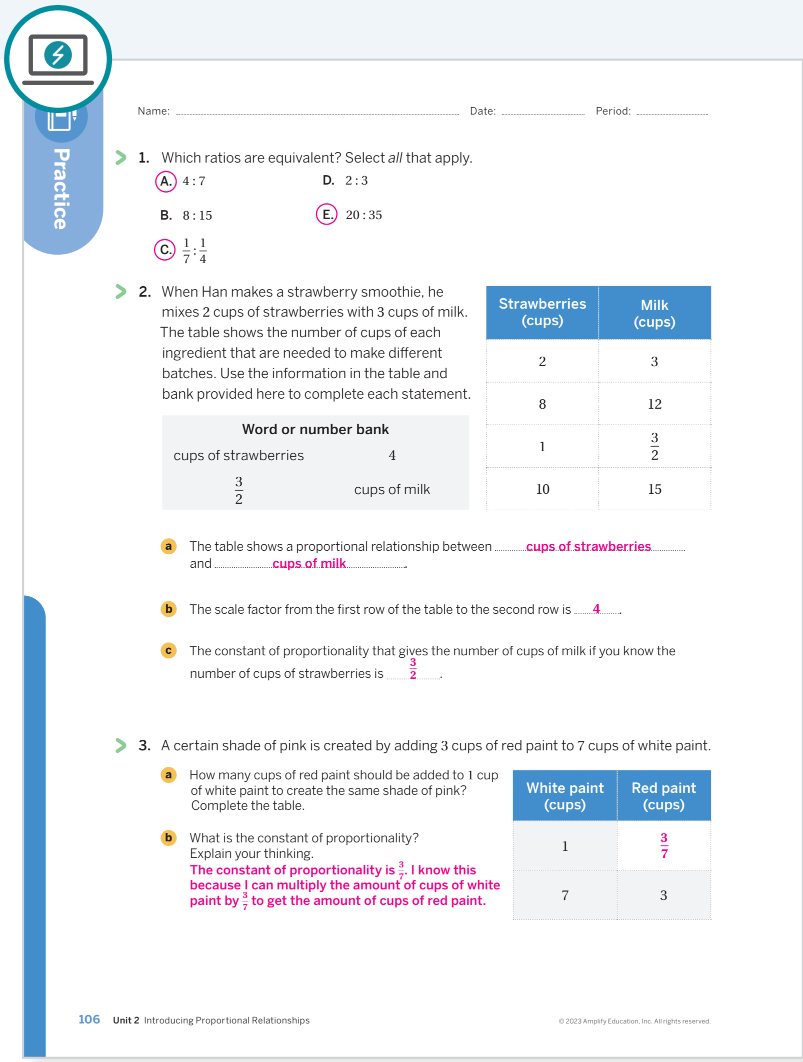
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- The instructional goal for this lesson was to understand that the term *proportional relationship* refers to when one quantity is related to another by multiplying by a constant of proportionality. How well did students accomplish this? What did you specifically do to help students accomplish it?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Which ratios are equivalent? Select *all* that apply.
   - [✓ A.] $4:7$
   - D. $2:3$
   - B. $8:15$
   - [✓ E.] $20:35$
   - [✓ C.] $\frac{1}{7}:\frac{1}{4}$

2. When Han makes a strawberry smoothie, he mixes 2 cups of strawberries with 3 cups of milk. The table shows the number of cups of each ingredient that are needed to make different batches. Use the information in the table and bank provided here to complete each statement.

Word or number bank

cups of strawberries	4
$\frac{3}{2}$	cups of milk

Strawberries (cups)	Milk (cups)
2	3
8	12
1	$\frac{3}{2}$
10	15

a. The table shows a proportional relationship between **cups of strawberries** and **cups of milk**.

b. The scale factor from the first row of the table to the second row is **4**.

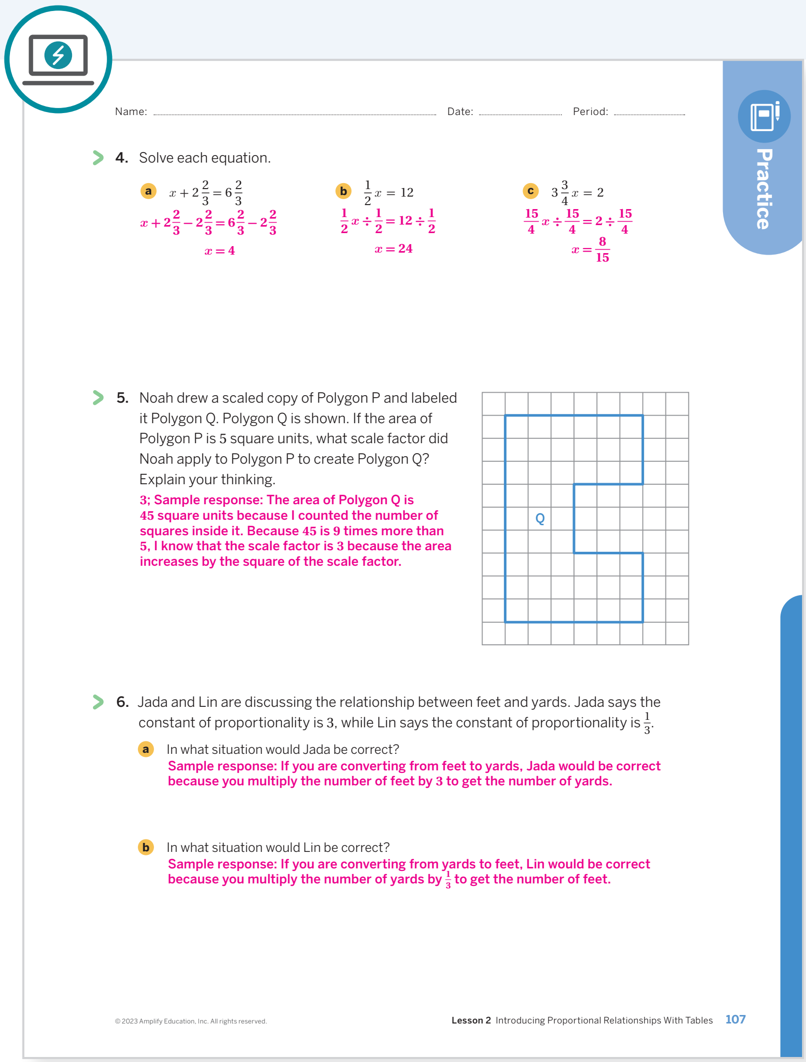
c. The constant of proportionality that gives the number of cups of milk if you know the number of cups of strawberries is $\frac{3}{2}$.

3. A certain shade of pink is created by adding 3 cups of red paint to 7 cups of white paint.

a. How many cups of red paint should be added to 1 cup of white paint to create the same shade of pink? Complete the table.

b. What is the constant of proportionality? Explain your thinking.
The constant of proportionality is $\frac{3}{7}$. I know this because I can multiply the amount of cups of white paint by $\frac{3}{7}$ to get the amount of cups of red paint.

White paint (cups)	Red paint (cups)
1	$\frac{3}{7}$
7	3



Name: ______ Date: ______ Period: ______

Practice

4. Solve each equation.

a. $x + 2\frac{2}{3} = 6\frac{2}{3}$

$x + 2\frac{2}{3} - 2\frac{2}{3} = 6\frac{2}{3} - 2\frac{2}{3}$

$x = 4$

b. $\frac{1}{2}x = 12$

$\frac{1}{2}x \div \frac{1}{2} = 12 \div \frac{1}{2}$

$x = 24$

c. $3\frac{3}{4}x = 2$

$\frac{15}{4}x \div \frac{15}{4} = 2 \div \frac{15}{4}$

$x = \frac{8}{15}$

5. Noah drew a scaled copy of Polygon P and labeled it Polygon Q. Polygon Q is shown. If the area of Polygon P is 5 square units, what scale factor did Noah apply to Polygon P to create Polygon Q? Explain your thinking.
3; Sample response: The area of Polygon Q is 45 square units because I counted the number of squares inside it. Because 45 is 9 times more than 5, I know that the scale factor is 3 because the area increases by the square of the scale factor.

6. Jada and Lin are discussing the relationship between feet and yards. Jada says the constant of proportionality is 3, while Lin says the constant of proportionality is $\frac{1}{3}$.

a. In what situation would Jada be correct?
Sample response: If you are converting from feet to yards, Jada would be correct because you multiply the number of feet by 3 to get the number of yards.

b. In what situation would Lin be correct?
Sample response: If you are converting from yards to feet, Lin would be correct because you multiply the number of yards by $\frac{1}{3}$ to get the number of feet.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-Up	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Grade 6	1
	5	Unit 1 Lesson 6	2
Formative ⚡	6	Unit 2 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 3**

More About the Constant of Proportionality

Let's solve more problems involving proportional relationships by using tables.

Focus

Goals

1. **Language Goal:** Understand and explain that there are two constants of proportionality for every relationship depending on the direction of the relationship, and that they are reciprocals. **(Speaking and Listening)**
2. **Language Goal:** Explain how to calculate missing values in a table that represents a proportional relationship. **(Speaking and Listening)**
3. **Language Goal:** Explain how to determine the constant of proportionality for a proportional relationship represented in a table. **(Speaking and Listening)**

Coherence

Today

Students continue their work with proportional relationships represented in tables by using currency conversions and scaled figures. Students apply their understanding that scaled figures have reciprocal scale factors, as they recognize that proportional relationships have reciprocal constants of proportionality by generalizing repeated operations in tables of values.

Previously

In Grade 6, students used equivalent ratios to determine missing values. In Unit 1 from Grade 7, they used scales and scale factors to determine the missing side lengths of scaled figures.

Coming Soon

In Lessons 5–8, students will formalize their use of the constant of proportionality to determine the missing values in proportional relationships by writing equations of the form $y = kx$.

Rigor

- Students build **conceptual understanding** of the constant of proportionality by comparing two tables of values for the same relationship.
- Students use the constant of proportionality to develop **fluency** in calculating unknown values in a table of values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	13 min	15 min	5 min	7 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- calculators

Math Language Development

Review words

- *constant of proportionality*
- *equivalent ratios*
- *proportional relationship*
- *reciprocal*
- *scale factor*
- *unit rate*

Amps | Featured Activity

Warm-up

See Student Thinking

Students are asked to explain their thinking on how they determined the missing values in a table of values. This gives you insight into the methods students are using when thinking about proportional relationships.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may begin to show signs of stress when they see more new currencies in Activity 1. Guide students to discuss how they have managed their stress previously in similar circumstances. Ask them to develop a plan to release their stress so that they can complete this activity. Using repeated multiplication or division will help to eliminate stress as students continually evaluate the reasonableness of their results.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

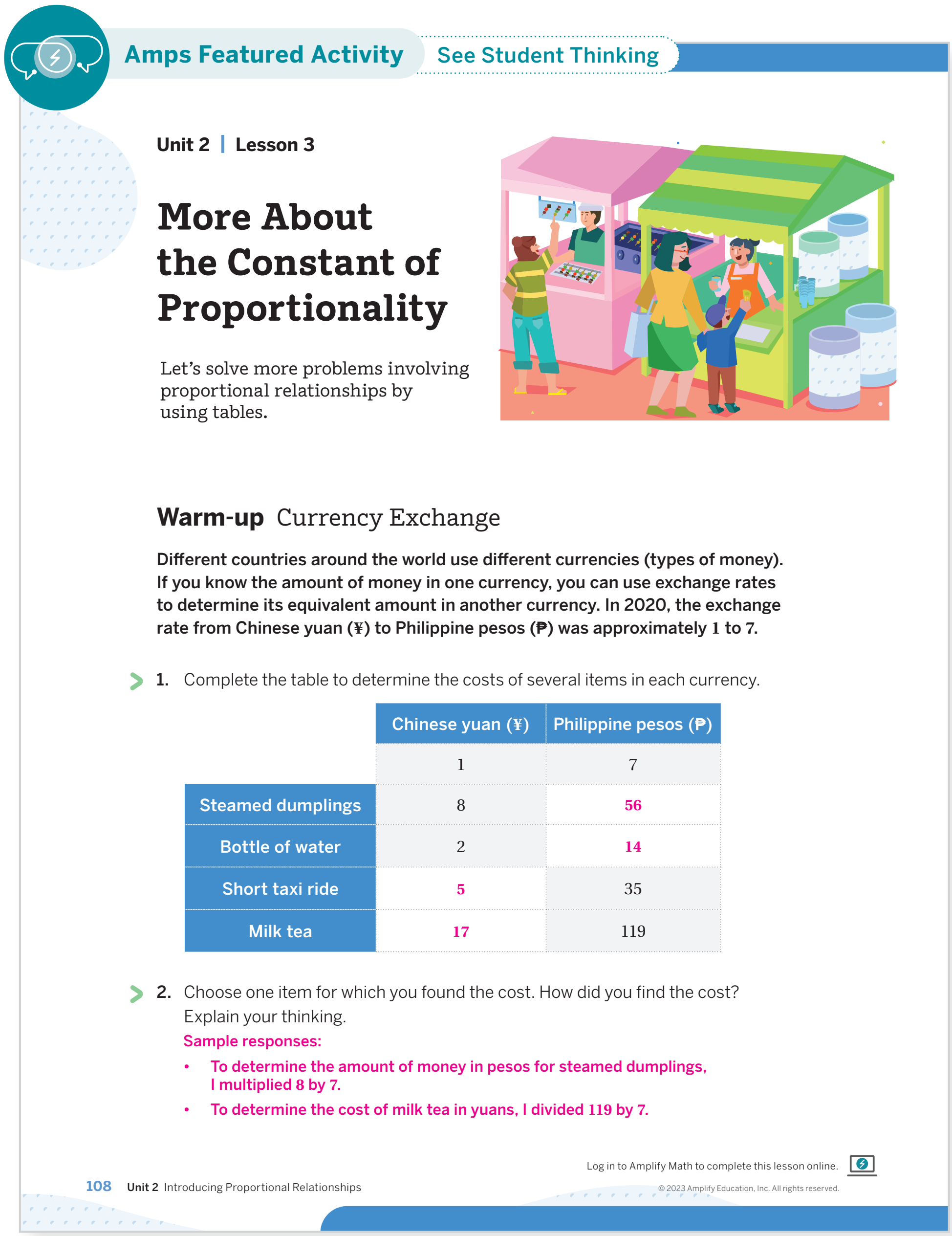
- The **Warm-up** may be omitted.
- In **Activity 1**, have each student in a pair complete only one table.
- In **Activity 2**, have students complete the table only. Then, mention that the scale factors are the constants of proportionality between the two scaled figures.

Independent | 5 min

Warm-up Currency Exchange

Students complete a table of values comparing Chinese yuan to Philippine pesos by using a given ratio to see its use in exchanging one currency for the other.

Amps Featured Activity See Student Thinking



Unit 2 | Lesson 3

More About the Constant of Proportionality

Let's solve more problems involving proportional relationships by using tables.

Warm-up Currency Exchange

Different countries around the world use different currencies (types of money). If you know the amount of money in one currency, you can use exchange rates to determine its equivalent amount in another currency. In 2020, the exchange rate from Chinese yuan (¥) to Philippine pesos (₱) was approximately 1 to 7.

1. Complete the table to determine the costs of several items in each currency.

	Chinese yuan (¥)	Philippine pesos (₱)
	1	7
Steamed dumplings	8	56
Bottle of water	2	14
Short taxi ride	5	35
Milk tea	17	119

2. Choose one item for which you found the cost. How did you find the cost? Explain your thinking.
Sample responses:
- To determine the amount of money in pesos for steamed dumplings, I multiplied 8 by 7.
- To determine the cost of milk tea in yuans, I divided 119 by 7.

Log in to Amplify Math to complete this lesson online.

108 Unit 2 Introducing Proportional Relationships

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1 Launch

Activate students' background knowledge by asking them if they have ever lived in or traveled to another country and used a currency other than U.S. dollars.

2 Monitor

Help students get started by asking, "How could you use equivalent ratios to solve this problem?"

Look for points of confusion:

- **Thinking they can add or subtract from row to row.** Have students apply this thinking to quarters and dollars and notice that when students increase dollars by 1, students must increase quarters by 4.

Look for productive strategies:

- Multiplying each value by 7 in the Chinese yuan column and dividing each value in the Philippine pesos column by 7 to determine the missing values.
- Multiplying the unit rate by the same number to determine missing values in a new row. For example, multiplying ¥1 and ₱7 by 5 to get the price of the short taxi ride.

3 Connect

Display the table of values.

Ask:

- "Is there more than one strategy you could use to determine the missing values in the table?"

Highlight that the constant of proportionality is seen in the first row (1, 7) and label an arrow from the yuan column to the pesos column with "constant of proportionality × 7". A price in pesos can be found from a price in yuan by multiplying by 7. Conversely, a price in yuan can be found from a price in pesos, by dividing by 7 or multiplying by $\frac{1}{7}$. Add an arrow from the pesos column to the yuan column to model this relationship. The price in pesos is proportional to the price in yuan, and the price in yuan is also proportional to the price in pesos.

Math Language Development

MLR8: Discussion Supports

During the Connect, model the process of determining the missing values in the table of values using a think-aloud approach. Model the steps by adding arrows vertically or horizontally, to show how to obtain each value using multiplication or division.

English Learners

Use gestures when modeling the language for horizontal and vertical arrows in the table.

Power-up

To power up students' ability to reason about the order of quantities in a unit rate:

Recall that a *unit rate* is a comparison of how much one unit changes when another changes by 1.

A store sells 5 lb of apples for $12.50.

a. What is the unit rate of cost per pound? $2.50 per pound

b. What is the unit rate of pounds per dollar? 0.40 pound per dollar

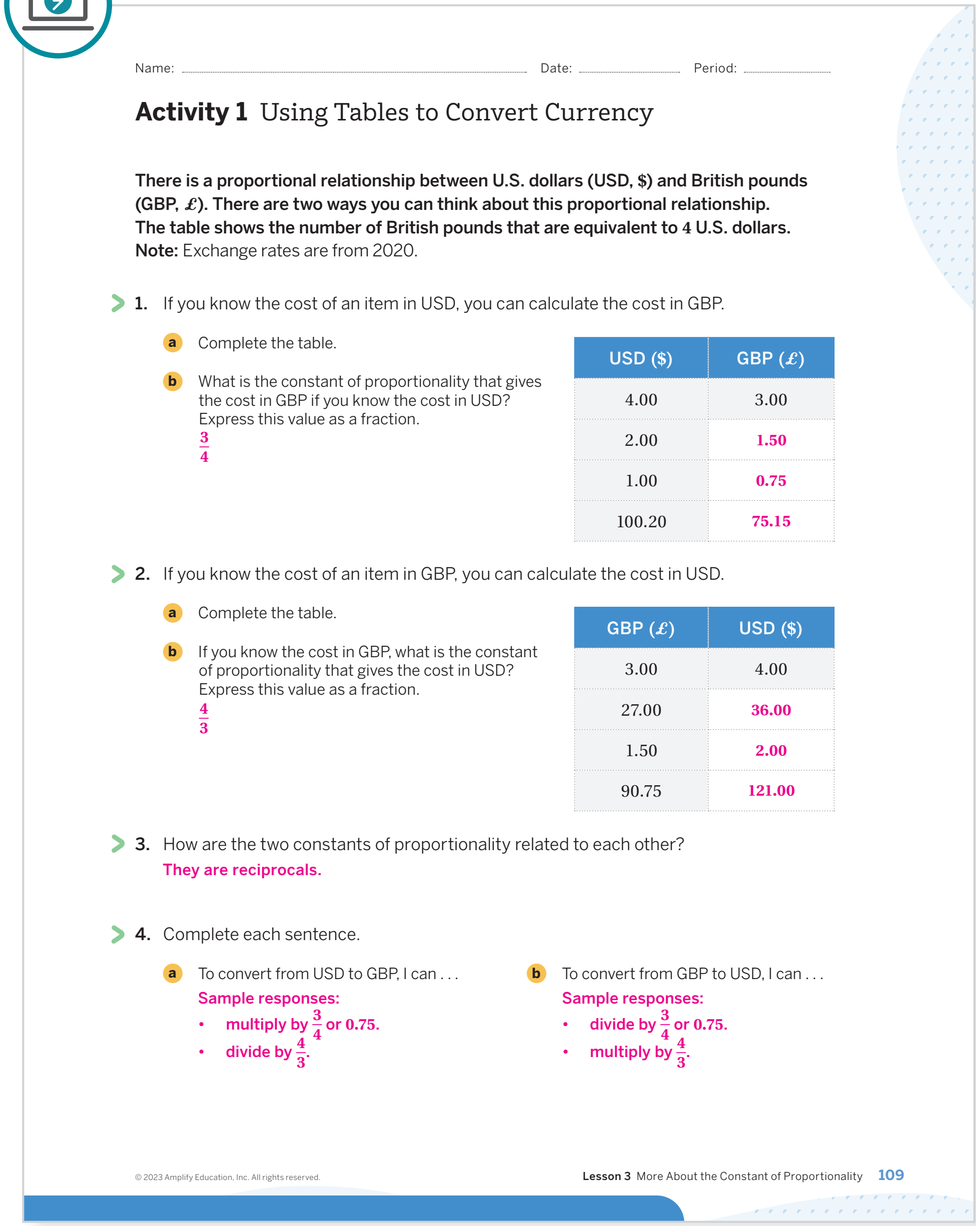
Use: Before Activity 1.

Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3.

Pairs | 13 min

Activity 1 Using Tables to Convert Currency

Students compare the same relationship in two tables of values to discover that the constants of proportionality are reciprocals.



Name: .... Date: .... Period: ....

Activity 1 Using Tables to Convert Currency

There is a proportional relationship between U.S. dollars (USD, \$) and British pounds (GBP, £). There are two ways you can think about this proportional relationship. The table shows the number of British pounds that are equivalent to 4 U.S. dollars.
Note: Exchange rates are from 2020.

1. If you know the cost of an item in USD, you can calculate the cost in GBP.

a Complete the table.

b What is the constant of proportionality that gives the cost in GBP if you know the cost in USD? Express this value as a fraction.
$\frac{3}{4}$

USD (\$)	GBP (£)
4.00	3.00
2.00	**1.50**
1.00	**0.75**
100.20	**75.15**

2. If you know the cost of an item in GBP, you can calculate the cost in USD.

a Complete the table.

b If you know the cost in GBP, what is the constant of proportionality that gives the cost in USD? Express this value as a fraction.
$\frac{4}{3}$

GBP (£)	USD (\$)
3.00	4.00
27.00	**36.00**
1.50	**2.00**
90.75	**121.00**

3. How are the two constants of proportionality related to each other?
They are reciprocals.

4. Complete each sentence.

a To convert from USD to GBP, I can . . .
Sample responses:
- **multiply by $\frac{3}{4}$ or 0.75.**
- **divide by $\frac{4}{3}$.**

b To convert from GBP to USD, I can . . .
Sample responses:
- **divide by $\frac{3}{4}$ or 0.75.**
- **multiply by $\frac{4}{3}$.**

1 Launch

Set an expectation for the amount of time students will have to work in pairs on this activity.

2 Monitor

Help students get started by asking them what it means for U.S. dollars (USD) and British pounds (GBP) to be in a proportional relationship.

Look for points of confusion:

- **Writing the constant of proportionality as its reciprocal.** Remind students it is the value the first quantity is multiplied by to get to the second quantity, not the ratio of first to second. Have students add the arrow from USD to GBP to visualize this relationship.

Look for productive strategies:

- Using repeated multiplication or division to determine missing values.
- Multiplying by the constant of proportionality to determine the missing value when repeated multiplication and division would not be appropriate.

3 Connect

Display the two tables of values.

Have pairs of students share their strategies for determining the missing values in each table.

Ask:

- "What method did you use to determine the last value in each table?"
- "For each constant of proportionality, what does it represent in context? Why is one value greater than 1 while the other is less than 1?"

Highlight that there are two constants of proportionality for every proportional relationship. They are reciprocal values since they are showing the relationship in opposite directions. Remind students that, if $(1, k)$ is not in their table of values, as in the second table, they can still determine the constant of proportionality by going from left to right in their table, or by determining the ratio of $y : x$.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Alter the given values so that they are multiples of 4 in Problem 1 and multiples of 3 in Problem 2. This will allow students to focus on the goal of the activity, which is to discover that the constants of proportionality are reciprocals, not necessarily operations with decimals.

Extension: *Math Enrichment*

Have students use the internet to look up other exchange rates, such as the following: USD to the Colombian peso, USD to the Egyptian pound, and USD to the Indian rupee. Have them determine how to use online exchange rate calculators to find the equivalent of 25 USD for each.

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share their strategies for determining the constant of proportionality, ask for details by requesting students to elaborate or give an example.

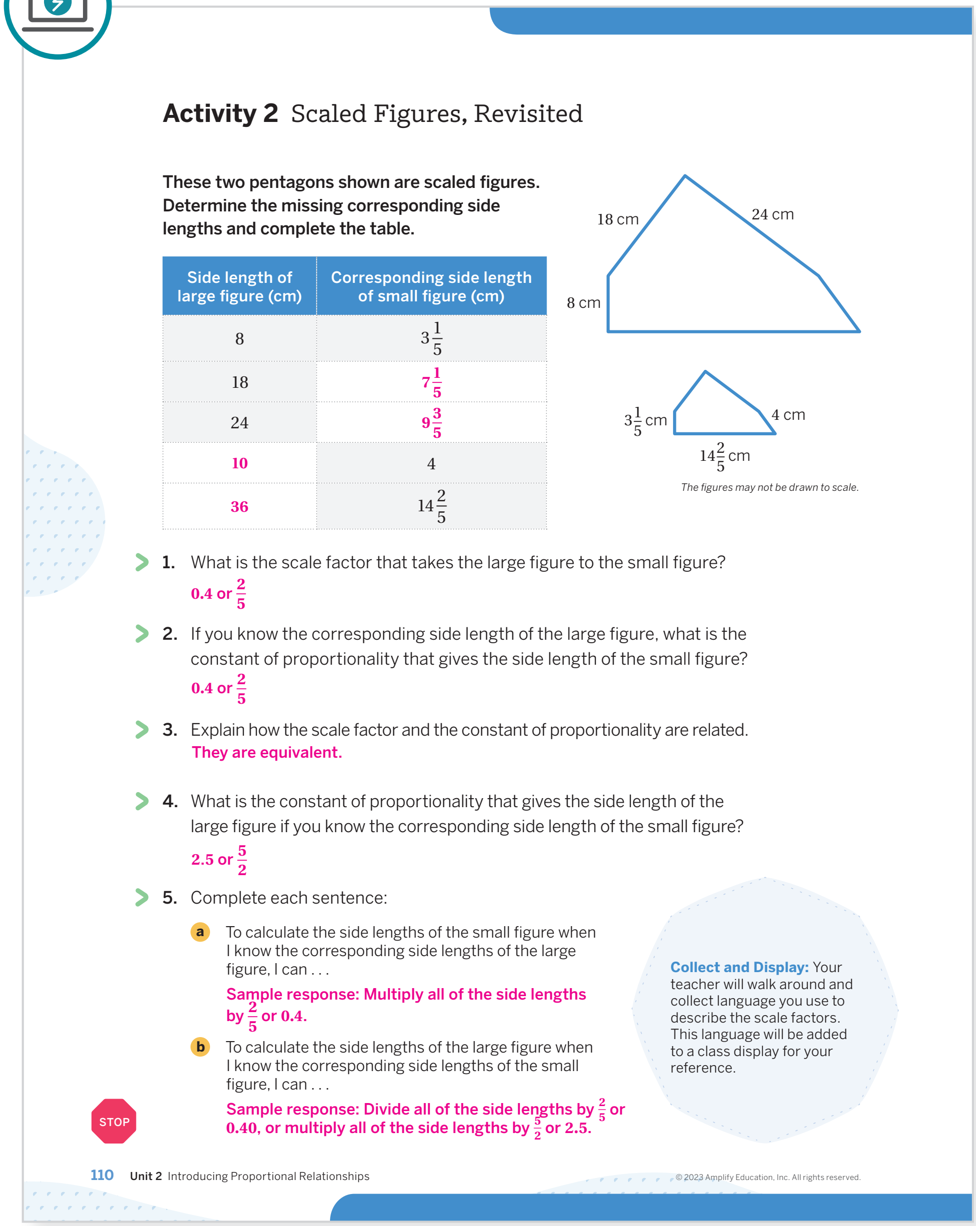
English Learners

As students share strategies, ask them to illustrate how the constant of proportionality is seen in each table by annotating how to determine its value.

Pairs | 15 min

Activity 2 Scaled Figures, Revisited

Students revisit scaled figures from Unit 1 and use a table of values to explore the relationship between the scale factor and the constant of proportionality.



Activity 2 Scaled Figures, Revisited

These two pentagons shown are scaled figures. Determine the missing corresponding side lengths and complete the table.

Side length of large figure (cm)	Corresponding side length of small figure (cm)
8	$3\frac{1}{5}$
18	$7\frac{1}{5}$
24	$9\frac{3}{5}$
10	4
36	$14\frac{2}{5}$

The figures may not be drawn to scale.

1. What is the scale factor that takes the large figure to the small figure?
   0.4 or $\frac{2}{5}$
2. If you know the corresponding side length of the large figure, what is the constant of proportionality that gives the side length of the small figure?
   0.4 or $\frac{2}{5}$
3. Explain how the scale factor and the constant of proportionality are related.
   They are equivalent.
4. What is the constant of proportionality that gives the side length of the large figure if you know the corresponding side length of the small figure?
   2.5 or $\frac{5}{2}$
5. Complete each sentence:
   - a To calculate the side lengths of the small figure when I know the corresponding side lengths of the large figure, I can . . .
     Sample response: Multiply all of the side lengths by $\frac{2}{5}$ or 0.4.
   - b To calculate the side lengths of the large figure when I know the corresponding side lengths of the small figure, I can . . .
     Sample response: Divide all of the side lengths by $\frac{2}{5}$ or 0.40, or multiply all of the side lengths by $\frac{5}{2}$ or 2.5.

Collect and Display: Your teacher will walk around and collect language you use to describe the scale factors. This language will be added to a class display for your reference.

STOP

110 **Unit 2** Introducing Proportional Relationships

1 Launch

Activate students' prior knowledge by asking what they recall about scaled figures.

2 Monitor

Help students get started by asking them how they determine the missing side lengths in the scaled figures.

Look for points of confusion:

- **Thinking that the two constants of proportionality are not reciprocals because they are in decimal form.** Ask students to change their constants of proportionality to a fraction form to represent this relationship.

3 Connect

Display the table and the similar figures.

Have pairs of students share how they determined the scale factors and the constants of proportionality.

Ask:

- "How is the constant of proportionality related to the scale factor?" Sample response: They are the same value.
- "How did you determine the constant of proportionality if 1 was not in your table or a side length of the large figure?" Sample response: I determined the ratio of the small figure to the large figure.
- "What does it represent for a scale factor, or a constant of proportionality, to be greater than 1? Less than 1?" Sample response: If it is greater than 1, then the resulting values are getting larger. If it is less than 1, then the resulting values are getting smaller.

Highlight that scale factors are the constants of proportionality between two figures. Add arrows in the the table and between figures to show the multiplication by $\frac{2}{5}$ from the large figure to the small figure. Also add arrows from the small figure to the large figure to show the multiplication by $\frac{5}{2}$. Bring attention to the fact that reversing the direction requires multiplying by the reciprocal value of the constant of proportionality.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Activate prior knowledge of scaled figures by illustrating how to determine the scale factor. Consider also altering the numerical measurements so that students work with whole numbers only. For example, scale the smaller figure by a factor of 10, so that the side lengths are 40 cm, 144 cm, and 32 cm. This figure will now become the larger figure, and the constants of proportionality will be 4 and $\frac{1}{4}$.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share how they determined the scale factors and constants of proportionality, collect language that they use to describe these terms. Place this language on a class display and encourage students to refer back to this display in future discussions.

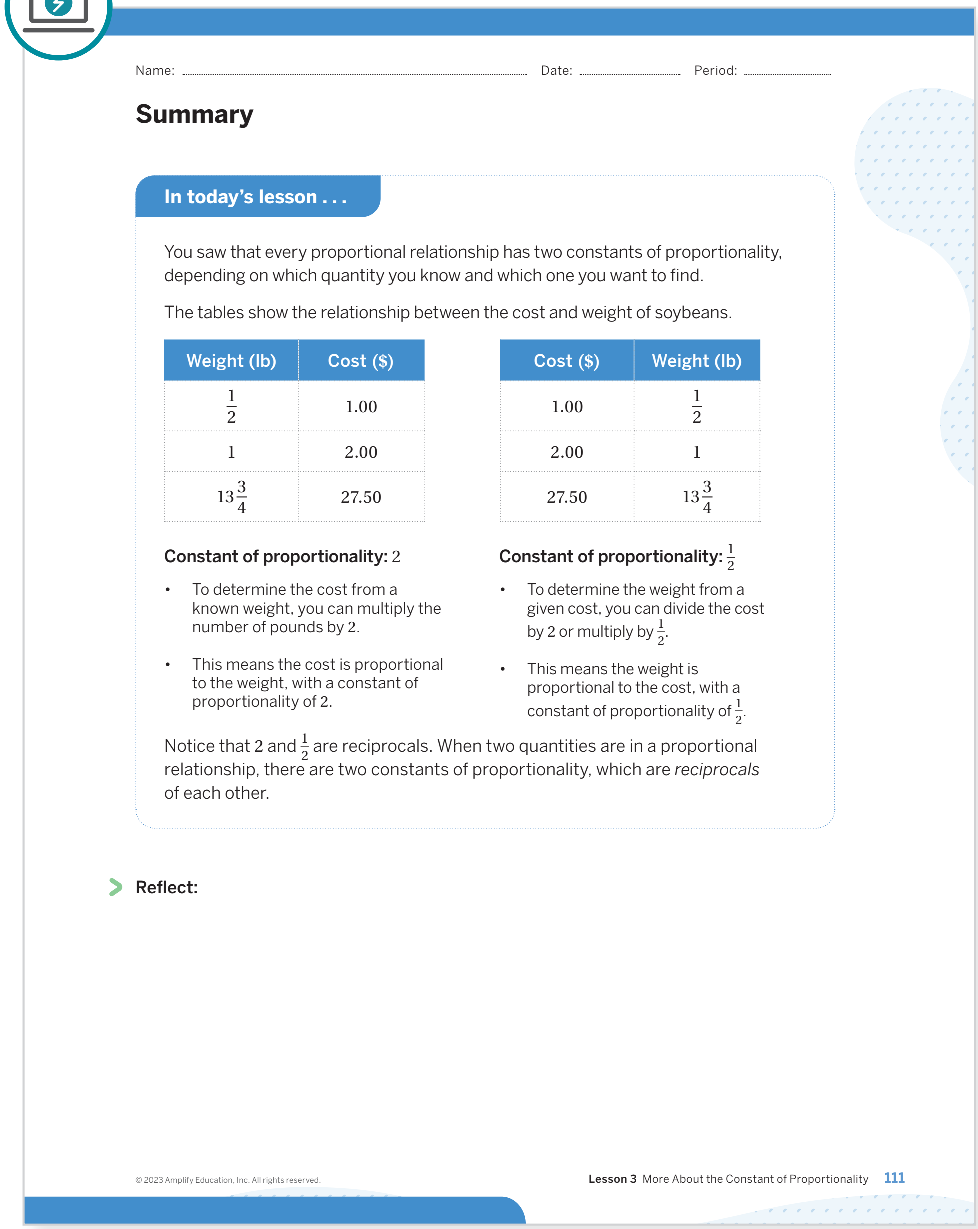
English Learners

Use diagrams to display how the scale factors are the constants of proportionality between two scaled figures.

Whole Class | 5 min

Summary

Review and synthesize why every proportional relationship has two constants of proportionality and summarize how to determine their values.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that every proportional relationship has two constants of proportionality, depending on which quantity you know and which one you want to find.

The tables show the relationship between the cost and weight of soybeans.

Weight (lb)	Cost ($)
$\frac{1}{2}$	1.00
1	2.00
$13\frac{3}{4}$	27.50

Cost ($)	Weight (lb)
1.00	$\frac{1}{2}$
2.00	1
27.50	$13\frac{3}{4}$

Constant of proportionality: 2

- To determine the cost from a known weight, you can multiply the number of pounds by 2.
- This means the cost is proportional to the weight, with a constant of proportionality of 2.

Constant of proportionality: $\frac{1}{2}$

- To determine the weight from a given cost, you can divide the cost by 2 or multiply by $\frac{1}{2}$.
- This means the weight is proportional to the cost, with a constant of proportionality of $\frac{1}{2}$.

Notice that 2 and $\frac{1}{2}$ are reciprocals. When two quantities are in a proportional relationship, there are two constants of proportionality, which are *reciprocals* of each other.

Reflect:

Lesson 3 More About the Constant of Proportionality 111

Synthesize

Display the two tables from the Summary.

Ask:

- "Does it make sense that the cost and weight of soybeans are in a proportional relationship?"
- "How is the constant of proportionality found in each table?" Sample response: Checking the row $(1, k)$ or simplifying $y : x$.
- "How are the two constants of proportionality related to each other?" Sample response: They are reciprocals of one another.
- "What is represented by each constant of proportionality in context?" Sample response: 2 represents $2 per pound, while $\frac{1}{2}$ represents $\frac{1}{2}$ lb for every $1.

Highlight that every proportional relationship has two constants of proportionality by adding arrows to each of the tables in the summary labeled with their respective constants of proportionality. Bring attention to the fact that because the x- and y-columns are reversed, the constants of proportionality are reciprocals. Add a second arrow to each table modeling how both constants of proportionality can be found in a table. Stress that for a table, the constant of proportionality is the value multiplied by x to get the corresponding value of y.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. To help them engage in meaningful reflection, consider asking:

- "What does it represent for two values or figures to be proportionally related? How can you tell?"

Math Language Development

MLR2: Collect and Display

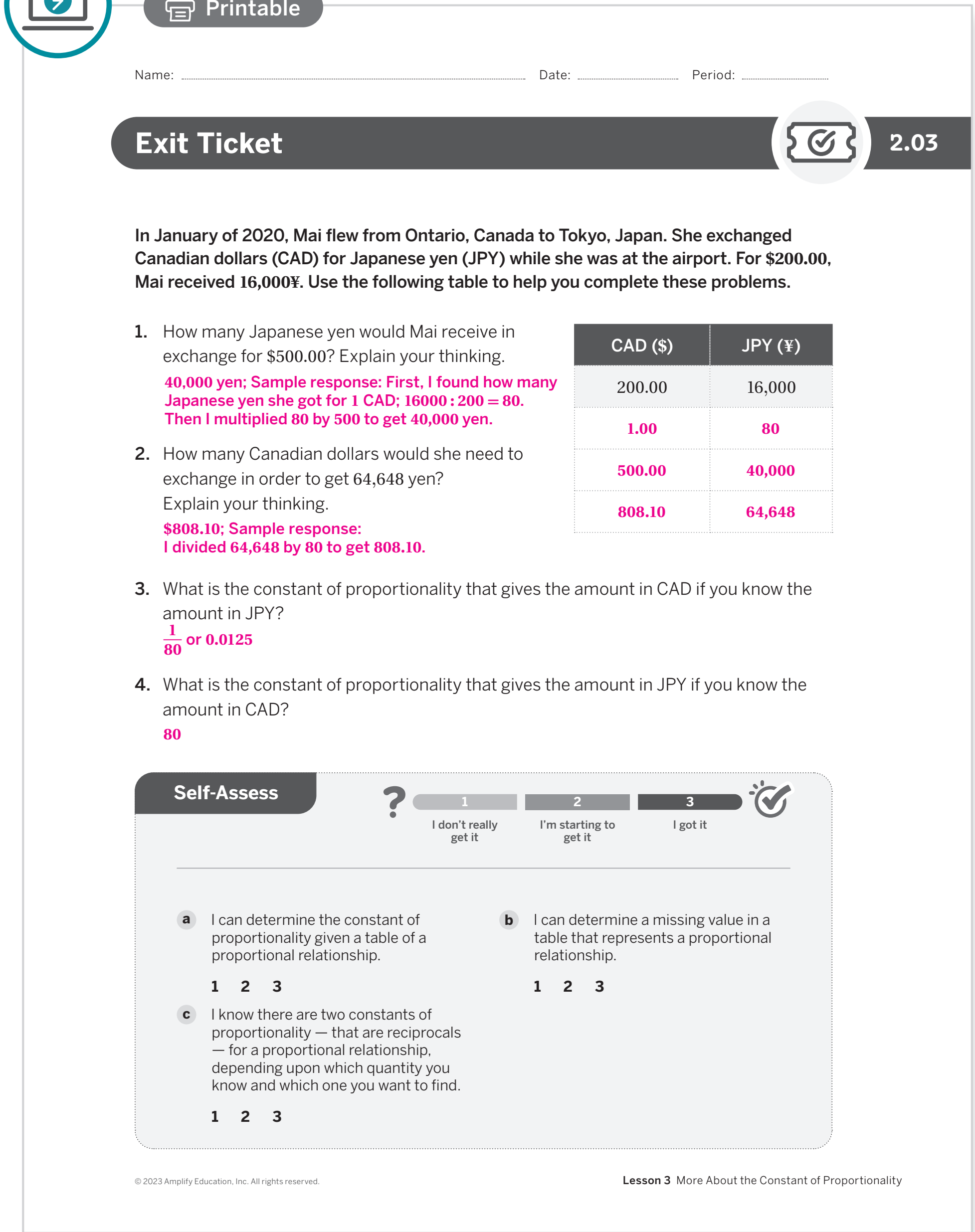
Students learned the terms proportional relationship and constant of proportionality in Lesson 2. As they continue to use these terms throughout this unit, ask them to refer to the class display that you started in Activity 2. Ask them to review and reflect on any terms and phrases related to the terms *proportional relationship* or *constant of proportionality.*

Independent | 7 min

Exit Ticket

Students demonstrate their understanding of how to determine two constants of proportionality by answering questions about currency conversion.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.03

In January of 2020, Mai flew from Ontario, Canada to Tokyo, Japan. She exchanged Canadian dollars (CAD) for Japanese yen (JPY) while she was at the airport. For $200.00, Mai received 16,000¥. Use the following table to help you complete these problems.

CAD ($)	JPY (¥)
200.00	16,000
1.00	**80**
500.00	**40,000**
808.10	**64,648**

1. How many Japanese yen would Mai receive in exchange for $500.00? Explain your thinking.
   40,000 yen; Sample response: First, I found how many Japanese yen she got for 1 CAD; $16000 : 200 = 80$. Then I multiplied 80 by 500 to get 40,000 yen.

2. How many Canadian dollars would she need to exchange in order to get 64,648 yen? Explain your thinking.
   $808.10; Sample response: I divided 64,648 by 80 to get 808.10.

3. What is the constant of proportionality that gives the amount in CAD if you know the amount in JPY?
   $\frac{1}{80}$ or 0.0125

4. What is the constant of proportionality that gives the amount in JPY if you know the amount in CAD?
   80

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a. I can determine the constant of proportionality given a table of a proportional relationship.
1 2 3

b. I can determine a missing value in a table that represents a proportional relationship.
1 2 3

c. I know there are two constants of proportionality — that are reciprocals — for a proportional relationship, depending upon which quantity you know and which one you want to find.
1 2 3

Lesson 3 More About the Constant of Proportionality

Success looks like . . .

- **Language Goal:** Understanding and explaining that there are two constants of proportionality for every relationship, depending on the direction of the relationship, and that they are reciprocals. **(Speaking and Listening, Writing)**
  - » Determining the consent of proportionality for each direction of the relationship in Problems 3 and 4.
- **Language Goal:** Explaining how to calculate missing values in a table that represents a proportional relationship. **(Speaking and Listening)**
- **Language Goal:** Explaining how to determine the constant of proportionality for a proportional relationship represented in a table. **(Speaking and Listening)**

Suggested next steps

If students reverse their responses for Problem 3 and Problem 4 consider:

- Reviewing that the constant of proportionality for a given situation depends on what you are given and what you want to know. It represents a multiplicative relationship between the quantities, in the form "what you want to know" $= k \cdot$ "what you are given".
- Assigning Practice Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

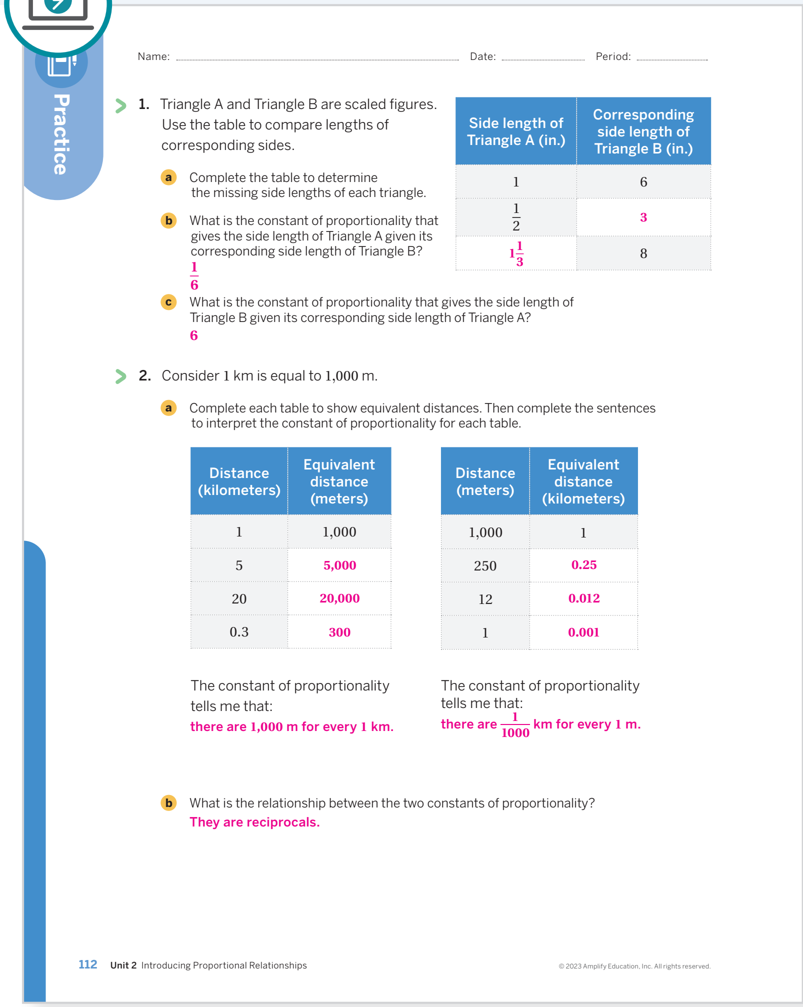
- What worked and didn't work today? Thinking about the questions you asked students today and what the students did as a result of the questions, which question was the most effective?
- When you compare and contrast today's work with the work that students did earlier this year on scaled figures, what similarities and differences do you see? What might you change the next time you teach this lesson?

Math Language Development

To support students' understanding of which constant of proportionality to use for a given situation, remind them that the constant of proportionality depends on two values, (1) what you are given and (2) what you want to know. It represents a multiplicative relationship between these quantities, in the form "what you want to know" $= k \cdot$ "what you are given." Consider displaying this multiplicative relationship for students to reference.

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Triangle A and Triangle B are scaled figures. Use the table to compare lengths of corresponding sides.

Side length of Triangle A (in.)	Corresponding side length of Triangle B (in.)
1	6
$\frac{1}{2}$	3
$1\frac{1}{3}$	8

a. Complete the table to determine the missing side lengths of each triangle.

b. What is the constant of proportionality that gives the side length of Triangle A given its corresponding side length of Triangle B?

$\frac{1}{6}$

c. What is the constant of proportionality that gives the side length of Triangle B given its corresponding side length of Triangle A?

6

2. Consider 1 km is equal to 1,000 m.

a. Complete each table to show equivalent distances. Then complete the sentences to interpret the constant of proportionality for each table.

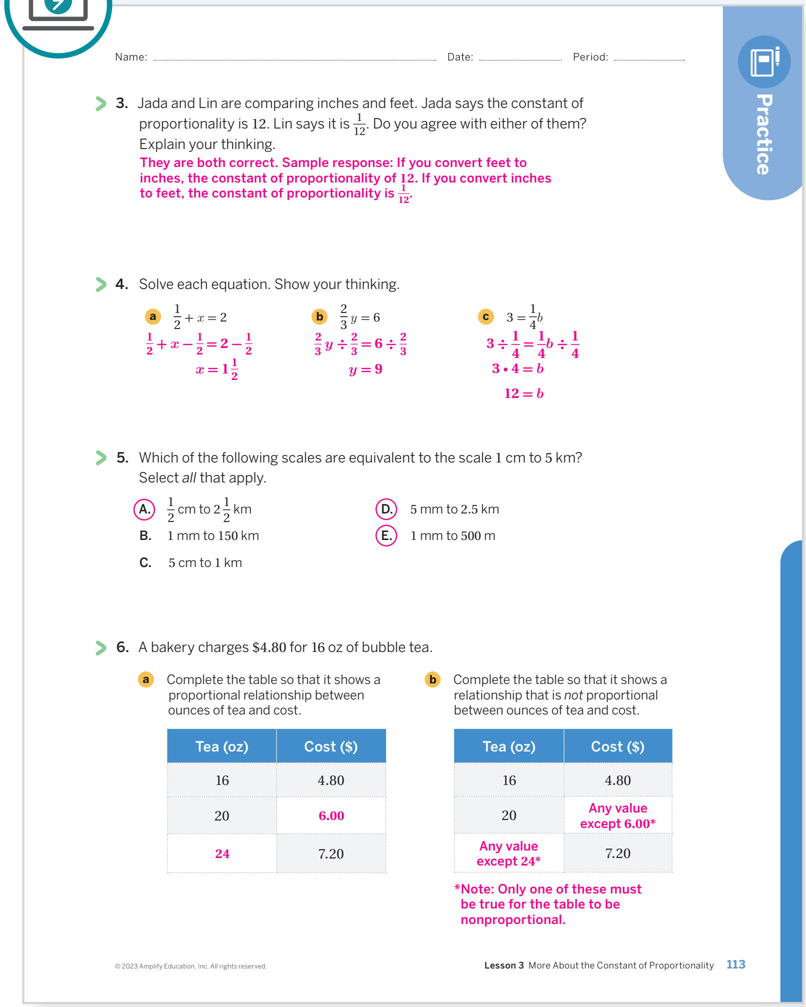
Distance (kilometers)	Equivalent distance (meters)
1	1,000
5	5,000
20	20,000
0.3	300

The constant of proportionality tells me that:
there are 1,000 m for every 1 km.

Distance (meters)	Equivalent distance (kilometers)
1,000	1
250	0.25
12	0.012
1	0.001

The constant of proportionality tells me that:
there are $\frac{1}{1000}$ km for every 1 m.

b. What is the relationship between the two constants of proportionality?
They are reciprocals.



Practice

Name: ________ Date: ________ Period: ________

3. Jada and Lin are comparing inches and feet. Jada says the constant of proportionality is 12. Lin says it is $\frac{1}{12}$. Do you agree with either of them? Explain your thinking.
They are both correct. Sample response: If you convert feet to inches, the constant of proportionality of 12. If you convert inches to feet, the constant of proportionality is $\frac{1}{12}$.

4. Solve each equation. Show your thinking.

a. $\frac{1}{2}+x=2$

$\frac{1}{2}+x-\frac{1}{2}=2-\frac{1}{2}$

$x=1\frac{1}{2}$

b. $\frac{2}{3}y=6$

$\frac{2}{3}y\div\frac{2}{3}=6\div\frac{2}{3}$

$y=9$

c. $3=\frac{1}{4}b$

$3\div\frac{1}{4}=\frac{1}{4}b\div\frac{1}{4}$

$3\cdot 4=b$

$12=b$

5. Which of the following scales are equivalent to the scale 1 cm to 5 km? Select *all* that apply.

[✓ A.] $\frac{1}{2}$ cm to $2\frac{1}{2}$ km
B. 1 mm to 150 km
C. 5 cm to 1 km
[✓ D.] 5 mm to 2.5 km
[✓ E.] 1 mm to 500 m

6. A bakery charges $4.80 for 16 oz of bubble tea.

a. Complete the table so that it shows a proportional relationship between ounces of tea and cost.

Tea (oz)	Cost ($)
16	4.80
20	6.00
24	7.20

b. Complete the table so that it shows a relationship that is *not* proportional between ounces of tea and cost.

Tea (oz)	Cost ($)
16	4.80
20	Any value except 6.00*
Any value except 24*	7.20

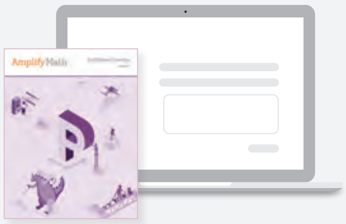
*Note: Only one of these must be true for the table to be nonproportional.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 1	3
Spiral	4	Grade 6	1
	5	Unit 1 Lesson 10	2
Formative ⚡	6	Unit 2 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 4**

Comparing Relationships With Tables

Let's explore how proportional relationships are different from other relationships.

Focus

Goals

1. **Language Goal:** Generate different examples of proportional relationships by using the same context and describe how the constant of proportionality relates to the context. **(Speaking and Listening)**
2. **Language Goal:** Justify whether the values in a given table could or could not represent a proportional relationship. **(Speaking and Listening)**

Coherence

Today

Students continue to develop their understanding of proportional relationships represented by a table of values. They determine whether a table of values models relationships that could be proportional. Students then create a table of values to represent a real-world situation and use the table to identify whether the relationship could be proportional or nonproportional.

Previously

In Lesson 3, students discovered that there are two constants of proportionality for every proportional relationship.

Coming Soon

In Lesson 5, students will further develop their understanding of proportionality to generalize relationships and to write equations that represent proportional relationships.

Rigor

- Students will build **conceptual understanding** of proportional and nonproportional relationships by comparing tables of values.
- Students develop **fluency** in determining the constant of proportionality for a proportional relationship represented by a table of values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	10 min	15 min	5 min	10 min
Independent	Pairs	Pairs	*Small Groups*	Whole Class	Independent

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For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, (as needed)
- Activity 1 PDF (answers, as needed)
- Activity 3 PDF, pre-cut cards (optional)
- Anchor Chart PDF, *Representing Proportional Relationships*
- calculators

Math Language Development

New word

- ***nonproportional relationship***

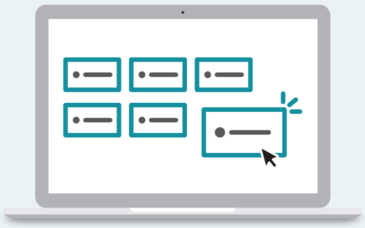
Review words

- *constant of proportionality*
- *equivalent ratios*
- *proportional relationship*

Amps | Featured Activity

Activity 3
Digital Card Sort

In Activity 3, students will sort cards showing tables of values into proportional and nonproportional relationships. They will then determine the constant of proportionality for the proportional relationships.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may want to give up right away if the way to distinguish between proportional and nonproportional relationships is not immediately clear. Model thinking out loud and responding to the thoughts of a partner as examples of how to work well with a partner to plan out what can be done together to make productive progress.

Modifications to Pacing

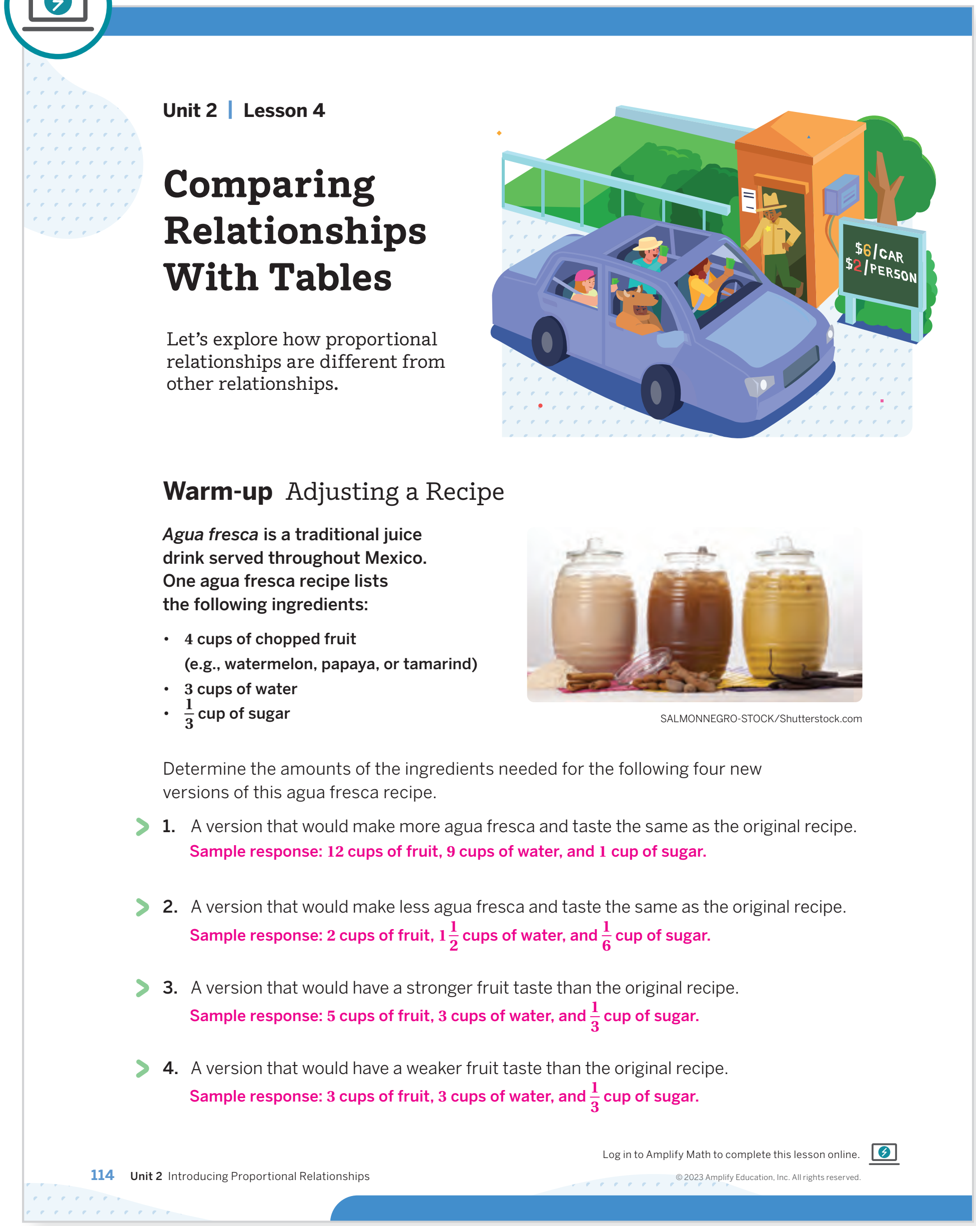
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, Problem 3 may be omitted.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up Adjusting a Recipe

Students adjust the amount of ingredients in a recipe to examine the effects of equivalent and nonequivalent ratios.



Unit 2 | Lesson 4

Comparing Relationships With Tables

Let's explore how proportional relationships are different from other relationships.

Warm-up Adjusting a Recipe

Agua fresca is a traditional juice drink served throughout Mexico. One agua fresca recipe lists the following ingredients:

- 4 cups of chopped fruit (e.g., watermelon, papaya, or tamarind)
- 3 cups of water
- $\frac{1}{3}$ cup of sugar

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Determine the amounts of the ingredients needed for the following four new versions of this agua fresca recipe.

1. A version that would make more agua fresca and taste the same as the original recipe.
   Sample response: 12 cups of fruit, 9 cups of water, and 1 cup of sugar.
2. A version that would make less agua fresca and taste the same as the original recipe.
   Sample response: 2 cups of fruit, $1\frac{1}{2}$ cups of water, and $\frac{1}{6}$ cup of sugar.
3. A version that would have a stronger fruit taste than the original recipe.
   Sample response: 5 cups of fruit, 3 cups of water, and $\frac{1}{3}$ cup of sugar.
4. A version that would have a weaker fruit taste than the original recipe.
   Sample response: 3 cups of fruit, 3 cups of water, and $\frac{1}{3}$ cup of sugar.

Log in to Amplify Math to complete this lesson online.

114 **Unit 2** Introducing Proportional Relationships

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1 Launch

Activate students' background knowledge by asking whether they have ever had to adjust a recipe. Allow access to calculators.

2 Monitor

Help students get started by asking them how they would create a recipe with more servings if there were only two ingredients, rather than three ingredients.

Look for points of confusion:

- **Thinking that they can add the same amount of each ingredient.** Ask students what it would mean to double a recipe. Have them consider how each ingredient would be affected.

Look for productive strategies:

- Multiplying and dividing all ingredients by the same factor in Problems 1 and 2.
- Adjusting only the amount of fruit in the recipe for Problems 3 and 4.

3 Connect

Have individual students share their strategies for adjusting the agua fresca recipe for each scenario along with their new recipe.

Highlight that in Problems 1 and 2, they are multiplying and dividing all of the amounts of the ingredients by the same value to maintain the ratio of ingredients. In the final two problems, they are no longer multiplying or dividing all of the amounts of the ingredients, because they no longer want to maintain the relationship between the ingredients.

Ask, "Which of the recipes are proportional to the original recipe?" The recipes from Problem 1 and 2.

Power-up

To power up students' ability to reason about equivalent ratios, have students complete:

Consider a recipe for agua fresca that calls for 4 cups of fruit for every 3 cups of water.

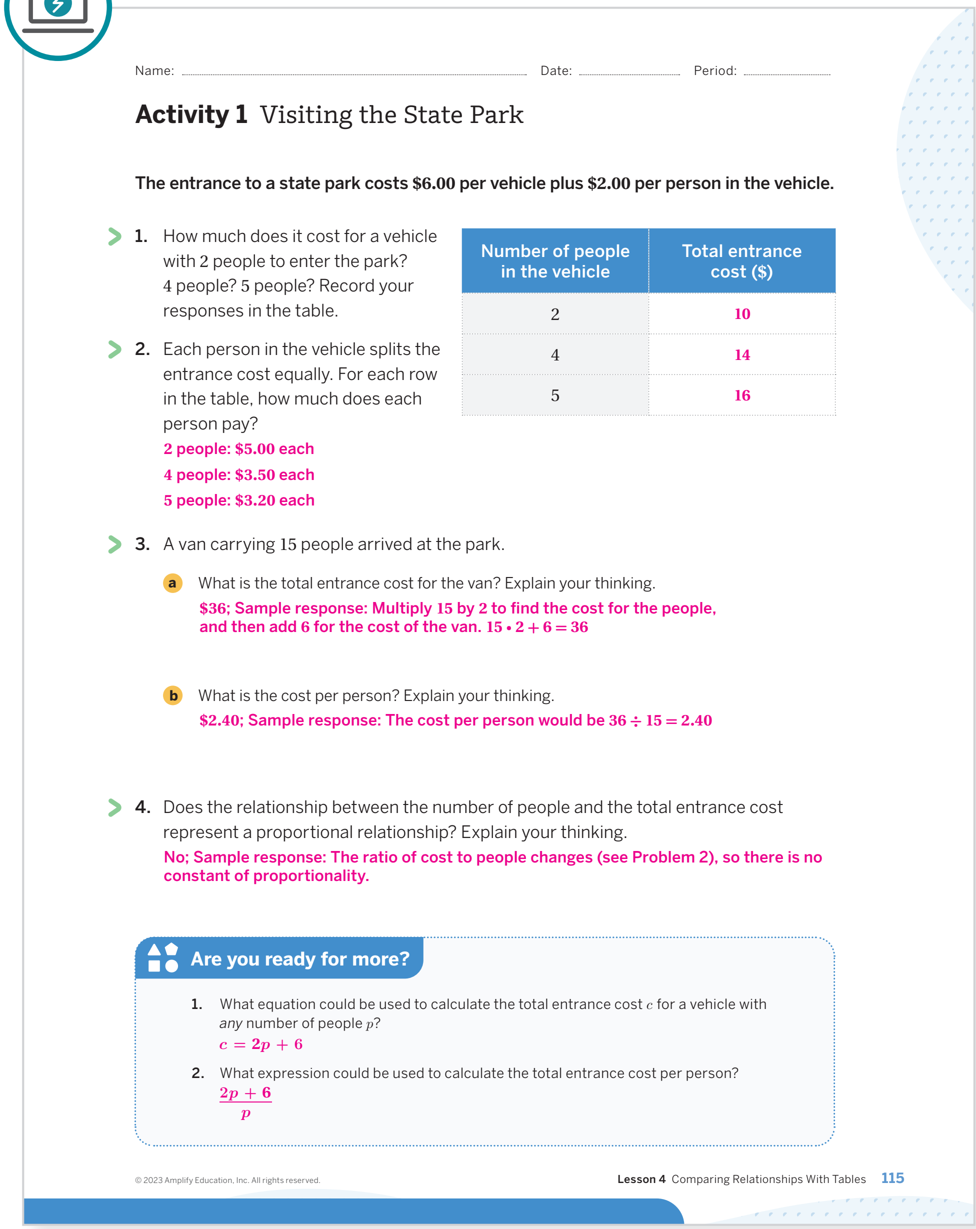
1. If you make a batch with 6 cups of fruit and 4.5 cups of water, would it taste the same? Explain your thinking. Sample response: Yes, the ratio of water to fruit is the same: 3 : 4.
2. If you make a batch with 5 cups of fruit and 4 cups of water, would it taste the same? Explain your thinking. Sample response: No, the ratio of water to fruit is not the same. Now it is 4 : 5.

Use: Before the Warm-up.
Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6.

Pairs | 15 min

Activity 1 Visiting the State Park

Students create a table of values to represent the cost of entering a park as an introduction to nonproportional relationships.



Name: ______ Date: ______ Period: ______

Activity 1 Visiting the State Park

The entrance to a state park costs \$6.00 per vehicle plus \$2.00 per person in the vehicle.

1. How much does it cost for a vehicle with 2 people to enter the park? 4 people? 5 people? Record your responses in the table.

Number of people in the vehicle	Total entrance cost (\$)
2	**10**
4	**14**
5	**16**

2. Each person in the vehicle splits the entrance cost equally. For each row in the table, how much does each person pay?

   2 people: \$5.00 each

   4 people: \$3.50 each

   5 people: \$3.20 each

3. A van carrying 15 people arrived at the park.

   a. What is the total entrance cost for the van? Explain your thinking.

   \$36; Sample response: Multiply 15 by 2 to find the cost for the people, and then add 6 for the cost of the van. $15 \cdot 2 + 6 = 36$

   b. What is the cost per person? Explain your thinking.

   \$2.40; Sample response: The cost per person would be $36 \div 15 = 2.40$

4. Does the relationship between the number of people and the total entrance cost represent a proportional relationship? Explain your thinking.

   No; Sample response: The ratio of cost to people changes (see Problem 2), so there is no constant of proportionality.

Are you ready for more?

1. What equation could be used to calculate the total entrance cost c for a vehicle with *any* number of people p?

   $c = 2p + 6$

2. What expression could be used to calculate the total entrance cost per person?

   $\frac{2p + 6}{p}$

Lesson 4 Comparing Relationships With Tables 115

1 Launch

Activate prior knowledge by asking if anyone has ever visited a state park. For Problem 1, follow the *Think-Pair-Share* routine, then students should work in pairs to complete Problems 2 and 3.

2 Monitor

Help students get started by reminding them that each group is being charged for their vehicle and for each person inside it.

Look for points of confusion:

- **Forgetting to account for the cost of the car.** Remind students that they need to account for the cost of the car along with the cost of each person.
- **Thinking that the number of cars increases with the number of people.** Ask students whether each person in the car is being charged for the vehicle, or whether the vehicle charge is a one-time fee.

3 Connect

Display the table from the Student Edition.

Have pairs of students share their solutions and explanations for Problems 2 and 4.

Highlight that the amount paid by each person changes depending on the number of people in the car. The ratios of $cost : number\ of\ people$ in the table are not equivalent, or *constant*, and therefore there is no constant of proportionality.

Define ***nonproportional relationship*** as a relationship between two quantities where the corresponding values do not have a constant ratio.

Ask, “Can you think of other examples of nonproportional relationships?”
Sample responses: cost of going to a drive-in movie, side length vs. area of a square, changing a recipe to make the fruit taste stronger.

Differentiated Support

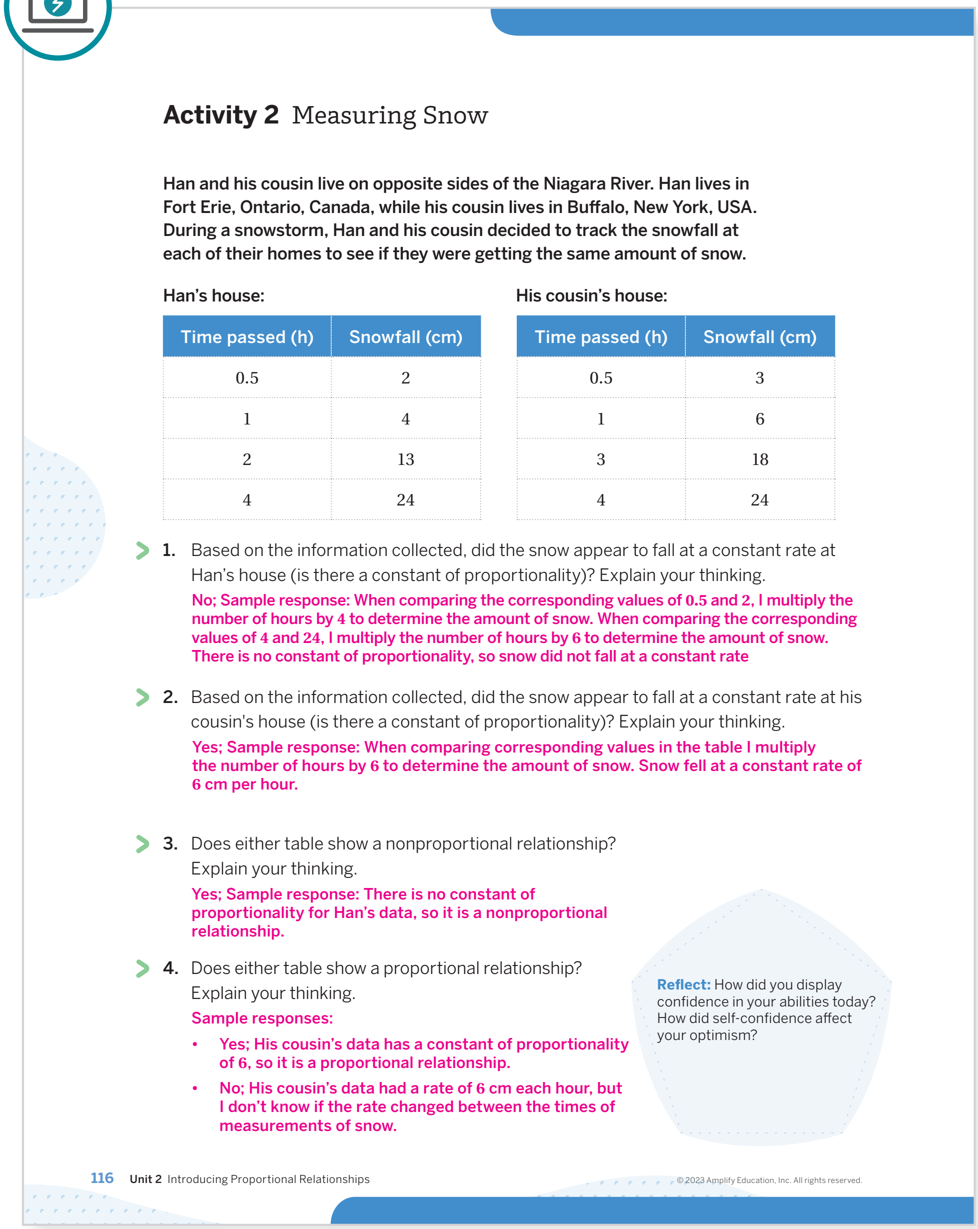
Accessibility: *Guide Processing and Visualization*

Provide copies of the Activity 1 PDF to help students organize their work and make sense of the problem. The table in the PDF will help them remember to account for the cost of the vehicle in the total entrance cost.

Pairs | 10 min

Activity 2 Measuring Snow

Students compare data from two tables of values on snowfall to determine whether they are modeling proportional or nonproportional relationships.



Activity 2 Measuring Snow

Han and his cousin live on opposite sides of the Niagara River. Han lives in Fort Erie, Ontario, Canada, while his cousin lives in Buffalo, New York, USA. During a snowstorm, Han and his cousin decided to track the snowfall at each of their homes to see if they were getting the same amount of snow.

Han's house:

Time passed (h)	Snowfall (cm)
0.5	2
1	4
2	13
4	24

His cousin's house:

Time passed (h)	Snowfall (cm)
0.5	3
1	6
3	18
4	24

1. Based on the information collected, did the snow appear to fall at a constant rate at Han's house (is there a constant of proportionality)? Explain your thinking.
   No; Sample response: When comparing the corresponding values of 0.5 and 2, I multiply the number of hours by 4 to determine the amount of snow. When comparing the corresponding values of 4 and 24, I multiply the number of hours by 6 to determine the amount of snow. There is no constant of proportionality, so snow did not fall at a constant rate

2. Based on the information collected, did the snow appear to fall at a constant rate at his cousin's house (is there a constant of proportionality)? Explain your thinking.
   Yes; Sample response: When comparing corresponding values in the table I multiply the number of hours by 6 to determine the amount of snow. Snow fell at a constant rate of 6 cm per hour.

3. Does either table show a nonproportional relationship? Explain your thinking.
   Yes; Sample response: There is no constant of proportionality for Han's data, so it is a nonproportional relationship.

4. Does either table show a proportional relationship? Explain your thinking.
   Sample responses:
   - **Yes; His cousin's data has a constant of proportionality of 6, so it is a proportional relationship.**
   - **No; His cousin's data had a rate of 6 cm each hour, but I don't know if the rate changed between the times of measurements of snow.**

Reflect: How did you display confidence in your abilities today? How did self-confidence affect your optimism?

116 **Unit 2** Introducing Proportional Relationships

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1 Launch

Activate students' background knowledge by asking them if they have ever measured snowfall or rainfall. Provide access to calculator for the remainder of the class.

2 Monitor

Help students get started by asking, "What is meant by 'at a constant rate'?"

Look for points of confusion:

- **Thinking that there is a constant of proportionality because the values in the first two rows of Han's data table are equivalent ratios.** Clarify that the ratio relating one value to the other must be the same for *all* the rows containing data.

3 Connect

Display the tables.

Have pairs of students share their reasoning of whether the data in each table is proportional. Focus on their explanations for Problems 3 and 4.

Highlight that a table could be representing a proportional relationship if there is a constant of proportionality.

Ask:

- "How many rows of a table of values must we check to determine whether it could represent a proportional relationship?" Sample response: All the rows must have the same ratio of $y : x$ to say that the table of values is proportional. If there are two rows that don't have the same ratio, then it's nonproportional.
- "How can you use the information in Han table to make a prediction about how much snow fell at other times during the storm?" Sample response: I could predict the amount of snow after 2 hours would be 12 cm.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students annotate a third column of values to each table for the "Snowfall per hour" to help them make sense of the problem and organize their work.

Extension: *Math Enrichment*

For the relationship that appears to be proportional, ask students to write an equation that determines the number of centimeters c of snow fallen given the number of hours h that it snowed. $c = 6h$

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the prompt and the tables. Ask students to work with their partner to generate 2–3 questions they could ask about the situation. Listen for and amplify questions involving proportional relationships or mathematical language, such as *constant rate*, *proportional*, or *constant of proportionality*.

English Learners

Model how to craft 1–2 questions for students before asking them to co-craft questions with their partners.

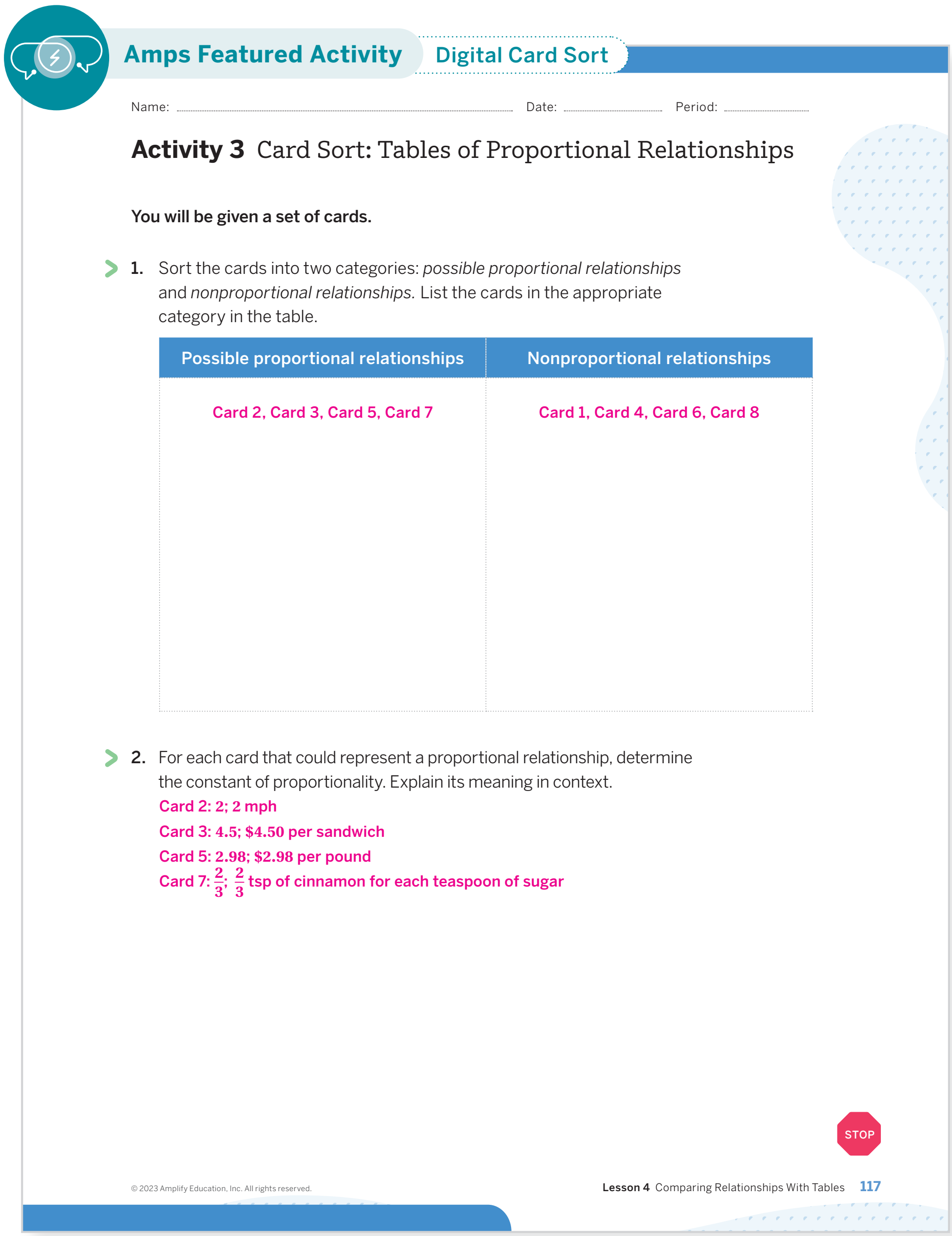
Optional

Small Groups | 15 min

Activity 3 Card Sort: Tables of Proportional Relationships

Students sort tables of values to distinguish between proportional and nonproportional relationships, which can be used as a formative assessment checkpoint.

Amps Featured Activity Digital Card Sort



Name: ______ Date: ______ Period: ______

Activity 3 Card Sort: Tables of Proportional Relationships

You will be given a set of cards.

1. Sort the cards into two categories: *possible proportional relationships* and *nonproportional relationships*. List the cards in the appropriate category in the table.

Possible proportional relationships	Nonproportional relationships
Card 2, Card 3, Card 5, Card 7	Card 1, Card 4, Card 6, Card 8

2. For each card that could represent a proportional relationship, determine the constant of proportionality. Explain its meaning in context.
   Card 2: 2; 2 mph
   Card 3: 4.5; $4.50 per sandwich
   Card 5: 2.98; $2.98 per pound
   Card 7: $\frac{2}{3}$; $\frac{2}{3}$ tsp of cinnamon for each teaspoon of sugar

STOP

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Lesson 4 Comparing Relationships With Tables 117

1 Launch

Distribute one set of pre-cut cards from the Activity 3 PDF to each small group and conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by asking them to explain how they can check whether a table is representing a proportional relationship.

Look for points of confusion:

- **Thinking that if the first two rows have the same ratio, the whole table is modeling a proportional relationship.** Remind students they must check *all* pairs of values in their tables.
- **Thinking all ratios must be different to be nonproportional.** Remind students that to be proportional, all ratios representing the relationship between x- and y-values must be the same, so there only needs to be one ratio that is different to be nonproportional.

Look for productive strategies:

- Adding a column to each table for the calculations of the constant of proportionality.

3 Connect

Display any cards that were topics of disagreement during group discussions.

Have groups of students share their strategies for grouping cards into proportional and nonproportional categories.

Highlight concrete methods for determining whether a table could be proportional. Note that determining a proportional relationship is based only on known information, and does not mean that the relationship is necessarily proportional.

Ask, "What do you look for to determine whether a table of values could represent a proportional relationship?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For Card 4, Card 5, and Card 7, change the values in the first column so that they are all whole numbers. Adjust their matching values proportionally to maintain the ratios of y to x for each table.

Extension: *Math Enrichment*

Have students edit the values in the tables so that the proportional relationships are no longer proportional. Have them justify their thinking.

Math Language Development

MLR1: Stronger and Clearer Each Time

Ask students to write a response to the question posed in the Connect. Have them share their response with a partner and each partner should ask questions and provide feedback to improve the response. Allow students time to revise their initial drafts.

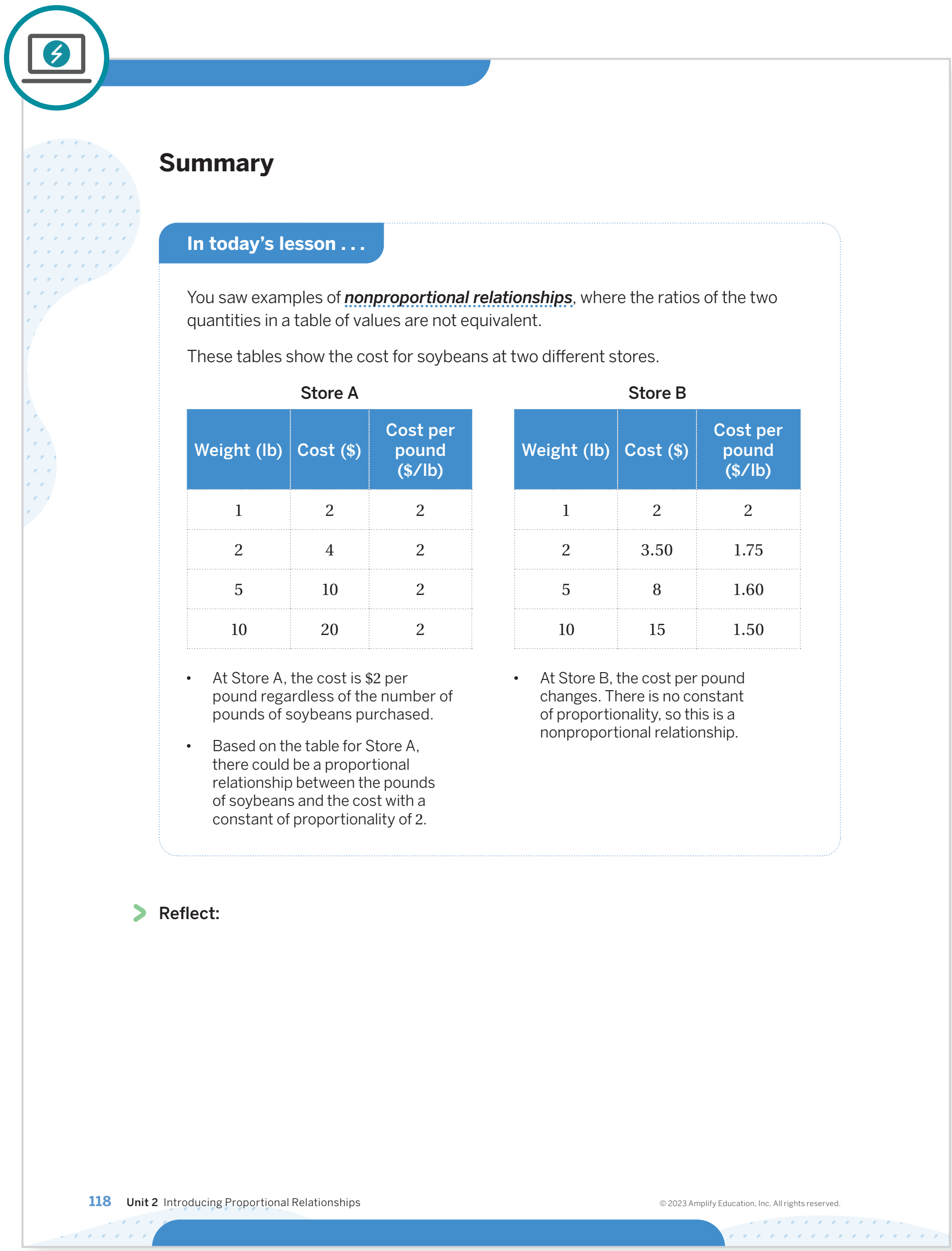
English Learners

Strategically pair students with peers who speak the same primary language in order to assist students in providing feedback and responding to the prompts.

Whole Class | 5 min

Summary

Review and synthesize how students can determine whether a relationship is proportional or nonproportional, based on a table of values.



Summary

In today's lesson . . .

You saw examples of ***nonproportional relationships***, where the ratios of the two quantities in a table of values are not equivalent.

These tables show the cost for soybeans at two different stores.

Store A

Weight (lb)	Cost ($)	Cost per pound ($/lb)
1	2	2
2	4	2
5	10	2
10	20	2

- At Store A, the cost is $2 per pound regardless of the number of pounds of soybeans purchased.
- Based on the table for Store A, there could be a proportional relationship between the pounds of soybeans and the cost with a constant of proportionality of 2.

Store B

Weight (lb)	Cost ($)	Cost per pound ($/lb)
1	2	2
2	3.50	1.75
5	8	1.60
10	15	1.50

- At Store B, the cost per pound changes. There is no constant of proportionality, so this is a nonproportional relationship.

Reflect:

118 **Unit 2** Introducing Proportional Relationships

Synthesize

Display the two tables from the Summary and the Anchor Chart PDF, *Representing Proportional Relationships*. Discuss and complete the Verbal Description section as a class.

Highlight that students can decide whether a table could represent a proportional relationship by calculating the quotient for the values in each row and checking for a constant of proportionality. Model this strategy by completing the table section of the Anchor Chart PDF and connecting it to the examples in the Summary. If there is no constant of proportionality, the relationship is *nonproportional*.

Formalize vocabulary: *nonproportional relationship.*

Ask, "What real-world examples could be represented by proportional relationships? Nonproportional relationships?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. To help them engage in meaningful reflection, consider asking:

- "What does it mean for two values to be proportionally related? How can you tell?"

Math Language Development

MLR2: Collect and Display

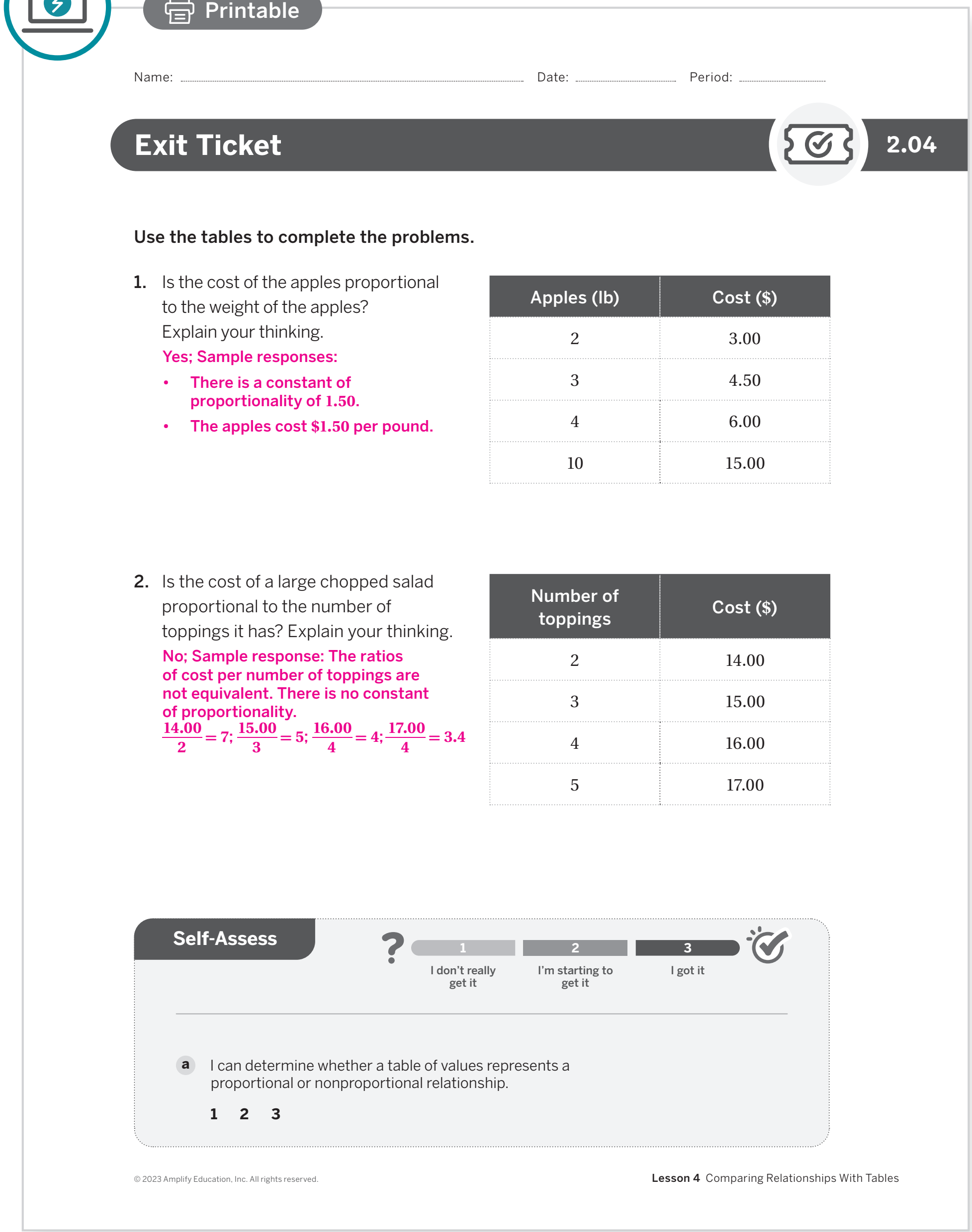
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the term *nonproportional relationship* that were added to the display during the lesson.

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by classifying tables of values as modeling proportional or nonproportional relationships.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 2.04

Use the tables to complete the problems.

1. Is the cost of the apples proportional to the weight of the apples? Explain your thinking.
Yes; Sample responses:
- **There is a constant of proportionality of 1.50.**
- **The apples cost $1.50 per pound.**

Apples (lb)	Cost ($)
2	3.00
3	4.50
4	6.00
10	15.00

2. Is the cost of a large chopped salad proportional to the number of toppings it has? Explain your thinking.
No; Sample response: The ratios of cost per number of toppings are not equivalent. There is no constant of proportionality.
$\frac{14.00}{2} = 7; \frac{15.00}{3} = 5; \frac{16.00}{4} = 4; \frac{17.00}{4} = 3.4$

Number of toppings	Cost ($)
2	14.00
3	15.00
4	16.00
5	17.00

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine whether a table of values represents a proportional or nonproportional relationship.

1 2 3

Lesson 4 Comparing Relationships With Tables

Success looks like . . .

- **Language Goal:** Generating different examples of proportional relationships by using the same context and describing how the constant of proportionality relates to the context. **(Listening and Speaking)**
  - » Explaining how the cost of apples is proportional to the weight of the apples in Problem 1.
- **Language Goal:** Justifying whether the values in a given table could or could not represent a proportional relationship. **(Listening and Speaking)**
  - » Explaining whether the relationship is proportional in Problems 1 and 2.

Suggested next steps

If students explain that the cost of chopped salad is proportional to the number of toppings because each topping increases the price by $1.00, consider:

- Reviewing that *proportional* in this scenario means that students can multiply the number of toppings by a constant to get the cost.
- Asking, "How is this problem similar to Activity 1?"
- Assigning Practice Problem 1.

Professional Learning

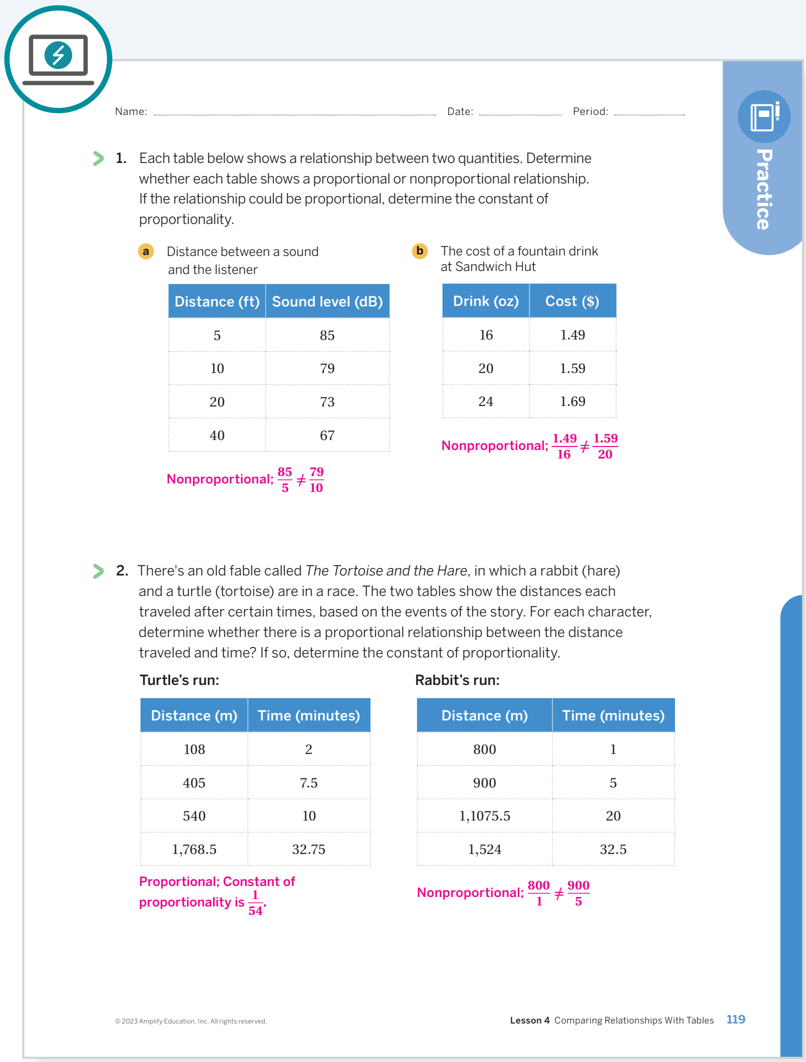
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas heard today?
- What challenges did students encounter as they work on Activity 1? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Each table below shows a relationship between two quantities. Determine whether each table shows a proportional or nonproportional relationship. If the relationship could be proportional, determine the constant of proportionality.

a Distance between a sound and the listener

Distance (ft)	Sound level (dB)
5	85
10	79
20	73
40	67

Nonproportional; $\frac{85}{5} \neq \frac{79}{10}$

b The cost of a fountain drink at Sandwich Hut

Drink (oz)	Cost ($)
16	1.49
20	1.59
24	1.69

Nonproportional; $\frac{1.49}{16} \neq \frac{1.59}{20}$

2. There's an old fable called *The Tortoise and the Hare*, in which a rabbit (hare) and a turtle (tortoise) are in a race. The two tables show the distances each traveled after certain times, based on the events of the story. For each character, determine whether there is a proportional relationship between the distance traveled and time? If so, determine the constant of proportionality.

Turtle's run:

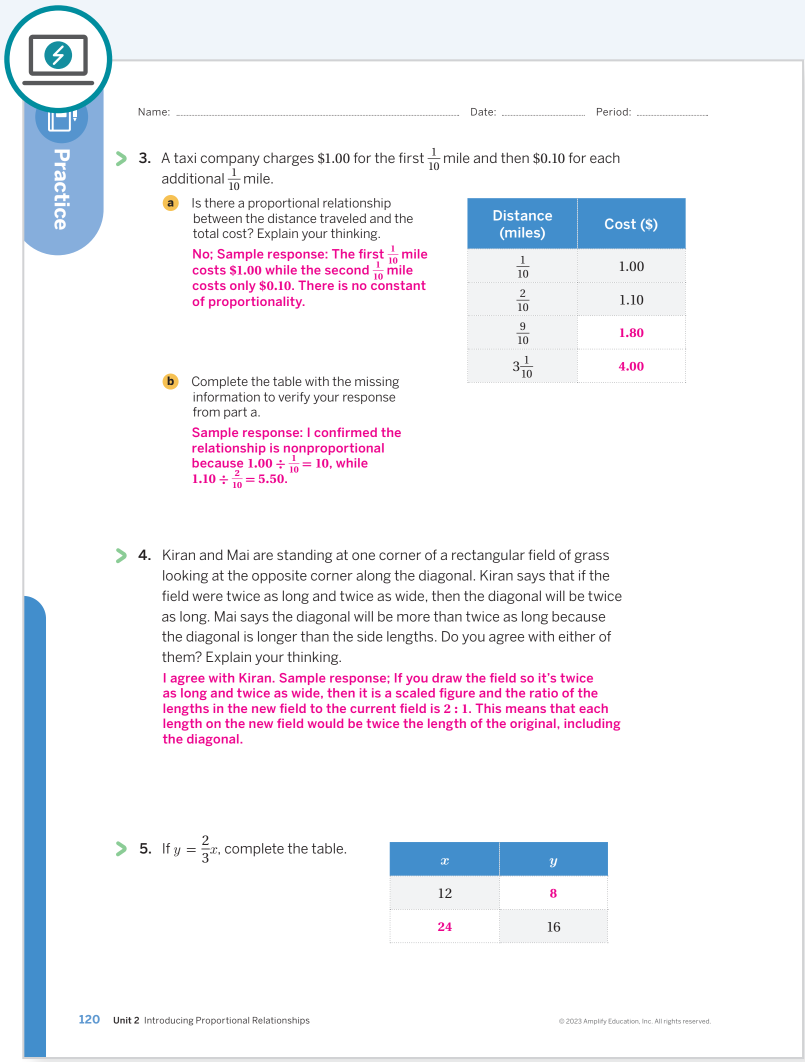
Distance (m)	Time (minutes)
108	2
405	7.5
540	10
1,768.5	32.75

Proportional; Constant of proportionality is $\frac{1}{54}$.

Rabbit's run:

Distance (m)	Time (minutes)
800	1
900	5
1,1075.5	20
1,524	32.5

Nonproportional; $\frac{800}{1} \neq \frac{900}{5}$



Name: Date: Period:

3. A taxi company charges $1.00 for the first $\frac{1}{10}$ mile and then $0.10 for each additional $\frac{1}{10}$ mile.

a Is there a proportional relationship between the distance traveled and the total cost? Explain your thinking.

No; Sample response: The first $\frac{1}{10}$ mile costs $1.00 while the second $\frac{1}{10}$ mile costs only $0.10. There is no constant of proportionality.

b Complete the table with the missing information to verify your response from part a.

Sample response: I confirmed the relationship is nonproportional because $1.00 \div \frac{1}{10} = 10$, while $1.10 \div \frac{2}{10} = 5.50$.

Distance (miles)	Cost ($)
$\frac{1}{10}$	1.00
$\frac{2}{10}$	1.10
$\frac{9}{10}$	**1.80**
$3\frac{1}{10}$	**4.00**

4. Kiran and Mai are standing at one corner of a rectangular field of grass looking at the opposite corner along the diagonal. Kiran says that if the field were twice as long and twice as wide, then the diagonal will be twice as long. Mai says the diagonal will be more than twice as long because the diagonal is longer than the side lengths. Do you agree with either of them? Explain your thinking.

I agree with Kiran. Sample response; If you draw the field so it's twice as long and twice as wide, then it is a scaled figure and the ratio of the lengths in the new field to the current field is 2 : 1. This means that each length on the new field would be twice the length of the original, including the diagonal.

5. If $y = \frac{2}{3}x$, complete the table.

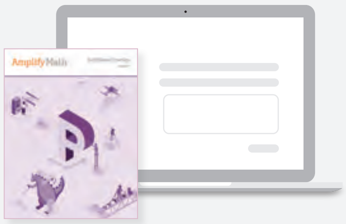
x	y
12	**8**
24	16

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 8	3
Formative ⚡	5	Unit 2 Lesson 5	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 5**

Proportional Relationships and Equations

Let's write equations describing proportional relationships.

Focus

Goals

1. **Language Goal:** Generalize a process for calculating missing values in a proportional relationship and justify why this can be abstracted as $y = kx$, where k is the constant of proportionality. **(Speaking and Listening)**
2. **Language Goal:** Generate an equation of the form $y = kx$ to represent a proportional relationship in a familiar context. **(Speaking and Listening)**

Coherence

Today

Students build on their work with tables, focusing on the idea that for each proportional relationship, the values in the table satisfy the equation $y = kx$ for the constant of proportionality, k. They also generalize that any proportional relationship, once the constant of proportionality is known, can be represented by the equation $y = kx$.

Previously

In Lessons 2 and 3, students determined the constant of proportionality from a table of values and used reasoning with the constant of proportionality to determine unknown values.

Coming Soon

In Lesson 6, students will apply their understanding of $y = kx$ to problems involving the relationship between distance, speed (rate), and time by using the equation $d = rt$.

Rigor

- Students generalize the relationship between values in a proportional relationship to build **conceptual understanding** of the equation $y = kx$.
- Students develop **procedural fluency** in writing equations to represent proportional relationships with and without the use of tables of values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	13 min	10 min	5 min	7 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Warm-up PDF (answers)

Math Language Development

Review words

- *constant of proportionality*
- *proportional relationship*

Amps | **Featured Activity**

Exit Ticket

Real-Time Exit Ticket

Check in real time if your students can represent a proportional relationship with an equation by using a digital Exit Ticket that is automatically scored.

Building Math Identity and Community

Connecting to Mathematical Practices

Because students are still working at recognizing the repeated reasoning used for proportional relationships, they might still see each problem as a unique task. Instill in students the importance of having a growth mindset, one where it is ok to make mistakes and learn from them for the future. Have them seek similarities with other problems they have done and point out how they can apply the same strategies.

Modifications to Pacing

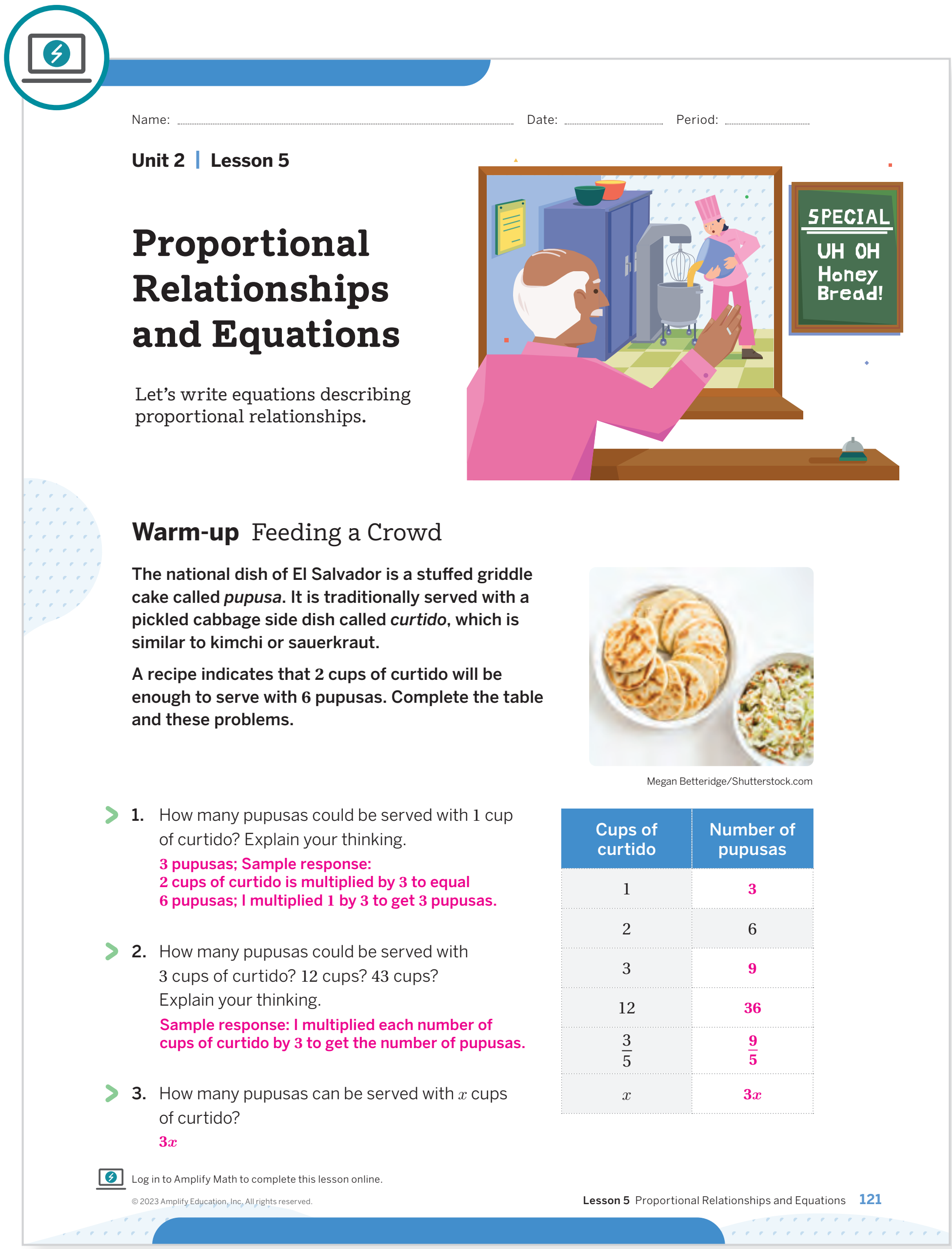
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, Problem 2 may be omitted.

Independent | 10 min

Warm-up Feeding a Crowd

Students will engage in repeated reasoning to generalize a proportional relationship as an equation.



Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 5

Proportional Relationships and Equations

Let's write equations describing proportional relationships.

Warm-up Feeding a Crowd

The national dish of El Salvador is a stuffed griddle cake called *pupusa*. It is traditionally served with a pickled cabbage side dish called *curtido*, which is similar to kimchi or sauerkraut.

A recipe indicates that 2 cups of curtido will be enough to serve with 6 pupusas. Complete the table and these problems.

Megan Betteridge/Shutterstock.com

1. How many pupusas could be served with 1 cup of curtido? Explain your thinking.
   3 pupusas; Sample response: 2 cups of curtido is multiplied by 3 to equal 6 pupusas; I multiplied 1 by 3 to get 3 pupusas.
2. How many pupusas could be served with 3 cups of curtido? 12 cups? 43 cups? Explain your thinking.
   Sample response: I multiplied each number of cups of curtido by 3 to get the number of pupusas.
3. How many pupusas can be served with x cups of curtido?
   $3x$

Cups of curtido	Number of pupusas
1	**3**
2	6
3	**9**
12	**36**
$\frac{3}{5}$	$\frac{9}{5}$
x	$3x$

Log in to Amplify Math to complete this lesson online.

Lesson 5 Proportional Relationships and Equations 121

1 Launch

Explain to students that this warm-up will build on their previous work of exploring proportional relationships and determining a constant of proportionality.

2 Monitor

Help students get started by asking them to explain the relationship between the number of cups of curtido and the number of pupusas.

Look for productive strategies:

- Seeing the pattern of multiplying by a constant of proportionality of 3 for every row and representing the number of people served by x cups as $3x$.
- Multiplying corresponding pairs by the same value.

3 Connect

Display the Warm-up PDF.

Have students share how they determined the expression representing the number of pupusas in the last row, when there are x cups of curtido.

Ask:

- "What does each part of the equation $y = 3x$ represent?"
- "How many rows of the table do you need to complete to determine the equation?"

Highlight, by adding arrows, that each cup of curtido is multiplied by the constant of proportionality, 3, to determine the number of pupusas. If x represents the number of cups of curtido, and y represents the number of pupusas, then this relationship can be represented by the equation $3x = y$ or $y = 3x$. Explain that for any proportional relationship, to determine y from x, students multiply the x value by the constant of proportionality, k. Thus, these relationships can be represented by the equation $y = kx$.

Math Language Development

MLR7: Compare and Connect

As students explain their different approaches to determining the expression representing the number of pupusas in the last row, ask, "What is similar? What is different?" about these approaches. Draw students' attention to the different ways the constant of proportionality is represented across the different strategies.

English Learners

Use arrows to highlight how the constant of proportionality is used to determine the number of pupusas.

Power-up

To power up students' ability to substitute a value into an equation and determine the value of the remaining variable, have students complete:

1. For $y = 3x$, if $x = 7$, then $y = \boxed{21}$.

2. For $y = \frac{2}{3} \cdot x$, if $x = 45$, then $y = \boxed{30}$.

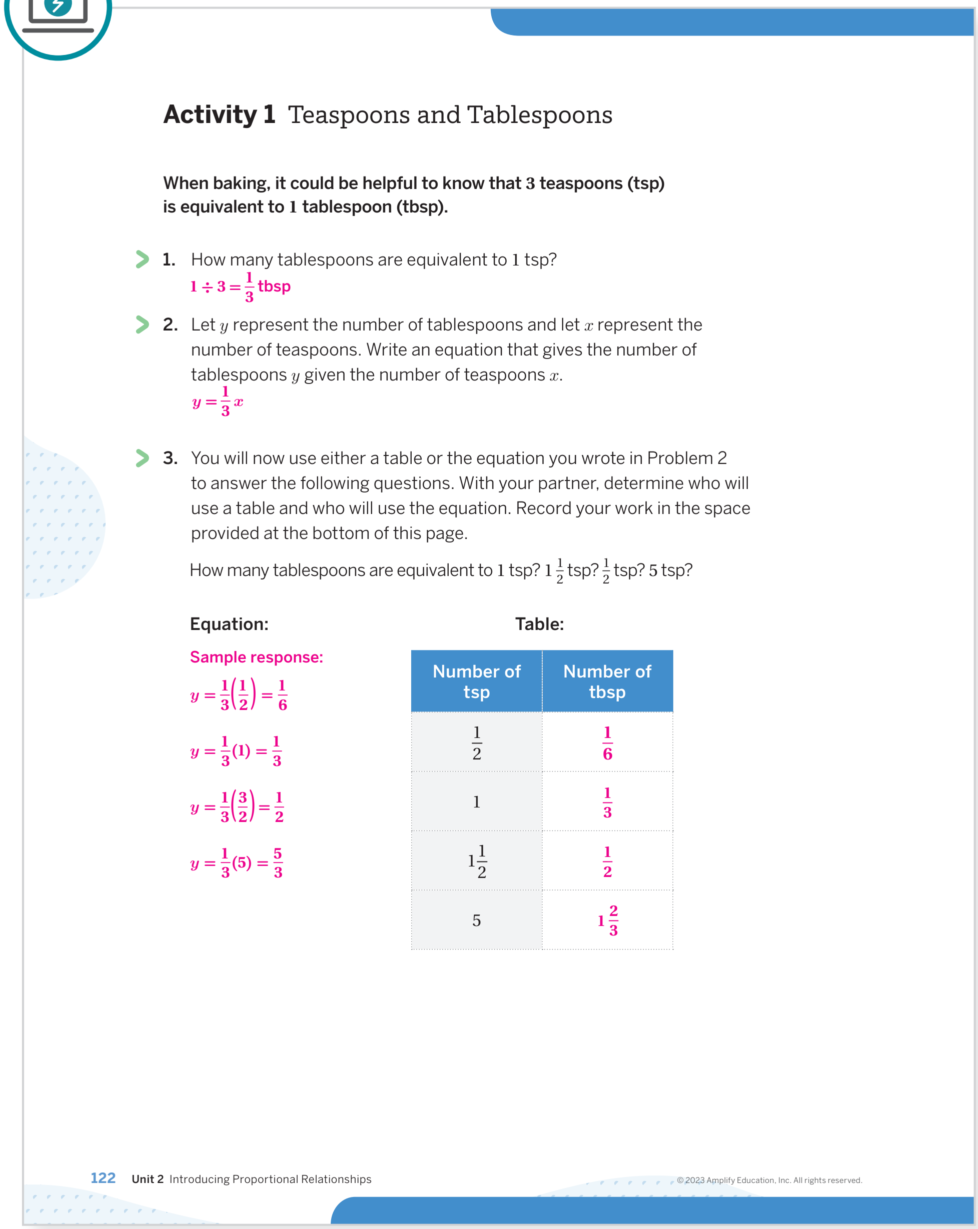
Use: Before Activity 1.

Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7.

Pairs | 13 min

Activity 1 Teaspoons and Tablespoons

Students write an equation based on the constant of proportionality, and then complete partner problems to see that each pair in the table satisfies the equation $y = kx$.



Activity 1 Teaspoons and Tablespoons

When baking, it could be helpful to know that 3 teaspoons (tsp) is equivalent to 1 tablespoon (tbsp).

1. How many tablespoons are equivalent to 1 tsp?
   $1 \div 3 = \frac{1}{3}$ tbsp
2. Let y represent the number of tablespoons and let x represent the number of teaspoons. Write an equation that gives the number of tablespoons y given the number of teaspoons x.
   $y = \frac{1}{3}x$
3. You will now use either a table or the equation you wrote in Problem 2 to answer the following questions. With your partner, determine who will use a table and who will use the equation. Record your work in the space provided at the bottom of this page.

   How many tablespoons are equivalent to 1 tsp? $1\frac{1}{2}$ tsp? $\frac{1}{2}$ tsp? 5 tsp?

Equation:

Sample response:

$y = \frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{6}$

$y = \frac{1}{3}(1) = \frac{1}{3}$

$y = \frac{1}{3}\left(\frac{3}{2}\right) = \frac{1}{2}$

$y = \frac{1}{3}(5) = \frac{5}{3}$

Table:

Number of tsp	Number of tbsp
$\frac{1}{2}$	$\frac{1}{6}$
1	$\frac{1}{3}$
$1\frac{1}{2}$	$\frac{1}{2}$
5	$1\frac{2}{3}$

1 Launch

Ask, "What is the relationship between teaspoons and tablespoons? Which is larger?" Explain to students that they will be working together for the first two problems, and that Problem 3 uses the ***Partner Problems*** routine.

2 Monitor

Help students get started by suggesting that they create a table to help them make sense of the relationship for Problems 1 and 2.

Look for points of confusion:

- **Thinking that the equation is $y = 3x$.** Ask, "Does it make sense that the number of tablespoons would be greater than the number of teaspoons?"

Look for productive strategies:

- Using repeated reasoning when multiplying each value of x by $\frac{1}{3}$ to determine each value of y in the table.

3 Connect

Ask:

- "How did you determine the constant of proportionality without using the table of values?"
- "What does each part of $y = \frac{1}{3}x$ represent?"

Have pairs of students share their methods for calculating the missing values in Problem 3.

Highlight that in both the table and the equation, the number of teaspoons is being multiplied by $\frac{1}{3}$ to calculate the number of tablespoons. Note that the number of tablespoons is less than the number of teaspoons, so in both the table and in the equation, the constant of proportionality is less than 1.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Students may confuse *teaspoons* with *tablespoons*. Use real measuring spoons and water to demonstrate how there are 3 teaspoons in 1 tablespoon. For students who are more familiar with metric measurements — or for any student — consider having them draw diagrams in their Student Edition that illustrate how the sizes of these measurements compare.

Extension: *Math Enrichment*

Have students determine the number of teaspoons that are equivalent to $2\frac{1}{2}$ tbsp. $7\frac{1}{2}$ tsp

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to describe what is similar and what is different about the various strategies used to determine the constant of proportionality without using the table. Highlight how the constant of proportionality appears in the equation and in the table.

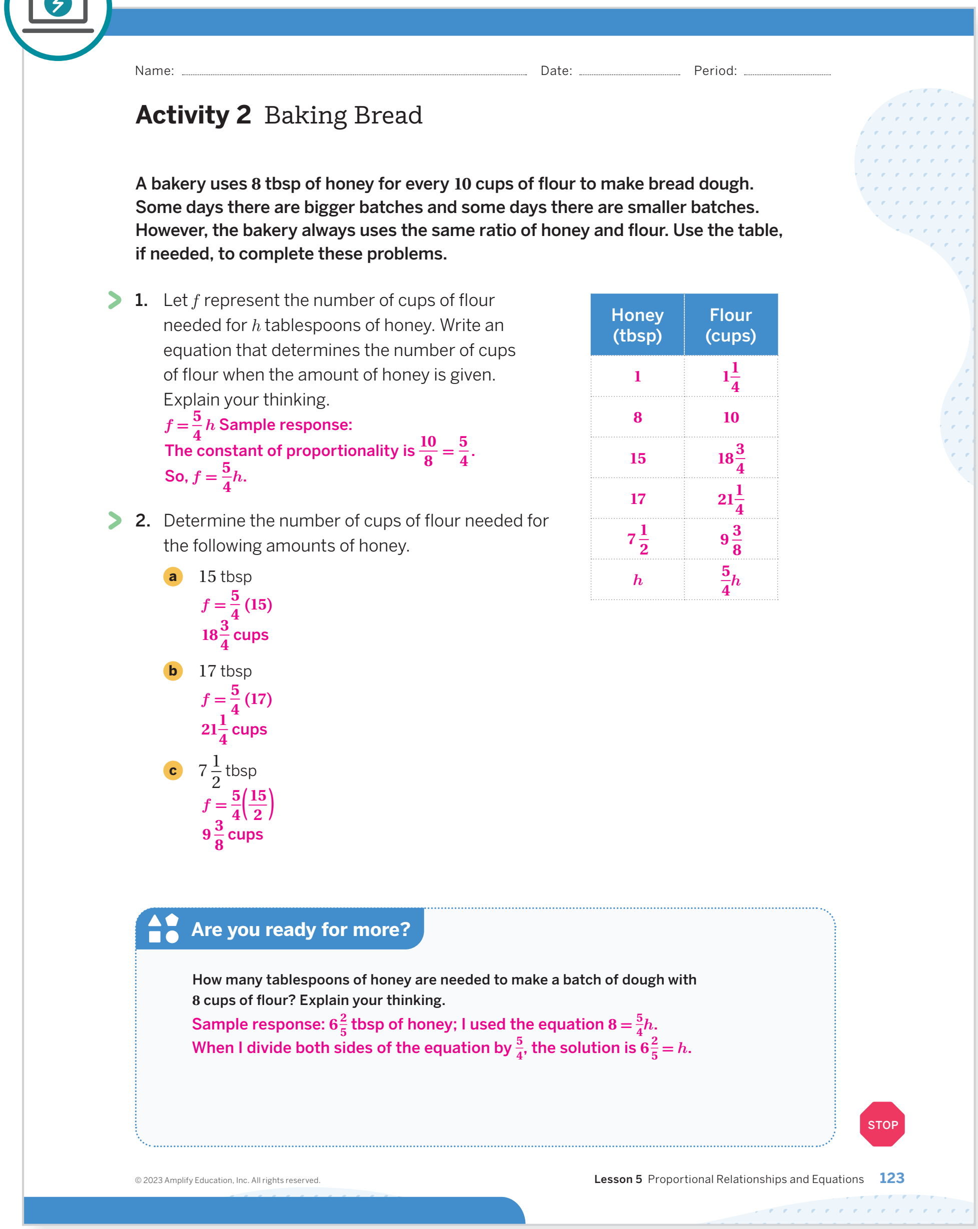
English Learners

Have students use colored pencils or highlighters to highlight the constant of proportionality in the equation and in the table. Have them write "constant of proportionality" and draw arrows from this term to each value they highlighted.

Pairs | 10 min

Activity 2 Baking Bread

Students write an equation to represent the proportional relationship between ingredients in a recipe, seeing that the equation is helpful for determining unknown quantities.



Name: ______ Date: ______ Period: ______

Activity 2 Baking Bread

A bakery uses 8 tbsp of honey for every 10 cups of flour to make bread dough. Some days there are bigger batches and some days there are smaller batches. However, the bakery always uses the same ratio of honey and flour. Use the table, if needed, to complete these problems.

1. Let f represent the number of cups of flour needed for h tablespoons of honey. Write an equation that determines the number of cups of flour when the amount of honey is given. Explain your thinking.
   $f = \frac{5}{4}h$ Sample response:
   The constant of proportionality is $\frac{10}{8} = \frac{5}{4}$.
   So, $f = \frac{5}{4}h$.

Honey (tbsp)	Flour (cups)
1	$1\frac{1}{4}$
8	10
15	$18\frac{3}{4}$
17	$21\frac{1}{4}$
$7\frac{1}{2}$	$9\frac{3}{8}$
h	$\frac{5}{4}h$

2. Determine the number of cups of flour needed for the following amounts of honey.
   - a. 15 tbsp
     $f = \frac{5}{4}(15)$
     $18\frac{3}{4}$ cups
   - b. 17 tbsp
     $f = \frac{5}{4}(17)$
     $21\frac{1}{4}$ cups
   - c. $7\frac{1}{2}$ tbsp
     $f = \frac{5}{4}\left(\frac{15}{2}\right)$
     $9\frac{3}{8}$ cups

Are you ready for more?

How many tablespoons of honey are needed to make a batch of dough with 8 cups of flour? Explain your thinking.

Sample response: $6\frac{2}{5}$ tbsp of honey; I used the equation $8 = \frac{5}{4}h$. When I divide both sides of the equation by $\frac{5}{4}$, the solution is $6\frac{2}{5} = h$.

STOP

1 Launch

Students will follow the ***Think-Pair-Share*** routine for Problem 1, and then complete Problem 2 in pairs.

2 Monitor

Help students get started by asking what information they need to know in order to write the equation. Encourage them to use Activity 1 for reference.

Look for points of confusion:

- **Thinking that the scenario translates into the equation $8h = 10f$.** Have students complete a row of the table, and then check to see whether their equation represents the relationship between the values.
- **Thinking the equation is $f = \frac{4}{5}h$.** Ask students whether the number of cups of flour is more, or less, than the number of tablespoons of honey. Based on this relationship, ask if the constant of proportionality should be greater than, or less than, 1.

3 Connect

Have pairs of students share their process for determining the equation in Problem 1.

Highlight that the number of cups of flour is greater than the corresponding number of tablespoons of honey so the constant of proportionality must be greater than 1. That is, it must be $\frac{5}{4}$, not $\frac{4}{5}$.

Ask:

- "What does each part of the equation $f = \frac{5}{4}h$ represent?" Sample response: f is the number of cups of flour, h is the number of tablespoons of honey, and $\frac{5}{4}$ represents $\frac{5}{4}$ cups of flour for every 1 tablespoon of honey.
- "How can you use the equation to determine unknown values?" Sample response: Substitute the given value for its corresponding variable and then solve for the unknown value.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Display the equation in the form $f =$ ____h and ask students what value completes the equation. Have students use the table to generate other equivalent ratios of honey to flour, such as $4 : 5$, $2 : 2\frac{1}{2}$, and $1 : 1\frac{1}{4}$ to help them determine this value.

Extension: *Math Enrichment*

Ask students to manipulate the equation $f = \frac{5}{4}h$ to solve the equation for h. Then have them explain what this equation represents. $h = \frac{4}{5}f$; This equation represents the number of teaspoons of honey given the amount of flour.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

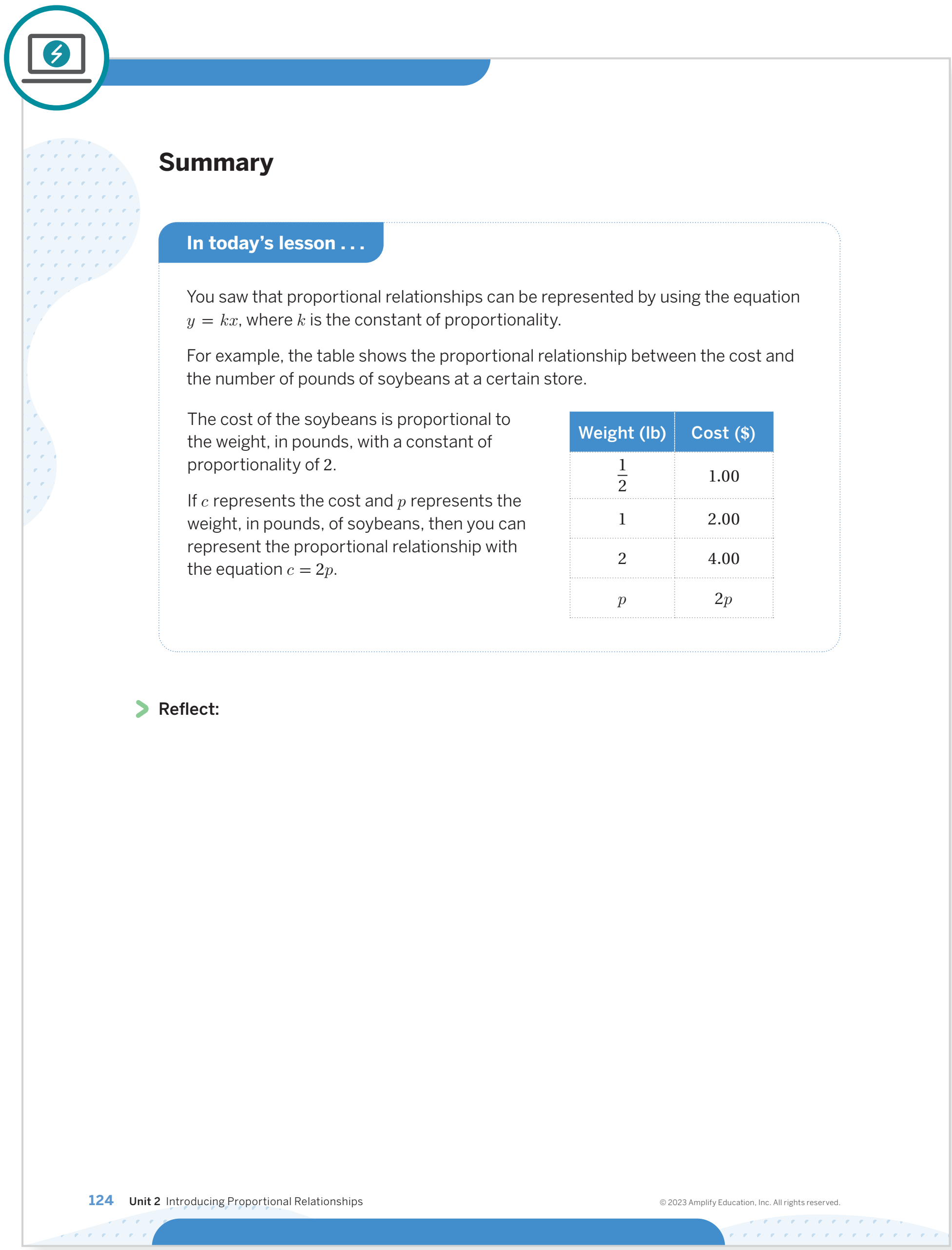
- **Read 1:** Students should understand that a bakery uses the same ratio of honey and flour to make smaller and larger batches of bread.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as 8 tbsp for every 10 cups of flour.
- **Read 3:** Ask students to plan their solution strategy for writing the equation in Problem 1.

English Learners

Have students highlight key phrases, such as *8 tbsp of honey for every 10 cups of flour and same ratio.*

Summary

Review and synthesize how proportional relationships can be represented by equations to determine an unknown value.



Summary

In today's lesson . . .

You saw that proportional relationships can be represented by using the equation $y = kx$, where k is the constant of proportionality.

For example, the table shows the proportional relationship between the cost and the number of pounds of soybeans at a certain store.

The cost of the soybeans is proportional to the weight, in pounds, with a constant of proportionality of 2.

If c represents the cost and p represents the weight, in pounds, of soybeans, then you can represent the proportional relationship with the equation $c = 2p$.

Weight (lb)	Cost ($)
$\frac{1}{2}$	1.00
1	2.00
2	4.00
p	$2p$

Reflect:

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Synthesize

Display the table from the Summary.

Highlight that the weight of soybeans is multiplied by 2 in each row to determine the cost, and model this relationship by adding the arrows from one column to the next column. Explain the relationship in this table can be generalized by writing the equation $c = 2p$ (or $y = 2x$). In general, if the constant of proportionality, k, is known for a proportional relationship, then the relationship can be modeled by the equation $y = kx$.

Ask:

- "How is an equation used to calculate the unknown value in a proportional relationship if one quantity is already known?" Sample response: Substitute the known value for its corresponding variable and then solve for the unknown value.
- "If a proportional relationship between the cost of apples and the number of pounds could be modeled by the equation $c = 2.99p$, what do you know?" Sample response: The constant of proportionality is 2.99. The cost of one pound of apples is $2.99.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. To help them engage in meaningful reflection, consider asking:

- "What does it mean for two values to be proportionally related? How can you tell?"

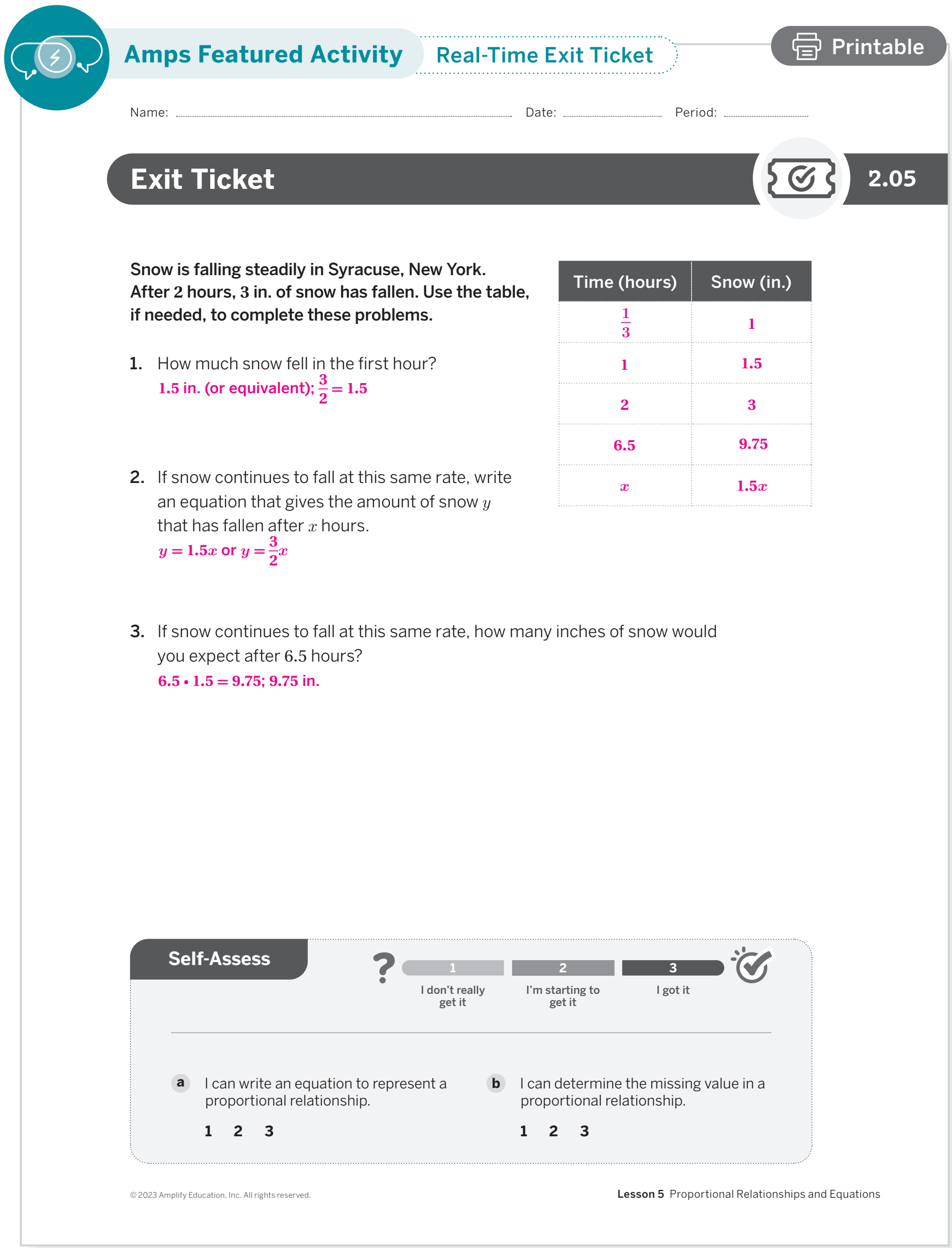
Independent | 7 min

Exit Ticket

Students demonstrate their understanding by writing an equation to represent a proportional relationship between an amount of snow and time.

Amps Featured Activity Real-Time Exit Ticket

Printable



Name: ______________________ Date: ____________ Period: ________

Exit Ticket 2.05

Snow is falling steadily in Syracuse, New York. After 2 hours, 3 in. of snow has fallen. Use the table, if needed, to complete these problems.

Time (hours)	Snow (in.)
$\frac{1}{3}$	1
1	1.5
2	3
6.5	9.75
x	$1.5x$

1. How much snow fell in the first hour?
   1.5 in. (or equivalent); $\frac{3}{2} = 1.5$

2. If snow continues to fall at this same rate, write an equation that gives the amount of snow y that has fallen after x hours.
   $y = 1.5x$ or $y = \frac{3}{2}x$

3. If snow continues to fall at this same rate, how many inches of snow would you expect after 6.5 hours?
   $6.5 \cdot 1.5 = 9.75$; 9.75 in.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can write an equation to represent a proportional relationship.
1 2 3

b I can determine the missing value in a proportional relationship.
1 2 3

Lesson 5 Proportional Relationships and Equations

Success looks like . . .

- **Language Goal:** Generalizing a process for calculating missing values in a proportional relationship and justifying why this can be abstracted as $y = kx$, where k is the constant of proportionality. **(Speaking and Listening)**
  - » Completing the table for the proportional relationship of time and snow.
- **Language Goal:** Generating an equation of the form $y = kx$ to represent a proportional relationship in a familiar context. **(Speaking and Listening)**
  - » Writing an equation for the amount of snow that has fallen after x hours in Problem 2.

Suggested next steps

If student write the equation $2x = 3y$, consider:

- Reviewing the use of the table to make sense of the relationship, specifically, drawing arrows with the constant of proportionality to model the direction of the relationship.
- Assigning Practice Problems 2 and 3.

If students rely on the table of values, consider:

- Assigning Practice Problem 3 and asking them to attempt it without using the table. Then allow students to use the table to check their work.

Professional Learning

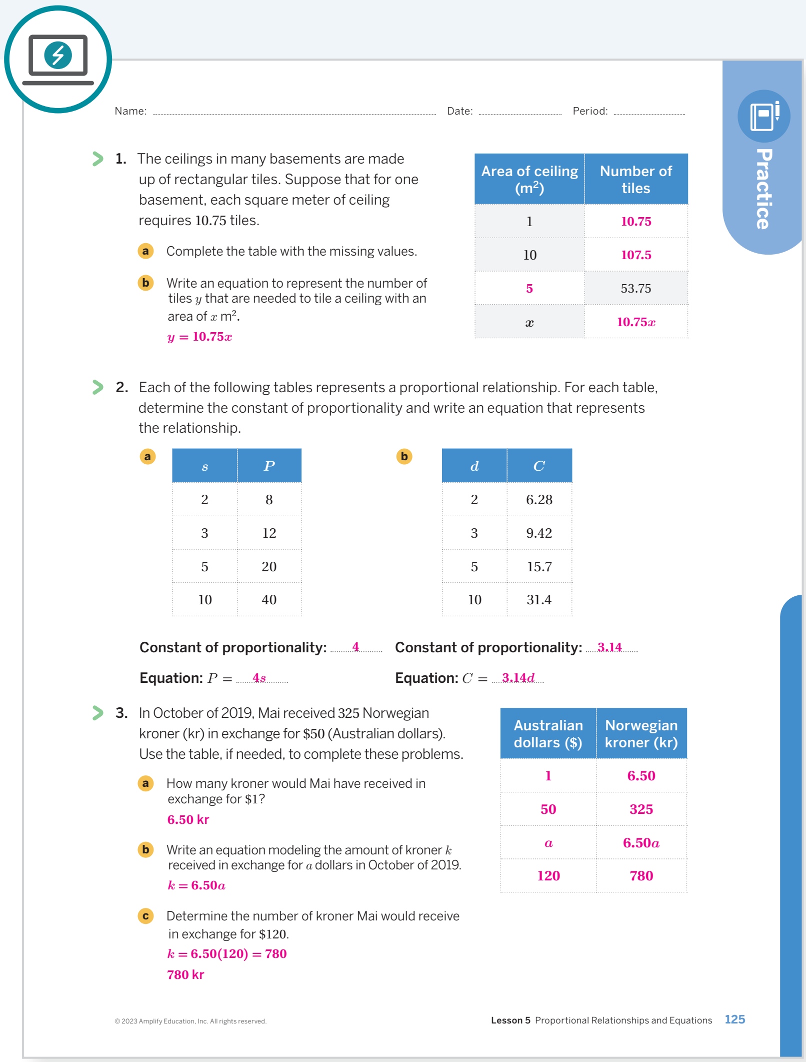
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students determined the constant of proportionality from tables. How did that support them in writing equations to represent proportional relationships?
- In what ways have your students improved at expressing regularity in repeated reasoning? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. The ceilings in many basements are made up of rectangular tiles. Suppose that for one basement, each square meter of ceiling requires 10.75 tiles.

Area of ceiling (m^2)	Number of tiles
1	**10.75**
10	**107.5**
5	53.75
x	**$10.75x$**

   a. Complete the table with the missing values.

   b. Write an equation to represent the number of tiles y that are needed to tile a ceiling with an area of x m^2.

   $y = 10.75x$

2. Each of the following tables represents a proportional relationship. For each table, determine the constant of proportionality and write an equation that represents the relationship.

a.

s	P
2	8
3	12
5	20
10	40

Constant of proportionality: **4**

Equation: $P =$ **$4s$**

b.

d	C
2	6.28
3	9.42
5	15.7
10	31.4

Constant of proportionality: **3.14**

Equation: $C =$ **$3.14d$**

3. In October of 2019, Mai received 325 Norwegian kroner (kr) in exchange for $50 (Australian dollars). Use the table, if needed, to complete these problems.

Australian dollars ($)	Norwegian kroner (kr)
1	**6.50**
50	**325**
a	**$6.50a$**
120	**780**

   a. How many kroner would Mai have received in exchange for $1?

   6.50 kr

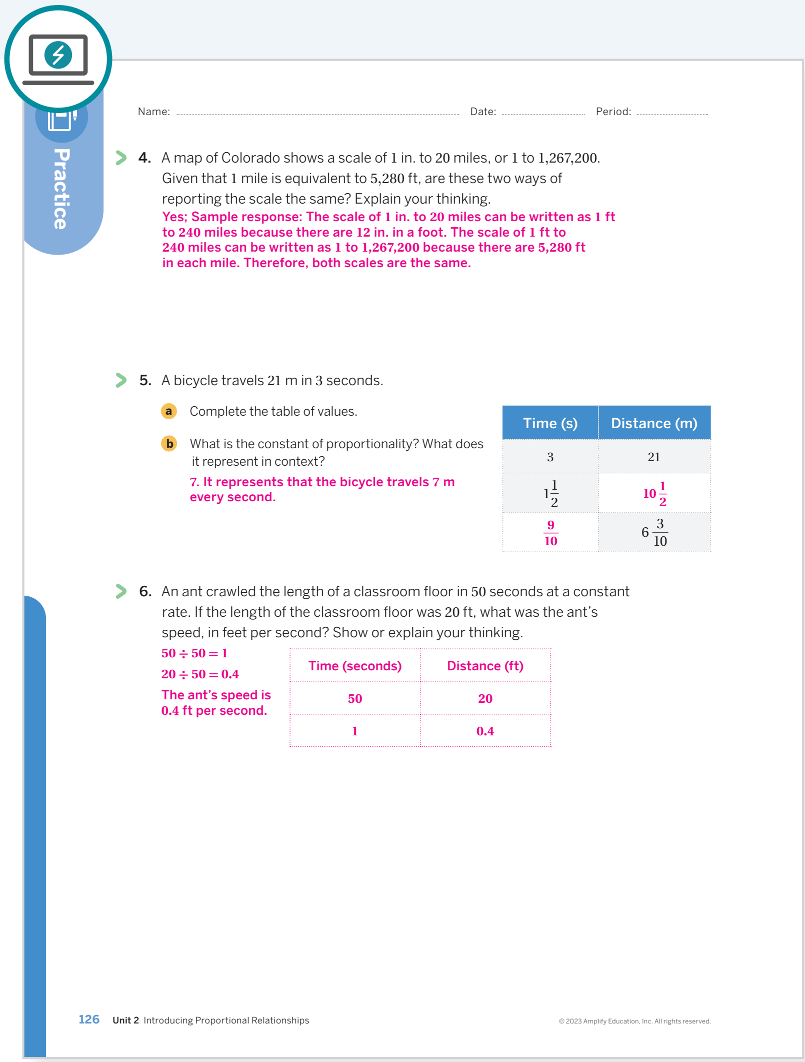
   b. Write an equation modeling the amount of kroner k received in exchange for a dollars in October of 2019.

   $k = 6.50a$

   c. Determine the number of kroner Mai would receive in exchange for $120.

   $k = 6.50(120) = 780$

   780 kr



Practice

Name: ______ Date: ______ Period: ______

4. A map of Colorado shows a scale of 1 in. to 20 miles, or 1 to 1,267,200. Given that 1 mile is equivalent to 5,280 ft, are these two ways of reporting the scale the same? Explain your thinking.

   Yes; Sample response: The scale of 1 in. to 20 miles can be written as 1 ft to 240 miles because there are 12 in. in a foot. The scale of 1 ft to 240 miles can be written as 1 to 1,267,200 because there are 5,280 ft in each mile. Therefore, both scales are the same.

5. A bicycle travels 21 m in 3 seconds.

Time (s)	Distance (m)
3	21
$1\frac{1}{2}$	**$10\frac{1}{2}$**
$\frac{9}{10}$	$6\frac{3}{10}$

   a. Complete the table of values.

   b. What is the constant of proportionality? What does it represent in context?

   7. It represents that the bicycle travels 7 m every second.

6. An ant crawled the length of a classroom floor in 50 seconds at a constant rate. If the length of the classroom floor was 20 ft, what was the ant's speed, in feet per second? Show or explain your thinking.

   $50 \div 50 = 1$

   $20 \div 50 = 0.4$

   The ant's speed is 0.4 ft per second.

Time (seconds)	Distance (ft)
50	**20**
1	**0.4**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 11	2
	5	Unit 2 Lesson 2	2
Formative ⚡	6	Unit 2 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 6**

Speed and Equations

Let's write equations describing proportional relationships involving speed.

Focus

Goals

1. Interpret the constant of proportionality for a relationship in the context of constant speed.
2. Generate an equation for a proportional relationship, given a verbal description of the situation, and without being given a table.
3. **Language Goal:** Interpret each part of an equation that represents a proportional relationship in an unfamiliar context. **(Speaking and Listening)**

Coherence

Today

Students extend their work representing proportional relationships using equations of the form $y = kx$, to contexts involving time, distance, and speed. They notice that speed is the constant of proportionality when distance is dependent on time. Students use equations to express the regularity of the repeated calculations of values in a table.

Previously

In Lesson 5, students generalized that for any proportional relationship, once the constant of proportionality is known, it could be represented by the equation $y = kx$.

Coming Soon

In Lesson 7, students will see that every proportional relationship can be represented by an equation written in two different ways.

Rigor

- Students build **conceptual understanding** of the form of an equation representing a proportional relationship between time and distance.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	12 min	15 min	5 min	8 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

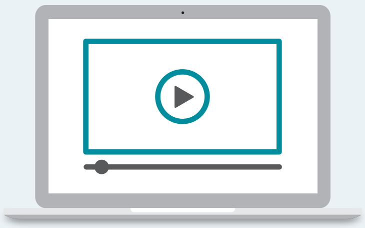
- *constant of proportionality*
- *proportional relationship*
- *unit rate*

Amps | Featured Activity

Activity 1

Instant Feedback

Students can check their tables and get instant feedback on whether they need to continue working.



Building Math Identity and Community

Connecting to Mathematical Practices

Students find themselves unable to regulate their emotions as they approach the Activity 1 quantities in context. Prepare students for the activity by discussing speed as it relates to them personally. Allowing them to share stories about how fast or how slow something was engages them in the topic and motivates them to pursue more understanding.

Modifications to Pacing

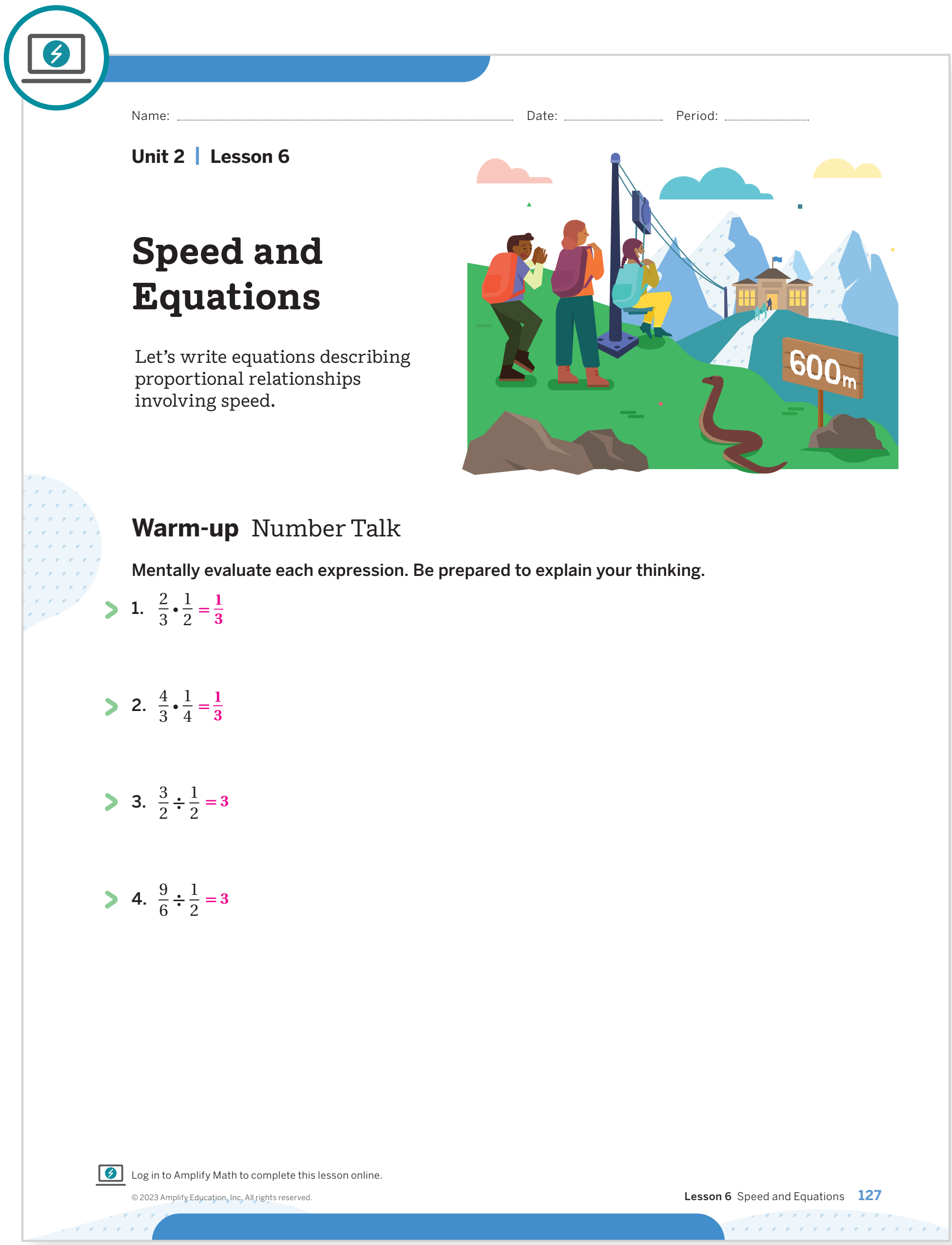
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, Problems 3 and 4 may be omitted.
- In **Activity 2**, Problem 4 may be omitted.

Independent | 5 min

Warm-up Number Talk

Students evaluate expressions with fractions to prepare for calculating rates involving fractions.



Name: ______________ Date: ________ Period: ________

Unit 2 | Lesson 6

Speed and Equations

Let's write equations describing proportional relationships involving speed.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

2. $\frac{4}{3} \cdot \frac{1}{4} = \frac{1}{3}$

3. $\frac{3}{2} \div \frac{1}{2} = 3$

4. $\frac{9}{6} \div \frac{1}{2} = 3$

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the ***Number Talk*** routine. Instruct students to consider only one problem at a time. Allow students about 30 seconds for each problem, and ask them to give a signal once they have a solution and a strategy.

2 Monitor

Look for points of confusion:

- **Taking the reciprocal of both fractions before multiplying.** Review strategies for dividing fractions.

Look for productive strategies:

- Using the relationships between the numbers in earlier expressions to reason about the products and quotients.

3 Connect

Display the expressions.

Have students share their strategies for evaluating each expression.

Ask:

- "Suppose for Problem 1, some students had $\frac{2}{6}$ and others had $\frac{1}{3}$. Which is correct?" Both answers are correct because they are equivalent fractions.
- "Who evaluated the expressions a different way?"
- "Do you agree with ___'s strategy? What are your reasons?"
- "Who can restate ___'s reasoning in their own words?"
- "What is similar about Problem 3 and Problem 4? What is different?" Even though $\frac{9}{6}$ is different from $\frac{3}{2}$, the answers are the same because they are equivalent fractions.

Highlight that when working with fractions, it can be useful to rewrite them in lowest terms before performing operations.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies for evaluating each expression, have them discuss what is similar to and different from the various strategies used to evaluate the expressions in Problems 3 and 4. Look for and amplify language used, such as *equivalent fractions*.

English Learners

Have students circle $\frac{3}{2}$ and $\frac{9}{6}$ in Problems 3 and 4 and write the phrase *equivalent fractions* with arrows pointing to each fraction.

Power-up

To power up students' ability to calculate the speed by relating it to the unit rate, have students complete:

Recall that a *unit rate* is a comparison of how much one unit changes when another changes by 1. Speed is a type of unit rate given as a distance for every one unit of time.

Kiran rode 7.5 miles in half an hour. Determine his speed in miles per hour.
15 mph

Use: Before Activity 1.

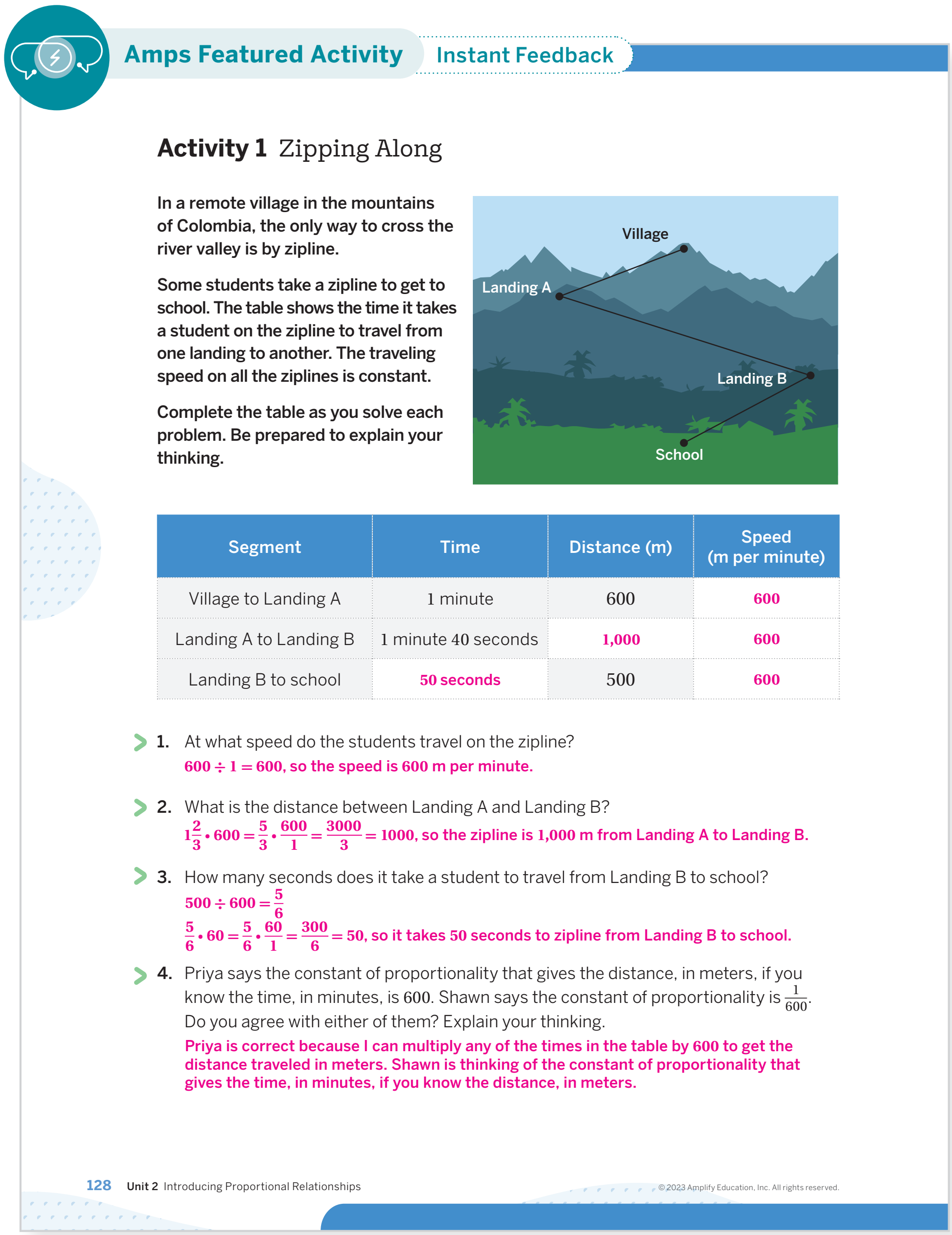
Informed by: Performance on Lesson 5, Practice Problem 6.

Pairs | 12 min

Activity 1 Zipping Along

Students analyze an object traveling at a steady speed, in a relationship between distance and time, to determine that speed is the constant of proportionality.

Amps Featured Activity Instant Feedback



Activity 1 Zipping Along

In a remote village in the mountains of Colombia, the only way to cross the river valley is by zipline.

Some students take a zipline to get to school. The table shows the time it takes a student on the zipline to travel from one landing to another. The traveling speed on all the ziplines is constant.

Complete the table as you solve each problem. Be prepared to explain your thinking.

Segment	Time	Distance (m)	Speed (m per minute)
Village to Landing A	1 minute	600	600
Landing A to Landing B	1 minute 40 seconds	1,000	600
Landing B to school	50 seconds	500	600

1. At what speed do the students travel on the zipline?
   $600 \div 1 = 600$, so the speed is 600 m per minute.

2. What is the distance between Landing A and Landing B?
   $1\frac{2}{3} \cdot 600 = \frac{5}{3} \cdot \frac{600}{1} = \frac{3000}{3} = 1000$, so the zipline is 1,000 m from Landing A to Landing B.

3. How many seconds does it take a student to travel from Landing B to school?
   $500 \div 600 = \frac{5}{6}$
   $\frac{5}{6} \cdot 60 = \frac{5}{6} \cdot \frac{60}{1} = \frac{300}{6} = 50$, so it takes 50 seconds to zipline from Landing B to school.

4. Priya says the constant of proportionality that gives the distance, in meters, if you know the time, in minutes, is 600. Shawn says the constant of proportionality is $\frac{1}{600}$. Do you agree with either of them? Explain your thinking.
   Priya is correct because I can multiply any of the times in the table by 600 to get the distance traveled in meters. Shawn is thinking of the constant of proportionality that gives the time, in minutes, if you know the distance, in meters.

128 Unit 2 Introducing Proportional Relationships

1 Launch

Activate students' background knowledge by asking for different ways students travel to school. Underline the sentence indicating the speed is constant, and remind students that this means the speed will stay the same while ziplining.

2 Monitor

Help students get started by reminding them that speed is a unit rate of distance per time.

Look for points of confusion:

- **Dividing the time value by the distance value.** Point out to students the unit for speed is meters per minute. Ask, "How can you calculate how far the student travels on the zipline in one minute?"

3 Connect

Display the completed table.

Ask, "How does knowing the speed is constant help you to solve problems like this one?" Once I calculate the speed for the first row, I can use that to help determine the missing information in the other two rows.

Highlight that, in a proportional relationship comparing distance to time, the speed is the constant of proportionality. While the other constant of proportionality is for time per distance, it does not represent speed

Note: In upcoming lessons, students will see that there are two constants of proportionality, and that Shawn's statement in Problem 4 gives the time, if the distance is known.

Differentiated Support

Accessibility: *Optimize Access to Technology, Guide Processing and Visualization*

Have students use the Amps slides for this activity, in which they can see the motion of the student traveling down the zipline. This helps illustrate the concepts of constant speed and the relationship between time and distance.

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to compare the quantities in the Speed column of the table with the constant of proportionality that gives the distance, if you know the time. Compare this to the other constant of proportionality — which is the reciprocal of the speed — and gives the time, if you know the distance.

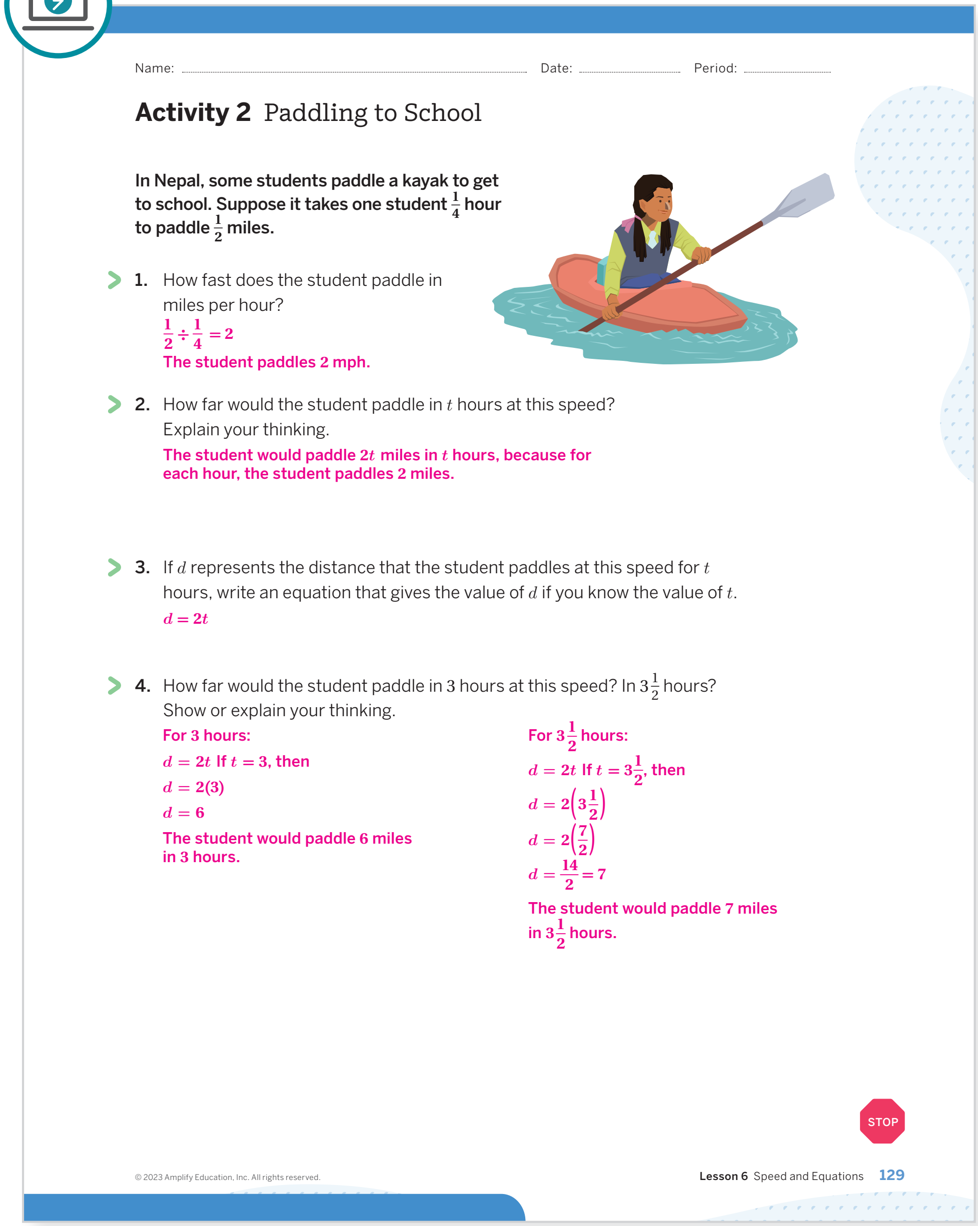
English Learners

Display the term *reciprocal* and the values 600 and $\frac{1}{600}$. Draw arrows from the term to the values.

Pairs | 15 min

Activity 2 Paddling to School

Students reason about a context, without a table, to develop their understanding of how to write equations for proportional relationships involving speed.



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Activity 2 Paddling to School

In Nepal, some students paddle a kayak to get to school. Suppose it takes one student $\frac{1}{4}$ hour to paddle $\frac{1}{2}$ miles.

1. How fast does the student paddle in miles per hour?

   $\frac{1}{2} \div \frac{1}{4} = 2$

   The student paddles 2 mph.

2. How far would the student paddle in t hours at this speed? Explain your thinking.

   The student would paddle $2t$ miles in t hours, because for each hour, the student paddles 2 miles.

3. If d represents the distance that the student paddles at this speed for t hours, write an equation that gives the value of d if you know the value of t.

   $d = 2t$

4. How far would the student paddle in 3 hours at this speed? In $3\frac{1}{2}$ hours? Show or explain your thinking.

   For 3 hours:

   $d = 2t$ **If** $t = 3$**, then**

   $d = 2(3)$

   $d = 6$

   The student would paddle 6 miles in 3 hours.

   For $3\frac{1}{2}$ hours:

   $d = 2t$ **If** $t = 3\frac{1}{2}$**, then**

   $d = 2\left(3\frac{1}{2}\right)$

   $d = 2\left(\frac{7}{2}\right)$

   $d = \frac{14}{2} = 7$

   The student would paddle 7 miles in $3\frac{1}{2}$ hours.

STOP

Lesson 6 Speed and Equations 129

1 Launch

Activate students' background knowledge by asking whether they have ever paddled a kayak. Ask, "Was it fast or slow? What was the estimated speed of the kayak?"

2 Monitor

Help students get started by prompting them to revisit how they found the speed of the student on the zipline in Activity 1.

Look for points of confusion:

- Multiplying or dividing $\frac{1}{4}$ by $\frac{1}{2}$. Have students draw a picture or use a table of values to represent the minutes and miles. Ask, "Which unit, hours or miles, do you need to become 1?"

Look for productive strategies:

- Creating a table and first determining other corresponding values representing time and distance before determining that $2t$ represents the distance for any amount of time t.

3 Connect

Display the table from Activity 1.

Ask:

- "What equation could you write for the relationship in Activity 1?"
- "How does an equation help you determine missing information in a relationship involving time and distance?"
- "What generalizations can you make about writing equations for relationships involving distance, time, and speed?"

Highlight that, in Activity 1, the speed was the constant of proportionality when relating time to distance in the table. Note that once the speed is known, students can place it in the general equation for proportional relationships as the constant of proportionality.

Fostering Diverse Thinking

How Students Travel to School

Ask students how long it took them to travel to school in the morning, and to compare these times. (Do any of them travel to school by kayak?) Ask what steps could be taken in order to help all students get to school more easily.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that some students in Nepal paddle a kayak to get to school.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as $\frac{1}{4}$ hour to paddle $\frac{1}{2}$ miles.
- **Read 3:** Ask students to plan their solution strategy to find the speed in miles per hour for Problem 1.

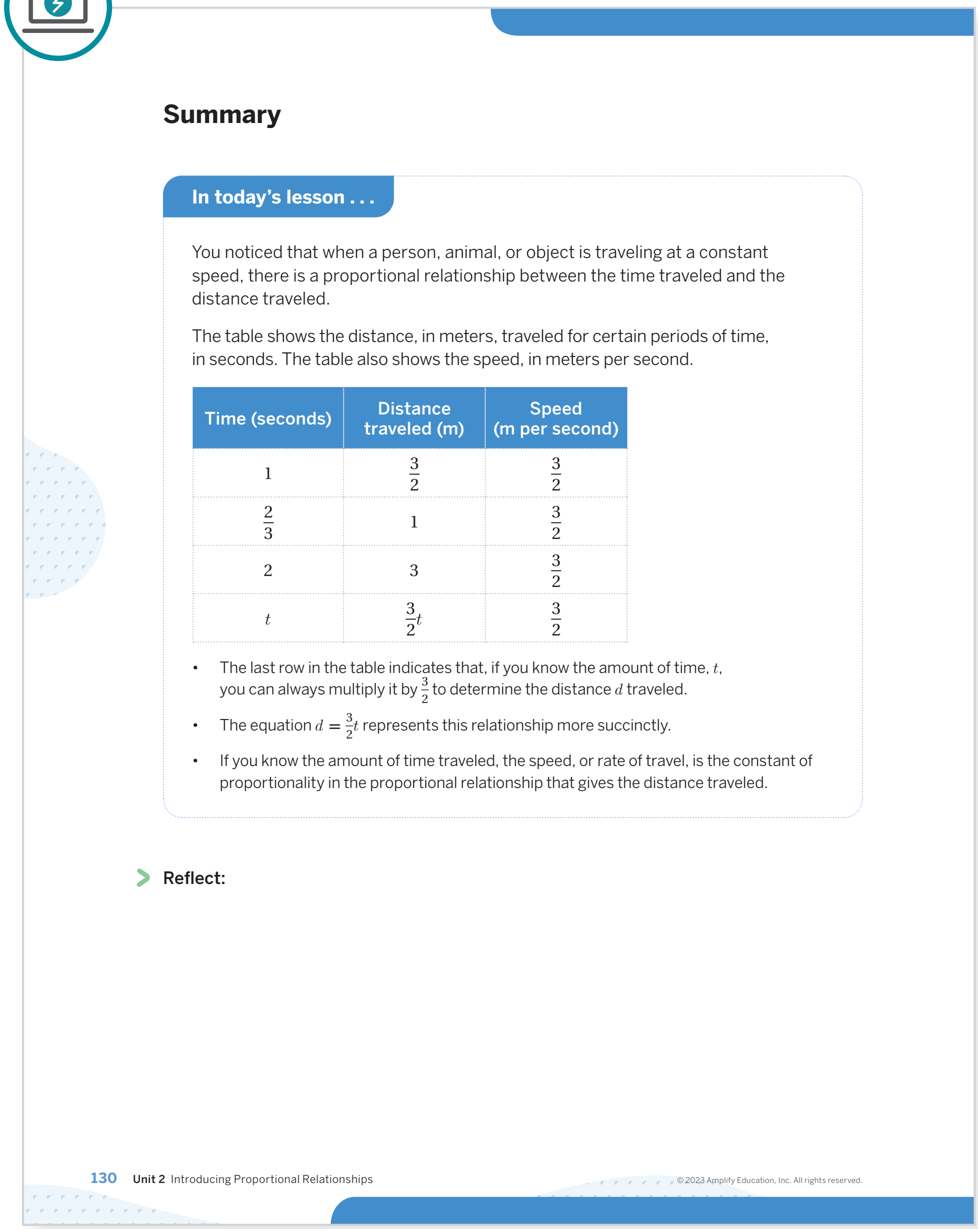
English Learners

Use hand gestures to illustrate what it means to paddle a kayak.

Whole Class | 5 min

Summary

Review and synthesize that in a relationship between distance and time, speed represents the constant of proportionality.



Summary

In today's lesson . . .

You noticed that when a person, animal, or object is traveling at a constant speed, there is a proportional relationship between the time traveled and the distance traveled.

The table shows the distance, in meters, traveled for certain periods of time, in seconds. The table also shows the speed, in meters per second.

Time (seconds)	Distance traveled (m)	Speed (m per second)
1	$\frac{3}{2}$	$\frac{3}{2}$
$\frac{2}{3}$	1	$\frac{3}{2}$
2	3	$\frac{3}{2}$
t	$\frac{3}{2}t$	$\frac{3}{2}$

- The last row in the table indicates that, if you know the amount of time, t, you can always multiply it by $\frac{3}{2}$ to determine the distance d traveled.
- The equation $d = \frac{3}{2}t$ represents this relationship more succinctly.
- If you know the amount of time traveled, the speed, or rate of travel, is the constant of proportionality in the proportional relationship that gives the distance traveled.

Reflect:

130 Unit 2 Introducing Proportional Relationships

Synthesize

Display the table from the Summary.

Highlight that the relationship between distance and time is a very common context. Speed can be calculated for any combination of time unit and distance unit, though some are more common, such as miles per hour and meters per second.

Ask, "When you represent proportional relationships with equations, you use the form $y = kx$. Which quantities (time, distance, or speed) match with the variables in the general equation?" k is the speed, x is the time, and y is the distance.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

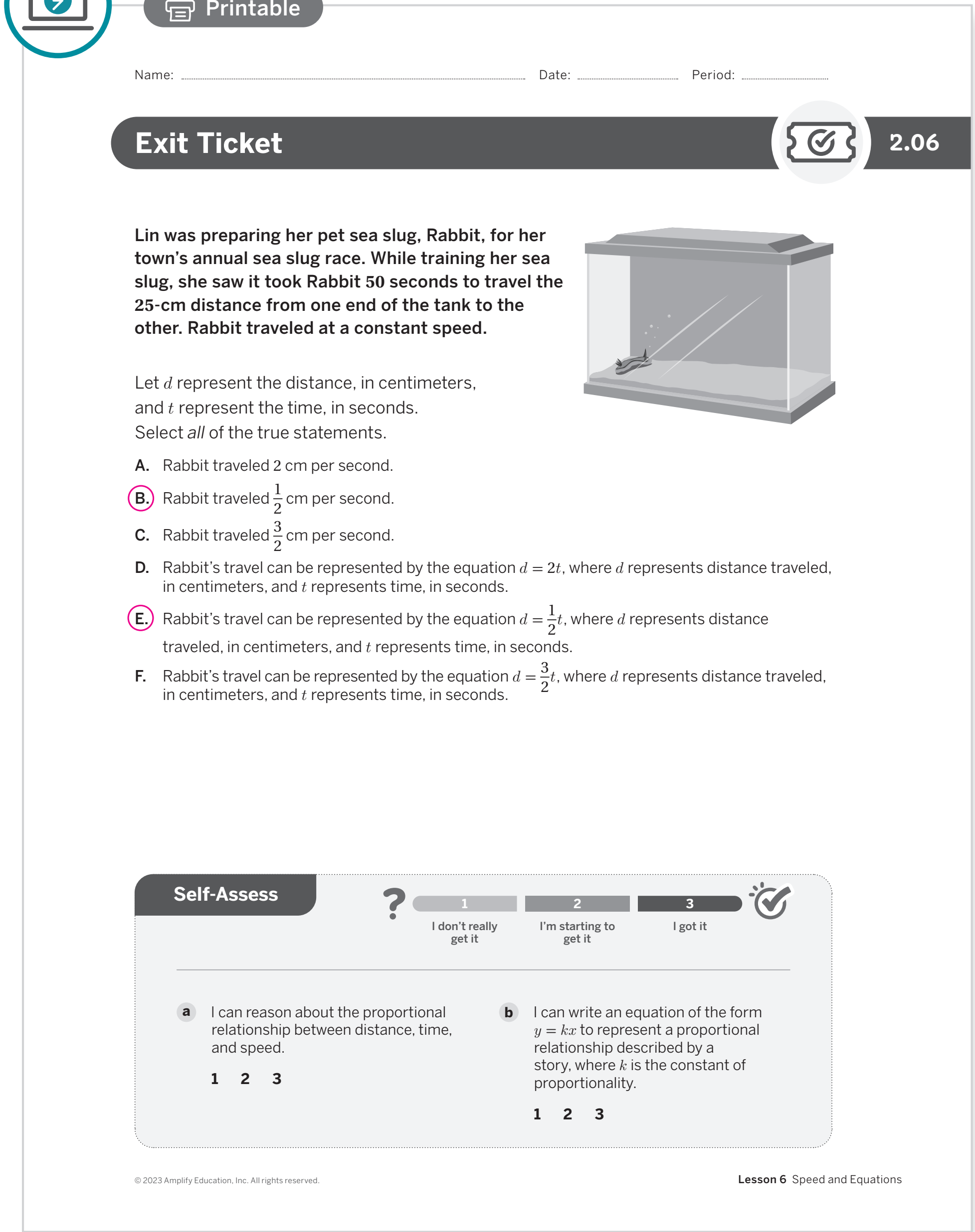
- "What were you able to generalize about the relationship between distance and time?"

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of how to write equations for situations involving constant speed, and then apply their understanding to solve problems.

Printable



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Exit Ticket 2.06

Lin was preparing her pet sea slug, Rabbit, for her town's annual sea slug race. While training her sea slug, she saw it took Rabbit 50 seconds to travel the 25-cm distance from one end of the tank to the other. Rabbit traveled at a constant speed.

Let d represent the distance, in centimeters, and t represent the time, in seconds. Select *all* of the true statements.

A. Rabbit traveled 2 cm per second.

[✓ B.] Rabbit traveled $\frac{1}{2}$ cm per second.

C. Rabbit traveled $\frac{3}{2}$ cm per second.

D. Rabbit's travel can be represented by the equation $d = 2t$, where d represents distance traveled, in centimeters, and t represents time, in seconds.

[✓ E.] Rabbit's travel can be represented by the equation $d = \frac{1}{2}t$, where d represents distance traveled, in centimeters, and t represents time, in seconds.

F. Rabbit's travel can be represented by the equation $d = \frac{3}{2}t$, where d represents distance traveled, in centimeters, and t represents time, in seconds.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can reason about the proportional relationship between distance, time, and speed.

1 2 3

b. I can write an equation of the form $y = kx$ to represent a proportional relationship described by a story, where k is the constant of proportionality.

1 2 3

Success looks like . . .

- **Goal:** Interpreting the constant of proportionality for a relationship in the context of constant speed.
  - » Selecting the correct choice that gives Rabbit's speed.
- **Goal:** Generating an equation for a proportional relationship, given a verbal description of the situation, and without being given a table.
  - » Selecting the correct equation that represents Rabbit's travel.
- **Language Goal:** Interpreting each part of an equation that represents a proportional relationship in an unfamiliar context. **(Speaking and Listening)**
  - » Selecting the correct equation along with the explanation of each variable in the equation.

Suggested next steps

If students select Choices A or C, consider:

- Having them make a table to organize how the values correspond to the units.

If students select Choices D or F, consider:

- Having them test their equation with the values from the problem to see whether the equation is a true statement.

Professional Learning

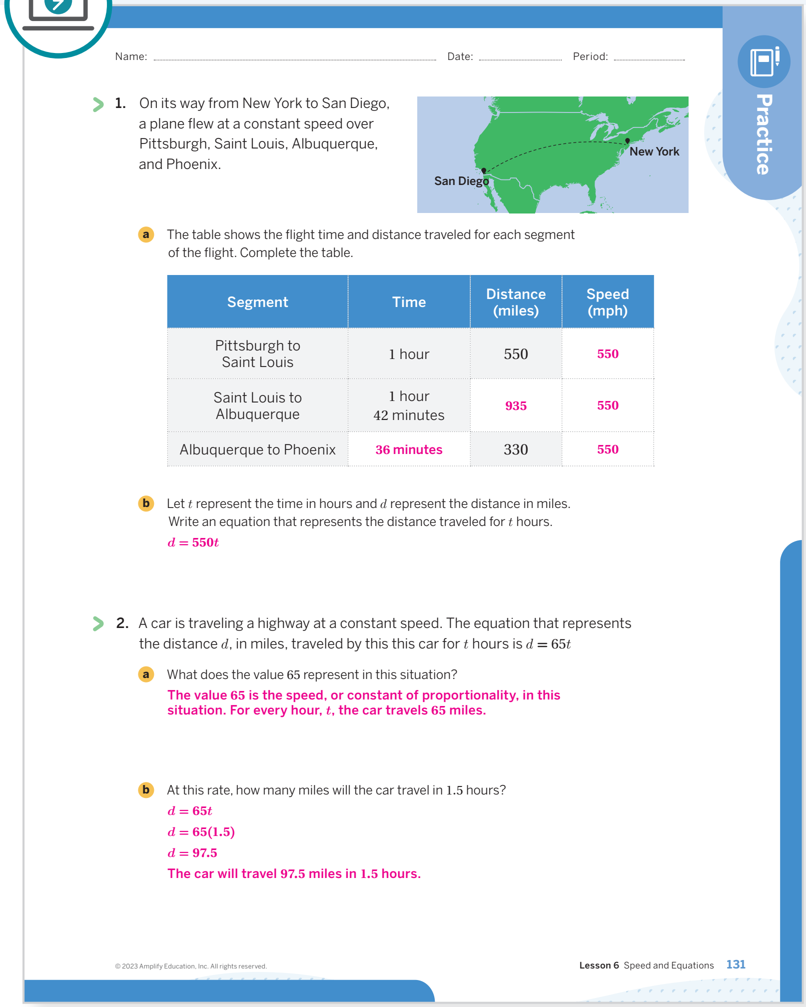
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach Activity 2?
- What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. On its way from New York to San Diego, a plane flew at a constant speed over Pittsburgh, Saint Louis, Albuquerque, and Phoenix.

   a. The table shows the flight time and distance traveled for each segment of the flight. Complete the table.

Segment	Time	Distance (miles)	Speed (mph)
Pittsburgh to Saint Louis	1 hour	550	**550**
Saint Louis to Albuquerque	1 hour 42 minutes	**935**	**550**
Albuquerque to Phoenix	**36 minutes**	330	**550**

   b. Let t represent the time in hours and d represent the distance in miles. Write an equation that represents the distance traveled for t hours.

   $d = 550t$

2. A car is traveling a highway at a constant speed. The equation that represents the distance d, in miles, traveled by this this car for t hours is $d = 65t$

   a. What does the value 65 represent in this situation?

   The value 65 is the speed, or constant of proportionality, in this situation. For every hour, t, the car travels 65 miles.

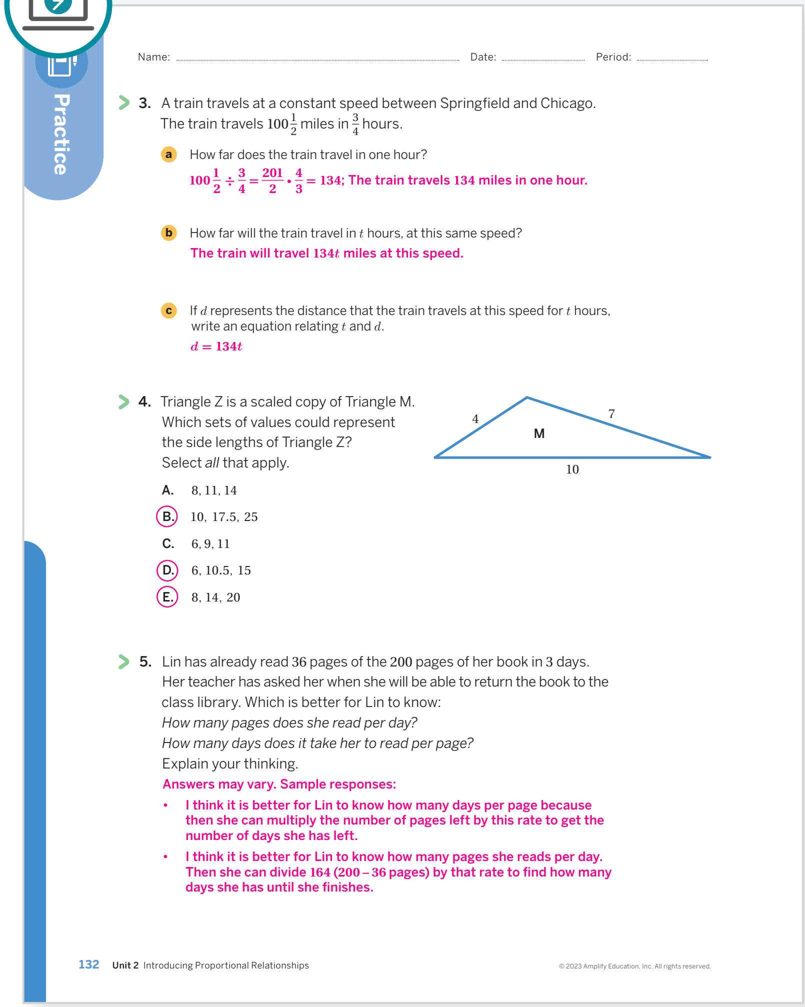
   b. At this rate, how many miles will the car travel in 1.5 hours?

   $d = 65t$

   $d = 65(1.5)$

   $d = 97.5$

   The car will travel 97.5 miles in 1.5 hours.



Name: Date: Period:

3. A train travels at a constant speed between Springfield and Chicago. The train travels $100\frac{1}{2}$ miles in $\frac{3}{4}$ hours.

   a. How far does the train travel in one hour?

   $100\frac{1}{2} \div \frac{3}{4} = \frac{201}{2} \cdot \frac{4}{3} = 134$; **The train travels 134 miles in one hour.**

   b. How far will the train travel in t hours, at this same speed?

   The train will travel 134t miles at this speed.

   c. If d represents the distance that the train travels at this speed for t hours, write an equation relating t and d.

   $d = 134t$

4. Triangle Z is a scaled copy of Triangle M. Which sets of values could represent the side lengths of Triangle Z? Select *all* that apply.

   A. 8, 11, 14

   [✓ B.] 10, 17.5, 25

   C. 6, 9, 11

   [✓ D.] 6, 10.5, 15

   [✓ E.] 8, 14, 20

5. Lin has already read 36 pages of the 200 pages of her book in 3 days. Her teacher has asked her when she will be able to return the book to the class library. Which is better for Lin to know:

   How many pages does she read per day?

   How many days does it take her to read per page?

   Explain your thinking.

   Answers may vary. Sample responses:

   - **I think it is better for Lin to know how many days per page because then she can multiply the number of pages left by this rate to get the number of days she has left.**
   - **I think it is better for Lin to know how many pages she reads per day. Then she can divide 164 (200 – 36 pages) by that rate to find how many days she has until she finishes.**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 5	2
Formative	5	Unit 2 Lesson 7	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 7**

Two Equations for Each Relationship

Let's investigate the equations that represent proportional relationships.

Focus

Goals

1. **Language Goal:** Use the word *reciprocal* to explain that there are two related constants of proportionality for a proportional relationship. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Write two equations that represent the same proportional relationship, i.e., $y = kx$ and $x = \frac{1}{k}y$, and explain what each equation represents. **(Speaking and Listening)**

Coherence

Today

Students write equations for the two ways a proportional relationship can be considered. They organize data in tables, write and solve equations to determine the constant of proportionality, and generalize from repeated calculations to arrive at an equation. After students write or use an equation, they interpret their answers in the context of the situation.

Previously

In Lesson 3, students saw that a proportional relationship can be viewed in two ways, depending on which quantity is regarded as being proportional to the other.

Coming Soon

In Lesson 8, students will use equations to solve problems. In Lesson 14, students see how both equations for a proportional relationship can be represented on a graph.

Rigor

- Students build **conceptual understanding** of the relationship between a constant of proportionality and its reciprocal.
- Students develop the **procedural skill** of determining the unit rate.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per group
- calculators

Math Language Development

Review words

- *constant of proportionality*
- *proportional relationship*
- *reciprocal*
- *unit rate*

Amps | Featured Activity

Activity 2
Using Work From Previous Slides

Equations students enter to represent proportional relationships are shown to them on a later slide to assist with their calculations.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Stress levels might rise as students are asked to represent the same problem with more than one equation. Assure students that they have the tools needed to do this, and ask them what strategies they have used in the past to settle themselves down. Suggest that they might take a few deep breaths or discuss the situation with a friend. They might have a new strategy that could help others, too, so have students share their calming methods.

Modifications to Pacing

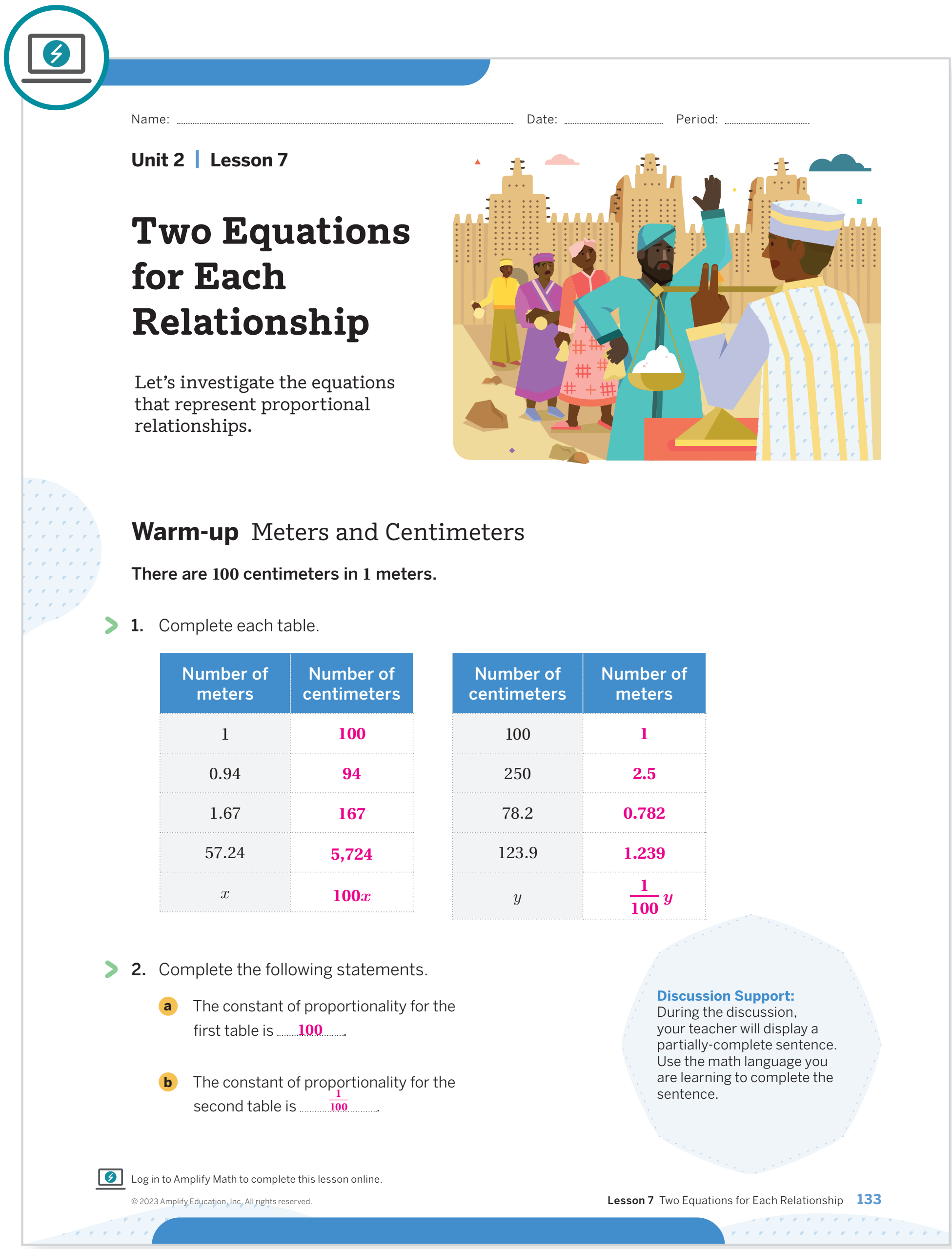
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, after students complete Problem 1, move directly to a discussion of the two equations that can be written to represent the relationship between the salt and gold.

Pairs | 5 min

Warm-up Meters and Centimeters

Students write two equations for a proportional relationship to notice that the constants of proportionality are reciprocals.



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Unit 2 | Lesson 7

Two Equations for Each Relationship

Let's investigate the equations that represent proportional relationships.

Warm-up Meters and Centimeters

There are 100 centimeters in 1 meters.

1. Complete each table.

Number of meters	Number of centimeters
1	**100**
0.94	**94**
1.67	**167**
57.24	**5,724**
x	**$100x$**

Number of centimeters	Number of meters
100	**1**
250	**2.5**
78.2	**0.782**
123.9	**1.239**
y	**$\frac{1}{100}y$**

2. Complete the following statements.
   - a. The constant of proportionality for the first table is **100**.
   - b. The constant of proportionality for the second table is **$\frac{1}{100}$**.

Discussion Support: During the discussion, your teacher will display a partially-complete sentence. Use the math language you are learning to complete the sentence.

Log in to Amplify Math to complete this lesson online.

Lesson 7 Two Equations for Each Relationship 133

1 Launch

Provide access to calculators. Call students' attention to the subtle difference between the headers in both tables.

2 Monitor

Help students get started by prompting them to use the information in the problem to complete the first row of each table.

Look for points of confusion:

- **Thinking that the constants of proportionality are the same.** Prompt students to annotate each table by adding an arrow from the left column to the right, and then labeling, with the constant of proportionality.

Look for productive strategies:

- Annotating the tables to show the multiplicative relationship from the left column to the right.

3 Connect

Display the completed tables.

Ask:

- "What is the relationship between the constants of proportionality?"
- "What equation could you write for the table on the left? How about for the table on the right?"

Highlight that in each table the same relationship is being considered in different ways. The first table helps students understand the number of centimeters per meter. The second table helps students understand the number of meters per centimeter. The constants of proportionality are reciprocals of each other.

Math Language Development

MLR8: Discussion Supports

Provide students with sentence frames, such as, "I found the constant of proportionality by _____," to help explain their thinking. Share student ideas to model mathematical language by restating a response as a question in order to clarify, and apply appropriate language. After clarifying any language, add terms or phrases to the class display/anchor chart.

English Learners

Reinforce the term *reciprocal* by having students circle 100 and $\frac{1}{100}$ in their responses to Problem 2, writing the term *reciprocal* in their Student Edition and drawing arrows to these values.

Power-up

To power up students' ability to compare unit rates for the same relationship, have students complete:

Mai writes two sentences to describe how quickly she wrote her essay.

I wrote 2 pages in 1 hour. In $\frac{1}{2}$ hour, I wrote 1 page of the essay.

1. Which statement is more useful to determine how many pages Mai wrote in 2 hours? The first statement.
2. Which statement is more useful to determine the amount of time it would take Mai to write 5 pages? The second statement.

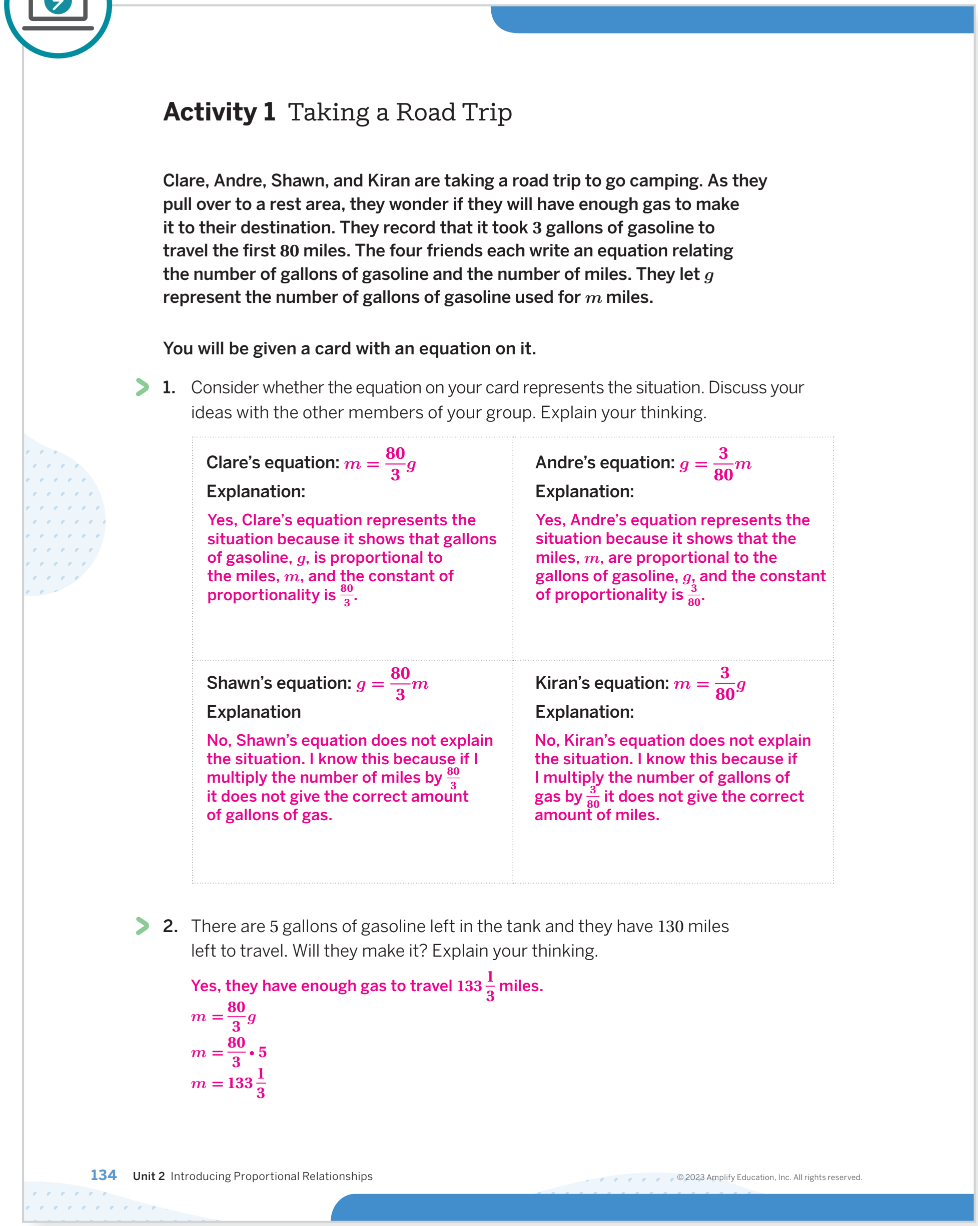
Use: Before Activity 1.

Informed by: Performance on Lesson 6, Practice Problem 5.

Small Groups | 15 min

Activity 1 Taking a Road Trip

Students discuss equations related to a story to reason that two different equations can represent the same proportional relationship.



Activity 1 Taking a Road Trip

Clare, Andre, Shawn, and Kiran are taking a road trip to go camping. As they pull over to a rest area, they wonder if they will have enough gas to make it to their destination. They record that it took 3 gallons of gasoline to travel the first 80 miles. The four friends each write an equation relating the number of gallons of gasoline and the number of miles. They let g represent the number of gallons of gasoline used for m miles.

You will be given a card with an equation on it.

1. Consider whether the equation on your card represents the situation. Discuss your ideas with the other members of your group. Explain your thinking.

Clare's equation: $m = \frac{80}{3}g$ Explanation: Yes, Clare's equation represents the situation because it shows that gallons of gasoline, g, is proportional to the miles, m, and the constant of proportionality is $\frac{80}{3}$.	Andre's equation: $g = \frac{3}{80}m$ Explanation: Yes, Andre's equation represents the situation because it shows that the miles, m, are proportional to the gallons of gasoline, g, and the constant of proportionality is $\frac{3}{80}$.
Shawn's equation: $g = \frac{80}{3}m$ Explanation No, Shawn's equation does not explain the situation. I know this because if I multiply the number of miles by $\frac{80}{3}$ it does not give the correct amount of gallons of gas.	Kiran's equation: $m = \frac{3}{80}g$ Explanation: No, Kiran's equation does not explain the situation. I know this because if I multiply the number of gallons of gas by $\frac{3}{80}$ it does not give the correct amount of miles.

2. There are 5 gallons of gasoline left in the tank and they have 130 miles left to travel. Will they make it? Explain your thinking.

Yes, they have enough gas to travel $133\frac{1}{3}$ miles.

$m = \frac{80}{3}g$

$m = \frac{80}{3} \cdot 5$

$m = 133\frac{1}{3}$

134 **Unit 2** Introducing Proportional Relationships

1 Launch

Activate students' background knowledge by asking whether they have ever taken a road trip where they have to stop for gas. Read the introduction with students, organize students in groups of four, and distribute the pre-cut cards from the Activity 1 PDF.

2 Monitor

Help students get started by asking, "What must you know about a proportional relationship to write an equation for it?"

Look for points of confusion:

- **Seeing the correct constant of proportionality in the equation without considering the variables.** Ask students, "Are the variables in your equation in the correct places? How do you know?"

Look for productive strategies:

- Discussing with their group how two equations with the same constant of proportionality, but different variables, cannot both be correct.
- Testing equations with the values from the problem.

3 Connect

Display all the equations.

Have groups of students share which equations explained the situation, which did not, and how they decided.

Ask, "How can knowing the equation for the proportional relationship in one way help you to determine it for the other way?"

Highlight that once the equation for the relationship is determined in one way, a new equation can be created by taking the reciprocal of the constant of proportionality and switching the variables. Because the relationship of the two values had switched, the variables in the equation must also switch.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For students who need more processing time, provide Clare's equation and Shawn's equation first. Have them analyze each equation and compare them, before giving them Andre's and Kiran's equations.

Extension: *Math Enrichment*

Have students complete the following problem:
If gasoline costs $2.79 per gallon, what is the cost of gasoline for the trip to go camping? Show or explain your thinking. About $21.97, $80 + 130 = 210$ total miles, $210 \div \frac{80}{3} = 7.875$, $7.875 \cdot \$2.79 \approx \$21.97$

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect equation, such as Kiran's equation. Provide a sample statement that Kiran might have used to support his equation, such as "It took 3 gallons to travel 80 miles, so the constant of proportionality is $\frac{3}{80}$." Ask students to critique Kiran's reasoning, correct his equation, and justify why their equation is correct.

English Learners

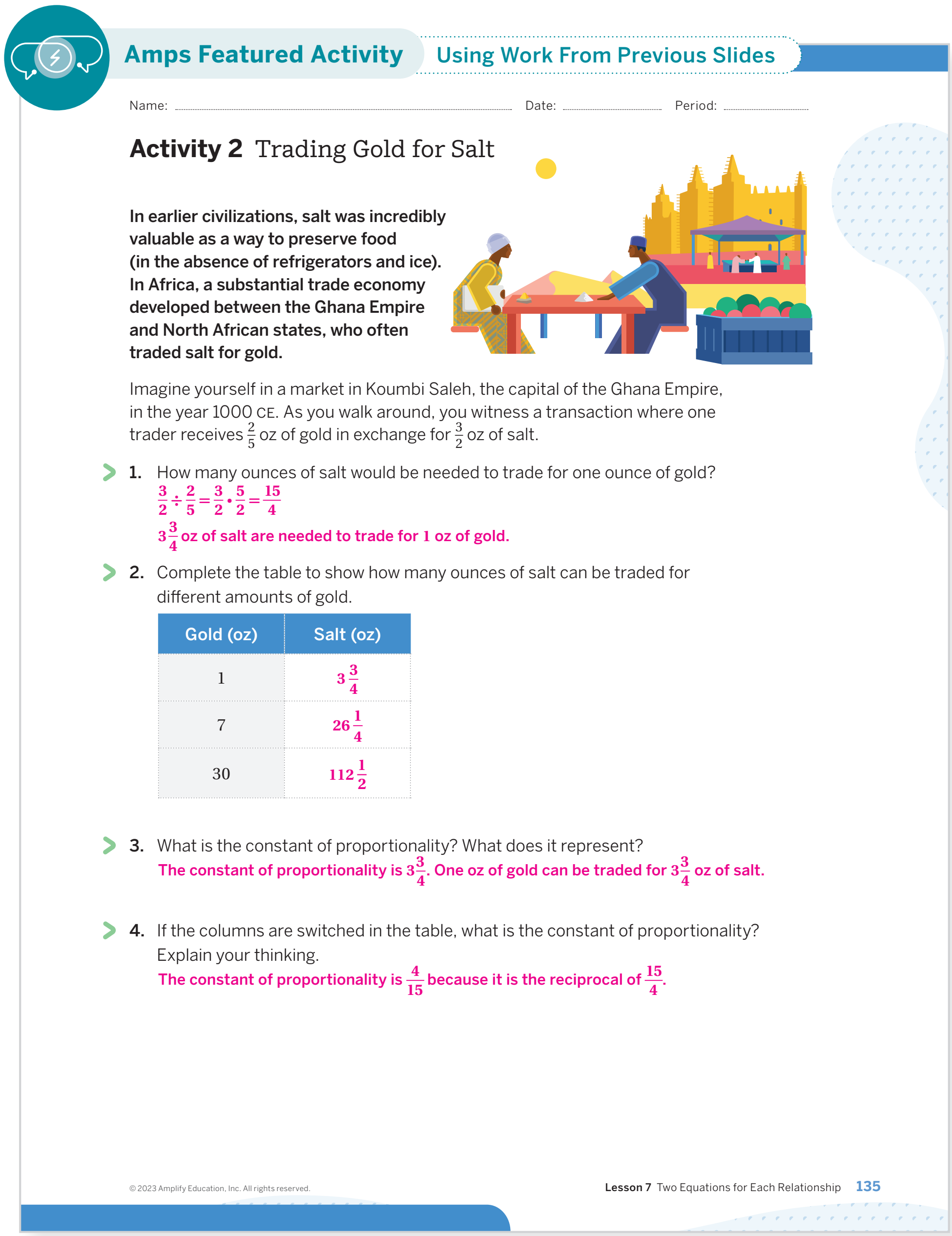
Provide students time to consult with a partner to critique Kiran's reasoning before sharing their responses.

Pairs | 15 min

Activity 2 Trading Gold for Salt

Students have an additional opportunity to represent a proportional relationship with two related equations to explore more complicated calculations.

Amps Featured Activity Using Work From Previous Slides



Name: ______________________ Date: __________ Period: __________

Activity 2 Trading Gold for Salt

In earlier civilizations, salt was incredibly valuable as a way to preserve food (in the absence of refrigerators and ice). In Africa, a substantial trade economy developed between the Ghana Empire and North African states, who often traded salt for gold.

Imagine yourself in a market in Koumbi Saleh, the capital of the Ghana Empire, in the year 1000 CE. As you walk around, you witness a transaction where one trader receives $\frac{2}{5}$ oz of gold in exchange for $\frac{3}{2}$ oz of salt.

1. How many ounces of salt would be needed to trade for one ounce of gold?
   $\frac{3}{2} \div \frac{2}{5} = \frac{3}{2} \bullet \frac{5}{2} = \frac{15}{4}$
   $3\frac{3}{4}$ oz of salt are needed to trade for 1 oz of gold.
2. Complete the table to show how many ounces of salt can be traded for different amounts of gold.

Gold (oz)	Salt (oz)
1	$3\frac{3}{4}$
7	$26\frac{1}{4}$
30	$112\frac{1}{2}$

3. What is the constant of proportionality? What does it represent?
   The constant of proportionality is $3\frac{3}{4}$. One oz of gold can be traded for $3\frac{3}{4}$ oz of salt.
4. If the columns are switched in the table, what is the constant of proportionality? Explain your thinking.
   The constant of proportionality is $\frac{4}{15}$ because it is the reciprocal of $\frac{15}{4}$.

1 Launch

Activate students' background knowledge by asking if they are aware of the historical civilizations that existed in Africa, and briefly discuss the value of the resources. Provide access to calculators.

2 Monitor

Help students get started by suggesting students add a row to the table in Problem 2 to represent the information in the introduction.

Look for points of confusion:

- **Writing the reciprocal of $3\frac{3}{4}$ as $3\frac{4}{3}$.** Prompt students to test their constant of proportionality by multiplying values from the salt column to see whether they yield the values in the gold column.
- **In Problem 5, representing the second equation by using division.** Let students know that while their equation may be true, it should follow the format of $y = kx$.

Look for productive strategies:

- Using the different equations to solve Problem 6.

Activity 2 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For students who need more processing time, consider having them focus on completing Problems 1–5, and only work on Problem 6 as they have time available. In Problem 4, consider displaying partially-completed equations, such as $s =$ _____g and $g =$ _____s for students to use.

Math Language Development

MLR5: Co-craft Questions

During the Launch, highlight this part of the sentence from the introduction: ". . . you witness a transaction where one trader receives $\frac{2}{5}$ oz of gold in exchange for $\frac{3}{2}$ oz of salt," and ask students to write down possible mathematical questions that they could ask about the situation.

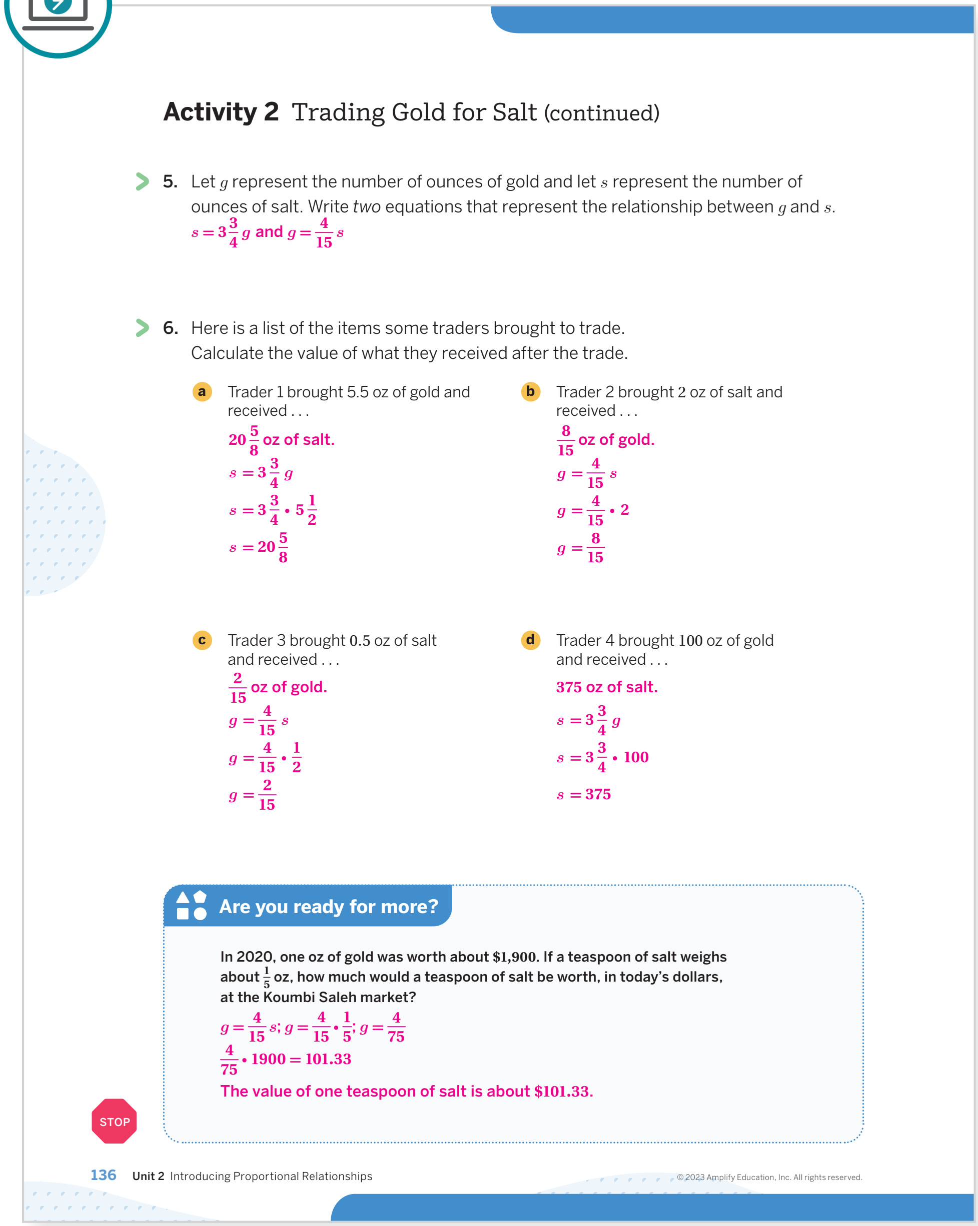
English Learners

Annotate this phrase by displaying $\frac{2}{5}$ oz gold $= \frac{3}{2}$ oz salt to help students make connections between the text and the relationship.

Pairs | 15 min

Activity 2 Trading Gold for Salt (continued)

Students have an additional opportunity to represent a proportional relationship with two related equations to explore more complicated calculations.



Activity 2 Trading Gold for Salt (continued)

5. Let g represent the number of ounces of gold and let s represent the number of ounces of salt. Write *two* equations that represent the relationship between g and s.
   $s = 3\frac{3}{4}g$ and $g = \frac{4}{15}s$

6. Here is a list of the items some traders brought to trade.
   Calculate the value of what they received after the trade.

   a. Trader 1 brought 5.5 oz of gold and received . . .
   $20\frac{5}{8}$ oz of salt.
   $s = 3\frac{3}{4}g$
   $s = 3\frac{3}{4} \cdot 5\frac{1}{2}$
   $s = 20\frac{5}{8}$

   b. Trader 2 brought 2 oz of salt and received . . .
   $\frac{8}{15}$ oz of gold.
   $g = \frac{4}{15}s$
   $g = \frac{4}{15} \cdot 2$
   $g = \frac{8}{15}$

   c. Trader 3 brought 0.5 oz of salt and received . . .
   $\frac{2}{15}$ oz of gold.
   $g = \frac{4}{15}s$
   $g = \frac{4}{15} \cdot \frac{1}{2}$
   $g = \frac{2}{15}$

   d. Trader 4 brought 100 oz of gold and received . . .
   375 oz of salt.
   $s = 3\frac{3}{4}g$
   $s = 3\frac{3}{4} \cdot 100$
   $s = 375$

Are you ready for more?

In 2020, one oz of gold was worth about \$1,900. If a teaspoon of salt weighs about $\frac{1}{5}$ oz, how much would a teaspoon of salt be worth, in today's dollars, at the Koumbi Saleh market?

$g = \frac{4}{15}s$; $g = \frac{4}{15} \cdot \frac{1}{5}$; $g = \frac{4}{75}$

$\frac{4}{75} \cdot 1900 = 101.33$

The value of one teaspoon of salt is about \$101.33.

STOP

136 Unit 2 Introducing Proportional Relationships

3 Connect

Ask, "When you are given a numerical comparison, such as the one at the start of this activity, what are some of the steps you should take on the way to determining both equations for the relationship?"

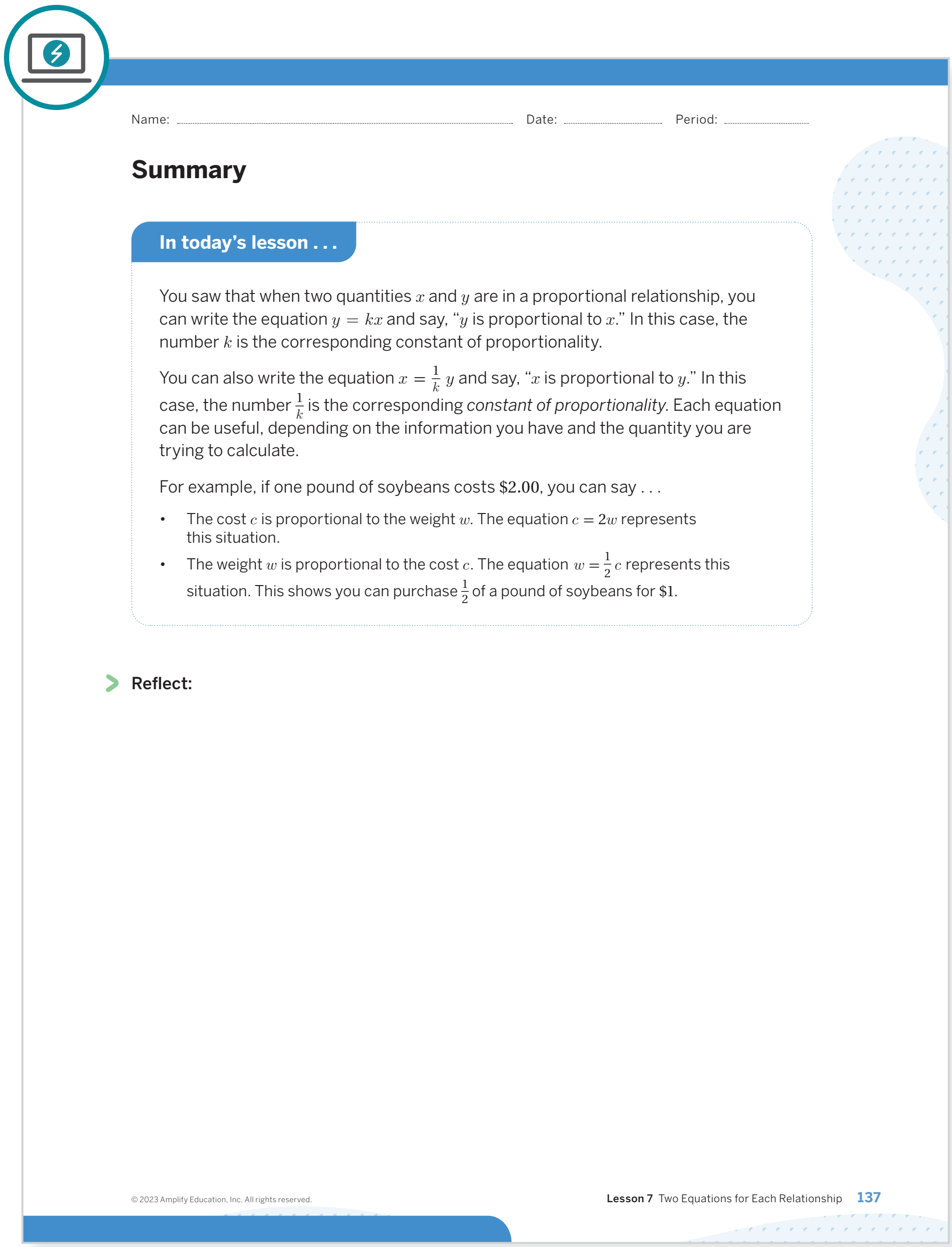
Have pairs of students share their ideas about the steps taken from reading about a proportional relationship to writing both equations for it.

Highlight how knowing both equations for a proportional relationship helps students solve problems where the given unit changes.

Whole Class | 5 min

Summary

Review and synthesize how every proportional relationship can be represented with two equations to understand the reciprocity in the constant of proportionality.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that when two quantities x and y are in a proportional relationship, you can write the equation $y = kx$ and say, "y is proportional to x." In this case, the number k is the corresponding constant of proportionality.

You can also write the equation $x = \frac{1}{k}y$ and say, "x is proportional to y." In this case, the number $\frac{1}{k}$ is the corresponding *constant of proportionality*. Each equation can be useful, depending on the information you have and the quantity you are trying to calculate.

For example, if one pound of soybeans costs $\$2.00$, you can say . . .

- The cost c is proportional to the weight w. The equation $c = 2w$ represents this situation.
- The weight w is proportional to the cost c. The equation $w = \frac{1}{2}c$ represents this situation. This shows you can purchase $\frac{1}{2}$ of a pound of soybeans for $\$1$.

Reflect:

Lesson 7 Two Equations for Each Relationship 137

Synthesize

Ask:

- "Why are you able to write two equations for any proportional relationship?"
- "How are the constants of proportionality related in the two equations?"

Highlight that the equation written for a proportional relationship should be determined by the unknown value. Sometimes, it is useful to write both equations to understand the relationship and how the given information can help select which equation is most useful.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

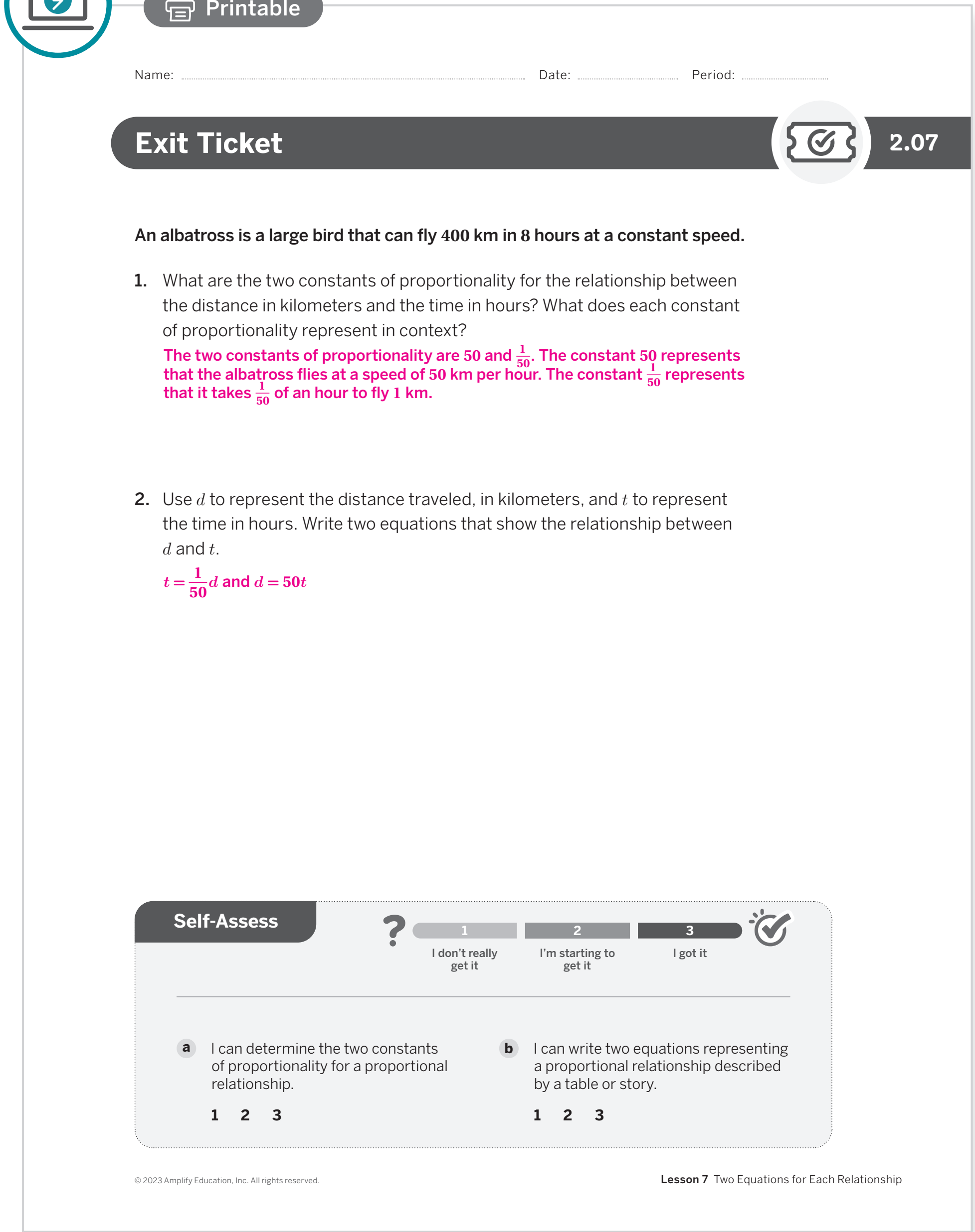
- "What is always true about the two equations that represent a proportional relationship?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of writing equations for a proportional relationship by analyzing the speed of an albatross.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.07

An albatross is a large bird that can fly 400 km in 8 hours at a constant speed.

1. What are the two constants of proportionality for the relationship between the distance in kilometers and the time in hours? What does each constant of proportionality represent in context?
   The two constants of proportionality are 50 and $\frac{1}{50}$. The constant 50 represents that the albatross flies at a speed of 50 km per hour. The constant $\frac{1}{50}$ represents that it takes $\frac{1}{50}$ of an hour to fly 1 km.

2. Use d to represent the distance traveled, in kilometers, and t to represent the time in hours. Write two equations that show the relationship between d and t.
   $t = \frac{1}{50}d$ **and** $d = 50t$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine the two constants of proportionality for a proportional relationship.
1 2 3

b I can write two equations representing a proportional relationship described by a table or story.
1 2 3

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Lesson 7 Two Equations for Each Relationship

Success looks like . . .

- **Language Goal:** Using the word *reciprocal* to explain that there are two related constants of proportionality for a proportional relationship. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Writing two equations that represent the same proportional relationship, i.e., $y = kx$ and $x = \frac{1}{k}\,y$, and explaining what each equation represents. **(Speaking and Listening)**
  - » Writing two equations showing the relationship between distance and time in Problem 2.

Suggested next steps

If students cannot determine the constant of proportionality, consider:

- Having them focus on the relationship of kilometers per hour first.

If students reverse the position of the appropriate variables for the equation, consider:

- Having them make a table of values with the variable in the last row.

Professional Learning

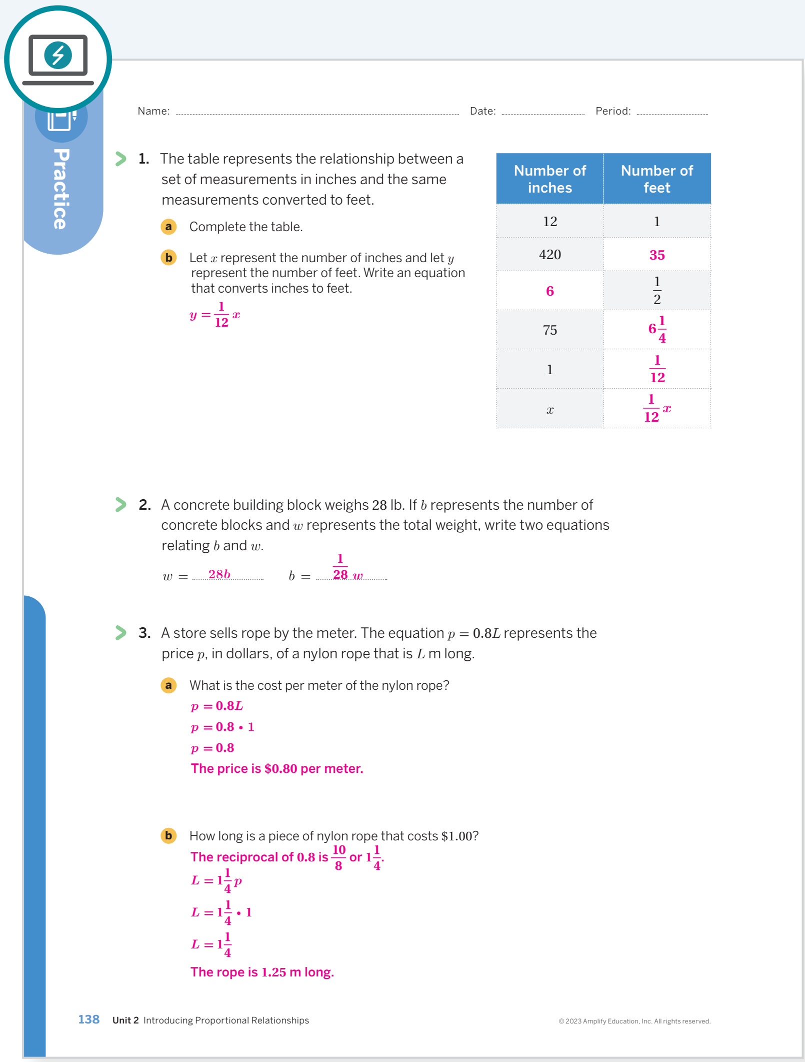
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Who participated and who didn't participate in the group discussion for Activity 1?
- What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. The table represents the relationship between a set of measurements in inches and the same measurements converted to feet.
   - **a** Complete the table.
   - **b** Let x represent the number of inches and let y represent the number of feet. Write an equation that converts inches to feet.

     $y = \frac{1}{12}x$

Number of inches	Number of feet
12	1
420	35
6	$\frac{1}{2}$
75	$6\frac{1}{4}$
1	$\frac{1}{12}$
x	$\frac{1}{12}x$

2. A concrete building block weighs 28 lb. If b represents the number of concrete blocks and w represents the total weight, write two equations relating b and w.

   $w =$ $28b$ $b =$ $\frac{1}{28}w$

3. A store sells rope by the meter. The equation $p = 0.8L$ represents the price p, in dollars, of a nylon rope that is L m long.
   - **a** What is the cost per meter of the nylon rope?

     $p = 0.8L$

     $p = 0.8 \cdot 1$

     $p = 0.8$

     The price is $0.80 per meter.
   - **b** How long is a piece of nylon rope that costs $1.00?

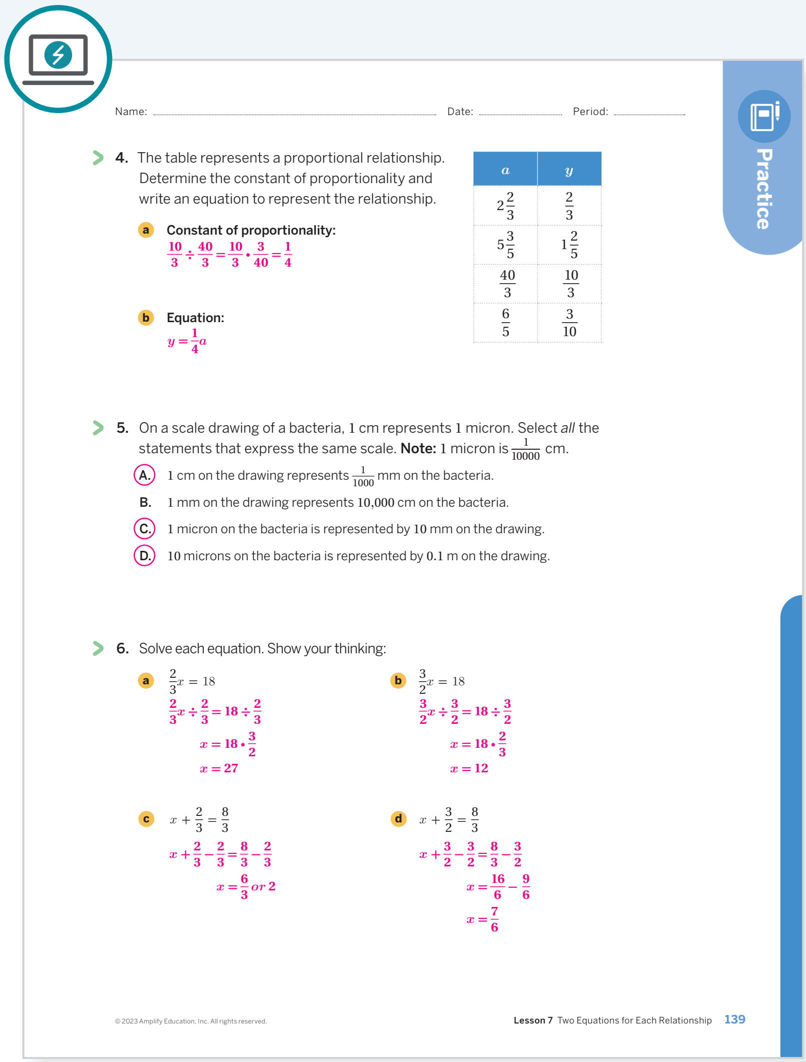
     The reciprocal of 0.8 is $\frac{10}{8}$ or $1\frac{1}{4}$.

     $L = 1\frac{1}{4}p$

     $L = 1\frac{1}{4} \cdot 1$

     $L = 1\frac{1}{4}$

     The rope is 1.25 m long.



Practice

Name: ________ Date: ________ Period: ________

4. The table represents a proportional relationship. Determine the constant of proportionality and write an equation to represent the relationship.
   - **a** **Constant of proportionality:**

     $\frac{10}{3} \div \frac{40}{3} = \frac{10}{3} \cdot \frac{3}{40} = \frac{1}{4}$
   - **b** **Equation:**

     $y = \frac{1}{4}a$

a	y
$2\frac{2}{3}$	$\frac{2}{3}$
$5\frac{3}{5}$	$1\frac{2}{5}$
$\frac{40}{3}$	$\frac{10}{3}$
$\frac{6}{5}$	$\frac{3}{10}$

5. On a scale drawing of a bacteria, 1 cm represents 1 micron. Select *all* the statements that express the same scale. **Note:** 1 micron is $\frac{1}{10000}$ cm.
   - [✓ A.] 1 cm on the drawing represents $\frac{1}{1000}$ mm on the bacteria.
   - B. 1 mm on the drawing represents 10,000 cm on the bacteria.
   - [✓ C.] 1 micron on the bacteria is represented by 10 mm on the drawing.
   - [✓ D.] 10 microns on the bacteria is represented by 0.1 m on the drawing.

6. Solve each equation. Show your thinking:
   - **a** $\frac{2}{3}x = 18$

     $\frac{2}{3}x \div \frac{2}{3} = 18 \div \frac{2}{3}$

     $x = 18 \cdot \frac{3}{2}$

     $x = 27$
   - **b** $\frac{3}{2}x = 18$

     $\frac{3}{2}x \div \frac{3}{2} = 18 \div \frac{3}{2}$

     $x = 18 \cdot \frac{2}{3}$

     $x = 12$
   - **c** $x + \frac{2}{3} = \frac{8}{3}$

     $x + \frac{2}{3} - \frac{2}{3} = \frac{8}{3} - \frac{2}{3}$

     $x = \frac{6}{3} \text{ or } 2$
   - **d** $x + \frac{3}{2} = \frac{8}{3}$

     $x + \frac{3}{2} - \frac{3}{2} = \frac{8}{3} - \frac{3}{2}$

     $x = \frac{16}{6} - \frac{9}{6}$

     $x = \frac{7}{6}$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 5	1
	5	Unit 1 Lesson 12	2
Formative ⚡	6	Unit 2 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Using Equations to Solve Problems

Let's use equations to solve problems involving proportional relationships.

Focus

Goals

1. **Language Goal:** Use an equation to solve problems involving a proportional relationship, and explain the reasoning. **(Speaking and Listening)**

Rigor

- Students build **conceptual understanding** that using an equation to solve a problem is more efficient than other methods.
- Students build **procedural skills** by writing equations from a story about a proportional relationship.

Coherence

Today

Students continue to write equations, especially for situations where using the equation is a more efficient way of solving problems than other methods, such as tables and equivalent ratios. Students use the abstract equation $y = kx$ to reason about quantitative situations.

Previously

In Lessons 6 and 7, students learned to represent proportional relationships with equations of the form $y = kx$.

Coming Soon

In Lesson 9, students will compare proportional and nonproportional relationships, focusing on the connection between the structure of the equation and the kind of relationship it represents.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	12 min	15 min	5 min	8 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (answers, for display)
- calculators

Math Language Development

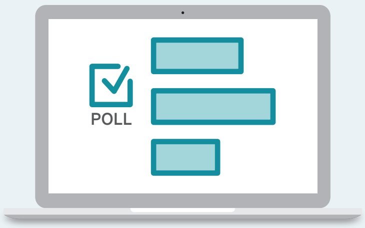
Review words

- *coefficient*
- *constant of proportionality*
- *equivalent ratios*
- *proportional relationship*
- *unit rate*

Amps | **Featured Activity**

Activity 2
Choose Your Vehicle

Leverage student choice as an engagement tool as they select the car they will take on their road trip.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might get confused by the different quantities represented in context. Ask them to read through each activity and pause to consider the units of measurement in context before actually representing them abstractly. Then have them identify ways to break down the relationships so that each part of the problem makes sense to them. Discuss how they will find the self-discipline to pause during an activity in order to decontextualize each situation.

Modifications to Pacing

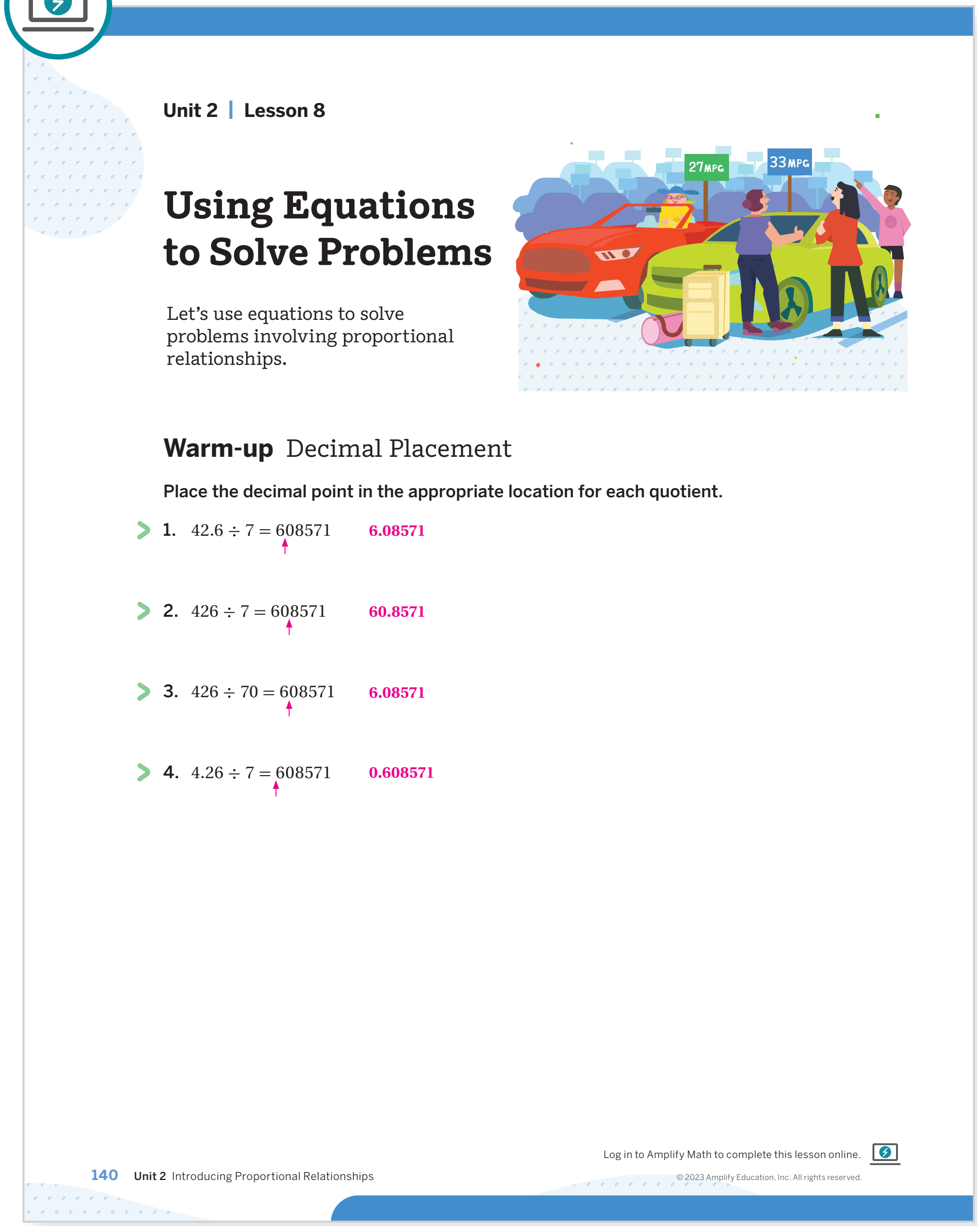
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, Problem 5 may be omitted.

Pairs | 5 min

Warm-up Decimal Placement

Students determine how a quotient changes when the decimal point in the divisor or dividend moves places, seeing the importance of checking for reasonableness in their answers.



Unit 2 | Lesson 8

Using Equations to Solve Problems

Let's use equations to solve problems involving proportional relationships.

Warm-up Decimal Placement

Place the decimal point in the appropriate location for each quotient.

1. $42.6 \div 7 = 608571$ 6.08571
2. $426 \div 7 = 608571$ 60.8571
3. $426 \div 70 = 608571$ 6.08571
4. $4.26 \div 7 = 608571$ 0.608571

Log in to Amplify Math to complete this lesson online.

140 **Unit 2** Introducing Proportional Relationships

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1 Launch

Display the first question. Allow students 1 minute of think time and ask them to give a signal when they are ready with reasoning to support their answer. Repeat for the rest of the problems.

2 Monitor

Help students get started by asking, "How might rounding the dividend to a friendlier number help you?"

Look for points of confusion:

- **Counting the decimal places in the dividend to determine the quotient.** Ask students if their answer makes sense and make a rounding suggestion.

Look for productive strategies:

- Noticing that if the dividend or divisor changes by a power of ten, the quotient will as well.

3 Connect

Have students share their reasoning for placing the decimal point where they did.

Ask:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ___'s strategy?"
- "Do you agree or disagree? Why?"

Highlight that fluency with decimal values and reasoning about the size of numbers is a skill that will continue to be important throughout all the units in Grade 7.

Math Language Development

MLR8: Discussion Supports

Pose the questions from the Connect section before students begin working on the Warm-up. This will help students know what to listen for during the conversation.

English Learners

Consider posting the questions from the Connect section somewhere in your classroom so that students can refer to these during future class discussions.

Power-up

To power up students' ability to solve equations of the form $x + p = q$ and $px = q$, have students complete:

Solve each equation:

a. $10x = 70$

$10x \div 10 = 70 \div 10$

$x = 7$

b. $\frac{1}{10}x = 70$

$\frac{1}{10}x \div \frac{1}{10} = 70 \div \frac{1}{10}$

$x = 70 \cdot 10$

$x = 700$

c. $x + 10 = 70$

$x + 10 - 10 = 70 - 10$

$x = 60$

d. $70 + 10 = x$

$80 = x$

Use: Before Activity 1.

Informed by: Performance on Lesson 7, Practice Problem 6. and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 12 min

Activity 1 Concert Tickets

Students work with a proportional relationship involving large numbers to encourage them to use an equation and to notice the efficiencies of doing so.

Name: ________ Date: ________ Period: ________

Activity 1 Concert Tickets

A performer expects to sell 5,000 tickets for an upcoming concert. They will make a total of $311,000 in sales from these tickets.

1. If all tickets have the same price, what is the price of one ticket?
   $311000 \div 5000 = 62.2$
   Each ticket costs $62.20.

2. How much will the performer make when 7,000 tickets are sold?
   $7000 \cdot 62.20 = 435400$
   The performer will make $435,400.

3. How much will the performer make if they sell . . .
   - a. 10,000 tickets?
     $10000 \cdot 62.20 = 622000$
     $622,000
   - b. 50,000 tickets?
     $50000 \cdot 62.20 = 3110000$
     $3,110,000
   - c. 120,000 tickets?
     $120000 \cdot 62.20 = 7464000$
     $7,464,000
   - d. a million tickets?
     $1000000 \cdot 62.20 = 62200000$
     $62,200,000
   - e. x tickets?
     $62.20x$

4. If the performer makes $404,300, how many tickets were sold? Let x represent the number of tickets sold. Write and solve an equation.
   $404300 = 62.20x$
   $404300 \div 62.20 = 6500$
   They have sold 6,500 tickets.

5. How many tickets will they have to sell to make $5,000,000? Let x represent the number of tickets sold. Write and solve an equation.
   $5000000 = 62.20x$
   $5000000 \div 62.20 \approx 80385.85$
   They would need to sell 80,386 tickets to make $5,000,000.

1 Launch

Activate background knowledge by asking if anyone has been to a concert before. Ask students to roughly estimate how much a ticket might cost: "Is it less than $10, between $10 and $100, or between $100 and $1,000?" Provide access to calculators.

2 Monitor

Help students get started by asking what operation they should do to calculate the price of one ticket.

Look for points of confusion:

- **Rounding the number of tickets in Problem 5 down to 80,385.** Have students test their value in the equation to check if the amount is greater than $5,000,000.

3 Connect

Have students share their responses. Sequence their explanations from less efficient and less organized to more efficient and more organized. Discuss how the solutions are the same and how they are different, and the advantages and disadvantages of each method. An important part of this discussion is making connections between different approaches.

Highlight the general equation $y = kx$, where k is the price per ticket. In the two equations, $404,300 = 62.20x$ and $5,000,000 = 62.20x$, the price 62.20 is known as the *coefficient* of x. In Grade 6, students learned that the coefficient is a number that is multiplied by a variable.

Ask, "How does writing an equation when solving a problem make solving your problem more efficient?"

Differentiated Support

Accessibility: *Optimize Access to Tools*

Allow students to create a table to help them organize their thinking. Consider providing blank tables for them to use. Guide them towards the use of an equation near the end of the activity.

Extension: *Math Enrichment*

After students have completed Problems 4 and 5, ask them to write an equation in the form $x =$ _____ y, where y represents the amount the performer makes. Ask them what this equation represents.

$x = \frac{1}{62.20}y$ or $x = \frac{y}{62.20}$; This equation represents the number of tickets x sold, if you are given the amount y the performer makes.

Math Language Development

MLR8: Discussion Supports—Restate It!

For each explanation that is shared during the Connect, ask students to restate what they heard using precise mathematical language, such as *coefficient* or *constant of proportionality*. Consider providing students time to restate what they heard to a partner before selecting one or two students to share with the class.

English Learners

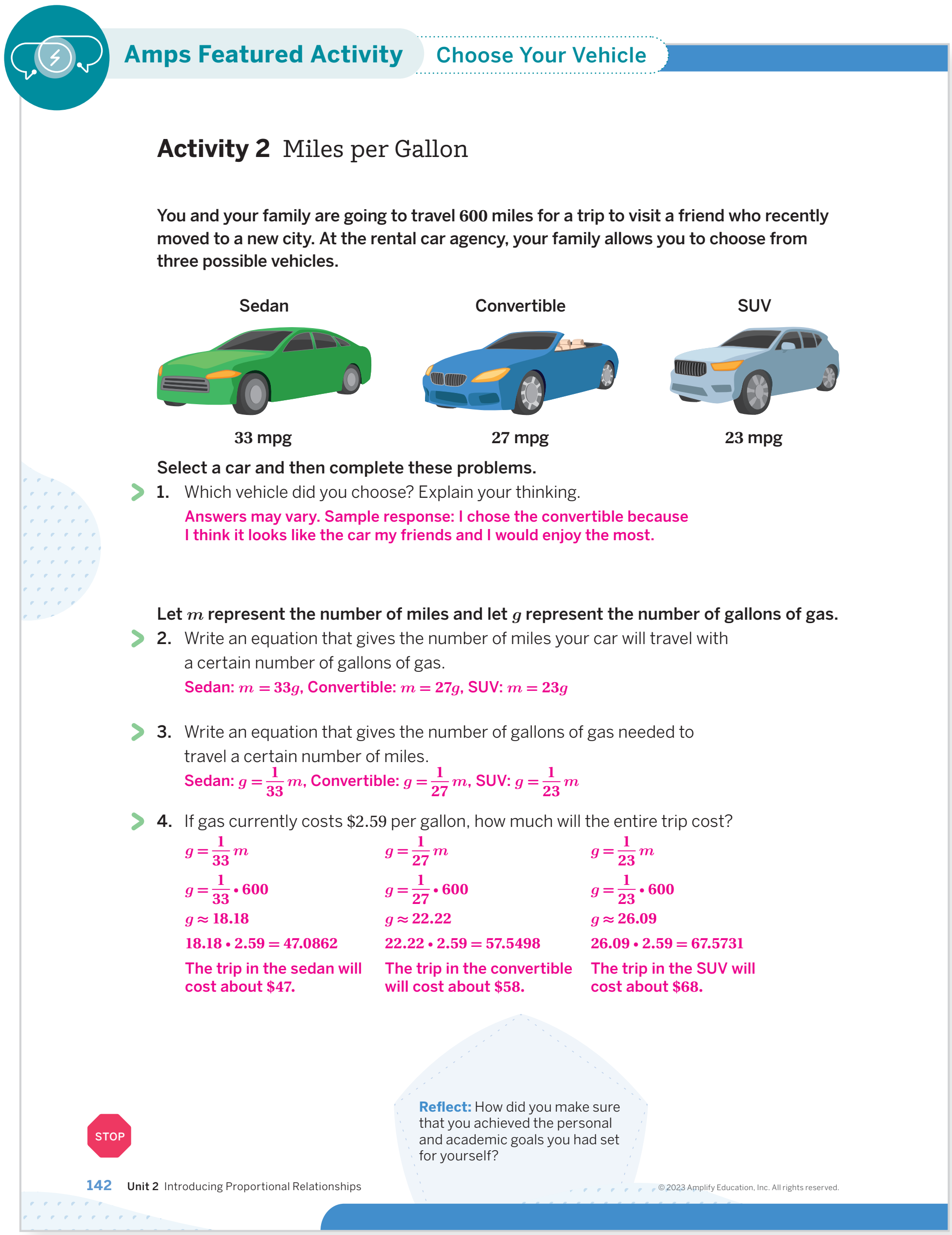
Encourage students to refer to the class display to assist them in using the appropriate mathematical language during the discussion.

Pairs | 15 min

Activity 2 Miles per Gallon

Students write equations for the relationship between miles and gallons of gas to help solve a problem relating to the cost of gas.

Amps Featured Activity Choose Your Vehicle



Activity 2 Miles per Gallon

You and your family are going to travel 600 miles for a trip to visit a friend who recently moved to a new city. At the rental car agency, your family allows you to choose from three possible vehicles.

Sedan	Convertible	SUV
33 mpg	27 mpg	23 mpg

Select a car and then complete these problems.

1. Which vehicle did you choose? Explain your thinking.
   Answers may vary. Sample response: I chose the convertible because I think it looks like the car my friends and I would enjoy the most.

Let m represent the number of miles and let g represent the number of gallons of gas.

2. Write an equation that gives the number of miles your car will travel with a certain number of gallons of gas.
   Sedan: $m = 33g$, Convertible: $m = 27g$, SUV: $m = 23g$
3. Write an equation that gives the number of gallons of gas needed to travel a certain number of miles.
   Sedan: $g = \frac{1}{33}m$, Convertible: $g = \frac{1}{27}m$, SUV: $g = \frac{1}{23}m$
4. If gas currently costs $2.59 per gallon, how much will the entire trip cost?

$g = \frac{1}{33}m$	$g = \frac{1}{27}m$	$g = \frac{1}{23}m$
$g = \frac{1}{33} \cdot 600$	$g = \frac{1}{27} \cdot 600$	$g = \frac{1}{23} \cdot 600$
$g \approx 18.18$	$g \approx 22.22$	$g \approx 26.09$
$18.18 \cdot 2.59 = 47.0862$	$22.22 \cdot 2.59 = 57.5498$	$26.09 \cdot 2.59 = 67.5731$
The trip in the sedan will cost about $47.	The trip in the convertible will cost about $58.	The trip in the SUV will cost about $68.

STOP

Reflect: How did you make sure that you achieved the personal and academic goals you had set for yourself?

1 Launch

Activate background knowledge by asking if students know which types of cars are more fuel efficient, and why. Remind students that *mpg* is the shorthand often used for "miles per gallon," and *mph* is used for "miles per hour."

2 Monitor

Help students get started by suggesting they create a table of values for the car they have selected.

Look for points of confusion:

- **Calculating the cost of the trip by multiplying the cost per gallon of gas by the number of miles driven.** Ask, "Is the cost of the gasoline given *per mile* or *per gallon*?"

Look for productive strategies:

- Writing and solving an equation that includes the rates for both miles per gallon and cost per gallon.

3 Connect

Have students share their solution methods for Problem 4.

Highlight the connections between the equations and methods students used to solve Problem 4.

Display the table in the Activity 2 PDF (answers).

Ask:

- "Which equation works better to determine the vehicle with the lowest cost of gas?" The equation that shows gallons per mile because the smallest coefficient corresponds to the smallest cost.
- If you were starting the trip over again, would you select a different car? Why or why not?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Represent the same information through different modalities, such as by using a table. For example, provide students with a table similar to the following:

Vehicle	mpg	$m = \_\_\_\_ g$	$g = \_\_\_\_ m$
Sedan	33		
Convertible	27		
SUV	23		

Math Language Development

MLR6: Three Reads

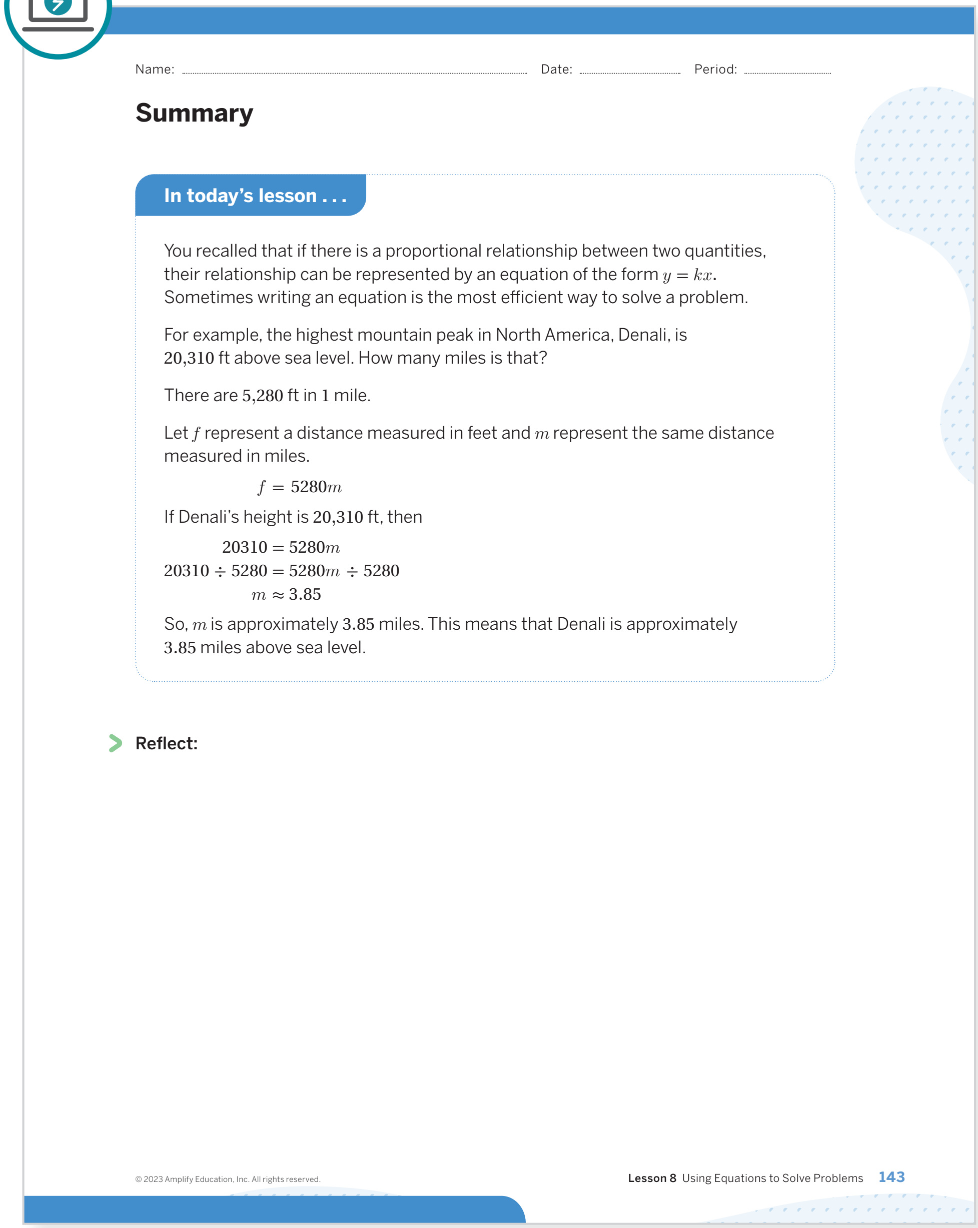
Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they can choose from three possible vehicles with varying gas mileages. Highlight that *mpg* means *miles per gallon* and measures how many miles a vehicle can travel on one gallon of gas.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the total trip is 600 miles.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problem 2.

Whole Class | 5 min

Summary

Review and synthesize how writing an equation can be a more efficient way of solving a problem.



Name: Date: Period:

Summary

In today's lesson . . .

You recalled that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the most efficient way to solve a problem.

For example, the highest mountain peak in North America, Denali, is $20{,}310$ ft above sea level. How many miles is that?

There are $5{,}280$ ft in 1 mile.

Let f represent a distance measured in feet and m represent the same distance measured in miles.

$$f = 5280m$$

If Denali's height is $20{,}310$ ft, then

$$\begin{aligned} 20310 &= 5280m \\ 20310 \div 5280 &= 5280m \div 5280 \\ m &\approx 3.85 \end{aligned}$$

So, m is approximately 3.85 miles. This means that Denali is approximately 3.85 miles above sea level.

Reflect:

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Lesson 8 Using Equations to Solve Problems 143

Synthesize

Have students share what they noticed about the contexts today that led them to solve equations more efficiently.

Highlight that now that students have a stronger understanding of proportional relationships and of the equations that represent them, they can often go directly from a context to the equation.

Ask, "Is every problem solved more efficiently by using an equation? Why or why not?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

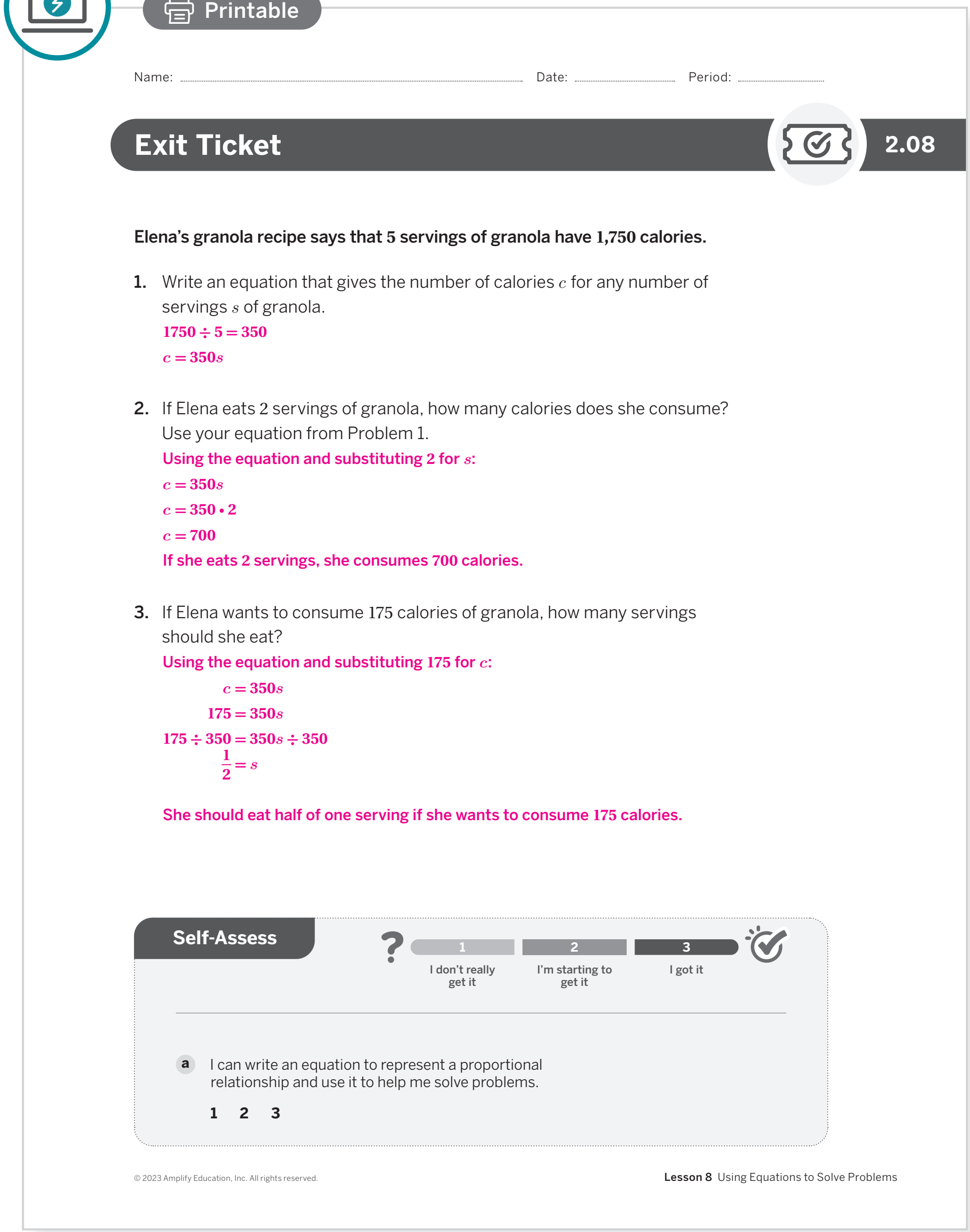
- "What strategies or tools did you find helpful today while using equations to solve problems? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of using equations to solve problems to determine the number of calories in a certain amount of granola.

Printable



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Exit Ticket 2.08

Elena's granola recipe says that 5 servings of granola have 1,750 calories.

1. Write an equation that gives the number of calories c for any number of servings s of granola.

   $1750 \div 5 = 350$

   $c = 350s$

2. If Elena eats 2 servings of granola, how many calories does she consume? Use your equation from Problem 1.

   Using the equation and substituting 2 for s:

   $c = 350s$

   $c = 350 \cdot 2$

   $c = 700$

   If she eats 2 servings, she consumes 700 calories.

3. If Elena wants to consume 175 calories of granola, how many servings should she eat?

   Using the equation and substituting 175 for c:

   $c = 350s$

   $175 = 350s$

   $175 \div 350 = 350s \div 350$

   $\frac{1}{2} = s$

   She should eat half of one serving if she wants to consume 175 calories.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can write an equation to represent a proportional relationship and use it to help me solve problems.

1 2 3

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Lesson 8 Using Equations to Solve Problems

Success looks like . . .

- **Language Goal:** Using an equation to solve problems involving a proportional relationship and explaining the reasoning. **(Speaking and Listening)**
  - » Solving problems using the equation representing a proportional relationship between calories and servings in Problems 2 and 3.

Suggested next steps

If students use a table before writing their equation, consider:

- Having them repeat the process of writing the equations for simpler contexts, until they can generalize how to determine the unit rate without a table.

If students write an equation without first determining the proper unit rate, consider:

- Having them estimate the number of calories in 2 servings of granola.

If students solve Problem 3 without using the equation, consider:

- Making a note about this choice of strategy and providing additional support during Unit 6 when solving equations.

Professional Learning

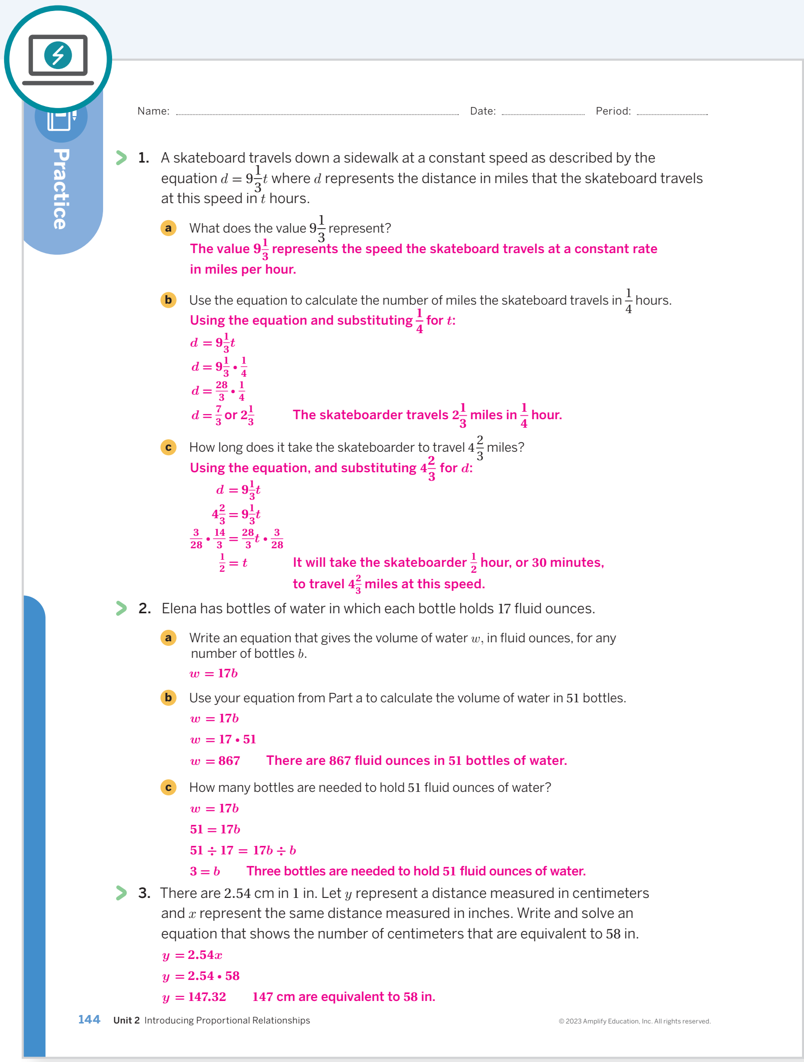
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students wrote equations without creating tables. How did that build on previous work determining the constant of proportionality?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ____________ Date: ________ Period: ________

Practice

1. A skateboard travels down a sidewalk at a constant speed as described by the equation $d = 9\frac{1}{3}t$ where d represents the distance in miles that the skateboard travels at this speed in t hours.

   a What does the value $9\frac{1}{3}$ represent?

   The value $9\frac{1}{3}$ represents the speed the skateboard travels at a constant rate in miles per hour.

   b Use the equation to calculate the number of miles the skateboard travels in $\frac{1}{4}$ hours.

   Using the equation and substituting $\frac{1}{4}$ for t:

   $d = 9\frac{1}{3}t$

   $d = 9\frac{1}{3} \cdot \frac{1}{4}$

   $d = \frac{28}{3} \cdot \frac{1}{4}$

   $d = \frac{7}{3}$ or $2\frac{1}{3}$ **The skateboarder travels $2\frac{1}{3}$ miles in $\frac{1}{4}$ hour.**

   c How long does it take the skateboarder to travel $4\frac{2}{3}$ miles?

   Using the equation, and substituting $4\frac{2}{3}$ for d:

   $d = 9\frac{1}{3}t$

   $4\frac{2}{3} = 9\frac{1}{3}t$

   $\frac{3}{28} \cdot \frac{14}{3} = \frac{28}{3}t \cdot \frac{3}{28}$

   $\frac{1}{2} = t$ **It will take the skateboarder $\frac{1}{2}$ hour, or 30 minutes, to travel $4\frac{2}{3}$ miles at this speed.**

2. Elena has bottles of water in which each bottle holds 17 fluid ounces.

   a Write an equation that gives the volume of water w, in fluid ounces, for any number of bottles b.

   $w = 17b$

   b Use your equation from Part a to calculate the volume of water in 51 bottles.

   $w = 17b$

   $w = 17 \cdot 51$

   $w = 867$ **There are 867 fluid ounces in 51 bottles of water.**

   c How many bottles are needed to hold 51 fluid ounces of water?

   $w = 17b$

   $51 = 17b$

   $51 \div 17 = 17b \div b$

   $3 = b$ **Three bottles are needed to hold 51 fluid ounces of water.**

3. There are 2.54 cm in 1 in. Let y represent a distance measured in centimeters and x represent the same distance measured in inches. Write and solve an equation that shows the number of centimeters that are equivalent to 58 in.

   $y = 2.54x$

   $y = 2.54 \cdot 58$

   $y = 147.32$ **147 cm are equivalent to 58 in.**

144 **Unit 2** Introducing Proportional Relationships

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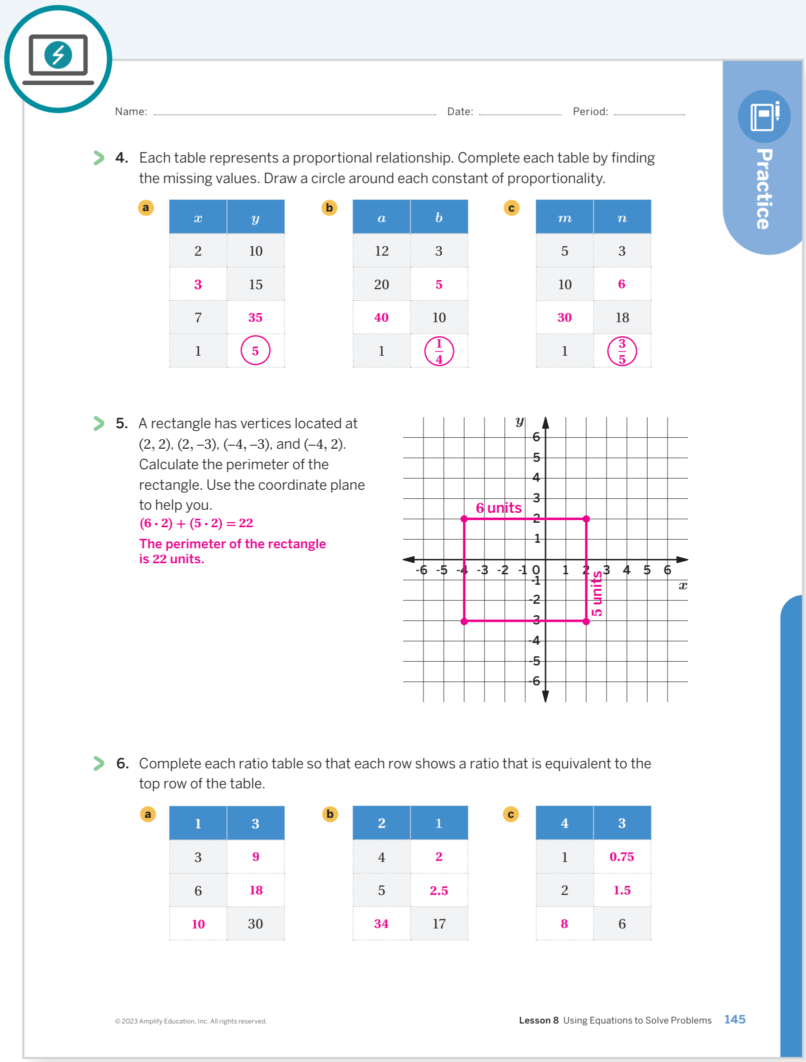

Name: ____________ Date: ________ Period: ________

Practice

4. Each table represents a proportional relationship. Complete each table by finding the missing values. Draw a circle around each constant of proportionality.

a

x	y
2	10
3	15
7	**35**
1	**⑤**

b

a	b
12	3
20	**5**
40	10
1	**$\frac{1}{4}$**

c

m	n
5	3
10	**6**
30	18
1	**$\frac{3}{5}$**

5. A rectangle has vertices located at (2, 2), (2, –3), (–4, –3), and (–4, 2). Calculate the perimeter of the rectangle. Use the coordinate plane to help you.

   $(6 \cdot 2) + (5 \cdot 2) = 22$

   The perimeter of the rectangle is 22 units.

6. Complete each ratio table so that each row shows a ratio that is equivalent to the top row of the table.

a

1	3
3	**9**
6	**18**
10	30

b

2	1
4	**2**
5	**2.5**
34	17

c

4	3
1	**0.75**
2	**1.5**
8	6

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Lesson 8 Using Equations to Solve Problems 145

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 2	1
	5	Grade 6	2
Formative ⚡	6	Unit 2 Lesson 9	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 9**

Comparing Relationships With Equations

Let's develop methods for deciding whether a relationship is proportional.

Focus

Goals

1. **Language Goal:** Compare and contrast equations that do and do not represent proportional relationships. **(Speaking and Listening)**
2. Generalize that an equation equivalent to the form $y = kx$ can represent a proportional relationship.
3. **Language Goal:** Use a table to determine whether a given equation represents a proportional relationship and justify the decision. **(Writing)**

Coherence

Today

Students return to comparing proportional and nonproportional relationships, focusing on the connection between the structure of the equation and the type of relationship it represents. By the end of this lesson, students should understand that an equation is of the form $y = kx$ is proportional, and determine the constant of proportionality by using $k = \frac{y}{x}$.

Previously

In Lessons 1–8, students explored and analyzed proportional relationships represented by tables and equations.

Coming Soon

Students will continue their work with proportional relationships, but will focus on graphical representations in Lessons 10–14.

Rigor

- Students develop **conceptual understanding** that if $\frac{y}{x}$ is the same for all y-values and their corresponding x-values, then the relationship is proportional.
- Students build **procedural fluency** in determining whether relationships are proportional by determining whether the ratio $\frac{y}{x}$ is constant.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (as needed)
- Activity 1 PDF (answers, for display)
- Activity 2 PDF (answers, for display)
- Anchor Chart PDF, *Representing Proportional Relationships*
- calculators
- snap cubes (optional)

Math Language Development

Review words

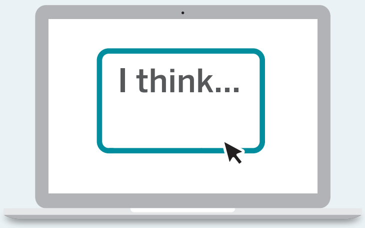
- *coefficient*
- *constant of proportionality*
- *equivalent ratios*
- *nonproportional relationship*
- *proportional relationship*
- *unit rate*

Amps | Featured Activity

Activities 1 and 2

See Student Thinking

Students explain whether relationships are proportional or nonproportional. These explanations are available to you digitally, in real time.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students might work through the activity without seeking to discern a pattern or structure. Challenge them to seek these patterns, and determine why an equation represents a proportional relationship. While the algebraic representation can seem abstract, its structure provides clues as to whether or not the relationship is proportional. Understanding this will help students feel more confident in their conclusions.

Modifications to Pacing

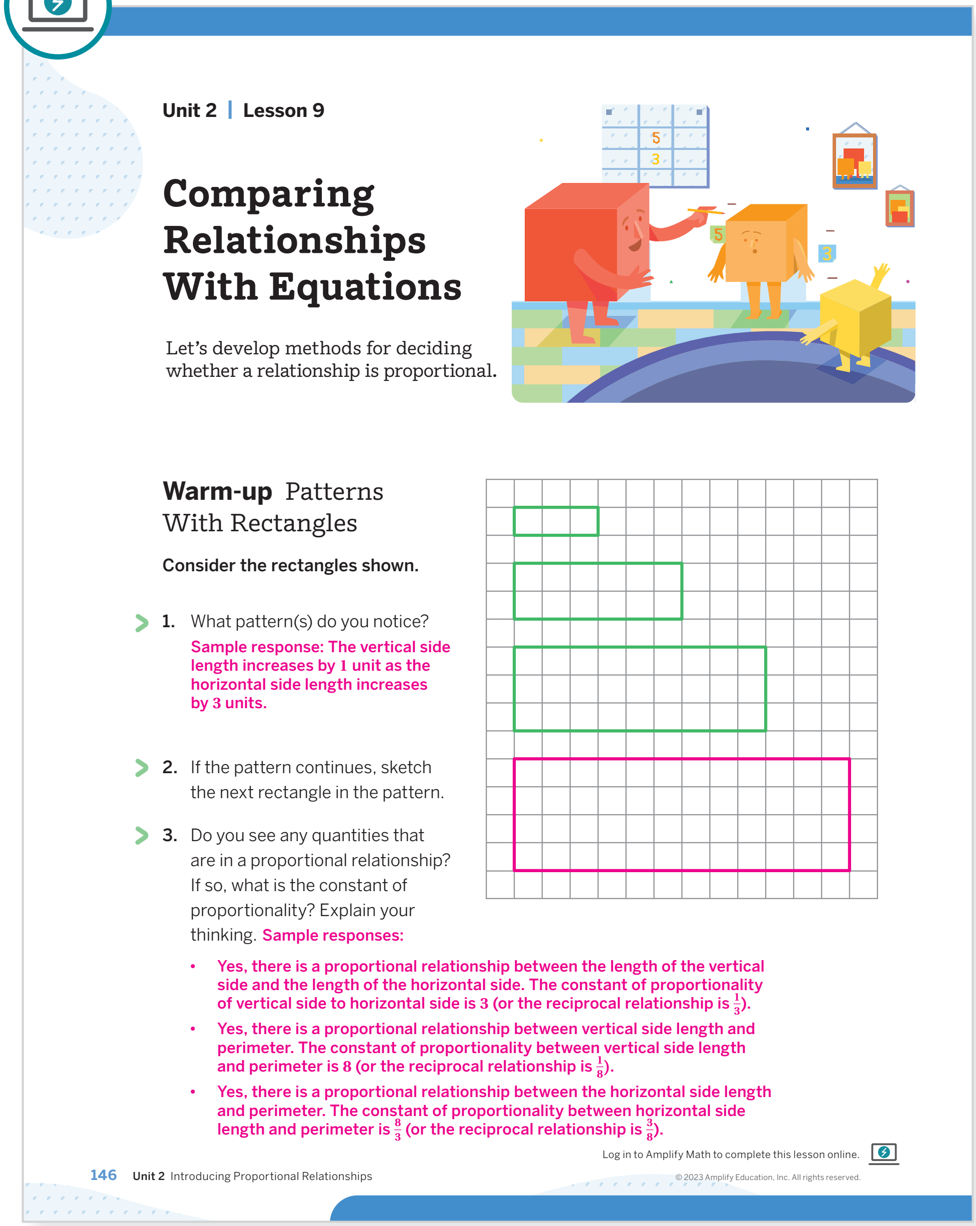
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Have each group member complete Problems 1, 2, or 3 in **Activity 1**, and then share their table values with the other group members.
- In **Activity 2**, have students complete one table, and then share their values with their group members.

Independent | 8 min

Warm-up Patterns With Rectangles

Students look for a pattern among a series of rectangles to determine whether there is a proportional relationship among the measurements.



Unit 2 | Lesson 9

Comparing Relationships With Equations

Let's develop methods for deciding whether a relationship is proportional.

Warm-up Patterns With Rectangles

Consider the rectangles shown.

1. What pattern(s) do you notice?
   Sample response: The vertical side length increases by 1 unit as the horizontal side length increases by 3 units.

2. If the pattern continues, sketch the next rectangle in the pattern.

3. Do you see any quantities that are in a proportional relationship? If so, what is the constant of proportionality? Explain your thinking. **Sample responses:**
   - **Yes, there is a proportional relationship between the length of the vertical side and the length of the horizontal side. The constant of proportionality of vertical side to horizontal side is 3 (or the reciprocal relationship is $\frac{1}{3}$).**
   - **Yes, there is a proportional relationship between vertical side length and perimeter. The constant of proportionality between vertical side length and perimeter is 8 (or the reciprocal relationship is $\frac{1}{8}$).**
   - **Yes, there is a proportional relationship between the horizontal side length and perimeter. The constant of proportionality between horizontal side length and perimeter is $\frac{8}{3}$ (or the reciprocal relationship is $\frac{3}{8}$).**

Log in to Amplify Math to complete this lesson online.

146 Unit 2 Introducing Proportional Relationships

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1 Launch

Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by activating prior knowledge and asking what can be measured on a rectangle. Prompt students until they think of length, width, area, and perimeter.

Look for points of confusion:

- Thinking only of the area. Remind students what is needed to measure the area to get them thinking about the dimensions.

Look for productive strategies:

- Noticing multiple patterns and determining whether there is a proportional relationship.

3 Connect

Display the rectangles.

Have students share their patterns and reasoning about proportionality.

Highlight some of the relationships (e.g., perimeter and area) do not have a proportional relationship. However, the length and the height of a rectangle *do* have a proportional relationship. Likewise, the length and perimeter are in a proportional relationship; however, length and area are *not* proportional.

Note: If time permits, display tables of values for the student-recognized patterns, and reference these examples throughout the other activities as additional examples of proportional or nonproportional relationships.

Ask, "If you compare height to width, what is the constant of proportionality? If you compare width to height, what is the constant of proportionality?"

Math Language Development

MLR2: Collect and Display

During the Connect, as you highlight that the length and height of a rectangle are in a proportional relationship, add this example to the class display/anchor chart as an example of a proportional relationship. Repeat this for the proportional relationship between length and perimeter, and the nonproportional relationships of length and area, and perimeter and area.

Power-up

To power up students' ability to determine unknown values in ratio tables, have student complete:

The work to determine the second row of the ratio table is shown. Use similar reasoning to complete two more rows of the table.

Sample responses shown

Use: Before Activity 1.

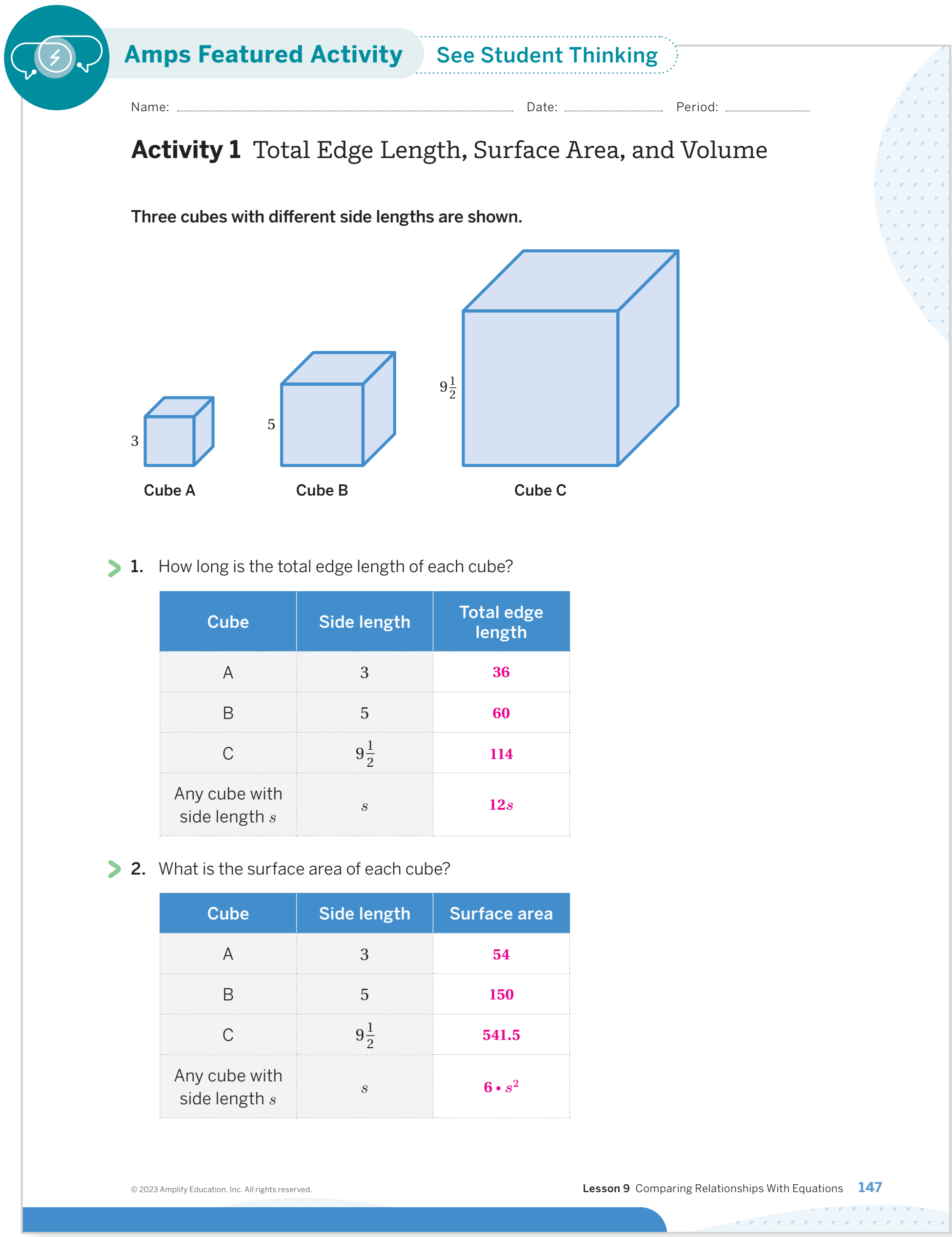
Informed by: Performance on Lesson 8, Practice Problem 6.

2 $\xrightarrow{\times 1.5}$	3
6 $\xrightarrow{\times 1.5}$	9
1	1.5
4	6

Activity 1 Total Edge Length, Surface Area, and Volume

Students compare measurements of cubes and write equations by using repetition in reasoning to determine which quantities have proportional relationships.

Amps Featured Activity See Student Thinking



Name: ______________________ Date: __________ Period: __________

Activity 1 Total Edge Length, Surface Area, and Volume

Three cubes with different side lengths are shown.

3
Cube A

5
Cube B

$9\frac{1}{2}$
Cube C

1. How long is the total edge length of each cube?

Cube	Side length	Total edge length
A	3	36
B	5	60
C	$9\frac{1}{2}$	114
Any cube with side length s	s	$12s$

2. What is the surface area of each cube?

Cube	Side length	Surface area
A	3	54
B	5	150
C	$9\frac{1}{2}$	541.5
Any cube with side length s	s	$6 \cdot s^2$

Lesson 9 Comparing Relationships With Equations 147

1 Launch

Activate prior knowledge by reviewing work from Grade 6 regarding surface area and volume of cubes. Ask questions such as:

- "How many edges does a cube have?"
- "How many faces does a cube have?"

2 Monitor

Help students get started by reminding them of the formulas for calculating surface area and volume of a cube.

Look for points of confusion:

- **Not calculating the measurements correctly for the cube with a side length of $9\frac{1}{2}$ units.** Provide access to calculators or have students change the side length to 10 units.
- **Not knowing how the equation in the last line shows whether the relationships are proportional or nonproportional.** Have students reference the summary from Lesson 5 or their equation work in previous lessons.

Look for productive strategies:

- Using the structure of the formulas (i.e., the formula of surface area takes the square of the side length, so it is nonproportional) to determine whether the relationship is proportional.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools, Activate Background Knowledge*

Provide students with copies of the Activity 1 PDF, which contains nets of the cubes in this activity. Have students cut out the nets, or provide nets already pre-cut. Students can use these nets to help them complete this activity. Consider reviewing how to find the surface area and volume of a cube, which students have learned in Grade 6. Display these formulas for students to reference during the activity.

Extension: *Math Enrichment*

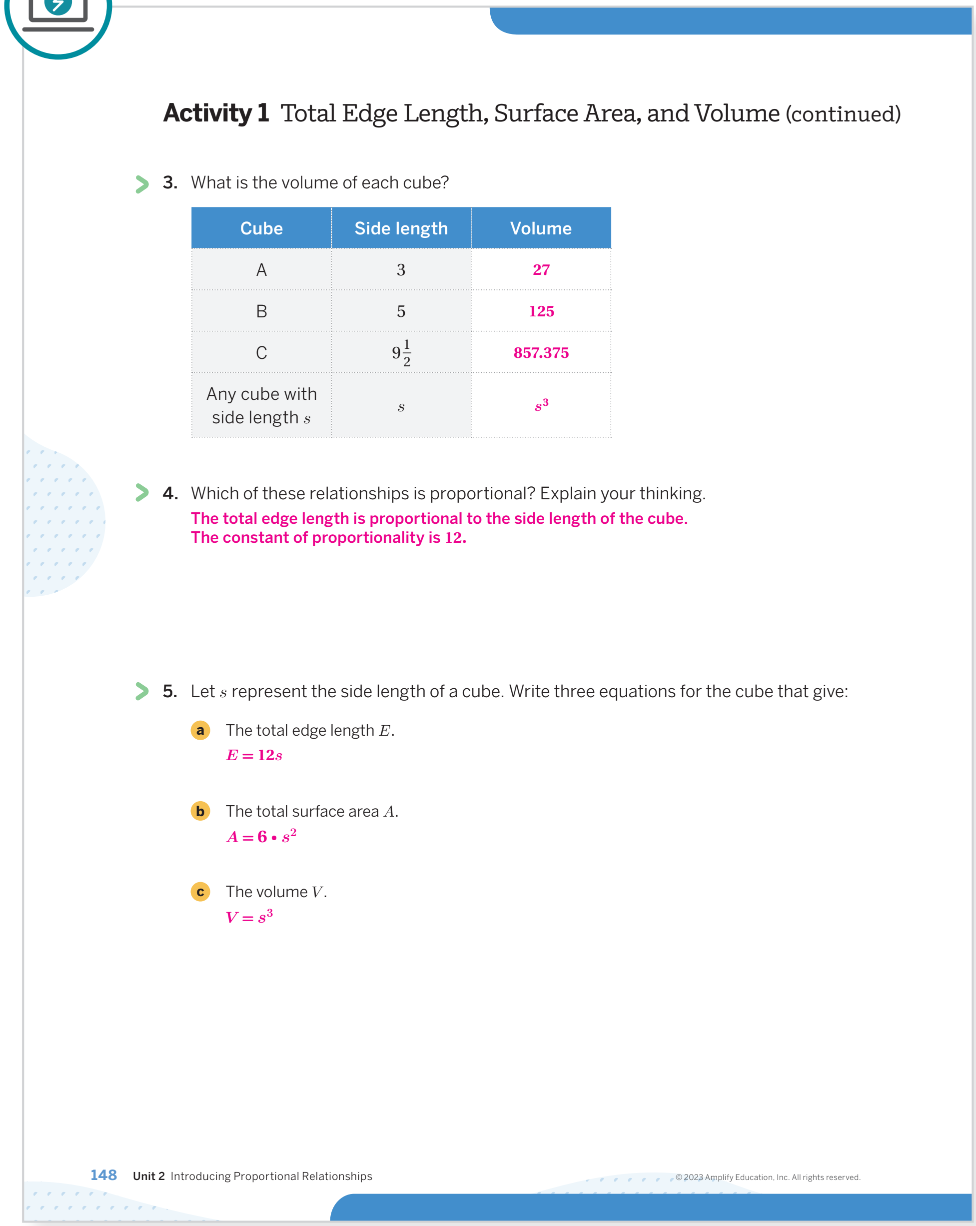
Guide students to understand Euler's polyhedron formula, $F + V - E = 2$, where F = the number of faces, V = the number of vertices, and E = the number of edges. Provide visuals of other polyhedra for students to use. Have them complete a table, similar to the one shown, and look for patterns among the values. Ask them to write an equation that illustrates the relationship between the faces, vertices, and edges.

Polyhedron	Number of faces	Number of vertices	Number of edges

Small Groups | 15 min

Activity 1 Total Edge Length, Surface Area, and Volume (continued)

Students compare measurements of cubes and write equations by using repetition in reasoning to determine which quantities have proportional relationships.



Activity 1 Total Edge Length, Surface Area, and Volume (continued)

3. What is the volume of each cube?

Cube	Side length	Volume
A	3	27
B	5	125
C	$9\frac{1}{2}$	857.375
Any cube with side length s	s	s^3

4. Which of these relationships is proportional? Explain your thinking.

The total edge length is proportional to the side length of the cube. The constant of proportionality is 12.

5. Let s represent the side length of a cube. Write three equations for the cube that give:

a. The total edge length E.

$E = 12s$

b. The total surface area A.

$A = 6 \cdot s^2$

c. The volume V.

$V = s^3$

3 Connect

Display the Activity 1 PDF (answers) and have students share their responses and reasoning.

Ask:

- "What are the possible units for the side lengths? What about surface area? Volume?"
- "How can a table be used to determine whether an equation is proportional?"

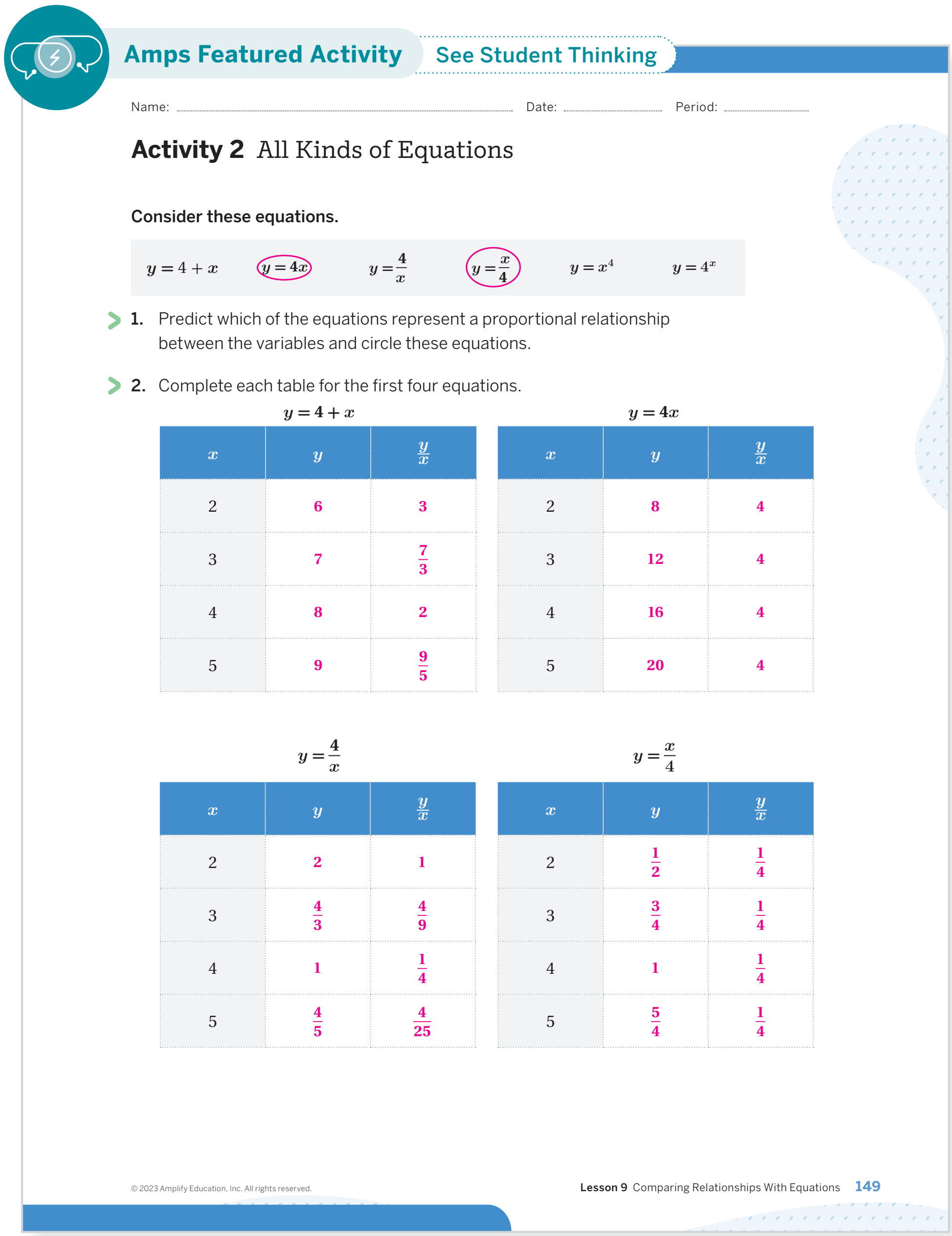
Highlight the connection between the constant of proportionality and the ratio of $\frac{y}{x}$. In this case, the total edge length divided by its corresponding side length is always 12, making the equation $y = 12x$. Explain how the units of measurements relate to the structure of the equation for each quantity: the side length and the units are raised to the same power.

Small Groups | 15 min

Activity 2 All Kinds of Equations

Students calculate the ratio $\frac{y}{x}$ for a series of simple equations to determine which equation structure represents a proportional relationship.

Amps Featured Activity See Student Thinking



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Activity 2 All Kinds of Equations

Consider these equations.

$y = 4 + x$ $\boxed{y = 4x}$ $y = \frac{4}{x}$ $\boxed{y = \frac{x}{4}}$ $y = x^4$ $y = 4^x$

1. Predict which of the equations represent a proportional relationship between the variables and circle these equations.
2. Complete each table for the first four equations.

$y = 4 + x$

x	y	$\frac{y}{x}$
2	6	3
3	7	$\frac{7}{3}$
4	8	2
5	9	$\frac{9}{5}$

$y = 4x$

x	y	$\frac{y}{x}$
2	8	4
3	12	4
4	16	4
5	20	4

$y = \frac{4}{x}$

x	y	$\frac{y}{x}$
2	2	1
3	$\frac{4}{3}$	$\frac{4}{9}$
4	1	$\frac{1}{4}$
5	$\frac{4}{5}$	$\frac{4}{25}$

$y = \frac{x}{4}$

x	y	$\frac{y}{x}$
2	$\frac{1}{2}$	$\frac{1}{4}$
3	$\frac{3}{4}$	$\frac{1}{4}$
4	1	$\frac{1}{4}$
5	$\frac{5}{4}$	$\frac{1}{4}$

Lesson 9 Comparing Relationships With Equations 149

1 Launch

Activate prior knowledge from Grade 6 of evaluating expressions by reviewing a few examples. Provide access to calculators. Let students know that, in Problem 1, they will make a prediction and it is acceptable to be unsure or incorrect.

2 Monitor

Help students get started by having them fill out the y columns first by substituting the x-value into the given equations.

Look for points of confusion:

- **Not noticing the proportional relationships are of the form $y = kx$.** Students do not need to accurately articulate this on their own; this synthesis should emerge in the whole-class discussion.
- **Not recognizing $y = \frac{x}{4}$ as $y = \frac{1}{4}x$.** Help students by writing a 1 as the coefficient and processing the similarity in the equations by showing $y = 0.25x$ produces the same table of values, or reviewing that dividing by 4 is the same as multiplying by the reciprocal .

Look for productive strategies:

- Understanding why $k = \frac{y}{x}$. Students may reason through the process comparing the proportional relationships to ratio tables, or they may manipulate the equation $y = kx$ to solve for k.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For students who need more processing time, allow them to complete tables for 2 of the equations, including either $y = 4x$ or $y = \frac{x}{4}$. Consider allowing them to choose which tables to complete. Provide copies of the Activity 2 PDF (answers) so that students can see all completed tables during the Connect discussion.

Accessibility: *Guide Processing and Visualization*

Have students use colored pencils or highlighters to highlight the constant of proportionality in the equation and table for the two proportional relationships. Illustrate how the equation $y = \frac{x}{4}$ can be written as $y = \frac{1}{4}x$.

Math Language Development

MLR3: Critique, Correct, Clarify

Before students share during the Connect, present both of the following incorrect statements:

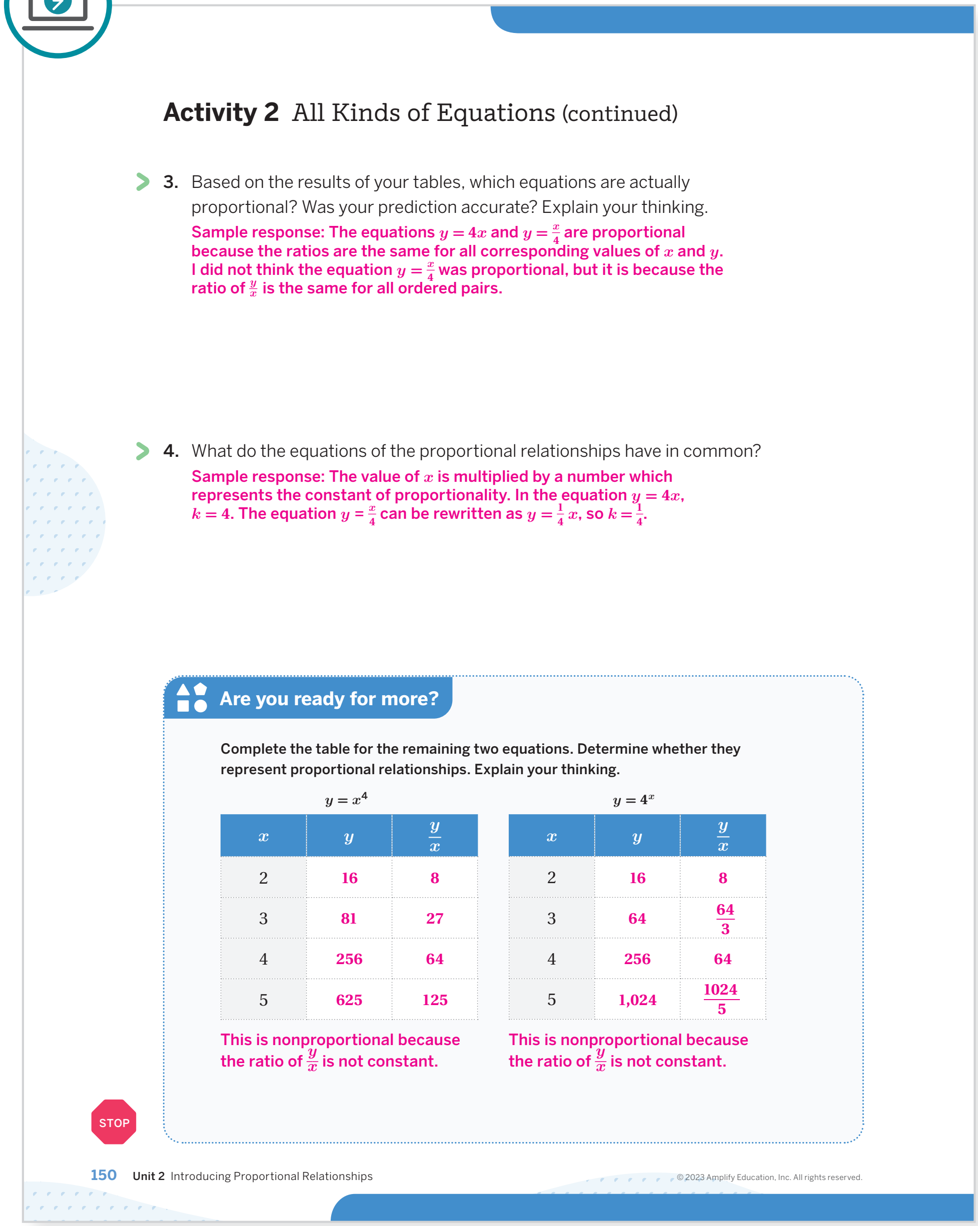
- "There is only one proportional relationship, $y = 4x$, because it is written in the form $y = kx$."
- There are three proportional relationships, $y = 4x$, $y = \frac{4}{x}$, and $y = \frac{x}{4}$. All of these relationships have a constant of proportionality of either 4 or $\frac{1}{4}$."

Ask students to critique these statements, write a corrected statement, and clarify their reasoning as to why and how they corrected it.

Small Groups | 15 min

Activity 2 All Kinds of Equations (continued)

Students calculate the ratio $\frac{y}{x}$ for a series of simple equations to determine which equation structure represents a proportional relationship.



Activity 2 All Kinds of Equations (continued)

3. Based on the results of your tables, which equations are actually proportional? Was your prediction accurate? Explain your thinking.
   Sample response: The equations $y = 4x$ and $y = \frac{x}{4}$ are proportional because the ratios are the same for all corresponding values of x and y. I did not think the equation $y = \frac{x}{4}$ was proportional, but it is because the ratio of $\frac{y}{x}$ is the same for all ordered pairs.

4. What do the equations of the proportional relationships have in common?
   Sample response: The value of x is multiplied by a number which represents the constant of proportionality. In the equation $y = 4x$, $k = 4$. The equation $y = \frac{x}{4}$ can be rewritten as $y = \frac{1}{4}x$, so $k = \frac{1}{4}$.

Are you ready for more?

Complete the table for the remaining two equations. Determine whether they represent proportional relationships. Explain your thinking.

$y = x^4$

x	y	$\frac{y}{x}$
2	16	8
3	81	27
4	256	64
5	625	125

This is nonproportional because the ratio of $\frac{y}{x}$ is not constant.

$y = 4^x$

x	y	$\frac{y}{x}$
2	16	8
3	64	$\frac{64}{3}$
4	256	64
5	1,024	$\frac{1024}{5}$

This is nonproportional because the ratio of $\frac{y}{x}$ is not constant.

STOP

150 Unit 2 Introducing Proportional Relationships

3 Connect

Display the Activity 2 PDF (answers).

Have students share their predictions and whether they were accurate.

Highlight that proportional relationships show the same ratio for every y-value and its corresponding x-value. The constant of proportionality is this ratio and is seen in the proportional relationship equations as the coefficient. Be explicit that proportional relationships are written as $y = kx$ or can include other variables, such as $d = 58t$.

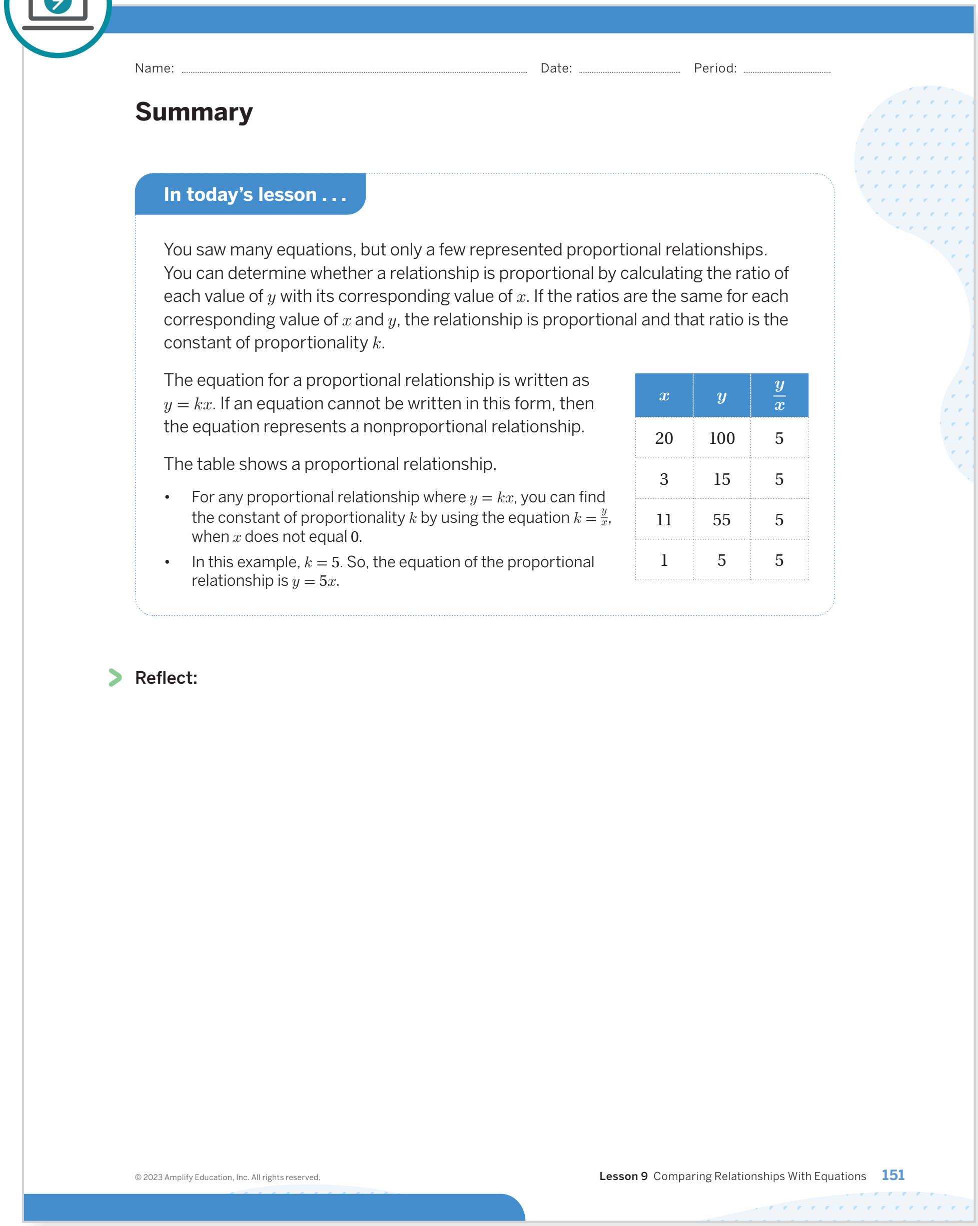
Ask:

- "How can you tell whether an equation represents a proportional relationship?" It is of the form $y = kx$.
- "In which part of the equation do you find the constant of proportionality?" It is the coefficient of x.
- "What does the ratio $\frac{y}{x}$ tell you?" It is the constant of proportionality.

Whole Class | 5 min

Summary

Review and synthesize that proportional relationships are of the form $y = kx$ and that k is found by the ratio of $\frac{y}{x}$.



Name: Date: Period:

Summary

In today's lesson . . .

You saw many equations, but only a few represented proportional relationships. You can determine whether a relationship is proportional by calculating the ratio of each value of y with its corresponding value of x. If the ratios are the same for each corresponding value of x and y, the relationship is proportional and that ratio is the constant of proportionality k.

The equation for a proportional relationship is written as $y = kx$. If an equation cannot be written in this form, then the equation represents a nonproportional relationship.

The table shows a proportional relationship.

- For any proportional relationship where $y = kx$, you can find the constant of proportionality k by using the equation $k = \frac{y}{x}$, when x does not equal 0.
- In this example, $k = 5$. So, the equation of the proportional relationship is $y = 5x$.

x	y	$\frac{y}{x}$
20	100	5
3	15	5
11	55	5
1	5	5

Reflect:

Synthesize

Display the Anchor Chart PDF, *Representing Proportional Relationships*, and complete the equation and constant of proportionality sections.

Have students share their strategies for determining whether an equation represents a proportional relationship.

Ask:

- "Equations representing proportional relationships all have what form?" $y = kx$
- "The coefficient of x represents which part of a proportional relationship?" the constant of proportionality
- "What is the constant of proportionality for $y = \frac{x}{5}$?" $\frac{1}{5}$
- "How can you determine the constant of proportionality when you are given an x-value and its corresponding y-value?" Calculate the ratio of $\frac{y}{x}$. **Note:** Let students know they cannot choose the origin to help determine the ratio because if $x = 0$, then $\frac{y}{0}$ is undefined.

Highlight that if an equation is not of the form $y = kx$, it does not represent a proportional relationship.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

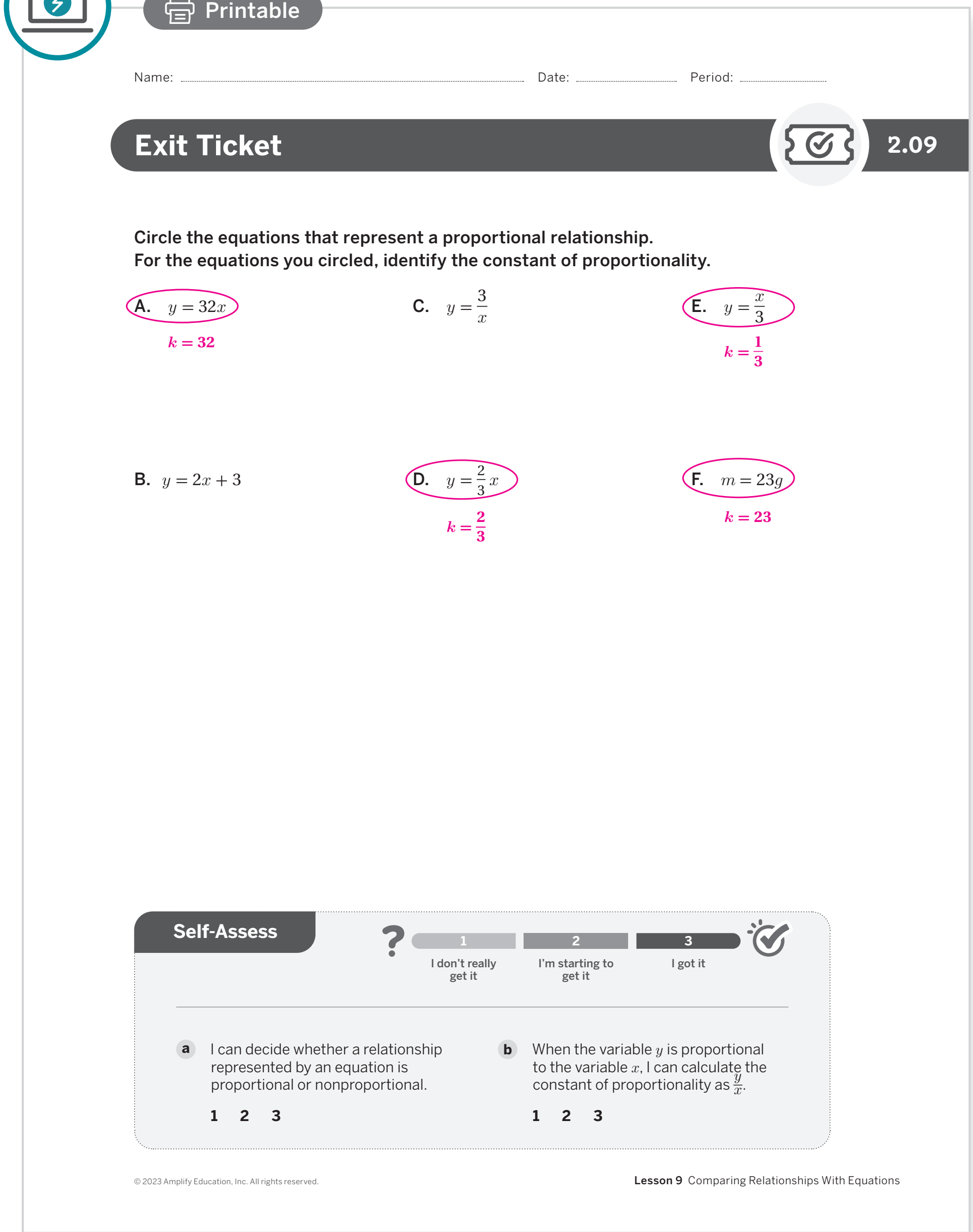
- "What does it mean for two things to be proportionally related? How can you tell?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying equations of proportional relationships.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 2.09

Circle the equations that represent a proportional relationship.
For the equations you circled, identify the constant of proportionality.

[✓ A. $y = 32x$]
$k = 32$

B. $y = 2x + 3$

C. $y = \frac{3}{x}$

[✓ D. $y = \frac{2}{3}x$]
$k = \frac{2}{3}$

[✓ E. $y = \frac{x}{3}$]
$k = \frac{1}{3}$

[✓ F. $m = 23g$]
$k = 23$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can decide whether a relationship represented by an equation is proportional or nonproportional.
1 2 3

b. When the variable y is proportional to the variable x, I can calculate the constant of proportionality as $\frac{y}{x}$.
1 2 3

Lesson 9 Comparing Relationships With Equations

Success looks like . . .

- **Language Goal:** Comparing and contrasting equations that do and do not represent proportional relationships. **(Speaking and Listening)**
  - » Comparing and contrasting all six equations to determine those that represent a proportional relationship.
- **Goal:** Generalizing that an equation equivalent to the form $y = kx$ can represent a proportional relationship.
- **Language Goal:** Using a table to determine whether a given equation represents a proportional relationship, and justifying the decision. **(Writing)**

Suggested next steps

If students do not select Choices A, D, E or F, consider:

- Reviewing Activity 2.
- Assigning Practice Problems 2 and 3.

If students show work creating an (x, y) table, consider:

- Congratulating them on producing quality work.
- Reminding them that the structure of an equation shows whether a relationship is a proportional relationship, which can save them time.

Professional Learning

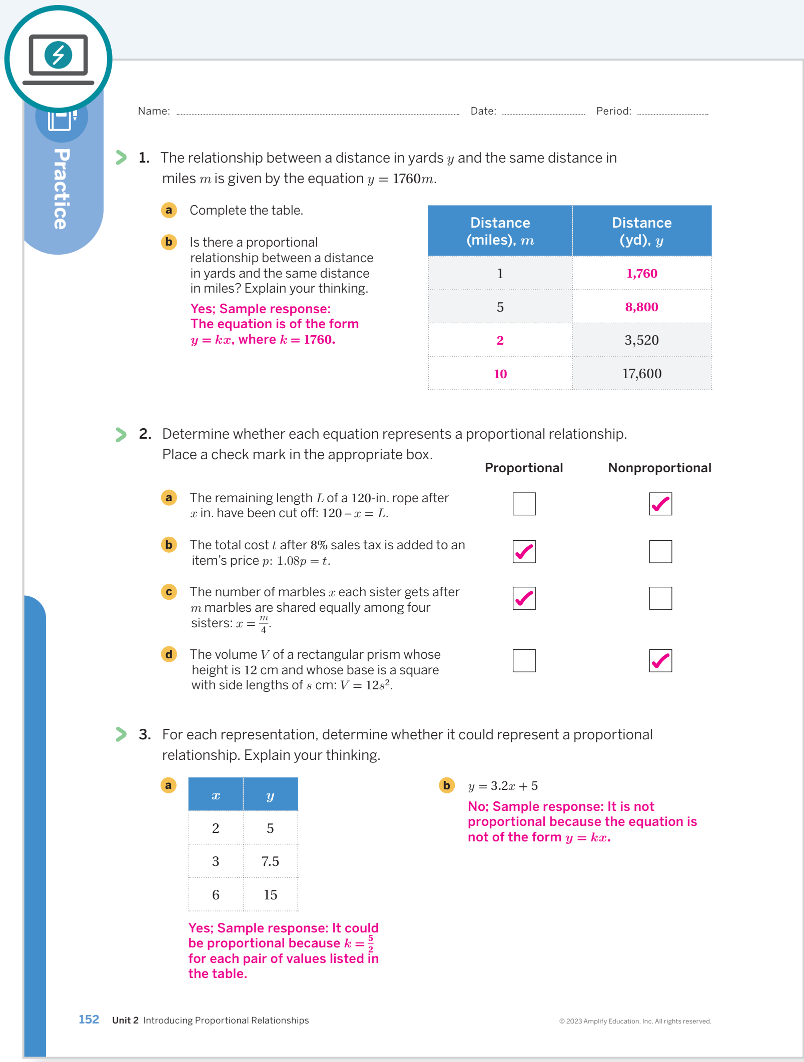
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Knowing where students need to be by the end of this unit, how did Activity 2 influence that future goal?
- Have you changed any former ideas about proportional relationships as a result of today's lesson? What might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. The relationship between a distance in yards y and the same distance in miles m is given by the equation $y = 1760m$.

a Complete the table.

Distance (miles), m	Distance (yd), y
1	**1,760**
5	**8,800**
2	3,520
10	17,600

b Is there a proportional relationship between a distance in yards and the same distance in miles? Explain your thinking.

Yes; Sample response: The equation is of the form $y = kx$, where $k = 1760$.

2. Determine whether each equation represents a proportional relationship. Place a check mark in the appropriate box.

	Proportional	Nonproportional
a The remaining length L of a 120-in. rope after x in. have been cut off: $120 - x = L$.	☐	☑
b The total cost t after 8% sales tax is added to an item's price p: $1.08p = t$.	☑	☐
c The number of marbles x each sister gets after m marbles are shared equally among four sisters: $x = \frac{m}{4}$.	☑	☐
d The volume V of a rectangular prism whose height is 12 cm and whose base is a square with side lengths of s cm: $V = 12s^2$.	☐	☑

3. For each representation, determine whether it could represent a proportional relationship. Explain your thinking.

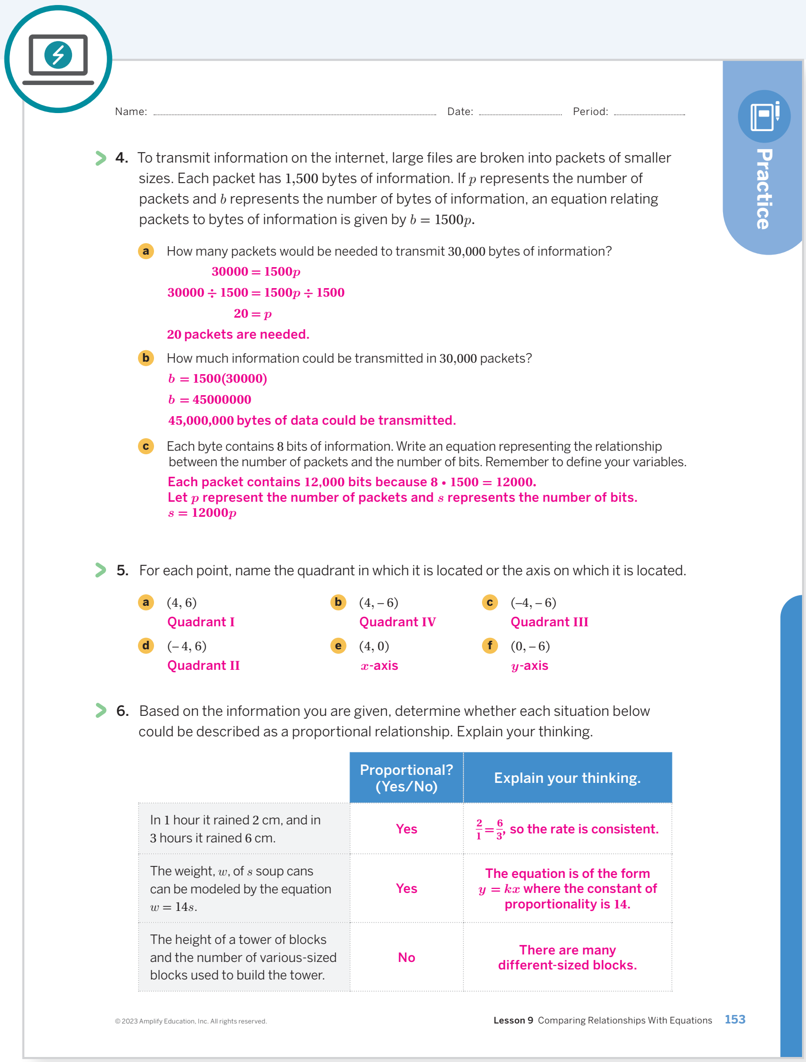
a

x	y
2	5
3	7.5
6	15

Yes; Sample response: It could be proportional because $k = \frac{5}{2}$ for each pair of values listed in the table.

b $y = 3.2x + 5$

No; Sample response: It is not proportional because the equation is not of the form $y = kx$.



Practice

Name: Date: Period:

4. To transmit information on the internet, large files are broken into packets of smaller sizes. Each packet has 1,500 bytes of information. If p represents the number of packets and b represents the number of bytes of information, an equation relating packets to bytes of information is given by $b = 1500p$.

a How many packets would be needed to transmit 30,000 bytes of information?

$$30000 = 1500p$$
$$30000 \div 1500 = 1500p \div 1500$$
$$20 = p$$

20 packets are needed.

b How much information could be transmitted in 30,000 packets?

$$b = 1500(30000)$$
$$b = 45000000$$

45,000,000 bytes of data could be transmitted.

c Each byte contains 8 bits of information. Write an equation representing the relationship between the number of packets and the number of bits. Remember to define your variables.

Each packet contains 12,000 bits because $8 \cdot 1500 = 12000$.
Let p represent the number of packets and s represents the number of bits.
$s = 12000p$

5. For each point, name the quadrant in which it is located or the axis on which it is located.

a $(4, 6)$ **Quadrant I**

b $(4, -6)$ **Quadrant IV**

c $(-4, -6)$ **Quadrant III**

d $(-4, 6)$ **Quadrant II**

e $(4, 0)$ **x-axis**

f $(0, -6)$ **y-axis**

6. Based on the information you are given, determine whether each situation below could be described as a proportional relationship. Explain your thinking.

	Proportional? (Yes/No)	Explain your thinking.
In 1 hour it rained 2 cm, and in 3 hours it rained 6 cm.	**Yes**	**$\frac{2}{1} = \frac{6}{3}$, so the rate is consistent.**
The weight, w, of s soup cans can be modeled by the equation $w = 14s$.	**Yes**	**The equation is of the form $y = kx$ where the constant of proportionality is 14.**
The height of a tower of blocks and the number of various-sized blocks used to build the tower.	**No**	**There are many different-sized blocks.**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 8	2
	5	Grade 6	1
Formative	6	Unit 2 Lesson 10	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 10**

Solving Problems About Proportional Relationships

Let's solve problems about proportional relationships

Focus

Goals

1. **Language Goal:** Decide whether it makes sense to represent a situation with a proportional relationship and explain the reason. **(Speaking and Listening)**
2. **Language Goal:** Determine, through inquiry, the information needed to solve a problem involving proportional relationships. **(Speaking and Listening)**
3. Write an equation to represent a proportional relationship and use it to solve problems in context.

Rigor

- Students develop **conceptual understanding** of the relationship between the constant of proportionality and constant rates.
- Students build **procedural fluency** in identifying proportional relationships by first deciding whether they have sufficient information, and then determining whether the ratio $\frac{y}{x}$ is constant.

Coherence

Today

Students learn to recognize proportional relationships when given information about a contextual situation. Students reason quantitatively about situations and tables of values to determine whether corresponding quantities represent proportional or nonproportional relationships.

Previously

In Lesson 4, students determined whether tables of values represented proportional relationships. In Lesson 9, students determined whether equations represented proportional relationships.

Coming Soon

In Lesson 11, students will build on their understanding of proportional relationships to determine whether a graph is modeling a proportional relationship.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	15 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- Activity 2 PDF, pre-cut cards, one set per group
- *Info Gap Routine* PDF (for display)

Math Language Development

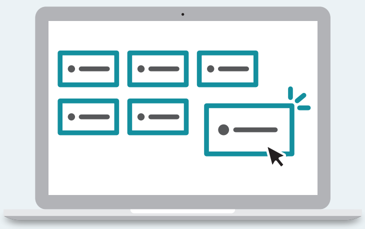
Review words

- *constant of proportionality*
- *nonproportional relationship*
- *proportional relationship*

Amps | Featured Activity

Activity 2
Digital Card Sort

Students match situations with whether they represent proportional or nonproportional relationships by dragging and connecting them on screen.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might disagree about what information they need to make sense of the problem. Before the activity, remind students that before they criticize anyone, they need to try to understand by taking on their perspective. Students' backgrounds might lead them to seek different information and they might bring new light to the situation.

Modifications to Pacing

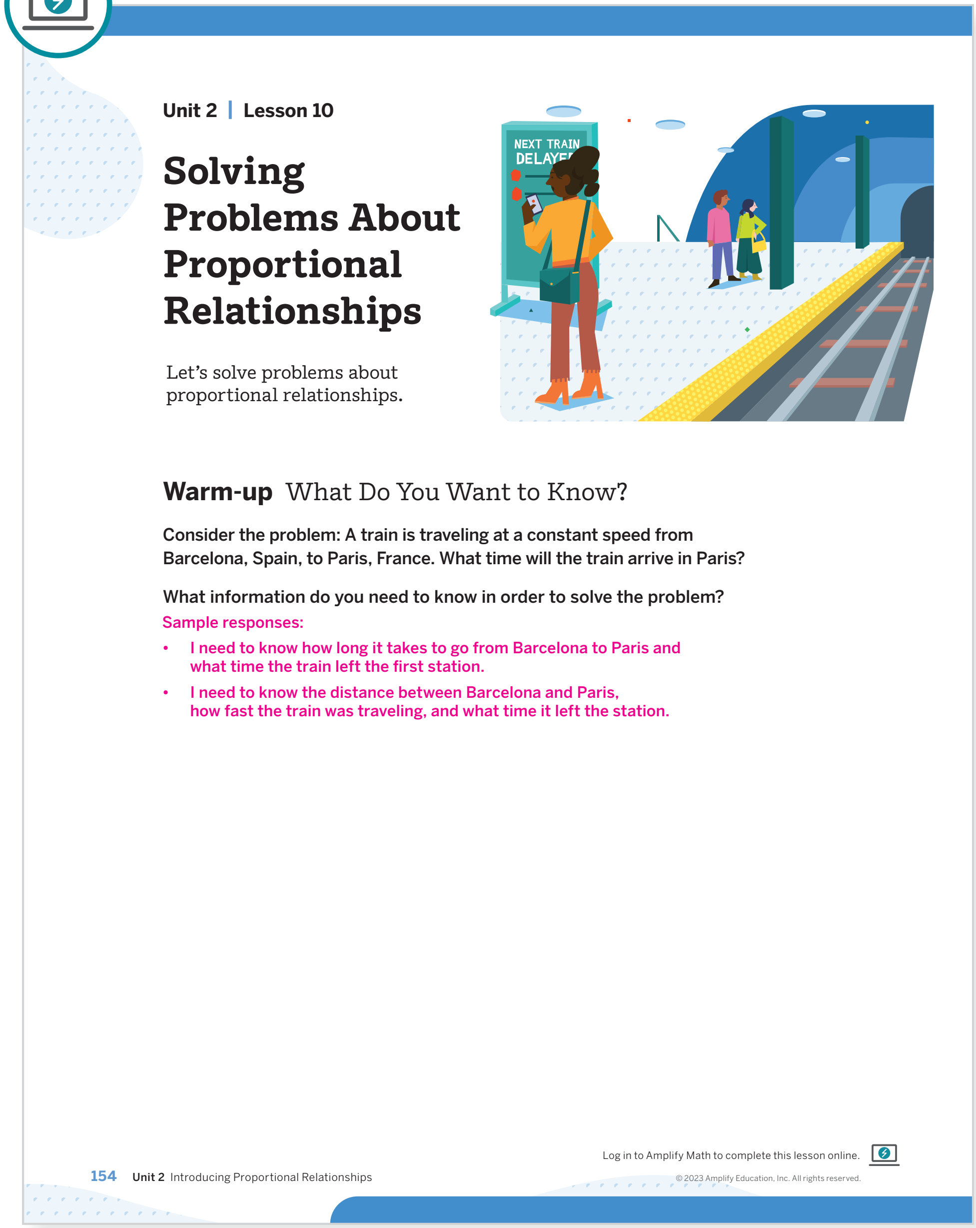
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, the second round of the ***Info Gap*** routine may be omitted.
- In **Activity 2**, distribute only the even-numbered cards.

Independent | 7 min

Warm-up What Do You Want to Know?

Students determine what information is needed to solve a problem involving constant speed.



Unit 2 | Lesson 10

Solving Problems About Proportional Relationships

Let's solve problems about proportional relationships.

Warm-up What Do You Want to Know?

Consider the problem: A train is traveling at a constant speed from Barcelona, Spain, to Paris, France. What time will the train arrive in Paris?

What information do you need to know in order to solve the problem?

Sample responses:

- I need to know how long it takes to go from Barcelona to Paris and what time the train left the first station.
- I need to know the distance between Barcelona and Paris, how fast the train was traveling, and what time it left the station.

Log in to Amplify Math to complete this lesson online.

154 Unit 2 Introducing Proportional Relationships

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1 Launch

Give students one minute to brainstorm what information they would need to know to solve the problem.

2 Monitor

Help students get started by asking them what it means for the train to be traveling at a constant speed.

3 Connect

Display the ***Info Gap Routine*** PDF. Explain that you will be modeling the routine as students ask questions.

Ask:

- "What specific information do you need?"
- "Why is that particular information important?"

Have individual students share their questions. Provide the following information only after a student has specifically asked for it.

- Barcelona and Paris are 830 km apart.
- The train left Barcelona at 9:15 a.m.
- The train travels an average of 132.8 km/hr.
- The trip takes 6.25 hours.
- In one hour, the train travels 132.8 km.

After each question, ask whether students have enough information to solve the problem. Once students agree they do have sufficient information, allow two minutes to work on a solution.

Highlight that students needed to ask multiple questions before having enough information to determine that the train would arrive in Paris at 3:30 p.m. Explain that they will be working in pairs for the next activity and will follow the same routine to solve problems.

Power-up

To power up students' ability to determine if an equation represents a proportional relationship, have students complete:

Identify which of the following equations represent proportional relationships. Select *all* that apply. If proportional, identify the constant of proportionality.

[✓ A.] $y = 2x$ $k = 2$

[✓ C.] $y = x$ $k = 1$

B. $y = x^2$

[✓ D.] $y = \frac{x}{2}$ $k = \frac{1}{2}$

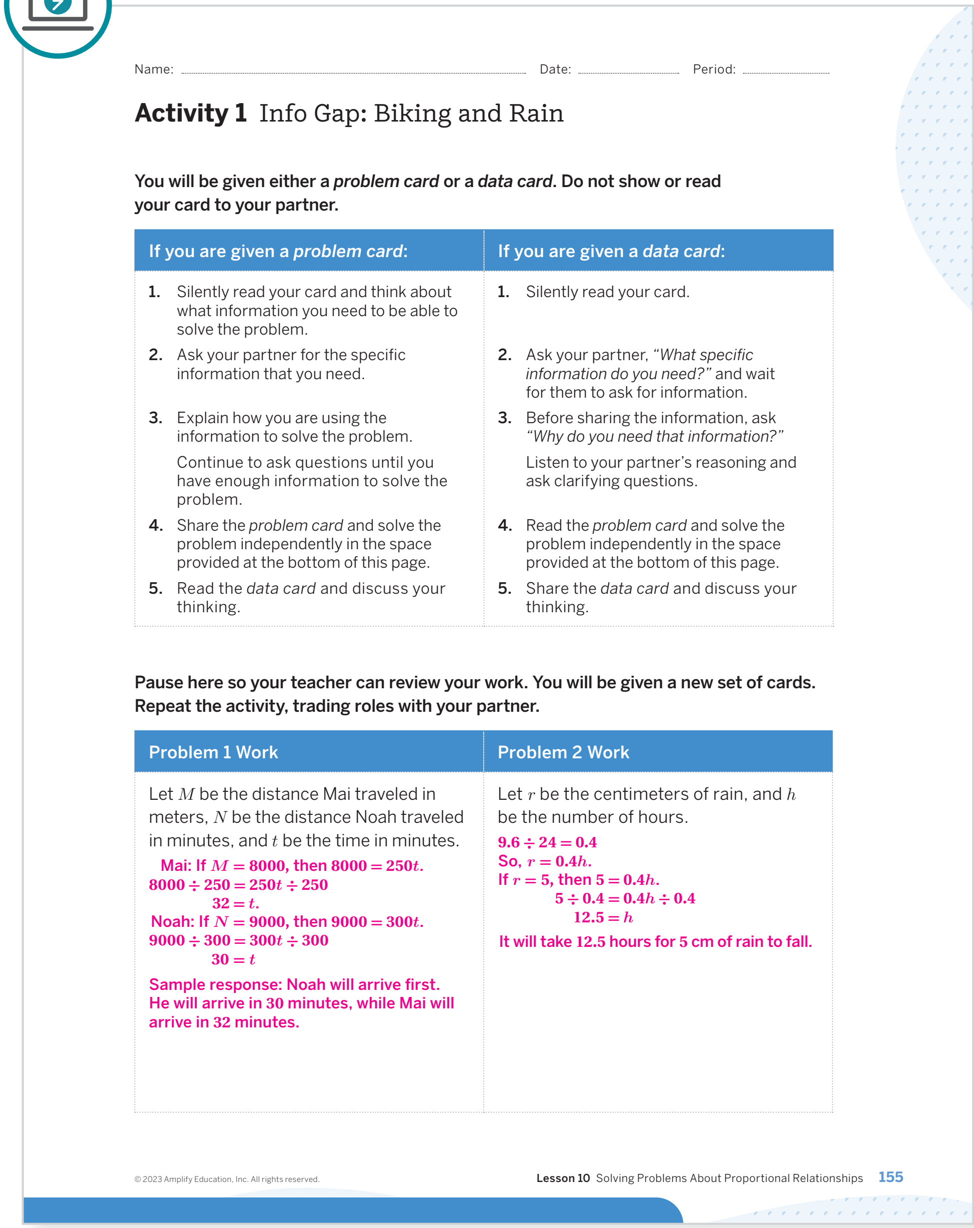
Use: Before the Activity 2.

Informed by: Performance on Lesson 9, Practice Problem 6 and Lesson 9 Exit Ticket.

Pairs | 15 min

Activity 1 Info Gap: Biking and Rain

Students will determine what information is necessary to write equations that represent proportional relationships and solve problems.



Name: ______ Date: ______ Period: ______

Activity 1 Info Gap: Biking and Rain

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	1. Silently read your card.
2. Ask your partner for the specific information that you need.	2. Ask your partner, *"What specific information do you need?"* and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	3. Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space provided at the bottom of this page.	4. Read the *problem card* and solve the problem independently in the space provided at the bottom of this page.
5. Read the *data card* and discuss your thinking.	5. Share the *data card* and discuss your thinking.

Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.

Problem 1 Work	Problem 2 Work
Let M be the distance Mai traveled in meters, N be the distance Noah traveled in minutes, and t be the time in minutes. **Mai: If $M = 8000$, then $8000 = 250t$.** $8000 \div 250 = 250t \div 250$ $32 = t$. **Noah: If $N = 9000$, then $9000 = 300t$.** $9000 \div 300 = 300t \div 300$ $30 = t$ **Sample response: Noah will arrive first. He will arrive in 30 minutes, while Mai will arrive in 32 minutes.**	Let r be the centimeters of rain, and h be the number of hours. $9.6 \div 24 = 0.4$ **So, $r = 0.4h$.** **If $r = 5$, then $5 = 0.4h$.** $5 \div 0.4 = 0.4h \div 0.4$ $12.5 = h$ **It will take 12.5 hours for 5 cm of rain to fall.**

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Lesson 10 Solving Problems About Proportional Relationships 155

1 Launch

Remind students that they will be following the ***Info Gap*** routine to determine the missing information. Each partner should have a turn with a *problem card* and with a *data card* from the Activity 1 PDF.

2 Monitor

Help students get started by asking them to brainstorm what information is needed to make sense of the problem. Have students consider why that particular information is necessary before they begin asking questions.

Look for points of confusion:

- **Thinking that they will have enough information after one or two questions.** Remind students that it may take multiple rounds of questioning to collect the information they need. Encourage them to ask precise questions.

Look for productive strategies:

- Rephrasing and refining questioning to elicit information from their partner.

3 Connect

Have pairs of students share their equations, predictions, and explanations of how each situation represents a proportional relationship.

Display the equations and process for solving both problems.

Highlight that each situation is describing a constant rate and is therefore proportional. Remind students the importance of being precise in questioning and listening carefully to others in order to determine the necessary information to solve each problem.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following questions during the think-aloud.

- "I wonder where each person lives, in relation to the park. I think I should ask how far each person lives from the park."
- "I wonder how fast each person is riding their bike. I think I should ask for their speeds, and whether or not they are each riding at a constant speed."

Math Language Development

MLR4: Information Gap

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

English Learners

Consider providing sample questions students could ask, such as the following for Problem Card 1:

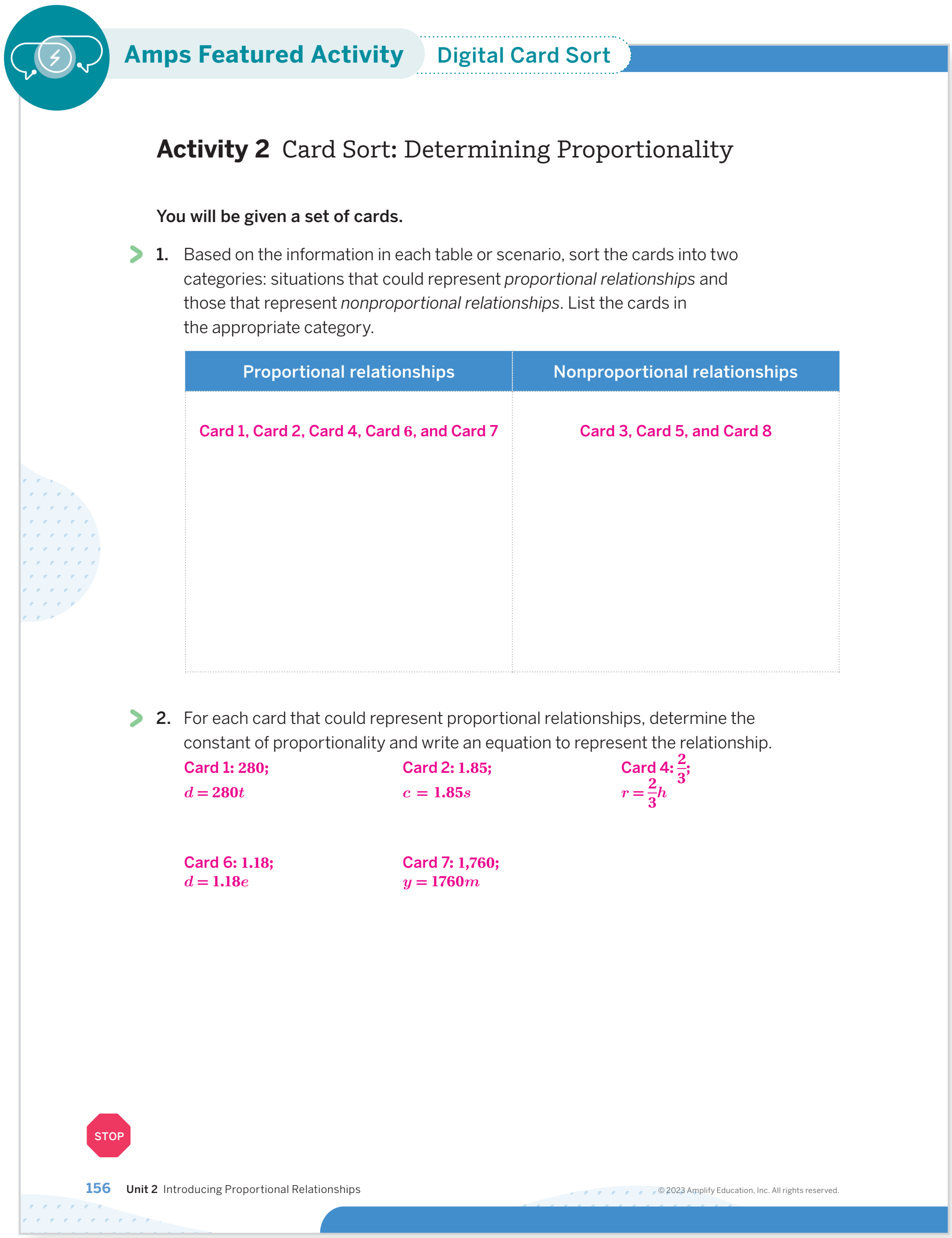
- How far does Mai live from the park? How far does Noah live from the park?
- How fast is Mai riding her bike? Is she riding at a constant speed?
- How fast is Noah riding his bike? Is he riding at a constant speed?

Small Groups | 13 min

Activity 2 Card Sort: Determining Proportionality

Students will categorize relationships represented in tables or as scenarios to determine whether two quantities are proportional, and identify the constant of proportionality, where possible.

Amps Featured Activity Digital Card Sort



Activity 2 Card Sort: Determining Proportionality

You will be given a set of cards.

1. Based on the information in each table or scenario, sort the cards into two categories: situations that could represent *proportional relationships* and those that represent *nonproportional relationships*. List the cards in the appropriate category.

Proportional relationships	Nonproportional relationships
Card 1, Card 2, Card 4, Card 6, and Card 7	Card 3, Card 5, and Card 8

2. For each card that could represent proportional relationships, determine the constant of proportionality and write an equation to represent the relationship.

Card 1: 280; $d = 280t$

Card 2: 1.85; $c = 1.85s$

Card 4: $\frac{2}{3}$; $r = \frac{2}{3}h$

Card 6: 1.18; $d = 1.18e$

Card 7: 1,760; $y = 1760m$

STOP

156 **Unit 2** Introducing Proportional Relationships

1 Launch

Distribute pre-cut cards from the Activity 2 PDF to small groups and conduct the *Card Sort* routine.

2 Monitor

Help students get started by asking what it means for two quantities to be in a proportional relationship.

Look for points of confusion:

- **Thinking that a relationship is proportional based on only two rows of values in a table.** Remind students that the relationship is based on all of the information in the table.
- **Thinking that the ratios in all the rows should be different to be nonproportional.** Remind students that they only need one ratio to be different in order to be nonproportional.

3 Connect

Display any cards that were topics of disagreement during the group discussions.

Have groups of students share their strategies for categorizing cards as proportional and nonproportional.

Highlight concrete methods for determining whether a situation represents a proportional relationship. Note that the decision is based on available information, and does not mean that the relationship is necessarily proportional for all values.

Ask:

- "What do you look for to determine whether a scenario could represent a proportional relationship?"
- "How do you find the constant of proportionality from a scenario? From a table?"

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide blank tables for students to use so that they can create tables of values to assist them for Cards 2–4. Consider providing students with two cards at a time (one proportional and one nonproportional). Tell them one of the relationships is proportional.

Extension: *Math Enrichment*

Have students alter the relationship in Card 3 so that the relationship is proportional. Sample response: The pizza shop could charge $33 to deliver two veggie pizzas ($16.50 per pizza), or they could charge $15.75 to deliver one veggie pizza ($15.75 per pizza).

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display a relationship that is nonproportional, such as Card 3, and an incorrect statement, such as "This relationship is proportional because the cost per pizza is always $16.50." Ask pairs of students to critique this statement, write a corrected statement, and clarify their reasoning as to why and how they corrected it.

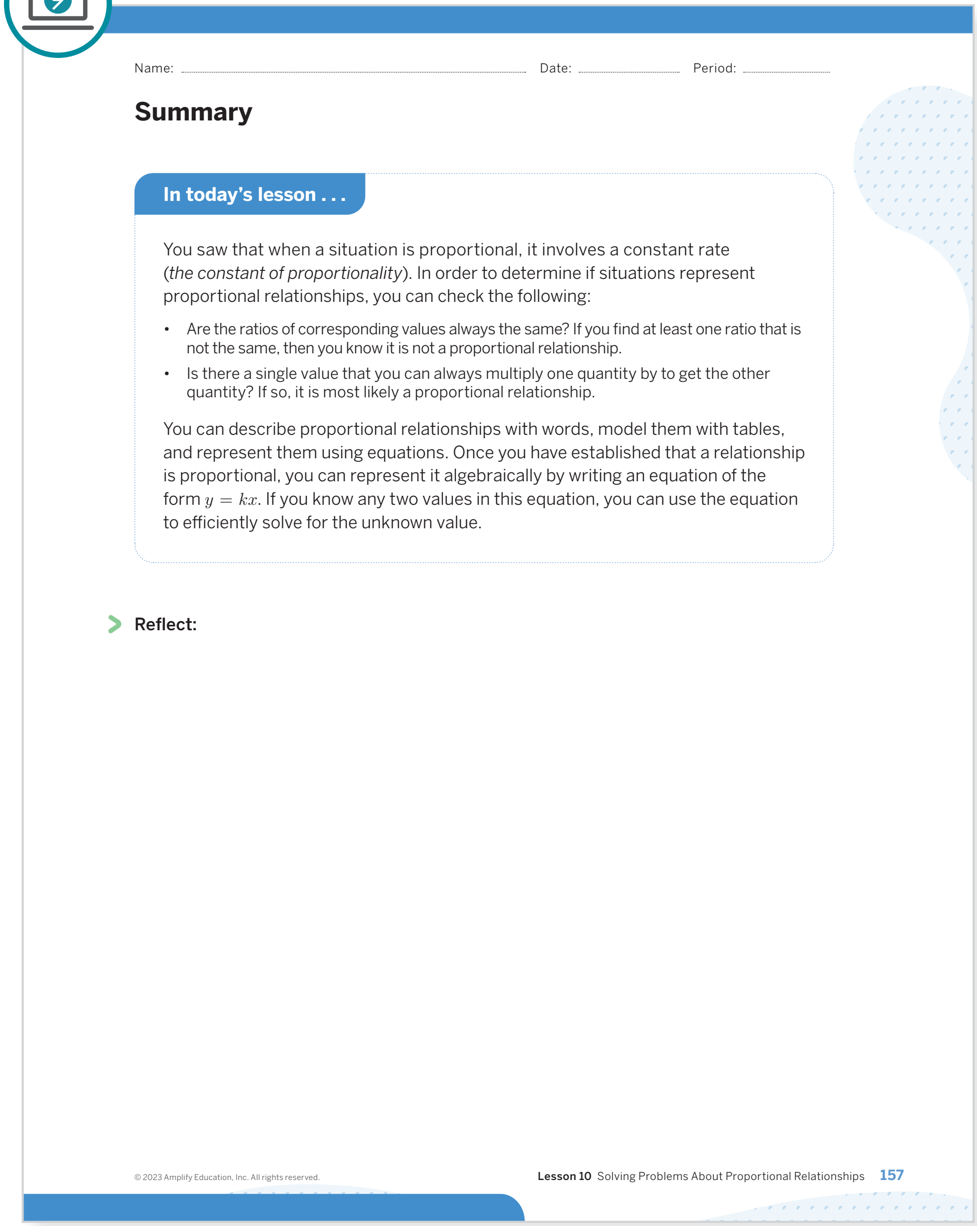
English Learners

Consider also presenting the information on Cards 2–4 in tables. Help students connect the words to the tables.

Whole Class | 5 min

Summary

Review and synthesize the information needed to determine whether two quantities are in a proportional relationship, and how to coordinate between tables and equations that represent proportional relationships.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You saw that when a situation is proportional, it involves a constant rate (*the constant of proportionality*). In order to determine if situations represent proportional relationships, you can check the following:

- Are the ratios of corresponding values always the same? If you find at least one ratio that is not the same, then you know it is not a proportional relationship.
- Is there a single value that you can always multiply one quantity by to get the other quantity? If so, it is most likely a proportional relationship.

You can describe proportional relationships with words, model them with tables, and represent them using equations. Once you have established that a relationship is proportional, you can represent it algebraically by writing an equation of the form $y = kx$. If you know any two values in this equation, you can use the equation to efficiently solve for the unknown value.

Reflect:

Synthesize

Ask:

- "What are some examples of situations where you have seen two proportional quantities?"
- "When a proportional relationship exists between two quantities, what information is needed to determine an equation?"
- "How can you decide whether a proportional relationship is a good representation of a particular situation?"
- "Equations are good tools to make predictions or decisions. When and how did you use an equation to make a prediction or a decision today?"

Highlight that whenever a situation is a proportional relationship, there is a constant rate between the quantities of interest. When it is proportional, it is helpful to use an equation to model the situation because an equation is often the most efficient way to solve problems.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

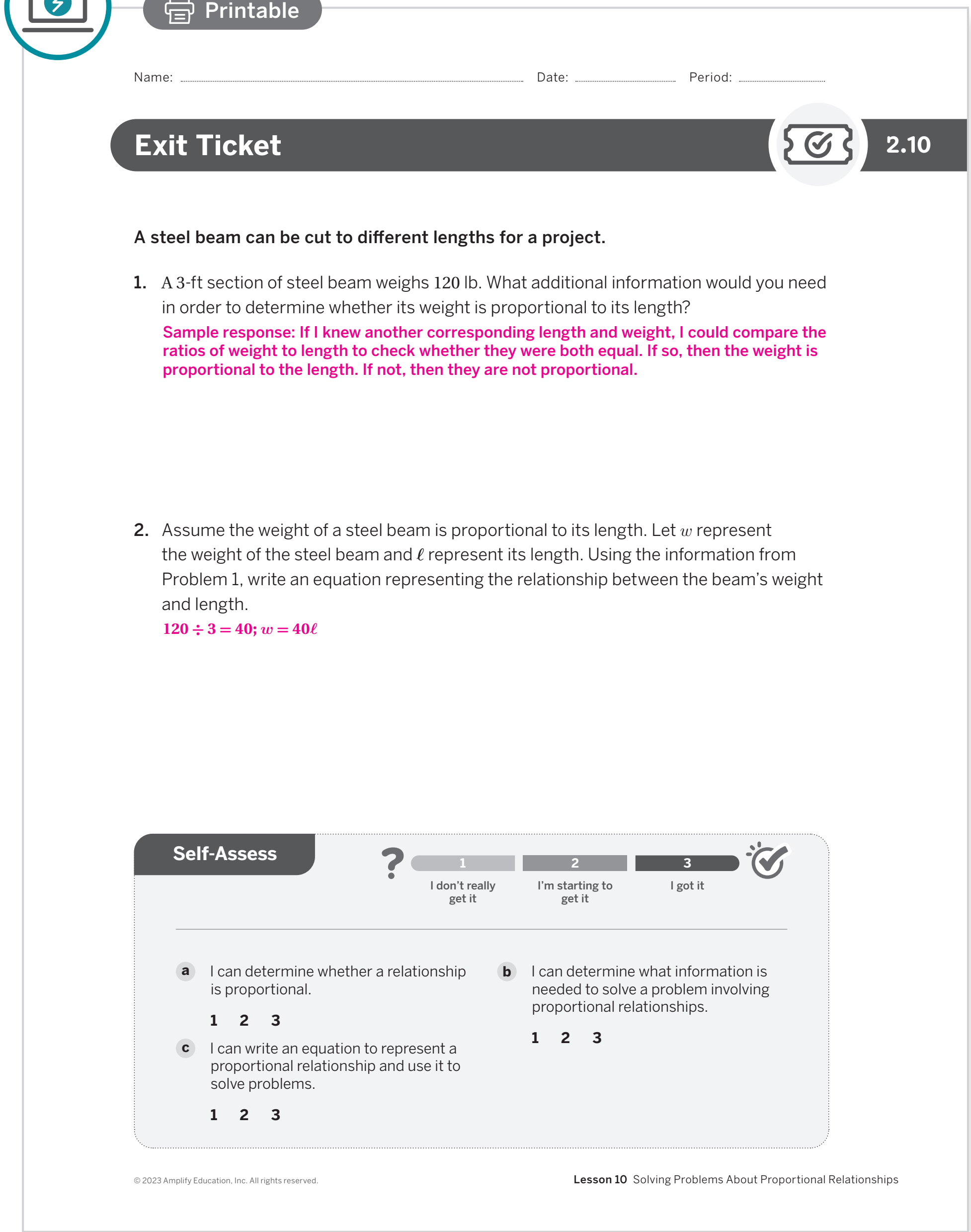
- "What does it mean for two things to be proportionally related? How can you tell?"

Independent | 5 min

Exit Ticket

Students demonstrate understanding by determining the missing information and writing an equation for the proportional relationship between the weight and the length of a steel beam.

Printable



Name: ______________ Date: ______ Period: ______

Exit Ticket 2.10

A steel beam can be cut to different lengths for a project.

1. A 3-ft section of steel beam weighs 120 lb. What additional information would you need in order to determine whether its weight is proportional to its length?

   Sample response: If I knew another corresponding length and weight, I could compare the ratios of weight to length to check whether they were both equal. If so, then the weight is proportional to the length. If not, then they are not proportional.

2. Assume the weight of a steel beam is proportional to its length. Let w represent the weight of the steel beam and ℓ represent its length. Using the information from Problem 1, write an equation representing the relationship between the beam's weight and length.

   $120 \div 3 = 40;\ w = 40\ell$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine whether a relationship is proportional.

1 2 3

b. I can determine what information is needed to solve a problem involving proportional relationships.

1 2 3

c. I can write an equation to represent a proportional relationship and use it to solve problems.

1 2 3

Lesson 10 Solving Problems About Proportional Relationships

Success looks like . . .

- **Language Goal:** Deciding whether it makes sense to represent a situation with a proportional relationship and explaining the reason. **(Speaking and Listening)**
- **Language Goal:** Determining, through inquiry, the information needed to solve a problem involving proportional relationships. **(Speaking and Listening)**
  - » Explaining the additional information to determine whether the steel beam's weight is proportional to its length in Problem 1.
- **Goal:** Writing an equation to represent a proportional relationship and using it to solve problems in context.
  - » Writing an equation representing the beam's weight and length in Problem 2.

Suggested next steps

If students have difficulty explaining the missing information to conclude weight and length are proportional, consider:

- Reviewing the definition of *proportional relationship*.
- Assigning Practice Problem 1.

If students have difficulty writing the equation to represent the proportional relationship, consider:

- Assigning Practice Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students attend to precision today? How are you helping students become aware of how they are progressing in this area?
- What did you see in the way some students approached the ***Info Gap*** that you would like other students to try? What might you change for the next time you teach this lesson?

Math Language Development

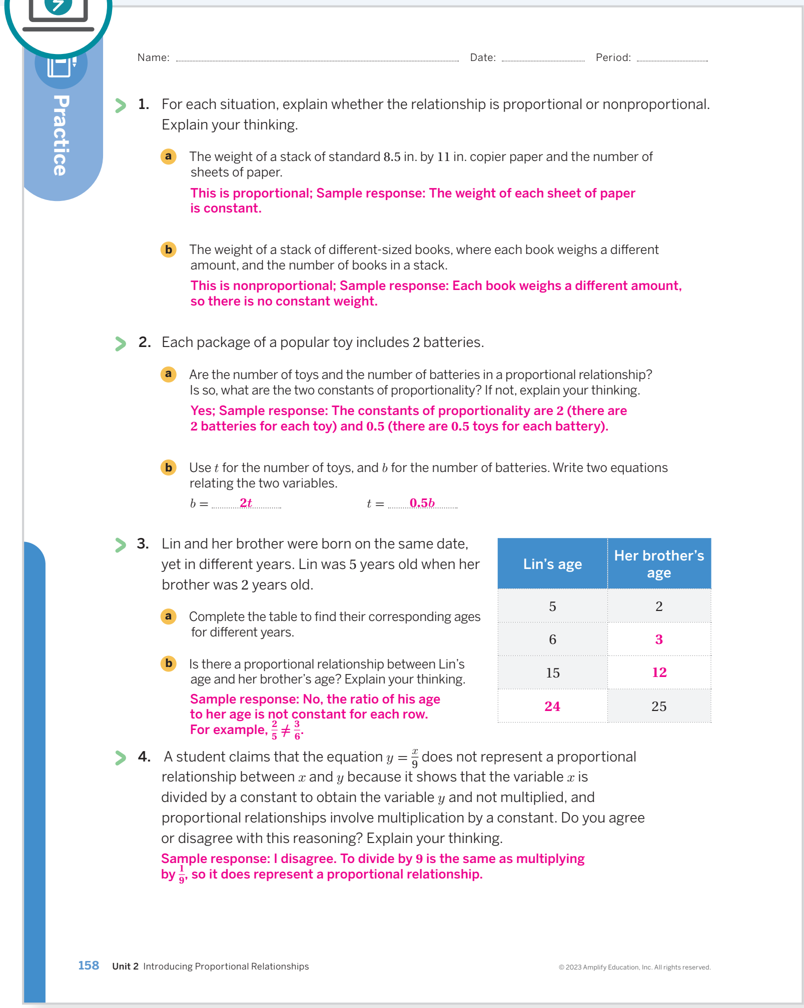
Language Goal: Deciding whether it makes sense to represent a situation with a proportional relationship and explaining the reason.

Reflect on students' language development toward this goal.

- How did using the ***Critique, Correct, Clarify*** routine in Activity 2 help students use their developing math language to compare and contrast proportional and nonproportional relationships?
- What support do they still need in order to be more precise in their justifications?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. For each situation, explain whether the relationship is proportional or nonproportional. Explain your thinking.

   a. The weight of a stack of standard 8.5 in. by 11 in. copier paper and the number of sheets of paper.

   This is proportional; Sample response: The weight of each sheet of paper is constant.

   b. The weight of a stack of different-sized books, where each book weighs a different amount, and the number of books in a stack.

   This is nonproportional; Sample response: Each book weighs a different amount, so there is no constant weight.

2. Each package of a popular toy includes 2 batteries.

   a. Are the number of toys and the number of batteries in a proportional relationship? Is so, what are the two constants of proportionality? If not, explain your thinking.

   Yes; Sample response: The constants of proportionality are 2 (there are 2 batteries for each toy) and 0.5 (there are 0.5 toys for each battery).

   b. Use t for the number of toys, and b for the number of batteries. Write two equations relating the two variables.

   $b =$ **$2t$** $t =$ **$0.5b$**

3. Lin and her brother were born on the same date, yet in different years. Lin was 5 years old when her brother was 2 years old.

   a. Complete the table to find their corresponding ages for different years.

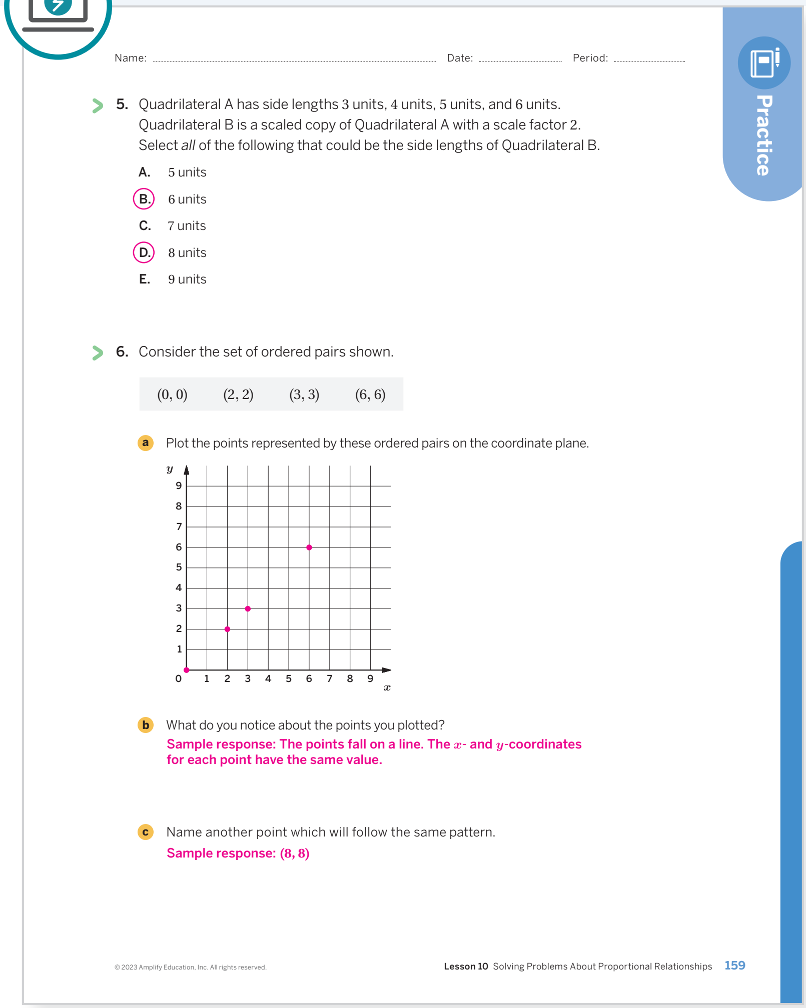
   b. Is there a proportional relationship between Lin's age and her brother's age? Explain your thinking.

   Sample response: No, the ratio of his age to her age is not constant for each row. For example, $\frac{2}{5} \neq \frac{3}{6}$.

Lin's age	Her brother's age
5	2
6	**3**
15	**12**
24	25

4. A student claims that the equation $y = \frac{x}{9}$ does not represent a proportional relationship between x and y because it shows that the variable x is divided by a constant to obtain the variable y and not multiplied, and proportional relationships involve multiplication by a constant. Do you agree or disagree with this reasoning? Explain your thinking.

   Sample response: I disagree. To divide by 9 is the same as multiplying by $\frac{1}{9}$, so it does represent a proportional relationship.



Name: ________ Date: ________ Period: ________

5. Quadrilateral A has side lengths 3 units, 4 units, 5 units, and 6 units. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor 2. Select *all* of the following that could be the side lengths of Quadrilateral B.

   A. 5 units
   [✓ B.] 6 units
   C. 7 units
   [✓ D.] 8 units
   E. 9 units

6. Consider the set of ordered pairs shown.

   (0, 0) (2, 2) (3, 3) (6, 6)

   a. Plot the points represented by these ordered pairs on the coordinate plane.

   b. What do you notice about the points you plotted?

   Sample response: The points fall on a line. The x- and y-coordinates for each point have the same value.

   c. Name another point which will follow the same pattern.

   Sample response: (8, 8)

Practice Problem Analysis

Type	Problem	Refer to	DOK
On Lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 9	2
	5	Unit 1 Lesson 3	1
Formative ⚡	6	Unit 2 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 2

Whole Class

Representing Proportional Relationships With Graphs

In this Sub-Unit, students notice that the graphs of proportional relationships have a certain look, and they reason about why this is.

SUB-UNIT

2 Representing Proportional Relationships With Graphs

Narrative Connections

What good is a graph?

In the previous lessons, you saw how tables and equations were useful for representing proportional relationships. Tables let you organize your data so you can find the **constant of proportionality**.

Meanwhile, the equation $y = kx$ gave you a way to express *any* proportional relationship.

These representations each have their own pros and cons. Tables offer a practical way to collect and organize data, but checking every entry for proportionality can be tedious.

Meanwhile, an equation may reveal the rules underpinning a proportional relationship, but getting useful data requires applying the equation several times.

In this next set of lessons you will learn about another kind of representation for proportional relationships: graphs!

Graphs can be a fast, intuitive way to understand and appreciate the data and mathematics underlying proportional relationships. With just one look, you will be able to tell if a relationship is proportional; find its constant of proportionality; and see all the solutions for that relationship.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore the advantages of using graphs to understand proportionality in the following places:

- **Lesson 11, Activity 1:** T-shirts for Sale
- **Lesson 12, Activities 1-2:** Making Tea With a Chai Wallah, Tyler's Job
- **Lesson 13, Activity 1:** Race to $\$1,000$
- **Lesson 15, Activities 1-2:** Tables, Graphs, and Equations, Finding the Constant of Proportionality

UNIT 2 | **LESSON 11**

Introducing Graphs of Proportional Relationships

Let's see how graphs of proportional relationships differ from graphs of other relationships.

Focus

Goals

1. **Language Goal:** Compare and contrast graphs of relationships. **(Speaking and Listening)**
2. **Language Goal:** Generalize that a proportional relationship can be represented on the coordinate plane by a line that includes the origin, or by a collection of points that lie on such a line. **(Speaking and Listening, Writing)**

Coherence

Today

Students begin working with graphs of proportional relationships. Through comparing graphs and critiquing each other's reasoning, they determine proportional relationships are represented as straight lines passing through the origin, or as points contained on such a line. During Activity 2, they use the structure of a graph to categorize a relationship as proportional or nonproportional.

Previously

Students used tables, equations, and verbal descriptions to determine whether relationships were proportional and if so, found the constant of proportionality.

Coming Soon

In Lesson 12, students will find the constant of proportionality from graphs of proportional relationships.

Rigor

- Students develop **conceptual understanding** of graphs of proportional relationships by comparing the structures of proportional and nonproportional graphs.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	**Activity 1**	**Activity 2**	**Summary**	**Exit Ticket**
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (answers, for display)
- Activity 2 PDF, pre-cut cards, one set per group
- rulers (optional)

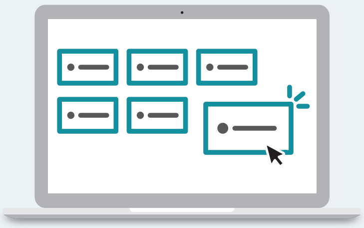
Math Language Development

- *constant of proportionality*
- *coordinate plane*
- *nonproportional relationship*
- *ordered pair*
- *origin*
- *proportional relationship*

Amps | **Featured Activity**

Activity 2
Digital Card Sort

Students sort graphs as to whether they are proportional or nonproportional by dragging and connecting them on screen.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might think that identifying one difference is sufficient for the task. Remind students that they can learn from each other. They should listen to others' arguments, too, as they build their sets of evidence. They also can help each other by looking for errors in the thinking so that a correction can be made. This requires engagement by all students.

Modifications to Pacing

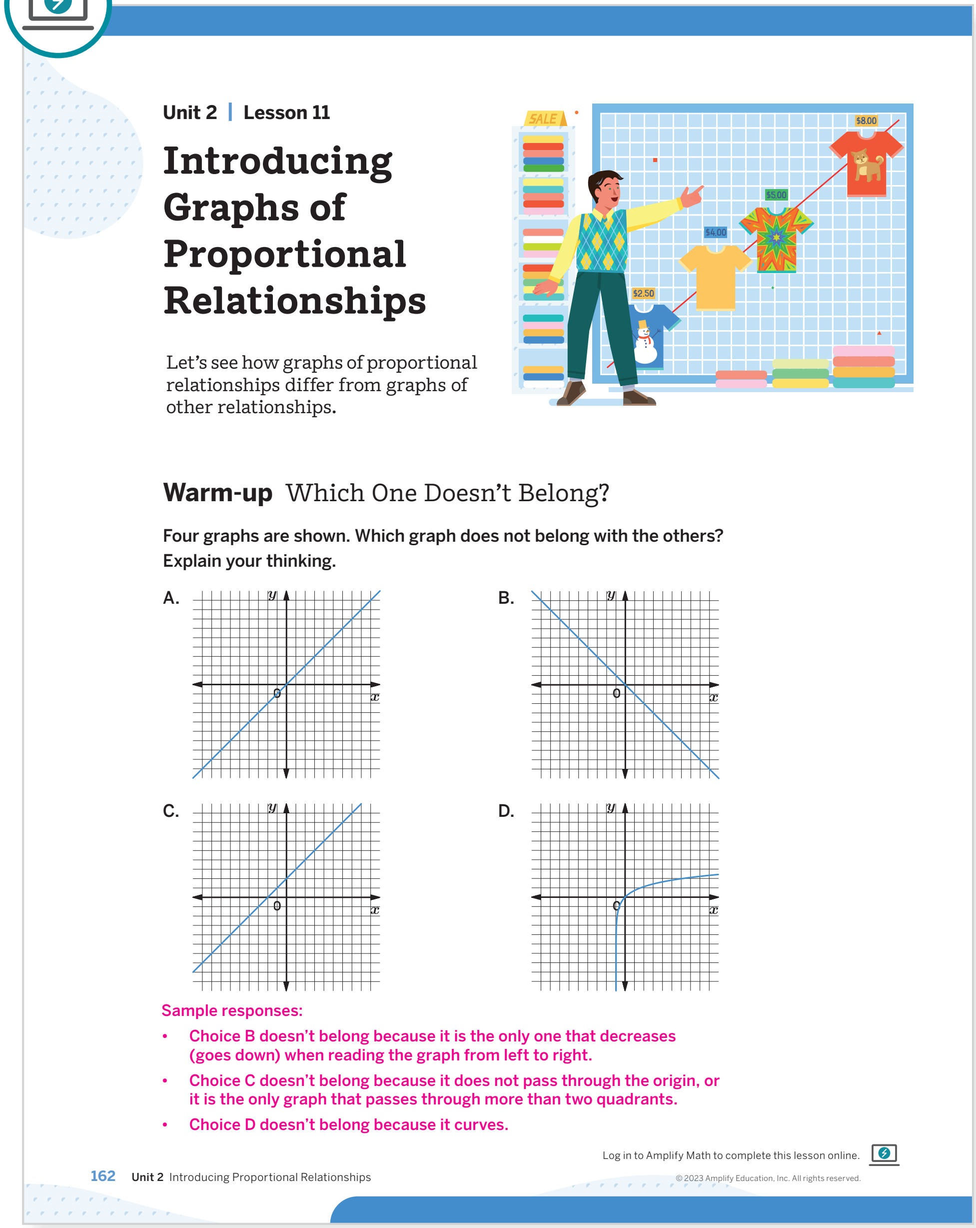
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up**.
- In **Activity 1**, complete Problems 3–5 as a whole-class discussion.
- In **Activity 2**, have students sort only Cards 1–8.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students analyze graphs to prepare them for understanding the characteristics of the graphs of proportional and nonproportional relationships.



Unit 2 | Lesson 11

Introducing Graphs of Proportional Relationships

Let's see how graphs of proportional relationships differ from graphs of other relationships.

Warm-up Which One Doesn't Belong?

Four graphs are shown. Which graph does not belong with the others? Explain your thinking.

Sample responses:

- **Choice B doesn't belong because it is the only one that decreases (goes down) when reading the graph from left to right.**
- **Choice C doesn't belong because it does not pass through the origin, or it is the only graph that passes through more than two quadrants.**
- **Choice D doesn't belong because it curves.**

Log in to Amplify Math to complete this lesson online.

162 **Unit 2** Introducing Proportional Relationships

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1 Launch

Conduct the ***Which One Doesn't Belong?*** routine.

2 Monitor

Help students get started by reminding them there is no right or wrong answer.

Look for points of confusion:

- **Having difficulty determining why Graph A does not belong.** If no one in the class can determine a reason, leave the question open and revisit Graph A during the Connect part of Activity 1.

Look for productive strategies:

- Determining reasons why each graph does not belong.
- Using the word *origin* to describe the point where the x- and y-axis meet.

3 Connect

Display the graphs.

Have students share their reasoning.

Highlight that when discussing graphs in mathematical situations, the terms *line* and *curve* are used to describe the graph. Anytime a graph is characterized as a line, it must be a straight line. Point out that Graph A is a relationship they will focus on this year. However, Graphs B and C are discussed in Grade 8 and Graph D is taught in high school. **Note:** The equations for the graphs are:

- Graph A: $y = x$
- Graph B: $y = -x$
- Graph C: $y = x + 2$,
- Graph D: $y = \ln(x + 1)$

Ask:

- "Which graph(s) are lines? Which graph(s) are curves?"
- "What is a coordinate plane? What is the origin? What is an ordered pair?"

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Provide copies of the graphs, pre-cut, so that students can sort them into various piles to help determine which one does not belong with the others. For example, they could place A, C, and D in one pile to help them see how B is the only graph that decreases from left to right. Consider asking these questions to help guide them:

- If A, C, and D belong together, what makes B different?
- If A, B, and C belong together, what makes D different?
- If A, B, and D belong together, what makes C different?

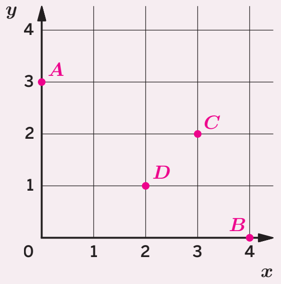
Power-up

To power up students' ability to plot points on a coordinate plane, have students complete:

Recall that the first number in a coordinate pair describes the horizontal distance from the origin, and the second number describes the vertical distance.

Plot each ordered pair on the coordinate plane.

A: $(0, 3)$ B: $(4, 0)$ C: $(3, 2)$ D: $(2, 1)$

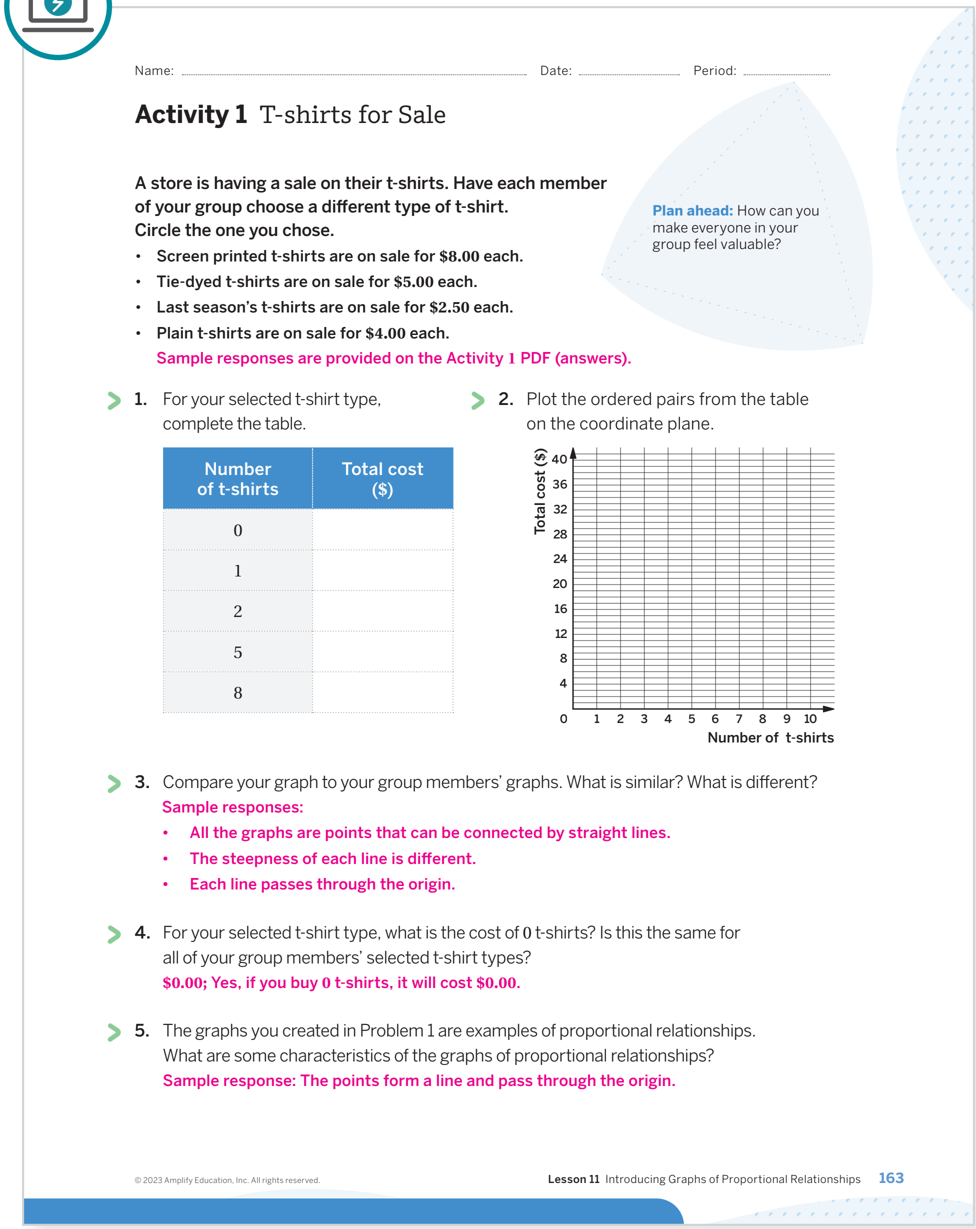


Use: Before Activity 1.
Informed by: Performance on Lesson 10, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5.

Small Groups | 15 min

Activity 1 T-shirts for Sale

Students plot points representing different t-shirt prices and compare graphical displays to determine the characteristics of graphs of proportional relationships.



Name: .... Date: .... Period: ....

Activity 1 T-shirts for Sale

A store is having a sale on their t-shirts. Have each member of your group choose a different type of t-shirt.
Circle the one you chose.

- **Screen printed t-shirts are on sale for $8.00 each.**
- **Tie-dyed t-shirts are on sale for $5.00 each.**
- **Last season's t-shirts are on sale for $2.50 each.**
- **Plain t-shirts are on sale for $4.00 each.**

Sample responses are provided on the Activity 1 PDF (answers).

Plan ahead: How can you make everyone in your group feel valuable?

1. For your selected t-shirt type, complete the table.

Number of t-shirts	Total cost ($)
0	
1	
2	
5	
8	

2. Plot the ordered pairs from the table on the coordinate plane.

3. Compare your graph to your group members' graphs. What is similar? What is different?
Sample responses:

- **All the graphs are points that can be connected by straight lines.**
- **The steepness of each line is different.**
- **Each line passes through the origin.**

4. For your selected t-shirt type, what is the cost of 0 t-shirts? Is this the same for all of your group members' selected t-shirt types?
$0.00; Yes, if you buy 0 t-shirts, it will cost $0.00.

5. The graphs you created in Problem 1 are examples of proportional relationships. What are some characteristics of the graphs of proportional relationships?
Sample response: The points form a line and pass through the origin.

Lesson 11 Introducing Graphs of Proportional Relationships 163

1 Launch

Activate background knowledge about students' familiarity with shopping at stores, specifically where items are displayed with the same price.

2 Monitor

Help students get started by asking, "How much would one t-shirt cost? Two t-shirts?"

Look for points of confusion:

- **Connecting the points.** At this time, it is allowed for students to connect the points. When your class is ready, mention when it is appropriate to connect the points or not.

3 Connect

Display the Activity 1 PDF (answers) and have students share their responses to Problem 3.

Ask:

- "What type of relationship do the tables represent?"
- "How do you know whether a graph represents a proportional relationship?" The points fall on a straight line that passes through the origin. This is because the ratios of the y-coordinates to their corresponding x-coordinates are constant ratios.
- "Could you buy 1.5 shirts, or $2\frac{3}{4}$ shirts?" No. I can't buy part of a shirt.

  Note: This scenario should be represented with discrete points; however, having students use a ruler to draw the line through the points may help students see the relationship easier.

Highlight that the graphs of proportional relationships are lines (or points contained on a line) passing through the origin.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Assign each group two of the four types of t-shirts and have them work together to complete Problems 1–2. For Problem 3, have each group visit another group who were assigned the remaining types of t-shirts.

Extension: *Math Enrichment*

Ask students to define variables and write an equation that represents the total cost of purchasing the *same number* of each type of t-shirt. Ask them if this relationship is also proportional. $y = 19.5x$, where y represents the total cost and x represents the number of each type of t-shirt purchased. This relationship is also proportional.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

During the Connect, as students share their responses to Problem 3, ask them to elaborate on their thinking by asking the following questions. Look for responses that include language such as *constant ratio of total cost to number of t-shirts, constant of proportionality*, etc.

- Why does it make sense that the points would fall on a straight line?
- Why does it make sense that this line would pass through the origin?

English Learners

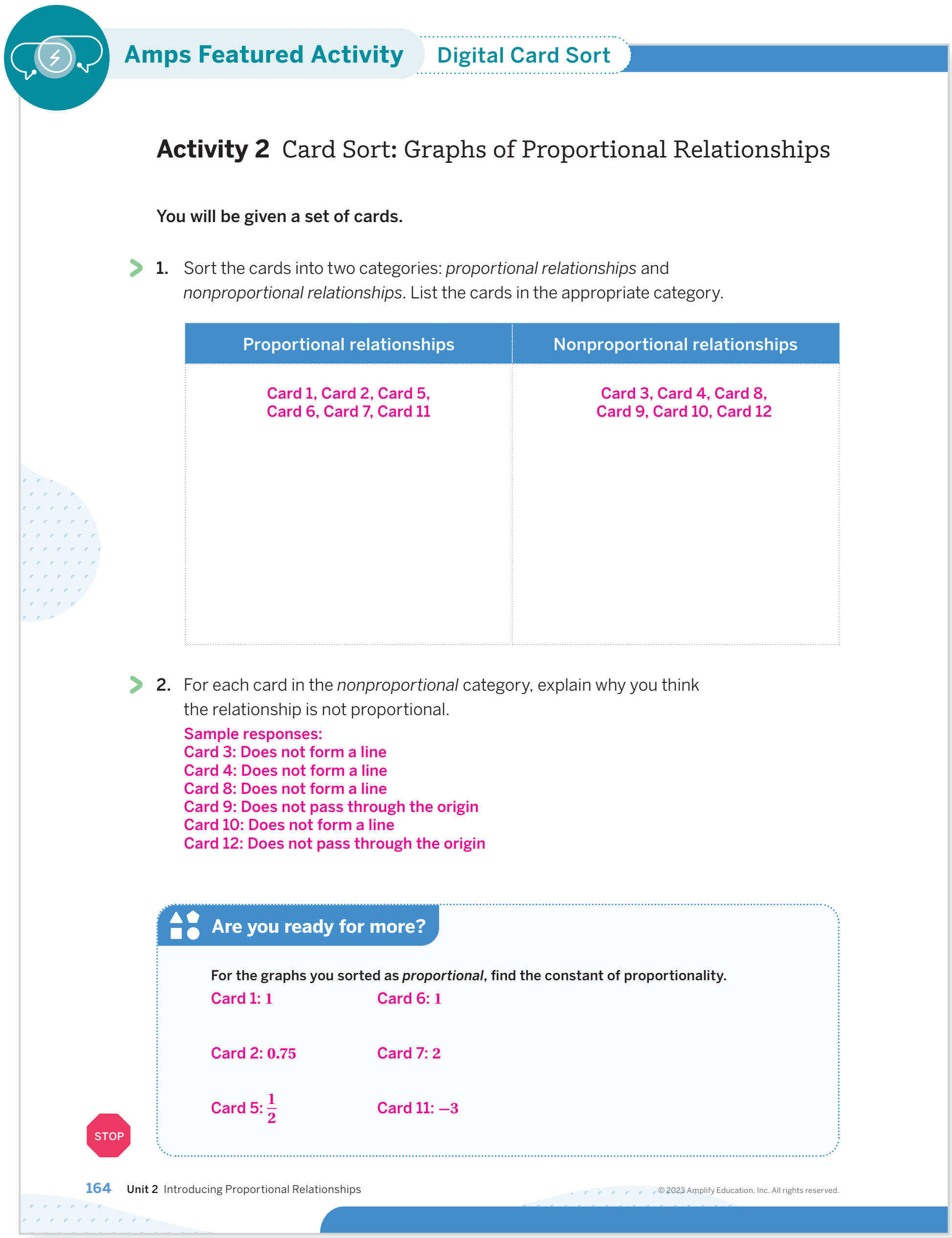
Consider using hand gestures, such as pointing to the points and moving your finger or hand to illustrate how these points form a straight line passing through the origin.

Small Groups | 15 min

Activity 2 Card Sort: Graphs of Proportional Relationships

Students sort graphs to distinguish between proportional and nonproportional relationships, which can be used as a formative assessment checkpoint.

Amps Featured Activity Digital Card Sort



Activity 2 Card Sort: Graphs of Proportional Relationships

You will be given a set of cards.

1. Sort the cards into two categories: *proportional relationships* and *nonproportional relationships*. List the cards in the appropriate category.

Proportional relationships	Nonproportional relationships
Card 1, Card 2, Card 5, Card 6, Card 7, Card 11	Card 3, Card 4, Card 8, Card 9, Card 10, Card 12

2. For each card in the *nonproportional* category, explain why you think the relationship is not proportional.

Sample responses:
Card 3: Does not form a line
Card 4: Does not form a line
Card 8: Does not form a line
Card 9: Does not pass through the origin
Card 10: Does not form a line
Card 12: Does not pass through the origin

Are you ready for more?

For the graphs you sorted as *proportional*, find the constant of proportionality.

Card 1: 1
Card 6: 1
Card 2: 0.75
Card 7: 2
Card 5: $\frac{1}{2}$
Card 11: -3

STOP

164 Unit 2 Introducing Proportional Relationships

1 Launch

Distribute the pre-cut cards from the Activity 2 PDF. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by asking what two characteristics a graph needs to show a proportional relationship.

Look for points of confusion:

- **Thinking that Card 7 is nonproportional.** Remind students that a line continues in both directions. Ask, "If this line was extended to the left, would it pass through the origin?"
- **Being unsure about Card 11.** Ask students if the graph has the characteristics necessary to make a proportional relationship. Let them know they will work more with these relationships in Grade 8.

3 Connect

Display any necessary cards to help with the discussion, and have students share their reasoning regarding their sorting, specifically how they knew whether a graph was nonproportional. Ask students to share any disagreements and their reasoning.

Highlight how the structure of a graph shows whether the relationship is proportional.

Ask:

- "If the graph is a curve, is it proportional?"
- "If the graph goes through the origin, is it proportional?"
- "Does it matter if the graph is a series of points or if it is a solid line?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For students who need more processing time, omit Cards 9–12 during the activity. Display these cards during the Connect discussion and use the ***Poll the Class*** routine as a quick formative assessment to see if students understand how they can use the structure of graphs to determine proportionality. Consider displaying the following as a checklist to assist students during this routine.

- Is the graph a straight line, or do the points fall on a straight line?
- Does the graph pass through the origin?

Tell students the answer to *both* of these questions must be "yes" in order for the relationship to be proportional.

Math Language Development

MLR3: Critique, Correct, Clarify

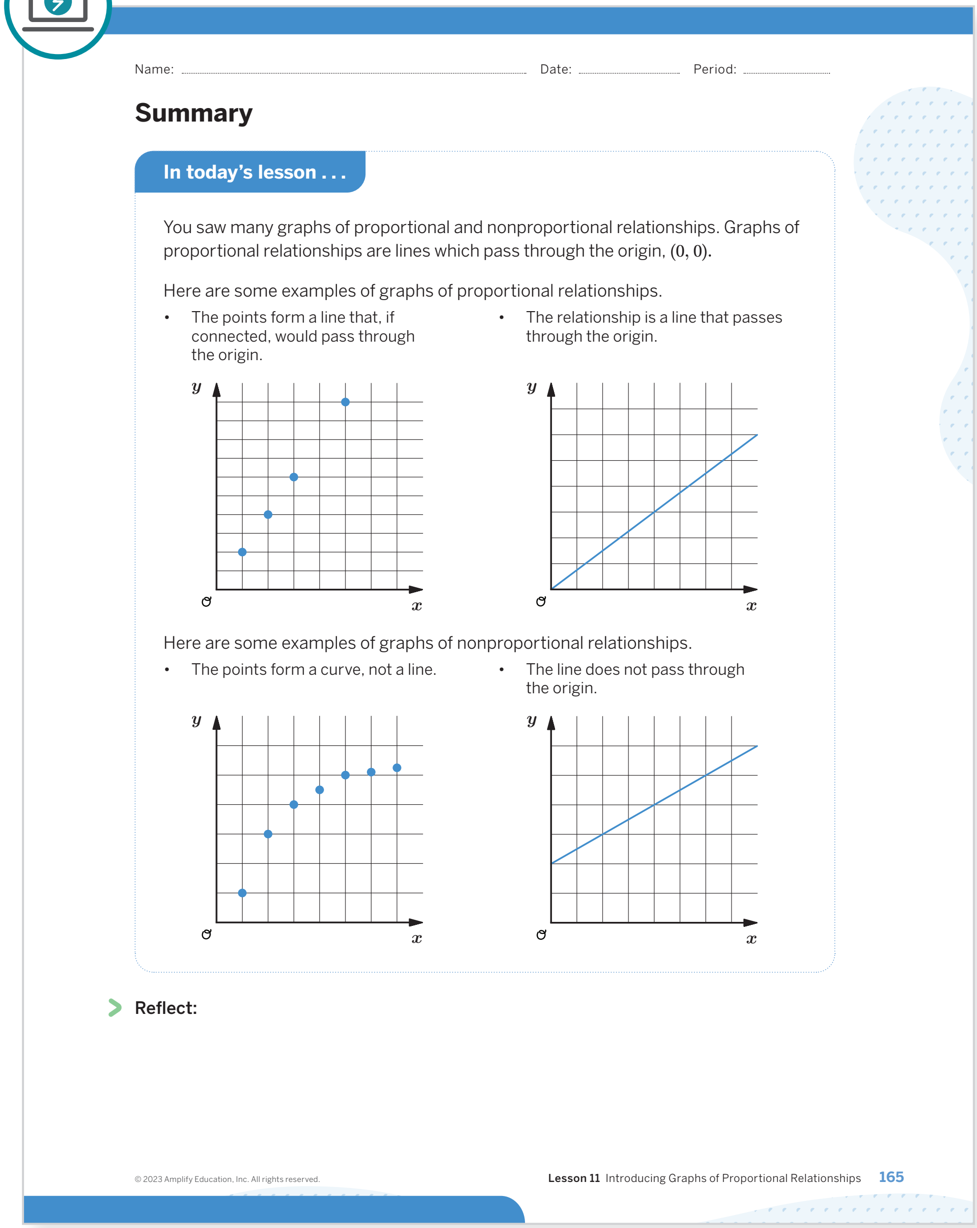
During the Connect, display a relationship that might represent a common misconception, such as Card 6, and an incorrect statement, such as "This relationship is not proportional because the points are not connected with a straight line." Ask pairs of students to critique this statement, write a corrected statement, and clarify their reasoning as to why and how they corrected it.

English Learners

Consider using hand gestures, such as your arm to illustrate a solid line, as in Card 5, and point to each dot in Card 6 to illustrate a series of points.

Summary

Review and synthesize the characteristics of the graphs of proportional relationships.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw many graphs of proportional and nonproportional relationships. Graphs of proportional relationships are lines which pass through the origin, $(0, 0)$.

Here are some examples of graphs of proportional relationships.

- The points form a line that, if connected, would pass through the origin.
- The relationship is a line that passes through the origin.

Here are some examples of graphs of nonproportional relationships.

- The points form a curve, not a line.
- The line does not pass through the origin.

Reflect:

Synthesize

Display examples of proportional and nonproportional graphs.

Have students share what they look for in determining whether a graph represents a proportional relationship.

Highlight that a proportional relationship is a line or a series of points in a line passing through the origin.

Ask, "Do you always connect the points to show the proportional relationship?" The context of the scenario might be one where the points should not be connected, but it is common to draw the line to see whether the relationship is proportional.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

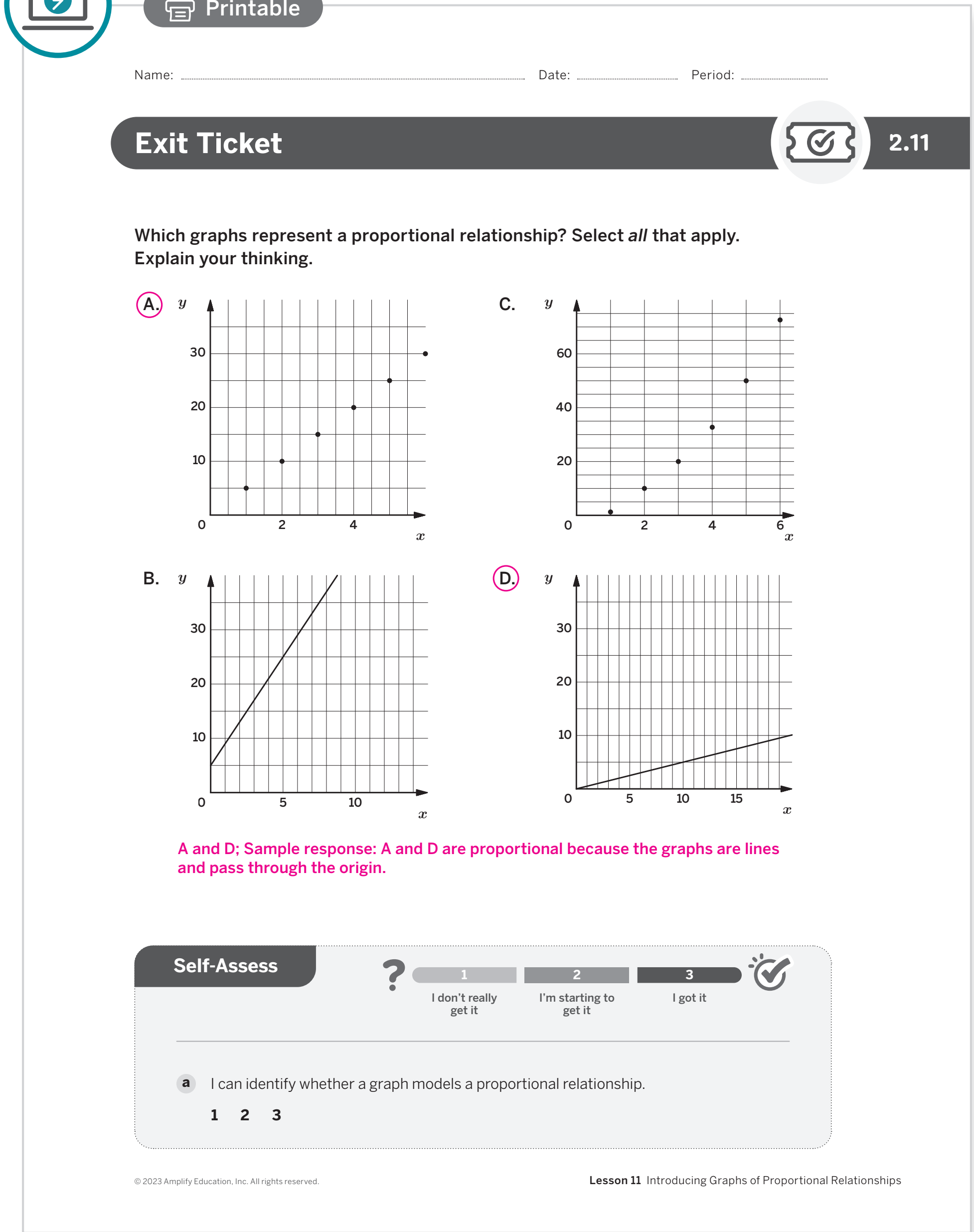
- "What are the different ways in which you can represent proportional relationships? How are the representations related?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the characteristics of graphs by selecting the proportional relationships.

Printable



Name: Date: Period:

Exit Ticket 2.11

Which graphs represent a proportional relationship? Select *all* that apply. Explain your thinking.

A and D; Sample response: A and D are proportional because the graphs are lines and pass through the origin.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can identify whether a graph models a proportional relationship.

1 2 3

Success looks like . . .

- **Language Goal:** Comparing and contrasting graphs of relationships. **(Speaking and Listening)**
  - » Comparing all four graphs to determine the ones that represent a proportional relationship.
- **Language Goal:** Generalizing that a proportional relationship can be represented on the coordinate plane by a line that includes the origin, or by a collection of points that lie on such a line. **(Speaking and Listening, Writing)**
  - » Selecting choices A and D as graphs representing proportional relationships.

Suggested next steps

If students select Choice B, consider:

- Reviewing Activity 1.
- Assigning Practice Problem 1.

If students select Choice C, consider:

- Having them use a ruler to connect the points.
- Assigning Practice Problem 1.

Professional Learning

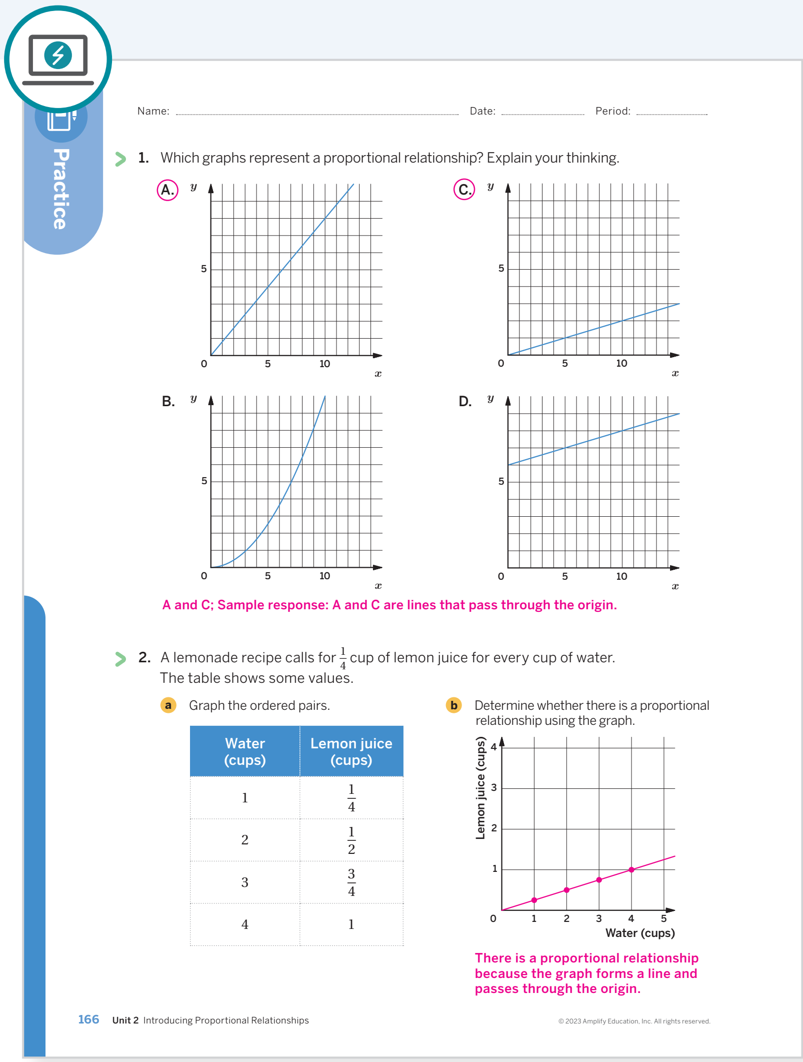
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students determined the characteristics of graphs of proportional relationships?
- What did the ***Card Sort*** reveal about your students as learners? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Which graphs represent a proportional relationship? Explain your thinking.

A and C; Sample response: A and C are lines that pass through the origin.

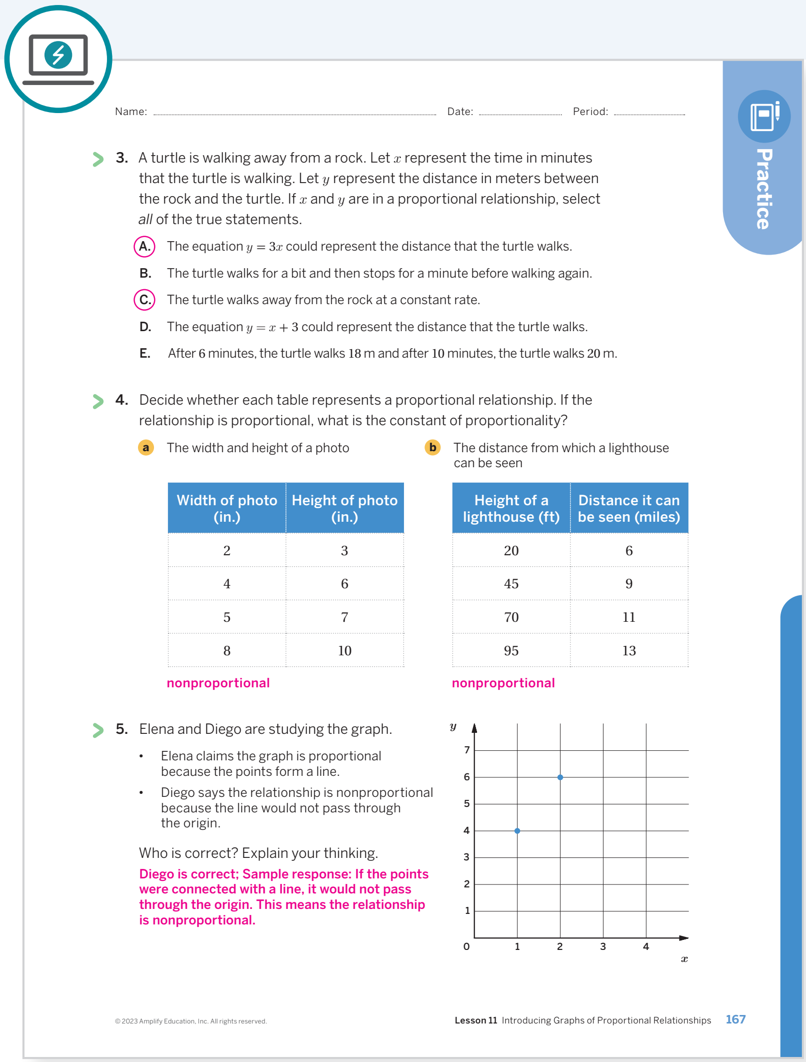
2. A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every cup of water. The table shows some values.

   a. Graph the ordered pairs.

Water (cups)	Lemon juice (cups)
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

   b. Determine whether there is a proportional relationship using the graph.

There is a proportional relationship because the graph forms a line and passes through the origin.



Practice

Name: Date: Period:

3. A turtle is walking away from a rock. Let x represent the time in minutes that the turtle is walking. Let y represent the distance in meters between the rock and the turtle. If x and y are in a proportional relationship, select *all* of the true statements.

   [✓ A.] The equation $y = 3x$ could represent the distance that the turtle walks.
   B. The turtle walks for a bit and then stops for a minute before walking again.
   [✓ C.] The turtle walks away from the rock at a constant rate.
   D. The equation $y = x + 3$ could represent the distance that the turtle walks.
   E. After 6 minutes, the turtle walks 18 m and after 10 minutes, the turtle walks 20 m.

4. Decide whether each table represents a proportional relationship. If the relationship is proportional, what is the constant of proportionality?

   a. The width and height of a photo

Width of photo (in.)	Height of photo (in.)
2	3
4	6
5	7
8	10

nonproportional

   b. The distance from which a lighthouse can be seen

Height of a lighthouse (ft)	Distance it can be seen (miles)
20	6
45	9
70	11
95	13

nonproportional

5. Elena and Diego are studying the graph.

   - Elena claims the graph is proportional because the points form a line.
   - Diego says the relationship is nonproportional because the line would not pass through the origin.

   Who is correct? Explain your thinking.

   Diego is correct; Sample response: If the points were connected with a line, it would not pass through the origin. This means the relationship is nonproportional.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On Lesson	1	Activity 2	2
	2	Activity 1	2
Spiral	3	Unit 2 Lesson 8	2
	4	Unit 2 Lesson 4	2
Formative ⚡	5	Unit 2 Lesson 12	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 12**

Interpreting Graphs of Proportional Relationships

Let's read stories from the graphs of proportional relationships.

Focus

Goals

1. Create the graph of a proportional relationship given only one pair of values, by drawing the line that connects the given point and $(0, 0)$.
2. Identify the constant of proportionality from the graph of a proportional relationship and write an equation.
3. **Language Goal:** Interpret points on the graph of a proportional relationship. **(Speaking and Listening, Writing)**

Coherence

Today

Students make connections between the graph and the context modeled by the proportional relationship, and between the graph and the equation it represents. Using the graph, they reason about the situation it represents. They interpret the meaning of the point $(1, k)$ on the graph, both in terms of the constant of proportionality k in the equation $y = kx$, and in terms of a constant rate in the context.

Previously

In Lesson 11, students learned a proportional relationship lies on a line through the origin, and they became comfortable using the term *origin* to describe the point $(0, 0)$.

Coming Soon

In Lessons 13–14, students continue their work with graphical representations of proportional relationships.

Rigor

- Students evolve their **conceptual understanding** of proportional relationships by focusing on the graphical representation.
- Students **apply** their understanding of the constant of proportionality in a table or equation to show how it is represented in the graph.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF (as needed)
- Anchor Chart PDF, *Representing Proportional Relationships*
- rulers

Math Language Development

Review words

- *constant of proportionality*
- *coordinate plane*
- *nonproportional relationship*
- *ordered pair*
- *origin*
- *proportional relationship*
- *unit rate*

Amps | Featured Activity

Activity 1

Interactive Graphs

Students draw a line through several points, creating a proportional relationship.

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Building Math Identity and Community

Connecting to Mathematical Practices

When describing their conclusions about graphs and proportional relationships, students might try to generalize their thinking or avoid new vocabulary. Praise efforts to use mathematically precise language. Ask questions that push them to release their inhibitions and use words that are not part of their everyday conversation.

Modifications to Pacing

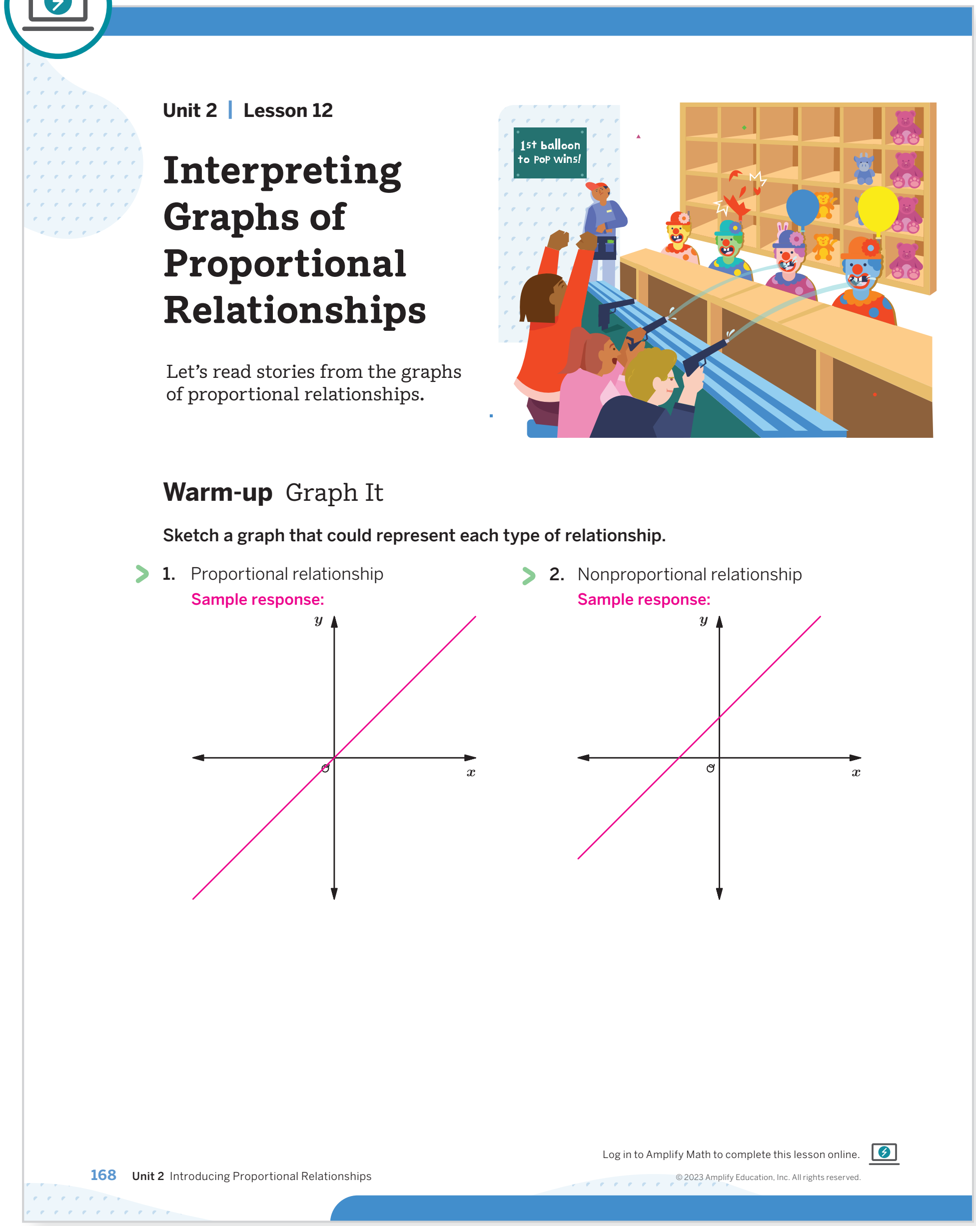
You may want to consider these additional modifications if you are short on time.

- **Activity 2** mirrors **Activity 1**. If your class needs an additional problem, assign **Activity 2**. If your class is ready to make their own proportional relationships, assign **Activity 3** instead.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up Graph It

Students sketch graphs to show their understanding of proportional and nonproportional graphs.



Unit 2 | Lesson 12

Interpreting Graphs of Proportional Relationships

Let's read stories from the graphs of proportional relationships.

Warm-up Graph It

Sketch a graph that could represent each type of relationship.

1. Proportional relationship
   Sample response:

2. Nonproportional relationship
   Sample response:

Log in to Amplify Math to complete this lesson online.

168 Unit 2 Introducing Proportional Relationships

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1 Launch

Activate prior knowledge by having students reference any material from Lesson 11 to review the characteristics of a graph of a proportional relationship. Provide access to rulers.

2 Monitor

Help students get started by asking how they know whether a graph is a proportional relationship.

Look for points of confusion:

- **Switching the graphs.** Review with the students that a proportional relationship is a line through the origin.

Look for productive strategies:

- Labeling the x- and y-axes with variables which belong in the requested relationship.
- Drawing a line through the origin with a negative slope for the proportional relationship.

3 Connect

Have students share their sketches and display them.

Highlight that *proportional relationships* are all lines (or points on a line) passing through the origin. *Nonproportional relationships* do not pass through the origin and/or they are not lines.

Ask, "By looking at a graph, you can tell whether the relationship is proportional, but how do you think we can determine what the *constant of proportionality* is?" **Note:** It is not expected students can articulate how to find the constant of proportionality at this time.

Math Language Development

MLR7: Compare and Connect

During the Connect, display all of the proportional relationships that students sketched. Ask students to compare and contrast these graphs. Then display all of the nonproportional relationships that students sketched. Ask students to compare and contrast these graphs. Look for and highlight language such as *straight line* or *pass through the origin*.

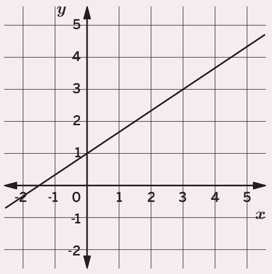
English Learners

As students or you use the phrase pass through the origin, point to or otherwise annotate the location of the origin on each graph.

Power-up

To power up students' ability to analyze a graph to determine whether it is proportional or nonproportional, have students complete:

Recall that a graph is proportional if it is a straight line through the origin. Determine if the graph is proportional or not by checking for each characteristic.



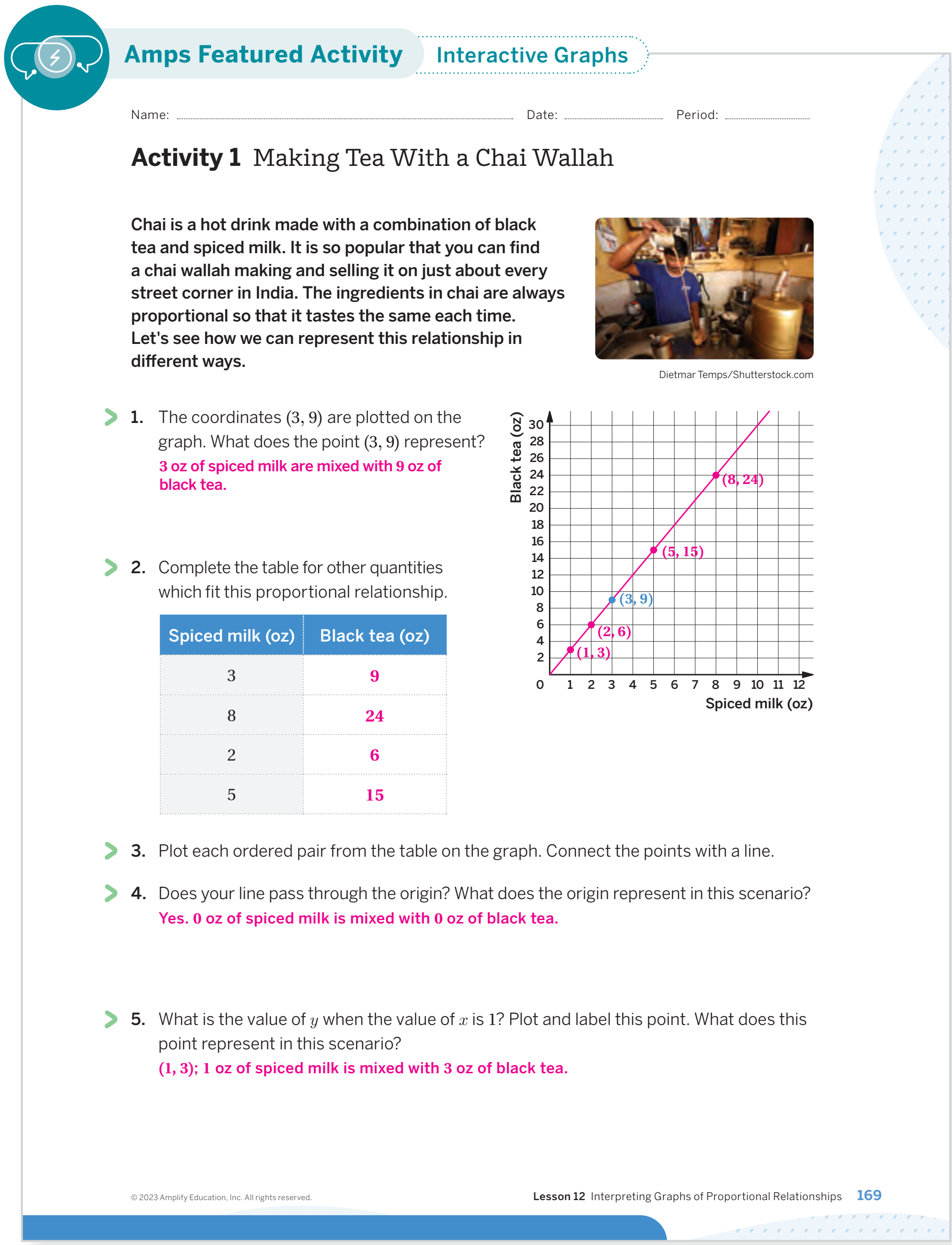
	Yes	No
Line	☑	☐
Origin	☐	☑
Proportional	☐	☑

Use: Before the Warm-up.
Informed by: Performance on Lesson 11, Practice Problem 5.

Small Groups | 15 min

Activity 1 Making Tea With a Chai Wallah

Students analyze a proportional relationship to find the constant of proportionality from a graph and interpret its meaning.



Amps Featured Activity Interactive Graphs

Name: ______________ Date: ________ Period: ________

Activity 1 Making Tea With a Chai Wallah

Chai is a hot drink made with a combination of black tea and spiced milk. It is so popular that you can find a chai wallah making and selling it on just about every street corner in India. The ingredients in chai are always proportional so that it tastes the same each time. Let's see how we can represent this relationship in different ways.

Dietmar Temps/Shutterstock.com

1. The coordinates $(3, 9)$ are plotted on the graph. What does the point $(3, 9)$ represent?
   3 oz of spiced milk are mixed with 9 oz of black tea.

2. Complete the table for other quantities which fit this proportional relationship.

Spiced milk (oz)	Black tea (oz)
3	9
8	24
2	6
5	15

3. Plot each ordered pair from the table on the graph. Connect the points with a line.

4. Does your line pass through the origin? What does the origin represent in this scenario?
   Yes. 0 oz of spiced milk is mixed with 0 oz of black tea.

5. What is the value of y when the value of x is 1? Plot and label this point. What does this point represent in this scenario?
   $(1, 3)$; 1 oz of spiced milk is mixed with 3 oz of black tea.

1 Launch

Activate students' background knowledge by asking whether their families have a traditional food or drink for their family or culture. Provide students access to rulers.

2 Monitor

Help students get started by explaining that the 3 represents the amount of spiced milk and asking what the 9 represents in the ordered pair $(3, 9)$.

Look for points of confusion:

- **Not drawing the line through the entire graph paper and thinking their line does not pass through the origin.** Remind students to use a ruler and draw the line beyond the points they plotted.

Look for productive strategies:

- Finding the constant of proportionality from the table to help explain how to find it from a graph.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Technology*

Provide a table and graph already completed and have students omit Problems 2 and 3. This will allow students to focus on analyzing and interpreting the points on the graph within the context of the scenario and then to make connections between the table, graph, and equation. Alternatively, have students use the Amps slides for this activity, in which they can use digital technology to create the line of the proportional relationship.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the completed table, graph, and equation and ask students how the constant of proportionality is illustrated by each representation. Listen for and amplify language, such as *constant ratio*, *ordered pair*, *the value of y when the value of x is 1*, *coefficient of x*, etc.

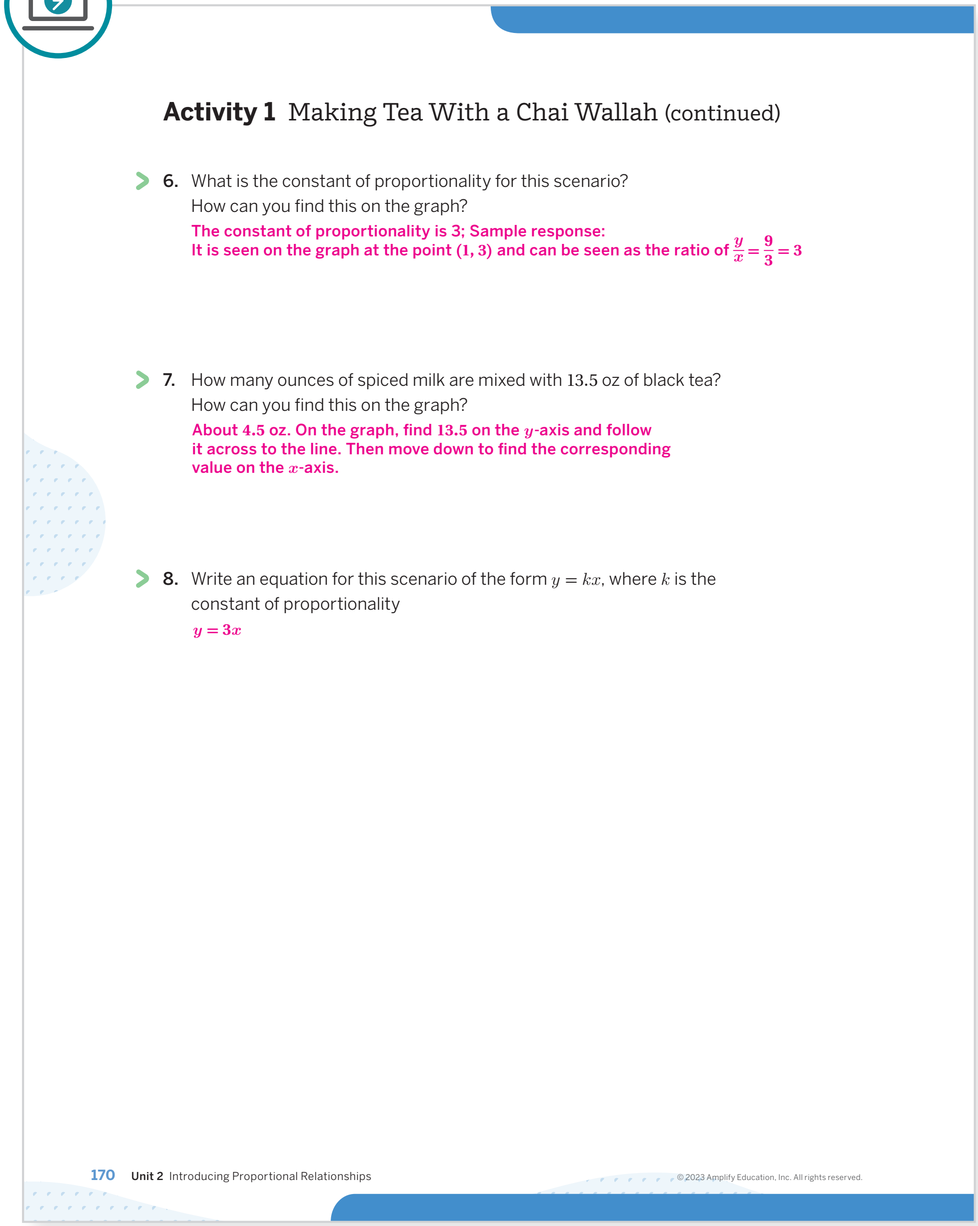
English Learners

Annotate where the constant of proportionality is seen in each representation by writing constant of proportionality with an arrow pointing to its value.

Small Groups | 15 min

Activity 1 Making Tea With a Chai Wallah (continued)

Students analyze a proportional relationship to find the constant of proportionality from a graph and interpret its meaning.



Activity 1 Making Tea With a Chai Wallah (continued)

6. What is the constant of proportionality for this scenario? How can you find this on the graph?
   The constant of proportionality is 3; Sample response: It is seen on the graph at the point (1, 3) and can be seen as the ratio of $\frac{y}{x} = \frac{9}{3} = 3$

7. How many ounces of spiced milk are mixed with 13.5 oz of black tea? How can you find this on the graph?
   About 4.5 oz. On the graph, find 13.5 on the y-axis and follow it across to the line. Then move down to find the corresponding value on the x-axis.

8. Write an equation for this scenario of the form $y = kx$, where k is the constant of proportionality
   $y = 3x$

170 **Unit 2** Introducing Proportional Relationships

3 Connect

Have students share their graphs, explanations, and reasoning, particularly paying attention to precise descriptions of the variables in the scenario.

Ask:

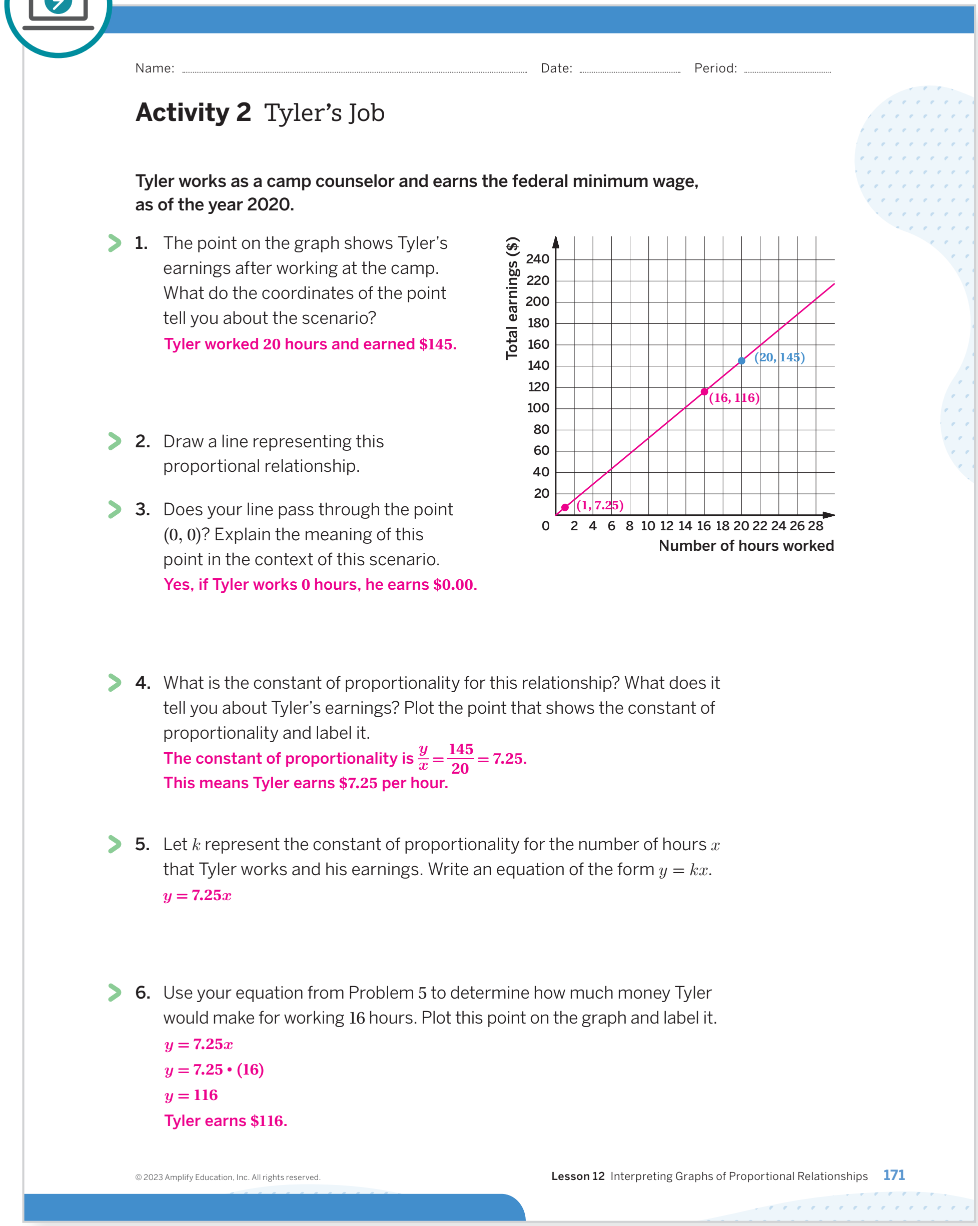
- "How can you use the equation from Problem 8 to check your answer from Problem 7?"
- "How can you find the constant of proportionality from a graph?"
- "Can you use any point on the line to find the constant of proportionality?"

Highlight that the graph is a visual display of the table and represents the equation students wrote in Problem 8. The graph of a proportional relationship models the same information as the rest of the representations. Finding the constant of proportionality from a graph is similar to that of a table: find the y-value paired with an x-value of 1 or find the ratio between the coordinates of an ordered pair.

Small Groups | 15 min

Activity 2 Tyler's Job

Students use a graph to model a proportional relationship, find the constant of proportionality, and interpret the meaning in context.



Name: ______ Date: ______ Period: ______

Activity 2 Tyler's Job

Tyler works as a camp counselor and earns the federal minimum wage, as of the year 2020.

1. The point on the graph shows Tyler's earnings after working at the camp. What do the coordinates of the point tell you about the scenario?
   Tyler worked 20 hours and earned $145.
2. Draw a line representing this proportional relationship.
3. Does your line pass through the point $(0, 0)$? Explain the meaning of this point in the context of this scenario.
   Yes, if Tyler works 0 hours, he earns $0.00.
4. What is the constant of proportionality for this relationship? What does it tell you about Tyler's earnings? Plot the point that shows the constant of proportionality and label it.
   The constant of proportionality is $\frac{y}{x} = \frac{145}{20} = 7.25$.
   This means Tyler earns $7.25 per hour.
5. Let k represent the constant of proportionality for the number of hours x that Tyler works and his earnings. Write an equation of the form $y = kx$.
   $y = 7.25x$
6. Use your equation from Problem 5 to determine how much money Tyler would make for working 16 hours. Plot this point on the graph and label it.
   $y = 7.25x$
   $y = 7.25 \cdot (16)$
   $y = 116$
   Tyler earns $116.

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Lesson 12 Interpreting Graphs of Proportional Relationships 171

1 Launch

Activate background knowledge by asking students if they know what the minimum wage is for their state. Let students know that the scales on the x- and y-axis are different.

2 Monitor

Help students get started by asking which axis explains the meaning of the 20 and which axis explains the meaning of the 145.

Look for points of confusion:

- **Thinking they do not have enough information to draw the line**. Ask students what the characteristics are of a graph or a proportional relationship, and then ensure they draw a line through the origin and through $(20, 145)$.

Look for productive strategies:

- Using $(1, k)$ or $\frac{y}{x}$ to find the constant of proportionality.

3 Connect

Display the graph.

Have students share their explanations and reasoning to the following questions. Encourage the use of their developing math language.

Ask:

- "What quantities are shown in the graph?"
- "What information do the coordinates on the graph give you?"
- "Why does the y-value paired with an x-value of 1 indicate the constant of proportionality?"

Highlight that the points on a graph can be interpreted in the context provided. The quotient of ordered pairs, excluding the point $(0, 0)$, is the constant of y-proportionality. If the x-coordinate is 1, then the corresponding coordinate is the constant of proportionality.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide a completed graph with the three points already labeled and have students omit Problem 2. This will allow students to focus on analyzing and interpreting the points on the graph within the context of the scenario and then to make connections between the graph and equation.

Extension: *Math Enrichment*

Ask students to solve their equation they wrote in Problem 6, so that it is in the form $x =$___y and ask them what this equation represents.
$x = \frac{y}{7.25}$ or $x = \frac{1}{7.25}y$; This equation gives the number of hours worked if you know how much money total Tyler made.

Math Language Development

MLR8: Discussion Supports—Revoicing

Using the questions from Connect, ask students to revoice or restate their peers' reasoning before they respond with their own reasoning. Encourage students to refer to the class display as they use developing mathematical language.

English Learners

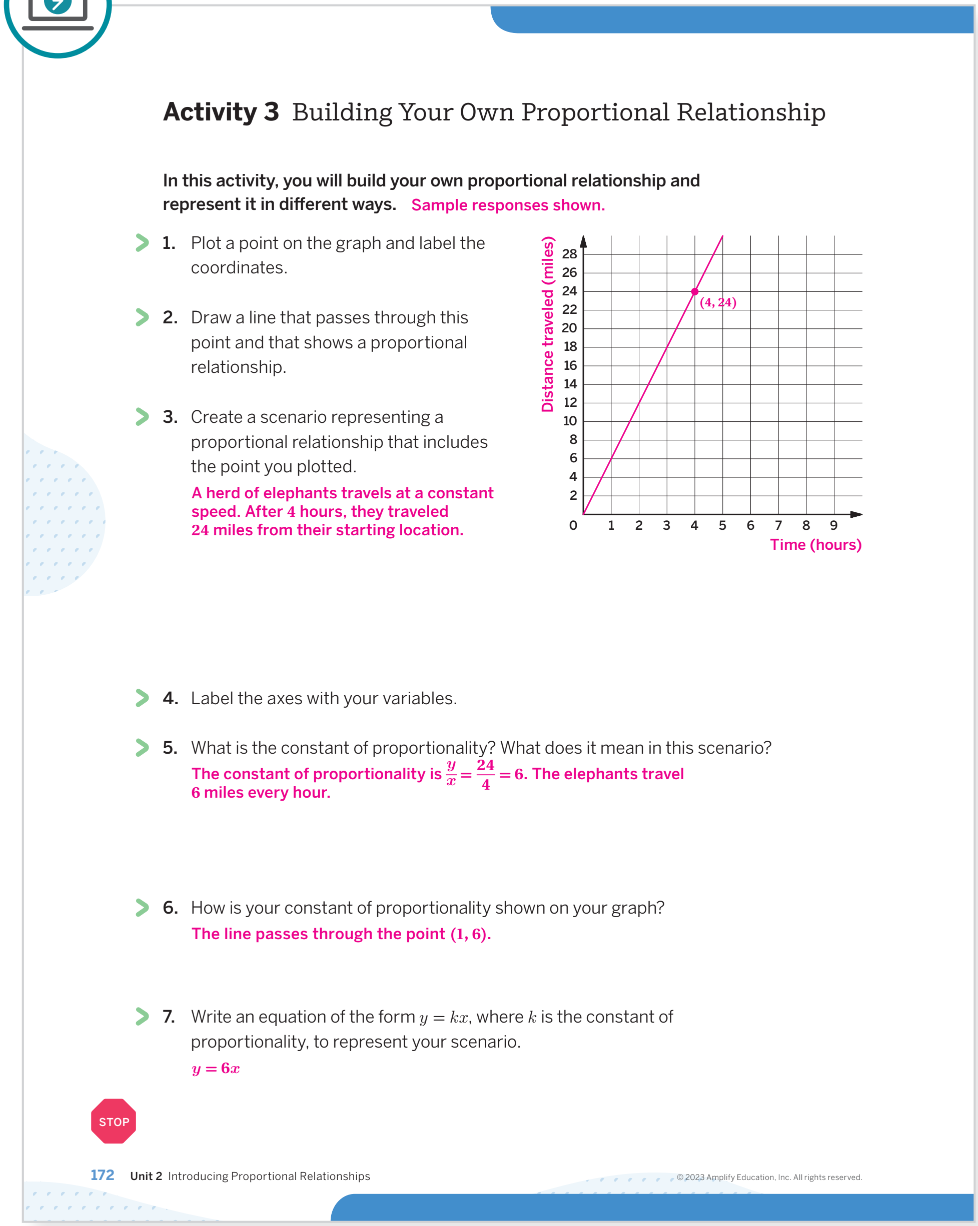
When asking the third question, "Why does the y-value paired with an x-value of 1 indicate the constant of proportionality," point to the ordered pair $(1, 7.25)$ and highlight 7.25 as you say y-value. Highlight 1 as you say x-value of 1.

Optional

Small Groups | 15 min

Activity 3 Building Your Own Proportional Relationship

Students plot a point on the coordinate plane and model a proportional relationship to practice interpreting the constant of proportionality.



Activity 3 Building Your Own Proportional Relationship

In this activity, you will build your own proportional relationship and represent it in different ways. Sample responses shown.

1. Plot a point on the graph and label the coordinates.
2. Draw a line that passes through this point and that shows a proportional relationship.
3. Create a scenario representing a proportional relationship that includes the point you plotted.
   A herd of elephants travels at a constant speed. After 4 hours, they traveled 24 miles from their starting location.

4. Label the axes with your variables.
5. What is the constant of proportionality? What does it mean in this scenario?
   The constant of proportionality is $\frac{y}{x} = \frac{24}{4} = 6$. The elephants travel 6 miles every hour.
6. How is your constant of proportionality shown on your graph?
   The line passes through the point $(1, 6)$.
7. Write an equation of the form $y = kx$, where k is the constant of proportionality, to represent your scenario.
   $y = 6x$

STOP

1 Launch

Activate prior knowledge by having students list everything they know about proportional relationships. Record their information and display for reference during this activity.

2 Monitor

Help students get started by asking what information they want their point to represent and have them label their axes with these variables.

Look for points of confusion:

- **Not being able to create a scenario.** Have students review past problems and use them as a guide to create their own scenario.
- **Not drawing the line through the origin.** Remind students about the characteristics of the graph of a proportional relationship.

Look for productive strategies:

- Recognizing whether their scenario only makes sense for whole-number values and plotting discrete points instead of a continuous line.

3 Connect

Have students share their proportional relationship graphs and ask the class to find the constant of proportionality.

Highlight that knowing one point is enough to draw a proportional relationship because the line must pass through the origin. The point plotted in Problem 1 is used to find the constant of proportionality by finding the ratio of the y-coordinate and the x-coordinate.

Ask:

- "How do you know your graph is proportional?"
- "How can you find the constant of proportionality from your graph?"

Differentiated Support

Accessibility: ***Activate Background Knowledge***

Ask students if they can think of any proportional relationships they may have encountered in their everyday lives. Consider providing some examples to help them get started, such as:

- Purchasing several items where each item costs the same amount
- Riding in a car/riding a bicycle/walking/running at a constant speed
- Following a recipe and wanting to make larger or smaller batches
- Working a part-time job and being paid the same amount per hour

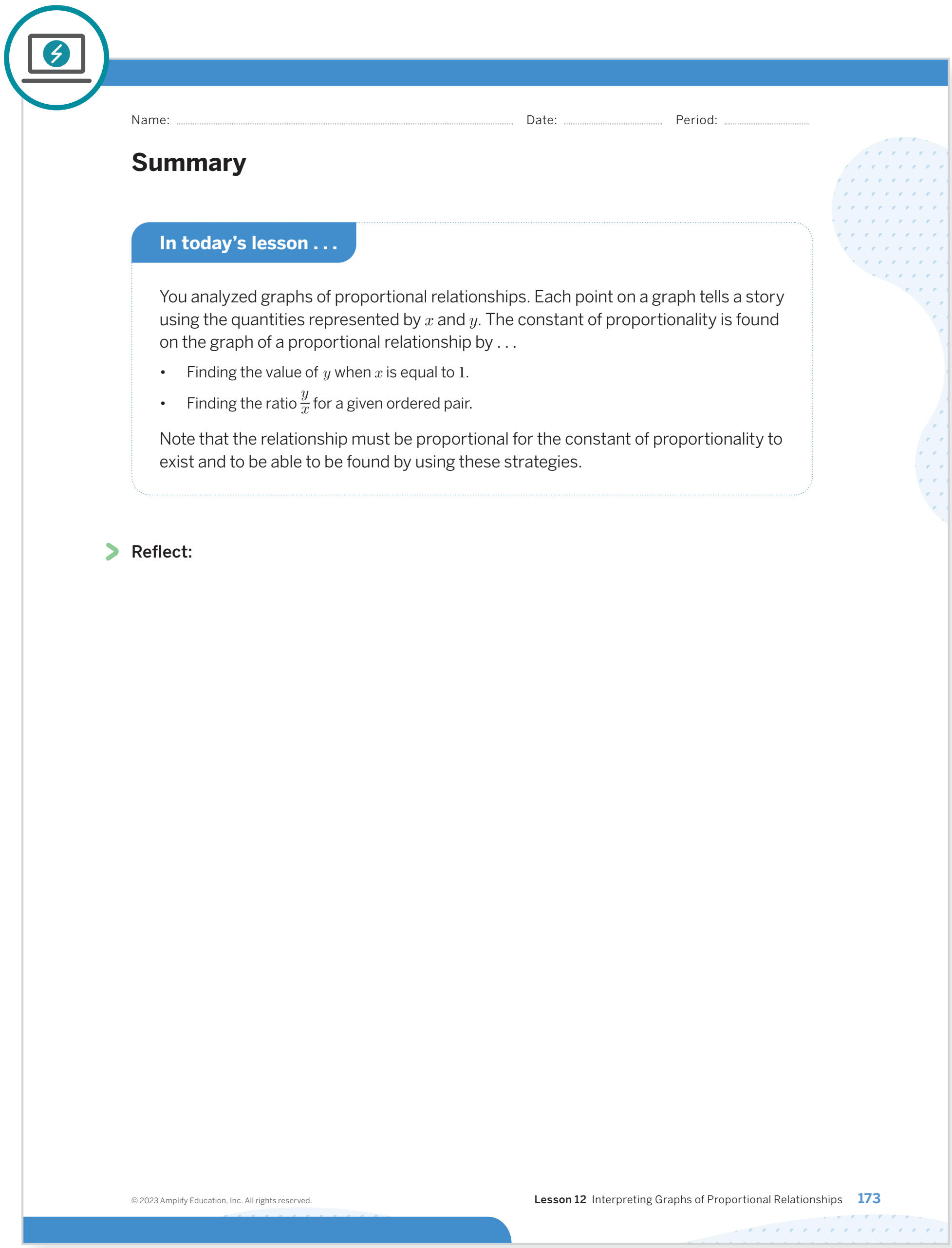
Accessibility: ***Vary Demands to Optimize Challenge***

Instead of having students come up with their own scenarios, have them choose variables from the different lists shown on the Activity 3 PDF. For example, they could choose to compare the distance, in centimeters, traveled by an earthworm related to the time, in seconds. They can estimate how many centimeters they think an earthworm can travel per second, or you could use the internet to help them research or provide this information to them, e.g., a large earthworm can travel about 2 cm per second.

Whole Class | 5 min

Summary

Review and synthesize how to find the constant of proportionality from a graph.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You analyzed graphs of proportional relationships. Each point on a graph tells a story using the quantities represented by x and y. The constant of proportionality is found on the graph of a proportional relationship by . . .

- Finding the value of y when x is equal to 1.
- Finding the ratio $\frac{y}{x}$ for a given ordered pair.

Note that the relationship must be proportional for the constant of proportionality to exist and to be able to be found by using these strategies.

Reflect:

Synthesize

Display the Anchor Chart PDF, *Representing Proportional Relationships*, and have students complete the graph section.

Highlight that every point on a graph tells a story based on the variables being measured. The constant of proportionality can be found from a graph by finding the ratio of $\frac{y}{x}$ for any ordered pair or by locating the point $(1, k)$.

Ask:

- "Why do you think the corresponding y-value is the constant of proportionality when x equals 1?" Sample response: It is known as the unit rate, which compares the y-value to 1 unit.
- "How can you use a graph to write an equation representing a proportional relationship?" Determine the constant of proportionality from an ordered pair, and write the equation using that value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

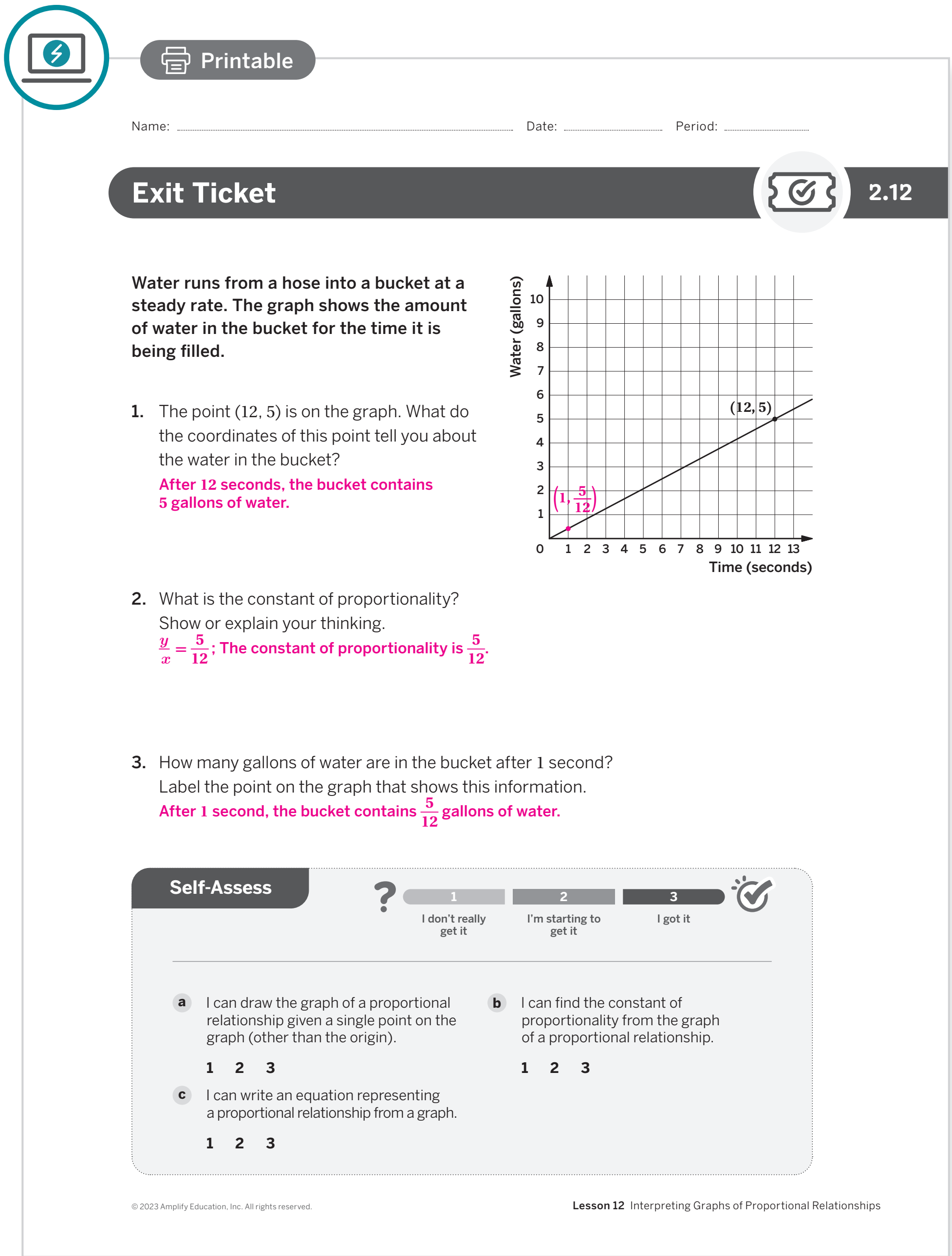
- "What are the different ways you can represent proportional relationships? How are the representations related?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of finding the constant of proportionality by analyzing a graph.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 2.12

Water runs from a hose into a bucket at a steady rate. The graph shows the amount of water in the bucket for the time it is being filled.

1. The point $(12, 5)$ is on the graph. What do the coordinates of this point tell you about the water in the bucket?
   After 12 seconds, the bucket contains 5 gallons of water.

2. What is the constant of proportionality? Show or explain your thinking.
   $\frac{y}{x} = \frac{5}{12}$; The constant of proportionality is $\frac{5}{12}$.

3. How many gallons of water are in the bucket after 1 second? Label the point on the graph that shows this information.
   After 1 second, the bucket contains $\frac{5}{12}$ gallons of water.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can draw the graph of a proportional relationship given a single point on the graph (other than the origin).
1 2 3

b I can find the constant of proportionality from the graph of a proportional relationship.
1 2 3

c I can write an equation representing a proportional relationship from a graph.
1 2 3

Success looks like . . .

- **Goal:** Creating the graph of a proportional relationship given only one pair of values, by drawing the line that connects the given point and $(0, 0)$.
- **Goal:** Identifying the constant of proportionality from the graph of a proportional relationship and writing an equation.
  - » Determining the constant of proportionality from the graph in Problem 2.
- **Language Goal:** Interpreting points on the graph of a proportional relationship. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of the point $(12,5)$ on the graph in Problem 1.

Suggested next steps

If students do not explain the meaning of the point in Problem 1, consider:

- Reviewing Problem 1 from Activity 1 or Activity 2.
- Assigning Practice Problems 1 and 2.

If students incorrectly find the constant of proportionality, consider:

- Reviewing Problem 6 from Activity 1.
- Assigning Practice Problem 2.

If students incorrectly answer Problem 3, consider:

- Reviewing Problem 5 from Activity 2.
- Assigning Practice Problems 1 and 2.

Professional Learning

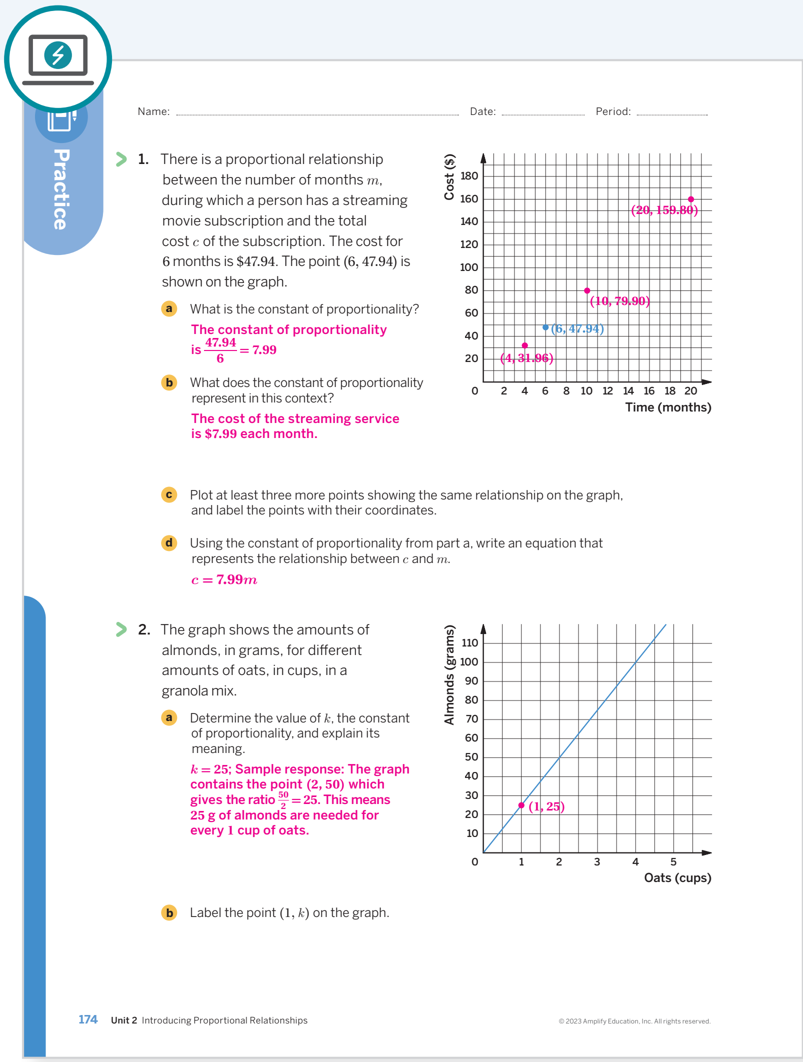
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What was especially satisfying about the class discussion from Activity 1?
- What did students find frustrating about Activity 3? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. There is a proportional relationship between the number of months m, during which a person has a streaming movie subscription and the total cost c of the subscription. The cost for 6 months is \$47.94. The point (6, 47.94) is shown on the graph.

   a. What is the constant of proportionality?

      The constant of proportionality is $\frac{47.94}{6} = 7.99$

   b. What does the constant of proportionality represent in this context?

      The cost of the streaming service is \$7.99 each month.

   c. Plot at least three more points showing the same relationship on the graph, and label the points with their coordinates.

   d. Using the constant of proportionality from part a, write an equation that represents the relationship between c and m.

      $c = 7.99m$

2. The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix.

   a. Determine the value of k, the constant of proportionality, and explain its meaning.

      $k = 25$; Sample response: The graph contains the point (2, 50) which gives the ratio $\frac{50}{2} = 25$. This means 25 g of almonds are needed for every 1 cup of oats.

   b. Label the point $(1, k)$ on the graph.

174 **Unit 2** Introducing Proportional Relationships

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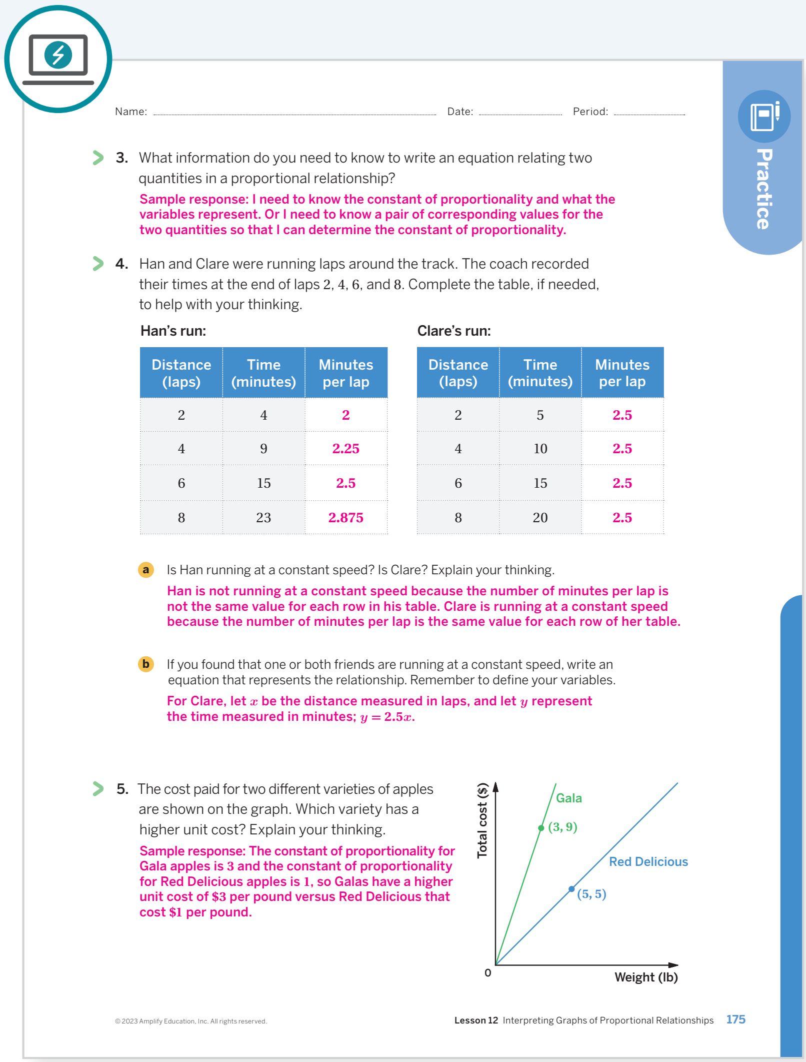

Name: ______ Date: ______ Period: ______

3. What information do you need to know to write an equation relating two quantities in a proportional relationship?

   Sample response: I need to know the constant of proportionality and what the variables represent. Or I need to know a pair of corresponding values for the two quantities so that I can determine the constant of proportionality.

4. Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8. Complete the table, if needed, to help with your thinking.

Han's run:

Distance (laps)	Time (minutes)	Minutes per lap
2	4	**2**
4	9	**2.25**
6	15	**2.5**
8	23	**2.875**

Clare's run:

Distance (laps)	Time (minutes)	Minutes per lap
2	5	**2.5**
4	10	**2.5**
6	15	**2.5**
8	20	**2.5**

   a. Is Han running at a constant speed? Is Clare? Explain your thinking.

      Han is not running at a constant speed because the number of minutes per lap is not the same value for each row in his table. Clare is running at a constant speed because the number of minutes per lap is the same value for each row of her table.

   b. If you found that one or both friends are running at a constant speed, write an equation that represents the relationship. Remember to define your variables.

      For Clare, let x be the distance measured in laps, and let y represent the time measured in minutes; $y = 2.5x$.

5. The cost paid for two different varieties of apples are shown on the graph. Which variety has a higher unit cost? Explain your thinking.

   Sample response: The constant of proportionality for Gala apples is 3 and the constant of proportionality for Red Delicious apples is 1, so Galas have a higher unit cost of \$3 per pound versus Red Delicious that cost \$1 per pound.

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Lesson 12 Interpreting Graphs of Proportional Relationships 175

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1, 2, and 3	2
	2	Activities 1, 2, and 3	2
Spiral	3	Unit 2 Lesson 5	2
	4	Unit 2 Lesson 9	2
Formative	5	Unit 2 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 13**

Using Graphs to Compare Relationships

Let's graph more than one relationship on the same coordinate plane.

Focus

Goals

1. Create and interpret graphs that represent two different proportional relationships on the same coordinate plane.
2. **Language Goal:** Generalize that when two different proportional relationships are graphed on the same coordinate plane, the steeper line has the greater constant of proportionality. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue their work with interpreting graphs of proportional relationships. They compare multiple proportional relationships on the same coordinate plane in order to interpret the steepness of each graph in context. Students reason abstractly to conclude that graphs can be used to compare constants of proportionality, whether or not the scale is specified on each axis.

Previously

In Lesson 12, students analyzed graphs of proportional relationships and interpreted the constant of proportionality in context.

Coming Soon

In Lesson 15, students will solidify their understanding of the different representations of proportional relationships.

Rigor

- Students build **conceptual understanding** of the relationship between the constant of proportionality and the steepness of a proportional relationship when graphed.
- Students build **fluency** in recognizing the relationship between steepness and constant of proportionality by comparing multiple proportional relationships on the same coordinate plane.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- calculators
- colored pencils (optional)
- rulers

Math Language Development

Review words

- *constant of proportionality*
- *coordinate plane*
- *ordered pair*
- *origin*
- *proportional relationship*

Amps | Featured Activity

Activity 1

Interactive Graphs

Students graph multiple proportional relationships on the same coordinate plane in order to make connections between steepness and the constant of proportionality.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, some students might need to pause and take some *think time* to determine what a graph without a scale tells them. The graph is an abstract representation that students will use to make a quantitative conclusion. By controlling their impulses, they will give themselves more time to reason and discover that they know more about the asteroids than they originally thought.

Modifications to Pacing

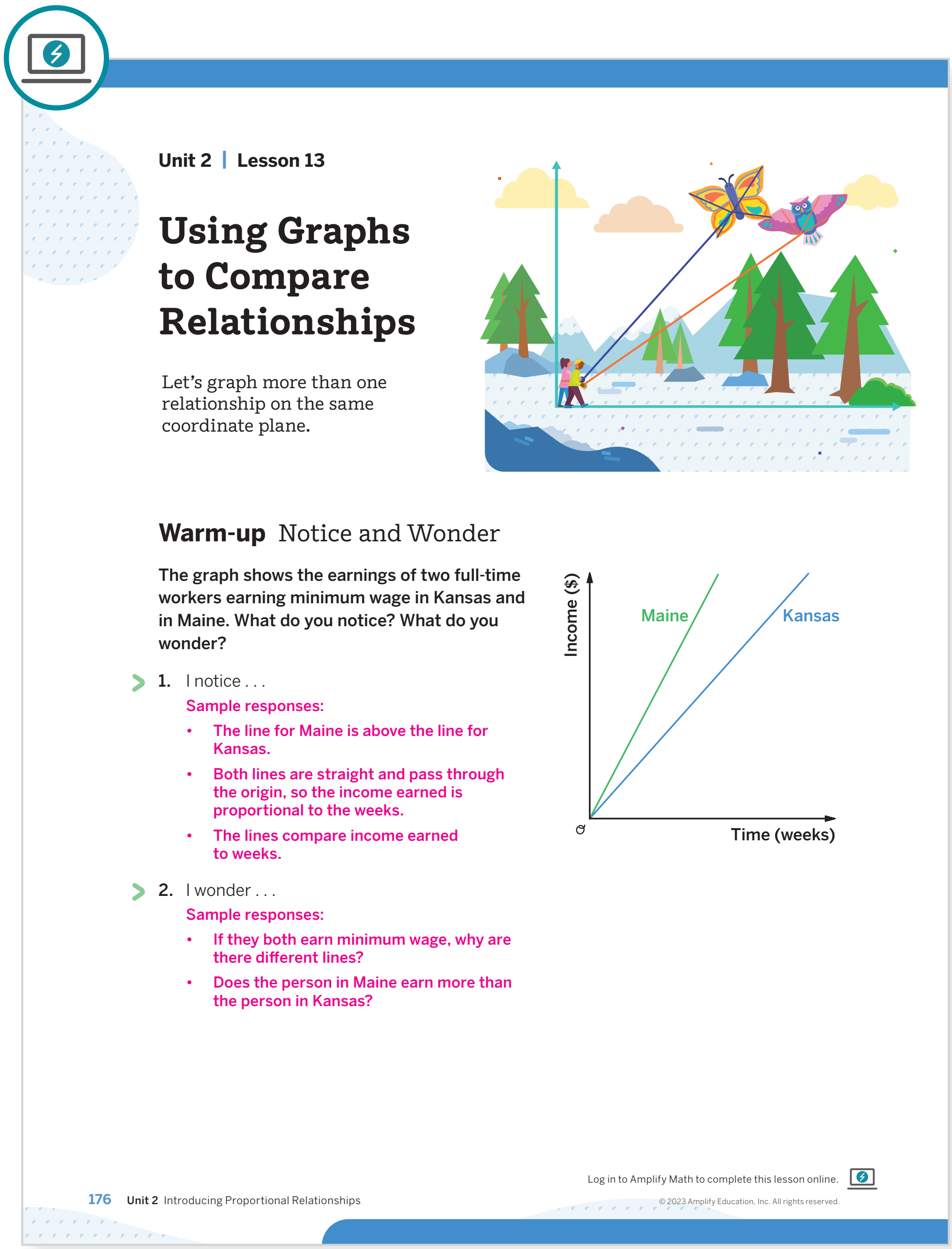
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1,** Problem 5 may be omitted.
- **Activity 2** may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students study the graphs of two proportional relationships, without axes scales, to prepare them for using graphs to compare more than one proportional relationship.



Unit 2 | Lesson 13

Using Graphs to Compare Relationships

Let's graph more than one relationship on the same coordinate plane.

Warm-up Notice and Wonder

The graph shows the earnings of two full-time workers earning minimum wage in Kansas and in Maine. What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - The line for Maine is above the line for Kansas.
   - Both lines are straight and pass through the origin, so the income earned is proportional to the weeks.
   - The lines compare income earned to weeks.
2. I wonder . . .
   Sample responses:
   - If they both earn minimum wage, why are there different lines?
   - Does the person in Maine earn more than the person in Kansas?

Log in to Amplify Math to complete this lesson online.

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176 Unit 2 Introducing Proportional Relationships

1 Launch

Follow the *Notice and Wonder* routine and explain that there is not one correct answer. Students will be sharing, observing, and brainstorming.

2 Monitor

Help students get started by saying, "Tell me what you see in the graph."

Look for points of confusion:

- **Thinking that if there are no units on the axes, then the relationships must not be proportional.** Ask students what characteristic we look for on a graph to determine whether it represents a proportional relationship.

Look for productive strategies:

- Adding number labels to the axes to help students reason about the relationships.

3 Connect

Display the graph.

Have individual students share their observations of the graph as well as what they are wondering.

Ask,

- "Are these graphs modeling proportional relationships? How do you know?
- "What might be different about Maine and Kansas that would cause these lines to be different?"

Highlight that the relationship between earnings and time is proportional for both Maine and Kansas. Unlike with tables of values, it is not necessary to determine the constant of proportionality for each relationship to have evidence of a proportional relationship; thus, the scales on each axis are not needed.

Note: Some students may share that Maine has a higher minimum wage than Kansas. Let them know as of 2020, the minimum wage in Maine was $12 per hour, while the minimum wage of Kansas was $7.25 per hour.

Power-up

To power up students' ability to determine the constant of proportionality from a graph:

Provide students with a copy of the Power-up PDF.

Use: Before Activity 1.

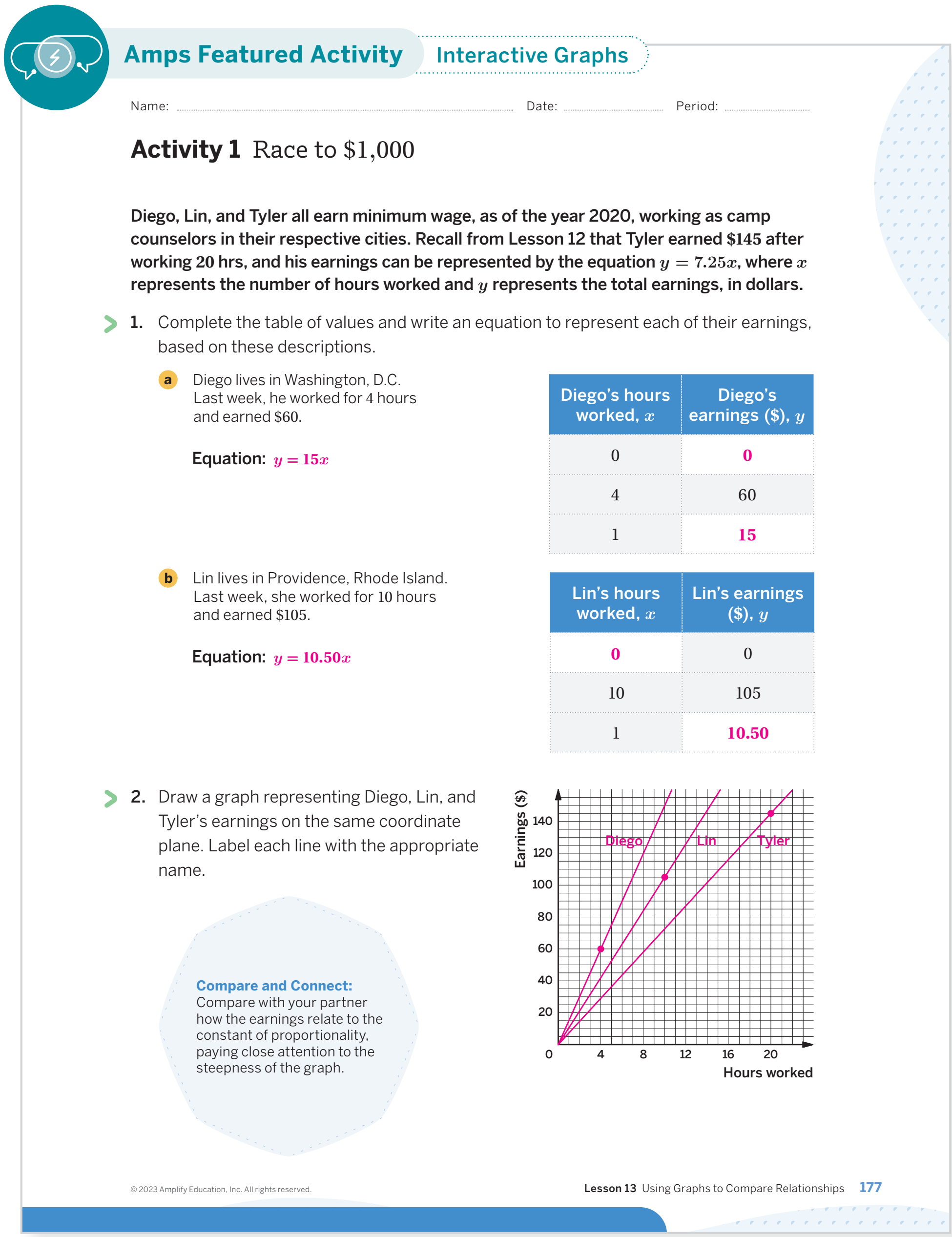
Informed by: Performance on Lesson 12, Practice Problem 6 and Lesson 12 Exit Ticket.

Pairs | 20 min

Activity 1 Race to $1,000

Students will compare the earnings of three workers in tables, equations, and graphs to see that a higher constant of proportionality results in a steeper line.

Amps Featured Activity Interactive Graphs



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Activity 1 Race to $1,000

Diego, Lin, and Tyler all earn minimum wage, as of the year 2020, working as camp counselors in their respective cities. Recall from Lesson 12 that Tyler earned $145 after working 20 hrs, and his earnings can be represented by the equation $y = 7.25x$, where x represents the number of hours worked and y represents the total earnings, in dollars.

1. Complete the table of values and write an equation to represent each of their earnings, based on these descriptions.

   a. Diego lives in Washington, D.C. Last week, he worked for 4 hours and earned $60.

   Equation: $y = 15x$

Diego's hours worked, x	Diego's earnings ($), y
0	0
4	60
1	15

   b. Lin lives in Providence, Rhode Island. Last week, she worked for 10 hours and earned $105.

   Equation: $y = 10.50x$

Lin's hours worked, x	Lin's earnings ($), y
0	0
10	105
1	10.50

2. Draw a graph representing Diego, Lin, and Tyler's earnings on the same coordinate plane. Label each line with the appropriate name.

Compare and Connect: Compare with your partner how the earnings relate to the constant of proportionality, paying close attention to the steepness of the graph.

1 Launch

Activate students' prior knowledge by explaining that this activity is tied to Lesson 12, Activity 2, *Tyler's Job*. Provide students with calculators and rulers.

2 Monitor

Help students get started by asking them how they were able to draw Tyler's graph in the previous lesson with only two pieces of information: the ordered pair and that his earning is proportional to the number of hours he works.

Look for points of confusion:

- **Connecting the three ordered pairs instead of creating three separate lines.** Have students focus on graphing one person's relationship at a time, using a different color for each line.
- **Not realizing that each constant of proportionality is the hourly wage.** Ask students to explain what they are comparing on the axes, and then ask them to explain the constant of proportionality in context.
- **Saying that it is impossible to answer Problem 4 because 1,000 is not on the y-axis.** Ask students to answer Problem 4 for $20. Then repeat for $100. Ask them to predict, based on those answers, who would be the first to make $1,000. Have them check their answers by using the equations from Problem 1.

Look for productive strategies:

- Plotting the given information (ordered pair) and the origin for each relationship, and connecting them with a ruler.
- Determining $(1, k)$ for each relationship and using that point to graph each line.
- Using the equations written in Problem 1 to aid students answering Problem 4.
- Adding additional rows of values to their tables to help students create their graph and/or solve Problem 4.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they use digital technology to graph multiple proportional relationships on the same coordinate plane. This will allow them to more quickly make connections between steepness and the constant of proportionality.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils or highlighters and encourage students to use color coding and annotations to highlight each person's graph in Problem 2, and then use the same color to circle or annotate their work or responses for each person: Tyler, Diego, Lin.

Math Language Development

MLR7: Compare and Connect

As students share their responses to Problems 3 and 4 during the Connect, draw connections that illustrate the following:

- The more money a person makes per hour, the greater the constant of proportionality.
- The greater the constant of proportionality, the steeper the line and the greater the coefficient of x in the equation $y = kx$.

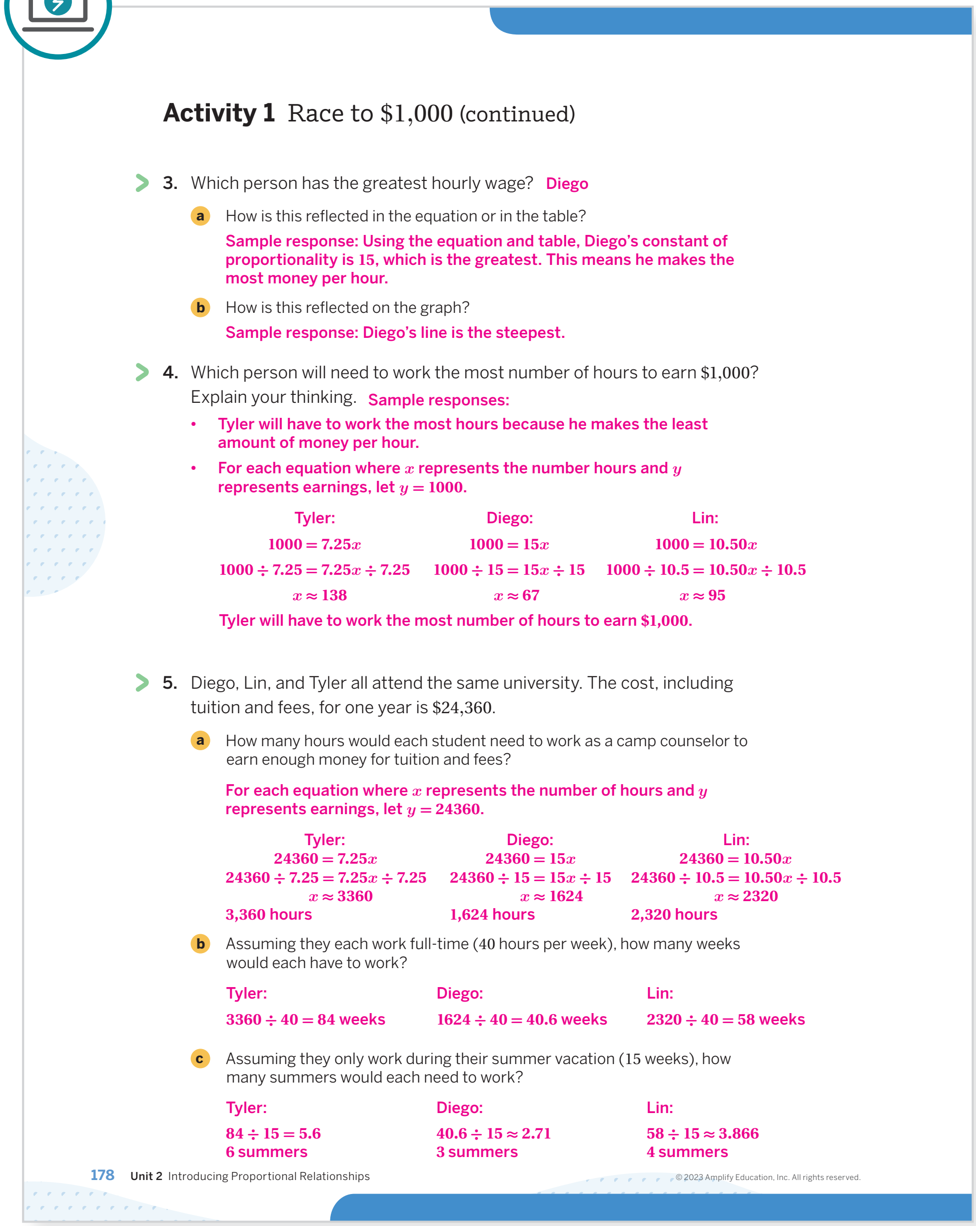
English Learners

Annotate the graph by writing "steepest" next to Diego's line. Use hand gestures to illustrate what it meant by "steep", "steepest", "steeper", or "less steep."

Pairs | 20 min

Activity 1 Race to $1,000 (continued)

Students will compare the earnings of three workers in tables, equations, and graphs to see that a higher constant of proportionality results in a steeper line.



Activity 1 Race to $1,000 (continued)

3. Which person has the greatest hourly wage? Diego
   - a How is this reflected in the equation or in the table?
     Sample response: Using the equation and table, Diego's constant of proportionality is 15, which is the greatest. This means he makes the most money per hour.
   - b How is this reflected on the graph?
     Sample response: Diego's line is the steepest.

4. Which person will need to work the most number of hours to earn $1,000? Explain your thinking. Sample responses:
   - Tyler will have to work the most hours because he makes the least amount of money per hour.
   - For each equation where x represents the number hours and y represents earnings, let $y = 1000$.

Tyler:	Diego:	Lin:
$1000 = 7.25x$	$1000 = 15x$	$1000 = 10.50x$
$1000 \div 7.25 = 7.25x \div 7.25$	$1000 \div 15 = 15x \div 15$	$1000 \div 10.5 = 10.50x \div 10.5$
$x \approx 138$	$x \approx 67$	$x \approx 95$

Tyler will have to work the most number of hours to earn $1,000.

5. Diego, Lin, and Tyler all attend the same university. The cost, including tuition and fees, for one year is $24,360.
   - a How many hours would each student need to work as a camp counselor to earn enough money for tuition and fees?
     For each equation where x represents the number of hours and y represents earnings, let $y = 24360$.

Tyler:	Diego:	Lin:
$24360 = 7.25x$	$24360 = 15x$	$24360 = 10.50x$
$24360 \div 7.25 = 7.25x \div 7.25$	$24360 \div 15 = 15x \div 15$	$24360 \div 10.5 = 10.50x \div 10.5$
$x \approx 3360$	$x \approx 1624$	$x \approx 2320$
3,360 hours	1,624 hours	2,320 hours

   - b Assuming they each work full-time (40 hours per week), how many weeks would each have to work?

Tyler:	Diego:	Lin:
$3360 \div 40 = 84$ weeks	$1624 \div 40 = 40.6$ weeks	$2320 \div 40 = 58$ weeks

   - c Assuming they only work during their summer vacation (15 weeks), how many summers would each need to work?

Tyler:	Diego:	Lin:
$84 \div 15 = 5.6$ 6 summers	$40.6 \div 15 \approx 2.71$ 3 summers	$58 \div 15 \approx 3.866$ 4 summers

3 Connect

Display the completed graph.

Have pairs of students share their responses to Problems 3 and 4 and their strategies for solving each problem.

Ask:

- "For each line, what is the constant of proportionality? How do you know?"
- "If you did not have a table or an equation, how could you use the graph to determine who makes the most amount of money per hour?"
- "How can you explain the relationship between the constant of proportionality and the steepness of each line?"

Highlight that the constant of proportionality in each case is the minimum wage for that person's state. In the graph, the higher the minimum wage, the steeper the line. If not brought up when students are sharing, note that because each of the representations have all of the information, students can use multiple methods to complete Problem 4. These methods include using the graph, comparing the constants of proportionality in context, solving each equation, and adding a row to each table. For Problem 5, facilitate a discussion about the discrepancy in minimum wage between each state.

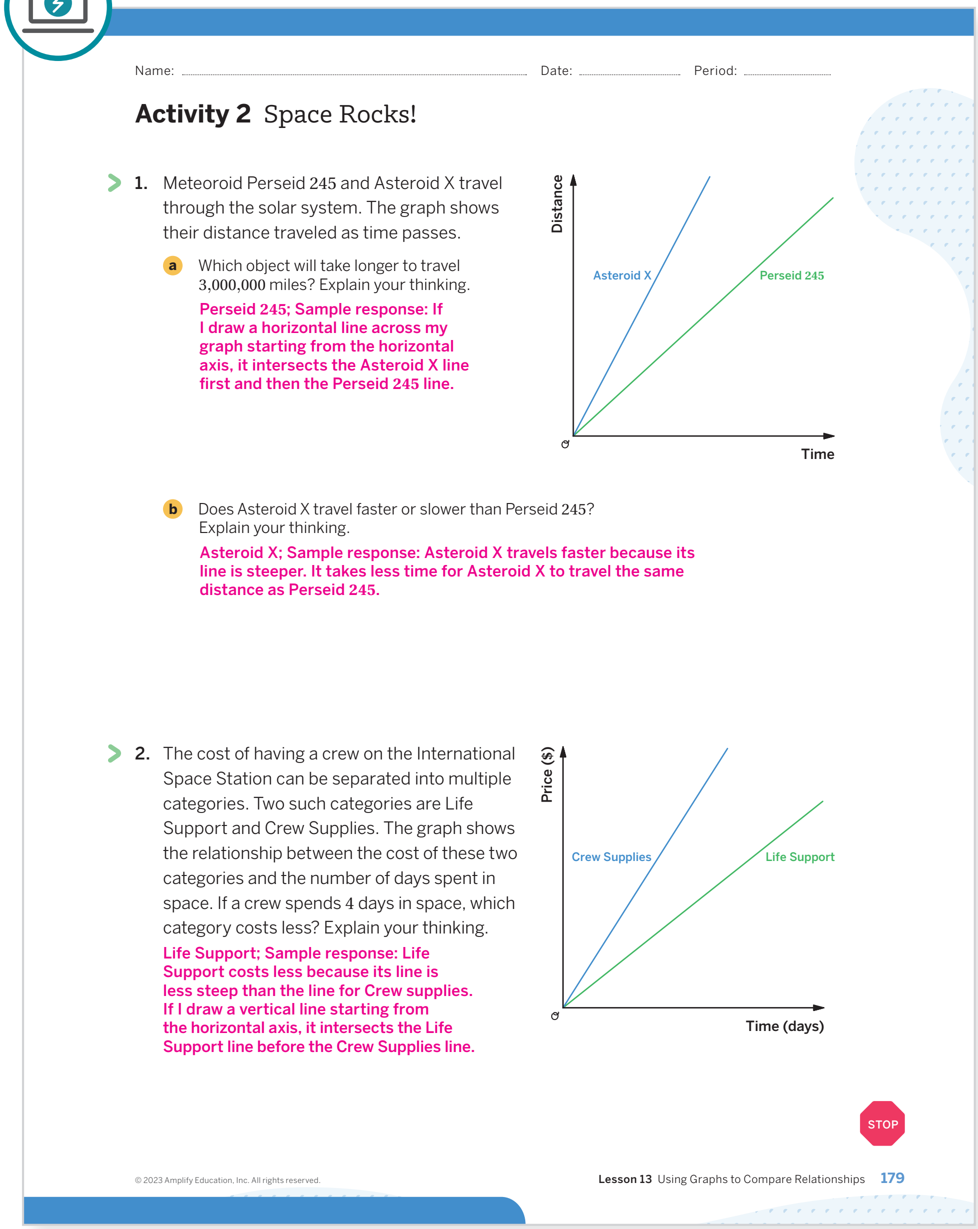
Ask:

- "Did anything surprise you in your answers to Problem 5?"
- "Why may Tyler, Diego, and Lin each get paid a different amount for minimum wage?"
- "What other things might impact Tyler, Diego, and Lin's amount of time needed to work to pay for their university?"

Pairs | 10 min

Activity 2 Space Rocks!

Students analyze the features of two graphs on the same coordinate plane to see that some questions can be answered even if there is no scale on the axes.



Name: ______ Date: ______ Period: ______

Activity 2 Space Rocks!

1. Meteoroid Perseid 245 and Asteroid X travel through the solar system. The graph shows their distance traveled as time passes.

   a. Which object will take longer to travel 3,000,000 miles? Explain your thinking.

   Perseid 245; Sample response: If I draw a horizontal line across my graph starting from the horizontal axis, it intersects the Asteroid X line first and then the Perseid 245 line.

   b. Does Asteroid X travel faster or slower than Perseid 245? Explain your thinking.

   Asteroid X; Sample response: Asteroid X travels faster because its line is steeper. It takes less time for Asteroid X to travel the same distance as Perseid 245.

2. The cost of having a crew on the International Space Station can be separated into multiple categories. Two such categories are Life Support and Crew Supplies. The graph shows the relationship between the cost of these two categories and the number of days spent in space. If a crew spends 4 days in space, which category costs less? Explain your thinking.

   Life Support; Sample response: Life Support costs less because its line is less steep than the line for Crew supplies. If I draw a vertical line starting from the horizontal axis, it intersects the Life Support line before the Crew Supplies line.

1 Launch

Activate students' background knowledge by asking what they know about the Perseid meteor shower (a meteor shower that can be observed between mid-July and mid-August).

2 Monitor

Help students get started by suggesting they choose a location on the y-axis to represent 3,000,000 miles.

Look for points of confusion:

- **Thinking that the graphs can't be compared without scales on the axes.** Have pairs of students make up their own scales to determine which object is going faster. They should then compare their answers to see that, no matter what scale is chosen, Asteroid x is faster.

3 Connect

Display both graphs.

Have pairs of students share their approach and reasoning for each graph.

Ask, "How do you know the steeper graph has a higher constant of proportionality if there are no values on the axes?"

Highlight that a steeper graph has a greater constant of proportionality. Use a ruler to draw a vertical line on the first graph to show that for any value of time the steeper graph has traveled a greater distance, so the speed will be greater. Use a ruler to add a horizontal line to the second graph to show that for any dollar amount the steeper graph will have fewer days, so the cost per day will be greater.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students draw a vertical line that intersects the horizontal axis and assign an arbitrary value to this location on the horizontal axis, such as 10 minutes. Have them examine where this line intersects each graph and ask:

- Which object — Perseid 245 or Asteroid X — has traveled a greater distance at 10 minutes? How do you know?
- How will this help you determine which object will take longer to travel any distance?
- Do you need to know the numerical values on the axes in order to compare the objects? Why or why not?

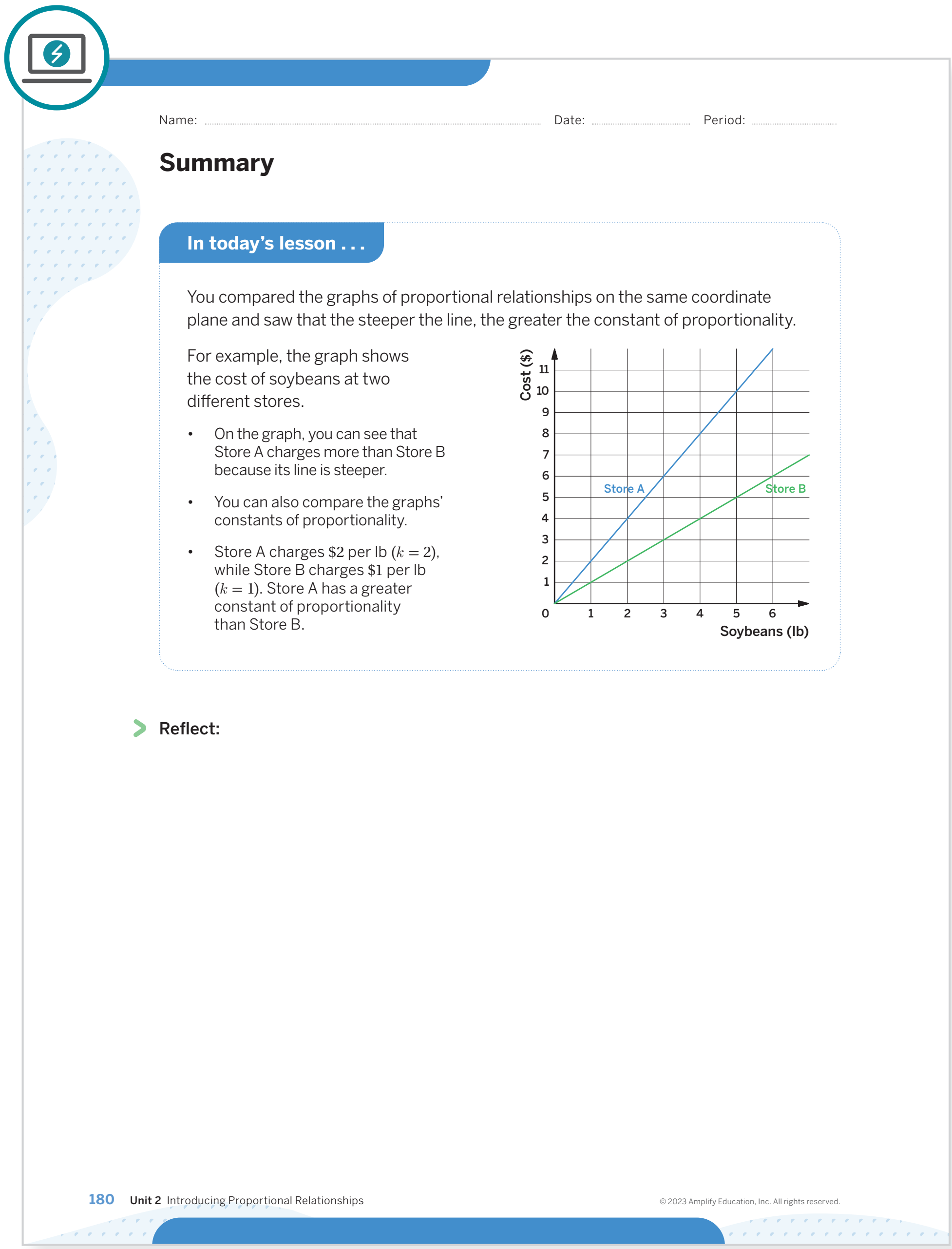
Extension: *Interdisciplinary Connections*

Preview the 3-minute video "How to Navigate the CNEOS" Website by NASA which provides context about CNEOS (the Center for Near Earth Object Studies) and decide if you would like your students to watch it. After watching the video, have students explore the online site by selecting Close Approaches, NEOs, and choosing how they would like to sort the table. Ask them to spend 5 minutes exploring the table and then have them share what they noticed. For example, in January 2021, there were over 125 close approaches of asteroids and comets to Earth! **(Science)**

Whole Class | 5 min

Summary

Review and synthesize how to compare proportional relationships on the same coordinate plane and the relationship between the constant of proportionality and steepness.



Name: Date: Period:

Summary

In today's lesson . . .

You compared the graphs of proportional relationships on the same coordinate plane and saw that the steeper the line, the greater the constant of proportionality.

For example, the graph shows the cost of soybeans at two different stores.

- On the graph, you can see that Store A charges more than Store B because its line is steeper.
- You can also compare the graphs' constants of proportionality.
- Store A charges $2 per lb ($k = 2$), while Store B charges $1 per lb ($k = 1$). Store A has a greater constant of proportionality than Store B.

Reflect:

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Synthesize

Display graph from the Summary.

Have students share what they observe about the relationships being modeled in the graph.

Highlight that the constants of proportionality can be compared by looking at the graph. Comparing steepness, it is clear Store A charges more than Store B per pound for soybeans. Since the axes are labeled, the constants of proportionality can be found by plotting the point $(1, k)$ on both lines.

Ask:

- "If the axes weren't labeled, and you had $20, could you determine which store to shop at to get the most soybeans?" Sample response: Yes. Store B's line is less steep so you would get more soybeans there than at Store A.
- "Would you be able to say exactly how many pounds of soybeans you could buy for $20?" Sample response: No. If the axes weren't labeled we wouldn't know their constants of proportionality. We would only know that you would get more soybeans at Store B than Store A.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

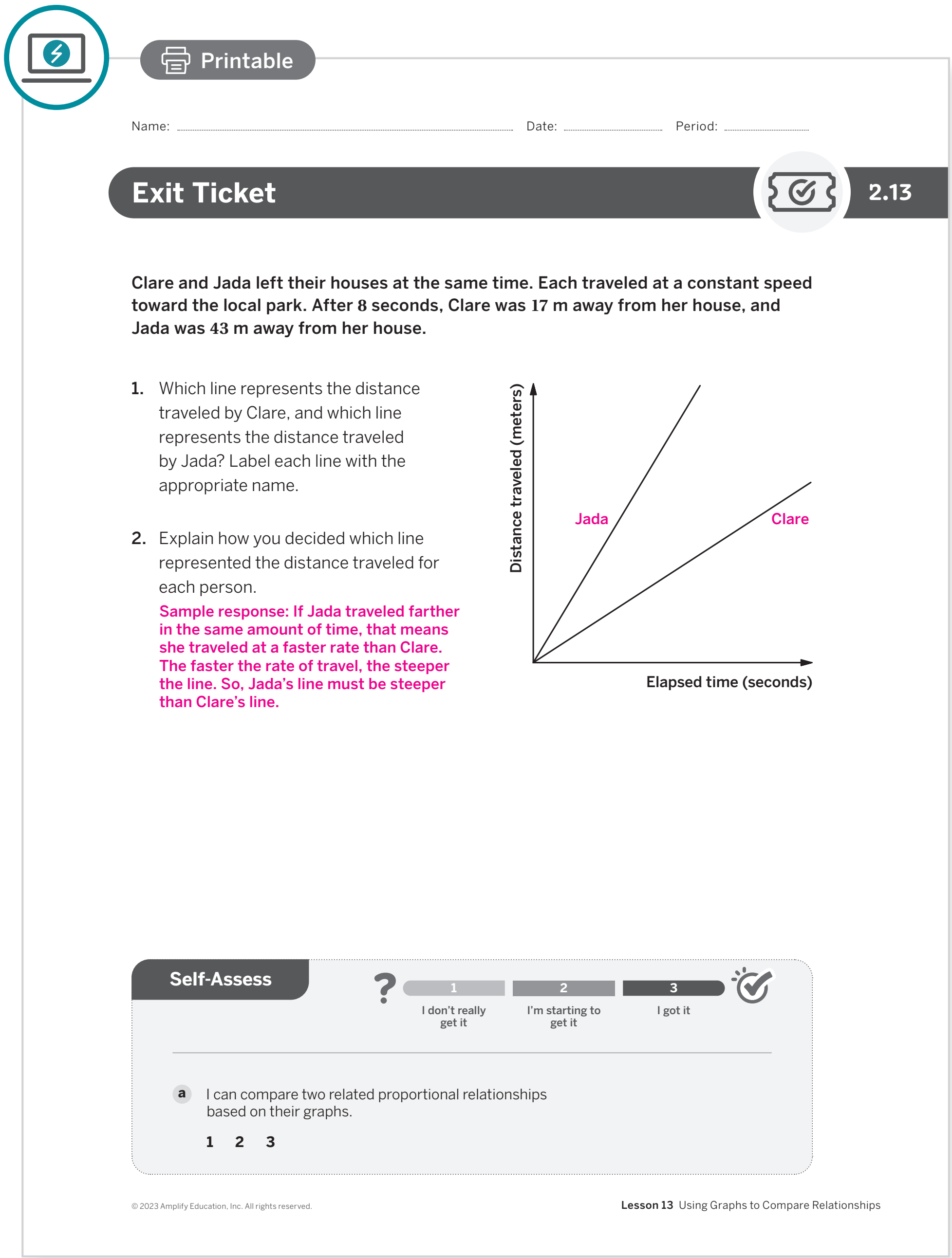
- "What strategies did you use to compare two proportional relationships on the same coordinate plane?"
- "Is there anything you still wonder about graphing two proportional relationships on the same coordinate plane?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of comparing constants of proportionality of two proportional relationships on the same coordinate plane.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 2.13

Clare and Jada left their houses at the same time. Each traveled at a constant speed toward the local park. After 8 seconds, Clare was 17 m away from her house, and Jada was 43 m away from her house.

1. Which line represents the distance traveled by Clare, and which line represents the distance traveled by Jada? Label each line with the appropriate name.

2. Explain how you decided which line represented the distance traveled for each person.
   Sample response: If Jada traveled farther in the same amount of time, that means she traveled at a faster rate than Clare. The faster the rate of travel, the steeper the line. So, Jada's line must be steeper than Clare's line.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can compare two related proportional relationships based on their graphs.
1 2 3

Lesson 13 Using Graphs to Compare Relationships

Success looks like . . .

- Creating and interpreting graphs that represent two different proportional relationships on the same coordinate plane.
  - » Identifying which line belongs to Clare and which line belongs to Jada in Problem 1.
- **Language Goal:** Generalizing that when two different proportional relationships are graphed on the same coordinate plane, the steeper line has the greater constant of proportionality. **(Speaking and Listening, Writing)**
  - » Determining that Jada's line is steeper than Clare's because Jada traveled at a faster rate in Problem 2.

Suggested next steps

If students mislabel Jada and Clare, consider:

- Asking students to show their work in calculating the constant of proportionality for each student based on the scenario.
- Asking students to plot the two ordered from the scenario onto a new graph. Then ask students to observe which line would be steeper when each ordered pair is connected to the origin.
- Assigning Practice Problem 1.

Professional Learning

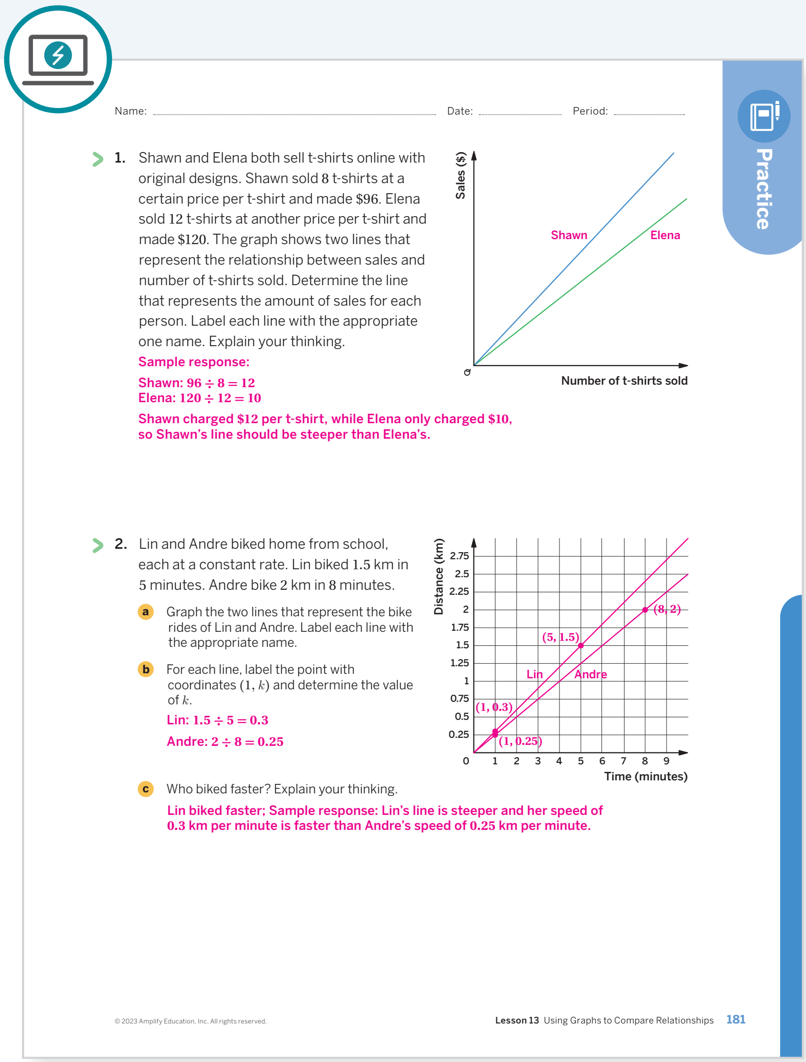
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which student's ideas were you able to highlight during the ***Notice and Wonder*** discussion?
- During the discussion about Activity 2, how did you encourage each student to share their understanding? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Shawn and Elena both sell t-shirts online with original designs. Shawn sold 8 t-shirts at a certain price per t-shirt and made $96. Elena sold 12 t-shirts at another price per t-shirt and made $120. The graph shows two lines that represent the relationship between sales and number of t-shirts sold. Determine the line that represents the amount of sales for each person. Label each line with the appropriate one name. Explain your thinking.

   Sample response:

   Shawn: $96 \div 8 = 12$
   Elena: $120 \div 12 = 10$

   Shawn charged $12 per t-shirt, while Elena only charged $10, so Shawn's line should be steeper than Elena's.

2. Lin and Andre biked home from school, each at a constant rate. Lin biked 1.5 km in 5 minutes. Andre bike 2 km in 8 minutes.

   a. Graph the two lines that represent the bike rides of Lin and Andre. Label each line with the appropriate name.

   b. For each line, label the point with coordinates $(1, k)$ and determine the value of k.

   Lin: $1.5 \div 5 = 0.3$

   Andre: $2 \div 8 = 0.25$

   c. Who biked faster? Explain your thinking.

   Lin biked faster; Sample response: Lin's line is steeper and her speed of 0.3 km per minute is faster than Andre's speed of 0.25 km per minute.

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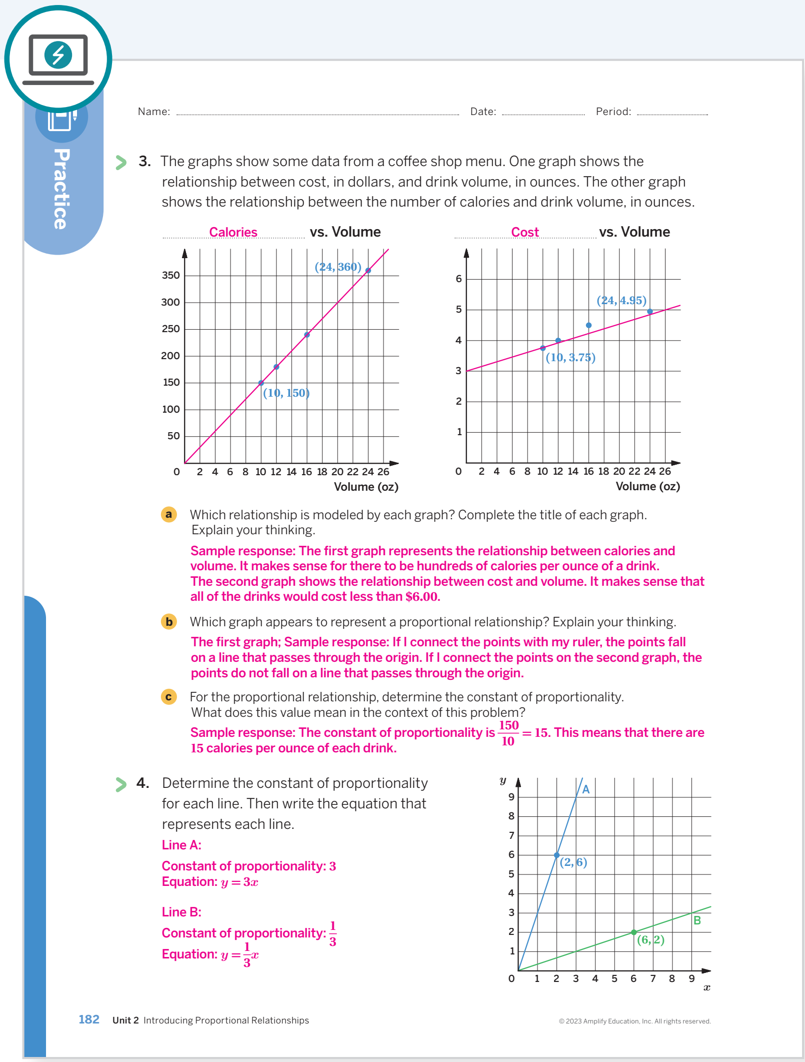
Lesson 13 Using Graphs to Compare Relationships 181

Practice



Practice

Name: Date: Period:

3. The graphs show some data from a coffee shop menu. One graph shows the relationship between cost, in dollars, and drink volume, in ounces. The other graph shows the relationship between the number of calories and drink volume, in ounces.

   a. Which relationship is modeled by each graph? Complete the title of each graph. Explain your thinking.

   Sample response: The first graph represents the relationship between calories and volume. It makes sense for there to be hundreds of calories per ounce of a drink. The second graph shows the relationship between cost and volume. It makes sense that all of the drinks would cost less than $6.00.

   b. Which graph appears to represent a proportional relationship? Explain your thinking.

   The first graph; Sample response: If I connect the points with my ruler, the points fall on a line that passes through the origin. If I connect the points on the second graph, the points do not fall on a line that passes through the origin.

   c. For the proportional relationship, determine the constant of proportionality. What does this value mean in the context of this problem?

   Sample response: The constant of proportionality is $\frac{150}{10} = 15$. This means that there are 15 calories per ounce of each drink.

4. Determine the constant of proportionality for each line. Then write the equation that represents each line.

   Line A:

   Constant of proportionality: 3
   Equation: $y = 3x$

   Line B:

   Constant of proportionality: $\frac{1}{3}$
   Equation: $y = \frac{1}{3}x$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
Spiral	3	Unit 2 Lesson 12	3
Formative ⚡	4	Unit 2 Lesson 14	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 14**

Two Graphs for Each Relationship

Let's use tables, equations, and graphs to solve problems about proportional relationships.

Focus

Goals

1. **Language Goal:** Interpret two different graphs that represent the same proportional relationship, but have reversed the quantities on the axes. **(Writing)**
2. Write an equation to represent a proportional relationship given only one pair of values or one point on the graph.
3. Graph a proportional relationship from a given equation by graphing an ordered pair and drawing the line through the point to the origin.

Coherence

Today

Students connect their work from the previous lessons on tables and equations to now see the two ways to graph a proportional relationship, determined by which quantity goes on which axis. Students connect the structure of the equation with features of the graph.

Previously

In Lessons 3 and 7, students reasoned about a single proportional relationship in two different ways, with the constant of proportionality of each representation being the reciprocal of the other.

Coming Soon

In Lessons 15 and 16, students will compare the different representations of proportional relationships.

Rigor

- Students build **conceptual understanding** of the two constants of proportionality in graphs of proportional relationships.
- Students gain **fluency** with writing equations of proportional relationships from a graph.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	15 min	5 min	10 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- calculators
- rulers

Math Language Development

Review words

- *constant of proportionality*
- *coordinate plane*
- *ordered pair*
- *origin*
- *proportional relationship*
- *reciprocal*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students digitally draw the line representing the proportional relationship.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost when they first encounter currency information presented solely on graphs. Encourage them to look for what they do know from the graph. Have students focus on one point at a time and encourage them to record what they know in another form if it better helps them determine the exchange rate. Challenge them to make connections between equations and graphs.

Modifications to Pacing

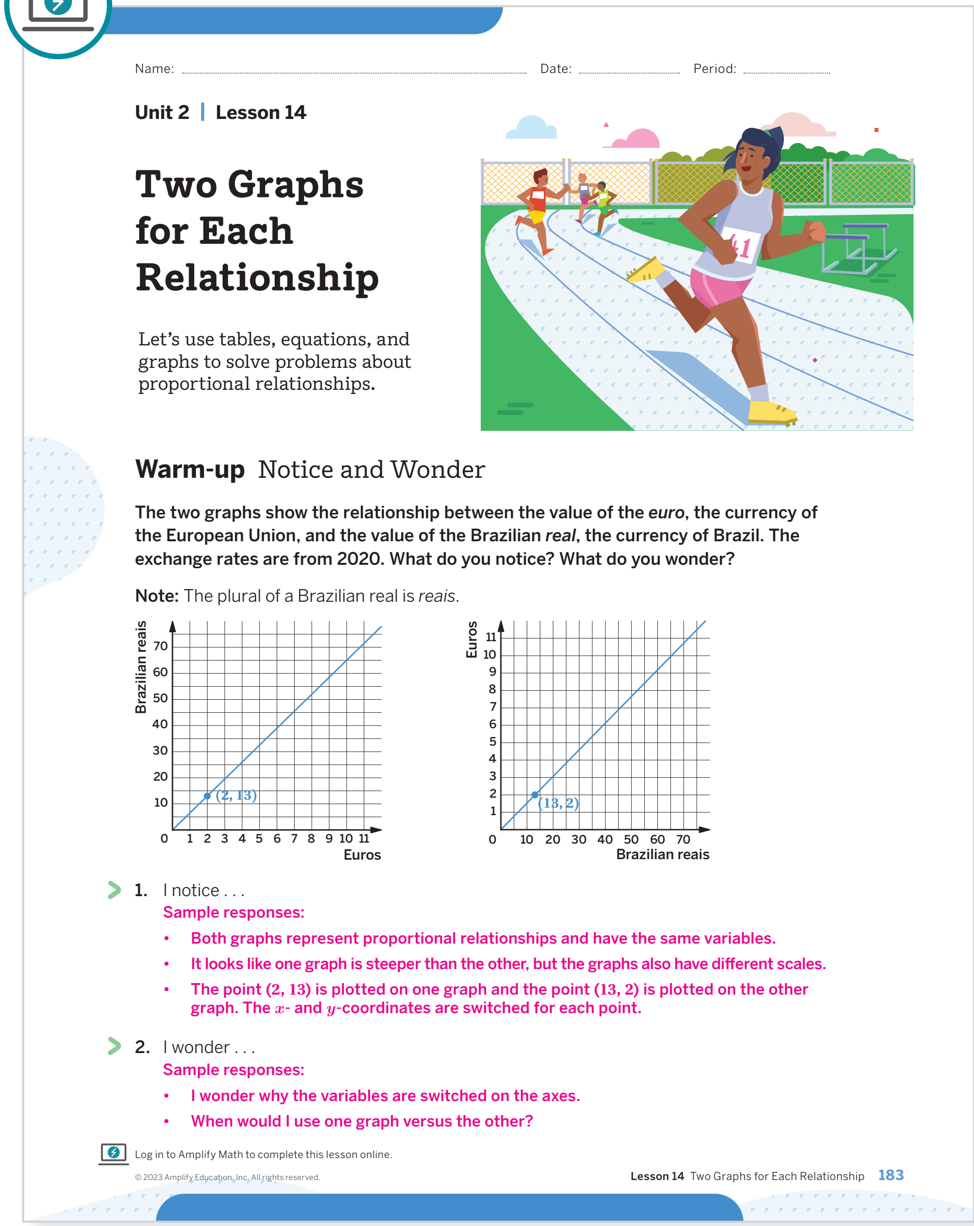
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- If your class understands why the two constants of proportionality for a relationship are reciprocals, **Activity 2** may be omitted; otherwise, use it as another example.

Independent | 5 min

Warm-up Notice and Wonder

Students analyze two graphs representing the same proportional relationship to bring attention to the variables that are placed on each axis.



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Unit 2 | Lesson 14

Two Graphs for Each Relationship

Let's use tables, equations, and graphs to solve problems about proportional relationships.

Warm-up Notice and Wonder

The two graphs show the relationship between the value of the *euro*, the currency of the European Union, and the value of the Brazilian *real*, the currency of Brazil. The exchange rates are from 2020. What do you notice? What do you wonder?

Note: The plural of a Brazilian real is *reais*.

1. I notice . . .
   Sample responses:
   - Both graphs represent proportional relationships and have the same variables.
   - It looks like one graph is steeper than the other, but the graphs also have different scales.
   - The point (2, 13) is plotted on one graph and the point (13, 2) is plotted on the other graph. The x- and y-coordinates are switched for each point.
2. I wonder . . .
   Sample responses:
   - I wonder why the variables are switched on the axes.
   - When would I use one graph versus the other?

Log in to Amplify Math to complete this lesson online.

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Lesson 14 Two Graphs for Each Relationship 183

1 Launch

Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking them to read the labels on the axes.

Look for points of confusion:

- **Thinking the graphs should look the same since they represent the same relationship.** Remind students they saw similar relationships in Lesson 3 and Lesson 7.

Look for productive strategies:

- Finding the constant of proportionality for each graph.

3 Connect

Display the graphs.

Have students share what they notice and wonder about the relationships.

Highlight how the variables on the axes change and the coordinates of the point plotted switch places between the two graphs.

Ask, "What does the ordered pair (2, 13) represent in the graph on the left? What does the ordered pair (13, 2) represent in the graph on the right?"

Power-up

To power up students' ability to write an equation for a proportional relationship from its graph:

Provide students with a copy of the Power-up PDF. Review how the constant of proportionality is modeled in the graph and in the equation for proportional relationships.

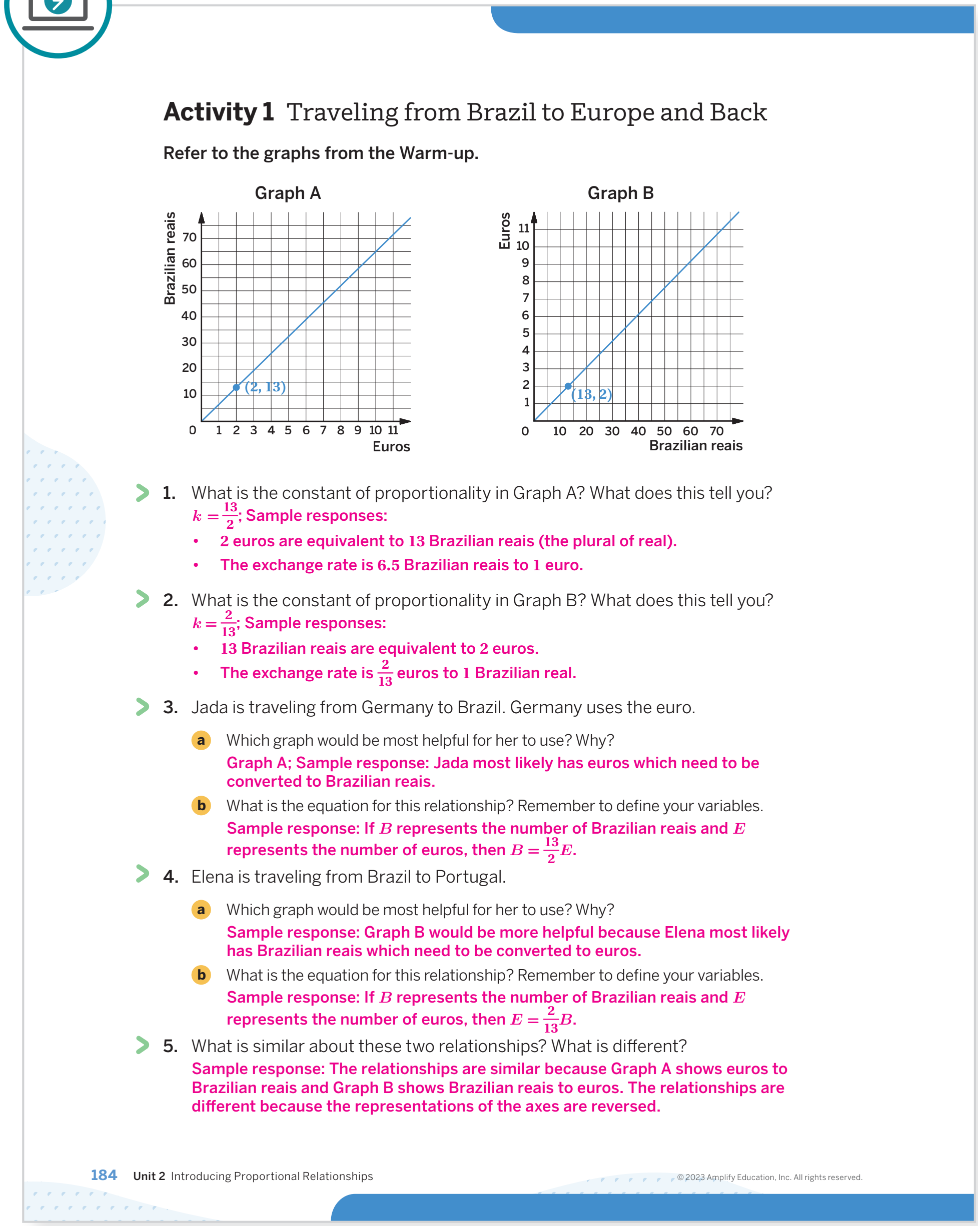
Use: Before Activity 1.

Informed by: Performance on Lesson 13, Practice Problem 4.

Pairs | 10 min

Activity 1 Traveling from Brazil to Europe and Back

Students make use of the structure of two graphs to discover that their constants of proportionality are reciprocals.



Activity 1 Traveling from Brazil to Europe and Back

Refer to the graphs from the Warm-up.

1. What is the constant of proportionality in Graph A? What does this tell you?
   $k = \frac{13}{2}$; Sample responses:
   - **2 euros are equivalent to 13 Brazilian reais (the plural of real).**
   - **The exchange rate is 6.5 Brazilian reais to 1 euro.**
2. What is the constant of proportionality in Graph B? What does this tell you?
   $k = \frac{2}{13}$; Sample responses:
   - **13 Brazilian reais are equivalent to 2 euros.**
   - **The exchange rate is $\frac{2}{13}$ euros to 1 Brazilian real.**
3. Jada is traveling from Germany to Brazil. Germany uses the euro.
   - a. Which graph would be most helpful for her to use? Why?
     Graph A; Sample response: Jada most likely has euros which need to be converted to Brazilian reais.
   - b. What is the equation for this relationship? Remember to define your variables.
     Sample response: If B represents the number of Brazilian reais and E represents the number of euros, then $B = \frac{13}{2}E$.
4. Elena is traveling from Brazil to Portugal.
   - a. Which graph would be most helpful for her to use? Why?
     Sample response: Graph B would be more helpful because Elena most likely has Brazilian reais which need to be converted to euros.
   - b. What is the equation for this relationship? Remember to define your variables.
     Sample response: If B represents the number of Brazilian reais and E represents the number of euros, then $E = \frac{2}{13}B$.
5. What is similar about these two relationships? What is different?
   Sample response: The relationships are similar because Graph A shows euros to Brazilian reais and Graph B shows Brazilian reais to euros. The relationships are different because the representations of the axes are reversed.

184 **Unit 2** Introducing Proportional Relationships

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1 Launch

Activate students' background knowledge and ask if any have lived in or visited another country where they had to exchange currencies.

2 Monitor

Help students get started by reviewing how to find the constant of proportionality from a point on the graph.

Look for points of confusion:

- **Having trouble defining the variables in Problems 3 and 4.** Encourage students to use variables relating to the unknown quantities (e.g., E for euros) rather than x and y.
- **Thinking that only one graph can answer Problem 3 or 4.** Students may have conflicting opinions on which graph to use for these problems. Let them know, because the graphs represent the same relationship, they could use either. **Note:** In Grade 8, students will learn about independent and dependent variables.

Look for productive strategies:

- Reasoning that either equation will work to exchange currency for Jada or Elena. Track students who think this for the class discussion.

3 Connect

Display the graphs and have students share their constants of proportionality and explanations.

Highlight that both graphs and equations could be used to exchange currencies because they represent the same relationship. For example, the Jada's amount of euros could be substituted for E in $B = \frac{13}{2}E$ and evaluated to find B, or Elena's amount of reais could be substituted for B and the equation could be solved for E to find the number of euros. The constant of proportionalities are reciprocals of each other.

Ask, "How would you use the graphs to exchange currencies? How would you use the equations?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide the constants of proportionality for Graphs A and B, and have students focus on completing Problems 3, 4, and 5. This will allow them to focus on connecting each constant of proportionality to each graph and allow them to have more processing time making the connection that the constants of proportionality are reciprocals.

Accessibility: *Guide Processing and Visualization*

Draw 2 rectangles on the board and 13 circles to illustrate the exchange rate of 2 euros for every 13 Brazilian reais. This will help students make sense of the relationship before starting the activity.

Math Language Development

MLR2: Collect and Display

As students share during the Connect, collect and display language that is used to describe the constant of proportionality and how it is represented in each graph and equation, such as *coefficient, ratio of y to x*, and *reciprocal*. Add the language, representations, and diagrams to the class display. Encourage students to refer to the display during discussions.

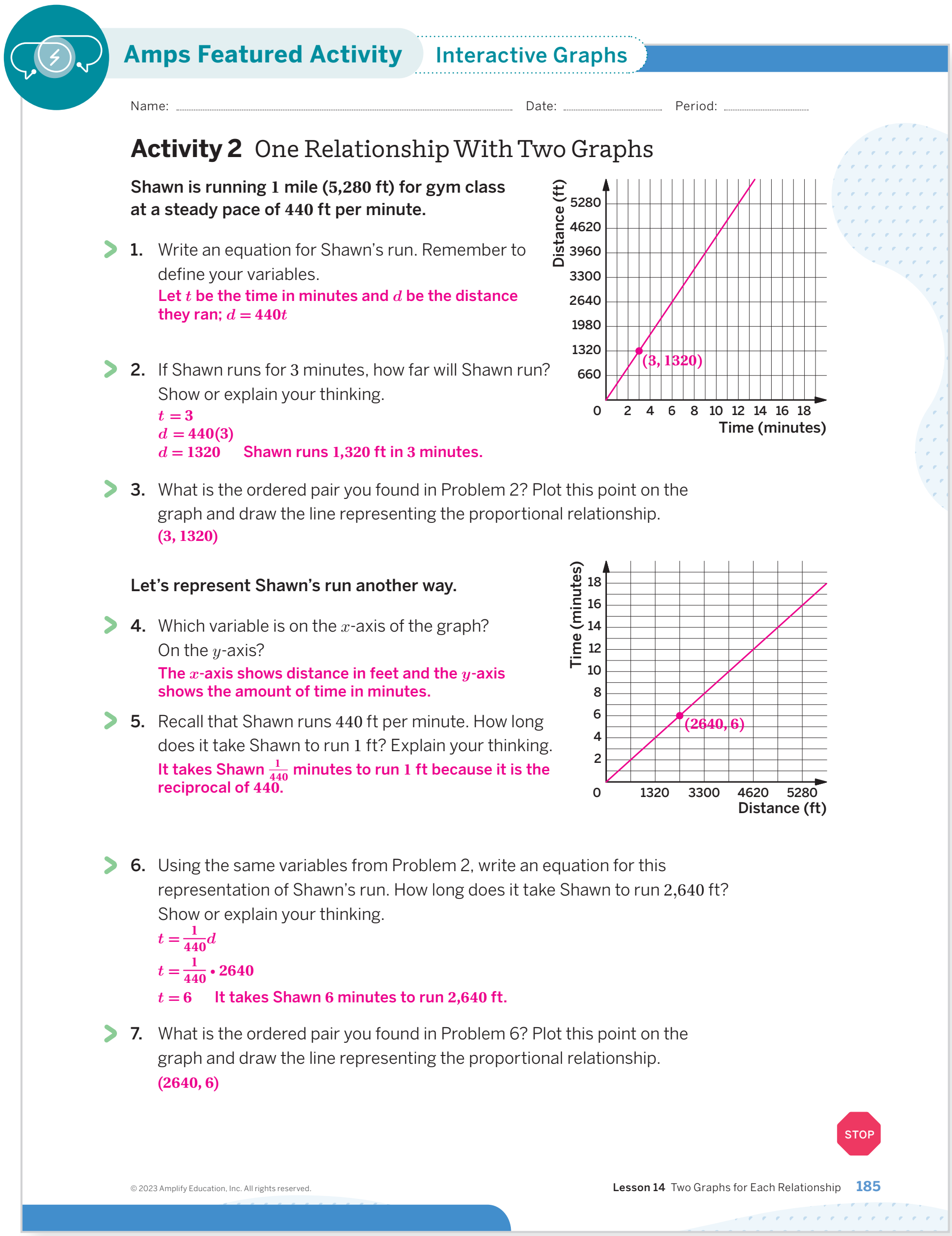
English Learners

Display the fractions $\frac{13}{2}$ and $\frac{2}{13}$ and annotate them with the term *reciprocal*. Use hand gestures to show how the axes labels and scales on Graphs A and B are reversed.

Pairs | 15 min

Activity 2 One Relationship With Two Graphs

Students analyze two graphs of the same relationship, just with the axes quantities reversed, to solidify their understanding that the constants of proportionality are reciprocals.



Amps Featured Activity Interactive Graphs

Name: ________ Date: ________ Period: ________

Activity 2 One Relationship With Two Graphs

Shawn is running 1 mile (5,280 ft) for gym class at a steady pace of 440 ft per minute.

1. Write an equation for Shawn's run. Remember to define your variables.
   Let t be the time in minutes and d be the distance they ran; $d = 440t$

2. If Shawn runs for 3 minutes, how far will Shawn run? Show or explain your thinking.
   $t = 3$
   $d = 440(3)$
   $d = 1320$ **Shawn runs 1,320 ft in 3 minutes.**

3. What is the ordered pair you found in Problem 2? Plot this point on the graph and draw the line representing the proportional relationship.
   (3, 1320)

Let's represent Shawn's run another way.

4. Which variable is on the x-axis of the graph? On the y-axis?
   The x-axis shows distance in feet and the y-axis shows the amount of time in minutes.

5. Recall that Shawn runs 440 ft per minute. How long does it take Shawn to run 1 ft? Explain your thinking.
   It takes Shawn $\frac{1}{440}$ minutes to run 1 ft because it is the reciprocal of 440.

6. Using the same variables from Problem 2, write an equation for this representation of Shawn's run. How long does it take Shawn to run 2,640 ft? Show or explain your thinking.
   $t = \frac{1}{440}d$
   $t = \frac{1}{440} \cdot 2640$
   $t = 6$ **It takes Shawn 6 minutes to run 2,640 ft.**

7. What is the ordered pair you found in Problem 6? Plot this point on the graph and draw the line representing the proportional relationship.
   (2640, 6)

STOP

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Lesson 14 Two Graphs for Each Relationship 185

1 Launch

Activate students' background knowledge by asking if any students run cross country or track, or know how quickly they run one mile. Provide access to rulers and calculators.

2 Monitor

Help students get started by reminding them that it is helpful to choose variables relating to the unknown quantities (i.e., *d* for distance) and referring them to the labels on the axes to help.

Look for points of confusion:

- **Writing the ordered pair incorrectly in Problems 3 and 7.** Remind students to carefully read the axes labels and ask which variable represents time.

3 Connect

Have students share their graphs and reasoning.

Highlight that only one point is needed to graph a proportional relationship and students can use their equation to find a point. Again, each constant of proportionality is a reciprocal of the other because the variables on the graphs are identical, just on different axes, meaning the ratio of $\frac{a}{b}$ becomes $\frac{b}{a}$.

Ask:

- "Do the graphs and equations tell the same story? How can you see the same information in both?"
- "How long did it take Shawn to run a mile? Which representation, the graph or the equation, helped you determine the answer?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they use digital technology to draw the lines representing the proportional relationships. This will allow them to focus on comparing the graphs and constants of proportionality to see that they represent the same proportional relationship — just with the axes reversed.

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide completed graphs for Shawn's run and access to colored pencils or highlighters. Have them use color coding to annotate how the two axes are reversed and the two constants of proportionality.

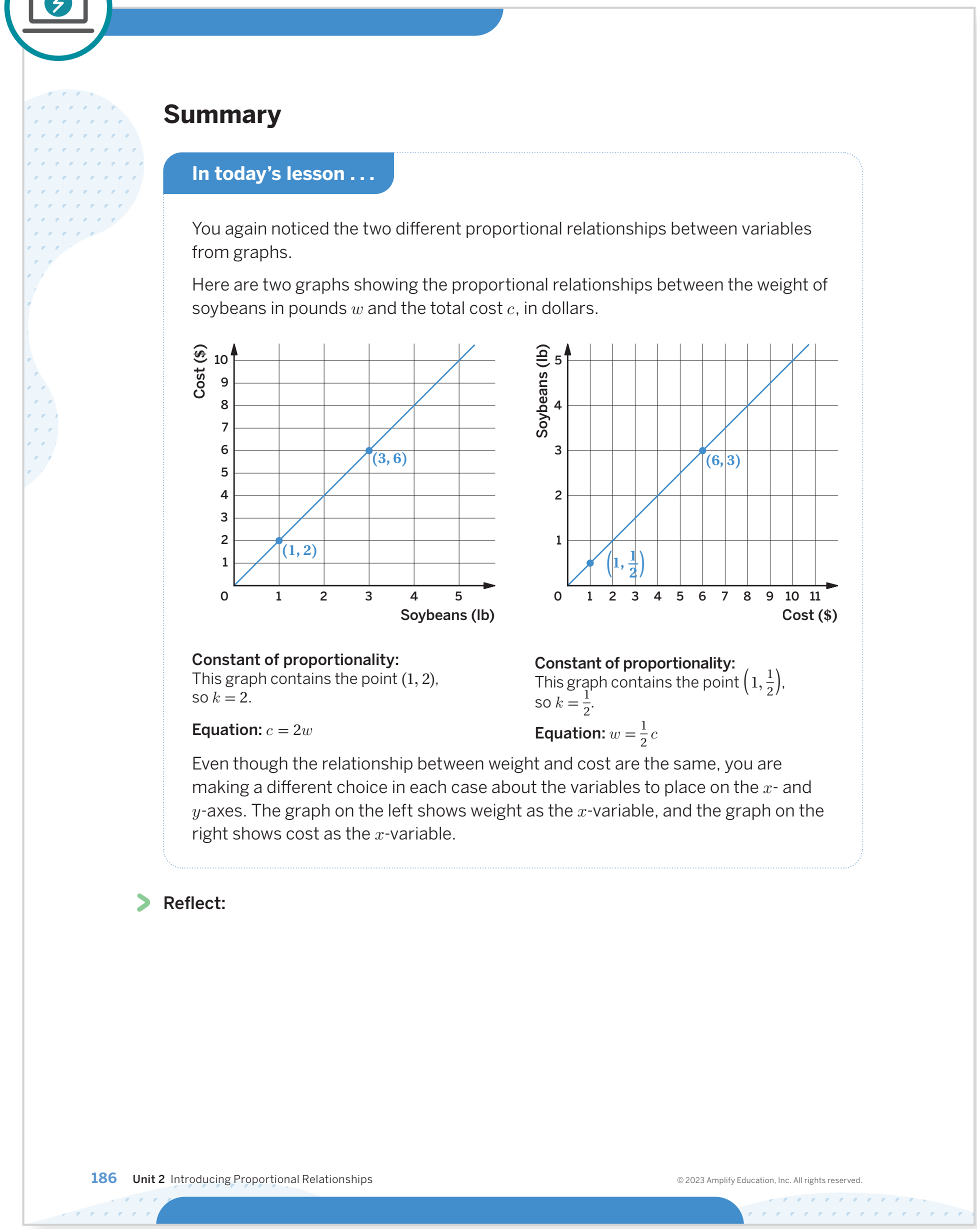
Extension: *Math Enrichment*

Have students complete the following problem:
If Andre runs 1,650 ft in 5 minutes, is he running slower, faster, or at the same speed as Shawn? Explain your thinking. Andre is running slower; Sample response: $\frac{1650}{5} = 330$, which means Andre's speed is 330 ft per minute.

Whole Class | 5 min

Summary

Review and synthesize why every proportional relationship has two constants of proportionality and how to determine their values from a graph.



Summary

In today's lesson . . .

You again noticed the two different proportional relationships between variables from graphs.

Here are two graphs showing the proportional relationships between the weight of soybeans in pounds w and the total cost c, in dollars.

Constant of proportionality:
This graph contains the point $(1, 2)$, so $k = 2$.

Equation: $c = 2w$

Constant of proportionality:
This graph contains the point $\left(1, \frac{1}{2}\right)$, so $k = \frac{1}{2}$.

Equation: $w = \frac{1}{2}c$

Even though the relationship between weight and cost are the same, you are making a different choice in each case about the variables to place on the x- and y-axes. The graph on the left shows weight as the x-variable, and the graph on the right shows cost as the x-variable.

Reflect:

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Synthesize

Highlight that when there are two quantities, x and y, in a proportional relationship, students have two choices for writing an equation to represent the relationship. If two students made different choices, i.e. one viewed x as proportional to y and the other viewed y as proportional to x, the representations are related and still provide the same information.

Ask, "If the constant of proportionality between two variables is 6, what would be the constant of proportionality for the related graph?"

$\frac{1}{6}$

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

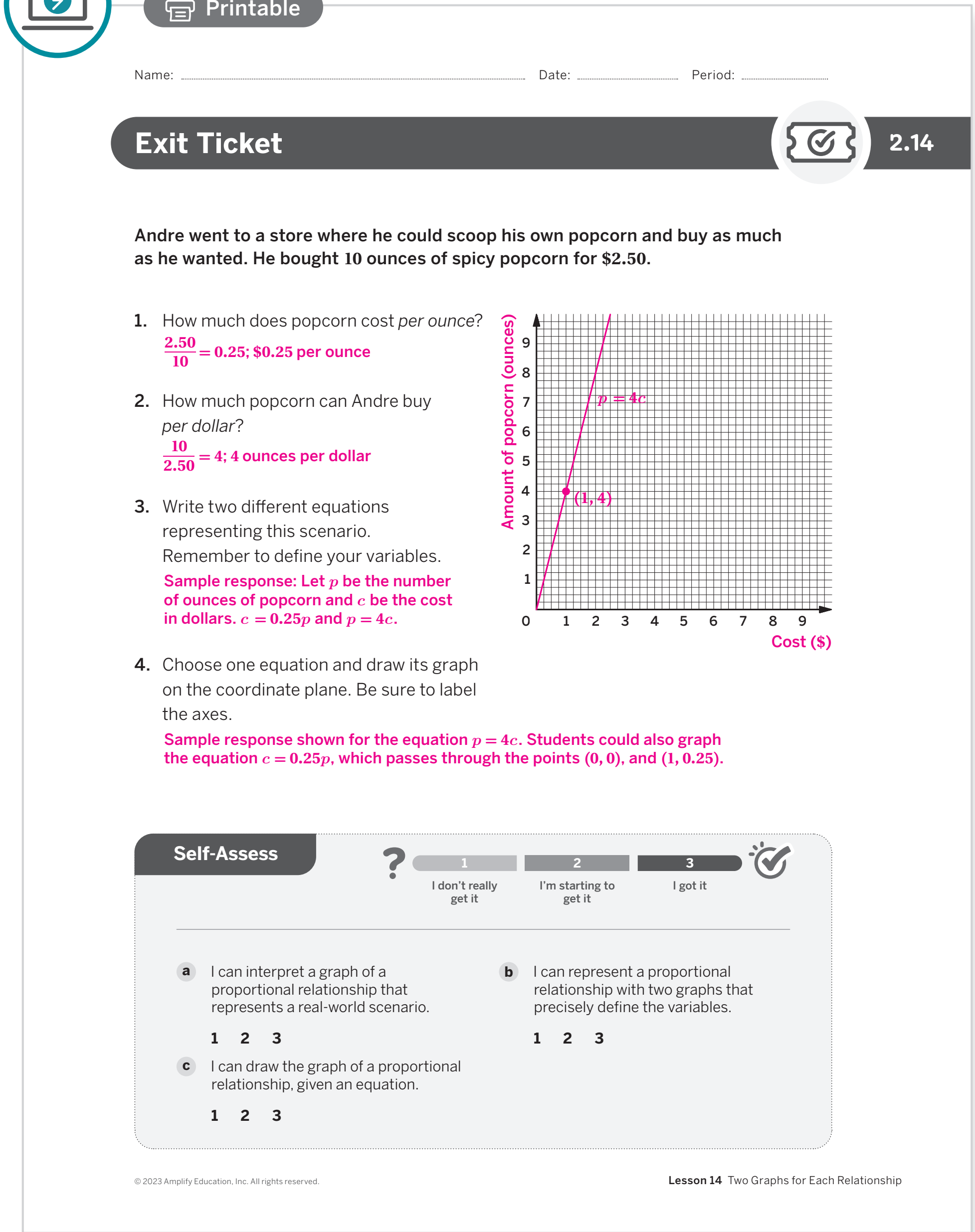
- "What does it mean for two things to be proportionally related? How can you tell?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by finding both constants of proportionality, writing their equations, and sketching graphs.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 2.14

Andre went to a store where he could scoop his own popcorn and buy as much as he wanted. He bought 10 ounces of spicy popcorn for $2.50.

1. How much does popcorn cost *per ounce*?
   $\frac{2.50}{10} = 0.25$; $0.25 per ounce
2. How much popcorn can Andre buy *per dollar*?
   $\frac{10}{2.50} = 4$; 4 ounces per dollar
3. Write two different equations representing this scenario. Remember to define your variables.
   Sample response: Let p be the number of ounces of popcorn and c be the cost in dollars. $c = 0.25p$ and $p = 4c$.
4. Choose one equation and draw its graph on the coordinate plane. Be sure to label the axes.
   Sample response shown for the equation $p = 4c$. Students could also graph the equation $c = 0.25p$, which passes through the points (0, 0), and (1, 0.25).

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can interpret a graph of a proportional relationship that represents a real-world scenario.
1 2 3

b. I can represent a proportional relationship with two graphs that precisely define the variables.
1 2 3

c. I can draw the graph of a proportional relationship, given an equation.
1 2 3

Lesson 14 Two Graphs for Each Relationship

Success looks like . . .

- **Language Goal:** Interpreting two different graphs that represent the same proportional relationship, but reversed the quantities on the axes. **(Writing)**
- **Goal:** Writing an equation to represent a proportional relationship given only one pair of values or one point on the graph.
- **Goal:** Graphing a proportional relationship from a given equation by graphing an ordered pair and drawing the line through the point to the origin.
  - » Graphing the equation $p = 4c$ in Problem 4.

Suggested next steps

If students switch the constants of proportionality, consider:

- Having students make tables of the two ways to understand the relationship and review Lesson 3.
- Reviewing Activity 1.
- Assigning Practice Problem 1.

If students reverse the equations, consider:

- Reviewing Lesson 7.
- Assigning Practice Problems 1 and 2.

If students cannot draw the graph, consider:

- Reminding them they only need one point to be able to graph a proportional relationship.
- Assigning Practice Problems 1 and 2.

Professional Learning

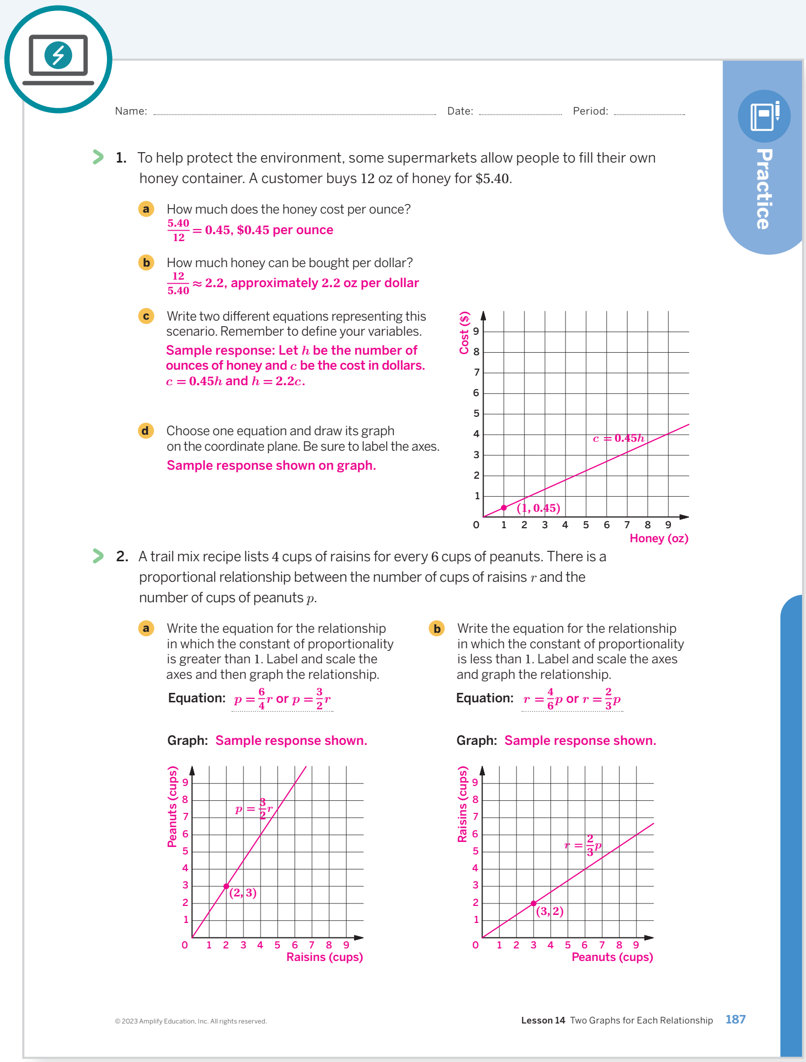
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? This lesson asked students to explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation. Where in your students' work today did you see or hear evidence of them doing this?
- How was Activity 1 similar to or different from activities in Lesson 3 or Lesson 7? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. To help protect the environment, some supermarkets allow people to fill their own honey container. A customer buys 12 oz of honey for $5.40.

   a. How much does the honey cost per ounce?
   $\frac{5.40}{12} = 0.45$, $0.45 per ounce

   b. How much honey can be bought per dollar?
   $\frac{12}{5.40} \approx 2.2$, approximately 2.2 oz per dollar

   c. Write two different equations representing this scenario. Remember to define your variables.
   Sample response: Let h be the number of ounces of honey and c be the cost in dollars. $c = 0.45h$ and $h = 2.2c$.

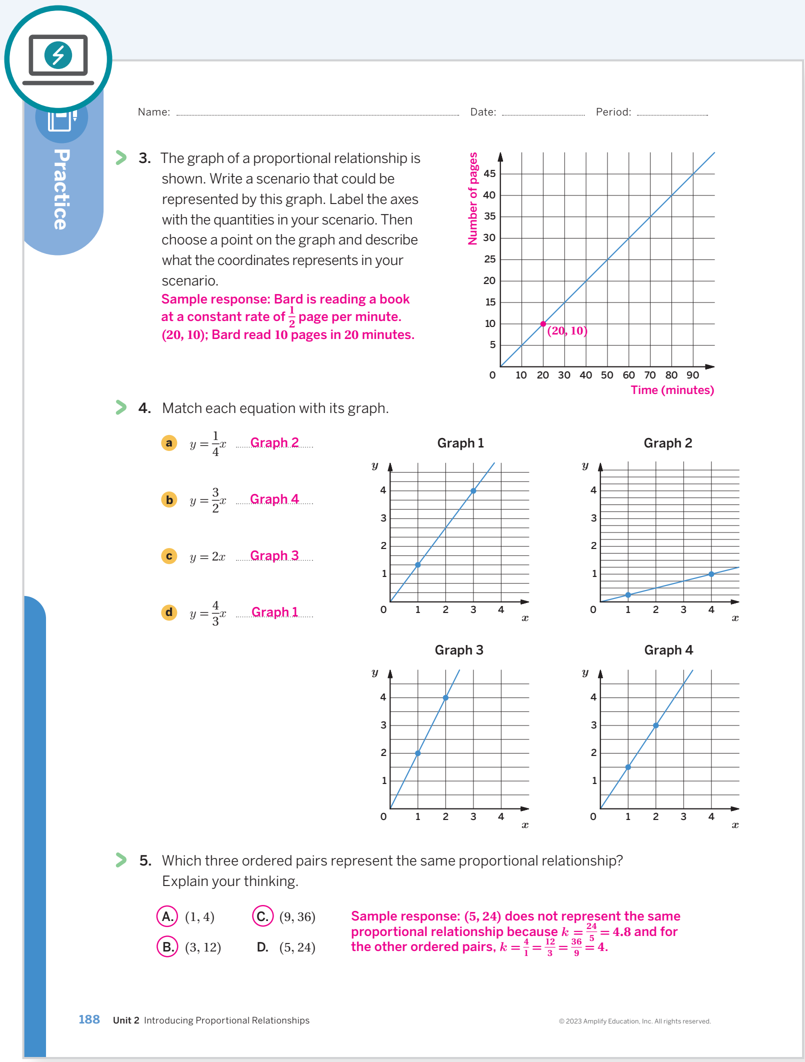
   d. Choose one equation and draw its graph on the coordinate plane. Be sure to label the axes.
   Sample response shown on graph.

2. A trail mix recipe lists 4 cups of raisins for every 6 cups of peanuts. There is a proportional relationship between the number of cups of raisins r and the number of cups of peanuts p.

   a. Write the equation for the relationship in which the constant of proportionality is greater than 1. Label and scale the axes and then graph the relationship.
   Equation: $p = \frac{6}{4}r$ or $p = \frac{3}{2}r$
   Graph: Sample response shown.

   b. Write the equation for the relationship in which the constant of proportionality is less than 1. Label and scale the axes and graph the relationship.
   Equation: $r = \frac{4}{6}p$ or $r = \frac{2}{3}p$
   Graph: Sample response shown.

Lesson 14 Two Graphs for Each Relationship 187



Name: Date: Period:

Practice

3. The graph of a proportional relationship is shown. Write a scenario that could be represented by this graph. Label the axes with the quantities in your scenario. Then choose a point on the graph and describe what the coordinates represents in your scenario.
Sample response: Bard is reading a book at a constant rate of $\frac{1}{2}$ page per minute. (20, 10); Bard read 10 pages in 20 minutes.

4. Match each equation with its graph.

   a. $y = \frac{1}{4}x$ Graph 2
   b. $y = \frac{3}{2}x$ Graph 4
   c. $y = 2x$ Graph 3
   d. $y = \frac{4}{3}x$ Graph 1

5. Which three ordered pairs represent the same proportional relationship? Explain your thinking.

   [✓ A.] $(1, 4)$
   [✓ B.] $(3, 12)$
   [✓ C.] $(9, 36)$
   D. $(5, 24)$

   Sample response: $(5, 24)$ does not represent the same proportional relationship because $k = \frac{24}{5} = 4.8$ and for the other ordered pairs, $k = \frac{4}{1} = \frac{12}{3} = \frac{36}{9} = 4$.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On Lesson	1	Activity 1	2
	2	Activity 1	2
Spiral	3	Unit 2 Lesson 12	2
	4	Unit 2 Lesson 13	1
Formative	5	Unit 2 Lesson 15	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

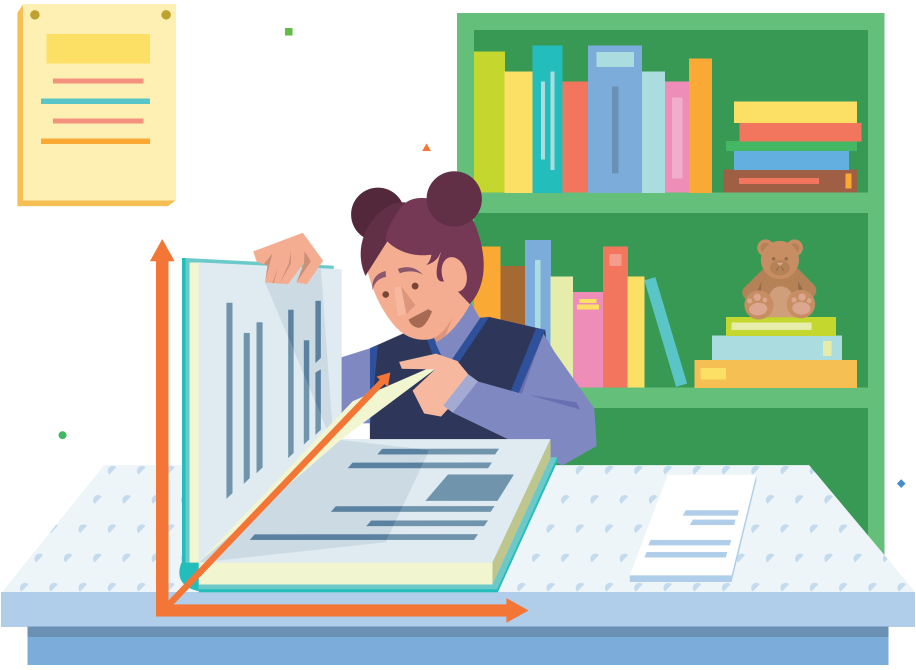
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 15**

Four Ways to Tell One Story (Part 1)

Let's find the constant of proportionality from different representations.



Focus

Goals

1. Create a proportional relationship given only one ordered pair.
2. **Language Goal:** Calculate the constant of proportionality for a proportional relationship in an unfamiliar context, and express it by using the correct units. **(Writing)**
3. **Language Goal:** Invent and describe proportional relationships by using tables, equations, graphs, and verbal descriptions. **(Writing)**

Coherence

Today

Students examine the connections between verbal descriptions, tables, equations, and graphs of proportional relationships. As they work, students pay close attention to the variables as they build contextual meaning of the constant of proportionality.

Previously

Throughout this unit, students studied various representations of proportional relationships and how to find the constant of proportionality.

Coming Soon

In Lesson 16, students will continue their work with multiple representations of proportional and nonproportional relationships.

Rigor

- Students develop their **fluency** of representing proportional relationships in various ways.
- Students strengthen their **procedural fluency** of finding the constant of proportionality from multiple representations.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one card per group
- Activity 1 PDF (answers)
- Anchor Chart PDF, *Representing Proportional Relationships*
- supplies needed to create posters (optional)
- rulers

Math Language Development

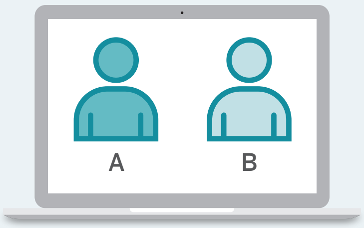
Review words

- *Associative Property*
- *coefficient*
- *constant of proportionality*
- *coordinate plane*
- *ordered pair*
- *origin*
- *proportional relationship*
- *unit rate*

Amps | Featured Activity

Activity 2
Digital Collaboration

Students are given one representation (verbal description, table, or graph) of a proportional relationship and find the constant of proportionality. They check each other's work and finish the activity together to determine if additional ordered pairs will fit their relationship.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, some students might feel frustrated in remembering how to accurately connect verbal descriptions, tables, equations, and graphs. Encourage students to brainstorm solutions to feel more confident such as making a graphic organizer that shows how each represents the same information.

Modifications to Pacing

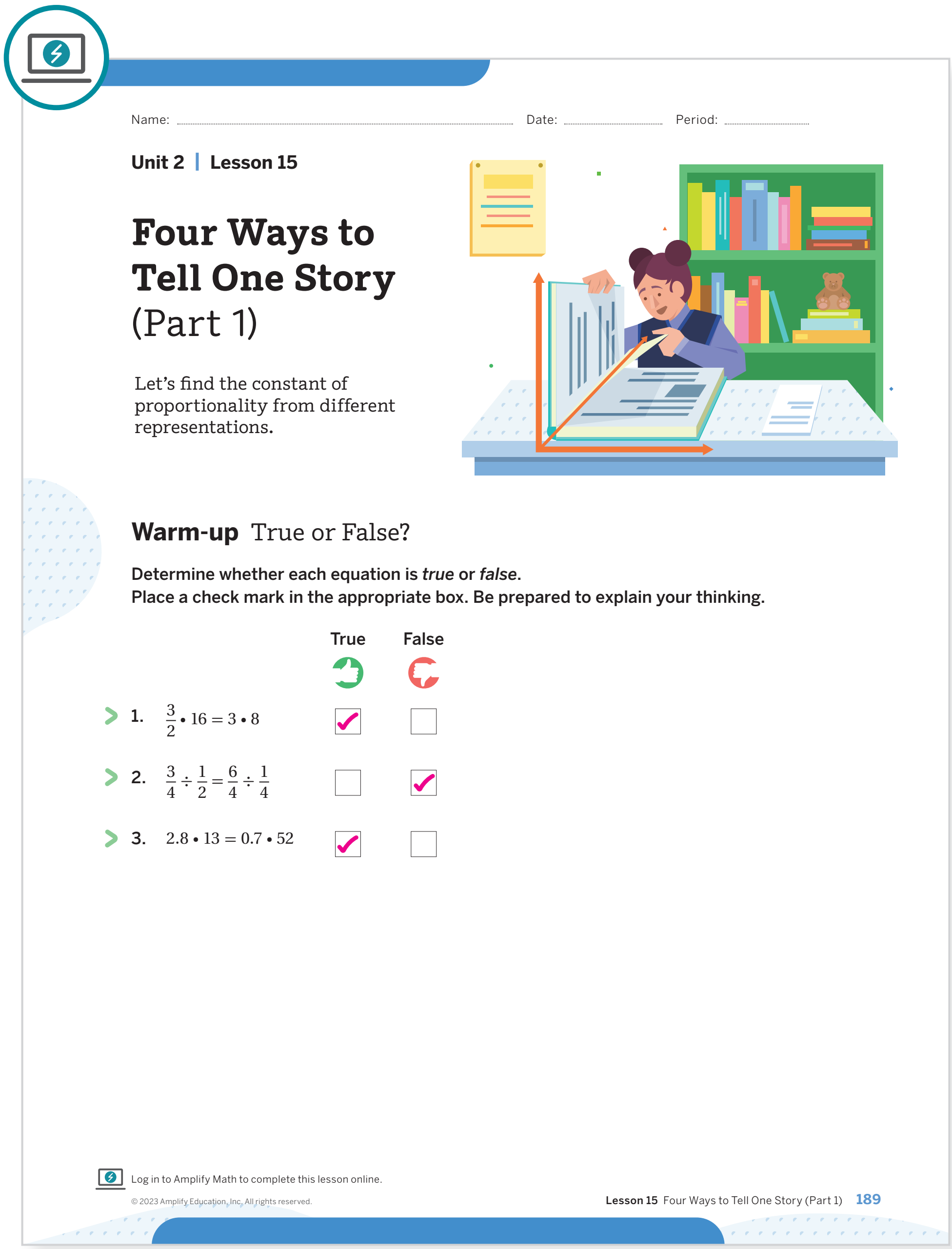
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, have all students find the constant proportionality from each representation in Problem 1, and then omit Problems 6–10.

Independent | 5 min

Warm-up True or False?

Students reason algebraically about various computational relationships to notice connections between operations with rational numbers.



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Unit 2 | Lesson 15

Four Ways to Tell One Story (Part 1)

Let's find the constant of proportionality from different representations.

Warm-up True or False?

Determine whether each equation is *true* or *false*.
Place a check mark in the appropriate box. Be prepared to explain your thinking.

	True	False
1. $\frac{3}{2} \cdot 16 = 3 \cdot 8$	☑	☐
2. $\frac{3}{4} \div \frac{1}{2} = \frac{6}{4} \div \frac{1}{4}$	☐	☑
3. $2.8 \cdot 13 = 0.7 \cdot 52$	☑	☐

Log in to Amplify Math to complete this lesson online.

1 Launch

Displaying one problem at a time, have students give a signal when they have determined a solution and a strategy. Have students discuss each problem.

2 Monitor

Help students get started by having them think about the relationships between the numbers. Ask, "Could you rewrite the left side of Problem 1 as $3 \cdot \frac{1}{2} \cdot 16$?"

Look for points of confusion:

- **Thinking they need to evaluate.** Although this is a valid strategy, remind them the goal is to test for equivalence, not find the value.
- **Thinking the same strategies that work for multiplication can be applied to division.** Have students describe what happens when a dividend is doubled or a divisor is halved. Or, remind students they can turn division into multiplication of the reciprocal.

3 Connect

Have students share their strategies for each problem. Involve more students in the discussion by asking, "Do you agree or disagree? Why?"

Highlight that, by using the Associative Property, students can show that the equations in Problem 1 and Problem 3 are true. For example, $2.8 \cdot 13 = (0.7 \cdot 4) \cdot 13 = 0.7 \cdot (4 \cdot 13) = 0.7 \cdot 52$; however, the Associative Property can not be used for division. In Problem 2, $\frac{1}{2}$ can be rewritten as $2 \cdot \frac{1}{4}$ and $\frac{6}{4}$ as $2 \cdot \frac{3}{4}$ however, $\frac{3}{4} \div \left(2 \cdot \frac{1}{4}\right)$ does not equal $\left(2 \cdot \frac{3}{4}\right) \div \frac{1}{4}$. Because the operation is division, this results in one side being 4 times the value of the other.

Ask, "How can you change Problem 2 so that it is true?"

Power-up

To power up students' ability to determine whether coordinate pairs are part of a proportional relationship, have students complete:

Which ordered pair does not fit the proportional relationship represented by $y = 8x$?

A. (2, 16) **B.** (3.2, 25.6) [✓ **C.** (3, 11)] **D.** (0.2, 1.6)

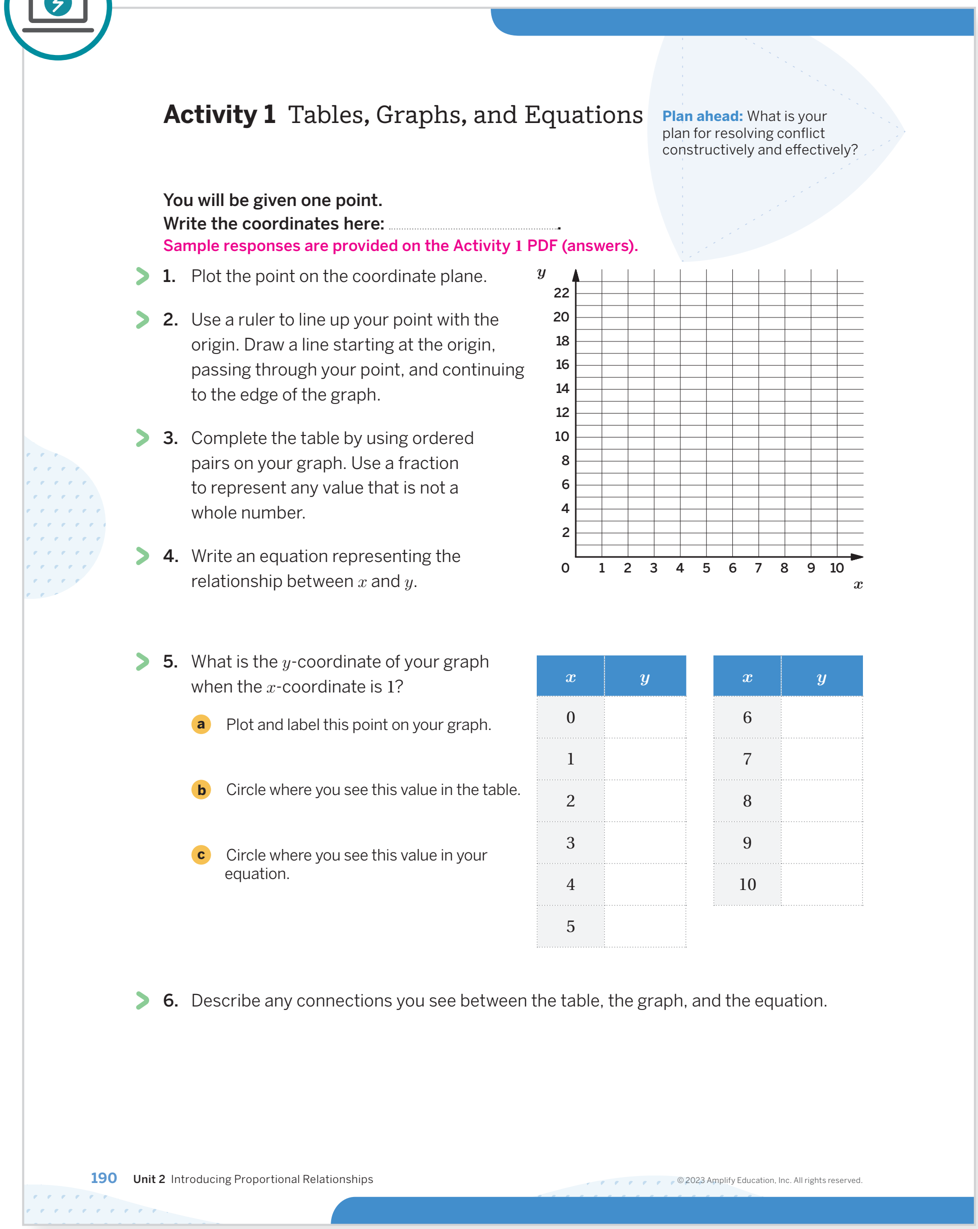
Use: Before Activity 2.

Informed by: Performance on Lesson 14, Practice Problem 5.

Small Groups | 15 min

Activity 1 Tables, Graphs, and Equations

Students create a table, graph, and an equation to demonstrate that a proportional relationship is defined by one ordered pair and the origin.



Activity 1 Tables, Graphs, and Equations

Plan ahead: What is your plan for resolving conflict constructively and effectively?

You will be given one point.
Write the coordinates here: ....

Sample responses are provided on the Activity 1 PDF (answers).

1. Plot the point on the coordinate plane.
2. Use a ruler to line up your point with the origin. Draw a line starting at the origin, passing through your point, and continuing to the edge of the graph.
3. Complete the table by using ordered pairs on your graph. Use a fraction to represent any value that is not a whole number.
4. Write an equation representing the relationship between x and y.

5. What is the y-coordinate of your graph when the x-coordinate is 1?
   - a. Plot and label this point on your graph.
   - b. Circle where you see this value in the table.
   - c. Circle where you see this value in your equation.

x	y
0	
1	
2	
3	
4	
5	

x	y
6	
7	
8	
9	
10	

6. Describe any connections you see between the table, the graph, and the equation.

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1 Launch

Provide groups with an ordered pair from the Activity 1 PDF. Provide access to rulers. If time permits, have students create a poster to display their graph, table, and equation.

2 Monitor

Help students get started by making sure the coordinates are plotted correctly and the line is drawn precisely.

Look for points of confusion:

- **Having a difficult time completing the table**. Have students determine the whole-number coordinates first, and then ask if they have other methods to determine the remaining ordered pairs.

3 Connect

Display different groups' proportional relationships and ask, "What is similar or different among the multiple representations of your relationship? What is similar or different among the various representations for other groups' relationships?"

Highlight the various structures of the proportional relationships in identifying the constant of proportionality:

- In a table, any pair of values (x, y) can be used by finding the multiplier from x to y or by finding the ratio of $\frac{y}{x}$.
- In a table, locate the y-value paired with the x-value of 1.
- In an equation $y = \frac{y}{x}x$, the constant of proportionality appears as the coefficient of x.
- Given a point $(x,\ y)$ other than the *origin* on the graph of a line through the origin, the constant of proportionality is always $\frac{y}{x}$.
- The constant of proportionality is the y-coordinate when x is 1, that is, $\left(1, \frac{y}{x}\right)$ is a point on the graph.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students create the graph and table, provide each group with a pre-created graph and table of values by cutting out the graph and table from copies of the Activity 1 PDF (answers) for each point. Have groups begin the activity with Problem 4.

Extension: *Math Enrichment*

Have students complete the following problem:
If $\left(1, \frac{y}{x}\right)$ is a point on the graph of a proportional relationship between y and x, what would be the ordered pair for a point on the graph of the same relationship if:

1. $x = 2$ $\left(2, \frac{2y}{x}\right)$ **2.** $x = 3$ $\left(3, \frac{3y}{x}\right)$ **3.** $x = n$ $\left(n, \frac{ny}{x}\right)$

Math Language Development

MLR7: Compare and Connect

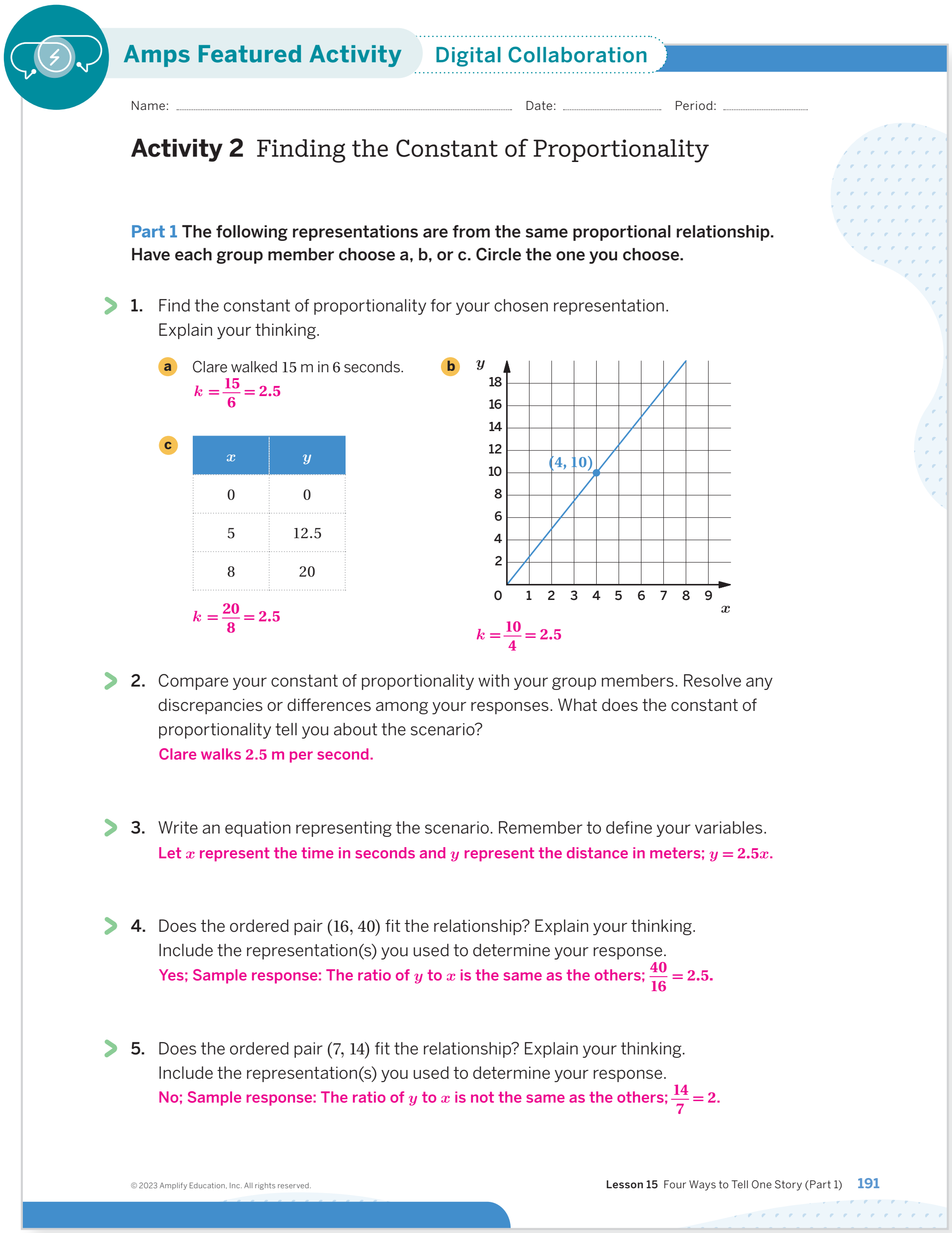
As students respond to the questions you pose during the Connect, select one group's graph, table, and equation and annotate how the constant of proportionality is represented in each. Model the use of precise mathematical language. For example, when annotating the constant of proportionality in the equation $y = kx$, both write and say the term *coefficient* as you highlight the value of k.

Small Groups | 15 min

Activity 2 Finding the Constant of Proportionality

Students practice finding the constant of proportionality from one representation and compare it with group members to analyze the connection between the multiple representations.

Amps Featured Activity Digital Collaboration



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Activity 2 Finding the Constant of Proportionality

Part 1 The following representations are from the same proportional relationship. Have each group member choose a, b, or c. Circle the one you choose.

1. Find the constant of proportionality for your chosen representation. Explain your thinking.

   a Clare walked 15 m in 6 seconds.
   $k = \frac{15}{6} = 2.5$

   b $k = \frac{10}{4} = 2.5$

   c

x	y
0	0
5	12.5
8	20

   $k = \frac{20}{8} = 2.5$

2. Compare your constant of proportionality with your group members. Resolve any discrepancies or differences among your responses. What does the constant of proportionality tell you about the scenario?
   Clare walks 2.5 m per second.

3. Write an equation representing the scenario. Remember to define your variables.
   Let x represent the time in seconds and y represent the distance in meters; $y = 2.5x$.

4. Does the ordered pair $(16, 40)$ fit the relationship? Explain your thinking. Include the representation(s) you used to determine your response.
   Yes; Sample response: The ratio of y to x is the same as the others; $\frac{40}{16} = 2.5$.

5. Does the ordered pair $(7, 14)$ fit the relationship? Explain your thinking. Include the representation(s) you used to determine your response.
   No; Sample response: The ratio of y to x is not the same as the others; $\frac{14}{7} = 2$.

1 Launch

Have students decide which letter they will be in their groups. Explain that for Problems 1 and 6, students only complete their part. Then they continue to work in groups for the remaining problems.

2 Monitor

Help students get started by having them reference Activity 1 on how to find the constant of proportionality for their representation.

Look for points of confusion:

- **Switching the x- and y-variables, particularly in Problem 6b**. Have students compare their constant of proportionality with their group members to determine if they have the same value or if they are reciprocals. Ask them how they can edit their constant of proportionality to match the rest of the group. Also, have students check the graph in Problem 6a for point $(7.5, 10)$ or $(10, 7.5)$ to help determine the variables.

Look for productive strategies:

- Using multiple strategies to check if an ordered pair fits the relationship.
- Labeling the axes on the graphs with the appropriate variables.

Activity 2 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on Part 1 of this activity. Part 2 includes a verbal description in which students need to find the ratio of two fractions.

Extension: *Math Enrichment*

After students complete Part 1, ask them which representation they would use to determine how far Clare will walk if she walks at this same rate for 2 minutes. Have them explain their thinking. Sample response: I would use the equation because I can efficiently substitute 120 for x (2 minutes = 120 seconds) to find the number of meters. Clare would walk 300 m in 2 minutes.

Math Language Development

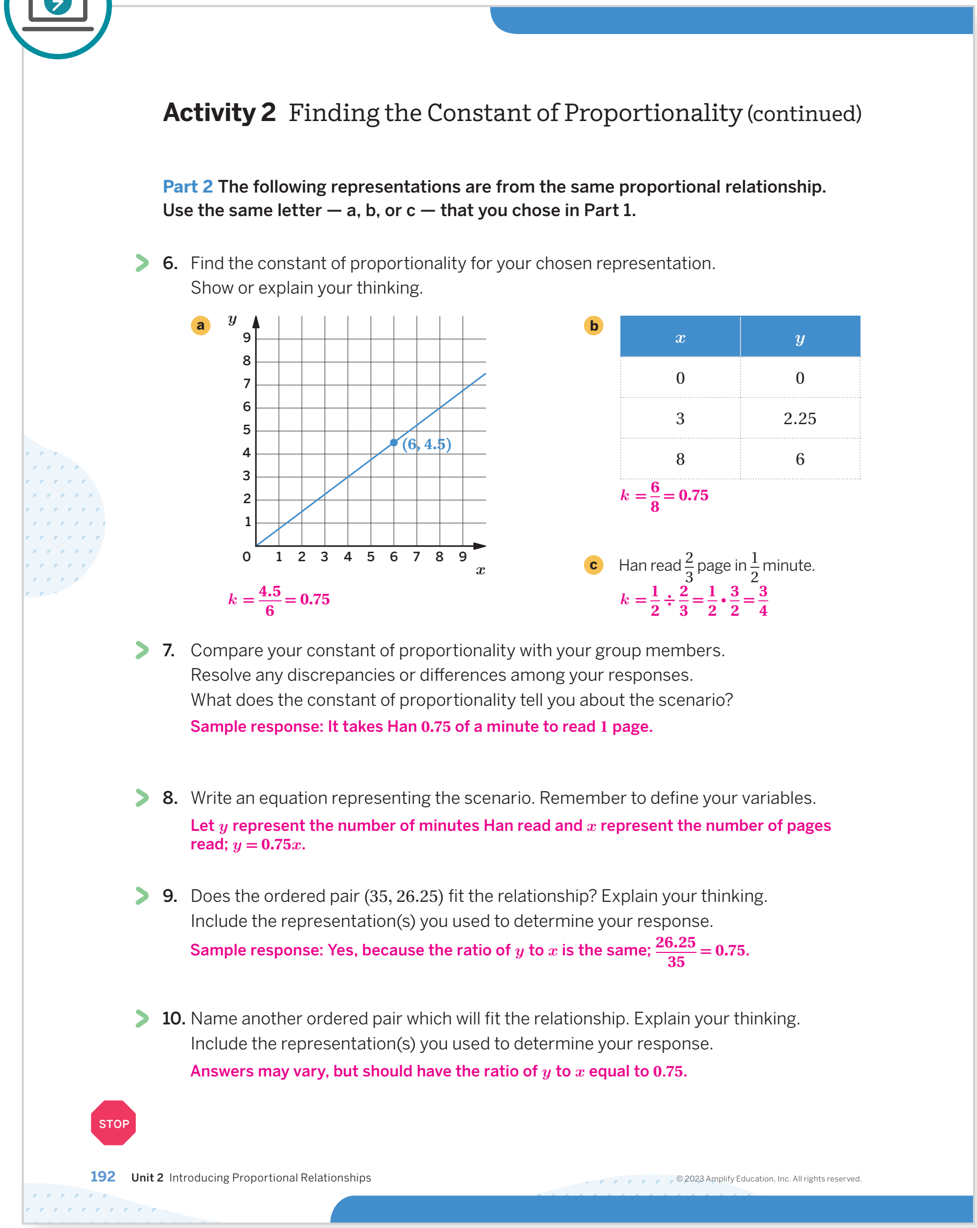
MLR2: Collect and Display

During the Connect, as you highlight that the *constant of proportionality* is the *unit rate* of a proportional relationship, add this language to the class display. Mention that although two different terms are used, they describe the same mathematical idea. While shopping, if customers calculate the cost per item, the term *unit cost* or *unit rate* will likely be used more often than the term *constant of proportionality*. The term *constant of proportionality* is used more often when referring to the value of k in the equation $y = kx$. However, as long as the relationship is proportional, these terms express the same value.

Activity 2 Finding the Constant of Proportionality (continued)

Small Groups | 15 min

Students practice finding the constant of proportionality from one representation and compare it with group members to analyze the connection between the multiple representations.



Activity 2 Finding the Constant of Proportionality (continued)

Part 2 The following representations are from the same proportional relationship. Use the same letter — a, b, or c — that you chose in Part 1.

6. Find the constant of proportionality for your chosen representation. Show or explain your thinking.

a $k = \frac{4.5}{6} = 0.75$

b

x	y
0	0
3	2.25
8	6

$k = \frac{6}{8} = 0.75$

c Han read $\frac{2}{3}$ page in $\frac{1}{2}$ minute.

$k = \frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

7. Compare your constant of proportionality with your group members. Resolve any discrepancies or differences among your responses. What does the constant of proportionality tell you about the scenario?

Sample response: It takes Han 0.75 of a minute to read 1 page.

8. Write an equation representing the scenario. Remember to define your variables.

Let y represent the number of minutes Han read and x represent the number of pages read; $y = 0.75x$.

9. Does the ordered pair (35, 26.25) fit the relationship? Explain your thinking. Include the representation(s) you used to determine your response.

Sample response: Yes, because the ratio of y to x is the same; $\frac{26.25}{35} = 0.75$.

10. Name another ordered pair which will fit the relationship. Explain your thinking. Include the representation(s) you used to determine your response.

Answers may vary, but should have the ratio of y to x equal to 0.75.

STOP

192 **Unit 2** Introducing Proportional Relationships

3 Connect

Display any necessary representations to support the discussion.

Have students share any disagreements they had regarding the constants of proportionality and how they used other representations to decide which constant of proportionality to use.

Highlight, when discussing Problems 2 and 7, that if there is a proportional relationship between two quantities with variables x and y, then the associated rates are expressed as number of ys per x. Remind students this is the unit rate. Clarify any points of confusion students may continue to have.

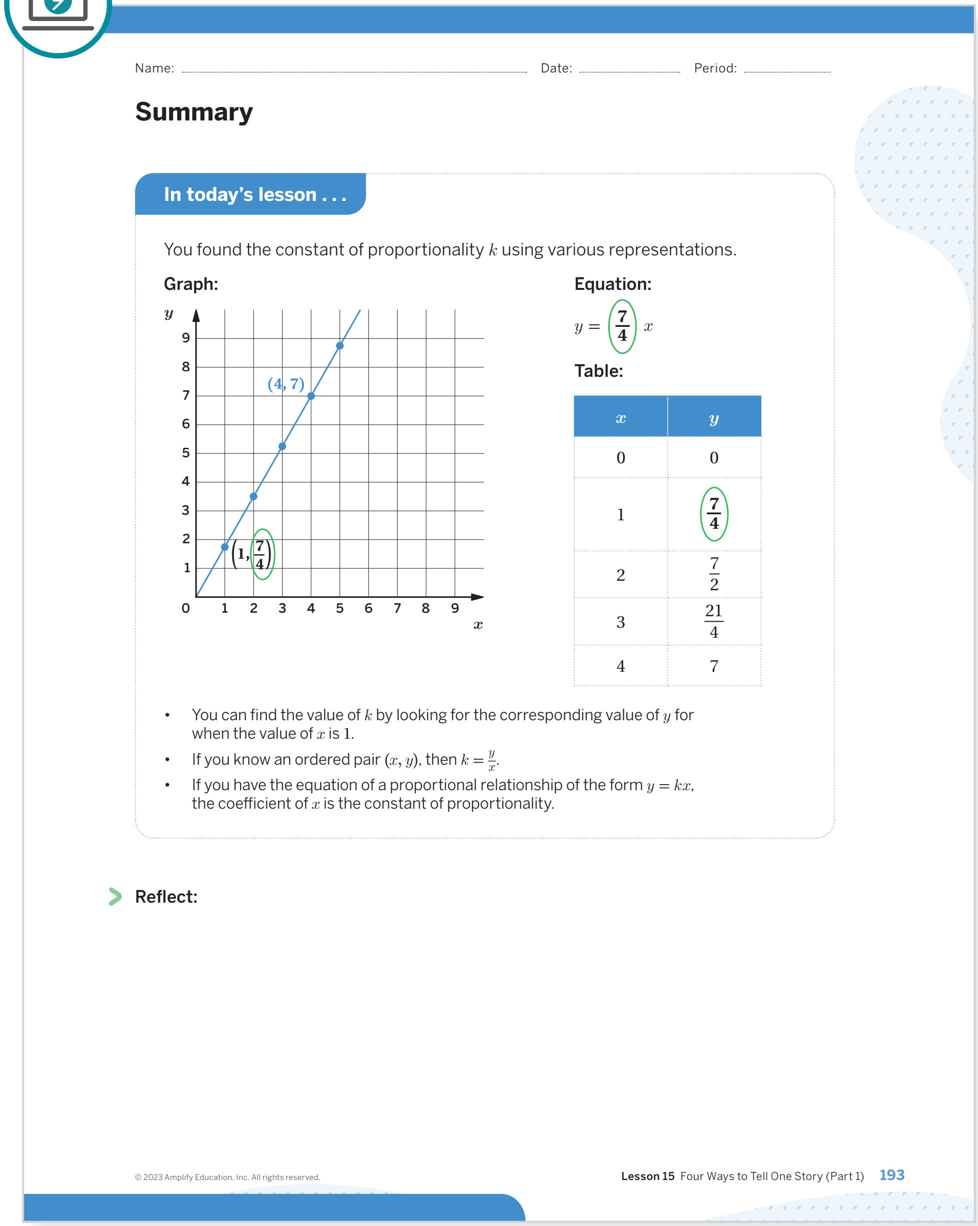
Ask:

- "How can you use one representation to help you understand another representation?"
- "Which representation did you find more helpful in completing Problems 4 and 5?"

Whole Class | 5 min

Summary

Review and synthesize how to find the constant of proportionality from the four representations of proportional relationships.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You found the constant of proportionality k using various representations.

Graph:

Equation:

$y = \frac{7}{4}x$

Table:

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

- You can find the value of k by looking for the corresponding value of y for when the value of x is 1.
- If you know an ordered pair (x, y), then $k = \frac{y}{x}$.
- If you have the equation of a proportional relationship of the form $y = kx$, the coefficient of x is the constant of proportionality.

Reflect:

Lesson 15 Four Ways to Tell One Story (Part 1) 193

Synthesize

Display the Anchor Chart PDF, *Representing Proportional Relationships* and complete any remaining sections.

Have students share the ways to find the constant of proportionality from each method.

Highlight that each form tells the same story, but in a different way: the table shows individual ordered pairs where the ratio can be evaluated, the equation gives a way to evaluate for known variables or solve for unknown variables, the graph displays the relationship, and the verbal description gives context to the scenario.

Ask, "When finding the constant of proportionality, we use an ordered pair and find the ratio of $\frac{y}{x}$. Why can you not use $(0, 0)$ as the ordered pair?" Students may reason mathematically and state dividing by 0 is not possible (undefined) or they may reason intuitively and say that because every proportional relationship contains that ordered pair, it cannot determine one ratio over another.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

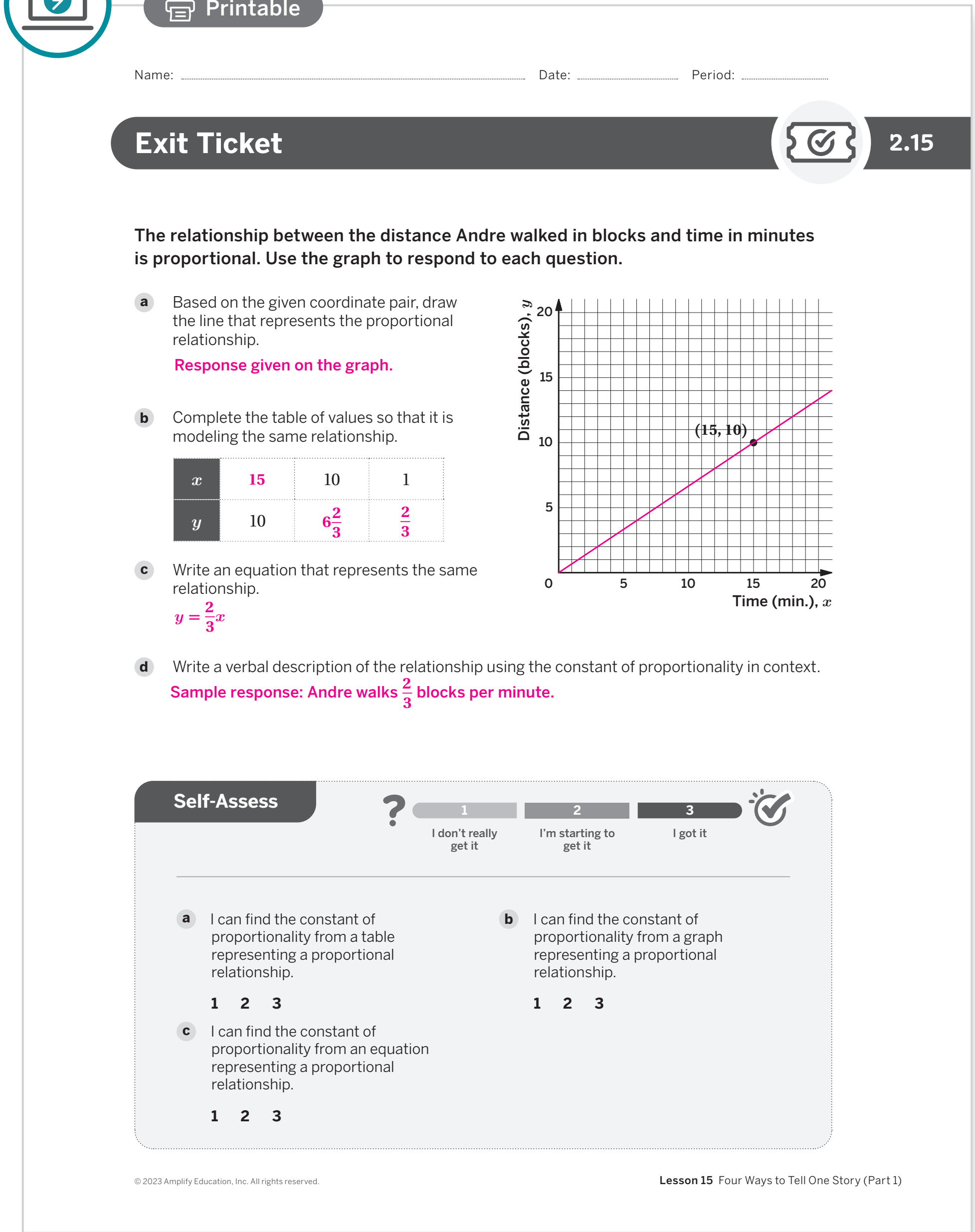
- "What are the different ways you can represent proportional relationships? How are the representations related?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing in their own words how to find the constant of proportionality from various representations.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 2.15

The relationship between the distance Andre walked in blocks and time in minutes is proportional. Use the graph to respond to each question.

a. Based on the given coordinate pair, draw the line that represents the proportional relationship.

Response given on the graph.

b. Complete the table of values so that it is modeling the same relationship.

x	15	10	1
y	10	$6\frac{2}{3}$	$\frac{2}{3}$

c. Write an equation that represents the same relationship.

$y = \frac{2}{3}x$

d. Write a verbal description of the relationship using the constant of proportionality in context.

Sample response: Andre walks $\frac{2}{3}$ blocks per minute.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can find the constant of proportionality from a table representing a proportional relationship.

1 2 3

b. I can find the constant of proportionality from a graph representing a proportional relationship.

1 2 3

c. I can find the constant of proportionality from an equation representing a proportional relationship.

1 2 3

Lesson 15 Four Ways to Tell One Story (Part 1)

Success looks like . . .

- **Goal:** Creating a proportional relationship given only one ordered pair.
  - » Creating a graph, table, and equation given the coordinate pair $(15, 10)$.
- **Language Goal:** Calculating the constant of proportionality for a proportional relationship in an unfamiliar context, and expressing it by using the correct units. **(Writing)**
  - » Determining the constant of proportionality and writing a statement describing it in terms of blocks per minute.
- **Language Goal:** Inventing and describing proportional relationships by using tables, equations, graphs, and verbal descriptions. **(Writing)**

Suggested next steps

If students incorrectly construct their graph in part a, consider:

- Reviewing Activity 1, Problem 2.

If students incorrectly complete their table of values in part b, consider:

- Reviewing Activity 1, Problem 3.

If students incorrectly write their equation in part c, consider:

- Reviewing Activity 2, Problems 1 and 3.

If students incorrectly define their constant of proportionality in part d, consider:

- Reviewing Activity 2, Problems 1 and 2.

Professional Learning

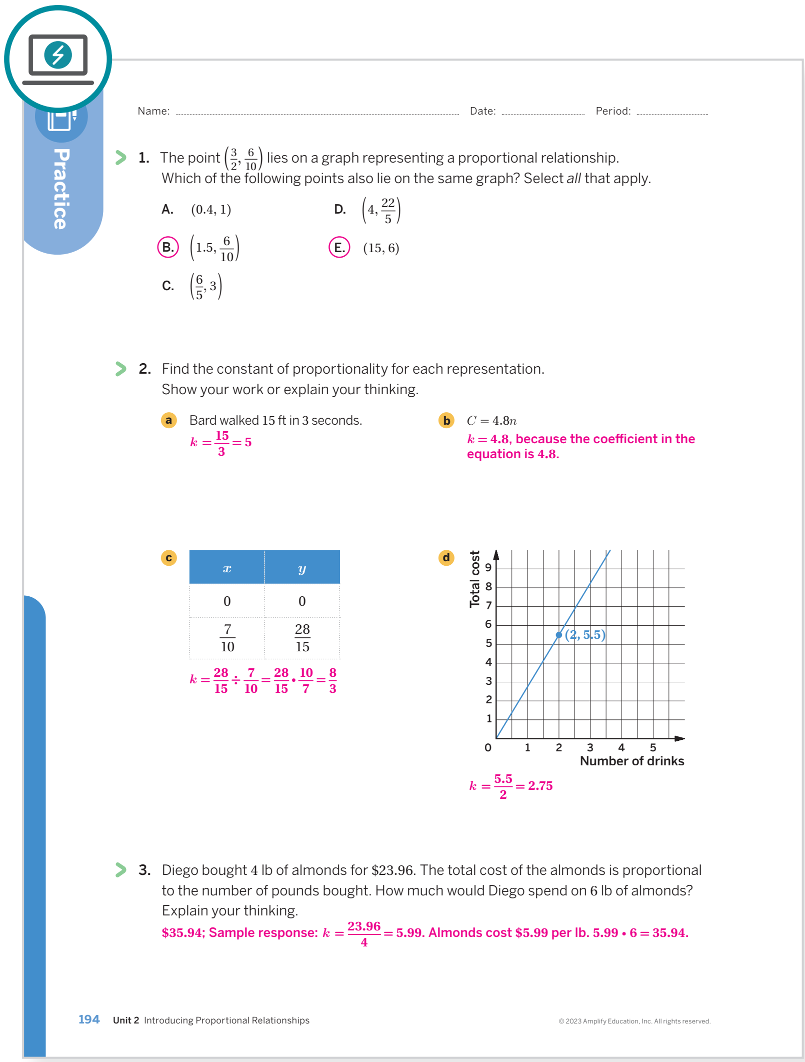
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was to find the constant of proportionality from different representations. How well did students make the connections?
- What resources did students use as they worked? Which resources were especially helpful? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ____________ Date: ________ Period: ________

1. The point $\left(\frac{3}{2}, \frac{6}{10}\right)$ lies on a graph representing a proportional relationship. Which of the following points also lie on the same graph? Select *all* that apply.

   A. $(0.4, 1)$

   [✓ B.] $\left(1.5, \frac{6}{10}\right)$

   C. $\left(\frac{6}{5}, 3\right)$

   D. $\left(4, \frac{22}{5}\right)$

   [✓ E.] $(15, 6)$

2. Find the constant of proportionality for each representation. Show your work or explain your thinking.

   a Bard walked 15 ft in 3 seconds.

   $k = \frac{15}{3} = 5$

   b $C = 4.8n$

   $k = 4.8$, because the coefficient in the equation is 4.8.

   c

x	y
0	0
$\frac{7}{10}$	$\frac{28}{15}$

   $k = \frac{28}{15} \div \frac{7}{10} = \frac{28}{15} \cdot \frac{10}{7} = \frac{8}{3}$

   d

   $k = \frac{5.5}{2} = 2.75$

3. Diego bought 4 lb of almonds for \$23.96. The total cost of the almonds is proportional to the number of pounds bought. How much would Diego spend on 6 lb of almonds? Explain your thinking.

   \$35.94; Sample response: $k = \frac{23.96}{4} = 5.99$. Almonds cost \$5.99 per lb. $5.99 \cdot 6 = 35.94$.

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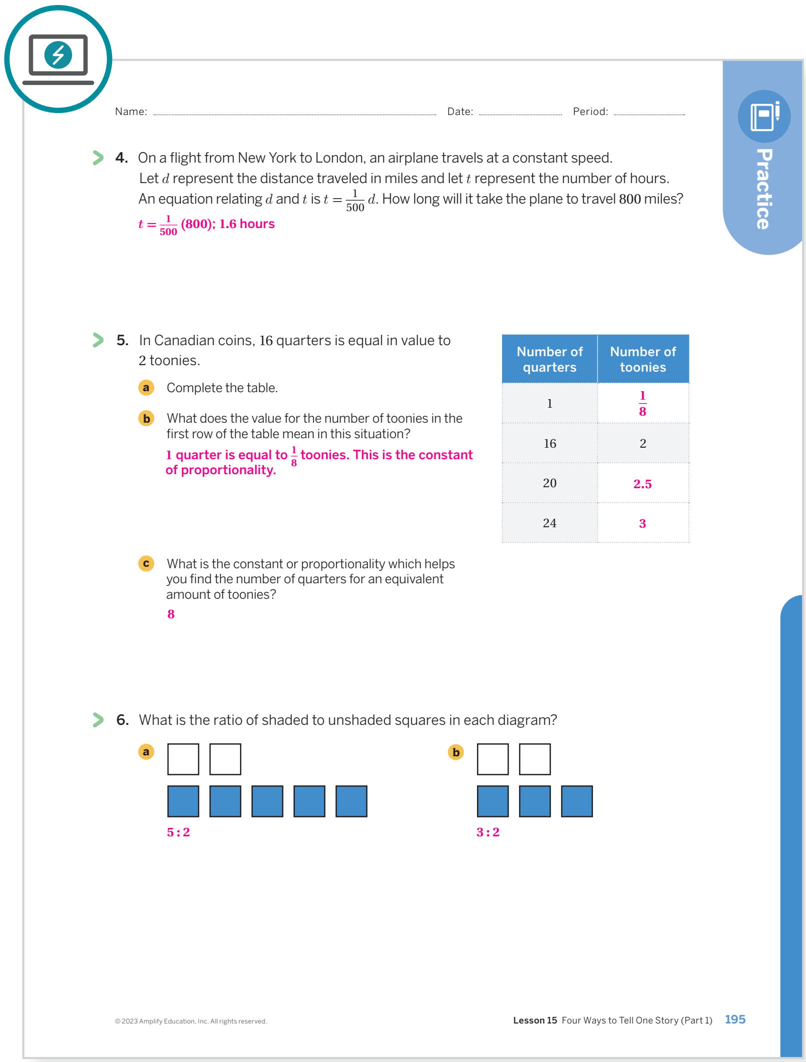

Name: ____________ Date: ________ Period: ________

4. On a flight from New York to London, an airplane travels at a constant speed. Let d represent the distance traveled in miles and let t represent the number of hours. An equation relating d and t is $t = \frac{1}{500}d$. How long will it take the plane to travel 800 miles?

   $t = \frac{1}{500}(800)$; 1.6 hours

5. In Canadian coins, 16 quarters is equal in value to 2 toonies.

   a Complete the table.

   b What does the value for the number of toonies in the first row of the table mean in this situation?

   1 quarter is equal to $\frac{1}{8}$ toonies. This is the constant of proportionality.

   c What is the constant or proportionality which helps you find the number of quarters for an equivalent amount of toonies?

   8

Number of quarters	Number of toonies
1	$\frac{1}{8}$
16	2
20	2.5
24	3

6. What is the ratio of shaded to unshaded squares in each diagram?

   a 5 : 2

   b 3 : 2

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Lesson 15 Four Ways to Tell One Story (Part 1) 195

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 2 Lesson 8	2
	4	Unit 2 Lesson 6	1
	5	Unit 2 Lesson 3	2
Formative ⚡	6	Unit 2 Lesson 16	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 2 | **LESSON 16**

Four Ways to Tell One Story (Part 2)

Let's compare relationships that are and are not proportional in four different ways.

Focus

Goal

1. **Language Goal:** Describe proportional relationships and nonproportional relationships by using various representations. **(Writing)**

Coherence

Today

Students use what they have learned about proportional relationships throughout the unit to match different representations of the same relationship together. Then they determine which relationships are proportional and which are not.

Previously

In Lesson 15, students found connections among all the representations of a single proportional relationship.

Coming Soon

In the final lesson of the unit, students will apply their knowledge of proportional relationships to mentor a new student from another country. Students will convert currency and create a map and itinerary to help welcome the new student to the area.

Rigor

- Students **apply** their knowledge of proportional and nonproportional relationships, tables, equations, graphs, and constants of proportionality to match multiple representations.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
5 min	30 min	5 min	5 min
Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, either one set for the class or one set per group
- Activity 1 PDF, *Four Representations* (answers)
- Activity 1 PDF, *Matching Four Representations* (optional)
- Activity 1 PDF, *Matching Four Representations* (answers)
- Anchor Chart PDF, *Representing Proportional Relationships*

Math Language Development

Review words

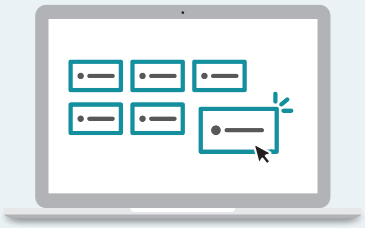
- *coefficient*
- *constant of proportionality*
- *coordinate plane*
- *nonproportional relationship*
- *ordered pair*
- *origin*
- *proportional relationship*

Amps | **Featured Activity**

Activity 1

Digital Card Sort

Students digitally sort cards and determine if the sets are proportional or nonproportional.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overwhelmed by the amount of information involved in Activity 1 and not be able to distinguish between proportional and nonproportional relationships. Have students focus on one card at a time, and as they move from one card to the next, model how they can ask themselves, "What is similar about this problem and ones I have already worked on? What is different or new about what it is asking me?" Encourage them to apply this reasoning to other cards, making the overall activity more manageable.

● Modifications to Pacing

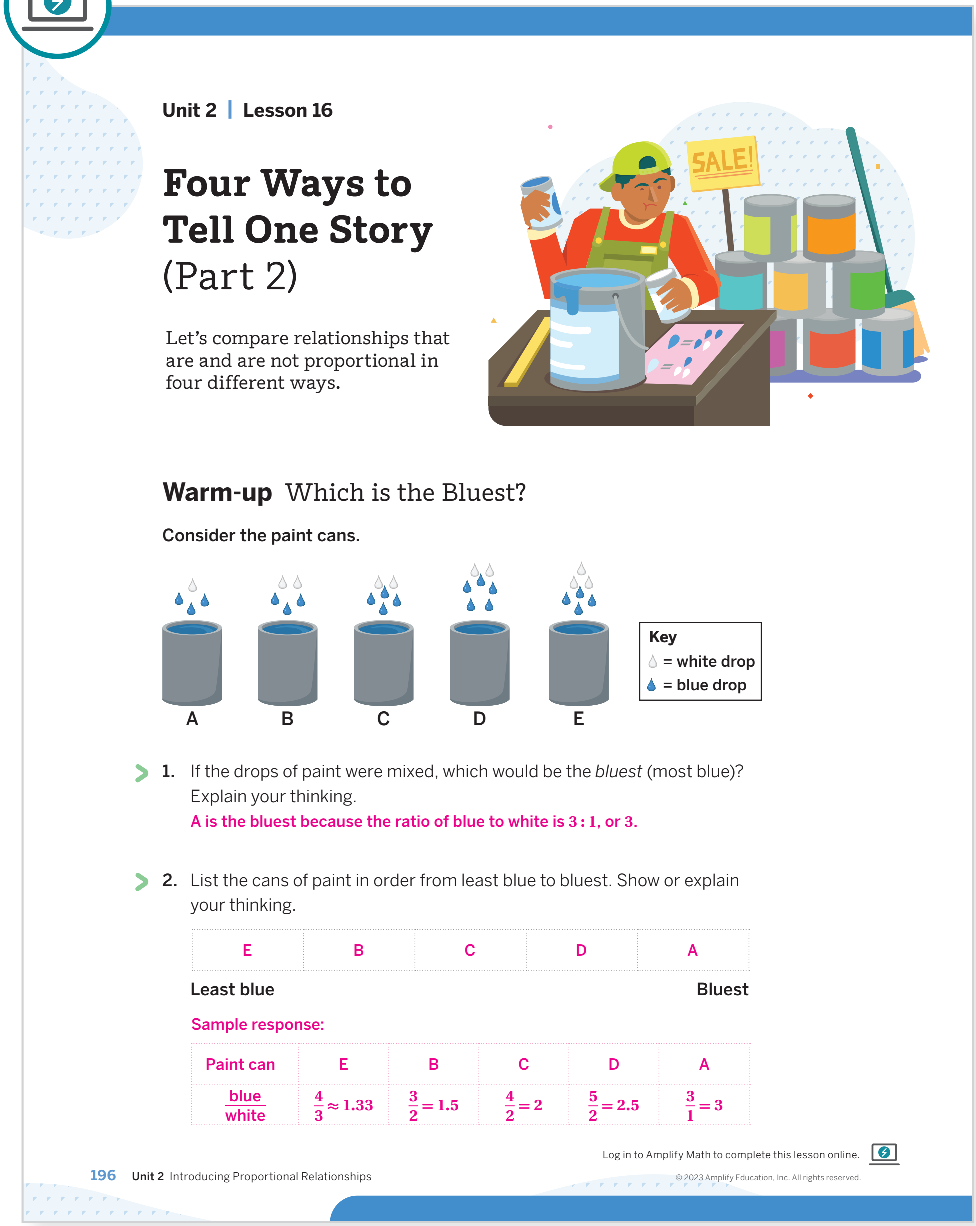
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- For **Activity 1,** limit the number of cards given to students, but ensure full sets (groups of 4 cards) of relationships are provided.

Independent | 5 min

Warm-up Which is the Bluest?

Students determine which can of paint will be the most blue to review and apply their knowledge of ratios. This Warm-up also prepares them for upcoming work in Unit 4.



Unit 2 | Lesson 16

Four Ways to Tell One Story (Part 2)

Let's compare relationships that are and are not proportional in four different ways.

Warm-up Which is the Bluest?

Consider the paint cans.

1. If the drops of paint were mixed, which would be the *bluest* (most blue)? Explain your thinking.
A is the bluest because the ratio of blue to white is 3 : 1, or 3.

2. List the cans of paint in order from least blue to bluest. Show or explain your thinking.

E	B	C	D	A

Least blue — Bluest

Sample response:

Paint can	E	B	C	D	A
$\frac{\text{blue}}{\text{white}}$	$\frac{4}{3} \approx 1.33$	$\frac{3}{2} = 1.5$	$\frac{4}{2} = 2$	$\frac{5}{2} = 2.5$	$\frac{3}{1} = 3$

Log in to Amplify Math to complete this lesson online.

196 Unit 2 Introducing Proportional Relationships

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by having students imagine the paint being mixed together and asking, "Which paint do you think will be the most blue? Now explain your choice by using math."

Look for points of confusion:

- **Saying Paint Can D is the bluest because it has 5 blue drops.** Remind students the colors are mixing and the white also will affect the color.
- **Canceling a white drop with a blue drop and counting the remaining blue drops.** Remind students that the colors will be mixed.

Look for productive strategies:

- Representing the amounts of paint by using a double number line or tape diagram.

3 Connect

Display the images.

Have students share their responses and reasoning.

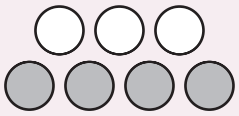
Highlight that when comparing multiple quantities, ratios are helpful. **Note:** This activity helps prepare students for Unit 4.

Ask, "If you wanted to double the amount of paint in Paint Can A, could you add 3 more blue drops and keep the same color?" No, the white will also need to be doubled.

Power-up

To power up students' ability to write ratios to represent a relationship shown by a diagram, have students complete:

Use the diagram to complete each problem:



1. How many shaded circles are there? 4
2. How many unshaded circles are there? 3
3. What is the ratio of shaded to unshaded circles? 4 : 3

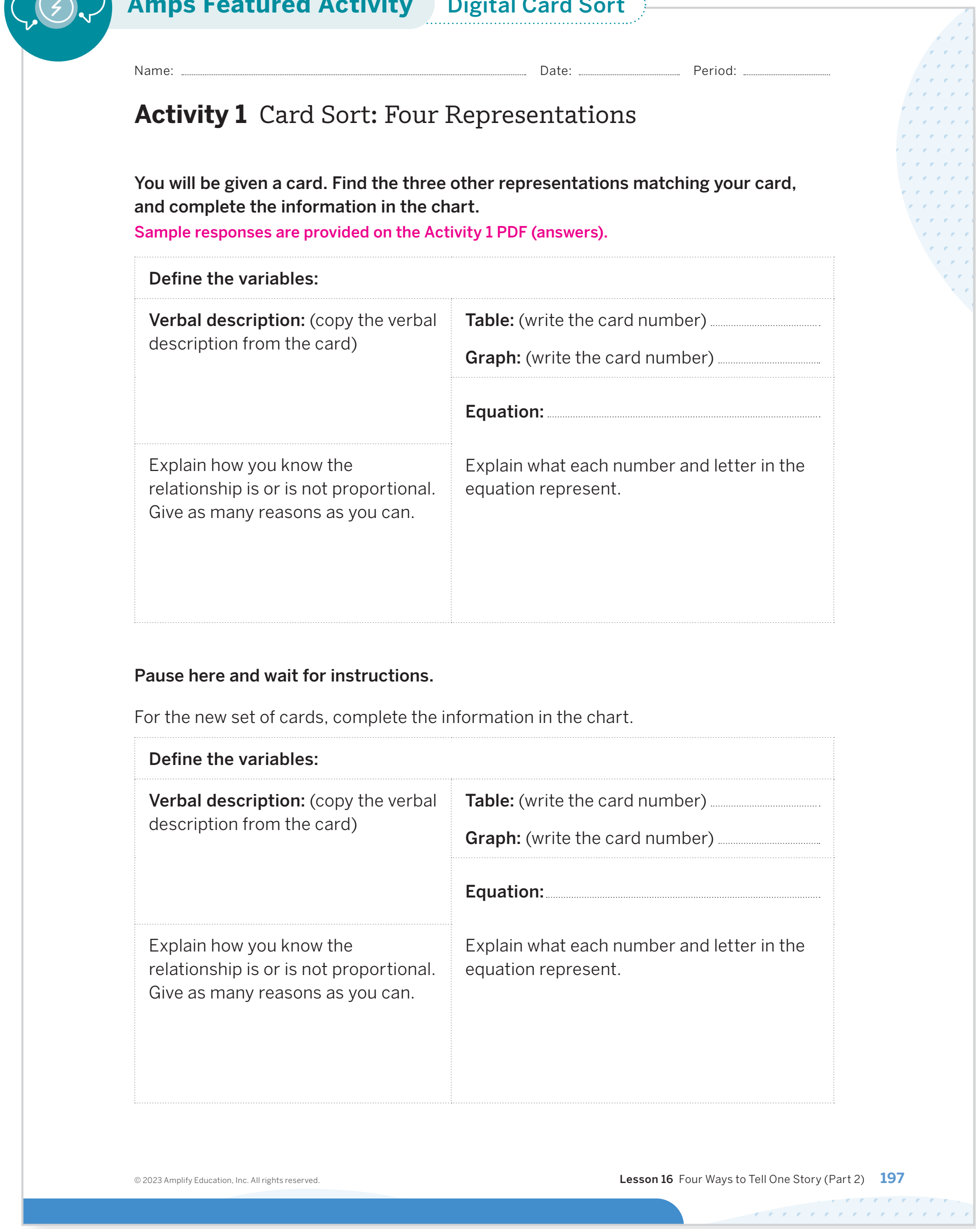
Use: Before the Warm-up.
Informed by: Performance on Lesson 15, Practice Problem 6.

Small Groups | 30 min

Activity 1 Card Sort: Four Representations

Students match multiple representations of proportional and nonproportional relationships to review the content of the unit.

Amps Featured Activity Digital Card Sort



Name: ______ Date: ______ Period: ______

Activity 1 Card Sort: Four Representations

You will be given a card. Find the three other representations matching your card, and complete the information in the chart.

Sample responses are provided on the Activity 1 PDF (answers).

Define the variables:	
Verbal description: (copy the verbal description from the card)	**Table:** (write the card number) ______ **Graph:** (write the card number) ______ **Equation:** ______
Explain how you know the relationship is or is not proportional. Give as many reasons as you can.	Explain what each number and letter in the equation represent.

Pause here and wait for instructions.

For the new set of cards, complete the information in the chart.

Define the variables:	
Verbal description: (copy the verbal description from the card)	**Table:** (write the card number) ______ **Graph:** (write the card number) ______ **Equation:** ______
Explain how you know the relationship is or is not proportional. Give as many reasons as you can.	Explain what each number and letter in the equation represent.

1 Launch

Display the Anchor Chart PDF, *Representing Proportional Relationships*. Provide each student with one pre-cut card from the Activity 1 PDF. Have students move around the classroom trying to find three other people with the other representations of their relationship. Once the groups are matched, have students record their information in the first table of the SE and answer the prompts. After a few minutes, rotate the sets of the cards and have students record the new information in the second table. Make sure groups who received a nonproportional relationship in Round 1 get a proportional relationship in Round 2. Continue the rotation process as many times as class time allows.

Note: Cards 7, 8, 15, 16, 23, 24, 31, and 32 are nonproportional relationships with which the students may be unfamiliar. Use your discretion as to whether to include this set of cards or not. If these cards are included, consider providing more support with matching these groups.

Note: If you prefer your students to be seated during this activity, make enough copies of the cards for each group to receive a set and have students complete a standard ***Card Sort*** routine with their groups. Use the optional Activity 1 PDF, *Matching Four Representations* for students to record their answers.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider using one of these alternative approaches to this activity.

- Limit the activity by having students only match the representations of the proportional relationships.
- Use the *I Have . . ., Who Has . . .* routine. Have one student share their card and everyone checks their individual cards for representations of the same relationship.
- There are 8 different relationships. Provide each group with 2 or 3 relationships (all representations of those relationships). Have students sort the cards into groups, where each group represents the same relationship.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

As you circulate and monitor students during this activity, ask these questions to help facilitate discussion:

- "Why is your relationship proportional or nonproportional?"
- "Does your card match your partner's card? If not, what is different?"

English Learners

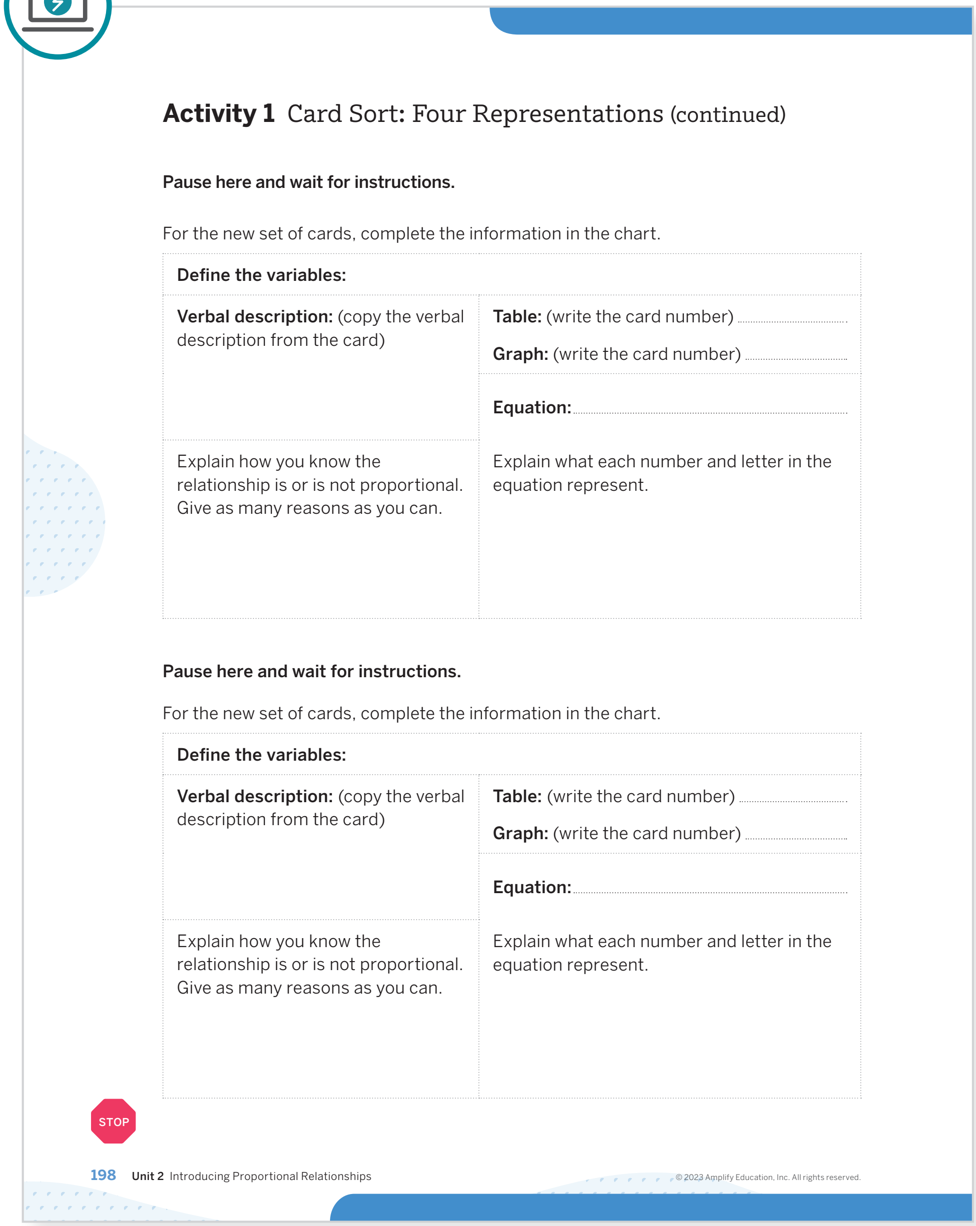
Provide sentence starters, such as:

- "My relationship is proportional because . . ."
- "My relationship is nonproportional because . . ."
- "The constant of proportionality is _____"
- "We match (do not match) because . . ."

Small Groups | 30 min

Activity 1 Card Sort: Four Representations (continued)

Students match multiple representations of proportional and nonproportional relationships to review the content of the unit.



Activity 1 Card Sort: Four Representations (continued)

Pause here and wait for instructions.

For the new set of cards, complete the information in the chart.

Define the variables:

Verbal description: (copy the verbal description from the card)

Table: (write the card number)

Graph: (write the card number)

Equation:

Explain how you know the relationship is or is not proportional. Give as many reasons as you can.

Explain what each number and letter in the equation represent.

Pause here and wait for instructions.

For the new set of cards, complete the information in the chart.

Define the variables:

Verbal description: (copy the verbal description from the card)

Table: (write the card number)

Graph: (write the card number)

Equation:

Explain how you know the relationship is or is not proportional. Give as many reasons as you can.

Explain what each number and letter in the equation represent.

STOP

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2 Monitor

Help students get started by asking them to reference the Anchor Chart PDF, *Representing Proportional Relationships*, for their particular representation.

Look for points of confusion:

- **Mixing up relationships with the same variables.** Remind students there are two constants of proportionality for every proportional relationship. They need to make sure their representations have the same constants of proportionality.

Look for productive strategies:

- Identifying the constant of proportionality when the relationship is proportional or knowing what the variables x and y represent before trying to determine that of their partner.

3 Connect

Display needed sets of cards to help with the discussion.

Have students share the strategies they used to determine their original matches.

Ask:

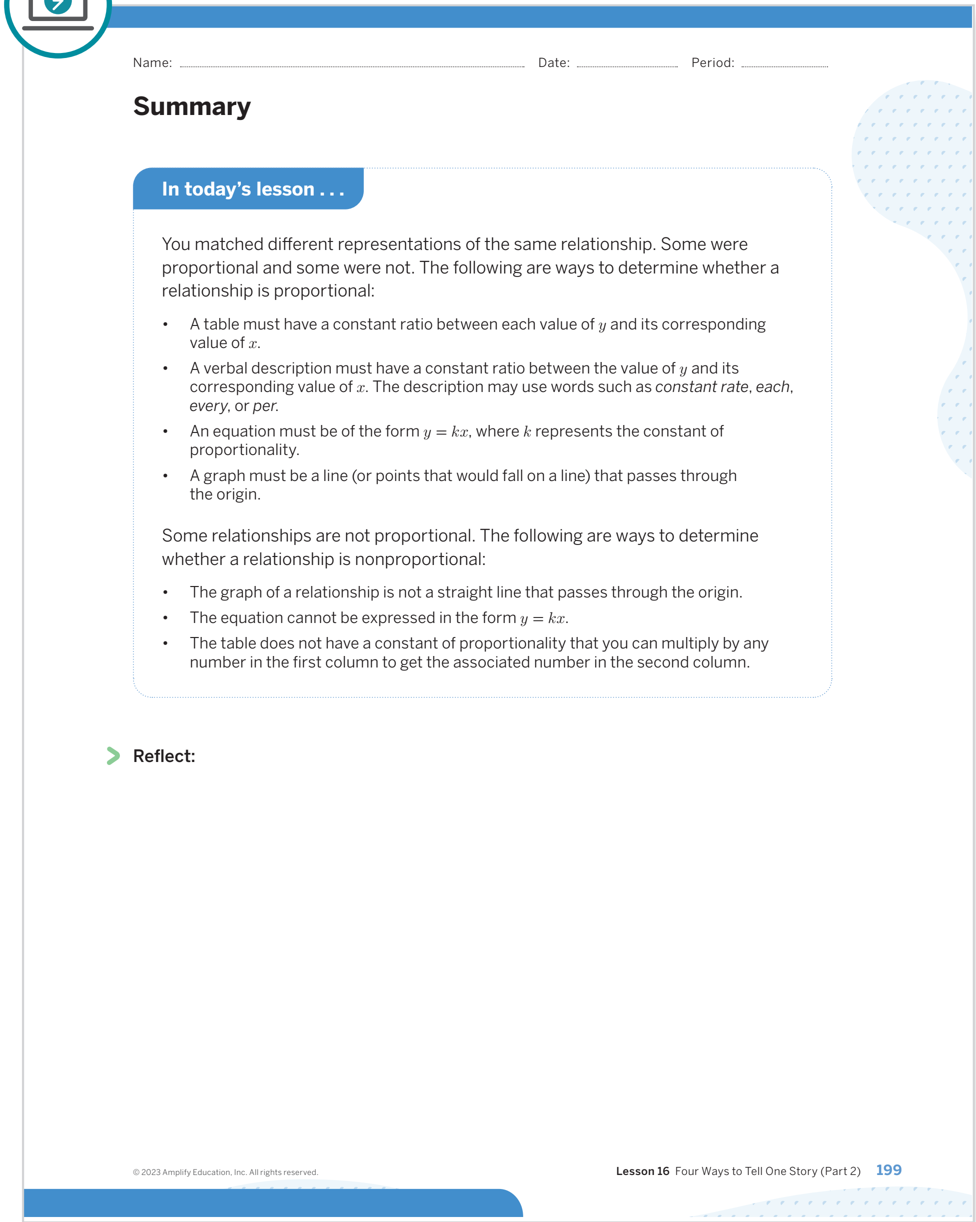
- "Did anyone mention a strategy that you did not use but that you would consider in the future?"
- "How did you know when the relationships were nonproportional?"

Highlight that knowing the variables and constant of proportionality is helpful when determining how to represent a relationship.

Whole Class | 5 min

Summary

Review and synthesize the methods for determining whether relationships are proportional from the various representations.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You matched different representations of the same relationship. Some were proportional and some were not. The following are ways to determine whether a relationship is proportional:

- A table must have a constant ratio between each value of y and its corresponding value of x.
- A verbal description must have a constant ratio between the value of y and its corresponding value of x. The description may use words such as *constant rate*, *each*, *every*, or *per*.
- An equation must be of the form $y = kx$, where k represents the constant of proportionality.
- A graph must be a line (or points that would fall on a line) that passes through the origin.

Some relationships are not proportional. The following are ways to determine whether a relationship is nonproportional:

- The graph of a relationship is not a straight line that passes through the origin.
- The equation cannot be expressed in the form $y = kx$.
- The table does not have a constant of proportionality that you can multiply by any number in the first column to get the associated number in the second column.

Reflect:

Lesson 16 Four Ways to Tell One Story (Part 2) 199

Synthesize

Display the Anchor Chart PDF, *Representing Proportional Relationships*.

Ask "How do you know whether a relationship is proportional by looking at the graph? The table? The story or verbal description? The equation?"

Sample responses:

- Graph: The graph is a straight line (or points that would fall on a line) that passes through the origin.
- Table: There is a constant ratio between each pair of values.
- Story or verbal description: The story must describe a constant ratio between corresponding pairs of values.
- Equation: The equation must be of the form $y = kx$, where k represents the constant of proportionality.

Have students share their strategies for determining whether a relationship is proportional using a verbal description, a table, an equation, and a graph.

Highlight the multiple ways to determine whether a relationship is proportional.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

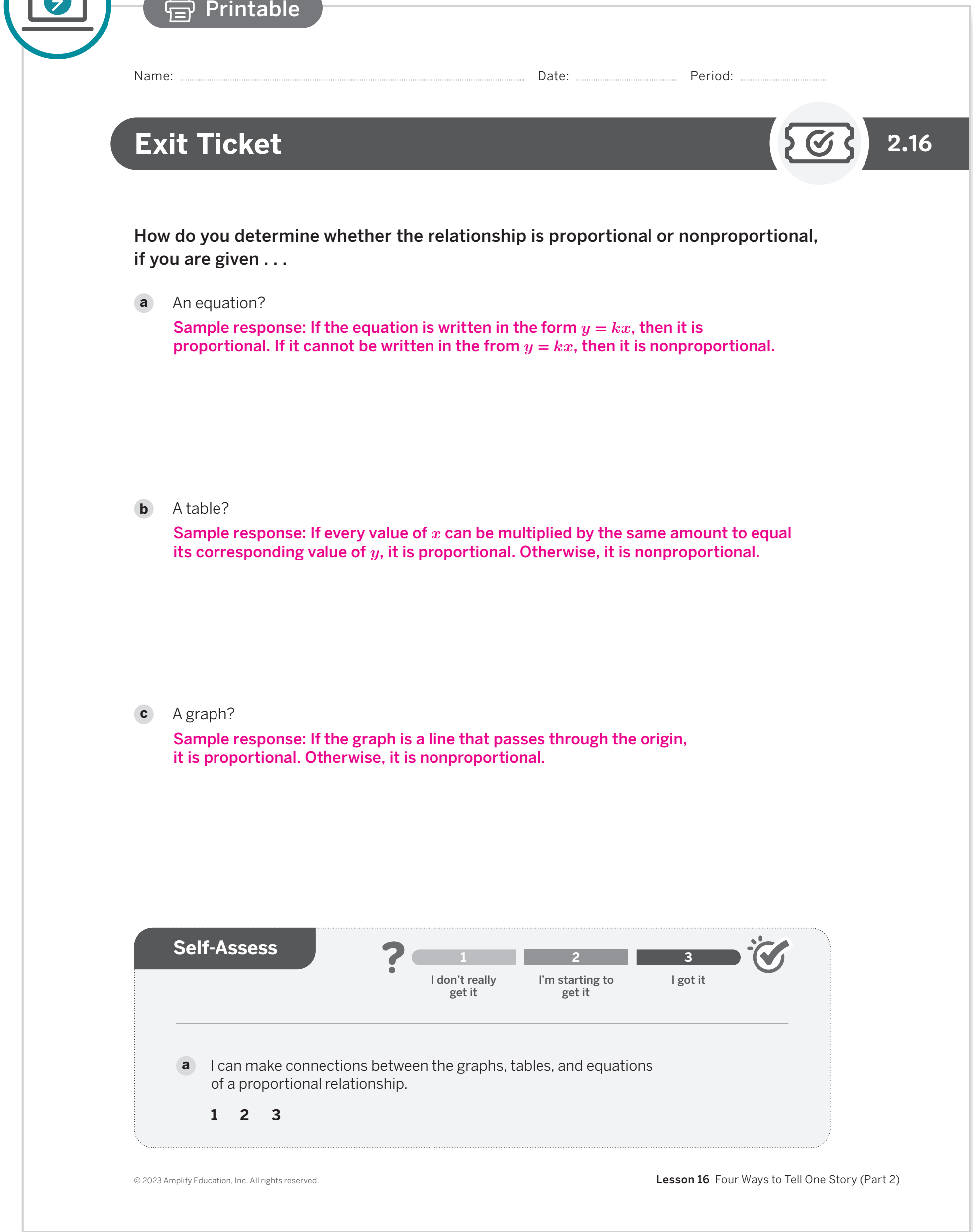
- "What are the different ways you can represent proportional relationships? How are the representations related?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing how to determine whether a relationship is proportional from the multiple representations.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.16

How do you determine whether the relationship is proportional or nonproportional, if you are given . . .

a An equation?

Sample response: If the equation is written in the form $y = kx$, then it is proportional. If it cannot be written in the from $y = kx$, then it is nonproportional.

b A table?

Sample response: If every value of x can be multiplied by the same amount to equal its corresponding value of y, it is proportional. Otherwise, it is nonproportional.

c A graph?

Sample response: If the graph is a line that passes through the origin, it is proportional. Otherwise, it is nonproportional.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can make connections between the graphs, tables, and equations of a proportional relationship.

1 2 3

Lesson 16 Four Ways to Tell One Story (Part 2)

Success looks like . . .

- **Language Goal:** Describing proportional relationships and nonproportional relationships by using various representations. **(Writing)**
  - » Describing how proportional and non proportional relationships are represented by an equation, a table, and a graph.

Suggested next steps

If students incorrectly identify how to determine whether a relationship is proportional, consider:

- Reviewing the appropriate representation from Activity 1.
- Reviewing the appropriate section of the Anchor Chart PDF, *Representing Proportional Relationships*.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did the ***Card Sort*** activity go as planned?
- In what ways did things happen that you did not expect? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Describing proportional relationships and nonproportional relationships by using various representations.

Reflect on students' language development toward this goal.

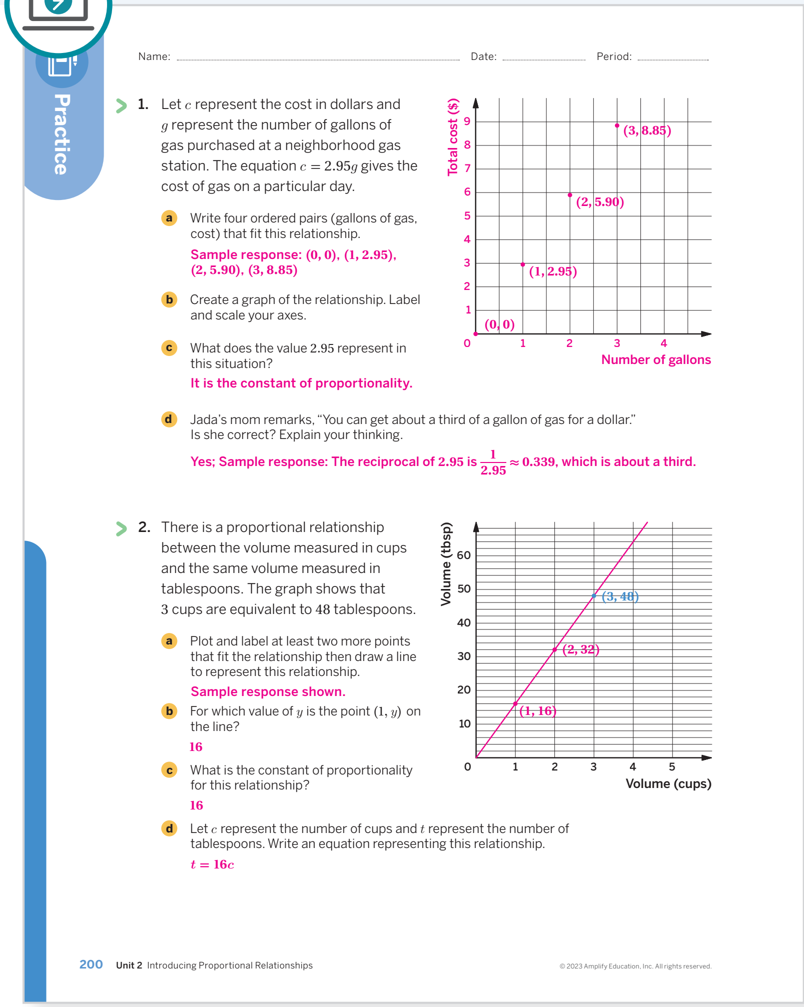
- How have students progressed toward justifying whether a relationship is proportional by studying tables, graphs, or equations?
- What su pport do they still need in order to be more precise in their justifications?

Sample justification for a graph:

Emerging	Expanding
It must be a straight line.	A straight line that passes through the origin represents a proportional relationship. Otherwise, the relationship is nonproportional.

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Let c represent the cost in dollars and g represent the number of gallons of gas purchased at a neighborhood gas station. The equation $c = 2.95g$ gives the cost of gas on a particular day.

- **a** Write four ordered pairs (gallons of gas, cost) that fit this relationship.

  Sample response: $(0, 0)$, $(1, 2.95)$, $(2, 5.90)$, $(3, 8.85)$

- **b** Create a graph of the relationship. Label and scale your axes.

- **c** What does the value 2.95 represent in this situation?

  It is the constant of proportionality.

- **d** Jada's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? Explain your thinking.

  Yes; Sample response: The reciprocal of 2.95 is $\frac{1}{2.95} \approx 0.339$, which is about a third.

2. There is a proportional relationship between the volume measured in cups and the same volume measured in tablespoons. The graph shows that 3 cups are equivalent to 48 tablespoons.

- **a** Plot and label at least two more points that fit the relationship then draw a line to represent this relationship.

  Sample response shown.

- **b** For which value of y is the point $(1, y)$ on the line?

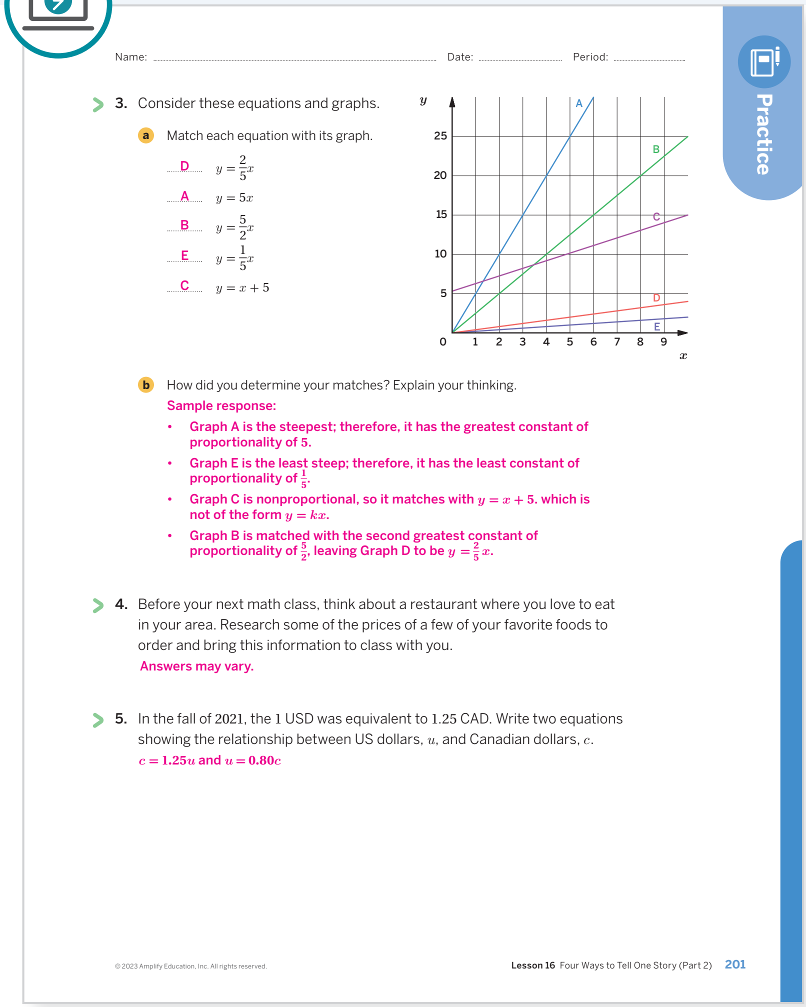
  16

- **c** What is the constant of proportionality for this relationship?

  16

- **d** Let c represent the number of cups and t represent the number of tablespoons. Write an equation representing this relationship.

  $t = 16c$



Practice

Name: ________ Date: ________ Period: ________

3. Consider these equations and graphs.

- **a** Match each equation with its graph.

  D $y = \frac{2}{5}x$
  A $y = 5x$
  B $y = \frac{5}{2}x$
  E $y = \frac{1}{5}x$
  C $y = x + 5$

- **b** How did you determine your matches? Explain your thinking.

  Sample response:
  - **Graph A is the steepest; therefore, it has the greatest constant of proportionality of 5.**
  - **Graph E is the least steep; therefore, it has the least constant of proportionality of $\frac{1}{5}$.**
  - **Graph C is nonproportional, so it matches with $y = x + 5$. which is not of the form $y = kx$.**
  - **Graph B is matched with the second greatest constant of proportionality of $\frac{5}{2}$, leaving Graph D to be $y = \frac{2}{5}x$.**

4. Before your next math class, think about a restaurant where you love to eat in your area. Research some of the prices of a few of your favorite foods to order and bring this information to class with you.

Answers may vary.

5. In the fall of 2021, the 1 USD was equivalent to 1.25 CAD. Write two equations showing the relationship between US dollars, u, and Canadian dollars, c.

$c = 1.25u$ and $u = 0.80c$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
Spiral	3	Unit 2 Lesson 13	2
Formative ⚡	4	Unit 2 Lesson 17	
	5	Unit 2 Lesson 17	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Welcoming Committee

Let's help a new student from another country feel welcomed and acquainted with your area.

Focus

Goals

1. Use an exchange rate to convert values from one currency to another.
2. **Language Goal:** Create a map with a scale and use it to calculate distance, speed, and time. **(Speaking and Listening)**
3. Use shared interests to help a new student feel welcomed.

Coherence

Today

Students explore the scenario of helping a new student in their school feel welcomed and acquainted with the area. They convert currencies for prices on a menu and create a map and itinerary for activities to do with the new student.

Previously

In Lesson 16, students matched multiple representations together and then determined which relationships were proportional and which were not.

Coming Soon

In Unit 3, students will begin their exploration of geometry involving circles, including learning about the special number π (pi) and how it represents an important ratio.

Rigor

- Students **apply** their understanding of proportional relationships while reasoning about real-world contexts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	10 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Restaurant Menu* (optional)
- Activity 2 PDF (as needed)
- Activity 2 PDF (answers)
- calculators
- rulers

Math Language Development

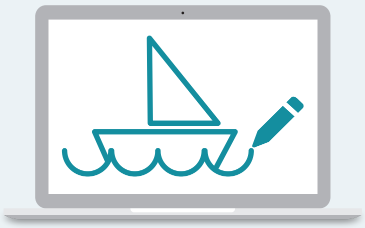
Review words

- *proportional relationship*
- *scale*
- *unit rate*

Amps | **Featured Activity AG**

Activity 2
Create a Map

Students use digital tools to help create a map of their area.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might become anxious about sharing their maps because it might be different than someone else's. Encourage students to celebrate these differences. They should all consider the similarities that connect them during the activity and express appreciation for the efforts of others.

Modifications to Pacing

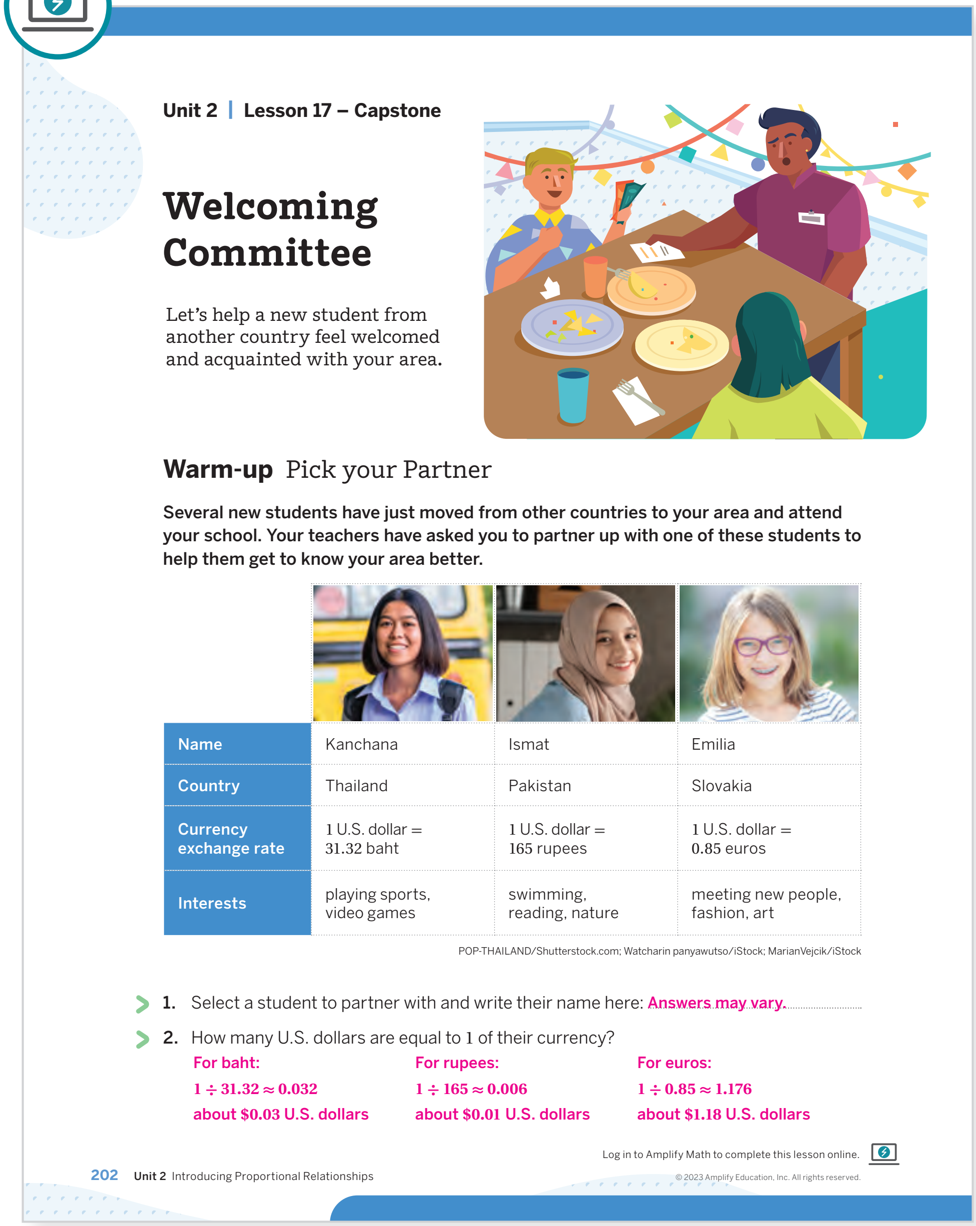
You may want to consider this additional modification if you are short on time.

- Either **Activity 1** or **Activity 2** may be omitted.

Independent | 8 min

Warm-up Pick Your Partner

Students use proportional reasoning to determine the unit rate for exchanging their country-of-origin's currency.



Unit 2 | Lesson 17 – Capstone

Welcoming Committee

Let's help a new student from another country feel welcomed and acquainted with your area.

Warm-up Pick your Partner

Several new students have just moved from other countries to your area and attend your school. Your teachers have asked you to partner up with one of these students to help them get to know your area better.

Name	Kanchana	Ismat	Emilia
Country	Thailand	Pakistan	Slovakia
Currency exchange rate	1 U.S. dollar = 31.32 baht	1 U.S. dollar = 165 rupees	1 U.S. dollar = 0.85 euros
Interests	playing sports, video games	swimming, reading, nature	meeting new people, fashion, art

POP-THAILAND/Shutterstock.com; Watcharin panyawutso/iStock; MarianVejcik/iStock

1. Select a student to partner with and write their name here: **Answers may vary.**
2. How many U.S. dollars are equal to 1 of their currency?

For baht:
$1 \div 31.32 \approx 0.032$
about \$0.03 U.S. dollars

For rupees:
$1 \div 165 \approx 0.006$
about \$0.01 U.S. dollars

For euros:
$1 \div 0.85 \approx 1.176$
about \$1.18 U.S. dollars

Log in to Amplify Math to complete this lesson online.

202 Unit 2 Introducing Proportional Relationships

1 Launch

To activate background knowledge, consider displaying a map and asking students to locate Thailand, Pakistan, and Slovakia. Read the introduction together as a class and give students time to look at the information in the table. Provide access to calculators.

2 Monitor

Help students get started by asking whether they share any hobbies with the new students.

Look for points of confusion:

- **Thinking that they can swap the values in the ratio to get the amount of dollars equal to one of the new students' currencies.** Have students think about the rupees and ask, "Will one rupee be worth more or less than one dollar?"

Look for productive strategies:

- Using the reciprocal for Problem 2.

3 Connect

Have students share which students they chose to partner with and their reasons why.

Ask:

- "How are euros different from both rupees and baht?"
- "Are any of these currencies easier to convert to dollars than the others? Why do you think that is?"
- "If you traveled to your partner's country of origin, how much would you expect to pay for lunch in their currency?

Highlight that when converting from U.S. dollars to one of the foreign currencies, students see the ratio in the table, but to convert from the foreign currency to U.S. dollars, students use the reciprocal of that ratio.

Differentiated Support

Extension: ***Interdisciplinary Connections***

Provide some background context on the different currencies mentioned in the Warm-up. Consider also displaying some images of the currency used in each country. **(Social Studies)**

- **Thailand:** The Thai baht is the official currency of Thailand. 1 baht is equal to 100 satangs.
- **Pakistan:** The Pakistani rupee is the official currency of Pakistan. 1 Pakistani rupee is equal to 100 paise.
- **Slovakia:** Slovakia adopted the euro in 2009 as its official currency. 1 euro is equal to 100 cents.

Power-up

To power up students' ability to write equations to represent proportional relationships, have students complete:

1. Use ratio reasoning to complete the missing values in the table.
2. Write two equations showing the relationship between milk and flour

$f = 2.5m$
$m = 0.4f$

Milk	Flour
1	2.5
m	$2.5m$
0.4	1
$0.4f$	f

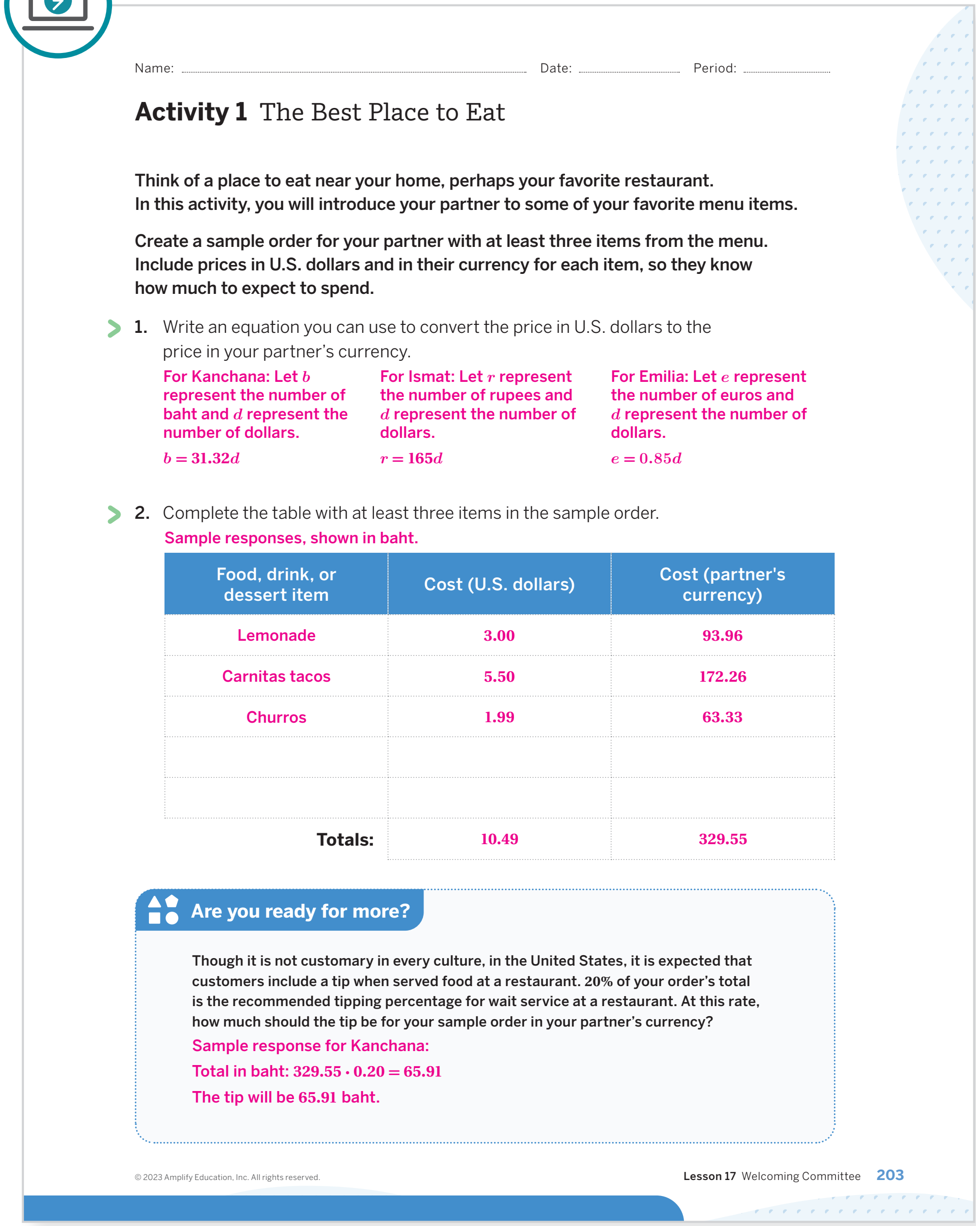
Use: Before Activity 1.

Informed by: Performance on Lesson 16, Practice Problem 5.

Pairs | 10 min

Activity 1 The Best Place to Eat

Students convert the prices of items on a menu from U.S. dollars to another currency to apply their understanding and skills about proportional relationships.



Name: ______ Date: ______ Period: ______

Activity 1 The Best Place to Eat

Think of a place to eat near your home, perhaps your favorite restaurant. In this activity, you will introduce your partner to some of your favorite menu items.

Create a sample order for your partner with at least three items from the menu. Include prices in U.S. dollars and in their currency for each item, so they know how much to expect to spend.

1. Write an equation you can use to convert the price in U.S. dollars to the price in your partner's currency.

   For Kanchana: Let b represent the number of baht and d represent the number of dollars.
   $b = 31.32d$

   For Ismat: Let r represent the number of rupees and d represent the number of dollars.
   $r = 165d$

   For Emilia: Let e represent the number of euros and d represent the number of dollars.
   $e = 0.85d$

2. Complete the table with at least three items in the sample order.
   Sample responses, shown in baht.

Food, drink, or dessert item	Cost (U.S. dollars)	Cost (partner's currency)
Lemonade	3.00	93.96
Carnitas tacos	5.50	172.26
Churros	1.99	63.33
Totals:	10.49	329.55

Are you ready for more?

Though it is not customary in every culture, in the United States, it is expected that customers include a tip when served food at a restaurant. 20% of your order's total is the recommended tipping percentage for wait service at a restaurant. At this rate, how much should the tip be for your sample order in your partner's currency?

Sample response for Kanchana:

Total in baht: $329.55 \cdot 0.20 = 65.91$

The tip will be 65.91 baht.

Lesson 17 Welcoming Committee 203

1 Launch

Prompt students to think of the prices of food at their favorite place to eat, which you may have had them prepare ahead of time. Provide access to the Activity 1 PDF, *Restaurant Menu*, for students who do not have the information ready.

2 Monitor

Help students get started by reminding them to define their variables before writing their equations.

Look for points of confusion:

- **Leaving the cost in their partner's currency unrounded to the nearest hundredth.** Ask, "Which decimal places are important for expressing a value in U.S. currency?"

Look for productive strategies:

- Substituting the cost for each item in U.S. dollars into the equation from Problem 1.

3 Connect

Have students share their orders with another student. Prompt them to explain why they chose the items they did, and have them check each other's work for accuracy.

Ask, "What relationship exists between the total costs?" The total cost in the partner's currency is related to the cost in U.S. dollars by the same constant of proportionality as the exchange rate.

Highlight that the totals for U.S. dollars and your partner's currency will be proportional as well.

Differentiated Support

Extension: *Math Around the Word*

Read the following text to students, and then have them complete the problem at the end.

Brazil's currency is called the real (plural: reais). Researchers have found that some children in Brazil start working at a young age as street merchants to survive. Although these children have received very limited formal education, they can mentally compute the cost of items they are selling by inventing their own strategies. For example, if one coconut costs 35 reais, how much would 10 coconuts cost?

Ask students to come up with different ways to mentally solve this problem, without using any rules they have learned in school.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect conversion equation, such as $d = 31.32b$ for Kanchana, where d represents U.S. dollars and b represents Thai baht. Provide an incorrect statement, such as "This equation is correct because there are 31.32 baht for every 1 U.S. dollar." Ask pairs of students to critique this statement, write a corrected statement, and clarify their reasoning as to why and how they corrected it.

English Learners

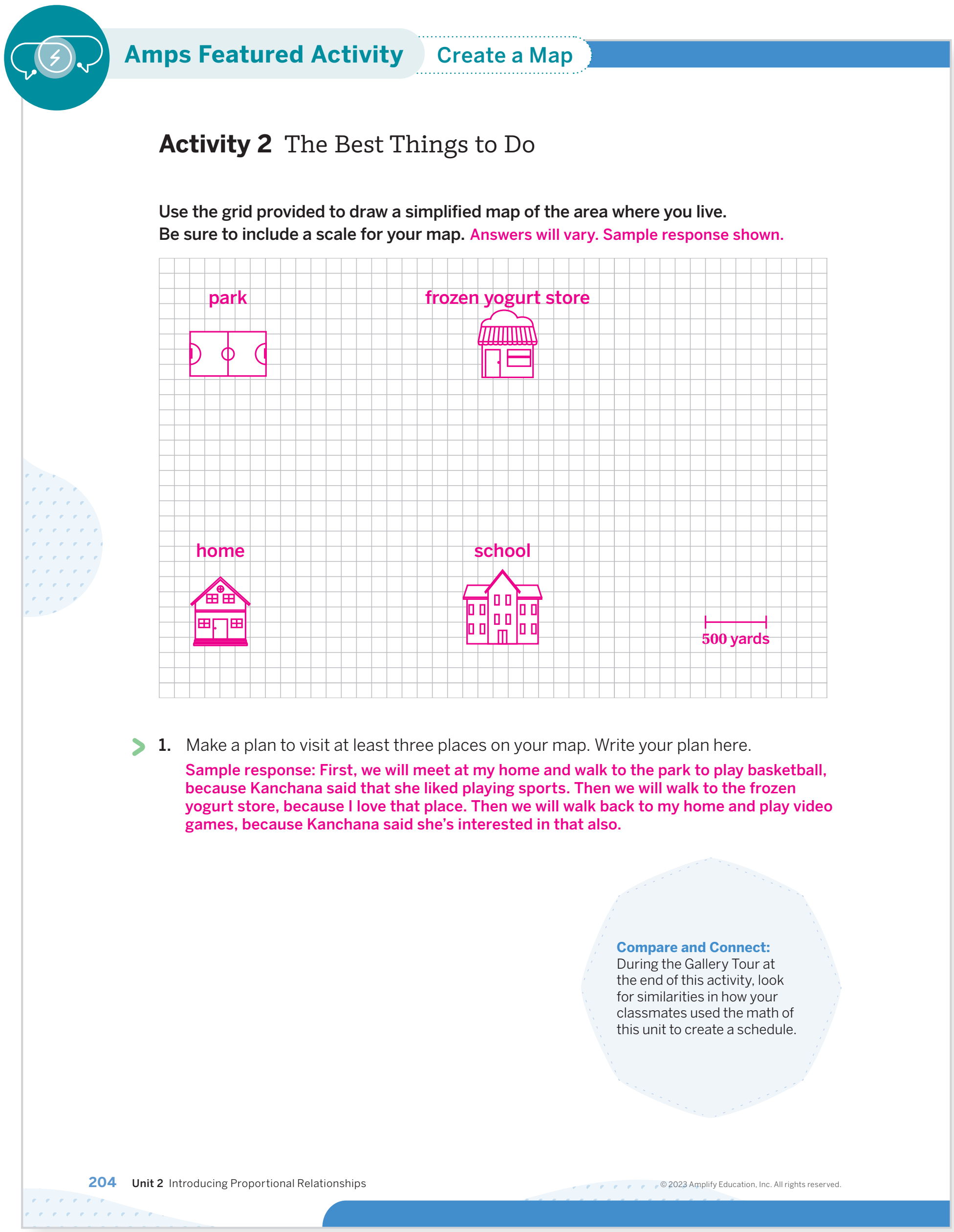
Annotate the variables in the equations with the quantities they represent.

Pairs | 20 min

Activity 2 The Best Things to Do

Students draw a map with a scale of their area and create an accurate itinerary based on distances between activities, to apply proportional reasoning when calculating distance, time, and speed.

Amps Featured Activity Create a Map



Activity 2 The Best Things to Do

Use the grid provided to draw a simplified map of the area where you live.
Be sure to include a scale for your map. Answers will vary. Sample response shown.

1. Make a plan to visit at least three places on your map. Write your plan here.
Sample response: First, we will meet at my home and walk to the park to play basketball, because Kanchana said that she liked playing sports. Then we will walk to the frozen yogurt store, because I love that place. Then we will walk back to my home and play video games, because Kanchana said she's interested in that also.

Compare and Connect: During the Gallery Tour at the end of this activity, look for similarities in how your classmates used the math of this unit to create a schedule.

204 Unit 2 Introducing Proportional Relationships

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1 Launch

Activate background knowledge by asking students about some of their favorite activities or places to visit in their area, within walking distance. Display a map of a local area to help students visualize what their map may look like. Provide access to rulers and the Activity 2 PDF as needed to support students in creating their schedules in Problem 2.

2 Monitor

Help students get started by mentioning that a common informal measurement unit for longer distances is a football field, which is 100 yards long.

Look for points of confusion:

- **Placing all of the locations on their map right next to each other.** Have students think about the relative distances between the locations. Ask, "Is it the same distance from the park to your home as it is from the school to your home?"
- **Not being able to imagine the number of lengths of a football field between locations.** Prompt students to reason about how long it takes them to walk from one place to another, and to estimate that they walk 100 yards per minute.
- **Choosing locations too far apart to walk between.** Suggest students choose any area they know where they can walk between locations.

Look for productive strategies:

- Using the grid lines to help measure vertical and horizontal distances.
- Writing an equation that converts time in seconds to time in minutes.

Activity 2 continued >

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Consider using one of these alternative approaches to this activity.

- Allow students to create their map using simpler distances rather than actual distances.
- Create and provide a sample map for students to use. Include at least 4 locations, such as a house or apartment, library, grocery store, or subway station.

Math Language Development

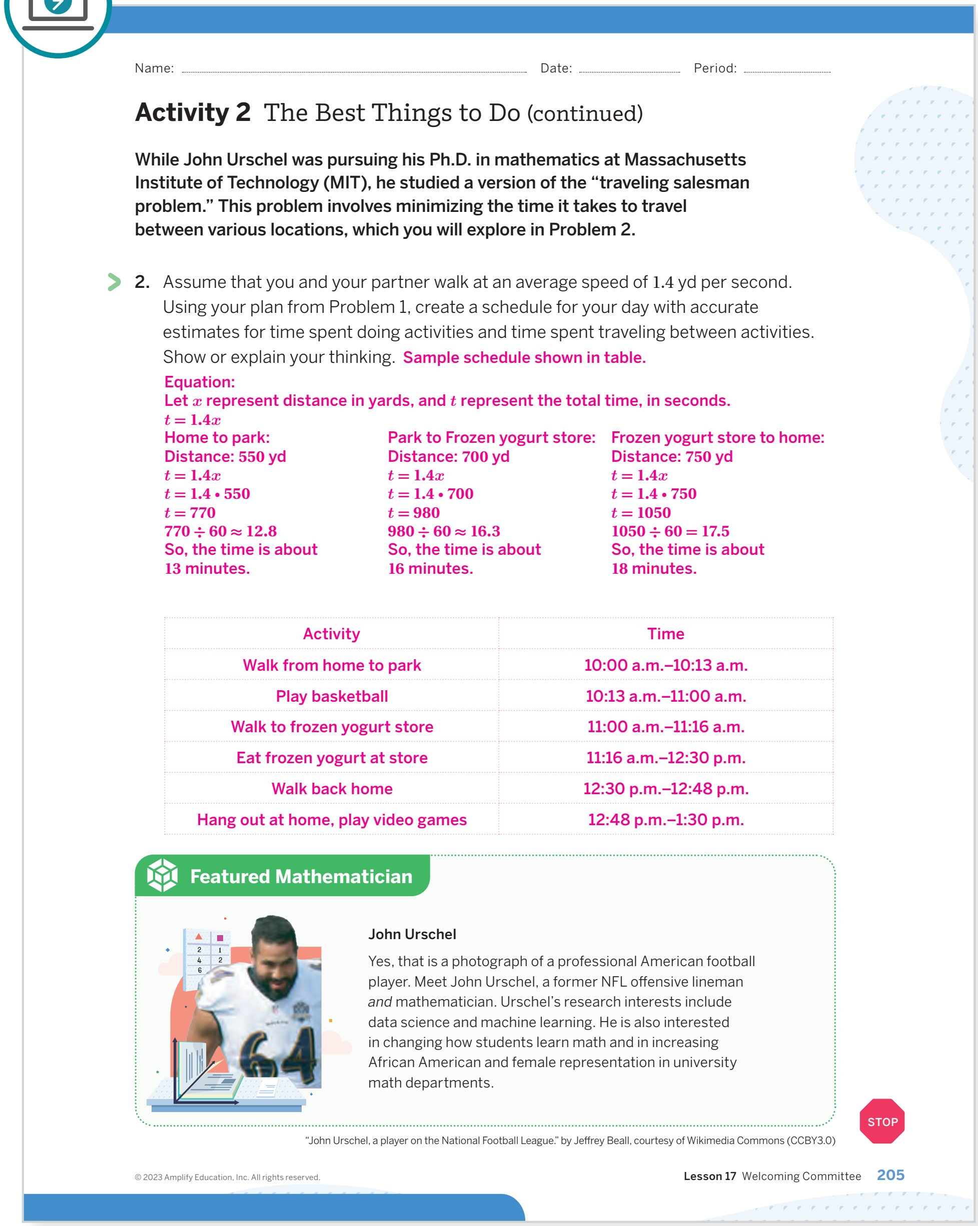
MLR7: Compare and Connect

Use this routine during the *Gallery Tour*. Provide students with sticky notes. Ask them to note any similarities among the strategies their classmates used to create a schedule, such as the use of equations, tables, and the constant of proportionality.

Pairs | 20 min

Activity 2 The Best Things to Do (continued)

Students draw a map with a scale of their area and create an accurate itinerary based on distances between activities, to apply proportional reasoning when calculating distance, time, and speed.



Name: ______ Date: ______ Period: ______

Activity 2 The Best Things to Do (continued)

While John Urschel was pursuing his Ph.D. in mathematics at Massachusetts Institute of Technology (MIT), he studied a version of the "traveling salesman problem." This problem involves minimizing the time it takes to travel between various locations, which you will explore in Problem 2.

2. Assume that you and your partner walk at an average speed of 1.4 yd per second. Using your plan from Problem 1, create a schedule for your day with accurate estimates for time spent doing activities and time spent traveling between activities. Show or explain your thinking. **Sample schedule shown in table.**

Equation:

Let x represent distance in yards, and t represent the total time, in seconds.

$t = 1.4x$

Home to park:	Park to Frozen yogurt store:	Frozen yogurt store to home:
Distance: 550 yd	Distance: 700 yd	Distance: 750 yd
$t = 1.4x$	$t = 1.4x$	$t = 1.4x$
$t = 1.4 \cdot 550$	$t = 1.4 \cdot 700$	$t = 1.4 \cdot 750$
$t = 770$	$t = 980$	$t = 1050$
$770 \div 60 \approx 12.8$	$980 \div 60 \approx 16.3$	$1050 \div 60 = 17.5$
So, the time is about 13 minutes.	So, the time is about 16 minutes.	So, the time is about 18 minutes.

Activity	Time
Walk from home to park	10:00 a.m.–10:13 a.m.
Play basketball	10:13 a.m.–11:00 a.m.
Walk to frozen yogurt store	11:00 a.m.–11:16 a.m.
Eat frozen yogurt at store	11:16 a.m.–12:30 p.m.
Walk back home	12:30 p.m.–12:48 p.m.
Hang out at home, play video games	12:48 p.m.–1:30 p.m.

Featured Mathematician

John Urschel

Yes, that is a photograph of a professional American football player. Meet John Urschel, a former NFL offensive lineman *and* mathematician. Urschel's research interests include data science and machine learning. He is also interested in changing how students learn math and in increasing African American and female representation in university math departments.

"John Urschel, a player on the National Football League." by Jeffrey Beall, courtesy of Wikimedia Commons (CCBY3.0)

STOP

Lesson 17 Welcoming Committee 205

3 Connect

Display student work using the ***Gallery Tour*** routine.

Have students share connections they noticed among the different maps in the class.

Ask, "Did anyone find that one of your calculations was either unreasonably big or small? How did you check whether you were correct?"

Highlight that creating a schedule of activities, or an itinerary, can require some careful calculations and often includes estimating distance, time, or speed. Tour guides must make schedules in a similar way, but other jobs, such as mapping package delivery routes, involve similar mathematical work.

Featured Mathematician

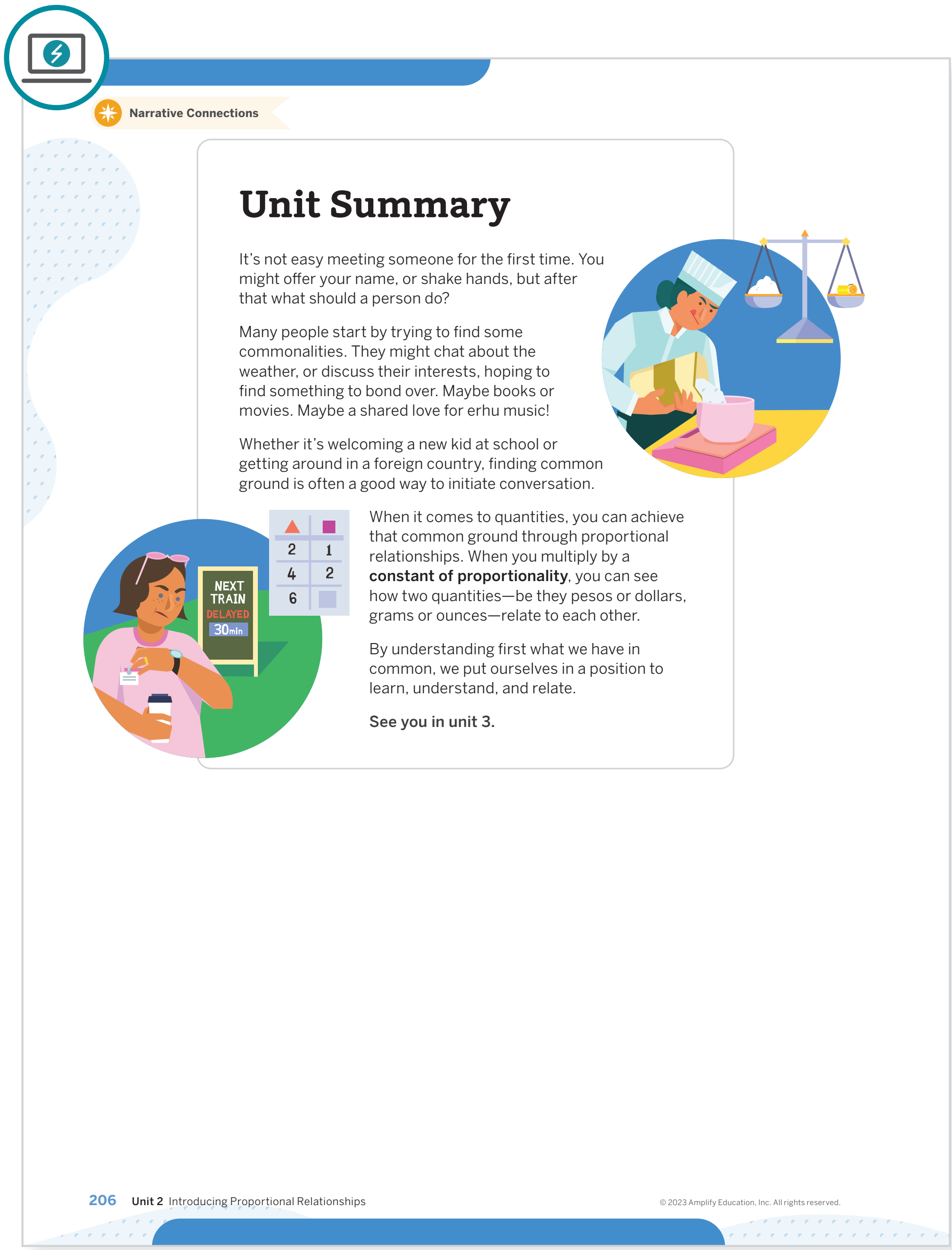
John Urschel

Have students read about featured mathematician John Urschel, who studied the "traveling salesman problem" and also happens to be a former professional American football player.

Whole Class | 5 min

Unit Summary

Review and synthesize how proportional reasoning relates to the narratives of this unit.



Narrative Connections

Unit Summary

It's not easy meeting someone for the first time. You might offer your name, or shake hands, but after that what should a person do?

Many people start by trying to find some commonalities. They might chat about the weather, or discuss their interests, hoping to find something to bond over. Maybe books or movies. Maybe a shared love for erhu music!

Whether it's welcoming a new kid at school or getting around in a foreign country, finding common ground is often a good way to initiate conversation.

▲	■
2	1
4	2
6	■

When it comes to quantities, you can achieve that common ground through proportional relationships. When you multiply by a **constant of proportionality**, you can see how two quantities—be they pesos or dollars, grams or ounces—relate to each other.

By understanding first what we have in common, we put ourselves in a position to learn, understand, and relate.

See you in unit 3.

206 **Unit 2** Introducing Proportional Relationships

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Ask:

- "Under what social circumstances would proportional reasoning be a useful tool in finding common ground?"
- "Under what social circumstances would proportional reasoning not be a useful tool in finding common ground?"

Highlight that it is very useful to spend some time reflecting on one's own learning at the conclusion of a unit. This helps students to retain information and to make connections that they may not have been able to make while focusing on each individual lesson.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "How has this unit allowed you to think about proportional relationships in a new way? What new connections were you able to make with how you use proportional relationships in your life?"

Fostering Diverse Thinking

The Meaning of Cultural Exchange

Exchange is a common real-world application of proportional reasoning, specifically when related to currency and units of measurement. Throughout this unit, students have seen several positive aspects of cultural exchange.

However, as the interaction between cultures has proven throughout history, "cultural exchange" is often an oversimplification. The word "exchange" itself implies a certain parity or balance, a "fair amount of this for a fair amount of that." Nevertheless, students may be aware of cultural interactions that are not fair exchanges.

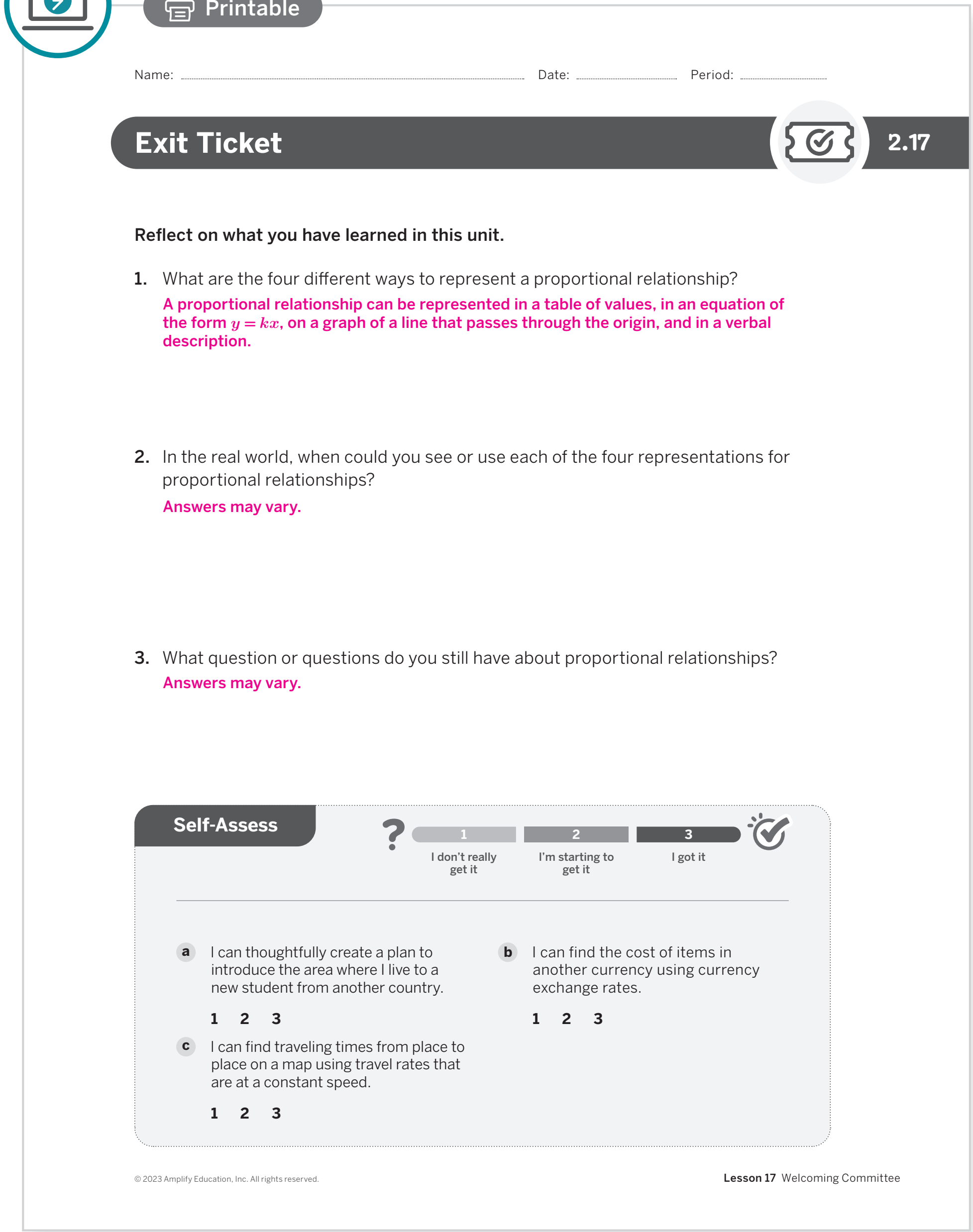
Have students think about different cultural interactions they are aware of, and describe whether they think they are fair or unfair.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of proportional reasoning by reflecting on what they learned and voicing any unresolved questions they may have.

Printable



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Exit Ticket 2.17

Reflect on what you have learned in this unit.

1. What are the four different ways to represent a proportional relationship?
   A proportional relationship can be represented in a table of values, in an equation of the form $y = kx$, on a graph of a line that passes through the origin, and in a verbal description.

2. In the real world, when could you see or use each of the four representations for proportional relationships?
   Answers may vary.

3. What question or questions do you still have about proportional relationships?
   Answers may vary.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can thoughtfully create a plan to introduce the area where I live to a new student from another country.
1 2 3

b. I can find the cost of items in another currency using currency exchange rates.
1 2 3

c. I can find traveling times from place to place on a map using travel rates that are at a constant speed.
1 2 3

Lesson 17 Welcoming Committee

Success looks like . . .

- **Goal:** Using an exchange rate to convert values from one currency to another.
- **Language Goal:** Creating a map with a scale and using it to calculate distance, speed, and time. **(Speaking and Listening)**
- **Goal:** Using shared interests to help a new student feel welcomed.

Suggested next steps

If students cannot think of all four representations of proportional relationships, consider:

- Allowing them to review the Summary from each lesson, and any notes they created while engaging in the Reflect section for those lessons.

If students can not think of when they may see or use each representation in the real world, consider:

- Changing the question to "In the real world, when may you see or use any of the four representations for proportional relationships?" and have them respond.

Professional Learning

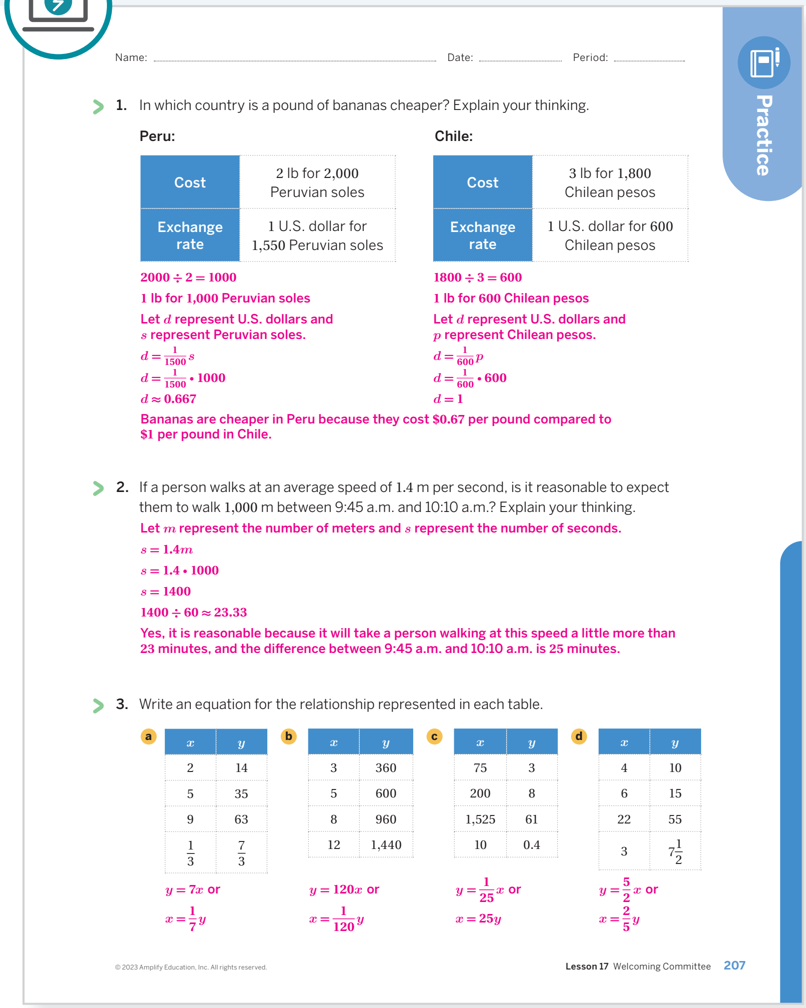
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What challenges did students encounter as they worked on drawing their map? How did they work through them?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. In which country is a pound of bananas cheaper? Explain your thinking.

Peru:

Cost	2 lb for 2,000 Peruvian soles
Exchange rate	1 U.S. dollar for 1,550 Peruvian soles

$2000 \div 2 = 1000$

1 lb for 1,000 Peruvian soles

Let d represent U.S. dollars and s represent Peruvian soles.

$d = \frac{1}{1500}s$

$d = \frac{1}{1500} \cdot 1000$

$d \approx 0.667$

Chile:

Cost	3 lb for 1,800 Chilean pesos
Exchange rate	1 U.S. dollar for 600 Chilean pesos

$1800 \div 3 = 600$

1 lb for 600 Chilean pesos

Let d represent U.S. dollars and p represent Chilean pesos.

$d = \frac{1}{600}p$

$d = \frac{1}{600} \cdot 600$

$d = 1$

Bananas are cheaper in Peru because they cost $0.67 per pound compared to $1 per pound in Chile.

2. If a person walks at an average speed of 1.4 m per second, is it reasonable to expect them to walk 1,000 m between 9:45 a.m. and 10:10 a.m.? Explain your thinking.

Let m represent the number of meters and s represent the number of seconds.

$s = 1.4m$

$s = 1.4 \cdot 1000$

$s = 1400$

$1400 \div 60 \approx 23.33$

Yes, it is reasonable because it will take a person walking at this speed a little more than 23 minutes, and the difference between 9:45 a.m. and 10:10 a.m. is 25 minutes.

3. Write an equation for the relationship represented in each table.

a

x	y
2	14
5	35
9	63
$\frac{1}{3}$	$\frac{7}{3}$

$y = 7x$ or

$x = \frac{1}{7}y$

b

x	y
3	360
5	600
8	960
12	1,440

$y = 120x$ or

$x = \frac{1}{120}y$

c

x	y
75	3
200	8
1,525	61
10	0.4

$y = \frac{1}{25}x$ or

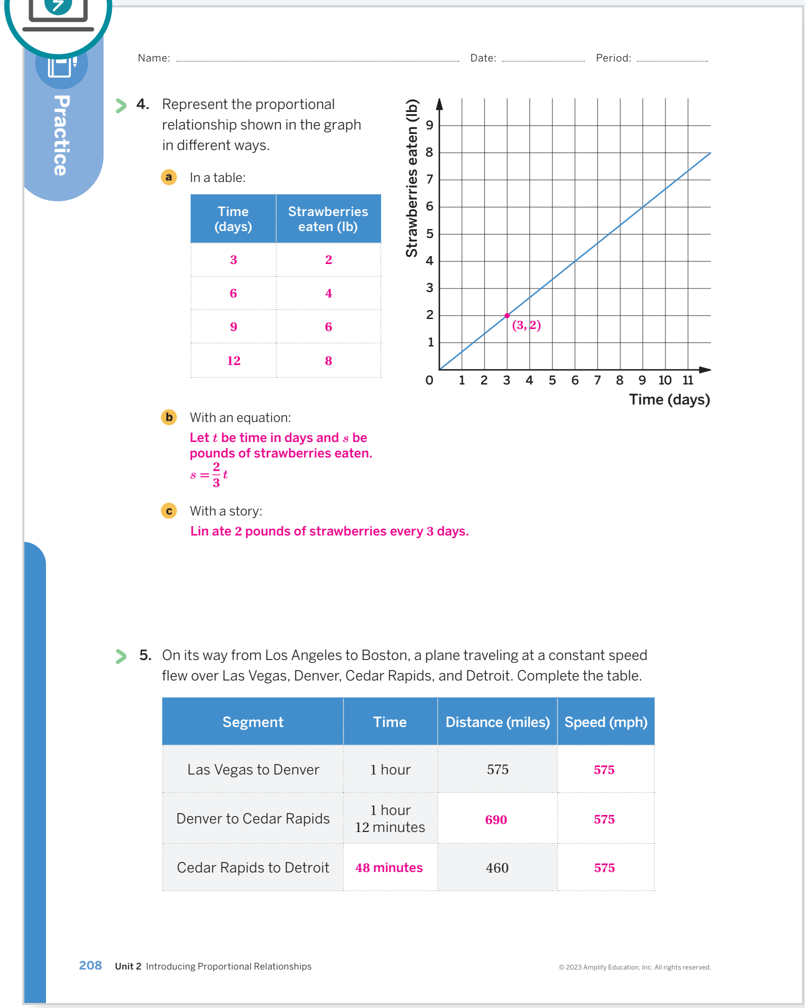
$x = 25y$

d

x	y
4	10
6	15
22	55
3	$7\frac{1}{2}$

$y = \frac{5}{2}x$ or

$x = \frac{2}{5}y$



Name: Date: Period:

Practice

4. Represent the proportional relationship shown in the graph in different ways.

a In a table:

Time (days)	Strawberries eaten (lb)
3	2
6	4
9	6
12	8

b With an equation:

Let t be time in days and s be pounds of strawberries eaten.

$s = \frac{2}{3}t$

c With a story:

Lin ate 2 pounds of strawberries every 3 days.

5. On its way from Los Angeles to Boston, a plane traveling at a constant speed flew over Las Vegas, Denver, Cedar Rapids, and Detroit. Complete the table.

Segment	Time	Distance (miles)	Speed (mph)
Las Vegas to Denver	1 hour	575	575
Denver to Cedar Rapids	1 hour 12 minutes	690	575
Cedar Rapids to Detroit	48 minutes	460	575

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 2 Lesson 9	2
	4	Unit 2 Lesson 16	2
	5	Unit 2 Lesson 6	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3

Measuring Circles

Identifying a circle may be straightforward, but measuring it is decidedly not. Students experience both the usefulness and challenges presented by this “perfect” shape.

Essential Questions

- How do we measure circles when all of our tools are straight?
- What is π and what does it have to do with circles?
- How can squares help you measure the space inside circles?
- *(By the way, does π really go on forever?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students learn to understand and use the term *circle* to mean the set of points that are equally distant from a point called the center. They gain an understanding of why the circumference of a circle is proportional to its diameter, with a constant of proportionality of π. They see informal derivations of the fact that the area of a circle is equal to π times the square of its radius. Students use the relationships of circumference, radius, diameter, and area of a circle to determine lengths and areas, expressing these in terms of π or using appropriate approximations of π to express them numerically.

Coherence

Previously . . .

Students identified and created scaled figures using a scale factor in Unit 1, which helps them to understand how all circles are scaled copies of each other. In Unit 2, students identified and wrote equations for proportional relationships, which helps them reason about the formula for the circumference of a circle. Unit 3 synthesizes many of the concepts learned in Units 1 and 2.

Coming soon . . .

Students will return to proportional relationships in the form of percentages. While learning about how percentages are used to communicate relative size and value, they also begin to reason about them algebraically.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students develop their understanding of π, where it comes from, and its role in the proportional relationship between the diameter and circumference of a circle (Lesson 4).

Procedural Fluency

Students have the opportunity to practice solving problems that build their fluency in using formulas for circumference and area of a circle (Lessons 6 and 10).

Application

Students must decide whether a real-world problem requires knowing the circumference or the area of a circle. They then estimate the size of the circle in question before using more precise measurements (Lesson 11).

‘Round and ‘Round We Go

SUB-UNIT

1

Lessons 2–7

Circumference of Circles

Students explore ways to measure a circle, beginning with the ***circumference***. After defining the ***radius*** and ***diameter***, they discover that some measures of a circle are proportional, revealing the value of π.

Narrative: Explore ways to measure a circle — a shape that has captivated humans for thousands of years.

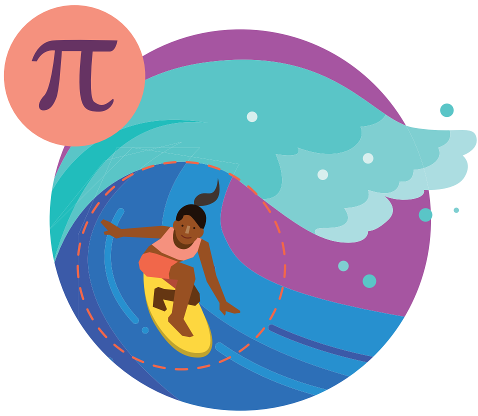
SUB-UNIT

2

Lessons 8–11

Area of Circles

Students use various approaches to approximate the area of circles, which reveal the relationship between the square of the radius and the area of the circle. They distinguish between real-world scenarios that use circumference or area.



Narrative: Discover how one of the most fascinating constants of proportionality, π, is related to the area of a circle.

Launch

Lesson 1

The Wandering Goat

Normally reserved for the puzzle page, the delightful and classic "goat problem" introduces many aspects of circles in a natural way. Students reason about rotation, ***radius***, and area, all while following around a friendly goat.

Capstone

Lesson 12

Capturing Space

In at least one way, the circle is an ideal shape. As students compete to capture more area than their opponent, they notice that equal perimeter does not mean equal area. Then, by calculating and comparing the areas of shapes with the same perimeter, they see that a circle will always have the maximum area.

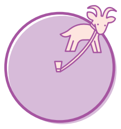
Unit at a Glance

Spoiler Alert: All scientific calculators have a π button, which simplifies calculations for circles and means you never need to memorize more than a few digits of π.

Assessment

A **Pre-Unit Readiness Assessment**

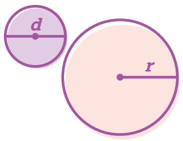
Launch Lesson



1 **The Wandering Goat**

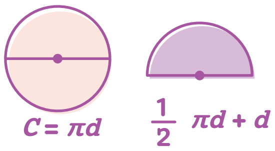
Solve a few versions of the classic goat problem to introduce a context for using circular regions.

Sub-Unit 1: Circumference of Circles



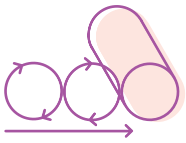
2 **Exploring Circles**

Develop the formal definition of a circle and learn to describe and measure a circle using the radius and diameter.



6 **Applying Circumference**

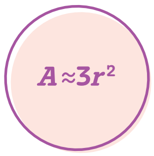
Build fluency with circumference by determining the perimeter of shapes composed of circular parts and solving for unknown lengths.



7 **Circumference and Wheels**

Investigate the proportional relationship between the number of times a wheel rotates and the distance the wheel travels.

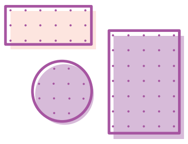
Sub-Unit 2: Area of Circles



8 **Exploring the Area of a Circle**

Explore how to find the area of a circle through two different approaches before concluding that $A \approx 3r^2$.

Capstone Lesson



12 **Capturing Space**

Play a game and compare polygons to discover that, among shapes with the same perimeter, a circle has the greatest area.

Assessment

A **End-of-Unit Assessment**

Key Concepts

Lesson 2: Define how the radius, center, and diameter relate to their corresponding circle.

Lesson 4: Discover the proportional relationship between circumference and diameter.

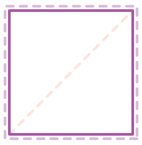
Lesson 9: Informally derive the formula for the area of a circle.

Pacing

12 Lessons: 45 min each
2 Assessments: 45 min each

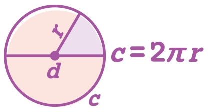
Full Unit: 14 days
● **Modified Unit:** 11 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



3 How Well Can You Measure?

Apply understanding of scaled figures and proportional relationships to explore the relationship between a square's perimeter and diagonal length.



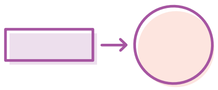
4 Exploring Circumference

Discover that there is a proportional relationship between the diameter and circumference of a circle.

$$\pi \approx 3.14 \approx \frac{22}{7}$$

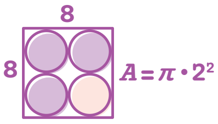
5 Understanding π

Explore historical approximations of π and determine how precise π should be for calculations of circumference in real-world contexts.



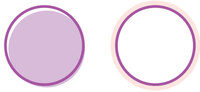
9 Relating Area to Circumference

Develop the formula for the area of a circle, $A = \pi r^2$, through informal dissection arguments.



10 Applying Area of Circles

Apply the area of a circle formula to solve problems involving the area of shapes composed of circular parts and polygons.



11 Distinguishing Circumference and Area ●

Decide whether a circle's circumference or area is needed to solve a problem, and critique the reasoning of others.

● Modifications to Pacing

Lesson 7: This lesson helps to connect the work from Unit 2 involving writing and using equations to solve problems with proportional relationships, but you may choose to skip if students already have a solid understanding of the concept.

Lesson 8: This lesson helps students understand how it is possible to use square units to measure the area of circles, and builds understanding for how the area formula is derived. However, Lesson 9 provides a more precise derivation and can be done in lieu of both lessons.

Lessons 11–12: While Lesson 12 reinforces how special circles are and gives students a chance to strategize and think critically, you might choose to complete only one of these lessons. Lesson 11 serves as a good opportunity to review both circumference and area before the unit closes.

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	center of a circle circle diameter radius
4	circumference pi (π)

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
1, 9	**MLR1:** Stronger and Clearer Each Time
1–4, 9, 11	**MLR2:** Collect and Display
6	**MLR3:** Critique, Correct, Clarify
2, 4, 7	**MLR5:** Co-craft Questions
5	**MLR6:** Three Reads
1, 4, 7–10, 12	**MLR7:** Compare and Connect
3–7, 9, 11	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket PDF
- Additional Practice Book

Additional required materials include:

Lesson(s)	Materials
3, 4, 5, 6, 7, 9, 10, 11*, 12	calculators
4*, 7	circular objects, for rolling
9, 10	colored pencils
4*	flexible measuring tape or lengths of string
9*	glue or tape
2, 10	geometry toolkits
1*	round head fasteners
3, 4*, 7,	rulers
1, 8, 9, 12	scissors
2, 7	sheet of paper
7*	soup can (with wrapper)
1, 12	string
1–9, 11	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.

* Optional materials

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
2, 6, 11	Card Sort
2, 3, 5, 8, 11	Poll the Class
2, 4, 5, 9, 12	Think-Pair-Share

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 12

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Social & Collaborative Digital Moments

Featured Activity

Covering a Circle

Put on your student hat and work through **Lesson 8, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- The Goat Problem **(Lesson 1)**
- Measuring Circumference and Diameter **(Lesson 4)**
- Rotations and Speed **(Lesson 7)**
- Capture the Dots Game **(Lesson 12)**

Activity 2: Covering a Circle

The side length of the square is the same as the radiu

Estimate how many of these squares would be neede circle entirely by responding to the following questior you can break the square up into smaller pieces, if ne

What number of squares do you think would be *too hi*

Unit Study
Professional Learning

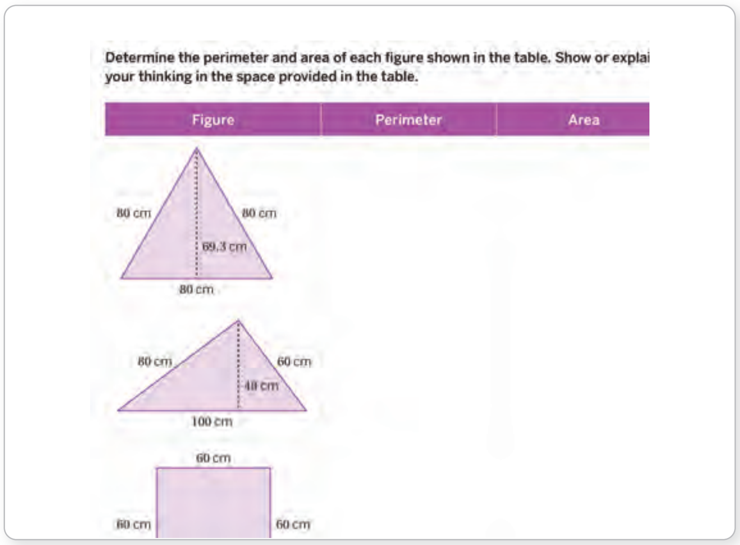
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces students to the area of a circle. Students explore by estimating the area of circles on square grids, reminding them that area is covering space in square units. Then students work on relating area to circumference, including differentiating the two quantities in real contexts. This work connects the area of a rectangle to the area of a circle, allowing students to see the derivation of πr^2 from $l \cdot w$. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 12, Activity 2:**



Determine the perimeter and area of each figure shown in the table. Show or explai your thinking in the space provided in the table.

Figure	Perimeter	Area

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

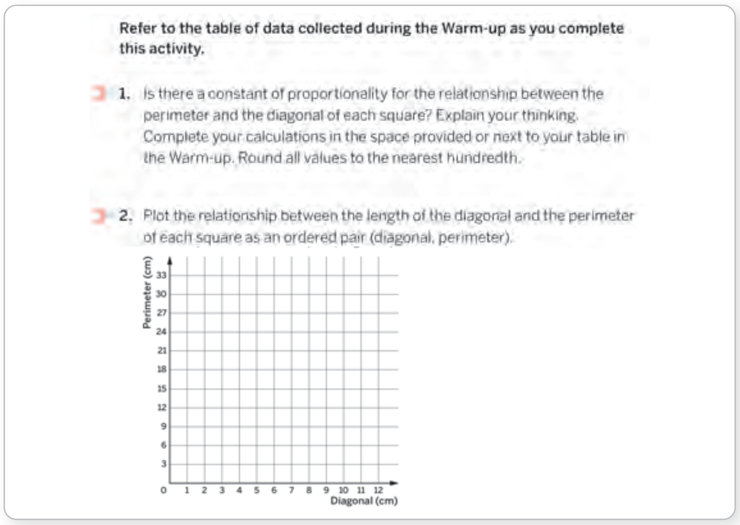
- What was it like to engage in this problem as a learner?
- Lesson 12's Warm-up "Same String, Different Shapes" hints at this activity. With a string, students try to form different shapes. How would you extend on this warm-up for further exploration of relating a fixed perimeter to various area possibilities?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Poll the Class

Rehearse . . .

How you'll facilitate the ***Poll the Class*** instructional routine in **Lesson 3, Activity 1:**



Refer to the table of data collected during the Warm-up as you complete this activity.

1. Is there a constant of proportionality for the relationship between the perimeter and the diagonal of each square? Explain your thinking. Complete your calculations in the space provided or next to your table in the Warm-up. Round all values to the nearest hundredth.

2. Plot the relationship between the length of the diagonal and the perimeter of each square as an ordered pair (diagonal, perimeter).

Points to Ponder . . .

- In *Poll the Class*, take a quick survey – public or private – of all students to gain insight into students thinking.

This routine . . .

- Allows students to share their initial thoughts or reactions to an idea in a low-risk manner.
- Gives students the opportunity to see what their peers are thinking, when done publicly.
- Is not a very reliable way to do formative assessment. Students can be easily influenced to adjust their responses to match others.
- Can help students practice using their metacognition.

Anticipate . . .

- Students may need to both see *and* hear the options in the poll.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Build procedural fluency from conceptual understanding.

This effective teaching practice . . .

- Begins with a foundation of deep understanding so that students develop sense-making skills, before procedural skills are introduced.
- Provides students with the opportunity to connect procedural skills with contextual or mathematical problems, strengthening their problem solving abilities.

Points to Ponder . . .

- Before introducing a formula or procedure, how will you ensure that your students have a solid understanding of the mathematical concepts?
- Do your students connect procedures to concepts, or are they reliant on memorization of formulas or procedural steps? How can you be sure they understand the "*why* behind the *what*"?

Math Language Development

MLR8: Discussion Supports

MLR8 appears in Lessons 3–7, 9, and 11.

- Throughout the unit, sentence frames are provided for you to display to your students. Students can use these prompts to help frame their responses and add structure and organization to their thinking.
- In Lesson 10, further probing questions are provided so that you can ask your students for further clarification or to press for details in their reasoning.
- **English Learners:** Provide wait time to allow students to formulate a response before sharing with others.

Point to Ponder . . .

- During class discussions in this unit, how will you know when to probe further to assess student understanding and encourage your students to use their developing mathematical vocabulary?

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Opportunities to provide visual support and guidance to help students process new information appear in Lessons 1–3, 6–12.

- Suggestions are provided throughout the unit to provide access to graphic organizers and anchor charts to help students make sense of new information. For example, In Lesson 5, provide students with the Graphic Organizer PDF, *Working With Circles (Part 1)*.
- In Lesson 9, demonstrate how to use color coding to shade corresponding parts of a circle to make sense of the relationship between the area of a rectangle and the circumference of a circle, where the height of the rectangle is the radius of the circle.
- In Lesson 10, suggest students create a table to help show their thinking for each decomposition.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when your students may benefit from visual support or suggested guidance? What clues will you gather from your students?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with having a precise understanding of the new vocabulary in the unit? Do you think your students will generally:
  - » have command over the distinctions between radius, diameter, and circumference?
  - » confuse which formula is used for area and which for circumference?
  - » remember the appropriate units for each measure?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their social awareness and self-management.

Points to Ponder . . .

- Are students able to control their impulses and stay focused on the tasks at hand? Can they set behavioral and academic goals that help them be more successful? How do they motivate themselves to achieve those goals?
- When working with materials in the classroom, do students consider their own safety and the safety of others? Are they able to identify the problem and then solve it? Do they make good decisions within their social interactions?

The Wandering Goat

Let's explore how far a goat on a rope can roam.

Focus

Goals

1. **Language Goal:** Describe the space an object can occupy while tethered to a fixed point. **(Speaking and Listening)**
2. Understand that when an object moves a given distance around a point, it creates a circular shape.

Coherence

Today

Students use a variety of tools to model and explore the space in which a goat can roam when tethered to different objects with a rope. They notice that in all cases, the final space in which the goat can roam is curved and is either circular, or includes circular sections.

Previously

In Units 1 and 2, students explored scaled figures and proportional relationships. This work prepared them for Unit 3 by providing the foundation for being able to construct their understanding of the relationships between parts of a circle.

Coming Soon

In Lesson 2, students will define what a circle is by exploring its characteristics.

Rigor

- Students build **conceptual understanding** of how circles are created by modeling movement about a fixed point.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

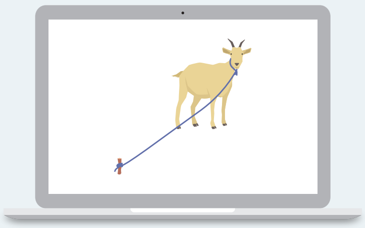
- Exit Ticket
- Additional Practice
- Activity 1 PDF
- Activity 1 PDF, *Are you ready for more?* (as needed)
- Activity 1 PDF, *Are you ready for more?* (answers)
- Activity 2 PDF
- round head fasteners, 3 per pair (optional)
- cardboard or paper (optional)
- scissors
- string, 2 long pieces per pair
- tape

Amps | Featured Activity

Warm-up

Digital Goat Problem

Students investigate the space in which a goat can roam by using a digital tool. They are able to move the goat while it is attached to a peg and observe its possible locations.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students might forget to attend to safety measures while modeling the problem in Activity 1. Talk about how to use the materials carefully so that they can keep not only themselves, and those around them, safe. This will also enable students to complete the activity correctly.

Modifications to Pacing

You may want to consider this additional modification if you are short on time:

- **Activity 1** may be omitted.
- In **Activities 1** and **2**, have students discuss their strategies in Problem 1 orally, without writing down their plans.

Pairs | 10 min

Warm-up The Goat Problem (Part 1)

Students act out the space in which a goat can roam while tethered to a peg, which prepares them for understanding circles in the upcoming unit.

Amps Featured Activity Digital Goat Problem

Unit 3 | Lesson 1 – Launch

The Wandering Goat

Let's explore how far a goat on a rope can roam.

Warm-up The Goat Problem (Part 1)

A famous recreational math problem — known as "The Goat Problem" — was first published in *The Ladies' Diary* in 1748. A variation of this problem asks the question, "If a goat were tethered by a rope to a peg, what space could the goat occupy (where could it roam)?"

1. Choose one person to act as the goat and one person to act as the peg. Using the string provided by your teacher, explore what space the goat could roam in while it is attached to the peg.
2. Sketch the shape of the space where the goat would be able to roam while tethered to a rope that is attached to the peg.

212 Unit 3 Measuring Circles

Log in to Amplify Math to complete this lesson online.

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1 Launch

Provide student pairs with one long piece of string. Have students play rock-paper-scissors to determine which role they will be assigned in this activity (goat or peg). Explain that they will switch roles in the next activity.

2 Monitor

Help students get started by encouraging the student acting as the goat to explore *all* the space they could occupy.

Look for points of confusion:

- **Thinking that they can only travel forward and backward.** Ask students whether they would expect a goat to move in a straight line, or wander around.

Look for productive strategies:

- Exploring all the space students could roam, including roaming close to or farther away from the peg.

3 Connect

Have pairs of students share how they modeled the movement of the goat and what they noticed about the space the goat was able to cover.

Ask:

- "How would you describe the section of the field in which the goat can roam?"
- "Where is the center of the space?"
- "Can the goat only travel along the outer edge of the space?"
- "What other tools could you have used to model this scenario?"

Highlight that there are multiple ways of exploring this scenario using different tools. Have a pair of students model how they used the long piece of string. Model how students could do the same exploration by taping a small piece of string to your board and rotating it, or by attaching a paper fastener to the cardboard, attaching the string to the fastener, and then rotating it.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing students with a pre-assembled "field" where the goat is tethered to a peg by tying the string to a round paper fastener and poking a hole into a piece of paper to represent the peg. Model how moving the string represents the goat's movement.

Math Language Development

MLR2: Collect and Display

As students complete the Warm-up, circulate and capture any informal language they use to describe the goat's movement. Add this language to a class display and encourage students to refer to the display throughout the lesson and unit.

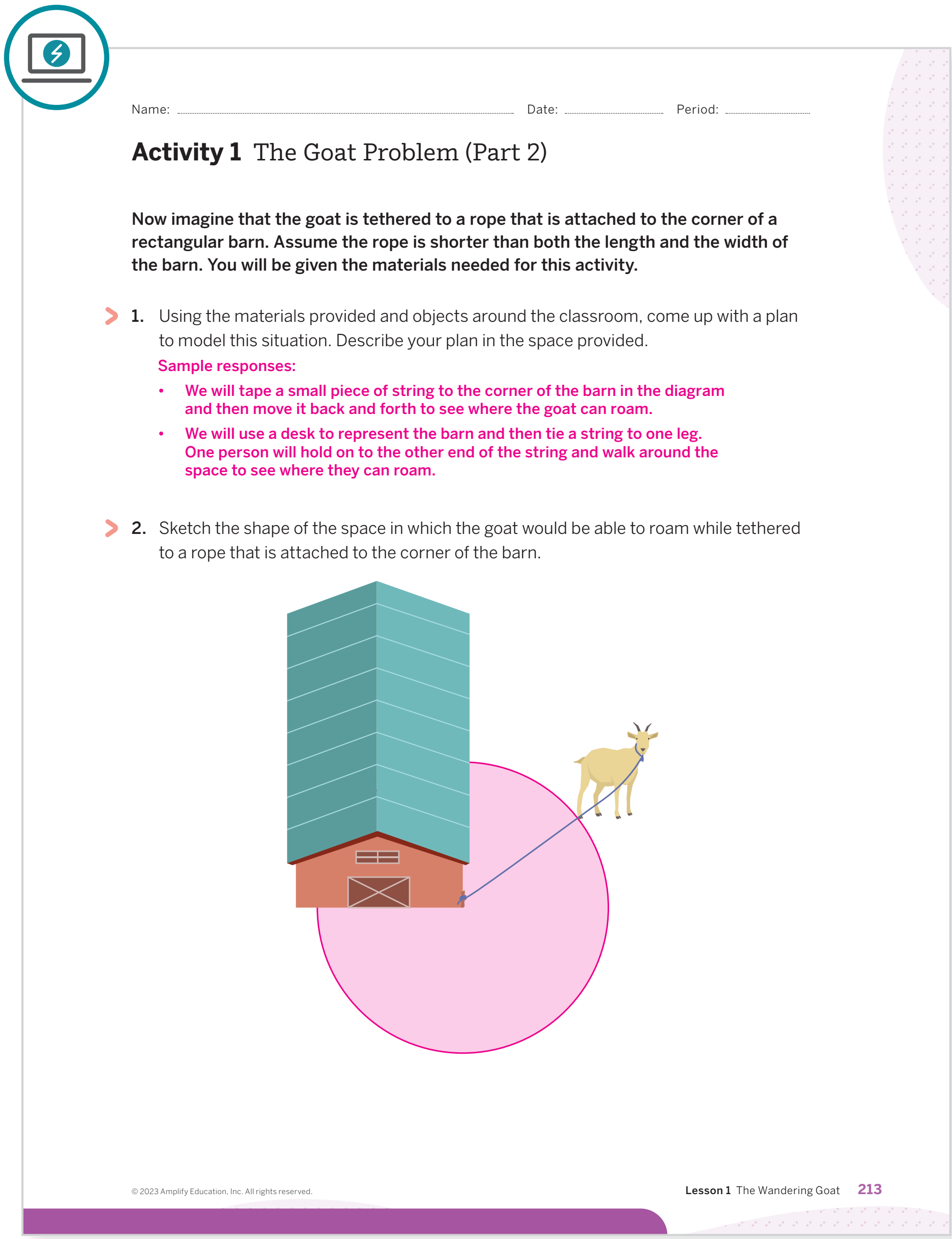
English Learners

Highlight how to model the goat's movement with the tools provided.

Pairs | 10 min

Activity 1 The Goat Problem (Part 2)

Students select tools to model the possible space in which a goat could roam while tethered to the corner of a building, reasoning about how the building restricts the circular path.



Name: ________________ Date: ________ Period: ________

Activity 1 The Goat Problem (Part 2)

Now imagine that the goat is tethered to a rope that is attached to the corner of a rectangular barn. Assume the rope is shorter than both the length and the width of the barn. You will be given the materials needed for this activity.

1. Using the materials provided and objects around the classroom, come up with a plan to model this situation. Describe your plan in the space provided.

   Sample responses:
   - **We will tape a small piece of string to the corner of the barn in the diagram and then move it back and forth to see where the goat can roam.**
   - **We will use a desk to represent the barn and then tie a string to one leg. One person will hold on to the other end of the string and walk around the space to see where they can roam.**

2. Sketch the shape of the space in which the goat would be able to roam while tethered to a rope that is attached to the corner of the barn.

Lesson 1 The Wandering Goat 213

1 Launch

Explain to students that you will provide them with a variety of tools to model the situation, including a long piece of string, scissors, round head fasteners, cardboard or paper, and tape. You may also choose to allow students to access other objects in your classroom that may help them better model the scenario.

2 Monitor

Help students get started by asking, "How can you update your strategy from the Warm-up for this new situation?"

Look for points of confusion:

- **Thinking that the goat can go into the barn or not taking the barn into consideration in their model.** Ask students to imagine they are on the corner of a building and ask them how that would change where they could walk when compared to being in the middle of a field.

Look for productive strategies:

- Using objects in the classroom, like a desk or a bookshelf, to represent the barn in their physical model, understanding how it would block the path of the goat.

3 Connect

Display the Activity 1 PDF.

Have pairs of students share or model the strategies they used to model the situation.

Highlight the space in which the goat could roam by drawing it on the displayed PDF.

Ask, "What similarities or differences do you notice between the two shapes of the travel areas in the Warm-up and in this activity?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

For students who would prefer not to use their bodies to model the scenario or have mobility restrictions, provide them with the Activity 1 PDF so that they have more space to model the scenario on grid paper.

Consider providing students with a rectangular prism, such as a tissue box, to help them make sense of the two-dimensional diagram of the barn.

Extension: *Math Enrichment*

Have students complete the Activity 1 PDF, *Are you ready for more?*

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, call attention to how the goat's travel area compares to the Warm-up. Consider asking:

- "What effect did the placement of the barn have on the space in which the goat could travel?"
- "If the goat was tethered to the opposite corner of the barn, what would the space look like in which the goat could travel?"

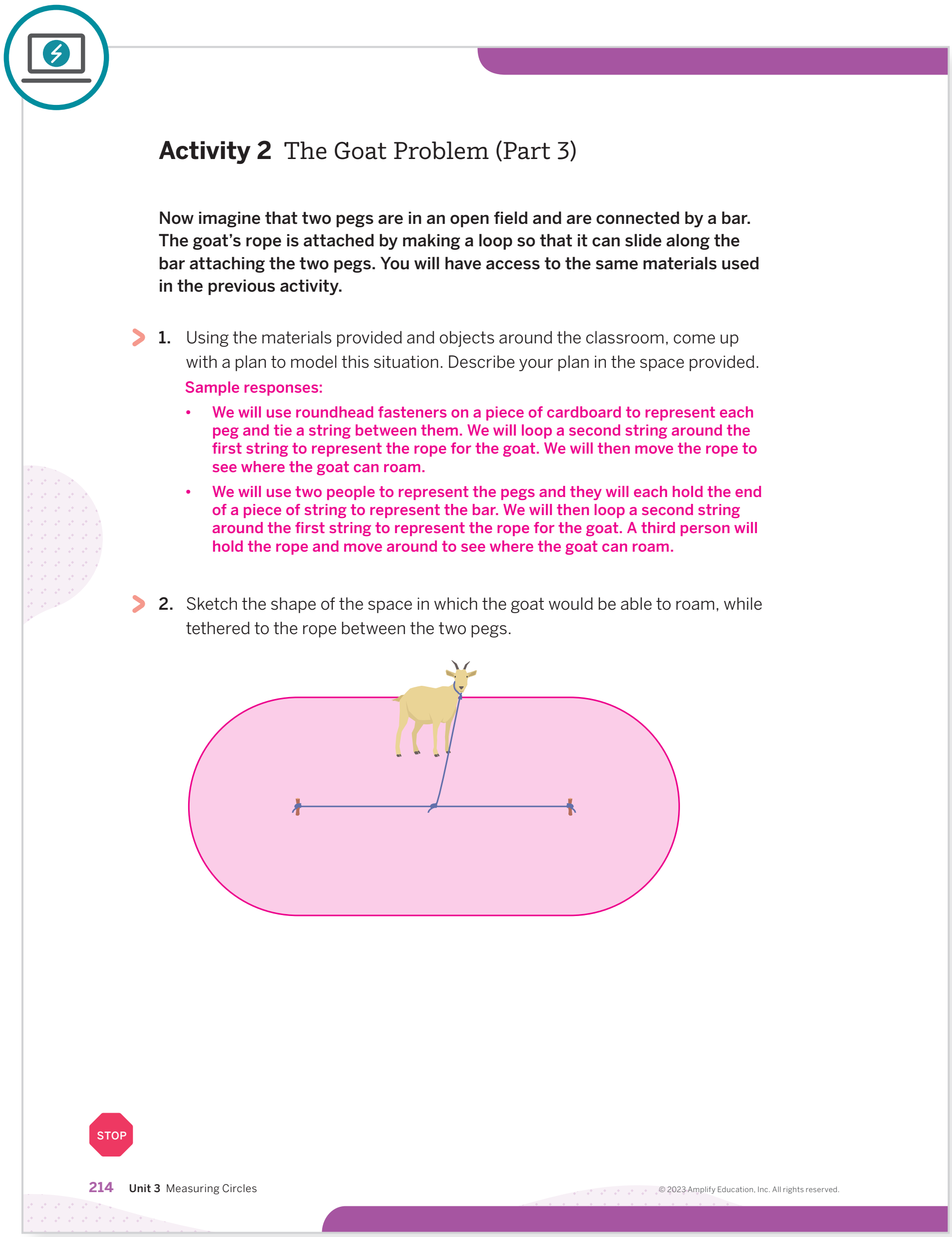
English Learners

Annotate on the displayed Activity 1 PDF how the space in which the goat could travel is a three-quarter circle, compared to the full circle from the Warm-up.

Small Groups | 15 min

Activity 2 The Goat Problem (Part 3)

Students select tools to model the possible space in which a goat could roam while tethered to a bar connecting two pegs, reasoning about how the goat's ability to roam is affected.



Activity 2 The Goat Problem (Part 3)

Now imagine that two pegs are in an open field and are connected by a bar. The goat's rope is attached by making a loop so that it can slide along the bar attaching the two pegs. You will have access to the same materials used in the previous activity.

1. Using the materials provided and objects around the classroom, come up with a plan to model this situation. Describe your plan in the space provided.
   Sample responses:
   - We will use roundhead fasteners on a piece of cardboard to represent each peg and tie a string between them. We will loop a second string around the first string to represent the rope for the goat. We will then move the rope to see where the goat can roam.
   - We will use two people to represent the pegs and they will each hold the end of a piece of string to represent the bar. We will then loop a second string around the first string to represent the rope for the goat. A third person will hold the rope and move around to see where the goat can roam.
2. Sketch the shape of the space in which the goat would be able to roam, while tethered to the rope between the two pegs.

STOP

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1 Launch

Remind students that they can use any tools from the previous activity, but that they will now be working in small groups and will have access to an additional piece of string.

2 Monitor

Help students get started by encouraging them to use their tools to model the bar between the two pegs before they determine their entire plan.

Look for points of confusion:

- **Not realizing that the rope can still move between the pegs.** Ask students to read aloud the prompt and examine what is meant by the phrase that says the rope "can slide along the bar attaching the two pegs".

Look for productive strategies:

- Having two students in the group represent the pegs while a third student represents the goat.
- Making a very loose loop around the string connecting the pegs so that the rope can easily move back and forth.

3 Connect

Display the Activity 2 PDF.

Have groups of students share the strategies they used to model the space in which the goat could roam. If appropriate, have students model their strategies for the class.

Highlight the space in which the goat could roam by drawing it on the displayed PDF.

Ask, "What are the similarities or differences you noticed between the three goat problems?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For students who would prefer not to use their bodies to model the scenario or have mobility restrictions, provide them with the Activity 2 PDF so that they have more space to model the scenario on grid paper.

Math Language Development

MLR1: Stronger and Clearer Each Time

Consider intentionally grouping students who experienced success in Activity 1 with students who may have needed extra support. After students have written a draft plan to model the situation in the activity, have them share their plan with 1–2 other groups to both give and receive feedback. After receiving feedback on their plan, allow students to make revisions to their plan.

English Learners

Encourage students to use drawings and diagrams to help them craft their explanation.

Whole Class | 5 min

Summary ‘Round and ‘Round We Go

Review and synthesize how rotating an object attached to a fixed point, or between two points, creates shapes that are curved, and sometimes shapes that are full circles.

Narrative Connections

Unit 3 Measuring Circles

‘Round and ‘Round We Go

Circles appear in countless places in the world around us, from the very large to the very small. When pioneering American astronauts like John Glenn and Sally Ride looked out of their spacecraft, their view was dominated by Earth’s circular profile. And when working with some of the earliest computers, mathematicians from John von Neumann to Edith Windsor were surrounded by the circular outlines of vacuum tubes, resistors, capacitors, and more.

At the same time, no shape is more elegant, more simple, or more symmetric than the circle.

Among indigenous peoples of the Great Plains of North America, there is the “sacred hoop” — a circle with spokes pointing in the four cardinal directions, representing the cycles of nature. In China, the Taoist concept of “yin” and “yang” — two opposing, but interconnected forces — come together to form a circle. And across the United States, circles can represent the eternity of marriage, from engagement rings to circular diamond pins.

Given the circle’s beauty and near-universal nature, it may be surprising that the mathematical systems you have seen (so far) don’t work well with circles. For example, it is far easier to draw a rectangle on graph paper than it is to draw a circle. And yet, from wheels to planetary orbits (but more on shapes called “ellipses” in high school!), circular shapes are central to humankind.

So, without further ado, let’s find out what, exactly, makes this shape go ‘round.

Welcome to Unit 3.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Highlight that during this unit, students will formalize their understanding of what makes circles so special as well as how circles can be described and measured.

Ask:

- “What do you already know about circles?” Sample response: They are round and they do not have straight edges.
- “If I asked you to create a circle on your paper, what tools would you use to help you draw it?” Sample response: I could tape a piece of string to my paper, and then use it to help me guide my pencil in a circle.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- “Which tools were most helpful in modeling the space in which the goat could roam?”

Fostering Diverse Thinking

Whose Images Are on U.S. Coins?

Most of the coins used around the world and in U.S. currency are circular and feature an image of a prominent person in the country’s history, an animal, or an object. Have students research whose faces are featured on U.S. coins that are still in circulation. Or provide this information:

Penny: Abraham Lincoln	**Nickel:** Thomas Jefferson
Dime: Franklin D. Roosevelt	**Quarter:** George Washington
Half-dollar: John F. Kennedy	**Dollar:** Sacagawea*

*Susan B. Anthony was featured on the coin until 1981.

Ask students what they notice and wonder about the people selected.

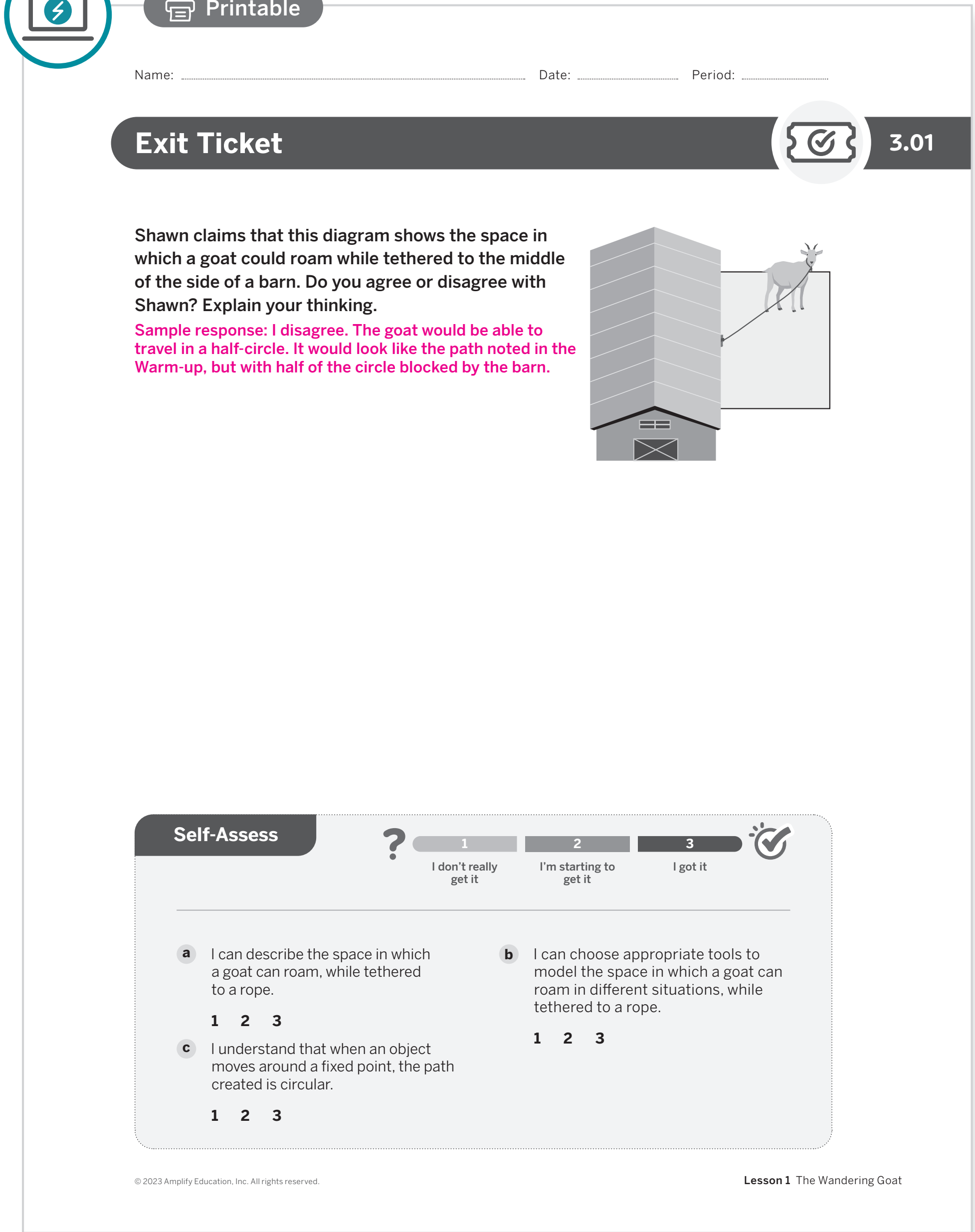
Tell students some of the history behind the dollar coin. In 1998, a committee recommended featuring Sacagawea, the Shoshone woman who guided the Lewis and Clark expedition, which students may study in a later history class. A committee including members of the Native American community, coin collectors, historians, and members of Congress narrowed down the designs. The final design included Sacagawea’s young son, Jean Baptiste Charbonneau.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining whether they agree with a student's claim about the space in which a goat could roam when tethered to a rope attached to the side of a barn.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.01

Shawn claims that this diagram shows the space in which a goat could roam while tethered to the middle of the side of a barn. Do you agree or disagree with Shawn? Explain your thinking.

Sample response: I disagree. The goat would be able to travel in a half-circle. It would look like the path noted in the Warm-up, but with half of the circle blocked by the barn.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can describe the space in which a goat can roam, while tethered to a rope.

1 2 3

b I can choose appropriate tools to model the space in which a goat can roam in different situations, while tethered to a rope.

1 2 3

c I understand that when an object moves around a fixed point, the path created is circular.

1 2 3

Success looks like . . .

- **Language Goal:** Describing the space an object can occupy while tethered to a fixed point. **(Speaking and Listening)**
  - » Explaining whether Shawn is correct about the diagram.
- **Goal:** Understanding that when an object moves a given distance around a point it creates a circular shape.
  - » Explaining that the goat could roam in a half-circle.

Suggested next steps

If students were unable to determine that the space in which the goat could roam would be half of a circle, consider:

- Providing them with a small piece of string to help them model the goat's movement on their paper.
- Assigning Practice Problems 1 and 2.

Professional Learning

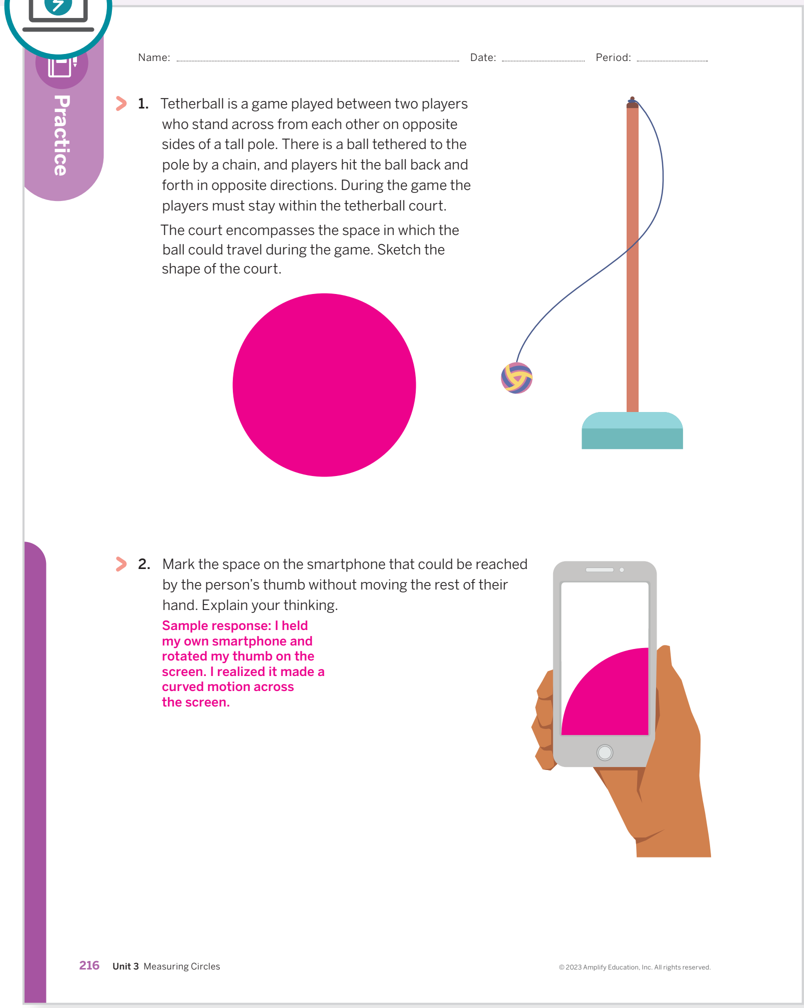
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In Activity 2, you may have used intentional grouping with MLR1 to group students who experienced different levels of success in Activity 1. What effect did this grouping strategy have on students' written plans? Would you change anything the next time you use MLR1?
- How did modeling the space in which the goat could roam set students up for success as they develop conceptual understanding of circles? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Tetherball is a game played between two players who stand across from each other on opposite sides of a tall pole. There is a ball tethered to the pole by a chain, and players hit the ball back and forth in opposite directions. During the game the players must stay within the tetherball court.

   The court encompasses the space in which the ball could travel during the game. Sketch the shape of the court.

2. Mark the space on the smartphone that could be reached by the person's thumb without moving the rest of their hand. Explain your thinking.

   Sample response: I held my own smartphone and rotated my thumb on the screen. I realized it made a curved motion across the screen.

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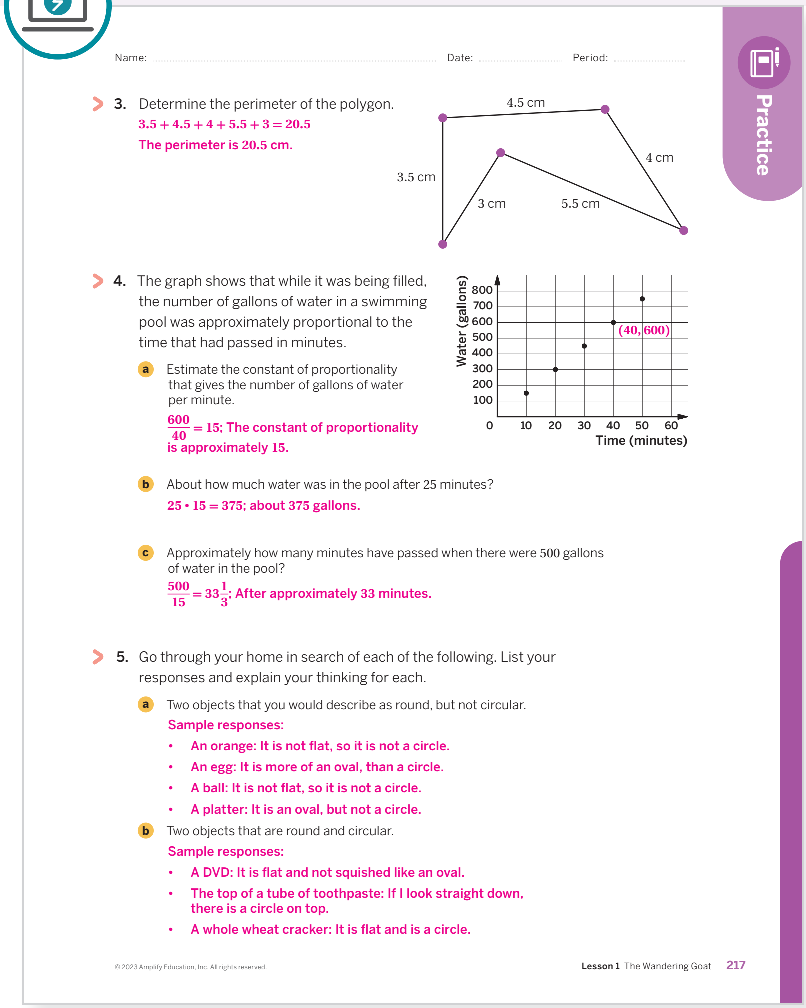

Name: Date: Period:

Practice

3. Determine the perimeter of the polygon.

   $3.5 + 4.5 + 4 + 5.5 + 3 = 20.5$

   The perimeter is 20.5 cm.

4. The graph shows that while it was being filled, the number of gallons of water in a swimming pool was approximately proportional to the time that had passed in minutes.

   a. Estimate the constant of proportionality that gives the number of gallons of water per minute.

   $\frac{600}{40} = 15$; The constant of proportionality is approximately 15.

   b. About how much water was in the pool after 25 minutes?

   $25 \cdot 15 = 375$; about 375 gallons.

   c. Approximately how many minutes have passed when there were 500 gallons of water in the pool?

   $\frac{500}{15} = 33\frac{1}{3}$; After approximately 33 minutes.

5. Go through your home in search of each of the following. List your responses and explain your thinking for each.

   a. Two objects that you would describe as round, but not circular.

   Sample responses:
   - **An orange: It is not flat, so it is not a circle.**
   - **An egg: It is more of an oval, than a circle.**
   - **A ball: It is not flat, so it is not a circle.**
   - **A platter: It is an oval, but not a circle.**

   b. Two objects that are round and circular.

   Sample responses:
   - **A DVD: It is flat and not squished like an oval.**
   - **The top of a tube of toothpaste: If I look straight down, there is a circle on top.**
   - **A whole wheat cracker: It is flat and is a circle.**

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 2	2
Spiral	3	Grade 5	1
	4	Unit 2 Lesson 12	2
Formative ⚡	5	Unit 3 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 1

Circumference of Circles

Whole Class

Mysterious, unique, and yet utterly common — the circle is a shape that has fascinated humans from the beginning of time. Though famously difficult to measure, students journey to find the best ways to do it, beginning with the circumference.

SUB-UNIT

1 Circumference of Circles

Narrative Connections

Why do aliens love circles?

In the United Kingdom, farmers have woken up to find portions of their field flattened into circular patterns. These patterns range anywhere from a single circle approximately 6 feet in diameter, to patterns with more than 400 circles spanning 1,000 feet.

These strange patterns are called crop circles. For decades, they have stirred up people's imaginations. Some consider them evidence of alien life. Others call them an elaborate hoax.

In 1991, artists Doug Bower and Dave Chorley took responsibility for at least 200 British crop circles dating back to 1978. But that still left thousands of circles unaccounted for around the world.

Most observers attribute these circles to copycat artists. Others still insist that aliens are among us!

Whoever the makers of intricate patterns are, circles have fascinated people—and any other intelligent beings that happen to be on this planet—for centuries.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore humanity's fascination with circles and real-world applications of circumference in the following places:

- **Lesson 2, Activity 3:** Art of the Circle
- **Lesson 4, Activity 1:** Refining Your Estimation
- **Lesson 5, Activities 1-2:** Approximating π, Approximating Circumference

UNIT 3 | **LESSON 2**

Exploring Circles

Let's explore circles.

Focus

Goals

1. **Language Goal:** Compare different ways to measure a circle and generalize the relationship between the radius and the diameter of a circle. **(Speaking and Listening)**
2. **Language Goal:** Understand and describe the characteristics of a circle using the terms *center*, *radius*, and *diameter* in reference to the parts of a circle. **(Speaking and Listening, Writing)**
3. Understand how to use a compass and a ruler to create a circle when given the length of the radius or diameter.

Coherence

Today

Students discover the characteristics of circles by examining and comparing examples and non-examples of circular figures. They develop the formal definition of *circle* as well as learn how to describe and measure a circle using the radius and the diameter. Finally, students gain experience creating circles based on given measurements using a compass and ruler.

Previously

In Lesson 1, students used string to explore the creation of circles and shapes containing parts of circles.

Coming Soon

In Lesson 4, students will define the term *circumference* and discover the relationship between circumference, diameter, and π.

Rigor

- Students build **conceptual understanding** of what is meant by the term *circle* by exploring its characteristics.
- Students gain **fluency** in identifying and determining the radii and diameters of circles from diagrams.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
3 min	10 min	8 min	15 min	5 min	5 min
Small Groups	Small Groups	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per group
- Activity 2 PDF (for display)
- Activity 3 PDF, one per student
- Activity 3 PDF (answers)
- Anchor Chart PDF, *Circles*
- geometry toolkits: compasses, rulers
- plain paper, one sheet per student

Math Language Development

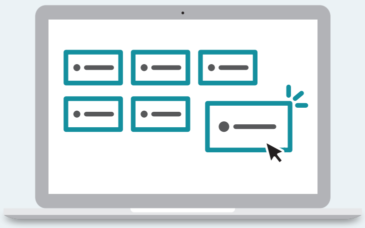
New words

- ***center of a circle***
- ***circle***
- ***diameter***
- ***radius***

Amps | Featured Activity

Activity 1
Digital Card Sort

Students will apply their prior knowledge of circles to determine whether given shapes are circular. They will compare and contrast their sorted shapes to develop the definition of *circle*.



Building Math Identity and Community

Connecting to Mathematical Practices

When working towards a definition of a circle in Activity 1, students might not agree with another person's definition. Review ways to show respect and emphasize that students can be respectful when disagreeing about a problem.

Modifications to Pacing

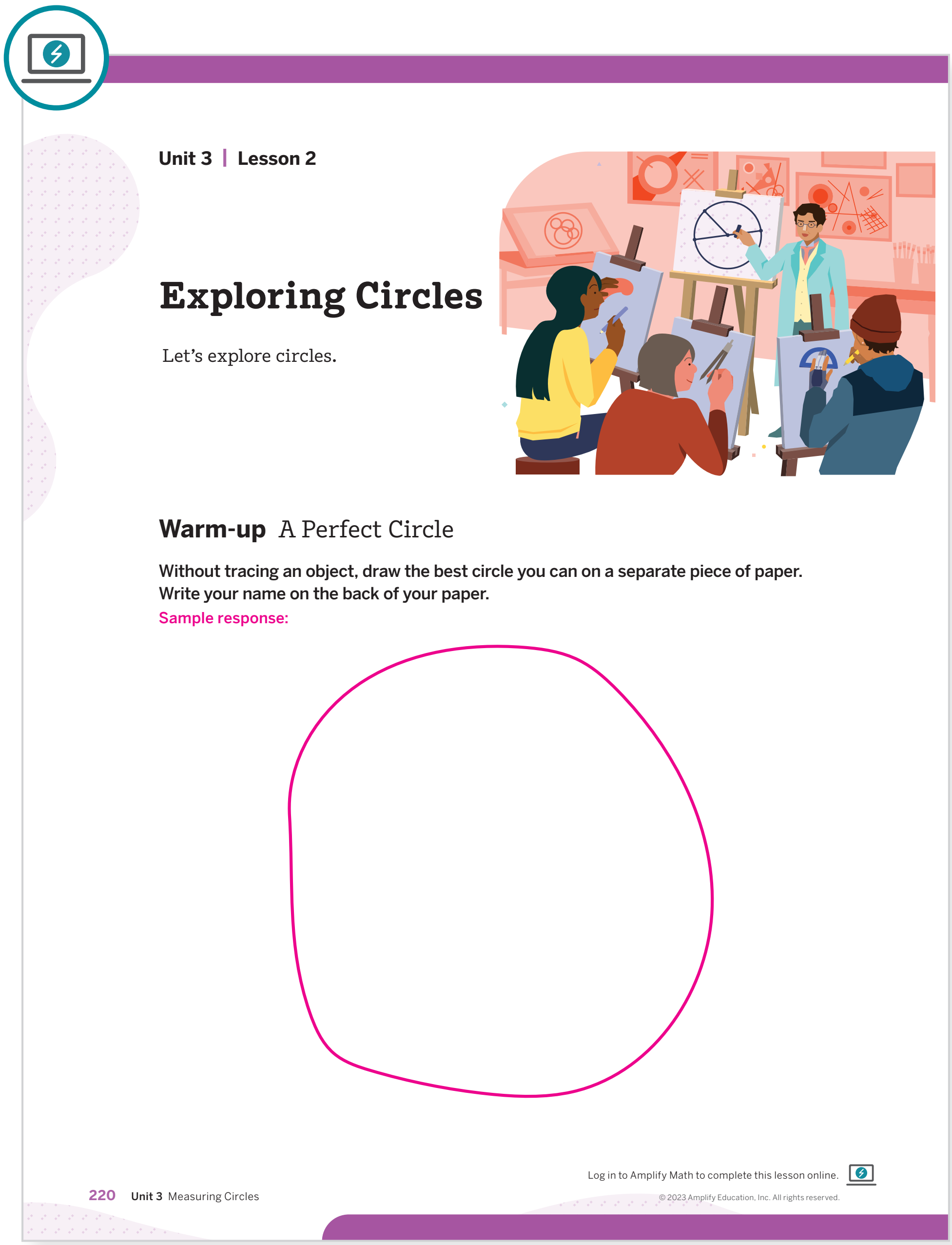
You may want to consider this additional modification if you are short on time:

- The **Warm-up** and the related vote in the Connect portion of **Activity 1** may be omitted.
- **Activity 3** may be assigned for homework. Alternatively, if students are already familiar with the using a compass to construct circles, it may be omitted entirely.

Small Groups | 3 min

Warm-up A Perfect Circle

Students draw a freehand circle to prepare them for discussing the characteristics of an actual circle.



Unit 3 | Lesson 2

Exploring Circles

Let's explore circles.

Warm-up A Perfect Circle

Without tracing an object, draw the best circle you can on a separate piece of paper. Write your name on the back of your paper.

Sample response:

Log in to Amplify Math to complete this lesson online.

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1 Launch

Activate students' prior knowledge by asking them what characteristics they think of when they hear the term *circle*. Distribute plain paper, and explain that students will draw their circles individually first and then compare them as a group.

2 Monitor

Help students get started by reminding them that their circles do not need to be perfect.

Look for productive strategies:

- Employing strategies from the Launch, such as using a piece of string to aid students in creating their circles. **Note:** As opposed to providing students tools for this activity, allow them to make their own determination as to whether they would like to use tools. Allow them to ask for certain tools.

3 Connect

Have individual students share their drawings with their group and vote on the one they think is the "best." Collect the "best" circle from each group and set them aside until the end of Activity 1.

Ask:

- "What characteristics were you thinking of when you drew your circle?" Sample response: A curved shape with no straight sides.
- "How did your group determine whose circle was the 'best'?" Sample response: We looked for shapes that were smooth with no "lumpy" parts and that did not look "egg-shaped."

Differentiated Support

***Accessibility:** Optimize Access to Tools, Guide Processing and Visualization*

Consider having one volunteer, or yourself, demonstrate how to draw a circle by hand, emphasizing that the circles do not have to be perfect. Provide access to writing implements with larger diameters, such as a bold marker, for students to use if they choose.

Power-up

To power up students' ability to identify characteristics of two-dimensional figures, have students complete:

Which of these objects is a circle?

A. An egg

[✓ **B.**] A coin

C. A pressed souvenir penny

D. A horseshoe

Use: Before Activity 1.

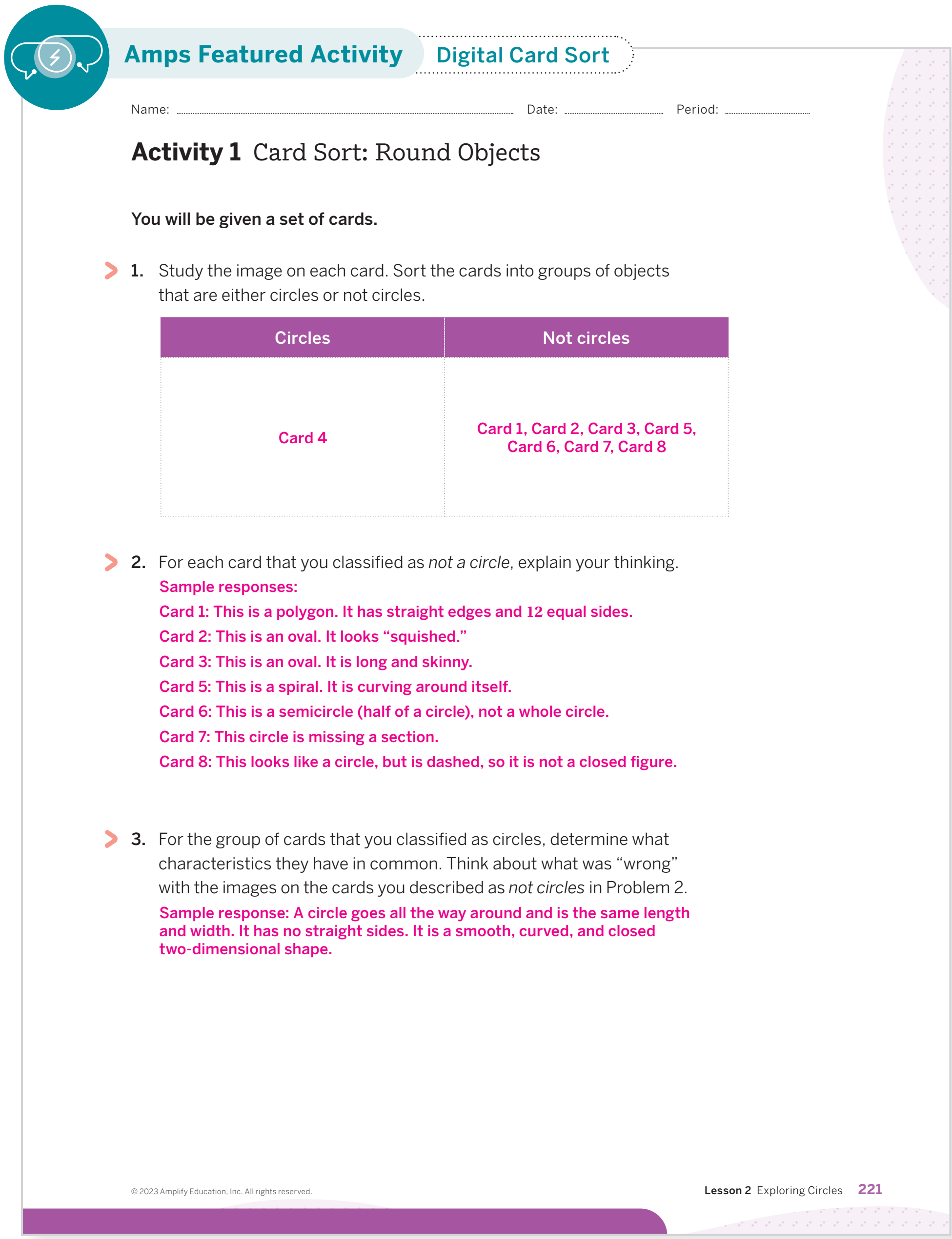
Informed by: Performance on Lesson 1, Practice Problem 5.

Small Groups | 10 min

Activity 1 Card Sort: Round Objects

Students build on their prior knowledge of circles to determine the characteristics that all circles have in common in order to develop the definition of the term *circle*.

Amps Featured Activity Digital Card Sort



Name: ______ Date: ______ Period: ______

Activity 1 Card Sort: Round Objects

You will be given a set of cards.

1. Study the image on each card. Sort the cards into groups of objects that are either circles or not circles.

Circles	Not circles
Card 4	Card 1, Card 2, Card 3, Card 5, Card 6, Card 7, Card 8

2. For each card that you classified as *not a circle*, explain your thinking.
   Sample responses:
   Card 1: This is a polygon. It has straight edges and 12 equal sides.
   Card 2: This is an oval. It looks "squished."
   Card 3: This is an oval. It is long and skinny.
   Card 5: This is a spiral. It is curving around itself.
   Card 6: This is a semicircle (half of a circle), not a whole circle.
   Card 7: This circle is missing a section.
   Card 8: This looks like a circle, but is dashed, so it is not a closed figure.

3. For the group of cards that you classified as circles, determine what characteristics they have in common. Think about what was "wrong" with the images on the cards you described as *not circles* in Problem 2.
   Sample response: A circle goes all the way around and is the same length and width. It has no straight sides. It is a smooth, curved, and closed two-dimensional shape.

Lesson 2 Exploring Circles 221

1 Launch

Distribute one set of cards from the Activity 1 PDF to each group and conduct the ***Card Sort*** routine. Explain that students should sort the cards by determining which figures are *exact* circles with the goal of determining their unique characteristics.

2 Monitor

Help students get started by suggesting they focus first on the cards they know are *not* circular.

Look for points of confusion:

- **Thinking that any shape with a curved edge is a circle.** Remind students shapes can have curved parts but not be true circles. Ask them which shapes are true circles.

3 Connect

Have groups of students share which cards they know are *not* circles as well as of which cards they were unsure. Facilitate a class discussion to determine the characteristics that define a circle. Highlight that Card 8 is not a circle because it is not a closed figure.

Define:

- A ***circle*** is a shape that is made up of *all* the points that are the same distance from a given point called the ***center of the circle***.
- The distance from the center of the circle to a point on the circle is called the ***radius***. The term *radius* also refers to the line segment that goes from the center of the circle to a point on the circle. The plural of radius is *radii*.

Display the circles from the Warm-up and conduct the ***Poll the Class*** routine to determine the paper on which the image is a true circle.

Ask, "How could you check how close this drawing is to being a true circle?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Suggest that students first group the cards into three categories: *Circles*, *Not Circles*, and *Possible Circles* or *Unsure*. Encourage them to look for characteristics between the figures that are not circles to help them make their final decision on the cards they grouped as Possible *Circles* or *Unsure*.

Math Language Development

MLR5: Co-craft Questions

After you distribute the cards, give a few moments for students to examine them. Ask them to write 2–3 mathematical questions they could ask about the information they are given. Ask students to share their questions with a partner and then invite a few students to share their questions aloud with the class before moving on to the activity.

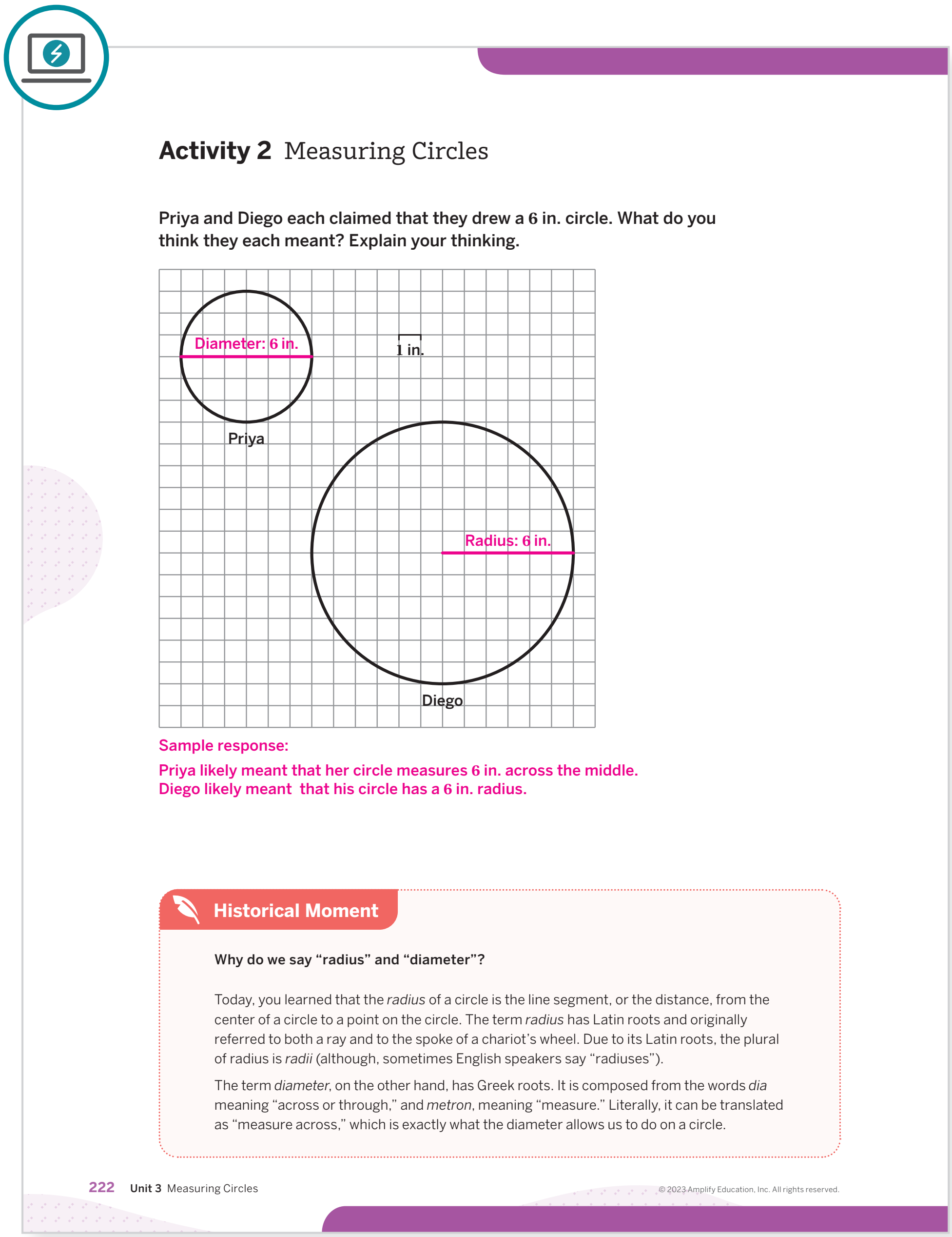
English Learners

Display a sample mathematical question, such as "Why are the shapes on Cards 5 and 8 "open", while the other cards are closed shapes?" or "Why do some cards have straight line segments, while others do not?"

Pairs | 8 min

Activity 2 Measuring Circles

Students examine circles on a grid to facilitate a discussion on how circles are measured, to solidify their understanding of the relationship between the terms *circle*, *radius*, and *diameter*.



Activity 2 Measuring Circles

Priya and Diego each claimed that they drew a 6 in. circle. What do you think they each meant? Explain your thinking.

Sample response:

Priya likely meant that her circle measures 6 in. across the middle.
Diego likely meant that his circle has a 6 in. radius.

Historical Moment

Why do we say "radius" and "diameter"?

Today, you learned that the *radius* of a circle is the line segment, or the distance, from the center of a circle to a point on the circle. The term *radius* has Latin roots and originally referred to both a ray and to the spoke of a chariot's wheel. Due to its Latin roots, the plural of radius is *radii* (although, sometimes English speakers say "radiuses").

The term *diameter*, on the other hand, has Greek roots. It is composed from the words *dia* meaning "across or through," and *metron*, meaning "measure." Literally, it can be translated as "measure across," which is exactly what the diameter allows us to do on a circle.

222 **Unit 3** Measuring Circles

1 Launch

Have students use the *Think-Pair-Share* routine.

2 Monitor

Help students get started by asking how they can use the grid to measure 6 in.

Look for points of confusion:

- **Using a ruler to measure 6 in., instead of the scale.** Remind students that they are looking at a scale drawing. Ask them what scale is being used.

Look for productive strategies:

- Drawing a segment that represents the radius or a segment that represents the diameter and determining its length.

3 Connect

Have pairs share their thinking for how each person claimed they drew a 6 in. circle.

Define the term ***diameter*** as a line segment with endpoints on the circle, that passes through the center. Explain that the word *diameter* also refers to the length of this segment. Ask students which person drew the circle with a 6 in. diameter. Priya

Display the Activity 2 PDF. Stress that circles are named by their center point, so the center of Circle A is point A.

Ask:

- "What is the radius of the circle?" Sample responses: AF, AB, AE, or 10 cm
- "What is the diameter of the circle?" EF or 20 cm
- "Why is segment CD neither a diameter nor a radius?"
- "What is a point that is on the circle? In the circle? Outside the circle?

Highlight that the diameter is composed of two radii and therefore is twice the length of the radius.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Annotate a circle on the class display with the terms *radius*, *diameter*, and *center*. Include examples of how these are named by points.

Extension: *Math Enrichment*

Priya claims that the longest segment she could draw connecting two points on the edge of her circle is 6 in. Is she correct? Explain your thinking. Yes; Sample response: If she draws a line above or below the diameter, it will be less than 6 in. long.

Math Language Development

MLR8: Discussion Supports

While students discuss the activity with their partner, or during the Connect, provide the following sentence frames to support them in their discussion:

- "I agree because . . ."
- "I disagree because . . ."
- "Another way to look at it is . . ."
- "Where does ____ show . . .?"

Historical Moment

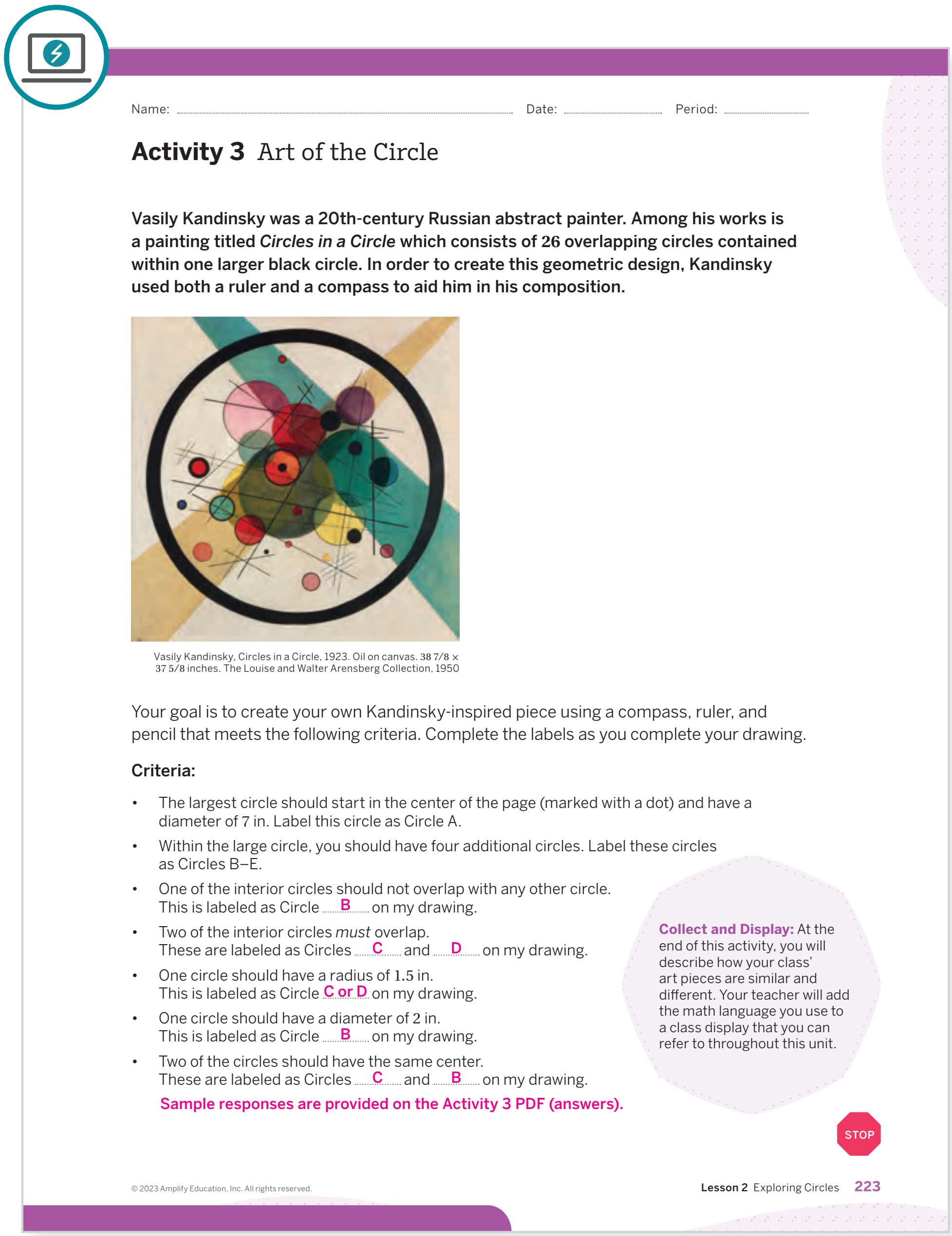
Why do we say "radius" and "diameter"?

Have students read about the history and etymology of the terms *radius* and *diameter*.

Independent | 15 min

Activity 3 Art of the Circle

Students use their geometry toolkits to create a Kandinsky-inspired design with given characteristics.



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Activity 3 Art of the Circle

Vasily Kandinsky was a 20th-century Russian abstract painter. Among his works is a painting titled *Circles in a Circle* which consists of 26 overlapping circles contained within one larger black circle. In order to create this geometric design, Kandinsky used both a ruler and a compass to aid him in his composition.

Vasily Kandinsky, Circles in a Circle, 1923. Oil on canvas. 38 7/8 × 37 5/8 inches. The Louise and Walter Arensberg Collection, 1950

Your goal is to create your own Kandinsky-inspired piece using a compass, ruler, and pencil that meets the following criteria. Complete the labels as you complete your drawing.

Criteria:

- The largest circle should start in the center of the page (marked with a dot) and have a diameter of 7 in. Label this circle as Circle A.
- Within the large circle, you should have four additional circles. Label these circles as Circles B–E.
- One of the interior circles should not overlap with any other circle. This is labeled as Circle B on my drawing.
- Two of the interior circles *must* overlap. These are labeled as Circles C and D on my drawing.
- One circle should have a radius of 1.5 in. This is labeled as Circle C or D on my drawing.
- One circle should have a diameter of 2 in. This is labeled as Circle B on my drawing.
- Two of the circles should have the same center. These are labeled as Circles C and B on my drawing.

Sample responses are provided on the Activity 3 PDF (answers).

Collect and Display: At the end of this activity, you will describe how your class' art pieces are similar and different. Your teacher will add the math language you use to a class display that you can refer to throughout this unit.

STOP

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Lesson 2 Exploring Circles 223

1 Launch

Distribute a geometry toolkit and the Activity 3 PDF to each student. Explain that students will be learning how to use mathematical tools to create their own works of art the same way Kandinsky used a compass and ruler to create *Circles in a Circle*. Display the Activity 3 PDF and model how to use a compass and ruler to create a circle with a diameter of 7 in. **Note:** If you are looking to provide students with the experience of using physical tools, complete this activity using the print edition.

2 Monitor

Help students get started by suggesting they practice with their compass on a separate piece of paper prior to completing Activity 3.

Look for points of confusion:

- **Not taking into account the difference between radius and diameter when creating their circles.** Suggest that students use one color to highlight the term *radius* and a second color to highlight the term *diameter* to draw their attention to the different measurements.

3 Connect

Have students share their art pieces with their group. Ask them to look for common features and differences between their creations.

Ask:

- "Why can you draw a 'perfect' circle using a compass?"
- "What did you find most challenging about using the compass and ruler?"
- "What strategies did you use to help you create a circle with a given radius or diameter?"

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to tools and assistive technologies such as $\frac{1}{4}$ in. graph paper or a compass applet. Some students may also benefit from a checklist of steps for using the compass.

Accessibility: *Guide Processing and Visualization*

For each circle, have students draw a line segment to represent the radius as their first step. They should align the two ends of their compass with the endpoints of the line segment before drawing their circle. Remind students to erase this line after they draw each circle.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their art pieces and look for common features and differences, collect any mathematical language they use and add it to the class display. For example, if students talk about *overlapping circles*, add and annotate a visual example to the class display.

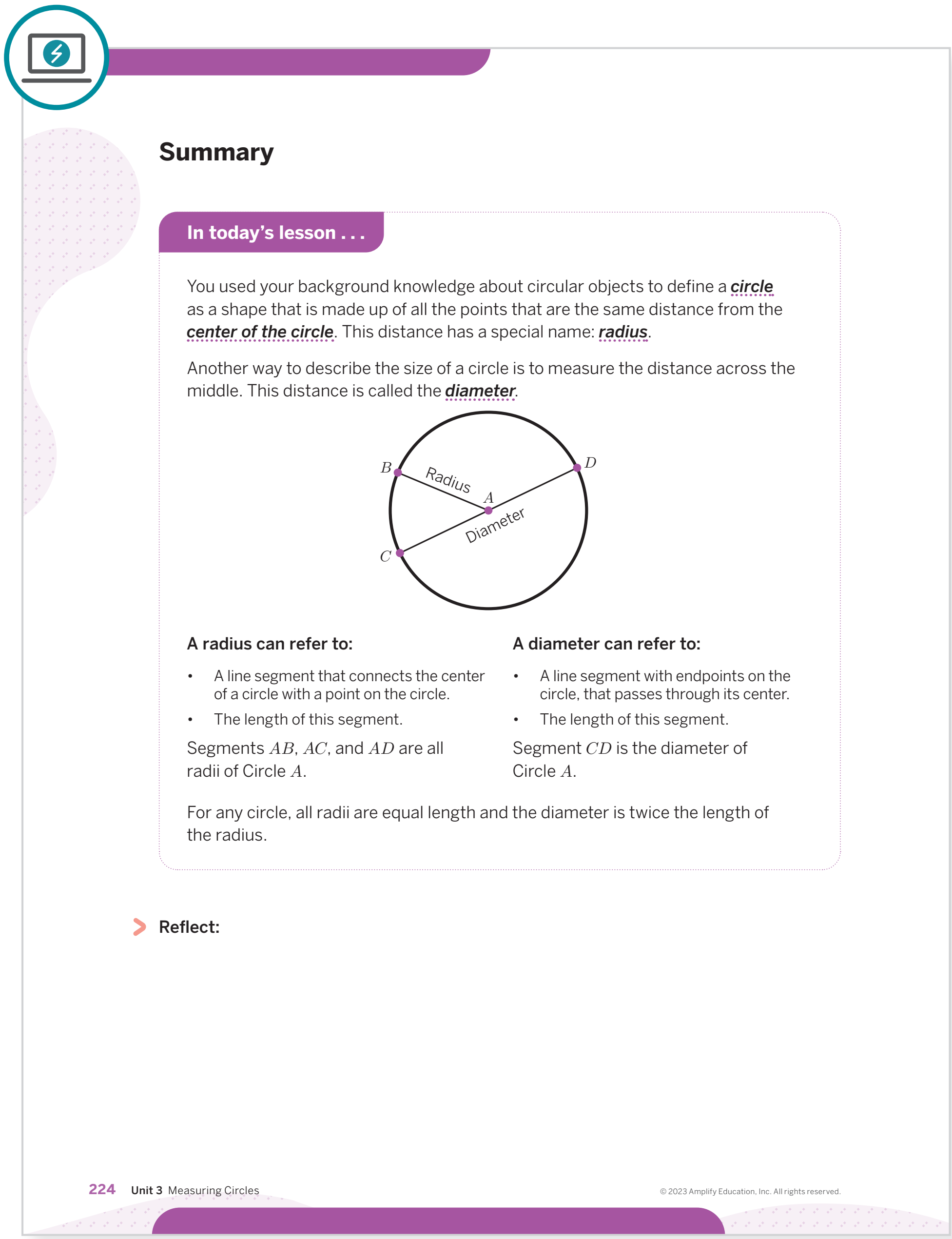
English Learners

Highlight any tool-specific language used, such as the *pointed leg of a compass*.

Whole Class | 5 min

Summary

Review and synthesize the characteristics of a circle, making connections between the definitions of *circle*, *radius*, and *diameter*.



Summary

In today's lesson . . .

You used your background knowledge about circular objects to define a ***circle*** as a shape that is made up of all the points that are the same distance from the ***center of the circle***. This distance has a special name: ***radius***.

Another way to describe the size of a circle is to measure the distance across the middle. This distance is called the ***diameter***.

A radius can refer to:	**A diameter can refer to:**
• A line segment that connects the center of a circle with a point on the circle. • The length of this segment.	• A line segment with endpoints on the circle, that passes through its center. • The length of this segment.
Segments AB, AC, and AD are all radii of Circle A.	Segment CD is the diameter of Circle A.

For any circle, all radii are equal length and the diameter is twice the length of the radius.

Reflect:

224 Unit 3 Measuring Circles

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Synthesize

Display the Anchor Chart PDF, *Circles*, and complete the sections on Radius and Diameter.

Formalize vocabulary:

- *center of a circle*
- *circle*
- *radius*
- *diameter*

Highlight that a circle can be drawn if the length of the radius or diameter is known. The relationship between the radius and diameter can be represented by the equations $d = 2r$ or $r = \frac{1}{2}d$.

Ask,

- "How were the Goat Problems from Lesson 1 related to our work with circles today?" Sample response: The peg represents the center of the circle and the rope represents the radius.
- "How are circles named? Sample response: By the name of the center point. If the center is point D, then the circle is named "Circle D".
- "If the radius of a circle is 5 cm, what is the diameter?" 10 cm
- "If the diameter of a circle is 5 cm, what is the radius?" 2.5 cm

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do we measure circles when all of our tools are straight?"

Math Language Development

MLR2: Collect and Display

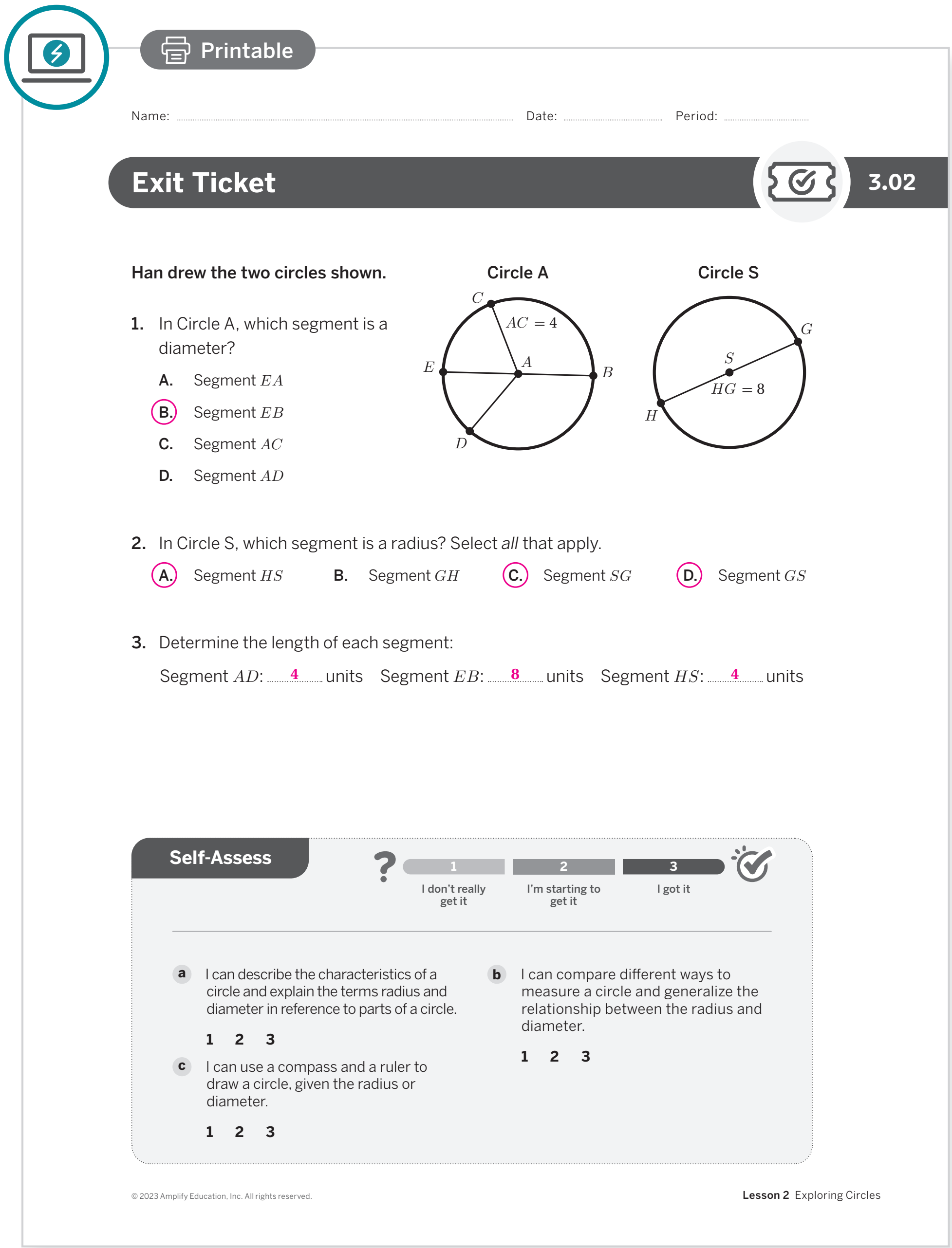
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *center of a circle*, *circle*, *diameter*, and *radius* that were added to the display during the lesson. Consider adding a section of phrases and terms that relate to a shape not being a circle, such as *polygon*, *straight edges*, "*squished*," *oval*, *spiral*, *not a whole circle*, *not a closed figure*, etc.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the characteristics of circles by comparing and contrasting measurements given on two circles.

Printable



Name: Date: Period:

Exit Ticket 3.02

Han drew the two circles shown.

Circle A

Circle S

1. In Circle A, which segment is a diameter?
   - A. Segment EA
   - [✓ B.] Segment EB
   - C. Segment AC
   - D. Segment AD

2. In Circle S, which segment is a radius? Select *all* that apply.
   - [✓ A.] Segment HS
   - B. Segment GH
   - [✓ C.] Segment SG
   - [✓ D.] Segment GS

3. Determine the length of each segment:

   Segment AD: 4 units Segment EB: 8 units Segment HS: 4 units

Self-Assess

? 1 I don't really get it 2 I'm starting to get it 3 I got it

a I can describe the characteristics of a circle and explain the terms radius and diameter in reference to parts of a circle.

1 2 3

b I can compare different ways to measure a circle and generalize the relationship between the radius and diameter.

1 2 3

c I can use a compass and a ruler to draw a circle, given the radius or diameter.

1 2 3

Lesson 2 Exploring Circles

Success looks like . . .

- **Language Goal:** Comparing different ways to measure a circle and generalizing the relationship between the *radius* and the *diameter* of a circle. **(Speaking and Listening)**
- **Language Goal:** Understanding and describing the characteristics of a circle using the terms *center*, *radius*, and *diameter* in reference to the parts of a circle. **(Speaking and Listening, Writing)**
  - » Applying knowledge about the parts of Circles A and S in Problems 1, 2, and 3.
- **Goal:** Understanding how to use a compass and a ruler to create a circle when given the length of the *radius* or the *diameter*.

Suggested next steps

If students identify any segment other than segment EB as a diameter of Circle A or segment GH as the diameter of Circle S, consider:

- Reviewing the definition of radius from Activity 1.
- Reviewing the definition of *diameter* from Activity 2.
- Assigning Practice Problems 2 and 3.

Professional Learning

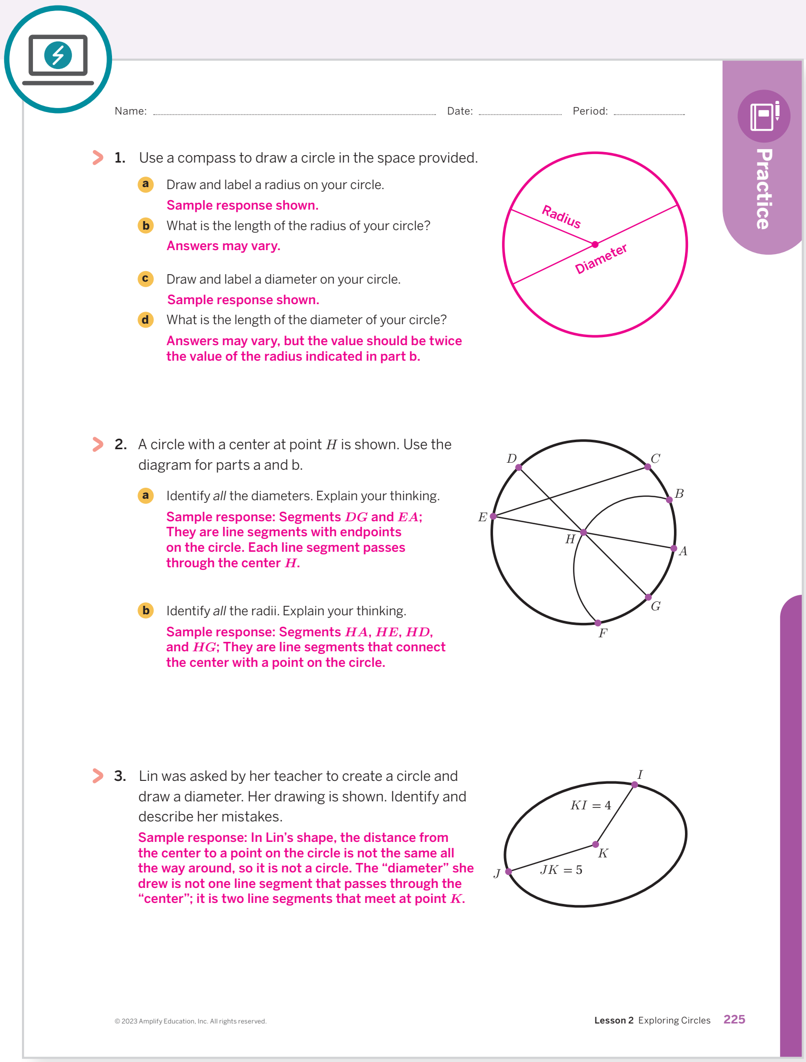
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students attend to precision today? How are you helping students become aware of how they are progressing in this area?
- What did you see in the way some students approached Activity 3 that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent

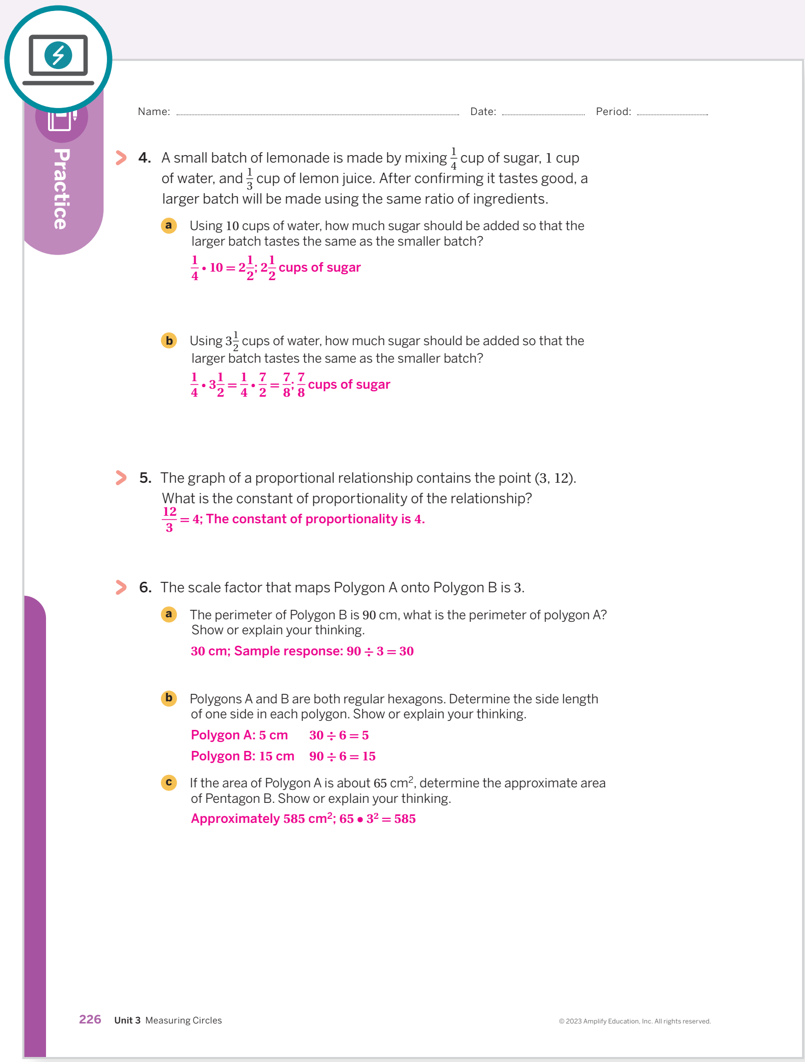


Name: Date: Period:

1. Use a compasss to draw a circle in the space provided.
   - a. Draw and label a radius on your circle.
     Sample response shown.
   - b. What is the length of the radius of your circle?
     Answers may vary.
   - c. Draw and label a diameter on your circle.
     Sample response shown.
   - d. What is the length of the diameter of your circle?
     Answers may vary, but the value should be twice the value of the radius indicated in part b.

2. A circle with a center at point H is shown. Use the diagram for parts a and b.
   - a. Identify *all* the diameters. Explain your thinking.
     Sample response: Segments DG and EA; They are line segments with endpoints on the circle. Each line segment passes through the center H.
   - b. Identify *all* the radii. Explain your thinking.
     Sample response: Segments HA, HE, HD, and HG; They are line segments that connect the center with a point on the circle.

3. Lin was asked by her teacher to create a circle and draw a diameter. Her drawing is shown. Identify and describe her mistakes.
   Sample response: In Lin's shape, the distance from the center to a point on the circle is not the same all the way around, so it is not a circle. The "diameter" she drew is not one line segment that passes through the "center"; it is two line segments that meet at point K.



Name: Date: Period:

4. A small batch of lemonade is made by mixing $\frac{1}{4}$ cup of sugar, 1 cup of water, and $\frac{1}{3}$ cup of lemon juice. After confirming it tastes good, a larger batch will be made using the same ratio of ingredients.
   - a. Using 10 cups of water, how much sugar should be added so that the larger batch tastes the same as the smaller batch?
     $\frac{1}{4} \bullet 10 = 2\frac{1}{2}$; $2\frac{1}{2}$ cups of sugar
   - b. Using $3\frac{1}{2}$ cups of water, how much sugar should be added so that the larger batch tastes the same as the smaller batch?
     $\frac{1}{4} \bullet 3\frac{1}{2} = \frac{1}{4} \bullet \frac{7}{2} = \frac{7}{8}$; $\frac{7}{8}$ cups of sugar

5. The graph of a proportional relationship contains the point $(3, 12)$. What is the constant of proportionality of the relationship?
   $\frac{12}{3} = 4$; The constant of proportionality is 4.

6. The scale factor that maps Polygon A onto Polygon B is 3.
   - a. The perimeter of Polygon B is 90 cm, what is the perimeter of polygon A? Show or explain your thinking.
     30 cm; Sample response: $90 \div 3 = 30$
   - b. Polygons A and B are both regular hexagons. Determine the side length of one side in each polygon. Show or explain your thinking.
     Polygon A: 5 cm $30 \div 6 = 5$
     Polygon B: 15 cm $90 \div 6 = 15$
   - c. If the area of Polygon A is about 65 cm^2, determine the approximate area of Pentagon B. Show or explain your thinking.
     Approximately 585 cm^2; $65 \bullet 3^2 = 585$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	1
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 2	1
	5	Unit 2 Lesson 12	1
Formative ⚡	6	Unit 3 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3 | **LESSON 3**

How Well Can You Measure?

Let's see how accurately you can measure.

Focus

Goals

1. Recognize that, when the quantities in a proportional relationship represent measurements, measurement error can result in the graph not being perfectly straight and the ratios not being exactly equivalent.
2. **Language Goal:** Justify whether the relationship shown on a graph could represent a proportional relationship with some measurement error. **(Speaking and Listening, Writing)**

Coherence

Today

Students apply their understanding of scaled figures and proportional relationships to explore the relationship between a square's perimeter and diagonal length. Despite observing some discrepancy between the graph and the constant of proportionality, they conclude the relationship is proportional. Students recognize the possibility of errors in measuring the squares manually and generalize the relationship between the perimeter and diagonal length using the equation $y = kx$.

Previously

In Unit 1, students noticed that if two figures are scaled figures, then both their side lengths and their perimeters are in a proportional relationship. In Unit 2, students studied proportional relationships and represented them in tables, graphs, and equations.

Coming Soon

In Lesson 4, students will apply their work with analyzing measurements of squares to analyze the relationship between the circumference and diameter of a circle to construct the formula $C = \pi d$.

Rigor

- Students build **conceptual understanding** of how errors in measurement affect representations of proportional relationships.
- Students **apply** their understanding of proportional relationships to analyze measurements of squares.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Small Groups	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, pre-cut cards, one card per student
- *Warm-up Table* PDF (for display)
- *Warm-up Table* PDF (answers, optional), one per student
- Activity 1 PDF (for display)
- calculators
- rulers

Math Language Development

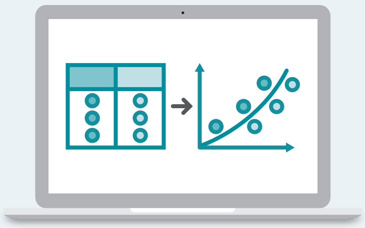
Review words

- *constant of proportionality*
- *diagonal*
- *perimeter*
- *proportional relationship*
- *scaled figures*

Amps | **Featured Activity**

Warm-up
Aggregating Student Data

Student measurement data in the Warm-up are aggregated into one table of class data that all students are able to access and analyze.



Building Math Identity and Community

Connecting to Mathematical Practices

As students seek to find regularity in the relationship between the perimeter of a square and the length of its diagonal, they might reach an unfeasible conclusion. Without discussing exact measurements, have students list what they know about the lengths of the sides of the squares and the diagonal. Ask students to identify ways they can check the reasonableness of their responses.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

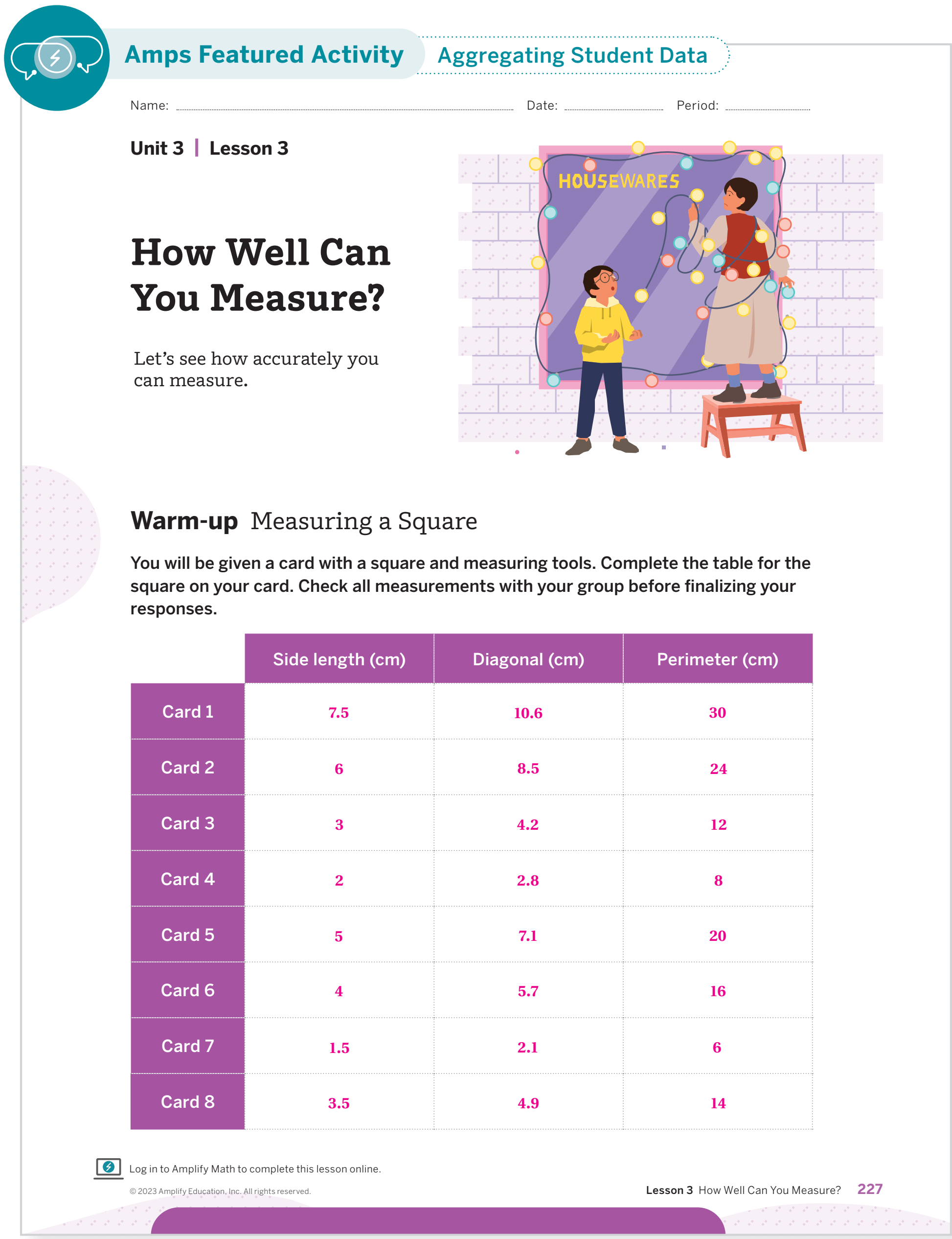
- In the **Warm-up**, during the Connect portion, provide students with a copy of the Warm-up PDF (answers), instead of completing the table as a class.
- Omit **Activity 2**. Instead, during the Connect in Activity 1, mention that knowing the constant of proportionality and one measurement for a square can help students to determine unknown measurements.

Small Groups | 10 min

Warm-up Measuring a Square

Students measure the side length, diagonal, and perimeter of a square and compare their measurement accuracy with their peers to create a set of data that they will analyze in the next activity.

Amps Featured Activity Aggregating Student Data



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Unit 3 | Lesson 3

How Well Can You Measure?

Let's see how accurately you can measure.

Warm-up Measuring a Square

You will be given a card with a square and measuring tools. Complete the table for the square on your card. Check all measurements with your group before finalizing your responses.

	Side length (cm)	Diagonal (cm)	Perimeter (cm)
Card 1	7.5	10.6	30
Card 2	6	8.5	24
Card 3	3	4.2	12
Card 4	2	2.8	8
Card 5	5	7.1	20
Card 6	4	5.7	16
Card 7	1.5	2.1	6
Card 8	3.5	4.9	14

Log in to Amplify Math to complete this lesson online.

1 Launch

Distribute the cards from the Warm-up PDF, rulers, and calculators. Explain that students should use their rulers to measure the square on their card individually, and then come to a consensus about each measurement as a group.

2 Monitor

Help students get started by asking them to trace each length with their finger to assess their understanding of side length, diagonal, and perimeter.

Look for points of confusion:

- **Measuring both diagonals and calculating their sum.** Remind students that like side length, where they only record the length of one side, the diagonal length is the length of one diagonal, not the sum.

Look for productive strategies:

- Measuring both diagonals and comparing their measurements to ensure they are equal and accurate.

3 Connect

Display the *Warm-up Table* PDF.

Have each group of students share how they came to a consensus on their measurements. As groups present, students should complete the missing values in their Warm-up table. **Note:** You may choose to distribute the *Warm-up Table* PDF (answers) instead of having students complete the table in their Student Edition.

Highlight that for some measurements, especially the diagonal, it may be challenging to determine the exact measurement. For example, for Card 4, it may have been challenging to decide whether the diagonal was 2.7, 2.8, or 2.9 cm.

Ask, "Do you notice any relationships between the values in the table? What relationships are shown?"

MLR Math Language Development

MLR8: Discussion Supports

While groups work on completing the Warm-up, display the following sentence stems to support them in coming to a consensus about the measurements:

- "We agree that . . ."
- "We need to know . . ."
- "Is it always true that . . .?"

English Learners

Annotate the visual of a square with the terms *side length, diagonal,* and *perimeter.*

Power-up

To power up students' ability to solve problems involving the perimeter of regular polygons, have students complete:

The perimeter of a regular hexagon is 12.2 cm. Diego says each side length is about 2 cm long, and Shawn says each side length is about 20.3 mm long. Explain why they are both correct.

Sample response: 12.2 divided by 6 is about 2.0333. Diego rounded to 2 cm, and Shawn rounded to 2.03 cm which is equal to 20.3 mm.

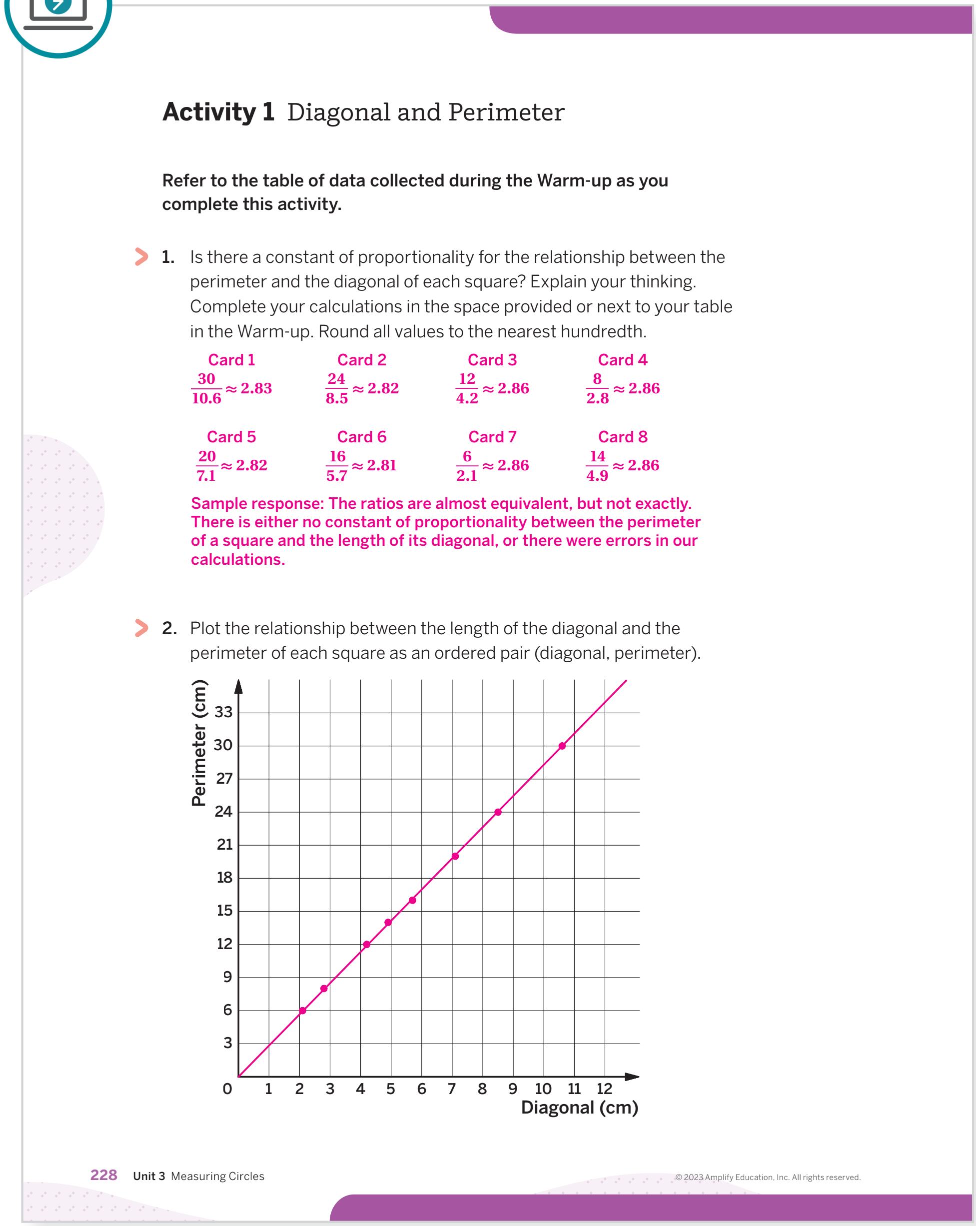
Use: Before Activity 1.

Informed by: Performance on Lesson 2, Practice Problem 6b and Pre-Unit Readiness Assessment, Problems 2 and 3.

Pairs | 15 min

Activity 1 Diagonal and Perimeter

Students analyze the data collected in the Warm-up to approximate the constant of proportionality in the relationship between the diagonal and perimeter of a square.



Activity 1 Diagonal and Perimeter

Refer to the table of data collected during the Warm-up as you complete this activity.

1. Is there a constant of proportionality for the relationship between the perimeter and the diagonal of each square? Explain your thinking. Complete your calculations in the space provided or next to your table in the Warm-up. Round all values to the nearest hundredth.

Card 1	Card 2	Card 3	Card 4
$\frac{30}{10.6} \approx 2.83$	$\frac{24}{8.5} \approx 2.82$	$\frac{12}{4.2} \approx 2.86$	$\frac{8}{2.8} \approx 2.86$
Card 5	Card 6	Card 7	Card 8
$\frac{20}{7.1} \approx 2.82$	$\frac{16}{5.7} \approx 2.81$	$\frac{6}{2.1} \approx 2.86$	$\frac{14}{4.9} \approx 2.86$

Sample response: The ratios are almost equivalent, but not exactly. There is either no constant of proportionality between the perimeter of a square and the length of its diagonal, or there were errors in our calculations.

2. Plot the relationship between the length of the diagonal and the perimeter of each square as an ordered pair (diagonal, perimeter).

228 Unit 3 Measuring Circles

1 Launch

Activate students' prior knowledge by asking, "Based on what you learned earlier this year about scaled figures, would you expect there to be a proportional relationship between the perimeter of a square and the length of its diameter?" Give students a minute of think time, and then use the ***Poll the Class*** routine to quickly assess their thinking. Explain that students will work in pairs, using the data collected in the Warm-up, to answer this question during this activity.

2 Monitor

Help students get started by asking what they remember about calculating the constant of proportionality. Clarify any confusion they have about the order of their ratios.

Look for points of confusion:

- **Using the incorrect columns in their table.** Have students use a scrap piece of paper to cover the side length column to help them better focus on the diagonal and perimeter columns.
- **Rounding the diagonal lengths prior to plotting them on the coordinate plane.** Remind students that they may not be able to plot each point exactly, but they should plot a close approximation.

Look for productive strategies:

- After plotting the points on their graph, using a ruler to determine if a straight line can approximate the points and pass through the origin.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital technology to plot the non-whole number values as they create their graphs.

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing the ratios and rounded values in Problem 1 and a pre-created graph in Problem 2. Have students begin the activity with Problem 3. This will allow them to access the main mathematical goal of the activity, which is to recognize the relationship between the diagonal and perimeter of a square is proportional.

Math Language Development

MLR2: Collect and Display

As students explain their thinking for Problem 4, collect and display mathematical language used, such as *scaled copies/figures, equivalent ratios, proportional,* and *constant of proportionality.* Continue adding to this display in the next activity and encourage students to borrow from the display during class discussions about proportionality.

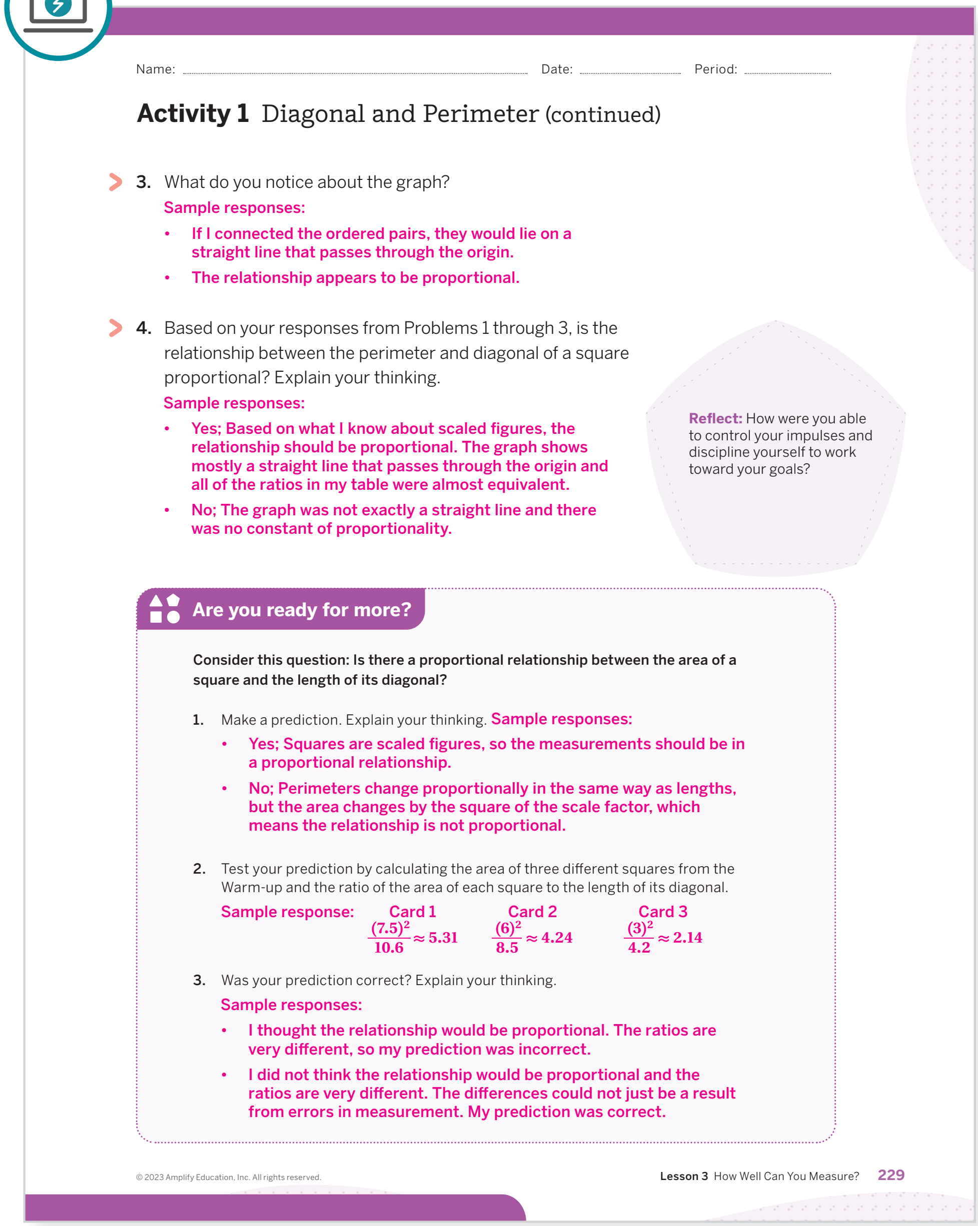
English Learners

Annotate the graph in Problem 2 with key phrases such as "straight line that passes through the origin" and "proportional."

Pairs | 15 min

Activity 1 Diagonal and Perimeter (continued)

Students analyze the data collected in the Warm-up to approximate the constant of proportionality in the relationship between the diagonal and perimeter of a square.



Name: ______________ Date: ________ Period: ________

Activity 1 Diagonal and Perimeter (continued)

3. What do you notice about the graph?

Sample responses:

- If I connected the ordered pairs, they would lie on a straight line that passes through the origin.
- The relationship appears to be proportional.

4. Based on your responses from Problems 1 through 3, is the relationship between the perimeter and diagonal of a square proportional? Explain your thinking.

Sample responses:

- Yes; Based on what I know about scaled figures, the relationship should be proportional. The graph shows mostly a straight line that passes through the origin and all of the ratios in my table were almost equivalent.
- No; The graph was not exactly a straight line and there was no constant of proportionality.

Reflect: How were you able to control your impulses and discipline yourself to work toward your goals?

Are you ready for more?

Consider this question: Is there a proportional relationship between the area of a square and the length of its diagonal?

1. Make a prediction. Explain your thinking. Sample responses:
   - Yes; Squares are scaled figures, so the measurements should be in a proportional relationship.
   - No; Perimeters change proportionally in the same way as lengths, but the area changes by the square of the scale factor, which means the relationship is not proportional.
2. Test your prediction by calculating the area of three different squares from the Warm-up and the ratio of the area of each square to the length of its diagonal.

   Sample response:

Card 1	Card 2	Card 3
$\frac{(7.5)^2}{10.6} \approx 5.31$	$\frac{(6)^2}{8.5} \approx 4.24$	$\frac{(3)^2}{4.2} \approx 2.14$

3. Was your prediction correct? Explain your thinking.

   Sample responses:
   - I thought the relationship would be proportional. The ratios are very different, so my prediction was incorrect.
   - I did not think the relationship would be proportional and the ratios are very different. The differences could not just be a result from errors in measurement. My prediction was correct.

Lesson 3 How Well Can You Measure? 229

3 Connect

Display the Activity 1 PDF. Ask, "Is there a proportional relationship between the perimeter of a square and the length of its diagonal?" Conduct the ***Poll the Class*** routine to assess student thinking.

Have pairs of students share their thinking. Encourage students to use mathematically precise language and give evidence to support their point of view.

Highlight that when measuring figures, there are likely to be small errors in measurement causing some differences in the ratios of corresponding lengths, although the relationship may actually be proportional. If the ratios are close, but not equivalent, plotting the values on a graph allows us to see whether the points lie on a line that passes through the origin. If so, we can conclude that the relationship is likely proportional. Using the measurements, we can estimate the constant of proportionality. In the case of the square, the constant of proportionality is approximately 2.83. **Note:** The exact value of the constant of proportionality is $2\sqrt{2}$, but students will not be introduced to irrational numbers until Grade 8.

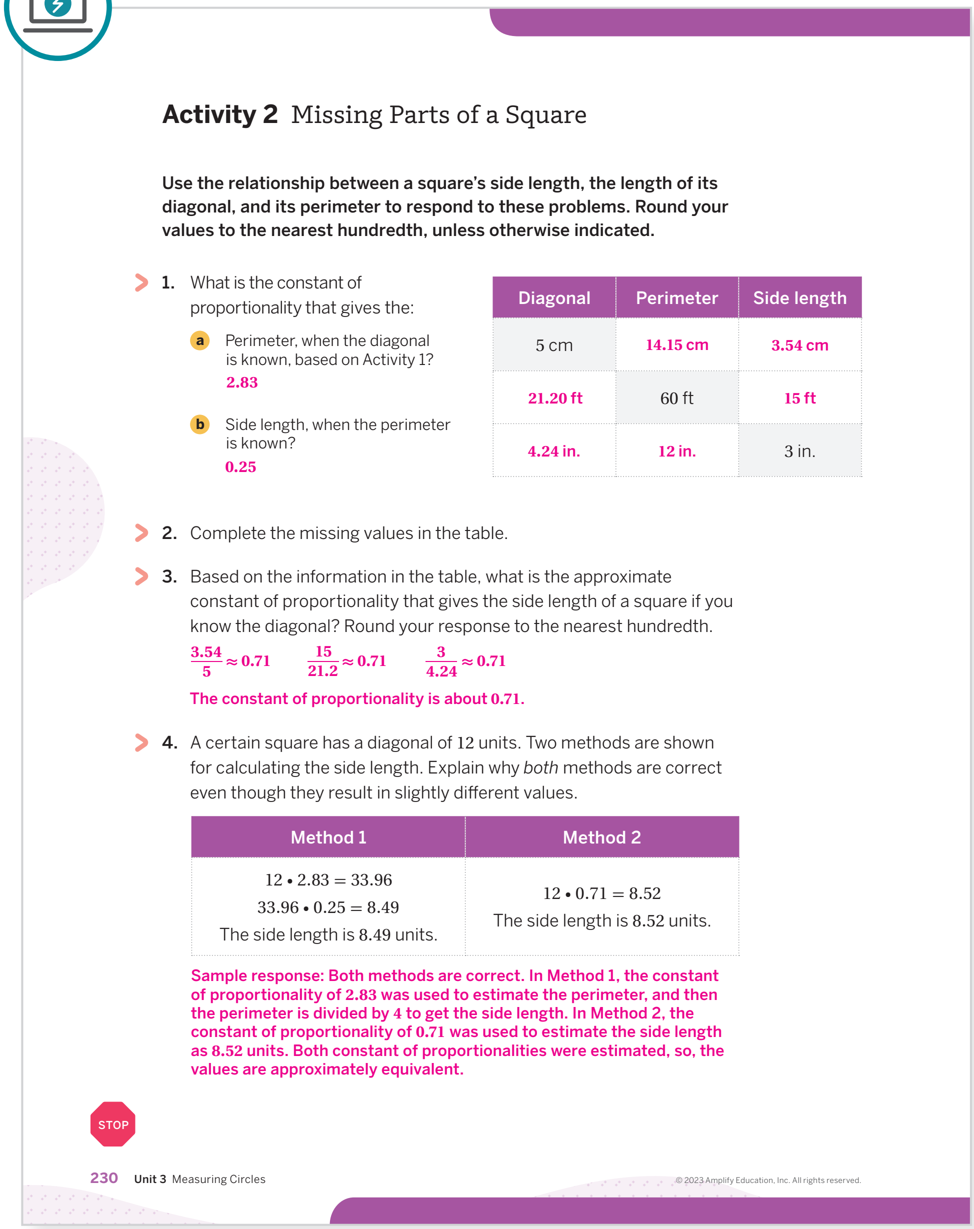
Ask:

- "Why might the values in Problem 1 be slightly different from one another, although the relationship is proportional?" There may be inaccurate measurements.
- "If you were to approximate the constant of proportionality for this relationship, what would it be?" An approximation would be 2.84, the average of my ratios.
- "What other measurements of the square should be in a proportional relationship?" Side length and perimeter.

Pairs | 10 min

Activity 2 Missing Parts of a Square

Students will apply their findings from Activity 1 to approximate the unknown lengths of a square, when given the length of the diagonal, side, or perimeter.



Activity 2 Missing Parts of a Square

Use the relationship between a square's side length, the length of its diagonal, and its perimeter to respond to these problems. Round your values to the nearest hundredth, unless otherwise indicated.

1. What is the constant of proportionality that gives the:
   - a. Perimeter, when the diagonal is known, based on Activity 1?
     2.83
   - b. Side length, when the perimeter is known?
     0.25

Diagonal	Perimeter	Side length
5 cm	**14.15 cm**	**3.54 cm**
21.20 ft	60 ft	**15 ft**
4.24 in.	**12 in.**	3 in.

2. Complete the missing values in the table.

3. Based on the information in the table, what is the approximate constant of proportionality that gives the side length of a square if you know the diagonal? Round your response to the nearest hundredth.

   $\frac{3.54}{5} \approx 0.71$ $\frac{15}{21.2} \approx 0.71$ $\frac{3}{4.24} \approx 0.71$

   The constant of proportionality is about 0.71.

4. A certain square has a diagonal of 12 units. Two methods are shown for calculating the side length. Explain why *both* methods are correct even though they result in slightly different values.

Method 1	Method 2
$12 \cdot 2.83 = 33.96$ $33.96 \cdot 0.25 = 8.49$ The side length is 8.49 units.	$12 \cdot 0.71 = 8.52$ The side length is 8.52 units.

Sample response: Both methods are correct. In Method 1, the constant of proportionality of 2.83 was used to estimate the perimeter, and then the perimeter is divided by 4 to get the side length. In Method 2, the constant of proportionality of 0.71 was used to estimate the side length as 8.52 units. Both constant of proportionalities were estimated, so, the values are approximately equivalent.

STOP

230 **Unit 3** Measuring Circles

1 Launch

Activate students' prior knowledge by asking what they recall about tables of proportional relationships.

2 Monitor

Help students get started by asking what they know about the relationship between a square's side length and its perimeter. If they are unsure, ask them how they determined the perimeter for their card in the Warm-up.

Look for points of confusion:

- **Using the incorrect constant of proportionality to determine the side length.** Review the table from the Warm-up and Activity 2. Ask, "How are these tables the same? How are they different? What is the relationship between side length and perimeter?"

Look for productive strategies:

- Adding arrows to the table to represent the relationship from diagonal to perimeter (× 2.83) and perimeter to side length (× 0.25).
- Attaching meaning to each value in the methods presented in Problem 4. For example, identifying that 12 is the diagonal length and 2.83 is the constant of proportionality (from diagonal to perimeter) to conclude that 33.96 units must represent the perimeter.

3 Connect

Have pairs of students share their explanations for Problem 4.

Highlight that throughout this unit, there are going to be many occasions where different methods of approaching a problem will result in slightly different values that can both be considered correct. Because students are working with approximations, final responses may not be exactly the same, which is why it is important to understand the relationships between measurements.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

In Problem 2, if students need more processing time, have them choose any two of the three rows of the table to complete.

Accessibility: *Guide Processing and Visualization*

In Problem 1, suggest that students find the ratios $\frac{\text{perimeter}}{\text{diagonal}}$ and $\frac{\text{side length}}{\text{perimeter}}$ to help them calculate the constant of proportionality. Consider displaying these ratios. In Problem 3, consider displaying the ratio $\frac{\text{side length}}{\text{diagonal}}$ to help students calculate the approximate constant of proportionality.

Math Language Development

MLR8: Discussion Supports

During the Connect, while discussing Problem 4, ask:

- "What makes an approximation or estimation reasonable?"
- "How can you determine if final responses are reasonably close?"
- "What information can you look at to determine reasonableness?"

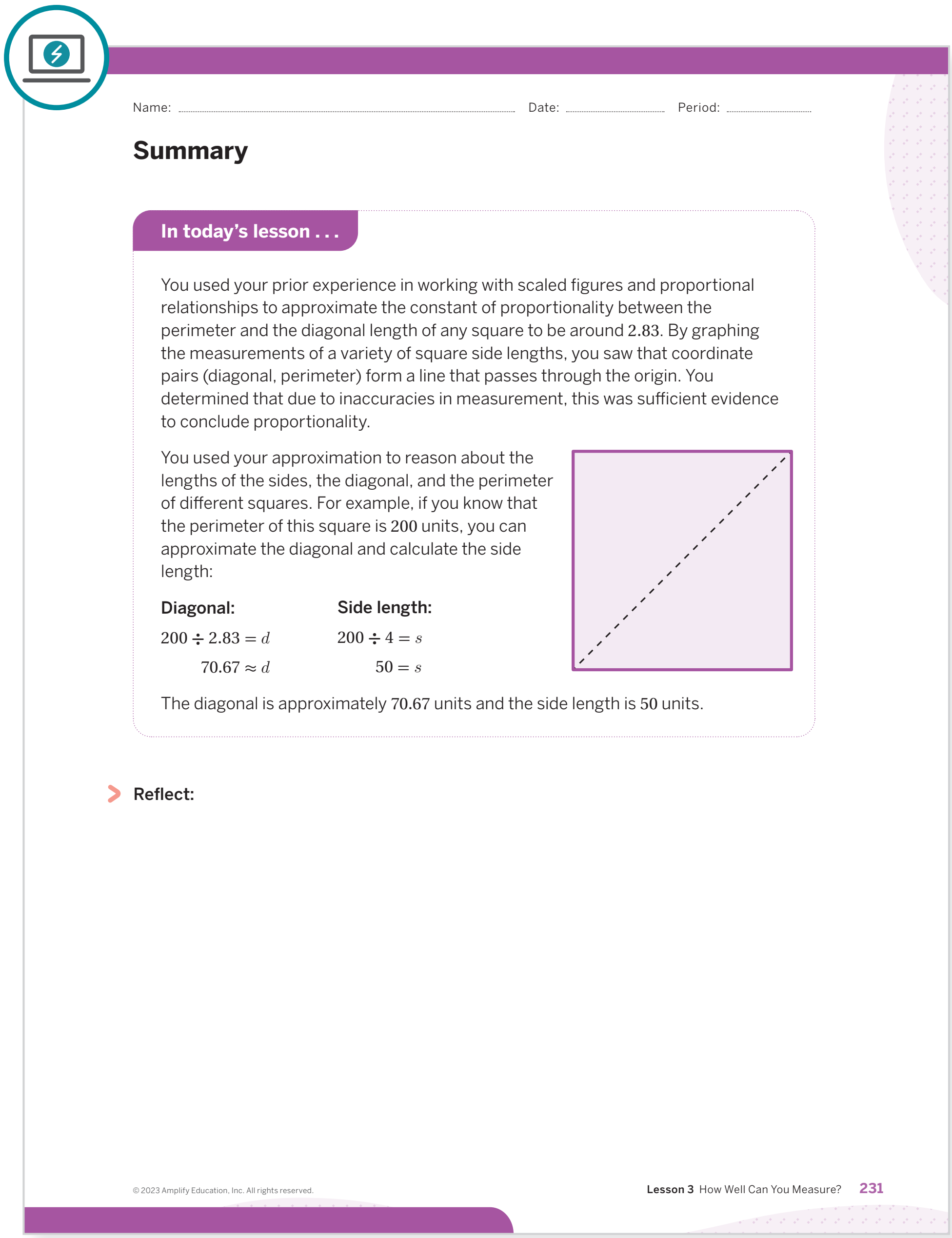
English Learners

Provide wait time to allow students to formulate a response before sharing with others.

Whole Class | 5 min

Summary

Review and synthesize that errors in measurements can result in the graphs of proportional relationships to not be perfectly straight and ratios that are not exactly equivalent.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used your prior experience in working with scaled figures and proportional relationships to approximate the constant of proportionality between the perimeter and the diagonal length of any square to be around 2.83. By graphing the measurements of a variety of square side lengths, you saw that coordinate pairs (diagonal, perimeter) form a line that passes through the origin. You determined that due to inaccuracies in measurement, this was sufficient evidence to conclude proportionality.

You used your approximation to reason about the lengths of the sides, the diagonal, and the perimeter of different squares. For example, if you know that the perimeter of this square is 200 units, you can approximate the diagonal and calculate the side length:

Diagonal:

$200 \div 2.83 = d$

$70.67 \approx d$

Side length:

$200 \div 4 = s$

$50 = s$

The diagonal is approximately 70.67 units and the side length is 50 units.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Ask:

- "Prior to today, how did you determine whether a relationship was proportional?" Sample responses:
  - » I graphed the relationship to see if it was a straight line that passes through the origin.
  - » I checked the ratios to see if there was a constant of proportionality.
- "Why were you able to conclude that the relationship between the diagonal and perimeter of a square is proportional, even though the ratios were not exactly equivalent?" Sample response: I looked at the graph and saw it was roughly a line passing through the origin, so I concluded that the ratios were not all exactly the same due to errors in measurement.

Highlight that when analyzing measurement data for lengths that should be proportionally related, measurement error may cause the ratios to not be exactly equivalent. Graphing the relationship can help determine whether the points are close enough to lying on a straight line that passes through the origin.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

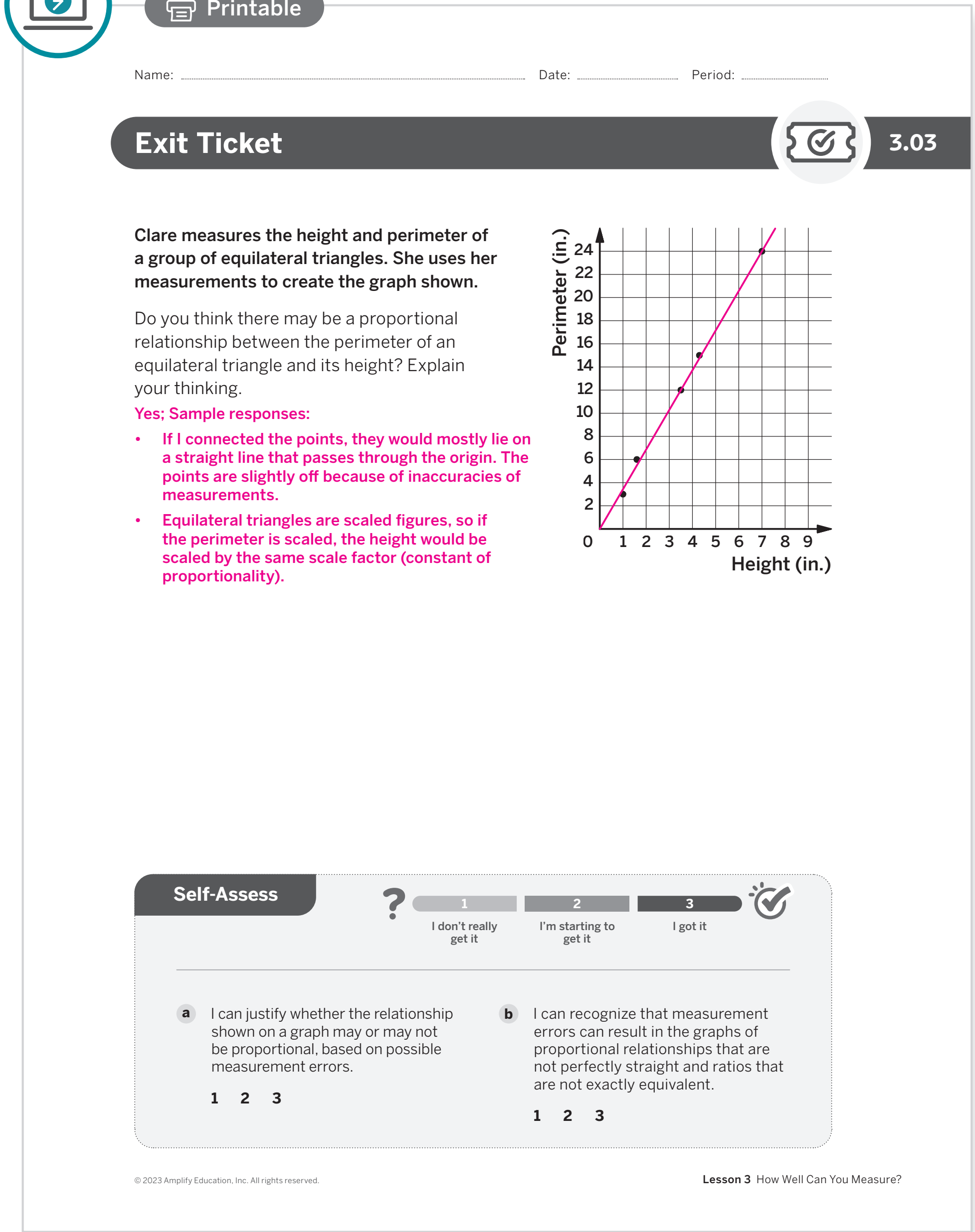
- "What did you find surprising about today's activity with squares?"
- "Make a prediction: Why do you think we analyzed squares today in a unit about circles?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of approximating proportional relationships by analyzing the graph relating the measured height and perimeter of equilateral triangles.

Printable



Name: Date: Period:

Exit Ticket 3.03

Clare measures the height and perimeter of a group of equilateral triangles. She uses her measurements to create the graph shown.

Do you think there may be a proportional relationship between the perimeter of an equilateral triangle and its height? Explain your thinking.

Yes; Sample responses:

- **If I connected the points, they would mostly lie on a straight line that passes through the origin. The points are slightly off because of inaccuracies of measurements.**
- **Equilateral triangles are scaled figures, so if the perimeter is scaled, the height would be scaled by the same scale factor (constant of proportionality).**

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can justify whether the relationship shown on a graph may or may not be proportional, based on possible measurement errors.

1 2 3

b I can recognize that measurement errors can result in the graphs of proportional relationships that are not perfectly straight and ratios that are not exactly equivalent.

1 2 3

Lesson 3 How Well Can You Measure?

Success looks like . . .

- **Goal:** Recognizing that, when the quantities in a proportional relationship represent measurements, measurement error can result in the graph not being perfectly straight and the ratios not being exactly equivalent.
  - » Explaining that there are some inaccuracies of measurements of the triangles.
- **Language Goal:** Justify whether the relationship shown on a graph could represent a proportional relationship with some measurement error. **(Speaking and Listening, Writing)**
  - » Explaining that the graph shows a proportional relationship between perimeter and height.

Suggested next steps

If students answer "No," consider:

- Reviewing Activity 1.
- Assigning Practice Problem 3.

If students answer "Yes," but their explanation is missing, incomplete, or inaccurate, consider:

- Reviewing the Summary.
- Assigning Practice Problem 2.

Professional Learning

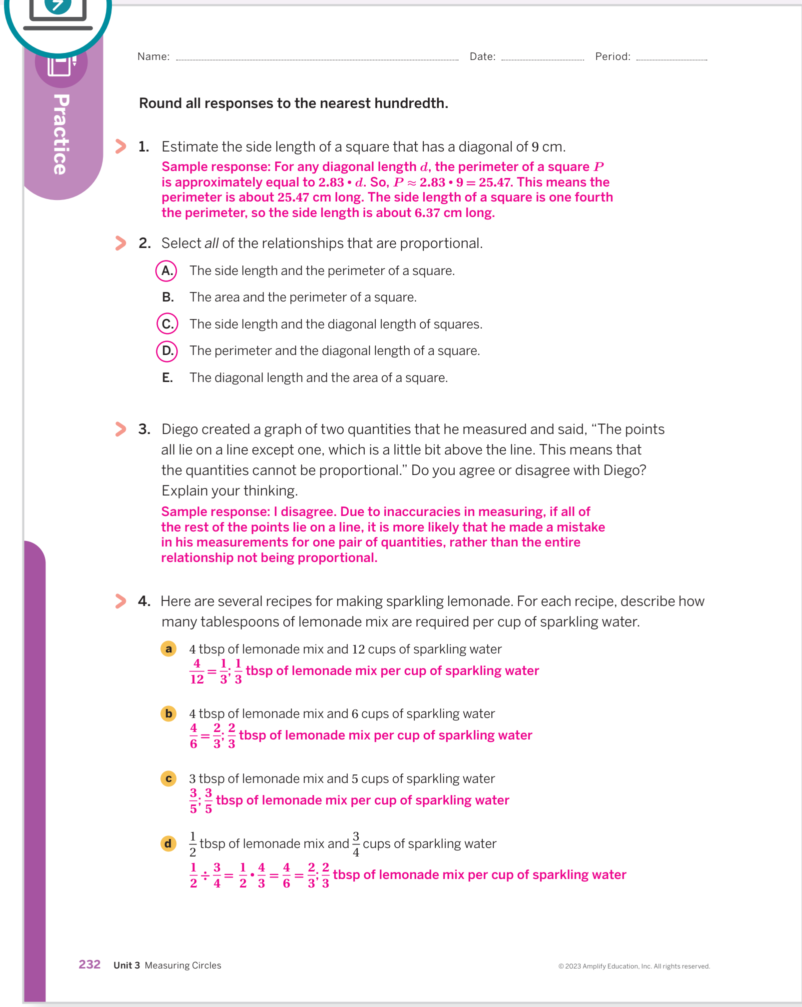
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- In this lesson, students saw that, when measuring, there may be errors that cause discrepancies when determining the constant of proportionality. How will that support them in developing their understanding of π? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Round all responses to the nearest hundredth.

1. Estimate the side length of a square that has a diagonal of 9 cm.

Sample response: For any diagonal length d, the perimeter of a square P is approximately equal to $2.83 \cdot d$. So, $P \approx 2.83 \cdot 9 = 25.47$. This means the perimeter is about 25.47 cm long. The side length of a square is one fourth the perimeter, so the side length is about 6.37 cm long.

2. Select *all* of the relationships that are proportional.

- [✓ **A.**] The side length and the perimeter of a square.
- **B.** The area and the perimeter of a square.
- [✓ **C.**] The side length and the diagonal length of squares.
- [✓ **D.**] The perimeter and the diagonal length of a square.
- **E.** The diagonal length and the area of a square.

3. Diego created a graph of two quantities that he measured and said, "The points all lie on a line except one, which is a little bit above the line. This means that the quantities cannot be proportional." Do you agree or disagree with Diego? Explain your thinking.

Sample response: I disagree. Due to inaccuracies in measuring, if all of the rest of the points lie on a line, it is more likely that he made a mistake in his measurements for one pair of quantities, rather than the entire relationship not being proportional.

4. Here are several recipes for making sparkling lemonade. For each recipe, describe how many tablespoons of lemonade mix are required per cup of sparkling water.

a 4 tbsp of lemonade mix and 12 cups of sparkling water

$\frac{4}{12} = \frac{1}{3}$; $\frac{1}{3}$ **tbsp of lemonade mix per cup of sparkling water**

b 4 tbsp of lemonade mix and 6 cups of sparkling water

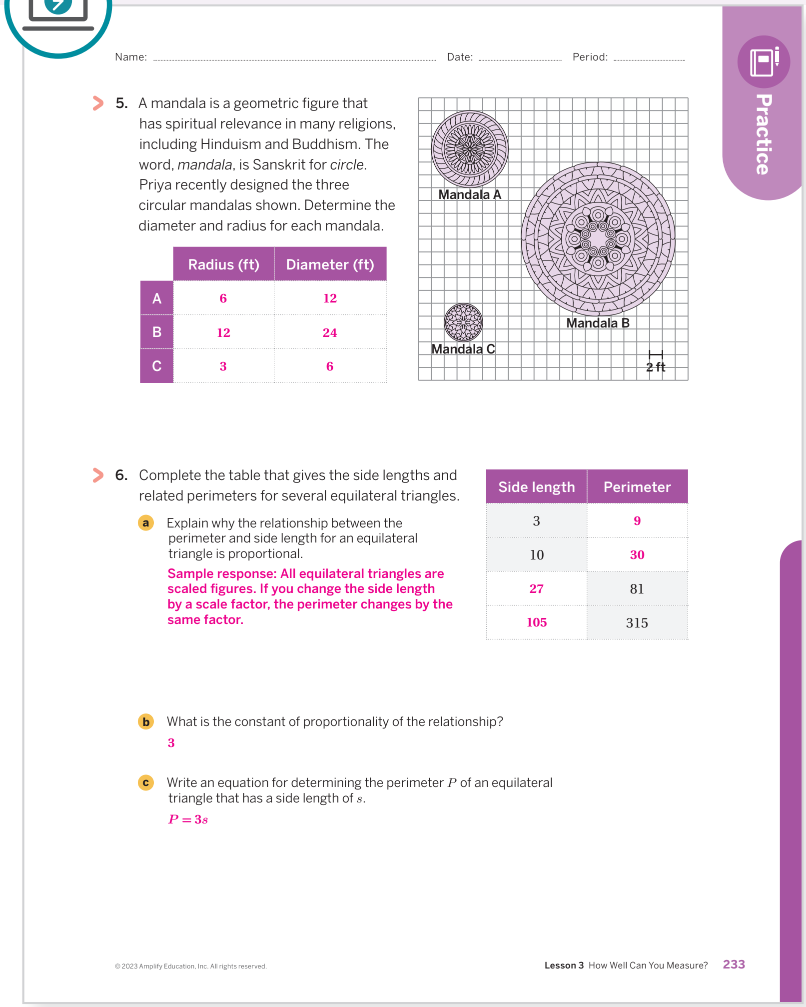
$\frac{4}{6} = \frac{2}{3}$; $\frac{2}{3}$ **tbsp of lemonade mix per cup of sparkling water**

c 3 tbsp of lemonade mix and 5 cups of sparkling water

$\frac{3}{5}$; $\frac{3}{5}$ **tbsp of lemonade mix per cup of sparkling water**

d $\frac{1}{2}$ tbsp of lemonade mix and $\frac{3}{4}$ cups of sparkling water

$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$; $\frac{2}{3}$ **tbsp of lemonade mix per cup of sparkling water**



Name: Date: Period:

5. A mandala is a geometric figure that has spiritual relevance in many religions, including Hinduism and Buddhism. The word, *mandala*, is Sanskrit for *circle*. Priya recently designed the three circular mandalas shown. Determine the diameter and radius for each mandala.

	Radius (ft)	Diameter (ft)
A	6	12
B	12	24
C	3	6

6. Complete the table that gives the side lengths and related perimeters for several equilateral triangles.

Side length	Perimeter
3	9
10	30
27	81
105	315

a Explain why the relationship between the perimeter and side length for an equilateral triangle is proportional.

Sample response: All equilateral triangles are scaled figures. If you change the side length by a scale factor, the perimeter changes by the same factor.

b What is the constant of proportionality of the relationship?

3

c Write an equation for determining the perimeter P of an equilateral triangle that has a side length of s.

$P = 3s$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 2	1
	5	Unit 3 Lesson 2	2
Formative	6	Unit 3 Lesson 4	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

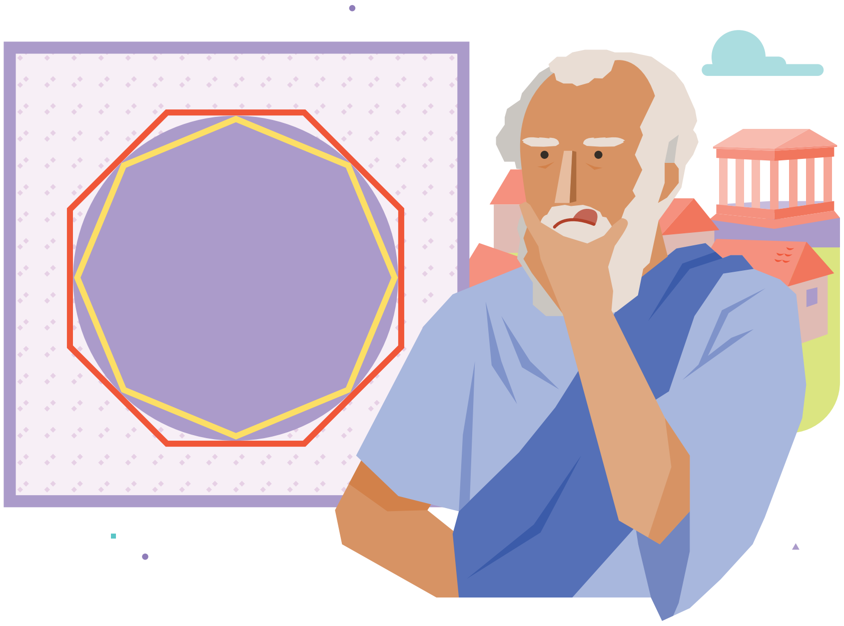
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3 | **LESSON 4**

Exploring Circumference

Let's explore the distance around a circle.



Focus

Goals

1. **Language Goal:** Understand the term *circumference* in reference to parts of a circle. **(Speaking and Listening)**
2. **Language Goal:** Understand the term *pi* and the symbol π refer to the constant of proportionality that gives the circumference of a circle, given its diameter, and generalize that its value is a little more than 3. **(Speaking and Listening)**
3. **Language Goal:** Create and describe graphs that show measurements of circles. **(Writing)**

Coherence

Today

Students use two methods to build and refine their understanding of circumference and π. They activate their prior understanding of the perimeter of regular polygons to make predictions about the circumference of a circle. They refine their prediction based on the observed proportional relationship between the circumference C and diameter d of a circle. Finally, students generalize their findings to conclude the relationship is represented by $C = \pi d$, where the value of the constant π is a little more than 3.

Previously

In Lesson 2, students defined the terms *circle*, *radius*, and *diameter*. In Lesson 3, students discovered that there is a proportional relationship between the perimeter and diagonal of a square.

Coming Soon

In Lesson 5, students will continue to refine their approximations of π. In Lesson 7, students will explore the relationship between rotations of circles and distance traveled.

Rigor

- Students build **conceptual understanding** of circumference and π by exploring the relationship between the diameter and circumference of a circle.
- Students **apply** their understanding of proportional relationships to circles to develop the formula $C = \pi d$.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- *Archimedes' Method* PDF (for display)
- Activity 1 PDF (answers, for display)
- Activity 2 PDF, pre-cut cards, one set per pair (or a variety of circular objects, four per pair)
- Anchor Chart PDF, *Circles* (as needed)
- calculators
- flexible measuring tape or lengths of string (optional)
- rulers (optional)

Math Language Development

New words

- *circumference*
- π *(pi)*

Review words

- *circle*
- *constant of proportionality*
- *diameter*
- *proportional relationship*
- *radius*
- *regular polygon*
- *scaled figures*

Amps | Featured Activity

Activity 2
Exploring Circumference and Diameter

Students use the digital circle tool to compare the circumferences and diameters of a variety of circles. Their collected data is displayed on a graph for them to analyze.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not notice that there is a pattern in the relationship between the circumference and the diameter of a circle. Point out that there are repeated calculations and encourage students to describe their work without using numbers. If their work is stalled in calculations, remind students to focus on the process more than the results to find the pattern.

Modifications to Pacing

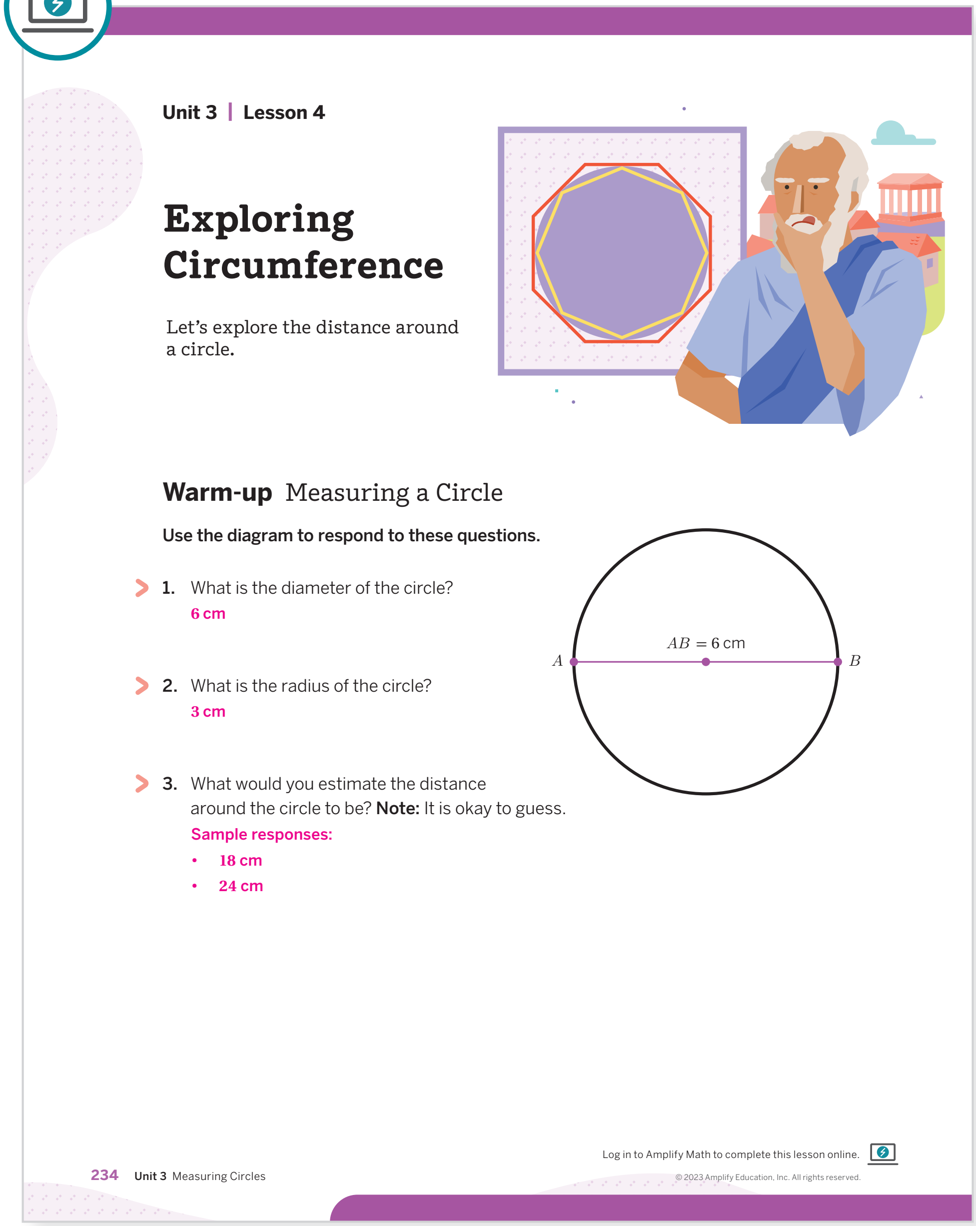
You may want to consider these additional modifications if you are short on time:

- Omit **Activity 1**. Instead, briefly discuss Archimedes' method for approximating the circumference of a circle.
- In **Activity 2,** omit Problems 1 and 2. These problems are optional if using the digital tool for Activity 2.
- In **Activity 2,** have each partner only measure two circles instead of four.

Independent | 5 min

Warm-up Measuring a Circle

Students will predict the distance around a circle based on the given diameter, preparing them to understand what the circumference of a circle means.



Unit 3 | Lesson 4

Exploring Circumference

Let's explore the distance around a circle.

Warm-up Measuring a Circle

Use the diagram to respond to these questions.

1. What is the diameter of the circle?
   6 cm
2. What is the radius of the circle?
   3 cm
3. What would you estimate the distance around the circle to be? **Note:** It is okay to guess.
   Sample responses:
   - **18 cm**
   - **24 cm**

Log in to Amplify Math to complete this lesson online.

234 **Unit 3** Measuring Circles

1 Launch

Activate students' prior knowledge by asking what they recall about radius and diameter from Lesson 2.

2 Monitor

Help students get started by asking, "What is a value you know that is too large for the distance around the circle? What is a value that you know is too small?"

Look for points of confusion:

- **Estimating the area of the circle instead of the distance around it (circumference).** Remind students that "distance around" is similar to perimeter.

Look for productive strategies:

- Realizing that the distance around the top half and bottom half of the circle each seem to be longer than its diameter, so the distance around the circle must be larger than 12 cm.

3 Connect

Display the Warm-up and review students' responses to Problems 1 and 2.

Define the distance around a circle as the ***circumference*** of the circle.

Ask,

- "What value do you know would be too small for the circumference of this circle? Why" Sample response: 6 cm, because the circumference is longer than the diameter.
- What value do you think would be too large for the circumference of this circle? Sample response: 30 cm, because the circumference looks like it's less than 5 diameters.

Have students share their estimations for the circumference of the circle and explain why they chose their value. Create a list of student responses. **Note:** Do not indicate whether estimates are correct at this point.

Math Language Development

MLR8: Discussion Supports

Provide students with a copy of the Anchor Chart PDF, *Circles* that displays the definitions of and visual examples for the terms *circle*, *radius*, and *diameter*. **Note:** Have them only refer to the top third of the anchor chart at this point in the unit. Ask them to label the given circle on the Anchor Chart with its radius (or radii) and diameter. Then have them complete the equations that relate the radius and diameter. Answers are provided on the Anchor Chart PDF, *Circles (answers)*.

Power-up

To power up students' ability to understand the proportional relationship between perimeter and side length, have students complete:

Two rectangles are scaled figures. The length and width of the first rectangle is 2 cm by 3 cm. The length and width of the second rectangle is 3 cm by 4.5 cm.

1. What is the perimeter of each rectangle? 10 cm and 15 cm
2. What is the ratio of the perimeter to the length for each rectangle? 10 : 2 and 15 : 3
3. What do you notice? Both ratios simplify to be 5.

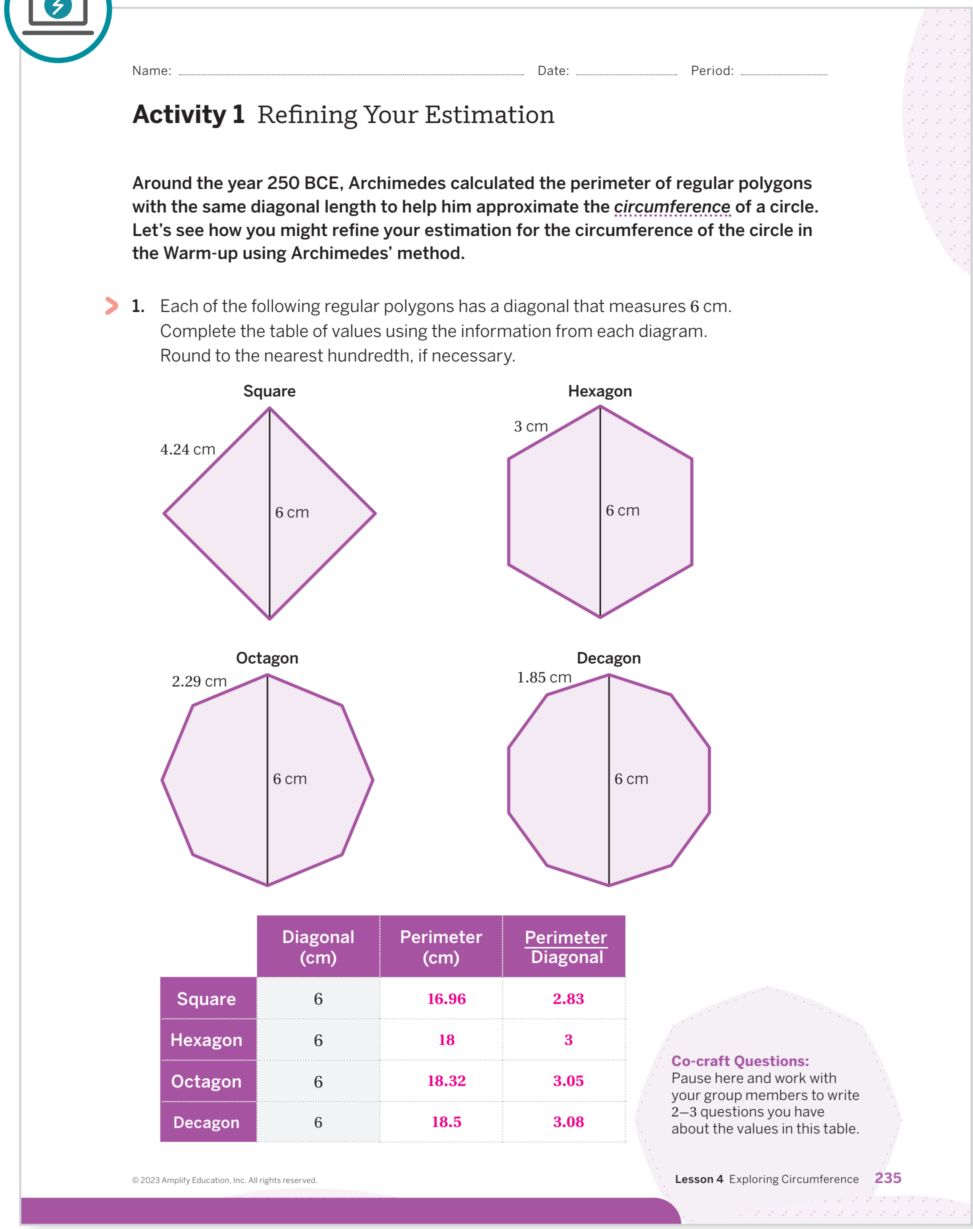
Use: Before Activity 1.

Informed by: Performance on Lesson 3, Practice Problem 6.

Small Groups | 10 min

Activity 1 Refining Your Estimation

Students will apply their prior understanding of calculating the perimeter of regular polygons to refine their estimation of the circumference of a circle with a given diameter.



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Activity 1 Refining Your Estimation

Around the year 250 BCE, Archimedes calculated the perimeter of regular polygons with the same diagonal length to help him approximate the *circumference* of a circle. Let's see how you might refine your estimation for the circumference of the circle in the Warm-up using Archimedes' method.

1. Each of the following regular polygons has a diagonal that measures 6 cm. Complete the table of values using the information from each diagram. Round to the nearest hundredth, if necessary.

	Diagonal (cm)	Perimeter (cm)	$\frac{\text{Perimeter}}{\text{Diagonal}}$
Square	6	**16.96**	**2.83**
Hexagon	6	**18**	**3**
Octagon	6	**18.32**	**3.05**
Decagon	6	**18.5**	**3.08**

Co-craft Questions:
Pause here and work with your group members to write 2–3 questions you have about the values in this table.

Lesson 4 Exploring Circumference 235

1 Launch

Activate students' prior knowledge by asking what they know about *regular polygons*. Display the *Archimedes' Method* PDF and ask which polygon has a perimeter length that is closer to the length of the circumference of the circle. Explain that students will be using the same method as Archimedes to refine their predictions of circumference from the Warm-up by using the perimeters of regular polygons. Distribute one calculator per student to use for the remainder of the class period.

2 Monitor

Help students get started by asking how they determined the perimeter of the square during Lesson 3.

Look for points of confusion:

- **Miscalculating the perimeters by increasing the number of sides by 1 between shapes instead of by 2.** Encourage students to count the number of sides if they are not familiar with the names of the shapes.

Look for productive strategies:

- Using multiplication to determine the perimeter of each shape rather than using repeated addition.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing a pre-completed table in Problem 1 and have students begin the activity with Problem 2. This will allow them more processing time to analyze the ratios in the table and refine their prediction from the Warm-up.

Math Language Development

MLR5: Co-craft Questions

After students complete Problem 1, have them pause and work with their group members to write 2–3 mathematical questions they may have about the values they recorded in the table.

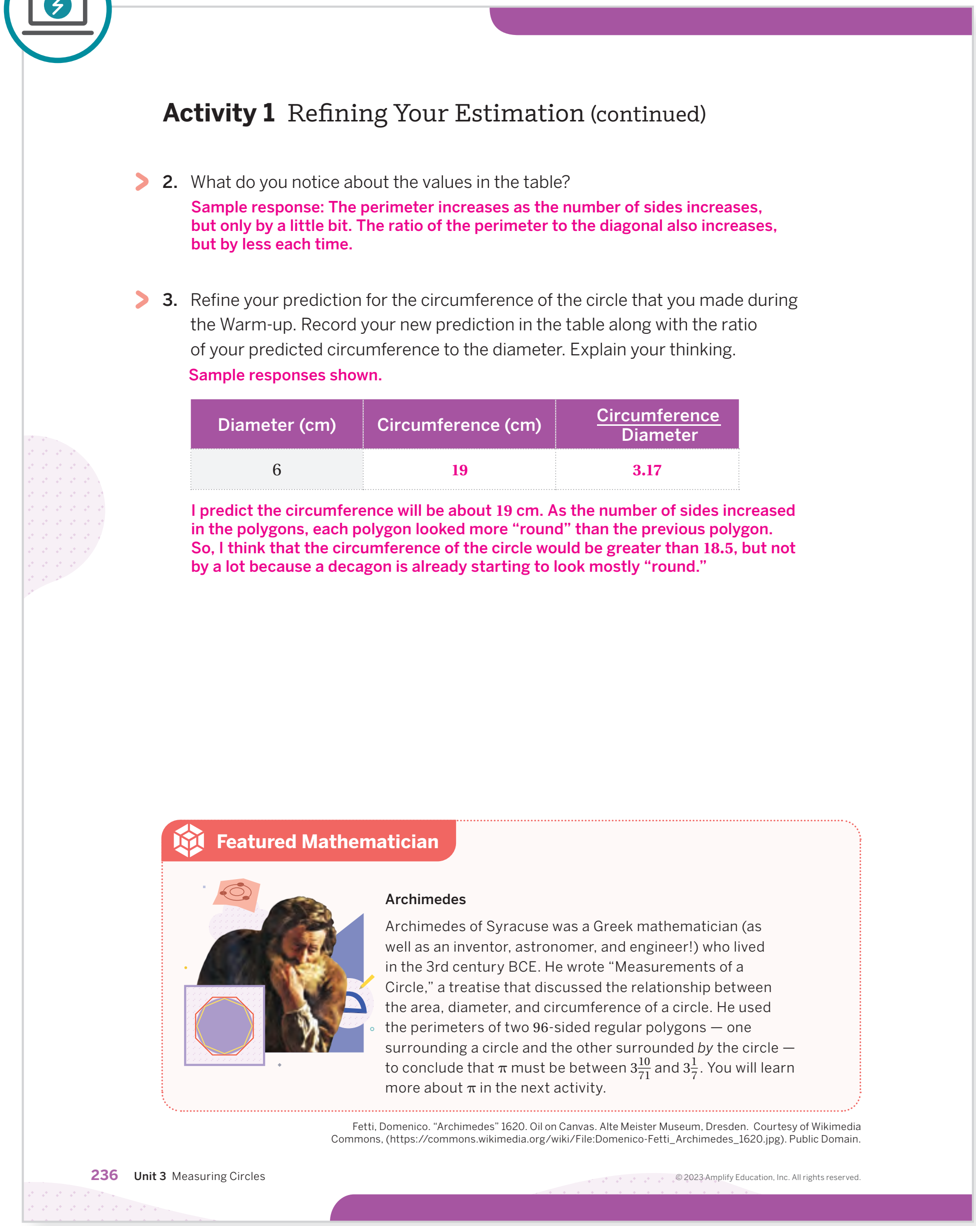
English Learners

Model for students an example of a question related to fairness based on the table. For example, "What is happening to the perimeter or the ratio $\frac{\text{perimeter}}{\text{diagonal}}$ as the number of sides of the polygon increases?"

Small Groups | 10 min

Activity 1 Refining Your Estimation (continued)

Students will apply their prior understanding of calculating the perimeter of regular polygons to refine their estimation of the circumference of a circle with a given diameter.



Activity 1 Refining Your Estimation (continued)

2. What do you notice about the values in the table?
Sample response: The perimeter increases as the number of sides increases, but only by a little bit. The ratio of the perimeter to the diagonal also increases, but by less each time.

3. Refine your prediction for the circumference of the circle that you made during the Warm-up. Record your new prediction in the table along with the ratio of your predicted circumference to the diameter. Explain your thinking.
Sample responses shown.

Diameter (cm)	Circumference (cm)	$\frac{\text{Circumference}}{\text{Diameter}}$
6	19	3.17

I predict the circumference will be about 19 cm. As the number of sides increased in the polygons, each polygon looked more "round" than the previous polygon. So, I think that the circumference of the circle would be greater than 18.5, but not by a lot because a decagon is already starting to look mostly "round."

Featured Mathematician

Archimedes

Archimedes of Syracuse was a Greek mathematician (as well as an inventor, astronomer, and engineer!) who lived in the 3rd century BCE. He wrote "Measurements of a Circle," a treatise that discussed the relationship between the area, diameter, and circumference of a circle. He used the perimeters of two 96-sided regular polygons — one surrounding a circle and the other surrounded *by* the circle — to conclude that π must be between $3\frac{10}{71}$ and $3\frac{1}{7}$. You will learn more about π in the next activity.

Fetti, Domenico. "Archimedes" 1620. Oil on Canvas. Alte Meister Museum, Dresden. Courtesy of Wikimedia Commons, (https://commons.wikimedia.org/wiki/File:Domenico-Fetti_Archimedes_1620.jpg). Public Domain.

236 **Unit 3** Measuring Circles

3 Connect

Display the Activity 1 PDF (answers).

Ask:

- "What do you notice about the values in the table?" Sample response: The perimeters are increasing and the ratios of the perimeter to the diagonal are also increasing.
- "Do you think the perimeter will continue to increase forever?" Sample response: No; the amount it is increasing by is a little less every time the number of sides increases.
- "How is determining the perimeter of these polygons related to estimating the circumference of your circle?" Sample response: The polygons look more and more like a circle so we can think of the diagonal like the diameter of a circle and use the perimeters to approximate the circumference.

Have groups of students share their new estimations for a circle with a diameter of 6 cm and explain how they determined their approximation.

Highlight that, while a circle is not a polygon, as the number of sides in a *regular polygon* increases, the shape of the polygon starts to look more like a circle. Historically, various civilizations used what they knew about polygons to help them to understand and measure circles.

Differentiated Support

Extension: *Math Enrichment*

Tell students that a 96-sided polygon is called a enneacontahexagon or enneacontakaihexagon. More generally, it is referred to as a 96-gon. Consider displaying an image of an enneacontahedron and ask students what they notice. Sample response: It looks like it is almost a circle.

Featured Mathematician

Archimedes

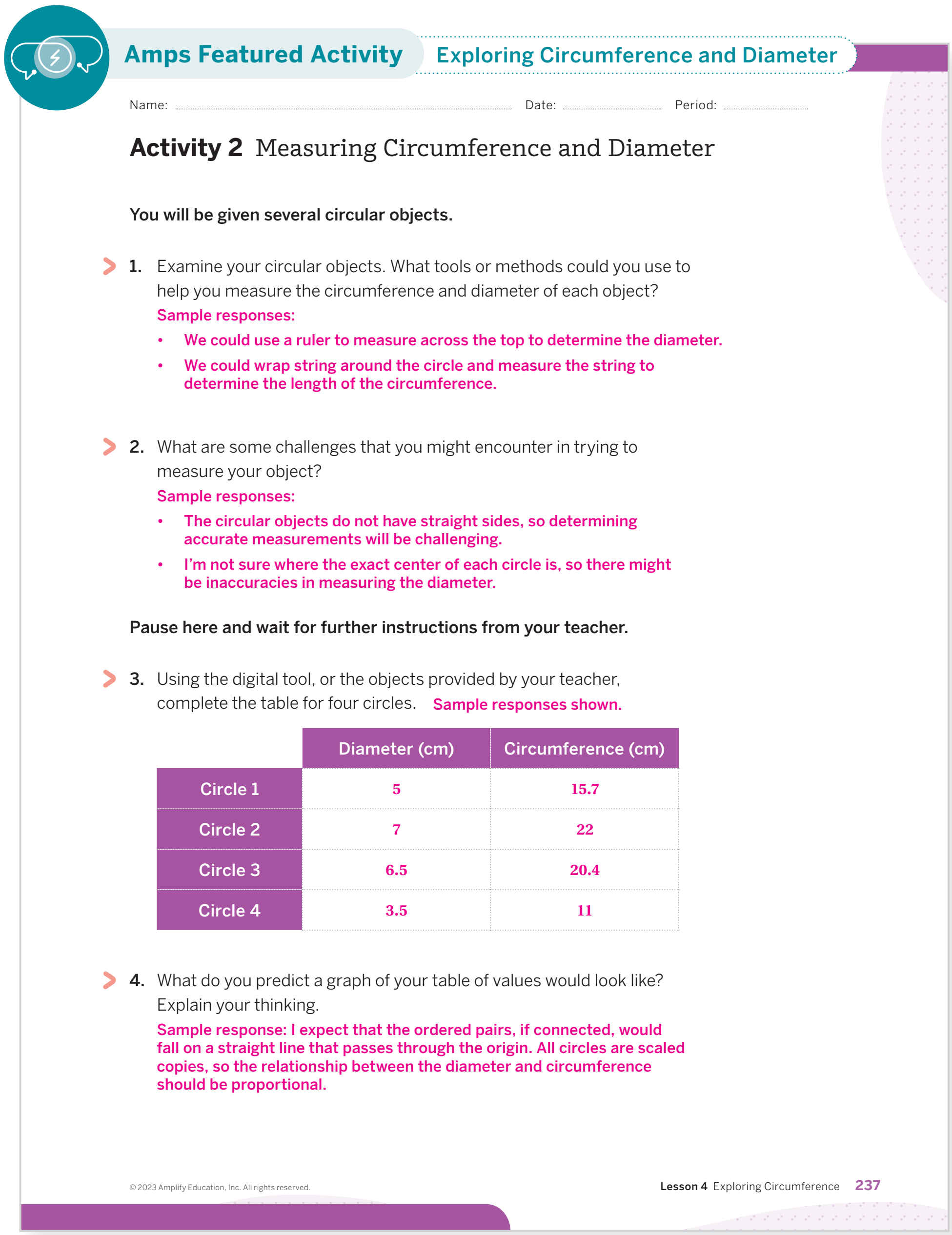
Have students read about featured mathematician, Archimedes of Syracuse. Archimedes was a Greek mathematician, inventor, astrologer, and engineer. He approximated the value of π by calculating the perimeters of two regular 96-agons; one inscribing a circle and one inscribed within the same circle.

Pairs | 20 min

Activity 2 Measuring Circumference and Diameter

Students will measure and graph the relationship between the circumference and diameter of circles to determine that they are in a proportional relationship.

Amps Featured Activity Exploring Circumference and Diameter



Name: ________ Date: ________ Period: ________

Activity 2 Measuring Circumference and Diameter

You will be given several circular objects.

1. Examine your circular objects. What tools or methods could you use to help you measure the circumference and diameter of each object?
   Sample responses:
   - We could use a ruler to measure across the top to determine the diameter.
   - We could wrap string around the circle and measure the string to determine the length of the circumference.

2. What are some challenges that you might encounter in trying to measure your object?
   Sample responses:
   - The circular objects do not have straight sides, so determining accurate measurements will be challenging.
   - I'm not sure where the exact center of each circle is, so there might be inaccuracies in measuring the diameter.

Pause here and wait for further instructions from your teacher.

3. Using the digital tool, or the objects provided by your teacher, complete the table for four circles. Sample responses shown.

	Diameter (cm)	Circumference (cm)
Circle 1	5	15.7
Circle 2	7	22
Circle 3	6.5	20.4
Circle 4	3.5	11

4. What do you predict a graph of your table of values would look like? Explain your thinking.
   Sample response: I expect that the ordered pairs, if connected, would fall on a straight line that passes through the origin. All circles are scaled copies, so the relationship between the diameter and circumference should be proportional.

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Lesson 4 Exploring Circumference 237

1 Launch

Distribute the cut-out circles from the Activity 2 PDF or circular objects to each pair of students. Conduct the ***Think-Pair-Share*** routine for Problems 1 and 2. After discussing their responses, explain that they will be using a digital tool instead of measuring real objects in order to minimize errors in measurement.

Note: You may choose to complete this activity using circular objects in lieu of the digital tool, but it will take additional time. Have each student measure one object and complete the table in small groups.

2 Monitor

Help students get started by modeling how to use the digital tool to approximate the circumference of a circle.

Look for points of confusion:

- **Thinking that the graph is not modeling a proportional relationship because it is not a perfectly straight line.** Remind students that, like the measuring squares activity in Lesson 3, there are going to be slight errors in measurement, even when using the applet.
- **Thinking that the verb *approximate* means to guess the constant of proportionality.** Explain to students that, mathematically, to *approximate* a value means to use a logical method for estimating that value, such as using the mean or median.

Look for productive strategies:

- Using the mean or median to approximate the constant of proportionality between the circumference and the diameter of their circles based on the ratios they calculated.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students do the actual measuring, provide the measurements of the circumference and diameter for the circular objects you choose to use for this activity. Display a table of the measurements and have students begin the activity with Problem 4.

Extension: *Math Enrichment*

Have students complete the following problem:
If a circle has a circumference of 63 units, what is a reasonable estimate for the circle's diameter? Sample response: About 20 units.

Math Language Development

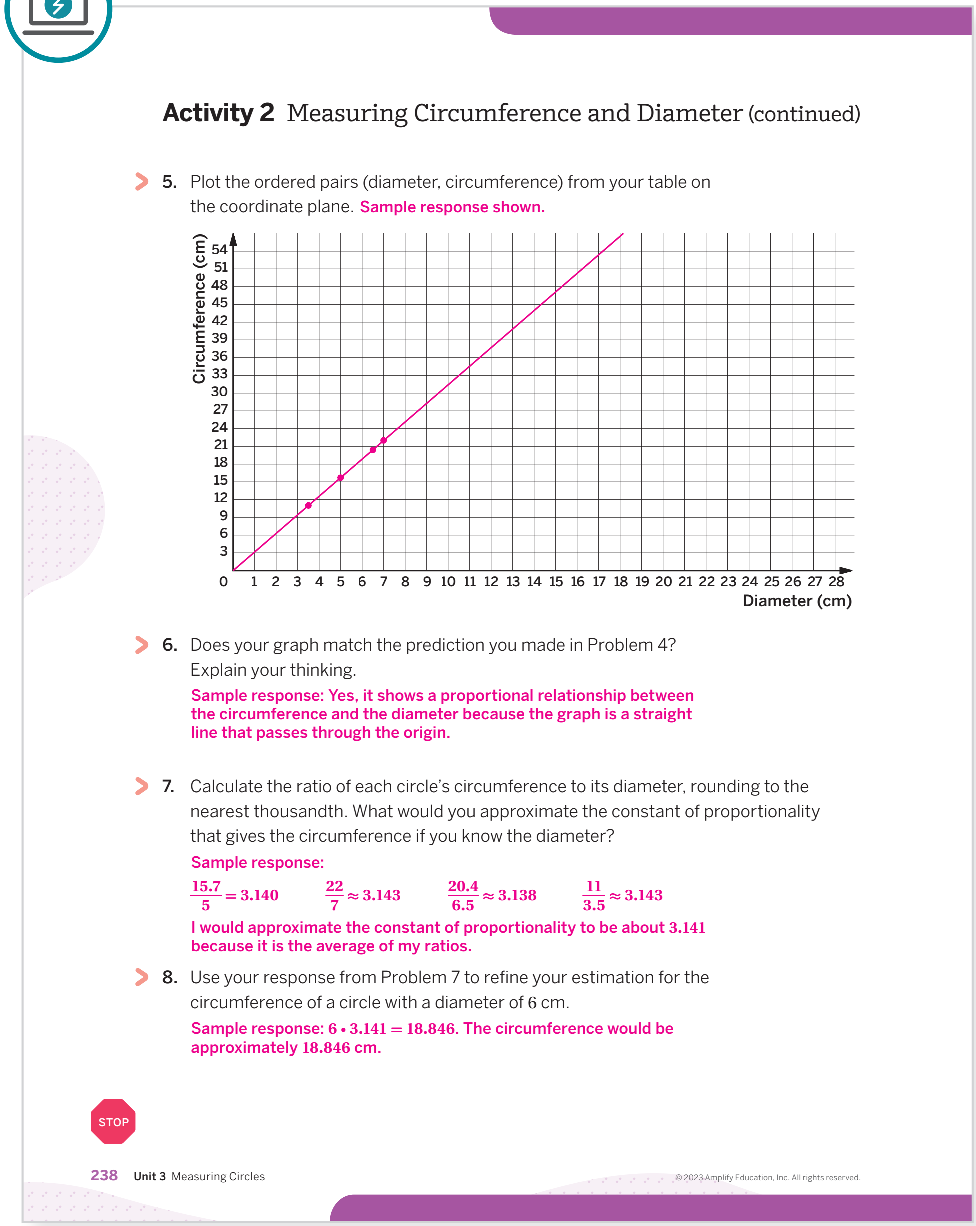
MLR7: Compare and Connect

After pairs of students have completed Problem 7, ask them to circulate among other pairs of students to compare their graphs and ratios. Ask students to look for commonalities among the graphs and ratios, and note any differences. Encourage them to use mathematical terms and phrases as they compare, such as *proportional*, *approximately equivalent ratios*, *straight line that passes through the origin*, and *constant of proportionality*.

Pairs | 20 min

Activity 2 Measuring Circumference and Diameter (continued)

Students will measure and graph the relationship between the circumference and diameter of circles to determine that they are in a proportional relationship.



Activity 2 Measuring Circumference and Diameter (continued)

5. Plot the ordered pairs (diameter, circumference) from your table on the coordinate plane. **Sample response shown.**

6. Does your graph match the prediction you made in Problem 4? Explain your thinking.

Sample response: Yes, it shows a proportional relationship between the circumference and the diameter because the graph is a straight line that passes through the origin.

7. Calculate the ratio of each circle's circumference to its diameter, rounding to the nearest thousandth. What would you approximate the constant of proportionality that gives the circumference if you know the diameter?

Sample response:

$\frac{15.7}{5} = 3.140 \qquad \frac{22}{7} \approx 3.143 \qquad \frac{20.4}{6.5} \approx 3.138 \qquad \frac{11}{3.5} \approx 3.143$

I would approximate the constant of proportionality to be about 3.141 because it is the average of my ratios.

8. Use your response from Problem 7 to refine your estimation for the circumference of a circle with a diameter of 6 cm.

Sample response: $6 \cdot 3.141 = 18.846$. The circumference would be approximately 18.846 cm.

238 **Unit 3** Measuring Circles

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3 Connect

Display the graph of the aggregated class data.

Have students share what they noticed or discovered about the relationship between the ***circumference*** and the diameter of a circle.

Define π (or ***pi***) as the ratio between the circumference and the diameter of a circle.

Ask:

- "If C represents the circumference and d represents the diameter, what equation represents the relationship between them?" $C = \pi d$
- "What is the approximate value of π from your data collection?" Values between 3.1 and 3.2
- "Does it make sense that the circumference and diameter would be in a proportional relationship? Why or why not?" Sample response: Yes, all circles are scaled figures so their lengths would be in proportion.
- "What is your final estimate for the circumference of the circle from the Warm-up?" Values between 18.6 and 19.2

Highlight that students only know an approximation of π that is slightly larger than 3. In the next lesson, they will continue to explore approximations of π both in historic times and in the current era.

Differentiated Support

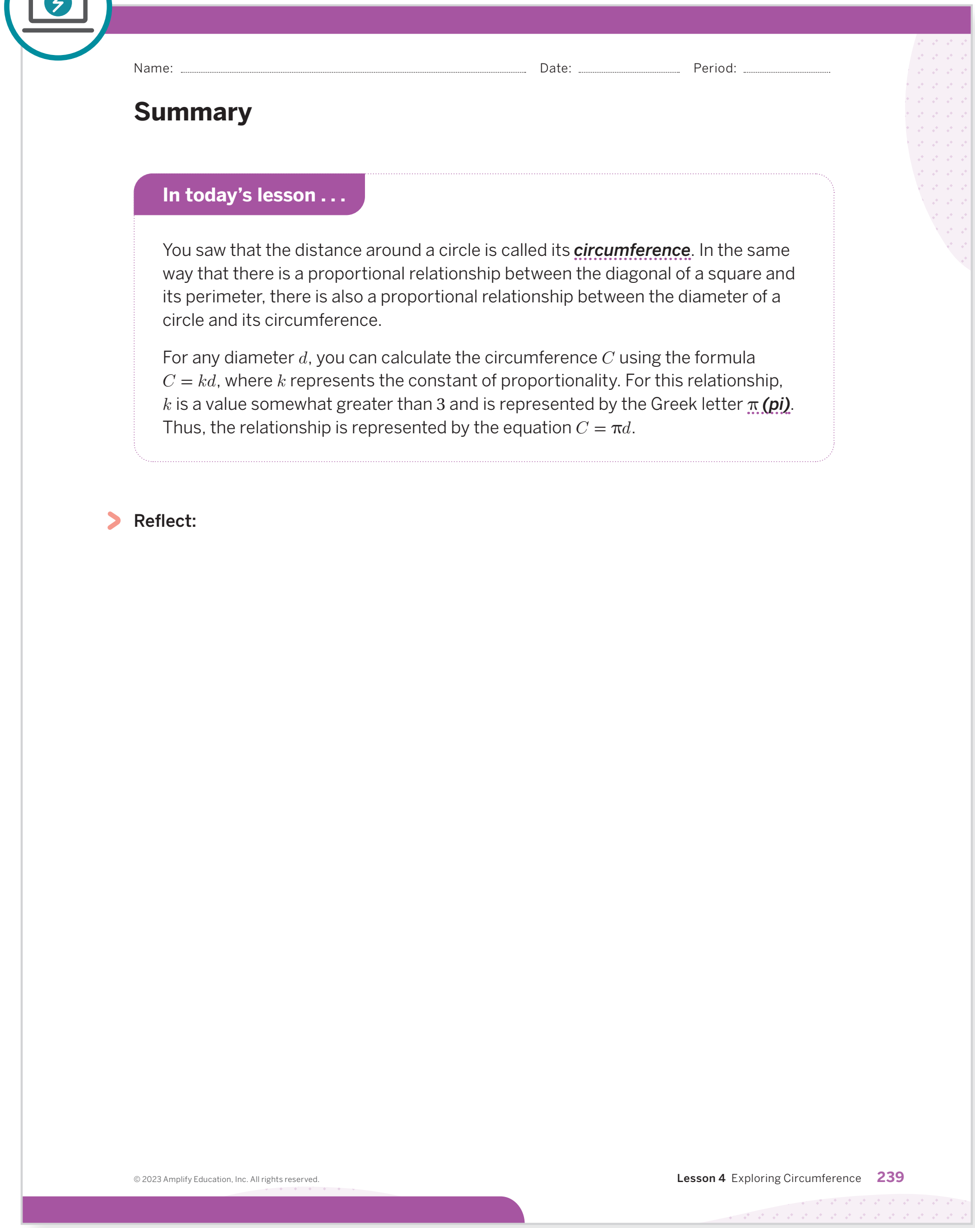
Extension: *Math Enrichment, Interdisciplinary Connections*

Ask students whether they are aware that, in 1897, the state of Indiana almost legally changed the mathematical definition of π! In 1882, German mathematician Ferdinand von Lindemann proved that pi was irrational. In doing so, he also proved that a classic Greek puzzle of squaring a circle – constructing a square with an area equal to that of a given circle – could *not* be done. Edward J. Goodwin, an Indiana physician who dabbled in math puzzles during his spare time, stated that you could square the circle, if you change the value of π to be 3.2. In 1897, Goodwin brought his new value of π to the Indiana state legislature and wanted the state to adopt the new value. Amazingly, the bill, now known as Indiana Bill. No. 246, passed in the Indiana House of Representatives unanimously. However, a mathematics professor at Purdue University, Clarence A. Waldo was present when the Senate was about to vote on the bill. Waldo was able to convince the senators that the value of π was a mathematical truth and its value could not be altered. In doing so, he helped prevent the bill from being passed. Ask students how this shows that mathematical understanding can be used in almost any discipline or career. **(History, Social Studies)**

Whole Class | 5 min

Summary

Review and synthesize that there is a proportional relationship between the circumference and diameter of a circle.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that the distance around a circle is called its ***circumference***. In the same way that there is a proportional relationship between the diagonal of a square and its perimeter, there is also a proportional relationship between the diameter of a circle and its circumference.

For any diameter d, you can calculate the circumference C using the formula $C = kd$, where k represents the constant of proportionality. For this relationship, k is a value somewhat greater than 3 and is represented by the Greek letter ***π (pi)***. Thus, the relationship is represented by the equation $C = \pi d$.

Reflect:

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Lesson 4 Exploring Circumference 239

Synthesize

Ask:

- "What is meant by *circumference*? How is it related to perimeter?" Sample response: It is the length around a circle. It measures the outside of a circle like perimeter measures the outside of a shape.
- "How is the circumference of a *circle* related to its *diameter*?" Sample response: The circumference is the diameter times π.
- "What is π? How is it related to *circumference* and *diameter*?" Sample response: It is the constant of proportionality when you go from diameter to circumference. It is a number slightly larger than 3.

Formalize vocabulary:

- ***circumference***
- ***π (pi)***

Highlight that the relationship between the *circumference* and the *diameter* of a circle is proportional and can be represented by the formula $C = \pi d$, where π is the *constant of proportionality* and is a value somewhat larger than 3.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on two of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do we measure circles when all of our tools are straight?"
- "What is π and what does it have to do with circles?"

Math Language Development

MLR2: Collect and Display

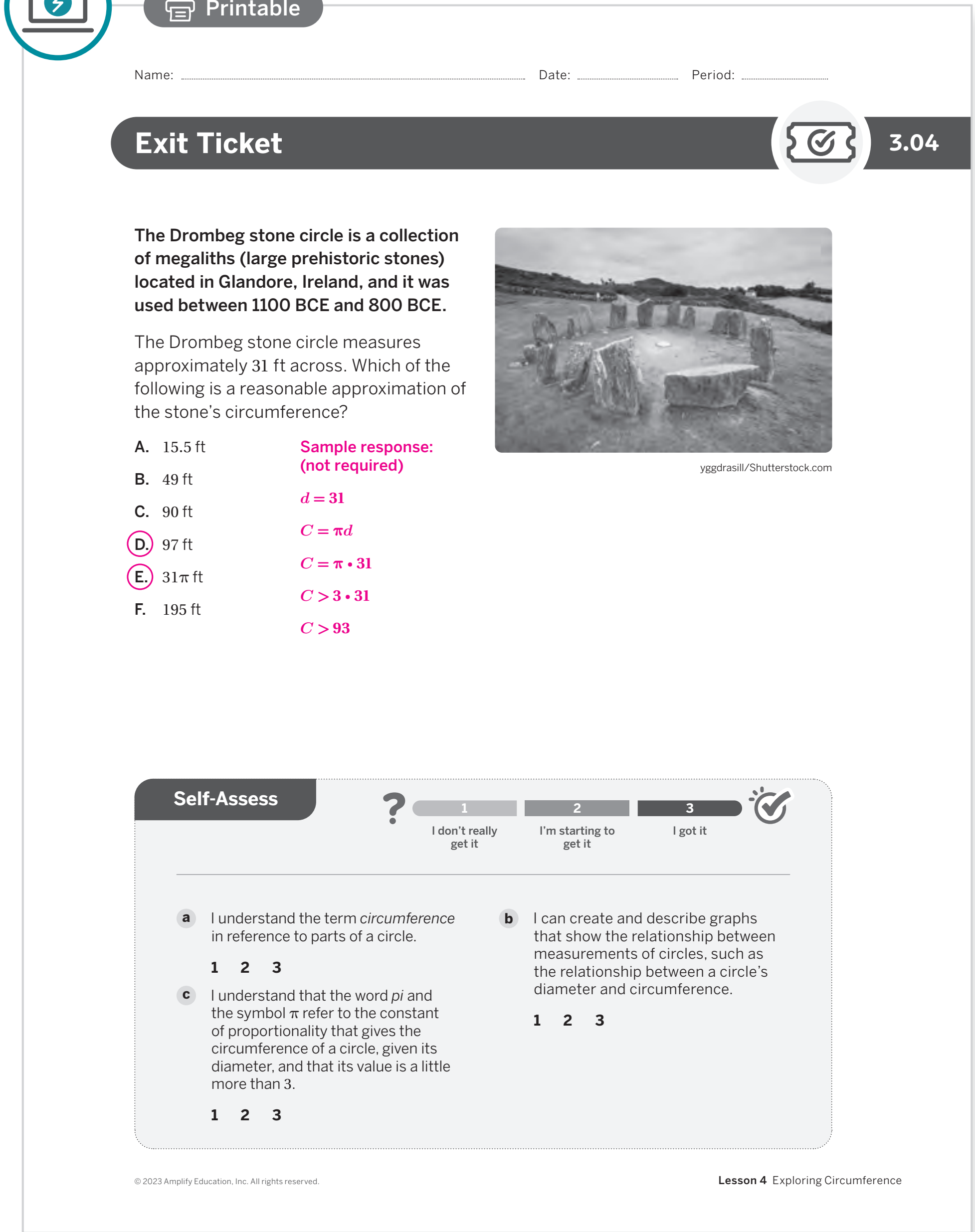
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *circumference* and π (*pi*) that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of $C = \pi d$ by determining reasonable approximations of a circumference for a given diameter.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.04

The Drombeg stone circle is a collection of megaliths (large prehistoric stones) located in Glandore, Ireland, and it was used between 1100 BCE and 800 BCE.

The Drombeg stone circle measures approximately 31 ft across. Which of the following is a reasonable approximation of the stone's circumference?

- A. 15.5 ft
- B. 49 ft
- C. 90 ft
- [✓ D.] 97 ft
- [✓ E.] 31π ft
- F. 195 ft

Sample response: (not required)

$d = 31$

$C = \pi d$

$C = \pi \cdot 31$

$C > 3 \cdot 31$

$C > 93$

yggdrasill/Shutterstock.com

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I understand the term *circumference* in reference to parts of a circle.

1 2 3

c. I understand that the word *pi* and the symbol π refer to the constant of proportionality that gives the circumference of a circle, given its diameter, and that its value is a little more than 3.

1 2 3

b. I can create and describe graphs that show the relationship between measurements of circles, such as the relationship between a circle's diameter and circumference.

1 2 3

Success looks like . . .

- **Language Goal:** Understanding the term *circumference* in reference to parts of a circle. **(Speaking and Listening)**
- **Language Goal:** Understanding the term *pi* and the symbol π refer to the constant of proportionality that gives the circumference of a circle, given its diameter, and generalizing that is a value somewhat larger than 3. **(Speaking and Listening)**
  - » Estimating the circumference of the Dromberg stone circle.
- **Language Goal:** Creating and describing graphs that show measurements of circles. **(Writing)**

Suggested next steps

If students answer A, B, or F, consider:

- Reviewing the definition of radius, diameter, and circumference as well as how they are related.
- Assigning Practice Problems 1 and 3.

If students answer C, consider:

- Reviewing the approximation of π. Students should multiply by a value slightly larger than 3, not slightly less than 3.
- Assigning Practice Problems 1 and 2.

If students answer D but not E, consider:

- Making a note to highlight this when students learn how to express their answer in terms of π in Lesson 6.

Professional Learning

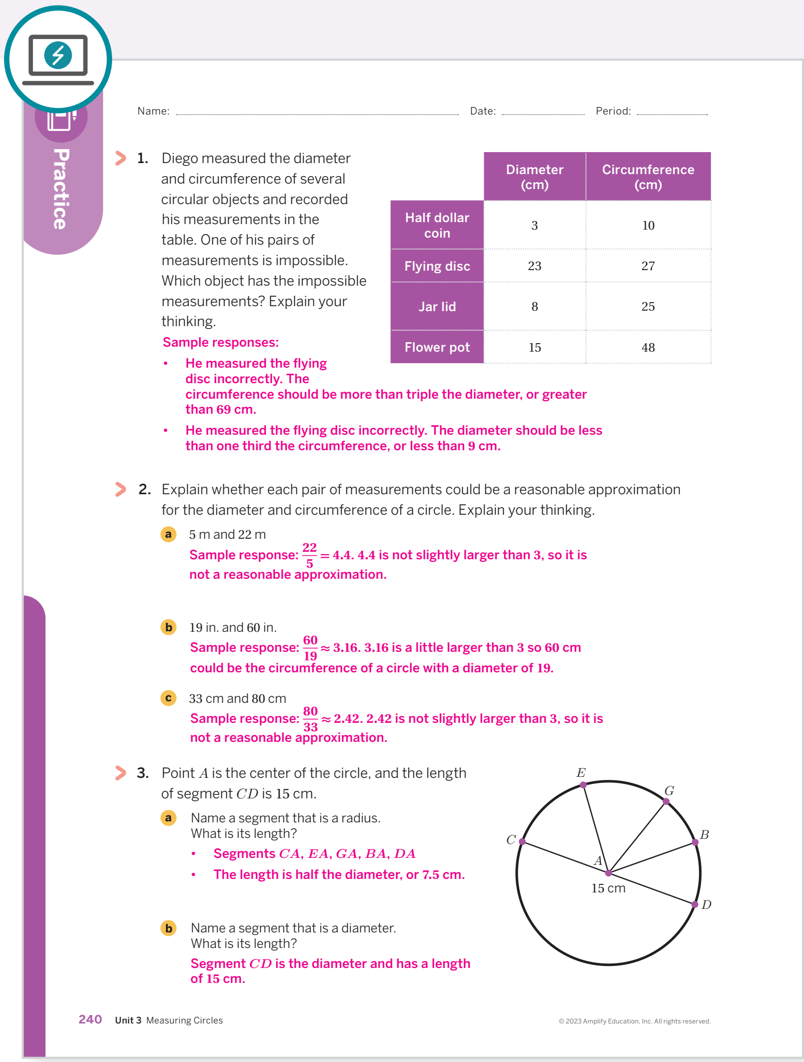
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about using polygons to estimate the circumference of a circle how did you encourage each student to share their understandings?
- How was Activity 2 similar to or different from the work students did with squares in Lesson 3? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Diego measured the diameter and circumference of several circular objects and recorded his measurements in the table. One of his pairs of measurements is impossible. Which object has the impossible measurements? Explain your thinking.

	Diameter (cm)	Circumference (cm)
Half dollar coin	3	10
Flying disc	23	27
Jar lid	8	25
Flower pot	15	48

Sample responses:
- **He measured the flying disc incorrectly. The circumference should be more than triple the diameter, or greater than 69 cm.**
- **He measured the flying disc incorrectly. The diameter should be less than one third the circumference, or less than 9 cm.**

2. Explain whether each pair of measurements could be a reasonable approximation for the diameter and circumference of a circle. Explain your thinking.

   a. 5 m and 22 m
   Sample response: $\frac{22}{5} = 4.4$. 4.4 is not slightly larger than 3, so it is not a reasonable approximation.

   b. 19 in. and 60 in.
   Sample response: $\frac{60}{19} \approx 3.16$. 3.16 is a little larger than 3 so 60 cm could be the circumference of a circle with a diameter of 19.

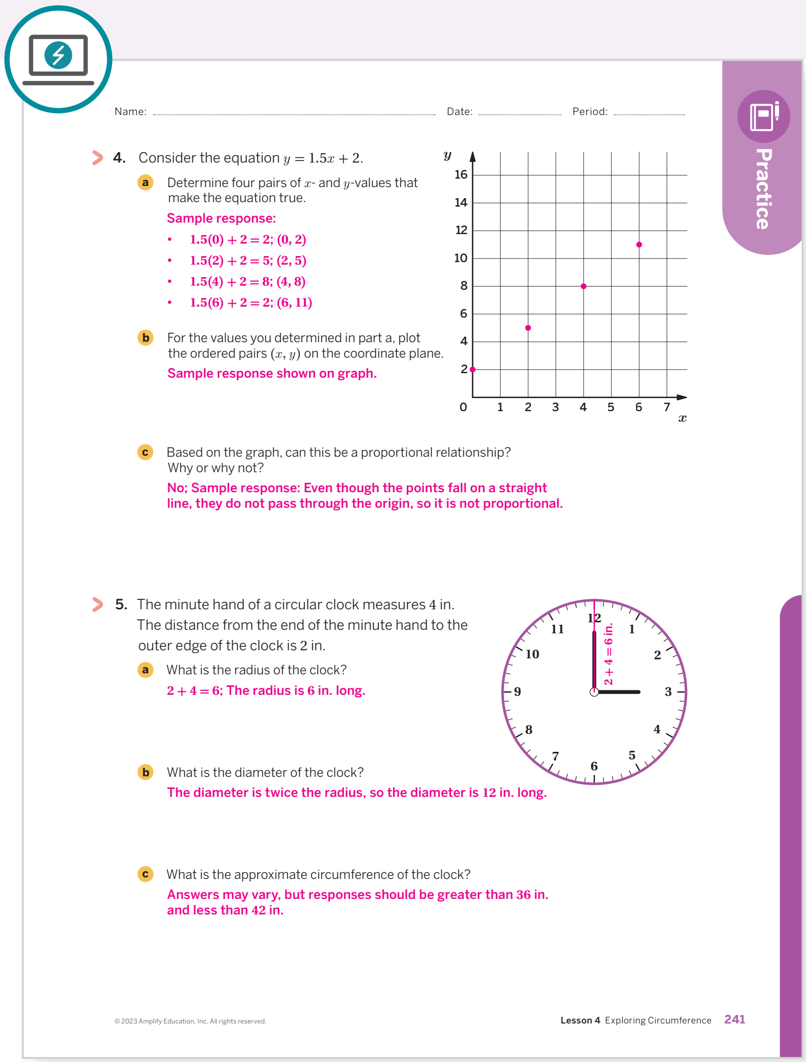
   c. 33 cm and 80 cm
   Sample response: $\frac{80}{33} \approx 2.42$. 2.42 is not slightly larger than 3, so it is not a reasonable approximation.

3. Point A is the center of the circle, and the length of segment CD is 15 cm.

   a. Name a segment that is a radius. What is its length?
   - **Segments CA, EA, GA, BA, DA**
   - **The length is half the diameter, or 7.5 cm.**

   b. Name a segment that is a diameter. What is its length?
   Segment CD is the diameter and has a length of 15 cm.

240 **Unit 3** Measuring Circles



Name: Date: Period:

4. Consider the equation $y = 1.5x + 2$.

   a. Determine four pairs of x- and y-values that make the equation true.
   Sample response:
   - $1.5(0) + 2 = 2$; $(0, 2)$
   - $1.5(2) + 2 = 5$; $(2, 5)$
   - $1.5(4) + 2 = 8$; $(4, 8)$
   - $1.5(6) + 2 = 2$; $(6, 11)$

   b. For the values you determined in part a, plot the ordered pairs (x, y) on the coordinate plane.
   Sample response shown on graph.

   c. Based on the graph, can this be a proportional relationship? Why or why not?
   No; Sample response: Even though the points fall on a straight line, they do not pass through the origin, so it is not proportional.

5. The minute hand of a circular clock measures 4 in. The distance from the end of the minute hand to the outer edge of the clock is 2 in.

   a. What is the radius of the clock?
   $2 + 4 = 6$; The radius is 6 in. long.

   b. What is the diameter of the clock?
   The diameter is twice the radius, so the diameter is 12 in. long.

   c. What is the approximate circumference of the clock?
   Answers may vary, but responses should be greater than 36 in. and less than 42 in.

Lesson 4 Exploring Circumference 241

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	**1**	Activity 2	2
	2	Activity 2	2
Spiral	**3**	Unit 3 Lesson 2	2
	4	Unit 2 Lesson 11	2
Formative	**5**	Unit 3 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

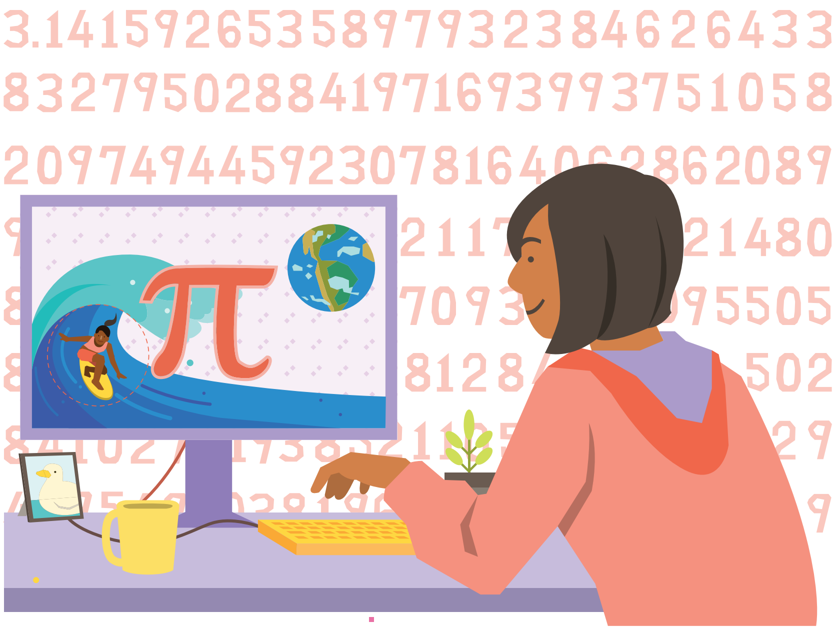
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3 | **LESSON 5**

Understanding π

Let's explore different approximations of π.



Focus

Goals

1. **Language Goal:** Compare and contrast values for the same measurement that were calculated using different approximations of π. **(Speaking and Listening)**
2. Approximate the circumference of a circle when given the length of either the radius or the diameter.

Coherence

Today

Students become flexible in determining the circumference of a circle when given either the diameter or radius, constructing the formula $C = 2\pi r$. They explore different historical approximations of π and determine the level of precision necessary for approximating the circumference in real-world contexts.

Previously

In Lesson 2, students defined radius and diameter and built their understanding of the relationship between them. In Lesson 4, students defined circumference and developed the formula $C = \pi d$, approximating π as a value somewhat greater than 3.

Coming Soon

In Lesson 6, students will build fluency in converting between the radius, diameter, and circumference of a circle.

Rigor

- Students will build **conceptual understanding** of π by discussing historical approximations.
- Students will gain **fluency** in calculating the circumference of circles using different approximations for π.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	15 min	12 min	5 min	7 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos ⁝ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per group
- Activity 1 PDF, (answers, for display)
- Activity 1 PDF, *Digits of* π (for display)
- Anchor Chart PDF, *Circles*
- Graphic Organizer PDF, *Working With Circles (Part 1)* (as needed)
- calculators

Math Language Development

Review words

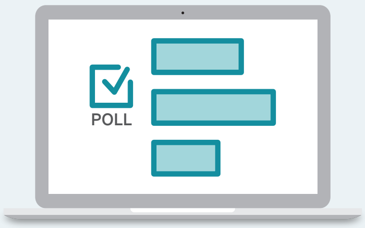
- *circle*
- *circumference*
- *diameter*
- π
- *radius*

Amps ⁝ Featured Activity

Warm-up and Activity 2

Poll the Class

In both the Warm-up and Activity 2, conduct the ***Poll the Class*** routine to quickly assess students' thinking about approximating with π.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might think that there is only one approximation for pi that can be used when finding the circumference of a circle. Before students share their reasoning, discuss ways to show active listening with the goal of personally making some general rules for when different estimates are most appropriate.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time:

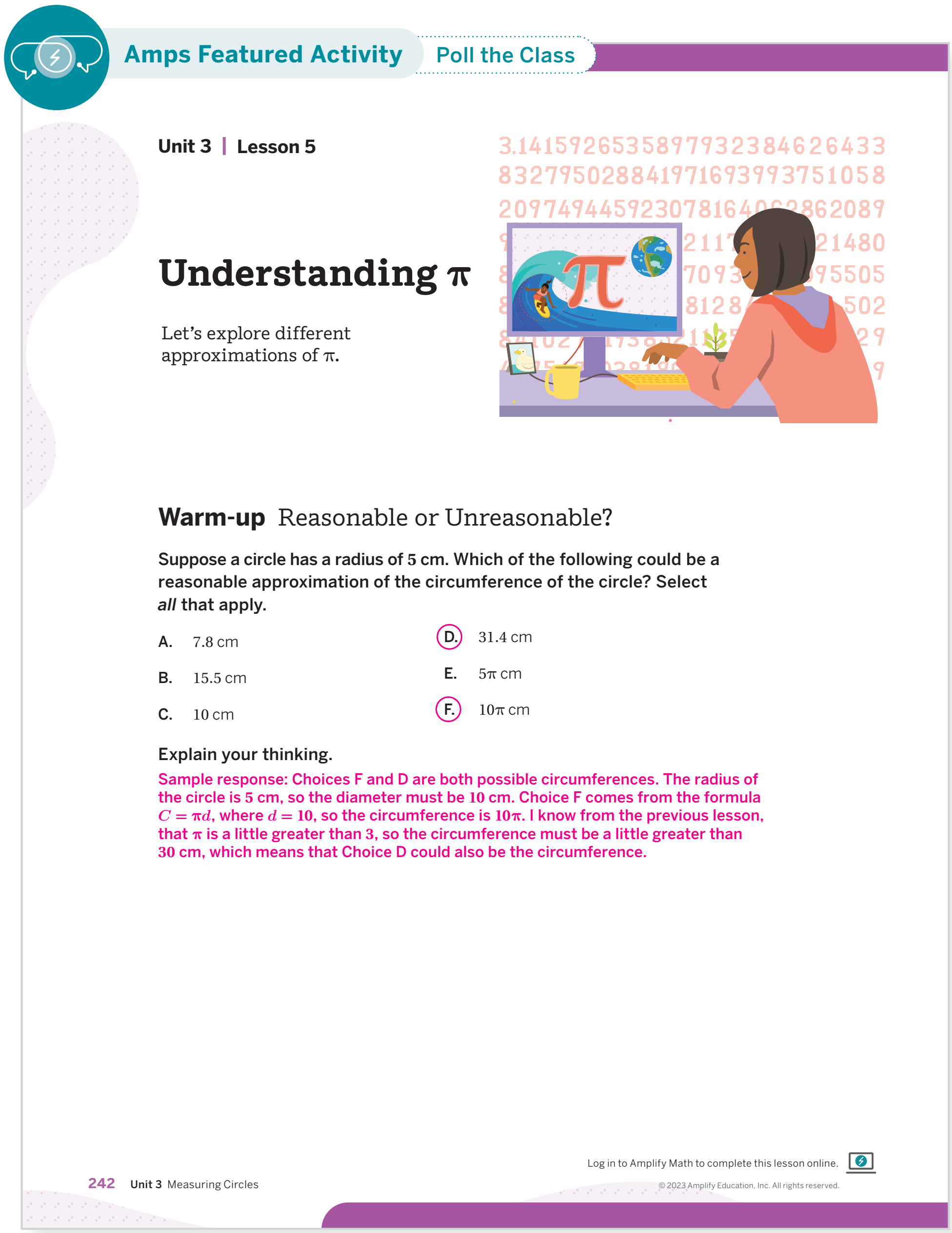
- In **Activity 1,** have students only complete the first and third rows of the table.
- In **Activity 2,** have students choose one problem to complete.

Pairs | 8 min

Warm-up Reasonable or Unreasonable?

Students will apply their understanding of π as a value slightly larger than 3 to determine reasonable approximations of the circumference of a circle.

Amps Featured Activity Poll the Class



Unit 3 | Lesson 5

Understanding π

Let's explore different approximations of π.

Warm-up Reasonable or Unreasonable?

Suppose a circle has a radius of 5 cm. Which of the following could be a reasonable approximation of the circumference of the circle? Select *all* that apply.

A. 7.8 cm	[✓ D.] 31.4 cm
B. 15.5 cm	E. 5π cm
C. 10 cm	[✓ F.] 10π cm

Explain your thinking.

Sample response: Choices F and D are both possible circumferences. The radius of the circle is 5 cm, so the diameter must be 10 cm. Choice F comes from the formula $C = \pi d$, where $d = 10$, so the circumference is 10π. I know from the previous lesson, that π is a little greater than 3, so the circumference must be a little greater than 30 cm, which means that Choice D could also be the circumference.

Log in to Amplify Math to complete this lesson online.

242 **Unit 3** Measuring Circles

1 Launch

Distribute calculators to all students for use throughout the class period. Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by asking, "What information do you know about the circle? What information are you looking for?"

Look for points of confusion:

- **Substituting the radius in the formula $C = \pi d$ for the diameter.** Ask students, "What is the relationship between the radius and diameter?"
- **Finding only one correct answer.** Remind students that they should select *all* the approximations that are reasonable and encourage them to check the remaining options.

Look for productive strategies:

- Determining the diameter and substituting that value into the formula $C = \pi d$.

3 Connect

Display the Warm-up. Conduct the *Poll the Class* routine.

Have pairs of students share how they determined reasonable values.

Highlight that when the radius is known, the first step in determining the *circumference* is converting the *radius* to the *diameter*.

Ask:

- "How could you rewrite the formula for circumference using the radius r in place of the diameter d?" $C = \pi(2r)$ or $C = 2\pi r$
- "There were two reasonable solutions, D and F. Which is more accurate?" F; D uses an approximation of π where F has the π symbol.

Math Language Development

MLR8: Discussion Supports

Provide students with a copy of the Anchor Chart PDF, *Circles* that displays a visual example for the term *circumference*. **Note:** Have them only refer to the circumference part of the anchor chart at this point in the unit. Ask them to complete the equations that relate the circumference, diameter, and radius. Answers are provided on the Anchor Chart PDF, *Circles* (answers).

Power-up

To power up students' ability to calculate the diameter and the circumference of a circle given its radius, have students complete:

Recall the two formulas from the previous lessons: $d = 2r$ and $C = \pi d$. Use these relationships to complete the missing values in the table.

Radius	Diameter	Circumference
1	2	6.28
5	10	31.4

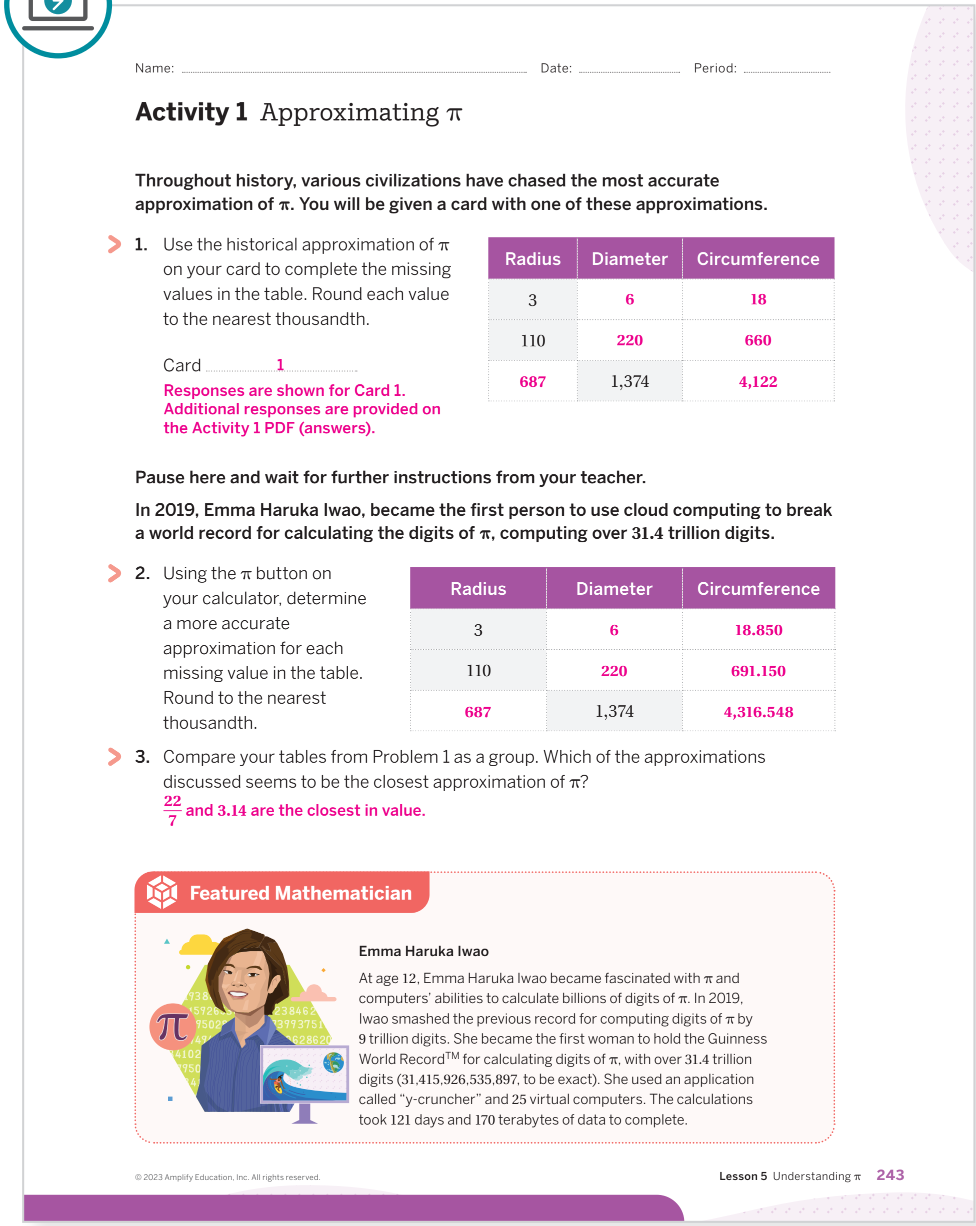
Use: Before the Warm-up.

Informed by: Performance on Lesson 4, Practice Problem 6.

Small Groups | 15 min

Activity 1 Approximating π

Students will explore historic approximations of π and compare and contrast their computations.



Name: ______ Date: ______ Period: ______

Activity 1 Approximating π

Throughout history, various civilizations have chased the most accurate approximation of π. You will be given a card with one of these approximations.

1. Use the historical approximation of π on your card to complete the missing values in the table. Round each value to the nearest thousandth.

Card 1

Responses are shown for Card 1. Additional responses are provided on the Activity 1 PDF (answers).

Radius	Diameter	Circumference
3	**6**	**18**
110	**220**	**660**
687	1,374	**4,122**

Pause here and wait for further instructions from your teacher.

In 2019, Emma Haruka Iwao, became the first person to use cloud computing to break a world record for calculating the digits of π, computing over 31.4 trillion digits.

2. Using the π button on your calculator, determine a more accurate approximation for each missing value in the table. Round to the nearest thousandth.

Radius	Diameter	Circumference
3	**6**	**18.850**
110	**220**	**691.150**
687	1,374	**4,316.548**

3. Compare your tables from Problem 1 as a group. Which of the approximations discussed seems to be the closest approximation of π?

$\frac{22}{7}$ and 3.14 are the closest in value.

Featured Mathematician

Emma Haruka Iwao

At age 12, Emma Haruka Iwao became fascinated with π and computers' abilities to calculate billions of digits of π. In 2019, Iwao smashed the previous record for computing digits of π by 9 trillion digits. She became the first woman to hold the Guinness World Record^{TM} for calculating digits of π, with over 31.4 trillion digits (31,415,926,535,897, to be exact). She used an application called "y-cruncher" and 25 virtual computers. The calculations took 121 days and 170 terabytes of data to complete.

Lesson 5 Understanding π 243

1 Launch

Distribute calculators and a set of cards from the Activity 1 PDF to each group, so that each student has a different card. Inform students to read their card and complete Problem 1 independently and, then Problems 2 and 3 as a group.

2 Monitor

Help students get started by asking, "What information do you know about the circle? What information are you looking for? What formula do you know that relates both pieces of information?"

Look for points of confusion:

- **Confusing the radius and the diameter.** Asking students to explain the relationship between the radius, diameter, and circumference. Clarify any confusion they may have.

3 Connect

Display the Activity 1 PDF, *Digits of π*. Explain that this is an estimation of π of only 200 digits. In order to print the estimation from Emma Haruka Iwao, you would need 15-billion sheets of paper! Since π is a decimal value that is never ending, you use estimations. Model for students how to use the π button on their calculator prior to completing Problems 2 and 3 with their groups.

Have groups of students share which approximation of π best matched the values from Problem 2. Display the Activity 1 PDF (answers) during this discussion.

Highlight that the most common approximations of π are 3, 3.14, and $\frac{22}{7}$.

Ask, "Can you think of a scenario when each approximation would be the 'best' choice?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of each student receiving a different card, distribute 1–2 cards to each group so that all of the cards are represented among the groups in the class. Have group members complete Problem 1 together for their assigned card(s).

Extension: *Math Enrichment*

Tell students that the Guiness World Record holder for the most number of decimal places of pi memorized is Rajveer Meena from India. She recalled 70,000 digits of pi accurately on March 21, 2015. She wore a blindfold while she recalled the digits, which took nearly 10 hours!

Featured Mathematician

Emma Haruka Iwao

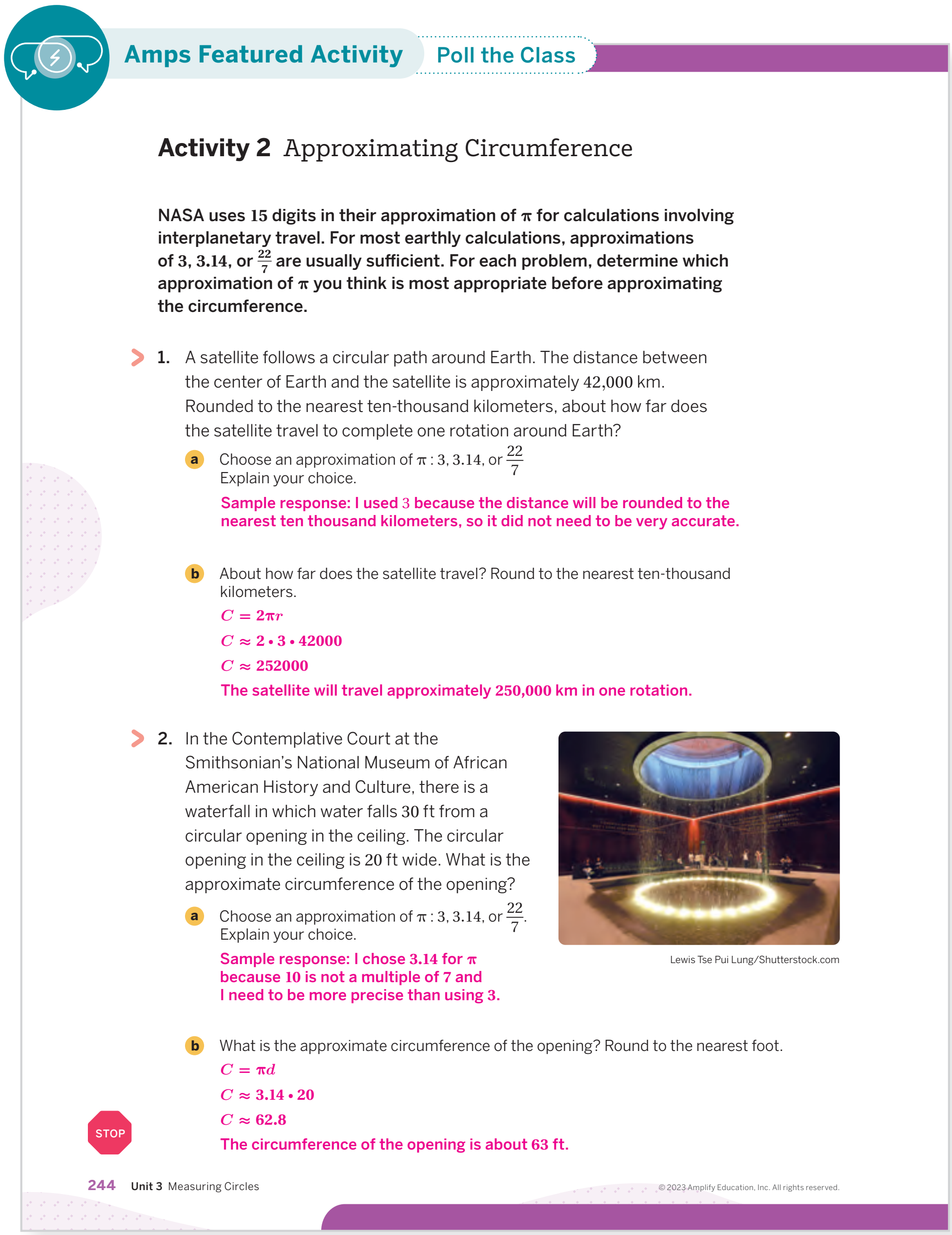
Have students read about featured mathematician, Emma Haruka Iwao. Emma is a Japanese computer scientist, who, in 2019, became the first female to hold the Guinness World Record in computing the digits of π.

Pairs | 12 min

Activity 2 Approximating Circumference

Students will choose the most appropriate approximation of π to solve real-world problems involving circumference, diameter, and radius.

Amps Featured Activity Poll the Class



Activity 2 Approximating Circumference

NASA uses 15 digits in their approximation of π for calculations involving interplanetary travel. For most earthly calculations, approximations of 3, 3.14, or $\frac{22}{7}$ are usually sufficient. For each problem, determine which approximation of π you think is most appropriate before approximating the circumference.

1. A satellite follows a circular path around Earth. The distance between the center of Earth and the satellite is approximately 42,000 km. Rounded to the nearest ten-thousand kilometers, about how far does the satellite travel to complete one rotation around Earth?

   a. Choose an approximation of π : 3, 3.14, or $\frac{22}{7}$. Explain your choice.

   Sample response: I used 3 because the distance will be rounded to the nearest ten thousand kilometers, so it did not need to be very accurate.

   b. About how far does the satellite travel? Round to the nearest ten-thousand kilometers.

   $C = 2\pi r$

   $C \approx 2 \cdot 3 \cdot 42000$

   $C \approx 252000$

   The satellite will travel approximately 250,000 km in one rotation.

2. In the Contemplative Court at the Smithsonian's National Museum of African American History and Culture, there is a waterfall in which water falls 30 ft from a circular opening in the ceiling. The circular opening in the ceiling is 20 ft wide. What is the approximate circumference of the opening?

   Lewis Tse Pui Lung/Shutterstock.com

   a. Choose an approximation of π : 3, 3.14, or $\frac{22}{7}$. Explain your choice.

   Sample response: I chose 3.14 for π because 10 is not a multiple of 7 and I need to be more precise than using 3.

   b. What is the approximate circumference of the opening? Round to the nearest foot.

   $C = \pi d$

   $C \approx 3.14 \cdot 20$

   $C \approx 62.8$

   The circumference of the opening is about 63 ft.

STOP

244 **Unit 3** Measuring Circles

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1 Launch

Activate students' prior knowledge of the three approximations of π discussed in Activity 1: 3, 3.14, and $\frac{22}{7}$. They will need to determine which approximation is the most appropriate for each problem and explain their thinking.

2 Monitor

Help students get started by asking, "Which approximation is the most accurate? Which is the easiest to use?"

Look for points of confusion:

- **Thinking that only one approximation of π is the "correct" choice.** Clarify that students can choose any approximation, but they must be able to justify *why* they chose that approximation.

Look for productive strategies:

- Noticing that the radius in Problem 1 is a multiple of 7, while the diameter in Problem 2 is not; therefore choosing to use $\frac{22}{7}$ for Problem 1, but not for Problem 2.

3 Connect

Display Activity 2 from the Student Edition. Conduct the ***Poll the Class*** routine to determine which approximations were used by pairs of students for each problem.

Have pairs of students share their reasoning for choosing each approximation and their final response to each question.

Ask:

- "What factors should you consider when determining which approximation of π to use?"
- "How were different students' responses related?"
- "Are all the responses reasonable although they are different?"

Highlight that because students are approximating π, their solutions may be close to equal, but not exactly equal.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Graphic Organizer PDF, *Working WIth Circles (Part 1)* to help them make sense of each problem.

Extension: *Math Enrichment, Math Around the World*

Remind students they previously learned about Archimedes' method for approximating π. Chinese mathematician Zu Chongzhi (429–401 BCE) used a similar approach. Chongzhi, who would not have had the opportunity to learn of Archimedes' work, approximated the value of π as $\frac{355}{113}$. To do so, he would have had to begin with an inscribed regular 25,576-gon and performed hundreds of complicated calculations.

Math Language Development

MLR6: Three Reads

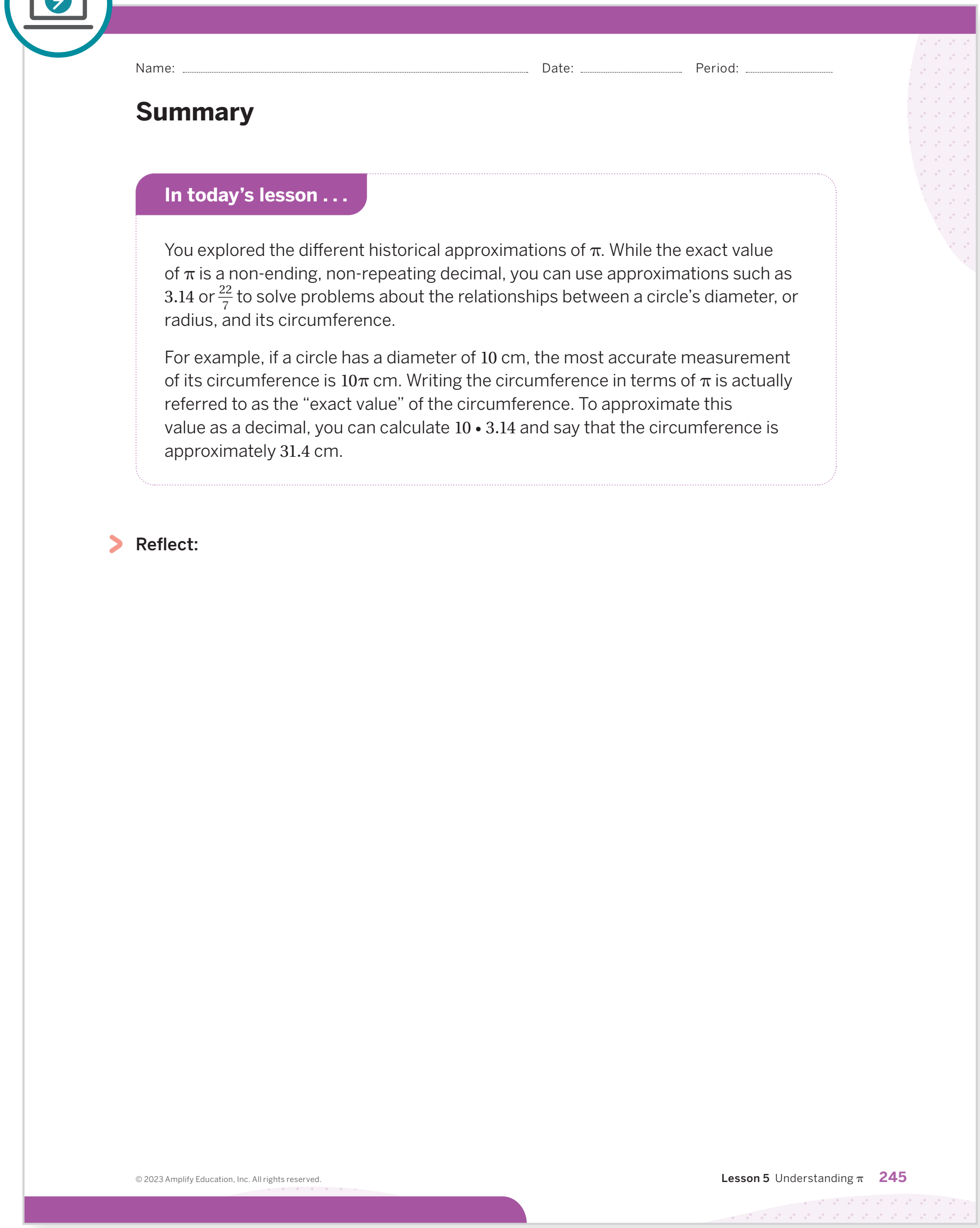
Use this routine to help students make sense of the introductory text in each problem. Use this example for Problem 1.

- **Read 1:** Students should understand that a satellite is traveling in a circular path around Earth.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the distance between the center of Earth and the satellite is approximately 42,000 km.
- **Read 3:** Ask students to plan their solution strategy and which approximation of π they will use to solve the problem.

Whole Class | 5 min

Summary

Review and synthesize that different approximations of π can be substituted into the formulas $C = \pi d$ and $C = 2\pi r$ to approximate the circumference of circles.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored the different historical approximations of π. While the exact value of π is a non-ending, non-repeating decimal, you can use approximations such as 3.14 or $\frac{22}{7}$ to solve problems about the relationships between a circle's diameter, or radius, and its circumference.

For example, if a circle has a diameter of 10 cm, the most accurate measurement of its circumference is 10π cm. Writing the circumference in terms of π is actually referred to as the "exact value" of the circumference. To approximate this value as a decimal, you can calculate $10 \cdot 3.14$ and say that the circumference is approximately 31.4 cm.

Reflect:

Synthesize

Display the Anchor Chart PDF, *Circles*.

Have students share one fact that they learned about π during today's lesson.

Highlight that π is a never-ending decimal, without a fractional equivalent, so the symbol π or approximations are used for calculations. In general, the three most common approximations are 3, 3.14, and $\frac{22}{7}$. Complete the remaining sections of the Anchor Chart as a class, except the formulas for area.

Ask:

- "If you know the diameter of a circle, how can you approximate the circumference?" Multiply the diameter by 3.14.
- "What if you only know the radius of a circle?" Multiply the radius by $\frac{22}{7}$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking.

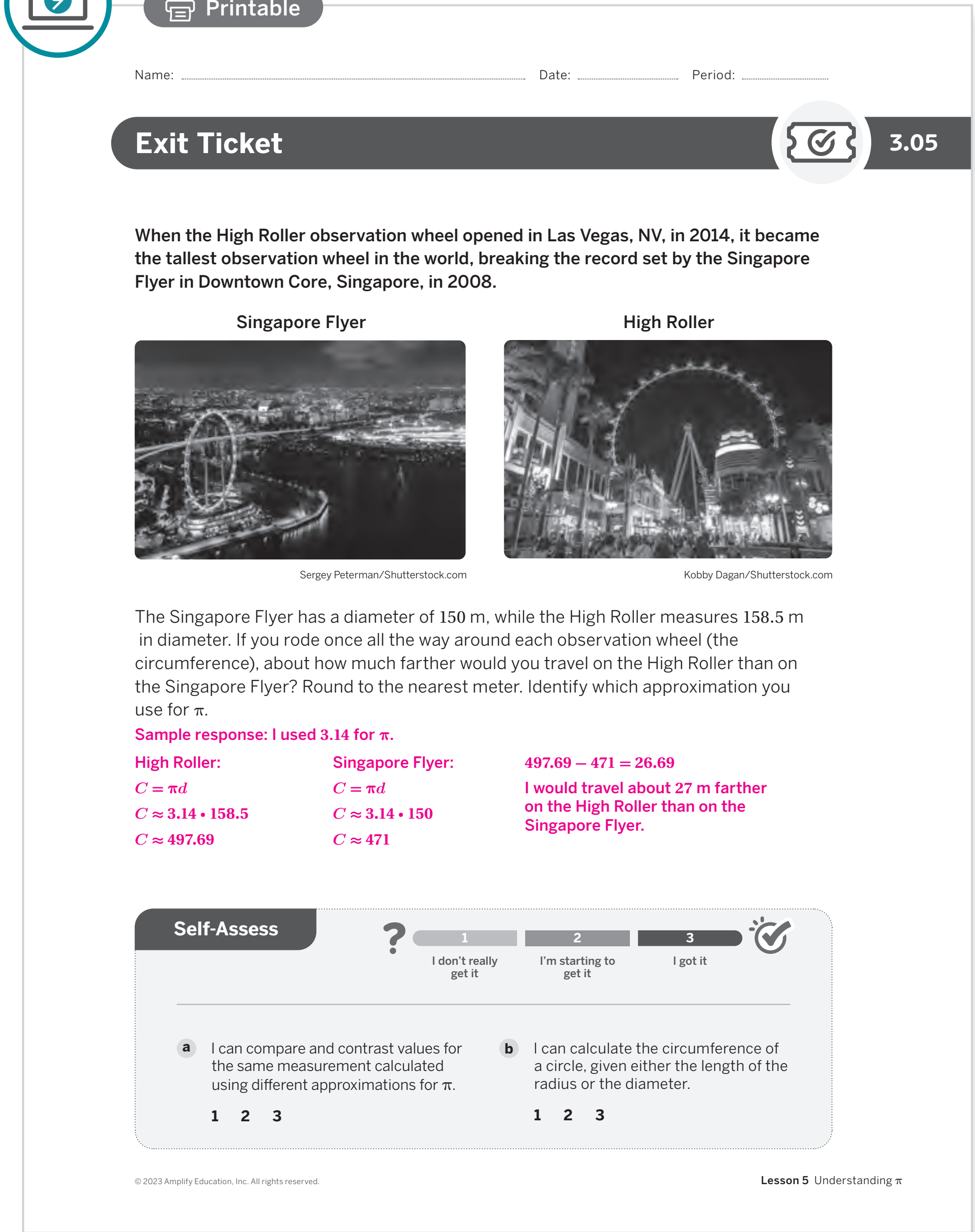
- "What is π and what does it have to do with circles?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding of approximations of π and circumference by approximating the difference between two circumferences when given the diameters.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 3.05

When the High Roller observation wheel opened in Las Vegas, NV, in 2014, it became the tallest observation wheel in the world, breaking the record set by the Singapore Flyer in Downtown Core, Singapore, in 2008.

Singapore Flyer

Sergey Peterman/Shutterstock.com

High Roller

Kobby Dagan/Shutterstock.com

The Singapore Flyer has a diameter of 150 m, while the High Roller measures 158.5 m in diameter. If you rode once all the way around each observation wheel (the circumference), about how much farther would you travel on the High Roller than on the Singapore Flyer? Round to the nearest meter. Identify which approximation you use for π.

Sample response: I used 3.14 for π.

High Roller:
$C = \pi d$
$C \approx 3.14 \cdot 158.5$
$C \approx 497.69$

Singapore Flyer:
$C = \pi d$
$C \approx 3.14 \cdot 150$
$C \approx 471$

$497.69 - 471 = 26.69$

I would travel about 27 m farther on the High Roller than on the Singapore Flyer.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can compare and contrast values for the same measurement calculated using different approximations for π.
1 2 3

b I can calculate the circumference of a circle, given either the length of the radius or the diameter.
1 2 3

Lesson 5 Understanding π

Success looks like . . .

- **Language Goal:** Comparing and contrasting values for the same measurement that were calculated using different approximations of π. **(Speaking and Listening)**
- **Goal:** Approximating the circumference of a circle when given the length of either the radius or the diameter.
  - » Approximating the circumferences of the High Roller and the Singapore Flyer.

Suggested next steps

If students subtracted the diameters, consider:

- Asking, "What part of the observation wheel do you travel on when you ride it?"
- Reviewing the definition of *diameter* and *circumference*.
- Assigning Practice Problem 3.

If students multiplied the difference of the diameters by an approximation of π, consider:

- Asking students to explain their strategy in order to ensure that they understand why this strategy works mathematically. At this point, students have not yet seen factoring (Unit 6), so they may not understand that $d_1\pi - d_2\pi = (d_1 - d_2)\pi$.

Professional Learning

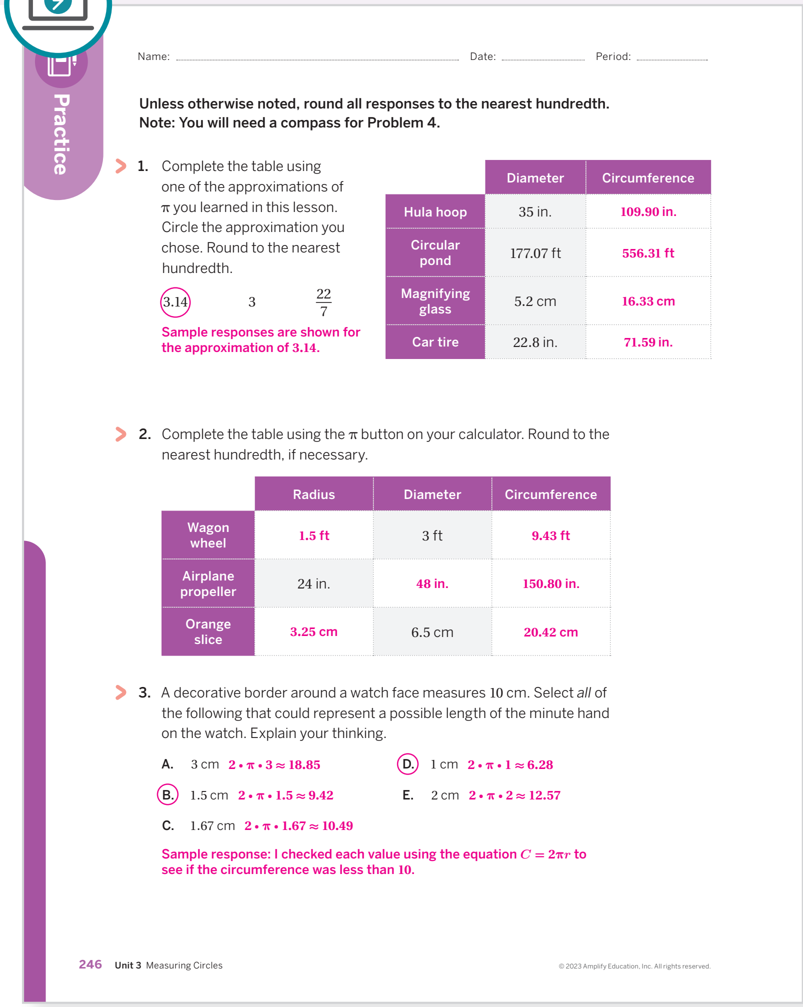
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- What different ways did students approach determining appropriate approximations of π? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

Unless otherwise noted, round all responses to the nearest hundredth.
Note: You will need a compass for Problem 4.

1. Complete the table using one of the approximations of π you learned in this lesson. Circle the approximation you chose. Round to the nearest hundredth.

   [✓ 3.14] 3 $\frac{22}{7}$

   Sample responses are shown for the approximation of 3.14.

	Diameter	Circumference
Hula hoop	35 in.	**109.90 in.**
Circular pond	177.07 ft	**556.31 ft**
Magnifying glass	5.2 cm	**16.33 cm**
Car tire	22.8 in.	**71.59 in.**

2. Complete the table using the π button on your calculator. Round to the nearest hundredth, if necessary.

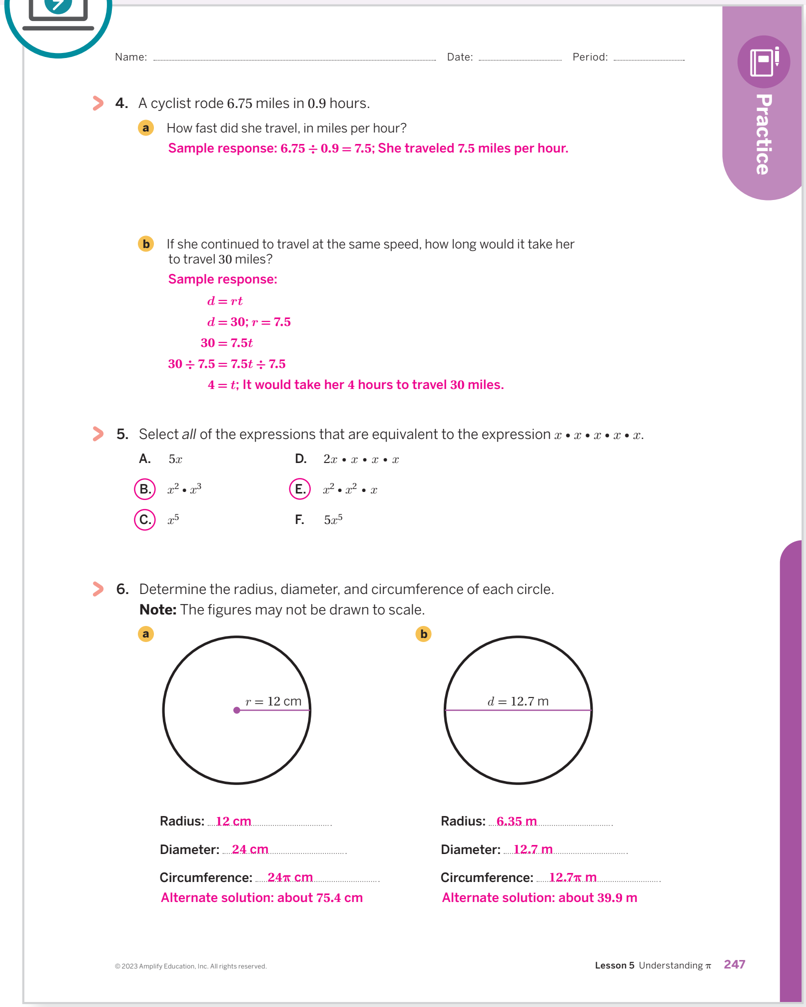
	Radius	Diameter	Circumference
Wagon wheel	**1.5 ft**	3 ft	**9.43 ft**
Airplane propeller	24 in.	**48 in.**	**150.80 in.**
Orange slice	**3.25 cm**	6.5 cm	**20.42 cm**

3. A decorative border around a watch face measures 10 cm. Select *all* of the following that could represent a possible length of the minute hand on the watch. Explain your thinking.

   A. 3 cm $2 \cdot \pi \cdot 3 \approx 18.85$
   [✓ B.] 1.5 cm $2 \cdot \pi \cdot 1.5 \approx 9.42$
   C. 1.67 cm $2 \cdot \pi \cdot 1.67 \approx 10.49$
   [✓ D.] 1 cm $2 \cdot \pi \cdot 1 \approx 6.28$
   E. 2 cm $2 \cdot \pi \cdot 2 \approx 12.57$

   Sample response: I checked each value using the equation $C = 2\pi r$ to see if the circumference was less than 10.

246 **Unit 3** Measuring Circles



Practice

Name: ______ Date: ______ Period: ______

4. A cyclist rode 6.75 miles in 0.9 hours.

   a. How fast did she travel, in miles per hour?

   Sample response: $6.75 \div 0.9 = 7.5$; She traveled 7.5 miles per hour.

   b. If she continued to travel at the same speed, how long would it take her to travel 30 miles?

   Sample response:

   $d = rt$
   $d = 30$; $r = 7.5$
   $30 = 7.5t$
   $30 \div 7.5 = 7.5t \div 7.5$
   $4 = t$; **It would take her 4 hours to travel 30 miles.**

5. Select *all* of the expressions that are equivalent to the expression $x \cdot x \cdot x \cdot x \cdot x$.

   A. $5x$
   [✓ B.] $x^2 \cdot x^3$
   [✓ C.] x^5
   D. $2x \cdot x \cdot x \cdot x$
   [✓ E.] $x^2 \cdot x^2 \cdot x$
   F. $5x^5$

6. Determine the radius, diameter, and circumference of each circle.
   Note: The figures may not be drawn to scale.

   a. $r = 12$ cm

   Radius: **12 cm**
   Diameter: **24 cm**
   Circumference: **24π cm**
   Alternate solution: about 75.4 cm

   b. $d = 12.7$ m

   Radius: **6.35 m**
   Diameter: **12.7 m**
   Circumference: **12.7π m**
   Alternate solution: about 39.9 m

Lesson 5 Understanding π 247

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 6	2
	5	Grade 6	2
Formative	6	Unit 3 Lesson 6	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

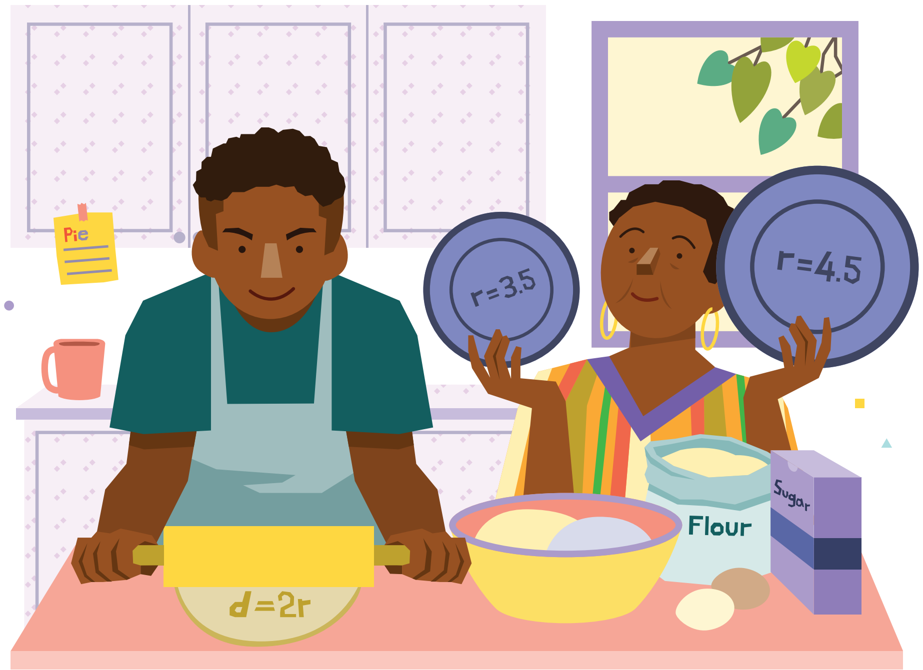
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3 | **LESSON 6**

Applying Circumference

Let’s use π to solve problems.



Focus

Goals

1. **Language Goal:** Explain how to calculate the radius, diameter, or circumference, given one of these three measurements. **(Speaking and Listening)**
2. **Language Goal:** Apply understanding of circumference to determine the perimeter of a shape that includes circular parts and explain the solution method including determining its exact and approximate measure. **(Speaking and Listening, Writing)**

Coherence

Today

Students gain fluency with circumference, radius, and diameter through determining the perimeter of shapes composed of circular parts and solving for unknown lengths in real-world scenarios. They reason that the most precise answer to problems that require the use of π is to leave their responses in terms of π rather than using an approximation.

Previously

In Lesson 5, students determined the circumference of a circle when given the radius or the diameter.

Coming Soon

In Lesson 7, students will apply their understanding of radius, diameter, and circumference to solve problems involving rotations and distance.

Rigor

- Students gain **fluency** in solving problems involving circumference, diameter, and radius of a circle.
- Students **apply** their understanding of circumference to determine the perimeter of shapes with circular parts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Small Groups	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per group
- Activity 2 PDF (answers, for display)
- Graphic Organizer PDF, *Working With Circles (Part 1)*, as needed
- calculators

Math Language Development

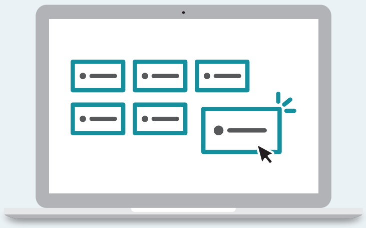
Review words

- *circle*
- *circumference*
- *diameter*
- *perimeter*
- π
- *radius*

Amps | **Featured Activity**

Activity 2
Digital Card Sort

Monitor student understanding of circumference, diameter, and radius by viewing their matches in the Card Sort in real-time.



Building Math Identity and Community

Connecting to Mathematical Practices

Working with other people in groups, as in Activity 2, can lead to conflict that does not help problem solving. Have students monitor the intensity of the interactions in their groups. If the intensity increases beyond an acceptable level, have students self-monitor and follow pre-designated steps to calm down before continuing with the activity.

Modifications to Pacing

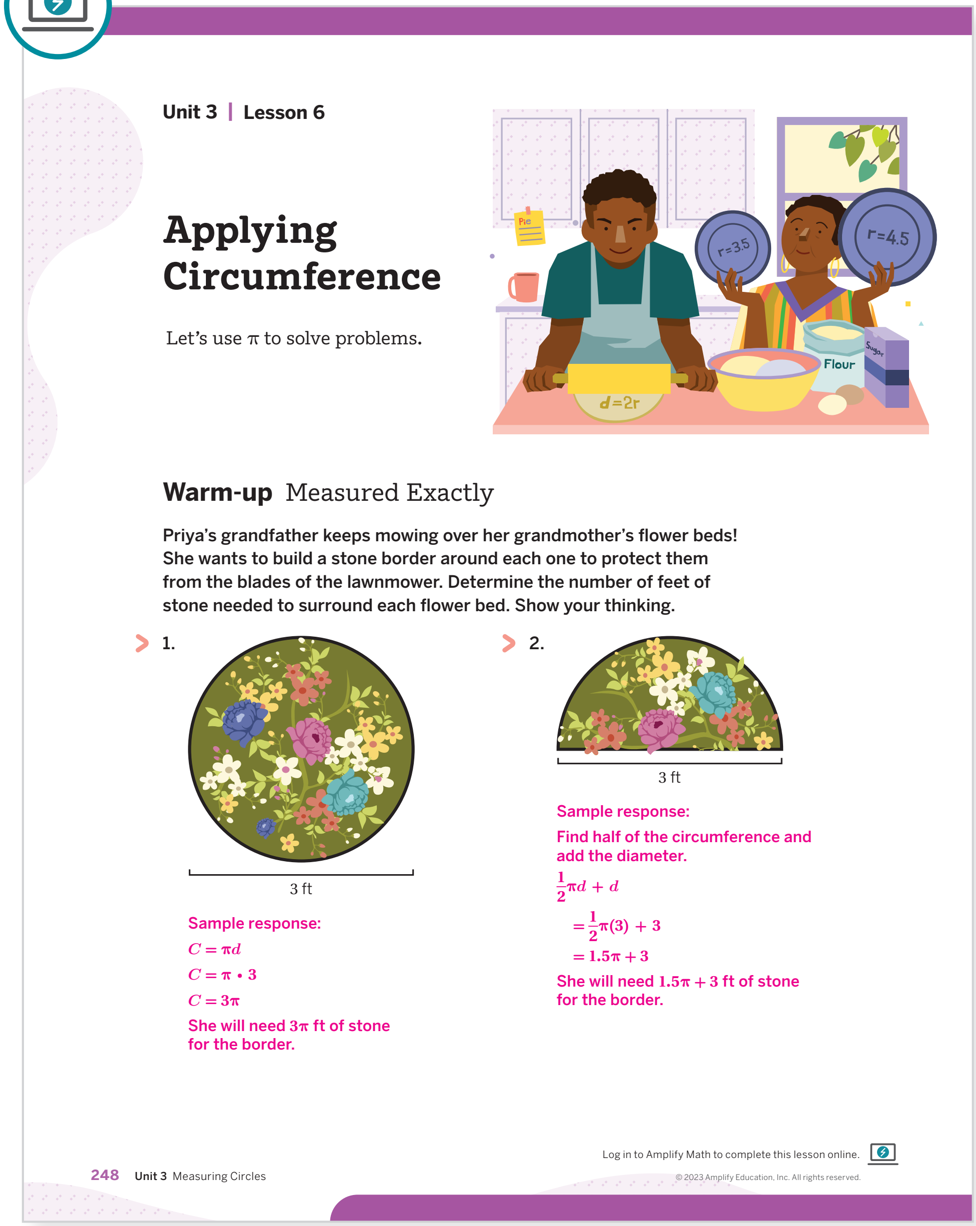
You may want to consider this additional modification if you are short on time:

- In the **Warm-up**, assign either Problem 1 or 2 only.
- In **Activity 2**, have students only complete calculations for one radius and one diameter card.

Small Groups | 10 min

Warm-up Measured Exactly

Students will apply their prior knowledge of circumference and perimeter to determine the distance around a circular and a semi-circular garden.



Unit 3 | **Lesson 6**

Applying Circumference

Let's use π to solve problems.

Warm-up Measured Exactly

Priya's grandfather keeps mowing over her grandmother's flower beds! She wants to build a stone border around each one to protect them from the blades of the lawnmower. Determine the number of feet of stone needed to surround each flower bed. Show your thinking.

1\. 3 ft

Sample response:

$C = \pi d$

$C = \pi \cdot 3$

$C = 3\pi$

She will need 3π ft of stone for the border.

2\. 3 ft

Sample response:

Find half of the circumference and add the diameter.

$\frac{1}{2}\pi d + d$

$= \frac{1}{2}\pi(3) + 3$

$= 1.5\pi + 3$

She will need $1.5\pi + 3$ ft of stone for the border.

248 Unit 3 Measuring Circles

Log in to Amplify Math to complete this lesson online.

1 Launch

Distribute calculators to each student. Explain that students will be working in small groups to determine the distance around both gardens.

2 Monitor

Help students get started by asking, "What information were you given about each garden? What are you trying to determine?"

Look for points of confusion:

- **Halving the distance around the circular garden to determine the distance around the semi-circular garden.** Have students trace around half of the circumference of the large circle. Ask whether that would include the straight edge of the semi-circular garden.

Look for productive strategies:

- Breaking the semi-circular garden into two pieces — the half circumference and the diameter.

3 Connect

Display observed student responses for Problem 1 or these sample responses: 3π, 9.42, 9, and 9.425. Ask students to order the responses from least precise to most precise.

Have groups of students share how they determined their ranking.

Highlight that 9 or 9.42 are the most practical responses to the problem, but the exact length of the fence for the circular fence is 3π. The other responses use an approximation that has been substituted for π while 3π does not. This is called responding "in terms of π."

Ask:

- "What is the exact distance around the semi-circular garden?"
- "Why is the exact distance *not* 4.5π?"

Power-up

To power up students' ability to determine the radius, diameter, or circumference of a circle, given its radius or diameter, have students complete:

Use the relationships $d = 2r$, $C = \pi d$, and $C = 2\pi r$ to complete the missing values in the table.

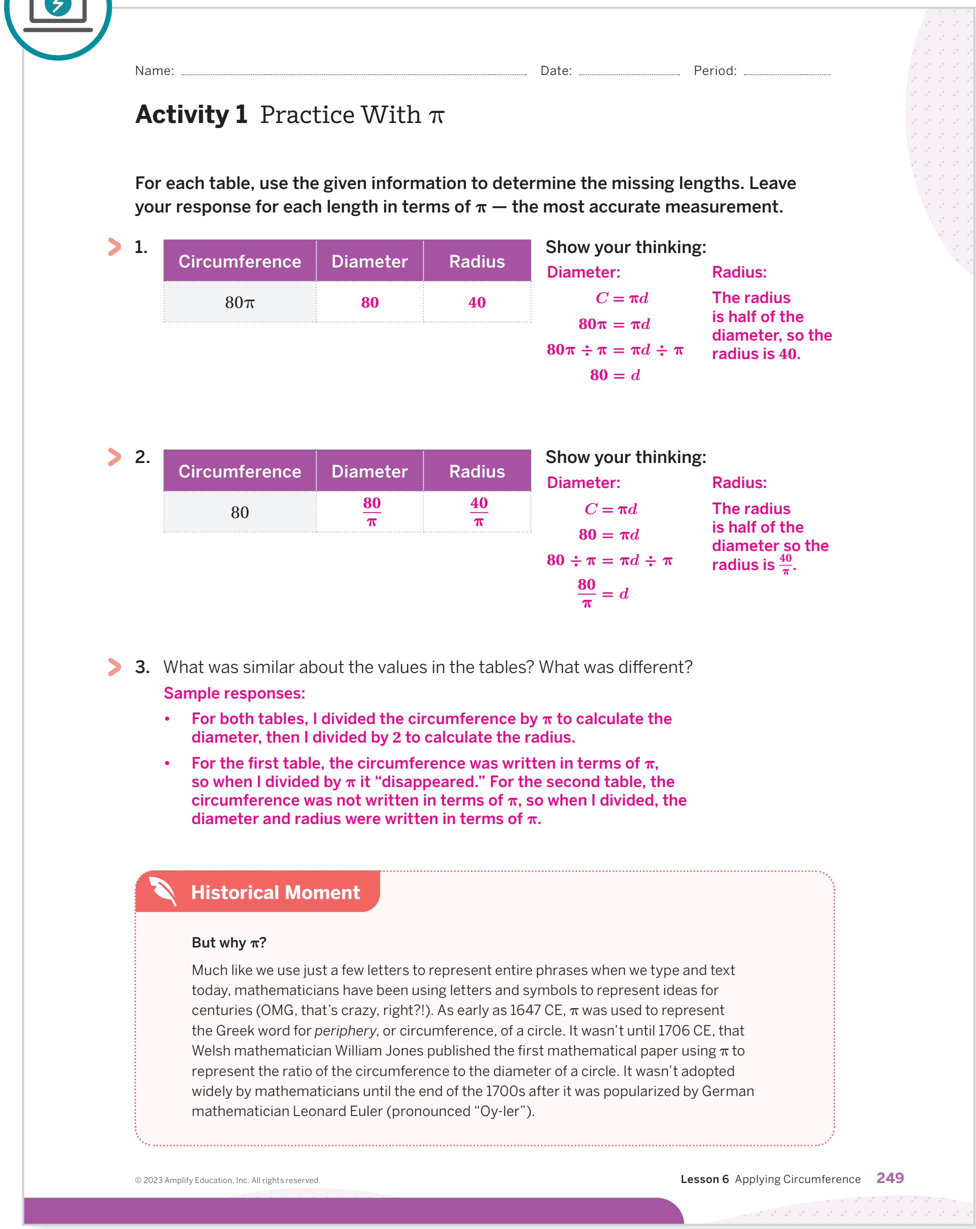
Radius	Diameter	Circumference
8	16	50.27
1.5	3	9.42

Use: Before Activity 1.
Informed by: Performance on Lesson 5, Practice Problem 6.

Pairs | 10 min

Activity 1 Practice With π

Students apply their understanding of the formulas relating the measurements of a circle to determine the exact length of the diameter and radius when given the circumference.



Name: ______ Date: ______ Period: ______

Activity 1 Practice With π

For each table, use the given information to determine the missing lengths. Leave your response for each length in terms of π — the most accurate measurement.

1.

Circumference	Diameter	Radius
80π	**80**	**40**

Show your thinking:

Diameter:

$$C = \pi d$$
$$80\pi = \pi d$$
$$80\pi \div \pi = \pi d \div \pi$$
$$80 = d$$

Radius:

The radius is half of the diameter, so the radius is 40.

2.

Circumference	Diameter	Radius
80	$\frac{80}{\pi}$	$\frac{40}{\pi}$

Show your thinking:

Diameter:

$$C = \pi d$$
$$80 = \pi d$$
$$80 \div \pi = \pi d \div \pi$$
$$\frac{80}{\pi} = d$$

Radius:

The radius is half of the diameter so the radius is $\frac{40}{\pi}$.

3. What was similar about the values in the tables? What was different?

Sample responses:

- **For both tables, I divided the circumference by π to calculate the diameter, then I divided by 2 to calculate the radius.**
- **For the first table, the circumference was written in terms of π, so when I divided by π it "disappeared." For the second table, the circumference was not written in terms of π, so when I divided, the diameter and radius were written in terms of π.**

Historical Moment

But why π?

Much like we use just a few letters to represent entire phrases when we type and text today, mathematicians have been using letters and symbols to represent ideas for centuries (OMG, that's crazy, right?!). As early as 1647 CE, π was used to represent the Greek word for *periphery*, or circumference, of a circle. It wasn't until 1706 CE, that Welsh mathematician William Jones published the first mathematical paper using π to represent the ratio of the circumference to the diameter of a circle. It wasn't adopted widely by mathematicians until the end of the 1700s after it was popularized by German mathematician Leonard Euler (pronounced "Oy-ler").

Lesson 6 Applying Circumference 249

1 Launch

Activate students' prior knowledge by asking, "How are the lengths of the *circumference*, *diameter*, and *radius* related in a circle?"

2 Monitor

Help students get started by asking, "What is the formula that relates circumference to diameter?"

Look for points of confusion:

- **Forgetting about π in the second problem.** Ask students to identify what is the same and what is different between the two problems.

Look for productive strategies:

- Using the value determined for the diameter to help them to determine the radius rather than solving $C = 2\pi r$.

3 Connect

Display the tables from Activity 1.

Have pairs of students share their solutions, including what similarities and differences they noticed in determining the unknown *diameters* and *radii*.

Highlight that in order to avoid making mistakes with π, students should always write down the formula they are using. Explain that the most common mistake is forgetting to divide by π when the *circumference* is written without π.

Ask:

- "When measuring the circumference of circles in the real-world, do you think the measurement should be expressed in terms of π?" No; you would approximate the length in feet, inches, etc.
- "Using the π button on your calculator, what are the approximate lengths of the diameter and radius from Problem 2?" Diameter: 25.46, Radius: 12.73
- "Is it more precise to say the diameter is $\frac{80}{\pi}$ or 25.46?" $\frac{80}{\pi}$

Differentiated Support

Accessibility: *Guide Processing and Visualization*

If students are unsure how to divide by π in Problem 1, ask them to rewrite the circumference as $80(3)$, substituting 3 for π. Then ask how they would solve the equation $80(3) = 3d$. Prompt them to use similar reasoning and steps to solve the equation $80\pi = \pi d$".

Math Language Development

MLR3: Critique, Correct, Clarify

After students complete Problems 1–2, display an incorrect solution, such as forgetting to divide the circumference by π when it is written without π. Have groups critique the error, correct it, and clarify why their solution is correct.

Historical Moment

But why π?

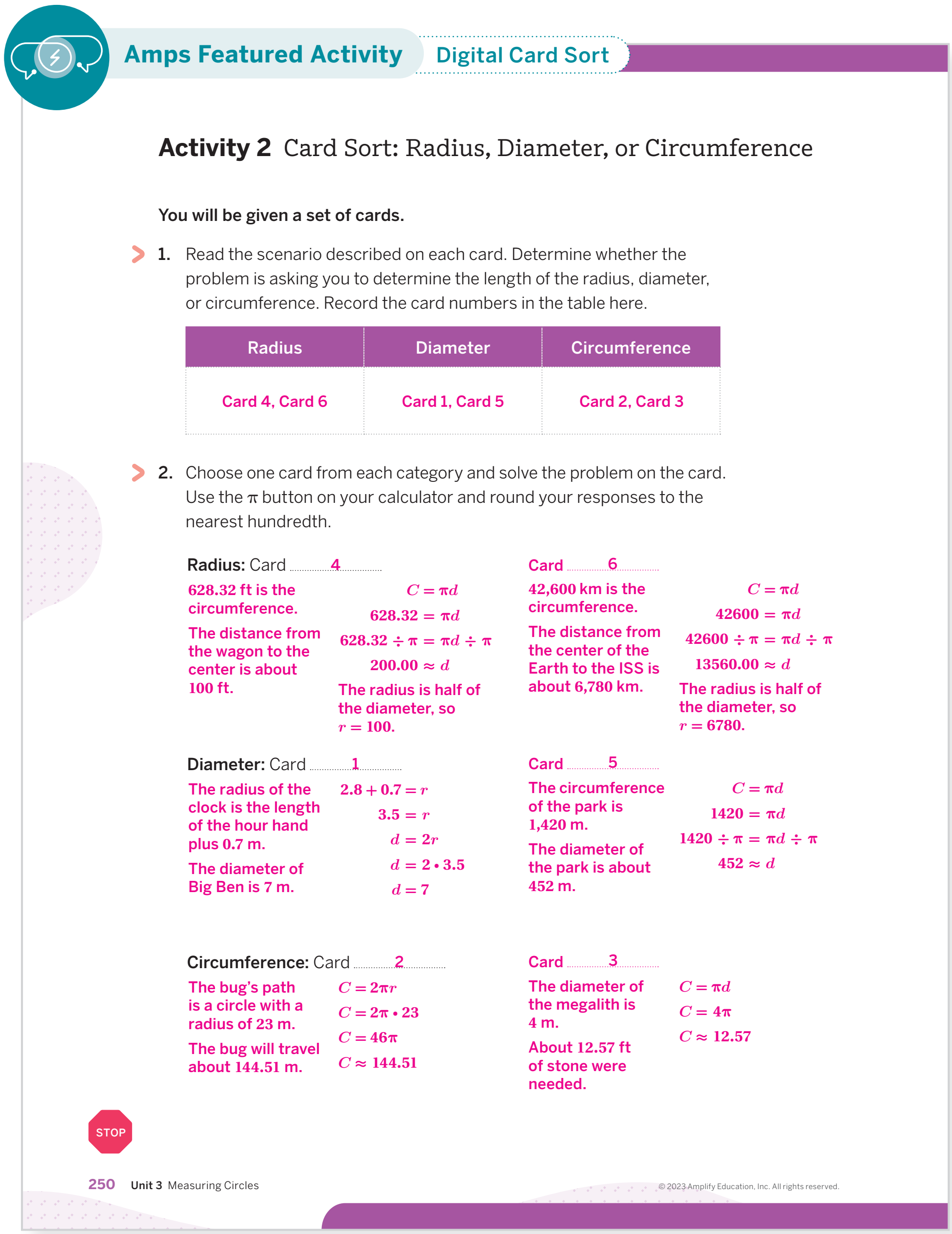
Have students read about the adoption of the symbol π to represent the ratio between the circumference and diameter of a circle.

Small Groups | 15 min

Activity 2 Card Sort: Radius, Diameter, or Circumference

Students sort scenarios according to the measurement they are describing (radius, diameter, or circumference), and then calculate the missing length.

Amps Featured Activity Digital Card Sort



Activity 2 Card Sort: Radius, Diameter, or Circumference

You will be given a set of cards.

1. Read the scenario described on each card. Determine whether the problem is asking you to determine the length of the radius, diameter, or circumference. Record the card numbers in the table here.

Radius	Diameter	Circumference
Card 4, Card 6	Card 1, Card 5	Card 2, Card 3

2. Choose one card from each category and solve the problem on the card. Use the π button on your calculator and round your responses to the nearest hundredth.

Radius: Card 4

628.32 ft is the circumference.

The distance from the wagon to the center is about 100 ft.

$$C = \pi d$$
$$628.32 = \pi d$$
$$628.32 \div \pi = \pi d \div \pi$$
$$200.00 \approx d$$

The radius is half of the diameter, so $r = 100$.

Card 6

42,600 km is the circumference.

The distance from the center of the Earth to the ISS is about 6,780 km.

$$C = \pi d$$
$$42600 = \pi d$$
$$42600 \div \pi = \pi d \div \pi$$
$$13560.00 \approx d$$

The radius is half of the diameter, so $r = 6780$.

Diameter: Card 1

The radius of the clock is the length of the hour hand plus 0.7 m.

The diameter of Big Ben is 7 m.

$$2.8 + 0.7 = r$$
$$3.5 = r$$
$$d = 2r$$
$$d = 2 \bullet 3.5$$
$$d = 7$$

Card 5

The circumference of the park is 1,420 m.

The diameter of the park is about 452 m.

$$C = \pi d$$
$$1420 = \pi d$$
$$1420 \div \pi = \pi d \div \pi$$
$$452 \approx d$$

Circumference: Card 2

The bug's path is a circle with a radius of 23 m.

The bug will travel about 144.51 m.

$$C = 2\pi r$$
$$C = 2\pi \bullet 23$$
$$C = 46\pi$$
$$C \approx 144.51$$

Card 3

The diameter of the megalith is 4 m.

About 12.57 ft of stone were needed.

$$C = \pi d$$
$$C = 4\pi$$
$$C \approx 12.57$$

STOP

250 Unit 3 Measuring Circles

1 Launch

Distribute one set of cards from the Activity 2 PDF to each group of students as well as calculators. Explain that they should follow the ***Card Sort*** routine, taking turns reading the scenarios, and then sorting the cards into groups based on the length they are being asked to determine: radius, diameter, or circumference. Once students sort the cards, they should complete one scenario from each category.

2 Monitor

Help students get started by asking, "What information were you given in your scenario? What are you being asked to determine?"

Look for points of confusion:

- **Confusing the information that is given with the information that they are being asked to determine.** Remind students that they are sorting their cards based on the *missing* information, not the given information.

Look for productive strategies:

- Writing down the formula that connects the known length to the unknown length as the first step.

3 Connect

Display the Activity 2 PDF (answers).

Have groups of students share their strategies for determining which measurement was given in each problem and which measurement was being determined.

Highlight key words that helped students to distinguish between *radius*, *diameter*, and *circumference*:

- Radius: center, minute/hour hand
- Diameter: width, height
- Circumference: orbit, surround, travel

Ask, "Are there any other words not mentioned in these scenarios that would represent radius, diameter, or circumference?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Graphic Organizer PDF, *Working With Circles, (Part 1)* to help them make sense of each scenario as they sort the cards and determine missing values.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their strategies, encourage them to reference the displays that have been created throughout the unit to help them remember important formulas, vocabulary, and visual examples.

English Learners

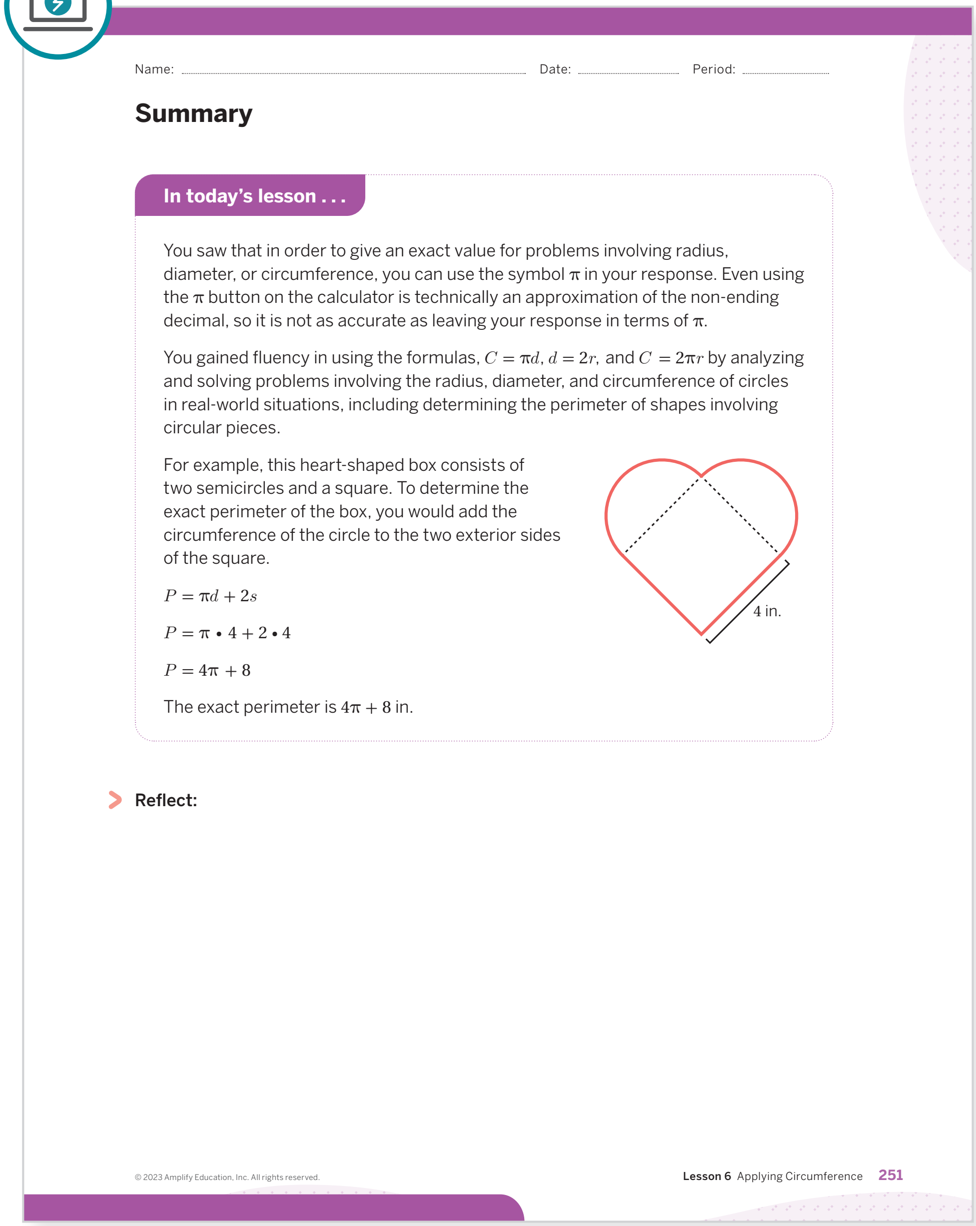
Display the following sentence frames to aid students in their discussion about classifying each scenario:

- "I know _____ because. . . ."
- "How do you know. . .?"
- "Both _____ and _____ are alike because. . ."
- "Both _____ and _____ are different because. . . ."

Whole Class | 5 min

Summary

Review and synthesize the relationship between circumference, diameter, and radius and how they can be used to solve problems involving circles and shapes with parts of circles.



Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that in order to give an exact value for problems involving radius, diameter, or circumference, you can use the symbol π in your response. Even using the π button on the calculator is technically an approximation of the non-ending decimal, so it is not as accurate as leaving your response in terms of π.

You gained fluency in using the formulas, $C = \pi d$, $d = 2r$, and $C = 2\pi r$ by analyzing and solving problems involving the radius, diameter, and circumference of circles in real-world situations, including determining the perimeter of shapes involving circular pieces.

For example, this heart-shaped box consists of two semicircles and a square. To determine the exact perimeter of the box, you would add the circumference of the circle to the two exterior sides of the square.

$P = \pi d + 2s$

$P = \pi \cdot 4 + 2 \cdot 4$

$P = 4\pi + 8$

The exact perimeter is $4\pi + 8$ in.

Reflect:

Synthesize

Display the Summary.

Highlight that if one measurement of *circumference, diameter*, or *radius* is known then it is possible to calculate the other two measurements. Students can also use what they know about these measurements to determine the *perimeter* around shapes with circular parts. Finally, if students want an exact measurement, they will need to leave their answers in terms of π.

Ask:

- "If you know the circumference of a circle, how do you determine the diameter and radius?"
- "Should you use the π button on your calculator when determining the exact length? Why or why not?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

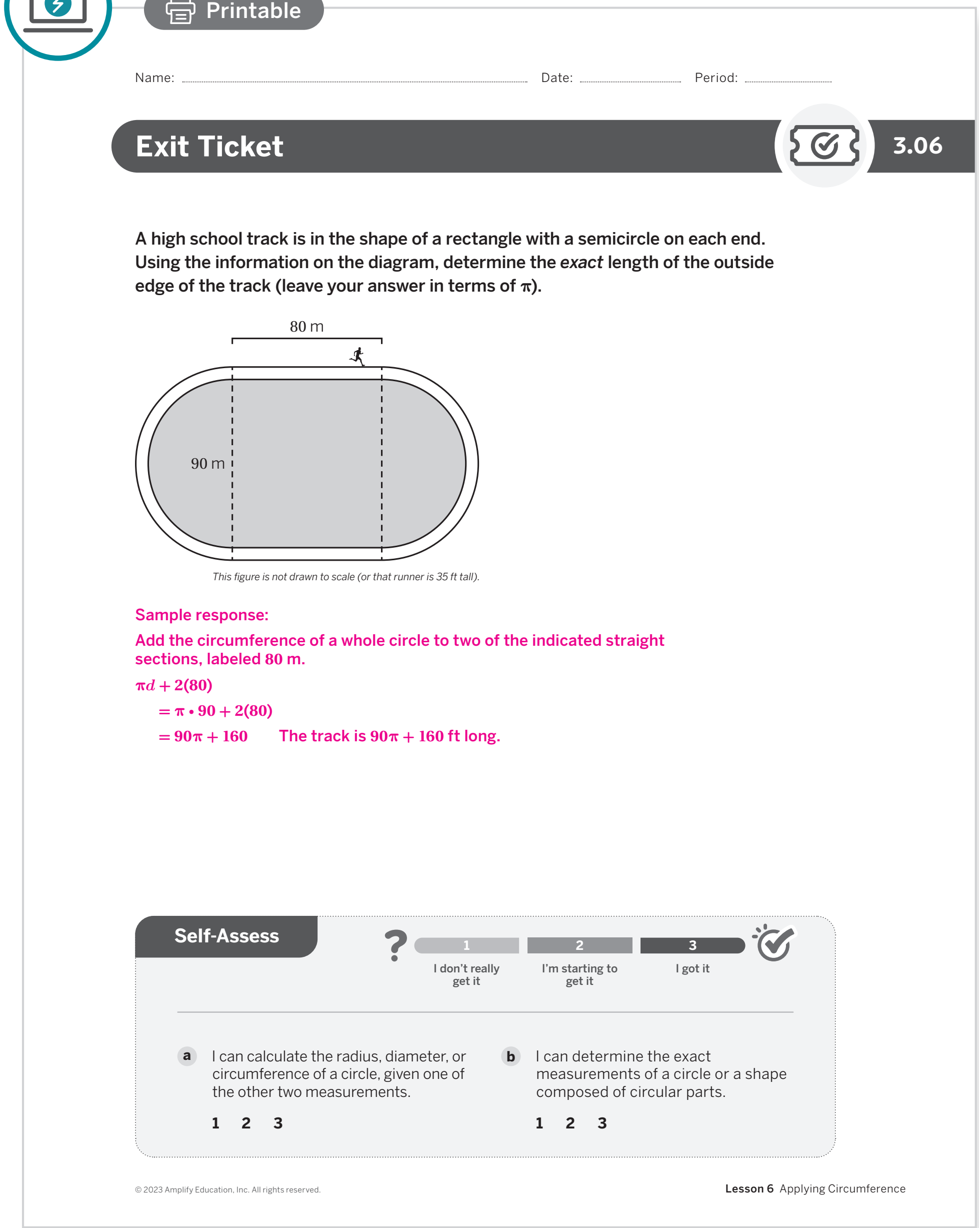
- "How are the *radius, diameter*, and *circumference* related?"
- "How do we ensure our responses are exact and not approximations when working with circles?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of measuring shapes with circular parts by determining the exact perimeter of a track in terms of π.

Printable



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Exit Ticket 3.06

A high school track is in the shape of a rectangle with a semicircle on each end. Using the information on the diagram, determine the *exact* length of the outside edge of the track (leave your answer in terms of π).

This figure is not drawn to scale (or that runner is 35 ft tall).

Sample response:

Add the circumference of a whole circle to two of the indicated straight sections, labeled 80 m.

$\pi d + 2(80)$

$= \pi \cdot 90 + 2(80)$

$= 90\pi + 160$ The track is $90\pi + 160$ ft long.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can calculate the radius, diameter, or circumference of a circle, given one of the other two measurements.

1 2 3

b. I can determine the exact measurements of a circle or a shape composed of circular parts.

1 2 3

Lesson 6 Applying Circumference

Success looks like . . .

- **Language Goal:** Explaining how to calculate the radius, diameter, or circumference given one of these three measurements. **(Speaking and Listening)**
- **Language Goal:** Applying understanding of circumference to determine the perimeter of a shape that includes circular parts and explaining the solution method, including determining its exact and approximate measure. **(Speaking and Listening, Writing)**.
  - » Calculating the perimeter of the outside edge of the track as an exact measure.

Suggested next steps

If students combine 90π and 160 as 250π, consider:

- Reviewing Problem 2 from the Warm-up.
- Assigning Practice Problem 3.
- Asking, "Is 160 also being multiplied by π?"

If students include the diameters in their calculations, consider:

- Reviewing Problem 2 from the Warm-up.
- Assigning Practice Problem 3.
- Asking, "Are the two diameters of 90 part of the track?"

If students give their response as an approximation and not in terms of π, consider:

- Reviewing Activity 1.
- Assigning Practice Problem 1.
- Asking, "What does it mean to give an *exact* answer?"

Make a note of any students that struggled with decomposing this shape and provide them additional support in Lesson 10, Activity 2 where they will work with a more complex version of this track.

Professional Learning

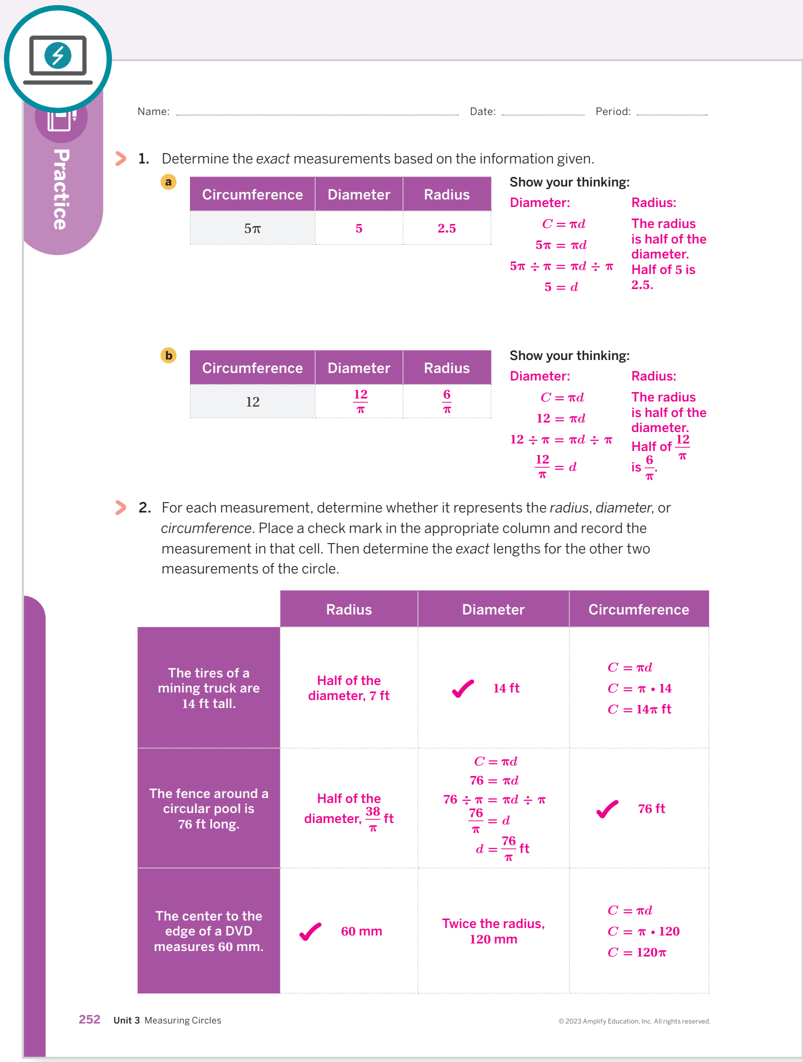
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students improved at making sense of problems and persevering in solving them?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Determine the *exact* measurements based on the information given.

a

Circumference	Diameter	Radius
5π	**5**	**2.5**

Show your thinking:

Diameter:

$C = \pi d$

$5\pi = \pi d$

$5\pi \div \pi = \pi d \div \pi$

$5 = d$

Radius:

The radius is half of the diameter. Half of 5 is 2.5.

b

Circumference	Diameter	Radius
12	$\frac{12}{\pi}$	$\frac{6}{\pi}$

Show your thinking:

Diameter:

$C = \pi d$

$12 = \pi d$

$12 \div \pi = \pi d \div \pi$

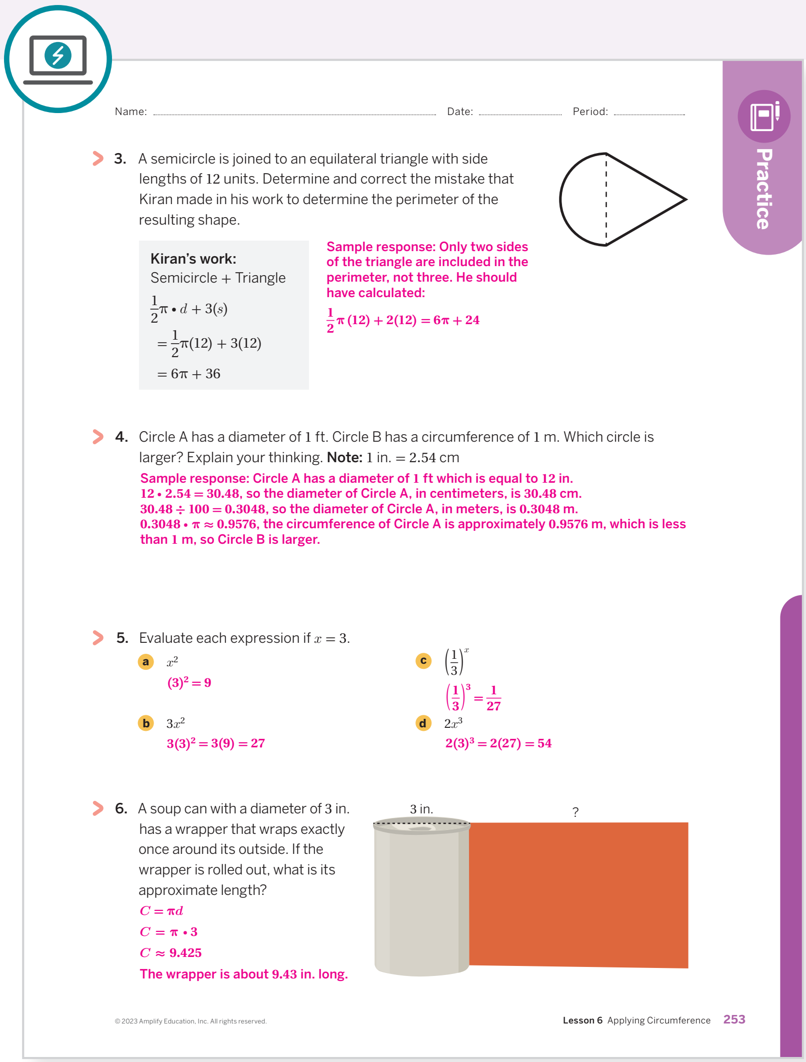
$\frac{12}{\pi} = d$

Radius:

The radius is half of the diameter. Half of $\frac{12}{\pi}$ is $\frac{6}{\pi}$.

2. For each measurement, determine whether it represents the *radius*, *diameter*, or *circumference*. Place a check mark in the appropriate column and record the measurement in that cell. Then determine the *exact* lengths for the other two measurements of the circle.

	Radius	Diameter	Circumference
The tires of a mining truck are 14 ft tall.	Half of the diameter, 7 ft	✓ 14 ft	$C = \pi d$ $C = \pi \cdot 14$ $C = 14\pi$ ft
The fence around a circular pool is 76 ft long.	Half of the diameter, $\frac{38}{\pi}$ ft	$C = \pi d$ $76 = \pi d$ $76 \div \pi = \pi d \div \pi$ $\frac{76}{\pi} = d$ $d = \frac{76}{\pi}$ ft	✓ 76 ft
The center to the edge of a DVD measures 60 mm.	✓ 60 mm	Twice the radius, 120 mm	$C = \pi d$ $C = \pi \cdot 120$ $C = 120\pi$



Name: Date: Period:

3. A semicircle is joined to an equilateral triangle with side lengths of 12 units. Determine and correct the mistake that Kiran made in his work to determine the perimeter of the resulting shape.

Kiran's work:
Semicircle + Triangle

$\frac{1}{2}\pi \cdot d + 3(s)$

$= \frac{1}{2}\pi(12) + 3(12)$

$= 6\pi + 36$

Sample response: Only two sides of the triangle are included in the perimeter, not three. He should have calculated:

$\frac{1}{2}\pi(12) + 2(12) = 6\pi + 24$

4. Circle A has a diameter of 1 ft. Circle B has a circumference of 1 m. Which circle is larger? Explain your thinking. **Note:** 1 in. = 2.54 cm

Sample response: Circle A has a diameter of 1 ft which is equal to 12 in. $12 \cdot 2.54 = 30.48$, so the diameter of Circle A, in centimeters, is 30.48 cm. $30.48 \div 100 = 0.3048$, so the diameter of Circle A, in meters, is 0.3048 m. $0.3048 \cdot \pi \approx 0.9576$, the circumference of Circle A is approximately 0.9576 m, which is less than 1 m, so Circle B is larger.

5. Evaluate each expression if $x = 3$.

a x^2
$(3)^2 = 9$

b $3x^2$
$3(3)^2 = 3(9) = 27$

c $\left(\frac{1}{3}\right)^x$
$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

d $2x^3$
$2(3)^3 = 2(27) = 54$

6. A soup can with a diameter of 3 in. has a wrapper that wraps exactly once around its outside. If the wrapper is rolled out, what is its approximate length?

$C = \pi d$

$C = \pi \cdot 3$

$C \approx 9.425$

The wrapper is about 9.43 in. long.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Warm-up	2
Spiral	4	Unit 3 Lesson 5	3
	5	Grade 6	2
Formative ⚡	6	Unit 3 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

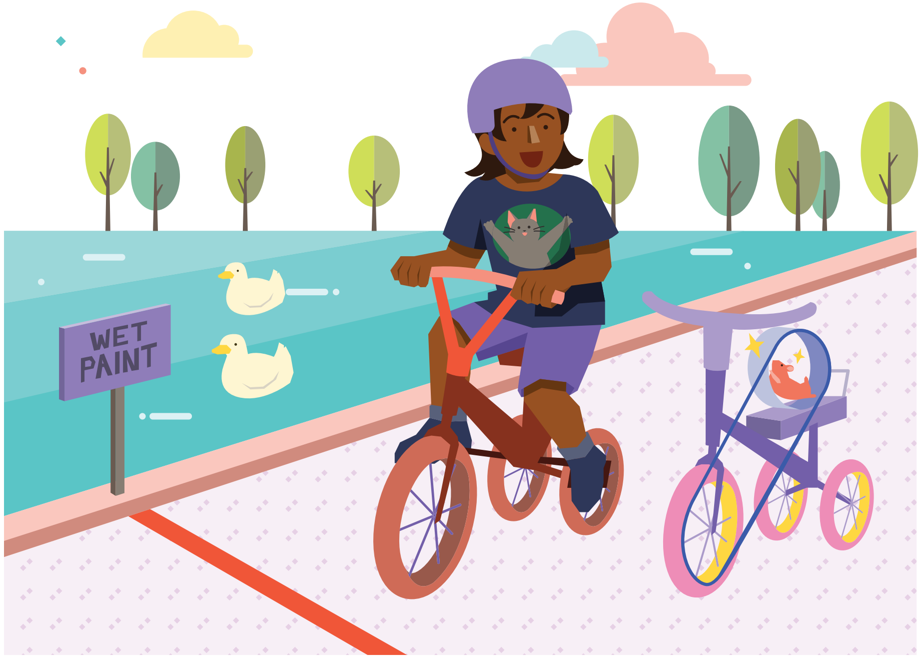
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3 | **LESSON 7**

Circumference and Wheels

Let's explore how far different wheels roll.



Focus

Goals

1. **Language Goal:** Compare wheels of different sizes and explain why a larger wheel needs fewer rotations to travel the same distance. **(Speaking and Listening)**
2. Generalize that the distance a wheel rolls in one rotation is equal to the circumference of the wheel.
3. Write an equation to represent the proportional relationship between the number of rotations and the distance a wheel travels.

Coherence

Today

Students notice that the distance a wheel rolls forward as it completes one rotation is equal to its circumference. They also see that there is also a proportional relationship between the number of times a wheel rotates and the distance the wheel travels.

Previously

In Lesson 4, students used an applet to unroll circles, and they related the radius and diameter to the circumference.

Coming Soon

In Lesson 8, students begin to explore how to determine the area of a circle.

Rigor

- Students **apply** their understanding of proportional relationships and the circumference of the circle to solve problems related to distance and speed of wheels.

Pacing Guide

Suggested Total Lesson Time ~ **45 min**

Warm-up	Activity 1 (optional)	Activity 2	Activity 3	Summary	Exit Ticket
10 min	15 min	10 min	15 min	5 min	5 min
Pairs	Small Groups	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per group (optional)
- rulers
- calculators
- paper, one sheet per small group
- soup can, with wrapper (optional)
- various circular objects for rolling

Math Language Development

Review words

- *circumference*
- *constant of proportionality*
- *diameter*
- π
- *proportional relationship*
- *radius*

Amps | Featured Activity

Activity 3

Interactive Hamster Tricycle

Students see Andre's hamster in action as she rides her tricycle around the neighborhood. They can stop, start, and reset reference points as they compare the rotation of the hamster wheel to the tricycle wheel.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not be able to discipline themselves to consider how their mathematical knowledge relates to rolling objects. Before the activity, have students determine how they will keep focused. Ask students to help each other regulate their behaviors so that the entire group can consider the structure of a circle and how it relates to distance rolled.

Modifications to Pacing

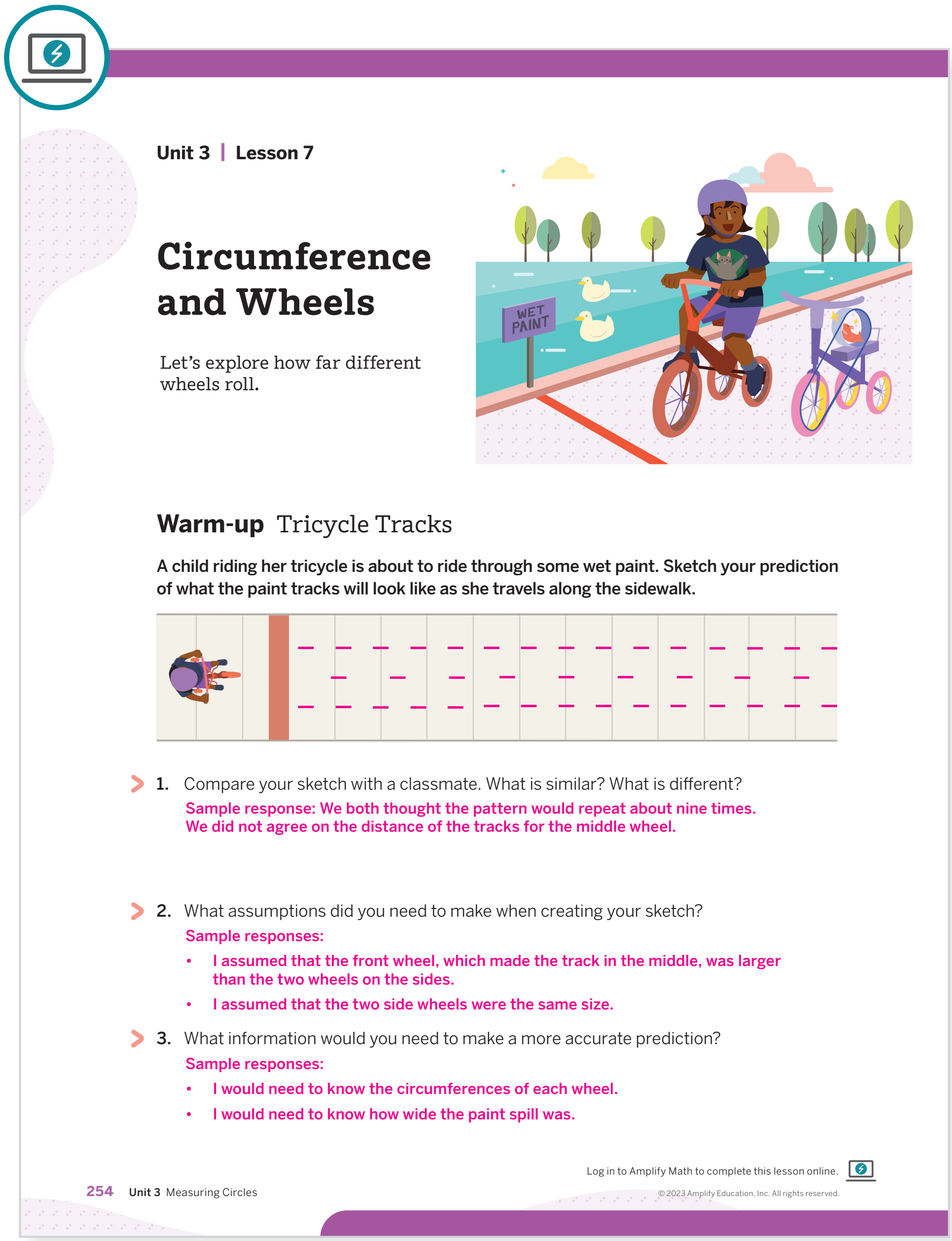
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 2 and 3 may be omitted.
- Optional **Activity 1** may be omitted.
- **Activity 3** may be omitted. The relationship between distance, speed, and time is also explored in Units 1, 2, and 5.

Pairs | 10 min

Warm-up Tricycle Tracks

Students predict the tracks of a wheel through paint to prepare for working with the distance a wheel travels per rotation.



Unit 3 | Lesson 7

Circumference and Wheels

Let's explore how far different wheels roll.

Warm-up Tricycle Tracks

A child riding her tricycle is about to ride through some wet paint. Sketch your prediction of what the paint tracks will look like as she travels along the sidewalk.

1. Compare your sketch with a classmate. What is similar? What is different?
   Sample response: We both thought the pattern would repeat about nine times. We did not agree on the distance of the tracks for the middle wheel.

2. What assumptions did you need to make when creating your sketch?
   Sample responses:
   - I assumed that the front wheel, which made the track in the middle, was larger than the two wheels on the sides.
   - I assumed that the two side wheels were the same size.

3. What information would you need to make a more accurate prediction?
   Sample responses:
   - I would need to know the circumferences of each wheel.
   - I would need to know how wide the paint spill was.

Log in to Amplify Math to complete this lesson online.

254 Unit 3 Measuring Circles

1 Launch

Activate background knowledge by asking whether students have ever left walking tracks with their shoes. Ask about any patterns they may have noticed.

2 Monitor

Help students get started by asking if the tracks will look the same for all of the wheels.

Look for points of confusion:

- **Drawing the tracks as a continuous line.** Ask, "Will the whole wheel get coated with paint when it rolls through the spill?"
- **Not taking into account the different sizes of the wheels on a tricycle.** Have an image of a tricycle available for those who may not be familiar with their design.

Look for productive strategies:

- Thinking about how the size of the wheel will affect the tracks.
- Considering that the wheel in the front of a tricycle is typically larger than the back wheels.

3 Connect

Display selected students' work.

Ask, "What assumptions did you make before drawing the tracks? Are these assumptions valid given what we know?"

Highlight how considering available information and making assumptions about unavailable information is an important part of the mathematical problem-solving process. Guesswork and estimation helps us to later consider whether our answer is reasonable or not.

Math Language Development

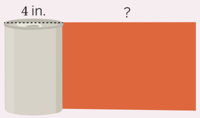
MLR7: Compare and Connect

As students complete the Warm-up and respond to Problems 1–2, consider displaying these questions to support them as they compare their sketches and assumptions.

- "Did your sketches show a pattern? If so, were the patterns similar or different? In what ways were they similar or different?"
- "Were the tracks for each wheel the same or different? If you thought they were different, do you think it makes sense that they are different? If you thought they were all the same, do you think it makes sense that they are all the same?"

Power-up

To power up students' ability to visualize the relationship between the circumference of a can and the distance around it, have students complete:



Use the diagram to complete each problem:

1. What is the diameter of the top of the can? 4 in.
2. What is the radius of the top of the can? 2 in.
3. What is the circumference of the top of the can? 4π in. or equivalent
4. Does the length of the label of the can represent the diameter, radius, or circumference? circumference

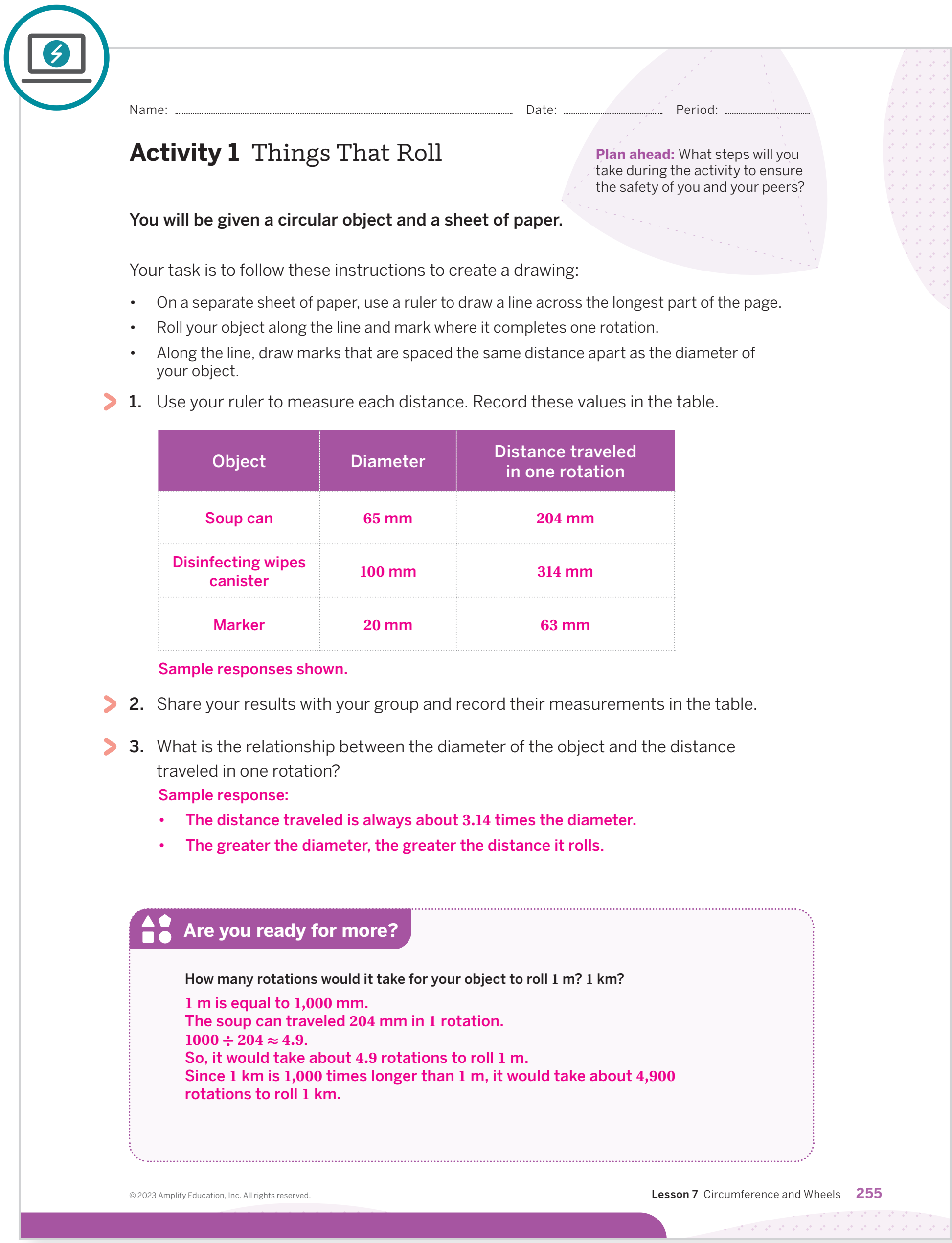
Use: Before the Warm-up.
Informed by: Performance on Lesson 6, Practice Problem 6.

Optional

Small Groups | 15 min

Activity 1 Things That Roll

Students roll circular objects in order to relate the distance of a rotation to the diameter of the object.



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Activity 1 Things That Roll

Plan ahead: What steps will you take during the activity to ensure the safety of you and your peers?

You will be given a circular object and a sheet of paper.

Your task is to follow these instructions to create a drawing:

- On a separate sheet of paper, use a ruler to draw a line across the longest part of the page.
- Roll your object along the line and mark where it completes one rotation.
- Along the line, draw marks that are spaced the same distance apart as the diameter of your object.

1. Use your ruler to measure each distance. Record these values in the table.

Object	Diameter	Distance traveled in one rotation
Soup can	65 mm	204 mm
Disinfecting wipes canister	100 mm	314 mm
Marker	20 mm	63 mm

Sample responses shown.

2. Share your results with your group and record their measurements in the table.

3. What is the relationship between the diameter of the object and the distance traveled in one rotation?

Sample response:

- The distance traveled is always about 3.14 times the diameter.
- The greater the diameter, the greater the distance it rolls.

Are you ready for more?

How many rotations would it take for your object to roll 1 m? 1 km?

1 m is equal to 1,000 mm.
The soup can traveled 204 mm in 1 rotation.
$1000 \div 204 \approx 4.9$.
So, it would take about 4.9 rotations to roll 1 m.
Since 1 km is 1,000 times longer than 1 m, it would take about 4,900 rotations to roll 1 km.

Lesson 7 Circumference and Wheels 255

1 Launch

Distribute the circular objects, pieces of paper, and rulers to each small group. Model how to follow the steps by using a circular object not provided to students as an example. Provide access to calculators for the rest of the lesson.

Note: You may use the cards in the Activity 1 PDF in place of objects, but note that they will likely be more difficult for students to roll.

2 Monitor

Help students get started by asking how a rotation is related to the circumference.

Look for points of confusion:

- **Thinking the distance of one rotation is equal to the diameter.** Ask, "What part of the circle will touch the surface as it rotates?"
- **Forgetting the conversions between millimeters, meters, and kilometers.** Write the metric conversions on a board so that the whole class can see them.

3 Connect

Display the completed table to the class.

Ask:

- "Which object covered the longest distance in one rotation?"
- "What relationships do you notice in the table?" I noticed the relationship between the diameter and the distance traveled in one rotation is proportional, but the relationship between the diameter and the number of rotations to travel a certain distance is not.

Highlight that the distance a wheel travels in one complete rotation is equal to the *circumference* of the wheel.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students roll physical objects or use the cards in the Activity 1 PDF, consider demonstrating the roll of one circular object and then providing students with pre-populated diameters and distances for the table in Problem 1. This will allow students to focus on visualizing and understanding the relationship between the diameter and distance rolled, instead of physically measuring the distances.

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Launch, to support students in understanding their task, provide them with an image that depicts what is being asked of them. For example, display a piece of paper that already has the line drawn and ask students to restate the directions for rolling their object in their own words. Ask, "Where should I draw the marks on the line?"

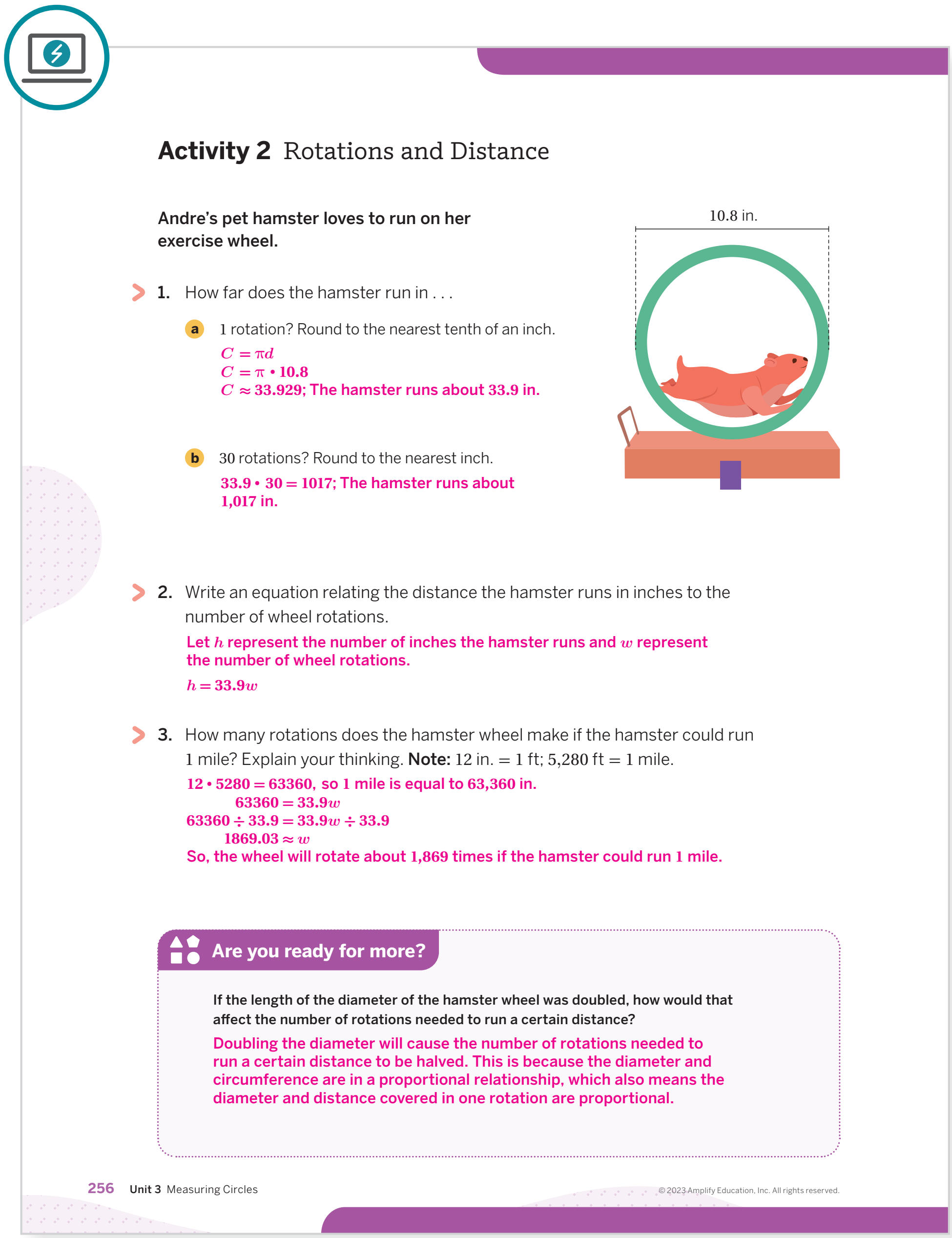
English Learners

Provide students with a partner who speaks the same primary language to support students in making sense of the directions to the task.

Pairs | 10 min

Activity 2 Rotations and Distance

Students write an equation relating the distance a wheel travels and the number of rotations it makes in order to find the number of rotations for any distance.



Activity 2 Rotations and Distance

Andre's pet hamster loves to run on her exercise wheel.

10.8 in.

1. How far does the hamster run in . . .

   a. 1 rotation? Round to the nearest tenth of an inch.

      $C = \pi d$

      $C = \pi \cdot 10.8$

      $C \approx 33.929$; The hamster runs about 33.9 in.

   b. 30 rotations? Round to the nearest inch.

      $33.9 \cdot 30 = 1017$; The hamster runs about 1,017 in.

2. Write an equation relating the distance the hamster runs in inches to the number of wheel rotations.

   Let h represent the number of inches the hamster runs and w represent the number of wheel rotations.

   $h = 33.9w$

3. How many rotations does the hamster wheel make if the hamster could run 1 mile? Explain your thinking. **Note:** 12 in. = 1 ft; 5,280 ft = 1 mile.

   $12 \cdot 5280 = 63360$, so 1 mile is equal to 63,360 in.

   $63360 = 33.9w$

   $63360 \div 33.9 = 33.9w \div 33.9$

   $1869.03 \approx w$

   So, the wheel will rotate about 1,869 times if the hamster could run 1 mile.

Are you ready for more?

If the length of the diameter of the hamster wheel was doubled, how would that affect the number of rotations needed to run a certain distance?

Doubling the diameter will cause the number of rotations needed to run a certain distance to be halved. This is because the diameter and circumference are in a proportional relationship, which also means the diameter and distance covered in one rotation are proportional.

256 **Unit 3** Measuring Circles

1 Launch

Activate students' background knowledge by asking whether they have ever seen how a hamster wheel works. You might choose to display a short video of a hamster running on an exercise wheel.

2 Monitor

Help students get started by having them imagine that the wheel is resting on the ground instead of turning on an axle.

Look for points of confusion:

- **Thinking they need to divide 5,280 by 12 to find the number of inches in a mile.** Ask, "Are there going to be more or less than 5,280 inches in a mile?"
- **Not defining variables in Problem 2.** Ask, "What does each variable represent? How would someone else know that?"

Look for productive strategies:

- Using the π button on the calculator.

3 Connect

Ask:

- "What can you tell about the relationship between the number of rotations and distance?"
- "What is the *constant of proportionality* for the relationship?" 33.9
- "If we double the *radius*, will we need more or fewer rotations to cover a certain distance?"

Highlight that there is a *proportional relationship* between the number of rotations of a wheel and the distance that wheel travels. We can say that $D = C \cdot R$, where D is the distance traveled, C is the *circumference*, and R is the number of rotations. There is also a proportional relationship between the radius of the circle and the distance traveled.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Chunk the task in Problem 3 into smaller, more manageable tasks, such as by asking students to do the following:

- Determine the number of inches in 1 mile. Or alternatively, alter the prompt in Problem 3 so that the number of inches is given, instead of 1 mile.
- Determine the number of wheel rotations that are made if the hamster could run this number of inches. Suggest students use their equation from Problem 2 if they are unsure how to proceed.

Math Language Development

MLR5: Co-craft Questions

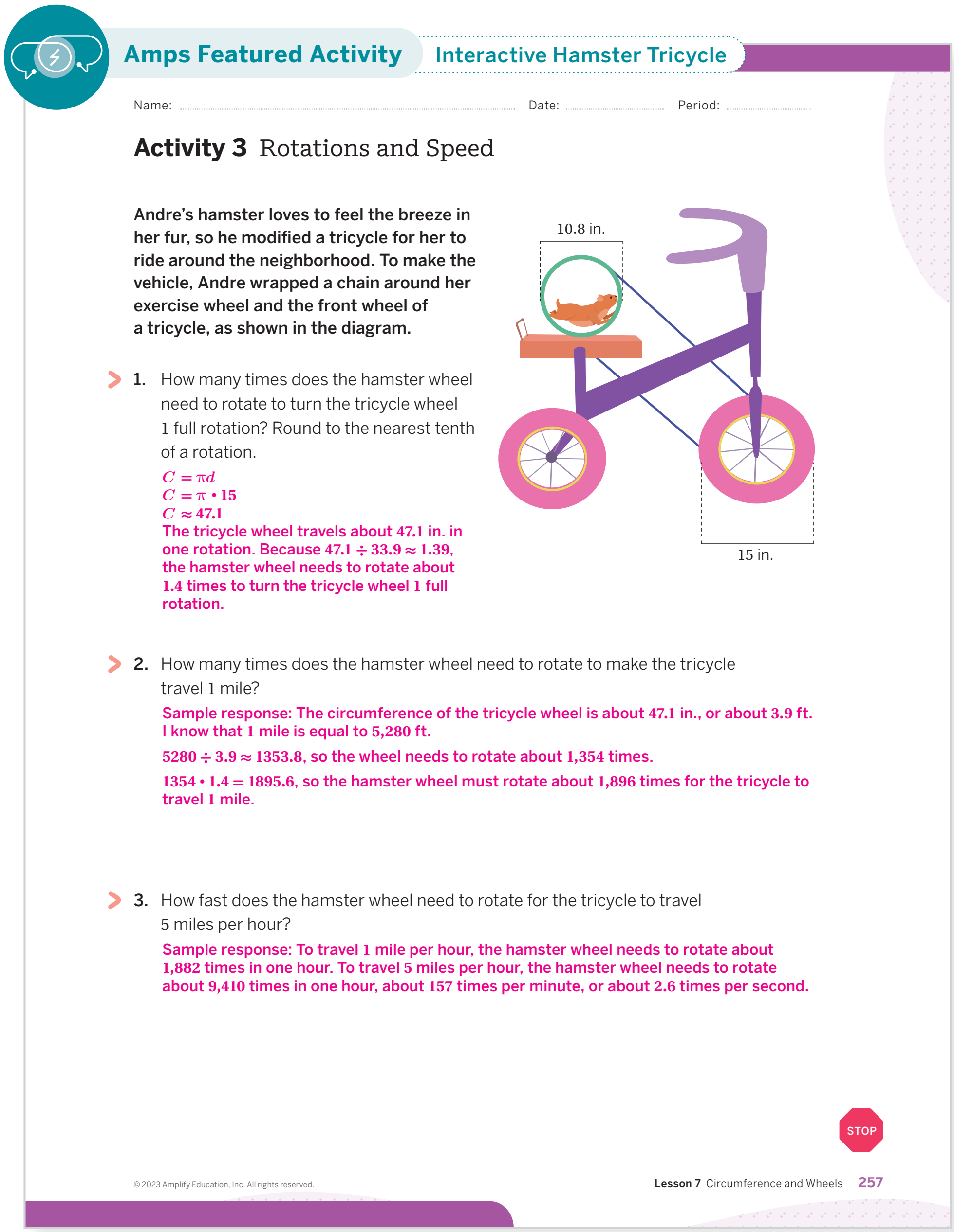
Show a short video clip of a hamster running on an exercise wheel to help students visualize the scenario. Ask students to generate 1–2 mathematical questions that could be asked about the scenario. Display a sample question to help students get started, such as, "How far does the hamster run in one rotation of the wheel?"

Pairs | 15 min

Activity 3 Rotations and Speed

Students relate the rotation of a circular object to the speed it would travel on the ground.

Amps Featured Activity Interactive Hamster Tricycle



Name: ________ Date: ________ Period: ________

Activity 3 Rotations and Speed

Andre's hamster loves to feel the breeze in her fur, so he modified a tricycle for her to ride around the neighborhood. To make the vehicle, Andre wrapped a chain around her exercise wheel and the front wheel of a tricycle, as shown in the diagram.

1. How many times does the hamster wheel need to rotate to turn the tricycle wheel 1 full rotation? Round to the nearest tenth of a rotation.

   $C = \pi d$
   $C = \pi \cdot 15$
   $C \approx 47.1$
   The tricycle wheel travels about 47.1 in. in one rotation. Because $47.1 \div 33.9 \approx 1.39$, the hamster wheel needs to rotate about 1.4 times to turn the tricycle wheel 1 full rotation.

2. How many times does the hamster wheel need to rotate to make the tricycle travel 1 mile?

   Sample response: The circumference of the tricycle wheel is about 47.1 in., or about 3.9 ft. I know that 1 mile is equal to 5,280 ft.

   $5280 \div 3.9 \approx 1353.8$, so the wheel needs to rotate about 1,354 times.

   $1354 \cdot 1.4 = 1895.6$, so the hamster wheel must rotate about 1,896 times for the tricycle to travel 1 mile.

3. How fast does the hamster wheel need to rotate for the tricycle to travel 5 miles per hour?

   Sample response: To travel 1 mile per hour, the hamster wheel needs to rotate about 1,882 times in one hour. To travel 5 miles per hour, the hamster wheel needs to rotate about 9,410 times in one hour, about 157 times per minute, or about 2.6 times per second.

STOP

Lesson 7 Circumference and Wheels 257

1 Launch

Show interactive demonstration of wheels connected by a chain. Ask, "What do you notice?" Highlight that the tricycle wheel is not turning as quickly as the hamster wheel. Have students pause and make a prediction before answering Problem 3.

2 Monitor

Help students get started by asking whether the hamster wheel will need to rotate more or less than one time to make the front tricycle wheel rotate completely.

Look for points of confusion:

- **Losing track of units.** Have students estimate the number of rotations needed to go one foot compared to one mile, and check if their answer is reasonable.
- **Not realizing they need to calculate the number of rotations of the tricycle wheel as their first step in Problem 2.** Remind students that the tricycle wheel is the wheel touching the ground and helping us to measure distance.

Look for productive strategies:

- Determining a unit rate first to reason about the speed in Problem 3.

3 Connect

Have students share whether their predictions prior to Problem 3 were accurate or not. Ask other students to analyze whether the assumptions were reasonable.

Ask, "Do you think it is possible for the hamster to run fast enough to make the tricycle go 5 miles per hour? Why or why not?"

Highlight the necessity of being careful and organized about units while making calculations throughout these problems.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Highlight that the diameter of the hamster wheel and the tricycle wheel are different, so they will not rotate at the same speed. Remind students they determined the length of one rotation of the hamster wheel in Activity 2, which was about 33.9 in. Ask students if they think the tricycle wheel will travel more than or less than 33.9 in. in one full rotation before beginning Problem 1.

Math Language Development

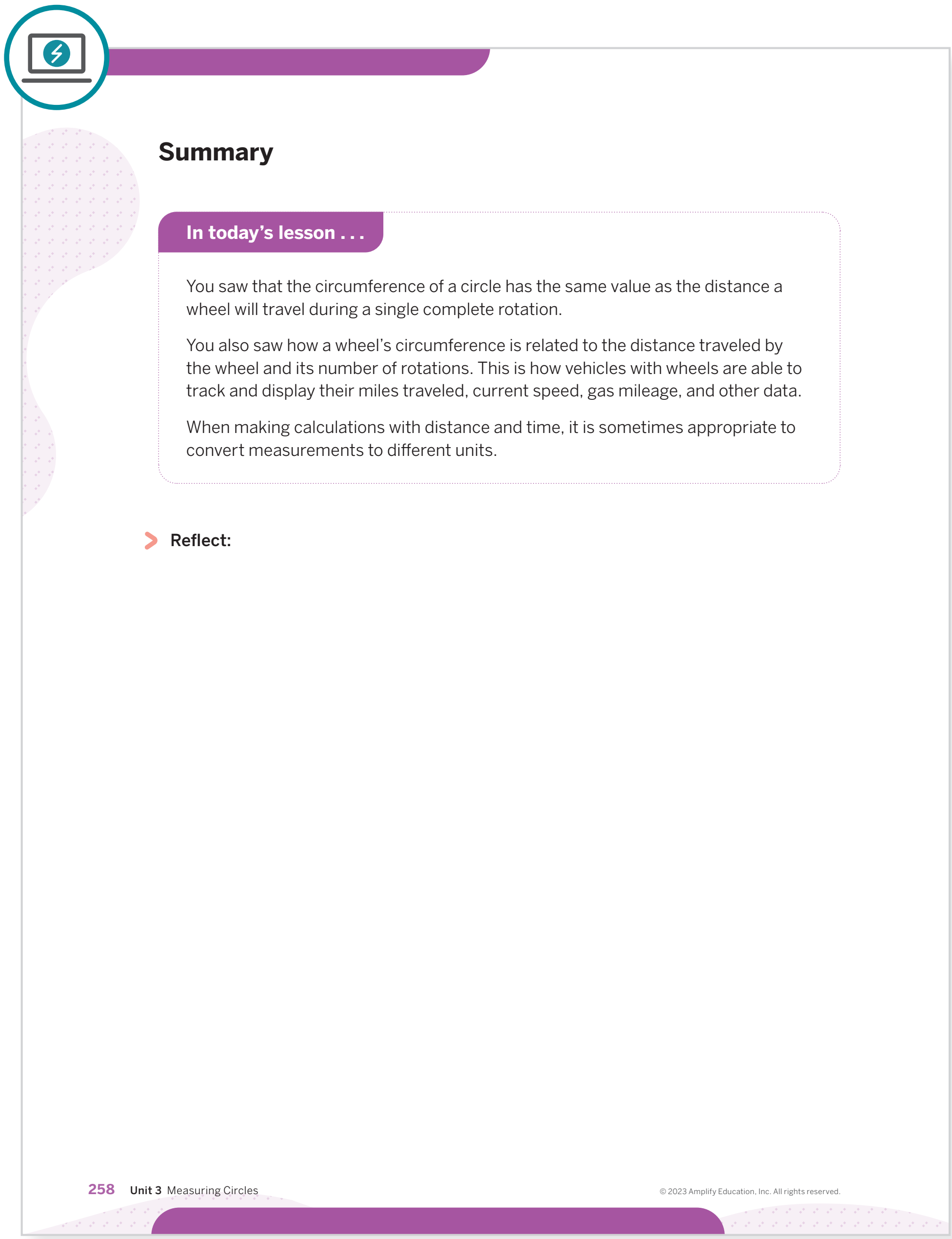
MLR7: Compare and Connect

Before the Connect, ask pairs of students to compare their responses for Problem 3 with another pair of students. Listen for and amplify any language that describes different strategies used, such as rotations per hour, per minute, or per second. Ask, "Which unit is the most helpful for determining whether it is reasonable for the tricycle to travel 5 miles per hour: rotations per hour, rotations per minute, or rotations per second? Why?"

Whole Class | 5 min

Summary

Review and synthesize how the distance a wheel travels is related to its circumference and the number of rotations.



Summary

In today's lesson . . .

You saw that the circumference of a circle has the same value as the distance a wheel will travel during a single complete rotation.

You also saw how a wheel's circumference is related to the distance traveled by the wheel and its number of rotations. This is how vehicles with wheels are able to track and display their miles traveled, current speed, gas mileage, and other data.

When making calculations with distance and time, it is sometimes appropriate to convert measurements to different units.

> **Reflect:**

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Synthesize

Ask, "Does the size of the wheels affect the speed of a vehicle? Why or why not?"

Highlight that knowing the exact size of a vehicle's wheel is important to calculating the speed and distance traveled.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

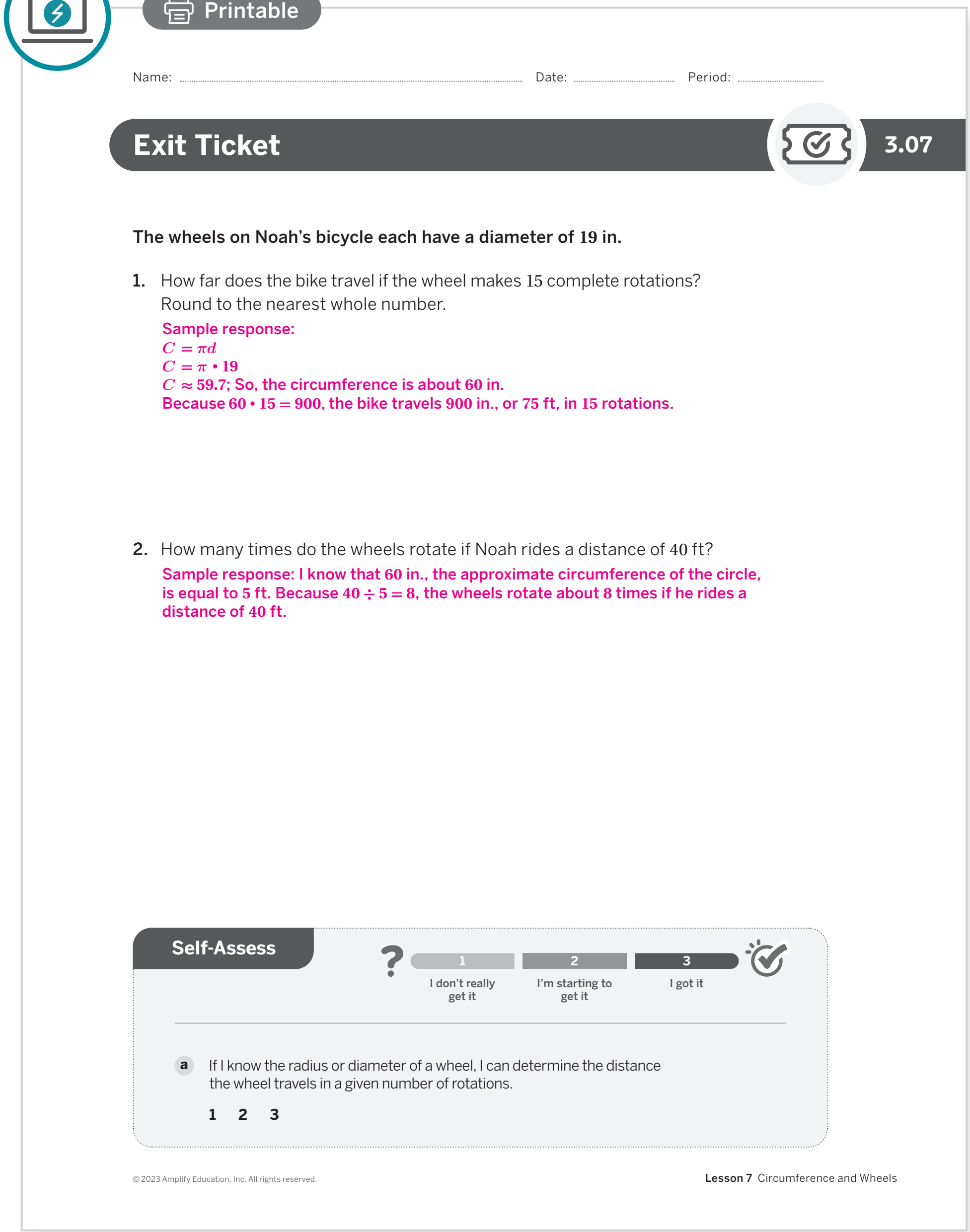
- "What strategies or tools did you find helpful today when determining the distance and speed of a wheel based on its rotation? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the relationship between the number of rotations of a wheel and the distance traveled by determining how far a bicycle will travel with a given size wheel.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.07

The wheels on Noah's bicycle each have a diameter of 19 in.

1. How far does the bike travel if the wheel makes 15 complete rotations? Round to the nearest whole number.

   Sample response:
   $C = \pi d$
   $C = \pi \cdot 19$
   $C \approx 59.7$; So, the circumference is about 60 in.
   Because $60 \cdot 15 = 900$, the bike travels 900 in., or 75 ft, in 15 rotations.

2. How many times do the wheels rotate if Noah rides a distance of 40 ft?

   Sample response: I know that 60 in., the approximate circumference of the circle, is equal to 5 ft. Because $40 \div 5 = 8$, the wheels rotate about 8 times if he rides a distance of 40 ft.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a If I know the radius or diameter of a wheel, I can determine the distance the wheel travels in a given number of rotations.

1 2 3

Lesson 7 Circumference and Wheels

Success looks like . . .

- **Language Goal:** Comparing wheels of different sizes and explaining why a larger wheel needs fewer rotations to travel the same distance. **(Speaking and Listening)**
- **Goal:** Generalizing that the distance a wheel rolls in one rotation is equal to the circumference of the wheel.
  - » Determining that the circumference of the wheel is one rotation in Problem 1.
- **Goal:** Writing an equation to represent the proportional relationship between the number of rotations and the distance a wheel travels.
  - » Writing an equation to determine how far Noah's bike travels in 15 complete rotations of the wheel in Problem 1.

Suggested next steps

If students find the distance traveled in one rotation, consider:

- Having them re-read the problem and underlining relevant information.
- Assigning Practice Problem 2.

If students do not account for the change in the distance unit in Problem 2, consider:

- Having them draw a scale diagram that compares the length of 60 in. with 40 ft.
- Assigning Practice Problem 2.

Professional Learning

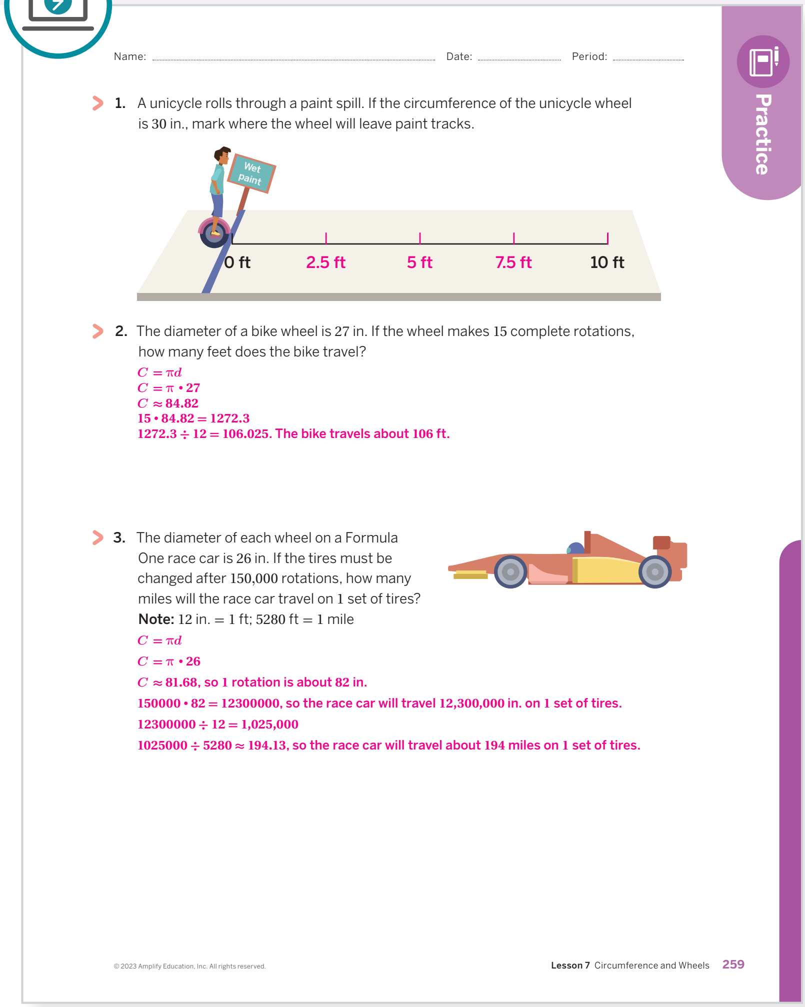
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Who participated and who didn't participate in the Warm-up today? What trends do you see in participation?
- Which students' ideas were you able to highlight during the Warm-up? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A unicycle rolls through a paint spill. If the circumference of the unicycle wheel is 30 in., mark where the wheel will leave paint tracks.

2. The diameter of a bike wheel is 27 in. If the wheel makes 15 complete rotations, how many feet does the bike travel?

$C = \pi d$
$C = \pi \cdot 27$
$C \approx 84.82$
$15 \cdot 84.82 = 1272.3$
$1272.3 \div 12 = 106.025$. The bike travels about 106 ft.

3. The diameter of each wheel on a Formula One race car is 26 in. If the tires must be changed after 150,000 rotations, how many miles will the race car travel on 1 set of tires?
Note: 12 in. = 1 ft; 5280 ft = 1 mile

$C = \pi d$
$C = \pi \cdot 26$
$C \approx 81.68$, so 1 rotation is about 82 in.
$150000 \cdot 82 = 12300000$, so the race car will travel 12,300,000 in. on 1 set of tires.
$12300000 \div 12 = 1{,}025{,}000$
$1025000 \div 5280 \approx 194.13$, so the race car will travel about 194 miles on 1 set of tires.

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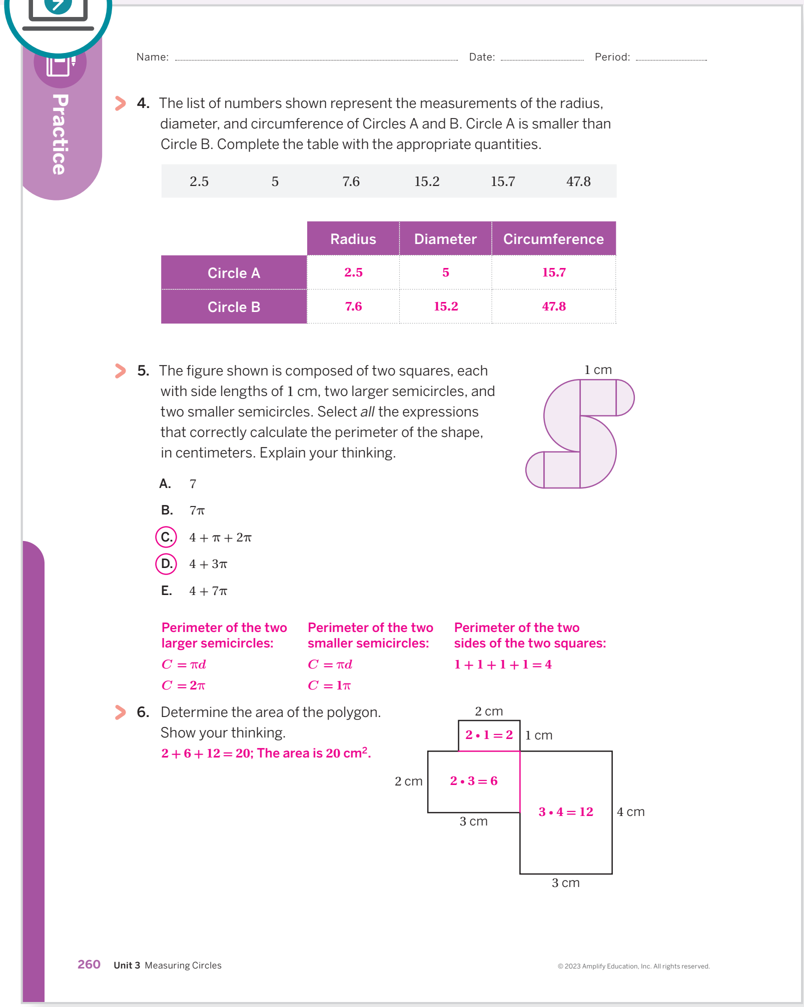
Lesson 7 Circumference and Wheels 259



Name: Date: Period:

4. The list of numbers shown represent the measurements of the radius, diameter, and circumference of Circles A and B. Circle A is smaller than Circle B. Complete the table with the appropriate quantities.

2.5	5	7.6	15.2	15.7	47.8

	Radius	Diameter	Circumference
Circle A	2.5	5	15.7
Circle B	7.6	15.2	47.8

5. The figure shown is composed of two squares, each with side lengths of 1 cm, two larger semicircles, and two smaller semicircles. Select *all* the expressions that correctly calculate the perimeter of the shape, in centimeters. Explain your thinking.

A. 7
B. 7π
[✓ C.] $4 + \pi + 2\pi$
[✓ D.] $4 + 3\pi$
E. $4 + 7\pi$

Perimeter of the two larger semicircles:	Perimeter of the two smaller semicircles:	Perimeter of the two sides of the two squares:
$C = \pi d$	$C = \pi d$	$1 + 1 + 1 + 1 = 4$
$C = 2\pi$	$C = 1\pi$	

6. Determine the area of the polygon. Show your thinking.
$2 + 6 + 12 = 20$; The area is 20 cm^2.

260 **Unit 3** Measuring Circles

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 2	2
	3	Activity 3	3
Spiral	4	Unit 3 Lesson 4	2
	5	Unit 3 Lesson 6	2
Formative ⚡	6	Unit 3 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 2
Area of Circles

Whole Class

Area is measured in square units; circles are definitively not squares. How can these be reconciled? Students confront the issue head-on using estimation, counting, cutting, rearranging, and covering.

SUB-UNIT

2 Area of Circles

Narrative Connections

What makes a circle so perfect?

People have admired circles for thousands of years. Ancient Babylonian mathematicians, as well as Chinese, Egyptian, and Greek mathematicians, further sought to understand the proportions of circles. They examined the ratio between the distance around a circle and the length across its middle. They found the ratio was always exactly the same number. A number so cool, it got its own name: pi (or in Greek: π)

But what is pi exactly? It's a little greater than three. To be precise, it's a little greater than 3.14 but a little less than 3.15. To be even more precise, it's close to 3.14159265359, but that's not quite it either. In fact, the digits after the decimal point go on forever, never terminating or settling into a repeating pattern.

Every circle you see contains this number within its proportions, whether it's a nickel or the Earth's equator. (Neither of which is a perfect circle, but they are both pretty close!)

People have gone to great lengths to show their affection for this special number. Retired engineer Akira Haraguchi memorized over 110,000 digits of pi. It took him more than half a day to recite them. In 2019, computer scientist Emma Haruka Iwao computed over 31 trillion digits of pi. The computation took 25 virtual machines four months to complete, and made Iwao a world-record holder. Of course, you'll never need that many digits of pi to actually do anything—the first few digits usually suffice.

Next time you're jingling coins in your pocket, or surfing through a cylindrical ocean wave, think of the beauty of circles, and that special ratio pi.

Sub-Unit 2 Area of Circles 261

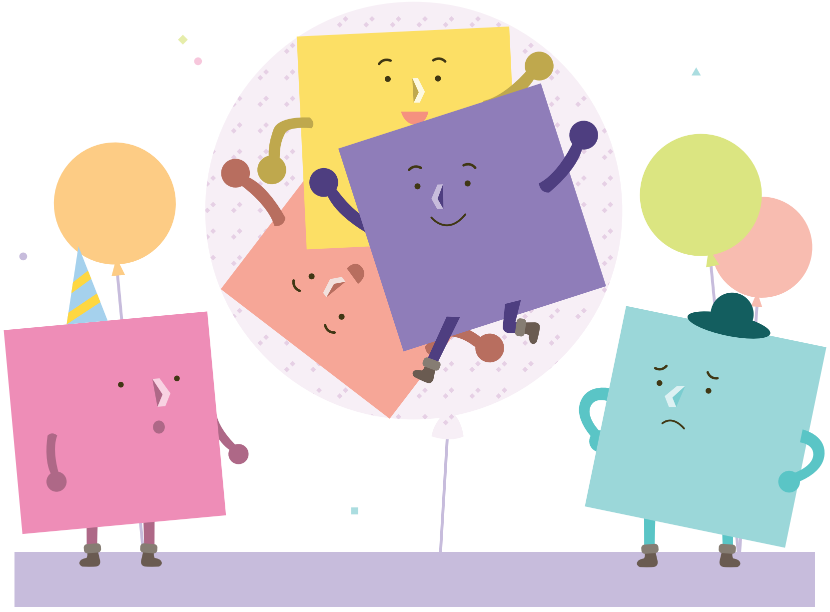
Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how the constant π is used to approximate the area of a circle in the following places:

- **Lesson 8, Activity 2:** Covering a Circle
- **Lesson 9, Activities 1-2:** Approximating a Circle, Relating Area to Circumference
- **Lesson 11, Activity 2:** Analyzing Circle Claims

UNIT 3 | **LESSON 8**

Exploring the Area of a Circle



Let's investigate the areas of circles.

Focus

Goals

1. Estimate the area of a circle on a grid by decomposing and approximating it with polygons.
2. Estimate the area of a circle with a radius of r by comparing it to the area of a square with a side length of r.

Rigor

- Students build **conceptual understanding** of how to approximate the area of a circle based on the relationship of its radius and a square with a side length equal to its radius.

Coherence

Today

Students explore how to find the area of a circle through different approaches. Students begin by estimating the area of a circle on a grid. Then they estimate the area of a circle by arranging squares inside the circle with side lengths equal to the length of the circle's radius.

Previously

In Lessons 4–6, students learned that the relationship between the circumference of a circle and its diameter is a proportional relationship with the constant of proportionality of π.

Coming Soon

In Lesson 9, students will relate the circumference of a circle to its area in order to find a more precise formula for calculating the area of a circle.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	15 min	5 min	10 min
Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one card per small group
- scissors

Math Language Development

Review words

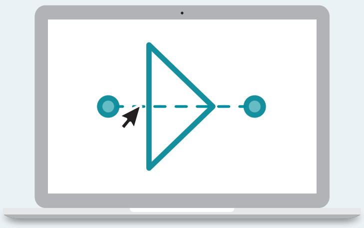
- *diameter*
- *radius*
- *regular polygon*

Amps | Featured Activity

Activity 2

Covering a Circle

This digital tool helps students efficiently partition and rearrange the squares to find how many fit inside the circle.



Building Math Identity and Community

Connecting to Mathematical Practices

Because the squares do not cover the circle perfectly, students might feel that they cannot explain how the squares are used to approximate the area of the circle. Remind them that the goal is to approximate the circle's area, not find the exact area and that each student could have a different argument for how they found the estimated area.

Modifications to Pacing

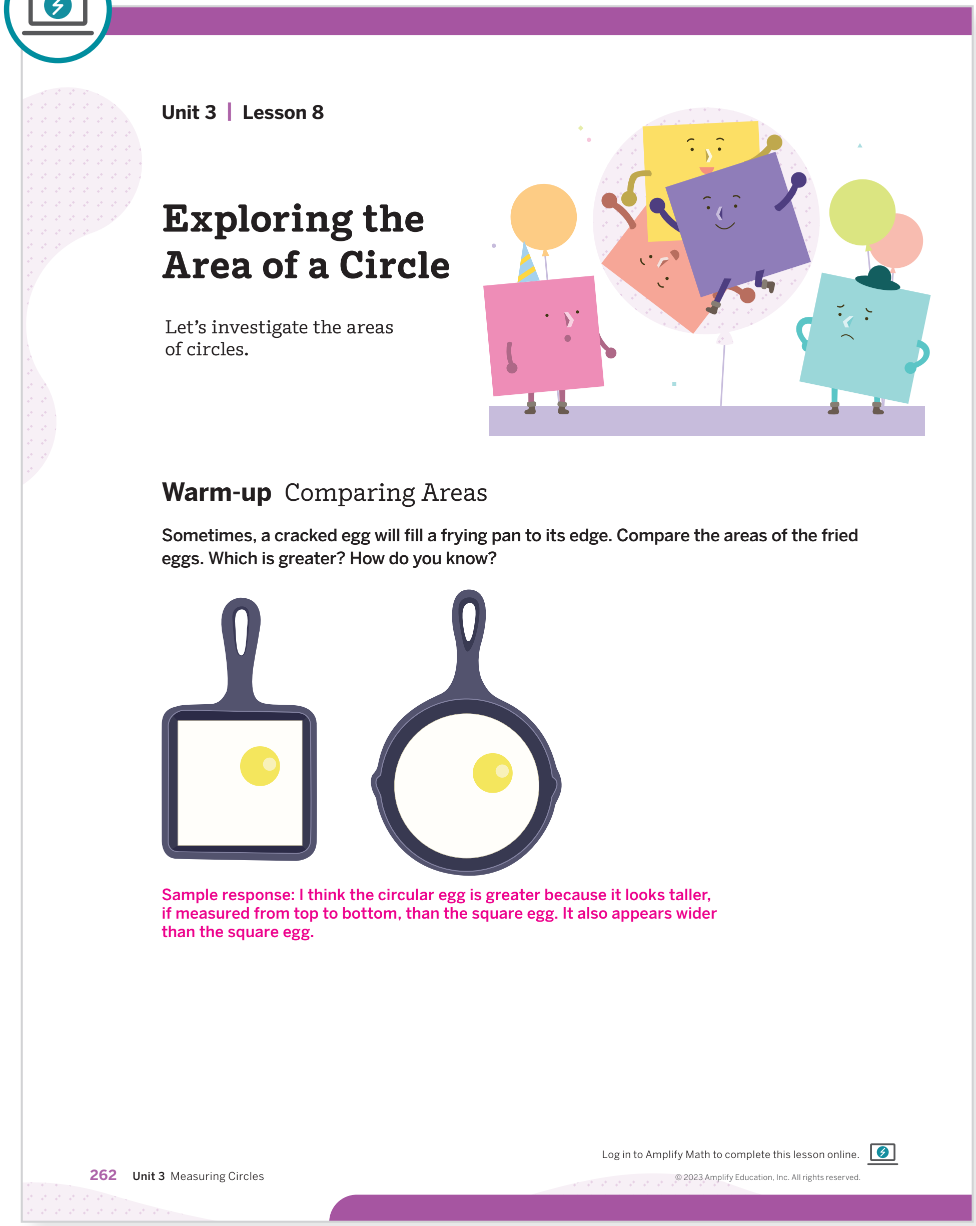
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, you may choose to go straight to the discussion instead of having students write their responses.
- In **Activity 1**, simply display the Figures A, B, and C and ask, "What makes determining the area of a circle more challenging than a rectangle or parallelogram?" Then move on to Activity 2.
- In **Activity 2**, you may omit Problem 1.

Pairs | 5 min

Warm-up Comparing Areas

Students use informal reasoning to compare the area of a square and circle.



Unit 3 | **Lesson 8**

Exploring the Area of a Circle

Let's investigate the areas of circles.

Warm-up Comparing Areas

Sometimes, a cracked egg will fill a frying pan to its edge. Compare the areas of the fried eggs. Which is greater? How do you know?

Sample response: I think the circular egg is greater because it looks taller, if measured from top to bottom, than the square egg. It also appears wider than the square egg.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Activate background knowledge by asking if students have ever fried eggs. Highlight that students should think about the area of the top surface of the egg in the frying pan.

2 Monitor

Help students get started by asking, "How have you found the areas of shapes in the past? What's different this time?"

Look for points of confusion:

- **Thinking that because it's the same number of eggs the area should be the same.** Say, "The surface of the egg will fill the pan, and you don't need to worry about how much egg is underneath."

Look for productive strategies:

- Sketching the square egg on top of the circular egg.
- Breaking the circle up into polygonal areas to compare.

3 Connect

Have pairs of students share their estimations for the areas of the eggs shown.

Ask:

- "What are some of the strategies you came up with to solve this problem?"
- "If your partner reached a different solution, what did you discuss in order to understand their strategy?"

Highlight that certain problems do not have a clear-cut answer, but it is important to have a problem-solving plan. Say, "As you further investigate the area of circles, you will see how it can be difficult to get an exact answer."

Power-up

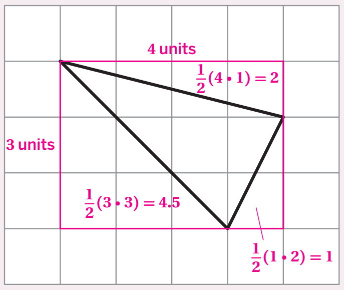
To power up students' ability to compose and decompose polygons in order to determine their areas, have students complete:

Determine the area of the triangle by first composing a rectangle that surrounds it.

$(4 \cdot 3) - (4.5 + 2 + 1)$

$= 12 - 7.5$

$= 4.5$; The triangle is 4.5 square units.



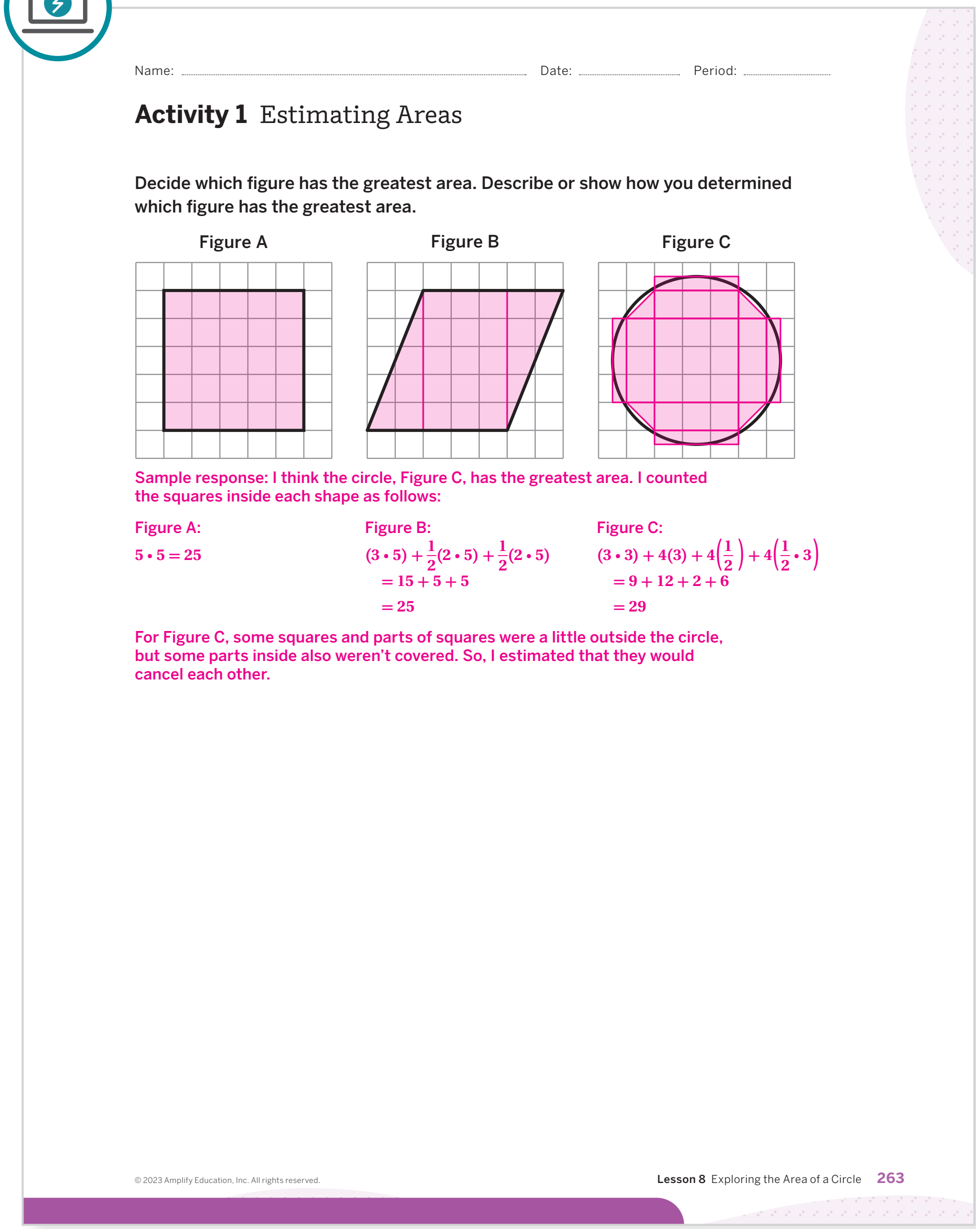
Use: Before Activity 1.

Informed by: Performance on Lesson 7, Practice Problem 6. and Pre-Unit Readiness Assessment, Problem 4.

Pairs | 10 min

Activity 1 Estimating Areas

Students use a grid of unit squares to find the area of a circle and consider the benefits and challenges of such an approach.



Name: ______________ Date: ________ Period: ________

Activity 1 Estimating Areas

Decide which figure has the greatest area. Describe or show how you determined which figure has the greatest area.

Sample response: I think the circle, Figure C, has the greatest area. I counted the squares inside each shape as follows:

Figure A:

$5 \cdot 5 = 25$

Figure B:

$$\begin{aligned}&(3 \cdot 5) + \frac{1}{2}(2 \cdot 5) + \frac{1}{2}(2 \cdot 5)\\ &= 15 + 5 + 5\\ &= 25\end{aligned}$$

Figure C:

$$\begin{aligned}&(3 \cdot 3) + 4(3) + 4\left(\frac{1}{2}\right) + 4\left(\frac{1}{2} \cdot 3\right)\\ &= 9 + 12 + 2 + 6\\ &= 29\end{aligned}$$

For Figure C, some squares and parts of squares were a little outside the circle, but some parts inside also weren't covered. So, I estimated that they would cancel each other.

1 Launch

Have students briefly look at Figures A, B, and C, and then have them cover the images. Ask students to make a conjecture, based on their intuition, about which one has the greatest area.

2 Monitor

Help students get started by asking, "How is comparing the areas in this problem different from the Warm-up? What strategies can help you?"

Look for points of confusion:

- **Difficulty finding the areas of partial squares.** Encourage students to estimate or find the dimensions of larger polygonal regions.
- **Thinking that the circle needs to be covered entirely by full squares.** Ask, "The area is the space *enclosed* by a shape. Is your estimate as accurate as possible?"

Look for productive strategies:

- Decomposing the circle into rectangular and triangular regions.
- Drawing Figure A over Figures B and C.

3 Connect

Display at least two different student strategies for determining the area of Figure C.

Ask:

- "Does anyone feel confident they found the exact area of Figure A? How about for Figure C? Why or why not?"
- "What makes finding the area of a circle more challenging than a rectangle or parallelogram?"
- "Are there any better approaches to finding a more accurate measure of the area of a circle?"

Highlight that finding an accurate measure of the area of a circle has been a problem that humans have grappled with for millenia.

Note: Throughout this sub-unit, references to the area of a circle can be taken to mean the area of the region *inside* of a circle.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they learned about the area of polygons, such as squares, rectangles, and parallelograms in prior grades. Display the area formulas for a square and parallelogram and consider annotating the base and height on a visual example of a parallelogram before students begin the activity.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you display student strategies for determining the area of Figure C, ask students to share what they found challenging about determining the area of the circle. Encourage students to compare strategies that were used and identify which strategies were the most efficient and/or the most accurate.

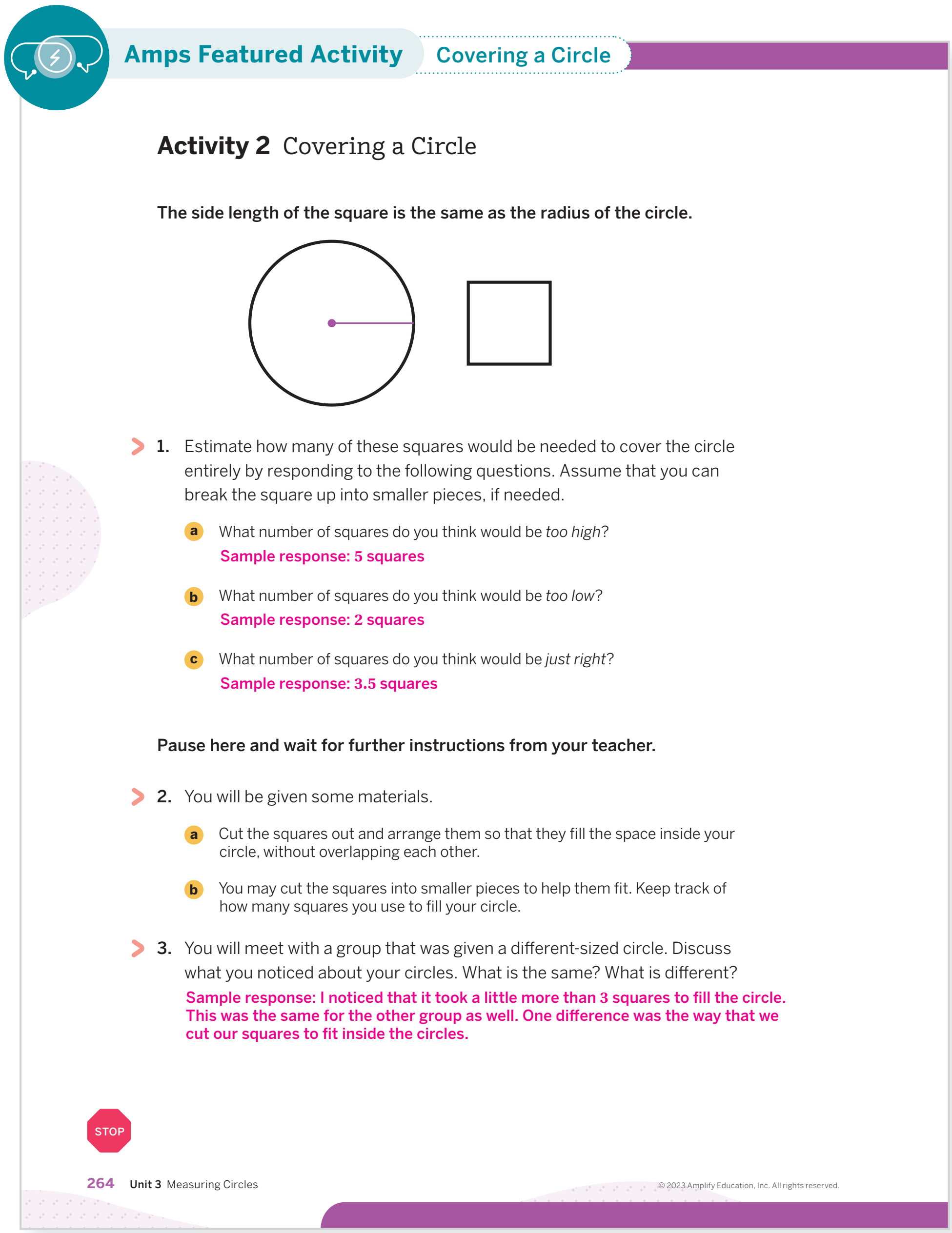
English Learners

Consider annotating the circle as students share their strategies. For example, for the strategy shown in the sample response, label the parts of squares that were *outside the circle* and the parts of the grid squares that *were not covered*.

Small Groups | 15 min

Activity 2 Covering a Circle

Students see how the square of a circle's radius relates to its area to generalize a formula.



Amps Featured Activity Covering a Circle

Activity 2 Covering a Circle

The side length of the square is the same as the radius of the circle.

1. Estimate how many of these squares would be needed to cover the circle entirely by responding to the following questions. Assume that you can break the square up into smaller pieces, if needed.
   - a What number of squares do you think would be *too high*? **Sample response: 5 squares**
   - b What number of squares do you think would be *too low*? **Sample response: 2 squares**
   - c What number of squares do you think would be *just right*? **Sample response: 3.5 squares**

Pause here and wait for further instructions from your teacher.

2. You will be given some materials.
   - a Cut the squares out and arrange them so that they fill the space inside your circle, without overlapping each other.
   - b You may cut the squares into smaller pieces to help them fit. Keep track of how many squares you use to fill your circle.
3. You will meet with a group that was given a different-sized circle. Discuss what you noticed about your circles. What is the same? What is different? **Sample response: I noticed that it took a little more than 3 squares to fill the circle. This was the same for the other group as well. One difference was the way that we cut our squares to fit inside the circles.**

STOP

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1 Launch

Read the introduction together to ensure students understand the connection between the radius of the circle and the side length of the square. Use the ***Poll the Class*** routine to elicit student responses to Problem 1, and then distribute the Activity 2 PDF and scissors.

2 Monitor

Help students get started by suggesting students position the cut-out squares on top of the circle.

Look for points of confusion:

- **Not breaking up and rearranging the leftover parts of the square.** Ask, "Are your squares covering the same space as the circle?"
- **Not keeping track of how many squares have been used to cover the circle.** Remind students that the goal is to find how many of the squares cover the circle.

3 Connect

Display two different-sized circles with a similar arrangement of the square pieces. Then, display same-sized circles with two different arrangements of squares.

Ask, "What conclusions can you draw from observing the number of squares that fit into each of these circles?" About 3 squares with a side length equal to the radius can fit inside the circle.

Highlight that in Lesson 9, students will explore a more accurate method for finding the area of a circle. However, for now, it is sufficient to know that the area is a little more than 3 times the size of a square with a side length equal to the circle's *radius*.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Alternatively, if you choose to use the Activity 2 PDF, have students cut their squares uniformly. For example, have them cut their squares into halves and then fourths. This will help them stay organized as they cut smaller pieces.

MLR Math Language Development

MLR7: Compare and Connect

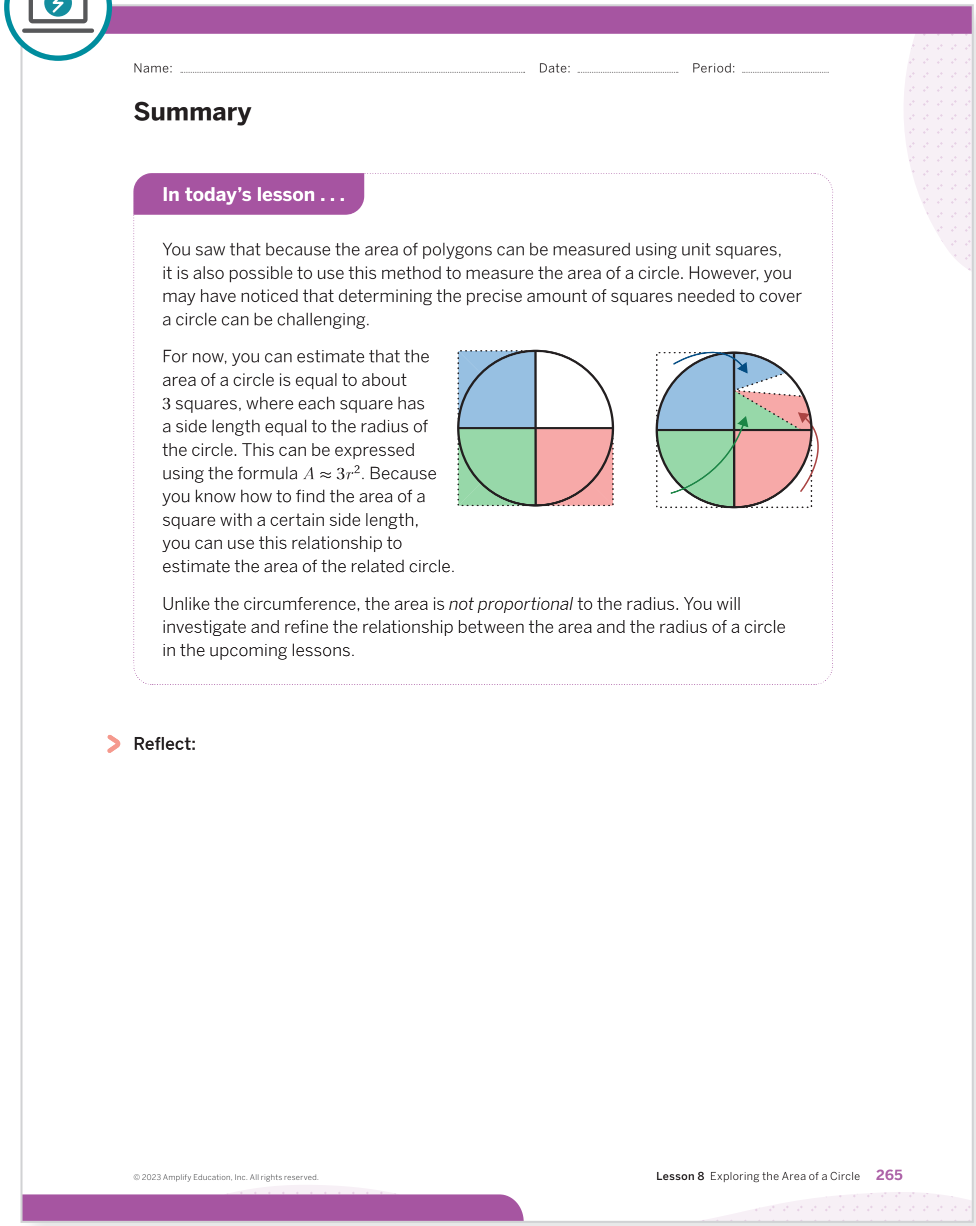
During the Connect, as students respond to the Ask question, draw their attention to the fact that the side length of each square used was the same length as the radius of the circle. Consider asking these probing questions to drive home the connection:

- "What does the area of one square represent, in terms of the length of the radius?" The area of one square is the square of the length of the radius, or $(\text{radius})^2$.
- "If a little more than 3 squares fit inside the circle, what is the relationship between the circle's area and the radius? The circle's area is a little more than 3 squares, where the area of each square represents $(\text{radius})^2$.

Whole Class | 5 min

Summary

Review and synthesize that the area of any circle is a little more than 3 times the area of a square whose side length is equal to the circle's radius.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that because the area of polygons can be measured using unit squares, it is also possible to use this method to measure the area of a circle. However, you may have noticed that determining the precise amount of squares needed to cover a circle can be challenging.

For now, you can estimate that the area of a circle is equal to about 3 squares, where each square has a side length equal to the radius of the circle. This can be expressed using the formula $A \approx 3r^2$. Because you know how to find the area of a square with a certain side length, you can use this relationship to estimate the area of the related circle.

Unlike the circumference, the area is *not proportional* to the radius. You will investigate and refine the relationship between the area and the radius of a circle in the upcoming lessons.

Reflect:

Lesson 8 Exploring the Area of a Circle 265

Synthesize

Highlight that, as with circumference, mathematicians throughout history have worked to solve the question of exactly how many squares with side lengths equal to the radius of a circle will fit inside that circle.

Ask, "Can anyone make a conjecture regarding the number of squares with a side length equal to the radius that will fit inside the circle?" The area of a circle is equal to approximately 3 squares, each with a side length that equals the radius of the circle.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

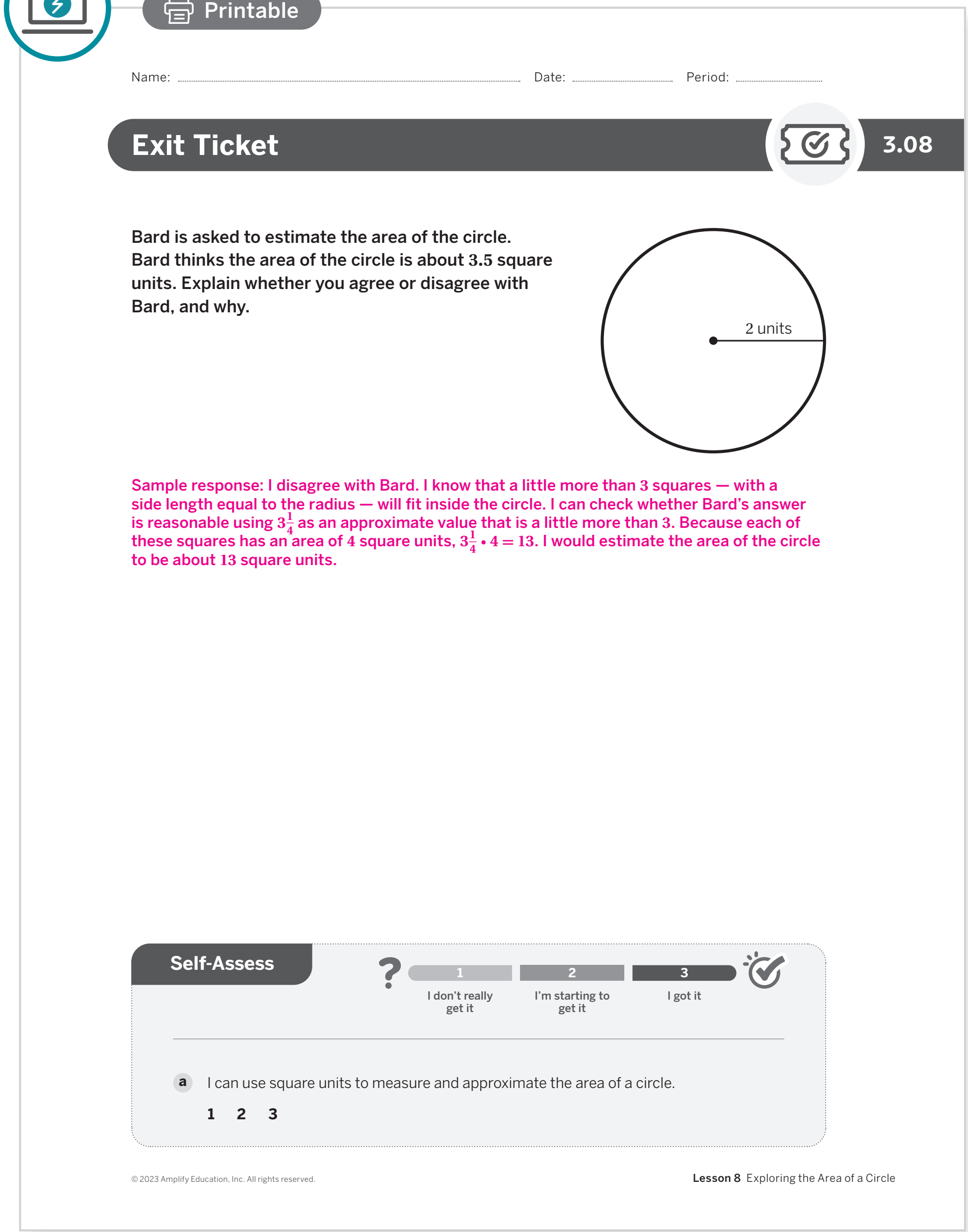
- "How can squares help you measure the space inside circles?

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of estimating the area of a circle by using its relationship to a square with a side length equal to the circle's radius.

Printable



Name: ____ Date: ____ Period: ____

Exit Ticket 3.08

Bard is asked to estimate the area of the circle. Bard thinks the area of the circle is about 3.5 square units. Explain whether you agree or disagree with Bard, and why.

Sample response: I disagree with Bard. I know that a little more than 3 squares — with a side length equal to the radius — will fit inside the circle. I can check whether Bard's answer is reasonable using $3\frac{1}{4}$ as an approximate value that is a little more than 3. Because each of these squares has an area of 4 square units, $3\frac{1}{4} \cdot 4 = 13$. I would estimate the area of the circle to be about 13 square units.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use square units to measure and approximate the area of a circle.

1 2 3

Success looks like . . .

- **Goal:** Estimating the area of a circle on a grid by decomposing and approximating it with polygons.
- **Goal:** Estimating the area of a circle with a radius of r by comparing it to the area of a square with a side length of r.
  - » Approximating the area of the circle by using squares with side lengths equal to 2 units.

Suggested next steps

If students think Bard is correct because Bard found that 3.5 is about the number of squares that fit inside the circle in Activity 2, consider:

- Asking students to calculate the area of the square first.

Professional Learning

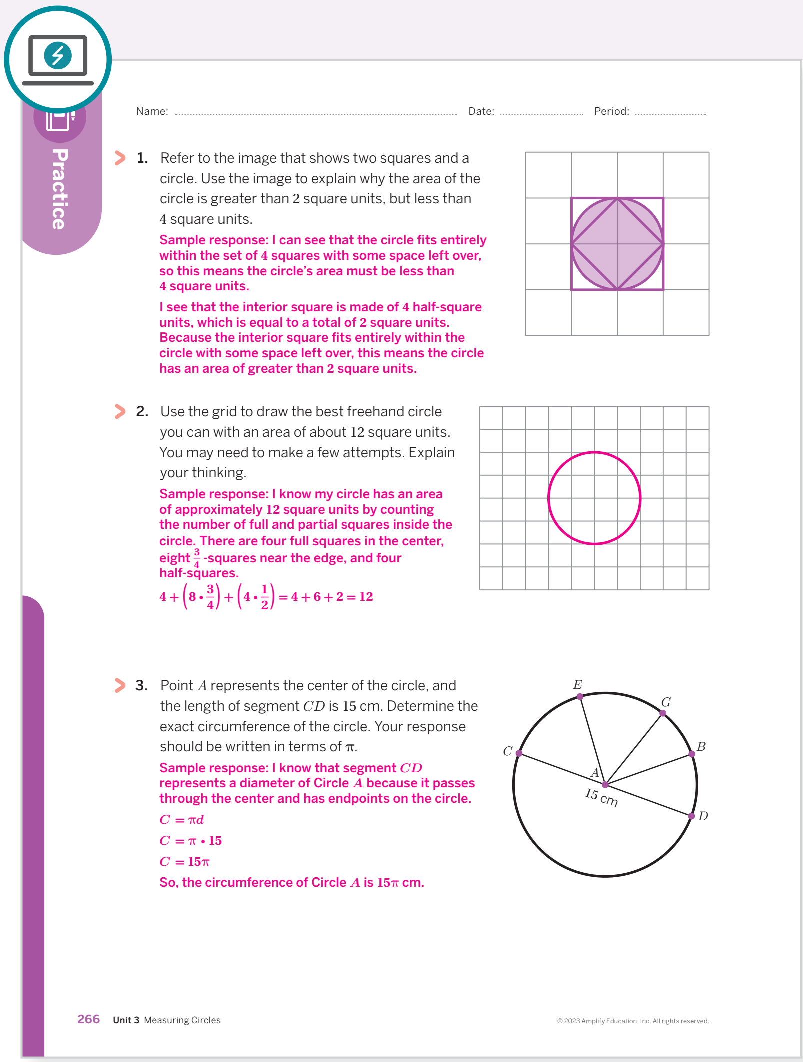
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find frustrating about Activity 2? What helped them work through this frustration?
- Have you changed any ideas you used to have about students exploring how to find the area of a circle as a result of today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Refer to the image that shows two squares and a circle. Use the image to explain why the area of the circle is greater than 2 square units, but less than 4 square units.

   Sample response: I can see that the circle fits entirely within the set of 4 squares with some space left over, so this means the circle's area must be less than 4 square units.

   I see that the interior square is made of 4 half-square units, which is equal to a total of 2 square units. Because the interior square fits entirely within the circle with some space left over, this means the circle has an area of greater than 2 square units.

2. Use the grid to draw the best freehand circle you can with an area of about 12 square units. You may need to make a few attempts. Explain your thinking.

   Sample response: I know my circle has an area of approximately 12 square units by counting the number of full and partial squares inside the circle. There are four full squares in the center, eight $\frac{3}{4}$-squares near the edge, and four half-squares.

   $4+\left(8 \cdot \frac{3}{4}\right)+\left(4 \cdot \frac{1}{2}\right)=4+6+2=12$

3. Point A represents the center of the circle, and the length of segment CD is 15 cm. Determine the exact circumference of the circle. Your response should be written in terms of π.

   Sample response: I know that segment CD represents a diameter of Circle A because it passes through the center and has endpoints on the circle.

   $C=\pi d$

   $C=\pi \cdot 15$

   $C=15\pi$

   So, the circumference of Circle A is 15π cm.

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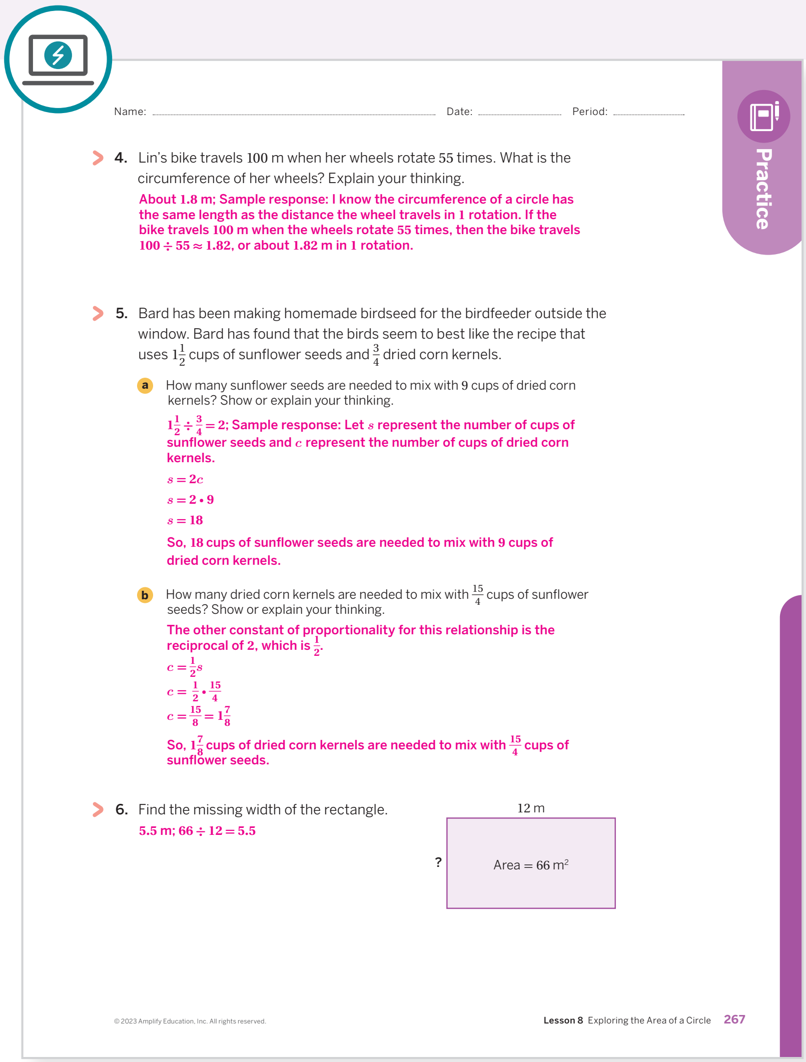


Practice

Name: Date: Period:

4. Lin's bike travels 100 m when her wheels rotate 55 times. What is the circumference of her wheels? Explain your thinking.

   About 1.8 m; Sample response: I know the circumference of a circle has the same length as the distance the wheel travels in 1 rotation. If the bike travels 100 m when the wheels rotate 55 times, then the bike travels $100 \div 55 \approx 1.82$, or about 1.82 m in 1 rotation.

5. Bard has been making homemade birdseed for the birdfeeder outside the window. Bard has found that the birds seem to best like the recipe that uses $1\frac{1}{2}$ cups of sunflower seeds and $\frac{3}{4}$ dried corn kernels.

   a. How many sunflower seeds are needed to mix with 9 cups of dried corn kernels? Show or explain your thinking.

      $1\frac{1}{2} \div \frac{3}{4}=2$; Sample response: Let s represent the number of cups of sunflower seeds and c represent the number of cups of dried corn kernels.

      $s=2c$

      $s=2 \cdot 9$

      $s=18$

      So, 18 cups of sunflower seeds are needed to mix with 9 cups of dried corn kernels.

   b. How many dried corn kernels are needed to mix with $\frac{15}{4}$ cups of sunflower seeds? Show or explain your thinking.

      The other constant of proportionality for this relationship is the reciprocal of 2, which is $\frac{1}{2}$.

      $c=\frac{1}{2}s$

      $c=\frac{1}{2} \cdot \frac{15}{4}$

      $c=\frac{15}{8}=1\frac{7}{8}$

      So, $1\frac{7}{8}$ cups of dried corn kernels are needed to mix with $\frac{15}{4}$ cups of sunflower seeds.

6. Find the missing width of the rectangle.

   5.5 m; $66 \div 12=5.5$

Lesson 8 Exploring the Area of a Circle 267

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	3
Spiral	3	Unit 3 Lesson 4	2
	4	Unit 3 Lesson 7	2
	5	Unit 2 Lesson 8	2
Formative	6	Unit 3 Lesson 9	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3 | **LESSON 9**

Relating Area to Circumference

Let’s rearrange circles to calculate their areas.

Focus

Goals

1. **Language Goal:** Show how a circle can be decomposed and rearranged to approximate a polygon, and justify that the area of this polygon is equal to half of the circle’s circumference multiplied by its radius. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Generalize a process for determining the area of a circle, and justify why this can be represented by πr^2. **(Speaking and Listening)**
3. Determine the area of a circle when given the circumference, diameter, or radius.

Coherence

Today

Students develop the formula for the area of a circle, $A = \pi r^2$, through informal dissection of arguments. In the opening activity, students cut and rearrange a rectangle into a shape that approximates a circle, and then reason about how the circumference and radius of the circle relate to the length and width of the rectangle.

Previously

In Lesson 8, students found that it takes a little more than 3 squares with side lengths equal to the circle’s radius to completely cover a circle. Students may have predicted that the area of a circle can be found by multiplying πr^2.

Coming Soon

In Lesson 10, students will apply their reasoning and formulas to find the area of shapes containing circular parts.

Rigor

- Students build **conceptual understanding** of how the formula for the area of a circle was derived and why it includes π.
- Students **apply** the area formula to solve problems about circles.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
10 min	10 min	10 min	10 min	5 min	5 min
Small Groups	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, pre-cut cards, one card per group
- Anchor Chart PDF, *Circles* (for display)
- Graphic Organizer PDF, *Working with Circles (Part 2)* (as needed)
- calculators
- colored pencils
- glue or tape (optional)
- scissors

Math Language Development

Review words

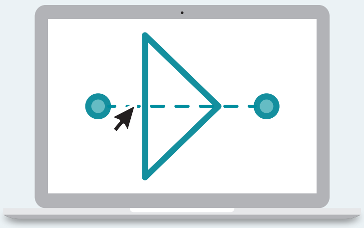
- *circumference*
- *diameter*
- π
- *radius*

Amps | Featured Activity

Activity 1

Polygon Rearrangement

Students explore how to rearrange a regular polygon into a rectangular shape, and vice versa.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might struggle to complete the connection between the model they use and the formula for area of a square. Challenge students to organize their thinking by labeling the parts of the model to make the symbolic representation more concrete. Labeling will connect the parts of the formula to the parts of the model.

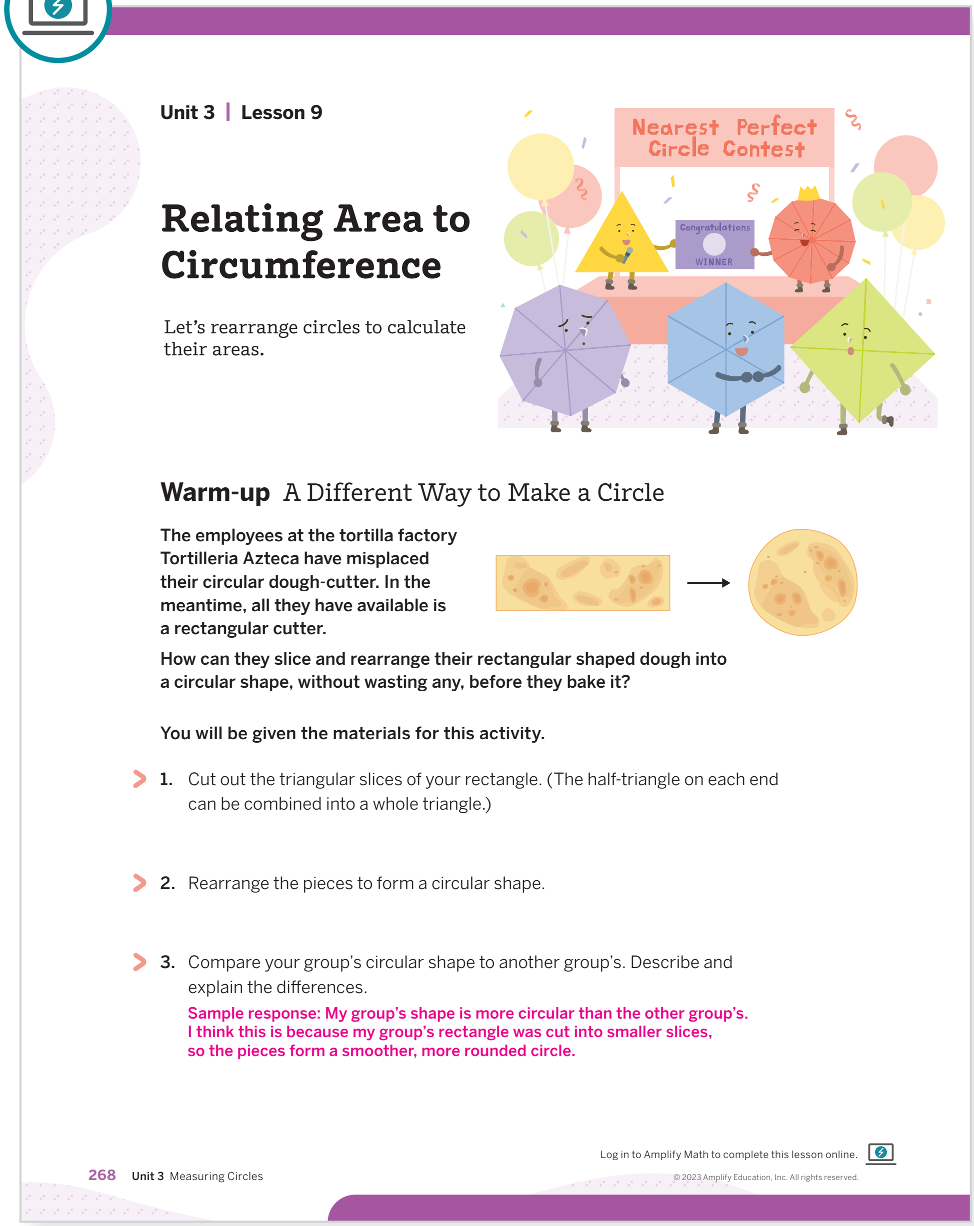
Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- **Activity 3** may be completed during Practice.

Warm-up A Different Way to Make a Circle

Students cut a rectangle into uniformly-sized slices and rearrange them into a circular shape to compare how different-sized slices form different regular polygons.



Unit 3 | Lesson 9

Relating Area to Circumference

Let's rearrange circles to calculate their areas.

Warm-up A Different Way to Make a Circle

The employees at the tortilla factory Tortilleria Azteca have misplaced their circular dough-cutter. In the meantime, all they have available is a rectangular cutter.

How can they slice and rearrange their rectangular shaped dough into a circular shape, without wasting any, before they bake it?

You will be given the materials for this activity.

1. Cut out the triangular slices of your rectangle. (The half-triangle on each end can be combined into a whole triangle.)
2. Rearrange the pieces to form a circular shape.
3. Compare your group's circular shape to another group's. Describe and explain the differences.
   Sample response: My group's shape is more circular than the other group's. I think this is because my group's rectangle was cut into smaller slices, so the pieces form a smoother, more rounded circle.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Ask, "Is it possible to slice and rearrange a rectangle into a circle?" Conduct the ***Think-Pair-Share*** routine. Distribute the pre-cut cards of the Warm-up PDF, scissors, and glue or tape (optional).

Note: Allowing students to glue or tape their shape down may make it easier to share during the Connect.

2 Monitor

Help students get started by prompting them to think of how a pizza is sliced and arranged.

Look for points of confusion:

- **Not orienting the triangles so that the narrowest vertex is pointing to the center.** Remind students that the goal is to arrange the pieces into a circular shape.

Look for productive strategies:

- Noticing that pointing the narrowest part of the triangle toward the center forms the most circle-like shape.
- Discussing how even more slices would create a more circle-like shape.

3 Connect

Have students share their arrangements with the rest of the class. Be sure to have each different set shared by at least one group.

Ask, "What caused some of the groups to have more circle-like shapes than others?" The skinnier triangles have shorter bases, and, therefore, the edges of the shape are smoother.

Highlight how in Lesson 8, students determined that the area of a circle is a little more than 3 squares with sides equal to the circle's radius. Say, "Today, we will explore a more precise method for finding the area of a circle."

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their arrangements and respond to the Ask question, highlight connections between the arrangements that created more circle-like shapes than other arrangements. Consider asking these follow-up questions:

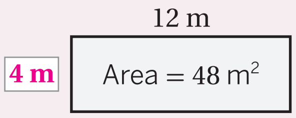
- "As the number of triangular slices increased, what happened to the shape that you could form?" The shape looked more like a circle.
- "What do you know about the area of the rectangle compared to the area of the shape you could form?" The areas are the same.

Power-up

To power up students' ability to determine unknown side length when given the area and one side length of a rectangle, have students complete:

1. Which formula would be helpful in determining the missing side length. Select *all* that apply:

   A. $\ell = w \cdot A$
   [✓ B.] $\ell \cdot w = A$
   [✓ C.] $\ell = A \div w$
   D. $P = 2\ell + 2w$
   E. $\ell = P - w$



2. Determine the missing side length. 4 m

Use: Before Activity 1.

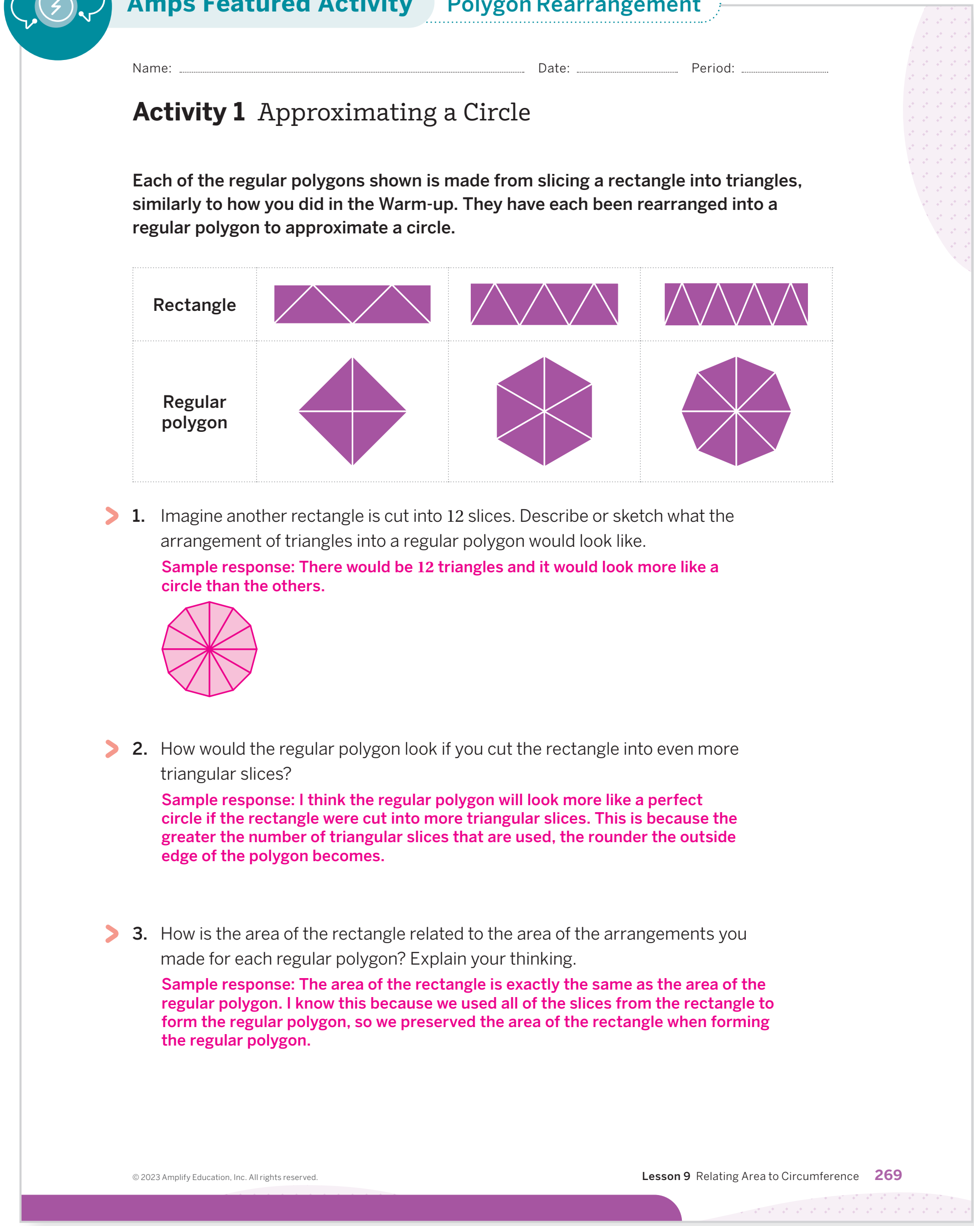
Informed by: Performance on Lesson 8, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6.

Pairs | 10 min

Activity 1 Approximating a Circle

Students reason that several regular polygons, sliced from the same-sized rectangle, become more circular as the number of triangular slices increases.

Amps Featured Activity Polygon Rearrangement



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Activity 1 Approximating a Circle

Each of the regular polygons shown is made from slicing a rectangle into triangles, similarly to how you did in the Warm-up. They have each been rearranged into a regular polygon to approximate a circle.

Rectangle			
Regular polygon			

1. Imagine another rectangle is cut into 12 slices. Describe or sketch what the arrangement of triangles into a regular polygon would look like.
   Sample response: There would be 12 triangles and it would look more like a circle than the others.

2. How would the regular polygon look if you cut the rectangle into even more triangular slices?
   Sample response: I think the regular polygon will look more like a perfect circle if the rectangle were cut into more triangular slices. This is because the greater the number of triangular slices that are used, the rounder the outside edge of the polygon becomes.

3. How is the area of the rectangle related to the area of the arrangements you made for each regular polygon? Explain your thinking.
   Sample response: The area of the rectangle is exactly the same as the area of the regular polygon. I know this because we used all of the slices from the rectangle to form the regular polygon, so we preserved the area of the rectangle when forming the regular polygon.

1 Launch

Conduct the ***Think-Pair-Share*** routine after reading Problem 1.

2 Monitor

Help students get started by asking them what they notice about how the shape of the regular polygon is changing as more and more slices are made to the rectangle.

Look for productive strategies:

- Drawing a step even further along in the pattern to help when answering Problem 2.

3 Connect

Have students share their explanations for why the area of the rectangle and the area of the regular polygons are the same. Students may choose to reason using their evidence from the Warm-up or Activity 1.

Ask:

- "Was any space lost or gained while cutting or rearranging?"
- "How many slices would you estimate there need to be before it would look like a circle?"
- "How was your work in this Activity similar to or different than using Archimedes' method for approximating the circumference of a circle?"

Highlight that if students had a pair of scissors sharp enough, and paper strong enough, they could continue to do this until the circle was almost perfect. While a circle is not a polygon, it can be useful to think of it as similar to a polygon with infinitely many sides.

Display the Polygon to Circle Transformer digital tool from the Amps slides.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, or display the Polygon to Circle Transformer digital tool from the Amps slides. This tool will help students visualize how the triangular pieces form a shape that becomes more circular as the number of pieces increases.

Extension: *Math Enrichment*

Ask students whether a parallelogram can be decomposed, sliced uniformly, and rearranged to approximate a circle. Have them create drawings or cut out and rearrange parallelograms to justify their response.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their explanations for why the areas are the same, highlight the reasoning or evidence they use in their responses. Listen for and amplify language students may use, such as "preserved the area of the rectangle," "did not add or subtract any pieces," "used all of the pieces," etc. Encourage students to revoice classmates' ideas before adding onto their ideas.

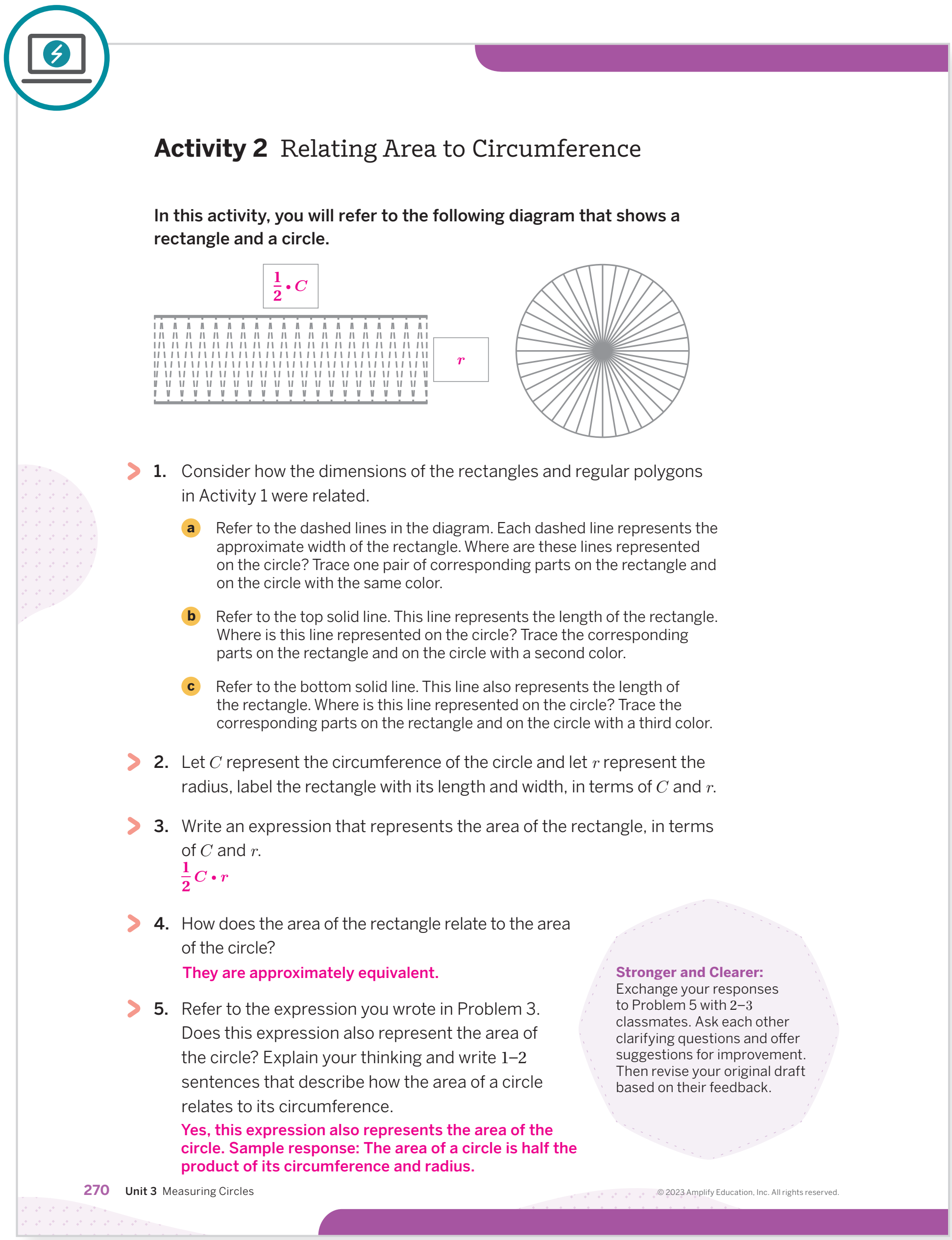
English Learners

Use wait time to provide students the opportunity to rehearse and formulate a response.

Pairs | 10 min

Activity 2 Relating Area to Circumference

Students use color-coding as they relate the parts of the circle to the parts of the rectangle. This allows students to derive the area formula for a circle from $A = l \cdot w$.



Activity 2 Relating Area to Circumference

In this activity, you will refer to the following diagram that shows a rectangle and a circle.

1. Consider how the dimensions of the rectangles and regular polygons in Activity 1 were related.
   - a. Refer to the dashed lines in the diagram. Each dashed line represents the approximate width of the rectangle. Where are these lines represented on the circle? Trace one pair of corresponding parts on the rectangle and on the circle with the same color.
   - b. Refer to the top solid line. This line represents the length of the rectangle. Where is this line represented on the circle? Trace the corresponding parts on the rectangle and on the circle with a second color.
   - c. Refer to the bottom solid line. This line also represents the length of the rectangle. Where is this line represented on the circle? Trace the corresponding parts on the rectangle and on the circle with a third color.
2. Let C represent the circumference of the circle and let r represent the radius, label the rectangle with its length and width, in terms of C and r.
3. Write an expression that represents the area of the rectangle, in terms of C and r.

   $\frac{1}{2}C \cdot r$
4. How does the area of the rectangle relate to the area of the circle?

   They are approximately equivalent.
5. Refer to the expression you wrote in Problem 3. Does this expression also represent the area of the circle? Explain your thinking and write 1–2 sentences that describe how the area of a circle relates to its circumference.

   Yes, this expression also represents the area of the circle. Sample response: The area of a circle is half the product of its circumference and radius.

Stronger and Clearer: Exchange your responses to Problem 5 with 2–3 classmates. Ask each other clarifying questions and offer suggestions for improvement. Then revise your original draft based on their feedback.

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1 Launch

Tell students that they should refer to their work from the Warm-up and Activity 1. Distribute colored pencils.

2 Monitor

Help students get started by suggesting they imagine moving each piece of the rectangle over to the circle.

Look for points of confusion:

- **Not noticing that the lengths of the rectangle will each make up exactly half the circle.** Have students count the number of sections of each color on the circumference.
- **Thinking the width of the rectangle is equal to the full circumference.** Ask, "How many of the triangular slices have their base at the top of the rectangle?"

Look for productive strategies:

- Using an alternating pattern to trace the edge colors on the circumference.

3 Connect

Have students share their answers.

Display the expression $\frac{1}{2}C \cdot r$ alongside $C = 2\pi r$.

Ask:

- "What would the expression look like if you substitute $2\pi r$ for C in the original expression?" $\frac{1}{2} \cdot 2\pi r \cdot r$
- "What can this new expression be simplified to?" πr^2
- "How is it helpful to use the area formula of a rectangle to derive the area formula of a circle?"

Highlight that, by determining how to arrange the pieces of a rectangle into the shape of a circle, students revealed the relationship between the areas of the rectangle and the circle. Because students know how to find the area of a rectangle, they can rewrite the formula for the area of a circle using the *radius:* $A = \pi r^2$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*
Complete Problem 1 together with students, demonstrating how to color code the corresponding parts of the circle.

Accessibility: *Optimize Access to Technology*
Have students use the Amps slides for this activity, or display the Polygon to Circle Transformer digital tool from the Amps slides. This tool will help students visualize how the triangular pieces form a shape that becomes more circular as the number of pieces increases.

Extension: *Math Enrichment*
Have students approximate the area of the rectangle, given the circumference of the circle is 100 in. About 400 in^2.

Math Language Development

MLR1: Stronger and Clearer Each Time

Provide students time to individually craft a draft response for Problem 5. Have them meet with 2–3 partners to give and receive feedback. Encourage partners to use these questions as they provide feedback:

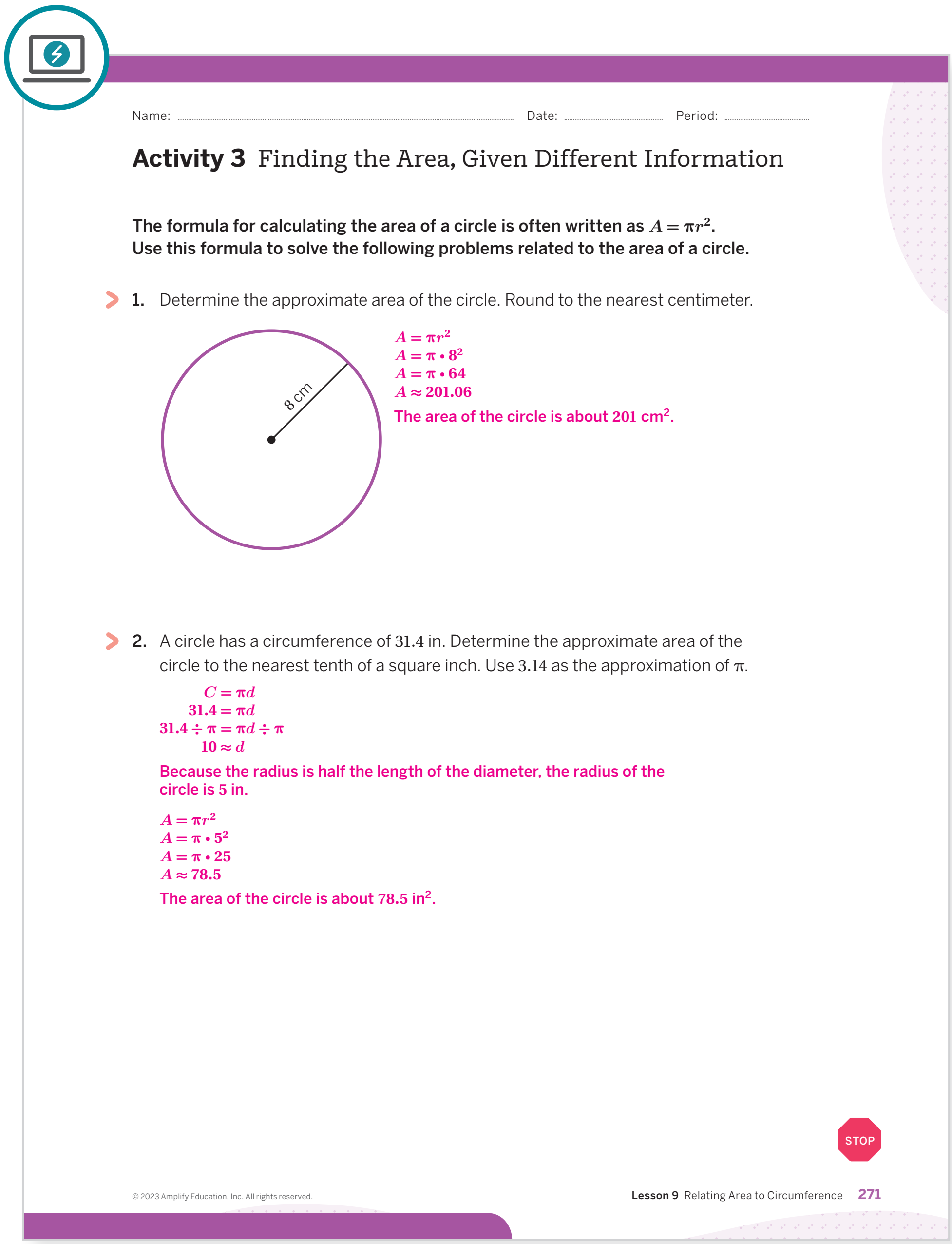
- "Does the response include how the expression does or does not represent the area of the circle?"
- "Does the response include 1–2 clear sentences that describe how the area of a circle relates to its circumference? Does the response make sense to you?"

Have students write an improved response, based on the feedback received.

Pairs | 10 min

Activity 3 Finding the Area, Given Different Information

Students apply the formula they derived in Activity 2 to practice approximating the area of a circle.



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Activity 3 Finding the Area, Given Different Information

The formula for calculating the area of a circle is often written as $A = \pi r^2$. Use this formula to solve the following problems related to the area of a circle.

1. Determine the approximate area of the circle. Round to the nearest centimeter.

$A = \pi r^2$
$A = \pi \cdot 8^2$
$A = \pi \cdot 64$
$A \approx 201.06$

The area of the circle is about 201 cm^2.

2. A circle has a circumference of 31.4 in. Determine the approximate area of the circle to the nearest tenth of a square inch. Use 3.14 as the approximation of π.

$C = \pi d$
$31.4 = \pi d$
$31.4 \div \pi = \pi d \div \pi$
$10 \approx d$

Because the radius is half the length of the diameter, the radius of the circle is 5 in.

$A = \pi r^2$
$A = \pi \cdot 5^2$
$A = \pi \cdot 25$
$A \approx 78.5$

The area of the circle is about 78.5 in^2.

STOP

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Lesson 9 Relating Area to Circumference 271

1 Launch

Provide access to calculators.

2 Monitor

Help students get started by reminding them to rewrite the formula needed to solve the problem.

Look for points of confusion:

- **Doubling the radius instead of squaring.** Have students rewrite the formula in words.
- **Not realizing they need to first determine the radius in Problem 2.** Ask, "How can knowing the circumference help you to determine other measures of the circle?"

Look for productive strategies:

- Writing the formula to be used first, before substituting any values.

3 Connect

Have pairs of students share their solutions for Problem 2.

Highlight that sometimes it will be necessary to use both the formula for the circumference of a circle and the formulas for the area of a circle in order to solve a problem. At the moment, there is not a single formula to find the area when given the circumference. For a problem where a single formula can not help determine a solution, it is helpful to solve the problem backwards. In Problem 2, it may be helpful to know the radius first in order to calculate the area. However, since the circumference is given, you can work backward to determine the diameter before using the radius to calculate the area.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Graphic Organizer PDF, *Working With Circles (Part 2)* to help them make sense of each problem and organize their thinking.

Math Language Development

MLR2: Collect and Display

Provide students with a copy of the Anchor Chart PDF, *Circles*. **Note:** Have them refer to the circle in the middle of the chart that is labeled area throughout. Have them complete the formula $A = \pi r^2$. Near this formula, write the formula in words, such as "Area = π • (radius)^2" or "Area = π • radius • radius".

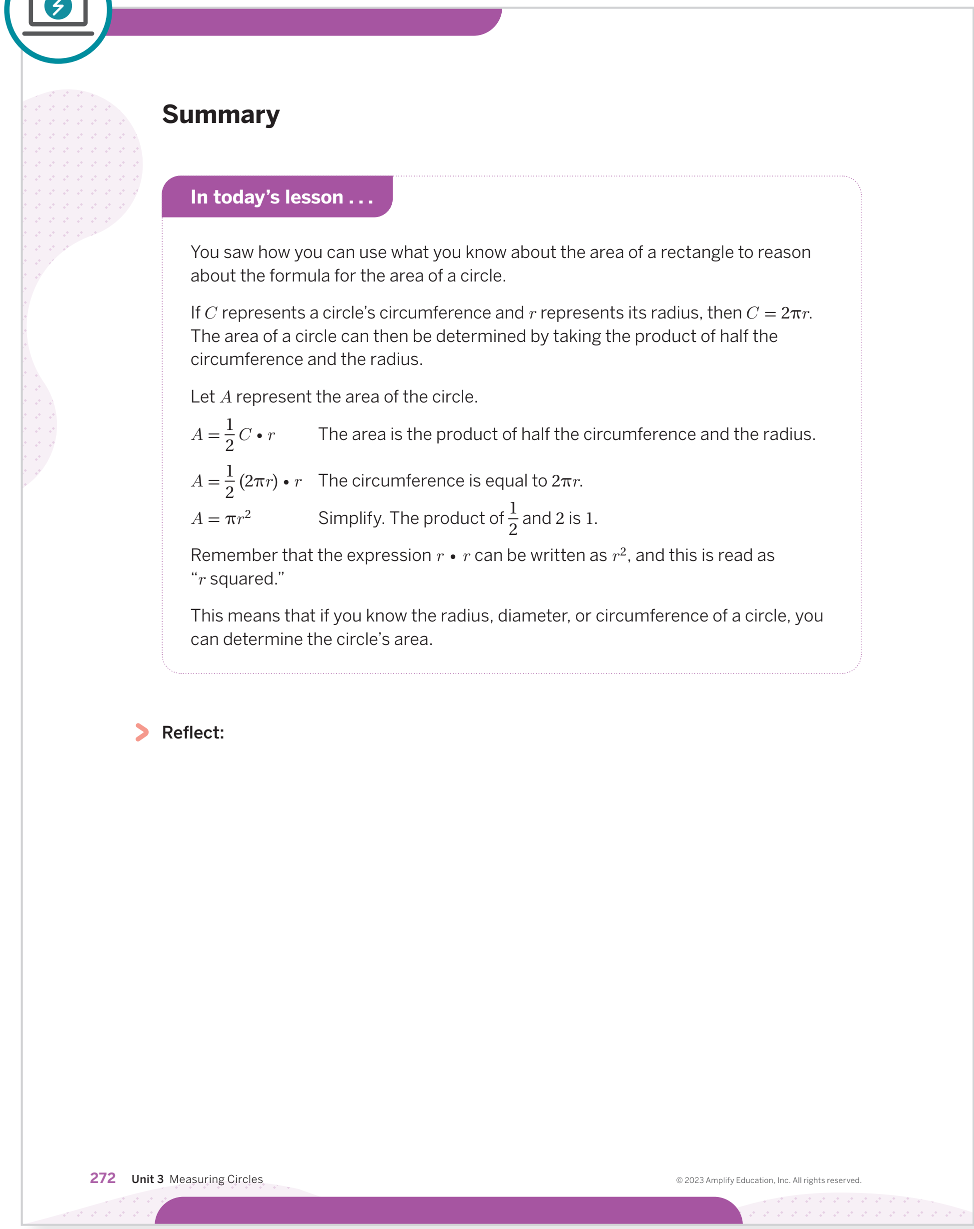
English Learners

Annotate the circle labeled "area" throughout with its radius.

Whole Class | 5 min

Summary

Review and synthesize how to relate the formulas for the area of a rectangle to the area of a circle.



Summary

In today's lesson . . .

You saw how you can use what you know about the area of a rectangle to reason about the formula for the area of a circle.

If C represents a circle's circumference and r represents its radius, then $C = 2\pi r$. The area of a circle can then be determined by taking the product of half the circumference and the radius.

Let A represent the area of the circle.

$A = \frac{1}{2}C \cdot r$	The area is the product of half the circumference and the radius.
$A = \frac{1}{2}(2\pi r) \cdot r$	The circumference is equal to $2\pi r$.
$A = \pi r^2$	Simplify. The product of $\frac{1}{2}$ and 2 is 1.

Remember that the expression $r \cdot r$ can be written as r^2, and this is read as "r squared."

This means that if you know the radius, diameter, or circumference of a circle, you can determine the circle's area.

> **Reflect:**

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Synthesize

Highlight that students now have two formulas for the measurement of circles. One of the challenges when learning a new formula is remembering when to use it. Say, "Let's add the new formula to the Anchor Chart."

Display the Anchor Chart PDF, *Circles*. Write the area formula on the chart.

Ask:

- "How was your approach to determining the area of a circle different in this lesson from the previous lesson?"
- "Which approach do you think is more accurate? Why?"
- "In Lesson 8, you determined that the area of a circle was approximately $3 \cdot r^2$. How does this new formula compare to this previous method used to determine the area of a circle?" This new formula is very close to the previous method, because the only difference is that the new formula uses the number π in place of the 3. But, we know that the value of π is quite close to 3, so this makes sense.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking,

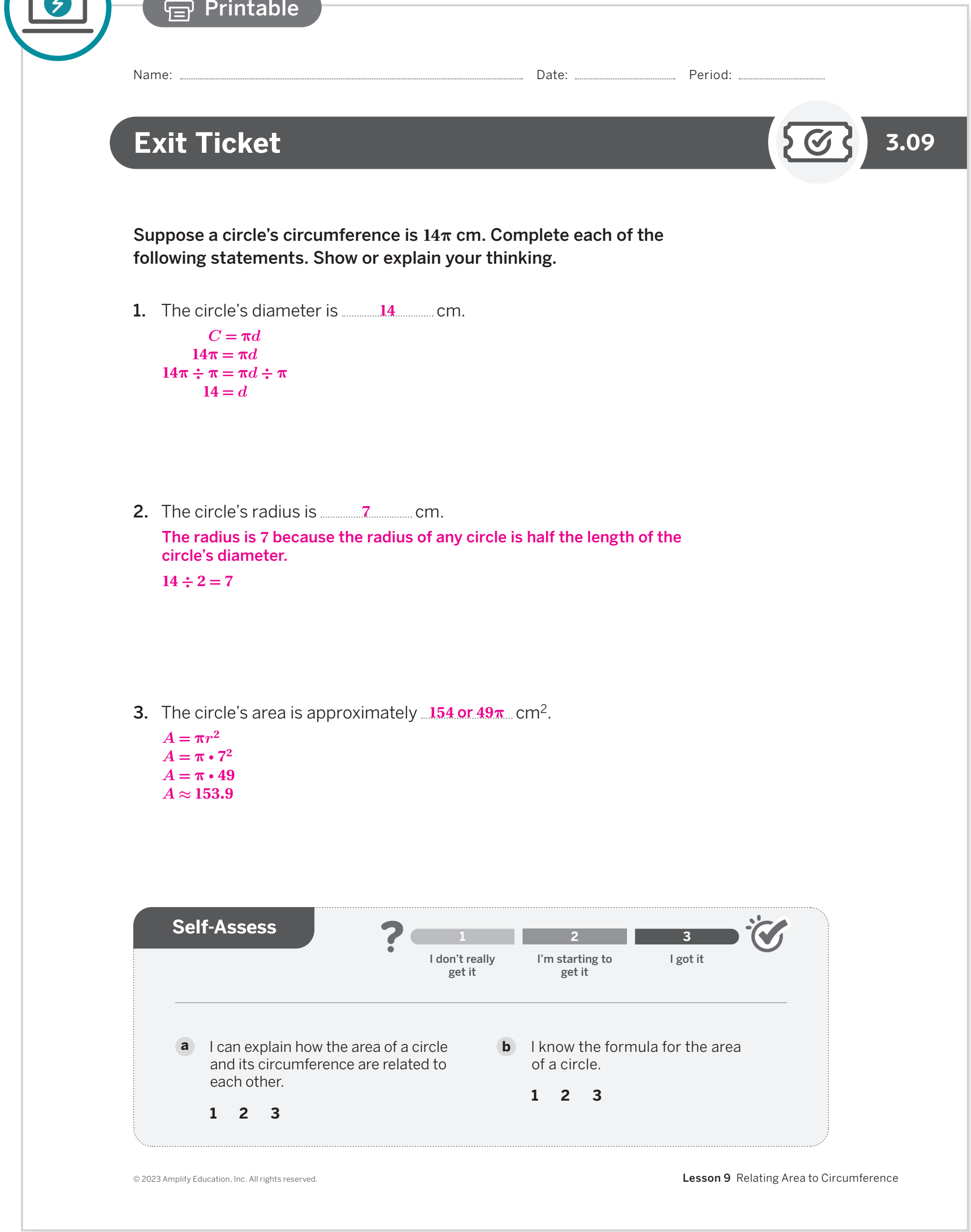
- "How can rectangles help you measure the space inside circles?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the relationship between a circle's radius, diameter, and area by completing statements given the circle's circumference.

Printable



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Exit Ticket 3.09

Suppose a circle's circumference is 14π cm. Complete each of the following statements. Show or explain your thinking.

1. The circle's diameter is 14 cm.

$$C = \pi d$$
$$14\pi = \pi d$$
$$14\pi \div \pi = \pi d \div \pi$$
$$14 = d$$

2. The circle's radius is 7 cm.

The radius is 7 because the radius of any circle is half the length of the circle's diameter.

$14 \div 2 = 7$

3. The circle's area is approximately 154 or 49π cm^2.

$$A = \pi r^2$$
$$A = \pi \cdot 7^2$$
$$A = \pi \cdot 49$$
$$A \approx 153.9$$

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can explain how the area of a circle and its circumference are related to each other.

1 2 3

b I know the formula for the area of a circle.

1 2 3

Lesson 9 Relating Area to Circumference

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students derived the formula for the circumference of a circle. How did that support deriving the formula for the area of a circle?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

Success looks like . . .

- **Language Goal:** Showing how a circle can be decomposed and rearranged to approximate a polygon, and justifying that the area of this polygon is equal to half of the circle's circumference multiplied by its radius. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Generalizing a process for determining the area of a circle, and justifying why this can be represented by πr^2. **(Speaking and Listening)**
- **Goal:** Determining the area of a circle when given the circumference, diameter, or radius.
  - » Determining the diameter, radius, and area when given the circumference.

Suggested next steps

If students use the incorrect formula to solve for the radius or diameter, consider:

- Having them rewrite all of the circle formulas and explain when each one is useful.
- Assigning Practice Problem 3.

If students are confused about which measurement is the diameter and which is the radius, consider:

- Having them sketch a diagram of the information in the Exit Ticket and referring them to the Anchor Chart PDF, *Circles*.
- Assigning Practice Problem 2.

If students determine the area to be 14π cm^2 or approximately 44 cm^2, consider:

- Asking them what having an exponent of 2 means.
- Assigning Practice Problem 3.

Math Language Development

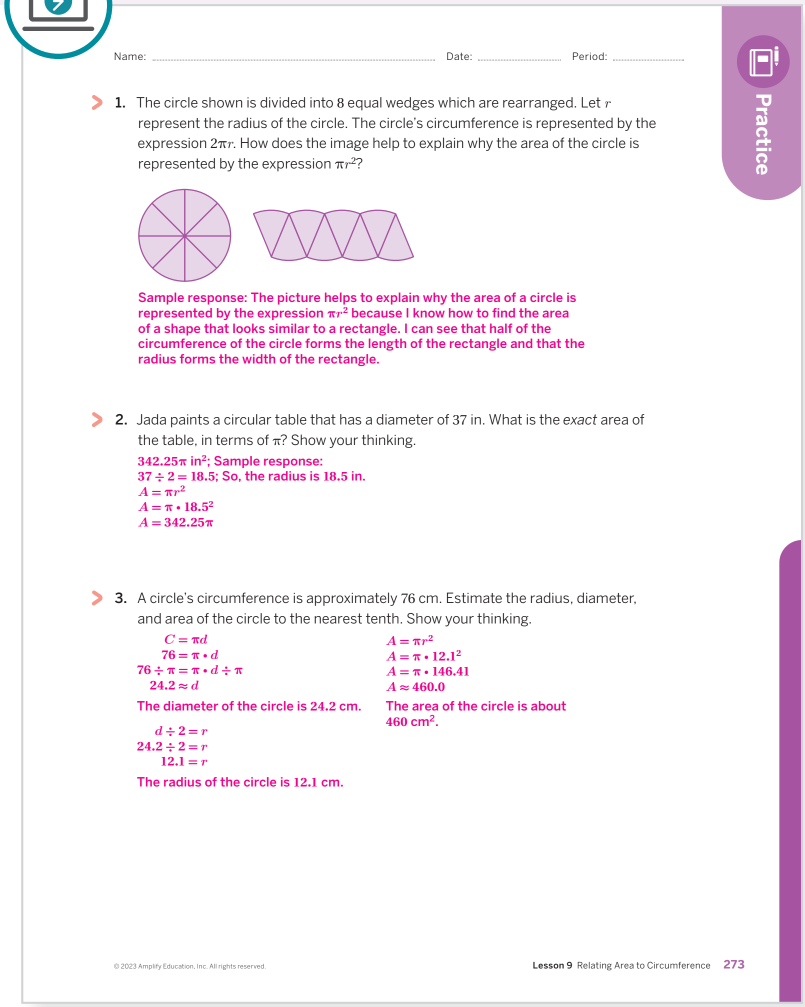
Language Goal: Showing how a circle can be decomposed and rearranged to approximate a polygon, and justifying that the area of this polygon is equal to half of the circle's circumference multiplied by its radius.

Reflect on students' language development toward this goal.

- In Activity 1, how did the ***Stronger and Clearer Each Time*** routine help students improve on their explanations for how the area of a circle relates to its circumference?
- Would you change anything the next time you use this routine?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. The circle shown is divided into 8 equal wedges which are rearranged. Let r represent the radius of the circle. The circle's circumference is represented by the expression $2\pi r$. How does the image help to explain why the area of the circle is represented by the expression πr^2?

Sample response: The picture helps to explain why the area of a circle is represented by the expression πr^2 because I know how to find the area of a shape that looks similar to a rectangle. I can see that half of the circumference of the circle forms the length of the rectangle and that the radius forms the width of the rectangle.

2. Jada paints a circular table that has a diameter of 37 in. What is the *exact* area of the table, in terms of π? Show your thinking.

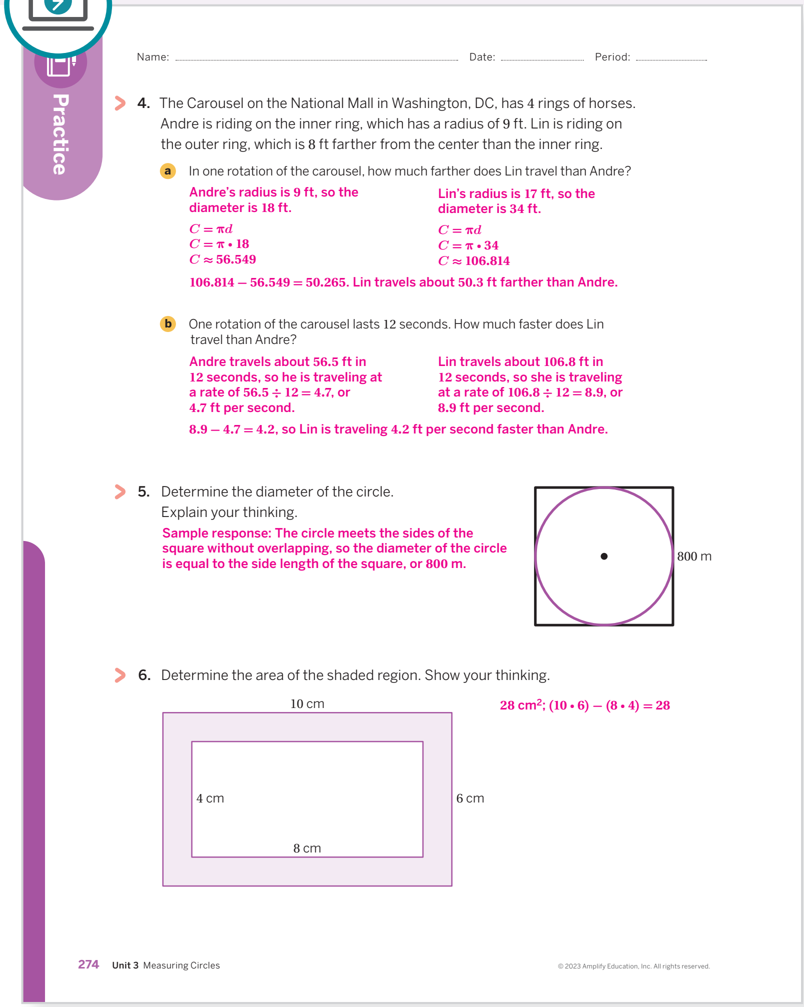
342.25π in^2; Sample response:
$37 \div 2 = 18.5$; So, the radius is 18.5 in.
$A = \pi r^2$
$A = \pi \cdot 18.5^2$
$A = 342.25\pi$

3. A circle's circumference is approximately 76 cm. Estimate the radius, diameter, and area of the circle to the nearest tenth. Show your thinking.

$C = \pi d$
$76 = \pi \cdot d$
$76 \div \pi = \pi \cdot d \div \pi$
$24.2 \approx d$
The diameter of the circle is 24.2 cm.

$d \div 2 = r$
$24.2 \div 2 = r$
$12.1 = r$
The radius of the circle is 12.1 cm.

$A = \pi r^2$
$A = \pi \cdot 12.1^2$
$A = \pi \cdot 146.41$
$A \approx 460.0$
The area of the circle is about 460 cm^2.



Name: ______ Date: ______ Period: ______

Practice

4. The Carousel on the National Mall in Washington, DC, has 4 rings of horses. Andre is riding on the inner ring, which has a radius of 9 ft. Lin is riding on the outer ring, which is 8 ft farther from the center than the inner ring.

   a. In one rotation of the carousel, how much farther does Lin travel than Andre?

   Andre's radius is 9 ft, so the diameter is 18 ft.
   $C = \pi d$
   $C = \pi \cdot 18$
   $C \approx 56.549$

   Lin's radius is 17 ft, so the diameter is 34 ft.
   $C = \pi d$
   $C = \pi \cdot 34$
   $C \approx 106.814$

   $106.814 - 56.549 = 50.265$. Lin travels about 50.3 ft farther than Andre.

   b. One rotation of the carousel lasts 12 seconds. How much faster does Lin travel than Andre?

   Andre travels about 56.5 ft in 12 seconds, so he is traveling at a rate of $56.5 \div 12 = 4.7$, or 4.7 ft per second.

   Lin travels about 106.8 ft in 12 seconds, so she is traveling at a rate of $106.8 \div 12 = 8.9$, or 8.9 ft per second.

   $8.9 - 4.7 = 4.2$, so Lin is traveling 4.2 ft per second faster than Andre.

5. Determine the diameter of the circle. Explain your thinking.

Sample response: The circle meets the sides of the square without overlapping, so the diameter of the circle is equal to the side length of the square, or 800 m.

6. Determine the area of the shaded region. Show your thinking.

28 cm^2; $(10 \cdot 6) - (8 \cdot 4) = 28$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	3
	2	Activity 3	2
	3	Activity 3	2
Spiral	4	Unit 3 Lesson 7	2
	5	Unit 3 Lesson 2	2
Formative ⚡	6	Unit 3 Lesson 10	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3 | **LESSON 10**

Applying Area of Circles

Let's find the areas of shapes that are made up of circles.

Focus

Goals

1. **Language Goal:** Determine or approximate the area of a shape that includes circular or semi-circular parts, and explain the solution method. **(Speaking and Listening, Reading and Writing)**
2. Comprehend and generate expressions in terms of π to express exact measurements related to a circle.

Coherence

Today

Students apply the area of a circle formula to solve problems involving the area of shapes composed of both circular parts and polygons. In working with decomposing complex diagrams, students make sense of problems and must persevere to solve them.

Note: Provide access to calculators throughout the entire lesson to take the focus off computation.

Previously

In Lessons 8 and 9, students estimated the area of circles on a grid and explored the relationship between the circumference and the area of a circle to determine that $A = \pi r^2$.

Coming Soon

In Lesson 11, students will make strategic choices about whether the solution to a problem involves the circumference or the area of a circle.

Rigor

- Students build **conceptual understanding** of how some objects can be abstracted and decomposed into geometric figures.
- Students **apply** the area of a circle formula to find the area of complex objects.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	12 min	5 min	8 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- calculators
- colored pencils (as needed)
- geometry toolkits: compasses, rulers (optional)

Math Language Development

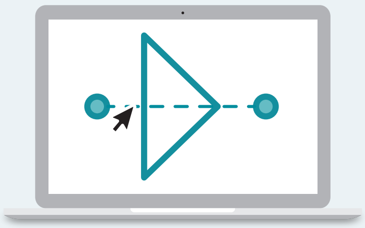
Review words

- *circumference*
- *diameter*
- π
- *radius*

Amps | **Featured Activity**

Warm-up
Radius or Diameter

Students can specify the radius or diameter and then see four corresponding circles drawn in a square sheet.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to decompose the figures and find the circles that are involved in each problem. Encourage students to highlight the circles in the images. They should label parts of the problem, even when the labels are not provided, to make clear what the problem is asking for.

Modifications to Pacing

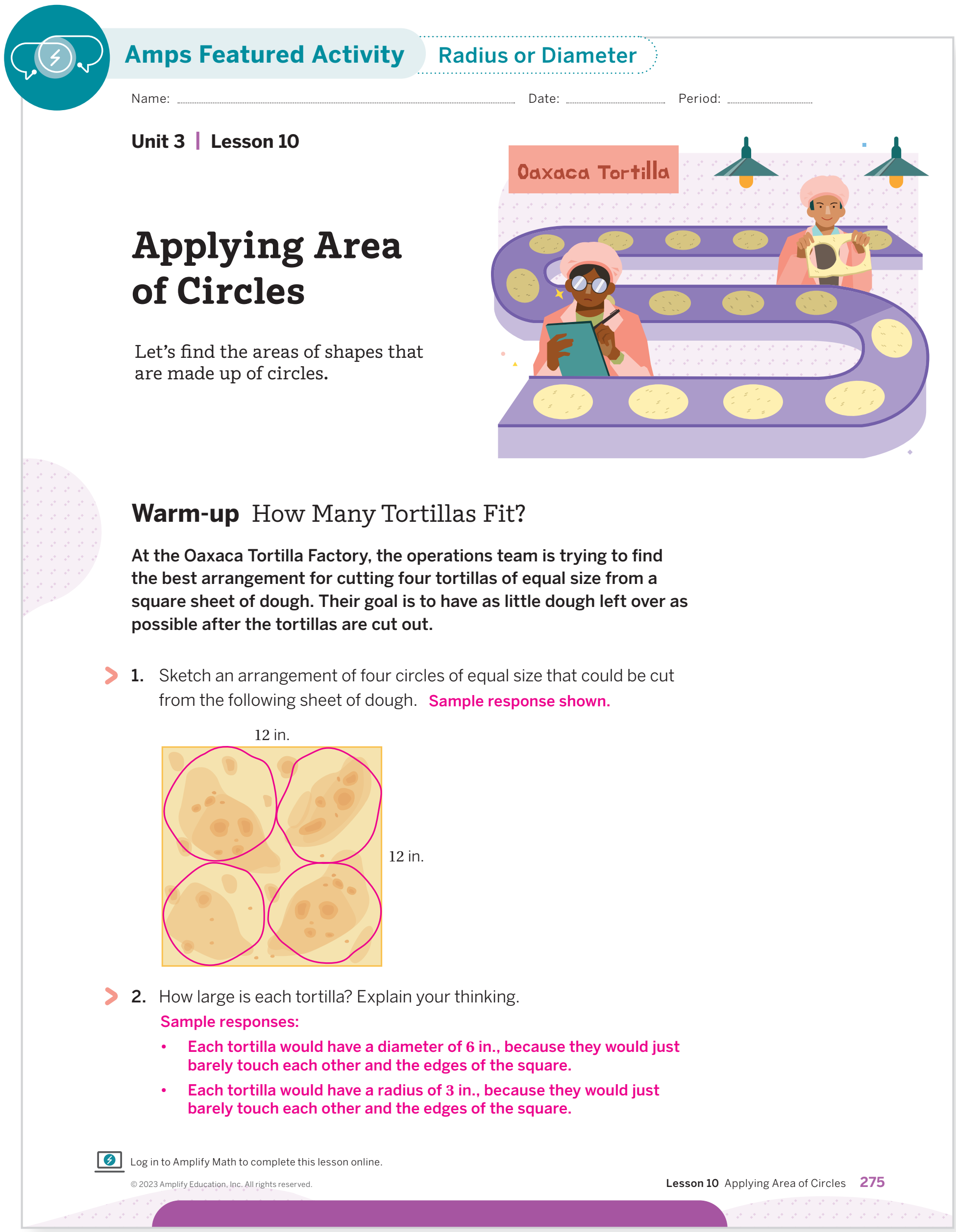
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- **Activity 2** may be completed during Practice.

Pairs | 5 min

Warm-up How Many Tortillas Fit?

Students sketch an arrangement of four circles inside a square to reason about how the side length of the square relates to the diameters and radii of the circles.



Amps Featured Activity Radius or Diameter

Name: Date: Period:

Unit 3 | Lesson 10

Applying Area of Circles

Let's find the areas of shapes that are made up of circles.

Warm-up How Many Tortillas Fit?

At the Oaxaca Tortilla Factory, the operations team is trying to find the best arrangement for cutting four tortillas of equal size from a square sheet of dough. Their goal is to have as little dough left over as possible after the tortillas are cut out.

1. Sketch an arrangement of four circles of equal size that could be cut from the following sheet of dough. **Sample response shown.**

2. How large is each tortilla? Explain your thinking.
   Sample responses:
   - **Each tortilla would have a diameter of 6 in., because they would just barely touch each other and the edges of the square.**
   - **Each tortilla would have a radius of 3 in., because they would just barely touch each other and the edges of the square.**

Log in to Amplify Math to complete this lesson online.

Lesson 10 Applying Area of Circles 275

1 Launch

Before students sketch, have them imagine how they will arrange their circles so there is as little dough left over as possible. Ask, "Is there any way to have *no* dough leftover?"

2 Monitor

Help students get started by suggesting they add smaller boxes or lines to help determine where the edges of their circles will go to.

Look for points of confusion:

- **Thinking that the radius of each circle is 6 in.** Have students draw the radii either vertically or horizontally.
- **Having the circles overlap.** Ask, "If the circles overlap, will they still be circles after they are cut out?"

3 Connect

Have pairs of students share their arrangements of circles in Problem 1. Share an arrangement that does not use the largest possible circles first.

Display a student's work that has the diameters drawn horizontally or vertically and the lengths labeled.

Ask, "Is there any arrangement of four circles that would cover the square entirely?"

Highlight that when circles are arranged inside a square, there will always be part of the square that is uncovered. In order to reduce the excess dough, be sure that the edges of the circle just meet without overlapping. Although it is possible to draw many diameters and radii on these circles, it is the horizontal and vertical segments that will provide the most helpful information relative to the square.

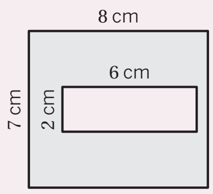
Differentiated Support

Accessibility: ***Optimize Access to Tools***

Provide access to students' geometry toolkits. Consider suggesting they may want to use a ruler to partition their square and a compass to sketch their circles.

Power-up

To power up students' ability to determine the area of a polygon with missing regions, have students complete:



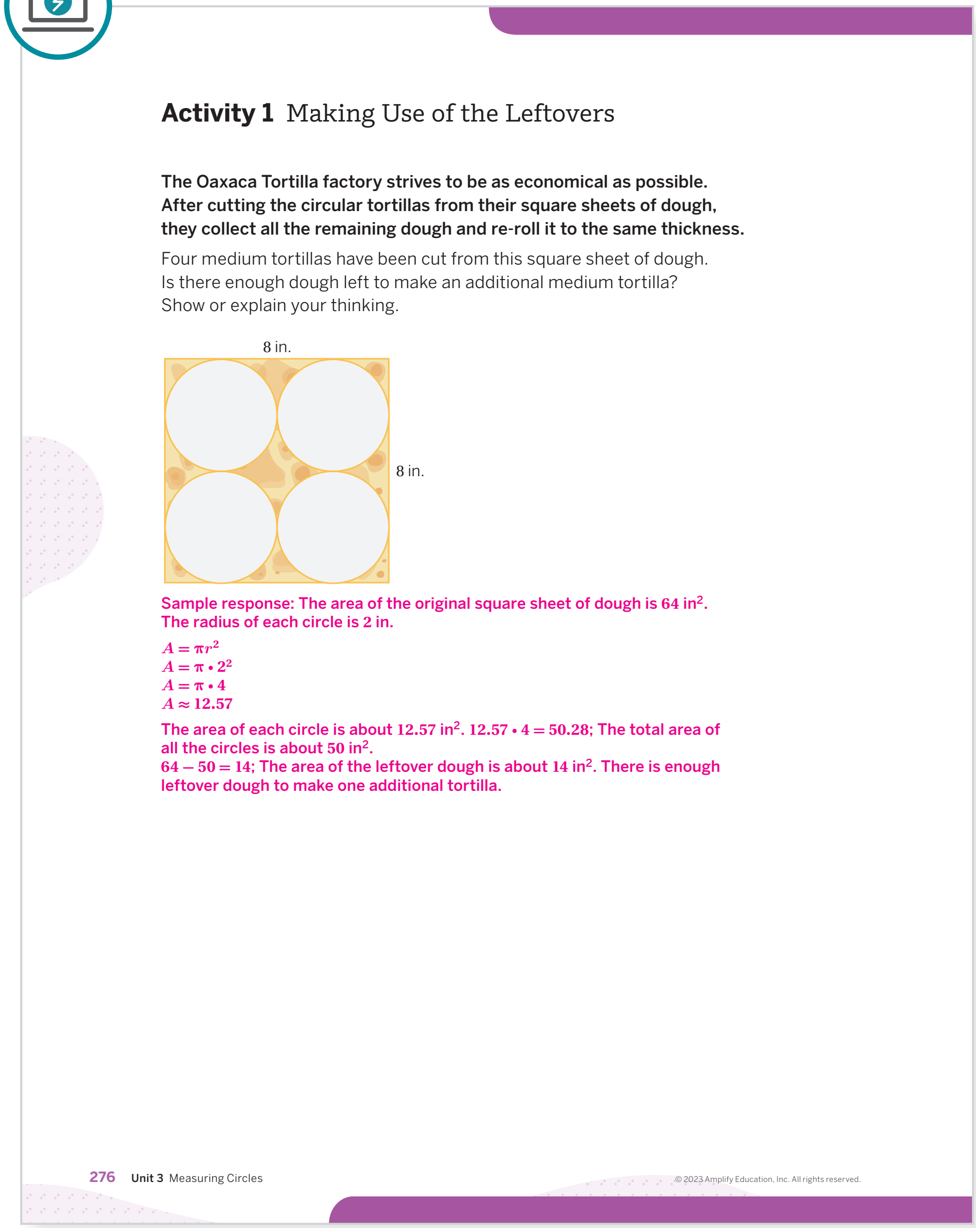
1. What is the area of the larger rectangle? The smaller rectangle? 56 cm^2; 12 cm^2
2. How could you use these two areas to determine the area of the shaded region? Sample response: Subtract the area of the small rectangle from the area of the large rectangle.

Use: Before Activity 1.
Informed by: Performance on Lesson 9, Practice Problem 6.

Pairs | 15 min

Activity 1 Making Use of the Leftovers

Students determine the area of a region not covered by circles to highlight the different strategies that can be used to solve area problems.



Activity 1 Making Use of the Leftovers

The Oaxaca Tortilla factory strives to be as economical as possible. After cutting the circular tortillas from their square sheets of dough, they collect all the remaining dough and re-roll it to the same thickness.

Four medium tortillas have been cut from this square sheet of dough. Is there enough dough left to make an additional medium tortilla? Show or explain your thinking.

8 in.

8 in.

Sample response: The area of the original square sheet of dough is 64 in^2. The radius of each circle is 2 in.

$A = \pi r^2$
$A = \pi \cdot 2^2$
$A = \pi \cdot 4$
$A \approx 12.57$

The area of each circle is about 12.57 in^2. $12.57 \cdot 4 = 50.28$; The total area of all the circles is about 50 in^2.
$64 - 50 = 14$; The area of the leftover dough is about 14 in^2. There is enough leftover dough to make one additional tortilla.

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1 Launch

Activate background knowledge by asking students if they have ever made anything from dough. Ask, "What can you do with the leftover dough after you have cut out the shape that you need?"

2 Monitor

Help students get started by asking, "What information would you like to know? What is a good first step to getting that information?"

Look for points of confusion:

- **Determining the area of the circles only.** Prompt the student to reread the problem. Ask, "Where will the leftover dough be in the picture?" Have students shade the region around the circles to help make sense of the difference between the space inside the circles compared to outside of the circles.

Look for productive strategies:

- Determining the leftover area surrounding one of the circles in one corner, then multiplying by 4.

3 Connect

Display a few different strategies for showing how students marked up the images.

Have students share their solution strategies, explaining the steps taken and their reasoning for each.

Ask, "Is there a way to use estimation and be sure that there is enough dough left over to make another tortilla?"

Highlight that sometimes finding the area of a space means finding the areas of shapes inside each other. This can also include *removing* the area of some of those shapes.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate or suggest that students draw vertical and horizontal lines representing the diameters of the circles. This will help them visualize the relationship between the diameter (and thus, the radius) of each circle and the side length of the square.

Extension: *Math Enrichment*

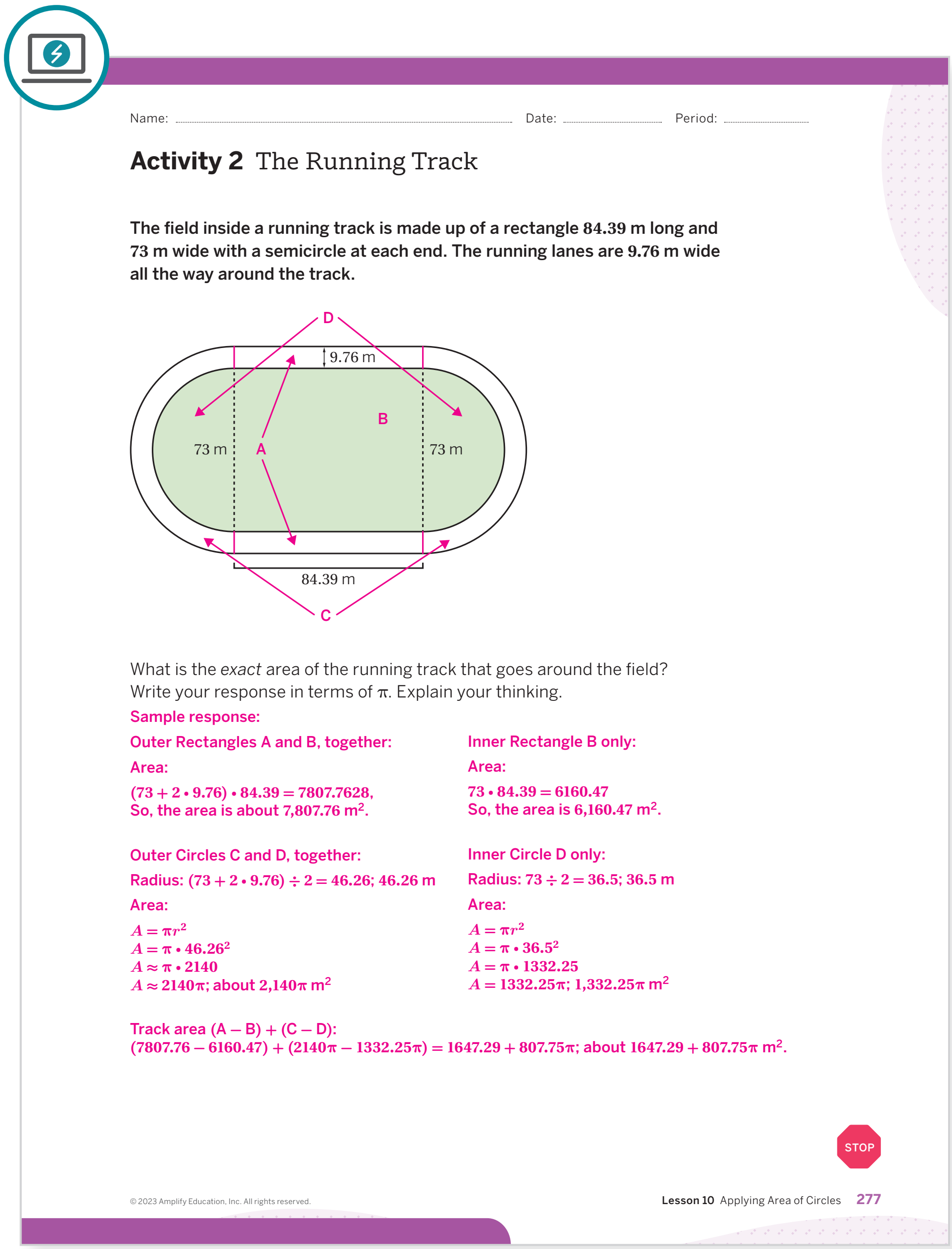
Have students complete the following problem:

If you were to cut 100 equally-sized, mini circles from this same square sheet of dough, how much dough would be left over? Explain your thinking. 13.7 in^2; There would be 10 rows and 10 columns of mini-circles. Each circle would have a radius of 0.4 in. The area of each circle is about 0.503 in^2. The total area of all 100 circles is about 50.3 in^2. The area of the leftover dough would be about $64 - 50.3$, or about 13.7 in.

Pairs | 12 min

Activity 2 The Running Track

Students decide how to decompose a real-world object into known shapes — including circular shapes and use the given information to determine areas.



Name: ______________ Date: ________ Period: ________

Activity 2 The Running Track

The field inside a running track is made up of a rectangle 84.39 m long and 73 m wide with a semicircle at each end. The running lanes are 9.76 m wide all the way around the track.

What is the *exact* area of the running track that goes around the field? Write your response in terms of π. Explain your thinking.

Sample response:

Outer Rectangles A and B, together:

Area:

$(73 + 2 \cdot 9.76) \cdot 84.39 = 7807.7628$,
So, the area is about 7,807.76 m^2.

Inner Rectangle B only:

Area:

$73 \cdot 84.39 = 6160.47$
So, the area is 6,160.47 m^2.

Outer Circles C and D, together:

Radius: $(73 + 2 \cdot 9.76) \div 2 = 46.26$; 46.26 m

Area:

$A = \pi r^2$
$A = \pi \cdot 46.26^2$
$A \approx \pi \cdot 2140$
$A \approx 2140\pi$; about 2,140π m^2

Inner Circle D only:

Radius: $73 \div 2 = 36.5$; 36.5 m

Area:

$A = \pi r^2$
$A = \pi \cdot 36.5^2$
$A = \pi \cdot 1332.25$
$A = 1332.25\pi$; 1,332.25π m^2

Track area (A – B) + (C – D):
$(7807.76 - 6160.47) + (2140\pi - 1332.25\pi) = 1647.29 + 807.75\pi$; about $1647.29 + 807.75\pi$ m^2.

STOP

1 Launch

Activate background knowledge by asking whether any students have run on a track. Ask, "Is it the same width all the way around?" Provide access to calculators throughout the activity.

2 Monitor

Help students get started by asking, "What shapes do you notice in the picture?"

Look for points of confusion:

- **Thinking there are no measurements given for the circular parts.** Have students redraw the circle (or half-circles) separately. Ask, "What does the image tell us about the size of the circle?"
- **Finding the area of the track and field together.** Have students shade the area that they found and prompt students to reread the problem.
- **Combining terms with and without π.** Have students confirm that $1 + 1\pi$ is not equal to 2π, and then ask if it makes sense to do this for their own values.

3 Connect

Have pairs of students share their strategies. Note students with different strategies. Then have students share their partner's strategy with the class.

Ask:

- "What does the quantity 2,140π represent in the diagram?" The area of the larger semicircles.
- "What does the quantity 1,332.25π represent in the diagram?" The area of the smaller semicircles.

Highlight that there are several ways to approach this problem. Say, "Some students will visualize the ends of the track as making one whole circle, and some will visualize it as two half-circles. Finding your own way to make sense of complex, real-world objects is important."

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Round the given values to the nearest ten to help simplify calculations. This will allow students to access the targeted goal of the activity, which is to apply the area formula for a circle in real-world problems that involve decomposing shapes. $1600 + 800\pi$ m^2

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest students create a table or other organized way to show their thinking. For example, suggest they color code each decomposition and determine the area of each using the same color.

Math Language Development

MLR7: Compare and Connect

Have students prepare a visual display showing how they determined the area of the running track. Look for and highlight different strategies for determining the areas of the curved parts of the track. As students investigate each other's displays, ask them to share what worked well in a particular approach.

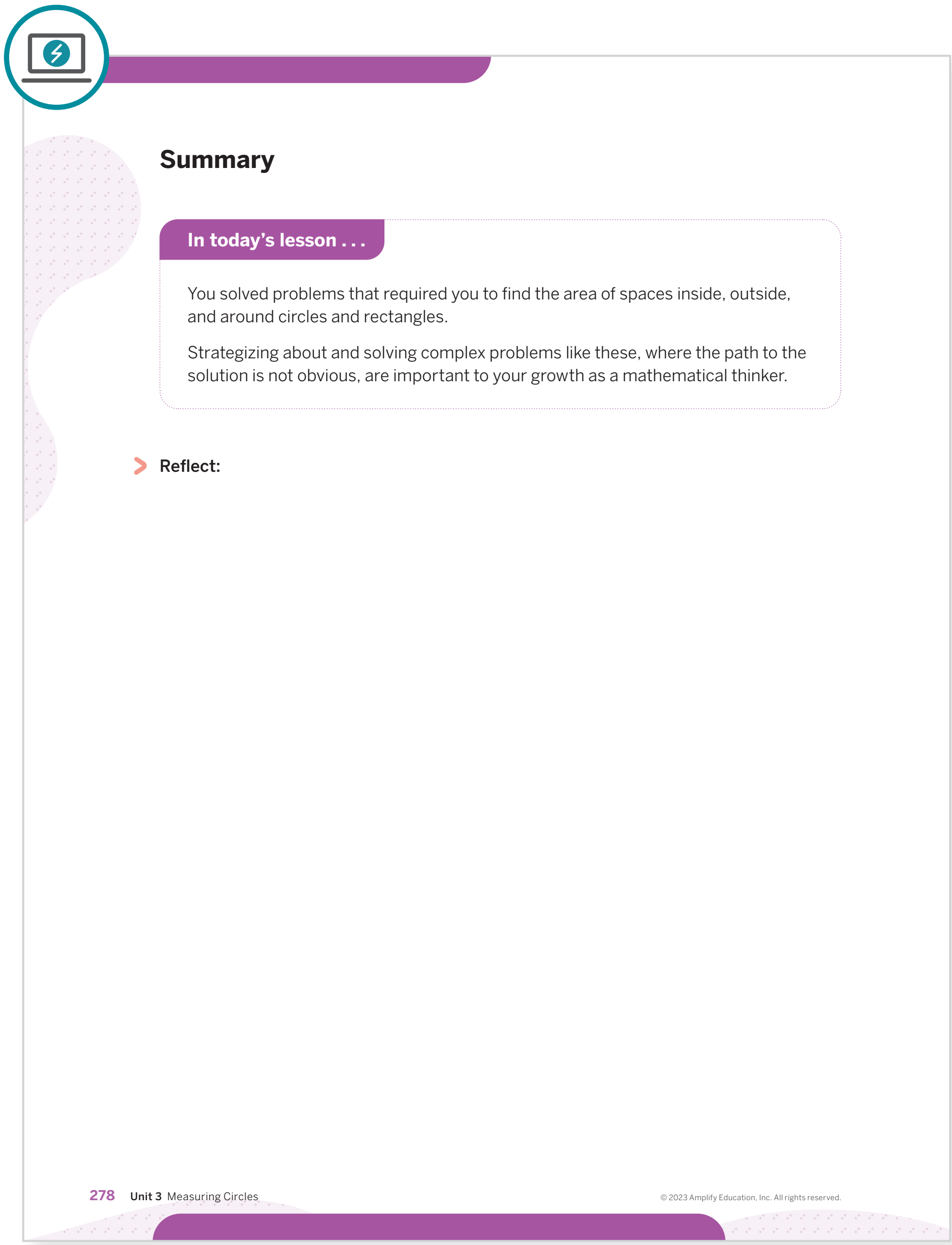
English Learners

Have students highlight the phrase "Write your response in terms of π" to help them connect that their final response should include the π symbol.

Whole Class | 5 min

Summary

Review and synthesize how complex problems involving area often have various solution pathways.



Summary

In today's lesson . . .

You solved problems that required you to find the area of spaces inside, outside, and around circles and rectangles.

Strategizing about and solving complex problems like these, where the path to the solution is not obvious, are important to your growth as a mathematical thinker.

Reflect:

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Synthesize

Display well-organized student work from Activity 1 or Activity 2.

Ask:

- "What strategies did this student use to organize their work? How did it help them to make sense of the problem?"
- "Why is it important to organize your work carefully when working on more complex problems?"

Have students share how the organization of the work supported the mathematical thinking of the student.

Highlight how organizing mathematical work carefully not only helps others understand the work, but often it can help make sense of the steps needed to solve the problem.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

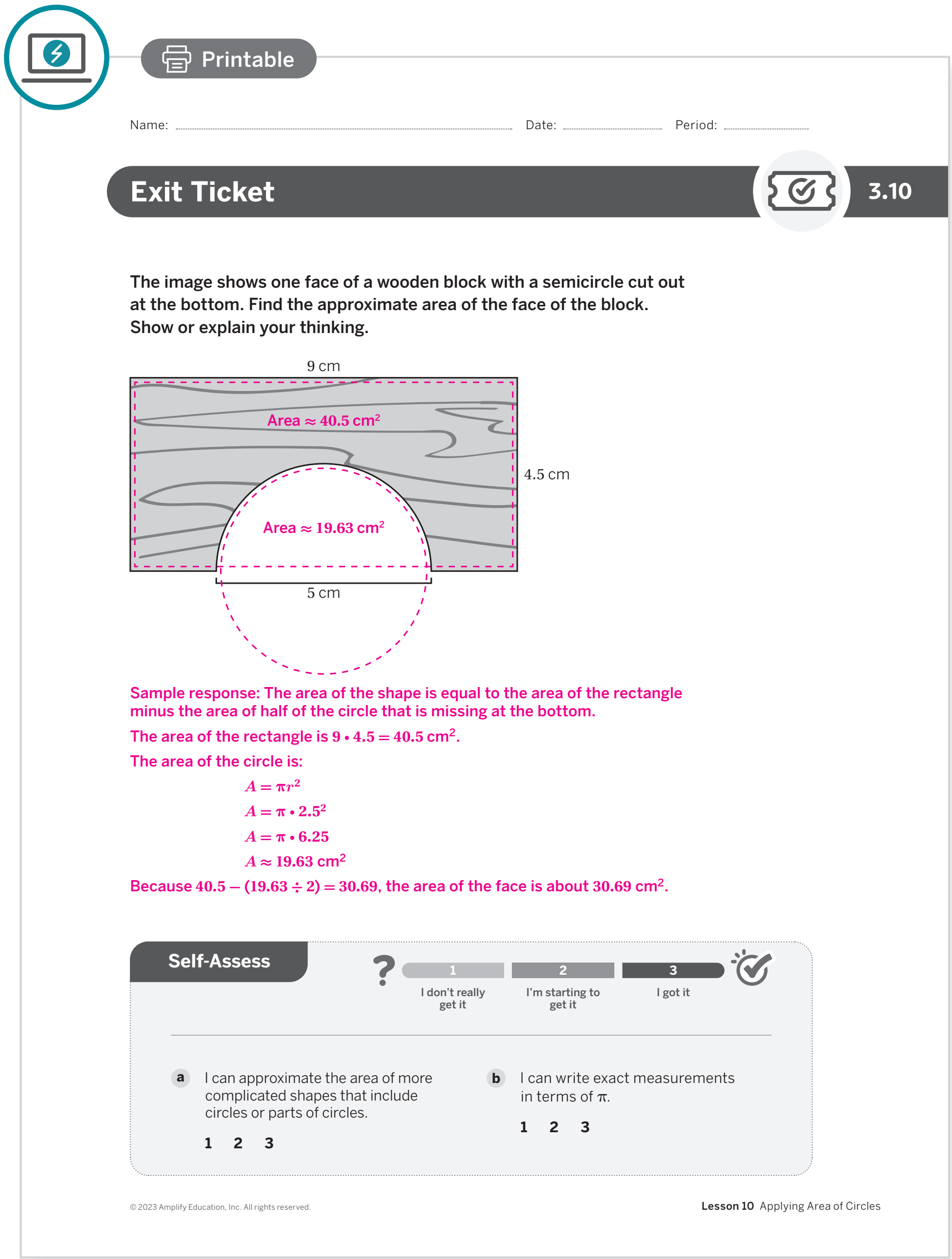
- "What strategies are helpful when finding the area of space that is left over?"
- "How can squares help to measure the space inside circles?"

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of finding the area of complex real-world objects by finding the area of a wood block with a circular cut-out.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 3.10

The image shows one face of a wooden block with a semicircle cut out at the bottom. Find the approximate area of the face of the block. Show or explain your thinking.

Sample response: The area of the shape is equal to the area of the rectangle minus the area of half of the circle that is missing at the bottom.

The area of the rectangle is $9 \cdot 4.5 = 40.5$ cm².

The area of the circle is:

$A = \pi r^2$

$A = \pi \cdot 2.5^2$

$A = \pi \cdot 6.25$

$A \approx 19.63$ cm²

Because $40.5 - (19.63 \div 2) = 30.69$, the area of the face is about 30.69 cm².

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a I can approximate the area of more complicated shapes that include circles or parts of circles.
1 2 3

b I can write exact measurements in terms of π.
1 2 3

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Lesson 10 Applying Area of Circles

Success looks like . . .

- **Language Goal:** Determining or approximating the area of a shape that includes circular or semi-circular parts, and explaining the solution method. **(Speaking and Listening, Reading and Writing)**
  - » Approximating the area of the block by subtracting the area of the half circle.
- **Goal:** Comprehending and generating expressions in terms of π to express exact measurements related to a circle.

Suggested next steps

If students subtract the area of a full circle with a 5 cm radius from the area of the rectangle, consider:

- Having them outline the shape of the rectangle and the circle.

If students use the wrong formula, for example, finding the circumference of the circle instead of the area, consider:

- Making a note of these students to revisit during Lesson 11, which helps to make appropriate decisions about which formula to use.
- Assigning Practice Problems 1 and 3.

If students find the area of each space properly, but do not subtract the area of the half-circle from the rectangle, consider:

- Having them label each part of their work with a description of what it represents.

Professional Learning

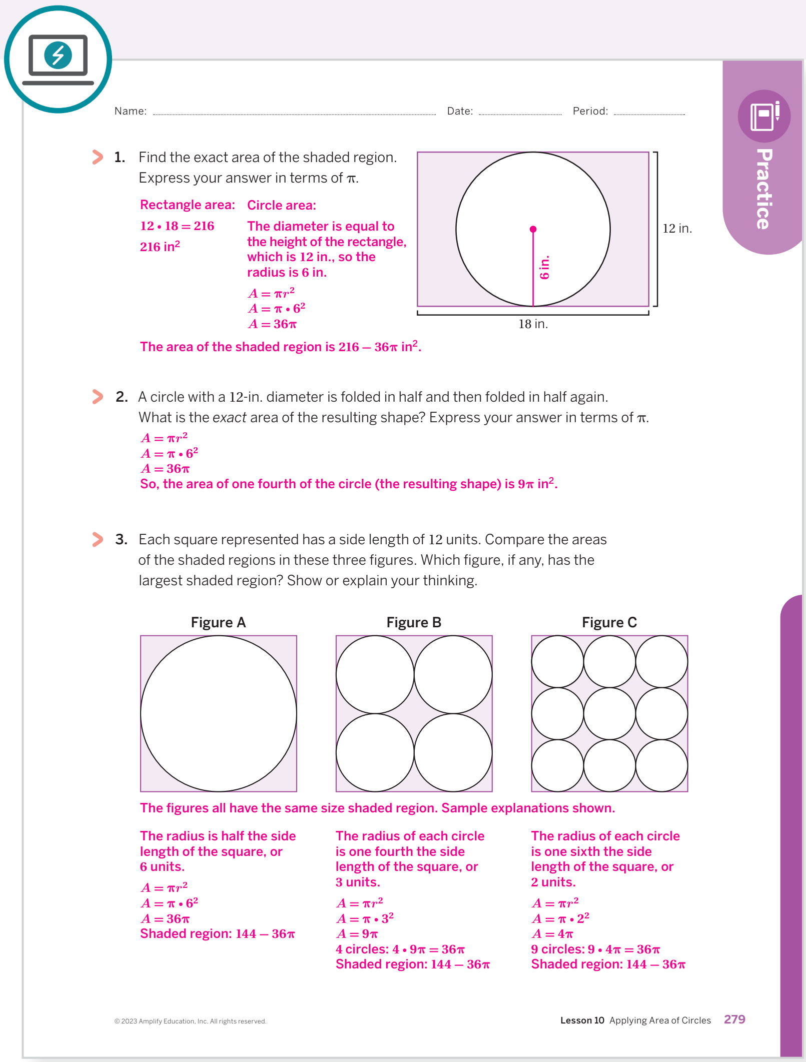
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which teacher actions made facilitating sharing of students' strategies strong?
- In what ways have your students gotten better at explaining their reasoning within their strategies? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Find the exact area of the shaded region. Express your answer in terms of π.

Rectangle area:

$12 \cdot 18 = 216$

216 in^2

Circle area:

The diameter is equal to the height of the rectangle, which is 12 in., so the radius is 6 in.

$A = \pi r^2$
$A = \pi \cdot 6^2$
$A = 36\pi$

The area of the shaded region is $216 - 36\pi$ in^2.

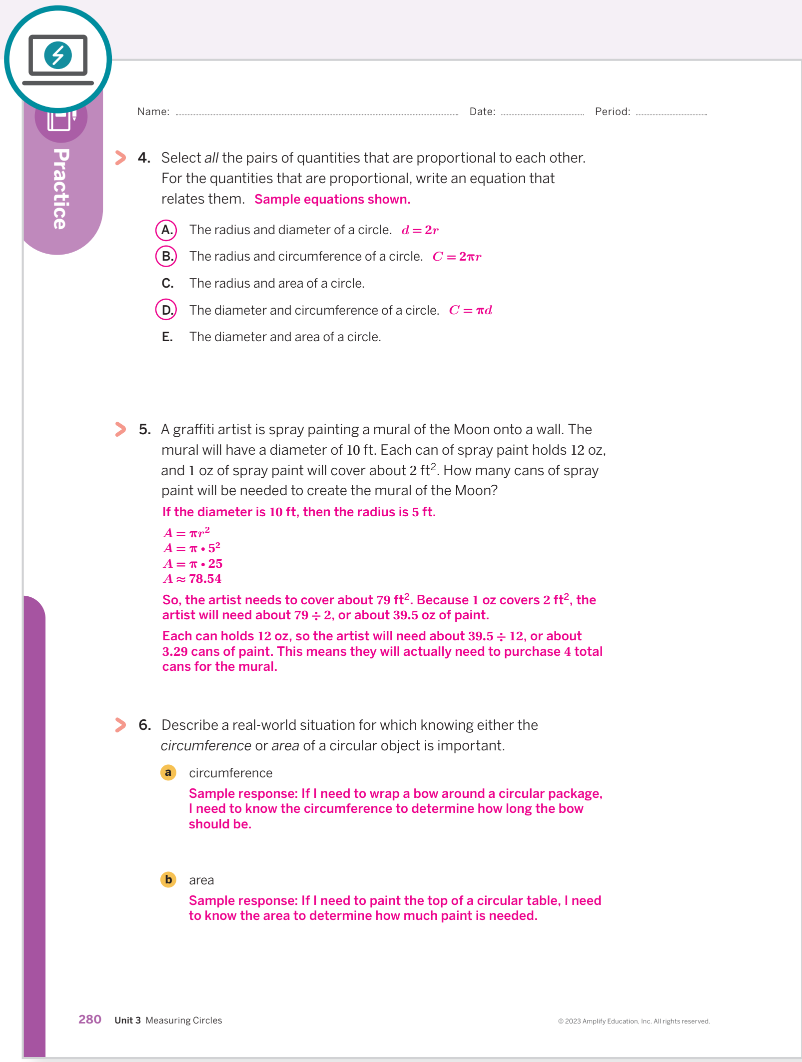
2. A circle with a 12-in. diameter is folded in half and then folded in half again. What is the *exact* area of the resulting shape? Express your answer in terms of π.

$A = \pi r^2$
$A = \pi \cdot 6^2$
$A = 36\pi$
So, the area of one fourth of the circle (the resulting shape) is 9π in^2.

3. Each square represented has a side length of 12 units. Compare the areas of the shaded regions in these three figures. Which figure, if any, has the largest shaded region? Show or explain your thinking.

The figures all have the same size shaded region. Sample explanations shown.

Figure A	Figure B	Figure C
The radius is half the side length of the square, or 6 units. $A = \pi r^2$ $A = \pi \cdot 6^2$ $A = 36\pi$ **Shaded region: $144 - 36\pi$**	**The radius of each circle is one fourth the side length of the square, or 3 units.** $A = \pi r^2$ $A = \pi \cdot 3^2$ $A = 9\pi$ **4 circles: $4 \cdot 9\pi = 36\pi$** **Shaded region: $144 - 36\pi$**	**The radius of each circle is one sixth the side length of the square, or 2 units.** $A = \pi r^2$ $A = \pi \cdot 2^2$ $A = 4\pi$ **9 circles: $9 \cdot 4\pi = 36\pi$** **Shaded region: $144 - 36\pi$**



Practice

Name: Date: Period:

4. Select *all* the pairs of quantities that are proportional to each other. For the quantities that are proportional, write an equation that relates them. **Sample equations shown.**

[✓ A.] The radius and diameter of a circle. $d = 2r$
[✓ B.] The radius and circumference of a circle. $C = 2\pi r$
C. The radius and area of a circle.
[✓ D.] The diameter and circumference of a circle. $C = \pi d$
E. The diameter and area of a circle.

5. A graffiti artist is spray painting a mural of the Moon onto a wall. The mural will have a diameter of 10 ft. Each can of spray paint holds 12 oz, and 1 oz of spray paint will cover about 2 ft^2. How many cans of spray paint will be needed to create the mural of the Moon?

If the diameter is 10 ft, then the radius is 5 ft.

$A = \pi r^2$
$A = \pi \cdot 5^2$
$A = \pi \cdot 25$
$A \approx 78.54$

So, the artist needs to cover about 79 ft^2. Because 1 oz covers 2 ft^2, the artist will need about $79 \div 2$, or about 39.5 oz of paint.

Each can holds 12 oz, so the artist will need about $39.5 \div 12$, or about 3.29 cans of paint. This means they will actually need to purchase 4 total cans for the mural.

6. Describe a real-world situation for which knowing either the *circumference* or *area* of a circular object is important.

a circumference

Sample response: If I need to wrap a bow around a circular package, I need to know the circumference to determine how long the bow should be.

b area

Sample response: If I need to paint the top of a circular table, I need to know the area to determine how much paint is needed.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 9	2
	5	Unit 3 Lesson 4	2
Formative	6	Unit 3 Lesson 11	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

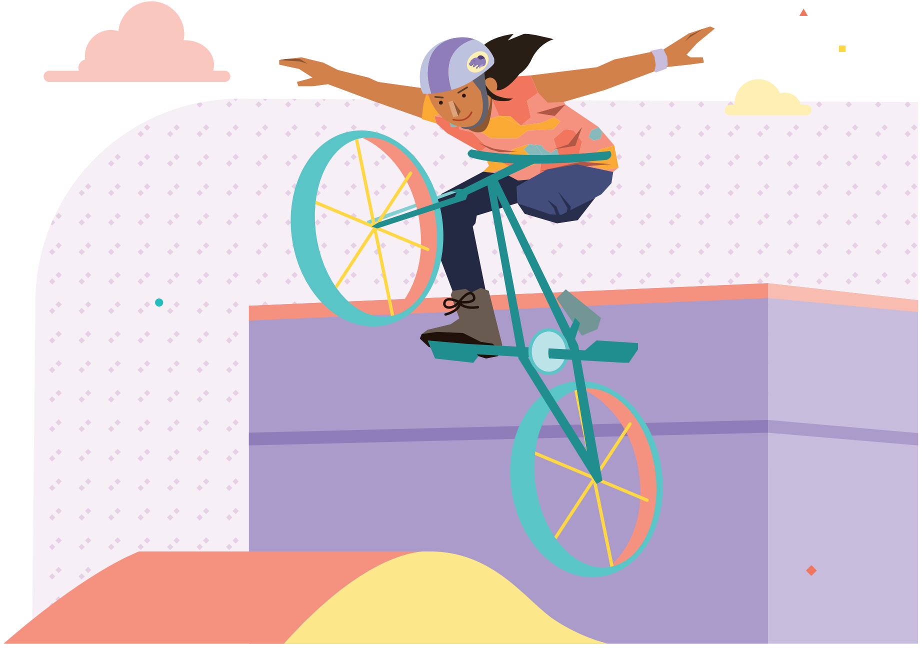
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 3 | **LESSON 11**

Distinguishing Circumference and Area

Let's contrast circumference and area.



Focus

Goals

1. **Language Goal:** Critique claims about the radius, diameter, circumference, or area of a circle in a real-world situation. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Decide whether to calculate the circumference or area of a circle to solve a problem in a real-world situation, and justify the decision. **(Speaking and Listening)**
3. **Language Goal:** Estimate measurements of a circle in a real-world situation, and explain the estimation strategy. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

In this lesson, both circumference and area problems are mixed together so that students are required to distinguish which measurement is called for in each problem situation.

Previously

In Lessons 4–7, students investigated circumference, and in Lessons 8–10, students investigated the area of circles. Students reasoned about the area and circumference of a circle to derive formulas and make sense of the relationship between parts of a circle.

Coming Soon

In Lesson 12, students will use a length of string to reason about which shape will maximize its area given a certain perimeter. (Hint: It's a circle!)

Rigor

- Students build **conceptual understanding** about which real-world situations require finding the circumference or the area of a circle.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	12 min	5 min	8 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut and folded cards, one set per pair
- Activity 1 PDF (answers)
- Anchor Chart PDF, *Circles*
- Graphic Organizer PDF, *Working With Circles (Part 2)* (as needed)
- calculators (optional)

Math Language Development

Review words

- *circumference*
- *diameter*
- π
- *radius*

Amps | Featured Activity

Activity 1
Digital Card Sort

The digital ***Card Sort*** experience allows you to have a window into student's thinking in real time and to provide instant feedback on their thinking.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students decide which response they agree with, they might become excited and forget to listen to their partner's thoughts. Remind students that, by listening well, each person can determine whether they need to seek or offer help from their partner in order to understand the problem. Review signals that indicate whether a person is actively listening and encourage students to practice them.

Modifications to Pacing

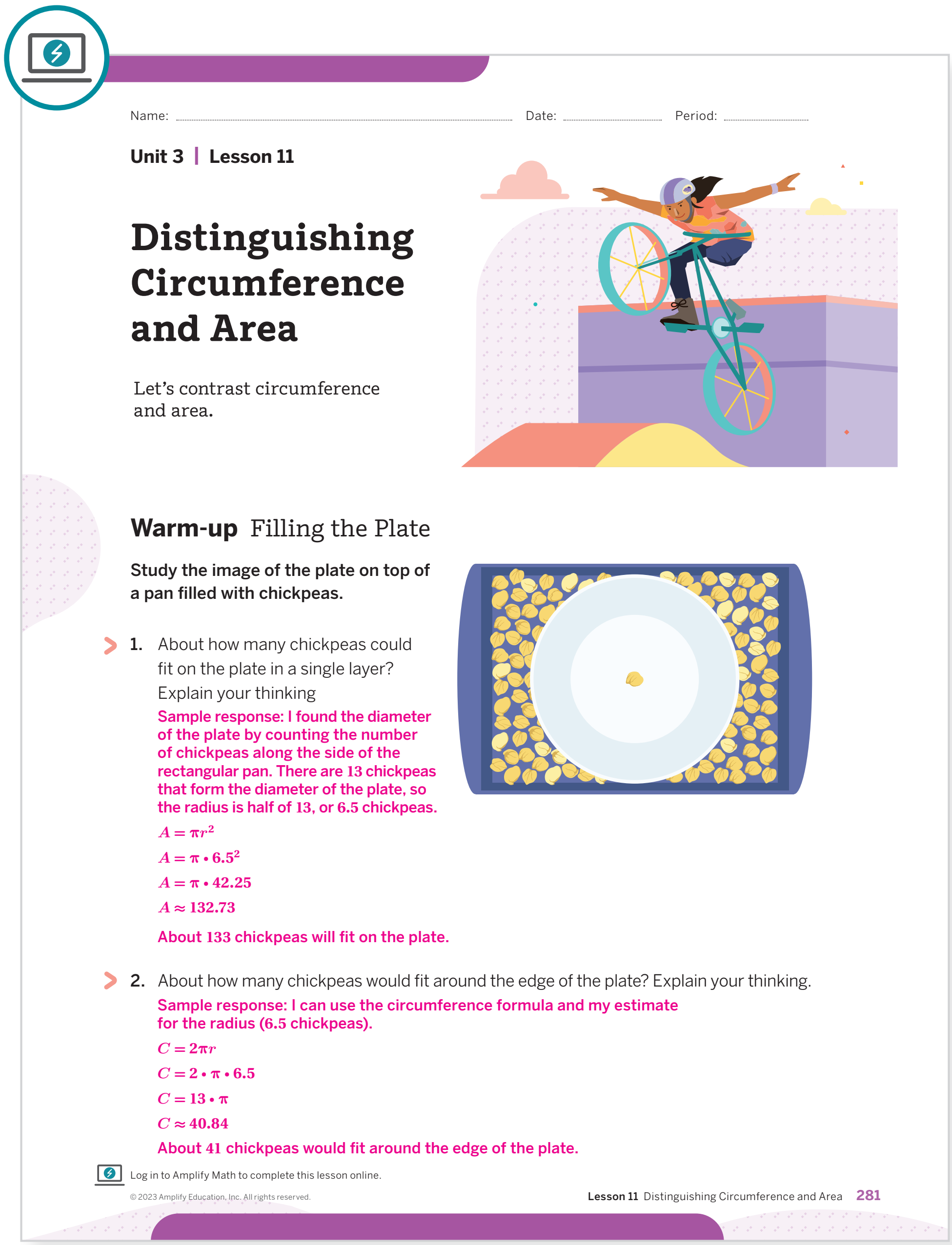
You may want to consider these additional modifications if you are short on time.

- Omit either problem or the entire **Warm-up**.
- In **Activity 1**, Problem 3 may be omitted.
- In **Activity 2**, choose one of the Problems for students to complete.

Pairs | 10 min

Warm-up Filling the Plate

Students estimate how many chickpeas will cover a circular plate to reason about which measurement — area or circumference — is needed.



Name: ____________ Date: ________ Period: ________

Unit 3 | Lesson 11

Distinguishing Circumference and Area

Let's contrast circumference and area.

Warm-up Filling the Plate

Study the image of the plate on top of a pan filled with chickpeas.

1. About how many chickpeas could fit on the plate in a single layer? Explain your thinking

   Sample response: I found the diameter of the plate by counting the number of chickpeas along the side of the rectangular pan. There are 13 chickpeas that form the diameter of the plate, so the radius is half of 13, or 6.5 chickpeas.

   $A = \pi r^2$

   $A = \pi \cdot 6.5^2$

   $A = \pi \cdot 42.25$

   $A \approx 132.73$

   About 133 chickpeas will fit on the plate.

2. About how many chickpeas would fit around the edge of the plate? Explain your thinking.

   Sample response: I can use the circumference formula and my estimate for the radius (6.5 chickpeas).

   $C = 2\pi r$

   $C = 2 \cdot \pi \cdot 6.5$

   $C = 13 \cdot \pi$

   $C \approx 40.84$

   About 41 chickpeas would fit around the edge of the plate.

Log in to Amplify Math to complete this lesson online.

Lesson 11 Distinguishing Circumference and Area 281

1 Launch

Prompt students to give an initial estimate of the number of chickpeas that will fit on the plate. Conduct the ***Poll the Class*** routine. Have students share their estimation strategy with a partner.

2 Monitor

Help students get started by prompting students to recall how they found the diameter of circles inside of squares in Lesson 10.

Look for points of confusion:

- **Thinking the chickpeas should only cover the center of the plate.** Specify that the problem is asking about covering the entire plate, including the rim.
- **Thinking they need measurements for the plate, pan, and chickpeas.** Suggest that students think of each chickpea as a unit of area.
- **Determining the circumference instead of the area. Ask,** "It is asking for the number of chickpeas that will cover the plate. Does this mean you are thinking about the space inside the circle or the distance around the edge of the circle?"

Look for productive strategies:

- Estimating by looking at groups of chickpeas and then drawing that area on top of the plate.

3 Connect

Have students share their strategies for estimating the number of chickpeas that will fit on the plate.

Ask,"What clues in each problem helped you decide whether it was asking for the area or the circumference?"

Highlight that since this problem was about both the area and circumference of a circle, it was necessary to think about which formula was needed and when. The area relates to covering the pan, while the circumference relates to how many chickpeas would surround the outer edge.

Power-up

To power up students' ability to describe scenarios in which they would use the circumference or the area of a circle, have student complete:

Determine whether each statement would be used to describe *circumference* or *area*:

1. Surround Circumference
2. Cover Area
3. Fill Area
4. Go around Circumference

Use: Before the Warm-up.

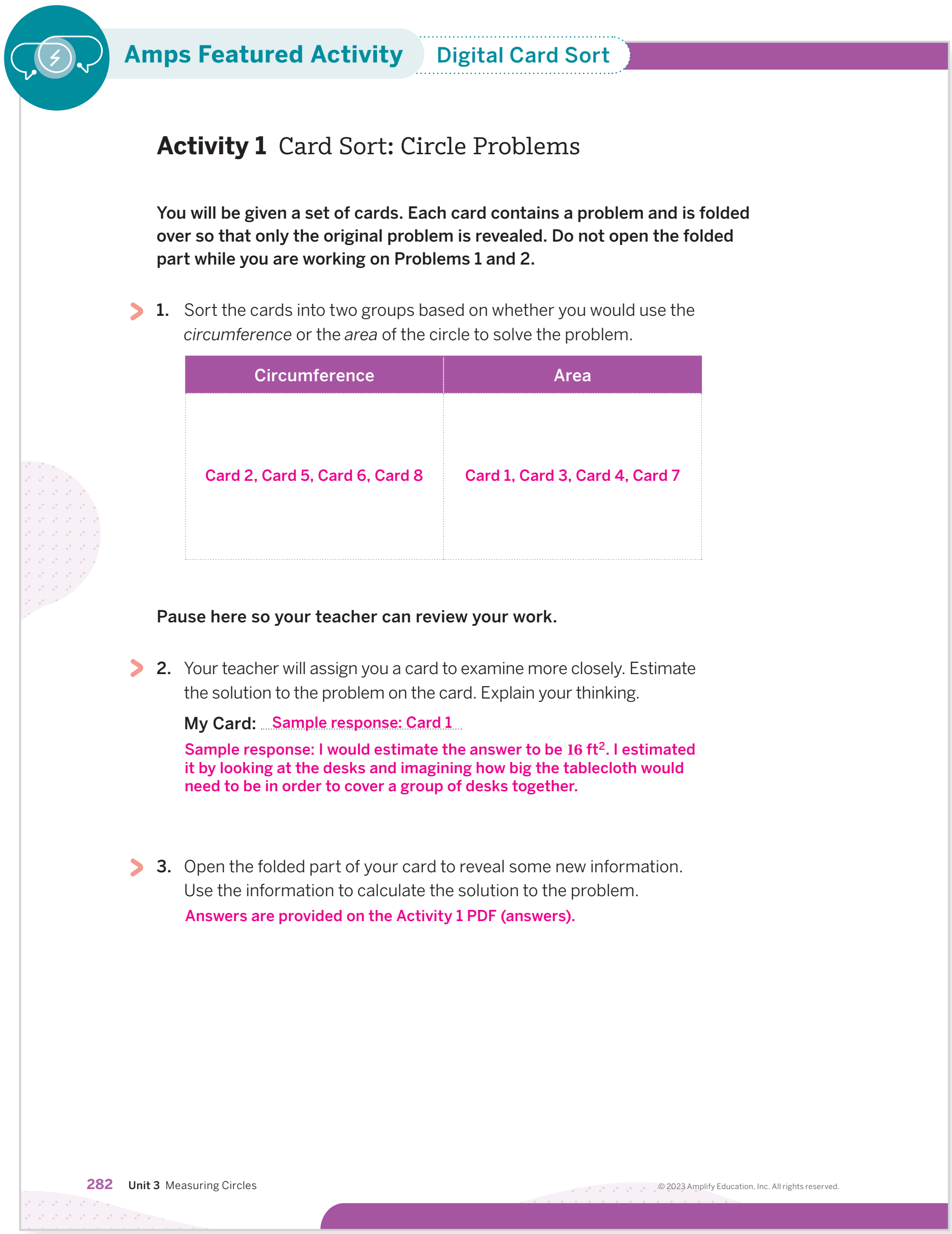
Informed by: Performance on Lesson 10, Practice Problem 6.

Pairs | 10 min

Activity 1 Card Sort: Circle Problems

Students interpret questions about circular, real-world objects to decide whether to calculate the circumference or area.

Amps Featured Activity Digital Card Sort



Activity 1 Card Sort: Circle Problems

You will be given a set of cards. Each card contains a problem and is folded over so that only the original problem is revealed. Do not open the folded part while you are working on Problems 1 and 2.

1. Sort the cards into two groups based on whether you would use the *circumference* or the *area* of the circle to solve the problem.

Circumference	Area
Card 2, Card 5, Card 6, Card 8	Card 1, Card 3, Card 4, Card 7

Pause here so your teacher can review your work.

2. Your teacher will assign you a card to examine more closely. Estimate the solution to the problem on the card. Explain your thinking.

   My Card: Sample response: Card 1

   Sample response: I would estimate the answer to be 16 ft^2. I estimated it by looking at the desks and imagining how big the tablecloth would need to be in order to cover a group of desks together.

3. Open the folded part of your card to reveal some new information. Use the information to calculate the solution to the problem.

   Answers are provided on the Activity 1 PDF (answers).

282 **Unit 3** Measuring Circles

1 Launch

Say, "You have become familiar with how to use the formulas for circles. Now it is time to make sure we know when to use each." Distribute pre-cut cards from the Activity 1 PDF to each pair. Instruct students to keep the folded part of the card closed.

2 Monitor

Help students get started by asking, "What does the circumference measure? What does the area measure?"

Look for points of confusion:

- **Forgetting the formulas for area and circumference.** Have students refer to the Anchor Chart PDF, *Circles*.
- **Being uncertain about which formula to use.** Provide the Graphic Organizer PDF, *Working With Circles (Part 2)*.

Look for productive strategies:

- Using objects in the classroom to help estimate the actual size of each object.

3 Connect

Have pairs of students share their work for Problems 2 and 3. Pair students who had the same card. Then have one student explain, for each card, whether it was more efficient to estimate the radius or the diameter.

Ask, "What key words or terms can help to distinguish an area problem from a circumference problem?"

Highlight that deciding whether to find the area or circumference of a circle requires thinking about what information is unknown and what the circular object is. Estimating the radius or diameter first is the best way to get an accurate estimate for the area or circumference of a circle.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display or provide the Anchor Chart PDF, *Circles*, and the Graphic Organizer PDF, *Working With Circles (Part 2)* for students to reference as they complete this activity.

Consider drawing visual sketches of the problem on each card, or have a student volunteer do so. This will allow students to access the language used in each problem. For example, in Card 4, draw a quick sketch of what a circular patch on the elbow of a jacket looks like so students can visualize the scenario.

Math Language Development

MLR2: Collect and Display

During the Connect, as students respond to the Ask question, collect and display the key words or terms students noticed in each problem that helped them distinguish an area problem from a circumference problem. For example, Card 3 uses the word *cover*, which indicates area. Card 6 uses the phrase *around the circle* and asks how far the ball travels; both of these phrases indicate circumference.

English Learners

Have students use color coding to highlight the key terms that indicate area in one color and the key terms that indicate circumference in another color.

Pairs | 12 min

Activity 2 Analyzing Circle Claims

Students revisit some of the circles from Activity 1 to analyze and critique claims about each situation.

Name: Date: Period:

Activity 2 Analyzing Circle Claims

Here are two students' responses to some problems similar to those you worked on in Activity 1. Do you agree with either of them? Show or explain your thinking.

1. How many feet are traveled by this child riding once around the merry-go-round?
   - **Clare:** "The radius of the merry-go-round is about 4 ft, so the distance around the edge is about 8π ft."
   - **Andre:** "The diameter of the merry-go-round is about 4 ft, so the distance around the edge is about 4π ft."

   Sample response: I agree with Clare because the radius looks to be about the same as the height of the child, which is about 4 ft. That would mean the circumference is 8π ft.

PeopleImages/iStock

2. How much room is there to spread peanut butter on the cracker shown?
   - **Clare:** "The radius of the cracker is about 3 cm, so the space for peanut butter is about 6π cm^2."
   - **Andre:** "The diameter of the cracker is about 3 in., so the space for peanut butter is about 2.25 in^2."

   Sample response: Neither is correct. I think both Clare and Andre have estimated the size of the cracker correctly, but they both made mistakes approximating the area of the cracker.

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3. Professional BMX rider Julian Molina needs to ride about 20 m to get enough speed to complete a BMX trick over an obstacle. How many times do his bike wheels rotate while traveling this distance?
   - **Clare:** "The diameter of the bike wheel is about 0.5 m. In 20 ft, the wheels will rotate completely about 12.7π times."
   - **Andre:** "I agree with Clare's estimate of the diameter, but the bike wheels will rotate completely about 6.4 times."

   Sample response: Neither is correct. If the diameter is 0.5 m, then the circumference is 0.5π. In 20 ft, the wheels would rotate completely about $20 \div 0.5\pi \approx 12.7$ times.

Reflect: How well were you able to communicate whether you agreed or disagreed with your classmates?

STOP

Lesson 11 Distinguishing Circumference and Area 283

1 Launch

Keep students in the same pairs as in Activity 1. Let students know they will now analyze how others estimated solutions to similar problems.

2 Monitor

Help students get started by having them consider the relative size of objects in each photo. For Problem 1, ask "How tall do you think the child is?"

Look for points of confusion:

- **Only checking whether the estimate is appropriate.** Say, "There seems to be four things we need to check for each problem: both students' estimates and both students' explanations."
- **Not realizing there is an error with both of the claims in Problems 2 and 3.** Let students know that it is possible for both Clare and Andre to be mistaken.

3 Connect

Have pairs of students share which person's claim, Clare's or Andre's, is more accurate.

Ask:

- "When would you need to determine the circumference of a circle?"
- "When would you need to determine the area?"
- "What do you need to know, the radius or the diameter, to determine the circumference of a circle? How about for the area?"

Highlight that the formula for the area of a circle has a squared term and the units for area are square units, while the formula for the circumference does not have a squared term and the units are linear units.

Note: Share an inspiring video of Julian Molina — the first adaptive athlete to compete in the Real BMX competition.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them only evaluate Clare's claim for each problem.

Extension: *Math Enrichment*

Have students write another claim for Problem 3 with an error that is different from the one that Clare or Andre made. Sample response: If the diameter of the bike is about 0.5 m, then the wheels will rotate completely about 31.4 times over a distance of 20 ft. This claim is incorrect because we need to find $20 \div 0.5\pi$, not $20 \cdot 0.5\pi$, to determine how many times the wheels will rotate.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask questions, press for details in their reasoning. For example, if a student says, "I need to know the circumference when I'm asked for a distance," ask for further clarification with these probing questions:

- "What do you mean by distance? Can you be more specific?"
- "A radius represents a distance. How do you know when you are asked to determine the radius or the circumference?"

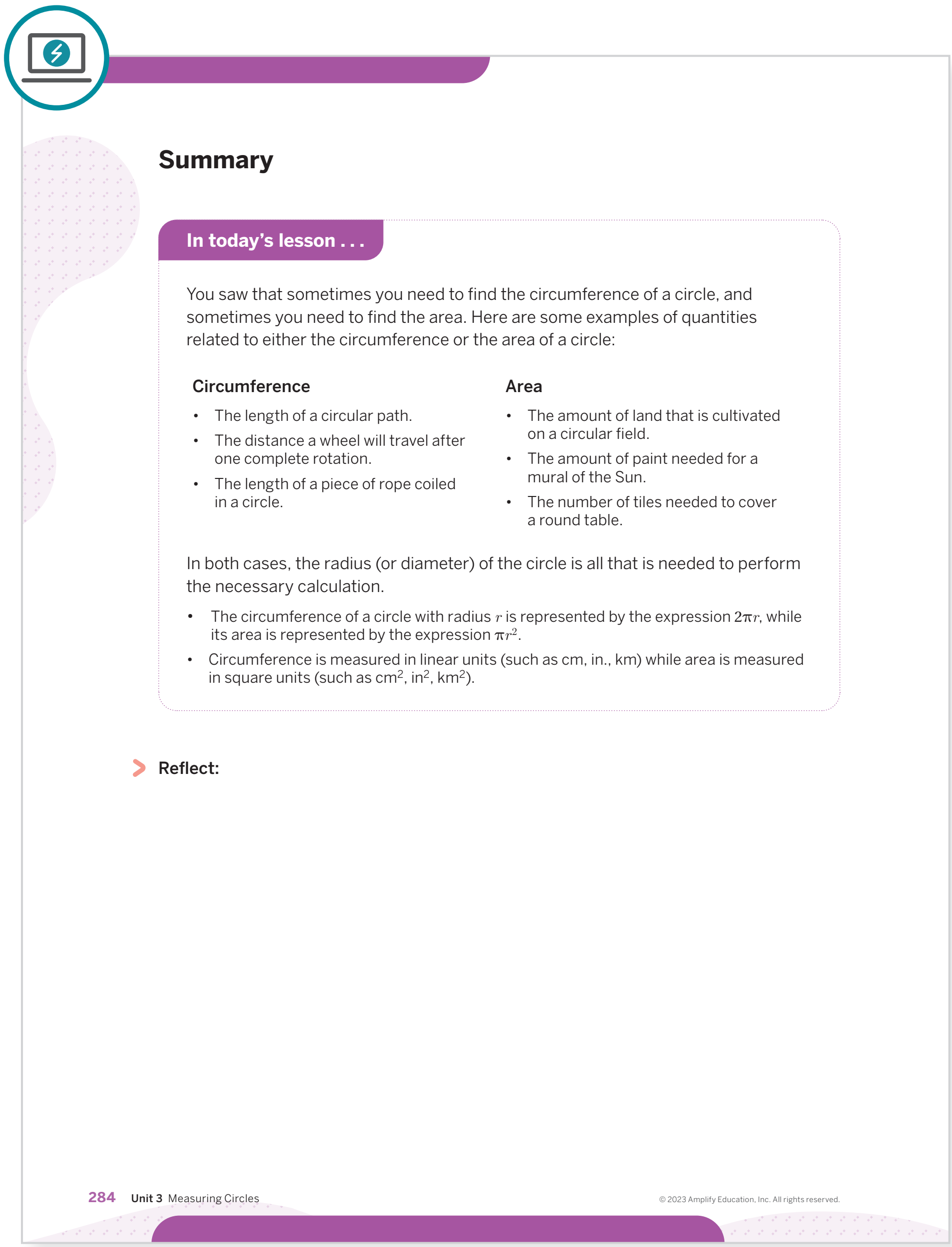
English Learners

Have students highlight key phrases that indicate circumference or area, such as *distance around or how much room (space).*

Whole Class | 5 min

Summary

Review and synthesize how to decide whether a situation about a circle has to do with area or circumference.



Summary

In today's lesson . . .

You saw that sometimes you need to find the circumference of a circle, and sometimes you need to find the area. Here are some examples of quantities related to either the circumference or the area of a circle:

Circumference	Area
The length of a circular path.	The amount of land that is cultivated on a circular field.
The distance a wheel will travel after one complete rotation.	The amount of paint needed for a mural of the Sun.
The length of a piece of rope coiled in a circle.	The number of tiles needed to cover a round table.

In both cases, the radius (or diameter) of the circle is all that is needed to perform the necessary calculation.

- The circumference of a circle with radius r is represented by the expression $2\pi r$, while its area is represented by the expression πr^2.
- Circumference is measured in linear units (such as cm, in., km) while area is measured in square units (such as cm^2, in^2, km^2).

Reflect:

284 Unit 3 Measuring Circles

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Synthesize

Display the Anchor Chart PDF, *Circles* to the class.

Have students share other real-world objects and situations, besides the ones described in this lesson, for which finding the circumference or area would be relevant.

Ask, "Why do you think circles have been an important part of mathematical discussions from some of our earliest recorded history?"

Highlight that circles are an integral part of nature and that humans have wanted to understand both how to make and measure those circles.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

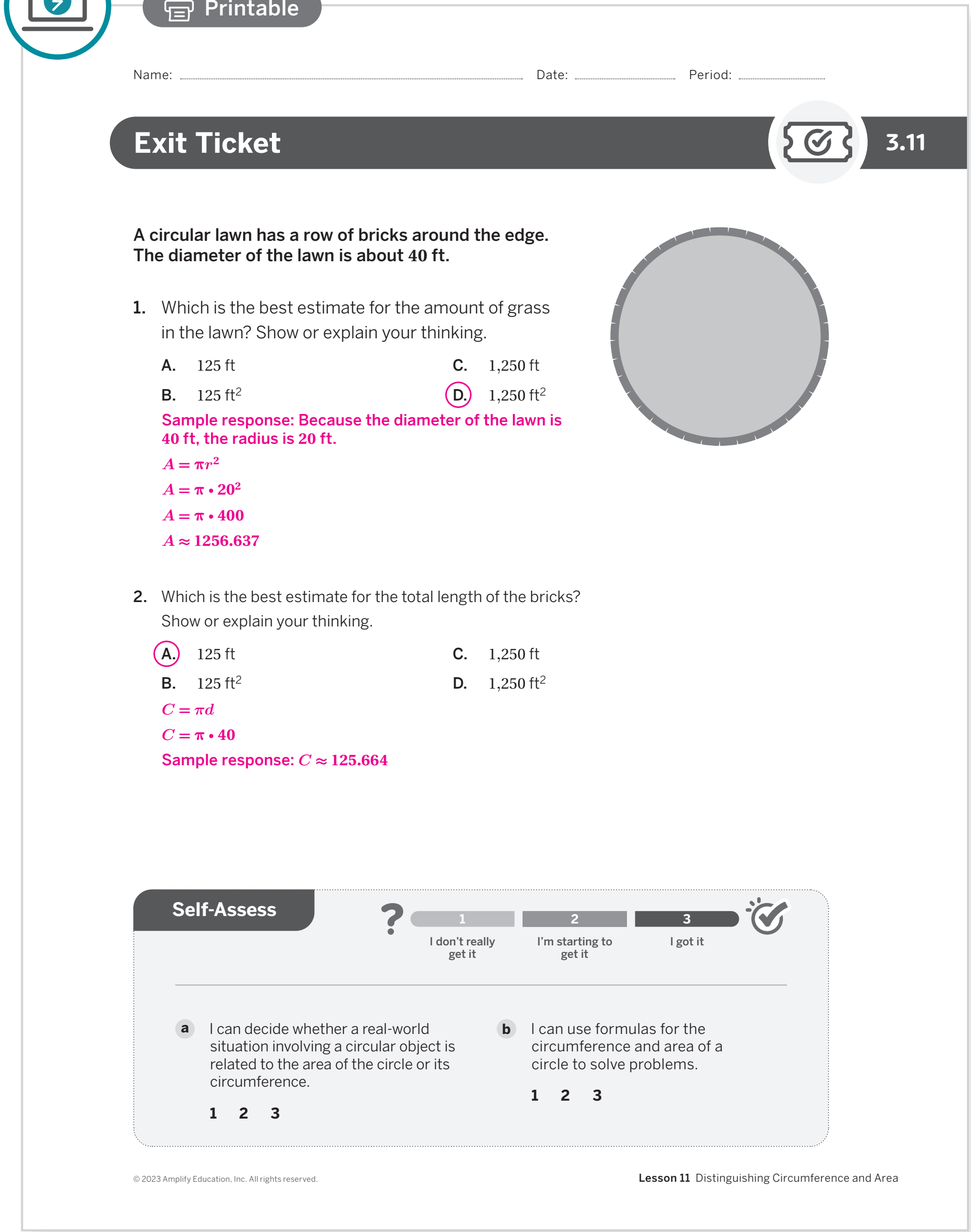
- "What tips or strategies would you give someone else to help them understand the difference between area and circumference?"

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of the distinction between the circumference and the area of a circle by finding the appropriate measures of a lawn.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 3.11

A circular lawn has a row of bricks around the edge. The diameter of the lawn is about 40 ft.

1. Which is the best estimate for the amount of grass in the lawn? Show or explain your thinking.

   A. 125 ft
   B. 125 ft^2
   C. 1,250 ft
   [✓ D.] 1,250 ft^2

   Sample response: Because the diameter of the lawn is 40 ft, the radius is 20 ft.

   $A = \pi r^2$
   $A = \pi \cdot 20^2$
   $A = \pi \cdot 400$
   $A \approx 1256.637$

2. Which is the best estimate for the total length of the bricks? Show or explain your thinking.

   [✓ A.] 125 ft
   B. 125 ft^2
   C. 1,250 ft
   D. 1,250 ft^2

   $C = \pi d$
   $C = \pi \cdot 40$
   Sample response: $C \approx 125.664$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can decide whether a real-world situation involving a circular object is related to the area of the circle or its circumference.
1 2 3

b I can use formulas for the circumference and area of a circle to solve problems.
1 2 3

Lesson 11 Distinguishing Circumference and Area

Success looks like . . .

- **Language Goal:** Critiquing claims about the radius, diameter, circumference, or area of a circle in a real-world situation. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Deciding whether to calculate the circumference or area of a circle to solve a problem in a real-world situation, and justifying the decision. **(Speaking and Listening)**
  - » Determining that the total length of the bricks represents the circumference in Problem 2.
- **Language Goal:** Estimating measurements of a circle in a real-world situation, and explaining the estimation strategy. **(Speaking and Listening, Reading and Writing)**
  - » Estimating the area of a circle in Problem 1 and the circumference in Problem 2.

Suggested next steps

If students select an answer choice with the incorrect unit but the correct numerical value, consider:

- Asking whether they needed to measure in regular units or square units.

If students find the wrong measure for each question, consider:

- Having them shade the part of the picture that the question addresses.
- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did estimating the size of circles in Activity 1 reveal about your students as learners?
- When you compare and contrast today's work with the work students did earlier this year on solving simple equations, what similarities and differences do you see? What might you change for the next time you teach this lesson?

Math Language Development

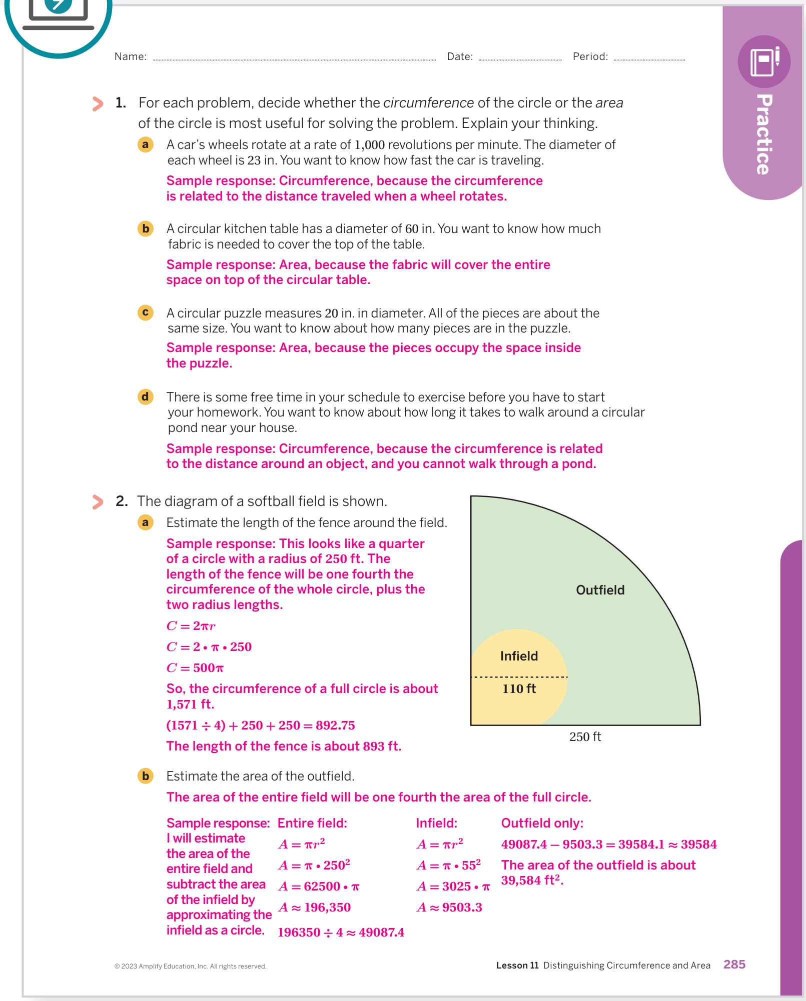
Language Goal: Deciding whether to calculate the circumference or area of a circle to solve a problem in a real-situation, and justifying the decision.

Reflect on students' language development toward this goal.

- During the *Card Sort activity,* how did students make their own determinations for whether each problem indicates circumference or area?
- How did the ***Stronger and Clearer Each*** *Time* routine during the Connect help students look for and use language to help make their determinations?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. For each problem, decide whether the *circumference* of the circle or the *area* of the circle is most useful for solving the problem. Explain your thinking.
   - a. A car's wheels rotate at a rate of 1,000 revolutions per minute. The diameter of each wheel is 23 in. You want to know how fast the car is traveling.

     Sample response: Circumference, because the circumference is related to the distance traveled when a wheel rotates.
   - b. A circular kitchen table has a diameter of 60 in. You want to know how much fabric is needed to cover the top of the table.

     Sample response: Area, because the fabric will cover the entire space on top of the circular table.
   - c. A circular puzzle measures 20 in. in diameter. All of the pieces are about the same size. You want to know about how many pieces are in the puzzle.

     Sample response: Area, because the pieces occupy the space inside the puzzle.
   - d. There is some free time in your schedule to exercise before you have to start your homework. You want to know about how long it takes to walk around a circular pond near your house.

     Sample response: Circumference, because the circumference is related to the distance around an object, and you cannot walk through a pond.

2. The diagram of a softball field is shown.
   - a. Estimate the length of the fence around the field.

     Sample response: This looks like a quarter of a circle with a radius of 250 ft. The length of the fence will be one fourth the circumference of the whole circle, plus the two radius lengths.

     $C = 2\pi r$

     $C = 2 \cdot \pi \cdot 250$

     $C = 500\pi$

     So, the circumference of a full circle is about 1,571 ft.

     $(1571 \div 4) + 250 + 250 = 892.75$

     The length of the fence is about 893 ft.
   - b. Estimate the area of the outfield.

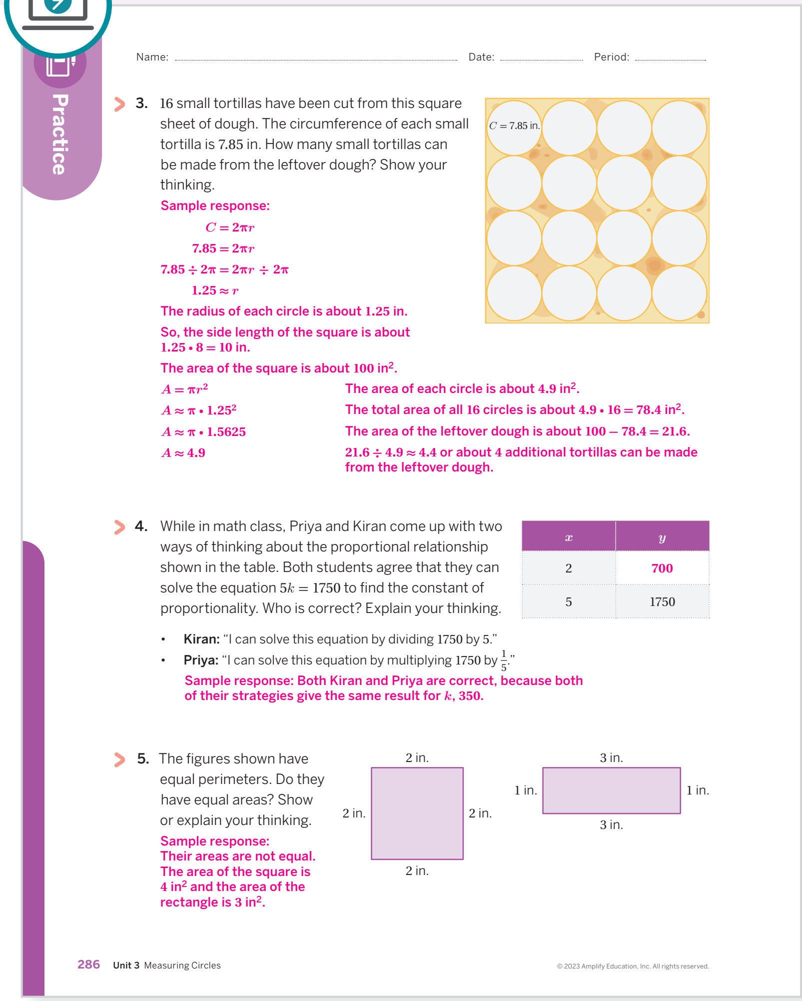
     The area of the entire field will be one fourth the area of the full circle.

     Sample response: I will estimate the area of the entire field and subtract the area of the infield by approximating the infield as a circle.

     Entire field:
     $A = \pi r^2$
     $A = \pi \cdot 250^2$
     $A = 62500 \cdot \pi$
     $A \approx 196{,}350$
     $196350 \div 4 \approx 49087.4$

     Infield:
     $A = \pi r^2$
     $A = \pi \cdot 55^2$
     $A = 3025 \cdot \pi$
     $A \approx 9503.3$

     Outfield only:
     $49087.4 - 9503.3 = 39584.1 \approx 39584$
     The area of the outfield is about 39,584 ft^2.



Name: ______ Date: ______ Period: ______

3. 16 small tortillas have been cut from this square sheet of dough. The circumference of each small tortilla is 7.85 in. How many small tortillas can be made from the leftover dough? Show your thinking.

   Sample response:

   $C = 2\pi r$

   $7.85 = 2\pi r$

   $7.85 \div 2\pi = 2\pi r \div 2\pi$

   $1.25 \approx r$

   The radius of each circle is about 1.25 in.

   So, the side length of the square is about $1.25 \cdot 8 = 10$ in.

   The area of the square is about 100 in^2.

   $A = \pi r^2$

   $A \approx \pi \cdot 1.25^2$

   $A \approx \pi \cdot 1.5625$

   $A \approx 4.9$

   The area of each circle is about 4.9 in^2.

   The total area of all 16 circles is about $4.9 \cdot 16 = 78.4$ in^2.

   The area of the leftover dough is about $100 - 78.4 = 21.6$.

   $21.6 \div 4.9 \approx 4.4$ or about 4 additional tortillas can be made from the leftover dough.

4. While in math class, Priya and Kiran come up with two ways of thinking about the proportional relationship shown in the table. Both students agree that they can solve the equation $5k = 1750$ to find the constant of proportionality. Who is correct? Explain your thinking.

x	y
2	700
5	1750

   - **Kiran:** "I can solve this equation by dividing 1750 by 5."
   - **Priya:** "I can solve this equation by multiplying 1750 by $\frac{1}{5}$."

   Sample response: Both Kiran and Priya are correct, because both of their strategies give the same result for k, 350.

5. The figures shown have equal perimeters. Do they have equal areas? Show or explain your thinking.

   Sample response: Their areas are not equal. The area of the square is 4 in^2 and the area of the rectangle is 3 in^2.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 3 Lesson 10	2
	4	Unit 2 Lesson 5	2
Formative	5	Unit 3 Lesson 12	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Capturing Space

Let’s find out which shape captures the most area.

Focus

Goals

1. Develop a strategy to capture the most space using a loop of string.
2. Compare the areas of different polygons with the same perimeter.
3. Recognize that a circle is a shape that has the greatest area among all the shapes with a given perimeter.

Rigor

- Students build **conceptual understanding** of how mathematical rigor can help prove a conjecture.
- Students develop **fluency** evaluating expressions with fractions and decimals.
- Students **apply** the formulas for the area and perimeter of both polygons and circles.

Coherence

Today

Students play a game and explore a set of polygons to discover one of the properties of circles that makes them so special: that among all shapes with the same perimeter, the circle has the greatest area.

Previously

Students have worked to both derive the formulas for the circumference and area of a circle and learned how and when to use them.

Coming Soon

In Unit 4, students will revisit proportional relationships in the context of percentages.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- 8-inch lengths of string, one per pair
- calculators
- scissors

Math Language Development

Review words

- *circumference*
- *diameter*
- *perimeter*
- π
- *radius*

Amps | Featured Activity

Activity 1

Capture the Dots Game

Students compete to find who can capture the most dots using a length of string. Students play multiple rounds and focus on optimizing their strategy rather than counting dots.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may not understand, on their own, how to start Activity 1 and then become frustrated. If necessary, have each pair of students explain the rules in their own words to each other as they work through the steps of the Activity. This will provide another layer of understanding of the structure of the Activity.

Modifications to Pacing

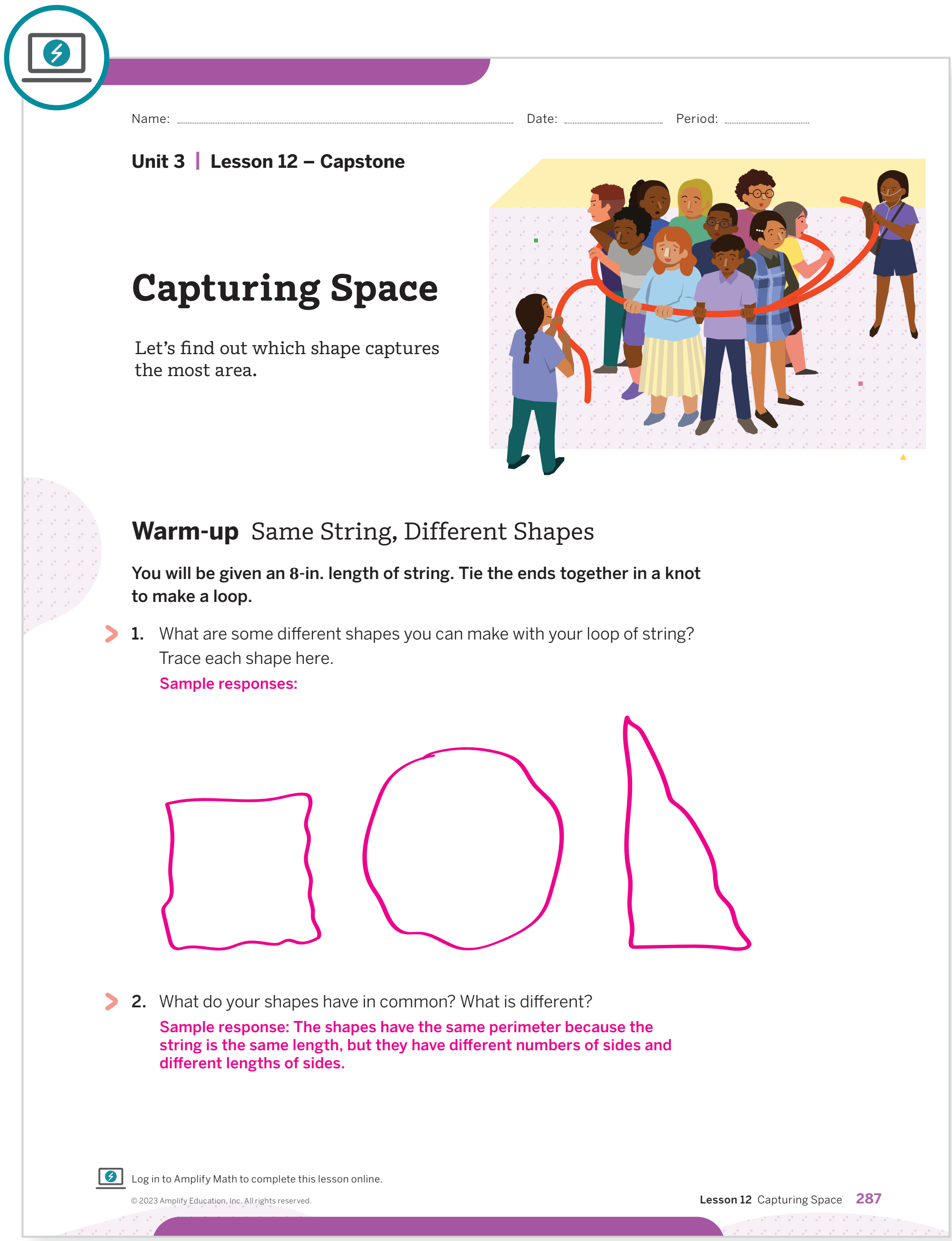
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, assign a different figure to each group, then share and discuss results as a class.

Pairs | 5 min

Warm-up Same String, Different Shapes

Students use a loop of string to make different shapes to connect how constraining the perimeter affects the area of different shapes within the set.



Name: ______________________ Date: __________ Period: __________

Unit 3 | Lesson 12 – Capstone

Capturing Space

Let's find out which shape captures the most area.

Warm-up Same String, Different Shapes

You will be given an 8-in. length of string. Tie the ends together in a knot to make a loop.

1. What are some different shapes you can make with your loop of string? Trace each shape here.
   Sample responses:

2. What do your shapes have in common? What is different?
   Sample response: The shapes have the same perimeter because the string is the same length, but they have different numbers of sides and different lengths of sides.

Log in to Amplify Math to complete this lesson online.

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Lesson 12 Capturing Space 287

1 Launch

Distribute the 8-in. lengths of string to each pair of students. Demonstrate how to make a simple knot. It may help to have students snip the loose ends.

Note: It is not important for all of the loops to be exactly the same size.

2 Monitor

Help students get started by allowing them to assist each other tying their knots.

Look for points of confusion:

- **Making amorphous shapes that are difficult to compare.** While it is acceptable for students to make any shape, encourage them to also try some common polygons.
- **Making sketches that do not match the size of the string.** Ask, "Is the distance around the outside of your sketches staying constant?"

Look for productive strategies:

- Making several versions of the same shape, e.g., rectangles.

3 Connect

Display a set of shapes created by one pair of students. This will ensure that observations can be made about the same length of string.

Have students share what is the same or different among the set of shapes being displayed. Ask the students whose work is being shared whether they agree or disagree with the observations.

Highlight that the reason we used a loop of string is that it forces us to maintain the same *perimeter* of our shapes. Say, "This will be an important rule you follow while you investigate the areas of different shapes in this lesson."

Math Language Development

MLR7: Compare and Connect

If possible, demonstrate tying the knot under a document camera. During the Connect, as students share what is the same or different among the set of shapes being displayed, highlight comparisons about whether the shapes are polygons, their number of sides, side lengths, perimeter (circumference), or other attributes. Revoice student comparisons that do not use mathematically precise language with the correct terms.

Power-up

To power up students' ability to compare a polygon's area to its perimeter, have students complete:

Recall that the formulas for area and perimeter of a rectangle are $A = l \cdot w$ and $P = 2l + 2w$, respectively. Determine the area and perimeter of a rectangle with a length of 3 units and width of 12 units.

$A = l \cdot w$

$A = 3 \cdot 12$

$A = 36$

$P = 2l + 2w$

$P = 2(3) + 2(12)$

$P = 6 + 24$

$P = 30$

Use: Before the Warm-up.

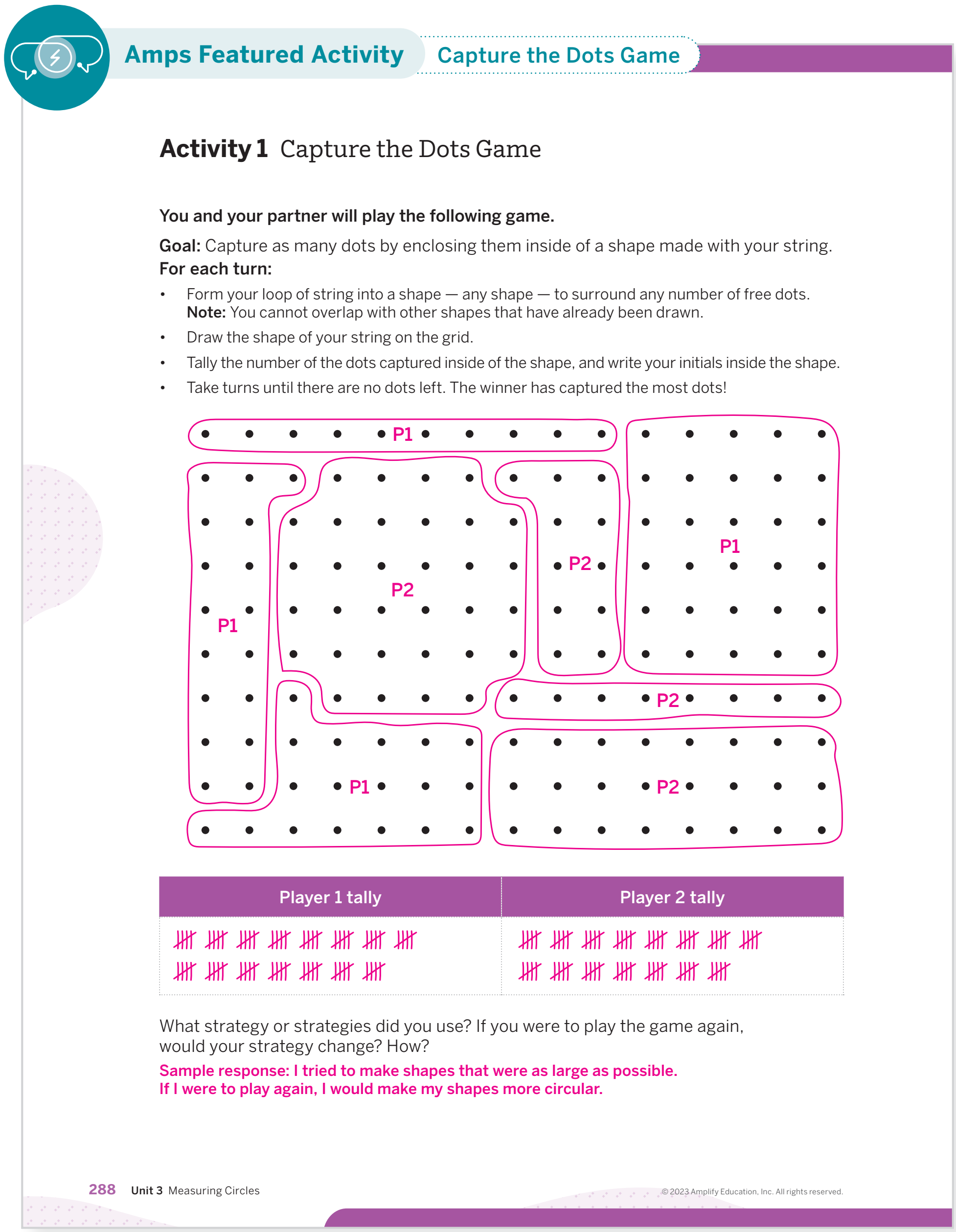
Informed by: Performance on Lesson 11, Practice Problem 5.

Pairs | 10 min

Activity 1 Capture the Dots Game

Students play a game to develop a strategy for maximizing the amount of area captured within a fixed perimeter.

Amps Featured Activity Capture the Dots Game



Activity 1 Capture the Dots Game

You and your partner will play the following game.

Goal: Capture as many dots by enclosing them inside of a shape made with your string.

For each turn:

- Form your loop of string into a shape — any shape — to surround any number of free dots.
  Note: You cannot overlap with other shapes that have already been drawn.
- Draw the shape of your string on the grid.
- Tally the number of the dots captured inside of the shape, and write your initials inside the shape.
- Take turns until there are no dots left. The winner has captured the most dots!

Player 1 tally	Player 2 tally
𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸

What strategy or strategies did you use? If you were to play the game again, would your strategy change? How?

Sample response: I tried to make shapes that were as large as possible. If I were to play again, I would make my shapes more circular.

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1 Launch

Read through the directions for how to play the game as a class. Model playing one round against a student. In the Connect, use the ***Think-Pair-Share*** routine to have students discuss their strategies.

2 Monitor

Help students get started by having them play a quick game of rock-paper-scissors to decide who will go first.

Look for points of confusion:

- **Laying the string directly over a dot.** Have students do their best to arrange the shape of their string so that it passes between points and makes deciding whether the dot is captured more clear.

Look for productive strategies:

- Noticing that certain shapes are capable of capturing greater amounts of dots, and trying to use those shapes as often as possible.
- Noticing that circles are the ideal shape to capture the greatest number of dots.

3 Connect

Have students share their strategies with a new partner.

Ask:

- "If you lost your game, how would you change your strategy for the next game?"
- "If you won your game, will you keep the same strategy or adjust it as well?"

Display a clear board and a loop of string. Show how a square captures more dots than a rectangle. Ask students why this might be.

Highlight that the perimeter of a shape does not have a fixed relationship to the area of the shape.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which the digital technology allows them to play multiple rounds and focus on optimizing their strategy rather than counting dots.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest students shade the areas they capture using a different color than their partner. This will make the distinctions between the areas more visible.

If time permits, consider allowing a few practice rounds before students officially start the game.

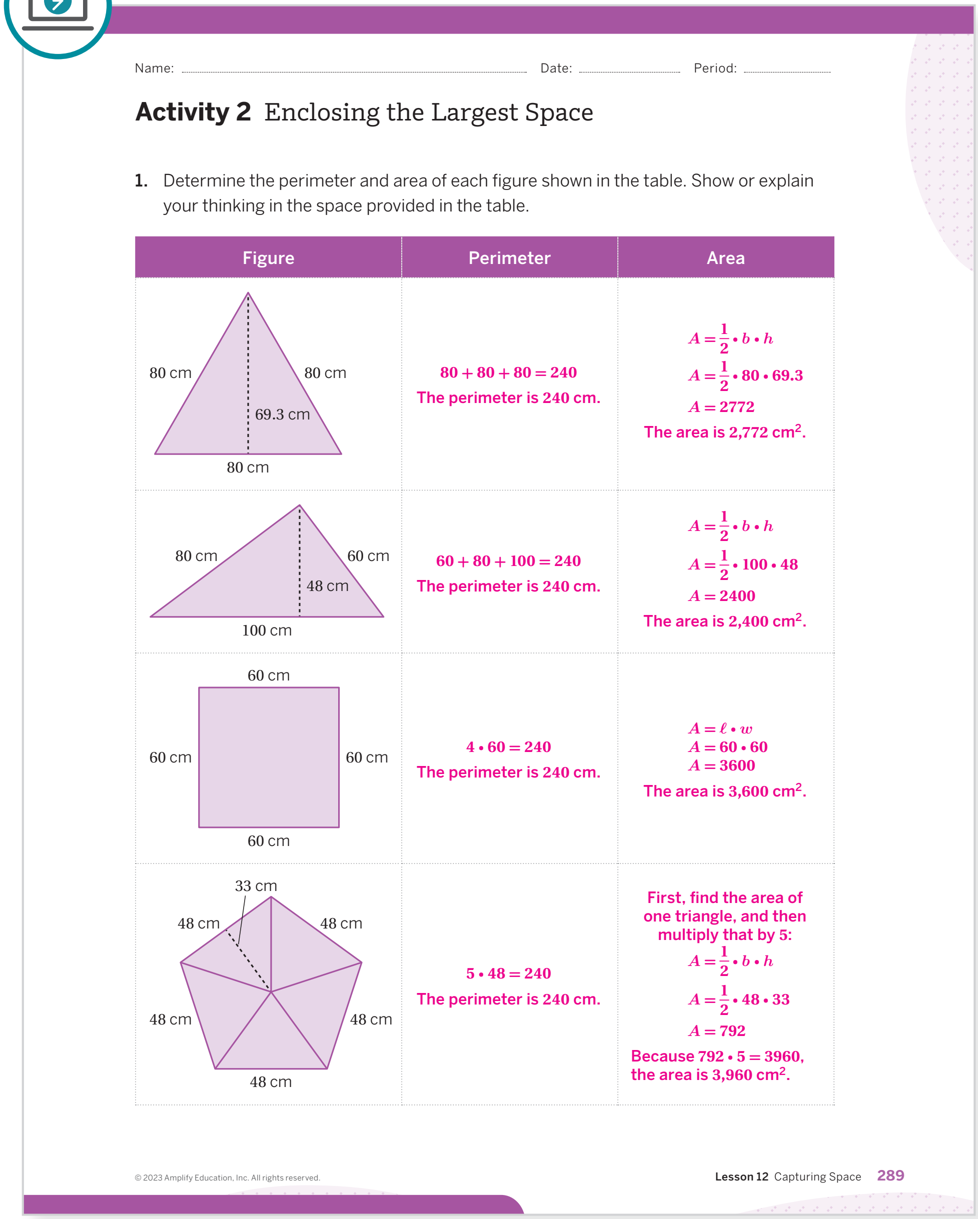
Extension: *Math Enrichment*

Share the definition of a "solved game", a game whose outcome can be predicted assuming both players play perfectly. *Tic-Tac-Toe* is a solved game. Ask, "Do you think *Capture the Dots* is a solved game?"

Small groups | 20 min

Activity 2 Enclosing the Largest Space

Students explore a set of polygons with equal perimeters and compare their areas to notice that the circle encloses the largest area.



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Activity 2 Enclosing the Largest Space

1. Determine the perimeter and area of each figure shown in the table. Show or explain your thinking in the space provided in the table.

Figure	Perimeter	Area
80 cm 80 cm 69.3 cm 80 cm	$80 + 80 + 80 = 240$ The perimeter is 240 cm.	$A = \frac{1}{2} \cdot b \cdot h$ $A = \frac{1}{2} \cdot 80 \cdot 69.3$ $A = 2772$ The area is 2,772 cm^2.
80 cm 60 cm 48 cm 100 cm	$60 + 80 + 100 = 240$ The perimeter is 240 cm.	$A = \frac{1}{2} \cdot b \cdot h$ $A = \frac{1}{2} \cdot 100 \cdot 48$ $A = 2400$ The area is 2,400 cm^2.
60 cm 60 cm 60 cm 60 cm	$4 \cdot 60 = 240$ The perimeter is 240 cm.	$A = \ell \cdot w$ $A = 60 \cdot 60$ $A = 3600$ The area is 3,600 cm^2.
33 cm 48 cm 48 cm 48 cm 48 cm 48 cm	$5 \cdot 48 = 240$ The perimeter is 240 cm.	First, find the area of one triangle, and then multiply that by 5: $A = \frac{1}{2} \cdot b \cdot h$ $A = \frac{1}{2} \cdot 48 \cdot 33$ $A = 792$ Because $792 \cdot 5 = 3960$, the area is 3,960 cm^2.

Lesson 12 Capturing Space 289

1 Launch

Activate prior knowledge by asking students for the perimeter formula for any polygon and the area formulas for triangles and quadrilaterals. Write these down in a place visible to the entire class. Provide access to calculators.

2 Monitor

Help students get started by encouraging small groups to divide up the task in an equitable way.

Look for points of confusion:

- **Not noticing that the perimeters are all the same.** Ask, "In the Warm-up and Activity 1, the perimeters remained the same for all shapes. Is that true here as well?"
- **Using a side length instead of the height when finding the area of a triangle.** Draw an anchor diagram with a generic triangle on a board and label the height of the triangle.

Look for productive strategies:

- Ensuring that each member of the group gets to work with a variety of the shapes and formulas.
- Discussing the answer to Problem 1 as a group before writing their responses.

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Preview the figures shown in the table and ask students to generate a list of the perimeter and area formulas they may need to use. Display these during the activity for students to use as a reference, clarifying the meaning of any variables given in the formulas.

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by having students first complete the table for the two triangles and the square. Have them share their results with another group and then complete the table for the pentagon, hexagon, and circle.

Extension: *Math Enrichment*

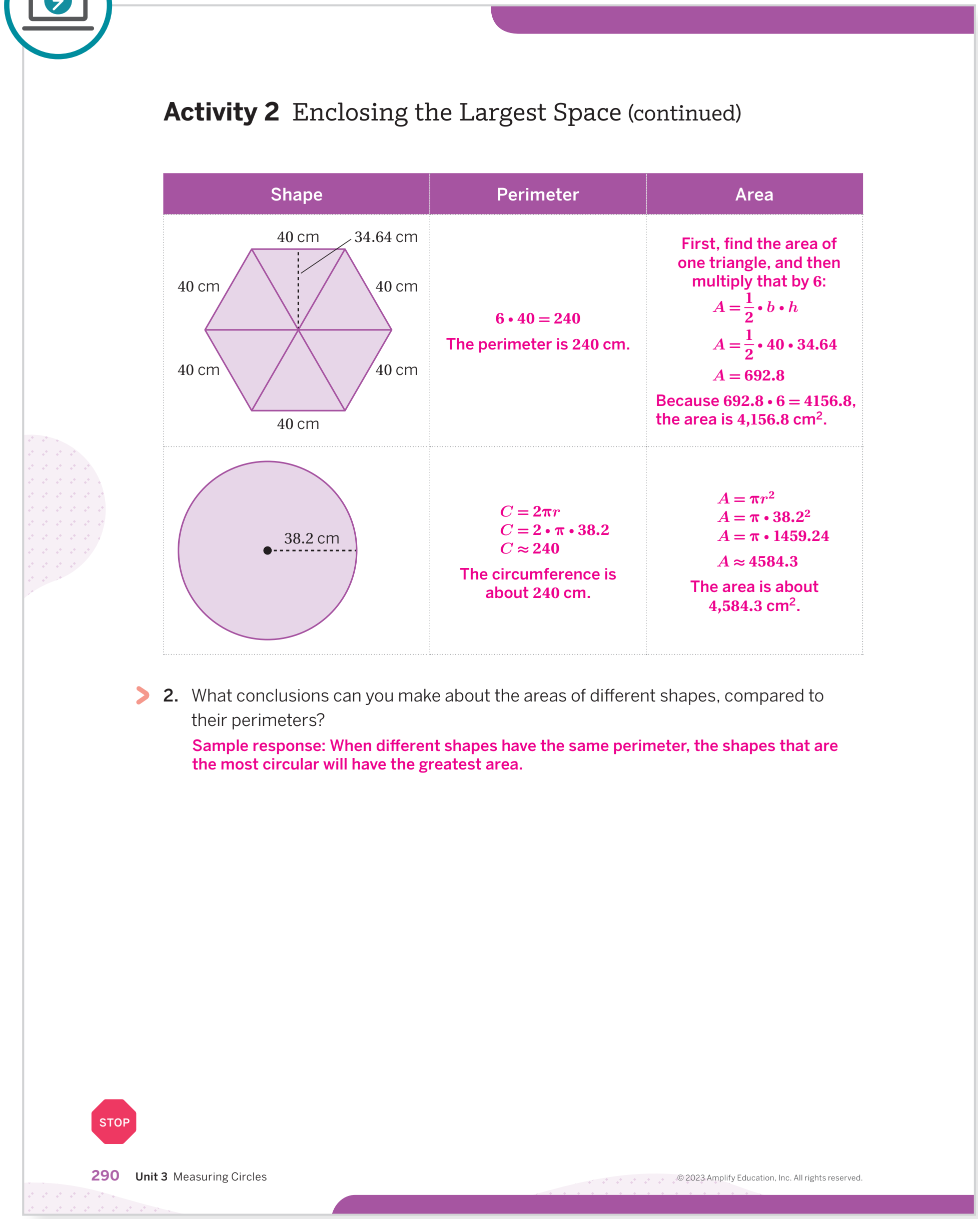
Let students know that this activity is similar to a concept called optimization. Ask students if they know what the terms optimal or optimist means. Optimal means "best." An optimist is a person who takes the "best" attitude in a situation. In this activity, the "best" or the optimal value was the greatest area. However, the optimal solution is not always the maximum value. Sometimes, the optional solution is the minimum value. Ask:

- "What is a real-world example of when you might want to maximize a particular value?" area, volume, money earned
- "What are some real-world examples of when you might want to minimize a particular value?" cost, number of errors, hours worked

Small groups | 20 min

Activity 2 Enclosing the Largest Space (continued)

Students explore a set of polygons with equal perimeters and compare their areas to notice that the circle encloses the largest area.



Activity 2 Enclosing the Largest Space (continued)

Shape	Perimeter	Area
40 cm 34.64 cm 40 cm 40 cm 40 cm 40 cm 40 cm	$6 \cdot 40 = 240$ The perimeter is 240 cm.	First, find the area of one triangle, and then multiply that by 6: $A = \frac{1}{2} \cdot b \cdot h$ $A = \frac{1}{2} \cdot 40 \cdot 34.64$ $A = 692.8$ Because $692.8 \cdot 6 = 4156.8$, the area is 4,156.8 cm^2.
38.2 cm	$C = 2\pi r$ $C = 2 \cdot \pi \cdot 38.2$ $C \approx 240$ The circumference is about 240 cm.	$A = \pi r^2$ $A = \pi \cdot 38.2^2$ $A = \pi \cdot 1459.24$ $A \approx 4584.3$ The area is about 4,584.3 cm^2.

2. What conclusions can you make about the areas of different shapes, compared to their perimeters?

Sample response: When different shapes have the same perimeter, the shapes that are the most circular will have the greatest area.

STOP

3 Connect

Display the activity with answers shown.

Have students share their conclusions from Problem 1.

Ask:

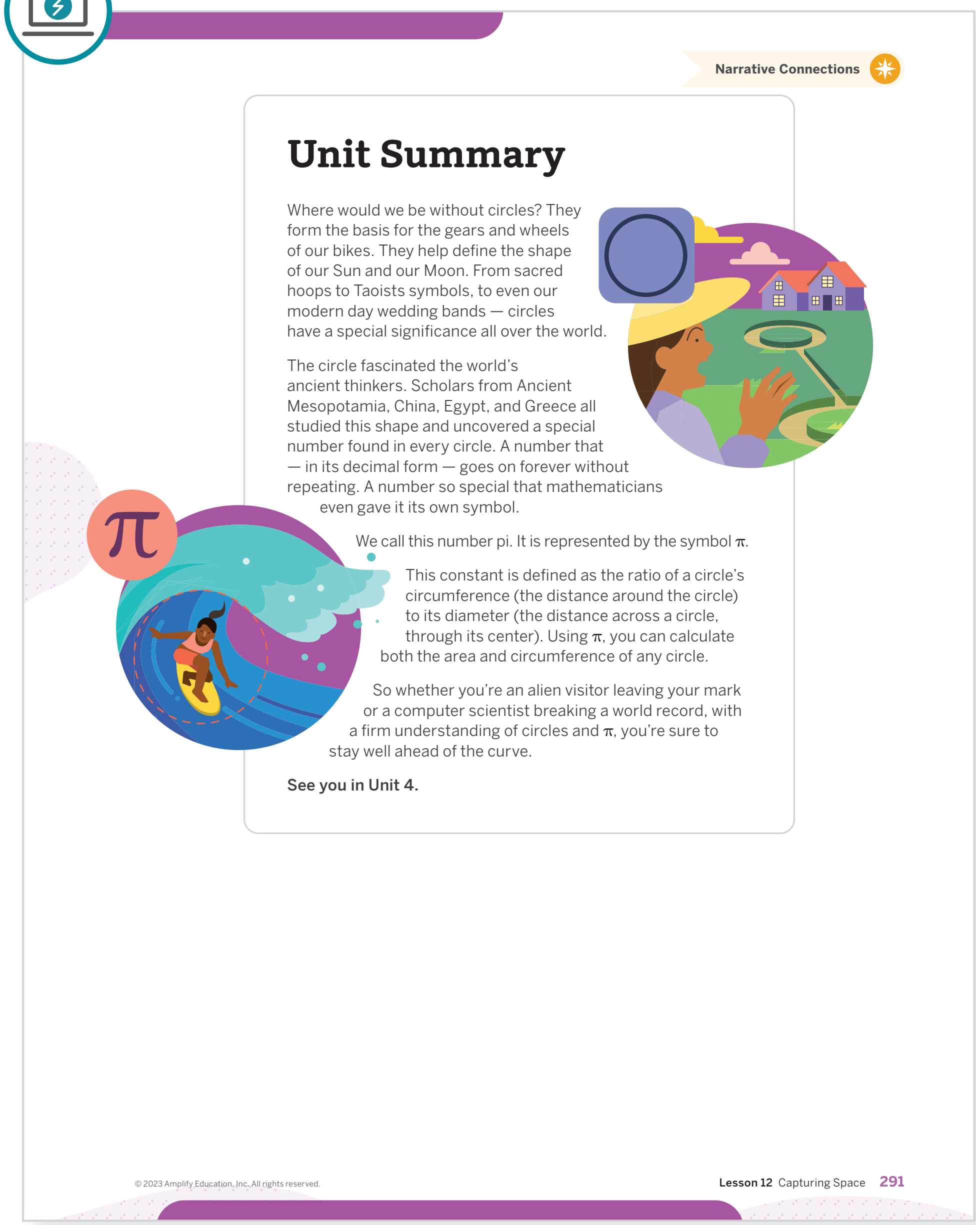
- "As you look for the shape with the greatest area, why is it important to keep the perimeters the same?" This is a way to make sure that the shapes being compared are similar. Without doing this, we could just make one shape larger than the other.
- "What objects can you think of that are designed in a circular shape because it has the greatest area?" Sample response: plates, cups, buckets, pots, pans.
- "How does knowing this fact about circles change your strategy for the *Capture the Dots* game?"

Highlight that this property makes the circle incredibly special. The circle is an ideal shape for certain applications where maximizing area is rewarded.

Whole Class | 5 min

Unit Summary

Review and synthesize what makes circles so special.



Narrative Connections

Unit Summary

Where would we be without circles? They form the basis for the gears and wheels of our bikes. They help define the shape of our Sun and our Moon. From sacred hoops to Taoists symbols, to even our modern day wedding bands — circles have a special significance all over the world.

The circle fascinated the world's ancient thinkers. Scholars from Ancient Mesopotamia, China, Egypt, and Greece all studied this shape and uncovered a special number found in every circle. A number that — in its decimal form — goes on forever without repeating. A number so special that mathematicians even gave it its own symbol.

We call this number pi. It is represented by the symbol π.

This constant is defined as the ratio of a circle's circumference (the distance around the circle) to its diameter (the distance across a circle, through its center). Using π, you can calculate both the area and circumference of any circle.

So whether you're an alien visitor leaving your mark or a computer scientist breaking a world record, with a firm understanding of circles and π, you're sure to stay well ahead of the curve.

See you in Unit 4.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Highlight that it is very useful to spend some time reflecting on one's own learning at the conclusion of a unit. This helps to retain information and make connections that students may not have been able to make while focusing on each individual lesson.

Ask students to take a few minutes to recall what they have learned about circles and their formulas throughout this unit.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for *reflect* around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Differentiated Support

Extension: *Math Enrichment*

Ask if students have heard of the constant tau, as it relates to circles. Tau, represented by the Greek letter τ is equivalent to 2π. While π represents the ratio of a circle's circumference to its diameter, τ represents the ratio of a circle's circumference to its radius. Some claim that tau is a more natural choice to represent the proportional relationship between circumference of a circle and it's linear dimension because:

- A circle is defined by its radius. A circle is the set of all points equidistant from a center point.
- Many equations involving circles, such as the area formula for a circle, are written in terms of their radius, not their diameter.

Many mathematicians and scientists celebrate Pi Day on March $14\left(\frac{3}{14}\right)$ of every year. Similarly, Tau Day, celebrated on June $28\left(\frac{6}{28}\right)$ is becoming increasingly more popular. Ask students to write the formulas for the circumference and area of the circle using τ, instead of π.

$C = r$

$A = \frac{1}{2}r^2$

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of circles and their formulas by reflecting on what they learned and voicing any unresolved questions they may have.

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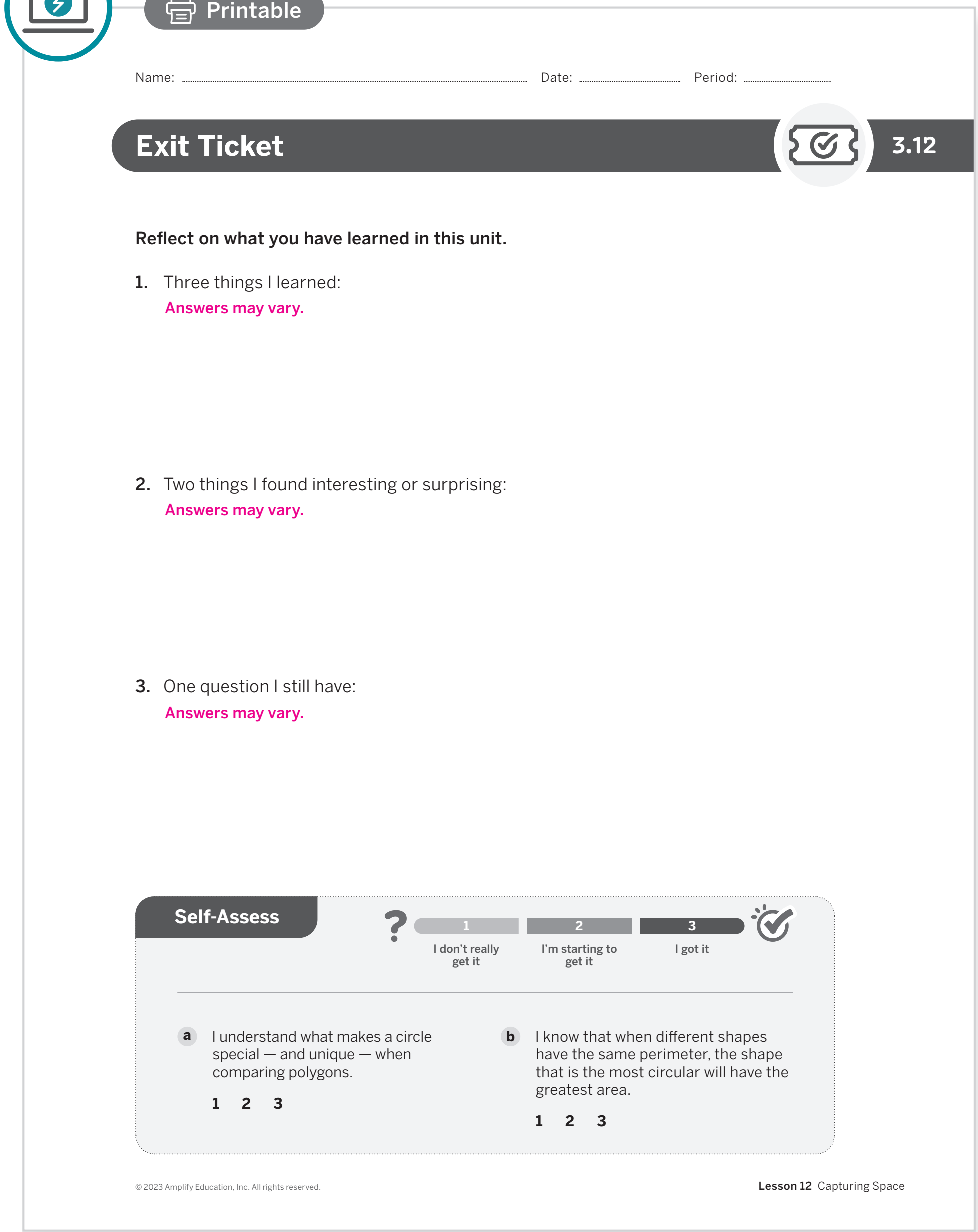


Name: Date: Period:

Exit Ticket 3.12

Reflect on what you have learned in this unit.

1. Three things I learned:
   Answers may vary.
2. Two things I found interesting or surprising:
   Answers may vary.
3. One question I still have:
   Answers may vary.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I understand what makes a circle special — and unique — when comparing polygons.

1 2 3

b I know that when different shapes have the same perimeter, the shape that is the most circular will have the greatest area.

1 2 3

Success looks like . . .

- **Goal:** Developing a strategy to capture the most space using a loop of string.
- **Goal:** Comparing the areas of different polygons with the same perimeter.
- **Goal:** Recognizing that a circle is a shape that has the greatest area among all the shapes with a given perimeter.
  - » Explaining this characteristic of a circle in Problem 1 or 2.

Suggested next steps

If students cannot think of three things they have learned, consider:

- Allowing them to review what they wrote in their Summary notes from each lesson.

If students cannot think of two things they found interesting or surprising, consider:

- Changing the prompt to "Two things that I enjoyed during this unit are . . ."

Professional Learning

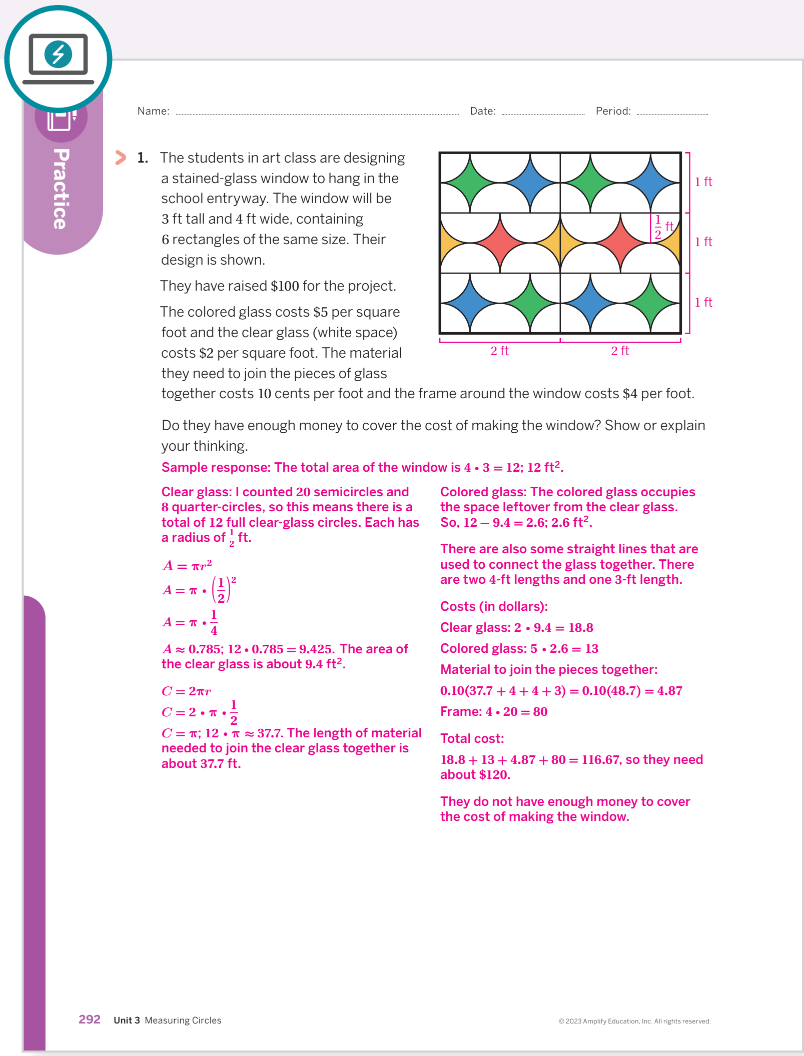
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about the optimal strategy for the Capture the Dots game, how did you encourage each student to listen to one another's strategies?
- What challenges did students encounter as they played the game? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. The students in art class are designing a stained-glass window to hang in the school entryway. The window will be 3 ft tall and 4 ft wide, containing 6 rectangles of the same size. Their design is shown.

   They have raised $100 for the project.

   The colored glass costs $5 per square foot and the clear glass (white space) costs $2 per square foot. The material they need to join the pieces of glass together costs 10 cents per foot and the frame around the window costs $4 per foot.

   Do they have enough money to cover the cost of making the window? Show or explain your thinking.

Sample response: The total area of the window is $4 \cdot 3 = 12$; 12 ft^2.

Clear glass: I counted 20 semicircles and 8 quarter-circles, so this means there is a total of 12 full clear-glass circles. Each has a radius of $\frac{1}{2}$ ft.

$A = \pi r^2$

$A = \pi \cdot \left(\frac{1}{2}\right)^2$

$A = \pi \cdot \frac{1}{4}$

$A \approx 0.785$; $12 \cdot 0.785 = 9.425$. The area of the clear glass is about 9.4 ft^2.

$C = 2\pi r$

$C = 2 \cdot \pi \cdot \frac{1}{2}$

$C = \pi$; $12 \cdot \pi \approx 37.7$. The length of material needed to join the clear glass together is about 37.7 ft.

Colored glass: The colored glass occupies the space leftover from the clear glass. So, $12 - 9.4 = 2.6$; 2.6 ft^2.

There are also some straight lines that are used to connect the glass together. There are two 4-ft lengths and one 3-ft length.

Costs (in dollars):

Clear glass: $2 \cdot 9.4 = 18.8$

Colored glass: $5 \cdot 2.6 = 13$

Material to join the pieces together:

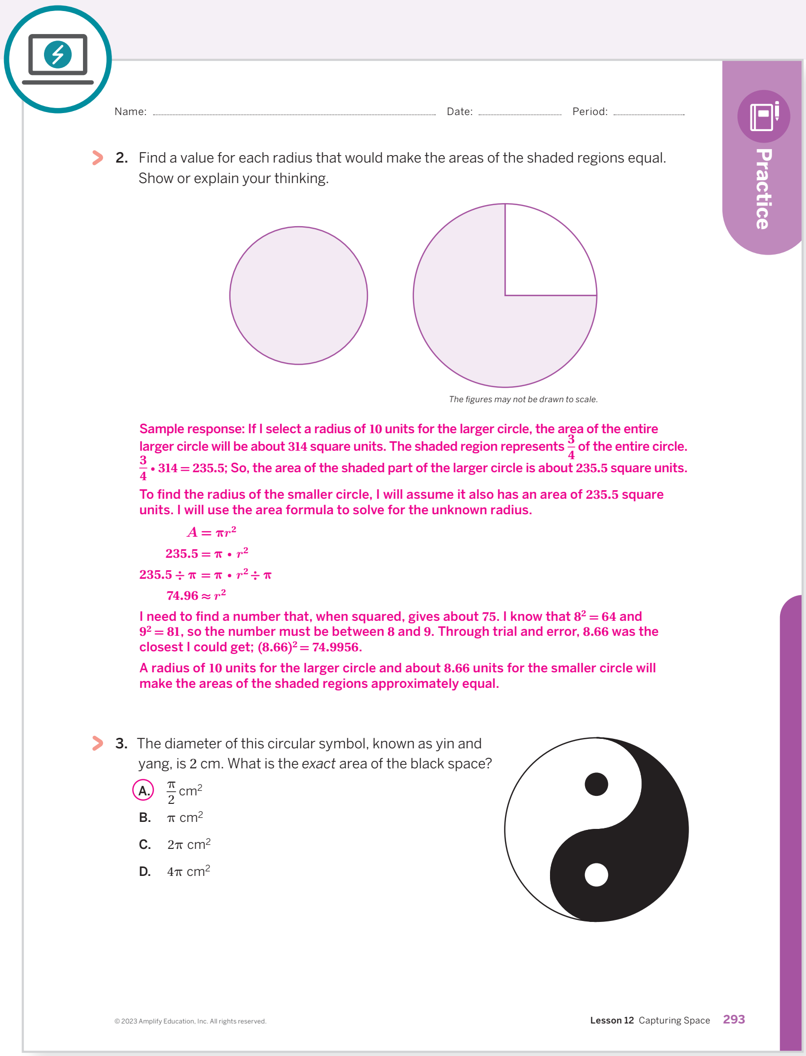
$0.10(37.7 + 4 + 4 + 3) = 0.10(48.7) = 4.87$

Frame: $4 \cdot 20 = 80$

Total cost:

$18.8 + 13 + 4.87 + 80 = 116.67$, so they need about $120.

They do not have enough money to cover the cost of making the window.



Name: Date: Period:

2. Find a value for each radius that would make the areas of the shaded regions equal. Show or explain your thinking.

The figures may not be drawn to scale.

Sample response: If I select a radius of 10 units for the larger circle, the area of the entire larger circle will be about 314 square units. The shaded region represents $\frac{3}{4}$ of the entire circle. $\frac{3}{4} \cdot 314 = 235.5$; So, the area of the shaded part of the larger circle is about 235.5 square units.

To find the radius of the smaller circle, I will assume it also has an area of 235.5 square units. I will use the area formula to solve for the unknown radius.

$$A = \pi r^2$$
$$235.5 = \pi \cdot r^2$$
$$235.5 \div \pi = \pi \cdot r^2 \div \pi$$
$$74.96 \approx r^2$$

I need to find a number that, when squared, gives about 75. I know that $8^2 = 64$ and $9^2 = 81$, so the number must be between 8 and 9. Through trial and error, 8.66 was the closest I could get; $(8.66)^2 = 74.9956$.

A radius of 10 units for the larger circle and about 8.66 units for the smaller circle will make the areas of the shaded regions approximately equal.

3. The diameter of this circular symbol, known as yin and yang, is 2 cm. What is the *exact* area of the black space?
   - [✓ A.] $\frac{\pi}{2}$ cm^2
   - B. π cm^2
   - C. 2π cm^2
   - D. 4π cm^2

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 3 Lesson 11	2
	2	Unit 3 Lesson 10	3
	3	Unit 3 Lesson 10	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

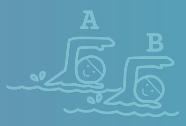
UNIT 4

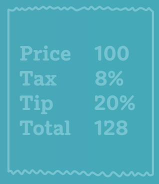
Percentages

From the supermarket to the stock market, percentages are relied on to communicate quickly about how much something has changed. Students build on their experience with proportional relationships while using percentages to compare quantities within the friendly confines of the number 100.

Essential Questions

- How are percentages related to proportional relationships?
- How are percentages used to represent change?
- When is it most helpful to use percentages?
- *(By the way, how come the price you see isn't always the price you pay?)*





Key Shifts in Mathematics

Focus

In this unit . . .

Students deepen their understanding of ratios and proportional relationships, using them to solve multi-step problems that are set in a wide variety of contexts that involve percentages. First, they consider situations for which percentages can be used to describe a change relative to some other amount. Later, they use their ability to find percentages to solve problems related to financial contexts and error.

Coherence

Previously . . .

In Grade 6, students began their work with percentages, although they did not explicitly refer to percentages as proportional relationships. Earlier this year, in Unit 2, students explored proportional relationships and the expressions and equations related to them. Additionally, the geometric scaling work from Unit 1 played a role in developing a visual understanding of proportionality — which can be extended here to percents.

Coming soon . . .

In Unit 6, students will return to working with expressions and equations to represent both proportional and nonproportional relationships. Students will also specifically return to the relationship between percent problems and the equations that represent them in this same unit.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students see representations of *percent of*, *percent increase*, and *percent decrease* situations side by side to notice what differentiates these types of problems (Lesson 3).

Procedural Fluency

A series of related markup and markdown problems, solved using equations, helps students notice the patterns inherent in this strategy. (Lesson 6).

Application

Students apply what they know about using equations to solve percentage problems and apply them to new contexts involving monetary transactions, such as sales tax and tipping (Lesson 8).

Keepin' it 100

SUB-UNIT

1

Lessons 2–7

Percent Increase and Decrease

Students use their understanding of percentages to solve problems involving ***percent change***, including when different given and unknown values are present. They use tape diagrams, expressions, and equations to reason about and solve problems involving ***percent increase*** and ***percent decrease***.

Narrative: Having a solid understanding of percentages can help you spot misleading news headlines.

SUB-UNIT

2

Lessons 8–12

Applying Percentages

Students recognize financial contexts — taxes, tips, discounts, markups, commission, and interest — as applications of percent change. They extend their understanding of solving problems involving percent increase and decrease to these types of financial transactions.

Narrative: Understand the importance of using precision when communicating financial aspects of percent change.

Launch

Lesson 1

(Re)Presenting the United States

Maps are incredibly rich sources of information. But when is a map not enough? Students use percentages and proportional relationships as they explore how to represent where the people are on a map — and notice how traditional maps often hide this information.

Capstone

Lesson 13

Writing Better Headlines

Skimming headlines is a useful skill for anyone who wants to be an informed global citizen, but sometimes headlines on their own can obscure the most important information. Students apply their understanding of percentages to highlight and stay true to the facts as they craft responsible and accurate headlines.

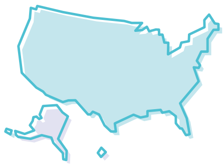
Unit at a Glance

Spoiler Alert: Percentages are just dressed-up proportional relationships. Many of the understandings from Unit 2 still apply, as long as students keep in mind the comparison is to 100.

Assessment

A Pre-Unit Readiness Assessment

Launch Lesson



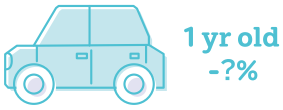
1 (Re)Presenting the United States

Use percentages to analyze land area and population to notice how population density is sometimes misleadingly represented on maps.

Sub-Unit 1: Percent Increase and Decrease

2 Understanding Percentages Involving Decimals

Build on understanding of percentages from Grade 6 to make sense of percentages that are not whole numbers.



6 Percent Increase and Decrease With Equations ●

Connect proportional relationships and percent change scenarios in order to write equations.

$100\% + 130\% = 230\%$

$x + 1.3x = 2.3x$

7 Using Equations to Solve Percent Problems ●

Represent percent increase and decrease problems with equations, and use them to solve for various unknown values.

Sub-Unit 2: Applying Percentages

Price	100
Tax	8%
Tip	20%
Total	128

8 Tax and Tip

Apply percent reasoning to contexts involving money — specifically sales tax and tips.

$100 \pm 1.5\%$ √

12 Error Intervals (optional) ●

Investigate real-world situations where error is expected and calculate values within a certain amount of error.

Capstone Lesson

13 Writing Better Headlines

Use percentages to craft responsible and accurate headlines for news stories.

Assessment

A End-of-Unit Assessment

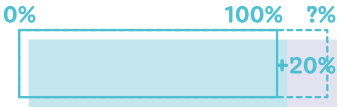
Key Concepts

Lesson 3: Identifying the original amount is crucial for percent change problems.
Lesson 6: Equations help make sense of the structure of percentage problems.
Lesson 9: Make strong connections between new percentage contexts and mathematical language used in those contexts.

Pacing

13 Lessons: 45 min each
2 Assessments: 45 min each
Full Unit: 15 days
● **Modified Unit:** 10 days

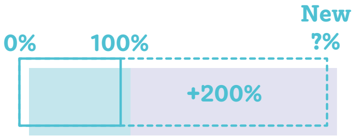
Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



3

Percent Increase and Decrease

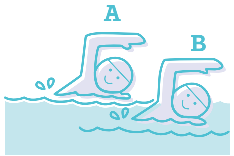
Model problems involving percent increase and decrease with tape diagrams and write expressions to represent the scenarios.



4

Determining 100%

Use tape diagrams to model percent increase and decrease to make sense of, and solve, problems that involve determining the original value.



5

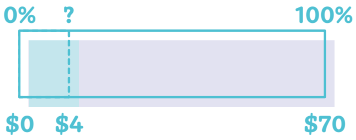
Determining Percent Change

Use tape diagrams to make sense of problems involving percent increase and decrease to determine the percent of change.

9

Percentage Contexts

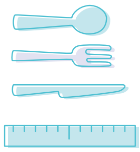
Continue to solve percent increase and decrease problems in contexts involving money, now including commission and simple interest.



10

Determining the Percentage ●

Solve a variety of multi-step percentage problems involving tax, tips, and discounts — including problems involving fractional percentages.



11

Measurement Error ●

Explore how to measure an amount of error using percentages and compare how percent error relates to percent increase and decrease.

● Modifications to Pacing

Lessons 6–7: These lessons support development of students' ability to reason about percentage situations using equations. However, this will be revisited in Unit 6, so this exploration could be reduced to a single lesson.

Lesson 10: This lesson provides further opportunity to practice determining missing percentages in context, but the concept is similar to work in Lesson 5 and can be omitted.

Lessons 11–12: You might opt to combine these lessons, both introducing the concept of percent error and working with error intervals.

Unit Supports

Math Language Development

Lesson	New Vocabulary
3	markdown markup percent decrease percent increase profit retail price
5	percent change
8	sales tax tip (gratuity)
9	commission simple interest
11	percent error
12	error interval

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
13	**MLR1:** Stronger and Clearer Each Time
3, 5, 6, 8, 9, 11	**MLR2:** Collect and Display
6, 9, 10	**MLR3:** Critique, Correct, Clarify
10, 12	**MLR4:** Information Gap
5, 6, 8, 12, 13	**MLR6:** Three Reads
2–7, 11	**MLR7:** Compare and Connect
1, 2	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Materials
1–12	calculators
8	index cards (optional)
1	markers
1, 3–13	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
3–5	sticky notes

Instructional Routines

Activities throughout this unit include these routines:

Lesson(s)	Instructional Routines
4, 5, 9	Card Sort
13	Gallery Tour
10, 12	Info Gap
1, 8	Notice and Wonder
7	Number Talk
7, 11	Partner Problems
2, 3, 4, 5, 6	Poll the Class
2, 3, 5, 6, 9, 11	Think-Pair-Share

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 13

Amps
POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

Analyzing State Data

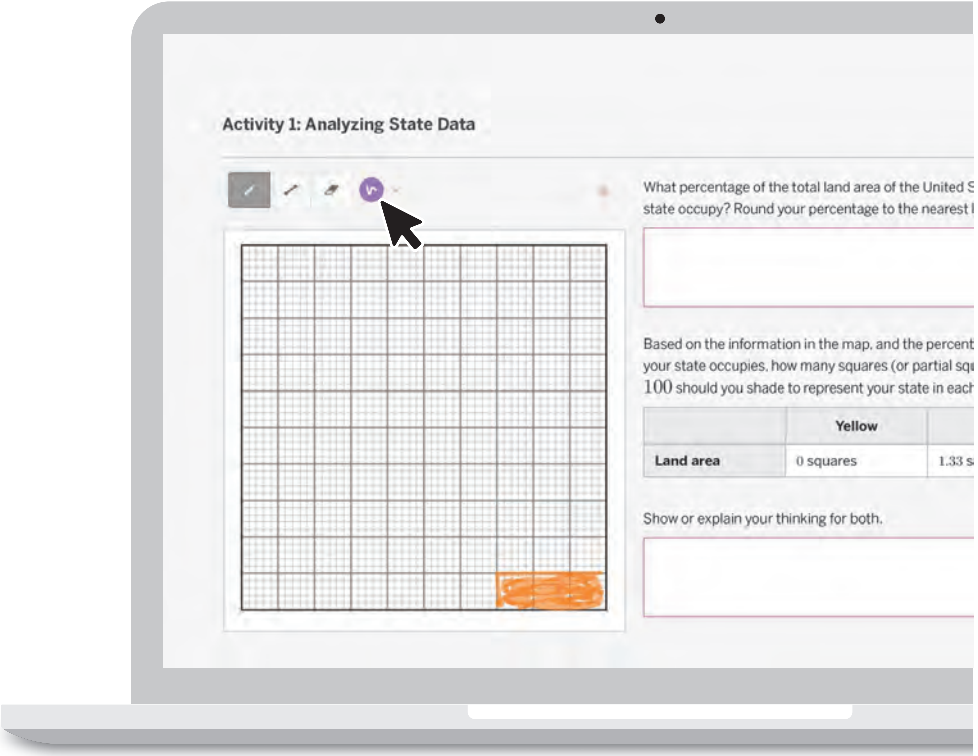
Put on your student hat and work through **Lesson 1, Activity 1:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Increase and Decrease Diagrams **(Lesson 3)**
- Third Place Books **(Lesson 8)**
- What is the Percentage? **(Lesson 10)**
- Reporting Responsibly **(Lesson 13)**



Activity 1: Analyzing State Data

What percentage of the total land area of the United S
state occupy? Round your percentage to the nearest

Based on the information in the map, and the percent
your state occupies, how many squares (or partial sq
100 should you shade to represent your state in each

	Yellow	
Land area	0 squares	1.33 s

Show or explain your thinking for both.

Unit Study
Professional Learning

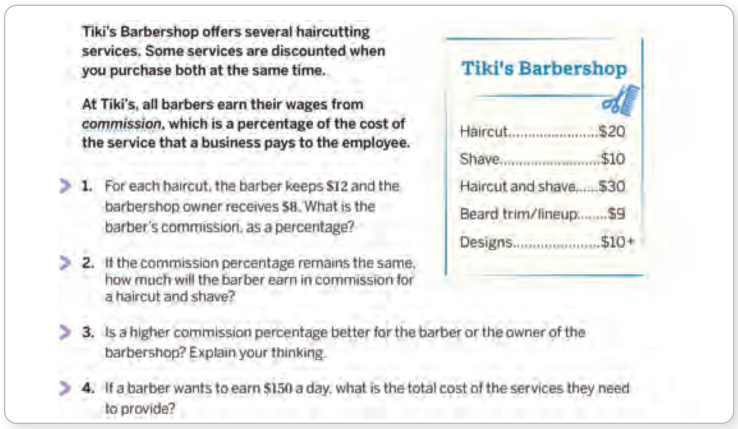
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces students to more applications of percentages in the contexts of current events. Students continue to use tape diagrams to help them find solutions. They examine and calculate markups and markdowns, percent increase and decrease, along with percent change. In addition to using tape diagrams, students write equations representing the situations and solve them. Students learn to examine news headlines more carefully, checking not only for mathematically correct figures but also discerning the importance of the messages being conveyed. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 9, Activity 1:



Tiki's Barbershop offers several haircutting services. Some services are discounted when you purchase both at the same time.

At Tiki's, all barbers earn their wages from *commission*, which is a percentage of the cost of the service that a business pays to the employee.

Tiki's Barbershop

Haircut	$20
Shave	$10
Haircut and shave	$30
Beard trim/lineup	$9
Designs	$10+

1. For each haircut, the barber keeps $12 and the barbershop owner receives $8. What is the barber's commission, as a percentage?
2. If the commission percentage remains the same, how much will the barber earn in commission for a haircut and shave?
3. Is a higher commission percentage better for the barber or the owner of the barbershop? Explain your thinking.
4. If a barber wants to earn $150 a day, what is the total cost of the services they need to provide?

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

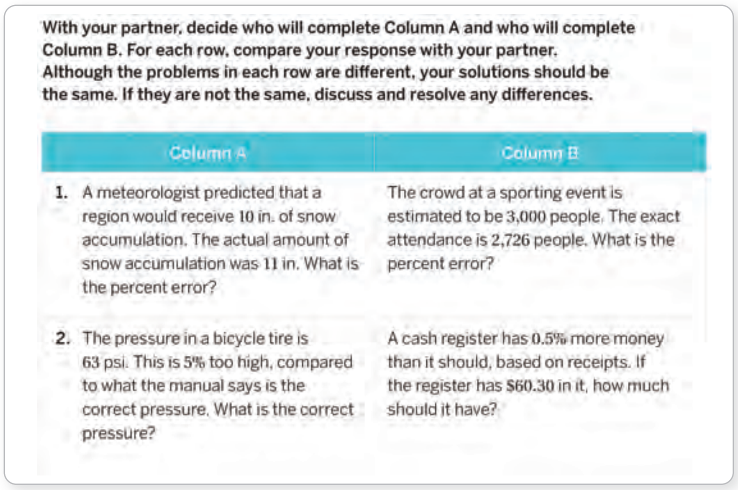
- What was it like to engage in this problem as a learner?
- Before asking the students to work on these questions, you might want to survey students on what their estimates are for a barber's commission. You can discuss the barbershop owners' expenses and the tips that customers give to barbers.
- The unit emphasizes the use of tape diagrams (among the tools stated in the CCSS) to handle percentage problems. Is this how you were taught also or is there a different strategy that you use?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Partner Problems

Rehearse . . .

How you'll facilitate the ***Partner Problems*** instructional routine in Lesson 11, Activity 2:



With your partner, decide who will complete Column A and who will complete Column B. For each row, compare your response with your partner. Although the problems in each row are different, your solutions should be the same. If they are not the same, discuss and resolve any differences.

Column A	Column B
1. A meteorologist predicted that a region would receive 10 in. of snow accumulation. The actual amount of snow accumulation was 11 in. What is the percent error?	The crowd at a sporting event is estimated to be 3,000 people. The exact attendance is 2,726 people. What is the percent error?
2. The pressure in a bicycle tire is 63 psi. This is 5% too high, compared to what the manual says is the correct pressure. What is the correct pressure?	A cash register has 0.5% more money than it should, based on receipts. If the register has $60.30 in it, how much should it have?

Points to Ponder . . .

- ***Partner Problems*** are set up for pairs of students to work on similar problems that always have the same solution.

This routine . . .

- ***Partner problems*** foster a natural forum and agenda for a rich math discussion between students.
- Students' work is re-positioned as its own answer key — a rearrangement of the hierarchy of power that typically lies within the teacher's answer key.
- New or unfamiliar partners are given automatic common ground while seeking a common solution.
- Comparing strategies and mathematical thinking is at the forefront of every ***Partner Problems*** routine.

Anticipate . . .

- If both students have misconceptions in their solution, it may lead to even deeper misconceptions.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Facilitate meaningful mathematical discourse.

This effective teaching practice . . .

- Ensures that there is a shared understanding of the mathematical ideas students have explored and discovered during each lesson's activities.
- Allows students to listen to and critique the strategies and conclusions of others.

Points to Ponder . . .

- How can you establish a classroom environment in which diverse approaches to solving problems are cultivated?
- Some students may not know how to dive deeper into discussions about mathematics. How can you model these discussions?

Math Language Development

MLR2: Collect and Display

MLR2 appears in Lessons 3, 5, 6, 8, 9, 11.

- Throughout the unit, as students formalize the new vocabulary they are learning, have them refer to the class display to continually review and reflect on new terms that are added.
- In Lesson 6, collect the different expressions or equations students use to represent percent increase and percent decrease and add these to the class display.
- **English Learners:** Add diagrams or illustrations to the class display so that students can visualize the terms or phrases. For example, add tape diagrams showing markups and markdowns where percentages are either added to 100% or subtracted from 100%.

Point to Ponder . . .

- How will you encourage or guide students toward using their developing math language to describe percentages as they solve problems involving percent change?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving multi-step percentage problems throughout the unit? Do you think your students will generally:
  - » miss the underlying concept of proportionality?
  - » struggle with identifying and organizing information appropriately?
  - » be ready to solve problems where the part is unknown, but face difficulty when the original amount is unknown?

Differentiated Support

Accessibility: *Optimize Access to Technology*

Support for Organization appears in Lessons 3, 4, and 8.

- Opportunities to provide physical manipulatives, the use of technology (through the Amps slides), or other tools appear in Lessons 1, 3, 8, 10–13.
- In Lesson 3, students can manipulate digital tape diagrams to support their conceptual understanding of percent increase and percent decrease.
- In Lesson 10, students strategically select digital tools to solve percentage problems.
- In the digital experience of Lesson 11, students only see their column of problems, allowing them to focus exclusively on their problems before comparing their work with their partner.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology, physical manipulatives, or other tools to deepen student understanding?

Building Math Identity and Community

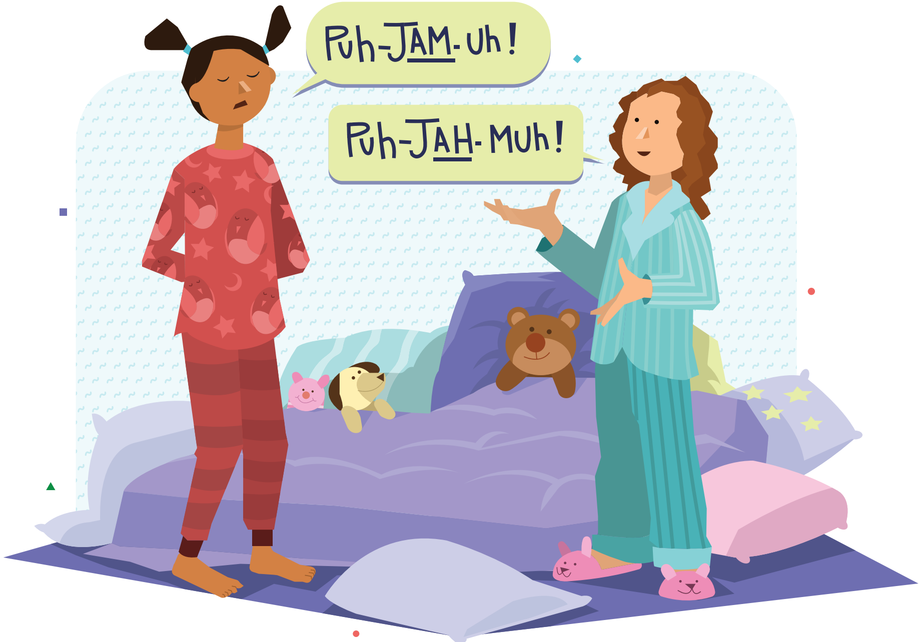
Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their social awareness and self-management.

Points to Ponder . . .

- Are students able to control their impulses and stay focused on the tasks at hand? Can they set behavioral and academic goals that help them be more successful? How do they motivate themselves to achieve those goals?
- Do students consider their responsibility in accurate reporting of information? Do they understand the consequences of poor decisions? On what do they base their decision making in order to demonstrate responsibility in their behaviors and communications?

UNIT 4 | **LESSON 1 — LAUNCH**



(Re)Presenting the United States

Let's use percentages to represent the United States.

Focus

Goals

1. **Language Goal:** Comprehend the term *percentage* and the symbol % to mean *rate per* 100. **(Speaking and Listening)**
2. Recognize that calculating how much per 100 (the *percentage*) is a useful strategy for comparing ratios and data.
3. **Language Goal:** Justify what information can be obtained from a 10-by-10 square grid. **(Speaking and Listening)**

Rigor

- Students **apply** their understanding of percentages to analyze land area and population data of the United States.

Coherence

Today

Students begin their study of percentages by analyzing information presented in a map of the United States. They represent the data in a new way by computing the percentage of total land area and the percentage of total population for individual state(s), and then analyzing the combined data as a class.

Previously

Students studied percentages in Grade 6 and ratios and proportions in Unit 2.

Coming Soon

In Lesson 2, students will expand on their understanding of whole-number percentages to solve problems involving percentages that are not whole numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	20 min	8 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, *Map of the United States* (as needed)
- Activity 1 PDF, pre-cut cards, two cards per student
- Activity 1 PDF, *Map Grid* (for display)
- Activity 1 PDF, *Map Grid* (answers, as needed)
- Activity 1 PDF, *State Data* (answers)
- calculators
- markers (purple and yellow/orange)

Math Language Development

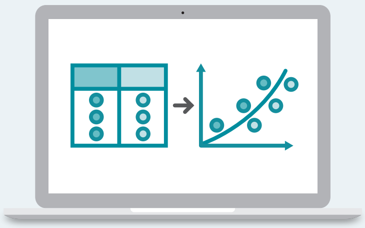
Review word

- *percentage*

Amps ⋮ **Featured Activity**

Activity 1
Multiple Representations

Students can toggle between tables and square grids as they represent populations.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might not immediately see the relationship between the grids and the percentages creating a self-doubt that they will be able to model the percentages precisely on the grids. Remind students that new activities are often intimidating because the solution is not obvious. Have them work with others to make a plan on how to determine the correct percentages first.

● Modifications to Pacing

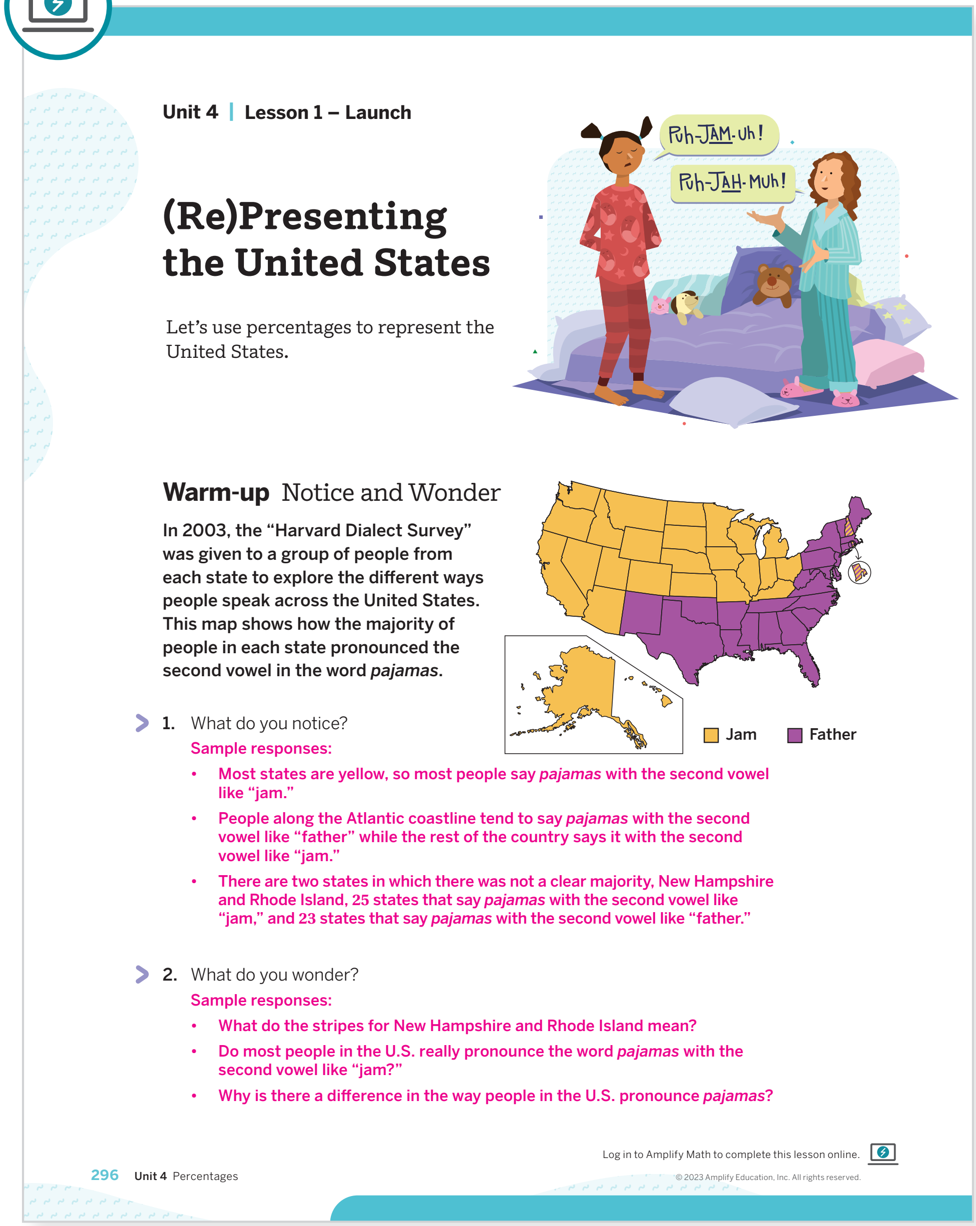
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, give each student one card (state). Pre-fill the Activity 1 PDF, *Map Grid* for the area and the population of the data on the unassigned cards.

Independent | 7 min

Warm-up Notice and Wonder

Students analyze a map of the United States to prepare for a discussion about the data being presented.



Unit 4 | **Lesson 1 – Launch**

(Re)Presenting the United States

Let's use percentages to represent the United States.

Warm-up Notice and Wonder

In 2003, the "Harvard Dialect Survey" was given to a group of people from each state to explore the different ways people speak across the United States. This map shows how the majority of people in each state pronounced the second vowel in the word *pajamas*.

1. What do you notice?

   Sample responses:

   - **Most states are yellow, so most people say *pajamas* with the second vowel like "jam."**
   - **People along the Atlantic coastline tend to say *pajamas* with the second vowel like "father" while the rest of the country says it with the second vowel like "jam."**
   - **There are two states in which there was not a clear majority, New Hampshire and Rhode Island, 25 states that say *pajamas* with the second vowel like "jam," and 23 states that say *pajamas* with the second vowel like "father."**

2. What do you wonder?

   Sample responses:

   - **What do the stripes for New Hampshire and Rhode Island mean?**
   - **Do most people in the U.S. really pronounce the word *pajamas* with the second vowel like "jam?"**
   - **Why is there a difference in the way people in the U.S. pronounce *pajamas*?**

Log in to Amplify Math to complete this lesson online.

296 **Unit 4** Percentages

1 Launch

Activate students' background knowledge by asking whether they have ever noticed different pronunciations for the same word. Conduct the *Notice and Wonder* routine using the map of the U.S.

2 Monitor

Help students get started by asking what they notice about the yellow states and then what they notice about the purple states.

Look for points of confusion:

- **Thinking that because most of the map is yellow, that most Americans pronounce *pajamas* with the second vowel sound like 'jam'.** Ask students if the size of a state is necessarily aligned with its population. Bring their attention to states like Alaska and New Jersey.

Look for productive strategies:

- Noticing *where* each pronunciation is used and making conclusions based on geography, not number of people.

3 Connect

Display the map from the Warm-up.

Have students share something they noticed and something they wondered. Create a list of the wonderings to be displayed and, if possible, answered throughout the class period.

Ask:

- "What conclusions can you make from this map?"
- "What conclusions can you *not* make from this map?"
- "What other information do you wish you had about this map?"

Highlight that the map can give us information about *where* people speak in a certain way but not *how many* people use each pronunciation. In order to make conclusions about which pronunciation is more popular, we need more data about each state.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

To assist students in the discussion of what they notice and wonder about the map of the United States shown in the Warm-up, distribute copies of the Warm-up PDF, *Map of the United States*, so that students can reference states by name. Allow them to use this map for the duration of this lesson.

Accessibility: *Activate Background Knowledge*

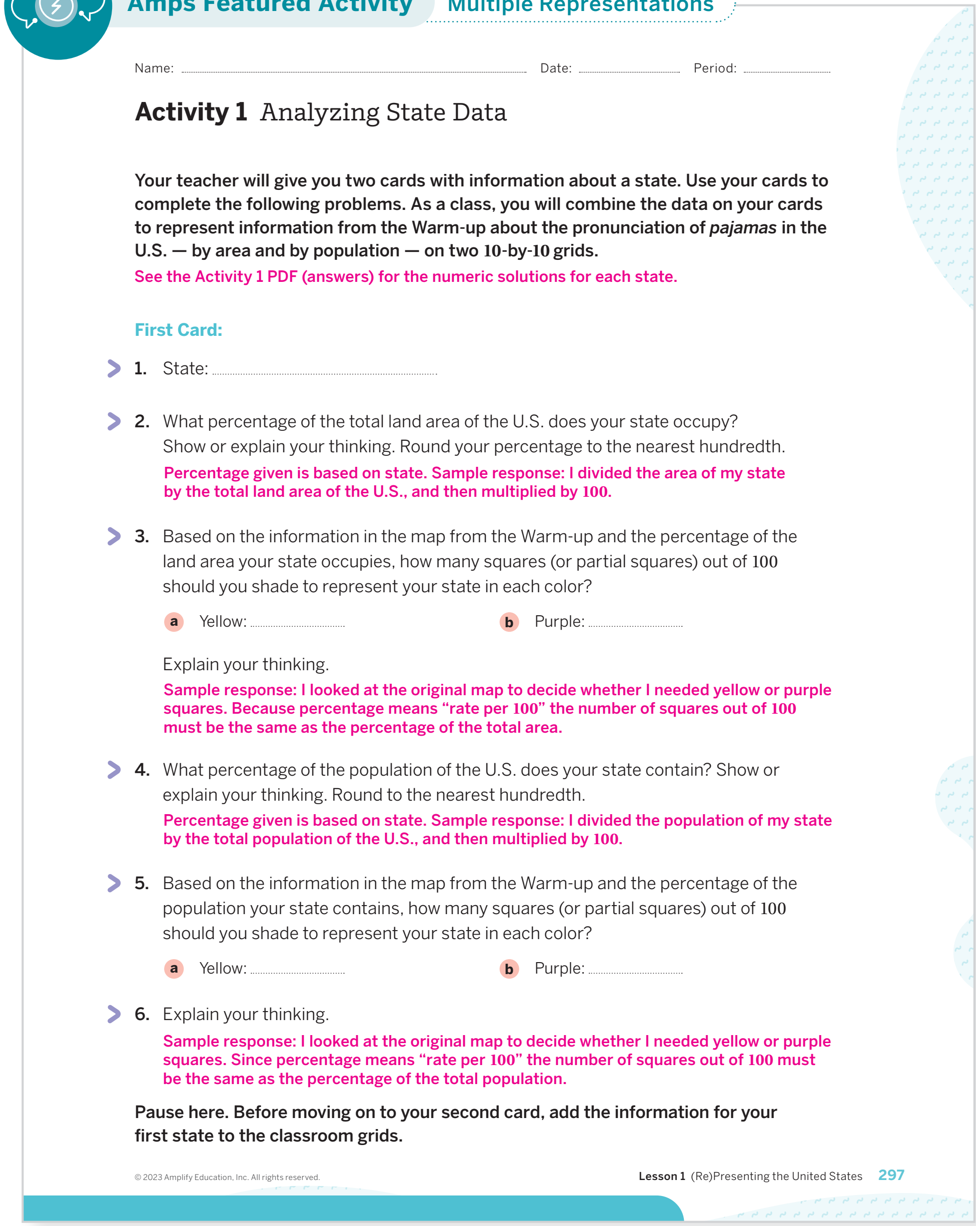
Use the *Poll the Class* routine to determine how students in your class pronounce the word *pajamas*. Consider adding a third alternative, those students who simply say *PJs* instead of saying the entire term.

Independent | 20 min

Activity 1 Analyzing State Data

Students apply their understanding of percentages to further analyze the information in the map of the United States from the Warm-up.

Amps Featured Activity Multiple Representations



Name: Date: Period:

Activity 1 Analyzing State Data

Your teacher will give you two cards with information about a state. Use your cards to complete the following problems. As a class, you will combine the data on your cards to represent information from the Warm-up about the pronunciation of *pajamas* in the U.S. — by area and by population — on two 10-by-10 grids.

See the Activity 1 PDF (answers) for the numeric solutions for each state.

First Card:

1. State:

2. What percentage of the total land area of the U.S. does your state occupy? Show or explain your thinking. Round your percentage to the nearest hundredth.
   Percentage given is based on state. Sample response: I divided the area of my state by the total land area of the U.S., and then multiplied by 100.

3. Based on the information in the map from the Warm-up and the percentage of the land area your state occupies, how many squares (or partial squares) out of 100 should you shade to represent your state in each color?
   a Yellow:
   b Purple:
   Explain your thinking.
   Sample response: I looked at the original map to decide whether I needed yellow or purple squares. Because percentage means "rate per 100" the number of squares out of 100 must be the same as the percentage of the total area.

4. What percentage of the population of the U.S. does your state contain? Show or explain your thinking. Round to the nearest hundredth.
   Percentage given is based on state. Sample response: I divided the population of my state by the total population of the U.S., and then multiplied by 100.

5. Based on the information in the map from the Warm-up and the percentage of the population your state contains, how many squares (or partial squares) out of 100 should you shade to represent your state in each color?
   a Yellow:
   b Purple:

6. Explain your thinking.
   Sample response: I looked at the original map to decide whether I needed yellow or purple squares. Since percentage means "rate per 100" the number of squares out of 100 must be the same as the percentage of the total population.

Pause here. Before moving on to your second card, add the information for your first state to the classroom grids.

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Lesson 1 (Re)Presenting the United States 297

1 Launch

Distribute up to two state cards and a calculator to each student. Explain that they will be analyzing area and population data from their states then adding their data to the Activity 1 PDF, *Map Grids* using purple and yellow markers. Instruct students to add their yellow data starting in the upper left corner and their purple data in the bottom right corner so that colors are grouped and the final grids are more easily analyzed. **Note:** If possible, create a larger version of each grid and place it in a visible location for students to fill in as they are ready.

2 Monitor

Help students get started by asking, "What do you remember about determining percentages?"

Look for points of confusion:

- **Forgetting to convert their decimals to percentages.** Remind students that to determine percent they can use the expression $\frac{\text{part}}{\text{whole}} \cdot 100$ from Grade 6.
- **Rounding all percentages to whole numbers.** Explain that they should be rounding their percentages to the nearest hundredth not the nearest whole number.
- **Rounding the decimal equivalent of each percent to the nearest hundredth prior to converting to a percent.** Remind students that they should convert their decimals to a percent first (with calculator assistance), then round to the nearest hundredth.
- **Not noticing the relationship between their percentages in Problems 2 and 4 and the number of squares in Problems 3 and 5.** Ask, "What does percent mean?"

Look for productive strategies:

- Realizing that each square is broken into 25 smaller squares, so that each small interior square must represent 0.04%.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which class data will aggregate into two 10-by-10 grids, displaying total percentage area and total percentage population for each pronunciation.

Accessibility: *Guide Processing and Visualization*

Display one of the grids from the Activity 1 PDF, *Map Grid.* Annotate the grid by showing that it is a square with 10 columns and 10 rows (a 10-by-10 grid). Each of these 100 squares is divided into 25 smaller squares. Ask students how each smaller square relates to the entire grid. Each smaller square represents 0.04 of the entire grid.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share how they determined the number of squares to shade, collect the language they use to describe percentages, such as "out of 100," "per 100," etc. Add this language to a class display, and invite students to add to and use the display throughout the unit.

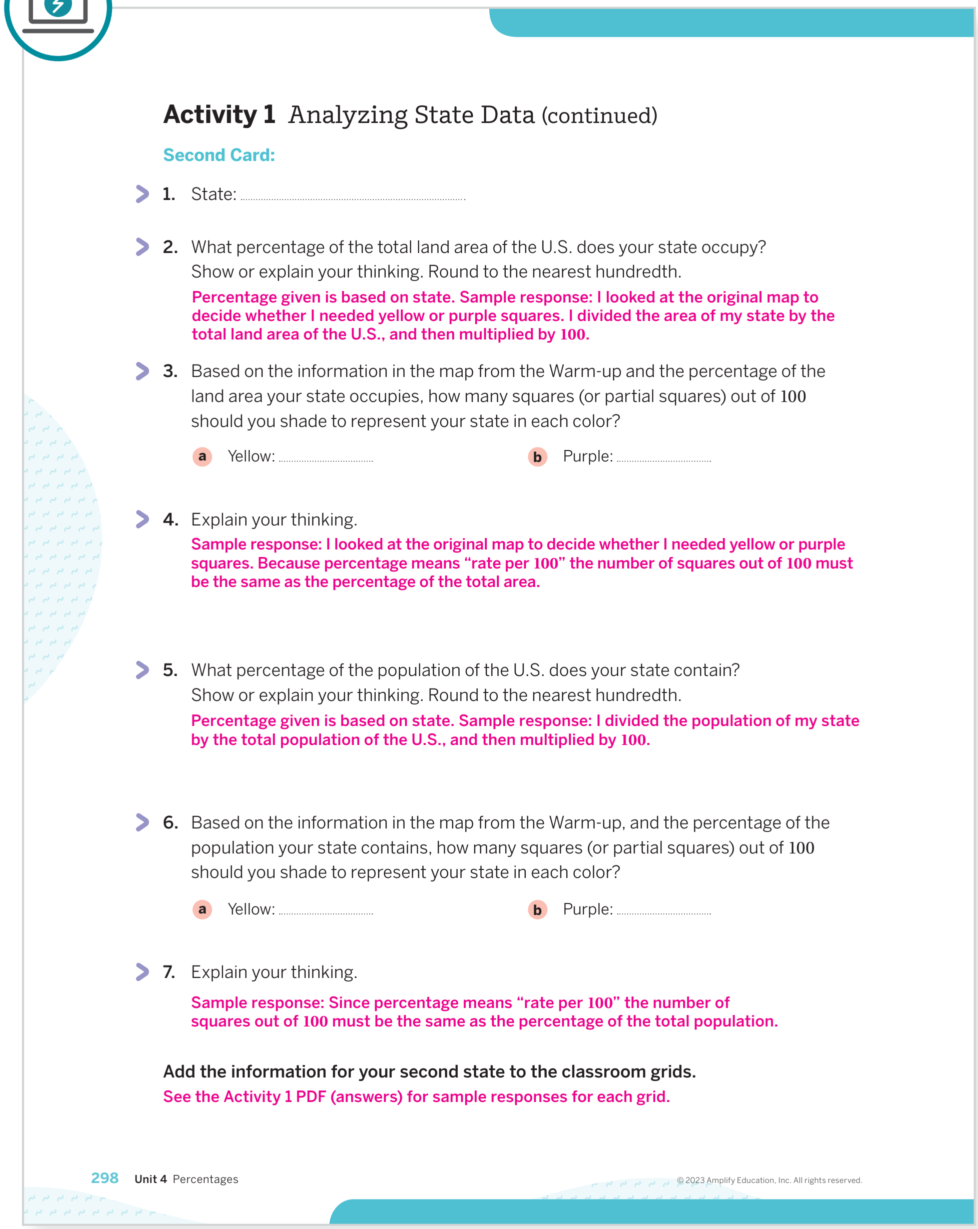
English Learners

Provide students time to think about other language or pronunciation nuances in their primary language. For example, there may be subtle differences in the Spanish language for different countries, e.g., Mexico and Spain.

Independent | 20 min

Activity 1 Analyzing State Data (continued)

Students apply their understanding of percentages to further analyze the information in the map of the United States from the Warm-up.



Activity 1 Analyzing State Data (continued)

Second Card:

1. State: ....

2. What percentage of the total land area of the U.S. does your state occupy? Show or explain your thinking. Round to the nearest hundredth.
   Percentage given is based on state. Sample response: I looked at the original map to decide whether I needed yellow or purple squares. I divided the area of my state by the total land area of the U.S., and then multiplied by 100.

3. Based on the information in the map from the Warm-up and the percentage of the land area your state occupies, how many squares (or partial squares) out of 100 should you shade to represent your state in each color?
   a Yellow: ....
   b Purple: ....

4. Explain your thinking.
   Sample response: I looked at the original map to decide whether I needed yellow or purple squares. Because percentage means "rate per 100" the number of squares out of 100 must be the same as the percentage of the total area.

5. What percentage of the population of the U.S. does your state contain? Show or explain your thinking. Round to the nearest hundredth.
   Percentage given is based on state. Sample response: I divided the population of my state by the total population of the U.S., and then multiplied by 100.

6. Based on the information in the map from the Warm-up, and the percentage of the population your state contains, how many squares (or partial squares) out of 100 should you shade to represent your state in each color?
   a Yellow: ....
   b Purple: ....

7. Explain your thinking.
   Sample response: Since percentage means "rate per 100" the number of squares out of 100 must be the same as the percentage of the total population.

Add the information for your second state to the classroom grids.
See the Activity 1 PDF (answers) for sample responses for each grid.

298 Unit 4 Percentages

3 Connect

Display the completed grids for area and for population.

Have students share how they determine how many squares to shade on each grid.

Highlight that since each grid is 100 square units that means that each large square represents 1% of the total land area or population.

Ask:

- "What surprised you when determining the percentages for land area and population for your states?"
- Were your percentages for land area and population close to equal or not? Why do you think that might be?"
- "What do you find surprising about each grid?"
- "Approximately what percent of the land area of the U.S. is populated by people who use each pronunciation of pajamas?
- "Approximately what percent of the population of the U.S. use each pronunciation of pajamas?

Note: A ***percentage*** is defined as the rate per 100 and when the percentage is known, such as 75%, it is read as "seventy-five *percent*." In the real world, people frequently use the term *percent* in place of *percentage*. Due to the subtle differences in the use of the two terms, it is appropriate to allow your students to use *percent* in place of *percentage* in their discussions and explanations (but not the reverse).

Differentiated Support

Extension: *Interdisciplinary Connections*

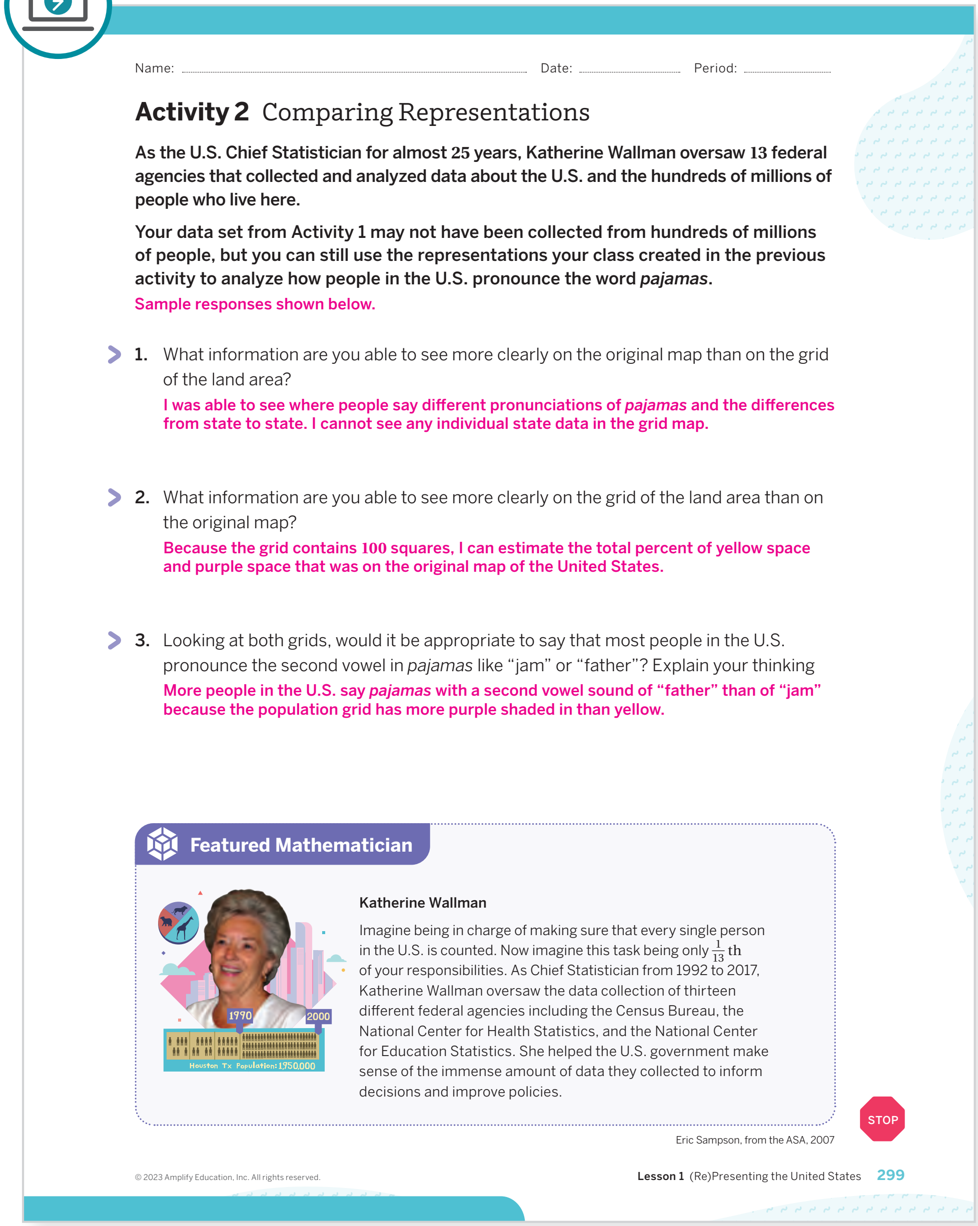
Let students know that the term *percent* comes from the Latin phrase *per centum*, which means "for each hundred." **(History)**

- Ancient Romans used the phrases *per cento*, *per* 100, *p* 100, and *p cento*, when referencing percentages.
- In the 17th century, European mathematicians used the symbol "o" to represent a percentage. This symbol later evolved into the percent symbol we use today, %.

Pairs | 8 min

Activity 2 Comparing Representations

Students compare and contrast the three representations of the data to make conclusions about what can and cannot be determined from each representation.



Name: ______ Date: ______ Period: ______

Activity 2 Comparing Representations

As the U.S. Chief Statistician for almost 25 years, Katherine Wallman oversaw 13 federal agencies that collected and analyzed data about the U.S. and the hundreds of millions of people who live here.

Your data set from Activity 1 may not have been collected from hundreds of millions of people, but you can still use the representations your class created in the previous activity to analyze how people in the U.S. pronounce the word *pajamas*.

Sample responses shown below.

1. What information are you able to see more clearly on the original map than on the grid of the land area?

   I was able to see where people say different pronunciations of *pajamas* and the differences from state to state. I cannot see any individual state data in the grid map.

2. What information are you able to see more clearly on the grid of the land area than on the original map?

   Because the grid contains 100 squares, I can estimate the total percent of yellow space and purple space that was on the original map of the United States.

3. Looking at both grids, would it be appropriate to say that most people in the U.S. pronounce the second vowel in *pajamas* like "jam" or "father"? Explain your thinking

   More people in the U.S. say *pajamas* with a second vowel sound of "father" than of "jam" because the population grid has more purple shaded in than yellow.

Featured Mathematician

Katherine Wallman

Imagine being in charge of making sure that every single person in the U.S. is counted. Now imagine this task being only $\frac{1}{13}$th of your responsibilities. As Chief Statistician from 1992 to 2017, Katherine Wallman oversaw the data collection of thirteen different federal agencies including the Census Bureau, the National Center for Health Statistics, and the National Center for Education Statistics. She helped the U.S. government make sense of the immense amount of data they collected to inform decisions and improve policies.

Eric Sampson, from the ASA, 2007

1 Launch

Display the map from the Warm-up and the two grids created as a class during Activity 1. **Note:** You may want to provide each pair of students with a copy of the Activity 1 PDF, *Map Grid* (answers). This will allow them to more readily compare the representations side by side, rather than having to refer to the display in the front of your classroom.

2 Monitor

Help students get started by asking, "What information do the map, the land area grid, and the population grid show?"

Look for points of confusion:

- **Not considering location when comparing the map of the U.S. with the area grid.** Ask students, "If we live in ______ (ex. Oklahoma) which pronunciation of *pajamas* do we likely use? Which representation gives us that information?"

3 Connect

Display all three representations of the data.

Have pairs of students share their thinking for Problems 1–3.

Highlight that each representation only gives us some of the information about who uses each pronunciation of pajamas.

Ask:

- "Why might you want to analyze the area and population data as a percent and not as the raw data?"
- "When might each of the three displays be most useful?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with copies of the Activity 1 PDF, *Map Grid* (answers) so that they can more readily compare the representations side by side, as opposed to just displaying one set for the class.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students describe their observations about each representation, ask them to challenge or elaborate on an idea that one of their classmates may have shared.

Provide these sentence frames for them to use to help structure their thinking:

- "______ and ______ are alike because . . ."
- "______ and ______ are different because . . ."

English Learners

Display a Venn Diagram to record similarities and differences.

Featured Mathematician

Katherine Wallman

Have students read about featured mathematician Katherine Wallman, the former U.S. Chief Statistician under Presidents George H.W. Bush, Bill Clinton, George W. Bush, and Barack Obama.

Whole Class | 5 min

Summary Keepin' it 100

Review and synthesize how to use different representations and percentages to compare and contrast data.

Unit 4 Percentages

Keepin' it 100

The verdict is in: People love percents!

We use them in everything — on our coupons, in department stores, and on our nutritional labels. Governments use them to describe tax rates, and the very clothing on your back is labeled with the percentages of different fabric types.

And as far as ratios go, percentages are *great* for communicating the way we feel. If a coach tells you to give 110% on the football field, it motivates you more than if they had asked you to give 22 out of 20. Percentages are also great for making comparisons. It's a lot quicker, for example, to choose a jar of peanut butter that contains 5% of your daily recommended sodium over a jar that contains 22% (blech!).

There's something comforting about seeing a value represented as the number of parts per one hundred. But things can get tricky once we start using percentages to describe *how things change*.

Our intuition for what those percentages mean start to get twisted. What does it mean for an item to be 20% off after a $15 rebate? How much more are we getting if a cereal box is now 15% larger?

Given how often we use percentages — in our current events, in our scientific research, in our reports on the economy, and as we describe changes in price — having a strong grasp of how to perform calculations and communicate with percents is key to being an informed decision-maker.

But don't worry. Stick with us, and we'll help you keep it 100.

Welcome to Unit 4.

300 Unit 4 Percentages

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Highlight that throughout this unit, students will use percentages to make comparisons between values.

Ask, "Can you think of examples of percentages that you see or use in your everyday life? "

Reflect

After to synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. To help them engage in meaningful reflection, consider asking:

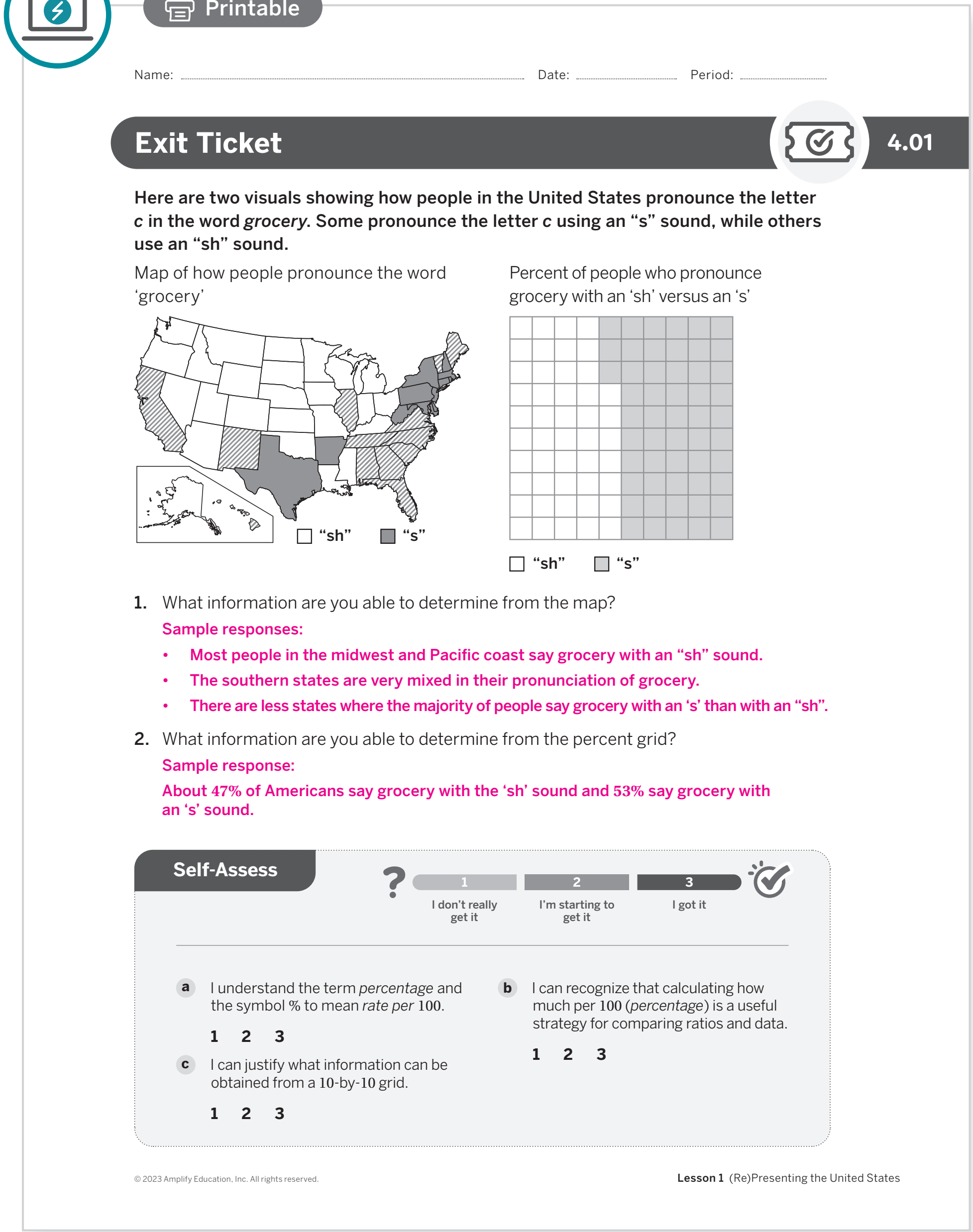
- "How are percentages related to proportional relationships?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of comparing representations by analyzing a map of the U.S. and a grid modeling the percent of the total population.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.01

Here are two visuals showing how people in the United States pronounce the letter *c* in the word *grocery*. Some pronounce the letter *c* using an "s" sound, while others use an "sh" sound.

Map of how people pronounce the word 'grocery'

☐ "sh" ▨ "s"

Percent of people who pronounce grocery with an 'sh' versus an 's'

☐ "sh" ▨ "s"

1. What information are you able to determine from the map?

   Sample responses:
   - Most people in the midwest and Pacific coast say grocery with an "sh" sound.
   - The southern states are very mixed in their pronunciation of grocery.
   - There are less states where the majority of people say grocery with an 's' than with an "sh".

2. What information are you able to determine from the percent grid?

   Sample response:

   About 47% of Americans say grocery with the 'sh' sound and 53% say grocery with an 's' sound.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I understand the term *percentage* and the symbol % to mean *rate per* 100.
1 2 3

b. I can recognize that calculating how much per 100 (*percentage*) is a useful strategy for comparing ratios and data.
1 2 3

c. I can justify what information can be obtained from a 10-by-10 grid.
1 2 3

Lesson 1 (Re)Presenting the United States

Success looks like . . .

- **Language Goal:** Comprehending the term *percentage* and the symbol % to mean *rate per 100*. **(Speaking and Listening)**
- **Goal:** Recognizing that calculating how much per 100 (the percentage) is a useful strategy for comparing ratios and data.
  - » Explaining the data represented by the percent grid in Problem 2.
- **Language Goal:** Justifying what information can be obtained from a 10-by-10 square grid. **(Speaking and Listening)**

Suggested next steps

If students do not mention location in their response to Problem 1, consider:

- Asking, "What information can you determine from a map of the United States that is not on the grid?"

If students respond that most people in the United States use "sh" in their pronunciation of grocery in Problem 1, consider:

- Asking, "Is the size of a state necessarily proportional to its population?"

If students miscalculate the percents, or do not mention percents, in their response to Problem 2, consider:

- Asking, "Why is it important that this grid has a total of 100 squares?"
- Assigning Practice Problem 1.

Professional Learning

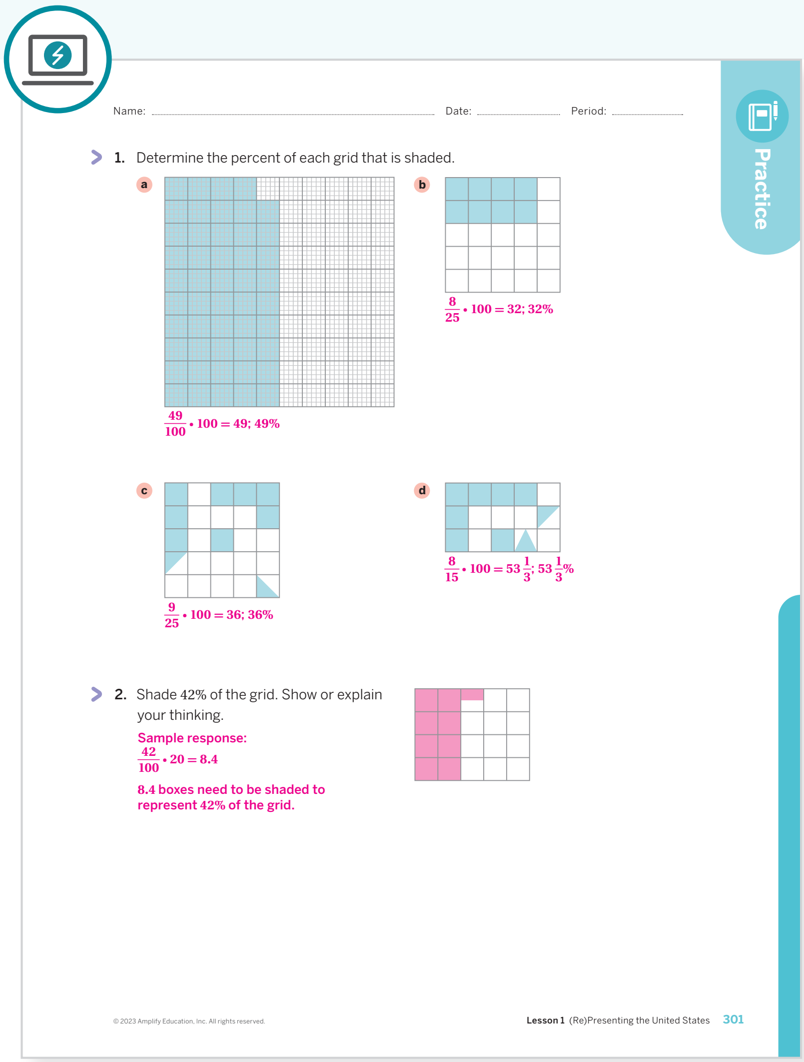
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which students' ideas were you able to highlight during Activity 1?
- What different ways did students approach Activity 2? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Determine the percent of each grid that is shaded.

a $\frac{49}{100} \cdot 100 = 49$; 49%

b $\frac{8}{25} \cdot 100 = 32$; 32%

c $\frac{9}{25} \cdot 100 = 36$; 36%

d $\frac{8}{15} \cdot 100 = 53\frac{1}{3}$; $53\frac{1}{3}$%

2. Shade 42% of the grid. Show or explain your thinking.

Sample response:
$\frac{42}{100} \cdot 20 = 8.4$

8.4 boxes need to be shaded to represent 42% of the grid.

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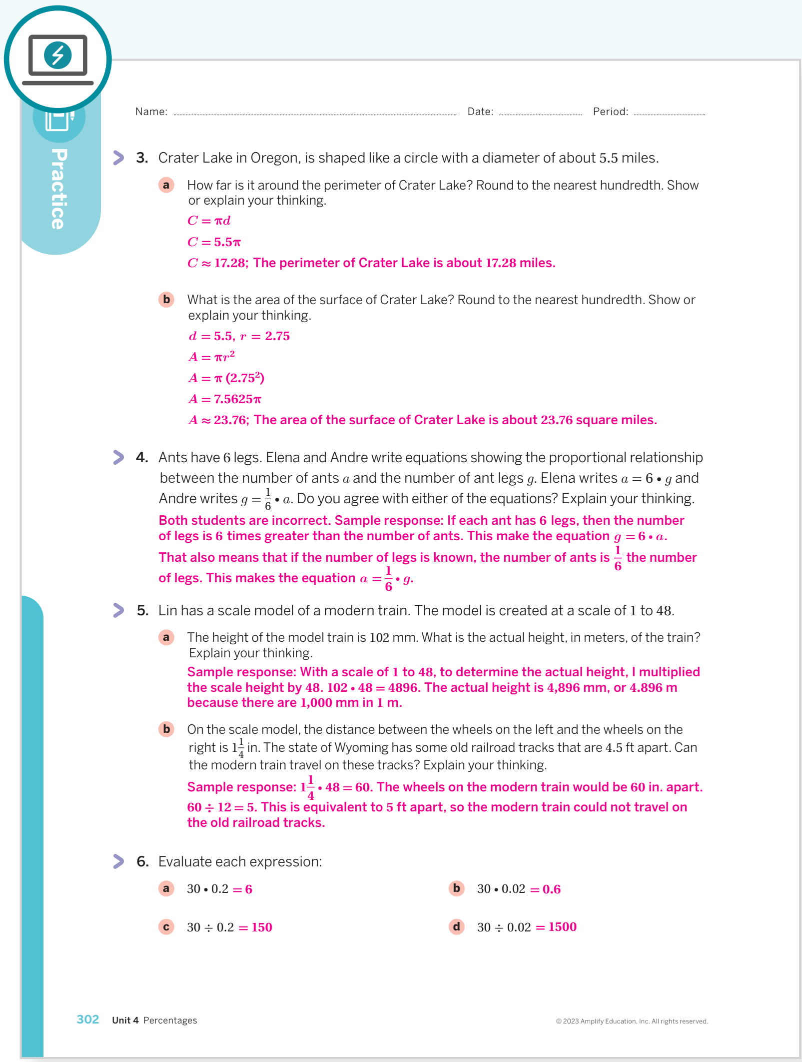

Name: Date: Period:

3. Crater Lake in Oregon, is shaped like a circle with a diameter of about 5.5 miles.

a How far is it around the perimeter of Crater Lake? Round to the nearest hundredth. Show or explain your thinking.

$C = \pi d$

$C = 5.5\pi$

$C \approx 17.28$; The perimeter of Crater Lake is about 17.28 miles.

b What is the area of the surface of Crater Lake? Round to the nearest hundredth. Show or explain your thinking.

$d = 5.5,\ r = 2.75$

$A = \pi r^2$

$A = \pi\,(2.75^2)$

$A = 7.5625\pi$

$A \approx 23.76$; The area of the surface of Crater Lake is about 23.76 square miles.

4. Ants have 6 legs. Elena and Andre write equations showing the proportional relationship between the number of ants a and the number of ant legs g. Elena writes $a = 6 \cdot g$ and Andre writes $g = \frac{1}{6} \cdot a$. Do you agree with either of the equations? Explain your thinking.

Both students are incorrect. Sample response: If each ant has 6 legs, then the number of legs is 6 times greater than the number of ants. This make the equation $g = 6 \cdot a$. That also means that if the number of legs is known, the number of ants is $\frac{1}{6}$ the number of legs. This makes the equation $a = \frac{1}{6} \cdot g$.

5. Lin has a scale model of a modern train. The model is created at a scale of 1 to 48.

a The height of the model train is 102 mm. What is the actual height, in meters, of the train? Explain your thinking.

Sample response: With a scale of 1 to 48, to determine the actual height, I multiplied the scale height by 48. $102 \cdot 48 = 4896$. The actual height is 4,896 mm, or 4.896 m because there are 1,000 mm in 1 m.

b On the scale model, the distance between the wheels on the left and the wheels on the right is $1\frac{1}{4}$ in. The state of Wyoming has some old railroad tracks that are 4.5 ft apart. Can the modern train travel on these tracks? Explain your thinking.

Sample response: $1\frac{1}{4} \cdot 48 = 60$. The wheels on the modern train would be 60 in. apart. $60 \div 12 = 5$. This is equivalent to 5 ft apart, so the modern train could not travel on the old railroad tracks.

6. Evaluate each expression:

a $30 \cdot 0.2 = 6$

b $30 \cdot 0.02 = 0.6$

c $30 \div 0.2 = 150$

d $30 \div 0.02 = 1500$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
Spiral	3	Unit 3 Lesson 9	2
	4	Unit 2 Lesson 7	2
	5	Unit 1 Lesson 11	2
Formative ⚡	6	Unit 4 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

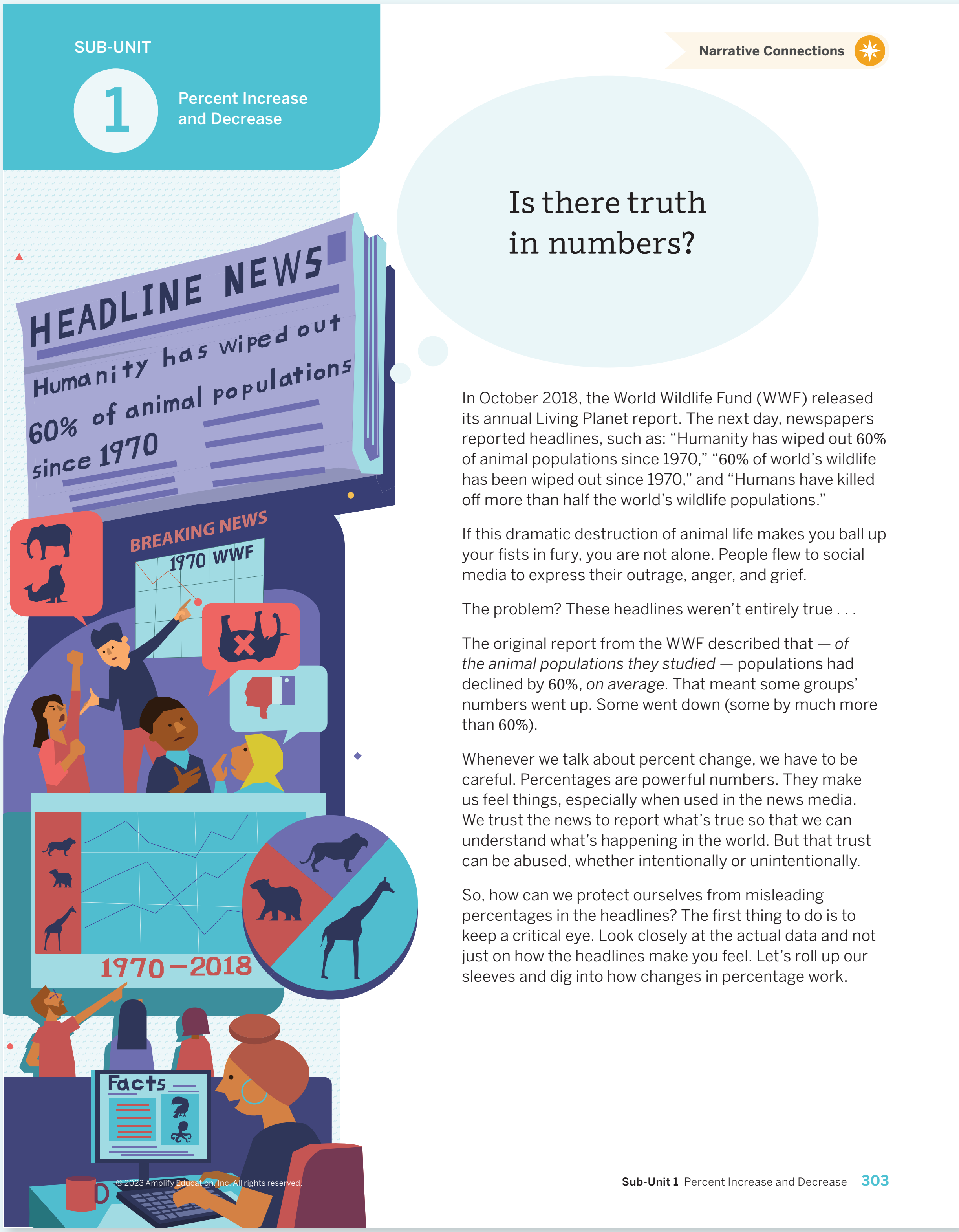
For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 1

Percent Increase and Decrease

Whole Class

In this Sub-Unit, students develop flexibility in solving problems around percent changes using tape diagrams, expressions, and equations to model and solve problems.



SUB-UNIT 1 Percent Increase and Decrease

Narrative Connections

Is there truth in numbers?

In October 2018, the World Wildlife Fund (WWF) released its annual Living Planet report. The next day, newspapers reported headlines, such as: “Humanity has wiped out 60% of animal populations since 1970,” “60% of world’s wildlife has been wiped out since 1970,” and “Humans have killed off more than half the world’s wildlife populations.”

If this dramatic destruction of animal life makes you ball up your fists in fury, you are not alone. People flew to social media to express their outrage, anger, and grief.

The problem? These headlines weren’t entirely true . . .

The original report from the WWF described that — *of the animal populations they studied* — populations had declined by 60%, *on average*. That meant some groups’ numbers went up. Some went down (some by much more than 60%).

Whenever we talk about percent change, we have to be careful. Percentages are powerful numbers. They make us feel things, especially when used in the news media. We trust the news to report what’s true so that we can understand what’s happening in the world. But that trust can be abused, whether intentionally or unintentionally.

So, how can we protect ourselves from misleading percentages in the headlines? The first thing to do is to keep a critical eye. Look closely at the actual data and not just on how the headlines make you feel. Let’s roll up our sleeves and dig into how changes in percentage work.

Narrative Connections

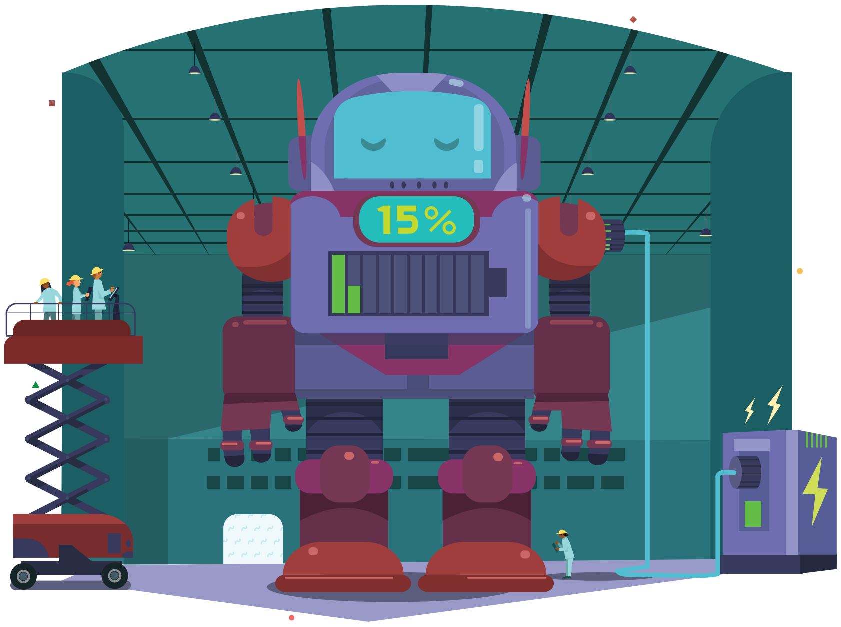
Read the narrative aloud as a class or have students read it individually. Students continue to explore real-world applications of percent change in the following places:

- **Lesson 3, Activities 1–2:** Increase or Decrease, Markup and Markdown
- **Lesson 4, Activity 1:** Population Growth and Decline
- **Lesson 5, Activity 1:** Changing Swimming Time
- **Lesson 6, Activity 1:** More Markup and Markdown
- **Lesson 7, Activity 1:** Analyzing Increase, Revisited

UNIT 4 | **LESSON 2**

Understanding Percentages Involving Decimals

Let's explore percentages that are not whole numbers.



Focus

Goals

1. **Language Goal:** Use reasoning about place value to calculate percentages that are not whole numbers, and explain the strategy. **(Speaking and Listening)**
2. **Language Goal:** Interpret and solve tape diagrams that represent situations that involve percentages that are not whole number values. **(Speaking and Listening)**

Coherence

Today

Students build on their understanding of percentages from Grade 6 to make sense of percentages that are not whole numbers. They connect percentages with their decimal equivalents and apply ratio, multiplicative, and additive reasoning to compute decimal percentages based on benchmark percentages (e.g. 10%, 5%, and 1%).

Previously

In Grade 6, students solved problems involving whole number percentages and used tape diagrams to solve problems involving percentages.

Coming Soon

In Lesson 3, students will use tape diagrams to represent scenarios involving percent increase and decrease.

Rigor

- Students build **conceptual understanding** of percentages that are not whole numbers.
- Students gain **fluency** in computing percentages that are not whole numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	13 min	13 min	5 min	7 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- calculators (as needed)

Math Language Development

Review words

- *percentage*
- *tape diagram*

Amps | Featured Activity

Exit Ticket
Real-Time Exit Ticket

Check in real-time whether your students can use non-whole number percentages to compare values by using a digital Exit Ticket that is automatically scored.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might feel their stress levels rise when they feel they are unable to make sense of the blank tape diagrams. Ask students how their partner in the *Think-Pair-Share* can help them with stress management. Encourage pairs to monitor each other and eliminate the stress by working together to make sense of and solve each problem.

Modifications to Pacing

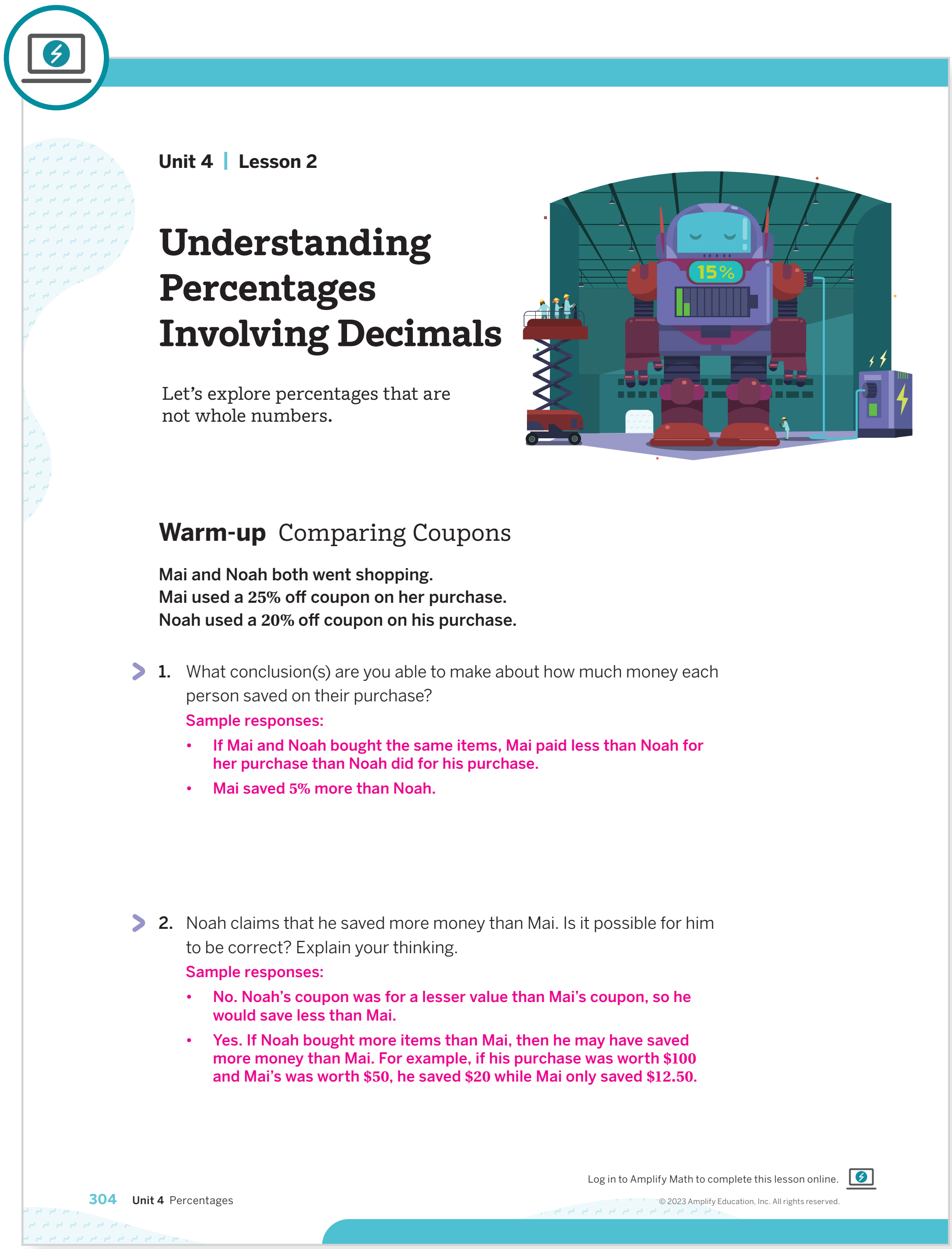
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students choose two percentages to calculate in Problem 3.

Pairs | 7 min

Warm-up Comparing Coupons

Students compare two coupons to determine what conclusions can and cannot be made.



Unit 4 | **Lesson 2**

Understanding Percentages Involving Decimals

Let's explore percentages that are not whole numbers.

Warm-up Comparing Coupons

Mai and Noah both went shopping.
Mai used a 25% off coupon on her purchase.
Noah used a 20% off coupon on his purchase.

1. What conclusion(s) are you able to make about how much money each person saved on their purchase?

   Sample responses:
   - If Mai and Noah bought the same items, Mai paid less than Noah for her purchase than Noah did for his purchase.
   - Mai saved 5% more than Noah.

2. Noah claims that he saved more money than Mai. Is it possible for him to be correct? Explain your thinking.

   Sample responses:
   - No. Noah's coupon was for a lesser value than Mai's coupon, so he would save less than Mai.
   - Yes. If Noah bought more items than Mai, then he may have saved more money than Mai. For example, if his purchase was worth $100 and Mai's was worth $50, he saved $20 while Mai only saved $12.50.

Log in to Amplify Math to complete this lesson online.

304 Unit 4 Percentages

1 Launch

Activate students' background knowledge by asking, "Have you ever used a coupon while shopping?" Then ask students to share examples. Conduct the ***Think-Pair-Share*** routine. Give students two minutes of think time prior to having a discussion with their partner.

2 Monitor

Help students get started by asking, "What information do you know about each purchase? What information is missing?"

Look for points of confusion:

- **Not considering that the price of the purchase for each person may be different.** Ask, "What if Mai spent $10 and Noah spent $1,000?"

Look for productive strategies:

- Brainstorming different purchase amounts that would result in different conclusions for which person saved more money with their coupon.

3 Connect

Have pairs of students share their conclusions from Problem 1. Conduct the ***Poll the Class*** routine for Problem 2 to assess student opinion prior to having students share their thinking.

Ask, "What conclusion could we make if Mai and Noah purchased the same items? If Mai spent $80 and Noah spent $75? If Mai spent $100 and Noah spent $200?"

Highlight that, in order to compare which person saved the most money, students must know the value of which they are determining the percentage.

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share their responses for Problem 2, press them for detail as to why they think Noah is either correct or incorrect in his reasoning. For example, if a student says, "Noah's coupon was for 20% off and 20% is less than 25%. So, of course, he did not save more money," ask:

- "Did Noah and Mai purchase the same item, at the same cost? How might this affect whether you think Noah did or did not save more money?"
- "Can you think of a purchase for which Noah would save more money? Would not save more money? Explain your thinking."

Power-up

To power up students' ability to determine the sum, difference, product, or quotient of decimal values, have students complete:

1. When solving problems involving decimals, in which cases do you need to line up the place values when evaluating? Select *all* that apply.

   [✓ Addition] [✓ Subtraction] Multiplication Division

2. Evaluate each expression:

   a. $3.2 + 0.05 = 3.25$ **b.** $3.2 - 0.05 = 3.15$

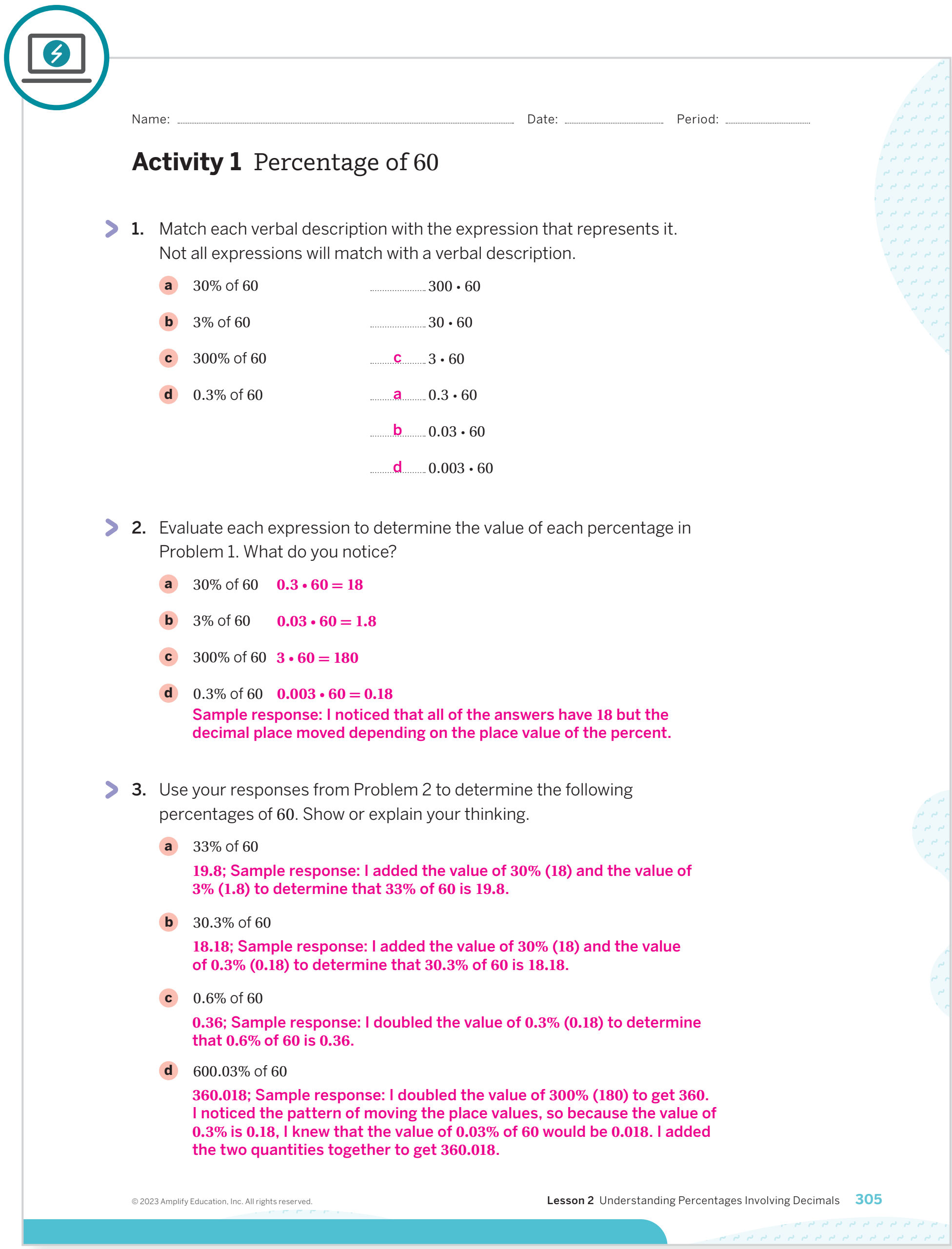
   c. $3.2 \cdot 0.05 = 0.16$ **d.** $3.2 \div 0.05 = 64$

Use: Before Activity 1.
Informed by: Performance on Lesson 1, Practice Problem 5 Pre-Unit Readiness Assessment, Problem 1.

Small groups | 13 min

Activity 1 Percentage of 60

Students apply their understanding of whole number value percentages and repeated reasoning to understand and evaluate percentages involving decimals.



Name: ________ Date: ________ Period: ________

Activity 1 Percentage of 60

1. Match each verbal description with the expression that represents it. Not all expressions will match with a verbal description.

a 30% of 60 .... 300 • 60
b 3% of 60 .... 30 • 60
c 300% of 60 ...c... 3 • 60
d 0.3% of 60 ...a... 0.3 • 60
...b... 0.03 • 60
...d... 0.003 • 60

2. Evaluate each expression to determine the value of each percentage in Problem 1. What do you notice?

a 30% of 60 **0.3 • 60 = 18**
b 3% of 60 **0.03 • 60 = 1.8**
c 300% of 60 **3 • 60 = 180**
d 0.3% of 60 **0.003 • 60 = 0.18**
Sample response: I noticed that all of the answers have 18 but the decimal place moved depending on the place value of the percent.

3. Use your responses from Problem 2 to determine the following percentages of 60. Show or explain your thinking.

a 33% of 60
19.8; Sample response: I added the value of 30% (18) and the value of 3% (1.8) to determine that 33% of 60 is 19.8.

b 30.3% of 60
18.18; Sample response: I added the value of 30% (18) and the value of 0.3% (0.18) to determine that 30.3% of 60 is 18.18.

c 0.6% of 60
0.36; Sample response: I doubled the value of 0.3% (0.18) to determine that 0.6% of 60 is 0.36.

d 600.03% of 60
360.018; Sample response: I doubled the value of 300% (180) to get 360. I noticed the pattern of moving the place values, so because the value of 0.3% is 0.18, I knew that the value of 0.03% of 60 would be 0.018. I added the two quantities together to get 360.018.

1 Launch

Activate students' prior knowledge by asking, "How are percentages represented as their decimal equivalents?" Use the ***Poll the Class*** routine after Problem 1 to assess understanding and address misconceptions prior to students moving on to Problems 2 and 3.

2 Monitor

Help students get started by asking, "What does the % symbol mean?"

Look for points of confusion:

- **Forgetting to convert the percentages to their decimal equivalents.** Ask, "How do you represent 25% as a decimal? What about 30%?"
- **Struggling to reason about decimal percentages.** Ask, "How can you use 3% of 60 to help you?"

Look for productive strategies:

- Using previous problems to solve new ones by applying ratio, additive, and multiplicative reasoning.

3 Connect

Have students share how they converted each percentage in Problem 1 to its decimal equivalent. Then have them share how they used ratio, multiplication, or additive reasoning to complete Problem 3 based on the response from Problems 1 and 2.

Highlight that to determine the percent of a number students can use repeated reasoning with a percent they know (e.g. to determine 3.3% they can determine 3% first and then add it to $\frac{1}{10}$ of 3%), or use the decimal equivalent and multiply.

Ask, "How could you determine 42.05% of 40?"
Sample responses:

- Multiply 0.4205 by 40.
- Add together multiples of 1%, 10%, and 5% of 40.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to think about the descriptions in Problem 1. Display a blank 10-by-10 grid and tell students that the entire grid represents a whole of 60. Ask:

- "How many squares would you need to shade to represent 30% (Problem 1a)? 30 squares
- "What fraction and decimal relate 30 squares out of 100 squares?" $\frac{30}{100}$ and 0.30 (or 0.3)

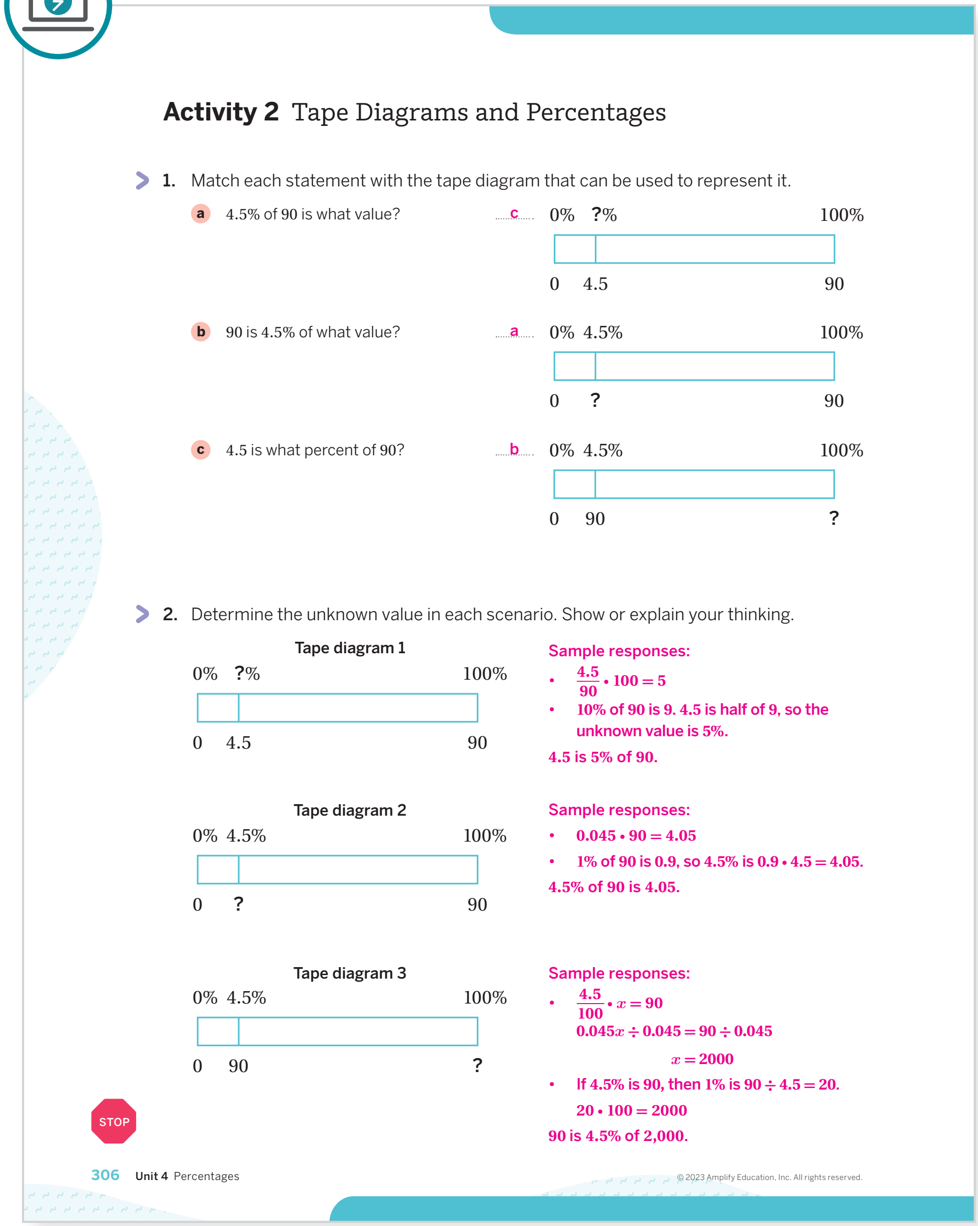
Extension: *Math Enrichment*

Ask students to explain how they could determine 911.1% of 60 without using a calculator. Sample response: Divide 33% of 60 (Problem 3a) and 0.3% of 60 (Problem 1d) each by 3 to determine 11% and 0.1%, respectively. Multiply 300% of 60 (Problem 1c) by 3 and then determine the sum, which is 546.66.

Pairs | 13 min

Activity 2 Tape Diagrams and Percentages

Students use tape diagrams to make sense of percent problems involving decimals.



Activity 2 Tape Diagrams and Percentages

1. Match each statement with the tape diagram that can be used to represent it.

   a 4.5% of 90 is what value? c

   b 90 is 4.5% of what value? a

   c 4.5 is what percent of 90? b

2. Determine the unknown value in each scenario. Show or explain your thinking.

   Tape diagram 1

   Sample responses:
   - $\frac{4.5}{90} \cdot 100 = 5$
   - 10% of 90 is 9. 4.5 is half of 9, so the unknown value is 5%.

   4.5 is 5% of 90.

   Tape diagram 2

   Sample responses:
   - $0.045 \cdot 90 = 4.05$
   - 1% of 90 is 0.9, so 4.5% is $0.9 \cdot 4.5 = 4.05$.

   4.5% of 90 is 4.05.

   Tape diagram 3

   Sample responses:
   - $\frac{4.5}{100} \cdot x = 90$
     $0.045x \div 0.045 = 90 \div 0.045$
     $x = 2000$
   - If 4.5% is 90, then 1% is $90 \div 4.5 = 20$.
     $20 \cdot 100 = 2000$

   90 is 4.5% of 2,000.

STOP

306 Unit 4 Percentages

1 Launch

Activate students' prior knowledge by asking what they remember about using tape diagrams to represent percent problems from Grade 6. Display the tables from Problem 1 and ask students what they notice prior to conducting the *Think-Pair-Share*. **Note:** You may choose to distribute calculators for this activity and the remainder of the lesson to give more time for discussion of the differences between the diagrams.

2 Monitor

Help students get started by asking them to identify which value matches with the 100% in each scenario.

Look for points of confusion:

- **Struggling to match values to the diagrams in Problem 1.** Ask, "What quantity should get a % symbol? What value is the whole (the one something is a *percent of*)?"
- **Misapplying an algorithm to determine percentages in Problem 2.** Have students check the reasonableness of their solutions against each diagram.

Look for productive strategies:

- In Problem 1, labeling the *part*, *whole*, or % to help them make sense of each scenario**.**
- In Problem 2, using ratio, multiplication, or additive reasoning to determine 4.5%.

3 Connect

Display Problem 1 from the Student Edition.

Have pairs of students share their strategies for matching each diagram to a scenario and for determining the unknown value in each diagram.

Highlight that for each diagram, the "whole" as discussed in Grade 6 matches with the 100%.

Ask, "How can the diagrams help you determine the unknown value in each scenario?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider changing the percent used in this activity from 4.5% to 5% to help students assess the reasonableness of their responses more readily.

Extension: *Math Enrichment*

Ask students to draw a tape diagram to represent 4.5% more than 90, and then determine the value. 94.05; Determine 4.5% of 90, and then add this amount to 90.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies for matching the tape diagrams to their corresponding statements, connect the tape diagrams to the language used in the statements. Ask:

- "Where do you see 90 in each diagram? In which diagram(s) is 90 the whole? In which statements is 90 the whole?"
- "Where do you see 4.5 in each diagram? In which diagram(s) is 4.5 the percent? In which statements is 4.5 the percent?"

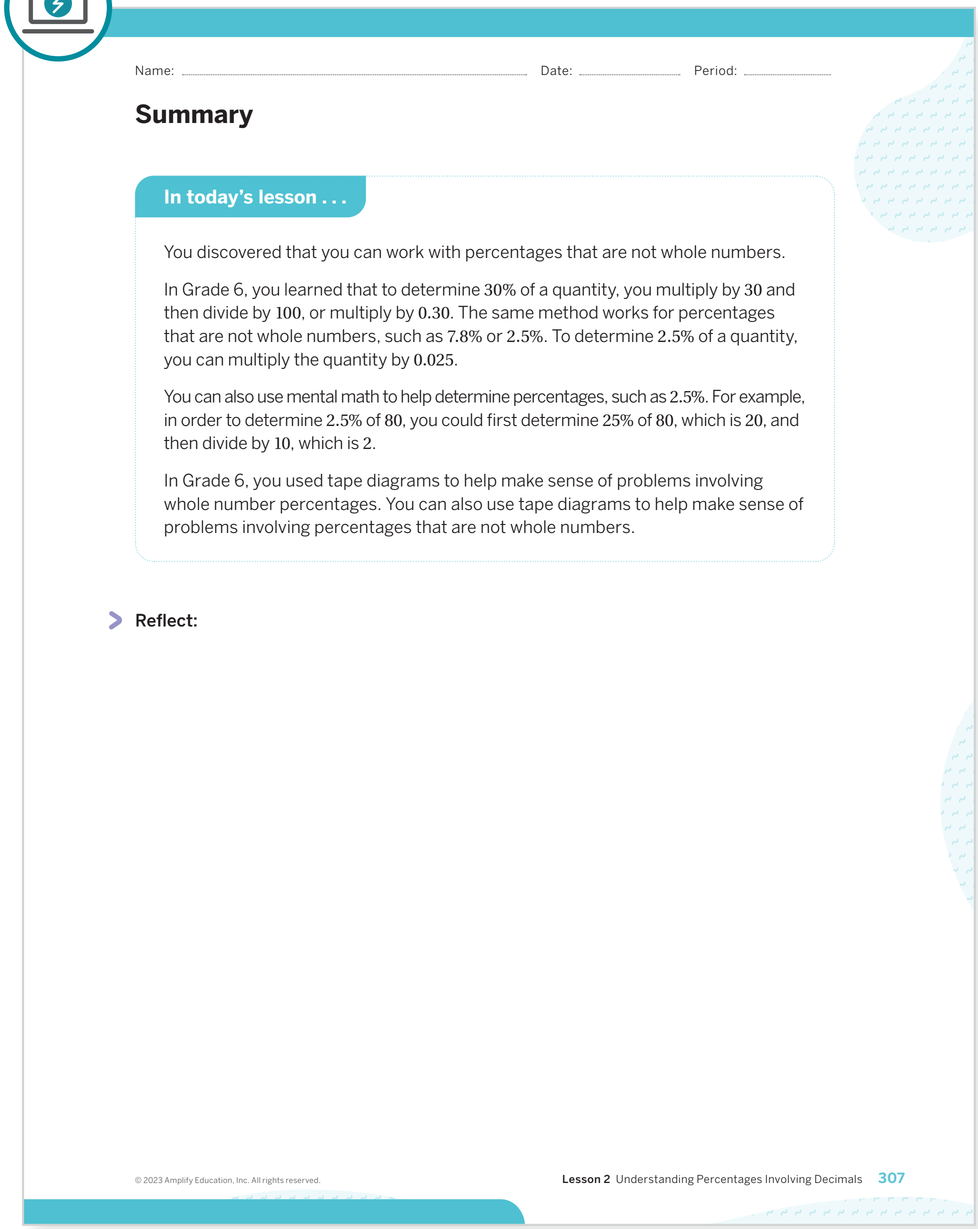
English Learners

Annotate the diagrams and statements with the terms part, percent, and whole.

Whole Class | 5 min

Summary

Review and synthesize how to calculate and compare percentages with decimals.



Name: Date: Period:

Summary

In today's lesson . . .

You discovered that you can work with percentages that are not whole numbers.

In Grade 6, you learned that to determine 30% of a quantity, you multiply by 30 and then divide by 100, or multiply by 0.30. The same method works for percentages that are not whole numbers, such as 7.8% or 2.5%. To determine 2.5% of a quantity, you can multiply the quantity by 0.025.

You can also use mental math to help determine percentages, such as 2.5%. For example, in order to determine 2.5% of 80, you could first determine 25% of 80, which is 20, and then divide by 10, which is 2.

In Grade 6, you used tape diagrams to help make sense of problems involving whole number percentages. You can also use tape diagrams to help make sense of problems involving percentages that are not whole numbers.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Highlight that there are multiple methods that can be used to determine percentages that are not whole numbers, including using percents with which students are more comfortable, and converting percentages to their decimal equivalents.

Ask, "How can you determine 7.5% of 12?"

Sample response:

- I can multiply 12 by 0.075.
- I know that 10% of 12 is 1.2, so I can determine 5% by dividing by 2, and 2.5% by dividing by 2 again. If I add 5% to 2.5%, then I will have 7.5%.

Reflect

After to synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful in working with percentages that were not whole numbers?"

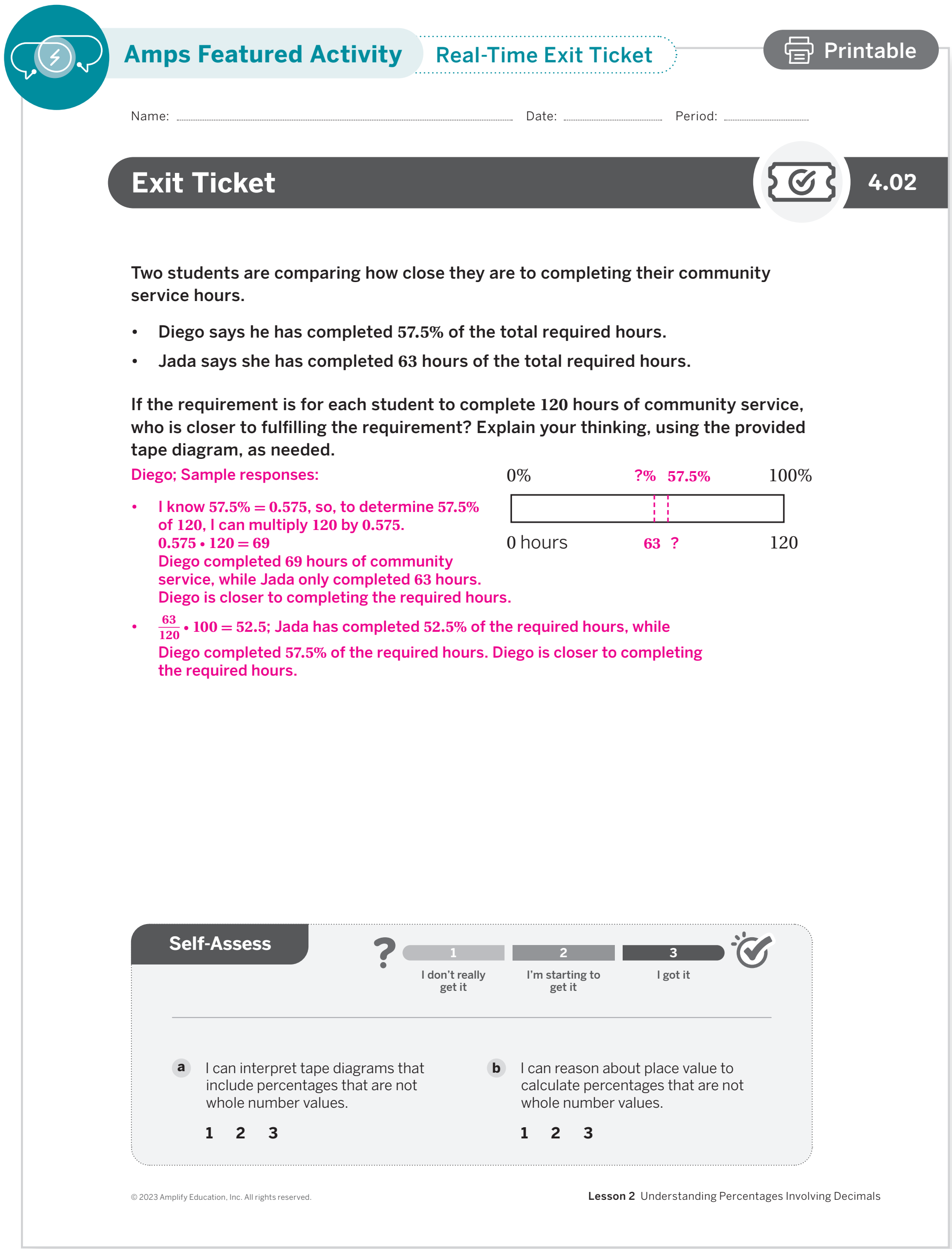
Independent | 7 min

Exit Ticket

Students demonstrate their understanding of decimal percentages by comparing raw data with a percentage to determine which is greater.

Amps Featured Activity Real-Time Exit Ticket

Printable



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Exit Ticket 4.02

Two students are comparing how close they are to completing their community service hours.

- Diego says he has completed 57.5% of the total required hours.
- Jada says she has completed 63 hours of the total required hours.

If the requirement is for each student to complete 120 hours of community service, who is closer to fulfilling the requirement? Explain your thinking, using the provided tape diagram, as needed.

Diego; Sample responses:

- I know $57.5\% = 0.575$, so, to determine 57.5% of 120, I can multiply 120 by 0.575.
  $0.575 \cdot 120 = 69$
  Diego completed 69 hours of community service, while Jada only completed 63 hours. Diego is closer to completing the required hours.
- $\frac{63}{120} \cdot 100 = 52.5$; Jada has completed 52.5% of the required hours, while Diego completed 57.5% of the required hours. Diego is closer to completing the required hours.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can interpret tape diagrams that include percentages that are not whole number values.

1 2 3

b I can reason about place value to calculate percentages that are not whole number values.

1 2 3

Lesson 2 Understanding Percentages Involving Decimals

Success looks like . . .

- **Language Goal:** Using reasoning about place value to calculate percentages that are not whole numbers, and explaining the strategy. **(Speaking and Listening)**
- **Language Goal:** Interpreting and solving tape diagrams that represent situations that involve percentages that are not whole number values. **(Speaking and Listening)**
  - » Explaining whether Diego or Jada is closer to fulfilling the service requirement using the tape diagram.

Suggested next steps

If students compare 57.5 to 63 without taking into consideration that one of the values is a percentage and the other is not, consider:

- Asking, "What does it mean for Diego to complete 57.5% of his hours? How is that different from completing 57.5 hours?"
- Assigning Practice Problem 2.

If students use calculators to aid in computations, consider:

- Challenging them to solve the problem using ratio, multiplication, or additive reasoning.

Professional Learning

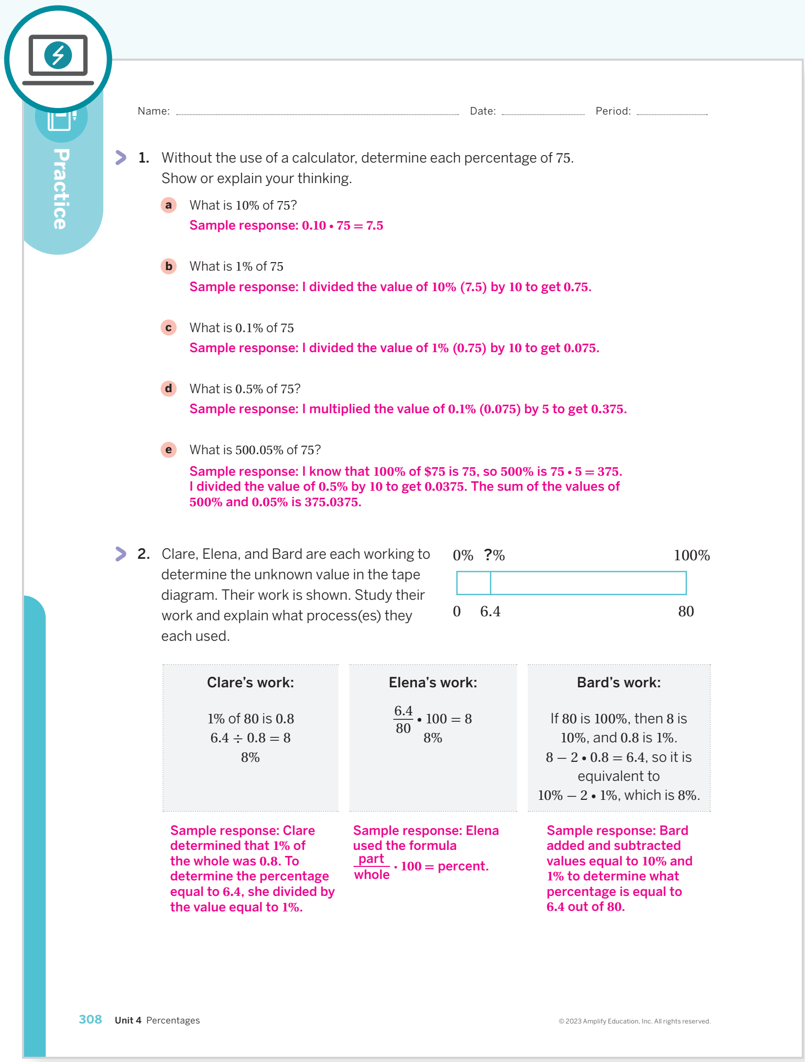
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about computing percentages that involve decimals, how did you encourage each student to share their understandings?
- In this lesson, students used tape diagrams to make sense of scenarios involving non-whole number percents. How will this understanding support them in using tape diagrams to represent scenarios involving percent increase or decrease? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Without the use of a calculator, determine each percentage of 75. Show or explain your thinking.

- **a** What is 10% of 75?
  Sample response: $0.10 \cdot 75 = 7.5$
- **b** What is 1% of 75
  Sample response: I divided the value of 10% (7.5) by 10 to get 0.75.
- **c** What is 0.1% of 75
  Sample response: I divided the value of 1% (0.75) by 10 to get 0.075.
- **d** What is 0.5% of 75?
  Sample response: I multiplied the value of 0.1% (0.075) by 5 to get 0.375.
- **e** What is 500.05% of 75?
  Sample response: I know that 100% of $75 is 75, so 500% is $75 \cdot 5 = 375$. I divided the value of 0.5% by 10 to get 0.0375. The sum of the values of 500% and 0.05% is 375.0375.

2. Clare, Elena, and Bard are each working to determine the unknown value in the tape diagram. Their work is shown. Study their work and explain what process(es) they each used.

0% ?% 100%
0 6.4 80

Clare's work:	Elena's work:	Bard's work:
1% of 80 is 0.8 $6.4 \div 0.8 = 8$ 8%	$\frac{6.4}{80} \cdot 100 = 8$ 8%	If 80 is 100%, then 8 is 10%, and 0.8 is 1%. $8 - 2 \cdot 0.8 = 6.4$, so it is equivalent to $10\% - 2 \cdot 1\%$, which is 8%.
Sample response: Clare determined that 1% of the whole was 0.8. To determine the percentage equal to 6.4, she divided by the value equal to 1%.	**Sample response: Elena used the formula $\frac{\text{part}}{\text{whole}} \cdot 100 = \text{percent}$.**	**Sample response: Bard added and subtracted values equal to 10% and 1% to determine what percentage is equal to 6.4 out of 80.**

308 **Unit 4** Percentages

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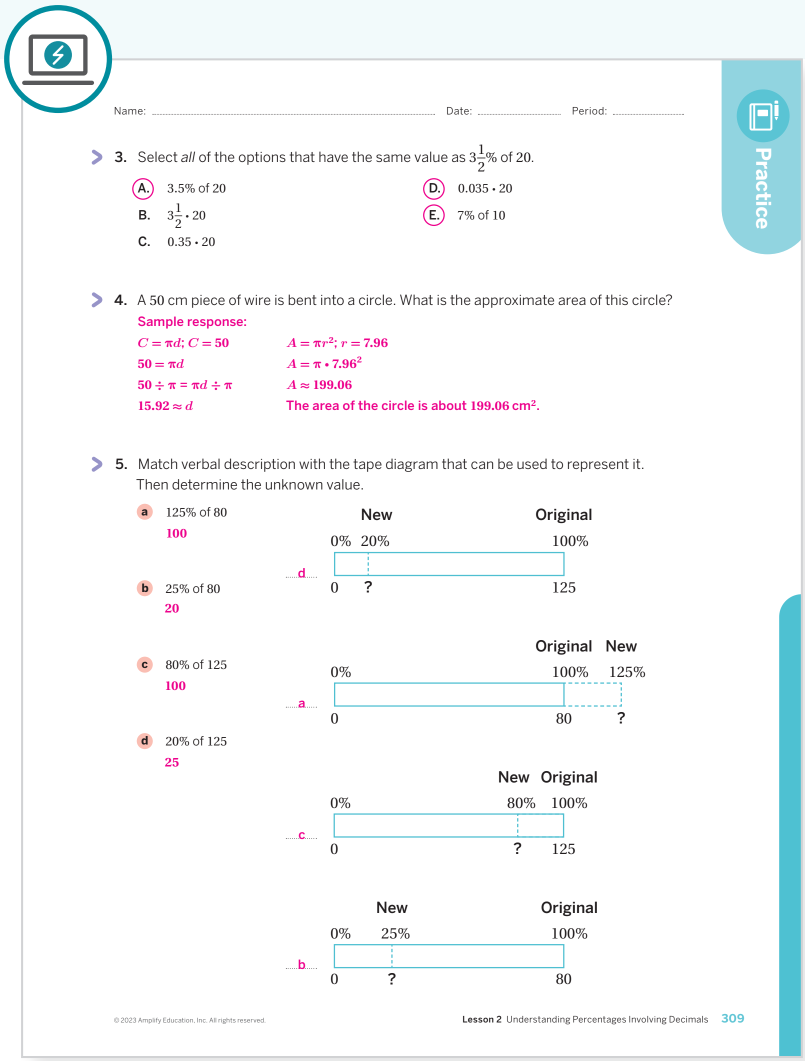

Practice

Name: Date: Period:

3. Select *all* of the options that have the same value as $3\frac{1}{2}$% of 20.

[✓ **A.**] 3.5% of 20
B. $3\frac{1}{2} \cdot 20$
C. $0.35 \cdot 20$
[✓ **D.**] $0.035 \cdot 20$
[✓ **E.**] 7% of 10

4. A 50 cm piece of wire is bent into a circle. What is the approximate area of this circle?

Sample response:

$C = \pi d$; $C = 50$	$A = \pi r^2$; $r = 7.96$
$50 = \pi d$	$A = \pi \cdot 7.96^2$
$50 \div \pi = \pi d \div \pi$	$A \approx 199.06$
$15.92 \approx d$	**The area of the circle is about 199.06 cm².**

5. Match verbal description with the tape diagram that can be used to represent it. Then determine the unknown value.

- **a** 125% of 80
  100
- **b** 25% of 80
  20
- **c** 80% of 125
  100
- **d** 20% of 125
  25

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Lesson 2 Understanding Percentages Involving Decimals 309

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 10	2
Formative ⚡	5	Unit 4 Lesson 3	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Percent Increase and Decrease

Let’s use percentages to describe increases and decreases.

Focus

Goals

1. Draw a tape diagram where 100% corresponds to the original value and represents a situation that involves adding or subtracting a percentage of the original value.
2. **Language Goal:** Explain how to calculate the new amount given the original amount and the percent increase or decrease. **(Speaking and Listening)**
3. **Language Goal:** Understand the terms *markup, markdown, retail price,* and *profit* as contexts that involve adding or subtracting a percentage of the original value. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students expand on their understanding of tape diagrams and percentages from Grade 6 to make sense of and model problems involving percent increase and decrease. Students write expressions to represent the scenarios modeled by their diagrams in order to determine the new value when given the original value and the percent of increase or decrease.

Previously

In Lesson 2, students used tape diagrams to model and solve problems involving percentages that were not whole numbers.

Coming Soon

In Lessons 4 and 5, students will use tape diagrams to help them determine the original amount and the percent change.

Rigor

- Students build **conceptual understanding** of percent increase and decrease.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	10 min	20 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Percent Decrease* (for display)
- Anchor Chart PDF, *Percent Decrease* (answers)
- Anchor Chart PDF, *Percent Increase* (for display)
- Anchor Chart PDF, *Percent Increase* (answers)
- calculators
- sticky notes

Math Language Development

New words

- ***markdown***
- ***markup***
- ***percent decrease***
- ***percent increase***
- ***profit***
- ***retail price***

Review word

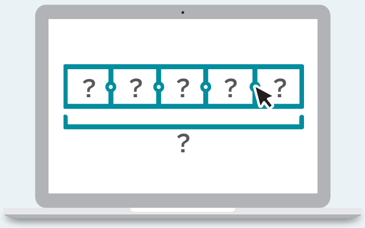
- *tape diagram*

Amps | Featured Activity

Activity 1

Digital Tape Diagrams

Students manipulate digital tape diagrams to support their conceptual understanding of percent increase and percent decrease.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel unable to reason abstractly about how to apply the new terms and new processes in Activity 2. To motivate them, explain that these are concepts that they will apply as they become consumers. Brainstorm reasons why these skills are important with the students and then have them each set a personal goal to work toward during the activity.

Modifications to Pacing

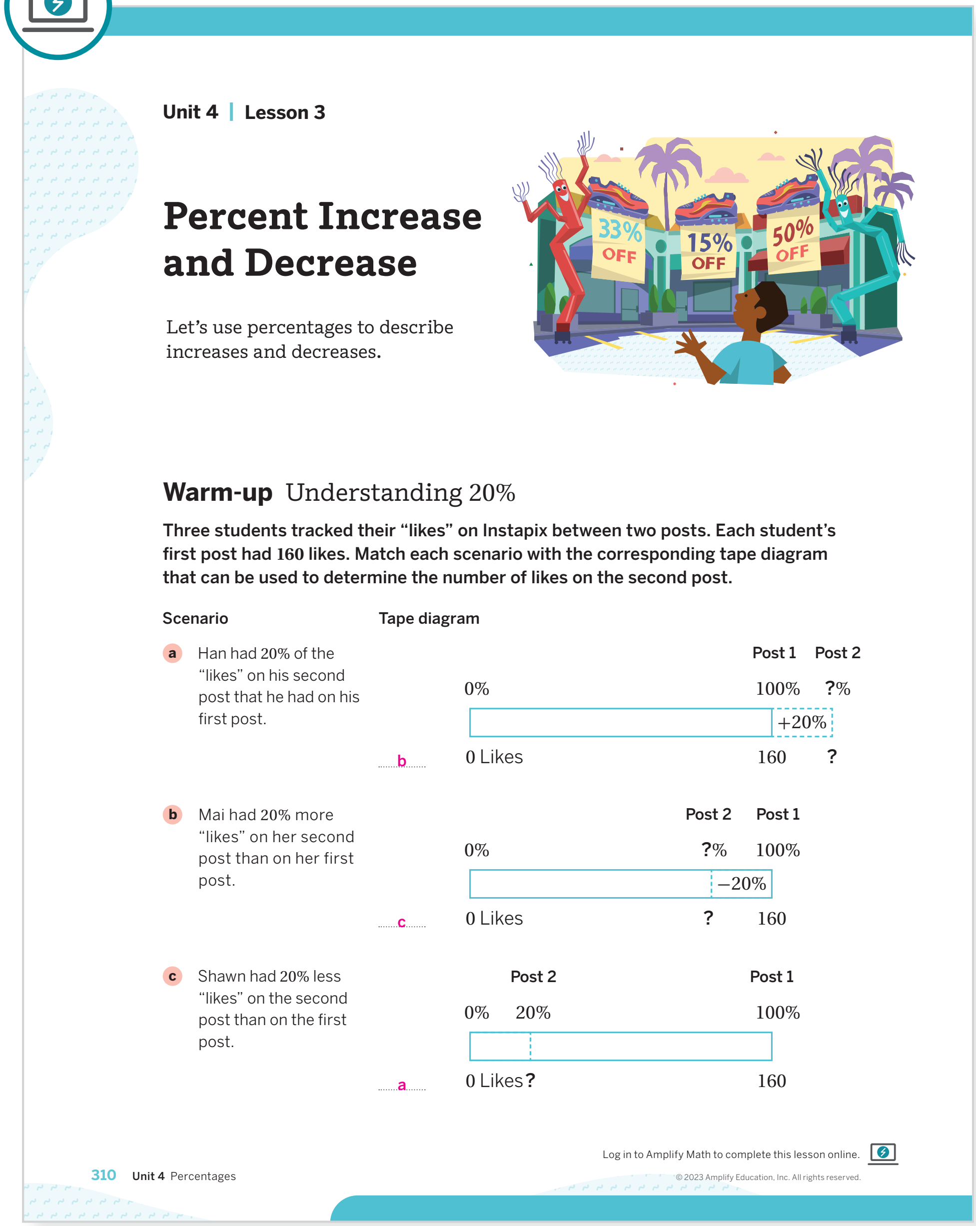
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, have each student in the group complete Problem 1, part a, b, or c, and then check each other's work.
- In **Activity 2**, for Problem 2 have each student compute the cost of the shoes at one store and then compare answers to determine the best price.

Pairs | 7 min

Warm-up Understanding 20%

Students analyze tape diagrams to build conceptual understanding of the differences between percent of, percent increase, and percent decrease.



Unit 4 | Lesson 3

Percent Increase and Decrease

Let's use percentages to describe increases and decreases.

Warm-up Understanding 20%

Three students tracked their "likes" on Instapix between two posts. Each student's first post had 160 likes. Match each scenario with the corresponding tape diagram that can be used to determine the number of likes on the second post.

Scenario		Tape diagram
a Han had 20% of the "likes" on his second post that he had on his first post.	b	Post 1, Post 2; 0%, 100%, ?%; +20%; 0 Likes, 160, ?
b Mai had 20% more "likes" on her second post than on her first post.	c	Post 2, Post 1; 0%, ?%, 100%; −20%; 0 Likes, ?, 160
c Shawn had 20% less "likes" on the second post than on the first post.	a	Post 2, Post 1; 0%, 20%, 100%; 0 Likes, ?, 160

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the *Think-Pair-Share* routine, giving students two minutes of think time before discussing with their partner.

2 Monitor

Help students get started by asking, "Without looking at the diagrams, what is the same and what is different about each student's number of 'likes'?"

Look for points of confusion:

- **Thinking that 20% and 20% less are the same.** Ask, "If you had $25 in your wallet, what would it mean for me to have $20 less than you? What operation would you use to determine how much money I have?"

3 Connect

Display the tape diagrams from the Warm-up in the Students' Edition. Conduct the *Poll the Class* routine to elicit the correct matches.

Ask:

- "How did you determine which scenario matched each diagram?"
- "Why do the tape diagrams for Shawn and Mai have two unknown values, but Han only has one unknown?"

Have pairs of students share how they would determine the unknown values in each tape diagram.

Highlight that in order to determine the unknown value of likes, students must first determine the unknown percent.

Define the following:

- *Percent increase* as the amount a value has gone up, expressed as a percentage of the original amount. Ask, "Which scenario models a percent increase?"
- *Percent decrease* as the amount a value has gone down, expressed as a percentage of the original amount. Ask, "Which scenario models a percent decrease?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies for matching the tape diagrams to their corresponding scenarios, connect the tape diagrams to the language used in the scenarios. Ask:

- "Where do you see 20% in each diagram?"
- "In which diagram(s) is 20% added to the whole? In which statements is 20% added to the whole?"
- "In which diagram(s) is 20% subtracted from the whole? In which statements is 20% subtracted from the whole?"

English Learners

Annotate the statements *20% more* and *20% less* and show how these are represented on the tape diagrams.

Power-up

To power up students' ability to determine the percent, part, and whole using a given tape diagram:

Provide students with a copy of the Power-up PDF.

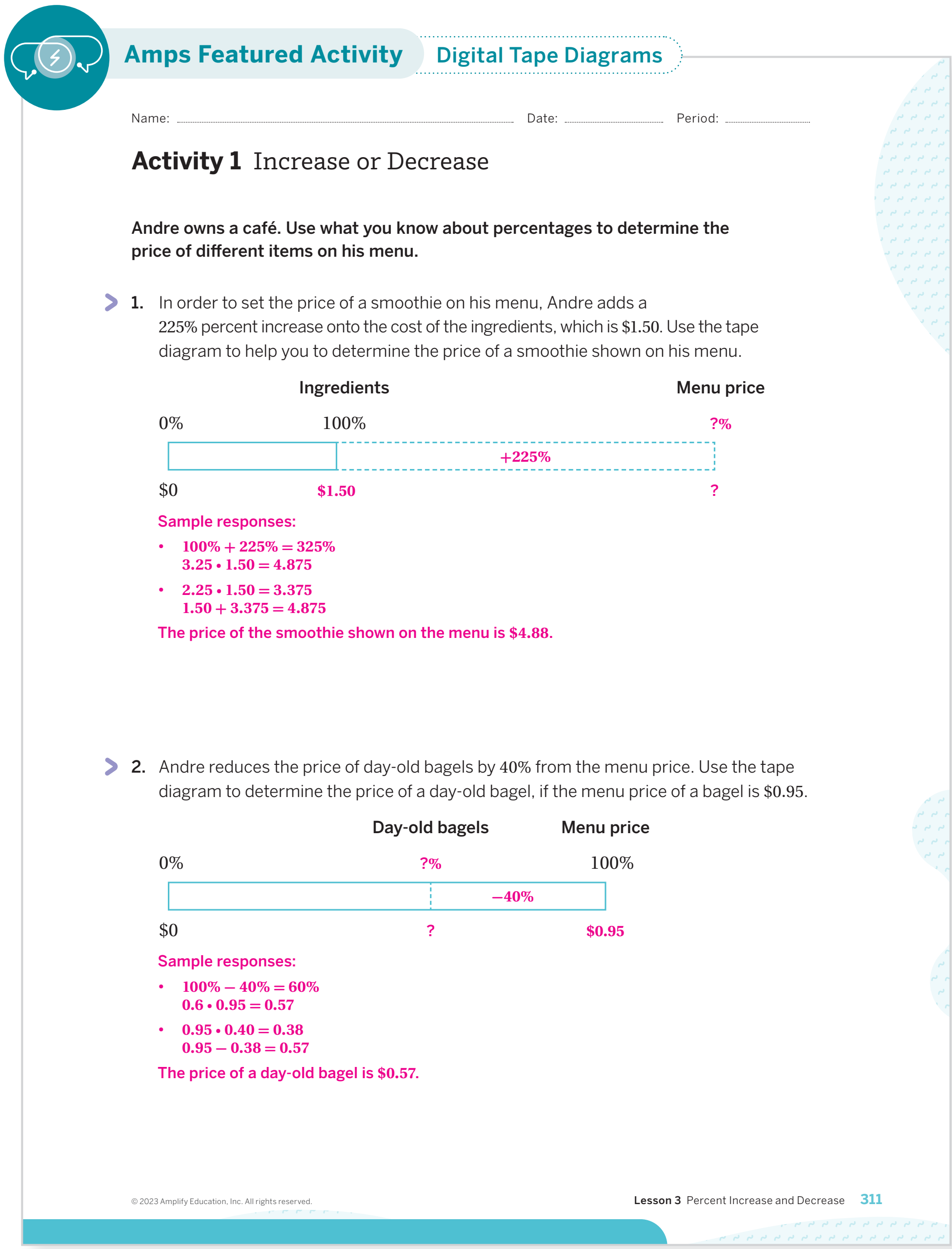
Use: Before the Warm-up.

Informed by: Performance on Lesson 2, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 5.

Activity 1 Increase or Decrease

Students use tape diagrams to make sense of problems involving percent increase and percent decrease.

Amps Featured Activity Digital Tape Diagrams



Name: Date: Period:

Activity 1 Increase or Decrease

Andre owns a café. Use what you know about percentages to determine the price of different items on his menu.

1. In order to set the price of a smoothie on his menu, Andre adds a 225% percent increase onto the cost of the ingredients, which is $1.50. Use the tape diagram to help you to determine the price of a smoothie shown on his menu.

Sample responses:

- $100\% + 225\% = 325\%$
$3.25 \cdot 1.50 = 4.875$
- $2.25 \cdot 1.50 = 3.375$
$1.50 + 3.375 = 4.875$

The price of the smoothie shown on the menu is $4.88.

2. Andre reduces the price of day-old bagels by 40% from the menu price. Use the tape diagram to determine the price of a day-old bagel, if the menu price of a bagel is $0.95.

Sample responses:

- $100\% - 40\% = 60\%$
$0.6 \cdot 0.95 = 0.57$
- $0.95 \cdot 0.40 = 0.38$
$0.95 - 0.38 = 0.57$

The price of a day-old bagel is $0.57.

1 Launch

Explain to students that they should use the tape diagrams from the Warm-up to aid them in completing each tape diagram. Distribute calculators to help with computations.

2 Monitor

Help students get started by asking, "What missing information in your diagram can you fill in first?"

Look for points of confusion:

- **Placing the percent of increase or decrease on the top of the diagram instead of within the diagram.** Ask, "Is that the percent that people will pay, or is that a percent that the original cost is being changed by?"

Look for productive strategies:

- Determining the percent first, and then multiplying by the original value to determine the new price.
- Determining the value of the percent increase or decrease, and then determining its sum or difference of the original price.

3 Connect

Display the tape diagrams from the Student Edition and have students share their methods for determining the unknown values.

Highlight that there are multiple methods for determining each unknown value. One method is to determine the new *percent of* by adding or subtracting the percent increase or decrease. Then students can use ratio reasoning and multiply by the constant of proportionality (the original value) to find the new value.

Define the following:

- ***Markup*** an amount, expressed as a percentage, added to the cost of an item.
- ***Markdown*** an amount, expressed as a percentage, subtracted from the cost of an item.

Ask, "Does a markup of 150% mean you pay 150% of the original price?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate digital tape diagrams to support their conceptual understanding of percent increase and percent decrease.

Extension: *Math Enrichment*

Have students refer to Problem 1. Have them determine the value that completes this statement and explain why the value is *not 225*.

"The price of the smoothie shown on the menu is _____ times the price of the ingredients."

3.25; The price of the smoothie increased by 225%, which means that it was added to 100% to obtain 325%.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share how they determined the prices of different items on the menu, collect and display language used to describe a *markup* and *markdown*, such as *percent increase*, *percent decrease*, *reduce*, etc. Add this language to the class display.

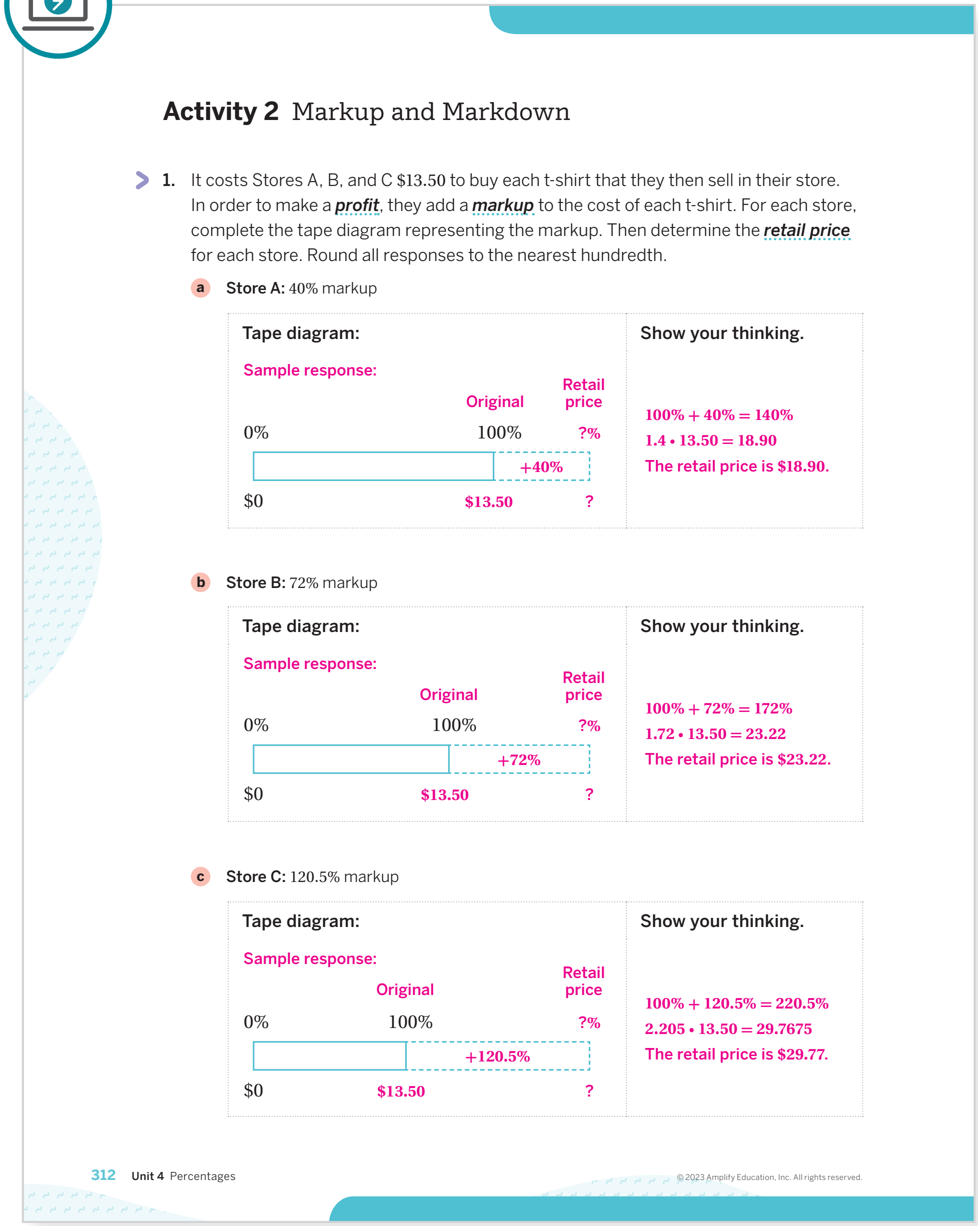
English Learners

Add examples of tape diagrams showing markups and markdowns where percentages are either added to 100% or subtracted from 100%.

Small groups | 20 min

Activity 2 Markup and Markdown

Students use tape diagrams and expressions to model percent increase and percent decrease problems involving markups and markdowns.



Activity 2 Markup and Markdown

1. It costs Stores A, B, and C $13.50 to buy each t-shirt that they then sell in their store. In order to make a **profit**, they add a **markup** to the cost of each t-shirt. For each store, complete the tape diagram representing the markup. Then determine the **retail price** for each store. Round all responses to the nearest hundredth.

   a. **Store A:** 40% markup

Tape diagram:	Show your thinking.
Sample response:	100% + 40% = 140% 1.4 • 13.50 = 18.90 The retail price is $18.90.

   b. **Store B:** 72% markup

Tape diagram:	Show your thinking.
Sample response:	100% + 72% = 172% 1.72 • 13.50 = 23.22 The retail price is $23.22.

   c. **Store C:** 120.5% markup

Tape diagram:	Show your thinking.
Sample response:	100% + 120.5% = 220.5% 2.205 • 13.50 = 29.7675 The retail price is $29.77.

1 Launch

Activate students' background knowledge by asking if they have ever seen the same item being sold for different prices at different stores. Define ***retail price*** and ***profit***. Explain to students that they will be working in small groups to determine the retail price and sale price of items at three different stores.

2 Monitor

Help students get started by asking, "Where is the original value represented in the diagram?"

Look for points of confusion:

- **Difficulty in labeling the diagrams.** Advise students to refer back to the diagrams in the Warm-up and in Activity 1. Ask, "How do you determine which value matches 100%.
- **Only calculating the value of the markup or the markdown and not the final value.** Ask, "What did you find? Have you answered the question being asked?"

Look for productive strategies:

- Calculating the percent decrease directly in the diagram.
- Determining the percent increase or percent decrease first, and then multiplying by the original value.
- Determining the monetary equivalent of the increase or decrease, and then adding or subtracting it from the original value.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider using "friendlier" numbers in this activity, such as changing the following for Problem 1:

- Change $13.50 to $10 or $15 for the cost of the t-shirt for the stores.
- Change 72% to 75% for Store B, and change 120.5% to 120% for Store C.

Extension: *Math Enrichment*

Have students complete the following problem:

What would the original cost of a pair of shoes be, if the sale price was $16.00 after a 20% markdown? $20.00

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies for determining the final cost of each item, draw their attention to the connections between the tape diagrams for each store in Problem 2b. Ask:

- "Where do you see the retail price in each diagram? What is the corresponding percent? Why does this make sense?"
- "Where do you see the sale price in each diagram? Why is it less than 100%?"

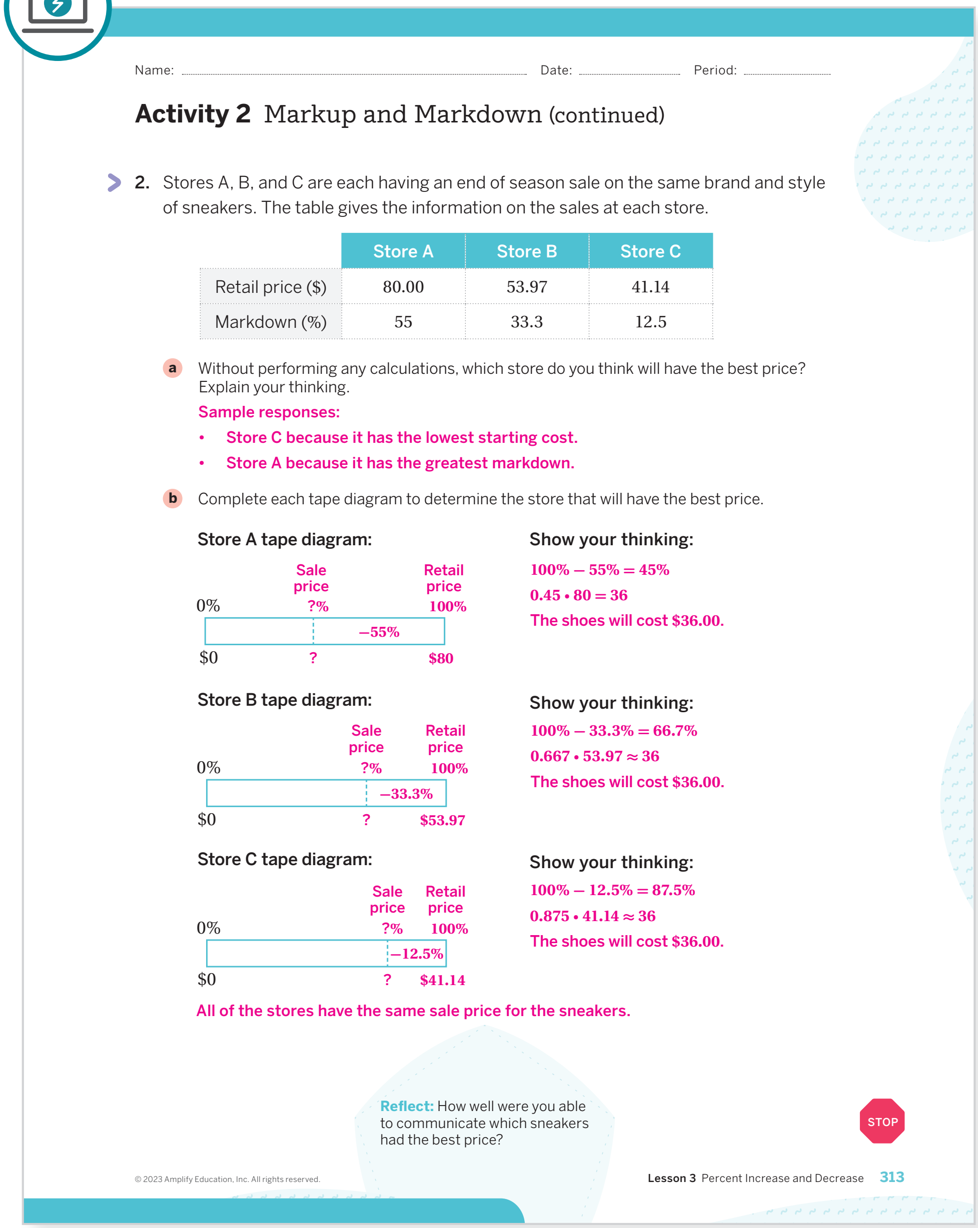
English Learners

Make sure students understand the difference between the terms *retail price* and *sale price*.

Small groups | 20 min

Activity 2 Markup and Markdown (continued)

Students use tape diagrams and expressions to model percent increase and percent decrease problems involving markups and markdowns.



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Activity 2 Markup and Markdown (continued)

2. Stores A, B, and C are each having an end of season sale on the same brand and style of sneakers. The table gives the information on the sales at each store.

	Store A	Store B	Store C
Retail price ($)	80.00	53.97	41.14
Markdown (%)	55	33.3	12.5

a Without performing any calculations, which store do you think will have the best price? Explain your thinking.

Sample responses:

- Store C because it has the lowest starting cost.
- Store A because it has the greatest markdown.

b Complete each tape diagram to determine the store that will have the best price.

Store A tape diagram:

0% | Sale price ?% | Retail price 100%

−55%

$0 | ? | $80

Show your thinking:

$100\% - 55\% = 45\%$

$0.45 \cdot 80 = 36$

The shoes will cost $36.00.

Store B tape diagram:

0% | Sale price ?% | Retail price 100%

−33.3%

$0 | ? | $53.97

Show your thinking:

$100\% - 33.3\% = 66.7\%$

$0.667 \cdot 53.97 \approx 36$

The shoes will cost $36.00.

Store C tape diagram:

0% | Sale price ?% | Retail price 100%

−12.5%

$0 | ? | $41.14

Show your thinking:

$100\% - 12.5\% = 87.5\%$

$0.875 \cdot 41.14 \approx 36$

The shoes will cost $36.00.

All of the stores have the same sale price for the sneakers.

Reflect: How well were you able to communicate which sneakers had the best price?

STOP

3 Connect

Display the tape diagrams from the Student Edition.

Have students share their strategies for placing the given information in each diagram as well as their methods for determining the final cost of each item.

Highlight that the original price always aligns with 100% in each diagram.

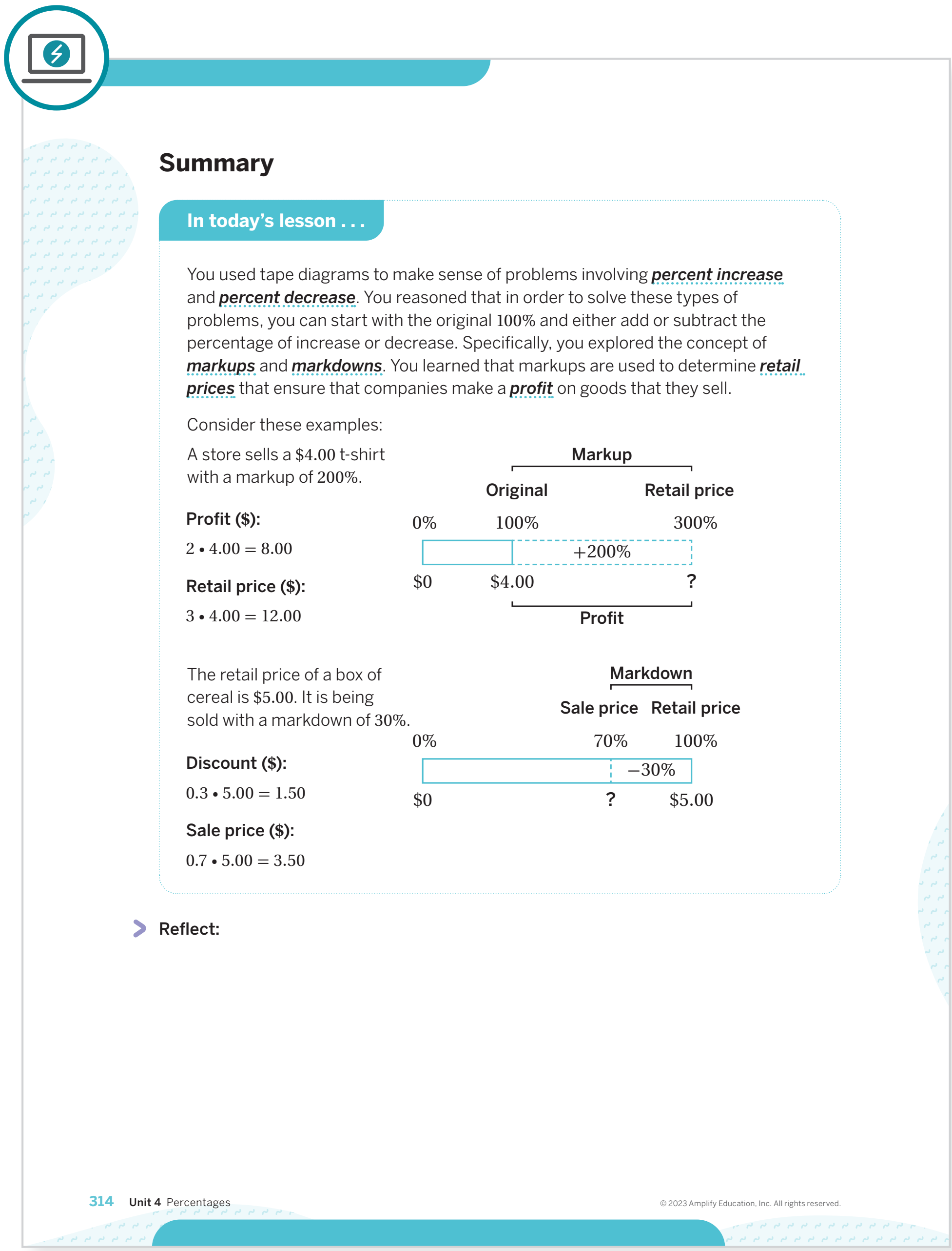
Ask:

- "How is it possible for all of the shoes to have the same price, even though the markdown at each store was different?"
- "How would you label a diagram to determine the original cost of a t-shirt, given that the retail cost of $15.00 includes a 150% markup?" Sample response: The $15.00 would be aligned with 250% (150% increase on 100%) and the unknown value would be aligned with 100%.

Whole Class | 5 min

Summary

Review and synthesize how to use tape diagrams and expressions to represent problems involving percent increase and percent decrease.



Summary

In today's lesson . . .

You used tape diagrams to make sense of problems involving ***percent increase*** and ***percent decrease***. You reasoned that in order to solve these types of problems, you can start with the original 100% and either add or subtract the percentage of increase or decrease. Specifically, you explored the concept of ***markups*** and ***markdowns***. You learned that markups are used to determine ***retail prices*** that ensure that companies make a ***profit*** on goods that they sell.

Consider these examples:

A store sells a $4.00 t-shirt with a markup of 200%.

Profit ($):

$2 \cdot 4.00 = 8.00$

Retail price ($):

$3 \cdot 4.00 = 12.00$

The retail price of a box of cereal is $5.00. It is being sold with a markdown of 30%.

Discount ($):

$0.3 \cdot 5.00 = 1.50$

Sale price ($):

$0.7 \cdot 5.00 = 3.50$

Reflect:

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Synthesize

Display the Anchor Chart PDF, *Percent Increase* and the Anchor Chart PDF, *Percent Decrease*. Use sticky notes to cover up the new value and its matching percentage in each scenario. Complete the "New Amount" section of each Anchor Chart.

Highlight that tape diagrams can help make sense or problems involving percent increase and decrease, including determining the amount of increase or decrease and the *new* amount.

Formalize vocabulary:

- ***markdown***
- ***markup***
- ***percent decrease***
- ***percent increase***
- ***profit***
- ***retail price***

Ask:

- "When have you seen examples of markup and markdown in real life?"
- "Can you think of examples of percent increase or decrease that do not involve money?"

Reflect

After to synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are percentages used to represent change (increase and decrease)?"

Math Language Development

MLR2: Collect and Display

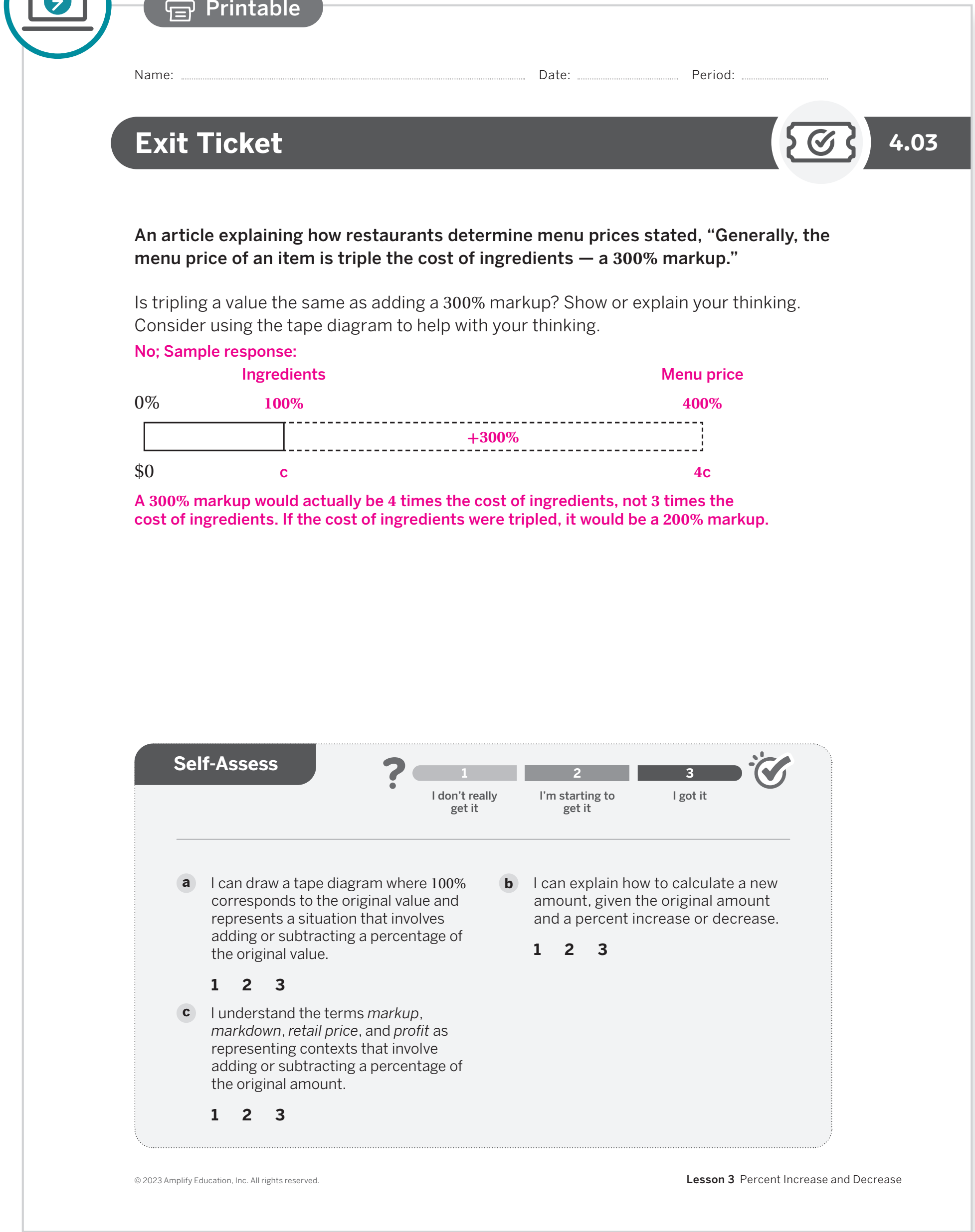
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the terms *markdown, markup, percent decrease, percent increase, profit,* and *retail price* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of percent increase by distinguishing between tripling a value and 300% of the value.

Printable



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Exit Ticket 4.03

An article explaining how restaurants determine menu prices stated, "Generally, the menu price of an item is triple the cost of ingredients — a 300% markup."

Is tripling a value the same as adding a 300% markup? Show or explain your thinking. Consider using the tape diagram to help with your thinking.

No; Sample response:

Ingredients — Menu price

0% — 100% — 400%

+300%

$0 — c — 4c

A 300% markup would actually be 4 times the cost of ingredients, not 3 times the cost of ingredients. If the cost of ingredients were tripled, it would be a 200% markup.

Self-Assess

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it ✓

a. I can draw a tape diagram where 100% corresponds to the original value and represents a situation that involves adding or subtracting a percentage of the original value.

1 2 3

b. I can explain how to calculate a new amount, given the original amount and a percent increase or decrease.

1 2 3

c. I understand the terms *markup*, *markdown*, *retail price*, and *profit* as representing contexts that involve adding or subtracting a percentage of the original amount.

1 2 3

Lesson 3 Percent Increase and Decrease

Success looks like . . .

- **Goal:** Drawing a tape diagram where 100% corresponds to the original value and represents a situation that involves adding or subtracting a percentage of the original value.
  - » Completing the tape diagram to represent and solve the problem about a menu price.
- **Language Goal:** Explaining how to calculate the new amount when given the original amount and the percent increase or decrease. **(Speaking and Listening)**
- **Language Goal:** Understanding the terms *markup*, *markdown*, *retail price*, and *profit* as contexts that involve adding or subtracting a percentage of the original amount. **(Speaking and Listening, Reading and Writing)**
  - » Comparing a 300% markup to tripling a value.

Suggested next steps

If students conclude that tripling is the same as an increase of 300% (markup), consider:

- Reviewing the definition of *markup*.
- Asking, "How could you model this on the tape diagram provided?"
- Assigning Practice Problems 1 and 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about the cost of shoes in Activity 2, how did you encourage each student to listen to one another's strategies?
- What other ways are there to try to model percent increase and decrease? What might you change for the next time you teach this lesson?

Math Language Development

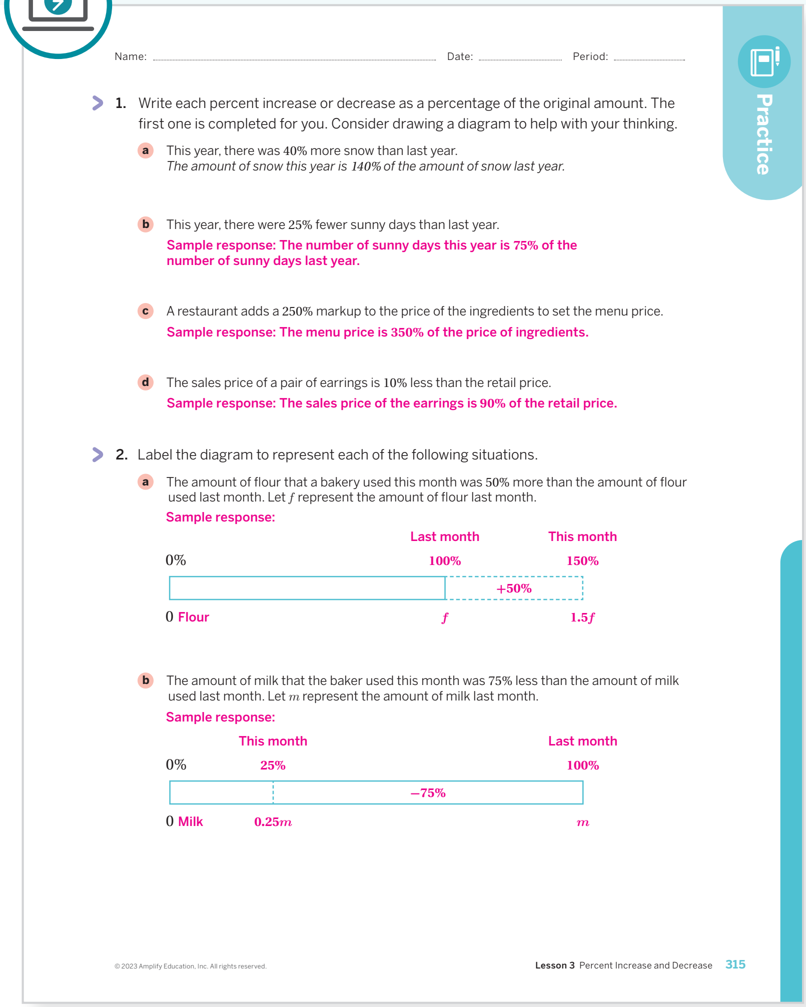
Language Goal: Explaining how to calculate the new amount when given the original amount and the percent increase or decrease.

Reflect on students' language development toward this goal.

- Do students' responses to the Exit Ticket problem demonstrate they understand that tripling the cost of an item is not the same as a 300% markup?
- How did using the math language routines in this lesson support students in their understanding of this distinction? What support can you provide to help them to be more precise in their explanations?

Practice

Independent



Name: Date: Period:

Practice

1. Write each percent increase or decrease as a percentage of the original amount. The first one is completed for you. Consider drawing a diagram to help with your thinking.
   - a This year, there was 40% more snow than last year.
     The amount of snow this year is 140% of the amount of snow last year.
   - b This year, there were 25% fewer sunny days than last year.
     Sample response: The number of sunny days this year is 75% of the number of sunny days last year.
   - c A restaurant adds a 250% markup to the price of the ingredients to set the menu price.
     Sample response: The menu price is 350% of the price of ingredients.
   - d The sales price of a pair of earrings is 10% less than the retail price.
     Sample response: The sales price of the earrings is 90% of the retail price.

2. Label the diagram to represent each of the following situations.
   - a The amount of flour that a bakery used this month was 50% more than the amount of flour used last month. Let f represent the amount of flour last month.
     Sample response:

   - b The amount of milk that the baker used this month was 75% less than the amount of milk used last month. Let m represent the amount of milk last month.
     Sample response:

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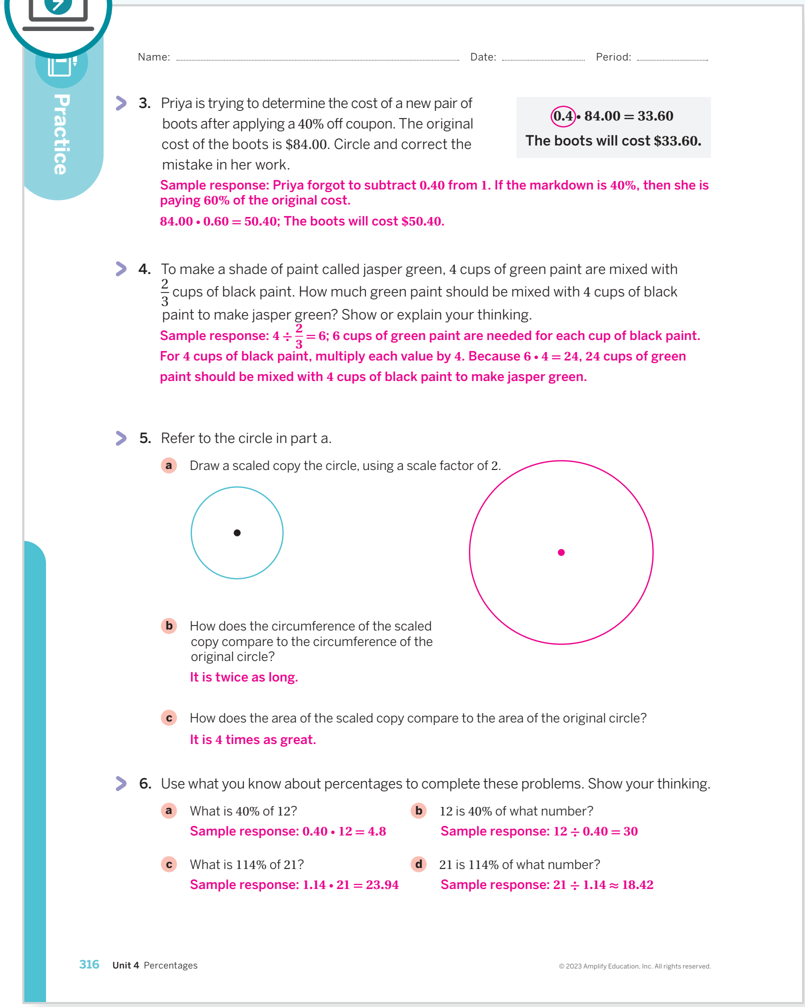
Lesson 3 Percent Increase and Decrease 315



Name: Date: Period:

Practice

3. Priya is trying to determine the cost of a new pair of boots after applying a 40% off coupon. The original cost of the boots is $84.00. Circle and correct the mistake in her work.

   $\textcircled{0.4} \cdot 84.00 = 33.60$
   The boots will cost $33.60.

   Sample response: Priya forgot to subtract 0.40 from 1. If the markdown is 40%, then she is paying 60% of the original cost.
   $84.00 \cdot 0.60 = 50.40$; The boots will cost $50.40.

4. To make a shade of paint called jasper green, 4 cups of green paint are mixed with $\frac{2}{3}$ cups of black paint. How much green paint should be mixed with 4 cups of black paint to make jasper green? Show or explain your thinking.
   Sample response: $4 \div \frac{2}{3} = 6$; 6 cups of green paint are needed for each cup of black paint. For 4 cups of black paint, multiply each value by 4. Because $6 \cdot 4 = 24$, 24 cups of green paint should be mixed with 4 cups of black paint to make jasper green.

5. Refer to the circle in part a.
   - a Draw a scaled copy the circle, using a scale factor of 2.
   - b How does the circumference of the scaled copy compare to the circumference of the original circle?
     It is twice as long.
   - c How does the area of the scaled copy compare to the area of the original circle?
     It is 4 times as great.

6. Use what you know about percentages to complete these problems. Show your thinking.
   - a What is 40% of 12?
     Sample response: $0.40 \cdot 12 = 4.8$
   - b 12 is 40% of what number?
     Sample response: $12 \div 0.40 = 30$
   - c What is 114% of 21?
     Sample response: $1.14 \cdot 21 = 23.94$
   - d 21 is 114% of what number?
     Sample response: $21 \div 1.14 \approx 18.42$

316 **Unit 4** Percentages

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	1
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Unit 2 Lesson 5	2
	5	Unit 3 Lesson 2	2
Formative ⚡	6	Unit 4 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

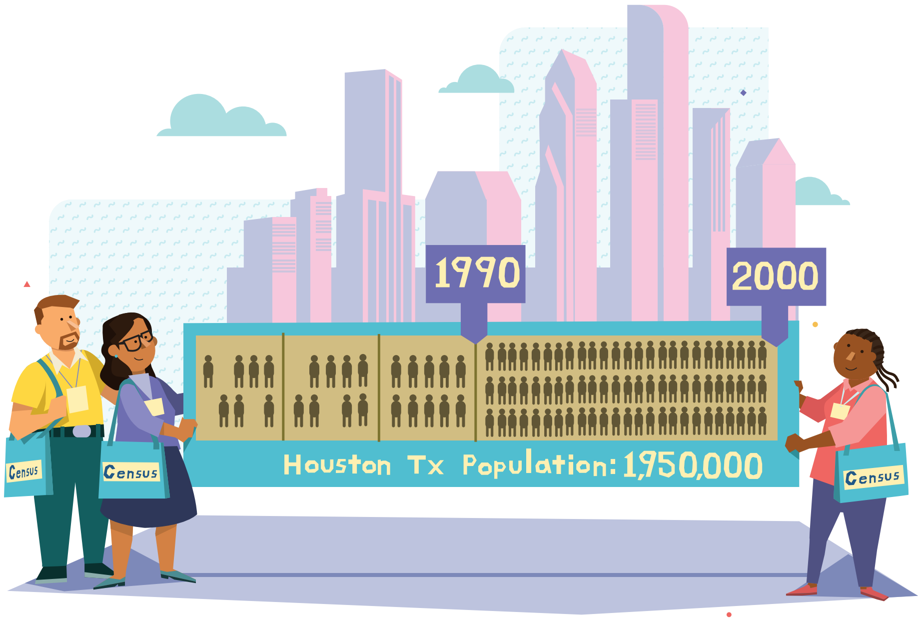
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 4 | **LESSON 4**

Determining 100%

Let's solve more problems about percent increase and decrease.



Focus

Goals

1. Determine whether a problem involving percent change is asking for the original value or the new value.
2. **Language Goal:** Explain how to calculate the original amount, given the new amount and the percent increase or decrease. **(Speaking and Listening)**
3. Draw and label a tape diagram to represent a situation that involves determining the original value after adding or subtracting a percentage.

Coherence

Today

Students build on their understanding of using tape diagrams to model percent of increase and decrease to make sense of, and solve, problems involving determining the original value. They reason about whether scenarios are representing a percentage of increase or decrease, as well as whether they are being asked to calculate the original amount or the new amount.

Previously

In Lesson 3, students used tape diagrams to determine the new amount when given the percent increase or decrease and the original amount.

Coming Soon

In Lesson 5, students will determine the percent change when given the original and the new amount.

Rigor

- Students build **conceptual understanding** of determining the original amount in percent increase and decrease scenarios.
- Students gain **fluency** in solving problems involving determining the new or original amount and percent increase or decrease.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	7 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per group
- Activity 2 PDF, *Sorted Cards* (for display)
- Anchor Chart PDF, *Percent Decrease* (for display)
- Anchor Chart PDF, *Percent Decrease* (answers)
- Anchor Chart PDF, *Percent Increase* (for display)
- Anchor Chart PDF, *Percent Increase* (answers)
- Graphic Organizer PDF, *Percentage Tape Diagrams* (as needed)
- calculators
- sticky notes

Math Language Development

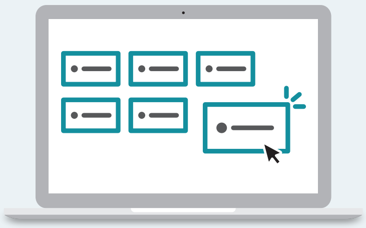
Review words

- *markdown*
- *markup*
- *percent decrease*
- *percent increase*
- *profit*
- *retail price*

Amps | Featured Activity

Activity 2
Digital Card Sort

In Activity 2, students will sort scenarios involving percent increase and decrease by whether they are being asked to determine the original amount or the new amount.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be a little too excited about working with a small group, causing themselves or others to not focus on making sense of the problems and solving them. Prior to arranging the groups, have students identify ways that they can set boundaries on their personal and group interactions so that they do not disturb others in the class. Have students form accountability pairs within the groups so that each student has someone who will gently remind them to work on solving the problems.

Modifications to Pacing

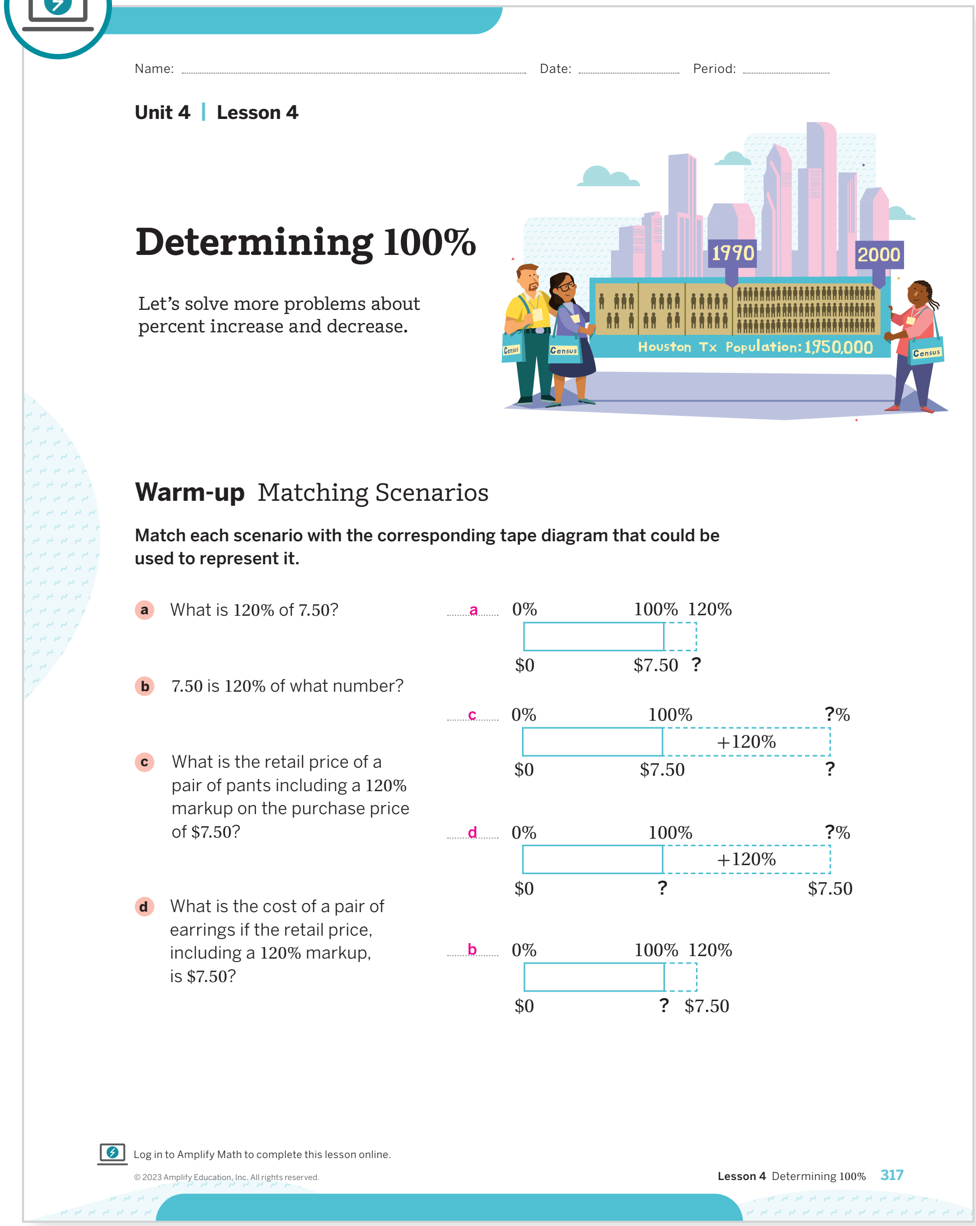
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, only distribute Cards 4–7. Students will identify two cards for each type of scenario.

Independent | 5 min

Warm-up Matching Scenarios

Students match scenarios with tape diagrams to make sense of determining the new and the original amount in scenarios involving percentages.



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Unit 4 | **Lesson 4**

Determining 100%

Let's solve more problems about percent increase and decrease.

Warm-up Matching Scenarios

Match each scenario with the corresponding tape diagram that could be used to represent it.

a. What is 120% of 7.50?

b. 7.50 is 120% of what number?

c. What is the retail price of a pair of pants including a 120% markup on the purchase price of \$7.50?

d. What is the cost of a pair of earrings if the retail price, including a 120% markup, is \$7.50?

Log in to Amplify Math to complete this lesson online.

Lesson 4 Determining 100% 317

1 Launch

Explain to students that they should be matching each scenario with the tape diagram they can use to determine the unknown value.

2 Monitor

Help students get started by asking, "What value matches with 100%".

Look for points of confusion

- **Thinking that "including a 120% markup" means that the total percent is 120%.** Ask, "How would you represent a 20% increase? A 120% increase?"

3 Connect

Display the Warm-up from the Student edition and conduct the *Poll the Class* routine to determine which diagram matches each scenario.

Have students share how they determined the matches for the Warm-up and made connections between the scenarios and the tape diagrams.

Ask:

- "For scenarios c and d, why are there two unknown values in each tape diagram?"
- "How could you determine the unknown value in scenarios a and b?" **Note:** Encourage students to write expressions to represent the unknown values.
- "How could you determine each missing value for scenarios c and d?" **Note:** Encourage students to write expressions to represent the unknown values.

Highlight that a tape diagram can help students make sense of whether they are trying to determine the new or the original value in a scenario involving percent increase or decrease. Similar to a table of values, when they go *backward* in a tape diagram, they divide, instead of multiplying. Model the multiplication or division by adding arrows to each tape diagram during the discussion.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they determined their matches, draw connections between the language used in each scenario and its corresponding tape diagram. Ask:

- "Which diagrams show 20% added to 100%? What language is used in their matching scenarios?"
- "Which diagrams show 120% added to 100%? What language is used in their matching scenarios?"

English Learners

Annotate key words and phrases in the scenarios that indicate whether 20% or 120% is added to 100%, such as "120% of ____" versus "120% markup."

Power-up

To power up students' ability to determine the percent, part, and whole without using a given tape diagram, have students complete:

Match each verbal statement with the expression that represents it:

a. What is 120% of 30? ......b...... $\frac{30}{120} \cdot 100$

b. 30 is what percent of 120? ......a...... $30 \cdot \frac{120}{100}$

c. 30 is 120% of what number? ......c...... $30 \cdot \frac{100}{120}$

d. 120 is what percent of 30? ......d...... $\frac{120}{30} \cdot 100$

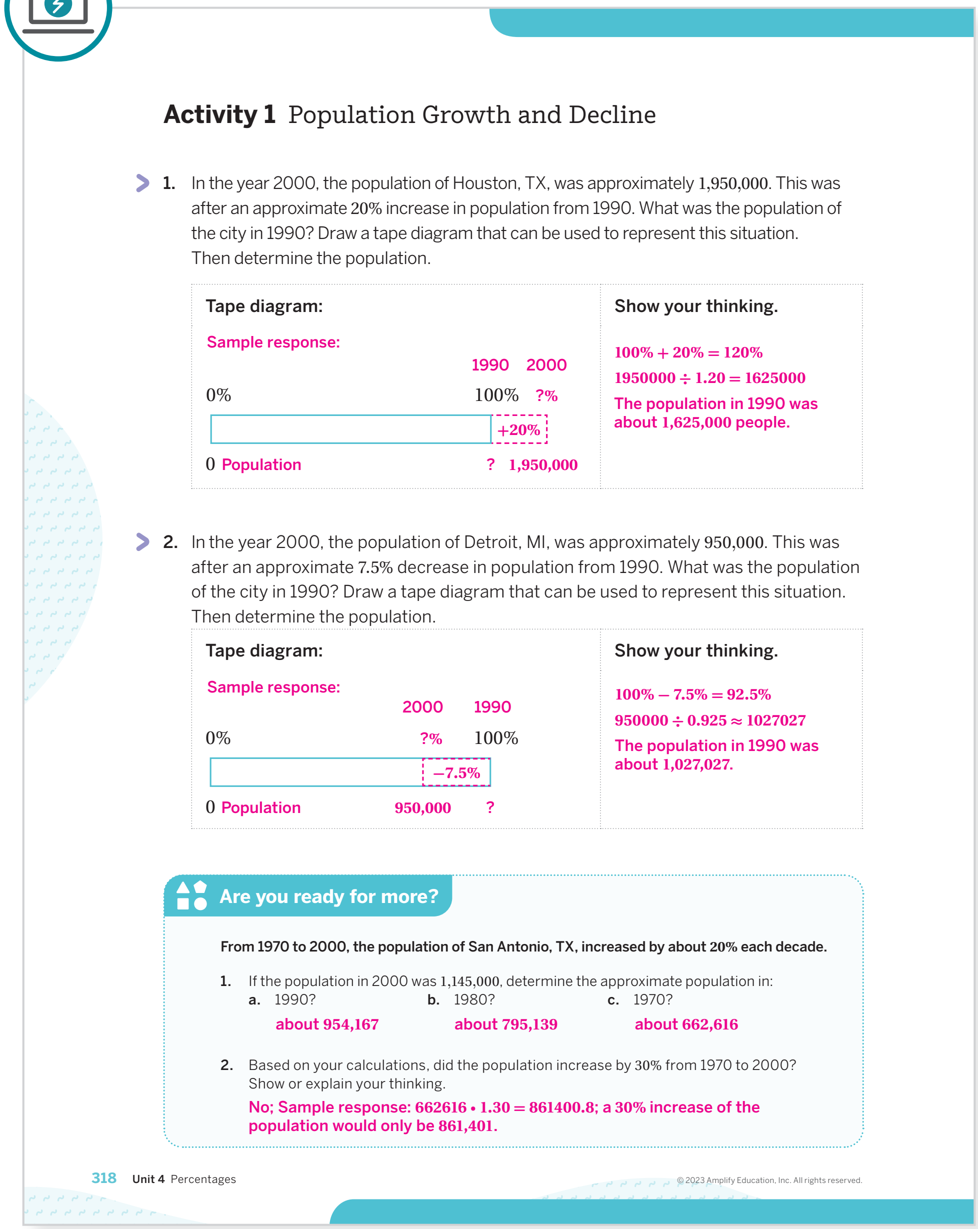
Use: Before Activity 1.

Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 4 and 6.

Pairs | 10 min

Activity 1 Population Growth and Decline

Students draw tape diagrams and write expressions to determine the original value when given the new value and the percent of increase or decrease.



Activity 1 Population Growth and Decline

1. In the year 2000, the population of Houston, TX, was approximately 1,950,000. This was after an approximate 20% increase in population from 1990. What was the population of the city in 1990? Draw a tape diagram that can be used to represent this situation. Then determine the population.

Tape diagram:

Sample response:

1990 2000

0% 100% ?%

+20%

0 Population ? 1,950,000

Show your thinking.

$100\% + 20\% = 120\%$

$1950000 \div 1.20 = 1625000$

The population in 1990 was about 1,625,000 people.

2. In the year 2000, the population of Detroit, MI, was approximately 950,000. This was after an approximate 7.5% decrease in population from 1990. What was the population of the city in 1990? Draw a tape diagram that can be used to represent this situation. Then determine the population.

Tape diagram:

Sample response:

2000 1990

0% ?% 100%

−7.5%

0 Population 950,000 ?

Show your thinking.

$100\% - 7.5\% = 92.5\%$

$950000 \div 0.925 \approx 1027027$

The population in 1990 was about 1,027,027.

Are you ready for more?

From 1970 to 2000, the population of San Antonio, TX, increased by about 20% each decade.

1. If the population in 2000 was 1,145,000, determine the approximate population in:
   - **a.** 1990? about 954,167
   - **b.** 1980? about 795,139
   - **c.** 1970? about 662,616
2. Based on your calculations, did the population increase by 30% from 1970 to 2000? Show or explain your thinking.
   No; Sample response: $662616 \cdot 1.30 = 861400.8$; a 30% increase of the population would only be 861,401.

318 **Unit 4** Percentages

1 Launch

Activate students' background knowledge by asking if they know of any city that has had a large increase or decrease in population. Explain that students will be working in pairs to determine the population in each city. Distribute calculators to help with computations.

2 Monitor

Help students get started by suggesting they use the tape diagrams from the Warm-up to help them make sense of each problem and draw their own diagrams.

Look for points of confusion:

- **Matching the given population with the 100%.** Ask students, "Is this the original population or the population after a change?"
- **In Problem 2, adding the 7.5%.** Ask, "Is the population getting larger or smaller?"
- **Determining the amount the population changed in each scenario but not the original amount.** Ask, "Looking at your tape diagram, does it make sense that your amount would match with 100%?"
- **Multiplying by the percentage instead of dividing.** Ask, "Does your original value make sense when compared to the new value?"

3 Connect

Display students' diagrams for each problem.

Have students share how they created their diagrams and wrote their expressions.

Highlight the connection between the scenarios, the diagrams, and the expressions.

Ask:

- "How are percentages represented in the expressions?" As their decimal equivalents.
- "Why is it necessary to divide when solving these problems instead of multiplying?"

Differentiated Support

Accessibility: *Activate Background Knowledge*

Consider altering this activity to use populations of two cities within your state, or a nearby state, where one city's population increased and another city's population decreased (if applicable). This will engage more students by examining the population of cities with which they are familiar.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their diagrams and how they wrote their expressions, draw connections between the diagrams and expressions. Ask:

- "Where in your expression for Problem 1 did you indicate a percent increase?"
- "Where in your expression for Problem 2 did you indicate a percent decrease?"
- "Where was the unknown value in the tape diagram? How did you use this location to tell you whether to multiply or divide by the decimal equivalent of the percentage?

English Learners

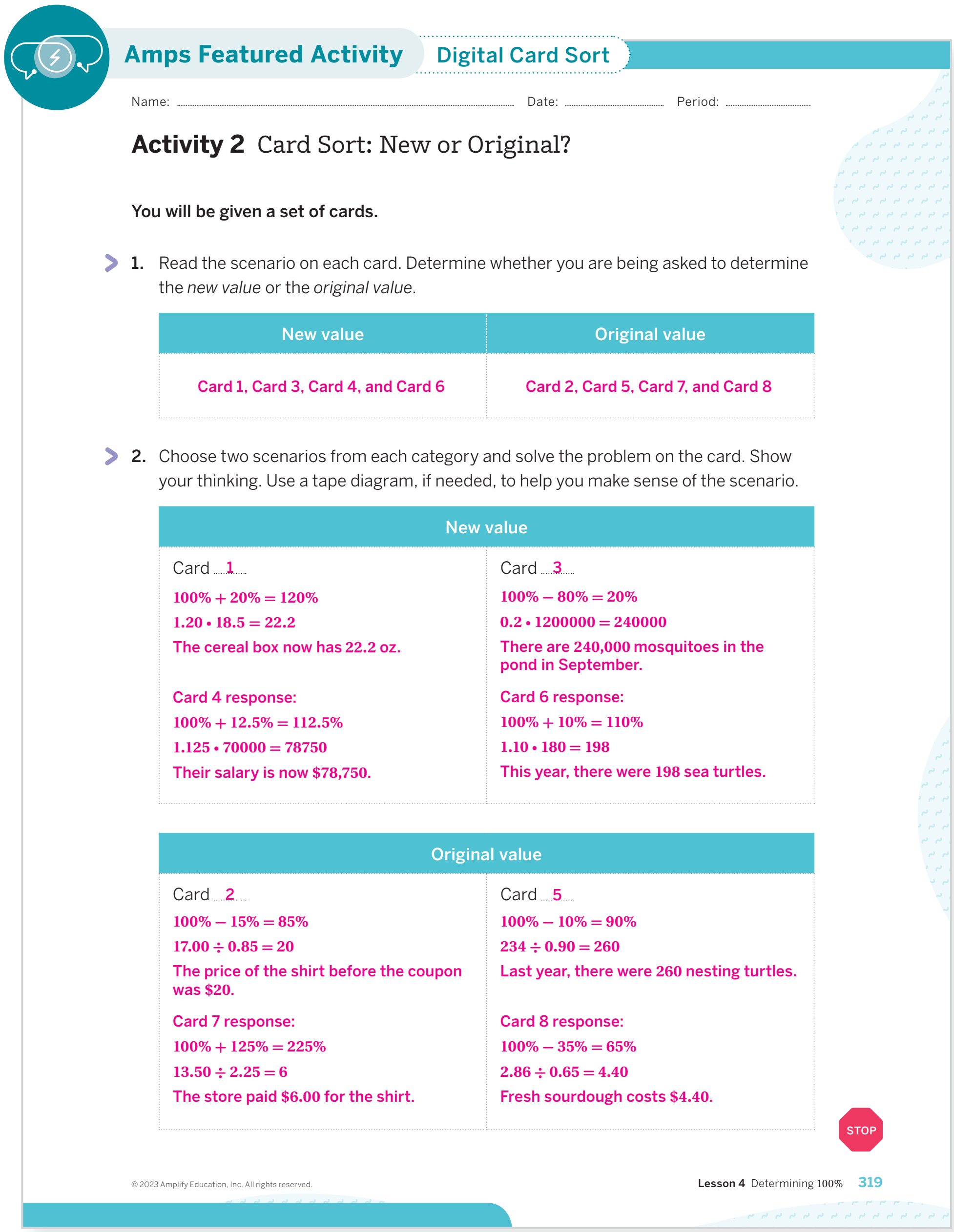
Annotate the key phrases that indicate percent increase or decrease in each problem. Then annotate the tape diagrams with the terms *increase* or *decrease* to highlight these connections.

Small groups | 20 min

Activity 2 Card Sort: New or Original?

Students determine whether scenarios involving percent increase or decrease are asking them to determine the 'new' amount or the 'original' amount.

Amps Featured Activity Digital Card Sort



Name: ________ Date: ________ Period: ________

Activity 2 Card Sort: New or Original?

You will be given a set of cards.

1. Read the scenario on each card. Determine whether you are being asked to determine the *new value* or the *original value*.

New value	Original value
Card 1, Card 3, Card 4, and Card 6	Card 2, Card 5, Card 7, and Card 8

2. Choose two scenarios from each category and solve the problem on the card. Show your thinking. Use a tape diagram, if needed, to help you make sense of the scenario.

New value	
Card 1 $100\% + 20\% = 120\%$ $1.20 \cdot 18.5 = 22.2$ The cereal box now has 22.2 oz. Card 4 response: $100\% + 12.5\% = 112.5\%$ $1.125 \cdot 70000 = 78750$ Their salary is now $78,750.	Card 3 $100\% - 80\% = 20\%$ $0.2 \cdot 1200000 = 240000$ There are 240,000 mosquitoes in the pond in September. Card 6 response: $100\% + 10\% = 110\%$ $1.10 \cdot 180 = 198$ This year, there were 198 sea turtles.

Original value	
Card 2 $100\% - 15\% = 85\%$ $17.00 \div 0.85 = 20$ The price of the shirt before the coupon was $20. Card 7 response: $100\% + 125\% = 225\%$ $13.50 \div 2.25 = 6$ The store paid $6.00 for the shirt.	Card 5 $100\% - 10\% = 90\%$ $234 \div 0.90 = 260$ Last year, there were 260 nesting turtles. Card 8 response: $100\% - 35\% = 65\%$ $2.86 \div 0.65 = 4.40$ Fresh sourdough costs $4.40.

STOP

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Lesson 4 Determining 100% 319

1 Launch

Distribute one set of pre-cut cards from the Activity 2 PDF to each small group and conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by asking, "The original amount always matches with what percent?"

Look for points of confusion:

- **Difficulty in determining whether they are looking for the original amount or the new amount.** Suggest students sketch diagrams to help them make sense of scenarios.

Look for productive strategies:

- Identifying what value (or unknown) matches 100% in each scenario as the first step.

3 Connect

Have students share how they determined when a scenario was asking them to determine the original amount or the new amount.

Display the Activity 2 PDF, *Sorted Cards*.

Ask, "What key words or phrases in each scenario helped you determine what quantity represented 100%?" Reflect student responses by annotating the cards focusing on words that reflect time, e.g. *before*, *after*, *years*, etc.

Highlight that, in order to determine the new amount, multiplication would be used, while in order to determine the original amount, division would be used.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* to help students make sense of, and model, each scenario.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask question, ask these follow-up questions to help highlight these connections. (Sample questions are provided for Card 3.)

- "How did you determine Card 3 was asking for the new value?" September comes after August and I knew the number decreased between August and September. The number for August was given.
- "Which value represents the whole? Explain your thinking?" 1,200,000; The number decreased by 80%, which means that 20% of this value is the September value.

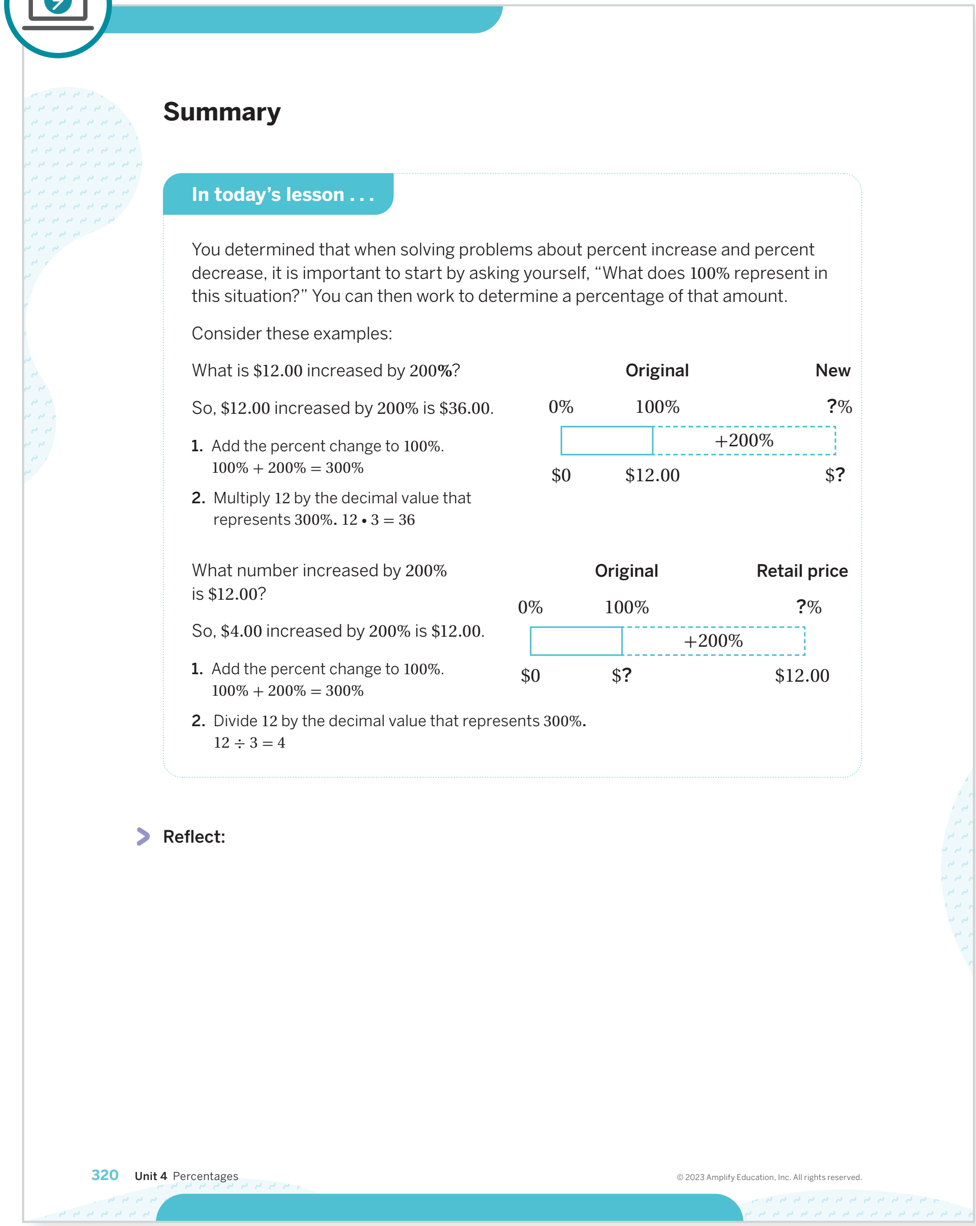
English Learners

Annotate key phrases that indicate time, such as *now, before the raise, last year, after applying the coupon,* etc., to help students understand whether they are looking for the new value or the original value.

Whole Class | 5 min

Summary

Review and synthesize how to determine the new or original amount when given one value and the percent increase or decrease.



Summary

In today's lesson . . .

You determined that when solving problems about percent increase and percent decrease, it is important to start by asking yourself, "What does 100% represent in this situation?" You can then work to determine a percentage of that amount.

Consider these examples:

What is $12.00 increased by 200%?

So, $12.00 increased by 200% is $36.00.

1. Add the percent change to 100%. $100\% + 200\% = 300\%$
2. Multiply 12 by the decimal value that represents 300%. $12 \cdot 3 = 36$

Original
New
0%
100%
?%
+200%
$0
$12.00
$?

What number increased by 200% is $12.00?

So, $4.00 increased by 200% is $12.00.

1. Add the percent change to 100%. $100\% + 200\% = 300\%$
2. Divide 12 by the decimal value that represents 300%. $12 \div 3 = 4$

Original
Retail price
0%
100%
?%
+200%
$0
$?
$12.00

Reflect:

320 **Unit 4** Percentages

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Synthesize

Display the Anchor Chart PDF, *Percent Increase* and the Anchor Chart PDF, *Percent Decrease*. Use sticky notes to cover up the original value and the percentage for the new value in each scenario. Complete the "Original Amount" section of each Anchor Chart.

Ask, "Can you think of a percent increase or decrease context that would ask you to determine the new amount? The original amount?"

Have students share their contexts with a partner. Then ask volunteers to share their contexts with the class.

Highlight the key words or phrases in each context that reflect that students are looking for either the new or original amount.

Reflect

After to synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

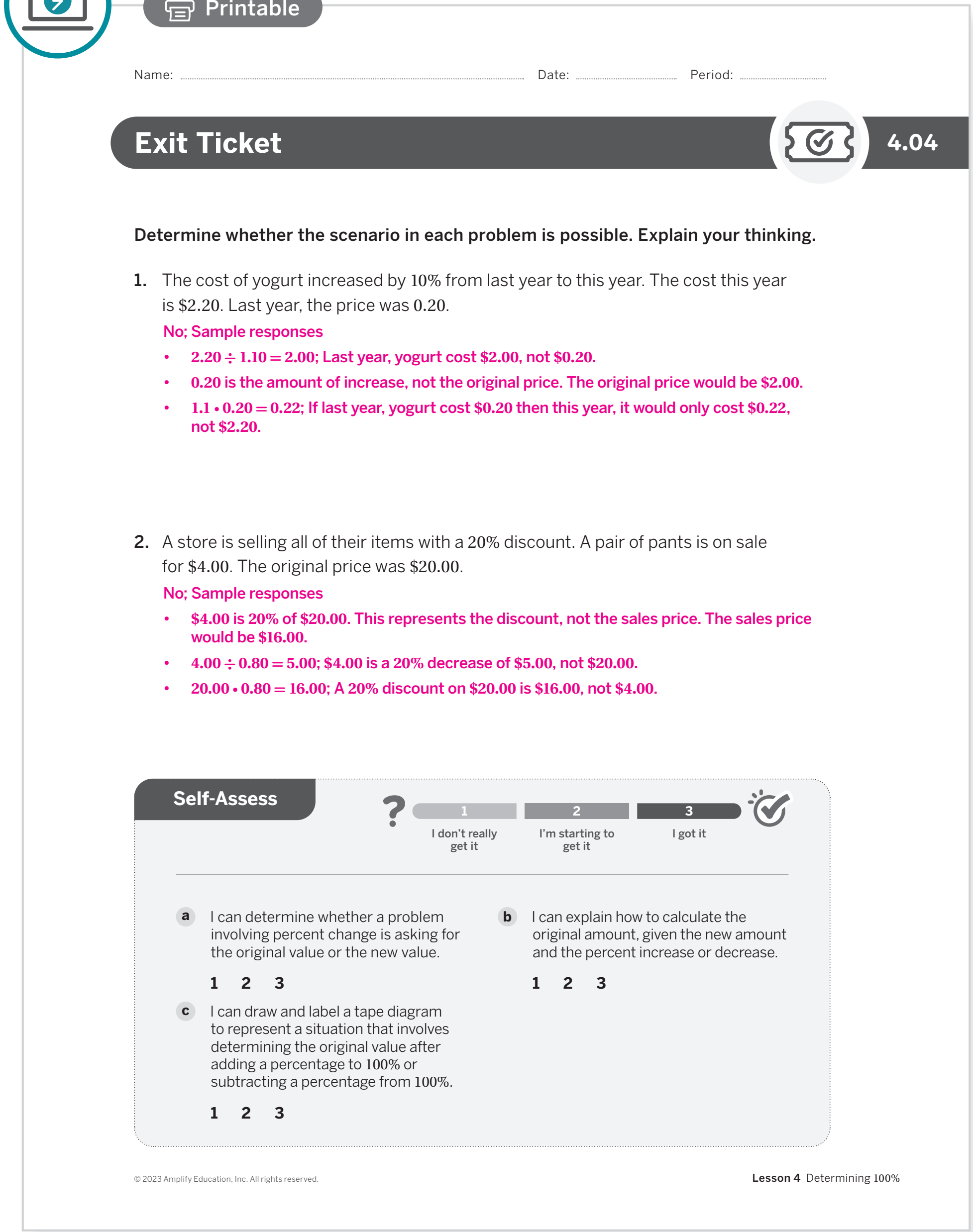
- "How are percentages used to represent change (increase and decrease)?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by determining whether the description of a percent increase or percent decrease is possible.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.04

Determine whether the scenario in each problem is possible. Explain your thinking.

1. The cost of yogurt increased by 10% from last year to this year. The cost this year is $2.20. Last year, the price was 0.20.

   No; Sample responses
   - $2.20 \div 1.10 = 2.00$; Last year, yogurt cost $2.00, not $0.20.
   - 0.20 is the amount of increase, not the original price. The original price would be $2.00.
   - $1.1 \cdot 0.20 = 0.22$; If last year, yogurt cost $0.20 then this year, it would only cost $0.22, not $2.20.

2. A store is selling all of their items with a 20% discount. A pair of pants is on sale for $4.00. The original price was $20.00.

   No; Sample responses
   - $4.00 is 20% of $20.00. This represents the discount, not the sales price. The sales price would be $16.00.
   - $4.00 \div 0.80 = 5.00$; $4.00 is a 20% decrease of $5.00, not $20.00.
   - $20.00 \cdot 0.80 = 16.00$; A 20% discount on $20.00 is $16.00, not $4.00.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine whether a problem involving percent change is asking for the original value or the new value.

1 2 3

b. I can explain how to calculate the original amount, given the new amount and the percent increase or decrease.

1 2 3

c. I can draw and label a tape diagram to represent a situation that involves determining the original value after adding a percentage to 100% or subtracting a percentage from 100%.

1 2 3

Lesson 4 Determining 100%

Success looks like . . .

- **Goal:** Determining whether a problem involving percent change is asking for the original value or the new value.
- **Language Goal:** Explaining how to calculate the original amount, given the new amount and the percent increase or decrease. **(Speaking and Listening)**
  - » Explaining how to determine the percent increase and the correct original amount in Problem 1.
- **Goal:** Drawing and labeling a tape diagram to represent a situation that involves determining the original value after adding or subtracting a percentage.

Suggested next steps

If students claim that both scenarios are possible, consider:

- Having students use a tape diagram to determine the correct cost for Problem 1 and the correct sale price for Problem 2.
- Assigning Practice Problems 2 and 3.

Professional Learning

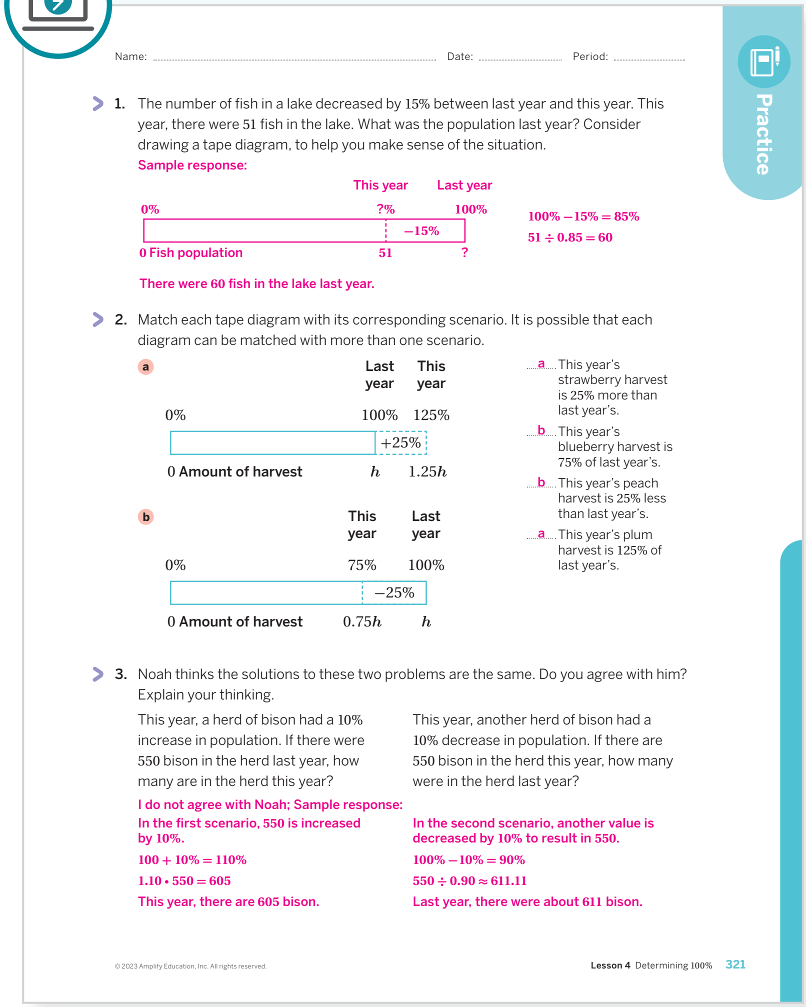
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During Activity 1, did anything happen that you did not expect?
- When you compare and contrast today's work with work students did earlier this year with tables and proportional relationships, what similarities and differences do you see? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. The number of fish in a lake decreased by 15% between last year and this year. This year, there were 51 fish in the lake. What was the population last year? Consider drawing a tape diagram, to help you make sense of the situation.
   Sample response:

   This year Last year
   0% ?% 100%
   −15%
   0 Fish population 51 ?

   $100\% - 15\% = 85\%$
   $51 \div 0.85 = 60$

   There were 60 fish in the lake last year.

2. Match each tape diagram with its corresponding scenario. It is possible that each diagram can be matched with more than one scenario.

   a
   Last year This year
   0% 100% 125%
   +25%
   0 Amount of harvest h $1.25h$

   b
   This year Last year
   0% 75% 100%
   −25%
   0 Amount of harvest $0.75h$ h

   a This year's strawberry harvest is 25% more than last year's.
   b This year's blueberry harvest is 75% of last year's.
   b This year's peach harvest is 25% less than last year's.
   a This year's plum harvest is 125% of last year's.

3. Noah thinks the solutions to these two problems are the same. Do you agree with him? Explain your thinking.

   This year, a herd of bison had a 10% increase in population. If there were 550 bison in the herd last year, how many are in the herd this year?

   This year, another herd of bison had a 10% decrease in population. If there are 550 bison in the herd this year, how many were in the herd last year?

   I do not agree with Noah; Sample response:

   In the first scenario, 550 is increased by 10%.
   $100 + 10\% = 110\%$
   $1.10 \cdot 550 = 605$
   This year, there are 605 bison.

   In the second scenario, another value is decreased by 10% to result in 550.
   $100\% - 10\% = 90\%$
   $550 \div 0.90 \approx 611.11$
   Last year, there were about 611 bison.

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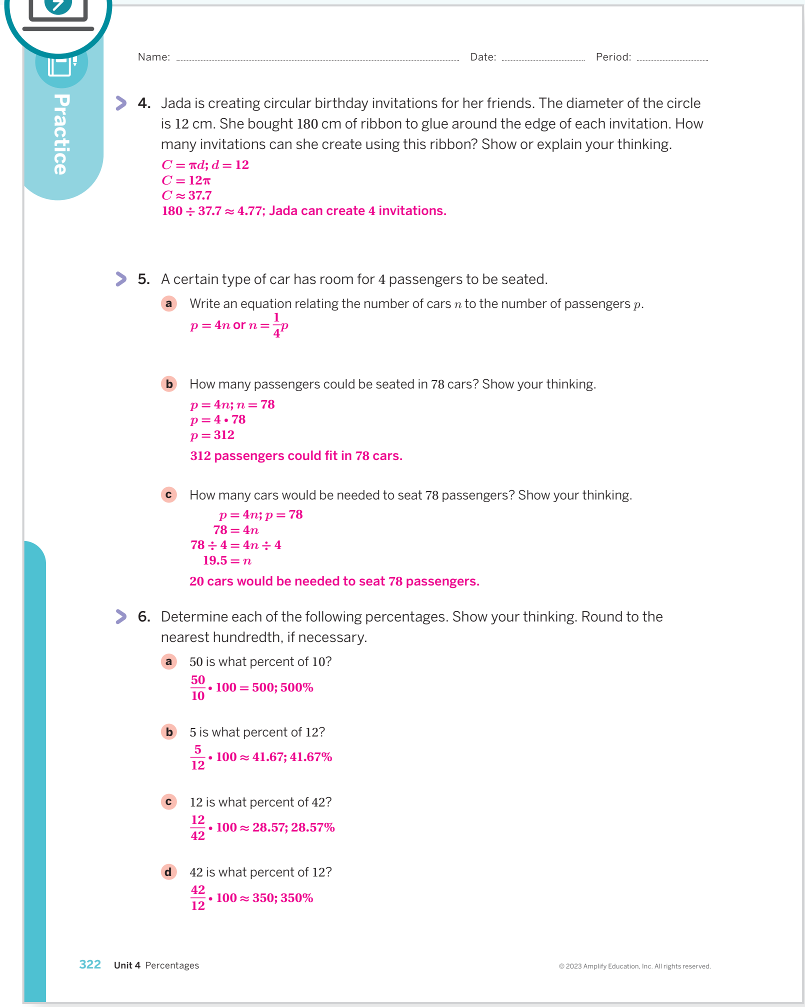
Lesson 4 Determining 100% 321



Practice

Name: Date: Period:

4. Jada is creating circular birthday invitations for her friends. The diameter of the circle is 12 cm. She bought 180 cm of ribbon to glue around the edge of each invitation. How many invitations can she create using this ribbon? Show or explain your thinking.
   $C = \pi d; d = 12$
   $C = 12\pi$
   $C \approx 37.7$
   $180 \div 37.7 \approx 4.77$; **Jada can create 4 invitations.**

5. A certain type of car has room for 4 passengers to be seated.
   - a Write an equation relating the number of cars n to the number of passengers p.
     $p = 4n$ or $n = \frac{1}{4}p$
   - b How many passengers could be seated in 78 cars? Show your thinking.
     $p = 4n; n = 78$
     $p = 4 \cdot 78$
     $p = 312$
     312 passengers could fit in 78 cars.
   - c How many cars would be needed to seat 78 passengers? Show your thinking.
     $p = 4n; p = 78$
     $78 = 4n$
     $78 \div 4 = 4n \div 4$
     $19.5 = n$
     20 cars would be needed to seat 78 passengers.

6. Determine each of the following percentages. Show your thinking. Round to the nearest hundredth, if necessary.
   - a 50 is what percent of 10?
     $\frac{50}{10} \cdot 100 = 500; 500\%$
   - b 5 is what percent of 12?
     $\frac{5}{12} \cdot 100 \approx 41.67; 41.67\%$
   - c 12 is what percent of 42?
     $\frac{12}{42} \cdot 100 \approx 28.57; 28.57\%$
   - d 42 is what percent of 12?
     $\frac{42}{12} \cdot 100 \approx 350; 350\%$

322 **Unit 4** Percentages

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Warm-up	1
	3	Activity 2	3
Spiral	4	Unit 3 Lesson 6	2
	5	Unit 2 Lesson 8	2
Formative	6	Unit 4 Lesson 5	1

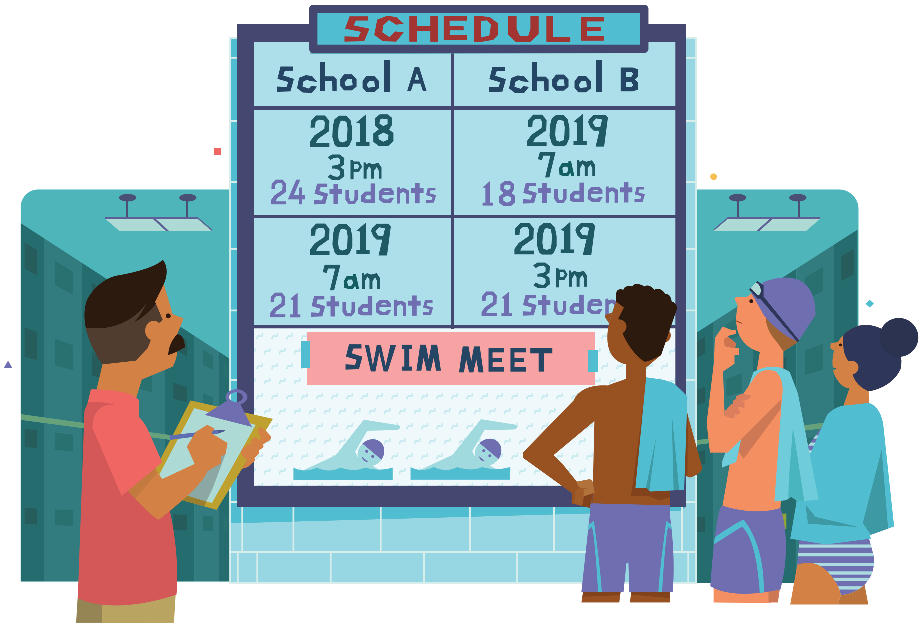
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Determining Percent Change

Let's determine the increase or decrease as a percent.



Focus

Goals

1. Identify the original value and the new amount in a problem involving percent increase or percent decrease.
2. Draw and label a tape diagram to represent a situation that involves determining the percent change when given the original amount and the new amount.
3. **Language Goal:** Describe how to determine the percent increase or decrease — percent change — when given the original amount and the new amount. **(Speaking and Listening)**.

Coherence

Today

Students expand their understanding of percent increase and decrease to determine the percent change between two values. They continue to use tape diagrams to make sense of scenarios, and reason about what information is given and what they are being asked to determine. They extend their understanding of percent change to make decisions about real-world contexts.

Previously

In Lessons 3 and 4, students determined the new or original amount when given one amount and the percent increase or percent decrease.

Coming Soon

In Lessons 6 and 7, students will use equations to model scenarios involving percent change.

Rigor

- Students build **conceptual understanding** of percent change.
- Students gain **fluency** in solving problems involving percent change.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
7 min	15 min	10 min	13 min	5 min	5 min
Pairs	Small Groups	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Activity 3 PDF, pre-cut cards, one set per group
- Anchor Chart PDF, *Percent Decrease* (for display)
- Anchor Chart PDF, *Percent Decrease* (answers)
- Anchor Chart PDF, *Percent Increase* (for display)
- Anchor Chart PDF, *Percent Increase* (answers)
- Graphic Organizer PDF, *Percentage Tape Diagrams* (as needed)
- calculators
- sticky notes

Math Language Development

New word

- ***percent change***

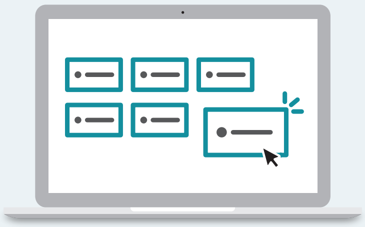
Review words

- *markdown*
- *markup*
- *percent decrease*
- *percent increase*

Amps | **Featured Activity**

Activity 3
Digital Card Sort

In Activity 3, students will sort scenarios involving percent increase and decrease by whether they are being asked to determine the original amount or the new amount.



Building Math Identity and Community

Connecting to Mathematical Practices

As students choose different methods, they might be tempted to show shock or disrespect to those who chose a method different from theirs. Before choosing their preferred method, have students work in pairs to describe and explain why each method could be beneficial. Understanding the perspectives of both methods can help them be more respectful of other students' ways of thinking.

Modifications to Pacing

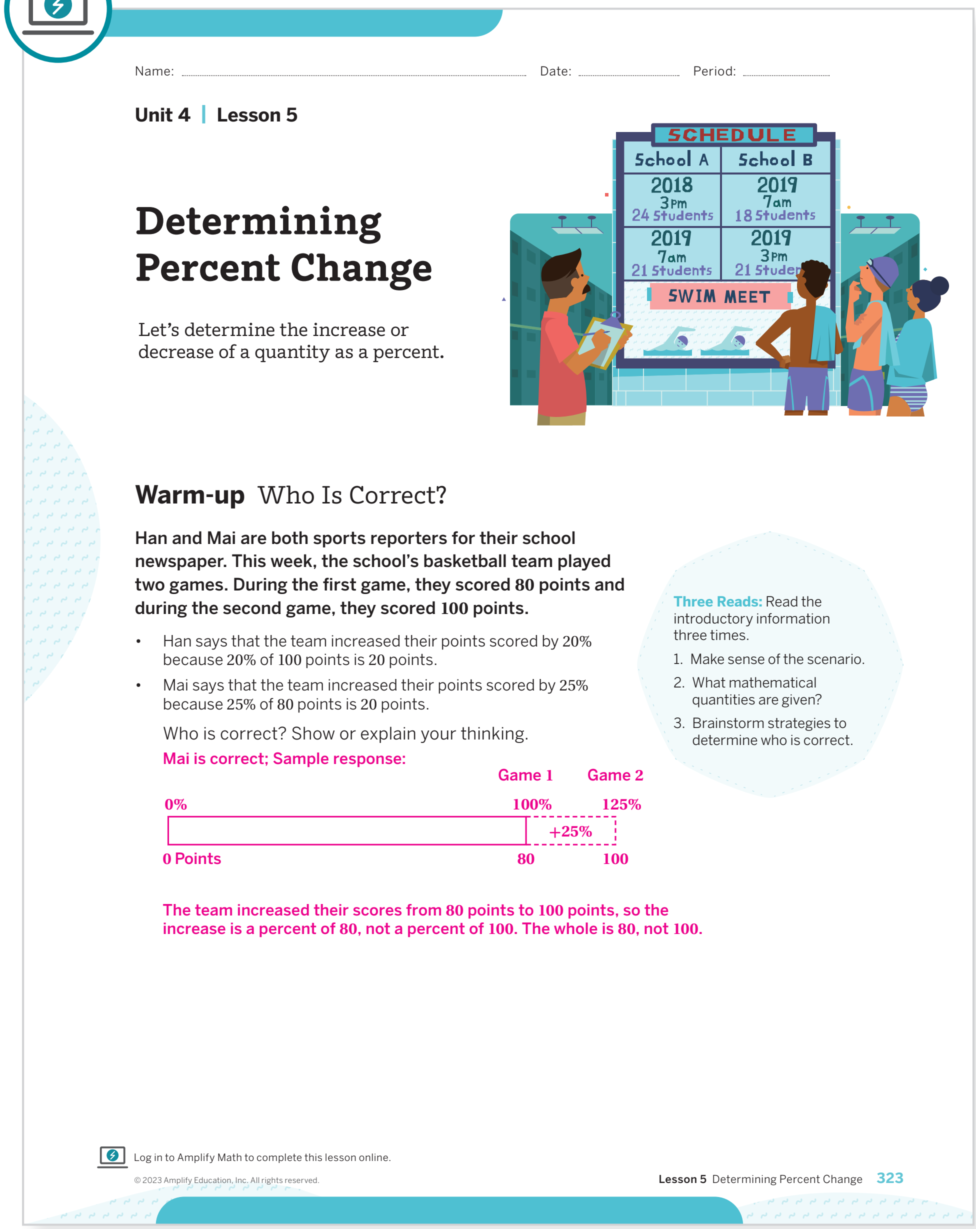
You may want to consider these additional modifications if you are short on time.

- Optional **Activity 2** may be omitted. You may choose to assign this activity as additional practice.
- In **Activity 3**, have students complete Problem 1 and omit Problem 2.

Pairs | 7 min

Warm-up Who Is Correct?

Students analyze two claims about the percent increase to determine which value matches the increase in points.



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Unit 4 | Lesson 5

Determining Percent Change

Let's determine the increase or decrease of a quantity as a percent.

Warm-up Who Is Correct?

Han and Mai are both sports reporters for their school newspaper. This week, the school's basketball team played two games. During the first game, they scored 80 points and during the second game, they scored 100 points.

- Han says that the team increased their points scored by 20% because 20% of 100 points is 20 points.
- Mai says that the team increased their points scored by 25% because 25% of 80 points is 20 points.

Who is correct? Show or explain your thinking.

Mai is correct; Sample response:

The team increased their scores from 80 points to 100 points, so the increase is a percent of 80, not a percent of 100. The whole is 80, not 100.

Three Reads: Read the introductory information three times.

1. Make sense of the scenario.
2. What mathematical quantities are given?
3. Brainstorm strategies to determine who is correct.

Log in to Amplify Math to complete this lesson online.

Lesson 5 Determining Percent Change 323

1 Launch

Conduct the ***Think-Pair-Share*** routine. Give students two minutes of 'think' time prior to their discussion with their partner.

2 Monitor

Help students get started by suggesting that they draw a tape diagram to help them make sense of the problem.

Look for points of confusion:

- **Not considering the order of the games.** Ask students, "What keywords do we look for to identify the original value?"

Look for productive strategies:

- Drawing a diagram or writing an expression to represent the problem.

3 Connect

Display the blank tape diagram from the Warm-up PDF. Conduct the ***Poll the Class*** routine to assess student opinion of whether Mai or Han is correct.

Have pairs of students share their reasoning for whether Mai or Han is correct. If possible, choose at least one pair that used the tape diagram to make sense of the scenario and reflect their thinking on the displayed tape diagram.

Highlight that when determining the *percent change*, the order of the values matter, because you are always taking the percent of the original amount.

Ask, "If the games swapped, and they scored 100 points in the first game and 80 points in the second game, would the percent decrease also be 25%?" No; Sample response: The original amount in this scenario is 100, and 20 is 20% of 100 not 25%.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that the basketball team increased their score from the first game to the second game.
- **Read 2:** Ask students to name or highlight the given quantities, such as *Han thinks they increased their points by* 20%.
- **Read 3:** Ask students to brainstorm strategies they can use to determine whether Han or Mai is correct.

English Learners

Have students annotate key phrases such as the different percentages by which Han and Mai think the team increased their points scored.

Power-up

To power up students' ability to determine the percent when given the new value and the original value, have students complete:

1. 20 is what percent of 100? 20%
2. 20 is what percent of 80? 25%
3. 20 is what percent of 60? $33\frac{1}{3}\%$
4. 20 is what percent of 8? 250%

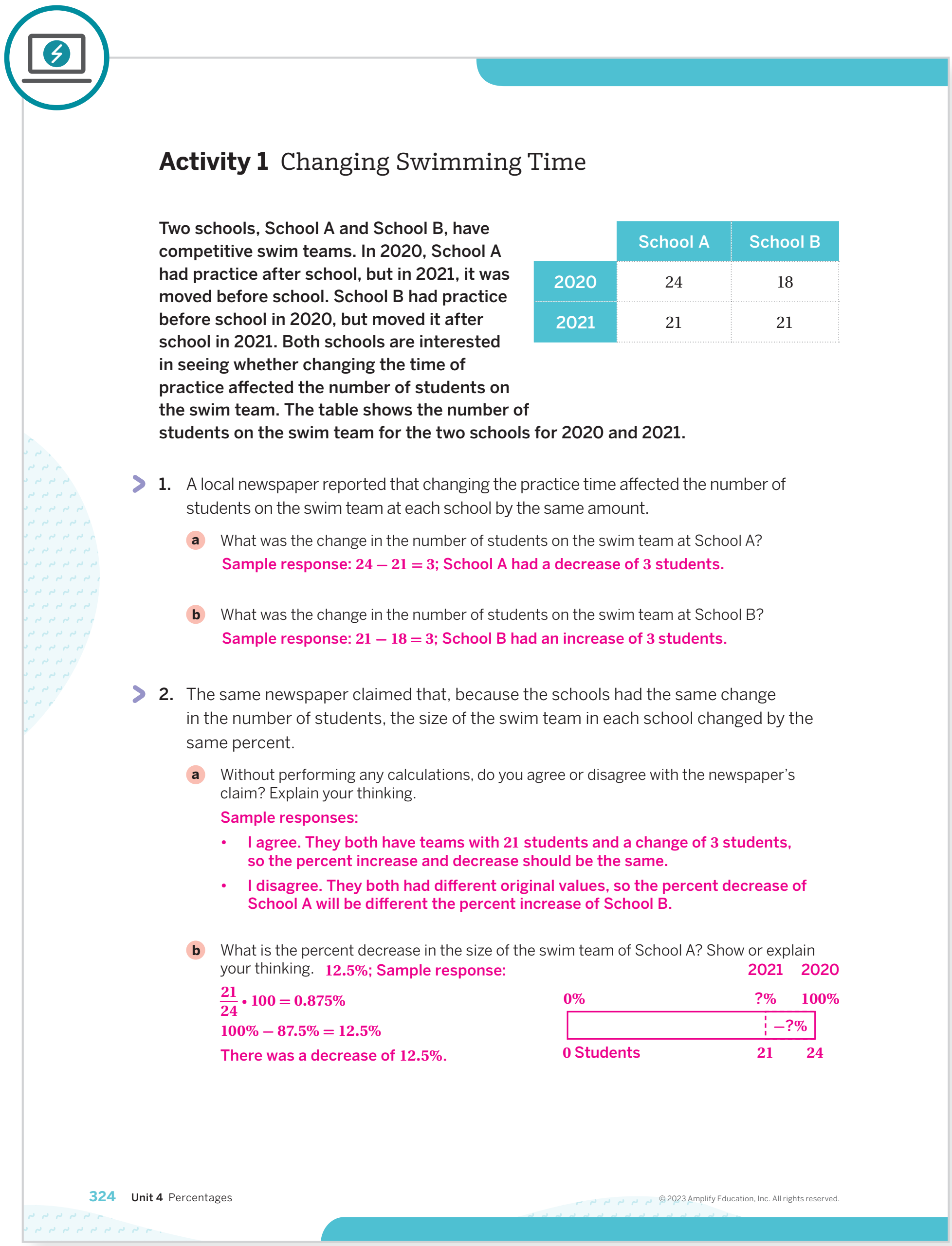
Use: Before the Warm-up.

Informed by: Performance on Lesson 4, Practice Problem 6.

Small groups | 15 min

Activity 1 Changing Swimming Time

Students compare the change in participation on swim teams at two schools to determine the percent increase or percent decrease of each team.



Activity 1 Changing Swimming Time

Two schools, School A and School B, have competitive swim teams. In 2020, School A had practice after school, but in 2021, it was moved before school. School B had practice before school in 2020, but moved it after school in 2021. Both schools are interested in seeing whether changing the time of practice affected the number of students on the swim team. The table shows the number of students on the swim team for the two schools for 2020 and 2021.

	School A	School B
2020	24	18
2021	21	21

1. A local newspaper reported that changing the practice time affected the number of students on the swim team at each school by the same amount.
   a. What was the change in the number of students on the swim team at School A?
   Sample response: $24 - 21 = 3$; School A had a decrease of 3 students.
   b. What was the change in the number of students on the swim team at School B?
   Sample response: $21 - 18 = 3$; School B had an increase of 3 students.

2. The same newspaper claimed that, because the schools had the same change in the number of students, the size of the swim team in each school changed by the same percent.
   a. Without performing any calculations, do you agree or disagree with the newspaper's claim? Explain your thinking.
   Sample responses:
      - I agree. They both have teams with 21 students and a change of 3 students, so the percent increase and decrease should be the same.
      - I disagree. They both had different original values, so the percent decrease of School A will be different the percent increase of School B.
   b. What is the percent decrease in the size of the swim team of School A? Show or explain your thinking. 12.5%; Sample response:
   $\frac{21}{24} \cdot 100 = 0.875\%$
   $100\% - 87.5\% = 12.5\%$
   There was a decrease of 12.5%.

324 Unit 4 Percentages

1 Launch

Give students a few minutes to answer Problem 1 and Problem 2a with their groups. Conduct the ***Poll the Class*** routine to assess student thinking prior to students continuing the activity. Distribute calculators to aid in computations.

2 Monitor

Help students get started by asking, "Which year matches 100% in this scenario?"

Look for points of confusion:

- **Calculating the percent 'of' instead of the percent change.** Ask students to create a tape diagram to model each change in swim team participation. Ask students to label the percent of and the percent increase or decrease.

Look for productive strategies:

- Determining the percent of students on the swim team in 2020 and 2021, and then determining the absolute difference from 100%.
- Determining the change in the participation (3 students) as a percent of the swim team in 2020.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* that students can use to partition pre-made blank tape diagrams.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their methods for calculating the percent change for each team, highlight similarities and differences between the two schools. Ask:

- "How do you know that the two schools had different quantities that represented the *whole?*" The number of students in 2020 was different for each school.
- "Why was the percent increase for School B not 12.5%? The whole was a different amount than for School A.

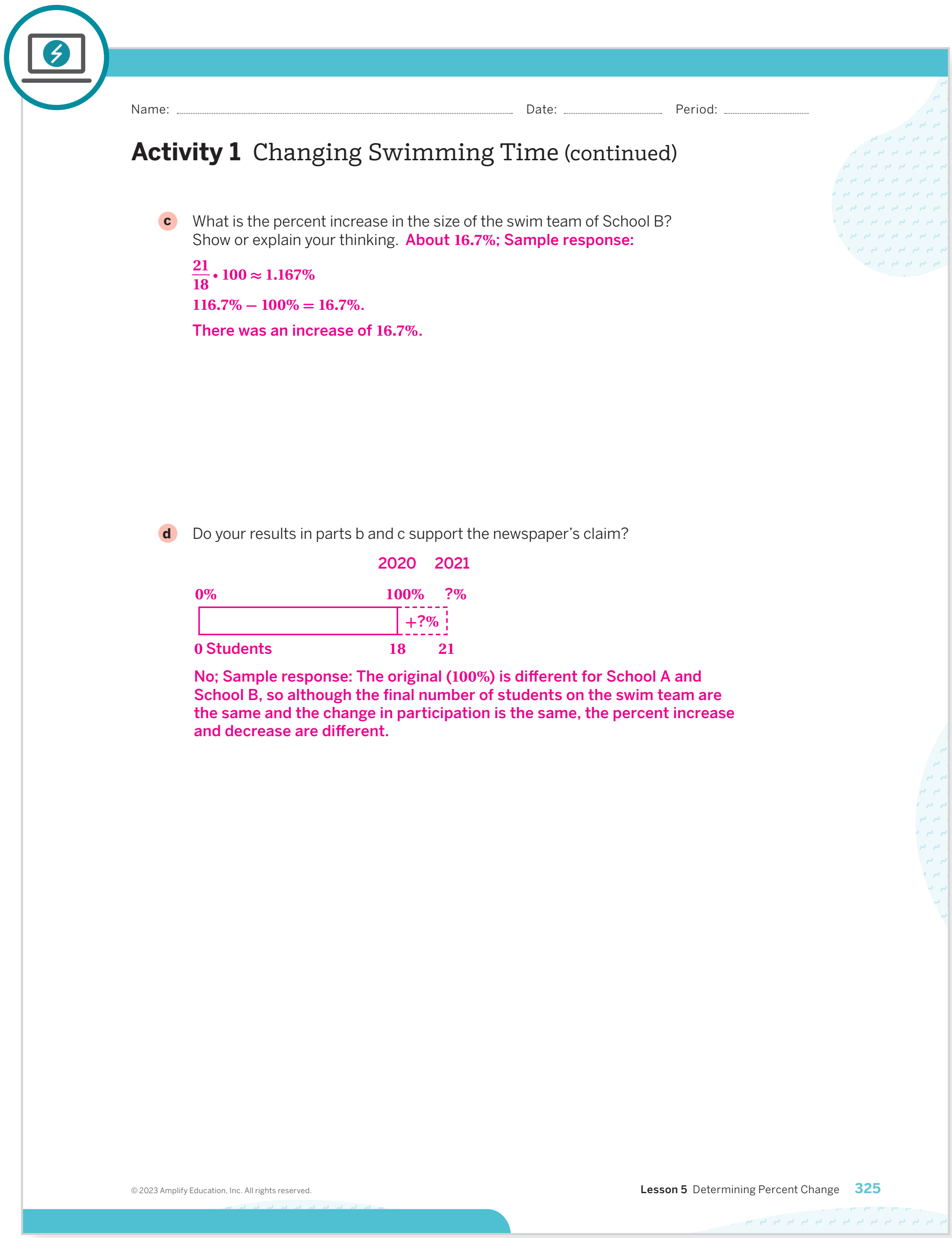
English Learners

Annotate the table by writing whole and 100% next to the row for 2020 and then writing increase or decrease next to the corresponding rows for 2021.

Small groups | 15 min

Activity 1 Changing Swimming Time (continued)

Students compare the change in participation on swim teams at two schools to determine the percent increase or percent decrease of each team.



Name: ______ Date: ______ Period: ______

Activity 1 Changing Swimming Time (continued)

c What is the percent increase in the size of the swim team of School B? Show or explain your thinking. About 16.7%; Sample response:

$\frac{21}{18} \cdot 100 \approx 1.167\%$

$116.7\% - 100\% = 16.7\%$.

There was an increase of 16.7%.

d Do your results in parts b and c support the newspaper's claim?

No; Sample response: The original (100%) is different for School A and School B, so although the final number of students on the swim team are the same and the change in participation is the same, the percent increase and decrease are different.

3 Connect

Have students share their methods for calculating the percent change for each team. If possible, choose groups that applied different strategies to share, and reflect those strategies visually for all students to compare side by side.

Define *percent change* as how much a quantity changed (increased or decreased), expressed as a percentage of the original amount.

Ask, "Which school had a greater percent change?" School B.

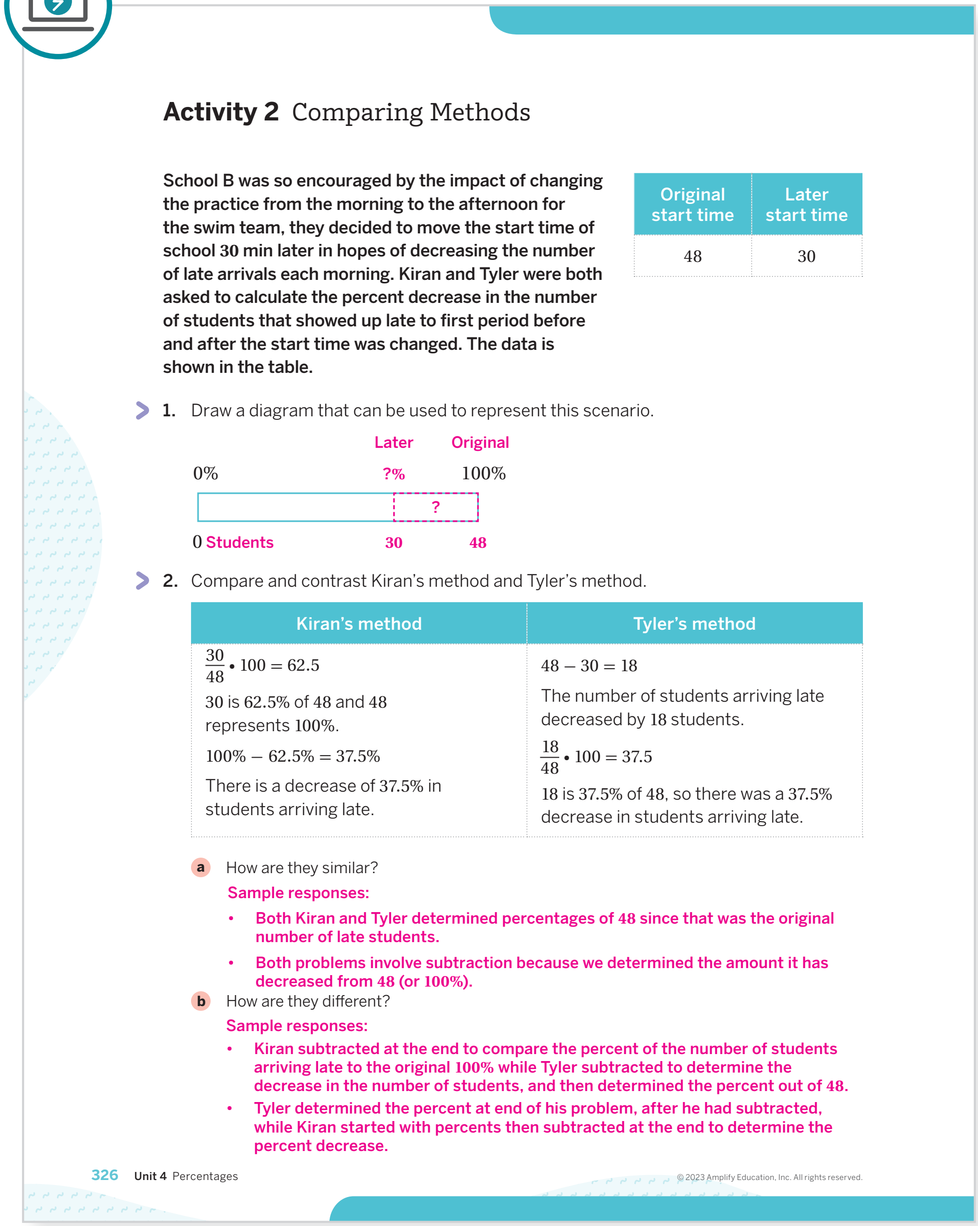
Highlight that when determining percent change, there will be two steps, much like when determining the new or original amount in previous problems. The first step is to determine the amount of change (whether an increase or decrease). The second step is to determine the percent change.

Optional

Pairs | 10 min

Activity 2 Comparing Methods

Students analyze two methods for determining the percent decrease to better understand differences in problem solving.



Activity 2 Comparing Methods

School B was so encouraged by the impact of changing the practice from the morning to the afternoon for the swim team, they decided to move the start time of school 30 min later in hopes of decreasing the number of late arrivals each morning. Kiran and Tyler were both asked to calculate the percent decrease in the number of students that showed up late to first period before and after the start time was changed. The data is shown in the table.

Original start time	Later start time
48	30

1. Draw a diagram that can be used to represent this scenario.

2. Compare and contrast Kiran's method and Tyler's method.

Kiran's method	Tyler's method
$\frac{30}{48} \cdot 100 = 62.5$ 30 is 62.5% of 48 and 48 represents 100%. $100\% - 62.5\% = 37.5\%$ There is a decrease of 37.5% in students arriving late.	$48 - 30 = 18$ The number of students arriving late decreased by 18 students. $\frac{18}{48} \cdot 100 = 37.5$ 18 is 37.5% of 48, so there was a 37.5% decrease in students arriving late.

a. How are they similar?

Sample responses:

- **Both Kiran and Tyler determined percentages of 48 since that was the original number of late students.**
- **Both problems involve subtraction because we determined the amount it has decreased from 48 (or 100%).**

b. How are they different?

Sample responses:

- **Kiran subtracted at the end to compare the percent of the number of students arriving late to the original 100% while Tyler subtracted to determine the decrease in the number of students, and then determined the percent out of 48.**
- **Tyler determined the percent at end of his problem, after he had subtracted, while Kiran started with percents then subtracted at the end to determine the percent decrease.**

326 Unit 4 Percentages

1 Launch

Activate students' prior knowledge by asking, "Can you think of a time when you solved a problem in a different way than your peer, but you were both correct?" Explain that students will be analyzing two methods for calculating the percent change.

2 Monitor

Help students get started by asking, "Which value would match 100% in the tape diagram?"

Look for points of confusion:

- **Thinking that the only thing Kiran and Tyler's methods have in common is the final answer.** Encourage students to look for similarities in the operations completed by each student.

Look for productive strategies:

- Annotating the diagram to match the work shown by each student.

3 Connect

Display Kiran's and Tyler's methods from the Student Edition.

Have pairs of students share what they noticed the two methods had in common and what was different.

Highlight that in both methods, students compared a value to the original amount of 48 students and the students found a difference. Model how the tape diagram can be used to aid students in either method.

Ask, "Which method makes the most sense to you?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Before students begin, ask them to explain why this scenario involves a percent decrease instead of a percent increase. Ask them to state which quantity represents the whole. The whole is the number of late arrivals recorded at the orginal start time. Because the number of late arrivals at the later start time is less than the number of late arrivals at the original start time, there is a decrease.

Extension: *Math Enrichment*

Ask students if they can think of another method to determine the percent decrease. Sample response: $\frac{48-30}{48}$ or $1 - \frac{30}{48}$ and then multiply by 100 and add the % symbol.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw connections between the methods by asking:

- "If Kiran had stopped after determining that 30 is 62.5% of 48, what would this percent represent?" The number of students arriving late after the start time was changed compared to the number of students arriving late before the start time was changed.
- "Is it reasonable to estimate that the number of students arriving late decreased by a little over $\frac{1}{3}$? Why or why not?" Yes, because 37.5% is 0.375, which is a little more than $\frac{1}{3}$.

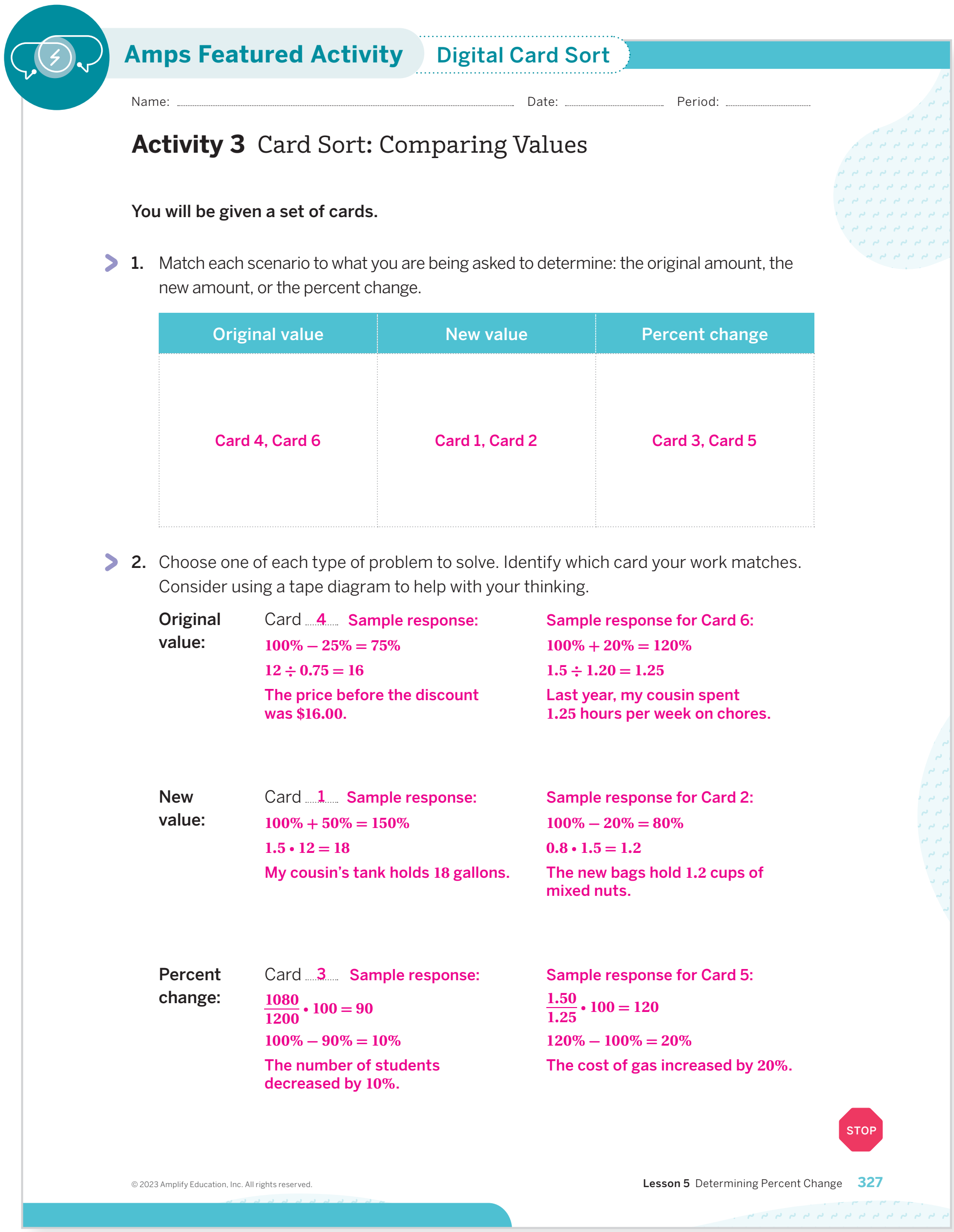
English Learners

Provide students time to rehearse and formulate what they will say before sharing with their partner.

Small groups | 13 min

Activity 3 Card Sort: Comparing Values

Students analyze scenarios to determine whether they are being asked to find the original amount, the new amount, or the percent change.


Amps Featured Activity Digital Card Sort

Name: Date: Period:

Activity 3 Card Sort: Comparing Values

You will be given a set of cards.

1. Match each scenario to what you are being asked to determine: the original amount, the new amount, or the percent change.

Original value	New value	Percent change
Card 4, Card 6	Card 1, Card 2	Card 3, Card 5

2. Choose one of each type of problem to solve. Identify which card your work matches. Consider using a tape diagram to help with your thinking.

Original value:

Card 4 Sample response:

$100\% - 25\% = 75\%$

$12 \div 0.75 = 16$

The price before the discount was $16.00.

Sample response for Card 6:

$100\% + 20\% = 120\%$

$1.5 \div 1.20 = 1.25$

Last year, my cousin spent 1.25 hours per week on chores.

New value:

Card 1 Sample response:

$100\% + 50\% = 150\%$

$1.5 \cdot 12 = 18$

My cousin's tank holds 18 gallons.

Sample response for Card 2:

$100\% - 20\% = 80\%$

$0.8 \cdot 1.5 = 1.2$

The new bags hold 1.2 cups of mixed nuts.

Percent change:

Card 3 Sample response:

$\frac{1080}{1200} \cdot 100 = 90$

$100\% - 90\% = 10\%$

The number of students decreased by 10%.

Sample response for Card 5:

$\frac{1.50}{1.25} \cdot 100 = 120$

$120\% - 100\% = 20\%$

The cost of gas increased by 20%.

STOP

Lesson 5 Determining Percent Change 327

1 Launch

Distribute one set of pre-cut cards from the Activity 3 PDF to each small group and conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by asking, "What key words did we discuss during the previous class that can help you to sort these cards?"

Look for points of confusion:

- **Difficulty in determining what they are being asked to find in each scenario.** Suggest students sketch diagrams to help them make sense of each scenario.
- **For Cards 3 and 5, determining the *percent of* instead of the percent change.** Ask, "What did you find? Did you answer the question being asked? What do you still need to do to answer this problem's question?"

Look for productive strategies:

- Determining the value (or unknown) that matches 100% in each scenario as a first step.

3 Connect

Display the table from the Student Edition. Conduct the ***Poll the Class*** routine to complete the chart.

Have students share their reasoning for the cards that did not have consensus. If appropriate, repeat the ***Poll the Class*** routine, after a discussion to achieve consensus.

Ask:

- "What was similar about the process of determining the original amount, the new amount, and the percent change?"
- "What was different about the process of determining each amount?"

Highlight that in all scenarios, students use proportional reasoning and either subtraction or addition, but the type and order of the operations changes depending on what value you are trying to determine.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* to help students make sense of, and model, each scenario.

Math Language Development

MLR7: Compare and Connect

During the Launch, suggest that students highlight or underline the question in each problem and then look for key words or phrases that indicate whether the quantity being asked for is the original value, new value, or percent change. Consider displaying Card 1 and using a think-aloud approach to demonstrate how to look for key words and phrases.

- "I know the gas tank in the first car holds 12 gallons."
- "The truck holds 50% more."
- "I'm asked for the number of gallons in the truck, so this is the new value."

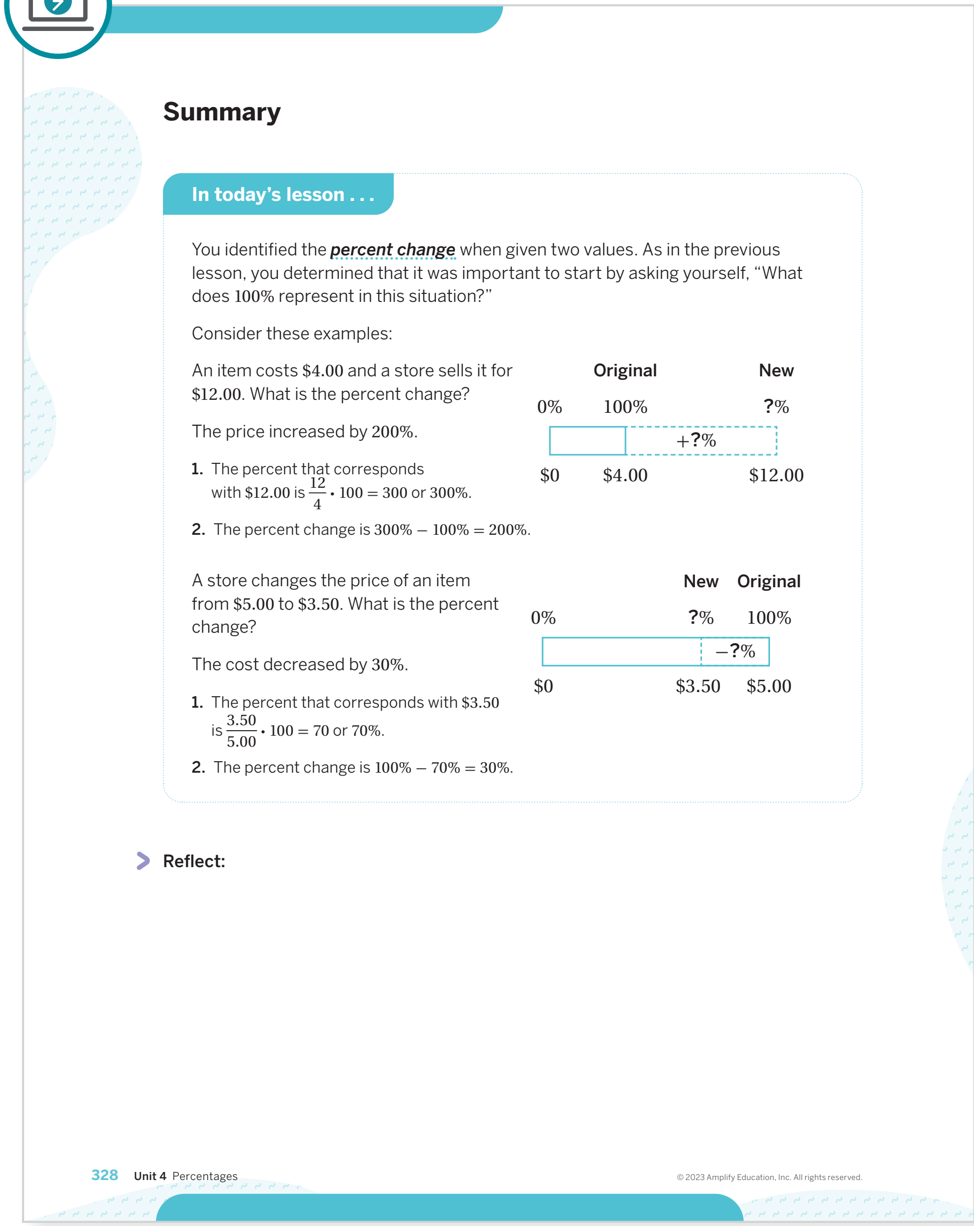
English Learners

Highlight the phrase 50% *more* and annotate this with the term *percent increase.*

Whole Class | 5 min

Summary

Review and synthesize methods for determining the percent change when given the original amount and the new amount.



Summary

In today's lesson . . .

You identified the ***percent change*** when given two values. As in the previous lesson, you determined that it was important to start by asking yourself, "What does 100% represent in this situation?"

Consider these examples:

An item costs $4.00 and a store sells it for $12.00. What is the percent change?

The price increased by 200%.

1. The percent that corresponds with $12.00 is $\frac{12}{4} \cdot 100 = 300$ or 300%.
2. The percent change is $300\% - 100\% = 200\%$.

A store changes the price of an item from $5.00 to $3.50. What is the percent change?

The cost decreased by 30%.

1. The percent that corresponds with $3.50 is $\frac{3.50}{5.00} \cdot 100 = 70$ or 70%.
2. The percent change is $100\% - 70\% = 30\%$.

Reflect:

328 Unit 4 Percentages

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Synthesize

Display the Anchor Chart PDF, *Percent Increase* and the Anchor Chart PDF, *Percent Decrease*. Use sticky notes to cover up the percent change and the percentage for the new value in each scenario. Complete the "Percent" section of each Anchor Chart.

Formalize vocabulary, *percent change*

Have students share scenarios in which they have encountered in the real world that reflect percent change.

Highlight that, when determining the percent change, it is important to identify which given value is the original value and which is the new value. If students confuse these, then the percent change will be different because the "whole" is a different value.

Ask, "What strategies do you know that you can use to determine the percent change?"

Sample responses:

- Drawing a tape diagram to model the situation.
- Determining the percent that the new value is part of the original amount, and then determining the change from 100%.
- Determining the difference between the new value and the original value, and then computing what percent that is of the original value.

Reflect

After to synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are percentages used to represent change?"

Math Language Development

MLR2: Collect and Display

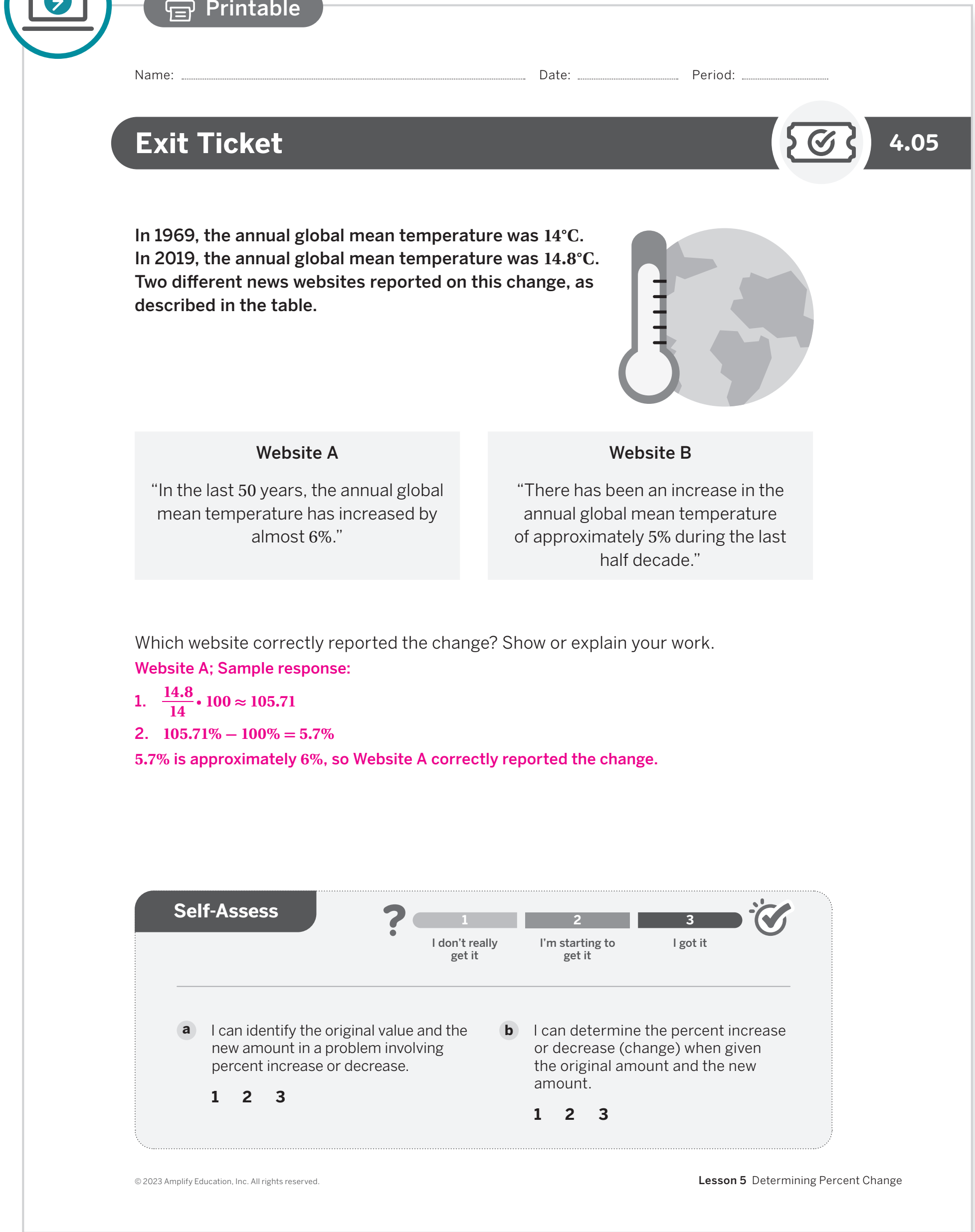
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the term *percent change* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of percent change by determining which website correctly reported a percent increase in the global mean temperature.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 4.05

In 1969, the annual global mean temperature was 14°C.
In 2019, the annual global mean temperature was 14.8°C.
Two different news websites reported on this change, as described in the table.

Website A	Website B
"In the last 50 years, the annual global mean temperature has increased by almost 6%."	"There has been an increase in the annual global mean temperature of approximately 5% during the last half decade."

Which website correctly reported the change? Show or explain your work.

Website A; Sample response:

1. $\frac{14.8}{14} \cdot 100 \approx 105.71$
2. $105.71\% - 100\% = 5.7\%$

5.7% is approximately 6%, so Website A correctly reported the change.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can identify the original value and the new amount in a problem involving percent increase or decrease.

1 2 3

b I can determine the percent increase or decrease (change) when given the original amount and the new amount.

1 2 3

Lesson 5 Determining Percent Change

Success looks like . . .

- **Goal:** Identifying the original value and the new amount in a problem involving percent increase or percent decrease.
- **Goal:** Drawing and labeling a tape diagram to represent a situation that involves determining the percent change when given the original amount and the new amount.
- **Language Goal:** Describing how to determine the percent increase or decrease — percent change — when given the original amount and the new amount. **(Speaking and Listening)**.
  - » Determining the website that reported the correct percent increase.

Suggested next steps

If students show the correct reasoning, but incorrectly round their percentage, consider:

- Reviewing rounding rules and providing additional practice.

If students identify that Website B is correct, consider:

- Asking, "Which temperature is the original global mean temperature?"
- Assigning Practice Problems 1 and 2.

If students say that both websites were incorrect and identified that the percent increase was approximately 106%, consider:

- Asking the students to draw a tape diagram to model the context.
- Asking, "Based on your diagram, is it reasonable to say the increase is 106%?"
- Assigning Practice Problem 3.

Professional Learning

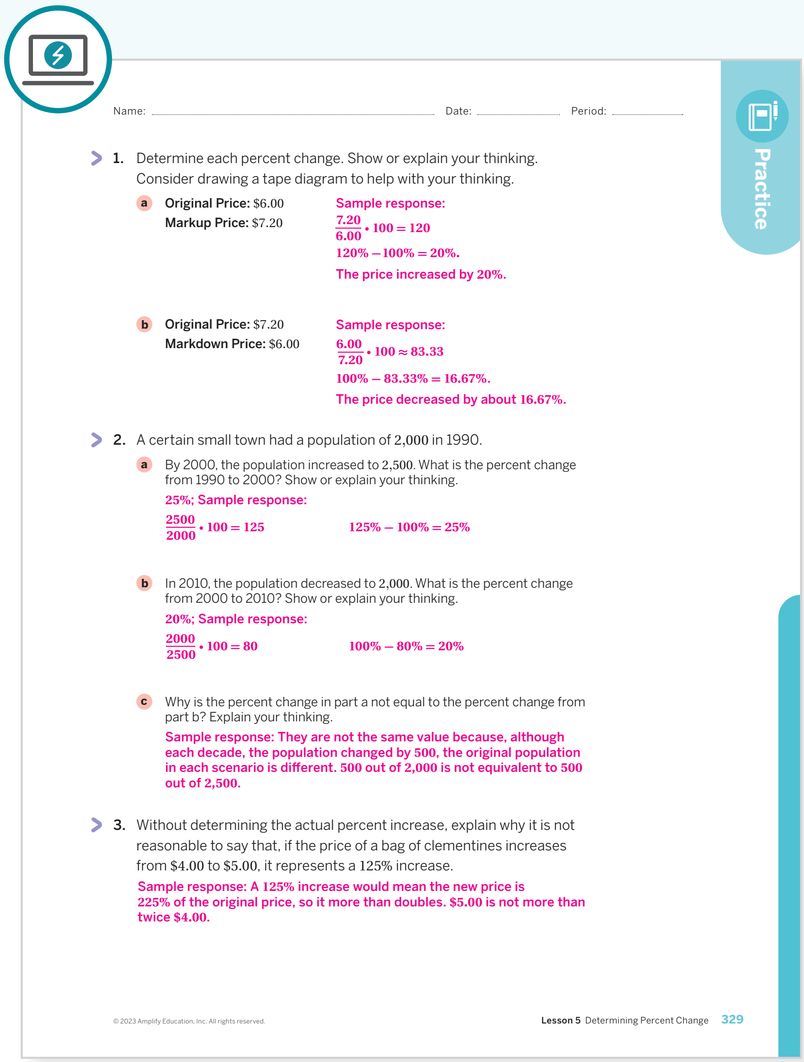
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- The focus of this lesson was for students to be able to determine the percent change between two values. How did this focus go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Determine each percent change. Show or explain your thinking. Consider drawing a tape diagram to help with your thinking.
   - a **Original Price:** \$6.00
     Markup Price: \$7.20
     Sample response:
     $\frac{7.20}{6.00} \cdot 100 = 120$
     $120\% - 100\% = 20\%$.
     The price increased by 20%.
   - b **Original Price:** \$7.20
     Markdown Price: \$6.00
     Sample response:
     $\frac{6.00}{7.20} \cdot 100 \approx 83.33$
     $100\% - 83.33\% = 16.67\%$.
     The price decreased by about 16.67%.
2. A certain small town had a population of 2,000 in 1990.
   - a By 2000, the population increased to 2,500. What is the percent change from 1990 to 2000? Show or explain your thinking.
     25%; Sample response:
     $\frac{2500}{2000} \cdot 100 = 125$ $125\% - 100\% = 25\%$
   - b In 2010, the population decreased to 2,000. What is the percent change from 2000 to 2010? Show or explain your thinking.
     20%; Sample response:
     $\frac{2000}{2500} \cdot 100 = 80$ $100\% - 80\% = 20\%$
   - c Why is the percent change in part a not equal to the percent change from part b? Explain your thinking.
     Sample response: They are not the same value because, although each decade, the population changed by 500, the original population in each scenario is different. 500 out of 2,000 is not equivalent to 500 out of 2,500.
3. Without determining the actual percent increase, explain why it is not reasonable to say that, if the price of a bag of clementines increases from \$4.00 to \$5.00, it represents a 125% increase.
   Sample response: A 125% increase would mean the new price is 225% of the original price, so it more than doubles. \$5.00 is not more than twice \$4.00.

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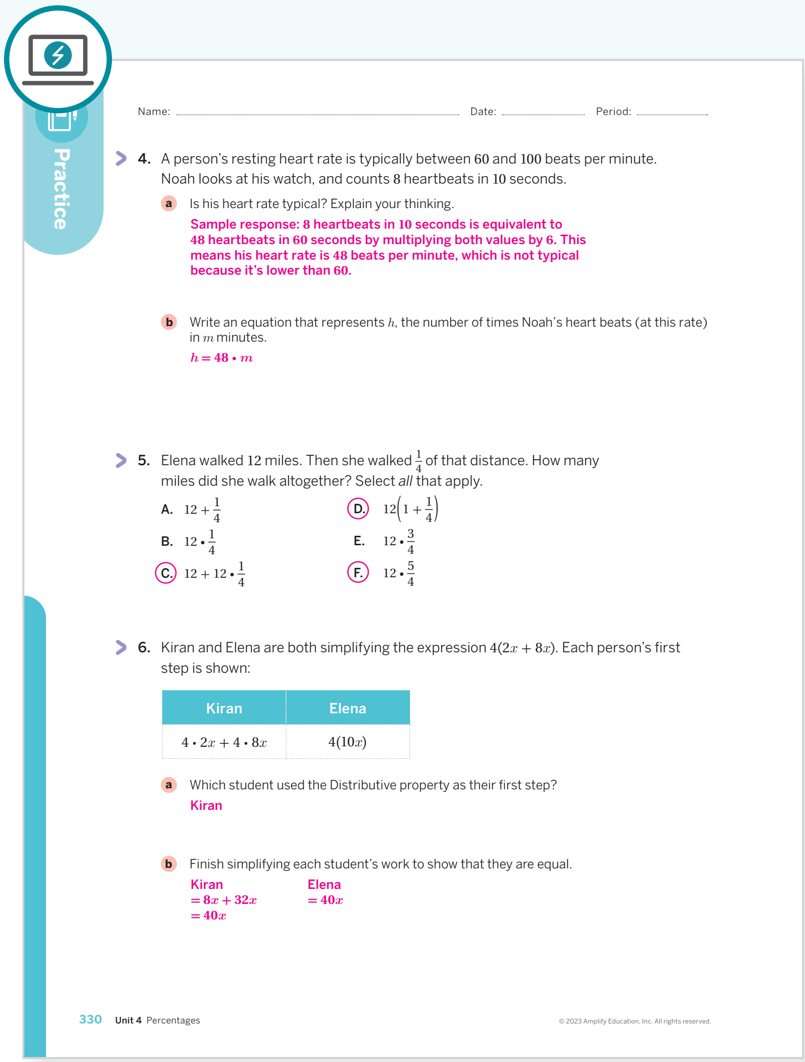
Lesson 5 Determining Percent Change 329



Name: ________ Date: ________ Period: ________

4. A person's resting heart rate is typically between 60 and 100 beats per minute. Noah looks at his watch, and counts 8 heartbeats in 10 seconds.
   - a Is his heart rate typical? Explain your thinking.
     Sample response: 8 heartbeats in 10 seconds is equivalent to 48 heartbeats in 60 seconds by multiplying both values by 6. This means his heart rate is 48 beats per minute, which is not typical because it's lower than 60.
   - b Write an equation that represents h, the number of times Noah's heart beats (at this rate) in m minutes.
     $h = 48 \cdot m$
5. Elena walked 12 miles. Then she walked $\frac{1}{4}$ of that distance. How many miles did she walk altogether? Select *all* that apply.
   - A. $12 + \frac{1}{4}$
   - B. $12 \cdot \frac{1}{4}$
   - [✓ C.] $12 + 12 \cdot \frac{1}{4}$
   - [✓ D.] $12\left(1 + \frac{1}{4}\right)$
   - E. $12 \cdot \frac{3}{4}$
   - [✓ F.] $12 \cdot \frac{5}{4}$
6. Kiran and Elena are both simplifying the expression $4(2x + 8x)$. Each person's first step is shown:

Kiran	Elena
$4 \cdot 2x + 4 \cdot 8x$	$4(10x)$

   - a Which student used the Distributive property as their first step?
     Kiran
   - b Finish simplifying each student's work to show that they are equal.
     Kiran
     $= 8x + 32x$
     $= 40x$
     Elena
     $= 40x$

330 **Unit 4** Percentages

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	1
	2	Activity 1	2
	3	Activity 1	3
Spiral	4	Unit 2 Lesson 10	2
	5	Grade 5	2
Formative	6	Unit 4 Lesson 6	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

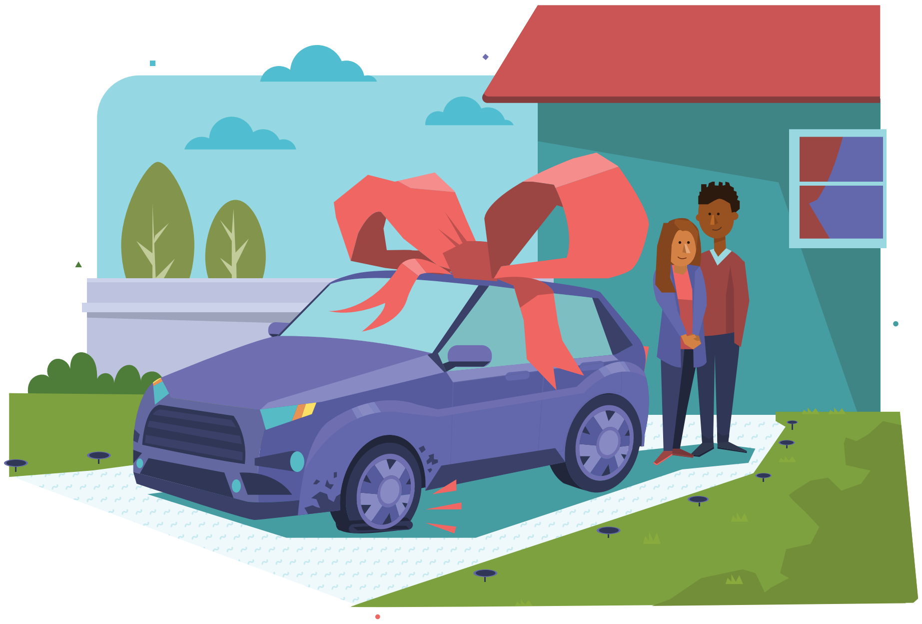
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 4 | **LESSON 6**

Percent Increase and Decrease With Equations

Let's use equations to represent scenarios involving percent increase and decrease.



Focus

Goals

1. **Language Goal:** Generate equations that represent a situation involving a given percent increase or decrease and justify the reasoning. **(Speaking and Listening)**
2. **Language Goal:** Explain how to use an equation to calculate the original amount or the new amount given one value and the percent increase or decrease. **(Speaking and Listening, Writing)**

Coherence

Today

Students make connections between proportional relationships and percent change scenarios in order to write equations that relate the original amount and the new amount when given a percentage or a percent change. They compare and contrast examples of their peers' work to determine that there are multiple equivalent ways to express percent change scenarios in equations. Students then write their own equations and use them to determine the new or the original amount when given the percent change and one value.

Previously

In Unit 2, students wrote equations of the form $y = kx$ to represent proportional relationships. In Lessons 3–5, students solved problems involving percent increase and decrease.

Coming Soon

In Lesson 7, students will write equations to determine the percent change when given the original and new amount.

Rigor

- Students build **conceptual understanding** of writing equations to represent scenarios involving percent change.
- Students develop **fluency** in using equations to determine the original or new amount when given one value and the percent change.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display, as needed)
- Warm-up PDF (answers)
- Graphic Organizer PDF, *Percentage Tape Diagrams* (as needed)
- calculators

Math Language Development

Review words

- *markdown*
- *markup*
- *percent change*
- *percent decrease*
- *percent increase*

Amps | Featured Activity

Exit Ticket
Real-Time Exit Ticket

Check in real time whether your students can identify equations that represent percent change scenarios using a Digital Exit Ticket.

Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students begin to model situations using equations, they might be impulsive, quickly writing an equation without understanding the repetition shown in the problem. Explain that looking for the patterns is important to understanding how to represent these scenarios algebraically, but that there is more than one pattern associated with them. Students will need to exert some self-discipline by identifying the different situations and matching them to an appropriate equation pattern.

Modifications to Pacing

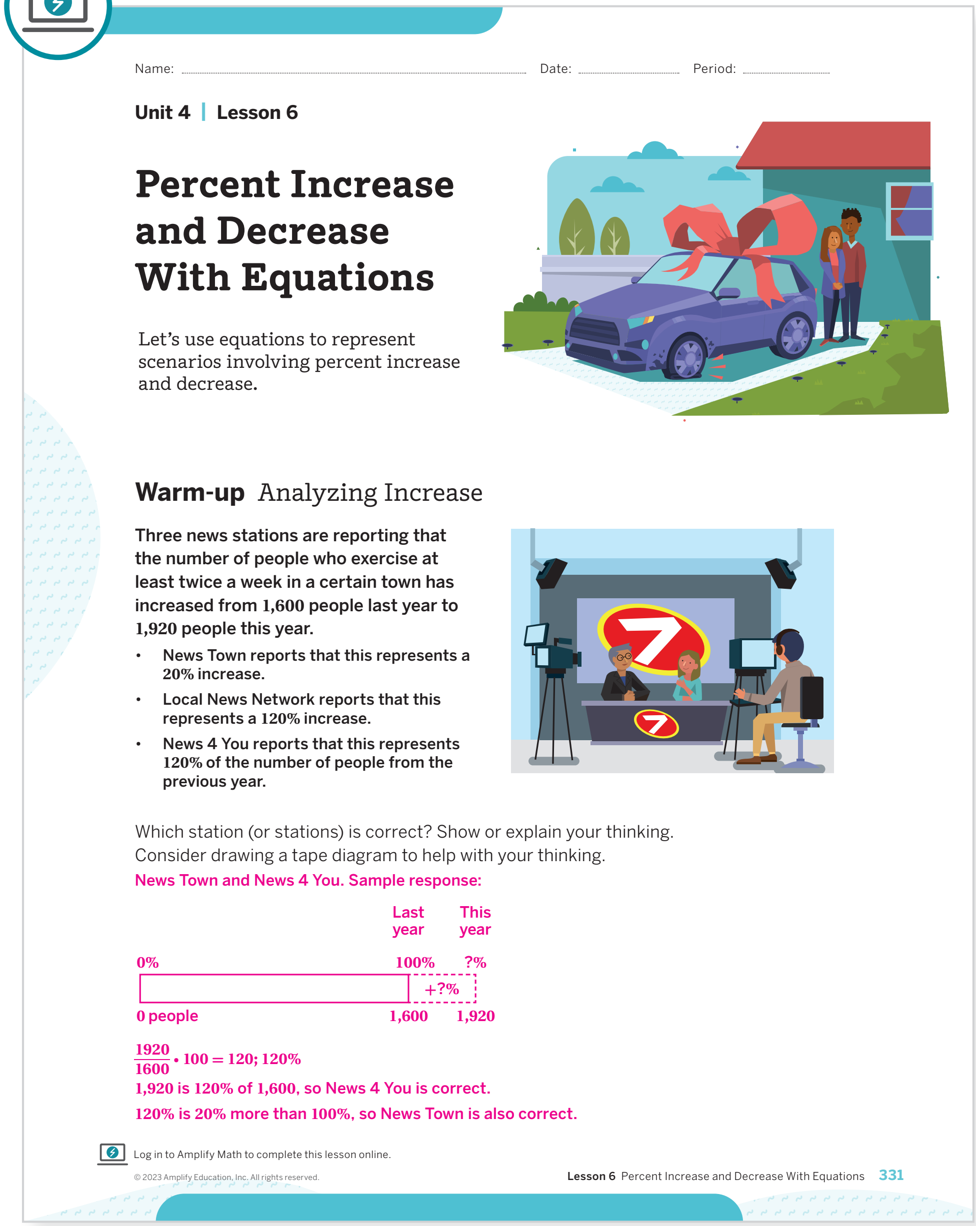
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Optional **Activity 2** may be omitted. You may choose to assign this activity as additional practice.
- In **Activity 3,** only have students write the equation(s) to represent each scenario.

Pairs | 5 min

Warm-up Analyzing Increase

Students analyze the claims made by three news stations to reason about percent change and percent of.



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Unit 4 | Lesson 6

Percent Increase and Decrease With Equations

Let's use equations to represent scenarios involving percent increase and decrease.

Warm-up Analyzing Increase

Three news stations are reporting that the number of people who exercise at least twice a week in a certain town has increased from 1,600 people last year to 1,920 people this year.

- **News Town reports that this represents a 20% increase.**
- **Local News Network reports that this represents a 120% increase.**
- **News 4 You reports that this represents 120% of the number of people from the previous year.**

Which station (or stations) is correct? Show or explain your thinking. Consider drawing a tape diagram to help with your thinking.

News Town and News 4 You. Sample response:

$\frac{1920}{1600} \cdot 100 = 120; 120\%$

1,920 is 120% of 1,600, so News 4 You is correct.

120% is 20% more than 100%, so News Town is also correct.

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the *Think-Pair-Share* routine. Give students two minutes of think-time prior to discussing their ideas with their partner. Distribute calculators for use throughout the lesson.

2 Monitor

Help students get started by suggesting students model the scenario on their own first, and then analyzing the claim made by each news station.

Look for points of confusion:

- **Thinking that Local News Network is correct.** Ask, "If there is a 120% increase, what percent of people exercise now compared to last year?"

Look for productive strategies:

- Drawing a tape diagram to model the increase in people exercising.
- Writing and solving an equation to determine the percent of or the percent increase.

3 Connect

Display the Warm-up PDF. Conduct the *Poll the Class* routine to assess student opinion on which news station(s) correctly reported the change in the number of people exercising at least twice a week.

Have pairs of students share how they determined which news stations correctly reported the change in population. Annotate the tape diagram to reflect and reinforce student understanding.

Highlight that a 20% increase is the same as a 120% of the original amount of people exercising.

Ask:

- "How could you represent the percent change as its decimal equivalent? What about the total *percent of*?"
- "If the problem hadn't given the options of 20% and 120%, what strategies could you use to determine the percent change?"

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that the number of people who exercised last year increased this year and that three different news stations reported the percentages differently.
- **Read 2:** Ask students to name or highlight the given quantities, such as "News Town says this is a 20% increase."
- **Read 3:** Ask students to brainstorm strategies they can use to determine whether each station reported this information correctly.

English Learners

Have students annotate key phrases, such as 120% *of the number of people* for News 4 You.

Power-up

To power up students' ability to identify equivalent expressions by applying the Distributive Property, have students complete:

Which of the following expressions are equivalent to $3(4 + 6)$? Select *all* that apply.

A. $12 + 6$
[✓ B.] $12 + 18$
[✓ C.] $3 \cdot 4 + 3 \cdot 6$
[✓ D.] $3 \cdot 10$
E. $3 + 10$

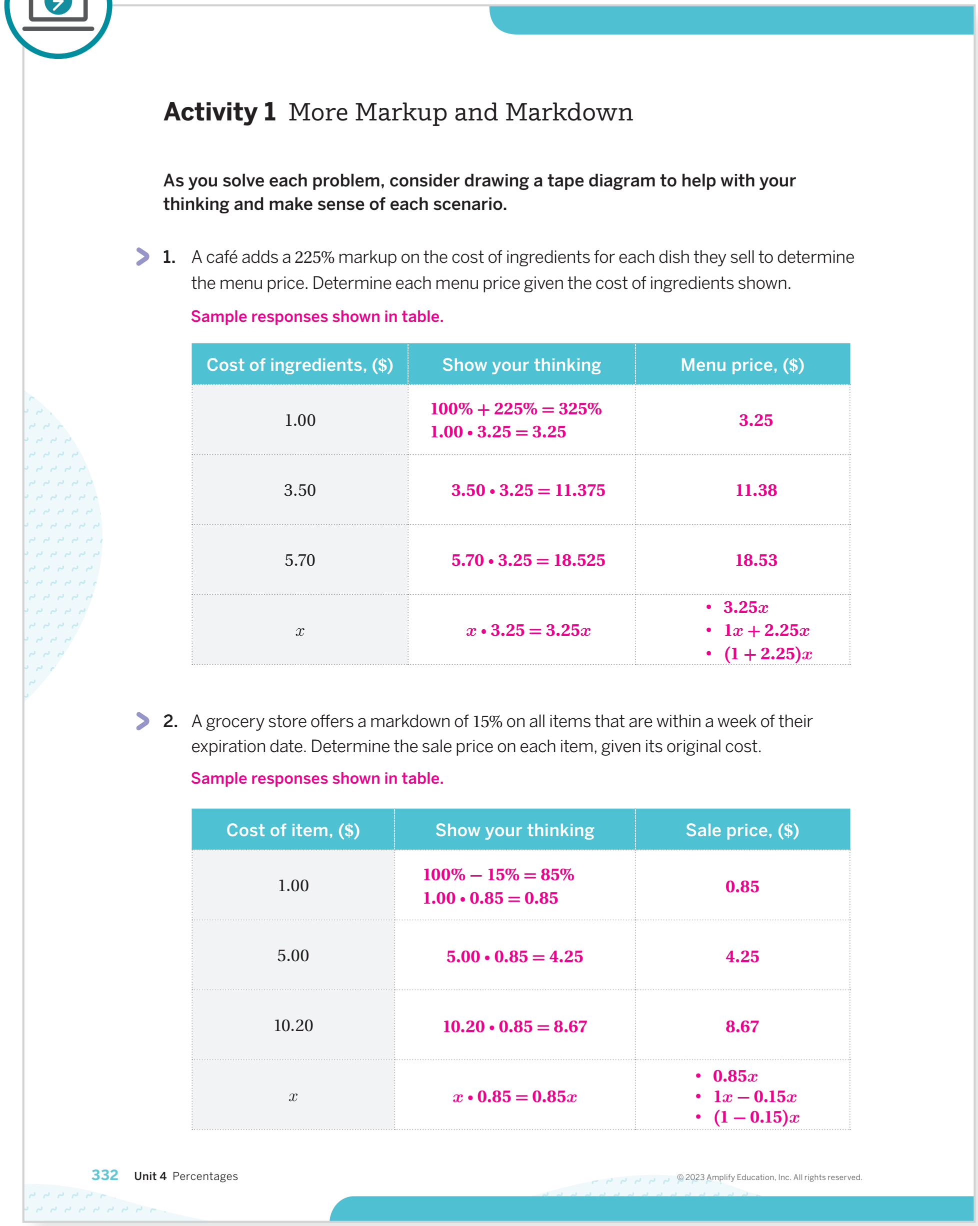
Use: Before Activity 1.

Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3.

Pairs | 15 min

Activity 1 More Markup and Markdown

Students use repeated reasoning to write expressions that represent new amounts after a percent increase or decrease.



Activity 1 More Markup and Markdown

As you solve each problem, consider drawing a tape diagram to help with your thinking and make sense of each scenario.

1. A café adds a 225% markup on the cost of ingredients for each dish they sell to determine the menu price. Determine each menu price given the cost of ingredients shown.

Sample responses shown in table.

Cost of ingredients, ($)	Show your thinking	Menu price, ($)
1.00	$100\% + 225\% = 325\%$ $1.00 \cdot 3.25 = 3.25$	3.25
3.50	$3.50 \cdot 3.25 = 11.375$	11.38
5.70	$5.70 \cdot 3.25 = 18.525$	18.53
x	$x \cdot 3.25 = 3.25x$	• $3.25x$ • $1x + 2.25x$ • $(1 + 2.25)x$

2. A grocery store offers a markdown of 15% on all items that are within a week of their expiration date. Determine the sale price on each item, given its original cost.

Sample responses shown in table.

Cost of item, ($)	Show your thinking	Sale price, ($)
1.00	$100\% - 15\% = 85\%$ $1.00 \cdot 0.85 = 0.85$	0.85
5.00	$5.00 \cdot 0.85 = 4.25$	4.25
10.20	$10.20 \cdot 0.85 = 8.67$	8.67
x	$x \cdot 0.85 = 0.85x$	• $0.85x$ • $1x - 0.15x$ • $(1 - 0.15)x$

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1 Launch

Activate students' prior knowledge by asking, "What do you know about markup and markdown?"

2 Monitor

Help students get started by asking, "Is the menu price the original amount or the new amount?"

Look for points of confusion:

- **Forgetting to determine the total percentage.** Have students draw a diagram to make sense of each problem.
- **Struggling to write an expression in terms of x.** Ask, "How could you write one expression that models your process for each value above?"

3 Connect

Display each scenario.

Have students share how they determined their expressions for each table. **Note:** If multiple expressions are discussed, skip Activity 2. If not, complete Activity 2, and part a of each problem in Activity 3.

Ask:

- "If m represents the menu price, what equation could you write to determine the menu price, given the cost of ingredients?"
- "Why is the value in the equation 3.25 and not 2.25 in Problem 1?"
- "If s represents the sale price, what equation could you write to determine the sale price, given the retail price?"
- "Why is the value in the equation 0.85 and not 0.15 in Problem 2?"
- "How are the equations for markup and markdown the same as or different from the equations you wrote to represent proportional relationships?"

Highlight that students can write equations relating the new price to the original price, given the percent increase or decrease.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* to help students model each scenario. Suggest that students underline the term *markup* in Problem 1 and *markdown* in Problem 2 to help them interpret each scenario.

Extension: *Math Enrichment*

Have students refer to Problem 1. Ask them for the greatest percent markup the café would need to add to the cost of ingredients if the menu price *cannot exceed* \$15.00 for ingredients that cost \$5.70.

About 163%; For a markup of 163%, the menu price would be \$14.99.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the different ways the percent increase or decrease is represented. Consider displaying something similar to the following, or add it to the class display:

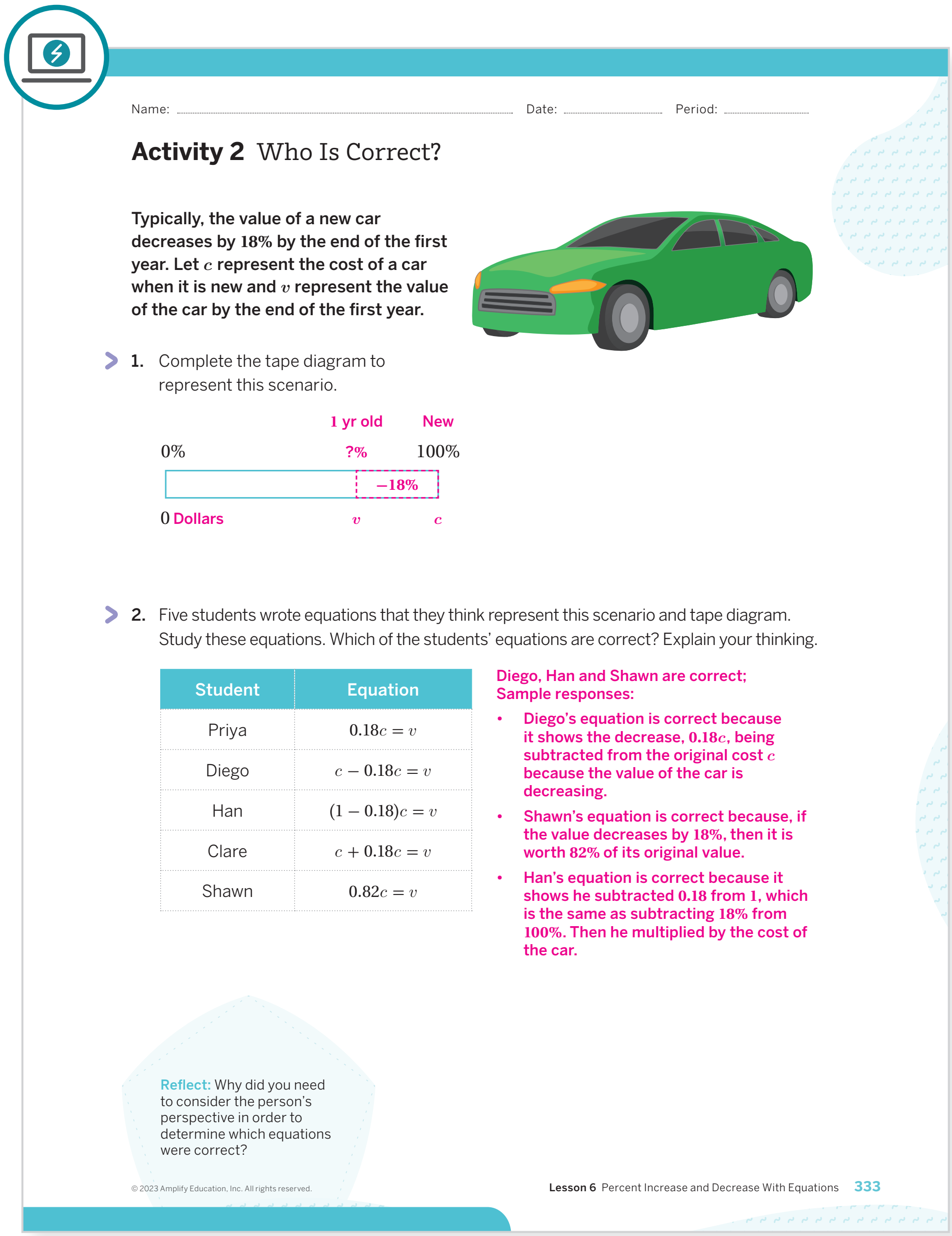
Suppose y is 20% more than x	
Percent increase	**Percent decrease**
$y = x + 0.20x$	$y = x - 0.20x$
$y = (1 + 0.20)x$	$y = (1 - 0.20)x$
$y = 1.20x$	$y = 0.80x$

Optional

Pairs | 10 min

Activity 2 Who Is Correct?

Students analyze equations written that represent percent to determine which equations accurately represent the scenario.



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Activity 2 Who Is Correct?

Typically, the value of a new car decreases by 18% by the end of the first year. Let c represent the cost of a car when it is new and v represent the value of the car by the end of the first year.

1. Complete the tape diagram to represent this scenario.

2. Five students wrote equations that they think represent this scenario and tape diagram. Study these equations. Which of the students' equations are correct? Explain your thinking.

Student	Equation
Priya	$0.18c = v$
Diego	$c - 0.18c = v$
Han	$(1 - 0.18)c = v$
Clare	$c + 0.18c = v$
Shawn	$0.82c = v$

Diego, Han and Shawn are correct; Sample responses:

- Diego's equation is correct because it shows the decrease, $0.18c$, being subtracted from the original cost c because the value of the car is decreasing.
- Shawn's equation is correct because, if the value decreases by 18%, then it is worth 82% of its original value.
- Han's equation is correct because it shows he subtracted 0.18 from 1, which is the same as subtracting 18% from 100%. Then he multiplied by the cost of the car.

Reflect: Why did you need to consider the person's perspective in order to determine which equations were correct?

Lesson 6 Percent Increase and Decrease With Equations 333

1 Launch

Conduct the ***Think-Pair-Share*** routine. Give students two minutes of think time prior to discussing their thinking with their partner.

2 Monitor

Help students get started by asking, "What if the original value of the car was $20,000? How would you determine the price at the end of the first year?"

Look for points of confusion:

- **Thinking that only one equation is correct.** Encourage students to think about what they remember about the Distributive Property and equivalent expressions from Grade 6 to identify equivalent equations.

Look for productive strategies:

- Referencing their tape diagram from Problem 1 and determining the unknown percentage that represents the value of the car at the end of the first year.

3 Connect

Display the table from Activity 2 in the Student Edition. Conduct the ***Poll the Class*** routine to assess student opinion on which equations accurately represent the worth of the car at the end of the first year.

Have pairs of students share how they determined which equations were correct.

Highlight that all three "correct" equations are equivalent, although they are written slightly differently.

Ask:

- "How is it possible for all three equations to be correct?" Sample response: They are all equivalent when simplified.
- "What does Priya's equation represent?" Sample response: The decrease in the worth of the car at the end of the first year.
- "What does Clare's equation represent?" Sample response: The value of the car if it had increased by 18%.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they know that the value of a new car decreases by a significant amount during the first year. To help them visualize what this means, tell them that the value of a new car is $20,000 and have them determine the value of this car after the first year. $16,400

Extension: *Math Enrichment*

Tell students that a brand new car actually loses between 9% and 11% of its value the moment you drive the car off the sales lot! Ask them to determine the value of a new car whose original value is $30,000 the moment it is driven off the lot. Between $26,700 and $27,300.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the following incorrect reasoning that Priya may have used, "The equation $0.18c = v$ represents this scenario because the car's value decreased by 18%." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Why or why not?"
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "How would you explain to Priya why her statement was not correct? How could you convince her that your statement is correct?"

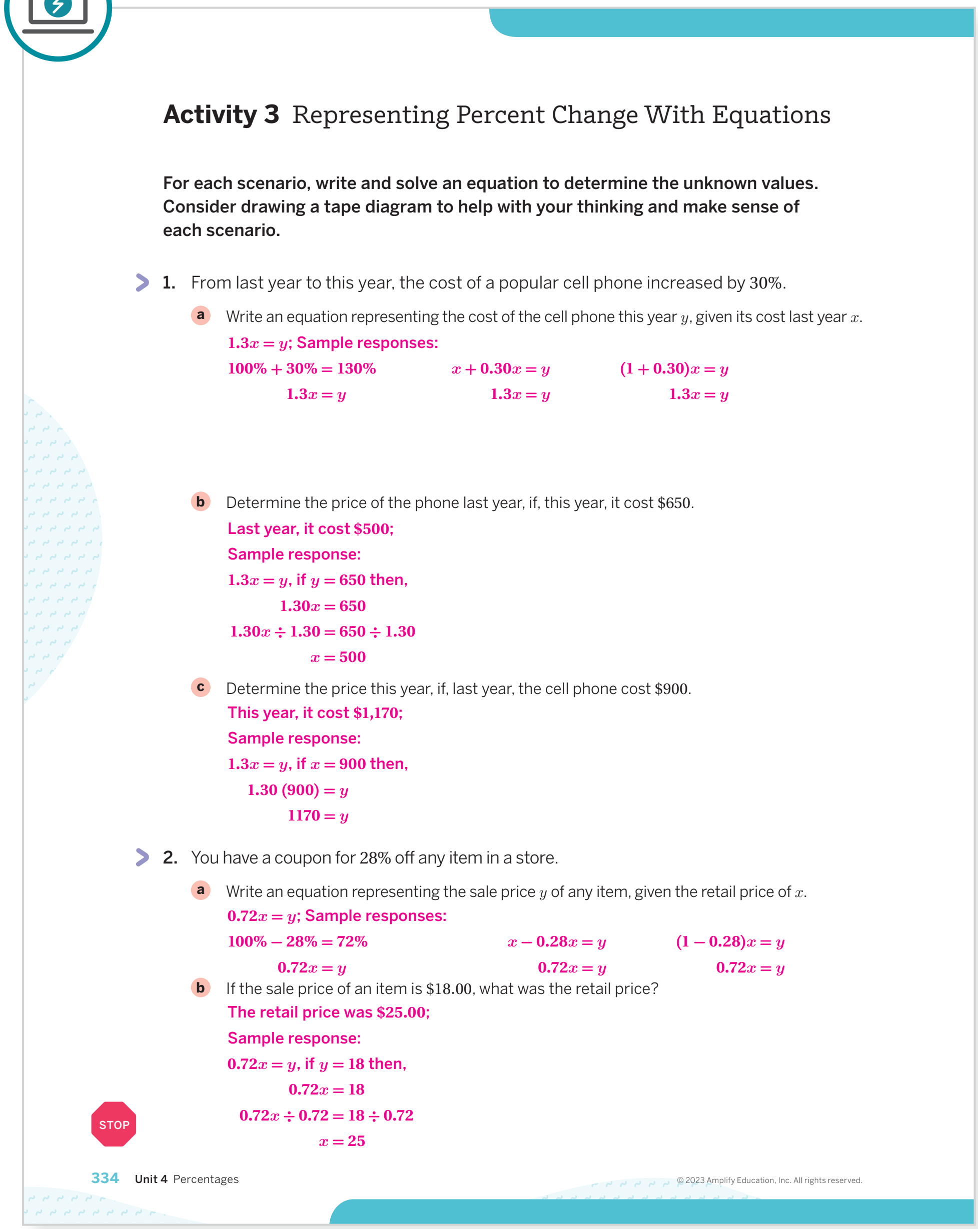
English Learners

Allow students time to rehearse what they will say with a partner before sharing with the whole class.

Small groups | 15 min

Activity 3 Representing Percent Change With Equations

Students represent scenarios involving percent change with equations to determine unknown values.



Activity 3 Representing Percent Change With Equations

For each scenario, write and solve an equation to determine the unknown values. Consider drawing a tape diagram to help with your thinking and make sense of each scenario.

1. From last year to this year, the cost of a popular cell phone increased by 30%.

   a. Write an equation representing the cost of the cell phone this year y, given its cost last year x.

   $1.3x = y$; Sample responses:

$100\% + 30\% = 130\%$ $1.3x = y$	$x + 0.30x = y$ $1.3x = y$	$(1 + 0.30)x = y$ $1.3x = y$

   b. Determine the price of the phone last year, if, this year, it cost $650.

   Last year, it cost $500;

   Sample response:

   $1.3x = y$, **if** $y = 650$ **then,**

   $$1.30x = 650$$
   $$1.30x \div 1.30 = 650 \div 1.30$$
   $$x = 500$$

   c. Determine the price this year, if, last year, the cell phone cost $900.

   This year, it cost $1,170;

   Sample response:

   $1.3x = y$, **if** $x = 900$ **then,**

   $$1.30\ (900) = y$$
   $$1170 = y$$

2. You have a coupon for 28% off any item in a store.

   a. Write an equation representing the sale price y of any item, given the retail price of x.

   $0.72x = y$; Sample responses:

$100\% - 28\% = 72\%$ $0.72x = y$	$x - 0.28x = y$ $0.72x = y$	$(1 - 0.28)x = y$ $0.72x = y$

   b. If the sale price of an item is $18.00, what was the retail price?

   The retail price was $25.00;

   Sample response:

   $0.72x = y$, **if** $y = 18$ **then,**

   $$0.72x = 18$$
   $$0.72x \div 0.72 = 18 \div 0.72$$
   $$x = 25$$

STOP

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1 Launch

Explain that students will work in small groups for this activity. They should try each problem individually first, then compare and discuss their answers to come to consensus as a group.

2 Monitor

Help students get started by asking, "Which variable represents the original amount? The new amount?"

Look for points of confusion:

- **Struggling to identify the original amount and the new amount.** Ask, "Which quantity matches 100%?"
- **Using the percent change in the calculation rather than percent of.** Suggest that students draw a diagram to make sense of each scenario.

Look for productive strategies:

- Checking their equations in part a of each problem with their group members prior to using the equations to determine the unknown original or new amount.

3 Connect

Display the scenarios from the Student Edition.

Have groups of students share the equations that they wrote for each scenario and explain their process in writing them. Discuss how each equation involves finding the percentage that corresponds with the new value instead of only the percent change.

Highlight that there are multiple ways to express each equation, but the most general way is: (percent of) • (original) = (new)

Ask:

- "What equation can be written to represent the value y after a 30% increase of x?" $1.30x = y$
- "What equation can be written to represent the value y after a 30% decrease of x?" $0.70x = y$

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* to help students make sense of, and model, each scenario. Suggest that students underline the term *increased by* 30% in Problem 1 and *coupon for* 28% in Problem 2 to help them interpret each scenario.

Accessibility: *Clarify Vocabulary and Symbols*

Be sure students understand the difference between *retail price* and *sale price* before they begin Problem 2. Consider displaying what these terms mean; the *retail price* is the original price of an item and the *sale price* is the final price of the item.

Math Language Development

MLR2: Collect and Display

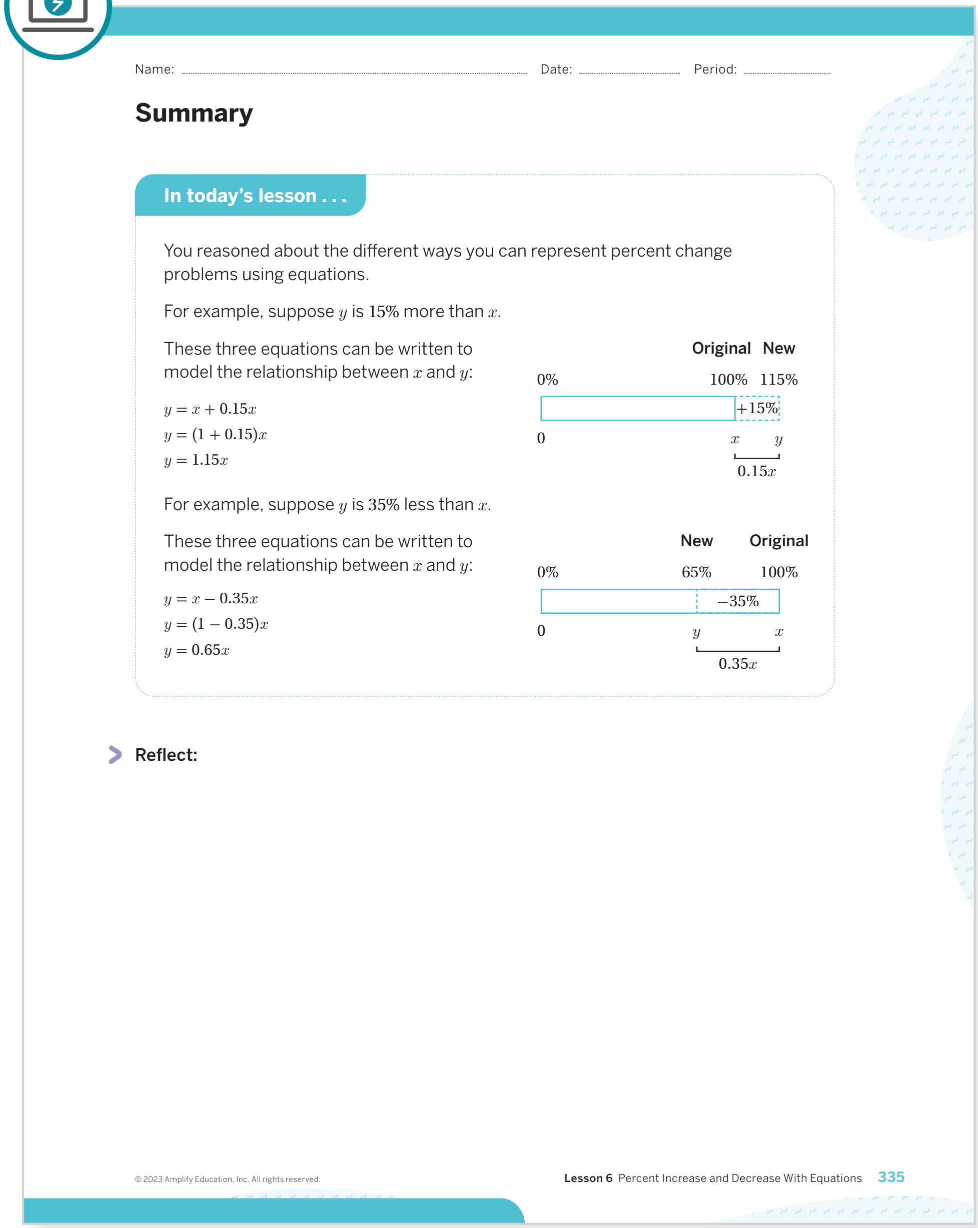
During the Connect, add the different equations students wrote for each scenario to the class display. If you have not done so already, add the general forms of the equations for percent increase and percent decrease to the class display.

Suppose b is 20% more than a:	
Percent increase	**Percent decrease**
$y = x + 0.20x$	$y = x - 0.20x$
$y = (1 + 0.20)x$	$y = (1 - 0.20)x$
$y = 1.20x$	$y = 0.80x$

Whole Class | 5 min

Summary

Review and synthesize how to write equations to represent problems involving percent increase and decrease.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You reasoned about the different ways you can represent percent change problems using equations.

For example, suppose y is 15% more than x.

These three equations can be written to model the relationship between x and y:

$y = x + 0.15x$
$y = (1 + 0.15)x$
$y = 1.15x$

For example, suppose y is 35% less than x.

These three equations can be written to model the relationship between x and y:

$y = x - 0.35x$
$y = (1 - 0.35)x$
$y = 0.65x$

Reflect:

Lesson 6 Percent Increase and Decrease With Equations 335

Synthesize

Display the Summary from the Student Edition.

Have students share what they notice about the connection between the tape diagram and each equation.

Highlight that all three equations for each example are equivalent by using the Distributive Property and combining like terms.

Ask:

- "What are the different ways you have learned to solve percent increase and decrease problems?"
- "Which representation do you prefer? Explain your thinking."

Reflect

After to synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are percentages related to proportional relationships?"

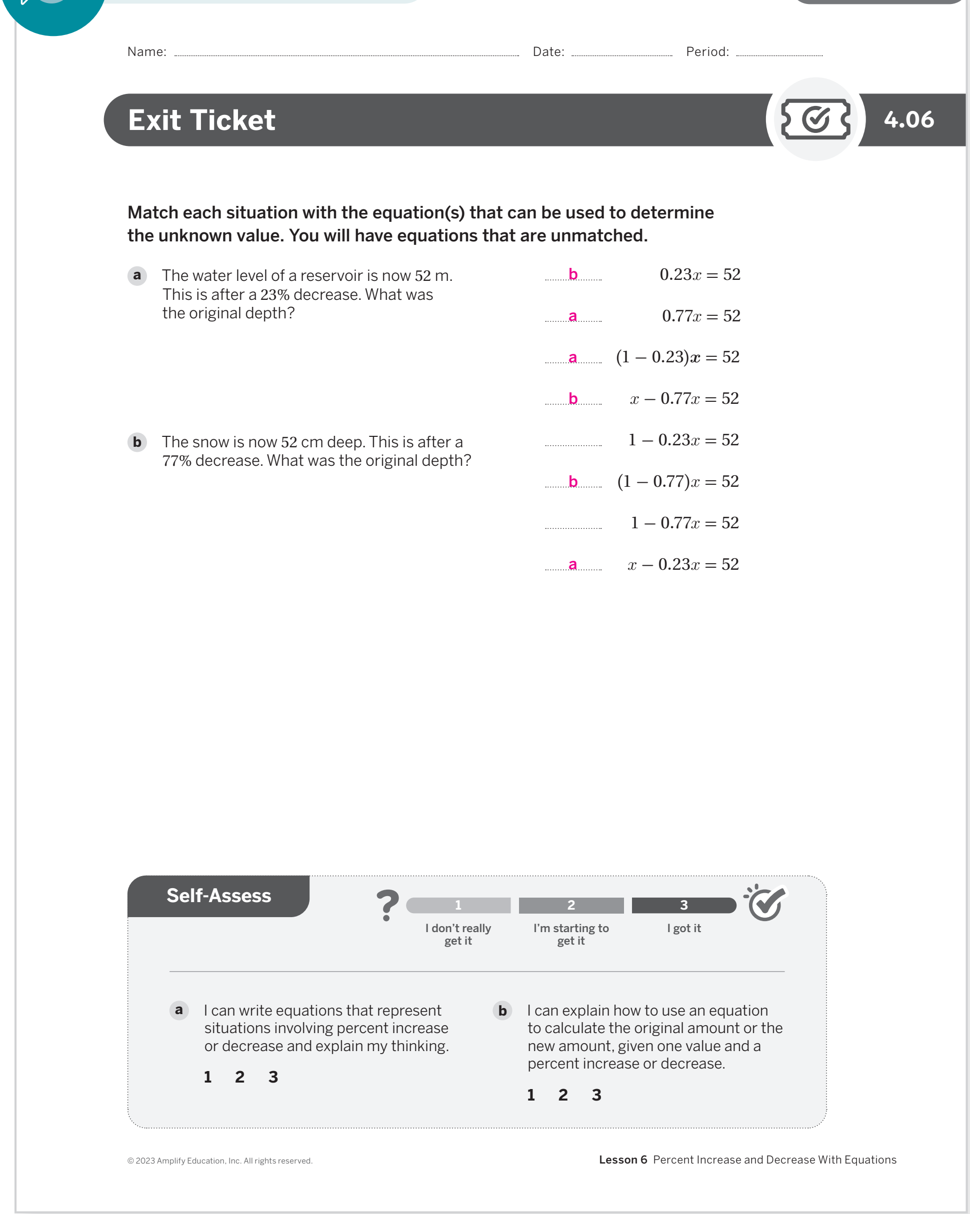
Independent | 5 min

Exit Ticket

Students demonstrate their understanding of using equations to represent percent change contexts by matching contexts with equations that represent them.

Amps Featured Activity Real-Time Exit Ticket

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 4.06

Match each situation with the equation(s) that can be used to determine the unknown value. You will have equations that are unmatched.

a The water level of a reservoir is now 52 m. This is after a 23% decrease. What was the original depth?

b The snow is now 52 cm deep. This is after a 77% decrease. What was the original depth?

b .... $0.23x = 52$

a .... $0.77x = 52$

a .... $(1 - 0.23)x = 52$

b .... $x - 0.77x = 52$

.... $1 - 0.23x = 52$

b .... $(1 - 0.77)x = 52$

.... $1 - 0.77x = 52$

a .... $x - 0.23x = 52$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can write equations that represent situations involving percent increase or decrease and explain my thinking.

1 2 3

b I can explain how to use an equation to calculate the original amount or the new amount, given one value and a percent increase or decrease.

1 2 3

Lesson 6 Percent Increase and Decrease With Equations

Success looks like . . .

- **Language Goal:** Generating equations that represent a situation involving a given percent increase or decrease and justifying the reasoning. **(Speaking and Listening)**
  - » Matching each situation with the equations that can calculate the original depth given the percent decrease.
- **Language Goal:** Explaining how to use an equation to calculate the original amount or the new amount given one value and the percent increase or decrease. **(Speaking and Listening, Writing)**

Suggested next steps

If students only identify one equation for each scenario, consider:

- Asking, "Are there any other equations that could represent this scenario?"
- Reviewing Activity 2.
- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was for students to write equations to represent percent change scenarios. How well did students accomplish this? What did you specifically do to help them accomplish it?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Math Language Development

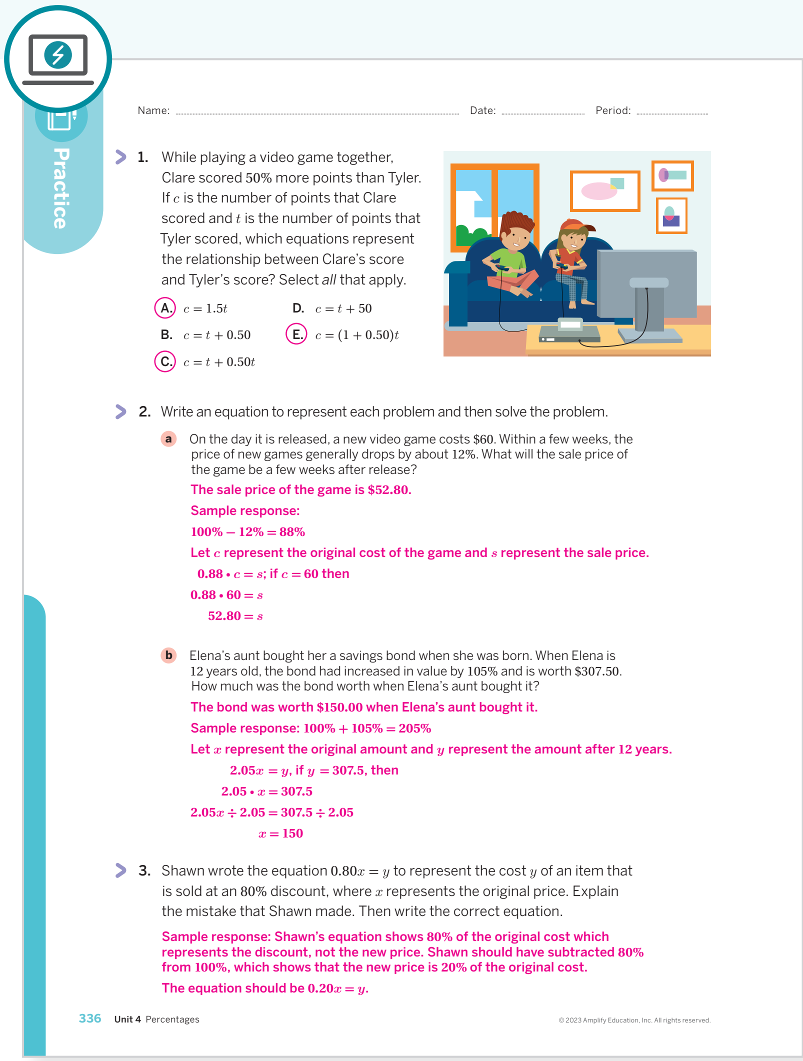
Language Goal: Explaining how to use an equation to calculate the original amount or the new amount given one value and the percent increase or decrease?

Reflect on students' language development toward this goal.

- How have students progressed so far in this unit in explaining different strategies for calculating the original amount or the new amount?
- How did using the math language routines in this lesson support students in understanding different ways percent change can be represented through equations? Would you change anything the next time you use these routines?

Practice

Independent



Practice

Name: Date: Period:

1. While playing a video game together, Clare scored 50% more points than Tyler. If c is the number of points that Clare scored and t is the number of points that Tyler scored, which equations represent the relationship between Clare's score and Tyler's score? Select *all* that apply.

   [✓ A.] $c = 1.5t$
   B. $c = t + 0.50$
   [✓ C.] $c = t + 0.50t$
   D. $c = t + 50$
   [✓ E.] $c = (1 + 0.50)t$

2. Write an equation to represent each problem and then solve the problem.

   a. On the day it is released, a new video game costs $60. Within a few weeks, the price of new games generally drops by about 12%. What will the sale price of the game be a few weeks after release?

   The sale price of the game is $52.80.

   Sample response:

   $100\% - 12\% = 88\%$

   Let c represent the original cost of the game and s represent the sale price.

   $0.88 \cdot c = s$; if $c = 60$ then

   $0.88 \cdot 60 = s$

   $52.80 = s$

   b. Elena's aunt bought her a savings bond when she was born. When Elena is 12 years old, the bond had increased in value by 105% and is worth $307.50. How much was the bond worth when Elena's aunt bought it?

   The bond was worth $150.00 when Elena's aunt bought it.

   Sample response: $100\% + 105\% = 205\%$

   Let x represent the original amount and y represent the amount after 12 years.

   $2.05x = y$, if $y = 307.5$, then

   $2.05 \cdot x = 307.5$

   $2.05x \div 2.05 = 307.5 \div 2.05$

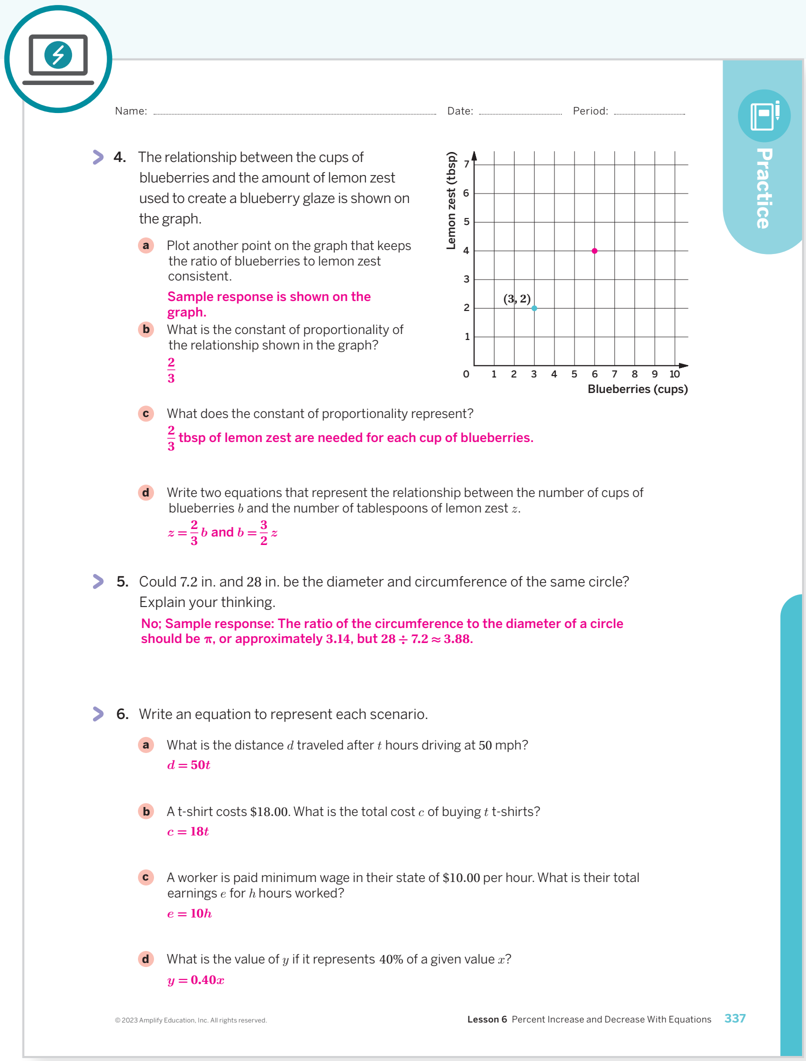
   $x = 150$

3. Shawn wrote the equation $0.80x = y$ to represent the cost y of an item that is sold at an 80% discount, where x represents the original price. Explain the mistake that Shawn made. Then write the correct equation.

   Sample response: Shawn's equation shows 80% of the original cost which represents the discount, not the new price. Shawn should have subtracted 80% from 100%, which shows that the new price is 20% of the original cost.

   The equation should be $0.20x = y$.

336 **Unit 4** Percentages



Practice

Name: Date: Period:

4. The relationship between the cups of blueberries and the amount of lemon zest used to create a blueberry glaze is shown on the graph.

   a. Plot another point on the graph that keeps the ratio of blueberries to lemon zest consistent.

   Sample response is shown on the graph.

   b. What is the constant of proportionality of the relationship shown in the graph?

   $\frac{2}{3}$

   c. What does the constant of proportionality represent?

   $\frac{2}{3}$ tbsp of lemon zest are needed for each cup of blueberries.

   d. Write two equations that represent the relationship between the number of cups of blueberries b and the number of tablespoons of lemon zest z.

   $z = \frac{2}{3}b$ and $b = \frac{3}{2}z$

5. Could 7.2 in. and 28 in. be the diameter and circumference of the same circle? Explain your thinking.

   No; Sample response: The ratio of the circumference to the diameter of a circle should be π, or approximately 3.14, but $28 \div 7.2 \approx 3.88$.

6. Write an equation to represent each scenario.

   a. What is the distance d traveled after t hours driving at 50 mph?

   $d = 50t$

   b. A t-shirt costs $18.00. What is the total cost c of buying t t-shirts?

   $c = 18t$

   c. A worker is paid minimum wage in their state of $10.00 per hour. What is their total earnings e for h hours worked?

   $e = 10h$

   d. What is the value of y if it represents 40% of a given value x?

   $y = 0.40x$

Lesson 6 Percent Increase and Decrease With Equations 337

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 3	2
	3	Activity 3	3
Spiral	4	Unit 2 Lesson 15	2
	5	Unit 3 Lesson 5	3
Formative ⚡	6	Unit 6 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 4 | **LESSON 7**

Using Equations to Solve Percent Problems

Let's use equations to solve problems about percent change.

Focus

Goals

1. **Language Goal:** Generate equations that represent situations involving percent increase and decrease when given new and original amounts, and justify the reasoning. **(Speaking and Listening)**
2. **Language Goal:** Explain how to use an equation to determine the percent change, given the new amount and the original amount. **(Speaking and Listening, Writing)**

Coherence

Today

Students extend their understanding of writing equations that represent percent increase and decrease problems to write equations that determine the unknown percent change when given the original and the new amount. Students analyze the two different methods of writing equations for determining percent change and complete ***Partner Problems*** to compare and contrast them. Finally, students identify which equations match different percent change scenarios, and then choose one equation for each scenario to solve for the unknown value.

Previously

In Lesson 6, students wrote and solved equations to determine the new or original amount when the percent change was known.

Coming Soon

In Lessons 8–12, students will apply their understanding of percent change to solve real-world problems including tax, tip, and commission.

Rigor

- Students build **conceptual understanding** of writing and solving equations to determine the percent change when the original and new amounts are known.
- Students develop **fluency** in using equations to solve problems involving percent change.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	13 min	15 min	15 min	5 min	7 min
Independent	Small Groups	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 1 PDF (answers)
- Graphic Organizer PDF, *Percentage Tape Diagrams* (as needed)
- calculators

Math Language Development

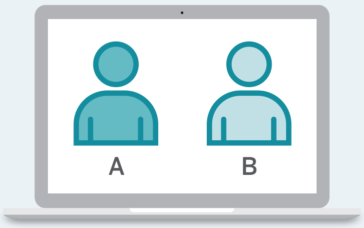
Review words

- *percent change*
- *percent decrease*
- *percent increase*

Amps | Featured Activity

Activity 2
Partner Problems

Monitor student understanding in real time as they work with a partner to solve the same problem using different methods and then compare their work and solutions.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might be so eager to tell others what they think that they forget to listen to others as they share their critiques. Encourage students to demonstrate active listening and use their body language in a positive way as others share their arguments or critique someone's reasoning. Have groups determine cues that will be used when someone is not listening, such as the speaker will pause until the focus is regained.

Modifications to Pacing

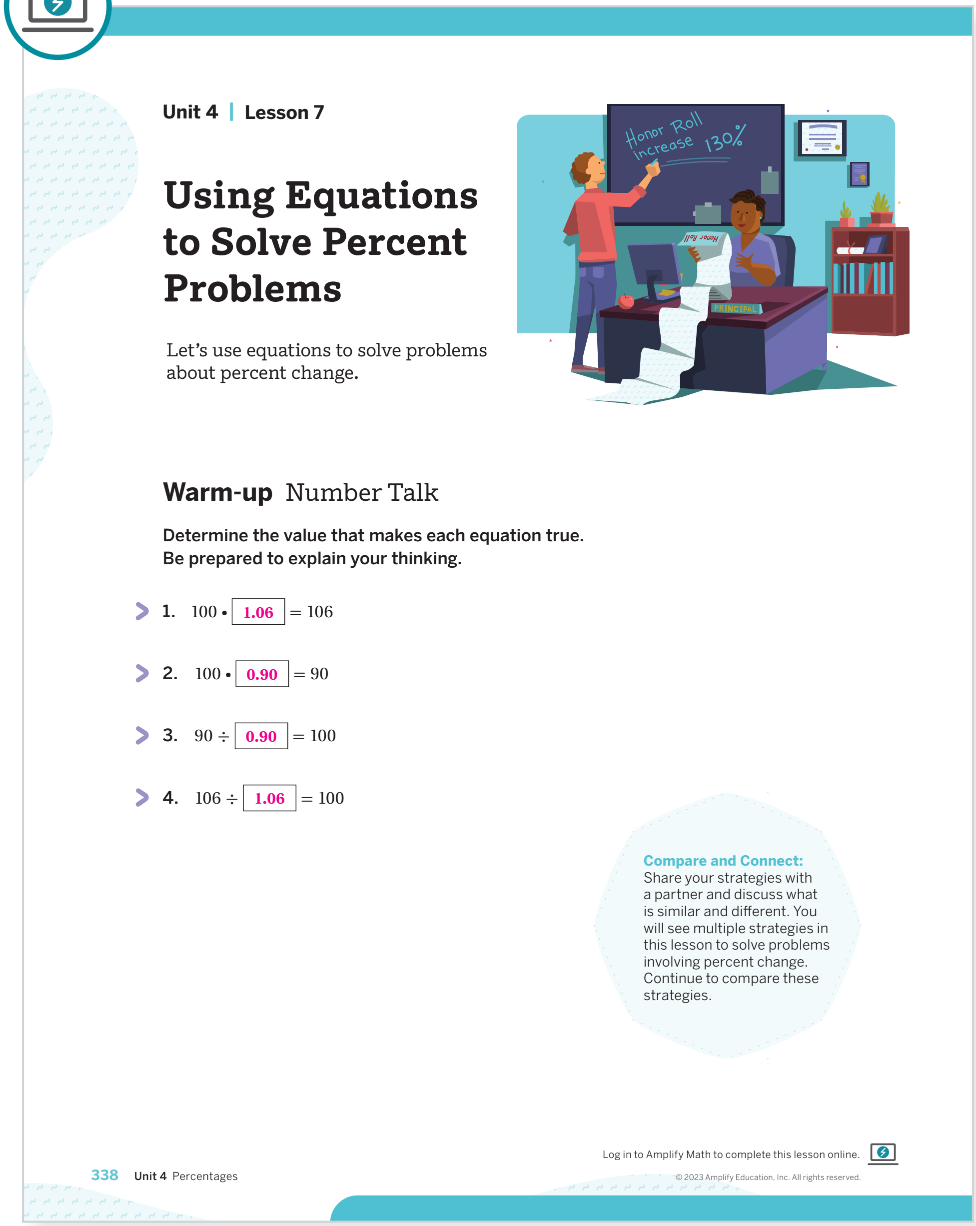
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have half of the students complete Problem 1 and the other half complete Problem 2, and then discuss both as a class.
- Optional **Activity 3** may be omitted. You may choose to assign this activity as additional practice.

Independent | 5 min

Warm-up Number Talk

Students reason about decimals and scale factors to determine what values to multiply by when converting from 100 and back to 100.



Unit 4 | Lesson 7

Using Equations to Solve Percent Problems

Let's use equations to solve problems about percent change.

Warm-up Number Talk

Determine the value that makes each equation true. Be prepared to explain your thinking.

1. $100 \cdot \boxed{1.06} = 106$
2. $100 \cdot \boxed{0.90} = 90$
3. $90 \div \boxed{0.90} = 100$
4. $106 \div \boxed{1.06} = 100$

Compare and Connect: Share your strategies with a partner and discuss what is similar and different. You will see multiple strategies in this lesson to solve problems involving percent change. Continue to compare these strategies.

Log in to Amplify Math to complete this lesson online.

338 Unit 4 Percentages

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1 Launch

Conduct the *Number Talk* routine, giving students the opportunity to discuss their strategy for each problem prior to the full class discussion.

2 Monitor

Help students get started by asking, "What if the problem was 1 times an unknown value, instead of 100?"

Look for points of confusion:

- **Struggling with the placement of the decimal point in the unknown quantity.** Ask, "Should the value you are multiplying or dividing by be approximately 1, 10, or 100?"

Look for productive strategies:

- Using inverse operations to determine the unknown value. For example, in Problems 1 and 2, students dividied each side of the equation by 100.
- Recognizing the inverse relationship between Problems 1 and 4, as well as Problems 2 and 3.

3 Connect

Display the Warm-up from the Student Edition.

Have students share their strategies for each problem. Record and display their responses.

Look for productive strategies:

- "What relationship do you notice between the values in these equations?"
- "How are these equations related to the work we've been doing with percent increase and decrease?"

Highlight that in order to get from the original value (100%) to the new value, multiplication is used. To get back to the original value, division is used.

Math Language Development

MLR7: Compare and Connect

During the Connect, have students share their strategies with a partner and discuss any similarities or differences. Some students may have divided both sides of the equations in Problems 1 and 2 by 100, while others may have reasoned about percent change. Amplify strategies where students noticed that 100 was present in all equations, and students could use reasoning about percent change.

English Learners

Display the sentence frames to help students in their discussions with their partners:

- "I noticed that ______, so I . . ."
- "I ______ because . . ."

Power-up

To power up students' ability to write equations to represent proportional relationships, have students complete:

Recall that an equation is proportional if it is of the form $y = kx$ where k is the constant of proportionality. Complete each equation to represent each proportional relationship.

ⓐ The number of feet f Diego travels in m minutes walking at speed of 300 ft/min.
$f = 300m$

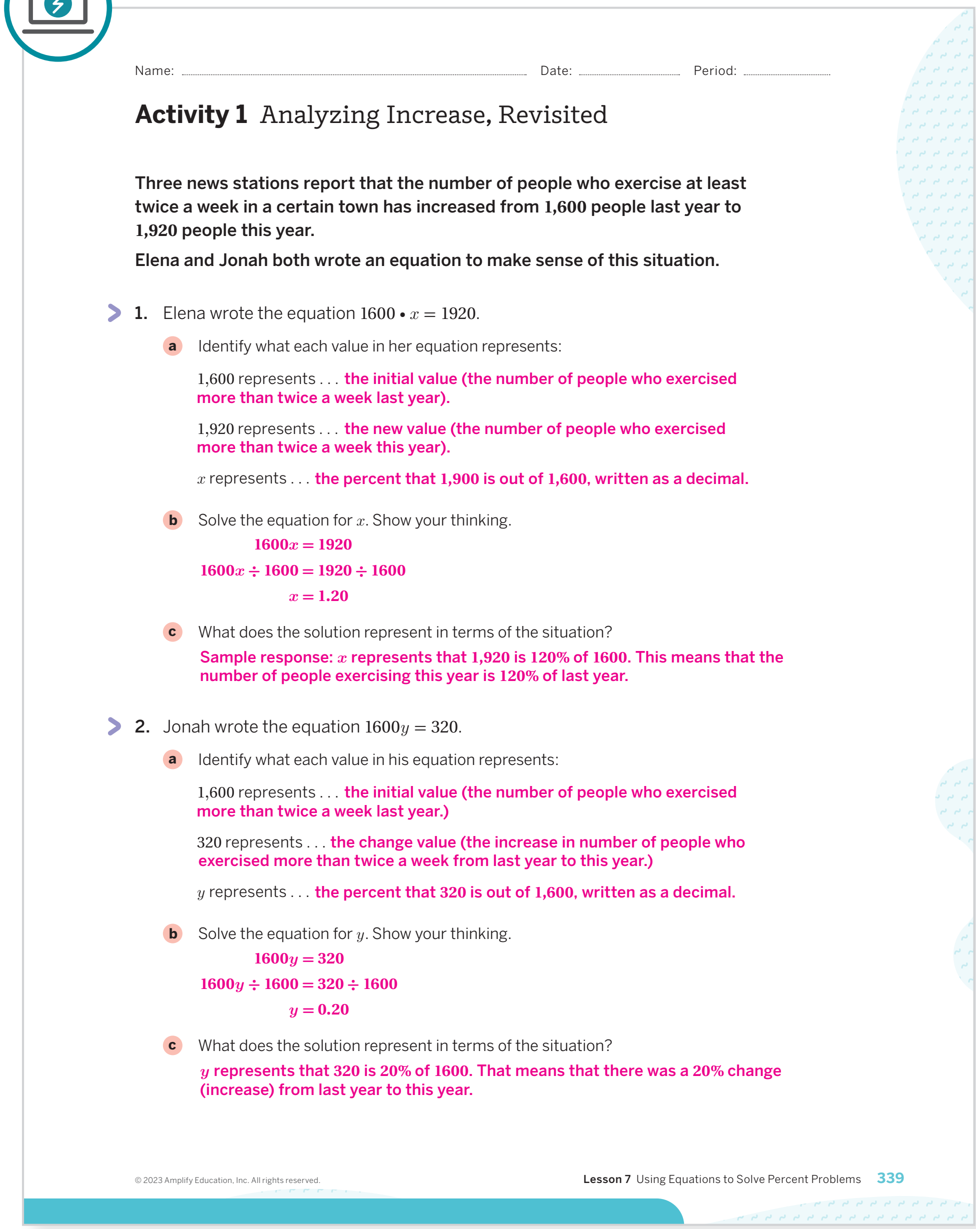
ⓑ The number of d dollars for q quarters.
$d = \frac{1}{4}q$

Use: Before Activity 1.
Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7.

Small groups | 13 min

Activity 1 Analyzing Increase, Revisited

Students analyze two equations used to determine the percent change from the Lesson 6 Warm-up to reason about multiple methods for representing percent change.



Name: ________ Date: ________ Period: ________

Activity 1 Analyzing Increase, Revisited

Three news stations report that the number of people who exercise at least twice a week in a certain town has increased from 1,600 people last year to 1,920 people this year.

Elena and Jonah both wrote an equation to make sense of this situation.

1. Elena wrote the equation $1600 \cdot x = 1920$.

   a. Identify what each value in her equation represents:

   1,600 represents . . . **the initial value (the number of people who exercised more than twice a week last year).**

   1,920 represents . . . **the new value (the number of people who exercised more than twice a week this year).**

   x represents . . . **the percent that 1,900 is out of 1,600, written as a decimal.**

   b. Solve the equation for x. Show your thinking.

   $$1600x = 1920$$
   $$1600x \div 1600 = 1920 \div 1600$$
   $$x = 1.20$$

   c. What does the solution represent in terms of the situation?

   Sample response: x represents that 1,920 is 120% of 1600. This means that the number of people exercising this year is 120% of last year.

2. Jonah wrote the equation $1600y = 320$.

   a. Identify what each value in his equation represents:

   1,600 represents . . . **the initial value (the number of people who exercised more than twice a week last year.)**

   320 represents . . . **the change value (the increase in number of people who exercised more than twice a week from last year to this year.)**

   y represents . . . **the percent that 320 is out of 1,600, written as a decimal.**

   b. Solve the equation for y. Show your thinking.

   $$1600y = 320$$
   $$1600y \div 1600 = 320 \div 1600$$
   $$y = 0.20$$

   c. What does the solution represent in terms of the situation?

   y represents that 320 is 20% of 1600. That means that there was a 20% change (increase) from last year to this year.

1 Launch

Activate students' prior knowledge by asking, "How did you use equations to represent percent change problems in the previous lesson?" Display the scenario from the Warm-up in Lesson 6, and explain that they will be analyzing two methods for using equations to determine the percent change, and then discussing their thoughts as a group. Distribute calculators to aid in computations.

2 Monitor

Help students get started by having them reread the scenario in the Warm-up then asking, "What do the two values in each equation represent?"

Look for points of confusion:

- **Struggling to make sense of the values in each equation.** Have students draw and label a tape diagram, or provide them with a copy of the Activity 1 PDF to annotate.

3 Connect

Display the Activity 1 PDF.

Have students share what each value in the equations represent. Annotate the tape diagram to reflect and reinforce student understanding, displaying the Activity 1 PDF (answers).

Highlight that in each equation, the variable represents the decimal equivalent of the percentage.

Ask:

- "If Elena wanted to determine the percent change, what additional step would she need to do?" Sample response: Elena would have to find the difference of 100% and 120% to determine the percent change.
- "What did Jonah do as his first step that Elena didn't do?" Sample response: Jonah found the difference of 1,920 and 1,600 to determine the change in the number of people exercising more than twice a week this year compared to last year.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider displaying the given information in another modality, such as by creating a table similar to the following:

Number of people who exercised at least twice a week	
Last year	This year
1,600	1,920

Accessibility: Optimize Access to Tools

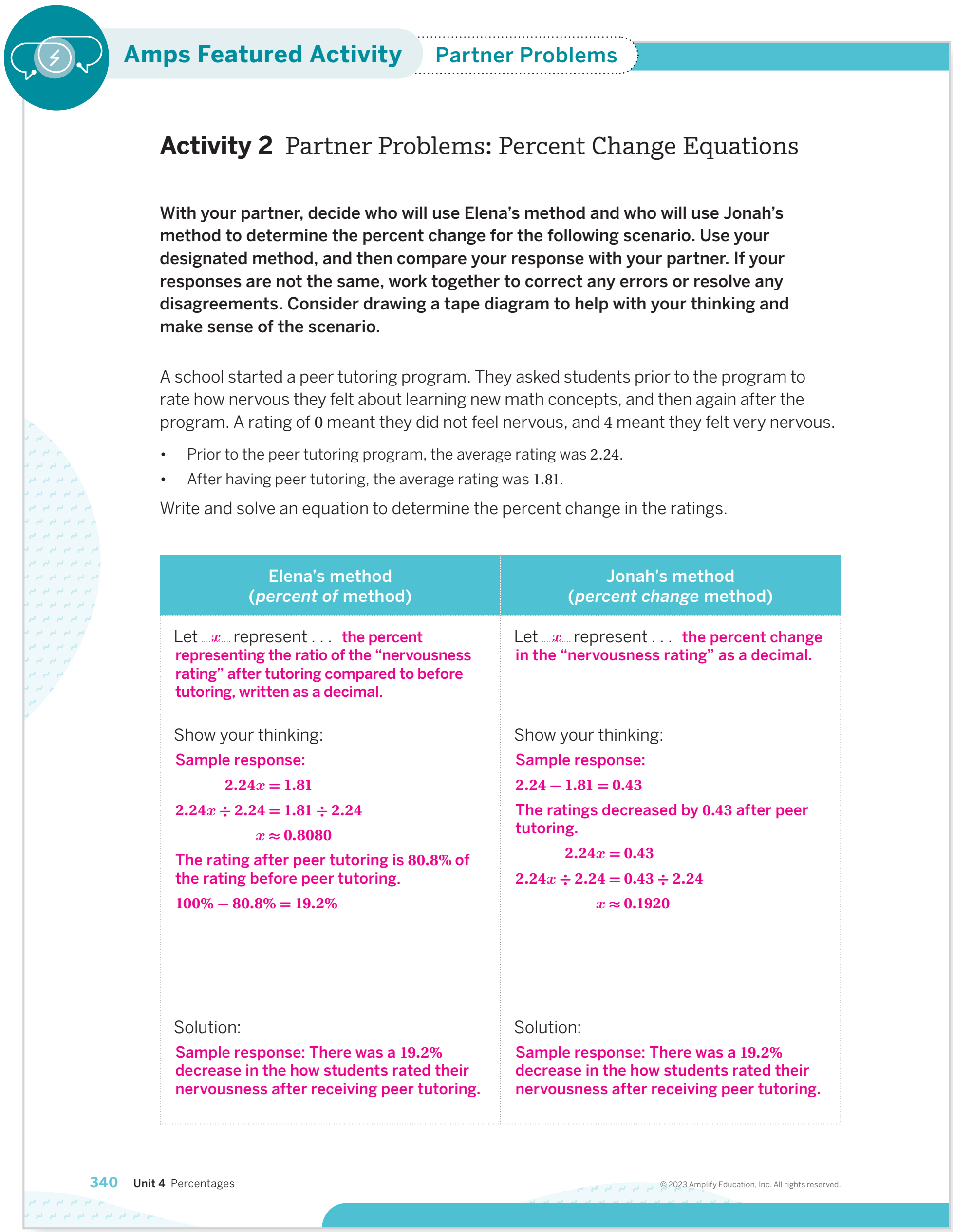
Provide students with a copy of the Activity 1 PDF, that shows a blank tape diagram. They can use this tape diagram to annotate their thinking.

Pairs | 15 min

Activity 2 Partner Problems: Percent Change Equations

Students compare and contrast two methods of writing equations for determining the percent change.

Amps Featured Activity Partner Problems



Activity 2 Partner Problems: Percent Change Equations

With your partner, decide who will use Elena's method and who will use Jonah's method to determine the percent change for the following scenario. Use your designated method, and then compare your response with your partner. If your responses are not the same, work together to correct any errors or resolve any disagreements. Consider drawing a tape diagram to help with your thinking and make sense of the scenario.

A school started a peer tutoring program. They asked students prior to the program to rate how nervous they felt about learning new math concepts, and then again after the program. A rating of 0 meant they did not feel nervous, and 4 meant they felt very nervous.

- Prior to the peer tutoring program, the average rating was 2.24.
- After having peer tutoring, the average rating was 1.81.

Write and solve an equation to determine the percent change in the ratings.

Elena's method (*percent of* method)	Jonah's method (*percent change* method)
Let x represent . . . **the percent representing the ratio of the "nervousness rating" after tutoring compared to before tutoring, written as a decimal.**	Let x represent . . . **the percent change in the "nervousness rating" as a decimal.**
Show your thinking: **Sample response:** $2.24x = 1.81$ $2.24x \div 2.24 = 1.81 \div 2.24$ $x \approx 0.8080$ **The rating after peer tutoring is 80.8% of the rating before peer tutoring.** $100\% - 80.8\% = 19.2\%$	Show your thinking: **Sample response:** $2.24 - 1.81 = 0.43$ **The ratings decreased by 0.43 after peer tutoring.** $2.24x = 0.43$ $2.24x \div 2.24 = 0.43 \div 2.24$ $x \approx 0.1920$
Solution: **Sample response: There was a 19.2% decrease in the how students rated their nervousness after receiving peer tutoring.**	Solution: **Sample response: There was a 19.2% decrease in the how students rated their nervousness after receiving peer tutoring.**

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1 Launch

Ask, "Have you helped a friend with their school work, or has a friend helped you? How did it make you feel?" Explain to students that with their partner they will determine who will use Elena's method and who will use Jonah's method. Conduct the ***Partner Problems*** routine.

2 Monitor

Help students get started by asking, "What is the original amount? The new amount?"

Look for points of confusion:

- **Not understanding that x represents a different percentage in each scenario.** Ask students to refer back to what x and y represented in Activity 1.
- **Forgetting that the value of x represents the percentage as its decimal equivalent.** Remind students that they need to convert from the decimal solution to a percentage.
- **Saying that the percent change is 80.8%.** Ask, "What did x represent in your equation?"

3 Connect

Display the scenario from Activity 2.

Have students share the equation they wrote for each method along with what the variable represents in their equation.

Highlight that it is important to keep track of what the variable represents when writing the equation. Students should ask themselves, "If I solve this equation, am I determining the *percent of* or the *percent change*?"

Ask:

- "How are the two methods similar?"
- "How are the two methods different?"
- "Which method do you prefer?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* to help students make sense of, and model, each scenario.

Math Language Development

MLR7: Compare and Connect

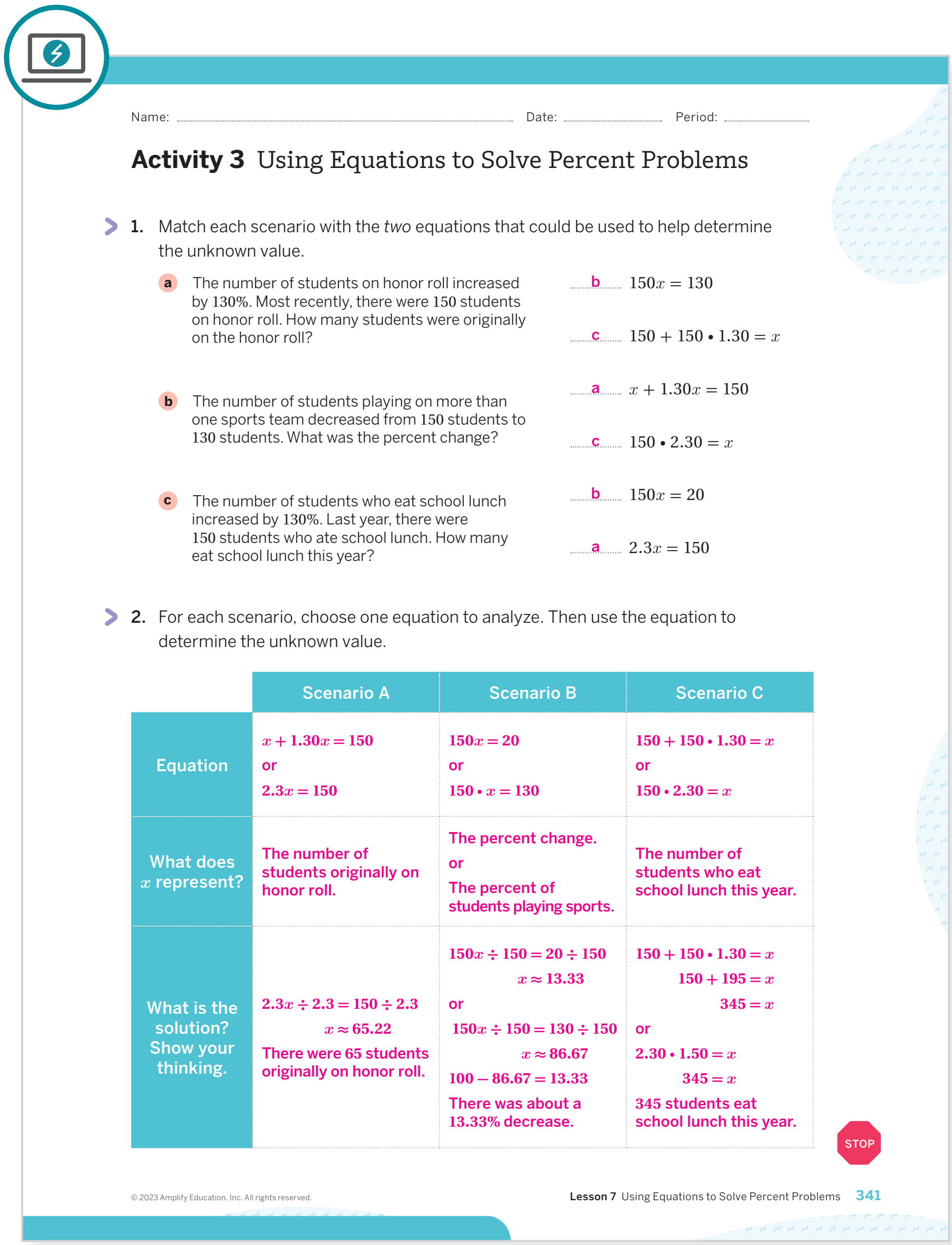
During the Connect, as students share the equations they wrote for each method, emphasize how the variable in each of the different methods refers to a different quantity. Ask students why it is important to define what the variable represents. Sample response: If I do not define the variable, then it might not be clear to me or anyone else what the solution represents in the context of the problem.

Optional

Small groups | 15 min

Activity 3 Using Equations to Solve Percent Problems

Students match scenarios involving the initial value, new value, or percent change to the equations that could be used to represent them.



Name: .... Date: .... Period: ....

Activity 3 Using Equations to Solve Percent Problems

1. Match each scenario with the *two* equations that could be used to help determine the unknown value.

   a. The number of students on honor roll increased by 130%. Most recently, there were 150 students on honor roll. How many students were originally on the honor roll?

   b. The number of students playing on more than one sports team decreased from 150 students to 130 students. What was the percent change?

   c. The number of students who eat school lunch increased by 130%. Last year, there were 150 students who ate school lunch. How many eat school lunch this year?

   b $150x = 130$

   c $150 + 150 \cdot 1.30 = x$

   a $x + 1.30x = 150$

   c $150 \cdot 2.30 = x$

   b $150x = 20$

   a $2.3x = 150$

2. For each scenario, choose one equation to analyze. Then use the equation to determine the unknown value.

	Scenario A	Scenario B	Scenario C
Equation	$x + 1.30x = 150$ or $2.3x = 150$	$150x = 20$ or $150 \cdot x = 130$	$150 + 150 \cdot 1.30 = x$ or $150 \cdot 2.30 = x$
What does x represent?	The number of students originally on honor roll.	The percent change. or The percent of students playing sports.	The number of students who eat school lunch this year.
What is the solution? Show your thinking.	$2.3x \div 2.3 = 150 \div 2.3$ $x \approx 65.22$ There were 65 students originally on honor roll.	$150x \div 150 = 20 \div 150$ $x \approx 13.33$ or $150x \div 150 = 130 \div 150$ $x \approx 86.67$ $100 - 86.67 = 13.33$ There was about a 13.33% decrease.	$150 + 150 \cdot 1.30 = x$ $150 + 195 = x$ $345 = x$ or $2.30 \cdot 1.50 = x$ $345 = x$ 345 students eat school lunch this year.

STOP

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Lesson 7 Using Equations to Solve Percent Problems 341

1 Launch

Give students three to five minutes to work as a group to complete Problem 1. After discussing their matches as a class, students should choose one equation for each scenario to solve. Encourage students in the same group to choose different equations, and then check their solutions together.

2 Monitor

Help students get started by asking, "What is the unknown value in this scenario: a percentage, the new value, or the original value?"

Look for points of confusion:

- **Thinking that there is only one equation that matches each scenario.** Explain that students should have a matching scenario for *every* equation.

Look for productive strategies:

- Identifying equivalent equations to help students match their scenarios to equations.

3 Connect

Display the scenarios and equations to the class.

Have students share which equation they solved for each scenario, along with what the variable represented in terms of the original scenario.

Highlight that, using the structure of a% of b is c for percent change problems, you can write the equation $a \cdot b = c$, where a can represent either the percent of or the percent change, and c can represent the new amount or the amount the original value b changed.

Ask, "What two equations can be written to determine the percent change between an original value of 8 and a new value of 10?" $a \cdot 8 = 10$ or $a \cdot 8 = 2$

Differentiated Support

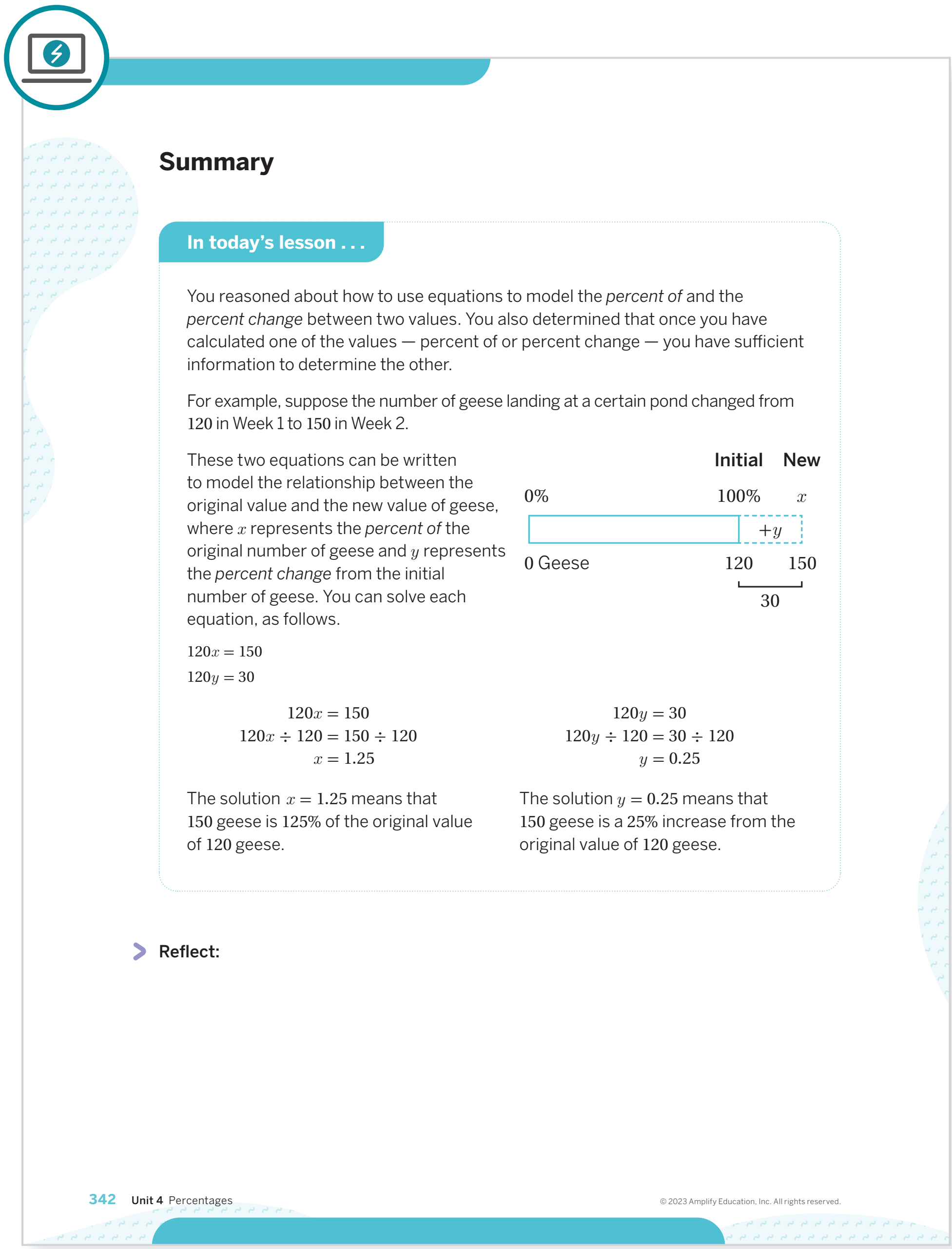
Accessibility: *Vary Demands to Optimize Challenge*

Chunk this activity into smaller, more manageable tasks by helping students focus on one problem at a time. In Problem 1, have them study the scenario in part a and limit the choices of the equations so that there are only four equations to choose from. Then use a similar approach for parts b and c.

Whole Class | 5 min

Summary

Review and synthesize how writing equations can help represent percent increase and decrease problems and how they can be used to determine the percent change.



Summary

In today's lesson . . .

You reasoned about how to use equations to model the *percent of* and the *percent change* between two values. You also determined that once you have calculated one of the values — percent of or percent change — you have sufficient information to determine the other.

For example, suppose the number of geese landing at a certain pond changed from 120 in Week 1 to 150 in Week 2.

These two equations can be written to model the relationship between the original value and the new value of geese, where x represents the *percent of* the original number of geese and y represents the *percent change* from the initial number of geese. You can solve each equation, as follows.

$120x = 150$

$120y = 30$

$$120x = 150$$
$$120x \div 120 = 150 \div 120$$
$$x = 1.25$$

The solution $x = 1.25$ means that 150 geese is 125% of the original value of 120 geese.

$$120y = 30$$
$$120y \div 120 = 30 \div 120$$
$$y = 0.25$$

The solution $y = 0.25$ means that 150 geese is a 25% increase from the original value of 120 geese.

Reflect:

342 Unit 4 Percentages

Synthesize

Display the Summary from the Student Edition.

Highlight that often writing the equation is the most efficient way of solving a problem involving percent increase or decrease. When writing the equation, it is important to keep in mind whether the variable is representing the *percent of* or the *percent change*.

Ask:

- "Could you determine the percent change using either equation? How?" Sample response: Yes; in the first equation, I would subtract 100% from 120% to determine the percent change.
- "Could you determine the percent of using either equation? How?" Sample response: Yes; in the first second equation, I would add 100% and 25% to determine the percent change.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

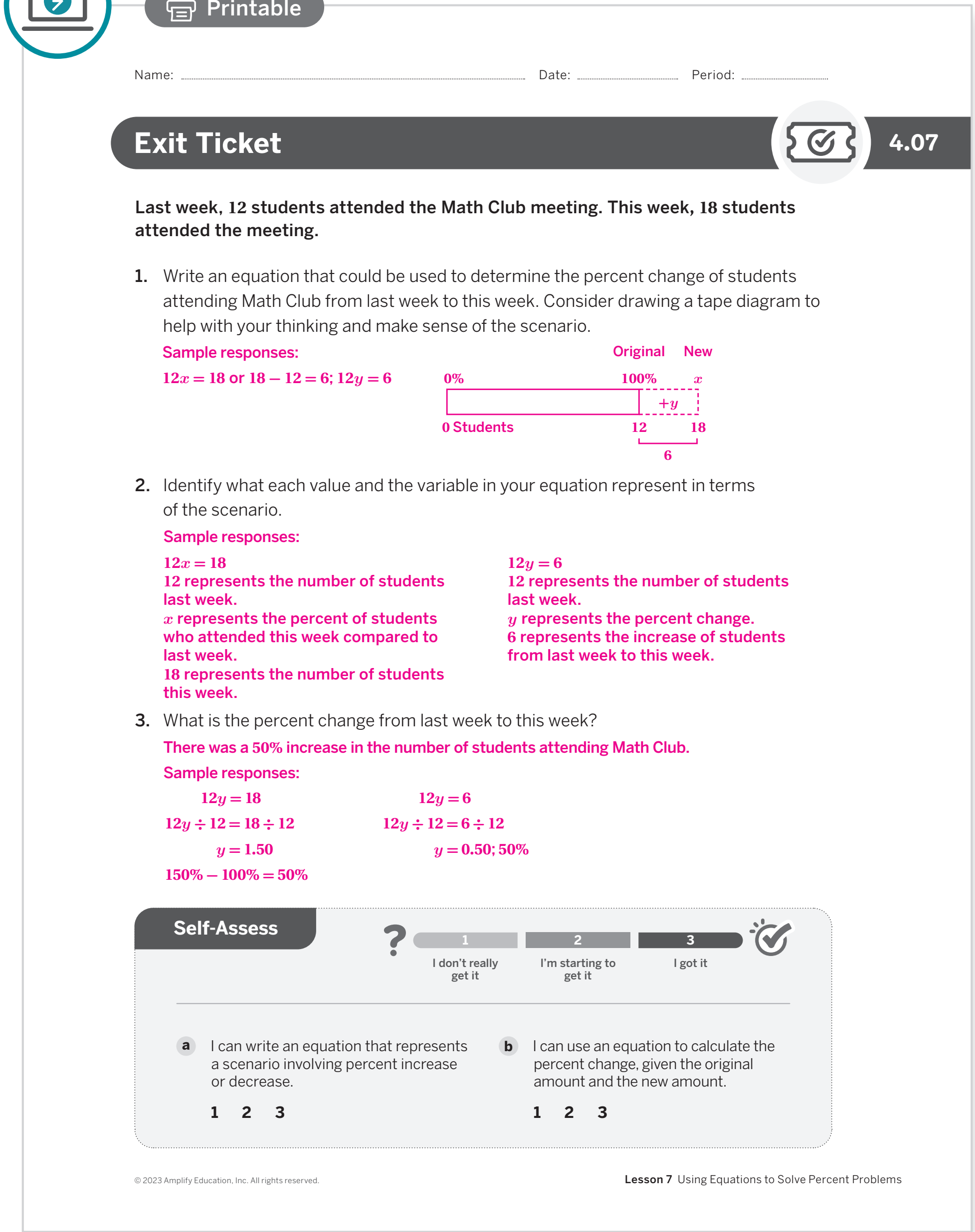
- "How are percentages related to proportional relationships?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by writing an equation to represent a percent change problem, using the equation to solve the problem, and interpreting the solution within the context of the problem.

Printable



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Exit Ticket 4.07

Last week, 12 students attended the Math Club meeting. This week, 18 students attended the meeting.

1. Write an equation that could be used to determine the percent change of students attending Math Club from last week to this week. Consider drawing a tape diagram to help with your thinking and make sense of the scenario.

   Sample responses:

   $12x = 18$ or $18 - 12 = 6$; $12y = 6$

2. Identify what each value and the variable in your equation represent in terms of the scenario.

   Sample responses:

$12x = 18$ 12 represents the number of students last week. x represents the percent of students who attended this week compared to last week. 18 represents the number of students this week.	$12y = 6$ 12 represents the number of students last week. y represents the percent change. 6 represents the increase of students from last week to this week.

3. What is the percent change from last week to this week?

   There was a 50% increase in the number of students attending Math Club.

   Sample responses:

$12y = 18$	$12y = 6$
$12y \div 12 = 18 \div 12$	$12y \div 12 = 6 \div 12$
$y = 1.50$	$y = 0.50$; 50%
$150\% - 100\% = 50\%$	

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can write an equation that represents a scenario involving percent increase or decrease.

1 2 3

b I can use an equation to calculate the percent change, given the original amount and the new amount.

1 2 3

Lesson 7 Using Equations to Solve Percent Problems

Success looks like . . .

- **Language Goal:** Generating equations that represent situations involving percent increase and decrease when given the new and original amounts, and justifying the reasoning. **(Speaking and Listening)**
  - » Writing the equation for the percent increase of students attending Math Club in Problem 1.
- **Language Goal:** Explaining how to use an equation to determine the percent change given the new amount and the original amount. **(Speaking and Listening, Writing)**
  - » Explaining how to use an equation to determine the percent increase from last week to this week in Problems 2 and 3.

Suggested next steps

If students identify the percent change as 150%, consider:

- Asking them if that value makes sense in terms of the original and new value.
- Asking, "What part of your work shows how the number of students changed?"
- Assigning Practice Problems 1 and 2.

Professional Learning

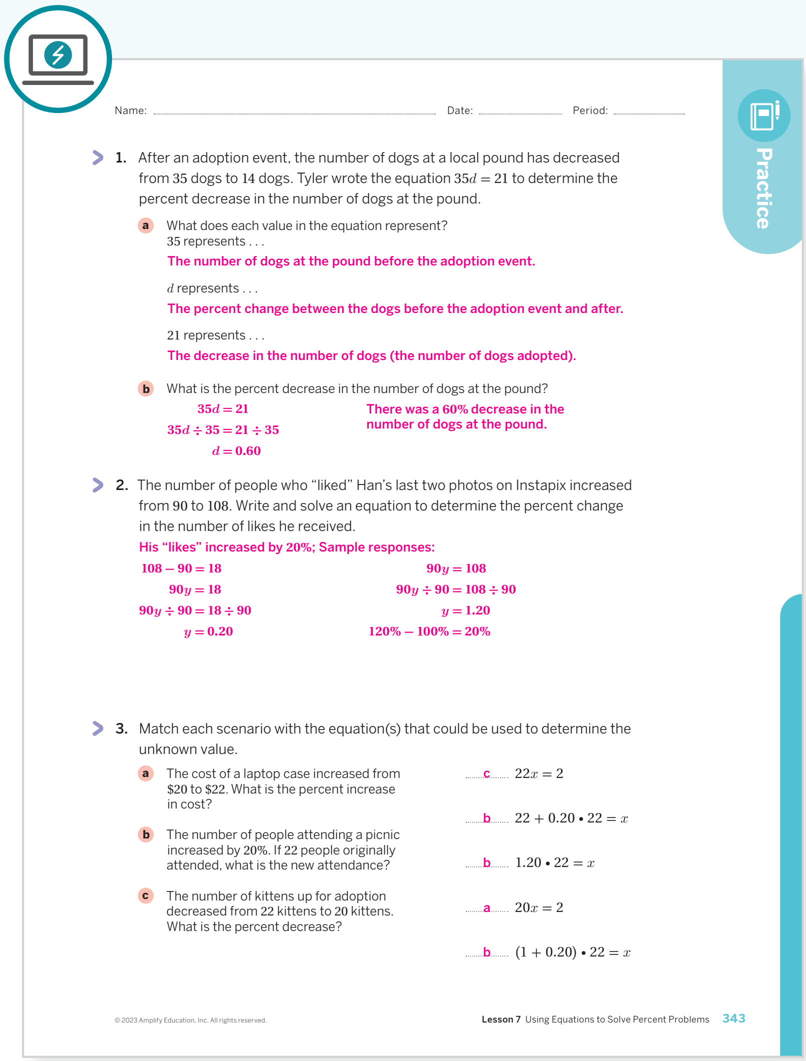
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- What did students find challenging about Activity 1? What helped them work through these challenges? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. After an adoption event, the number of dogs at a local pound has decreased from 35 dogs to 14 dogs. Tyler wrote the equation $35d = 21$ to determine the percent decrease in the number of dogs at the pound.
   - **a** What does each value in the equation represent?
     35 represents . . .
     The number of dogs at the pound before the adoption event.
     d represents . . .
     The percent change between the dogs before the adoption event and after.
     21 represents . . .
     The decrease in the number of dogs (the number of dogs adopted).
   - **b** What is the percent decrease in the number of dogs at the pound?
     $35d = 21$
     $35d \div 35 = 21 \div 35$
     $d = 0.60$
     There was a 60% decrease in the number of dogs at the pound.

2. The number of people who “liked” Han’s last two photos on Instapix increased from 90 to 108. Write and solve an equation to determine the percent change in the number of likes he received.
   His “likes” increased by 20%; Sample responses:

$108 - 90 = 18$	$90y = 108$
$90y = 18$	$90y \div 90 = 108 \div 90$
$90y \div 90 = 18 \div 90$	$y = 1.20$
$y = 0.20$	$120\% - 100\% = 20\%$

3. Match each scenario with the equation(s) that could be used to determine the unknown value.
   - **a** The cost of a laptop case increased from \$20 to \$22. What is the percent increase in cost?
   - **b** The number of people attending a picnic increased by 20%. If 22 people originally attended, what is the new attendance?
   - **c** The number of kittens up for adoption decreased from 22 kittens to 20 kittens. What is the percent decrease?

   c $22x = 2$
   b $22 + 0.20 \cdot 22 = x$
   b $1.20 \cdot 22 = x$
   a $20x = 2$
   b $(1 + 0.20) \cdot 22 = x$

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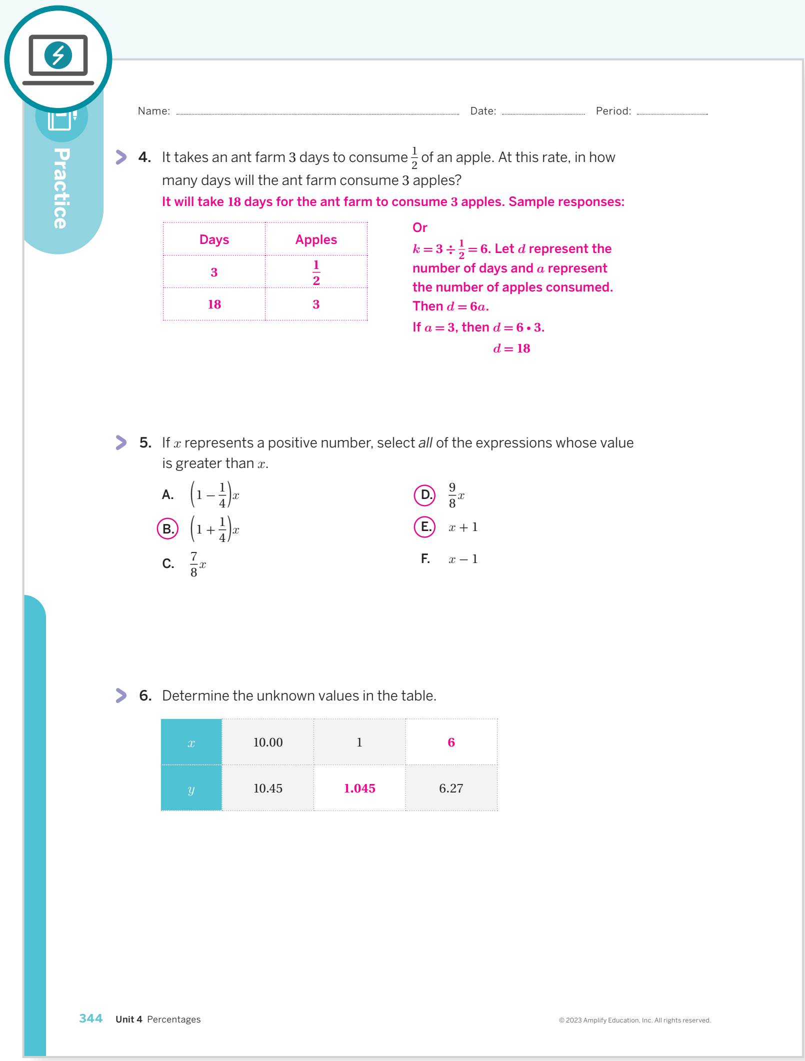
Lesson 7 Using Equations to Solve Percent Problems 343



Practice

Name: Date: Period:

4. It takes an ant farm 3 days to consume $\frac{1}{2}$ of an apple. At this rate, in how many days will the ant farm consume 3 apples?
   It will take 18 days for the ant farm to consume 3 apples. Sample responses:

Days	Apples
3	$\frac{1}{2}$
18	3

Or
$k = 3 \div \frac{1}{2} = 6$. **Let d represent the number of days and a represent the number of apples consumed.**
Then $d = 6a$.
If $a = 3$, then $d = 6 \cdot 3$.
$d = 18$

5. If x represents a positive number, select *all* of the expressions whose value is greater than x.
   - A. $\left(1 - \frac{1}{4}\right)x$
   - [✓ B.] $\left(1 + \frac{1}{4}\right)x$
   - C. $\frac{7}{8}x$
   - [✓ D.] $\frac{9}{8}x$
   - [✓ E.] $x + 1$
   - F. $x - 1$

6. Determine the unknown values in the table.

x	10.00	1	6
y	10.45	1.045	6.27

344 Unit 4 Percentages

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 3	2
Spiral	4	Unit 2 Lesson 2	1
	5	Grade 6	2
Formative	6	Unit 4 Lesson 8	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

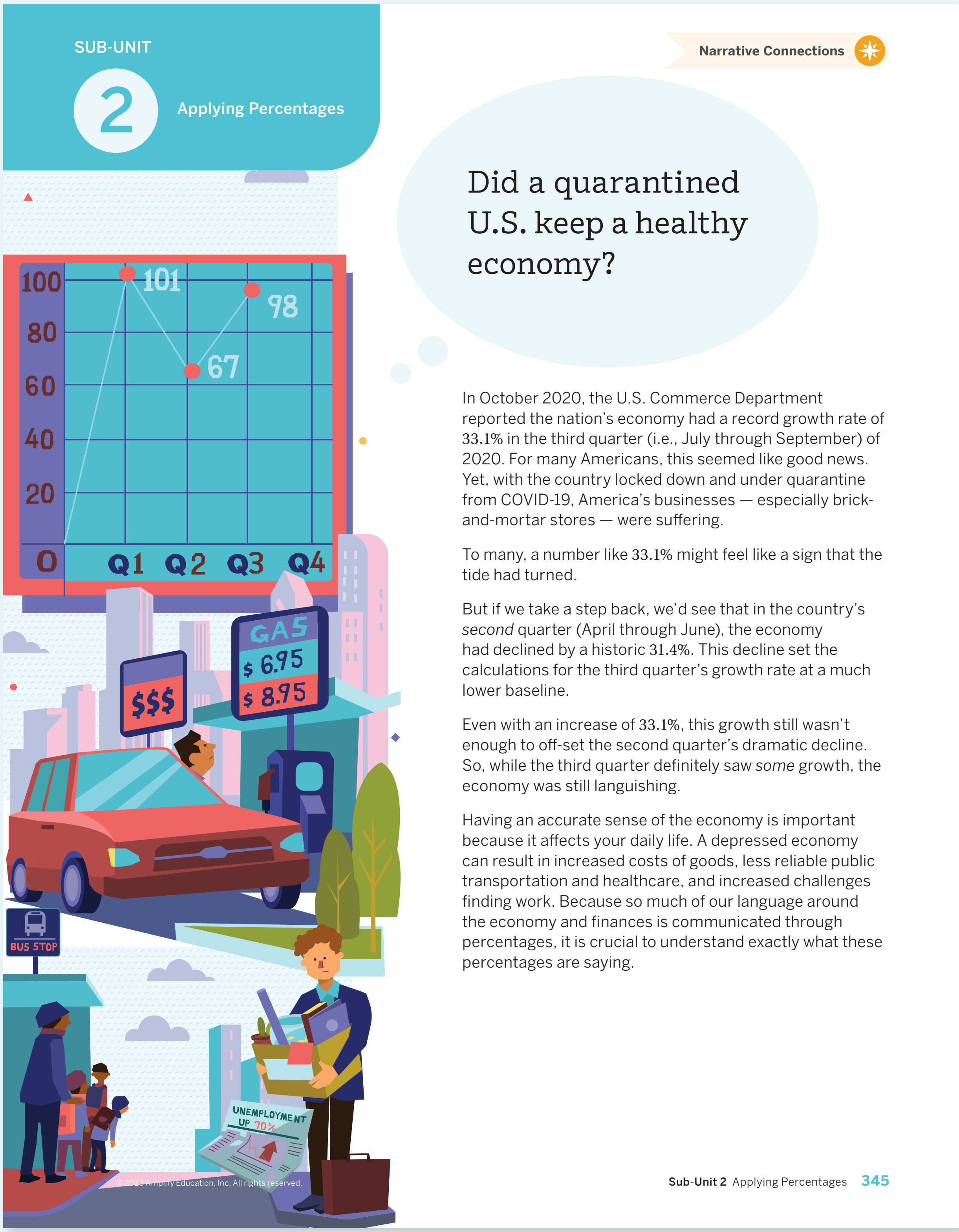
For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 2

Applying Percentages

Whole Class

In this Sub-Unit, students extend their understanding of percentages from the first Sub-Unit, as they apply percent reasoning to financial contexts.



SUB-UNIT

2 Applying Percentages

Narrative Connections

Did a quarantined U.S. keep a healthy economy?

In October 2020, the U.S. Commerce Department reported the nation's economy had a record growth rate of 33.1% in the third quarter (i.e., July through September) of 2020. For many Americans, this seemed like good news. Yet, with the country locked down and under quarantine from COVID-19, America's businesses — especially brick-and-mortar stores — were suffering.

To many, a number like 33.1% might feel like a sign that the tide had turned.

But if we take a step back, we'd see that in the country's *second* quarter (April through June), the economy had declined by a historic 31.4%. This decline set the calculations for the third quarter's growth rate at a much lower baseline.

Even with an increase of 33.1%, this growth still wasn't enough to off-set the second quarter's dramatic decline. So, while the third quarter definitely saw *some* growth, the economy was still languishing.

Having an accurate sense of the economy is important because it affects your daily life. A depressed economy can result in increased costs of goods, less reliable public transportation and healthcare, and increased challenges finding work. Because so much of our language around the economy and finances is communicated through percentages, it is crucial to understand exactly what these percentages are saying.

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Sub-Unit 2 Applying Percentages 345

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore financial applications of percent change and the need for precision in the following places:

- **Lesson 8, Activities 1–2:** Third Place Books, Tipping
- **Lesson 9, Activities 1–2:** Commission at the Barbershop, In Whose Interest is Simple Interest?
- **Lesson 10, Activities 1–2:** What Is the Percentage?, Info Gap: Fair Trade Produce
- **Lesson 11, Activity 1:** Budgeting Tolerance
- **Lesson 12, Activities 1–2:** Editing Headlines, Reporting Responsibly

Tax and Tip

Let's learn about sales tax and tips.

Focus

Goals

1. Comprehend sales tax and tip as two contexts that involve adding a percentage of the original amount.
2. **Language Goal:** Explain how to calculate the total cost including a tax or tip, given the subtotal and the percentage. **(Speaking and Listening)**
3. **Language Goal:** Explain how to determine what percentage of the subtotal is a tax or tip. **(Speaking and Listening)**

Coherence

Today

Students are introduced to contexts involving sales tax and tips. By repeatedly calculating the tax for different prices and then generalizing the process, students are engaging in expressing regularity in repeated reasoning. They are encouraged to use expressions to model a percentage situation and an equation of the form $y = kx$ to solve for unknowns.
Note: While some students may be ready to approach these new contexts using equations, others may prefer to revert to more familiar strategies.

Previously

In Lesson 7, students reasoned about how to use equations to model the *percent of* and the *percent change* between two values.

Coming Soon

Lessons 9–12 will continue to give students ample opportunity to work with percentages in contexts — some familiar but many likely unfamiliar in their experience.

Rigor

- Students build **procedural skills** determining percentages of values, especially with decimals.
- Students **apply** their understanding of writing equations to solve percentage problems involving sales tax and tips.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	12 min	15 min	5 min	8 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Percentage Contexts* (for display)
- Anchor Chart, *Percentage Contexts* (answers)
- Graphic Organizer PDF, *Percentage Tape Diagrams* (as needed)
- calculators
- index cards (optional)

Math Language Development

New words

- ***sales tax***
- ***tip (gratuity)****

*Students may confuse the mathematical term *tip* with its other meanings, such as, "a piece of advice" or "the pointed edge of an object." Be ready to address the differences between them.

Amps | Featured Activity

Activity 1
Multiple Representations

Students can use a table and sketch tape diagrams to organize their thinking around sales tax.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel unmotivated when faced with the task of calculating tips in Activity 2. Have students set a personal goal that relates to tipping that they can work towards. For example, they might decide to be the one who calculates the tips when their family eats out or to create a tip table to help their parents determine how much to leave based on the total bill.

Modifications to Pacing

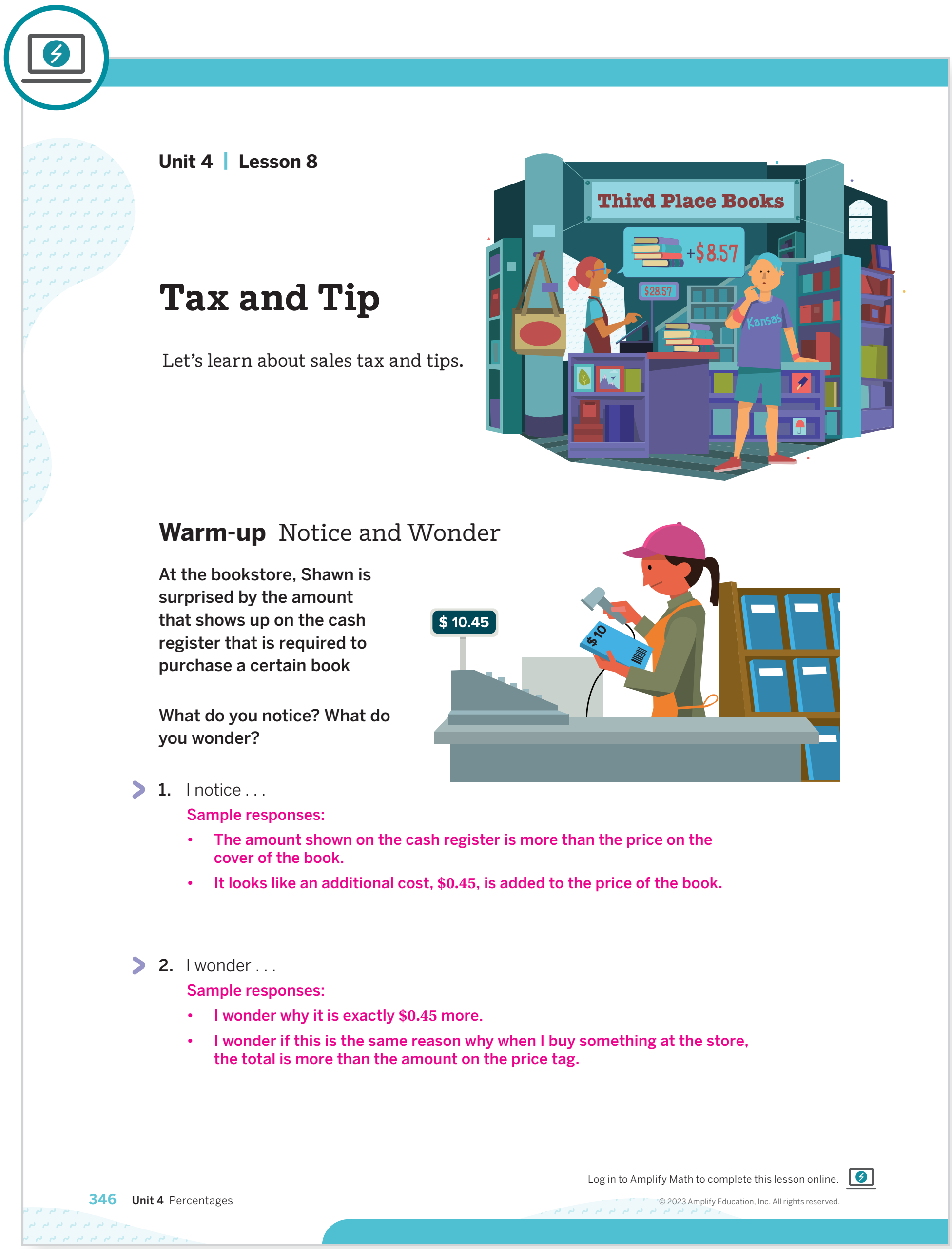
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, assign one table to each student in a pair.
- In **Activity 2**, Problem 2 may be omitted.

Pairs | 5 min

Warm-up Notice and Wonder

Students notice that the marked price of an item is not always the same as the price paid, which prompts a discussion about sales tax.



Unit 4 | Lesson 8

Tax and Tip

Let's learn about sales tax and tips.

Warm-up Notice and Wonder

At the bookstore, Shawn is surprised by the amount that shows up on the cash register that is required to purchase a certain book

What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - The amount shown on the cash register is more than the price on the cover of the book.
   - It looks like an additional cost, $0.45, is added to the price of the book.

2. I wonder . . .
   Sample responses:
   - I wonder why it is exactly $0.45 more.
   - I wonder if this is the same reason why when I buy something at the store, the total is more than the amount on the price tag.

Log in to Amplify Math to complete this lesson online.

1 Launch

Say, "This lesson will introduce some contexts where percents are used in everyday situations involving money." Conduct the *Notice and Wonder* routine.

2 Monitor

Help students get started by having them compare the price on the book in the cashier's hand to the price on the register.

Look for points of confusion:

- **Thinking that the extra money is money that the store earns.** Let students know that they will learn more about how this added amount works throughout the lesson.

Look for productive strategies:

- Deducing that the extra $0.45 is related to a tax, which students may have experience with at stores.

3 Connect

Have students share times they have noticed they had to pay more than the posted price for something at a store.

Define ***sales tax*** as an additional cost, as a rate to the cost of certain goods and services, applied by the government.

Highlight that sales tax always represents an increase to the price. The total cost of the item will be the original price plus the sales tax. Even though sales tax is paid at the time of purchase, the store eventually passes the money to the government. Say, "To calculate sales tax, determine the appropriate percent of the price of an item, and then add the amount of sales tax to get the total cost."

Ask:

- "What was the amount of sales tax that was added to the cost of the book?" $0.45
- "What percent of the original cost is the sales tax?" 4.5%

Power-up

To power up students' ability to determine unknown values in a ratio table, have students complete:

The relationship in the table is proportional.

x	8	6	9.6
y	10	7.5	12

1. What is the constant of proportionality in the table?
   $\frac{5}{4}$ or equivalent
2. Use the constant of proportionality to determine the unknown values.

Use: Before Activity 1.

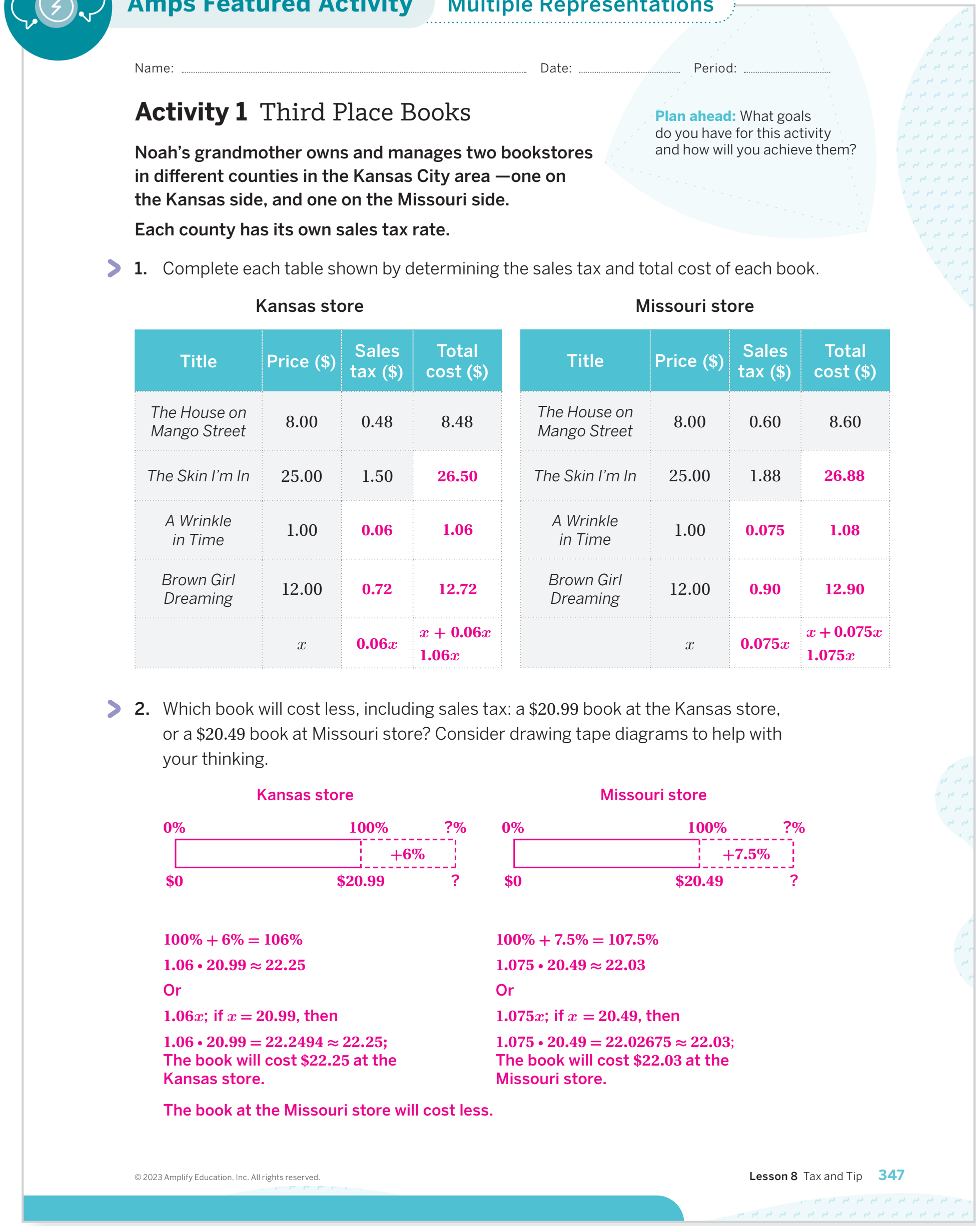
Informed by: Performance on Lesson 7, Practice Problem 6. and Pre-Unit Readiness Assessment, Problem 2.

Pairs | 12 min

Activity 1 Third Place Books

Students compare sales tax at locations with different rates to write equations involving sales tax.

Amps Featured Activity Multiple Representations



Name: ______ Date: ______ Period: ______

Activity 1 Third Place Books

Plan ahead: What goals do you have for this activity and how will you achieve them?

Noah's grandmother owns and manages two bookstores in different counties in the Kansas City area —one on the Kansas side, and one on the Missouri side.

Each county has its own sales tax rate.

1. Complete each table shown by determining the sales tax and total cost of each book.

Kansas store

Title	Price ($)	Sales tax ($)	Total cost ($)
The House on Mango Street	8.00	0.48	8.48
The Skin I'm In	25.00	1.50	**26.50**
A Wrinkle in Time	1.00	**0.06**	**1.06**
Brown Girl Dreaming	12.00	**0.72**	**12.72**
	x	$0.06x$	$x + 0.06x$ $1.06x$

Missouri store

Title	Price ($)	Sales tax ($)	Total cost ($)
The House on Mango Street	8.00	0.60	8.60
The Skin I'm In	25.00	1.88	**26.88**
A Wrinkle in Time	1.00	**0.075**	**1.08**
Brown Girl Dreaming	12.00	**0.90**	**12.90**
	x	$0.075x$	$x + 0.075x$ $1.075x$

2. Which book will cost less, including sales tax: a $20.99 book at the Kansas store, or a $20.49 book at Missouri store? Consider drawing tape diagrams to help with your thinking.

$100\% + 6\% = 106\%$

$1.06 \cdot 20.99 \approx 22.25$

Or

$1.06x$; if $x = 20.99$, then

$1.06 \cdot 20.99 = 22.2494 \approx 22.25$;
The book will cost $22.25 at the Kansas store.

$100\% + 7.5\% = 107.5\%$

$1.075 \cdot 20.49 \approx 22.03$

Or

$1.075x$; if $x = 20.49$, then

$1.075 \cdot 20.49 = 22.02675 \approx 22.03$;
The book will cost $22.03 at the Missouri store.

The book at the Missouri store will cost less.

Lesson 8 Tax and Tip 347

1 Launch

Provide access to calculators throughout the lesson. Make sure students understand the situation by asking, "How much would you have to pay for *The House on Mango Street* in the Kansas store? And for *The Skin I'm In* in the Missouri Store?"

2 Monitor

Help students get started by asking, "Is the sales tax at the Kansas store more or less than 10% of the original price?"

Look for points of confusion:

- **Multiplying the sales tax by the price to get the total cost.** Ask, "Does that match what the first row in the table shows?"
- **Writing $1.06 + x$ as the total cost for the Kansas store expression.** Have students test their expression for one of the rows where they have already calculated the total cost.

Look for productive strategies:

- Using the value of the sales tax in the third row of the tables to help write the expression in the fifth row.

3 Connect

Display student work showing the answers to both tables.

Have students share their solutions for Problem 2, including both a tape diagram and an equation strategy.

Highlight that a problem might ask you to find the amount of the sales tax, or it might ask for the total cost, which is the price plus the sales tax. Sales tax is typically less than 10%.

Ask:

- "How is working with sales tax similar to or different from finding percent increase?"
- "Why might sales tax be calculated as a 6% increase, instead of a $6 increase?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter values into a digital table and see their responses validated in real time. This empowers them to make any needed adjustments to their own thinking while they progress through the activity.

Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on the table for the Kansas store. This table covers the same understandings as the Missouri store, without the fractional percentage. In Problem 2, have them determine the final cost of the book at the Kansas store. Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* if students choose to draw a tape diagram.

Math Language Development

MLR6: Three Reads

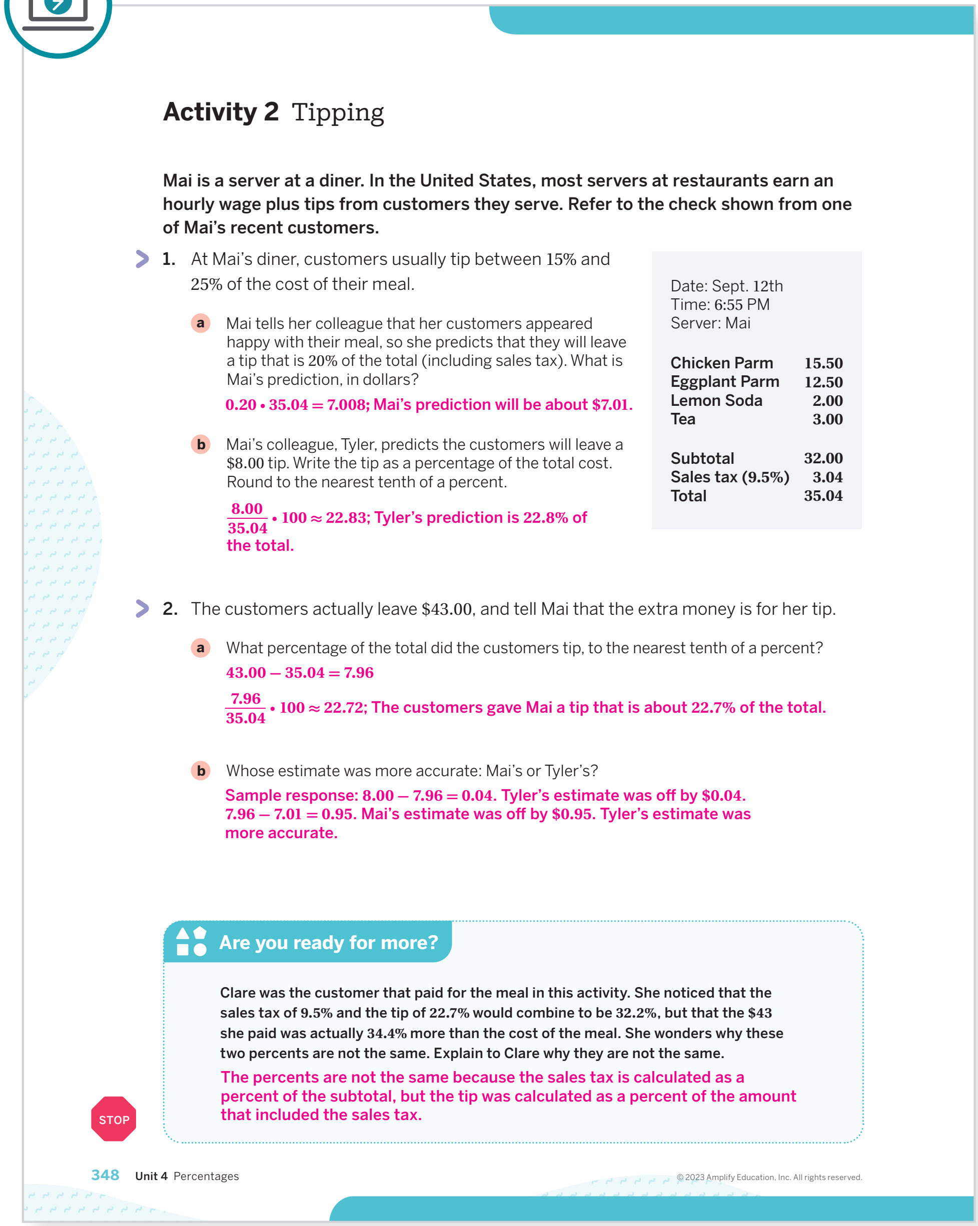
Use this routine to help students make sense of the introductory text and the two tables in Problem 1.

- **Read 1:** Students should understand that there are two bookstores and each bookstore has its own sales tax rate.
- **Read 2:** Ask students to name or highlight the given quantities, such as "The House on Mango Street costs more at the Missouri bookstore because the sales tax is a greater amount."
- **Read 3:** Ask students to brainstorm strategies they can use to complete the table and determine which book will cost less in Problem 2.

Pairs | 15 min

Activity 2 Tipping

Students calculate percentages and dollar amounts within the context of tipping, and work with fractional parts of a percent.



Activity 2 Tipping

Mai is a server at a diner. In the United States, most servers at restaurants earn an hourly wage plus tips from customers they serve. Refer to the check shown from one of Mai's recent customers.

Date: Sept. 12th
Time: 6:55 PM
Server: Mai

Chicken Parm	15.50
Eggplant Parm	12.50
Lemon Soda	2.00
Tea	3.00
Subtotal	32.00
Sales tax (9.5%)	3.04
Total	35.04

1. At Mai's diner, customers usually tip between 15% and 25% of the cost of their meal.

   a. Mai tells her colleague that her customers appeared happy with their meal, so she predicts that they will leave a tip that is 20% of the total (including sales tax). What is Mai's prediction, in dollars?

   $0.20 \cdot 35.04 = 7.008$; Mai's prediction will be about \$7.01.

   b. Mai's colleague, Tyler, predicts the customers will leave a \$8.00 tip. Write the tip as a percentage of the total cost. Round to the nearest tenth of a percent.

   $\frac{8.00}{35.04} \cdot 100 \approx 22.83$; Tyler's prediction is 22.8% of the total.

2. The customers actually leave \$43.00, and tell Mai that the extra money is for her tip.

   a. What percentage of the total did the customers tip, to the nearest tenth of a percent?

   $43.00 - 35.04 = 7.96$

   $\frac{7.96}{35.04} \cdot 100 \approx 22.72$; The customers gave Mai a tip that is about 22.7% of the total.

   b. Whose estimate was more accurate: Mai's or Tyler's?

   Sample response: $8.00 - 7.96 = 0.04$. Tyler's estimate was off by \$0.04. $7.96 - 7.01 = 0.95$. Mai's estimate was off by \$0.95. Tyler's estimate was more accurate.

Are you ready for more?

Clare was the customer that paid for the meal in this activity. She noticed that the sales tax of 9.5% and the tip of 22.7% would combine to be 32.2%, but that the \$43 she paid was actually 34.4% more than the cost of the meal. She wonders why these two percents are not the same. Explain to Clare why they are not the same.

The percents are not the same because the sales tax is calculated as a percent of the subtotal, but the tip was calculated as a percent of the amount that included the sales tax.

STOP

348 **Unit 4** Percentages

1 Launch

Activate background knowledge by asking students if their families have ever asked them to help determine the tip at a restaurant.

2 Monitor

Help students get started by asking which values on the receipt are relevant to the problem.

Look for points of confusion:

- **Calculating the tip on the subtotal amount in Problem 2.** Have students reread part a to identify what they are finding 20% of.

Look for productive strategies:

- Using an equation, including a decimal representation of the percentage, to reason about Problem 2.

3 Connect

Have students share a strategy that includes using an equation. Ask students who did not use an equation to identify the meaning of each term in the equation, and label them for the class to see.

Ask, "How would the amount of the tip change if it was calculated based on the pre-tax subtotal amount? Why?"

Define *tip*, or ***gratuity*** (as it is sometimes referred), as an amount given to a server at a restaurant that is calculated as a percentage of the bill.

Highlight that customers can choose to tip on the total before or after sales tax. Have students be on the lookout for what the problem prompts them to do. Explain that, even though servers often earn a base wage plus tips, the wage is often less than minimum wage.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students what methods their families or friends have used to determine the tip at a restaurant. Many students may say that they use a calculator or that the bill often includes suggested amounts for different percentages.

During the Connect, mention that the term *tip* is the term most often used, but some places will use the term *gratuity*. Students from other countries may have different experiences with tips and some countries may have different tipping practices. Ask students, who are comfortable to do so, to share different tipping practices with which they may be familiar.

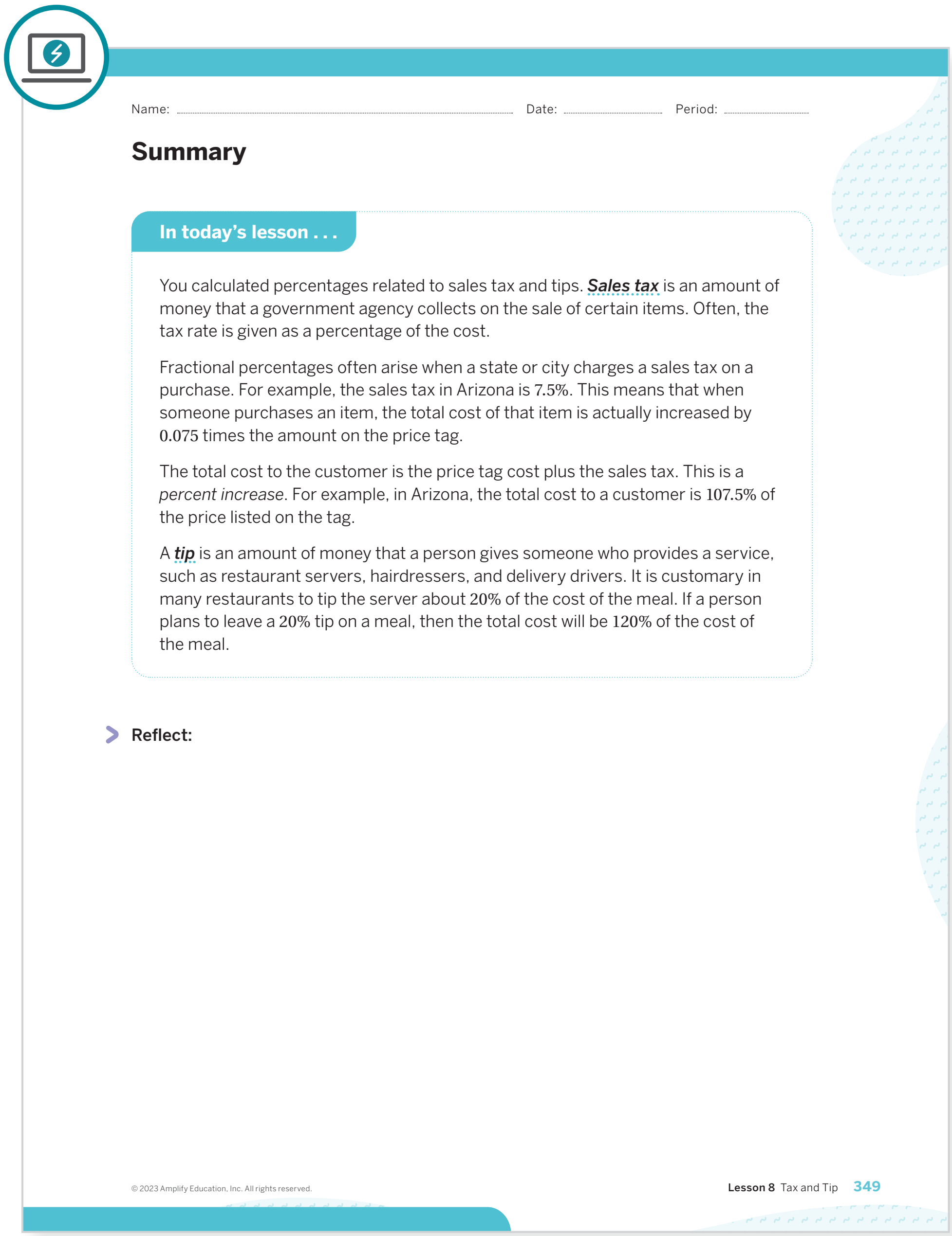
Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problem 2. Provide them with an index card or sticky note to cover up information they do not need in order to solve the problem. Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* if they choose to draw tape diagrams.

Whole Class | 5 min

Summary

Review and synthesize that sales tax and tips are two examples of contexts involving percent increase that relate to money.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You calculated percentages related to sales tax and tips. ***Sales tax*** is an amount of money that a government agency collects on the sale of certain items. Often, the tax rate is given as a percentage of the cost.

Fractional percentages often arise when a state or city charges a sales tax on a purchase. For example, the sales tax in Arizona is 7.5%. This means that when someone purchases an item, the total cost of that item is actually increased by 0.075 times the amount on the price tag.

The total cost to the customer is the price tag cost plus the sales tax. This is a *percent increase*. For example, in Arizona, the total cost to a customer is 107.5% of the price listed on the tag.

A ***tip*** is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. It is customary in many restaurants to tip the server about 20% of the cost of the meal. If a person plans to leave a 20% tip on a meal, then the total cost will be 120% of the cost of the meal.

Reflect:

Synthesize

Formalize vocabulary:

- *sales tax*
- *tip (gratuity)*

Display the Anchor Chart PDF, *Percentage Contexts*.

Highlight that students have seen and used various strategies for solving percentage problems throughout the unit so far. Now that they will be dealing with new contexts, often involving multiple steps, it may be more efficient to model the percentage context using equations to reduce the number of steps needed to solve the problem. Add suggested content for the sales tax and tip rows from the Anchor Chart PDF, *Percentage Contexts* (answers) to the student-facing version.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can using an equation to model a percentage context help to solve problems involving sales tax and tip?"

Math Language Development

MLR2: Collect and Display

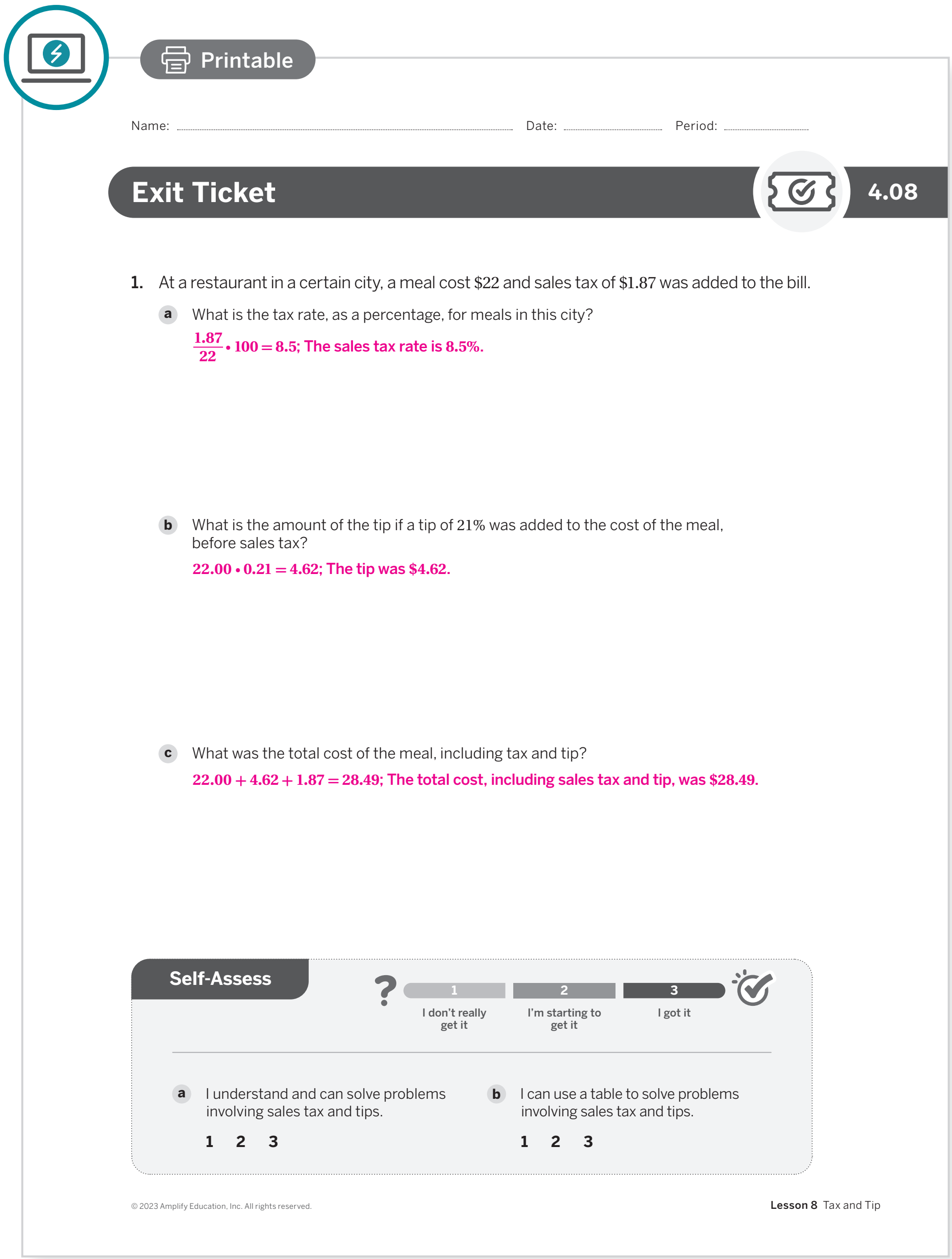
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the terms sales tax and *tip (gratuity)* that were added to the display during the lesson.

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of sales tax and tips by finding the total cost of a meal at a restaurant.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket

4.08

1. At a restaurant in a certain city, a meal cost \$22 and sales tax of \$1.87 was added to the bill.

   a What is the tax rate, as a percentage, for meals in this city?

   $\frac{1.87}{22} \cdot 100 = 8.5$; The sales tax rate is 8.5%.

   b What is the amount of the tip if a tip of 21% was added to the cost of the meal, before sales tax?

   $22.00 \cdot 0.21 = 4.62$; The tip was \$4.62.

   c What was the total cost of the meal, including tax and tip?

   $22.00 + 4.62 + 1.87 = 28.49$; The total cost, including sales tax and tip, was \$28.49.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I understand and can solve problems involving sales tax and tips.

1 2 3

b I can use a table to solve problems involving sales tax and tips.

1 2 3

Success looks like . . .

- **Goal:** Comprehending sales tax and tip as two contexts that involve adding a percentage of the original amount.
  - » Calculating the tax rate and tip in parts a and b and then adding them together with the cost of the meal in part c.
- **Language Goal:** Explaining how to calculate the total cost including a tax or tip, given the subtotal and the percentage. **(Speaking and Listening)**
  - » Calculating the total cost of the meal in part c.
- **Language Goal:** Explaining how to determine what percentage of the subtotal is a tax or tip. **(Speaking and Listening)**
  - » Determining the tax rate for meals in the city in part a.

Suggested next steps

If students give the tax rate as 0.085% in Problem 1a, consider:

- Having them determine 1% of \$22 and comparing that to the value of \$1.87.
- Assigning Practice Problem 1.

If students calculate the tip based on the cost of the meal including sales tax, consider:

- Reviewing that a tip can be calculated either before or after tax is added.

If students forget to add both the sales tax and the tip to determine the total cost, consider:

- Reviewing Problem 3 in Activity 2.

Professional Learning

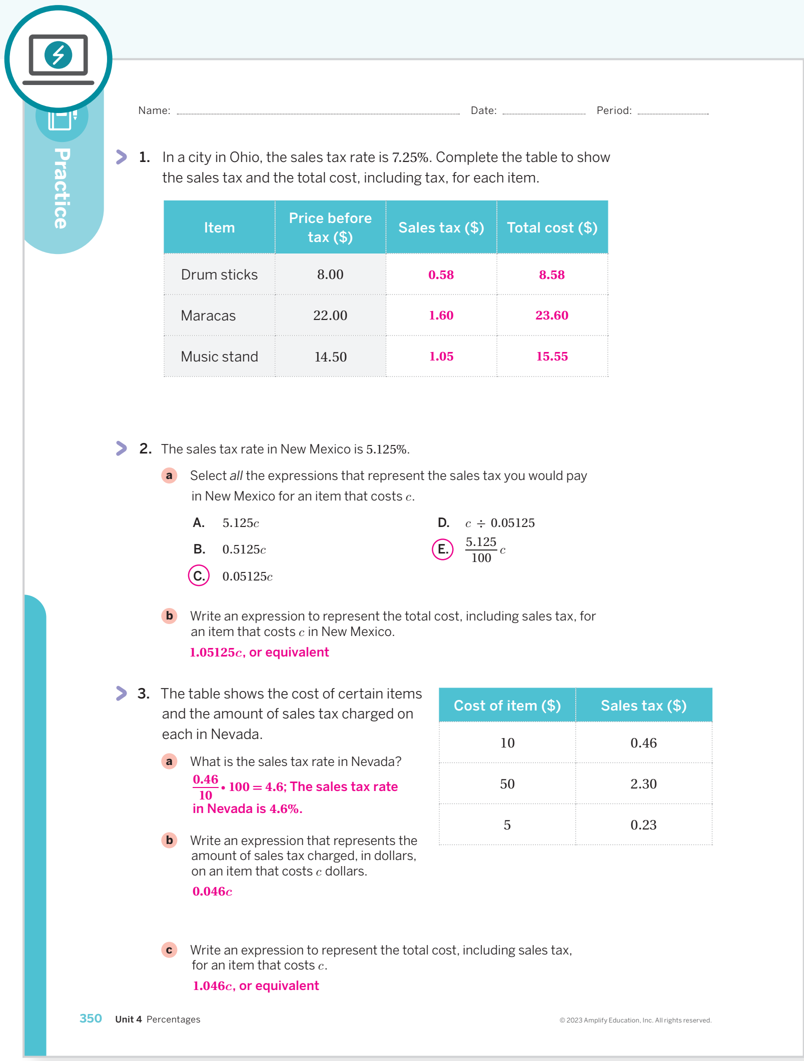
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students worked with sales tax and tip. How did that build on the earlier work they did involving percent increase and decrease?
- In what ways have your students shown improvement in generalizing an algebraic expression using repeated reasoning? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. In a city in Ohio, the sales tax rate is 7.25%. Complete the table to show the sales tax and the total cost, including tax, for each item.

Item	Price before tax ($)	Sales tax ($)	Total cost ($)
Drum sticks	8.00	**0.58**	**8.58**
Maracas	22.00	**1.60**	**23.60**
Music stand	14.50	**1.05**	**15.55**

2. The sales tax rate in New Mexico is 5.125%.
   - a Select *all* the expressions that represent the sales tax you would pay in New Mexico for an item that costs c.
     - A. $5.125c$
     - B. $0.5125c$
     - [✓ C.] $0.05125c$
     - D. $c \div 0.05125$
     - [✓ E.] $\frac{5.125}{100}c$
   - b Write an expression to represent the total cost, including sales tax, for an item that costs c in New Mexico.

     $1.05125c$, or equivalent

3. The table shows the cost of certain items and the amount of sales tax charged on each in Nevada.

Cost of item ($)	Sales tax ($)
10	0.46
50	2.30
5	0.23

   - a What is the sales tax rate in Nevada?

     $\frac{0.46}{10} \cdot 100 = 4.6$; The sales tax rate in Nevada is 4.6%.
   - b Write an expression that represents the amount of sales tax charged, in dollars, on an item that costs c dollars.

     $0.046c$
   - c Write an expression to represent the total cost, including sales tax, for an item that costs c.

     $1.046c$, or equivalent

350 **Unit 4** Percentages

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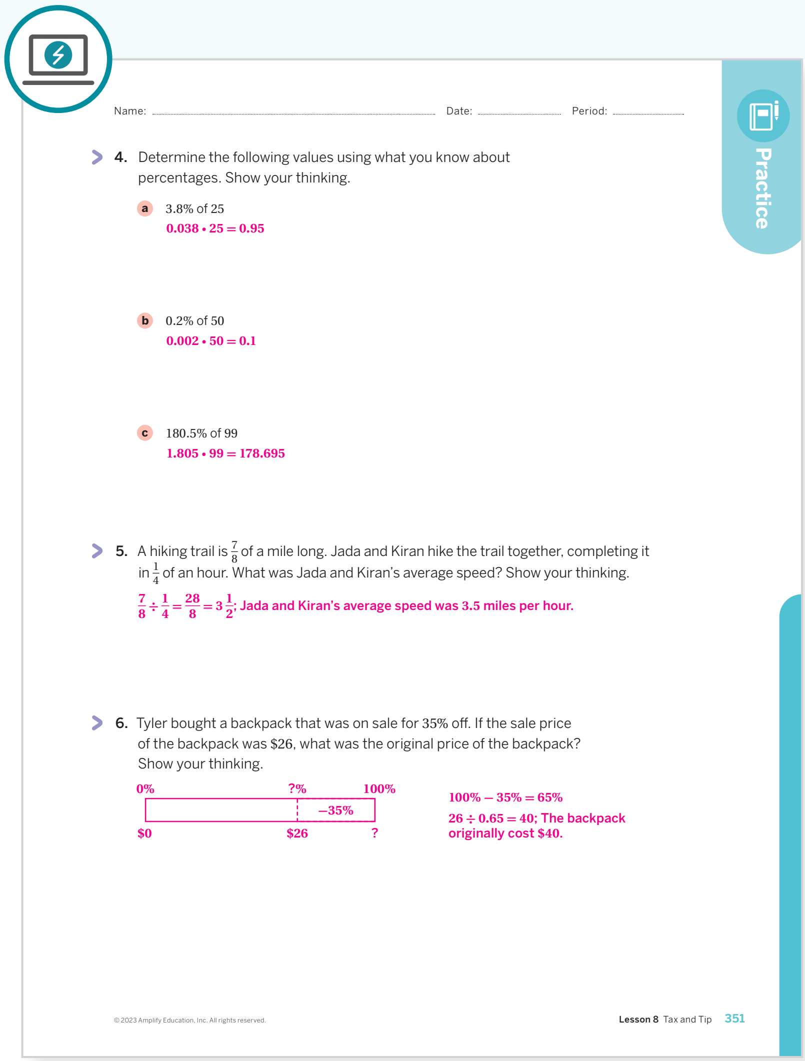

Name: Date: Period:

4. Determine the following values using what you know about percentages. Show your thinking.
   - a 3.8% of 25

     $0.038 \cdot 25 = 0.95$
   - b 0.2% of 50

     $0.002 \cdot 50 = 0.1$
   - c 180.5% of 99

     $1.805 \cdot 99 = 178.695$

5. A hiking trail is $\frac{7}{8}$ of a mile long. Jada and Kiran hike the trail together, completing it in $\frac{1}{4}$ of an hour. What was Jada and Kiran's average speed? Show your thinking.

   $\frac{7}{8} \div \frac{1}{4} = \frac{28}{8} = 3\frac{1}{2}$; Jada and Kiran's average speed was 3.5 miles per hour.

6. Tyler bought a backpack that was on sale for 35% off. If the sale price of the backpack was \$26, what was the original price of the backpack? Show your thinking.

   $100\% - 35\% = 65\%$

   $26 \div 0.65 = 40$; The backpack originally cost \$40.

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Lesson 8 Tax and Tip 351

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 2	1
	5	Unit 2 Lesson 6	2
Formative ⚡	6	Unit 4 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

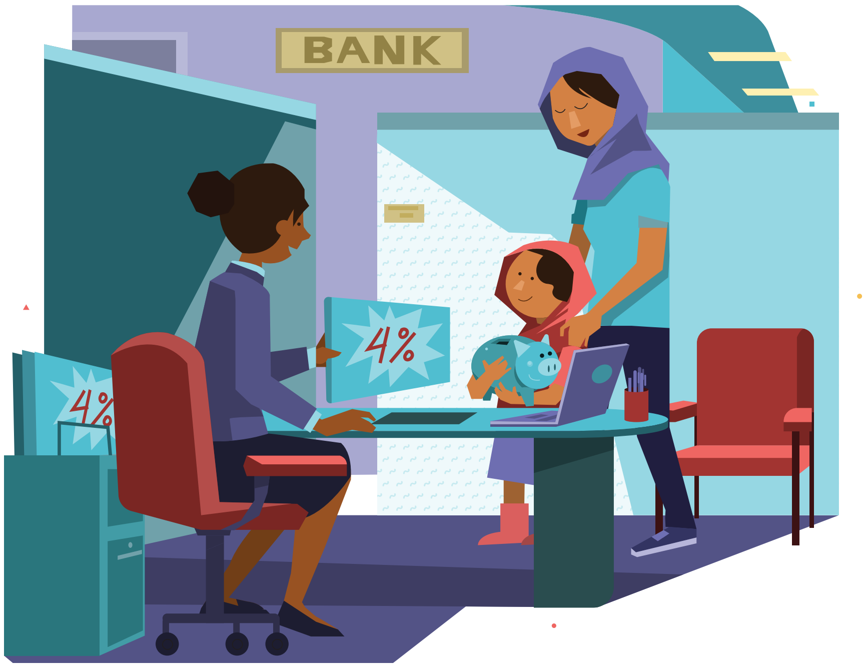
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 4 | **LESSON 9**

Percentage Contexts

Let’s learn about more situations that involve percentages.



Focus

Goals

1. Comprehend interest and commission as other contexts that involve adding or subtracting a percentage of the initial amount.
2. **Language Goal:** Explain how to calculate a new amount after commission or interest is applied. **(Speaking and Listening)**
3. Solve multi-step problems involving commission or interest.

Coherence

Today

Students dive further into percentage contexts, now involving commission and simple interest. As these financial contexts are often related to having a job or a bank account, students are less likely to have direct experience with them, but may perhaps be considering getting one or both soon. Students continue to develop their reasoning for solving percent increase and decrease problems in context, especially using equations.

Previously

Students were introduced to sales tax and tip in Lesson 8.

Coming Soon

Students will further develop their ability to reason about percentages in contexts involving money in Lesson 10, which cumulatively incorporates all of the contexts learned within the unit.

Rigor

- Students continue to develop **procedural skills** determining unknown values and percentages within real-world contexts.
- Students **apply** their understanding of solving percentage problems to new contexts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one copy per pair
- Anchor Chart PDF, *Percentage Contexts* (for display)
- Anchor Chart PDF, *Percentage Contexts* (answers)
- Graphic Organizer PDF, *Percentage Tape Diagrams (as needed)*
- calculators

Math Language Development

New words

- ***commission***
- ***simple interest***

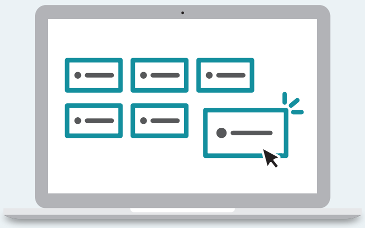
Review words

- *markdown*
- *markup*
- *sales tax*
- *tip (gratuity)*

Amps | Featured Activity

Activity 3
Digital Card Sort

Monitor student understanding of vocabulary terms related to percentages as they complete a card sort matching real-world scenarios to mathematical concepts.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not think that they will ever hold a commission job, so they might not think that the context in Activity 1 is applicable to them. Encourage students to appreciate the diversity of the types of jobs and payment structures. By taking on the perspective of those trying to earn a living through commissions as they make sense of these problems and persevere in understanding them, students will recognize and value the hard work those individuals undertake.

Modifications to Pacing

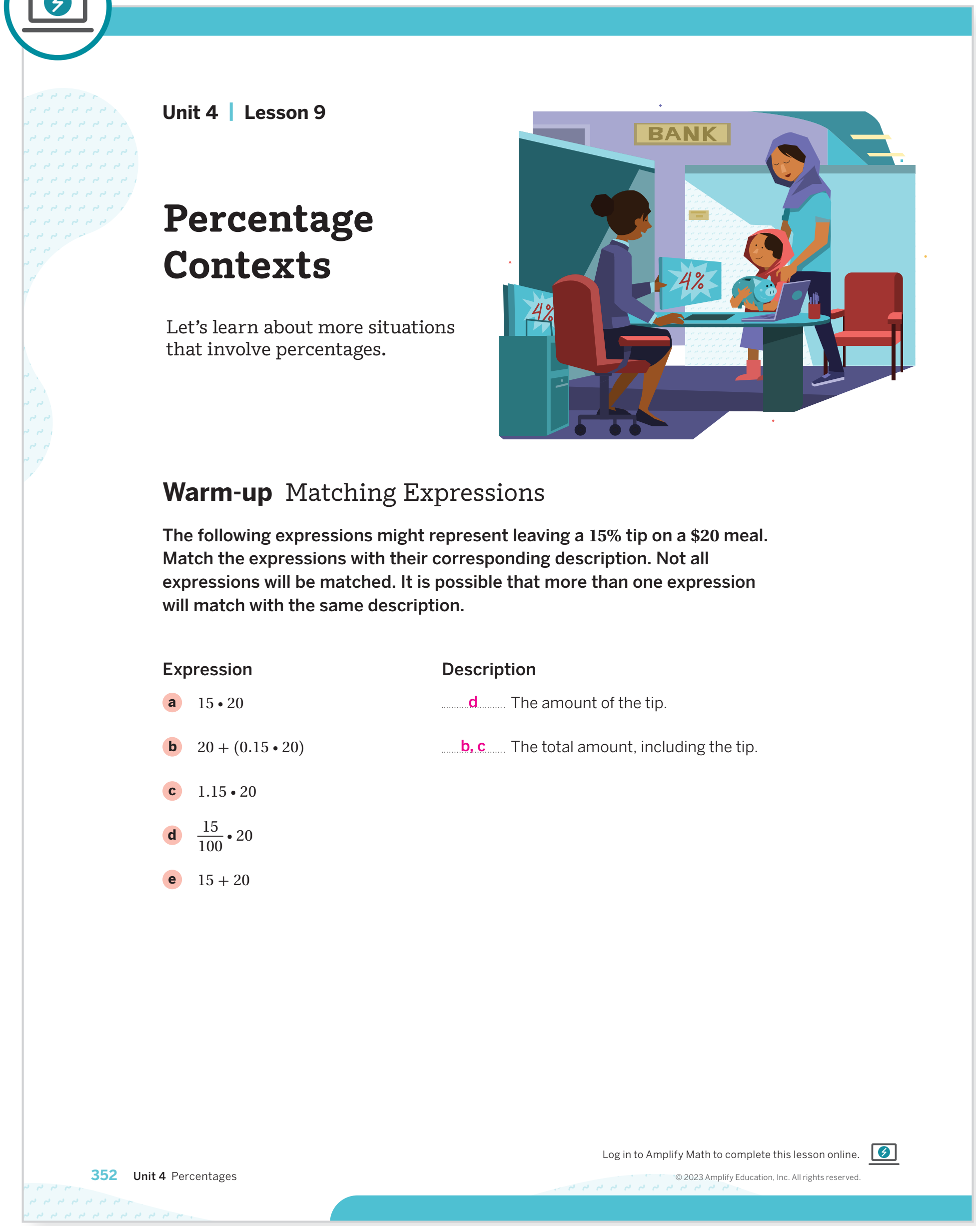
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 3 may be omitted
- In **Activity 2**, Problem 2 may be omitted.
- Optional **Activity 3** may be omitted.

Pairs | 5 min

Warm-up Matching Expressions

Students make deeper connections between the context of tipping and their prior work with percents, now using more efficient ways of finding this percent increase.



Unit 4 | Lesson 9

Percentage Contexts

Let's learn about more situations that involve percentages.

Warm-up Matching Expressions

The following expressions might represent leaving a 15% tip on a $20 meal. Match the expressions with their corresponding description. Not all expressions will be matched. It is possible that more than one expression will match with the same description.

Expression

a $15 \cdot 20$

b $20 + (0.15 \cdot 20)$

c $1.15 \cdot 20$

d $\frac{15}{100} \cdot 20$

e $15 + 20$

Description

d The amount of the tip.

b, c The total amount, including the tip.

352 Unit 4 Percentages

Log in to Amplify Math to complete this lesson online.

1 Launch

Provide access to calculators throughout the lesson. Conduct the *Think-Pair-Share* routine. Have students work with a partner and prompt them to discuss the structure of the expressions.

2 Monitor

Help students get started by prompting them to think about whether the amount of the tip and the total amount will be more or less than $20.

Look for points of confusion:

- **Thinking that multiplying by 15 is the same as finding 15% of a number.** Ask, "Will 15% of 20 be more or less than 20?" Then have students evaluate the expression to see if it matches their intuition.

Look for productive strategies:

- Evaluating each expression.
- Reasoning using estimation, e.g., "The amount of the tip must be less that 20, so it cannot be a, b, c, or e."

3 Connect

Have pairs of students share why expressions a and e do not fit either of the descriptions.

Ask:

- "How can estimation help to make sense of expressions related to percentage contexts?"
- "What evidence is there of percentages in some of the expressions? Will this always be the case?"

Highlight that expressions involving percentages will often involve a quantity between 0 and 1. Be sure to note that this will not always be the case — percentages of greater than 100% will be greater than 1 in these expressions.

Power-up

To power up students' ability to determine the original amount, given a percent and the partial amount, have students complete:

Mai buys a new basketball for 20% off. The sales price was $14.

1. Is the original price of the basketball *greater than* or *less than* $14? Greater than $14.
2. Which expressions or equations could be used to determine the original amount? Select *all* that apply.

[✓ A.] $0.70x = 14$
B. $0.70 \cdot 14$
C. $14x = 0.70$
[✓ D.] $14 \div 0.70$

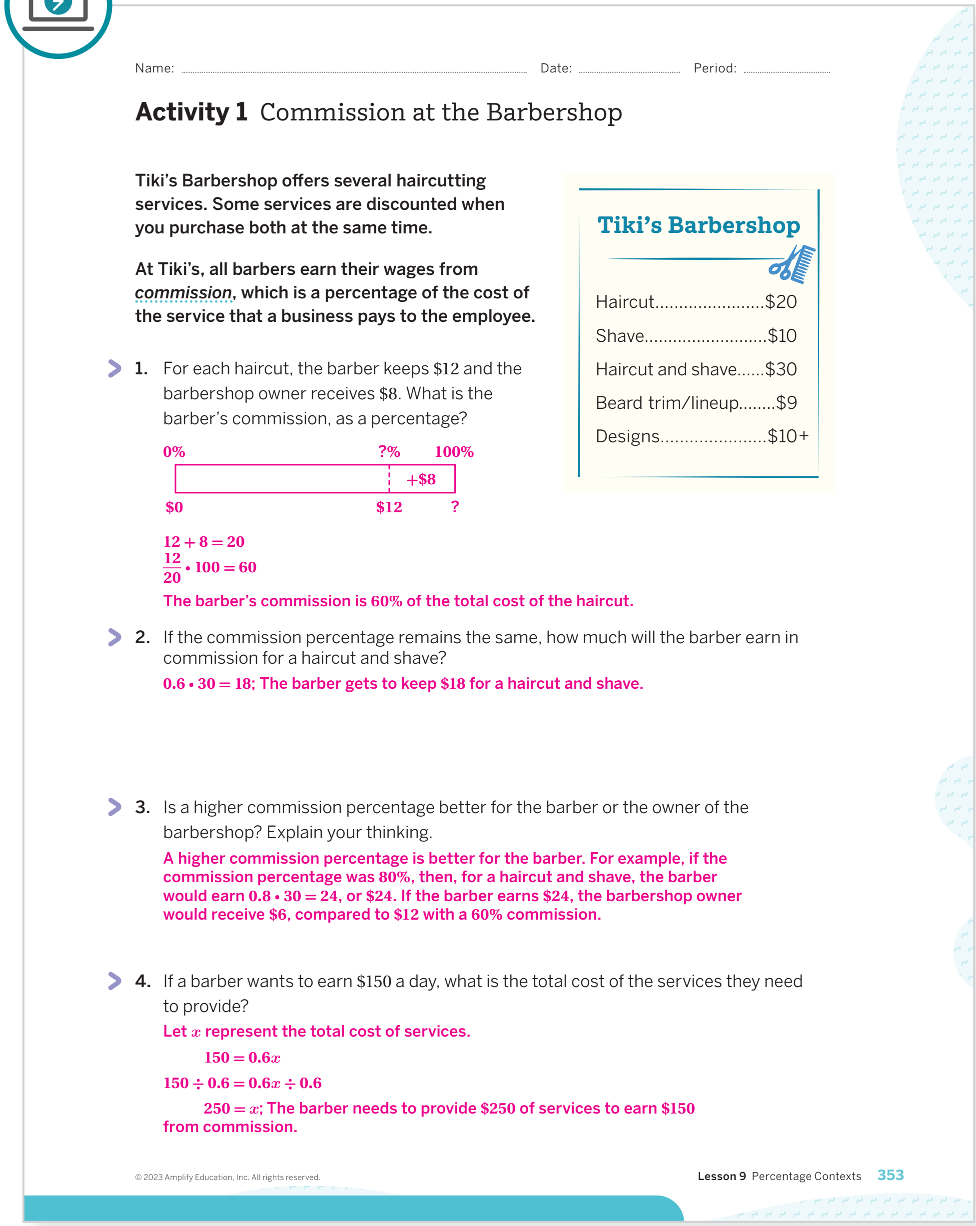
Use: Before Activity 1.

Informed by: Performance on Lesson 8, Practice Problem 6.

Pairs | 15 min

Activity 1 Commission at the Barbershop

Students are introduced to the concept of a commission and solve percentage problems in that context.



Name: ______ Date: ______ Period: ______

Activity 1 Commission at the Barbershop

Tiki's Barbershop offers several haircutting services. Some services are discounted when you purchase both at the same time.

At Tiki's, all barbers earn their wages from *commission*, which is a percentage of the cost of the service that a business pays to the employee.

Tiki's Barbershop

Haircut	$20
Shave	$10
Haircut and shave	$30
Beard trim/lineup	$9
Designs	$10+

1. For each haircut, the barber keeps $12 and the barbershop owner receives $8. What is the barber's commission, as a percentage?

$12 + 8 = 20$

$\frac{12}{20} \cdot 100 = 60$

The barber's commission is 60% of the total cost of the haircut.

2. If the commission percentage remains the same, how much will the barber earn in commission for a haircut and shave?

$0.6 \cdot 30 = 18$; The barber gets to keep $18 for a haircut and shave.

3. Is a higher commission percentage better for the barber or the owner of the barbershop? Explain your thinking.

A higher commission percentage is better for the barber. For example, if the commission percentage was 80%, then, for a haircut and shave, the barber would earn $0.8 \cdot 30 = 24$, or $24. If the barber earns $24, the barbershop owner would receive $6, compared to $12 with a 60% commission.

4. If a barber wants to earn $150 a day, what is the total cost of the services they need to provide?

Let x represent the total cost of services.

$150 = 0.6x$

$150 \div 0.6 = 0.6x \div 0.6$

$250 = x$; The barber needs to provide $250 of services to earn $150 from commission.

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Lesson 9 Percentage Contexts 353

1 Launch

Activate background knowledge by asking students if, when shopping, they have ever been asked, "Did someone help you today?" Explain that *commission* is money paid to a worker, and is calculated as a percentage of the money earned from their work. It is typical for salespeople and certain service workers to earn commission.

2 Monitor

Help students get started by encouraging them to draw a tape diagram for Problem 1.

Look for points of confusion:

- **Thinking they need to find what percent of 8 is 12.** Explain that commission is a percentage of the total cost and ask, "What is the total cost in this case?"
- **In Problem 4, finding 60% of $150.** Ask, "Does it make sense that the barber earns more than the services they provided?"

Look for productive strategies:

- Noticing that the $12 and $8 are two parts of a whole.

3 Connect

Define *commission* as a fee paid for services, usually a percentage of the total cost.

Ask, "Is commission similar or different to percent increase and decrease problems? How?"

Highlight that commission typically works like this: A customer pays a business for something. The business keeps track of who was the salesperson, or who provided the service. The business then pays the worker with a percentage of the amount paid by the customer.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* if students choose to draw tape diagrams.

Extension: *Math Enrichment*

Have students write an equation that can be used to determine the number of haircuts h needed for the barber to earn m dollars.

$0.6(20h) = m$

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the following incorrect statement for Problem 2, For the haircut and shave, the barber would earn $12." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Why or why not?"
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "What was the most likely misunderstanding of the person who wrote this incorrect statement?" They found 60% of $30, which is $18, but then they subtracted this amount from $30, thinking it was a percent change problem.

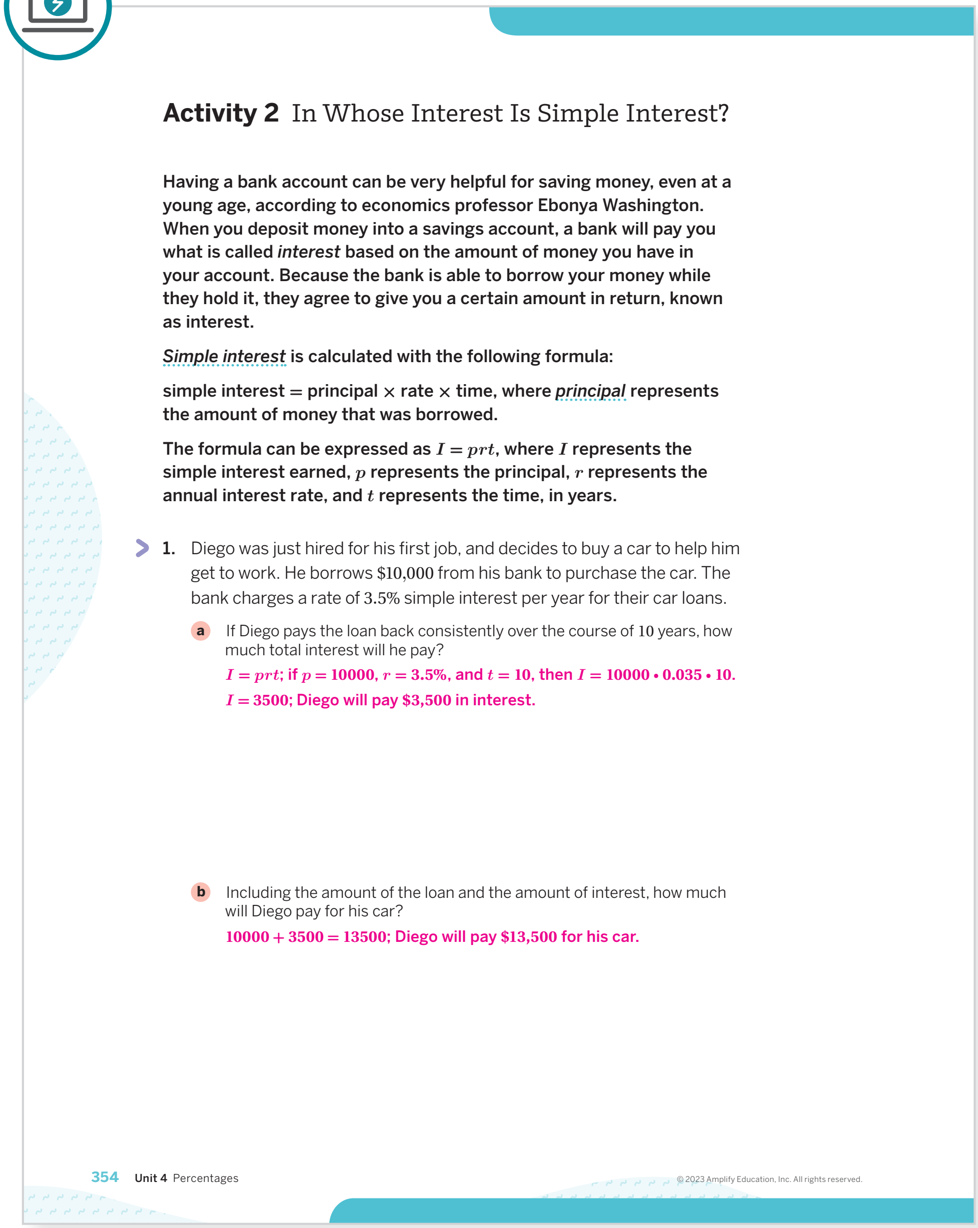
English Learners

Allow students time to rehearse what they will say with a partner before sharing with the whole class.

Pairs | 15 min

Activity 2 In Whose Interest Is Simple Interest?

Students are introduced to two forms of simple interest — the kind a bank pays and also the kind a bank collects — and become more familiar with this context.



Activity 2 In Whose Interest Is Simple Interest?

Having a bank account can be very helpful for saving money, even at a young age, according to economics professor Ebonya Washington. When you deposit money into a savings account, a bank will pay you what is called *interest* based on the amount of money you have in your account. Because the bank is able to borrow your money while they hold it, they agree to give you a certain amount in return, known as interest.

***Simple interest* is calculated with the following formula:**

simple interest = principal × rate × time, where *principal* represents the amount of money that was borrowed.

The formula can be expressed as $I = prt$, where I represents the simple interest earned, p represents the principal, r represents the annual interest rate, and t represents the time, in years.

1. Diego was just hired for his first job, and decides to buy a car to help him get to work. He borrows $10,000 from his bank to purchase the car. The bank charges a rate of 3.5% simple interest per year for their car loans.

   a. If Diego pays the loan back consistently over the course of 10 years, how much total interest will he pay?

   $I = prt$; if $p = 10000$, $r = 3.5\%$, and $t = 10$, then $I = 10000 \cdot 0.035 \cdot 10$. $I = 3500$; Diego will pay $3,500 in interest.

   b. Including the amount of the loan and the amount of interest, how much will Diego pay for his car?

   $10000 + 3500 = 13500$; Diego will pay $13,500 for his car.

354 Unit 4 Percentages

1 Launch

Read through the introduction as a class. Explain that the *principal* is the amount of money at the start — either borrowed or put into a bank account. While this word is not considered new vocabulary, it will help to have students write the meaning down.

2 Monitor

Help students get started by having them annotate Problem 1 with the letters p, r, and t for the corresponding values within the situation.

Look for points of confusion:

- **Forgetting that the total amount paid, or saved, will be the principal plus the interest**. Have students draw a tape diagram to represent the situation.
- **Feeling unsure of how to get started on Problem 2**. Help students see that by depositing $275, Diego is, in a way, lending this money to the bank. Therefore, they can consider this amount the *principal*.

Look for productive strategies:

- For Problem 2, writing the equation out fully, substituting in known values, and solving for the unknown value.

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they have ever opened a bank account or are familiar with how bank accounts work. Consider demonstrating a simple example of depositing $100 into a bank account that earns a certain amount of simple interest each year and illustrate how the bank account grows each year, assuming no deposits or withdrawals.

Similarly, before students attempt Problem 1, ask them whether they are familiar with how loans work. Tell them that it is a similar process to earning money from your bank account; yet with a loan, you now owe the bank because you are borrowing from them.

Math Language Development

MLR2: Collect and Display

During the Connect, add the term *simple interest* to the class display and collect language students use to discuss simple interest in the contexts of the problems in this activity. Display the formula for simple interest and give students time to ask questions and make sense of the structure of the formula. Be sure students understand that time in the formula represents the number of years, and that both the time and how the interest rate is calculated needs to be considered in context of the problem.

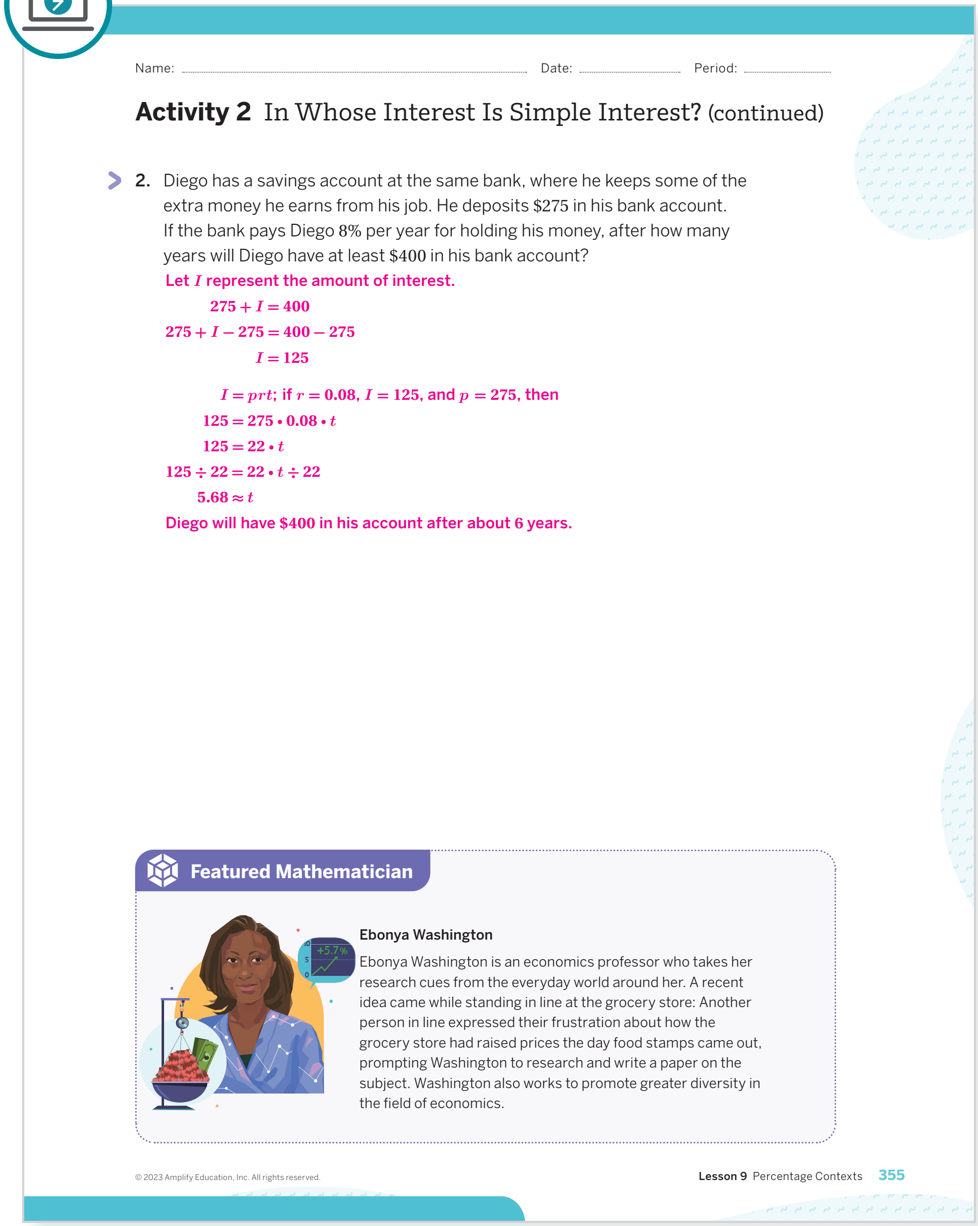
English Learners

Annotate the formula on the class display with each variable defined and spelled out.

Pairs | 15 min

Activity 2 In Whose Interest Is Simple Interest? (continued)

Students are introduced to two forms of simple interest — the kind a bank pays and also the kind a bank collects — and become more familiar with this context.



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Activity 2 In Whose Interest Is Simple Interest? (continued)

2. Diego has a savings account at the same bank, where he keeps some of the extra money he earns from his job. He deposits $275 in his bank account. If the bank pays Diego 8% per year for holding his money, after how many years will Diego have at least $400 in his bank account?

Let I represent the amount of interest.

$$275 + I = 400$$
$$275 + I - 275 = 400 - 275$$
$$I = 125$$

$I = prt$; if $r = 0.08$, $I = 125$, and $p = 275$, then

$$125 = 275 \cdot 0.08 \cdot t$$
$$125 = 22 \cdot t$$
$$125 \div 22 = 22 \cdot t \div 22$$
$$5.68 \approx t$$

Diego will have $400 in his account after about 6 years.

Featured Mathematician

Ebonya Washington

Ebonya Washington is an economics professor who takes her research cues from the everyday world around her. A recent idea came while standing in line at the grocery store: Another person in line expressed their frustration about how the grocery store had raised prices the day food stamps came out, prompting Washington to research and write a paper on the subject. Washington also works to promote greater diversity in the field of economics.

3 Connect

Have pairs of students share their solution strategies for Problem 2.

Define *simple interest* as an amount of money that is added on to an original amount, usually meant to be paid back to a bank savings account holder.

Ask:

- "When does simple interest earn the customer additional money?"
- "When does simple interest require the customer to pay additional money?"

Highlight that the equation for simple interest is fairly straightforward. However, students should be careful to pay attention to the rate — is it given per month, or per year? This may require converting the amount of time between units in some cases. You may also choose to engage in a discussion about loans.

Featured Mathematician

Ebonya Washington

Have students read about Ebonya Washington, an economist and a professor at Yale who is studying the impact of public policy on everyday life.

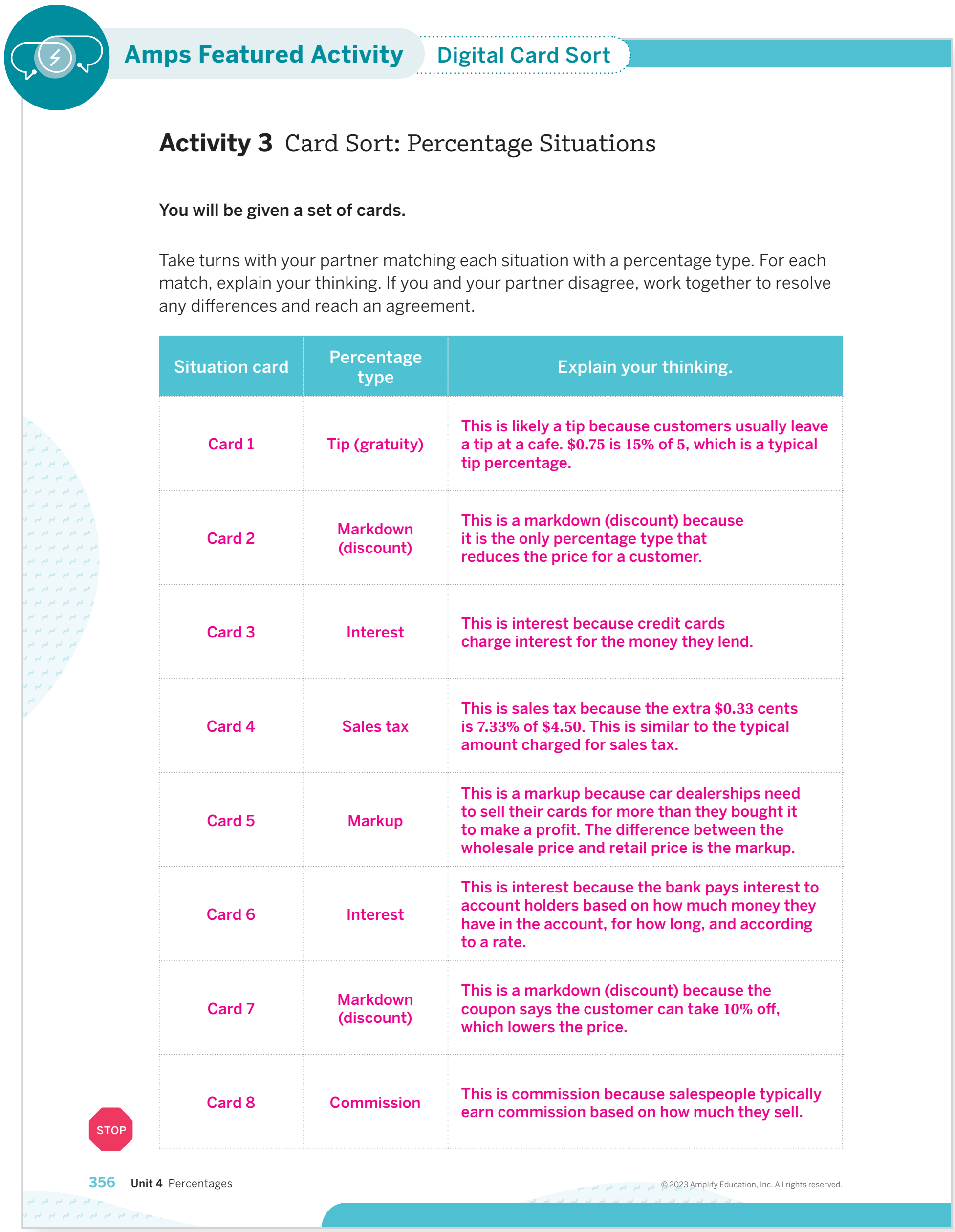
Optional

Small Groups | 15 min

Activity 3 Card Sort: Percentage Situations

Students sort scenarios to different descriptors using the images, sentences or questions to practice various vocabulary terms related to percentages.

Amps Featured Activity Digital Card Sort



Activity 3 Card Sort: Percentage Situations

You will be given a set of cards.

Take turns with your partner matching each situation with a percentage type. For each match, explain your thinking. If you and your partner disagree, work together to resolve any differences and reach an agreement.

Situation card	Percentage type	Explain your thinking.
Card 1	Tip (gratuity)	This is likely a tip because customers usually leave a tip at a cafe. $0.75 is 15% of 5, which is a typical tip percentage.
Card 2	Markdown (discount)	This is a markdown (discount) because it is the only percentage type that reduces the price for a customer.
Card 3	Interest	This is interest because credit cards charge interest for the money they lend.
Card 4	Sales tax	This is sales tax because the extra $0.33 cents is 7.33% of $4.50. This is similar to the typical amount charged for sales tax.
Card 5	Markup	This is a markup because car dealerships need to sell their cards for more than they bought it to make a profit. The difference between the wholesale price and retail price is the markup.
Card 6	Interest	This is interest because the bank pays interest to account holders based on how much money they have in the account, for how long, and according to a rate.
Card 7	Markdown (discount)	This is a markdown (discount) because the coupon says the customer can take 10% off, which lowers the price.
Card 8	Commission	This is commission because salespeople typically earn commission based on how much they sell.

STOP

356 Unit 4 Percentages

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1 Launch

Arrange students in pairs. Distribute the Activity 3 PDF, and explain that students will sort the scenarios into one of six categories. Demonstrate how students can take turns placing a scenario under a category and productive ways to disagree. Conduct the *Card Sort* routine.

2 Monitor

Help students get started by suggesting they begin with the contexts with which they feel most familiar.

Look for points of confusion:

- **Feeling challenged by identifying the proper context**. Ask, "How can the question at the bottom of the card help you make sense about the context?"

Look for productive strategies:

- Using the vocabulary: *tip*, *sales tax*, *gratuity*, *commission*, *markup*, *markdown*, *interest*, and *discount*.

3 Connect

Display the completed table.

Have students share which situations they sorted under each word, particularly students who were careful to use proper vocabulary during the activity.

Highlight that there is a copy of the Anchor Chart PDF, *Percentage Contexts* in the Student Edition at the end of the lesson that students can use as a reference tool during future lessons.

Ask:

- "What made you decide to put these situations under this descriptor?"
- "Were there any situations that you were really unsure of? What made you decide on where to sort them?"

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge, Guide Processing and Visualization***

Chunk this activity into smaller, more manageable tasks by first distributing Cards 1–4 to students. Consider first having students sort the cards by whether they show an increase or decrease and then for each additional category.

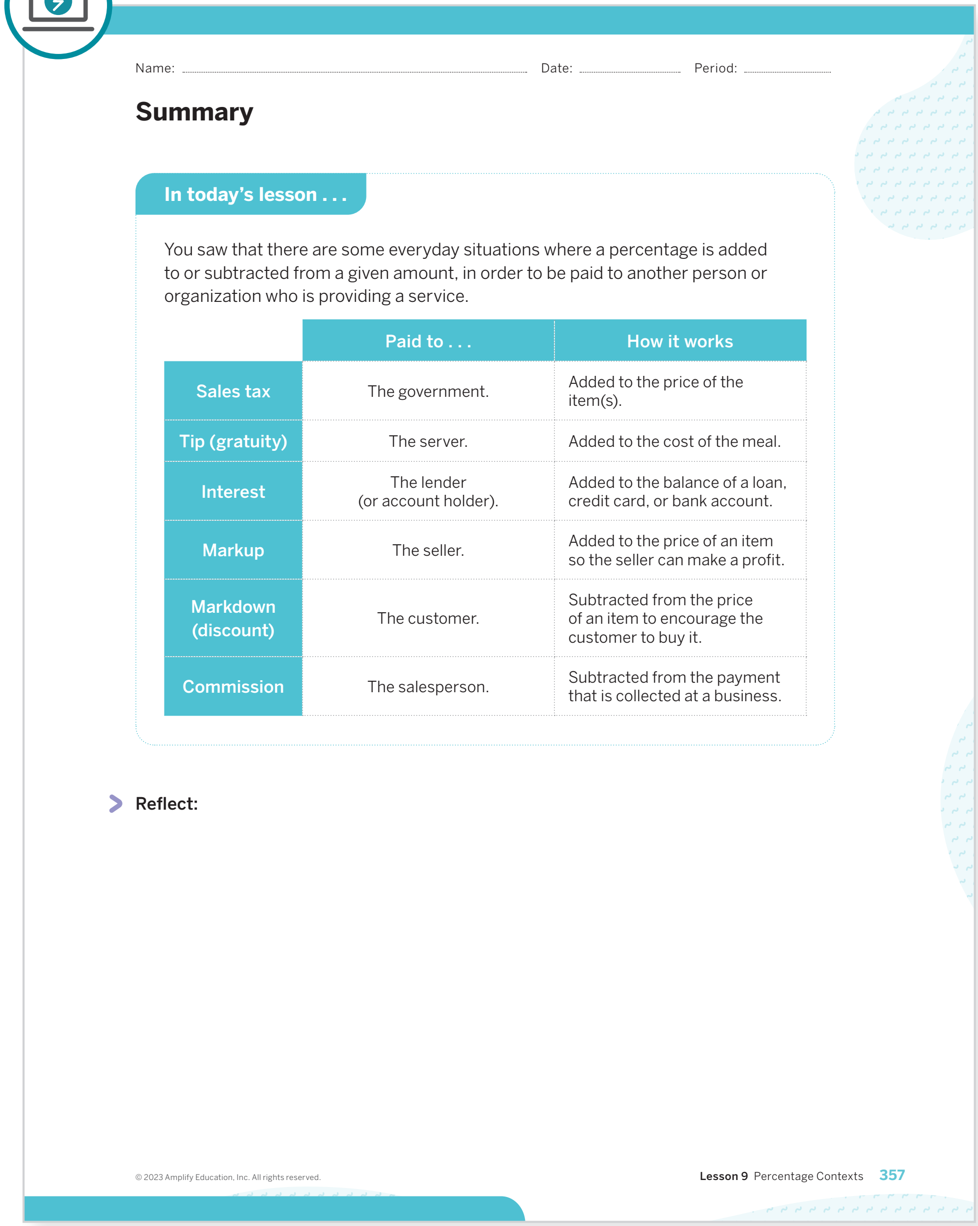
Extension: ***Math Enrichment***

Display Card 5. Have students determine the percentage of the markup. Then tell them that the car will decrease in value by 18% of the retail price by the end of the first year. Ask them to determine the value of the car after 1 year. The markup percent is 15.5%. The car's value will be 8,523.90 after 1 year.

Whole Class | 5 min

Summary

Review and synthesize how important it is to understand the vocabulary of different percentage types.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that there are some everyday situations where a percentage is added to or subtracted from a given amount, in order to be paid to another person or organization who is providing a service.

	Paid to . . .	How it works
Sales tax	The government.	Added to the price of the item(s).
Tip (gratuity)	The server.	Added to the cost of the meal.
Interest	The lender (or account holder).	Added to the balance of a loan, credit card, or bank account.
Markup	The seller.	Added to the price of an item so the seller can make a profit.
Markdown (discount)	The customer.	Subtracted from the price of an item to encourage the customer to buy it.
Commission	The salesperson.	Subtracted from the payment that is collected at a business.

> **Reflect:**

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Lesson 9 Percentage Contexts 357

Synthesize

Display the Anchor Chart PDF, *Percentage Contexts*.

Highlight that there are many everyday situations where a percentage of an amount of money is added to or subtracted from that amount. One of the most significant challenges is remembering how each situation is affected by the percentage of change. Add suggested content from the Anchor Chart PDF, *Percentage Contexts* (answers) for the commission and interest rows to the Anchor Chart PDF, *Percentage Contexts*.

Formalize vocabulary:

- ***commission***
- ***simple interest***

Ask:

- "What are some situations in life in which people encounter percentages?"
- "Give examples of situations where you would encounter tax, tip, markup, markdown, or commission."

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "With which percentage context do you feel most comfortable? Which feels the most challenging?"

Math Language Development

MLR2: Collect and Display

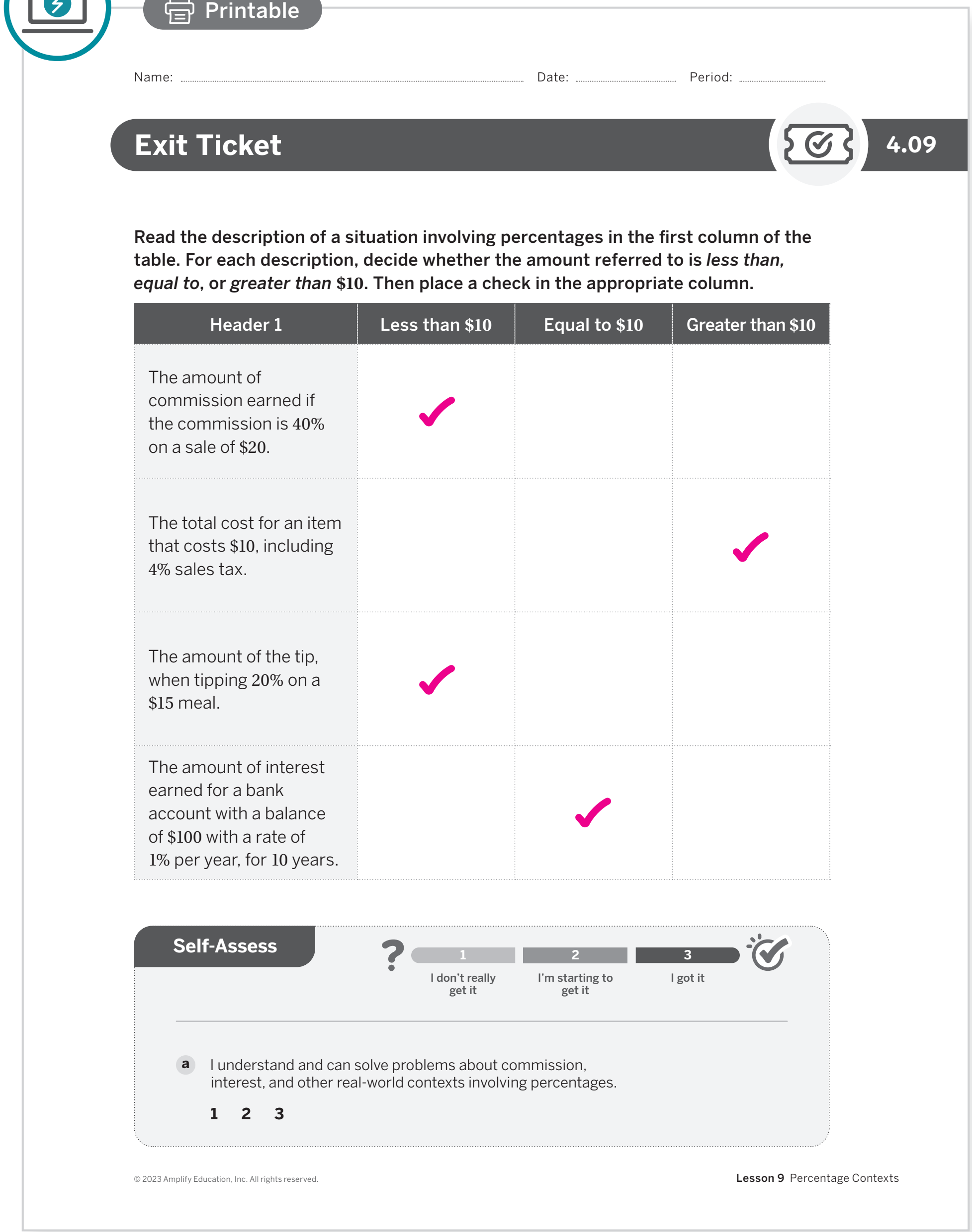
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the terms *commission* and *simple interest* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of different percentage contexts by reasoning about the effect of those percentage types on given values.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

4.09

Read the description of a situation involving percentages in the first column of the table. For each description, decide whether the amount referred to is *less than*, *equal to*, or *greater than* $10. Then place a check in the appropriate column.

Header 1	Less than $10	Equal to $10	Greater than $10
The amount of commission earned if the commission is 40% on a sale of $20.	✓		
The total cost for an item that costs $10, including 4% sales tax.			✓
The amount of the tip, when tipping 20% on a $15 meal.	✓		
The amount of interest earned for a bank account with a balance of $100 with a rate of 1% per year, for 10 years.		✓	

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I understand and can solve problems about commission, interest, and other real-world contexts involving percentages.

1 2 3

Lesson 9 Percentage Contexts

Success looks like . . .

- **Goal:** Comprehending interest and commission as other contexts that involve adding or subtracting a percentage of the initial amount.
  - » Determining the interest or commission in the first and last rows of the table.
- **Language Goal:** Explaining how to calculate the new dollar amount after commission. **(Speaking and Listening)**
- **Goal:** Solving multi-step problems involving commission or interest.
  - » Estimating amounts of commission, total cost, tip, and interest in the table.

Suggested next steps

If students can find the correct value, but misunderstand how the context affects the solution, consider:

- Having students complete the Exit Ticket while having access to the Lesson 9 Summary.

If students are facing challenges calculating the relevant values, consider:

- Assigning Practice Problem 4.

Professional Learning

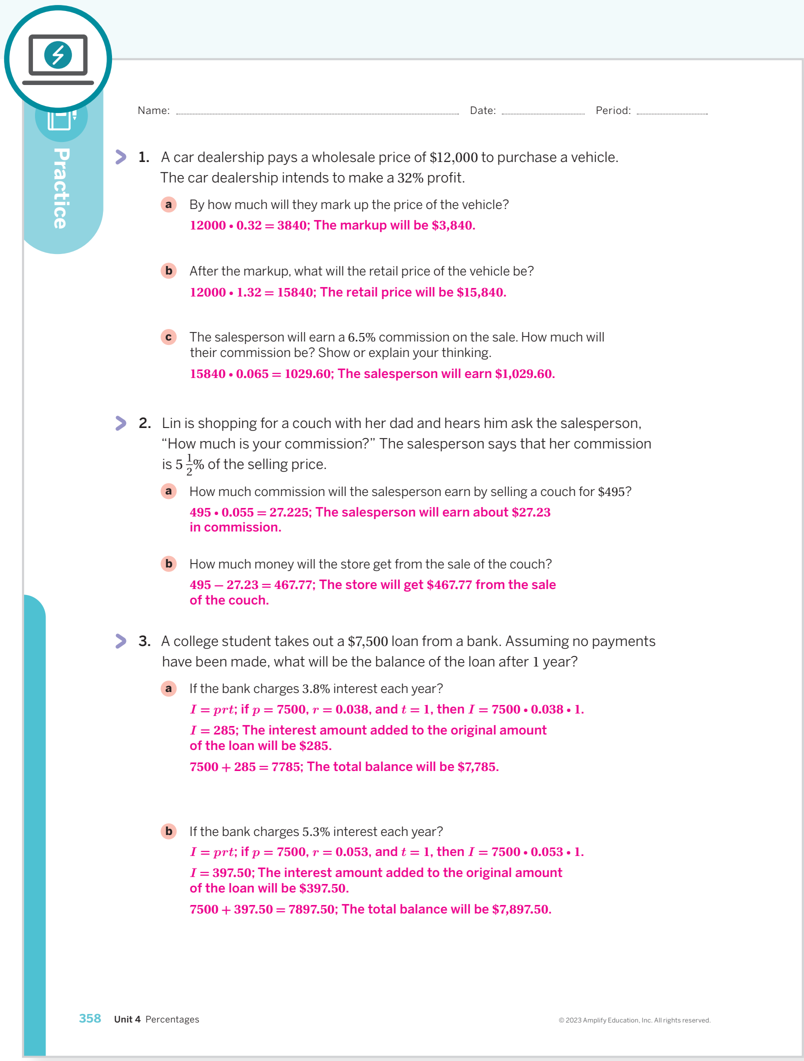
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was introducing new percentage contexts similar to or different from the contexts that were introduced in Lesson 8?
- What did you see in some of the approaches students used to parse important information in word problems? Which of those methods would you like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A car dealership pays a wholesale price of $12,000 to purchase a vehicle. The car dealership intends to make a 32% profit.

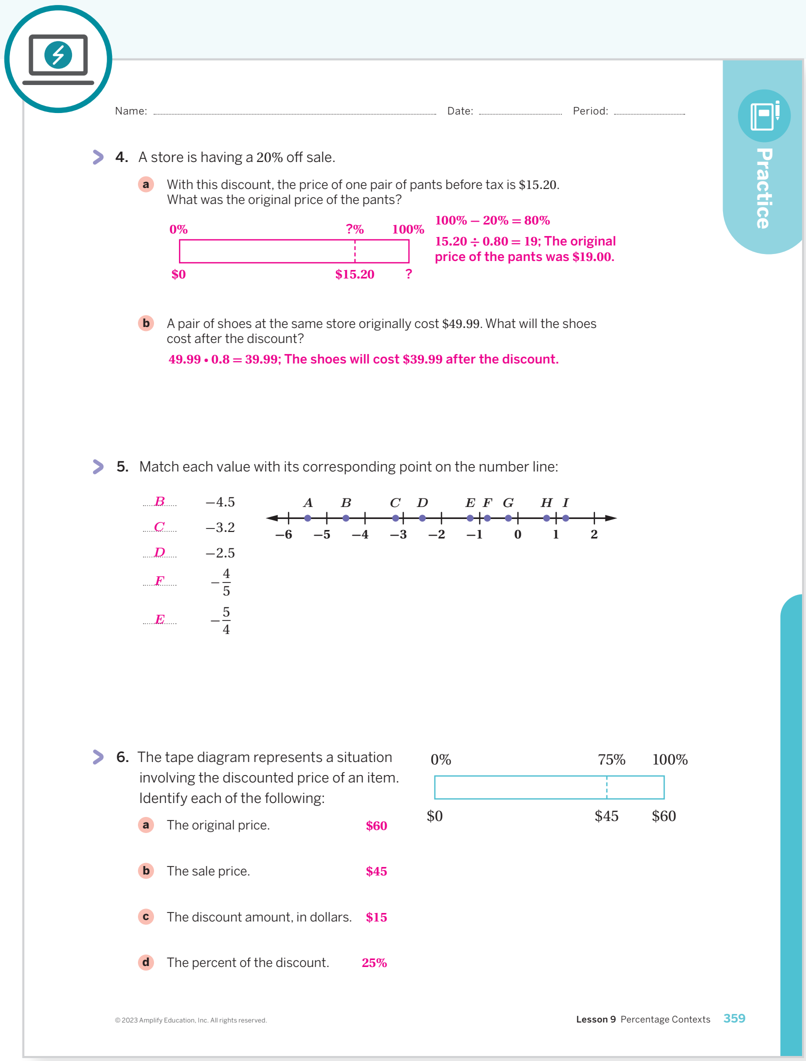
- **a** By how much will they mark up the price of the vehicle?
  12000 • 0.32 = 3840; The markup will be $3,840.
- **b** After the markup, what will the retail price of the vehicle be?
  12000 • 1.32 = 15840; The retail price will be $15,840.
- **c** The salesperson will earn a 6.5% commission on the sale. How much will their commission be? Show or explain your thinking.
  15840 • 0.065 = 1029.60; The salesperson will earn $1,029.60.

2. Lin is shopping for a couch with her dad and hears him ask the salesperson, "How much is your commission?" The salesperson says that her commission is $5\frac{1}{2}$% of the selling price.

- **a** How much commission will the salesperson earn by selling a couch for $495?
  495 • 0.055 = 27.225; The salesperson will earn about $27.23 in commission.
- **b** How much money will the store get from the sale of the couch?
  495 – 27.23 = 467.77; The store will get $467.77 from the sale of the couch.

3. A college student takes out a $7,500 loan from a bank. Assuming no payments have been made, what will be the balance of the loan after 1 year?

- **a** If the bank charges 3.8% interest each year?
  $I = prt$; if $p = 7500$, $r = 0.038$, and $t = 1$, then $I = 7500 \cdot 0.038 \cdot 1$.
  $I = 285$; The interest amount added to the original amount of the loan will be $285.
  7500 + 285 = 7785; The total balance will be $7,785.
- **b** If the bank charges 5.3% interest each year?
  $I = prt$; if $p = 7500$, $r = 0.053$, and $t = 1$, then $I = 7500 \cdot 0.053 \cdot 1$.
  $I = 397.50$; The interest amount added to the original amount of the loan will be $397.50.
  7500 + 397.50 = 7897.50; The total balance will be $7,897.50.



Name: Date: Period:

4. A store is having a 20% off sale.

- **a** With this discount, the price of one pair of pants before tax is $15.20. What was the original price of the pants?

100% – 20% = 80%
15.20 ÷ 0.80 = 19; The original price of the pants was $19.00.

- **b** A pair of shoes at the same store originally cost $49.99. What will the shoes cost after the discount?
  49.99 • 0.8 = 39.99; The shoes will cost $39.99 after the discount.

5. Match each value with its corresponding point on the number line:

B –4.5
C –3.2
D –2.5
F $-\frac{4}{5}$
E $-\frac{5}{4}$

6. The tape diagram represents a situation involving the discounted price of an item. Identify each of the following:

- **a** The original price. **$60**
- **b** The sale price. **$45**
- **c** The discount amount, in dollars. **$15**
- **d** The percent of the discount. **25%**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 4	2
	5	Grade 6	2
Formative ⚡	6	Unit 4 Lesson 10	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 4 | **LESSON 10**

Determining the Percentage

Let's determine unknown percentages.

Focus

Goals

1. Determine what information is needed to solve a problem involving sales tax and discounts. Ask questions to elicit that information.
2. **Language Goal:** Explain how to calculate the percentage, given the dollar amounts before and after a sales tax, tip, or discount. **(Speaking and Listening)**
3. **Language Goal**: Interpret tape diagrams that represent situations involving a sales tax, tip, or discount. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students consolidate what they have learned over the last few lessons and solve a variety of multi-step percentage problems involving taxes, tips, and discounts, including problems involving fractional percentages. They continue to move towards using equations to represent problems, which enable them to see the common underlying structure behind different problems.

Previously

In Lessons 8 and 9, students were introduced to and solved problems related to percentage contexts involving monetary transactions, such as sales tax, tips, commission, and simple interest.

Coming Soon

In Lessons 11 and 12, students will learn about measurement error, how to quantify the amount of error as a percent, and acceptable error intervals.

Rigor

- Students build **procedural skills** determining the percentage when given the original amount and the new amount.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	8 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF pre-cut cards, one set per pair
- Graphic Organizer PDF, *Solving Percent Problems* (as needed)
- Info Gap Routine PDF (as needed)
- calculators

Math Language Development

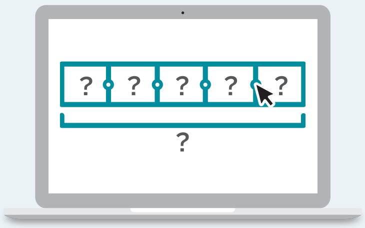
Review words

- *commission*
- *markdown*
- *markup*
- *sales tax*
- *simple interest*
- *tip (gratuity)*

Amps | **Featured Activity**

Activity 1
Digital Tape Diagrams

Students manipulate digital tape diagrams to model problems with percentage.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not feel confident as they attempt to make sense of Activity 1 because they have many calculations in order to complete the task. Have students read through the activity, first circling the questions that they know they have the ability to complete and then highlighting those for which they might need help. Ask them to identify ways in which they can both get and give help, thus addressing both their limitations and their strengths during the lesson.

Modifications to Pacing

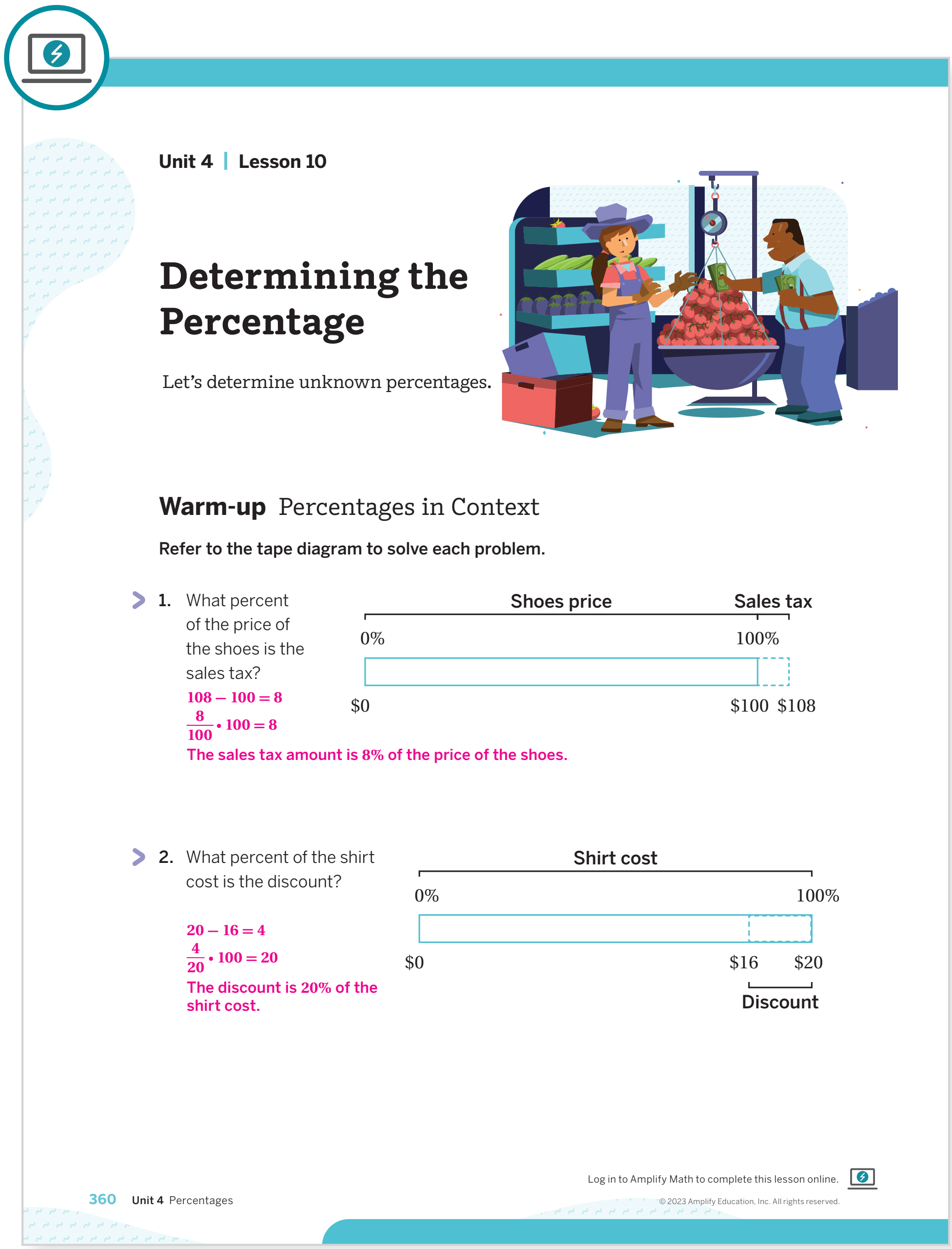
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, problem 3 may be omitted.
- In **Activity 2**, have students only complete one round of the *Info Gap* problems.

Independent | 5 min

Warm-up Percentages in Context

Students use tape diagrams to reason about parts of a whole in the context of tips, taxes, and discounts and compare the types of problems.



Unit 4 | **Lesson 10**

Determining the Percentage

Let's determine unknown percentages.

Warm-up Percentages in Context

Refer to the tape diagram to solve each problem.

1. What percent of the price of the shoes is the sales tax?

Shoes price **Sales tax**
0% 100%
$0 $100 $108

$108 - 100 = 8$

$\frac{8}{100} \cdot 100 = 8$

The sales tax amount is 8% of the price of the shoes.

2. What percent of the shirt cost is the discount?

Shirt cost
0% 100%
$0 $16 $20
Discount

$20 - 16 = 4$

$\frac{4}{20} \cdot 100 = 20$

The discount is 20% of the shirt cost.

Log in to Amplify Math to complete this lesson online.

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360 **Unit 4** Percentages

1 Launch

Provide access to calculators throughout the lesson. Say, "This Warm-up and lesson will give you opportunities to try problem-solving strategies that may feel like a stretch now." **Note:** Having students work independently provides an opportunity to assess who still benefits from using a tape diagram strategy.

2 Monitor

Help students get started by asking, "How much did the price of the shoes increase by?"

Look for points of confusion:

- **Finding the difference in cost when asked for the percentage.** Have students annotate the tape diagram with a question mark to indicate what value they are finding.
- **Not knowing which value to determine.** Have students add a question mark to the diagram to represent the value they need to find.

Look for productive strategies:

- Marking other relevant locations on the tape diagrams to reason about percent change.

3 Connect

Have pairs of students share the similarities and differences between the problems.

Ask:

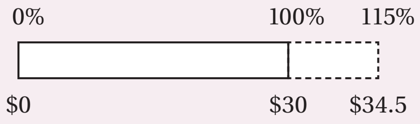
- "What clues can the tape diagram give you about the type of percentage you are working with?"
- "What new strategies might you try today?"

Highlight that throughout the unit, students may have felt most comfortable sticking with one strategy, or perhaps avoiding equations. Say, "As there are no new contexts in this lesson, this is a good opportunity to get more comfortable with strategies unfamiliar to you."

Power-up

To power up students' ability to identify what the values in a tape diagram represent, have students complete:

Recall that, in a tape diagram, 100% always matches with the original amount. Determine each value based on the tape diagram:



1. The original price. $30
2. The markup price. $34.5
3. The amount of markup. $4.5
4. The percent of markup. 15%

Use: Before the Warm-up.

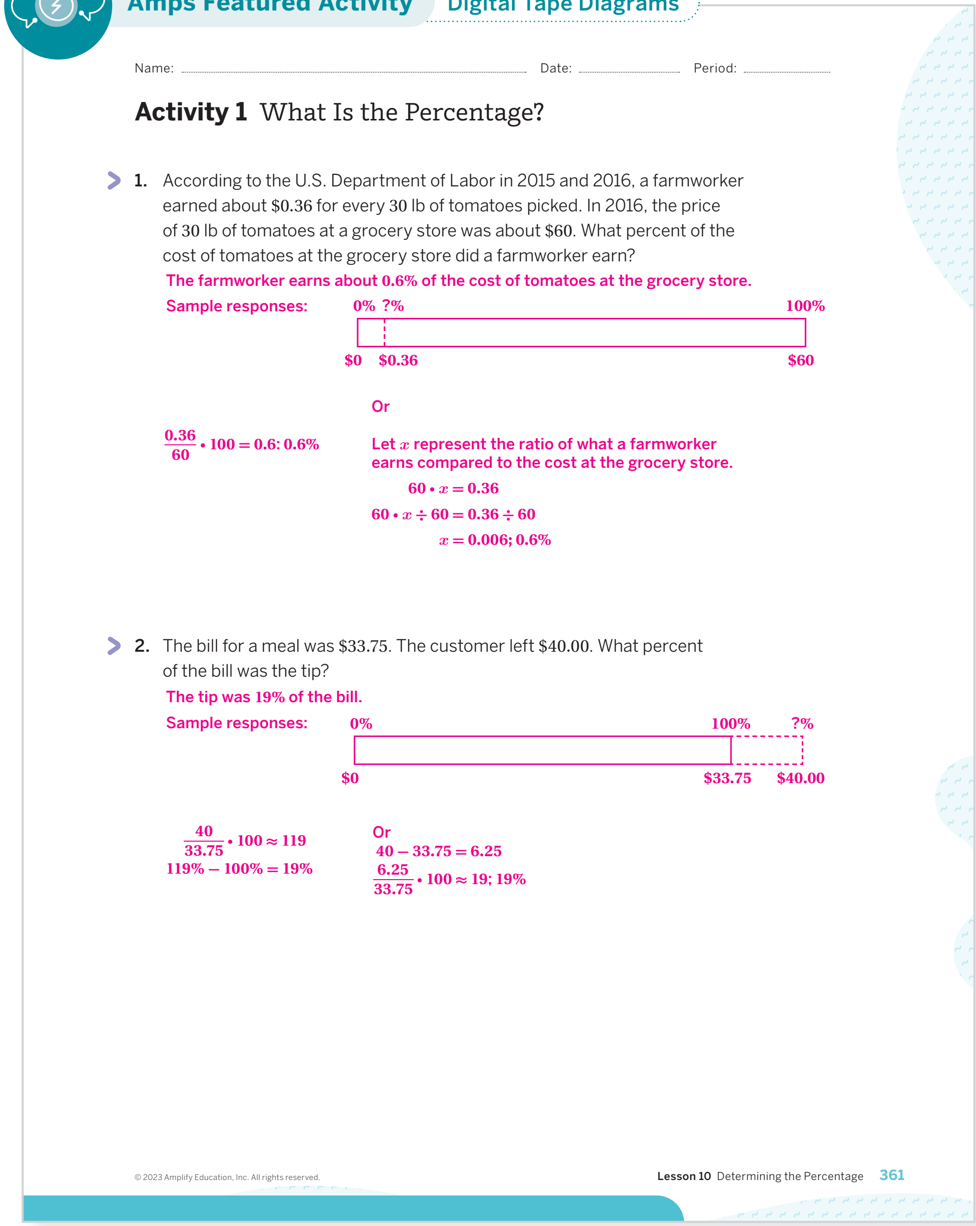
Informed by: Performance on Lesson 9, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 15 min

Activity 1 What Is the Percentage?

Students practice finding percentages from dollar amounts including commission, tip, and markdown and move further toward using equations to represent percent problems.

Amps Featured Activity Digital Tape Diagrams



Name: Date: Period:

Activity 1 What Is the Percentage?

1. According to the U.S. Department of Labor in 2015 and 2016, a farmworker earned about $0.36 for every 30 lb of tomatoes picked. In 2016, the price of 30 lb of tomatoes at a grocery store was about $60. What percent of the cost of tomatoes at the grocery store did a farmworker earn?

   The farmworker earns about 0.6% of the cost of tomatoes at the grocery store.

   Sample responses:

   Or

   $\frac{0.36}{60} \cdot 100 = 0.6\text{: } 0.6\%$

   Let x represent the ratio of what a farmworker earns compared to the cost at the grocery store.

   $60 \cdot x = 0.36$

   $60 \cdot x \div 60 = 0.36 \div 60$

   $x = 0.006\text{; } 0.6\%$

2. The bill for a meal was $33.75. The customer left $40.00. What percent of the bill was the tip?

   The tip was 19% of the bill.

   Sample responses:

   $\frac{40}{33.75} \cdot 100 \approx 119$

   $119\% - 100\% = 19\%$

   Or

   $40 - 33.75 = 6.25$

   $\frac{6.25}{33.75} \cdot 100 \approx 19\text{; } 19\%$

1 Launch

Read through Problem 1 as a class. Ask students, "What do we know? What do we want to know?" Encourage students to begin their work by writing an equation to represent the situation in each problem.

2 Monitor

Help students get started by having them draw and label a tape diagram to help them to make sense of the problem.

Look for points of confusion:

- **Thinking they need to add or subtract the quantities before finding the percentage.** Have students refer to the Summary from Lesson 9 to review the relevant context.
- **In Problem 1, thinking the weight of the tomatoes is needed to solve.** Ask, "Are you comparing the same amount of tomatoes? Does the relationship change if both were doubled?"
- **Calculating the tip amount in dollars instead of the tip percentage.** Have students reread the title of the Activity to help remind them of the importance of reading all information related to a problem, and to ground them in the goal of the Activity.
- **Forgetting to multiply by 100 to get the percentage.** Have students work backward with the value they found as the percent, and have them check if they return to the same value they started with.

Look for productive strategies:

- Writing and solving equations for each situation.
- Drawing a tape diagram and then writing an equation.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can strategically select digital tools to solve these percentage problems. By doing so, they engage in meta-cognitive skills as they consider which strategy and tool best suits the problem.

Accessibility: *Guide Processing and Visualization*

Provide copies of the Graphic Organizer PDF, *Percentage Tape Diagrams* if students choose to draw tape diagrams to represent each problem.

Math Language Development

MLR3: Critique, Correct, Clarify

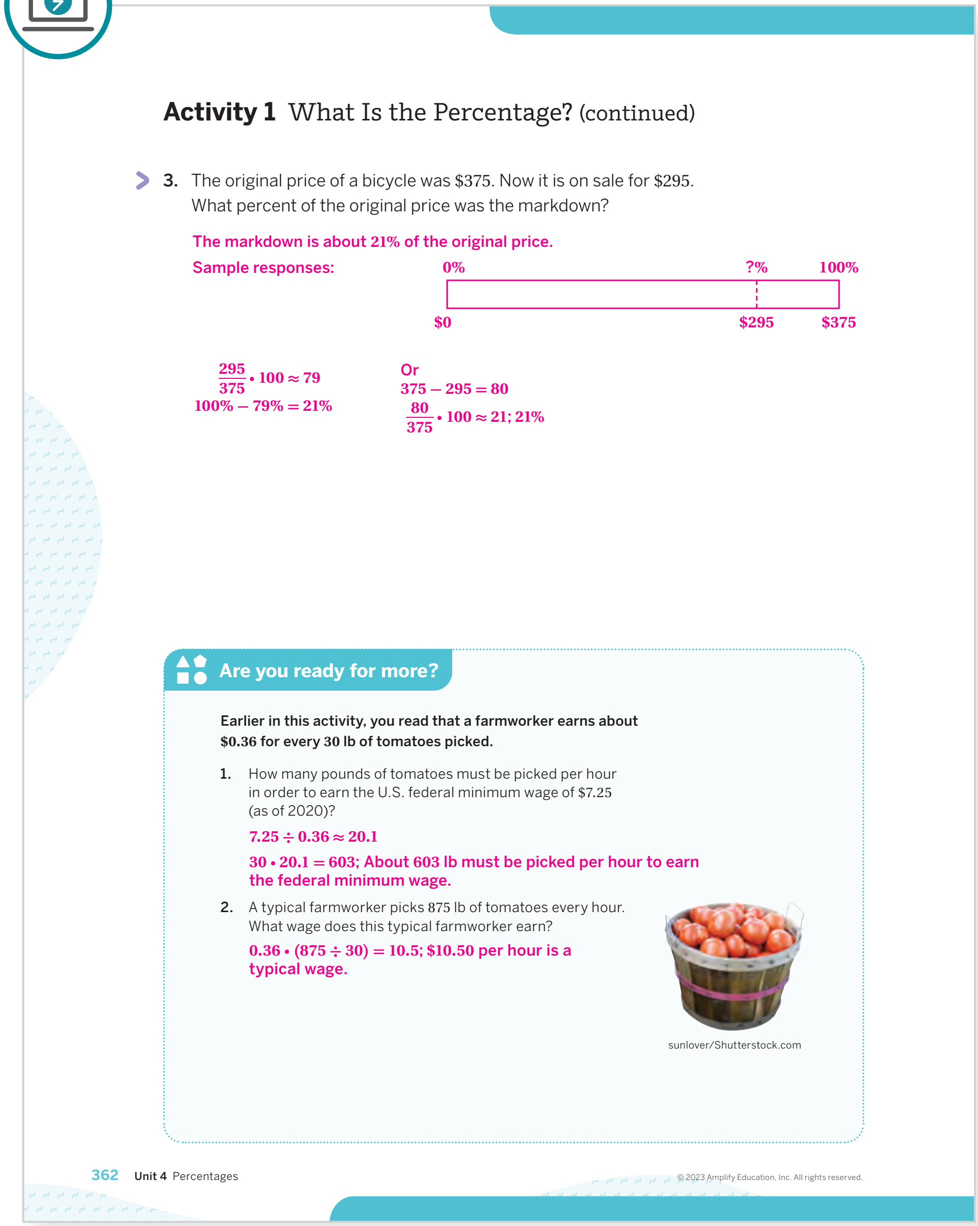
During the Connect, present an incorrect response to Problem 3 that demonstrates a common misunderstanding. For example, display, "Priya believed that the bicycle was marked down about 27% because the difference, $80 is about 27% of $295." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Why or why not?" Listen for students who recognize that the whole is the original price, not the sale price.
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "How would you explain to Priya why her statement was not correct? How could you convince her that your statement is correct?"

Pairs | 15 min

Activity 1 What Is the Percentage? (continued)

Students practice finding percentages from dollar amounts including commission, tip, and markdown and move further toward using equations to represent percent problems.



Activity 1 What Is the Percentage? (continued)

3. The original price of a bicycle was $375. Now it is on sale for $295. What percent of the original price was the markdown?

The markdown is about 21% of the original price.

Sample responses:

$\frac{295}{375} \cdot 100 \approx 79$

$100\% - 79\% = 21\%$

Or

$375 - 295 = 80$

$\frac{80}{375} \cdot 100 \approx 21; 21\%$

Are you ready for more?

Earlier in this activity, you read that a farmworker earns about $0.36 for every 30 lb of tomatoes picked.

1. How many pounds of tomatoes must be picked per hour in order to earn the U.S. federal minimum wage of $7.25 (as of 2020)?

   $7.25 \div 0.36 \approx 20.1$

   $30 \cdot 20.1 = 603$; About 603 lb must be picked per hour to earn the federal minimum wage.

2. A typical farmworker picks 875 lb of tomatoes every hour. What wage does this typical farmworker earn?

   $0.36 \cdot (875 \div 30) = 10.5$; $10.50 per hour is a typical wage.

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3 Connect

Display students' work with different solution strategies, including at least one equation, for one of the problems. Ideally, choose a problem where students showed the greatest number of misconceptions. **Note:** You may want to prepare a worked solution that includes an equation for one of the problems ahead of time if you are concerned you may not see this in your students' strategies.

Have students share similarities and differences between the equation and the other strategies. Annotate the equation to highlight what each part of the equation represents.

Ask:

- "What is your preferred solution strategy? Why?"
- "Did your solution strategy change for any of the problems?"
- "Did the answer to any of the problems surprise you? Why?"

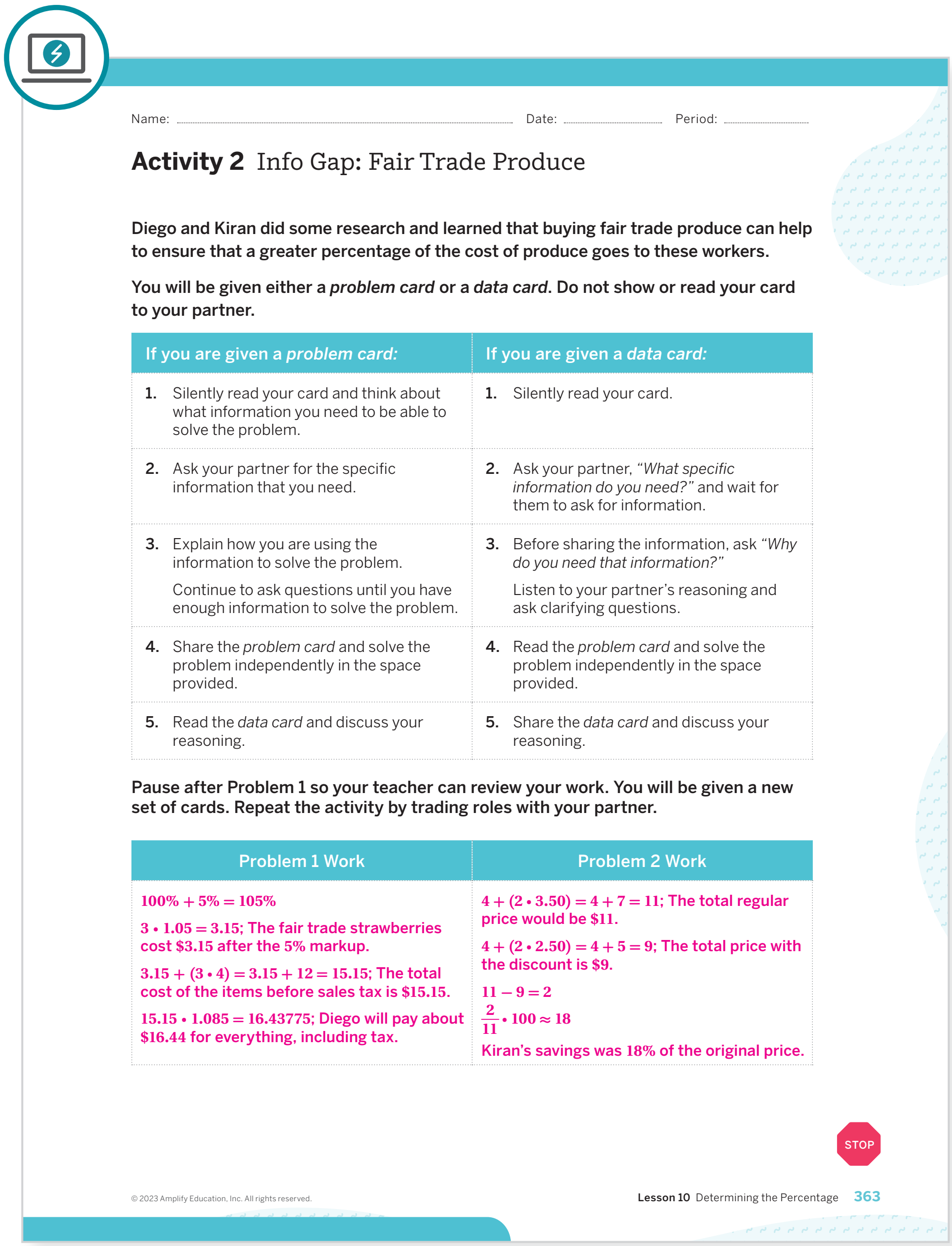
Highlight that finding the *percent change* in a situation involving a monetary transaction can allow for multiple solution strategies. Encourage students to combine their strategy of choice with an equation strategy to prepare for work in upcoming units.

Ask, "Were you surprised by the solution to Problem 1? What surprised you?"

Pairs | 15 min

Activity 2 Info Gap: Fair Trade Produce

Students collaborate and identify the essential information needed to determine the total cost and savings after markups and discounts are applied to different items.



Name: ______ Date: ______ Period: ______

Activity 2 Info Gap: Fair Trade Produce

Diego and Kiran did some research and learned that buying fair trade produce can help to ensure that a greater percentage of the cost of produce goes to these workers.

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner, *"What specific information do you need?"* and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space provided.	**4.** Read the *problem card* and solve the problem independently in the space provided.
5. Read the *data card* and discuss your reasoning.	**5.** Share the *data card* and discuss your reasoning.

Pause after Problem 1 so your teacher can review your work. You will be given a new set of cards. Repeat the activity by trading roles with your partner.

Problem 1 Work	Problem 2 Work
$100\% + 5\% = 105\%$ $3 \cdot 1.05 = 3.15$; The fair trade strawberries cost \$3.15 after the 5% markup. $3.15 + (3 \cdot 4) = 3.15 + 12 = 15.15$; The total cost of the items before sales tax is \$15.15. $15.15 \cdot 1.085 = 16.43775$; Diego will pay about \$16.44 for everything, including tax.	$4 + (2 \cdot 3.50) = 4 + 7 = 11$; The total regular price would be \$11. $4 + (2 \cdot 2.50) = 4 + 5 = 9$; The total price with the discount is \$9. $11 - 9 = 2$ $\frac{2}{11} \cdot 100 \approx 18$ Kiran's savings was 18% of the original price.

STOP

Lesson 10 Determining the Percentage 363

1 Launch

Activate background knowledge by asking, "Have you noticed how there can be two or three different kinds of bananas at a grocery store? Why do you think this is?" Have students continue working in pairs, and distribute the Activity 2 PDF. Conduct the ***Info Gap*** routine.

2 Monitor

Help students get started by having them consider and write down questions they could ask.

Look for points of confusion:

- **Not noticing that Diego and Kiran buy multiple packages of certain things.** Ask students, "Are you given the cost for one or multiple packages?"
- **Applying a 5% markup to Diego's full purchase.** Ask, "Is everything Diego buys marked up?"

Look for productive strategies:

- Making an organized list to track the purchases and the percentages applied to each.

3 Connect

Have students share their responses and ask students to discuss the different ways they solved this problem.

Ask:

- "How did you determine the total cost after tax for Diego's purchases?" Multiply the total by 1.085 or multiply by 0.085 and add to the original cost.
- "Was there information given that you did not need to use?"

Highlight that often real-world percentage problems involve multiple steps which need to be done in the proper order. Students organizing their work into a series of steps can help them keep track of when and where to apply the proper percentages.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following questions during the think-aloud.

- "I wonder how much the regular strawberries and bunches of bananas cost. I will ask for these prices."
- "I wonder what the markup is for fair trade strawberries or fair trade bananas. I will ask for these percentages."
- "I am not given the sales tax percentage, so I will ask for it."

Math Language Development

MLR4: Information Gap

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

English Learners

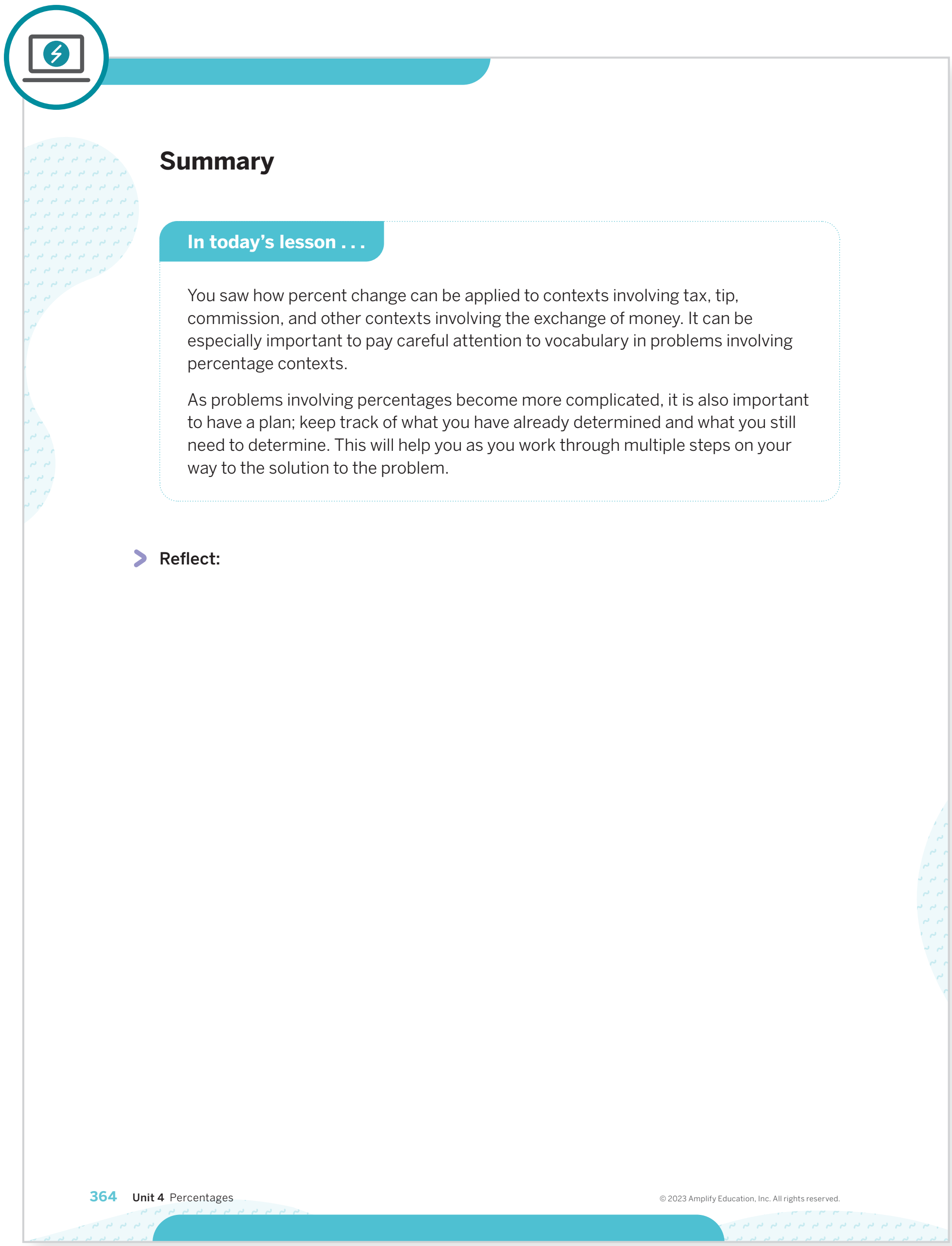
Consider providing sample questions students could ask, such as the following for Problem Card 1:

- How much do the regular strawberries cost? Bananas?
- What is the percentage of markup on these for buying fair trade?
- What is the sales tax percentage?

Whole Class | 5 min

Summary

Review and synthesize how to determine percentages for contexts involving sales tax, tips, and discounts.



Summary

In today's lesson . . .

You saw how percent change can be applied to contexts involving tax, tip, commission, and other contexts involving the exchange of money. It can be especially important to pay careful attention to vocabulary in problems involving percentage contexts.

As problems involving percentages become more complicated, it is also important to have a plan; keep track of what you have already determined and what you still need to determine. This will help you as you work through multiple steps on your way to the solution to the problem.

> **Reflect:**

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Synthesize

Highlight that, when solving percentage problems involving monetary transactions, it can be very helpful to be as organized as possible. Because there are often multiple values and steps, labeling words and numbers in both the problem and solution steps can help keep students on the right track.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

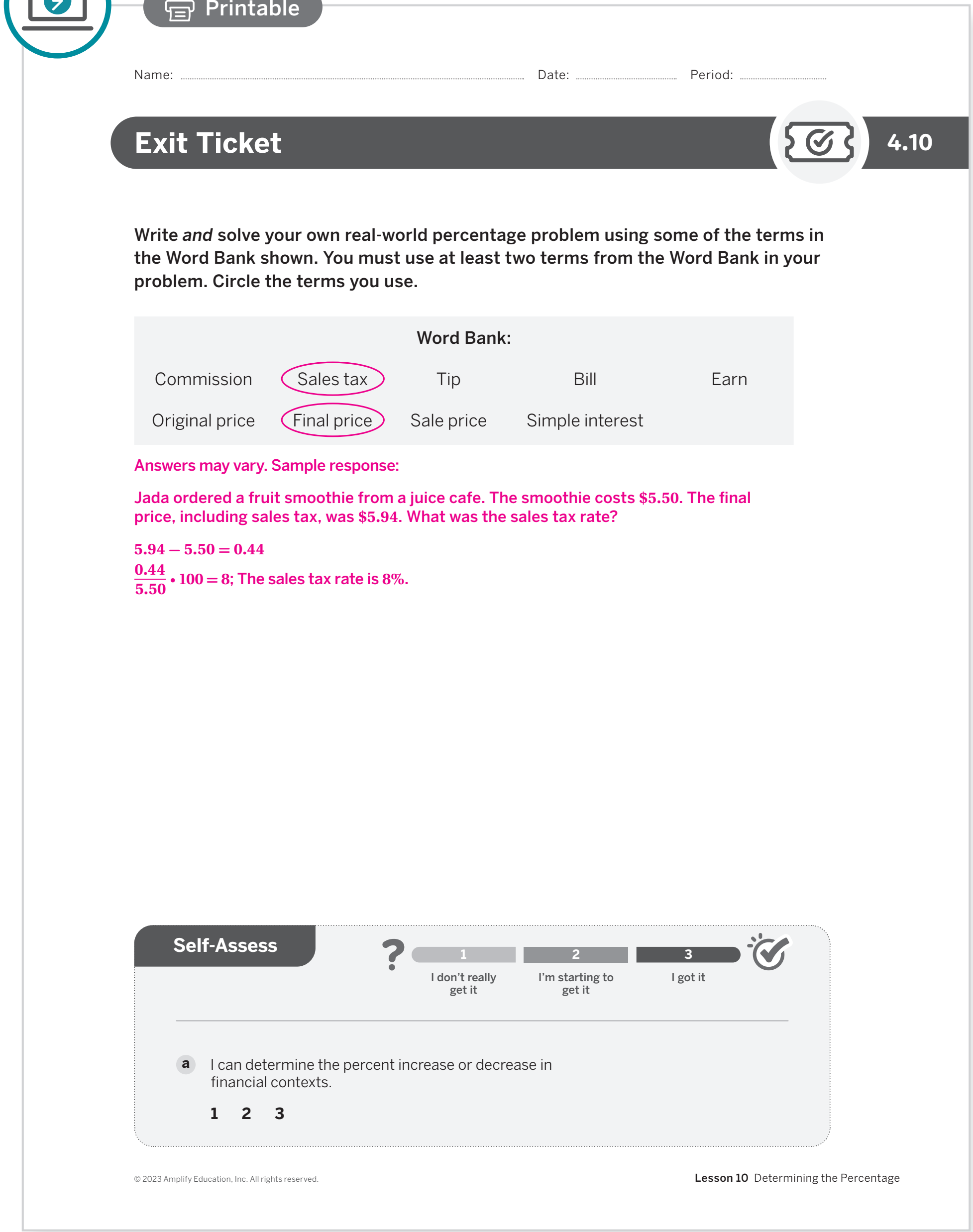
- "Were you able to try a strategy that initially felt outside of your comfort zone? How do you feel after trying it?"

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of percentage contexts involving monetary transactions by writing their own problem given specific vocabulary requirements.

Printable



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Exit Ticket 4.10

Write *and* solve your own real-world percentage problem using some of the terms in the Word Bank shown. You must use at least two terms from the Word Bank in your problem. Circle the terms you use.

Word Bank:

Commission	[✓ Sales tax]	Tip	Bill	Earn
Original price	[✓ Final price]	Sale price	Simple interest	

Answers may vary. Sample response:

Jada ordered a fruit smoothie from a juice cafe. The smoothie costs $5.50. The final price, including sales tax, was $5.94. What was the sales tax rate?

$5.94 - 5.50 = 0.44$

$\frac{0.44}{5.50} \cdot 100 = 8$; The sales tax rate is 8%.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can determine the percent increase or decrease in financial contexts.

1 2 3

Lesson 10 Determining the Percentage

Success looks like . . .

- **Goal:** Determining what information is needed to solve a problem involving sales tax and discounts. Asking questions to elicit that information.
- **Language Goal:** Explaining how to calculate the percentage, given the dollar amounts before and after a sales tax, tip, or discount. **(Speaking and Listening)**
  - » Writing and solving their own real-world percentage problem.
- **Language Goal:** Interpreting tape diagrams that represent situations involving a sales tax, tip, or discount. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students do not know how to write their own problem, consider:

- Encouraging them to select some numbers first, and then create a problem to match.

If students misuse a term from the Word Bank, consider:

- Having them review the Summary from Lesson 9.

If students do not use an equation to represent the solution to their problem, consider:

- Revisiting and reassigning this problem after Unit 6.

Professional Learning

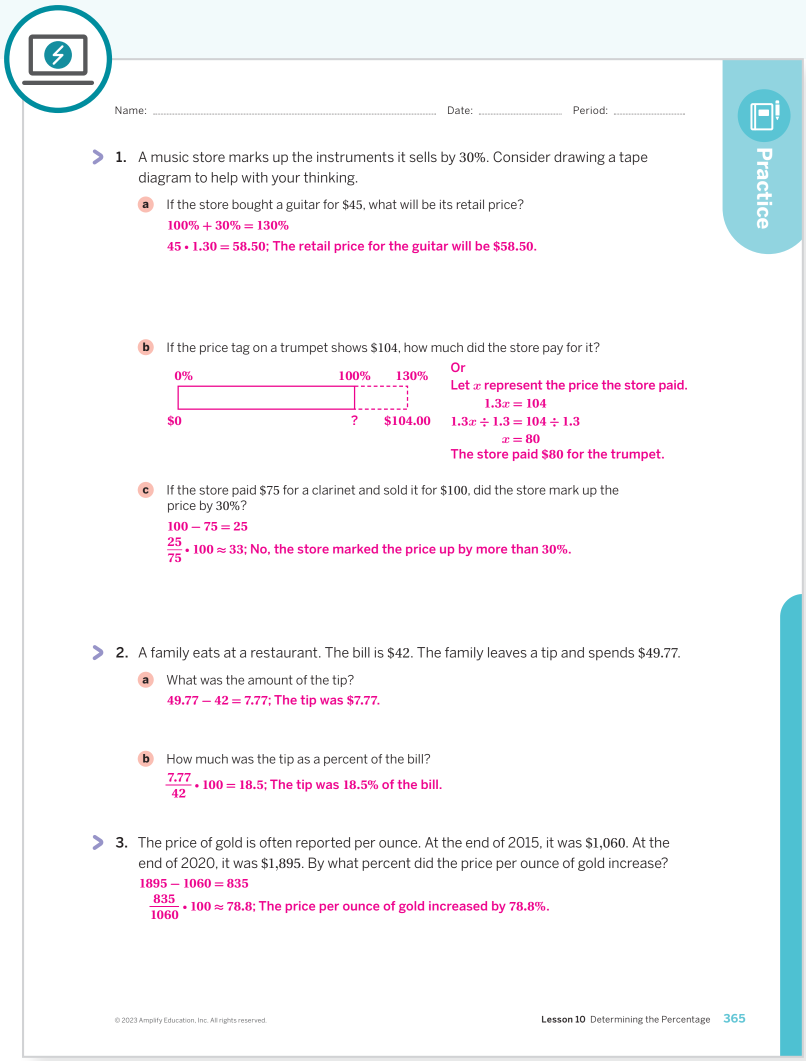
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on Activity 1?
- What challenges did students encounter as they worked on Activity 2? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A music store marks up the instruments it sells by 30%. Consider drawing a tape diagram to help with your thinking.

   a. If the store bought a guitar for \$45, what will be its retail price?

   $100\% + 30\% = 130\%$

   $45 \cdot 1.30 = 58.50$; The retail price for the guitar will be \$58.50.

   b. If the price tag on a trumpet shows \$104, how much did the store pay for it?

   Or

   Let x represent the price the store paid.

   $$1.3x = 104$$
   $$1.3x \div 1.3 = 104 \div 1.3$$
   $$x = 80$$

   The store paid \$80 for the trumpet.

   c. If the store paid \$75 for a clarinet and sold it for \$100, did the store mark up the price by 30%?

   $100 - 75 = 25$

   $\frac{25}{75} \cdot 100 \approx 33$; No, the store marked the price up by more than 30%.

2. A family eats at a restaurant. The bill is \$42. The family leaves a tip and spends \$49.77.

   a. What was the amount of the tip?

   $49.77 - 42 = 7.77$; The tip was \$7.77.

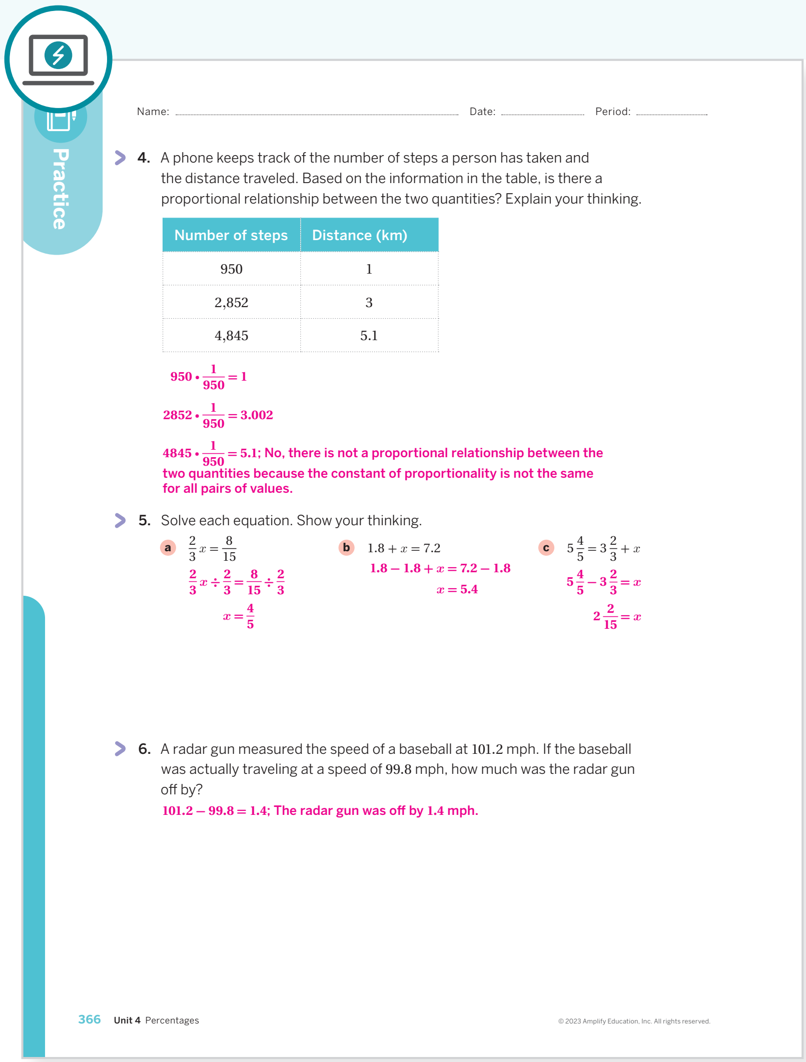
   b. How much was the tip as a percent of the bill?

   $\frac{7.77}{42} \cdot 100 = 18.5$; The tip was 18.5% of the bill.

3. The price of gold is often reported per ounce. At the end of 2015, it was \$1,060. At the end of 2020, it was \$1,895. By what percent did the price per ounce of gold increase?

   $1895 - 1060 = 835$

   $\frac{835}{1060} \cdot 100 \approx 78.8$; The price per ounce of gold increased by 78.8%.



Name: Date: Period:

4. A phone keeps track of the number of steps a person has taken and the distance traveled. Based on the information in the table, is there a proportional relationship between the two quantities? Explain your thinking.

Number of steps	Distance (km)
950	1
2,852	3
4,845	5.1

   $950 \cdot \frac{1}{950} = 1$

   $2852 \cdot \frac{1}{950} = 3.002$

   $4845 \cdot \frac{1}{950} = 5.1$; No, there is not a proportional relationship between the two quantities because the constant of proportionality is not the same for all pairs of values.

5. Solve each equation. Show your thinking.

   a. $\frac{2}{3}x = \frac{8}{15}$

   $$\frac{2}{3}x \div \frac{2}{3} = \frac{8}{15} \div \frac{2}{3}$$
   $$x = \frac{4}{5}$$

   b. $1.8 + x = 7.2$

   $$1.8 - 1.8 + x = 7.2 - 1.8$$
   $$x = 5.4$$

   c. $5\frac{4}{5} = 3\frac{2}{3} + x$

   $$5\frac{4}{5} - 3\frac{2}{3} = x$$
   $$2\frac{2}{15} = x$$

6. A radar gun measured the speed of a baseball at 101.2 mph. If the baseball was actually traveling at a speed of 99.8 mph, how much was the radar gun off by?

   $101.2 - 99.8 = 1.4$; The radar gun was off by 1.4 mph.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 4	2
	5	Grade 6	2
Formative ⚡	6	Unit 4 Lesson 11	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Measurement Error

Let's use percentages to describe how accurately we can measure.

Focus

Goals

1. **Language Goal:** Compare and contrast multiple measurements of the same length that result from using rulers with different levels of precision. **(Speaking and Listening)**
2. **Language Goal:** Calculate the percent error or correct amount, and explain the solution method. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Compare and contrast strategies used for solving problems about percent error with strategies used for solving problems about percent increase or decrease. **(Speaking and Listening)**

Coherence

Today

In this lesson students see how measurement error can arise in two different ways: from the level of precision in the measurement device, and from human error. Students are asked to compare how percent error relates to percent increase and decrease to build on their schema from earlier in the unit.

Previously

In Lessons 9 and 10, students worked with some new contexts involving percents, especially those related to monetary transactions.

Coming Soon

In Lesson 12, students continue their work with percent error as they determine that acceptable error can mean an interval both above and below an ideal value.

Rigor

- Students build **conceptual understanding** of other useful contexts for percentages.
- Students continue to develop **procedural skills** for organizing and solving multi-step percentage problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	13 min	12 min	5 min	8 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, pre-cut rulers, one pair per student
- calculators

Math Language Development

New word

- *percent error*

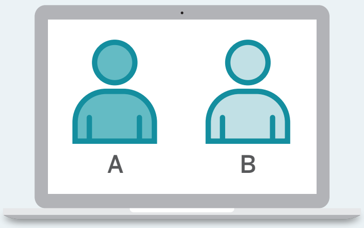
Review word

- *percent change*

Amps | Featured Activity

Activity 2
Partner Problems

Assigning Partner Problems is quick and easy with Amps. Students see only their column of problems, allowing them to focus exclusively on their problems before comparing their work with their partner.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students may experience a conflict with their partner over each other's critiques and not know how to manage it. Remind students to first listen to their partner and seek to understand what their partner's viewpoint is before voicing their own opinion. They might hear something that they had not previously considered and learn that they are incorrect. Or, they might convince the partners to change their minds. Either way, handling conflict peacefully and with grace will build healthier relationships.

Modifications to Pacing

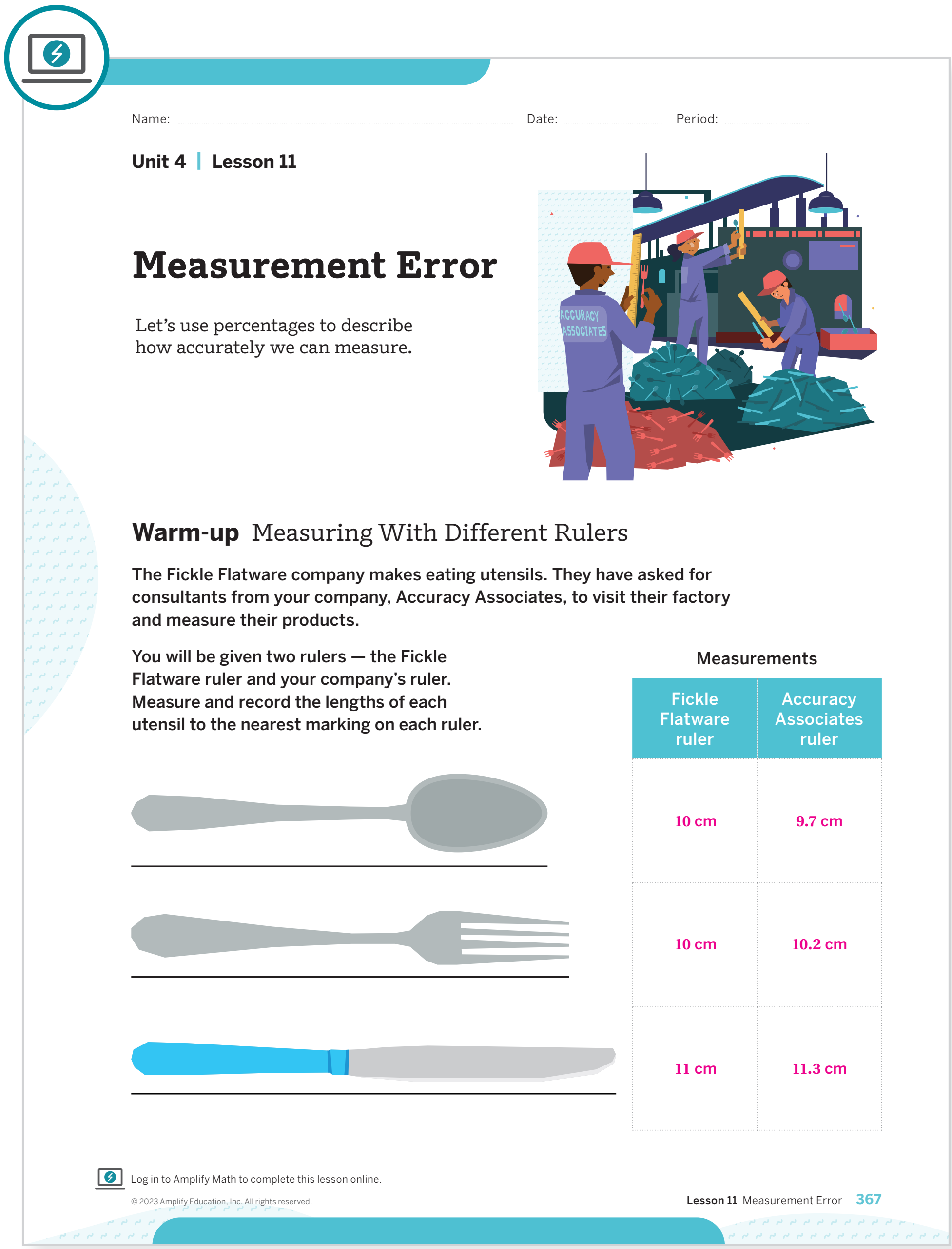
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up** and **Activity 1**, measuring and calculating for one of the utensils may be omitted.

Pairs | 8 min

Warm-up Measuring With Different Rulers

Students measure the same lengths with two different rulers — one showing only centimeters and the other with millimeters — to compare their accuracy.



Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 11

Measurement Error

Let's use percentages to describe how accurately we can measure.

Warm-up Measuring With Different Rulers

The Fickle Flatware company makes eating utensils. They have asked for consultants from your company, Accuracy Associates, to visit their factory and measure their products.

You will be given two rulers — the Fickle Flatware ruler and your company's ruler. Measure and record the lengths of each utensil to the nearest marking on each ruler.

Measurements

Fickle Flatware ruler	Accuracy Associates ruler
10 cm	9.7 cm
10 cm	10.2 cm
11 cm	11.3 cm

Log in to Amplify Math to complete this lesson online.

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Lesson 11 Measurement Error 367

1 Launch

Distribute the rulers from the Warm-up PDF. Remind students to first measure all three utensils — using the segment underneath — with the first ruler, then switch to the second. For each ruler, students should measure to the nearest marking. Provide access to calculators throughout the lesson. Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by helping ensure they are lining up their ruler precisely at the start of each segment.

Look for points of confusion:

- **Thinking they should estimate to the nearest millimeter while using the Fickle Flatware ruler.** Let students know that the goal is to compare the accuracy, and that they should measure to the nearest marking.

Look for productive strategies:

- Partners comparing their measurements and remeasuring the flatware to resolve any discrepancies between their values.

3 Connect

Have students share their measurements. Decide as a class on which measurements to use as the correct values.

Ask:

- "Why might it be helpful to have both partners measure independently?"
- "Which ruler is better for measuring more accurately? How do you know?"
- "Is it possible to measure even more accurately than with the Accuracy Associates ruler?

Highlight that students can say the Accuracy Associates ruler is more accurate because they are measuring to a more precise value. In the next activity, students will formalize that there is a way to represent how much more accurate one measure is than the other.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

After distributing the rulers, conduct the *Notice and Wonder* routine using the rulers. Ask:

- "Examine each ruler. What do you notice?"
  Sample response: The ruler for Fickle Flatware only includes whole number increments. The ruler for Accuracy Associates includes increments in between the whole numbers.
- "What do you wonder or what questions do you have?"
  Sample response: I wonder which ruler will give a more precise measurement.

Power-up

To power up students' ability to determine the different of decimal values, have student complete:

Recall that in order to subtract decimal values, the place values and decimal point must be aligned.

Evaluate each difference:

a. $120 - 108 \quad = 12$

b. $12 - 10.8 \quad = 1.2$

c. $1.2 - 1.08 \quad = 0.12$

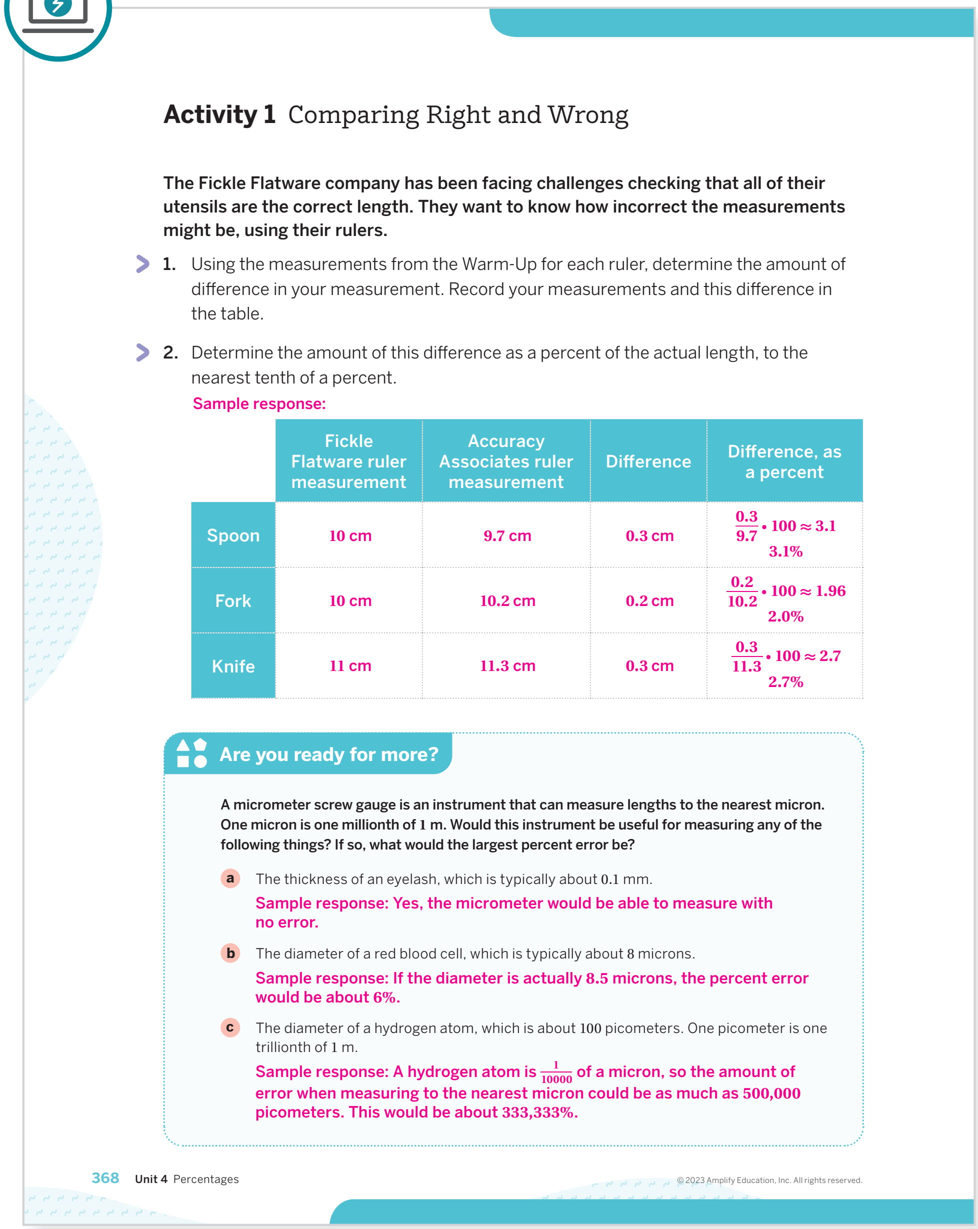
Use: Before Activity 1.

Informed by: Performance on Lesson 10, Practice Problem 6.

Pairs | 13 min

Activity 1 Comparing Right and Wrong

Students compare the difference between two values — one correct and one incorrect — to find the percent error of the incorrect value.



Activity 1 Comparing Right and Wrong

The Fickle Flatware company has been facing challenges checking that all of their utensils are the correct length. They want to know how incorrect the measurements might be, using their rulers.

1. Using the measurements from the Warm-Up for each ruler, determine the amount of difference in your measurement. Record your measurements and this difference in the table.
2. Determine the amount of this difference as a percent of the actual length, to the nearest tenth of a percent.

Sample response:

	Fickle Flatware ruler measurement	Accuracy Associates ruler measurement	Difference	Difference, as a percent
Spoon	10 cm	9.7 cm	0.3 cm	$\frac{0.3}{9.7} \cdot 100 \approx 3.1$ 3.1%
Fork	10 cm	10.2 cm	0.2 cm	$\frac{0.2}{10.2} \cdot 100 \approx 1.96$ 2.0%
Knife	11 cm	11.3 cm	0.3 cm	$\frac{0.3}{11.3} \cdot 100 \approx 2.7$ 2.7%

Are you ready for more?

A micrometer screw gauge is an instrument that can measure lengths to the nearest micron. One micron is one millionth of 1 m. Would this instrument be useful for measuring any of the following things? If so, what would the largest percent error be?

a. The thickness of an eyelash, which is typically about 0.1 mm.

Sample response: Yes, the micrometer would be able to measure with no error.

b. The diameter of a red blood cell, which is typically about 8 microns.

Sample response: If the diameter is actually 8.5 microns, the percent error would be about 6%.

c. The diameter of a hydrogen atom, which is about 100 picometers. One picometer is one trillionth of 1 m.

Sample response: A hydrogen atom is $\frac{1}{10000}$ of a micron, so the amount of error when measuring to the nearest micron could be as much as 500,000 picometers. This would be about 333,333%.

368 Unit 4 Percentages

1 Launch

Say, "Have you ever wanted to measure *how incorrect* a measurement might be?" Explain that *percent error* is the difference between an approximate value and an exact value, and is always compared to the exact value. Ask, "What might a formula for this calculation look like?" and encourage students to write the agreed-upon formula above the table.

2 Monitor

Help students get started by ensuring they are properly finding the difference of the two measures.

Look for points of confusion:

- **Thinking they always have to subtract Fickle from Accuracy or vice versa.** Remind students to subtract the smaller measurement from the larger measurement.
- **Dividing by the Fickle Flatware measurement.** Ask, "Which ruler did we agree should represent the exact measurement?"

Look for productive strategies:

- Using one equation, with the difference in the numerator of the percent ratio.

3 Connect

Display student work that uses two separate equations and one that uses one equation.

Define ***percent error*** as the difference between approximate and exact values, as a percentage of the exact value.

Highlight that finding *percent error* is similar to determining a percent change because we are comparing a part to a whole, and in both calculations it is important to be clear about which value represents the part and which represents the whole.

Ask, "When might *percent error* be more useful than simply knowing the measurement error?"

MLR Math Language Development

MLR7: *Compare and Connect*

During the Connect, as you display student work, draw students' attention to the connections between the *measurement*, the *measurement error*, and the *percent error*. Ask:

- "How is the measurement error related to the measurement?" Sample response: The measurement error is the difference between the actual measurement and the correct measurement.
- "How is the percent error related to the measurement error?" Sample response: The percent error describes the ratio of the measurement error to the correct measurement, as a percentage.

Help students make the connection between *percent change* and *percent error* more explicit by asking, "How is determining percent error similar to determining percent change?" Sample response: First, find the difference and then determine the percentage.

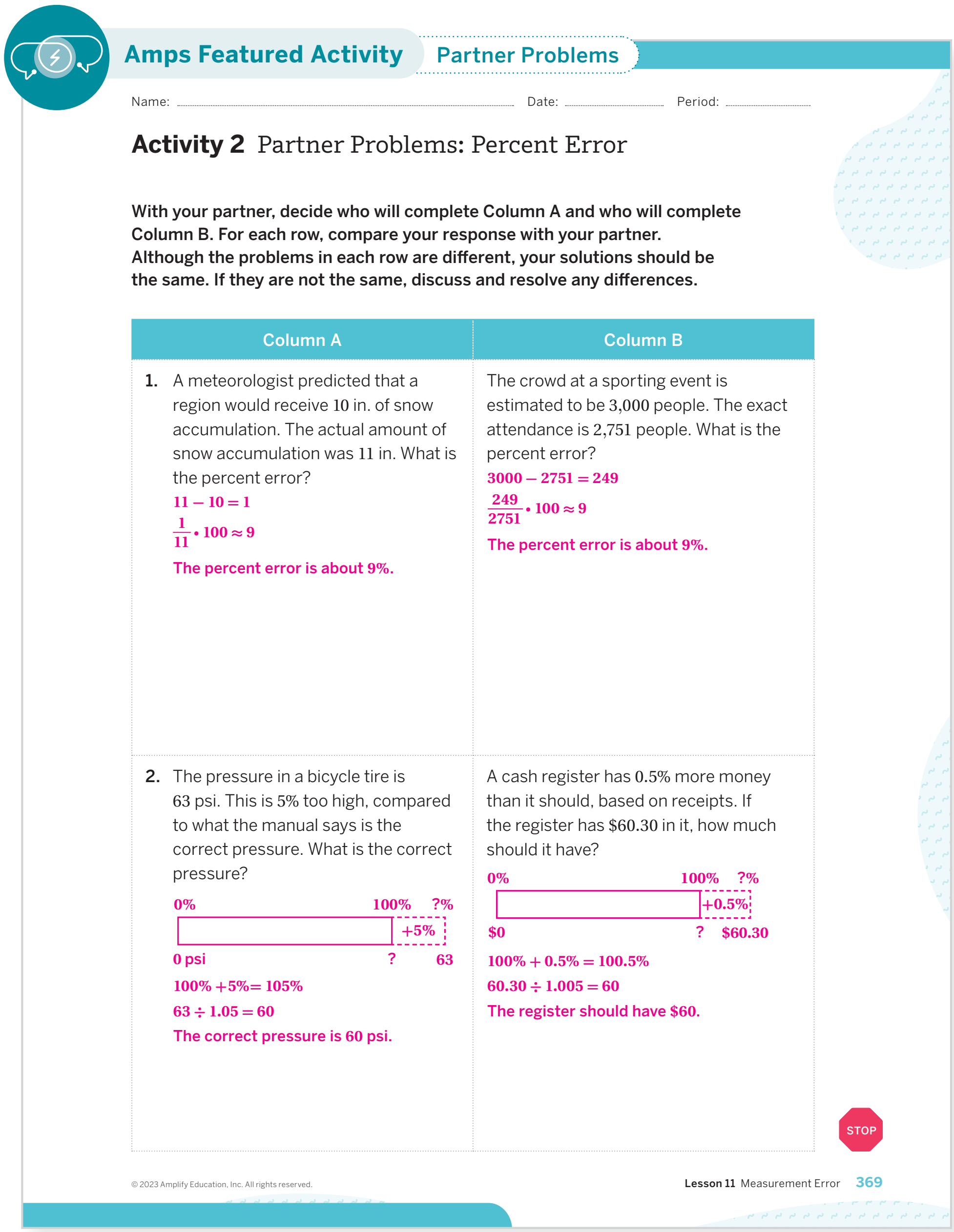
English Learners

Annotate the table in this activity using the terms *measurement* for the first two columns, *measurement error* for the third column, and *percent error* for the fourth column.

Pairs | 12 min

Activity 2 Partner Problems: Percent Error

Students solve percent error problems by identifying the erroneous value and the correct value to compare the size of the difference using a percent.



Amps Featured Activity Partner Problems

Name: ______________ Date: ________ Period: ________

Activity 2 Partner Problems: Percent Error

With your partner, decide who will complete Column A and who will complete Column B. For each row, compare your response with your partner. Although the problems in each row are different, your solutions should be the same. If they are not the same, discuss and resolve any differences.

Column A	Column B
1. A meteorologist predicted that a region would receive 10 in. of snow accumulation. The actual amount of snow accumulation was 11 in. What is the percent error? $11 - 10 = 1$ $\frac{1}{11} \cdot 100 \approx 9$ **The percent error is about 9%.**	The crowd at a sporting event is estimated to be 3,000 people. The exact attendance is 2,751 people. What is the percent error? $3000 - 2751 = 249$ $\frac{249}{2751} \cdot 100 \approx 9$ **The percent error is about 9%.**
2. The pressure in a bicycle tire is 63 psi. This is 5% too high, compared to what the manual says is the correct pressure. What is the correct pressure? 0% 100% ?% +5% 0 psi ? 63 $100\% + 5\% = 105\%$ $63 \div 1.05 = 60$ **The correct pressure is 60 psi.**	A cash register has 0.5% more money than it should, based on receipts. If the register has $60.30 in it, how much should it have? 0% 100% ?% +0.5% $0 ? $60.30 $100\% + 0.5\% = 100.5\%$ $60.30 \div 1.005 = 60$ **The register should have $60.**

STOP

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Lesson 11 Measurement Error 369

1 Launch

Say, "*Percent error* can be used in situations besides approximate and exact measurements. In general, it is used to compare any two values where one is correct and the other is incorrect." Let them know that an important task is to identify which value is which by reading carefully and relating words to values. Conduct the *Partner Problems* routine.

2 Monitor

Help students get started by suggesting they circle the numerical values and label them as *correct* or *incorrect*.

Look for points of confusion:

- **Thinking that the first value students encounter in a problem is the "correct" value.** Have students identify the words related to each value. Ask, "Does *predicted* indicate the correct or incorrect value?"
- **In Problem 2, not relating the values with their appropriate meaning.** Suggest students draw a tape diagram to help reason about the meaning of each value.

3 Connect

Have students share how they identified which value was the incorrect value and which was the correct value.

Ask:

- "What was different about the first set of problems from the second set?"

Highlight that terms indicating correct or incorrect values should not be considered a rule, but a guide. It is important that students make sense of the problem and reason about which is the correct value. The correct or exact value will match with 100%. Say, "Because percent error is a relative measurement, the percent change in Problem 1 is the same even though the values are very different."

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they see only their column of problems, allowing them to focus exclusively on their problems before comparing their work with their partner.

Accessibility: *Guide Processing and Visualization*

To help students get started and remain organized throughout the activity, provide students with the following checklist to keep track of their work:

- Identify the two measurements and determine which measurement is the actual measurement.
- Determine the amount of the error.
- Determine the percent error by determining the percentage of the amount of the error to the actual measurement.

Whole Class | 5 min

Summary

Review and synthesize the similarities between percent error and percent change problems.

Summary

In today's lesson . . .

You saw that ***percent error*** can be used to describe any situation where there is a correct value and an incorrect value, and you want to describe the relative difference between them. For example, suppose a milk carton manufactured by a company is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces:

- The measurement error is 1 fluid ounce.
- The percent error is 6.25% because $\frac{1}{16} \cdot 100 = 6.25$.

It is important to remember that the amount of the error is always compared to the actual or correct value to determine the percent error. You can use the following formula.

$$\text{percent error} = \frac{\text{(difference between correct and incorrect value)}}{\text{correct value}} \cdot 100$$

Reflect:

370 Unit 4 Percentages

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Synthesize

Formalize vocabulary, *percent error*

Ask:

- "When might you want to know how much error was involved in a situation?"
- "When might it be useful to compare the percent error from two situations?"

Highlight that multi-step percent situations often involve determining the amount of some change — in this case the change is the amount of the error — and comparing it to some starting value. When determining percent error, students may often start from an estimate or approximate value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why are percentages especially helpful to use when measuring error?"

Math Language Development

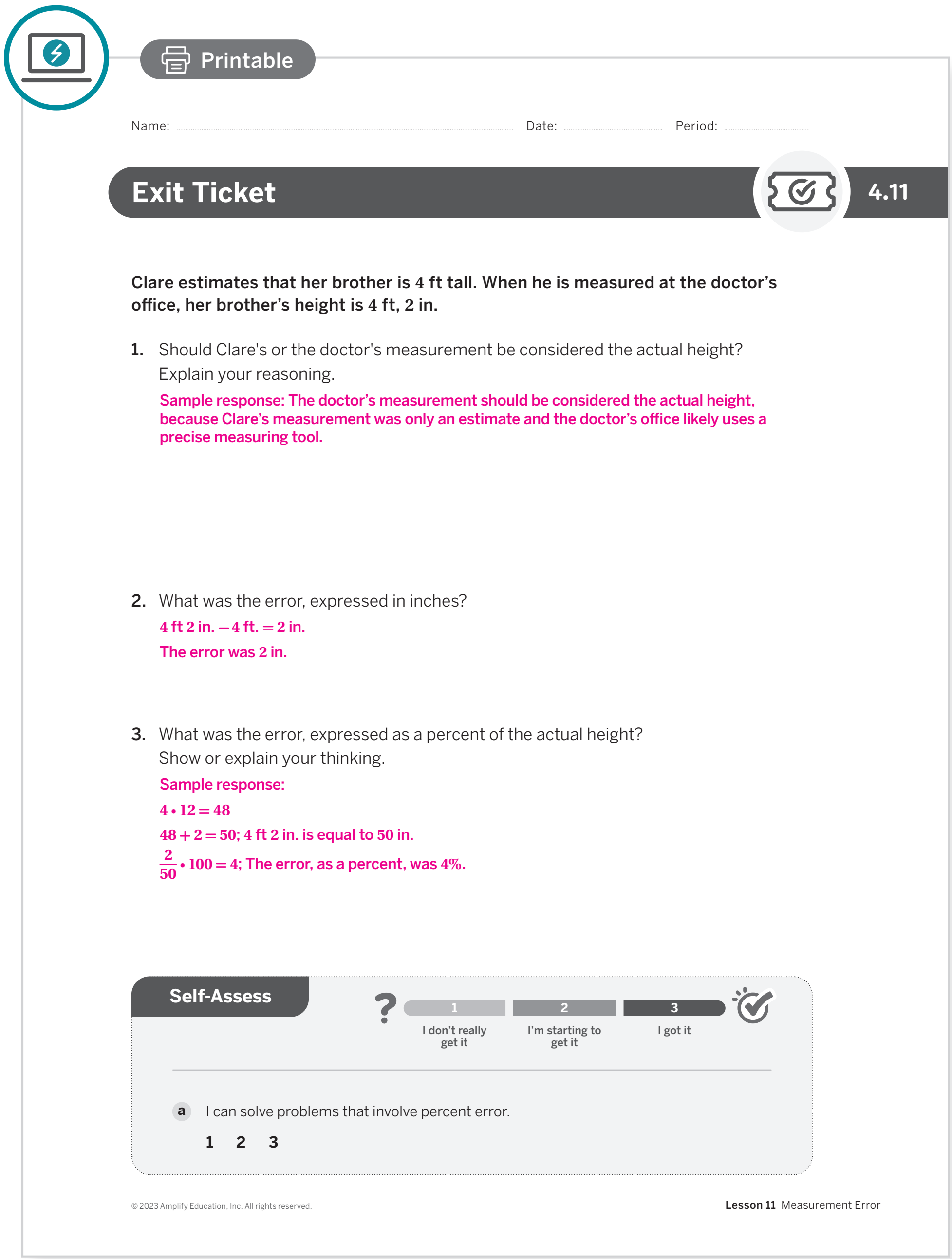
MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started earlier in this unit. Ask them to review and reflect on any terms and phrases related to the term *percent error* that were added to the display during the lesson.

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of percent error by comparing an estimated height of a person to the height of that person measured at a doctor's office.



Printable

Name: ______________________ Date: ____________ Period: ____________

Exit Ticket 4.11

Clare estimates that her brother is 4 ft tall. When he is measured at the doctor's office, her brother's height is 4 ft, 2 in.

1. Should Clare's or the doctor's measurement be considered the actual height? Explain your reasoning.

   Sample response: The doctor's measurement should be considered the actual height, because Clare's measurement was only an estimate and the doctor's office likely uses a precise measuring tool.

2. What was the error, expressed in inches?

   4 ft 2 in. − 4 ft. = 2 in.

   The error was 2 in.

3. What was the error, expressed as a percent of the actual height? Show or explain your thinking.

   Sample response:

   $4 \cdot 12 = 48$

   $48 + 2 = 50$; 4 ft 2 in. is equal to 50 in.

   $\frac{2}{50} \cdot 100 = 4$; The error, as a percent, was 4%.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can solve problems that involve percent error.

1 2 3

Lesson 11 Measurement Error

Success looks like . . .

- **Language Goal:** Comparing and contrasting multiple measurements of the same length that result from using rulers with different levels of precision. **(Speaking and Listening)**
- **Language Goal:** Calculating the percent error, correct amount, or erroneous amount, given the other two of these three quantities, and explaining the solution method. **(Speaking and Listening, Reading and Writing)**
  - » Determining the error and the percent error in Problems 2 and 3.
- **Language Goal:** Comparing and contrasting strategies used for solving problems about percent error with strategies used for solving problems about percent increase or decrease. **(Speaking and Listening)**

Suggested next steps

If students determine the difference to be 2 ft, consider:

- Asking them to use their hands to demonstrate what 2 ft looks like.

If students represent 4 ft 2 in. as 4.2, consider:

- Asking whether it makes sense that the percent error is so high.

If students use the estimate as the correct value, consider:

- Asking, "Which is more likely to be accurate, Clare's estimate or the measurement at the doctor's office?"
- Assigning Practice Problem 1.

Professional Learning

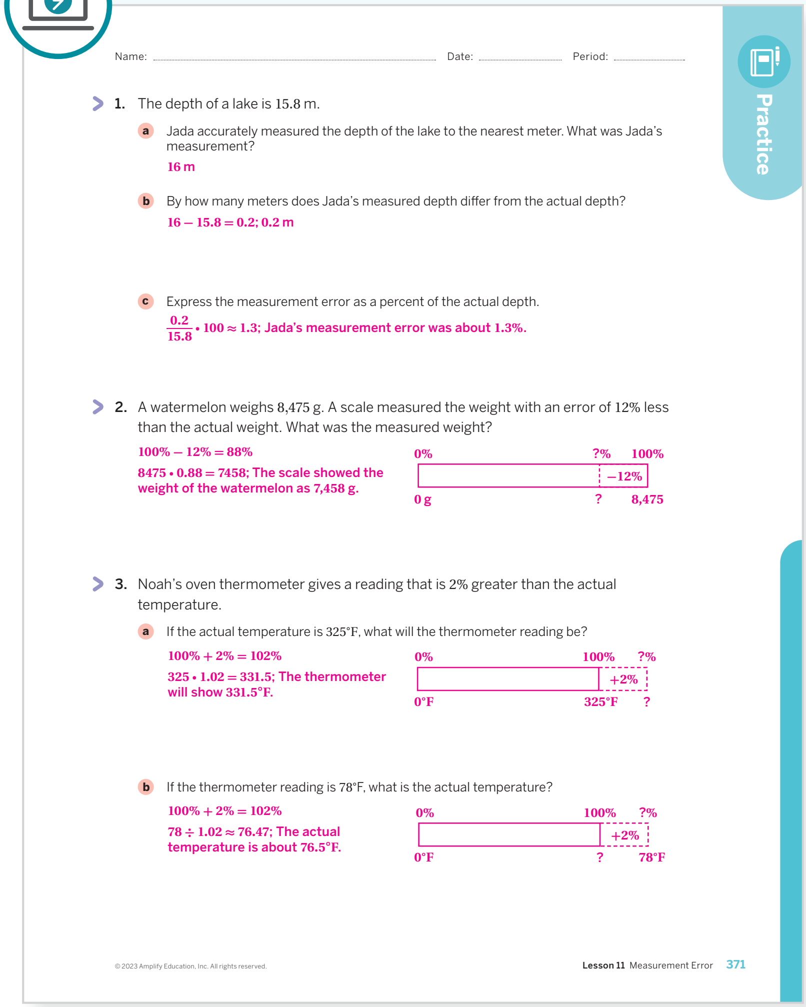
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students worked with percent increase and decrease. How did that support finding percent error?
- What did students find frustrating about finding percent error? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. The depth of a lake is 15.8 m.

a Jada accurately measured the depth of the lake to the nearest meter. What was Jada's measurement?

16 m

b By how many meters does Jada's measured depth differ from the actual depth?

$16 - 15.8 = 0.2$; 0.2 m

c Express the measurement error as a percent of the actual depth.

$\frac{0.2}{15.8} \cdot 100 \approx 1.3$; Jada's measurement error was about 1.3%.

2. A watermelon weighs 8,475 g. A scale measured the weight with an error of 12% less than the actual weight. What was the measured weight?

$100\% - 12\% = 88\%$

$8475 \cdot 0.88 = 7458$; The scale showed the weight of the watermelon as 7,458 g.

3. Noah's oven thermometer gives a reading that is 2% greater than the actual temperature.

a If the actual temperature is 325°F, what will the thermometer reading be?

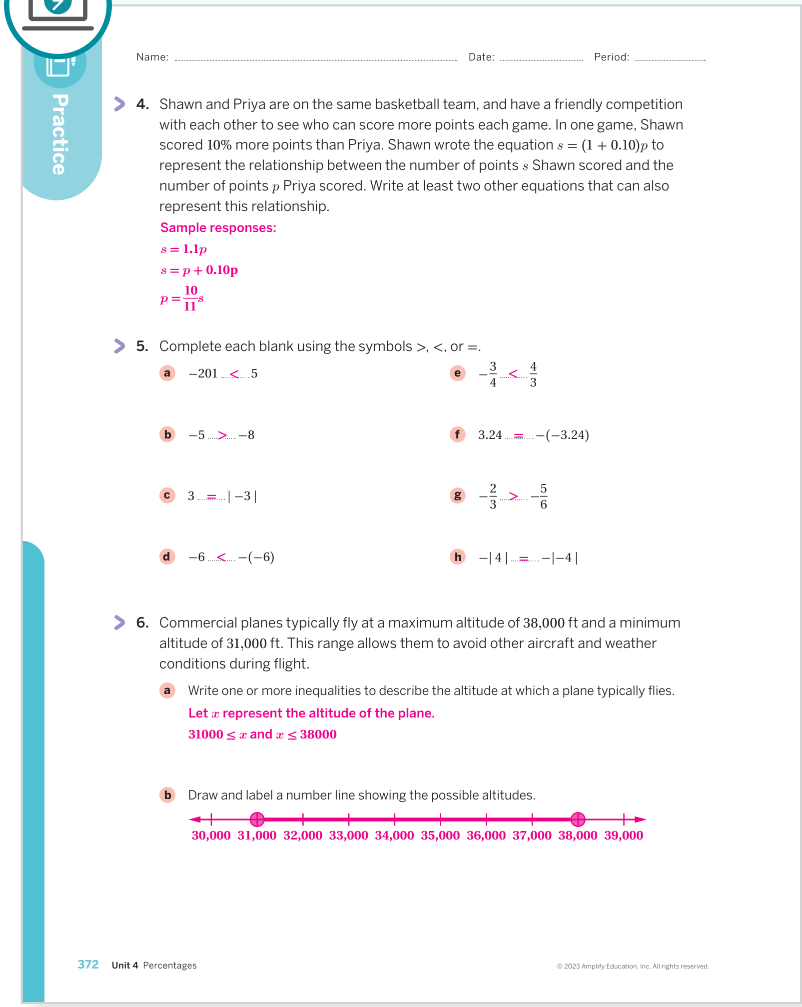
$100\% + 2\% = 102\%$

$325 \cdot 1.02 = 331.5$; The thermometer will show 331.5°F.

b If the thermometer reading is 78°F, what is the actual temperature?

$100\% + 2\% = 102\%$

$78 \div 1.02 \approx 76.47$; The actual temperature is about 76.5°F.



Name: ________ Date: ________ Period: ________

4. Shawn and Priya are on the same basketball team, and have a friendly competition with each other to see who can score more points each game. In one game, Shawn scored 10% more points than Priya. Shawn wrote the equation $s = (1 + 0.10)p$ to represent the relationship between the number of points s Shawn scored and the number of points p Priya scored. Write at least two other equations that can also represent this relationship.

Sample responses:

$s = 1.1p$

$s = p + 0.10\text{p}$

$p = \frac{10}{11}s$

5. Complete each blank using the symbols >, <, or =.

a -201 < 5

b -5 > -8

c 3 = $|-3|$

d -6 < $-(-6)$

e $-\frac{3}{4}$ < $\frac{4}{3}$

f 3.24 = $-(-3.24)$

g $-\frac{2}{3}$ > $-\frac{5}{6}$

h $-|4|$ = $-|-4|$

6. Commercial planes typically fly at a maximum altitude of 38,000 ft and a minimum altitude of 31,000 ft. This range allows them to avoid other aircraft and weather conditions during flight.

a Write one or more inequalities to describe the altitude at which a plane typically flies.

Let x represent the altitude of the plane.

$31000 \le x$ and $x \le 38000$

b Draw and label a number line showing the possible altitudes.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 6	2
	5	Grade 6	2
Formative ⚡	6	Unit 4 Lesson 12	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Optional

UNIT 4 | **LESSON 12**

Error Intervals

Let's determine how much error is acceptable.

Focus

Goals

1. Comprehend that manufacturers often define a maximum acceptable percent error for characteristics of their products.
2. **Language Goal:** Determine what information is needed to solve a problem involving percent error. Ask questions to elicit that information. **(Speaking and Listening)**
3. Generate values that fall within the acceptable range for a measurement, given a maximum percent error.

Coherence

Today

Students investigate real-world situations where error is not only acceptable — it is expected. When factories are responsible for producing thousands of products, or governments are spending thousands of dollars, it becomes virtually impossible to avoid error entirely. Students will explore how to calculate values within a certain amount of error.

Previously

In Lesson 11, students were introduced to percent error. They found that this context is related to their work with percent increase and decrease, but also differs in meaningful ways.

Coming Soon

In Lesson 13, students will conclude the unit by applying their understanding of percentages to headline writing. They will see how headlines that use percentages can be misleading and they will learn about the importance of accuracy and responsibility when writing headlines.

Rigor

- Students develop **conceptual understanding** of how percentages can be of greater use than absolute comparisons when considering acceptable error amounts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	12 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- *Info Gap Routine* PDF
- calculators

Math Language Development

New word

- ***error interval***

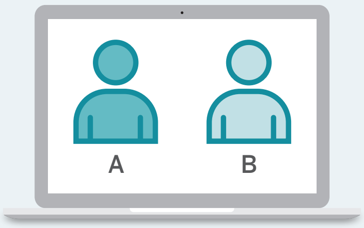
Review word

- *percent error*

Amps | Featured Activity

Activity 2
Digital Info Gap

This routine is essential to helping your students become collaborative problem solvers. The digital experience helps ensure students are clear about their roles and the logistics of which student receives which card at what time is automatically managed.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students work through the activities in this lesson, emphasize that acceptable amounts of spending compared to the targeted budget can be above or below the targeted budget. Some students may think that spending more than the targeted budget is not permissible when there is a percent error allowance.

Modifications to Pacing

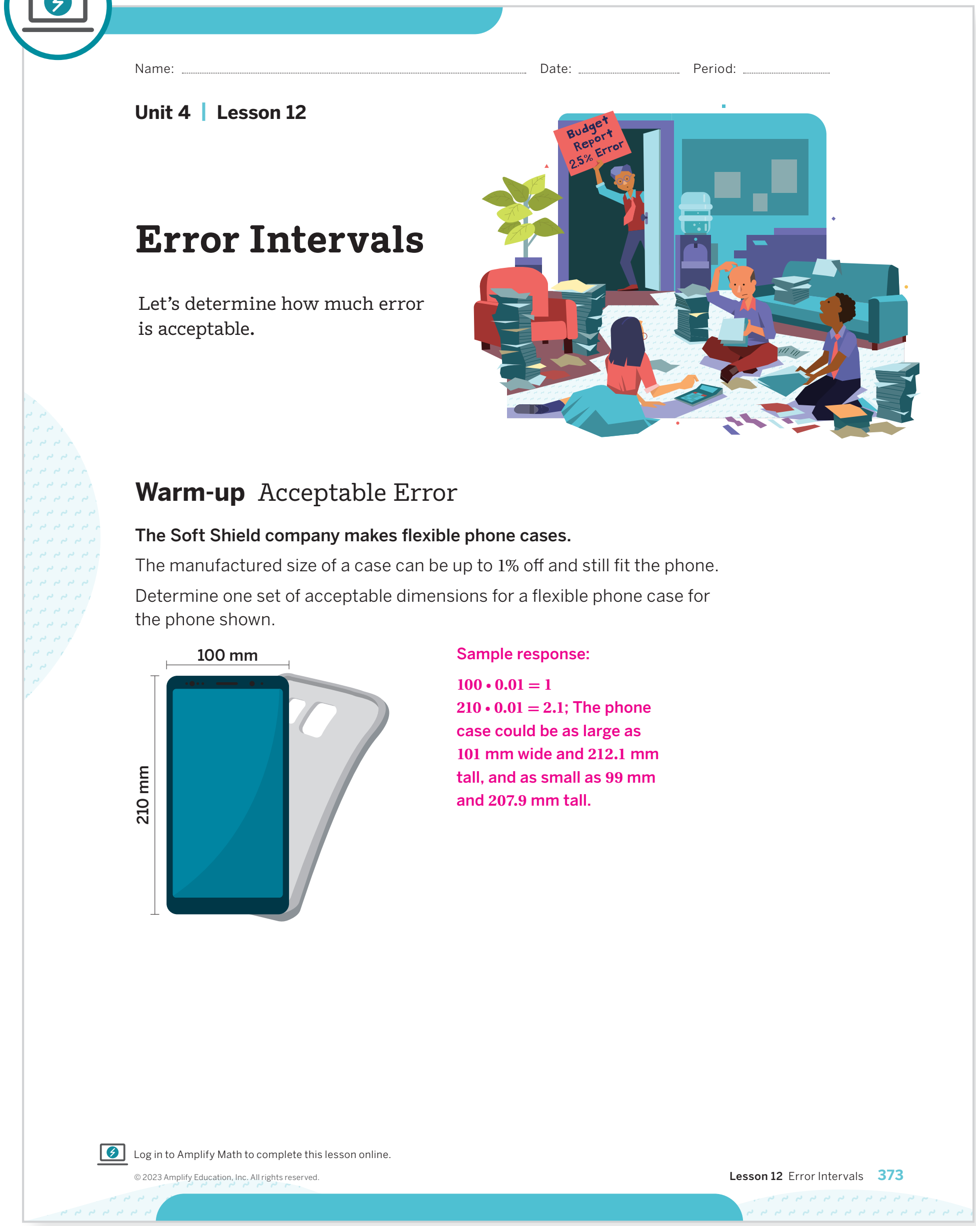
You may want to consider these additional modifications if you are short on time.

- The Warm-up may be omitted. Then, prior to **Activity 1**, mention that allowing for some error in measuring can mean both above and below a target value.
- In **Activity 1**, have students only complete the first two rows of the table.
- In **Activity 2**, have students only complete Problem 1.

Pairs | 8 min

Warm-up Acceptable Error

Students give possible lengths for a phone case to understand the idea that a maximum percent error defines an interval of values that a quantity can lie within.



Name: ____________ Date: ________ Period: ________

Unit 4 | Lesson 12

Error Intervals

Let's determine how much error is acceptable.

Warm-up Acceptable Error

The Soft Shield company makes flexible phone cases.

The manufactured size of a case can be up to 1% off and still fit the phone.

Determine one set of acceptable dimensions for a flexible phone case for the phone shown.

Sample response:

$100 \cdot 0.01 = 1$

$210 \cdot 0.01 = 2.1$; The phone case could be as large as 101 mm wide and 212.1 mm tall, and as small as 99 mm and 207.9 mm tall.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate background knowledge by asking, "Have you ever thought — perhaps even in math class — that you deserved credit for being 'close enough' to the right answer? What made you think you were 'close enough'?" Provide access to calculators throughout the lesson.

2 Monitor

Help students get started by asking what it means for something to be "a little off?"

Look for points of confusion:

- **Thinking that the only acceptable dimensions are exactly 1% above and below the given length.** Ask, "Do you think it would be okay if the phone case were exactly the lengths in the diagram? What else could work?"

Look for productive strategies:

- Noticing that one length is 100, making the calculations of percentages very straightforward.
- Selecting a length, and then checking that it is within 1% of the target.

3 Connect

Have students share all of the dimensions they calculated that they think are acceptable, and make a list for the class to see. Allow other students to comment on whether they think the dimensions are valid or not.

Ask:

- "What conclusions can you draw from this list?"
- "Have we listed all of the possible acceptable values? How do you know?"

Highlight that we call this range of values above and below an exact value, expressed as a percentage, the *error interval*. Say, "We can represent a range of values using the inequality symbols <, >, ≤, and ≥."

Differentiated Support

Extension: ***Math Enrichment***

Have students complete the following problem:

If the back of the phone case is covered with material that costs $0.03 per square centimeter, what are the minimum and maximum costs to cover the back of the phone case with this material? Do not subtract the material for the holes for the camera. The minimum cost is about $6.43 and the maximum cost is about $7.00.

Power-up

To power up students' ability to represent a range of possible values as an inequality, have students complete:

Children that are at least 3 years old but less than 16 years old qualify for a child ticket at a certain movie theater.

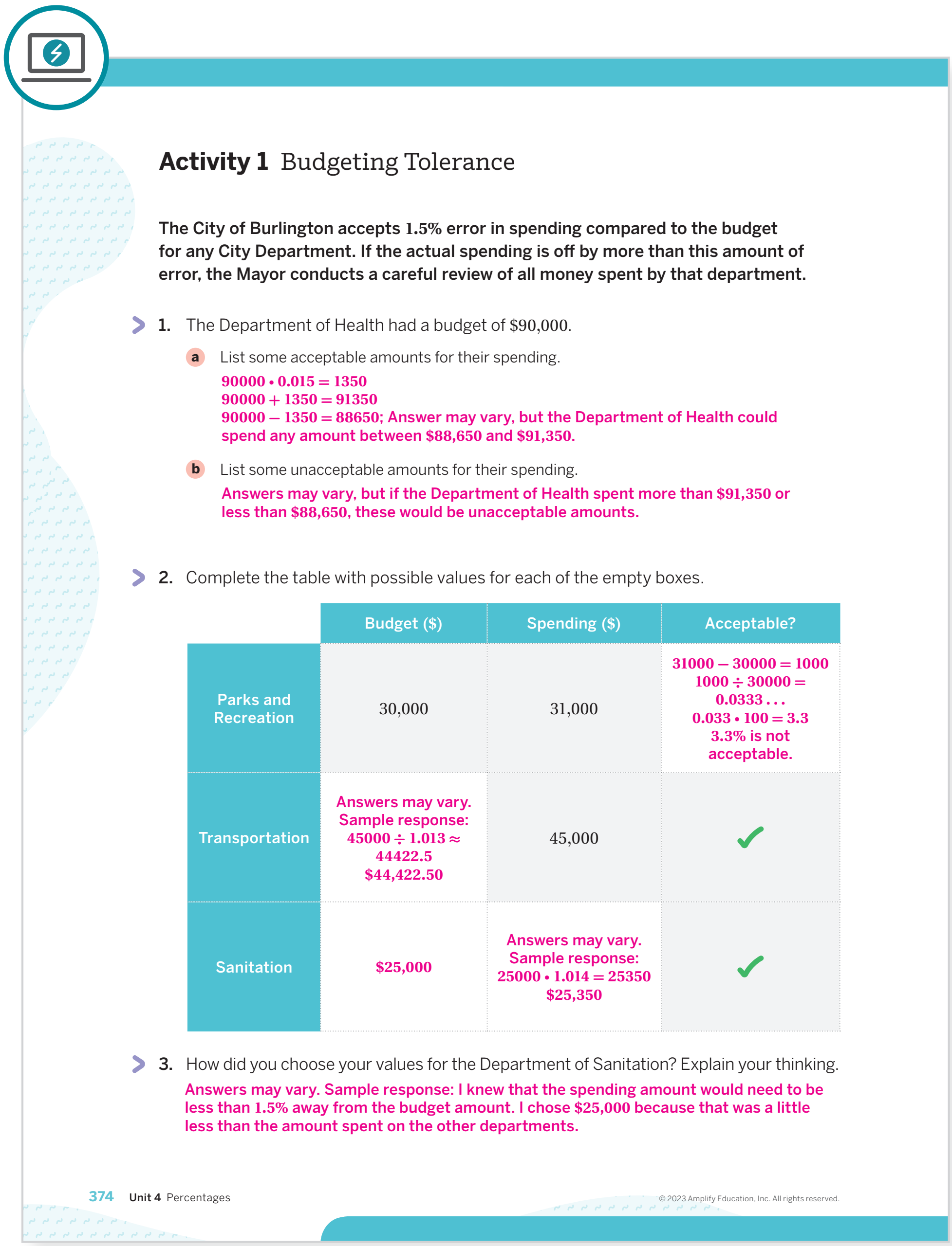
1. What are three possible ages that qualify for a child ticket?
   Answers may vary, but should satisfy the inequality $3 \le x < 16$.
2. What are three ages that would not qualify for a child ticket? Answers may vary, but should *not* satisfy the inequality $3 \le x < 16$.
3. Complete the two inequalities to represent the ages that qualify for a child ticket.
   $3 \boxed{\le} x$ and $x \boxed{<} 16$

Use: Before the Warm-up.
Informed by: Performance on Lesson 11, Practice Problem 6.

Pairs | 12 min

Activity 1 Budgeting Tolerance

Students analyze the annual budget of a city and compare the actual spending to what was planned, determining whether the amount spent was within the targeted interval.



Activity 1 Budgeting Tolerance

The City of Burlington accepts 1.5% error in spending compared to the budget for any City Department. If the actual spending is off by more than this amount of error, the Mayor conducts a careful review of all money spent by that department.

1. The Department of Health had a budget of $90,000.

   a. List some acceptable amounts for their spending.

   $90000 \cdot 0.015 = 1350$
   $90000 + 1350 = 91350$
   $90000 - 1350 = 88650$; Answer may vary, but the Department of Health could spend any amount between $88,650 and $91,350.

   b. List some unacceptable amounts for their spending.

   Answers may vary, but if the Department of Health spent more than $91,350 or less than $88,650, these would be unacceptable amounts.

2. Complete the table with possible values for each of the empty boxes.

	Budget ($)	Spending ($)	Acceptable?
Parks and Recreation	30,000	31,000	$31000 - 30000 = 1000$ $1000 \div 30000 = 0.0333...$ $0.033 \cdot 100 = 3.3$ 3.3% is not acceptable.
Transportation	Answers may vary. Sample response: $45000 \div 1.013 \approx 44422.5$ $44,422.50	45,000	✓
Sanitation	$25,000	Answers may vary. Sample response: $25000 \cdot 1.014 = 25350$ $25,350	✓

3. How did you choose your values for the Department of Sanitation? Explain your thinking.

   Answers may vary. Sample response: I knew that the spending amount would need to be less than 1.5% away from the budget amount. I chose $25,000 because that was a little less than the amount spent on the other departments.

374 **Unit 4** Percentages

1 Launch

Activate prior knowledge by asking students to recall that sales tax is paid to the government. Ask, "What are some things the government spends this money on?" Explain that a budget is a plan for what a person or group expects to spend over a certain period of time.

2 Monitor

Help students get started by asking how this problem is similar to the Warm-up problem.

Look for points of confusion:

- **Thinking that acceptable amounts are only below the target, but still within the percentage given.** Explain that the percent error allows for the percentage both above and below the target.
- **Having difficulty determining both a budget and spending for the Sanitation Department.** Supply a value for students to work from for the budget; $25,000 works well.

3 Connect

Display a student's completed table.

Have students share whether they agree with the values in the table shown.

Ask:

- "Why do you think there are acceptable amounts of error both above and below the budget target?"
- "Why does it make sense that each department is given a percentage to stay within rather than a fixed dollar amount?

Highlight that making a budget is an essential function of government. Often, the amount budgeted communicates both what is important to the people who work in government and also that it is capable of effective management.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Discuss what some of the roles of each department listed in the table might be for any particular city: Parks and Recreation, Transportation, and Sanitation. Have students brainstorm what some of the costs might be for each department. This will help engage them in the context of this task.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they will compare differences in budgeted amounts versus actual spending.
- **Read 2:** Ask students to identify the important quantities and relationships, such as, "Burlington accepts 1.5% error in spending." Encourage students to explain what each of the quantities represent, including any units.
- **Read 3:** Ask students to plan their solution strategies for each problem.

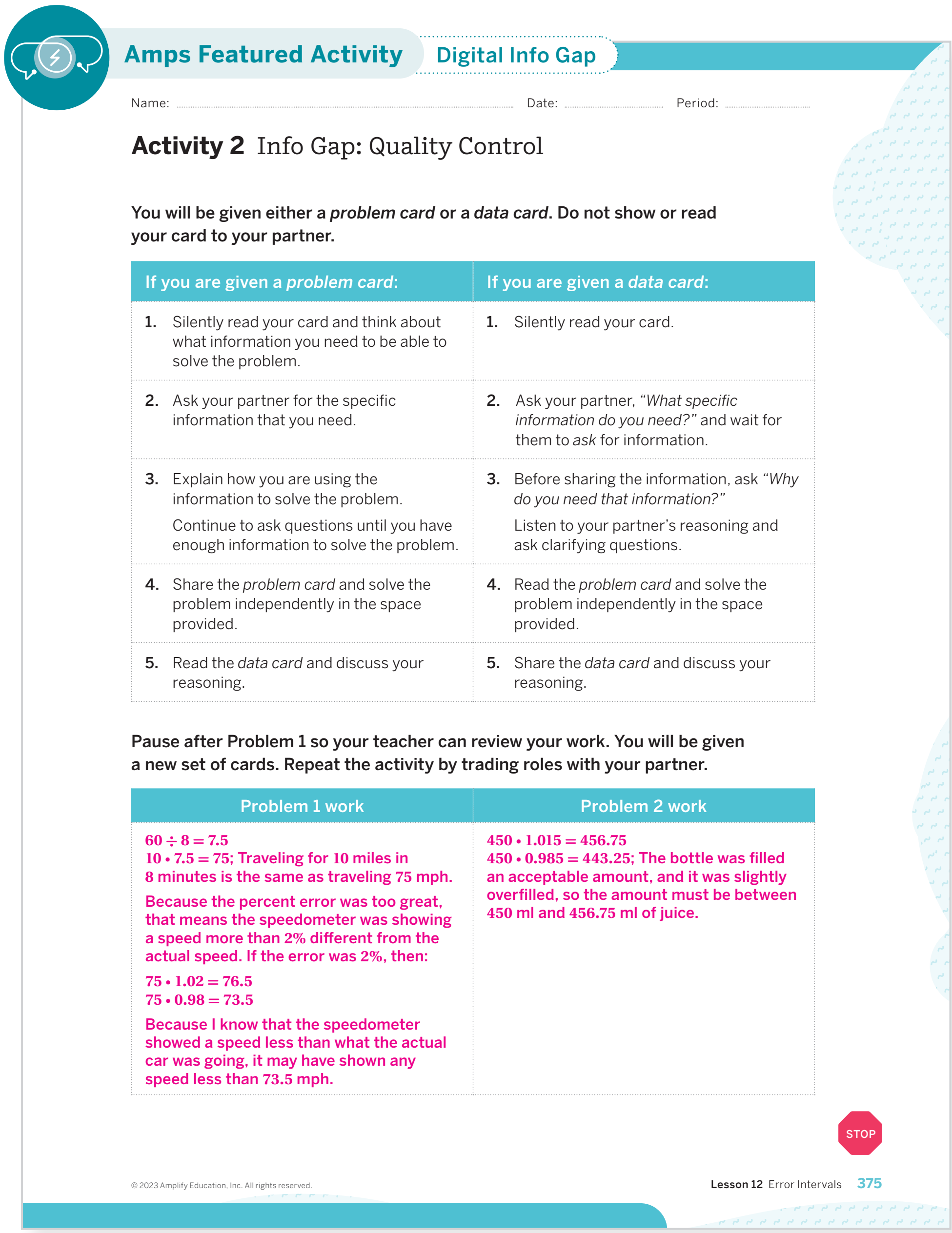
English Learners

Discuss what the terms *budget* and *acceptable* and *unacceptable amounts of spending* mean within the context of this text.

Pairs | 15 min

Activity 2 Info Gap: Quality Control

Students make sense of a problem involving mass factory production by determining what information is necessary to solve it, and then asking for that information.



Amps Featured Activity Digital Info Gap

Name: ______ Date: ______ Period: ______

Activity 2 Info Gap: Quality Control

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	1. Silently read your card.
2. Ask your partner for the specific information that you need.	2. Ask your partner, *"What specific information do you need?"* and wait for them to *ask* for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	3. Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space provided.	4. Read the *problem card* and solve the problem independently in the space provided.
5. Read the *data card* and discuss your reasoning.	5. Share the *data card* and discuss your reasoning.

Pause after Problem 1 so your teacher can review your work. You will be given a new set of cards. Repeat the activity by trading roles with your partner.

Problem 1 work	Problem 2 work
$60 \div 8 = 7.5$ $10 \cdot 7.5 = 75$; Traveling for 10 miles in 8 minutes is the same as traveling 75 mph. Because the percent error was too great, that means the speedometer was showing a speed more than 2% different from the actual speed. If the error was 2%, then: $75 \cdot 1.02 = 76.5$ $75 \cdot 0.98 = 73.5$ Because I know that the speedometer showed a speed less than what the actual car was going, it may have shown any speed less than 73.5 mph.	$450 \cdot 1.015 = 456.75$ $450 \cdot 0.985 = 443.25$; The bottle was filled an acceptable amount, and it was slightly overfilled, so the amount must be between 450 ml and 456.75 ml of juice.

STOP

1 Launch

Tell students they will continue to work with percent errors in realistic scenarios. Conduct the *Info Gap* routine. Distribute a *problem card* to one student per pair and a *data card* to the other student. After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

2 Monitor

Help students get started by asking what information the speedometer on a car gives.

Look for points of confusion:

- **Miscalculating the actual speed of the car.** Ask, "Was the car travelling more than or less than 60 mph?"
- **Thinking that the "percent error was too large" means the actual value was over the target.** Say, "In Activity 1, we saw that the error could be above or below the target. What does it mean for the percentage to be too large?"

Look for productive strategies:

- Finding the interval for the speed of the car first before working to find the exact answer.

3 Connect

Have pairs of students share a re-enactment of their discussion, for either problem, in front of the class.

Highlight that percent error is useful for expressing a range of values because it provides a rule that is applicable in multiple situations.

Ask, "What inequality statement could you write to represent the acceptable amounts of juice in Problem 2?" $443.25 \leq x$ and $x \leq 456.75$

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which the digital experience helps ensure they are clear about their roles and the logistics of which student receives which card at what time is automatically managed.

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. For example:

- "I wonder what the acceptable percent error is for this context. I will ask for the range for acceptable percent error."
- "I wonder what the speed of the car was. I can also ask for the time and distance the car traveled, and then I can calculate the speed."

Math Language Development

MLR4: Information Gap

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

English Learners

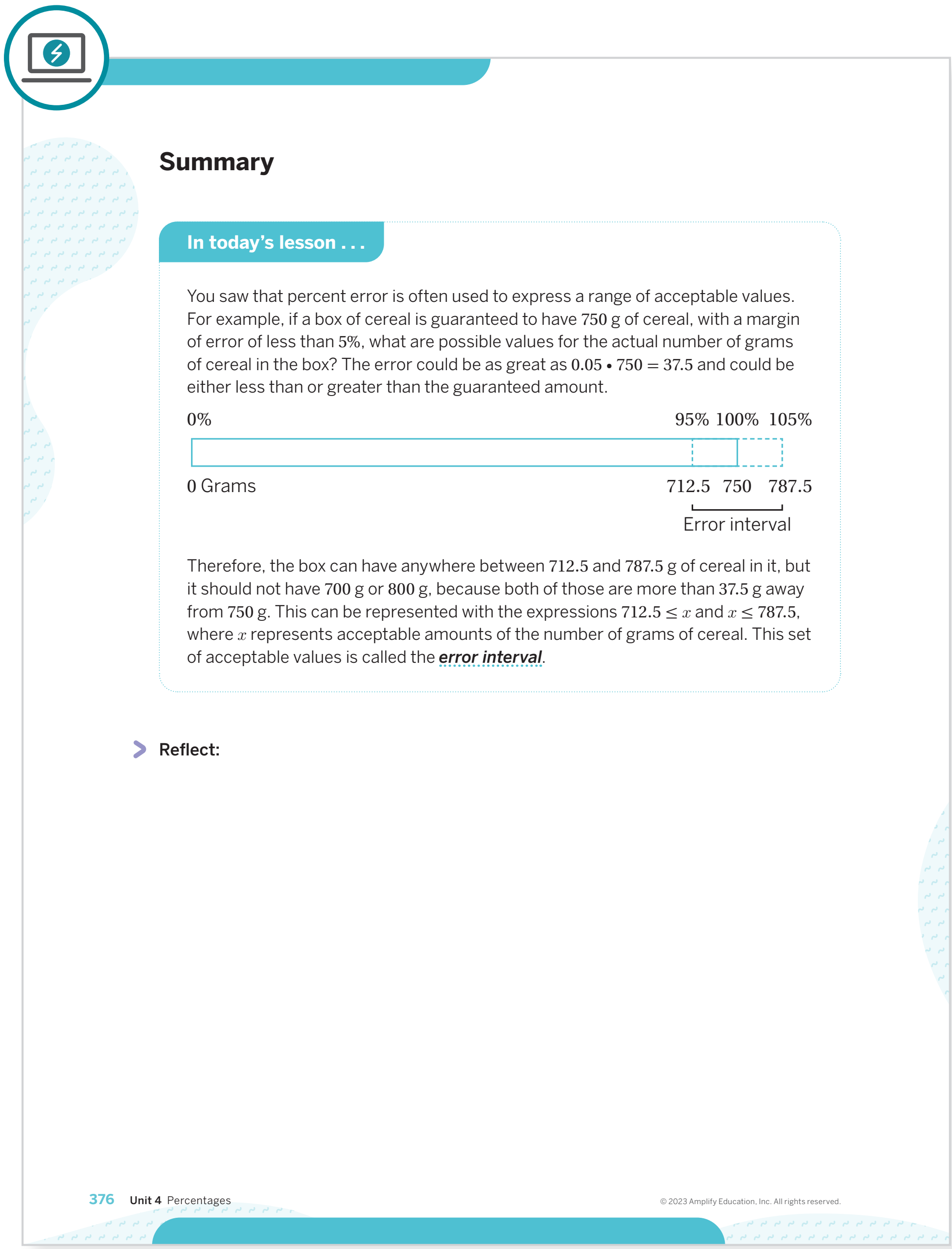
Consider providing sample questions students could ask, such as the following for Problem Card 1:

- How far did the car travel? How long, in terms of time, did the car travel?
- What is the range for acceptable percent error?

Whole Class | 5 min

Summary

Review and synthesize how to find the range of possible values within a percentage of error interval.



Summary

In today's lesson . . .

You saw that percent error is often used to express a range of acceptable values. For example, if a box of cereal is guaranteed to have 750 g of cereal, with a margin of error of less than 5%, what are possible values for the actual number of grams of cereal in the box? The error could be as great as $0.05 \cdot 750 = 37.5$ and could be either less than or greater than the guaranteed amount.

Therefore, the box can have anywhere between 712.5 and 787.5 g of cereal in it, but it should not have 700 g or 800 g, because both of those are more than 37.5 g away from 750 g. This can be represented with the expressions $712.5 \le x$ and $x \le 787.5$, where x represents acceptable amounts of the number of grams of cereal. This set of acceptable values is called the ***error interval***.

Reflect:

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Synthesize

Ask:

- "How is determining percent error similar to determining a percent increase or decrease?"
- How is determining percent error different from determining a percent increase or decrease?"

Formalize vocabulary: error interval

Highlight that determining a percent error is similar to determining a percent increase and decrease at the same time. Using the same percentage to determine a value both above and below a target helps to specify a range of acceptable values. Because perfection is rarely attainable by humans or machines, it can be useful to know when it is okay to get close to a target rather than hit it exactly.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

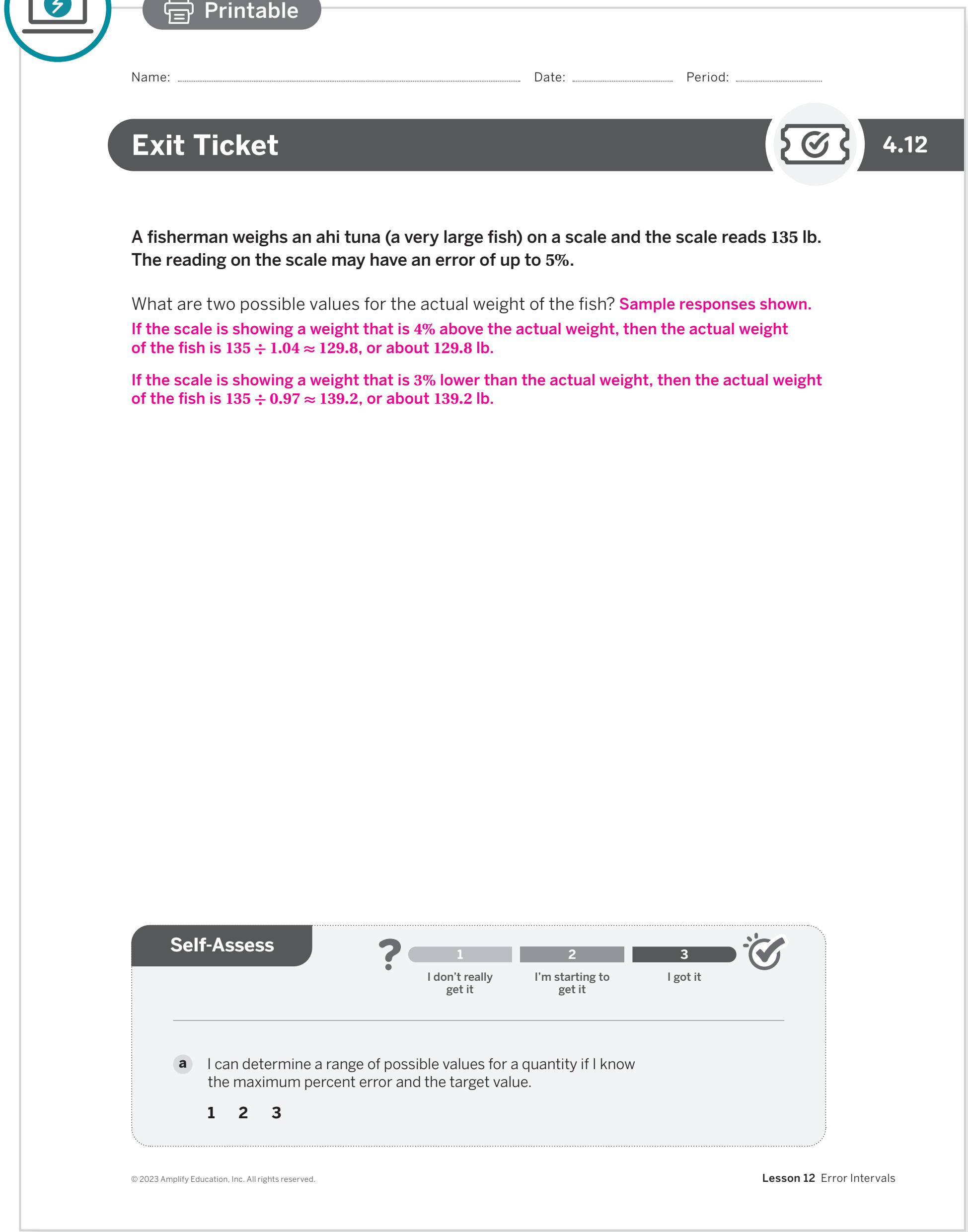
- "When is it most helpful to use percentages?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of percent error intervals by determining possible actual values when given an erroneous value and the possible percent error.

Printable



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Exit Ticket 4.12

A fisherman weighs an ahi tuna (a very large fish) on a scale and the scale reads 135 lb. The reading on the scale may have an error of up to 5%.

What are two possible values for the actual weight of the fish? **Sample responses shown.**

If the scale is showing a weight that is 4% above the actual weight, then the actual weight of the fish is $135 \div 1.04 \approx 129.8$, or about 129.8 lb.

If the scale is showing a weight that is 3% lower than the actual weight, then the actual weight of the fish is $135 \div 0.97 \approx 139.2$, or about 139.2 lb.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can determine a range of possible values for a quantity if I know the maximum percent error and the target value.

1 2 3

Lesson 12 Error Intervals

Success looks like . . .

- **Goal:** Comprehending that manufacturers often define a maximum acceptable percent error for characteristics of their products.
- **Language Goal:** Determining what information is needed to solve a problem involving percent error. Asking questions to elicit that information. **(Speaking and Listening)**
- **Goal:** Generating values that fall within the acceptable range for a measurement, given a maximum percent error.
    - » Determining two possible values for the actual weight of the fish.

Suggested next steps

If students use 135 lb as the actual value, consider:

- Having them draw a tape diagram to represent the situation.
- Asking, "Do you know whether the scale is accurate or not?"
- Assigning Practice Problem 2.

If students determine two values on the same side of the interval, consider:

- Mentioning that, while this is acceptable, it is worth reminding that the values could be on *either* side of the actual value.

Professional Learning

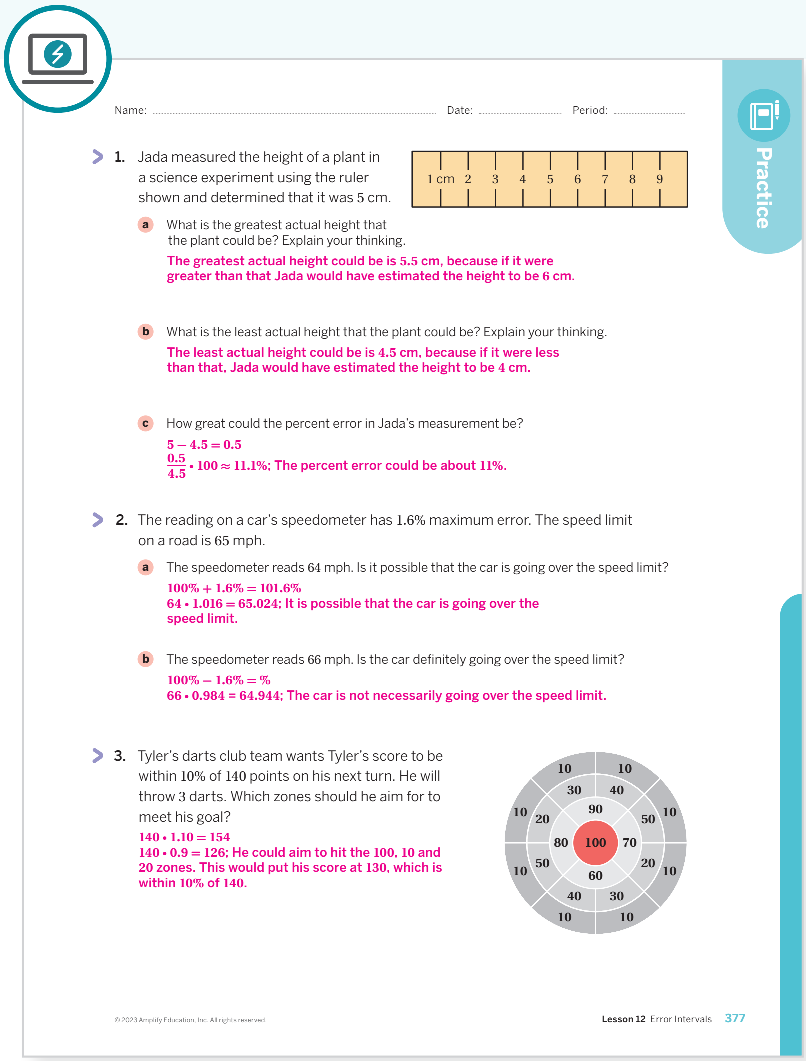
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did the ***Info Gap*** routine go as planned?
- How did the ***Info Gap*** routine support students in determining what information is needed to solve a problem involving percent error? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Jada measured the height of a plant in a science experiment using the ruler shown and determined that it was 5 cm.

a. What is the greatest actual height that the plant could be? Explain your thinking.

The greatest actual height could be is 5.5 cm, because if it were greater than that Jada would have estimated the height to be 6 cm.

b. What is the least actual height that the plant could be? Explain your thinking.

The least actual height could be is 4.5 cm, because if it were less than that, Jada would have estimated the height to be 4 cm.

c. How great could the percent error in Jada's measurement be?

$5 - 4.5 = 0.5$

$\frac{0.5}{4.5} \cdot 100 \approx 11.1\%$; **The percent error could be about 11%.**

2. The reading on a car's speedometer has 1.6% maximum error. The speed limit on a road is 65 mph.

a. The speedometer reads 64 mph. Is it possible that the car is going over the speed limit?

$100\% + 1.6\% = 101.6\%$

$64 \cdot 1.016 = 65.024$; **It is possible that the car is going over the speed limit.**

b. The speedometer reads 66 mph. Is the car definitely going over the speed limit?

$100\% - 1.6\% = \%$

$66 \cdot 0.984 = 64.944$; **The car is not necessarily going over the speed limit.**

3. Tyler's darts club team wants Tyler's score to be within 10% of 140 points on his next turn. He will throw 3 darts. Which zones should he aim for to meet his goal?

$140 \cdot 1.10 = 154$

$140 \cdot 0.9 = 126$; **He could aim to hit the 100, 10 and 20 zones. This would put his score at 130, which is within 10% of 140.**

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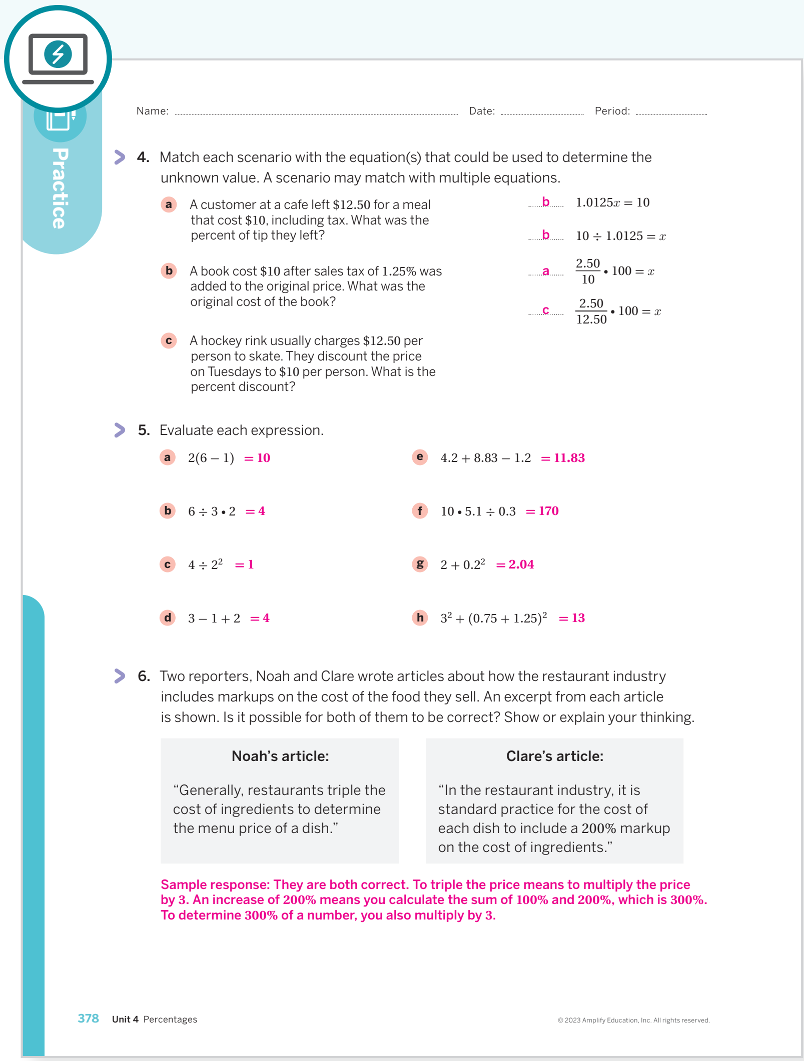
Lesson 12 Error Intervals 377



Name: Date: Period:

Practice

4. Match each scenario with the equation(s) that could be used to determine the unknown value. A scenario may match with multiple equations.

a. A customer at a cafe left \$12.50 for a meal that cost \$10, including tax. What was the percent of tip they left?

b. A book cost \$10 after sales tax of 1.25% was added to the original price. What was the original cost of the book?

c. A hockey rink usually charges \$12.50 per person to skate. They discount the price on Tuesdays to \$10 per person. What is the percent discount?

b $1.0125x = 10$

b $10 \div 1.0125 = x$

a $\frac{2.50}{10} \cdot 100 = x$

c $\frac{2.50}{12.50} \cdot 100 = x$

5. Evaluate each expression.

a. $2(6 - 1)$ **$= 10$**

b. $6 \div 3 \cdot 2$ **$= 4$**

c. $4 \div 2^2$ **$= 1$**

d. $3 - 1 + 2$ **$= 4$**

e. $4.2 + 8.83 - 1.2$ **$= 11.83$**

f. $10 \cdot 5.1 \div 0.3$ **$= 170$**

g. $2 + 0.2^2$ **$= 2.04$**

h. $3^2 + (0.75 + 1.25)^2$ **$= 13$**

6. Two reporters, Noah and Clare wrote articles about how the restaurant industry includes markups on the cost of the food they sell. An excerpt from each article is shown. Is it possible for both of them to be correct? Show or explain your thinking.

Noah's article:

"Generally, restaurants triple the cost of ingredients to determine the menu price of a dish."

Clare's article:

"In the restaurant industry, it is standard practice for the cost of each dish to include a 200% markup on the cost of ingredients."

Sample response: They are both correct. To triple the price means to multiply the price by 3. An increase of 200% means you calculate the sum of 100% and 200%, which is 300%. To determine 300% of a number, you also multiply by 3.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	3
Spiral	4	Unit 4 Lesson 8	2
	5	Grades 5 and 6	2
Formative	6	Unit 4 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 4 | **LESSON 13 – CAPSTONE**

Writing Better Headlines

Let's write responsible and accurate headlines.

Focus

Goals

1. Comprehend that a responsible and accurate headline represents the most honest way to interpret some information.
2. **Language Goal:** Analyze a data set and write questions that can be answered by this analysis. **(Reading and Writing)**
3. **Language Goal:** Write headlines that responsibly and accurately represent the data in a data set. **(Reading and Writing)**

Rigor

- Students build **conceptual understanding** of how percentages can be used to convey large amounts of information succinctly.
- Students **apply** their understanding of which types of percentages are more appropriate to use for different situations.

Coherence

Today

Students take on the role of editor in a newsroom, and apply their skills with calculating and reasoning about percent to craft headlines. Throughout the lesson, the importance of responsible and accurate reporting is emphasized. Students both edit misleading headlines and write their own after analyzing a data set.

Previously

In Unit 4, students solved multi-step percent problems and became familiar with all of the ways percents can be used to compare and represent change.

Coming Soon

In Unit 5, students will build on their work with integers from Grade 6 and explore the set of rational numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	12 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (Data sets), one per student
- Activity 2 PDF (answers)

Math Language Development

Review words

- *percent change*

Amps | Featured Activity

Warm-up

Poll the Class

Students choose which headline best fits a piece of news, and justify their thinking. You can aggregate these results and present them back to the class.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activities 1 and 2, students might become frustrated as they learn that headlines can be misleading. Have a large-group discussion about honesty and integrity. Explain that writing a "technically honest" headline could be misleading to their readers. Encourage discussions about ethical responsibility and how to be sure a headline is not misleading.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

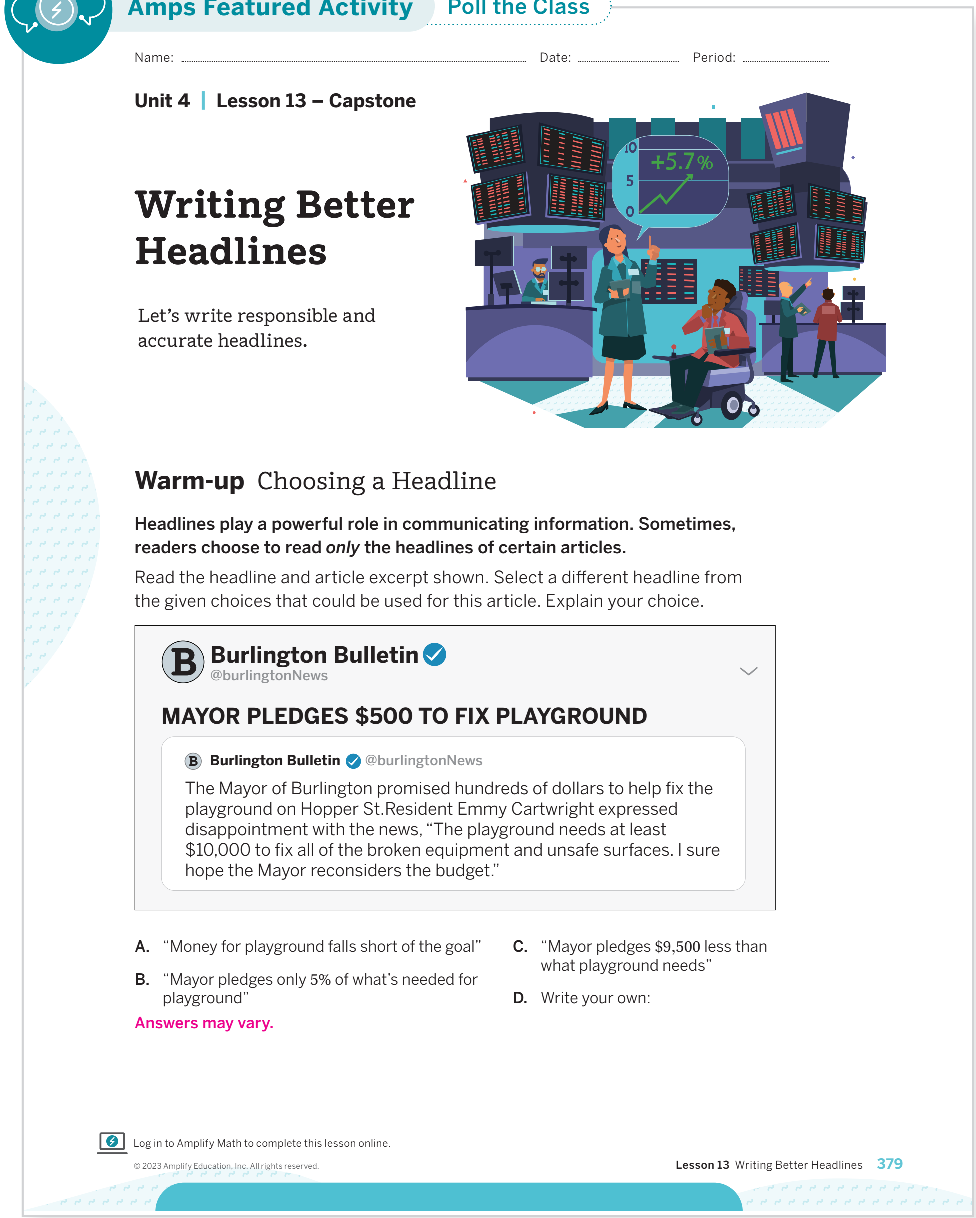
- In the **Warm-up**, you may have students skip writing their explanations in favor of a quick share with a partner.
- In **Activity 1**, you may choose to have students only complete one of the problems.
- In **Activity 2**, Problem 2 may be omitted. Instead, after Problem 1, mention that a responsible reporter would then carefully analyze the data set in order to answer the question.

Pairs | 8 min

Warm-up Choosing a Headline

Students read a short article excerpt to determine which headline best represents the information responsibly and accurately.

Amps Featured Activity Poll the Class



Name: Date: Period:

Unit 4 | Lesson 13 – Capstone

Writing Better Headlines

Let's write responsible and accurate headlines.

Warm-up Choosing a Headline

Headlines play a powerful role in communicating information. Sometimes, readers choose to read *only* the headlines of certain articles.

Read the headline and article excerpt shown. Select a different headline from the given choices that could be used for this article. Explain your choice.

Burlington Bulletin @burlingtonNews

MAYOR PLEDGES $500 TO FIX PLAYGROUND

Burlington Bulletin @burlingtonNews

The Mayor of Burlington promised hundreds of dollars to help fix the playground on Hopper St.Resident Emmy Cartwright expressed disappointment with the news, "The playground needs at least $10,000 to fix all of the broken equipment and unsafe surfaces. I sure hope the Mayor reconsiders the budget."

A. "Money for playground falls short of the goal"

B. "Mayor pledges only 5% of what's needed for playground"

C. "Mayor pledges $9,500 less than what playground needs"

D. Write your own:

Answers may vary.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate background knowledge by asking if students have ever noticed when a headline did not seem to tell the truth. Explain that numbers — and especially percentages — are often used in headlines to make an argument.

2 Monitor

Help students get started by asking, "How do you feel after just reading the headline? Is that different from how you feel after reading the article excerpt?"

Look for points of confusion:

- **Thinking that the headline is false, rather than misleading.** Let students know that only in rare instances are headlines typically untrue. This lesson will focus on headlines that are misleading.

Look for productive strategies:

- Selecting choice B because it uses a percentage, and realizing that a relative comparison is useful because $500 is rather small compared to $10,000.

3 Connect

Have pairs of students share their headline choice with each other, discussing the reason behind their choices.

Highlight that, as a relative comparison, percents can be very useful to convey information quickly. In a headline, this quickness is important, since you only have a few words to tell the reader about what is important.

Ask:

- "When is $500 a relatively large amount of money?" When you are thinking about the price of food, or amounts in the tens of dollars.
- "When is $500 a relatively small amount of money?" When you are thinking about the price of a car or a home, or amounts in the tens of thousands of dollars.

Power-up

To power up students' ability to understand how change can be described as both a percent increase and as multiplication, have students complete.

The original cost of an item is $40.

1. In Store A the item has a markup of 150%. What is the price after the markup?
   $100; $40 + 1.5(40) = 100$
2. In Store B the item's price is 2.5 times greater than the original cost. What is the price at Store B? $100; $40 \cdot 2.5 = 100$
3. Are the costs the same or different? The same.

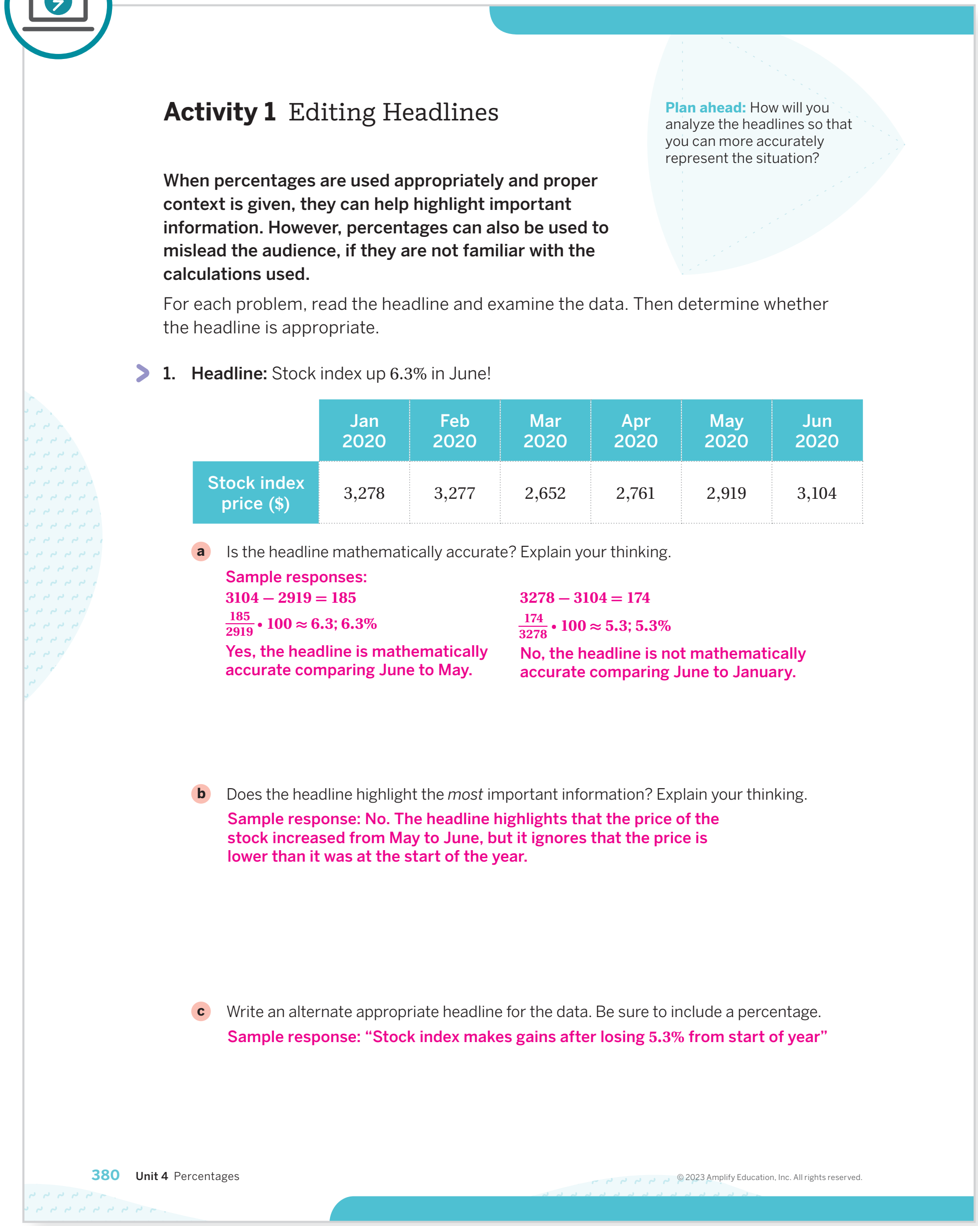
Use: Before Activity 1.

Informed by: Performance on Lesson 12, Practice Problem 6.

Pairs | 12 min

Activity 1 Editing Headlines

Students analyze small tables of data to consider how an existing headline, containing a percentage, can be improved to be more accurate and responsible.



Activity 1 Editing Headlines

Plan ahead: How will you analyze the headlines so that you can more accurately represent the situation?

When percentages are used appropriately and proper context is given, they can help highlight important information. However, percentages can also be used to mislead the audience, if they are not familiar with the calculations used.

For each problem, read the headline and examine the data. Then determine whether the headline is appropriate.

1. **Headline:** Stock index up 6.3% in June!

	Jan 2020	Feb 2020	Mar 2020	Apr 2020	May 2020	Jun 2020
Stock index price ($)	3,278	3,277	2,652	2,761	2,919	3,104

a. Is the headline mathematically accurate? Explain your thinking.

Sample responses:

$3104 - 2919 = 185$

$\frac{185}{2919} \cdot 100 \approx 6.3; 6.3\%$

Yes, the headline is mathematically accurate comparing June to May.

$3278 - 3104 = 174$

$\frac{174}{3278} \cdot 100 \approx 5.3; 5.3\%$

No, the headline is not mathematically accurate comparing June to January.

b. Does the headline highlight the *most* important information? Explain your thinking.

Sample response: No. The headline highlights that the price of the stock increased from May to June, but it ignores that the price is lower than it was at the start of the year.

c. Write an alternate appropriate headline for the data. Be sure to include a percentage.

Sample response: "Stock index makes gains after losing 5.3% from start of year"

1 Launch

Activate prior knowledge by asking students what it means to edit writing. Tell students that news organizations employ people strictly as editors to help ensure that the writing is clear to the reader. In this activity, their job will be to edit some headlines that are misleading in some way.

2 Monitor

Help students get started by having them compare the percent change in price from May 2020 to June 2020.

Look for points of confusion:

- **Not understanding why the headline in Problem 1 could be considered misleading.** Have students compare the price in June to the price in January. Ask, "Is there more to the story of the stock index price for the year?"

Look for productive strategies:

- Trying to first identify how the percentage could be accurate, then looking at the data more broadly.
- Noticing that headlines that use exclamation points may be trying to sensationalize the news, valuing excitement over accuracy.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

It may be helpful for some students to begin with Problem 2. Even though the first column contains rather large quantities, students may be able to reason about the other columns more readily than in Problem 1.

Extension: *Math Enrichment*

Have students research other headlines that they think might not be mathematically accurate and share those with the class.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they will analyze how percentages can be used in misleading headlines.
- **Read 2:** Ask students to identify the important quantities and relationships in each headline, such as, "The stock index is up 6.3% in June." Encourage students to explain what each of the quantities represent, including any units.
- **Read 3:** Ask students to plan their solution strategies for determining whether each headline is mathematically accurate.

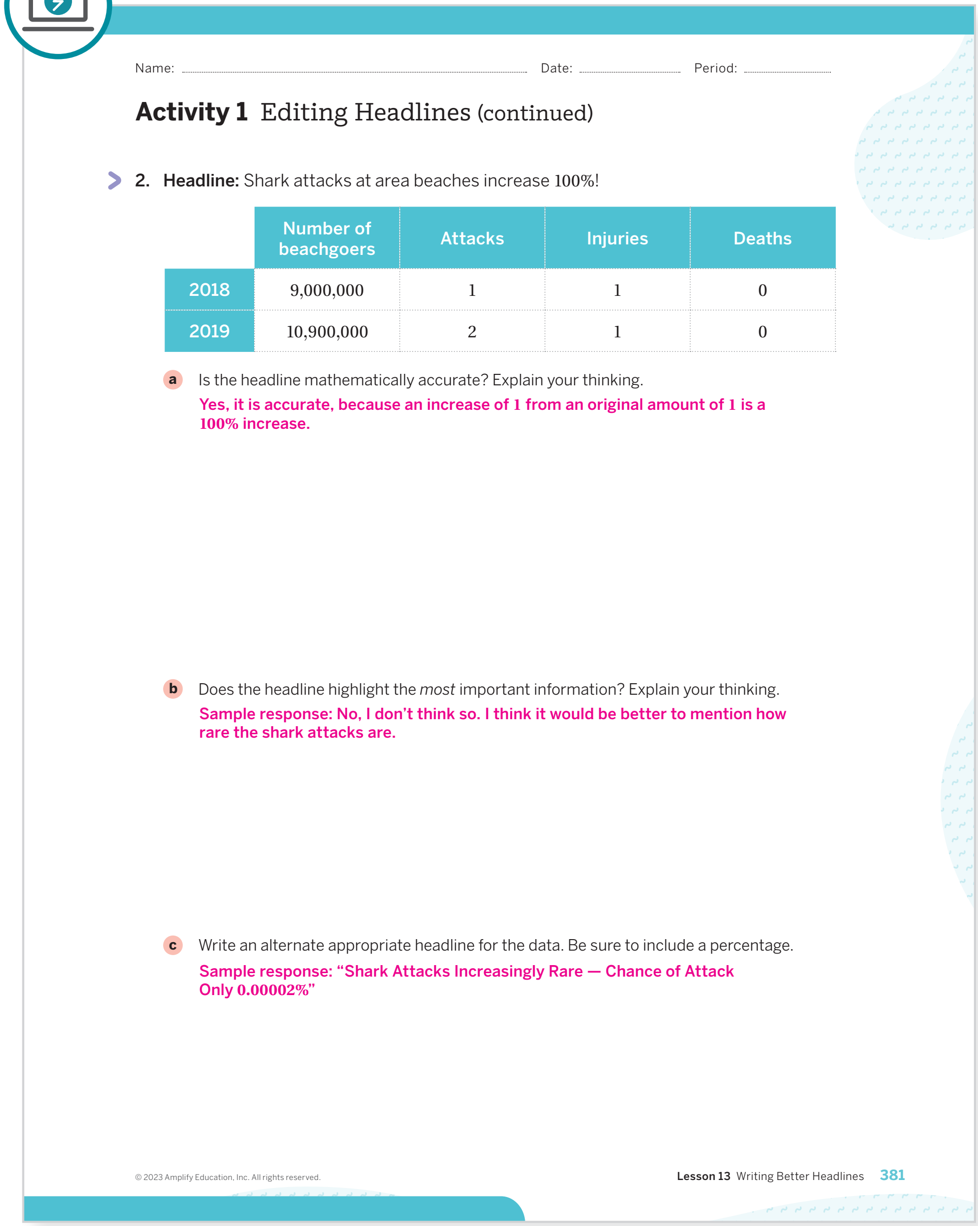
English Learners

Annotate the headline in Problem 1 with the term *percent increase* to make the connection between the term *up* and an increase in percentage.

Pairs | 12 min

Activity 1 Editing Headlines (continued)

Students analyze small tables of data to consider how an existing headline, containing a percentage, can be improved to be more accurate and responsible.



Name: ______ Date: ______ Period: ______

Activity 1 Editing Headlines (continued)

2. **Headline:** Shark attacks at area beaches increase 100%!

	Number of beachgoers	Attacks	Injuries	Deaths
2018	9,000,000	1	1	0
2019	10,900,000	2	1	0

a. Is the headline mathematically accurate? Explain your thinking.
Yes, it is accurate, because an increase of 1 from an original amount of 1 is a 100% increase.

b. Does the headline highlight the *most* important information? Explain your thinking.
Sample response: No, I don't think so. I think it would be better to mention how rare the shark attacks are.

c. Write an alternate appropriate headline for the data. Be sure to include a percentage.
Sample response: "Shark Attacks Increasingly Rare — Chance of Attack Only 0.00002%"

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3 Connect

Display one of the problems. Consider choosing a problem that will more deeply engage your students.

Have students share how they showed that the existing headlines were mathematically accurate.

Highlight that it is important to understand how the same numbers can be represented in different ways. Say, "As you have seen throughout this unit, the same percentage situation can be represented several different ways."

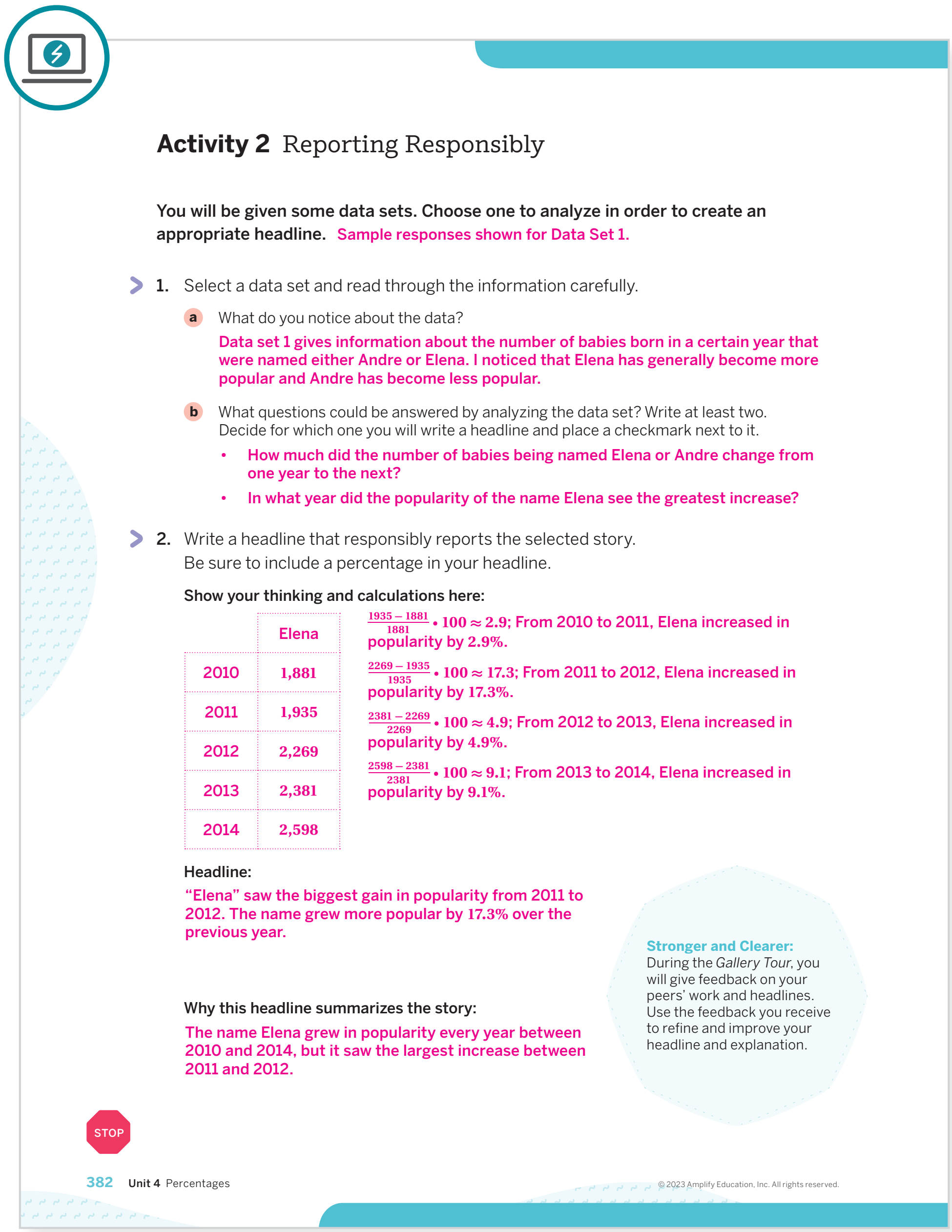
Ask:

- "Is it more accurate to report the change in shark attacks or the rarity of shark attacks?" They are both accurate.
- "Is it more responsible to report the change in shark attacks or the rarity of shark attacks?" It is probably more responsible to report the rarity. Reporting the change can make people scared, even though there is a very, very small chance of being attacked.

Pairs | 15 min

Activity 2 Reporting Responsibly

Students analyze a larger data set to identify a question that can be answered and a headline that can be written from the data.



Activity 2 Reporting Responsibly

You will be given some data sets. Choose one to analyze in order to create an appropriate headline. Sample responses shown for Data Set 1.

1. Select a data set and read through the information carefully.
   - a. What do you notice about the data?
     Data set 1 gives information about the number of babies born in a certain year that were named either Andre or Elena. I noticed that Elena has generally become more popular and Andre has become less popular.
   - b. What questions could be answered by analyzing the data set? Write at least two. Decide for which one you will write a headline and place a checkmark next to it.
     - **How much did the number of babies being named Elena or Andre change from one year to the next?**
     - **In what year did the popularity of the name Elena see the greatest increase?**
2. Write a headline that responsibly reports the selected story. Be sure to include a percentage in your headline.

Show your thinking and calculations here:

	Elena
2010	1,881
2011	1,935
2012	2,269
2013	2,381
2014	2,598

$\frac{1935 - 1881}{1881} \bullet 100 \approx 2.9$; From 2010 to 2011, Elena increased in popularity by 2.9%.

$\frac{2269 - 1935}{1935} \bullet 100 \approx 17.3$; From 2011 to 2012, Elena increased in popularity by 17.3%.

$\frac{2381 - 2269}{2269} \bullet 100 \approx 4.9$; From 2012 to 2013, Elena increased in popularity by 4.9%.

$\frac{2598 - 2381}{2381} \bullet 100 \approx 9.1$; From 2013 to 2014, Elena increased in popularity by 9.1%.

Headline:

"Elena" saw the biggest gain in popularity from 2011 to 2012. The name grew more popular by 17.3% over the previous year.

Why this headline summarizes the story:

The name Elena grew in popularity every year between 2010 and 2014, but it saw the largest increase between 2011 and 2012.

Stronger and Clearer: During the *Gallery Tour*, you will give feedback on your peers' work and headlines. Use the feedback you receive to refine and improve your headline and explanation.

STOP

382 Unit 4 Percentages

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1 Launch

Tell students that they will receive four tables, each with a different data set and short introduction to the data. Suggest that they discuss and decide with their partner which set of data they will work with. Distribute the Activity 2 PDF (Data sets).

2 Monitor

Help students get started by telling them they do not need to use all of the data in the data set for their analysis. They can choose as much or as little as they need to complete the problem.

Look for points of confusion:

- **Comparing the temperature in different locations for Data Set 3.** Have students identify what the heading of each column indicates. Ask, "Would you expect the temperature in different parts of the country to change in similar ways?"

Look for productive strategies:

- Creating a plan of action for how to divide responsibilities for each partner.

3 Connect

Have students share their work and headlines by conducting the ***Gallery Tour*** routine.

Highlight that even pairs who analyzed the same data set came up with different questions and different headlines. Say, "The work you did today is similar to the work of a journalist. You collected information, made a decision about a question you could answer, and thought about how to responsibly and accurately convey that information to your reader."

Ask:

- "How are percentages helpful when analyzing a data set?"
- "How are percentages helpful when writing headlines?"
- "How can percentages be misused when writing headlines?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which students can share their work in a virtual forum that allows for rapid connections to be made and includes digital options for providing feedback.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, use the ***Gallery Tour*** routine for students to both give and receive feedback on their work and headlines. Provide sticky notes for students to use and encourage students to leave feedback on at least 2–3 peers' work and headlines. After receiving feedback, allow students time to revise and improve their headlines.

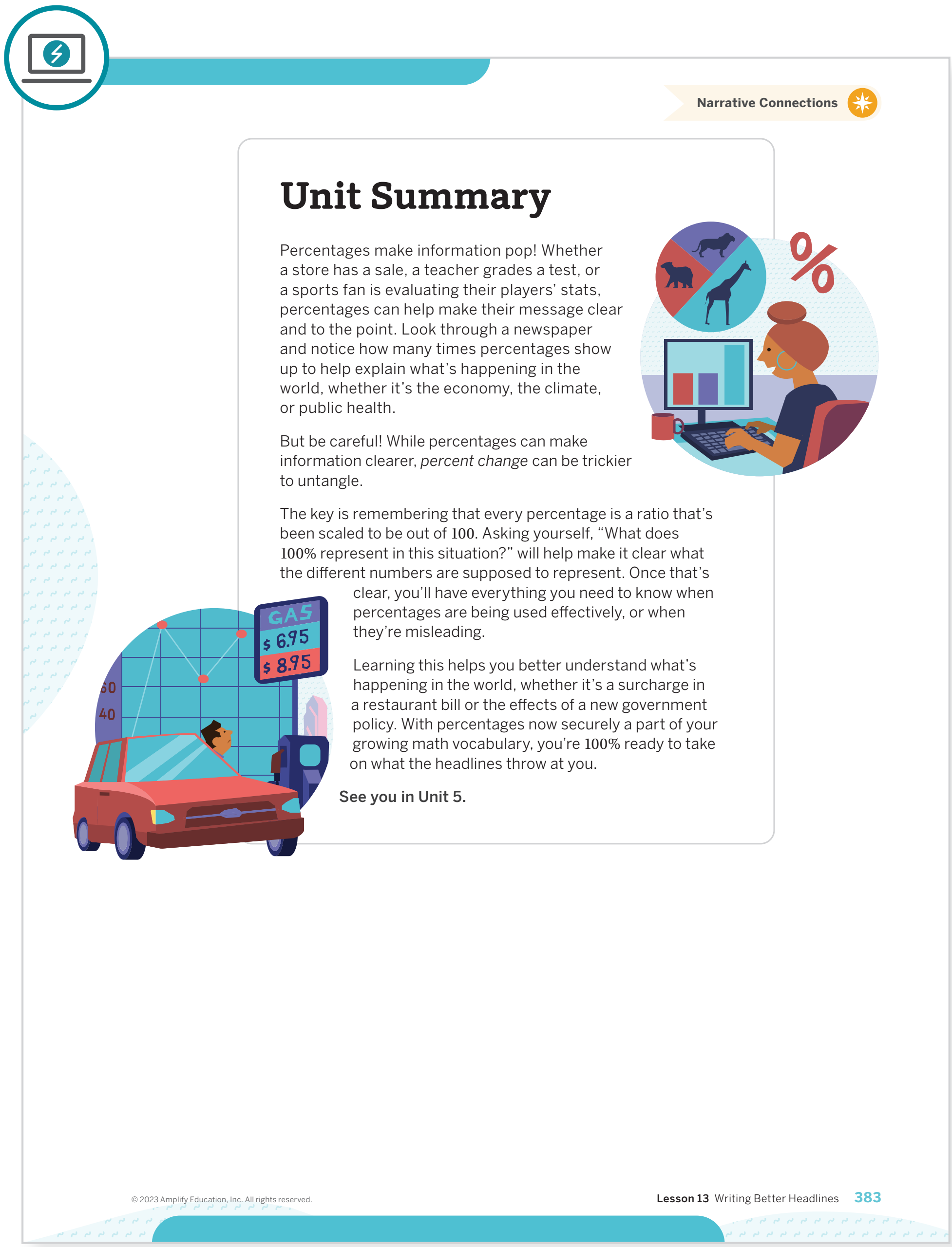
English Learners

Have students participate in the ***Gallery Tour*** with a partner who speaks the same primary language. This will help support students' conversations about their observations and feedback in a supportive environment.

Whole Class | 5 min

Unit Summary

Review and synthesize the power of using percentages for comparisons.



Narrative Connections

Unit Summary

Percentages make information pop! Whether a store has a sale, a teacher grades a test, or a sports fan is evaluating their players' stats, percentages can help make their message clear and to the point. Look through a newspaper and notice how many times percentages show up to help explain what's happening in the world, whether it's the economy, the climate, or public health.

But be careful! While percentages can make information clearer, *percent change* can be trickier to untangle.

The key is remembering that every percentage is a ratio that's been scaled to be out of 100. Asking yourself, "What does 100% represent in this situation?" will help make it clear what the different numbers are supposed to represent. Once that's clear, you'll have everything you need to know when percentages are being used effectively, or when they're misleading.

Learning this helps you better understand what's happening in the world, whether it's a surcharge in a restaurant bill or the effects of a new government policy. With percentages now securely a part of your growing math vocabulary, you're 100% ready to take on what the headlines throw at you.

See you in Unit 5.

Lesson 13 Writing Better Headlines 383

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary and have students read it as a class or independently.

Highlight that it is very useful to spend some time reflecting on one's own learning at the conclusion of a unit. This helps to retain information and make connections that students may not have been able to make while focusing on each individual lesson.

Ask students to take a few minutes to recall what they have learned about calculating percentages in various contexts.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

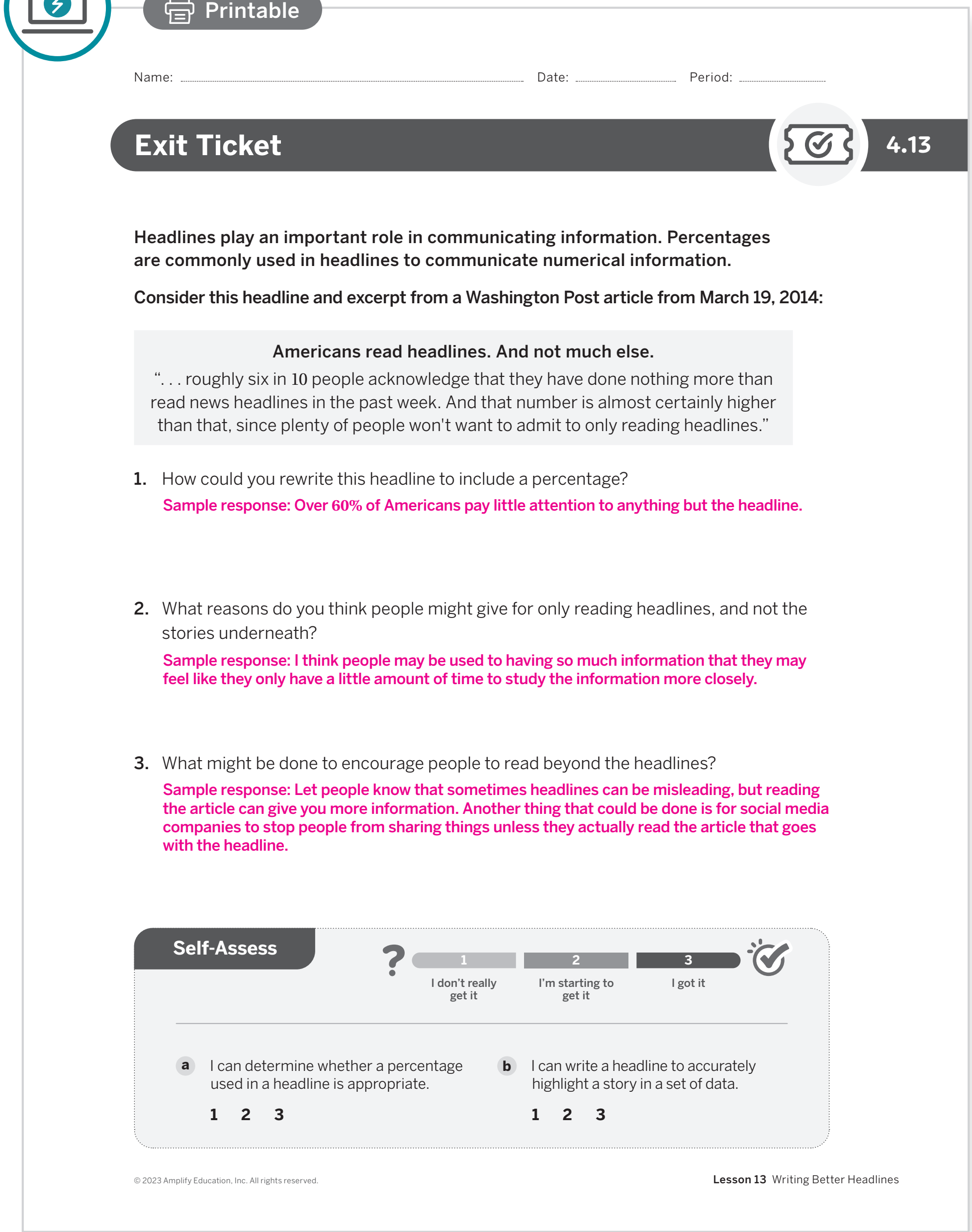
- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of calculating percentages in various contexts by reflecting on what they have learned and voicing any unresolved questions they may have.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.13

Headlines play an important role in communicating information. Percentages are commonly used in headlines to communicate numerical information.

Consider this headline and excerpt from a Washington Post article from March 19, 2014:

> **Americans read headlines. And not much else.**
> ". . . roughly six in 10 people acknowledge that they have done nothing more than read news headlines in the past week. And that number is almost certainly higher than that, since plenty of people won't want to admit to only reading headlines."

1. How could you rewrite this headline to include a percentage?
   Sample response: Over 60% of Americans pay little attention to anything but the headline.

2. What reasons do you think people might give for only reading headlines, and not the stories underneath?
   Sample response: I think people may be used to having so much information that they may feel like they only have a little amount of time to study the information more closely.

3. What might be done to encourage people to read beyond the headlines?
   Sample response: Let people know that sometimes headlines can be misleading, but reading the article can give you more information. Another thing that could be done is for social media companies to stop people from sharing things unless they actually read the article that goes with the headline.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can determine whether a percentage used in a headline is appropriate.
1 2 3

b. I can write a headline to accurately highlight a story in a set of data.
1 2 3

Lesson 13 Writing Better Headlines

Success looks like . . .

- **Goal:** Comprehending that a responsible and accurate headline represents the most honest way to interpret some information.
- **Language Goal:** Analyzing a data set and writing questions that can be answered by this analysis. **(Reading and Writing)**
- **Language Goal:** Writing headlines that responsibly and accurately represent the data in a data set. **(Reading and Writing)**
  - » Writing a headline to include a percentage from data in Problem 1.

Suggested next steps

If students cannot think of what can be done to encourage people to read beyond the headlines, consider:

- Having them review Activity 1 and ask, "How did your understanding change after you gained more information?"

Professional Learning

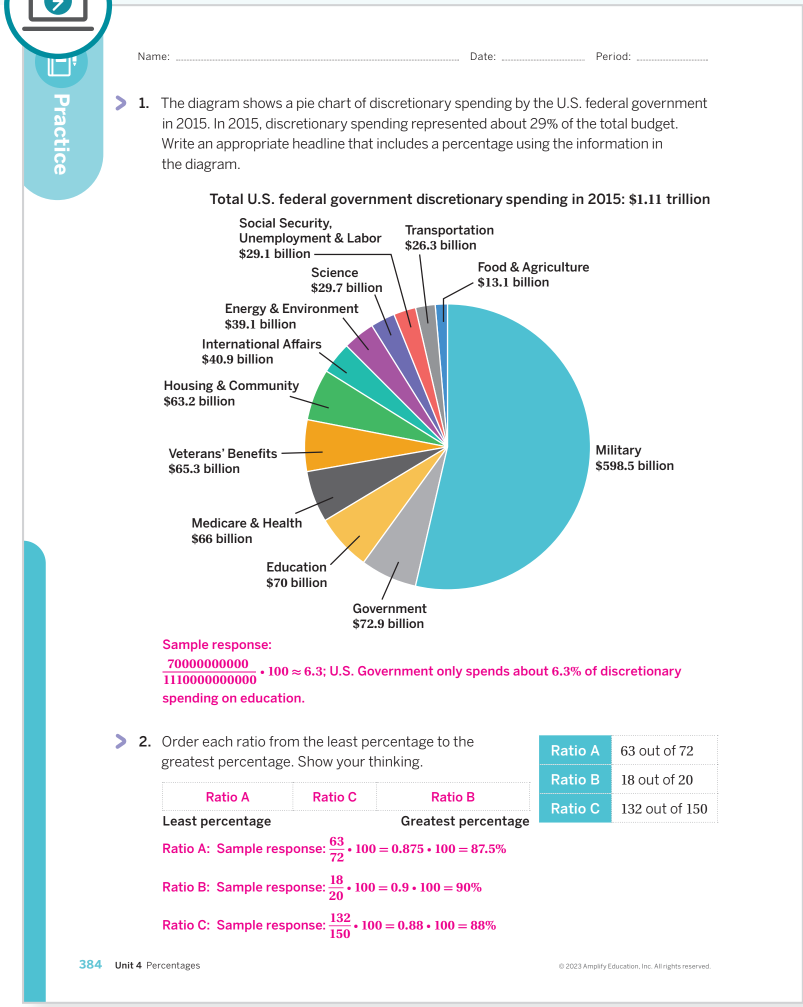
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students critique the reasoning of others today? How are you helping them become aware of how they are progressing in this area?
- What did choosing from among various data sets reveal about your students as learners? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. The diagram shows a pie chart of discretionary spending by the U.S. federal government in 2015. In 2015, discretionary spending represented about 29% of the total budget. Write an appropriate headline that includes a percentage using the information in the diagram.

Total U.S. federal government discretionary spending in 2015: $1.11 trillion

Social Security, Unemployment & Labor $29.1 billion
Transportation $26.3 billion
Science $29.7 billion
Food & Agriculture $13.1 billion
Energy & Environment $39.1 billion
International Affairs $40.9 billion
Housing & Community $63.2 billion
Veterans' Benefits $65.3 billion
Military $598.5 billion
Medicare & Health $66 billion
Education $70 billion
Government $72.9 billion

Sample response:

$\frac{70000000000}{1110000000000} \cdot 100 \approx 6.3$; **U.S. Government only spends about 6.3% of discretionary spending on education.**

2. Order each ratio from the least percentage to the greatest percentage. Show your thinking.

Ratio A	63 out of 72
Ratio B	18 out of 20
Ratio C	132 out of 150

Ratio A	Ratio C	Ratio B
Least percentage		Greatest percentage

Ratio A: Sample response: $\frac{63}{72} \cdot 100 = 0.875 \cdot 100 = 87.5\%$

Ratio B: Sample response: $\frac{18}{20} \cdot 100 = 0.9 \cdot 100 = 90\%$

Ratio C: Sample response: $\frac{132}{150} \cdot 100 = 0.88 \cdot 100 = 88\%$

384 **Unit 4** Percentages

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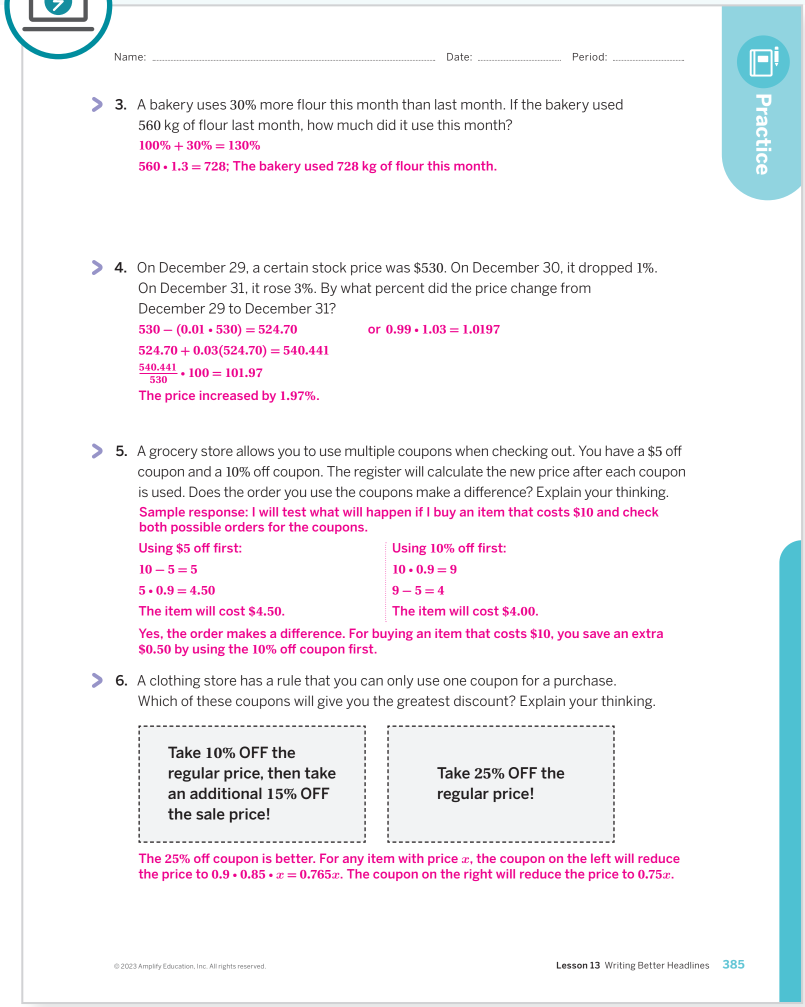

Name: Date: Period:

3. A bakery uses 30% more flour this month than last month. If the bakery used 560 kg of flour last month, how much did it use this month?

$100\% + 30\% = 130\%$

$560 \cdot 1.3 = 728$; **The bakery used 728 kg of flour this month.**

4. On December 29, a certain stock price was $530. On December 30, it dropped 1%. On December 31, it rose 3%. By what percent did the price change from December 29 to December 31?

$530 - (0.01 \cdot 530) = 524.70$ or $0.99 \cdot 1.03 = 1.0197$

$524.70 + 0.03(524.70) = 540.441$

$\frac{540.441}{530} \cdot 100 = 101.97$

The price increased by 1.97%.

5. A grocery store allows you to use multiple coupons when checking out. You have a $5 off coupon and a 10% off coupon. The register will calculate the new price after each coupon is used. Does the order you use the coupons make a difference? Explain your thinking.

Sample response: I will test what will happen if I buy an item that costs $10 and check both possible orders for the coupons.

Using $5 off first:	Using 10% off first:
$10 - 5 = 5$	$10 \cdot 0.9 = 9$
$5 \cdot 0.9 = 4.50$	$9 - 5 = 4$
The item will cost $4.50.	The item will cost $4.00.

Yes, the order makes a difference. For buying an item that costs $10, you save an extra $0.50 by using the 10% off coupon first.

6. A clothing store has a rule that you can only use one coupon for a purchase. Which of these coupons will give you the greatest discount? Explain your thinking.

Take 10% OFF the regular price, then take an additional 15% OFF the sale price!

Take 25% OFF the regular price!

The 25% off coupon is better. For any item with price x, the coupon on the left will reduce the price to $0.9 \cdot 0.85 \cdot x = 0.765x$. The coupon on the right will reduce the price to $0.75x$.

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Lesson 13 Writing Better Headlines 385

Practice Problem Analysis

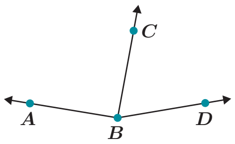
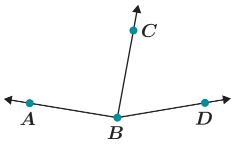
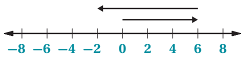
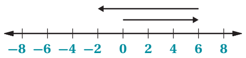
Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
Spiral	2	Unit 4 Lesson 2	2
	3	Unit 4 Lesson 5	2
	4	Unit 4 Lesson 10	2
	5	Unit 4 Lesson 5	3
	6	Unit 4 Lesson 5	3

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Glossary/Glosario

A

English	Español
absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $\lvert -3 \rvert = 3$.	**valor absoluto** Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $\lvert -3 \rvert = 3$.
Addition Property of Equality A property stating that, if $a = b$, then $a + c = b + c$.	**Propiedad de igualdad en la suma** Propiedad que establece que si $a = b$, entonces $a + c = b + c$.
additive inverse The additive inverse of a number a is the number that, when added to a, gives a sum of zero. It is the number's opposite.	**inverso aditivo** El inverso aditivo de un número a es el número que, cuando se suma a a, resulta en cero. Es el opuesto del número.
adjacent angles Angles that share a common side and vertex. For example, $\angle ABC$ and $\angle CBD$ are adjacent angles.  	**ángulos adyacentes** Ángulos que comparten un lado y un vértice. Por ejemplo, $\angle ABC$ y $\angle CBD$ son ángulos adyacentes. 
area The number of unit squares needed to fill a two-dimensional figure without gaps or overlaps.	**área** Número de unidades cuadradadas necesario para llenar una figura bidimensional sin dejar espacios vacíos ni superposiciones.
arrow diagram A model used in combination with a number line to show positive and negative numbers and operations on them.  	**diagrama de flechas** Modelo que se utiliza en combinación con una línea numérica para mostrar números positivos y negativos, y operaciones sobre estos. 
Associative Property of Addition A property stating that how addends are grouped does not change the result. For example, $(a + b) + c = a + (b + c)$.	**Propiedad asociativa de la suma** Propiedad que establece que la forma en que se agrupan los sumandos en una suma no cambia el resultado. Por ejemplo, $(a + b) + c = a + (b + c)$.
Associative Property of Multiplication A property stating that how factors are grouped in multiplication does not change the product. For example, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.	**Propiedad asociativa de la multiplicación** Propiedad que establece que la forma en que se agrupan los factores en una multiplicación no cambia el producto. Por ejemplo, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

B

balance The amount that represents the difference between positive and negative amounts of money in an account.	**balance** Cantidad que representa la diferencia entre cantidades positivas y negativas de dinero en una cuenta bancaria.
bar notation Notation that indicates the repeated part of a repeating decimal. For example, $0.\overline{6} = 0.66666\ldots.$	**notación de barras** Notación que indica la parte repetida de un número decimal periódico. Por ejemplo, $0.\overline{6} = 0.66666\ldots.$
base (of a prism) Either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.	**base (de un prisma)** Cualquiera de las dos caras idénticas y paralelas de un prisma que están conectadas por un conjunto de caras rectangulares.
base (of a pyramid) The face of a pyramid that is opposite from the vertex, where all the triangular faces connect.	**base (de una pirámide)** La cara de una pirámide que se opone al vértice, donde todas las caras triangulares se conectan.

Glossary/Glosario

English

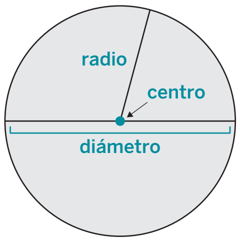
C

center of a circle The point that is the same distance from all points on the circle.

certain A certain event is an event that is sure to happen. (The probability of the event happening is 1.)

chance experiment An experiment that can be performed multiple times, in which the outcome may be different each time.

circle A shape that is made up of all of the points that are the same distance from a given point.



circumference The distance around a circle.

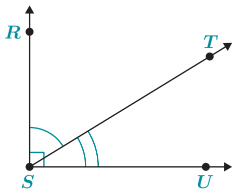
coefficient A number that is multiplied by a variable, typically written in front of or “next to” the variable, often without a multiplication symbol.

commission A fee paid for services, usually as a percentage of the total cost.

common factor A number that divides evenly into each of two or more given numbers.

commutative property Changing the order in which numbers are either added or multiplied does not change the value of the sum or product.

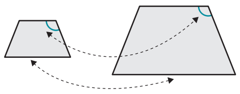
complementary angles Two angles whose measures add up to 90 degrees. For example, $\angle RST$ and $\angle TSU$ are complementary angles.



constant of proportionality The number in a proportional relationship by which the value of one quantity is multiplied to get the value of the other quantity.

coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both represent values that are positive, negative, or zero.

corresponding parts Parts of two scaled copies that match up, or “correspond” with each other. These corresponding parts could be points, segments, angles, or lengths.



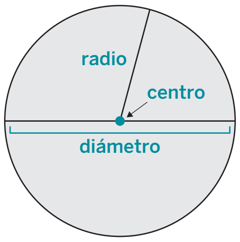
Español

centro de un círculo Punto que está a la misma distancia de todos los puntos del círculo.

seguro Un evento seguro es un evento que ocurrirá con certeza. (La probabilidad de que el evento ocurra es 1.)

experimento aleatorio Experimento que puede ser llevado a cabo muchas veces, en cada una de las cuales el resultado será diferente.

círculo Forma compuesta de todos los puntos que están a la misma distancia de un punto dado.



circunferencia Distancia alrededor de un círculo.

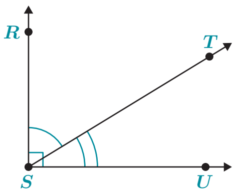
coeficiente Número por el cual una variable es multiplicada, escrito comúnmente frente o junto a la variable.

comisión Pago realizado a cambio de algún servicio, usualmente como porcentaje del costo total.

factor común Número que divide en partes iguales cada número de entre dos o más números dados.

propiedad conmutativa Cambiar el orden de los operandos en una suma o multiplicación no cambia el valor final de la suma o el producto.

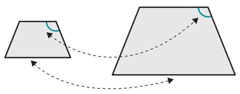
ángulos complementarios Dos ángulos cuyas medidas suman 90 grados. Por ejemplo, $\angle RST$ y $\angle TSU$ son ángulos complementarios.

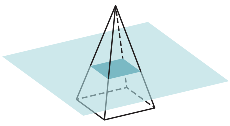
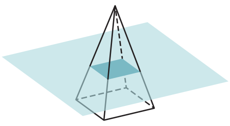


constante de proporcionalidad En una relación proporcional, el número por el cual el valor de una cantidad es multiplicado para obtener el valor de otra cantidad.

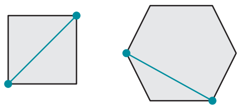
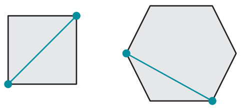
plano de coordenadas Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.

partes correspondientes Partes de dos copias a escala que coinciden, o “se corresponden” entre sí. Estas partes correspondientes pueden ser puntos, segmentos, ángulos o longitudes.



English	Español
cross section A cross section is the new face seen when slicing through a three-dimensional figure. For example, a rectangular pyramid that is sliced parallel to the base has a smaller rectangle as the cross section. 	**corte transversal** Un corte transversal es la nueva cara que aparece cuando una figura tridimensional es rebanada. Por ejemplo, una pirámide rectangular que es rebanada en forma paralela a la base tiene un rectángulo más pequeño como corte transversal. 

D

debt Amount of money that has been borrowed and owed to the person or bank from which it was borrowed.	**deuda** Cantidad de dinero que ha sido pedida prestada y se le debe a la persona o al banco que la prestó.
deposit Money put into an account.	**depósito** Dinero colocado en una cuenta.
diagonal A line segment connecting two vertices on different sides of a polygon. The *diagonal* of a square connects opposite vertices. 	**diagonal** Segmento de una línea que conecta dos vértices en lados diferentes de un polígono. La *diagonal* de un cuadrado conecta vértices opuestos. 
diameter The distance across a circle through its center. The line segment with endpoints on the circle, that passes through its center. (See also *circle*.)	**diámetro** Distancia a través de un círculo que atraviesa su centro. Segmento de línea cuyos extremos limitan con el círculo y que pasa por su centro. (Ver también *círculo*.)
discount A reduction in the price of an item, typically due to a sale.	**descuento** Reducción del precio de un artículo, usualmente debido a una venta de rebaja.
Distributive Property A property that states the product of a number and a sum of numbers is equal to the sum of two products: $a\,(b + c) = ab + ac$.	**Propiedad distributiva** Propiedad que establece que el producto de un número y una suma de números es igual a la suma de dos productos: $a\,(b + c) = ab + ac$.

E

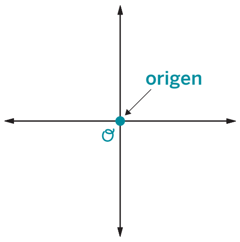
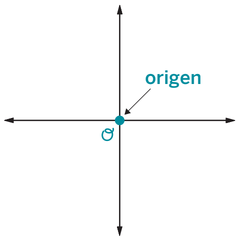
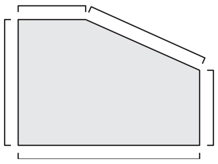
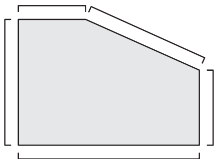
equally likely as not An event that has equal chances of occurring and not occurring. (The probability of the event happening is exactly $\frac{1}{2}$.)	**tan probable como improbable** Evento que tiene las mismas posibilidades de ocurrir que de no ocurrir. (La probabilidad de que ocurra es exactamente $\frac{1}{2}$.)
equation Two expressions with an equal sign between them. When the two expressions are equal, the equation is true. An equation can also be false, when the values of the two expressions are not equal.	**ecuación** Dos expresiones con un signo igual entre sí. Cuando las dos expresiones son iguales, la ecuación es verdadera. Una ecuación también puede ser falsa, cuando los valores de las dos expresiones no son iguales.
equivalent equations Equations that have the same solution.	**ecuaciones equivalentes** Ecuaciones que tienen la misma solución.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
equivalent ratios Any two ratios in which the values for one quantity in each ratio can be multiplied or divided by the same number to get the values for the other quantity in each ratio.	**razones equivalentes** Dos razones entre las cuales los valores de una cantidad en cada razón pueden ser multiplicados o divididos por el mismo número para obtener así los valores de la otra cantidad en cada razón.
equivalent scales Different scales (relating scaled and actual measurements) that have the same scale factor.	**escalas equivalentes** Diferentes escalas (que relacionan medidas a escala y reales) que tienen el mismo factor de escala.
error interval A range of values above and below an exact value, expressed as a percentage.	**intervalo de error** Rango de valores por sobre y por debajo de un valor exacto, expresado como porcentaje.

Glossary/Glosario

English	Español
event A set of one or more outcomes in a chance experiment.	**evento** Conjunto de uno o más resultados de un experimento aleatorio.
expand To expand an expression means to use the Distributive Property to rewrite a product as a sum. The new expression is equivalent to the original expression.	**expandir** Expandir una expresión significa usar la Propiedad distributiva para volver a escribir un producto como una suma. La nueva expresión es equivalente a la expresión original.
F	
factor To factor an expression means to use the Distributive Property to rewrite a sum as a product. The new expression is equivalent to the original expression.	**factorizar** Factorizar una expresión significa usar la Propiedad distributiva para volver a escribir una suma como un producto. La nueva expresión es equivalente a la expresión original.
G	
gratuity See the definition for *tip*.	**gratificación** Ver *propina*.
greater than or equal to $x \geq a$, x is greater than a or x is equal to a.	**mayor o igual a** $x \geq a$, x es mayor que a o x es igual a a.
H	
hanger diagram A model in which quantities are represented as weights attached to either side of a hanger. When the hanger is balanced, the sum of the quantities on either side must be equal.	**diagrama de colgador** Modelo en el cual ciertas cantidades son representadas como pesos sujetos a cada lado de un colgador. Cuando el colgador está en equilibrio, la suma de las cantidades en cualquiera de los lados debe ser igual.
I	
impossible An impossible event is an event that has no chance of occurring. The probability of the event happening is 0.	**imposible** Un evento imposible es un evento que no tiene posibilidad de que ocurra. La probabilidad de que ocurra es 0.
inequality A statement relating two numbers or expressions that are not equal. The phrases *less than*, *less than or equal to*, *greater than*, and *greater than* or *equal to* describe inequalities.	**desigualdad** Enunciado que relaciona dos números o expresiones que no son iguales. Las expresiones "menor que", "menor o igual a", "mayor que" o "mayor o igual a" describen desigualdades.
integers Whole numbers and their opposites.	**enteros** Números completos y sus opuestos.
inverse operations Operations that "undo" each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.	**operaciones inversas** Operaciones que se cancelan entre sí. La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

English	Español
L	
less than or equal to $x \le a$, x is less than a or x is equal to a.	**menor o igual a** $x \le a$, x es menor que a o x es igual a a.
like terms Terms in an expression that have the same variables and can be combined, such as $7x$ and $9x$.	**términos semejantes** Partes de una expresión que tiene la misma variable y que pueden ser sumadas, tales como $7x$ and $9x$.
likely A likely event is an event that has a greater chance of occurring than not occurring. (The probability of happening is more than $\frac{1}{2}$.)	**probable** Un evento probable es un evento que tiene más posibilidad de ocurrir que de no ocurrir. (La probabilidad de que ocurra es mayor que $\frac{1}{2}$.)
long division A method that shows the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right. $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{-2\ 4} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$	**división larga** Método que muestra los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha. $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{-2\ 4} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$
M	
magnitude The absolute value of a number, or the distance of a number from 0 on the number line.	**magnitud** Valor absoluto de un número, o distancia de un número con respecto a 0 en la línea numérica.
markdown An amount, expressed as a percentage, subtracted from the cost of an item.	**descuento** Monto, expresado como porcentaje, que se resta al costo de un producto.
markup An amount, expressed as a percentage, added to the cost of an item.	**sobreprecio** Monto, expresado como porcentaje, que se agrega al costo de un producto.
multi-step event When an experiment consists of two or more events, it is called a multi-step event.	**evento de varios pasos** Cuando un experimento consiste en dos o más eventos, es llamado un evento de varios pasos.
multiplicative inverse Another name for the reciprocal of a number; The multiplicative inverse of a number a is the number that, when multiplied by a, gives a product of 1.	**inverso multiplicativo** Otro nombre para el recíproco de un número. El inverso multiplicativo de un número a es el número que, cuando se multiplica por a, tiene como producto 1.
N	
negative numbers Numbers whose values are less than zero.	**números negativos** Números cuyos valores son menores que cero.
nonproportional relationship A relationship between two quantities in which the corresponding values do not have a constant ratio. (In other words, a relationship that is *not* a proportional relationship.)	**relación no proporcional** Relación entre dos cantidades, en la cual los valores correspondientes no tienen una razón constante. (En otras palabras, una relación que *no* es una relación proporcional.)

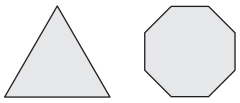
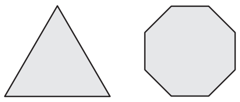
Glossary/Glosario

English	Español
O	**O**
opposites Two numbers that are the same distance from 0, but are on different sides of the number line.	**opuestos** Dos números que están a la misma distancia de 0, pero que están en lados diferentes de la línea numérica.
order of operations When an expression has multiple operations, they are applied in a consistent order (the "order of operations") so that the expression is evaluated the same way by everyone.	**orden de las operaciones** Cuando una expresión contiene múltiples operaciones, estas se aplican en cierto orden consistente (el "orden de las operaciones") de forma que la expresión sea evaluada de la misma manera por todas las personas.
ordered pair Two values, written as (x, y), that represent a point on the coordinate plane.	**par ordenado** Dos valores, escritos como (x, y), que representan un punto en el plano de coordenadas.
origin The point represented by the ordered pair $(0, 0)$ on the coordinate plane. The *origin* is where the x- and y-axes intersect. 	**origen** Punto representado por el par ordenado $(0, 0)$ en el plano de coordenadas. El *origen* es donde los ejes x y y se intersecan. 
outcome One of the possible results that can happen when an experiment is performed. For example, the possible outcomes of tossing a coin are heads and tails.	**resultado** El resultado de un experimento aleatorio es una de las cosas que pueden ocurrir cuando se realiza el experimento. Por ejemplo, los posibles resultados de tirar una moneda al aire son cara o cruz.
P	**P**
percent change How much a quantity changed (increased or decreased), expressed as a percentage of the original amount.	**cambio porcentual** Cuánto ha cambiado una cantidad (aumentado o disminuido), expresado en un porcentaje del monto original.
percent decrease The amount a value has gone down, expressed as a percentage of the original amount.	**disminución porcentual** Cantidad en que un valor ha disminuido, expresada como porcentaje del monto original.
percent error The difference between approximate and exact values, as a percentage of the exact value.	**error porcentual** Diferencia entre valores aproximados y valores exactos, expresada como porcentaje del valor exacto.
percent increase The amount a value has gone up, expressed as a percentage of the original amount.	**aumento porcentual** Monto en que un valor ha incrementado, expresado como porcentaje del monto original.
percentage A rate per 100. (A specific *percentage* is also called a *percent*, such as "70 percent.")	**porcentaje** Tasa por cada 100. (Un *porcentaje* específico también es llamado *por ciento*, como por ejemplo "70 por ciento.")
perimeter The total distance around the sides of a two-dimensional figure. 	**perímetro** Distancia total alrededor de los lados de una forma bidimensional. 

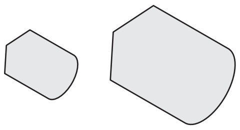
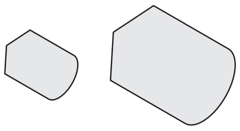
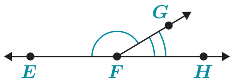
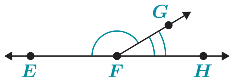
English	Español
pi, or π The ratio between the circumference and the diameter of a circle.	**pi, o π** Razón entre la circunferencia y el diámetro de un círculo.
polygon A closed, two-dimensional shape with straight sides that do not cross each other.	**porcentaje** Tasa por cada 100. (Un porcentaje específico también es llamado “por ciento”, como por ejemplo “70 por ciento”.)
population A set of people or objects that are to be studied. For example, if the heights of people on different sports teams are studied, the population would be all the people on the teams.	**población** Una población es un conjunto de personas o cosas por estudiar. Por ejemplo, si se estudia la altura de las personas en diferentes equipos deportivos, la población constaría de todas las personas que conforman los equipos.
population proportion A number in statistics, between 0 and 1 that represents the fraction of the data that fits into the desired category.	**proporción de la población** En estadística, número entre 0 y 1 que representa la fracción de los datos que cabe en la categoría deseada.
positive numbers Numbers whose values are greater than zero.	**números positivos** Números cuyos valores son mayores que cero.
prism A three-dimensional figure with two parallel, identical faces (called *bases*) that are connected by a set of rectangular faces.	**prisma** Forma tridimensional con dos caras iguales y paralelas (llamadas *bases*) que se conectan entre sí a través de un conjunto de caras rectangulares.
probability The ratio of the number of favorable outcomes to the total possible number of outcomes. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen.	**probabilidad** La razón entre el número de resultados favorables y el número total posible de resultados. Una probabilidad de 1 significa que el evento siempre ocurrirá. Una probabilidad de 0 significa que el evento nunca va a ocurrir.
profit The amount of money earned, minus expenses.	**ganancia** Monto del dinero obtenido, menos los gastos.
properties of equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that, if an equation is true, then applying the same operation to both sides will give a new equation that is also true.	**propiedades de igualdad** Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al aplicar la misma operación a ambos lados se obtendrá una nueva ecuación que también es verdadera.
proportional relationship A relationship in which the values for one quantity are each multiplied by the same number (the *constant of proprtionality*) to get the values for the other quantity.	**relación proporcional** Relación en la que los valores de una cantidad se multiplican cada uno por el mismo número (la *constante de proporcionalidad*) para encontrar los valores de la otra cantidad.
pyramid A three-dimensional figure with one base and a set of triangular faces that meet at a singular vertex.	**pirámide** Forma tridimensional con una base y un conjunto de caras triangulares que se intersecan en un solo vértice.

Glossary/Glosario

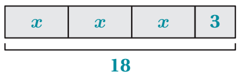
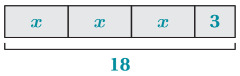
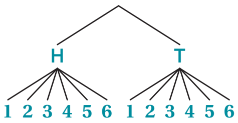
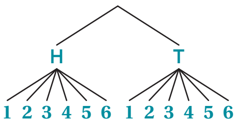
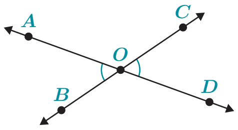
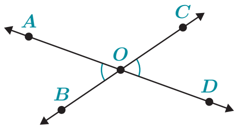
R

English	Español
radius A line segment that connects the center of a circle with a point on the circle. The term *radius* can also refer to the length of this segment. (See also *circle*.)	**radio** Segmento de una línea que conecta el centro de un círculo con un punto del círculo. *Radio* también puede referirse a la longitud de este segmento. (Ver también *círculo*.)
random sample A sample that has an equal chance of being selected from the population as any sample of the same size	**muestra al azar** Muestra que tiene la misma posibilidad de ser seleccionada de entre la población que cualquier otra muestra del mismo tamaño.
rate A comparison of how two quantities change together.	**tasa** Comparación de cuánto cambian dos cantidades en conjunto.
ratio A comparison of two quantities by multiplication or division.	**razón** Comparación de dos cantidades a través de la multiplicación o la división.
rational numbers The set of all numbers, positive and negative, that can be written as fractions. For example, any whole number is a rational number.	**números racionales** Conjunto de todos los números positivos y negativos que pueden ser escritos como fracciones. Por ejemplo, todo número entero es un número racional.
reciprocal Two numbers whose product is 1 are *reciprocals* of each other. (For example, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals.)	**recíproco/a** Dos números cuyo producto es 1 son *recíprocos* entre sí. (Por ejemplo, $\frac{3}{5}$ y $\frac{5}{3}$ son recíprocos.)
regular polygon A polygon whose sides all have the same length and whose angles all have the same measure. 	**polígono regular** Polígono cuyos lados tienen todos la misma longitud y cuyos ángulos tienen todos la misma medida. 
relative frequency The relative frequency is the ratio of the number of times an outcome occurs in a set of data. The relative frequency can be written as a fraction, a decimal, or a percentage.	**frecuencia relativa** La frecuencia relativa es la razón del número de veces que ocurre un resultado en un conjunto de datos. Se puede escribir como una fracción, un decimal o un porcentaje.
repeating decimal A decimal in which there is a sequence of non-zero digits that repeat indefinitely.	**número decimal periódico** Decimal que tiene una secuencia de dígitos distintos de cero que se repite de manera indefinida.
representative sample A sample is representative of a population if its distribution resembles the population's distribution in center, shape, and spread.	**muestra representativa** Una muestra es representativa de una población si su distribución asemeja la distribución de la población en centro, forma y extensión.
retail price The price a store typically charges for an item	**precio de venta al público** Precio que una tienda comercial usualmente cobra por un producto.
right angle An angle whose measure is 90 degrees.	**ángulo recto** Ángulo cuya medida es de 90 grados.

S

English	Español
sales tax An additional cost, as a rate to the cost of certain goods and services, applied by the government.	**impuesto de venta** Costo adicional, como una tasa del costo de ciertos bienes y servicios, aplicado por el gobierno.
sample Part of a population. For example, a population could be all the seventh graders at one school. One sample of that population is all the seventh graders who are in band.	**interés simple** Monto de dinero que se agrega a un monto original, usualmente pagado al titular o a la titular de una cuenta bancaria de ahorros.
sample space A list of every possible outcome for a chance experiment.	**espacio de muestra** Lista de cada resultado posible de un experimento aleatorio.
scale A ratio, sometimes shown as a segment, that indicates how the measurements in a scale drawing represent the actual measurements of the object shown.	**escala** Razón, a veces mostrada como segmento, que indica de qué forma las medidas de un dibujo a escala representan las verdaderas medidas del objeto mostrado.
scale drawing A drawing that represents an actual place, object, or person. All of the measurements in the scale drawing correspond to the measurements of the actual object by the same scale.	**dibujo a escala** Dibujo que representa un lugar, objeto o persona real. Todas las medidas en el dibujo a escala corresponden en la misma escala a las medidas del objeto real.
scale factor The value that side lengths are multiplied by to produce a certain scaled copy.	**factor de escala** Valor por el cual las longitudes de cada lado se multiplican para producir cierta copia a escala.
scaled copy A copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy. 	**copia a escala** Copia de una figura donde cada longitud de la figura original es multiplicada por el mismo valor, para producir longitudes correspondientes en la copia. 
simple interest An amount of money that is added on to an original amount, usually paid to the holder of a bank savings account.	**muestra** Una muestra es una parte de la población. Por ejemplo, una población podría ser todos/as los/as estudiantes de séptimo grado en una escuela. Una muestra de esa población son todos/as los/as estudiantes de séptimo grado que están en una banda.
simulation An experiment that is used to estimate the probability of a real-world event.	**simulación** Un experimento que es utilizado para estimar la probabilidad de un evento en el mundo real.
solution to an equation A value that will make an equation true when substituted into the equation.	**solución a una ecuación** Número que puede sustituir una variable para volver verdadera una ecuación.
solution to an inequality A value that will make an inequality a true statement when substituted into the inequality.	**solución a una desigualdad** Cualquier número que puede sustituir una variable para volver verdadera una desigualdad.
straight angle An angle whose measure is 180 degrees. For example, $\angle EFH$ is a straight angle. 	**ángulo llano** Ángulo cuya medida es de 180 grados.Por ejemplo, $\angle EFH$ es un ángulo llano. 
supplementary angles Two angles whose measures add up to 180 degrees. For example, $\angle EFG$ and $\angle GFH$ are supplementary angles. 	**ángulos suplementarios** Dos ángulos cuyas medidas suman 180 grados. Por ejemplo, $\angle EFG$ y $\angle GFH$ son ángulos suplementarios. 
surface area The number of unit squares needed to cover all of the faces of a three-dimensional figure without gaps or overlaps.	**área de superficie** Número de unidades cuadradas necesarias para cubrir todas las caras de una figura tridimensional sin dejar espacios vacíos ni superposiciones.

Glossary/Glosario

English	Español
T	
tape diagram A model in which quantities are represented as lengths (of tape) placed end-to-end, and which can be used to show addition, subtraction, multiplication, and division. 	**diagrama de cinta** Modelo en el cual las cantidades están representadas como longitudes (de cinta) colocadas de forma continua, y que pueden ser usadas para mostrar suma, resta, multiplicación y división. 
term A term is a part of an expression. It can be a number, a variable, or a product of a number and a variable.	**término** Un término es una parte de una expresión. Puede ser un número individual, una variable o el producto de un número y una variable.
terminating decimal A decimal that ends at a specific place value.	**decimal exacto** Un decimal que termina en un valor posicional específico.
tip An amount given to a server at a restaurant (or other service provider) that is calculated as a percentage of the bill.	**propina** Cantidad dada a un mesero o mesera en un restaurante (o a una persona que presta cualquier otro servicio) que se calcula como porcentaje de la cuenta.
tree diagram A diagram that represent all the possible outcomes in an experiment. 	**diagrama de árbol** Diagrama que representa todos los resultados posibles. 
U	
unit rate How much one quantity changes when the other changes by 1.	**tasa unitaria** Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.
unlikely An unlikely event is an event that has small chance of occurring. (The probablity of the event happening is less than $\frac{1}{2}$.)	**improbable** Un evento improbable es un evento que tiene pocas posibilidades de ocurrir. (La probabilidad de que ocurra es menor que $\frac{1}{2}$.)
V	
variable A letter that represents an unknown number in an expression or equation.	**variable** Letra que representa un número desconocido en una expresión o ecuación.
velocity A quantity that represents the speed and the direction of motion. In general, speed, like distance, is always positive, but velocity can be either positive or negative.	**velocidad** Cantidad que representa la rapidez y la dirección de un movimiento. En general, la rapidez, como la distancia, es siempre positiva, pero la velocidad puede ser tanto positiva como negativa.
vertical angles Opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal. For example, $\angle AOB$ and $\angle COD$ are vertical angles. 	**ángulos verticales** Ángulos opuestos que comparten el mismo vértice. Están compuestos de un par de líneas que se intersecan. Sus medidas de ángulo son iguales. Por ejemplo, $\angle AOB$ y $\angle COD$ son ángulos verticales. 
volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.	**volumen** Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.
W	
withdrawal Money taken out of an account.	**retiro** Dinero que es extraído de una cuenta.

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