Amplify Math
TENNESSEE

Teacher Edition

Grade 7 | Volume 2

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Amplify Math

Grade 7

Volume 2: Units 5–8

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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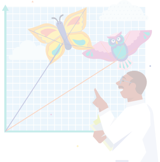
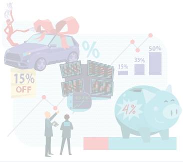
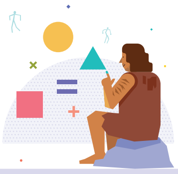
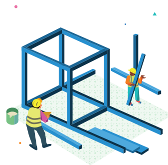
Program Scope and Sequence

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 160 days total	Area and Surface Area 20 Instructional Days 3 Assessment Days 23 days total	Introducing Ratios 20 Instructional Days 2 Assessment Days 22 days total	Rates and Percentages 15 Instructional Days 2 Assessment Days 17 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 153 days total	Scale Drawings 13 Instructional Days 2 Assessment Days 15 days total	Introducing Proportional Relationships 17 Instructional Days 2 Assessment Days 19 days total	Measuring Circles 12 Instructional Days 2 Assessment Days 14 days total	Percentages 13 Instructional Days 2 Assessment Days 15 days total
Grade 8 145 days total	Rigid Transformation and Congruence 18 Instructional Days 3 Assessment Days 21 days total	Dilations and Similarity 12 Instructional Days 2 Assessment Days 14 days total	Linear Relationships 19 Instructional Days 2 Assessment Days 21 days total	Linear Equations and Systems of Linear Equations 17 Instructional Days 2 Assessment Days 19 days total
Algebra 1 157 days total	Linear Equations, Inequalities, and Systems 26 Instructional Days 3 Assessment Days 29 days total	Data Analysis and Statistics 22 Instructional Days 3 Assessment Days 25 days total	Functions and Their Graphs 22 Instructional Days 3 Assessment Days 25 days total	Introducing Exponential Functions 22 Instructional Days 3 Assessment Days 25 days total





Volume 2

Unit 5	Unit 6	Unit 7	Unit 8
Arithmetic in Base Ten 14 Instructional Days 2 Assessment Days 16 days total	Expressions and Equations 19 Instructional Days 2 Assessment Days 21 days total	Rational Numbers 19 Instructional Days 2 Assessment Days 21 days total	Data Sets and Distributions 17 Instructional Days 3 Assessment Days 20 days total
 **Rational Number Arithmetic** **20 Instructional Days** **3 Assessment Days** **23 days total**	 **Expressions, Equations, and Inequalities** **23 Instructional Days** **3 Assessment Days** **26 days total**	 **Angles, Triangles, and Prisms** **18 Instructional Days** **3 Assessment Days** **21 days total**	 **Probability and Sampling** **17 Instructional Days** **3 Assessment Days** **20 days total**
Functions and Volume 21 Instructional Days 3 Assessment Days 24 days total	Exponents and Scientific Notation 15 Instructional Days 2 Assessment Days 17 days total	Irrationals and the Pythagorean Theorem 16 Instructional Days 2 Assessment Days 18 days total	Associations in Data 9 Instructional Days 2 Assessment Days 11 days total
Introducing Quadratic Functions 23 Instructional Days 3 Assessment Days 26 days total	Quadratic Equations 24 Instructional Days 3 Assessment Days 27 days total		

Volume 1

Unit 1 Scale Drawings

Certain objects in our universe exist at sizes and distances that are impossible for our eyes to see (such as a red blood cell, or Jupiter). In this unit, students harness the power of scaling — bringing large and small objects to a manageable size without distorting them.

Unit Narrative:
Life in the
Little Big City

Sub-Unit Narrative:
How do you get the perfect fit?

If we are making a larger or smaller copy of something, it needs to look right. The key is the scale factor.

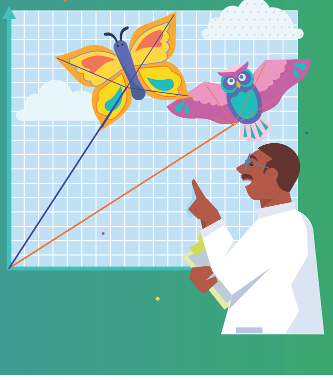
Sub-Unit Narrative:
Who was the King of Monsters?

We use maps and other scale drawings to help simplify large, complex places. Interpreting them is about knowing the scale and how to measure.

Unit 2 Introducing Proportional Relationships

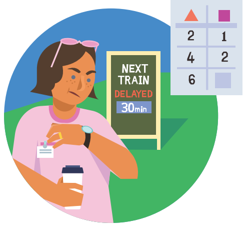
When we exchange money from one currency to another, there is a rate that helps us find the amount of one currency equal in value to the other. Students see that a rate is at the heart of every proportional relationship as they encounter problems across cultures where two quantities are directly related.

Unit Narrative:
The World in Proportion



Sub-Unit 1 Representing Proportional Relationships With Tables and Equations 101

Sub-Unit Narrative:
Who was the original globetrotter?
Tables help keep us organized, but equations tell an entire story with just a few symbols. We'll use both of them to represent proportional relationships.



Sub-Unit 2 Representing Proportional Relationships With Graphs 161

Sub-Unit Narrative:
Narrative: What good is a graph?
We turn to drawing, interpreting, and comparing proportional relationships in graphs, and notice what is particular to these types of graphs.

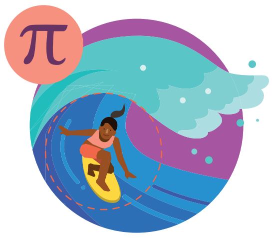
Unit 3 Measuring Circles

Identifying a circle may be straightforward, but measuring it is decidedly not. Students experience both the usefulness and challenges presented by this "perfect" shape.

Unit Narrative:
'Round and
'Round We Go

Sub-Unit Narrative:
Why do aliens love circles?

Circles are famously difficult to measure precisely, but that won't stop us from trying. Let's see how close we can get.



Sub-Unit Narrative:
What makes a circle so perfect?

Squares and circles may not have much in common, but we'll need both to measure a circle's area.

Unit 4 Percentages

From the supermarket to the stock market, percents are relied on to communicate quickly about how much something has changed. Students build on their experience with proportional relationships while using percentages to compare quantities within the friendly confines of the number 100.

Unit Narrative:
Keepin' it 100

Sub-Unit Narrative:
Is there truth in numbers?

Numbers never lie, but should we always believe them? Percentages can show how something changes – if we pay careful attention to the original amount.

Sub-Unit Narrative:
Did a quarantined U.S. keep a healthy economy?

See why percentages are used to calculate taxes, tips, interest, and other amounts when spending or saving money.

Volume 2

Unit 5 Rational Number Arithmetic

Students discover the need to work with both positive and negative values to describe the vastness of the world around them. With the entire set of rational numbers and all four operations now at their disposal, the sky (or the sea floor) is the limit.

Unit Narrative:
A World of Opposites

Sub-Unit Narrative:
What was Jeanne Baret's big secret?
Sure, you've probably been adding and subtracting for many years, but have you ever tried to take something away when you had less than zero to start with?

Sub-Unit Narrative:
Who was the toughest Grandma to ever hike the Appalachian Trail?
Travel forwards and backwards in time to help make sense of multiplication and division of negative numbers.

Sub-Unit Narrative:
How do you climb the world's most dangerous mountain?
Put it all together — adding, subtracting, multiplying, and dividing with rational numbers — while exercising your algebraic thinking muscles in a sneak preview of the next unit.

Unit 6 Expressions, Equations, and Inequalities

Students return to the study of algebra and focus on how representation plays such a large role in communicating mathematical ideas. In this unit, the symbols, language, and drawings students use will help them tell the stories they see in the numbers.

Unit Narrative:
Solving One Step at a Time

Sub-Unit Narrative:
What are the first words you learn in "Caveman"?
Dog walking, tools of early civilization, and hangers all come together to help you explore new ways of solving equations.

Sub-Unit Narrative:
Who were the VIPs of ancient Egypt?
Solving word problems is about making meaning of the quantities, and tape diagrams return to help.

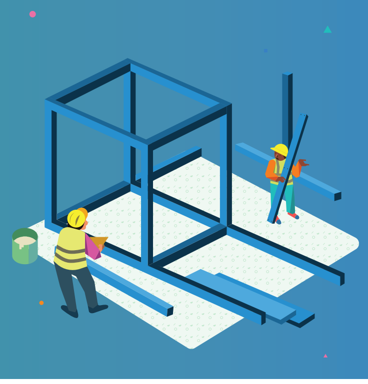
Sub-Unit Narrative:
Did a member of the School of Night infiltrate your math class?
Expressions are not always equal, so we must reckon with inequalities. Thankfully, finding their solutions will feel familiar.

Sub-Unit Narrative:
Which three blockheads did NASA send into space?
Find efficiencies for simplifying expressions like the Distributive Property and combining like terms.

Unit 7 Angles, Triangles, and Prisms

This unit is about the math of what can be seen and what can be held. Through constructing and drawing, students explore relationships among angles, lines, surfaces, and solids.

Unit Narrative:
Journey to the Third Dimension



PRE-UNIT READINESS ASSESSMENT

LAUNCH

Sub-Unit 1 Angle Relationships 701

Sub-Unit Narrative:
Did radio kill the aviation star?

As you'll see, some angles were just meant to go together. Here, you'll be introduced to complementary, supplementary, and vertical angles.

Sub-Unit 2 Drawing Polygons With Given Conditions 741

Sub-Unit Narrative:
How did triangles help win a war?

In this Sub-Unit, you will find that constructing polygons with specific lengths and angle measures can have dramatically different results.

MID-UNIT ASSESSMENT

Sub-Unit 3 Solid Geometry 777

Sub-Unit Narrative:
This machine will slice, but will it dice?

You've studied the surfaces of three-dimensional figures and the spaces inside them. Now, let's see what happens when we slice them open.

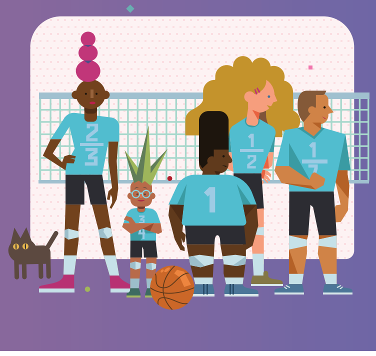
CAPSTONE

END-OF-UNIT ASSESSMENT

Unit 8 Probability and Sampling

For the first time, students encounter how to quantify the chances of something happening. Though the future is unwritten, probability and statistics help us make better predictions and thus better decisions.

Unit Narrative:
Winning Chance



Sub-Unit 1 Probabilities of Single-Step Events 827

Sub-Unit Narrative:
How did the women of Bletchley Park save the free world?

Welcome to probability, the math of games and chance. Discover how probability can reveal hidden information, even secret codes.

Sub-Unit 2 Probabilities of Multi-Step Events 861

Sub-Unit Narrative:
How did a blazing shoal bring the Philadelphia Convention Center to its feet?

When predicting the chances gets complicated, a simulation can help make predictions.

Sub-Unit 3 Sampling 889

Sub-Unit Narrative:
What's on your mind?

Not all data is created equal. It is important to know how to identify when a sample is representative of a population.

UNIT 5

Rational Number Arithmetic

Students discover the need to work with both positive and negative values to describe the vastness of the world around them. With the entire set of rational numbers and all four operations now at their disposal, the sky (or the sea floor) is the limit.

Essential Questions

- How do you represent addition, subtraction, or multiplication of rational numbers on a number line?
- How is solving problems with rational numbers the same or different from solving problems with only non-negative rational numbers?
- How can rational numbers be used to represent real-world situations?
- *(By the way, do two negatives always make a positive?)*

$2 - (-3) = 2 + 3 = 5$

Key Shifts in Mathematics

Focus

In this unit . . .

Students interpret rational numbers in contexts (e.g., temperature, elevation, payments and debts, position, direction, speed and velocity) together with their sums, differences, products, and quotients. Students use arrow diagrams to represent sums and differences of rational numbers and in contexts such as temperature or elevation change. They view situations in which objects are traveling at a constant speed, use multiplication equations to represent changes in position, and interpret positive and negative velocities in context. They become more fluent writing different multiplication and division equations for the same relationship.

Coherence

Previously . . .

In Grade 6, students learned that rational numbers comprise positive and negative fractions. They plotted rational numbers on the number line and plotted pairs of rational numbers on the coordinate plane. In Unit 2, students explored proportional relationships exclusively with positive rational constants of proportionality.

Coming soon . . .

In Unit 6, students apply their understanding of the entire set of rational numbers to expressions and equations. In Grade 8, they learn that there are numbers that are not rational (beyond π), which can only be approximated with rational numbers on the number line.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Operations with rational numbers are first explored in context, so students start with a concrete framework on which to build their understanding (Lessons 3 and 10).

Procedural Fluency

Students compare numerical expressions with all four operations, determining equivalent pairs using knowledge of additive and multiplicative inverses (Lesson 15).

Application

Students apply their understanding of rational number arithmetic to interpret negative quantities, such as negative time or rates of change (Lesson 17).

A World of Opposites

SUB-UNIT

1

Lessons 2–9

Adding and Subtracting Rational Numbers

Students revisit rational numbers from Grade 6, including how to represent them on the number line. They extend their understanding by observing how values change in context, such as temperature and elevation changes, and generalize rules for the signs of the sums and differences of rational numbers.

Narrative: From botany to temperature and elevation, rational numbers are everywhere!

SUB-UNIT

2

Lessons 10–14

Multiplying and Dividing Rational Numbers

Students consider problems about position, direction, constant speed, and constant velocity, and represent these quantities using arrow diagrams and numerical expressions containing rational numbers. They interpret products of rational numbers in terms of position and direction and use the relationship between multiplication and division to divide rational numbers.

Narrative: Discover how hiking the Appalachian trail relates to rational numbers.

Launch

Lesson 1

Target: Zero

Students warm back up to positive and negative values by playing a card game with the goal of being the closest to 0. They notice the benefits of getting opposite values on their cards, and they compare how far their scores are from zero — from both above and below.

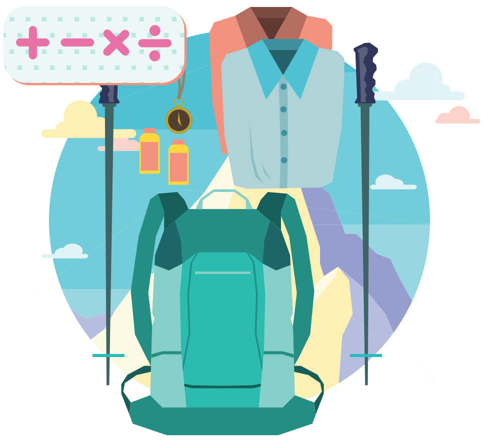
SUB-UNIT

3

Lessons 15–19

Four Operations With Rational Numbers

Students engage in a smorgasbord of problem solving with rational numbers that buttons up work from throughout the unit. Ample opportunity is provided to both practice and extend understanding of how all four operations interact with rational numbers.



Narrative: Rational numbers can help you make preparations for climbing Mt. Everest.

Capstone

Lesson 20

Summiting Everest

Let's end the unit the way we started: with a game. Now equipped with how to deal with rational numbers in various situations and contexts, students put themselves to the ultimate test: budgeting. Working with rational number rates, students plan for climbing Mt. Everest by strategizing about how quickly their limited resources will deplete.

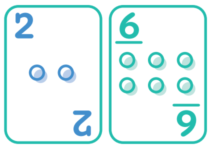
Unit at a Glance

Spoiler Alert: Even though there are four operations, every subtraction and division problem can be rewritten as an addition or multiplication problem, respectively.

Assessment

A **Pre-Unit Readiness Assessment**

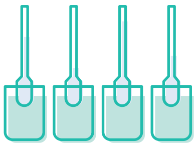
Launch Lesson



1 **Target: Zero**

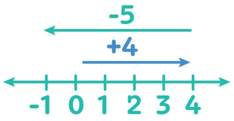
A reintroduction to positive and negative values in the form of a competitive card game.

Sub-Unit 1: Adding and Subtracting



2 **Interpreting Negative Numbers** ●

Revisit number lines to show positive and negative values, primarily in the contexts of temperature and elevation.



$4 - 5 = -1$

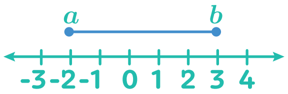
6 **Representing Subtraction**

Extend understanding of adding rational numbers to subtraction.

$$2 - (-3) = 2 + 3 = 5$$

7 **Subtracting Rational Numbers** (Part 1) ●

Subtract rational numbers to compare differences and notice that the order of subtraction changes the sign of the difference.



$$|a - b| = |b - a|$$

8 **Subtracting Rational Numbers** (Part 2) ●

Observe that the difference between two numbers can be positive or negative, but the distance between them is always positive.

Sub-Unit 2: Multiplying and Dividing Rational Numbers

$$\begin{matrix} a \cdot b \\ (-a) \cdot (-b) \end{matrix} = ab$$

$$a \cdot (-b) = -ab$$

11 **Multiplying Rational Numbers**

Apply understanding of the distance formula, $d = rt$, to make observations about the rules for multiplying rational numbers.

$$\boxed{?} \cdot 12 = -48$$

$$? = \text{negative}$$

12 **Multiply!**

Reason about the sign of the product when multiplying more than two rational numbers and use properties to evaluate expressions.

$$2 \cdot (-3) = -6$$

$$-6 \div 2 = -3$$

13 **Dividing Rational Numbers**

Use the relationship between multiplication and division to develop rules for dividing rational numbers.

Key Concepts

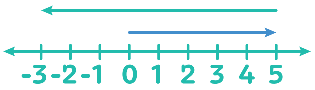
Lesson 6: Subtracting a number is equivalent to adding the additive inverse.
Lesson 10: Multiplying a negative by a negative results in a positive product.
Lesson 13: Dividing by a number is equivalent to multiplying by the multiplicative inverse.

Pacing

20 Lessons: 45 min each
3 Assessments: 45 min each
Full Unit: 23 days
● **Modified Unit:** 21 days

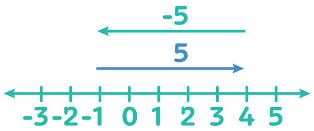
Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Rational Numbers



3 Changing Temperatures ●

Represent addition of rational numbers on a number line with arrow diagrams.



4 Adding Rational Numbers

Explore adding rational numbers and generalize rules about the sign of the sum.

5 Money and Debts

Use negative numbers in the context of money to represent an amount that is owed.

$-4 + 5 - 6 + (-9) - (-3) + 11$
$= \boxed{?}$

9 Adding and Subtracting Rational Numbers

Apply knowledge of addition and subtraction of rational numbers to real-world contexts.

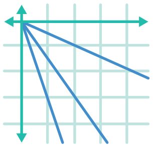
Assessment

A Mid-Unit Assessment

$$2 \cdot (-\frac{3}{2}) = -3$$

10 Position, Speed, and Time

Formalize that the product of a positive and negative number is negative, by relating multiplication to repeated addition.



14 Negative Rates

Explore negative rates of change and how to represent them in equations and on graphs.

Sub-Unit 3: Four Operations With Rational Numbers

$$10000 - 99999 < 0$$

15 Expressions With Rational Numbers

Address common misconceptions that can arise about expressions involving variables, such as $-x$ must always be a negative number.

$$\frac{1}{2} = 0.5$$

16 Say It With Decimals

Use long division to express fractions as decimals and see how calculations sometimes repeat, resulting in a repeating decimal.

Unit at a Glance

Spoiler Alert: Even though there are four operations, every subtraction and division problem can be rewritten as an addition or multiplication problem, respectively.

continued

+ — × ÷

$$x + a = b$$
$$px = 9$$

17 Solving Problems With Rational Numbers

Synthesize understanding of rational number arithmetic and interpreting negative quantities, such as rates of change.

18 Solving Equations With Rational Numbers ●

Solve equations of the forms $p + x = q$ and $px = q$ with rational values.

19 Representing Contexts With Equations ●

Build on the work from Lesson 18 to connect, write, and solve equations that represent real-world scenarios.

Key Concepts

Lesson 6: Subtracting a number is equivalent to adding the additive inverse.
Lesson 10: Multiplying a negative by a negative results in a positive product.
Lesson 13: Dividing by a number is equivalent to multiplying by the multiplicative inverse.

Pacing

20 Lessons: 45 min each
3 Assessments: 45 min each

Full Unit: 23 days
● **Modified Unit:** 21 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Captstone Lesson

20 **Summiting Everest** ●

Use rational number rates while role playing as mountain climbers attempting to summit Mt. Everest.

Assessment

A **End-of-Unit Assessment**

● Modifications to Pacing

Lessons 2 and 3: If students enter with a very strong foundation in their understanding of placing rational numbers on the number line, it is possible to omit these lessons, or combine them into a 1-day lesson.

Lessons 7 and 8: It is possible to combine Lessons 7 and 8 into a 1-day lesson. Any activities not completed in class could be assigned for homework to provide extra practice.

Lessons 18–20: Consider spending 3 days on Lessons 18 and 19, instead of 2. To gain a day back, omit the Capstone.

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	rational numbers
3	arrow diagram
4	additive inverse
5	balance charge credit debt deposit withdrawal
11	velocity
13	multiplicative inverse
16	bar notation repeating decimal terminating decimal

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
4	**MLR1:** Stronger and Clearer Each Time
1–5, 10, 11, 13, 16	**MLR2:** Collect and Display
6, 7, 18	**MLR3:** Critique, Correct, Clarify
2, 3, 5	**MLR5:** Co-craft Questions
9, 17, 19	**MLR6:** Three Reads
1–5, 9–13, 15, 17–19	**MLR7:** Compare and Connect
1, 6–8, 12–14, 18	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Materials
5, 20	calculators
4	collection of small, short objects
19	materials for creating a poster
20	number cubes
10, 11	paper clips, snap cubes, or other objects to move on the number line
1, 2, 4–13, 15–19	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
1, 12	standard deck of playing cards
1	sticky notes

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
15	Card Sort
19	Gallery Tour
16	Notice and Wonder
7, 12, 18	Number Talk
6, 7, 12	Partner Problems
2, 7, 9, 12, 18, 19	Poll the Class
4, 5, 9–11, 16, 19	Think-Pair-Share
15	True or False?
3, 17	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 9
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 20

Amps

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Social & Collaborative Digital Moments

Featured Activity

Target: Zero, Part 2

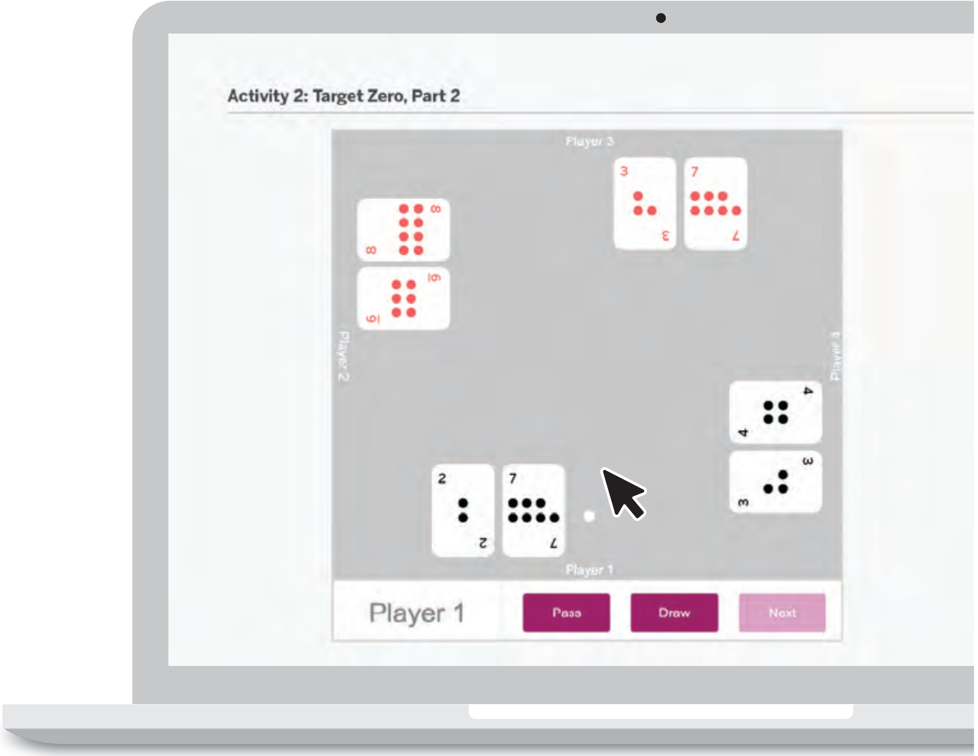
Put on your student hat and work through **Lesson 1, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Backward and Forward in Time **(Lesson 10)**
- How Close Can You Get? **(Lesson 13)**
- Greatest Product **(Lesson 15)**
- The Summit Attempt **(Lesson 20)**



Unit Study
Professional Learning

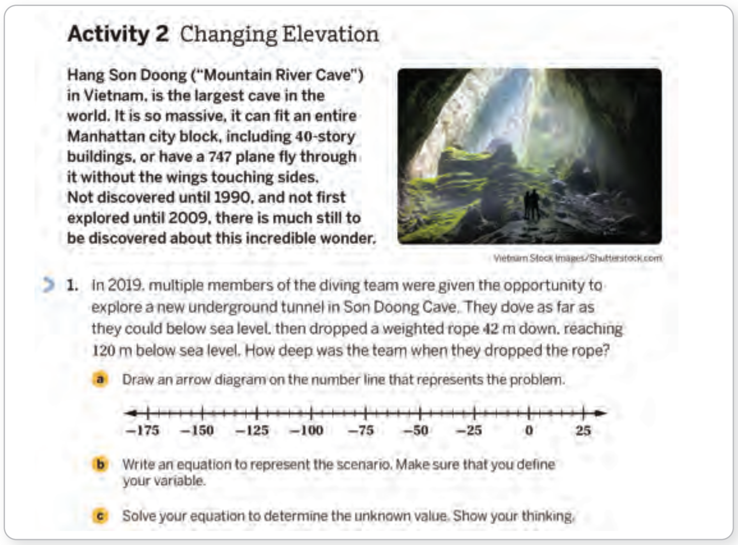
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 3 has all four operations of adding, subtracting, multiplying, and dividing with rational numbers. Students use arrow diagrams and number lines to practice adding and subtracting rational numbers. They work on real-world examples of changing temperatures and elevation. Students interpret multiplication and division with rational numbers in the context of velocity and temperature change over time. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 19, Activity 2:



Activity 2 Changing Elevation

Hang Son Doong ("Mountain River Cave") in Vietnam, is the largest cave in the world. It is so massive, it can fit an entire Manhattan city block, including 40-story buildings, or have a 747 plane fly through it without the wings touching sides. Not discovered until 1990, and not first explored until 2009, there is much still to be discovered about this incredible wonder.

Vietnam Stock Images/Shutterstock.com

1. In 2019, multiple members of the diving team were given the opportunity to explore a new underground tunnel in Son Doong Cave. They dove as far as they could below sea level, then dropped a weighted rope 42 m down, reaching 120 m below sea level. How deep was the team when they dropped the rope?

   a. Draw an arrow diagram on the number line that represents the problem.

   b. Write an equation to represent the scenario. Make sure that you define your variable.

   c. Solve your equation to determine the unknown value. Show your thinking.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

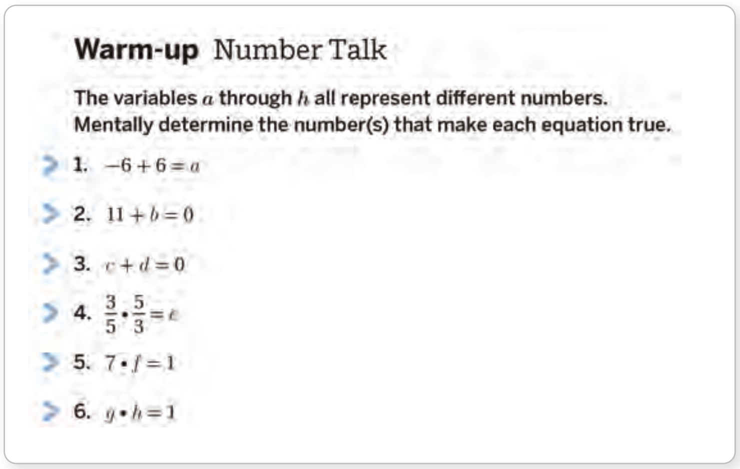
- What was it like to engage in this problem as a learner?
- Students may not fully appreciate how deep 42 meters (in question 1) is, it might be interesting to ask for their estimates of the heights of different buildings on the school campus or around town.
- Arrow diagrams are used throughout this unit, what other diagrams might you share with students to aid their understanding?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Number Talk

Rehearse . . .

How you'll facilitate the ***Number Talk*** instructional routine in Lesson 18, Warm-up:



Warm-up Number Talk

The variables a through h all represent different numbers. Mentally determine the number(s) that make each equation true.

1. $-6+6=a$
2. $11+b=0$
3. $c+d=0$
4. $\frac{3}{5} \cdot \frac{5}{3}=e$
5. $7 \cdot f=1$
6. $g \cdot h=1$

Points to Ponder . . .

- ***Number Talk*** routines offer a chance to slow down and focus on the process of reasoning about a mathematical problem, instead of jumping straight to the solution.

This routine . . .

- Encourages divergent thinking as you solicit different approaches to the same solution.
- Helps to lift up voices that may not always want to be first to share.
- Shows that even a seemingly straightforward problem can hold complexity worthy of discussion.
- Requires students to be able to explain their thinking rather than just give an answer.

Anticipate . . .

- More students may want to share than you have time for. Have a plan for how much time you have for each problem and which are the most impactful to discuss further.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Elicit and use evidence of student thinking.

This effective teaching practice . . .

- Helps you assess student progress toward the mathematical goals and objectives of the lessons and units. By knowing where your students are at, you can help them get to where they need to be!
- Allows you to adjust your instruction, based upon student responses, so that you can support your students and extend their learning.

Points to Ponder . . .

- How and when will I use student responses from this unit's assessments to help adjust my instruction?
- Where are there moments during the lessons' activities in which I can informally elicit evidence of my students' thinking? How will I approach these moments?

Math Language Development

MLR5: Co-craft Questions

MLR5 appears in Lessons 2, 3, and 5.

- In Lesson 2, after you display the images of the thermometers, ask students to work with their partner to co-craft questions they have about the thermometers. Sample questions are provided.
- In Lesson 3, ask students to examine the map before revealing the problems of the activity. Generating their own questions about the map will help them make sense of the scenario before diving in.
- **English Learners:** DIsplay 1 or 2 sample questions and allow students to say their questions aloud as opposed to writing them, or vice versa.

Point to Ponder . . .

- As students work with each other to create mathematical questions they have about graphs or scenarios, how can you model for them how to use their developing mathematical vocabulary?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with reasoning about rational numbers in new and unfamiliar contexts throughout the unit? Do you think your students will generally:
    - Overgeneralize or misapply rules for operations with rational numbers?
    - Face challenges keeping track of the signs or numbers in multi-step problems?
    - Have a strong understanding when working with numerical expressions, but lose the thread when dealing with algebraic expressions?

Students With Disabilities

Accessibility: *Guide Processing and Visualization*

Opportunities to provide visual support and guidance to help students process new information appear in Lessons 1–7, 9–11, and 14–20.

- In Lesson 5, display or provide a checklist to help students complete the energy costs statement. A sample checklist is provided.
- In Lesson 10, use a think-aloud to demonstrate how the hiker's speed was used to determine their position at various times.
- In Lesson 11, ask student volunteers to demonstrate the bikers traveling at the same speed, but in opposite directions, passing an object at the same time.
- In selected lessons, display the Anchor Chart PDFs, *Operations With Rational Numbers and Solving Equations With Rational Numbers*, and the Graphic Organizer PDF, *Blank Number Lines* for students to use as references.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when your students may benefit from visual support or suggested guidance? What clues will you gather from your students?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind

In this unit, pay particular attention to supporting students in building their relationship skills and self-management skills.

Points to Ponder . . .

- Do students establish and maintain healthy relationships? Are they able to get along and work productively with others? Do students communicate clearly with each other? Are they able to negotiate through conflicts effectively?
- Do students show self-discipline while working? Are they able to stay organized? Do they set goals, both personal and academic and find paths towards achieving those goals? Can students manage their stress levels? Do they control their impulses?

Target: Zero

Let’s aim for zero in this card game.

Focus

Goals

1. Identify negative numbers.
2. Comprehend that pairs of integers can combine to produce a sum of 0.
3. **Language Goal:** Explain which card values will produce a sum closer to 0. **(Reading and Writing)**

Coherence

Today

In this Launch lesson, students are re-introduced to positive and negative values while keeping score in a card game. They compete and strategize to get closer to zero than their opponents. While the lesson avoids symbolic representation of operations with integers, students reason about opposite quantities by calculating scores resulting from combining positive and negative numbers.

Previously

Units 1–4 have focused mainly on proportional reasoning, so the last known contact students had with the Number System domain stretches to Grade 6.

Coming Soon

As students progress through this unit, they will encounter addition, subtraction, multiplication, and division — in roughly that order — with the set of rational numbers. Some additional work with expressions and equations using rational numbers will appear as well, serving as a runway to Unit 6.

Rigor

- Students build **conceptual understanding** of the effects of adding positive and negative values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	12 min	10 min	5 min	5 min
Pairs	Small Groups	Small Groups	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Game Cards* (optional)
- standard deck of playing cards with face cards removed, one per small group (optional)
- sticky notes

Note: Activity 1 PDF is provided if you would rather use printed cards instead playing cards. You do not need both. If printing in grayscale, show students how to identify the "red" cards.

Math Language Development

Review words

- *negative numbers*
- *positive numbers*

Amps | Featured Activity

Activity 2
Play Target: Zero

Students play a social card game to determine who can generate a score closest to zero. As they play, they reason about positive and negative values.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be overly-competitive as they play *Target: Zero* in Activity 1 or Activity 2. While discussing the rules of the game, also discuss appropriate social behavior. Remind students that, while trying to win, they are also trying to learn. Ask them to be respectful of others as they try to learn, too.

Modifications to Pacing

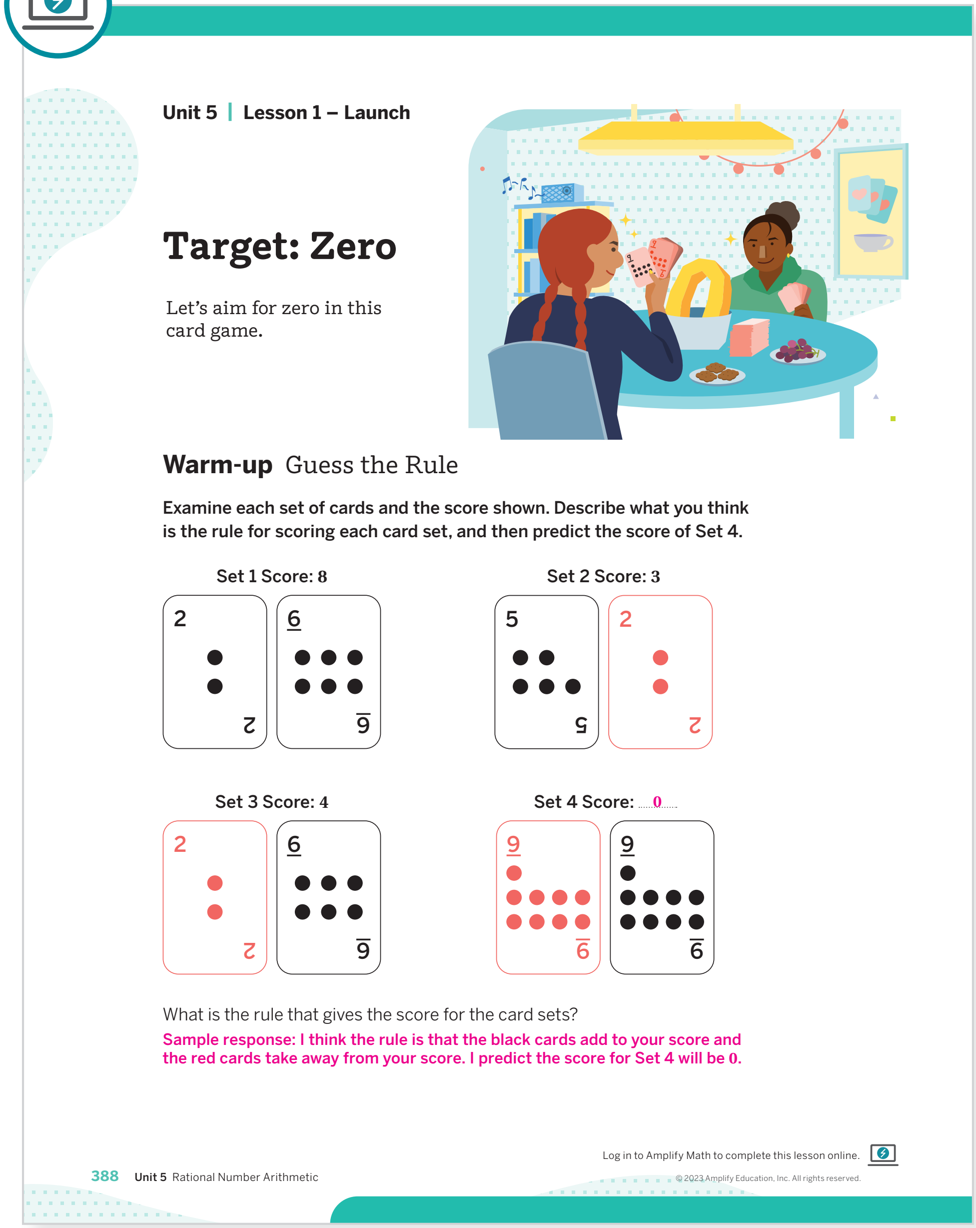
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. During the Launch for Activity 1, tell students that black cards increase their score and red cards decrease their score.
- **Activity 3** may be omitted. You may consider assigning this Activity as Additional Practice.

Pairs | 5 min

Warm-up Guess the Rule

Students examine sets of cards with given, but ambiguous, scores. They work to determine a rule that fits all the sets of cards.



Unit 5 | **Lesson 1 – Launch**

Target: Zero

Let's aim for zero in this card game.

Warm-up Guess the Rule

Examine each set of cards and the score shown. Describe what you think is the rule for scoring each card set, and then predict the score of Set 4.

Set 1 Score: 8

Set 2 Score: 3

Set 3 Score: 4

Set 4 Score: 0

What is the rule that gives the score for the card sets?

Sample response: I think the rule is that the black cards add to your score and the red cards take away from your score. I predict the score for Set 4 will be 0.

Log in to Amplify Math to complete this lesson online.

388 **Unit 5** Rational Number Arithmetic

1 Launch

Tell students they will be playing a card game during the lesson, but first they need to help reveal one of the rules. Point out that the numbers on the cards displaying a 6 or a 9 will be underlined to show their orientation.

2 Monitor

Help students get started by suggesting they think about what the different colors of the cards might mean.

Look for points of confusion:

- **Thinking that Set 2 means you subtract the second card from the first.** Have students apply this reasoning to Set 3 and check whether it gives the same score.

3 Connect

Have students share the score they predicted for Set 4, and the rule they used to make their prediction. Confirm with the class that the rule is that black cards increase your score by the number written on the card and red cards decrease your score by the number written on the card.

Display a set of cards showing a red 2 and a red 6.

Ask:

- "What would be your score for these two cards, according to the rule?" −8
- "What is another set of cards, other than Set 4, that would give a score of 0?" Sample response: Red 5 and black 5

Highlight that during the *Target: Zero* game, black cards will always increase your score and red cards will always decrease your score by the amount shown on the card.

Math Language Development

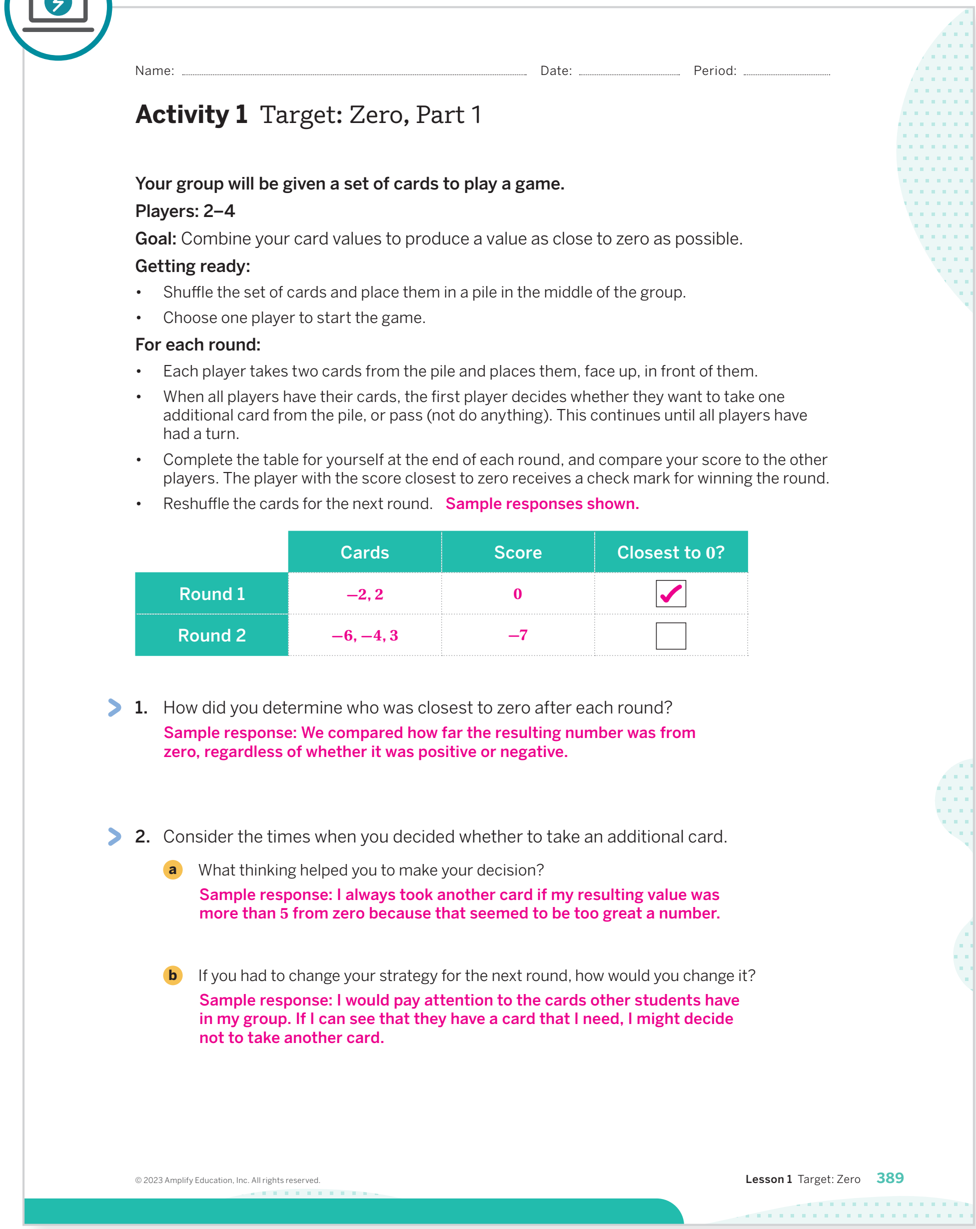
MLR2: Collect and Display

Listen carefully for the language students begin to use as they encounter opposite or negative values. Collect these words and phrases and add them to a class display so they can be revisited and refined — or adopted more widely — as you continue through the unit.

Small Groups | 12 min

Activity 1 Target: Zero, Part 1

Students play a simplified version of the *Target: Zero* game to get acquainted with the rules.



Name: ______ Date: ______ Period: ______

Activity 1 Target: Zero, Part 1

Your group will be given a set of cards to play a game.

Players: 2–4

Goal: Combine your card values to produce a value as close to zero as possible.

Getting ready:

- Shuffle the set of cards and place them in a pile in the middle of the group.
- Choose one player to start the game.

For each round:

- Each player takes two cards from the pile and places them, face up, in front of them.
- When all players have their cards, the first player decides whether they want to take one additional card from the pile, or pass (not do anything). This continues until all players have had a turn.
- Complete the table for yourself at the end of each round, and compare your score to the other players. The player with the score closest to zero receives a check mark for winning the round.
- Reshuffle the cards for the next round. **Sample responses shown.**

	Cards	Score	Closest to 0?
Round 1	$-2, 2$	0	☑
Round 2	$-6, -4, 3$	-7	☐

1. How did you determine who was closest to zero after each round?
Sample response: We compared how far the resulting number was from zero, regardless of whether it was positive or negative.

2. Consider the times when you decided whether to take an additional card.
   a. What thinking helped you to make your decision?
   Sample response: I always took another card if my resulting value was more than 5 from zero because that seemed to be too great a number.
   b. If you had to change your strategy for the next round, how would you change it?
   Sample response: I would pay attention to the cards other students have in my group. If I can see that they have a card that I need, I might decide not to take another card.

Lesson 1 Target: Zero 389

1 Launch

Distribute either the decks of playing cards or the cards from the Activity 1 PDF to groups of students. Play a mock round with a student for the whole class to observe the game in action.

2 Monitor

Help students get started by reminding them that the first round is meant to be played collaboratively. Encourage them to be as helpful to each other as possible.

Look for points of confusion:

- **Adding or subtracting values indiscriminately.** Have students first work with any black cards, and then consider red cards.
- **Not sure what to do in the event of a tie.** Have students decide on a fair way to break the tie, and encourage them to share their solution during the Connect.

Look for productive strategies:

- Representing card values with + and – signs.
- Using a number line to support determining their score.

3 Connect

Display two cards: a red 4 and a red 6. Then show an additional black 10 as a third card.

Have students share how they would determine the score for these cards.

Ask:

- "If you have two cards of the same color, will getting another card of the same color ever help your score?"
- "What rules for ties were created that could be helpful for other groups?"

Highlight that, in the next activity, students will play the same game with some updated rules. Importantly, the values of the cards do not change, so students can continue to use similar strategies for calculating their scores.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to two-sided counters – or a similar physical manipulative with two colors – for students to choose to use as tactile representations of addition and subtraction.

Extension: *Math Enrichment*

Have students respond to the following question:
If you drew a red 7 as your first card, what is the exact value and color of the second card you would need to draw to get a score of exactly zero? Exactly 2? black 7, black 9

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share how they would determine the score for the cards displayed (red 4, red 6, black 10), press for details in their reasoning.
For example:

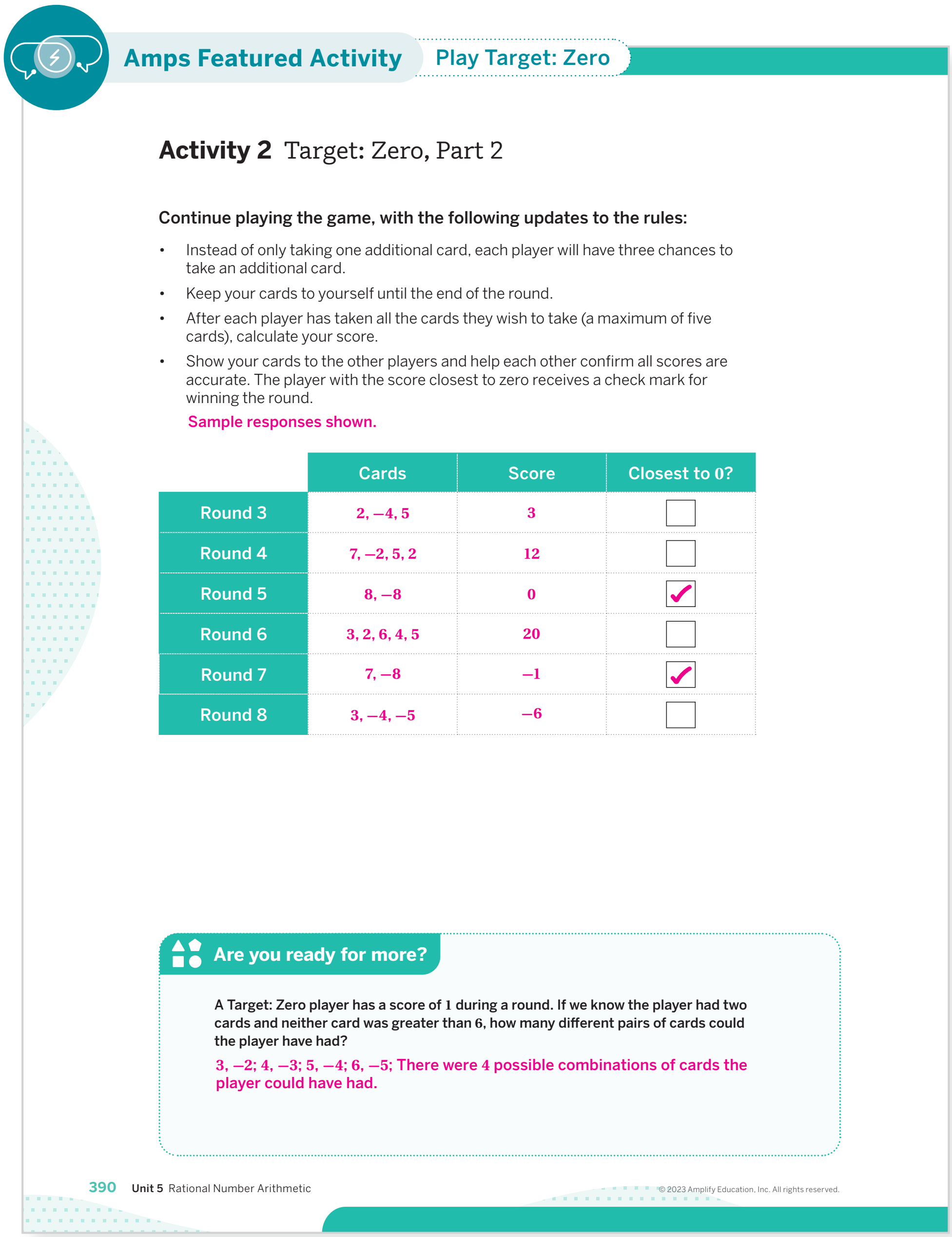
If a student says . . .	Press for details by asking . . .
"I added and then subtracted."	"Which numbers did you add? How did you know to add them? Which number(s) did you subtract? How did you know to subtract?"

Small Groups | 12 min

Activity 2 Target: Zero, Part 2

Students play a more competitive version of the Target: Zero game to solidify their strategy and practice with determining the sum of positive and negative numbers.

Amps Featured Activity Play Target: Zero



Activity 2 Target: Zero, Part 2

Continue playing the game, with the following updates to the rules:

- Instead of only taking one additional card, each player will have three chances to take an additional card.
- Keep your cards to yourself until the end of the round.
- After each player has taken all the cards they wish to take (a maximum of five cards), calculate your score.
- Show your cards to the other players and help each other confirm all scores are accurate. The player with the score closest to zero receives a check mark for winning the round.

Sample responses shown.

	Cards	Score	Closest to 0?
Round 3	2, −4, 5	3	☐
Round 4	7, −2, 5, 2	12	☐
Round 5	8, −8	0	☑
Round 6	3, 2, 6, 4, 5	20	☐
Round 7	7, −8	−1	☑
Round 8	3, −4, −5	−6	☐

Are you ready for more?

A Target: Zero player has a score of 1 during a round. If we know the player had two cards and neither card was greater than 6, how many different pairs of cards could the player have had?

3, −2; 4, −3; 5, −4; 6, −5; There were 4 possible combinations of cards the player could have had.

390 **Unit 5** Rational Number Arithmetic

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1 Launch

Have students read through the new rules for playing the game, and then ask someone to explain the changes in their own words. (You might pretend to not understand the new rules yourself). Ask, "How might the new rules affect your strategy?"

2 Monitor

Help students get started by asking, "What is the current value of your cards?"

Look for points of confusion:

- **Thinking they need to add their cards in the order they received them.** Ask, "Does the order in which you add numbers in an expression affect the sum?"

Look for productive strategies:

- Representing the total value of the cards with an expression.
- Using a number line to support determining their score.

3 Connect

Display the following cards: black 4, black 10, red 4, black 2.

Have students share the current score for this set of cards, and how they determined it. Listen for a strategy that first combines the black and red cards displaying a 4, and then adds the black 10 and 2 cards to result in a total of 12.

Ask:

- "How many other ways can we calculate the score for this same set of cards?"
- "Are you adding or subtracting in your calculations?"
- "Does the order of your cards affect your score? How do you know?"

Highlight that red cards function similarly to negative numbers and black cards represent positive numbers.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can play a social card game to determine who can generate a score closest to zero.

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide access to sticky notes and suggest that students keep a running tally of their score on a sticky note or in the open space in their Student Edition. Continue to provide access to two-sided counters – or a similar physical manipulative with two colors – for students to choose to use as tactile representations of addition and subtraction.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask questions, have them compare the various ways the score can be calculated for the same set of cards. Consider displaying the different strategies, such as:

One strategy:	Another strategy:
1. Red 4 + Black 4 = 0	1. Combine the black cards for a total of 4 + 10 + 2 = 16.
2. Black 10 + Black 2 = 12	2. Subtract the red 4.
3. 0 + 12 = 12	3. 16 − 4 = 12

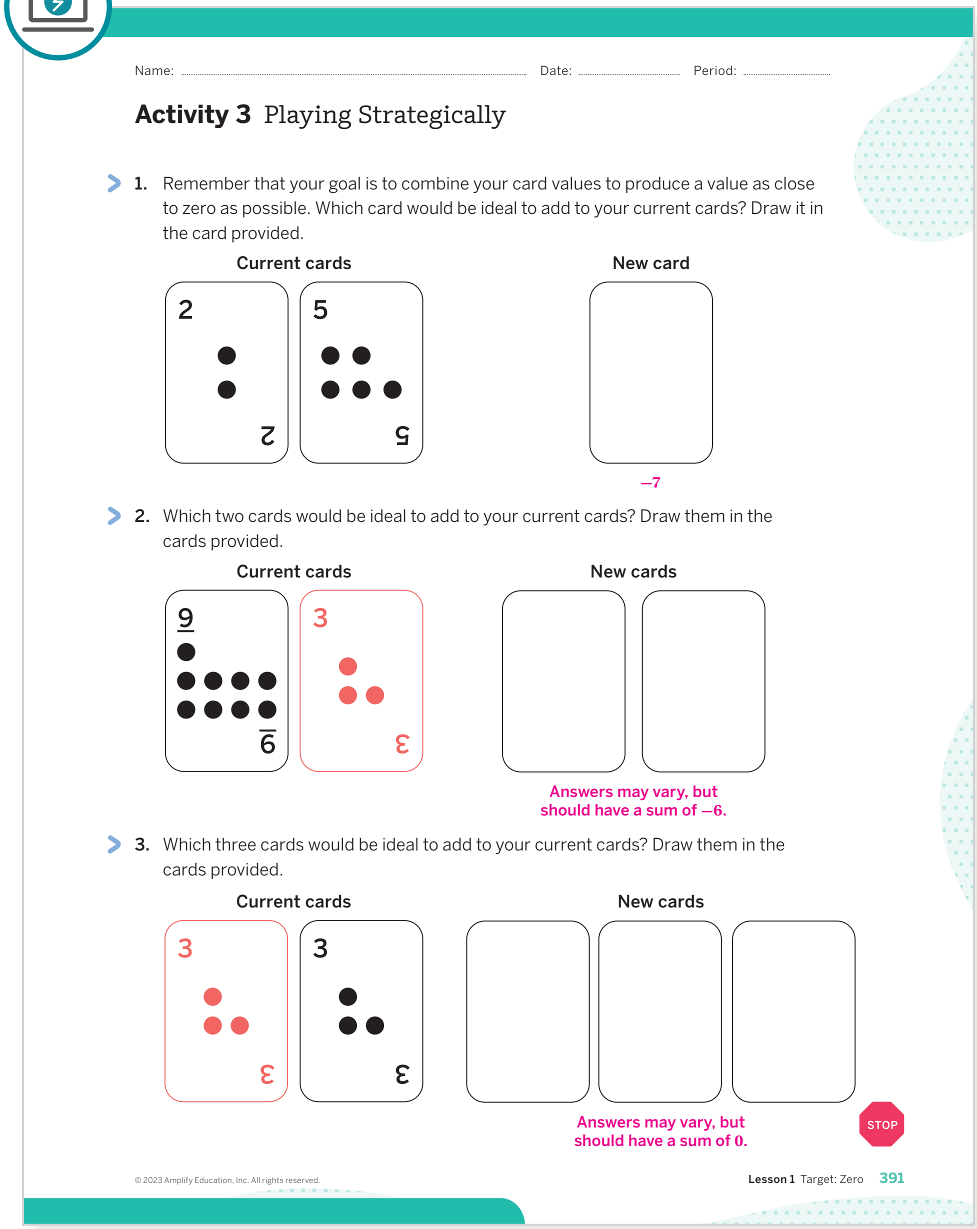
English Learners

Physically group the related cards together that show which values are added or subtracted.

Independent | 10 min

Activity 3 Playing Strategically

Given some cards, students determine an ideal set of cards to add to produce a number as close to 0 as possible.



Name: Date: Period:

Activity 3 Playing Strategically

1. Remember that your goal is to combine your card values to produce a value as close to zero as possible. Which card would be ideal to add to your current cards? Draw it in the card provided.

Current cards New card

2. Which two cards would be ideal to add to your current cards? Draw them in the cards provided.

Current cards New cards

Answers may vary, but should have a sum of –6.

3. Which three cards would be ideal to add to your current cards? Draw them in the cards provided.

Current cards New cards

Answers may vary, but should have a sum of 0.

1 Launch

Let students know that they are still playing with an imaginary single deck of cards, though they do not know what any other player has. Tell students they do not need to make the pictures of the cards look accurate, as long as the values shown are clear.

2 Monitor

Help students get started by having them determine the score of the current cards shown first.

Look for points of confusion:

- **Choosing cards without paying attention to their color.** Have students add a plus or minus sign to their cards to indicate their value.
- **Not using all of the current cards shown.** Tell students they must consider all of the cards shown.

Look for productive strategies:

- In Problem 2, drawing the two opposite cards of the current cards shown.

3 Connect

Ask, "A player has two cards that you cannot see. They take one more card, which is a black 1, and they say 'I've got zero!' What cards might they have had at the start?"

Highlight that many different combinations of cards can result in a given sum. Say, "As you continue throughout this unit, you will notice that there are often many different ways to write an expression that will give you the same result."

Differentiated Support

Accessibility: *Optimize Access to Tools*

Continue to provide access to two-sided counters – or a similar physical manipulative with two colors – for students to choose to use as tactile representations of addition and subtraction.

Extension: *Math Enrichment*

Have students complete the following problem:
Suppose you have a set of cards that are labeled with numbers from 1 to 10 and consist of both black and red cards. Your current cards are a black 10 and a red 3. How many ways can you select two more cards of the same color to have a total score of 0? The current score is 7. Possible ways to select two cards of the same color: red 1 and red 6, red 2 and red 5, red 3 and red 4 (in any order).

Whole Class | 5 min

Summary A World of Opposites

Review and synthesize how to produce a score of 0 with differently numbered cards.

Narrative Connections

Unit 5 Rational Number Arithmetic

A World of Opposites

Opposites have played a major part in how we think about our world.

The philosopher Heraclitus believed that everything in the universe had an opposite. These opposites were bound together in a continuous cycle. Lao Tze, in the classic Chinese text "Tao Te Ching," argued that every idea arises from its own opposite. Good cannot exist without evil. Beauty cannot exist without ugliness.

And in 1947, the physicist Neils Bohr designed his own coat-of-arms for the King of Denmark. Instead of the usual lions and crosses, Bohr drew a yin and yang symbol with the phrase "contraria sunt complementa." In English, the phrase translates to "opposites are complementary."

Around the globe, there are many kinds of opposites. There are the vast depths of the Mariana Trench and the breathtaking heights of Mount Everest. The lush, humid jungles of the Amazon Rainforest could not be more different than the arid winds of the Atacama Desert.

Throughout our planet, you can find opposites in temperatures, locations, heights, sizes, and even in measurements of time. In order to describe and understand all the characteristics of our world, we'll need the full breadth of both the positive and the negative.

Welcome to Unit 5.

392 Unit 5 Rational Number Arithmetic

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the summary or have a student volunteer to read it aloud.

Ask:

- "Were you already familiar with any of the people or places mentioned in the Summary? What else can you tell us?"
- "What must be true about two cards that result in 0 by themselves?"
- "What must be true about a set of three cards that result in 0?"

Highlight that cards that have the same number, but different colors result in zero automatically. This is because they represent opposite values.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

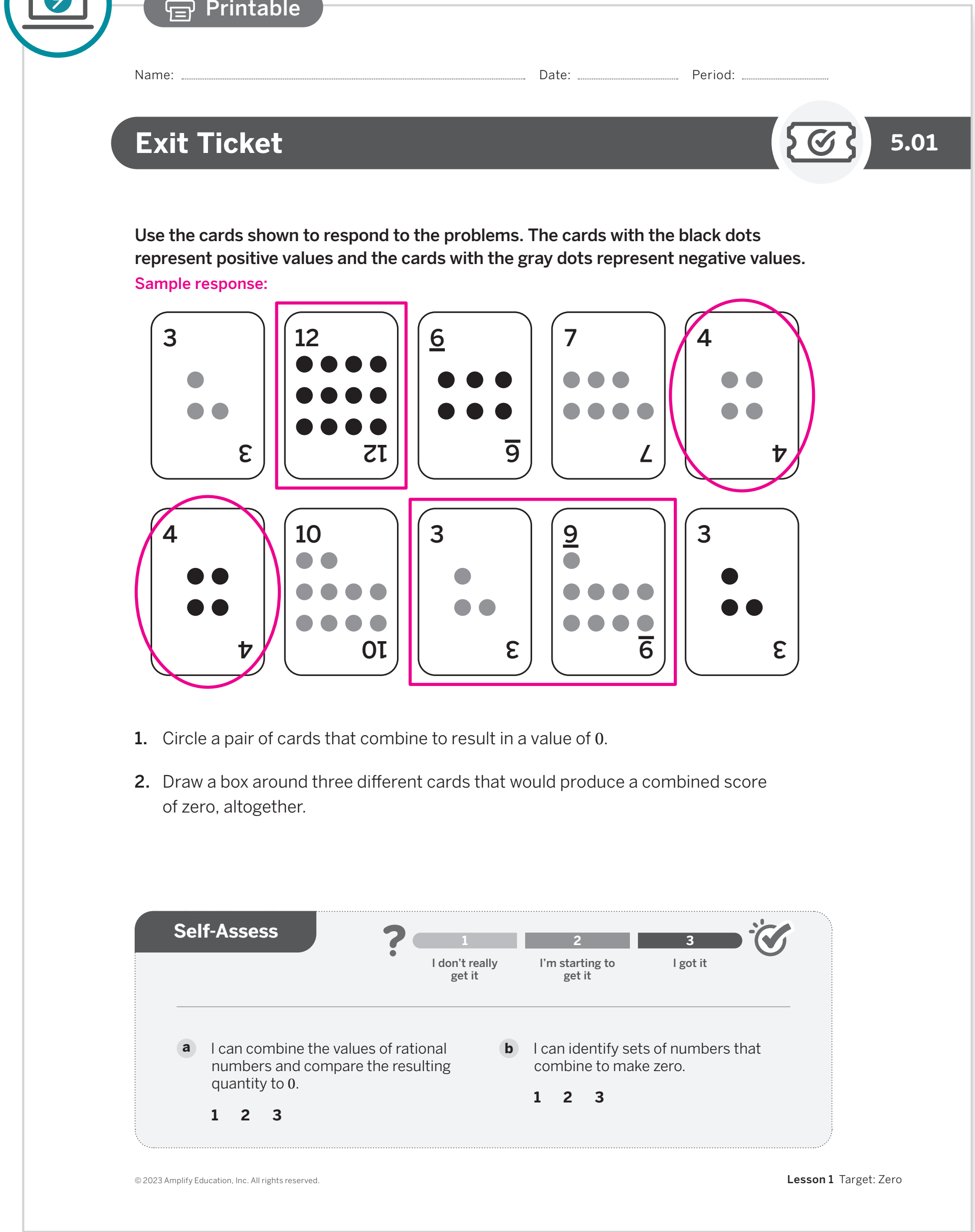
- "What are some things that feel like opposites to you in your own life?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of combining the value of positive and negative quantities by selecting sets of cards that will produce a score of 0.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 5.01

Use the cards shown to respond to the problems. The cards with the black dots represent positive values and the cards with the gray dots represent negative values.

Sample response:

1. Circle a pair of cards that combine to result in a value of 0.
2. Draw a box around three different cards that would produce a combined score of zero, altogether.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can combine the values of rational numbers and compare the resulting quantity to 0.
1 2 3

b I can identify sets of numbers that combine to make zero.
1 2 3

Lesson 1 Target: Zero

Success looks like . . .

- **Goal:** Identifying negative numbers.
- **Goal:** Comprehending that pairs of integers can combine to produce a sum of 0.
  - » Selecting a pair of cards that combine to give 0.
- **Language Goal:** Explaining which card values will produce a sum closer to 0. **(Reading and Writing)**

Suggested next steps

If students select two cards that are the same color for Problem 1, consider:

- Reviewing Activity 3.

If students use more than three cards for Problem 2, consider:

- Asking them to revise their thinking to determine whether they have any cards they can eliminate.
- Assigning Practice Problem 3.

Professional Learning

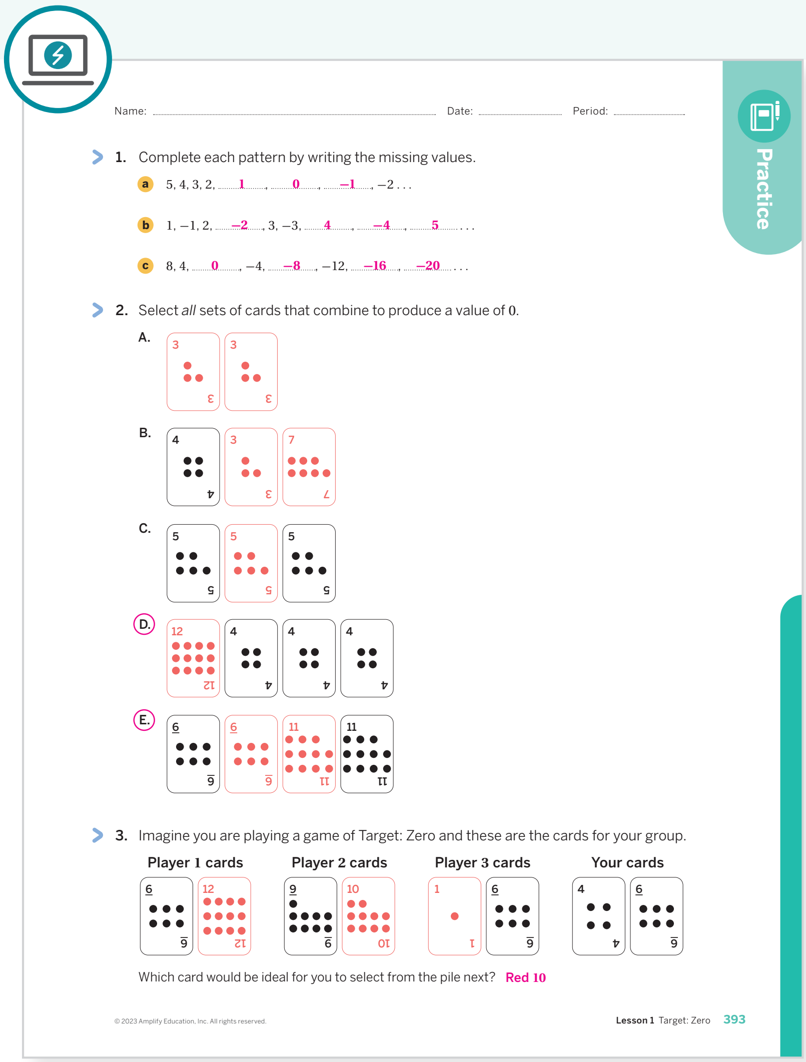
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students played the Target: zero game?
- What resources did students use as they played the game? Which resources were especially helpful? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

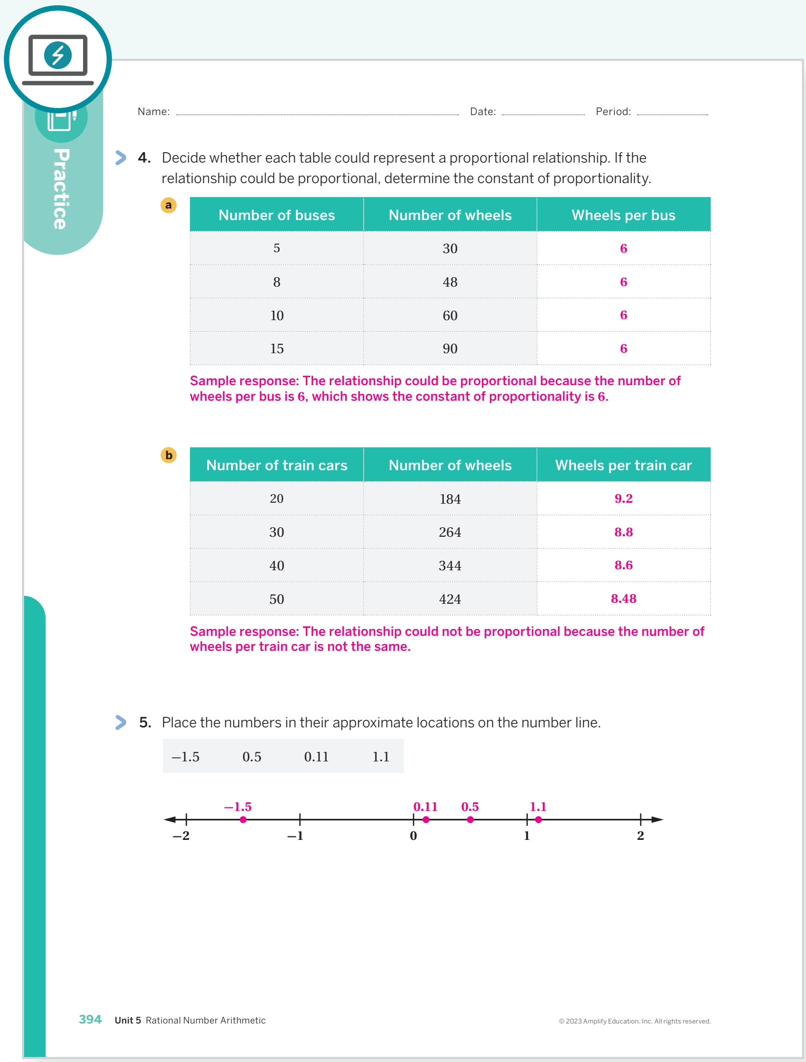
Practice

1. Complete each pattern by writing the missing values.
   - a 5, 4, 3, 2, 1, 0, −1, −2 . . .
   - b 1, −1, 2, −2, 3, −3, 4, −4, 5 . . .
   - c 8, 4, 0, −4, −8, −12, −16, −20 . . .

2. Select *all* sets of cards that combine to produce a value of 0.
   - A.
   - B.
   - [✓ D.]
   - [✓ E.]

3. Imagine you are playing a game of Target: Zero and these are the cards for your group.

Which card would be ideal for you to select from the pile next? Red 10



Name: Date: Period:

Practice

4. Decide whether each table could represent a proportional relationship. If the relationship could be proportional, determine the constant of proportionality.

a

Number of buses	Number of wheels	Wheels per bus
5	30	6
8	48	6
10	60	6
15	90	6

Sample response: The relationship could be proportional because the number of wheels per bus is 6, which shows the constant of proportionality is 6.

b

Number of train cars	Number of wheels	Wheels per train car
20	184	9.2
30	264	8.8
40	344	8.6
50	424	8.48

Sample response: The relationship could not be proportional because the number of wheels per train car is not the same.

5. Place the numbers in their approximate locations on the number line.

−1.5 0.5 0.11 1.1

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 2	3
Spiral	3	Grade 6	2
	4	Unit 2 Lesson 4	2
Formative ⚡	5	Unit 5 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 1

Adding and Subtracting Rational Numbers

Whole Class

In this Sub-Unit, students generalize the rules for determining the sums and differences of rational numbers by reasoning about rational values in real-world contexts and modeling changes in values using arrow diagrams.

SUB-UNIT

1 Adding and Subtracting Rational Numbers

Narrative Connections

What was Jeanne Baret's big secret?

In 1765, the botanist Philibert Commerçon was invited on an expedition to sail around the world. Commerçon wanted to bring his assistant and housekeeper, Jeanne Baret.

Baret had grown up in the French Loire Valley. There, she had become well versed in the native plant life, and was an herbal healer for her community. Baret would have been the perfect choice to join Commerçon. But there was a problem: Women were not allowed on French Navy ships. So Commerçon and Baret hatched a plan. Baret would disguise herself as a man, changing her name to "Jean."

For more than two years, "Jean" and Commerçon kept up their ruse. Together, they explored the rugged terrains Montevideo, Rio de Janeiro, and Tierra del Fuego. When Commerçon fell ill, Baret took over as chief botanist, collecting samples and lugging them through rough wilderness back to the ship for further study.

Imagine the sights she must have seen. She had grown up impoverished in the French countryside. Now, she was seeing sights no French woman had ever seen before: looming mountains, dense jungles, pebble beaches. Between Commerçon and Baret, the pair had collected over 6,000 specimens, many of which had never been seen before by a European.

For botanists like Baret, rational numbers are useful. They can be used to track the starting and ending points of one's journey, describe the differences of elevation of where different samples are collected, or compare the various climates of different locales. In these next lessons we will look at how to handle these rational numbers when adding and subtracting.

Read and discuss

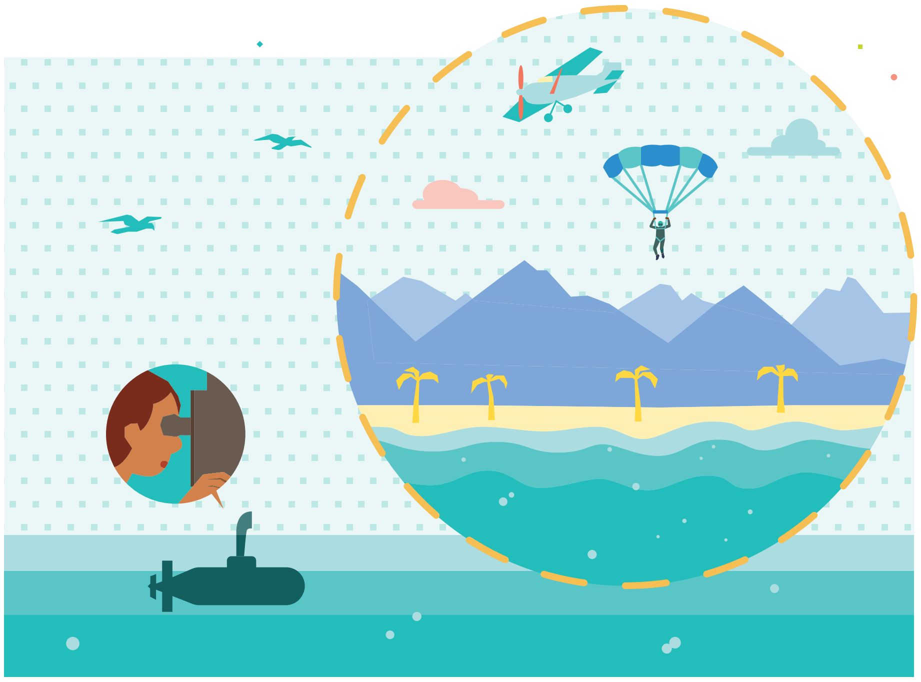
Read the narrative aloud as a class or have students read it individually. Students continue to explore real-world applications of adding and subtracting rational numbers in the following places:

- **Lesson 2, Activity 1:** Exploring the Extremes
- **Lesson 3, Activities 1-2:** Warmer and Colder, Comparing Temperatures
- **Lesson 4, Activities 1-2:** Above or Below the Water?, Energy Supply
- **Lesson 5, Activity 1:** Dealing With Debt
- **Lesson 7, Activity 2:** Expressions With Temperature
- **Lesson 9, Activity 1:** Writing Expressions

UNIT 5 | **LESSON 2**

Interpreting Negative Numbers

Let's review what we know about negative numbers.



Focus

Goals

1. Interpret rational numbers in the contexts of temperature and elevation.
2. **Language Goal:** Order rational numbers, and justify the comparisons. **(Speaking and Listening)**
3. Plot points on a vertical or horizontal number line to represent rational numbers.
4. Comprehend that the term *opposite* refers to numbers with the same magnitude but different signs.

Coherence

Today

Students reacquaint themselves with the number line model, which is important for reasoning about the position and difference of rational number values. This lesson serves as both a general reintroduction to a model that will be used often throughout the unit and to basic operations with rational numbers. The well-known contexts of temperature and elevation help ease students into this work. Activity 2 is a wonderful social moment that can serve as an anchor for the remainder of the unit.

Previously

In Lesson 1, students revisited positive and negative values. In Grade 6, students positioned and compared integer values on number lines.

Coming Soon

In Lessons 3 and 4, students will add positive and negative numbers — including fractions and decimals — using number lines and arrow diagrams.

Rigor

- Students build **conceptual understanding** of the relative positions and values of negative numbers on horizontal and vertical number lines.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per group
- Activity 2 PDF (answers)
- Anchor Chart PDF, *Wall Number Line*

Note: The wall-length number line may be a challenge to prepare, but it is worth it. Students will benefit from this visual, tactile, and social experience throughout the unit.

Math Language Development

New word

- *rational numbers**

Review words

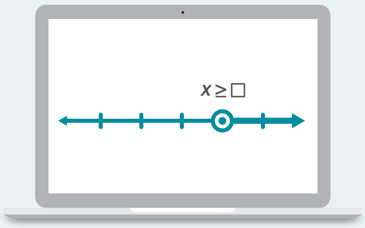
- *absolute value*
- *magnitude*
- *opposites*

*Students may confuse the mathematical term *rational* with the term's everyday meaning as it relates to reason and logic. Be ready to address how a *rational number* gets its name because it can be written as a ratio of two integers.

Amps | Featured Activity

Activity 1
Interactive Number Line

Students explore a large interactive number line and learn about the elevations of some of Earth's most extreme heights and depths.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students try to work together to place their numbers on a number line in Activity 2, they might not communicate well with each other. Remind students that effective communication is required if the group is to succeed. Set some guidelines for conflict resolution. Differences and mistakes will occur, so the group needs to work together to resolve them

Modifications to Pacing

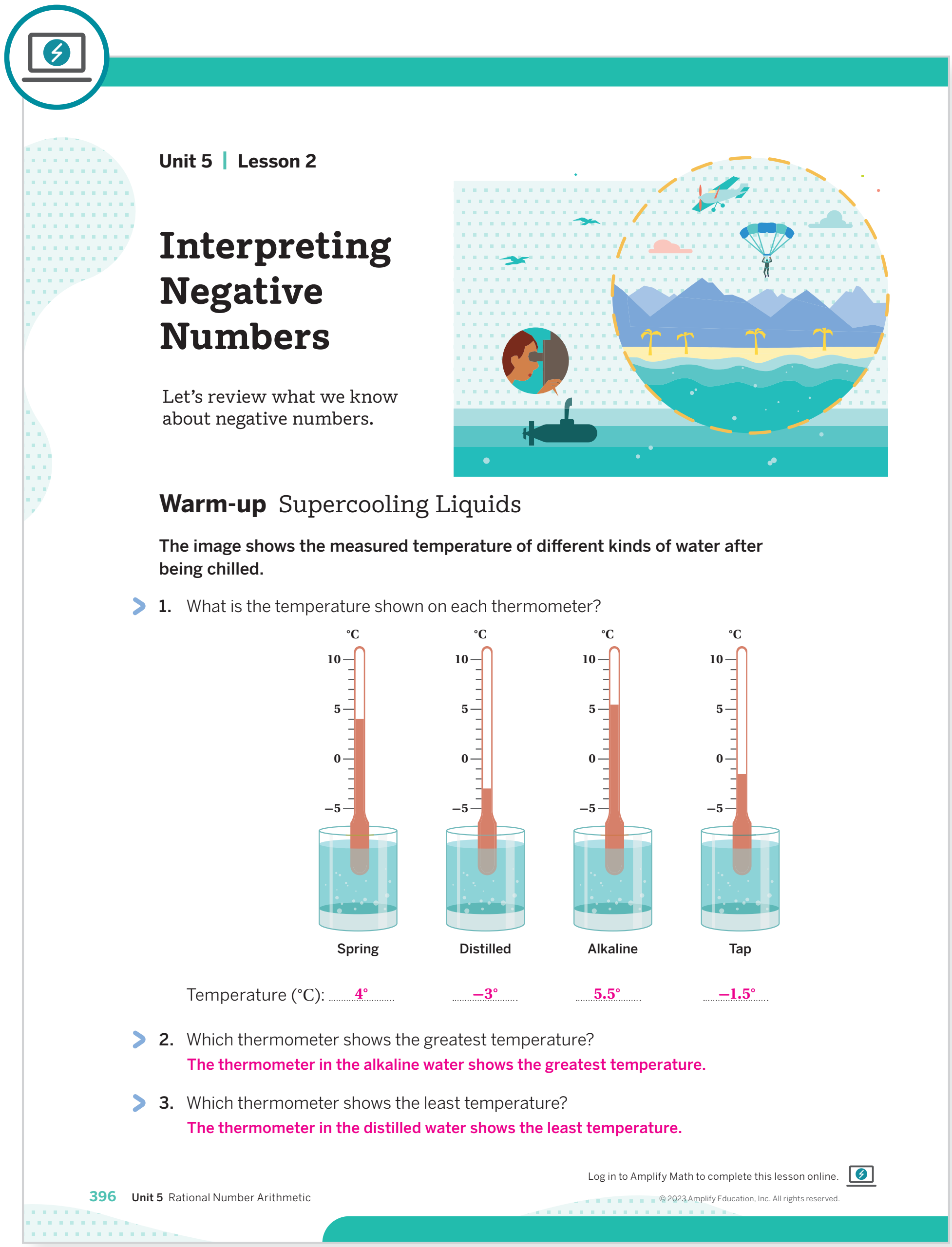
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 3 may be omitted.
- In **Activity 2**, have groups of students discuss and then place their numbers directly on the number line, rather than create a model on their page.

Pairs | 5 min

Warm-up Supercooling Liquids

Students read and compare temperatures on a thermometer to review identifying positive and negative values on a number line.



Unit 5 | Lesson 2

Interpreting Negative Numbers

Let's review what we know about negative numbers.

Warm-up Supercooling Liquids

The image shows the measured temperature of different kinds of water after being chilled.

1. What is the temperature shown on each thermometer?

Temperature (°C): 4° −3° 5.5° −1.5°

2. Which thermometer shows the greatest temperature?
   The thermometer in the alkaline water shows the greatest temperature.

3. Which thermometer shows the least temperature?
   The thermometer in the distilled water shows the least temperature.

Log in to Amplify Math to complete this lesson online.

396 **Unit 5** Rational Number Arithmetic

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1 Launch

Say, "Did you know it is possible to cool water to below its freezing point while remaining a liquid? Under certain conditions, it is!" Remind students that they worked with negative numbers in Grade 6, and that temperatures are a common context, where negative values are used in the real world.

2 Monitor

Help students get started by demonstrating how to start from zero and count up or down to the temperature mark. You might also have students label each tick mark with its value.

Look for points of confusion:

- **Thinking, for example, that one mark *above* -5 is -6.** Ask, "Between which two labeled marks is the temperature? Does your answer still make sense?"
- **Thinking -1.5 is the lowest temperature.** Have students compare the height of the temperature mark across all four thermometers. Ask, "Which is the lowest?"

Look for productive strategies:

- Marking the intermediate values on one of the thermometers.

3 Connect

Ask, "Suppose the temperature of another liquid is $-4°$. Is that colder or warmer than the coldest temperature shown?" $-4°$ is colder than the coldest temperature shown.

Highlight that temperatures below 0 can be referred to using a negative sign, such as -3 degrees, or by saying 3 degrees below 0. Point out that the latter example does not include a negative sign because the words sufficiently describe its position.

Math Language Development

MLR5: Co-Craft Questions

During the Launch, display the introductory text and the images of the four thermometers. Ask students to work with their partner to write 2–3 mathematical questions they could ask about the situation and the thermometers shown. Listen for how students use the idea of numbers being above or below zero. Ask pairs to share their questions with the whole class. Sample questions shown.

- Which container shows the least water temperature? The greatest?
- Which container(s) show temperatures above zero? Below zero?
- What does a zero temperature mean in this context?

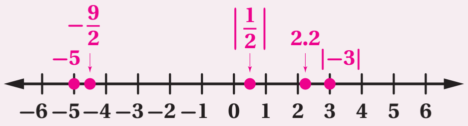
English Learners

Display the thermometers and color code the numbers above and below zero as students describe them.

Power-up

To power up students' ability to plot values on a number line, have students complete:

1. Place and label the numbers on the number line:
   $-5, 2.2, -\frac{9}{2}, |-3|, \left|\frac{1}{2}\right|$



2. Complete the statements with any values from part 1:
   -5 < $-\frac{9}{2}$ and $|-3|$ > $\left|\frac{1}{2}\right|$

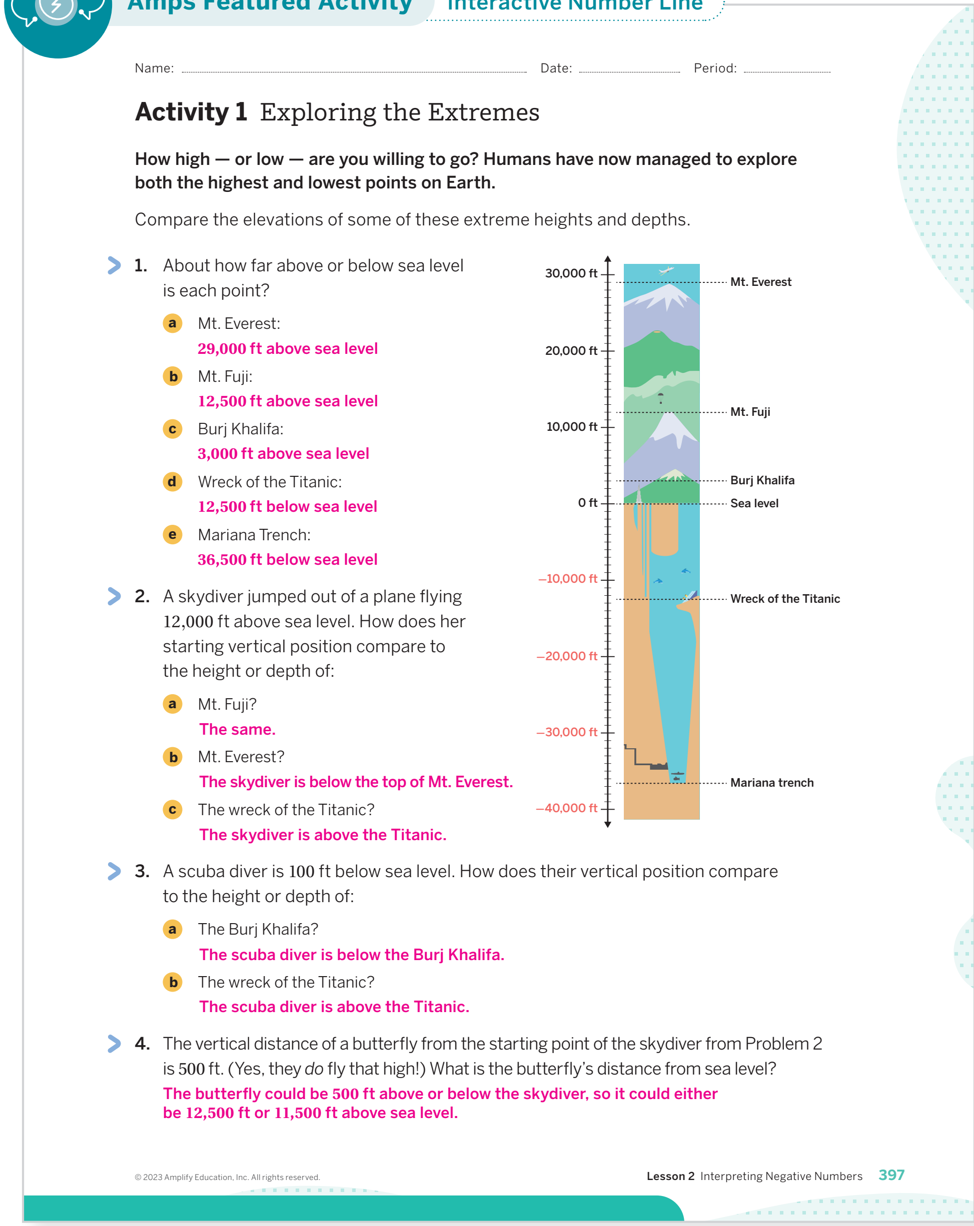
Use: Before Activity 2.
Informed by: Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 2, 3, and 4.

Pairs | 10 min

Activity 1 Exploring the Extremes

Students study elevations of well-known geographic landmarks next to a number line to identify their elevations and compare their relative positions.

Amps Featured Activity Interactive Number Line



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Activity 1 Exploring the Extremes

How high — or low — are you willing to go? Humans have now managed to explore both the highest and lowest points on Earth.

Compare the elevations of some of these extreme heights and depths.

1. About how far above or below sea level is each point?
   - a Mt. Everest:
     29,000 ft above sea level
   - b Mt. Fuji:
     12,500 ft above sea level
   - c Burj Khalifa:
     3,000 ft above sea level
   - d Wreck of the Titanic:
     12,500 ft below sea level
   - e Mariana Trench:
     36,500 ft below sea level

2. A skydiver jumped out of a plane flying 12,000 ft above sea level. How does her starting vertical position compare to the height or depth of:
   - a Mt. Fuji?
     The same.
   - b Mt. Everest?
     The skydiver is below the top of Mt. Everest.
   - c The wreck of the Titanic?
     The skydiver is above the Titanic.
3. A scuba diver is 100 ft below sea level. How does their vertical position compare to the height or depth of:
   - a The Burj Khalifa?
     The scuba diver is below the Burj Khalifa.
   - b The wreck of the Titanic?
     The scuba diver is above the Titanic.
4. The vertical distance of a butterfly from the starting point of the skydiver from Problem 2 is 500 ft. (Yes, they *do* fly that high!) What is the butterfly's distance from sea level?
   The butterfly could be 500 ft above or below the skydiver, so it could either be 12,500 ft or 11,500 ft above sea level.

Lesson 2 Interpreting Negative Numbers 397

1 Launch

Ask, "Which do you think is greater, the height of the highest point on earth or the depth of the lowest point?" Conduct the ***Poll the Class*** routine. Have students explore the elevation chart, noting the important point that an elevation of 0 ft is commonly referred to as *sea level*. You may choose to pause the class after Problem 1 to have a discussion about the elevations of each marked point.

2 Monitor

Help students get started by asking them where 15,000 ft above sea level would be on the number line, and having them label it.

Look for points of confusion:

- **Using numbers without a negative sign or appropriate description for elevations below sea level.** Ask, "How would someone know whether this value is above or below sea level?"
- **Thinking the Titanic is 8,000 ft below sea level.** Ask, "The Titanic is between which two marked values on the number line?"

3 Connect

Have students share how they reasoned about the butterfly's distance from sea level in Problem 3.

Ask:

- "Which elevations could also be written using negative numbers?"
- "Which elevations are the same distance from 0, or sea level?"

Highlight that the *absolute value* of a number is its distance from 0 on the number line. In the case of the elevations, Mt. Fuji, the skydiver, and the wreck of the Titanic are all 12,000 ft away from sea level, so they all have the same absolute value. Say, "You might recall from Grade 6 that we write 'the absolute value of 12,000' using this notation: $|12,000|$."

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can explore a large interactive number line to learn about the elevations of some of Earth's most extreme heights and depths.

Extension: *Math Enrichment*

Have students estimate the distance between the peak of Mt. Everest and the Mariana trench. About 65,000 ft.

Math Language Development

MLR2: Collect and Display

During the Connect, as you highlight the term *absolute value*, ask students to recall this term from Grade 6. Draw connections between this term and the words and phrases used in this activity, such as above and below sea level. Add these words and phrases to the class display. For example:

Distance	Absolute value
12,000 ft *above sea level*	$|12,000| = 12,000$
12,000 ft *below sea level*	$|-12,000| = -12,000$

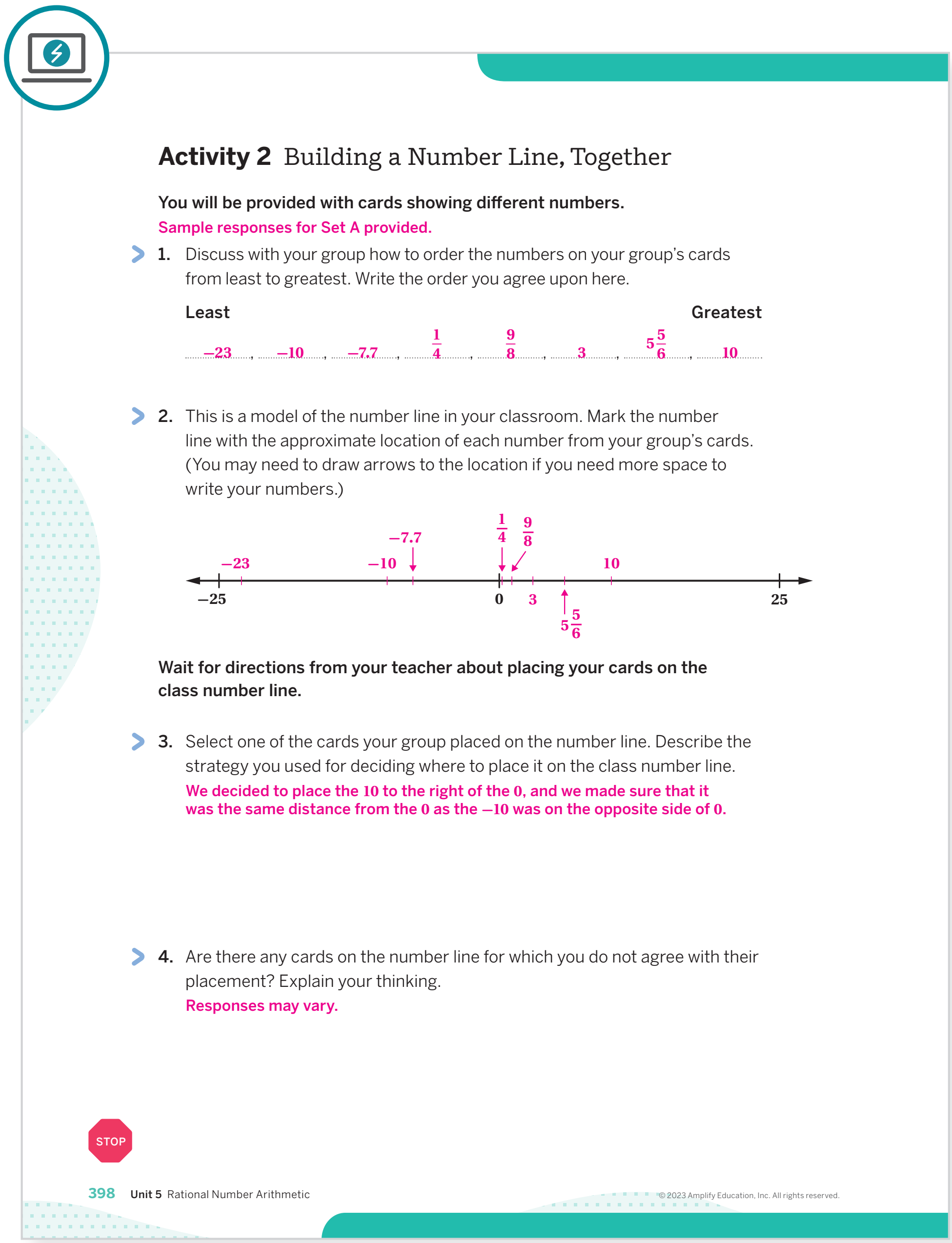
English Learners

Use gestures as students use the language of *above* and *below*.

Small Groups | 20 min

Activity 2 Building a Number Line, Together

Small groups of students work together to place their numbers on a number line while reasoning about relative distances between rational numbers on both sides of 0.



Activity 2 Building a Number Line, Together

You will be provided with cards showing different numbers.

Sample responses for Set A provided.

1. Discuss with your group how to order the numbers on your group's cards from least to greatest. Write the order you agree upon here.

Least **Greatest**

-23, -10, -7.7, $\frac{1}{4}$, $\frac{9}{8}$, 3, $5\frac{5}{6}$, 10

2. This is a model of the number line in your classroom. Mark the number line with the approximate location of each number from your group's cards. (You may need to draw arrows to the location if you need more space to write your numbers.)

Wait for directions from your teacher about placing your cards on the class number line.

3. Select one of the cards your group placed on the number line. Describe the strategy you used for deciding where to place it on the class number line.

We decided to place the 10 to the right of the 0, and we made sure that it was the same distance from the 0 as the −10 was on the opposite side of 0.

4. Are there any cards on the number line for which you do not agree with their placement? Explain your thinking.

Responses may vary.

STOP

398 Unit 5 Rational Number Arithmetic

1 Launch

Say, "While it is useful to use a vertical number line for contexts, such as temperature and elevation, we also know that number lines can be placed horizontally. For this activity, your group will receive a set of cards that you will eventually place on the class number line." Distribute sets of cards from the Activity 2 PDF.

2 Monitor

Help students get started by asking, "Which cards will be to the left of 0? Which will be to the right? How can you tell?"

Look for points of confusion:

- **Placing numbers to the correct side of the other numbers, but not in the proper position on the whole number line.** Ask for the student to explain the position of a friendly value, such as 15, relative to 0 and 30 on the number line.

Look for productive strategies:

- Using opposite values to match distances from 0.

3 Connect

Have students share pairs of numbers that are the same distance from 0, but on opposite sides of the number line.

Highlight that these numbers have the same absolute value, which is always represented as a positive number because absolute value represents distance from zero. Distance is always positive. Numbers with the same value on opposite sides of the number line are called *opposites*.

Ask:

- "How can you tell, when comparing two numbers, which number will be farther from zero?"
- "How can you tell, when comparing two numbers, whether they will be on the same side or opposite sides of 0?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students first sort the cards into two categories: positive numbers and negative numbers. Then have them sort each group according to their absolute values. Remind them that a negative number with a greater absolute value is actually less than a negative number with a lesser absolute value. For example, $-5 < -3$ and $|-5| > |-3|$.

Math Language Development

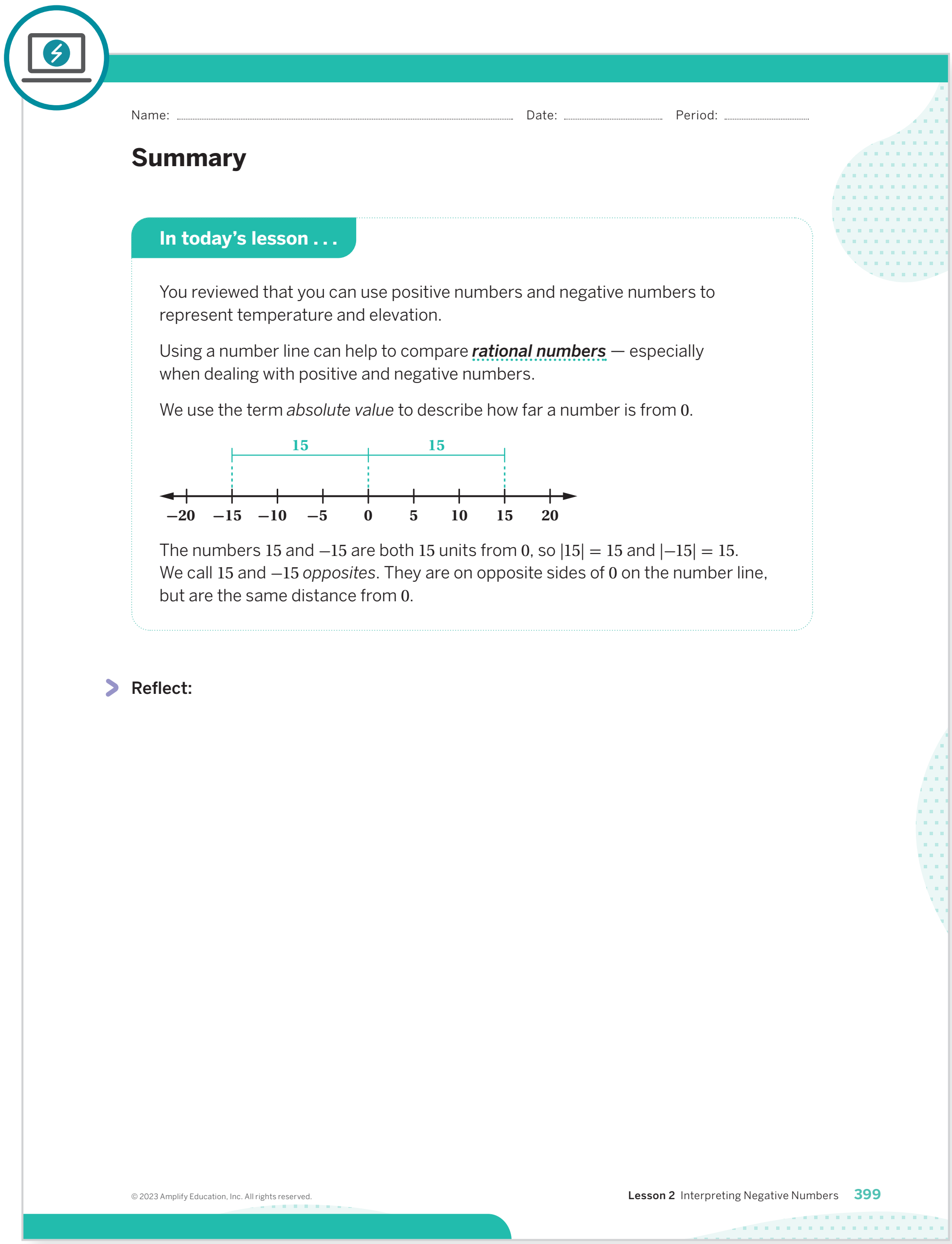
MLR7: Compare and Connect

During the Connect, as students share pairs of numbers that are the same distance from 0, but on opposite sides, draw connections between *opposites* and *absolute value*. Display the following sentence frame and ask students to complete it. Then add this statement to the class display, along with an example, such as 10 and −10.

Numbers that are _____ have the same _____ because they are the same distance from zero. opposites; absolute value

Summary

Review and synthesize that number lines can be used to compare temperatures, elevations, and the relative value of positive and negative numbers.



Name: Date: Period:

Summary

In today's lesson . . .

You reviewed that you can use positive numbers and negative numbers to represent temperature and elevation.

Using a number line can help to compare ***rational numbers*** — especially when dealing with positive and negative numbers.

We use the term *absolute value* to describe how far a number is from 0.

The numbers 15 and -15 are both 15 units from 0, so $|15| = 15$ and $|-15| = 15$. We call 15 and -15 *opposites*. They are on opposite sides of 0 on the number line, but are the same distance from 0.

Reflect:

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Lesson 2 Interpreting Negative Numbers 399

Synthesize

Display the class number line from Activity 2.

Have students share where to find the least and greatest value numbers on a horizontal and vertical number line.

Define *rational numbers* as the set of all numbers, positive and negative, that can be written as fractions. For example, any whole number is a rational number.

Highlight that several important features of a number line can help when comparing positive and negative numbers: the number's distance from 0, the distance from other numbers near it, and the side of 0 it is on.

Ask:

- "Can two numbers have the same absolute value if they have the same sign?"
- "Can two numbers with the same sign be opposites?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How was building a number line as a group more or less challenging than doing it by yourself?"

Math Language Development

MLR2: Collect and Display

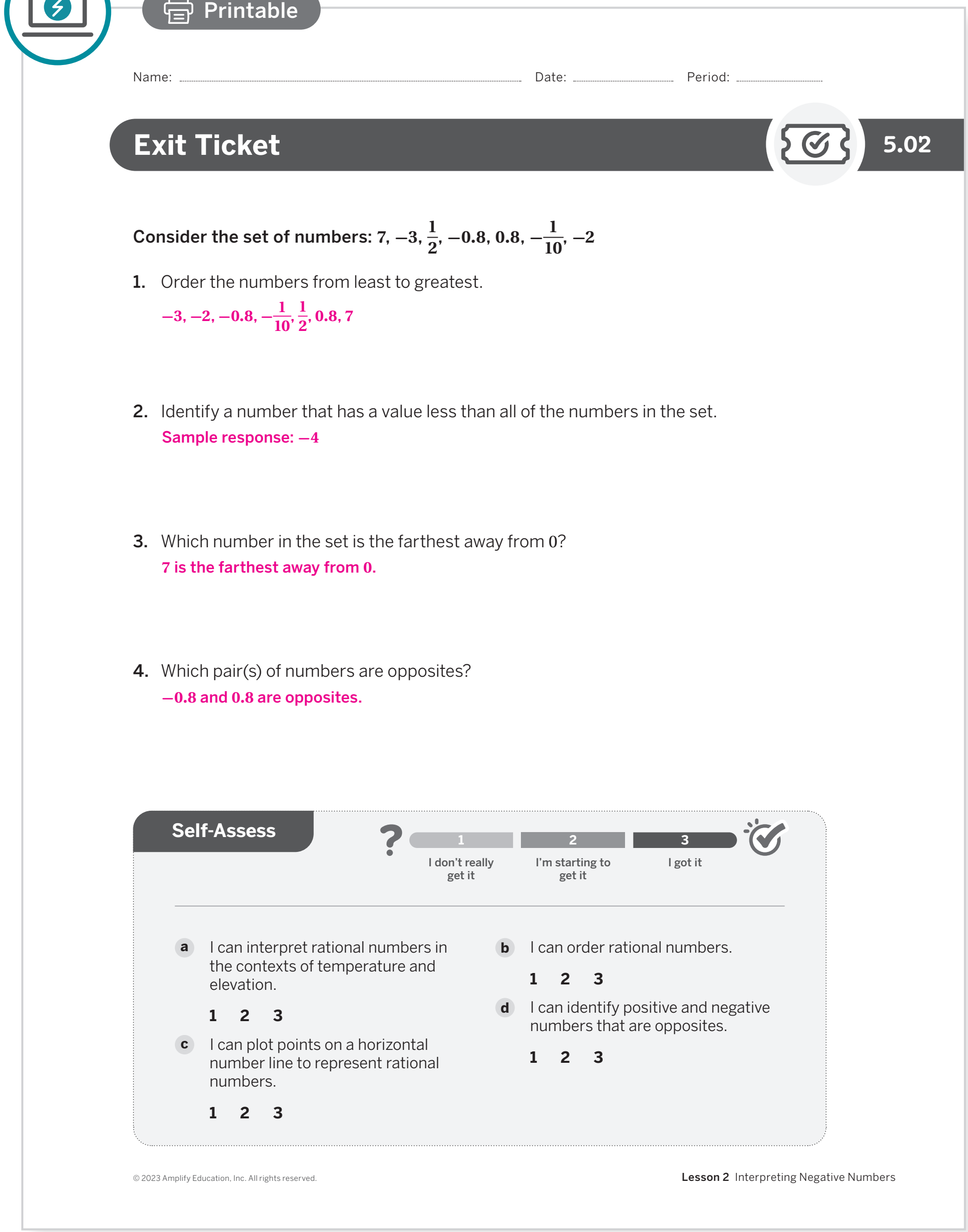
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on the terms and phrases related to the term *rational numbers* that were added to the display during the lesson. Highlight that the term *rational* comes from the term *ratio*; numbers that are rational can be written as the ratio of two integers.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the value of positive and negative numbers by comparing and ordering a set of rational numbers.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.02

Consider the set of numbers: $7, -3, \frac{1}{2}, -0.8, 0.8, -\frac{1}{10}, -2$

1. Order the numbers from least to greatest.

   $-3, -2, -0.8, -\frac{1}{10}, \frac{1}{2}, 0.8, 7$

2. Identify a number that has a value less than all of the numbers in the set.

   Sample response: -4

3. Which number in the set is the farthest away from 0?

   7 is the farthest away from 0.

4. Which pair(s) of numbers are opposites?

   -0.8 and 0.8 are opposites.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can interpret rational numbers in the contexts of temperature and elevation.
1 2 3

b I can order rational numbers.
1 2 3

c I can plot points on a horizontal number line to represent rational numbers.
1 2 3

d I can identify positive and negative numbers that are opposites.
1 2 3

Lesson 2 Interpreting Negative Numbers

Success looks like . . .

- **Goal:** Interpreting rational numbers in the contexts of temperature and elevation.
- **Language Goal:** Ordering rational numbers, and justifying the comparisons. **(Speaking and Listening)**
  - » Ordering rational numbers from least to greatest in Problem 1.
- **Goal:** Plotting points on a vertical or horizontal number line to represent rational numbers.
- **Goal:** Comprehending that the term *opposite* refers to numbers with the same magnitude but different signs.
  - » Identifying pairs of opposites in Problem 4.

Suggested next steps

If students identify $-\frac{1}{10}$ as the least value, consider:

- Reviewing the class number line from Activity 2.
- Assigning Practice Problem 1.
- Asking, "Which number would be farther from 0, -3 or $-\frac{1}{10}$?"

If students do not order the numbers properly, consider:

- Having students say each number aloud as they consider where it should be placed on the number line.
- Having students mark other friendly values on their number line, such as 1 and -1.

Professional Learning

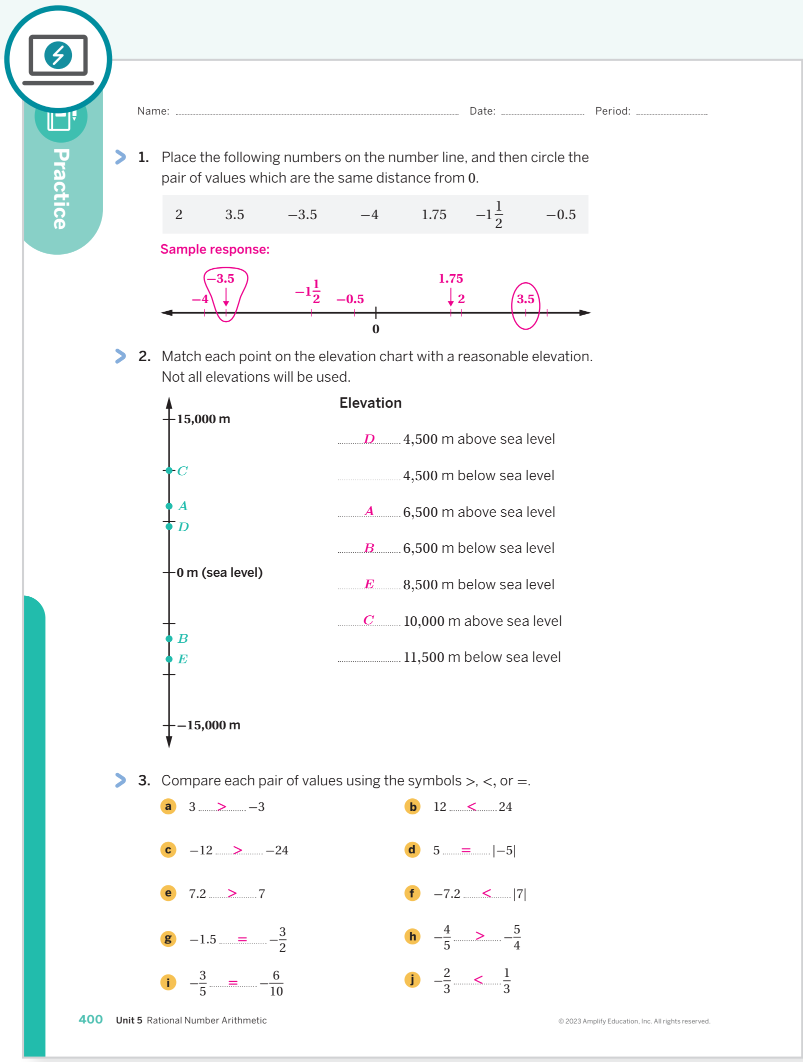
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did building the number line go as planned?
- Have you changed any ideas you used to have about building number sense around rational numbers as a result of today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Place the following numbers on the number line, and then circle the pair of values which are the same distance from 0.

2	3.5	-3.5	-4	1.75	$-1\frac{1}{2}$	-0.5

Sample response:

2. Match each point on the elevation chart with a reasonable elevation. Not all elevations will be used.

Elevation

D 4,500 m above sea level

4,500 m below sea level

A 6,500 m above sea level

B 6,500 m below sea level

E 8,500 m below sea level

C 10,000 m above sea level

11,500 m below sea level

3. Compare each pair of values using the symbols >, <, or =.

a. $3 > -3$

b. $12 < 24$

c. $-12 > -24$

d. $5 = |-5|$

e. $7.2 > 7$

f. $-7.2 < |7|$

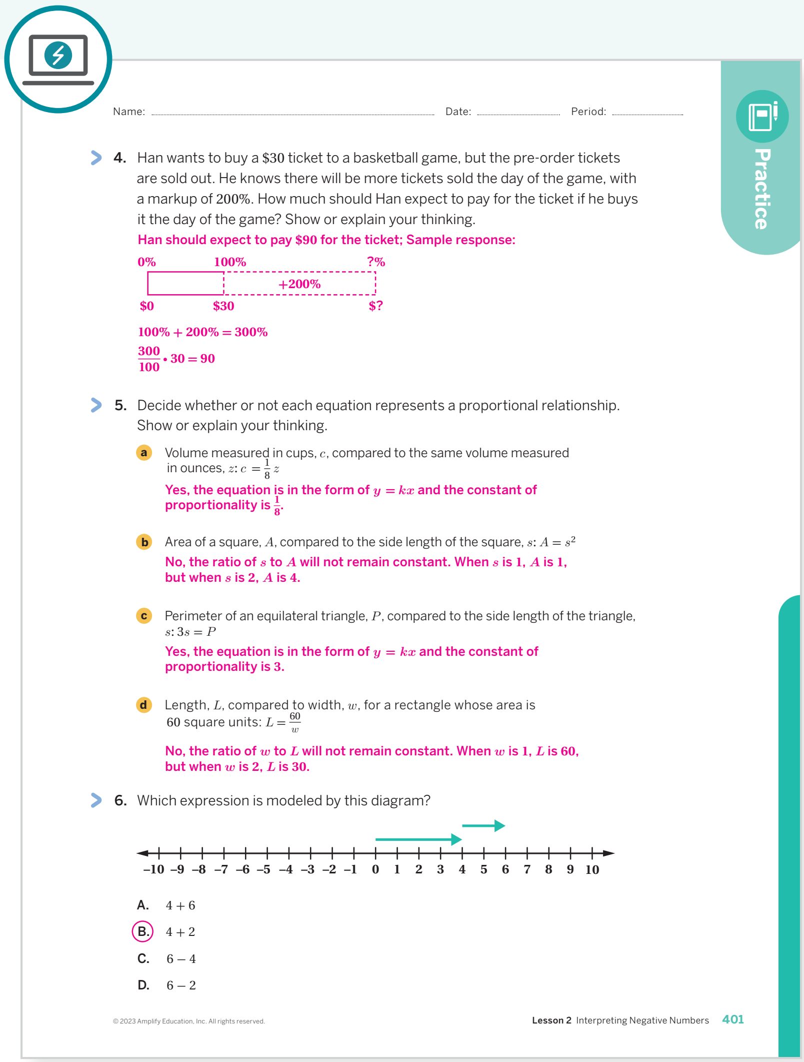
g. $-1.5 = -\frac{3}{2}$

h. $-\frac{4}{5} > -\frac{5}{4}$

i. $-\frac{3}{5} = -\frac{6}{10}$

j. $-\frac{2}{3} < \frac{1}{3}$

400 **Unit 5** Rational Number Arithmetic



Practice

Name: Date: Period:

4. Han wants to buy a $30 ticket to a basketball game, but the pre-order tickets are sold out. He knows there will be more tickets sold the day of the game, with a markup of 200%. How much should Han expect to pay for the ticket if he buys it the day of the game? Show or explain your thinking.

Han should expect to pay $90 for the ticket; Sample response:

$100\% + 200\% = 300\%$

$\frac{300}{100} \cdot 30 = 90$

5. Decide whether or not each equation represents a proportional relationship. Show or explain your thinking.

a. Volume measured in cups, c, compared to the same volume measured in ounces, z: $c = \frac{1}{8}z$

Yes, the equation is in the form of $y = kx$ and the constant of proportionality is $\frac{1}{8}$.

b. Area of a square, A, compared to the side length of the square, s: $A = s^2$

No, the ratio of s to A will not remain constant. When s is 1, A is 1, but when s is 2, A is 4.

c. Perimeter of an equilateral triangle, P, compared to the side length of the triangle, s: $3s = P$

Yes, the equation is in the form of $y = kx$ and the constant of proportionality is 3.

d. Length, L, compared to width, w, for a rectangle whose area is 60 square units: $L = \frac{60}{w}$

No, the ratio of w to L will not remain constant. When w is 1, L is 60, but when w is 2, L is 30.

6. Which expression is modeled by this diagram?

A. $4 + 6$
[✓ B.] $4 + 2$
C. $6 - 4$
D. $6 - 2$

Lesson 2 Interpreting Negative Numbers 401

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 3	2
	5	Unit 2 Lesson 9	2
Formative	6	Unit 5 Lesson 3	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 3**

Changing Temperatures

Let's add positive and negative numbers.

Focus

Goals

1. **Language Goal:** Determine the final temperature, given the starting temperature and the change in temperature, and explain the solution method. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain how to create a number line diagram that represents adding rational numbers. **(Speaking and Listening, Writing)**
3. Write an addition equation to represent a situation involving a temperature increase or decrease.

Coherence

Today

Students represent addition of rational numbers on a number line using arrows. There are different ways to do this; in this unit, the convention is that each addend is represented by an arrow and the sum is represented as a point on the number line. Positive addends are represented by arrows that point to the right, and negative addends are represented by arrows that point to the left. The starting point is always zero; the next arrow starts where the first arrow ends. The sum is represented by a point on the number line where the arrow for the last addend ends.

Previously

In Lesson 2, students became reacquainted with using vertical and horizontal numbers lines to position and compare rational numbers.

Coming Soon

In Lesson 4, students will further practice adding rational numbers in context, and they will also formulate some rules to better understand the sign of a sum.

Rigor

- Students build the **conceptual understanding** that rational numbers have both a direction and a magnitude.
- Students develop **procedural skills** by using number lines and equations simultaneously to represent addition of rational numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	12 min	10 min	5 min	10 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New word

- *arrow diagram*

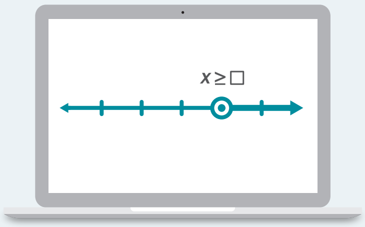
Review words

- *absolute value*
- *negative numbers*
- *positive numbers*
- *rational numbers*

Amps | Featured Activity

Activity 1
Interactive Arrow Diagrams

Students represent changing temperatures on number lines using arrow diagrams. The digital environment facilitates the ability to quickly manipulate the arrows.



Building Math Identity and Community

Connecting to Mathematical Practices

Because of their understanding of addition of whole numbers, students might impulsively draw their own conclusions about how to add rational numbers. Explain to students that they are using so many different representations of the addition in Activity 1 so that they can reason abstractly about the correct rules for adding signed integers from their work.

Modifications to Pacing

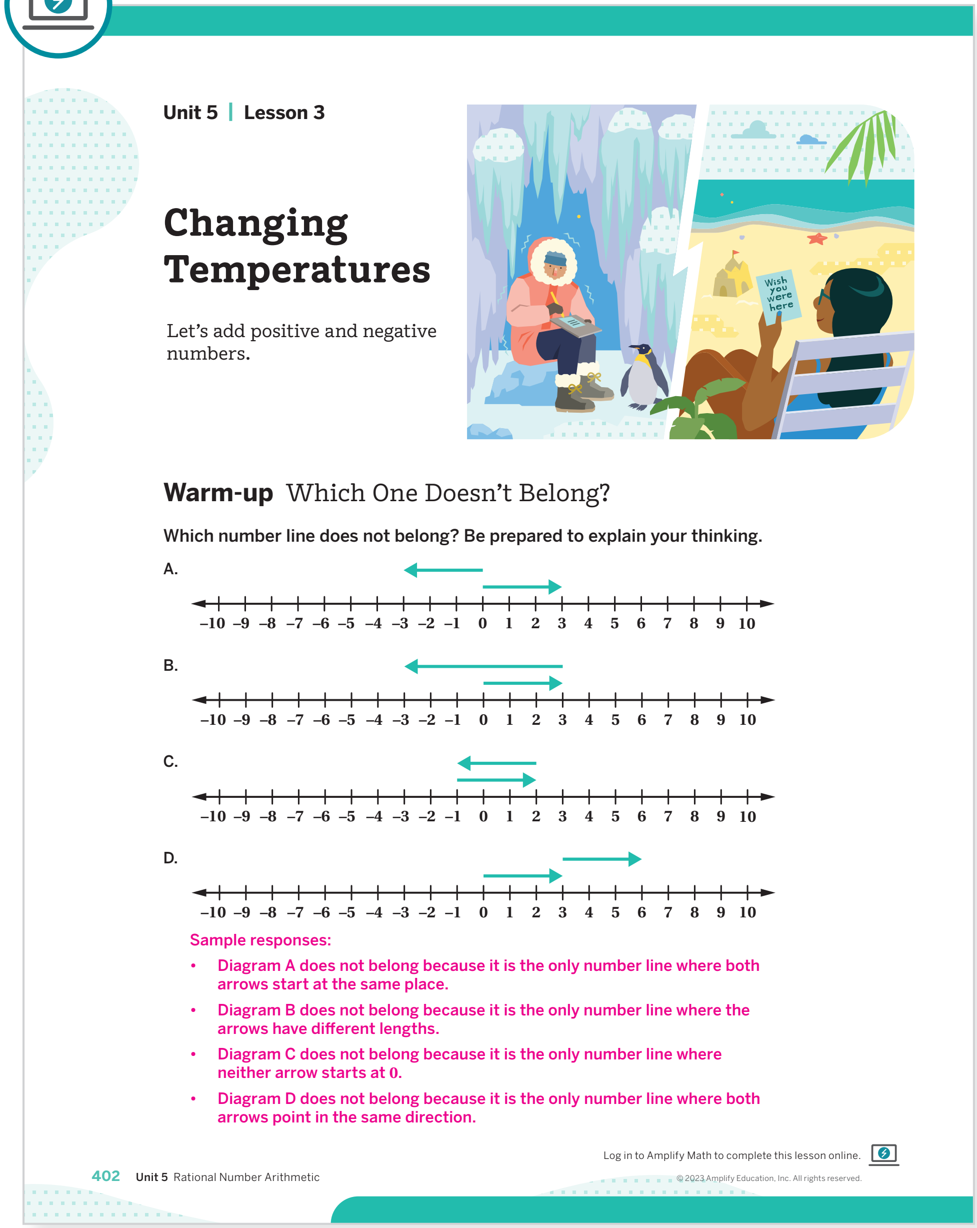
You may want to consider these additional modifications if you are short on time.

- In **Activity 1,** provide student choice by allowing them to choose any 4 or 5 problems to complete, then discussing all problems as a class.
- In **Activity 2,** have students only complete the first 2 or 3 problems.

Independent | 8 min

Warm-up Which One Doesn't Belong?

Students explore unfamiliar, but intuitive, number line diagrams with arrows to discover a new representation for adding rational numbers on the number line.



Unit 5 | Lesson 3

Changing Temperatures

Let's add positive and negative numbers.

Warm-up Which One Doesn't Belong?

Which number line does not belong? Be prepared to explain your thinking.

A.

B.

C.

D.

Sample responses:

- Diagram A does not belong because it is the only number line where both arrows start at the same place.
- Diagram B does not belong because it is the only number line where the arrows have different lengths.
- Diagram C does not belong because it is the only number line where neither arrow starts at 0.
- Diagram D does not belong because it is the only number line where both arrows point in the same direction.

Log in to Amplify Math to complete this lesson online.

402 Unit 5 Rational Number Arithmetic

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1 Launch

Conduct the *Which One Doesn't Belong?* routine. Encourage students to jump right in by examining all of the diagrams and noting differences between them. Have students who finish early try to create a rule for why each diagram does not belong.

2 Monitor

Help students get started by having them draw lines from the start and end of each arrow down to the number line.

Look for points of confusion:

- **Thinking that Diagram A shows 3 + (–3).** Ask, "Would a similar diagram work for 4 + 3?"

Look for productive strategies:

- Annotating the arrows with their length and direction.

3 Connect

Display Diagram B, alongside the expression 3 + (–6).

Ask:

- "How does this expression represent the diagram?"
- "What expressions could you write to represent Diagram D?"

Have students share their expressions for representing Diagram D.

Define an ***arrow diagram*** as a model used within a number line to show positive and negative numbers and operations on them.

Highlight that the second arrow is always placed slightly above the first arrow. This helps indicate which is the first, or starting, value.

Ask, "Why are Diagrams A and C impossible representation for adding two numbers?

Differentiated Support

Accessibility: *Guide Processing and Visualization*

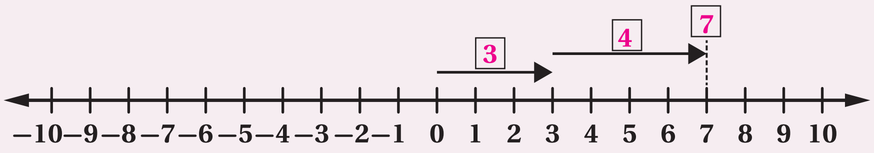
Suggest that students ask themselves these questions as they compare the number line diagrams.

- Where do the arrows start?
- Where do the arrows end?
- In which direction does each arrow point?
- What is the length of each arrow?

Power-up

To power up students' ability to interpret arrow diagrams, have students complete:

1. Add the numbers 3, 4, and 7 to the appropriate boxes in the given arrow diagram.



2. Write the equation that modeled by the arrow diagram using the numbers 3, 4, and 7.
   3 + 4 = 7

Use: Before the Warm-up.

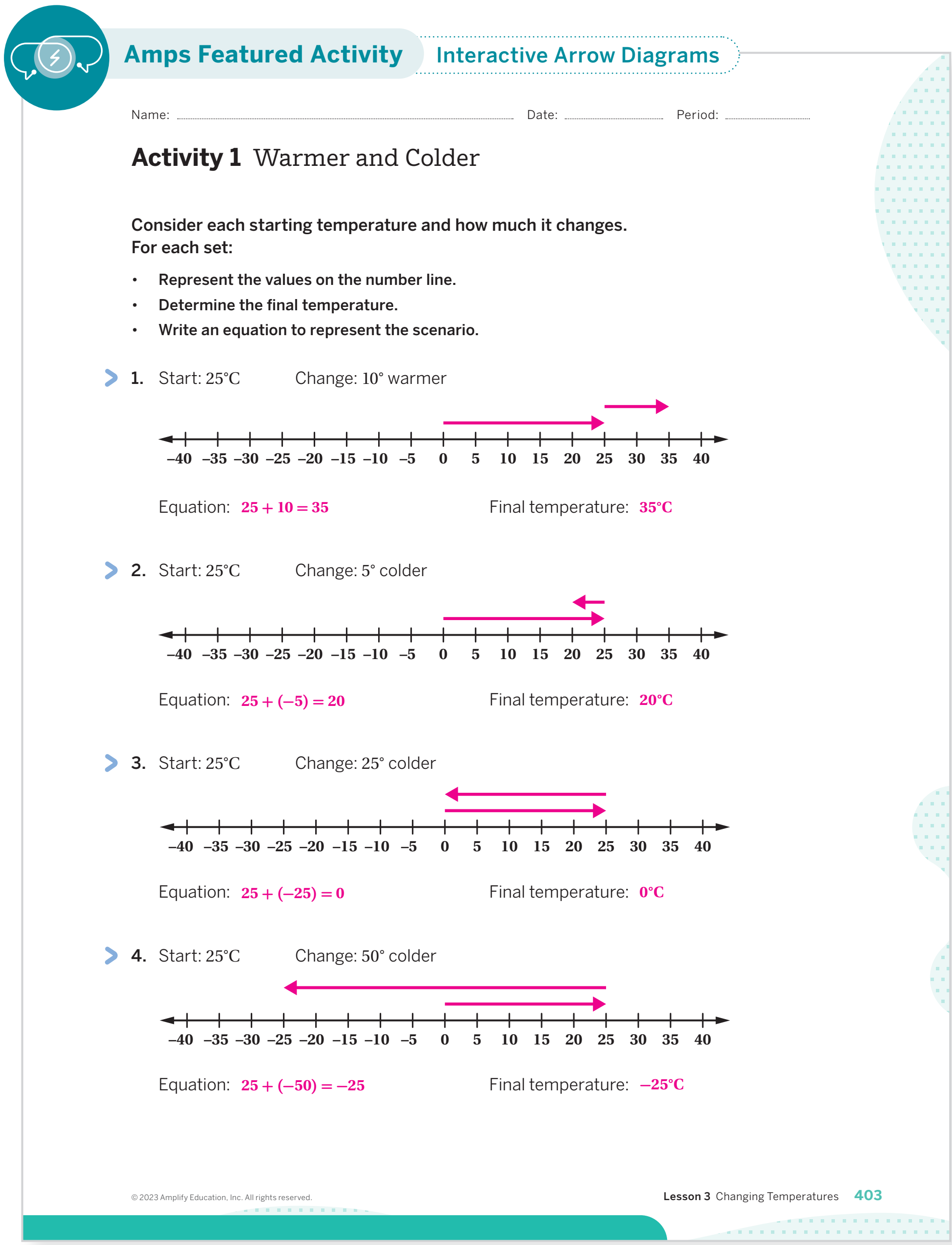
Informed by: Performance on Lesson 2, Practice Problem 6.

Pairs | 12 min

Activity 1 Warmer and Colder

The context of temperature is used to help students make sense of adding rational numbers, writing equations, and representing using them arrow diagrams.

Amps Featured Activity Interactive Arrow Diagrams



Name: ________ Date: ________ Period: ________

Activity 1 Warmer and Colder

Consider each starting temperature and how much it changes. For each set:

- Represent the values on the number line.
- Determine the final temperature.
- Write an equation to represent the scenario.

1. Start: 25°C Change: 10° warmer

Equation: $25 + 10 = 35$ Final temperature: 35°C

2. Start: 25°C Change: 5° colder

Equation: $25 + (-5) = 20$ Final temperature: 20°C

3. Start: 25°C Change: 25° colder

Equation: $25 + (-25) = 0$ Final temperature: 0°C

4. Start: 25°C Change: 50° colder

Equation: $25 + (-50) = -25$ Final temperature: −25°C

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Lesson 3 Changing Temperatures 403

1 Launch

Ask, "If the temperature is $40°$ and it becomes $10°$ colder, what temperature is it?" Demonstrate representing this situation on a number line using arrows and write the equation $40 + (-10) = 30$. Tell students that placing the -10 inside parentheses is a convention that helps to separate the $+$ operator from the negative sign. They do not need the parentheses when the negative number is first, or when a positive number follows the addition sign, such as in the expression $-3 + 4$.

2 Monitor

Help students get started by saying, "The first arrow should always start at 0."

Look for points of confusion:

- **Drawing only one arrow — the change from the value of the first number.** Say, "That is another way to represent the situation, but it will be more helpful for future situations to represent both values with arrows."
- **Drawing both arrows starting from 0.** Say, "Your second arrow should end at the new temperature after the change. Is that true for your diagram?"

Look for productive strategies:

- Drawing dots at the start and end of each arrow to help organize the representation.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can represent changing temperatures on number lines using interactive arrow diagrams. The digital environment facilitates the ability to quickly manipulate the arrows.

Accessibility: *Vary Demands to Optimize Challenge*

Students may visually benefit from drawing jumps on the number line instead of drawing straight arrows. Allow them to do so and connect the length of all of their jumps in one direction to the length of a straight arrow.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw connections between the visual representations of adding positive and negative numbers with numerical expressions. For example, for Problem 5, write the expression $-20 + 35$. Ask:

- "Which arrow represents the first addend in the expression?"
- "How is adding a positive number represented on the number line? In Problem 7, how is adding a negative number represented?"
- "Where is the sum represented on the number line?"

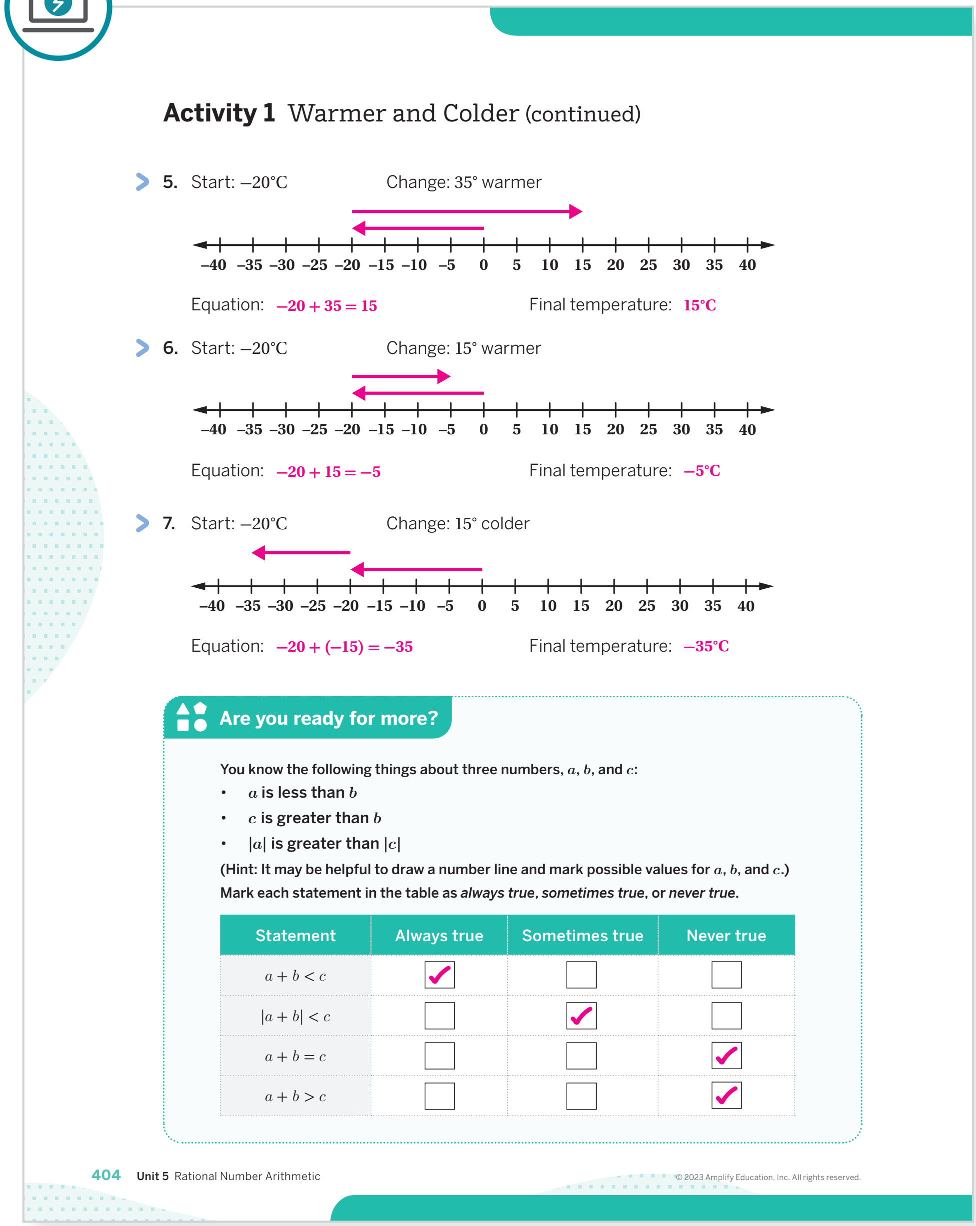
English Learners

Add annotations to the number lines, such as labeling "warmer" as "add a positive number" and "colder" as "add a negative number."

Pairs | 12 min

Activity 1 Warmer and Colder (continued)

The context of temperature is used to help students make sense of adding rational numbers, writing equations, and representing using them arrow diagrams.



Activity 1 Warmer and Colder (continued)

5. Start: –20°C Change: 35° warmer

Equation: $-20 + 35 = 15$ Final temperature: 15°C

6. Start: –20°C Change: 15° warmer

Equation: $-20 + 15 = -5$ Final temperature: –5°C

7. Start: –20°C Change: 15° colder

Equation: $-20 + (-15) = -35$ Final temperature: –35°C

Are you ready for more?

You know the following things about three numbers, a, b, and c:

- a is less than b
- c is greater than b
- $|a|$ is greater than $|c|$

(Hint: It may be helpful to draw a number line and mark possible values for a, b, and c.)

Mark each statement in the table as *always true*, *sometimes true*, or *never true*.

Statement	Always true	Sometimes true	Never true
$a + b < c$	☑	☐	☐
$\|a + b\| < c$	☐	☑	☐
$a + b = c$	☐	☐	☑
$a + b > c$	☐	☐	☑

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3 Connect

Have students share their thought process as they walk through their solution to one of the problems.

Display two of the problems so that both are visible at the same time.

Ask:

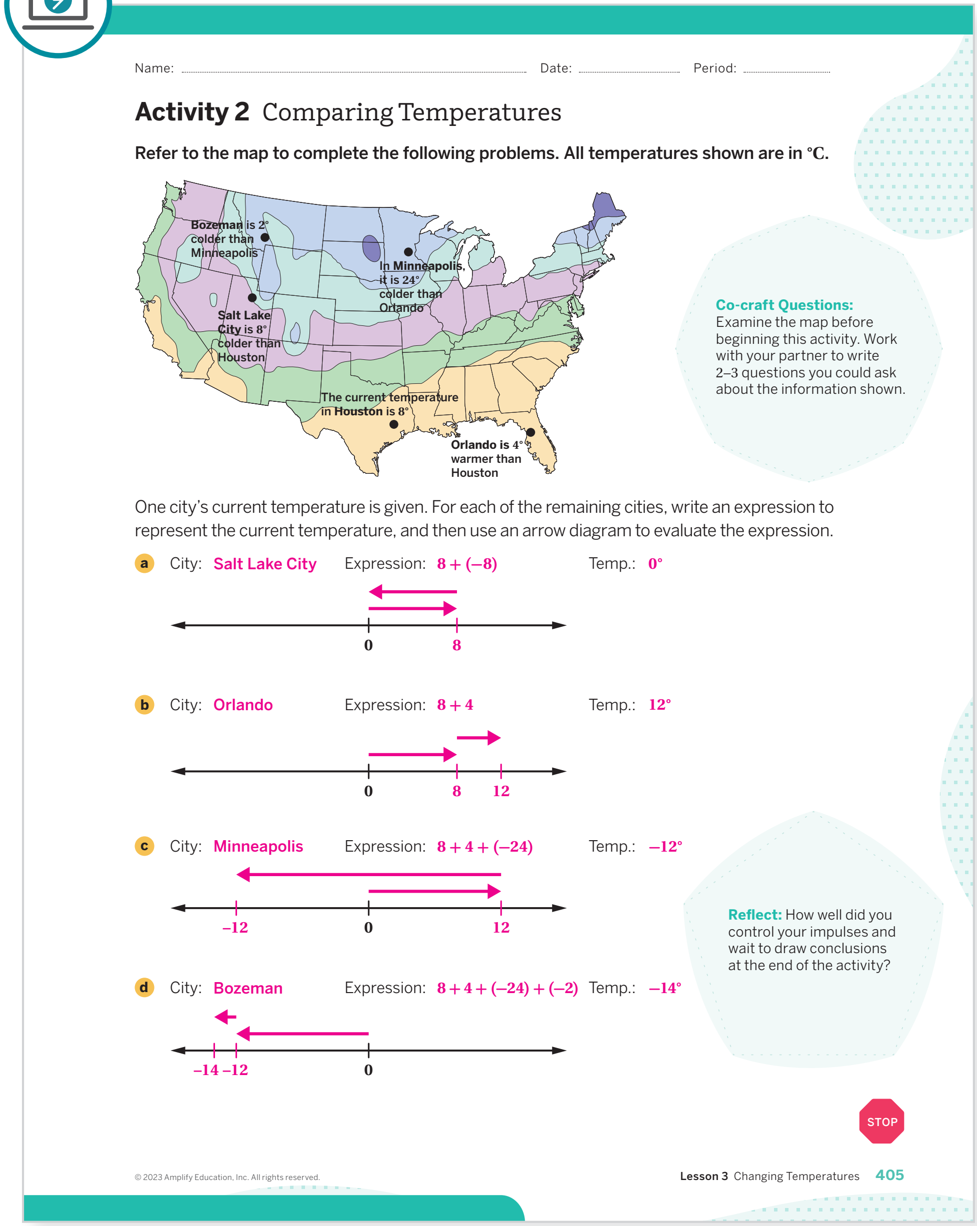
- "How is the addition of a positive number represented on a number line?"
  An arrow pointing to the right.
- "How is the addition of a negative number represented on a number line?"
  An arrow pointing to the left.
- "Where does the second arrow always start?"
  The second arrow always starts where the first arrow ends. It is drawn slightly above the first arrow.

Highlight that the end of the second arrow is the sum of both numbers.

Pairs | 10 min

Activity 2 Comparing Temperatures

Students use their understanding from the previous activity to determine temperature differences and connect them to addition equations.



Name: ______ Date: ______ Period: ______

Activity 2 Comparing Temperatures

Refer to the map to complete the following problems. All temperatures shown are in °C.

Co-craft Questions: Examine the map before beginning this activity. Work with your partner to write 2–3 questions you could ask about the information shown.

One city's current temperature is given. For each of the remaining cities, write an expression to represent the current temperature, and then use an arrow diagram to evaluate the expression.

a City: Salt Lake City Expression: $8 + (-8)$ Temp.: 0°

b City: Orlando Expression: $8 + 4$ Temp.: 12°

c City: Minneapolis Expression: $8 + 4 + (-24)$ Temp.: −12°

d City: Bozeman Expression: $8 + 4 + (-24) + (-2)$ Temp.: −14°

Reflect: How well did you control your impulses and wait to draw conclusions at the end of the activity?

STOP

Lesson 3 Changing Temperatures 405

1 Launch

Ask, "Which place on the map would you expect to be the warmest? The coldest?" Let students know that because the number lines are unmarked, it will be up to them to determine reasonable lengths for their arrows.

2 Monitor

Help students get started by helping them locate Houston, where the temperature is given.

Look for points of confusion:

- **Thinking that the temperature is the number in the text after the city name.** Have students read the text carefully, then ask, "What does that mean? Do you have enough information yet?"
- **Starting with Bozeman, because it's at the top and left of the map.** Say, "It says Bozeman is 2° colder than Minneapolis. Do you know the temperature of Minneapolis? Is there another city that might make more sense to reference as a starting point?"

Look for productive strategies:

- Planning an order of which remaining temperatures to find.
- Marking each number line with the 0 point.

3 Connect

Display two different strategies for Minneapolis — one that shows $8 + 4 + (-24)$ and one that shows $12 + (-24)$. Ask, "Which do you think is more accurate? Which better represents the statement?"

Highlight that both expressions accurately model the situation, because both use numbers that come from the other relevant cities. Either expression may be more helpful depending on how students prefer to add the values.

Ask, "How can using a number line — even a blank one — help make sense of what the final temperature should be?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students draw arrows from each city to the other city that helps them determine the temperature. For example, draw an arrow from Salt Lake City to Houston, and draw an arrow from Bozeman to Minneapolis.

Extension: *Math Enrichment*

Tell students that Detroit is 6° colder than St. Louis and St. Louis is 8° warmer than Denver. Ask them to write a statement that only compares the temperatures of Detroit and Denver. Denver is 2° colder than Detroit, or Detroit is 2° warmer than Denver.

Math Language Development

MLR5: Co-Craft Questions

During the Launch, display the map. Ask students to work with their partner to write 2–3 mathematical questions they could ask about the information shown. Ask pairs to share their questions with the whole class. Sample questions shown.

- How can I determine the temperature for each city?
- In which order should I determine the temperature for each city?
- How does the temperature of Bozeman compare to Salt Lake City?

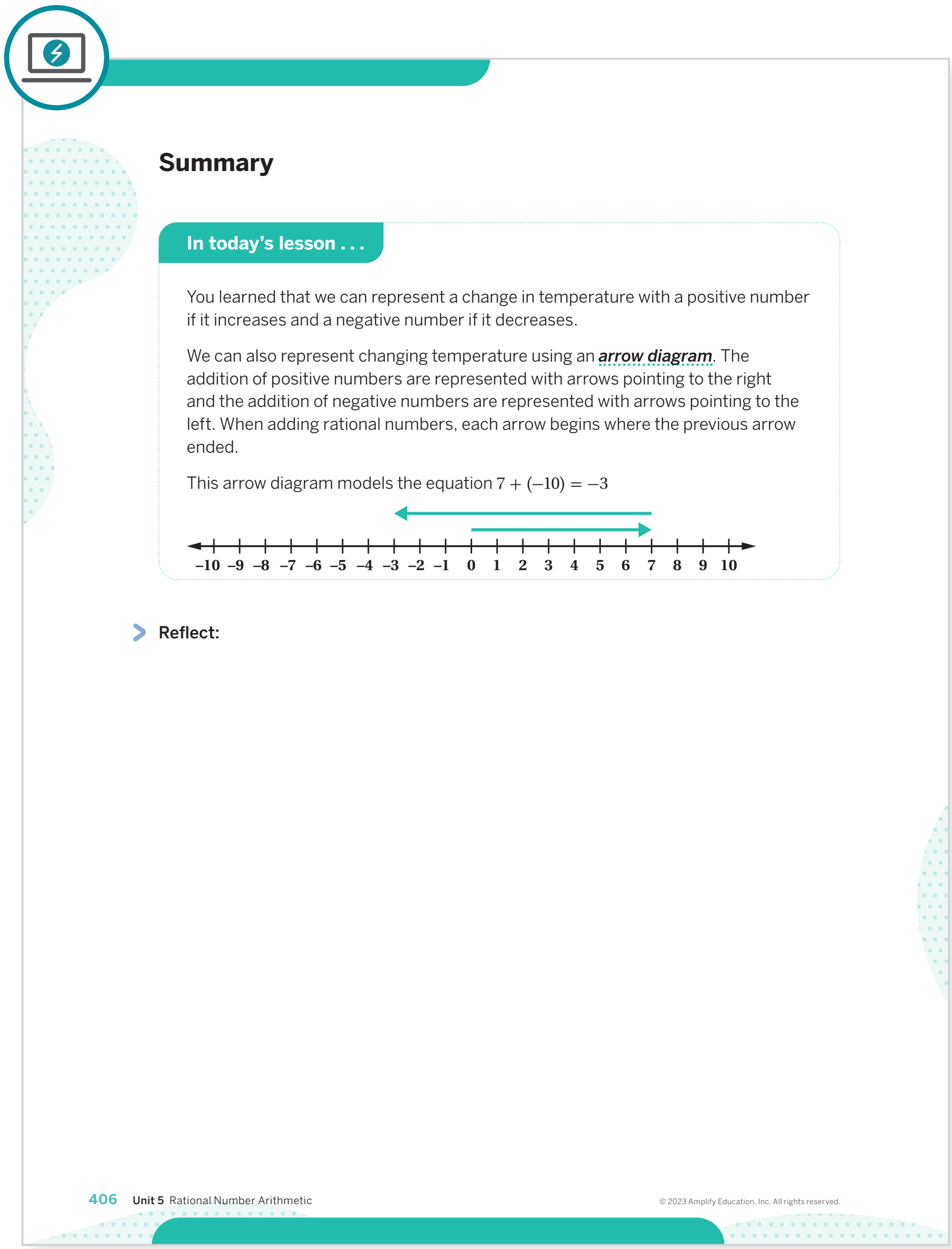
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Whole Class | 5 min

Summary

Review and synthesize that changing temperature can be represented using arrow diagrams and expressions with positive and negative numbers.



Summary

In today's lesson . . .

You learned that we can represent a change in temperature with a positive number if it increases and a negative number if it decreases.

We can also represent changing temperature using an ***arrow diagram***. The addition of positive numbers are represented with arrows pointing to the right and the addition of negative numbers are represented with arrows pointing to the left. When adding rational numbers, each arrow begins where the previous arrow ended.

This arrow diagram models the equation $7 + (-10) = -3$

Reflect:

406 **Unit 5** Rational Number Arithmetic

Synthesize

Display the arrow diagram from the Summary in the Student Edition.

Formalize vocabulary: *arrow diagram*

Ask:

- "What expression could be written to represent this arrow diagram?" $7 + (-10)$
- "How many different expressions could be written?" Only one. There are other expressions that are equivalent to this, but there is only one way to write the expression that is modeled by this arrow diagram.

Highlight that using arrow diagrams and equations in conjunction with one another strengthens and reinforces both visual and symbolic reasoning about adding rational numbers.

Ask:

- "How can you represent an increase or decrease in temperature using an addition equation?"
- "How can you represent an addition equation on a number line?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which representation — expressions or arrow diagrams — helps you to make sense of adding positive and negative numbers?"

Math Language Development

MLR2: Collect and Display

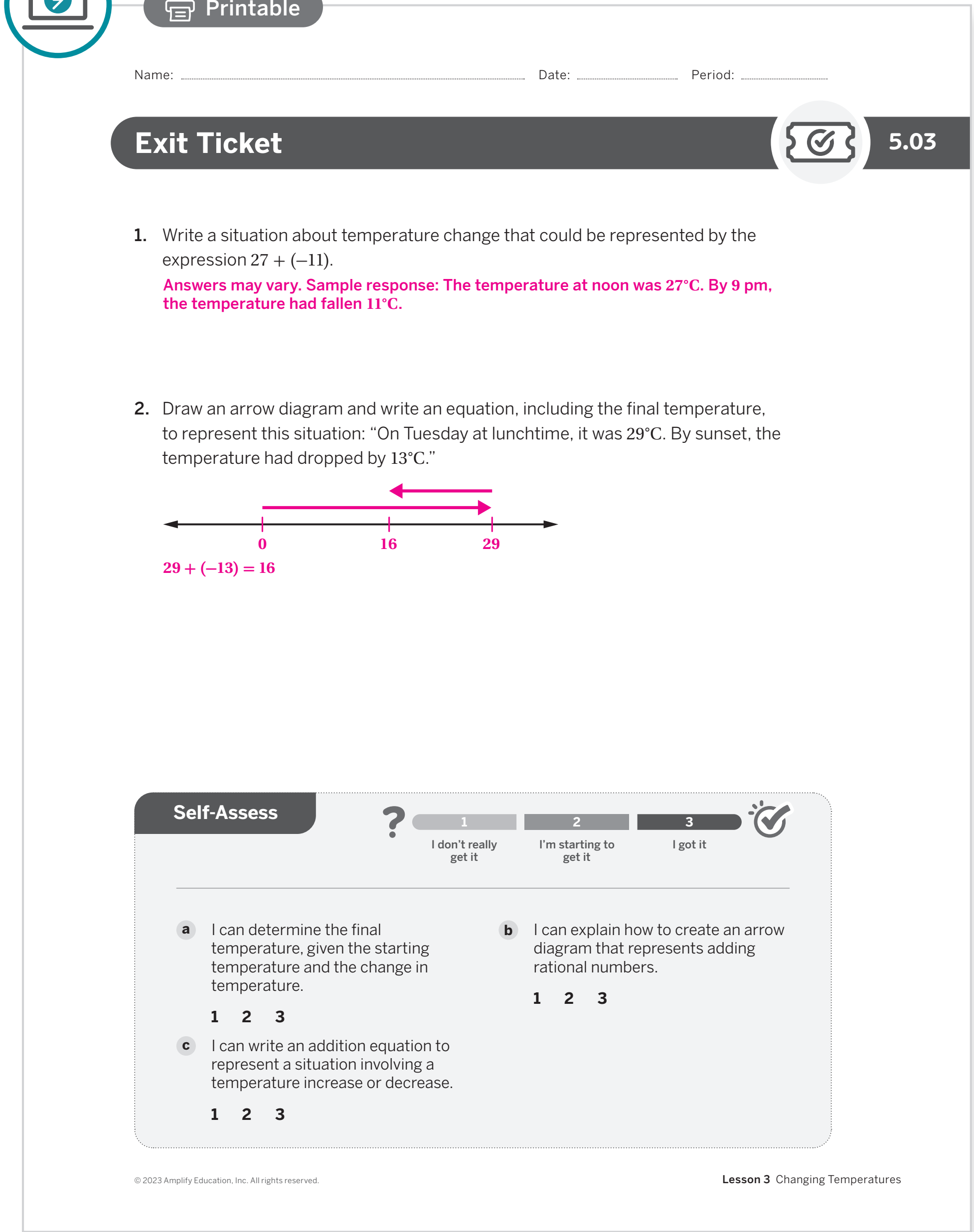
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on the terms and phrases related to the term *arrow diagram* that were added to the display during the lesson.

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of adding rational numbers by creating, representing, and solving a temperature change problem.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 5.03

1. Write a situation about temperature change that could be represented by the expression 27 + (−11).
   Answers may vary. Sample response: The temperature at noon was 27°C. By 9 pm, the temperature had fallen 11°C.

2. Draw an arrow diagram and write an equation, including the final temperature, to represent this situation: "On Tuesday at lunchtime, it was 29°C. By sunset, the temperature had dropped by 13°C."

$29 + (-13) = 16$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine the final temperature, given the starting temperature and the change in temperature.
1 2 3

b. I can explain how to create an arrow diagram that represents adding rational numbers.
1 2 3

c. I can write an addition equation to represent a situation involving a temperature increase or decrease.
1 2 3

Lesson 3 Changing Temperatures

Success looks like . . .

- **Language Goal:** Determining the final temperature, given the starting temperature and the change in temperature, and explaining the solution method. (**Speaking and Listening, Writing)**
  - » Drawing an arrow diagram to determine the final temperature in Problem 2.
- **Language Goal:** Explaining how to create a number line diagram that represents adding rational numbers. **(Speaking and Listening, Writing)**
- **Goal:** Writing an addition equation to represent a situation involving a temperature increase or decrease.
  - » Writing an equation to determine a temperature decrease in Problem 2.

Suggested next steps

If students struggle to write a coherent story for Problem 1, consider:

- Reviewing the contexts from Activity 2.
- Assigning Practice Problem 2 or 3.
- Asking, "How can you use times of day to help show how something has changed?"

If students show a change to 13°C, consider:

- Having them reread the last sentence aloud.

Professional Learning

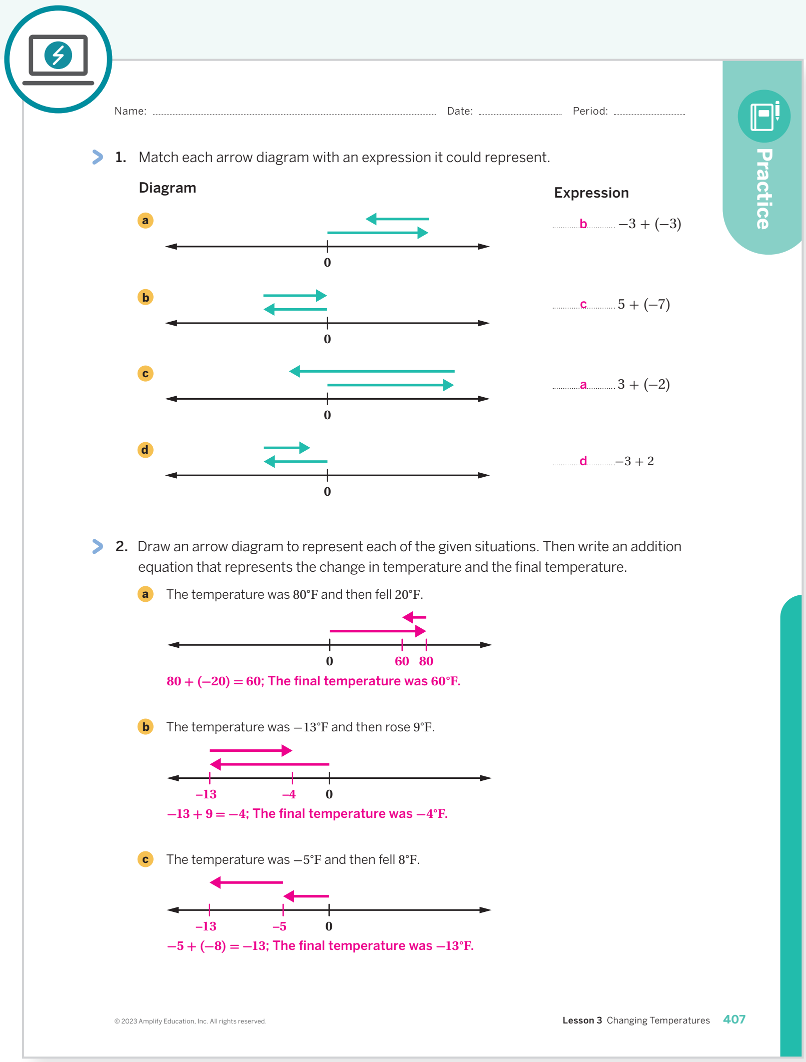
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. Match each arrow diagram with an expression it could represent.

Diagram

Expression

b $-3 + (-3)$

c $5 + (-7)$

a $3 + (-2)$

d $-3 + 2$

2. Draw an arrow diagram to represent each of the given situations. Then write an addition equation that represents the change in temperature and the final temperature.

a The temperature was 80°F and then fell 20°F.

$80 + (-20) = 60$; The final temperature was 60°F.

b The temperature was -13°F and then rose 9°F.

$-13 + 9 = -4$; The final temperature was -4°F.

c The temperature was -5°F and then fell 8°F.

$-5 + (-8) = -13$; The final temperature was -13°F.

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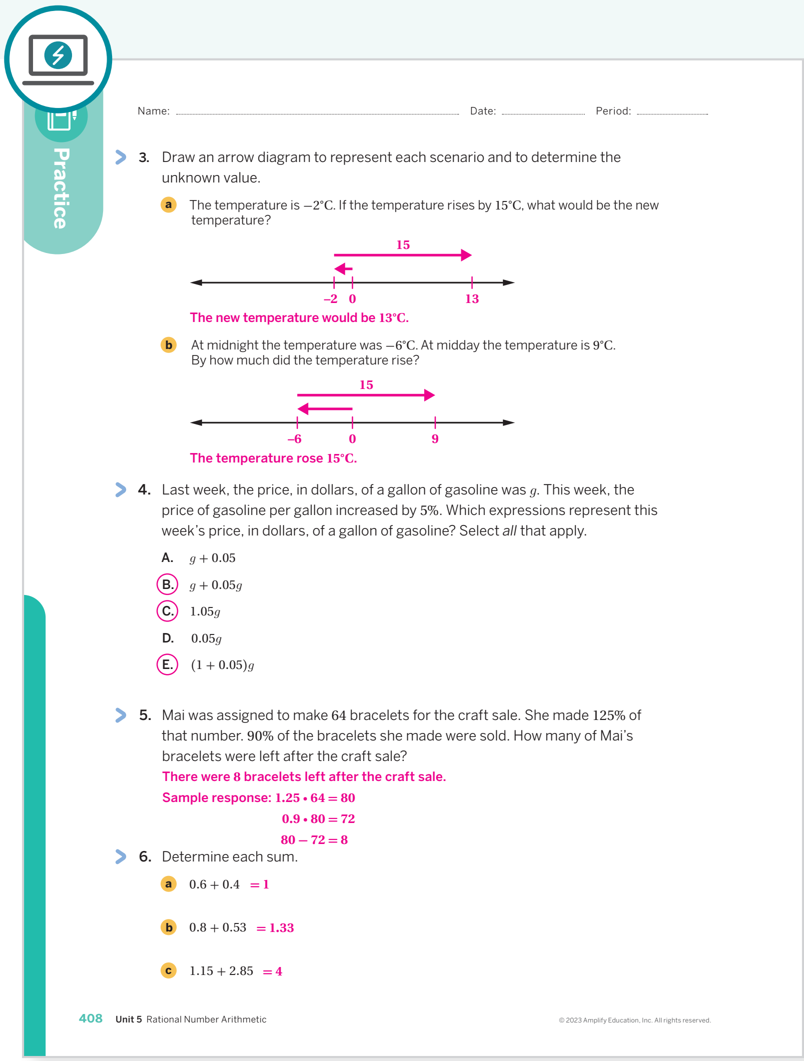
Lesson 3 Changing Temperatures 407



Name: .... Date: .... Period: ....

3. Draw an arrow diagram to represent each scenario and to determine the unknown value.

a The temperature is -2°C. If the temperature rises by 15°C, what would be the new temperature?

The new temperature would be 13°C.

b At midnight the temperature was -6°C. At midday the temperature is 9°C. By how much did the temperature rise?

The temperature rose 15°C.

4. Last week, the price, in dollars, of a gallon of gasoline was g. This week, the price of gasoline per gallon increased by 5%. Which expressions represent this week's price, in dollars, of a gallon of gasoline? Select *all* that apply.

A. $g + 0.05$

[✓ B.] $g + 0.05g$

[✓ C.] $1.05g$

D. $0.05g$

[✓ E.] $(1 + 0.05)g$

5. Mai was assigned to make 64 bracelets for the craft sale. She made 125% of that number. 90% of the bracelets she made were sold. How many of Mai's bracelets were left after the craft sale?

There were 8 bracelets left after the craft sale.

Sample response: $1.25 \cdot 64 = 80$
$0.9 \cdot 80 = 72$
$80 - 72 = 8$

6. Determine each sum.

a $0.6 + 0.4 \quad = 1$

b $0.8 + 0.53 \quad = 1.33$

c $1.15 + 2.85 \quad = 4$

408 **Unit 5** Rational Number Arithmetic

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 6	2
	5	Unit 4 Lesson 7	2
Formative ⚡	6	Unit 5 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 4**

Adding Rational Numbers

Let's solve problems about adding rational numbers.

Focus

Goals

1. Create and interpret equations and diagrams that represent adding rational numbers in the context of elevation.
2. **Language Goal:** Generalize a method for determining the sum of two rational numbers. **(Speaking and Listening)**

Coherence

Today

In this lesson, students build fluency adding rational numbers. Using the structure of opposites on the number line, they see that when adding two numbers with different signs, the sign of the sum will match the sign of the addend with the greater magnitude. Students then practice adding rational numbers with decimals.

Previously

In Lesson 3, students represented addition of positive and negative numbers using arrow diagrams.

Coming Soon

In Lesson 5, students will encounter adding positive and negative numbers within the context of money.

Rigor

- Students build **conceptual understanding** of the structure of addition problems that give a positive or negative result.
- Students **apply** their rules for adding positive and negative numbers in context with climbing up and down.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (Optional)	Summary	Exit Ticket
5 min	15 min	13 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos ⋮ Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Operations, Part 1* (for display)
- Anchor Chart PDF, *Operations, Part 1* (answers)
- a collection of small, short objects (e.g. erasers, paper clips, pen caps)

Math Language Development

New word

- ***additive inverse***

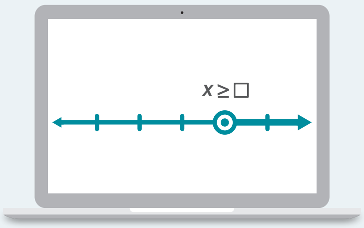
Review words

- *absolute value*
- *arrow diagram*
- *opposite*
- *rational numbers*

Amps ⋮ Featured Activity

Activity 3
School Supply Number Line

Students select objects of their choice and arrange them on a digital number line to compare the values of rational-number variable expressions.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students complete the ***Think-Pair-Share*** routine in Activity 1, they might not persevere in problem solving with their partner. As students are probably well aware of their strengths, during an exercise such as this, they should also lean on their partner to help them stretch beyond what they may think is a personal limitation. Working together will help accomplish solving the problems efficiently.

Modifications to Pacing

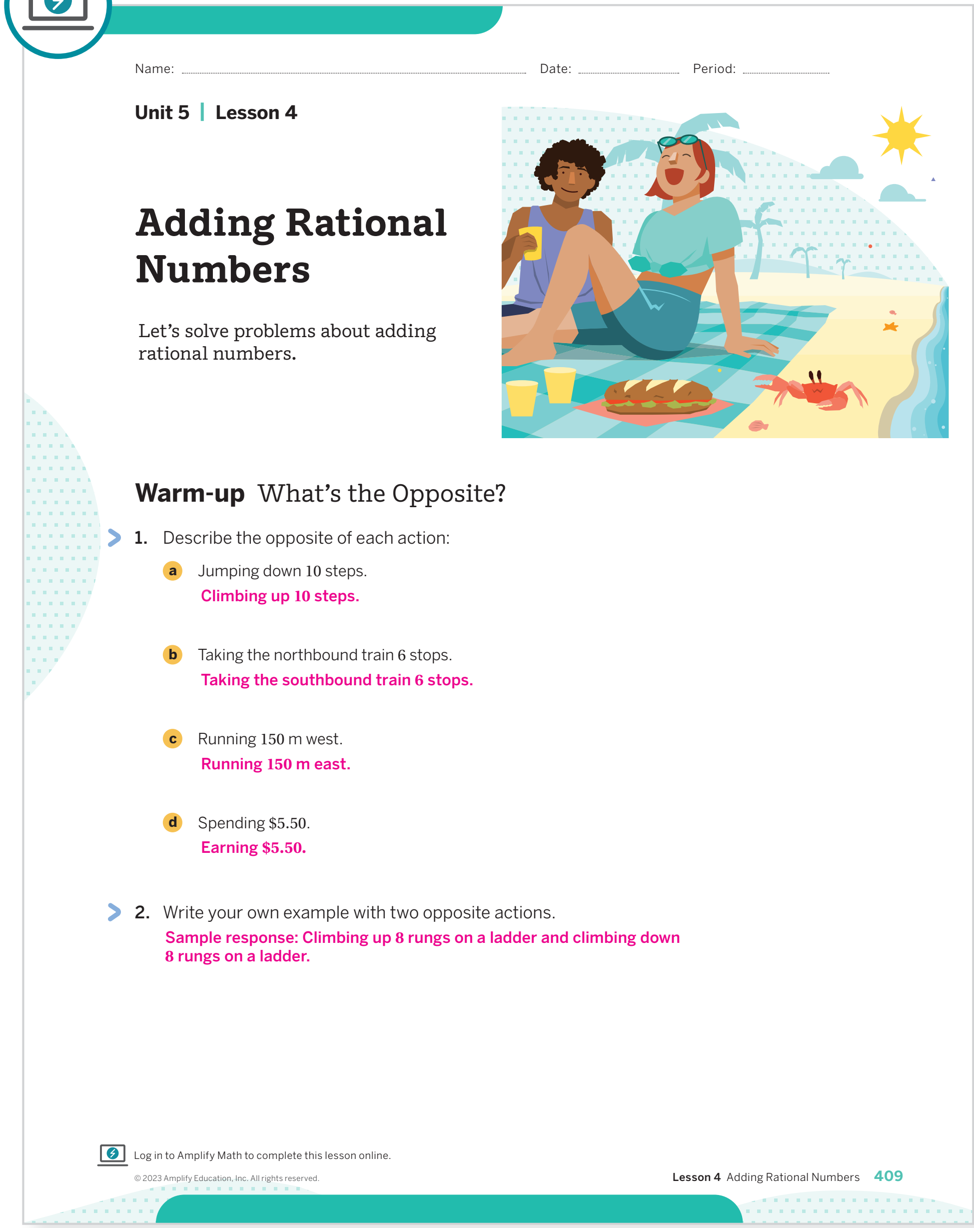
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- During **Activity 2**, instead of having students write responses for Problem 2, ask this question during the discussion.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up What's the Opposite?

Students explain, based on their background knowledge, what the opposite of a certain action is to better understand how opposites can appear in context.



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Unit 5 | Lesson 4

Adding Rational Numbers

Let's solve problems about adding rational numbers.

Warm-up What's the Opposite?

1. Describe the opposite of each action:
   - a) Jumping down 10 steps.
     Climbing up 10 steps.
   - b) Taking the northbound train 6 stops.
     Taking the southbound train 6 stops.
   - c) Running 150 m west.
     Running 150 m east.
   - d) Spending \$5.50.
     Earning \$5.50.
2. Write your own example with two opposite actions.
   Sample response: Climbing up 8 rungs on a ladder and climbing down 8 rungs on a ladder.

Log in to Amplify Math to complete this lesson online.

Lesson 4 Adding Rational Numbers 409

1 Launch

Activate background knowledge by asking students what it means to "retrace your steps." Prompt students to describe opposite directions. You might ask students to give you directions to a specific location in the classroom (your desk, for example) and then give directions for how to get back to your original spot.

2 Monitor

Help students get started by having students sketch what is happening in the action described in each problem.

Look for points of confusion:

- **Only giving a direction or a number, but not both.** Ask, "Would someone know you are doing *exactly* the opposite of the action described?"
- **In part c, saying walking is the opposite of running.** Ask, "Will changing the speed affect the position at the end of the opposite action?"

Look for productive strategies:

- Identifying the important directional word in the action first.

3 Connect

Have students share their own examples of opposite actions. After a few examples, conduct the ***Think-Pair-Share*** routine while asking, "Are these clear examples of opposite actions? How do you know"

Ask, "Why is traveling 100 steps north, and then 100 steps west *not* an example of opposite actions?"

Highlight that combining two opposite actions will always bring the action back to where it started.

Define the ***additive inverse*** of a number a as the number such that, when added to a, results in a sum of zero. It is the number's opposite.

Math Language Development

MLR2: Collect and Display

During the Connect, collect informal student language used to make sense of the additive inverse, such as "It is the opposite of a number," "It returns to the start," and "It makes zero." Add the language students use to the class display and remind them to continue to refer to and use the display during class discussions.

English Learners

Display or draw a compass rose annotated with *north*, *south*, *east*, and *west* to help students reason about opposite directions in parts b and c.

Power-up

To power up students' ability to reason about how the placement of the decimal point affects the sum of decimal values, have students complete:

Determine whether each sum is *greater than* or *less than* 1.

1. 80 cents plus 53 cents. Greater than 1.
2. $0.80 + 0.53$ Greater than 1.
3. $0.08 + 0.53$ Less than 1.

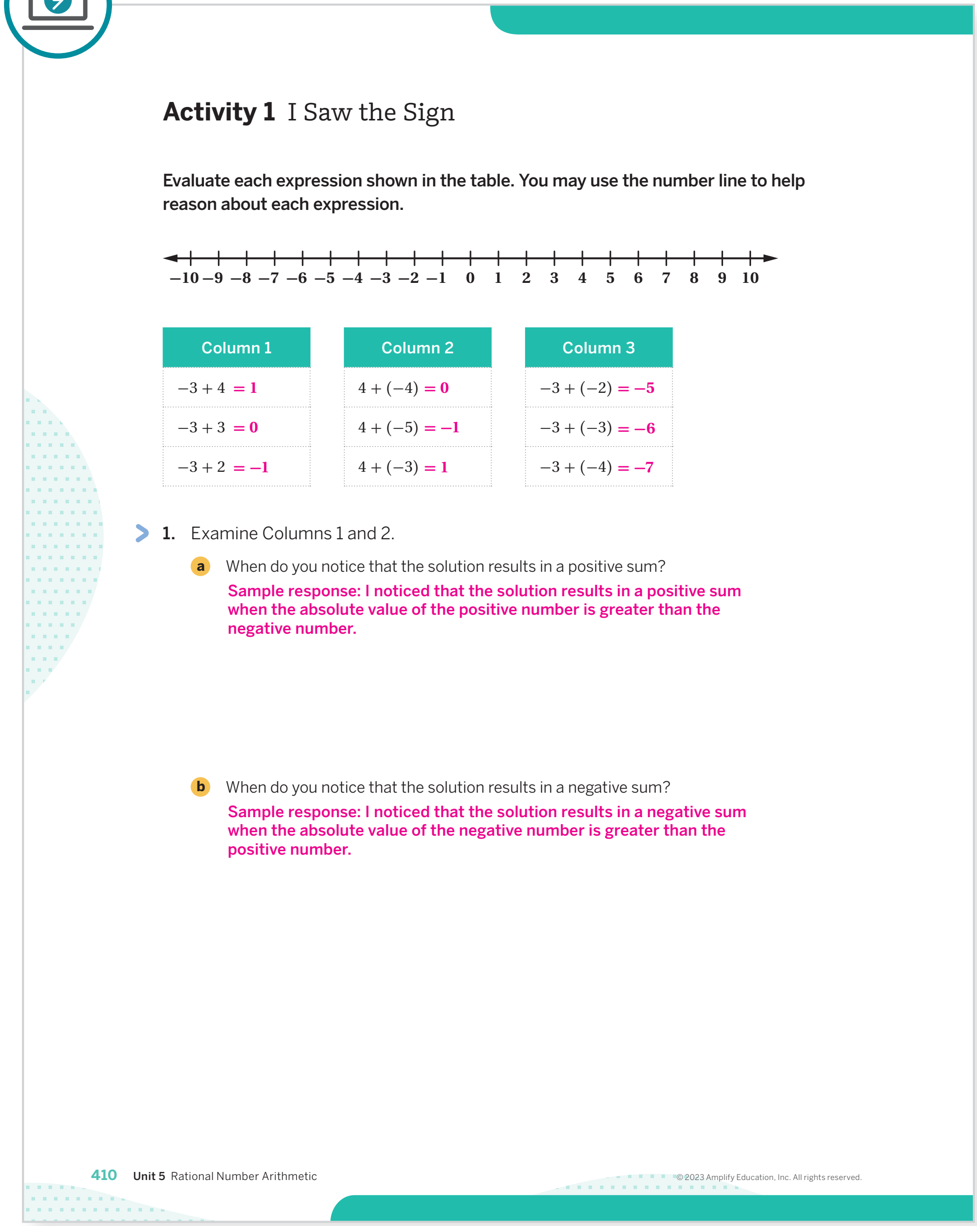
Use: Before Activity 2.

Informed by: Performance on Lesson 3, Practice Problem 6.

Pairs | 15 min

Activity 1 I Saw the Sign

Students examine patterns in repeated calculations to develop a rule for the sign of a sum of an expression including positive and negative numbers.



Activity 1 I Saw the Sign

Evaluate each expression shown in the table. You may use the number line to help reason about each expression.

Column 1	Column 2	Column 3
$-3 + 4$ **$= 1$**	$4 + (-4)$ **$= 0$**	$-3 + (-2)$ **$= -5$**
$-3 + 3$ **$= 0$**	$4 + (-5)$ **$= -1$**	$-3 + (-3)$ **$= -6$**
$-3 + 2$ **$= -1$**	$4 + (-3)$ **$= 1$**	$-3 + (-4)$ **$= -7$**

1. Examine Columns 1 and 2.

a. When do you notice that the solution results in a positive sum?

Sample response: I noticed that the solution results in a positive sum when the absolute value of the positive number is greater than the negative number.

b. When do you notice that the solution results in a negative sum?

Sample response: I noticed that the solution results in a negative sum when the absolute value of the negative number is greater than the positive number.

410 **Unit 5** Rational Number Arithmetic

1 Launch

Ask, "What similarities or differences do you notice between Columns 1, 2, and 3?" Conduct the ***Think-Pair-Share*** routine. Point out that for Problem 1, students are only considering Columns 1 and 2. Remind students that the *absolute value* means "the distance from zero."

2 Monitor

Help students get started by suggesting they draw an arrow diagram for the first expression in Column 1.

Look for points of confusion:

- Not **thinking that the results for positive and negative sums can be generalized to larger numbers.** Ask students to write an additional expression that fits the pattern for each column.

Look for productive strategies:

- Noticing that the numbers in Columns 1 and 2 have different signs, but Column 3 has numbers with only negative signs.
- Using the term *absolute value* when comparing the size of numbers with different signs.

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

During the Launch, consider providing the following hints as students note the similarities and differences between the columns.

- "What is true about signs of the addends in Column 1? Column 2? Column 3?"
- "How are the addends in Columns 1 and 2 similar? How are they different?"

While students complete Problems 1 and 2, suggest they record the absolute value of each number to help them notice any patterns.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "Do your rules include examples of when the addends are positive or negative?"
- "What math language can you use in your response?"
- "How do you know your rules are always true? Can you include examples to support them?"

Have students revise their responses, as needed.

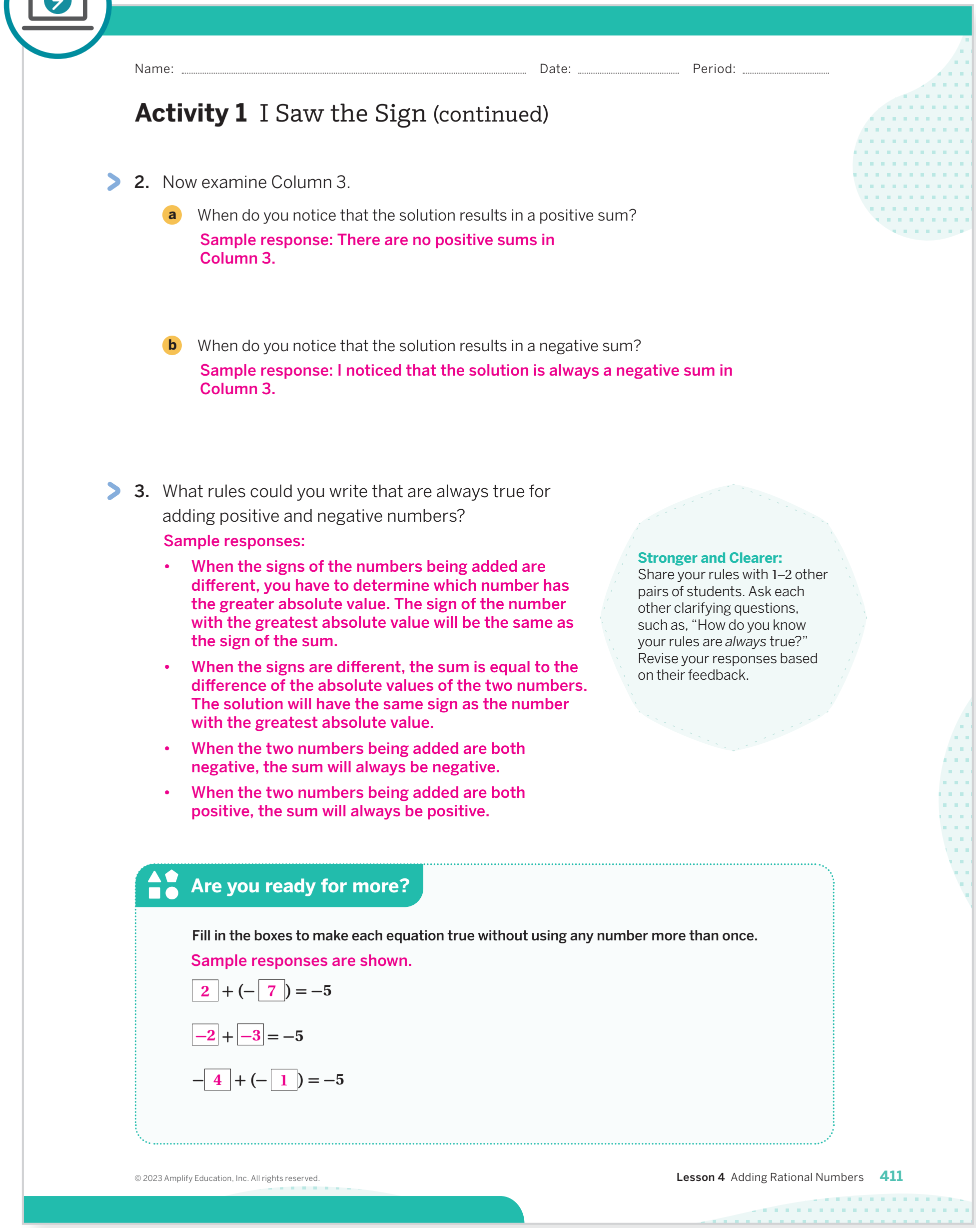
English Learners

Consider chunking the feedback and revision portion of this routine. For example, after students meet with their first pair of students, have them refine their rules based on this initial feedback before meeting with the second pair of students.

Pairs | 15 min

Activity 1 I Saw the Sign (continued)

Students examine patterns in repeated calculations to develop a rule for the sign of a sum of an expression including positive and negative numbers.



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Activity 1 I Saw the Sign (continued)

2. Now examine Column 3.

   a. When do you notice that the solution results in a positive sum?

   Sample response: There are no positive sums in Column 3.

   b. When do you notice that the solution results in a negative sum?

   Sample response: I noticed that the solution is always a negative sum in Column 3.

3. What rules could you write that are always true for adding positive and negative numbers?

   Sample responses:

   - When the signs of the numbers being added are different, you have to determine which number has the greater absolute value. The sign of the number with the greatest absolute value will be the same as the sign of the sum.
   - When the signs are different, the sum is equal to the difference of the absolute values of the two numbers. The solution will have the same sign as the number with the greatest absolute value.
   - When the two numbers being added are both negative, the sum will always be negative.
   - When the two numbers being added are both positive, the sum will always be positive.

Stronger and Clearer: Share your rules with 1–2 other pairs of students. Ask each other clarifying questions, such as, "How do you know your rules are *always* true?" Revise your responses based on their feedback.

Are you ready for more?

Fill in the boxes to make each equation true without using any number more than once.

Sample responses are shown.

$\boxed{2} + (-\boxed{7}) = -5$

$\boxed{-2} + \boxed{-3} = -5$

$-\boxed{4} + (-\boxed{1}) = -5$

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Lesson 4 Adding Rational Numbers 411

3 Connect

Display the expressions from Columns 1, 2, and 3.

Have students share their observations from Problems 1 and 2. After each share, ask whether anyone in the class disagrees.

Ask:

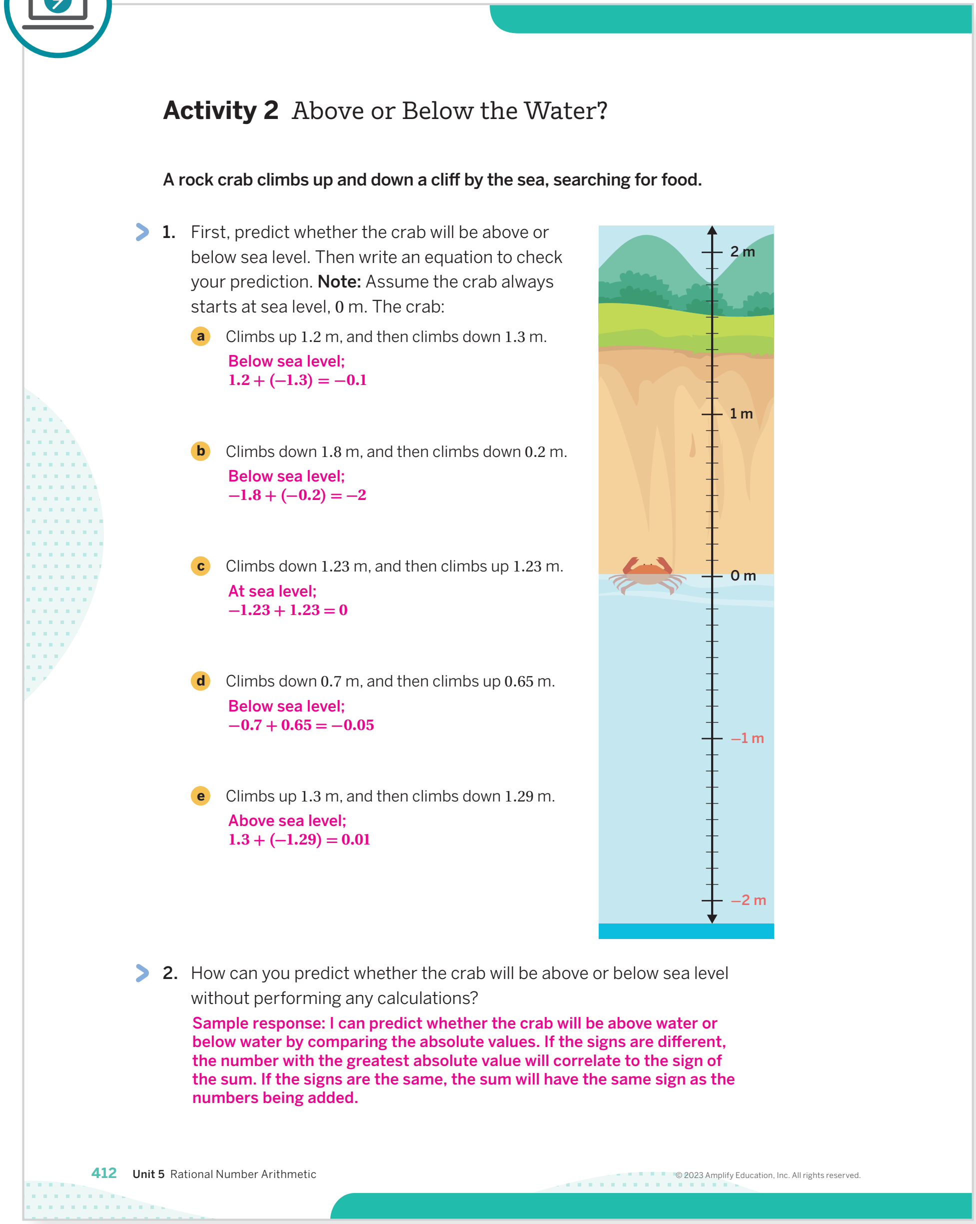
- "How can you tell, just from looking at an expression, whether the sum will be positive or negative?"
- "If you know that $|a|$ is greater than $|b|$, can you predict the sign of the sum of $a + b$? What else do you need to know?"
- "What rules did you write for Problem 3? Would your rules work when adding more than two numbers?"

Highlight that it is possible to predict the sign of the sum of two numbers based on only a couple of assumptions. When the signs of the addends are the same, determine the sum of the absolute values, and then give the sum the same sign as the addends. If the signs of the addends are different, determine the difference of their absolute values, and then give the result the same sign as the addend with the greater absolute value.

Pairs | 13 min

Activity 2 Above or Below the Water?

Students predict the final position of a crab by comparing distances with a given direction. This helps them reason about the sum of rational numbers in an elevation context.



Activity 2 Above or Below the Water?

A rock crab climbs up and down a cliff by the sea, searching for food.

1. First, predict whether the crab will be above or below sea level. Then write an equation to check your prediction. **Note:** Assume the crab always starts at sea level, 0 m. The crab:
   - **a** Climbs up 1.2 m, and then climbs down 1.3 m.
     Below sea level;
     $1.2 + (-1.3) = -0.1$
   - **b** Climbs down 1.8 m, and then climbs down 0.2 m.
     Below sea level;
     $-1.8 + (-0.2) = -2$
   - **c** Climbs down 1.23 m, and then climbs up 1.23 m.
     At sea level;
     $-1.23 + 1.23 = 0$
   - **d** Climbs down 0.7 m, and then climbs up 0.65 m.
     Below sea level;
     $-0.7 + 0.65 = -0.05$
   - **e** Climbs up 1.3 m, and then climbs down 1.29 m.
     Above sea level;
     $1.3 + (-1.29) = 0.01$

2. How can you predict whether the crab will be above or below sea level without performing any calculations?
   Sample response: I can predict whether the crab will be above water or below water by comparing the absolute values. If the signs are different, the number with the greatest absolute value will correlate to the sign of the sum. If the signs are the same, the sum will have the same sign as the numbers being added.

1 Launch

Have students examine the number line. Ask, “How is the number line partitioned? About how high is the top of the cliff from the water line?”

2 Monitor

Help students get started by having them annotate each part of Problem 1, noting the specific direction of each value.

Look for points of confusion:

- **Not knowing how to write the equation.** For now, allow students to reason about the final position of the crab however they feel comfortable. They will have opportunities to practice writing equations in later lessons in the unit.

Look for productive strategies:

- First identifying which absolute value is greater, and then comparing the directions to decide whether the crab will be above or below the water line.

3 Connect

Display the number line.

Ask, “Suppose the crab climbs 0.9 m up the wall. How far does it need to climb down to be *under* the water?”

Have students share their responses to Problem 2.

Ask:

- “Does the order of the signs matter?”
- “Are there differences for determining the sign of the sum when the numbers are decimals, compared to when they are only integers?”

Highlight that comparing the magnitude of rational numbers is helpful when adding them. Knowing which number is greater gives a good reasonable check for whether the result makes sense.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Because this activity involves decimal values, suggest that students label the tick marks on their number line before beginning the activity. Consider having students make all of their predictions for each part of Problem 1 first, before going back to write the equations.

Extension: *Math Enrichment*

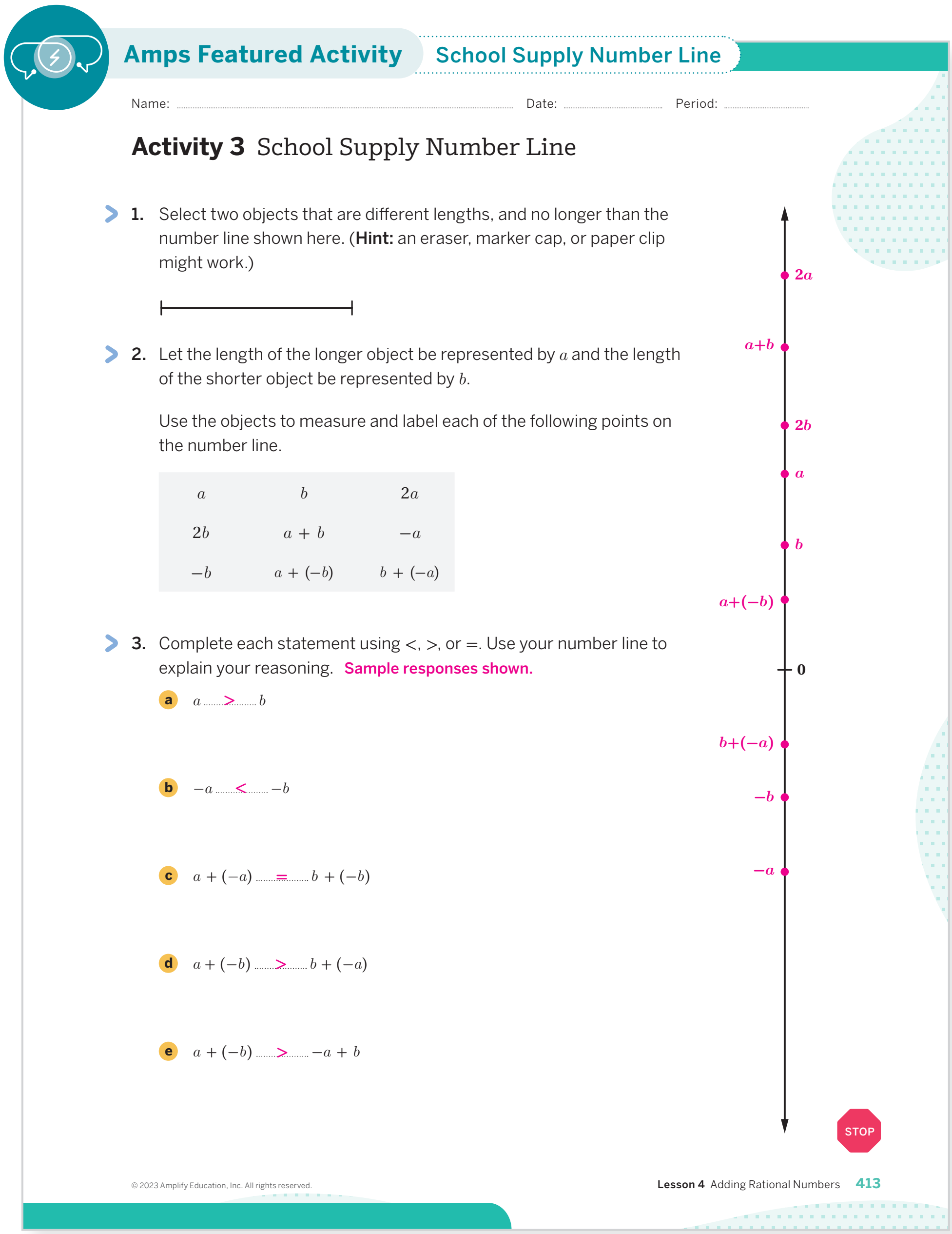
As a follow-up to Problem 2, have students determine whether the final location of the crab will be above or below sea level, if it starts at sea level, climbs up 1.4 m, climbs down 1.6 m, and then climbs up 1 m. Ask them to explain their thinking, without using the number line. Above sea level; Sample response: The absolute value of the difference of 1.4 and 1.6 is 0.2. Because the absolute value of 1.6 is greater than the absolute value of 1.4, the crab is below sea level by 0.2 m. To climb up 1 m means the crab’s final location will be above sea level, by 0.8 m.

Optional

Pairs | 20 min

Activity 3 School Supply Number Line

Students see that even without actual numbers, knowing the signs and magnitudes of two numbers is enough to determine whether their sum will be positive or negative.



Amps Featured Activity School Supply Number Line

Name: ____________ Date: ________ Period: ________

Activity 3 School Supply Number Line

1. Select two objects that are different lengths, and no longer than the number line shown here. (**Hint:** an eraser, marker cap, or paper clip might work.)

2. Let the length of the longer object be represented by a and the length of the shorter object be represented by b.

   Use the objects to measure and label each of the following points on the number line.

a	b	$2a$
$2b$	$a + b$	$-a$
$-b$	$a + (-b)$	$b + (-a)$

3. Complete each statement using <, >, or =. Use your number line to explain your reasoning. Sample responses shown.

   a. a .... > .... b

   b. $-a$ .... < .... $-b$

   c. $a + (-a)$ .... = ... $b + (-b)$

   d. $a + (-b)$ .... > .... $b + (-a)$

   e. $a + (-b)$ .... > .... $-a + b$

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Lesson 4 Adding Rational Numbers 413

1 Launch

Have students gather, or provide them access to, two small, short objects that are no longer than the length given in Problem 1. Point out that Problem 2 specifies the longer object will be represented by a and the shorter will be represented by b.

2 Monitor

Help students get started by demonstrating how to lay the objects on the number line to mark the positions of a and b.

Look for points of confusion:

- **Confusing the comparison symbols.** Have students write "less than" and "greater than" above their respective symbols.
- **Being confused between comparing the value of the expression and the magnitude of the expression.** Explain that the number to the left or lower on a number line has the lesser value but greater magnitude because it is further from 0.

3 Connect

Display a completed number line to the class. Have students compare the number line on display to their own.

Ask:

- "Is the order of the expressions on this number line the same or different than on yours?"
- "Would it be possible for $a + b$ to ever be greater than $2a$? Why or why not?"
- "What must always be true about $2a$ and $-2a$?"

Highlight that, even without knowing the actual numbers, knowing how the signs and magnitudes of two numbers compare is enough to determine whether their sum will be positive or negative.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can select objects of their choice and arrange them on a digital number line to compare the values of rational-number variable expressions.

Math Language Development

MLR7: Compare and Connect

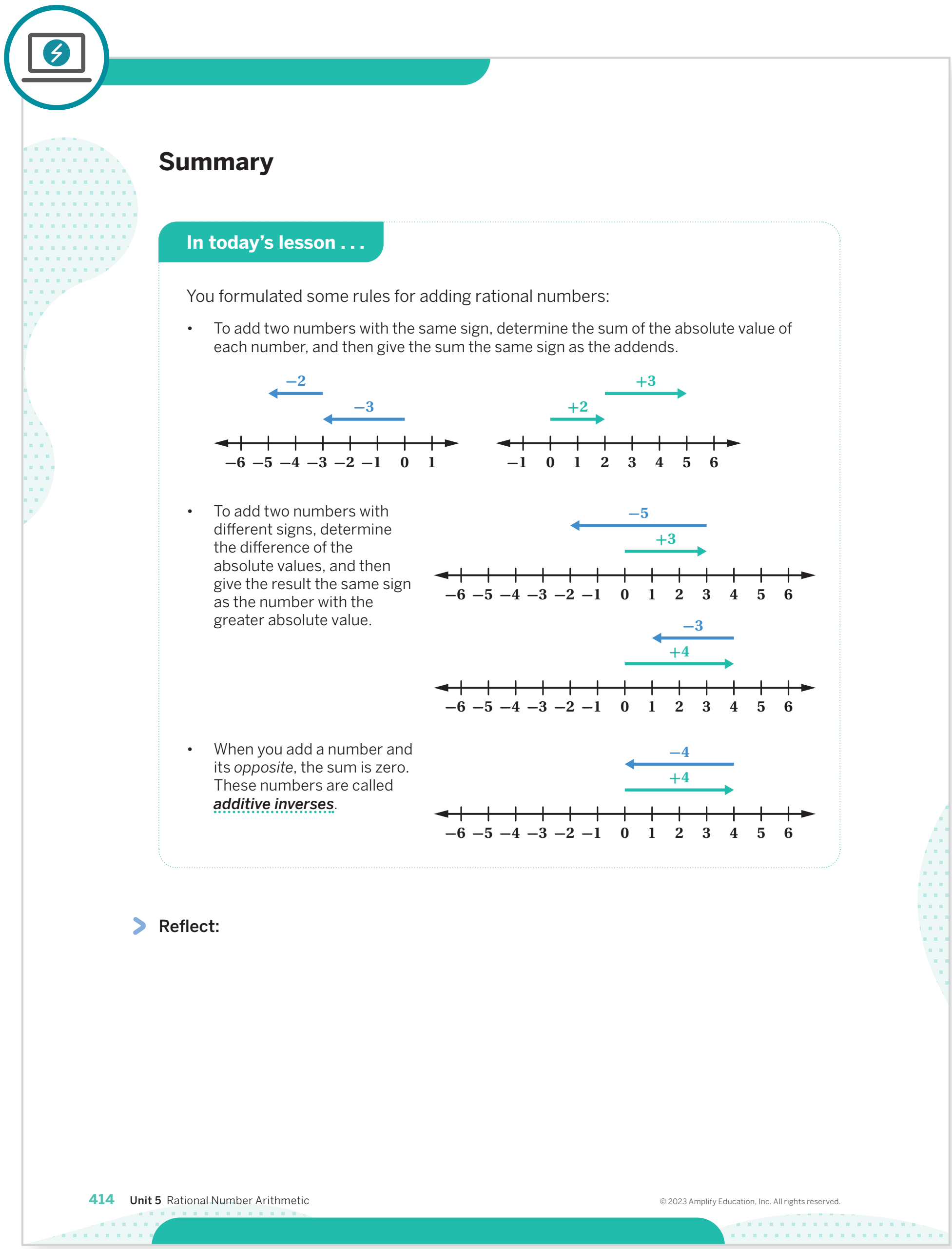
During the Connect, have students compare verbal descriptions with the algebraic comparison statements in Problem 3. For example, consider displaying a table similar to the one shown. Highlight math words and phrases, such as *additive inverse* or *opposite*.

a is *longer* than b. .... a is *greater* than b. The *length* of a is *greater* than the length of b.	$a > b$
The *opposite* of a is *less* than the *opposite* of b.	$-a < -b$
The sum of a and its *opposite* is equal to the sum of b and its *opposite*.	$a + (-a) = b + (-b)$

Whole Class | 5 min

Summary

Review and synthesize how to add rational numbers.



Summary

In today's lesson . . .

You formulated some rules for adding rational numbers:

- To add two numbers with the same sign, determine the sum of the absolute value of each number, and then give the sum the same sign as the addends.
- To add two numbers with different signs, determine the difference of the absolute values, and then give the result the same sign as the number with the greater absolute value.
- When you add a number and its *opposite*, the sum is zero. These numbers are called ***additive inverses***.

Reflect:

414 **Unit 5** Rational Number Arithmetic

Synthesize

Display the Anchor Chart PDF, *Rational Numbers (Part 1)*. Obtain the missing information from your class and complete the chart together.

Highlight that, when adding two numbers with different signs, the sign of the sum will match the sign of the number with the greater magnitude.

Formalize vocabulary: ***additive inverse***

Ask:

- "What is the opposite of 5? Of -8? Of $\frac{1}{3}$? Of -0.6?"
- "What is the sum of a number and its opposite?"
- "Are the numbers 3 and 3 additive inverses? Are the numbers 4 and -4.1 additive inverses? Why or why not?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do you add two numbers with the same sign? How do you add two numbers with different signs?"

Math Language Development

MLR2: Collect and Display

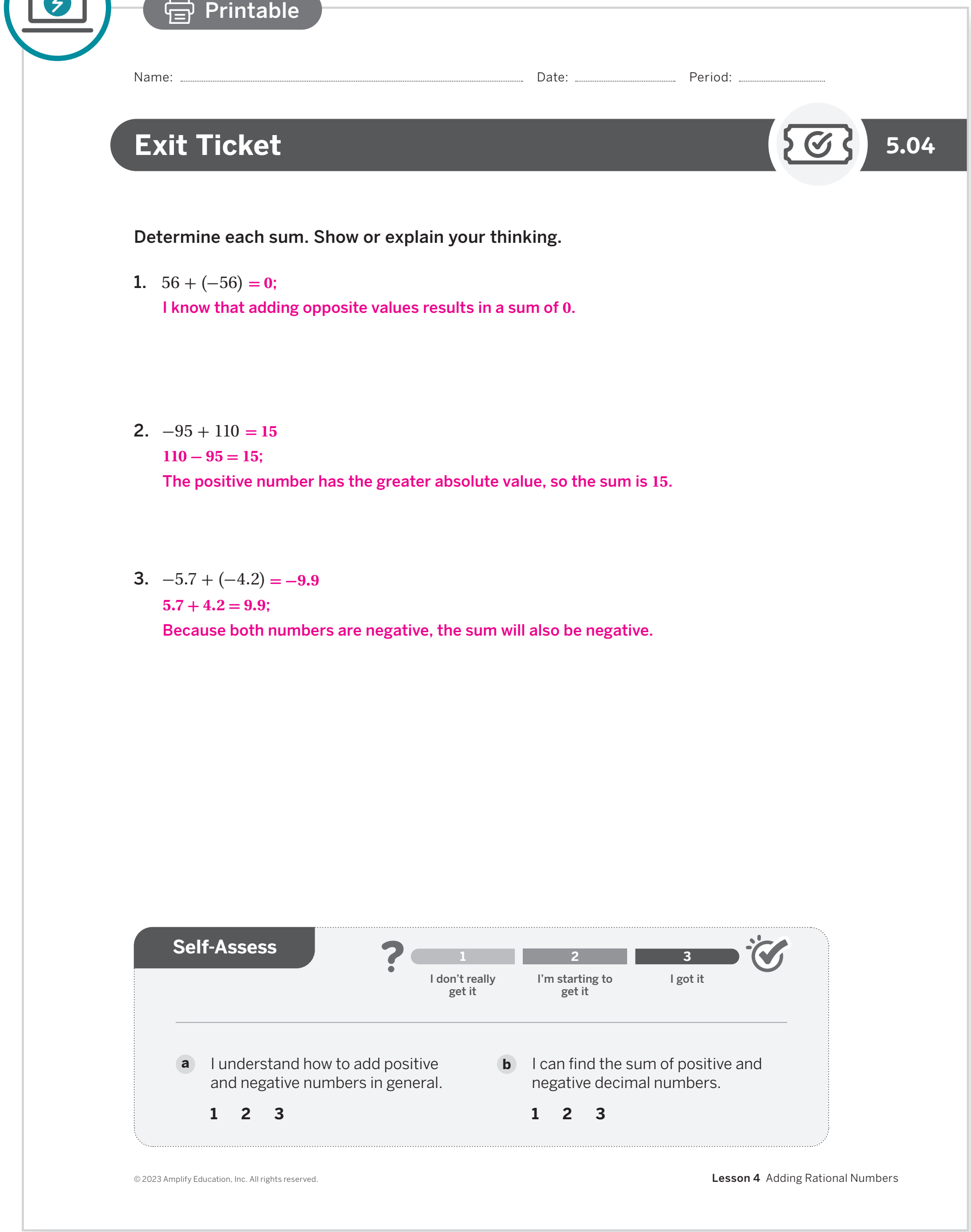
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on the terms and phrases related to the term *additive inverse* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to add rational numbers.

Printable



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Exit Ticket 5.04

Determine each sum. Show or explain your thinking.

1. $56 + (-56) = 0;$
   I know that adding opposite values results in a sum of 0.

2. $-95 + 110 = 15$
   $110 - 95 = 15;$
   The positive number has the greater absolute value, so the sum is 15.

3. $-5.7 + (-4.2) = -9.9$
   $5.7 + 4.2 = 9.9;$
   Because both numbers are negative, the sum will also be negative.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I understand how to add positive and negative numbers in general.
1 2 3

b I can find the sum of positive and negative decimal numbers.
1 2 3

Lesson 4 Adding Rational Numbers

Success looks like . . .

- **Goal:** Creating and interpreting equations and diagrams that represent adding rational numbers in the context of elevation.
- **Language Goal:** Generalizing a method for determining the sum of two rational numbers. **(Speaking and Listening)**
  - » Determining sums of two rational numbers and explaining how to calculate the sums in Problems 1–3.

Suggested next steps

If students are confused about the rules for adding numbers with the same versus different signs, consider:

- Suggesting they draw a number line to represent the problem.
- Assigning Practice Problem 1 to help connect the computations with context.

If students add the absolute values, but give the assign the sign of the number with the greater magnitude incorrectly, consider:

- Having them represent the problem on a number line with directional arrows.

Note: If students are showing facility with the computations, you may choose to not require them to explain their thinking.

Professional Learning

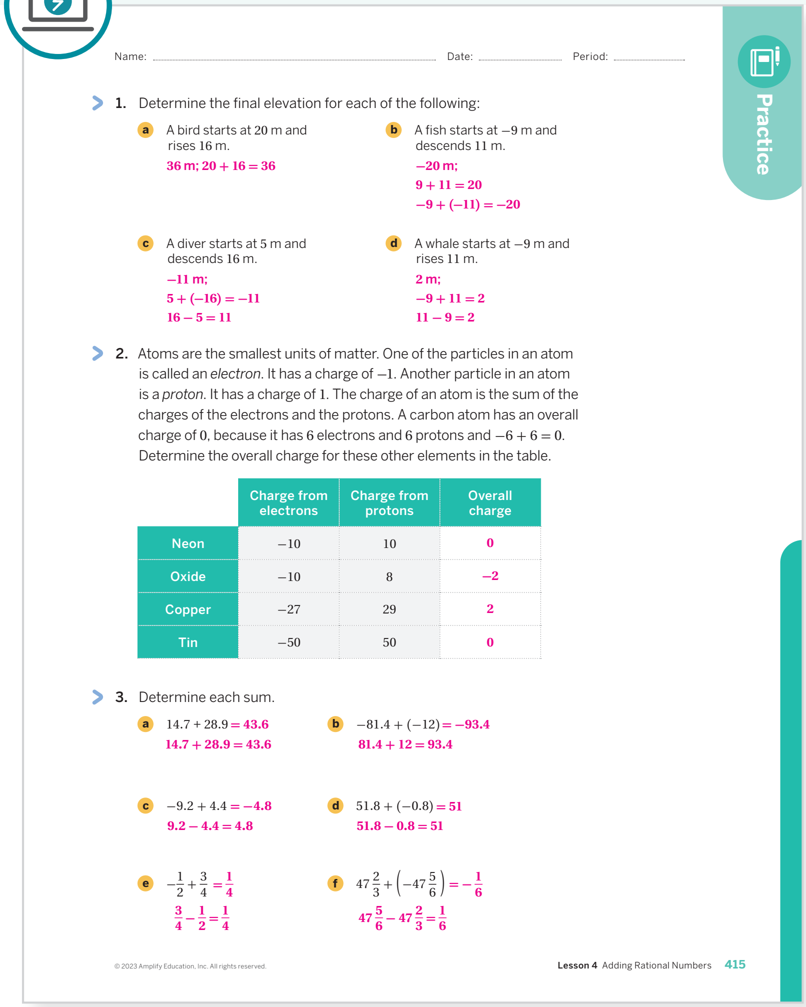
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did asking students to generate rules themselves reveal about your students as learners?
- Who participated and who didn't participate in Activity 3 today? What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

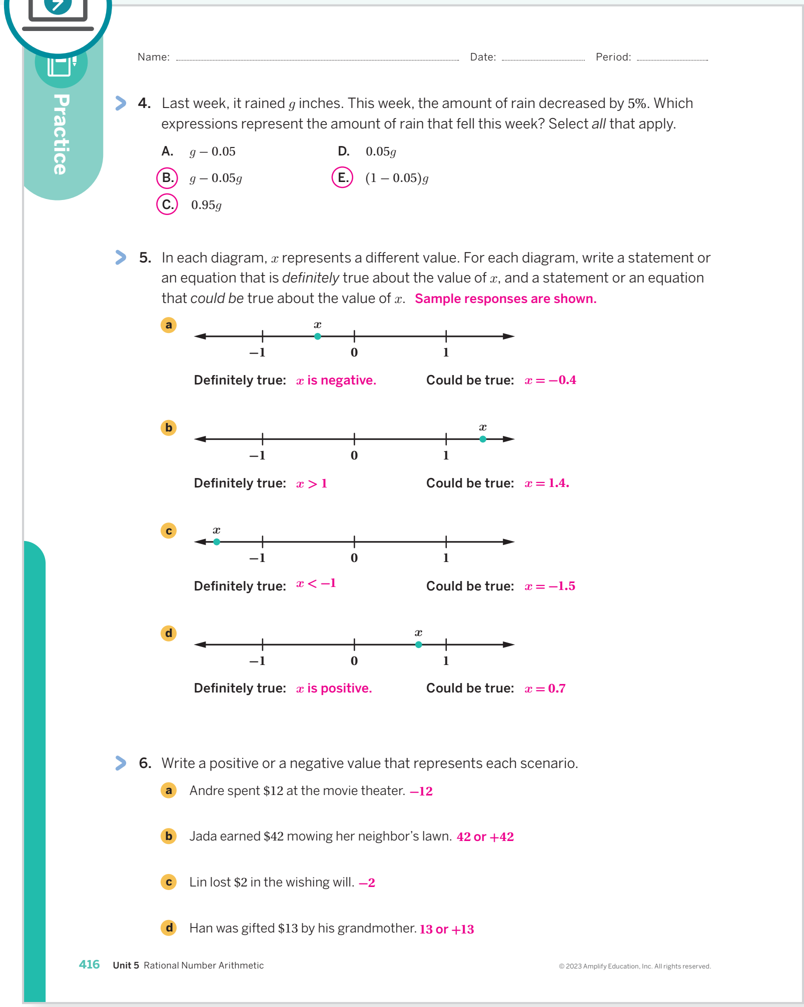
1. Determine the final elevation for each of the following:
   - a. A bird starts at 20 m and rises 16 m.
     36 m; $20 + 16 = 36$
   - b. A fish starts at -9 m and descends 11 m.
     -20 m;
     $9 + 11 = 20$
     $-9 + (-11) = -20$
   - c. A diver starts at 5 m and descends 16 m.
     -11 m;
     $5 + (-16) = -11$
     $16 - 5 = 11$
   - d. A whale starts at -9 m and rises 11 m.
     2 m;
     $-9 + 11 = 2$
     $11 - 9 = 2$

2. Atoms are the smallest units of matter. One of the particles in an atom is called an *electron*. It has a charge of -1. Another particle in an atom is a *proton*. It has a charge of 1. The charge of an atom is the sum of the charges of the electrons and the protons. A carbon atom has an overall charge of 0, because it has 6 electrons and 6 protons and $-6 + 6 = 0$. Determine the overall charge for these other elements in the table.

	Charge from electrons	Charge from protons	Overall charge
Neon	-10	10	0
Oxide	-10	8	-2
Copper	-27	29	2
Tin	-50	50	0

3. Determine each sum.
   - a. $14.7 + 28.9 = 43.6$
     $14.7 + 28.9 = 43.6$
   - b. $-81.4 + (-12) = -93.4$
     $81.4 + 12 = 93.4$
   - c. $-9.2 + 4.4 = -4.8$
     $9.2 - 4.4 = 4.8$
   - d. $51.8 + (-0.8) = 51$
     $51.8 - 0.8 = 51$
   - e. $-\frac{1}{2} + \frac{3}{4} = \frac{1}{4}$
     $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$
   - f. $47\frac{2}{3} + \left(-47\frac{5}{6}\right) = -\frac{1}{6}$
     $47\frac{5}{6} - 47\frac{2}{3} = \frac{1}{6}$

Lesson 4 Adding Rational Numbers 415



Name: ________ Date: ________ Period: ________

Practice

4. Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select *all* that apply.
   - A. $g - 0.05$
   - [✓ B.] $g - 0.05g$
   - [✓ C.] $0.95g$
   - D. $0.05g$
   - [✓ E.] $(1 - 0.05)g$

5. In each diagram, x represents a different value. For each diagram, write a statement or an equation that is *definitely* true about the value of x, and a statement or an equation that *could be* true about the value of x. Sample responses are shown.
   - a. **Definitely true:** x is negative. **Could be true:** $x = -0.4$
   - b. **Definitely true:** $x > 1$ **Could be true:** $x = 1.4.$
   - c. **Definitely true:** $x < -1$ **Could be true:** $x = -1.5$
   - d. **Definitely true:** x is positive. **Could be true:** $x = 0.7$

6. Write a positive or a negative value that represents each scenario.
   - a. Andre spent $12 at the movie theater. -12
   - b. Jada earned $42 mowing her neighbor's lawn. 42 or $+42$
   - c. Lin lost $2 in the wishing will. -2
   - d. Han was gifted $13 by his grandmother. 13 or $+13$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	1
Spiral	4	Unit 4 Lesson 6	2
	5	Unit 5 Lesson 2	2
Formative ⚡	6	Unit 5 Lesson 5	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

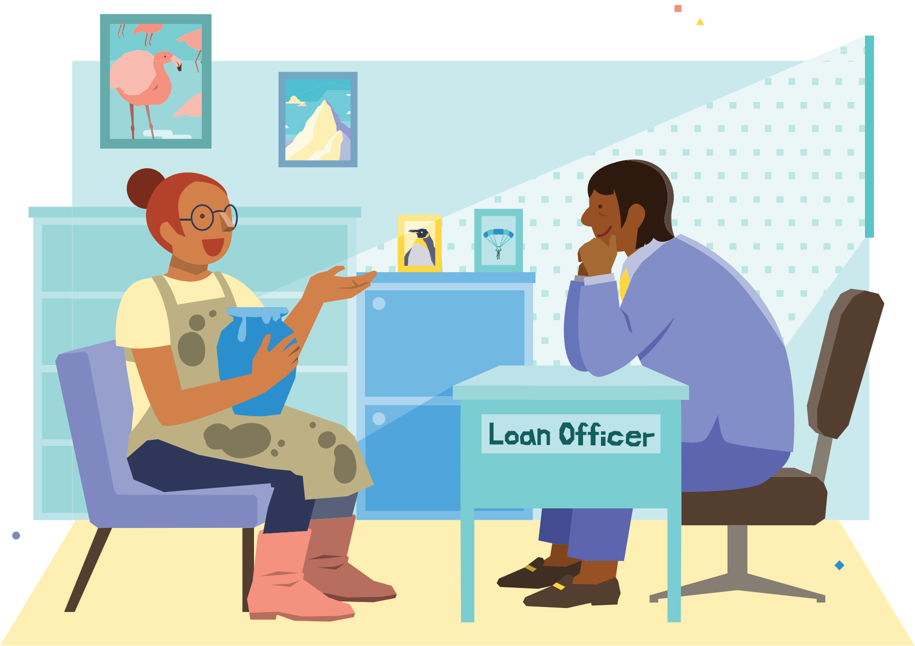
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 5**

Money and Debts

Let's apply what we know about positive and negative numbers to money.



Focus

Goals

1. **Language Goal:** Apply addition of rational numbers to calculate an account balance after a deposit or withdrawal, and explain the solution method. **(Speaking and Listening)**
2. **Language Goal:** Explain how rational numbers can be used to represent situations involving money, including deposits or withdrawals and assets or debts. **(Speaking and Listening, Reading and Writing)**
3. Write an equation with an unknown addend to represent a situation where the amount of change is unknown.

Coherence

Today

Students are introduced to using negative numbers in the context of money to represent debts or debits. One point that often gets overlooked is that it is a *convention* that we do this, rather than a necessity, so be prepared to handle students' curiosities about *why* it is done this way. Using a mathematical structure (rational numbers) to represent a context (a balance with an energy company) is an example of modeling with mathematics.

Previously

In Lessons 2–4 in this unit, students worked with and reasoned about adding rational numbers in primarily distance contexts.

Coming Soon

Students will begin to subtract rational numbers in Lesson 6, realizing that this operation is closely linked with addition, even within the set of rational numbers.

Rigor

- Students **apply** their understanding of adding rational numbers to the contexts involving money — debts, bills, and banking.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	12 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Are you ready for more?* (as needed)
- Activity 1 PDF, *Are you ready for more?* (answers)
- calculators

Math Language Development

New words

- *balance**
- *charge*
- *credit*
- *debt*
- *deposit*
- *withdrawal*

*Students may confuse the term *balance* with the many other variations of meaning for the term. Be ready to address the similarities and differences between the various definitions of this term.

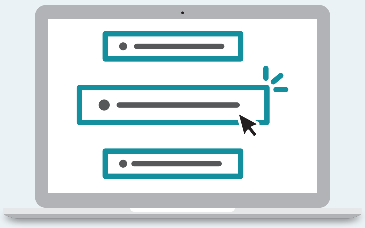
Review words

- *additive inverse*
- *commutative property*

Amps | **Featured Activity**

Activity 2
Using Work From Previous Slides

Charges and credits students enter in a table help them calculate balances later on.



Amps
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Building Math Identity and Community

Connecting to Mathematical Practices

Students might not see any real-world application of the new skills and, therefore, might lack motivation to complete Activity 2. However, applying addition of rational numbers as a model should grab students' attention. Have students set two goals for this activity. The first goal should be about the process of adding rational numbers. The second goal should be about interpreting the rational numbers and the sums in a real-world context.

Modifications to Pacing

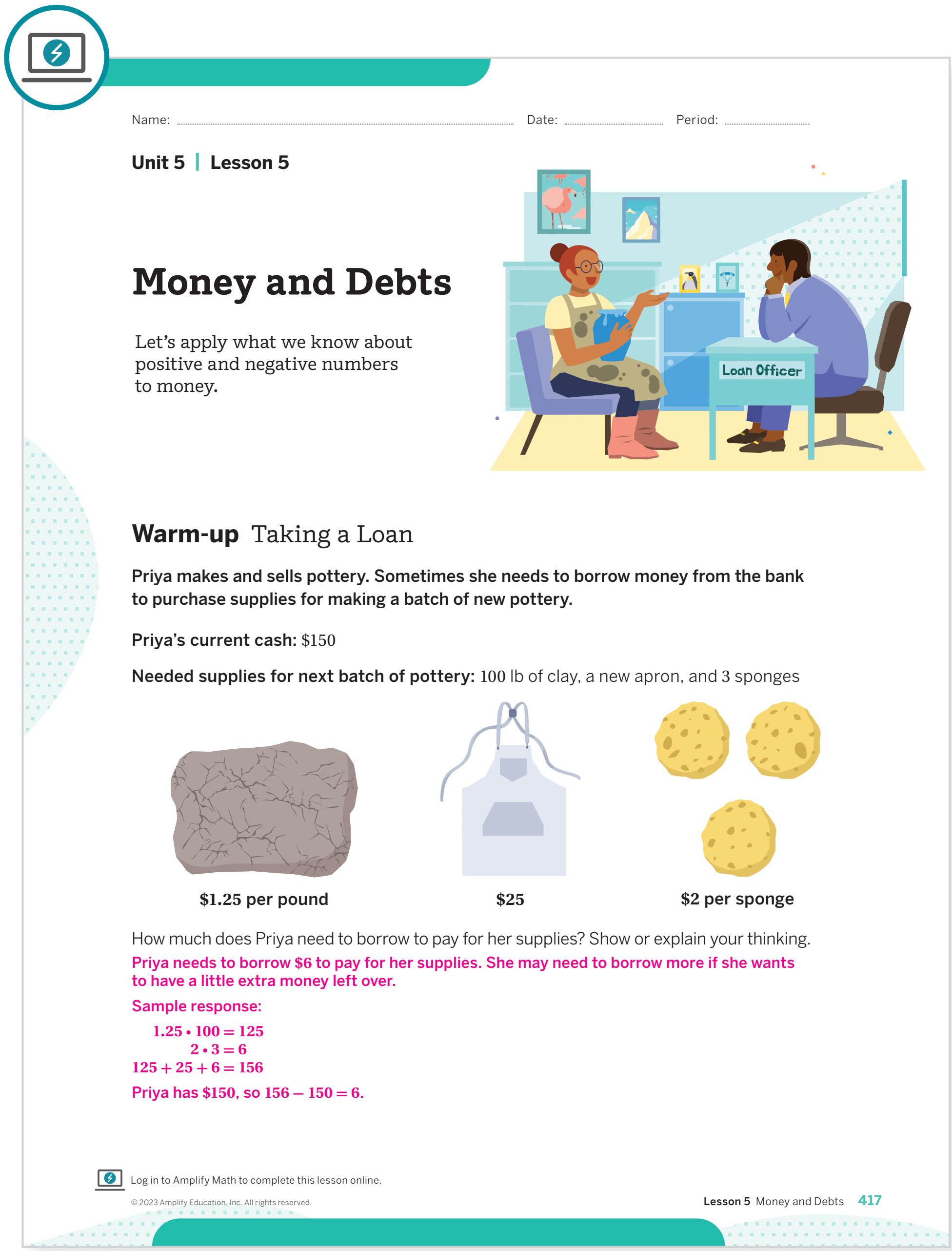
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, have different pairs of students each complete one row, and then complete the rest of the table and balance column as a class.
- In **Activity 2**, Problem 2 may be omitted.

Pairs | 7 min

Warm-up Taking a Loan

Students explore a context where it is necessary to borrow money and go into debt to understand why this value can be represented with a negative number.



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Unit 5 | **Lesson 5**

Money and Debts

Let's apply what we know about positive and negative numbers to money.

Warm-up Taking a Loan

Priya makes and sells pottery. Sometimes she needs to borrow money from the bank to purchase supplies for making a batch of new pottery.

Priya's current cash: $150

Needed supplies for next batch of pottery: 100 lb of clay, a new apron, and 3 sponges

$1.25 per pound **$25** **$2 per sponge**

How much does Priya need to borrow to pay for her supplies? Show or explain your thinking.

Priya needs to borrow $6 to pay for her supplies. She may need to borrow more if she wants to have a little extra money left over.

Sample response:

$1.25 \cdot 100 = 125$

$2 \cdot 3 = 6$

$125 + 25 + 6 = 156$

Priya has $150, so $156 - 150 = 6$.

Log in to Amplify Math to complete this lesson online.

Lesson 5 Money and Debts 417

1 Launch

Activate background knowledge by prompting students to think of a time they borrowed money to pay for something they wanted. Ask, "What did you have to do after you borrowed the money?" Provide access to calculators throughout the lesson.

2 Monitor

Help students get started by asking, "WIll Priya spend $1.25 for all of her clay?"

Look for points of confusion:

- **Thinking Priya needs to borrow the total cost of the supplies.** Remind students that Priya's current cash is given in the introduction to the problem.

Look for productive strategies:

- Noticing that the combination of $125 + $25 for the clay and apron is equal to how much cash Priya has, and determining that she needs to borrow the cost of the sponges.

3 Connect

Display different student strategies. Look for an expression that shows the opposite values of 150 and -150.

Have students share which strategy they believe helps to make the calculations more efficient.

Highlight that under certain circumstances, people or businesses need to borrow more money than they currently have. Banks can help with this. The money that you need to pay back, or that you owe, is called a ***debt***.

Ask:

- "What do you already know about *debt*?"
- "Is a *debt* always a bad thing? Is it ever a good thing?"
- "Why might we typically represent debt as a negative number?"

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory text and images. Have pairs examine Priya's current amount of cash and the supplies she needs to buy. Ask them to work with their partner to write 2–3 mathematical questions that could be answered based on the information given.

English Learners

After students have had time to craft one of their own questions, provide an exemplar question for students to use as a reference, such as "What is the total cost of the clay that she needs to purchase?" Encourage students to use this exemplar question as a model for writing 1–2 more mathematical questions.

Power-up

To power up students' ability to identify the meaning of positive and negative values in the context of money, have students complete:

Determine whether each scenario would best be represented by a *positive* or a *negative* value.

1. A loss of $4.	Negative
2. An earnings of $53.	Positive
3. A debt of $82.	Negative
4. Receiving a gift of $120.	Positive

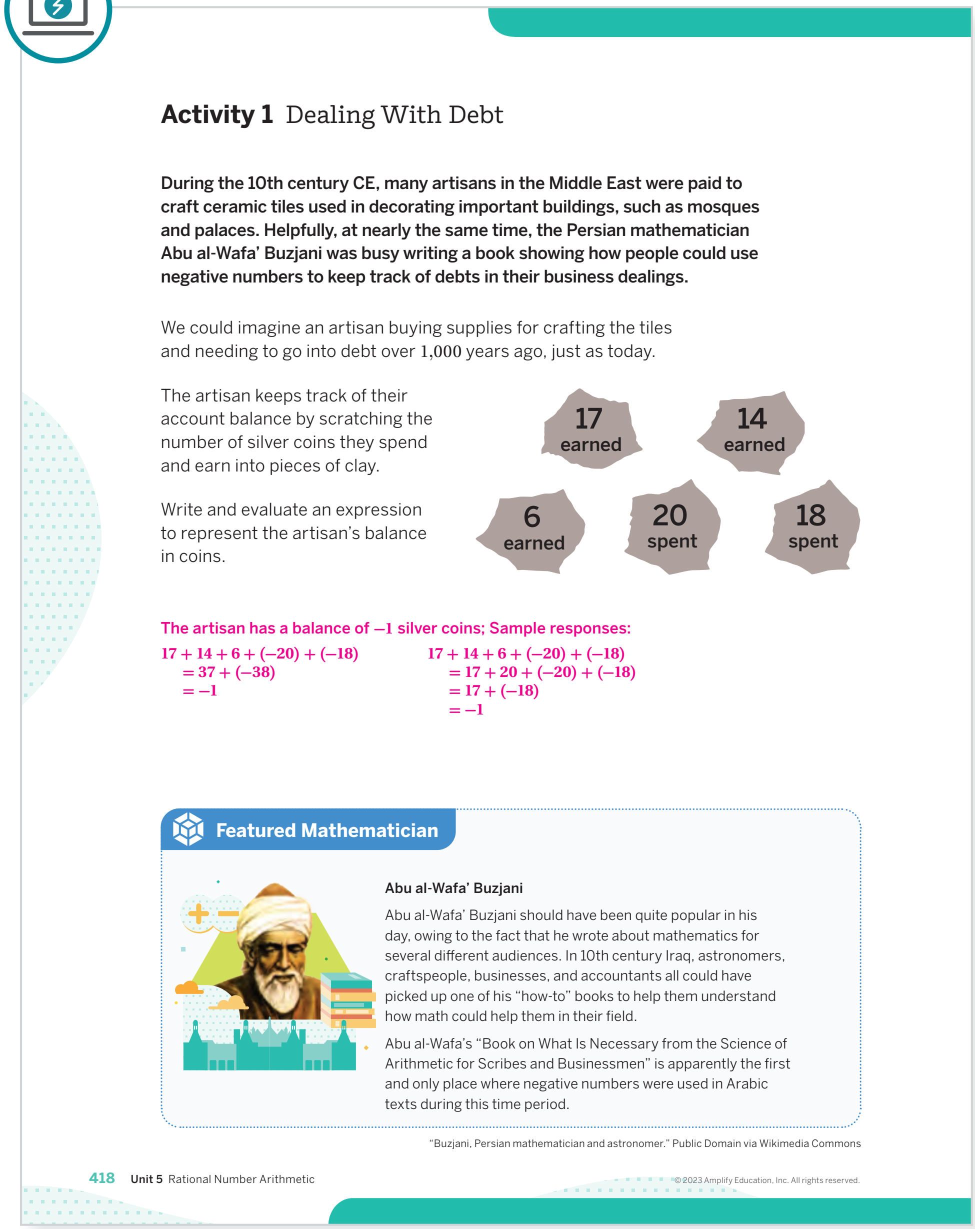
Use: Before Activity 1.

Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment Problem 5.

Pairs | 12 min

Activity 1 Dealing With Debt

Students consider a set of transactions to determine whether an account has a negative balance and see how the commutative property can help make calculations more efficient.



Activity 1 Dealing With Debt

During the 10th century CE, many artisans in the Middle East were paid to craft ceramic tiles used in decorating important buildings, such as mosques and palaces. Helpfully, at nearly the same time, the Persian mathematician Abu al-Wafa' Buzjani was busy writing a book showing how people could use negative numbers to keep track of debts in their business dealings.

We could imagine an artisan buying supplies for crafting the tiles and needing to go into debt over 1,000 years ago, just as today.

The artisan keeps track of their account balance by scratching the number of silver coins they spend and earn into pieces of clay.

Write and evaluate an expression to represent the artisan's balance in coins.

17 earned

14 earned

6 earned

20 spent

18 spent

The artisan has a balance of –1 silver coins; Sample responses:

$$\begin{aligned} &17 + 14 + 6 + (-20) + (-18) \\ &\quad = 37 + (-38) \\ &\quad = -1 \end{aligned}$$

$$\begin{aligned} &17 + 14 + 6 + (-20) + (-18) \\ &\quad = 17 + 20 + (-20) + (-18) \\ &\quad = 17 + (-18) \\ &\quad = -1 \end{aligned}$$

Featured Mathematician

Abu al-Wafa' Buzjani

Abu al-Wafa' Buzjani should have been quite popular in his day, owing to the fact that he wrote about mathematics for several different audiences. In 10th century Iraq, astronomers, craftspeople, businesses, and accountants all could have picked up one of his "how-to" books to help them understand how math could help them in their field.

Abu al-Wafa's "Book on What Is Necessary from the Science of Arithmetic for Scribes and Businessmen" is apparently the first and only place where negative numbers were used in Arabic texts during this time period.

"Buzjani, Persian mathematician and astronomer." Public Domain via Wikimedia Commons

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1 Launch

Read the introduction to the activity together as a class. Introduce the term ***balance*** as the current amount of money in an account, either positive or negative to represent the amount of money available or owed. A *negative balance* would indicate a *debt*.

2 Monitor

Help students get started by mentioning that there may be multiple ways to solve this problem.

Look for points of confusion:

- **Being unsure of how to incorporate the values of the counting rods.** Let students know that the counting rods are just another way to express the value that is at the top of the bit of paper.

Look for productive strategies:

- Noticing that the combination of \$14 + \$6 earned is the opposite of \$20 spent.

3 Connect

Display several student strategies. Be prepared with examples of expressions that show different methods for ordering and combining the values.

Have students share their observations about similarities and differences among the strategies.

Ask, "How might it be helpful to look over all the values in a set of positives and negatives before adding them?"

Highlight that it can be helpful to first consider the values before operating with them. Just as the pieces of clay could be rearranged into a more helpful order, the addends in an addition sentence can be reordered using the commutative property.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can rearrange the values into an order that aids their calculations, by using the commutative property.

Extension: *Math Enrichment*

Have students complete the Activity 1 PDF, *Are you ready for more?*

Math Language Development

MLR7: Compare and Connect

During the Connect, after displaying several student strategies, use a ***Think-Pair-Share*** routine to encourage partners to compare the various strategies. Ask:

- "What do the strategies have in common? How are they different?"
- "How much was earned? How much was spent? How can grouping these together aid calculations?"

Featured Mathematician

Abu al-Wafa' Buzjani

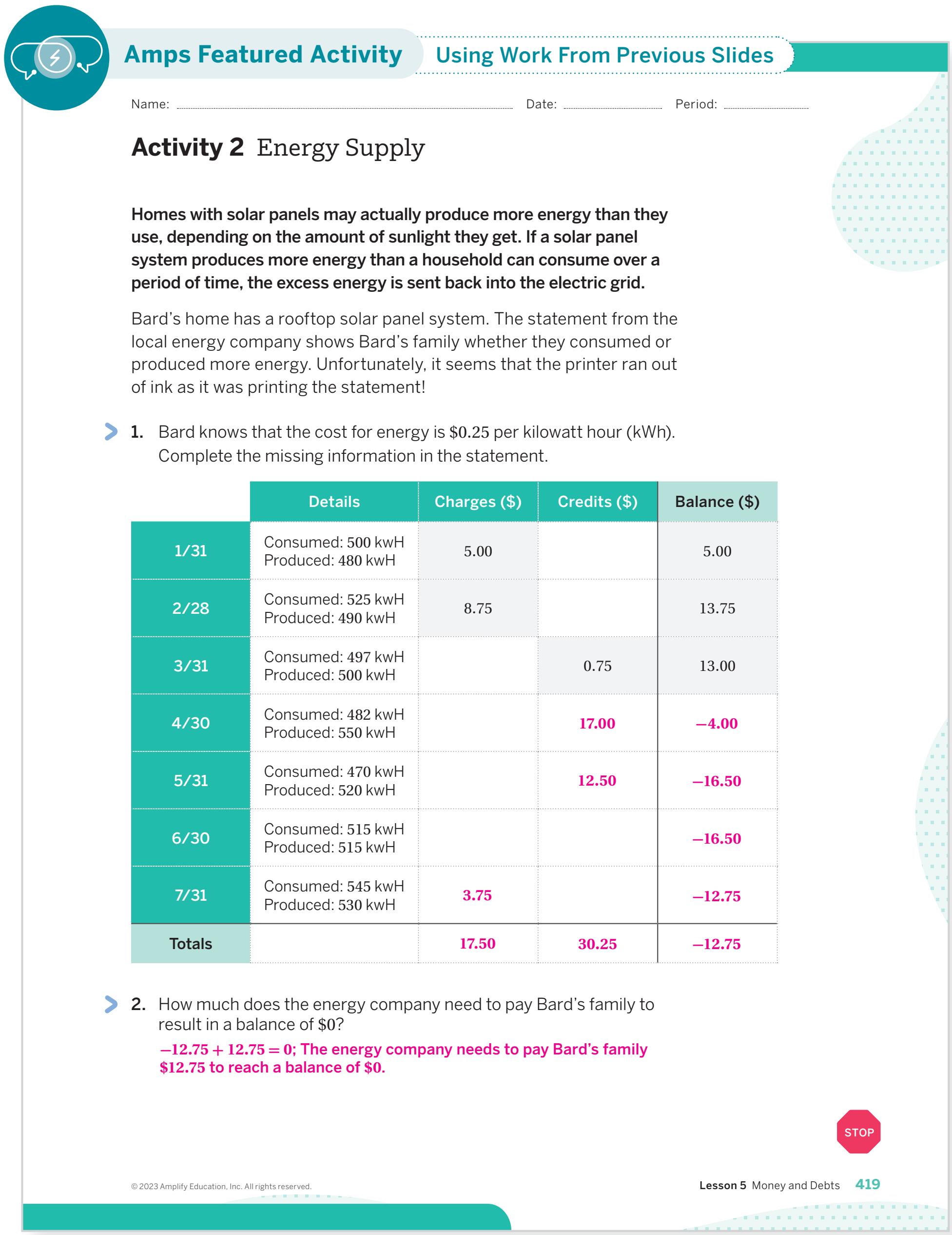
Have students read about featured mathematician Abu al-Wafa' Buzjani, who was the first of his time to include a discussion of negative numbers in an Arabic math text during the 10th century.

Pairs | 15 min

Activity 2 Energy Supply

Students complete a balance statement with missing credits and charges to make sense of a common real-world context and work with values that include decimals.

Amps Featured Activity Using Work From Previous Slides



Name: ______ Date: ______ Period: ______

Activity 2 Energy Supply

Homes with solar panels may actually produce more energy than they use, depending on the amount of sunlight they get. If a solar panel system produces more energy than a household can consume over a period of time, the excess energy is sent back into the electric grid.

Bard's home has a rooftop solar panel system. The statement from the local energy company shows Bard's family whether they consumed or produced more energy. Unfortunately, it seems that the printer ran out of ink as it was printing the statement!

1. Bard knows that the cost for energy is $0.25 per kilowatt hour (kWh). Complete the missing information in the statement.

	Details	Charges ($)	Credits ($)	Balance ($)
1/31	Consumed: 500 kwH Produced: 480 kwH	5.00		5.00
2/28	Consumed: 525 kwH Produced: 490 kwH	8.75		13.75
3/31	Consumed: 497 kwH Produced: 500 kwH		0.75	13.00
4/30	Consumed: 482 kwH Produced: 550 kwH		17.00	−4.00
5/31	Consumed: 470 kwH Produced: 520 kwH		12.50	−16.50
6/30	Consumed: 515 kwH Produced: 515 kwH			−16.50
7/31	Consumed: 545 kwH Produced: 530 kwH	3.75		−12.75
Totals		17.50	30.25	−12.75

2. How much does the energy company need to pay Bard's family to result in a balance of $0?

   $-12.75 + 12.75 = 0$; The energy company needs to pay Bard's family $12.75 to reach a balance of $0.

STOP

1 Launch

Have students look over the statement in Problem 1. Then have them share anything they think is important or anything that they have questions about. Point out that each line will display only a ***charge*** or a ***credit*** and take time to explain the meaning of each term.

2 Monitor

Help students get started by asking, "What does the information in the *Details* column tell you? How much more energy was consumed than produced in the first month?"

Look for points of confusion:

- **Not finding the difference of the energy consumed and produced.** Have students check, for the first few months, if their strategy works to determine the charge or credit given.
- **Being confused about why the balance for the second month isn't $8.75.** Remind students that balance refers to the current amount, so it is a running total that includes the previous months.

3 Connect

Display a completed table.

Have students share at least two different ways for determining the final balance at the bottom of the statement.

Ask, "Does the negative balance mean that Bard's family owes money or that the energy company owes them money? Why do you think that is?"

Highlight that sometimes the sign of a number is conveyed by the word or words used to describe it. For example, someone might say that they "owe $3" or that they "have a balance of −$3." However, it would not make sense to say that someone "owes −$3."

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider displaying or providing students with a checklist to help them complete the statement, such as the following:

- Determine the difference between the energy consumed and produced.
- Deciding whether the customer will be charged or given a credit.
- Use the given rate to determine the total cost.

Provide access to colored pencils and suggest that students color code the cells in the Details column with whether it represents a charge or a credit.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the image of the statement. Ask students to work with their partner to write 2–3 mathematical questions they could ask about the information shown. Ask pairs to share their questions with the whole class. Sample questions shown.

- What does it mean when more energy is consumed than produced?
- How can I determine the amount charged or credited?
- What does a positive balance mean? A negative balance?

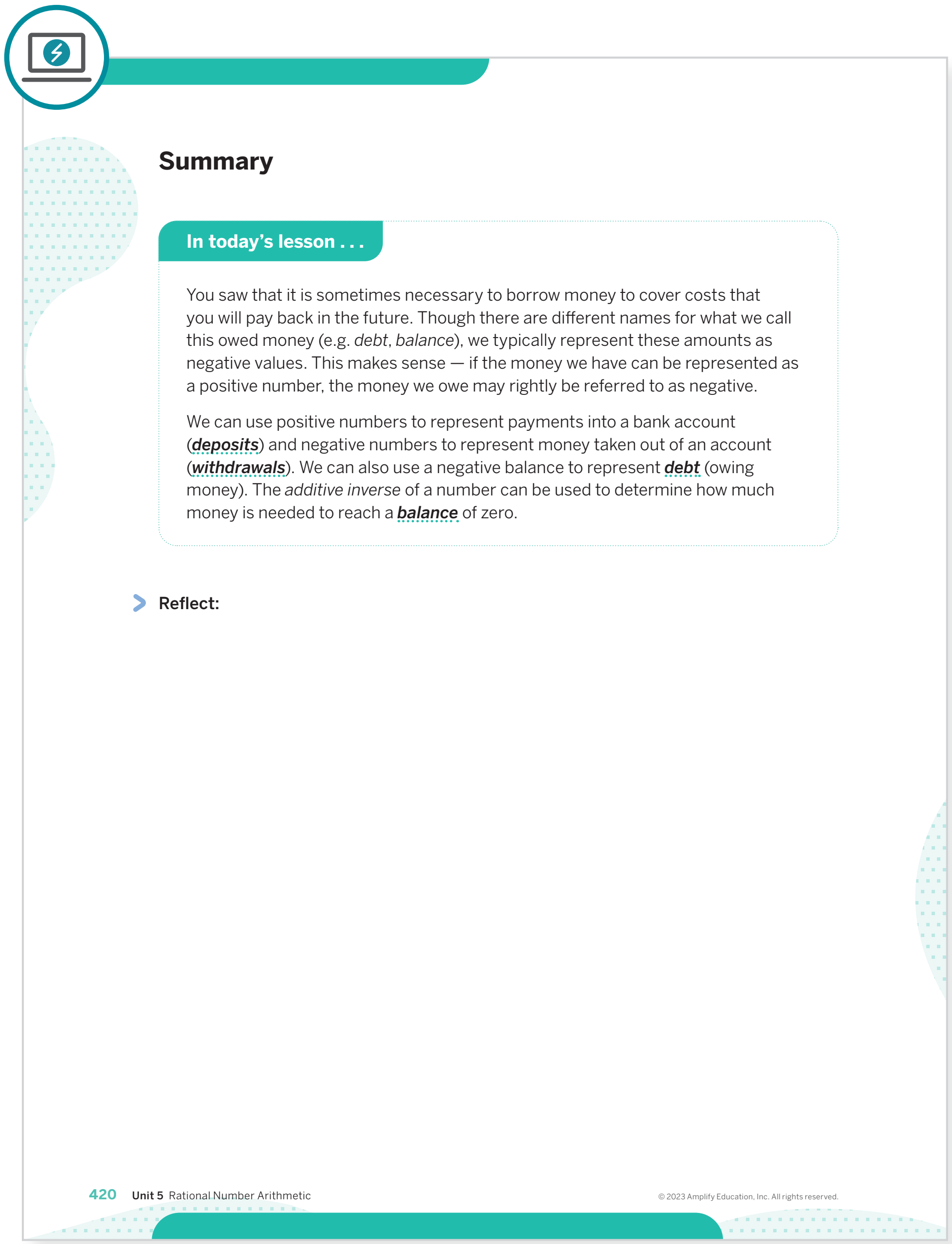
English Learners

Clarify the meanings of the terms in the statement (e.g., *credit*, *charge*, *balance*, *consumed*, and *produced*) in this context.

Whole Class | 5 min

Summary

Review and synthesize the vocabulary and contexts that may be involved when dealing with money and debt.



Summary

In today's lesson . . .

You saw that it is sometimes necessary to borrow money to cover costs that you will pay back in the future. Though there are different names for what we call this owed money (e.g. *debt*, *balance*), we typically represent these amounts as negative values. This makes sense — if the money we have can be represented as a positive number, the money we owe may rightly be referred to as negative.

We can use positive numbers to represent payments into a bank account (***deposits***) and negative numbers to represent money taken out of an account (***withdrawals***). We can also use a negative balance to represent ***debt*** (owing money). The *additive inverse* of a number can be used to determine how much money is needed to reach a ***balance*** of zero.

Reflect:

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Synthesize

Highlight that sometimes the sign of the number depends on who is writing the number. If a company owes you money, they might consider the amount a negative, whereas you might consider it a positive (because it is money you will receive). When adding positive and negative amounts, you can use the commutative property to rearrange the order of the amounts for easier calculations.

Formalize vocabulary:

- *balance*
- *charge*
- *credit*
- *debt*
- *deposit*
- *withdrawal*

Ask:

- "If you borrow $5 from your friend, would you consider that amount to be a positive value or a negative value? How might your friend consider it?"
- "What does it mean to have a balance of $0?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why might the same amount of money be represented as a negative value to one person and a positive value to another?"

Math Language Development

MLR2: Collect and Display

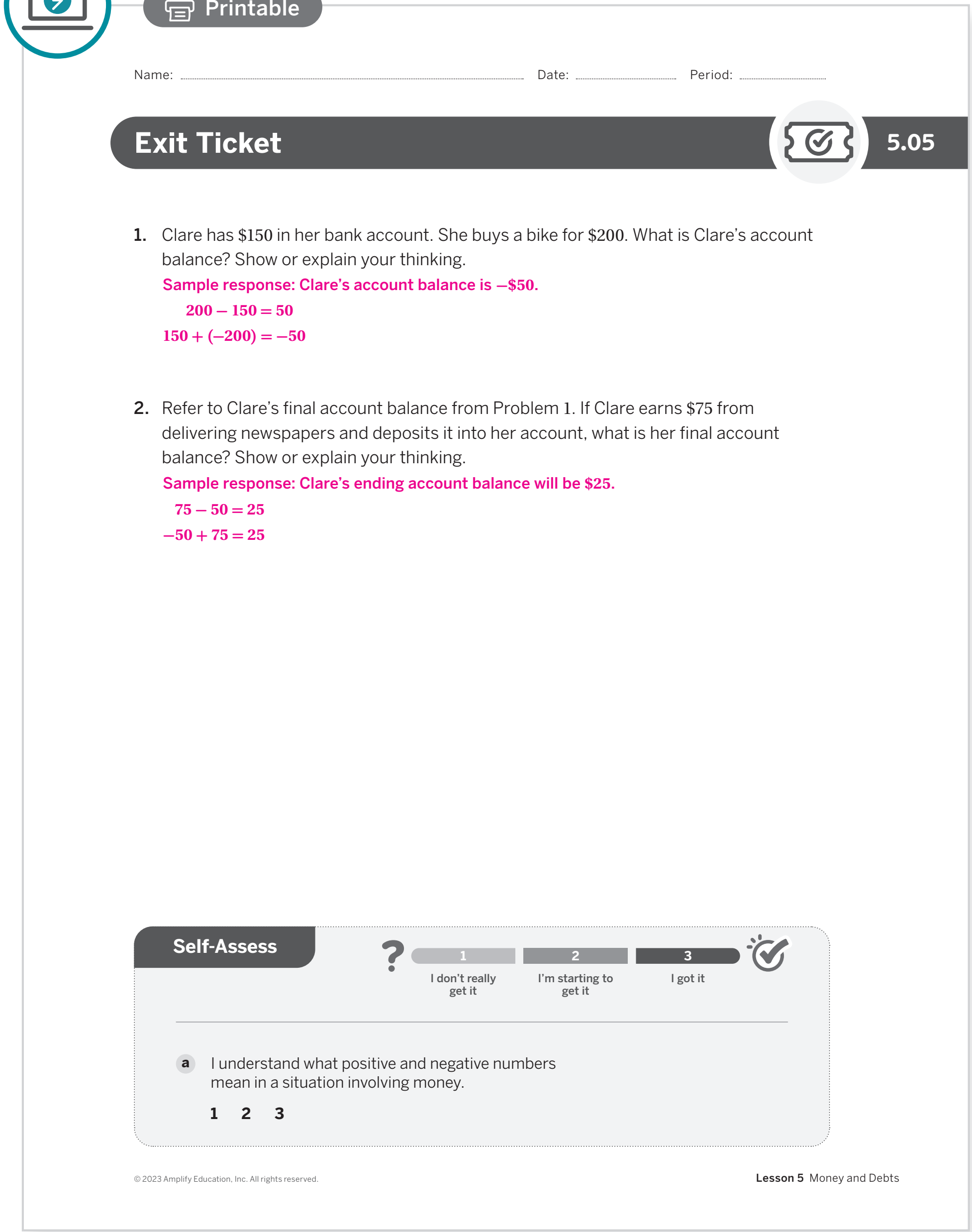
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on the terms and phrases related to the terms *balance*, *charge*, *credit*, *debt*, *deposit*, and *withdrawal* that were used during the lesson. Add these new terms, along with any informal student language or visual examples, to the class display.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of money and debt contexts by finding the account balance after various transactions.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.05

1. Clare has $150 in her bank account. She buys a bike for $200. What is Clare's account balance? Show or explain your thinking.

   Sample response: Clare's account balance is –$50.

   $200 - 150 = 50$

   $150 + (-200) = -50$

2. Refer to Clare's final account balance from Problem 1. If Clare earns $75 from delivering newspapers and deposits it into her account, what is her final account balance? Show or explain your thinking.

   Sample response: Clare's ending account balance will be $25.

   $75 - 50 = 25$

   $-50 + 75 = 25$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I understand what positive and negative numbers mean in a situation involving money.

1 2 3

Success looks like . . .

- **Language Goal:** Applying addition of rational numbers to calculate an account balance after a deposit or withdrawal, and explaining the solution method. **(Speaking and Listening)**
  - » Determining Clare's account balance and explaining how to determine it in Problems 1 and 2.
- **Language Goal:** Explaining how rational numbers can be used to represent situations involving money, including deposits or withdrawals, and assets or debts. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Writing an equation with an unknown addend to represent a situation where the amount of change is unknown.

Suggested next steps

If students add the cost of the bike to the amount in Clare's bank account, consider:

- Assigning Practice Problem 3.
- Asking, "If Clare buys a bike, did she earn or spend money?"

If students correctly find the difference of 150 and 200, but forget to represent the difference as a negative value, consider:

- Asking, "Would Clare have any money left in her bank account after buying the bike? How do you know?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach writing expressions for adding several positive and negative numbers? What does that tell you about the similarities and differences among your students?
- Which teacher actions made interpreting negative numbers in the context of money and debt strong? What might you change for the next time you teach this lesson?

Math Language Development

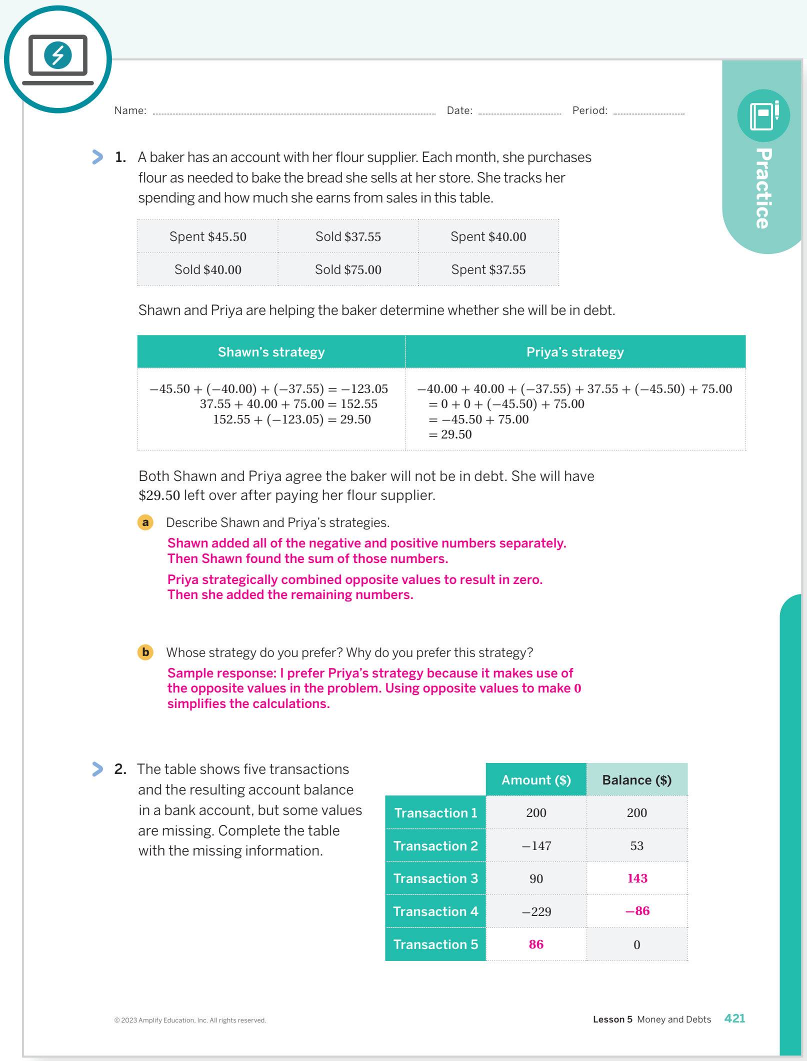
Language Goal: Explaining how rational numbers can be used to represent situations involving money, including deposits or withdrawals, and assets or debts.

Reflect on students' language development toward this goal.

- How did using the ***Co-craft Questions*** routine during Activity 2 help students make sense of positive and negative balances?
- How have students progressed in their comfort using the terms *balance, charge, credit, debt, deposit,* and *withdrawal* that were used in this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. A baker has an account with her flour supplier. Each month, she purchases flour as needed to bake the bread she sells at her store. She tracks her spending and how much she earns from sales in this table.

Spent \$45.50	Sold \$37.55	Spent \$40.00
Sold \$40.00	Sold \$75.00	Spent \$37.55

Shawn and Priya are helping the baker determine whether she will be in debt.

Shawn's strategy	Priya's strategy
$-45.50 + (-40.00) + (-37.55) = -123.05$ $37.55 + 40.00 + 75.00 = 152.55$ $152.55 + (-123.05) = 29.50$	$-40.00 + 40.00 + (-37.55) + 37.55 + (-45.50) + 75.00$ $= 0 + 0 + (-45.50) + 75.00$ $= -45.50 + 75.00$ $= 29.50$

Both Shawn and Priya agree the baker will not be in debt. She will have \$29.50 left over after paying her flour supplier.

a. Describe Shawn and Priya's strategies.

Shawn added all of the negative and positive numbers separately. Then Shawn found the sum of those numbers.

Priya strategically combined opposite values to result in zero. Then she added the remaining numbers.

b. Whose strategy do you prefer? Why do you prefer this strategy?

Sample response: I prefer Priya's strategy because it makes use of the opposite values in the problem. Using opposite values to make 0 simplifies the calculations.

2. The table shows five transactions and the resulting account balance in a bank account, but some values are missing. Complete the table with the missing information.

	Amount (\$)	Balance (\$)
Transaction 1	200	200
Transaction 2	−147	53
Transaction 3	90	**143**
Transaction 4	−229	**−86**
Transaction 5	**86**	0

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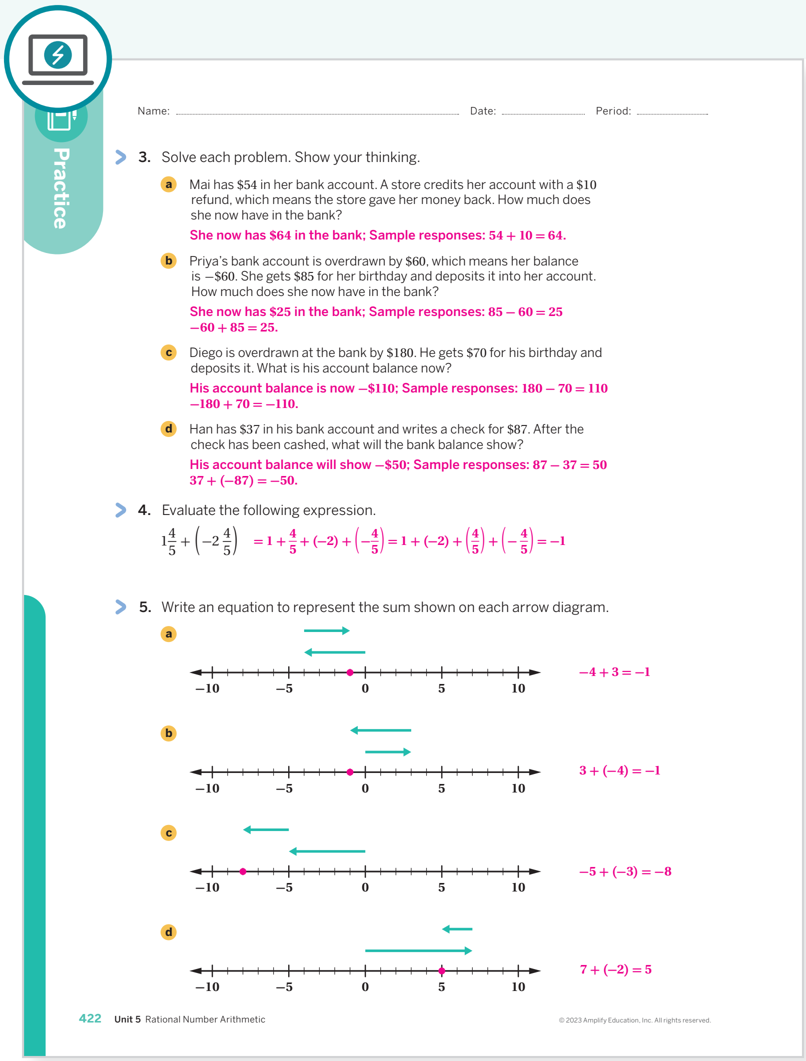

Name: Date: Period:

Practice

3. Solve each problem. Show your thinking.

a. Mai has \$54 in her bank account. A store credits her account with a \$10 refund, which means the store gave her money back. How much does she now have in the bank?

She now has \$64 in the bank; Sample responses: $54 + 10 = 64$.

b. Priya's bank account is overdrawn by \$60, which means her balance is −\$60. She gets \$85 for her birthday and deposits it into her account. How much does she now have in the bank?

She now has \$25 in the bank; Sample responses: $85 - 60 = 25$ $-60 + 85 = 25$.

c. Diego is overdrawn at the bank by \$180. He gets \$70 for his birthday and deposits it. What is his account balance now?

His account balance is now −\$110; Sample responses: $180 - 70 = 110$ $-180 + 70 = -110$.

d. Han has \$37 in his bank account and writes a check for \$87. After the check has been cashed, what will the bank balance show?

His account balance will show −\$50; Sample responses: $87 - 37 = 50$ $37 + (-87) = -50$.

4. Evaluate the following expression.

$$1\frac{4}{5} + \left(-2\frac{4}{5}\right) \quad = 1 + \frac{4}{5} + (-2) + \left(-\frac{4}{5}\right) = 1 + (-2) + \left(\frac{4}{5}\right) + \left(-\frac{4}{5}\right) = -1$$

5. Write an equation to represent the sum shown on each arrow diagram.

a. $-4 + 3 = -1$

b. $3 + (-4) = -1$

c. $-5 + (-3) = -8$

d. $7 + (-2) = 5$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	3
	2	Activity 2	2
	3	Exit Ticket	2
Spiral	4	Unit 5 Lesson 4	1
Formative ⚡	5	Unit 5 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

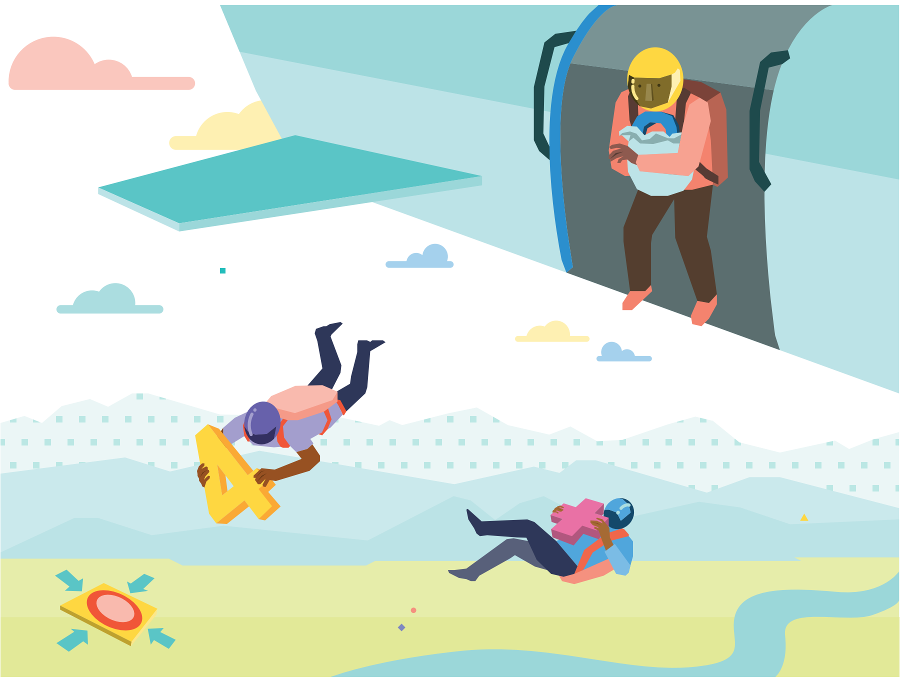
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 6**

Representing Subtraction

Let's subtract rational numbers.



Focus

Goals

1. **Language Goal:** Use a number line to determine the difference of rational numbers, and explain the reasoning. **(Speaking and Listening)**
2. **Language Goal:** Generalize that subtracting a number from a given value results in the same value as adding the additive inverse of that number. **(Speaking and Listening, Writing)**

Coherence

Today

Students extend their understanding of adding rational numbers to subtraction. They begin by applying the zero sum property to relate addition of opposites to subtracting a value from itself both on number lines and in equations. Students build on their observation that subtracting a value is the same as adding its opposite value (or *additive inverse*) when the sum or difference is zero to other values. They generalize this relationship, concluding that subtracting a number results in the same value as adding the *additive inverse*.

Previously

In Lessons 2–5, students generalized rules for adding rational numbers with and without the use of a number line.

Coming Soon

In Lesson 7, students connect their understanding of subtraction to determine the change in elevation and temperature.

Rigor

- Students build **conceptual understanding** of subtracting rational numbers using a number line.
- Students build **conceptual understanding** of the relationship between subtracting and adding rational numbers.
- Students **apply** their understanding of determining the sum of rational numbers to determining the difference of rational numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	13 min	12 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Operations, Part 2* (for display)
- Anchor Chart PDF, *Operations, Part 2* (answers)
- calculator (as needed)

Math Language Development

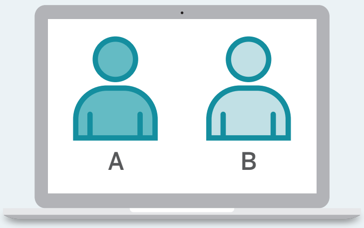
Review words

- *additive inverse*
- *negative numbers*
- *opposites*
- *positive numbers*
- *rational numbers*

Amps | Featured Activity

Activity 2
Digital Partner Problems

Students individually complete a series of problems digitally. After each problem, students check their response with their partner to come to a consensus before moving to the next one.



Building Math Identity and Community

Connecting to Mathematical Practices

Defining subtraction in terms of addition might confuse some students in Activity 1 and make them question whether this new skill is really necessary. Challenge students to focus on the patterns that they see when rewriting subtraction as addition. Remind them that they know that addition and subtraction are inverse operations and that any nonzero number has an opposite. Therefore, they are simply combining previous skills into one that for now looks more complicated.

Modifications to Pacing

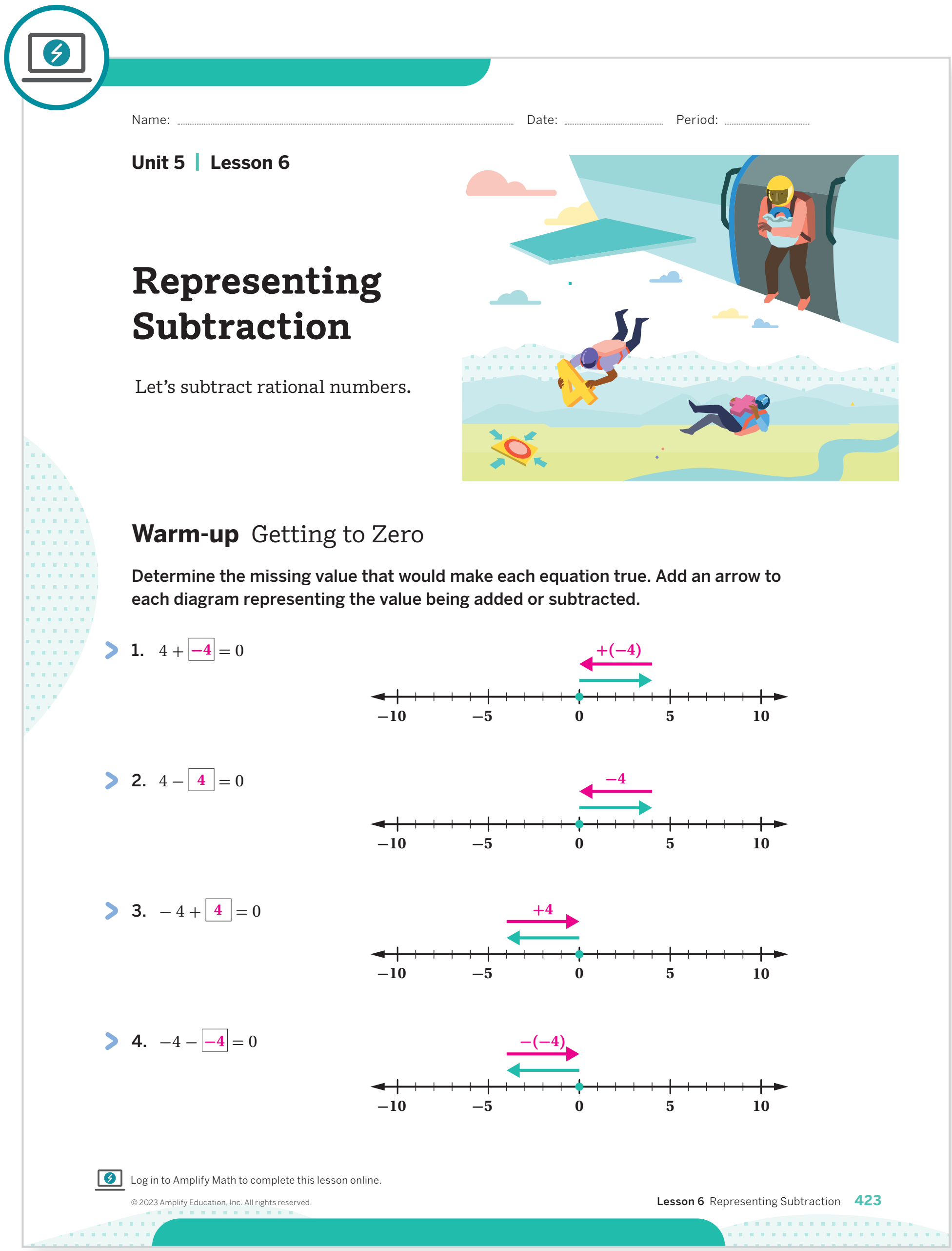
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, omit part a.
- In **Activity 2**, have students complete any three rows of the table. Encourage pairs who are prepared for more challenging examples to complete the final three rows.

Independent | 10 min

Warm-up Getting to Zero

Students apply their understanding of a sum or difference of 0 to determine the missing rational value in equations.



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Unit 5 | Lesson 6

Representing Subtraction

Let's subtract rational numbers.

Warm-up Getting to Zero

Determine the missing value that would make each equation true. Add an arrow to each diagram representing the value being added or subtracted.

1. $4 + \boxed{-4} = 0$

2. $4 - \boxed{4} = 0$

3. $-4 + \boxed{4} = 0$

4. $-4 - \boxed{-4} = 0$

Log in to Amplify Math to complete this lesson online.

Lesson 6 Representing Subtraction 423

1 Launch

Activate students' prior knowledge by asking, "How do you use number lines to represent addition of rational numbers?"

2 Monitor

Help students get started by asking, "If two numbers have a sum of zero, what do you know about their signs?"

Look for points of confusion:

- **Thinking that the missing value for Problem 4 is 4.** Ask, "How is Problem 4 similar or different from Problem 2?"

Look for productive strategies:

- Recognizing that an arrow of the same length, but opposite direction, needs to be added to each diagram to return to zero.

3 Connect

Display samples of student diagrams.

Have students share their method for determining the missing value in each equation.

Ask:

- "In general, what is true about two values whose sum is zero?" They are opposites.
- "In general, for any value, what do you need to subtract from it to result in zero?" The same value.
- "Which diagrams are drawn the same?"

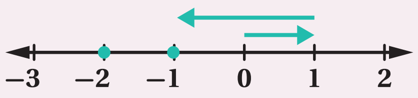
Highlight that Problems 1 and 2 model the same relationship, and Problems 3 and 4 model the same relationship.

Power-up

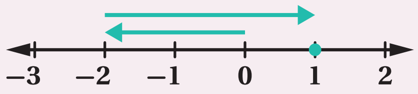
To power up students' ability to write equations to match an arrow diagram, have students complete:

Match each diagram with the equation it models.

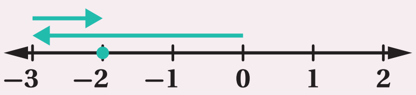
a.



b.



c.



b $-2 + 3 = 1$

a $1 + (-2) = -1$

c $-3 + 1 = -2$

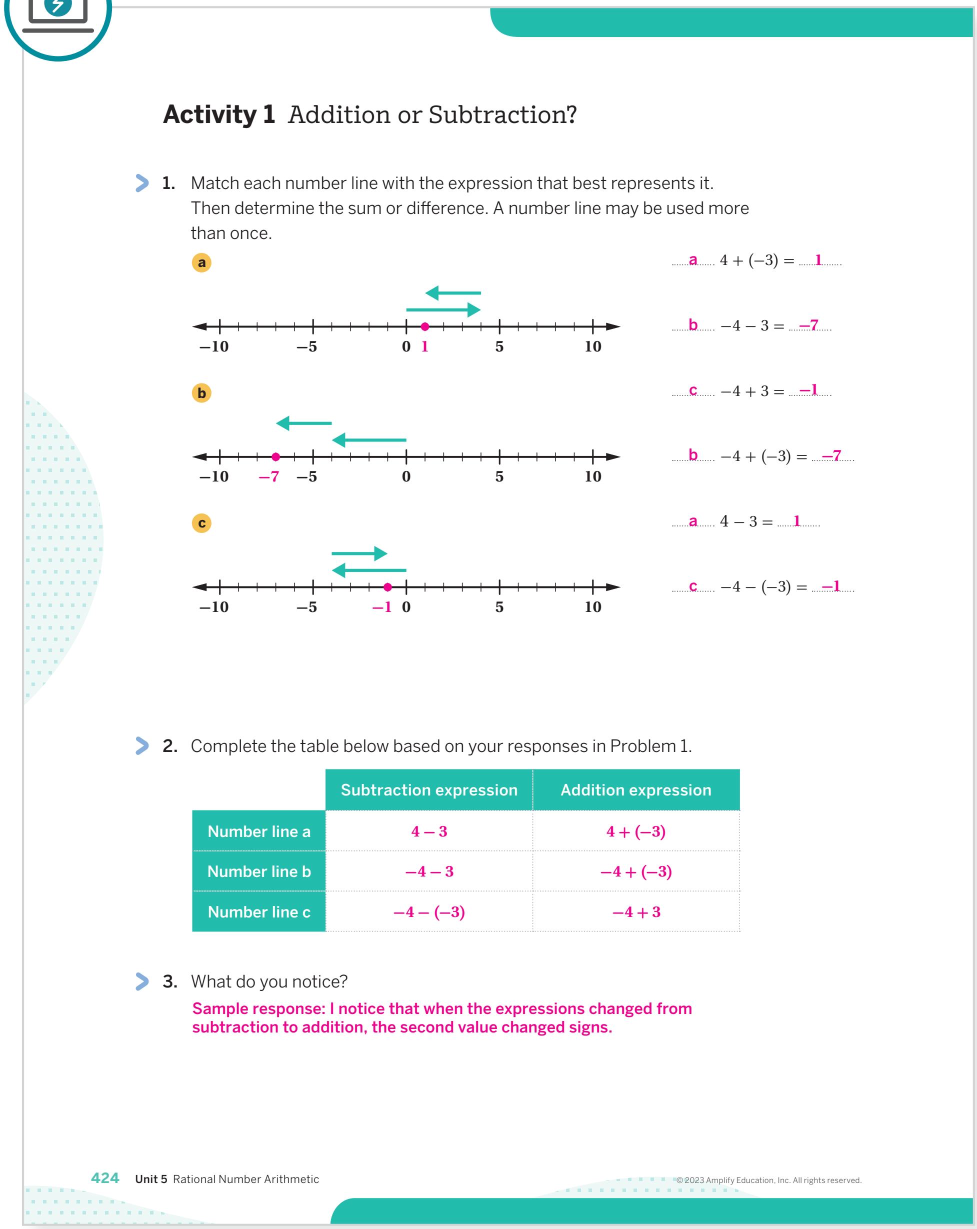
Use: Before Activity 1.

Informed by: Performance on Lesson 5, Practice Problem 5.

Small Groups | 13 min

Activity 1 Addition or Subtraction?

Students use number lines to make connections between equivalent addition and subtraction expressions involving rational numbers.



Activity 1 Addition or Subtraction?

1. Match each number line with the expression that best represents it. Then determine the sum or difference. A number line may be used more than once.

a $4 + (-3) =$ 1

b $-4 - 3 =$ −7

c $-4 + 3 =$ −1

b $-4 + (-3) =$ −7

a $4 - 3 =$ 1

c $-4 - (-3) =$ −1

2. Complete the table below based on your responses in Problem 1.

	Subtraction expression	Addition expression
Number line a	$4 - 3$	$4 + (-3)$
Number line b	$-4 - 3$	$-4 + (-3)$
Number line c	$-4 - (-3)$	$-4 + 3$

3. What do you notice?

Sample response: I notice that when the expressions changed from subtraction to addition, the second value changed signs.

424 Unit 5 Rational Number Arithmetic

1 Launch

Explain that each number line should correspond to two expressions.

2 Monitor

Help students get started by suggesting they first match the addition expressions to number lines.

Look for points of confusion:

- **Thinking that $-4 - (-3)$ is represented by Diagram c and not b.** Ask, "How is $-4 - 3$ similar to or different from $-4 - (-3)$? How are their representations on the number line the same or different?"

Look for productive strategies:

- Comparing the number lines in the Warm-up to the number lines in Problem 1.

3 Connect

Display the completed table for Problem 2.

Have groups of students share how they matched each expression to a number line.

Highlight that the corresponding diagrams have the same initial value, but when the operation is inverse (addition versus subtraction) the second value is also inverse (opposite sign).

Ask:

- "How would you represent $8 - 10$ using addition?" $8 + (-10)$
- "How would you represent $-6 + 8$ using subtraction?" $-6 - (-8)$
- "Can you think of any real-world examples of when you would subtract a negative value?" Sample response: Removing a debt.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display these questions that students can ask themselves as they complete the matching in Problem 1.

- What number represents the starting number?
- In which direction is addition of a positive number represented? Addition of a negative number?
- In which direction is subtraction of a positive number represented? Subtraction of a negative number?

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement about adding and subtracting numbers that reflects a possible misunderstanding, such as "$8 - 10$ has the same value as $-8 + 10$, because I can change subtraction to addition of the opposite." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

English Learners

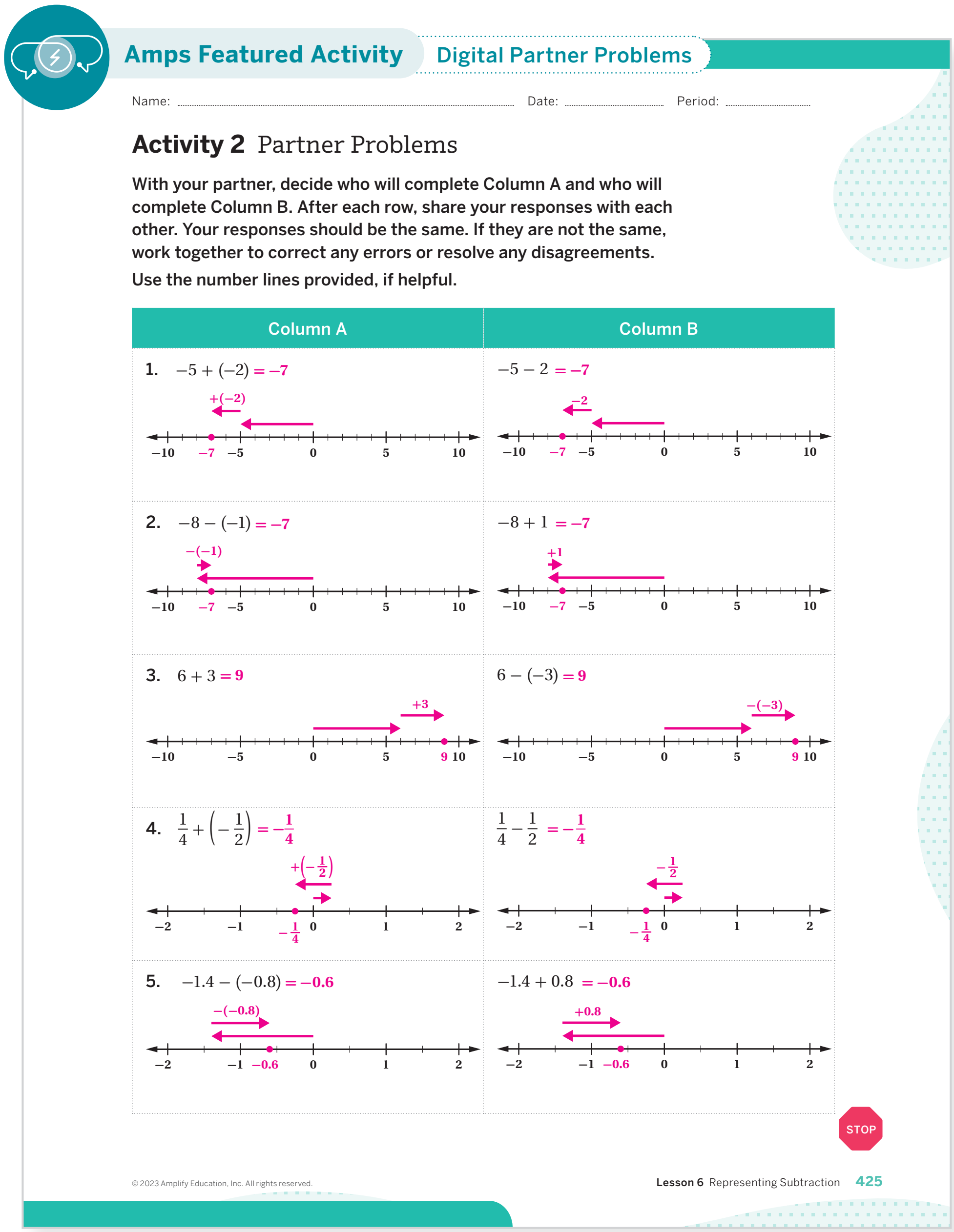
Annotate a clear example showing subtraction being rewritten as addition of the opposite, where the second addend becomes the opposite (or additive inverse) and the first addend stays the same.

Pairs | 12 min

Activity 2 Partner Problems

Students solidify their understanding of equivalent addition and subtraction problems by comparing their solutions with a partner who has the inverse representation.

Amps Featured Activity Digital Partner Problems



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Activity 2 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with each other. Your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

Use the number lines provided, if helpful.

Column A	Column B
1. $-5 + (-2) = -7$	$-5 - 2 = -7$
2. $-8 - (-1) = -7$	$-8 + 1 = -7$
3. $6 + 3 = 9$	$6 - (-3) = 9$
4. $\frac{1}{4} + \left(-\frac{1}{2}\right) = -\frac{1}{4}$	$\frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$
5. $-1.4 - (-0.8) = -0.6$	$-1.4 + 0.8 = -0.6$

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Lesson 6 Representing Subtraction 425

1 Launch

Explain and conduct the ***Partner Problems*** routine.

2 Monitor

Help students get started by suggesting they use the provided number lines to help them to make sense of their problems.

Look for points of confusion:

- **Changing subtraction to addition of the same value.** Ask students if that relationship is true for a difference with simple whole numbers. For example, if 3 – 2 is the equivalent to 3 + 2. Ask, "What would be equivalent to 3 – 2?"

Look for productive strategies:

- Changing subtraction expressions into adding the inverse value.

3 Connect

Display the partner problems.

Have pairs of students share their reasoning for the last two rows of the table.

Highlight that for any subtraction expression, it can be rewritten as addition using the *additive inverse* of the second term. This means students will not need to learn a new procedure for subtracting rational numbers. They can rewrite the problem using addition, and then use the rules they already know.

Ask, "How would you determine the value of $-10 - (-5)$?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them only complete calculations for Problems 1–3, and then in Problems 4 and 5, only determine whether the sum or difference will be positive or negative. This will still allow them access to the main goal of the activity.

Extension: *Math Enrichment*

Have students rewrite the following expression so that it only uses the operation of addition.

$-11 - 3\frac{1}{2} + (-4.6) - 1\frac{2}{3} + 9$

$-11 + \left(-3\frac{1}{2}\right) + (-4.6) + \left(-1\frac{2}{3}\right) + 9$

Math Language Development

MLR8: Discussion Supports—Revoicing

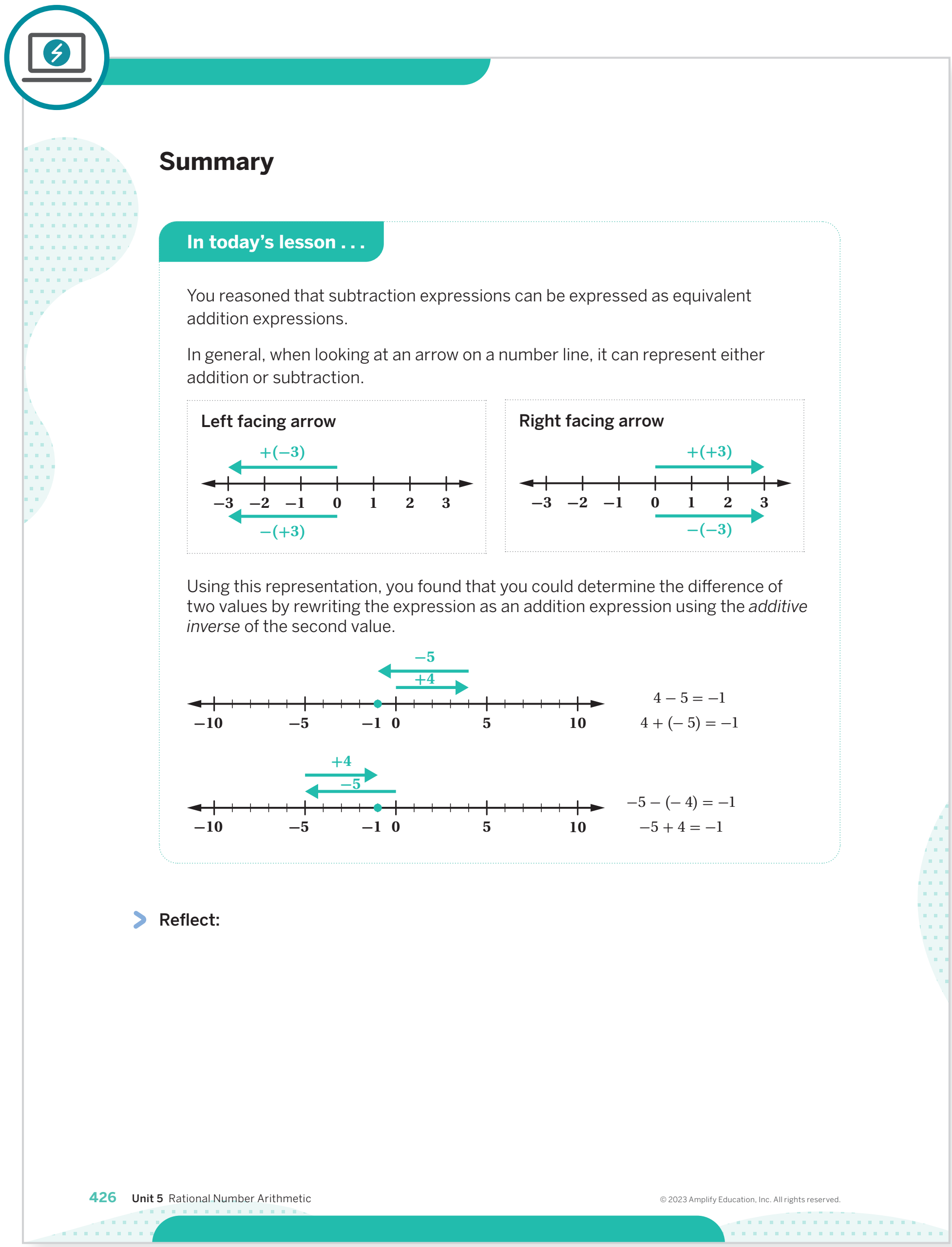
While students work, encourage partners to revoice each other's reasoning before correcting or resolving disagreements or errors. This will help foster productive mathematical discussions. Display sentence frames and/or questions students could ask each other, such as:

- "I hear you say that subtracting _____ is represented by _____ but when I use addition, I find that . . ."
- "What ideas do you have?"
- "Do you agree or disagree?"
- "Can you explain what you mean by . . .?"

Whole Class | 5 min

Summary

Review and synthesize how to determine the difference of two rational values by rewriting a subtraction expression as an addition expression using the additive inverse.



Summary

In today's lesson . . .

You reasoned that subtraction expressions can be expressed as equivalent addition expressions.

In general, when looking at an arrow on a number line, it can represent either addition or subtraction.

Using this representation, you found that you could determine the difference of two values by rewriting the expression as an addition expression using the *additive inverse* of the second value.

$4 - 5 = -1$

$4 + (-5) = -1$

$-5 - (-4) = -1$

$-5 + 4 = -1$

Reflect:

426 Unit 5 Rational Number Arithmetic

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Synthesize

Display the Anchor Chart PDF, *Operations With Rational Numbers, Part 2.* Obtain the missing information from your class and complete the chart together.

Ask, "What is the relationship between addition and subtraction of rational numbers?"

Highlight that subtracting a value is the same as adding the additive inverse of that value. This can be represented on a number line and in an expression.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

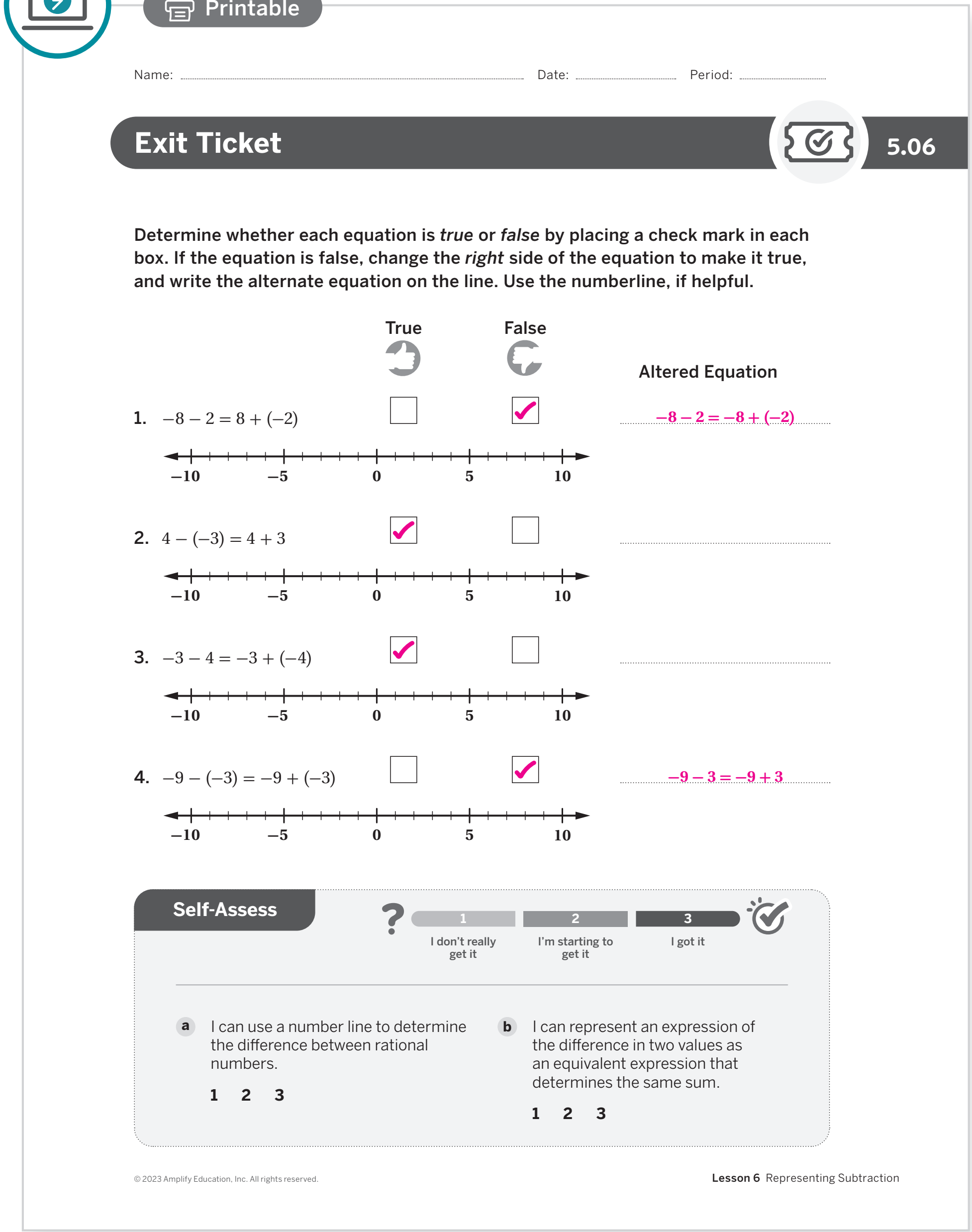
- "How do you represent subtraction of rational numbers on a number line?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding subtracting rational numbers by determining whether two expressions showing addition and subtraction are equivalent.

Printable



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Exit Ticket 5.06

Determine whether each equation is *true* or *false* by placing a check mark in each box. If the equation is false, change the *right* side of the equation to make it true, and write the alternate equation on the line. Use the numberline, if helpful.

	True	False	Altered Equation
1. $-8 - 2 = 8 + (-2)$	☐	☑	$-8 - 2 = -8 + (-2)$
2. $4 - (-3) = 4 + 3$	☑	☐	
3. $-3 - 4 = -3 + (-4)$	☑	☐	
4. $-9 - (-3) = -9 + (-3)$	☐	☑	$-9 - 3 = -9 + 3$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can use a number line to determine the difference between rational numbers.
1 2 3

b I can represent an expression of the difference in two values as an equivalent expression that determines the same sum.
1 2 3

Success looks like . . .

- **Language Goal:** Using a number line to determine the difference of rational numbers, and explaining the reasoning. **(Speaking and Listening)**
  - » Using a number line to determine whether each difference of rational numbers is true and changing the equation, if needed, in Problems 1–4.
- **Language Goal:** Generalizing that subtracting a number from a given value results in the same value as adding the additive inverse of that number. **(Speaking and Listening, Writing)**
  - » Determining whether the correct additive inverse is added in Problems 1–4.

Suggested next steps

If students incorrectly identify the equations that are true or false, consider:

- Reviewing Problem 1 from Activity 1.
- Asking, "How do you rewrite a subtraction expression as an equivalent addition expression?"
- Assigning Practice Problems 1 and 2.

Professional Learning

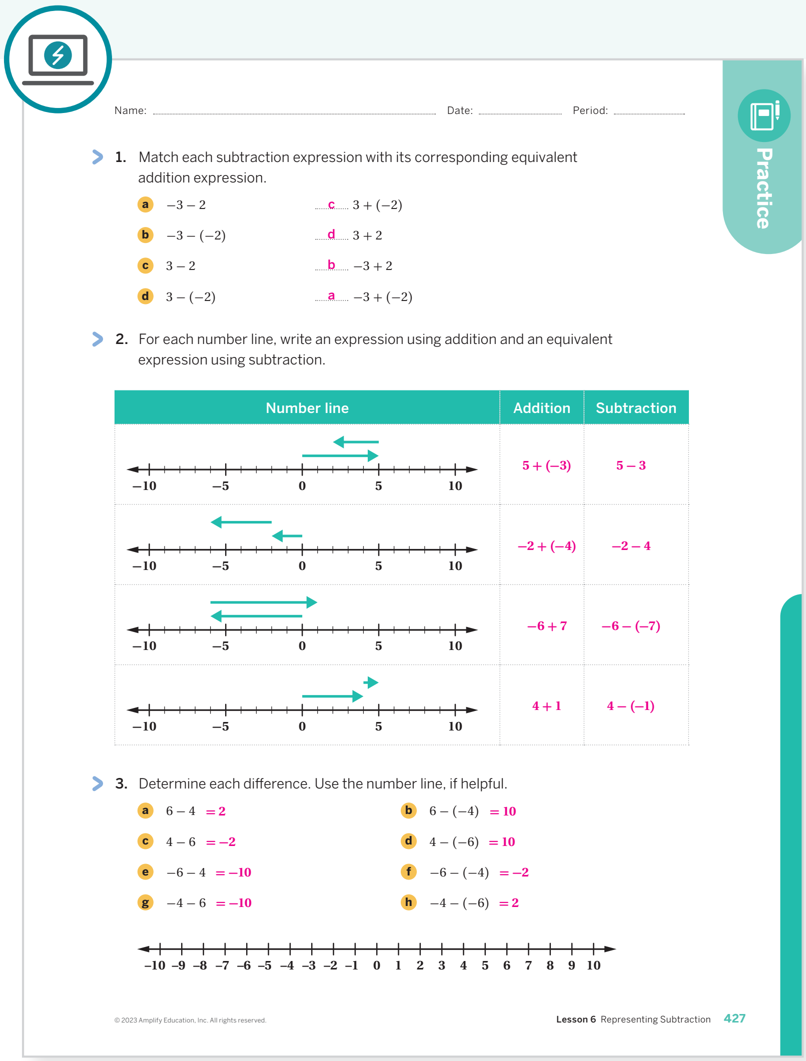
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion in the ***Partner Problems*** routine, how did you encourage each student to listen to one another's strategies?
- In this lesson, students represented subtraction of integers on a number line. How did that build on the earlier work students did with adding rational numbers? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Match each subtraction expression with its corresponding equivalent addition expression.

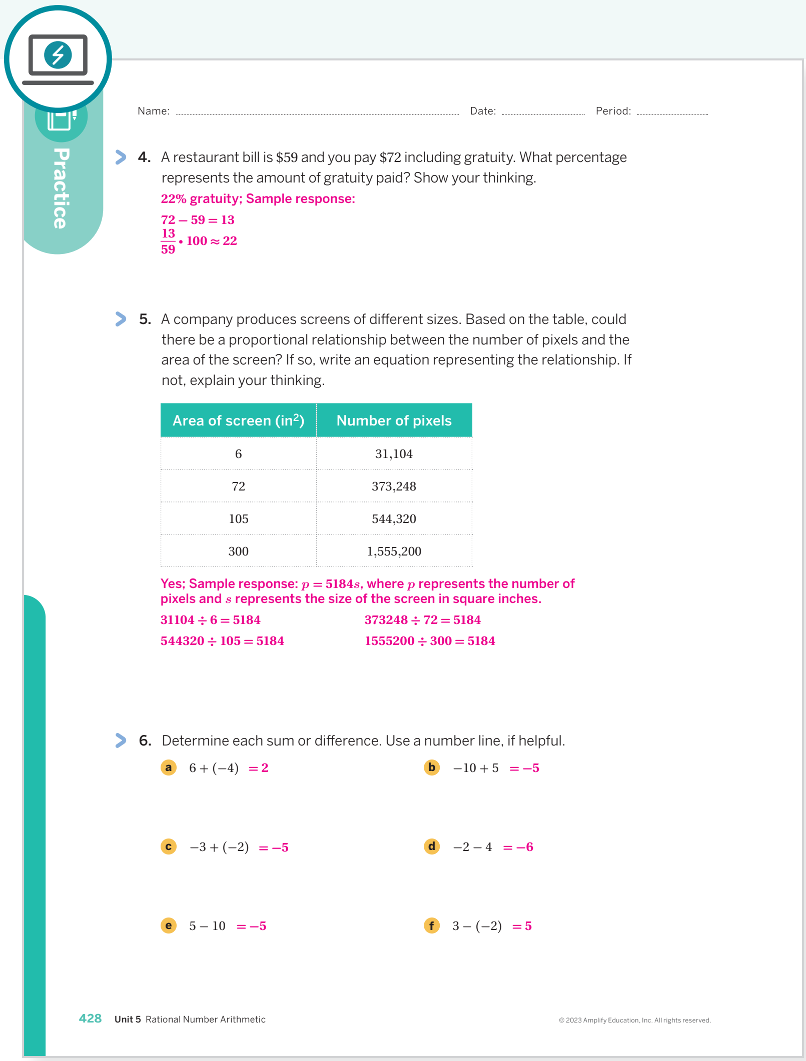
- **a** $-3-2$ — c $3+(-2)$
- **b** $-3-(-2)$ — d $3+2$
- **c** $3-2$ — b $-3+2$
- **d** $3-(-2)$ — a $-3+(-2)$

2. For each number line, write an expression using addition and an equivalent expression using subtraction.

Number line	Addition	Subtraction
	$5+(-3)$	$5-3$
	$-2+(-4)$	$-2-4$
	$-6+7$	$-6-(-7)$
	$4+1$	$4-(-1)$

3. Determine each difference. Use the number line, if helpful.

- **a** $6-4$ $=2$
- **b** $6-(-4)$ $=10$
- **c** $4-6$ $=-2$
- **d** $4-(-6)$ $=10$
- **e** $-6-4$ $=-10$
- **f** $-6-(-4)$ $=-2$
- **g** $-4-6$ $=-10$
- **h** $-4-(-6)$ $=2$



Name: Date: Period:

4. A restaurant bill is $59 and you pay $72 including gratuity. What percentage represents the amount of gratuity paid? Show your thinking.

22% gratuity; Sample response:

$72-59=13$

$\frac{13}{59}\cdot 100\approx 22$

5. A company produces screens of different sizes. Based on the table, could there be a proportional relationship between the number of pixels and the area of the screen? If so, write an equation representing the relationship. If not, explain your thinking.

Area of screen (in²)	Number of pixels
6	31,104
72	373,248
105	544,320
300	1,555,200

Yes; Sample response: $p=5184s$, where p represents the number of pixels and s represents the size of the screen in square inches.

$31104\div 6=5184$ $373248\div 72=5184$

$544320\div 105=5184$ $1555200\div 300=5184$

6. Determine each sum or difference. Use a number line, if helpful.

- **a** $6+(-4)$ $=2$
- **b** $-10+5$ $=-5$
- **c** $-3+(-2)$ $=-5$
- **d** $-2-4$ $=-6$
- **e** $5-10$ $=-5$
- **f** $3-(-2)$ $=5$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 8	2
	5	Unit 2 Lesson 5	2
Formative ⚡	6	Unit 5 Lesson 7	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

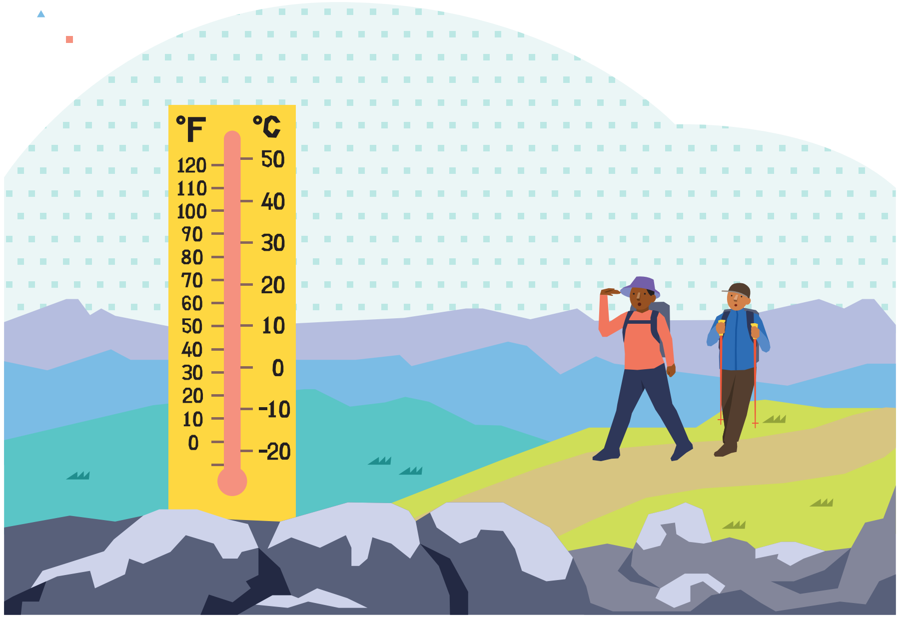
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 7**

Subtracting Rational Numbers (Part 1)

Let’s determine the difference of rational numbers.



Focus

Goals

1. Recognize that the difference of two numbers can be positive or negative depending on the order given in the expression.
2. **Language Goal:** Subtract rational numbers, and explain the reasoning. **(Speaking and Listening)**
3. **Language Goal:** Compare subtraction expressions that give the same numbers in the reverse order. **(Speaking and Listening)**

Coherence

Today

Students apply their understanding of subtracting rational numbers to compare differences and notice that, unlike addition, the order of subtraction changes the sign of the difference. Students apply this understanding to determine the change in temperature over time representing increases as a positive change and decreases as a negative change.

Previously

In Lesson 6, students discovered that a subtraction expression can be written as an equivalent addition expression using the additive inverse of the second term.

Coming Soon

In Lesson 8, students will apply their understanding of subtraction to reason about difference and distance on number lines and on the coordinate plane.

Rigor

- Students gain **fluency** in subtraction of rational numbers by comparing subtraction expressions with the same values in reverse order.
- Students **apply** their understanding of subtraction of rational values to determine change as a rational value.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Are you ready for more?* (as needed)
- Graphic Organizer PDF, *Blank Number Lines* (as needed)

Math Language Development

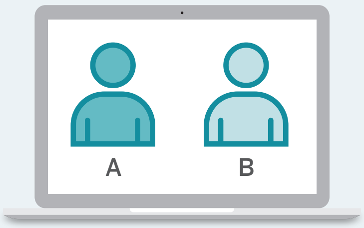
Review words

- *additive inverse*
- *commutative property*
- *difference*

Amps | Featured Activity

Activity 1
Digital Partner Problems

Students individually complete a series of problems digitally. After each problem, students check their response with their partner to come to a consensus before moving to the next one.



Building Math Identity and Community

Connecting to Mathematical Practices

While working with a partner to determine whether the differences are equivalent in Activity 1, students might not take the time to be precise when writing subtraction as addition, which may cause conflict with their partner. Students should recognize that it is beneficial to both of them to cooperate and help each other when needed. They should both be willing to provide help if it guides the other person, and thus their team, to success.

Modifications to Pacing

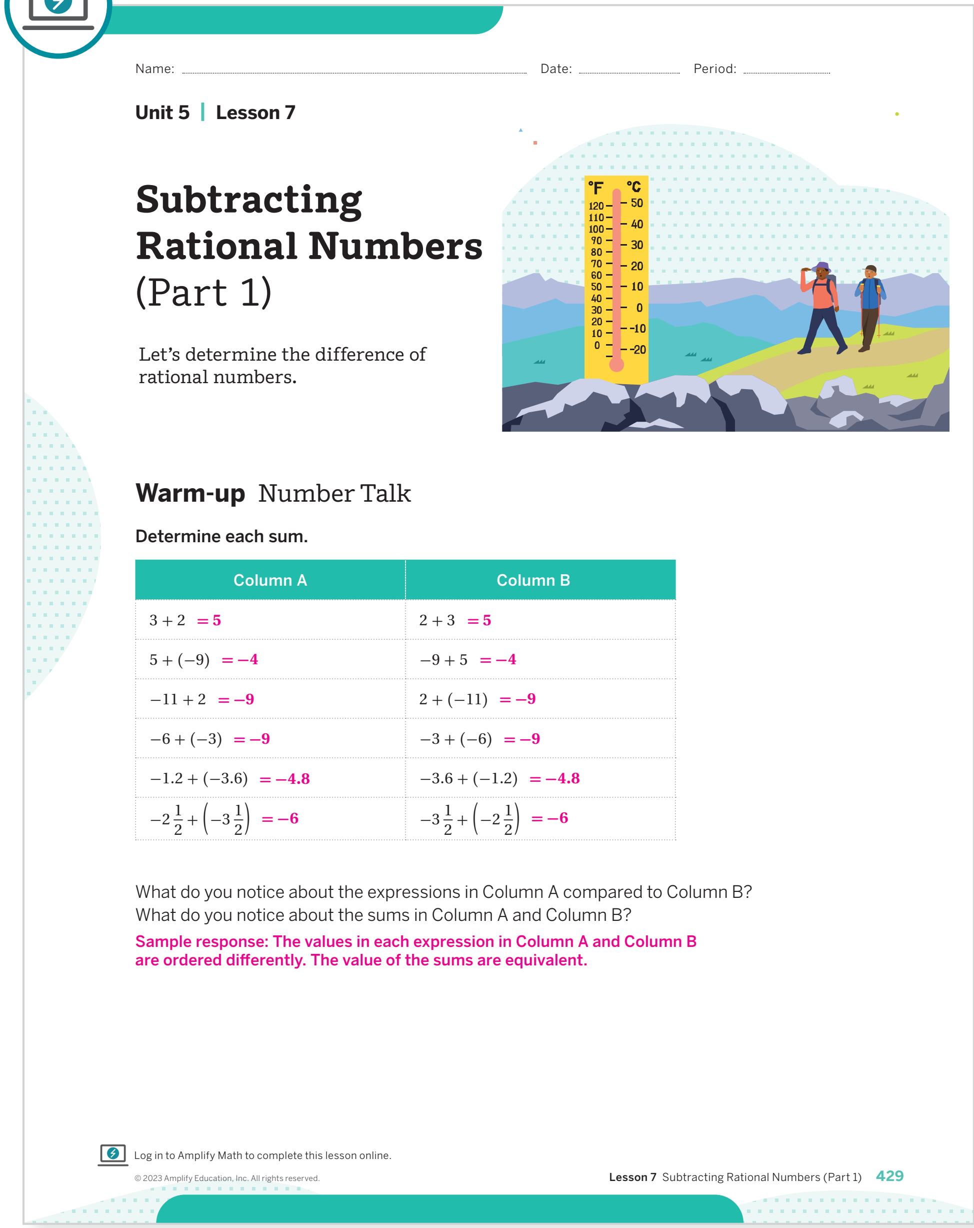
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up,** have students only complete the first four rows of the table.
- In **Activity 1,** have students complete any three rows of the table. Encourage students who are ready for more challenging examples to complete the last two rows.

Pairs | 10 min

Warm-up Number Talk

Students compare the sums of pairs of values expressed in the reversed order.



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Unit 5 | Lesson 7

Subtracting Rational Numbers (Part 1)

Let's determine the difference of rational numbers.

Warm-up Number Talk

Determine each sum.

Column A	Column B
$3 + 2$ $= 5$	$2 + 3$ $= 5$
$5 + (-9)$ $= -4$	$-9 + 5$ $= -4$
$-11 + 2$ $= -9$	$2 + (-11)$ $= -9$
$-6 + (-3)$ $= -9$	$-3 + (-6)$ $= -9$
$-1.2 + (-3.6)$ $= -4.8$	$-3.6 + (-1.2)$ $= -4.8$
$-2\frac{1}{2} + \left(-3\frac{1}{2}\right)$ $= -6$	$-3\frac{1}{2} + \left(-2\frac{1}{2}\right)$ $= -6$

What do you notice about the expressions in Column A compared to Column B? What do you notice about the sums in Column A and Column B?

Sample response: The values in each expression in Column A and Column B are ordered differently. The value of the sums are equivalent.

Log in to Amplify Math to complete this lesson online.

Lesson 7 Subtracting Rational Numbers (Part 1) 429

1 Launch

Conduct the *Number Talk* routine.

2 Monitor

Help students get started by covering up all but the first row of expressions. Suggest they uncover one row at a time.

Look for points of confusion:

- **Thinking that the sign of the first addend determines the sign of the sum.** Remind students that the sign of the sum matches the sign of the addend with the greater absolute value.
- **Adding the magnitudes regardless of the original signs of each value.** Ask students to represent $5 + (-9)$ on a number line and to compare their sum from the number line with the sum from adding the magnitudes.

Look for productive strategies:

- Splitting the expressions in Columns A and B among their group, and then comparing their sums.

3 Connect

Display the table from the Warm-up in the Student Edition.

Have students share what they notice about each pair of sums.

Highlight that, by the Commutative Property of Addition, the sum of two numbers is the same regardless of the order of the addends.

Ask, "Do you think the commutative property holds true for subtraction?" Conduct the *Poll the Class* routine to assess student thinking. Do not ask students to explain their thinking, instead inform the class that they will be answering this question in the next activity.

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share what they noticed, press them for details in their reasoning by asking:

- "Does the order of the addends matter when adding two numbers?"
- "How do you know?"
- "Does it always work this way?"
- "Can you provide an example or a counterexample?"

Highlight student reasoning that demonstrates the importance of mathematical structure when adding negative numbers.

Power-up

To power up students' ability to write equations to determine the sum and difference of integers, have students complete:

Recall that subtracting in an integer is the same as adding its opposite. For example, $-3 - 4 = -3 + (-4)$.

Determine each sum or difference. Use a number line if helpful.

1. $-9 + 5 = -4$
2. $5 + (-9) = -4$
3. $5 - 9 = -4$

Use: Before the Warm-up.

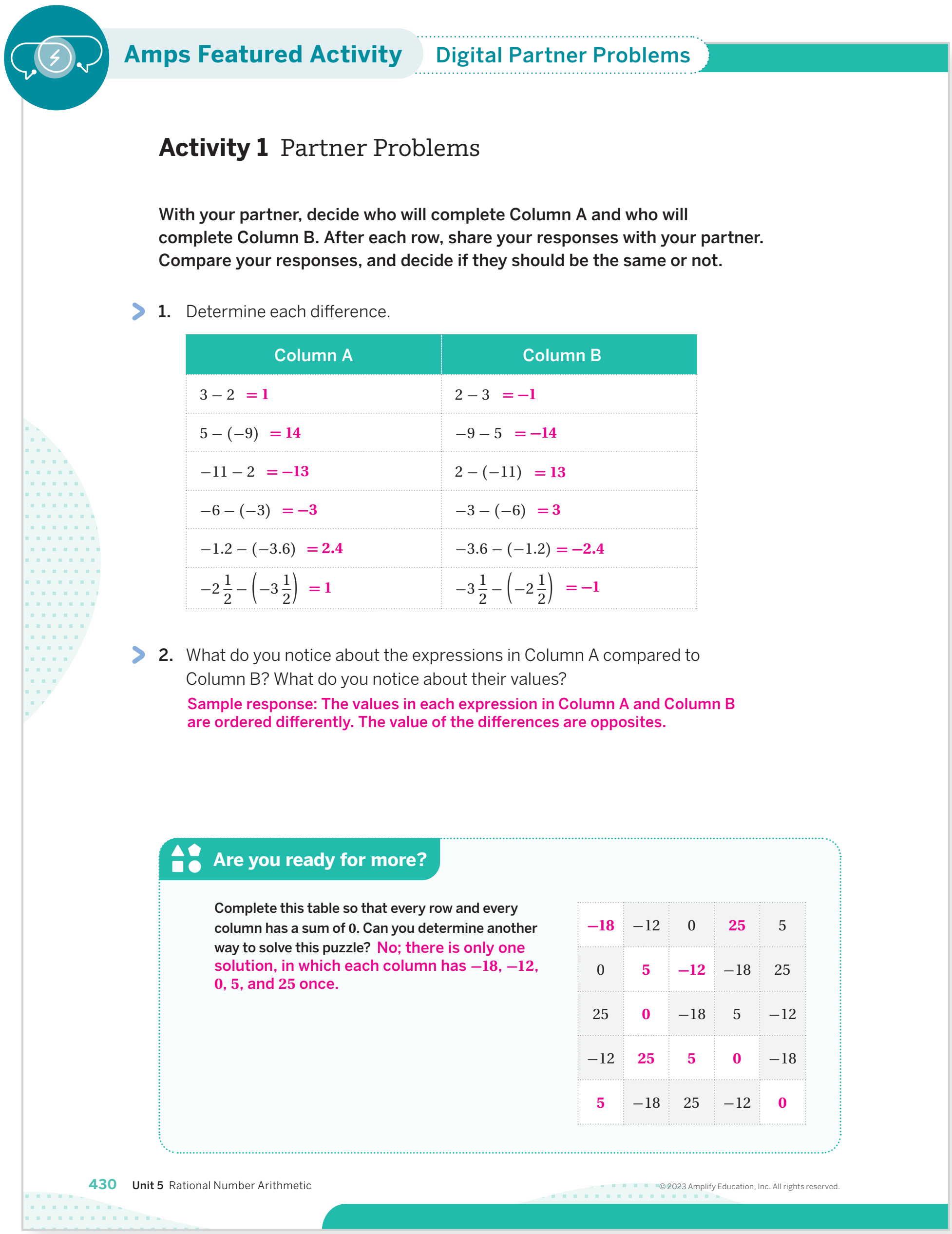
Informed by: Performance on Lesson 6, Practice Problem 6.

Small groups | 15 min

Activity 1 Partner Problems

Students compare differences of pairs of values in the reversed order to determine whether the differences are equivalent.

Amps Featured Activity Digital Partner Problems



Activity 1 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with your partner. Compare your responses, and decide if they should be the same or not.

1. Determine each difference.

Column A	Column B
$3 - 2$ **$= 1$**	$2 - 3$ **$= -1$**
$5 - (-9)$ **$= 14$**	$-9 - 5$ **$= -14$**
$-11 - 2$ **$= -13$**	$2 - (-11)$ **$= 13$**
$-6 - (-3)$ **$= -3$**	$-3 - (-6)$ **$= 3$**
$-1.2 - (-3.6)$ **$= 2.4$**	$-3.6 - (-1.2)$ **$= -2.4$**
$-2\frac{1}{2} - \left(-3\frac{1}{2}\right)$ **$= 1$**	$-3\frac{1}{2} - \left(-2\frac{1}{2}\right)$ **$= -1$**

2. What do you notice about the expressions in Column A compared to Column B? What do you notice about their values?
Sample response: The values in each expression in Column A and Column B are ordered differently. The value of the differences are opposites.

Are you ready for more?

Complete this table so that every row and every column has a sum of 0. Can you determine another way to solve this puzzle? **No; there is only one solution, in which each column has −18, −12, 0, 5, and 25 once.**

−18	−12	0	**25**	5
0	**5**	**−12**	−18	25
25	**0**	−18	5	−12
−12	**25**	**5**	**0**	−18
5	−18	25	−12	**0**

430 Unit 5 Rational Number Arithmetic

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1 Launch

Conduct the *Partner Problems* routine, explaining to students that in this modified version, they should determine whether their differences are equivalent.

2 Monitor

Help students get started by suggesting that they rewrite the subtraction expressions as addition expressions using the additive inverse of the second term.

Look for points of confusion:

- **Thinking that the differences have to be equal because they have the same values.** Have students sketch their differences from the first row on a number line and compare. Ask, "What do you notice about the two differences?"
- **Dropping a negative sign when rewriting subtraction expressions as addition.** Remind students that any expression $a - b$ is equivalent to adding the additive inverse, or $a + (-b)$.

Look for productive strategies:

- Comparing the equivalent addition expressions and noticing that when they change the order of subtraction that the addends in the addition expressions have opposite values.

3 Connect

Display the table from the Student Edition.

Have students share what they noticed about the differences between Columns A and B.

Ask, "How could you represent the expressions in the first row as addition?" $3 + (-2)$ and $2 + (-3)$

Highlight that, unlike addition, the commutative property does not apply to subtraction. Compare the addition expressions for the first row, noting that they are adding opposite addends when the order of subtraction is reversed so the differences are opposite values.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide copies of the Graphic Organizer PDF, *Blank Number Lines* for students to choose to use to help them organize their thinking and make sense of subtracting rational numbers.

Extension: *Math Enrichment*

Have students complete the table puzzles from the *Are you ready for more?* PDF.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

During the Connect, as students share what they noticed about the differences between Columns A and B, display the following prompt:

- "Changing the order of the numbers in a subtraction expression does/does not change the value because . . ."

Ask students to complete the prompt, encouraging them to justify their reasoning using examples and nonexamples.

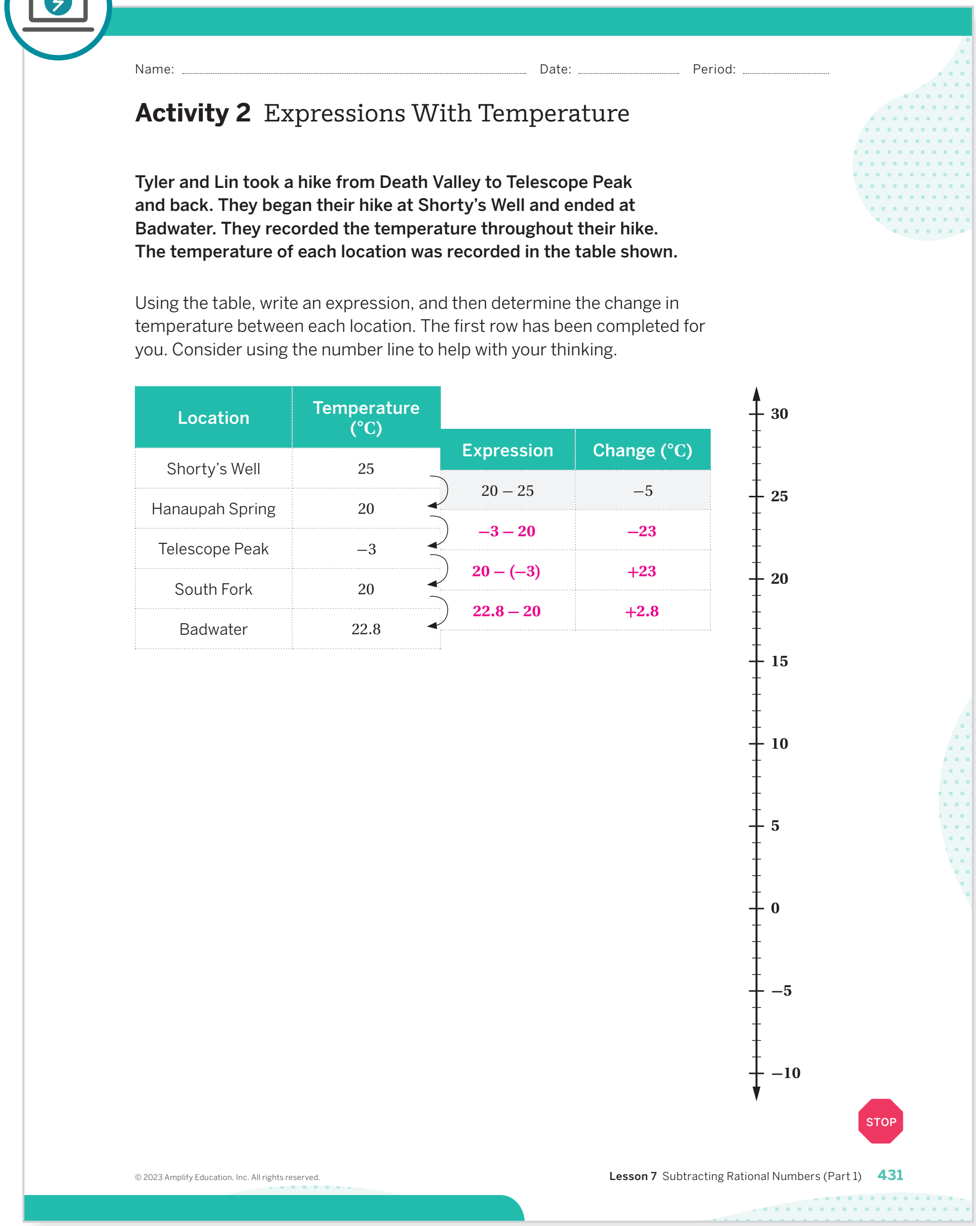
English Learners

Encourage students to respond to each other's ideas using the frames, "I agree or I disagree with ____ because . . ."

Pairs | 10 min

Activity 2 Expressions With Temperature

Students compare temperatures to connect the relationship between differences in rational numbers and how far apart they are on the number line.



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Activity 2 Expressions With Temperature

Tyler and Lin took a hike from Death Valley to Telescope Peak and back. They began their hike at Shorty's Well and ended at Badwater. They recorded the temperature throughout their hike. The temperature of each location was recorded in the table shown.

Using the table, write an expression, and then determine the change in temperature between each location. The first row has been completed for you. Consider using the number line to help with your thinking.

Location	Temperature (°C)
Shorty's Well	25
Hanaupah Spring	20
Telescope Peak	−3
South Fork	20
Badwater	22.8

Expression	Change (°C)
20 − 25	−5
−3 − 20	−23
20 − (−3)	+23
22.8 − 20	+2.8

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Lesson 7 Subtracting Rational Numbers (Part 1) 431

1 Launch

Activate prior knowledge by asking, "What is the first row in this table modeling?" Give students a chance to discuss with their partners prior to discussing as a class. Model the change on the number line by drawing an arrow from 25 to 20 and labeling it with −5.

2 Monitor

Help students get started by having them place one finger on the temperature for Hanaupah Spring and second finger on the temperature for Telescope Park on their number line to help them to make sense of the chart.

Look for points of confusion:

- **Giving the change as only a positive value.** Bring their attention back to the first row of the table. Ask, "Why was this change expressed as a negative value?"
- **Reversing the order when determining change.** Remind students that the expression to determine change is *final value − initial value*.

Look for productive strategies:

- Rewriting each subtraction expression as addition to determine the change in temperature.

3 Connect

Have students share their strategies for determining the change in temperature between locations.

Ask, "What does it mean when the change in temperature is positive? Negative?"

Highlight that the change in temperature reflects how far apart the two values are as well as if the temperature was increasing or decreasing. Draw attention to Rows 2 and 3 of the table to highlight how the order of subtraction changed the sign of the change in temperature.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how the first row in the second table was completed to serve as a reference for students. Ask, "Why is 20 the first number in the expression, and not 25? Why is the change negative, not positive?"

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present an incorrect statement that reflects a potential misunderstanding such as, "If the final temperature is −3°C and the starting temperature is 20°C, then the difference is 17°C." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking." Listen for students who reason that −3 and 20 are on opposite sides of zero on the number line.
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

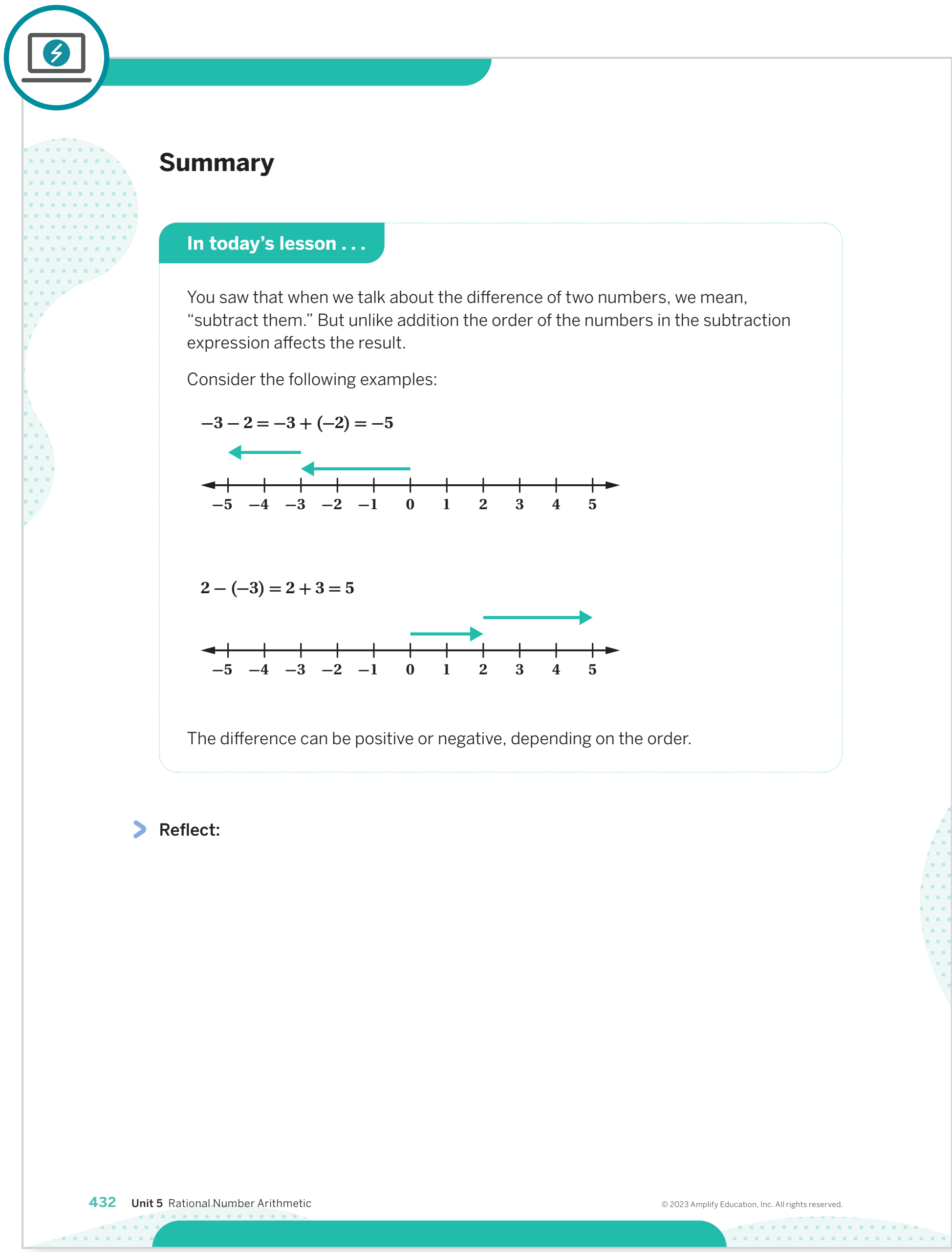
English Learners

Listen for ways students use the words, "negative," "difference," and "subtract," and clarify the meanings of these words.

Whole Class | 5 min

Summary

Review and synthesize how the order of subtraction affects the difference of two values.



Summary

In today's lesson . . .

You saw that when we talk about the difference of two numbers, we mean, “subtract them.” But unlike addition the order of the numbers in the subtraction expression affects the result.

Consider the following examples:

$-3 - 2 = -3 + (-2) = -5$

$2 - (-3) = 2 + 3 = 5$

The difference can be positive or negative, depending on the order.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Ask:

- “Does changing the order of the values in an addition expression change the sum? Why?”
- “Does changing the order of the values in a subtraction expression change the difference? Why?”

Highlight that the difference between two numbers can be positive or negative depending on the order. If the order is changed, then the difference becomes the opposite value.

Have students share real-world scenarios where they would want to determine the change between two rational values and would want to know whether the change was positive or negative.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

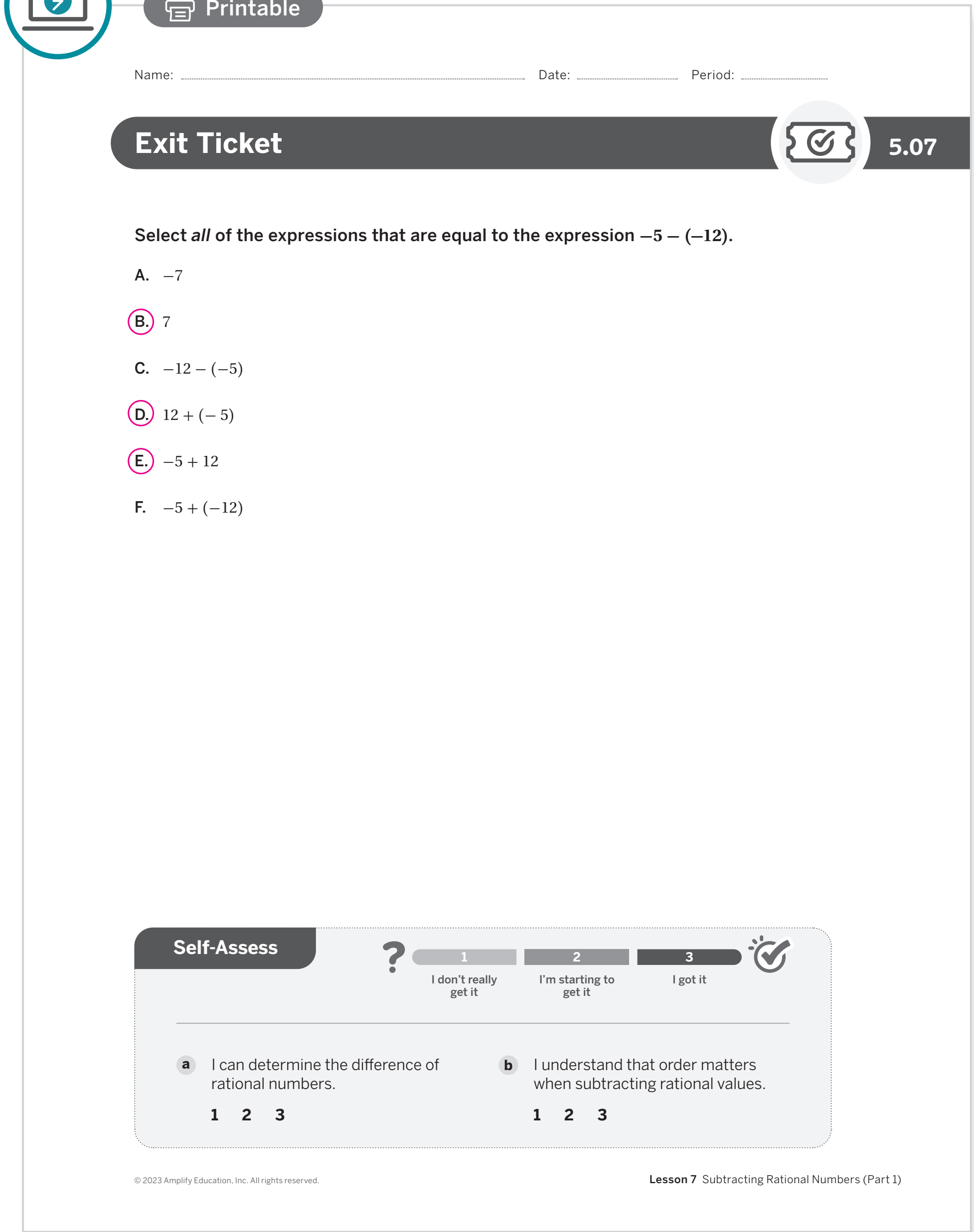
- “How did using the number line in Activity 2 help you determine the change in temperature?”

Independent | 5 min

Exit Ticket

Students demonstrate their understanding determining the difference of two rational numbers by identifying equivalent subtraction and addition expressions.

Printable



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Exit Ticket 5.07

Select *all* of the expressions that are equal to the expression $-5-(-12)$.

A. -7

[✓ B.] 7

C. $-12-(-5)$

[✓ D.] $12+(-5)$

[✓ E.] $-5+12$

F. $-5+(-12)$

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I can determine the difference of rational numbers.
1 2 3

b I understand that order matters when subtracting rational values.
1 2 3

Lesson 7 Subtracting Rational Numbers (Part 1)

Success looks like . . .

- **Goal:** Recognizing that the difference of two numbers can be positive or negative depending on the order given in the expression.
- **Language Goal:** Subtracting rational numbers, and explaining the reasoning. **(Speaking and Listening)**
  - » Selecting all the expressions equal to $-5-(-12)$.
- **Language Goal:** Comparing subtraction expressions that give the same numbers in the reverse order. **(Speaking and Listening)**
  - » Recognizing that $12+(-5)$ is equal to $-5-(-12)$.

Suggested next steps

If students select Choice C, consider:

- Reviewing Activity 1.
- Assigning Practice Problem 1.
- Asking, "What is $5-3$? What about $3-5$?"

If students select Choice E, but not Choice D, consider:

- Reviewing the Warm-up.
- Asking, "Does order matter when adding?"

Professional Learning

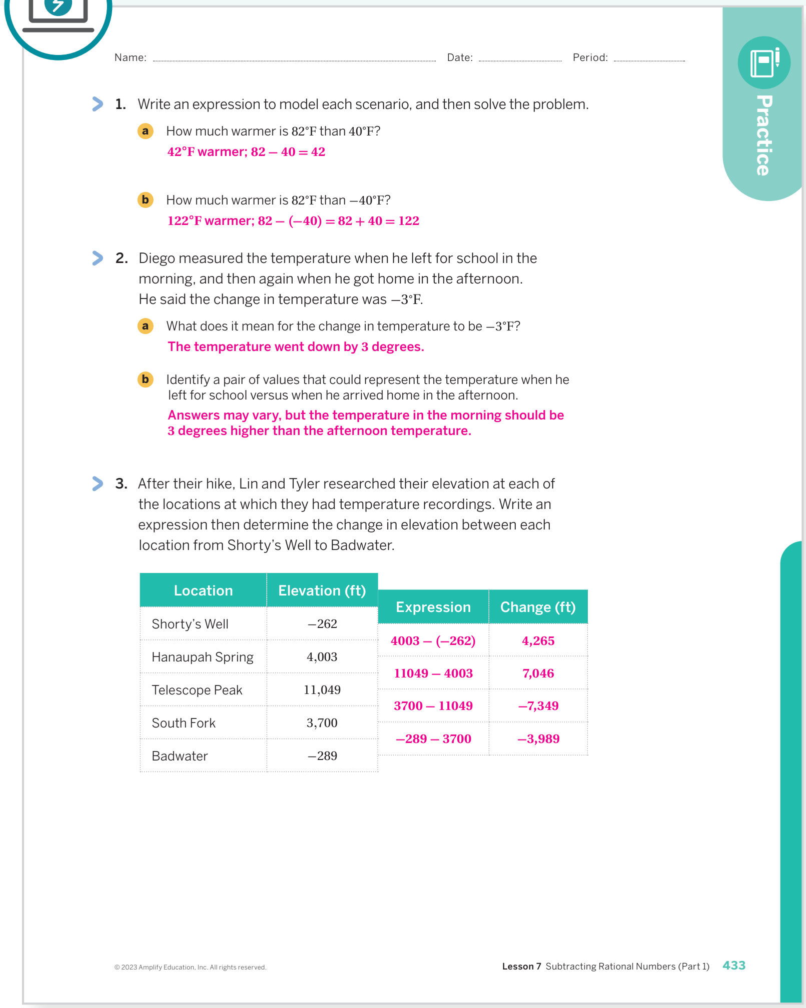
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways in Activity 1 did things happen that you did not expect?
- How did the ***Partner Problems*** routine support students in understanding how order affects the sign when determining the difference of two values? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Write an expression to model each scenario, and then solve the problem.

   a. How much warmer is 82°F than 40°F?

   42°F warmer; $82 - 40 = 42$

   b. How much warmer is 82°F than −40°F?

   122°F warmer; $82 - (-40) = 82 + 40 = 122$

2. Diego measured the temperature when he left for school in the morning, and then again when he got home in the afternoon. He said the change in temperature was −3°F.

   a. What does it mean for the change in temperature to be −3°F?

   The temperature went down by 3 degrees.

   b. Identify a pair of values that could represent the temperature when he left for school versus when he arrived home in the afternoon.

   Answers may vary, but the temperature in the morning should be 3 degrees higher than the afternoon temperature.

3. After their hike, Lin and Tyler researched their elevation at each of the locations at which they had temperature recordings. Write an expression then determine the change in elevation between each location from Shorty's Well to Badwater.

Location	Elevation (ft)
Shorty's Well	−262
Hanaupah Spring	4,003
Telescope Peak	11,049
South Fork	3,700
Badwater	−289

Expression	Change (ft)
4003 − (−262)	**4,265**
11049 − 4003	**7,046**
3700 − 11049	**−7,349**
−289 − 3700	**−3,989**

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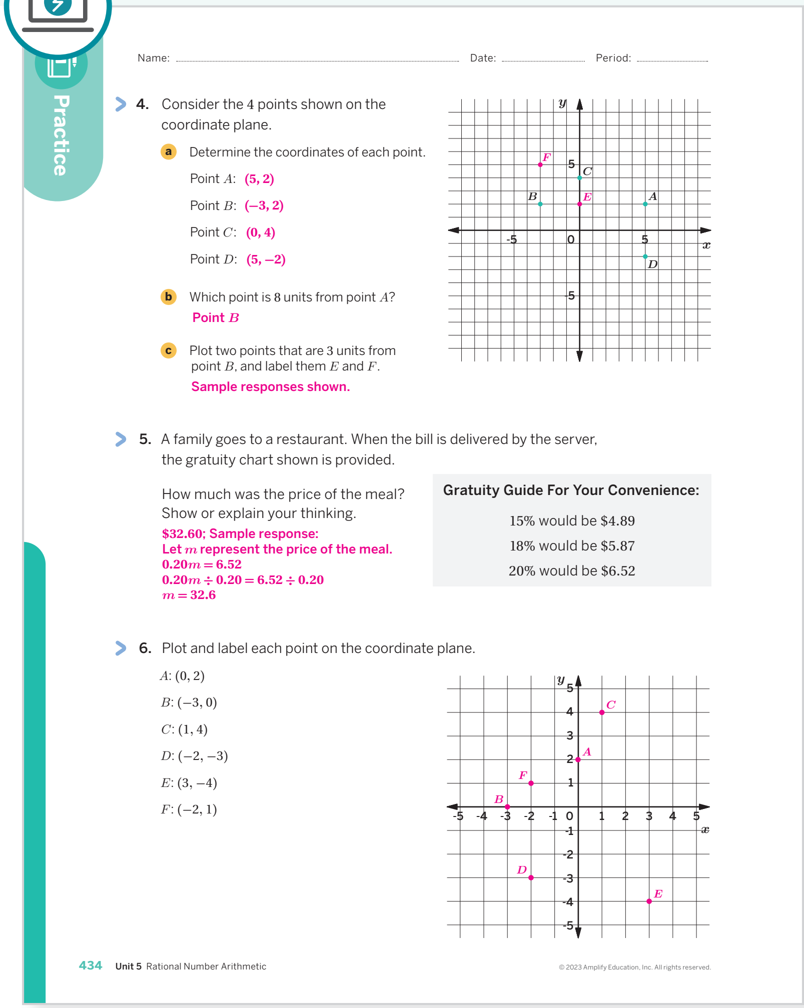
Lesson 7 Subtracting Rational Numbers (Part 1) 433



Name: ______ Date: ______ Period: ______

Practice

4. Consider the 4 points shown on the coordinate plane.

   a. Determine the coordinates of each point.

   Point A: **(5, 2)**

   Point B: **(−3, 2)**

   Point C: **(0, 4)**

   Point D: **(5, −2)**

   b. Which point is 8 units from point A?

   Point B

   c. Plot two points that are 3 units from point B, and label them E and F.

   Sample responses shown.

5. A family goes to a restaurant. When the bill is delivered by the server, the gratuity chart shown is provided.

   How much was the price of the meal? Show or explain your thinking.

   \$32.60; Sample response:
   Let m represent the price of the meal.
   $0.20m = 6.52$
   $0.20m \div 0.20 = 6.52 \div 0.20$
   $m = 32.6$

Gratuity Guide For Your Convenience:

15% would be \$4.89

18% would be \$5.87

20% would be \$6.52

6. Plot and label each point on the coordinate plane.

   A: (0, 2)
   B: (−3, 0)
   C: (1, 4)
   D: (−2, −3)
   E: (3, −4)
   F: (−2, 1)

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Grade 6	2
	5	Unit 4 Lesson 8	2
Formative ⚡	6	Unit 5 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Subtracting Rational Numbers (Part 2)

Let's get more practice subtracting rational numbers.

Focus

Goals

1. **Language Goal:** Recognize and explain that the *difference* of two numbers can be positive or negative depending on the order of an expression, while the *distance* between two numbers is always positive. **(Speaking and Listening)**
2. Determine the distance between values on a number line and points on a coordinate plane.
3. Determine the location of events on a number line by determining the change from the current point in time.

Coherence

Today

Students see that the *difference* of two numbers can be positive or negative, but the *distance* between two numbers is always positive. Using both a number line and points on a coordinate plane, they relate the value of the difference of two points to the distance between them concluding that the distance is the absolute value of the difference. Finally, they apply their understanding of difference and distance to create a number line of their lifetime, given that 0 represents today.

Previously

In Lesson 7, students used a number line to determine the difference between two values concluding that the order of subtraction affects the sign of the difference.

Coming Soon

In Lesson 9, students will apply their understanding of addition and subtraction to solve a variety of real-world problems.

Rigor

- Students build **conceptual understanding** of the distance between two points.
- Students gain **fluency** in solving subtraction problems related to difference and distance.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	10 min	20 min	5 min	5 min
Pairs	Small Groups	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 2 PDF, one per student
- Activity 2 PDF (answers)

Math Language Development

Review words

- *absolute value*
- *difference*
- *distance*
- *magnitude*

Amps | Featured Activity

Activity 2
Digital Timeline

Students create a digital timeline of important moments in their lifetime by determining the difference between their current age and the age at which the event happened or is anticipated to happen.

Building Math Identity and Community

Connecting to Mathematical Practices

The concept of representing today as 0 on a timeline might not sit well with students who think of today as having a date that is composed of numbers. Rather than give up before even getting started, students should persevere in their attempts to make sense of this problem. Ask them to identify the emotion they feel at the beginning of the problem and describe what they will do to turn it into a growth mentality.

Modifications to Pacing

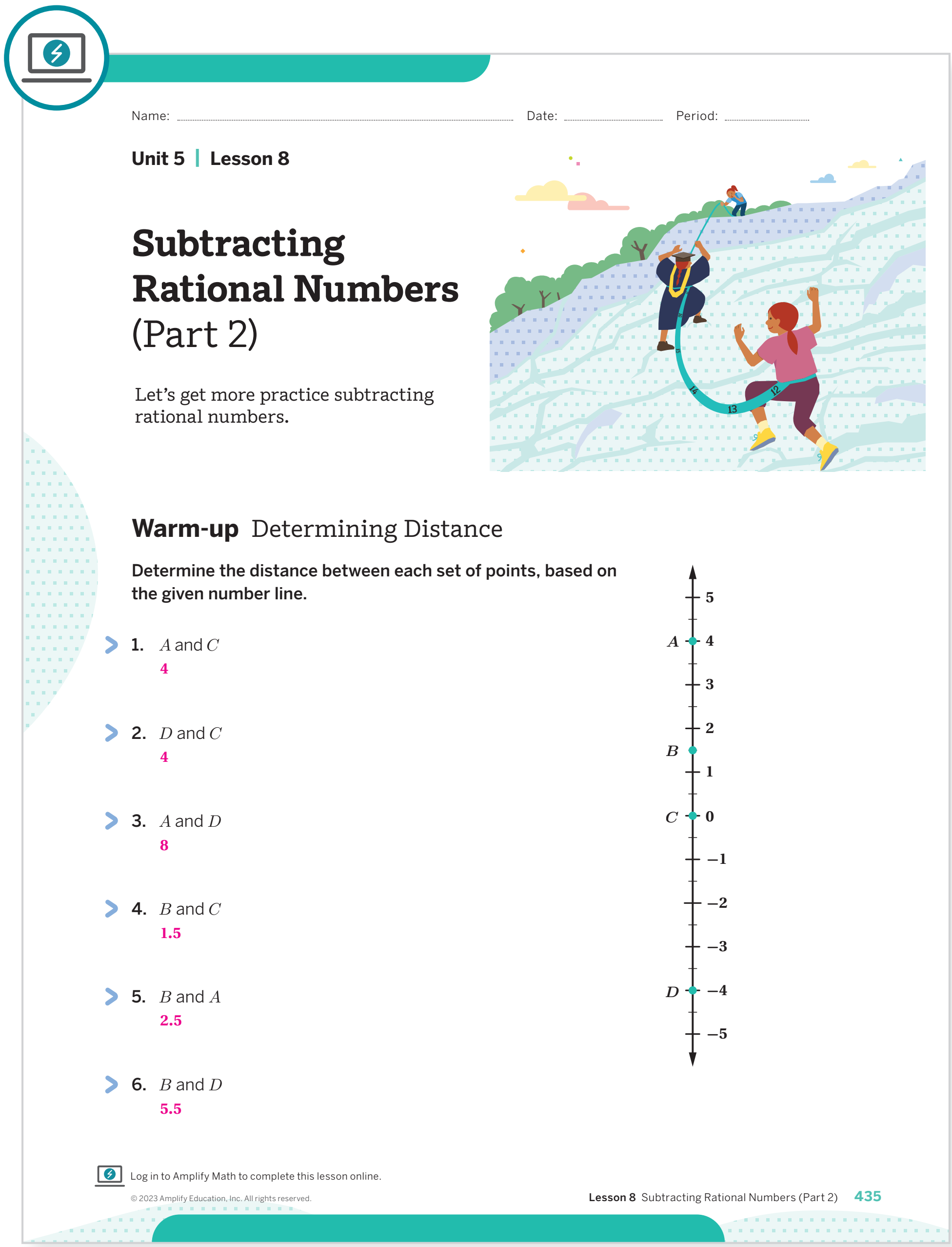
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up,** Problems 4–6 may be omitted.
- In **Activity 2,** have students only choose two events in Problems 1 and 4. Alternatively, Activity 2 may be omitted entirely or assigned outside of class time. It makes for a great display in a classroom!

Pairs | 7 min

Warm-up Determining Distance

Students use a number line to determine the distance between two points.



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Unit 5 | Lesson 8

Subtracting Rational Numbers (Part 2)

Let's get more practice subtracting rational numbers.

Warm-up Determining Distance

Determine the distance between each set of points, based on the given number line.

1. A and C
   4
2. D and C
   4
3. A and D
   8
4. B and C
   1.5
5. B and A
   2.5
6. B and D
   5.5

Log in to Amplify Math to complete this lesson online.

Lesson 8 Subtracting Rational Numbers (Part 2) 435

1 Launch

Activate background knowledge by asking if any students know the distance around a track (400 m or $\frac{1}{4}$ mile). Ask, "Does the direction you run around the track change the distance?" Explain that regardless of the direction you run around the track the *distance* is always the same.

2 Monitor

Help students get started by saying, "Imagine the number line shown were a ruler, how far apart are points A and B?"

Look for points of confusion:

- **Giving the distance as a negative value.** Remind students about the distance around the track example from the launch. Ask, "Would it make sense to say that the distance around a track is −400 meters?"

Look for productive strategies:

- Counting the spaces between values on the number line.
- Writing subtraction expressions to represent the difference between each value to aid in determining the distance.

3 Connect

Display the number line from the Student Edition.

Have students share their methods for determining the distance between the points, including writing expressions.

Define *distance* as the length between two points. In general, the distance between two values a and b is $|a - b|$.

Highlight that the distance is the magnitude of the difference.

Ask, "Does order matter when determining the distance between two points?"

Power-up

To power up students' ability to plot points on the coordinate plane:

Provide students with a copy of the Power-up PDF.

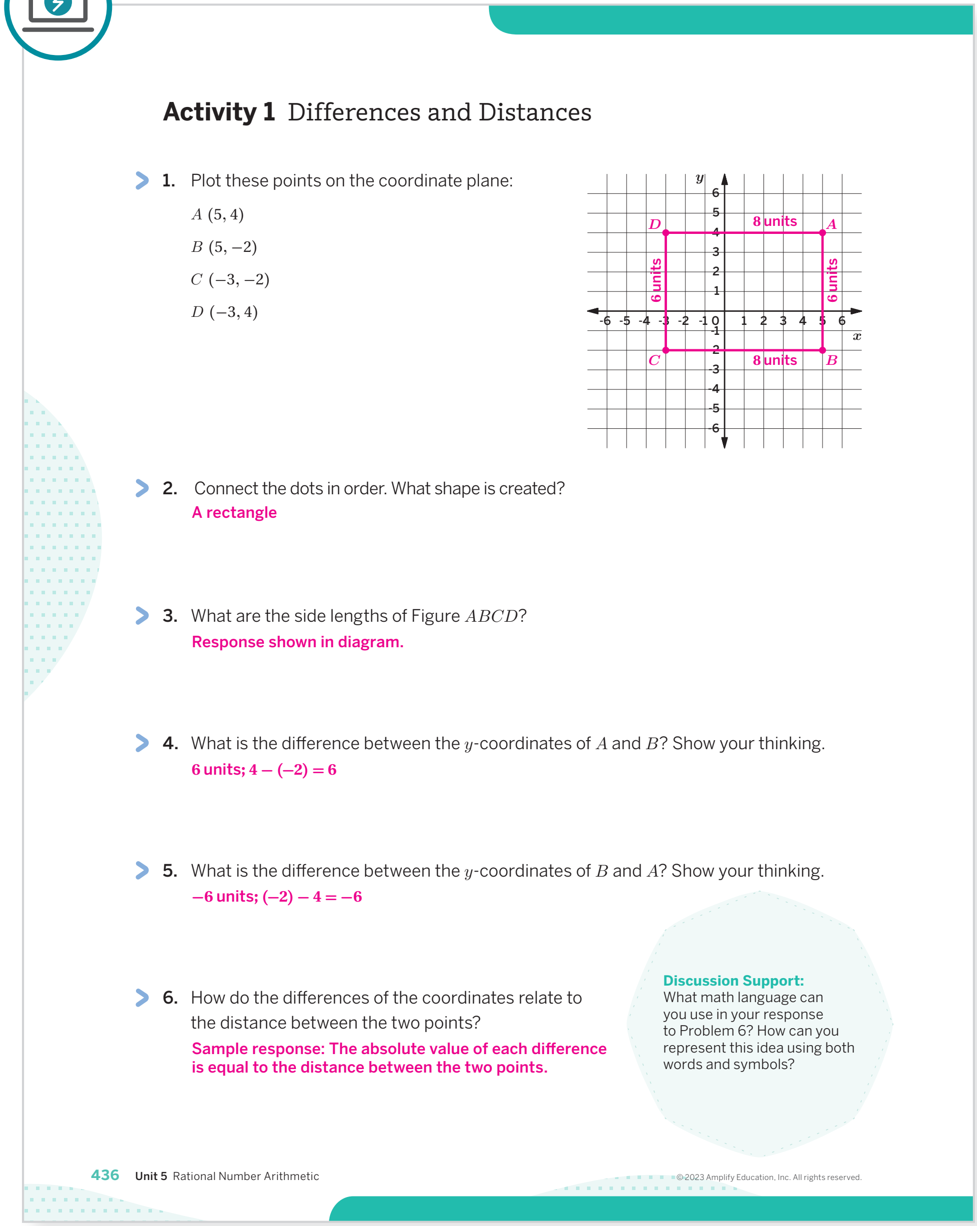
Use: Before Activity 1.

Informed by: Performance on Lesson 7, Practice Problem 6. and Pre-Unit Readiness Assessment, Problem 6.

Small groups | 10 min

Activity 1 Differences and Distances

Students connect their work with determining distance on the coordinate plane in Grade 6 to subtracting coordinate values.



Activity 1 Differences and Distances

1. Plot these points on the coordinate plane:
   - A (5, 4)
   - B (5, –2)
   - C (–3, –2)
   - D (–3, 4)

2. Connect the dots in order. What shape is created?
   A rectangle

3. What are the side lengths of Figure $ABCD$?
   Response shown in diagram.

4. What is the difference between the y-coordinates of A and B? Show your thinking.
   6 units; $4 - (-2) = 6$

5. What is the difference between the y-coordinates of B and A? Show your thinking.
   -6 units; $(-2) - 4 = -6$

6. How do the differences of the coordinates relate to the distance between the two points?
   Sample response: The absolute value of each difference is equal to the distance between the two points.

Discussion Support: What math language can you use in your response to Problem 6? How can you represent this idea using both words and symbols?

1 Launch

Activate prior knowledge by asking, "What does each value in the point (5, 4) tell you about the location of the point on the coordinate plane?"

2 Monitor

Help students get started by asking, "What do you remember about plotting points on the coordinate plane?" Model how to determine the location of point A.

Look for points of confusion:

- **Mixing up the x and y-values when plotting points.** Remind students the first value is the horizontal movement and the second value is the vertical movement.
- **Not realizing that Problems 4 and 5 should give opposite values.** Ask, "How does changing the order of the values in an expression affect the difference between them?"

3 Connect

Display a student sample of the completed coordinate plane.

Have students share their thinking for Problem 6.

Highlight that the values of the answers for Problems 4 and 5 are opposites. *Distance* is the magnitude of the difference. The distance between points A and B is the same regardless of the direction it is measured because the magnitude is the same. Generally, for any two values a and b, the distance between them is the same regardless of the order given in an expression; $|a - b| = |b - a|$.

Ask:

- "When you are determining the distance between two points on the coordinate plane, how do you determine when to look at the x-coordinate or the y-coordinate?"
- "How can you determine the distance between the points (3, 6) and (–5, 6)?"

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously plotted points in all four quadrants of the coordinate plane in Grade 6. Students may benefit from a review of the four quadrants of the coordinate plane and how to plot points that have positive and negative coordinates.

Extension: *Math Enrichment*

Without plotting, have students determine the distance between each pair of points.

- $(-4, 1)$ and $(2, 1)$ 6
- $(3, 6)$ and $(3, -3)$ 9
- (a, b) and (c, b) $|c - a|$
- (m, n) and (m, p) $|p - n|$

Math Language Development

MLR8: Discussion Supports

While students complete Problem 6, ask them to think about how they can use the math language they have been learning in their responses. During the Connect, have students share their responses. Listen for and amplify the ways students describe the distinction between *distance* and *difference*. Add this language to the class display and connect it to the language collected during the Warm-up.

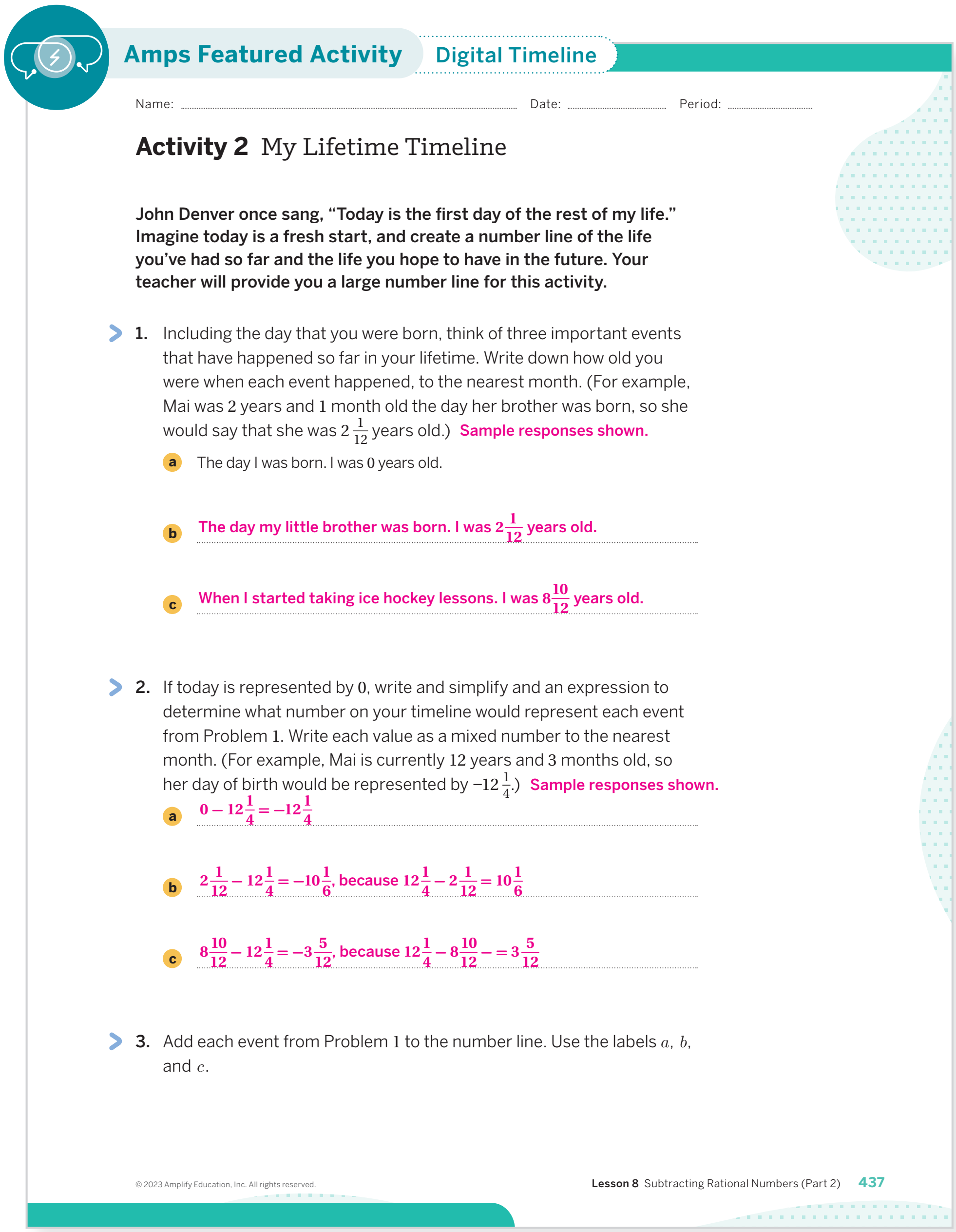
English Learners

Add the coordinate plane to the display and add annotations to highlight the distinction students make between *distance* and *difference*.

Independent | 20 min

Activity 2 My Lifetime Timeline

Students expand their understanding of subtracting rational numbers to create a number line of their life where "today" is represented by 0.



Amps Featured Activity Digital Timeline

Name: ______ Date: ______ Period: ______

Activity 2 My Lifetime Timeline

John Denver once sang, "Today is the first day of the rest of my life." Imagine today is a fresh start, and create a number line of the life you've had so far and the life you hope to have in the future. Your teacher will provide you a large number line for this activity.

1. Including the day that you were born, think of three important events that have happened so far in your lifetime. Write down how old you were when each event happened, to the nearest month. (For example, Mai was 2 years and 1 month old the day her brother was born, so she would say that she was $2\frac{1}{12}$ years old.) **Sample responses shown.**
   - a The day I was born. I was 0 years old.
   - b **The day my little brother was born. I was $2\frac{1}{12}$ years old.**
   - c **When I started taking ice hockey lessons. I was $8\frac{10}{12}$ years old.**
2. If today is represented by 0, write and simplify and an expression to determine what number on your timeline would represent each event from Problem 1. Write each value as a mixed number to the nearest month. (For example, Mai is currently 12 years and 3 months old, so her day of birth would be represented by $-12\frac{1}{4}$.) **Sample responses shown.**
   - a $0 - 12\frac{1}{4} = -12\frac{1}{4}$
   - b $2\frac{1}{12} - 12\frac{1}{4} = -10\frac{1}{6}$, **because** $12\frac{1}{4} - 2\frac{1}{12} = 10\frac{1}{6}$
   - c $8\frac{10}{12} - 12\frac{1}{4} = -3\frac{5}{12}$, **because** $12\frac{1}{4} - 8\frac{10}{12} - = 3\frac{5}{12}$
3. Add each event from Problem 1 to the number line. Use the labels a, b, and c.

1 Launch

Explain that students will be using what they know about rational numbers and differences to create a timeline of their life where "today" is represented by 0. Distribute the Activity 2 PDF to each student.

2 Monitor

Help students get started by asking, "Can you share an interesting fact about yourself?"

Look for points of confusion:

- **Plotting the age (or the negative of the age) that each even happened, rather than the difference from their current age.** Ask, "If something happened when you were 5, how would you represent how many years ago that was compared to today?"
- **Difficulty in converting negative mixed numbers into improper fractions.** Remind students that they can convert the magnitude of the mixed number into an improper fraction first, and then incorporate the sign.

Activity 2 continued ›

Differentiated Support

Accessibility: ***Optimize Access to Technology***

Have students use the Amps slides for this activity, in which they can create a digital timeline of important moments in their lifetime by determining the difference between their current age and the age at which the event happened or is anticipated to happen.

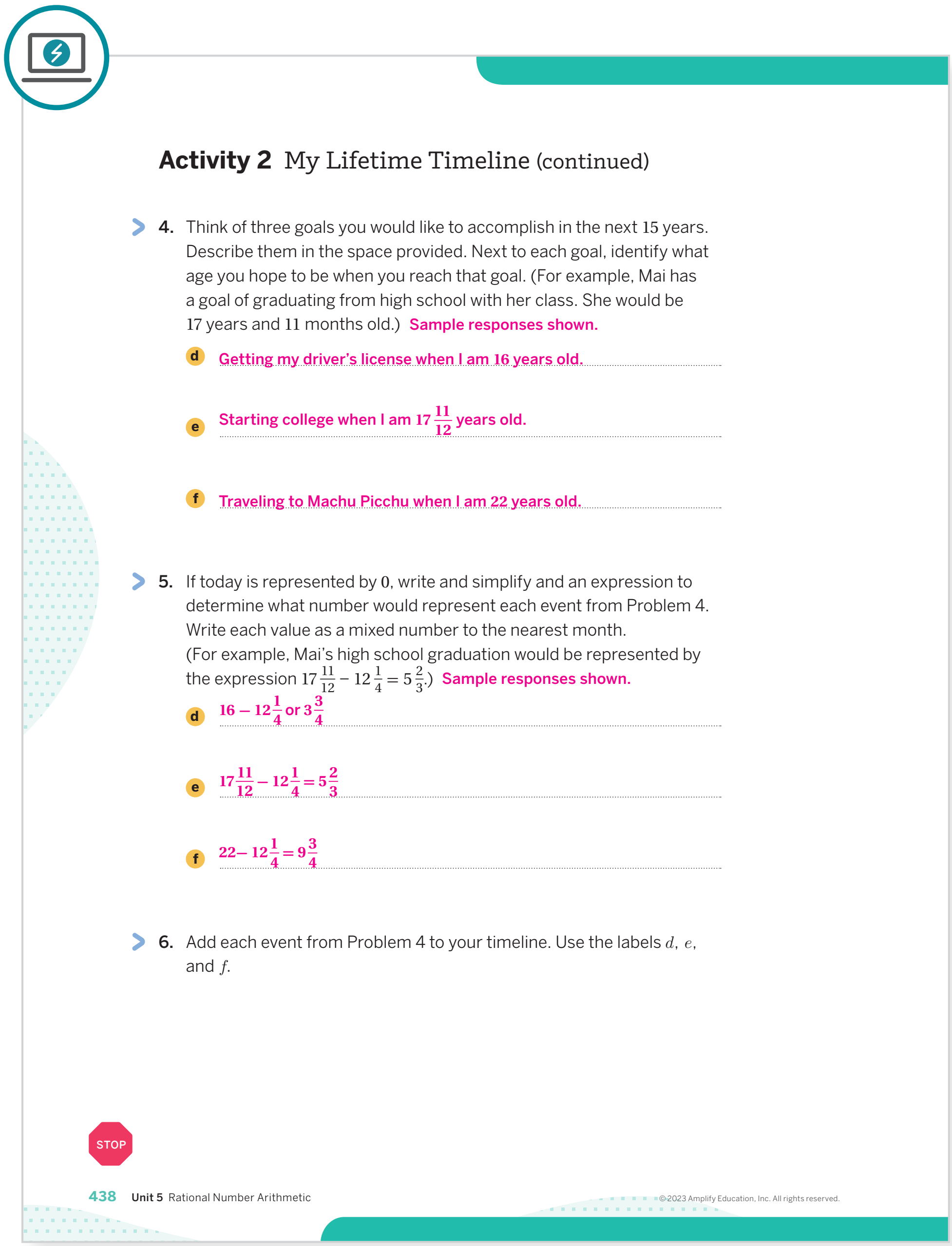
Accessibility: ***Vary Demands to Optimize Challenge***

Allow students to approximate their ages to the nearest 6 months, or $\frac{1}{2}$ year, instead of the nearest month.

Independent | 20 min

Activity 2 My Lifetime Timeline (continued)

Students expand their understanding of subtracting rational numbers to create a number line of their life where “today” is represented by 0.



Activity 2 My Lifetime Timeline (continued)

4. Think of three goals you would like to accomplish in the next 15 years. Describe them in the space provided. Next to each goal, identify what age you hope to be when you reach that goal. (For example, Mai has a goal of graduating from high school with her class. She would be 17 years and 11 months old.) Sample responses shown.

   d Getting my driver's license when I am 16 years old.

   e Starting college when I am 17 $\frac{11}{12}$ years old.

   f Traveling to Machu Picchu when I am 22 years old.

5. If today is represented by 0, write and simplify and an expression to determine what number would represent each event from Problem 4. Write each value as a mixed number to the nearest month. (For example, Mai's high school graduation would be represented by the expression $17\frac{11}{12} - 12\frac{1}{4} = 5\frac{2}{3}$.) Sample responses shown.

   d $16 - 12\frac{1}{4}$ or $3\frac{3}{4}$

   e $17\frac{11}{12} - 12\frac{1}{4} = 5\frac{2}{3}$

   f $22 - 12\frac{1}{4} = 9\frac{3}{4}$

6. Add each event from Problem 4 to your timeline. Use the labels d, e, and f.

STOP

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3 Connect

Display examples of students’ number lines.

Have students share how they determined the values that would represent each life event on their timeline.

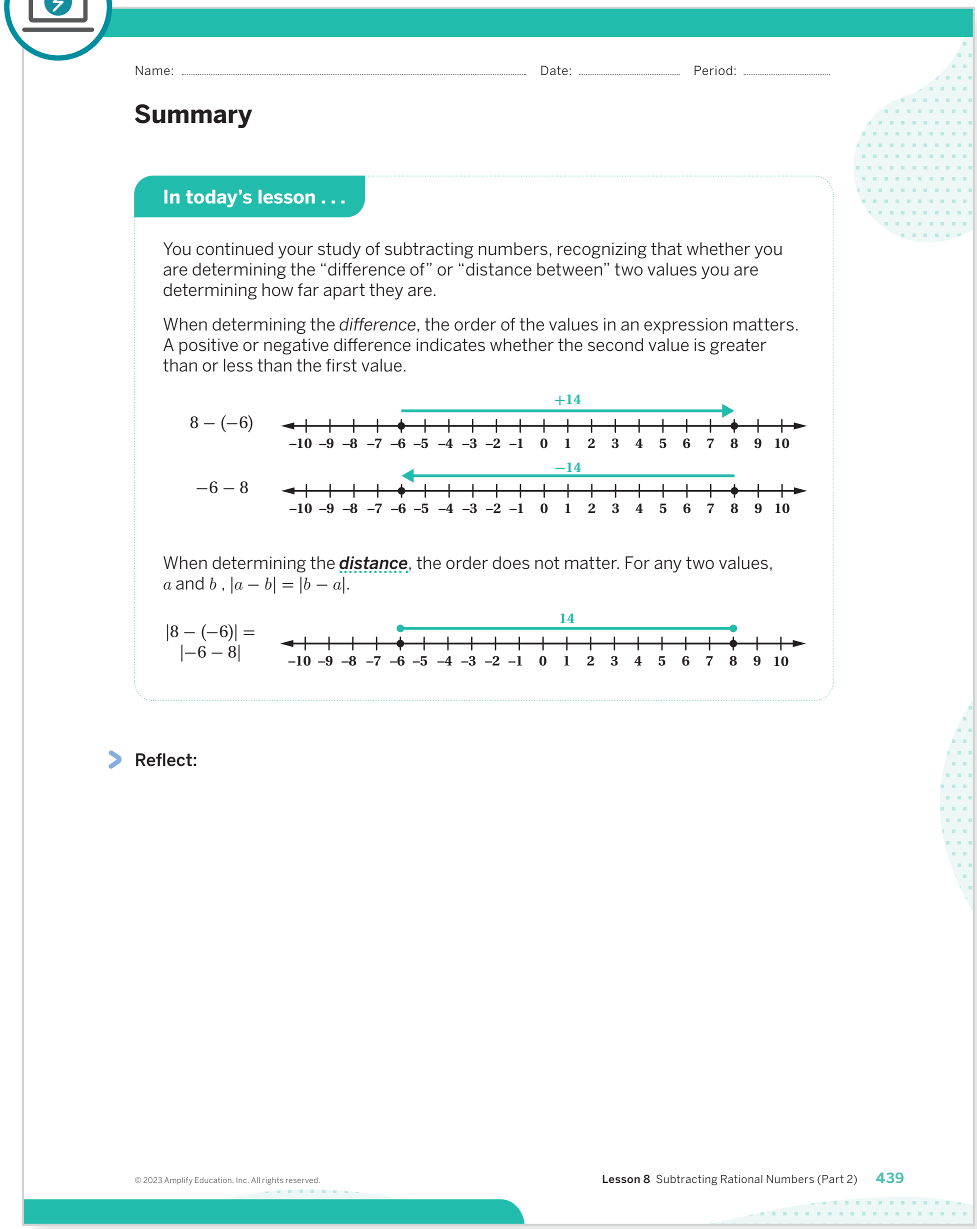
Highlight that, in this scenario, each value represents the *difference* between their current age and the age at which the event occurred or is projected to occur.

Ask, “If you add your fourth and a half birthday on your number line, what value would represent it?” Answers may vary.

Whole Class | 5 min

Summary

Review and synthesize the relationship between determining the *distance* between two points and the *difference* between two values.



Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You continued your study of subtracting numbers, recognizing that whether you are determining the "difference of" or "distance between" two values you are determining how far apart they are.

When determining the *difference*, the order of the values in an expression matters. A positive or negative difference indicates whether the second value is greater than or less than the first value.

$8 - (-6)$

$-6 - 8$

When determining the ***distance***, the order does not matter. For any two values, a and b , $|a - b| = |b - a|$.

$|8 - (-6)| =$
$|-6 - 8|$

Reflect:

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Lesson 8 Subtracting Rational Numbers (Part 2) 439

Synthesize

Display the Summary from the Student Edition.

Ask:

- "Does the order of two numbers being subtracted matter when determining the difference?"
- "Does the order of the numbers being subtracted matter when determining the distance?"
- "How are difference and distance related?"

Highlight that *difference* can be positive or negative, and *distance* is the magnitude of the difference, therefore is always positive.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

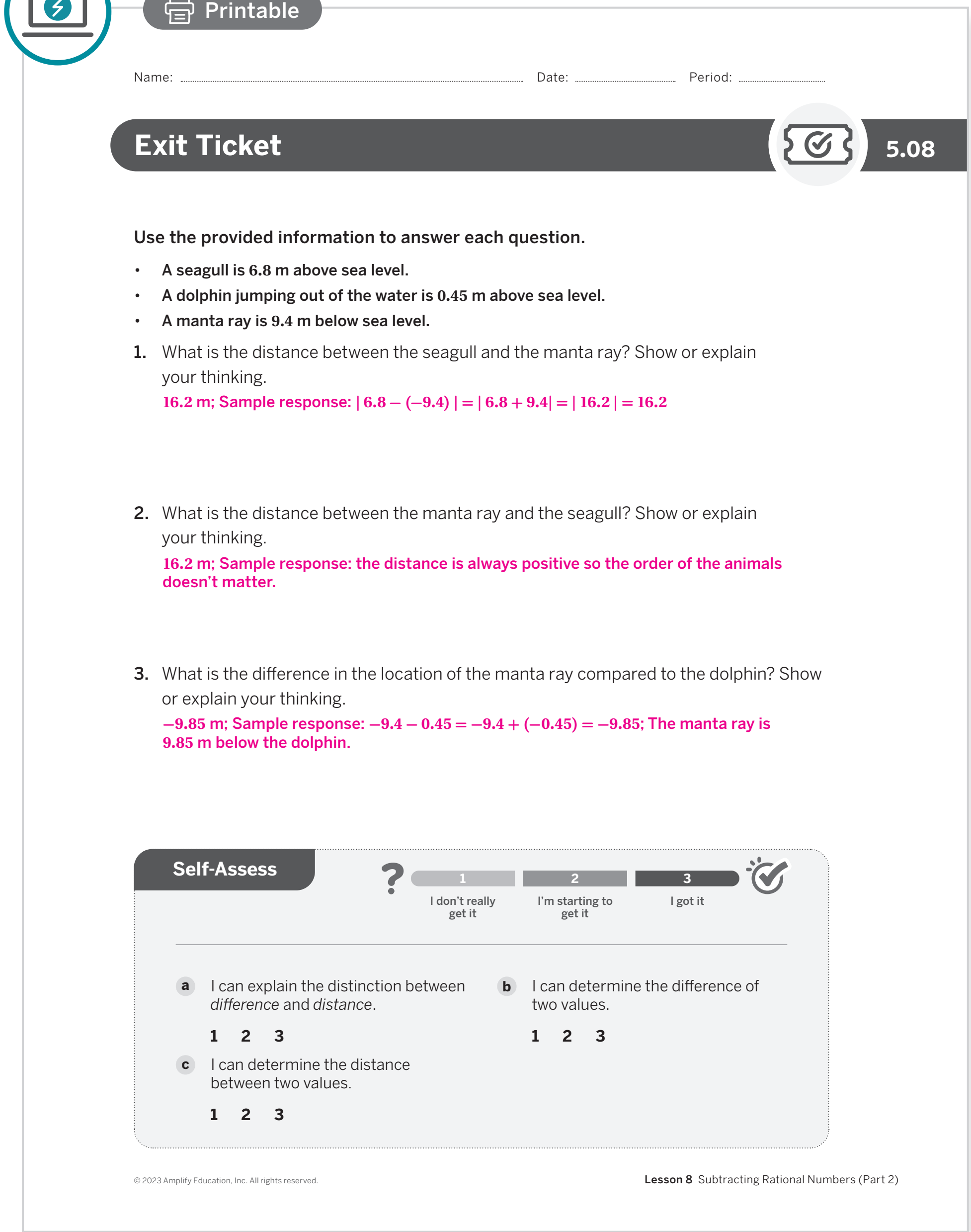
- "How are *difference* and *distance* similar? How are they distinct?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of determining the difference and distance by analyzing and representing real-world scenarios.

Printable



Name: __________ Date: __________ Period: __________

Exit Ticket 5.08

Use the provided information to answer each question.

- **A seagull is 6.8 m above sea level.**
- **A dolphin jumping out of the water is 0.45 m above sea level.**
- **A manta ray is 9.4 m below sea level.**

1. What is the distance between the seagull and the manta ray? Show or explain your thinking.
   16.2 m; Sample response: $|6.8 - (-9.4)| = |6.8 + 9.4| = |16.2| = 16.2$
2. What is the distance between the manta ray and the seagull? Show or explain your thinking.
   16.2 m; Sample response: the distance is always positive so the order of the animals doesn't matter.
3. What is the difference in the location of the manta ray compared to the dolphin? Show or explain your thinking.
   -9.85 m; Sample response: $-9.4 - 0.45 = -9.4 + (-0.45) = -9.85$; The manta ray is 9.85 m below the dolphin.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain the distinction between *difference* and *distance*.
1 2 3

b I can determine the difference of two values.
1 2 3

c I can determine the distance between two values.
1 2 3

Lesson 8 Subtracting Rational Numbers (Part 2)

Success looks like . . .

- **Language Goal:** Recognizing and explaining that the *difference* of two numbers can be positive or negative depending on the order of an expression, while the *distance* between two numbers is always positive. **(Speaking and Listening)**
- **Goal:** Determining the distance between values on a number line and points on a coordinate plane.
  - » Determining the distance between pairs of animals above sea level in Problems 1–3.
- **Goal:** Determining the location of events on a number line by determining the change from the current point in time.

Suggested next steps

If students provide a negative distance for Problem 1 or 2, consider:

- Reviewing the definition of *distance*.
- Assigning Practice Problem 1.

Professional Learning

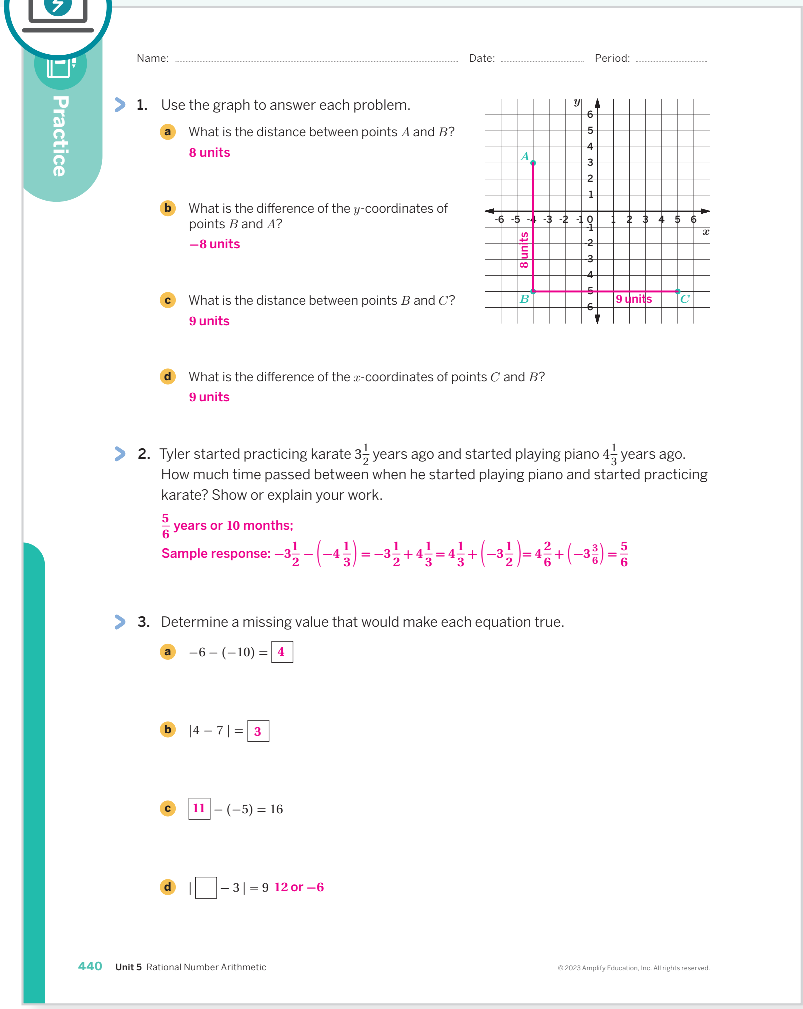
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What challenges did students encounter as they worked on creating their timelines? How did they work through them?
- The instructional goal for this lesson was for students to understand the relationship between *distance* and *difference*. How well did students accomplish this? What did you specifically do to help students accomplish it? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Use the graph to answer each problem.
   - **a** What is the distance between points A and B?
     8 units
   - **b** What is the difference of the y-coordinates of points B and A?
     −8 units
   - **c** What is the distance between points B and C?
     9 units
   - **d** What is the difference of the x-coordinates of points C and B?
     9 units

2. Tyler started practicing karate $3\frac{1}{2}$ years ago and started playing piano $4\frac{1}{3}$ years ago. How much time passed between when he started playing piano and started practicing karate? Show or explain your work.

   $\frac{5}{6}$ years or 10 months;
   Sample response: $-3\frac{1}{2}-\left(-4\frac{1}{3}\right)=-3\frac{1}{2}+4\frac{1}{3}=4\frac{1}{3}+\left(-3\frac{1}{2}\right)=4\frac{2}{6}+\left(-3\frac{3}{6}\right)=\frac{5}{6}$

3. Determine a missing value that would make each equation true.
   - **a** $-6-(-10)=\boxed{4}$
   - **b** $|4-7|=\boxed{3}$
   - **c** $\boxed{11}-(-5)=16$
   - **d** $|\boxed{}-3|=9$ **12 or −6**

440 **Unit 5** Rational Number Arithmetic

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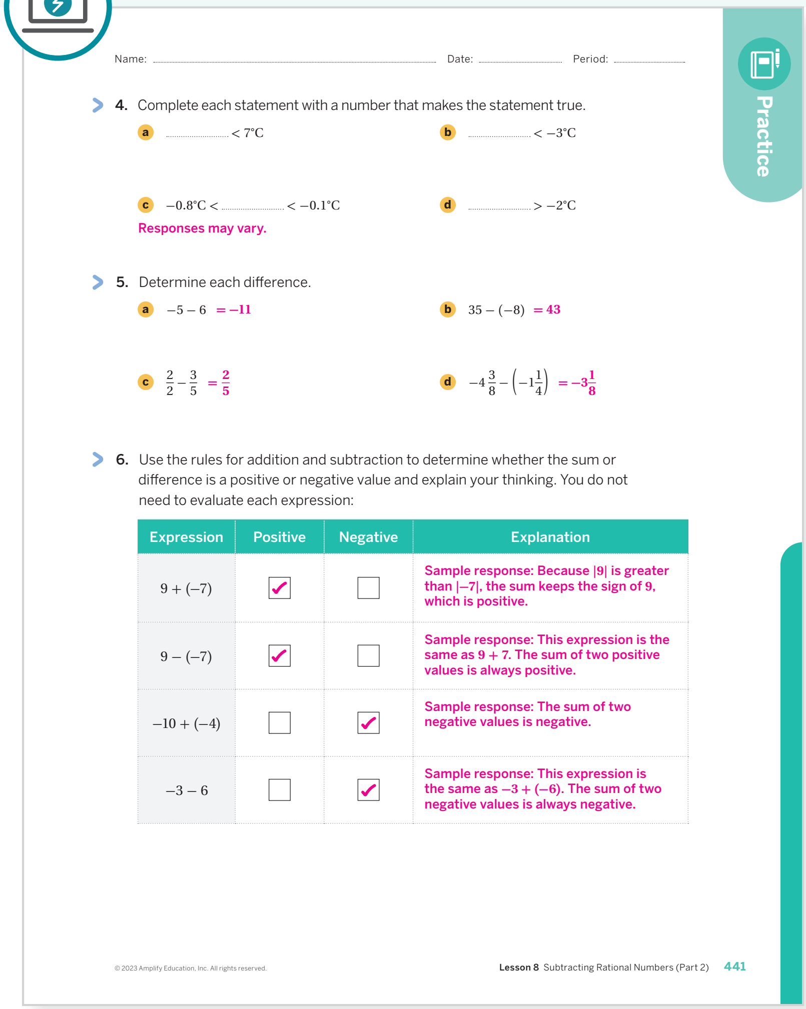

Name: Date: Period:

Practice

4. Complete each statement with a number that makes the statement true.
   - **a** .... $< 7°\text{C}$
   - **b** .... $< -3°\text{C}$
   - **c** $-0.8°\text{C} <$ .... $< -0.1°\text{C}$
   - **d** .... $> -2°\text{C}$

   Responses may vary.

5. Determine each difference.
   - **a** $-5-6$ $=-11$
   - **b** $35-(-8)$ $=43$
   - **c** $\frac{2}{2}-\frac{3}{5}$ $=\frac{2}{5}$
   - **d** $-4\frac{3}{8}-\left(-1\frac{1}{4}\right)$ $=-3\frac{1}{8}$

6. Use the rules for addition and subtraction to determine whether the sum or difference is a positive or negative value and explain your thinking. You do not need to evaluate each expression:

Expression	Positive	Negative	Explanation
$9+(-7)$	☑	☐	**Sample response: Because |9| is greater than |−7|, the sum keeps the sign of 9, which is positive.**
$9-(-7)$	☑	☐	**Sample response: This expression is the same as 9 + 7. The sum of two positive values is always positive.**
$-10+(-4)$	☐	☑	**Sample response: The sum of two negative values is negative.**
$-3-6$	☐	☑	**Sample response: This expression is the same as −3 + (−6). The sum of two negative values is always negative.**

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Lesson 8 Subtracting Rational Numbers (Part 2) 441

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 5 Lesson 2	1
	5	Unit 5 Lesson 6	1
Formative	6	Unit 5 Lesson 9	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Adding and Subtracting Rational Numbers

Let's determine the sum and difference of rational numbers.

Focus

Goals

1. **Language Goal:** Apply addition and subtraction of rational numbers to solve problems in unfamiliar contexts, and explain the solution method. **(Writing, Speaking and Listening)**
2. **Language Goal:** Evaluate expressions involving both addition and subtraction, and explain the reasoning. **(Speaking and Listening)**
3. **Language Goal:** Determine whether the sum or difference of two rational numbers is positive or negative, and explain the reasoning . **(Speaking and Listening)**

Coherence

Today

Students put their knowledge of addition and subtraction of rational numbers to use in real world contexts. They make sense of scenarios, writing expressions to determine the unknown quantity and reason about whether it would be a positive or negative value. Students continue working with expressions by writing their own scenarios to match a given expression and receiving feedback from a peer. Finally, students compare and contrast methods for simplifying expressions that include more than two rational values.

Previously

In Lessons 2–8, students gained fluency in adding and subtracting rational numbers.

Coming Soon

In Lesson 10, students will extend their understanding of adding rational numbers to multiplication.

Rigor

- Students **apply** their understanding of addition and subtraction of rational numbers to write and evaluate expressions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
7 min	15 min	10 min	10 min	5 min	8 min
Pairs	Small Groups	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Graphic Organizer PDF, *Labeled Number Lines* (as needed)

Math Language Development

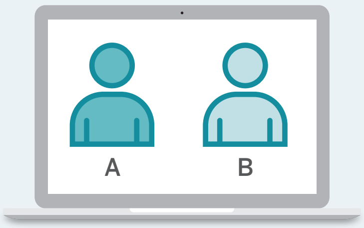
Review words

- *absolute value*
- *associative property*
- *commutative property*
- *difference*
- *distance*
- *magnitude*
- *rational numbers*

Amps | Featured Activity

Activity 2
Writing Scenarios

Students choose an expression and write a scenario that matches it. After trading scenarios with their partners, display multiple scenarios that match each expression and facilitate a discussion about the similarities and differences between them.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students determine whether they agree or disagree with each evaluation method in Activity 3, they might forget to consider others' backgrounds. Math is universal, and, at this level, there is usually one correct answer, but the approaches can be varied, especially if students received part of their mathematics training in other parts of the world. Ask students to take on the perspective of the person speaking and work diligently to understand what they are describing.

Modifications to Pacing

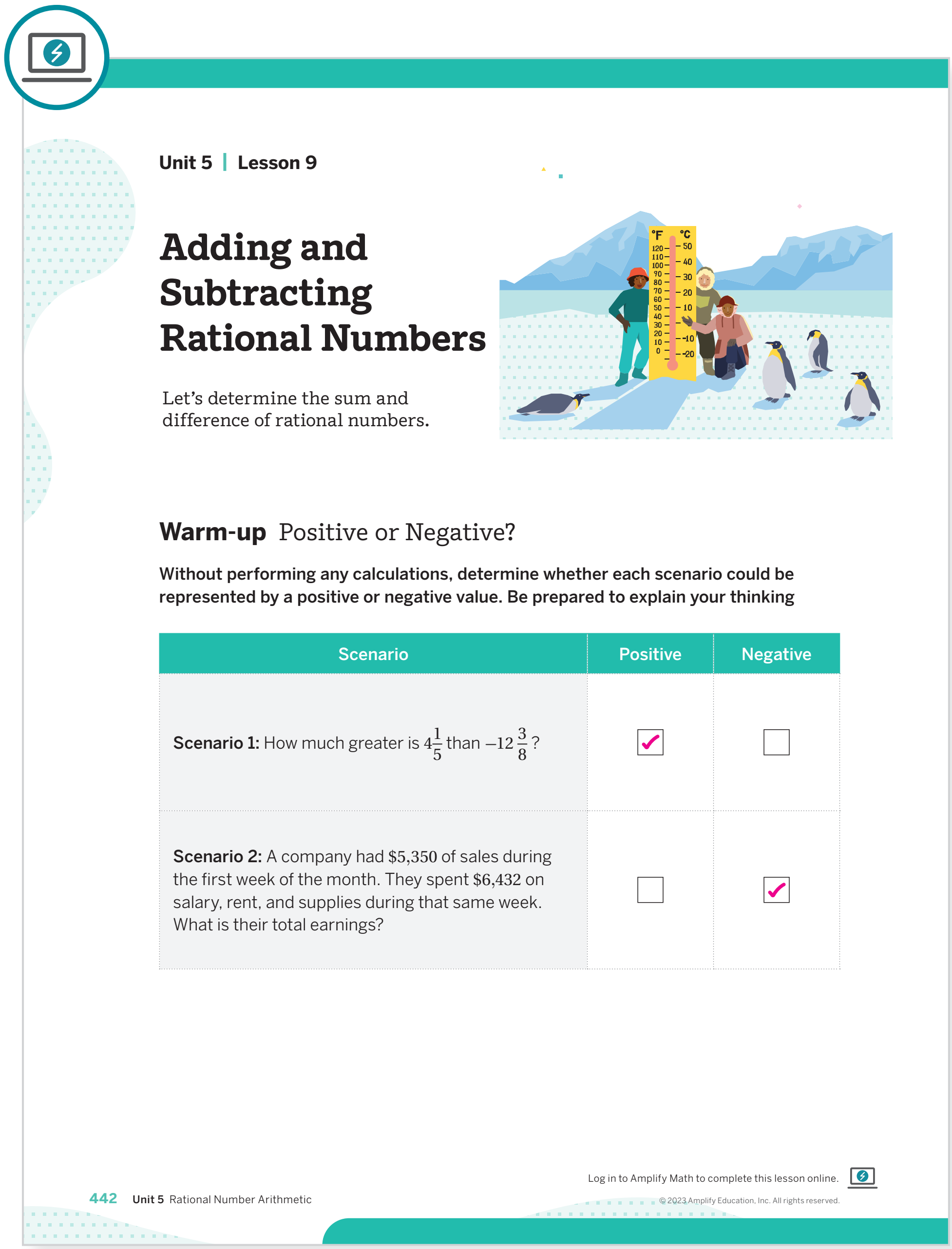
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1,** have students choose any two scenarios to analyze, write expressions for, and answer.
- Optional **Activity 2,** may be omitted.
- In **Activity 3,** have students discuss Problem 2 without writing their responses on their paper.

Pairs | 7 min

Warm-up Positive or Negative?

Students reason about whether scenarios are representing positive or negative values without doing any calculations to prepare them to write expressions to represent them.



Unit 5 | Lesson 9

Adding and Subtracting Rational Numbers

Let's determine the sum and difference of rational numbers.

Warm-up Positive or Negative?

Without performing any calculations, determine whether each scenario could be represented by a positive or negative value. Be prepared to explain your thinking

Scenario	Positive	Negative
Scenario 1: How much greater is $4\frac{1}{5}$ than $-12\frac{3}{8}$?	☑	☐
Scenario 2: A company had \$5,350 of sales during the first week of the month. They spent \$6,432 on salary, rent, and supplies during that same week. What is their total earnings?	☐	☑

442 Unit 5 Rational Number Arithmetic

Log in to Amplify Math to complete this lesson online.

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1 Launch

Conduct the ***Think-Pair-Share*** routine, asking students to read through each scenario twice prior to their partner discussion.

2 Monitor

Help students get started by asking, "What do you need to do to determine how much greater one value is than another?"

Look for points of confusion:

- **Taking the time to solve each scenario.** Remind students that they should be making sense of each scenario to reason about the sign of the solution, and they are not being asked to determine the solution.

Look for productive strategies:

- Comparing the magnitude of the two given values in each scenario as well as the relationship between the values to help them to determine the sign of each solution.

3 Connect

Display the scenarios one at a time. Conduct the ***Poll the Class*** routine to determine the sign of each solution.

Have students share how they determined whether each solution would be positive or negative without doing any calculations.

Ask:

- "What expression could you write to represent the first scenario?" $4\frac{1}{5} - \left(-12\frac{3}{8}\right)$
- "What expression could you write to represent the second scenario?" $5350 - 6432$
- "Do you feel that knowing the sign of the solution helped in writing the expressions?"

Highlight that when solving real-world problems, it is helpful to consider what type of solutions are possible prior to completing any calculations as a way to check the reasonableness of your answer.

Power-up

To power up students' ability to determine whether the sum or difference of rational numbers is positive or negative, have students complete:

Without calculating, determine whether each sum or difference is *positive* or *negative*.

1. $4 + 13$ Positive
2. $4 - (-13)$ Positive
3. $-4 + 13$ Positive
4. $-4 - 13$ Negative
5. $4 - 13$ Negative
6. $-4 - (-13)$ Positive

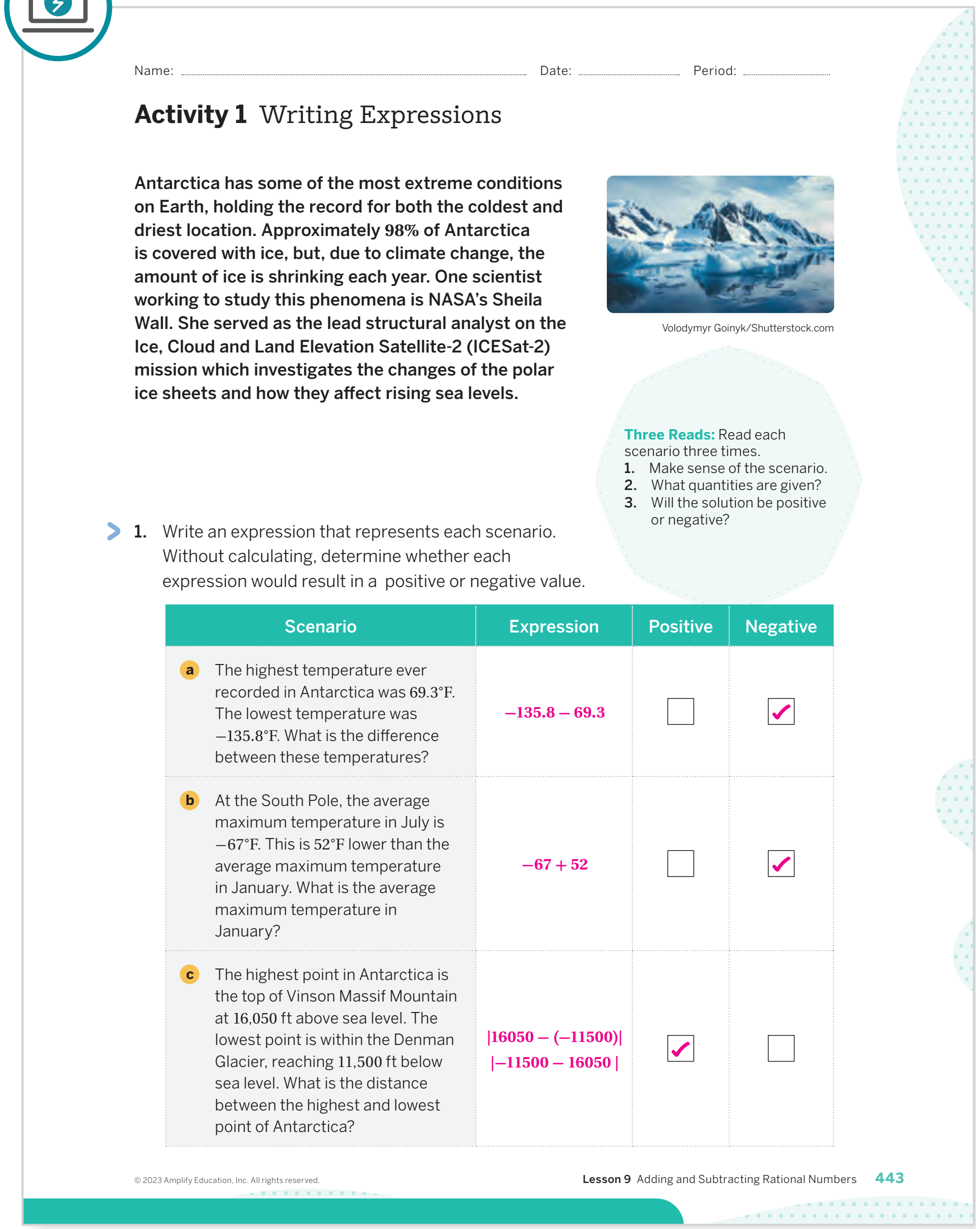
Use: Before the Warm-up.

Informed by: Performance on Lesson 8, Practice Problem 6.

Small groups | 15 min

Activity 1 Writing Expressions

Students apply their understanding of adding and subtracting rational numbers to write and evaluate expressions that represent scenarios.



Name: ______ Date: ______ Period: ______

Activity 1 Writing Expressions

Antarctica has some of the most extreme conditions on Earth, holding the record for both the coldest and driest location. Approximately 98% of Antarctica is covered with ice, but, due to climate change, the amount of ice is shrinking each year. One scientist working to study this phenomena is NASA's Sheila Wall. She served as the lead structural analyst on the Ice, Cloud and Land Elevation Satellite-2 (ICESat-2) mission which investigates the changes of the polar ice sheets and how they affect rising sea levels.

Volodymyr Goinyk/Shutterstock.com

Three Reads: Read each scenario three times.
1. Make sense of the scenario.
2. What quantities are given?
3. Will the solution be positive or negative?

1. Write an expression that represents each scenario. Without calculating, determine whether each expression would result in a positive or negative value.

Scenario	Expression	Positive	Negative
a The highest temperature ever recorded in Antarctica was 69.3°F. The lowest temperature was −135.8°F. What is the difference between these temperatures?	−135.8 − 69.3	☐	☑
b At the South Pole, the average maximum temperature in July is −67°F. This is 52°F lower than the average maximum temperature in January. What is the average maximum temperature in January?	−67 + 52	☐	☑
c The highest point in Antarctica is the top of Vinson Massif Mountain at 16,050 ft above sea level. The lowest point is within the Denman Glacier, reaching 11,500 ft below sea level. What is the distance between the highest and lowest point of Antarctica?	$\|16050 - (-11500)\|$ $\|-11500 - 16050\|$	☑	☐

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Lesson 9 Adding and Subtracting Rational Numbers 443

1 Launch

Activate students' background knowledge by asking what they know about Antarctica. Ask, "It is colder in Antarctica in July than it is in December. Can anyone explain why?" Explain that they will be using what they know about addition and subtraction of rational numbers to write and evaluate expressions about the extreme conditions in Antarctica.

2 Monitor

Help students get started by asking, "Which operation would you use to model each scenario?"

Look for points of confusion:

- **Struggling to write an equation.** Suggest students sketch a number line to help them make sense of each scenario.
- **Thinking that they have to use subtraction for all of the expressions.** Remind students that they may use either addition or subtraction to best represent each scenario.
- **Thinking that there is only one correct expression.** Encourage students to check whether their expressions are equivalent.

Look for productive strategies:

- Rewriting subtraction expressions as addition to determine the sign of the solution.
- Checking the reasonableness of the solution with the original scenario.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students draw number lines to help them visualize what is happening in each scenario.

Extension: *Math Enrichment, Interdisciplinary Connections*

Let students know the highest point on Earth is Mt. Everest, at about 29,000 ft above sea level. The deepest point on Earth is the Mariana Trench, in the Pacific Ocean, with a depth of about 36,000 ft below sea level. Have students compare the distance between the highest and lowest points of Antarctica to the distance between highest and lowest points on Earth. (Geography) The distance between the highest and lowest points on Antarctica is 27,500 ft, which is about 24%of the distance between the highest and lowest points on Earth, 65,000 ft.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of each scenario.

- **Read 1:** Students should understand the basic context for each scenario and what they are asked to determine.
- **Read 2:** Ask students to name or highlight the given quantities and relationships from each of the scenarios.
- **Read 3:** Ask students to plan their solution strategy and predict whether the answers will be positive or negative.

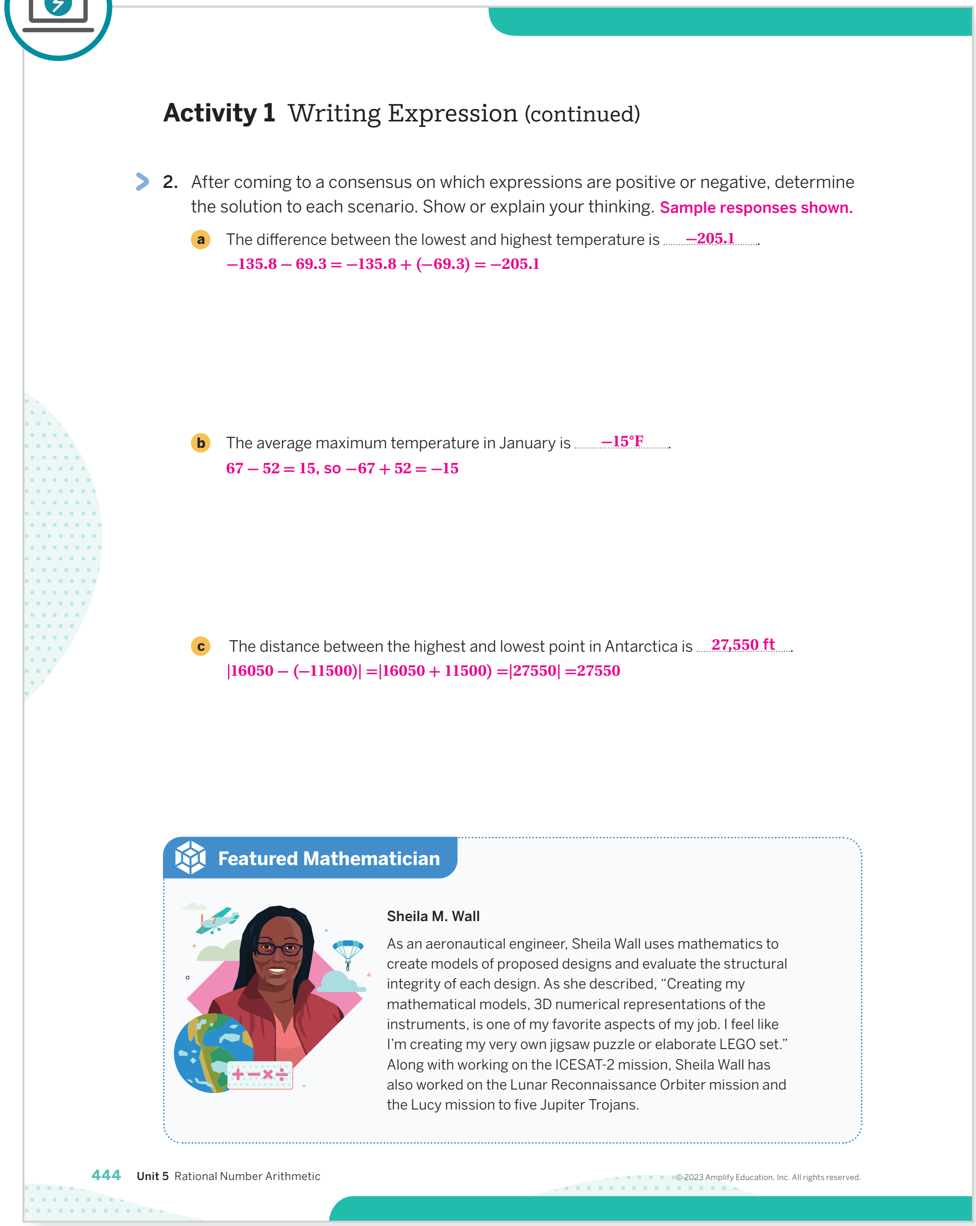
English Learners

After each read, provide students an opportunity to discuss their understanding with a partner to ensure comprehension.

Small groups | 15 min

Activity 1 Writing Expressions (continued)

Students apply their understanding of adding and subtracting rational numbers to write and evaluate expressions that represent scenarios.



Activity 1 Writing Expression (continued)

2. After coming to a consensus on which expressions are positive or negative, determine the solution to each scenario. Show or explain your thinking. **Sample responses shown.**
   - a The difference between the lowest and highest temperature is **−205.1**.
     $-135.8 - 69.3 = -135.8 + (-69.3) = -205.1$
   - b The average maximum temperature in January is **−15°F**.
     $67 - 52 = 15$, so $-67 + 52 = -15$
   - c The distance between the highest and lowest point in Antarctica is **27,550 ft**.
     $|16050 - (-11500)| = |16050 + 11500) = |27550| = 27550$

Featured Mathematician

Sheila M. Wall

As an aeronautical engineer, Sheila Wall uses mathematics to create models of proposed designs and evaluate the structural integrity of each design. As she described, "Creating my mathematical models, 3D numerical representations of the instruments, is one of my favorite aspects of my job. I feel like I'm creating my very own jigsaw puzzle or elaborate LEGO set." Along with working on the ICESAT-2 mission, Sheila Wall has also worked on the Lunar Reconnaissance Orbiter mission and the Lucy mission to five Jupiter Trojans.

444 Unit 5 Rational Number Arithmetic

3 Connect

Display each scenario to the class, one at a time.

Have students share their expressions for each one and how they determined whether the solution would be positive or negative.

Ask:

- "How did you determine which operation to use when writing your expressions?"
- "How did thinking of the sign prior to evaluating your expression help you to determine whether your answer was reasonable?"

Highlight that, for each scenario, considering the sign of the solution can aid in writing the expression that represents the scenario, as well as to assess whether the solution after evaluating the expression is reasonable.

Featured Mathematician

Sheila Wall

Have students read about the featured mathematician, Sheila Wall, an aerospace engineer working for NASA. Her designs have been used to study Earth, the Moon, and Jupiter trojans (large asteroids that follow the same orbit as Jupiter around the Sun).

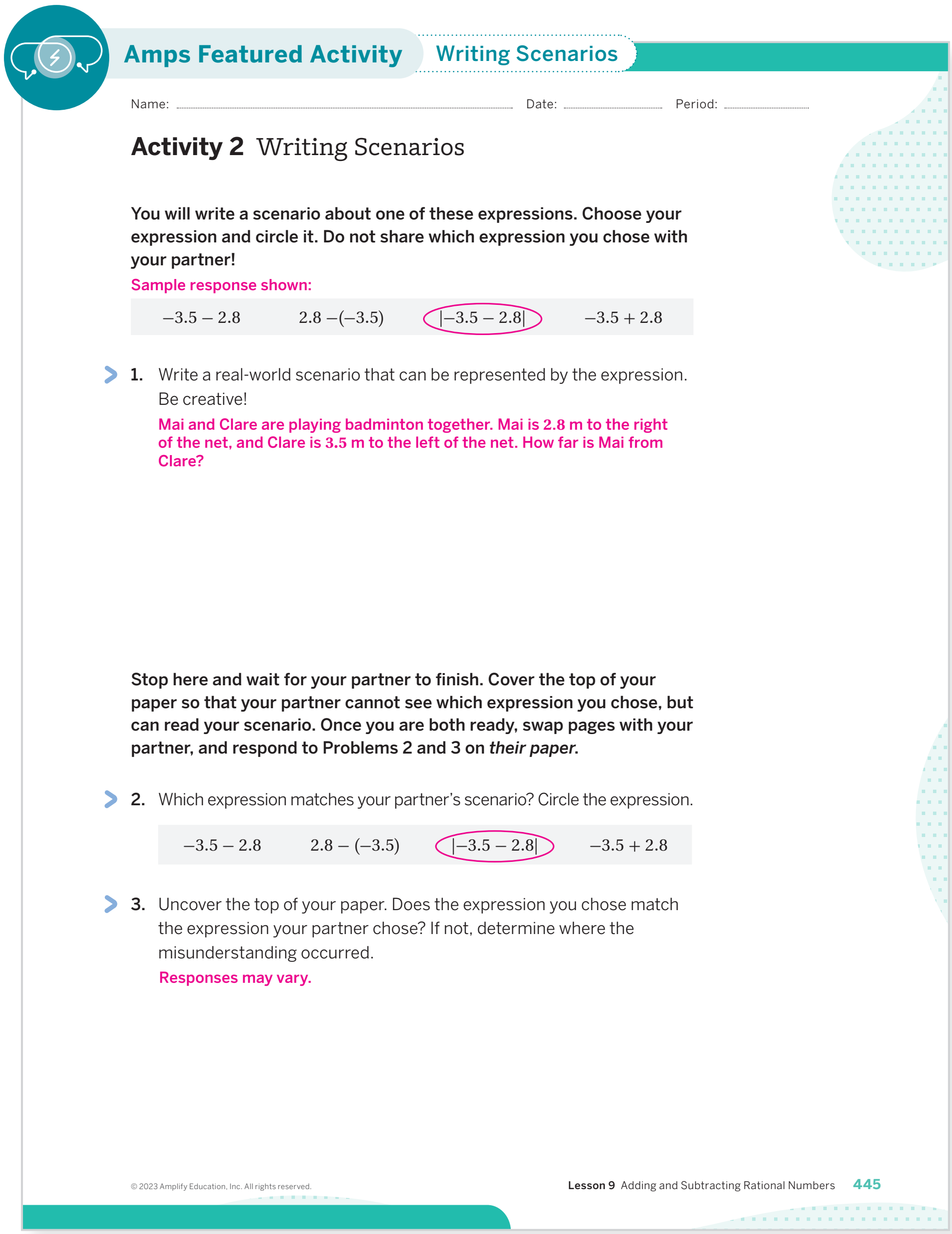
Optional

Pairs | 10 min

Activity 2 Writing Scenarios

Students write a real-world scenario to represent an expression with rational numbers and confirm with a peer.

Amps Featured Activity Writing Scenarios



Name: ____________ Date: ________ Period: ________

Activity 2 Writing Scenarios

You will write a scenario about one of these expressions. Choose your expression and circle it. Do not share which expression you chose with your partner!

Sample response shown:

$-3.5 - 2.8$ $2.8 -(-3.5)$ [✓ $|-3.5 - 2.8|$] $-3.5 + 2.8$

1. Write a real-world scenario that can be represented by the expression. Be creative!

   Mai and Clare are playing badminton together. Mai is 2.8 m to the right of the net, and Clare is 3.5 m to the left of the net. How far is Mai from Clare?

Stop here and wait for your partner to finish. Cover the top of your paper so that your partner cannot see which expression you chose, but can read your scenario. Once you are both ready, swap pages with your partner, and respond to Problems 2 and 3 on *their paper*.

2. Which expression matches your partner's scenario? Circle the expression.

   $-3.5 - 2.8$ $2.8 - (-3.5)$ [✓ $|-3.5 - 2.8|$] $-3.5 + 2.8$

3. Uncover the top of your paper. Does the expression you chose match the expression your partner chose? If not, determine where the misunderstanding occurred.

   Responses may vary.

Lesson 9 Adding and Subtracting Rational Numbers 445

1 Launch

Explain to students that they will begin by writing their own scenario to match one of the given expressions. They should not share which expression they chose. Once all students have written their scenarios, they should cover the expression they chose, and swap papers with their partner.

2 Monitor

Help students get started by asking, "What are the two values being added or subtracted in each expression?"

Look for points of confusion:

- **Writing a literal translation of the expression (e.g. the difference between −3.5 and 2.8).** Remind students that their goal is to write a real-world scenario. Ask, "Can you think of a real-world scenario that would involve these values?"

3 Connect

Display the four expressions.

Have students share the scenarios they wrote. Try to have at least three scenarios for each expression.

Ask:

- "What similarities do you notice about the scenarios that were read aloud for the first expression?" Repeat this question for each expression.
- "Was there any confusion between the scenario that was written and the expression chosen?"
- "What was the most difficult part about writing your own scenario?"

Highlight that there are multiple scenarios for each expression. Although they all are unrelated in narrative, the description of the relationship between the values (e.g. difference, distance, or change) is similar.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can choose an expression and write a scenario that matches it. You can display multiple scenarios that match each expression and facilitate a discussion about the similarities and differences between them.

Accessibility: *Guide Processing and Visualization*

Provide a sample real-world scenario that can be represented by an expression not shown, such as $|2.8 - (-3.8)|$ representing the scenario: Priya climbed 2.8 m up a mountain at the same time Andre descended 3.8 m of a cave. How far apart are they? Have students refer to this sample as a reference.

Math Language Development

MLR7: Compare and Connect

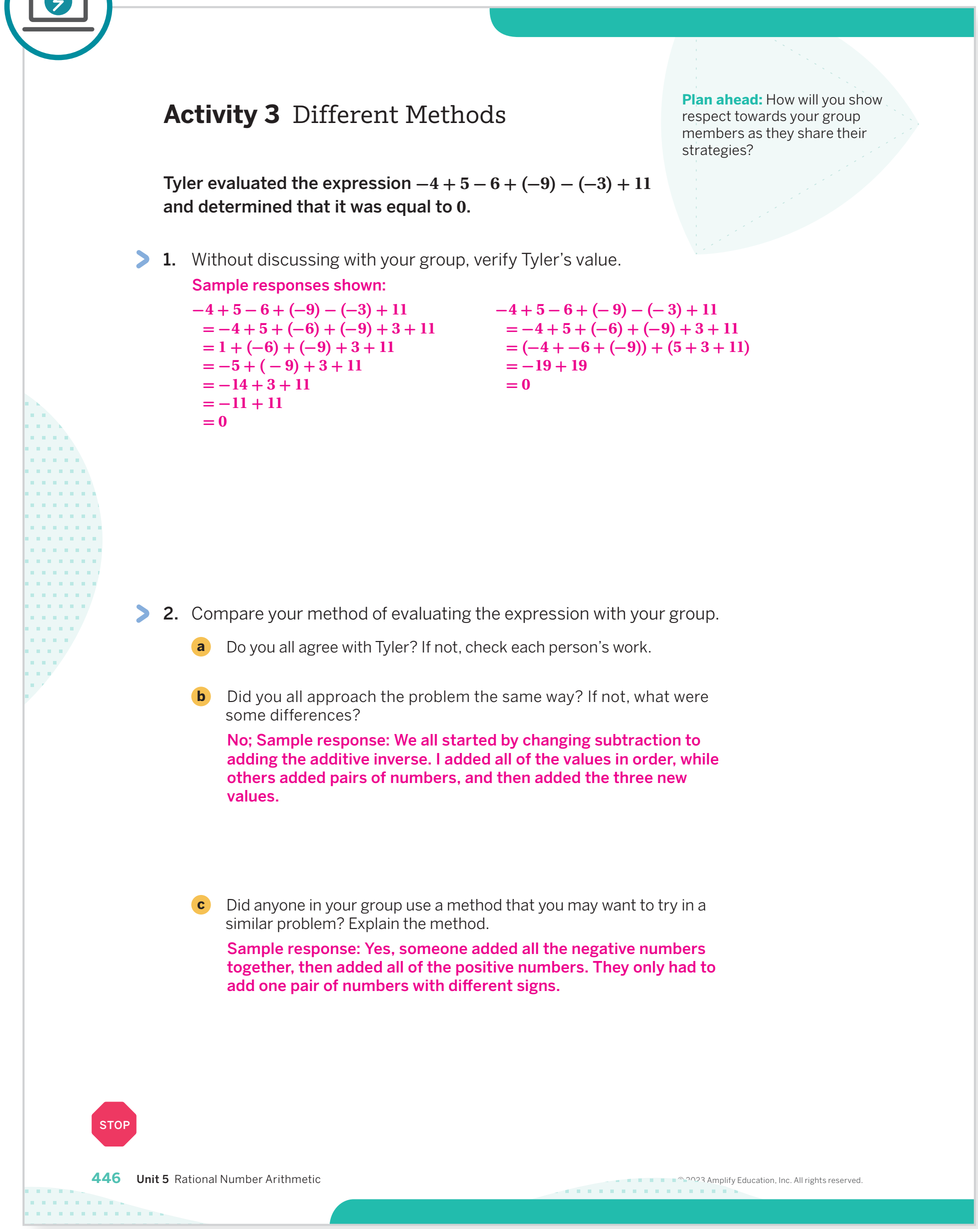
During the Connect, as students share their scenarios, draw their attention to the similarities and differences between different scenarios represented by the same expression. Ask:

- "How did you choose the quantities in your scenario? How do you know they matched the expression?"
- "Did you use the terms *distance* and/or *difference* in your scenarios? How did the expressions indicate to you to use these terms?"
- "If you chose an expression with absolute value bars, how was that described in your scenario?"

Small groups | 10 min

Activity 3 Different Methods

Students explore different methods for simplifying expressions with both addition and subtraction by comparing and contrasting their method with their group.



Activity 3 Different Methods

Plan ahead: How will you show respect towards your group members as they share their strategies?

Tyler evaluated the expression $-4 + 5 - 6 + (-9) - (-3) + 11$ and determined that it was equal to 0.

1. Without discussing with your group, verify Tyler's value.

Sample responses shown:

$$\begin{aligned} &-4 + 5 - 6 + (-9) - (-3) + 11 \\ &= -4 + 5 + (-6) + (-9) + 3 + 11 \\ &= 1 + (-6) + (-9) + 3 + 11 \\ &= -5 + (-9) + 3 + 11 \\ &= -14 + 3 + 11 \\ &= -11 + 11 \\ &= 0 \end{aligned}$$

$$\begin{aligned} &-4 + 5 - 6 + (-9) - (-3) + 11 \\ &= -4 + 5 + (-6) + (-9) + 3 + 11 \\ &= (-4 + -6 + (-9)) + (5 + 3 + 11) \\ &= -19 + 19 \\ &= 0 \end{aligned}$$

2. Compare your method of evaluating the expression with your group.
   - a Do you all agree with Tyler? If not, check each person's work.
   - b Did you all approach the problem the same way? If not, what were some differences?

     No; Sample response: We all started by changing subtraction to adding the additive inverse. I added all of the values in order, while others added pairs of numbers, and then added the three new values.
   - c Did anyone in your group use a method that you may want to try in a similar problem? Explain the method.

     Sample response: Yes, someone added all the negative numbers together, then added all of the positive numbers. They only had to add one pair of numbers with different signs.

STOP

446 Unit 5 Rational Number Arithmetic

1 Launch

Explain that students will be responding to Problem 1 independently, and then they will compare their methods as a group.

2 Monitor

Help students get started by covering all but the first three values. Ask, "What would be your first step in simplifying this expression with three values?"

Look for points of confusion:

- **Completing all of the addition, prior to subtracting.** Remind students that addition and subtraction are done from left to right according to the order of the operations.

Look for productive strategies:

- Rewriting all of the subtraction in the original expression as adding the additive inverse.
- Using the associative and commutative properties to make the addition more efficient.

3 Connect

Display student samples of correct methods for evaluating the expression. Ask students to determine whether they agree or disagree with each method.

Have students share what they notice are similar or different between the methods displayed.

Ask, "Did any of the methods use a strategy or idea that you did not consider?"

Highlight that when the expression is rewritten using only addition, the values can be added in any order.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide copies of the Graphic Organizer PDF, *Blank Number Lines* for students to choose to use to help them organize their thinking and make sense of the expression and the operations involved.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share what they notice about the different methods, draw their attention to the connections between them. Ask:

- "Did anyone rewrite the subtraction as addition? How do you know that this is a valid approach?"
- "Did anyone group the positive numbers (or negative numbers) together and add them separately? What property allows you to do so?"

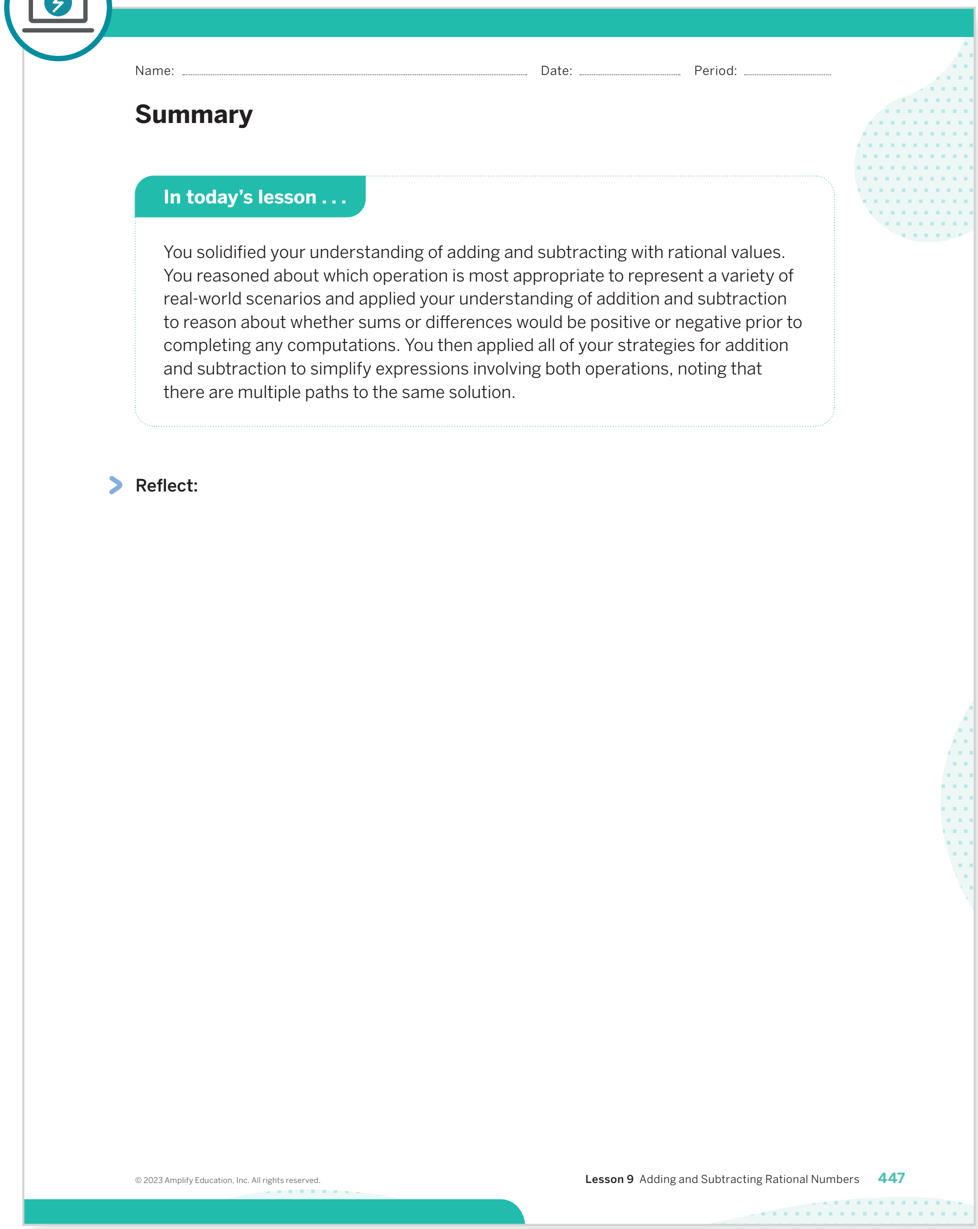
English Learners

Encourage students to refer to and use the class display to support their use of appropriate mathematical language.

Whole Class | 5 min

Summary

Review and synthesize methods for writing and evaluating expressions involving adding and subtracting rational numbers.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You solidified your understanding of adding and subtracting with rational values. You reasoned about which operation is most appropriate to represent a variety of real-world scenarios and applied your understanding of addition and subtraction to reason about whether sums or differences would be positive or negative prior to completing any computations. You then applied all of your strategies for addition and subtraction to simplify expressions involving both operations, noting that there are multiple paths to the same solution.

Reflect:

Synthesize

Have students share strategies they used throughout class today to determine the sum or difference of rational numbers.

Highlight that rational numbers can be used to represent problems in context. To solve problems in these situations, students have to understand what it means when the quantity is positive, when it is negative, and what it means to add and subtract them

Ask, "What are some other situations where adding and subtracting rational numbers can help solve problems?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can rational numbers be used to represent real-world situations?"

Differentiated Support

Extension: *Math Around the World, Interdisciplinary Connections*

Ask students if they have ever wondered when the rules for addition and subtraction with rational numbers first appeared in the history of mathematics. Tell them that in his writings around 620 CE, Indian mathematician Brahmagupta first used a special sign for negative values and described the rules for adding and subtracting with negative values. The need to operate with negative numbers in India likely arose out of necessity of their highly developed and cosmopolitan civilization that began four to five thousands years ago. They traded with other civilizations including some from thousands of miles away which required them to grapple with concepts like assets, debts, revenues, expenses, and income.

Brahmagupta used the idea of "fortunes" as representing positive values and "debts" as representing negative values. He stated:

- A debt minus zero is a debt.
- A fortune minus zero is a fortune.
- Zero minus zero is a zero.
- A debt subtracted from a fortune is a fortune.
- A fortune subtracted from zero is a debt.

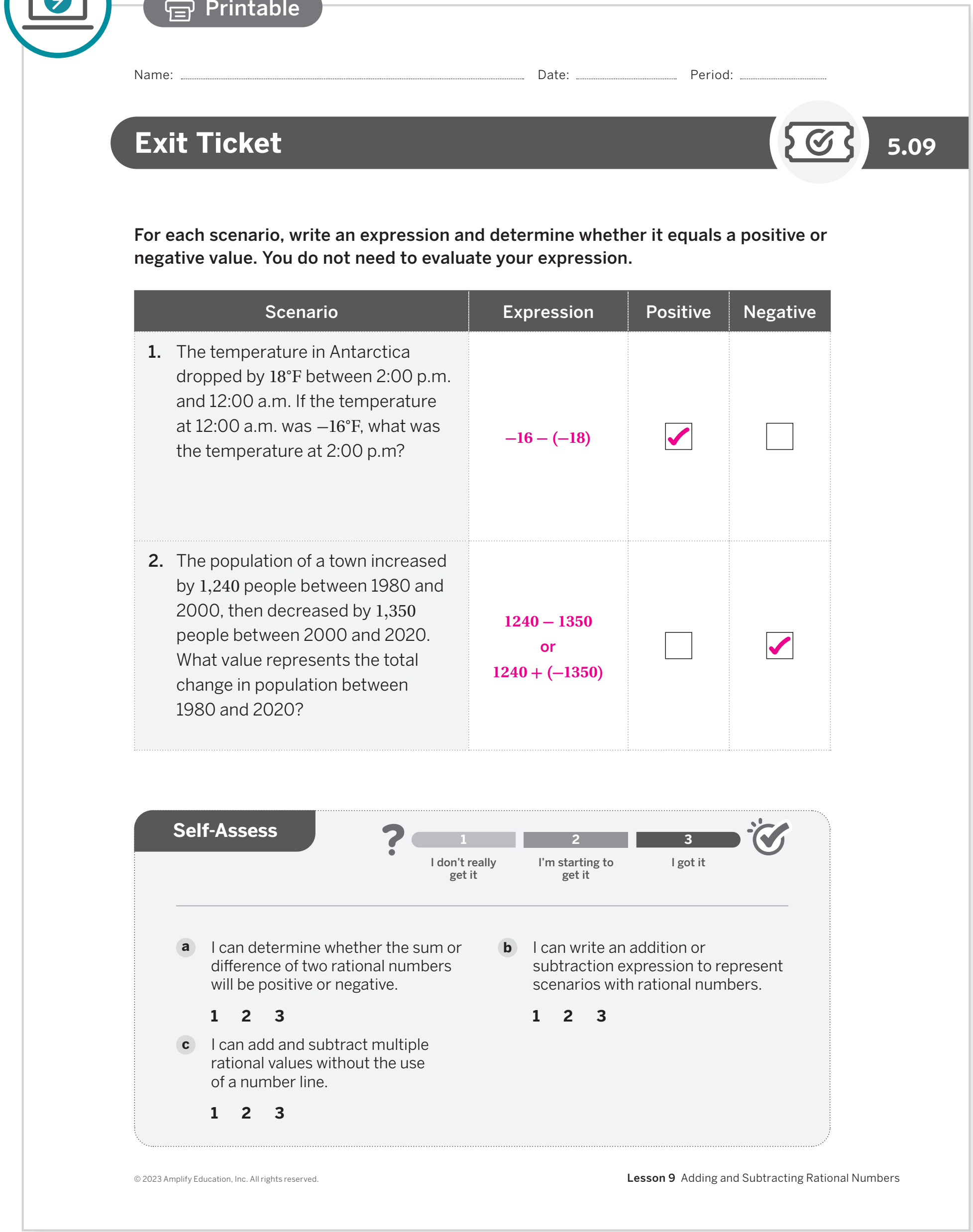
Ask students to rewrite Brahmagupta's rules using the terms "positive number" and "negative number." Then ask them to provide numerical examples that illustrate Brahmagupta's rules.

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of expressions with rational numbers by writing expressions to represent scenarios and determining the sign of the solution.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 5.09

For each scenario, write an expression and determine whether it equals a positive or negative value. You do not need to evaluate your expression.

Scenario	Expression	Positive	Negative
1. The temperature in Antarctica dropped by 18°F between 2:00 p.m. and 12:00 a.m. If the temperature at 12:00 a.m. was −16°F, what was the temperature at 2:00 p.m?	$-16 - (-18)$	☑	☐
2. The population of a town increased by 1,240 people between 1980 and 2000, then decreased by 1,350 people between 2000 and 2020. What value represents the total change in population between 1980 and 2020?	$1240 - 1350$ or $1240 + (-1350)$	☐	☑

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can determine whether the sum or difference of two rational numbers will be positive or negative.

1 2 3

b. I can write an addition or subtraction expression to represent scenarios with rational numbers.

1 2 3

c. I can add and subtract multiple rational values without the use of a number line.

1 2 3

Lesson 9 Adding and Subtracting Rational Numbers

Success looks like . . .

- **Language Goal:** Applying addition and subtraction of rational numbers to solve problems in unfamiliar contexts, and explaining the solution method. **(Writing, Speaking and Listening)**
  - » Solving the problems of temperature and population by adding and subtracting rational numbers in Problems 1 and 2.
- **Language Goal:** Evaluating expressions involving both addition and subtraction, and explaining the reasoning. **(Speaking and Listening)**
- **Language Goal:** Determining whether the sum or difference of two rational numbers is positive or negative, and explaining the reasoning. **(Speaking and Listening)**

Suggested next steps

If students incorrectly write the expression, consider:

- Reviewing Activity 1.
- Suggesting that students model the scenario on a number line prior to writing an expression.
- Assigning Practice Problem 1.

If students correctly write the expressions but incorrectly identify the sum or difference as positive, consider:

- Reviewing the rules for adding and subtracting rational numbers.
- Suggesting that students model the expression on a number line.
- Assigning Practice Problem 2.

Professional Learning

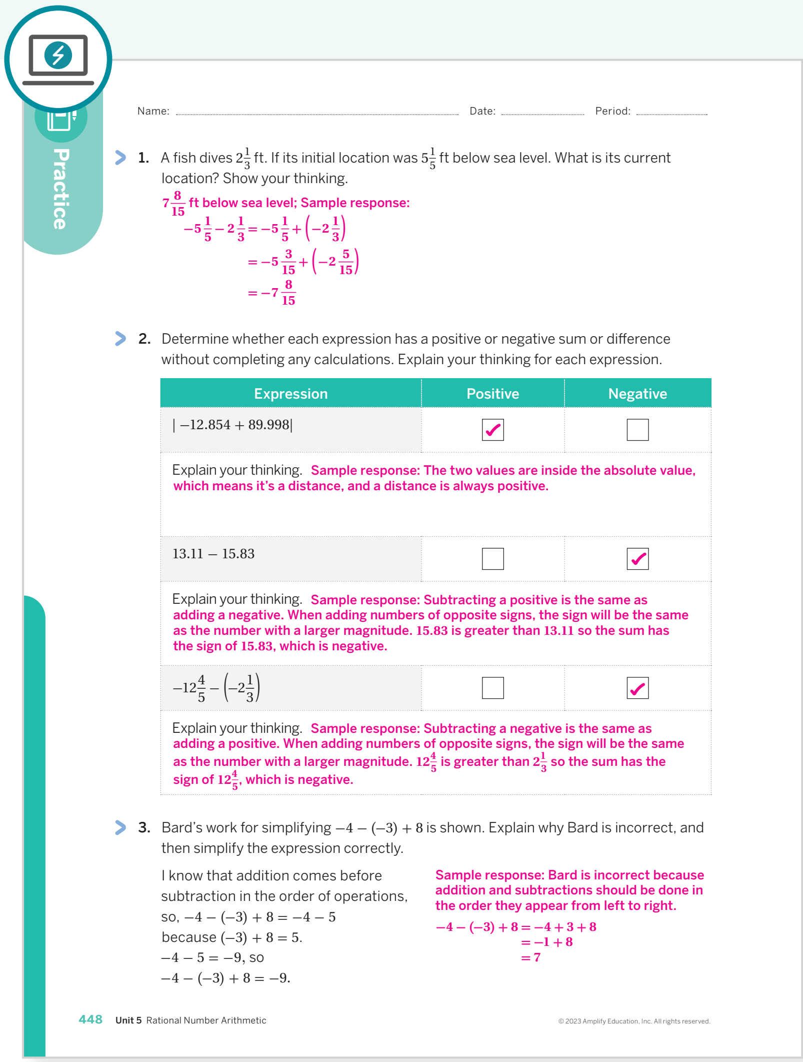
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students become better at making sense of problems and persevering in solving them?
- During the discussion about Activity 3 how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. A fish dives $2\frac{1}{3}$ ft. If its initial location was $5\frac{1}{5}$ ft below sea level. What is its current location? Show your thinking.

   $7\frac{8}{15}$ ft below sea level; Sample response:

   $$\begin{aligned} -5\frac{1}{5} - 2\frac{1}{3} &= -5\frac{1}{5} + \left(-2\frac{1}{3}\right) \\ &= -5\frac{3}{15} + \left(-2\frac{5}{15}\right) \\ &= -7\frac{8}{15} \end{aligned}$$

2. Determine whether each expression has a positive or negative sum or difference without completing any calculations. Explain your thinking for each expression.

Expression	Positive	Negative
$\lvert -12.854 + 89.998 \rvert$	☑	☐
Explain your thinking. **Sample response: The two values are inside the absolute value, which means it's a distance, and a distance is always positive.**		
$13.11 - 15.83$	☐	☑
Explain your thinking. **Sample response: Subtracting a positive is the same as adding a negative. When adding numbers of opposite signs, the sign will be the same as the number with a larger magnitude. 15.83 is greater than 13.11 so the sum has the sign of 15.83, which is negative.**		
$-12\frac{4}{5} - \left(-2\frac{1}{3}\right)$	☐	☑
Explain your thinking. **Sample response: Subtracting a negative is the same as adding a positive. When adding numbers of opposite signs, the sign will be the same as the number with a larger magnitude. $12\frac{4}{5}$ is greater than $2\frac{1}{3}$ so the sum has the sign of $12\frac{4}{5}$, which is negative.**		

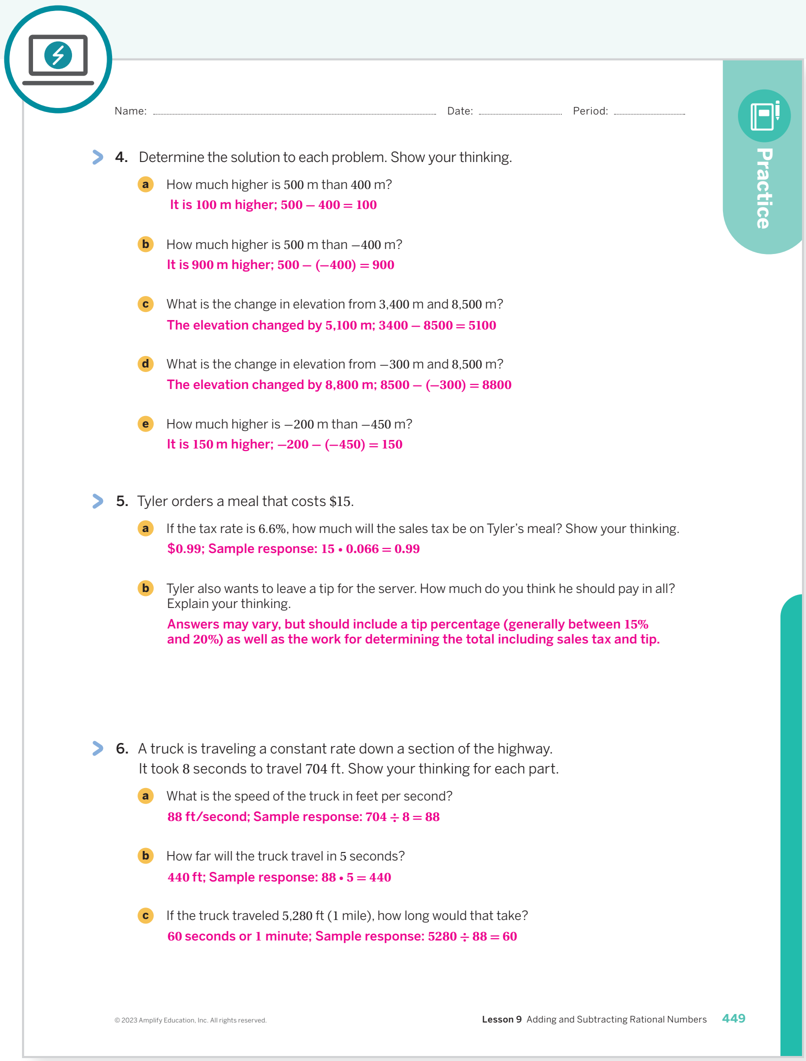
3. Bard's work for simplifying $-4 - (-3) + 8$ is shown. Explain why Bard is incorrect, and then simplify the expression correctly.

   I know that addition comes before subtraction in the order of operations,
   so, $-4 - (-3) + 8 = -4 - 5$
   because $(-3) + 8 = 5$.
   $-4 - 5 = -9$, so
   $-4 - (-3) + 8 = -9$.

   Sample response: Bard is incorrect because addition and subtractions should be done in the order they appear from left to right.

   $$\begin{aligned} -4 - (-3) + 8 &= -4 + 3 + 8 \\ &= -1 + 8 \\ &= 7 \end{aligned}$$

448 **Unit 5** Rational Number Arithmetic © 2023 Amplify Education, Inc. All rights reserved.



Name: ______ Date: ______ Period: ______

4. Determine the solution to each problem. Show your thinking.
   - a. How much higher is 500 m than 400 m?
     It is 100 m higher; $500 - 400 = 100$
   - b. How much higher is 500 m than −400 m?
     It is 900 m higher; $500 - (-400) = 900$
   - c. What is the change in elevation from 3,400 m and 8,500 m?
     The elevation changed by 5,100 m; $3400 - 8500 = 5100$
   - d. What is the change in elevation from −300 m and 8,500 m?
     The elevation changed by 8,800 m; $8500 - (-300) = 8800$
   - e. How much higher is −200 m than −450 m?
     It is 150 m higher; $-200 - (-450) = 150$

5. Tyler orders a meal that costs $15.
   - a. If the tax rate is 6.6%, how much will the sales tax be on Tyler's meal? Show your thinking.
     $0.99; Sample response: $15 \cdot 0.066 = 0.99$
   - b. Tyler also wants to leave a tip for the server. How much do you think he should pay in all? Explain your thinking.
     Answers may vary, but should include a tip percentage (generally between 15% and 20%) as well as the work for determining the total including sales tax and tip.

6. A truck is traveling a constant rate down a section of the highway. It took 8 seconds to travel 704 ft. Show your thinking for each part.
   - a. What is the speed of the truck in feet per second?
     88 ft/second; Sample response: $704 \div 8 = 88$
   - b. How far will the truck travel in 5 seconds?
     440 ft; Sample response: $88 \cdot 5 = 440$
   - c. If the truck traveled 5,280 ft (1 mile), how long would that take?
     60 seconds or 1 minute; Sample response: $5280 \div 88 = 60$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	1
	3	Activity 3	2
Spiral	4	Unit 5 Lesson 7	2
	5	Unit 4 Lesson 8	3
Formative ⚡	6	Unit 5 Lesson 10	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 2

Multiplying and Dividing Rational Numbers

Whole Class

In this Sub-Unit, students build on their prior knowledge of distance, rate, and time, to model and interpret products and quotients of rational values in terms of position and direction, ultimately developing the rules for multiplying and dividing rational numbers.

SUB-UNIT

2 Multiplying and Dividing Rational Numbers

Narrative Connections

How many Mt. Everests can a grandma climb between Georgia and Maine?

Emma Gatewood stood atop Mt. Oglethorpe. The Georgia wilderness lay before the 67-year-old great-grandmother — hundreds of mountains, trees, rivers, and streams. She had told her family back in Ohio that she was going for a walk. What she didn't tell them was that the walk was going to cover 2,168 miles along the length of the Appalachian Trail.

The Appalachian Trail follows the Appalachian Mountains across nearly a dozen states. Established in 1937, the trail was a place anyone could go to reconnect with nature. The trail linked work, study, and farming camps that were set up along the mountain range. Every year, "thru-hikers" like Gatewood take up the challenge of hiking its total length.

In the summer of 1955, equipped with barely any supplies and a pair of tennis shoes, Gatewood set to conquer the trail. Over the course of a season, she faced rocky passes, dizzying heights, floods, storms, and rattlesnakes. She ate off the land and slept rough in improvised shelters. Soon, newspapers caught wind of her story, and strangers started offering her food and a place to stay. Finally, after 146 days, across 14 states, she emerged from the trail at Maine's Mount Katahdin.

By the time her trip was done, she had trekked the equivalent of climbing up and down Mt. Everest 16 times. Not only was she the oldest hiker to thru-hike the trail, she was also the first woman. When reporters asked her why she set out on this task, she simply answered, "For the heck of it!"

To determine that Gatewood hiked the equivalent of 16 times the height of Mt. Everest (up and down), we need to understand how to multiply and divide rational numbers.

Read and discuss

Read the narrative aloud as a class or have students read it individually. Students continue to explore real-world applications of multiplying and dividing rational numbers in the following places:

- **Lesson 10, Activity 1:** Backward and Forward in Time
- **Lesson 11, Activity 1:** Velocity and Time
- **Lesson 14, Activities 1-2:** Drilling a Well, Diving With the Ama

UNIT 5 | **LESSON 10**

Position, Speed, and Time

Let’s use rational numbers to represent time and movement.

Focus

Goals

1. **Language Goal:** Explain how rational numbers can be used to represent elapsed time and distance before and after a chosen reference point. **(Speaking and Listening, Writing)**
2. Write a multiplication expression to represent a situation involving constant speed and time.
3. **Language Goal:** Generalize that the product of a positive number and a negative number is negative. **(Speaking and Listening)**

Coherence

Today

Students explore the product of a negative number and a positive number by analyzing the location of a person traveling at a constant speed given a positive or negative time. They model movement on a number line, noting that when going forward in time the location on the number line is positive and when moving backward in time the location is negative. Students formalize that the product of a positive number with a negative number is negative, by relating multiplication expressions to repeated addition.

Previously

In Lessons 3–9, students developed and practiced the rules for adding and subtracting rational numbers.

Coming Soon

In Lesson 11, students will generalize the rules for multiplying rational numbers.

Rigor

- Students use number lines and repeated addition to build **conceptual understanding** of multiplying a positive and negative value.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	15 min	13 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Graphic Organizer PDF, *Blank Number Lines* (as needed)
- paper clips, snap cubes, or other objects to move on the number line

Math Language Development

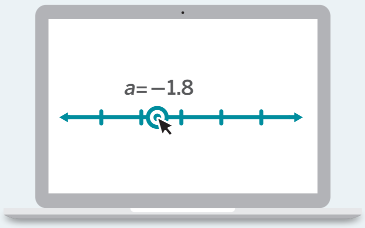
Review words

- *absolute value*
- *magnitude*
- *negative numbers*
- *positive numbers*
- *rational numbers*

Amps | Featured Activity

Activity 1
Interactive Number Line

Students use interactive number lines to determine the location of vehicles as they move at constant speeds forward and backward in time.



Building Math Identity and Community

Connecting to Mathematical Practices

While working on Activity 1, students might be stressed with the different concepts all being modeled in one problem. Prior to starting, have students brainstorm ways that they might lower their stress levels in order to focus on the task at hand. They might need to take a quick break, do some deep breathing, or get a drink of water. It might help them to color code their work to help them focus on the different goals of the Activity.

Modifications to Pacing

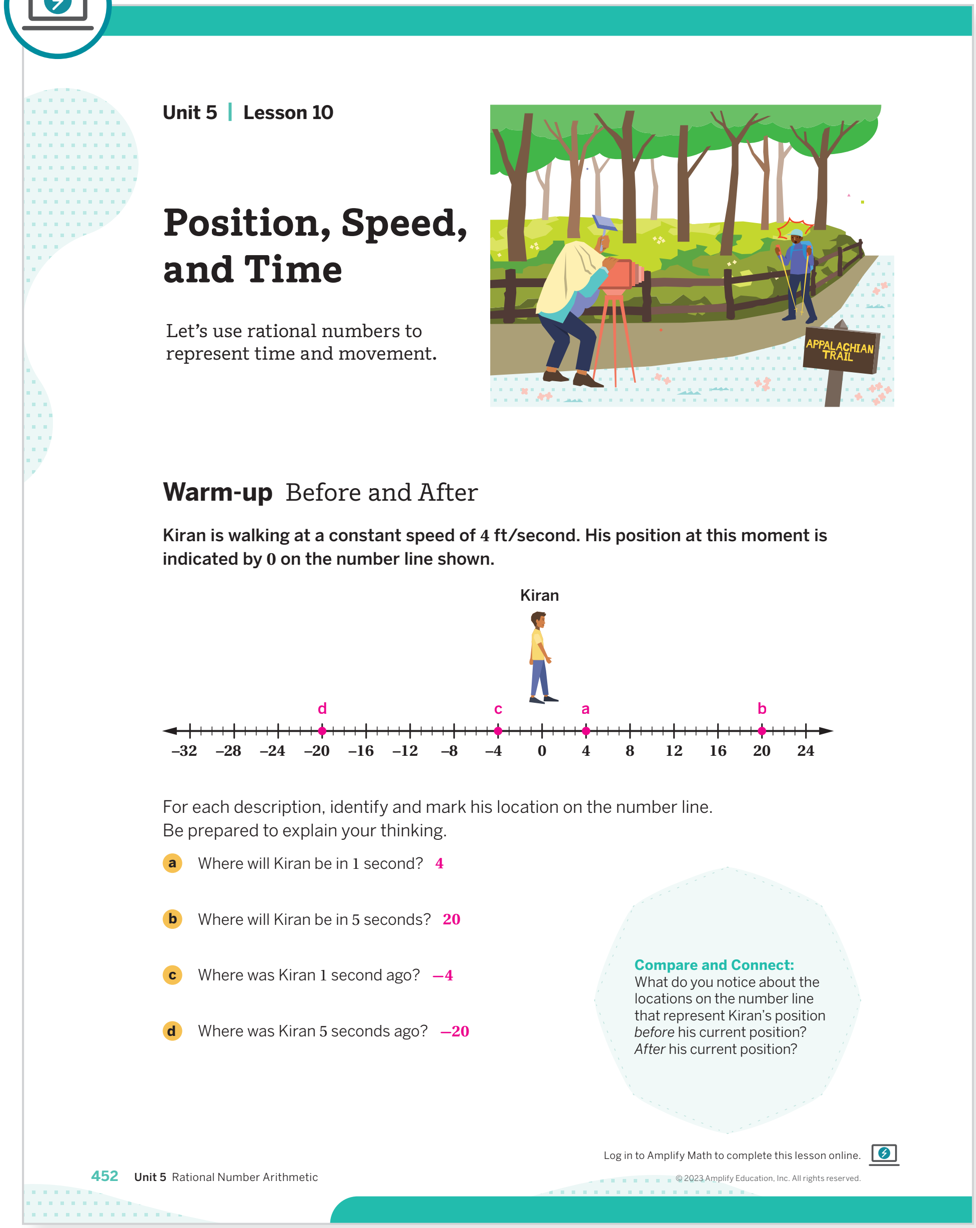
You may want to consider these additional modifications if you are short on time.

- **In Activity 1**, ask students to discuss Problem 3 with their group, without writing down their responses.
- **In Activity 2**, have students choose any three rows to complete along with the final row. Have students discuss Problem 2 as a group without writing down their responses.

Pairs | 7 min

Warm-up Before and After

Students reason about location in the past and future using positive and negative values for a person traveling at a constant speed.



Unit 5 | Lesson 10

Position, Speed, and Time

Let's use rational numbers to represent time and movement.

Warm-up Before and After

Kiran is walking at a constant speed of 4 ft/second. His position at this moment is indicated by 0 on the number line shown.

For each description, identify and mark his location on the number line. Be prepared to explain your thinking.

a. Where will Kiran be in 1 second? 4

b. Where will Kiran be in 5 seconds? 20

c. Where was Kiran 1 second ago? –4

d. Where was Kiran 5 seconds ago? –20

Compare and Connect: What do you notice about the locations on the number line that represent Kiran's position *before* his current position? *After* his current position?

Log in to Amplify Math to complete this lesson online.

452 **Unit 5** Rational Number Arithmetic

1 Launch

Conduct the ***Think-Pair-Share*** routine. Provide students with an object (block, paper clip, etc.) to represent Kiran moving on the number line as well as the hiker moving in the next activity.

2 Monitor

Help students get started by asking them to stand and model what it would look like for them to walk forward in time and backward in time to help them make sense of the number line model.

Look for points of confusion:

- **Not taking Kiran's walking speed into consideration and placing his location at 1 second at 1 on the number line.** Bring students attention to Kiran's speed of 4 ft/second. Ask, "Without using the number line, think about his walking rate. How far would he travel in 1 second? 2 seconds?"

3 Connect

Display a large number line in the front of the room and ask for a student to represent Kiran.

Have students share how they determined the location of Kiran on the number line with the student volunteer acting out the movements.

Highlight that students can skip count by Kiran's walking speed, where each movement of 4 to the right represents moving forward in time and 4 to the left represents moving backward in time.

Ask, "What integer value could be used to represent 5 seconds in the future? 5 seconds in the past?" 5; –5

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to share their responses to the questions posed in the Student Edition:

- "What do you notice about the locations on the number line that represent Kiran's position *before* his current position? *After* his current position?"

Encourage students for details about their reasoning by asking:

- "How did your solution to part a help you with part c?"
- "How did your solution to part b help with part d?"

English Learners

Use the number line to demonstrate Kiran moving forward and backward in time. Clarify the meaning of the word *ago*.

Power-up

To power up students' ability to use proportional reasoning to solve problems involving distance, rate, and time, have students complete:

Mai travels at a rate of 8 mph when cross-country skiing.

1. Her total distance traveled can be modeled by the equation $d = 8t$. Match each part of the equation with what it represents.
   a. d b The speed.
   b. 8 c The time in hours.
   c. t a The distance in miles.
2. What is her distance traveled after half an hour? Show your thinking.
   She traveled 4 miles; $t = 0.5$; $d = 8(0.5)$
   $d = 4$

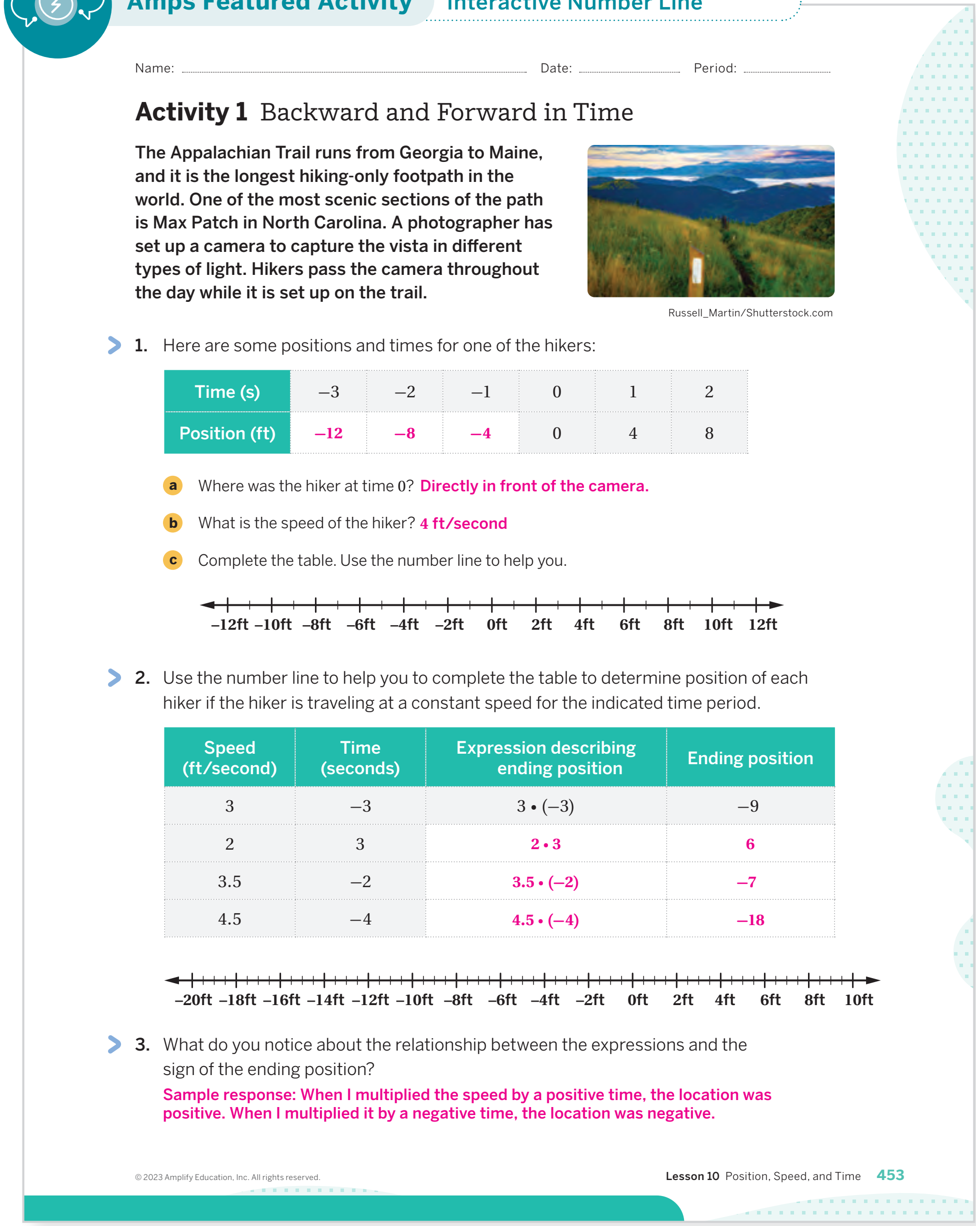
Use: Before the Warm-up.
Informed by: Performance on Lesson 9, Practice Problem 6.

Small groups | 15 min

Activity 1 Backward and Forward in Time

Students explore the relationship between speed and time noticing that the product of positive and negative values seems to be negative.

Amps Featured Activity Interactive Number Line



Name: ________ Date: ________ Period: ________

Activity 1 Backward and Forward in Time

The Appalachian Trail runs from Georgia to Maine, and it is the longest hiking-only footpath in the world. One of the most scenic sections of the path is Max Patch in North Carolina. A photographer has set up a camera to capture the vista in different types of light. Hikers pass the camera throughout the day while it is set up on the trail.

Russell_Martin/Shutterstock.com

1. Here are some positions and times for one of the hikers:

Time (s)	–3	–2	–1	0	1	2
Position (ft)	**–12**	**–8**	**–4**	0	4	8

a. Where was the hiker at time 0? **Directly in front of the camera.**

b. What is the speed of the hiker? **4 ft/second**

c. Complete the table. Use the number line to help you.

2. Use the number line to help you to complete the table to determine position of each hiker if the hiker is traveling at a constant speed for the indicated time period.

Speed (ft/second)	Time (seconds)	Expression describing ending position	Ending position
3	–3	$3 \cdot (-3)$	–9
2	3	**$2 \cdot 3$**	**6**
3.5	–2	**$3.5 \cdot (-2)$**	**–7**
4.5	–4	**$4.5 \cdot (-4)$**	**–18**

3. What do you notice about the relationship between the expressions and the sign of the ending position?

Sample response: When I multiplied the speed by a positive time, the location was positive. When I multiplied it by a negative time, the location was negative.

Lesson 10 Position, Speed, and Time 453

1 Launch

Activate students' background knowledge by asking whether they have heard of the Appalachian Trail. Explain that students will be working together to determine the location of a variety of hikers traveling at different speeds in relation to a camera. Provide access to objects to use to represent the hikers moving in time on the number line.

2 Monitor

Help students get started by covering the first three columns of the table. Ask, "What do you notice about the values in the table?"

Look for points of confusion:

- **Skip counting by 2 instead of by 4 because that is the unit on the number line.** Ask, "What is the speed of the car?"

Look for productive strategies:

- Noticing a pattern in the sign of the values of the ending position, and then checking the pattern by using the number line.

3 Connect

Display the completed table from Problem 2.

Ask:

- "What does the negative value represent in each expression?"
- "What does the negative value represent in the ending position?"

Have students share what they noticed about the relationship between each expression and the ending position of each hiker.

Highlight that it appears that the product of a positive and a negative value is negative. Explain that students will explore whether this is true outside of speed and time in the next activity.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive number lines to determine the location of vehicles as they move at constant speeds both forward and backward in time.

Accessibility: *Guide Processing and Visualization*

Use a think-aloud to demonstrate how to respond to Problems 1a and 1b and how the hiker's speed was used to determine their position at 1 second and 2 seconds. Then have students work backward to complete the rest of the table. Ask, "What does a negative position mean?" The hiker was moving away from the camera, in the opposite direction.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the connections between the signs of the factors in the expression and the sign of the product. Ask:

- "What do you notice about the ending position when one of the factors is negative?"
- "Why does this make sense in this context?"

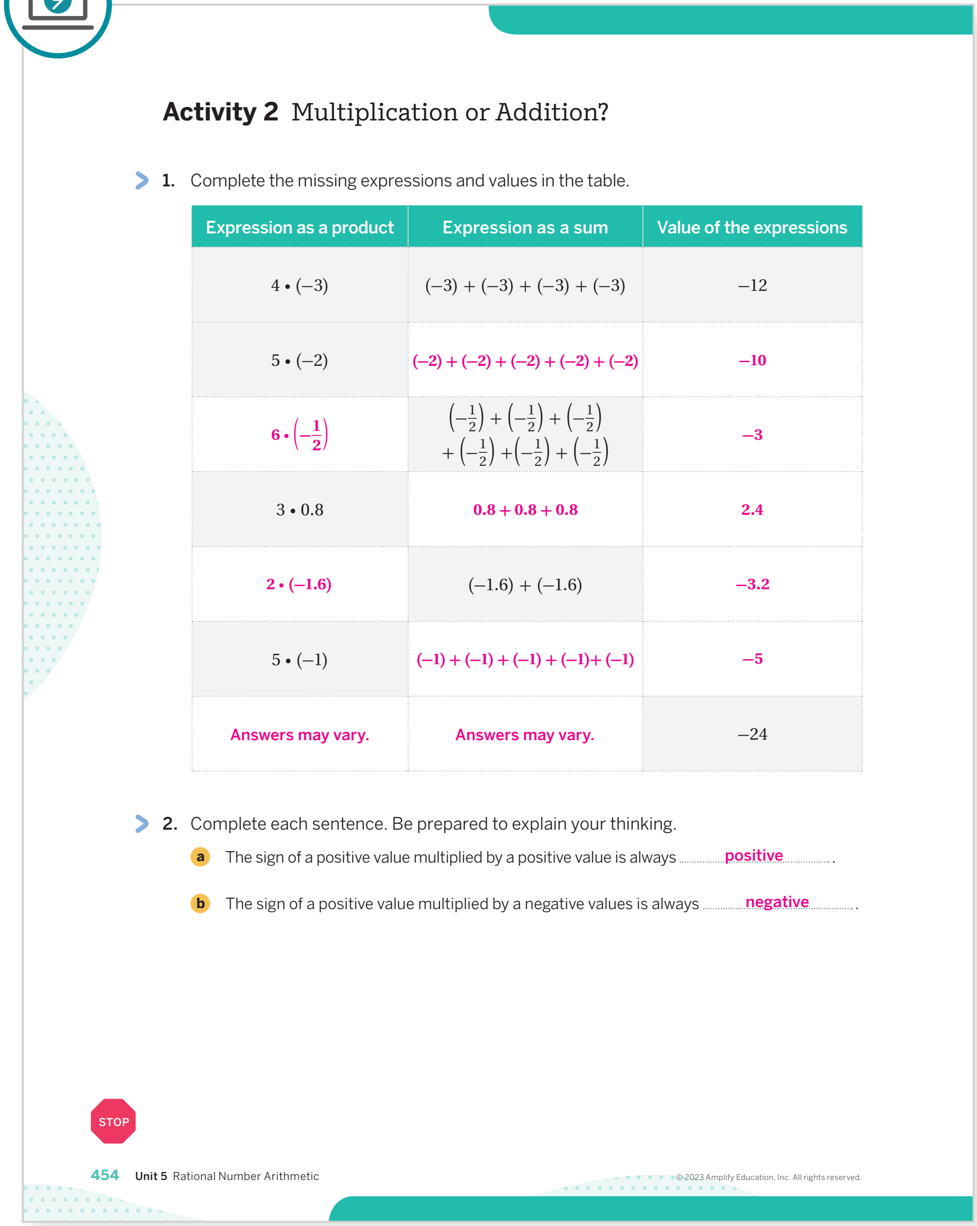
English Learners

Annotate the number line by writing "camera" at 0 ft, "moving away from the camera in one direction" for positive distances, and "moving away from the camera in the opposite direction" for negative distances.

Small groups | 13 min

Activity 2 Multiplication or Addition?

Students use repeated addition to determine that the product of a positive and negative will always be negative.



Activity 2 Multiplication or Addition?

1. Complete the missing expressions and values in the table.

Expression as a product	Expression as a sum	Value of the expressions
$4 \cdot (-3)$	$(-3) + (-3) + (-3) + (-3)$	-12
$5 \cdot (-2)$	$(-2) + (-2) + (-2) + (-2) + (-2)$	-10
$6 \cdot \left(-\frac{1}{2}\right)$	$\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)$ $+ \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)$	-3
$3 \cdot 0.8$	$0.8 + 0.8 + 0.8$	2.4
$2 \cdot (-1.6)$	$(-1.6) + (-1.6)$	-3.2
$5 \cdot (-1)$	$(-1) + (-1) + (-1) + (-1)+ (-1)$	-5
Answers may vary.	Answers may vary.	-24

2. Complete each sentence. Be prepared to explain your thinking.
   a. The sign of a positive value multiplied by a positive value is always positive.
   b. The sign of a positive value multiplied by a negative values is always negative.

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1 Launch

Activate students' prior knowledge by asking, "How could we model what $10 \cdot 2$ means?" After eliciting responses, illustrate that $10 \cdot 2$ represents 10 groups of 2.

2 Monitor

Help students get started by asking, "Can you explain why the two expressions in the first row are equivalent?"

Look for points of confusion:

- **Thinking that $a \cdot b$ is b groups of a.** Remind students that it is a groups of b, or b being added to itself a times.

3 Connect

Display correct student responses for the last row of the table. Give students a minute to look at the different responses.

Have students share what they notice about all of the responses as well as what conclusion, or rule, they can make about the product of a positive and a negative number.

Highlight that, in each case of multiplying a whole number by a negative value, it could be represented by repeated addition of the negative value resulting in a negative sum. In general, when multiplying a positive number by a negative number, first determine the product of the absolute value of the numbers, and then make the value negative.

Ask:

- "How would you determine the value of $2 \cdot (-4)$?" Write the expression as a sum of two groups of -4; $(-4) + (-4) = -8$.
- "What is the value of $\frac{1}{2} \cdot (-4)$?" -2; Sample response: Think of this as half of one group of -4, which is -2.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide copies of the Graphic Organizer PDF, *Blank Number Lines* for students to choose to use to help them organize their thinking and make sense of the expression and the operations involved.

Extension: *Math Enrichment*

Have students explain whether they could use similar reasoning to determine the value of the expression $\frac{1}{2} \cdot (-6)$. Sample response: Yes, this represents half of one group of -6, which is -3.

Math Language Development

MLR2: Collect and Display

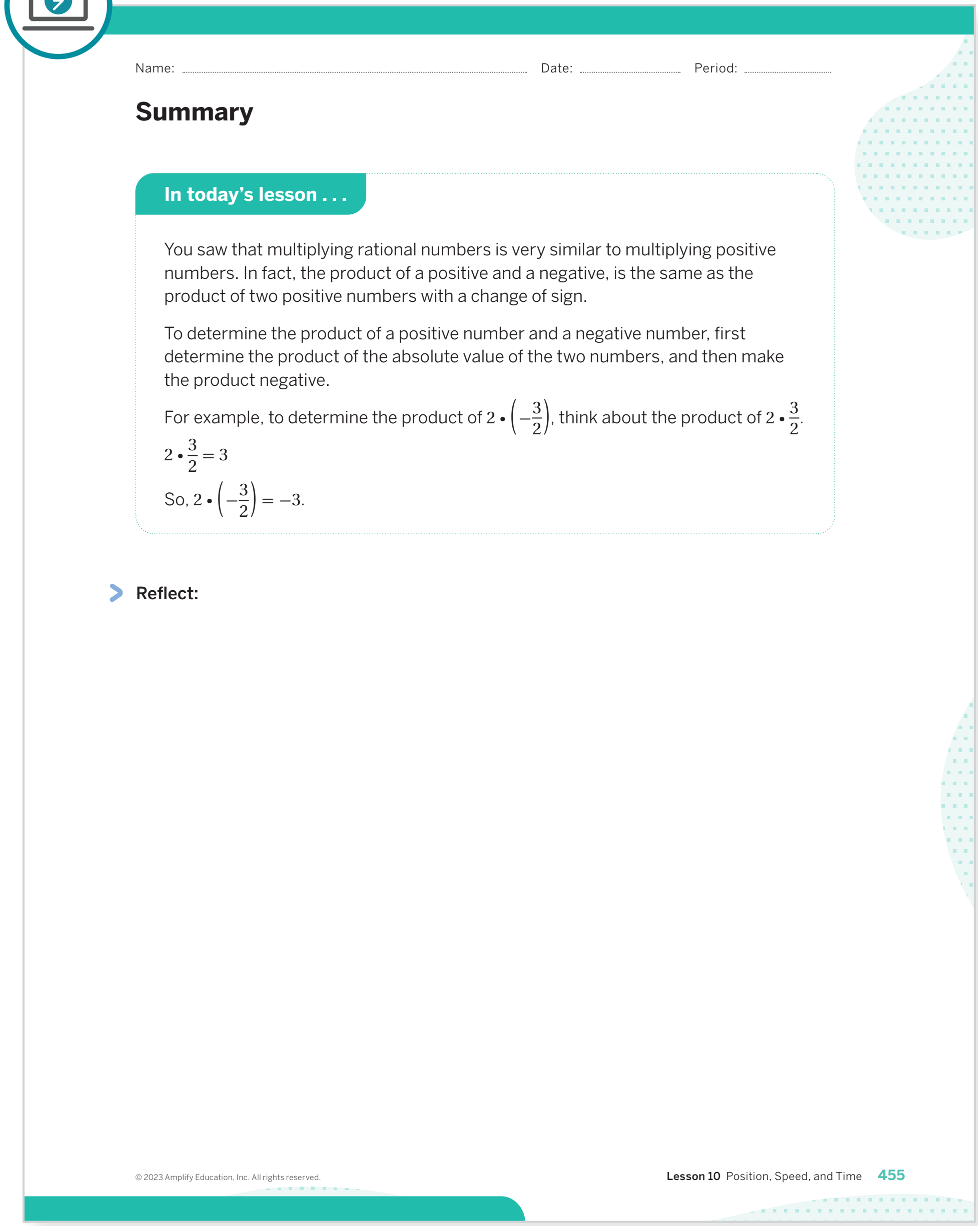
During the Connect, as students share their responses to Problem 2, add these statements — or a condensed version of them — to the class display. Include examples of multiplication expressions written as a sum, such as:

The product of two positive numbers is always positive.	$4 \cdot 3 = 3 + 3 + 3 + 3 = 12$ Four groups of positive $3 =$ positive 12.
The product of a positive number and a negative number is always negative.	$4 \cdot (-3) = (-3) + (-3) + (-3) + (-3) = -12$ Four groups of negative $3 =$ negative 12.

Whole Class | 5 min

Summary

Review and synthesize that the product of a positive and a negative is always positive.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that multiplying rational numbers is very similar to multiplying positive numbers. In fact, the product of a positive and a negative, is the same as the product of two positive numbers with a change of sign.

To determine the product of a positive number and a negative number, first determine the product of the absolute value of the two numbers, and then make the product negative.

For example, to determine the product of $2 \cdot \left(-\frac{3}{2}\right)$, think about the product of $2 \cdot \frac{3}{2}$.

$2 \cdot \frac{3}{2} = 3$

So, $2 \cdot \left(-\frac{3}{2}\right) = -3$.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Ask:

- "What kind of value do you get when you multiply a positive and a negative value?"
- "How can a number line be used to represent multiplication?"
- "Can you think of any scenarios other than speed and time that could be represented by the product of a positive and a negative value?"

Highlight that the product of a positive and a negative is always negative for any pair of rational values; integers, fractions, and decimals.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

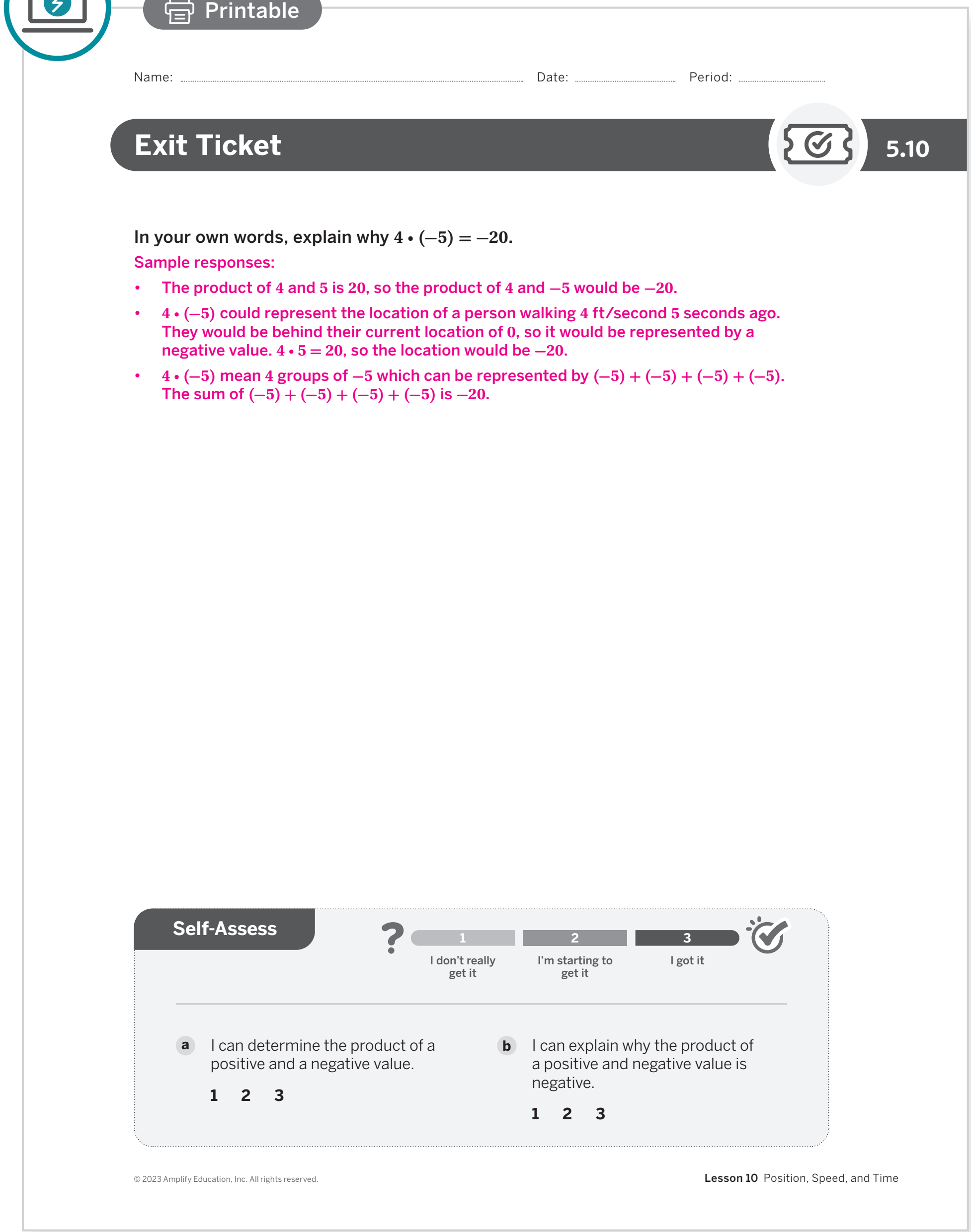
- "How do you represent multiplication of rational numbers on a number line?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of determining the product of a positive and a negative by explaining why the product must be negative in their own words.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.10

In your own words, explain why $4 \cdot (-5) = -20$.

Sample responses:

- The product of 4 and 5 is 20, so the product of 4 and –5 would be –20.
- $4 \cdot (-5)$ could represent the location of a person walking 4 ft/second 5 seconds ago. They would be behind their current location of 0, so it would be represented by a negative value. $4 \cdot 5 = 20$, so the location would be –20.
- $4 \cdot (-5)$ mean 4 groups of –5 which can be represented by $(-5) + (-5) + (-5) + (-5)$. The sum of $(-5) + (-5) + (-5) + (-5)$ is –20.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a) I can determine the product of a positive and a negative value.

1 2 3

b) I can explain why the product of a positive and negative value is negative.

1 2 3

Lesson 10 Position, Speed, and Time

Success looks like . . .

- **Language Goal:** Explaining how rational numbers can be used to represent elapsed time and distance before and after a chosen reference point. **(Speaking and Listening, Writing)**
- **Goal:** Writing a multiplication expression to represent a situation involving constant speed and time.
- **Language Goal:** Generalizing that the product of a positive number and a negative number is negative. **(Speaking and Listening)**

Suggested next steps

If students use the rule as their reasoning, consider:

- Encouraging them to use one of the other two methods discussed in class from Activity 1 or Activity 2 to explain why the rule makes sense.
- Assigning Practice Problem 3.

Professional Learning

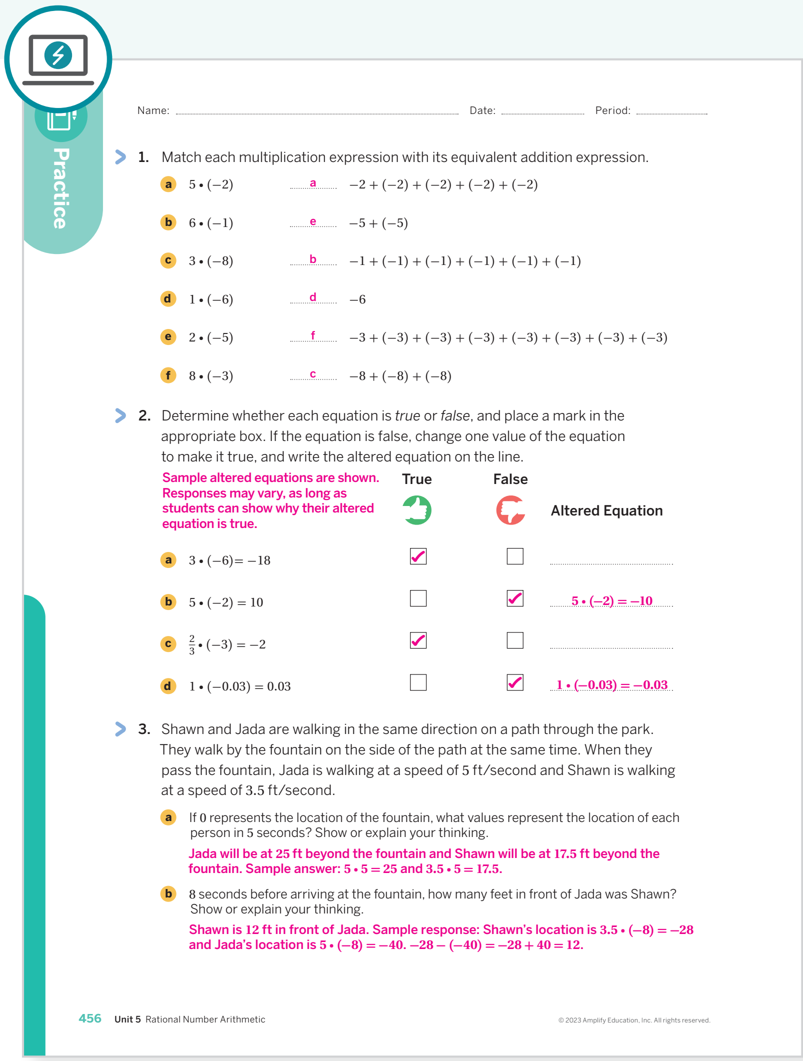
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students added and subtracted values on the number line. How did that support students' understanding of moving at a constant rate on the number line?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Match each multiplication expression with its equivalent addition expression.

- a $5 \cdot (-2)$ ___a___ $-2 + (-2) + (-2) + (-2) + (-2)$
- b $6 \cdot (-1)$ ___e___ $-5 + (-5)$
- c $3 \cdot (-8)$ ___b___ $-1 + (-1) + (-1) + (-1) + (-1) + (-1)$
- d $1 \cdot (-6)$ ___d___ -6
- e $2 \cdot (-5)$ ___f___ $-3 + (-3) + (-3) + (-3) + (-3) + (-3) + (-3) + (-3)$
- f $8 \cdot (-3)$ ___c___ $-8 + (-8) + (-8)$

2. Determine whether each equation is *true* or *false*, and place a mark in the appropriate box. If the equation is false, change one value of the equation to make it true, and write the altered equation on the line.

Sample altered equations are shown. Responses may vary, as long as students can show why their altered equation is true.

	True	False	Altered Equation
a $3 \cdot (-6) = -18$	☑	☐	
b $5 \cdot (-2) = 10$	☐	☑	$5 \cdot (-2) = -10$
c $\frac{2}{3} \cdot (-3) = -2$	☑	☐	
d $1 \cdot (-0.03) = 0.03$	☐	☑	$1 \cdot (-0.03) = -0.03$

3. Shawn and Jada are walking in the same direction on a path through the park. They walk by the fountain on the side of the path at the same time. When they pass the fountain, Jada is walking at a speed of 5 ft/second and Shawn is walking at a speed of 3.5 ft/second.

a. If 0 represents the location of the fountain, what values represent the location of each person in 5 seconds? Show or explain your thinking.

Jada will be at 25 ft beyond the fountain and Shawn will be at 17.5 ft beyond the fountain. Sample answer: $5 \cdot 5 = 25$ and $3.5 \cdot 5 = 17.5$.

b. 8 seconds before arriving at the fountain, how many feet in front of Jada was Shawn? Show or explain your thinking.

Shawn is 12 ft in front of Jada. Sample response: Shawn's location is $3.5 \cdot (-8) = -28$ and Jada's location is $5 \cdot (-8) = -40$. $-28 - (-40) = -28 + 40 = 12$.

456 **Unit 5** Rational Number Arithmetic

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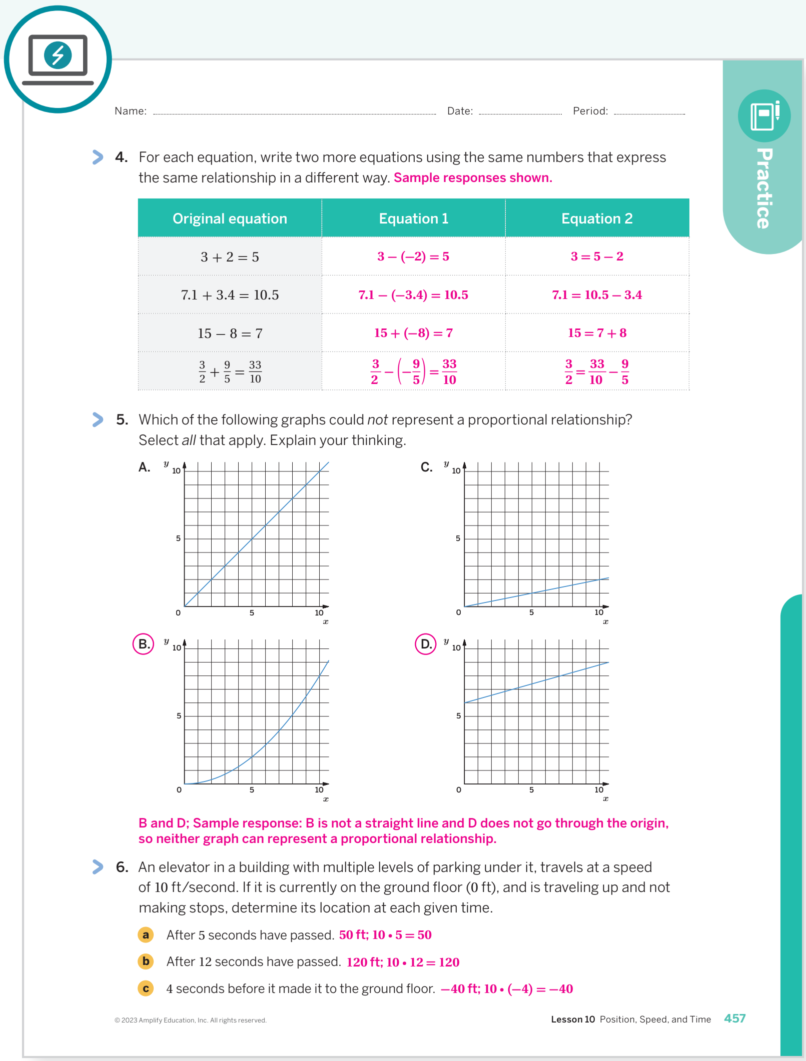

Name: ______ Date: ______ Period: ______

4. For each equation, write two more equations using the same numbers that express the same relationship in a different way. **Sample responses shown.**

Original equation	Equation 1	Equation 2
$3 + 2 = 5$	$3 - (-2) = 5$	$3 = 5 - 2$
$7.1 + 3.4 = 10.5$	$7.1 - (-3.4) = 10.5$	$7.1 = 10.5 - 3.4$
$15 - 8 = 7$	$15 + (-8) = 7$	$15 = 7 + 8$
$\frac{3}{2} + \frac{9}{5} = \frac{33}{10}$	$\frac{3}{2} - \left(-\frac{9}{5}\right) = \frac{33}{10}$	$\frac{3}{2} = \frac{33}{10} - \frac{9}{5}$

5. Which of the following graphs could *not* represent a proportional relationship? Select *all* that apply. Explain your thinking.

B and D; Sample response: B is not a straight line and D does not go through the origin, so neither graph can represent a proportional relationship.

6. An elevator in a building with multiple levels of parking under it, travels at a speed of 10 ft/second. If it is currently on the ground floor (0 ft), and is traveling up and not making stops, determine its location at each given time.

a. After 5 seconds have passed. **50 ft; $10 \cdot 5 = 50$**

b. After 12 seconds have passed. **120 ft; $10 \cdot 12 = 120$**

c. 4 seconds before it made it to the ground floor. **-40 ft; $10 \cdot (-4) = -40$**

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Lesson 10 Position, Speed, and Time 457

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1	3
Spiral	4	Unit 5 Lesson 8	2
	5	Unit 2 Lesson 11	2
Formative ⚡	6	Unit 5 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

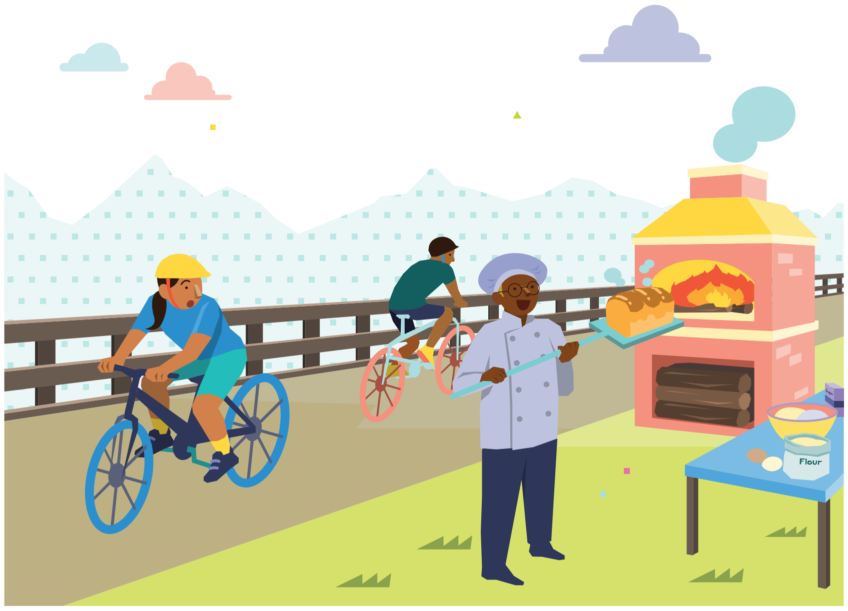
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 11**

Multiplying Rational Numbers

Let's multiply rational numbers.



Focus

Goals

1. **Language Goal:** Explain how rational numbers can be used to represent position and speeds in opposite directions. **(Speaking and Listening)**
2. **Language Goal:** Generalize that the product of two negative numbers is positive. **(Speaking and Listening)**
3. **Language Goal:** Generalize that the product of two rational numbers with different signs is negative. **(Speaking and Listening)**

Coherence

Today

Students apply their understanding of the distance formula, $d = rt$, to make observations about the rules for multiplying rational numbers. They connect their understanding from the previous lesson that time can be represented by positive and negative values with objects moving in opposite directions. Students determine that a negative velocity and a negative time gives a positive position, and a negative velocity and a positive time result in a negative position. Finally, students generalize the rules for multiplying rational numbers by applying the order of operations and the Distributive Property to expressions in the form $a\ (b + c)$.

Previously

In Lesson 10, students modeled *before* and *after* as negative and positive time. They concluded that the product of a positive number and a negative number is negative.

Coming Soon

In Lesson 12, students will gain fluency in multiplication of rational numbers, determining the products of three or more rational values.

Rigor

- Students build **conceptual understanding** of multiplying rational numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	18 min	12 min	5 min	5 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Graphic Organizer PDF, *Blank Number Lines* (as needed)
- paper clips, snap cubes, or other objects to move on the number line

Math Language Development

New word

- ***velocity***

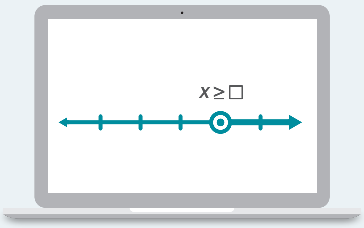
Review words

- *absolute value*
- *Distributive Property*
- *magnitude*
- *order of operations*
- *rational numbers*

Amps | Featured Activity

Activity 1
Interactive Number Lines

Students use interactive number lines to determine the location of bicyclists as they move at constant speeds forward and backward in time and in opposite directions.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students predict the location of the bicycles and they might struggle to see any sort of repeated reasoning in the problem. Encourage students to continually evaluate their results, determining whether or not they are reasonable. Remind them that predictions could be incorrect, but they should apply a growth mindset and recognize that they have not determined the correct locations yet.

Modifications to Pacing

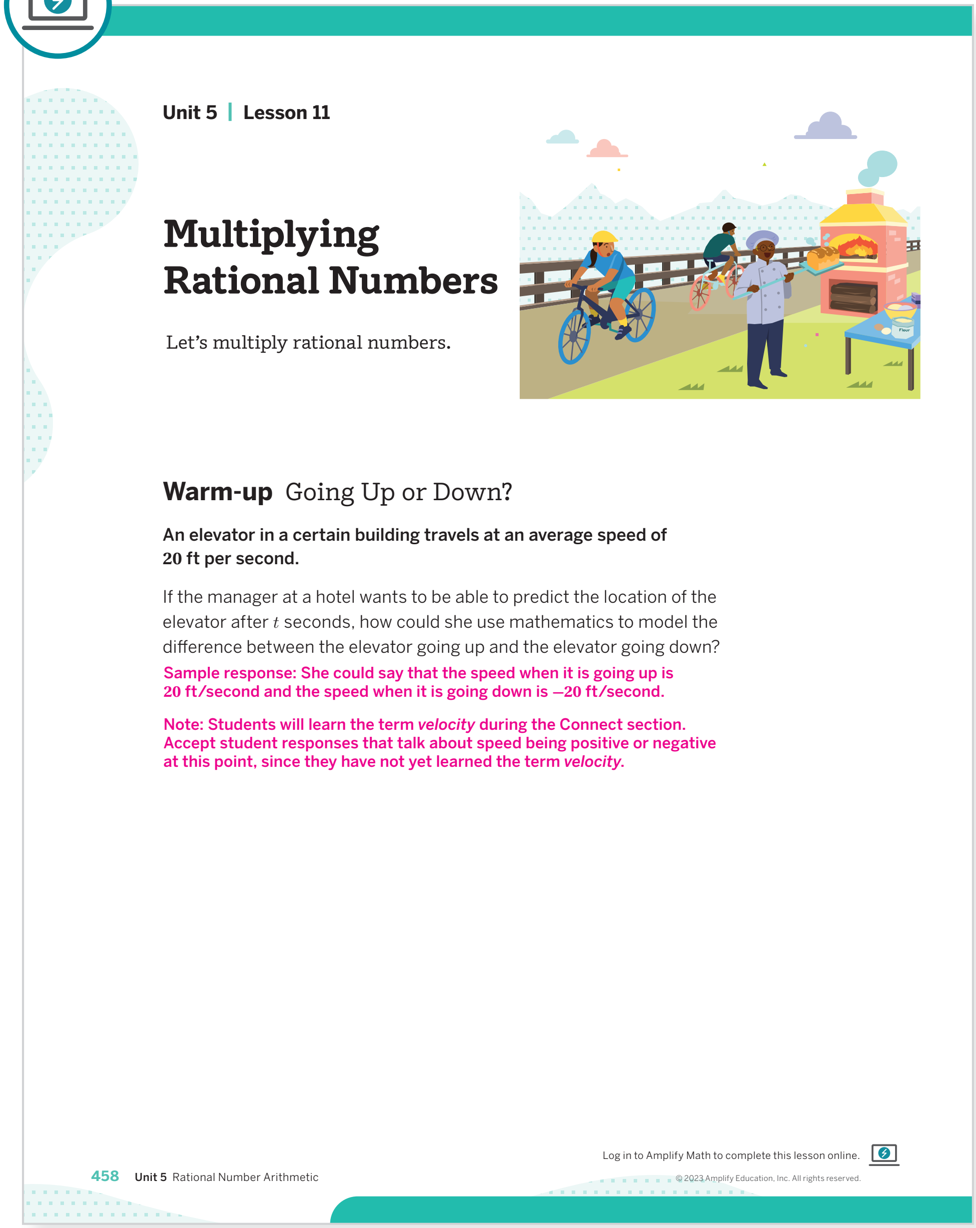
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. Instead, share the definition of velocity during the Launch for Activity 1.
- In **Activity 2**, have students work in pairs on Problem 3. One student should use the Distributive Property, while the other student uses the order of operations. Then they should compare their values.

Pairs | 5 min

Warm-up Going Up or Down?

Students reason about representing the speed and direction of an elevator moving within a building.



Unit 5 | Lesson 11

Multiplying Rational Numbers

Let's multiply rational numbers.

Warm-up Going Up or Down?

An elevator in a certain building travels at an average speed of 20 ft per second.

If the manager at a hotel wants to be able to predict the location of the elevator after t seconds, how could she use mathematics to model the difference between the elevator going up and the elevator going down?

Sample response: She could say that the speed when it is going up is 20 ft/second and the speed when it is going down is -20 ft/second.

Note: Students will learn the term *velocity* during the Connect section. Accept student responses that talk about speed being positive or negative at this point, since they have not yet learned the term *velocity*.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Activate prior knowledge by asking, "How did you use rational numbers to represent moving forward and backward in time in the previous lesson?" Have a student read the scenario, and then conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by having them sketch a picture of how an elevator moves.

Look for points of confusion:

- Not considering using positive and negative numbers. Ask, "How can you use mathematics to represent direction?"

3 Connect

Have pairs of students share their responses for how to represent the direction the elevator is traveling.

Highlight that 20 can be used to represent the speed and direction of an elevator going up and -20 can be used to represent the speed and direction of an elevator going down.

Define *velocity* as a quantity that represents the speed *and* the direction of motion. In general, speed, like distance, is always positive but velocity can be either positive or negative.

Ask, "Can you think of any other scenarios where you would want to compare velocity and not just speed?"

Power-up

To power up students' ability to determine location with positive and negative times, have student complete:

Complete each statement with the values -10, -2, 2, or 10.
A man is jogging at a constant speed of 5 mph. His current location can be represented by 0 on a number line.

1. $\underline{\ 2\ }$ represents the time after 2 seconds have passed.
2. $\underline{\ -2\ }$ represents the time 2 seconds before his current location.
3. $\underline{\ 10\ }$ represents the location of the runner after 2 seconds have passed.
4. $\underline{\ -10\ }$ represents the location of the runner 2 seconds before.

Use: Before Activity 1.

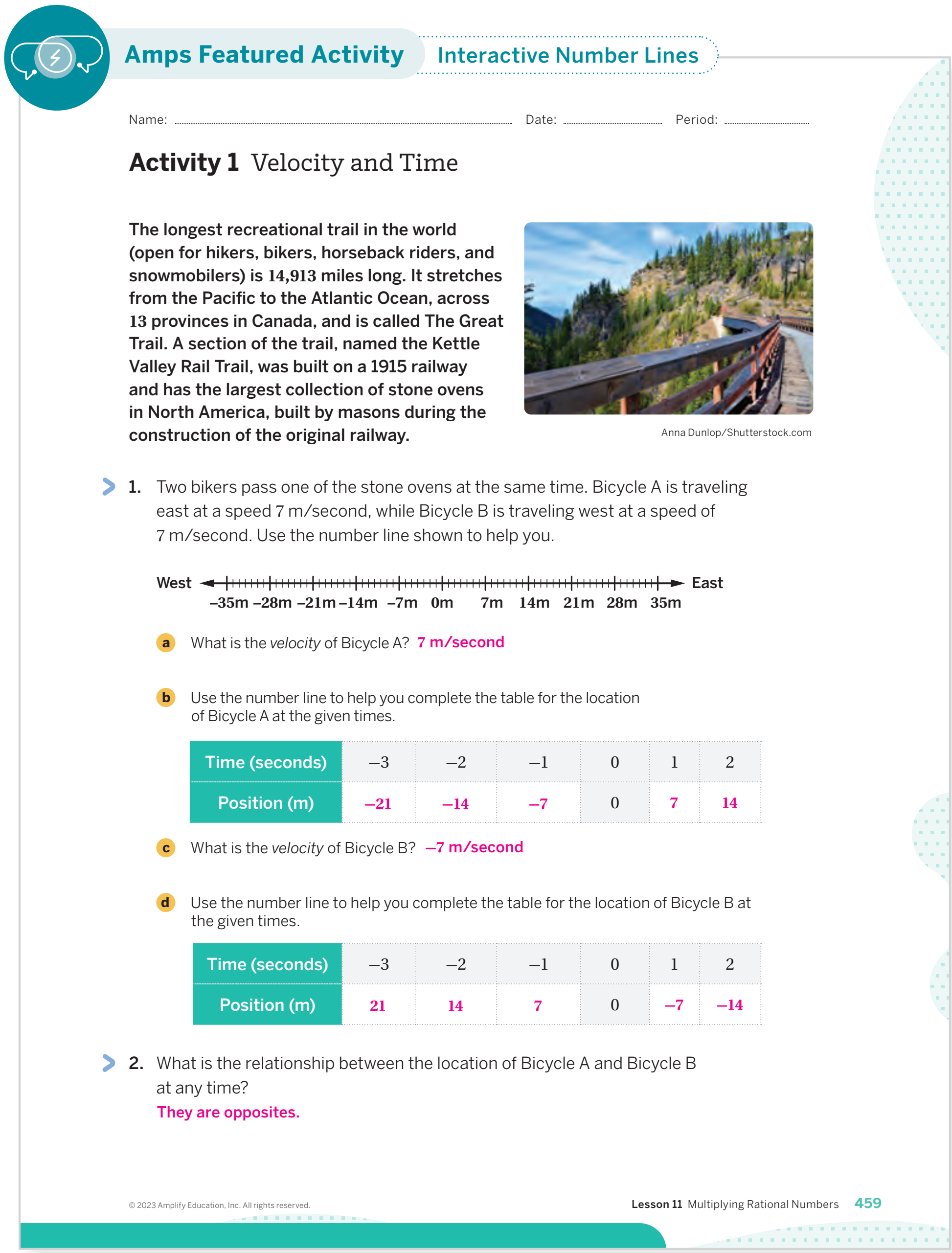
Informed by: Performance on Lesson 10, Practice Problem 6.

Small groups | 18 min

Activity 1 Velocity and Time

Students use number lines and expressions to explore and represent the location of people traveling in opposite directions forward and backward in time.

Amps Featured Activity Interactive Number Lines



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Activity 1 Velocity and Time

The longest recreational trail in the world (open for hikers, bikers, horseback riders, and snowmobilers) is 14,913 miles long. It stretches from the Pacific to the Atlantic Ocean, across 13 provinces in Canada, and is called The Great Trail. A section of the trail, named the Kettle Valley Rail Trail, was built on a 1915 railway and has the largest collection of stone ovens in North America, built by masons during the construction of the original railway.

Anna Dunlop/Shutterstock.com

1. Two bikers pass one of the stone ovens at the same time. Bicycle A is traveling east at a speed 7 m/second, while Bicycle B is traveling west at a speed of 7 m/second. Use the number line shown to help you.

West –35m –28m –21m –14m –7m 0m 7m 14m 21m 28m 35m East

a. What is the *velocity* of Bicycle A? 7 m/second

b. Use the number line to help you complete the table for the location of Bicycle A at the given times.

Time (seconds)	–3	–2	–1	0	1	2
Position (m)	–21	–14	–7	0	7	14

c. What is the *velocity* of Bicycle B? –7 m/second

d. Use the number line to help you complete the table for the location of Bicycle B at the given times.

Time (seconds)	–3	–2	–1	0	1	2
Position (m)	21	14	7	0	–7	–14

2. What is the relationship between the location of Bicycle A and Bicycle B at any time?
They are opposites.

1 Launch

Activate students' background knowledge by asking if they've ever been hiking or biking on a trail. Explain that in this activity they will build on their work from the previous lesson with hikers traveling on the Appalachian Trail, but now they will have people moving in opposite directions. Provide access to objects (paper clip, block, etc.) that could represent the vehicles moving in time on the number line.

2 Monitor

Help students get started by covering the first three columns of each table in Problem 1 and asking them to reason about the positive times first.

Look for points of confusion:

- **Not starting their 'bicycles' at 0 for each scenario in Problem 1.** Encourage students to place their "bicycle" at 0; then model one side of the table, and then the other (forward in time, then backward in time).
- **Thinking that, because Bicycles A and B have the same speed, they also have the same velocity.** Review the definition of velocity from the Warm-up.
- **Thinking that Bicycle B would have a negative position for negative time.** Ask, "What does the velocity tell you about the direction the bicycle is traveling? What direction would it be going if they were going backward in time?"
- **For Problem 2, applying rules of addition to multiplication to determine the sign of the final position.** Ask students to use the number line from Problem 1 to check their values.

Look for productive strategies:

- Using a noticed pattern to predict the location of Bicycle A and Bicycle B then using the number line to check their predictions.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive number lines to determine the location of bicyclists as they move at constant speeds forward and backward in time and in opposite directions.

Accessibility: *Guide Processing and Visualization*

Consider asking two student volunteers to demonstrate the bikers traveling at the same speed, but in opposite directions. Have them pass an object, such as a desk, to represent passing the stone oven at the same time.

Math Language Development

MLR2: Collect and Display

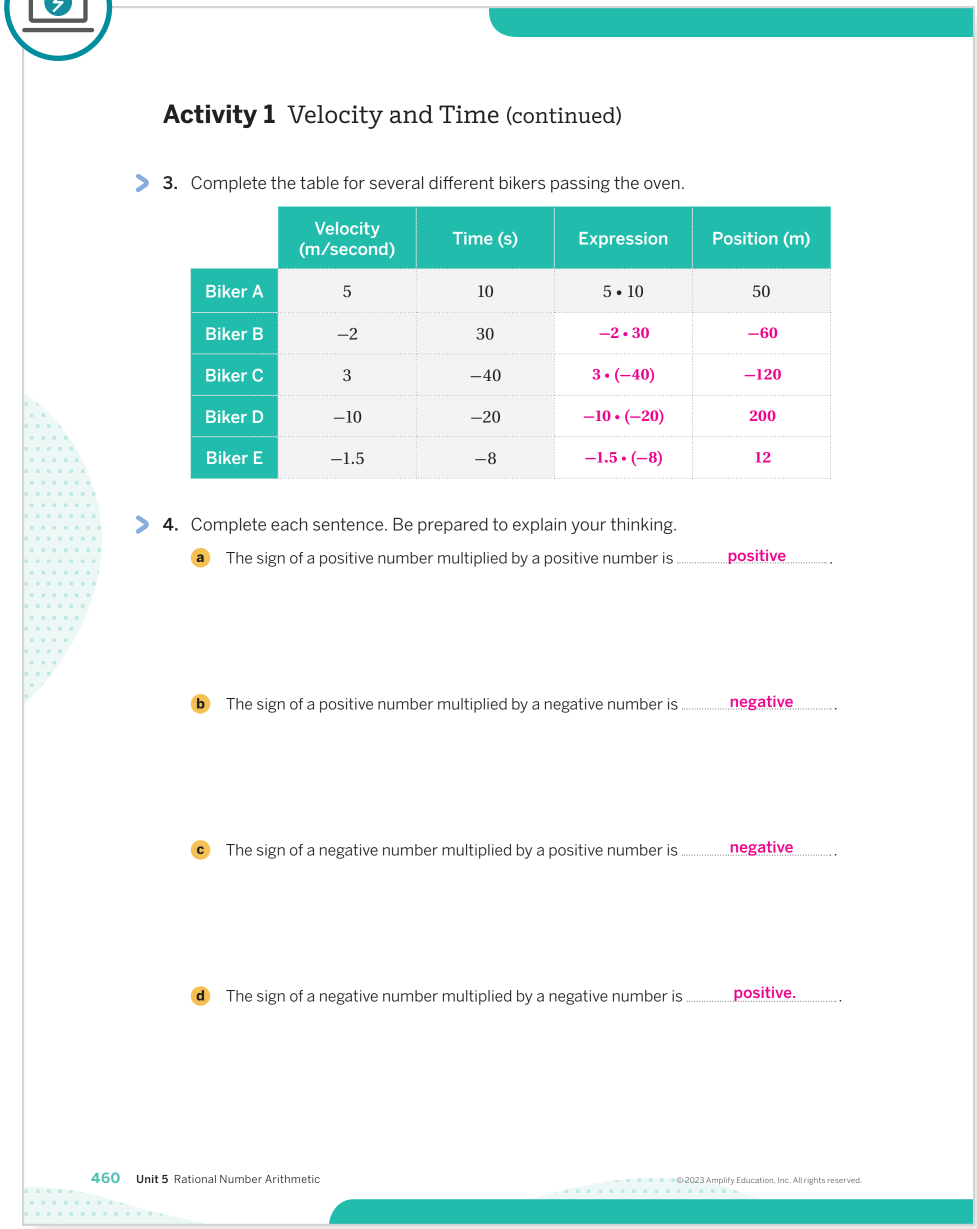
During the Connect, as students share their responses to Problem 4, add a new statement about the product of two negative numbers to the class display, such as the one shown here.

The product of two negative numbers is always positive.	$-4 \cdot (-3) = 12$ A biker is traveling at 4 m/second in the opposite direction. Three seconds ago, the biker traveled a distance of 12 m.

Small groups | 18 min

Activity 1 Velocity and Time (continued)

Students use number lines and expressions to explore and represent the location of people traveling in opposite directions forward and backward in time.



Activity 1 Velocity and Time (continued)

3. Complete the table for several different bikers passing the oven.

	Velocity (m/second)	Time (s)	Expression	Position (m)
Biker A	5	10	5 • 10	50
Biker B	–2	30	–2 • 30	–60
Biker C	3	–40	3 • (–40)	–120
Biker D	–10	–20	–10 • (–20)	200
Biker E	–1.5	–8	–1.5 • (–8)	12

4. Complete each sentence. Be prepared to explain your thinking.

a. The sign of a positive number multiplied by a positive number is positive.

b. The sign of a positive number multiplied by a negative number is negative.

c. The sign of a negative number multiplied by a positive number is negative.

d. The sign of a negative number multiplied by a negative number is positive..

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3 Connect

Display a large number line. Ask two students to represent Bicycles A and B and model the bicycles moving in opposite directions forward and backward in time.

Have students share what they noticed about the relationship between speed, time, and location.

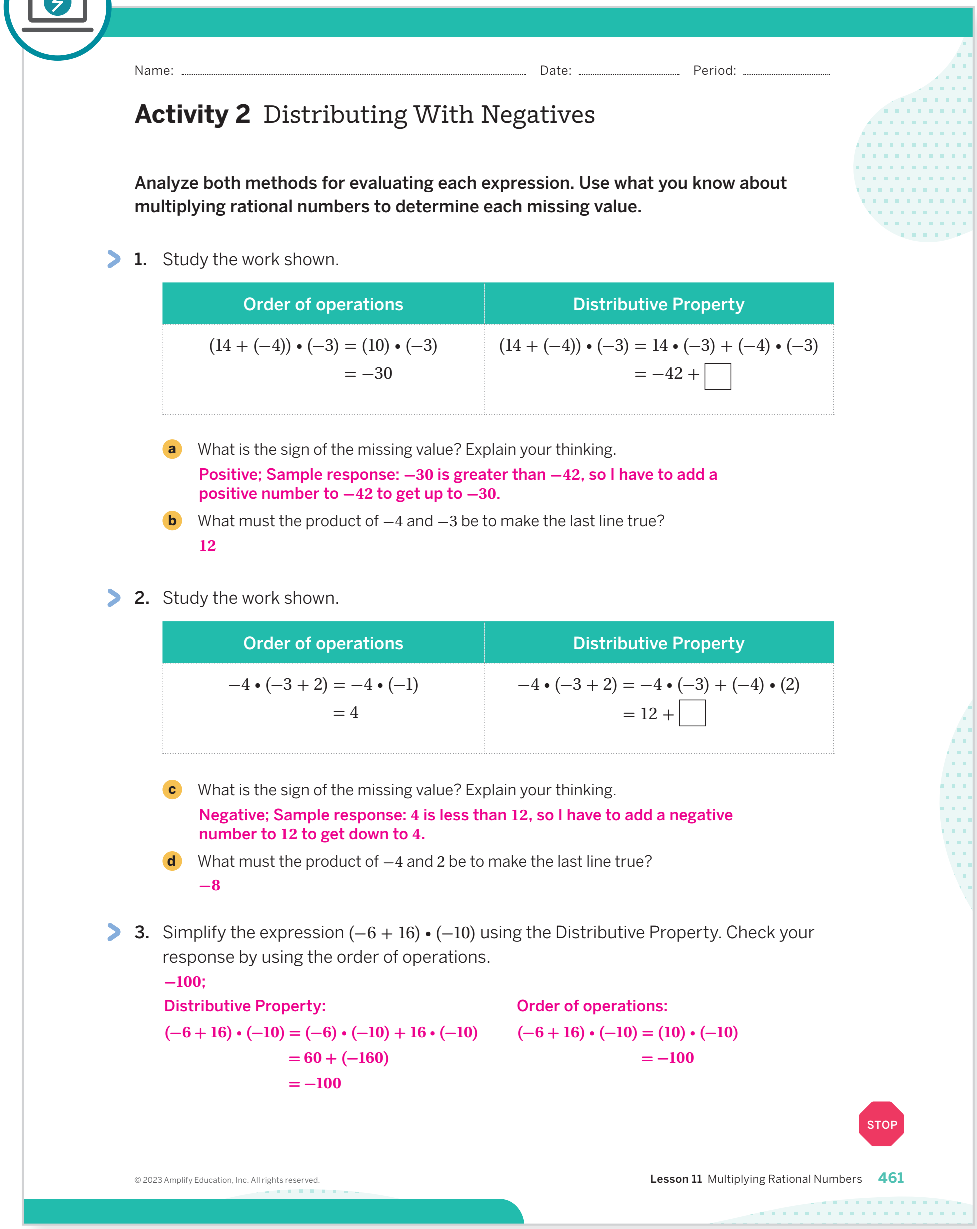
Highlight that a negative velocity and positive time result in a "negative" location. A negative velocity and a negative time result in a "positive" location. Explain that students will explore if this is true outside of velocity and time in the next activity.

Ask, "Can you think of any other real-world examples of multiplying two negative values?" Sample response: Determining a previous temperature if it has been decreasing at a constant rate.

Pairs | 12 min

Activity 2 Distributing With Negatives

Students apply their understanding of order of operations and the Distributive Property to simplify expressions and solidify their understanding of the product of rational numbers.



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Activity 2 Distributing With Negatives

Analyze both methods for evaluating each expression. Use what you know about multiplying rational numbers to determine each missing value.

1. Study the work shown.

Order of operations	Distributive Property
$(14 + (-4)) \cdot (-3) = (10) \cdot (-3)$ $= -30$	$(14 + (-4)) \cdot (-3) = 14 \cdot (-3) + (-4) \cdot (-3)$ $= -42 + \square$

a What is the sign of the missing value? Explain your thinking.

Positive; Sample response: −30 is greater than −42, so I have to add a positive number to −42 to get up to −30.

b What must the product of −4 and −3 be to make the last line true?

12

2. Study the work shown.

Order of operations	Distributive Property
$-4 \cdot (-3 + 2) = -4 \cdot (-1)$ $= 4$	$-4 \cdot (-3 + 2) = -4 \cdot (-3) + (-4) \cdot (2)$ $= 12 + \square$

c What is the sign of the missing value? Explain your thinking.

Negative; Sample response: 4 is less than 12, so I have to add a negative number to 12 to get down to 4.

d What must the product of −4 and 2 be to make the last line true?

−8

3. Simplify the expression $(-6 + 16) \cdot (-10)$ using the Distributive Property. Check your response by using the order of operations.

−100;

Distributive Property:

$$(-6 + 16) \cdot (-10) = (-6) \cdot (-10) + 16 \cdot (-10)$$
$$= 60 + (-160)$$
$$= -100$$

Order of operations:

$$(-6 + 16) \cdot (-10) = (10) \cdot (-10)$$
$$= -100$$

STOP

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Lesson 11 Multiplying Rational Numbers 461

1 Launch

Activate prior knowledge by asking, "What are the two methods for simplifying $(1 + 3) \cdot 3$?" Verify that students are comfortable with simplifying Distributive Property expressions in the form $(a + b) \cdot c$.

2 Monitor

Help students get started by asking, "How is $(14 + (-4)) \cdot (-3)$ related to $-3(14 + (-4))$?"

Look for points of confusion:

- **Dropping the signs of the values when showing their work in Problem 3.** Remind students to organize their work and check it carefully to ensure they aren't leaving off any negative signs**.**

3 Connect

Display samples of student work from Problem 3.

Ask, "If you were to write a rule for multiplying integers, what would you write?"

Have students share their rules with their group, and then ask groups to share with the class.

Highlight that, in this activity, students observed that the product of two integers with the same sign is alway positive. The product of two integers with different signs is always negative. Explain that this rule is true for all rational values (including fractions and decimals), not just integers.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide copies of the Graphic Organizer PDF, *Blank Number Lines* for students to choose to use to help them organize their thinking and make sense of the expression and the operations involved.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses to Problem 3, draw their attention to the connections between the two methods: the Distributive Property and the order of operations. Ask:

- "When using the Distributive Property, what do you do first?"
- "When using the order of operations, what do you do first?"
- "Compare the results of these first steps: $(-6) \cdot (-10) + 16 \cdot (-10)$ and $10 \cdot (-10)$. Are they equivalent expressions? Explain your thinking."

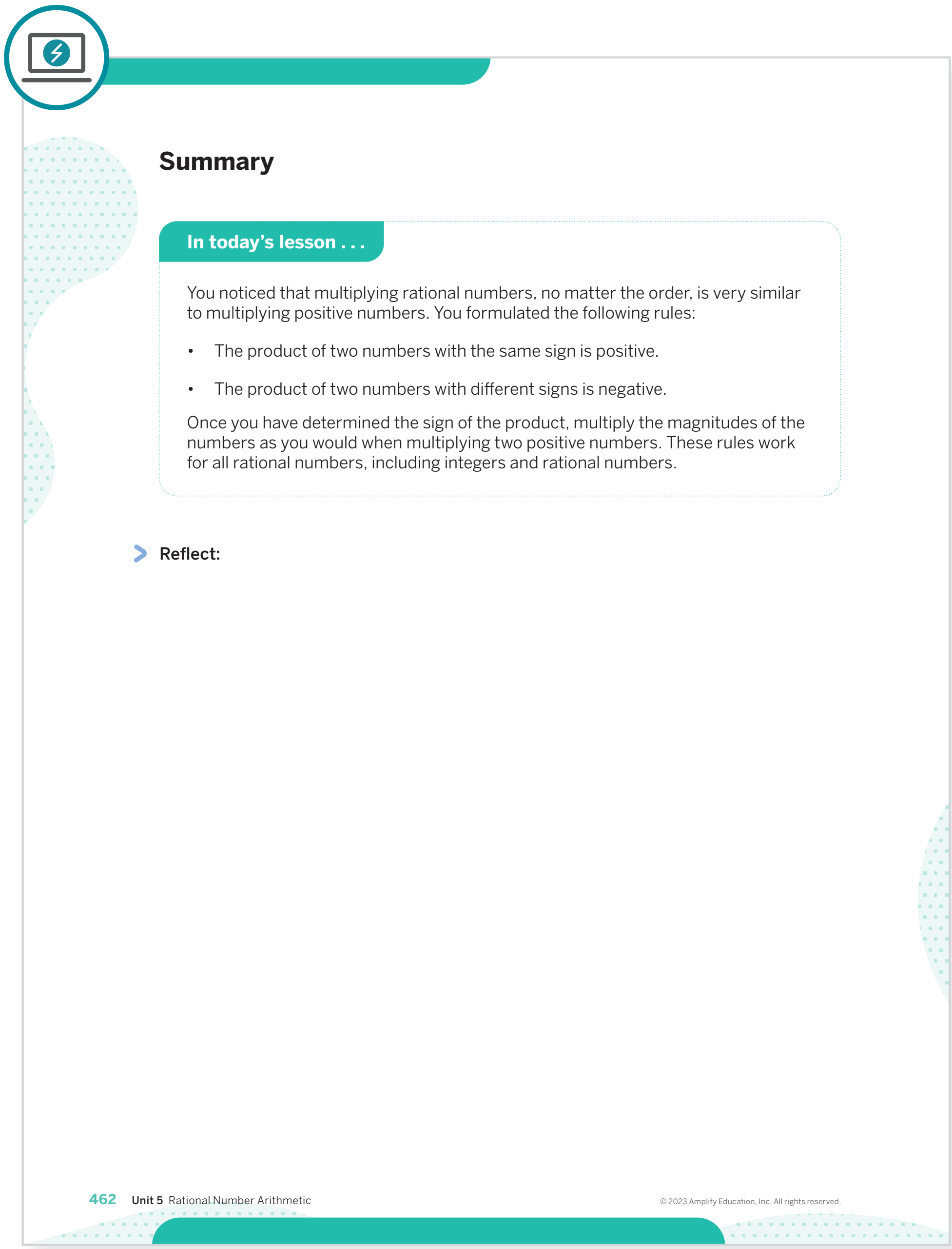
English Learners

Use color coding to annotate the parts of the expression students describe. This will help students visualize the relationships being discussed.

Whole Class | 5 min

Summary

Review and synthesize the rules for multiplying rational numbers.



Summary

In today's lesson . . .

You noticed that multiplying rational numbers, no matter the order, is very similar to multiplying positive numbers. You formulated the following rules:

- The product of two numbers with the same sign is positive.
- The product of two numbers with different signs is negative.

Once you have determined the sign of the product, multiply the magnitudes of the numbers as you would when multiplying two positive numbers. These rules work for all rational numbers, including integers and rational numbers.

Reflect:

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Synthesize

Display the Summary from the Student Edition.

Have students share real-world examples of when they may want to determine the product of rational numbers.

Highlight that unlike adding rational numbers, the sign of the product is not based on the sign of the factor with the greater magnitude.

Formalize vocabulary: ***velocity***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do you represent multiplication of rational numbers on a number line?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on the terms and phrases related to the term *velocity* that were added to the display during the lesson. Highlight that *velocity* can be either positive or negative, while speed is always positive. Velocity represents both the magnitude and direction, while speed only represents the magnitude.

Consider adding a table to the class display, such as the one shown here.

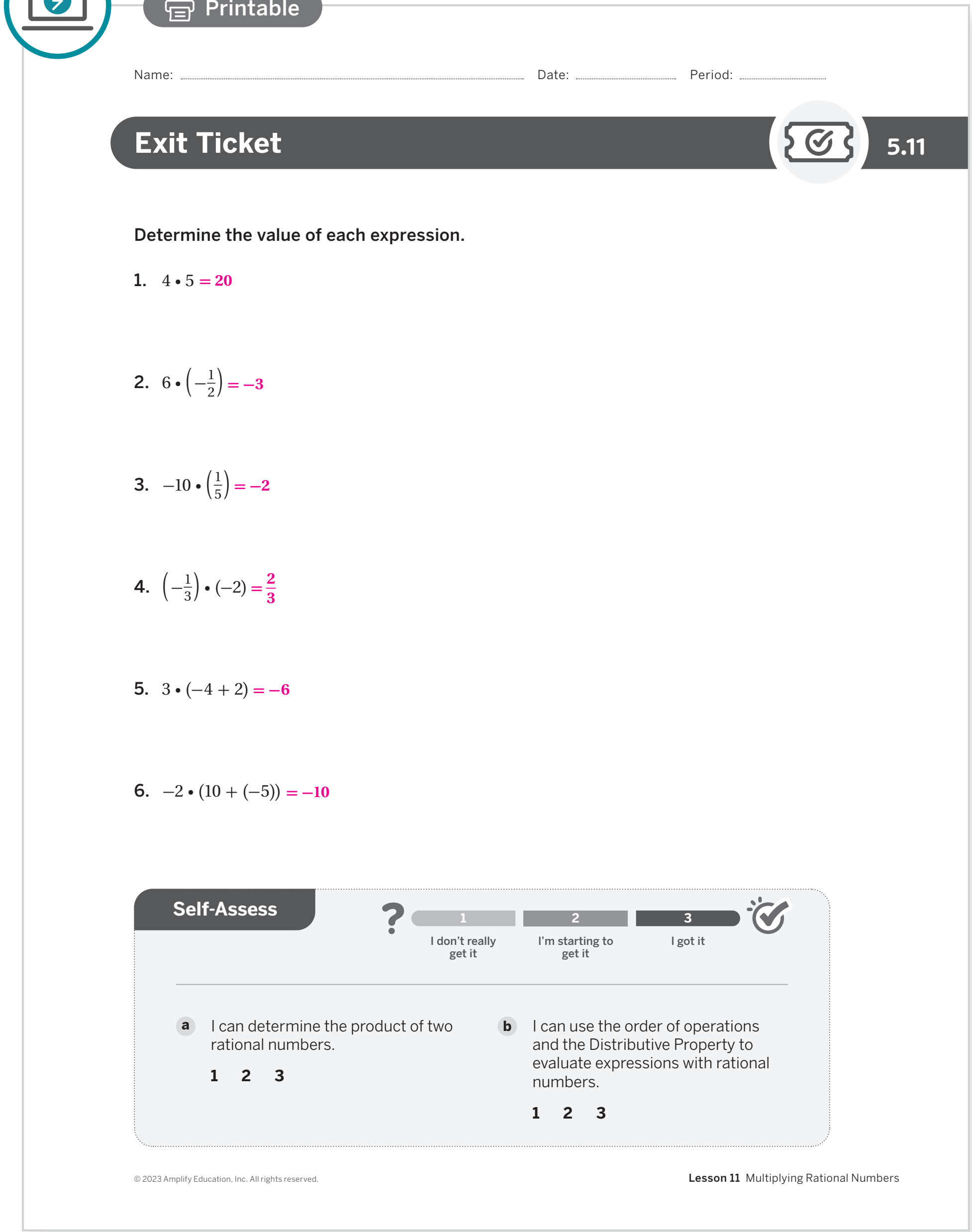
Speed	Velocity
Represents the magnitude.	Represents both the magnitude and the direction.
Always positive.	Can be positive or negative. A negative velocity represents an object traveling in the opposite direction.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of multiplying rational numbers by evaluating expressions.

Printable



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Exit Ticket 5.11

Determine the value of each expression.

1. $4 \cdot 5 = 20$
2. $6 \cdot \left(-\frac{1}{2}\right) = -3$
3. $-10 \cdot \left(\frac{1}{5}\right) = -2$
4. $\left(-\frac{1}{3}\right) \cdot (-2) = \frac{2}{3}$
5. $3 \cdot (-4 + 2) = -6$
6. $-2 \cdot (10 + (-5)) = -10$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can determine the product of two rational numbers.
1 2 3

b I can use the order of operations and the Distributive Property to evaluate expressions with rational numbers.
1 2 3

Lesson 11 Multiplying Rational Numbers

Success looks like . . .

- **Language Goal:** Explaining how signed numbers can be used to represent position and speeds in opposite directions. **(Speaking and Listening)**
- **Language Goal:** Generalizing that the product of two negative numbers is positive. **(Speaking and Listening)**
- **Language Goal:** Generalizing that the product of two rational numbers with different signs is negative. **(Speaking and Listening)**
  - » Determining the product of two rational numbers with different signs in Problems 2, 3 and 5.

Suggested next steps

If students use the Distributive Property for Problems 5 and 6, consider:

- Asking them to evaluate one of those expressions using the order of operations.

If students use the order of operations for Problems 5 and 6, consider:

- Asking them to evaluate one of those expressions using the Distributive Property.

If students struggle with determining the sign of an evaluated expression, consider:

- Reviewing Activity 2.
- Assigning Practice Problem 1.
- Asking them to reread the Summary in the Student Edition.

Professional Learning

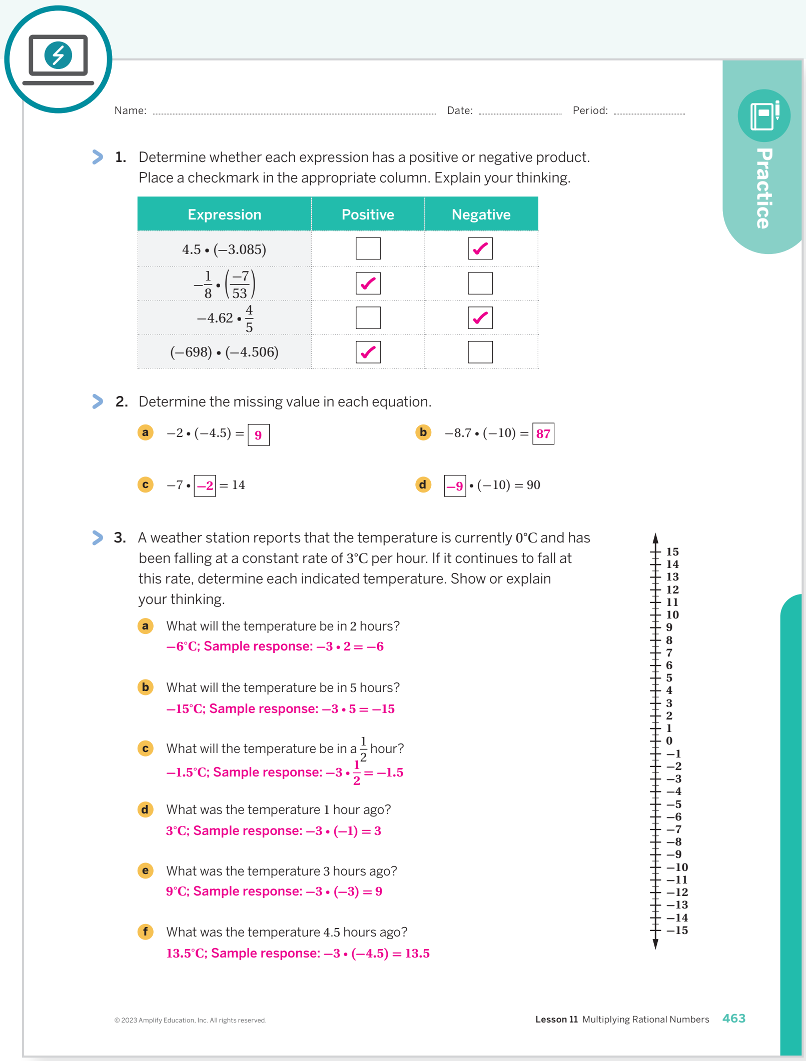
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find frustrating about Activity 1? What helped them work through this frustration?
- The focus of this lesson was developing the rules for multiplying rational numbers. How did this focus go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Determine whether each expression has a positive or negative product. Place a checkmark in the appropriate column. Explain your thinking.

Expression	Positive	Negative
$4.5 \cdot (-3.085)$	☐	☑
$-\frac{1}{8} \cdot \left(\frac{-7}{53}\right)$	☑	☐
$-4.62 \cdot \frac{4}{5}$	☐	☑
$(-698) \cdot (-4.506)$	☑	☐

2. Determine the missing value in each equation.

a $-2 \cdot (-4.5) = \boxed{9}$

b $-8.7 \cdot (-10) = \boxed{87}$

c $-7 \cdot \boxed{-2} = 14$

d $\boxed{-9} \cdot (-10) = 90$

3. A weather station reports that the temperature is currently $0°C$ and has been falling at a constant rate of $3°C$ per hour. If it continues to fall at this rate, determine each indicated temperature. Show or explain your thinking.

a What will the temperature be in 2 hours?
$-6°C$; Sample response: $-3 \cdot 2 = -6$

b What will the temperature be in 5 hours?
$-15°C$; Sample response: $-3 \cdot 5 = -15$

c What will the temperature be in a $\frac{1}{2}$ hour?
$-1.5°C$; Sample response: $-3 \cdot \frac{1}{2} = -1.5$

d What was the temperature 1 hour ago?
$3°C$; Sample response: $-3 \cdot (-1) = 3$

e What was the temperature 3 hours ago?
$9°C$; Sample response: $-3 \cdot (-3) = 9$

f What was the temperature 4.5 hours ago?
$13.5°C$; Sample response: $-3 \cdot (-4.5) = 13.5$

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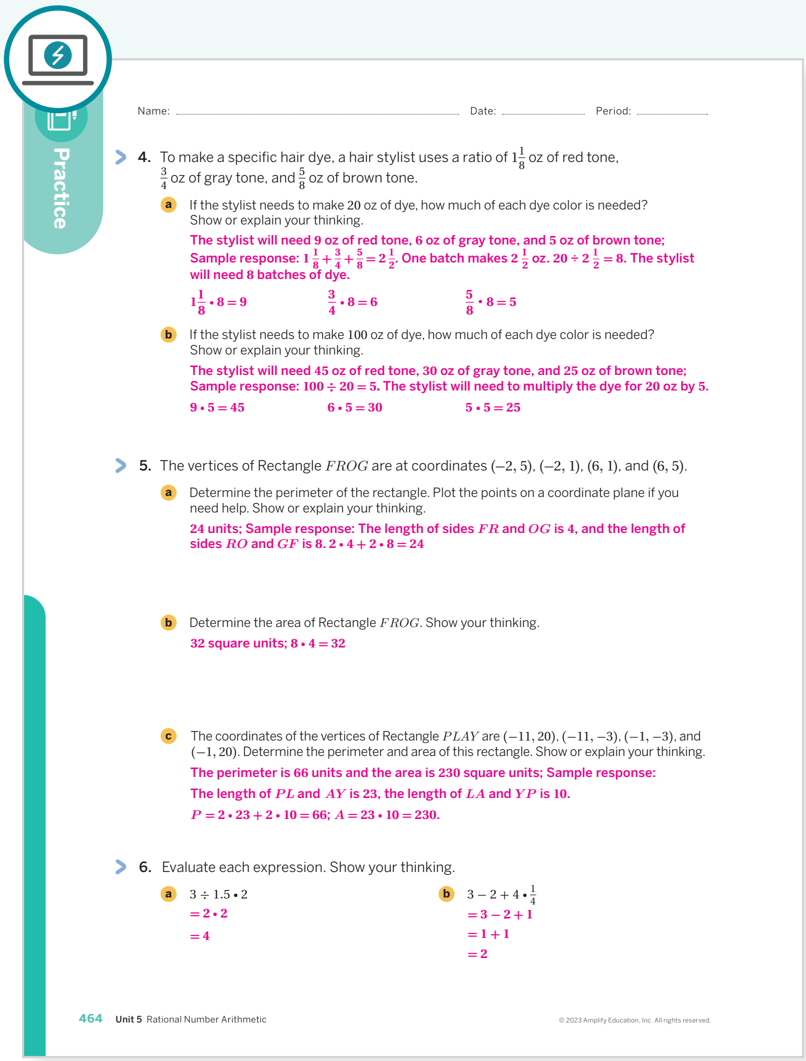
Lesson 11 Multiplying Rational Numbers 463



Name: Date: Period:

Practice

4. To make a specific hair dye, a hair stylist uses a ratio of $1\frac{1}{8}$ oz of red tone, $\frac{3}{4}$ oz of gray tone, and $\frac{5}{8}$ oz of brown tone.

a If the stylist needs to make 20 oz of dye, how much of each dye color is needed? Show or explain your thinking.
The stylist will need 9 oz of red tone, 6 oz of gray tone, and 5 oz of brown tone; Sample response: $1\frac{1}{8} + \frac{3}{4} + \frac{5}{8} = 2\frac{1}{2}$. One batch makes $2\frac{1}{2}$ oz. $20 \div 2\frac{1}{2} = 8$. The stylist will need 8 batches of dye.

$1\frac{1}{8} \cdot 8 = 9$ $\frac{3}{4} \cdot 8 = 6$ $\frac{5}{8} \cdot 8 = 5$

b If the stylist needs to make 100 oz of dye, how much of each dye color is needed? Show or explain your thinking.
The stylist will need 45 oz of red tone, 30 oz of gray tone, and 25 oz of brown tone; Sample response: $100 \div 20 = 5$. The stylist will need to multiply the dye for 20 oz by 5.

$9 \cdot 5 = 45$ $6 \cdot 5 = 30$ $5 \cdot 5 = 25$

5. The vertices of Rectangle $FROG$ are at coordinates $(-2, 5)$, $(-2, 1)$, $(6, 1)$, and $(6, 5)$.

a Determine the perimeter of the rectangle. Plot the points on a coordinate plane if you need help. Show or explain your thinking.
24 units; Sample response: The length of sides FR and OG is 4, and the length of sides RO and GF is 8. $2 \cdot 4 + 2 \cdot 8 = 24$

b Determine the area of Rectangle $FROG$. Show your thinking.
32 square units; $8 \cdot 4 = 32$

c The coordinates of the vertices of Rectangle $PLAY$ are $(-11, 20)$, $(-11, -3)$, $(-1, -3)$, and $(-1, 20)$. Determine the perimeter and area of this rectangle. Show or explain your thinking.
The perimeter is 66 units and the area is 230 square units; Sample response: The length of PL and AY is 23, the length of LA and YP is 10.
$P = 2 \cdot 23 + 2 \cdot 10 = 66$; $A = 23 \cdot 10 = 230$.

6. Evaluate each expression. Show your thinking.

a $3 \div 1.5 \cdot 2$
$= 2 \cdot 2$
$= 4$

b $3 - 2 + 4 \cdot \frac{1}{4}$
$= 3 - 2 + 1$
$= 1 + 1$
$= 2$

464 **Unit 5** Rational Number Arithmetic

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 2	2
	5	Unit 5 Lesson 8	2
Formative	6	Unit 5 Lesson 12	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 12**

Multiply!

Let’s get more practice multiplying rational numbers.

Focus

Goals

1. **Language Goal:** Identify multiplication expressions that are equal, and justify that they are equal. **(Speaking and Listening)**
2. **Language Goal:** Multiply rational numbers, including expressions with three factors, and explain the reasoning. **(Writing, Speaking and Listening)**

Coherence

Today

Students reason about multiplying more than two rational numbers. First, they return to the cards from the Launch lesson, now with the goal of producing the greatest product. They must strategize about how the sign of their cards impact the final product and when taking an additional card is or is not necessary. Next, students apply their understanding of the commutative, associative, and Distributive Properties to evaluate expressions, comparing and contrasting equivalent expressions with their partner.

Previously

In Lessons 10 and 11, students applied their understanding of the distance formula to reason about and generalize the rules for multiplying rational numbers.

Coming Soon

In Lesson 13, students will extend their understanding of multiplying rational numbers to generate the rules for dividing rational numbers.

Rigor

- Students gain **fluency** in determining the product of two or more rational numbers.
- Students **apply** their understanding of addition, subtraction, and multiplication of rational numbers to evaluate and compare expressions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	18 min	13 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Lesson 1, Activity 1 PDF, *Game Cards*, pre-cut cards, one set per group (optional)
- Activity 1 PDF (for display)
- Activity 2 PDF (for display)
- Anchor Chart PDF, *Operations With Rational Numbers (Part 3)* (for display)
- Anchor Chart PDF, *Operations With Rational Numbers (Part 3)* (answers)
- standard deck of playing cards with face cards removed, one per small group (optional)

Note: Activity 1 PDF is provided if you would rather use printed cards instead playing cards. You do not need both. If printing in grayscale, show students how to identify the "red" cards.

Math Language Development

Review words

- *absolute value*
- *associative property*
- *commutative property*
- *Distributive Property*
- *magnitude*
- *opposites*
- *rational numbers*

Amps | **Featured Activity**

Activity 1

Formative Feedback for Students

Students revisit the cards from the Launch lesson, now working to obtain the greatest product. They are able to compare their hands and get immediate feedback on whether their calculated product is correct.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might miss any regularity in the multiplication of rational numbers if they do not approach the process with great organization, documenting each step along the way. Encourage students to not only write the equations of the problems they solve, but also write equation skeletons where they just use the signs of the numbers (+/–) and the multiplication and equals symbols. Looking at these skeletons will help students draw valid conclusions about the signs of products.

Modifications to Pacing

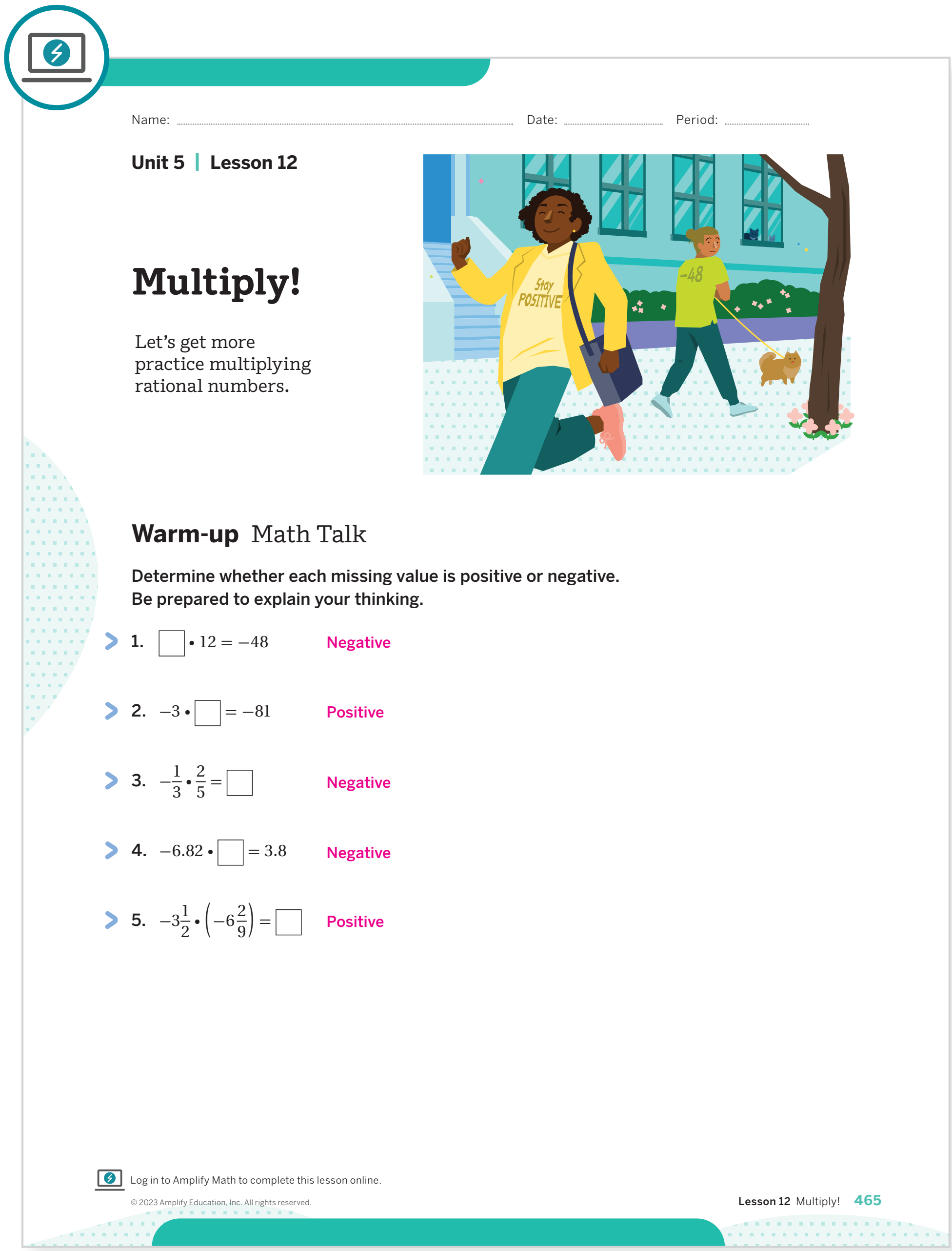
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students only complete two rounds of the game.
- In **Activity 2**, have pairs of students complete only 2 or 3 of the rows.

Independent | 5 min

Warm-up Math Talk

Students reason about the sign of missing value in equations that represent the product of rational numbers.



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Unit 5 | Lesson 12

Multiply!

Let's get more practice multiplying rational numbers.

Warm-up Math Talk

Determine whether each missing value is positive or negative. Be prepared to explain your thinking.

1. $\square \cdot 12 = -48$ Negative
2. $-3 \cdot \square = -81$ Positive
3. $-\frac{1}{3} \cdot \frac{2}{5} = \square$ Negative
4. $-6.82 \cdot \square = 3.8$ Negative
5. $-3\frac{1}{2} \cdot \left(-6\frac{2}{9}\right) = \square$ Positive

Log in to Amplify Math to complete this lesson online.

Lesson 12 Multiply! 465

① Launch

Activate students' prior knowledge by asking, "How do you determine the sign of the product of two rational numbers?" Explain to students that they are only responsible for determining the *sign* of each missing value. Conduct the ***Number Talk*** routine.

② Monitor

Help students get started by asking, "What do you know about the sign of each value in the first equation?"

Look for points of confusion:

- **Trying to determine the value of the missing number, rather than just its sign.** Remind students that they are not responsible for solving for the missing amount.

Look for productive strategies:

- Annotating the sign above each value in an equation so they are focusing on the signs and not the values.

③ Connect

Display the expressions.

Have students share their strategies for determining the missing sign for each problem. Focus the discussion on the problems that did not have consensus.

Highlight that, for a product of two factors to be positive, the two factors must have the same sign. For the product of two factors to be negative the factors must have different signs.

Ask:

- "Do you think that this is true if there are more than two factors?"
- "What would the sign of the value of the expression $(-1) \cdot (-2) \cdot (-3) \cdot \ldots \cdot (-2021)$ be?"

Note: Do not confirm student responses at this point. You will address these questions throughout the lesson.

Power-up

To power up students' ability to evaluate expressions with non-negative rational numbers, have students complete:

Recall that you follow the order of operations to evaluate expressions. For each expression, circle the step that would be completed first based on the order of operations.

1. $3 \div 1.5 \cdot 4$ [✓ $3 \div 1.5$] $1.5 \cdot 4$
2. $3 + 4 \cdot 5 - 8$ $3 + 4$ [✓ $4 \cdot 5$]
3. $4 - 2 + 6 + 7$ [✓ $4 - 2$] $2 + 6$

Use: Before Activity 2.

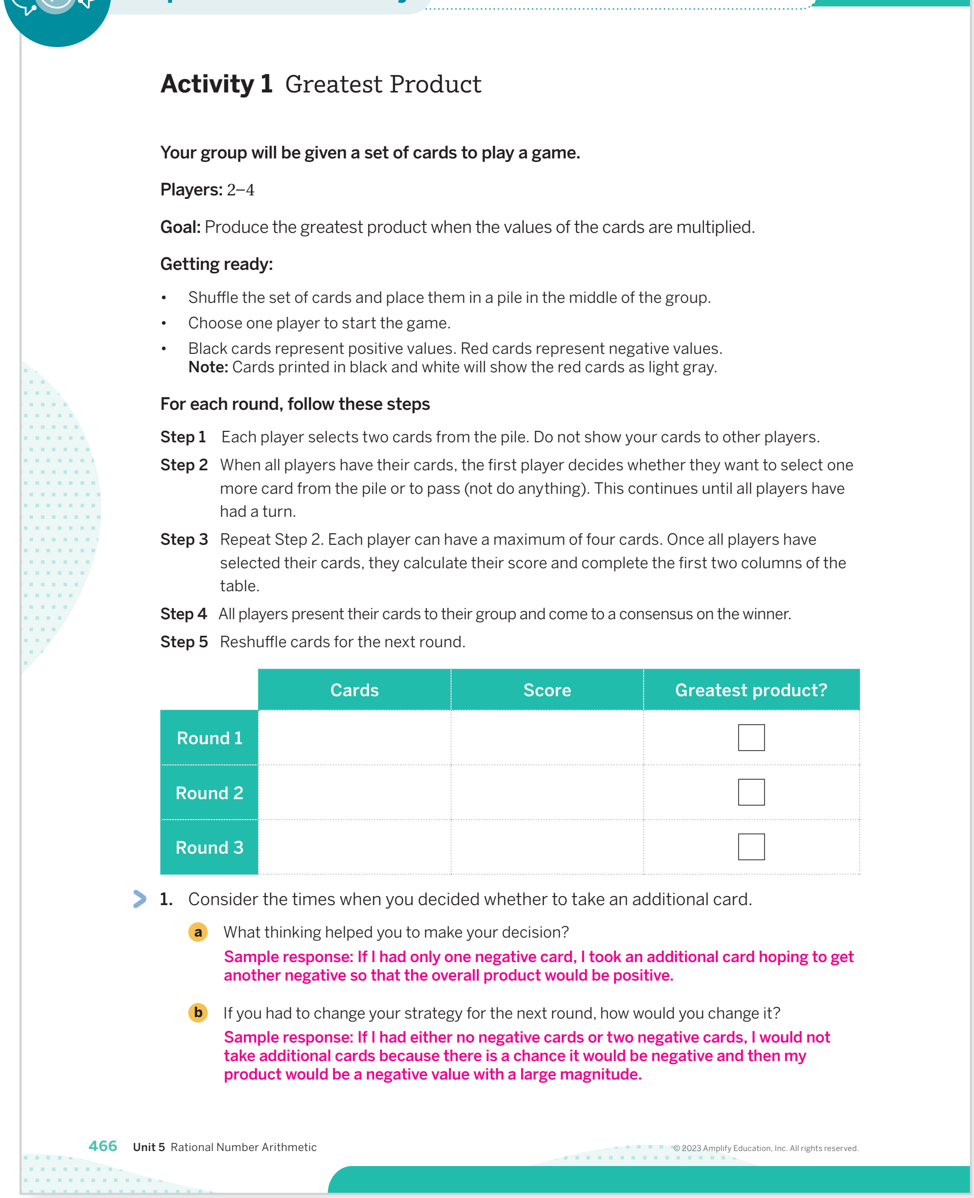
Informed by: Performance on Lesson 11, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1.

Small groups | 18 min

Activity 1 Greatest Product

Students reason about products of rational numbers by determining a strategy to obtain the greatest product from up to four positive and negative numbers.

Amps Featured Activity Formative Feedback for Students



Activity 1 Greatest Product

Your group will be given a set of cards to play a game.

Players: 2–4

Goal: Produce the greatest product when the values of the cards are multiplied.

Getting ready:

- Shuffle the set of cards and place them in a pile in the middle of the group.
- Choose one player to start the game.
- Black cards represent positive values. Red cards represent negative values. **Note:** Cards printed in black and white will show the red cards as light gray.

For each round, follow these steps

Step 1 Each player selects two cards from the pile. Do not show your cards to other players.

Step 2 When all players have their cards, the first player decides whether they want to select one more card from the pile or to pass (not do anything). This continues until all players have had a turn.

Step 3 Repeat Step 2. Each player can have a maximum of four cards. Once all players have selected their cards, they calculate their score and complete the first two columns of the table.

Step 4 All players present their cards to their group and come to a consensus on the winner.

Step 5 Reshuffle cards for the next round.

	Cards	Score	Greatest product?
Round 1			☐
Round 2			☐
Round 3			☐

1. Consider the times when you decided whether to take an additional card.

   a. What thinking helped you to make your decision?

   Sample response: If I had only one negative card, I took an additional card hoping to get another negative so that the overall product would be positive.

   b. If you had to change your strategy for the next round, how would you change it?

   Sample response: If I had either no negative cards or two negative cards, I would not take additional cards because there is a chance it would be negative and then my product would be a negative value with a large magnitude.

466 **Unit 5** Rational Number Arithmetic

1 Launch

Invite one or more students to read the directions aloud for the class and model a round of the game with two players. Assess understanding by asking, "If Student A had -4, 1, and 5 and Student B had -1, 2 and 3, who would win the round?" Student B

2 Monitor

Help students get started by asking, "For the cards you currently have, is their product positive or negative?"

Look for points of confusion:

- **Thinking that they are comparing the magnitudes of the products, not the products.** Remind students that they want the greatest product, not the product with the greatest absolute value.

3 Connect

Display the top half of the Activity 1 PDF. Conduct the ***Poll the Class*** routine asking, "Would you take a fourth card in this scenario?" Repeat for the bottom half of the PDF.

Have students share their strategies for determining when they should or should not take an additional card.

Ask, "If you have more than two cards (factors), how do you determine whether the product is positive or negative?"

Highlight that when three negative values are being multiplied, the product is negative. When four negative values are being multiplied, the product is positive. Ask, "What do you think would happen with 5 negative factors?" Generalize that if there are an odd number of negative factors, the product will be negative. Otherwise, the product is positive.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can receive immediate feedback on whether their calculated product is correct.

Extension: *Math Enrichment*

Students can play more rounds of the game, as time allows. Consider asking them to alter the rules to make the game more challenging. For example, each player can have a maximum of three cards, instead of four.

Math Language Development

MLR8: Discussion Supports

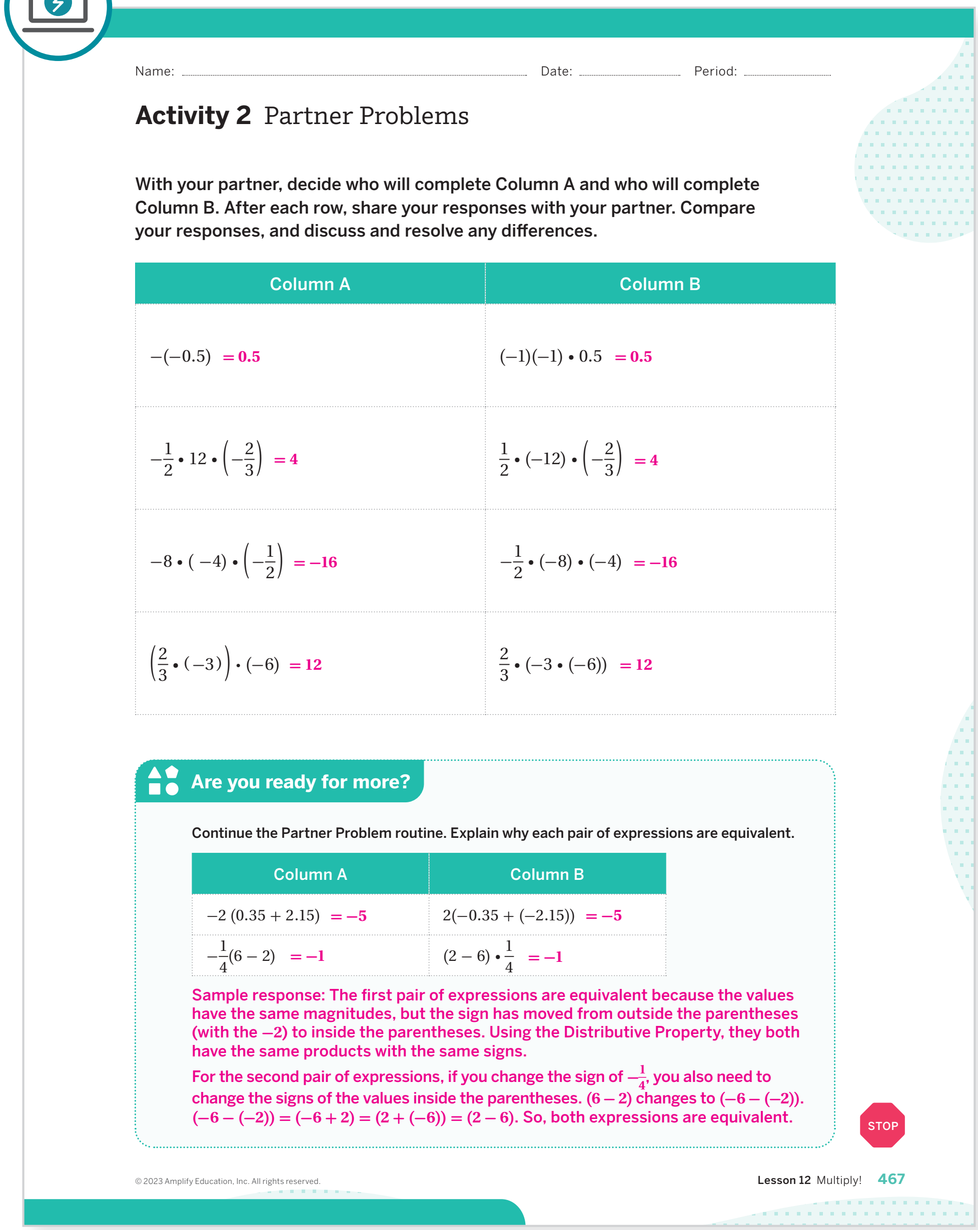
As you play the mock round during the Launch of this activity, talk through the directions as you go. Make sure to verbalize your thinking for any decisions that you make as you play. For example:

- [If one of your cards was negative] "Right now, I have one positive card and one negative card, so the product is negative. In order to obtain a positive product, I need one more negative card."
- [If both of your cards were positive, or if both are negative] "Right now, the product is positive. If I select one more card, I might select a negative card, which would result in the product being negative.

Pairs | 13 min

Activity 2 Partner Problems

Students simplify expressions involving rational numbers, addition, subtraction, and multiplication to come to a consensus with their partner.



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Activity 2 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with your partner. Compare your responses, and discuss and resolve any differences.

Column A	Column B
$-(-0.5)$ $= \mathbf{0.5}$	$(-1)(-1) \bullet 0.5$ $= \mathbf{0.5}$
$-\frac{1}{2} \bullet 12 \bullet \left(-\frac{2}{3}\right)$ $= \mathbf{4}$	$\frac{1}{2} \bullet (-12) \bullet \left(-\frac{2}{3}\right)$ $= \mathbf{4}$
$-8 \bullet (-4) \bullet \left(-\frac{1}{2}\right)$ $= \mathbf{-16}$	$-\frac{1}{2} \bullet (-8) \bullet (-4)$ $= \mathbf{-16}$
$\left(\frac{2}{3} \bullet (-3)\right) \bullet (-6)$ $= \mathbf{12}$	$\frac{2}{3} \bullet (-3 \bullet (-6))$ $= \mathbf{12}$

Are you ready for more?

Continue the Partner Problem routine. Explain why each pair of expressions are equivalent.

Column A	Column B
$-2\,(0.35 + 2.15)$ $= \mathbf{-5}$	$2(-0.35 + (-2.15))$ $= \mathbf{-5}$
$-\frac{1}{4}(6 - 2)$ $= \mathbf{-1}$	$(2 - 6) \bullet \frac{1}{4}$ $= \mathbf{-1}$

Sample response: The first pair of expressions are equivalent because the values have the same magnitudes, but the sign has moved from outside the parentheses (with the -2) to inside the parentheses. Using the Distributive Property, they both have the same products with the same signs.

For the second pair of expressions, if you change the sign of $-\frac{1}{4}$, you also need to change the signs of the values inside the parentheses. $(6 - 2)$ changes to $(-6 - (-2))$. $(-6 - (-2)) = (-6 + 2) = (2 + (-6)) = (2 - 6)$. So, both expressions are equivalent.

STOP

Lesson 12 Multiply! 467

1 Launch

Conduct the ***Partner Problems*** routine.

2 Monitor

Help students get started by suggesting they cover up all of the expressions, except the one they are evaluating.

Look for points of confusion:

- **Thinking that any expression with at least two negatives must be positive.** Remind students that, in the previous activity, they saw that the product of three negative cards resulted in a negative value.

3 Connect

Display the Activity 2 PDF.

Ask:

- "How would you read the first expression, $-(-0.5)$ out loud?" The opposite of negative 0.5 or positive 0.5.
- "What if the expression was $-(-(-0.5))$? How else could the expression be represented?"

Highlight that when there are multiple "–" signs in front of a number, it can be thought of taking the opposite multiple times. The opposite of any value is that value multiplied by -1, so the expression can be rewritten as the product of multiple factors of -1.

Have students share what other relationships they notice in the other pairs of expressions. Encourage them to use vocabulary, such as associative and commutative properties.

Highlight that, in the remaining expressions, the magnitudes of the values are the same, but the placement of the negative sign or the order of the values is different between the expressions. Generalize that the sign of the product is dependent on the number of negative factors.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to choose three of the four rows to complete. Offering them the power of choice can lead to greater engagement and ownership of the task.

Extension: *Math Enrichment*

Have students determine whether the product of each of the following descriptions of expressions will be positive or negative, without performing any calculations.

- The number -1 is multiplied by itself thirty times. Positive
- The number -5 is multiplied by itself seventeen times. Negative

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight how the sign of the product is dependent upon the number of negative factors, display these sentence frames and have students complete them. Add these completed statements to the class display.

In a multiplication expression, if the number of negative factors is . . .

- odd, the sign of the product will be _____.
- even, the sign of the product will be _____.

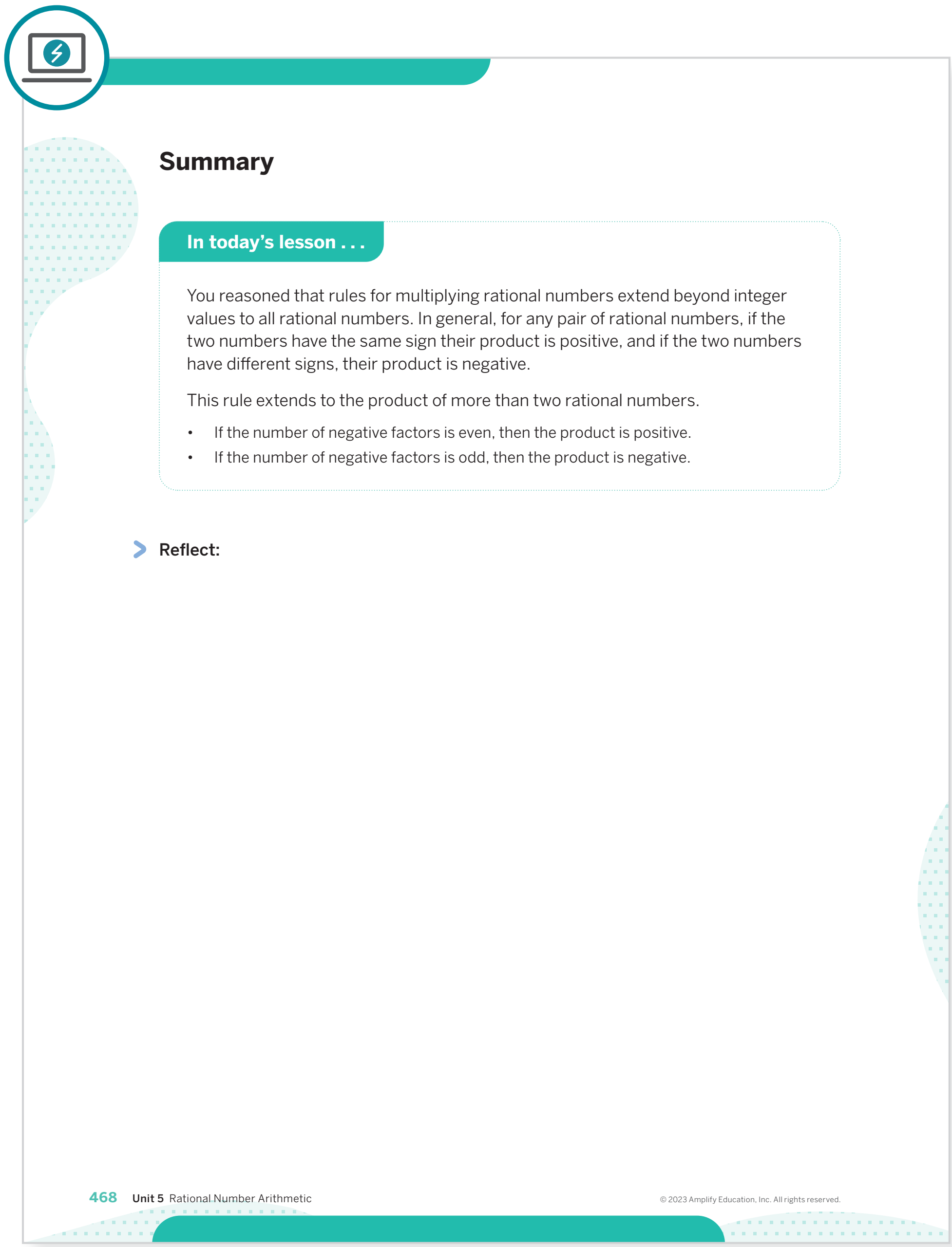
English Learners

Include examples of expressions and clarify the meanings of the terms *odd* and *even*.

Whole Class | 5 min

Summary

Review and synthesize that the sign of the product of rational numbers is dependent on the number of negative factors.



Summary

In today's lesson . . .

You reasoned that rules for multiplying rational numbers extend beyond integer values to all rational numbers. In general, for any pair of rational numbers, if the two numbers have the same sign their product is positive, and if the two numbers have different signs, their product is negative.

This rule extends to the product of more than two rational numbers.

- If the number of negative factors is even, then the product is positive.
- If the number of negative factors is odd, then the product is negative.

Reflect:

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Synthesize

Display the Anchor Chart PDF, *Operations With Rational Numbers (Part 3)*. As a class, complete the section on multiplication.

Ask:

- "What would the sign of $-(-(-(-(-(-6)))))$ be?" Positive
- "What would the sign of $(-1) \cdot (-2) \cdot (-3) \cdot \ldots \cdot (-2021)$ be?" Negative

Have students share how they determined the sign of each problem with the class.

Highlight that, in order to determine the product of multiple rational numbers, first determine the product of the absolute value of each factor. Then determine the number of factors that are negative.

- If there is an odd number of negative factors, the product is negative.
- If there is an even number of negative factors, the product is positive.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

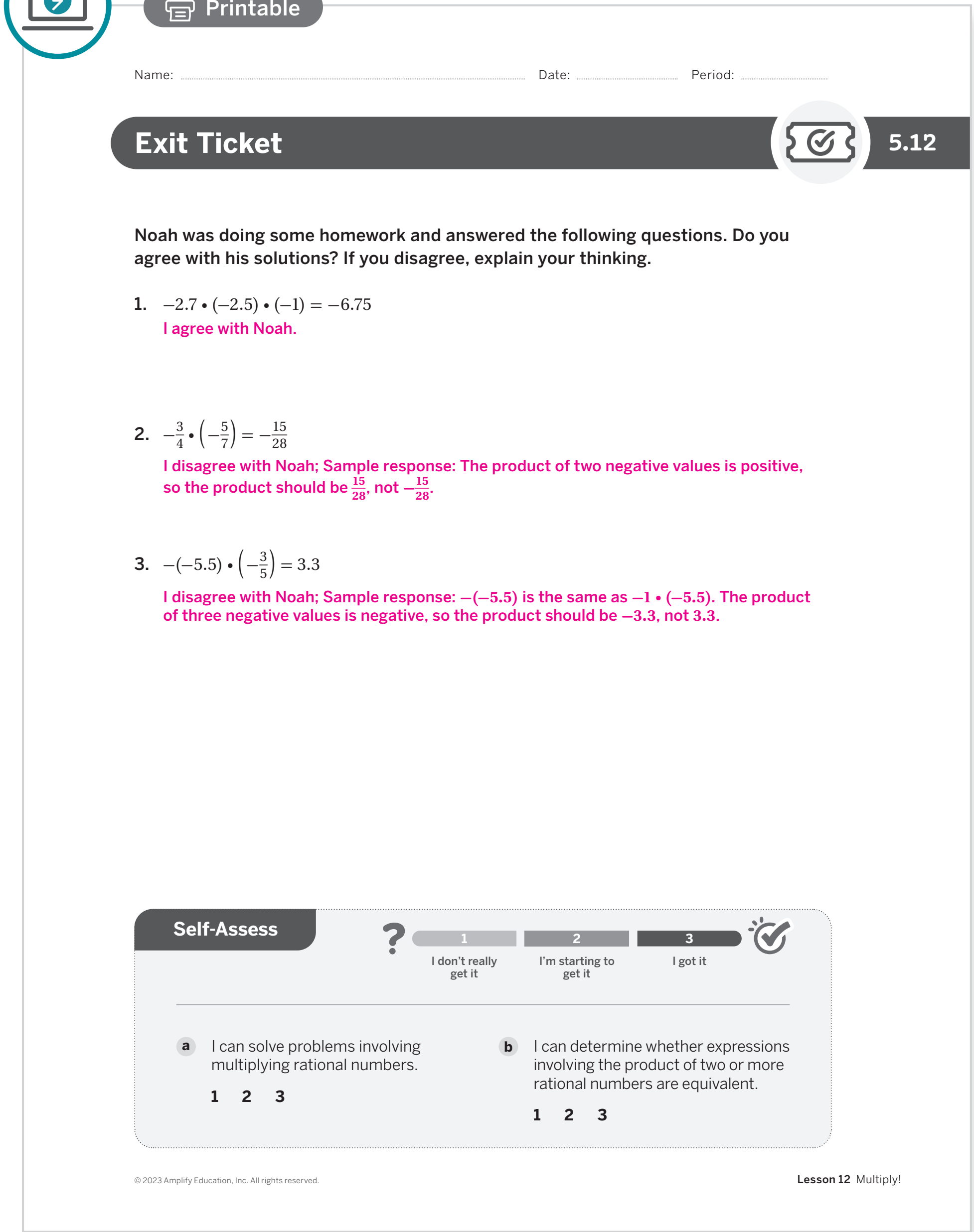
- "How do you determine the sign of the product of two or more rational numbers?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding multiplying multiple rational numbers by assessing the reasonableness of each other's work.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.12

Noah was doing some homework and answered the following questions. Do you agree with his solutions? If you disagree, explain your thinking.

1. $-2.7 \bullet (-2.5) \bullet (-1) = -6.75$

   I agree with Noah.

2. $-\frac{3}{4} \bullet \left(-\frac{5}{7}\right) = -\frac{15}{28}$

   I disagree with Noah; Sample response: The product of two negative values is positive, so the product should be $\frac{15}{28}$, not $-\frac{15}{28}$.

3. $-(-5.5) \bullet \left(-\frac{3}{5}\right) = 3.3$

   I disagree with Noah; Sample response: $-(-5.5)$ is the same as $-1 \bullet (-5.5)$. The product of three negative values is negative, so the product should be -3.3, not 3.3.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can solve problems involving multiplying rational numbers.
1 2 3

b. I can determine whether expressions involving the product of two or more rational numbers are equivalent.
1 2 3

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Lesson 12 Multiply!

Success looks like . . .

- **Language Goal:** Identifying multiplication expressions that are equal, and justifying that they are equal. **(Speaking and Listening)**
- **Language Goal:** Multiplying rational numbers, including expressions with three factors, and explaining the reasoning. **(Writing, Speaking and Listening)**
  - » Multiplying rational numbers and explaining whether Noah was correct in Problems 1–3.

Suggested next steps

If students incorrectly identify that Noah was correct for Problem 2, consider:

- Reviewing the rules for multiplying a pair of rational numbers.
- Assigning Practice Problem 1.

If students incorrectly identify that Noah was incorrect for Problem 1 or correct for Problem 3, consider:

- Reviewing the rules for multiplying more than two rational numbers.
- Assigning Practice Problems 2 and 3.

Professional Learning

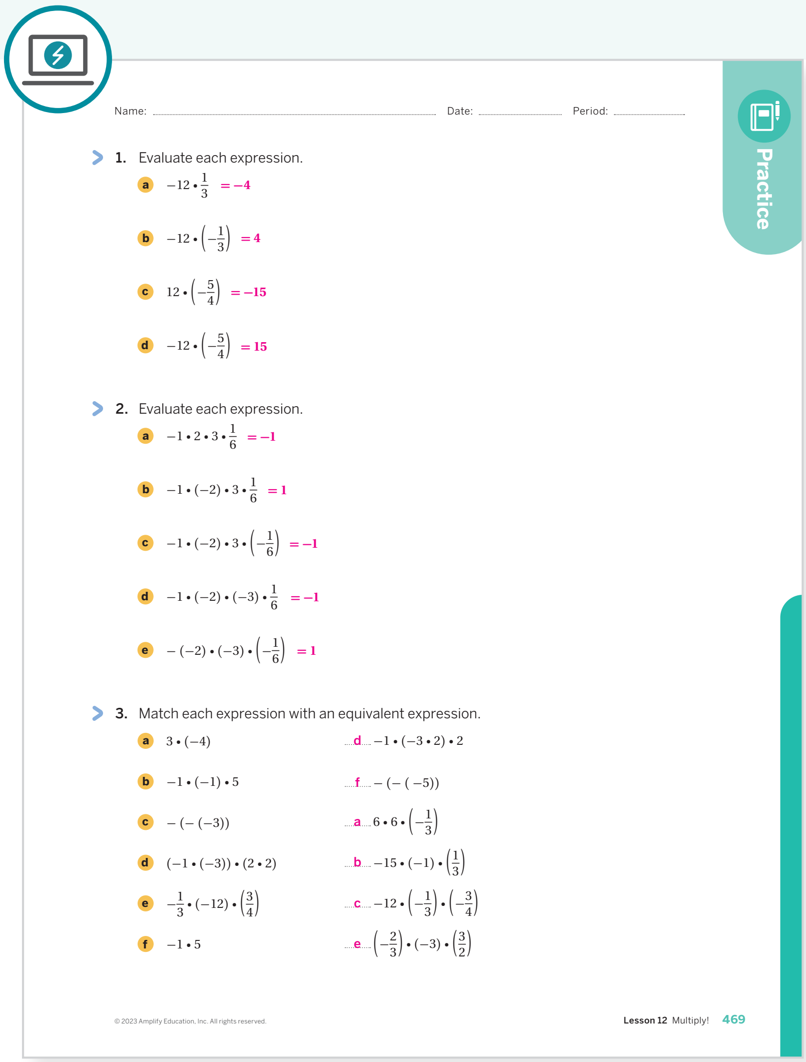
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What routines enabled all students to do math in today's lesson?
- Which students' ideas were you able to highlight during Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Evaluate each expression.
   - a $-12 \cdot \frac{1}{3}$ $= -4$
   - b $-12 \cdot \left(-\frac{1}{3}\right)$ $= 4$
   - c $12 \cdot \left(-\frac{5}{4}\right)$ $= -15$
   - d $-12 \cdot \left(-\frac{5}{4}\right)$ $= 15$

2. Evaluate each expression.
   - a $-1 \cdot 2 \cdot 3 \cdot \frac{1}{6}$ $= -1$
   - b $-1 \cdot (-2) \cdot 3 \cdot \frac{1}{6}$ $= 1$
   - c $-1 \cdot (-2) \cdot 3 \cdot \left(-\frac{1}{6}\right)$ $= -1$
   - d $-1 \cdot (-2) \cdot (-3) \cdot \frac{1}{6}$ $= -1$
   - e $-(-2) \cdot (-3) \cdot \left(-\frac{1}{6}\right)$ $= 1$

3. Match each expression with an equivalent expression.

a $3 \cdot (-4)$	d $-1 \cdot (-3 \cdot 2) \cdot 2$
b $-1 \cdot (-1) \cdot 5$	f $-(-(-5))$
c $-(-(-3))$	a $6 \cdot 6 \cdot \left(-\frac{1}{3}\right)$
d $(-1 \cdot (-3)) \cdot (2 \cdot 2)$	b $-15 \cdot (-1) \cdot \left(\frac{1}{3}\right)$
e $-\frac{1}{3} \cdot (-12) \cdot \left(\frac{3}{4}\right)$	c $-12 \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{3}{4}\right)$
f $-1 \cdot 5$	e $\left(-\frac{2}{3}\right) \cdot (-3) \cdot \left(\frac{3}{2}\right)$

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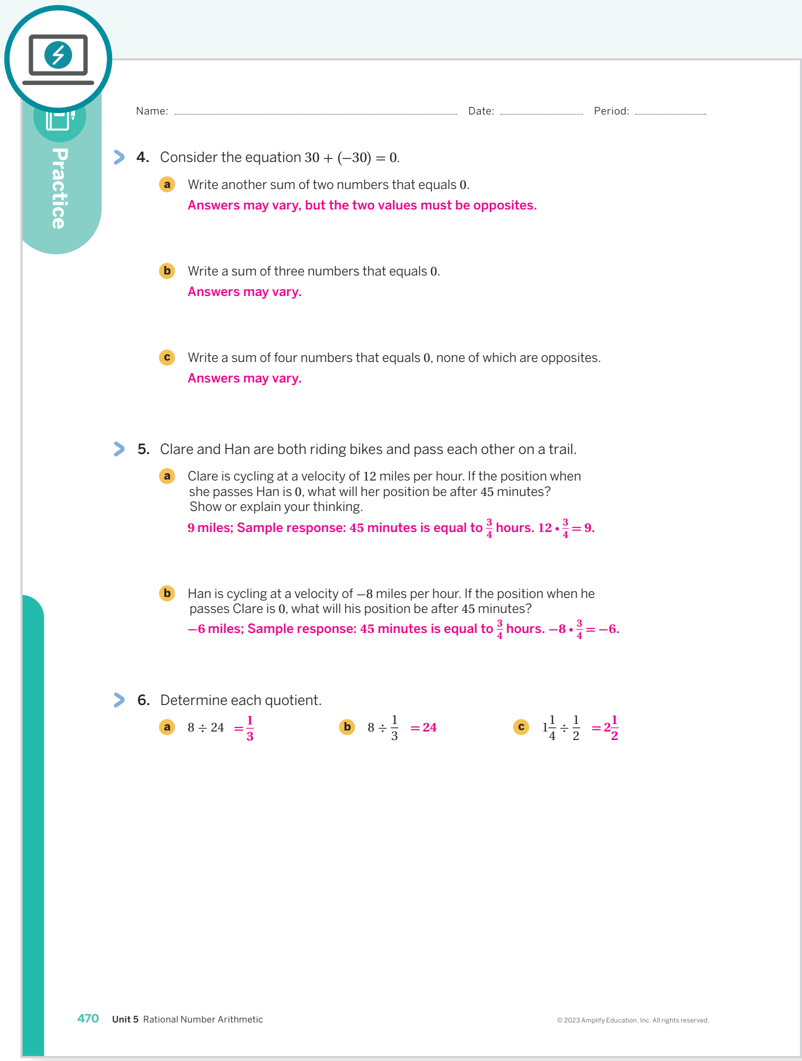
Lesson 12 Multiply! 469



Practice

Name: Date: Period:

4. Consider the equation $30 + (-30) = 0$.
   - a Write another sum of two numbers that equals 0.
     Answers may vary, but the two values must be opposites.
   - b Write a sum of three numbers that equals 0.
     Answers may vary.
   - c Write a sum of four numbers that equals 0, none of which are opposites.
     Answers may vary.

5. Clare and Han are both riding bikes and pass each other on a trail.
   - a Clare is cycling at a velocity of 12 miles per hour. If the position when she passes Han is 0, what will her position be after 45 minutes? Show or explain your thinking.
     9 miles; Sample response: 45 minutes is equal to $\frac{3}{4}$ hours. $12 \cdot \frac{3}{4} = 9$.
   - b Han is cycling at a velocity of -8 miles per hour. If the position when he passes Clare is 0, what will his position be after 45 minutes?
     −6 miles; Sample response: 45 minutes is equal to $\frac{3}{4}$ hours. $-8 \cdot \frac{3}{4} = -6$.

6. Determine each quotient.
   - a $8 \div 24$ $= \frac{1}{3}$
   - b $8 \div \frac{1}{3}$ $= 24$
   - c $1\frac{1}{4} \div \frac{1}{2}$ $= 2\frac{1}{2}$

470 **Unit 5** Rational Number Arithmetic

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 5	2
	5	Unit 5 Lesson 11	2
Formative ⚡	6	Unit 5 Lesson 13	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 13**

Dividing Rational Numbers

Let's divide rational numbers.

Focus

Goals

1. **Language Goal:** Generalize a method for determining the quotient of two rational numbers. **(Speaking and Listening)**
2. Generate a division equation that represents the same relationship as a given multiplication equation with rational numbers.
3. Apply the order of operations while working with rational numbers.

Coherence

Today

Students complete their work extending all four operations to signed numbers by studying division. They use the relationship between multiplication and division to develop rules for dividing rational numbers. They practice applying the rules for the order of operations and using the multiplicative inverse to evaluate expressions involving all four operations with rational numbers.

Previously

In Lessons 10-12, students explored multiplication with rational numbers.

Coming Soon

In Lesson 14, students will apply their understanding of rational numbers to contexts with negative rates.

Rigor

- Students build **conceptual understanding** of why the rules for one set of operations also apply to another set.
- Students develop **procedural skills** multiplying and dividing rational numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (answers, for display)
- Anchor Chart PDF, *Rational Numbers, Part 3* (for display)
- Anchor Chart PDF, *Rational Numbers, Part 3* (answers)

Math Language Development

New word

- ***multiplicative inverse***

Review words

- *inverse operations*
- *rational numbers*
- *solution*

Amps | Featured Activity

Activity 2

Track Your Path

Students move across a game board, evaluating rational number expressions, sometimes with multiple operations. Using the digital version allows them to efficiently track their path and revise their thinking.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might want to draw a conclusion for the sign of quotients without completing the activity, but explain that using the structure of the relationship between multiplication and division will help them verify or alter their conjecture. By the end of the activity, students should note that the pattern for the signs in division is the same as with multiplication.

Modifications to Pacing

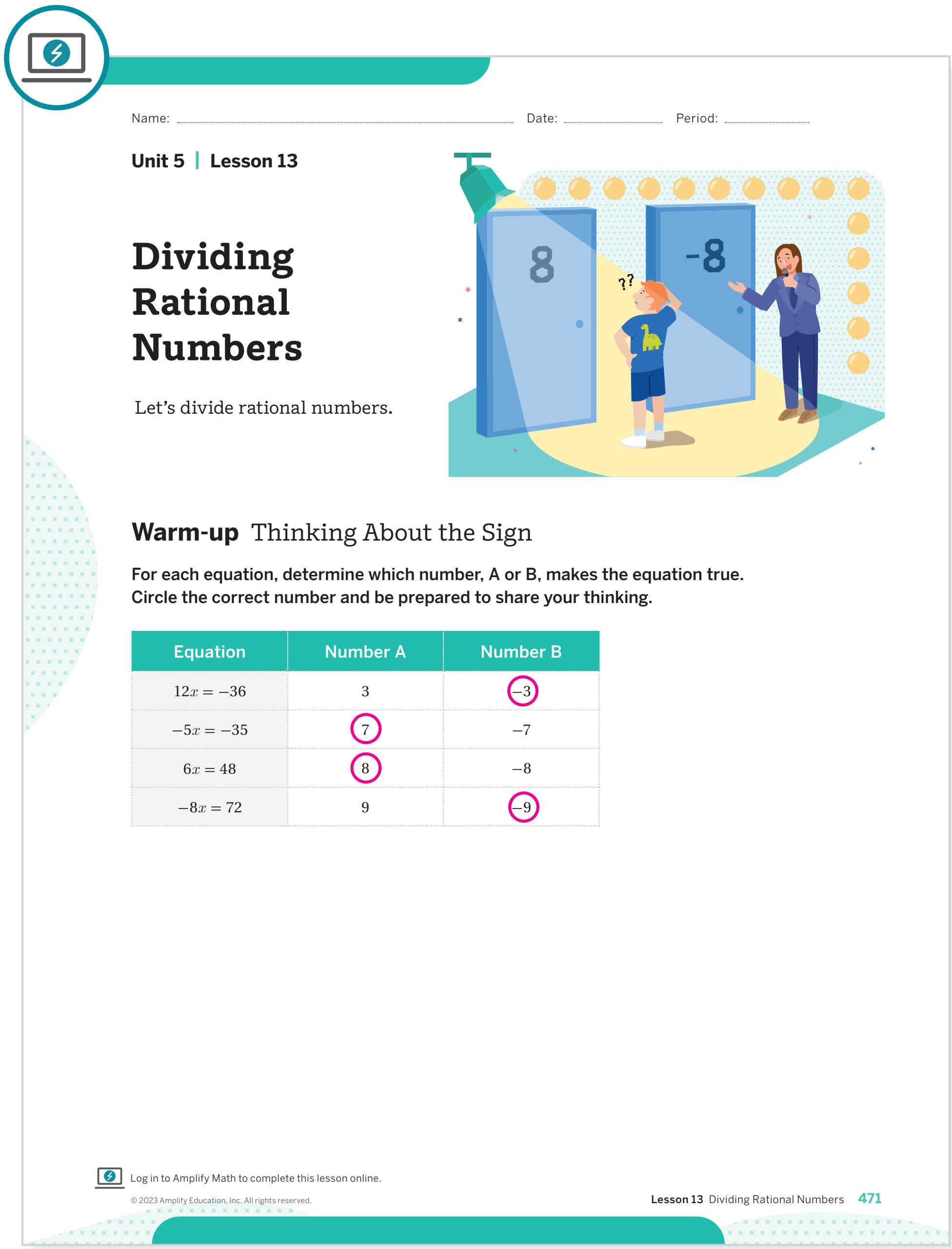
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1,** Problem 2 may be omitted.

Independent | 5 min

Warm-up Thinking About the Sign

Students use what they know about multiplication with rational numbers to reason about the sign of the solution to an equation involving negative numbers.



Name: ______________ Date: ________ Period: ________

Unit 5 | **Lesson 13**

Dividing Rational Numbers

Let's divide rational numbers.

Warm-up Thinking About the Sign

For each equation, determine which number, A or B, makes the equation true. Circle the correct number and be prepared to share your thinking.

Equation	Number A	Number B
$12x = -36$	3	[✓ −3]
$-5x = -35$	[✓ 7]	−7
$6x = 48$	[✓ 8]	−8
$-8x = 72$	9	[✓ −9]

Log in to Amplify Math to complete this lesson online.

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Lesson 13 Dividing Rational Numbers 471

1 Launch

Have students examine the table for 30 seconds. Ask, "What do you notice about the numbers in the last two columns? What parts of the equations are most important for you to pay attention to?"

2 Monitor

Help students get started by asking, "What types of numbers would need to be multiplied to produce a negative product?"

Look for points of confusion:

- **Thinking that x can't be negative because it doesn't have a negative sign in front of it in the equation.** Have students substitute a negative number into the equations.

Look for productive strategies:

- Dividing the product by the given factor, noticing that the same rules for multiplication of negative numbers also apply to division.

3 Connect

Display students' responses to the activity.

Have students share their thinking for how they found which number makes each equation true.

Highlight that this lesson will use multiplication equations, similar to those in the Warm-up, to help students make sense of dividing positive and negative numbers. Have students recall that the number that makes each equation true is called the *solution*.

Ask, "How are multiplication and division related to each other?" They are inverse operations. You can think of one of these operations undoing the other operation.

Math Language Development

MLR8: Discussion Supports

During the Launch, read the first equation aloud by saying, "12 times an unknown number equals negative 36" or "The product of 12 and an unknown number is negative 36." Encourage students to say verbal statements about the remaining equations to help them make sense of their structure.

Power-up

To power up students' ability to divide when the divisor is a fraction, have students complete:

Determine each quotient.

1. $8 \div 4 = 2$
2. $8 \div 2 = 4$
3. $8 \div 1 = 8$
4. $8 \div \frac{1}{2} = 16$

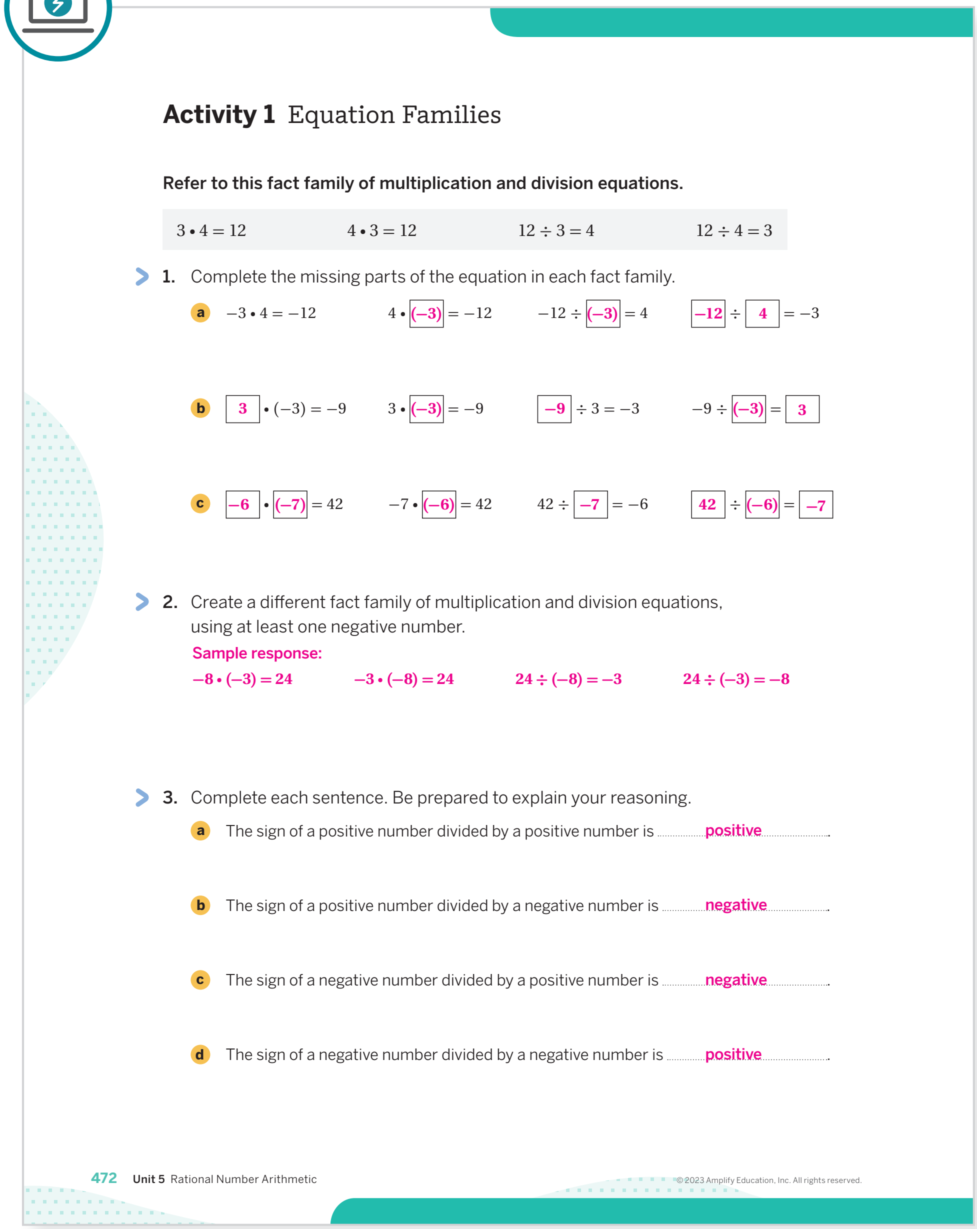
Use: Before Activity 2.

Informed by: Performance on Lesson 12, Practice Problem 6.

Pairs | 15 min

Activity 1 Equation Families

Students work through several equation families relating multiplication to division to articulate a rule for the sign of a quotient based on the signs of the dividend and divisor.



Activity 1 Equation Families

Refer to this fact family of multiplication and division equations.

$3 \bullet 4 = 12$ $\quad$ $4 \bullet 3 = 12$ $\quad$ $12 \div 3 = 4$ $\quad$ $12 \div 4 = 3$

1. Complete the missing parts of the equation in each fact family.

a $-3 \bullet 4 = -12$ $\quad$ $4 \bullet \boxed{(-3)} = -12$ $\quad$ $-12 \div \boxed{(-3)} = 4$ $\quad$ $\boxed{-12} \div \boxed{4} = -3$

b $\boxed{3} \bullet (-3) = -9$ $\quad$ $3 \bullet \boxed{(-3)} = -9$ $\quad$ $\boxed{-9} \div 3 = -3$ $\quad$ $-9 \div \boxed{(-3)} = \boxed{3}$

c $\boxed{-6} \bullet \boxed{(-7)} = 42$ $\quad$ $-7 \bullet \boxed{(-6)} = 42$ $\quad$ $42 \div \boxed{-7} = -6$ $\quad$ $\boxed{42} \div \boxed{(-6)} = \boxed{-7}$

2. Create a different fact family of multiplication and division equations, using at least one negative number.

Sample response:

$-8 \bullet (-3) = 24$ $\quad$ $-3 \bullet (-8) = 24$ $\quad$ $24 \div (-8) = -3$ $\quad$ $24 \div (-3) = -8$

3. Complete each sentence. Be prepared to explain your reasoning.

a The sign of a positive number divided by a positive number is positive.

b The sign of a positive number divided by a negative number is negative.

c The sign of a negative number divided by a positive number is negative.

d The sign of a negative number divided by a negative number is positive.

472 **Unit 5** Rational Number Arithmetic

1 Launch

Read through the introduction to the activity as a class. Ask, "What do you notice about these equations that make them belong to the same fact family?" Note that for the purposes of this activity, the sign of a number must stay the same throughout the fact family of equations.

2 Monitor

Help students get started by asking, "Which number from the first equation in part a is missing from the second?"

Look for points of confusion:

- **Thinking for part c they can make their own factors because there are two blanks in the first equation.** Have students check the rest of the equations in the family to see which factors should be used.

Look for productive strategies:

- In Problem 3, noticing that the rules are the same for division of rational numbers as they are for multiplication.

3 Connect

Have students share their responses to Problem 3.

Highlight that students took two things they knew to be true — multiplication and division as inverse operations, and the rules for multiplication of rational numbers — and combined them into a new understanding. They were able to conclude that the same rules for multiplying rational numbers also apply to the division of rational numbers.

Ask:

- "How can you predict the sign of the quotient of a division problem?"
- "Why did none of the equations in the families have 3 negative numbers? Would that ever be possible?"

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously learned about fact families in prior grades. A fact family consists of three numbers that are used together to create a set of math facts. Those math facts can be addition, subtraction, multiplication, or division.

Math Language Development

MLR2: Collect and Display

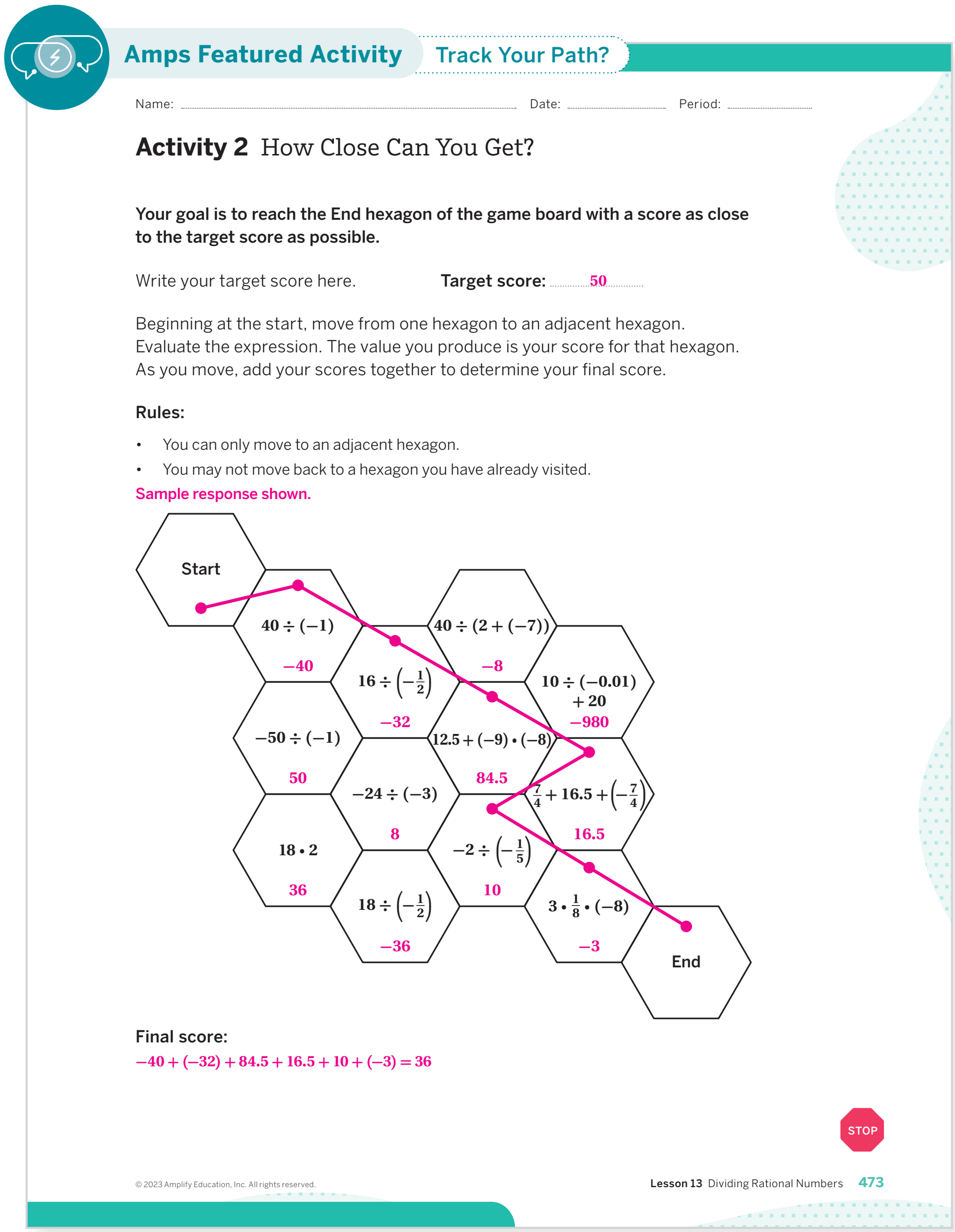
During the Connect, as students share their responses to Problem 3, add these statements — or a condensed version of them — to the class display. Compare them to the multiplication rules for rational numbers that students previously formulated.

Multiplication: The product of . . .	Division: The quotient of . . .
Two positive numbers is always positive.	Two positive numbers is always positive.
One positive number and one negative number is always negative.	One positive number and one negative number is always negative.
Two negative numbers is always positive.	Two negative numbers is always positive.

Pairs | 15 min

Activity 2 How Close Can You Get?

Students choose their path through a gameboard of expressions to practice order of operations and explore the effect of the multiplicative inverse with rational numbers.



Amps Featured Activity Track Your Path?

Name: Date: Period:

Activity 2 How Close Can You Get?

Your goal is to reach the End hexagon of the game board with a score as close to the target score as possible.

Write your target score here. **Target score:** 50

Beginning at the start, move from one hexagon to an adjacent hexagon. Evaluate the expression. The value you produce is your score for that hexagon. As you move, add your scores together to determine your final score.

Rules:

- You can only move to an adjacent hexagon.
- You may not move back to a hexagon you have already visited.

Sample response shown.

Final score:

$-40 + (-32) + 84.5 + 16.5 + 10 + (-3) = 36$

STOP

Lesson 13 Dividing Rational Numbers 473

1 Launch

Read through the instructions together as a class and let students know the target score they will aim for. **Note:** Suggestions for target scores: Choose 0 as a target to focus on balancing positive and negative values; Choose $-1{,}000$ to help kids focus on reasoning about signs; Or let your students choose their own target number.

2 Monitor

Help students get started by modelling how a student might think about choosing which spot to jump to next.

Look for points of confusion:

- **Thinking dividing by $\frac{1}{2}$ is the same as dividing by 2.** Help students recall that dividing by a fraction is the same as multiplying by its reciprocal.
- **Performing operations from left to right regardless of type.** Have students record the order of operations on their paper.

Look for productive strategies:

- Noticing that some expressions can be evaluated by using the properties of operations.

3 Connect

Display the Activity 2 PDF (answers) and discuss any questions students might have.

Have students share how they made their decisions for the route they took. Highlight student reasoning where division by a fraction is rewritten as multiplication by the fraction's reciprocal.

Highlight that when evaluating expressions with multiple operations and groupings, even with negative numbers, the order of operations remains the same.

Define ***multiplicative inverse*** as another name for the reciprocal of a number.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can move across a digital game board, evaluating rational number expressions, sometimes with multiple operations. Using the digital version allows them to efficiently track their path and revise their thinking.

Accessibility: *Vary Demands to Optimize Challenge*

Have students select a pathway first (in lightly drawn pencil), and then they evaluate the expressions along that path. If they would like to change their path, they can go back and redraw it.

Math Language Development

MLR7: Compare and Connect

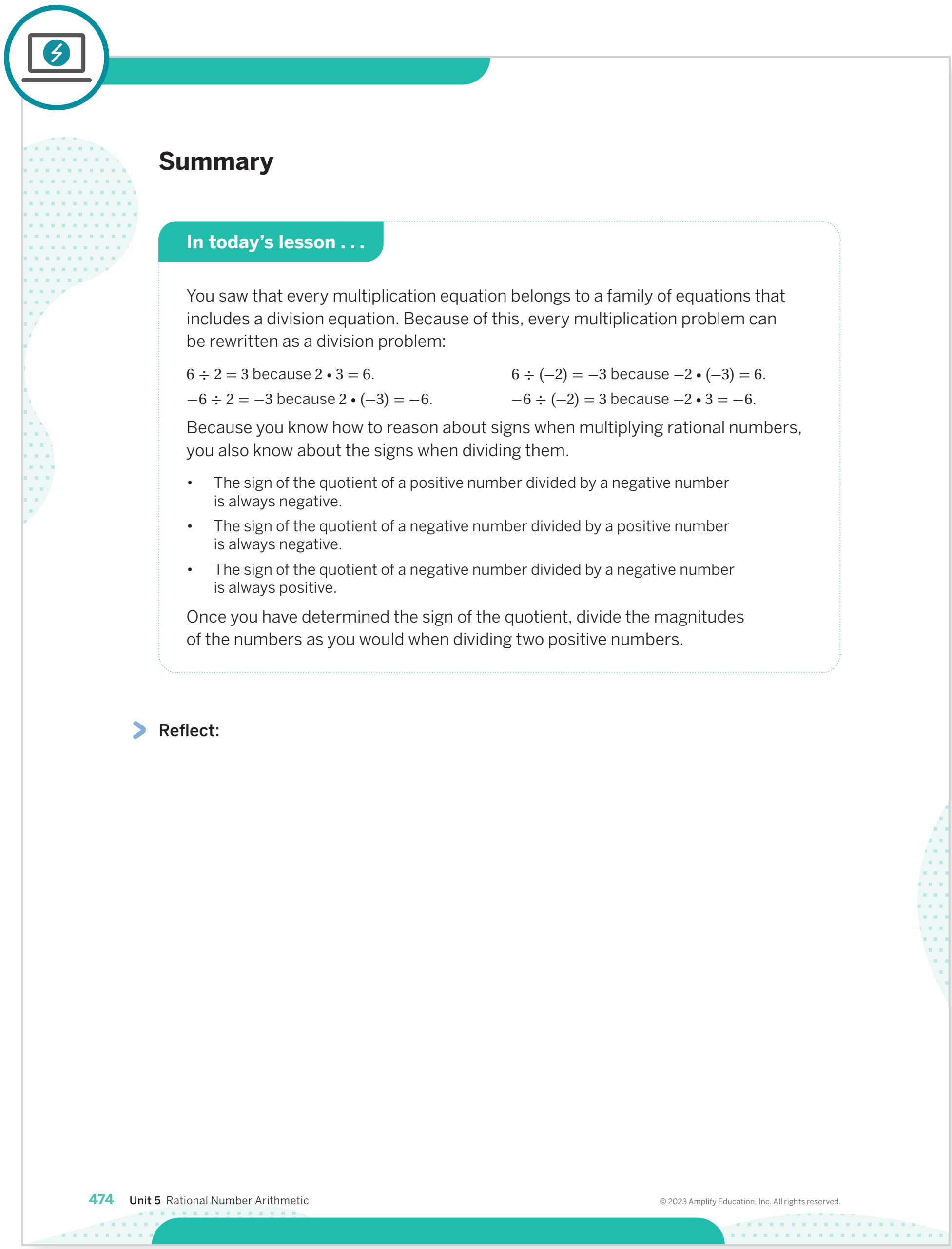
During the Connect, as you define the term *multiplicative inverse*, draw students' attention to how the *multiplicative inverse* of a number compares to its *additive inverse*. Consider displaying a table like the following, or add it to the class display.

Additive inverse	Multiplicative inverse
The sum of a number and its additive inverse is 0.	The product of a number and its multiplicative inverse is 1.
Also called the *opposite* of a number.	Also called the *reciprocal* of a number.
3 and −3 are additive inverses (opposites) of each other.	3 and $\frac{1}{3}$ are multiplicative inverses (reciprocals) of each other.

Whole Class | 5 min

Summary

Review and synthesize the relationship between multiplication and division to understand the rules for dividing rational numbers.



Summary

In today's lesson . . .

You saw that every multiplication equation belongs to a family of equations that includes a division equation. Because of this, every multiplication problem can be rewritten as a division problem:

$6 \div 2 = 3$ because $2 \cdot 3 = 6$.

$6 \div (-2) = -3$ because $-2 \cdot (-3) = 6$.

$-6 \div 2 = -3$ because $2 \cdot (-3) = -6$.

$-6 \div (-2) = 3$ because $-2 \cdot 3 = -6$.

Because you know how to reason about signs when multiplying rational numbers, you also know about the signs when dividing them.

- The sign of the quotient of a positive number divided by a negative number is always negative.
- The sign of the quotient of a negative number divided by a positive number is always negative.
- The sign of the quotient of a negative number divided by a negative number is always positive.

Once you have determined the sign of the quotient, divide the magnitudes of the numbers as you would when dividing two positive numbers.

Reflect:

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Synthesize

Highlight that students were able to generalize their own rule for dividing rational numbers using what they already knew about multiplication of rational numbers.

Display the Anchor Chart PDF, *Rational Numbers (Part 2)*. Obtain the missing information from your class and complete the chart together.

Formalize vocabulary: *multiplicative inverse*

Ask:

- "What kind of number do you get when you divide a negative number by a positive number? Use a multiplication equation to explain why this makes sense."
- "What kind of number do you get when you divide a negative number by a negative number? Use a multiplication equation to explain why this makes sense."
- "What is the sign of the quotient of $-3 \div 4$?" Negative
- "What is the magnitude of the quotient of $-3 \div 4$?" 0.75 or $\frac{3}{4}$
- "What is the quotient of $-3 \div 4$?" -0.75 or $-\frac{3}{4}$

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is dividing rational numbers similar to or different from multiplying rational numbers?"

Differentiated Support

Extension: ***Math Around the World, Interdisciplinary Connections***

Tell students that, in his writings around 620 CE, Indian mathematician Brahmagupta described the rules for multiplying and dividing with negative values. He again used the idea of "fortunes" as representing positive values and "debts" as representing negative values and stated:

- The product or quotient of two fortunes is one fortune.
- The product or quotient of two debts is one fortune.
- The product or quotient of a debt and a fortune is a debt.
- The product or quotient of a fortune and a debt is a debt.

Ask students to rewrite Brahmagupta's rules using the terms "positive number" and "negative number." Then ask them to provide numerical examples that illustrate Brahmagupta's rules.

Math Language Development

MLR2: Collect and Display

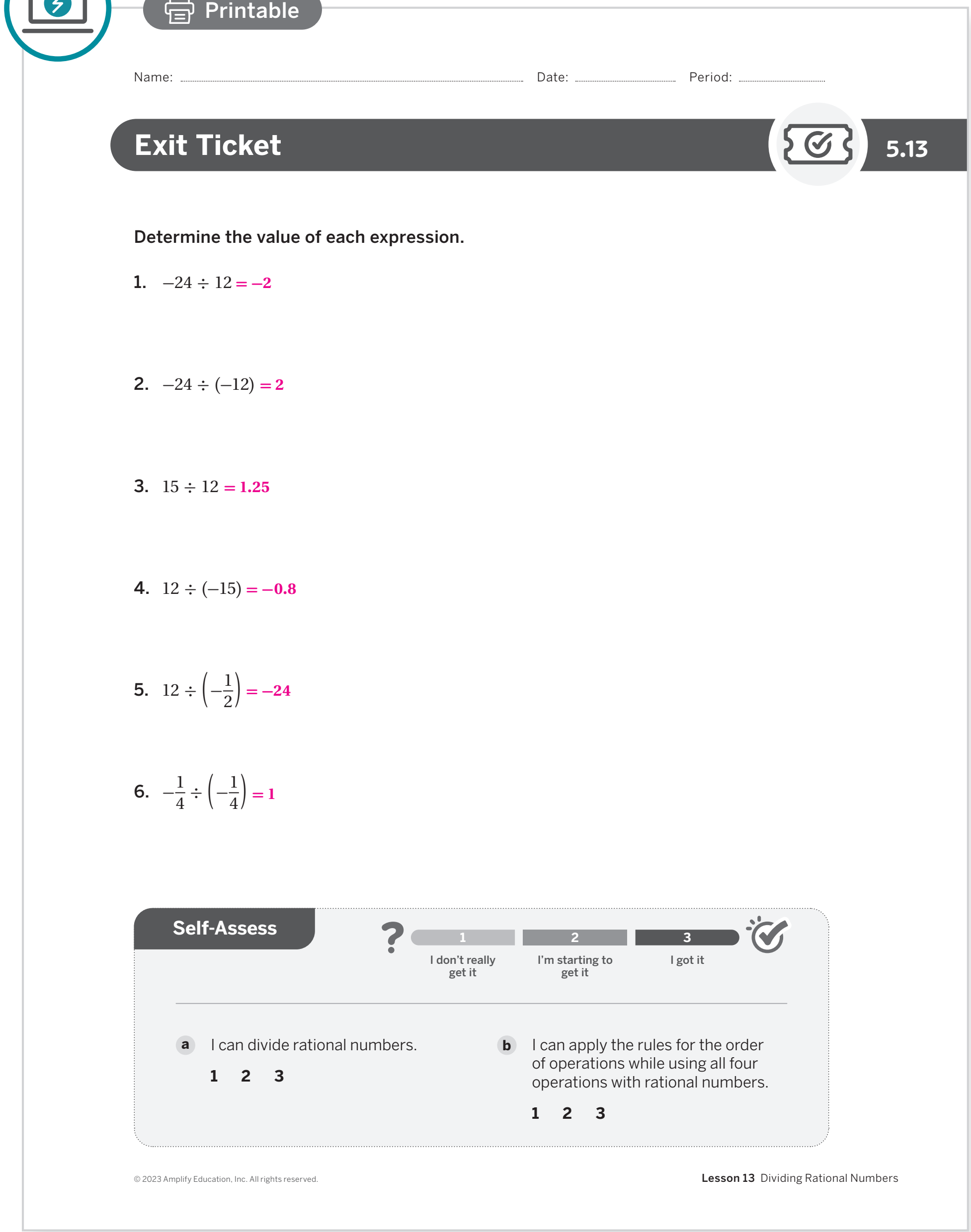
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on the terms and phrases related to the term *multiplicative inverse* that were added to the display during the lesson. Compare the terms *additive inverse* and *multiplicative inverse* and highlight how they are similar and different on the class display.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of dividing rational numbers.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 5.13

Determine the value of each expression.

1. $-24 \div 12 = -2$

2. $-24 \div (-12) = 2$

3. $15 \div 12 = 1.25$

4. $12 \div (-15) = -0.8$

5. $12 \div \left(-\frac{1}{2}\right) = -24$

6. $-\frac{1}{4} \div \left(-\frac{1}{4}\right) = 1$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can divide rational numbers.
1 2 3

b I can apply the rules for the order of operations while using all four operations with rational numbers.
1 2 3

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Lesson 13 Dividing Rational Numbers

Success looks like . . .

- **Language Goal:** Generalizing a method for determining the quotient of two rational numbers. **(Speaking and Listening)**
  - » Dividing two rational numbers in Problems 1–6.
- **Goal:** Generating a division equation that represents the same relationship as a given multiplication equation with rational numbers.
- **Goal:** Applying the order of operations while working with rational numbers.

Suggested next steps

If students use the wrong sign for the quotient in any problem, consider:

- Reviewing the rules in the Summary.

If students have difficulty with Problems 5 or 6, consider:

- Revisiting the Power-up for this lesson.

Professional Learning

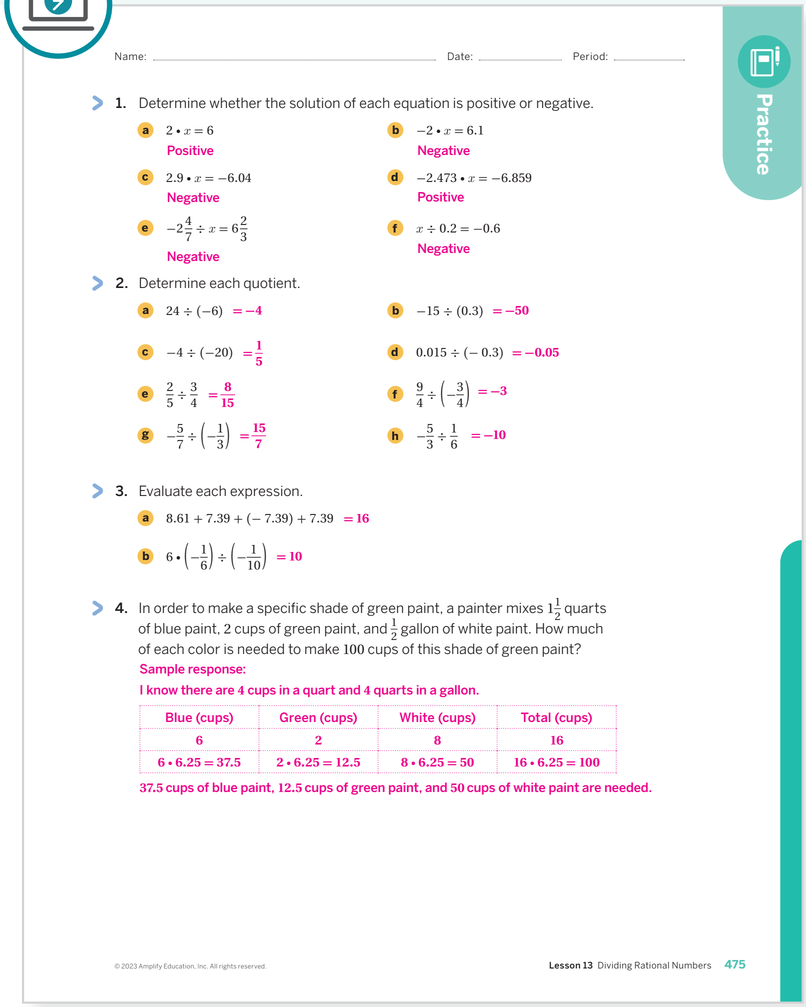
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- What was especially satisfying about seeing students create their own rule for division of rational numbers? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Determine whether the solution of each equation is positive or negative.

   a. $2 \cdot x = 6$
   Positive

   b. $-2 \cdot x = 6.1$
   Negative

   c. $2.9 \cdot x = -6.04$
   Negative

   d. $-2.473 \cdot x = -6.859$
   Positive

   e. $-2\frac{4}{7} \div x = 6\frac{2}{3}$
   Negative

   f. $x \div 0.2 = -0.6$
   Negative

2. Determine each quotient.

   a. $24 \div (-6)$ $= -4$

   b. $-15 \div (0.3)$ $= -50$

   c. $-4 \div (-20)$ $= \frac{1}{5}$

   d. $0.015 \div (-0.3)$ $= -0.05$

   e. $\frac{2}{5} \div \frac{3}{4}$ $= \frac{8}{15}$

   f. $\frac{9}{4} \div \left(-\frac{3}{4}\right)$ $= -3$

   g. $-\frac{5}{7} \div \left(-\frac{1}{3}\right)$ $= \frac{15}{7}$

   h. $-\frac{5}{3} \div \frac{1}{6}$ $= -10$

3. Evaluate each expression.

   a. $8.61 + 7.39 + (-7.39) + 7.39$ $= 16$

   b. $6 \cdot \left(-\frac{1}{6}\right) \div \left(-\frac{1}{10}\right)$ $= 10$

4. In order to make a specific shade of green paint, a painter mixes $1\frac{1}{2}$ quarts of blue paint, 2 cups of green paint, and $\frac{1}{2}$ gallon of white paint. How much of each color is needed to make 100 cups of this shade of green paint?

   Sample response:

   I know there are 4 cups in a quart and 4 quarts in a gallon.

Blue (cups)	Green (cups)	White (cups)	Total (cups)
6	2	8	16
$6 \cdot 6.25 = 37.5$	$2 \cdot 6.25 = 12.5$	$8 \cdot 6.25 = 50$	$16 \cdot 6.25 = 100$

   37.5 cups of blue paint, 12.5 cups of green paint, and 50 cups of white paint are needed.

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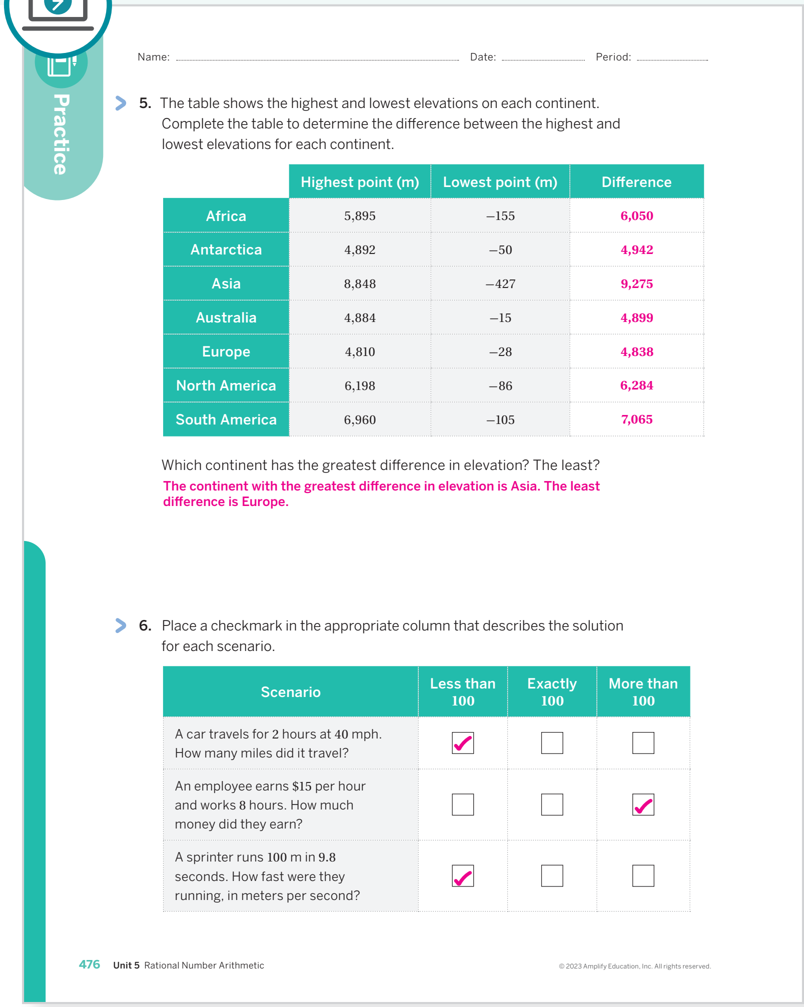
Lesson 13 Dividing Rational Numbers 475



Name: Date: Period:

Practice

5. The table shows the highest and lowest elevations on each continent. Complete the table to determine the difference between the highest and lowest elevations for each continent.

	Highest point (m)	Lowest point (m)	Difference
Africa	5,895	−155	6,050
Antarctica	4,892	−50	4,942
Asia	8,848	−427	9,275
Australia	4,884	−15	4,899
Europe	4,810	−28	4,838
North America	6,198	−86	6,284
South America	6,960	−105	7,065

   Which continent has the greatest difference in elevation? The least?

   The continent with the greatest difference in elevation is Asia. The least difference is Europe.

6. Place a checkmark in the appropriate column that describes the solution for each scenario.

Scenario	Less than 100	Exactly 100	More than 100
A car travels for 2 hours at 40 mph. How many miles did it travel?	☑	☐	☐
An employee earns $15 per hour and works 8 hours. How much money did they earn?	☐	☐	☑
A sprinter runs 100 m in 9.8 seconds. How fast were they running, in meters per second?	☑	☐	☐

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 2	2
	5	Unit 5 Lesson 3	2
Formative ⚡	6	Unit 5 Lesson 14	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 14**

Negative Rates

Let's apply what we know about rational numbers.

Focus

Goals

1. **Language Goal:** Apply operations with rational numbers to solve problems involving constant rates, and explain the solution method. **(Speaking and Listening)**
2. **Language Goal:** Explain how rational numbers can be used to represent situations involving constant rates. **(Speaking and Listening, Reading and Writing)**
3. Write an equation of the form $y = kx$ to represent a situation that involves descending at a constant rate.

Coherence

Today

Students are introduced to negative rates of change and their representations in equations and on graphs. They apply their understanding of operating with rational numbers to solve problems in context. The first problem is about drilling a water well. The second problem deals with the famous pearl divers, known as *ama*, in Japan. Through these real-world contexts, students reason quantitatively about what it means for a rate to be negative.

Previously

In Unit 2, students encountered writing equations for proportional relationships, almost exclusively with positive values.

Coming Soon

In Lesson 18, students will solve equations with negative coefficients, similar to the equations they formulate in this lesson.

Rigor

- Students build **conceptual understanding** of proportional relationships that have negative rates.
- Students **apply** their previous understanding of the constant of proportionality to include rational numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	7 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *constant of proportionality*
- *proportional relationships*
- *rate*

Amps | Featured Activity

Activity 1
Interactive Graphs

Students see negative rates in action as they plot the height of a drill over time. The digital environment allows them to plot the points on the graph and notice the direction of the line, which helps them to see what makes these graphs special.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be frustrated that the graph of a proportional relationship with a negative rate looks different than a graph of a proportional relationship with a positive rate. To alleviate some of the frustration, have students identify how the graphs are the same before concentrating on the differences. In order to completely understand why the slopes are different, students will need to use the context of the problem to reason quantitatively. By making the connection between the scenario, the numbers, and the shape of the graph, students will become more comfortable with proportional graphs with negative rates.

● Modifications to Pacing

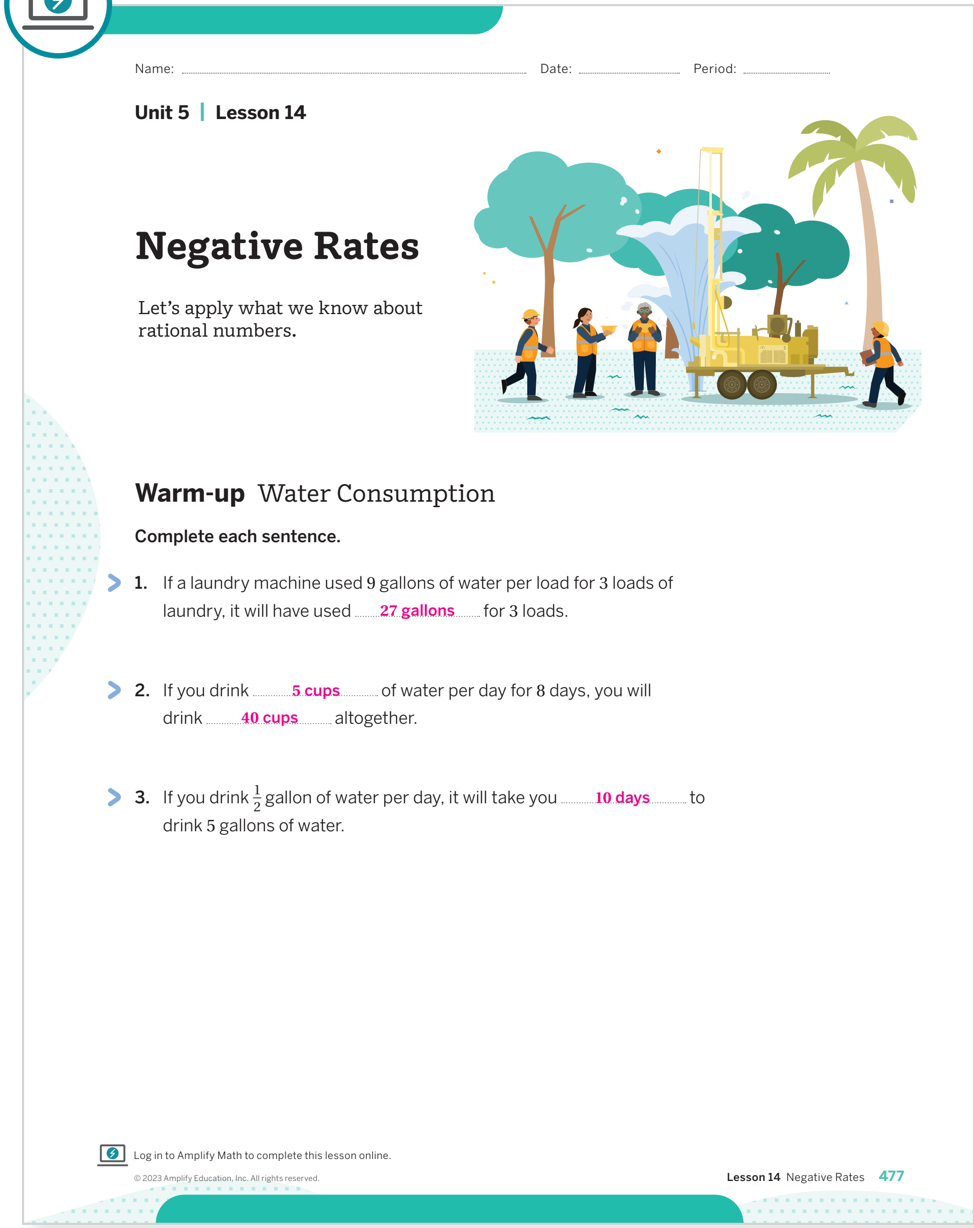
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, Problem 2 may be omitted.
- Alternatively, **Activity 1** may be omitted entirely. Instead, mention during the Activity 2 Launch that constants of proportionality can be negative.

Pairs | 5 min

Warm-up Water Consumption

Students are reacquainted with *per* language to prepare them for working with rates in this lesson.



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Unit 5 | Lesson 14

Negative Rates

Let's apply what we know about rational numbers.

Warm-up Water Consumption

Complete each sentence.

1. If a laundry machine used 9 gallons of water per load for 3 loads of laundry, it will have used 27 gallons for 3 loads.

2. If you drink 5 cups of water per day for 8 days, you will drink 40 cups altogether.

3. If you drink $\frac{1}{2}$ gallon of water per day, it will take you 10 days to drink 5 gallons of water.

Log in to Amplify Math to complete this lesson online.

Lesson 14 Negative Rates 477

1 Launch

Activate students' prior knowledge by asking, "What do you remember about proportional relationships?" Then ask, "What does the lesson title make you think of?"

2 Monitor

Help students get started by suggesting students model the relationship between gallons and loads using a table or an expression.

Look for points of confusion:

- **Thinking that for Problem 3, that they need to multiply 5 and $\frac{1}{2}$.** Ask, "How is this problem different from Problems 1 and 2?"

Look for productive strategies:

- Writing an equation to model Problems 1 and 2.

3 Connect

Display responses to each problem.

Have students share which value is the rate in each problem.

Ask:

- "If you were to write an equation to model each of these, what would they look like?"
- "How did the structure of each problem change?"

Highlight that students had been working with positive rates in prior grades, but that today's lesson will introduce an important new kind of rate — negative rates.

Power-up

To power up students' ability to reason about proportional relationships, have students complete:

Suppose a car is traveling at 40 mph.

1. Determine the missing values in the ratio table.

Hours	1	2	x
Miles	40	80	$40x$

2. Write an equation representing the number of miles y traveled in x hours. $y = 40x$

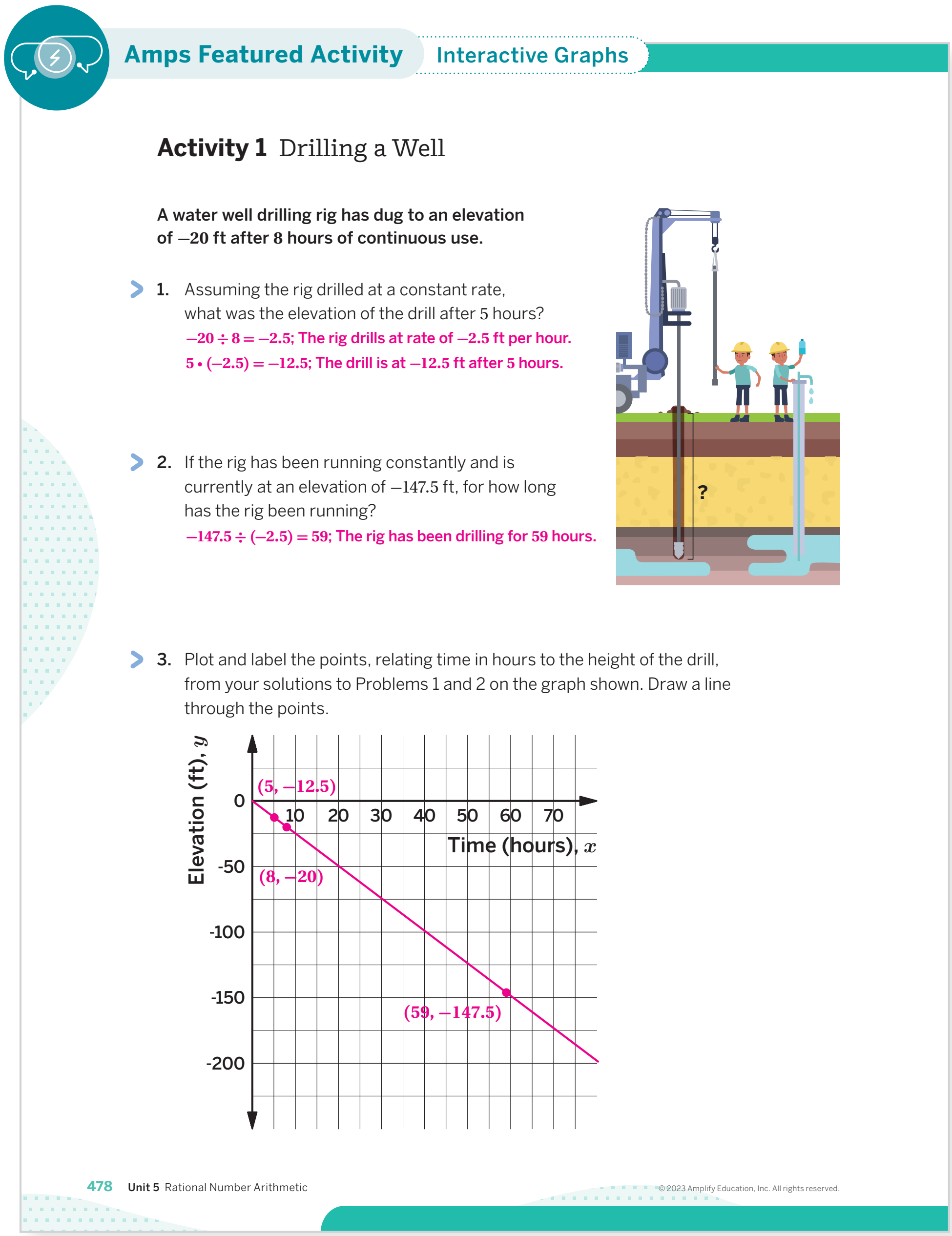
Use: Before Activity 2.
Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 15 min

Activity 1 Drilling a Well

Students use their skills of multiplying and dividing rational numbers to represent and solve problems in a new context, involving a decreasing rate.

Amps Featured Activity Interactive Graphs



Activity 1 Drilling a Well

A water well drilling rig has dug to an elevation of –20 ft after 8 hours of continuous use.

1. Assuming the rig drilled at a constant rate, what was the elevation of the drill after 5 hours?
   $-20 \div 8 = -2.5$; The rig drills at rate of –2.5 ft per hour.
   $5 \cdot (-2.5) = -12.5$; The drill is at –12.5 ft after 5 hours.

2. If the rig has been running constantly and is currently at an elevation of –147.5 ft, for how long has the rig been running?
   $-147.5 \div (-2.5) = 59$; The rig has been drilling for 59 hours.

3. Plot and label the points, relating time in hours to the height of the drill, from your solutions to Problems 1 and 2 on the graph shown. Draw a line through the points.

478 **Unit 5** Rational Number Arithmetic

1 Launch

Activate students' background knowledge by asking what they already know about drilling. Say, "Water wells are dug deep into the earth until the hole reaches the groundwater. Sometimes, it is impossible to know how deep the groundwater lies until you drill." Remind students that they can model positions below the surface with negative values.

2 Monitor

Help students get started by asking, "How might knowing the unit rate help you to solve Problem 1?"

Look for points of confusion:

- **Thinking the drill's rate is 0.4 ft per hour.** Ask, "If the drill is at –20 ft after 8 hours, is the magnitude of the rate more or less than 1 ft per hour?"
- **Using a positive number to represent the rate.** Ask, "What equation could you write to find the elevation of the drill after any amount of time?"

Look for productive strategies:

- Writing an equation or using a table to model the relationships between the elevation of the drill and the time elapsed.

3 Connect

Display a graph with the drill's elevation over various times plotted.

Have pairs of students share observations about the graph.

Ask:

- "Does this graph show a proportional relationship? How do you know?"
- "What is the constant of proportionality for this relationship?"

Highlight that, because students can now complete any calculation with any rational number, they can extend constants of proportionality, k, to include negative values.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see negative rates in action as they plot the height of a drill over time. The digital environment allows them to manipulate the points on the graph and notice the direction of the line, which helps them to see what makes these graphs special.

Accessibility: *Guide Processing and Visualization*

Suggest that students create a table of values relating the height of the drill for various times to help them respond to Problems 1 and 2, and graph the corresponding values on the graph in Problem 3.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their observations, encourage the use of mathematical language by reviewing the terms *proportional relationship* and *constant of proportionality*. Remind students of the general equation for a proportional relationship, $y = kx$, and ask:

- "What is the value of k in this context?" –2.5
- "What does it mean, in this context, that the constant of proportionality is negative?" The drill is below the ground.

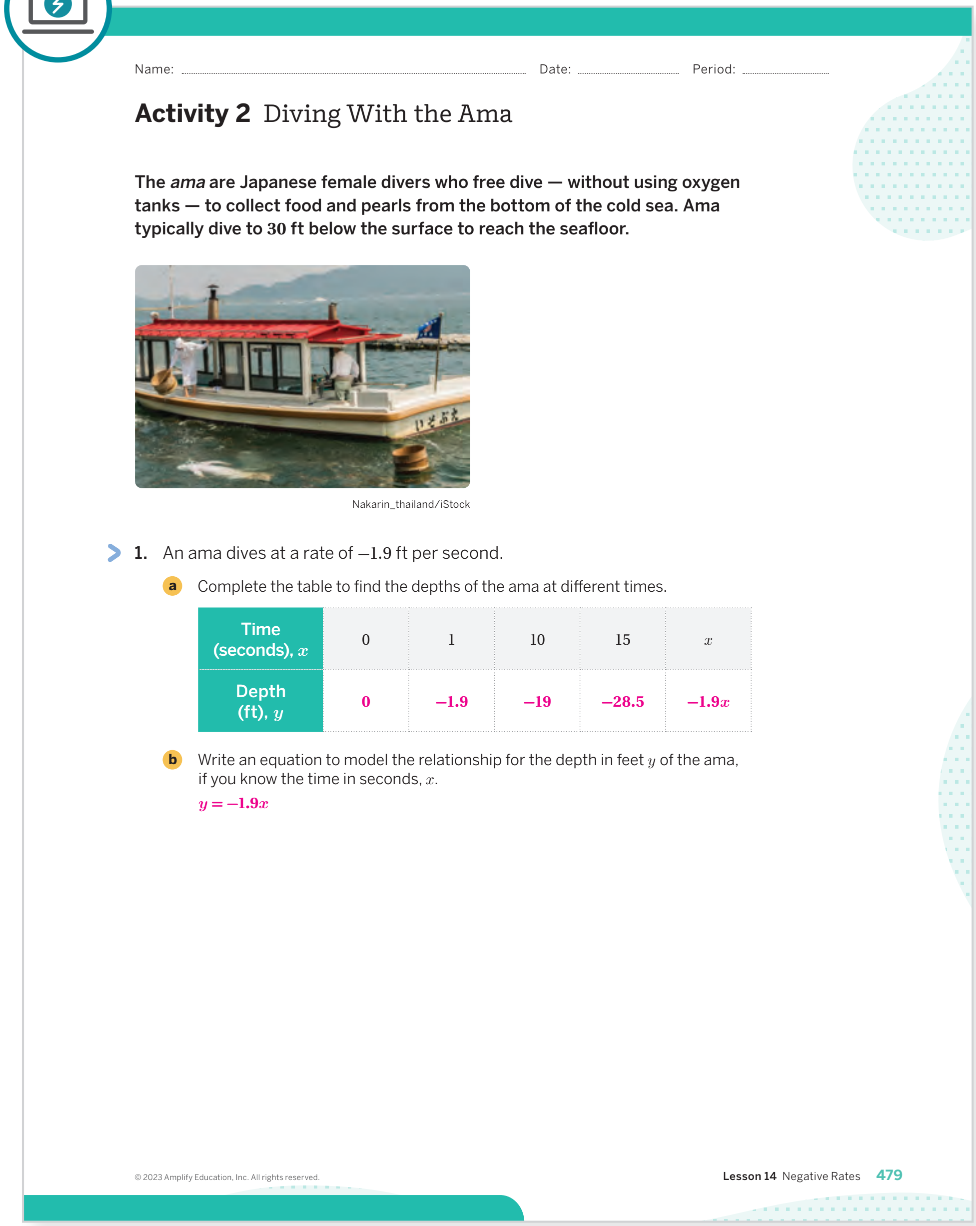
English Learners

Clarify the meaning of continuous use in the introductory text. Tell students that this means the drilling rig did not stop during these 8 hours.

Pairs | 15 min

Activity 2 Diving With the Ama

Students model the rate of a Japanese pearl diver to build on the previous work with proportional relationships and understanding of multiplying and dividing rational numbers.



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Activity 2 Diving With the Ama

The *ama* are Japanese female divers who free dive — without using oxygen tanks — to collect food and pearls from the bottom of the cold sea. Ama typically dive to 30 ft below the surface to reach the seafloor.

Nakarin_thailand/iStock

1. An ama dives at a rate of -1.9 ft per second.

   a Complete the table to find the depths of the ama at different times.

Time (seconds), x	0	1	10	15	x
Depth (ft), y	0	-1.9	-19	-28.5	$-1.9x$

   b Write an equation to model the relationship for the depth in feet y of the ama, if you know the time in seconds, x.

   $y = -1.9x$

1 Launch

Activate students' background knowledge by asking, "How deep can you dive? How long can you hold your breath underwater?" Read through the introduction as a class. Let students know that, for the purpose of this activity, they will assume the divers are moving at a constant rate.

2 Monitor

Help students get started by saying that the ama is at sea level at the start of the dive. Ask, "What number is usually used to represent sea level?"

Look for points of confusion:

- **Not knowing how to write an equation.** Suggest students use the form $y = kx$, where k is the unit rate.

Look for productive strategies:

- Scaling up from the unit rate to calculate other columns in the table.
- Using the unit rate from the table to help write the equation for the relationship.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students use the rate -2 ft per second, instead of -1.9 ft per second, to aid their calculations. By doing so, they will still be able to access the targeted goal of the activity.

Extension: *Math Enrichment*

Ask students to explain how to use the table, graph, and equation to determine the depth of the ama after 6 seconds. Sample response: Scale the table from -1.9 ft at 1 second by multiplying both values by 6. In the equation, substitute 6 for x. On the graph, draw a line to connect the points and estimate the coordinates of a point on the line whose x-coordinate is 6.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask questions, demonstrate the use of mathematical language by reviewing the meanings of the word *descending*. Connect this to the negative unit rate and have students recall that for a proportional relationship, *the unit* rate has the same value as the *constant of proportionality*.

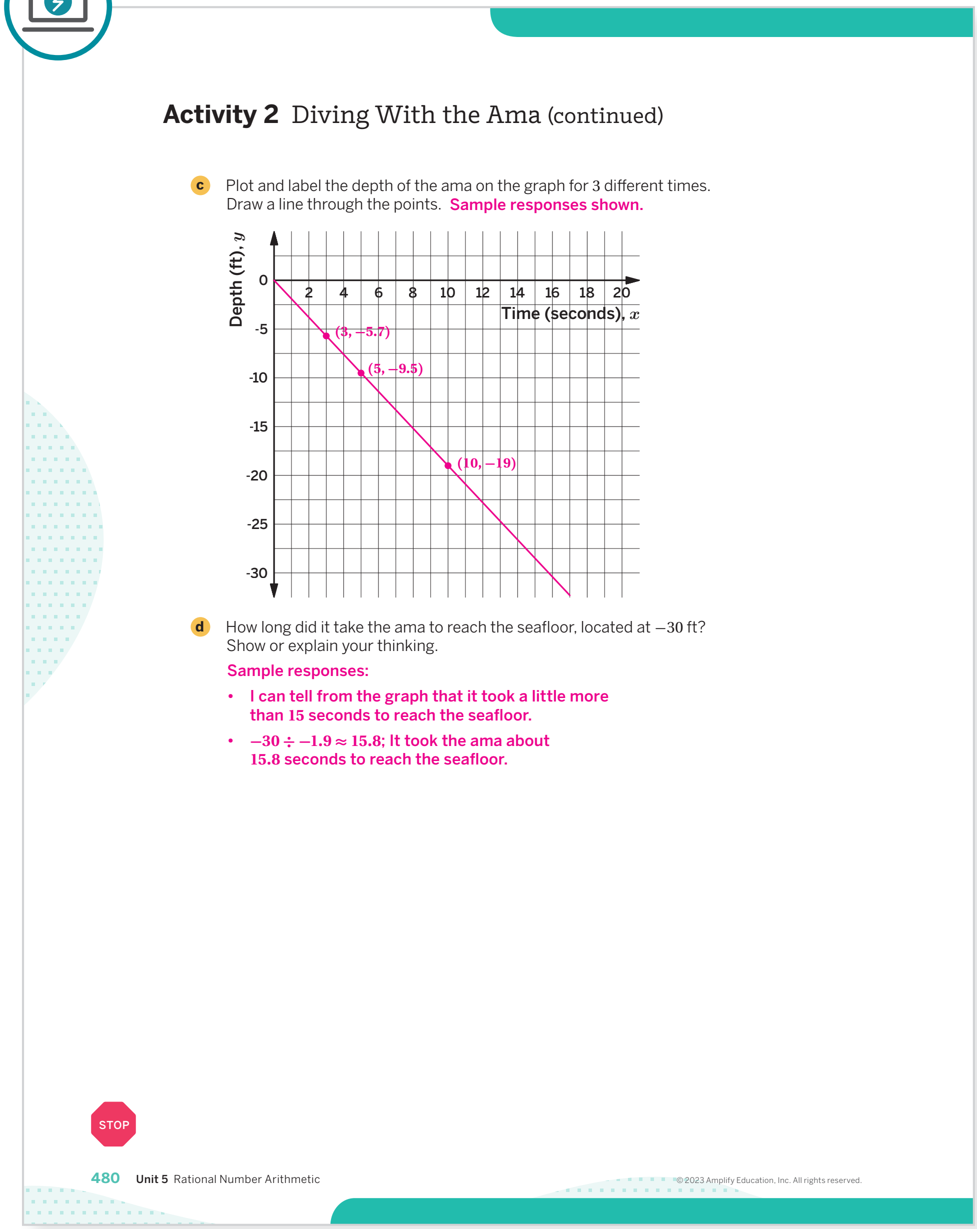
English Learners

Use pointing gestures to illustrate where the table, graph, and equation indicate that the ama are descending.

Pairs | 15 min

Activity 2 Diving With the Ama (continued)

Students model the rate of a Japanese pearl diver to build on the previous work with proportional relationships and understanding of multiplying and dividing rational numbers.



Activity 2 Diving With the Ama (continued)

c Plot and label the depth of the ama on the graph for 3 different times. Draw a line through the points. Sample responses shown.

d How long did it take the ama to reach the seafloor, located at -30 ft? Show or explain your thinking.

Sample responses:

- I can tell from the graph that it took a little more than 15 seconds to reach the seafloor.
- $-30 \div -1.9 \approx 15.8$; It took the ama about 15.8 seconds to reach the seafloor.

STOP

3 Connect

Display the graph with the equation written next to the line on the graph.

Have students share their solutions for part d. Select students who can share strategies that used both the graph and the equation to reason about the solution for part d.

Highlight that a graph of a proportional relationship, even where the value of k is negative, will still be a straight line that passes through the origin.

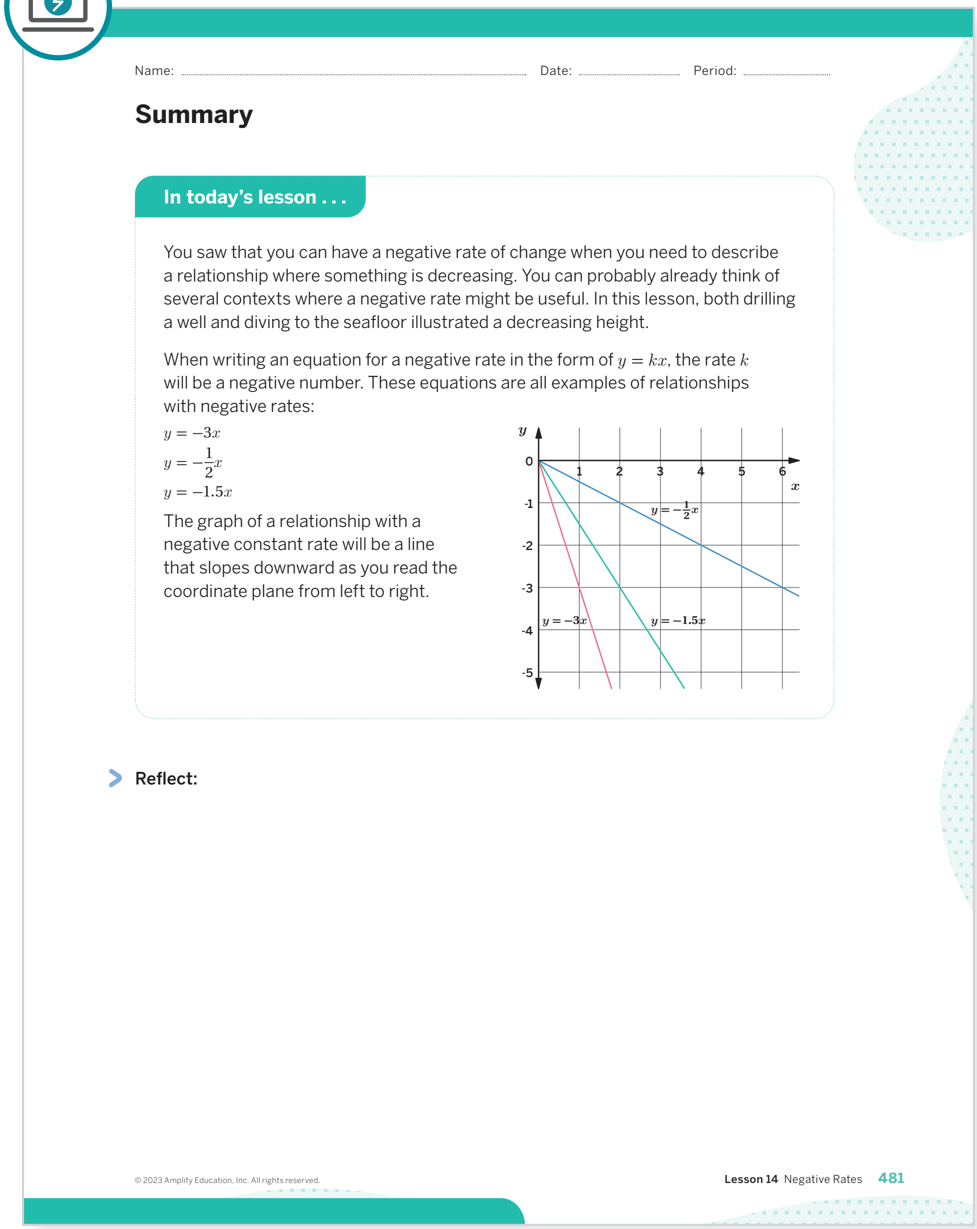
Ask:

- "How can you tell from the graph that the ama are descending?" The graph of the line is moving downward.
- "How can you tell from the equation that the ama are descending?" The equation has a negative sign.
- "Where on the graph can you find the constant of proportionality? $(1, -1.9)$ Where can you find it in the equation?" The number -1.9 in front of x
- "What is different about the graph of a positive proportional relationship from a negative proportional relationship?"

Whole Class | 5 min

Summary

Review and synthesize how negative rates appear in equations and graphs.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that you can have a negative rate of change when you need to describe a relationship where something is decreasing. You can probably already think of several contexts where a negative rate might be useful. In this lesson, both drilling a well and diving to the seafloor illustrated a decreasing height.

When writing an equation for a negative rate in the form of $y = kx$, the rate k will be a negative number. These equations are all examples of relationships with negative rates:

$y = -3x$

$y = -\frac{1}{2}x$

$y = -1.5x$

The graph of a relationship with a negative constant rate will be a line that slopes downward as you read the coordinate plane from left to right.

Reflect:

Lesson 14 Negative Rates 481

Synthesize

Display the Summary from the Student Edition.

Have students share observations about the equations and graphed lines.

Ask, "What do the three lines have in common? What makes them different?"

Highlight that velocity is used to represent speed with the added component of direction, using rational numbers. This is also true for vertical movement (in fact with any rate). In an equation, this negative value will appear as the constant of proportionality, as it did with positive proportional relationships. When relationships with negative constants of proportionality are represented on graphs, the lines slope downward as the coordinate plane is read from left to right.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

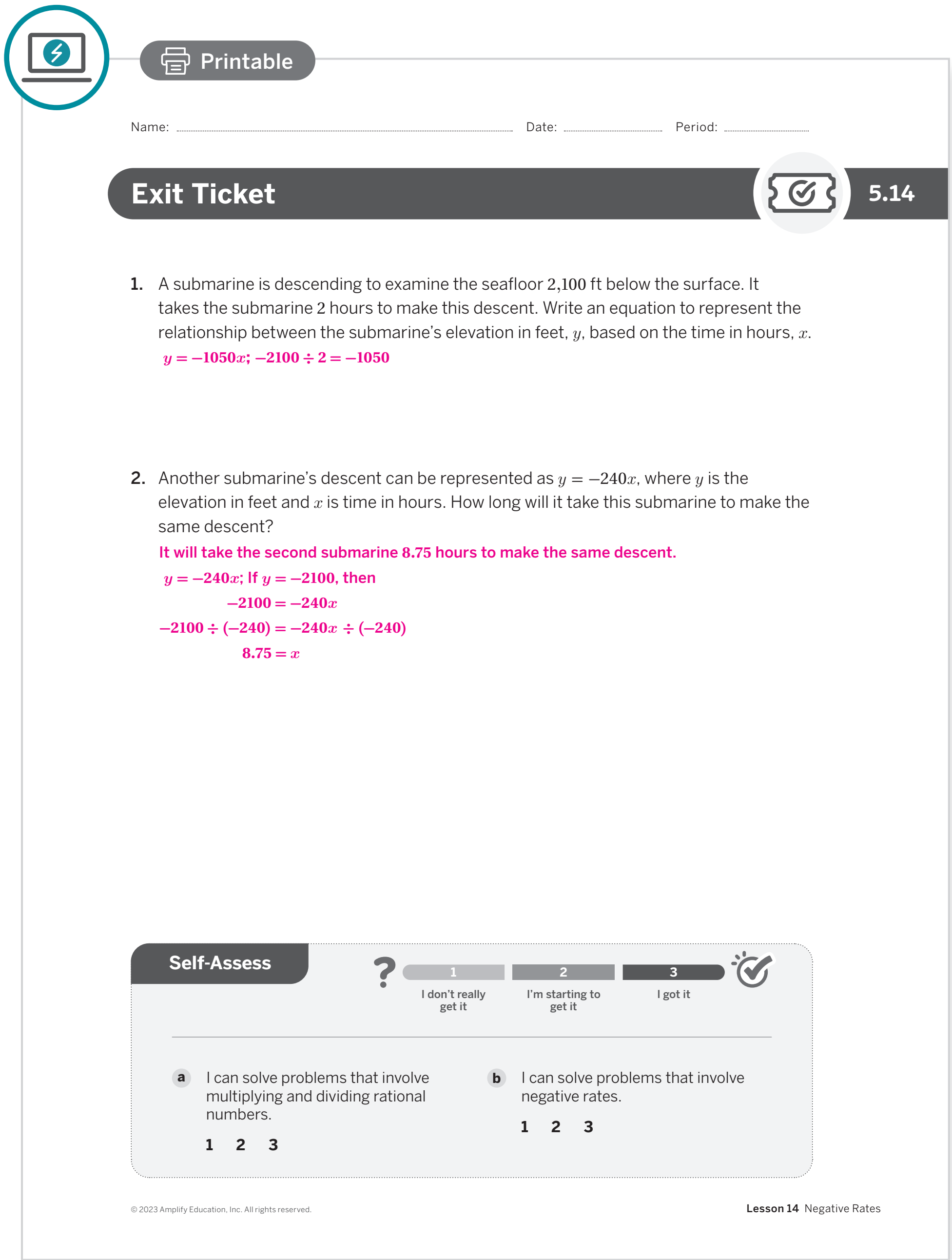
- "What other rates have you encountered where it makes sense to have positive and negative values?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of negative rates of change by solving problems involving a vehicle descending at a constant rate.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.14

1. A submarine is descending to examine the seafloor 2,100 ft below the surface. It takes the submarine 2 hours to make this descent. Write an equation to represent the relationship between the submarine's elevation in feet, y, based on the time in hours, x.

   $y = -1050x$; $-2100 \div 2 = -1050$

2. Another submarine's descent can be represented as $y = -240x$, where y is the elevation in feet and x is time in hours. How long will it take this submarine to make the same descent?

   It will take the second submarine 8.75 hours to make the same descent.

   $y = -240x$; If $y = -2100$, then

   $$-2100 = -240x$$
   $$-2100 \div (-240) = -240x \div (-240)$$
   $$8.75 = x$$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can solve problems that involve multiplying and dividing rational numbers.

1 2 3

b. I can solve problems that involve negative rates.

1 2 3

Success looks like . . .

- **Language Goal:** Applying operations with rational numbers to solve problems involving constant rates, and explaining the solution method. **(Speaking and Listening)**
  - » Solving problems about the descent of submarines in Problems 1 and 2.
- **Language Goal:** Explaining how rational numbers can be used to represent situations involving constant rates. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Writing an equation of the form $y = kx$ to represent a situation that involves descending at a constant rate.

Suggested next steps

If students use –2,100 as the constant of proportionality, consider:

- Reviewing how to determine a unit rate.
- Assigning Practice Problem 3.

If students multiply –240 by –2,100 to solve Problem 2, consider:

- Asking, "What does the variable y represent in the equation? What does the variable x represent? Where should you substitute 2,100?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier this year on proportional reasoning, what similarities and differences do you see?
- How was Activity 2 similar to or different from graphing proportional relationships in Unit 2? What might you change for the next time you teach this lesson?

Math Language Development

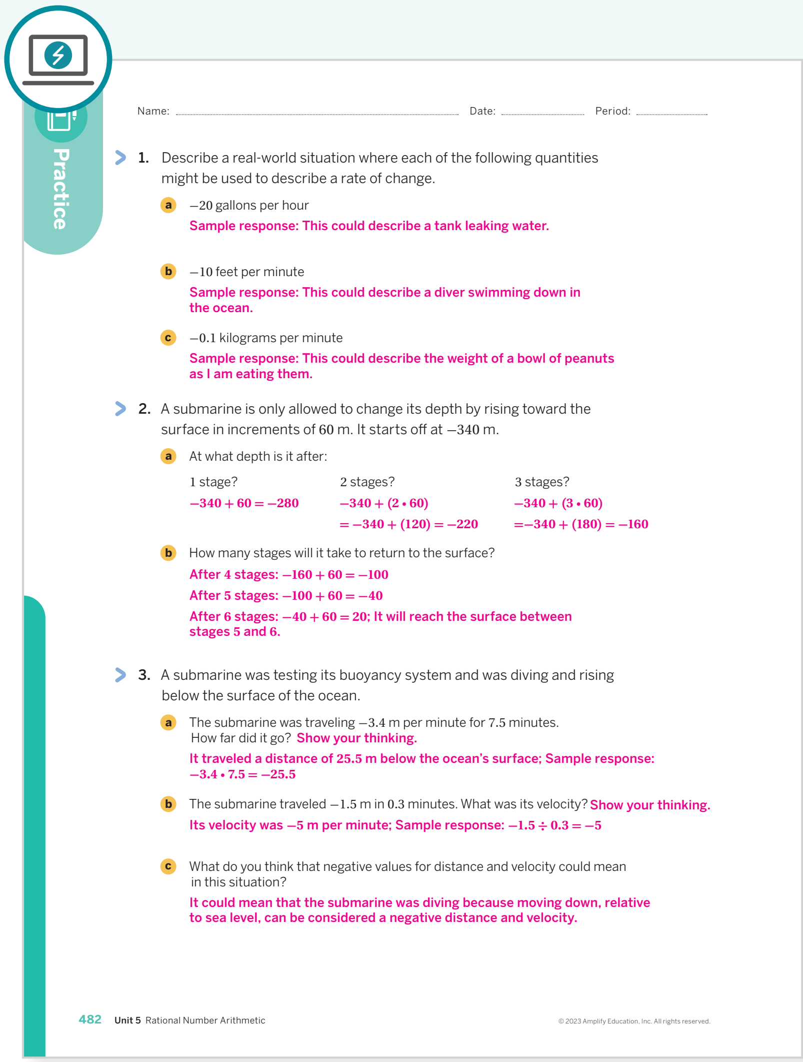
Language Goal: Explaining how rational numbers can be used to represent situations involving rates.

Reflect on students' language development toward this goal.

- How did using the ***Discussion Supports*** routines in Activities 1 and 2 help students use mathematical language, such as *proportional relationship, constant of proportionality,* and *unit rate?*
- How did these routines support their understanding of what a negative constant of proportionality or negative unit rate means in context?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. Describe a real-world situation where each of the following quantities might be used to describe a rate of change.

   a. -20 gallons per hour

   Sample response: This could describe a tank leaking water.

   b. -10 feet per minute

   Sample response: This could describe a diver swimming down in the ocean.

   c. -0.1 kilograms per minute

   Sample response: This could describe the weight of a bowl of peanuts as I am eating them.

2. A submarine is only allowed to change its depth by rising toward the surface in increments of 60 m. It starts off at -340 m.

   a. At what depth is it after:

1 stage?	2 stages?	3 stages?
$-340 + 60 = -280$	$-340 + (2 \cdot 60)$ $= -340 + (120) = -220$	$-340 + (3 \cdot 60)$ $=-340 + (180) = -160$

   b. How many stages will it take to return to the surface?

   After 4 stages: $-160 + 60 = -100$

   After 5 stages: $-100 + 60 = -40$

   After 6 stages: $-40 + 60 = 20$; It will reach the surface between stages 5 and 6.

3. A submarine was testing its buoyancy system and was diving and rising below the surface of the ocean.

   a. The submarine was traveling -3.4 m per minute for 7.5 minutes. How far did it go? **Show your thinking.**

   It traveled a distance of 25.5 m below the ocean's surface; Sample response: $-3.4 \cdot 7.5 = -25.5$

   b. The submarine traveled -1.5 m in 0.3 minutes. What was its velocity? **Show your thinking.**

   Its velocity was -5 m per minute; Sample response: $-1.5 \div 0.3 = -5$

   c. What do you think that negative values for distance and velocity could mean in this situation?

   It could mean that the submarine was diving because moving down, relative to sea level, can be considered a negative distance and velocity.

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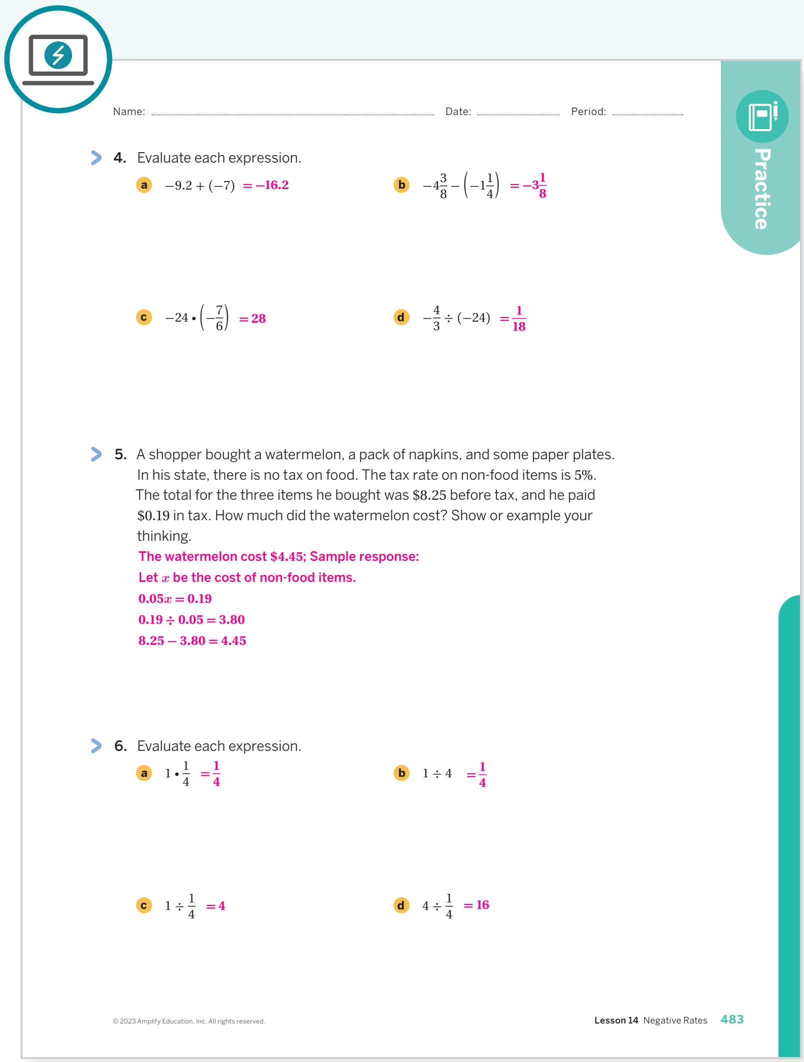

Name: ________ Date: ________ Period: ________

Practice

4. Evaluate each expression.

   a. $-9.2 + (-7)$ **$= -16.2$**

   b. $-4\frac{3}{8} - \left(-1\frac{1}{4}\right)$ **$= -3\frac{1}{8}$**

   c. $-24 \cdot \left(-\frac{7}{6}\right)$ **$= 28$**

   d. $-\frac{4}{3} \div (-24)$ **$= \frac{1}{18}$**

5. A shopper bought a watermelon, a pack of napkins, and some paper plates. In his state, there is no tax on food. The tax rate on non-food items is 5%. The total for the three items he bought was \$8.25 before tax, and he paid \$0.19 in tax. How much did the watermelon cost? Show or example your thinking.

   The watermelon cost \$4.45; Sample response:

   Let x be the cost of non-food items.

   $0.05x = 0.19$

   $0.19 \div 0.05 = 3.80$

   $8.25 - 3.80 = 4.45$

6. Evaluate each expression.

   a. $1 \cdot \frac{1}{4}$ **$= \frac{1}{4}$**

   b. $1 \div 4$ **$= \frac{1}{4}$**

   c. $1 \div \frac{1}{4}$ **$= 4$**

   d. $4 \div \frac{1}{4}$ **$= 16$**

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 13	1
	5	Unit 4 Lesson 8	2
Formative	6	Unit 5 Lesson 15	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

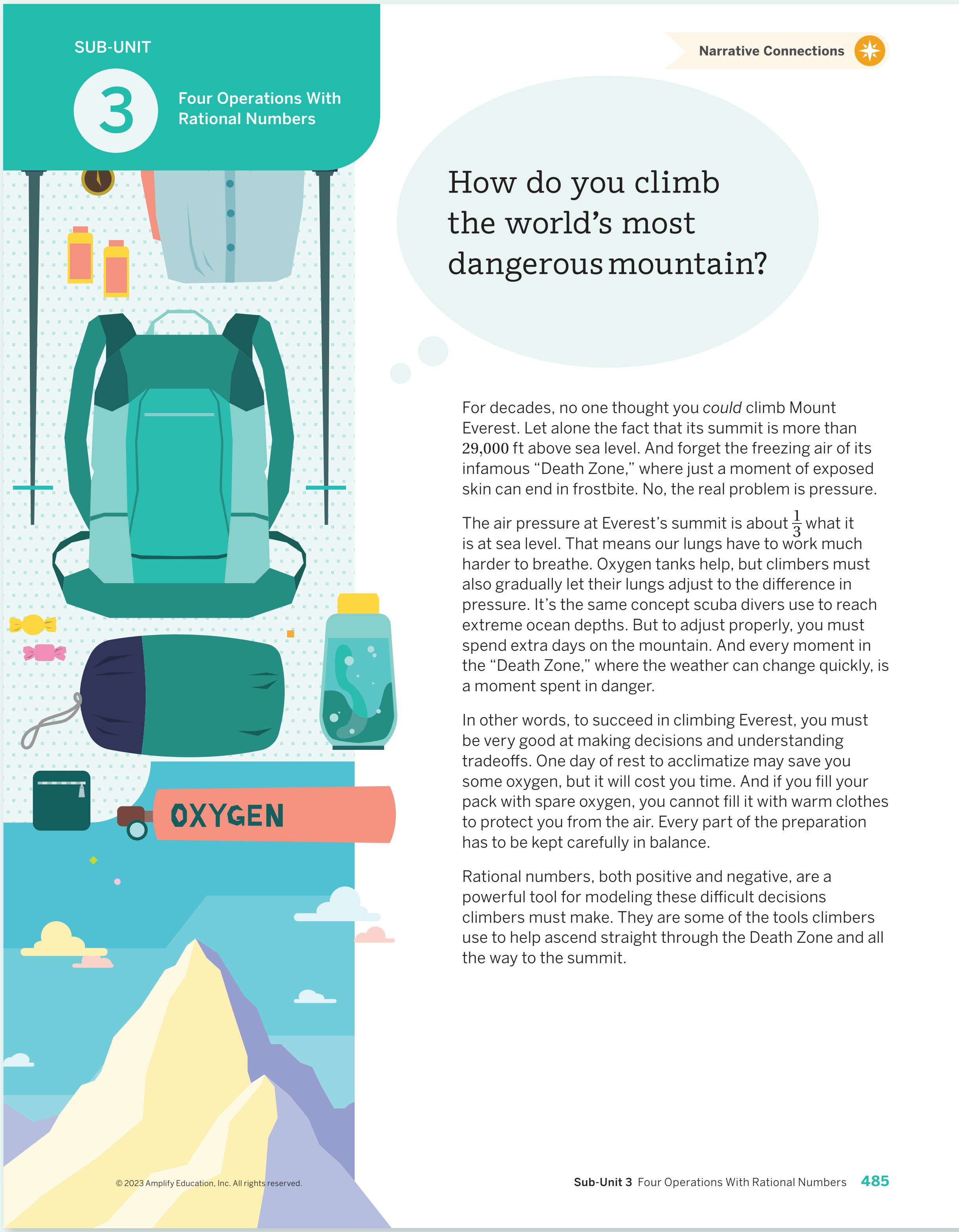
For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 3

Four Operations With Rational Numbers

Whole Class

In this Sub-Unit, students apply their understanding of the four operation on rational numbers to solve problems involving more than one operation as well as to solve equations with rational coefficients or addends.



SUB-UNIT

3 Four Operations With Rational Numbers

Narrative Connections

How do you climb the world's most dangerous mountain?

For decades, no one thought you *could* climb Mount Everest. Let alone the fact that its summit is more than 29,000 ft above sea level. And forget the freezing air of its infamous "Death Zone," where just a moment of exposed skin can end in frostbite. No, the real problem is pressure.

The air pressure at Everest's summit is about $\frac{1}{3}$ what it is at sea level. That means our lungs have to work much harder to breathe. Oxygen tanks help, but climbers must also gradually let their lungs adjust to the difference in pressure. It's the same concept scuba divers use to reach extreme ocean depths. But to adjust properly, you must spend extra days on the mountain. And every moment in the "Death Zone," where the weather can change quickly, is a moment spent in danger.

In other words, to succeed in climbing Everest, you must be very good at making decisions and understanding tradeoffs. One day of rest to acclimatize may save you some oxygen, but it will cost you time. And if you fill your pack with spare oxygen, you cannot fill it with warm clothes to protect you from the air. Every part of the preparation has to be kept carefully in balance.

Rational numbers, both positive and negative, are a powerful tool for modeling these difficult decisions climbers must make. They are some of the tools climbers use to help ascend straight through the Death Zone and all the way to the summit.

Sub-Unit 3 Four Operations With Rational Numbers 485

Narrative Connections

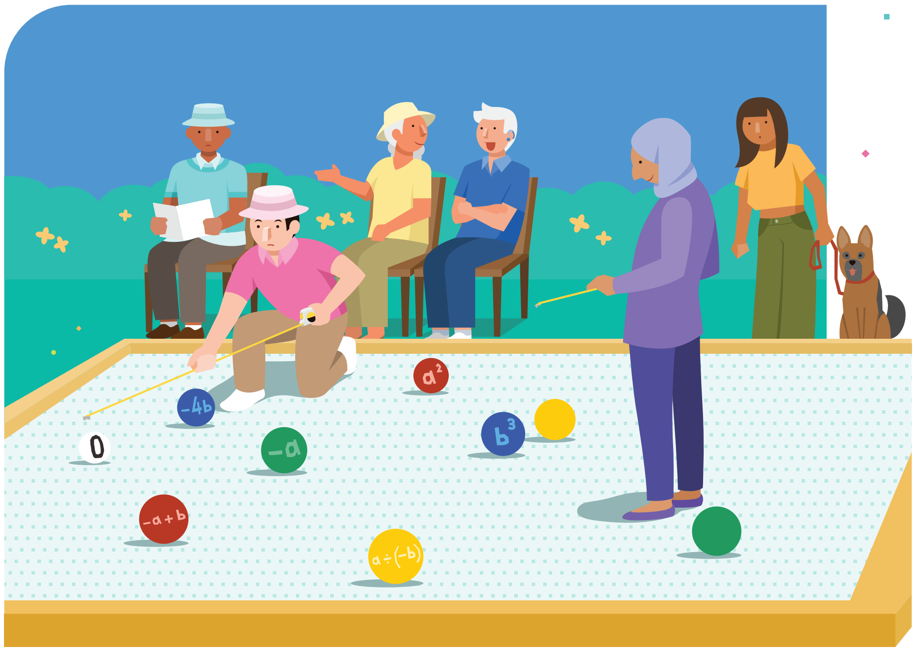
Read the narrative aloud as a class or have students read it individually. Students continue to explore connections between rational numbers and hiking or recreational trails in the following places:

- **Lesson 17, Activities 1-2:** Energy Supply, Revisited, Deep Ocean Exploration
- **Lesson 19, Activity 2:** Changing Elevation
- **Lesson 20, Activities 1-2:** Making Preparations, The Summit Attempt

UNIT 5 | **LESSON 15**

Expressions With Rational Numbers

Let’s develop our number sense with rational numbers.



Focus

Goals

1. **Language Goal:** Evaluate an expression for given values of the variable, including negative values, and compare the resulting values of the expression. **(Speaking and Listening)**
2. **Language Goal:** Generalize about the relationship between additive inverses and about the relationship between multiplicative inverses. **(Speaking and Listening)**
3. **Language Goal:** Identify numerical expressions that are equal, and justify that they are equal. **(Speaking and Listening)**

Coherence

Today

The purpose of this lesson is to help students make sense of expressions, such as whether a number is positive or negative, which of two numbers is greater, or whether two expressions represent the same number. Students work through common misconceptions that can arise about expressions involving variables, for example the misconception that $-x$ must always be a negative number. When students look at a numerical expression and see without calculation that it must be positive because it is a product of two negative numbers, they are making use of structure.

Previously

In Lesson 4, students related the lengths of school supplies to algebraic expressions and noticed that certain relationships stayed the same no matter the values of the variables, but found other relationships change.

Coming Soon

In Lesson 16, students will revisit long division in order to represent fractions as decimals.

Rigor

- Students build **conceptual understanding** of inverse operations.
- Students develop **fluency** with operations involving rational numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair

Math Language Development

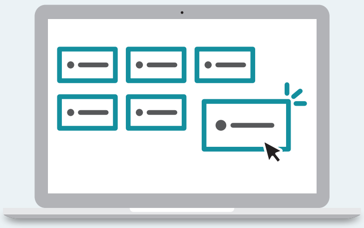
Review words

- *additive inverse*
- *inverse operations*
- *multiplicative inverse*

Amps | Featured Activity

Activity 1
Digital Card Sort

Digital card sorts save set-up time, allowing students to spend more time thinking about and discussing the relationships among the expressions on the cards.



Building Math Identity and Community

Connecting to Mathematical Practices

As students have been building understanding of operating with rational numbers, they might not feel completely confident in their ability to do so in Activity 1. In order to gain confidence, students need to focus on the structure of the signs for each operation. The more easily they can verbalize these patterns, the more confident they will feel.

Modifications to Pacing

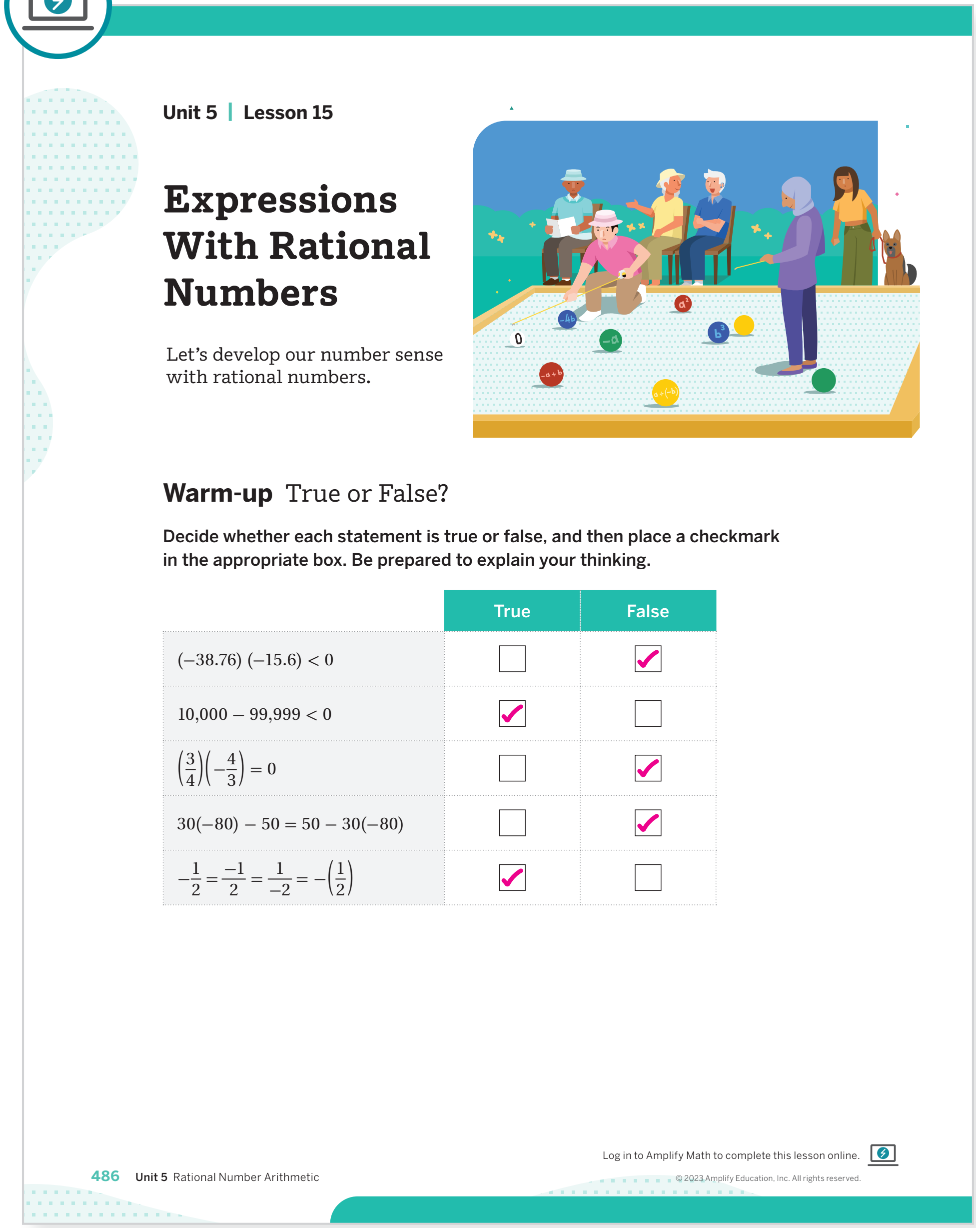
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, Problem 4 may be omitted.

Pairs | 5 min

Warm-up True or False?

Students reason about numeric expressions using what they know about operations with negative and positive numbers.



Unit 5 | Lesson 15

Expressions With Rational Numbers

Let's develop our number sense with rational numbers.

Warm-up True or False?

Decide whether each statement is true or false, and then place a checkmark in the appropriate box. Be prepared to explain your thinking.

	True	False
$(-38.76)\ (-15.6) < 0$	☐	☑
$10{,}000 - 99{,}999 < 0$	☑	☐
$\left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 0$	☐	☑
$30(-80) - 50 = 50 - 30(-80)$	☐	☑
$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2} = -\left(\frac{1}{2}\right)$	☑	☐

Log in to Amplify Math to complete this lesson online.

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1 Launch

Display one problem at a time. Give students 1 minute of think-time per problem and ask them to give a signal when they have completed each problem. Remind students that sometimes the multiplication dot is not included before parentheses and that it is assumed the operation is multiplication.

2 Monitor

Help students get started by asking, "How else could you refer to *any number that is less than zero*?"

Look for points of confusion:

- **Not using the order of operations properly in Row 4.** Ask, "Which operation — multiplication or subtraction — comes first in the order of operations?"

Look for productive strategies:

- Thinking of the fractions in Row 5 as division problems.

3 Connect

Display the statement in Row 5 for all to see.

Have students share what all the fractional expressions have in common.

Highlight that students can think of any fractional expression as a division problem. If each of these fractions were rewritten as division, they would see that the end result will always be a negative number. Because the magnitude of all the fractions is the same, they are all equivalent.

Ask, "How could you alter each false expression to make it true?"

Power-up

To power up students' ability to divide with fractions, have students complete:

Recall that division expressions can be rewritten as multiplying by the reciprocal. For example, $3 \div \frac{3}{2} = 3 \cdot \frac{2}{3}$.

Rewrite each division expression as an equivalent multiplication expression.

1. $8 \div 4 = 8 \cdot \frac{1}{4}$
2. $9 \div \frac{1}{3} = 9 \cdot 3$
3. $\frac{2}{3} \div 3 = \frac{2}{3} \cdot \frac{1}{3}$
4. $\frac{1}{8} \div \frac{1}{4} = \frac{1}{8} \cdot 4$

Use: Before Activity 1.

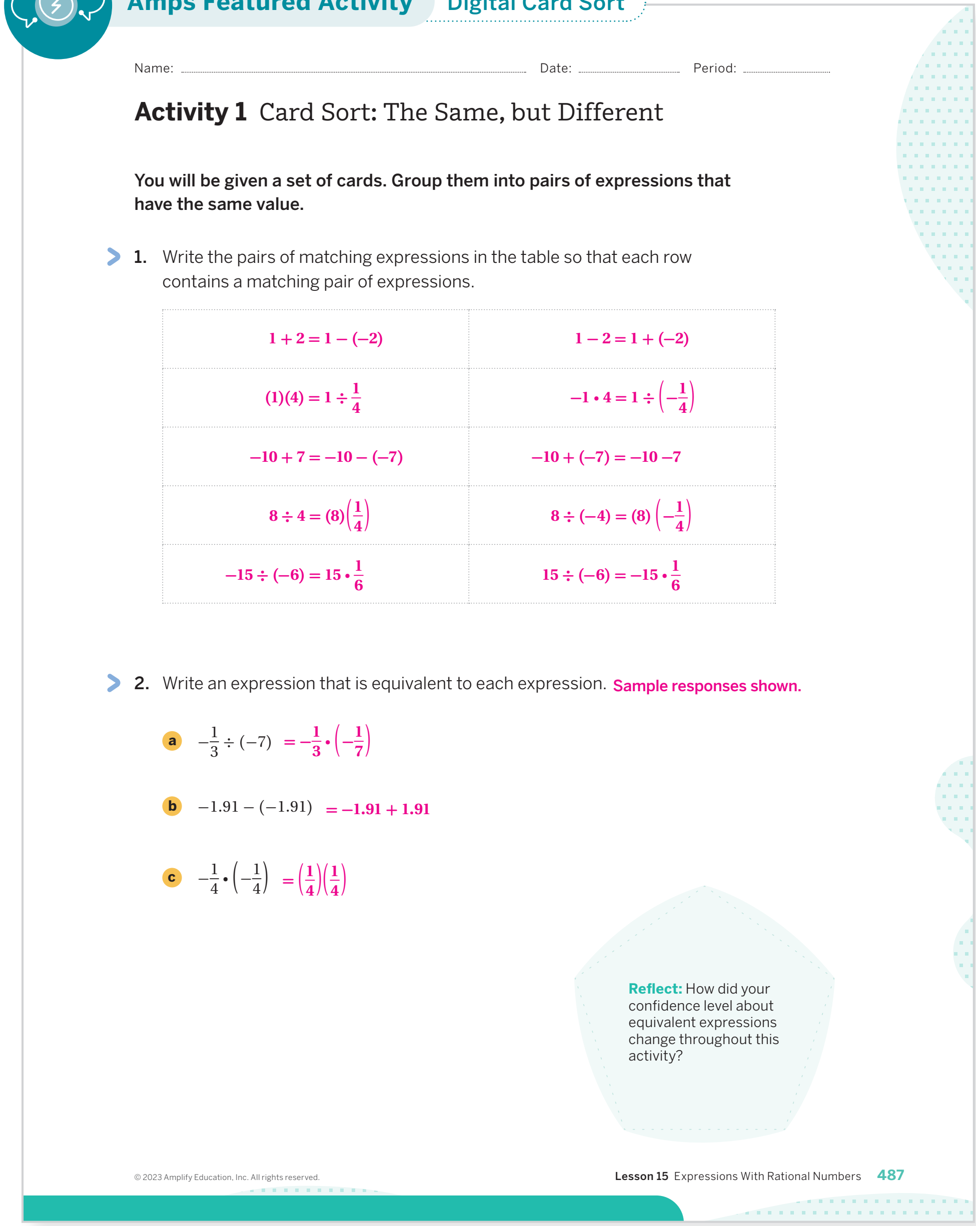
Informed by: Performance on Lesson 14, Practice Problem 6.

Pairs | 15 min

Activity 1 Card Sort: The Same, but Different

Students match different expressions that have the same value to build fluency operating with signed numbers.

Amps Featured Activity Digital Card Sort



Name: ______ Date: ______ Period: ______

Activity 1 Card Sort: The Same, but Different

You will be given a set of cards. Group them into pairs of expressions that have the same value.

1. Write the pairs of matching expressions in the table so that each row contains a matching pair of expressions.

$1 + 2 = 1 - (-2)$	$1 - 2 = 1 + (-2)$
$(1)(4) = 1 \div \frac{1}{4}$	$-1 \cdot 4 = 1 \div \left(-\frac{1}{4}\right)$
$-10 + 7 = -10 - (-7)$	$-10 + (-7) = -10 - 7$
$8 \div 4 = (8)\left(\frac{1}{4}\right)$	$8 \div (-4) = (8)\left(-\frac{1}{4}\right)$
$-15 \div (-6) = 15 \cdot \frac{1}{6}$	$15 \div (-6) = -15 \cdot \frac{1}{6}$

2. Write an expression that is equivalent to each expression. Sample responses shown.

a $-\frac{1}{3} \div (-7) = -\frac{1}{3} \cdot \left(-\frac{1}{7}\right)$

b $-1.91 - (-1.91) = -1.91 + 1.91$

c $-\frac{1}{4} \cdot \left(-\frac{1}{4}\right) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$

Reflect: How did your confidence level about equivalent expressions change throughout this activity?

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Lesson 15 Expressions With Rational Numbers 487

1 Launch

Distribute sets of cards from the Activity 1 PDF to each pair of students. Let students know the sets of matching expressions do not need to be in any particular order in the table.

2 Monitor

Help students get started by having them focus on one set of values first (for example, the cards with 1 and 2 on them).

Look for points of confusion:

- **Struggling to determine matches.** Encourage them to think of the operations in different ways. Ask, "How else can you think of (addition, subtraction, multiplication, division)?"

Look for productive strategies:

- Grouping cards into sets that have similar values on them.

3 Connect

Display the matching sets of expressions.

Ask, "What patterns do you see in the matched sets of expressions?" One pattern I see is that division problems can be rewritten as multiplication problems by changing the operation to multiplcation and using the multiplicative inverse of the divisor.

Have students share their responses for Problem 2 with a partner. Have them discuss until they agree that all expressions are, in fact, equivalent to the original.

Highlight that students can rewrite a division problem as a multiplication problem if they change the divisor to the multiplicative inverse (or reciprocal). They can also change a subtraction problem into an addition problem if they change the number being subtracted into its additive inverse (or opposite).

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can spend more time thinking about and discussing the relationships among the expressions on the cards. Digital card sorts save set-up time.

Accessibility: *Vary Demands to Optimize Challenge*

Distribute Cards 1–10 first and have students work with this subset of cards to determine any possible matches. After they have determined all possible matches, distribute the remaining cards.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share the patterns they see in the matched expressions, highlight the inverse relationships between operations. Display these sentence frames and have students complete them. Add these statements to the class display.

- Subtracting a number is the same as _____ the _____. adding; additive inverse (or opposite)
- Dividing by a number is the same as _____ by that number's _____. multiplying; multiplicative inverse (or reciprocal)

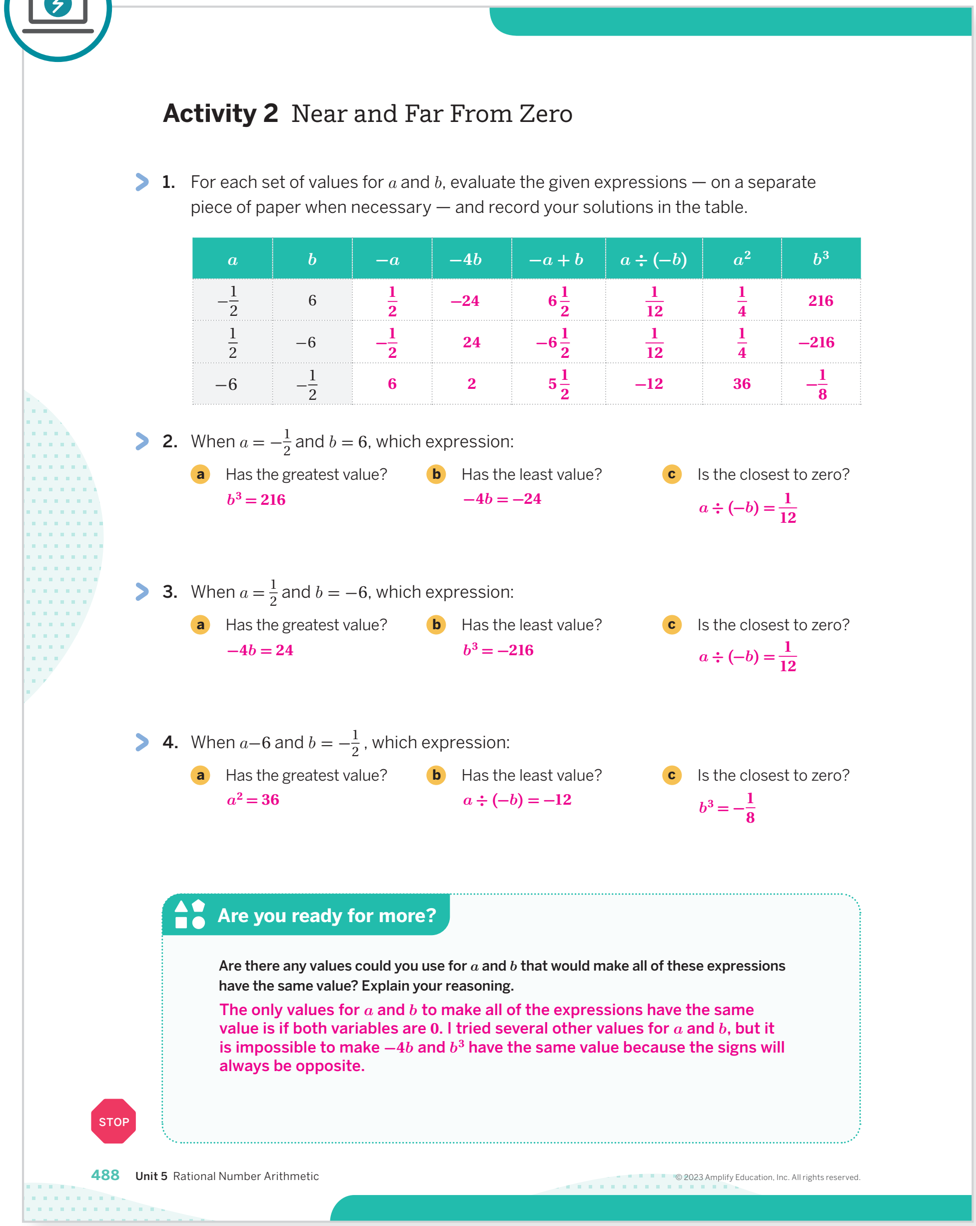
English Learners

Add examples to the class display and annotate the additive inverse and multiplicative inverse with "additive inverse or opposite" and "multiplicative inverse or reciprocal."

Pairs | 15 min

Activity 2 Near and Far From Zero

Students see that when comparing two expressions with the same variable, it is not possible to know which expression is larger or smaller (without knowing the values of the variables).



Activity 2 Near and Far From Zero

1. For each set of values for a and b, evaluate the given expressions — on a separate piece of paper when necessary — and record your solutions in the table.

a	b	$-a$	$-4b$	$-a+b$	$a \div (-b)$	a^2	b^3
$-\frac{1}{2}$	6	$\frac{1}{2}$	-24	$6\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{4}$	216
$\frac{1}{2}$	-6	$-\frac{1}{2}$	24	$-6\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{4}$	-216
-6	$-\frac{1}{2}$	6	2	$5\frac{1}{2}$	-12	36	$-\frac{1}{8}$

2. When $a = -\frac{1}{2}$ and $b = 6$, which expression:

   a. Has the greatest value? $b^3 = 216$

   b. Has the least value? $-4b = -24$

   c. Is the closest to zero? $a \div (-b) = \frac{1}{12}$

3. When $a = \frac{1}{2}$ and $b = -6$, which expression:

   a. Has the greatest value? $-4b = 24$

   b. Has the least value? $b^3 = -216$

   c. Is the closest to zero? $a \div (-b) = \frac{1}{12}$

4. When a–6 and $b = -\frac{1}{2}$, which expression:

   a. Has the greatest value? $a^2 = 36$

   b. Has the least value? $a \div (-b) = -12$

   c. Is the closest to zero? $b^3 = -\frac{1}{8}$

Are you ready for more?

Are there any values could you use for a and b that would make all of these expressions have the same value? Explain your reasoning.

The only values for a and b to make all of the expressions have the same value is if both variables are 0. I tried several other values for a and b, but it is impossible to make $-4b$ and b^3 have the same value because the signs will always be opposite.

STOP

488 Unit 5 Rational Number Arithmetic

1 Launch

Display the expressions from the first row of the table. Ask, "Which expression do you think will have the greatest value? Which will have the least? Which will be closest to zero? Is it possible to say if a or $-a$ is greater without knowing the value of a?"

2 Monitor

Help students get started by having them substitute the original value into the expression first, and then simplifying as a second step.

Look for points of confusion:

- **Struggling to find the greatest value, least value, or value closest to zero in the set.** Encourage students to create a number line to help them reason about the positions of different expressions.

Look for productive strategies:

- Noticing that the sign of a^2 will always be positive, but the sign of b^3 will match the original sign of b.

3 Connect

Display the completed table.

Have students share their values that are the greatest, least, and closest to zero from each set and explain their reasoning.

Ask:

- "Were you surprised by any of the results? Which ones?"
- "Why is b^3 not always the greatest value?"

Highlight that when substituting values into algebraic expressions, it is important to pay attention to the signs in both the expression and the number being substituted in. If a negative number is substituted into a variable with a negative sign, the value of the expression will be positive. For example, if $a = -3$, then $-a = -(-3)$, which is 3.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students first focus on substituting the values into the expressions. After they have done so, they can go back and evaluate them.

Extension: *Math Enrichment*

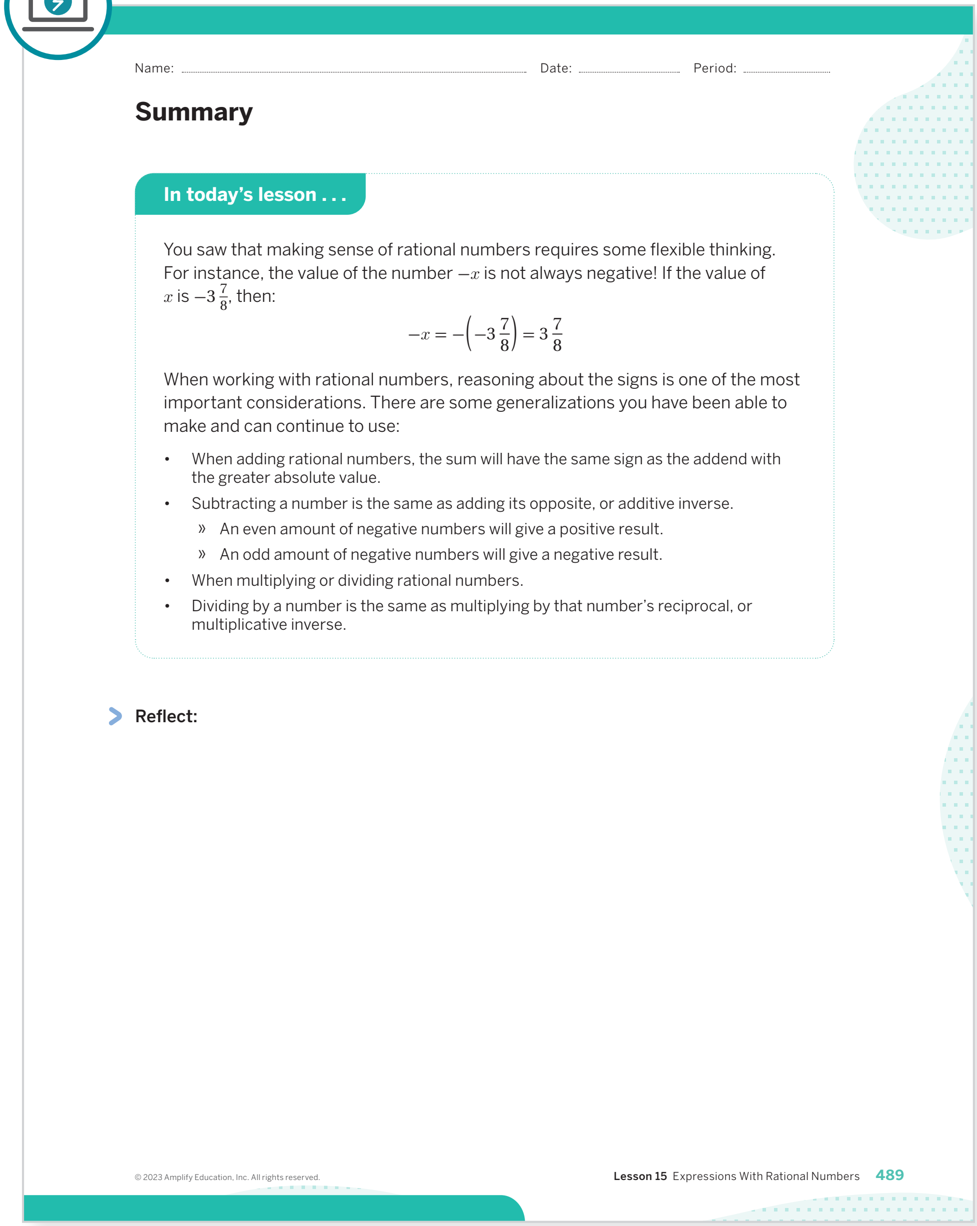
Have students respond to the following questions.

- When $|a|$ is greater than 1, which is farther from zero: a or a^2? Why? a^2 is farther from zero because a^2 has a greater magnitude than a.
- When $|a|$ is between 0 and 1, which is farther from zero: a or a^2? Why? a is farther from zero because a has a greater magnitude than a^2.

Whole Class | 5 min

Summary

Review and synthesize the ways in which expressions with various operations can be written to have the same value.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that making sense of rational numbers requires some flexible thinking. For instance, the value of the number $-x$ is not always negative! If the value of x is $-3\frac{7}{8}$, then:

$$-x = -\left(-3\frac{7}{8}\right) = 3\frac{7}{8}$$

When working with rational numbers, reasoning about the signs is one of the most important considerations. There are some generalizations you have been able to make and can continue to use:

- When adding rational numbers, the sum will have the same sign as the addend with the greater absolute value.
- Subtracting a number is the same as adding its opposite, or additive inverse.
  - » An even amount of negative numbers will give a positive result.
  - » An odd amount of negative numbers will give a negative result.
- When multiplying or dividing rational numbers.
- Dividing by a number is the same as multiplying by that number's reciprocal, or multiplicative inverse.

Reflect:

Synthesize

Highlight that addition and subtraction and multiplication and division are considered inverse operations because they are able to undo each other.

Ask, "Can you give an example of a number whose additive inverse is the same as its multiplicative inverse? Why not?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

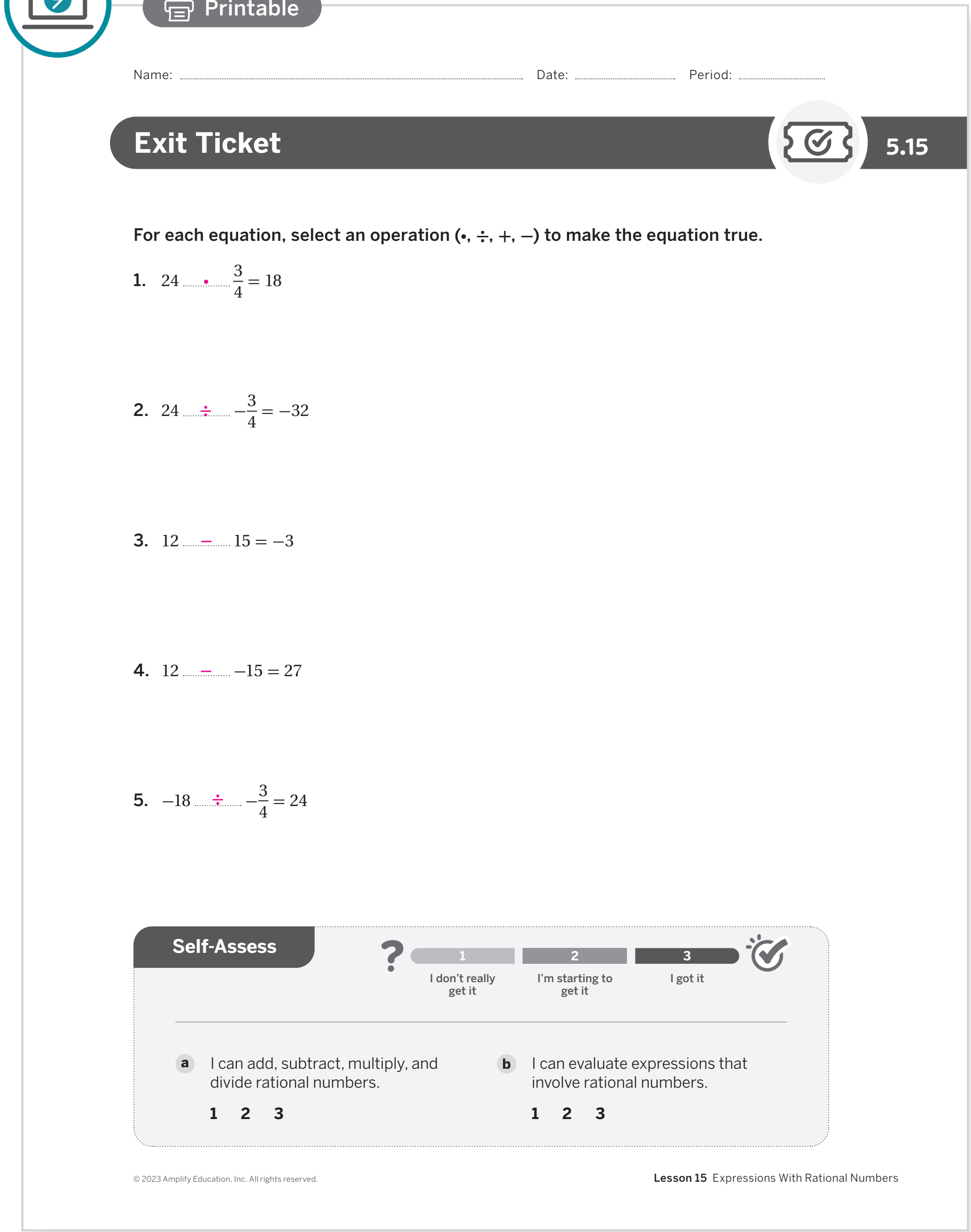
- "What does it mean for something to be the *inverse* of something else?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of performing operations with rational numbers.

Printable



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Exit Ticket 5.15

For each equation, select an operation (•, ÷, +, −) to make the equation true.

1. $24 \cdot \frac{3}{4} = 18$
2. $24 \div -\frac{3}{4} = -32$
3. $12 - 15 = -3$
4. $12 - -15 = 27$
5. $-18 \div -\frac{3}{4} = 24$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can add, subtract, multiply, and divide rational numbers. 1 2 3

b I can evaluate expressions that involve rational numbers. 1 2 3

Lesson 15 Expressions With Rational Numbers

Success looks like . . .

- **Language Goal:** Evaluating an expression for given values of the variable, including negative values, and comparing the resulting values of the expression. **(Speaking and Listening)**
- **Language Goal:** Generalizing about the relationship between additive inverses and about the relationship between multiplicative inverses. **(Speaking and Listening)**
- **Language Goal:** Identifying numerical expressions that are equal, and justifying that they are equal. **(Speaking and Listening)**

Suggested next steps

If students use multiplication for Problems 2 and 5, consider:

- Asking, "Does multiplication always give a greater product than both factors? Is it possible for a quotient to be greater than the dividend and divisor?"

If students struggle with Problem 4, consider:

- Asking, "What happens when you add a negative number? When you subtract a negative number?"

Professional Learning

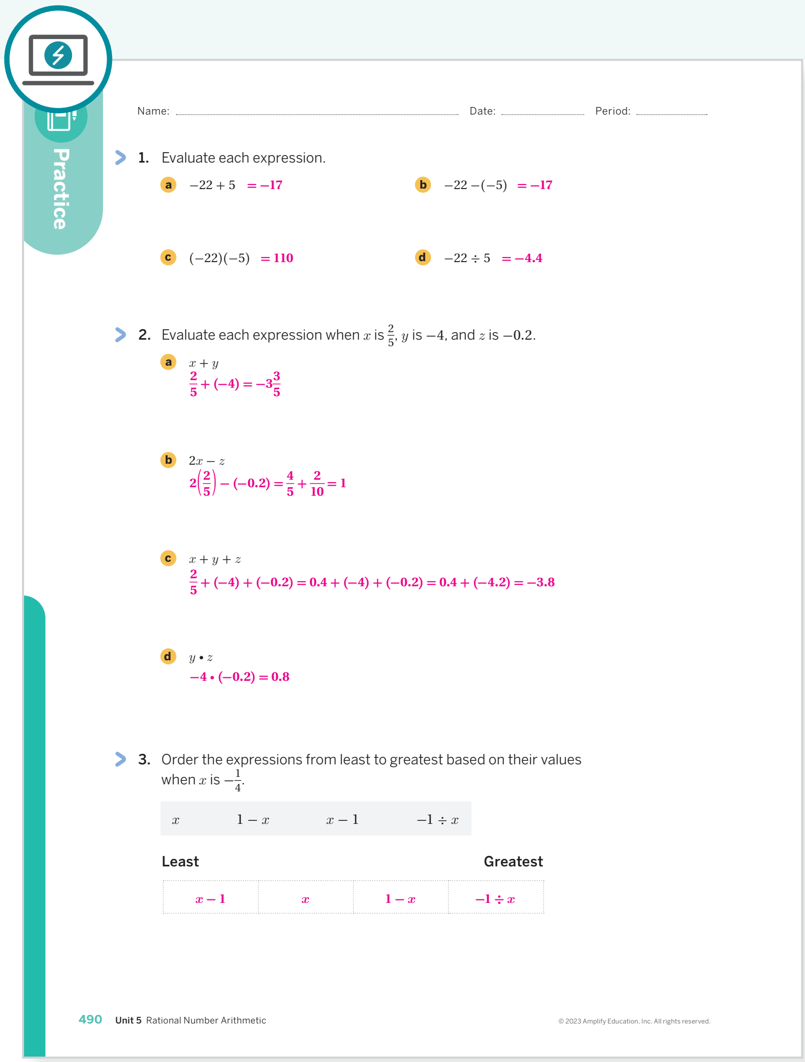
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What routines enabled all students to do math in today's lesson?
- During the discussion about Activity 1, how did you encourage each student to listen to one another's strategies? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Evaluate each expression.

a $-22 + 5$ $= -17$

b $-22 - (-5)$ $= -17$

c $(-22)(-5)$ $= 110$

d $-22 \div 5$ $= -4.4$

2. Evaluate each expression when x is $\frac{2}{5}$, y is -4, and z is -0.2.

a $x + y$

$\frac{2}{5} + (-4) = -3\frac{3}{5}$

b $2x - z$

$2\left(\frac{2}{5}\right) - (-0.2) = \frac{4}{5} + \frac{2}{10} = 1$

c $x + y + z$

$\frac{2}{5} + (-4) + (-0.2) = 0.4 + (-4) + (-0.2) = 0.4 + (-4.2) = -3.8$

d $y \bullet z$

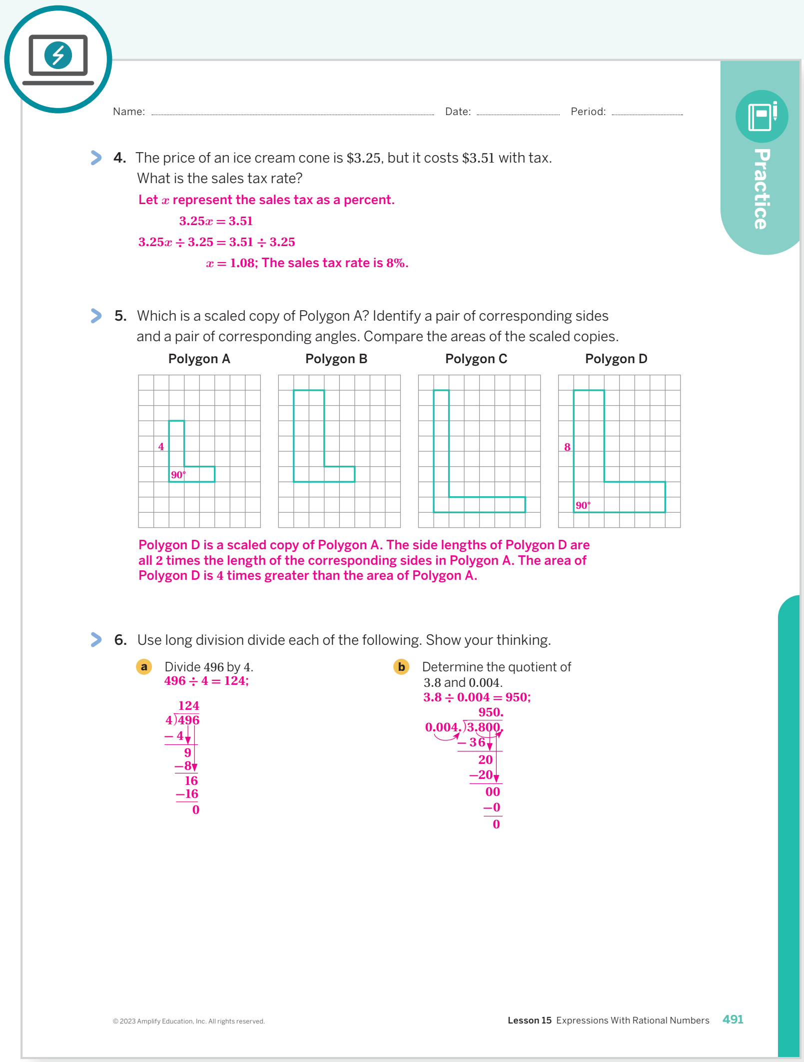
$-4 \bullet (-0.2) = 0.8$

3. Order the expressions from least to greatest based on their values when x is $-\frac{1}{4}$.

x	$1 - x$	$x - 1$	$-1 \div x$

Least **Greatest**

$x - 1$	x	$1 - x$	$-1 \div x$



Name: ________ Date: ________ Period: ________

Practice

4. The price of an ice cream cone is \$3.25, but it costs \$3.51 with tax. What is the sales tax rate?

Let x represent the sales tax as a percent.

$3.25x = 3.51$

$3.25x \div 3.25 = 3.51 \div 3.25$

$x = 1.08$; The sales tax rate is 8%.

5. Which is a scaled copy of Polygon A? Identify a pair of corresponding sides and a pair of corresponding angles. Compare the areas of the scaled copies.

Polygon D is a scaled copy of Polygon A. The side lengths of Polygon D are all 2 times the length of the corresponding sides in Polygon A. The area of Polygon D is 4 times greater than the area of Polygon A.

6. Use long division divide each of the following. Show your thinking.

a Divide 496 by 4.

$496 \div 4 = 124$;

$$\begin{array}{r} 124 \\ 4\overline{)496} \\ -4 \\ \hline 9 \\ -8 \\ \hline 16 \\ -16 \\ \hline 0 \end{array}$$

b Determine the quotient of 3.8 and 0.004.

$3.8 \div 0.004 = 950$;

$$\begin{array}{r} 950. \\ 0.004.\overline{)3.800.} \\ -36 \\ \hline 20 \\ -20 \\ \hline 00 \\ -0 \\ \hline 0 \end{array}$$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 8	2
	5	Unit 1 Lesson 3	2
Formative ⚡	6	Unit 5 Lesson 16	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

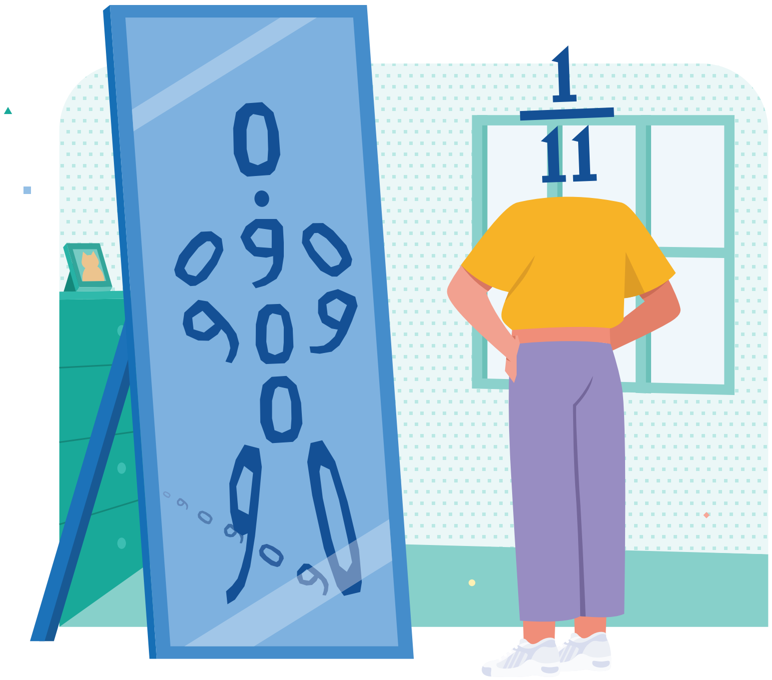
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 16**

Say It With Decimals

Let's represent fractions with decimals.



Focus

Goals

1. **Language Goal:** Understand and use the term *repeating decimal* when describing a decimal expansion does not terminate, and represent a repeating decimal expansion with bar notation. **(Speaking and Listening, Writing)**
2. **Language Goal:** Use long division to generate a decimal representation of a fraction, and describe the decimal that results. **(Writing)**
3. Represent a complex fraction as a fraction with integers in simplest form.

Coherence

Today

Students use long division to express fractions as decimals. They see how the calculations can sometimes be repeated resulting in a repeating decimal. Students also evaluate complex fractions to a single fraction in simplest form.

Previously

In Lesson 15, students performed operations involving rational numbers.

Coming Soon

In Lesson 17, students solve problems with rational numbers.

Rigor

- Students practice **procedural skills** of long division to represent fractions as decimals, including repeating decimals.
- Students **apply** their knowledge of operations with fractions to express complex fractions as a single fraction in lowest terms.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, *A Different Calculator View* (for display)
- Power-up PDF
- Power-up PDF (answers)
- Anchor Chart PDF, *Examples of Division Methods* (from Grade 6)

Math Language Development

New words

- ***bar notation***
- ***repeating decimal***
- ***terminating decimal***

Review words

- *integer*
- *long division*
- *rational numbers*

Amps | **Featured Activity**

Exit Ticket
Real-Time Exit Ticket

Check in real time if your students can express a fraction as a repeating decimal using a digital Exit Ticket that is automatically scored.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may not recognize when a decimal begins to repeat. They need to recognize when the process they are using to divide continually will result in the same number(s) as they determine the value of a fraction expressed as a repeated decimal. It might help students to whisper aloud what they are doing in order to hear the repetition of the process. As soon as they can identify the repetition, they can write the decimal using bar notation.

Modifications to Pacing

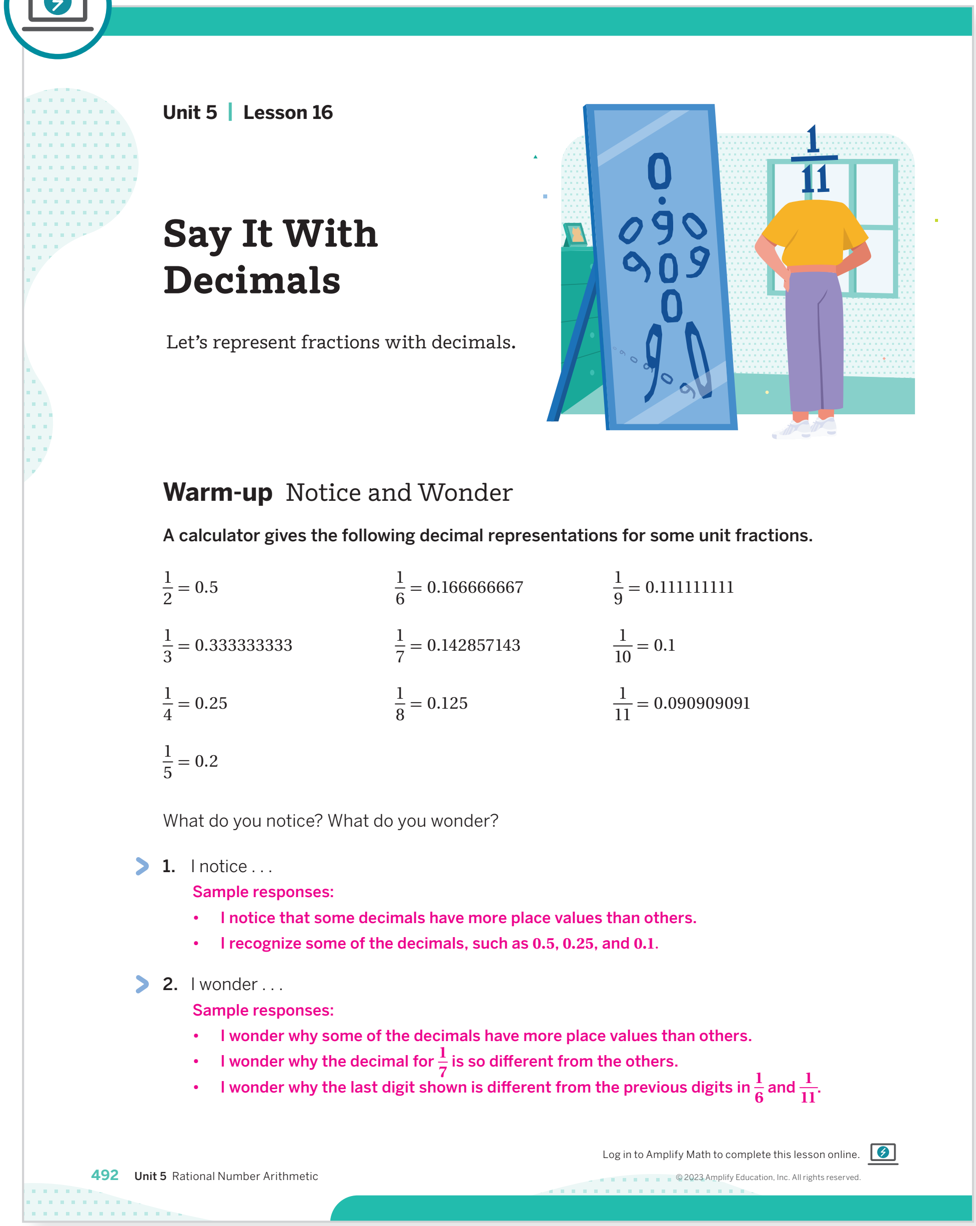
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up**. Instead, share the definitions for repeating decimals and terminating decimals during the Activity 1 Connect.
- In **Activity 2**, give students a time limit and allow them to complete as many problems as possible during that time.
- Omit Problem 3 in **Activity 3**.

Independent | 5 min

Warm-up Notice and Wonder

Students look at decimal expansions of unit fractions to prepare for the upcoming activities involving repeating decimals.



Unit 5 | **Lesson 16**

Say It With Decimals

Let's represent fractions with decimals.

Warm-up Notice and Wonder

A calculator gives the following decimal representations for some unit fractions.

$\frac{1}{2} = 0.5$

$\frac{1}{3} = 0.333333333$

$\frac{1}{4} = 0.25$

$\frac{1}{5} = 0.2$

$\frac{1}{6} = 0.166666667$

$\frac{1}{7} = 0.142857143$

$\frac{1}{8} = 0.125$

$\frac{1}{9} = 0.111111111$

$\frac{1}{10} = 0.1$

$\frac{1}{11} = 0.090909091$

What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - **I notice that some decimals have more place values than others.**
   - **I recognize some of the decimals, such as 0.5, 0.25, and 0.1.**
2. I wonder . . .
   Sample responses:
   - **I wonder why some of the decimals have more place values than others.**
   - **I wonder why the decimal for $\frac{1}{7}$ is so different from the others.**
   - **I wonder why the last digit shown is different from the previous digits in $\frac{1}{6}$ and $\frac{1}{11}$.**

Log in to Amplify Math to complete this lesson online.

492 **Unit 5** Rational Number Arithmetic

1 Launch

Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking whether they recognize or are familiar with any of the decimals.

Look for points of confusion:

- **Trying to find a pattern among the list of decimals.** Having a numerator of 1 in common does not yield a pattern among the decimal expansions.

Look for productive strategies:

- Wondering if some of the decimals are rounded.

3 Connect

Display the Warm-up PDF, *A Different Calculator View* and ask students if these expanded decimals help answer any of their questions.

Ask, "How did the calculator get these decimal representations? The numbers were divided.

Have students share what they noticed and what they wondered.

Highlight that calculators cannot always show the entire decimal value because it may continue forever. Calculators round; therefore, students will need to be aware there may be repetition even if the last digit is different. If there is a pattern of repeating digits, it is known as a repeating decimal. In the next activity, students will learn how to write those numbers more precisely than a calculator.

Define *repeating decimal* as a decimal that has the same sequences of non-zero digits that repeat indefinitely. ***Terminating decimals*** are decimals that end at a specific place value.

Power-up

To power up students' ability to determine the quotient of two values using long division:

Provide students with a copy of the Power-up PDF.

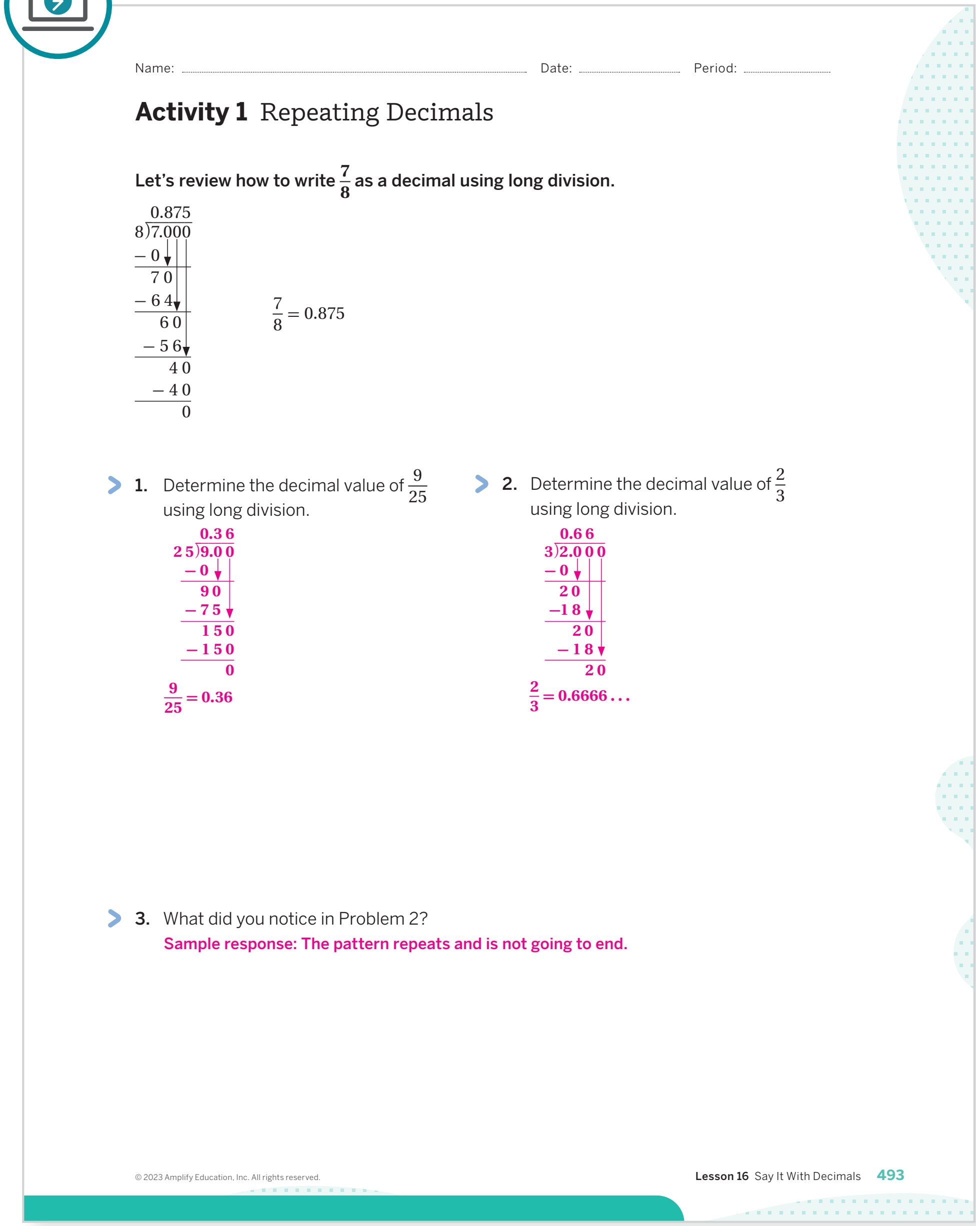
Use: Before Activity 1.

Informed by: Performance on Lesson 15, Practice Problem 6.

Pairs | 10 min

Activity 1 Repeating Decimals

Students use long division to rewrite fractions as decimals and use bar notation to represent digits which repeat forever.



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Activity 1 Repeating Decimals

Let's review how to write $\frac{7}{8}$ as a decimal using long division.

0.875
$8\overline{)7.000}$
-0
70
-64
60
-56
40
-40
0

$\frac{7}{8} = 0.875$

1. Determine the decimal value of $\frac{9}{25}$ using long division.

0.36
$25\overline{)9.00}$
-0
90
-75
150
-150
0

$\frac{9}{25} = 0.36$

2. Determine the decimal value of $\frac{2}{3}$ using long division.

0.66
$3\overline{)2.000}$
-0
20
-18
20
-18
20

$\frac{2}{3} = 0.6666\ldots$

3. What did you notice in Problem 2?
Sample response: The pattern repeats and is not going to end.

1 Launch

Activate prior knowledge and review that fractions represent division between two whole numbers. Discuss the long division example. Consider displaying the Long Division example from the Anchor Chart PDF, *Examples of Division Methods* from Grade 6.

2 Monitor

Help students get started by reminding them there are infinitely many zeros after a decimal point and they can add as many as they need to expand their number.

Look for points of confusion:

- **Setting up their long division with the divisor and dividend in the incorrect places.** Prompt them to think about what is being divided and what it is being divided by. Consider writing the fraction as a division problem with the obelus symbol (÷) to help them determine which is the divisor and/or dividend.
- **Misaligning their digits.** Remind students to be very careful and methodical when performing long division.

Look for productive strategies:

- Recognizing the pattern continues forever when dividing 2 by 3.

3 Connect

Display any problems or solutions necessary to facilitate class discussion.

Highlight the example in Problem 2 is known as a *repeating decimal* and can be represented with bar notation, such as $\frac{2}{3} = 0.666666\ldots = 0.\overline{6}$. The ***bar notation*** above the 6 represents the 6 repeats forever, as seen in the long division in Problem 2. Also, show how the ellipses (. . .) can be used to show repetition if a few iterations of the pattern are shown.

Ask, "How could you write $0.4545454545\ldots$ using bar notation? How could you write the decimals in the Warm-up using the bar notation?"

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Review with students how to write fractions as decimals using long division. Consider walking through the given example, $\frac{7}{8}$. Ask a student volunteer to demonstrate each step, using a think-aloud to illustrate what is happening at each step.

Math Language Development

MLR2: Collect and Display

As students share what patterns they notice, capture and define language related to repeating and terminating decimals and add this to the class display. For example, consider adding the following to the class display:

Repeating decimals	Terminating decimals
The long division results in a repeating, nonzero digit. Bar notation can represent the repeating digit, for example, $0.\overline{6}$.	The long division results in 0.

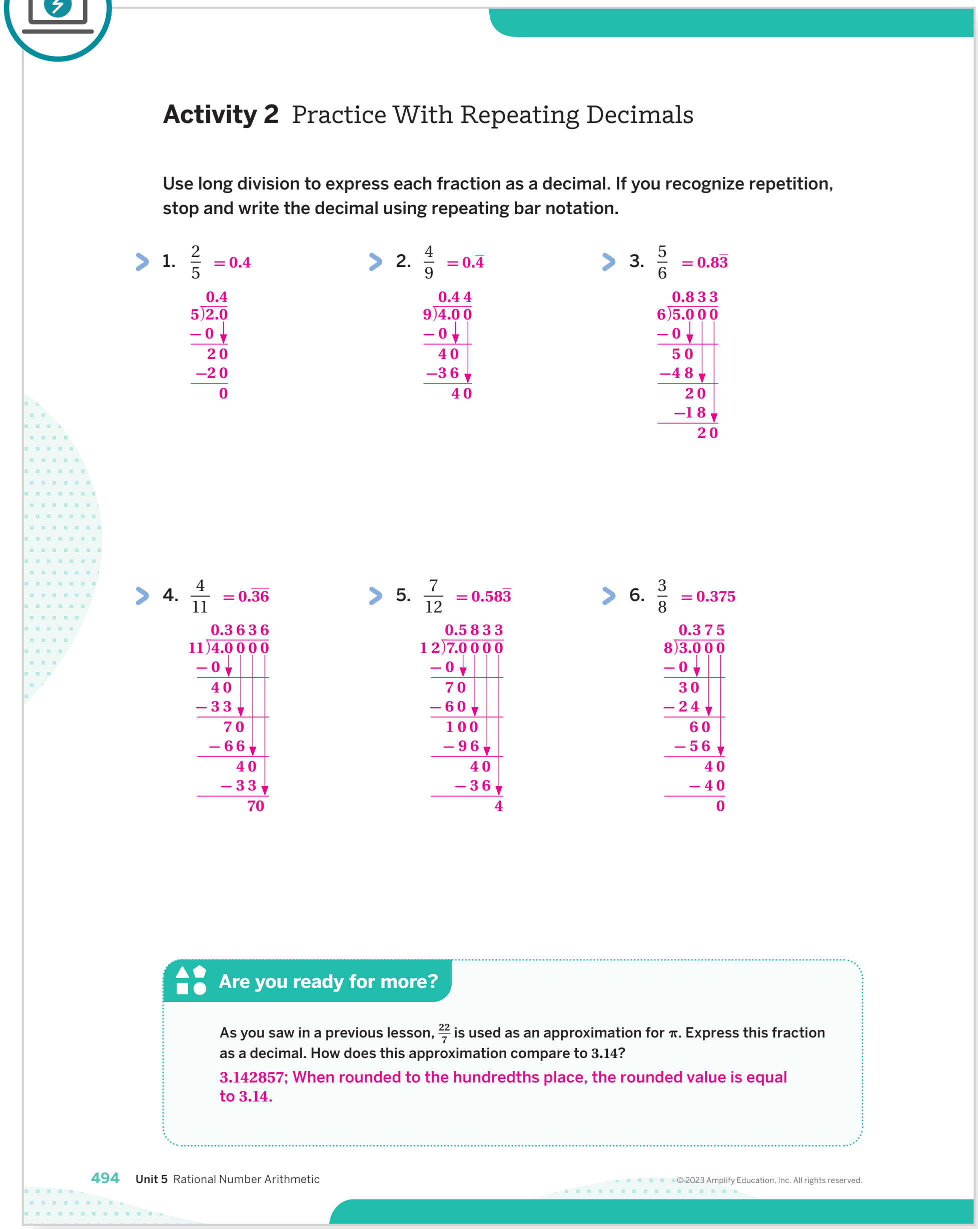
English Learners

Provide examples of fractions that lead to terminating and repeating decimals.

Pairs | 10 min

Activity 2 Practice With Repeating Decimals

Students express rational numbers as decimals to build fluency.



Activity 2 Practice With Repeating Decimals

Use long division to express each fraction as a decimal. If you recognize repetition, stop and write the decimal using repeating bar notation.

1. $\frac{2}{5}$ $= 0.4$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ -0 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

2. $\frac{4}{9}$ $= 0.\overline{4}$

$$\begin{array}{r} 0.44 \\ 9\overline{)4.00} \\ -0 \\ \hline 40 \\ -36 \\ \hline 40 \end{array}$$

3. $\frac{5}{6}$ $= 0.8\overline{3}$

$$\begin{array}{r} 0.833 \\ 6\overline{)5.000} \\ -0 \\ \hline 50 \\ -48 \\ \hline 20 \\ -18 \\ \hline 20 \end{array}$$

4. $\frac{4}{11}$ $= 0.\overline{36}$

$$\begin{array}{r} 0.3636 \\ 11\overline{)4.0000} \\ -0 \\ \hline 40 \\ -33 \\ \hline 70 \\ -66 \\ \hline 40 \\ -33 \\ \hline 70 \end{array}$$

5. $\frac{7}{12}$ $= 0.58\overline{3}$

$$\begin{array}{r} 0.5833 \\ 12\overline{)7.0000} \\ -0 \\ \hline 70 \\ -60 \\ \hline 100 \\ -96 \\ \hline 40 \\ -36 \\ \hline 4 \end{array}$$

6. $\frac{3}{8}$ $= 0.375$

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -0 \\ \hline 30 \\ -24 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Are you ready for more?

As you saw in a previous lesson, $\frac{22}{7}$ is used as an approximation for π. Express this fraction as a decimal. How does this approximation compare to 3.14?

3.142857; When rounded to the hundredths place, the rounded value is equal to 3.14.

1 Launch

Have students conduct the ***Think-Pair-Share*** routine as they work through the problem set.

2 Monitor

Help students get started by having them refer to the example and problems in Activity 1.

Look for points of confusion:

- **Writing the repeating bar over both the 8 and the 3 in Problem 3.** Remind students that the bar only is above the digit or digits which repeat over and over.
- **Not stopping when they see repetition.** Remind students that these numbers will continue forever, so they will need to stop.

3 Connect

Display any problems and solutions which will help facilitate class discussion.

Highlight the many patterns of repetition, such as in Problem 1, there was one digit that did not repeat, Problem 4 has two digits that repeat, and Problem 5 has two non-repeating digits with one repeating digit.

Ask:

- "When did you decide to stop the division? Why?"
- "Order the numbers from least to greatest. Did you compare the fractions or the decimals to help you order the numbers?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to choose three of the six problems to complete. Offering them the power of choice can lead to greater engagement and ownership of the task.

Extension: *Math Enrichment*

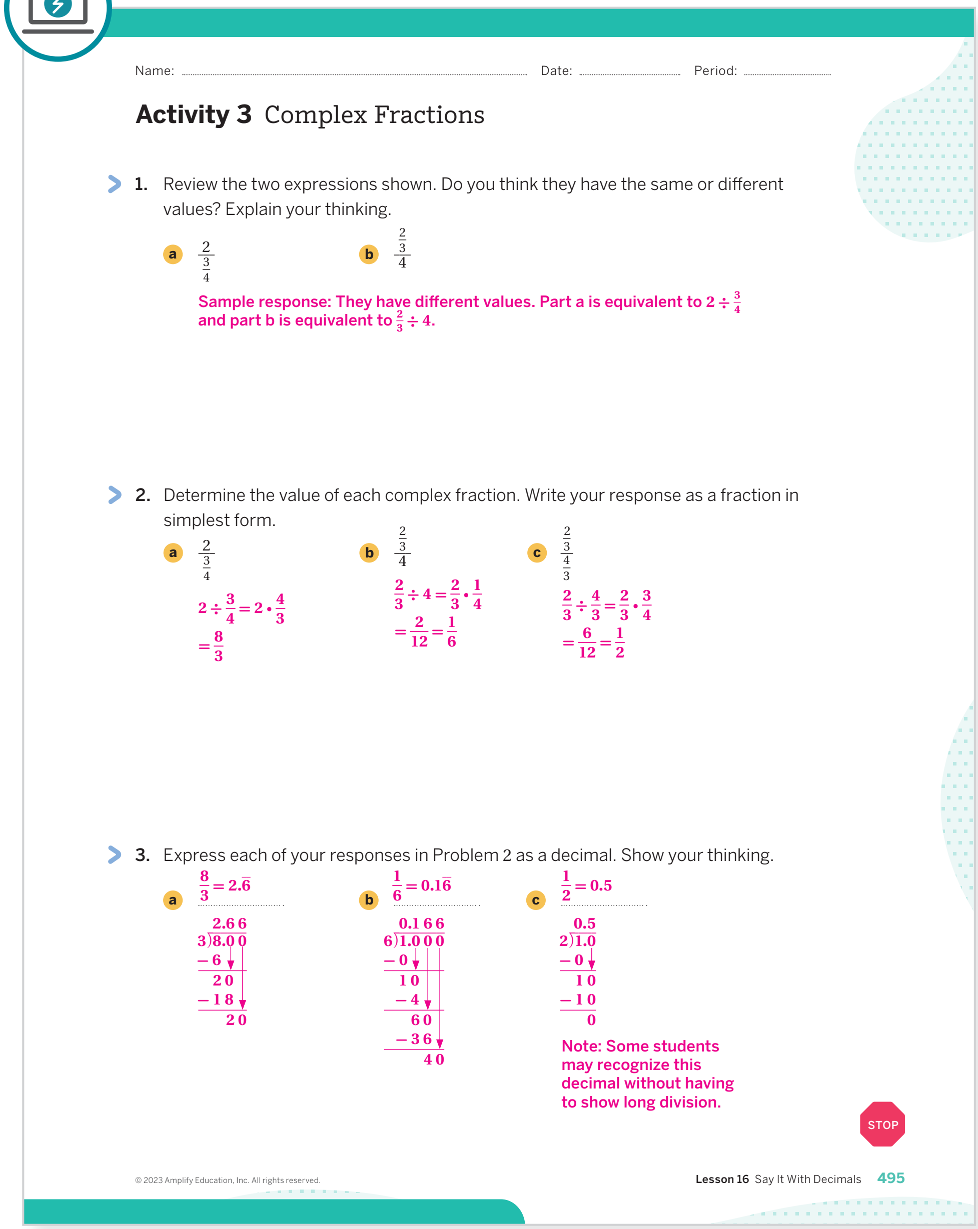
Ask students to create a graphic organizer of common fractions and whether their decimal representations are repeating or terminating. Have them consider fractions, such as the following:

- Denominators of 2, 4, 5, 8, 10 Terminating
- Denominators of 3, 6, 9 Some are repeating, while others are terminating. The ones that terminate can be written in simpler forms, such as $\frac{3}{6} = \frac{1}{2}$.

Pairs | 30 min

Activity 3 Complex Fractions

Students use division of fractions to rewrite complex fractions as a single fraction in lowest terms.



Name: ________ Date: ________ Period: ________

Activity 3 Complex Fractions

1. Review the two expressions shown. Do you think they have the same or different values? Explain your thinking.

   a $\dfrac{2}{\frac{3}{4}}$ b $\dfrac{\frac{2}{3}}{4}$

   Sample response: They have different values. Part a is equivalent to $2 \div \frac{3}{4}$ and part b is equivalent to $\frac{2}{3} \div 4$.

2. Determine the value of each complex fraction. Write your response as a fraction in simplest form.

   a $\dfrac{2}{\frac{3}{4}}$

   $2 \div \frac{3}{4} = 2 \cdot \frac{4}{3}$

   $= \frac{8}{3}$

   b $\dfrac{\frac{2}{3}}{4}$

   $\frac{2}{3} \div 4 = \frac{2}{3} \cdot \frac{1}{4}$

   $= \frac{2}{12} = \frac{1}{6}$

   c $\dfrac{\frac{2}{3}}{\frac{4}{3}}$

   $\frac{2}{3} \div \frac{4}{3} = \frac{2}{3} \cdot \frac{3}{4}$

   $= \frac{6}{12} = \frac{1}{2}$

3. Express each of your responses in Problem 2 as a decimal. Show your thinking.

   a $\frac{8}{3} = 2.\overline{6}$

   $$\begin{array}{r} 2.66 \\ 3\overline{)8.00} \\ -6 \\ \hline 20 \\ -18 \\ \hline 20 \end{array}$$

   b $\frac{1}{6} = 0.1\overline{6}$

   $$\begin{array}{r} 0.166 \\ 6\overline{)1.000} \\ -0 \\ \hline 10 \\ -4 \\ \hline 60 \\ -36 \\ \hline 40 \end{array}$$

   c $\frac{1}{2} = 0.5$

   $$\begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ -0 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

   Note: Some students may recognize this decimal without having to show long division.

STOP

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Lesson 16 Say It With Decimals 495

1 Launch

Activate background knowledge that when dividing two integers the result will either be a terminating or repeating decimal. Today, students will see what happens when they divide two rational numbers. Remind them that dividing by a fraction is equivalent to multiplying by the reciprocal of that fraction.

2 Monitor

Help students get started by helping them rewrite the complex fractions using the obelus (÷) symbol.

Look for points of confusion:

- **Not reciprocating the second fraction in Problem 2.** Have students reference an example of how dividing fractions is equivalent to multiplying by the reciprocal.

Look for productive strategies:

- Recognizing $\frac{8}{3}$ is $2\frac{2}{3}$ and $\frac{2}{3} = 0.\overline{6}$ without having to perform the long division calculations.

3 Connect

Have students share their thinking and solutions to the problems.

Highlight how *complex fractions* are called complex because they have multiple fraction bars, not because they are complicated. Students already possess the skills necessary to evaluate these numbers. They need to pay attention to where the longest fraction bar is located, because this will represent the division of the two rational numbers.

Ask:

- "How do you evaluate a complex fraction?" Rewrite the division problem as multiplication of the reciprocal of the number in the denominator of the complex fraction.
- "Evaluate $1 + \frac{1}{1 + \frac{1}{2}}$."

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Help students make sense of the structure of a complex fraction by having them think of a complex fraction as "a fraction within a fraction." Provide access to colored pencils and have them color code the longest fraction bar in each problem. This indicates the "overall fraction." Then ask them to describe whether the numerator, denominator, or both of each overall fraction are also fractions.

$$\dfrac{\square}{\square}$$

☐ ◄—— The numerator could also be a fraction.

☐ ◄—— The denominator could also be a fraction.

Extension: *Math Enrichment*

Have students examine the structure of the following complex fractions and evaluate them, without performing long division. Have them explain their thinking.

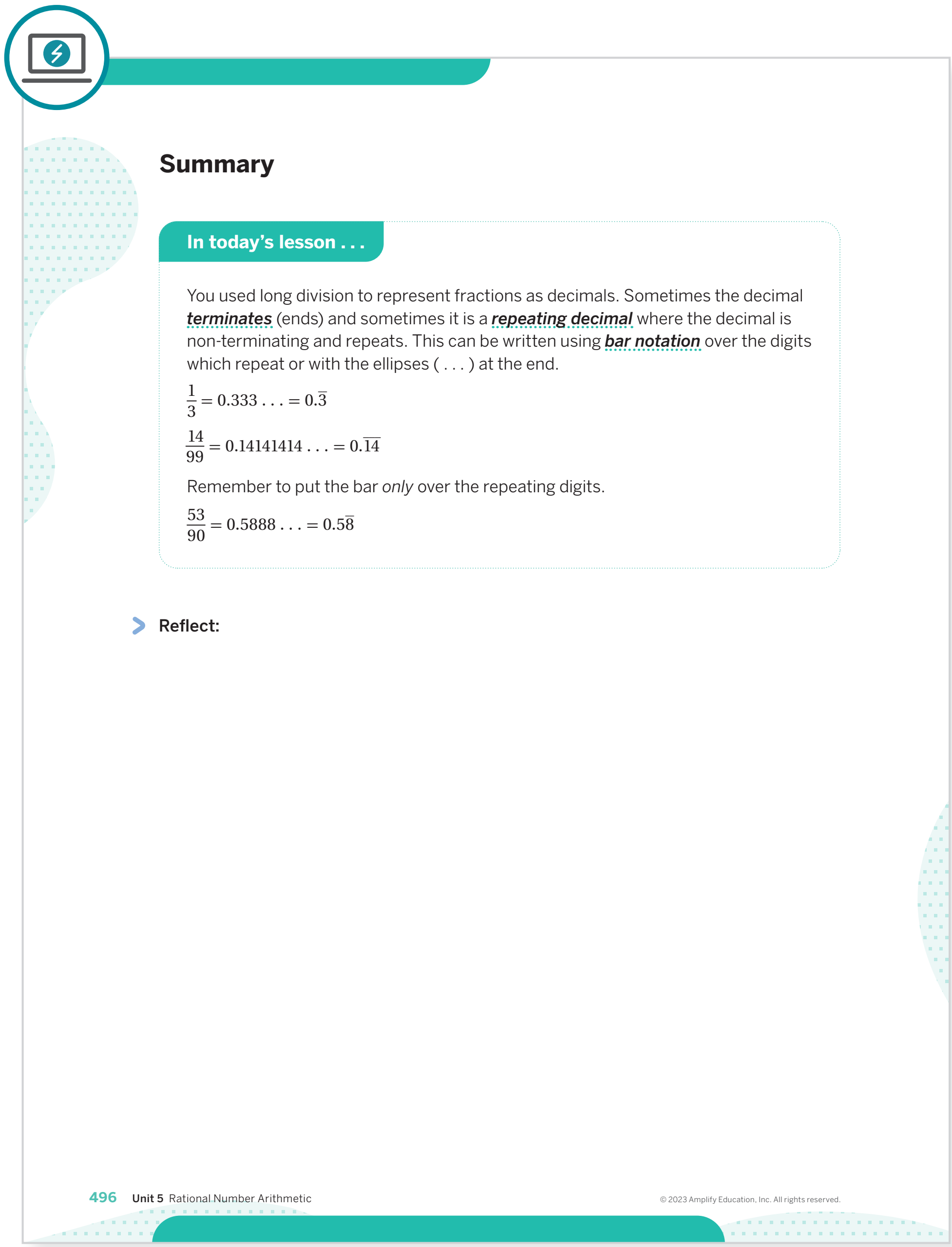
$\dfrac{\frac{1}{3}}{\frac{5}{3}}$ $\frac{1}{5}$; One third out of five thirds is the same as one out of five.

$\dfrac{\frac{3}{8}}{\frac{7}{8}}$ $\frac{3}{7}$; Three eighths out of seven eighths is the same as three out of seven.

Whole Class | 5 min

Summary

Review and synthesize how to rewrite fractions as terminating or repeating decimals.



Summary

In today's lesson . . .

You used long division to represent fractions as decimals. Sometimes the decimal ***terminates*** (ends) and sometimes it is a ***repeating decimal*** where the decimal is non-terminating and repeats. This can be written using ***bar notation*** over the digits which repeat or with the ellipses (. . .) at the end.

$$\frac{1}{3} = 0.333\ldots = 0.\overline{3}$$

$$\frac{14}{99} = 0.14141414\ldots = 0.\overline{14}$$

Remember to put the bar *only* over the repeating digits.

$$\frac{53}{90} = 0.5888\ldots = 0.5\overline{8}$$

Reflect:

496 Unit 5 Rational Number Arithmetic

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Synthesize

Display the Summary from the Student Edition.

Formalize vocabulary:

- ***bar notation***
- ***repeating decimal***
- ***terminating decimal***

Highlight rational numbers are numbers which can be expressed as a ratio of two integers which can either be expressed as terminating decimals or repeating decimals.

Ask, "Did you recognize any patterns in what type of fractions produced repeating decimals versus ones that produced terminating decimals?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do you rewrite a fraction as a decimal?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on the terms and phrases related to the terms *bar notation*, *repeating decimal*, and *terminating decimal* that were added to the display during the lesson.

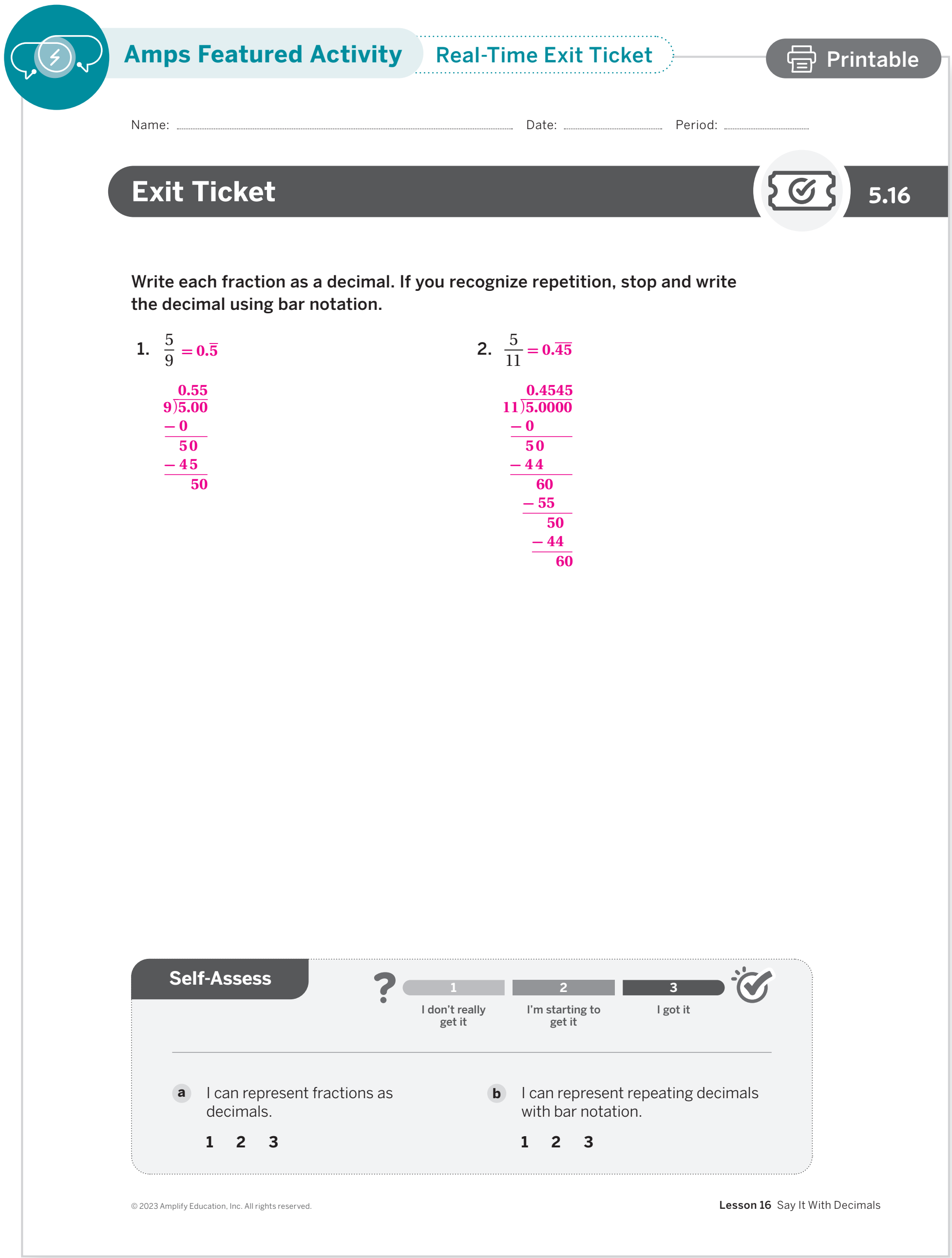
Independent | 5 min

Exit Ticket

Students demonstrate their understanding by rewriting fractions as repeating decimals using proper notation.

Amps Featured Activity Real-Time Exit Ticket

Printable



Name: ______________ Date: ______ Period: ______

Exit Ticket 5.16

Write each fraction as a decimal. If you recognize repetition, stop and write the decimal using bar notation.

1. $\frac{5}{9} = 0.\overline{5}$

$$\begin{array}{r} 0.55 \\ 9\overline{)5.00} \\ -0 \\ \hline 50 \\ -45 \\ \hline 50 \end{array}$$

2. $\frac{5}{11} = 0.\overline{45}$

$$\begin{array}{r} 0.4545 \\ 11\overline{)5.0000} \\ -0 \\ \hline 50 \\ -44 \\ \hline 60 \\ -55 \\ \hline 50 \\ -44 \\ \hline 60 \end{array}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can represent fractions as decimals.
1 2 3

b I can represent repeating decimals with bar notation.
1 2 3

Lesson 16 Say It With Decimals

Success looks like . . .

- **Language Goal:** Understanding and using the term *repeating decimal* when describing a decimal expansion does not terminate, and representing a repeating decimal expansion with bar notation. **(Speaking and Listening, Writing)**
- **Language Goal:** Using long division to generate a decimal representation of a fraction, and describing the decimal that results. **(Writing)**
    - » Using long division to find the decimal representation of $\frac{5}{9}$ and $\frac{5}{11}$ in Problems 1 and 2.
- **Goal:** Representing a complex fraction as a fraction with integers in simplest form.

Suggested next steps

If students write the repeating bar above the incorrect digits, consider:

- Reviewing Activity 2.
- Assigning Practice Problems 1 and 2.

Note: Students will revisit this content in Grade 8.

Professional Learning

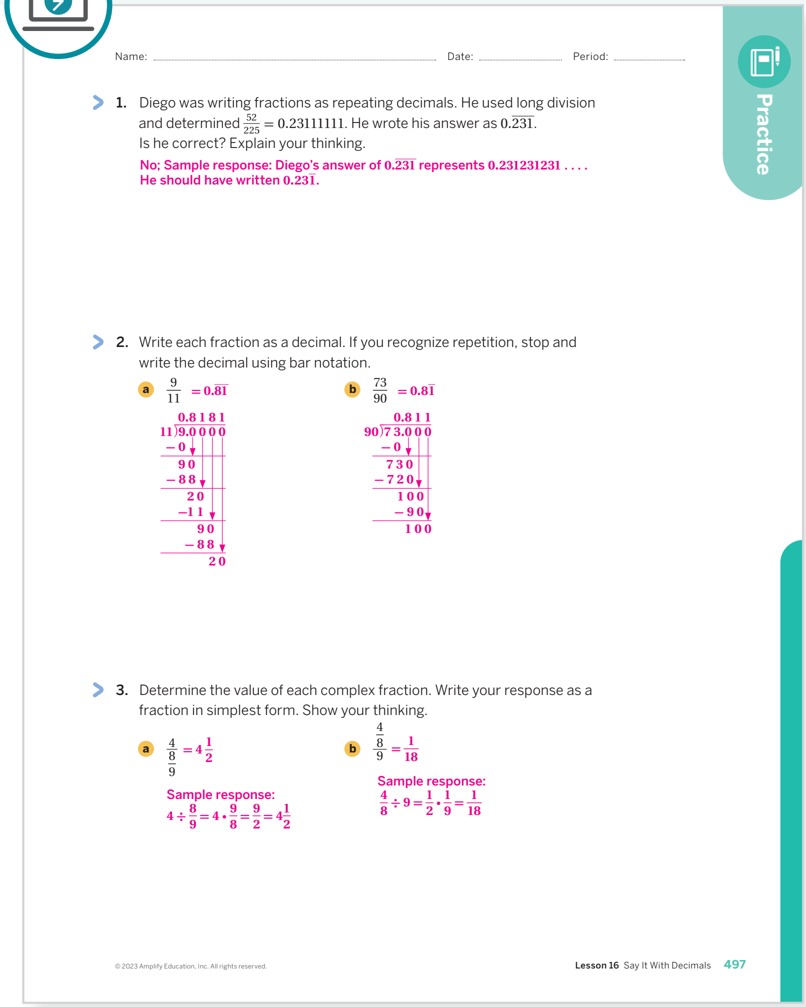
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did things happen that you did not expect?
- The focus of this lesson was rewriting fractions as repeated decimals. How did that go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Diego was writing fractions as repeating decimals. He used long division and determined $\frac{52}{225} = 0.23111111$. He wrote his answer as $0.\overline{231}$. Is he correct? Explain your thinking.
   No; Sample response: Diego's answer of $0.\overline{231}$ represents $0.231231231\ldots$. He should have written $0.23\overline{1}$.

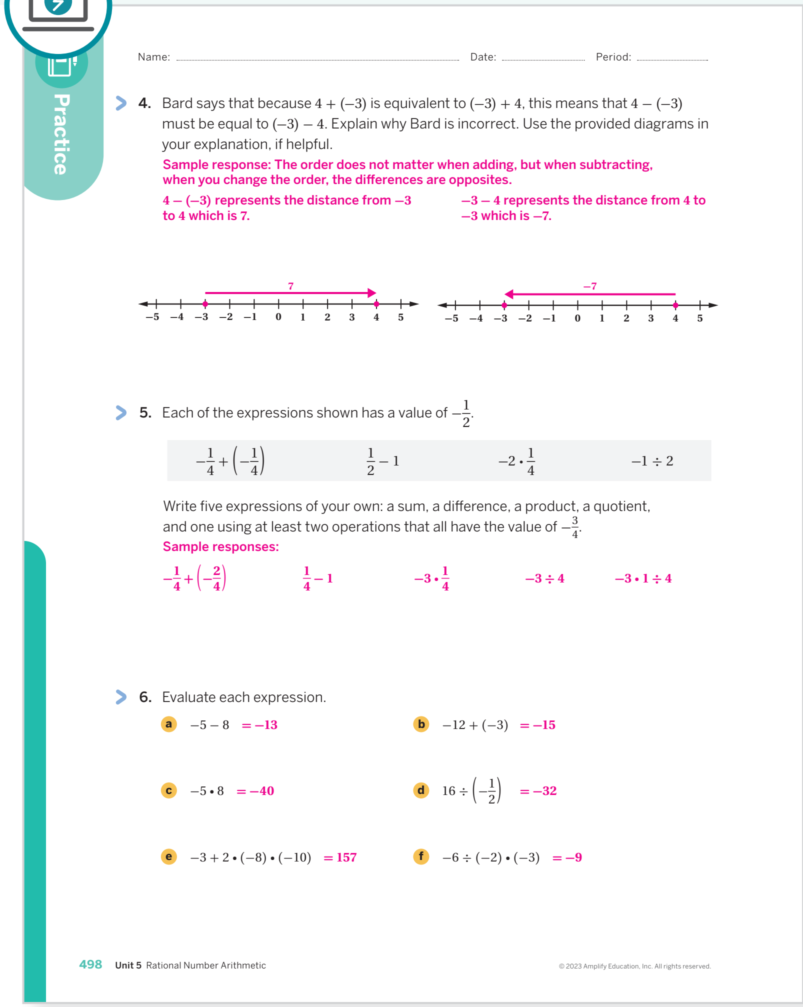
2. Write each fraction as a decimal. If you recognize repetition, stop and write the decimal using bar notation.
   - a $\frac{9}{11}$ **$= 0.\overline{81}$**

     $$\begin{array}{r} 0.8181 \\ 11\overline{)9.0000} \\ -0 \\ \hline 90 \\ -88 \\ \hline 20 \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \end{array}$$

   - b $\frac{73}{90}$ **$= 0.8\overline{1}$**

     $$\begin{array}{r} 0.811 \\ 90\overline{)73.000} \\ -0 \\ \hline 730 \\ -720 \\ \hline 100 \\ -90 \\ \hline 100 \end{array}$$

3. Determine the value of each complex fraction. Write your response as a fraction in simplest form. Show your thinking.
   - a $\frac{4}{\frac{8}{9}}$ **$= 4\frac{1}{2}$**
     Sample response:
     $4 \div \frac{8}{9} = 4 \cdot \frac{9}{8} = \frac{9}{2} = 4\frac{1}{2}$
   - b $\frac{\frac{4}{8}}{9}$ **$= \frac{1}{18}$**
     Sample response:
     $\frac{4}{8} \div 9 = \frac{1}{2} \cdot \frac{1}{9} = \frac{1}{18}$



Name: ________ Date: ________ Period: ________

4. Bard says that because $4 + (-3)$ is equivalent to $(-3) + 4$, this means that $4 - (-3)$ must be equal to $(-3) - 4$. Explain why Bard is incorrect. Use the provided diagrams in your explanation, if helpful.
   Sample response: The order does not matter when adding, but when subtracting, when you change the order, the differences are opposites.

   $4 - (-3)$ represents the distance from -3 to 4 which is 7.

   $-3 - 4$ represents the distance from 4 to -3 which is -7.

5. Each of the expressions shown has a value of $-\frac{1}{2}$.

$-\frac{1}{4} + \left(-\frac{1}{4}\right)$	$\frac{1}{2} - 1$	$-2 \cdot \frac{1}{4}$	$-1 \div 2$

   Write five expressions of your own: a sum, a difference, a product, a quotient, and one using at least two operations that all have the value of $-\frac{3}{4}$.
   Sample responses:
   $-\frac{1}{4} + \left(-\frac{2}{4}\right)$ $\frac{1}{4} - 1$ $-3 \cdot \frac{1}{4}$ $-3 \div 4$ $-3 \cdot 1 \div 4$

6. Evaluate each expression.
   - a $-5 - 8$ **$= -13$**
   - b $-12 + (-3)$ **$= -15$**
   - c $-5 \cdot 8$ **$= -40$**
   - d $16 \div \left(-\frac{1}{2}\right)$ **$= -32$**
   - e $-3 + 2 \cdot (-8) \cdot (-10)$ **$= 157$**
   - f $-6 \div (-2) \cdot (-3)$ **$= -9$**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 3	1
Spiral	4	Unit 5 Lesson 7	2
	5	Unit 5 Lesson 15	2
Formative ⚡	6	Unit 5 Lesson 17	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 17**

Solving Problems With Rational Numbers

Let's use all four operations with rational numbers to solve problems.

Focus

Goal

1. **Language Goal:** Apply operations with rational numbers to solve problems, and present the solution method. **(Writing, Speaking and Listening)**

Coherence

Today

Students put together what they have learned about rational number arithmetic and interpretation of negative quantities, such as negative time or rates of change. The problems are designed so that it is natural to solve them by completing tables or making numeral calculations to set the foundation for writing and solving equations with rational numbers.

Previously

In Lesson 15, students evaluated expressions with rational numbers and all four operations.

Coming Soon

In Lesson 19, students will write equations with rational numbers to solve problems.

Rigor

- Students **apply** their understanding of the four operations with rational numbers to solve real-world problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	18 min	5 min	7 min
Small Groups	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Operations with Rational Numbers, Parts 1–3* (answers, as needed)

Math Language Development

Review words

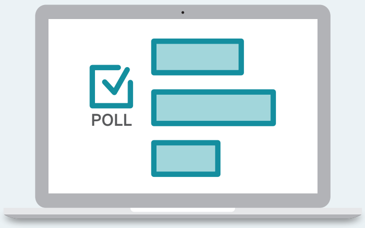
- *associate property*
- *commutative property*
- *Distributive Property*
- *rational numbers*

Amps | Featured Activity

Warm-up

Poll the Class

Assess, in real-time, which equation(s) students do not believe belong with the others. Use the results to facilitate a discussion about why each equation might not belong.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students should keep in mind that good relationships are an important part of math practice. Just as they must be precise when working with signed numbers, students must also be precise in their communication. They must clearly communicate when they are contributing to the group task, as well as when they need something from the group.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

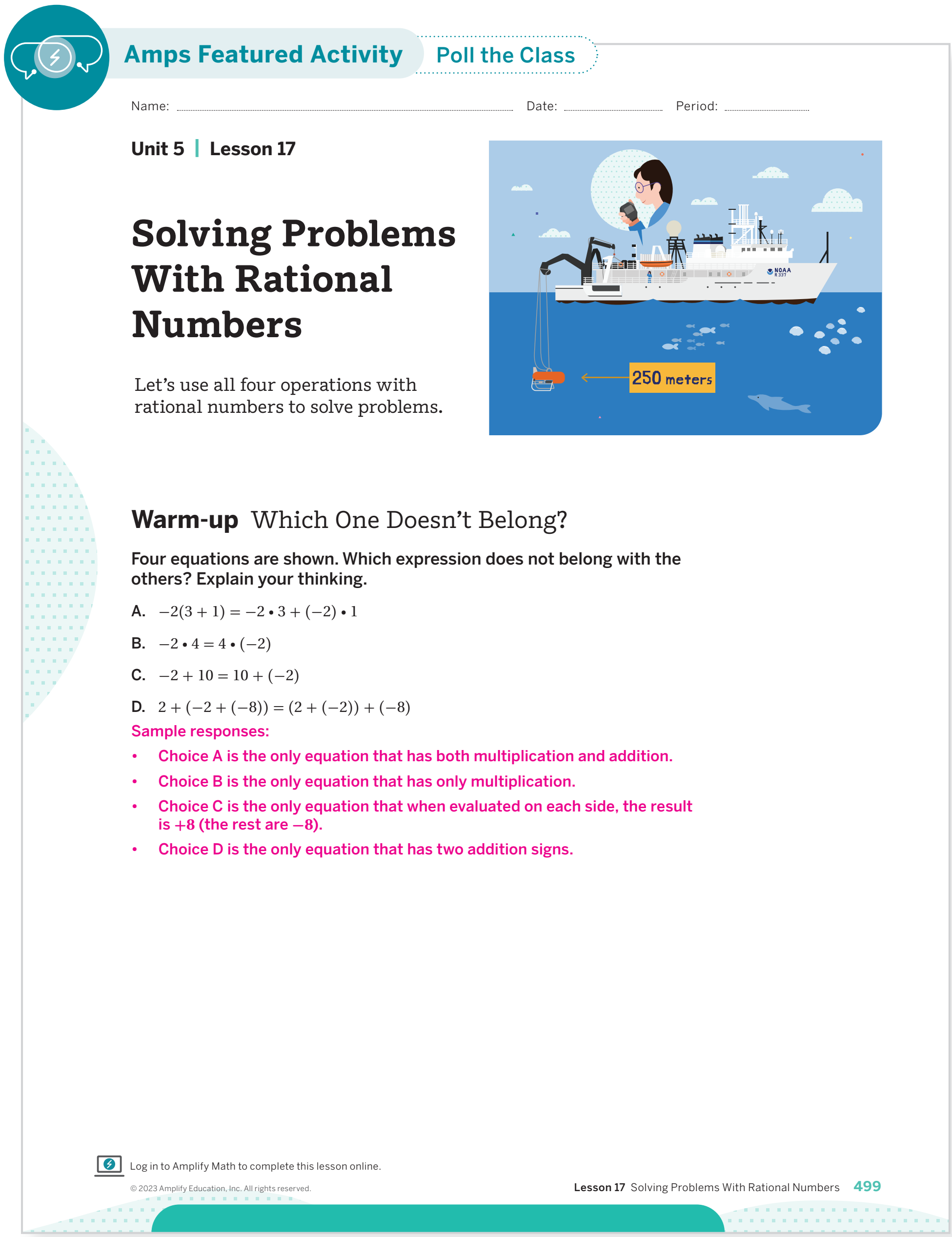
- Omit the **Warm-up**.
- Omit **Activity 1**. Consider assigning it as Additional Practice.

Small Groups | 5 min

Warm-up Which One Doesn't Belong?

Students apply their understanding of the four operations on rational numbers to analyze equations representing properties of addition and multiplication.

Amps Featured Activity Poll the Class



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Unit 5 | Lesson 17

Solving Problems With Rational Numbers

Let's use all four operations with rational numbers to solve problems.

Warm-up Which One Doesn't Belong?

Four equations are shown. Which expression does not belong with the others? Explain your thinking.

A. $-2(3 + 1) = -2 \cdot 3 + (-2) \cdot 1$

B. $-2 \cdot 4 = 4 \cdot (-2)$

C. $-2 + 10 = 10 + (-2)$

D. $2 + (-2 + (-8)) = (2 + (-2)) + (-8)$

Sample responses:

- **Choice A is the only equation that has both multiplication and addition.**
- **Choice B is the only equation that has only multiplication.**
- **Choice C is the only equation that when evaluated on each side, the result is +8 (the rest are −8).**
- **Choice D is the only equation that has two addition signs.**

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the ***Which One Doesn't Belong?*** routine.

2 Monitor

Help students get started by asking, "What do you notice about each equation?"

Look for points of confusion:

- **Thinking that there is only one correct answer.** Remind students that there are multiple correct answers, but they must be able to justify their reasoning.
- **Citing a reason that eliminates more than one equation.** Ask, "Is this the only equation that has that quality?"

Look for productive strategies:

- Comparing the operations used in each equation.
- Comparing the value of each equation once evaluated on each side.

3 Connect

Display the four equations.

Have students share which equation they chose and their reasoning. Ask the class whether they agree or disagree and to justify their reasoning.

Ask, "What property of addition or multiplication does each equation model?"

- Equation A: Distributive Property
- Equation B: Commutative Property of Multiplication
- Equation C: Commutative Property of Addition
- Equation D: Associative Property of Addition

Highlight that the associative and commutative properties, as well as the Distributive Property, can be helpful when simplifying expressions that represent problems involving rational numbers.

Power-up

To power up students' ability to evaluate expressions with rational values, have students complete:

Evaluate each expression. Show your thinking.

1. $1 - 8 + 2$
$= -7 + 2$
$= -5$

2. $-3 - 9 \cdot (-2)$
$= -3 - (-18)$
$= -3 + 18$
$= 15$

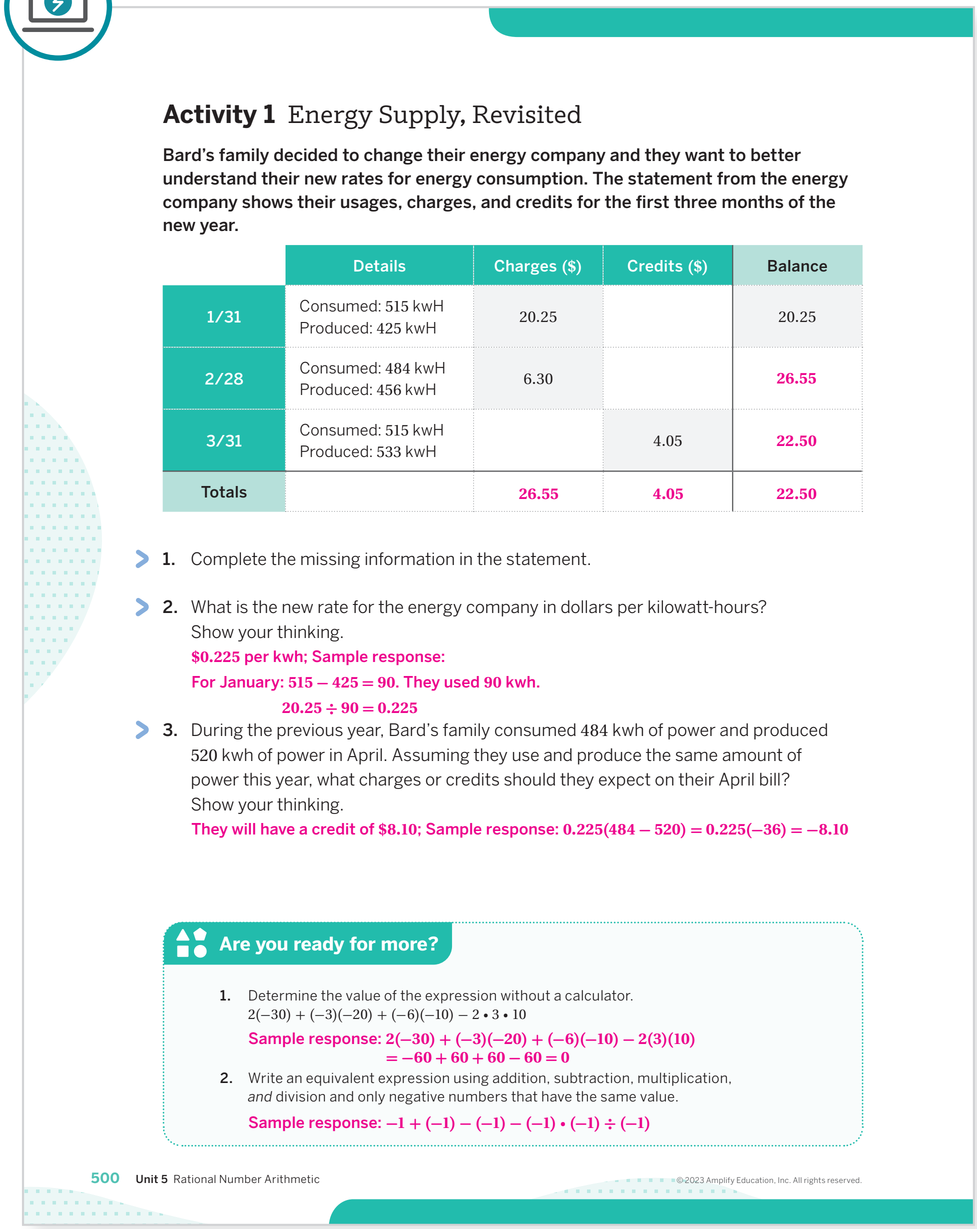
Use: Before Activity 2.

Informed by: Performance on Lesson 16, Practice Problem 6.

Pairs | 10 min

Activity 1 Energy Supply, Revisited

Students apply their understanding of rational numbers and operations to solve problems involving energy generation.



Activity 1 Energy Supply, Revisited

Bard's family decided to change their energy company and they want to better understand their new rates for energy consumption. The statement from the energy company shows their usages, charges, and credits for the first three months of the new year.

	Details	Charges ($)	Credits ($)	Balance
1/31	Consumed: 515 kwH Produced: 425 kwH	20.25		20.25
2/28	Consumed: 484 kwH Produced: 456 kwH	6.30		26.55
3/31	Consumed: 515 kwH Produced: 533 kwH		4.05	22.50
Totals		26.55	4.05	22.50

1. Complete the missing information in the statement.
2. What is the new rate for the energy company in dollars per kilowatt-hours? Show your thinking.
   $0.225 per kwh; Sample response:
   For January: $515 - 425 = 90$. They used 90 kwh.
   $20.25 \div 90 = 0.225$
3. During the previous year, Bard's family consumed 484 kwh of power and produced 520 kwh of power in April. Assuming they use and produce the same amount of power this year, what charges or credits should they expect on their April bill? Show your thinking.
   They will have a credit of $8.10; Sample response: $0.225(484 - 520) = 0.225(-36) = -8.10$

Are you ready for more?

1. Determine the value of the expression without a calculator.
   $2(-30) + (-3)(-20) + (-6)(-10) - 2 \cdot 3 \cdot 10$
   Sample response: $2(-30) + (-3)(-20) + (-6)(-10) - 2(3)(10)$
   $= -60 + 60 + 60 - 60 = 0$
2. Write an equivalent expression using addition, subtraction, multiplication, *and* division and only negative numbers that have the same value.
   Sample response: $-1 + (-1) - (-1) - (-1) \cdot (-1) \div (-1)$

500 **Unit 5** Rational Number Arithmetic

1 Launch

Activate prior knowledge by asking what students remember about how the cost of energy is calculated for families who have solar panels. Remind students of the work they did in Lesson 5, Activity 2.

2 Monitor

Help students get started by asking, "How does the energy company determine whether Bard's family is charged or whether they earn a credit?"

Look for points of confusion:

- **Forgetting that the balance is not the amount due that month.** Remind students that the balance is the running total or the sum of the current month's charges with the previous month's balance.

Look for productive strategies:

- Determining the difference between the kwh used and generated each month and checking that the cost per kwh is consistent from month to month.

3 Connect

Display the table from the Student Edition.

Have students share how they used the values in the table to determine the missing values and the cost per kwh for energy.

Ask, "For Problem 3, what expression can you write to determine the cost for April?" Sample response: $0.225(484 - 520)$

Highlight that, when working with problems with rational numbers, students must keep in mind the signs of the values, the operations that represent the scenario, and the order for each operation.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

Ask students to look back at their work from Lesson 5, Activity 2 and ask, "What is similar about these activities? What is different?" Guide them to see that the new rate is not known in this activity, as it was in Lesson 5, Activity 2.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

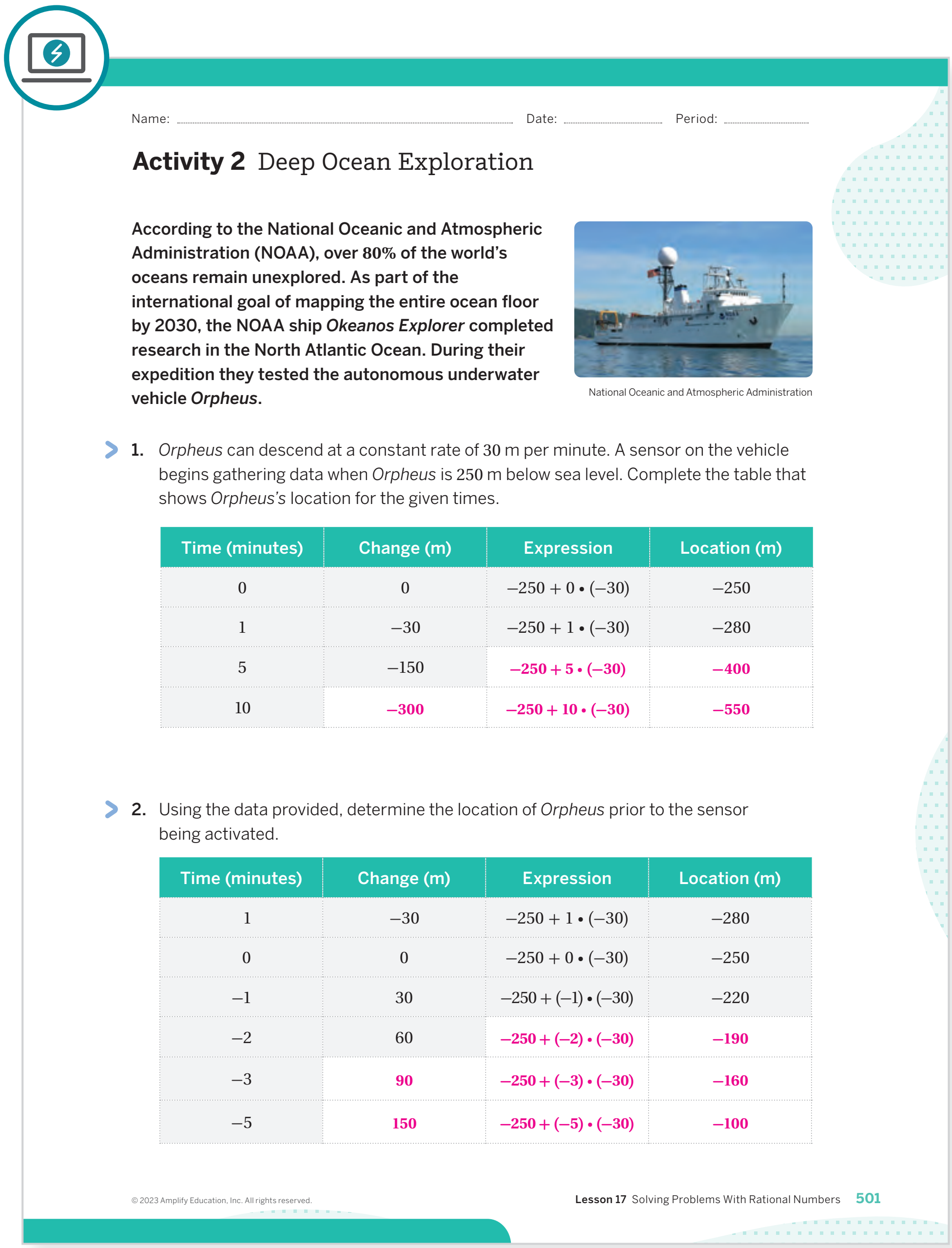
- **Read 1:** Students read the problem with the goal of comprehending the situation (e.g., A utility company charges for energy use.)
- **Read 2:** Ask students to name or highlight the given quantities and relationships, without focusing on specific values. For example, ask, "Was more energy consumed or produced in February? How do you know?"
- **Read 3:** Ask students to plan their solution strategy as to how they will complete the statement in Problem 1.

English Learners

Clarify the meaning of the terms *consumed* and *produced* for students.

Activity 2 Deep Ocean Exploration

Students investigate the location of an automated underwater vehicle to reason about a real-world situation involving the four operations on rational numbers.



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Activity 2 Deep Ocean Exploration

According to the National Oceanic and Atmospheric Administration (NOAA), over 80% of the world's oceans remain unexplored. As part of the international goal of mapping the entire ocean floor by 2030, the NOAA ship *Okeanos Explorer* completed research in the North Atlantic Ocean. During their expedition they tested the autonomous underwater vehicle *Orpheus*.

National Oceanic and Atmospheric Administration

1. *Orpheus* can descend at a constant rate of 30 m per minute. A sensor on the vehicle begins gathering data when *Orpheus* is 250 m below sea level. Complete the table that shows *Orpheus's* location for the given times.

Time (minutes)	Change (m)	Expression	Location (m)
0	0	$-250 + 0 \cdot (-30)$	−250
1	−30	$-250 + 1 \cdot (-30)$	−280
5	−150	$-250 + 5 \cdot (-30)$	−400
10	−300	$-250 + 10 \cdot (-30)$	−550

2. Using the data provided, determine the location of *Orpheus* prior to the sensor being activated.

Time (minutes)	Change (m)	Expression	Location (m)
1	−30	$-250 + 1 \cdot (-30)$	−280
0	0	$-250 + 0 \cdot (-30)$	−250
−1	30	$-250 + (-1) \cdot (-30)$	−220
−2	60	$-250 + (-2) \cdot (-30)$	−190
−3	90	$-250 + (-3) \cdot (-30)$	−160
−5	150	$-250 + (-5) \cdot (-30)$	−100

1 Launch

Activate students' background knowledge by asking what they know about ocean exploration or automated underwater vehicles (AUVs).

2 Monitor

Help students get started by asking, "How does each given expression in the table represent the situation being described?"

Look for points of confusion:

- **Not following the order of operations when simplifying their expressions.** Remind students that they should evaluate multiplication prior to addition.
- **Confusing the rules for multiplication and addition when they are in the same expression.** Have students reference the posted Anchor Chart PDF, *Operations on Rational Numbers*.
- **Struggling to determine the time it took *Orpheus* to reach 250 m below sea level from the surface (0).** Encourage students to apply repeated reasoning to the table in Problem 2.

Look for productive strategies:

- Applying their understanding of the equation $d = rt$ to Problem 3, and dividing −250 by 30 to determine the time to get to 250 m below sea level from the surface.

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

Provide students with copies of the Anchor Chart PDFs, *Operations with Rational Numbers, Parts 1–3* (answers) to reference throughout the activity.

Math Language Development

MLR7: Compare and Connect

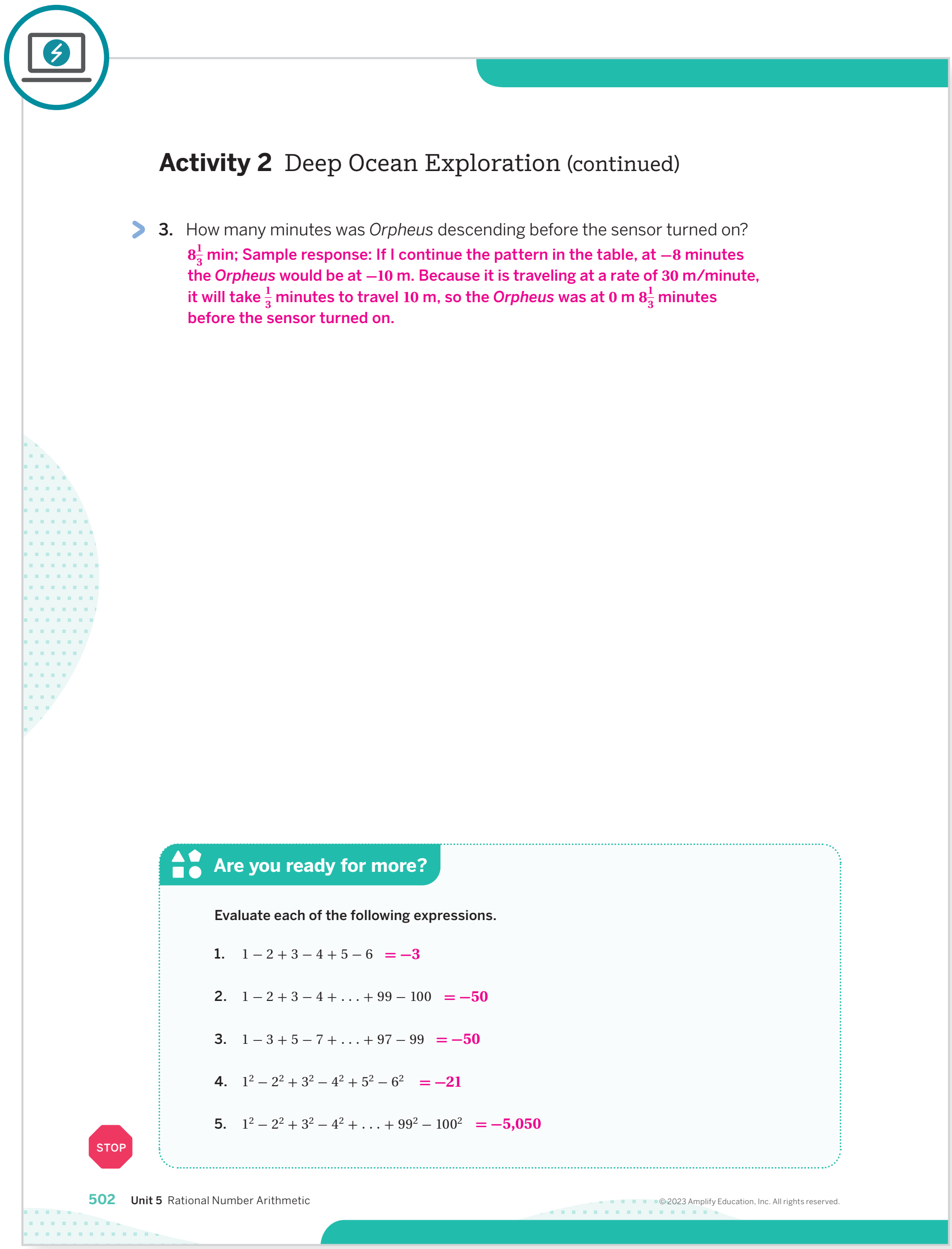
During the Connect, as you display completed tables and have students respond to the Ask questions, draw their attention to the connections between the structure of the expressions. Ask these follow-up questions:

- "Where do you see the change in location in your expression?"
- "How is the time represented in your expression?"
- "How do repeated computations appear in your expression?"
- "Why does the time in the second table start at 1 minute and decrease? What do the negative times mean in this context?"
- "Why do the changes in location become more negative in the first table, but become more positive in the second table?"

Small Groups | 18 min

Activity 2 Deep Ocean Exploration (continued)

Students investigate the location of an automated underwater vehicle to reason about a real-world situation involving the four operations on rational numbers.



Activity 2 Deep Ocean Exploration (continued)

3. How many minutes was *Orpheus* descending before the sensor turned on?

   $8\frac{1}{3}$ min; Sample response: If I continue the pattern in the table, at -8 minutes the *Orpheus* would be at -10 m. Because it is traveling at a rate of 30 m/minute, it will take $\frac{1}{3}$ minutes to travel 10 m, so the *Orpheus* was at 0 m $8\frac{1}{3}$ minutes before the sensor turned on.

Are you ready for more?

Evaluate each of the following expressions.

1. $1 - 2 + 3 - 4 + 5 - 6$ $= -3$
2. $1 - 2 + 3 - 4 + \ldots + 99 - 100$ $= -50$
3. $1 - 3 + 5 - 7 + \ldots + 97 - 99$ $= -50$
4. $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2$ $= -21$
5. $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2 - 100^2$ $= -5,050$

STOP

502 Unit 5 Rational Number Arithmetic

3 Connect

Display tables completed by students for Problems 1 and 2.

Ask:

- "What do the positive and negative values in each expression represent?"
- "How does each expression match the scenario?"

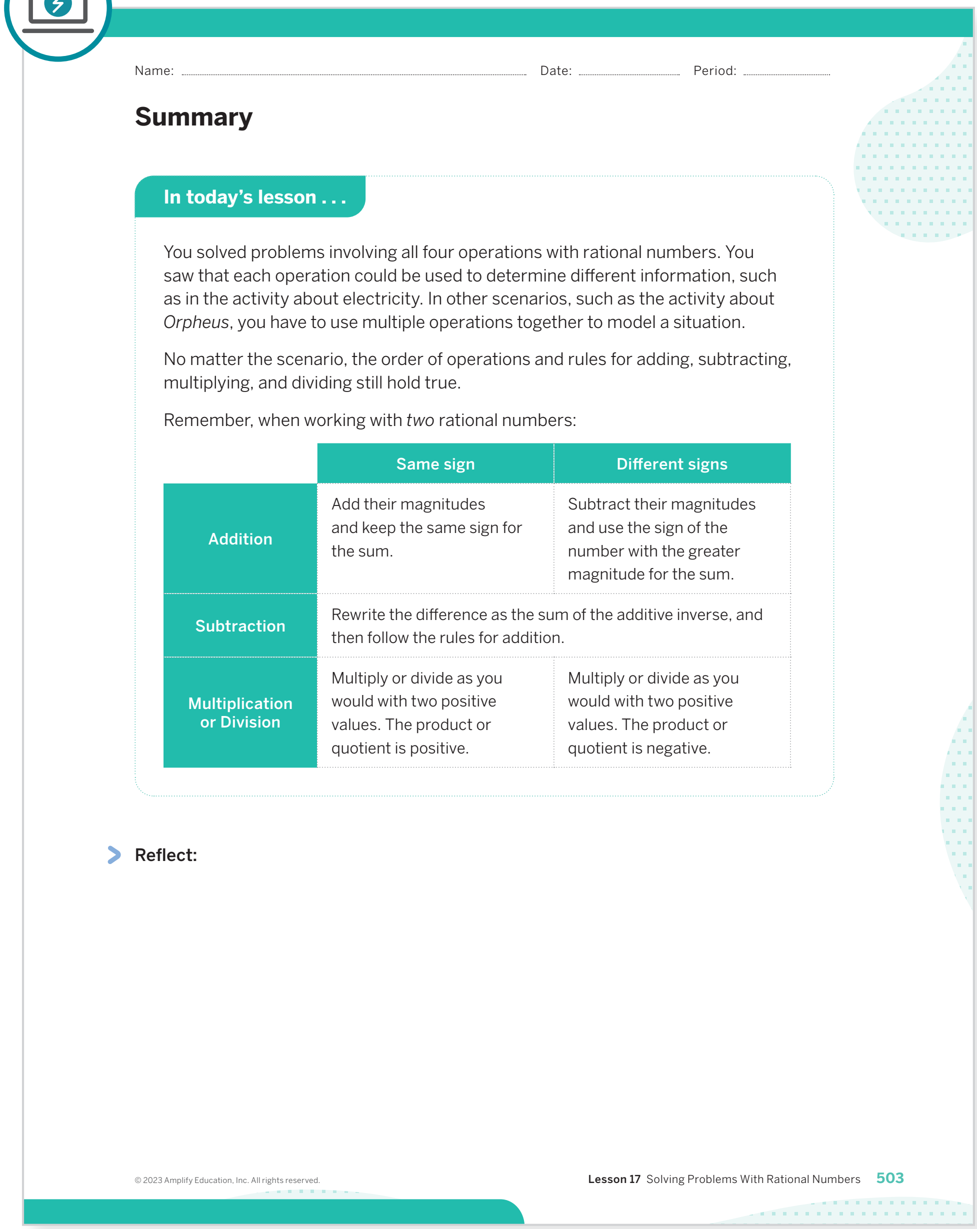
Have students share their reasoning for Problem 3. If possible, have students share strategies with and without expanding the table from Problem 2.

Highlight that writing expressions can help to see and reason about repeated operations and make sense of problems. When simplifying these expressions, it is important that the order of operations is followed.

Whole Class | 5 min

Summary

Review and synthesize how to use the four operations with rational numbers to solve problems.



Name: Date: Period:

Summary

In today's lesson . . .

You solved problems involving all four operations with rational numbers. You saw that each operation could be used to determine different information, such as in the activity about electricity. In other scenarios, such as the activity about *Orpheus*, you have to use multiple operations together to model a situation.

No matter the scenario, the order of operations and rules for adding, subtracting, multiplying, and dividing still hold true.

Remember, when working with *two* rational numbers:

	Same sign	Different signs
Addition	Add their magnitudes and keep the same sign for the sum.	Subtract their magnitudes and use the sign of the number with the greater magnitude for the sum.
Subtraction	Rewrite the difference as the sum of the additive inverse, and then follow the rules for addition.	
Multiplication or Division	Multiply or divide as you would with two positive values. The product or quotient is positive.	Multiply or divide as you would with two positive values. The product or quotient is negative.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Ask, "How did you determine which operations to use and in what order when solving problems in the activities today?"

Highlight that the order in which computations are completed depends on the scenario. In Activity 1, students needed to subtract prior to multiplying. However, in Activity 2, they needed to multiply by the rate of change first and then add that to -250 to determine the final location.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

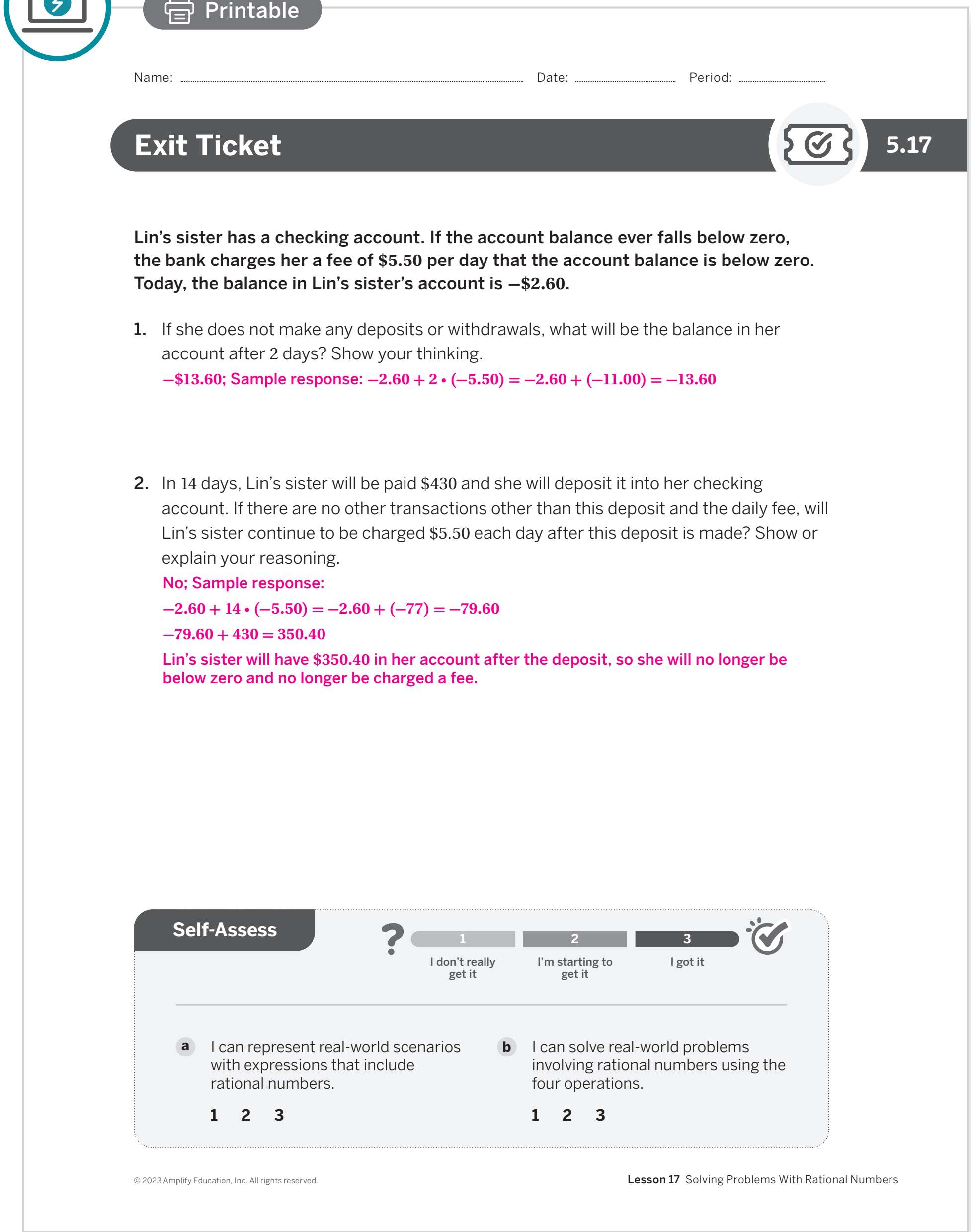
- "How is solving problems with negative rational numbers similar to or different from solving problems with only non-negative rational numbers?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding of how to use rational numbers and the four operations to solve problems.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.17

Lin's sister has a checking account. If the account balance ever falls below zero, the bank charges her a fee of $5.50 per day that the account balance is below zero. Today, the balance in Lin's sister's account is –$2.60.

1. If she does not make any deposits or withdrawals, what will be the balance in her account after 2 days? Show your thinking.
   –$13.60; Sample response: $-2.60 + 2 \cdot (-5.50) = -2.60 + (-11.00) = -13.60$

2. In 14 days, Lin's sister will be paid $430 and she will deposit it into her checking account. If there are no other transactions other than this deposit and the daily fee, will Lin's sister continue to be charged $5.50 each day after this deposit is made? Show or explain your reasoning.
   No; Sample response:
   $-2.60 + 14 \cdot (-5.50) = -2.60 + (-77) = -79.60$
   $-79.60 + 430 = 350.40$
   Lin's sister will have $350.40 in her account after the deposit, so she will no longer be below zero and no longer be charged a fee.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can represent real-world scenarios with expressions that include rational numbers.
1 2 3

b I can solve real-world problems involving rational numbers using the four operations.
1 2 3

Lesson 17 Solving Problems With Rational Numbers

Success looks like . . .

- **Language Goal:** Applying operations with rational numbers to solve problems, and presenting the solution method. **(Writing, Speaking and Listening)**
  - » Using rational numbers to solve problems in the context of account balances in Problems 1 and 2.

Suggested next steps

If students struggle to make sense of each problem, consider:

- Asking, "How would you determine how much money is in Lin's account after one day? Two days? Ten days?"
- Reviewing strategies from Activity 2.
- Assigning Practice Problems 1 and 3.

Professional Learning

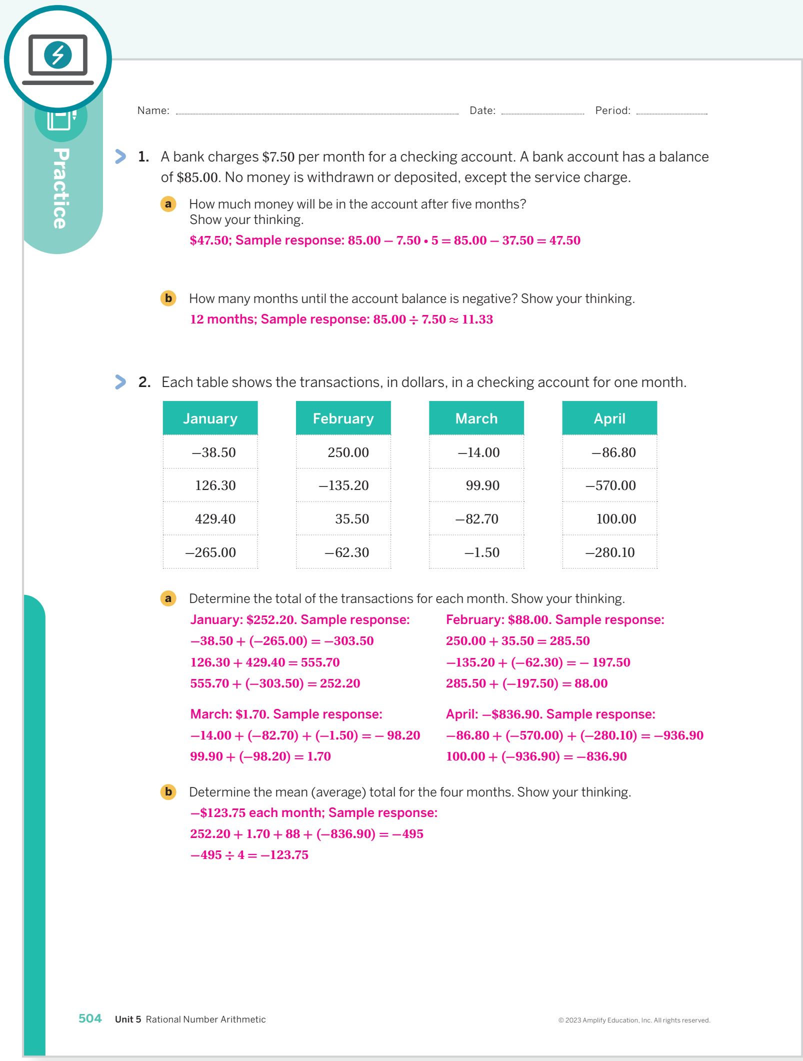
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- This lesson asked students to apply their understanding of operations on rational numbers to solve problems. Where in your students' work today did you see or hear evidence of them doing this? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. A bank charges \$7.50 per month for a checking account. A bank account has a balance of \$85.00. No money is withdrawn or deposited, except the service charge.

   a. How much money will be in the account after five months? Show your thinking.

   \$47.50; Sample response: $85.00 - 7.50 \cdot 5 = 85.00 - 37.50 = 47.50$

   b. How many months until the account balance is negative? Show your thinking.

   12 months; Sample response: $85.00 \div 7.50 \approx 11.33$

2. Each table shows the transactions, in dollars, in a checking account for one month.

January	February	March	April
–38.50	250.00	–14.00	–86.80
126.30	–135.20	99.90	–570.00
429.40	35.50	–82.70	100.00
–265.00	–62.30	–1.50	–280.10

   a. Determine the total of the transactions for each month. Show your thinking.

   January: \$252.20. Sample response:
   $-38.50 + (-265.00) = -303.50$
   $126.30 + 429.40 = 555.70$
   $555.70 + (-303.50) = 252.20$

   February: \$88.00. Sample response:
   $250.00 + 35.50 = 285.50$
   $-135.20 + (-62.30) = -197.50$
   $285.50 + (-197.50) = 88.00$

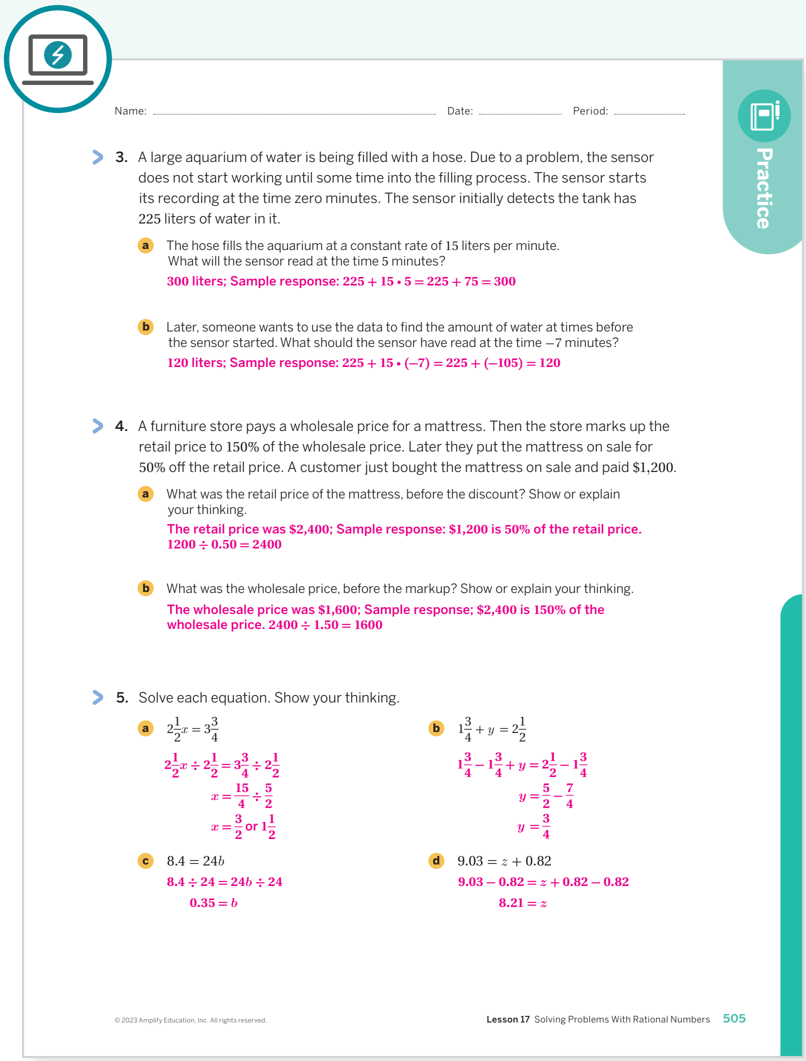
   March: \$1.70. Sample response:
   $-14.00 + (-82.70) + (-1.50) = -98.20$
   $99.90 + (-98.20) = 1.70$

   April: –\$836.90. Sample response:
   $-86.80 + (-570.00) + (-280.10) = -936.90$
   $100.00 + (-936.90) = -836.90$

   b. Determine the mean (average) total for the four months. Show your thinking.

   –\$123.75 each month; Sample response:
   $252.20 + 1.70 + 88 + (-836.90) = -495$
   $-495 \div 4 = -123.75$

504 **Unit 5** Rational Number Arithmetic



Name: .... Date: .... Period: ....

3. A large aquarium of water is being filled with a hose. Due to a problem, the sensor does not start working until some time into the filling process. The sensor starts its recording at the time zero minutes. The sensor initially detects the tank has 225 liters of water in it.

   a. The hose fills the aquarium at a constant rate of 15 liters per minute. What will the sensor read at the time 5 minutes?

   300 liters; Sample response: $225 + 15 \cdot 5 = 225 + 75 = 300$

   b. Later, someone wants to use the data to find the amount of water at times before the sensor started. What should the sensor have read at the time –7 minutes?

   120 liters; Sample response: $225 + 15 \cdot (-7) = 225 + (-105) = 120$

4. A furniture store pays a wholesale price for a mattress. Then the store marks up the retail price to 150% of the wholesale price. Later they put the mattress on sale for 50% off the retail price. A customer just bought the mattress on sale and paid \$1,200.

   a. What was the retail price of the mattress, before the discount? Show or explain your thinking.

   The retail price was \$2,400; Sample response: \$1,200 is 50% of the retail price. $1200 \div 0.50 = 2400$

   b. What was the wholesale price, before the markup? Show or explain your thinking.

   The wholesale price was \$1,600; Sample response; \$2,400 is 150% of the wholesale price. $2400 \div 1.50 = 1600$

5. Solve each equation. Show your thinking.

   a. $2\frac{1}{2}x = 3\frac{3}{4}$

   $$2\frac{1}{2}x \div 2\frac{1}{2} = 3\frac{3}{4} \div 2\frac{1}{2}$$
   $$x = \frac{15}{4} \div \frac{5}{2}$$
   $$x = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

   b. $1\frac{3}{4} + y = 2\frac{1}{2}$

   $$1\frac{3}{4} - 1\frac{3}{4} + y = 2\frac{1}{2} - 1\frac{3}{4}$$
   $$y = \frac{5}{2} - \frac{7}{4}$$
   $$y = \frac{3}{4}$$

   c. $8.4 = 24b$

   $$8.4 \div 24 = 24b \div 24$$
   $$0.35 = b$$

   d. $9.03 = z + 0.82$

   $$9.03 - 0.82 = z + 0.82 - 0.82$$
   $$8.21 = z$$

Lesson 17 Solving Problems With Rational Numbers 505

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 4	2
Formative	5	Unit 5 Lesson 18	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 18**

Solving Equations With Rational Numbers

Let's solve equations that include negative numbers.

Focus

Goals

1. **Language Goal:** Explain how to solve an equation of the form $x + p = q$ or $px = q$, where p, q, and x are rational numbers. **(Speaking and Listening, Writing)**
2. **Language Goal:** Generalize the usefulness of additive inverses and multiplicative inverses for solving equations of the forms $x + p = q$ and $px = q$. **(Speaking and Listening)**

Coherence

Today

Students expand on their understanding of solving equations of the forms $p + x = q$ and $px = q$, where all values are positive, to solving equations of the same form with rational values. They determine that additive and multiplicative inverses can be used whether the values are positive or negative.

Previously

In Grade 6, students solved equations of the forms $p + x = q$ and $px = q$, where p, q, and x were all non-negative values.

Coming Soon

In Unit 6, students will build on their understanding of solving equations of the forms $p + x = q$ and $px = q$ with rational values to solve equations of the forms $px + r = q$ and $p(x + q) = r$.

Rigor

- Students build **conceptual understanding** of solving equations of the forms $x + p = q$ and $px = q$ with rational values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	8 min	13 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor chart PDF, *Solving Equations* (for display)
- Anchor chart PDF, *Solving Equations* (answers)

Math Language Development

Review words

- *additive inverse*
- *inverse operation*
- *multiplicative inverse*
- *solution*

Amps | Featured Activity

Exit Ticket
Real-Time Exit Ticket

Check in real time that your students understand methods for solving equations with rational numbers by using a digital Exit Ticket that is automatically scored.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not yet see patterns when solving equations in Activity 3. They should focus on the structures they know that help them keep the equations true throughout the process. They might reach a point where they are unsure what to do, but, if they are organized and have applied the structure of recording each step of the solution process, others can easily see their thinking and guide them to the next steps.

Modifications to Pacing

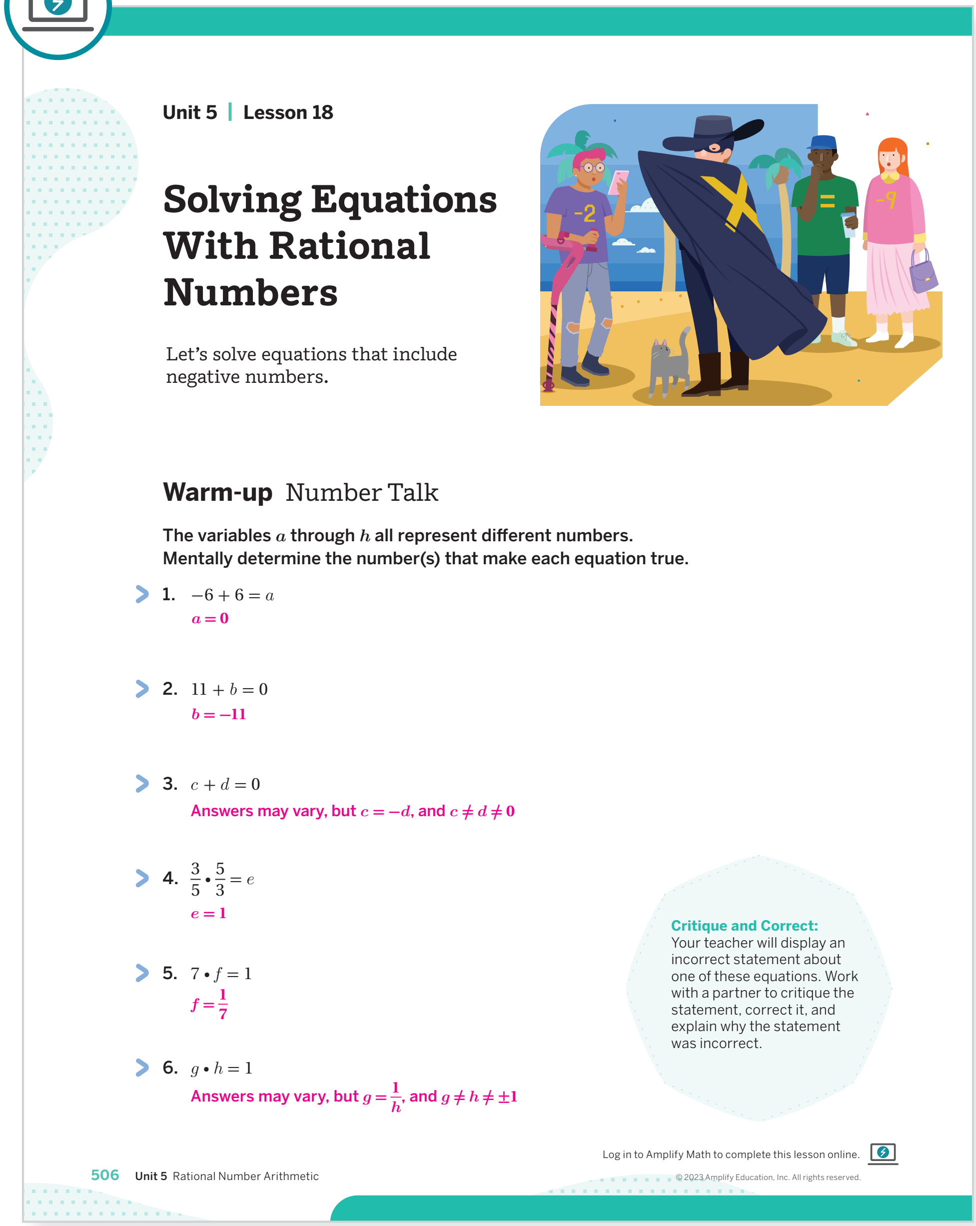
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 3**, have students match the inverse operations without solving the equations. Assign the final column as a practice problem.

Independent | 5 min

Warm-up Number Talk

Students determine the missing value(s) in equations by using their prior knowledge of additive and multiplicative inverses.



Unit 5 | **Lesson 18**

Solving Equations With Rational Numbers

Let's solve equations that include negative numbers.

Warm-up Number Talk

The variables a through h all represent different numbers. Mentally determine the number(s) that make each equation true.

1. $-6+6=a$
   $a=0$
2. $11+b=0$
   $b=-11$
3. $c+d=0$
   Answers may vary, but $c=-d$, and $c \neq d \neq 0$
4. $\frac{3}{5} \cdot \frac{5}{3}=e$
   $e=1$
5. $7 \cdot f=1$
   $f=\frac{1}{7}$
6. $g \cdot h=1$
   Answers may vary, but $g=\frac{1}{h}$, and $g \neq h \neq \pm 1$

Critique and Correct: Your teacher will display an incorrect statement about one of these equations. Work with a partner to critique the statement, correct it, and explain why the statement was incorrect.

Log in to Amplify Math to complete this lesson online.

506 **Unit 5** Rational Number Arithmetic

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① Launch

Conduct the ***Number Talk*** routine, displaying one equation at a time.

② Monitor

Help students get started by asking, "What does it mean to make an equation true?"

Look for points of confusion:

- **Using inverse operations to solve Problems 2 and 5.** Ask students to reason about what they notice given the values in each equation.

Look for productive strategies:

- Choosing additive inverses for Problem 3 and multiplicative inverses for Problem 6.

③ Connect

Have students share their reasoning for each problem. Record and display the responses.

Ask:

- "What strategies or relationships did you use to help you to determine the unknown values?"
- "Who can restate ___'s reasoning in a different way?"
- "Does anyone want to add on to ___'s strategy?"
- "Do you agree or disagree? Why?"

Highlight that the sum of a number and its opposite is 0. To determine the number added to a number that will produce 0 as a sum, use its opposite (additive inverse). The product of a number and its reciprocal is 1. To determine the number multiplied by a value to produce 1 as a product, use its reciprocal (multiplicative inverse).

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement, such as "The value of e in Problem 1 is 0 because $\frac{3}{5}$ and $\frac{5}{3}$ are inverses." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking." Listen for students who recognize that these fractions are *multiplicative inverses* (not additive), which means their product is 1, not 0.
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

Power-up

To power up students' ability to solve equations of the form $x+p=q$ and $px=q$, have students complete:

For each equation, circle the operation that could be done on both sides to solve it.

1. $6+x=12$ Subtract 12 [✓ Subtract 6]
2. $0.2x=5.4$ Subtract 0.2 [✓ Divide by 0.2]
3. $4=\frac{2}{3}y$ [✓ Divide by $\frac{2}{3}$] Multiply by $\frac{2}{3}$

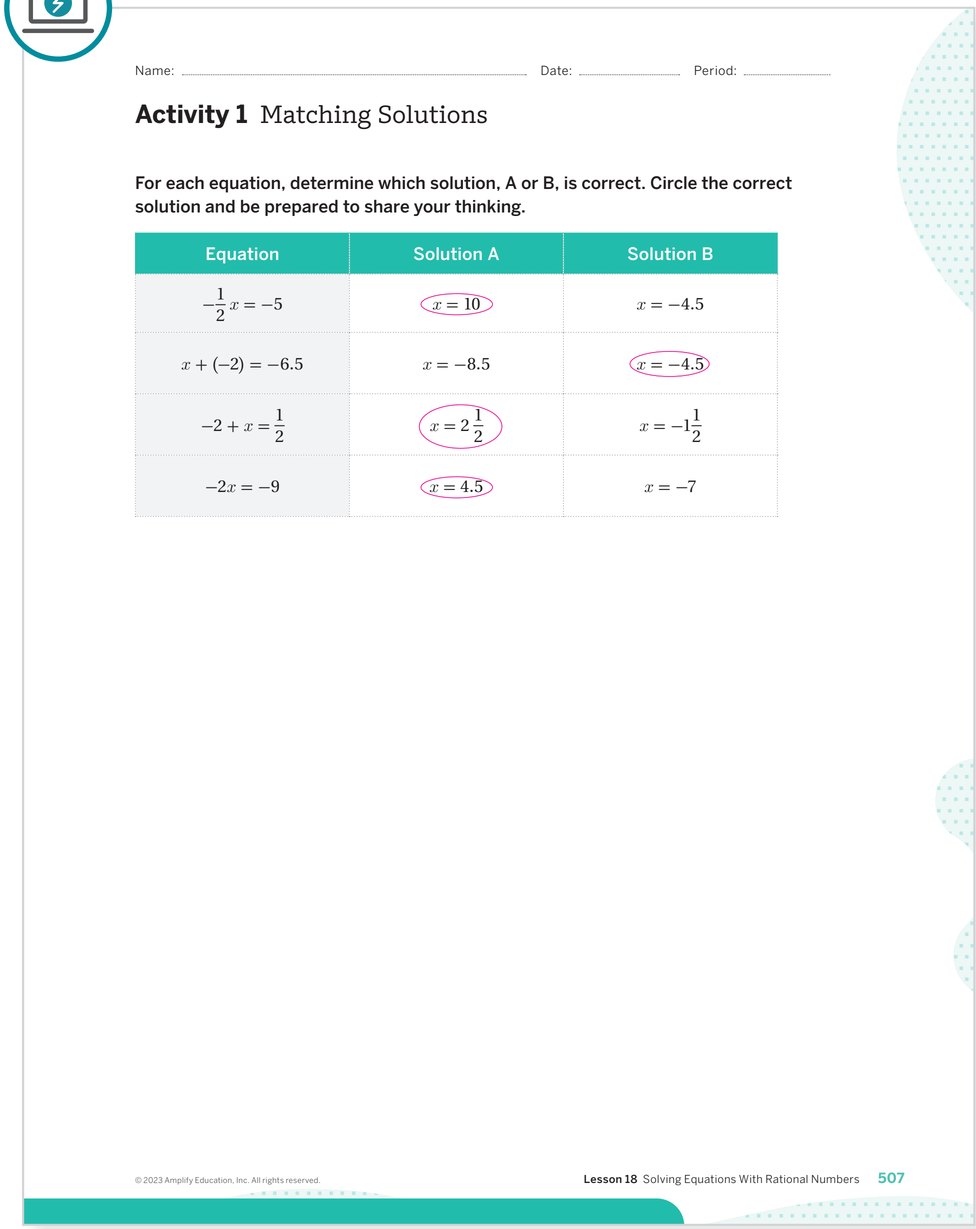
Use: Before the Warm-up.

Informed by: Performance on Lesson 17, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 7.

Pairs | 8 min

Activity 1 Matching Solutions

Students connect their understanding of solving equations of the forms $x + p = q$ and $px = q$ with rational numbers by determining what value makes each equation true.



Name: ________ Date: ________ Period: ________

Activity 1 Matching Solutions

For each equation, determine which solution, A or B, is correct. Circle the correct solution and be prepared to share your thinking.

Equation	Solution A	Solution B
$-\frac{1}{2}x = -5$	[✓ $x = 10$]	$x = -4.5$
$x + (-2) = -6.5$	$x = -8.5$	[✓ $x = -4.5$]
$-2 + x = \frac{1}{2}$	[✓ $x = 2\frac{1}{2}$]	$x = -1\frac{1}{2}$
$-2x = -9$	[✓ $x = 4.5$]	$x = -7$

Lesson 18 Solving Equations With Rational Numbers 507

1 Launch

Activate students' prior knowledge by asking, "What does it mean for a value to be a solution to an equation?" Explain that students will work in pairs to determine the correct solution to each equation.

2 Monitor

Help students get started by asking what strategies they know for determining the solution to an equation.

Look for points of confusion:

- **Adding p to both sides of the equations of the form $px = q$ where p is a negative value.** Ask, "What operation is 'connecting' p and q?"
- **Mixing up the rules of signs for addition and multiplication.** Remind students the rules for determining the sign of the sum or product of two rational values.

3 Connect

Display each equation to the class. Conduct the *Poll the Class* routine to determine the correct solution for each equation.

Have students share their methods for determining the correct solution with the class.

Ask, "Without using substitution, how can you solve equations of the form $x + p = q$? What about equations of the form $px = q$?"

Highlight that to solve an equation of the form $x + p = q$ using inverse operations, subtract p from both sides. To solve equations of the form $px = q$ using inverse operations, divide both sides by p. This is the same as multiplying by $\frac{1}{p}$, the reciprocal. Model the two methods for multiplication using the first and last equation in the table.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously learned to solve one-step equations involving positive numbers in Grade 6. Ask them what strategies they would use to solve the equation $\frac{1}{2}x = 5$, and how those strategies can help them solve the similar equation shown in the table, now with negative signs added.

Accessibility: *Clarify Vocabulary and Symbols*

Provide access to colored pencils and suggest that students color code the equations that are of the form $x + p = q$ in one color and equations of the form $px = q$ in another color.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students compare the various methods used, consider adding a table similar to the following to the class display.

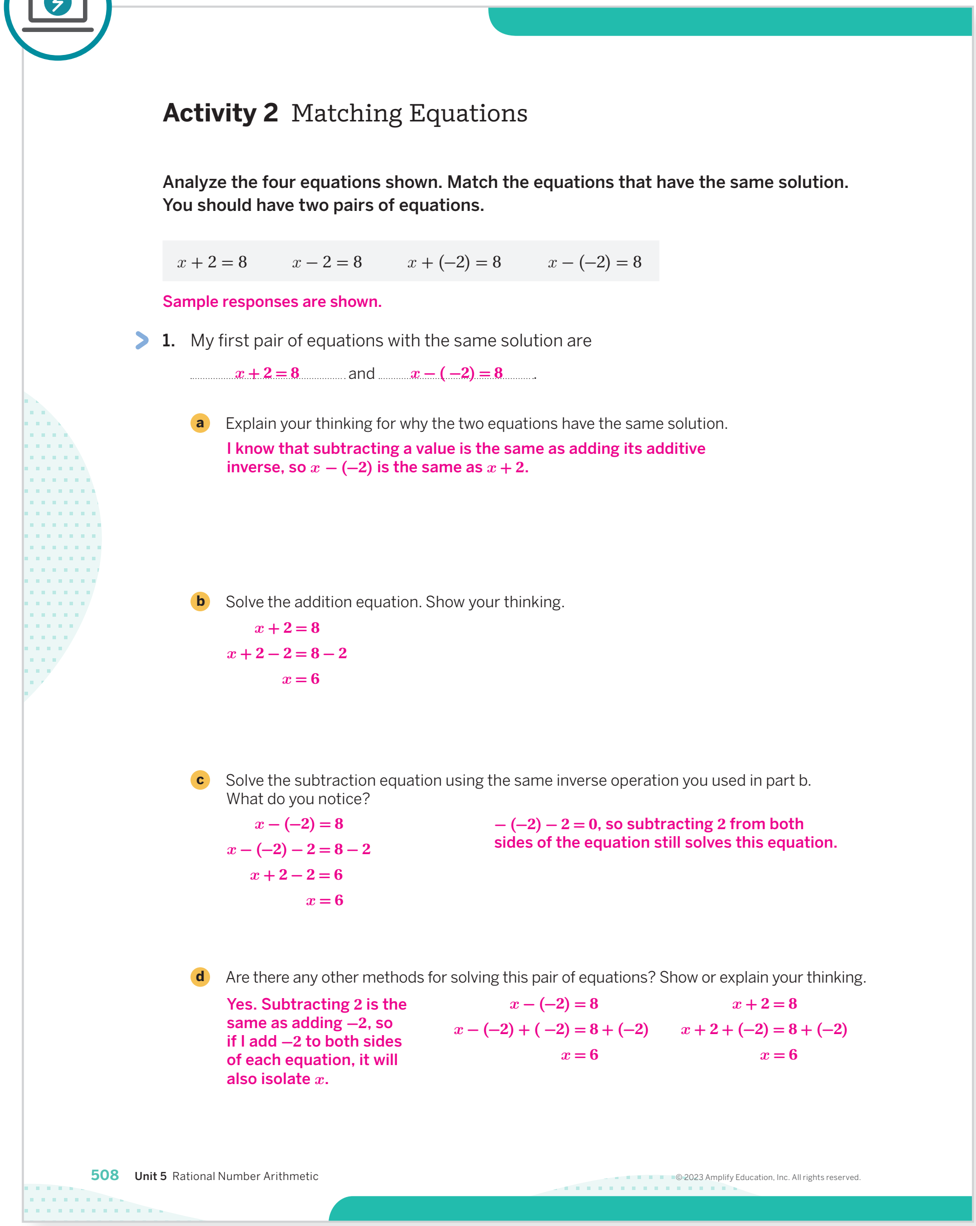
Solving equations of the forms $x + p = q$ and $px = q$:

Arithmetic method:	Algebraic method:
Substitute values for the variable to determine which value makes the equation true.	Use inverse operations. • For equations of the form $x + p = q$, subtract p from each side. • For equations of the form $px = q$, divide both sides by p.

Pairs | 13 min

Activity 2 Matching Equations

Students match equivalent equations to develop the process for solving equations in the form $x - p = q$.



Activity 2 Matching Equations

Analyze the four equations shown. Match the equations that have the same solution. You should have two pairs of equations.

$x + 2 = 8$	$x - 2 = 8$	$x + (-2) = 8$	$x - (-2) = 8$

Sample responses are shown.

1. My first pair of equations with the same solution are $x + 2 = 8$ and $x - (-2) = 8$.

   a. Explain your thinking for why the two equations have the same solution.

   I know that subtracting a value is the same as adding its additive inverse, so $x - (-2)$ is the same as $x + 2$.

   b. Solve the addition equation. Show your thinking.

   $$\begin{aligned} x + 2 &= 8 \\ x + 2 - 2 &= 8 - 2 \\ x &= 6 \end{aligned}$$

   c. Solve the subtraction equation using the same inverse operation you used in part b. What do you notice?

   $$\begin{aligned} x - (-2) &= 8 \\ x - (-2) - 2 &= 8 - 2 \\ x + 2 - 2 &= 6 \\ x &= 6 \end{aligned}$$

   $-(-2) - 2 = 0$, so subtracting 2 from both sides of the equation still solves this equation.

   d. Are there any other methods for solving this pair of equations? Show or explain your thinking.

   Yes. Subtracting 2 is the same as adding –2, so if I add –2 to both sides of each equation, it will also isolate x.

   $$\begin{aligned} x - (-2) &= 8 \\ x - (-2) + (-2) &= 8 + (-2) \\ x &= 6 \end{aligned}$$

   $$\begin{aligned} x + 2 &= 8 \\ x + 2 + (-2) &= 8 + (-2) \\ x &= 6 \end{aligned}$$

508 **Unit 5** Rational Number Arithmetic

1 Launch

Activate students' prior knowledge by asking, "What is another way you can write the expression $5 - (-2)$?" $5 + 2$ Explain that students will use their understanding of equivalent expressions to match equations with the same solution, prior to solving them.

2 Monitor

Help students get started by asking, "How do you change a subtraction expression to an addition expression?"

Look for points of confusion:

- **Matching the equations that have the same operation, not the equations that have the same solution.** Ask, "How would you solve $x + 2 = 8$? Is that the same way that you would solve $x + (-2) = 8$?"

Look for productive strategies:

- Writing each equation with subtraction as addition in order to determine the equations with the same solution.
- Recognizing that subtracting a value is the same as adding its additive inverse to determine that there are two methods for solving equations of the form $x + p = q$.

Activity 2 continued >

Math Language Development

MLR8: Discussion Supports

During the Connect, display sentence frames to support students as they explain their reasoning for each match. For example:

- "___ matches ___ because . . ."
- "I know that ___ and ___ are additive inverses because . . ."

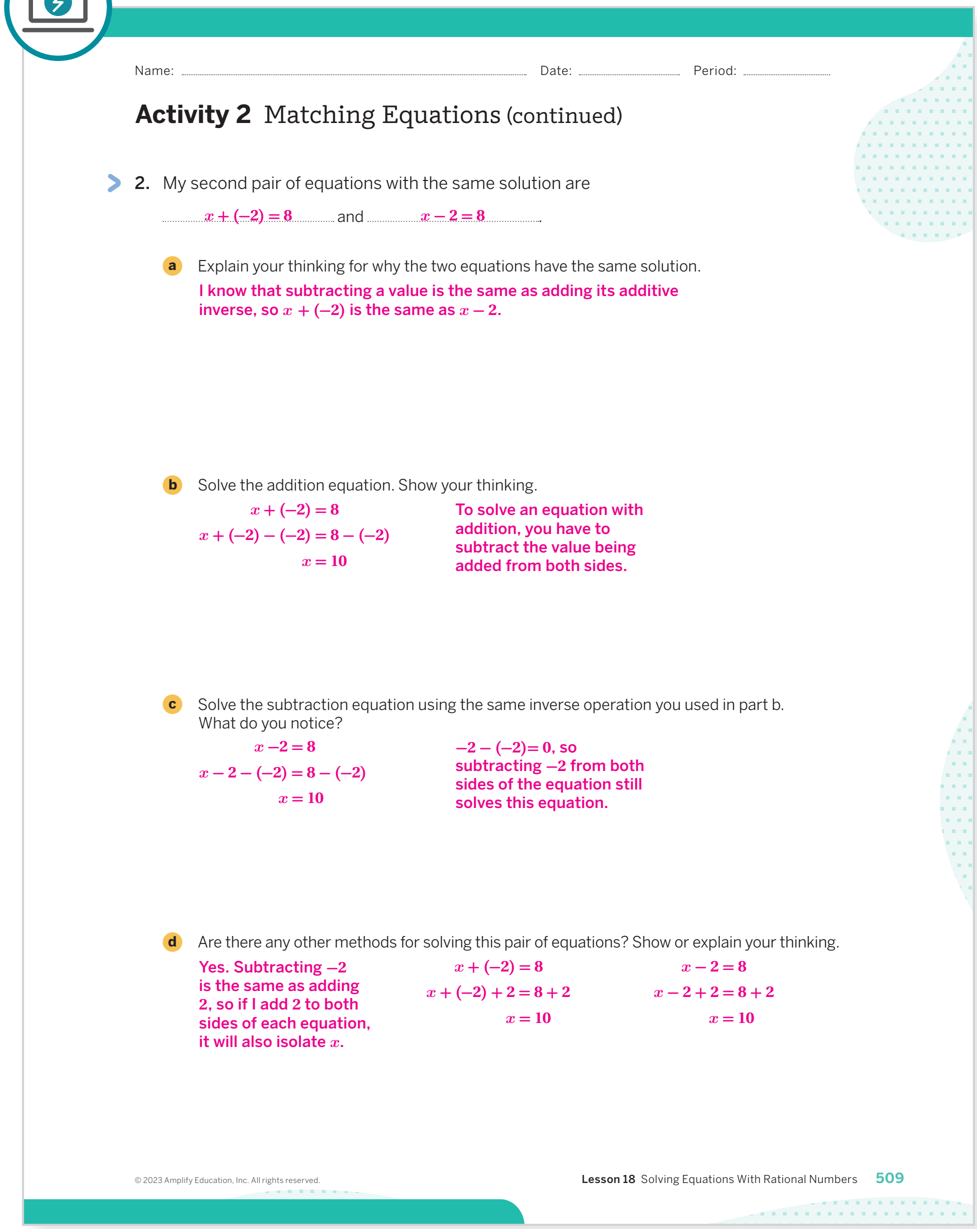
Encourage students to respond to the matches their partner makes using:

- "I agree, because . . ."
- "I disagree, because . . ."

Pairs | 13 min

Activity 2 Matching Equations (continued)

Students match equivalent equations to develop the process for solving equations in the form $x - p = q$.



Name: ________ Date: ________ Period: ________

Activity 2 Matching Equations (continued)

2. My second pair of equations with the same solution are $x + (-2) = 8$ and $x - 2 = 8$.

   a. Explain your thinking for why the two equations have the same solution.
   I know that subtracting a value is the same as adding its additive inverse, so $x + (-2)$ is the same as $x - 2$.

   b. Solve the addition equation. Show your thinking.

   $$x + (-2) = 8$$
   $$x + (-2) - (-2) = 8 - (-2)$$
   $$x = 10$$

   To solve an equation with addition, you have to subtract the value being added from both sides.

   c. Solve the subtraction equation using the same inverse operation you used in part b. What do you notice?

   $$x -2 = 8$$
   $$x - 2 - (-2) = 8 - (-2)$$
   $$x = 10$$

   $-2 - (-2) = 0$, so subtracting -2 from both sides of the equation still solves this equation.

   d. Are there any other methods for solving this pair of equations? Show or explain your thinking.
   Yes. Subtracting -2 is the same as adding 2, so if I add 2 to both sides of each equation, it will also isolate x.

   $$x + (-2) = 8$$
   $$x + (-2) + 2 = 8 + 2$$
   $$x = 10$$

   $$x - 2 = 8$$
   $$x - 2 + 2 = 8 + 2$$
   $$x = 10$$

3 Connect

Display the two pairs of matching equations to the class.

Have students share how they determined their matches as well as methods for solving each equation.

Highlight that to solve equations of the form $x + p = q$, there are two methods. Students can either subtract p, or add $(-p)$. When p is a negative value, it is generally more efficient to solve the equation $x - (-p) = q$ by adding $(-p)$ to both sides.

Likewise, there are two methods for solving equations of the form $x - p = q$. Students can either add p to both sides, or subtract $(-p)$. When p is a negative value, it is generally more efficient to solve the equation $x + -(-p) = q$ by subtracting $(-p)$ to both sides.

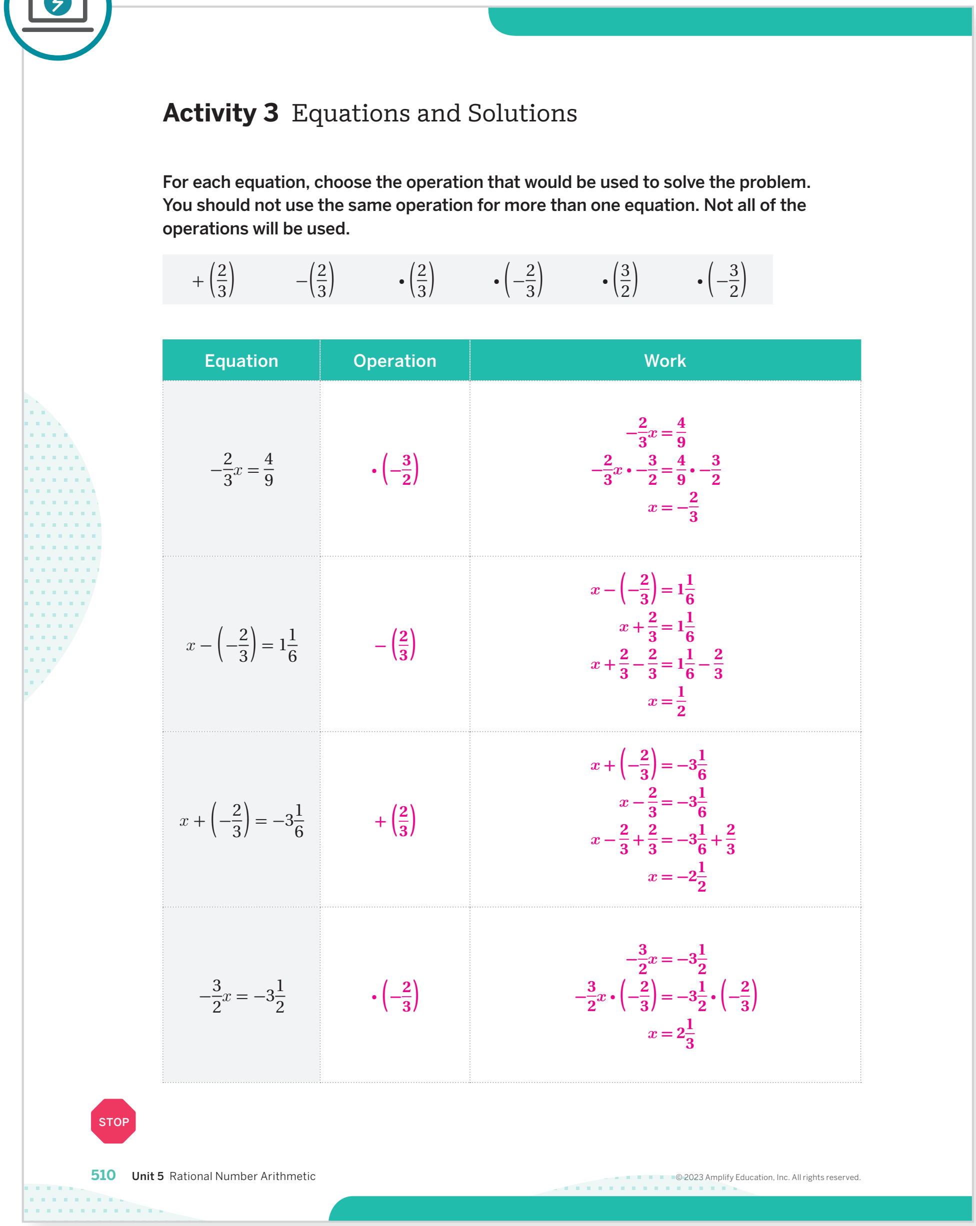
Ask:

- "How can you solve the equation $x - 3 = 6$?"
  Sample response: Add 3 or subtract (–3) from both sides.
- "How can you solve the equation $x - (-3) = 6$?"
  Sample response: Rewrite the equation as $x + 3 = 6$, then subtract 3 from or add (–3) to both sides.
- "How can you solve the equation $x + (-3) = 6$?"
  Sample response: Add 3 or subtract (–3) from both sides.

Small Groups | 10 min

Activity 3 Equations and Solutions

Students match equations involving rational numbers to the inverse operations needed to solve them prior to determining the solution.



Activity 3 Equations and Solutions

For each equation, choose the operation that would be used to solve the problem. You should not use the same operation for more than one equation. Not all of the operations will be used.

$+\left(\frac{2}{3}\right)$ $-\left(\frac{2}{3}\right)$ $\cdot\left(\frac{2}{3}\right)$ $\cdot\left(-\frac{2}{3}\right)$ $\cdot\left(\frac{3}{2}\right)$ $\cdot\left(-\frac{3}{2}\right)$

Equation	Operation	Work
$-\frac{2}{3}x = \frac{4}{9}$	$\cdot\left(-\frac{3}{2}\right)$	$-\frac{2}{3}x = \frac{4}{9}$ $-\frac{2}{3}x \cdot -\frac{3}{2} = \frac{4}{9} \cdot -\frac{3}{2}$ $x = -\frac{2}{3}$
$x - \left(-\frac{2}{3}\right) = 1\frac{1}{6}$	$-\left(\frac{2}{3}\right)$	$x - \left(-\frac{2}{3}\right) = 1\frac{1}{6}$ $x + \frac{2}{3} = 1\frac{1}{6}$ $x + \frac{2}{3} - \frac{2}{3} = 1\frac{1}{6} - \frac{2}{3}$ $x = \frac{1}{2}$
$x + \left(-\frac{2}{3}\right) = -3\frac{1}{6}$	$+\left(\frac{2}{3}\right)$	$x + \left(-\frac{2}{3}\right) = -3\frac{1}{6}$ $x - \frac{2}{3} = -3\frac{1}{6}$ $x - \frac{2}{3} + \frac{2}{3} = -3\frac{1}{6} + \frac{2}{3}$ $x = -2\frac{1}{2}$
$-\frac{3}{2}x = -3\frac{1}{2}$	$\cdot\left(-\frac{2}{3}\right)$	$-\frac{3}{2}x = -3\frac{1}{2}$ $-\frac{3}{2}x \cdot \left(-\frac{2}{3}\right) = -3\frac{1}{2} \cdot \left(-\frac{2}{3}\right)$ $x = 2\frac{1}{3}$

STOP

510 **Unit 5** Rational Number Arithmetic

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1 Launch

Explain that students will be matching each equation with the inverse operation that can be used to solve it. Clarify that each operation will only be used once, and there will be operations remaining.

2 Monitor

Help students get started asking, "Are there any equations that you feel confident in solving?" Remind them that they do not need to solve the equations in the order they were written.

Look for points of confusion:

- **Forgetting that multiplication equations can be solved by using the multiplicative inverse.** Ask, "How would you simplify $6 \div \frac{2}{3}$? How can you apply this strategy to solving equations?"

Look for productive strategies:

- Rewriting the second and third equation using the additive inverse, and then solving.

3 Connect

Display each equation to the class. Use the *Poll the Class* routine to come to consensus on which operation is used for each equation.

Ask, "How did you determine which operation matched each equation?"

Have students share how they used each operation to solve the equations.

Highlight multiplicative inverses require that the numbers have the same sign in order for the product to be positive. This means that negative numbers require a negative multiplicative inverse and positive numbers require a positive inverse. Contrast this with additive inverses, which must have opposite signs in order for their sum to be 0.

Differentiated Support

Accessibility: *Guide Processing and Visualizatio*

Suggest that students focus on the second and third equation first. Ask, "How are these similar? How are they different?" Suggest that it may be more efficient to solve subtraction equations if they first write them using the additive inverse.

Math Language Development

MLR8: Discussion Supports

During the Connect, as you use the *Poll the Class* routine, encourage students to work through their disagreements until they come to a consensus. Display sentence frames to support students as they explain their reasoning for each operation they chose. For example:

- "I chose the operation ___ because . . ."
- The equation is of the form ___, so I . . ."
- "To solve an addition/multiplication equation, use the inverse operation, which is . . ."

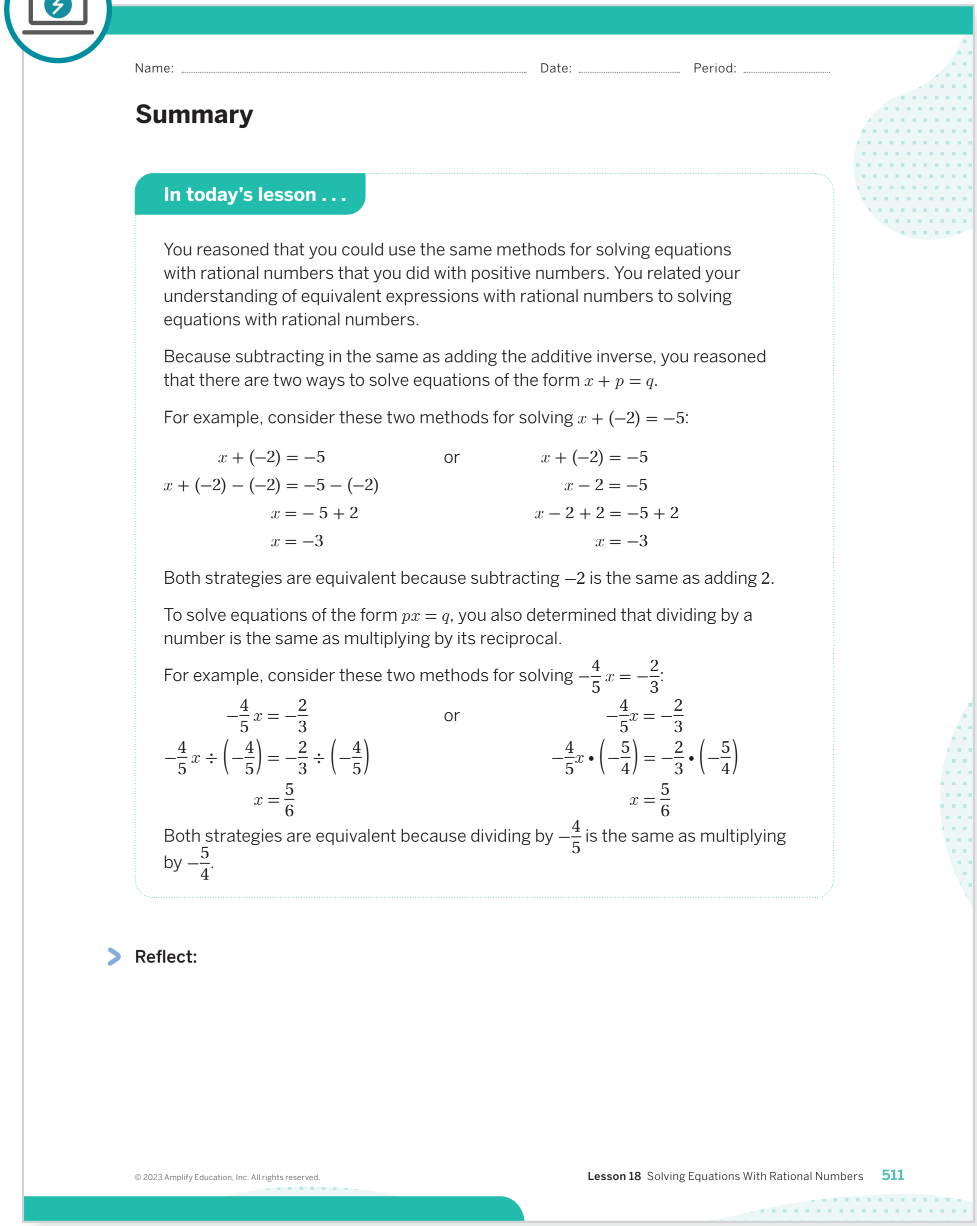
Encourage students to respond with whether they agree by using:

- "I agree, because . . ."
- "I disagree, because . . ."

Whole Class | 5 min

Summary

Review and synthesize that there are two methods for solving equations of the forms $x + p = q$ and $px = q$ when p, x, and q are rational numbers.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You reasoned that you could use the same methods for solving equations with rational numbers that you did with positive numbers. You related your understanding of equivalent expressions with rational numbers to solving equations with rational numbers.

Because subtracting in the same as adding the additive inverse, you reasoned that there are two ways to solve equations of the form $x + p = q$.

For example, consider these two methods for solving $x + (-2) = -5$:

$$\begin{aligned} x + (-2) &= -5 \\ x + (-2) - (-2) &= -5 - (-2) \\ x &= -5 + 2 \\ x &= -3 \end{aligned}$$

or

$$\begin{aligned} x + (-2) &= -5 \\ x - 2 &= -5 \\ x - 2 + 2 &= -5 + 2 \\ x &= -3 \end{aligned}$$

Both strategies are equivalent because subtracting –2 is the same as adding 2.

To solve equations of the form $px = q$, you also determined that dividing by a number is the same as multiplying by its reciprocal.

For example, consider these two methods for solving $-\frac{4}{5}x = -\frac{2}{3}$:

$$\begin{aligned} -\frac{4}{5}x &= -\frac{2}{3} \\ -\frac{4}{5}x \div \left(-\frac{4}{5}\right) &= -\frac{2}{3} \div \left(-\frac{4}{5}\right) \\ x &= \frac{5}{6} \end{aligned}$$

or

$$\begin{aligned} -\frac{4}{5}x &= -\frac{2}{3} \\ -\frac{4}{5}x \bullet \left(-\frac{5}{4}\right) &= -\frac{2}{3} \bullet \left(-\frac{5}{4}\right) \\ x &= \frac{5}{6} \end{aligned}$$

Both strategies are equivalent because dividing by $-\frac{4}{5}$ is the same as multiplying by $-\frac{5}{4}$.

Reflect:

Synthesize

Display each equation from the Anchor Chart PDF, *Solving Equations* one at a time.

Ask:

- "What is another way to write the first equation?" $x - 2 = -5$
- "What are the two ways to solve the first equation?" Subtract –2 or add 2 to both sides of the equation.
- "What are the two ways to solve the second equation?" Subtract 3 or add –3 to both sides of the equation.
- "What are the two ways to solve the second equation?" Divide by $-\frac{2}{3}$ or multiply by $-\frac{3}{2}$.

Complete the missing information in the chart as a class.

Highlight that when solving equations with rational numbers, it can be helpful to use the additive inverse or the multiplicative inverse to isolate the variable.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is solving problems with rational numbers the same or different from solving problems with non-negative numbers?"

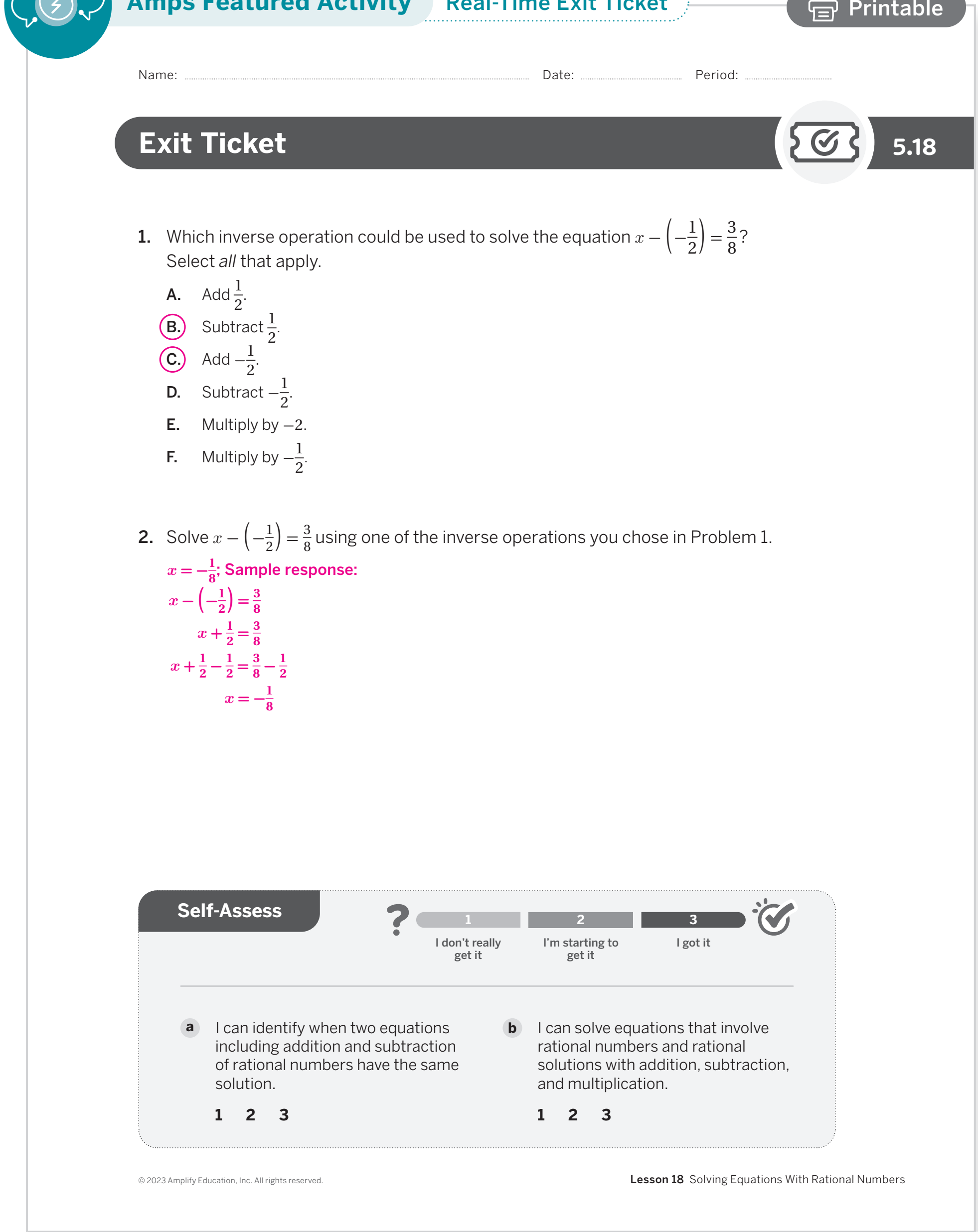
Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to solve equations with rational numbers.

Amps Featured Activity Real-Time Exit Ticket

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.18

1. Which inverse operation could be used to solve the equation $x-\left(-\frac{1}{2}\right)=\frac{3}{8}$? Select *all* that apply.
   - A. Add $\frac{1}{2}$.
   - [✓ B.] Subtract $\frac{1}{2}$.
   - [✓ C.] Add $-\frac{1}{2}$.
   - D. Subtract $-\frac{1}{2}$.
   - E. Multiply by -2.
   - F. Multiply by $-\frac{1}{2}$.

2. Solve $x-\left(-\frac{1}{2}\right)=\frac{3}{8}$ using one of the inverse operations you chose in Problem 1.

   $x=-\frac{1}{8}$; **Sample response:**

   $x-\left(-\frac{1}{2}\right)=\frac{3}{8}$

   $x+\frac{1}{2}=\frac{3}{8}$

   $x+\frac{1}{2}-\frac{1}{2}=\frac{3}{8}-\frac{1}{2}$

   $x=-\frac{1}{8}$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can identify when two equations including addition and subtraction of rational numbers have the same solution.

1 2 3

b I can solve equations that involve rational numbers and rational solutions with addition, subtraction, and multiplication.

1 2 3

Lesson 18 Solving Equations With Rational Numbers

Success looks like . . .

- **Language Goal:** Explaining how to solve an equation of the form $x+p=q$ or $px=q$, where p, q, and x are rational numbers. **(Speaking and Listening, Writing)**
  - » Selecting the inverse operations that could be used to solve the equation $x-(-\frac{1}{2})=\frac{3}{8}$ in Problem 1.
- **Language Goal:** Generalizing the usefulness of additive inverses and multiplicative inverses for solving equations of the forms $x+p=q$ and $px=q$. **(Speaking and Listening)**

Suggested next steps

If students incorrectly identify the operation that could be used to solve the equation in Problem 1, consider:

- Reviewing the strategies from Activity 2.
- Assigning Practice Problems 1 and 2.

If students incorrectly use their operation to solve the equation in Problem 2, consider:

- Reviewing the strategies from Activity 3.
- Assigning Practice Problems 2 and 3.

Professional Learning

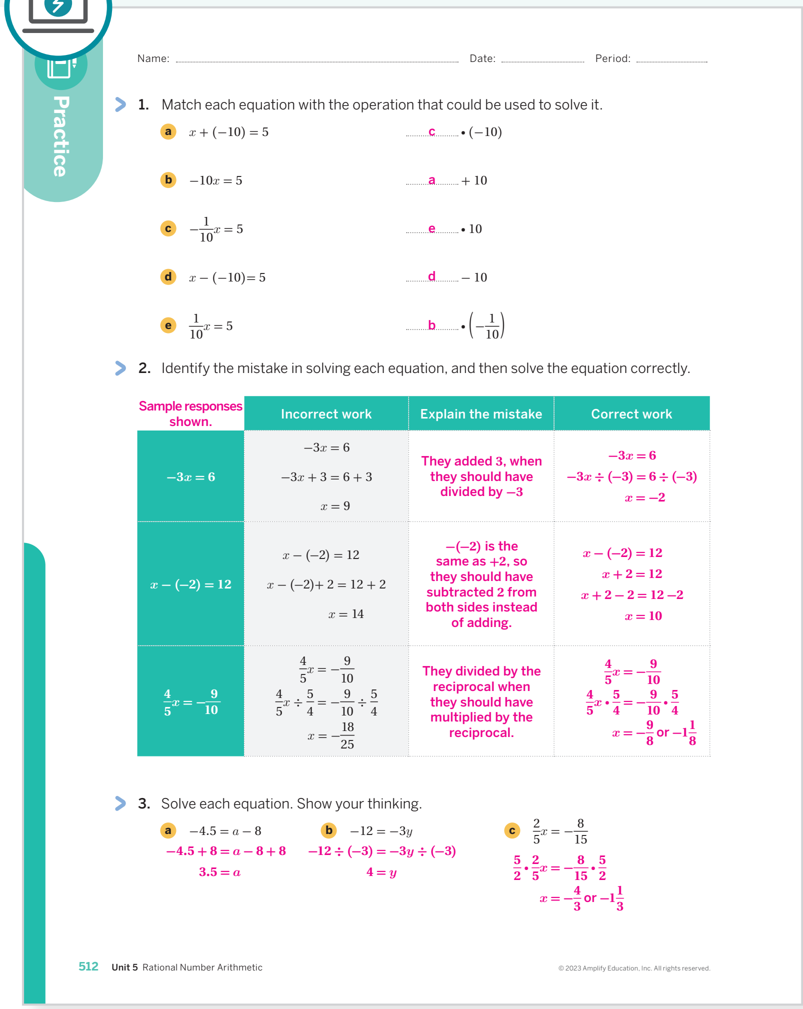
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did you see in the way some students approached Activity 1 that you would like other students to try?
- How did students attend to precision today? How are you helping students become aware of how they are progressing in this area? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Match each equation with the operation that could be used to solve it.

a	$x + (-10) = 5$	c	$\cdot (-10)$
b	$-10x = 5$	a	$+ 10$
c	$-\frac{1}{10}x = 5$	e	$\cdot 10$
d	$x - (-10) = 5$	d	$- 10$
e	$\frac{1}{10}x = 5$	b	$\cdot \left(-\frac{1}{10}\right)$

2. Identify the mistake in solving each equation, and then solve the equation correctly.

Sample responses shown.	Incorrect work	Explain the mistake	Correct work
$-3x = 6$	$-3x = 6$ $-3x + 3 = 6 + 3$ $x = 9$	They added 3, when they should have divided by −3	$-3x = 6$ $-3x \div (-3) = 6 \div (-3)$ $x = -2$
$x - (-2) = 12$	$x - (-2) = 12$ $x - (-2) + 2 = 12 + 2$ $x = 14$	−(−2) is the same as +2, so they should have subtracted 2 from both sides instead of adding.	$x - (-2) = 12$ $x + 2 = 12$ $x + 2 - 2 = 12 - 2$ $x = 10$
$\frac{4}{5}x = -\frac{9}{10}$	$\frac{4}{5}x = -\frac{9}{10}$ $\frac{4}{5}x \div \frac{5}{4} = -\frac{9}{10} \div \frac{5}{4}$ $x = -\frac{18}{25}$	They divided by the reciprocal when they should have multiplied by the reciprocal.	$\frac{4}{5}x = -\frac{9}{10}$ $\frac{4}{5}x \cdot \frac{5}{4} = -\frac{9}{10} \cdot \frac{5}{4}$ $x = -\frac{9}{8}$ or $-1\frac{1}{8}$

3. Solve each equation. Show your thinking.

a. $-4.5 = a - 8$

$-4.5 + 8 = a - 8 + 8$

$3.5 = a$

b. $-12 = -3y$

$-12 \div (-3) = -3y \div (-3)$

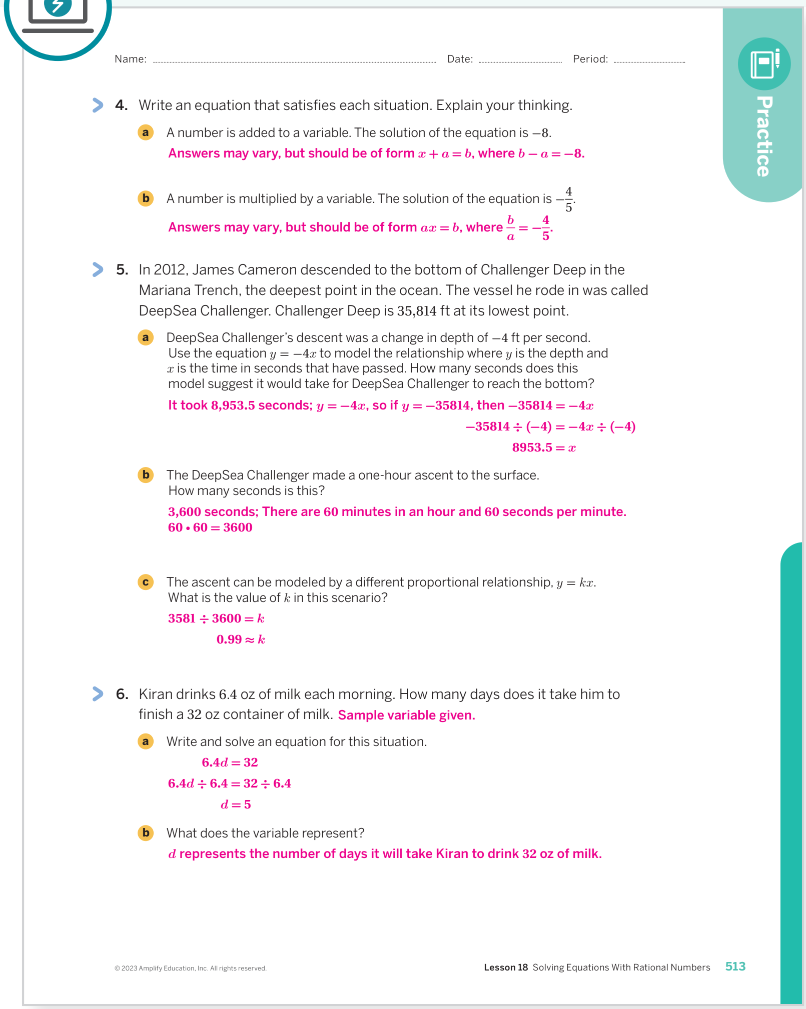
$4 = y$

c. $\frac{2}{5}x = -\frac{8}{15}$

$\frac{5}{2} \cdot \frac{2}{5}x = -\frac{8}{15} \cdot \frac{5}{2}$

$x = -\frac{4}{3}$ or $-1\frac{1}{3}$

512 **Unit 5** Rational Number Arithmetic



Name: ______ Date: ______ Period: ______

4. Write an equation that satisfies each situation. Explain your thinking.

a. A number is added to a variable. The solution of the equation is −8.

Answers may vary, but should be of form $x + a = b$, where $b - a = -8$.

b. A number is multiplied by a variable. The solution of the equation is $-\frac{4}{5}$.

Answers may vary, but should be of form $ax = b$, where $\frac{b}{a} = -\frac{4}{5}$.

5. In 2012, James Cameron descended to the bottom of Challenger Deep in the Mariana Trench, the deepest point in the ocean. The vessel he rode in was called DeepSea Challenger. Challenger Deep is 35,814 ft at its lowest point.

a. DeepSea Challenger's descent was a change in depth of −4 ft per second. Use the equation $y = -4x$ to model the relationship where y is the depth and x is the time in seconds that have passed. How many seconds does this model suggest it would take for DeepSea Challenger to reach the bottom?

It took 8,953.5 seconds; $y = -4x$, so if $y = -35814$, then $-35814 = -4x$

$-35814 \div (-4) = -4x \div (-4)$

$8953.5 = x$

b. The DeepSea Challenger made a one-hour ascent to the surface. How many seconds is this?

3,600 seconds; There are 60 minutes in an hour and 60 seconds per minute.
$60 \cdot 60 = 3600$

c. The ascent can be modeled by a different proportional relationship, $y = kx$. What is the value of k in this scenario?

$3581 \div 3600 = k$

$0.99 \approx k$

6. Kiran drinks 6.4 oz of milk each morning. How many days does it take him to finish a 32 oz container of milk. Sample variable given.

a. Write and solve an equation for this situation.

$6.4d = 32$

$6.4d \div 6.4 = 32 \div 6.4$

$d = 5$

b. What does the variable represent?

d represents the number of days it will take Kiran to drink 32 oz of milk.

Lesson 18 Solving Equations With Rational Numbers 513

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	1
	2	Activity 3	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 15	2
	5	Unit 5 Lesson 14	2
Formative ⚡	6	Unit 5 Lesson 19	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 19**

Representing Contexts With Equations

Let's write equations that represent scenarios.

Focus

Goals

1. **Language Goal:** Coordinate verbal descriptions, equations, and diagrams that represent the same scenario involving an unknown amount. **(Writing, Speaking and Listening)**
2. **Language Goal:** Write equations of the forms $x + p = q$ and $px = q$ to represent and solve a problem in an unfamiliar context.

Rigor

- Students build **conceptual understanding** of representing real-world scenarios involving rational numbers with equations.
- Students gain **fluency** in solving equations of the forms $x + p = q$ and $px = q$ with rational numbers.

Coherence

Today

Students are encouraged to think critically about equations to determine if the solution would be positive or negative without solving it. They build on the work in the previous lesson to match, write, and solve equations that represent real-world scenarios.

Previously

In Lesson 18, students solved equations of the forms $x + p = q$ and $px = q$ with rational values.

Coming Soon

In Unit 6, students will build on their understanding of solving equations of the forms $p + x = q$ and $px = q$ with rational values to solve equations of the forms $px + r = q$ and $p(x + q) = r$.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	15 min	5 min	5 min
Pairs	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (as needed, for display)
- Activity 1 PDF (answers)
- Activity 3 PDF, pre-cut cards, one card per group (optional)
- Activity 3 PDF (answers, optional)
- Anchor Chart PDF, *Solving Equations With Rational Numbers* (answers, as needed)
- materials for creating a poster (optional)

Math Language Development

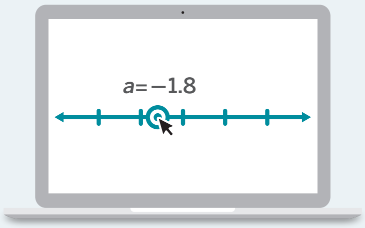
Review words

- *additive inverse*
- *inverse operation*
- *multiplicative inverse*
- *solution*

Amps ⋮ **Featured Activity**

Activity 2
Digital Arrow Diagrams

Students create digital arrow diagrams to represent real-world scenarios. You can monitor students' creations and display their diagrams during the whole-class discussion.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might struggle to find the equations that match the scenario and become frustrated or distracted. Students need to manage their energy to focus on finding them. Discuss strategies that they can use to help them maintain focus. Explain that through the abstract reasoning they can represent the scenario algebraically and by quantitative reasoning they can solve the equations.

Modifications to Pacing

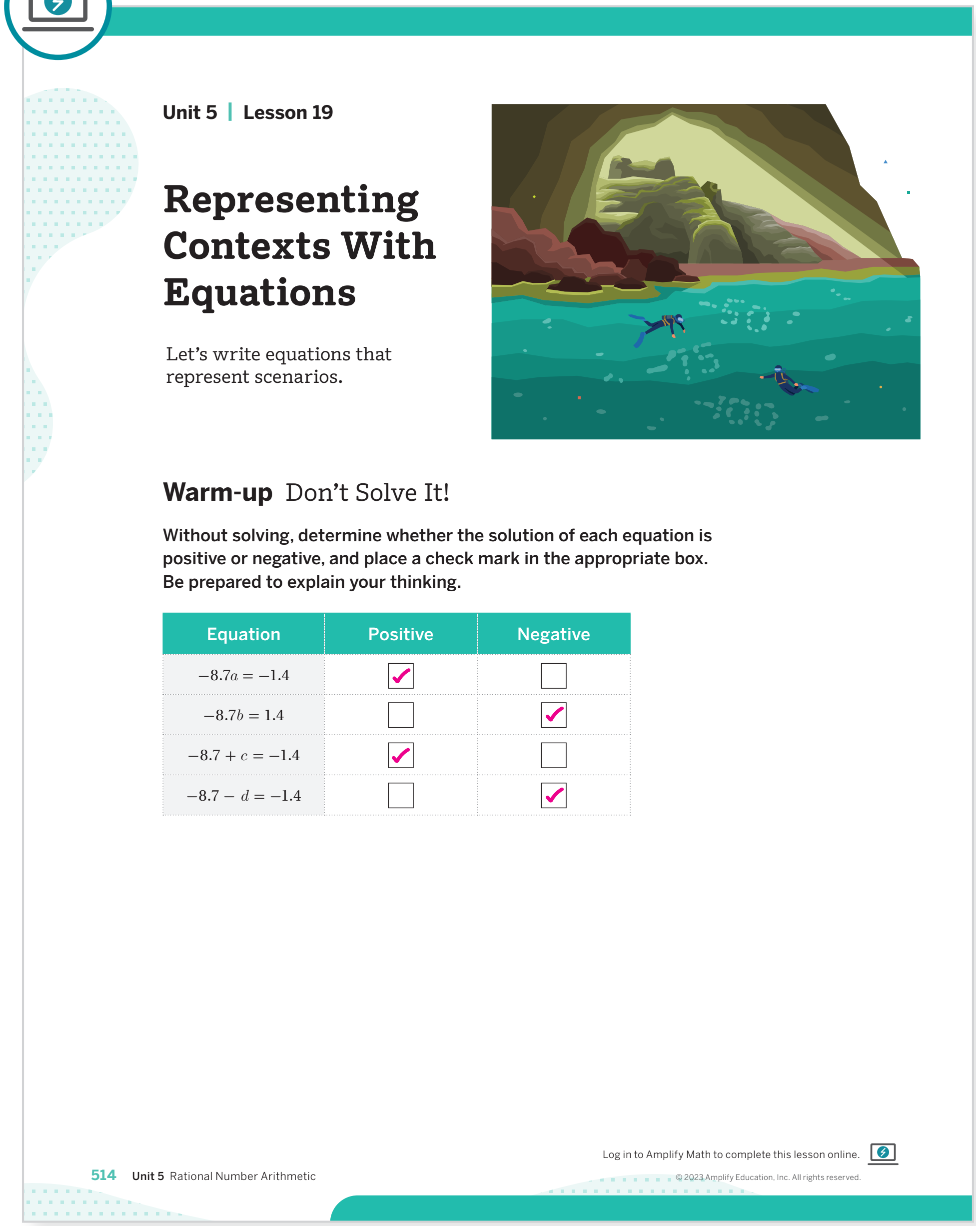
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students choose two scenarios to complete fully.
- In **Activity 2**, split the class such that half of the groups model each problem. Display and discuss both scenarios in the Connect.
- Optional **Activity 3** may be omitted.

Pairs | 5 min

Warm-up Don't Solve It!

Students apply their understanding of rational numbers to reason about whether the solution to an equation is positive or negative.



Unit 5 | Lesson 19

Representing Contexts With Equations

Let's write equations that represent scenarios.

Warm-up Don't Solve It!

Without solving, determine whether the solution of each equation is positive or negative, and place a check mark in the appropriate box. Be prepared to explain your thinking.

Equation	Positive	Negative
$-8.7a = -1.4$	☑	☐
$-8.7b = 1.4$	☐	☑
$-8.7 + c = -1.4$	☑	☐
$-8.7 - d = -1.4$	☐	☑

Log in to Amplify Math to complete this lesson online.

514 **Unit 5** Rational Number Arithmetic

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1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by asking, "When is the sign of the product of two rational numbers negative?"

Look for points of confusion:

- **Thinking that they need to solve each equation.** Remind students that they only need to determine the sign of the solution, not the solution.
- **Thinking that the penultimate equation must have a negative solution because the sum of two negatives is negative.** Ask, "Can you add two values with different signs and have a negative solution?"

Look for productive strategies:

- Focusing on the signs and magnitudes of the values in each equation to reason about the sign of the missing value.

3 Connect

Display each equation one at a time. Conduct the ***Poll the Class*** routine to determine if each solution is positive or negative.

Have students share how they determined the sign of each variable.

Ask:

- "How did determining the sign of c aid in determining the sign of d?"
- "How are the values of a and b related?"
- "How are the values of c and d related?"

Highlight that focusing on the signs of the numbers and the relative magnitudes without actually computing is helpful in determining whether the solution calculated is reasonable.

Power-up

To power up students' ability to write and use equations that represent proportional relationships, have students complete:

Write an equation that represents each scenario. Define what the variable represents in each equation. Sample responses shown.

1. Kiran walks at a speed of 2 mph. How far will he travel in 3.5 hours?
   $2 \cdot 3.5 = d$, where d represents distance in miles.
2. Han jogs at a speed of 5 mph. How long will it take him to jog 3 miles?
   $5t = 3$, where t represent the time in hours.

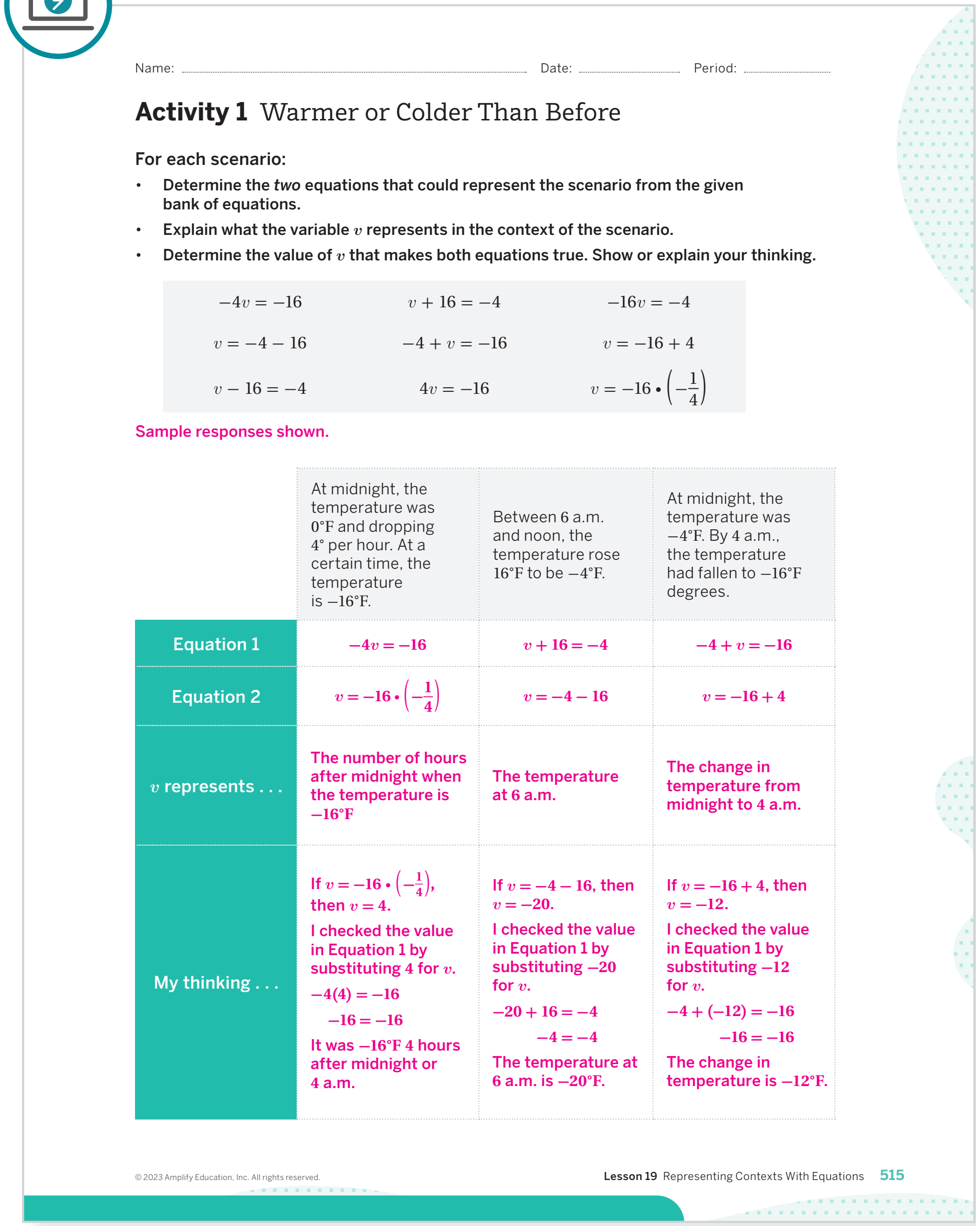
Use: Before Activity 1.

Informed by: Performance on Lesson 18, Practice Problem 6.

Small Groups | 15 min

Activity 1 Warmer or Colder Than Before?

Students build on their understanding of solving equations with rational numbers by matching scenarios with equations that represent them.



Name: ______ Date: ______ Period: ______

Activity 1 Warmer or Colder Than Before

For each scenario:

- **Determine the *two* equations that could represent the scenario from the given bank of equations.**
- **Explain what the variable v represents in the context of the scenario.**
- **Determine the value of v that makes both equations true. Show or explain your thinking.**

$-4v = -16$	$v + 16 = -4$	$-16v = -4$
$v = -4 - 16$	$-4 + v = -16$	$v = -16 + 4$
$v - 16 = -4$	$4v = -16$	$v = -16 \cdot \left(-\frac{1}{4}\right)$

Sample responses shown.

	At midnight, the temperature was 0°F and dropping 4° per hour. At a certain time, the temperature is −16°F.	Between 6 a.m. and noon, the temperature rose 16°F to be −4°F.	At midnight, the temperature was −4°F. By 4 a.m., the temperature had fallen to −16°F degrees.
Equation 1	$-4v = -16$	$v + 16 = -4$	$-4 + v = -16$
Equation 2	$v = -16 \cdot \left(-\frac{1}{4}\right)$	$v = -4 - 16$	$v = -16 + 4$
v represents . . .	**The number of hours after midnight when the temperature is −16°F**	**The temperature at 6 a.m.**	**The change in temperature from midnight to 4 a.m.**
My thinking . . .	**If $v = -16 \cdot \left(-\frac{1}{4}\right)$, then $v = 4$.** **I checked the value in Equation 1 by substituting 4 for v.** $-4(4) = -16$ $-16 = -16$ **It was −16°F 4 hours after midnight or 4 a.m.**	**If $v = -4 - 16$, then $v = -20$.** **I checked the value in Equation 1 by substituting −20 for v.** $-20 + 16 = -4$ $-4 = -4$ **The temperature at 6 a.m. is −20°F.**	**If $v = -16 + 4$, then $v = -12$.** **I checked the value in Equation 1 by substituting −12 for v.** $-4 + (-12) = -16$ $-16 = -16$ **The change in temperature is −12°F.**

Lesson 19 Representing Contexts With Equations 515

1 Launch

Invite a student volunteer to read the directions for the activity aloud, while the rest of the class reads along.

2 Monitor

Help students get started by asking, "What operation would model each scenario?"

Look for points of confusion:

- **Struggling to find both equations that match the scenario.** If they found the equation that matches the scenario exactly, ask, "What would the first step in solving this equation be?"
- **Not realizing that −(−4) is the same as +4.** If students are looking for the equation $x = -16 - (-4)$ ask, "Is there another way you could represent this equation using addition?"

3 Connect

Display the Activity 1 PDF. Ask, "How could you represent each scenario on a number line (thermometer)?"

Have students share how they would use the number line to model each scenario. Encourage them to explain how their model matches.

Ask:

- "Which equation in each scenario matches the scenario as written?" Equation 1 in the Activity 1 PDF (answers)
- "Which equation in each scenario represents a strategy for solving?" Equation 2 in the Activity 1 PDF (answers)
- "How did you determine that the two equations could both be used to determine the unknown value?"

Highlight that to determine both equations students must know how to represent the scenario algebraically and also how to use inverse operations to isolate the variable in each equation.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide students with a copy of the Activity 1 PDF and allow them to use the number line to make sense of each scenario.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of each scenario.

- **Read 1:** Students should understand that in each scenario the temperature is either increasing or decreasing.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as in the first scenario, the temperature was dropping 4° per hour.
- **Read 3:** Ask students to plan their solution strategy as to how they will determine the two equations that could represent each scenario.

English Learners

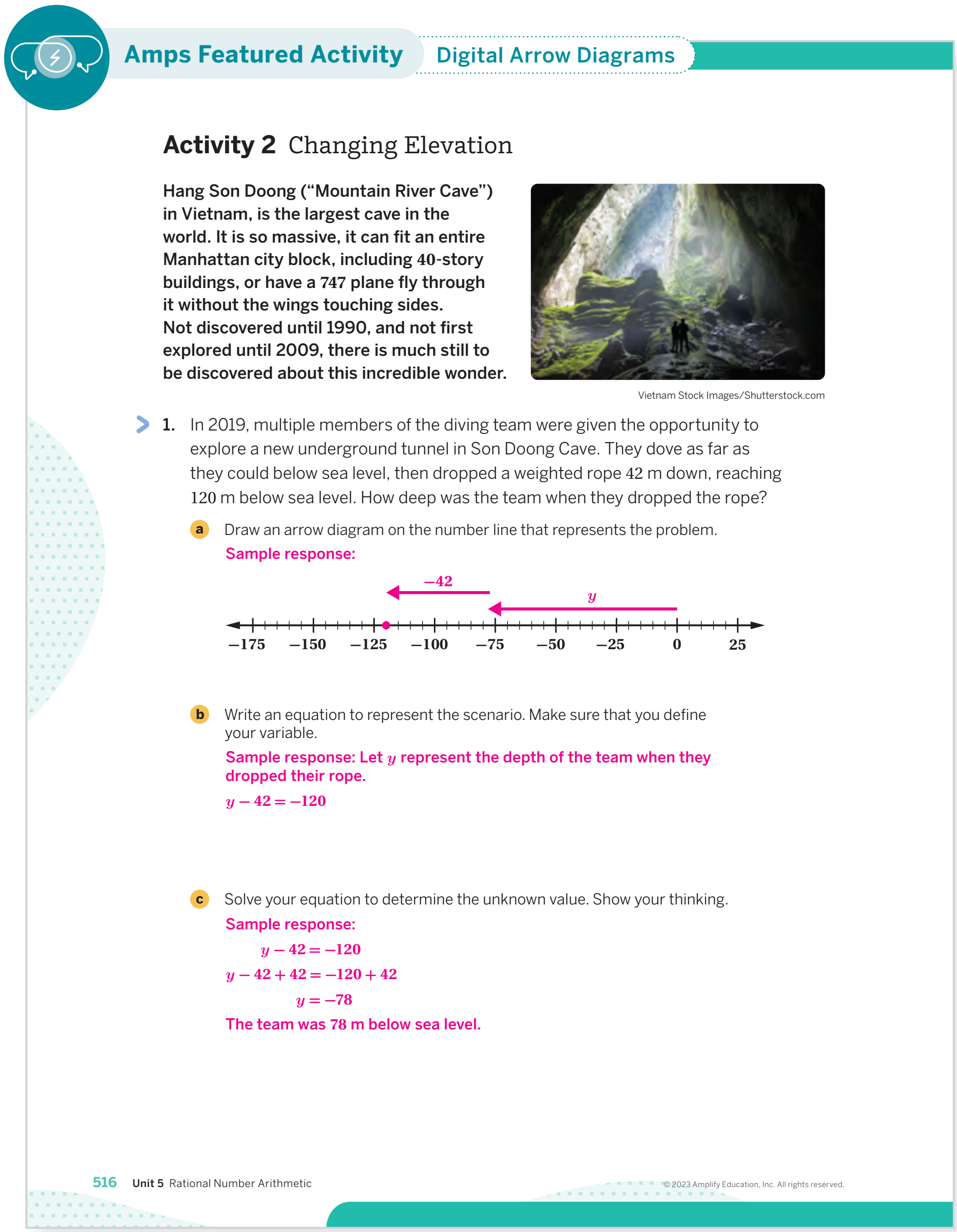
Clarify the meaning of the words "dropping," "rose," and "had fallen" as they indicate the temperature increasing or decreasing.

Small Groups | 15 min

Activity 2 Changing Elevation

Students model scenarios on number lines to aid them in writing and solving equations involving rational numbers.

Amps Featured Activity Digital Arrow Diagrams



Activity 2 Changing Elevation

Hang Son Doong ("Mountain River Cave") in Vietnam, is the largest cave in the world. It is so massive, it can fit an entire Manhattan city block, including 40-story buildings, or have a 747 plane fly through it without the wings touching sides. Not discovered until 1990, and not first explored until 2009, there is much still to be discovered about this incredible wonder.

Vietnam Stock Images/Shutterstock.com

1. In 2019, multiple members of the diving team were given the opportunity to explore a new underground tunnel in Son Doong Cave. They dove as far as they could below sea level, then dropped a weighted rope 42 m down, reaching 120 m below sea level. How deep was the team when they dropped the rope?

a Draw an arrow diagram on the number line that represents the problem.

Sample response:

b Write an equation to represent the scenario. Make sure that you define your variable.

Sample response: Let y represent the depth of the team when they dropped their rope.

$y - 42 = -120$

c Solve your equation to determine the unknown value. Show your thinking.

Sample response:

$$y - 42 = -120$$
$$y - 42 + 42 = -120 + 42$$
$$y = -78$$

The team was 78 m below sea level.

516 Unit 5 Rational Number Arithmetic

1 Launch

Activate students' background knowledge by asking what they know about caves. Explain that students will be drawing diagrams and writing equations to solve problems about divers in the world's largest cave (Hang Son Doong).

2 Monitor

Help students get started by asking them to read the first problem out loud. Ask, "How would you represent each value in your arrow diagram?"

Look for points of confusion:

- **Writing an expression to determine the unknown value without writing an equation that represents the scenario.** Refer to the Activity 1 PDF. Remind them that the first equation they write should match the scenario before they isolate the unknown value.
- **Calculating a negative time for Problem 2.** Ask students what velocity represents the divers' rate of descent.

Look for productive strategies:

- In group discussion, recognizing that there is more than one correct method for solving each equation by using inverse operations or by using the additive or multiplicative inverse.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital arrow diagrams to represent real-world scenarios. You can monitor students' creations and display their diagrams during the whole-class discussion.

Extension: *Activate Prior Knowledge*

Provide students with a copy of the Anchor Chart PDF, *Solving Equations With Rational Numbers* (answers) to use as a reference for solving equations.

Math Language Development

MLR7: Compare and Connect

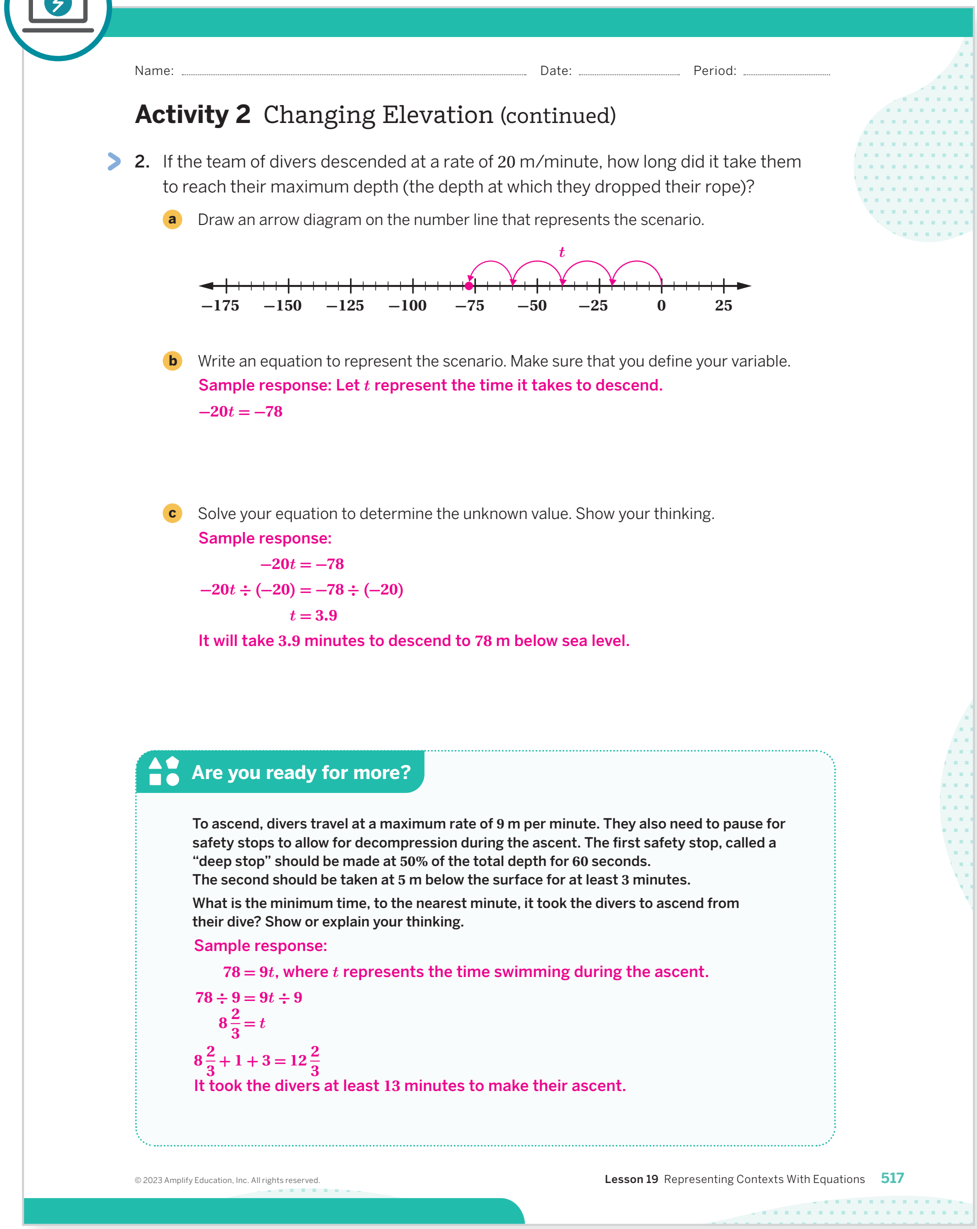
During the Connect, as students share how their arrow diagrams helped them write an equation, draw their attention to the connections between the arrow diagrams, equations, and scenarios. Sample questions are shown for Problem 1.

- "Where did you begin labeling the number line?"
- "What information was missing that you needed to determine? How is this value represented in your equation?"
- "How did you know where you needed to end up on the number line? How is this value represented in your equation?"

Small Groups | 15 min

Activity 2 Changing Elevation (continued)

Students model scenarios on number lines to aid them in writing and solving equations involving rational numbers.



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Activity 2 Changing Elevation (continued)

2. If the team of divers descended at a rate of 20 m/minute, how long did it take them to reach their maximum depth (the depth at which they dropped their rope)?

   a. Draw an arrow diagram on the number line that represents the scenario.

   b. Write an equation to represent the scenario. Make sure that you define your variable.

   Sample response: Let t represent the time it takes to descend.

   $-20t = -78$

   c. Solve your equation to determine the unknown value. Show your thinking.

   Sample response:

   $$-20t = -78$$
   $$-20t \div (-20) = -78 \div (-20)$$
   $$t = 3.9$$

   It will take 3.9 minutes to descend to 78 m below sea level.

Are you ready for more?

To ascend, divers travel at a maximum rate of 9 m per minute. They also need to pause for safety stops to allow for decompression during the ascent. The first safety stop, called a "deep stop" should be made at 50% of the total depth for 60 seconds.
The second should be taken at 5 m below the surface for at least 3 minutes.

What is the minimum time, to the nearest minute, it took the divers to ascend from their dive? Show or explain your thinking.

Sample response:

$78 = 9t$, where t represents the time swimming during the ascent.

$$78 \div 9 = 9t \div 9$$
$$8\frac{2}{3} = t$$
$$8\frac{2}{3} + 1 + 3 = 12\frac{2}{3}$$

It took the divers at least 13 minutes to make their ascent.

3 Connect

Display student-created examples of the number arrow diagrams for each scenario.

Have students share how they used their arrow diagrams to write an equation to represent each scenario.

Ask:

- "Is this the only equation that could be written to match the scenario?" Sample response: In the first problem we could have written the equation $v - 42 = -120$ or $v + (-42) = -120$.
- "What are the two ways you could solve each equation?" Sample response: For the first equation add 42 or subtract –42. For the second equation, divide by –20 or multiply by $-\frac{1}{20}$.

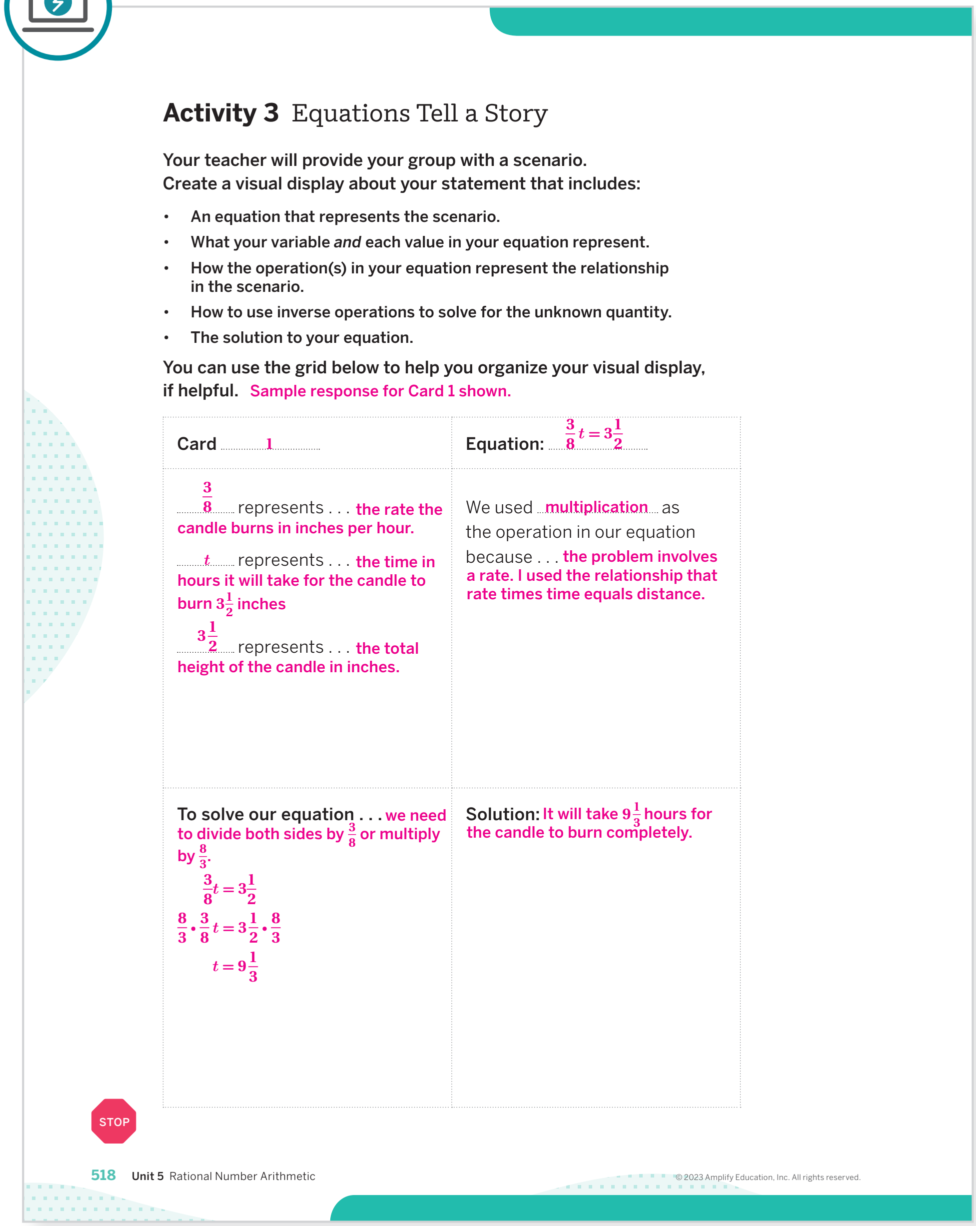
Highlight that writing equations can help students determine unknown values in scenarios involving rational numbers. Drawing an arrow diagram can help in making sense of the problem and aid in writing the equation.

Optional

Small Groups | 15 min

Activity 3 Equations Tell a Story

Students work in small groups to generate an equation to represent a scenario then complete a *Gallery Tour* to compare and contrast the process for each group's scenario.



Activity 3 Equations Tell a Story

Your teacher will provide your group with a scenario.
Create a visual display about your statement that includes:

- An equation that represents the scenario.
- What your variable *and* each value in your equation represent.
- How the operation(s) in your equation represent the relationship in the scenario.
- How to use inverse operations to solve for the unknown quantity.
- The solution to your equation.

You can use the grid below to help you organize your visual display, if helpful. Sample response for Card 1 shown.

Card 1	Equation: $\frac{3}{8}t = 3\frac{1}{2}$
$\frac{3}{8}$ represents . . . the rate the candle burns in inches per hour. t represents . . . the time in hours it will take for the candle to burn $3\frac{1}{2}$ inches $3\frac{1}{2}$ represents . . . the total height of the candle in inches.	We used multiplication as the operation in our equation because . . . the problem involves a rate. I used the relationship that rate times time equals distance.
To solve our equation . . . we need to divide both sides by $\frac{3}{8}$ or multiply by $\frac{8}{3}$. $\frac{3}{8}t = 3\frac{1}{2}$ $\frac{8}{3} \cdot \frac{3}{8}t = 3\frac{1}{2} \cdot \frac{8}{3}$ $t = 9\frac{1}{3}$	Solution: It will take $9\frac{1}{3}$ hours for the candle to burn completely.

STOP

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1 Launch

Group students homogeneously. Distribute one card to each group from the Activity 3 PDF, as well as materials for creating a poster. **Note:** The difficulty of representing each scenario increases as the card number increases, (e.g., Card 1 is the least challenging and Card 8 is the most challenging).

2 Monitor

Help students get started by asking them to read their scenario out loud. Ask, "What operation is described by this scenario?"

Look for points of confusion:

- **Difficulty in writing an equation without a visual aid.** Suggest that students draw their own diagram on a piece of paper to aid them in making sense of their problem.

3 Connect

Have students share their scenarios and methods for representing and solving them using the ***Gallery Tour*** routine. Have half of the students rotate the room while the other half answers questions about their scenario. Switch and repeat with the other half of the class.

Ask:

- "When writing an equation to represent a situation, how do you decide what your variable represents?"
- "How do you solve the equation?"

Highlight that writing equations can help determine unknown values in scenarios involving rational numbers. The variable represents the value that is unknown in each scenario.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Provide students with a copy of the Anchor Chart PDF, *Solving Equations With Rational Numbers* (answers) to use as a reference for solving equations.

Extension: *Activate Prior Knowledge*

Provide students with a copy of the Anchor Chart PDF, *Solving Equations With Rational Numbers* (answers) to use as a reference for solving equations.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their scenarios and how they represented them with equations, draw their attention to the connections between the scenarios and the equations. Sample questions are shown for Card 1.

- "What is the rate at which the candle burns? How is this value represented in your equation?"
- "What is the initial height of the candle? How is this value represented in your equation?"
- "What are you asked to determine? How is this value represented in your equation?"

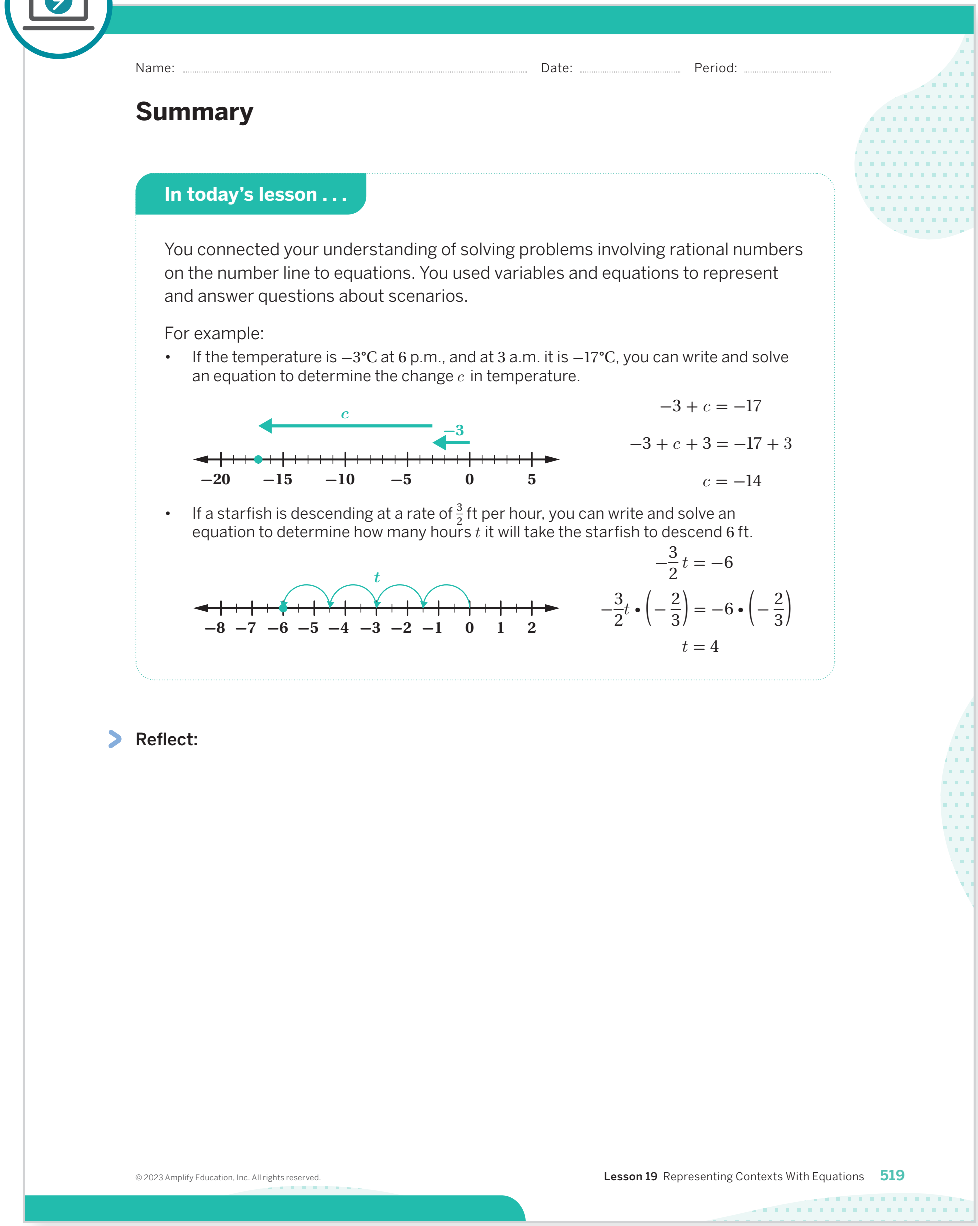
English Learners

Have students circle or underline the questions in each scenario and annotate them with "variable" or "unknown."

Whole Class | 5 min

Summary

Review and synthesize that writing and solving equations can help to model and determine unknown values in real-world scenarios involving rational numbers.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You connected your understanding of solving problems involving rational numbers on the number line to equations. You used variables and equations to represent and answer questions about scenarios.

For example:

- If the temperature is -3°C at 6 p.m., and at 3 a.m. it is -17°C, you can write and solve an equation to determine the change c in temperature.

$$-3 + c = -17$$
$$-3 + c + 3 = -17 + 3$$
$$c = -14$$

- If a starfish is descending at a rate of $\frac{3}{2}$ ft per hour, you can write and solve an equation to determine how many hours t it will take the starfish to descend 6 ft.

$$-\frac{3}{2}t = -6$$
$$-\frac{3}{2}t \cdot \left(-\frac{2}{3}\right) = -6 \cdot \left(-\frac{2}{3}\right)$$
$$t = 4$$

Reflect:

Lesson 19 Representing Contexts With Equations 519

Synthesize

Display the scenarios and matching arrow diagrams from the student edition one at a time.

Ask:

- "What does the variable represent in this scenario?"
- "What equation could be written to represent the scenario?"
- "How would you solve this equation?"
- "What does the solution represent?"

Highlight that writing and solving equations can help to model and determine unknown values in real-world scenarios involving rational numbers.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

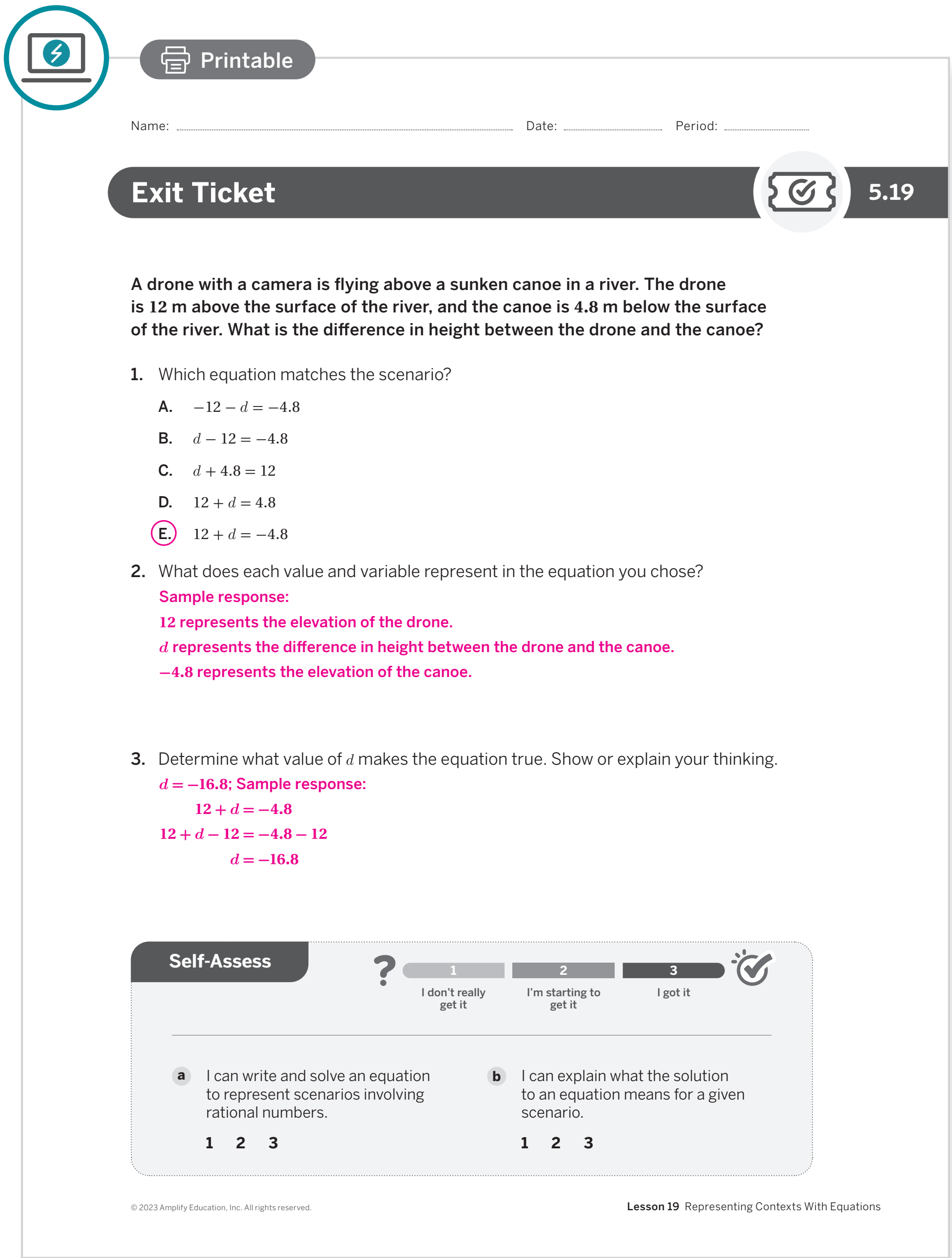
- "How can rational numbers be used to represent real-world situations?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to write and solve equations that represent real-world scenarios including identifying what each term represents.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.19

A drone with a camera is flying above a sunken canoe in a river. The drone is 12 m above the surface of the river, and the canoe is 4.8 m below the surface of the river. What is the difference in height between the drone and the canoe?

1. Which equation matches the scenario?
   - A. $-12 - d = -4.8$
   - B. $d - 12 = -4.8$
   - C. $d + 4.8 = 12$
   - D. $12 + d = 4.8$
   - [✓ E.] $12 + d = -4.8$
2. What does each value and variable represent in the equation you chose?
   Sample response:
   12 represents the elevation of the drone.
   d represents the difference in height between the drone and the canoe.
   -4.8 represents the elevation of the canoe.
3. Determine what value of d makes the equation true. Show or explain your thinking.
   $d = -16.8$; Sample response:
   $$12 + d = -4.8$$
   $$12 + d - 12 = -4.8 - 12$$
   $$d = -16.8$$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can write and solve an equation to represent scenarios involving rational numbers.
1 2 3

b I can explain what the solution to an equation means for a given scenario.
1 2 3

Lesson 19 Representing Contexts With Equations

Success looks like . . .

- **Language Goal:** Coordinating verbal descriptions, equations, and diagrams that represent the same scenario involving an unknown amount. **(Writing, Speaking and Listening)**
  - » Selecting the equation that represents finding the distance between the drone and the sunken canoe in Problem 1.
- **Language Goal:** Writing equations of the forms $x + p = q$ and $px = q$ to represent and solve a problem in an unfamiliar context.
  - » Selecting the equation that matches the scenario of the drone and the canoe and solving the equation in Problems 1 and 3.

Suggested next steps

If students identify the incorrect equation in Problem 1, consider:

- Reviewing matching equations with scenarios in Activity 1.
- Encouraging them to draw a diagram representing the scenario.
- Assigning Practice Problem 2.

If students incorrectly solve the equation in Problem 3, consider:

- Reviewing matching the two equations that match each scenario in Activity 1.
- Assigning Practice Problem 1.

Professional Learning

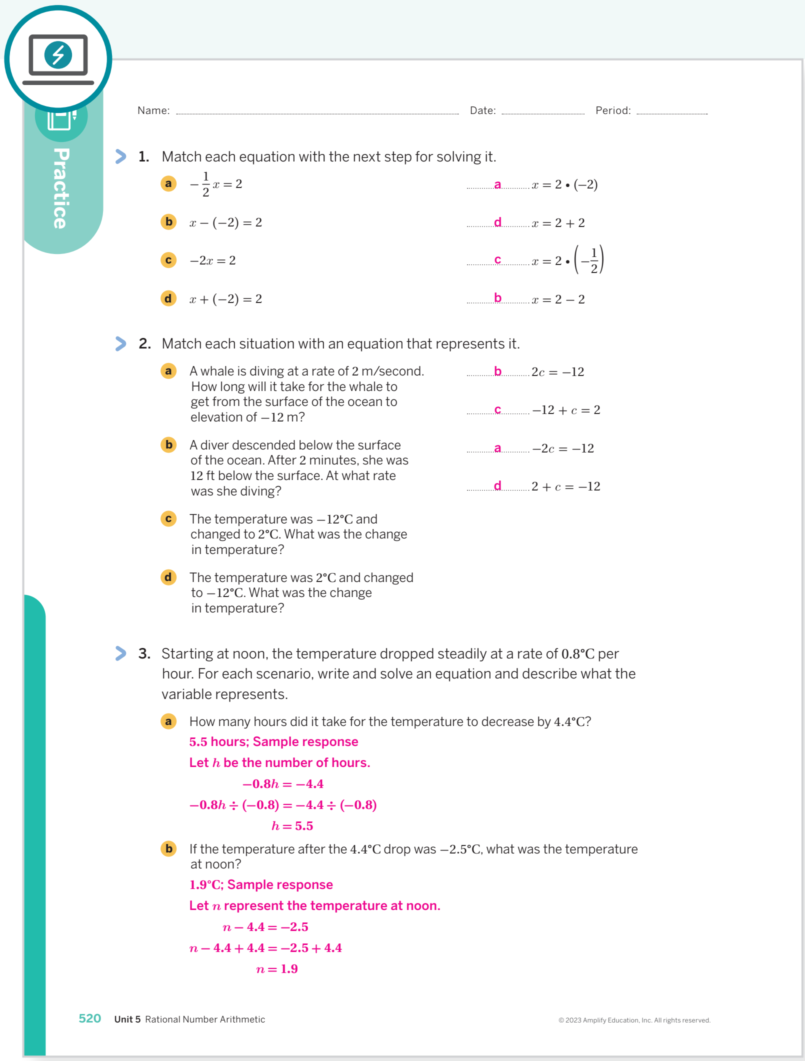
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did writing and solving equations of the forms $x + p = q$ and $px = q$ set students up to develop their understanding of solving equations of the forms $px + q = r$ and $p(x + q) = r$?
- Who participated and who didn't participate in Activity 1 today? What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Match each equation with the next step for solving it.
   - a $-\frac{1}{2}x = 2$
   - b $x - (-2) = 2$
   - c $-2x = 2$
   - d $x + (-2) = 2$

   a $x = 2 \cdot (-2)$
   d $x = 2 + 2$
   c $x = 2 \cdot \left(-\frac{1}{2}\right)$
   b $x = 2 - 2$

2. Match each situation with an equation that represents it.
   - a A whale is diving at a rate of 2 m/second. How long will it take for the whale to get from the surface of the ocean to elevation of -12 m?
   - b A diver descended below the surface of the ocean. After 2 minutes, she was 12 ft below the surface. At what rate was she diving?
   - c The temperature was -12°C and changed to 2°C. What was the change in temperature?
   - d The temperature was 2°C and changed to -12°C. What was the change in temperature?

   b $2c = -12$
   c $-12 + c = 2$
   a $-2c = -12$
   d $2 + c = -12$

3. Starting at noon, the temperature dropped steadily at a rate of 0.8°C per hour. For each scenario, write and solve an equation and describe what the variable represents.
   - a How many hours did it take for the temperature to decrease by 4.4°C?

     5.5 hours; Sample response

     Let h be the number of hours.

     $$-0.8h = -4.4$$
     $$-0.8h \div (-0.8) = -4.4 \div (-0.8)$$
     $$h = 5.5$$
   - b If the temperature after the 4.4°C drop was -2.5°C, what was the temperature at noon?

     1.9°C; Sample response

     Let n represent the temperature at noon.

     $$n - 4.4 = -2.5$$
     $$n - 4.4 + 4.4 = -2.5 + 4.4$$
     $$n = 1.9$$

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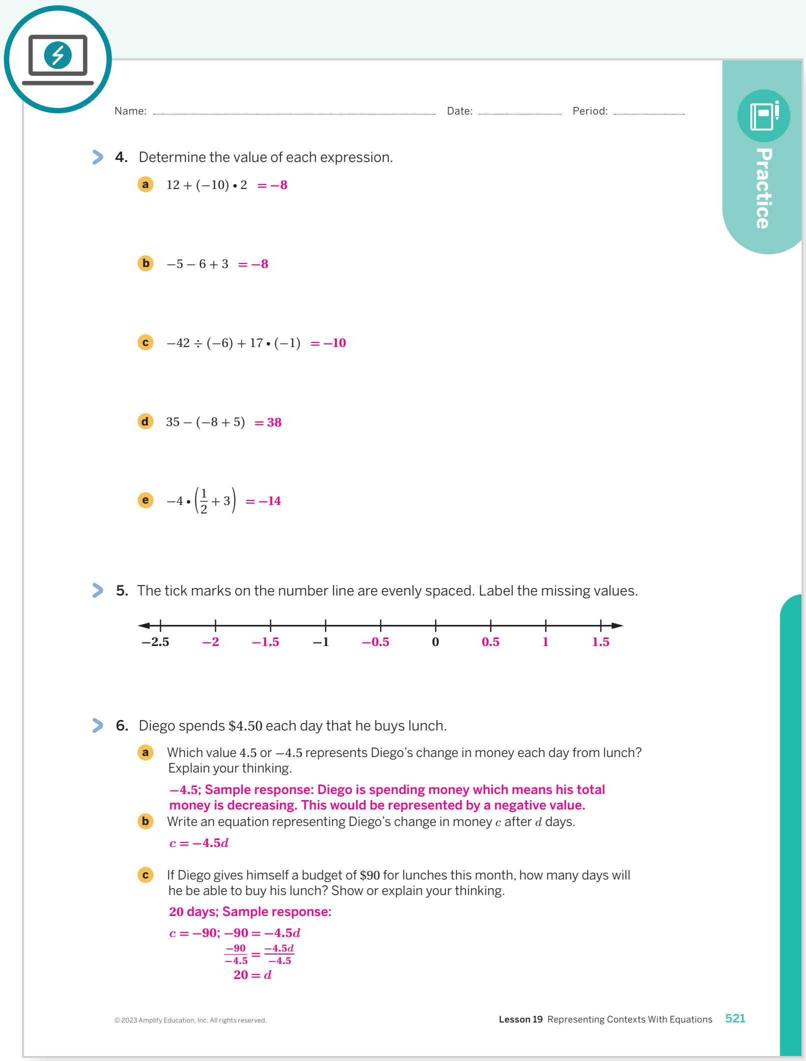

Practice

Name: ______ Date: ______ Period: ______

4. Determine the value of each expression.
   - a $12 + (-10) \cdot 2$ $= -8$
   - b $-5 - 6 + 3$ $= -8$
   - c $-42 \div (-6) + 17 \cdot (-1)$ $= -10$
   - d $35 - (-8 + 5)$ $= 38$
   - e $-4 \cdot \left(\frac{1}{2} + 3\right)$ $= -14$

5. The tick marks on the number line are evenly spaced. Label the missing values.

   -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5

6. Diego spends $4.50 each day that he buys lunch.
   - a Which value 4.5 or -4.5 represents Diego's change in money each day from lunch? Explain your thinking.

     -4.5; Sample response: Diego is spending money which means his total money is decreasing. This would be represented by a negative value.
   - b Write an equation representing Diego's change in money c after d days.

     $c = -4.5d$
   - c If Diego gives himself a budget of $90 for lunches this month, how many days will he be able to buy his lunch? Show or explain your thinking.

     20 days; Sample response:

     $$c = -90;\ -90 = -4.5d$$
     $$\frac{-90}{-4.5} = \frac{-4.5d}{-4.5}$$
     $$20 = d$$

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Lesson 19 Representing Contexts With Equations 521

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 15	1
	5	Unit 5 Lesson 10	1
Formative ⚡	6	Unit 5 Lesson 20	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 5 | **LESSON 20 – CAPSTONE**

Summiting Everest

Let's play a game to imagine what it's like to prepare to summit Mt. Everest.

Focus

Goals

1. Apply operations with rational numbers while playing a game preparing to climb Mt. Everest.
2. Write expressions and equations to model situations involving rates with rational numbers.

Coherence

Today

Students role play as mountain climbers attempting to summit Mt. Everest. As they make a plan for how to best prepare for this journey, they must consider the values of various rational number rates. Students experience the unpredictable nature of climbing Mt. Everest as they roll number cubes to determine how certain random events impact their climb. They notice how careful preparation helps give a better chance of success, though success is never guaranteed.

Previously

In Lesson 14, students saw how to express negative rates with rational numbers. In Lessons 18 and 19, students wrote and solved equations with rational numbers.

Coming Soon

In Unit 6, students will focus on working with expressions and equations, including rational numbers.

Rigor

- Students develop **fluency** working with negative rates.
- Students **apply** their understanding of rational number operations and writing expressions with rational number rates to solve a multi-step real-world problem.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Pairs	Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- calculators
- number cubes, one pair per small group

Math Language Development

Review word

- *rational numbers*

Amps | Featured Activity

Activity 1
Making Preparations Digitally

The digital environment allows students to make changes to their preparation calendar quickly and efficiently. Thus, they will be able to spend more time reasoning about their strategies.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might be timid about sharing why they think they are best prepared to climb the mountain. The diversity of individuals' perspectives should be celebrated. Make sure that groups set a goal to make everyone feel safe to share within the group. Encourage students to listen to each other's reasons. Seek the positive and support each other in it.

Modifications to Pacing

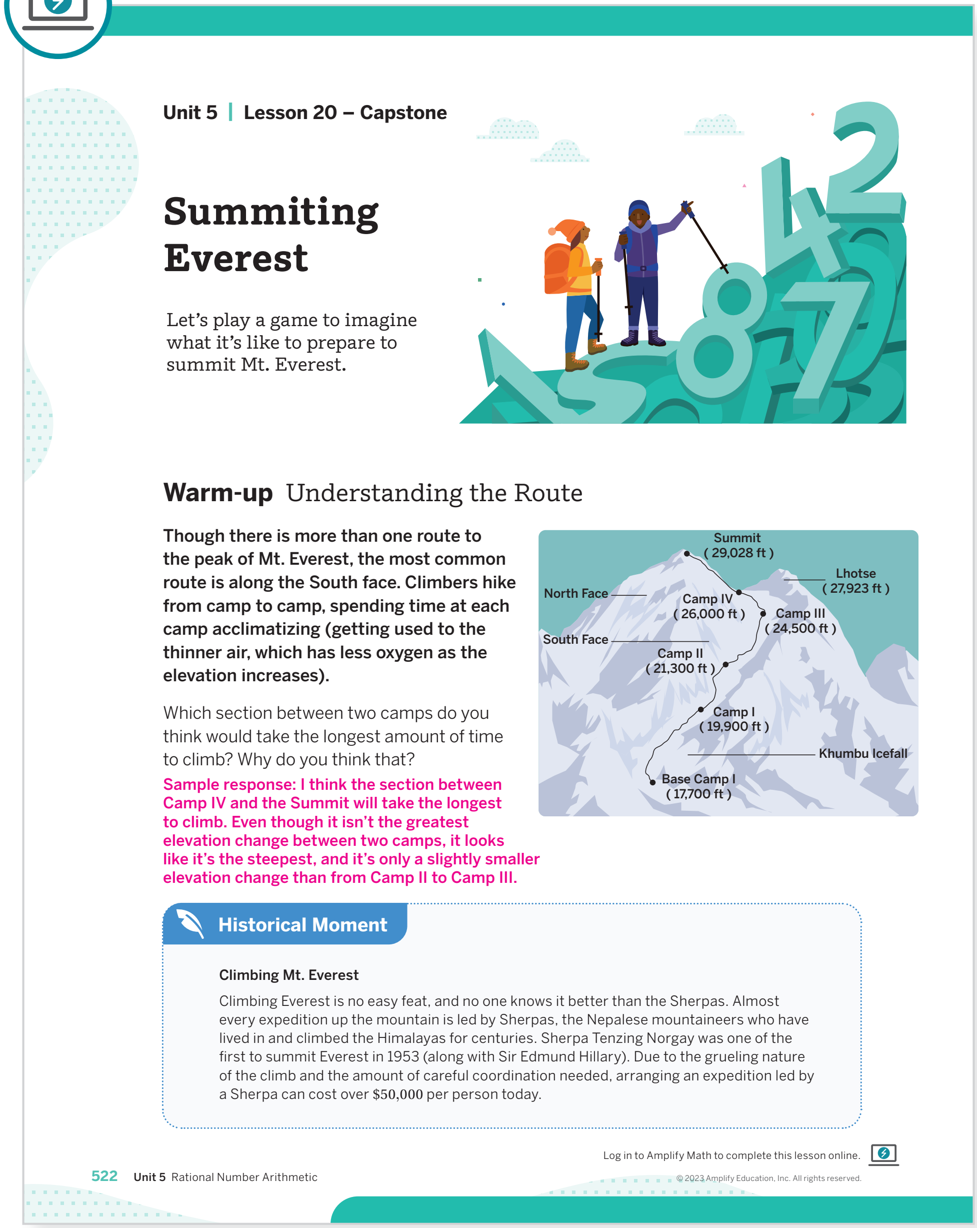
You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Pairs | 5 min

Warm-up Understanding the Route

Students are introduced to a route map for Mt. Everest and consider the distance and difficulty of each section to reason about how long they take to climb.



Unit 5 | Lesson 20 – Capstone

Summiting Everest

Let's play a game to imagine what it's like to prepare to summit Mt. Everest.

Warm-up Understanding the Route

Though there is more than one route to the peak of Mt. Everest, the most common route is along the South face. Climbers hike from camp to camp, spending time at each camp acclimatizing (getting used to the thinner air, which has less oxygen as the elevation increases).

Which section between two camps do you think would take the longest amount of time to climb? Why do you think that?

Sample response: I think the section between Camp IV and the Summit will take the longest to climb. Even though it isn't the greatest elevation change between two camps, it looks like it's the steepest, and it's only a slightly smaller elevation change than from Camp II to Camp III.

Historical Moment

Climbing Mt. Everest

Climbing Everest is no easy feat, and no one knows it better than the Sherpas. Almost every expedition up the mountain is led by Sherpas, the Nepalese mountaineers who have lived in and climbed the Himalayas for centuries. Sherpa Tenzing Norgay was one of the first to summit Everest in 1953 (along with Sir Edmund Hillary). Due to the grueling nature of the climb and the amount of careful coordination needed, arranging an expedition led by a Sherpa can cost over $50,000 per person today.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Activate students' background knowledge by asking, "Has anyone been hiking before? What factors affect how long a hike will take?" Point out that the measurements shown indicate each camp's elevation. Let students know that each camp has tents for which climbers can rest.

2 Monitor

Help students get started by suggesting to consider the steepness, distance, and elevation change between each set of camps.

Look for points of confusion:

- **Thinking they can determine the distance between camps by comparing the elevations.** Ask, "What does elevation measure?"

Look for productive strategies:

- Finding the difference in elevation for each set of camps.

3 Connect

Display the route map.

Have students share their answers, and invite other students to respond to or ask questions of each other.

Ask:

- "Why might a climber have to climb in this way — back-and-forth — rather than climbing straight to the top?"
- "What else do you know about climbing Mt. Everest?"
- "Why might strength and perseverance be important attributes for a climber?"

Highlight that students will play a game in Activities 1 and 2. Activity 1 involves making strategic choices for increasing their readiness to climb. Activity 2 is about attempting to get to the summit — and seeing whether their preparations were sufficient.

Power-up

To power up students' ability to reason about negative rates, have students complete:

Determine whether each rate would be represented by a positive or negative value.

1. Bard loses $100 per month. Negative
2. Andre earns $40 per hour. Positive
3. Shawn dives 30 m per hour. Negative
4. Priya runs 9 mph. Positive

Use: Before Activity 1.

Informed by: Performance on Lesson 19, Practice Problem 6.

Historical Moment

Climbing Mt. Everest

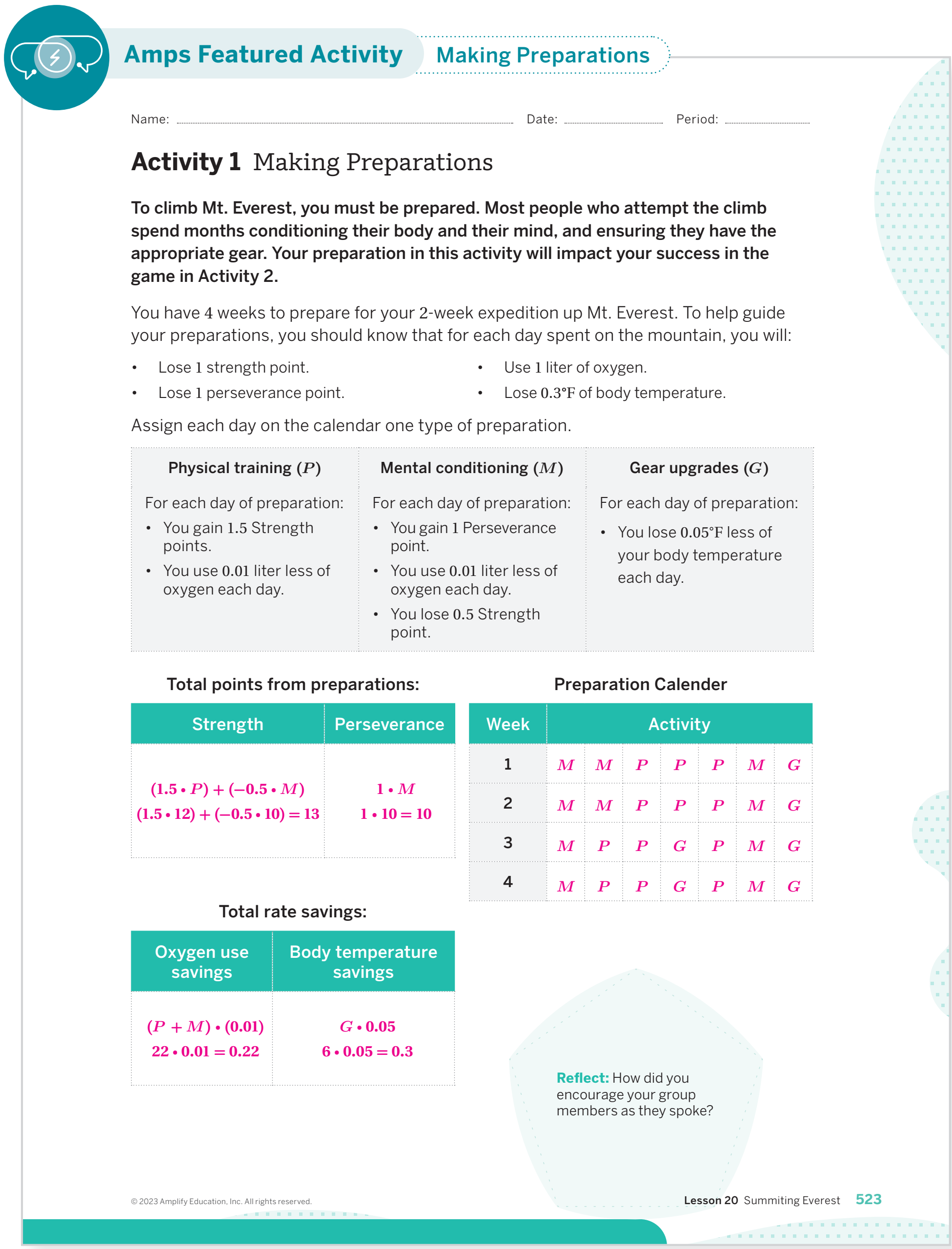
Have students read about The Sherpa people, the Nepalese ethnic group who live in the Himalayas and are famous for their unparalleled climbing abilities.

Independent | 15 min

Activity 1 Making Preparations

Students consider how the length of their climb up Mt. Everest will decrease a climber's physical and mental fitness in order to prepare properly for the climb.

Amps Featured Activity Making Preparations



Name: ______ Date: ______ Period: ______

Activity 1 Making Preparations

To climb Mt. Everest, you must be prepared. Most people who attempt the climb spend months conditioning their body and their mind, and ensuring they have the appropriate gear. Your preparation in this activity will impact your success in the game in Activity 2.

You have 4 weeks to prepare for your 2-week expedition up Mt. Everest. To help guide your preparations, you should know that for each day spent on the mountain, you will:

- Lose 1 strength point.
- Lose 1 perseverance point.
- Use 1 liter of oxygen.
- Lose 0.3°F of body temperature.

Assign each day on the calendar one type of preparation.

Physical training (P)	Mental conditioning (M)	Gear upgrades (G)
For each day of preparation: • You gain 1.5 Strength points. • You use 0.01 liter less of oxygen each day.	For each day of preparation: • You gain 1 Perseverance point. • You use 0.01 liter less of oxygen each day. • You lose 0.5 Strength point.	For each day of preparation: • You lose 0.05°F less of your body temperature each day.

Total points from preparations:

Strength	Perseverance
$(1.5 \cdot P) + (-0.5 \cdot M)$ $(1.5 \cdot 12) + (-0.5 \cdot 10) = 13$	$1 \cdot M$ $1 \cdot 10 = 10$

Preparation Calender

Week	Activity						
1	M	M	P	P	P	M	G
2	M	M	P	P	P	M	G
3	M	P	P	G	P	M	G
4	M	P	P	G	P	M	G

Total rate savings:

Oxygen use savings	Body temperature savings
$(P + M) \cdot (0.01)$ $22 \cdot 0.01 = 0.22$	$G \cdot 0.05$ $6 \cdot 0.05 = 0.3$

Reflect: How did you encourage your group members as they spoke?

1 Launch

Read through the introduction together as a class. Point out that students have three choices for their type of preparation each day: Physical training, Mental conditioning, or Gear upgrades. Say, "Each type offers different advantages, but you need to consider whether you will have enough Strength and Perseverance to make it all 14 days." Let students know that the rate savings works differently than Strength and Perseverance. Strength and Perseverance affect the total the climber starts with, while rate savings will slow down the speed of losing oxygen and body temperature.

2 Monitor

Help students get started by having them plan one week, and calculating how many points and how much rate savings they have for that one week.

3 Connect

Display work that includes an expression for calculating strength points. If no student used variables in their expression, ask the class to write one together.

Have students share why they think they are prepared to climb the mountain.

Ask:

- "Why do you need more than one variable in the expression?"
- "How has your work with rational numbers in this unit helped you in this activity?"

Highlight that certain factors that will affect students' climb in Activity 2 are out of their control — similar to real life. Let students know they will need to refer back to the values from the end of Activity 1 in Activity 2.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which the digital environment allows students to make changes to their preparation calendar quickly and efficiently. Thus, they will be able to spend more time reasoning about their strategies.

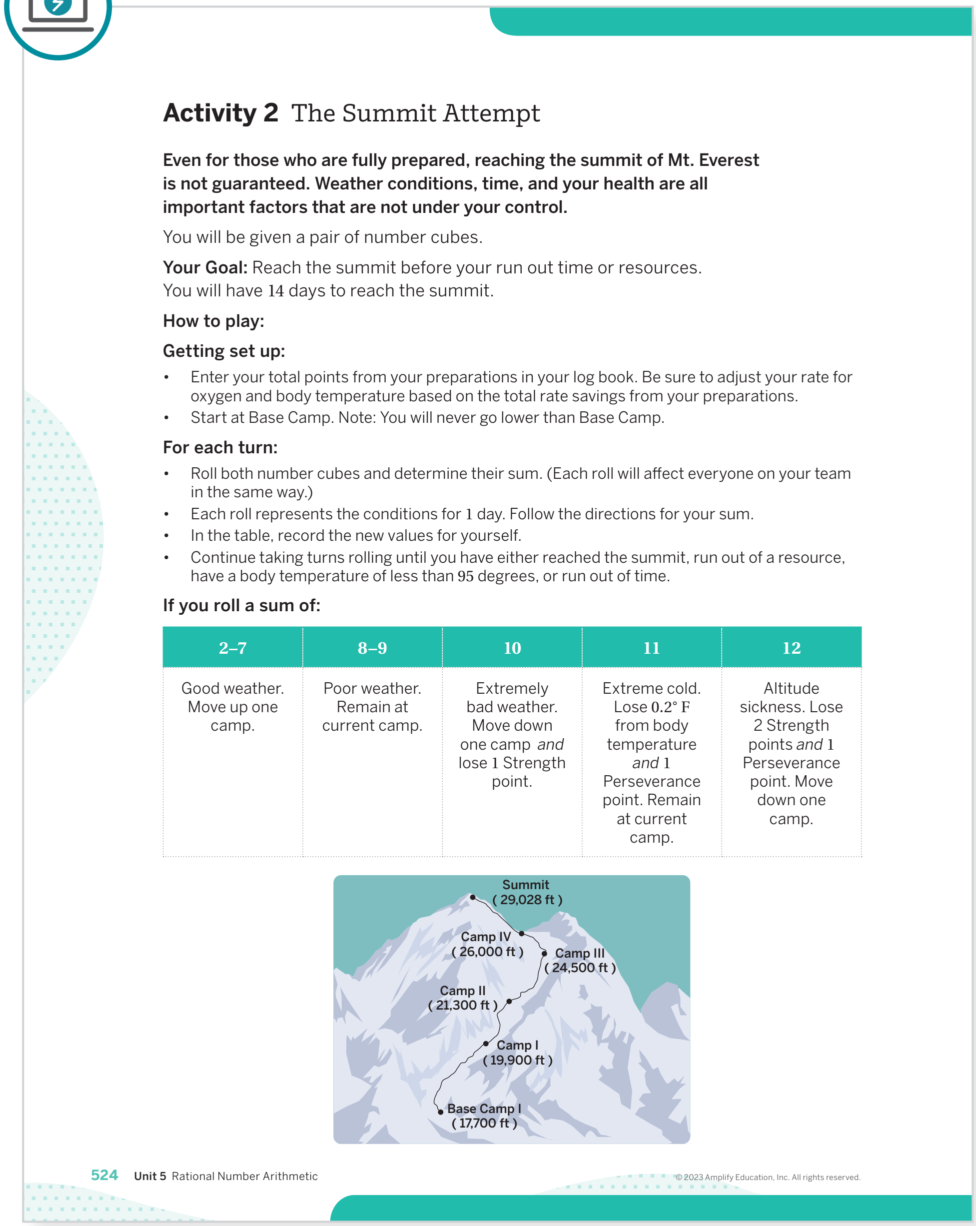
Accessibility: *Guide Processing and Visualization*

Suggest that students try a straightforward pattern for their preparation on the calendar, such as a pattern that repeats for each week. Because they can modify the calendar up through the end of the activity, encourage them to use "first-draft thinking" as they begin.

Small Groups | 20 min

Activity 2 The Summit Attempt

Students track the decrease in their resources and reason about negative rates as they roll dice to determine their movement up and down Mt. Everest.



Activity 2 The Summit Attempt

Even for those who are fully prepared, reaching the summit of Mt. Everest is not guaranteed. Weather conditions, time, and your health are all important factors that are not under your control.

You will be given a pair of number cubes.

Your Goal: Reach the summit before your run out time or resources.
You will have 14 days to reach the summit.

How to play:

Getting set up:

- Enter your total points from your preparations in your log book. Be sure to adjust your rate for oxygen and body temperature based on the total rate savings from your preparations.
- Start at Base Camp. Note: You will never go lower than Base Camp.

For each turn:

- Roll both number cubes and determine their sum. (Each roll will affect everyone on your team in the same way.)
- Each roll represents the conditions for 1 day. Follow the directions for your sum.
- In the table, record the new values for yourself.
- Continue taking turns rolling until you have either reached the summit, run out of a resource, have a body temperature of less than 95 degrees, or run out of time.

If you roll a sum of:

2–7	8–9	10	11	12
Good weather. Move up one camp.	Poor weather. Remain at current camp.	Extremely bad weather. Move down one camp *and* lose 1 Strength point.	Extreme cold. Lose 0.2° F from body temperature *and* 1 Perseverance point. Remain at current camp.	Altitude sickness. Lose 2 Strength points *and* 1 Perseverance point. Move down one camp.

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1 Launch

Read through the directions for the game together as a class. Have students go to the second page of the activity to record their values for Strength and Perseverance from Activity 1. If students had oxygen or body temperature rate savings, they should record their new, smaller loss rate for each; model how to represent this in the log book table. Distribute pairs of number cubes to each small group.

2 Monitor

Help students get started by demonstrating how to play a couple of rounds; roll the number cubes and record the updated values in the log book table.

Look for points of confusion:

- **Increasing the rate of loss for oxygen or body temperature.** Ask, "In your preparations, your physical training will have made your body more efficient using oxygen. Would it make sense for you to be using more or less oxygen?"

Look for productive strategies:

- Completing the table until one of the resources is exhausted, then rolling the number cubes for each day and hoping to reach the summit before the last day.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider demonstrating how to play the game for one roll of the number cubes, which represents one day. Show how to record the results of the roll in the table, and whether you can move up or down camps and whether you lose Strength, Temperature, or Perseverance points.

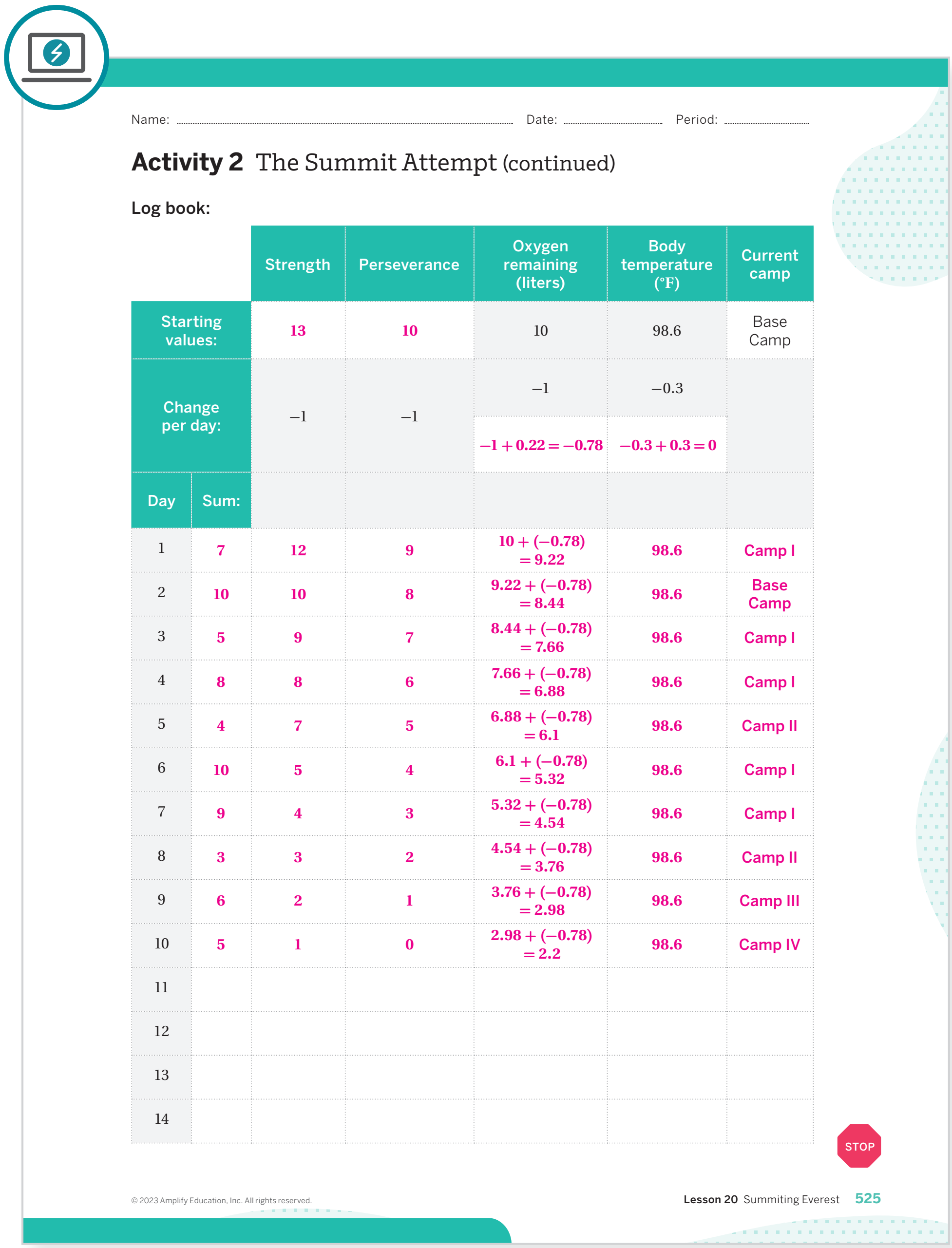
Extension: *Math Enrichment*

Have students return to their preparation calendar from Activity 1 and choose a different set of preparations. Allow them to replay the game in Activity 2, as time permits, and discuss how their new preparations affected their likelihood of success.

Small Groups | 20 min

Activity 2 The Summit Attempt (continued)

Students track the decrease in their resources and reason about negative rates as they roll dice to determine their movement up and down Mt. Everest.



Name: ______ Date: ______ Period: ______

Activity 2 The Summit Attempt (continued)

Log book:

		Strength	Perseverance	Oxygen remaining (liters)	Body temperature (°F)	Current camp
Starting values:		13	10	10	98.6	Base Camp
Change per day:		−1	−1	−1 $-1 + 0.22 = -0.78$	−0.3 $-0.3 + 0.3 = 0$	
Day	Sum:					
1	7	12	9	$10 + (-0.78) = 9.22$	98.6	Camp I
2	10	10	8	$9.22 + (-0.78) = 8.44$	98.6	Base Camp
3	5	9	7	$8.44 + (-0.78) = 7.66$	98.6	Camp I
4	8	8	6	$7.66 + (-0.78) = 6.88$	98.6	Camp I
5	4	7	5	$6.88 + (-0.78) = 6.1$	98.6	Camp II
6	10	5	4	$6.1 + (-0.78) = 5.32$	98.6	Camp I
7	9	4	3	$5.32 + (-0.78) = 4.54$	98.6	Camp I
8	3	3	2	$4.54 + (-0.78) = 3.76$	98.6	Camp II
9	6	2	1	$3.76 + (-0.78) = 2.98$	98.6	Camp III
10	5	1	0	$2.98 + (-0.78) = 2.2$	98.6	Camp IV
11						
12						
13						
14						

STOP

3 Connect

Ask, "Did you make it to the summit? If not, what went wrong?"

Have students share what changes they would make to their preparation schedule, if any, based on the outcome of their game.

Display a student's log book table.

Ask:

- "What equation could you write to model how much oxygen is left if you know how many days have passed?"
- "Could you predict how many days you have before any of your resources reach 0? How could you do this?"
- "What other situations can you think of where a negative rate would help predict when a certain resource might run out?"

Highlight how understanding rates, especially negative rates, can help plan for a situation involving using up a limited resource.

Whole Class | 5 min

Unit Summary

Review and synthesize the important concepts related to operations with rational numbers.

Narrative Connections

Unit Summary

It's easy to forget how large our planet is. Beyond the comforts of home, there are towering cliffs, wide oceans, and miles and miles of rocky terrain. These hard-to-reach places have tempted adventurers for centuries. From the breath-stealing heights of Mt. Everest to the sub-zero temperatures of Antarctica, there's no shortage of places for people to test their strength and resolve.

Many of these challenges can be described using rational numbers. Rational numbers show not only *how much* of something there is, but also in what *direction*. This is especially useful when discussing elevation and temperature, where quantities can be above or below the starting point of zero.

But we can use rational numbers for more than just position. Rational numbers open a world where we acknowledge what it means to lose something, to owe something, and to go below where we started. In the fields of accounting and finance, positive and negative numbers show up as credit and debt. Operating with these numbers can give you the big picture on your total gains and losses.

While addition and subtraction may seem relatively simple with rational numbers, be careful when multiplying or dividing. A misplaced sign can mean the difference between climbing a triumphant peak or plummeting into the deepest depths.

See you in Unit 6.

526 Unit 5 Rational Number Arithmetic

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Unit Summary from the Student Edition. Have students read the summary or have a student volunteer to read it aloud.

Highlight that it is very useful to spend some time reflecting on one's own learning at the conclusion of a unit. This helps to retain information and make connections that students may not have been able to make while focusing on each individual lesson.

Ask students to take a few minutes to recall what they have learned about working with rational numbers within all four operations.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

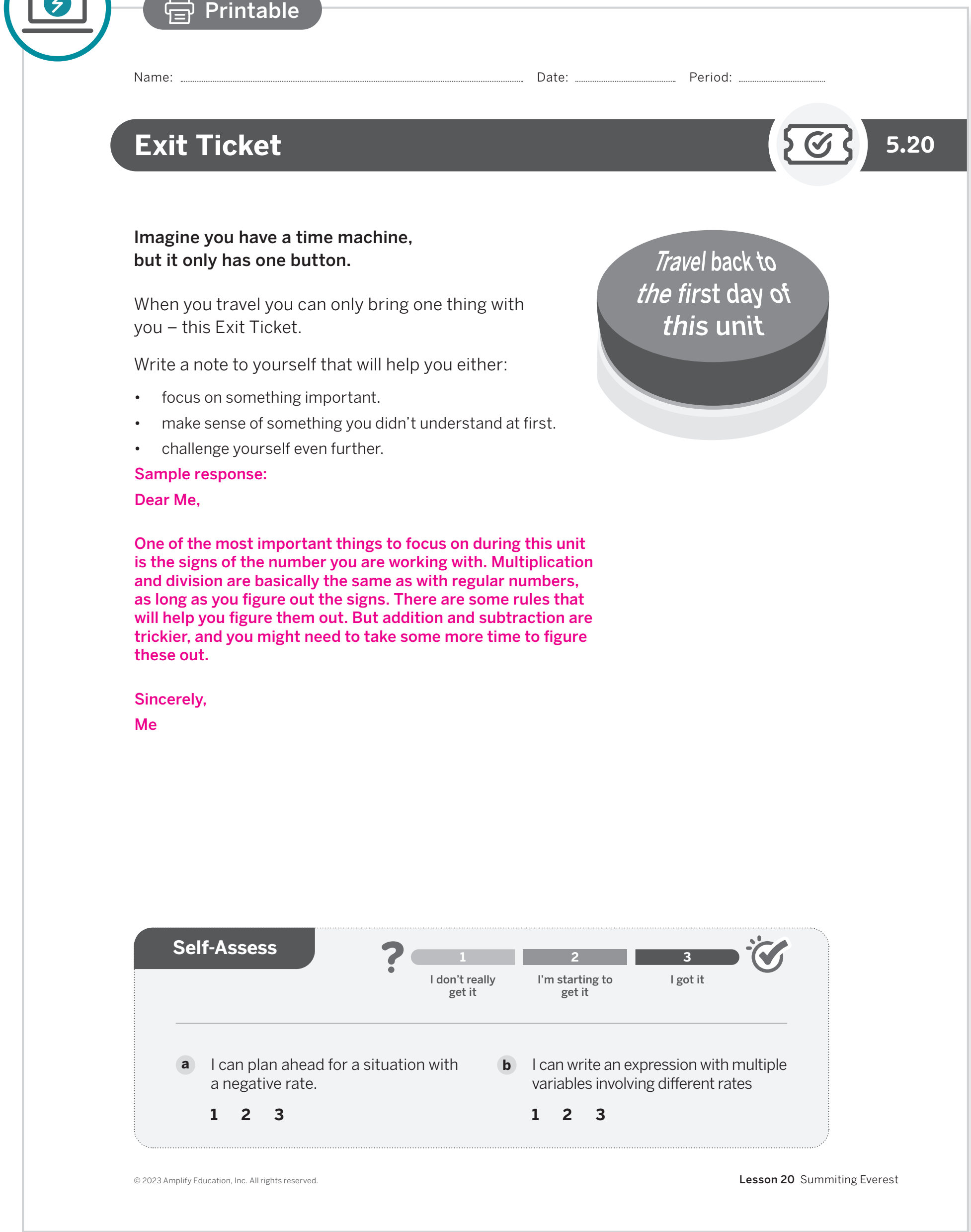
- "Did anything surprise you while reading the narratives of this unit?"
- "Is there any topic you would like to learn move about? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the most important concepts in operations with rational numbers by writing a note to their past selves at the beginning of the unit.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.20

Imagine you have a time machine, but it only has one button.

When you travel you can only bring one thing with you – this Exit Ticket.

Write a note to yourself that will help you either:

- focus on something important.
- make sense of something you didn't understand at first.
- challenge yourself even further.

Travel back to the first day of this unit

Sample response:

Dear Me,

One of the most important things to focus on during this unit is the signs of the number you are working with. Multiplication and division are basically the same as with regular numbers, as long as you figure out the signs. There are some rules that will help you figure them out. But addition and subtraction are trickier, and you might need to take some more time to figure these out.

Sincerely,

Me

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can plan ahead for a situation with a negative rate.
1 2 3

b I can write an expression with multiple variables involving different rates
1 2 3

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Lesson 20 Summiting Everest

Success looks like . . .

- **Goal:** Applying operations with rational numbers while playing a game preparing to climb Mt. Everest.
- **Goal:** Writing expressions and equations to model situations involving rates with rational numbers.

Suggested next steps

If students cannot recall moments that match the prompts in the Exit Ticket, consider:

- Encouraging them to look back through their work from the unit to jog their memory.

Professional Learning

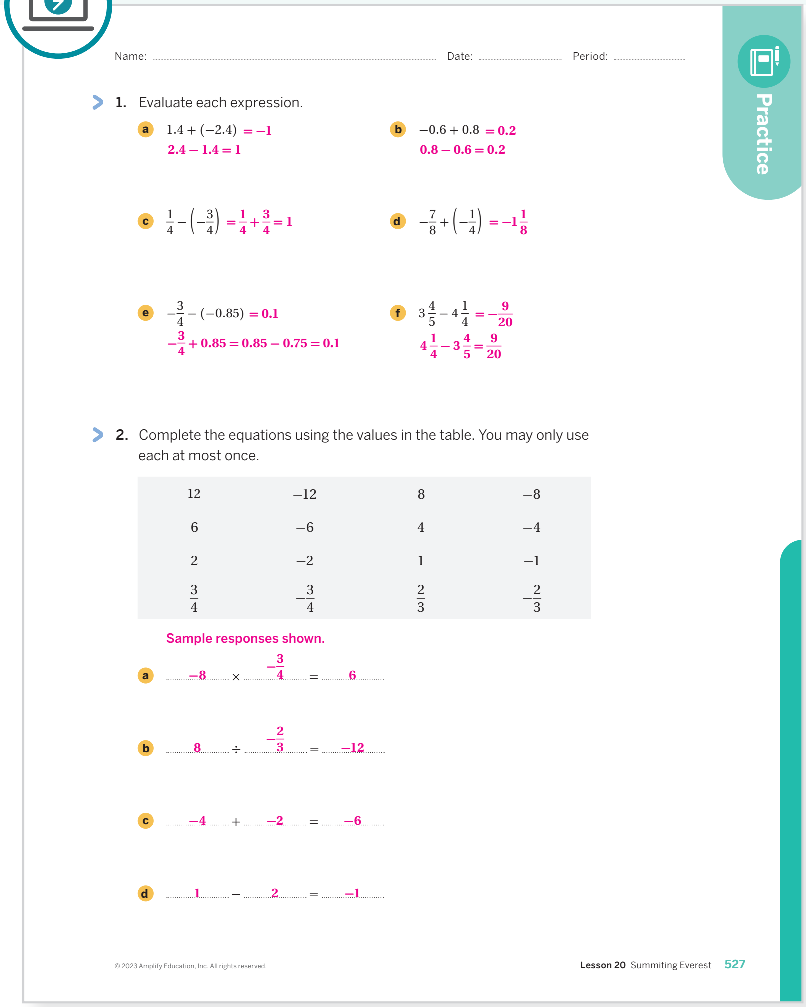
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. Evaluate each expression.

a $1.4 + (-2.4)$ $= -1$
$2.4 - 1.4 = 1$

b $-0.6 + 0.8$ $= 0.2$
$0.8 - 0.6 = 0.2$

c $\frac{1}{4} - \left(-\frac{3}{4}\right)$ $= \frac{1}{4} + \frac{3}{4} = 1$

d $-\frac{7}{8} + \left(-\frac{1}{4}\right)$ $= -1\frac{1}{8}$

e $-\frac{3}{4} - (-0.85)$ $= 0.1$
$-\frac{3}{4} + 0.85 = 0.85 - 0.75 = 0.1$

f $3\frac{4}{5} - 4\frac{1}{4}$ $= -\frac{9}{20}$
$4\frac{1}{4} - 3\frac{4}{5} = \frac{9}{20}$

2. Complete the equations using the values in the table. You may only use each at most once.

12	-12	8	-8
6	-6	4	-4
2	-2	1	-1
$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{2}{3}$	$-\frac{2}{3}$

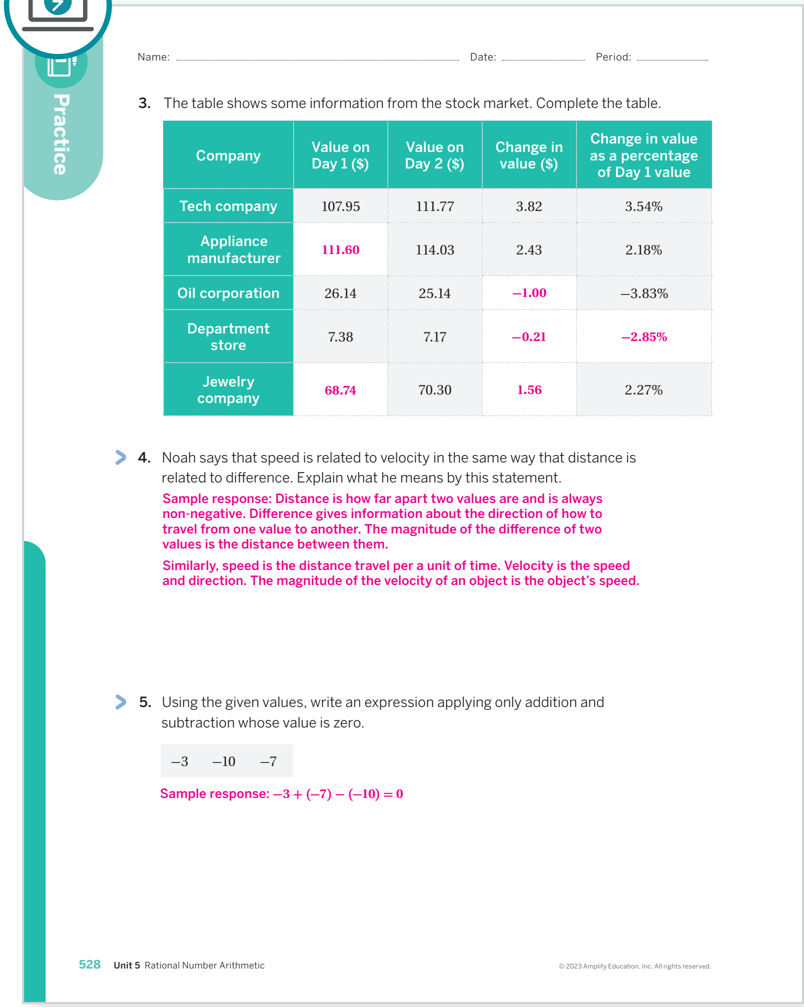
Sample responses shown.

a $-8 \times -\frac{3}{4} = 6$

b $8 \div -\frac{2}{3} = -12$

c $-4 + -2 = -6$

d $1 - 2 = -1$



Name: .... Date: .... Period: ....

3. The table shows some information from the stock market. Complete the table.

Company	Value on Day 1 ($)	Value on Day 2 ($)	Change in value ($)	Change in value as a percentage of Day 1 value
Tech company	107.95	111.77	3.82	3.54%
Appliance manufacturer	111.60	114.03	2.43	2.18%
Oil corporation	26.14	25.14	-1.00	-3.83%
Department store	7.38	7.17	-0.21	-2.85%
Jewelry company	68.74	70.30	1.56	2.27%

4. Noah says that speed is related to velocity in the same way that distance is related to difference. Explain what he means by this statement.

Sample response: Distance is how far apart two values are and is always non-negative. Difference gives information about the direction of how to travel from one value to another. The magnitude of the difference of two values is the distance between them.

Similarly, speed is the distance travel per a unit of time. Velocity is the speed and direction. The magnitude of the velocity of an object is the object's speed.

5. Using the given values, write an expression applying only addition and subtraction whose value is zero.

-3	-10	-7

Sample response: $-3 + (-7) - (-10) = 0$

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 5 Lesson 9	1
	2	Unit 5 Lesson 15	3
	3	Unit 5 Lesson 17	2
	4	Unit 5 Lesson 11	3
	5	Unit 5 Lesson 9	3

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6

Expressions, Equations, and Inequalities

Students return to the study of algebra and focus on how representation plays such a large role in communicating mathematical ideas. In this unit, the symbols, language, and drawings students use will help them tell the stories they see in the numbers.

Essential Questions

- Which representations best help you make sense of certain mathematical scenarios?
- Which strategies that worked for solving simple equations or inequalities can be put to use when solving more complex ones?
- How can you increase your efficiency when solving mathematical problems?
- *(By the way, what does dog walking have to do with mathematics?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students incorporate their new awareness of the set of rational numbers into their previous experience with solving equations and inequalities. They interpret and make connections among various representations of the relationship between two quantities, including tape diagrams, hanger diagrams, area models, and algebraic equations.

Coherence

Previously . . .

Two important areas that have been studied in the past come together in this unit: work with expressions and equations from Grade 6 and the set of rational numbers from earlier in Grade 7.

Coming soon . . .

Students will revisit working with expressions and equations in Grade 8 while solving systems of equations. Additionally, their work with manipulating equations will be important when rewriting linear equations in more helpful forms in Grade 8.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Hanger diagrams help students make sense of the need for balance when solving equations (Lesson 2). Inequalities are presented as an extension of their work with equations, which helps to ease the transition (Lesson 13).

Procedural Fluency

Students formalize their understanding of solving equations algebraically with activities like the Equation Chain (Lesson 7).

Application

Students make sense of how inequalities can model the relationships between quantities in their world (Lesson 18).

Solving One Step at a Time

SUB-UNIT

1

Lessons 2–7

Solving Two-Step Equations

Students move to a more formal representation of balance: hanger diagrams. In considering how this concrete representation connects to the abstract algebraic representation, students build schema they can call on throughout the rest of the unit when solving for unknown values.

Narrative: Symbols and diagrams can help you to make sense of equations, and even to solve them.

SUB-UNIT

2

Lessons 8–12

Solving Real-World Problems Using Two-Step Equations

Now understanding how to solve for unknown values in multi-step equations, students think about where these equations come from. What, exactly, do they represent? Students interpret situations with various quantitative relationships and see that they can be modeled with algebraic expressions and equations.

Narrative: From ancient Egypt to the modern world, solving equations can help you solve problems.

SUB-UNIT

3

Lessons 13–18

Inequalities

Students are reacquainted with the equation's close (and beloved) relative: the inequality. They'll notice that the apple doesn't fall too far from the tree. Students experience a similar progression as they did with equations, learning to solve them before learning how they can use them to model situations.

Narrative: Inequalities are more than symbols. And you already have the tools to solve them.

Launch

Lesson 1

Keeping the Balance

The concept of balance is central to solving equations and inequalities. The dog walker reminds students how there can be many ways to balance the same quantities, all of which are equivalent.

SUB-UNIT

4

Lessons 19–22

Equivalent Expressions

Equipped with a deeper understanding of solving equations and inequalities, students wade into the water a bit further and begin to consider efficiency. They see that expressions can be simplified into fewer terms, ultimately making their work easier.

Narrative: Technology can make life a little easier. So can changing the structure of a mathematical expression.

Capstone

Lesson 23

Pattern Thinking

This lesson helps students see the expressions they have been manipulating in a new light. Working with visual, growing patterns, students find their own ways of seeing the numbers, variables, and expressions in current and future steps.

Unit at a Glance

Spoiler Alert: Solving equations with multiple steps — even with negative numbers and fractions — involves the same reasoning as solving simpler equations. Just perform the same operation with the same number to both sides, like always.

Assessment

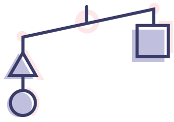
A **Pre-Unit Readiness Assessment**

Launch Lesson

1 **Keeping the Balance** ●

Illustrate balance and the concept of equality using a brand new model: the dog walker.

Sub-Unit 1: Solving Two-Step Equations



2 **Balanced and Unbalanced**

Find unknown values on balanced hanger diagrams that model two-step equations.

$p(x+q) = r$ DISTRIBUTE

OR

DIVIDE $x + q = \frac{r}{p}$

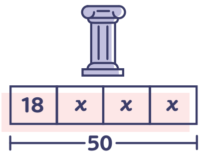
6 **Two Ways to Solve One Equation**

Can it be? Two ways? Practice solving equations of the form $p(x+q) = r$, focusing on the structure of the equation to determine the most efficient solution method.

$5+3x = 12 \Rightarrow \square = \frac{12-5}{3}$

7 **Practice Solving Equations** ●

Build fluency in algebraic manipulation by solving a variety of equations.



8 **Reasoning With Tape Diagrams**

Use tape diagrams and equations of the form $px + q = r$ and $p(x+q) = r$ to describe relationships in real-world scenarios.

12 **Solving Percent Problems in New Ways**

Represent percent increase and decrease using tape diagrams and equations.

Assessment

A **Mid-Unit Assessment**

Sub-Unit 3: Inequalities

< OR = IS ≤

13 **Reintroducing Inequalities**

Students met them last year. Reintroduce inequalities and how they can be used to represent real-world scenarios.

Key Concepts

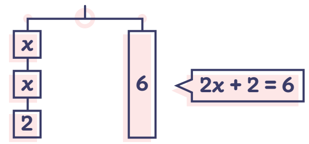
Lesson 2: Hanger diagrams can be used to model equations and help with solving them.
Lesson 11: Real-world scenarios can be represented by writing equations.
Lesson 14: Meet the equation's beloved relative. Solving inequalities is just like solving equations.

Pacing

23 Lessons: 45 min each
3 Assessments: 45 min each

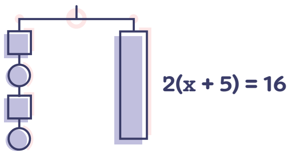
Full Unit: 26 days
● **Modified Unit:** 23 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



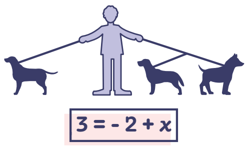
3 Reasoning About Solving Equations (Part 1)

Match hanger diagrams with corresponding equations, and use them to reason about solutions.



4 Reasoning About Solving Equations (Part 2)

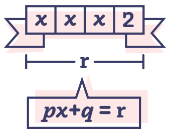
Connect the Distributive Property with solving equations of the form $p(x + q) = r$, using hanger diagrams to assist.



5 Dealing With Negative Numbers

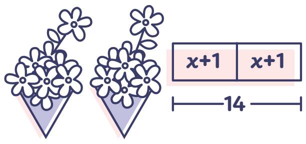
A new twist. Or is it? Introduce equations that have negative values, showing how the concept of balance still applies.

Sub-Unit 2: Solving Real-World Problems Using Two-Step Equations



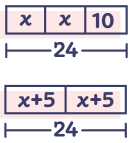
9 Reasoning About Equations and Tape Diagrams (Part 1)

Create tape diagrams, write equations, and solve real-world problems, focusing on equations of the form $px + q = r$.



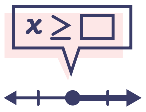
10 Reasoning About Equations and Tape Diagrams (Part 2)

Create tape diagrams, write equations, and solve real-world problems, focusing on equations of the form $p(x + q) = r$.



11 Using Equations to Solve Problems

Decide which type of equation, $px + q = r$ or $p(x + q) = r$, describes the relationships in a real-world story problem.

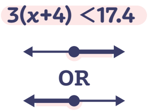


14 Solving Inequalities

Equations can help! Write and solve equations to help find the solutions of inequalities, which may include negative values.

15 Finding Solutions to Inequalities in Context

Solve contextual problems involving inequalities using the strategies from previous lessons.



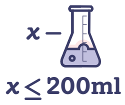
16 Efficiently Solving Inequalities

Solve inequalities of the forms $px + q < r$ and $p(x + q) < r$ by first writing and solving a related equation.

Unit at a Glance

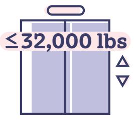
Spoiler Alert: Solving equations with multiple steps — even with negative numbers and fractions — involves the same reasoning as solving simpler equations. Just perform the same operation with the same number to both sides, like always.

‹ continued



17 **Interpreting Inequalities**

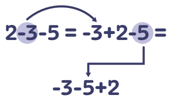
Interpret and solve inequalities that represent real-world situations, while making sense of quantities and their relationships in the problem.



18 **Modeling With Inequalities**

Focusing on the modeling process, by first proposing a question and then deciding how to represent the situation mathematically.

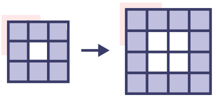
Sub-Unit 4: Equivalent Expressions



19 **Subtraction in Equivalent Expressions**

Highlight common errors when rewriting expressions containing subtraction and/or negative signs.

Capstone Lesson



23 **Pattern Thinking** ●

Make connections between equivalent expressions, non-proportional linear relationships, and pattern growth.

Assessment

A **End-of-Unit Assessment**

● Modifications to Pacing

Lesson 1: The Launch lesson provides a re-entry point to balance and solving equations, but may be omitted if needed.

Lesson 7: This fluency-focused lesson may be omitted. Fluency practice for solving equations can be found in the Practice Problems of lessons in Sub-Unit 1 and in the Additional Practice.

Lesson 23: This Capstone lesson challenges students to apply algebraic reasoning to pattern growth, but may be omitted, as it is not directly tied to the major work of the grade.

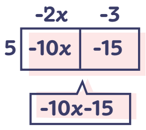
Key Concepts

Lesson 2: Hanger diagrams can be used to model equations and help with solving them.
Lesson 11: Real-world scenarios can be represented by writing equations.
Lesson 14: Meet the equation's beloved relative. Solving inequalities is just like solving equations.

Pacing

23 Lessons: 45 min each
3 Assessments: 45 min each
Full Unit: 26 days
● **Modified Unit:** 23 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



20 Expanding and Factoring

Use the Distributive Property to expand and factor expressions that include subtraction and negative values.

$$3a + (-2c) = \boxed{3a - 2c}$$

21 Combining Like Terms (Part 1)

Use the properties of operations to understand how like terms can be combined to write an equivalent expression with fewer terms.

$$16x - 12 - 24x + 4 = \boxed{-8x - 8}$$

22 Combining Like Terms (Part 2)

Combine like terms to write equivalent expressions with fewer terms, now including negative coefficients and parentheses.

Unit Supports

Math Language Development

Lesson	New Vocabulary
5	equivalent equations
13	greater than or equal to less than or equal to solution to an inequality
20	expand factor
21	like terms

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
11	**MLR1:** Stronger and Clearer Each Time
1, 5, 11, 13, 16, 20, 21	**MLR2:** Collect and Display
1, 13, 22	**MLR3:** Critique, Correct, Clarify
18	**MLR4:** Information Gap
2, 15	**MLR5:** Co-craft Questions
7, 11, 12, 17	**MLR6:** Three Reads
1–4, 5, 8–10, 12–14, 18–20, 23	**MLR7:** Compare and Connect
4, 5, 9, 10, 12, 16, 20, 21	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Materials
8, 14	colored pencils, markers, or highlighters
8	glue or tape
15	number lines (optional)
11	sticky notes
	tools for creating a visual display (chart paper, markers, etc.)
5–8, 13, 14, 16, 18–20, 22	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.

Instructional Routines

Activities throughout Unit 6 include the following instructional routines:

Lesson(s)	Instructional Routine
4	Algebra Talk
8, 13, 20	Card Sort
9, 10	Equation String
11	Gallery Tour
18	Info Gap
1, 8	Notice and Wonder
20	Number Talk
6, 13, 18, 20–22	Poll the Class
1, 2, 14, 16, 20, 22, 23	Think-Pair-Share
14	True or False
11	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 12
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 23

Amps
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Social & Collaborative Digital Moments

Featured Activity

Walking Dogs LIke a Pro

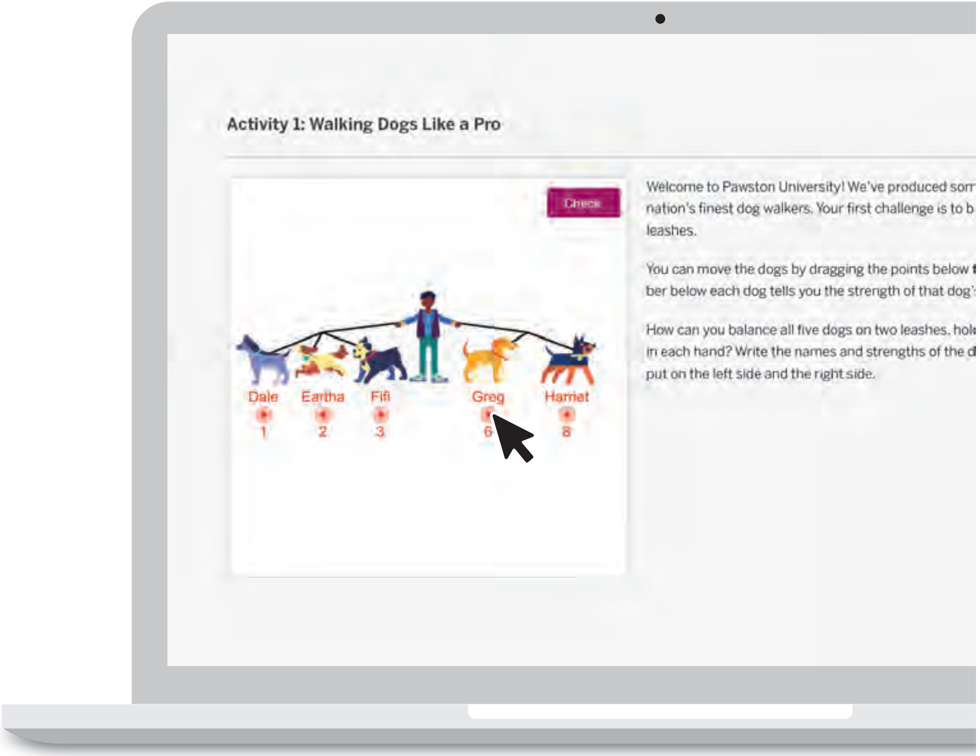
Put on your student hat and work through **Lesson 1, Activity 1:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities

- Dynamic Hanger Diagrams **(Lesson 2)**
- The Roller Coaster **(Lesson 13)**
- Robot Recharge **(Lesson 19)**
- Pool Border Problem **(Lesson 23)**



Activity 1: Walking Dogs Like a Pro

Welcome to Pawston University! We've produced sor
nation's finest dog walkers. Your first challenge is to b
leashes.

You can move the dogs by dragging the points below t
ber below each dog tells you the strength of that dog'

How can you balance all five dogs on two leashes, hol
in each hand? Write the names and strengths of the d
put on the left side and the right side.

Unit Study
Professional Learning

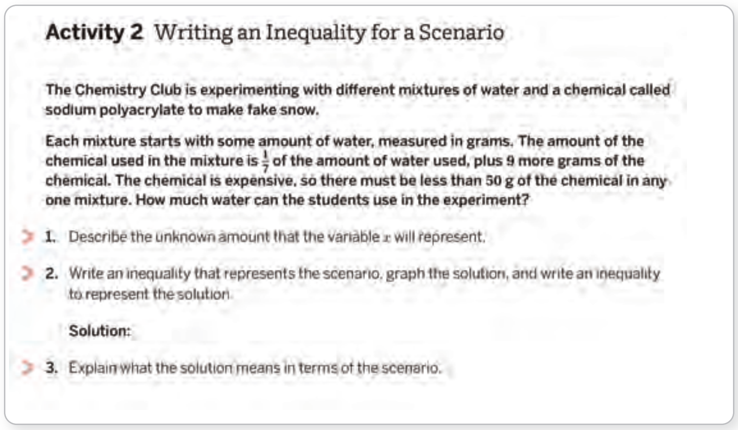
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 3 introduces students to inequalities by having students continue to solve these as if they were equations initially. They plot the solution on the number line, then check the inequality to see whether the solution is true at that point, and whether it's also true when greater than or less than that point. Students learn to write inequalities from a scenario, define what the variable represents and explain what the solution means in context. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 17, Activity 2:**



Activity 2 Writing an Inequality for a Scenario

The Chemistry Club is experimenting with different mixtures of water and a chemical called sodium polyacrylate to make fake snow.

Each mixture starts with some amount of water, measured in grams. The amount of the chemical used in the mixture is $\frac{1}{7}$ of the amount of water used, plus 9 more grams of the chemical. The chemical is expensive, so there must be less than 50 g of the chemical in any one mixture. How much water can the students use in the experiment?

1. Describe the unknown amount that the variable x will represent.
2. Write an inequality that represents the scenario, graph the solution, and write an inequality to represent the solution.

   Solution:
3. Explain what the solution means in terms of the scenario.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

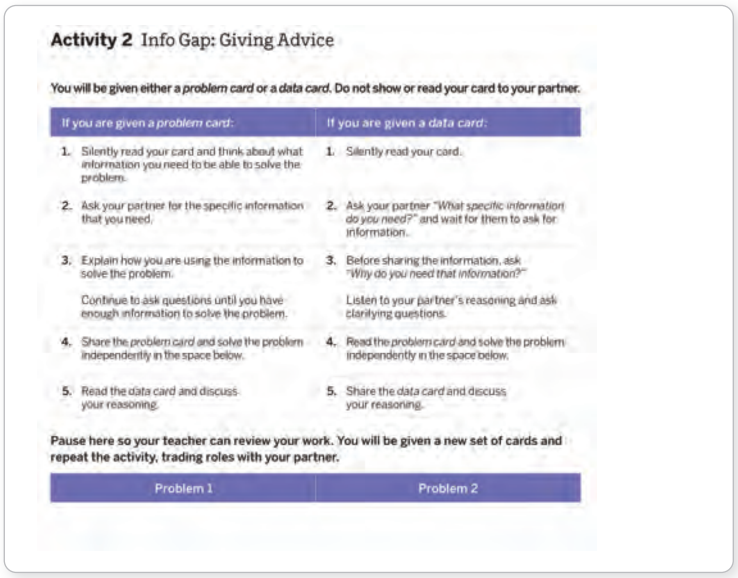
- What was it like to engage in this problem as a learner?
- Sodium polyacrylate is a polymer found in many common products. Have students research this and whether it can be found in their own household.
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Information Gap (Info Gap)

Rehearse . . .

How you'll facilitate student interaction during the ***Info Gap*** instructional routine in **Lesson 18, Activity 2:**



Activity 2 Info Gap: Giving Advice

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	1. Silently read your card.
2. Ask your partner for the specific information that you need.	2. Ask your partner *"What specific information do you need?"* and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	3. Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space below.	4. Read the *problem card* and solve the problem independently in the space below.
5. Read the *data card* and discuss your reasoning.	5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. You will be given a new set of cards and repeat the activity, trading roles with your partner.

Problem 1	Problem 2

Point to Ponder . . .

- Students may need to be patient while their partner processes the information on their card — what thinking job can you give students while they wait?

This routine . . .

- Encourages socialization and interdependency.
- Positions students as knowledge-givers and knowledge-seekers.
- Models the nonlinear nature of mathematical problem solving.

Anticipate . . .

- Students being confused about the order in which to present their information during their first experience with this routine.
- How might your students share discussion time equitably?
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What do you want to refine?

Strengthening Your Effective Teaching Practices

Use and connect mathematical representations.

This effective teaching practice . . .

- Deepens student understanding of mathematical concepts and procedures and builds a toolkit for problem solving.
- Facilitates meaningful mathematical discourse by drawing connections between the different representations used and how each one illustrates the same mathematical ideas.

Points to Ponder . . .

- What representations will be presented in this unit?
- Where do you see opportunities to make connections among the different representations used, and when should students be allowed choice of the representation used?

Math Language Development

MLR6: Three Reads

MLR6 appears in Lessons 7, 11, 12, and 17.

- Encourage students to read introductory text multiple times before jumping into a task. By doing so, they will have more opportunities to understand the task and the quantities and relationships presented. The ***Three Reads*** routine asks students to focus on the following for each read:
  - **Read 1:** Make sense of the overall information or scenario, without focusing on specific quantities.
  - **Read 2:** Look for specific quantities and relationships and make note of them.
  - **Read 3:** Brainstorm strategies for how to approach the task.
- **English Learners:** Annotate or highlight key words and phrases in the introductory text to help students understand the relationships between quantities.

Point to Ponder . . .

- Some students may resist reading information multiple times. How will you help them see the benefits to doing so before jumping into the actual task?

Unit Assessments

- Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with writing and solving equations and inequalities throughout the unit? Do you think your students will generally:
  - Understand how to deal with negative and fractional values?
  - Be ready to apply what they have learned about solving equations to solving inequalities?

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Technology*

Opportunities to provide visual support, guide student processing, or provide the use of technology (through the Amps slides) appear in Lessons 1–19.

- In Lessons 2–4, students can manipulate a digital hanger diagram which animates and provides real-time feedback showing whether the hanger is balanced.
- In Lessons 9 and 10, students can create and edit tape diagrams using digital tools.
- Display or provide copies of the Anchor Chart PDFs, *Solving Equations*, *Solving Inequalities*, and *Writing Equivalent Expressions*, for students to reference throughout the unit.
- Use color coding and annotations to connect key words and phrases from text and verbal descriptions and how they are represented in equations and inequalities.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to display or provide an anchor chart, use color coding, or use the Amps slides to deepen students' understanding of the concepts of the unit?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

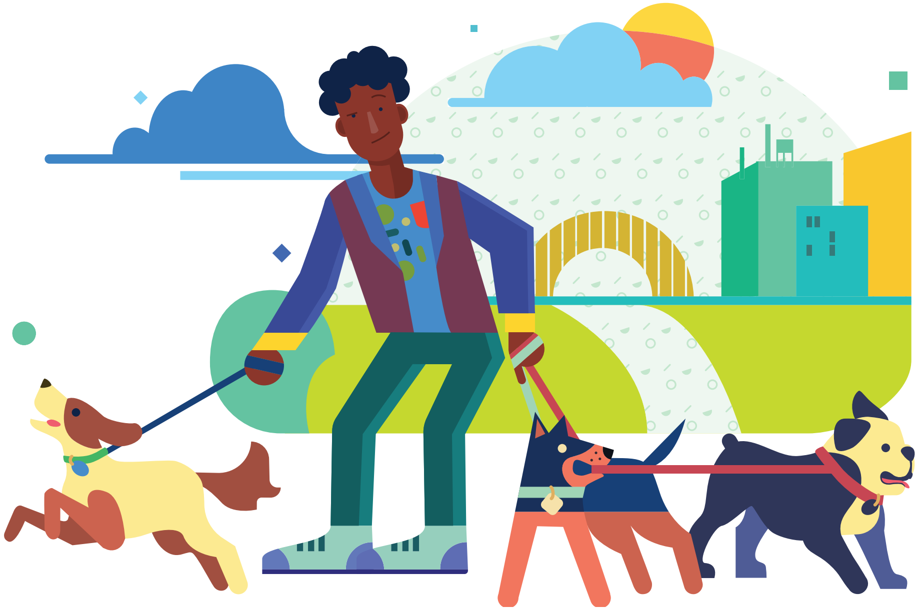
In this unit, pay particular attention to supporting students in building their relationship skills and self-management.

Points to Ponder . . .

- Are students able to resolve conflict without damaging relationships? Can they effectively communicate their opinion about a mathematical practice? Do they work well as a team, making sure each person is able to offer and seek help when needed?
- Are students able to control their stress levels when working on a new skill or task? Do they have the organizational skills required to accomplish their goals? Can they muster enough self-discipline to take the steps necessary to solve equations and inequalities? How do they keep themselves motivated?

Keeping the Balance

Let's walk some dogs.



Focus

Goals

1. **Language Goal:** Generalize that adjusting the amount of strength equally on each side of a dog walker diagram keeps it balanced. **(Speaking)**
2. **Language Goal:** Explain how to use a balanced dog walker model to solve an equation of the form $px + q = r$. **(Speaking)**

Coherence

Today

Students learn how a dog walker model can be used to illustrate balance and the concept of equality. This understanding is essential in solving for unknown values in equations.

Previously

In Grade 6, students wrote and solved one-step equations to represent word problems.

Coming Soon

In the next few lessons, students will work with hanger diagrams to expand their toolbox of strategies for solving multi-step equations.

Rigor

- Students build **conceptual understanding** of equality by balancing the pull of dogs on either side of a dog walker.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *Addition Property of Equality*
- *Division Property of Equality*
- *equation*
- *Multiplication Property of Equality*
- *Subtraction Property of Equality*

Amps | Featured Activity

Activity 1

Dynamic Dog Leashes

Students can manipulate the position of the dogs on either side of the dog walker, while trying to balance them.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not take the naming in Activity 3 seriously. They might impulsively give silly names without any consideration for the quantitative reasoning involved. In order to help students focus, have them prepare to explain their terms to someone else, connecting what they know about the properties to their names.

Modifications to Pacing

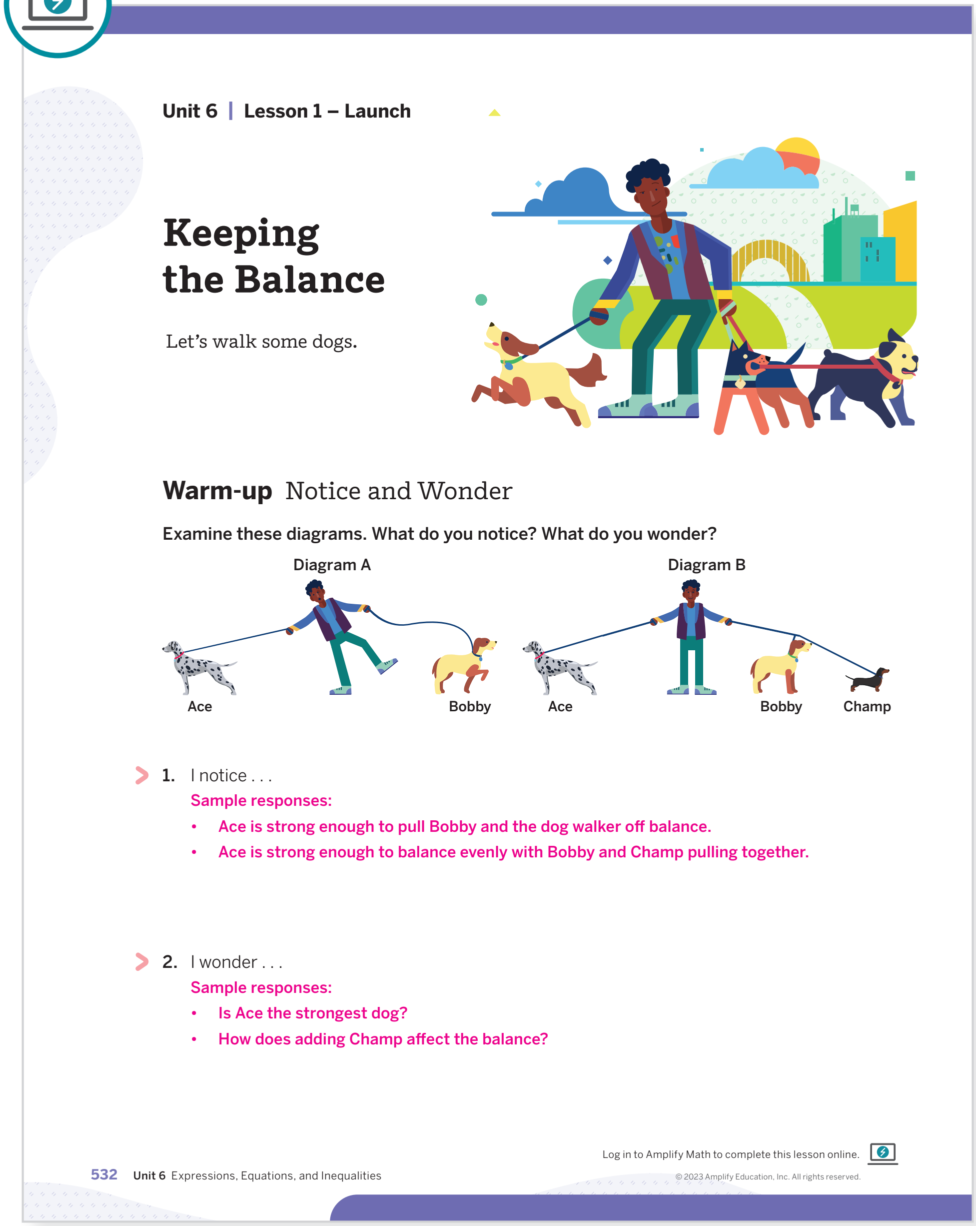
You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Pairs | 5 min

Warm-up Notice and Wonder

Students look at two pictures representing the dog walker model to gain a sense of balanced and unbalanced situations.



Unit 6 | Lesson 1 – Launch

Keeping the Balance

Let's walk some dogs.

Warm-up Notice and Wonder

Examine these diagrams. What do you notice? What do you wonder?

Diagram A

Ace

Bobby

Diagram B

Ace

Bobby

Champ

1. I notice . . .

   Sample responses:
   - Ace is strong enough to pull Bobby and the dog walker off balance.
   - Ace is strong enough to balance evenly with Bobby and Champ pulling together.

2. I wonder . . .

   Sample responses:
   - Is Ace the strongest dog?
   - How does adding Champ affect the balance?

532 Unit 6 Expressions, Equations, and Inequalities

Log in to Amplify Math to complete this lesson online.

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1 Launch

Conduct the *Notice and Wonder* routine. Suggest that students share their thinking with their partner before writing their responses.

2 Monitor

Help students get started by saying, "Tell me about what you see happening in the first picture."

Look for points of confusion:

- **Thinking the size of the dog matters.** Ask, "Can a smaller dog be stronger than a larger dog?"
- **Not noticing that Diagram A represents imbalance.** Ask, "What does it mean if the dog walker is being pulled in one direction?"
- **Saying Champ is the strongest because he is pulling Bobby and Ace.** Say, "Bobby is pulling the same direction as Champ. Does that work for or against Champ?"

3 Connect

Have students share their observations about the diagrams.

Highlight that Diagram B represents a balanced dog walker. Balance and equality will be important themes throughout this unit.

Ask:

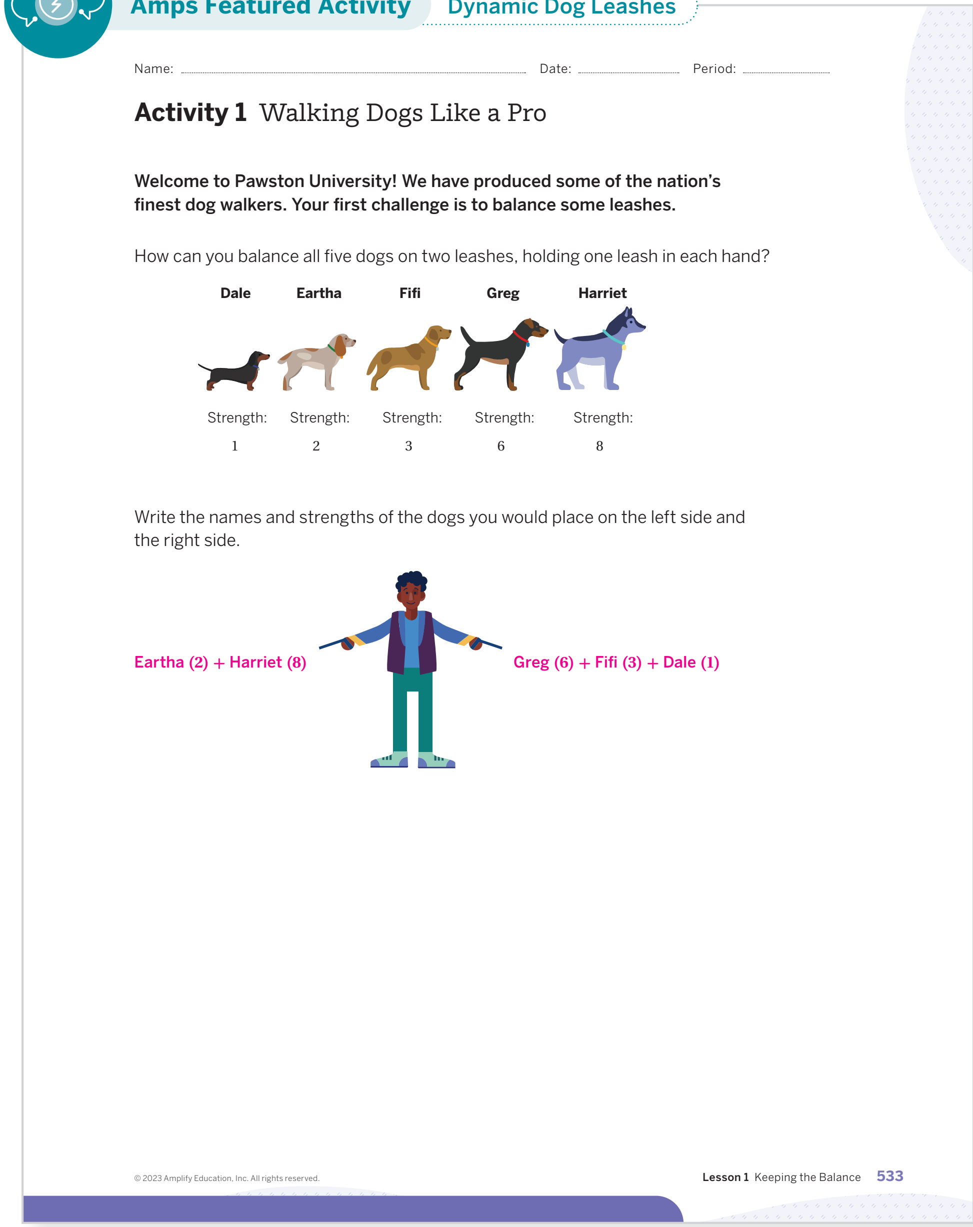
- "What is an example of something in your own life that you feel you must 'balance'?" Answers will vary, but students may refer to balancing responsibilities, or friendships.
- "How do we represent balance in math?" Using an equal sign.

Pairs | 10 min

Activity 1 Walking Dogs Like a Pro

Students balance dogs of varying strengths on opposite sides of the dog walker, connecting this model with balance in equations.

Amps Featured Activity Dynamic Dog Leashes



Name: ______________ Date: ________ Period: ________

Activity 1 Walking Dogs Like a Pro

Welcome to Pawston University! We have produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

How can you balance all five dogs on two leashes, holding one leash in each hand?

Dale	**Eartha**	**Fifi**	**Greg**	**Harriet**
Strength: 1	Strength: 2	Strength: 3	Strength: 6	Strength: 8

Write the names and strengths of the dogs you would place on the left side and the right side.

Eartha (2) + Harriet (8)

Greg (6) + Fifi (3) + Dale (1)

1 Launch

Conduct the ***Think-Pair-Share*** routine. Give students a few minutes to work independently, and then have them compare their work with a partner. Explain that this activity lends itself well to trial and error. Suggest students begin by using rough-draft thinking.

2 Monitor

Help students get started by suggesting they write the strengths of the dogs on small pieces of paper and manipulate them by hand.

Look for points of confusion:

- **Not using all five dogs.** Say, "The question asks to balance all five dogs."
- **Finding a combination that is not precisely balanced.** Ask, "If this dog walker were on roller skates, what would happen if one side was just a little stronger than the other?"

3 Connect

Have students share where they placed the dogs. Record student responses so the class can see. Be sure to include a student who reversed the sides of the dogs, which is equally valid.

Display the equality statement that represents each response that students shared.

Highlight by saying "This is also true about equations, if you think of the dog walker as the equal sign. Both sides must be balanced in order for it to be an equation. The equal sign tells you it is balanced."

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate the position of the dogs on either side of the dog walker, while trying to balance them.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present a set of dogs that would not be balanced, such as Dale, Eartha, and Greg on the left and Harriet and Fifi on the right.

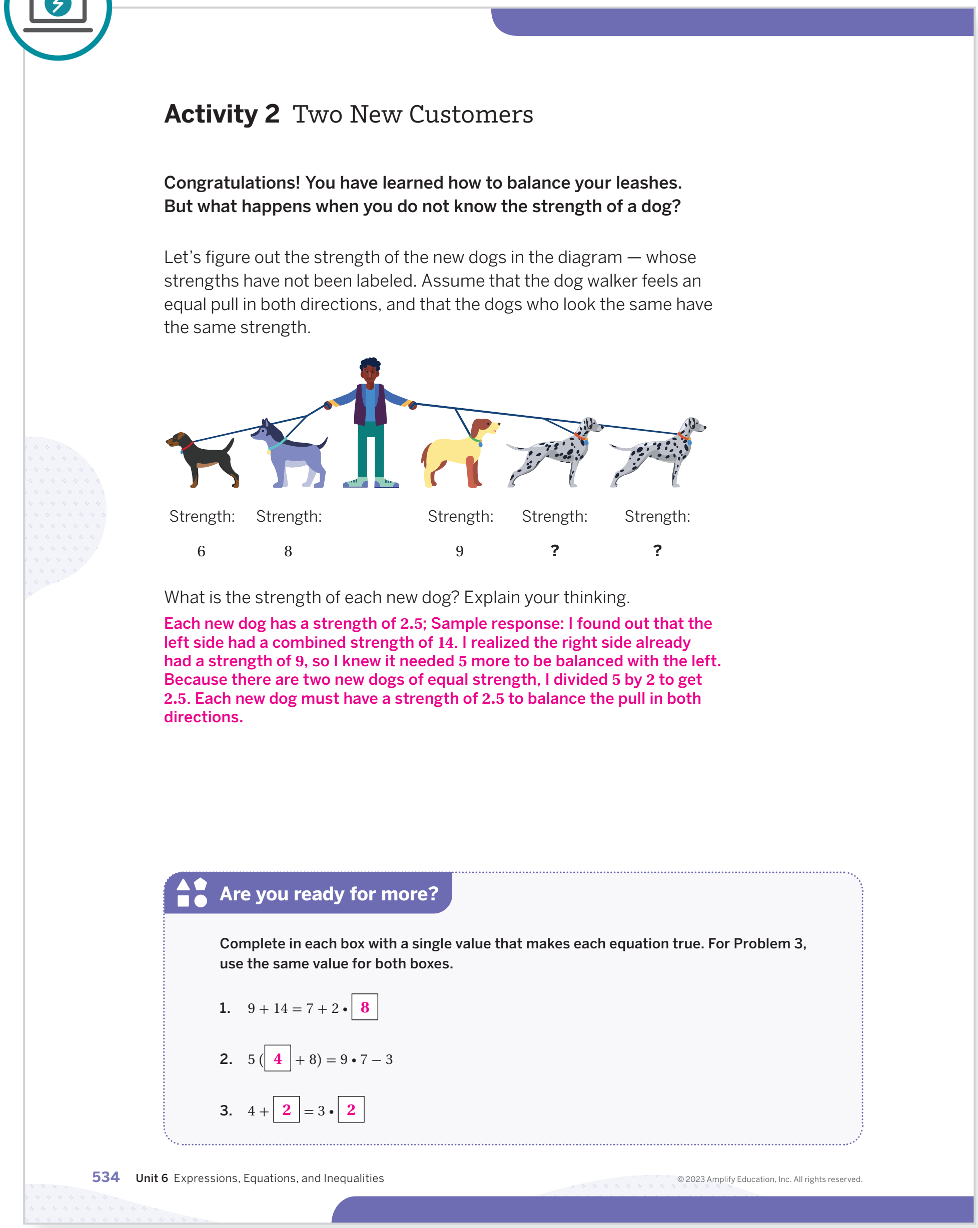
Ask:

- ***Critique:*** "Do you agree or disagree with this set of dogs? Explain your thinking."
- ***Correct:*** "How would you correct this set of dogs? Would you add or subtract dogs to balance both sides? Or both?"
- ***Clarify:*** "How can you convince someone that your corrected set of dogs is now balanced?"

Pairs | 10 min

Activity 2 Two New Customers

Students find the strength of an unknown dog in a balanced dog walker model to reintroduce the concept of variables in equations.



Activity 2 Two New Customers

Congratulations! You have learned how to balance your leashes. But what happens when you do not know the strength of a dog?

Let's figure out the strength of the new dogs in the diagram — whose strengths have not been labeled. Assume that the dog walker feels an equal pull in both directions, and that the dogs who look the same have the same strength.

Strength:	Strength:	Strength:	Strength:	Strength:
6	8	9	**?**	**?**

What is the strength of each new dog? Explain your thinking.

Each new dog has a strength of 2.5; Sample response: I found out that the left side had a combined strength of 14. I realized the right side already had a strength of 9, so I knew it needed 5 more to be balanced with the left. Because there are two new dogs of equal strength, I divided 5 by 2 to get 2.5. Each new dog must have a strength of 2.5 to balance the pull in both directions.

Are you ready for more?

Complete in each box with a single value that makes each equation true. For Problem 3, use the same value for both boxes.

1. $9 + 14 = 7 + 2 \cdot \boxed{8}$
2. $5(\boxed{4} + 8) = 9 \cdot 7 - 3$
3. $4 + \boxed{2} = 3 \cdot \boxed{2}$

534 **Unit 6** Expressions, Equations, and Inequalities

① Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

② Monitor

Help students get started by asking, "What is the strength of the dogs on the left? What must be true about the dogs on the right?"

Look for points of confusion:

- **Thinking each new dog has a strength of 5**. Say, "Try adding the values on the right side, and make sure they are balanced with the values on the left."

Look for productive strategies:

- **Using trial and error to find missing values.** Note this as Strategy 1.
- **Using counting on from 9 to find that the missing combined value is 5.** Note this as Strategy 2.
- **Subtracting 9 from 14 to find the difference that accounts for the unknown combined value.** Note this as Strategy 3.

③ Connect

Have students share the strategies they used. Depending on the strategy most students use, you may choose to share either Strategy 2 or Strategy 3 from above. Strategy 3 is ideal because it will prepare students for the work later on in the unit.

Ask, "What equation could you write to match this diagram?" $6 + 8 = 9 + x + x$

Highlight the connection between the dog walker scenario and equations.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Cover up the two unknown dogs. Ask, "What must be the strength remaining on the right side of the dog walker?" 5 Then uncover the two unknown dogs and ask, "If the total strength of these two dogs is 5, what must the strength of each dog be?" 2.5

Extension: *Math Enrichment*

As you highlight how the equation $6 + 8 = 9 + x + x$ represents the diagram, ask, "Is there a shorter way to write this equation?" $6 + 8 = 9 + 2x$ Some students may be familiar with combining like terms, without necessarily using that terminology. Like terms will be introduced later in this unit.

Math Language Development

MLR7: Compare and Connect

During the Connect, display Strategies 1–3. If no student mentioned one of these strategies, still display it and ask students to determine if it is a valid strategy. Have students compare and contrast the strategies by asking:

- "Are all of these strategies valid? Explain your thinking."
- "Do any strategies appear to be more efficient than others? Explain your thinking."

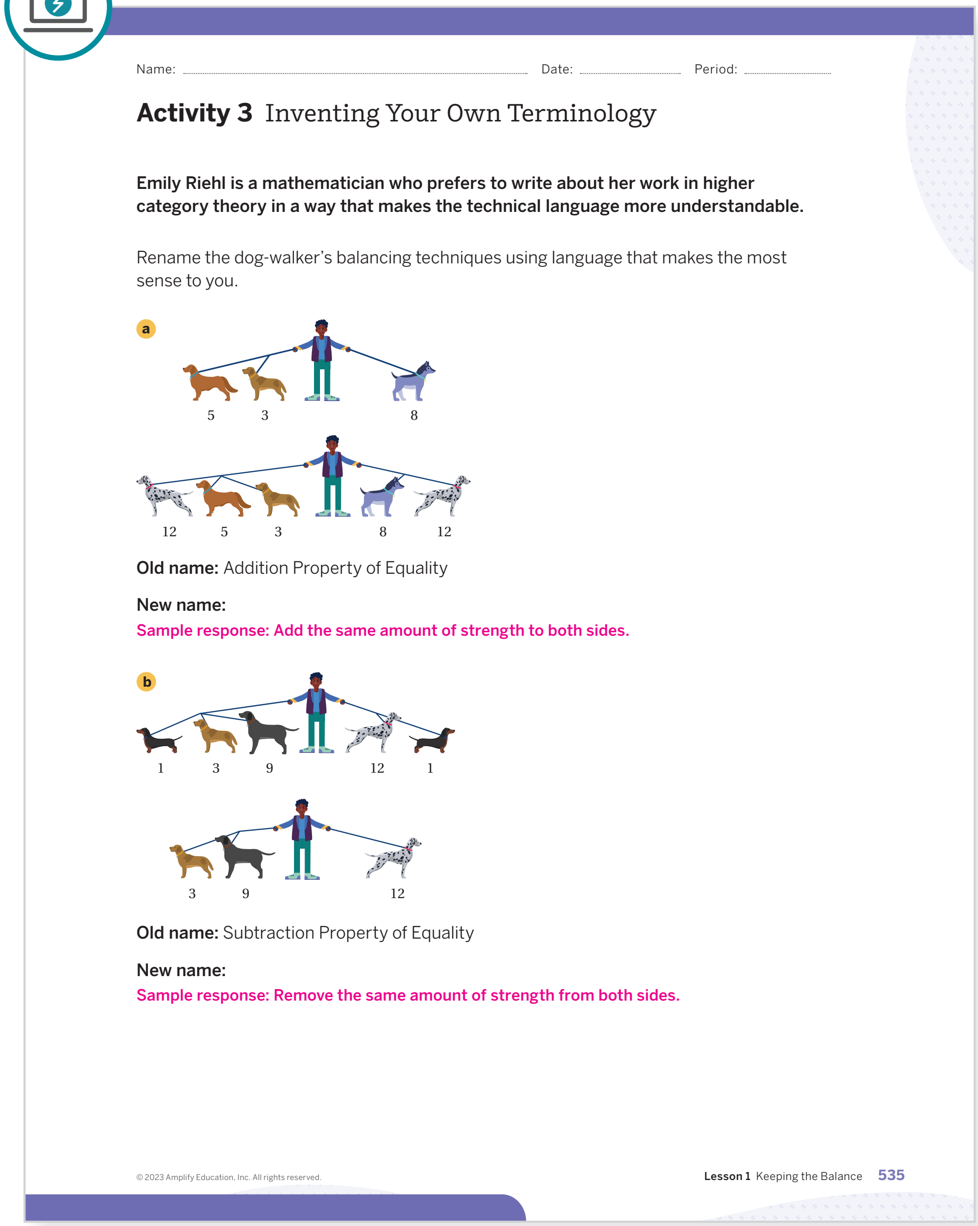
English Learners

Annotate the diagram to highlight how the equation $6 + 8 = 9 + x + x$ matches the diagram.

Pairs | 10 min

Activity 3 Inventing Your Own Terminology

Students engage with new vocabulary by using their own language to create new names for the properties of equality.



Name: ______ Date: ______ Period: ______

Activity 3 Inventing Your Own Terminology

Emily Riehl is a mathematician who prefers to write about her work in higher category theory in a way that makes the technical language more understandable.

Rename the dog-walker's balancing techniques using language that makes the most sense to you.

a

5 3 8

12 5 3 8 12

Old name: Addition Property of Equality

New name:

Sample response: Add the same amount of strength to both sides.

b

1 3 9 12 1

3 9 12

Old name: Subtraction Property of Equality

New name:

Sample response: Remove the same amount of strength from both sides.

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Lesson 1 Keeping the Balance 535

1 Launch

Complete part a as a whole class. Give students 1 minute to brainstorm a new name for the arrangement. Encourage them to use whatever wording they would like. Suggest, "Add the same amount of strength to both sides." as an example for part a.

2 Monitor

Help students get started by suggesting they give the dogs names so they can more easily recognize their positioning.

Look for points of confusion:

- **Trying to use words from the "old name."** Cover up the old name. Have students describe the arrangement in their own words.
- **Thinking new names need to be short.** Say, "Don't be afraid to have multiple drafts of your new names. Shorter is not necessarily better."

Activity 3 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge*

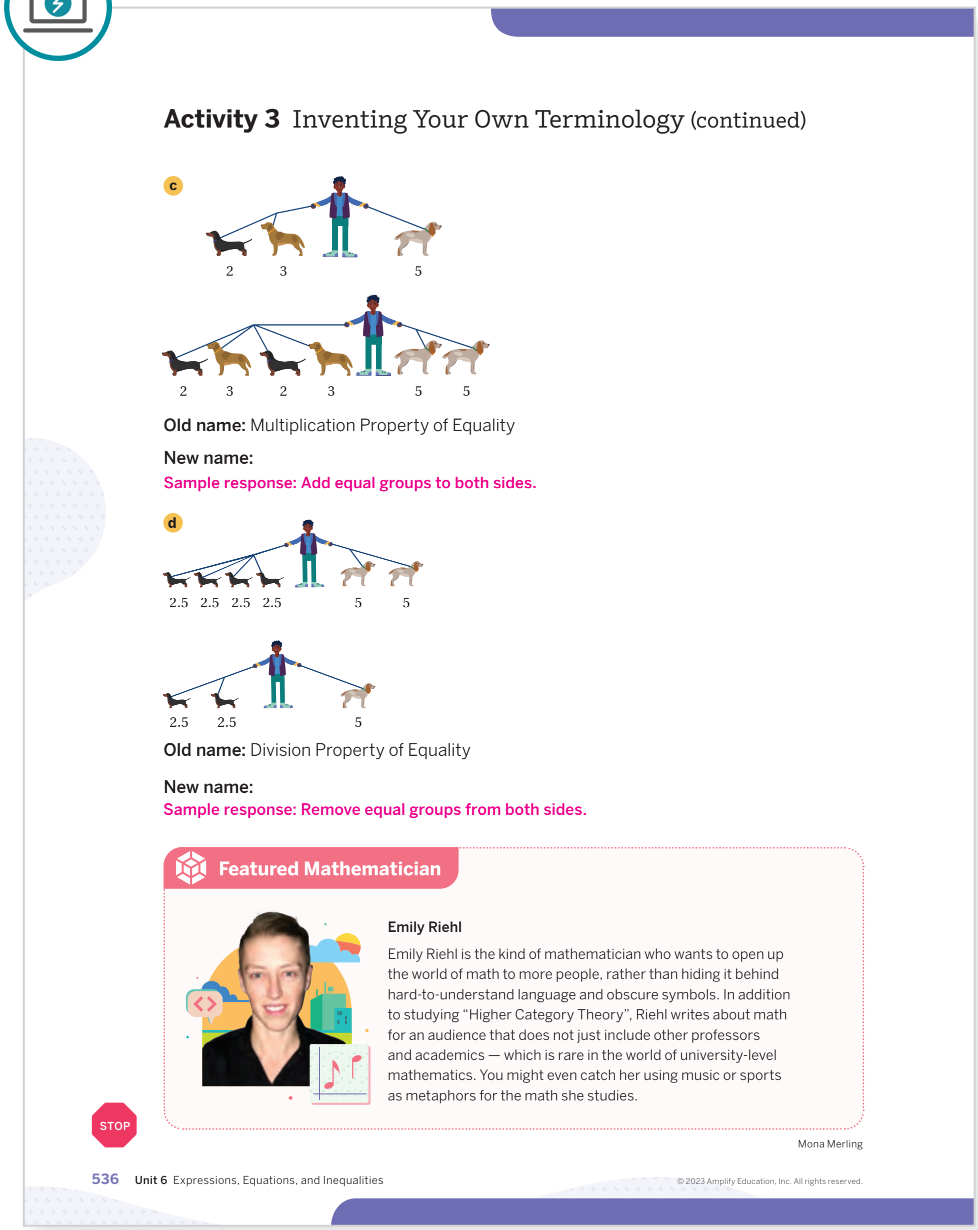
Remind students they previously learned the names of the properties given in this activity. Some students may benefit from a quick review of these properties. Consider providing them with these sentence frames shown to complete. Ask students to generate their own examples, such as, if $a = b$, then $a + 3 = b + 3$, to illustrate the Additional Property of Equality.

- **Addition Property of Equality:** If you ______ the same quantity to both sides of an equation, the equation remains ______.
- **Subtraction Property of Equality:** If you ______ the same quantity from both sides of an equation, the equation remains ______.
- **Multiplication Property of Equality:** If you ______ both sides of an equation by the same quantity, the equation remains ______.
- **Division Property of Equality:** If you ______ both sides of an equation by the same quantity, the equation remains ______.

Pairs | 10 min

Activity 3 Inventing Your Own Terminology (continued)

Students engage with new vocabulary by using their own language to create new names for the properties of equality.



Activity 3 Inventing Your Own Terminology (continued)

c

Old name: Multiplication Property of Equality

New name:

Sample response: Add equal groups to both sides.

d

Old name: Division Property of Equality

New name:

Sample response: Remove equal groups from both sides.

Featured Mathematician

Emily Riehl

Emily Riehl is the kind of mathematician who wants to open up the world of math to more people, rather than hiding it behind hard-to-understand language and obscure symbols. In addition to studying "Higher Category Theory", Riehl writes about math for an audience that does not just include other professors and academics — which is rare in the world of university-level mathematics. You might even catch her using music or sports as metaphors for the math she studies.

STOP

Mona Merling

536 Unit 6 Expressions, Equations, and Inequalities

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3 Connect

Display the pictures on a poster, showing both the "old name" and a "new name" for each property.

Highlight "rough-draft thinking" as a part of the mathematical reasoning process. Say, "You can think of addition and subtraction as similar properties, and multiplication and division as similar properties."

Featured Mathematician

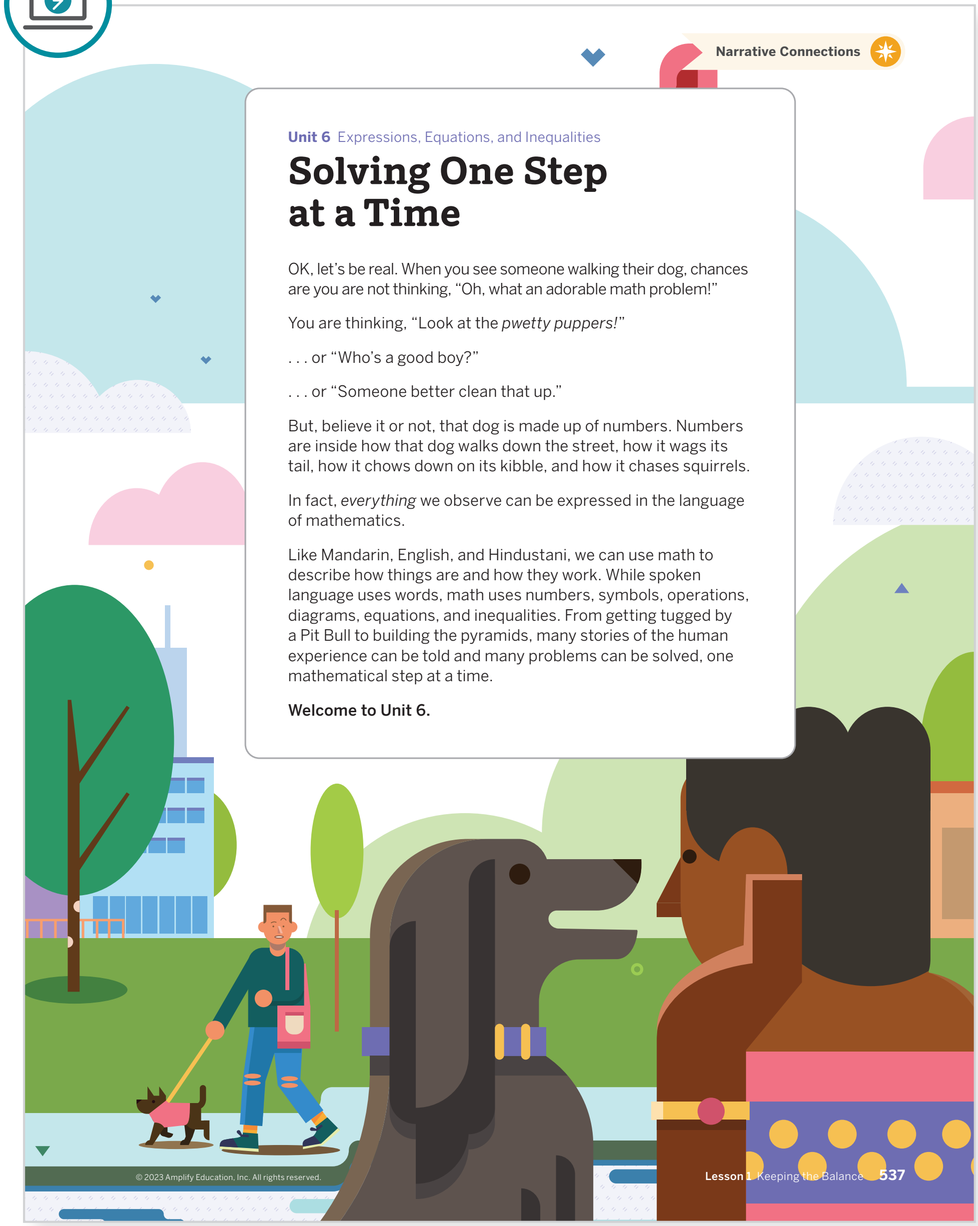
Emily Riehl

Have students read about *Featured Mathematician* Emily Riehl, who writes about complex mathematical theories in a way that can be understood by a wider audience.

Whole Class | 5 min

Summary Solving One Step at a Time

Review and synthesize how dog walkers, just like equations, prefer to stay in balance.



Unit 6 Expressions, Equations, and Inequalities

Solving One Step at a Time

OK, let's be real. When you see someone walking their dog, chances are you are not thinking, "Oh, what an adorable math problem!"

You are thinking, "Look at the *pwetty puppers!*"

. . . or "Who's a good boy?"

. . . or "Someone better clean that up."

But, believe it or not, that dog is made up of numbers. Numbers are inside how that dog walks down the street, how it wags its tail, how it chows down on its kibble, and how it chases squirrels.

In fact, *everything* we observe can be expressed in the language of mathematics.

Like Mandarin, English, and Hindustani, we can use math to describe how things are and how they work. While spoken language uses words, math uses numbers, symbols, operations, diagrams, equations, and inequalities. From getting tugged by a Pit Bull to building the pyramids, many stories of the human experience can be told and many problems can be solved, one mathematical step at a time.

Welcome to Unit 6.

Lesson 1 Keeping the Balance 537

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary. Have students read the Summary or have a student volunteer read it aloud.

Ask:

- "Can you think of any other times this year we have used numbers, symbols, operators, and diagrams to represent real-world scenarios mathematically?"
- "How is analyzing dog walkers related to mathematics?"

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

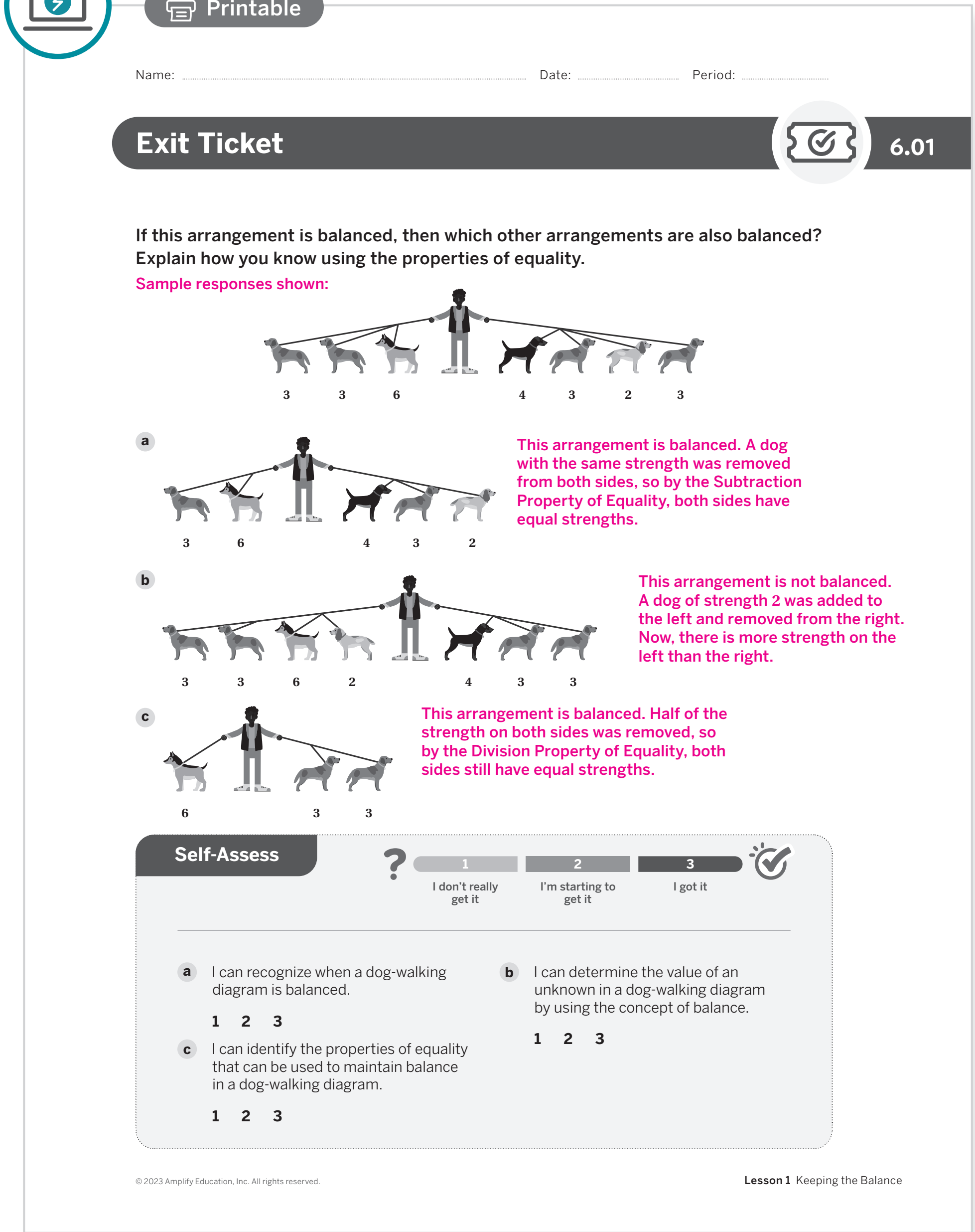
- "What strategies did you use in order to help balance the dog walker in each scenario?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by transferring their understanding of the properties of equality to equations.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 6.01

If this arrangement is balanced, then which other arrangements are also balanced? Explain how you know using the properties of equality.

Sample responses shown:

a

This arrangement is balanced. A dog with the same strength was removed from both sides, so by the Subtraction Property of Equality, both sides have equal strengths.

b

This arrangement is not balanced. A dog of strength 2 was added to the left and removed from the right. Now, there is more strength on the left than the right.

c

This arrangement is balanced. Half of the strength on both sides was removed, so by the Division Property of Equality, both sides still have equal strengths.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can recognize when a dog-walking diagram is balanced.

1 2 3

b I can determine the value of an unknown in a dog-walking diagram by using the concept of balance.

1 2 3

c I can identify the properties of equality that can be used to maintain balance in a dog-walking diagram.

1 2 3

Lesson 1 Keeping the Balance

Success looks like . . .

- **Language Goal:** Generalizing that adjusting the amount of strength equally on each side of a dog walker diagram keeps it balanced. **(Speaking)**
  - » Explaining whether each arrangement is balanced.
- **Language Goal:** Explaining how to use a balanced dog walker model to solve an equation of the form $px + q = r$. **(Speaking)**

Suggested next steps

If students struggle to identify which arrangements work, consider:

- Having them find the total strength on each side, and compare these strengths to determine if the dog walker is balanced.
- Assigning Practice Problem 2.

If students struggle to identify which property of equality applies to Problems 1 and 3, consider:

- Taking another look at the examples in Activity 3, and seeing if students can use them to identify the properties shown in the Exit Ticket.

Professional Learning

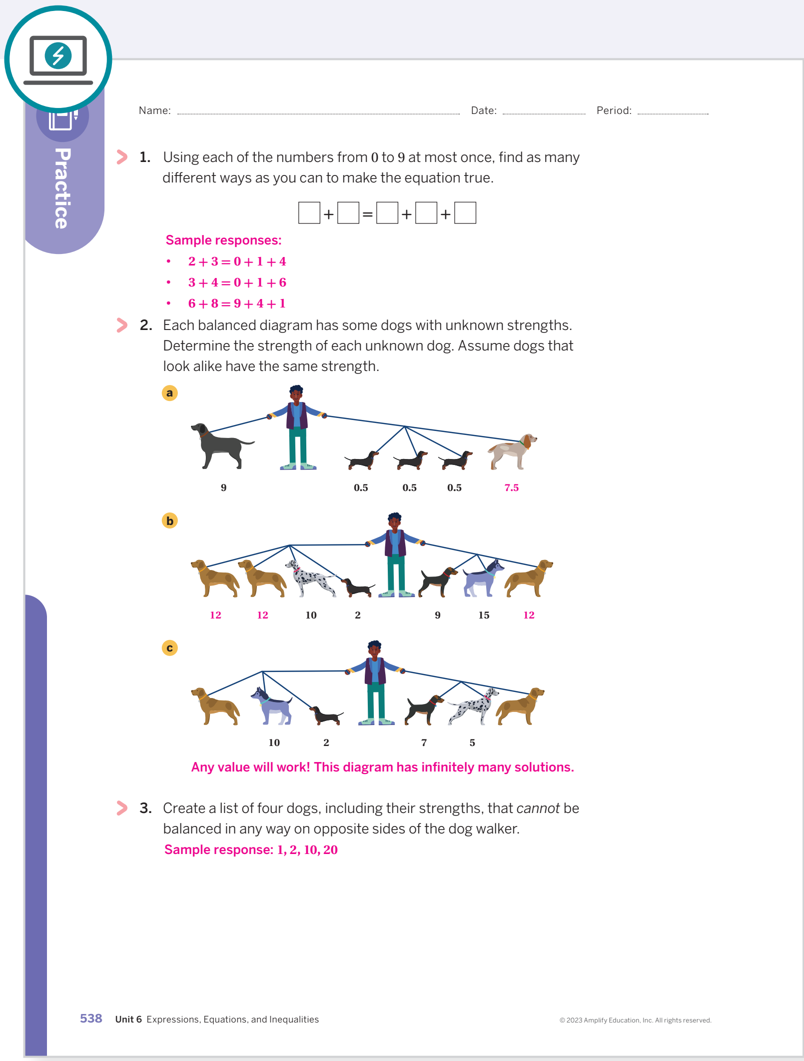
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students balanced the Strength of the dogs on both sides of a dog walker. How will that support students' understanding of balancing equations?
- Knowing where students need to be by the end of this unit, how did engaging in a discussion about the properties of equality (Activity 3) influence that future goal? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Using each of the numbers from 0 to 9 at most once, find as many different ways as you can to make the equation true.

$\square + \square = \square + \square + \square$

Sample responses:

- $2 + 3 = 0 + 1 + 4$
- $3 + 4 = 0 + 1 + 6$
- $6 + 8 = 9 + 4 + 1$

2. Each balanced diagram has some dogs with unknown strengths. Determine the strength of each unknown dog. Assume dogs that look alike have the same strength.

Any value will work! This diagram has infinitely many solutions.

3. Create a list of four dogs, including their strengths, that *cannot* be balanced in any way on opposite sides of the dog walker.

Sample response: 1, 2, 10, 20

538 **Unit 6** Expressions, Equations, and Inequalities

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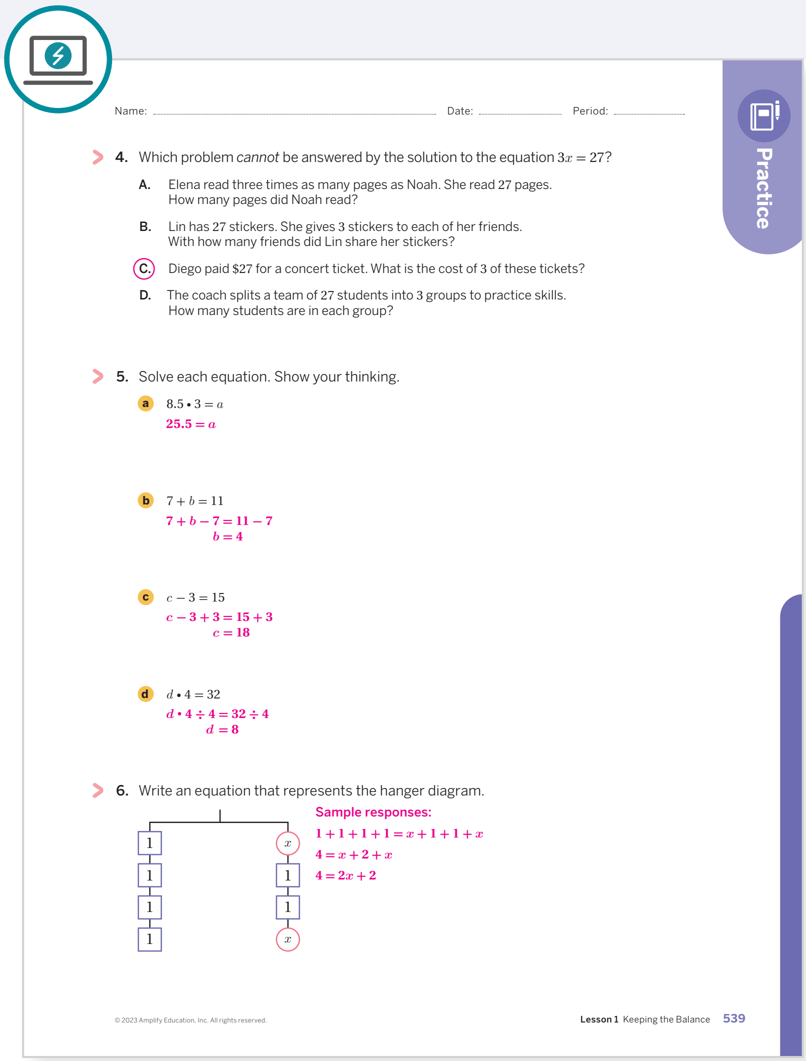

Practice

Name: Date: Period:

4. Which problem *cannot* be answered by the solution to the equation $3x = 27$?

A. Elena read three times as many pages as Noah. She read 27 pages. How many pages did Noah read?

B. Lin has 27 stickers. She gives 3 stickers to each of her friends. With how many friends did Lin share her stickers?

[✓ **C.**] Diego paid $27 for a concert ticket. What is the cost of 3 of these tickets?

D. The coach splits a team of 27 students into 3 groups to practice skills. How many students are in each group?

5. Solve each equation. Show your thinking.

a. $8.5 \cdot 3 = a$

$25.5 = a$

b. $7 + b = 11$

$7 + b - 7 = 11 - 7$

$b = 4$

c. $c - 3 = 15$

$c - 3 + 3 = 15 + 3$

$c = 18$

d. $d \cdot 4 = 32$

$d \cdot 4 \div 4 = 32 \div 4$

$d = 8$

6. Write an equation that represents the hanger diagram.

Sample responses:

$1 + 1 + 1 + 1 = x + 1 + 1 + x$

$4 = x + 2 + x$

$4 = 2x + 2$

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Lesson 1 Keeping the Balance 539

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	3
Spiral	4	Grade 6	2
	5	Grade 6	1
Formative	6	Unit 6 Lesson 2	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 1
Solving Two-Step Equations

Whole Class

In this Sub-Unit, students translate their understanding of hanger diagrams to the more-abstract algebraic representations as they learn to solve equations with more than one step.

SUB-UNIT

1 Solving Two-Step Equations

Narrative Connections

What are the first words you learn in “Caveman”?

Strictly speaking, there is no language called “Caveman.” In fact, scientists are pretty sure our cave-dwelling ancestors did not speak any language at all. But that does not mean early humans had nothing to say!

Archaeologists have discovered that many ancient cave paintings were drawn near “acoustic hotspots." These hotspots made anyone standing in that spot, loud and clear for others in the cave to hear. Scientists theorize that early humans connected the sounds made in those hotspots to the paintings on the walls. So, if one of our cave-dwelling ancestors painted a horse in full gallop or a trumpeting mammoth, these images may have represented similar sounds they heard in that very spot.

We do this kind of symbolic thinking every time we use letters (“H-O-R-S-E”) to represent a word (“horse”). This kind of thinking, they believe, helped bring about language as we know it today.

For thousands of years, our species has used pictures and symbols to express how we see the world. As a species we have come a long way from cave drawings. Today we can use words, emojis, pictures, and diagrams to express ourselves. These symbols help us connect, whether we are in the same classroom or on the other side of human history. And as you'll see, they can also help us to express and understand mathematical thinking.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to use diagrams and visual models to model their mathematical thinking in the following places:

- **Lesson 2, Activity 1:** Hanging Out
- **Lesson 3, Activity 1:** Matching Hanger Diagrams and Equations
- **Lesson 5, Warm-up:** Dogs in Different Directions

UNIT 6 | **LESSON 2**

Balanced and Unbalanced

Let's see how hanger diagrams can represent balanced relationships.

Focus

Goals

1. **Language Goal:** Generalize that performing the same operations to each side of a hanger diagram keeps it balanced. **(Speaking)**
2. Find a missing weight on a hanger diagram.

Rigor

- Students build **conceptual understanding** of balancing an equation by analyzing hanger diagrams.

Coherence

Today

Students find unknown values on balanced hanger diagrams that model two-step equations. They use the properties of equality to manipulate the diagrams, ensuring they remain balanced.

Previously

In Lesson 1, students used a dog walker model to review the properties of equality.

Coming Soon

In Lesson 3, students will use hanger diagrams to solve two-step equations, specifically of the form $px + q = r$.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

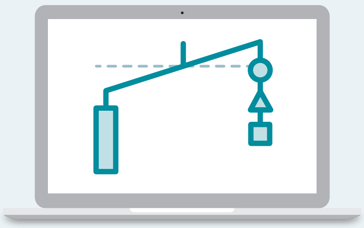
Review words

- *Addition Property of Equality*
- *Division Property of Equality*
- *equation*
- *hanger diagram*
- *Multiplication Property of Equality*
- *Subtraction Property of Equality*

Amps | **Featured Activity**

Activities 2 and 3:
Dynamic Hanger Diagrams

When students remove weights from a balanced hanger diagram, the hanger will animate, giving real-time feedback that shows whether the hanger is balanced.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

While using a hanger diagram, students must apply both abstract and quantitative reasoning, but they might lack the organizational skills to keep their explanations accurate. Encourage students to mark up the diagrams with information that they know. By listing what they know for sure, they can proceed to determine what could also be true and what definitely is not true.

Modifications to Pacing

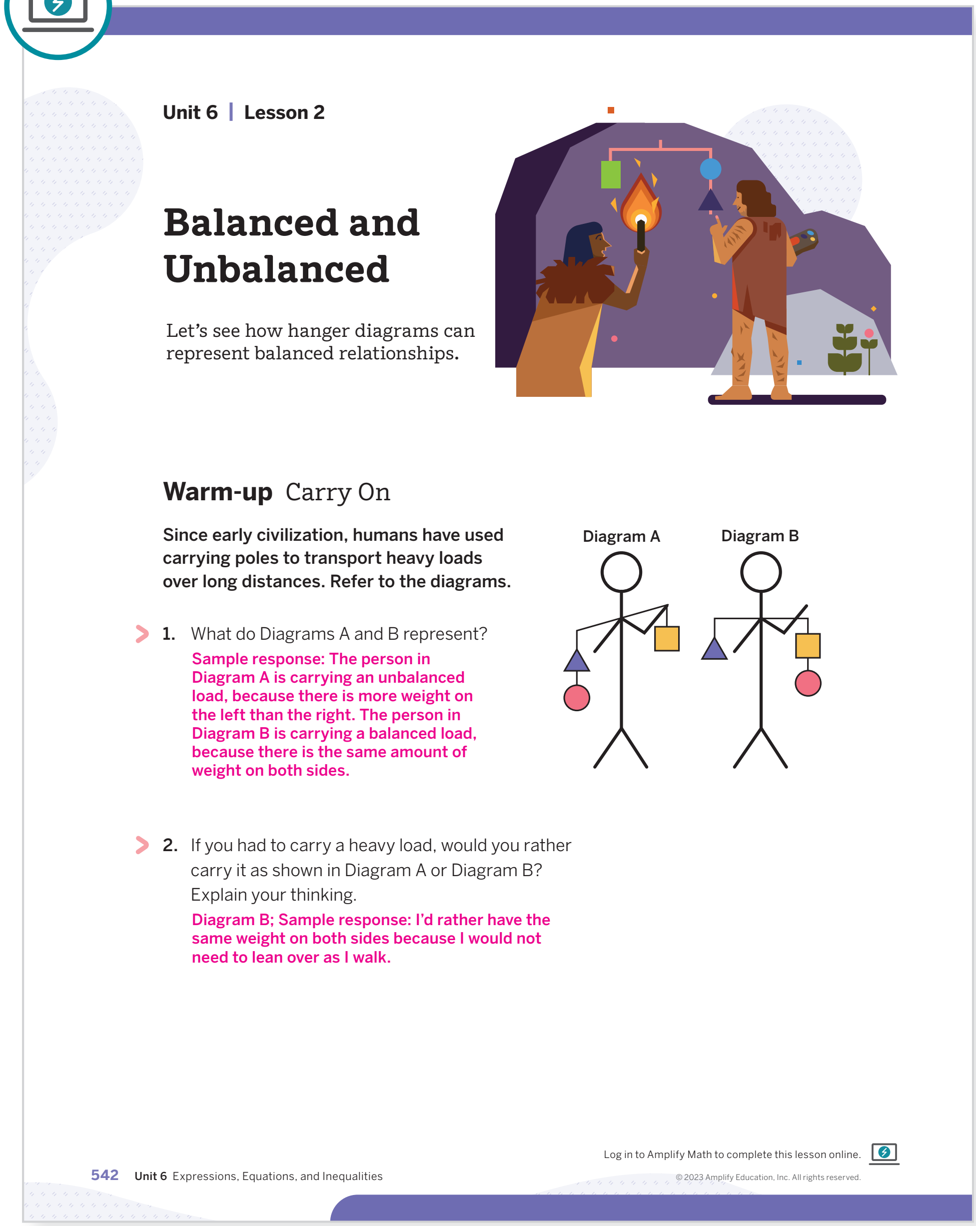
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. Instead, discuss during the Activity 1 Launch which hanger diagram is balanced and which is unbalanced.
- In **Activities 2 and 3**, have students discuss their strategies for Problem 1 orally without writing down their responses.

Independent | 5 min

Warm-up Carry On

Students consider whether carrying balanced or unbalanced loads is more stable, as preparation for understanding hanger diagrams.



Unit 6 | Lesson 2

Balanced and Unbalanced

Let's see how hanger diagrams can represent balanced relationships.

Warm-up Carry On

Since early civilization, humans have used carrying poles to transport heavy loads over long distances. Refer to the diagrams.

Diagram A

Diagram B

1. What do Diagrams A and B represent?

   Sample response: The person in Diagram A is carrying an unbalanced load, because there is more weight on the left than the right. The person in Diagram B is carrying a balanced load, because there is the same amount of weight on both sides.

2. If you had to carry a heavy load, would you rather carry it as shown in Diagram A or Diagram B? Explain your thinking.

   Diagram B; Sample response: I'd rather have the same weight on both sides because I would not need to lean over as I walk.

Log in to Amplify Math to complete this lesson online.

542 Unit 6 Expressions, Equations, and Inequalities

① Launch

Conduct the ***Think-Pair-Share*** routine. Give students 1 minute to respond to the problems individually, and 1 minute to discuss their thinking with a partner.

② Monitor

Help students get started by asking them to imagine themselves as the people in the illustrations, and to consider which load seems easier to carry.

Look for points of confusion:

- **Not understanding how the carrying pole works.** Consider having a broom or meter stick that students can place over their shoulders to better envision the situation.

③ Connect

Display the diagrams showing the balanced and unbalanced loads.

Have students share their thinking about the balanced and unbalanced loads.

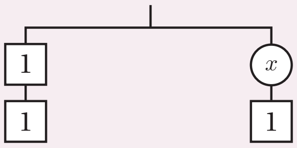
Ask, "What, exactly, is being balanced?" weight

Highlight that a carrying pole is another representation of balance. When the weights on both sides are equal, the pole is balanced. When one side is heavier than the other, the pole is unbalanced.

Power-up

To power up students' ability to reason about hanger diagrams, have students complete:

Which equation represents the hanger diagram?



A. $2 = x \cdot 1$

B. $1 \cdot 1 = x \cdot 1$

[✓ C.] $2 = x + 1$

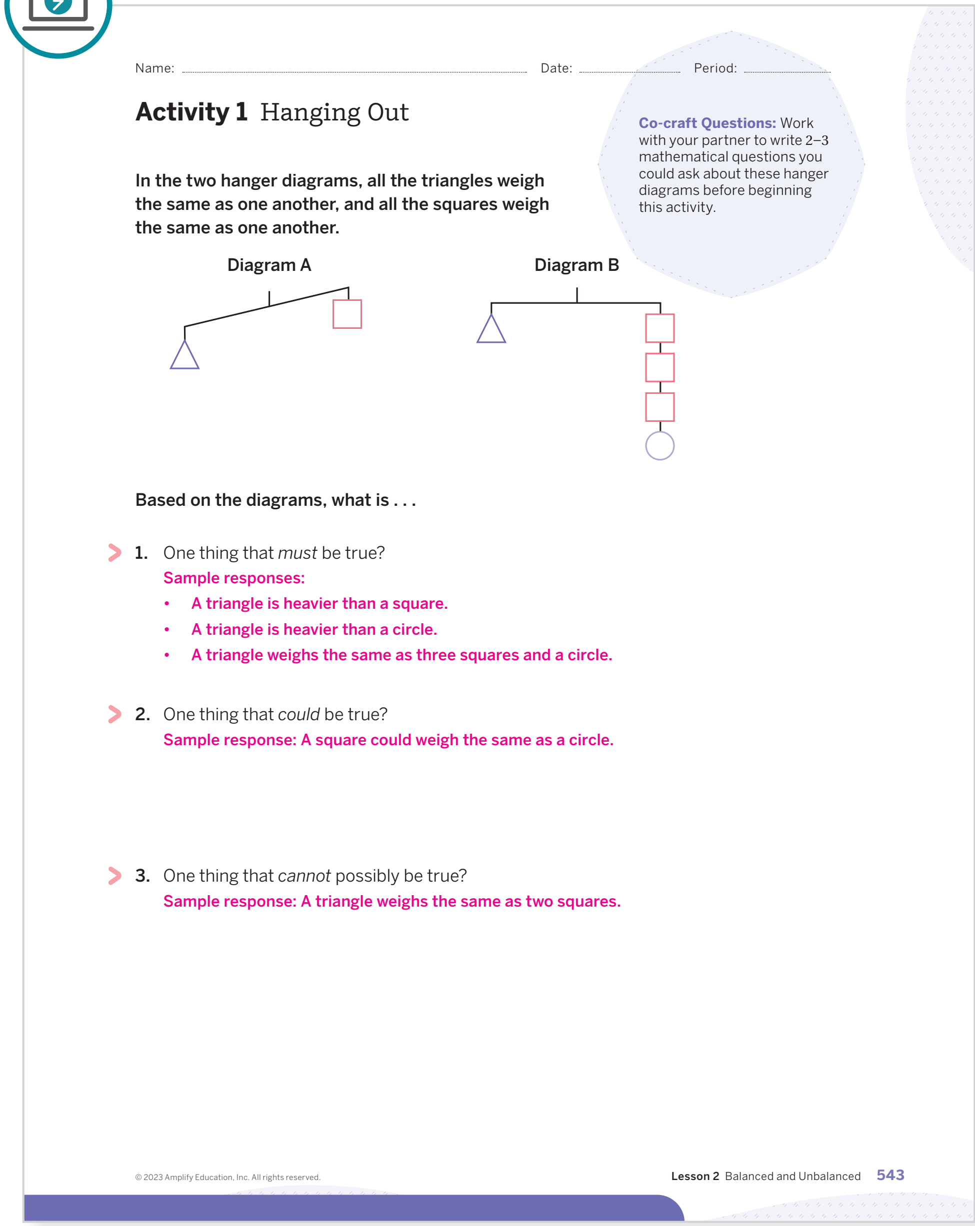
D. $3 = x$

Use: Before Activity 1

Informed by: Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7

Activity 1 Hanging Out

Students investigate relationships shown in hanger diagrams, to prepare them for solving for missing weights on a hanger diagram.



Name: ______ Date: ______ Period: ______

Activity 1 Hanging Out

Co-craft Questions: Work with your partner to write 2–3 mathematical questions you could ask about these hanger diagrams before beginning this activity.

In the two hanger diagrams, all the triangles weigh the same as one another, and all the squares weigh the same as one another.

Diagram A

Diagram B

Based on the diagrams, what is . . .

1. One thing that *must* be true?
   Sample responses:
   - **A triangle is heavier than a square.**
   - **A triangle is heavier than a circle.**
   - **A triangle weighs the same as three squares and a circle.**

2. One thing that *could* be true?
   Sample response: A square could weigh the same as a circle.

3. One thing that *cannot* possibly be true?
   Sample response: A triangle weighs the same as two squares.

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Lesson 2 Balanced and Unbalanced 543

1 Launch

Conduct the ***Think-Pair-Share*** routine. Give students 2 minutes to consider the diagrams and complete the problems individually. Then have them share their thinking with a partner.

Note: For each activity all squares/triangles/circles and so on weight the same, but the weight can be different from activity to activity.

2 Monitor

Help students get started by asking, "Which diagram is balanced? Which diagram is not balanced? What does this tell you about the weights of the shapes?" Consider discussing the meaning of *must* (always, for any weights), *could* (sometimes), and *cannot* (there is no example) in this context.

Look for productive strategies:

- Comparing the weights of the shapes (e.g., saying that a triangle is heavier than a square).

3 Connect

Display the two hanger diagrams.

Have students share what *must*, *could*, and *cannot* be true about the shapes and the diagrams.

Highlight how students justify their conclusions about the diagrams.

Ask, "How do you know that what you stated must/could/cannot be true is correct?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

To demonstrate how the weight of the triangle compares to the weight of the square in Diagram A, consider bringing in two items of differing weights and hold one in each hand. Show how the heavier weight would cause the balance to be lower on that side than the side with the lighter weight.

Extension: *Math Enrichment*

Ask students to assign possible weights to the triangle, square, and circle, based on the relationships shown in the diagram. Sample response: A triangle could weigh 7 g, a square could weigh 2 g, and a circle could weigh 1 g; $7 > 2$ and $7 = 3(2) + 1$.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the hanger diagrams and have students work with their partner to write 2–3 mathematical questions they could ask about the hanger diagrams. Sample questions shown.

- Which weighs more, a triangle or a square?
- What does it mean that Diagram B is balanced?
- How does the weight of a square compare to the weight of a circle?

English Learners

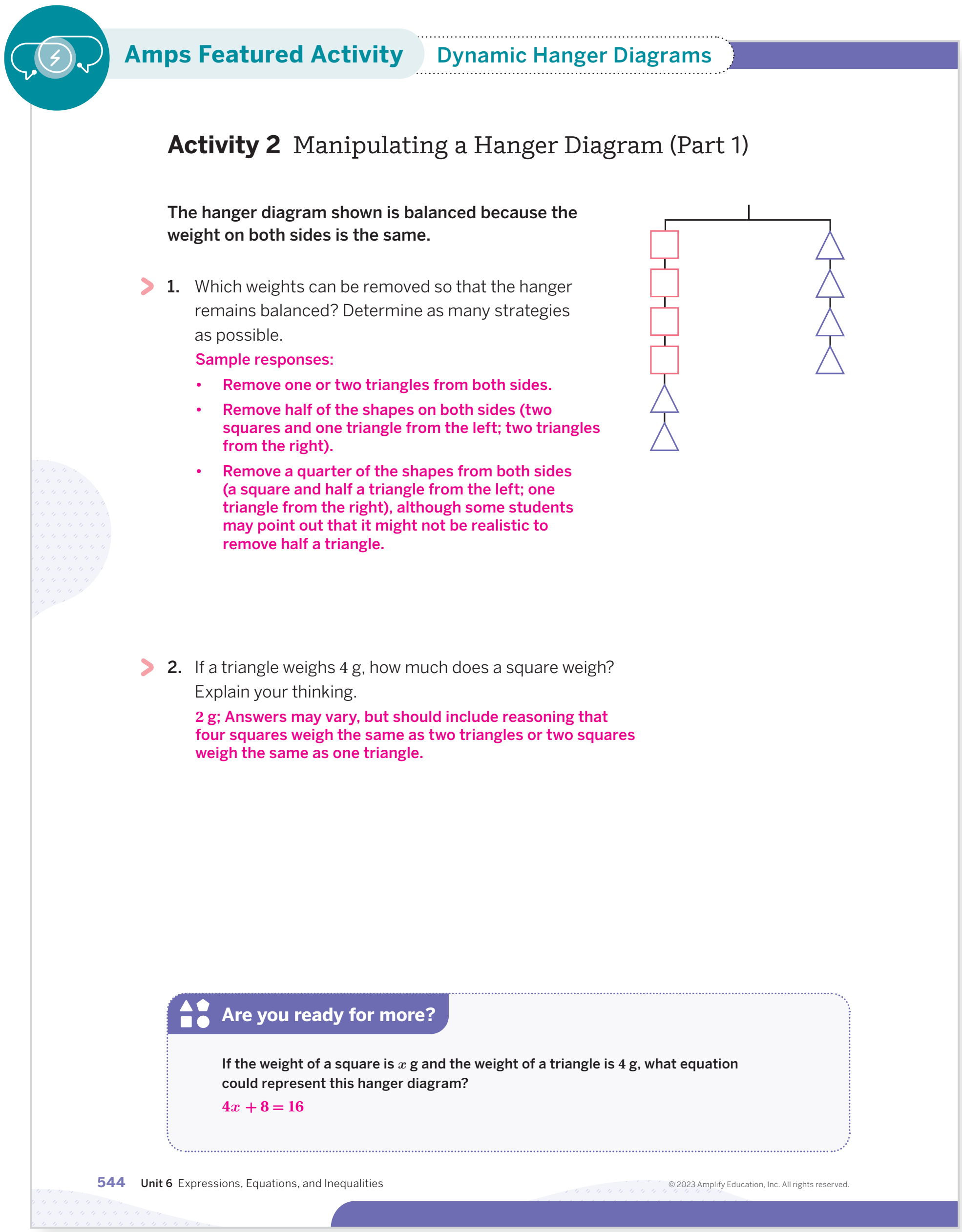
To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 10 min

Activity 2 Manipulating a Hanger Diagram (Part 1)

Students manipulate a balanced hanger diagram that models an equation of the form $px + q = r$ to determine an unknown weight.

Amps Featured Activity Dynamic Hanger Diagrams



Activity 2 Manipulating a Hanger Diagram (Part 1)

The hanger diagram shown is balanced because the weight on both sides is the same.

1. Which weights can be removed so that the hanger remains balanced? Determine as many strategies as possible.

   Sample responses:
   - Remove one or two triangles from both sides.
   - Remove half of the shapes on both sides (two squares and one triangle from the left; two triangles from the right).
   - Remove a quarter of the shapes from both sides (a square and half a triangle from the left; one triangle from the right), although some students may point out that it might not be realistic to remove half a triangle.

2. If a triangle weighs 4 g, how much does a square weigh? Explain your thinking.

   2 g; Answers may vary, but should include reasoning that four squares weigh the same as two triangles or two squares weigh the same as one triangle.

Are you ready for more?

If the weight of a square is x g and the weight of a triangle is 4 g, what equation could represent this hanger diagram?

$4x + 8 = 16$

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1 Launch

Suggest students mark the hanger diagram (or sketch new ones) when considering Problems 1 and 2.

2 Monitor

Help students get started by asking, "If you remove one triangle from each side, will the hanger stay balanced? What if you remove one triangle from one side and two triangles from the other side?"

Look for points of confusion:

- **Removing unequal amounts of weight from each side.** Remind students that they must perform the same action to each side in order to keep the hanger balanced.

Look for productive strategies:

- Marking the diagram or drawing a new one to show the weights being removed.
- Labeling the triangles with 4 (Problem 2).
- Writing and solving the equation $4x = 8$ to determine the weight of a square.

3 Connect

Display the hanger diagram shown in the problem.

Have students share how they removed weights. Ask a student to explain how they found the weight of a square, and see if anyone else used a different strategy.

Highlight the connection between Problem 1 and the properties of equality. For example, removing two triangles from each side is allowed by the Subtraction Property of Equality, while the Division Property of Equality allows the removal of half of the shapes (for each type of shape) from each side.

Ask, "How could you use the Addition and Multiplication Properties of Equality to create new balanced hanger diagrams?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which the hanger will animate as they remove weights, providing them with real-time feedback that shows whether the hanger is balanced.

Accessibility: *Vary Demands to Optimize Challenge*

In Problem 1, ask students to determine only one possible response, instead of determining as many as possible.

Math Language Development

MLR7: Compare and Connect

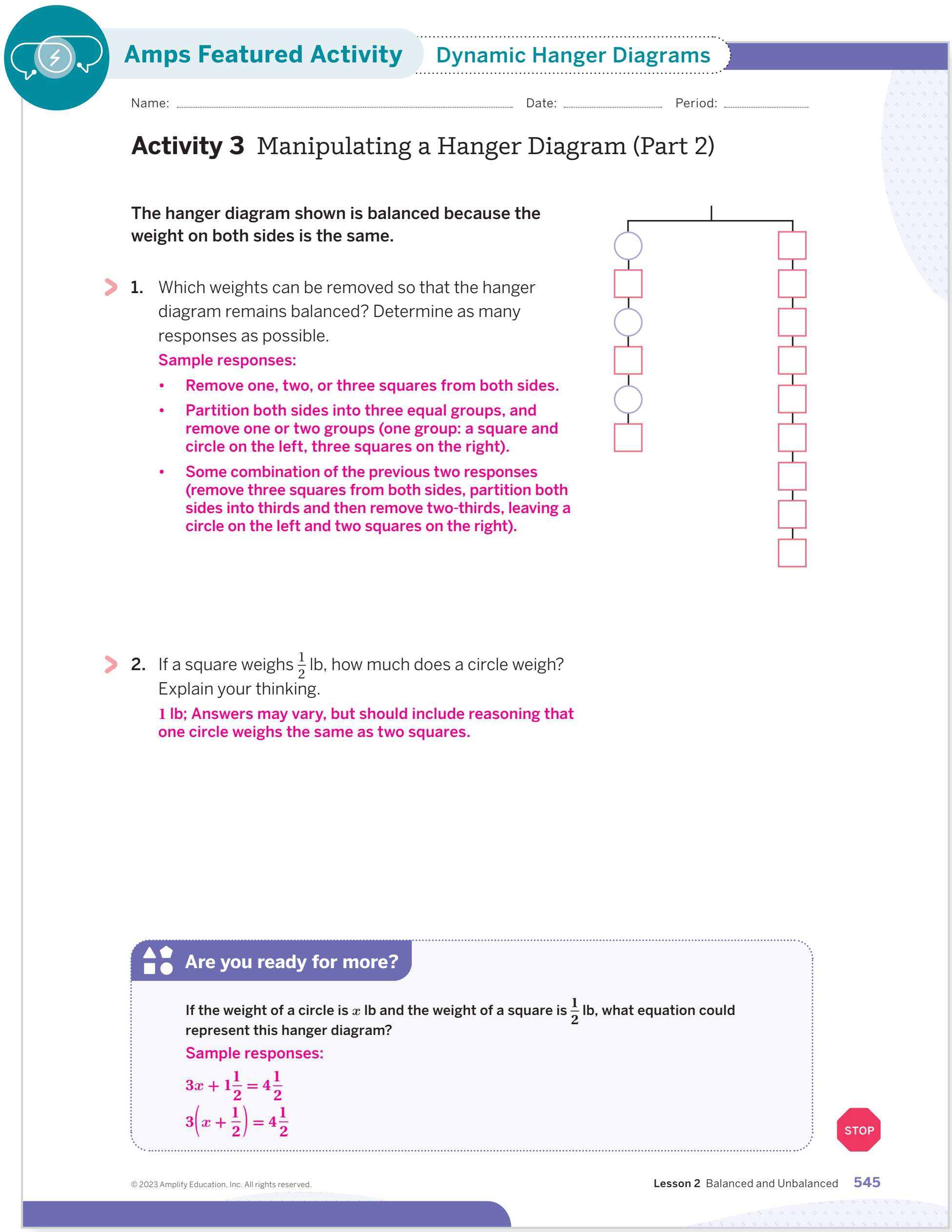
During the Connect, as students share their responses to Problem 1, draw their attention to the connections between removing weights and the properties of equality. For example:

If a student says . . .	Ask . . .
"I can remove two triangles from both sides."	"What property allows you to do that? How do you know the resulting hanger is balanced?"
"I can remove half of the shapes from each side."	"Which shapes can you remove from the left side? How do you know this is 'half of the shapes'? What property allows you to do this?"

Pairs | 10 min

Activity 3 Manipulating a Hanger Diagram (Part 2)

Students manipulate a balanced hanger diagram that models an equation of the form $p(x + q) = r$ to determine an unknown weight.



Amps Featured Activity Dynamic Hanger Diagrams

Name: ____________ Date: ________ Period: ________

Activity 3 Manipulating a Hanger Diagram (Part 2)

The hanger diagram shown is balanced because the weight on both sides is the same.

1. Which weights can be removed so that the hanger diagram remains balanced? Determine as many responses as possible.
   Sample responses:
   - **Remove one, two, or three squares from both sides.**
   - **Partition both sides into three equal groups, and remove one or two groups (one group: a square and circle on the left, three squares on the right).**
   - **Some combination of the previous two responses (remove three squares from both sides, partition both sides into thirds and then remove two-thirds, leaving a circle on the left and two squares on the right).**

2. If a square weighs $\frac{1}{2}$ lb, how much does a circle weigh? Explain your thinking.
   1 lb; Answers may vary, but should include reasoning that one circle weighs the same as two squares.

Are you ready for more?

If the weight of a circle is x lb and the weight of a square is $\frac{1}{2}$ lb, what equation could represent this hanger diagram?

Sample responses:

$3x + 1\frac{1}{2} = 4\frac{1}{2}$

$3\left(x + \frac{1}{2}\right) = 4\frac{1}{2}$

STOP

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Lesson 2 Balanced and Unbalanced 545

1 Launch

Discuss that the arrangement of shapes on this hanger diagram is slightly different from the one in Activity 2, and note that the weight on each side does not depend on the relative arrangement of the shapes.

2 Monitor

Help students get started by suggesting they consider which moves from Activity 2 will also work with this hanger diagram.

Look for productive strategies:

- Removing three squares on each side to see that the weight of three circles is equal to the weight of six squares.
- Partitioning both sides into three equal groups, and then removing all but one group.

3 Connect

Display the hanger diagram.

Have students share their strategies for removing weights. Look for examples of students using combinations of moves.

Highlight two distinct strategies for removing weights: 1) removing three squares from each side and partitioning the remaining shapes into three equal groups; 2) partitioning each side into three equal groups, and then removing one square from each group. Discuss the properties of equality that prove these moves keep the hanger balanced. Note that both strategies result in discovering that one circle weighs the same as two squares, and have students use this to complete Problem 2.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which the hanger will animate as they remove weights, providing them with real-time feedback that shows whether the hanger is balanced.

Accessibility: *Vary Demands to Optimize Challenge*

Consider replacing $\frac{1}{2}$ with 1 in Problem 2, which will still allow students to access the targeted goal for this activity, but remove the added task of reasoning about fractional values.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight the strategies for removing weights, draw students' attention to the connections between the strategies. Ask:

- "What similarities do you see among these two strategies?" Sample response: Both strategies involve partitioning into equal groups and removing squares.
- "What differences do you see?" The order in which these actions take place. If the partitioning is done first, then only one square is removed (from each group).
- "What properties of equality are illustrated by these strategies?" Division Property of Equality and Subtraction Property of Equality

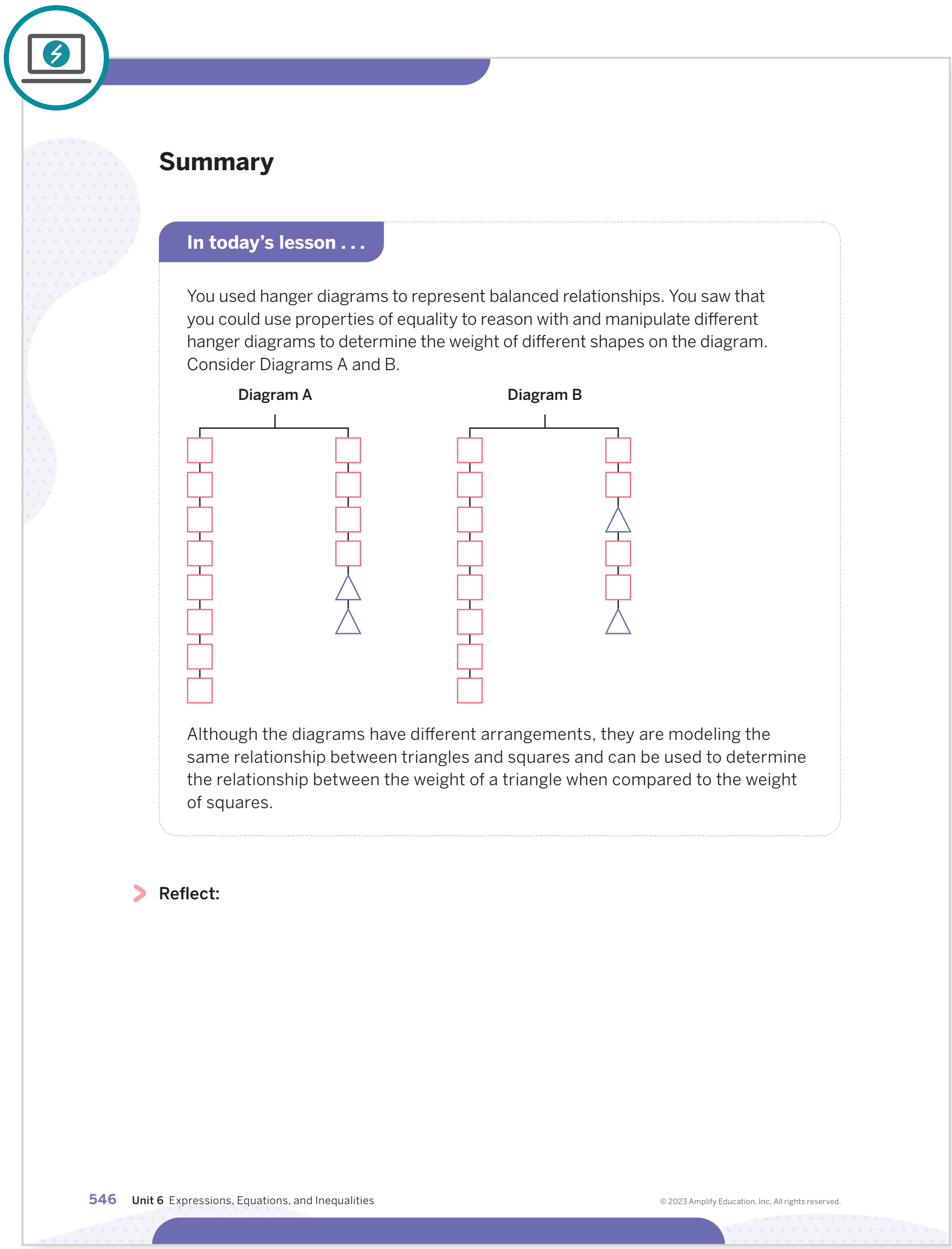
English Learners

Annotate the diagram to show the weights being removed to help students begin to visualize the equation solving process.

Whole Class | 5 min

Summary

Review and synthesize how the properties of equality can be used to manipulate hanger diagrams.



Summary

In today's lesson . . .

You used hanger diagrams to represent balanced relationships. You saw that you could use properties of equality to reason with and manipulate different hanger diagrams to determine the weight of different shapes on the diagram. Consider Diagrams A and B.

Diagram A

Diagram B

Although the diagrams have different arrangements, they are modeling the same relationship between triangles and squares and can be used to determine the relationship between the weight of a triangle when compared to the weight of squares.

Reflect:

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Synthesize

Display the diagram from the Summary and give students one minute to compare the two hanger diagrams.

Ask:

- "What is the same?" Both diagrams show that eight squares weigh the same as four squares and two triangles.
- "What is different?" The arrangement of the shapes on the right.
- "How could you determine the relationship between the weight of a triangle compared to the weight of the squares?" Use properties of equality to balance both sides of the hanging diagram.

Have students share their thinking with a partner.

Highlight that the arrangement of Diagram A makes it easier to remove four squares from each side (Subtraction Property of Equality) and then divide both sides into two groups (Division Property of Equality), to see that one triangle equals one-and-a-half squares. Meanwhile, the arrangement of Diagram B makes it easier to start by dividing both sides into two groups (Division Property of Equality) and then removing two squares from each side (Subtraction Property of Equality), to see that one triangle equals one-and-a-half squares. Note that both strategies are valid and result in the same equivalency.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

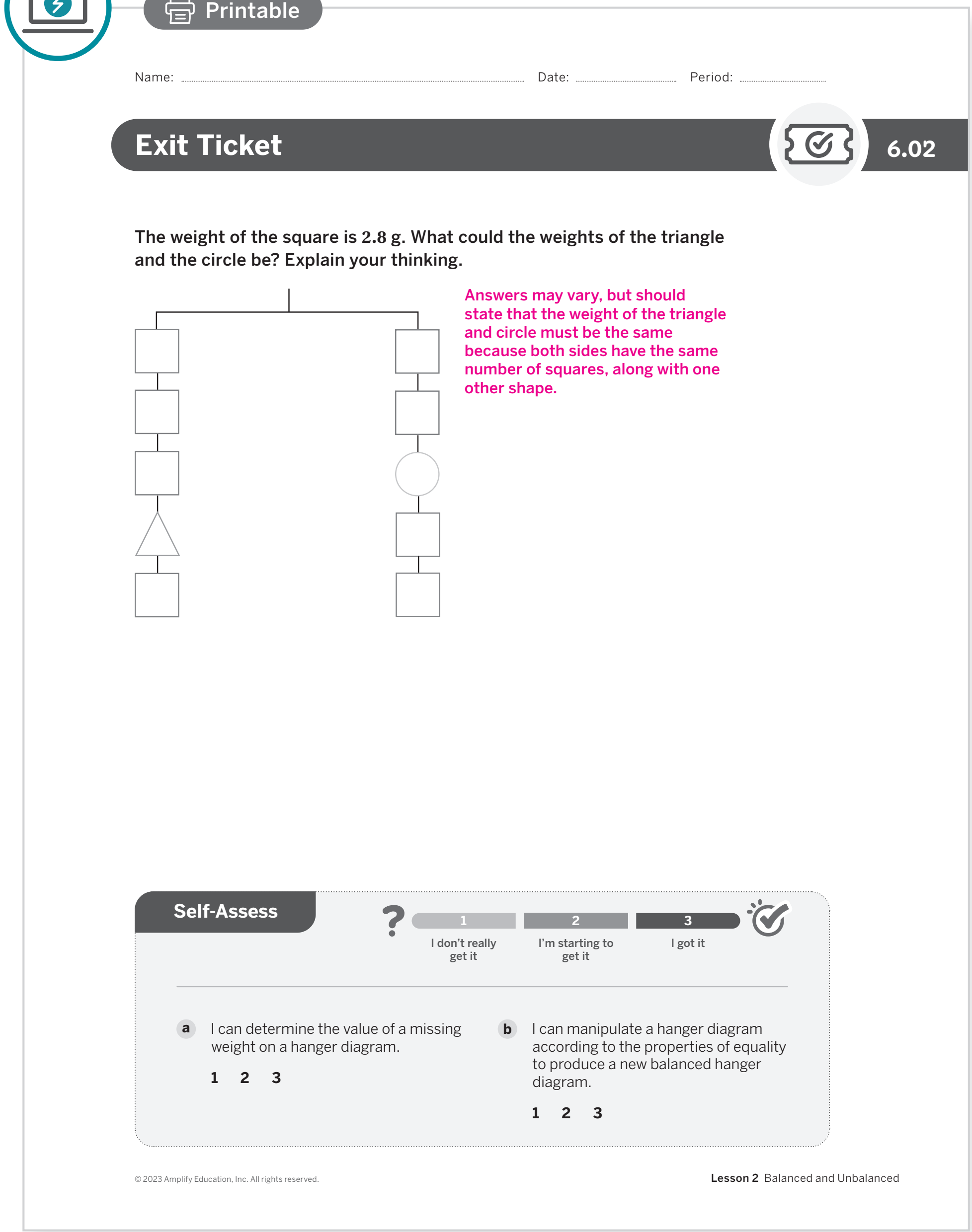
- "How is balancing the hanger diagram similar and different than balancing the dog walker from the previous lesson?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reasoning about the relationships shown in a hanger diagram.

Printable



Name: Date: Period:

Exit Ticket 6.02

The weight of the square is 2.8 g. What could the weights of the triangle and the circle be? Explain your thinking.

Answers may vary, but should state that the weight of the triangle and circle must be the same because both sides have the same number of squares, along with one other shape.

Self-Assess

? 1 I don't really get it 2 I'm starting to get it 3 I got it

a I can determine the value of a missing weight on a hanger diagram.
1 2 3

b I can manipulate a hanger diagram according to the properties of equality to produce a new balanced hanger diagram.
1 2 3

Lesson 2 Balanced and Unbalanced

Success looks like . . .

- **Language Goal:** Generalizing that performing the same operations to each side of a hanger diagram keeps it balanced. **(Speaking)**
- **Goal:** Finding a missing weight on a hanger diagram.
  - » Determining the weights of the triangle and the circle in the hanger diagram.

Suggested next steps

If students give a different weight for the square than the circle, consider:

- Reviewing how they reasoned about removing weights in the hanger diagram to arrive at that conclusion.
- Assigning Practice Problems 1 and 2.

If students give only one possible weight for the square and circle, consider:

- Asking whether this is the only possible weight for these shapes. If they say it is, demonstrate why, as long as the square and circle have the same weight, the diagram will be balanced.
- Assigning Practice Problem 3.

Professional Learning

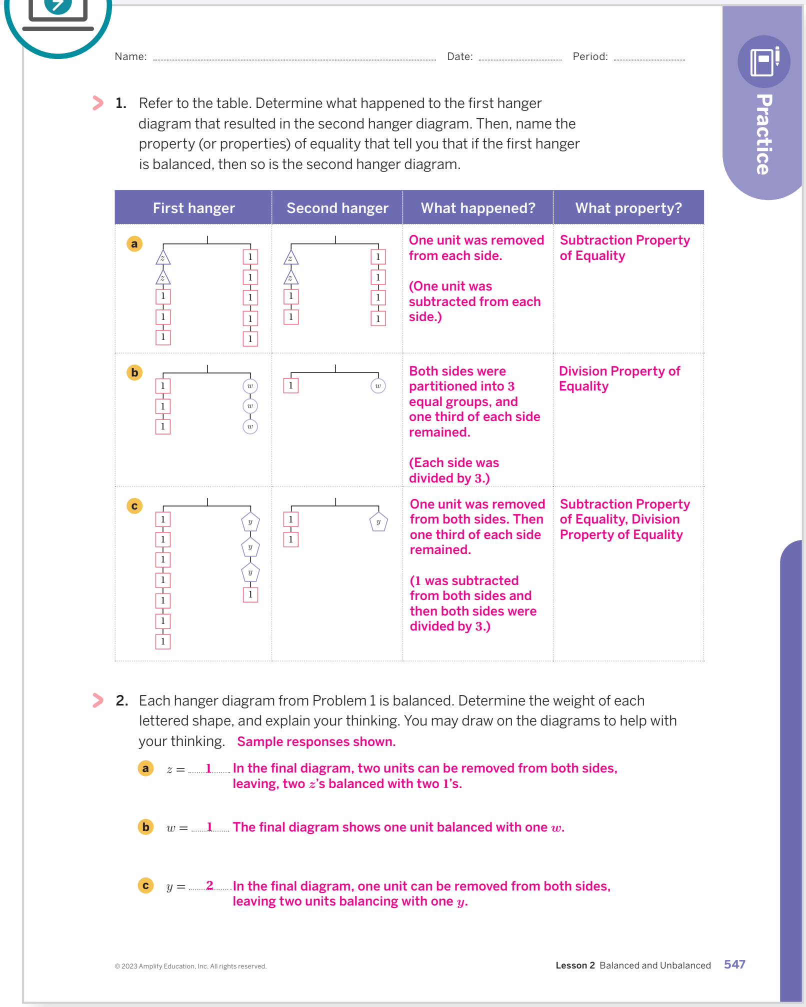
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students gotten better at reasoning abstractly and quantitatively?
- What did the process of balancing hanger diagrams reveal about your students as learners? What might you change for the next time you teach this lesson?

Practice

Independent

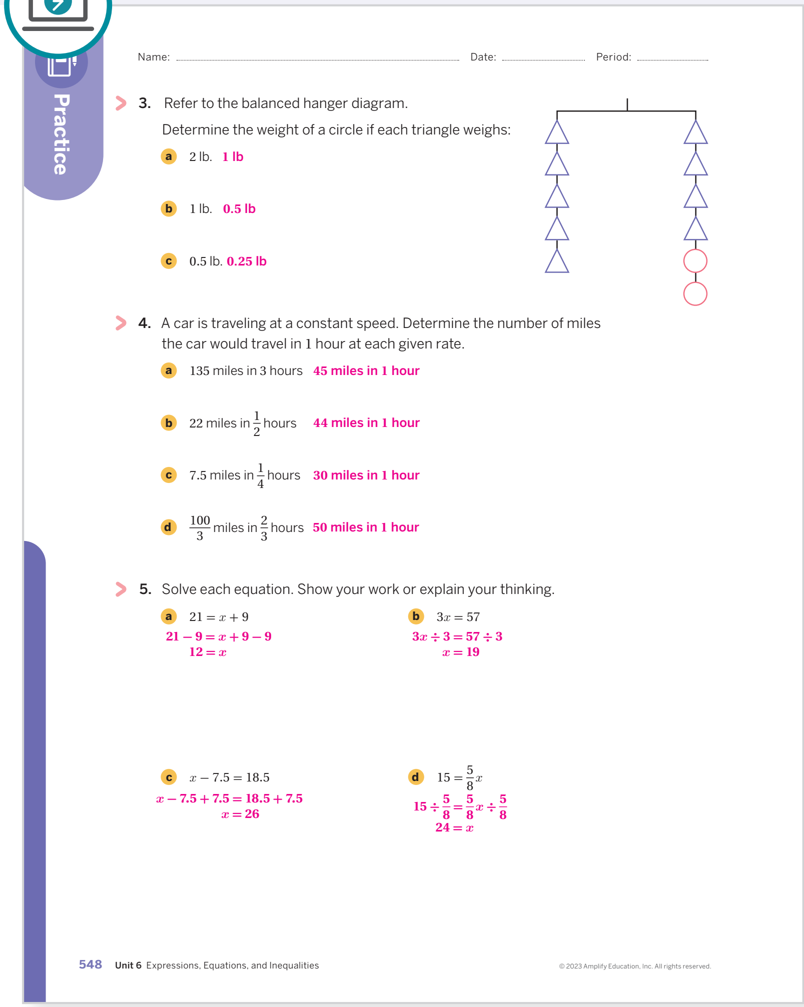


Name: .... Date: .... Period: ....

1. Refer to the table. Determine what happened to the first hanger diagram that resulted in the second hanger diagram. Then, name the property (or properties) of equality that tell you that if the first hanger is balanced, then so is the second hanger diagram.

	First hanger	Second hanger	What happened?	What property?
a			One unit was removed from each side. (One unit was subtracted from each side.)	Subtraction Property of Equality
b			Both sides were partitioned into 3 equal groups, and one third of each side remained. (Each side was divided by 3.)	Division Property of Equality
c			One unit was removed from both sides. Then one third of each side remained. (1 was subtracted from both sides and then both sides were divided by 3.)	Subtraction Property of Equality, Division Property of Equality

2. Each hanger diagram from Problem 1 is balanced. Determine the weight of each lettered shape, and explain your thinking. You may draw on the diagrams to help with your thinking. **Sample responses shown.**
   - a $z =$ **1** **In the final diagram, two units can be removed from both sides, leaving, two z's balanced with two 1's.**
   - b $w =$ **1** **The final diagram shows one unit balanced with one w.**
   - c $y =$ **2** **In the final diagram, one unit can be removed from both sides, leaving two units balancing with one y.**



Name: .... Date: .... Period: ....

3. Refer to the balanced hanger diagram.
   Determine the weight of a circle if each triangle weighs:
   - a 2 lb. **1 lb**
   - b 1 lb. **0.5 lb**
   - c 0.5 lb. **0.25 lb**

4. A car is traveling at a constant speed. Determine the number of miles the car would travel in 1 hour at each given rate.
   - a 135 miles in 3 hours **45 miles in 1 hour**
   - b 22 miles in $\frac{1}{2}$ hours **44 miles in 1 hour**
   - c 7.5 miles in $\frac{1}{4}$ hours **30 miles in 1 hour**
   - d $\frac{100}{3}$ miles in $\frac{2}{3}$ hours **50 miles in 1 hour**

5. Solve each equation. Show your work or explain your thinking.
   - a $21 = x + 9$
     $21 - 9 = x + 9 - 9$
     $12 = x$
   - b $3x = 57$
     $3x \div 3 = 57 \div 3$
     $x = 19$
   - c $x - 7.5 = 18.5$
     $x - 7.5 + 7.5 = 18.5 + 7.5$
     $x = 26$
   - d $15 = \frac{5}{8}x$
     $15 \div \frac{5}{8} = \frac{5}{8}x \div \frac{5}{8}$
     $24 = x$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 6	1
Formative ⚡	5	Unit 6 Lesson 3	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 3**

Reasoning About Solving Equations (Part 1)

Let's see how a balanced hanger diagram is like an equation and how moving its weights is like solving an equation.

Focus

Goals

1. Interpret a balanced hanger diagram, and write an equation of the form $px + q = r$ to represent the relationship shown.
2. **Language Goal:** Explain how to solve an equation of the form $px + q = r$. **(Speaking and Listening, Writing)**

Coherence

Today

Students connect hanger diagrams and two-step equations of the form $px + q = r$. Students match hanger diagrams with corresponding equations, and use them to reason about solutions.

Previously

In Grade 6, students wrote and solved one-step equations. Students review finding solutions to one-step equations during the Warm-up.

Coming Soon

In Lesson 4, students will continue working with hanger diagrams to understand and solve different two-step equations of the form $px + q = r$.

Rigor

- Students interpret hanger diagrams to build **conceptual understanding** of solving equations of the form $px + q = r$.
- Students develop **procedural fluency** in solving equations of the form $px + q = r$ with and without the use of hanger diagrams.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	15 min	13 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

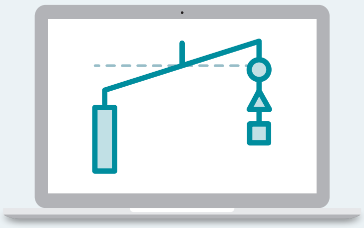
- *constant*
- *coefficient*
- *equation*
- *hanger diagram*
- *properties of equality*
- *solution to an equation*
- *variable*

Amps ⋮ **Featured Activity**

Activity 1

Dynamic Hanger Diagrams

When students enter a weight for a variable in a hanger diagram, the hanger will animate, giving real-time feedback that shows whether the hanger is balanced.



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Building Math Identity and Community

Connecting to Mathematical Practices

Responsible decision making: As students solve equations using hanger diagrams, they must consider the consequences of each step they take. Working outside the structure that the properties of equality provide, students will solve the equation incorrectly. Compare this to decisions that they make in real life. Have them explain what structures are set for them so that they have positive, instead of negative, consequences.

Modifications to Pacing

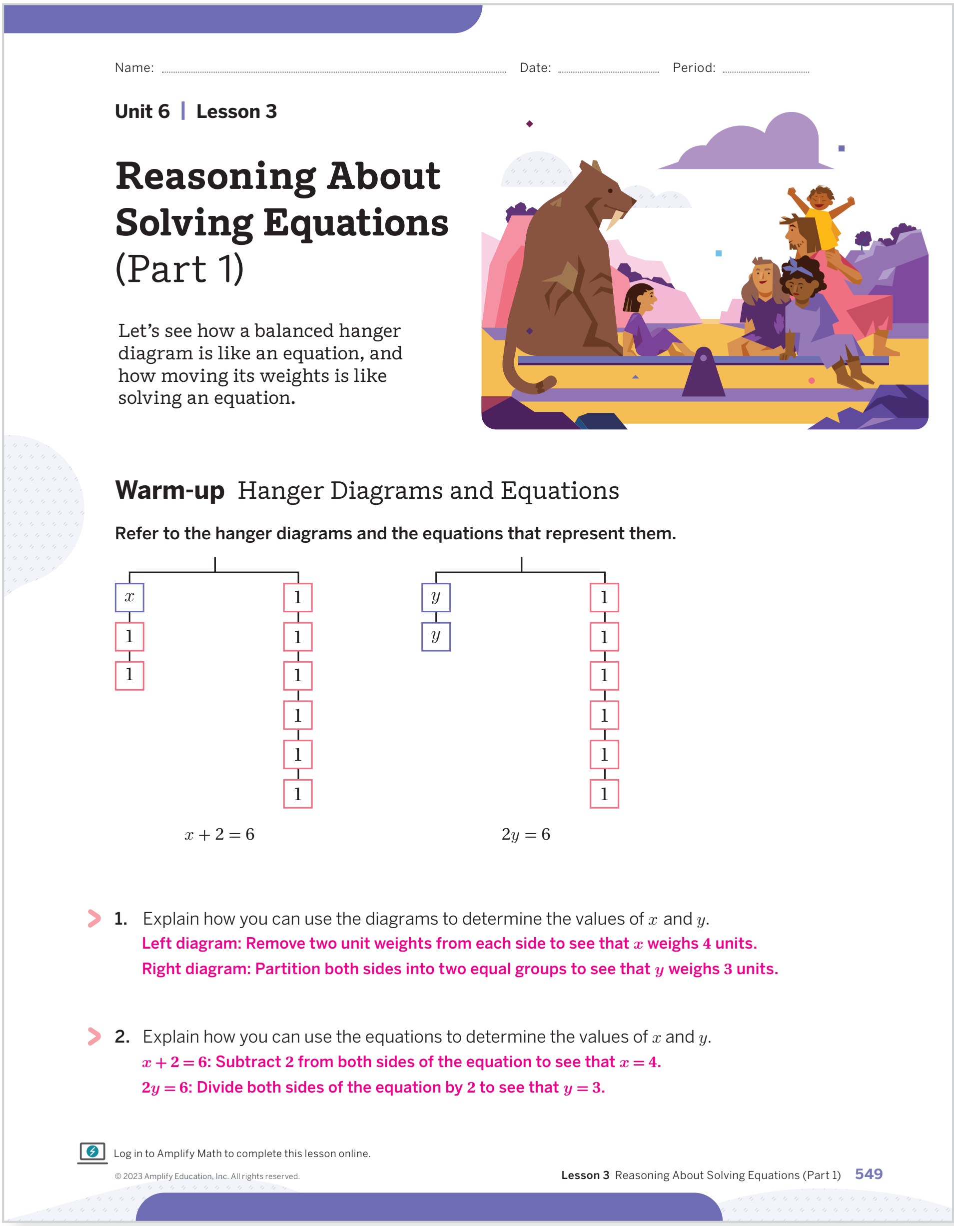
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, after matching the equations to the hanger diagrams, have students choose only two equations to solve.

Independent | 7 min

Warm-up Hanger Diagrams and Equations

Students solve two pairs of one-step equations using hanger diagrams in preparation for solving two-step equations using hanger diagrams.



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Unit 6 | Lesson 3

Reasoning About Solving Equations (Part 1)

Let's see how a balanced hanger diagram is like an equation, and how moving its weights is like solving an equation.

Warm-up Hanger Diagrams and Equations

Refer to the hanger diagrams and the equations that represent them.

$x + 2 = 6$

$2y = 6$

1. Explain how you can use the diagrams to determine the values of x and y.
Left diagram: Remove two unit weights from each side to see that x weighs 4 units.
Right diagram: Partition both sides into two equal groups to see that y weighs 3 units.

2. Explain how you can use the equations to determine the values of x and y.
$x + 2 = 6$: Subtract 2 from both sides of the equation to see that $x = 4$.
$2y = 6$: Divide both sides of the equation by 2 to see that $y = 3$.

Log in to Amplify Math to complete this lesson online.

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Lesson 3 Reasoning About Solving Equations (Part 1) 549

1 Launch

Explain to students that they can write equations to represent hanger diagrams. Suggest that while students complete the problems they consider *how* the equations represent the hanger diagrams.

Ask, "what 1, x, and y represent in each diagram? What does each equation represent?"

2 Monitor

Help students get started by asking, "How can you adjust the hangers while still keeping them balanced? Which values of the variables will make each hanger balanced and each equation true?"

Look for productive strategies:

- Using the guess-and-check strategy to find the unknown weights in the diagrams. Review how to manipulate a hanger diagram to find the unknown weight.
- Using the guess-and-check strategy to solve the equations. Review solving one-step equations using inverse operations.

3 Connect

Have students share how they determined the unknown weight on each diagram, and how they solved each equation.

Highlight how the expressions $2y$ and $x + 2$ are represented in the diagrams, and remind students that $2y = 2 \cdot y = y + y$. Discuss why they subtract to solve an addition equation and divide to solve a multiplication equation, using the diagrams.

Ask, "How is determining the missing weight on the diagram similar to solving the equation?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter a weight for a variable in a hanger diagram. By doing so, the hanger will animate, providing them with real-time feedback that shows whether the hanger is balanced.

Power-up

To power up students' ability to solve one-step equations, have students complete:

Match each equation with the operation that could be used to solve it.

a. $2x = 8$
b. $x + 2 = 8$
c. $x + (-2) = 8$
d. $x \div 2 = 8$

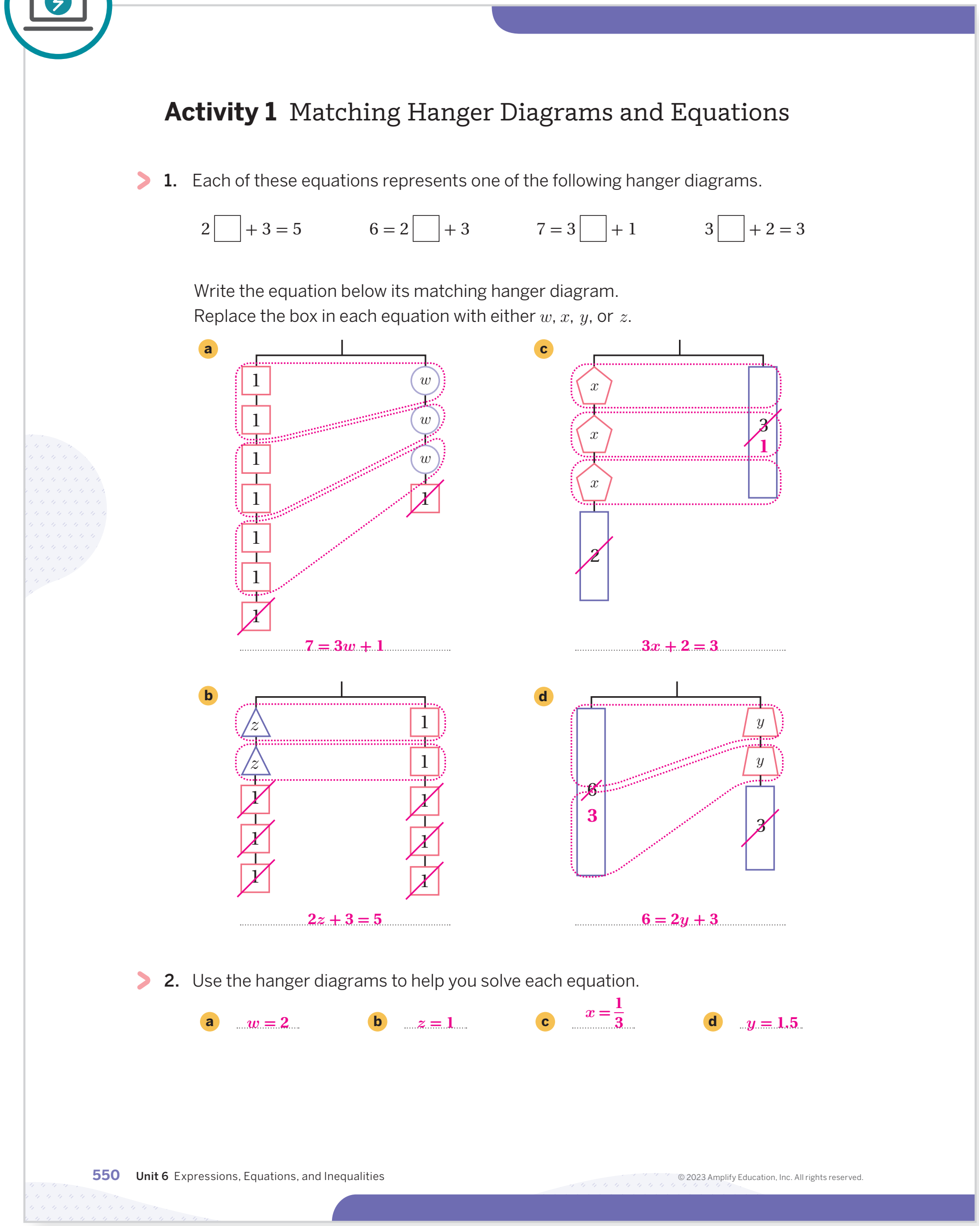
c Add 2
b Subtract 2
d Multiply by 2
a Divide by 2

Use: Before the Warm-up.
Informed by: Performance on Lesson 2, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 2.

Pairs | 15 min

Activity 1 Matching Hanger Diagrams and Equations

Students match equations to diagrams, analyzing the relationship between each pair to find the unknown values.



Activity 1 Matching Hanger Diagrams and Equations

1. Each of these equations represents one of the following hanger diagrams.

$2\boxed{}+3=5$ $\quad 6=2\boxed{}+3$ $\quad 7=3\boxed{}+1$ $\quad 3\boxed{}+2=3$

Write the equation below its matching hanger diagram.
Replace the box in each equation with either w, x, y, or z.

a $7=3w+1$

c $3x+2=3$

b $2z+3=5$

d $6=2y+3$

2. Use the hanger diagrams to help you solve each equation.

a $w=2$ b $z=1$ c $x=\frac{1}{3}$ d $y=1.5$

550 **Unit 6** Expressions, Equations, and Inequalities

1 Launch

Explain that each equation has a box where the variable will go. Say, "On the diagrams, each shape labeled with a letter has an unknown weight, and shapes labeled with the same letter have the same weight."

2 Monitor

Help students get started by going over the first equation together.

Look for points of confusion:

- **Confusing which parts of the diagram model addition or multiplication.** Refer to the equations and diagrams from the Warm-up.
- **Being unsure how to work with the shapes labeled with numbers other than 1.** Demonstrate how to partition these shapes, in order, to remove the same amount from both sides.

Look for productive strategies:

- Determining the value of the variables using the guess-and-check strategy. Check that they know how to manipulate the hanger diagram to find the unknown values.

3 Connect

Have students share strategies for matching the equations and diagrams, and how they found the values of w and x.

Highlight the structure of these equations ($px+q=r$, where p, q, and r are specific given numbers), and compare them to the equations in the Warm-up. Referring to the diagram, generalize that, to solve these equations, subtract the constant from each side and then divide each side by the coefficient. Demonstrate how the movements in the diagram can be written algebraically as steps to solve the equation.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

If students have difficulty working with the rectangles labeled with numbers other than 1, suggest they draw a related hanger diagram with smaller rectangles, where each rectangle is labeled 1. For example, in part c, have them draw two smaller rectangles on the left side and three smaller rectangles on the right side.

Extension: *Math Enrichment*

Challenge students to write the steps for solving a two-step equation, based on the relationship between the hanger diagrams and how to solve one-step equations.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the connections between the structure of the diagram and the structure of its corresponding equation. Ask:

- "How are these equations different from the one-step equations in the Warm-up?"
- "How is the coefficient of the variable illustrated in the hanger diagram?"
- "Where do you see division in both the hanger diagram and the equation?"

English Learners

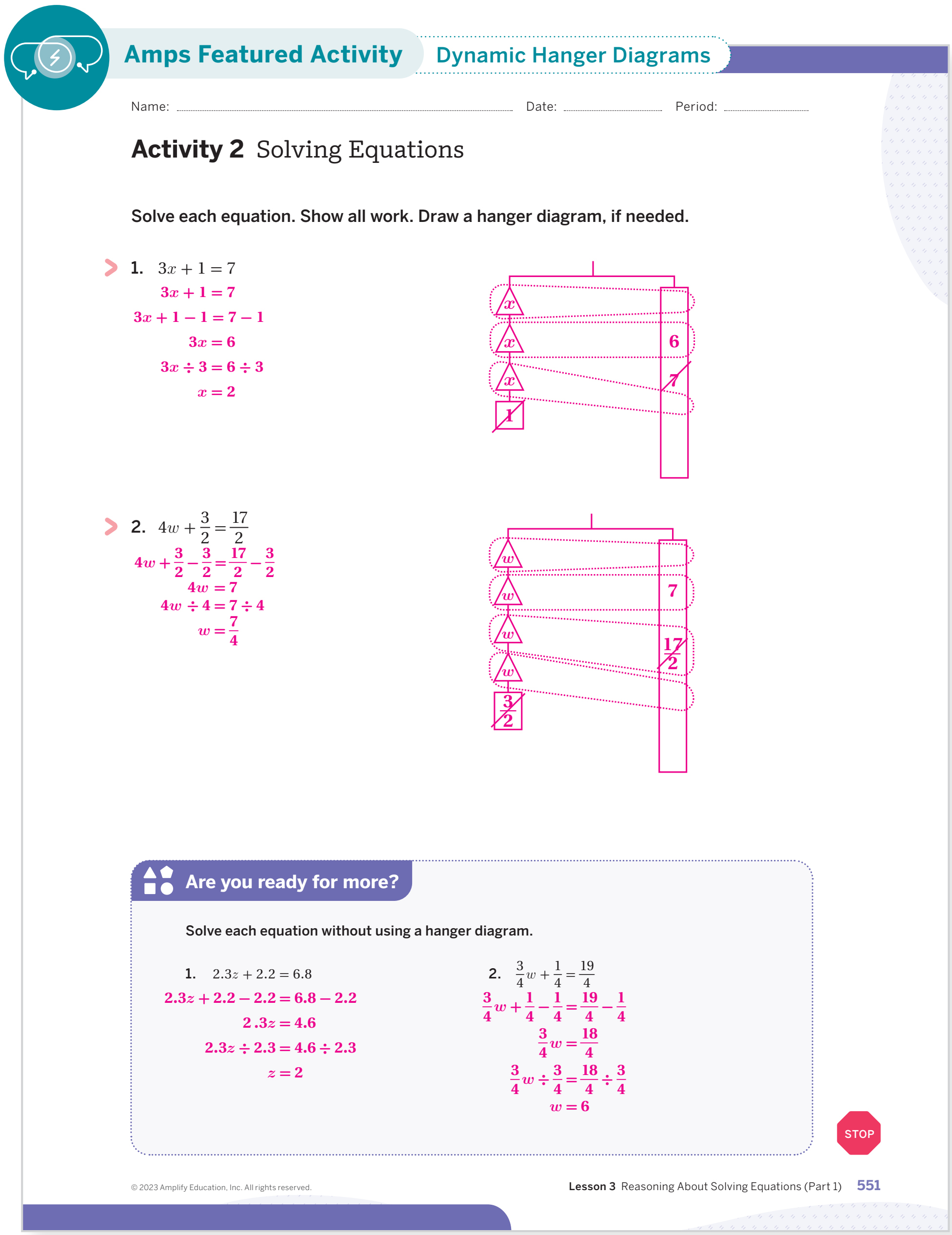
Use different colors to annotate the corresponding parts of the diagram and equation.

Pairs | 13 min

Activity 2 Solving Equations

Students use the formal process of solving equations (with hanger diagrams as needed) to solve two-step equations.

Amps Featured Activity Dynamic Hanger Diagrams



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Activity 2 Solving Equations

Solve each equation. Show all work. Draw a hanger diagram, if needed.

1. $3x + 1 = 7$

$3x + 1 = 7$

$3x + 1 - 1 = 7 - 1$

$3x = 6$

$3x \div 3 = 6 \div 3$

$x = 2$

2. $4w + \frac{3}{2} = \frac{17}{2}$

$4w + \frac{3}{2} - \frac{3}{2} = \frac{17}{2} - \frac{3}{2}$

$4w = 7$

$4w \div 4 = 7 \div 4$

$w = \frac{7}{4}$

Are you ready for more?

Solve each equation without using a hanger diagram.

1. $2.3z + 2.2 = 6.8$

$2.3z + 2.2 - 2.2 = 6.8 - 2.2$

$2 .3z = 4.6$

$2.3z \div 2.3 = 4.6 \div 2.3$

$z = 2$

2. $\frac{3}{4}w + \frac{1}{4} = \frac{19}{4}$

$\frac{3}{4}w + \frac{1}{4} - \frac{1}{4} = \frac{19}{4} - \frac{1}{4}$

$\frac{3}{4}w = \frac{18}{4}$

$\frac{3}{4}w \div \frac{3}{4} = \frac{18}{4} \div \frac{3}{4}$

$w = 6$

STOP

Lesson 3 Reasoning About Solving Equations (Part 1) 551

1 Launch

Let students know that hanger diagrams are not provided, but they may draw their own. However, since not all equations can be easily represented with hanger diagrams, students should be encouraged to practice using the formal solving process.

2 Monitor

Help students get started by asking "How can you represent this equation with a hanger diagram? What is the first step for solving this type of equation?"

Look for productive strategies:

- Drawing a hanger diagram to solve. Check that they are drawing diagrams correctly.
- Solving the equations algebraically, without a diagram. Check that they are solving correctly. If not, suggest they draw a diagram.
- Writing correct solutions without showing work. Explain that even if they can reason about the answer mentally, they need to show they understand the process for solving equations. This will help with more difficult equations later.

3 Connect

Have students share their work for each problem. Include examples where students drew diagrams, and others where they solved the equations without diagrams.

Highlight how to draw a diagram from an equation. Review the steps for solving equations of the form $px + q = r$. Say, "For some equations (e.g., *Are you ready for more?*), drawing a hanger diagram is impractical, which is why the properties of equality are used." Review how to verify that a value is the solution to an equation.

Differentiated Support

Extension: ***Math Around the World***

Tell students that the Persian mathematician Muḥammad ibn Mūsā al-Khwārizmī is considered by many mathematicians to be the Father of Algebra, who lived in the late 8th century to early 9th century in what is modern day Baghdad, Iraq. The term *algebra* is derived from the title of a book Al-Khwārizmī wrote that described general rules for how to solve problems. He described general rules for "reduction," "completion," and "balancing." Ask students to generate their own examples that illustrate these terms.

Reduction:	Completion:	Balancing:
Writing expressions in simpler, equivalent forms.	Moving a negative quantity from one side of an equation to the other, and reversing its sign.	Performing the same operations to both sides of an equation, maintaining equivalence.

Math Language Development

MLR7: Compare and Connect

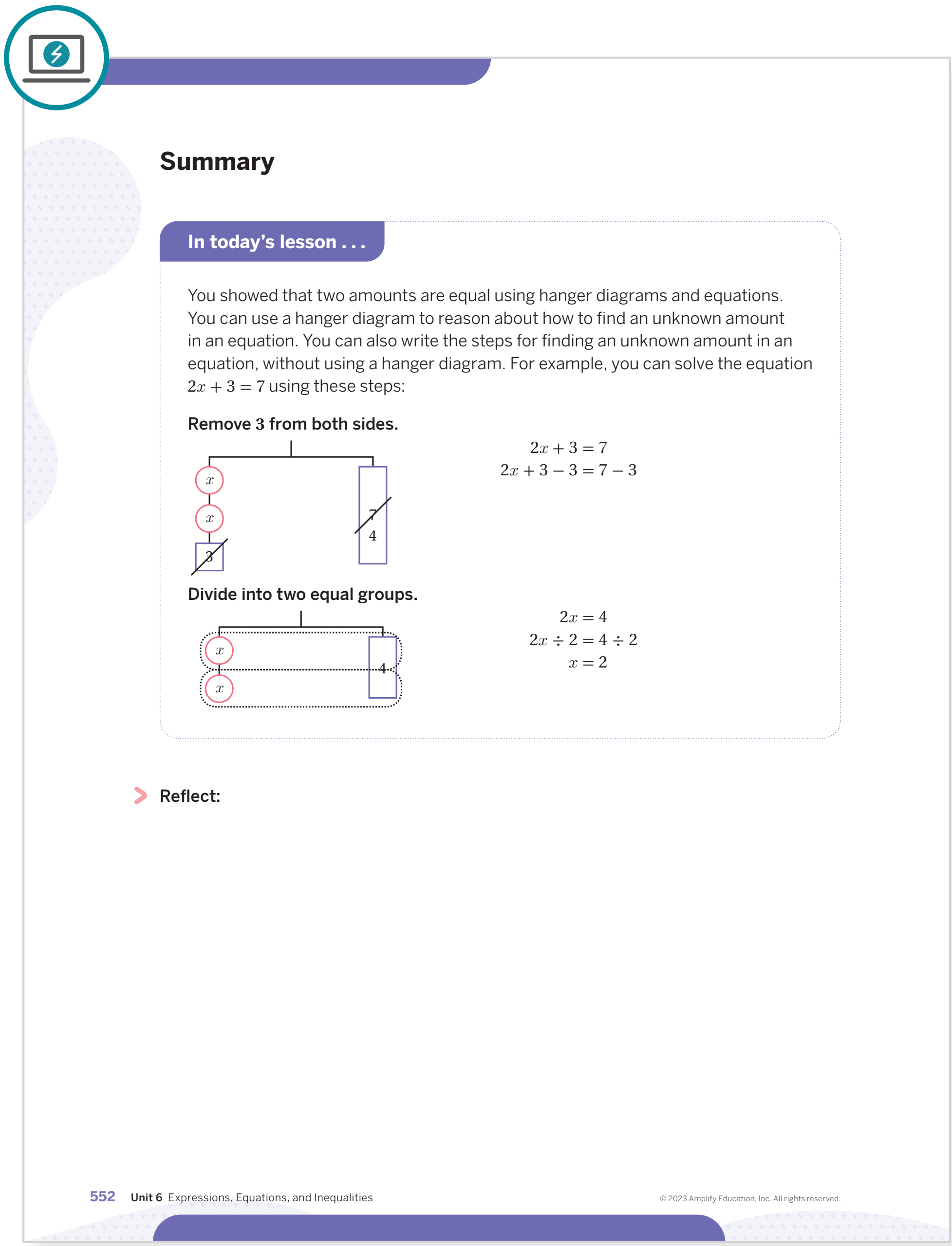
During the Connect, draw correspondences between the equations and any hanger diagrams that students draw. Guide them toward using equations, but allow them to draw hanger diagrams, as needed. Ask:

- "What steps, and in what order, did you use to solve each equation?"
- "How are those same steps illustrated by using a hanger diagram?"

Whole Class | 5 min

Summary

Review and synthesize how to solve an equation of the form $px + r = q$.



Summary

In today's lesson . . .

You showed that two amounts are equal using hanger diagrams and equations. You can use a hanger diagram to reason about how to find an unknown amount in an equation. You can also write the steps for finding an unknown amount in an equation, without using a hanger diagram. For example, you can solve the equation $2x + 3 = 7$ using these steps:

Remove 3 from both sides.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3$$

Divide into two equal groups.

$$2x = 4$$
$$2x \div 2 = 4 \div 2$$
$$x = 2$$

Reflect:

552 Unit 6 Expressions, Equations, and Inequalities

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Synthesize

Highlight each step for solving the equation, both algebraically and using the diagram. Highlight the connections between the methods. Review the words *constant* and *coefficient* and how to verify that a solution is correct.

Ask, "Compare the two strategies you have used: drawing and reasoning about hanger diagrams, and solving equations algebraically without using diagrams. How are they similar and how are they different?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

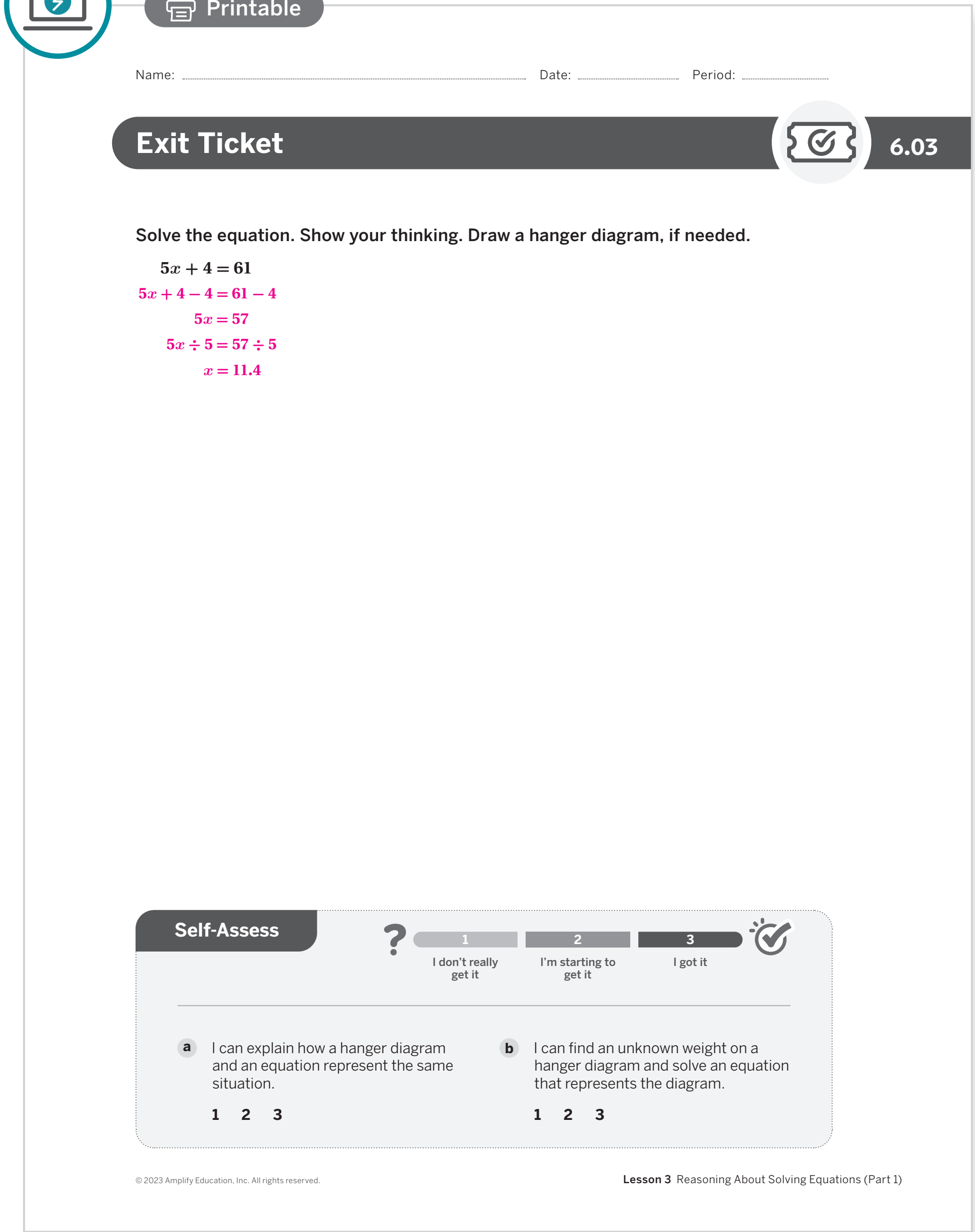
- "Which strategies for solving simple equations can be put to use when solving more complex ones?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving an equation of the form $px + q = r$, algebraically.

Printable



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Exit Ticket 6.03

Solve the equation. Show your thinking. Draw a hanger diagram, if needed.

$5x + 4 = 61$

$5x + 4 - 4 = 61 - 4$

$5x = 57$

$5x \div 5 = 57 \div 5$

$x = 11.4$

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I can explain how a hanger diagram and an equation represent the same situation.

1 2 3

b I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.

1 2 3

Lesson 3 Reasoning About Solving Equations (Part 1)

Success looks like . . .

- **Goal:** Interpreting a balanced hanger diagram, and writing an equation of the form $px + q = r$ to represent the relationship shown.
- **Language Goal:** Explaining how to solve an equation of the form $px + q = r$. **(Speaking and Listening, Writing)**
  - » Solving the equation $5x + 4 = 61$ and explaining by showing the steps or drawing a hanger diagram.

Suggested next steps

If students find an incorrect solution with an incorrect or missing hanger diagram, consider:

- Practicing drawing a hanger diagram to represent the equation. Then check whether students can find the solution using the diagram. Then consider reviewing how to solve an equation of the form $px + q = r$ by subtracting the constant from each side and then dividing each side by the coefficient.
- Assigning Practice Problem 3.

If students find an incorrect solution with the correct hanger diagram, consider:

- Referring to the hanger diagram for reference, reviewing how to solve an equation of the form $px + q = r$ by subtracting the constant from each side, and then dividing each side by the coefficient.
- Assigning Practice Problem 2.

Professional Learning

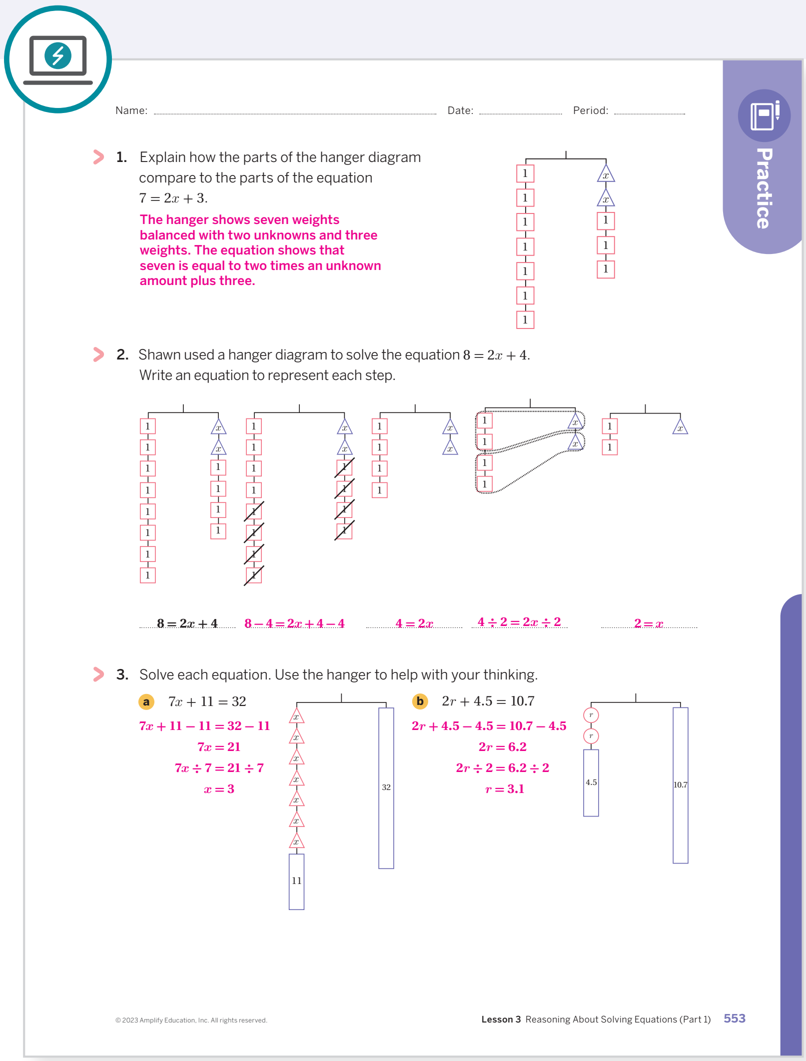
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach Activity 1? What does that tell you about similarities and differences among your students?
- In what ways in Activity 2 did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Explain how the parts of the hanger diagram compare to the parts of the equation $7 = 2x + 3$.

   The hanger shows seven weights balanced with two unknowns and three weights. The equation shows that seven is equal to two times an unknown amount plus three.

2. Shawn used a hanger diagram to solve the equation $8 = 2x + 4$. Write an equation to represent each step.

   $8 = 2x + 4$ | $\mathbf{8 - 4 = 2x + 4 - 4}$ | $\mathbf{4 = 2x}$ | $\mathbf{4 \div 2 = 2x \div 2}$ | $\mathbf{2 = x}$

3. Solve each equation. Use the hanger to help with your thinking.

   a. $7x + 11 = 32$

   $\mathbf{7x + 11 - 11 = 32 - 11}$

   $\mathbf{7x = 21}$

   $\mathbf{7x \div 7 = 21 \div 7}$

   $\mathbf{x = 3}$

   b. $2r + 4.5 = 10.7$

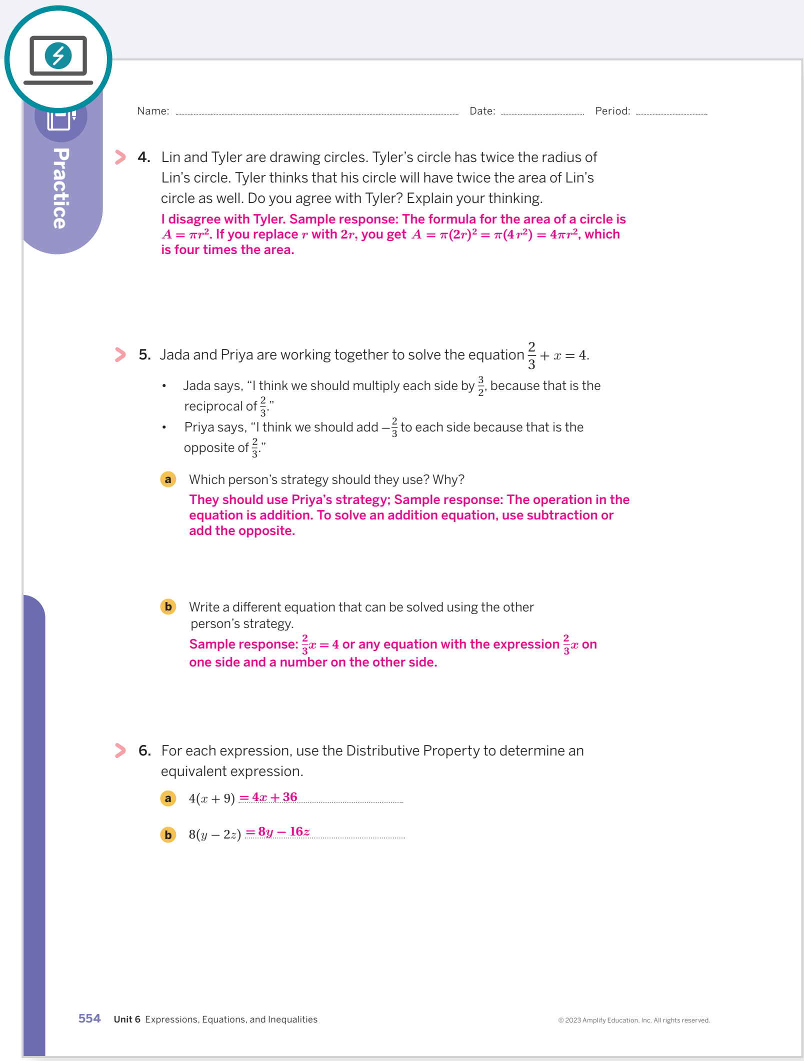
   $\mathbf{2r + 4.5 - 4.5 = 10.7 - 4.5}$

   $\mathbf{2r = 6.2}$

   $\mathbf{2r \div 2 = 6.2 \div 2}$

   $\mathbf{r = 3.1}$

Lesson 3 Reasoning About Solving Equations (Part 1) 553



Practice

Name: ______ Date: ______ Period: ______

4. Lin and Tyler are drawing circles. Tyler's circle has twice the radius of Lin's circle. Tyler thinks that his circle will have twice the area of Lin's circle as well. Do you agree with Tyler? Explain your thinking.

   I disagree with Tyler. Sample response: The formula for the area of a circle is $A = \pi r^2$. If you replace r with $2r$, you get $A = \pi(2r)^2 = \pi(4r^2) = 4\pi r^2$, which is four times the area.

5. Jada and Priya are working together to solve the equation $\frac{2}{3} + x = 4$.

   - Jada says, "I think we should multiply each side by $\frac{3}{2}$, because that is the reciprocal of $\frac{2}{3}$."
   - Priya says, "I think we should add $-\frac{2}{3}$ to each side because that is the opposite of $\frac{2}{3}$."

   a. Which person's strategy should they use? Why?

   They should use Priya's strategy; Sample response: The operation in the equation is addition. To solve an addition equation, use subtraction or add the opposite.

   b. Write a different equation that can be solved using the other person's strategy.

   Sample response: $\frac{2}{3}x = 4$ or any equation with the expression $\frac{2}{3}x$ on one side and a number on the other side.

6. For each expression, use the Distributive Property to determine an equivalent expression.

   a. $4(x + 9)$ $\mathbf{= 4x + 36}$

   b. $8(y - 2z)$ $\mathbf{= 8y - 16z}$

554 **Unit 6** Expressions, Equations, and Inequalities

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Unit 3 Lesson 10	3
	5	Unit 5 Lesson 18	2
Formative ⚡	6	Unit 6 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 4**

Reasoning About Solving Equations (Part 2)

Let's use hangers to understand two different ways of solving equations with parentheses.

Focus

Goals

1. **Language Goal:** Compare and contrast (orally) different strategies for solving an equation of the form $p(x+q)=r$. **(Speaking and Listening)**
2. **Language Goal:** Explain how to solve an equation of the form $p(x+q)=r$. **(Speaking and Listening, Writing)**

Coherence

Today

Students connect the Distributive Property with solving equations and learn how to solve equations of the form $p(x+q)=r$ using hanger diagrams to assist them in making sense of each equation. Students look for and use the structure of the hanger diagrams and the equations to develop efficient methods of solving equations.

Previously

In Grade 6, students identified when two expressions were equivalent, simplified expressions involving the Distributive Property, and solved simple equations. So far in this unit, students established the properties of solving equations using hanger diagrams and solved equations of the form $px+q=r$.

Coming Soon

In Lesson 5, students will solve equations using negative values. In Lesson 6, students will focus on the structure of the equation and practice solving equations of the form $p(x+q)=r$, using either the Distributive Property first or dividing by the factor in front of the parentheses first.

Rigor

- Students interpret hanger diagrams to build **conceptual understanding** of solving equations of the form $p(x+q)=r$.
- Students develop **procedural fluency** in solving equations of the form $p(x+q)=r$ with and without the use of hanger diagrams.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Small Group	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

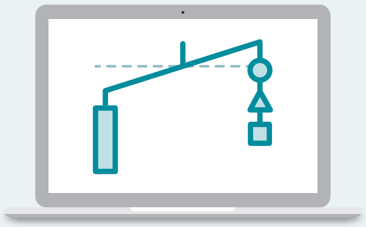
- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *hanger diagram*
- *properties of equality*
- *solution to an equation*
- *variable*

Amps | **Featured Activity**

Activities 1 and 2

Dynamic Hanger Diagrams

When students enter a weight for a variable in a hanger diagram, the hanger will animate, giving real-time feedback that shows whether the hanger is balanced.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might find themselves feeling stressed about having to solve equations. Remind them that the hanger diagram represents a structure that they can use to solve the equations correctly. Compare working in structure to working in chaos. Ask students to identify which relieves stress and which causes stress.

Modifications to Pacing

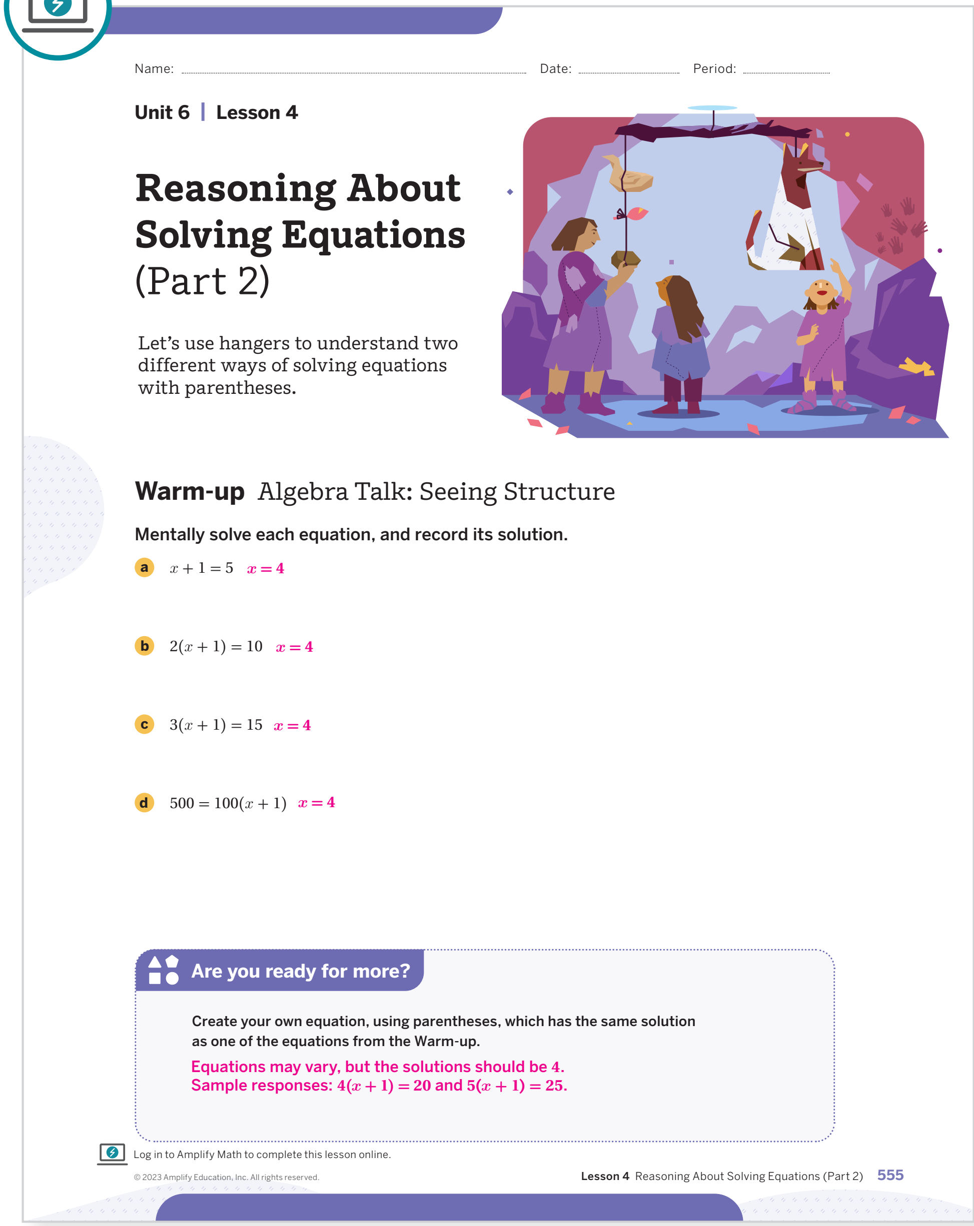
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up**. Then, during the Activity 1 Connect, discuss how the Distributive Property could explain the equivalency of the two equations.
- In **Activity 2**, have groups divide the problems so that half of the group solves each one
- In **Activity 3**, provide students with a choice, having them solve only two equations. Encourage students who need extra support to choose the first two problems while encouraging students who are comfortable to focus on Problem 3.

Independent | 5 min

Warm-up Algebra Talk: Seeing Structure

Students look for structure among a set of equations, noticing that in each subsequent equation, each side has been multiplied by the same value.



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Unit 6 | Lesson 4

Reasoning About Solving Equations (Part 2)

Let's use hangers to understand two different ways of solving equations with parentheses.

Warm-up Algebra Talk: Seeing Structure

Mentally solve each equation, and record its solution.

a $x + 1 = 5$ $x = 4$

b $2(x + 1) = 10$ $x = 4$

c $3(x + 1) = 15$ $x = 4$

d $500 = 100(x + 1)$ $x = 4$

Are you ready for more?

Create your own equation, using parentheses, which has the same solution as one of the equations from the Warm-up.

Equations may vary, but the solutions should be 4. Sample responses: $4(x + 1) = 20$ and $5(x + 1) = 25$.

Log in to Amplify Math to complete this lesson online.

Lesson 4 Reasoning About Solving Equations (Part 2) 555

1 Launch

Conduct the ***Algebra Talk*** routine. Prompt students to look for the structure of each subsequent equation and how it compares to the first equation.

2 Monitor

Help students get started by asking, "What does it mean to find the solution to an equation?" "In the first equation, can you determine what x should be to make the statement true? Do you see a connection between the first two equations? What is similar? Different?"

Look for productive strategies:

- Applying the Distributive Property to the second and third equations. Note any students who do this and have them explain their thinking.
- Noticing commonalities among the third and fourth equations. Note any students who notice the similar structure and have them explain their thinking.

3 Connect

Display the equations.

Have students share their solutions to each equation. Once the class understands the solutions are the same value, 4, have select students share their strategies for solving the equations.

Highlight a student's strategy that used the Distributive Property. Then have a student who noticed the left and right sides contained common multiples explain their strategy. If no one solved using this strategy, highlight the similarities in structure of the equations and discuss the efficiency of solving

Ask, "Why does each equation have the same solution?"

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their solutions, ask, "How did you use the equation in part a to help you in part b? (Repeat for part c) What was different about part d compared to the others? What do you notice about the values on the right side of the equation?"

English Learners

Display these sentence frames to support students as they explain their strategy.

- "First, I _____ because . . ."
- "I noticed _____, so I . . ."
- "The equation in part a is similar to the equation in part _____, because . . ."

Power-up

To power up students' ability to apply the Distributive Property to create equivalent expressions, have students complete:

Use the Distributive Property to write an expression equivalent to each given expression.

a. $5(x + 2)$ $= 5x + 10$

b. $3(5 - x)$ $= 15 - 3x$ (or equivalent)

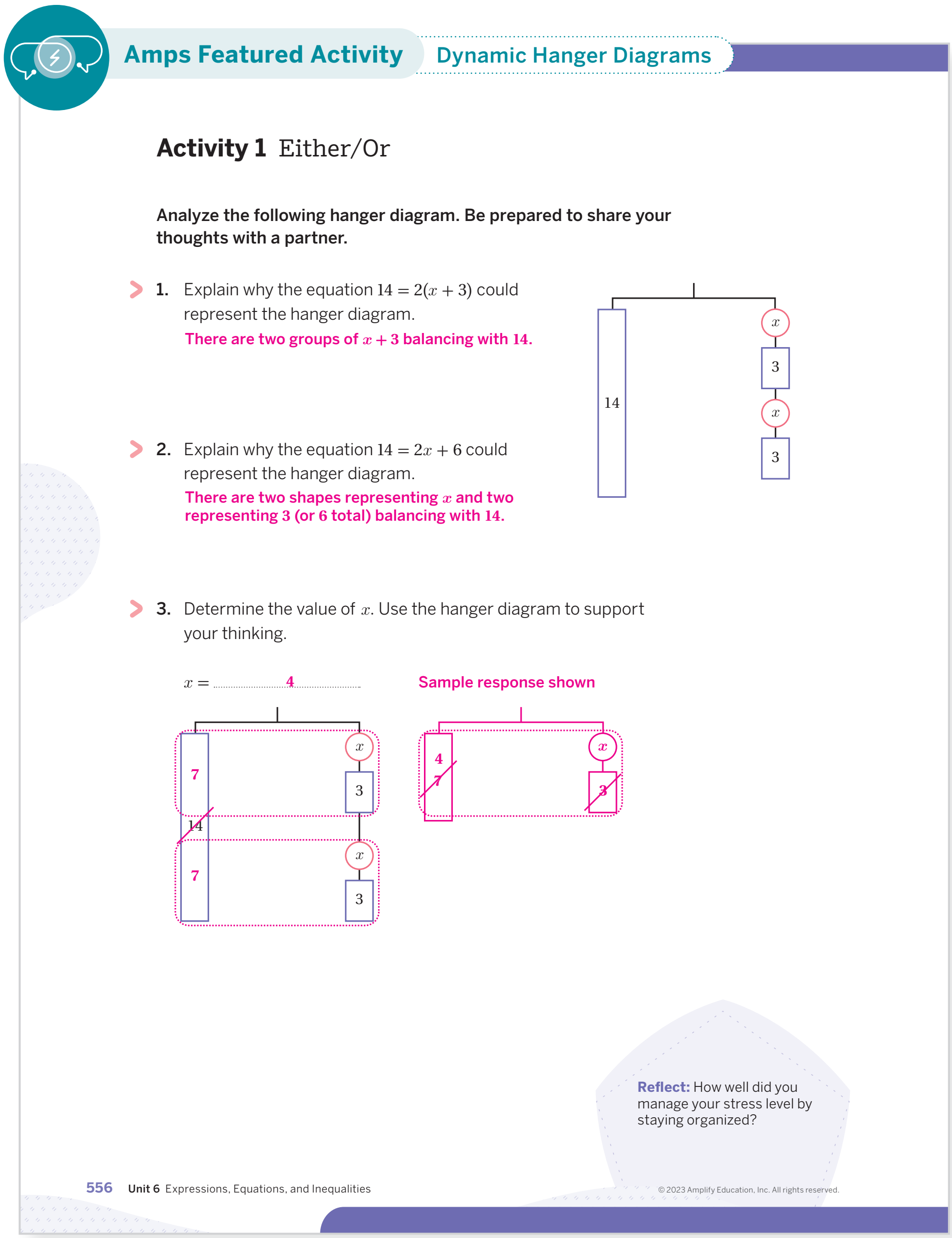
Use: Before Activity 1.
Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5.

Pairs | 10 min

Activity 1 Either/Or

Students develop strategies to solve equations by working with hanger diagrams to show multiple groups representing the terms in parentheses.

Amps Featured Activity Dynamic Hanger Diagrams



Activity 1 Either/Or

Analyze the following hanger diagram. Be prepared to share your thoughts with a partner.

1. Explain why the equation $14 = 2(x + 3)$ could represent the hanger diagram.
   There are two groups of $x + 3$ balancing with 14.
2. Explain why the equation $14 = 2x + 6$ could represent the hanger diagram.
   There are two shapes representing x and two representing 3 (or 6 total) balancing with 14.
3. Determine the value of x. Use the hanger diagram to support your thinking.

$x =$ 4

Sample response shown

Reflect: How well did you manage your stress level by staying organized?

556 Unit 6 Expressions, Equations, and Inequalities

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1 Launch

Display the hanger diagram. Have students discuss with a partner what they see in the diagram, before beginning the activity.

2 Monitor

Help students get started by asking, "How do you understand the right side of the balance: two groups of $x + 3$ or $2x$ and 6?" Depending on their response, have them complete the corresponding problem first, and share responses with their partner.

Look for productive strategies:

- Using the guess-and-check strategy. This is a valid strategy; however, encourage students to use the diagram to support their guesses.
- Solving by subtracting 6 from 14, then dividing the result by 2. Note students who begin this way.
- Solving by dividing the 14 into 2 groups of 7, then subtracting 3. Note students who begin this way.
- Solving the equation without using the diagram. Make sure students understand the diagrams in relation to the structure of the equation.

3 Connect

Have students share their strategies. Select a student who distributed first to share their strategy first, followed by a student who divided first. If one of these strategies is not mentioned, demonstrate it and show how the diagram supports either strategy.

Highlight the proper use of the properties of equality. Both strategies arrive at the same solution and each could be helpful depending on the situation.

Ask, "Which method do you prefer? Why?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter a weight for a variable in a hanger diagram. By doing so, the hanger will animate, providing them with real-time feedback that shows whether the hanger is balanced.

Accessibility: *Guide Processing and Visualization*

To help students make sense of the equations in Problems 1 and 2, ask:

- "Which equation shows that 14 is equal to *the sum of two groups of x and 6*?"
- "Which equation shows that 14 is equal to *two groups of the sum of x and 3*?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, draw their attention to the connections between the various strategies used. For example, display the following, and have students describe how the strategies are similar and different.

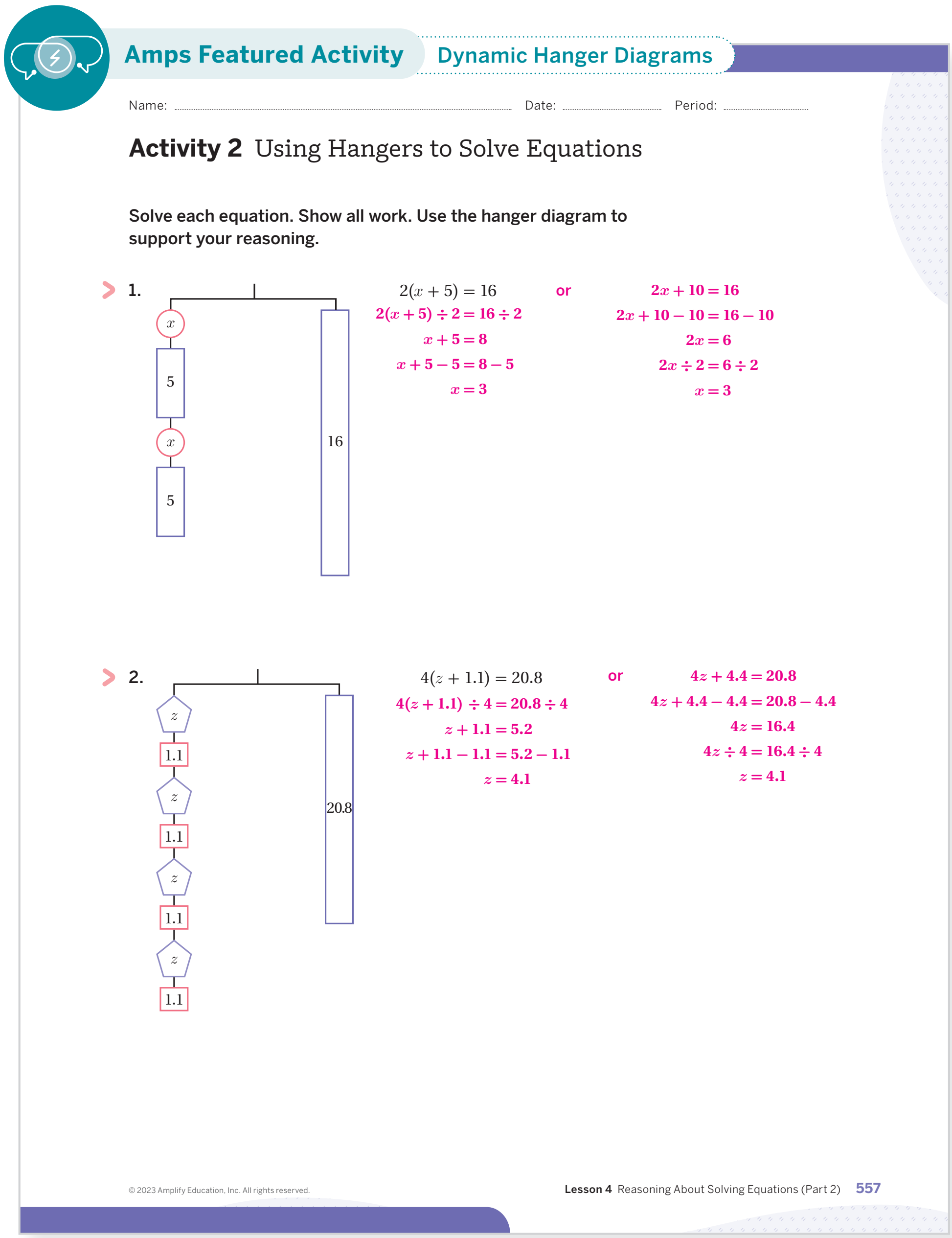
Distribute first.		Divide first.	
$14 = 2(x + 3)$		$14 = 2(x + 3)$	
$14 = 2x + 6$	Distributive Property	$7 = x + 3$	Divide by 2.
$8 = 2x$	Subtract 6.	$4 = x$	Subtract 3.
$4 = x$	Divide by 2.		

Small Groups | 10 min

Activity 2 Using Hangers to Solve Equations

Students formalize the algebraic steps to solving equations by using hanger diagrams to support their reasoning.

Amps Featured Activity Dynamic Hanger Diagrams



Name: __________ Date: ________ Period: ________

Activity 2 Using Hangers to Solve Equations

Solve each equation. Show all work. Use the hanger diagram to support your reasoning.

1.

$2(x + 5) = 16$ or $2x + 10 = 16$

$2(x + 5) \div 2 = 16 \div 2$

$x + 5 = 8$

$x + 5 - 5 = 8 - 5$

$x = 3$

$2x + 10 = 16$

$2x + 10 - 10 = 16 - 10$

$2x = 6$

$2x \div 2 = 6 \div 2$

$x = 3$

2.

$4(z + 1.1) = 20.8$ or $4z + 4.4 = 20.8$

$4(z + 1.1) \div 4 = 20.8 \div 4$

$z + 1.1 = 5.2$

$z + 1.1 - 1.1 = 5.2 - 1.1$

$z = 4.1$

$4z + 4.4 = 20.8$

$4z + 4.4 - 4.4 = 20.8 - 4.4$

$4z = 16.4$

$4z \div 4 = 16.4 \div 4$

$z = 4.1$

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Lesson 4 Reasoning About Solving Equations (Part 2) 557

1 Launch

Encourage students to look back at their work in Activity 1 to help them in this activity.

2 Monitor

Help students get started by asking, “How many groups of $x + 5$ are in the hanger? What does one group of $x + 5$ equal?”

Look for productive strategies:

- Using the Distributive Property first. Make sure students write $2x + 10 = 16$ for Problem 1 and $4z + 4.4 = 20.8$ for Problem 2.
- Dividing by the coefficient first. Make sure students write $x + 5 = 8$ for Problem 1 and $z + 1.1 = 5.2$ for Problem 2.

3 Connect

Display the hanger diagrams and equations.

Have students share their methods. Select a student who first divided by the factor outside the parentheses to share. Then select a student who first distributed to share.

Highlight that using the Distributive Property is a valid approach. Analyzing the structure of the equation can help make the solution process more efficient. Have students formalize the process for solving by first dividing by the factor outside the parentheses.

Ask, “How does dividing first simplify the equation, especially in Problem 2?”

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter a weight for a variable in a hanger diagram. By doing so, the hanger will animate, providing them with real-time feedback that shows whether the hanger is balanced.

Extension: *Math Enrichment*

Challenge students to draw a hanger diagram to represent the equation $\frac{1}{2}(x + 6) = 9$. Then have them solve the equation. $x = 12$.

Math Language Development

MLR7: Compare and Connect

While students work, display these questions and have small groups discuss them as they solve the equations.

- “How does the hanger diagram illustrate the equation?”
- “Will you distribute first or divide first? Why?”
- “Solve the equation using the other method. If your solutions are not the same, discuss and resolve any errors.”

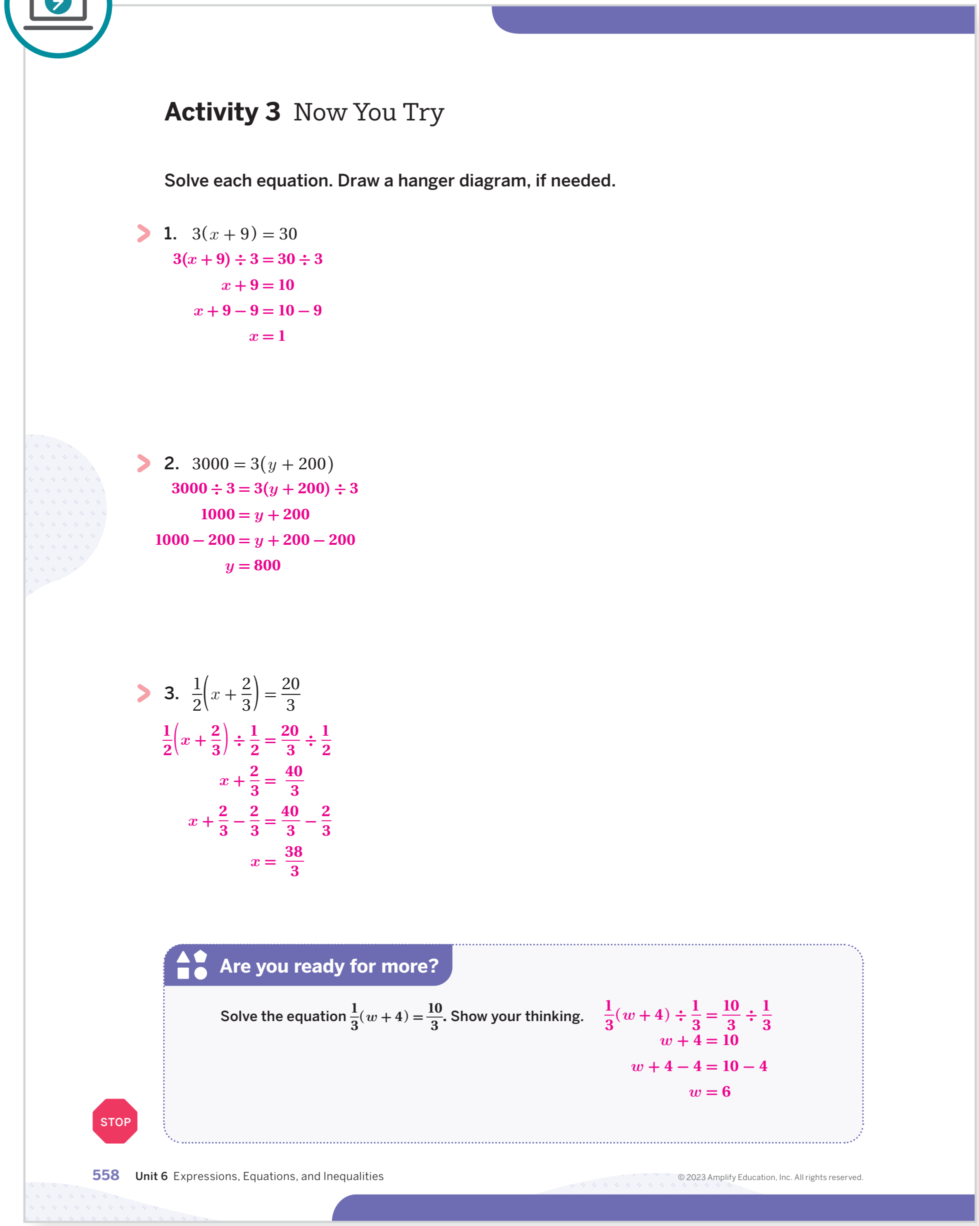
English Learners

Encourage students to annotate their work with the terms or properties that describe each step.

Independent | 10 min

Activity 3 Now You Try

Students solidify the algebraic steps of solving equations of the form $p(x+q)=r$ to become more efficient in the solving process.



Activity 3 Now You Try

Solve each equation. Draw a hanger diagram, if needed.

1. $3(x+9)=30$

$$\begin{aligned} 3(x+9)\div 3 &= 30\div 3 \\ x+9 &= 10 \\ x+9-9 &= 10-9 \\ x &= 1 \end{aligned}$$

2. $3000=3(y+200)$

$$\begin{aligned} 3000\div 3 &= 3(y+200)\div 3 \\ 1000 &= y+200 \\ 1000-200 &= y+200-200 \\ y &= 800 \end{aligned}$$

3. $\frac{1}{2}\left(x+\frac{2}{3}\right)=\frac{20}{3}$

$$\begin{aligned} \frac{1}{2}\left(x+\frac{2}{3}\right)\div\frac{1}{2} &= \frac{20}{3}\div\frac{1}{2} \\ x+\frac{2}{3} &= \frac{40}{3} \\ x+\frac{2}{3}-\frac{2}{3} &= \frac{40}{3}-\frac{2}{3} \\ x &= \frac{38}{3} \end{aligned}$$

Are you ready for more?

Solve the equation $\frac{1}{3}(w+4)=\frac{10}{3}$. Show your thinking.

$$\begin{aligned} \frac{1}{3}(w+4)\div\frac{1}{3} &= \frac{10}{3}\div\frac{1}{3} \\ w+4 &= 10 \\ w+4-4 &= 10-4 \\ w &= 6 \end{aligned}$$

STOP

558 **Unit 6** Expressions, Equations, and Inequalities

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1 Launch

Let students know they can start by drawing a hanger diagram, but the goal is to practice the algebraic steps.

2 Monitor

Help students get started by drawing hanger diagrams and referencing equations from Activity 2.

Look for points of confusion:

- **Struggling to create hanger diagrams for Problem 3.** Have students reference the algebraic steps in Activity 2.
- **Dividing fractions incorrectly.** Review that dividing fractions is the same as multiplying by the reciprocal.

Look for productive strategies:

- Multiplying by the denominator of the factor outside the parentheses in Problem 3. This is a valid technique and will alleviate fractions; however, you may wish to save this technique for a later discussion.

3 Connect

Have students share their solutions. If time is limited, discuss solutions to Problems 1 and 3.

Highlight that the Distributive Property can be used first. However, in Problems 2 and 3, students may notice that dividing each side by the factor outside the parentheses simplifies the math. For some equations (e.g., Problem 3), drawing a hanger diagram is impractical, which is why solving the equation algebraically is preferred. Continue to highlight the meaning of the *solution* to an equation; the value which makes the mathematical statement true

Ask, "How can you check these solutions?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on solving the equations in Problems 1 and 2. As time permits, they can work on solving the equation in Problem 3.

Extension: *Math Enrichment*

Tell students that another strategy they can use to solve the equation in Problem 3 is to multiply each term by the least common denominator of the fractions. This will eliminate the fractions. Have them use this strategy to solve the equation in Problem 3 and compare the solution to the solution they already determined. They should note it is the same.

Math Language Development

MLR7: Compare and Connect

While students work, display the following questions (similar to the ones from Activity 2) that students can ask themselves as they solve each equation.

- (If they draw a hanger diagram) "How does the hanger diagram illustrate the equation?"
- "Will you distribute first or divide first? Why?"
- "Solve the equation using the other method. If your solutions are not the same, check your work and resolve any errors."

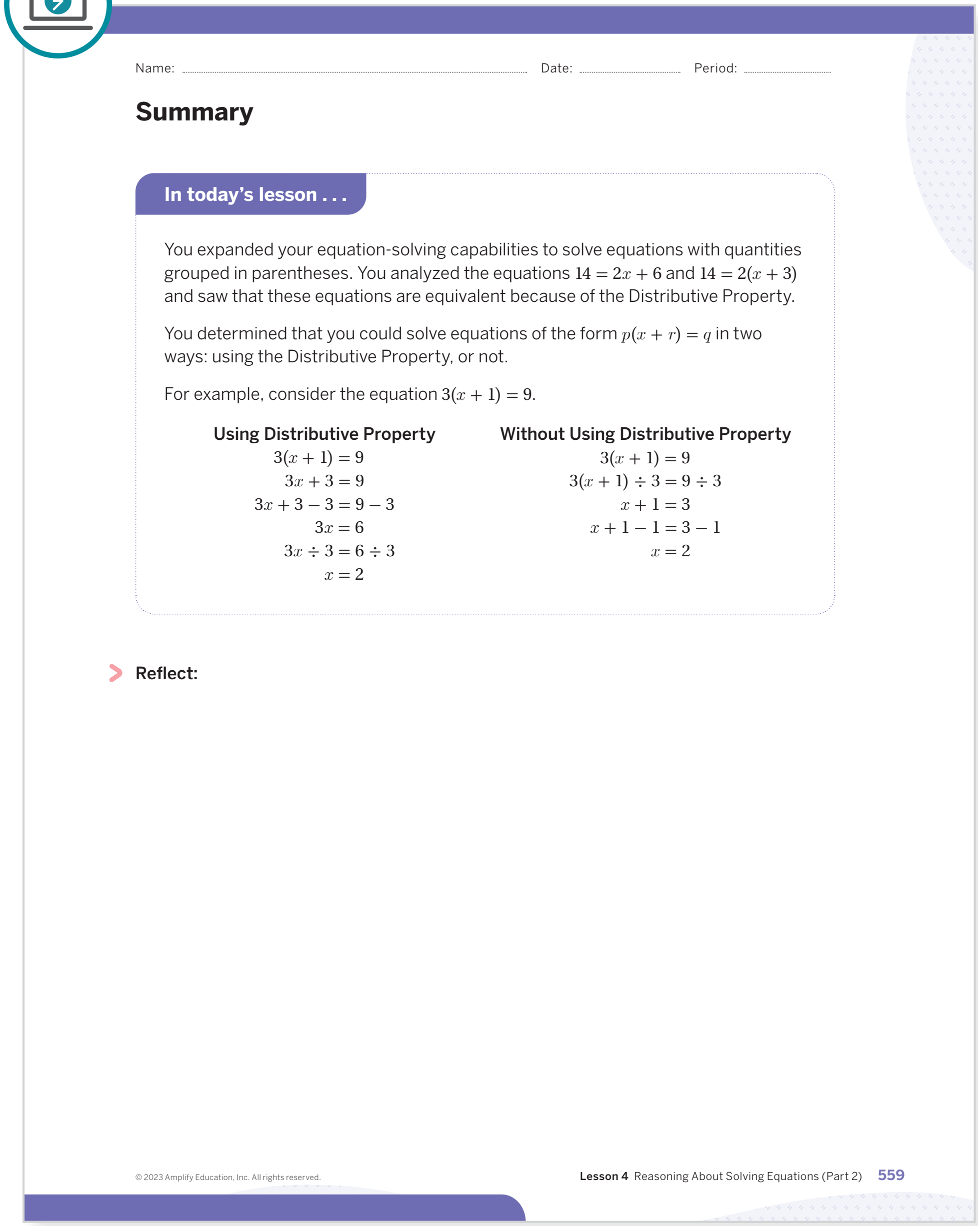
English Learners

Encourage students to annotate their work with the terms or properties that describe each step.

Whole Class | 5 min

Summary

Review and synthesize how to solve an equation of the form $p(x+q)=r$.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You expanded your equation-solving capabilities to solve equations with quantities grouped in parentheses. You analyzed the equations $14=2x+6$ and $14=2(x+3)$ and saw that these equations are equivalent because of the Distributive Property.

You determined that you could solve equations of the form $p(x+r)=q$ in two ways: using the Distributive Property, or not.

For example, consider the equation $3(x+1)=9$.

Using Distributive Property	Without Using Distributive Property
$3(x+1)=9$	$3(x+1)=9$
$3x+3=9$	$3(x+1)\div 3=9\div 3$
$3x+3-3=9-3$	$x+1=3$
$3x=6$	$x+1-1=3-1$
$3x\div 3=6\div 3$	$x=2$
$x=2$	

Reflect:

Synthesize

Display the equation $4(x+7)=40$.

Have students share the steps for solving the equation by using the Distributive Property. Have another student explain the steps for solving the equation by dividing both sides by the factor outside the parentheses first. Discuss methods showing the algebraic steps using the properties of equality. Draw hanger diagrams, if needed. The solution is $x=3$.

Highlight both strategies are valid; however, attending to the structure of the equation will help identify which strategy is more efficient. This idea will be continued in Lesson 6.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

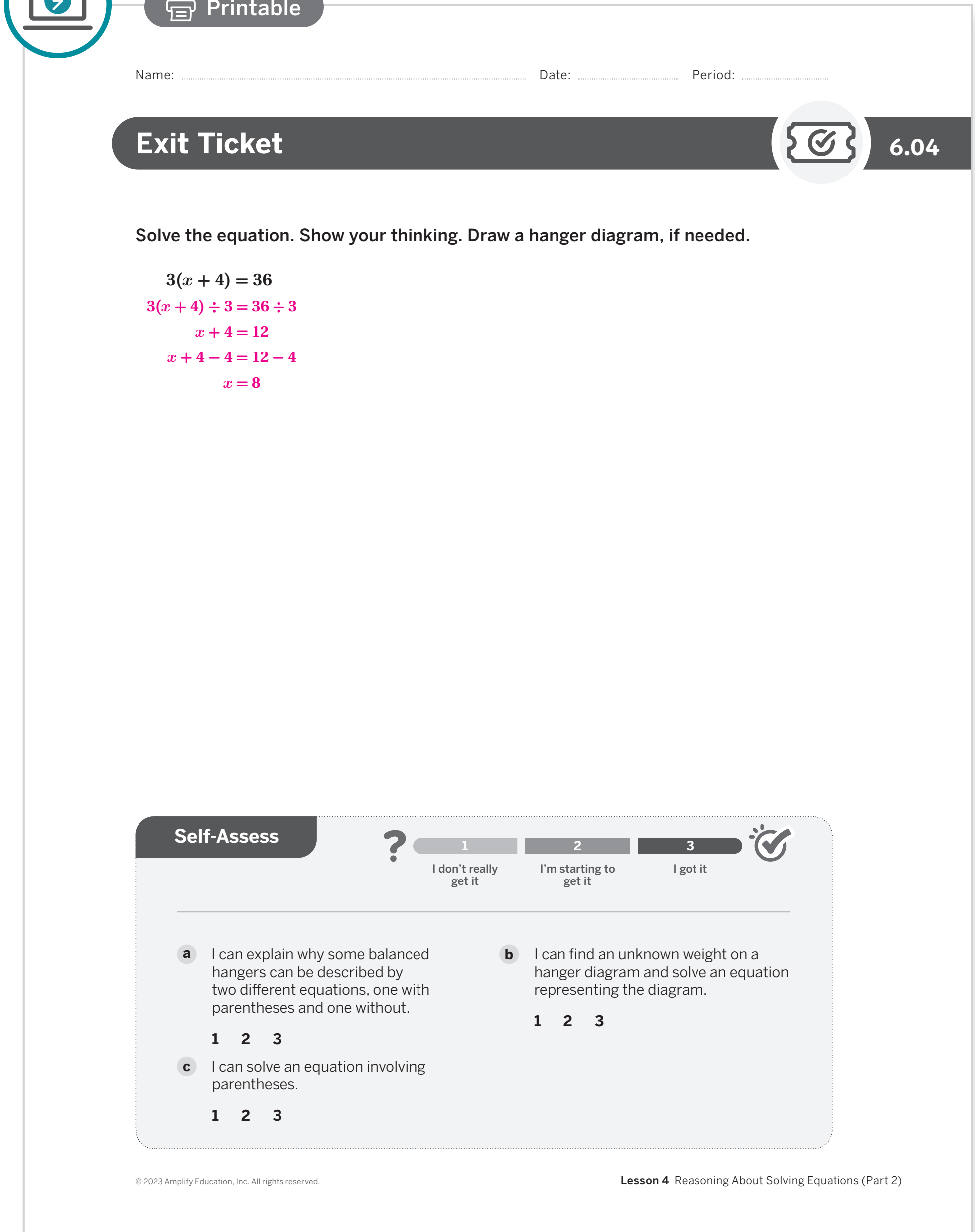
- "Which representations best help you to make sense of certain mathematical scenarios?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving an equation of the form $p(x + q) = r$.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.04

Solve the equation. Show your thinking. Draw a hanger diagram, if needed.

$3(x + 4) = 36$

$3(x + 4) \div 3 = 36 \div 3$

$x + 4 = 12$

$x + 4 - 4 = 12 - 4$

$x = 8$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain why some balanced hangers can be described by two different equations, one with parentheses and one without.

1 2 3

b. I can find an unknown weight on a hanger diagram and solve an equation representing the diagram.

1 2 3

c. I can solve an equation involving parentheses.

1 2 3

Lesson 4 Reasoning About Solving Equations (Part 2)

Success looks like . . .

- **Language Goal:** Comparing and contrasting different strategies for solving an equation of the form $p(x + q) = r$. **(Listening and Speaking)**
- **Language Goal:** Explaining how to solve an equation of the form $p(x + q) = r$. **(Listening and Speaking, Writing)**
  - » Solving the equation $3(x + 4) = 36$ and showing the steps or hanger diagram used for solving.

Suggested next steps

If the hanger diagram is incorrectly drawn, but the equation is solved correctly, consider:

- Letting students know to only use the hanger diagram if it makes the math more efficient for them.
- Assigning Practice Problem 2.

If the hanger diagram is correct, but the algebraic steps are incorrect, consider:

- Reminding students to use the hanger diagram to support their algebraic reasoning.
- Assigning Practice Problems 2 and 3.

Professional Learning

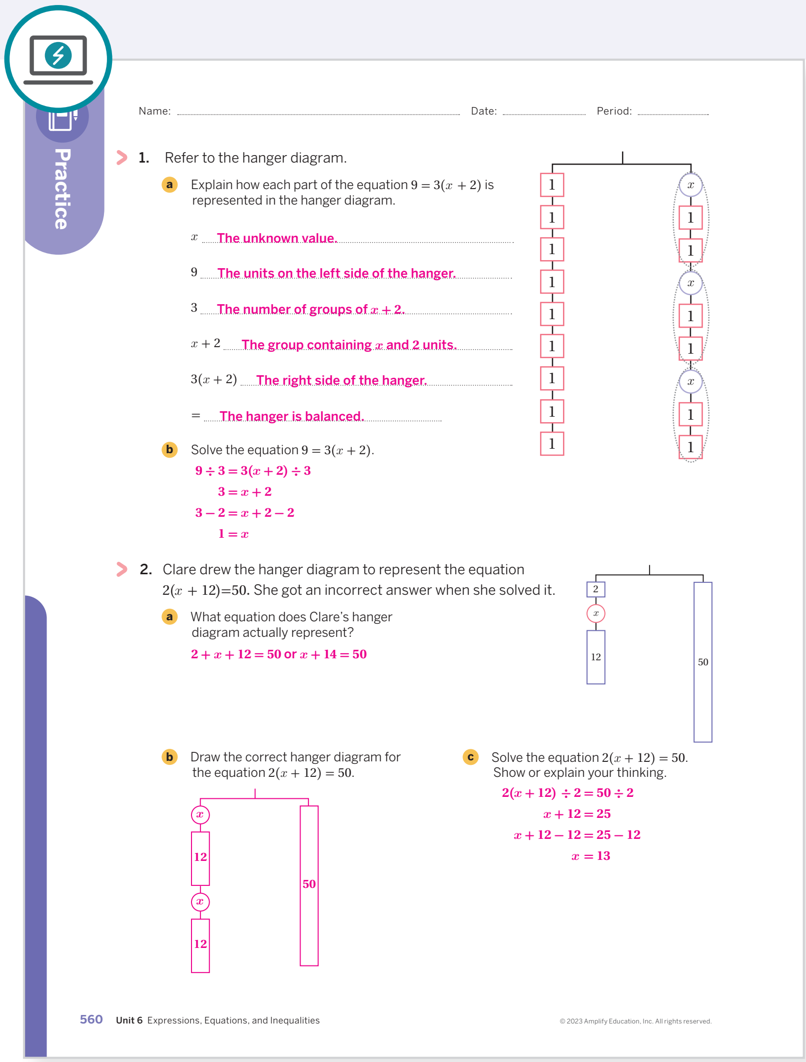
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students look for and make use of structure today? How are you helping students become aware of how they are progressing in this area?
- What challenges did students encounter as they worked on Activity 2? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Refer to the hanger diagram.

a Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger diagram.

x The unknown value.

9 The units on the left side of the hanger.

3 The number of groups of $x + 2$.

$x + 2$ The group containing x and 2 units.

$3(x + 2)$ The right side of the hanger.

$=$ The hanger is balanced.

b Solve the equation $9 = 3(x + 2)$.

$$9 \div 3 = 3(x + 2) \div 3$$
$$3 = x + 2$$
$$3 - 2 = x + 2 - 2$$
$$1 = x$$

2. Clare drew the hanger diagram to represent the equation $2(x + 12)$=50. She got an incorrect answer when she solved it.

a What equation does Clare's hanger diagram actually represent?

$2 + x + 12 = 50$ or $x + 14 = 50$

b Draw the correct hanger diagram for the equation $2(x + 12) = 50$.

c Solve the equation $2(x + 12) = 50$. Show or explain your thinking.

$$2(x + 12) \div 2 = 50 \div 2$$
$$x + 12 = 25$$
$$x + 12 - 12 = 25 - 12$$
$$x = 13$$

560 Unit 6 Expressions, Equations, and Inequalities

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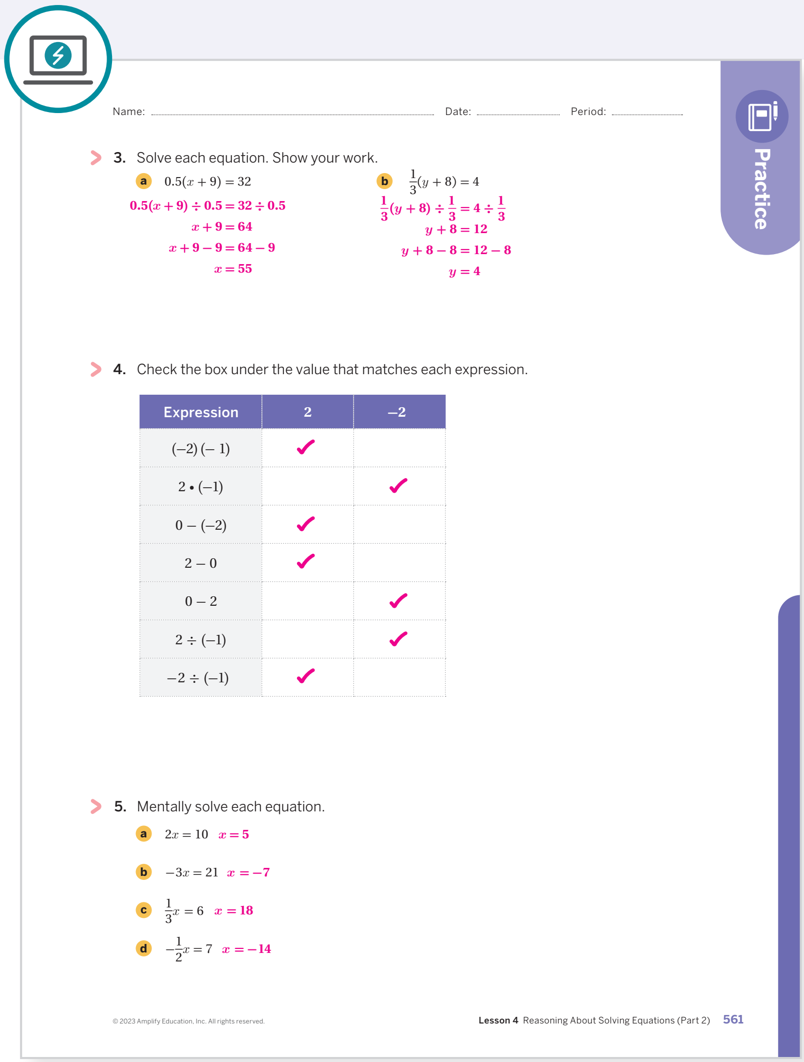

Name: Date: Period:

3. Solve each equation. Show your work.

a $0.5(x + 9) = 32$

$$0.5(x + 9) \div 0.5 = 32 \div 0.5$$
$$x + 9 = 64$$
$$x + 9 - 9 = 64 - 9$$
$$x = 55$$

b $\frac{1}{3}(y + 8) = 4$

$$\frac{1}{3}(y + 8) \div \frac{1}{3} = 4 \div \frac{1}{3}$$
$$y + 8 = 12$$
$$y + 8 - 8 = 12 - 8$$
$$y = 4$$

4. Check the box under the value that matches each expression.

Expression	2	–2
(–2) (– 1)	✓	
2 • (–1)		✓
0 – (–2)	✓	
2 – 0	✓	
0 – 2		✓
2 ÷ (–1)		✓
–2 ÷ (–1)	✓	

5. Mentally solve each equation.

a $2x = 10$ $x = 5$

b $-3x = 21$ $x = -7$

c $\frac{1}{3}x = 6$ $x = 18$

d $-\frac{1}{2}x = 7$ $x = -14$

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Lesson 4 Reasoning About Solving Equations (Part 2) 561

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activities 1 and 2	2
	3	Activity 3	1
Spiral	4	Unit 5 Lesson 17	1
Formative ⚡	5	Unit 6 Lesson 5	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 5**

Dealing With Negative Numbers

Let’s show that performing the same operations to each side of an equation also works for equations with negative numbers.

Focus

Goals

1. **Language Goal:** Generalize that performing the same operations to each side of an equation generates an equivalent equation. **(Speaking and Listening)**
2. **Language Goal:** Solve equations of the form $px + q = r$ or $p(x + q) = r$ that involve negative numbers, and explain the solution method. **(Speaking and Listening, Writing)**

Coherence

Today

Students are introduced to equations with negative values. They see that the strategies they learned for solving equations with positive values still work now that they are solving equations with negative values but that they need to take extra care with the sign of each value.

Previously

In Unit 5, students were introduced to solving one-step equations with negative values. In Lessons 3 and 4, students have learned to solve equations by performing the same operation to both sides.

Coming Soon

Students will gain further understanding of valid strategies for solving equations in Lesson 7. Students will build toward making sense of how negative values in equations could represent real-world contexts.

Rigor

- Students **apply** their understanding of inverse operations and equality to solve equations with negative numbers.
- Students develop **procedural fluency** in solving equations of the form $px + q = r$ and $p(x + q) = r$ with positive and negative values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	10 min	5 min	10 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Solving Equations* (for display)
- Anchor Chart PDF, *Solving Equations* (answers)

Math Language Development

New word

- ***equivalent equations***

Review words

- *coefficient*
- *constant*
- *equation*
- *properties of equality*
- *solution to an equation*
- *variable*

Amps | **Featured Activity**

Activity 1

Dynamic Dog Leashes

Students manipulate the positions of dogs on either side of the dog walker. If the dogs (and the equation they represent) are out of balance, the dog walker is in for a wild ride.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may struggle with using negative values in an equation in Activity 1. Remind them that the dog-walking diagram can be a source of structure. After discussing the diagram, have students explain how it helps them feel confident in their work. If they are still struggling, encourage a growth mindset where students are able to express that they believe they will understand, but it just hasn't happened yet.

Modifications to Pacing

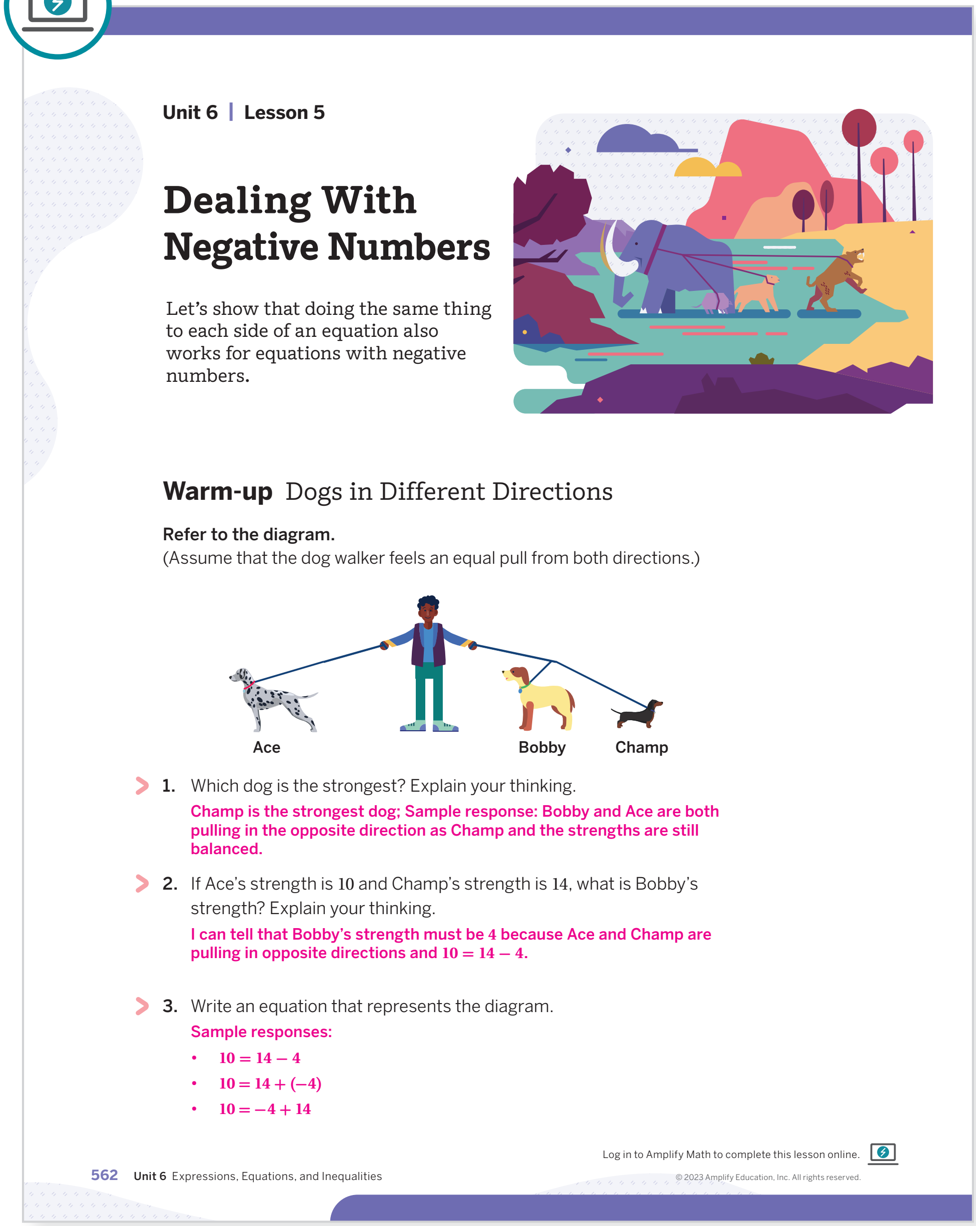
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students identify the equivalent equations but do not have them solve equations. Consider assigning the remainder of the activity as additional practice.

Pairs | 10 min

Warm-up Dogs in Different Directions

Students make sense of an illustration of the dog walker with a dog walking in the opposite direction to understand the relationship between negative values and balance.



Unit 6 | Lesson 5

Dealing With Negative Numbers

Let's show that doing the same thing to each side of an equation also works for equations with negative numbers.

Warm-up Dogs in Different Directions

Refer to the diagram.
(Assume that the dog walker feels an equal pull from both directions.)

1. Which dog is the strongest? Explain your thinking.
   Champ is the strongest dog; Sample response: Bobby and Ace are both pulling in the opposite direction as Champ and the strengths are still balanced.
2. If Ace's strength is 10 and Champ's strength is 14, what is Bobby's strength? Explain your thinking.
   I can tell that Bobby's strength must be 4 because Ace and Champ are pulling in opposite directions and $10 = 14 - 4$.
3. Write an equation that represents the diagram.
   Sample responses:
   - $10 = 14 - 4$
   - $10 = 14 + (-4)$
   - $10 = -4 + 14$

Log in to Amplify Math to complete this lesson online.

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① Launch

Introduce students to the illustration, pointing out that Bobby and Champ are pulling in opposite directions.

② Monitor

Help students get started by asking, "What do you notice is happening in the illustration? What would happen if the walker lets go of Ace? Bobby? Champ?"

Look for points of confusion:

- **Thinking that Ace is strongest because it seems that Ace (one dog) balances with two dogs.** Ask, "Is it important that Bobby is pulling in the same direction as Ace?"

Look for productive strategies:

- Thinking that Champ is the strongest. Note students with this response.

③ Connect

Have students share a response that indicates Champ is the strongest.

Highlight that students haven't seen dog walking illustrations in prior lessons where the dogs on the same side were pulling in opposite directions.

Remind students that the negative sign and the minus sign for the operation look the same and mention the convention of omitting the positive sign. For example, $a - (+1)$ is written as $a - 1$.

Ask, "What do you think it might mean that Bobby is on the same side as Champ, but is pulling in the opposite direction?" Bobby could represent a negative value.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses, use the *Poll the Class* routine to determine whether more students think Ace, Bobby, or Champ is the strongest dog. Ask students to justify their thinking and display the following sentence frames to help them organize their thoughts.

- "_____ is the strongest dog because . . ."
- "I know that _____ is stronger than _____ because . . ."
- "I know that _____ is the weakest dog because . . ."

Power-up

To power up students' ability to solve equations with negative values, have students complete:

Determine whether the solution to each equation is a *positive* or a *negative* value.

Equation	Positive solution	Negative solution
$-3x = 12$	☐	☑
$x + (-3) = 12$	☑	☐
$-10 = x - (-3)$	☐	☑

Use: Before Activity 1
Informed by: Performance on Lesson 4, Practice Problem 5 and Pre-Unit Readiness Assessment Problem 8

Pairs | 10 min

Activity 1 New Solutions, Old Ways

Students make sense of a negative value in an *equation* and connect the equation to the dog walking diagram, in order to understand how to solve for the unknown.

Amps Featured Activity Dynamic Dog Leashes

Name: Date: Period:

Activity 1 New Solutions, Old Ways

1. Write an equation that represents the diagram.
   Sample responses: $3 = x - 2$ or $3 = -2 + x$

2. How else could the dogs be arranged so that the strengths are still balanced? Sketch a diagram of the new arrangement.
   Answers may vary, but the sketch could include two dogs with strengths 3 and 2 on the left, and a dog with strength x on the right.

3. Write an equation that represents *your* diagram.
   Sample response: $3 + 2 = x$

4. Determine the value of x for both of the equations you wrote.
   $x = 5$ for both

Collect and Display: As you describe how your arrangements and equations are similar and different, your teacher will add the language you use to a class display that you can refer to during this unit.

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Lesson 5 Dealing With Negative Numbers 563

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What's different about this illustration compared to the Warm-up?" "What do the numbers refer to? What does x refer to?

Look for points of confusion:

- **Writing the expression $3 - 2 + x$.** Ask, "What does the term 'equation' mean?"

Look for productive strategies:

- **Writing $3 = x - 2$; $3 = -2 + x$; $3 + 2 = x$** or other equivalent equations. Note students with these responses.

3 Connect

Display the diagram.

Have students share the different equations they generated.

Highlight that each of these equations will have the same solution because they describe the same situation.

Define *equivalent equations* as equations that have the same solution. Say, "We know they have the same solution the same action or operation was performed on each side."

Ask, "What happens to the sign of the number 3 if the dog switches to the other side?" It should switch, too, because he will be adding to the strength on that side.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate the position of dogs on either side of the dog walker. If the dogs — and the equation they represent — are out of balance, the dog walker is in for a wild ride.

Extension: *Math Enrichment*

Have students complete the following problem:
If each dog was exactly twice as strong as they are now, would the dog walker still feel balanced? Explain your thinking. Yes; Sample response: The dog on the left would have a strength of 6, and the dogs on the right would have a total strength of $-4 + 10$, which is 6.

Math Language Development

MLR2: Collect and Display

During the Connect, as students describe how their arrangements and equations are similar and different, collect and display the terms and phrases they use to describe *equivalent equations*. Add these terms along with an example of equivalent equations to the class display. For example, students may say "they have the same solution," "they are generated using properties of equality," or "the same operation was done to each side."

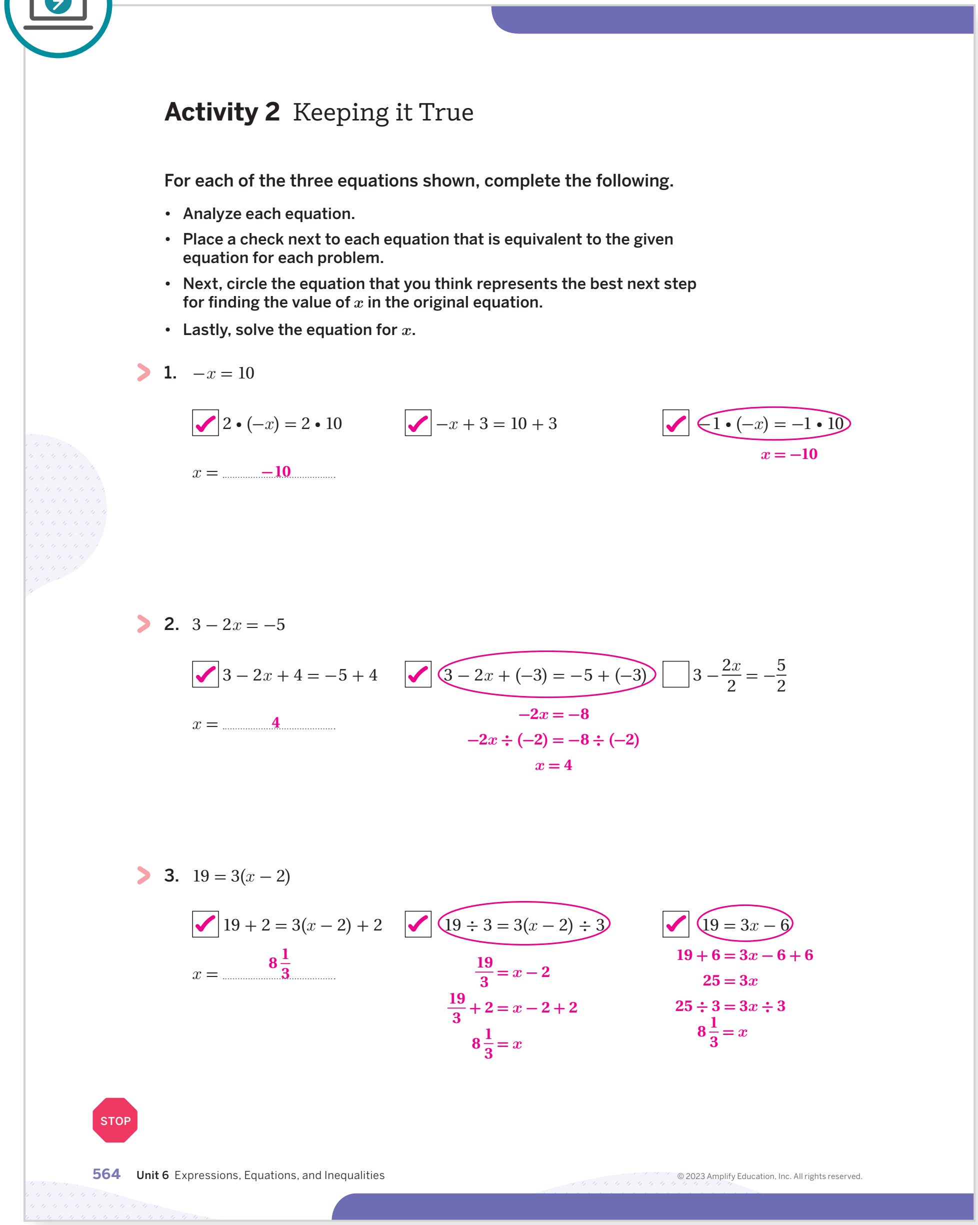
English Learners

Provide examples of equivalent equations, such as $3 = x - 2$ and $3 + 2 = x$, to the class display.

Pairs | 10 min

Activity 2 Keeping it True

Students think strategically about equivalent equations by exploring possible next steps in solving them.



Activity 2 Keeping it True

For each of the three equations shown, complete the following.

- Analyze each equation.
- Place a check next to each equation that is equivalent to the given equation for each problem.
- Next, circle the equation that you think represents the best next step for finding the value of x in the original equation.
- Lastly, solve the equation for x.

1. $-x = 10$

☑ $2 \bullet (-x) = 2 \bullet 10$ ☑ $-x + 3 = 10 + 3$ ☑ $-1 \bullet (-x) = -1 \bullet 10$

$x = -10$

$x =$ -10

2. $3 - 2x = -5$

☑ $3 - 2x + 4 = -5 + 4$ ☑ $3 - 2x + (-3) = -5 + (-3)$ ☐ $3 - \frac{2x}{2} = -\frac{5}{2}$

$-2x = -8$

$-2x \div (-2) = -8 \div (-2)$

$x = 4$

$x =$ 4

3. $19 = 3(x - 2)$

☑ $19 + 2 = 3(x - 2) + 2$ ☑ $19 \div 3 = 3(x - 2) \div 3$ ☑ $19 = 3x - 6$

$x =$ $8\frac{1}{3}$

$\frac{19}{3} = x - 2$

$\frac{19}{3} + 2 = x - 2 + 2$

$8\frac{1}{3} = x$

$19 + 6 = 3x - 6 + 6$

$25 = 3x$

$25 \div 3 = 3x \div 3$

$8\frac{1}{3} = x$

STOP

564 Unit 6 Expressions, Equations, and Inequalities

1 Launch

Review the directions for this activity with the whole class. Set an expectation for the amount of time students will have in pairs to work on the activity.

2 Monitor

Help students get started by asking, "Did you perform the same operation to each side? If yes, then the equations are equivalent and still balanced." Consider going over Problem 1 together with the students.

Look for points of confusion:

- **Only choosing some, but not all correct answers.** Ask, "How do you know which equation(s) are equivalent to the original? How can you check to make sure you carefully looked at the others?"
- **Not finding any that are equivalent.** Have students annotate the new equations to show what changed from the original equation.
- **Having difficulty solving for x.** Suggest students show a simplified next step. Then they can try substituting values.

3 Connect

Have students share in their own words how they chose which equation represented the "best next step."

Highlight that a "next best step" is one that gets students closer to isolating the variable. As they found in previous lessons, sometimes there are multiple steps that are equally helpful.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completed Problems 1 and 2, as they are sufficient for developing the core understanding for this activity.

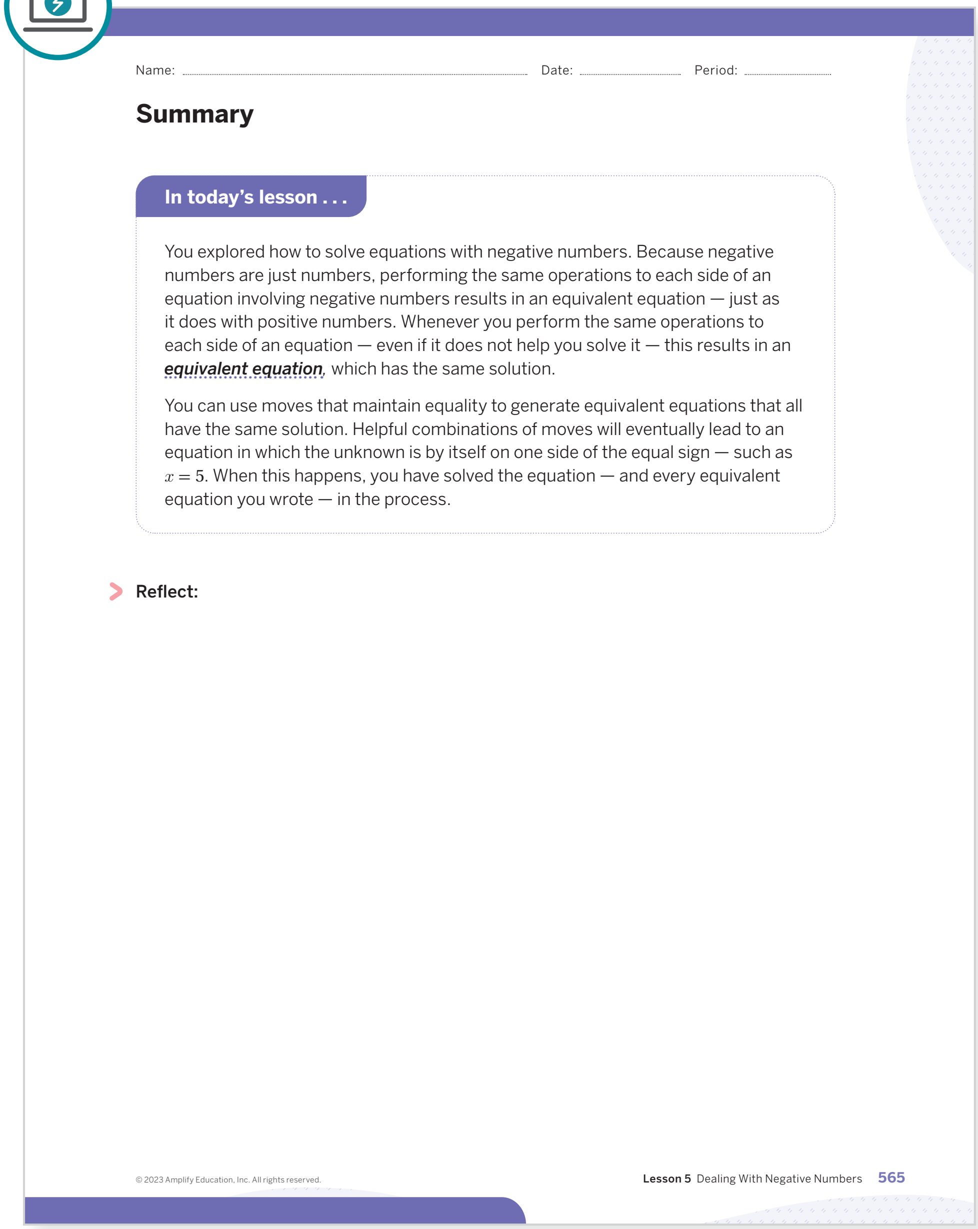
Extension: *Math Enrichment*

Challenge students to generate three different equations that are equivalent to the equation $p(x + q) = r$. Sample response: $px + pq = r$, $px = r - pq$, and $x + q = \frac{r}{p}$.

Whole Class | 5 min

Summary

Review and synthesize the idea that performing the same operations to each side of an equation creates an equivalent equation.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You explored how to solve equations with negative numbers. Because negative numbers are just numbers, performing the same operations to each side of an equation involving negative numbers results in an equivalent equation — just as it does with positive numbers. Whenever you perform the same operations to each side of an equation — even if it does not help you solve it — this results in an ***equivalent equation***, which has the same solution.

You can use moves that maintain equality to generate equivalent equations that all have the same solution. Helpful combinations of moves will eventually lead to an equation in which the unknown is by itself on one side of the equal sign — such as $x = 5$. When this happens, you have solved the equation — and every equivalent equation you wrote — in the process.

> **Reflect:**

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Lesson 5 Dealing With Negative Numbers 565

Synthesize

Formalize vocabulary: ***equivalent equations***

Display the Anchor Chart PDF, *Solving Equations* and complete the top section.

Ask, "What would the next best step be in solving this equation."

Highlight the steps for solving the given equation, bringing attention to the fact that each new equation is equivalent to the original equation.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is solving equations with negative values similar to or different from solving equations with only positive values?"

Math Language Development

MLR2: Collect and Display

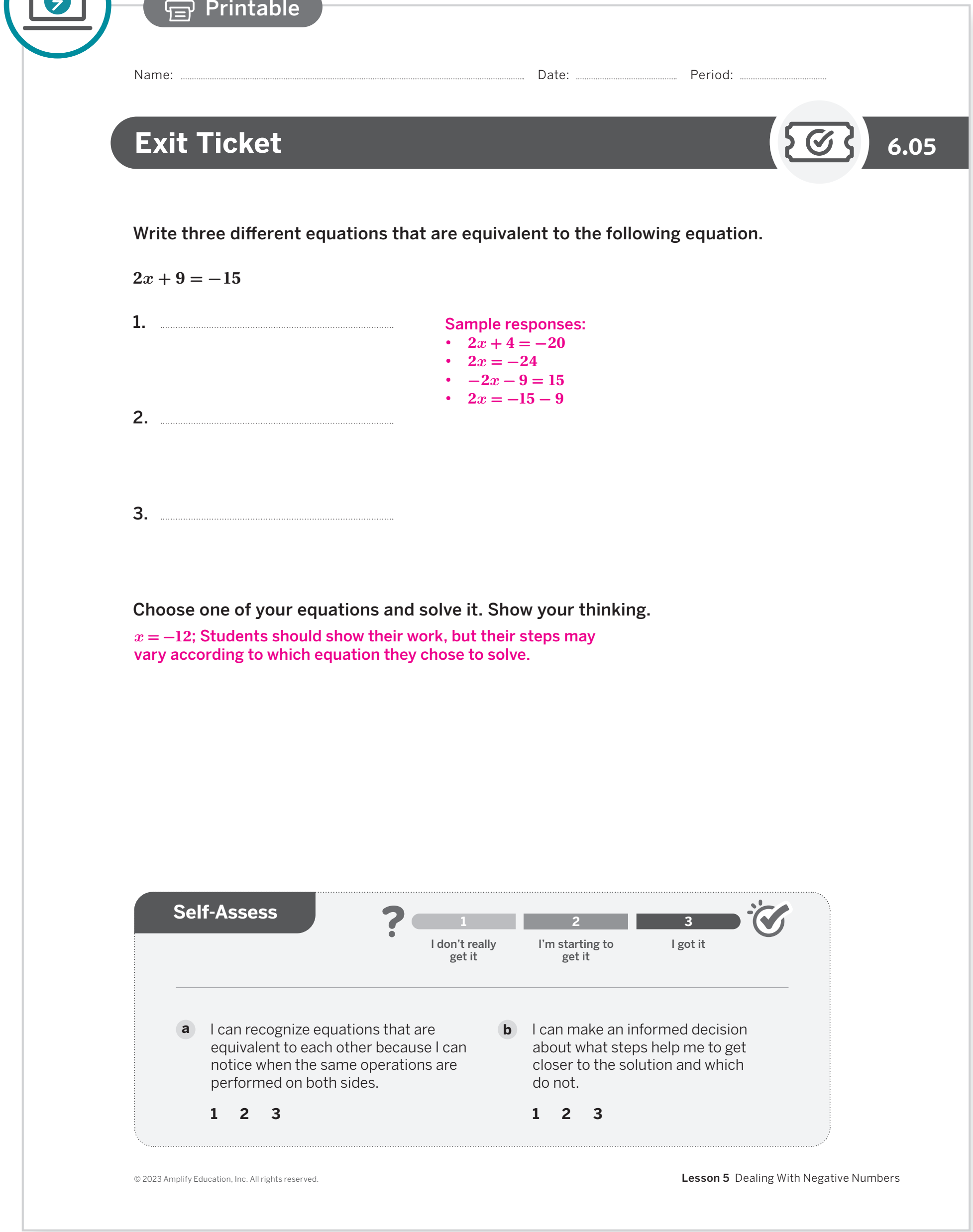
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *equivalent equations to* that were added to the display during the lesson.

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by showing that they can create equivalent equations strategically.

Printable



Name: Date: Period:

Exit Ticket 6.05

Write three different equations that are equivalent to the following equation.

$2x + 9 = -15$

1.

2.

3.

Sample responses:

- $2x + 4 = -20$
- $2x = -24$
- $-2x - 9 = 15$
- $2x = -15 - 9$

Choose one of your equations and solve it. Show your thinking.

$x = -12$; Students should show their work, but their steps may vary according to which equation they chose to solve.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can recognize equations that are equivalent to each other because I can notice when the same operations are performed on both sides.

1 2 3

b I can make an informed decision about what steps help me to get closer to the solution and which do not.

1 2 3

Lesson 5 Dealing With Negative Numbers

Success looks like . . .

- **Language Goal:** Generalizing that performing the same operations to each side of an equation generates an equivalent equation. **(Speaking and Listening)**
  - » Generating three different equivalent equations for the given equation.
- **Language Goal:** Solving equations of the form $px + q = r$ or $p(x + q) = r$ that involve negative numbers, and explaining the solution method. **(Speaking and Listening, Writing)**
  - » Solving one of the equivalent equations.

Suggested next steps

If students have difficulty with the mechanics of solving equations, consider:

- Continuing to practice solving simple two-step equations in the next lessons.

If students have difficulty writing an equivalent equation, consider:

- Reviewing the definition of *equivalent equations*, and revisit Activity 2, Problem 2.

Professional Learning

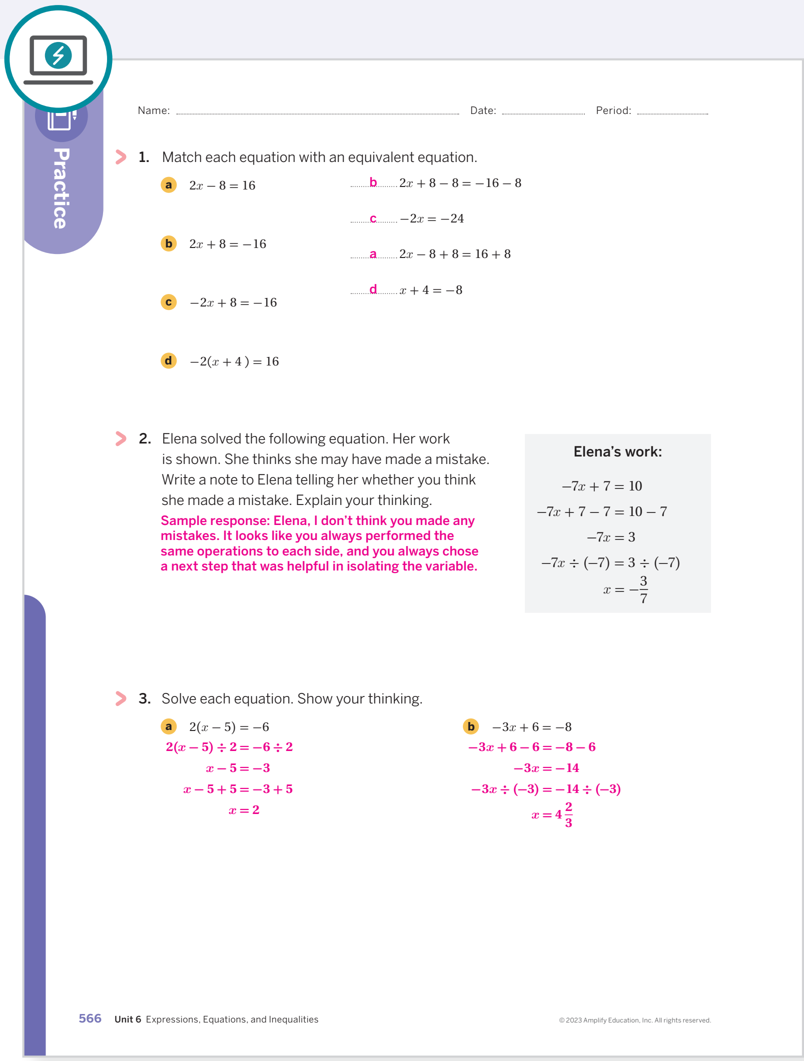
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about Activity 2 how did you encourage each student to share their understandings?
- The focus of this lesson was to solve equations of the form $px + q = r$ and $p(x + q) = r$ with negative numbers. How did this focus go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. Match each equation with an equivalent equation.

   a $2x - 8 = 16$

   b $2x + 8 = -16$

   c $-2x + 8 = -16$

   d $-2(x + 4) = 16$

   ......b...... $2x + 8 - 8 = -16 - 8$

   ......c...... $-2x = -24$

   ......a...... $2x - 8 + 8 = 16 + 8$

   ......d...... $x + 4 = -8$

2. Elena solved the following equation. Her work is shown. She thinks she may have made a mistake. Write a note to Elena telling her whether you think she made a mistake. Explain your thinking.

   Sample response: Elena, I don't think you made any mistakes. It looks like you always performed the same operations to each side, and you always chose a next step that was helpful in isolating the variable.

   Elena's work:

   $$-7x + 7 = 10$$
   $$-7x + 7 - 7 = 10 - 7$$
   $$-7x = 3$$
   $$-7x \div (-7) = 3 \div (-7)$$
   $$x = -\frac{3}{7}$$

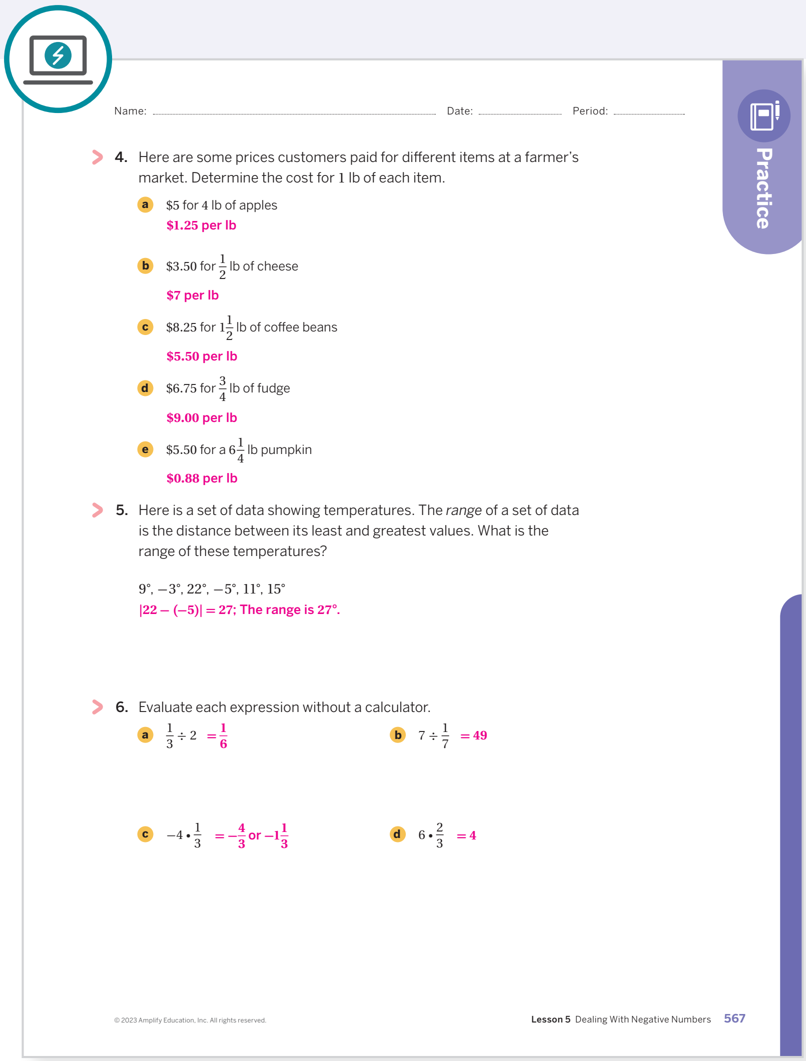
3. Solve each equation. Show your thinking.

   a $2(x - 5) = -6$

   $$\mathbf{2(x - 5) \div 2 = -6 \div 2}$$
   $$\mathbf{x - 5 = -3}$$
   $$\mathbf{x - 5 + 5 = -3 + 5}$$
   $$\mathbf{x = 2}$$

   b $-3x + 6 = -8$

   $$\mathbf{-3x + 6 - 6 = -8 - 6}$$
   $$\mathbf{-3x = -14}$$
   $$\mathbf{-3x \div (-3) = -14 \div (-3)}$$
   $$\mathbf{x = 4\frac{2}{3}}$$



Name: .... Date: .... Period: ....

4. Here are some prices customers paid for different items at a farmer's market. Determine the cost for 1 lb of each item.

   a $5 for 4 lb of apples

   $1.25 per lb

   b $3.50 for $\frac{1}{2}$ lb of cheese

   $7 per lb

   c $8.25 for $1\frac{1}{2}$ lb of coffee beans

   $5.50 per lb

   d $6.75 for $\frac{3}{4}$ lb of fudge

   $9.00 per lb

   e $5.50 for a $6\frac{1}{4}$ lb pumpkin

   $0.88 per lb

5. Here is a set of data showing temperatures. The *range* of a set of data is the distance between its least and greatest values. What is the range of these temperatures?

   9°, −3°, 22°, −5°, 11°, 15°

   $|22 - (-5)| = 27$; The range is 27°.

6. Evaluate each expression without a calculator.

   a $\frac{1}{3} \div 2$ $\mathbf{= \frac{1}{6}}$

   b $7 \div \frac{1}{7}$ $\mathbf{= 49}$

   c $-4 \bullet \frac{1}{3}$ $\mathbf{= -\frac{4}{3}}$ **or** $\mathbf{-1\frac{1}{3}}$

   d $6 \bullet \frac{2}{3}$ $\mathbf{= 4}$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	1
Spiral	4	Unit 2 Lesson 5	1
	5	Unit 5 Lesson 8	1
Formative ⚡	6	Unit 6 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

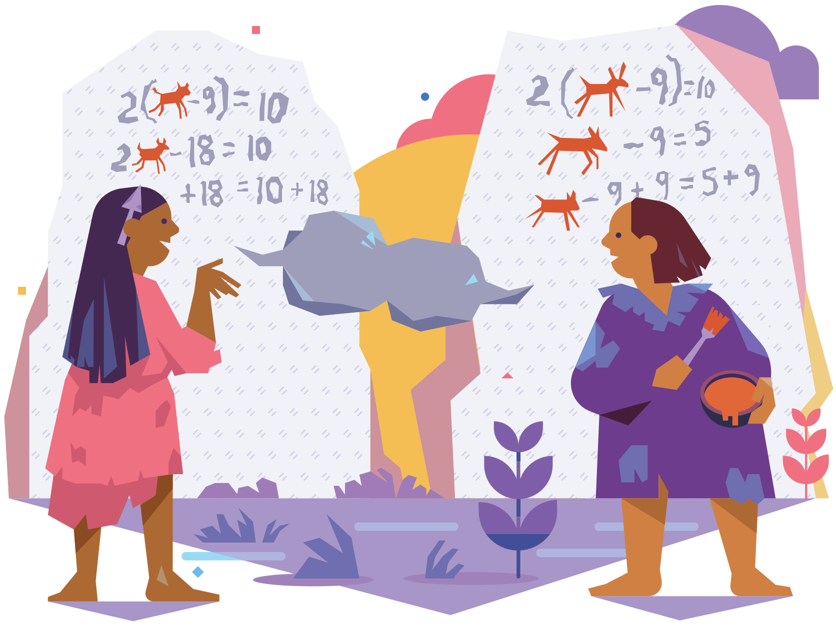
Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 6**

Two Ways to Solve One Equation

Let’s think about efficient ways to solve equations with parentheses.



Focus

Goals

1. Recognize there are two common approaches for solving an equation of the form $p(x + q) = r$; (1) expanding using the Distributive Property or (2) dividing each side by p.
2. **Language Goal:** Critique a given solution method for an equation of the form $p(x + q) = r$. **(Speaking and Listening, Writing)**
3. **Language Goal:** Evaluate the usefulness of different approaches for solving a given equation of the form $p(x + q) = r$. **(Speaking and Listening, Writing)**

Coherence

Today

Students practice solving equations of the form $p(x + q) = r$, focusing on the structure of the equation to determine which method; (1) applying the Distributive Property or (2) dividing by the factor outside the parentheses will produce the most efficient strategy.

Previously

In Lesson 3, 4, and 5, students learned how to solve equations of the form $px + q = r$ and $p(x + q) = r$ with rational numbers.

Coming Soon

In Lesson 8, students will use these strategies to solve real-world and mathematical problems.

Rigor

- Students analyze and practice multiple methods of solving equations of the form $p(x + q) = r$ to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Solving Equations* (for display)
- Anchor Chart PDF, *Solving Equations* (answers)

Math Language Development

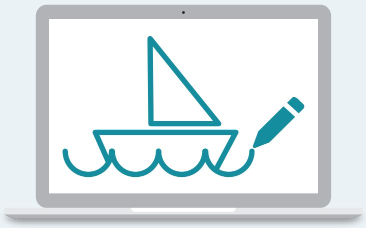
Review words

- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *properties of equality*
- *solution to an equation*
- *substitute*
- *variable*

Amps | **Featured Activity**

Activity 1
Sketch Box

Students are able to show their work solving an equation by using the sketch tool.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might disagree with their partners about which solution method is preferred. Remind students that there is more than one way to solve the equation. There is, however, an incorrect way of handling this disagreement. Students should show respect for their partner, trying to understand why they chose the method that they did.

Modifications to Pacing

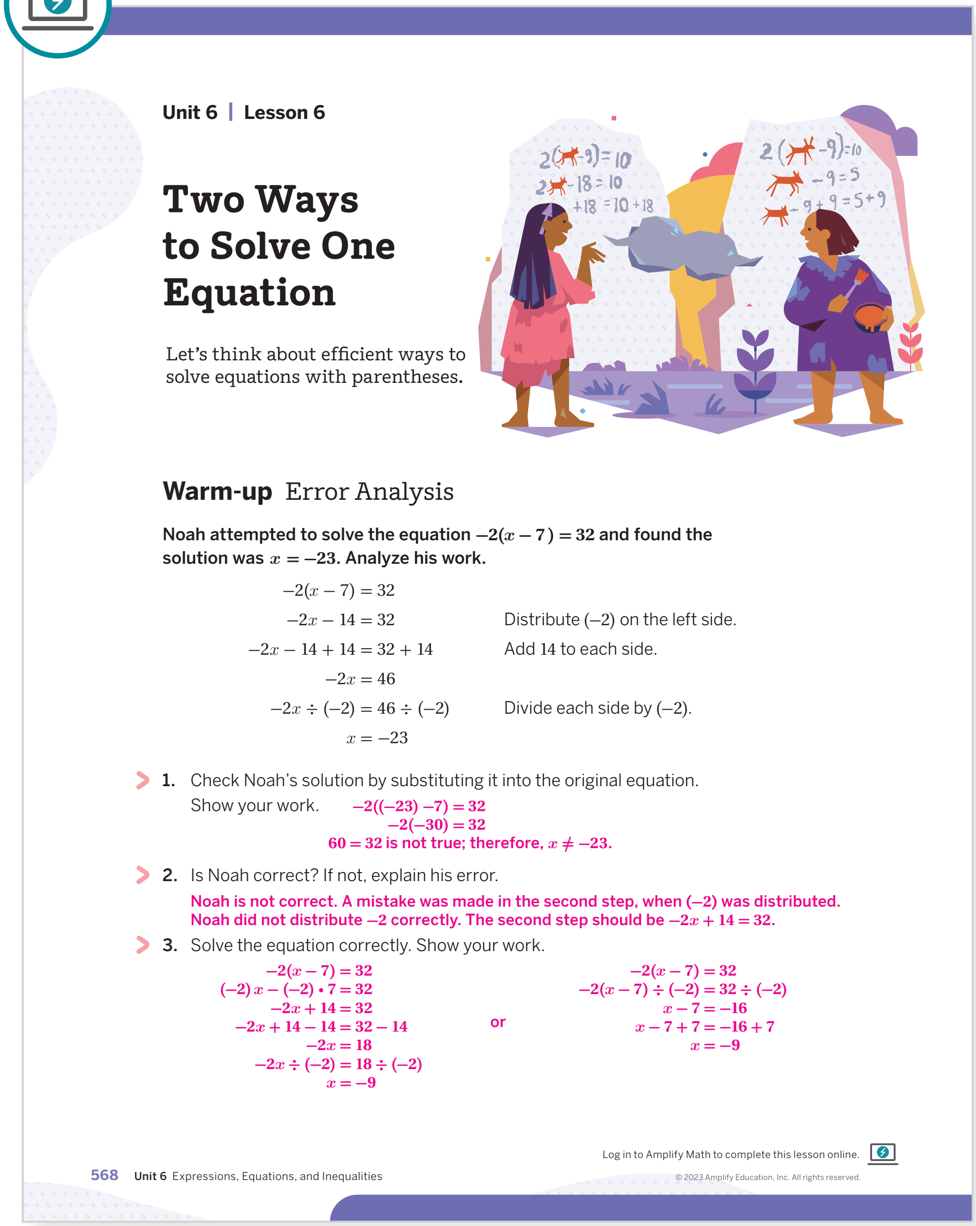
You may want to consider these additional modifications if you are short on time.

- During the **Warm-up**, let students know there is an error in the solution and ask them to identify it. Discuss as a class.
- In **Activity 1**, have partners divide the work so that one student solves using Elena's method and the other student solves using Han's method. Then have partners compare their work.
- In **Activity 2**, provide students with choice by having them solve only two equations. Then discuss all three as a class.

Independent | 5 min

Warm-up Error Analysis

Students analyze another's work to determine an error in distribution (of a negative number) and correct the solution process.



Unit 6 | Lesson 6

Two Ways to Solve One Equation

Let's think about efficient ways to solve equations with parentheses.

Warm-up Error Analysis

Noah attempted to solve the equation $-2(x-7)=32$ and found the solution was $x=-23$. Analyze his work.

$-2(x-7)=32$	
$-2x-14=32$	Distribute (–2) on the left side.
$-2x-14+14=32+14$	Add 14 to each side.
$-2x=46$	
$-2x\div(-2)=46\div(-2)$	Divide each side by (–2).
$x=-23$	

1. Check Noah's solution by substituting it into the original equation. Show your work.

$$-2((-23)-7)=32$$
$$-2(-30)=32$$

60 = 32 is not true; therefore, $x \neq -23$.

2. Is Noah correct? If not, explain his error.

Noah is not correct. A mistake was made in the second step, when (–2) was distributed. Noah did not distribute –2 correctly. The second step should be $-2x+14=32$.

3. Solve the equation correctly. Show your work.

$$\begin{aligned}-2(x-7)&=32\\(-2)x-(-2)\cdot 7&=32\\-2x+14&=32\\-2x+14-14&=32-14\\-2x&=18\\-2x\div(-2)&=18\div(-2)\\x&=-9\end{aligned}$$

or

$$\begin{aligned}-2(x-7)&=32\\-2(x-7)\div(-2)&=32\div(-2)\\x-7&=-16\\x-7+7&=-16+7\\x&=-9\end{aligned}$$

Log in to Amplify Math to complete this lesson online.

568 Unit 6 Expressions, Equations, and Inequalities

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1 Launch

Tell students that they will analyze and critique Noah's work in solving the equation.

2 Monitor

Help students get started by asking, "How can you check whether –23 is a solution?"

Look for points of confusion:

- **Thinking Noah is correct.** Show the Distributive Property by drawing arrows and/or show this step: $(-2)x-(-2)\cdot 7=32$.
- **Thinking $-(-2)\cdot 7=-14$.** Remind students that a negative on the outside of parenthesis is the same as multiplying by -1. So, $-(-2)=-1\cdot(-2)=+2$.

3 Connect

Display Noah's work.

Highlight that Noah's mistake is a common mistake that anyone can make. Checking the solution each time is an important part of the equation solving process.

Ask, "Who has made a mistake similar to Noah's? What can you do to make sure you do not make it again?"

Power-up

To power up students' ability to evaluate expressions involving multiplying or dividing by a fraction, have students complete:

Recall that when dividing by a fraction, you can rewrite the expression as multiplying by the reciprocal. For example $8\div\frac{2}{3}=8\cdot\frac{3}{2}$.

Evaluate each expression.

1. $2\div\frac{1}{3}$ $=6$
2. $4\cdot\frac{1}{3}$ $=\frac{4}{3}$
3. $\frac{5}{6}\div\frac{2}{3}$ $=1\frac{1}{4}$

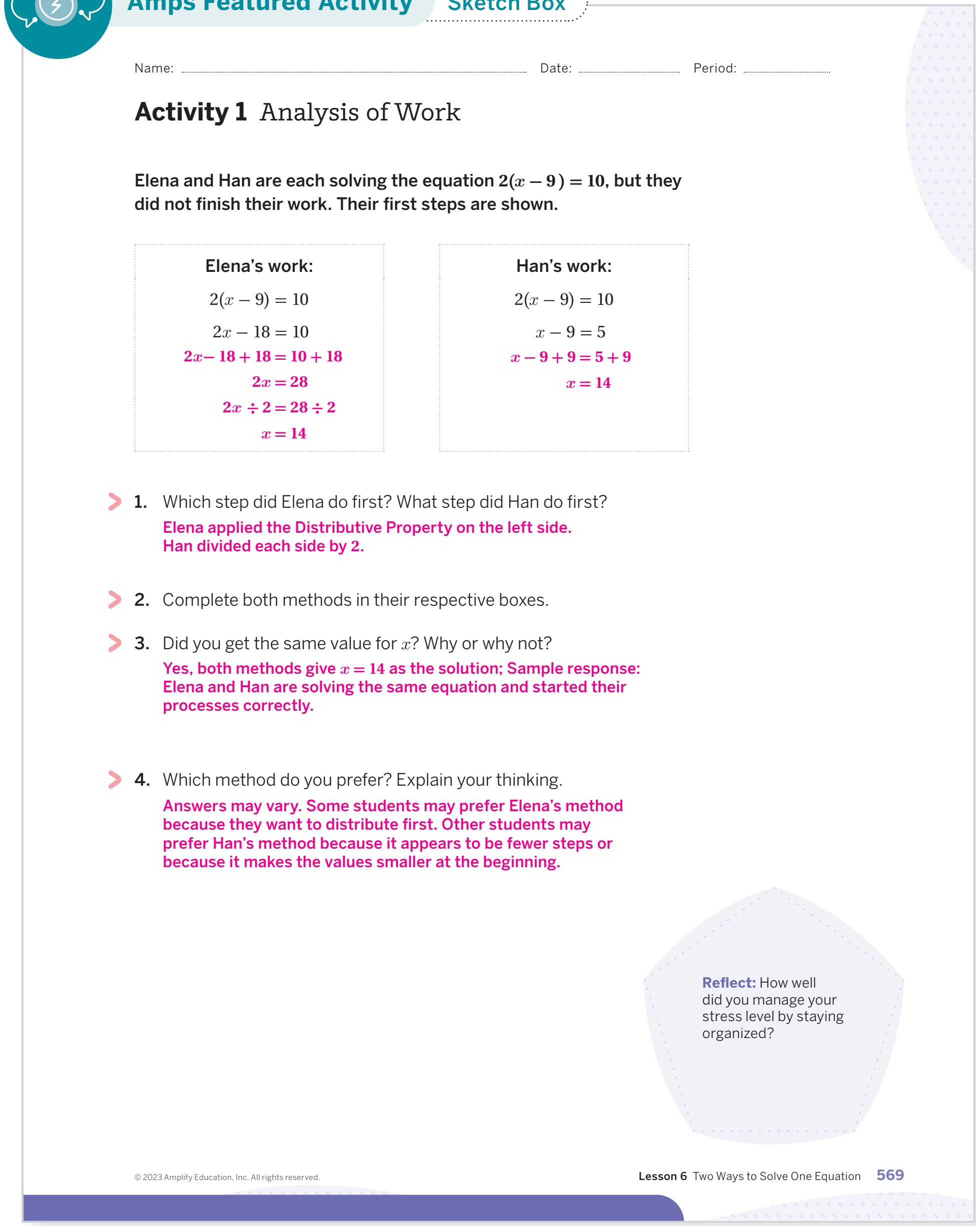
Use: Before Activity 2
Informed by: Performance on Lesson 5, Practice Problem 6

Pairs | 15 min

Activity 1 Analysis of Work

Students analyze two different solution methods for solving an equation of the form $p(x+q)=r$ and determine which method they prefer.

Amps Featured Activity Sketch Box



Name: ____________ Date: ________ Period: ________

Activity 1 Analysis of Work

Elena and Han are each solving the equation $2(x-9)=10$, but they did not finish their work. Their first steps are shown.

Elena's work:	Han's work:
$2(x-9)=10$	$2(x-9)=10$
$2x-18=10$	$x-9=5$
$2x-18+18=10+18$	$x-9+9=5+9$
$2x=28$	$x=14$
$2x\div 2=28\div 2$	
$x=14$	

1. Which step did Elena do first? What step did Han do first?
   Elena applied the Distributive Property on the left side. Han divided each side by 2.
2. Complete both methods in their respective boxes.
3. Did you get the same value for x? Why or why not?
   Yes, both methods give $x=14$ as the solution; Sample response: Elena and Han are solving the same equation and started their processes correctly.
4. Which method do you prefer? Explain your thinking.
   Answers may vary. Some students may prefer Elena's method because they want to distribute first. Other students may prefer Han's method because it appears to be fewer steps or because it makes the values smaller at the beginning.

Reflect: How well did you manage your stress level by staying organized?

1 Launch

Let students know they are to analyze each method shown and determine which method they prefer, based on the structure of the equation.

2 Monitor

Help students get started by asking, "What would your first step be in solving this equation?" Have students analyze Elena's method first if they want to distribute, or have them analyze Han's method first if they want to divide first.

Look for points of confusion:

- **Solving the equations incorrectly.** Have students check their answers by substituting the value in the original equation.

3 Connect

Display Elena's and Han's work.

Have students share and explain Elena's and Han's methods, pointing out the similarities and differences between them. Then have 3–4 students share which method they prefer and why.

Highlight that, when solving equations, both methods work because they produce equivalent equations. However, it may be easier to use one method instead of the other, based on the structure of the equation. Knowing both methods is helpful.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital sketch tool to show their work solving an equation.

Extension: *Math Enrichment*

Ask students to write an equation in which they would want to distribute first, and have them explain why. Then ask them to write an equation in which they would want to divide by the factor outside the parentheses first, and have them explain why.

Math Language Development

MLR7: Compare and Connect

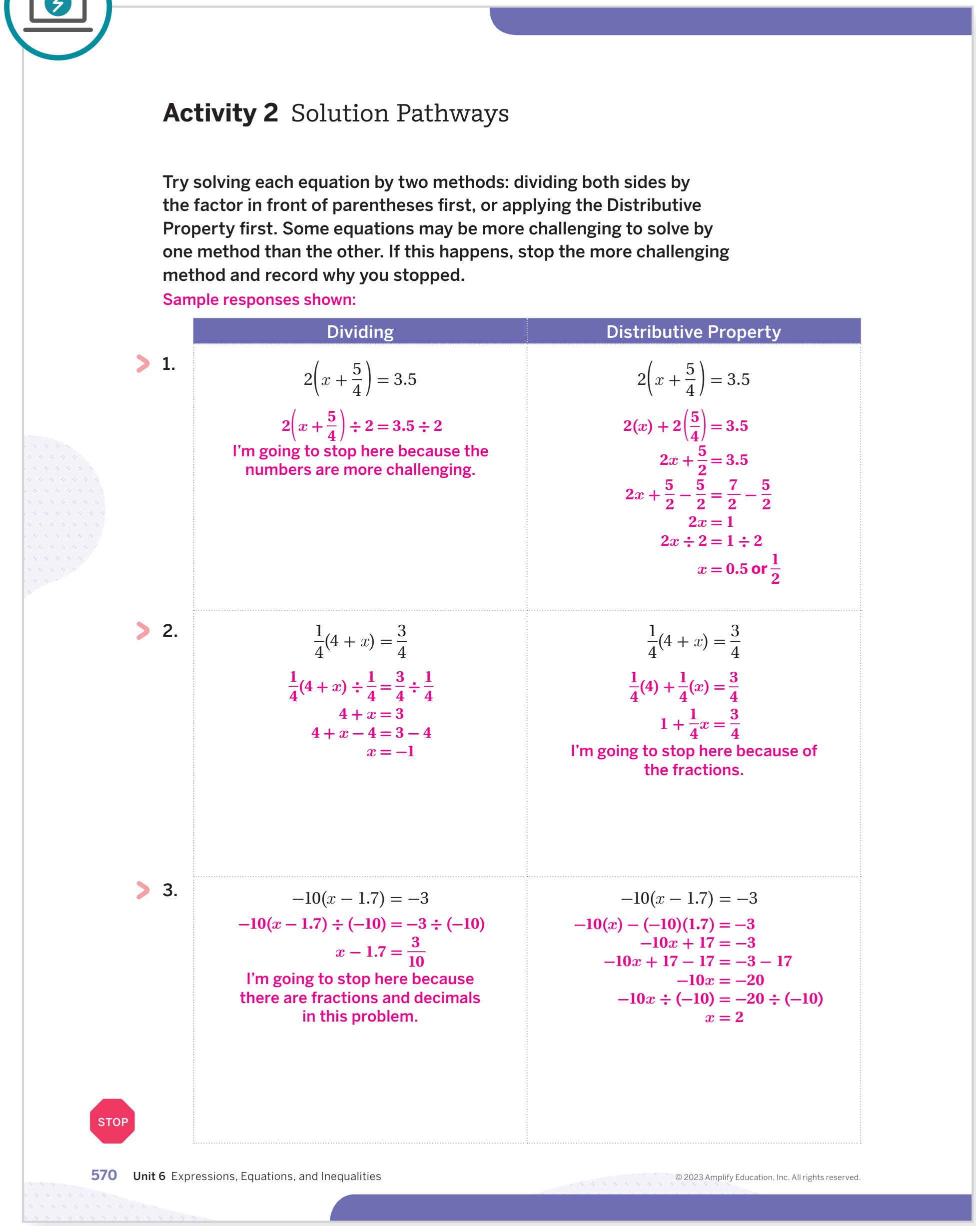
During the Connect, as students compare and contrast Elena's and Han's methods, draw their attention to how an equation's structure can indicate which strategies will be helpful for solving it. Ask:

- "If the parentheses were not part of this equation and the equation was instead $2x-9=10$, could you solve it by dividing both sides by 2? Explain your thinking. You could divide both sides by 2, but that won't necessarily help you solve the equation.
- "Suppose a classmate states that you can solve this equation by thinking 'two times a quantity is 10, which means that quantity is 5.' Is this a valid approach? What would 'the quantity' represent in this instance? Yes; this is a valid approach. The quantity is $x-9$.

Independent | 15 min

Activity 2 Solution Pathways

Students solve equations using both methods (distributing first vs. dividing first) to decide which method to use, based on the structure of the equation.



Activity 2 Solution Pathways

Try solving each equation by two methods: dividing both sides by the factor in front of parentheses first, or applying the Distributive Property first. Some equations may be more challenging to solve by one method than the other. If this happens, stop the more challenging method and record why you stopped.

Sample responses shown:

	Dividing	Distributive Property
1.	$2\left(x+\frac{5}{4}\right)=3.5$ $2\left(x+\frac{5}{4}\right)\div 2=3.5\div 2$ I'm going to stop here because the numbers are more challenging.	$2\left(x+\frac{5}{4}\right)=3.5$ $2(x)+2\left(\frac{5}{4}\right)=3.5$ $2x+\frac{5}{2}=3.5$ $2x+\frac{5}{2}-\frac{5}{2}=\frac{7}{2}-\frac{5}{2}$ $2x=1$ $2x\div 2=1\div 2$ $x=0.5$ or $\frac{1}{2}$
2.	$\frac{1}{4}(4+x)=\frac{3}{4}$ $\frac{1}{4}(4+x)\div\frac{1}{4}=\frac{3}{4}\div\frac{1}{4}$ $4+x=3$ $4+x-4=3-4$ $x=-1$	$\frac{1}{4}(4+x)=\frac{3}{4}$ $\frac{1}{4}(4)+\frac{1}{4}(x)=\frac{3}{4}$ $1+\frac{1}{4}x=\frac{3}{4}$ I'm going to stop here because of the fractions.
3.	$-10(x-1.7)=-3$ $-10(x-1.7)\div(-10)=-3\div(-10)$ $x-1.7=\frac{3}{10}$ I'm going to stop here because there are fractions and decimals in this problem.	$-10(x-1.7)=-3$ $-10(x)-(-10)(1.7)=-3$ $-10x+17=-3$ $-10x+17-17=-3-17$ $-10x=-20$ $-10x\div(-10)=-20\div(-10)$ $x=2$

STOP

570 Unit 6 Expressions, Equations, and Inequalities

1 Launch

Allow students to check their solutions with a partner after solving each equation independently. Point out that while they might choose a different method from their partner, the solutions should be the same. This activity helps students decide which method is more efficient, based on structure.

2 Monitor

Help students get started by asking, "What would the equation look like after applying the Distributive Property?" or "What would the equation look like after dividing by the factor first?"

Look for points of confusion:

- **Misunderstanding the directions.** Model the expectation of starting both methods and deciding which to finish for Problem 1.
- **Incorrectly operating with the fractions.** Work with students on these skills.

3 Connect

Display the solution to each equation. Conduct the ***Poll the Class*** routine to determine which method students preferred for each equation.

Have students share which method — distribute first or divide first — they preferred. Discuss any disagreements.

Highlight that there is no right or wrong method for each equation. Some students might prefer to eliminate fractions/decimals as early as possible, while some might want to minimize the number of computations.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on solving the equations in Problems 1 and 2 first. As time allows, they can solve the equation in Problem 3.

Extension: *Math Enrichment*

Tell students that another strategy they can use to solve the equation in Problems 1 and 2 is to multiply each term by the least common denominator of the fractions. This will eliminate the fractions. Have them use this strategy and compare the solutions to the solutions they already determined. They should note the solutions are the same.

Math Language Development

MLR7: Compare and Connect

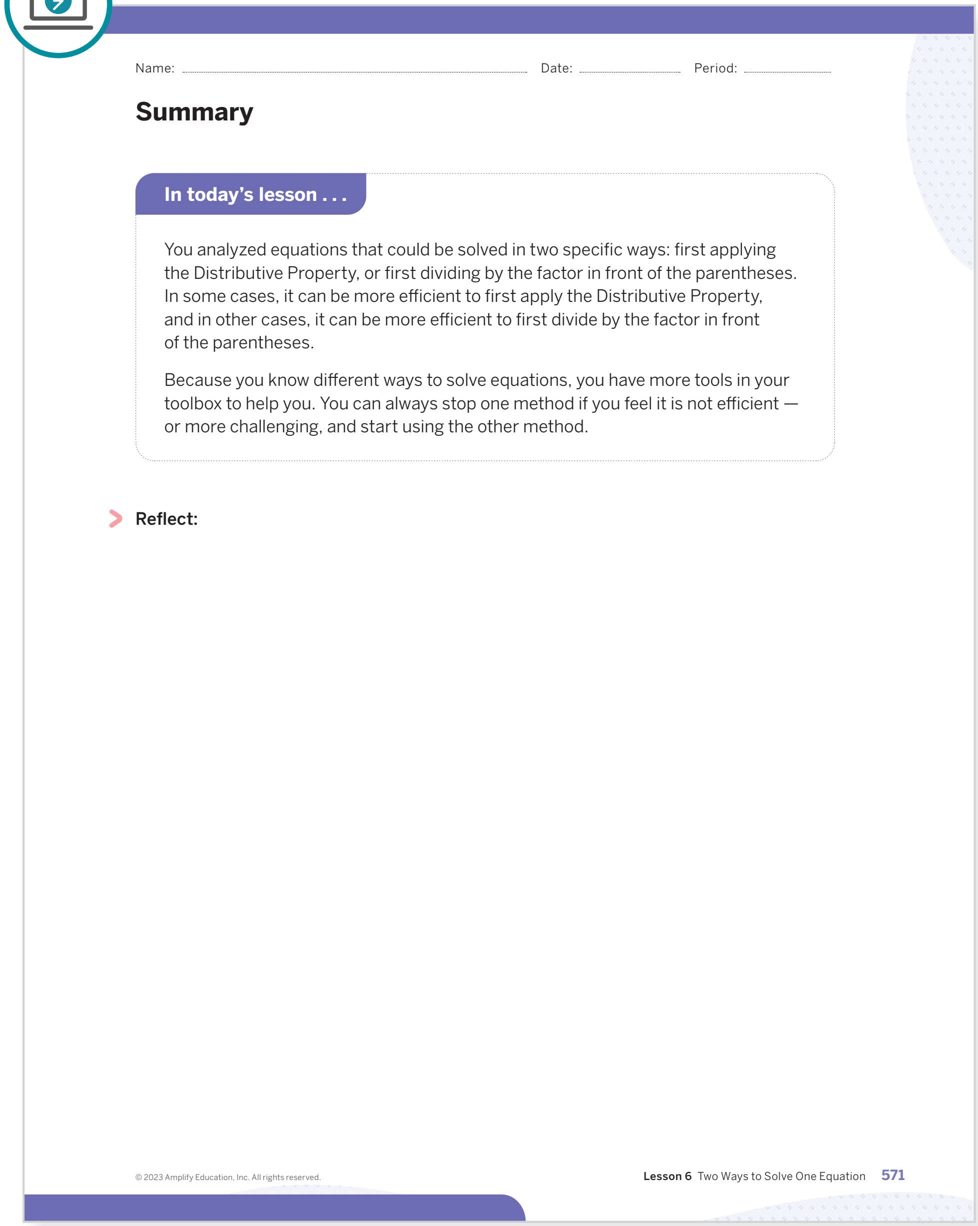
During the Connect, as students compare and contrast the strategies, draw their attention to which strategy may be more efficient in each case, and how analyzing the equation can help them determine the more efficient strategy. For example, in Problem 2, ask:

- "Why might dividing both sides of the equation by the factor outside the parentheses be helpful here? What is it about the numbers that indicate this may be a useful strategy?"
- "Dividing both sides by $\frac{1}{4}$ is the same as multiplying both sides by what number?"

Whole Class | 5 min

Summary

Review and synthesize how equations of the form $p(x + q) = r$ can be solved using two methods, and how students can choose an efficient method of dividing first or distributing first, based on the structure of the equation.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You analyzed equations that could be solved in two specific ways: first applying the Distributive Property, or first dividing by the factor in front of the parentheses. In some cases, it can be more efficient to first apply the Distributive Property, and in other cases, it can be more efficient to first divide by the factor in front of the parentheses.

Because you know different ways to solve equations, you have more tools in your toolbox to help you. You can always stop one method if you feel it is not efficient — or more challenging, and start using the other method.

Reflect:

Synthesize

Ask:

- "What are the two main ways you can approach solving equations like the ones you saw today?" Divide first or distribute first.
- "What kinds of things could you look for to decide which approach is more efficient?" Operations that result in whole numbers, moves that will eliminate fractions or decimals, etc.
- "How can you check if your answer is a solution to the equation?" Substitute the answer for the variable and see whether it makes the equation true

Display the Anchor Chart, *Solving Equations* and complete the bottom section.

Have students share how to solve the equation using each method.

Highlight both methods are valid, useful, and important to know.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

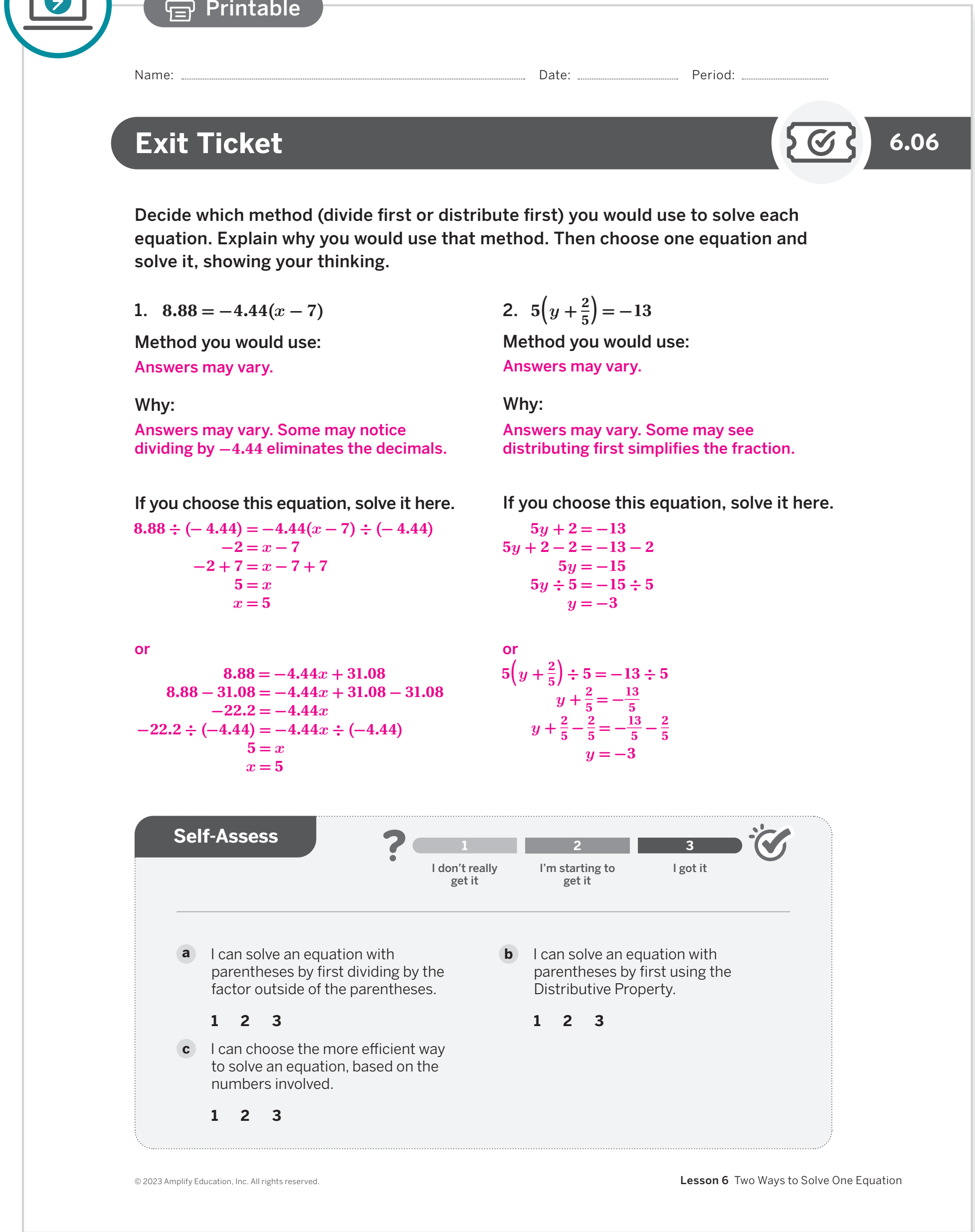
- "How can you increase your efficiency in solving mathematical problems?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by choosing the most efficient method for solving equations of the form $p(x + q) = r$ and justifying their reasoning.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.06

Decide which method (divide first or distribute first) you would use to solve each equation. Explain why you would use that method. Then choose one equation and solve it, showing your thinking.

1. $8.88 = -4.44(x - 7)$

Method you would use:

Answers may vary.

Why:

Answers may vary. Some may notice dividing by -4.44 eliminates the decimals.

If you choose this equation, solve it here.

$$\begin{aligned} 8.88 \div (-4.44) &= -4.44(x - 7) \div (-4.44) \\ -2 &= x - 7 \\ -2 + 7 &= x - 7 + 7 \\ 5 &= x \\ x &= 5 \end{aligned}$$

or

$$\begin{aligned} 8.88 &= -4.44x + 31.08 \\ 8.88 - 31.08 &= -4.44x + 31.08 - 31.08 \\ -22.2 &= -4.44x \\ -22.2 \div (-4.44) &= -4.44x \div (-4.44) \\ 5 &= x \\ x &= 5 \end{aligned}$$

2. $5\left(y + \frac{2}{5}\right) = -13$

Method you would use:

Answers may vary.

Why:

Answers may vary. Some may see distributing first simplifies the fraction.

If you choose this equation, solve it here.

$$\begin{aligned} 5y + 2 &= -13 \\ 5y + 2 - 2 &= -13 - 2 \\ 5y &= -15 \\ 5y \div 5 &= -15 \div 5 \\ y &= -3 \end{aligned}$$

or

$$\begin{aligned} 5\left(y + \tfrac{2}{5}\right) \div 5 &= -13 \div 5 \\ y + \tfrac{2}{5} &= -\tfrac{13}{5} \\ y + \tfrac{2}{5} - \tfrac{2}{5} &= -\tfrac{13}{5} - \tfrac{2}{5} \\ y &= -3 \end{aligned}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can solve an equation with parentheses by first dividing by the factor outside of the parentheses.

1 2 3

b. I can solve an equation with parentheses by first using the Distributive Property.

1 2 3

c. I can choose the more efficient way to solve an equation, based on the numbers involved.

1 2 3

Lesson 6 Two Ways to Solve One Equation

Success looks like . . .

- **Goal:** Recognizing there are two common approaches for solving an equation of the form $p(x + q) = r$; (1) expanding using the Distributive Property or (2) dividing each side by p.
  - » Solving each equation either by using the Distributive Property or dividing by the coefficient in front of the parentheses in Problems 1 and 2.
- **Language Goal:** Critiquing a given solution method for an equation of the form $p(x + q) = r$. **(Speaking and Listening, Writing)**
  - » Explaining why a given solution method was used in Problems 1 and 2.
- **Language Goal:** Evaluating the usefulness of different approaches for solving a given equation of the form $p(x + q) = r$. **(Speaking and Listening, Writing)**
  - » Explaining how the chosen method is useful in Problems 1 and 2.

Suggested next steps

If students do not know which method is more efficient, consider

- Assigning Practice Problem 1, in which students analyze the two methods.

If students need more practice solving similar equations, consider

- Assigning Practice Problem 2.
- Move on to the next lesson, which provides students with additional practice.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- How did the Error Analysis support students in recognizing that there are two approaches for solving an equation of the form $p(x + q) = r$? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Evaluating the usefulness of different approaches for solving a given equation of the form $p(x + q) = r$.

Reflect on students' language development toward this goal.

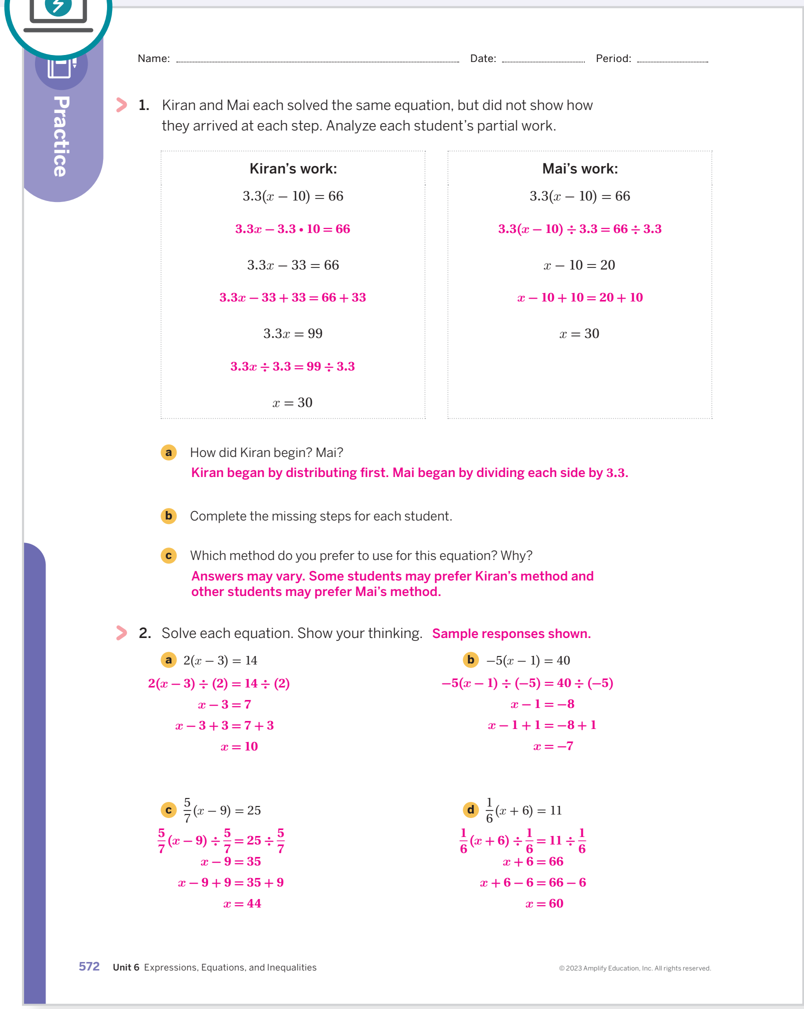
- How did using the *Compare and Connect* routine in Activity 2 help students evaluate the usefulness of the two methods (divide first or distribute first)?
- How can you help them explain when they might want to use one method over another?

Sample explanations shown for Problem 2:

Emerging	Expanding
5 times $\frac{2}{5}$ is $\frac{10}{5}$, or 2.	Distributing the 5 eliminates the fraction.

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Kiran and Mai each solved the same equation, but did not show how they arrived at each step. Analyze each student's partial work.

Kiran's work:

$3.3(x - 10) = 66$

$3.3x - 3.3 \cdot 10 = 66$

$3.3x - 33 = 66$

$3.3x - 33 + 33 = 66 + 33$

$3.3x = 99$

$3.3x \div 3.3 = 99 \div 3.3$

$x = 30$

Mai's work:

$3.3(x - 10) = 66$

$3.3(x - 10) \div 3.3 = 66 \div 3.3$

$x - 10 = 20$

$x - 10 + 10 = 20 + 10$

$x = 30$

a. How did Kiran begin? Mai?

Kiran began by distributing first. Mai began by dividing each side by 3.3.

b. Complete the missing steps for each student.

c. Which method do you prefer to use for this equation? Why?

Answers may vary. Some students may prefer Kiran's method and other students may prefer Mai's method.

2. Solve each equation. Show your thinking. **Sample responses shown.**

a. $2(x - 3) = 14$

$2(x - 3) \div (2) = 14 \div (2)$

$x - 3 = 7$

$x - 3 + 3 = 7 + 3$

$x = 10$

b. $-5(x - 1) = 40$

$-5(x - 1) \div (-5) = 40 \div (-5)$

$x - 1 = -8$

$x - 1 + 1 = -8 + 1$

$x = -7$

c. $\frac{5}{7}(x - 9) = 25$

$\frac{5}{7}(x - 9) \div \frac{5}{7} = 25 \div \frac{5}{7}$

$x - 9 = 35$

$x - 9 + 9 = 35 + 9$

$x = 44$

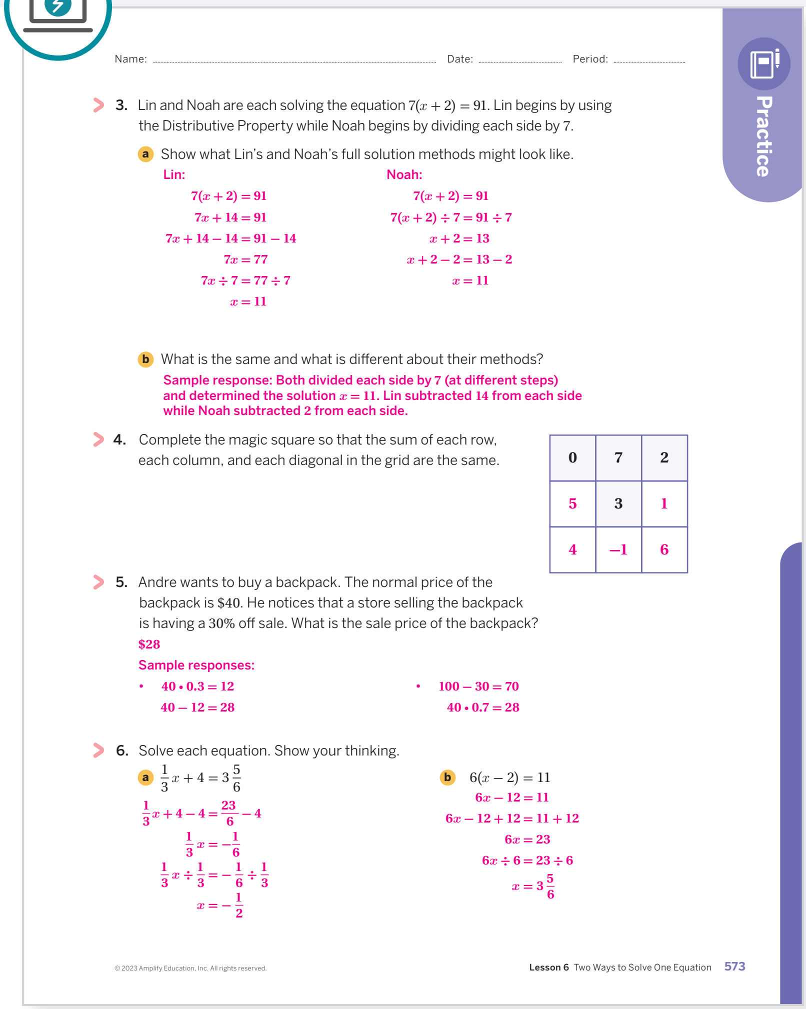
d. $\frac{1}{6}(x + 6) = 11$

$\frac{1}{6}(x + 6) \div \frac{1}{6} = 11 \div \frac{1}{6}$

$x + 6 = 66$

$x + 6 - 6 = 66 - 6$

$x = 60$



Practice

Name: ______ Date: ______ Period: ______

3. Lin and Noah are each solving the equation $7(x + 2) = 91$. Lin begins by using the Distributive Property while Noah begins by dividing each side by 7.

a. Show what Lin's and Noah's full solution methods might look like.

Lin:

$7(x + 2) = 91$

$7x + 14 = 91$

$7x + 14 - 14 = 91 - 14$

$7x = 77$

$7x \div 7 = 77 \div 7$

$x = 11$

Noah:

$7(x + 2) = 91$

$7(x + 2) \div 7 = 91 \div 7$

$x + 2 = 13$

$x + 2 - 2 = 13 - 2$

$x = 11$

b. What is the same and what is different about their methods?

Sample response: Both divided each side by 7 (at different steps) and determined the solution $x = 11$. Lin subtracted 14 from each side while Noah subtracted 2 from each side.

4. Complete the magic square so that the sum of each row, each column, and each diagonal in the grid are the same.

0	7	2
5	3	1
4	–1	6

5. Andre wants to buy a backpack. The normal price of the backpack is \$40. He notices that a store selling the backpack is having a 30% off sale. What is the sale price of the backpack?

\$28

Sample responses:

- $40 \cdot 0.3 = 12$
  $40 - 12 = 28$
- $100 - 30 = 70$
  $40 \cdot 0.7 = 28$

6. Solve each equation. Show your thinking.

a. $\frac{1}{3}x + 4 = 3\frac{5}{6}$

$\frac{1}{3}x + 4 - 4 = \frac{23}{6} - 4$

$\frac{1}{3}x = -\frac{1}{6}$

$\frac{1}{3}x \div \frac{1}{3} = -\frac{1}{6} \div \frac{1}{3}$

$x = -\frac{1}{2}$

b. $6(x - 2) = 11$

$6x - 12 = 11$

$6x - 12 + 12 = 11 + 12$

$6x = 23$

$6x \div 6 = 23 \div 6$

$x = 3\frac{5}{6}$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	1
	3	Activity 1	3
Spiral	4	Unit 5	2
	5	Unit 4	1
Formative	6	Unit 6 Lesson 7	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 7**

Practice Solving Equations

Let's practice.

Focus

Goal

1. Solve equations of the forms $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers.

Rigor

- Students develop **procedural fluency** in solving equations of the form $px + q = r$ and $p(x + q) = r$.

Coherence

Today

In today's lesson, students practice solving equations in an equation chain, where each solution is used to create the next equation. Students need to keep in mind signs and operations and are encouraged to check their solutions to ensure accuracy within the chain. Students also create their own equation for their partner to solve.

Previously

In Lessons 1–6, students developed the skills of solving equations of the forms $px + q = r$ and $p(x + q) = r$.

Coming Soon

In Lessons 8–11, students continue solving equations of the forms $px + q = r$ and $p(x + q) = r$ in context.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1 (Optional)	Activity 2	Summary	Exit Ticket
5 min	15 min	30 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF
- Activity 2 PDF (answers)

Math Language Development

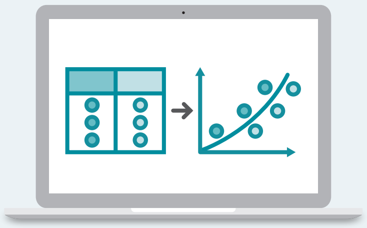
Review words

- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *properties of equality*
- *solution to an equation*
- *substitute*
- *variable*

Amps | **Featured Activity**

Activity 1
Digital Equation Chain

Students' responses from previous screens populate the appropriate spaces on the next screen.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel overly confident as they write an equation with a given solution. They might not even realize that they need help. When working with a partner, it is important that students recognize when to seek and offer help. Create a code word for students to use when they think their partner needs help but does not know it.

Modifications to Pacing

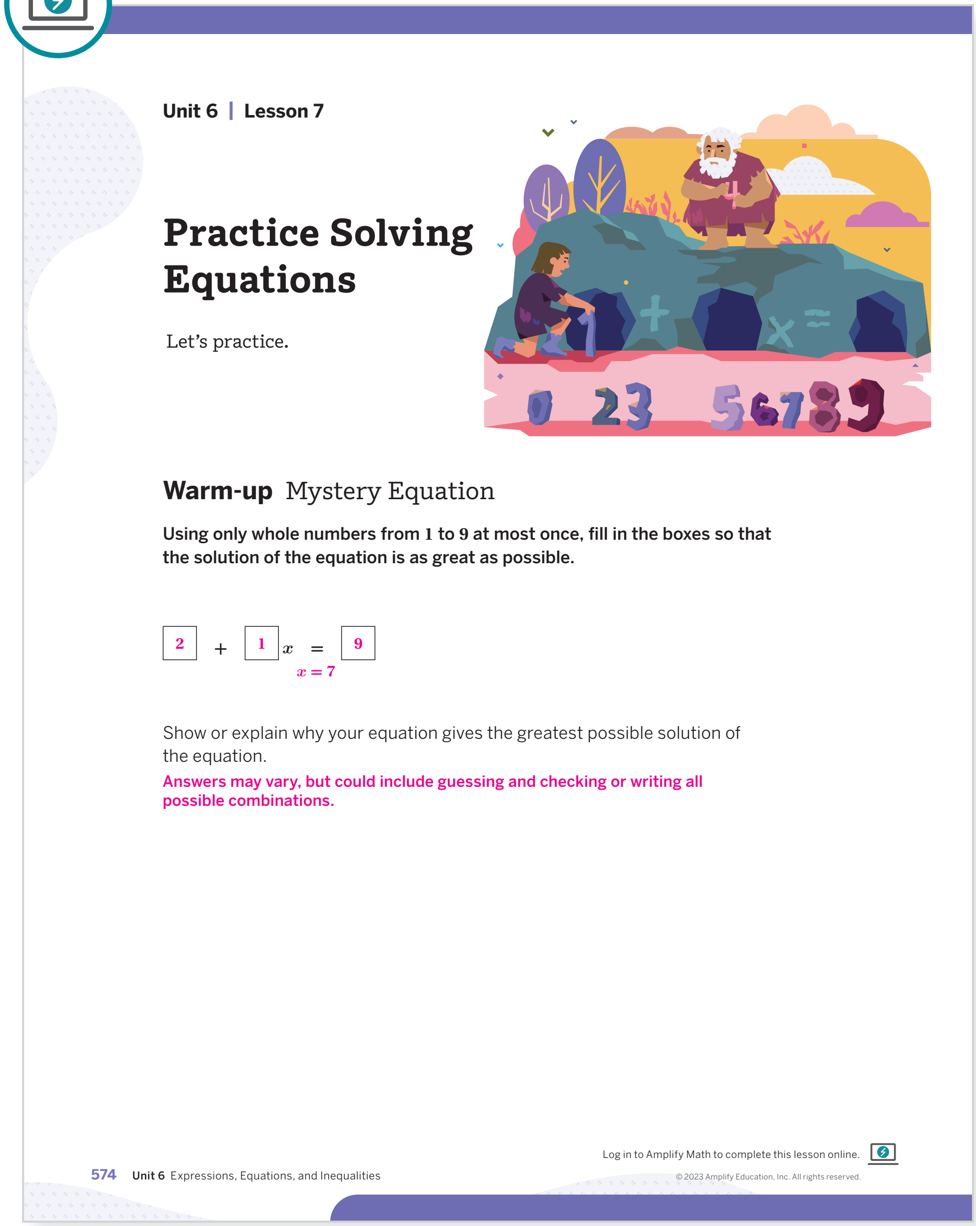
You may want to consider these additional modifications if you are short on time.

- Optional **Activity 1** might be omitted.
- Have pairs of students swap only one equation in **Activity 2**. Provide an opportunity for student choice, allowing pairs to decide whether they would rather use Equation 1 or Equation 2.

Independent | 5 min

Warm-up Mystery Equation

Students use specific values to create an equation allowing the solution to be the largest possible number. This task helps students further develop their understanding of equations and number sense.



Unit 6 | Lesson 7

Practice Solving Equations

Let's practice.

Warm-up Mystery Equation

Using only whole numbers from 1 to 9 at most once, fill in the boxes so that the solution of the equation is as great as possible.

$\boxed{2} + \boxed{1}x = \boxed{9}$

$x = 7$

Show or explain why your equation gives the greatest possible solution of the equation.

Answers may vary, but could include guessing and checking or writing all possible combinations.

574 **Unit 6** Expressions, Equations, and Inequalities

Log in to Amplify Math to complete this lesson online.

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1 Launch

Be sure students understand that they can only use each digit one time. They may use the space at the bottom of the page to show their thinking.

2 Monitor

Help students get started by saying, "Place the digits in the boxes and solve for x. Once you reach the solution, rearrange the digits to see whether the solution is greater."

Look for productive strategies:

- Using the guess-and-check strategy. This is a valid problem solving technique and should be encouraged.

3 Connect

Display the mystery equation.

Have students share their strategies to test equations and/or for knowing when an equation was not going to produce a greater solution.

Highlight the correct arrangement to yield the greatest solution.

Ask, "Is there anything interesting about the arrangement of the digits?" Sample response: The coefficient is the smallest value.

Power-up

To power up students' ability to solve equations with more than one step, have students complete:

Match each equation with the most efficient first step for solving it.

c $2x + 1 = 8$
b $2(x + 1) = 8$
a $2x - 1 = 8$
d $2x - 2 = 8$

a. Add 1 to both sides.
b. Divide by 2 both sides.
c. Subtract 1 from both sides.
d. Add 2 to both sides.

Use: Before the Warm-up
Informed by: Performance on Lesson 6, Practice Problem 6

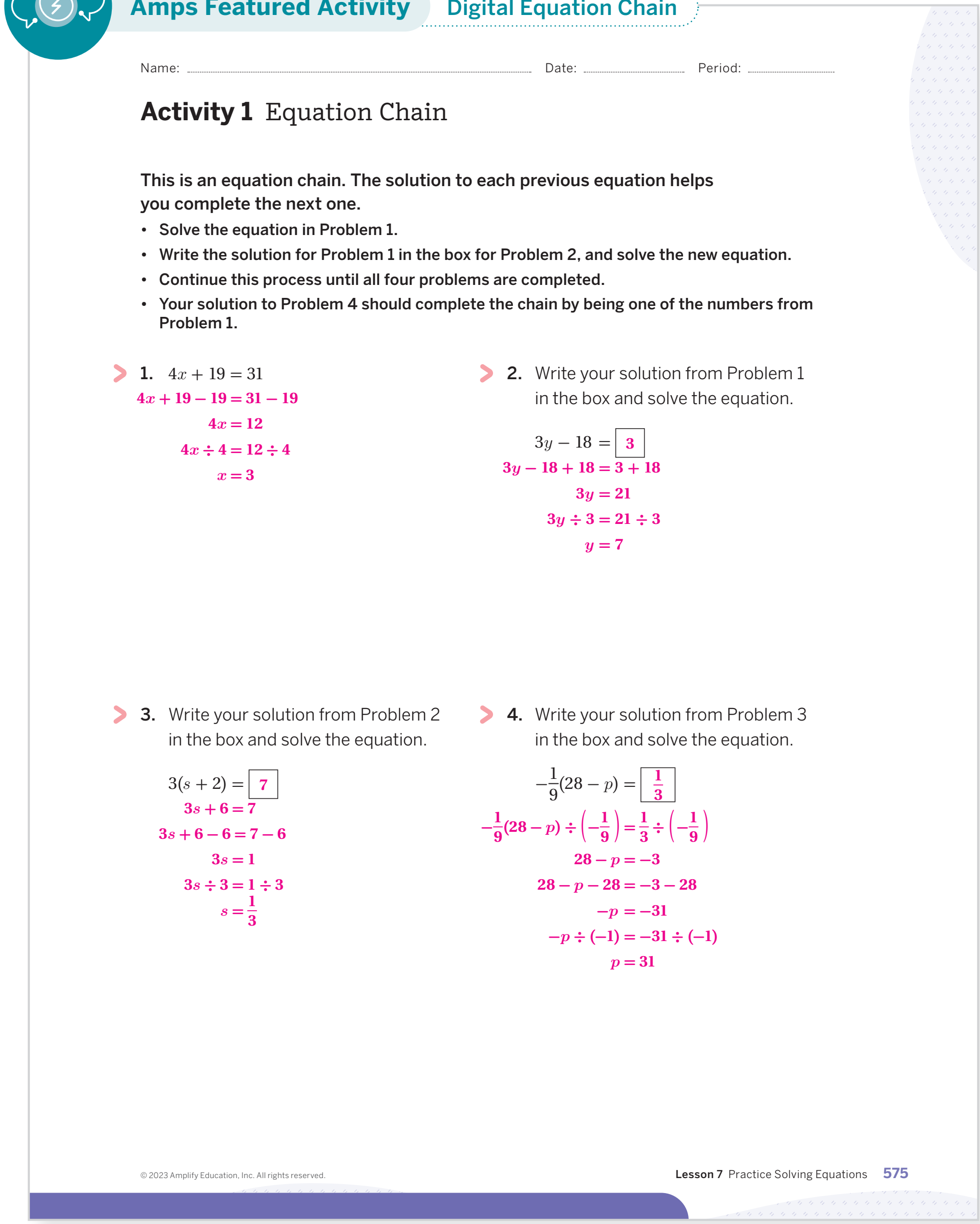
Optional

Independent | 15 min

Activity 1 Equation Chain

Students solve equations using the solution from the previous equation to help them solve the next one.

Amps Featured Activity Digital Equation Chain



Name: ____________ Date: ________ Period: ________

Activity 1 Equation Chain

This is an equation chain. The solution to each previous equation helps you complete the next one.

- **Solve the equation in Problem 1.**
- **Write the solution for Problem 1 in the box for Problem 2, and solve the new equation.**
- **Continue this process until all four problems are completed.**
- **Your solution to Problem 4 should complete the chain by being one of the numbers from Problem 1.**

1. $4x + 19 = 31$

$4x + 19 - 19 = 31 - 19$

$4x = 12$

$4x \div 4 = 12 \div 4$

$x = 3$

2. Write your solution from Problem 1 in the box and solve the equation.

$3y - 18 = \boxed{3}$

$3y - 18 + 18 = 3 + 18$

$3y = 21$

$3y \div 3 = 21 \div 3$

$y = 7$

3. Write your solution from Problem 2 in the box and solve the equation.

$3(s + 2) = \boxed{7}$

$3s + 6 = 7$

$3s + 6 - 6 = 7 - 6$

$3s = 1$

$3s \div 3 = 1 \div 3$

$s = \frac{1}{3}$

4. Write your solution from Problem 3 in the box and solve the equation.

$-\frac{1}{9}(28 - p) = \boxed{\frac{1}{3}}$

$-\frac{1}{9}(28 - p) \div \left(-\frac{1}{9}\right) = \frac{1}{3} \div \left(-\frac{1}{9}\right)$

$28 - p = -3$

$28 - p - 28 = -3 - 28$

$-p = -31$

$-p \div (-1) = -31 \div (-1)$

$p = 31$

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Lesson 7 Practice Solving Equations 575

1 Launch

Have students check their solutions to each equation before moving on to the next equation.

2 Monitor

Help students get started by asking, "What is the first step in solving the equation in Problem 1?" or saying, "Once you find the solution, check it to make sure it works. Then write it in the box for Problem 2."

Look for points of confusion:

- **Solving the equations incorrectly.** Show students how to check their solution by substituting the value for x in the original equation and evaluating it. The chain only works if the correct solutions are used.
- **Difficulty solving the last equation.** Walk through the process step by step, reminding students when they divide by a fraction, they multiply by its reciprocal.

3 Connect

Display the solutions to the equations.

Ask, "What are some ways you can make sure you complete the equation chain?"

Have students share their solution methods if another student requests a problem to be shown.

Highlight there are multiple ways to solve equations, such as the ones in Problems 3 and 4. Help them to remember that they can divide first or distribute first. In Problem 4, students might decide to multiply by 9 first to help alleviate the fractions.

Differentiated Support

Accessibility: ***Optimize Access to Technology***

Have students use the Amps slides for this activity, in which their responses from previous slides populate the corresponding spaces on the next screen.

Extension: ***Math Enrichment***

Have students refer to Problem 4 and ask them how the equation solving process would be different if they first multiplied each side by 9 versus −9.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the directions for the Equation Chain.

- **Read 1:** Students should understand that the solution to each equation goes in the box for the next equation.
- **Read 2:** Ask students to identify which equations are written in the form $px + q = r$ and which are written in the form $p(x + q) = r$.
- **Read 3:** Ask students to brainstorm what strategies they can use to solve the equation in Problem 1.

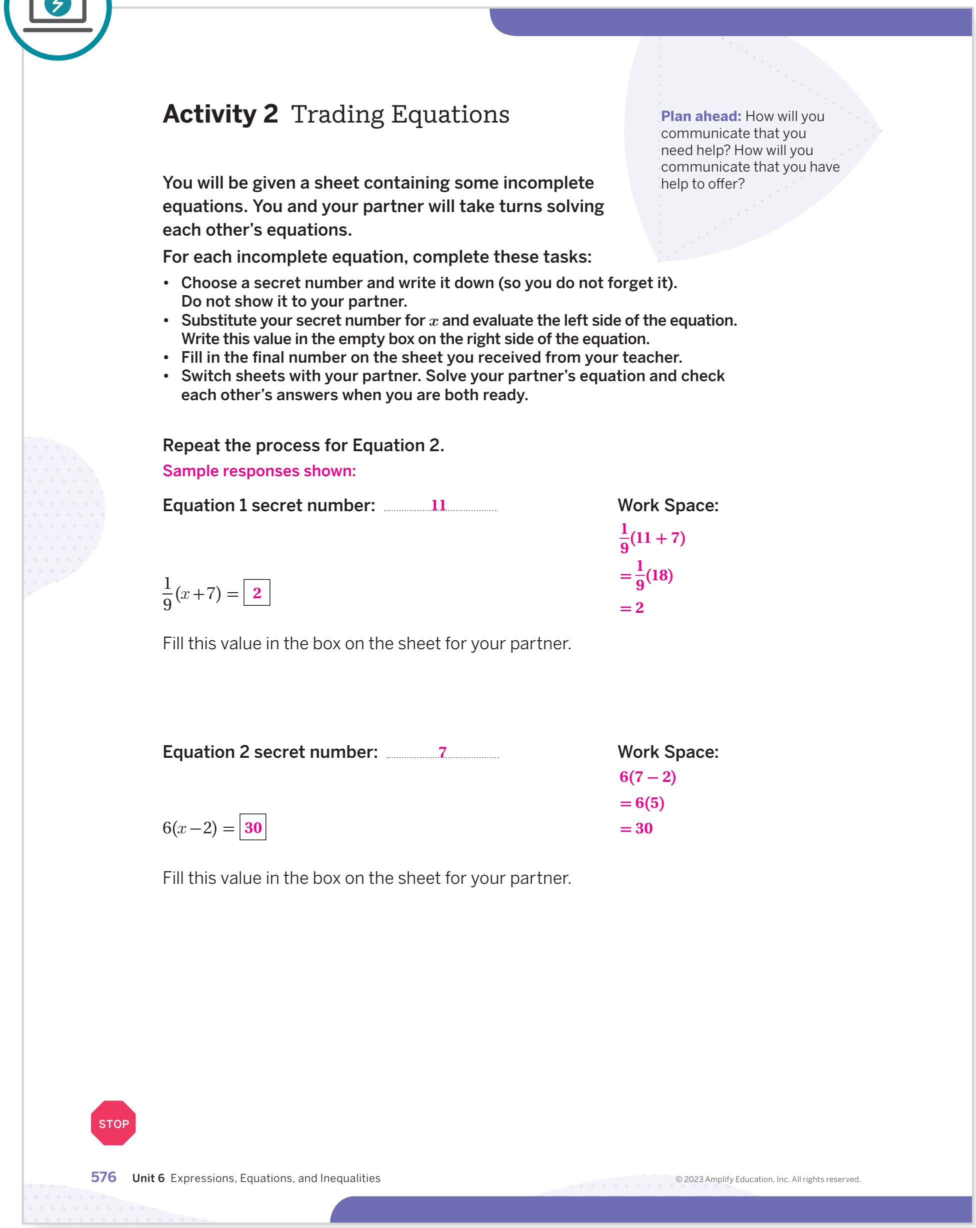
English Learners

Have students annotate the box in each of the equations in Problems 2–4 with "solution from Problem _____" to help them understand the Equation Chain directions.

Pairs | 30 min

Activity 2 Trading Equations

Students continue to practice solving equations, this time by creating equations with a certain solution, and then trading and solve equations with a partner.



Activity 2 Trading Equations

Plan ahead: How will you communicate that you need help? How will you communicate that you have help to offer?

You will be given a sheet containing some incomplete equations. You and your partner will take turns solving each other's equations.

For each incomplete equation, complete these tasks:

- Choose a secret number and write it down (so you do not forget it). Do not show it to your partner.
- Substitute your secret number for x and evaluate the left side of the equation. Write this value in the empty box on the right side of the equation.
- Fill in the final number on the sheet you received from your teacher.
- Switch sheets with your partner. Solve your partner's equation and check each other's answers when you are both ready.

Repeat the process for Equation 2.

Sample responses shown:

Equation 1 secret number: 11

$\frac{1}{9}(x+7) = \boxed{2}$

Work Space:

$\frac{1}{9}(11+7)$

$=\frac{1}{9}(18)$

$=2$

Fill this value in the box on the sheet for your partner.

Equation 2 secret number: 7

$6(x-2) = \boxed{30}$

Work Space:

$6(7-2)$

$=6(5)$

$=30$

Fill this value in the box on the sheet for your partner.

STOP

576 Unit 6 Expressions, Equations, and Inequalities

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1 Launch

Distribute the Activity 2 PDF. Review the directions with the whole class, and check for understanding about the order of the steps.

2 Monitor

Help students get started by demonstrating the procedures using your own secret number.

Look for points of confusion:

- **Evaluating their own expression incorrectly.** This will leave them with an incorrect "solution" to their equation. If this happens, encourage students to discuss and prove to their partner where an error was made.
- **Taking vastly different times to solve their equations.** Suggest partners agree on a set of numbers to use as their solution, e.g. integers, fractions.

3 Connect

Have students share with a partner, something they realized about equations after doing this activity.

Display an equation that has a negative solution.

Ask, "What clues do you have that the solution may be a negative number?" You know that a negative times a positive will give a negative value.

Highlight that sometimes an equation can reveal clues about its solution even before students solve it. The signs of numbers can be one of those clues.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students choose a secret number that simplifies the equation. For example, ask, "If you chose the number 11, what would the sum inside the parentheses be? What is $\frac{1}{9}$ of that sum?"

Extension: *Math Enrichment*

Ask students to challenge themselves to use more complicated secret numbers, such as decimals, fractions, or multi-digit numbers.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the directions for the activity.

- **Read 1:** Students should understand that they will choose a secret number for the solution and their partner will solve the equation to determine the solution.
- **Read 2:** Ask students to identify whether the equations are written in the form $px + q = r$ or $p(x + q) = r$.
- **Read 3:** Ask students to brainstorm what strategies they can use to determine what secret number they will use in Equation 1.

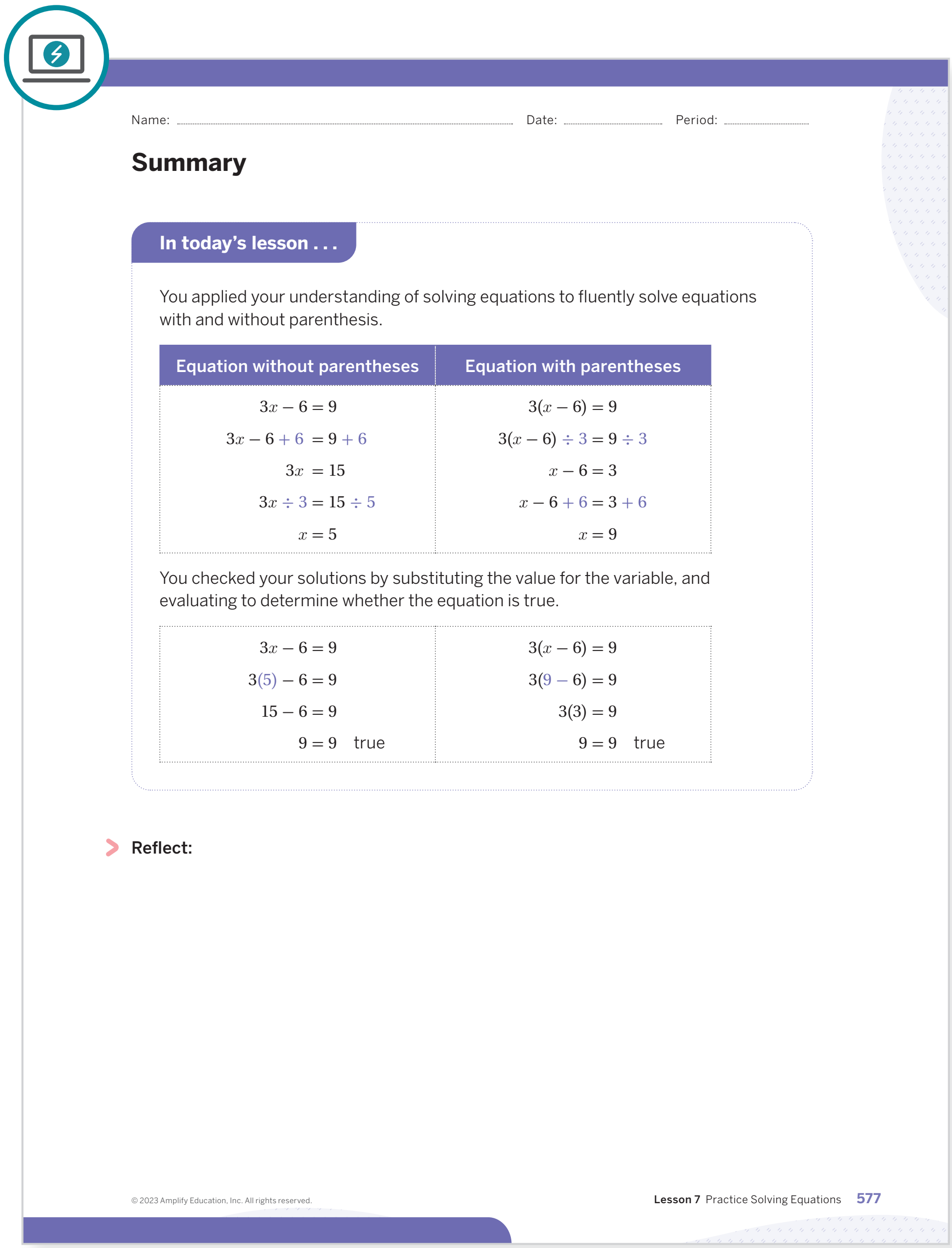
English Learners

Consider demonstrating choosing a secret number, such as 1 and show how to substitute it into the equation to determine the number that would go in the box.

Whole Class | 5 min

Summary

Review and synthesize different methods for solving equations of the forms $px + q = r$ and $p(x + q) = r$.



Name: Date: Period:

Summary

In today's lesson . . .

You applied your understanding of solving equations to fluently solve equations with and without parenthesis.

Equation without parentheses	Equation with parentheses
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 5$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

You checked your solutions by substituting the value for the variable, and evaluating to determine whether the equation is true.

$3x - 6 = 9$ $3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ true	$3(x - 6) = 9$ $3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ true

Reflect:

Synthesize

Display the equations $px + q = r$ and $p(x + q) = r$.

Have students share with a partner using a *turn-and-talk* routine. Have one student explain to their partner how to solve the first equation for x. The other student explains how to solve the second equation for x.

Highlight any strategies or statements presented by students which would benefit the class. Discuss options for the second equation: depending on the values of p, q, and r, students may want to divide by p first or distribute p.

Ask, "Did anyone's partner explain it in a way that made sense to you? Can you share any new ways of explaining which might benefit the whole class?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

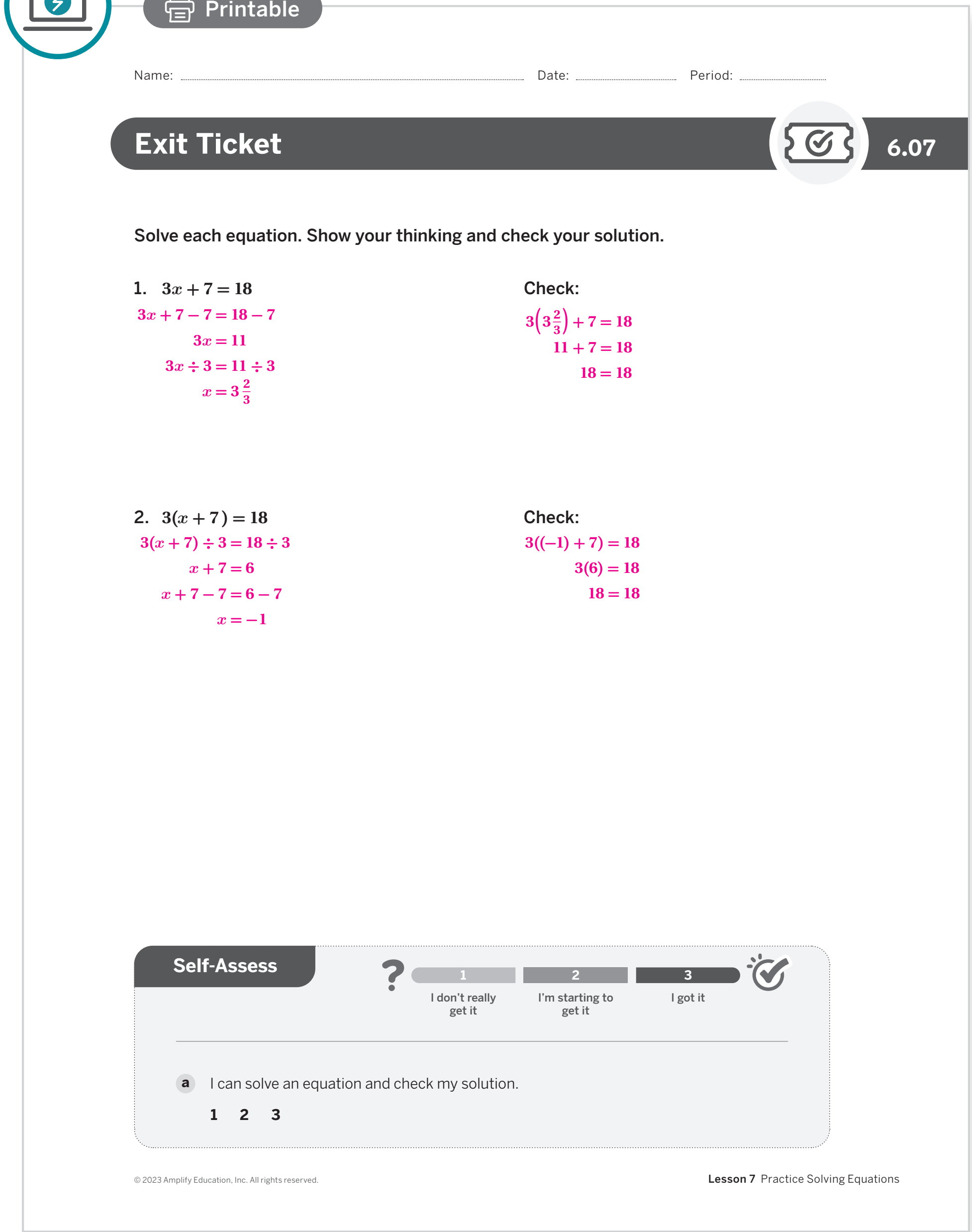
- "How can you increase your efficiency in solving mathematical problems?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving two equations, one of the form $px + q = r$ and the other of the form $p(x + q) = r$.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.07

Solve each equation. Show your thinking and check your solution.

1. $3x + 7 = 18$

$3x + 7 - 7 = 18 - 7$

$3x = 11$

$3x \div 3 = 11 \div 3$

$x = 3\frac{2}{3}$

Check:

$3\left(3\frac{2}{3}\right) + 7 = 18$

$11 + 7 = 18$

$18 = 18$

2. $3(x + 7) = 18$

$3(x + 7) \div 3 = 18 \div 3$

$x + 7 = 6$

$x + 7 - 7 = 6 - 7$

$x = -1$

Check:

$3((-1) + 7) = 18$

$3(6) = 18$

$18 = 18$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can solve an equation and check my solution.

1 2 3

Lesson 7 Practice Solving Equations

Success looks like . . .

- **Goal:** Solving equations of the forms $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers.
  - » Solving each caution in Problems 1 and 2.

Suggested next steps

If students make computational errors, consider:

- Reminding them to check their solutions as they work to notice errors as soon as possible.

If there are errors in understanding the steps for solving equations of the form $px + q = r$, consider:

- Assigning Practice Problem 1.

If there are errors in understanding the steps for solving equations of the form $p(x + q) = r$, consider:

- Assigning Practice Problem 2.

Professional Learning

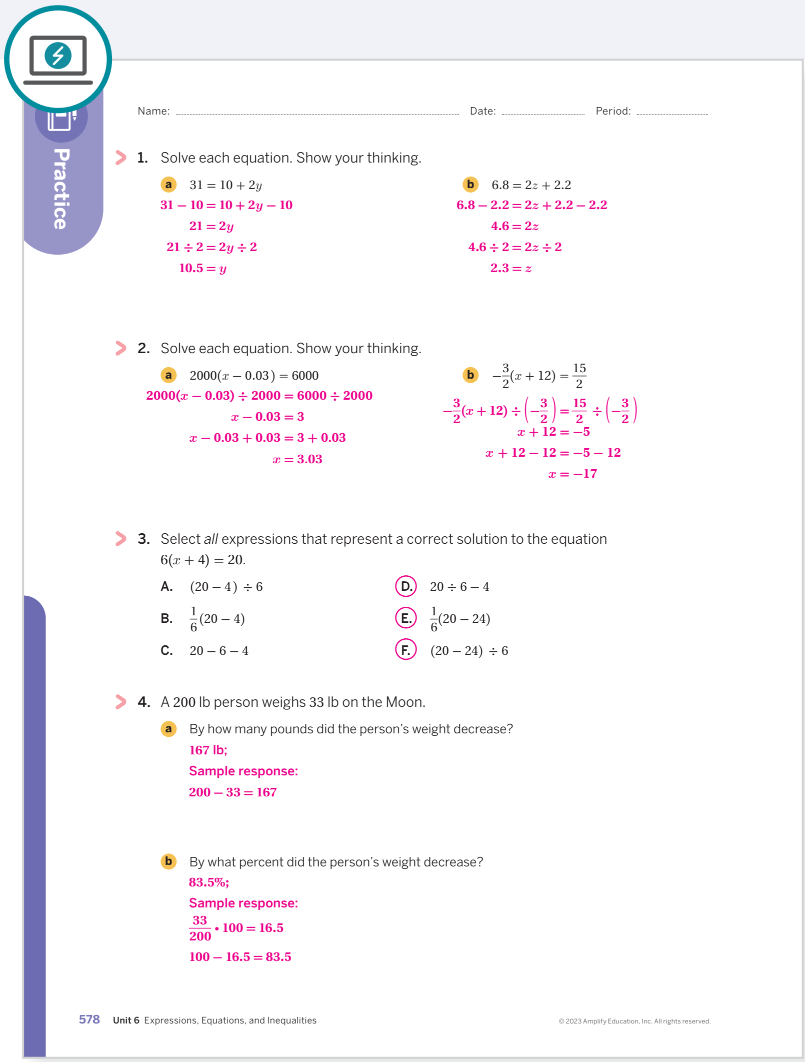
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and did not have their ideas seen and heard today?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Solve each equation. Show your thinking.

a $31 = 10 + 2y$

$31 - 10 = 10 + 2y - 10$

$21 = 2y$

$21 \div 2 = 2y \div 2$

$10.5 = y$

b $6.8 = 2z + 2.2$

$6.8 - 2.2 = 2z + 2.2 - 2.2$

$4.6 = 2z$

$4.6 \div 2 = 2z \div 2$

$2.3 = z$

2. Solve each equation. Show your thinking.

a $2000(x - 0.03) = 6000$

$2000(x - 0.03) \div 2000 = 6000 \div 2000$

$x - 0.03 = 3$

$x - 0.03 + 0.03 = 3 + 0.03$

$x = 3.03$

b $-\frac{3}{2}(x + 12) = \frac{15}{2}$

$-\frac{3}{2}(x + 12) \div \left(-\frac{3}{2}\right) = \frac{15}{2} \div \left(-\frac{3}{2}\right)$

$x + 12 = -5$

$x + 12 - 12 = -5 - 12$

$x = -17$

3. Select *all* expressions that represent a correct solution to the equation $6(x + 4) = 20$.

A. $(20 - 4) \div 6$

B. $\frac{1}{6}(20 - 4)$

C. $20 - 6 - 4$

[✓ D.] $20 \div 6 - 4$

[✓ E.] $\frac{1}{6}(20 - 24)$

[✓ F.] $(20 - 24) \div 6$

4. A 200 lb person weighs 33 lb on the Moon.

a By how many pounds did the person's weight decrease?

167 lb;

Sample response:

$200 - 33 = 167$

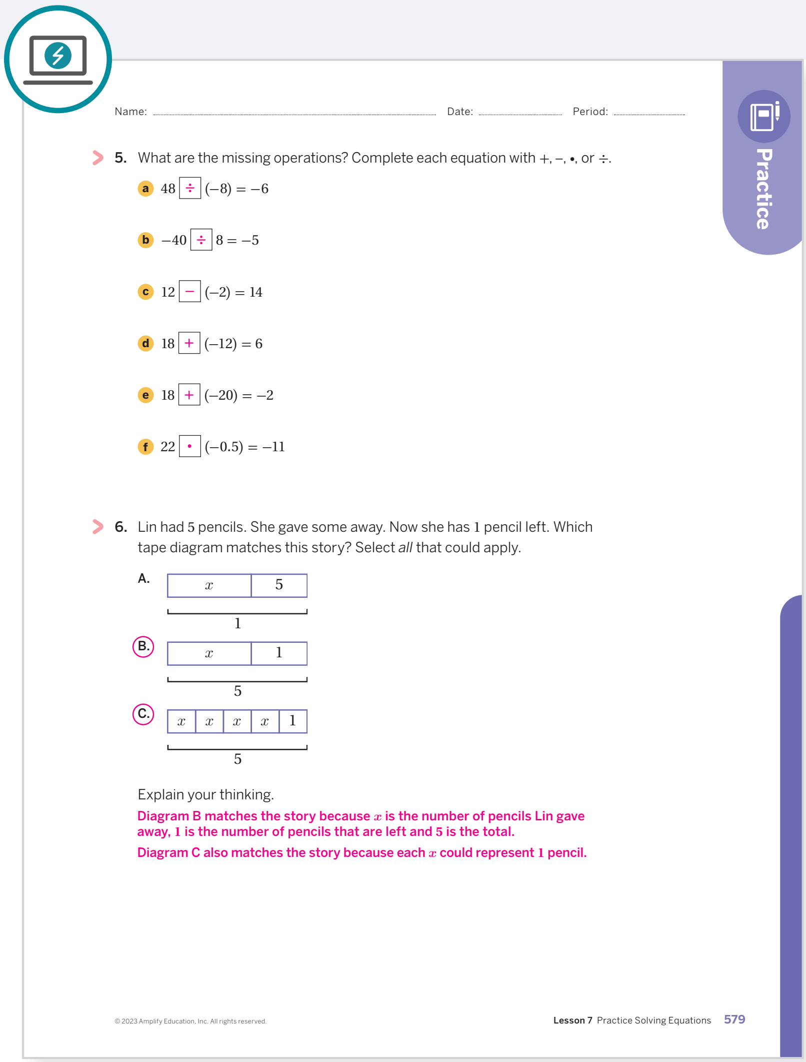
b By what percent did the person's weight decrease?

83.5%;

Sample response:

$\frac{33}{200} \cdot 100 = 16.5$

$100 - 16.5 = 83.5$



Name: ______ Date: ______ Period: ______

Practice

5. What are the missing operations? Complete each equation with +, –, •, or ÷.

a $48 \boxed{\div} (-8) = -6$

b $-40 \boxed{\div} 8 = -5$

c $12 \boxed{-} (-2) = 14$

d $18 \boxed{+} (-12) = 6$

e $18 \boxed{+} (-20) = -2$

f $22 \boxed{\cdot} (-0.5) = -11$

6. Lin had 5 pencils. She gave some away. Now she has 1 pencil left. Which tape diagram matches this story? Select *all* that could apply.

A.

[✓ B.]

[✓ C.]

Explain your thinking.

Diagram B matches the story because x is the number of pencils Lin gave away, 1 is the number of pencils that are left and 5 is the total.

Diagram C also matches the story because each x could represent 1 pencil.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	1
	2	Activities 1 and 2	1
	3	Activities 1 and 2	2
Spiral	4	Unit 4 Lesson 3	1
	5	Unit 5 Lesson 17	2
Formative ⚡	6	Unit 6 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

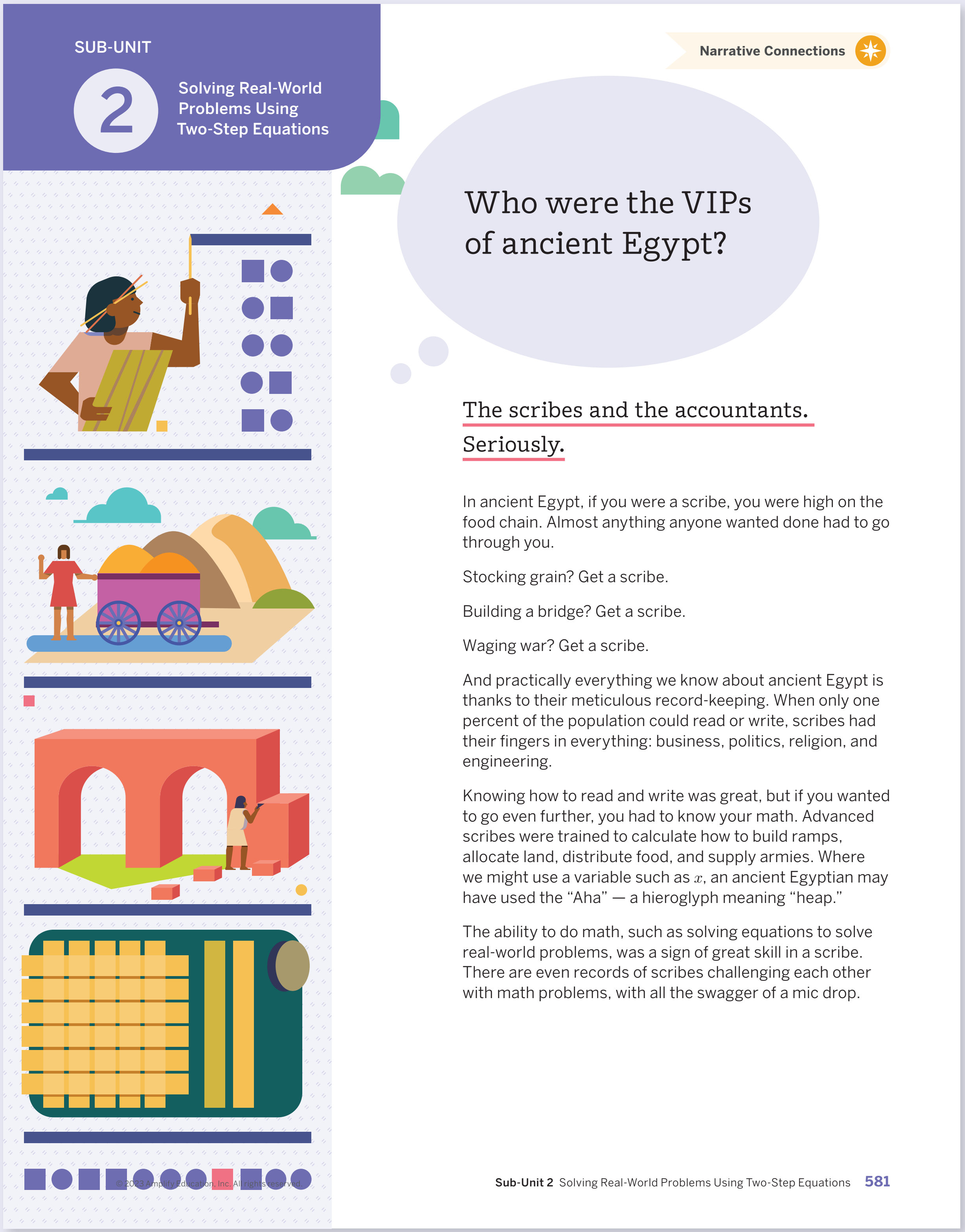
For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 2

Whole Class

Solving Real-World Problems Using Two-Step Equations

In this Sub-Unit, students interpret situations with various quantitative relationships and see that they can be modeled with algebraic expressions and equations.



SUB-UNIT

2 Solving Real-World Problems Using Two-Step Equations

Narrative Connections

Who were the VIPs of ancient Egypt?

The scribes and the accountants. Seriously.

In ancient Egypt, if you were a scribe, you were high on the food chain. Almost anything anyone wanted done had to go through you.

Stocking grain? Get a scribe.

Building a bridge? Get a scribe.

Waging war? Get a scribe.

And practically everything we know about ancient Egypt is thanks to their meticulous record-keeping. When only one percent of the population could read or write, scribes had their fingers in everything: business, politics, religion, and engineering.

Knowing how to read and write was great, but if you wanted to go even further, you had to know your math. Advanced scribes were trained to calculate how to build ramps, allocate land, distribute food, and supply armies. Where we might use a variable such as x, an ancient Egyptian may have used the "Aha" — a hieroglyph meaning "heap."

The ability to do math, such as solving equations to solve real-world problems, was a sign of great skill in a scribe. There are even records of scribes challenging each other with math problems, with all the swagger of a mic drop.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to use equations to represent stories (albeit, not in ancient Egypt) in the following places:

- **Lesson 8, Activity 1:** Every Picture Tells a Story
- **Lesson 11, Activity 2:** Science Club
- **Lesson 12, Activity 2:** Selling Shoes

Reasoning With Tape Diagrams

Let's use tape diagrams to write equations for different scenarios.

Focus

Goals

1. Select a tape diagram to represent relationships between quantities in a situation.
2. Coordinate tape diagrams and equations of the form $px + q = r$ or $p(x + q) = r$.
3. **Language Goal:** Identify equivalent equations and justify that they are equivalent. **(Speaking and Listening, Writing)**

Coherence

Today

Students use tape diagrams and equations of the form $px + q = r$ and $p(x + q) = r$ to represent relationships in real-world scenarios.

Previously

In Unit 5, students wrote and solved one-step equations to represent real-world scenarios involving rational numbers.

Coming Soon

In the next few lessons, students will continue to work with tape diagrams and two-step equations to represent and eventually solve real-world problems.

Rigor

- Students use tape diagrams to build **conceptual understanding** of writing equations from verbal descriptions of the form $px + q = r$ and $p(x + q) = r$.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	7 min	13 min	5 min	5 min
Pairs	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one per group
- Activity 2 PDF, *Are you ready for more?*, pre-cut cards, one per group (as needed)
- colored pencils, markers, or highlighters (as needed)
- glue or tape

Math Language Development

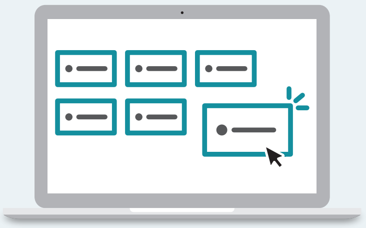
Review words

- *equivalent equations*
- *variable*

Amps | **Featured Activity**

Activity 2
Digital Card Sort

Students group tape diagrams into two categories of their choosing by dragging and connecting them on screen.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students try to sort and match tape diagrams in Activities 2–3, they might talk over others in their group, particularly if they are excited about their choice and the justification. Prior to starting the lesson, ask students to set some group rules for how they will communicate clearly and effectively. They also need to determine how they will communicate to someone who is not following those guidelines.

Modifications to Pacing

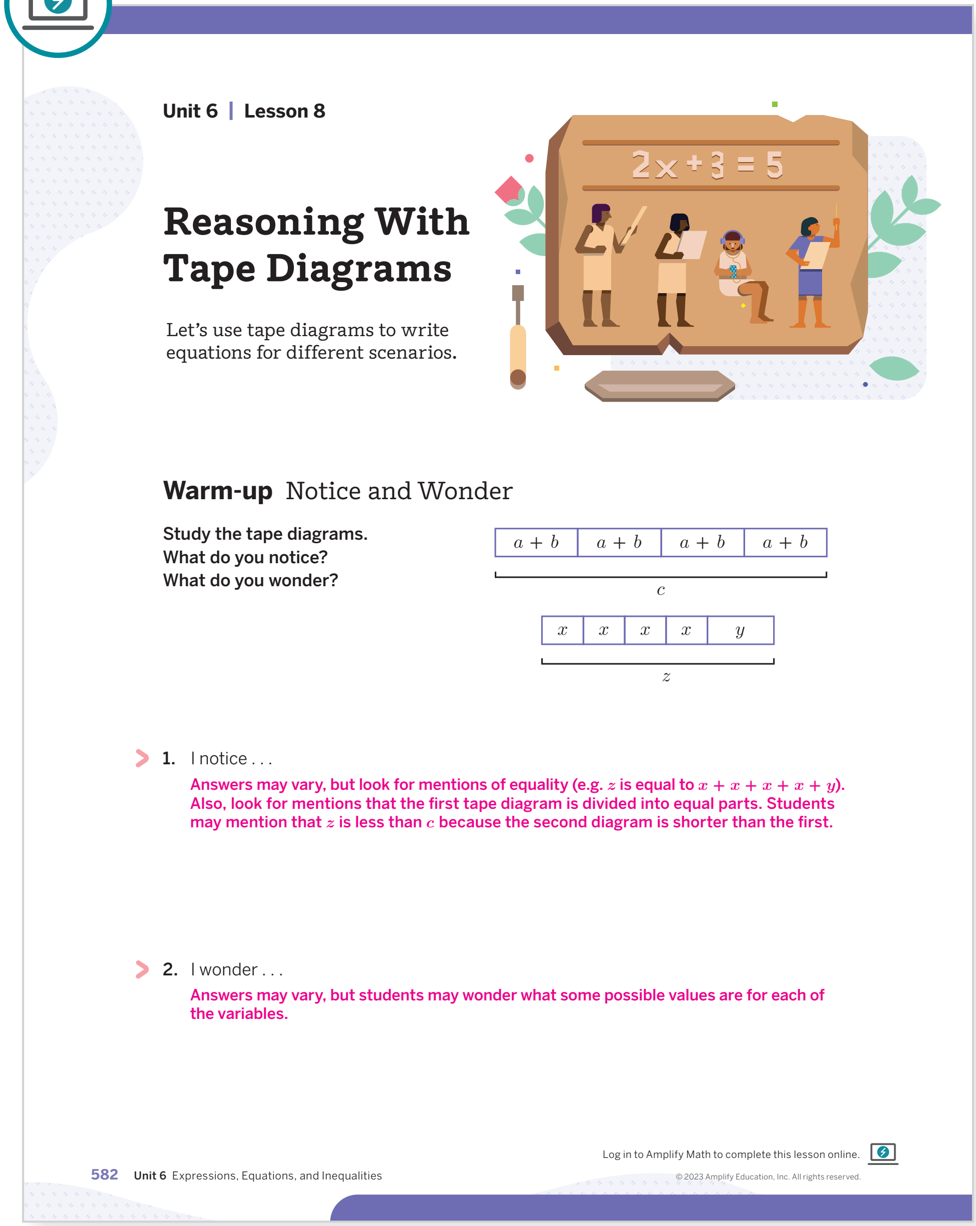
You may want to consider this additional modification if you are short on time.

- **Activity 2** may be omitted. Consider assigning the activity as additional practice using the digital card sort.

Pairs | 5 min

Warm-up Notice and Wonder

Students are re-introduced to tape diagrams as representations of relationships between quantities in preparation for using them to write equations that represent and solve real-world problems.



Unit 6 | **Lesson 8**

Reasoning With Tape Diagrams

Let's use tape diagrams to write equations for different scenarios.

Warm-up Notice and Wonder

Study the tape diagrams. What do you notice? What do you wonder?

$$\underbrace{\boxed{a+b}\boxed{a+b}\boxed{a+b}\boxed{a+b}}_{c}$$

$$\underbrace{\boxed{x}\boxed{x}\boxed{x}\boxed{x}\boxed{\;y\;}}_{z}$$

1. I notice . . .

   Answers may vary, but look for mentions of equality (e.g. z is equal to $x + x + x + x + y$). Also, look for mentions that the first tape diagram is divided into equal parts. Students may mention that z is less than c because the second diagram is shorter than the first.

2. I wonder . . .

   Answers may vary, but students may wonder what some possible values are for each of the variables.

Log in to Amplify Math to complete this lesson online.

582 **Unit 6** Expressions, Equations, and Inequalities

1 Launch

Conduct the ***Notice and Wonder*** routine. Set an expectation for the amount of time students have to work in pairs on the activity. Provide students some time to think independently before sharing what they notice with their partner.

2 Monitor

Look for points of confusion:

- **Choosing values for the variables that don't match what is shown in the tape diagrams.** Ask students to be sure that the values they chose describe the relationships shown in the tape diagrams.

Look for productive strategies:

- Starting from the total (c and z) and selecting appropriate values for a and b and, x and y. If students are struggling with this strategy, suggest they choose values for the other variables (a and b, and x and y) and use them to calculate the total.

3 Connect

Have students share what they noticed and wondered about the tape diagrams. Ask a few students to share the values they chose for the variables in each diagram.

Highlight that sections or parts labeled with the same variable or expression have the same value. Show how to check that the chosen values are valid by substituting them into the diagram and checking that total equals the sum of the parts.

Ask, "How does a tape diagram show that two values are equal? Which values in each diagram are equal?"

Power-up

To power up students' ability to determine when a tape diagram represents a scenario, have students complete:

Determine which scenarios are represented by the tape diagram. Select *all* that apply.

$$\underbrace{\boxed{\quad x \quad}\boxed{\;2\;}}_{5}$$

[✓ **A.**] Andre had 5 carrot sticks. He gave some away and had 2 left.

[✓ **B.**] Diego had 2 stickers. Jada gave him some more stickers and now he has 5.

C. Noah had 5 erasers. He gave some away and had 3 left.

D. Priya had 2 markers. Han gave her 5 more.

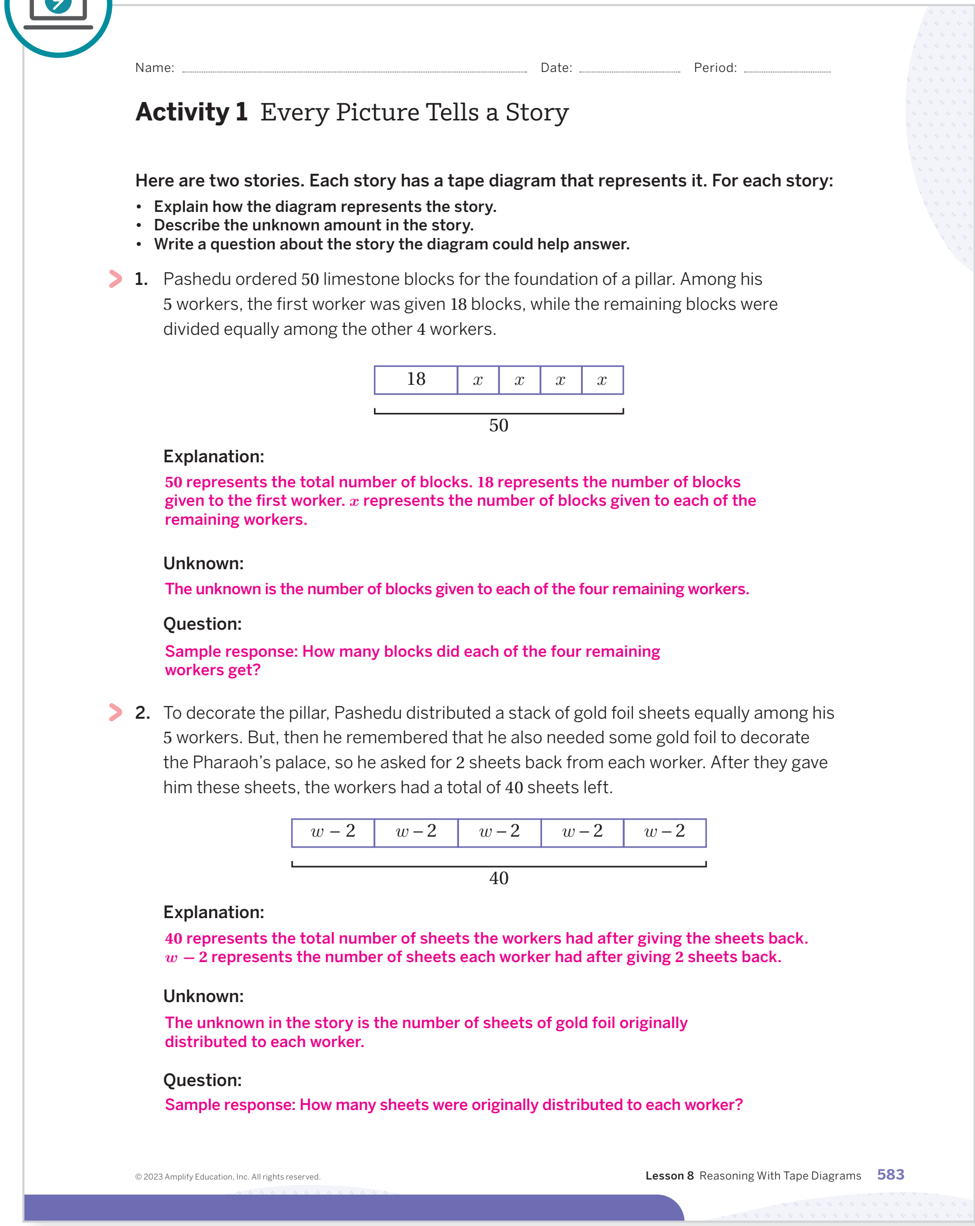
Use: Before the Warm-up

Informed by: Performance on Lesson 7, Practice Problem 6. and Pre-Unit Readiness Assessment, Problem 6

Small Groups | 10 min

Activity 1 Every Picture Tells a Story

Students analyze how a tape diagram can describe a story. Then they use the tape diagram to formulate a question about the story.



Name: ______________ Date: ________ Period: ________

Activity 1 Every Picture Tells a Story

Here are two stories. Each story has a tape diagram that represents it. For each story:

- **Explain how the diagram represents the story.**
- **Describe the unknown amount in the story.**
- **Write a question about the story the diagram could help answer.**

1. Pashedu ordered 50 limestone blocks for the foundation of a pillar. Among his 5 workers, the first worker was given 18 blocks, while the remaining blocks were divided equally among the other 4 workers.

18	x	x	x	x

50

Explanation:

50 represents the total number of blocks. 18 represents the number of blocks given to the first worker. x represents the number of blocks given to each of the remaining workers.

Unknown:

The unknown is the number of blocks given to each of the four remaining workers.

Question:

Sample response: How many blocks did each of the four remaining workers get?

2. To decorate the pillar, Pashedu distributed a stack of gold foil sheets equally among his 5 workers. But, then he remembered that he also needed some gold foil to decorate the Pharaoh's palace, so he asked for 2 sheets back from each worker. After they gave him these sheets, the workers had a total of 40 sheets left.

$w-2$	$w-2$	$w-2$	$w-2$	$w-2$

40

Explanation:

40 represents the total number of sheets the workers had after giving the sheets back. $w-2$ represents the number of sheets each worker had after giving 2 sheets back.

Unknown:

The unknown in the story is the number of sheets of gold foil originally distributed to each worker.

Question:

Sample response: How many sheets were originally distributed to each worker?

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Lesson 8 Reasoning With Tape Diagrams 583

1 Launch

Read the directions as a class, emphasizing that there are three tasks for each story.

2 Monitor

Look for points of confusion:

- **Struggling to understand the stories.** Discuss each story with students and make sure they know what each of these terms mean: limestone block, pillar, sheet of gold foil.
- **Suggesting alternate diagrams to describe the stories.** Explain that their task is to understand how these diagrams describe each story.
- **Struggling to make connections between the stories and the tape diagrams.** Provide students with colored pencils or highlighters and have them color code the connections between each story and the tape diagram that represents it.

3 Connect

Have students share their responses for each story. Ask if anyone thought about each diagram a different way (one additional line of reasoning for each diagram is probably sufficient) or came up with a different question.

Highlight how the diagrams represent each story by asking:

- "Where in the diagrams do you see equal parts? How do you know they are equal?"
- "For the first story, where in the diagram do you see the remaining blocks?"
- "Why don't you see the number 4 in the first diagram to show the 4 remaining workers?"
- "In the second diagram, where are the five workers represented?"
- "How did the diagrams help you to determine the value of the unknown quantities?"

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they worked with tape diagrams and used them to represent one-step equations in Grade 6. Consider showing a tape diagram that would represent the one-step equation $x + 3 = 10$ and ask students to explain how the tape diagram represents the equation.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the quantities in each story and how they are represented in the tape diagram. For example, in Problem 1, they could color code 50 limestone blocks in the story in one color, and use the same color to circle the total of 50 in the tape diagram.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight how the diagrams represent each story, use color coding to annotate the story and the diagram as students respond to the questions. For example, use one color to draw the outlines for each of the five rectangles labeled $w-2$ and use the same color to annotate 5 workers in the story.

English Learners

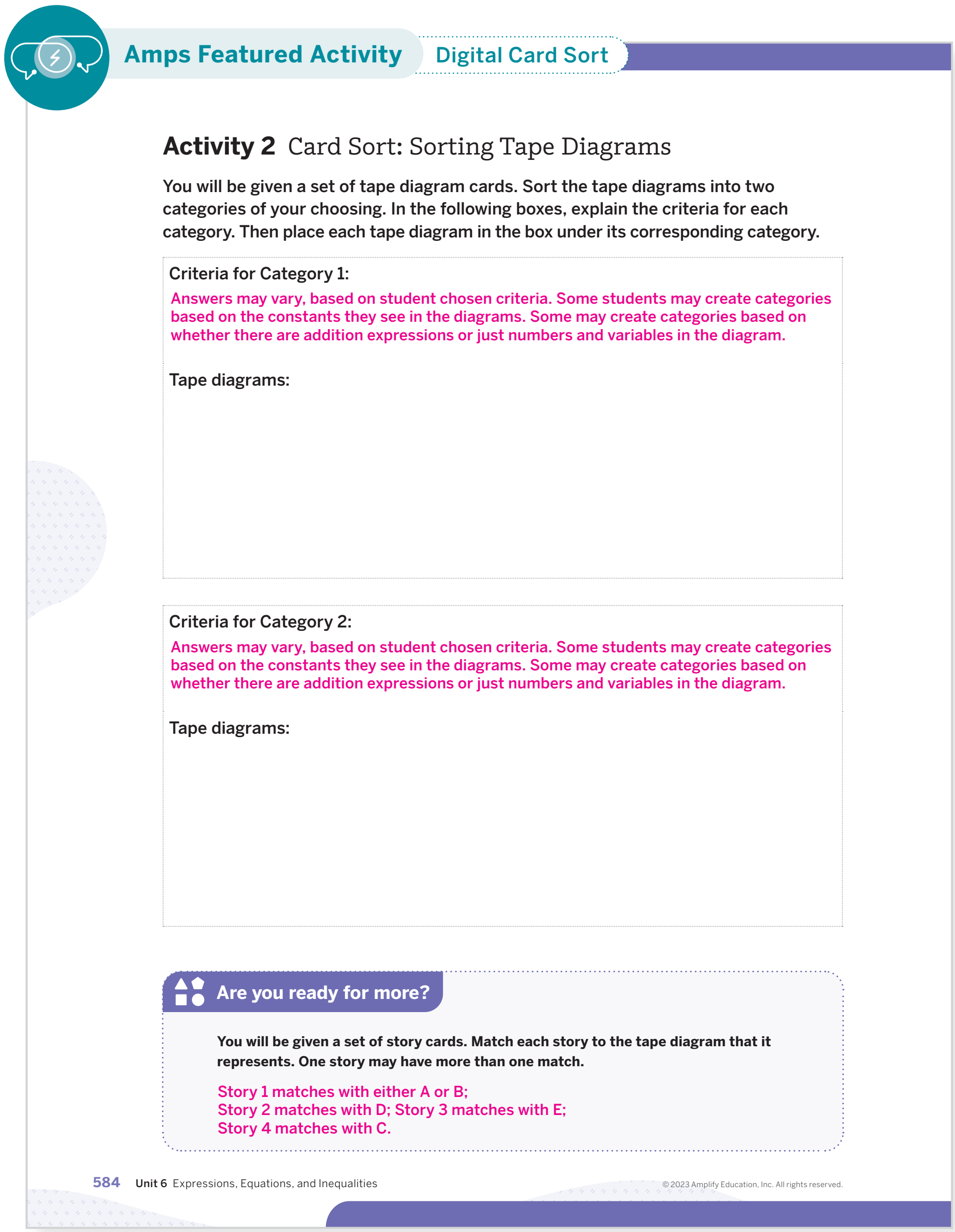
Show or draw images of what a pillar looks like, as well as blocks and sheets of gold foil, to help students access the words used in these stories.

Small Groups | 7 min

Activity 2 Card Sort: Sorting Tape Diagrams

Students sort tape diagrams into two categories of their choosing in preparation for identifying the two forms of equations they will use to solve real-world problems.

Amps Featured Activity Digital Card Sort



Activity 2 Card Sort: Sorting Tape Diagrams

You will be given a set of tape diagram cards. Sort the tape diagrams into two categories of your choosing. In the following boxes, explain the criteria for each category. Then place each tape diagram in the box under its corresponding category.

Criteria for Category 1:

Answers may vary, based on student chosen criteria. Some students may create categories based on the constants they see in the diagrams. Some may create categories based on whether there are addition expressions or just numbers and variables in the diagram.

Tape diagrams:

Criteria for Category 2:

Answers may vary, based on student chosen criteria. Some students may create categories based on the constants they see in the diagrams. Some may create categories based on whether there are addition expressions or just numbers and variables in the diagram.

Tape diagrams:

Are you ready for more?

You will be given a set of story cards. Match each story to the tape diagram that it represents. One story may have more than one match.

Story 1 matches with either A or B;
Story 2 matches with D; Story 3 matches with E;
Story 4 matches with C.

584 Unit 6 Expressions, Equations, and Inequalities

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1 Launch

Distributive a set of cards from Activity 2 PDF to each group, along with glue or tape so that students may attach them to the categories on their student page. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by asking them to identify what is the same and what is different between the tape diagrams.

Look for productive strategies:

- Creating categories based on the constants in each diagram. Note who uses this strategy.
- Creating categories distinguishing Diagrams A, B, and C from Diagrams D and E. Note who uses this strategy.

3 Connect

Display the set of tape diagrams.

Have students share the criteria for each of their categories. If possible, include a student with categories distinguishing Diagrams A, B, and C from Diagrams D and E.

Highlight that Diagrams D and E are each divided into equal parts, and the other diagrams are not divided into equal parts. However, Diagrams A and B represent the same relationship between x, 5, and 19. In other words the sum of two xs and 5 is 19 in each diagram.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can group tape diagrams into two categories of their choosing by dragging and connecting them on screen. By using the digital card sort, you can eliminate the prep work needed for this activity.

Accessibility: *Guide Processing and Visualization*

Consider providing some sample categories that students can think about how to sort the tape diagrams. For example, consider suggesting the following categories: Individual rectangles containing single numbers or variables versus the individual rectangles containing a sum of numbers and variables.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight how the diagrams compare, consider displaying a table similar to the following:

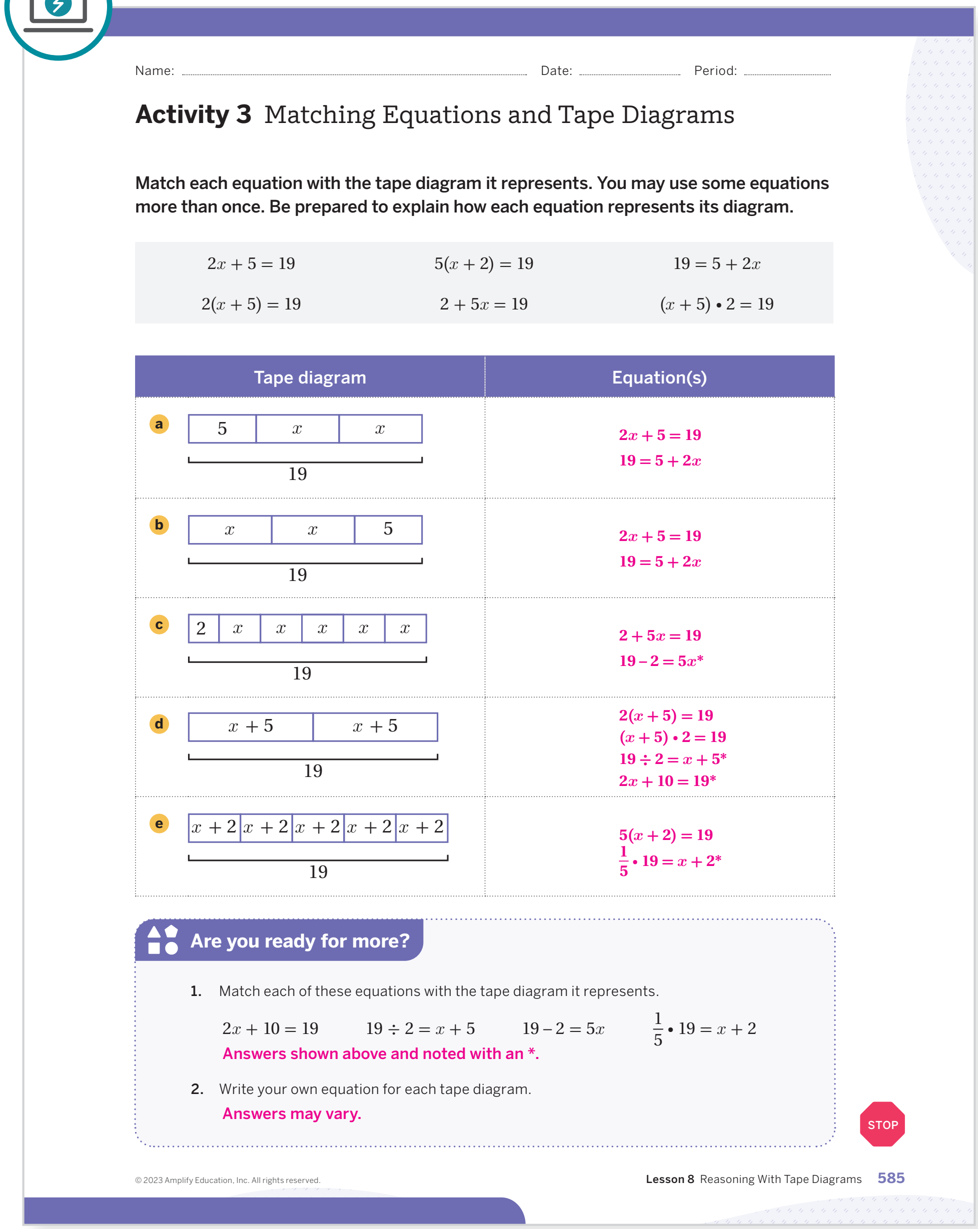
Divided into equal parts.	Diagrams D and E
Divided into equal parts.	Diagrams A, B, C
Represent the equation, $2x + 5 = 19$.	Diagrams A and B

Ask students to explain why Diagram D does not represent the equation $2x + 5 = 19$.

Small Groups | 13 min

Activity 3 Matching Equations and Tape Diagrams

Students match equations to tape diagrams to identify equivalent equations and prepare for writing equations from stories.



Name: ______ Date: ______ Period: ______

Activity 3 Matching Equations and Tape Diagrams

Match each equation with the tape diagram it represents. You may use some equations more than once. Be prepared to explain how each equation represents its diagram.

$2x + 5 = 19$	$5(x + 2) = 19$	$19 = 5 + 2x$
$2(x + 5) = 19$	$2 + 5x = 19$	$(x + 5) \cdot 2 = 19$

Tape diagram	Equation(s)
a 5 | x | x 19	$2x + 5 = 19$ $19 = 5 + 2x$
b x | x | 5 19	$2x + 5 = 19$ $19 = 5 + 2x$
c 2 | x | x | x | x | x 19	$2 + 5x = 19$ $19 - 2 = 5x$*
d $x + 5$ | $x + 5$ 19	$2(x + 5) = 19$ $(x + 5) \cdot 2 = 19$ $19 \div 2 = x + 5$* $2x + 10 = 19$*
e $x + 2$ | $x + 2$ | $x + 2$ | $x + 2$ | $x + 2$ 19	$5(x + 2) = 19$ $\frac{1}{5} \cdot 19 = x + 2$*

Are you ready for more?

1. Match each of these equations with the tape diagram it represents.

$2x + 10 = 19$ $19 \div 2 = x + 5$ $19 - 2 = 5x$ $\frac{1}{5} \cdot 19 = x + 2$

Answers shown above and noted with an *.

2. Write your own equation for each tape diagram.

Answers may vary.

STOP

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Lesson 8 Reasoning With Tape Diagrams 585

1 Launch

Tell students that they will match equations to the tape diagrams they categorized in Activity 2. Note that more than one equation may match a diagram and that some equations may match more than one diagram.

2 Monitor

Help students get started by encouraging them to describe the diagrams and equations in words. For example, the diagram in part d could be described as "two groups of $x + 5$ equal 19," which is represented by the equation $2(x + 5) = 19$.

Look for productive strategies:

- Matching the same equations to the diagrams in parts a and b. Note students who do this and ask them to explain their thinking during the class discussion.

3 Connect

Have students share how they matched the tape diagrams and equations, and ask if others agree or disagree with the matches. Include the equations from the *Are you ready for more?*, if any students completed those problems.

Highlight why the same equations can be paired with both diagrams in parts a and b, noting that these diagrams both show that 19 is equal to $2x + 5$, so the equations are *equivalent*. Explain when two equations can be written to describe the same tape diagram, those equations are equivalent. Ask students to write additional equivalent equations for each diagram.

Ask, "How do the categories you identified in Activity 2 relate to the different types of equations in this activity?" The diagrams in parts a, b, and c can all be modeled with equations of the form $px + q = r$ and the diagrams in parts d and e can both be modeled with equations of the form $p(x + q) = r$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the quantities in each equation and how they are represented in the tape diagram. Consider also suggesting that they circle the equations and tape diagrams that represent adding equal groups of a sum, such as the equation $2(x + 5) = 19$.

Math Language Development

MLR7: Compare and Connect

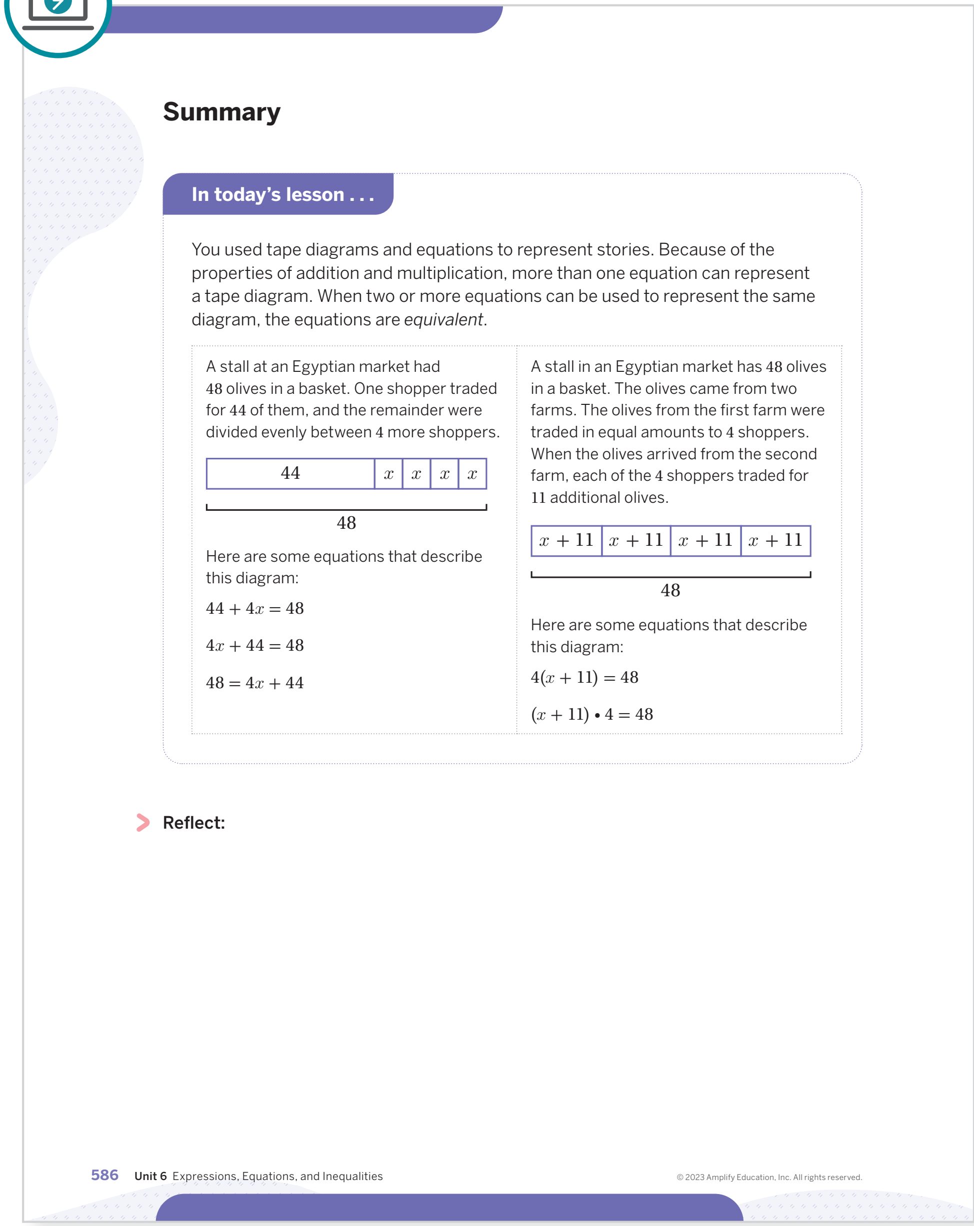
During the Connect, as students share their matches and you highlight the equivalent equations, draw their attention to how the tape diagrams show equations of the form $px + q = r$ or equations of the form $p(x + q) = r$. Ask:

- "Which tape diagrams show adding equal groups of a sum? How is this illustrated in their corresponding equations?"
- "Which tape diagrams show adding the sum of a number and variables? How is this illustrated in their corresponding equations?"

Whole Class | 5 min

Summary

Review and synthesize how tape diagrams and equivalent equations can represent stories.



Summary

In today's lesson . . .

You used tape diagrams and equations to represent stories. Because of the properties of addition and multiplication, more than one equation can represent a tape diagram. When two or more equations can be used to represent the same diagram, the equations are *equivalent*.

A stall at an Egyptian market had 48 olives in a basket. One shopper traded for 44 of them, and the remainder were divided evenly between 4 more shoppers.	A stall in an Egyptian market has 48 olives in a basket. The olives came from two farms. The olives from the first farm were traded in equal amounts to 4 shoppers. When the olives arrived from the second farm, each of the 4 shoppers traded for 11 additional olives.
Here are some equations that describe this diagram: $44 + 4x = 48$ $4x + 44 = 48$ $48 = 4x + 44$	Here are some equations that describe this diagram: $4(x + 11) = 48$ $(x + 11) \cdot 4 = 48$

Reflect:

Synthesize

Display the Summary.

Highlight how each tape diagram represents its corresponding story and how the equations match the tape diagrams. Note that these stories/diagrams/equations illustrate two categories. In the first, a portion of the whole is partitioned and the remaining is divided into some number of equal groups. The equations that represent this category are equivalent to an equation of the form $px + q = r$. In the second, the whole is partitioned into equal groups and then the same amount is added to/subtracted from each group. The equations that represent this category are equivalent to an equation of the form $p(x + q) = r$. Note that these are just some of the equivalent equations that represent each diagram. For example, the equation $48 \div 4 = x + 11$ also represents the second story.

Ask, "How do you know all the equations in each group are equivalent?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

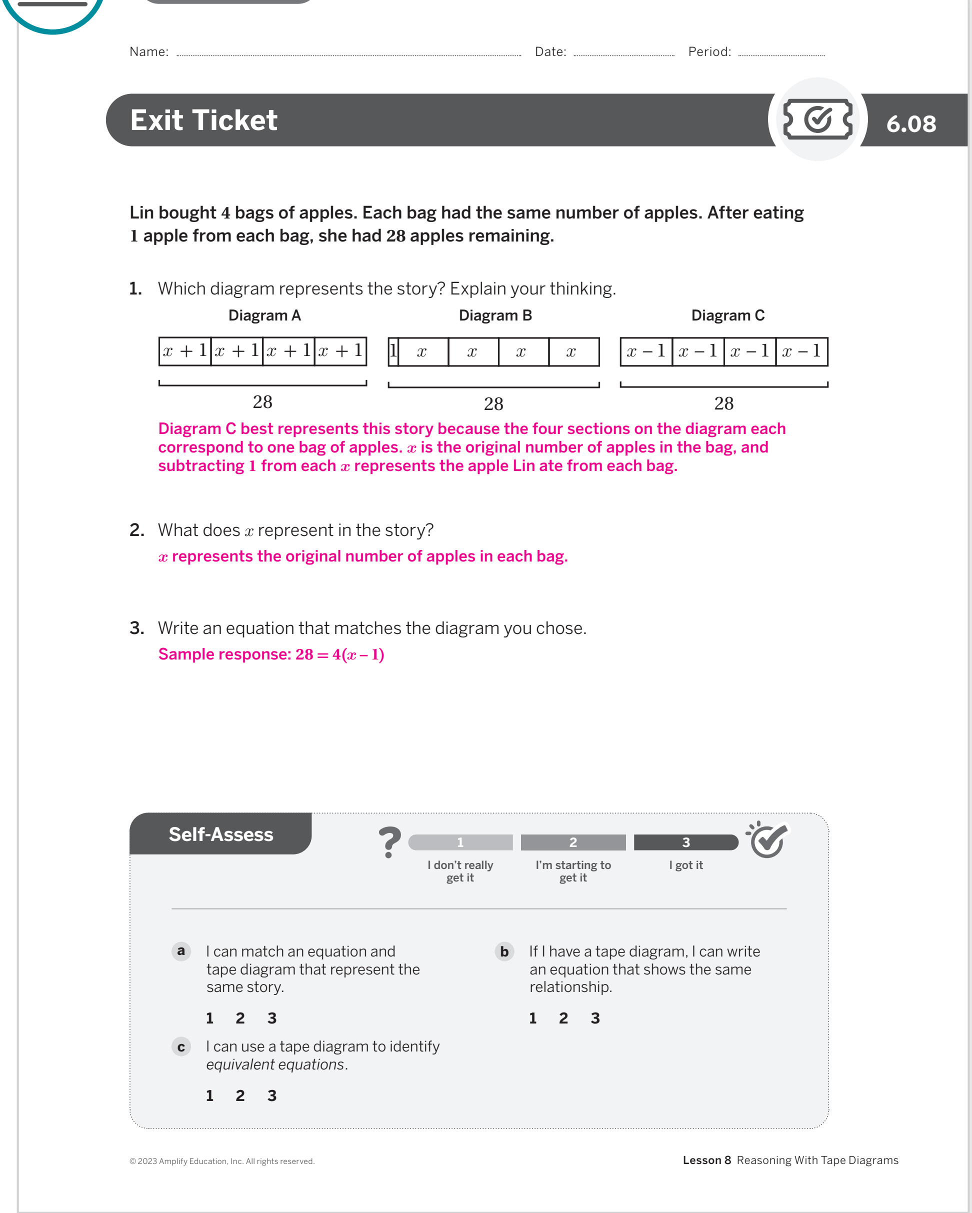
- "Which representations best help you to make sense of certain mathematical scenarios?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by choosing a tape diagram to represents a story and using the diagram to write a corresponding equation.

Printable



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Exit Ticket 6.08

Lin bought 4 bags of apples. Each bag had the same number of apples. After eating 1 apple from each bag, she had 28 apples remaining.

1. Which diagram represents the story? Explain your thinking.

Diagram A

$x+1$	$x+1$	$x+1$	$x+1$

28

Diagram B

1	x	x	x	x

28

Diagram C

$x-1$	$x-1$	$x-1$	$x-1$

28

Diagram C best represents this story because the four sections on the diagram each correspond to one bag of apples. x is the original number of apples in the bag, and subtracting 1 from each x represents the apple Lin ate from each bag.

2. What does x represent in the story?

x represents the original number of apples in each bag.

3. Write an equation that matches the diagram you chose.

Sample response: $28 = 4(x - 1)$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can match an equation and tape diagram that represent the same story.

1 2 3

b If I have a tape diagram, I can write an equation that shows the same relationship.

1 2 3

c I can use a tape diagram to identify *equivalent equations*.

1 2 3

Success looks like . . .

- **Goal:** Selecting a tape diagram to represent relationships between quantities in a situation.
  - » Selecting the tape diagram that represents the 4 bags of apples after Lin ate 1 apple from each begin Problem 1.
- **Goal:** Coordinating tape diagrams and equations of the form $px + q = r$ or $p(x + q) = r$.
  - » Writing an equation for Diagram C in Problem 3.
- **Language Goal:** Identifying equivalent equations and justifying that they are equivalent. **(Speaking and Listening, Writing)**

Suggested next steps

If students choose an incorrect tape diagram to represent the story, consider:

- Reviewing why choice C best represents the story.
- Assigning Practice Problems 1 and 2.

If students are unable to describe what the variable represents in the story, consider:

- Explaining that each section in the diagram represents a bag of apples and that the "minus one" represents the apple Lin ate from each bag, so the variable represents the original number of apples in each bag.
- Assigning Practice Problem 1 and adding an additional step for them to explain what x represents in each story.

If students are unable to write an equation to describe the story, consider:

- Working together to describe the relationships in the tape diagram (four groups of $x - 1$ equal 28) and represent it as an equation.
- Assigning Practice Problem 3, part a.

Professional Learning

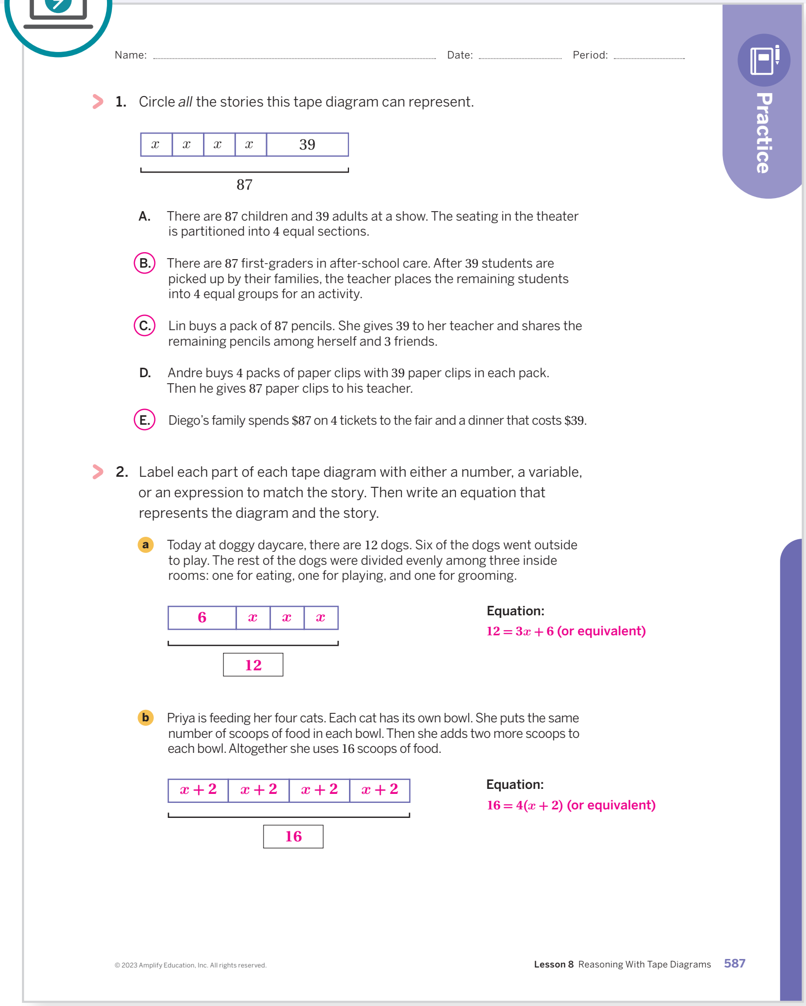
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In Unit 4, students used tape diagrams to understand problems about percentages. How did that support their use of tape diagrams to make sense of writing equations to describe real-world scenarios?
- Which students' ideas were you able to highlight during Activity 3? What might you change for the next time you teach this lesson?

Practice

Independent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Practice

1. Circle *all* the stories this tape diagram can represent.

x	x	x	x	39

87

A. There are 87 children and 39 adults at a show. The seating in the theater is partitioned into 4 equal sections.

[✓ B.] There are 87 first-graders in after-school care. After 39 students are picked up by their families, the teacher places the remaining students into 4 equal groups for an activity.

[✓ C.] Lin buys a pack of 87 pencils. She gives 39 to her teacher and shares the remaining pencils among herself and 3 friends.

D. Andre buys 4 packs of paper clips with 39 paper clips in each pack. Then he gives 87 paper clips to his teacher.

[✓ E.] Diego's family spends \$87 on 4 tickets to the fair and a dinner that costs \$39.

2. Label each part of each tape diagram with either a number, a variable, or an expression to match the story. Then write an equation that represents the diagram and the story.

a. Today at doggy daycare, there are 12 dogs. Six of the dogs went outside to play. The rest of the dogs were divided evenly among three inside rooms: one for eating, one for playing, and one for grooming.

6	x	x	x

12

Equation:

$12 = 3x + 6$ (or equivalent)

b. Priya is feeding her four cats. Each cat has its own bowl. She puts the same number of scoops of food in each bowl. Then she adds two more scoops to each bowl. Altogether she uses 16 scoops of food.

$x + 2$	$x + 2$	$x + 2$	$x + 2$

16

Equation:

$16 = 4(x + 2)$ (or equivalent)

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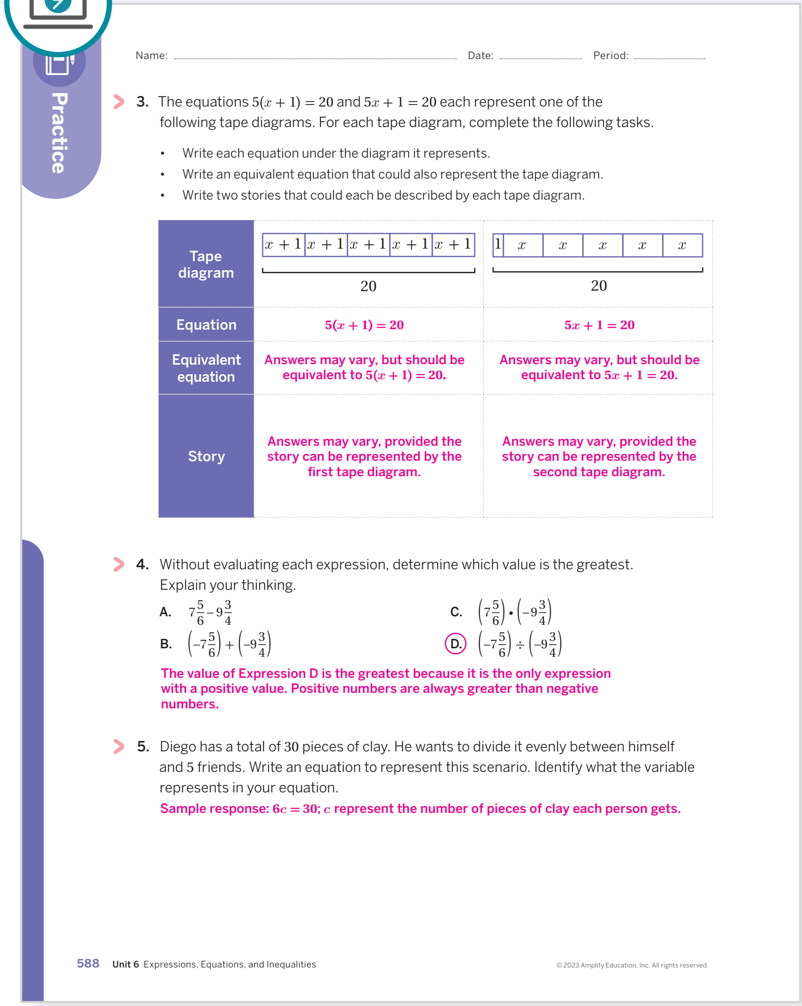
Lesson 8 Reasoning With Tape Diagrams 587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Practice

3. The equations $5(x + 1) = 20$ and $5x + 1 = 20$ each represent one of the following tape diagrams. For each tape diagram, complete the following tasks.
- Write each equation under the diagram it represents.
- Write an equivalent equation that could also represent the tape diagram.
- Write two stories that could each be described by each tape diagram.

Tape diagram	$x + 1$ | $x + 1$ | $x + 1$ | $x + 1$ | $x + 1$ 20	1 | x | x | x | x | x 20
Equation	**$5(x + 1) = 20$**	**$5x + 1 = 20$**
Equivalent equation	**Answers may vary, but should be equivalent to $5(x + 1) = 20$.**	**Answers may vary, but should be equivalent to $5x + 1 = 20$.**
Story	**Answers may vary, provided the story can be represented by the first tape diagram.**	**Answers may vary, provided the story can be represented by the second tape diagram.**

4. Without evaluating each expression, determine which value is the greatest. Explain your thinking.

A. $7\frac{5}{6} - 9\frac{3}{4}$

C. $\left(7\frac{5}{6}\right) \cdot \left(-9\frac{3}{4}\right)$

B. $\left(-7\frac{5}{6}\right) + \left(-9\frac{3}{4}\right)$

[✓ D.] $\left(-7\frac{5}{6}\right) \div \left(-9\frac{3}{4}\right)$

The value of Expression D is the greatest because it is the only expression with a positive value. Positive numbers are always greater than negative numbers.

5. Diego has a total of 30 pieces of clay. He wants to divide it evenly between himself and 5 friends. Write an equation to represent this scenario. Identify what the variable represents in your equation.

Sample response: $6c = 30$; c represent the number of pieces of clay each person gets.

588 **Unit 6** Expressions, Equations, and Inequalities

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 3	2
	3	Activity 3	3
Spiral	4	Unit 5 Lesson 17	2
Formative ⚡	5	Unit 6 Lesson 9	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 9**

Reasoning About Equations and Tape Diagrams (Part 1)

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

Focus

Goals

1. Coordinate tape diagrams, equations of the form $px + q = r$, and verbal descriptions of the situations.
2. **Language Goal:** Solve the equation that represents a situation and interpret the solution in the context of the situation. **(Writing)**

Coherence

Today

Students create tape diagrams, write equations, and solve real-world scenarios, particularly focusing on equations of the form $px + q = r$. Students also connect the meaning of the equation's solution to the context of the real-world situation.

Previously

Students created tape diagrams to help write equations in Lesson 8.

Coming Soon

In Lesson 10, students will continue to create tape diagrams, write equations, and solve real-world problems, but will focus on equations of the form $p(x + q) = r$.

Rigor

- Students develop **procedural fluency** in writing equations from verbal descriptions of the form $px + q = r$ with and without the use of tape diagrams.
- Students **apply** their understanding of solving equations of the form $p(x + q) = r$ to the context of real-world scenarios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	7 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

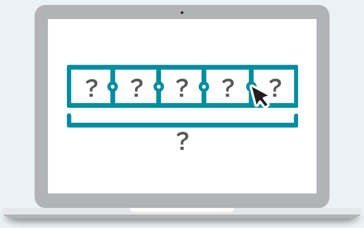
Review words

- *equation*
- *equivalent equations*
- *solution to an equation*
- *variable*

Amps | Featured Activity

Activity 1
Sketching Tape Diagrams

Students can create their tape diagrams digitally. They must reason through the scenario to determine the number of segments and how to label them.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might feel unmotivated to put forth the effort with mathematical reasoning to find three different representations of the same thing. Point out that students are reviewing two of the ways, so they should go quickly. Set an academic goal for relating the equation to the other forms.

Modifications to Pacing

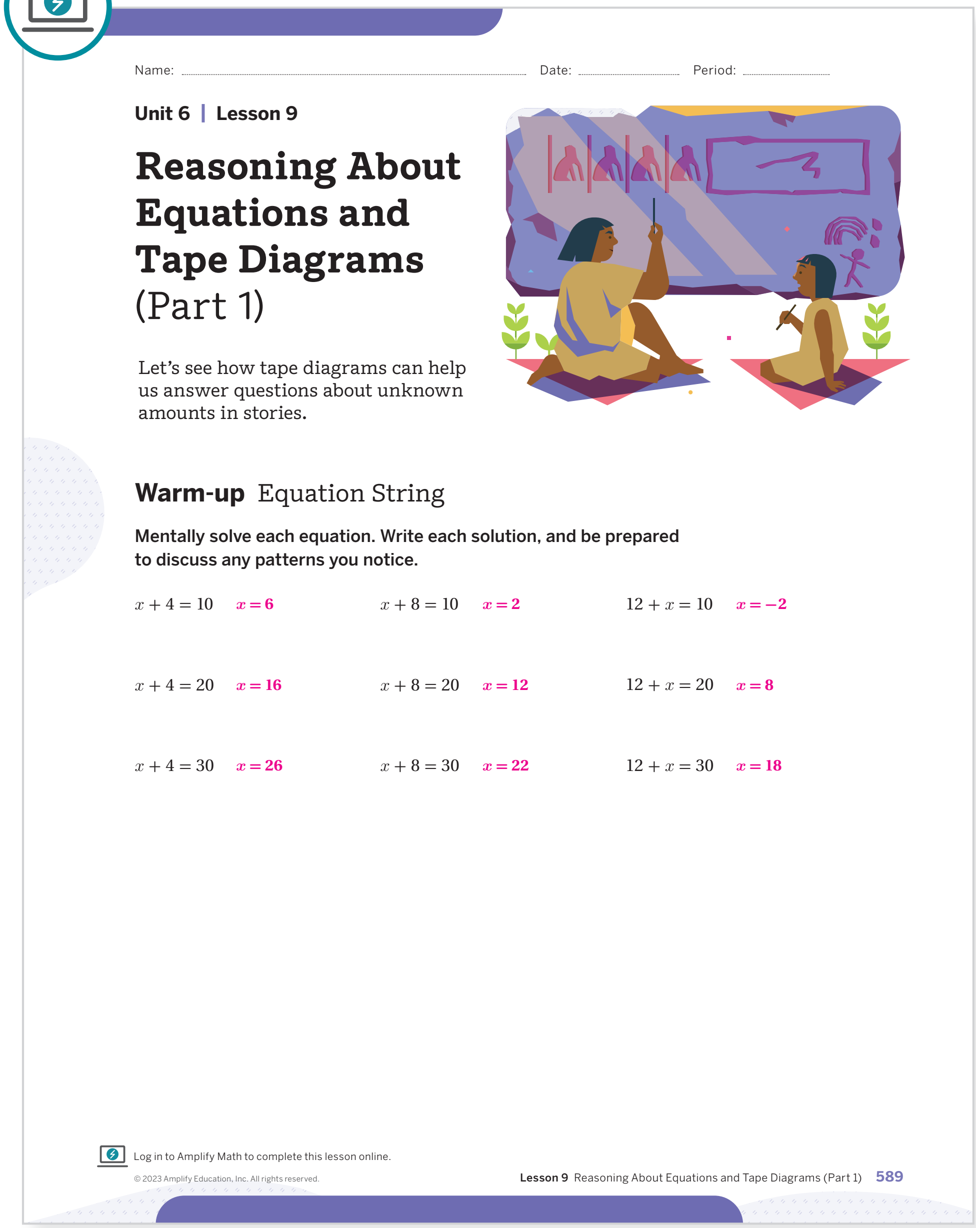
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students only complete the first two columns of equations. Alternatively, the entire Warm-up may be omitted.
- In **Activity 1**, provide student choice by having them complete any three scenarios. Discuss all scenarios as a class.

Independent | 5 min

Warm-up Equation String

Students solve a string of related equations to observe the patterns connecting them.



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Unit 6 | Lesson 9

Reasoning About Equations and Tape Diagrams (Part 1)

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

Warm-up Equation String

Mentally solve each equation. Write each solution, and be prepared to discuss any patterns you notice.

$x + 4 = 10$	$x = 6$	$x + 8 = 10$	$x = 2$	$12 + x = 10$	$x = -2$
$x + 4 = 20$	$x = 16$	$x + 8 = 20$	$x = 12$	$12 + x = 20$	$x = 8$
$x + 4 = 30$	$x = 26$	$x + 8 = 30$	$x = 22$	$12 + x = 30$	$x = 18$

Log in to Amplify Math to complete this lesson online.

1 Launch

Let students know they should solve each equation mentally, by looking for structure and patterns among the three sets of equations. If they are unable to solve an equation, have them skip it and move to the next equation.

2 Monitor

Help students get started by asking, "What is similar with the equations in the first column?"

Look for points of confusion:

- **Not noticing any patterns in the equations.** Help students by asking, "What do you notice that stays the same? What do you notice that is changing?"
- **Hesitating with the equations in the third column.** Have students try the patterns they mentioned in the previous columns.

Look for productive strategies:

- Noticing these patterns:
  - » The equations in each column are similar in structure. Each set is an addition equation where the left side of the equal sign remains the same. The number on the right side increases by 10.
  - » The solutions to the equations in each set also increase by 10.
- **Wanting to solve the equations algebraically.** Remind students the goal is to notice patterns in the equation string.

3 Connect

Display the equation string.

Ask, "What patterns did you notice in the equations, solutions, or both?"

Highlight the similarity between the structure of the equations and the patterns in their solutions.

Math Language Development

MLR8: Discussion Supports

During the Connect, help students make use of the structure of the equations by asking:

- "In each column, what stayed the same? What changed?"
- "How were the solutions affected by what changed?"
- "What will be the next solution in each column, if the pattern continues? What will be the next equation?"
- "What would have been the previous solution in each column, if the pattern continues? The previous equation?"

English Learners

Annotate parts of the equations that are similar and different using different colors.

Power-up

To power up students' ability to write equations to represent a real-world situation, have students complete:

Match each scenario with the equation that represents it.

a. Elena is collecting bugs. After collecting three more, she has a total of 45. a $x + 3 = 45$
b. Andre is collecting baseball cards. After giving three away, he has a total of 45. c $3x = 45$
c. Noah is collecting shells. After a trip to the beach, he tripled his collection. He has a total of 45. b $x - 3 = 45$

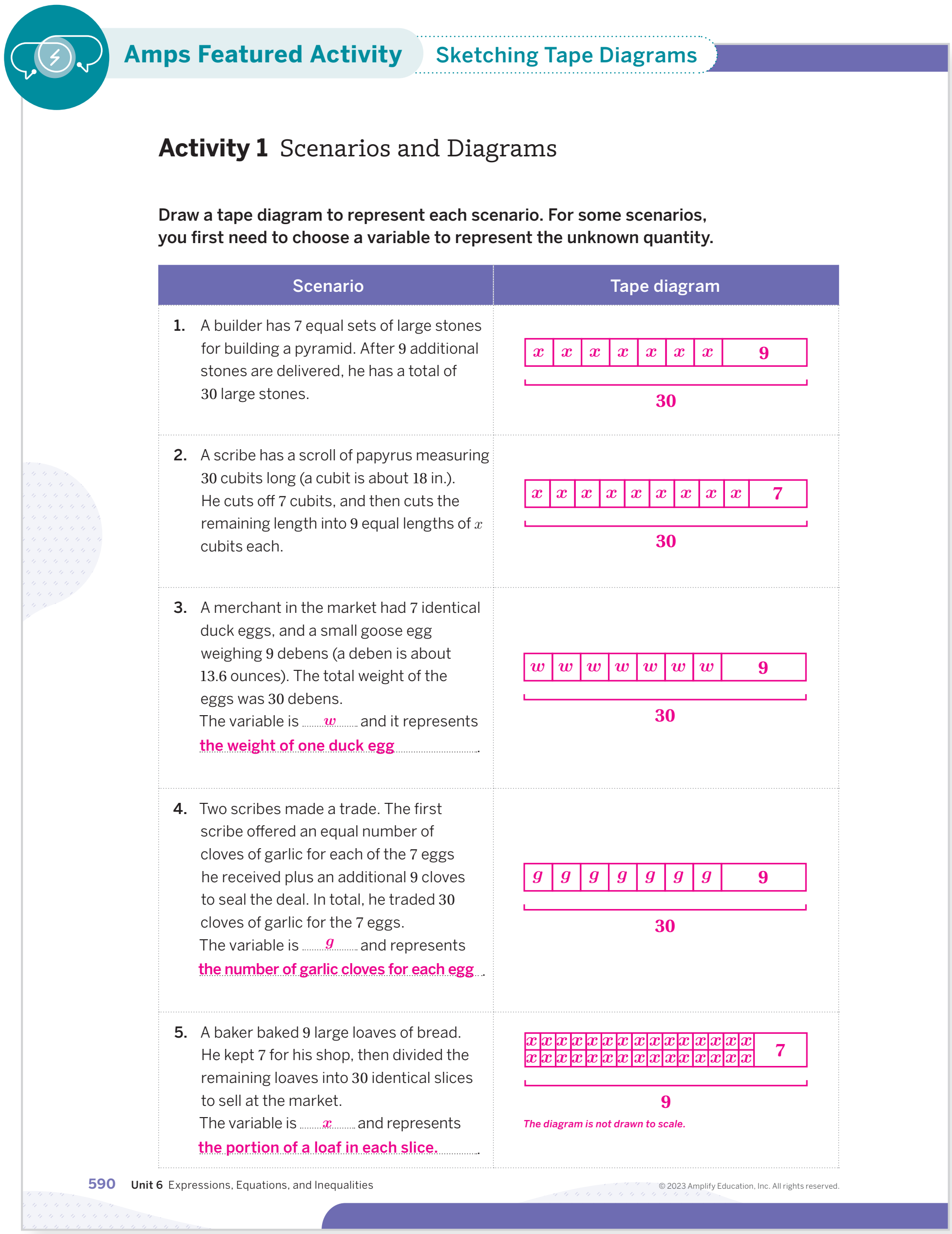
Use: Before Activity 1
Informed by: Performance on Lesson 8, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 15 min

Activity 1 Scenarios and Diagrams

Students draw tape diagrams to represent scenarios, being careful to define variables to represent the unknown quantities.

Amps Featured Activity Sketching Tape Diagrams



Activity 1 Scenarios and Diagrams

Draw a tape diagram to represent each scenario. For some scenarios, you first need to choose a variable to represent the unknown quantity.

Scenario	Tape diagram
1. A builder has 7 equal sets of large stones for building a pyramid. After 9 additional stones are delivered, he has a total of 30 large stones.	x x x x x x x 9 30
2. A scribe has a scroll of papyrus measuring 30 cubits long (a cubit is about 18 in.). He cuts off 7 cubits, and then cuts the remaining length into 9 equal lengths of x cubits each.	x x x x x x x x x 7 30
3. A merchant in the market had 7 identical duck eggs, and a small goose egg weighing 9 debens (a deben is about 13.6 ounces). The total weight of the eggs was 30 debens. The variable is w and it represents the weight of one duck egg.	w w w w w w w 9 30
4. Two scribes made a trade. The first scribe offered an equal number of cloves of garlic for each of the 7 eggs he received plus an additional 9 cloves to seal the deal. In total, he traded 30 cloves of garlic for the 7 eggs. The variable is g and represents the number of garlic cloves for each egg.	g g g g g g g 9 30
5. A baker baked 9 large loaves of bread. He kept 7 for his shop, then divided the remaining loaves into 30 identical slices to sell at the market. The variable is x and represents the portion of a loaf in each slice.	x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x 7 9 *The diagram is not drawn to scale.*

590 **Unit 6** Expressions, Equations, and Inequalities

1 Launch

Be sure students understand that their task is to draw a tape diagram representing each scenario. Let them know there is no requirement to write or solve an equation.

2 Monitor

Help students get started by asking, "What do you *know* in each scenario?" and "What do you *want* to know in each scenario?"

Look for points of confusion:

- **Thinking because *all* the scenarios use the *same* quantities, they *all* have the same tape diagrams.** Have students check their tape diagram with the scenario to check whether it is reasonable.
- **Struggling to represent Scenario 5 with a tape diagram.** Explain that students will look for more efficient ways to represent these types of scenarios.

3 Connect

Have students share their tape diagrams for each scenario.

Ask:

- "For the situations with no variable, how did you decide what quantity to represent with a variable?"
- "Did any situations have the same diagrams?"
- "How could you tell from the stories that the diagrams would be the same?"
- "How is the last scenario different from the others?"

Highlight how all the scenarios use the same values, but do not all have the same tape diagrams. Note, however, there are some scenarios which use the same tape diagram.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create and edit their tape diagrams using digital tools.

Accessibility: *Guide Processing and Visualization*

Have students complete Problems 1, 3, and 4 first, as they can be represented by the same tape diagrams. Then have them complete Problems 2 and 5 and ask, "How are these scenarios different from the scenarios in Problems 1, 3, and 4?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask questions, press for details in their reasoning by asking these follow-up questions:

"Which scenario(s) show that there are . . .

- 7 groups of the variable?" Scenarios 1, 3, and 4
- 9 groups of the variable?" Scenario 2
- 30 groups of the variable?" Scenario 5

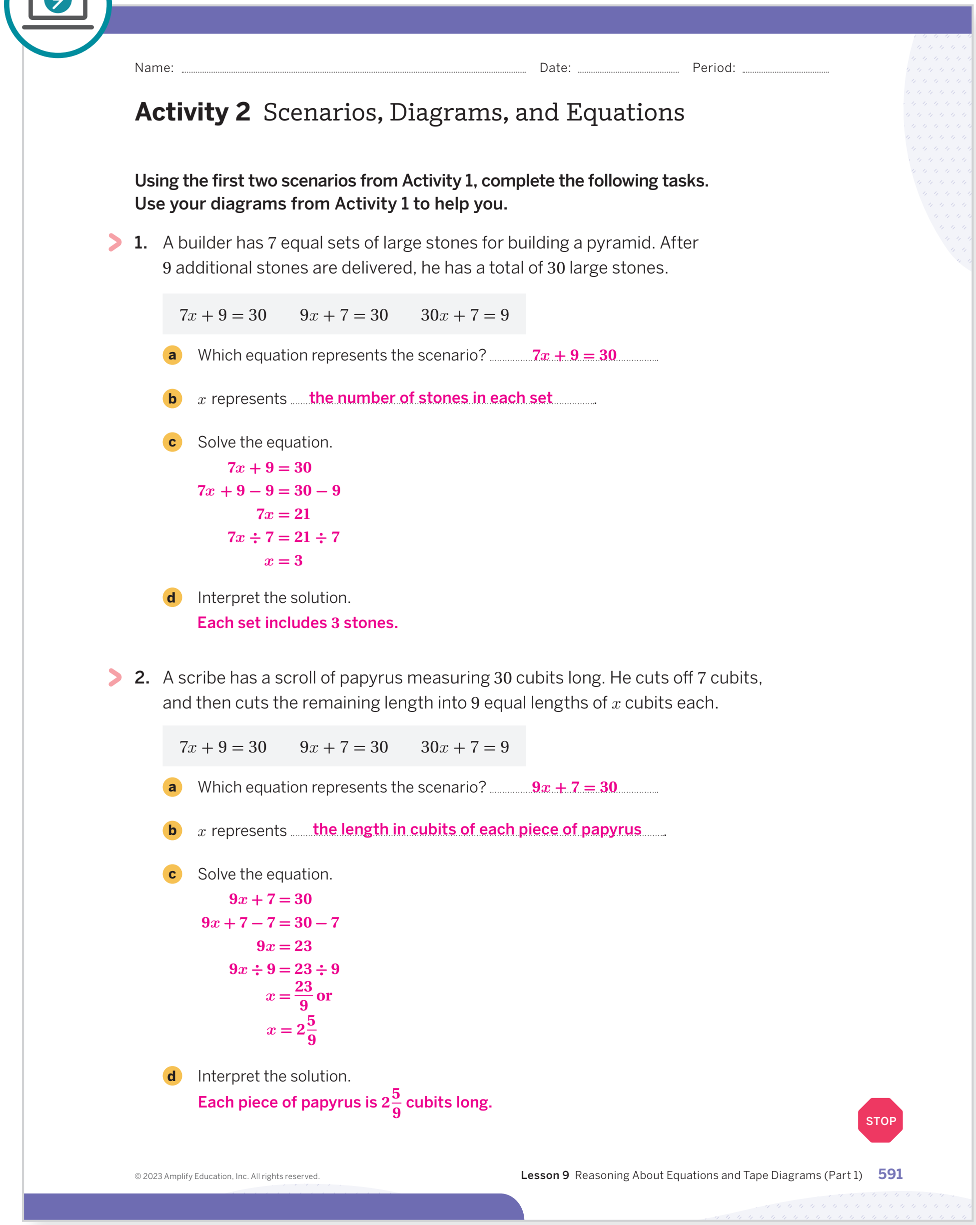
English Learners

Annotate key words in the text that indicate the number of equal groups of the variable, such as 9 *equal lengths of x cubits each*.

Pairs | 15 min

Activity 2 Scenarios, Diagrams, and Equations

Students make connections between the scenarios, tape diagrams, equations, variables, and solutions to build an understanding between the different representations.



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Activity 2 Scenarios, Diagrams, and Equations

Using the first two scenarios from Activity 1, complete the following tasks. Use your diagrams from Activity 1 to help you.

1. A builder has 7 equal sets of large stones for building a pyramid. After 9 additional stones are delivered, he has a total of 30 large stones.

   $7x + 9 = 30$ $9x + 7 = 30$ $30x + 7 = 9$

   a. Which equation represents the scenario? $7x + 9 = 30$

   b. x represents the number of stones in each set.

   c. Solve the equation.

   $$\begin{aligned} 7x + 9 &= 30 \\ 7x + 9 - 9 &= 30 - 9 \\ 7x &= 21 \\ 7x \div 7 &= 21 \div 7 \\ x &= 3 \end{aligned}$$

   d. Interpret the solution.
   Each set includes 3 stones.

2. A scribe has a scroll of papyrus measuring 30 cubits long. He cuts off 7 cubits, and then cuts the remaining length into 9 equal lengths of x cubits each.

   $7x + 9 = 30$ $9x + 7 = 30$ $30x + 7 = 9$

   a. Which equation represents the scenario? $9x + 7 = 30$

   b. x represents the length in cubits of each piece of papyrus.

   c. Solve the equation.

   $$\begin{aligned} 9x + 7 &= 30 \\ 9x + 7 - 7 &= 30 - 7 \\ 9x &= 23 \\ 9x \div 9 &= 23 \div 9 \\ x &= \frac{23}{9} \text{ or} \\ x &= 2\frac{5}{9} \end{aligned}$$

   d. Interpret the solution.
   Each piece of papyrus is $2\frac{5}{9}$ cubits long.

STOP

Lesson 9 Reasoning About Equations and Tape Diagrams (Part 1) 591

1 Launch

Students may reference their responses from Activity 1. Point out that not all the equations from Activity 1 are present in Activity 2. However, the scenarios come from two of the scenarios in Activity 1.

2 Monitor

Help students get started by asking, "What similarities and differences do you notice between the equations?"

Look for points of confusion:

- **Matching the equations incorrectly.** Say, "I notice the numbers from the equation match your scenario, but can you show me where each quantity is seen in the equation? For example, how did you represent, '9 additional stones are delivered'?" Have students refer to their tape diagrams from Activity 1.

3 Connect

Have students share their work for the most challenging tasks of the activity. If matching the equations was the most challenging, discuss in detail how to represent each quantity in an equation. If solving the equations was the most challenging, discuss strategies for solving the equations.

Highlight students who use precise language to describe the variable (x represents the *number* of stones vs. x represents stones). Reinforce the difference between *finding the solution* and *describing the meaning of the solution*.

Ask, "What does each number and letter in the equation represent?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students refer to the tape diagrams they matched with each scenario from Activity 1 to assist them with this activity.

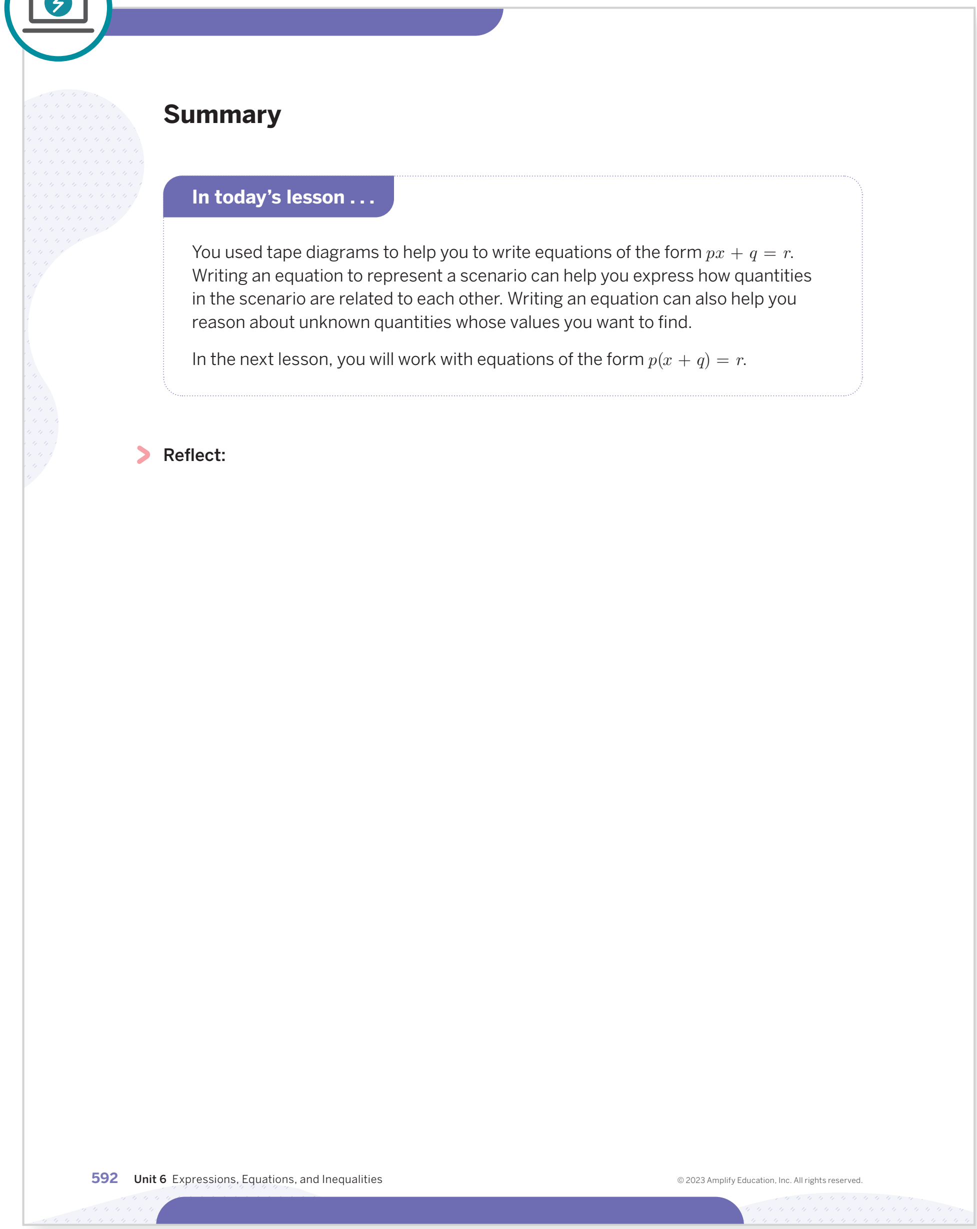
Extension: *Math Enrichment*

Ask students to choose one of the two scenarios in this activity and explain how the scenario would be altered if each of the other two equations correctly represented it. For example, in Problem 1, the equation $9x + 7 = 30$ would represent the scenario if a builder has 9 equal sets of large stones for building a pyramid and 7 additional stones are delivered to the builder.

Whole Class | 7 min

Summary

Review and synthesize the connections between scenarios, tape diagrams, and corresponding equations, and how tape diagrams and equations can help you reason about unknown quantities in the scenarios.



Summary

In today's lesson . . .

You used tape diagrams to help you to write equations of the form $px + q = r$. Writing an equation to represent a scenario can help you express how quantities in the scenario are related to each other. Writing an equation can also help you reason about unknown quantities whose values you want to find.

In the next lesson, you will work with equations of the form $p(x + q) = r$.

Reflect:

592 **Unit 6** Expressions, Equations, and Inequalities

Synthesize

Display the problem, "An architect is drafting plans for a new supermarket. There will be a dedicated space that is 144 in. long for rows of nested shopping carts. The first cart is 34 in. long and each nested cart adds another 10 in. The architect wants to know how many shopping carts will fit in each row."

Have students share what the tape diagram would look like and their strategies for solving the corresponding equation $10x + 34 = 144$.

Ask:

- "What does each number and variable in the tape diagram or equation represent in the scenario?"
- "What is the reason for the operations addition and multiplication used in the equation?"
- "What is the solution to the equation?" $x = 11$
- "What does it mean to be a *solution* to an equation?"
- "What does the solution represent in the scenario?"

Highlight the entire process for determining what the variable represents, drawing a tape diagram, writing an equation, solving the equation, and explaining the meaning of the solution in context.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

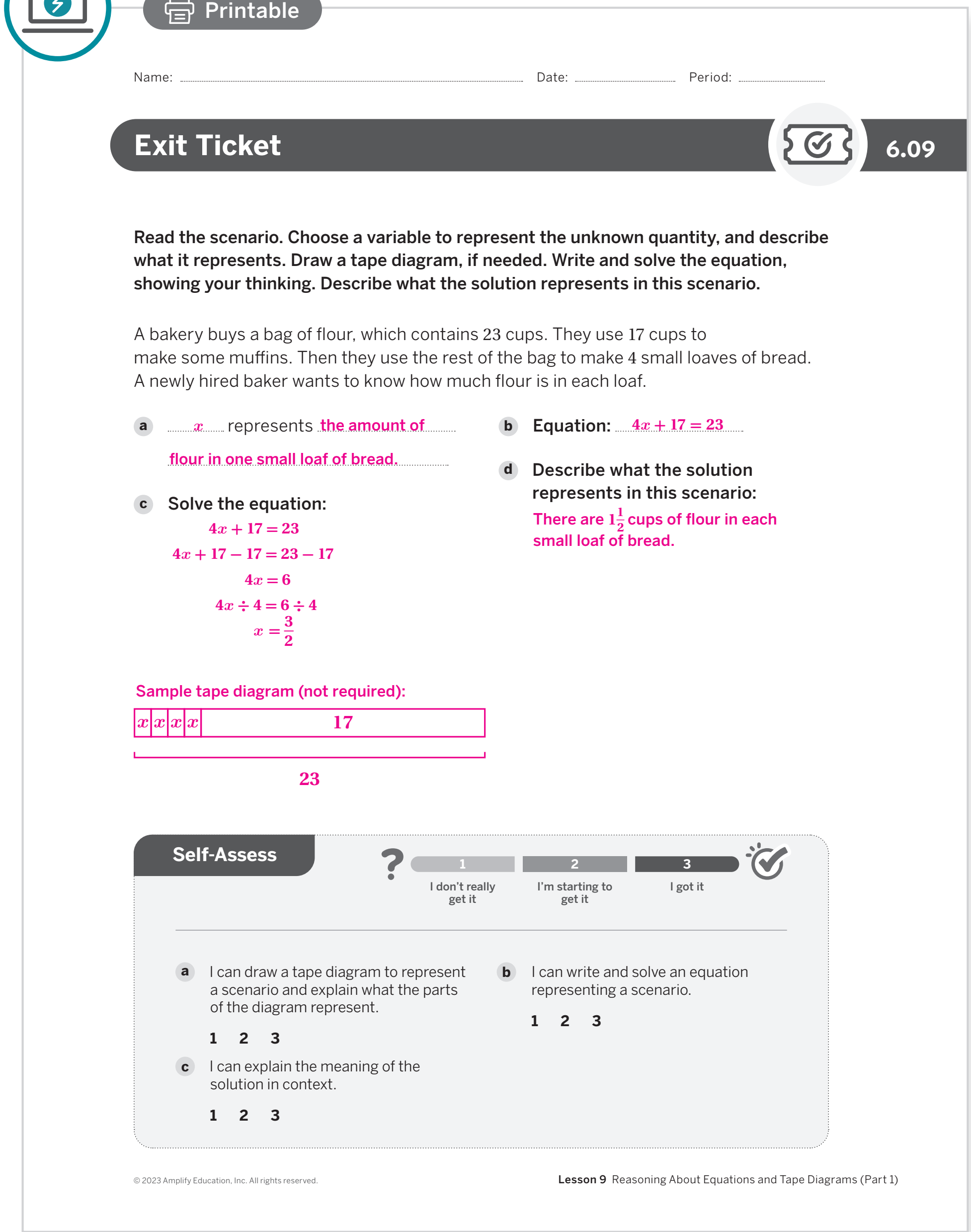
- "Which representations best help you to make sense of certain mathematical scenarios?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing and solving an equation of the form $px + r = q$ to match a scenario that involves an unknown quantity.

Printable



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Exit Ticket 6.09

Read the scenario. Choose a variable to represent the unknown quantity, and describe what it represents. Draw a tape diagram, if needed. Write and solve the equation, showing your thinking. Describe what the solution represents in this scenario.

A bakery buys a bag of flour, which contains 23 cups. They use 17 cups to make some muffins. Then they use the rest of the bag to make 4 small loaves of bread. A newly hired baker wants to know how much flour is in each loaf.

a x represents the amount of flour in one small loaf of bread.

b **Equation:** $4x + 17 = 23$

c **Solve the equation:**

$$4x + 17 = 23$$
$$4x + 17 - 17 = 23 - 17$$
$$4x = 6$$
$$4x \div 4 = 6 \div 4$$
$$x = \frac{3}{2}$$

d **Describe what the solution represents in this scenario:**
There are $1\frac{1}{2}$ cups of flour in each small loaf of bread.

Sample tape diagram (not required):

$$\underbrace{\boxed{x}\boxed{x}\boxed{x}\boxed{x}\boxed{\quad 17 \quad}}_{23}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can draw a tape diagram to represent a scenario and explain what the parts of the diagram represent.
1 2 3

b I can write and solve an equation representing a scenario.
1 2 3

c I can explain the meaning of the solution in context.
1 2 3

Lesson 9 Reasoning About Equations and Tape Diagrams (Part 1)

Success looks like . . .

- **Goal:** Coordinating tape diagrams, equations of the form $px + q = r$, and verbal descriptions of the situations.
  - » Writing the equation to determine how much flour is in each loaf in part b.
- **Language Goal:** Solving the equation that represents a situation and interpreting the solution in the context of the situation. **(Writing)**
  - » Solving the equation and describing how the solution represents the amount of flour in each loaf of bread in parts c and d.

Suggested next steps

If errors with tape diagrams are present, consider:

- Assigning Practice Problem 2.

If errors with defining the variable are present, consider:

- Continuing to model defining the variable for every problem.

If errors occur with writing the equation, Consider:

- Assigning Practice Problem 3.

If errors with solving the equation and/or describing the meaning of the solution, consider:

- Assigning Practice Problem 1.

Professional Learning

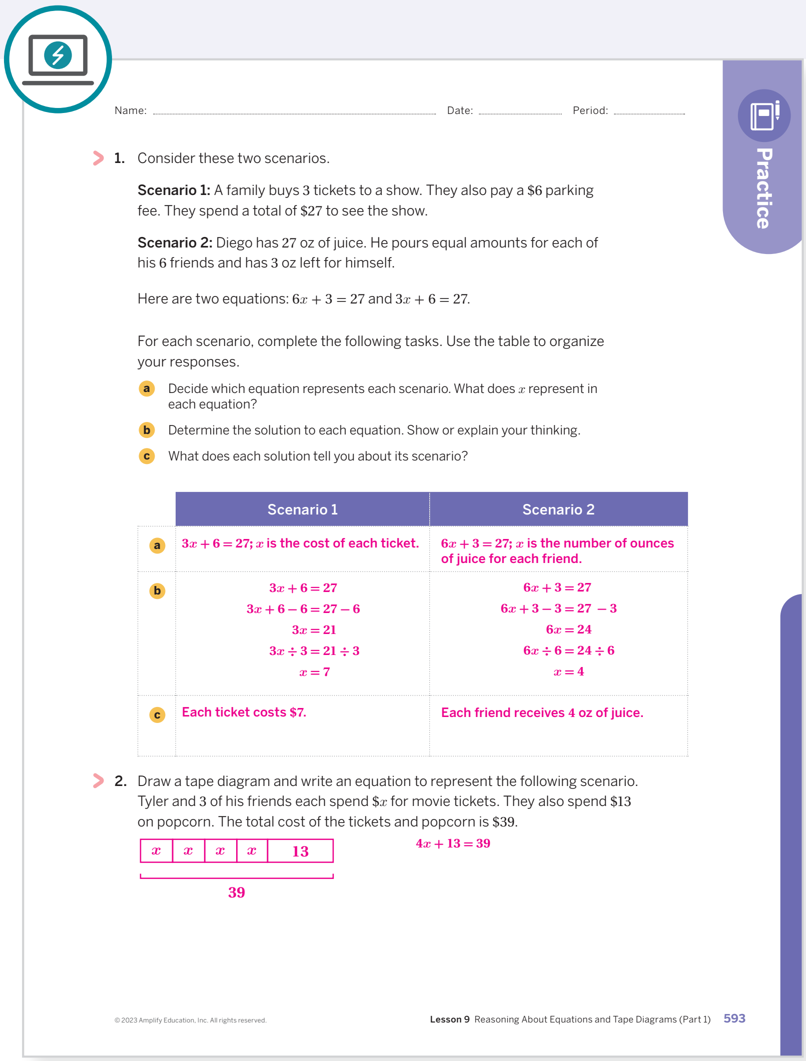
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did you see in the way some students approached Activity 1 that you would like other students to try?
- The instructional goal for this lesson was for students to coordinate tape diagrams, equations, and verbal scenarios to represent and interpret solutions in the context of real-world scenarios. How well did students accomplish this? What did you specifically do to help students accomplish it? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Consider these two scenarios.

Scenario 1: A family buys 3 tickets to a show. They also pay a \$6 parking fee. They spend a total of \$27 to see the show.

Scenario 2: Diego has 27 oz of juice. He pours equal amounts for each of his 6 friends and has 3 oz left for himself.

Here are two equations: $6x + 3 = 27$ and $3x + 6 = 27$.

For each scenario, complete the following tasks. Use the table to organize your responses.

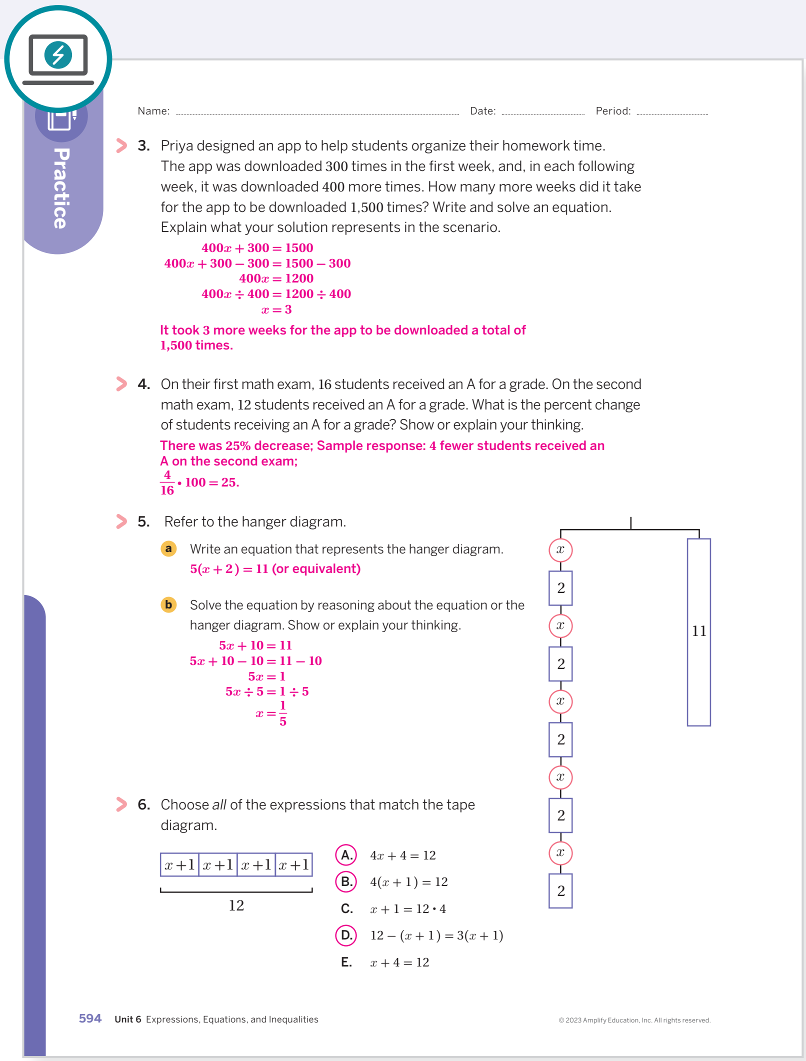
- a Decide which equation represents each scenario. What does x represent in each equation?
- b Determine the solution to each equation. Show or explain your thinking.
- c What does each solution tell you about its scenario?

	Scenario 1	Scenario 2
a	$3x + 6 = 27$; x is the cost of each ticket.	$6x + 3 = 27$; x is the number of ounces of juice for each friend.
b	$3x + 6 = 27$ $3x + 6 - 6 = 27 - 6$ $3x = 21$ $3x \div 3 = 21 \div 3$ $x = 7$	$6x + 3 = 27$ $6x + 3 - 3 = 27 - 3$ $6x = 24$ $6x \div 6 = 24 \div 6$ $x = 4$
c	Each ticket costs \$7.	Each friend receives 4 oz of juice.

2. Draw a tape diagram and write an equation to represent the following scenario. Tyler and 3 of his friends each spend \$$x$ for movie tickets. They also spend \$13 on popcorn. The total cost of the tickets and popcorn is \$39.

$$\underbrace{\boxed{x}\boxed{x}\boxed{x}\boxed{x}\boxed{\quad 13 \quad}}_{39}$$

$4x + 13 = 39$



Practice

Name: Date: Period:

3. Priya designed an app to help students organize their homework time. The app was downloaded 300 times in the first week, and, in each following week, it was downloaded 400 more times. How many more weeks did it take for the app to be downloaded 1,500 times? Write and solve an equation. Explain what your solution represents in the scenario.

$$400x + 300 = 1500$$
$$400x + 300 - 300 = 1500 - 300$$
$$400x = 1200$$
$$400x \div 400 = 1200 \div 400$$
$$x = 3$$

It took 3 more weeks for the app to be downloaded a total of 1,500 times.

4. On their first math exam, 16 students received an A for a grade. On the second math exam, 12 students received an A for a grade. What is the percent change of students receiving an A for a grade? Show or explain your thinking.

There was 25% decrease; Sample response: 4 fewer students received an A on the second exam;

$\frac{4}{16} \cdot 100 = 25.$

5. Refer to the hanger diagram.

a Write an equation that represents the hanger diagram.

$5(x + 2) = 11$ (or equivalent)

b Solve the equation by reasoning about the equation or the hanger diagram. Show or explain your thinking.

$$5x + 10 = 11$$
$$5x + 10 - 10 = 11 - 10$$
$$5x = 1$$
$$5x \div 5 = 1 \div 5$$
$$x = \frac{1}{5}$$

6. Choose *all* of the expressions that match the tape diagram.

$$\underbrace{\boxed{x+1}\boxed{x+1}\boxed{x+1}\boxed{x+1}}_{12}$$

[✓ A.] $4x + 4 = 12$
[✓ B.] $4(x + 1) = 12$
C. $x + 1 = 12 \cdot 4$
[✓ D.] $12 - (x + 1) = 3(x + 1)$
E. $x + 4 = 12$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 3	1
	5	Unit 6 Lesson 4	2
Formative ⚡	6	Unit 6 Lesson 10	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 10**

Reasoning About Equations and Tape Diagrams (Part 2)

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

Focus

Goals

1. Coordinate tape diagrams, equations of the form $p(x+q)=r$, and verbal descriptions of the situations.
2. **Language Goal:** Solve the equation that represents a situation and interpret the solution in the context of the situation. **(Writing)**

Coherence

Today

Students use tape diagrams and equations of the form $p(x+q)=r$ to describe relationships in real-world story problems and solve them algebraically. Students connect the meaning of the equation's solution to the context of the story.

Previously

In Lesson 9, students used tape diagrams and equations of the form $px+q=r$ to describe relationships in real-world problems.

Coming Soon

In Lesson 11, students will use tape diagrams and reasoning to decide which type of equation, $px+q=r$ or $p(x+q)=r$, describes the relationships in a real-world story problem. Then they will write and solve the equation algebraically.

Rigor

- Students develop **procedural fluency** in writing equations from verbal descriptions of the form $p(x+q)=r$ with and without the use of tape diagrams.
- Students **apply** their understanding of solving equations of the form $p(x+q)=r$ to the context of real-world scenarios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

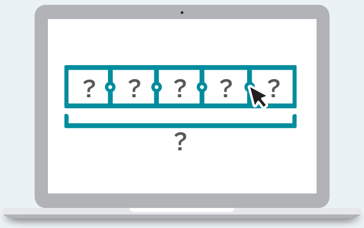
Review words

- *equation*
- *equivalent equations*
- *solution to an equation*
- *variable*

Amps | Featured Activity

Activity 1
Dynamic Tape Diagrams

Students can create digital tape diagrams, and you can overlay them all to see similarities and differences at a glance.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Because Activity 2 looks familiar, students might lack the self-discipline to exercise the mental reasoning required to achieve the best results. Since students are working in pairs, ask them to work together to encourage each other to show focus and determination throughout the task.

Modifications to Pacing

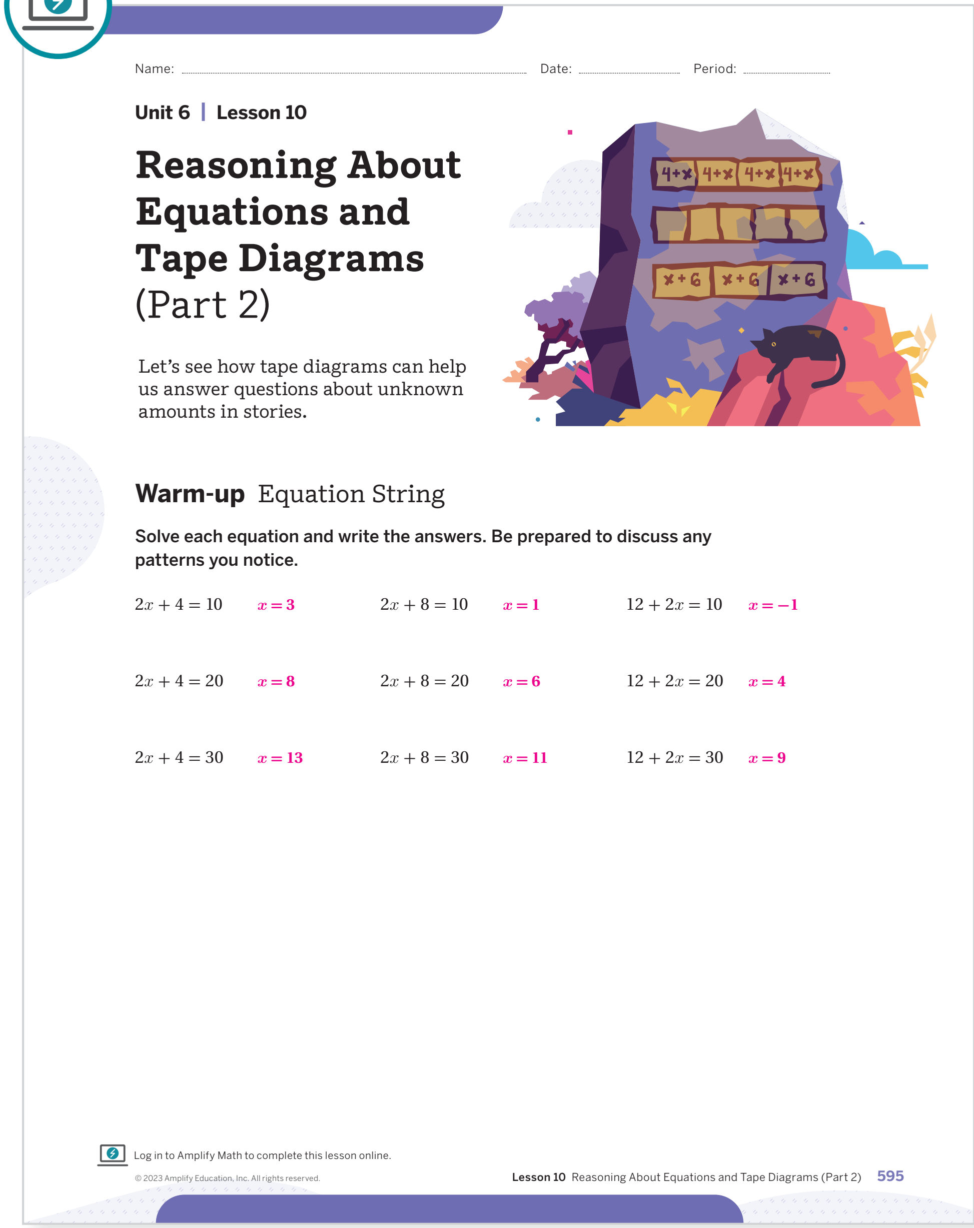
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students only complete the first two columns of equations. Alternatively, the entire Warm-up may be omitted.
- In **Activity 1**, provide student choice by allowing students to create diagrams for any three scenarios. Discuss all scenarios as a class.

Independent | 5 min

Warm-up Equation String

Students solve a string of related equations to build fluency and pattern recognition.



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Unit 6 | Lesson 10

Reasoning About Equations and Tape Diagrams (Part 2)

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

Warm-up Equation String

Solve each equation and write the answers. Be prepared to discuss any patterns you notice.

$2x + 4 = 10$ $x = 3$

$2x + 8 = 10$ $x = 1$

$12 + 2x = 10$ $x = -1$

$2x + 4 = 20$ $x = 8$

$2x + 8 = 20$ $x = 6$

$12 + 2x = 20$ $x = 4$

$2x + 4 = 30$ $x = 13$

$2x + 8 = 30$ $x = 11$

$12 + 2x = 30$ $x = 9$

Log in to Amplify Math to complete this lesson online.

Lesson 10 Reasoning About Equations and Tape Diagrams (Part 2) 595

1 Launch

Let students know they should solve each equation mentally, by looking for structure and patterns among the three sets of equations. If they are unable to solve an equation, have them skip it and move to the next equation.

2 Monitor

Help students get started by saying, "Think of the variable as a mystery number. What number should go in place of the variable to make the left side equal to the right side?"

Look for points of confusion:

- **Not noticing any patterns in the equations:**. Ask, "What do you notice that stays the same? What do you notice that is changing?"
- **Getting stuck trying to solve algebraically.** Have them solve the equation using any strategy. The goal is to notice patterns and structure.

Look for productive strategies:

- Noticing these patterns:
  - » As the right side value increases, the value of the variable increases by half of that amount.
  - » Corresponding solutions in each set are two less than solutions in the previous set.

3 Connect

Display the list of equations and their solutions.

Ask, "What patterns did you notice in the equations, solutions, or both?"
I used the same solution steps for equations that were grouped together.

Have students share their individual observations of patterns.

Highlight the similarity in the structure of the equations and the patterns in their solutions.

Math Language Development

MLR8: Discussion Supports

During the Connect, help students make use of the structure of the equations by asking:

- "In each column, what stayed the same? What changed?"
- "How were the solutions affected by what changed?"
- "What will be the next solution in each column, if the pattern continues? What will be the next equation?"
- "What would have been the previous solution in each column, if the pattern continues? The previous equation?"

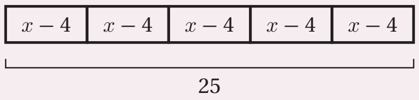
English Learners

Annotate parts of the equations that are similar and different using different colors.

Power-up

To power up students' ability to identify equations that match tape diagrams which involve grouping, have students complete:

Determine which equation does *not* represent the tape diagram.



A. $5(x - 4) = 25$

[✓ **B.**] $5x - 4 = 25$

C. $5x - 20 = 25$

D. $x - 4 = 5$

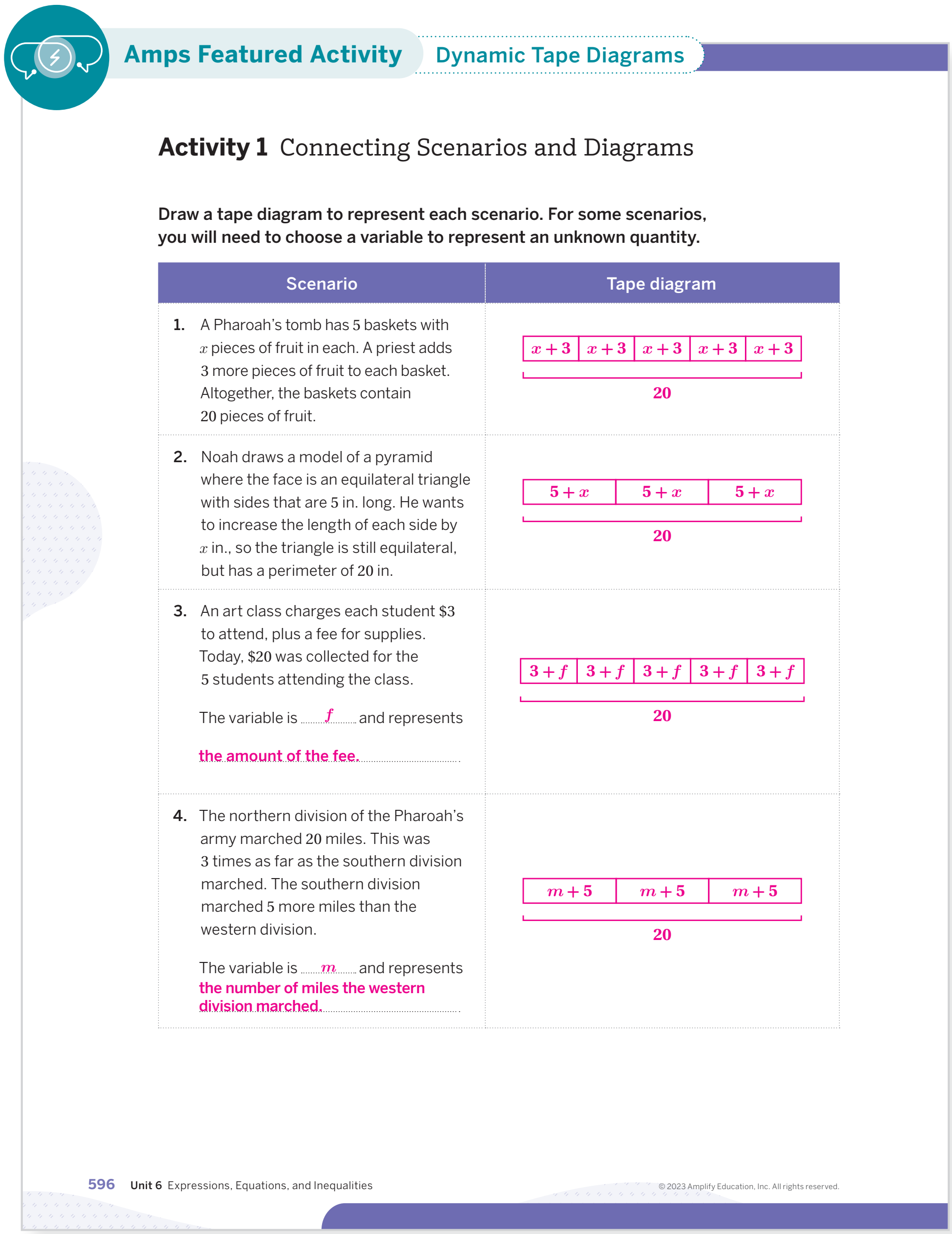
Use: Before Activity 1.
Informed by: Performance on Lesson 9, Practice Problem 6.

Pairs | 15 min

Activity 1 Connecting Scenarios and Diagrams

Students draw tape diagrams to represent scenarios, being careful to define variables to represent the unknown quantities.

Amps Featured Activity Dynamic Tape Diagrams



Activity 1 Connecting Scenarios and Diagrams

Draw a tape diagram to represent each scenario. For some scenarios, you will need to choose a variable to represent an unknown quantity.

Scenario	Tape diagram
1. A Pharoah's tomb has 5 baskets with x pieces of fruit in each. A priest adds 3 more pieces of fruit to each basket. Altogether, the baskets contain 20 pieces of fruit.	$x+3$ | $x+3$ | $x+3$ | $x+3$ | $x+3$ 20
2. Noah draws a model of a pyramid where the face is an equilateral triangle with sides that are 5 in. long. He wants to increase the length of each side by x in., so the triangle is still equilateral, but has a perimeter of 20 in.	$5+x$ | $5+x$ | $5+x$ 20
3. An art class charges each student \$3 to attend, plus a fee for supplies. Today, \$20 was collected for the 5 students attending the class. The variable is f and represents the amount of the fee.	$3+f$ | $3+f$ | $3+f$ | $3+f$ | $3+f$ 20
4. The northern division of the Pharoah's army marched 20 miles. This was 3 times as far as the southern division marched. The southern division marched 5 more miles than the western division. The variable is m and represents the number of miles the western division marched.	$m+5$ | $m+5$ | $m+5$ 20

596 **Unit 6** Expressions, Equations, and Inequalities

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1 Launch

Be sure students understand that their task is to define the variable and draw a tape diagram representing each scenario. Let them know there is no requirement to write or solve an equation.

2 Monitor

Help students get started by asking, "What do you know in each scenario? What do you want to know in each scenario?"

Look for points of confusion:

- **Thinking because *all* the scenarios use the same quantities, they all have the same tape diagrams.** Have students check their tape diagram with the scenario to see if it makes sense.

3 Connect

Have students share their correct diagrams.

Ask:

- "For the situations with no variable, how did you decide what quantity to represent with a variable?" I thought about which number is not known.
- "Did any scenarios have the same diagrams?"
- "How could you tell from the scenarios that the diagrams would be the same?"

Highlight how the equal groups of sums in the scenarios are shown in the tape diagrams.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create tape diagrams using digital tools. You can overlay them all to see similarities and differences at a glance.

Accessibility: *Guide Processing and Visualization*

Have students complete Problems 1 and 3 first, as they can be represented by the same tape diagrams. Then have students complete Problems 2 and 4, as they can be represented by the same diagram. Ask, "What do you notice about the tape diagrams you drew?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask questions, press for details in their reasoning by asking these follow-up questions:

"Which scenario(s) show that there are . . .

- 5 equal groups of a sum? What key words indicate this?" Scenarios 1 and 3
- 3 equal groups of a sum? What key words indicate this?" Scenarios 2 and 4

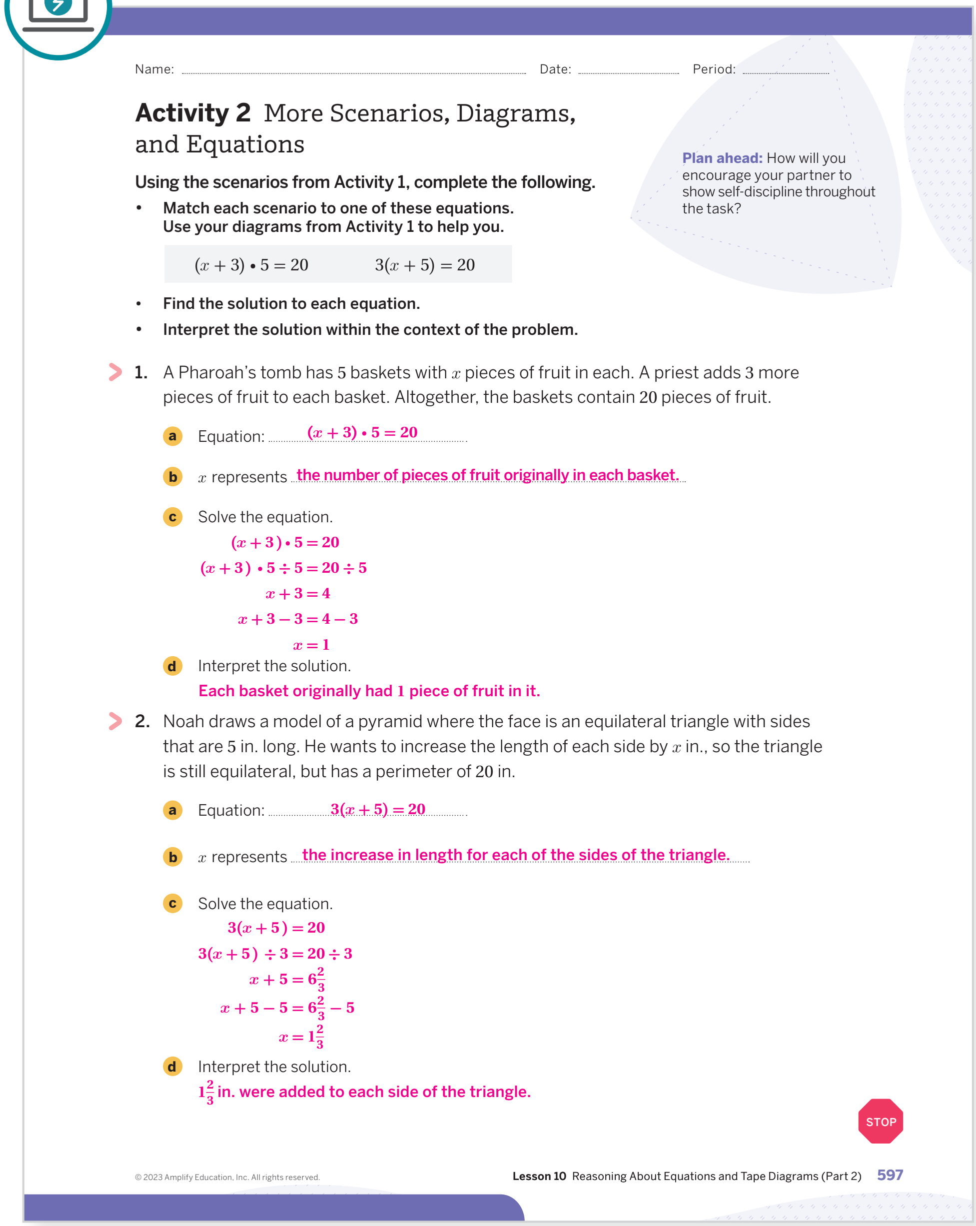
English Learners

Annotate key words in the text that indicate adding equal groups of a sum, such as *5 baskets with x pieces of fruit in each and adds 3 more pieces of fruit to each basket*

Pairs | 15 min

Activity 2 More Scenarios, Diagrams, and Equations

Students make connections between the scenarios, tape diagrams, and equations to deepen understanding of their relationships.



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Activity 2 More Scenarios, Diagrams, and Equations

Using the scenarios from Activity 1, complete the following.

- **Match each scenario to one of these equations. Use your diagrams from Activity 1 to help you.**

$(x+3) \cdot 5 = 20$ $3(x+5) = 20$

- **Find the solution to each equation.**
- **Interpret the solution within the context of the problem.**

Plan ahead: How will you encourage your partner to show self-discipline throughout the task?

1. A Pharoah's tomb has 5 baskets with x pieces of fruit in each. A priest adds 3 more pieces of fruit to each basket. Altogether, the baskets contain 20 pieces of fruit.

   a. Equation: $(x+3) \cdot 5 = 20$

   b. x represents the number of pieces of fruit originally in each basket.

   c. Solve the equation.

   $$(x+3) \cdot 5 = 20$$
   $$(x+3) \cdot 5 \div 5 = 20 \div 5$$
   $$x+3 = 4$$
   $$x+3-3 = 4-3$$
   $$x = 1$$

   d. Interpret the solution.

   Each basket originally had 1 piece of fruit in it.

2. Noah draws a model of a pyramid where the face is an equilateral triangle with sides that are 5 in. long. He wants to increase the length of each side by x in., so the triangle is still equilateral, but has a perimeter of 20 in.

   a. Equation: $3(x+5) = 20$

   b. x represents the increase in length for each of the sides of the triangle.

   c. Solve the equation.

   $$3(x+5) = 20$$
   $$3(x+5) \div 3 = 20 \div 3$$
   $$x+5 = 6\tfrac{2}{3}$$
   $$x+5-5 = 6\tfrac{2}{3} - 5$$
   $$x = 1\tfrac{2}{3}$$

   d. Interpret the solution.

   $1\frac{2}{3}$ in. were added to each side of the triangle.

STOP

Lesson 10 Reasoning About Equations and Tape Diagrams (Part 2) 597

1 Launch

Have students reference their answers from Activity 1. Point out that this activity uses two of the scenarios from Activity 1.

2 Monitor

Help students get started by asking, "What similarities and differences do you notice between the equations?"

Look for points of confusion:

- **Not matching equations appropriately.** Say, "The numbers from the equations seem to match your scenario. How is each quantity in the scenario represented by the equation? For example, how did you represent *a priest adds 3 more pencils to each basket*?"
- **Struggling to solve the equations.** Refer students to strategies used to solve the Warm-up equations.

3 Connect

Have students share their work for the two most challenging tasks for your class. If your class had difficulty identifying the variable, ask students what they think the unknown quantity is for each scenario.

Highlight the use of precise language to describe the variable (e.g. "x represents the number of pieces of fruit vs. x represents fruit.") and what the solution represents in terms of the scenario.

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Suggest that students refer to the tape diagrams they matched with each scenario from Activity 1 to assist them with this activity.

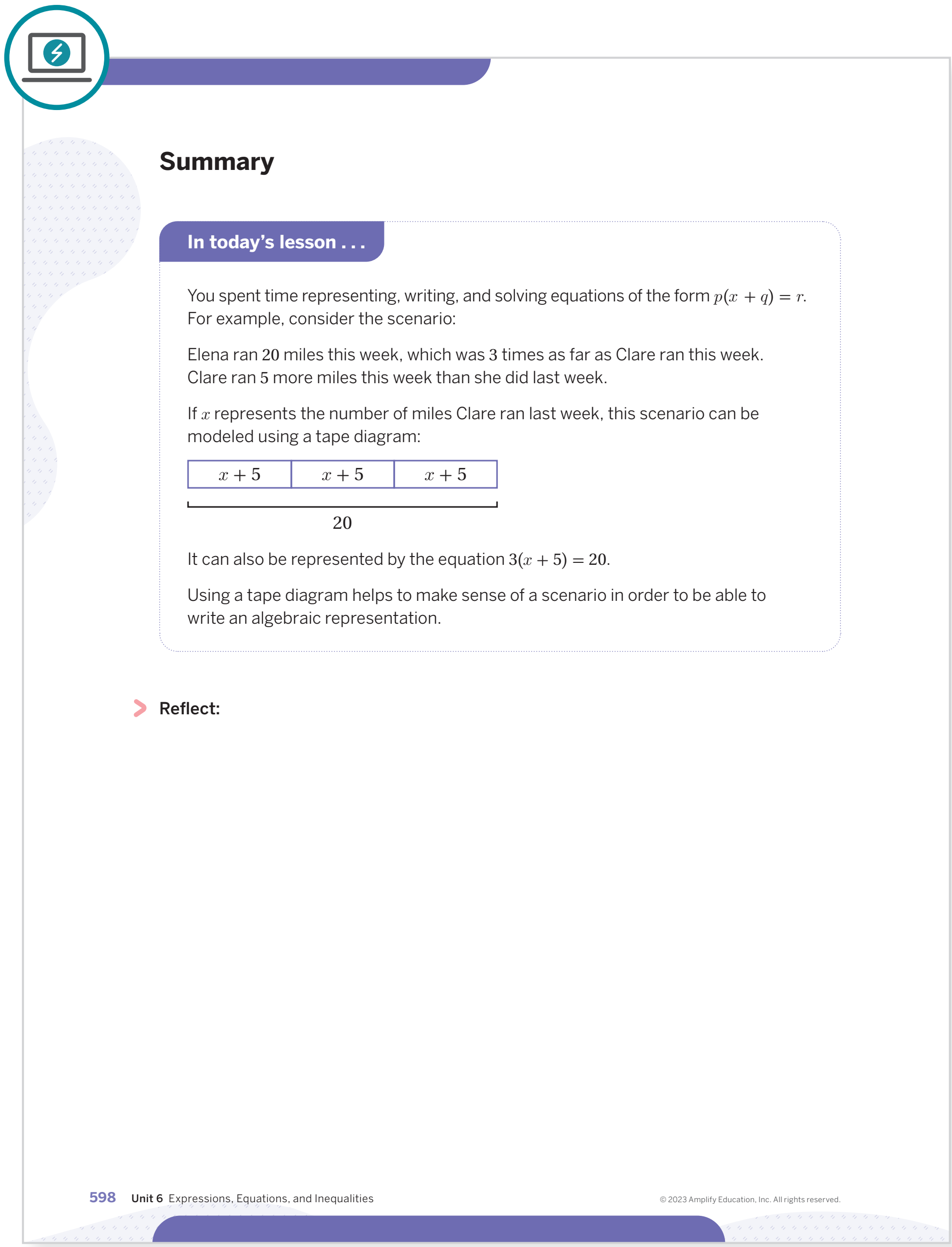
Extension: ***Math Enrichment***

Ask students to choose one of the two scenarios in this activity and explain how the scenario would be altered if the other equation correctly represented it. For example, in Problem 1, the equation $3(x+5)$ would represent the scenario if there were 3 baskets with each containing x pieces of fruit and the priest adding 5 more pieces of fruit to each basket.

Whole Class | 5 min

Summary

Review and synthesize the connections between scenarios, tape diagrams, and corresponding equations of the form $p(x + q) = r$, and how these connections can help students reason about unknown quantities in the scenarios.



Summary

In today's lesson . . .

You spent time representing, writing, and solving equations of the form $p(x + q) = r$. For example, consider the scenario:

Elena ran 20 miles this week, which was 3 times as far as Clare ran this week. Clare ran 5 more miles this week than she did last week.

If x represents the number of miles Clare ran last week, this scenario can be modeled using a tape diagram:

It can also be represented by the equation $3(x + 5) = 20$.

Using a tape diagram helps to make sense of a scenario in order to be able to write an algebraic representation.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Ask:

- What does each number and variable in the equation represent in the situation?"
- "What is the reason for the operations of multiplication or addition used in the equation?"
- "What is the solution to the equation?"
- "What does it mean to be a *solution* to an equation?"
- "What does the solution represent in the situation?"

Highlight that the total value always represents the entire sum of all the sections of the tape diagram. Remind students to look carefully at what the unknown value is, which values are grouped together, and how many times they are grouped together.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

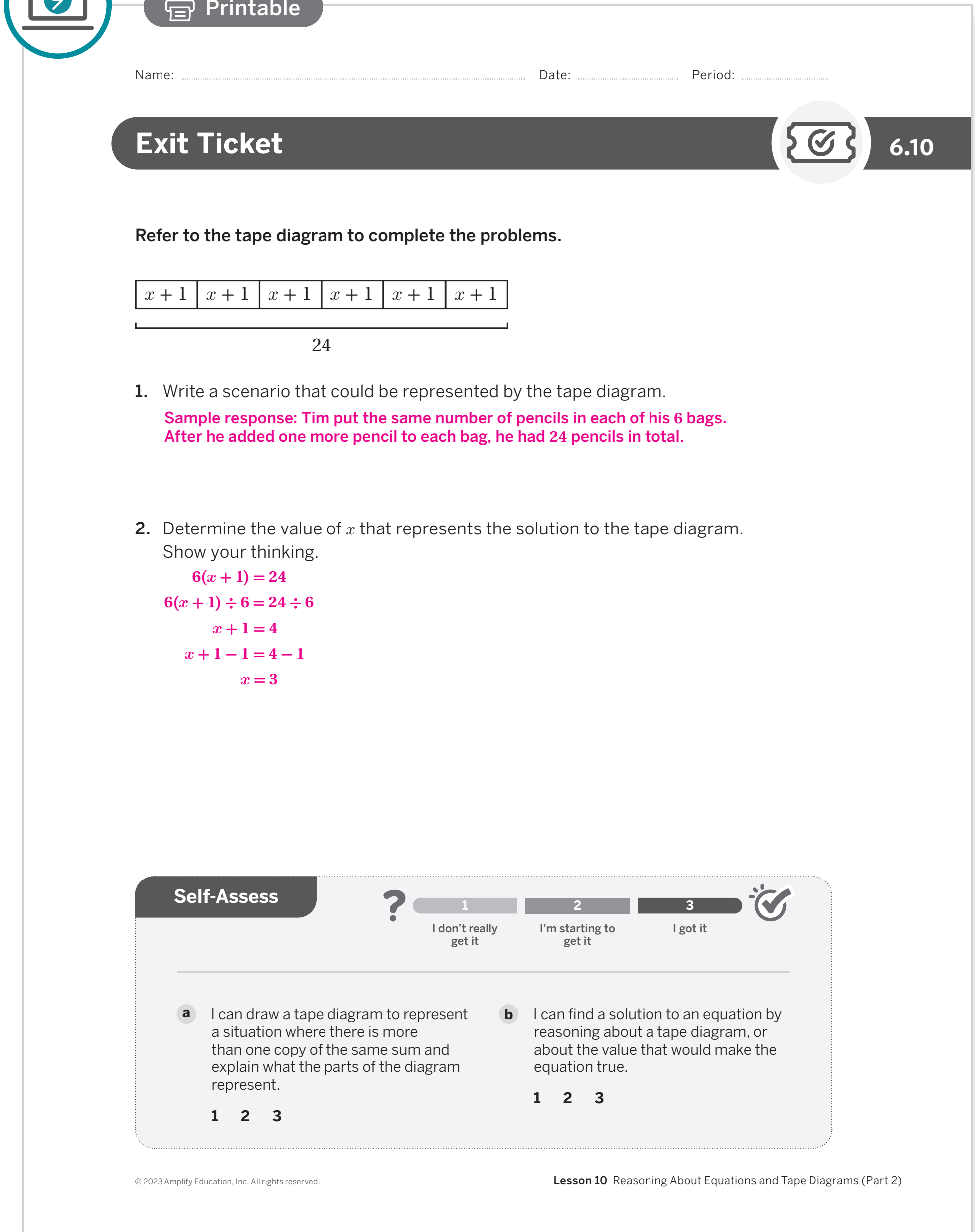
- "Which representations best help you to make sense of certain mathematical scenarios?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing a scenario to match a given tape diagram.

Printable



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Exit Ticket 6.10

Refer to the tape diagram to complete the problems.

$x+1$	$x+1$	$x+1$	$x+1$	$x+1$	$x+1$

24

1. Write a scenario that could be represented by the tape diagram.
   Sample response: Tim put the same number of pencils in each of his 6 bags. After he added one more pencil to each bag, he had 24 pencils in total.

2. Determine the value of x that represents the solution to the tape diagram. Show your thinking.

$$6(x+1)=24$$
$$6(x+1)\div 6=24\div 6$$
$$x+1=4$$
$$x+1-1=4-1$$
$$x=3$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can draw a tape diagram to represent a situation where there is more than one copy of the same sum and explain what the parts of the diagram represent.

1 2 3

b I can find a solution to an equation by reasoning about a tape diagram, or about the value that would make the equation true.

1 2 3

Lesson 10 Reasoning About Equations and Tape Diagrams (Part 2)

Success looks like . . .

- **Goal:** Coordinating tape diagrams, equations of the form $p(x+q)=r$, and verbal descriptions of the situations.
  - » Writing a verbal description to represent the given tape diagram in Problem 1.
- **Language Goal:** Solving the equation that represents a situation and interpreting the solution in the context of the situation. **(Writing)**
  - » Solving the equation that represents the tape diagram in Problem 2.

Suggested next steps

If students struggle to write a scenario, consider:

- Having students review Activity 1. Encourage them to simply substitute numbers while they get comfortable with the structure of these scenarios.
- Assigning Practice Problem 2.

If students forget to distribute the number 6 to all terms inside parentheses, consider:

- Reviewing Lesson 6.
- Assigning Practice Problem 3

Professional Learning

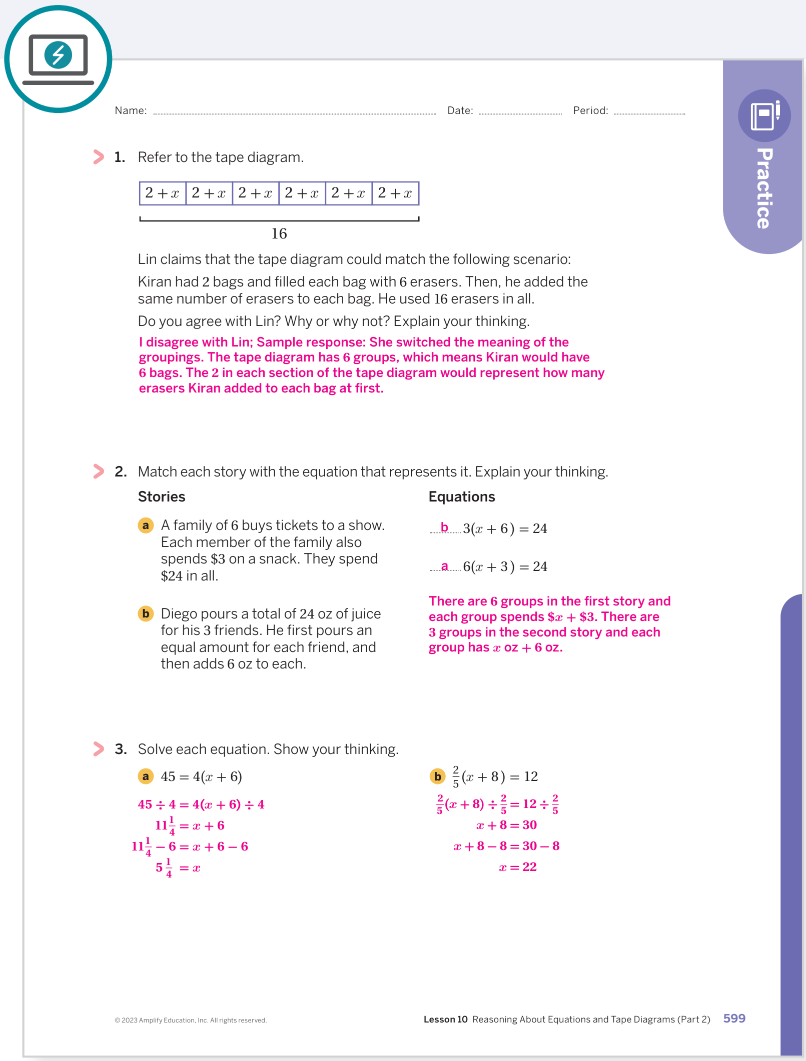
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on Activity 1?
- How were the activities from today's lesson similar to or different from the activities students completed in the previous lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Refer to the tape diagram.

$2+x$	$2+x$	$2+x$	$2+x$	$2+x$	$2+x$

16

Lin claims that the tape diagram could match the following scenario:

Kiran had 2 bags and filled each bag with 6 erasers. Then, he added the same number of erasers to each bag. He used 16 erasers in all.

Do you agree with Lin? Why or why not? Explain your thinking.

I disagree with Lin; Sample response: She switched the meaning of the groupings. The tape diagram has 6 groups, which means Kiran would have 6 bags. The 2 in each section of the tape diagram would represent how many erasers Kiran added to each bag at first.

2. Match each story with the equation that represents it. Explain your thinking.

Stories	Equations
a A family of 6 buys tickets to a show. Each member of the family also spends \$3 on a snack. They spend \$24 in all.	**b** $3(x+6)=24$
b Diego pours a total of 24 oz of juice for his 3 friends. He first pours an equal amount for each friend, and then adds 6 oz to each.	**a** $6(x+3)=24$

There are 6 groups in the first story and each group spends \$$x$ + \$3. There are 3 groups in the second story and each group has x oz + 6 oz.

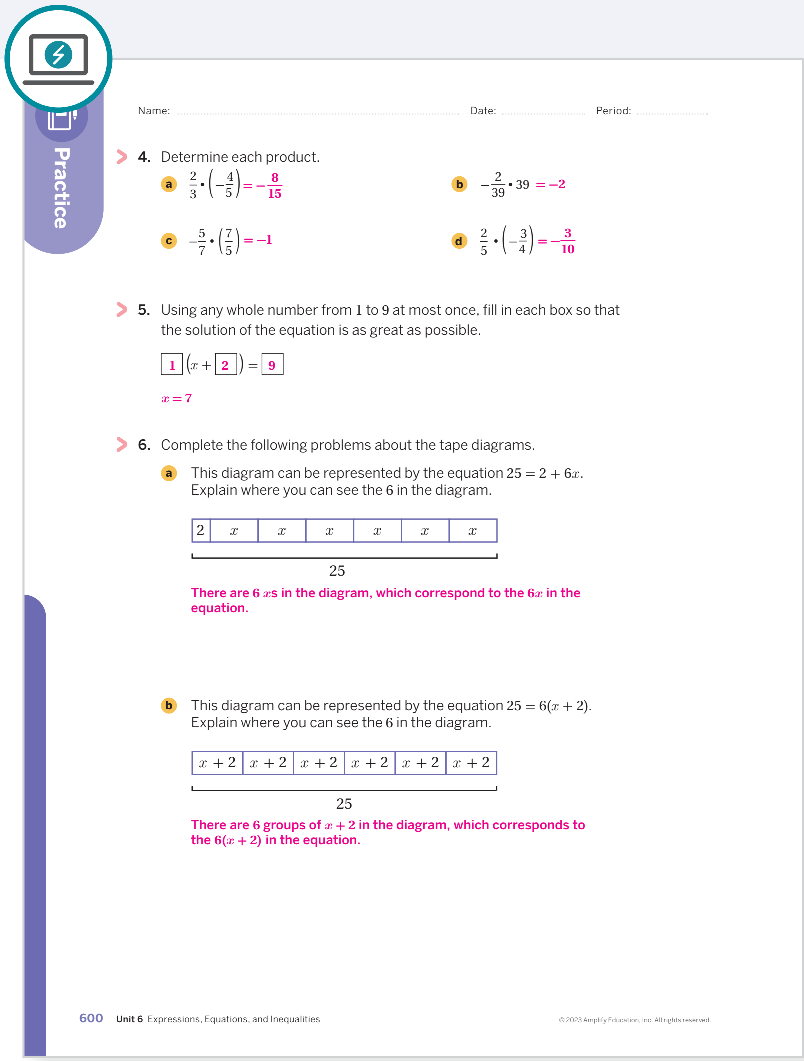
3. Solve each equation. Show your thinking.

a $45=4(x+6)$

$$45 \div 4 = 4(x+6) \div 4$$
$$11\frac{1}{4} = x + 6$$
$$11\frac{1}{4} - 6 = x + 6 - 6$$
$$5\frac{1}{4} = x$$

b $\frac{2}{5}(x+8)=12$

$$\frac{2}{5}(x+8) \div \frac{2}{5} = 12 \div \frac{2}{5}$$
$$x + 8 = 30$$
$$x + 8 - 8 = 30 - 8$$
$$x = 22$$



Practice

Name: ______ Date: ______ Period: ______

4. Determine each product.

a $\frac{2}{3} \cdot \left(-\frac{4}{5}\right) = -\frac{8}{15}$

b $-\frac{2}{39} \cdot 39 = -2$

c $-\frac{5}{7} \cdot \left(\frac{7}{5}\right) = -1$

d $\frac{2}{5} \cdot \left(-\frac{3}{4}\right) = -\frac{3}{10}$

5. Using any whole number from 1 to 9 at most once, fill in each box so that the solution of the equation is as great as possible.

$\boxed{1}\left(x+\boxed{2}\right)=\boxed{9}$

$x=7$

6. Complete the following problems about the tape diagrams.

a This diagram can be represented by the equation $25=2+6x$. Explain where you can see the 6 in the diagram.

2	x	x	x	x	x	x

25

There are 6 xs in the diagram, which correspond to the $6x$ in the equation.

b This diagram can be represented by the equation $25=6(x+2)$. Explain where you can see the 6 in the diagram.

$x+2$	$x+2$	$x+2$	$x+2$	$x+2$	$x+2$

25

There are 6 groups of $x+2$ in the diagram, which corresponds to the $6(x+2)$ in the equation.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	1
Spiral	4	Unit 5 Lesson 11	1
	5	Unit 6 Lesson 9	3
Formative ⚡	6	Unit 6 Lesson 11	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

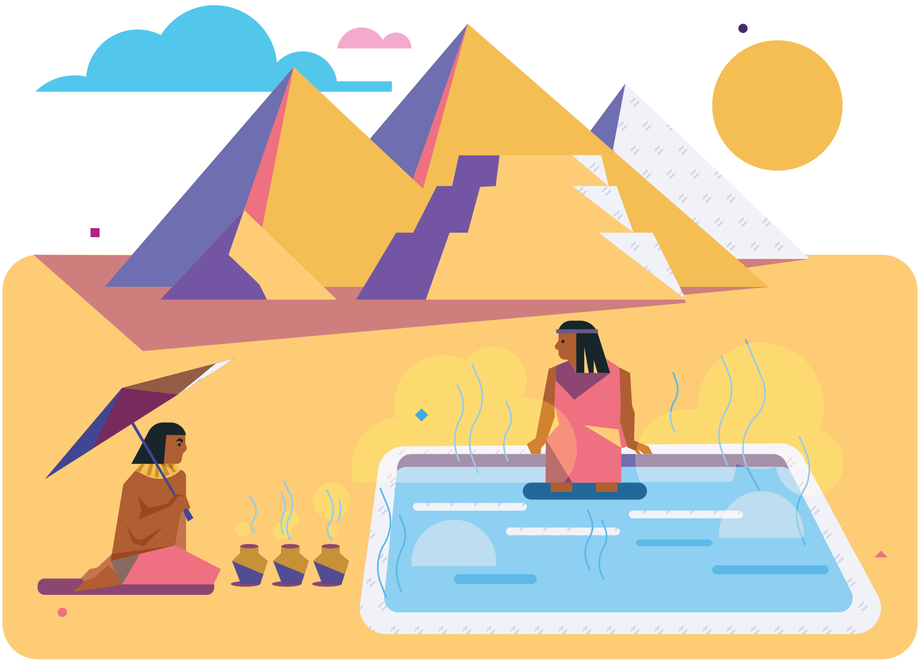
Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 11**

Using Equations to Solve Problems

Let's use equations with and without parentheses to solve problems.



Focus

Goals

1. **Language Goal:** Interpret a written description of a situation and write and solve an equation of the form $px + q = r$ or $p(x + q) = r$ to represent it. **(Reading)**
2. Write an equation of the form $px + q = r$ or $p(x + q) = r$ to represent a situation involving signed numbers.

Coherence

Today

Students use tape diagrams and reasoning to decide whether $px + q = r$ or $p(x + q) = r$ describes the relationships in a real-world story problem. Then they write and solve the equation algebraically.

Previously

In Lesson 9, students used tape diagrams and equations of the form $px + q = r$ to describe relationships in real-world problems. In Lesson 10, they did the same with equations of the form $p(x + q) = r$.

Coming Soon

In Lessons 16, 17, and 18, students will write inequalities of the form $px + q < r$ and $p(x + q) < r$ to describe relationships in real-world problems and solve them algebraically.

Rigor

- Students use tape diagrams to build **conceptual understanding** of writing equations from verbal descriptions of the form $px + q = r$ and $p(x + q) = r$ involving negative numbers.
- Students develop **procedural fluency** in writing and solving equations from verbal descriptions of the form $px + q = r$ and $p(x + q) = r$ with negative numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	12 min	5 min	8 min
Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- sticky notes
- tools for creating a visual display (chart paper, markers, etc)

Math Language Development

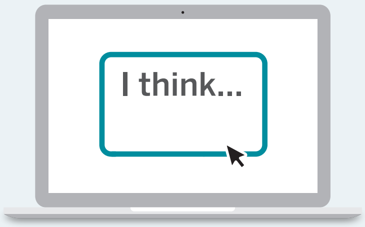
Review words

- *equation*
- *equivalent equations*
- *solution to an equation*
- *variable*

Amps | Featured Activity

Warm-up
See Student Thinking

Students are asked to explain their thinking behind choosing an equation that doesn't belong, and these explanations are passed to you.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Before starting the solution process in Activity 2, students must take time to define the variable. Defining the variable is similar to identifying the problem in a situation. When students understand what problem they are trying to solve or what variable they are trying to solve for, they can make better decisions about how to approach the task.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

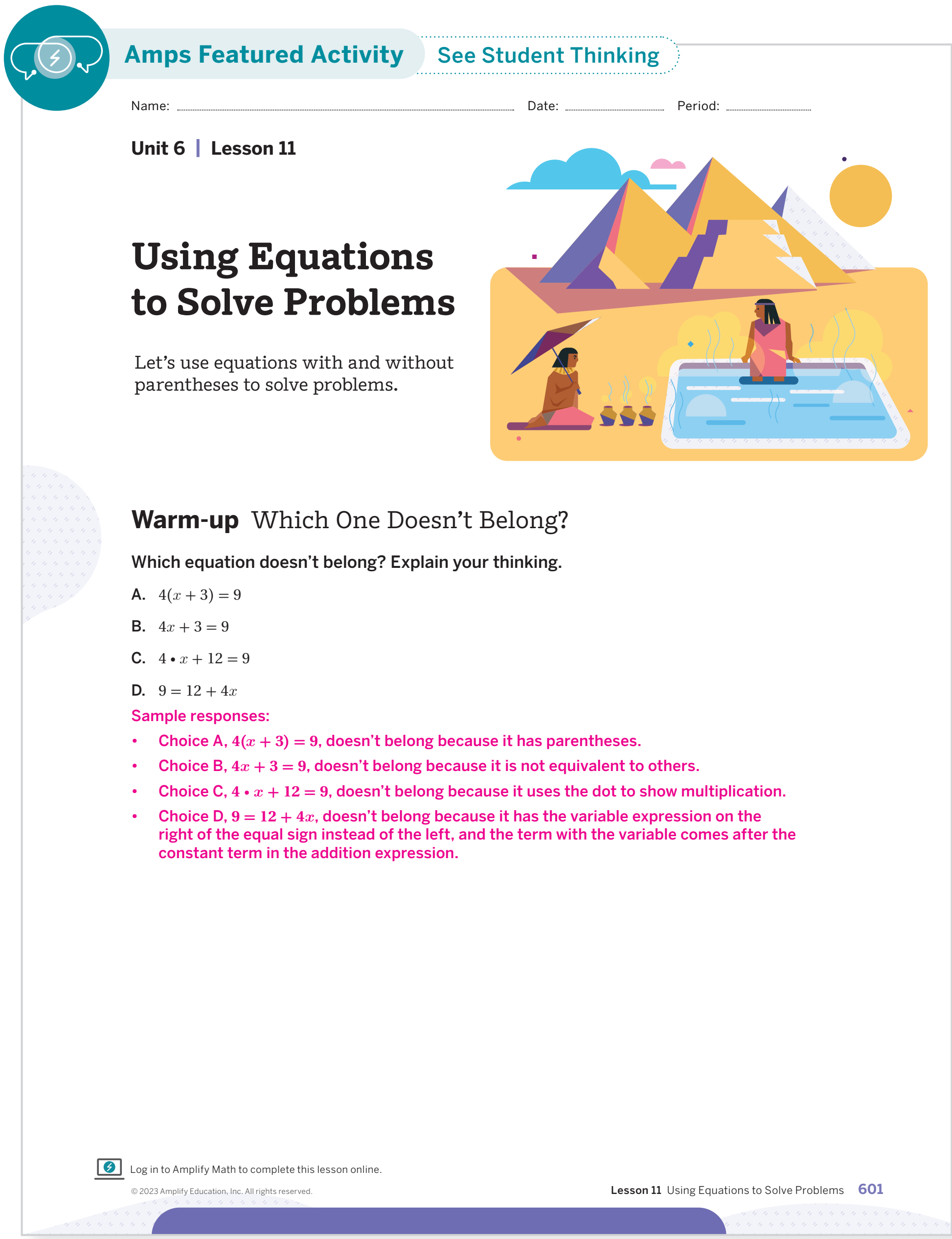
- The **Warm-up** may be omitted.
- For Problems 2 and 3 in **Activity 1**, have students choose one scenario.
- In **Activity 2**, have half of each group complete each problem and then compare their strategies as a group.

Small Groups | 5 min

Warm-up Which One Doesn't Belong?

Students analyze four equations to determine which one doesn't belong.

Amps Featured Activity See Student Thinking



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Unit 6 | Lesson 11

Using Equations to Solve Problems

Let's use equations with and without parentheses to solve problems.

Warm-up Which One Doesn't Belong?

Which equation doesn't belong? Explain your thinking.

A. $4(x+3)=9$

B. $4x+3=9$

C. $4 \cdot x+12=9$

D. $9=12+4x$

Sample responses:

- Choice A, $4(x+3)=9$, doesn't belong because it has parentheses.
- Choice B, $4x+3=9$, doesn't belong because it is not equivalent to others.
- Choice C, $4 \cdot x+12=9$, doesn't belong because it uses the dot to show multiplication.
- Choice D, $9=12+4x$, doesn't belong because it has the variable expression on the right of the equal sign instead of the left, and the term with the variable comes after the constant term in the addition expression.

Log in to Amplify Math to complete this lesson online.

Lesson 11 Using Equations to Solve Problems 601

1 Launch

Conduct the ***Which One Doesn't Belong*** routine.

2 Monitor

Look for points of confusion:

- **Thinking that there is only one correct answer.** Challenge students who claim they are finished after finding one reason why one equation doesn't belong, and challenge them to find reasons why other equations don't belong.

3 Connect

Display the four equations.

Have students share one reason why a particular equation doesn't belong. Ask the rest of the class whether they agree or disagree. Since there is no single correct answer, attend to students' explanations and ensure the reasons they give are correct. Students should explain the meaning of any mathematical terminology they use, such as *coefficient* or *solution*.

Highlight that Equation B is the only one that is not equivalent to the others by asking:

- "Imagine a group of students wrote these equations to match a story. Which equation doesn't describe the same story as the others? Why not?" Equation B; because it is not equivalent to the others.
- "Imagine a student started with Equation A, distributed, and ended up with Equation B. What mistake did they make?" They forgot to multiply 4 by both terms in the expression.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their reasons for why a particular equation does not belong, collect and display the language they use, such as *parentheses, equivalent, expressions, variable term, multiplication dot*, and *constant term*. Add these terms and phrases to the class display.

English Learners

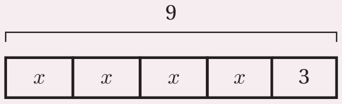
Annotate the equations with the language used to describe them.

Power-up

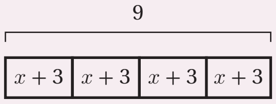
To power up students' ability to reason about how tape diagrams model equations, have students complete:

Write each equation under the tape diagram that matches it.

$4(x+3)=9$ $4x+3=9$



$4x+3=9$



$4(x+3)=9$

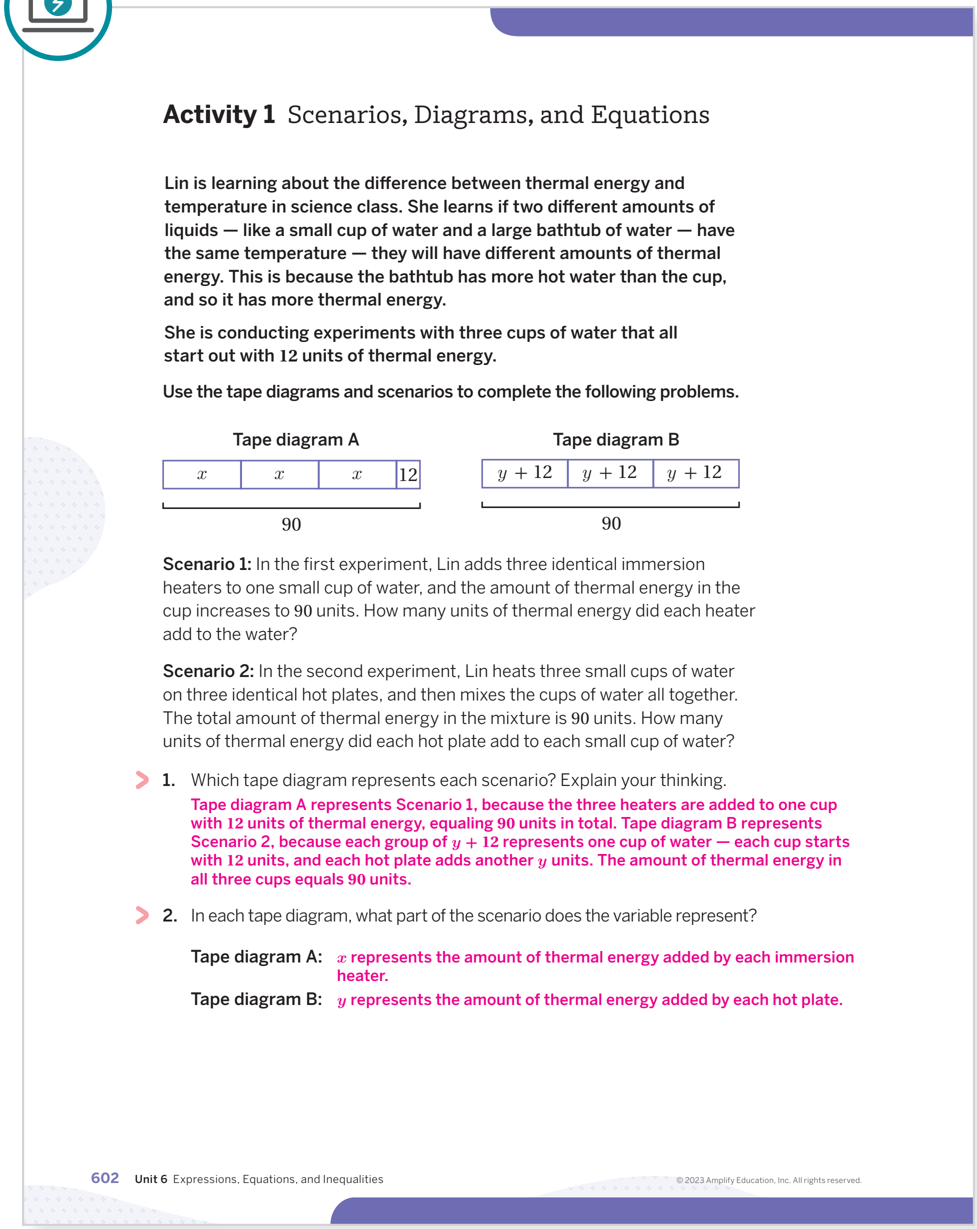
Use: Before Activity 1

Informed by: Performance on Lesson 10, Practice Problem 6

Small Groups | 15 min

Activity 1 Scenarios, Diagrams, and Equations

Students make connections between tape diagrams, equations, and real-world story problems to contrast the two main types of equations studied in this unit.



Activity 1 Scenarios, Diagrams, and Equations

Lin is learning about the difference between thermal energy and temperature in science class. She learns if two different amounts of liquids — like a small cup of water and a large bathtub of water — have the same temperature — they will have different amounts of thermal energy. This is because the bathtub has more hot water than the cup, and so it has more thermal energy.

She is conducting experiments with three cups of water that all start out with 12 units of thermal energy.

Use the tape diagrams and scenarios to complete the following problems.

Tape diagram A

x	x	x	12

90

Tape diagram B

$y + 12$	$y + 12$	$y + 12$

90

Scenario 1: In the first experiment, Lin adds three identical immersion heaters to one small cup of water, and the amount of thermal energy in the cup increases to 90 units. How many units of thermal energy did each heater add to the water?

Scenario 2: In the second experiment, Lin heats three small cups of water on three identical hot plates, and then mixes the cups of water all together. The total amount of thermal energy in the mixture is 90 units. How many units of thermal energy did each hot plate add to each small cup of water?

1. Which tape diagram represents each scenario? Explain your thinking.
   Tape diagram A represents Scenario 1, because the three heaters are added to one cup with 12 units of thermal energy, equaling 90 units in total. Tape diagram B represents Scenario 2, because each group of $y + 12$ represents one cup of water — each cup starts with 12 units, and each hot plate adds another y units. The amount of thermal energy in all three cups equals 90 units.

2. In each tape diagram, what part of the scenario does the variable represent?

   Tape diagram A: **x represents the amount of thermal energy added by each immersion heater.**

   Tape diagram B: **y represents the amount of thermal energy added by each hot plate.**

602 **Unit 6** Expressions, Equations, and Inequalities

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1 Launch

Read and discuss the passage as a class to ensure students understand the situation before working. Then give students a few minutes of individual work time before discussing the problems with their group.

2 Monitor

Help students get started by asking, "How are the tape diagrams alike? How are they different? How are the scenarios alike/different?"

Look for points of confusion:

- **Struggling to explain what the variables represent.** Suggest students ask themselves what the unknown variable is in each situation.

Look for productive strategies:

- Solving the second equation using the Distributive Property. Note students who use this strategy.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Present the two tape diagrams and before students read the scenarios, ask the following questions to help students make sense of the tape diagrams.

- "Which tape diagram shows adding equal groups of a sum?" Tape diagram B
- "Which tape diagram shows adding equal groups of a variable plus one number?" Tape diagram A

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of each scenario.

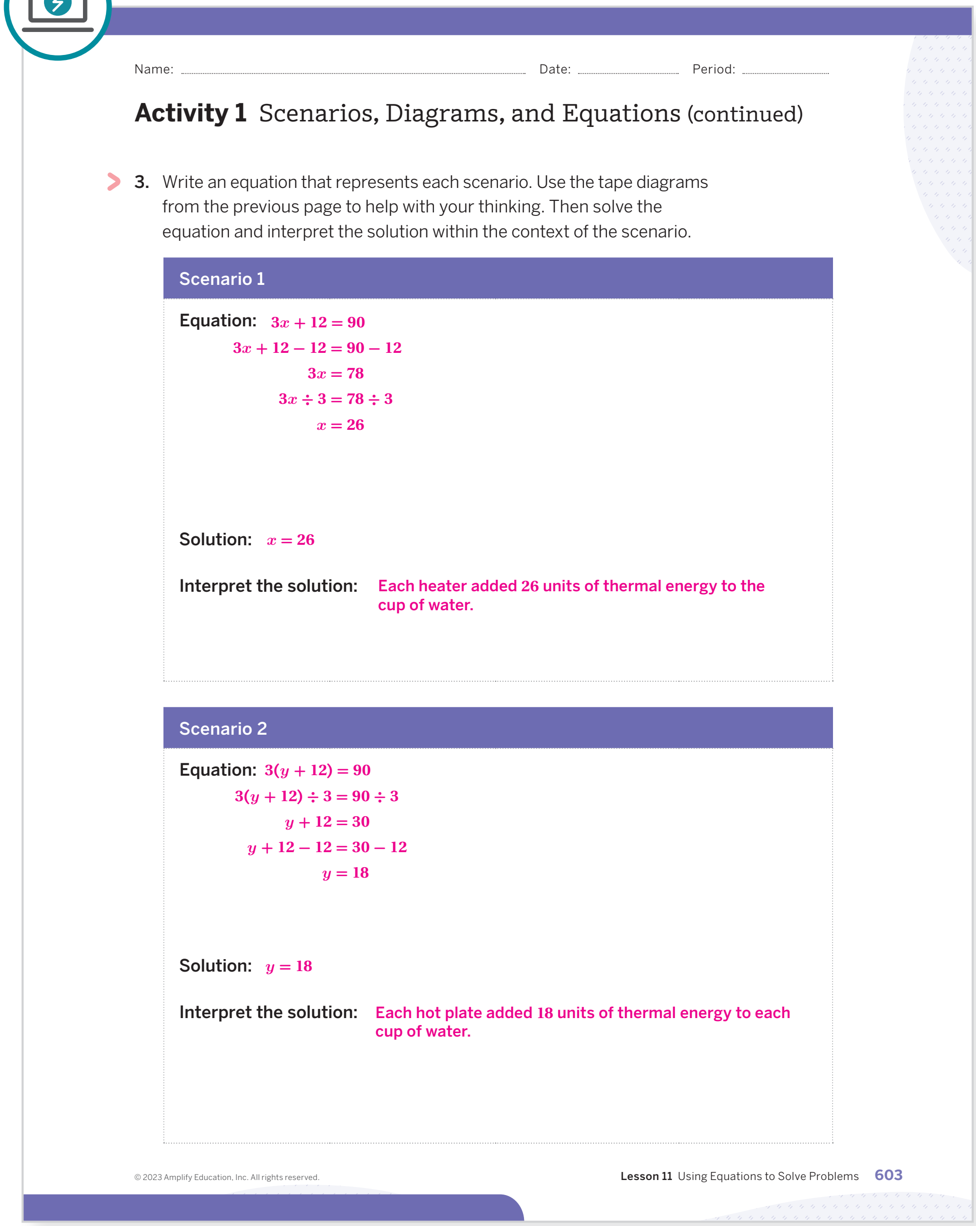
- **Read 1:** Students should understand the basics of each experiment, without paying attention to the quantities.
- **Read 2:** Ask students to identify the important quantities of the problem, such as Lin adding three identical heaters to one cup of water in Scenario 1.
- **Read 3:** Ask students to brainstorm strategies for how they can use key words from the text to determine which tape diagram represents each scenario.

English Learners

Have students annotate key words and phrases, such as *three heaters*, *one cup* in Scenario 1, and *three hot plates, three cups* in Scenario 2.

Activity 1 Scenarios, Diagrams, and Equations (continued)

Students make connections between tape diagrams, equations, and real-world story problems to contrast the two main types of equations studied in this unit.



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Activity 1 Scenarios, Diagrams, and Equations (continued)

3. Write an equation that represents each scenario. Use the tape diagrams from the previous page to help with your thinking. Then solve the equation and interpret the solution within the context of the scenario.

Scenario 1

Equation: $3x + 12 = 90$

$3x + 12 - 12 = 90 - 12$

$3x = 78$

$3x \div 3 = 78 \div 3$

$x = 26$

Solution: $x = 26$

Interpret the solution: Each heater added 26 units of thermal energy to the cup of water.

Scenario 2

Equation: $3(y + 12) = 90$

$3(y + 12) \div 3 = 90 \div 3$

$y + 12 = 30$

$y + 12 - 12 = 30 - 12$

$y = 18$

Solution: $y = 18$

Interpret the solution: Each hot plate added 18 units of thermal energy to each cup of water.

Lesson 11 Using Equations to Solve Problems 603

3 Connect

Have students share how they matched each tape diagram to each scenario, and have them share the equations they wrote to represent them.

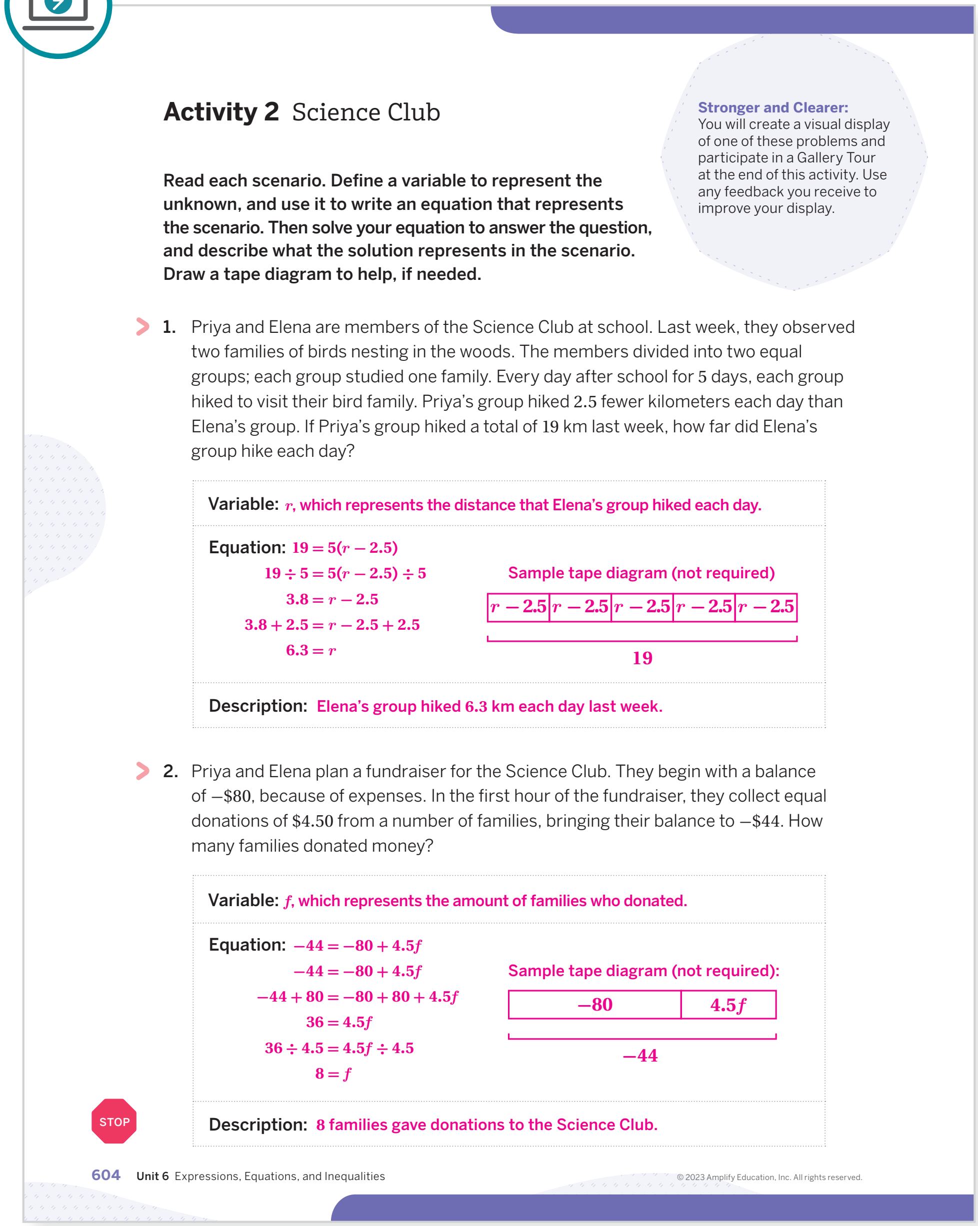
Highlight the aspects of each scenario that result in it being represented by one equation rather than another. Note the difference between the solution to the equation and the answer to the question in the scenario.

Ask, "What parts of the story made you think that one of the two diagrams represented it?"

Small Groups | 12 min

Activity 2 Science Club

Students write and solve equations that include subtraction and negative numbers to represent real-world scenarios.



Activity 2 Science Club

Stronger and Clearer: You will create a visual display of one of these problems and participate in a Gallery Tour at the end of this activity. Use any feedback you receive to improve your display.

Read each scenario. Define a variable to represent the unknown, and use it to write an equation that represents the scenario. Then solve your equation to answer the question, and describe what the solution represents in the scenario. Draw a tape diagram to help, if needed.

1. Priya and Elena are members of the Science Club at school. Last week, they observed two families of birds nesting in the woods. The members divided into two equal groups; each group studied one family. Every day after school for 5 days, each group hiked to visit their bird family. Priya's group hiked 2.5 fewer kilometers each day than Elena's group. If Priya's group hiked a total of 19 km last week, how far did Elena's group hike each day?

Variable: r, which represents the distance that Elena's group hiked each day.

Equation: $19 = 5(r - 2.5)$

$19 \div 5 = 5(r - 2.5) \div 5$

$3.8 = r - 2.5$

$3.8 + 2.5 = r - 2.5 + 2.5$

$6.3 = r$

Sample tape diagram (not required)

$$\underbrace{\boxed{r - 2.5}\boxed{r - 2.5}\boxed{r - 2.5}\boxed{r - 2.5}\boxed{r - 2.5}}_{19}$$

Description: Elena's group hiked 6.3 km each day last week.

2. Priya and Elena plan a fundraiser for the Science Club. They begin with a balance of −\$80, because of expenses. In the first hour of the fundraiser, they collect equal donations of \$4.50 from a number of families, bringing their balance to −\$44. How many families donated money?

Variable: f, which represents the amount of families who donated.

Equation: $-44 = -80 + 4.5f$

$-44 = -80 + 4.5f$

$-44 + 80 = -80 + 80 + 4.5f$

$36 = 4.5f$

$36 \div 4.5 = 4.5f \div 4.5$

$8 = f$

Sample tape diagram (not required):

$$\underbrace{\boxed{\quad -80 \quad}\boxed{4.5f}}_{-44}$$

Description: 8 families gave donations to the Science Club.

STOP

604 Unit 6 Expressions, Equations, and Inequalities

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1 Launch

Alert students that the equations they write for this activity may include subtraction or negative numbers. Suggest they annotate the text and draw a tape diagram to describe each scenario. If time allows, have students create a visual display of one of the problems and and conduct the ***Gallery Tour*** routine.

2 Monitor

Help students get started by suggesting they draw a tape diagram to represent each situation.

Look for points of confusion:

- **Making errors in calculations.** Remind students to check if their solutions are reasonable as a strategy to catch calculation errors.

Look for productive strategies:

- Determining the solution without writing an equation. Suggest students think about the steps they took to find the solution and consider an equation that could be solved using the same steps.

3 Connect

Display students' visual displays.

Have students share their work through a ***Gallery Tour***. Provide sticky notes. Have students silently view their classmates' work and leave comments about what they see. Then give each group time to review the notes left on their work and discuss with, or ask for clarification from, the class. If time allows, each group can also give a short presentation about their work.

Highlight that, despite the fact the first equation uses subtraction instead of addition and the second equation uses negative numbers, the equations in this activity still fit the forms $px + q = r$ and $p(x + q) = r$ because the variables p, q, and r can have negative or positive values and students can rewrite subtraction as addition of a negative number.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code or annotate the quantities and relationships in the text that help them write an equation or draw a tape diagram. For example, in Problem 1, they could color code 5 days in one color and 19 km in another color.

Extension: *Math Enrichment*

Have students explain how the equation, tape diagram (if they drew one), and solution would be altered for Problem 1 if Priya's group hiked half of the distance each day that Elena's group hiked, but everything else remained the same. The equation would become $19 = 5(0.5r)$ and the solution would be 7.6 km.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, as students participate in the ***Gallery Tour***, give them time to discuss the sticky notes their classmates added to their work. Have them ask each other clarifying questions. Then have groups of students revise and improve their displays before presenting to the class.

English Learners

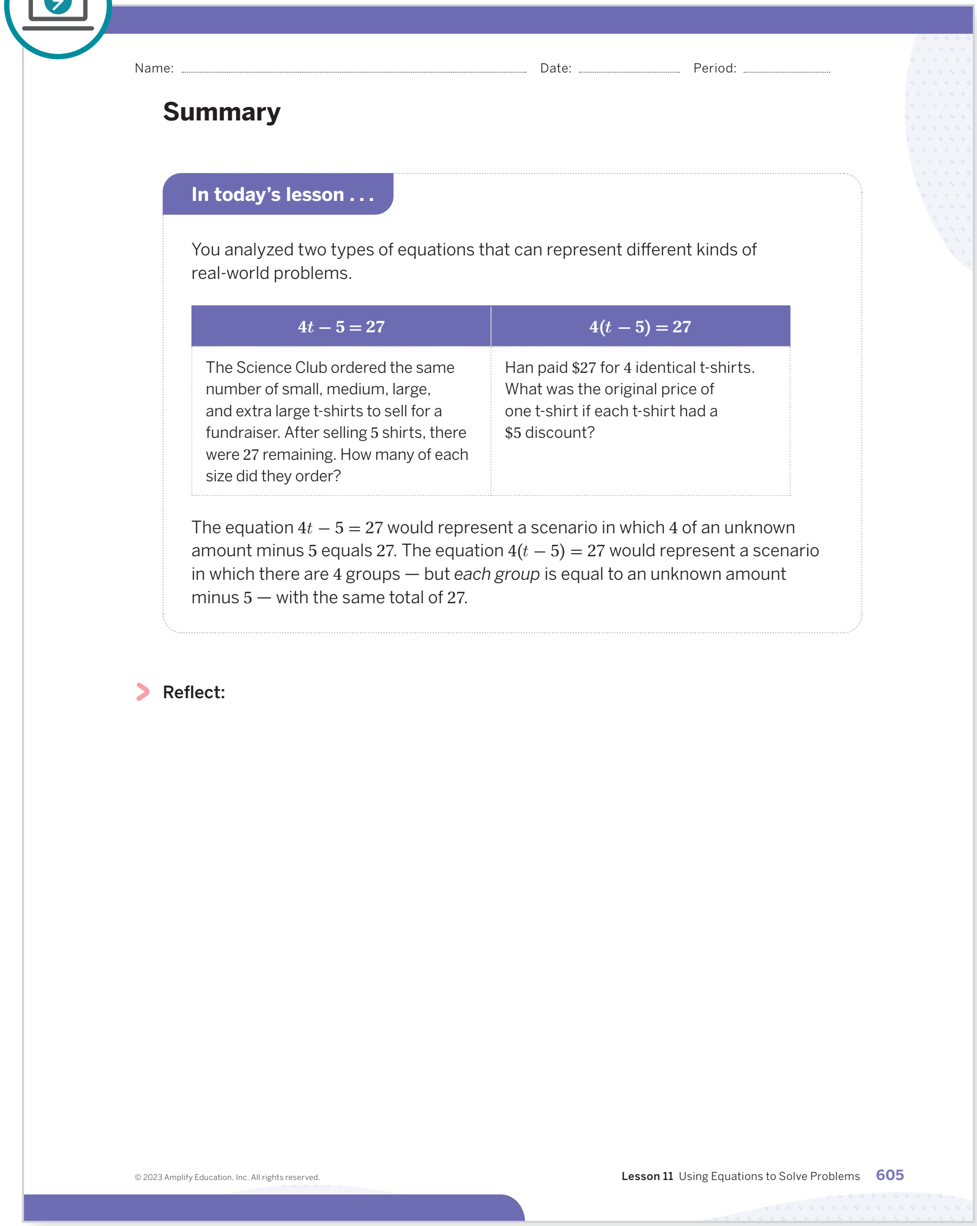
Provide samples of clarifying questions students could ask each other, such as:

- "How did you define the variable?"
- "How did you solve the equation? What strategy did you use?"
- "What clues from the scenario helped you interpret the meaning of the solution?"

Whole Class | 5 min

Summary

Review and synthesize how equations of the form $px + q = r$ represents "(p groups of x) added to q equals r," and equations of the form $p(x + q) = r$ represents "p groups of the quantity $(x + q)$ equals r."



Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You analyzed two types of equations that can represent different kinds of real-world problems.

$4t - 5 = 27$	$4(t - 5) = 27$
The Science Club ordered the same number of small, medium, large, and extra large t-shirts to sell for a fundraiser. After selling 5 shirts, there were 27 remaining. How many of each size did they order?	Han paid \$27 for 4 identical t-shirts. What was the original price of one t-shirt if each t-shirt had a \$5 discount?

The equation $4t - 5 = 27$ would represent a scenario in which 4 of an unknown amount minus 5 equals 27. The equation $4(t - 5) = 27$ would represent a scenario in which there are 4 groups — but *each group* is equal to an unknown amount minus 5 — with the same total of 27.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Highlight the connection between the equations and word problems by annotating each scenario and its accompanying equation to show how the equation represents the scenario. Discuss which aspects of each scenario result in an equation of the form $px + q = r$ or $p(x + q) = r$. Note that the first equation fits the form $px + q = r$ even though the operation is subtraction instead of addition. This can be shown by rewriting the equation as $4t + (-5) = 27$.

If time allows, challenge students to edit each scenario so that it would be modeled with the other type of equation. For example, in the first problem, if the science club sold five of each size of the t-shirt, instead of five t-shirts, then the equation representing the problem would be of the other form.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

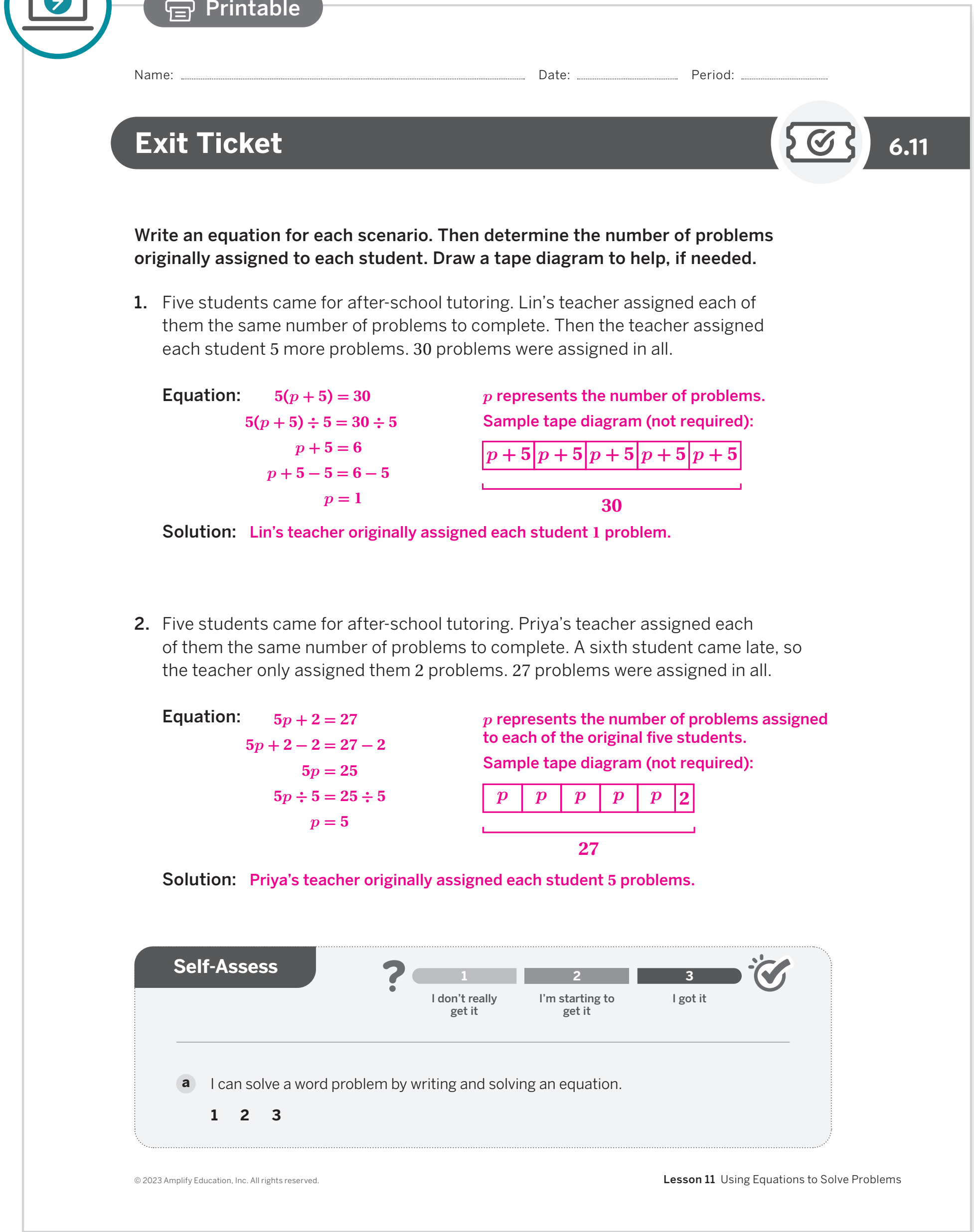
- "What did you notice during your ***Gallery Tour***? Did any other students use the same strategies as you? Did anyone use a different strategy?"

Independent | 8 min

Exit Ticket

Students demonstrate their understanding by writing and solving an equation to answer a question about a real-world scenario.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 6.11

Write an equation for each scenario. Then determine the number of problems originally assigned to each student. Draw a tape diagram to help, if needed.

1. Five students came for after-school tutoring. Lin's teacher assigned each of them the same number of problems to complete. Then the teacher assigned each student 5 more problems. 30 problems were assigned in all.

Equation:
$5(p+5)=30$
$5(p+5)\div 5=30\div 5$
$p+5=6$
$p+5-5=6-5$
$p=1$

p represents the number of problems.

Sample tape diagram (not required):

$p+5$	$p+5$	$p+5$	$p+5$	$p+5$

$$\underbrace{\hphantom{p+5\;p+5\;p+5\;p+5\;p+5}}_{30}$$

Solution: Lin's teacher originally assigned each student 1 problem.

2. Five students came for after-school tutoring. Priya's teacher assigned each of them the same number of problems to complete. A sixth student came late, so the teacher only assigned them 2 problems. 27 problems were assigned in all.

Equation:
$5p+2=27$
$5p+2-2=27-2$
$5p=25$
$5p\div 5=25\div 5$
$p=5$

p represents the number of problems assigned to each of the original five students.

Sample tape diagram (not required):

p	p	p	p	p	2

$$\underbrace{\hphantom{p\;p\;p\;p\;p\;2}}_{27}$$

Solution: Priya's teacher originally assigned each student 5 problems.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can solve a word problem by writing and solving an equation.
1 2 3

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Lesson 11 Using Equations to Solve Problems

Success looks like . . .

- **Language Goal:** Interpreting a written description of a situation and writing and solving an equation of the form $px+q=r$ or $p(x+q)=r$ to represent it. **(Reading)**
  - » Solving the written equation and interpreting the solution in Problems 1 and 2.
- **Goal:** Writing an equation of the form $px+q=r$ or $p(x+q)=r$ to represent a situation involving signed numbers.
  - » Writing equations to represent both scenarios in Problems 1 and 2.

Suggested next steps

If students write the same form of equation for both problems, consider:

- Having them reread the scenarios and identify the difference between them. Then they can draw a tape diagram for each scenario, considering the differences they identified, and write an equation for each diagram.
- Assigning Practice Problem 1.

If students solve the problem, but neglect to write an equation, consider:

- Having them draw a tape diagram for each problem and write the equation based on their diagram.
- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about Activity 1, how did you encourage each student to listen to one another's strategies?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Math Language Development

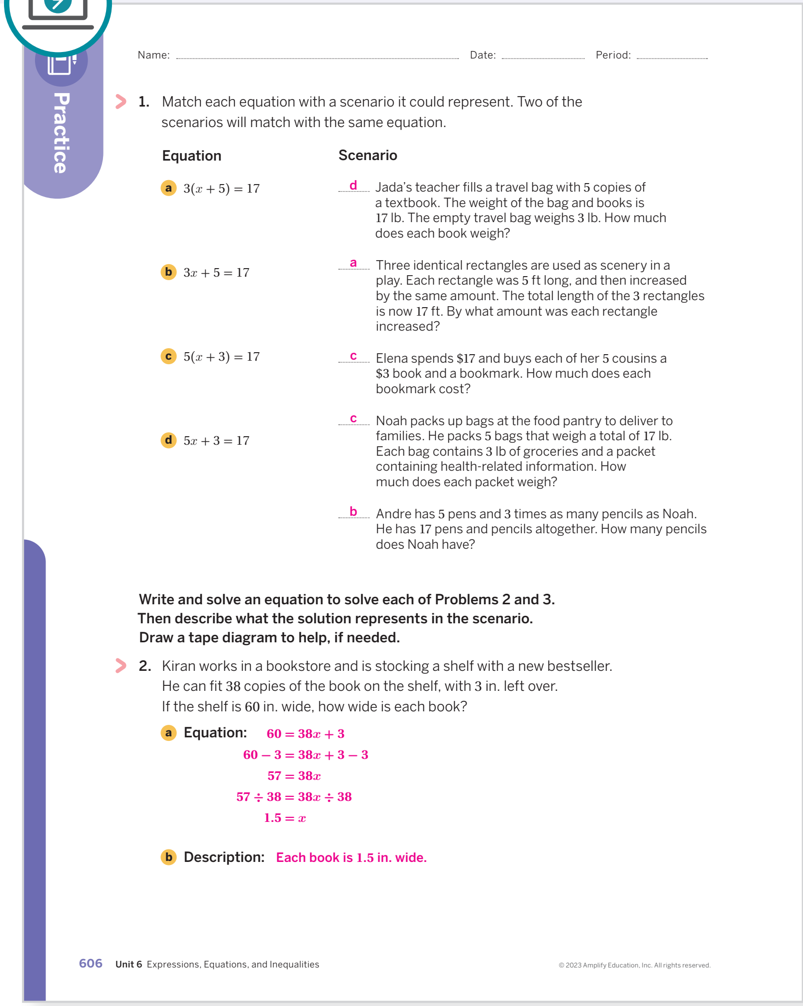
Language Goal: Interpreting a written description of a situation and writing and solving an equation of the form $px+q=r$ or $p(x+q)=r$ to represent it.

Reflect on students' language development toward this goal.

- How did using the *Three Reads* and *Stronger and Clearer Each Time* routines in Activities 1 and 2 help students make sense of written descriptions of situations and how to define variables?
- How are students looking for key words in the written descriptions to help them write an equation to represent the situation?

Practice

Independent



Name: Date: Period:

1. Match each equation with a scenario it could represent. Two of the scenarios will match with the same equation.

Equation	Scenario
a $3(x+5)=17$	d Jada's teacher fills a travel bag with 5 copies of a textbook. The weight of the bag and books is 17 lb. The empty travel bag weighs 3 lb. How much does each book weigh?
b $3x+5=17$	a Three identical rectangles are used as scenery in a play. Each rectangle was 5 ft long, and then increased by the same amount. The total length of the 3 rectangles is now 17 ft. By what amount was each rectangle increased?
c $5(x+3)=17$	c Elena spends $17 and buys each of her 5 cousins a $3 book and a bookmark. How much does each bookmark cost?
d $5x+3=17$	c Noah packs up bags at the food pantry to deliver to families. He packs 5 bags that weigh a total of 17 lb. Each bag contains 3 lb of groceries and a packet containing health-related information. How much does each packet weigh?
	b Andre has 5 pens and 3 times as many pencils as Noah. He has 17 pens and pencils altogether. How many pencils does Noah have?

Write and solve an equation to solve each of Problems 2 and 3. Then describe what the solution represents in the scenario. Draw a tape diagram to help, if needed.

2. Kiran works in a bookstore and is stocking a shelf with a new bestseller. He can fit 38 copies of the book on the shelf, with 3 in. left over. If the shelf is 60 in. wide, how wide is each book?

a **Equation:** $60 = 38x + 3$

$60 - 3 = 38x + 3 - 3$

$57 = 38x$

$57 \div 38 = 38x \div 38$

$1.5 = x$

b **Description:** Each book is 1.5 in. wide.

606 Unit 6 Expressions, Equations, and Inequalities

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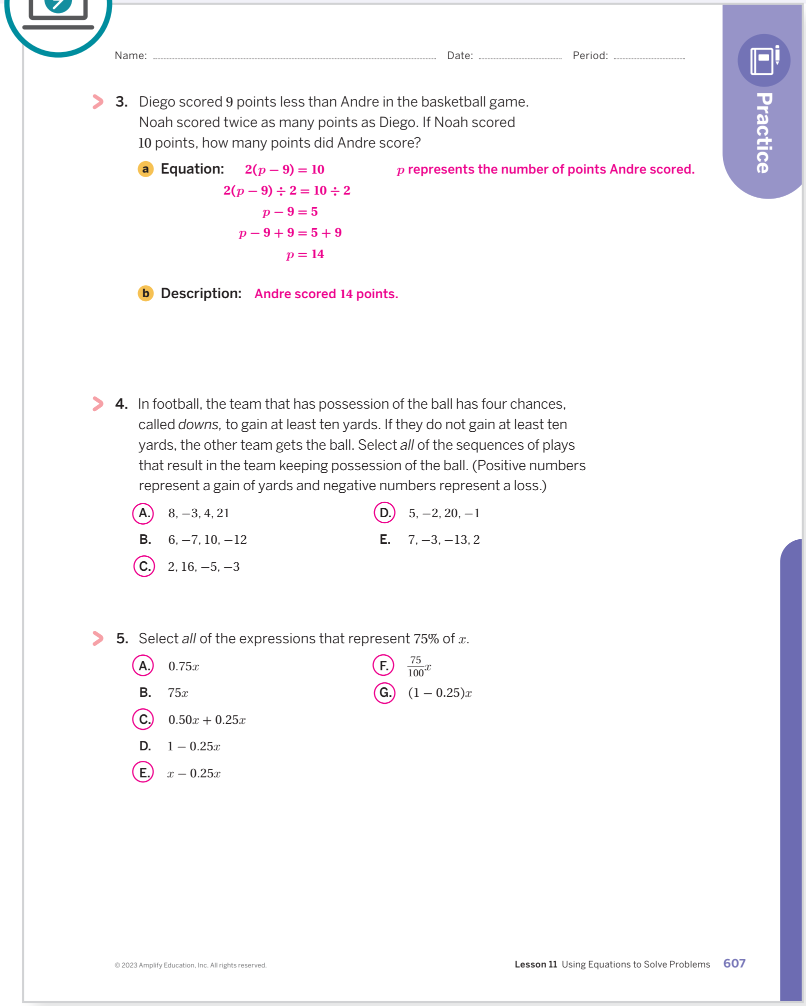

Name: Date: Period:

3. Diego scored 9 points less than Andre in the basketball game. Noah scored twice as many points as Diego. If Noah scored 10 points, how many points did Andre score?

a **Equation:** $2(p-9)=10$ p represents the number of points Andre scored.

$2(p-9) \div 2 = 10 \div 2$

$p - 9 = 5$

$p - 9 + 9 = 5 + 9$

$p = 14$

b **Description:** Andre scored 14 points.

4. In football, the team that has possession of the ball has four chances, called *downs,* to gain at least ten yards. If they do not gain at least ten yards, the other team gets the ball. Select *all* of the sequences of plays that result in the team keeping possession of the ball. (Positive numbers represent a gain of yards and negative numbers represent a loss.)

[✓ A.] $8, -3, 4, 21$
B. $6, -7, 10, -12$
[✓ C.] $2, 16, -5, -3$
[✓ D.] $5, -2, 20, -1$
E. $7, -3, -13, 2$

5. Select *all* of the expressions that represent 75% of x.

[✓ A.] $0.75x$
B. $75x$
[✓ C.] $0.50x + 0.25x$
D. $1 - 0.25x$
[✓ E.] $x - 0.25x$
[✓ F.] $\frac{75}{100}x$
[✓ G.] $(1 - 0.25)x$

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Lesson 11 Using Equations to Solve Problems 607

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 4	2
Formative ⚡	5	Unit 6 Lesson 12	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

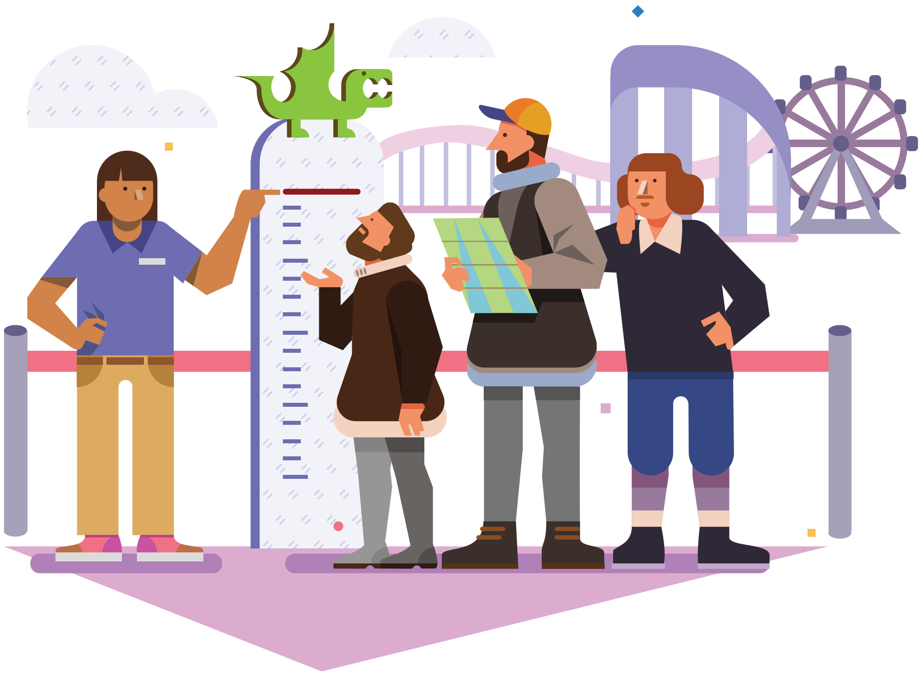
Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 12**

Solving Percent Problems in New Ways

Let's use tape diagrams, equations, and reasoning to solve problems with negative numbers and percents.



Focus

Goals

1. **Language Goal:** Solve word problems leading to equations of the form $px + q = r$ or $p(x + q) = r$. **(Reading)**
2. **Language Goal:** Solve word problems involving percentages leading to equations of the form $px + q = r$ or $p(x + q) = r$. **(Reading)**

Coherence

Today

Today students learn to represent multi-step percent increase and decrease problems using tape diagrams and equations. They then use these representations to solve problems.

Previously

In Lesson 11, students used tape diagrams to represent scenarios leading to equations of the form $px + q = r$ and $p(x + q) = r$. In Unit 4, students used tape diagrams and equations to solve percent problems.

Coming Soon

In Sub-Unit 3, students will apply their understanding of solving equations to solving inequalities.

Rigor

- Students **apply** their understanding of solving equations of the form $px + q = r$ and $p(x + q) = r$ to problems involving percent increase and decrease.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

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For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *discount*
- *equation*
- *equivalent equations*
- *percent*
- *solution to an equation*
- *variable*

Amps | Featured Activity

Exit Ticket
Choose Your Own Adventure

Students choose their preferred solution strategy for the Exit Ticket and are guided appropriately.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel bored with learning about another tool that they can use to help better understand equations. Remind students that they will be better equipped to solve any problem with more tools in the mathematical toolbox. Encourage them to look for similarities and differences, and while they might have a particular favorite, stress that they should not discount any of the tools, for they can all serve a good purpose.

Modifications to Pacing

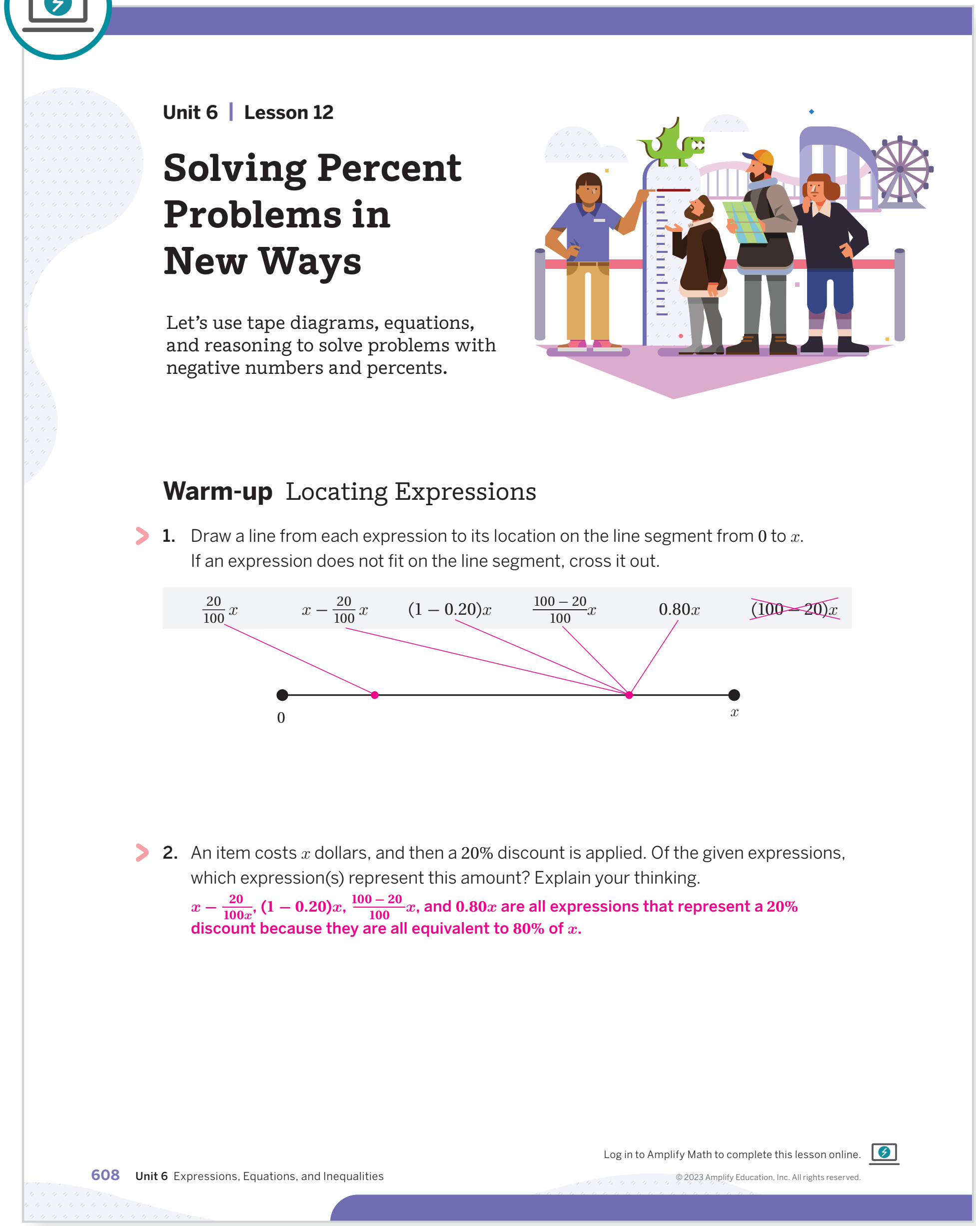
You may want to consider these additional modifications if you are short on time.

- During the **Warm-up**, instead of having students write their response to Problem 2, discuss the question during the Connect.
- Have students choose one problem to complete in **Activity 2**.

Pairs | 10 min

Warm-up Locating Expressions

Students place expressions on a number line in order to identify equivalent expressions and make connections with percents.



Unit 6 | Lesson 12

Solving Percent Problems in New Ways

Let's use tape diagrams, equations, and reasoning to solve problems with negative numbers and percents.

Warm-up Locating Expressions

1. Draw a line from each expression to its location on the line segment from 0 to x. If an expression does not fit on the line segment, cross it out.

$\frac{20}{100}x$ $\quad x - \frac{20}{100}x$ $\quad (1 - 0.20)x$ $\quad \frac{100 - 20}{100}x$ $\quad 0.80x$ $\quad$ ~~$(100 - 20)x$~~

0 $\quad$ x

2. An item costs x dollars, and then a 20% discount is applied. Of the given expressions, which expression(s) represent this amount? Explain your thinking.

$x - \frac{20}{100x}$, $(1 - 0.20)x$, $\frac{100 - 20}{100}x$, and $0.80x$ are all expressions that represent a 20% discount because they are all equivalent to 80% of x.

Log in to Amplify Math to complete this lesson online.

608 Unit 6 Expressions, Equations, and Inequalities

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1 Launch

Ensure students notice that the number line has 0 on the left and x on the right.

2 Monitor

Help students get started by saying, "x could have any value, but it may help to give it a value of your choice to understand where the other expressions should be placed in relationship to x."

Look for points of confusion:

- **Getting stuck on computations.** Ask, "What if you substitute a friendly value for x. How about 10?"
- **Having difficulty understanding the values on the number line.** Ask, "If you think of this like a percent bar, where would $\frac{1}{2}x$ be placed?"
- **Disregarding the difference between $0.80x$ and $80x$.** Ask, "What if you thought about those numbers as money? Would they have the same value?"
- **Equating "20% of" a price with "20% off."** Ask, "What does it mean to take 'money off' of a price?"

3 Connect

Have students share reasoning for why the expressions $(1 - 0.20)x$, $x - \frac{20}{100}x$, $0.80x$, and $\frac{100 - 20}{100}x$ are equivalent.

Ask, "How is this number line similar to a percent bar? Where would 0% be placed? Where would 100% be placed? What percentages could you write about each group of expressions?"

Highlight that there are multiple expressions that can represent the same amount. When dealing with percents, the percent can be expressed in different ways.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

During the Connect, as students share their reasoning for why the expressions they determined in Problem 2 are equivalent, press for details in their reasoning. For example:

If a student says . . .	Ask . . .
"They have the same value."	"How do you know they have the same value? What property or operations allow you to make this claim?"

Power-up

To power up students' ability to identify equivalent expressions involving percents, have students complete:

Recall that *percent* means out of 100 and can be represented as a fraction or a decimal. For example, 40% can also be represented as $\frac{40}{100}$ and 0.40.

Identify *all* of the expressions that are equivalent to 20% of p.

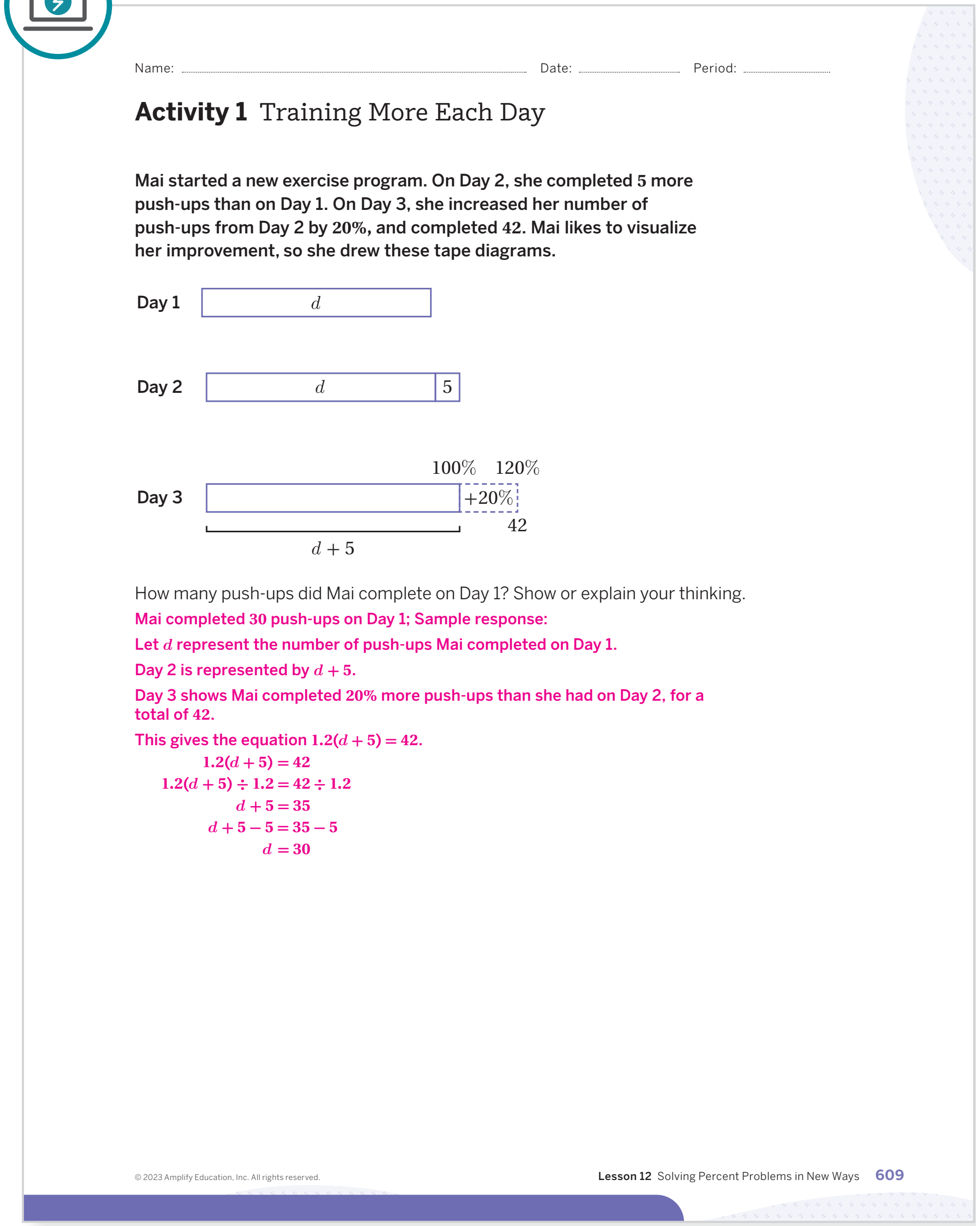
[✓ A.] $0.20p$
[✓ B.] $\frac{20}{100}p$
C. $20p$
[✓ D.] $\frac{1}{5}p$
[✓ E.] $p(1 - 0.80)$
F. $1 - 0.80p$
[✓ G.] $p - 0.80p$
[✓ H.] $\frac{p}{5}$

Use: Before the Warm-up
Informed by: Performance on Lesson 11, Practice Problem 5

Pairs | 10 min

Activity 1 Training More Each Day

Students interpret a diagram that is similar to the tape diagrams they have seen, preparing them to consider representing percent change in equation form.



Name: ______ Date: ______ Period: ______

Activity 1 Training More Each Day

Mai started a new exercise program. On Day 2, she completed 5 more push-ups than on Day 1. On Day 3, she increased her number of push-ups from Day 2 by 20%, and completed 42. Mai likes to visualize her improvement, so she drew these tape diagrams.

How many push-ups did Mai complete on Day 1? Show or explain your thinking.

Mai completed 30 push-ups on Day 1; Sample response:

Let d represent the number of push-ups Mai completed on Day 1.

Day 2 is represented by $d + 5$.

Day 3 shows Mai completed 20% more push-ups than she had on Day 2, for a total of 42.

This gives the equation $1.2(d + 5) = 42$.

$$\begin{aligned} 1.2(d + 5) &= 42 \\ 1.2(d + 5) \div 1.2 &= 42 \div 1.2 \\ d + 5 &= 35 \\ d + 5 - 5 &= 35 - 5 \\ d &= 30 \end{aligned}$$

1 Launch

Read the scenario aloud and be sure students understand how the tape diagram represents the scenario. Activate prior knowledge by asking, "How are these tape diagrams similar to or different from the tape diagrams you created to represent percent change earlier this year?"

2 Monitor

Help students get started by asking, "For which day do you have the most information?"

Look for points of confusion:

- **Trying to estimate the value of x.** Say, "That is a good place to start. Can you be more precise?"
- **Not noticing that Day 3 is modeling a percent increase.** Ask, "What do you notice about the bar for Day 3? How are the values on the top of the bar related to the values on the bottom of the bar?"

Look for productive strategies:

- Writing an equation or reasoning about the tape diagram. Note which students use each strategy.

3 Connect

Have students share their strategies for solving the problem. Bring attention to the connection between the tape diagram, equations, and the original scenario.

Ask, "What similarities do you see when you compare an equation with the values in the diagram?"

Highlight the connections between the diagrams. Point out that by comparing the diagrams for Days 2 and 3, students can assign the Day 2 diagram as representing 100%.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Help students brainstorm a checklist that they can use to solve this problem. A sample checklist for writing and solving an equation is shown.

☐ Decide on a variable and explain what it represents.
☐ How does the number of push-ups on Day 2 compare to Day 1? Represent this in your equation.
☐ How does the number of push-ups on Day 3 compare to Day 2? Represent this in your equation.
☐ What was the total number of push-ups she completed on Day 3? Represent this in your equation.

Math Language Development

MLR7: Compare and Connect

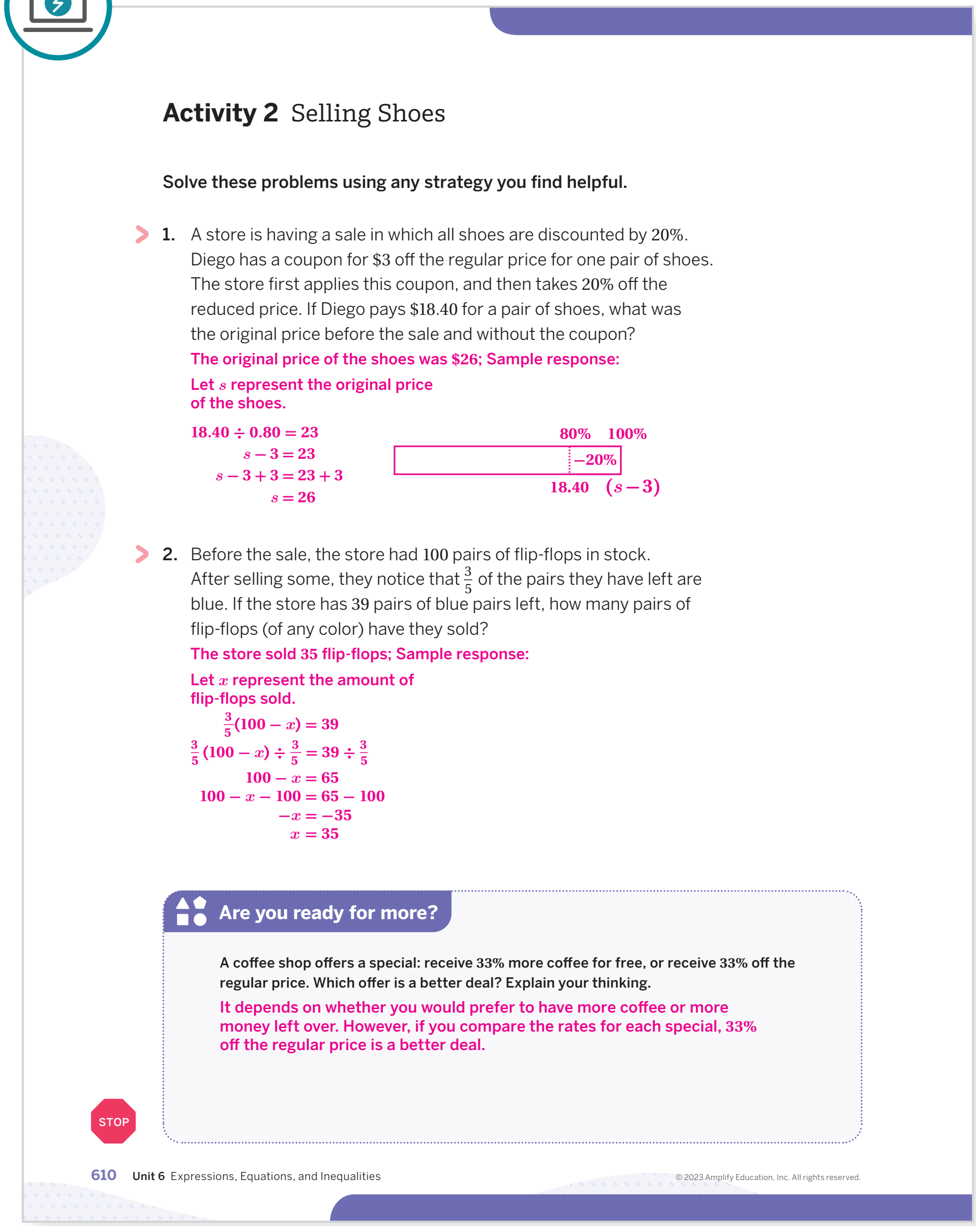
During the Connect, as you highlight the connections between the tape diagrams, ask these questions to help students understand how the diagram represents a percent increase scenario.

- "What happened between Days 1 and 2? How is this illustrated on the diagram for Day 2?"
- "What happened between Days 2 and 3? How is this illustrated on the diagram for Day 3?"
- "Why is 20% added to the diagram for Day 3?"
- "Why does 42 represent 120% of the length $d + 5$ and not 100% of the length $d + 5$?"

Pairs | 15 min

Activity 2 Selling Shoes

Students solve percent problems in context without any visual models provided. This helps students think strategically about their choice of representation.



Activity 2 Selling Shoes

Solve these problems using any strategy you find helpful.

1. A store is having a sale in which all shoes are discounted by 20%. Diego has a coupon for \$3 off the regular price for one pair of shoes. The store first applies this coupon, and then takes 20% off the reduced price. If Diego pays \$18.40 for a pair of shoes, what was the original price before the sale and without the coupon?

   The original price of the shoes was \$26; Sample response:

   Let s represent the original price of the shoes.

   $$18.40 \div 0.80 = 23$$
   $$s - 3 = 23$$
   $$s - 3 + 3 = 23 + 3$$
   $$s = 26$$

2. Before the sale, the store had 100 pairs of flip-flops in stock. After selling some, they notice that $\frac{3}{5}$ of the pairs they have left are blue. If the store has 39 pairs of blue pairs left, how many pairs of flip-flops (of any color) have they sold?

   The store sold 35 flip-flops; Sample response:

   Let x represent the amount of flip-flops sold.

   $$\frac{3}{5}(100 - x) = 39$$
   $$\frac{3}{5}(100 - x) \div \frac{3}{5} = 39 \div \frac{3}{5}$$
   $$100 - x = 65$$
   $$100 - x - 100 = 65 - 100$$
   $$-x = -35$$
   $$x = 35$$

Are you ready for more?

A coffee shop offers a special: receive 33% more coffee for free, or receive 33% off the regular price. Which offer is a better deal? Explain your thinking.

It depends on whether you would prefer to have more coffee or more money left over. However, if you compare the rates for each special, 33% off the regular price is a better deal.

STOP

610 **Unit 6** Expressions, Equations, and Inequalities

1 Launch

Have students refer back to Activity 1 for support as they complete this activity.

2 Monitor

Help students get started by asking, "What is the unknown quantity in each problem? How will you represent that quantity?"

Look for points of confusion:

- **Not knowing which operation to do first.** Say, "Reread the whole scenario. Are there any clues about what operation happens first?"
- **Struggling to write the equations, using a particular form.** Encourage students to try the other form of the equation to see if it helps them to make more sense of the problem.

Look for productive strategies:

- Using a tape diagram. Invite students who use this strategy to share during the Connect.

3 Connect

Display a solved equation and corresponding tape diagram for each problem.

Have students share their equations and tape diagrams, and explain how each represents the information in the problem. Have students explain how they chose to use a particular strategy or tool.

Highlight how each representation (words, equation, and tape diagram) illustrates the same information. Annotate these representations as students share.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose to complete either Problem 1 or Problem 2. Allowing them to choose which problem to complete can increase their engagement and ownership of the task.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of each scenario.

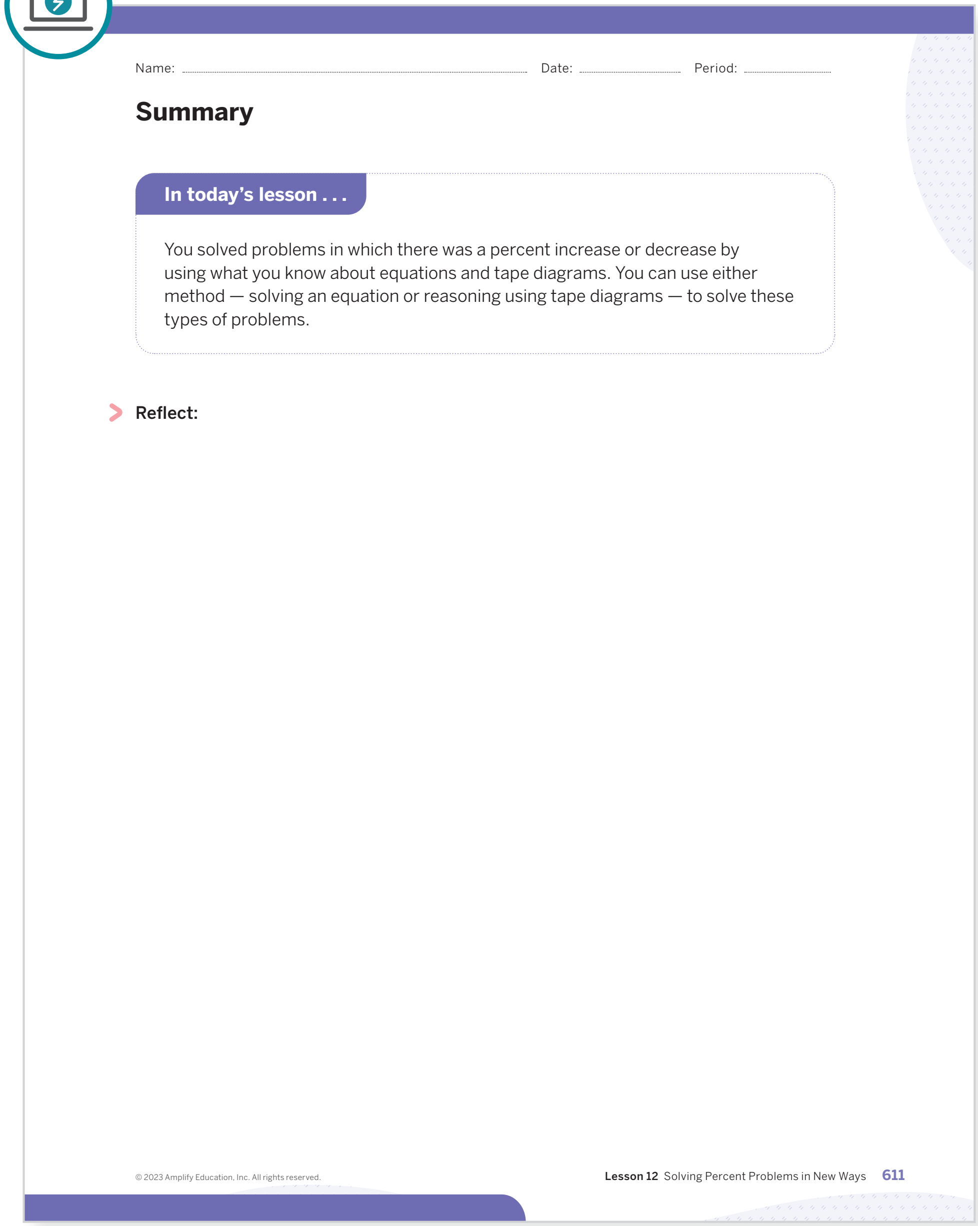
- **Read 1:** Students should understand the context of each scenario, without paying attention to the quantities or relationships. For example, in Problem 1, they should understand that Diego is buying a pair of shoes with a coupon and the store is also having a sale.
- **Read 2:** Ask students to name the important quantities in each scenario, such as the shoes are discounted by \$20% in Problem 1.
- **Read 3:** Ask students to brainstorm possible strategies for solving the problems, such as using a tape diagram or solving an equation.

English Learners

Suggest that students annotate or highlight key words and phrases in the text, such as "the store first applies the coupon" before the discount is applied.

Summary

Review and synthesize how to solve problems with percents using visual representations and equations.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You solved problems in which there was a percent increase or decrease by using what you know about equations and tape diagrams. You can use either method — solving an equation or reasoning using tape diagrams — to solve these types of problems.

Reflect:

Lesson 12 Solving Percent Problems in New Ways 611

Synthesize

Display these questions and ask:

- "What strategies have you learned so far in this unit?"
- "What kinds of problems can you solve now that you were not able to solve previously?"

Have students share their responses to these questions with a partner.

Highlight student responses. Select three students to share what their partner said. Record their answers so that the whole class can see.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

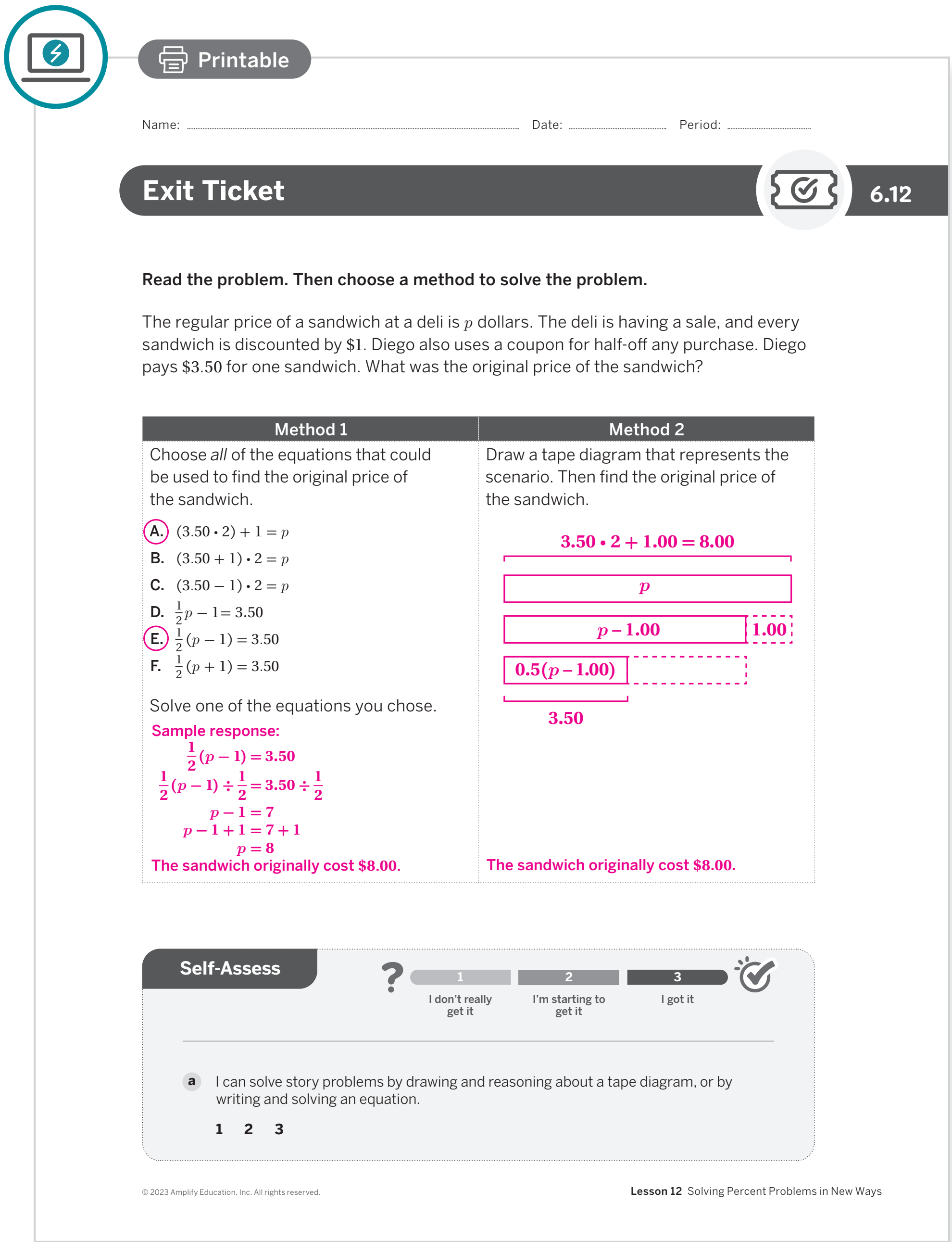
- "Which representations best help you to make sense of certain mathematical scenarios?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving a problem either with a tape diagram or an equation.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 6.12

Read the problem. Then choose a method to solve the problem.

The regular price of a sandwich at a deli is p dollars. The deli is having a sale, and every sandwich is discounted by $1. Diego also uses a coupon for half-off any purchase. Diego pays $3.50 for one sandwich. What was the original price of the sandwich?

Method 1	Method 2
Choose *all* of the equations that could be used to find the original price of the sandwich. [✓ A.] $(3.50 \cdot 2) + 1 = p$ B. $(3.50 + 1) \cdot 2 = p$ C. $(3.50 - 1) \cdot 2 = p$ D. $\frac{1}{2}p - 1 = 3.50$ [✓ E.] $\frac{1}{2}(p - 1) = 3.50$ F. $\frac{1}{2}(p + 1) = 3.50$ Solve one of the equations you chose. Sample response: $\frac{1}{2}(p - 1) = 3.50$ $\frac{1}{2}(p - 1) \div \frac{1}{2} = 3.50 \div \frac{1}{2}$ $p - 1 = 7$ $p - 1 + 1 = 7 + 1$ $p = 8$ The sandwich originally cost $8.00.	Draw a tape diagram that represents the scenario. Then find the original price of the sandwich. The sandwich originally cost $8.00.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can solve story problems by drawing and reasoning about a tape diagram, or by writing and solving an equation.

1 2 3

Success looks like . . .

- **Language Goal:** Solving word problems leading to equations of the form $px + q = r$ or $p(x + q) = r$. **(Reading)**
  - » Determining the solution to the word problem for the original price of the sandwich by selecting the correct equations and solving one of them.
- **Language Goal:** Solving word problems involving percentages leading to equations of the form $px + q = r$ or $p(x + q) = r$. **(Reading)**

Suggested next steps

If students are having difficulty in choosing a method, consider:

- Identifying whether the issue is in representing the scenario or determining the solution. It is possible students may struggle to represent the scenario with an equation or diagram, but still find a valid solution.

If students cannot properly identify the operations or the order in which they occur in the scenario, consider:

- Having them reread the scenario and identify each step separately.
- Assigning Practice Problems 2 and 3.

If students cannot properly identify the unknown amount, consider:

- Having them reread the scenario.
- Asking, "What value do you want to find?"
- Assigning Practice Problems 2 and 3.

If students cannot successfully solve for the value of the unknown, consider:

- Having students refer to previous lessons in this unit where they practiced solving equations of similar types.
- Assigning Practice Problems 2 and 3.

Professional Learning

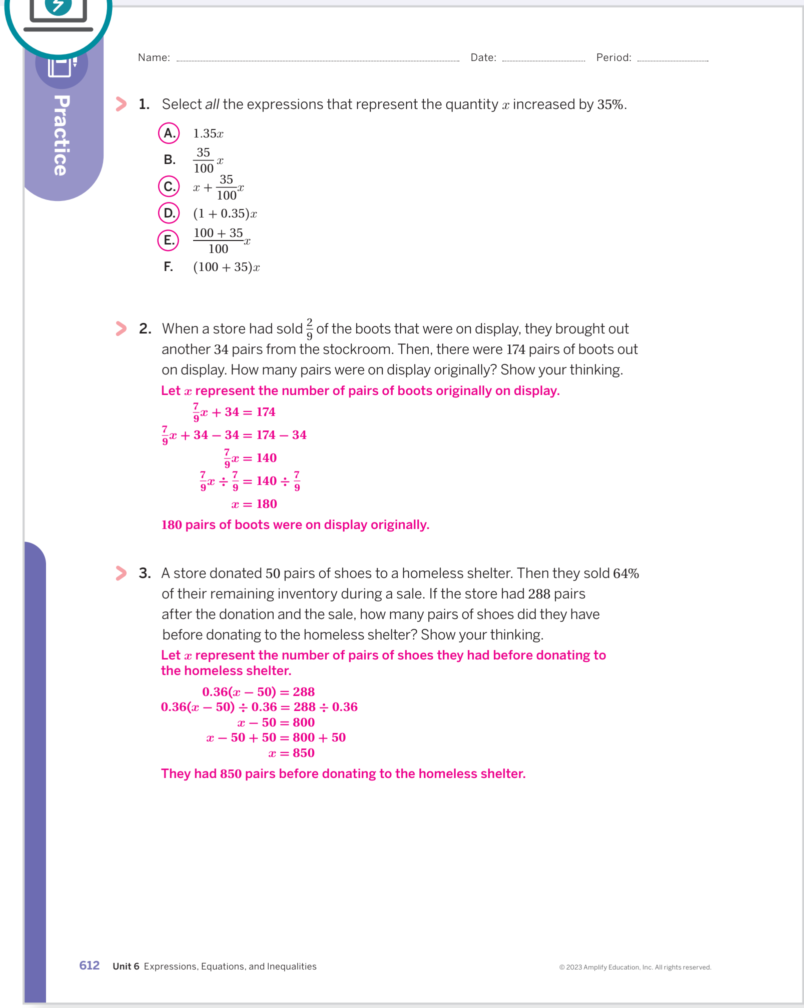
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What resources did students use as they worked on Activity 2? Which resources were especially helpful?
- When you compare and contrast today's work with work students did earlier this year on solving problems involving percentages, what similarities and differences do you see? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Select *all* the expressions that represent the quantity x increased by 35%.
   - [✓ A.] $1.35x$
   - B. $\frac{35}{100}x$
   - [✓ C.] $x + \frac{35}{100}x$
   - [✓ D.] $(1 + 0.35)x$
   - [✓ E.] $\frac{100 + 35}{100}x$
   - F. $(100 + 35)x$

2. When a store had sold $\frac{2}{9}$ of the boots that were on display, they brought out another 34 pairs from the stockroom. Then, there were 174 pairs of boots out on display. How many pairs were on display originally? Show your thinking.

   Let x represent the number of pairs of boots originally on display.

$$\begin{aligned} \tfrac{7}{9}x + 34 &= 174 \\ \tfrac{7}{9}x + 34 - 34 &= 174 - 34 \\ \tfrac{7}{9}x &= 140 \\ \tfrac{7}{9}x \div \tfrac{7}{9} &= 140 \div \tfrac{7}{9} \\ x &= 180 \end{aligned}$$

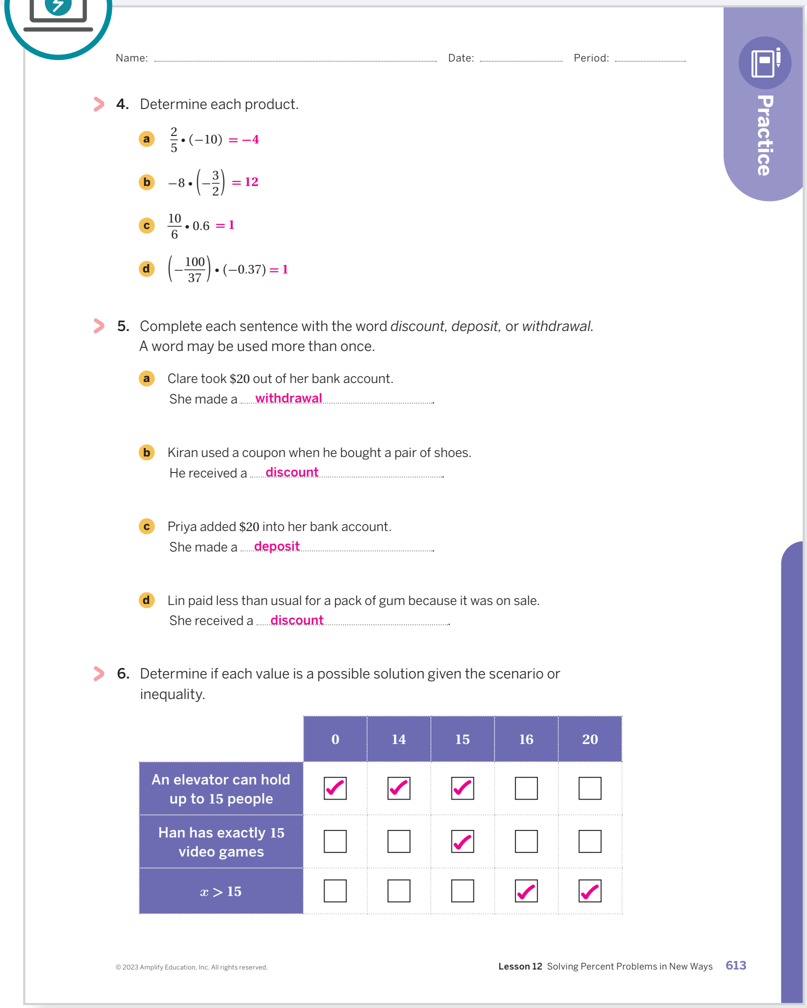
   180 pairs of boots were on display originally.

3. A store donated 50 pairs of shoes to a homeless shelter. Then they sold 64% of their remaining inventory during a sale. If the store had 288 pairs after the donation and the sale, how many pairs of shoes did they have before donating to the homeless shelter? Show your thinking.

   Let x represent the number of pairs of shoes they had before donating to the homeless shelter.

$$\begin{aligned} 0.36(x - 50) &= 288 \\ 0.36(x - 50) \div 0.36 &= 288 \div 0.36 \\ x - 50 &= 800 \\ x - 50 + 50 &= 800 + 50 \\ x &= 850 \end{aligned}$$

   They had 850 pairs before donating to the homeless shelter.



Name: ________ Date: ________ Period: ________

4. Determine each product.
   - a $\frac{2}{5} \cdot (-10)$ $= -4$
   - b $-8 \cdot \left(-\frac{3}{2}\right)$ $= 12$
   - c $\frac{10}{6} \cdot 0.6$ $= 1$
   - d $\left(-\frac{100}{37}\right) \cdot (-0.37)$ $= 1$

5. Complete each sentence with the word *discount, deposit,* or *withdrawal.* A word may be used more than once.
   - a Clare took $20 out of her bank account.
     She made a **withdrawal**.
   - b Kiran used a coupon when he bought a pair of shoes.
     He received a **discount**.
   - c Priya added $20 into her bank account.
     She made a **deposit**.
   - d Lin paid less than usual for a pack of gum because it was on sale.
     She received a **discount**.

6. Determine if each value is a possible solution given the scenario or inequality.

	0	14	15	16	20
An elevator can hold up to 15 people	☑	☑	☑	☐	☐
Han has exactly 15 video games	☐	☐	☑	☐	☐
$x > 15$	☐	☐	☐	☑	☑

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 11	1
	5	Unit 4 Lesson 9	1
Formative ⚡	6	Unit 6 Lesson 13	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

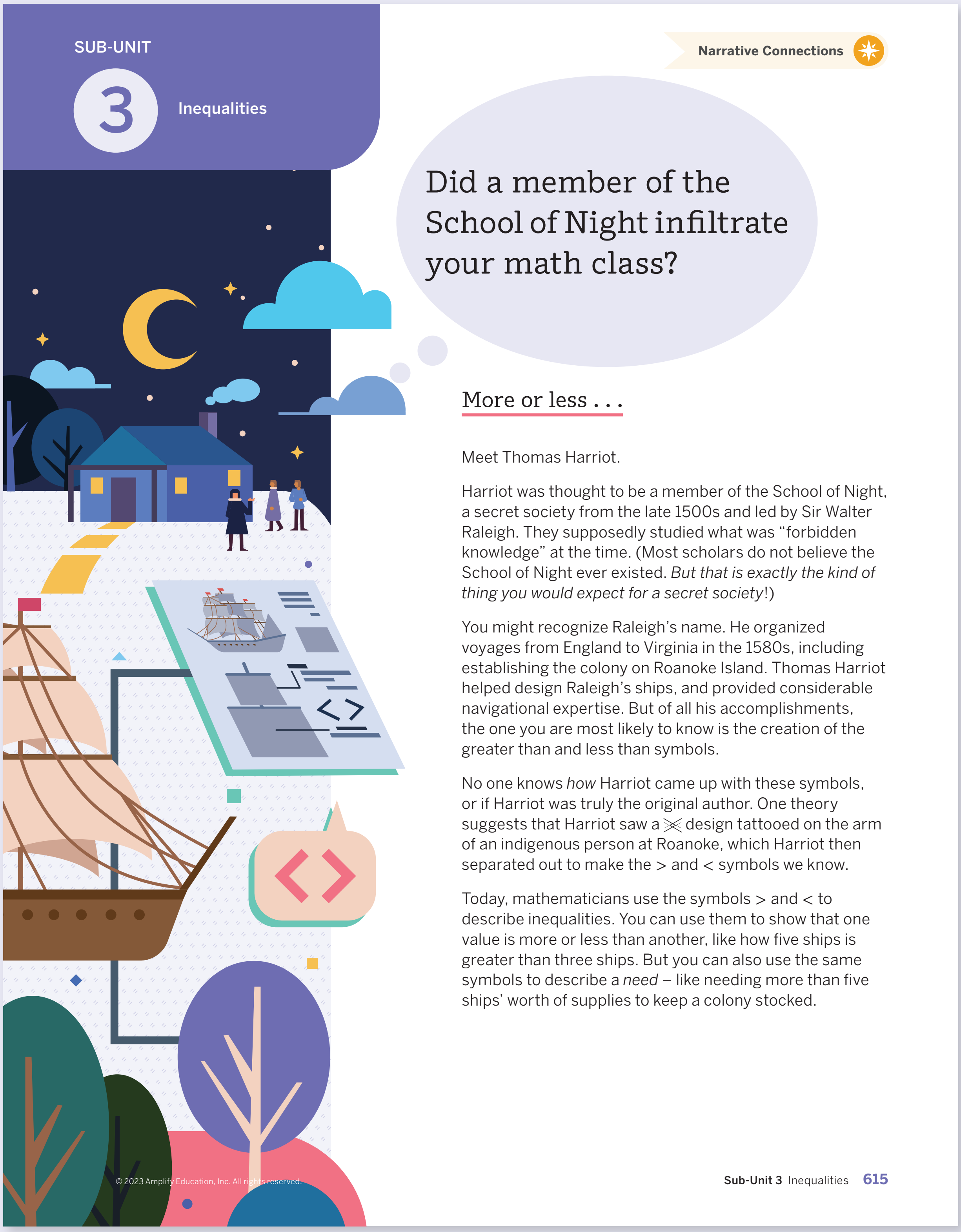
For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 3

Inequalities

Whole Class

In this Sub-Unit, students are reintroduced to inequalities, and they learn to write and solve inequalities in a similar way as with equations.



SUB-UNIT 3 Inequalities

Narrative Connections

Did a member of the School of Night infiltrate your math class?

More or less . . .

Meet Thomas Harriot.

Harriot was thought to be a member of the School of Night, a secret society from the late 1500s and led by Sir Walter Raleigh. They supposedly studied what was “forbidden knowledge” at the time. (Most scholars do not believe the School of Night ever existed. *But that is exactly the kind of thing you would expect for a secret society*!)

You might recognize Raleigh’s name. He organized voyages from England to Virginia in the 1580s, including establishing the colony on Roanoke Island. Thomas Harriot helped design Raleigh’s ships, and provided considerable navigational expertise. But of all his accomplishments, the one you are most likely to know is the creation of the greater than and less than symbols.

No one knows *how* Harriot came up with these symbols, or if Harriot was truly the original author. One theory suggests that Harriot saw a ⪥ design tattooed on the arm of an indigenous person at Roanoke, which Harriot then separated out to make the > and < symbols we know.

Today, mathematicians use the symbols > and < to describe inequalities. You can use them to show that one value is more or less than another, like how five ships is greater than three ships. But you can also use the same symbols to describe a *need* – like needing more than five ships’ worth of supplies to keep a colony stocked.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to think carefully about how the inequality symbol connects to the mathematics in the following places:

- **Lesson 13, Activity 1:** The Roller Coaster
- **Lesson 13, Activity 2:** Understanding Inequalities
- **Lesson 16, Activity 1:** Which Side Has the Solutions?
- **Lesson 18, Activity 1:** Loading an Elevator

UNIT 6 | **LESSON 13**

Reintroducing Inequalities

Let's work with inequalities.

Focus

Goals

1. **Language Goal:** Comprehend the terms *less than or equal to* and *greater than or equal to* and the symbols ≤ and ≥. **(Speaking and Listening, Reading and Writing)**
2. Represent solutions to an inequality on a number line.
3. Recognize that more than one value for a variable makes the same inequality true.

Coherence

Today

Students write inequalities to represent scenarios, test values to determine whether they are solutions, and reason about solving one-step inequalities.The inequalities of *less than or equal to* and *greater than or equal to* are introduced.

Previously

In Grade 6, students reasoned about and represented solutions to inequalities of the forms $x > a$ and $x < a$.

Coming Soon

In Lessons 14–18, students will formalize the process to solve inequalities and notice similarities to solving equations.

Rigor

- Students discuss real-world scenarios to build **conceptual understanding** of *less than or equal to* and *greater than or equal to*.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Solving Inequalities* (for display)
- Anchor Chart PDF, *Solving Inequalities* (answers)

Math Language Development

New words

- ***greater than or equal to***
- ***less than or equal to***
- ***solution to an inequality***

Review words

- *greater than*
- *inequality*
- *less than*
- *solution to an equation*

Amps | **Featured Activity**

Warm-up

Dynamic Dog Walking Diagrams

Dog walking diagrams and measures of strength will automatically update as students change their input values.

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Building Math Identity and Community

Connecting to Mathematical Practices

Students might not spend enough time analyzing the inequalities in Activity 3. Ask students to list some steps that they should take in their analysis in order to match inequalities to their graphed solutions. Ask them to explain which steps are absolutely necessary for solving the problem correctly and which steps could be beneficial, but might not be critical.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

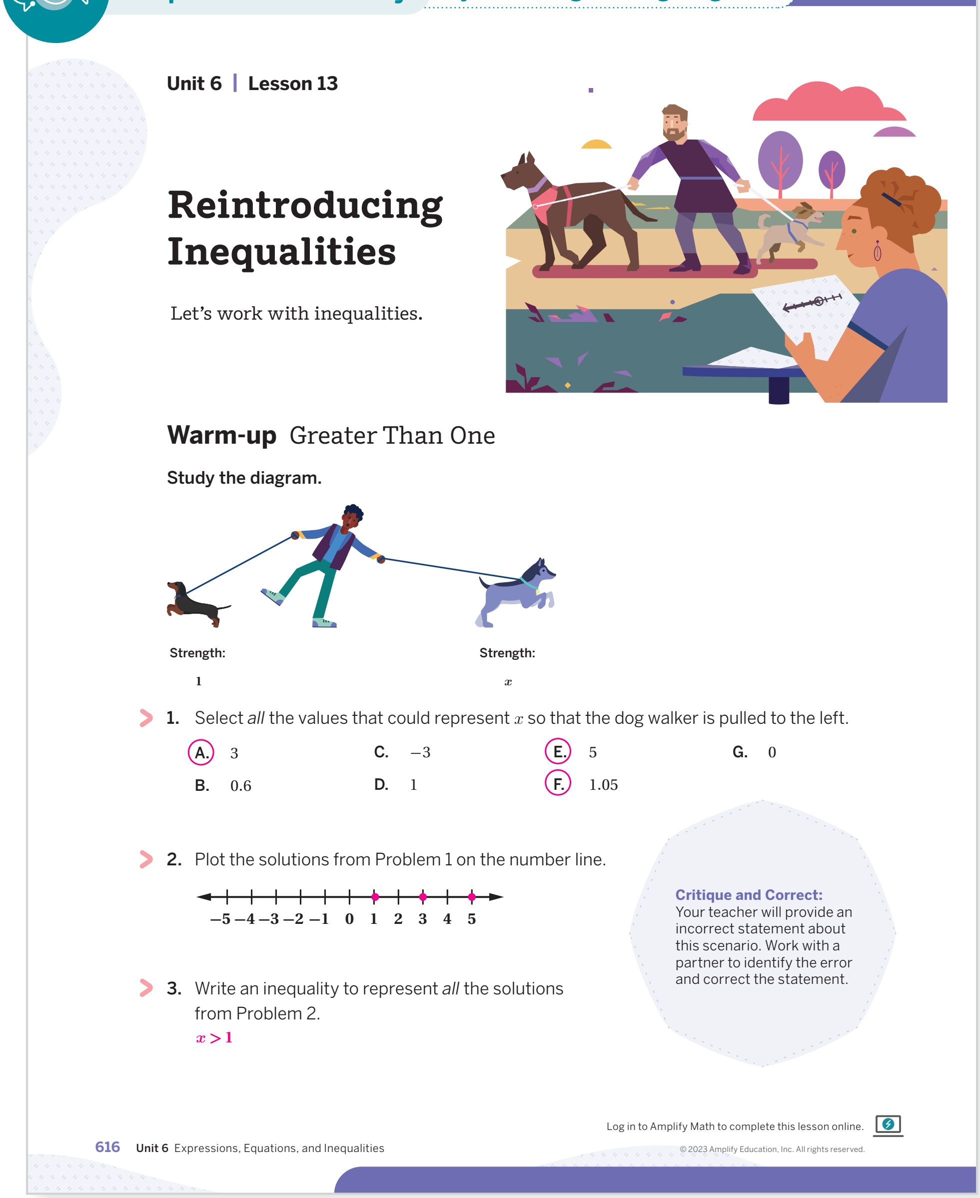
- **Activity 3** may be omitted. Consider assigning the activity as Additional Practice using the Digital Card Sort.

Independent | 5 min

Warm-up Greater Than One

Students make connections between a dog walking diagram and all possible solutions that satisfy a given criteria, as an introduction to writing and solving inequalities.

Amps Featured Activity Dynamic Dog Walking Diagrams



Unit 6 | Lesson 13

Reintroducing Inequalities

Let's work with inequalities.

Warm-up Greater Than One

Study the diagram.

Strength: 1

Strength: x

1. Select *all* the values that could represent x so that the dog walker is pulled to the left.

[✓ A.] 3
B. 0.6
C. -3
D. 1
[✓ E.] 5
[✓ F.] 1.05
G. 0

2. Plot the solutions from Problem 1 on the number line.

$-5\ -4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5$

3. Write an inequality to represent *all* the solutions from Problem 2.

$x > 1$

Critique and Correct: Your teacher will provide an incorrect statement about this scenario. Work with a partner to identify the error and correct the statement.

Log in to Amplify Math to complete this lesson online.

616 Unit 6 Expressions, Equations, and Inequalities

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1 Launch

Set an expectation for the amount of time students will have to work independently on the activity.

2 Monitor

Help students get started by asking, "What numbers listed will pull the dog walker to the left, i.e., are they *less than* or *greater than* 1?"

Look for points of confusion:

- **Difficulty understanding which values are greater than 1.** Identify where the numbers are on the number line.
- **Thinking 700 is not a solution because it is not visible on the number line.** Continue the number line until students realize it will eventually reach 700.

3 Connect

Have students share responses to Problem 1 by conducting the ***Poll the Class*** routine. Ask students to explain their thinking.

Highlight:

- Substituting the value in the inequality and reading it aloud can help students determine if the statement is true. Say, "$-3 > 1$ is not true; therefore, -3 is not a solution."
- Comparing this statement to the location of -3 on the number line.
- The boundary value of 1 is not included because 1 is *not greater than* 1 and is represented with an open circle.
- Write $1 < x$, read it, and discuss if this inequality is the same as or different from the inequality $x > 1$.

Define *solution to an inequality*: a value that will make an inequality a true statement when substituted into the inequality.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement about this scenario, such as "The value of x could be any number that is 1 or greater." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking." Listen for students who reason that the strength of the dog on the left cannot be equal to 1.
- ***Correct:*** "How would you correct this statement?"
- ***Clarify:*** "How can you convince someone that your statement is true?"

Power-up

To power up students' ability to determine which values make an inequality true, have students complete:

Apples from a certain orchard are always picked before they weigh 0.5 lbs. Select *all* of the values that are less than 0.5.

A. 0.5
[✓ B.] 0.25
[✓ C.] $\frac{1}{3}$
D. $\frac{2}{3}$
[✓ E.] 0
F. 0.9

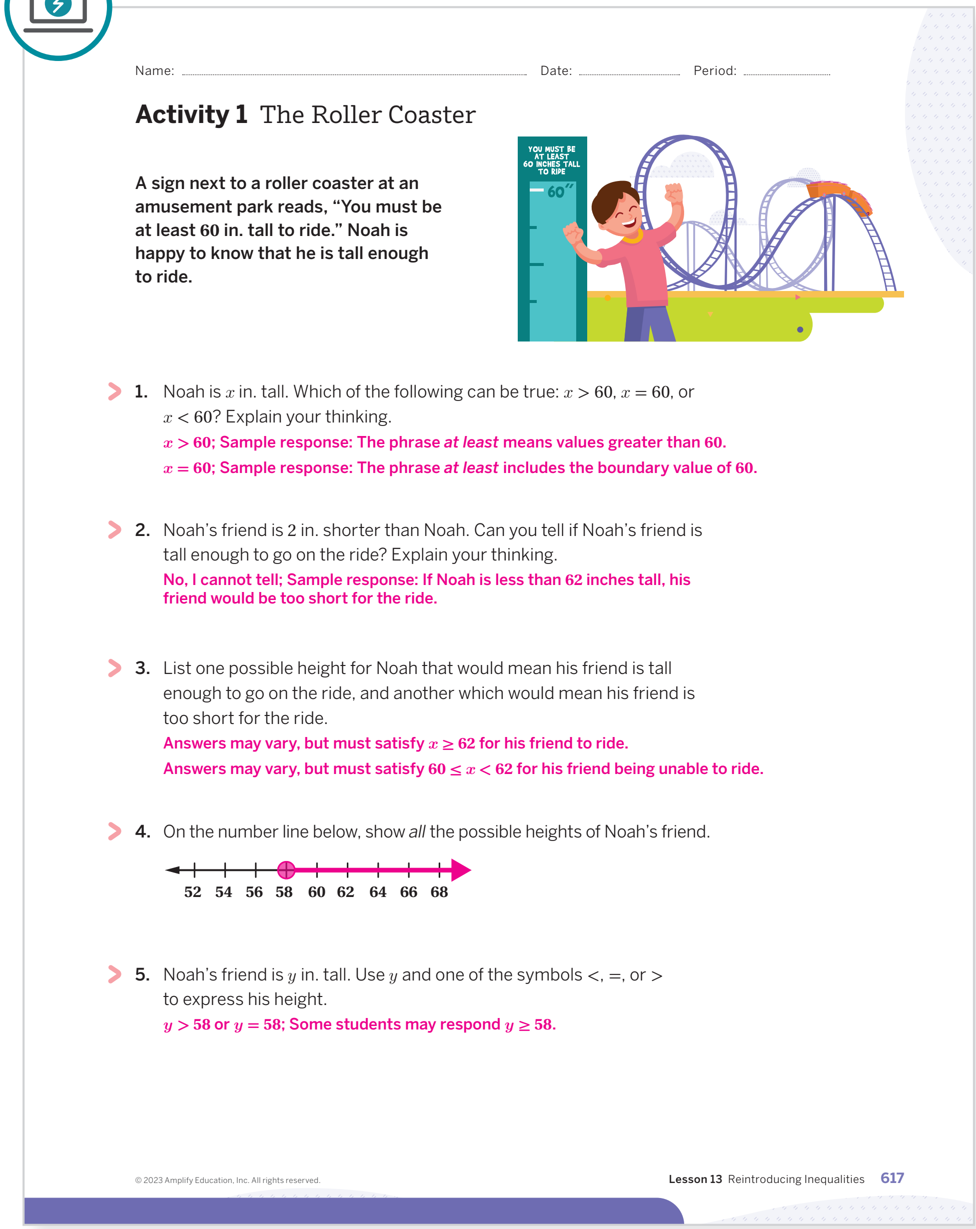
Use: Before the Warm-up

Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

Pairs | 10 min

Activity 1 The Roller Coaster

Students review the meanings of the symbols $<$ and $>$ and then read a scenario which facilitates the necessity for the new inequality symbols $\leq$ and $\geq$.



Name: Date: Period:

Activity 1 The Roller Coaster

A sign next to a roller coaster at an amusement park reads, “You must be at least 60 in. tall to ride.” Noah is happy to know that he is tall enough to ride.

1. Noah is x in. tall. Which of the following can be true: $x > 60$, $x = 60$, or $x < 60$? Explain your thinking.
   $x > 60$; Sample response: The phrase *at least* means values greater than 60.
   $x = 60$; Sample response: The phrase *at least* includes the boundary value of 60.

2. Noah’s friend is 2 in. shorter than Noah. Can you tell if Noah’s friend is tall enough to go on the ride? Explain your thinking.
   No, I cannot tell; Sample response: If Noah is less than 62 inches tall, his friend would be too short for the ride.

3. List one possible height for Noah that would mean his friend is tall enough to go on the ride, and another which would mean his friend is too short for the ride.
   Answers may vary, but must satisfy $x \geq 62$ for his friend to ride.
   Answers may vary, but must satisfy $60 \leq x < 62$ for his friend being unable to ride.

4. On the number line below, show *all* the possible heights of Noah’s friend.

5. Noah’s friend is y in. tall. Use y and one of the symbols $<$, $=$, or $>$ to express his height.
   $y > 58$ or $y = 58$; Some students may respond $y \geq 58$.

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Lesson 13 Reintroducing Inequalities 617

1 Launch

Activate background knowledge by asking students whether they have been on a roller coaster or a ride at the fair and whether they experienced being with someone who was not able to ride because of their height.

2 Monitor

Help students get started by asking, “If Noah is ___ inches tall, can he ride?” Continue with similar examples until students can continue on their own.

Look for points of confusion:

- **Not knowing they can pick two options for Problem 1.** Discuss why Noah can either be *greater than* 60 or *equal to* 60.
- **Struggling to represent their responses to Problems 1–3.** Have them use the number line to help identify solutions.
- **Not knowing how to show that 58 is included.** Ask students how they would plot the point 58 on a number line. Discuss that a closed circle is used because it represents shading the values that make the inequality true.

3 Connect

Have students share possible heights for Noah’s friend and mark the solutions on the number line. Ask students to share their responses to Problem 5. Students may write, “$y = 58$ or $y > 58$.”

Highlight that there is a symbol meaning *greater than or equal to* and introduce the notation $\geq$. Since Noah’s friend can be 58 in. tall, have students mark 58 with a closed circle on the number line.

Define:

- ***greater than or equal to***
- ***less than or equal to***

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Before students begin the activity, ask them to generate possible heights for Noah that would allow him to ride the roller coaster. Then ask them to generate 1 or 2 heights for Noah that would mean he would not be allowed to ride the roller coaster.

Extension: *Math Enrichment*

Have students complete the following problem:

Suppose Noah’s sister will be allowed to ride the roller coaster when her height increases by at least 8 in. Define a variable and write an inequality that represents Noah’s sister’s current height. Sample response: Let z represent Noah’s sister’s current height; $z + 8 \geq 60$.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you introduce the $\geq$ and $\leq$ symbols, add them to the class display, along with common words and phrases that can indicate them. For example, consider displaying the following table.

$\geq$	$\leq$
Greater than or equal to At least	Less than or equal to At most

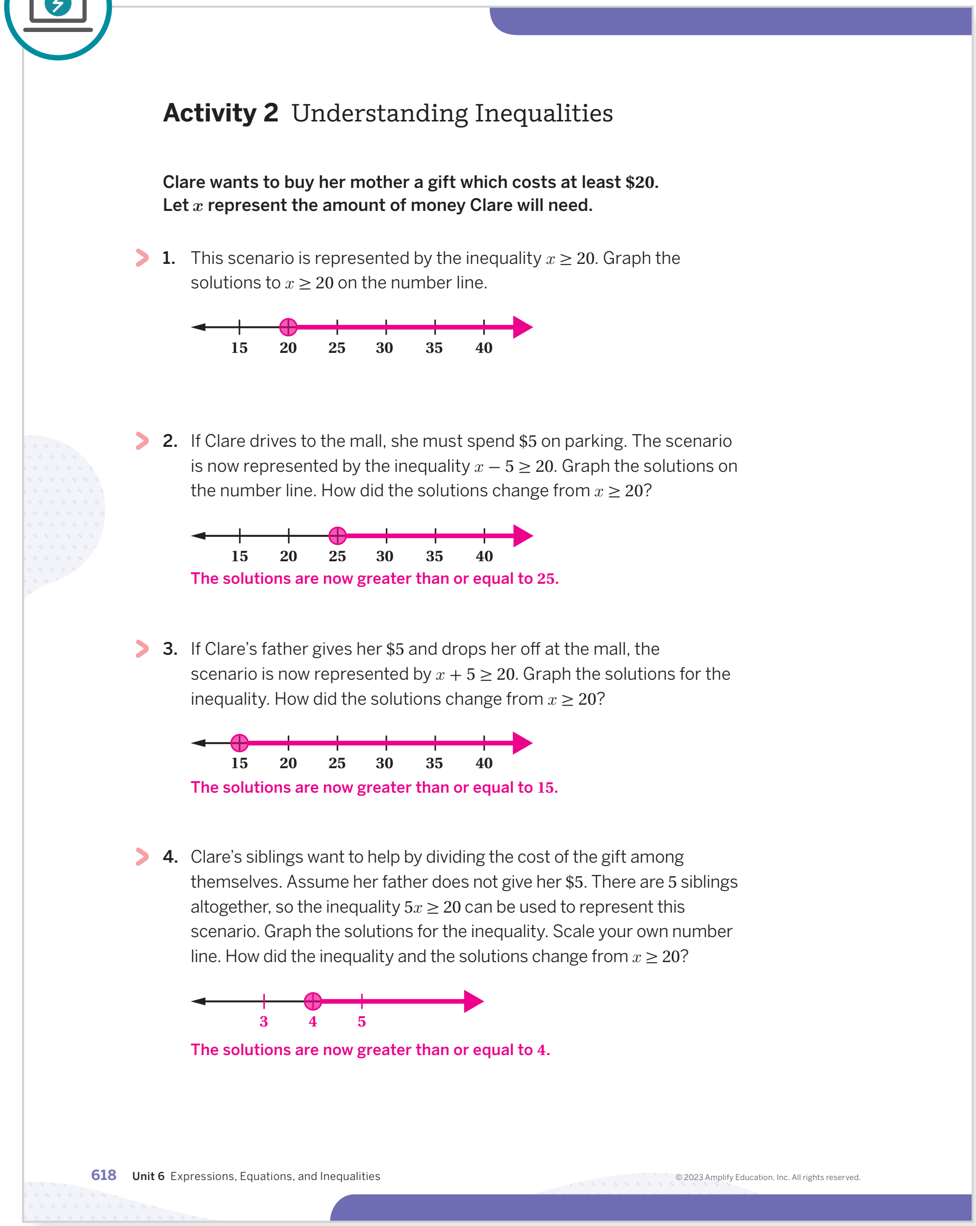
English Learners

Provide examples of phrases, such as “I own at least 4 baseball hats, which means the number of baseball hats I own is greater than or equal to 4.”

Pairs | 10 min

Activity 2 Understanding Inequalities

Students solve one-step inequalities by reasoning about the solutions. They are not expected to formally solve inequalities in this activity.



Activity 2 Understanding Inequalities

Clare wants to buy her mother a gift which costs at least $20. Let x represent the amount of money Clare will need.

1. This scenario is represented by the inequality $x \geq 20$. Graph the solutions to $x \geq 20$ on the number line.

2. If Clare drives to the mall, she must spend $5 on parking. The scenario is now represented by the inequality $x - 5 \geq 20$. Graph the solutions on the number line. How did the solutions change from $x \geq 20$?

The solutions are now greater than or equal to 25.

3. If Clare's father gives her $5 and drops her off at the mall, the scenario is now represented by $x + 5 \geq 20$. Graph the solutions for the inequality. How did the solutions change from $x \geq 20$?

The solutions are now greater than or equal to 15.

4. Clare's siblings want to help by dividing the cost of the gift among themselves. Assume her father does not give her $5. There are 5 siblings altogether, so the inequality $5x \geq 20$ can be used to represent this scenario. Graph the solutions for the inequality. Scale your own number line. How did the inequality and the solutions change from $x \geq 20$?

The solutions are now greater than or equal to 4.

618 **Unit 6** Expressions, Equations, and Inequalities

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1 Launch

Be sure students understand what the symbols $<$, $>$, $\leq$, and $\geq$ represent.

2 Monitor

Help students get started by asking, "What does $\geq$ mean?" or "Is ___ greater than or equal to 20?"

Look for points of confusion:

- **Wanting to shade the direction in which the inequality points (i.e., $<$ shades left because the arrow points in that direction).** Encourage students to test values in the inequality to determine in which direction to shade. To further emphasize this point, show an inequality such as $3 < x$, and discuss the solutions.

Look for productive strategies:

- Wanting to solve the inequalities algebraically. Although this is not the goal of this lesson, encourage students to test values before shading the number line. This process will be formalized in the next lesson.

3 Connect

Highlight that students can test values to determine whether the inequality is true. Students are not expected to solve inequalities during this lesson.

Ask, "Why is the boundary point shaded?" It makes the inequality true and is a solution.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1–3. As time allows, they can work on Problem 4.

Accessibility: *Guide Processing and Visualization*

Test possible solutions from Problem 1 into the inequality in Problem 2, starting with values that are true for both inequalities, such as 25 and 30. Then test a value that is only a solution to Problem 1, such as 20 or 24. Ask students why these values are not solutions to the inequality in Problem 2.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the four inequalities and ask students how the inequalities represent each scenario. Ask them to look for the key words and phrases that indicate why the inequality symbol is always $\geq$, and what operation(s) are performed on the variable. Ask:

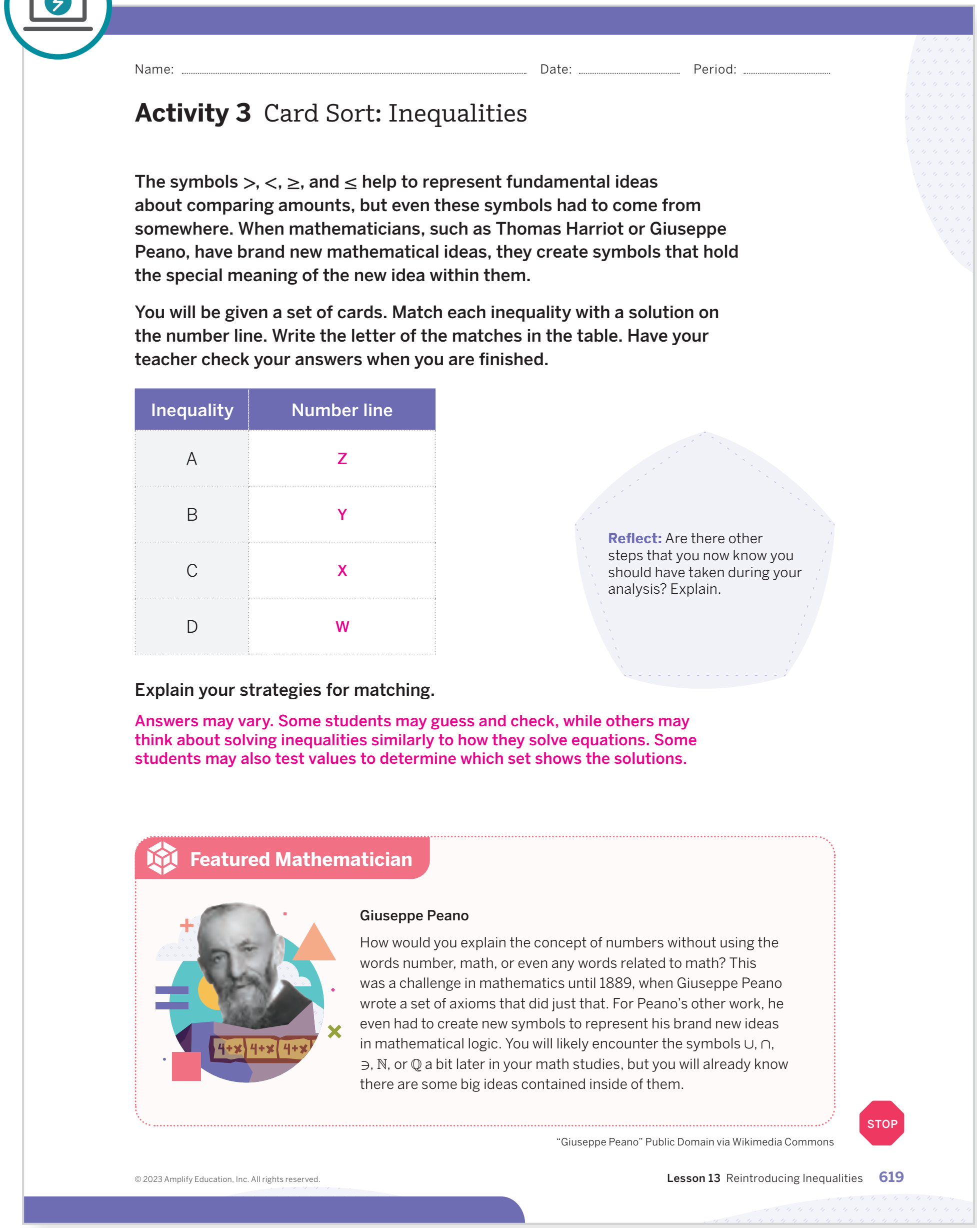
- "What does x represent in these scenarios? What does the phrase 'at least' $20 tell you?"
- "If Clare spends $5 on parking, what does this mean for the amount of money she will need? Why is 5 subtracted from x?"

Ask similar questions for the two remaining scenarios.

Pairs | 10 min

Activity 3 Card Sort: Inequalities

Students sort cards to match inequalities with solutions on number lines. This will help further their understanding of testing values to determine whether inequalities are true.



Name: ______________ Date: ________ Period: ________

Activity 3 Card Sort: Inequalities

The symbols >, <, ≥, and ≤ help to represent fundamental ideas about comparing amounts, but even these symbols had to come from somewhere. When mathematicians, such as Thomas Harriot or Giuseppe Peano, have brand new mathematical ideas, they create symbols that hold the special meaning of the new idea within them.

You will be given a set of cards. Match each inequality with a solution on the number line. Write the letter of the matches in the table. Have your teacher check your answers when you are finished.

Inequality	Number line
A	Z
B	Y
C	X
D	W

Reflect: Are there other steps that you now know you should have taken during your analysis? Explain.

Explain your strategies for matching.

Answers may vary. Some students may guess and check, while others may think about solving inequalities similarly to how they solve equations. Some students may also test values to determine which set shows the solutions.

Featured Mathematician

Giuseppe Peano

How would you explain the concept of numbers without using the words number, math, or even any words related to math? This was a challenge in mathematics until 1889, when Giuseppe Peano wrote a set of axioms that did just that. For Peano's other work, he even had to create new symbols to represent his brand new ideas in mathematical logic. You will likely encounter the symbols ∪, ∩, ∋, ℕ, or ℚ a bit later in your math studies, but you will already know there are some big ideas contained inside of them.

STOP

"Giuseppe Peano" Public Domain via Wikimedia Commons

1 Launch

Distribute one set of cards from Activity 3 PDF to each pair of students. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by testing values near the boundary point.

Look for points of confusion:

- **Mismatching cards.** Remind them to test values on both sides of the boundary point including ones close to the boundary point.

3 Connect

Have students share their strategies for matching cards.

Ask, "Why is the boundary value not included in this set of inequalities?" It does not make the inequality true.

Highlight how the solutions change as the value is added, subtracted, and multiplied on the left side of the inequality.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Distribute the cards with addition or subtraction inequalities and their solutions first. After these are matched, distribute the cards with multiplication inequalities and their solutions.

Extension: *Interdisciplinary Connections*

Ask students, "Have you ever wondered where the symbols for =, >, or < came from?" Tell them that the first use of the equal sign, =, is attributed to Robert Recorde who was a Welsh physician and mathematician. He intentionally used two parallel lines to represent equality because they are always the same distance apart. The works of British mathematician Thomas Harriot included the first inequality symbols. These were actually introduced by his book's editor, who altered the original triangular symbols Harriot used. Ask students to think of and create their own symbols they would use to indicate equality, greater than, or less than, and explain why they chose the symbols they did. **(History)**

Featured Mathematician

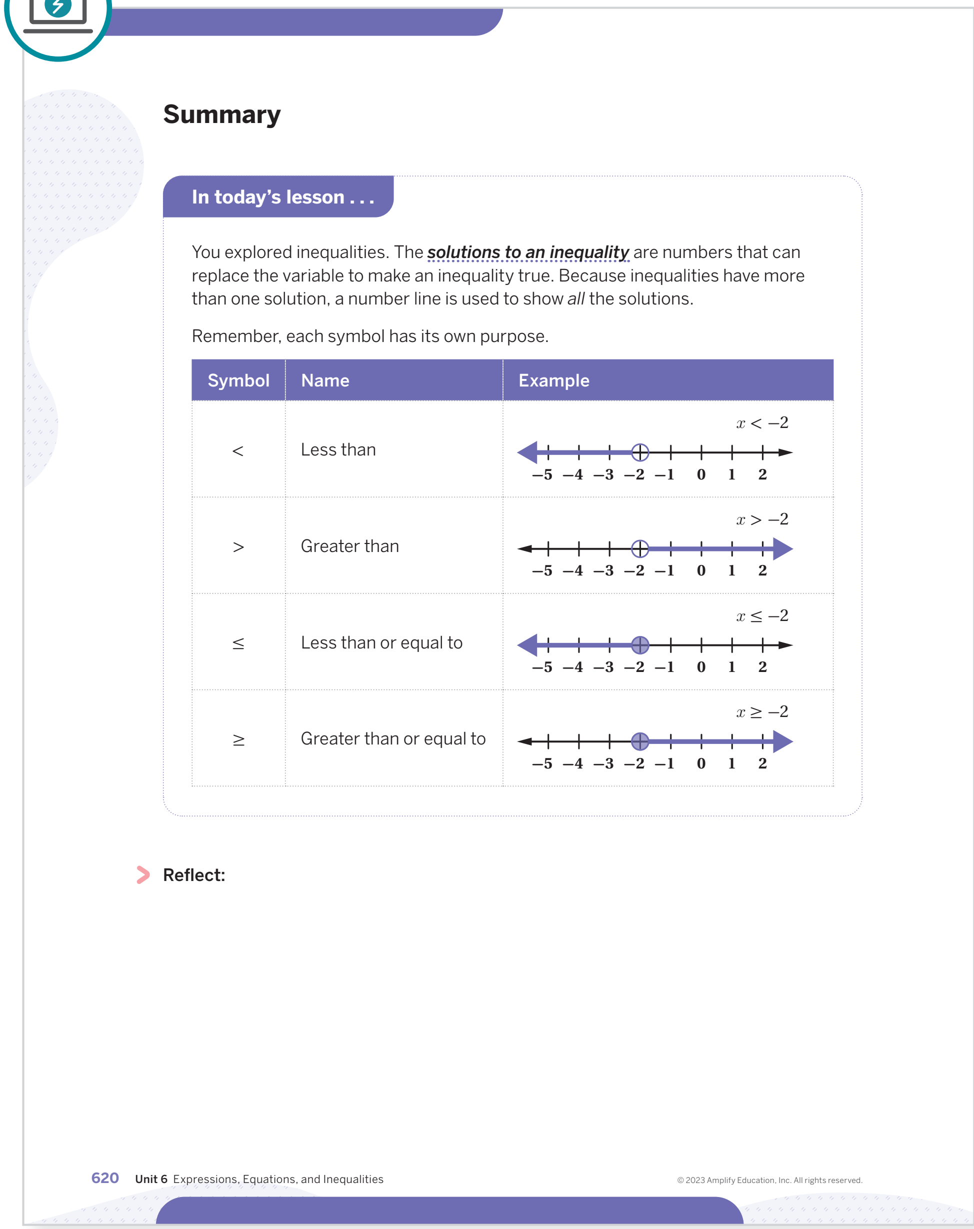
Giuseppe Peano

Have students read about *Featured Mathematician* Giuseppe Peano, the creator of several symbols for mathematical logic.

Whole Class | 5 min

Summary

Review and synthesize how inequalities can be used to represent real-world situations, and how testing values can help determine whether inequalities are true.



Summary

In today's lesson . . .

You explored inequalities. The ***solutions to an inequality*** are numbers that can replace the variable to make an inequality true. Because inequalities have more than one solution, a number line is used to show *all* the solutions.

Remember, each symbol has its own purpose.

Symbol	Name	Example
<	Less than	$x < -2$ −5 −4 −3 −2 −1 0 1 2
>	Greater than	$x > -2$ −5 −4 −3 −2 −1 0 1 2
≤	Less than or equal to	$x \leq -2$ −5 −4 −3 −2 −1 0 1 2
≥	Greater than or equal to	$x \geq -2$ −5 −4 −3 −2 −1 0 1 2

Reflect:

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Synthesize

Display the Anchor Chart PDF, Solving Inequalities and complete the top section as a class.

Ask:

- "Which inequality symbol should be place in each box?"
- "How are the two inequalities for each number lines similar? How are they different?"
- "What clue from the number line lets you know when to use > versus ≥?"
- "Are inequalities with ≤ and < always shaded to the left? Why or why not?"

Highlight that the location of the variable in an inequality, on the left or the right, impacts the side of the number line which is shaded. Bring attention to the fact that when the value and the variable swap sides the direction of the inequality symbol also changes direction (e.g. $x > 2$ is the same as $2 < x$).

Formalize vocabulary:

- ***less than or equal to***
- ***greater than or equal to***
- ***solution to an inequality***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean to be a solution to an inequality?"
- "Does it make sense that an inequality can have more than one solution? Why or why not?"

Math Language Development

MLR2: Collect and Display

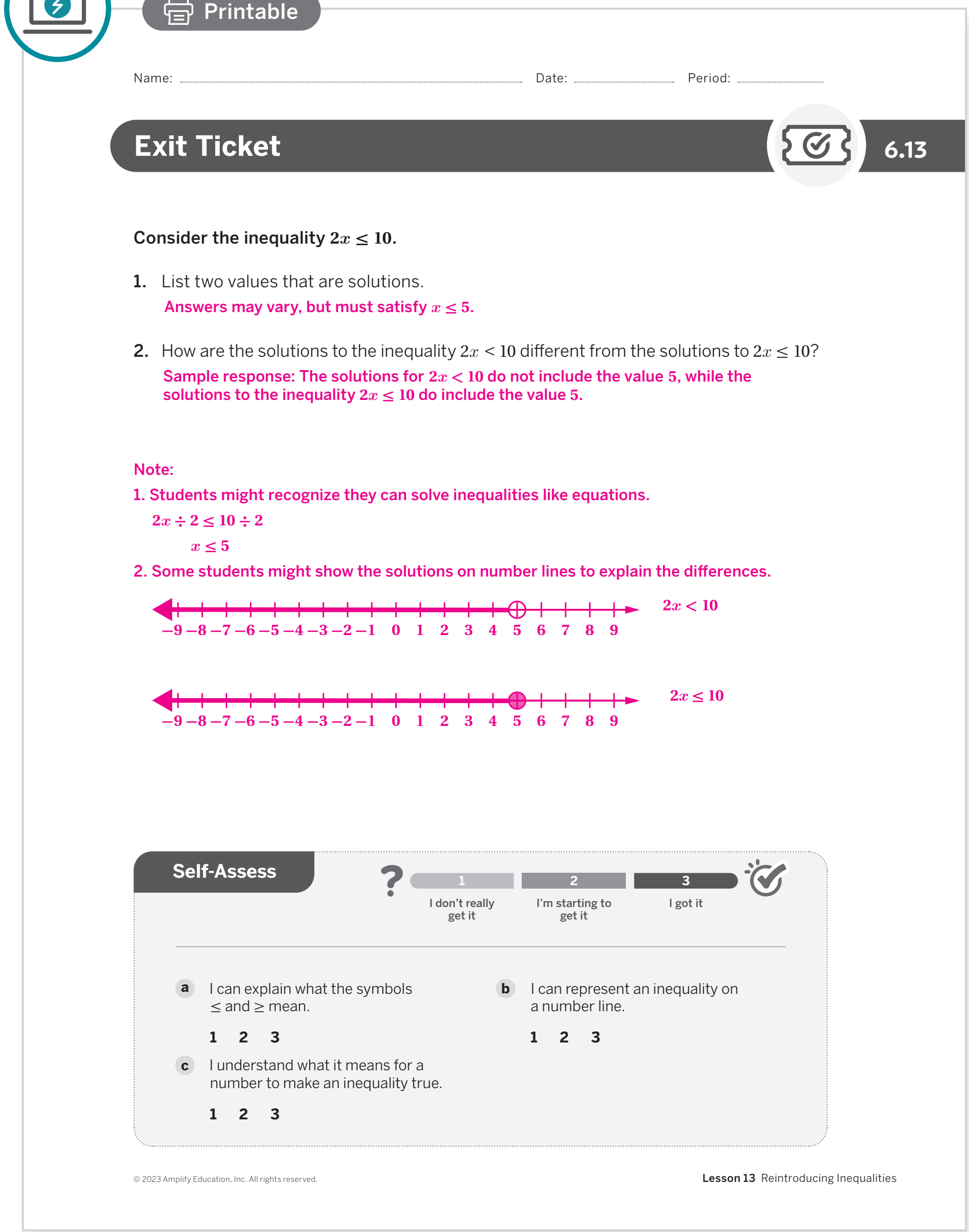
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *solution to an inequality*, *less than*, *greater than*, *less than or equal to*, and *greater than or equal to* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by giving solutions to an inequality and determining how solutions change when the term *equal to* is added to an inequality.

Printable



Name: ____________________ Date: __________ Period: __________

Exit Ticket 6.13

Consider the inequality $2x \leq 10$.

1. List two values that are solutions.
   Answers may vary, but must satisfy $x \leq 5$.

2. How are the solutions to the inequality $2x < 10$ different from the solutions to $2x \leq 10$?
   Sample response: The solutions for $2x < 10$ do not include the value 5, while the solutions to the inequality $2x \leq 10$ do include the value 5.

Note:

1. Students might recognize they can solve inequalities like equations.

$2x \div 2 \leq 10 \div 2$

$x \leq 5$

2. Some students might show the solutions on number lines to explain the differences.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain what the symbols ≤ and ≥ mean.
1 2 3

b. I can represent an inequality on a number line.
1 2 3

c. I understand what it means for a number to make an inequality true.
1 2 3

Lesson 13 Reintroducing Inequalities

Success looks like . . .

- **Language Goal:** Comprehending the terms *less than or equal to* and *greater than or equal to*, and the symbols ≤ and ≥. **(Speaking and Listening, Reading and Writing)**
  - » Explaining how the solutions to the inequality $2x < 10$ are different from the solutions to $2x \leq 10$ in Problem 2.
- **Goal:** Representing solutions to an inequality on a number line.
- **Goal:** Recognizing that more than one value for a variable makes the same inequality true.

Suggested next steps

If students determine solutions satisfying $x \geq 5$, they most likely understand the relationship with 2 and 10. If students possibly did not test their points to determine whether they made the inequality true, consider:

- Assigning Practice Problem 3.

If students need more practice with understanding the inequality symbols, consider:

- Assigning Practice Problem 1.
- Assigning Practice Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did the Card Sort set up students to develop understanding of solutions of inequalities?
- In what ways did the Warm-up go as planned? What might you change for the next time you teach this lesson?

Practice

Independent

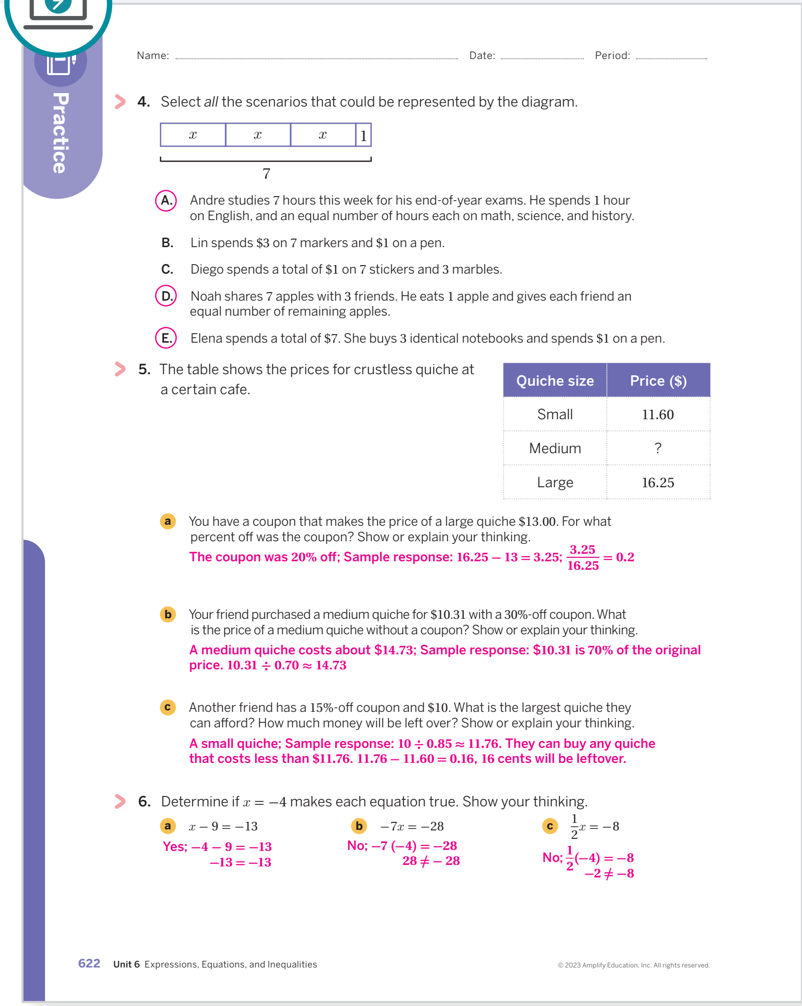
Name: Date: Period:

Practice

1. Express each statement as an inequality, and write two values which will make the inequality true.
   - **a** x is less than or equal to 15.
     $x \leq 15$; Accept any two values which satisfy $x \leq 15$.
   - **b** a is greater than or equal to 0.1.
     $a \geq 0.1$; Accept any two values which satisfy $a \geq 0.1$.
   - **c** 3 is equal to or less than t.
     $3 \leq t$ or $t \geq 3$; Accept any two values which satisfy $t \geq 3$.
   - **d** r is no more than 12.
     $r \leq 12$; Accept any two values which satisfy $r \leq 12$.
   - **e** p is at least $\frac{3}{4}$.
     $p \geq \frac{3}{4}$; Accept any two values which satisfy $p \geq \frac{3}{4}$.

2. Write an inequality for each scenario.
   - **a** A movie ticket costs a minimum of $7.
     $x \geq 7$
   - **b** For safety reasons, an elevator cannot exceed a 1,500-kg capacity.
     $x \leq 1500$
   - **c** The temperature is higher than 40°.
     $x > 40$
   - **d** Diego has no more than 3 cousins.
     $x \leq 3$

3. Consider the inequality $-3x > 18$.
   - **a** List four values for x that would make this inequality true.
     Answers may vary, but should satisfy $x < -6$.
   - **b** How are the solutions to the inequality $-3x \geq 18$ different from the solutions to $-3x > 18$? Explain your thinking.
     Solutions to the inequality $-3x \geq 18$ now include -6.



Practice

Name: Date: Period:

4. Select *all* the scenarios that could be represented by the diagram.

[✓ A.] Andre studies 7 hours this week for his end-of-year exams. He spends 1 hour on English, and an equal number of hours each on math, science, and history.
B. Lin spends $3 on 7 markers and $1 on a pen.
C. Diego spends a total of $1 on 7 stickers and 3 marbles.
[✓ D.] Noah shares 7 apples with 3 friends. He eats 1 apple and gives each friend an equal number of remaining apples.
[✓ E.] Elena spends a total of $7. She buys 3 identical notebooks and spends $1 on a pen.

5. The table shows the prices for crustless quiche at a certain cafe.

Quiche size	Price ($)
Small	11.60
Medium	?
Large	16.25

   - **a** You have a coupon that makes the price of a large quiche $13.00. For what percent off was the coupon? Show or explain your thinking.
     The coupon was 20% off; Sample response: $16.25 - 13 = 3.25$; $\frac{3.25}{16.25} = 0.2$
   - **b** Your friend purchased a medium quiche for $10.31 with a 30%-off coupon. What is the price of a medium quiche without a coupon? Show or explain your thinking.
     A medium quiche costs about $14.73; Sample response: $10.31 is 70% of the original price. $10.31 \div 0.70 \approx 14.73$
   - **c** Another friend has a 15%-off coupon and $10. What is the largest quiche they can afford? How much money will be left over? Show or explain your thinking.
     A small quiche; Sample response: $10 \div 0.85 \approx 11.76$. They can buy any quiche that costs less than $11.76. $11.76 - 11.60 = 0.16$, 16 cents will be leftover.

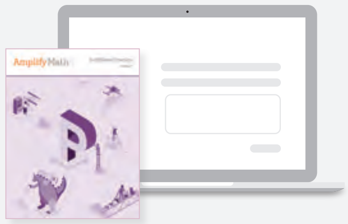
6. Determine if $x = -4$ makes each equation true. Show your thinking.
   - **a** $x - 9 = -13$
     Yes; $-4 - 9 = -13$
     $-13 = -13$
   - **b** $-7x = -28$
     No; $-7\,(-4) = -28$
     $28 \neq -28$
   - **c** $\frac{1}{2}x = -8$
     No; $\frac{1}{2}(-4) = -8$
     $-2 \neq -8$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 8	1
	5	Unit 4 Lesson 9	2
Formative	6	Unit 6 Lesson 14	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 14**

Solving Inequalities

Let's solve more complicated inequalities.

Focus

Goals

1. Use substitution to determine whether a given value for a variable makes an inequality true.
2. Generalize that it is possible to solve an inequality of the form $x + q > r$ or $x + q < r$ by solving the equation $x + q = r$ and then testing a value to determine the direction of the inequality in the solution.
3. Generalize that it is possible to solve an inequality of the form $qx > r$ or $qx < r$ by solving the equation $qx = r$ and then testing a value to determine the direction of the inequality in the solution.

Coherence

Today

Students write and solve equations and use those solutions to help them determine the solutions of corresponding one-step inequalities that may include negative values.

Previously

In Lesson 13, students wrote one-step inequalities to represent real-world scenarios and tested values to determine if they were solutions.

Coming Soon

In Lesson 15, students will write and solve two-step inequalities.

Rigor

- Students use substitution to build **conceptual understanding** of what is meant by a solution to an inequality.
- Students use tables of values to build their **conceptual understanding** of solution sets of one-step inequalities.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (Optional)	Summary	Exit Ticket
7 min	13 min	15 min	10 min	5 min	5 min
Pairs	Small Group	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- two colors of colored pencils, markers, or highlighters for each group

Math Language Development

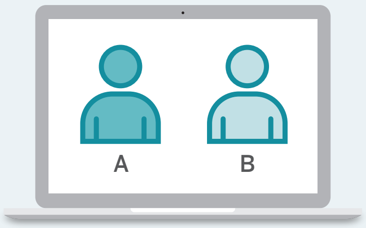
Review words

- *greater than*
- *greater than or equal to*
- *inequality*
- *less than*
- *less than or equal to*
- *solution to an equation*
- *solutions to an inequality*

Amps ⋮ **Featured Activity**

Activity 1
Overlay Student Work

Students individually plot points on a number line and can then see all their classmates' data shown together on one number line.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 3, students write possible inequalities that have given solutions and then check their work with a partner. Remind them to communicate clearly and precisely as they share the inequalities they wrote, and why they believe they are correct.

Modifications to Pacing

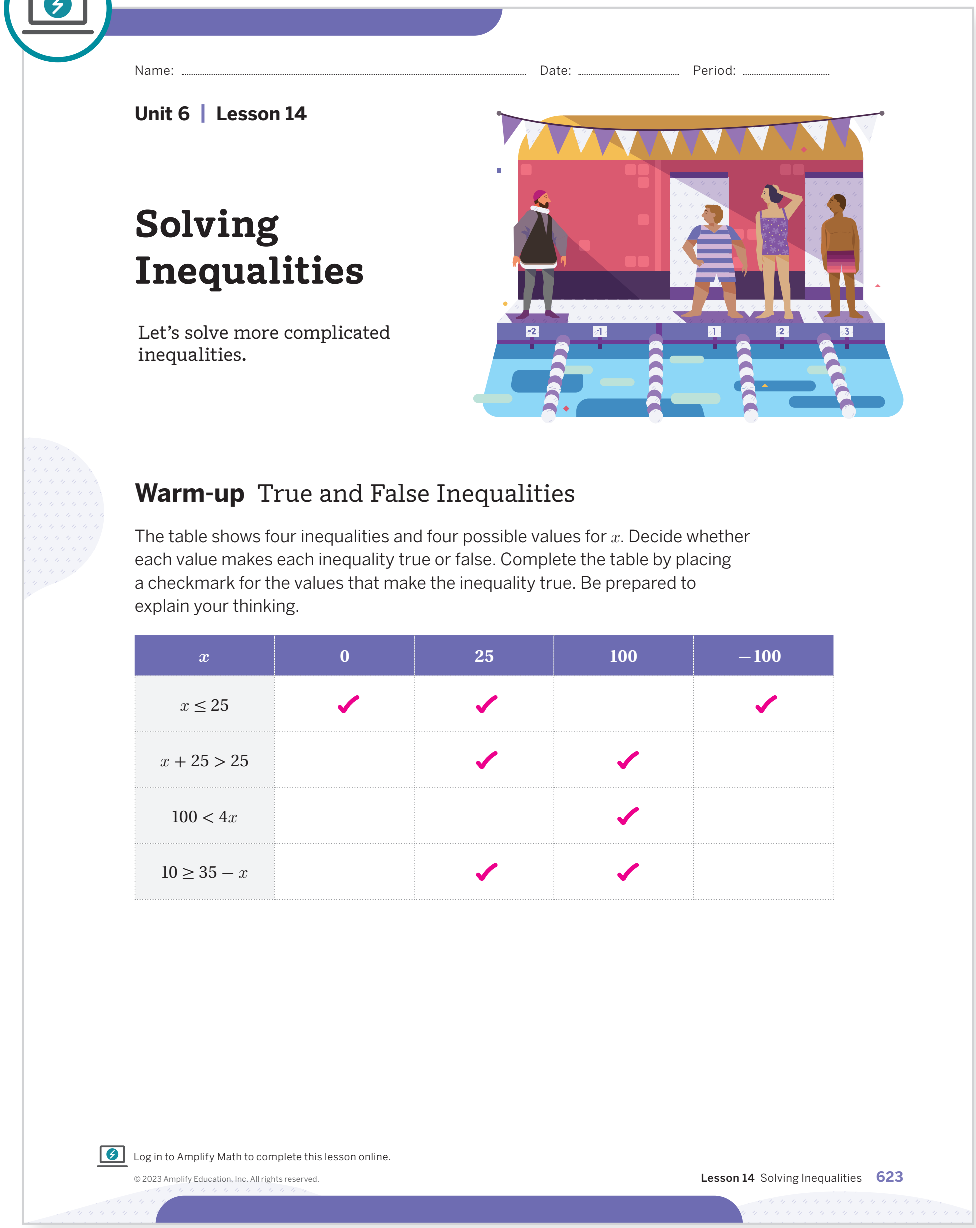
You may want to consider these additional modifications if you are short on time.

- During the **Warm-up**, choose only one row for the class to complete.
- In **Activity 1**, Problem 3 may be omitted.
- Optional **Activity 3** may be omitted, or students may choose one problem to complete.

Pairs | 7 min

Warm-up True and False Inequalities

Students complete a table by determining which values make an inequality true or false to practice identifying solutions to inequalities.



Name: Date: Period:

Unit 6 | Lesson 14

Solving Inequalities

Let's solve more complicated inequalities.

Warm-up True and False Inequalities

The table shows four inequalities and four possible values for x. Decide whether each value makes each inequality true or false. Complete the table by placing a checkmark for the values that make the inequality true. Be prepared to explain your thinking.

x	0	25	100	-100
$x \le 25$	✓	✓		✓
$x + 25 > 25$		✓	✓	
$100 < 4x$			✓	
$10 \ge 35 - x$		✓	✓	

Log in to Amplify Math to complete this lesson online.

Lesson 14 Solving Inequalities 623

1 Launch

Conduct the ***Think-Pair-Share*** routine. Give students 2 minutes of silent work time. Then give them 2 minutes to compare their work with a partner.

2 Monitor

Help students get started by modeling how to reason about one of the inequalities in the table. Choose an inequality and model for students how to substitute each value into the inequality to determine if the inequality is true or false.

Look for points of confusion:

- **Misinterpreting inequalities with the variable on the right (e.g., assuming $100 < 4x$ is equivalent to $x < 25$).** Suggest substituting values for x to see the difference between "x is less than 25" and "25 is less than x."
- **Forgetting that inequalities with ≥ and ≤ signs are *true when both sides are equal*.** Ask students to explain the difference between </> and ≤/≥.

3 Connect

Have students share their entries for each row of the table and the strategies they used.

Highlight the differences between substituting 25 for x in $100 < 4x$ and $10 \ge 35 - x$. Note that inequalities using the symbols ≤ and ≥ are considered true when both sides are equal, whereas < and > inequalities are considered *false* when both sides are equal. Emphasize that substituting a value for x and checking if the resulting inequality is true is the most direct way to check whether the value is a solution.

Power-up

To power up students' ability to determine whether a given value makes an equation true, have students complete:

Match each equation with the value that makes it true.

a. $x + 0 = 25$

b. $x + 25 = 25$

c. $100 = 4x$.

d. $10 = x + 35$

a, c $x = 25$

d $x = -25$

b $x = 0$

Use: Before the Warm-up.

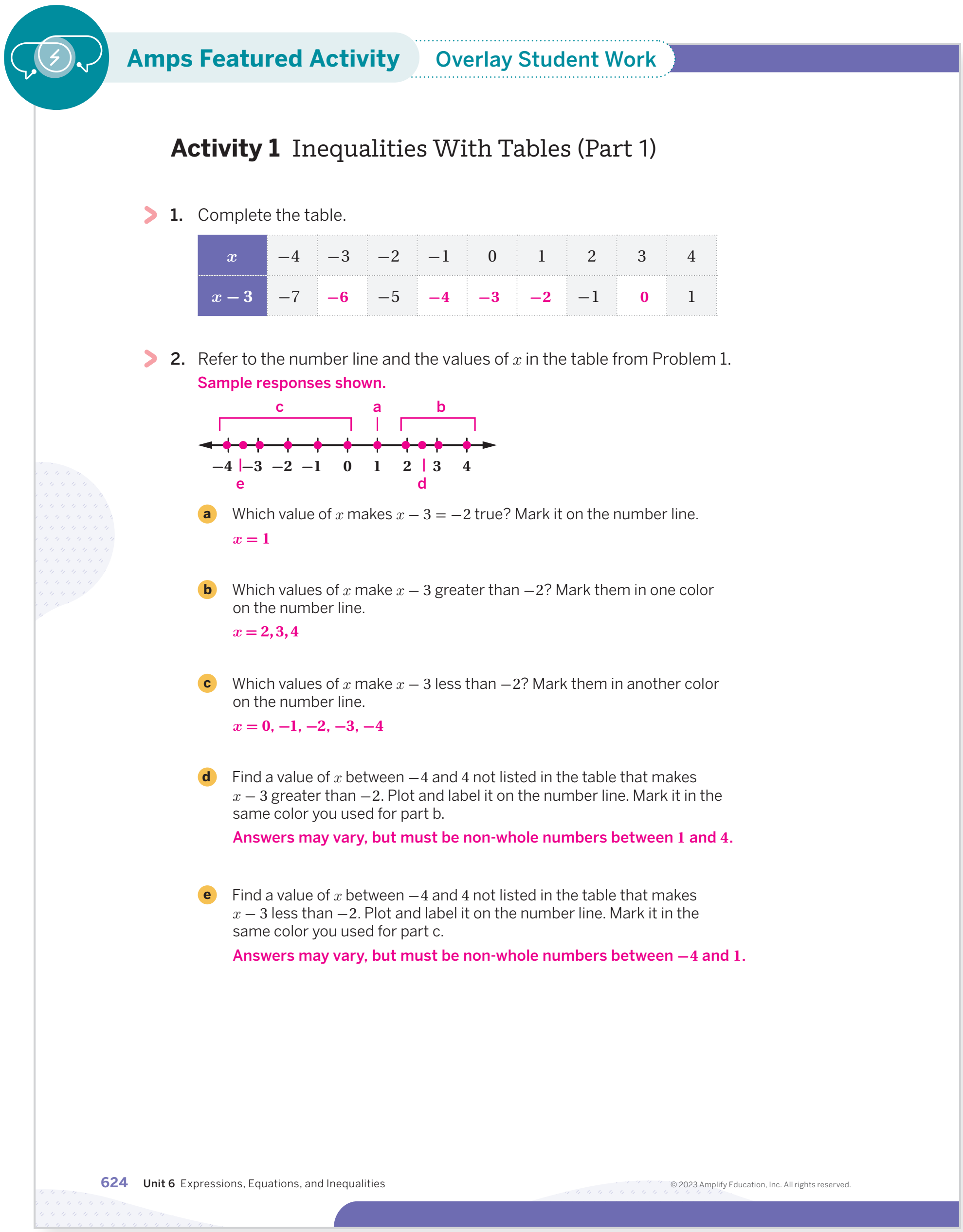
Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

Small Group | 13 min

Activity 1 Inequalities with Tables (Part 1)

Students complete and analyze a table of values in order to better understand the relationship between inequalities and their solutions.

Amps Featured Activity Overlay Student Work



Activity 1 Inequalities With Tables (Part 1)

1. Complete the table.

x	-4	-3	-2	-1	0	1	2	3	4
$x-3$	-7	-6	-5	-4	-3	-2	-1	0	1

2. Refer to the number line and the values of x in the table from Problem 1.

Sample responses shown.

a Which value of x makes $x - 3 = -2$ true? Mark it on the number line.

$x = 1$

b Which values of x make $x - 3$ greater than -2? Mark them in one color on the number line.

$x = 2, 3, 4$

c Which values of x make $x - 3$ less than -2? Mark them in another color on the number line.

$x = 0, -1, -2, -3, -4$

d Find a value of x between -4 and 4 not listed in the table that makes $x - 3$ greater than -2. Plot and label it on the number line. Mark it in the same color you used for part b.

Answers may vary, but must be non-whole numbers between 1 and 4.

e Find a value of x between -4 and 4 not listed in the table that makes $x - 3$ less than -2. Plot and label it on the number line. Mark it in the same color you used for part c.

Answers may vary, but must be non-whole numbers between −4 and 1.

624 Unit 6 Expressions, Equations, and Inequalities

1 Launch

Display the Activity 1 PDF. Ask, "How are the numbers in the top row and bottom row related? Think about the equation $x + 2 = -2$. What value of x makes this true? Where do you see that in the table? How about the inequality $x + 2 > 3$?" Distribute two different colored pencils, markers, or highlighters to each group.

2 Monitor

Look for points of confusion:

- **Struggling to identify values not listed in the table for Problem 2, parts d and e.** Suggest students consider non-integer values.
- **Saying the solution of $x - 3 > -2$ is $x > 2$, based on the integer values.** Suggest they check whether any values between 1 and 2 make the inequality true.
- **Making mistakes in Problem 3 when deciding if the circle on each graph is open or closed.** Ask them to consider if each inequality is true when $x = 1$.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can individually plot points on a digital number line and then see all of their classmate's data shown together on one number line.

Extension: *Math Enrichment*

Ask students to explain how the solutions to the following inequalities are similar to and different from the solutions to the inequalities in Problem 3.

$x + 1 > -2$

$x + 1 \geq -2$

$x + 1 < -2$

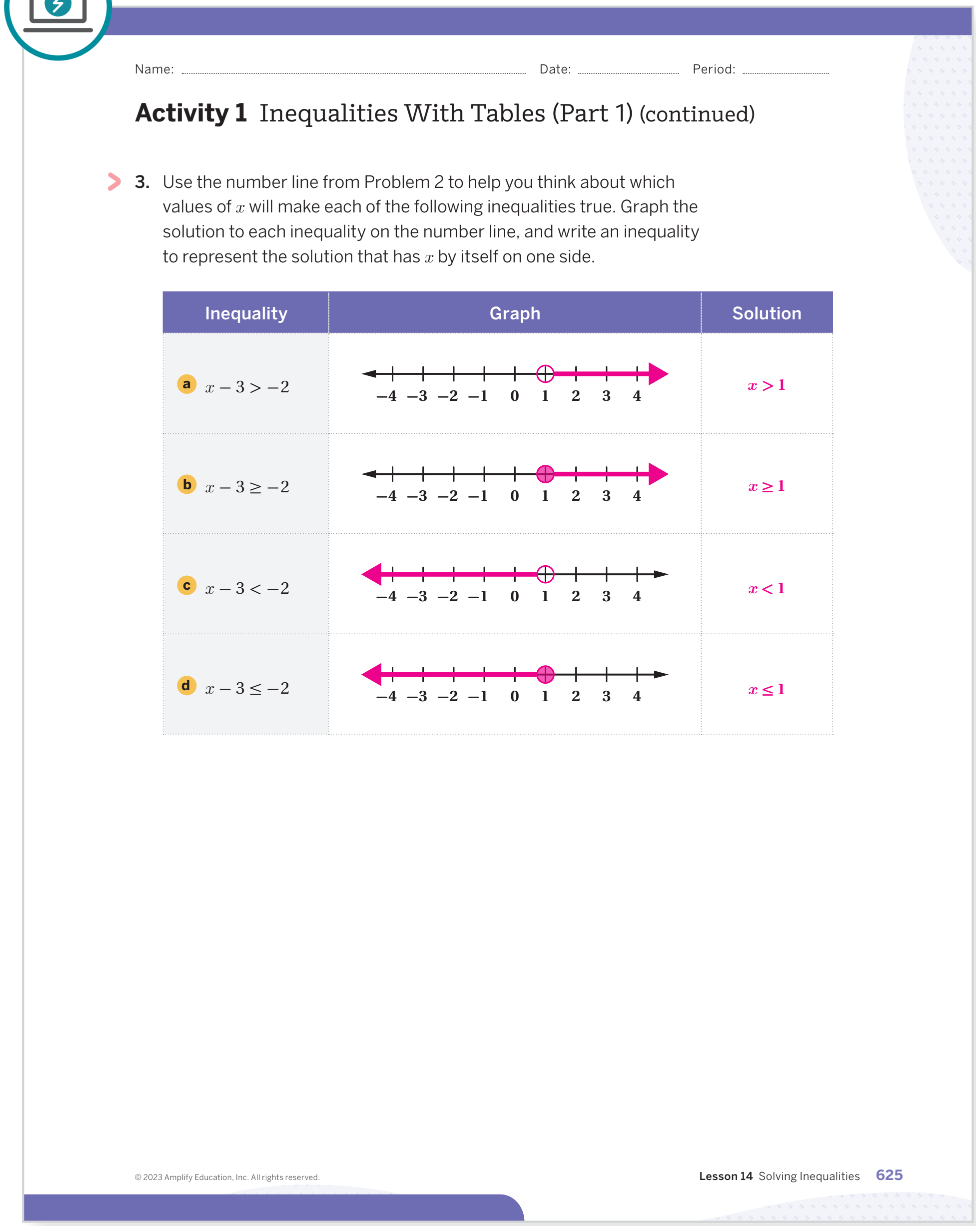
$x + 1 \leq -2$

Sample response: The direction in which the inequality is shaded will remain the same. Whether the boundary value is an open or closed circle will remain the same. The boundary values will change to be 4 less than they were in Problem 3; the boundary values will now be −3.

Pairs | 15 min

Activity 1 Inequalities with Tables (Part 1) (continued)

Students complete and analyze a table of values in order to better understand the relationship between inequalities and their solutions.



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Activity 1 Inequalities With Tables (Part 1) (continued)

3. Use the number line from Problem 2 to help you think about which values of x will make each of the following inequalities true. Graph the solution to each inequality on the number line, and write an inequality to represent the solution that has x by itself on one side.

Inequality	Graph	Solution
a $x - 3 > -2$	−4 −3 −2 −1 0 1 2 3 4	$x > 1$
b $x - 3 \geq -2$	−4 −3 −2 −1 0 1 2 3 4	$x \geq 1$
c $x - 3 < -2$	−4 −3 −2 −1 0 1 2 3 4	$x < 1$
d $x - 3 \leq -2$	−4 −3 −2 −1 0 1 2 3 4	$x \leq 1$

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Lesson 14 Solving Inequalities 625

3 Connect

Display the number line from Problem 2, marked according to the instructions in parts a–c.

Have students share their points from Problem 2, parts d and e. Ask each group to plot them on the number line.

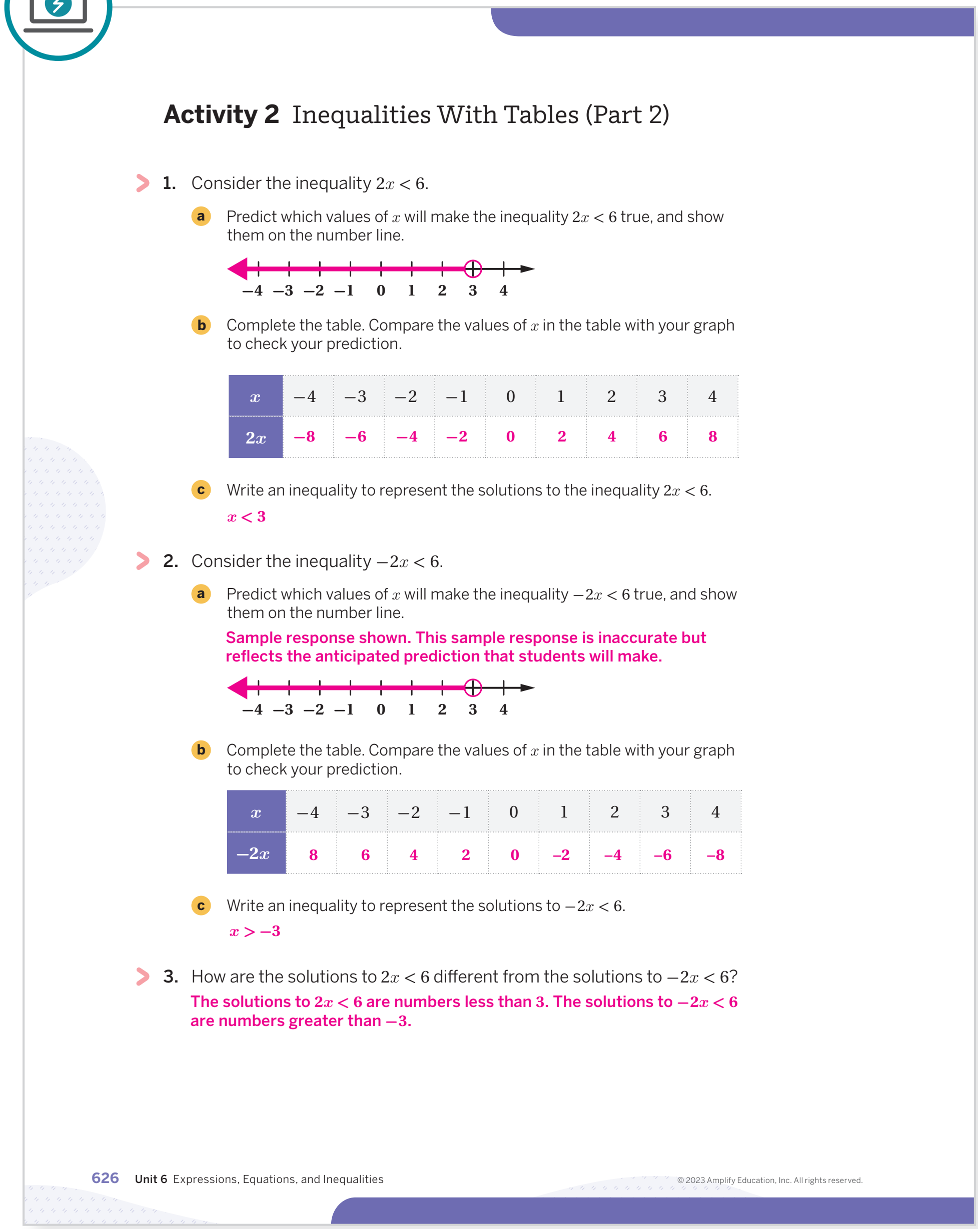
Highlight how to use the number line in Problem 2 to find the solution to the inequalities in Problem 3. Referring to the number line displayed, discuss which parts are included in the solution to each inequality and write an inequality to describe each solution. Note the inequalities for which the solution $x = 1$ is or is not included in the solution.

Ask, "How does the equation relate to the inequalities? Why does it make sense that the solution to an inequality is also an inequality?"

Small Group | 13 min

Activity 2 Inequalities With Tables (Part 2)

Students use tables and numbers lines to reason about the solutions of inequalities, including those with negative coefficients, in preparation for solving two-step inequalities.



Activity 2 Inequalities With Tables (Part 2)

1. Consider the inequality $2x < 6$.

   a. Predict which values of x will make the inequality $2x < 6$ true, and show them on the number line.

   b. Complete the table. Compare the values of x in the table with your graph to check your prediction.

x	−4	−3	−2	−1	0	1	2	3	4
$2x$	**−8**	**−6**	**−4**	**−2**	**0**	**2**	**4**	**6**	**8**

   c. Write an inequality to represent the solutions to the inequality $2x < 6$.

   $x < 3$

2. Consider the inequality $-2x < 6$.

   a. Predict which values of x will make the inequality $-2x < 6$ true, and show them on the number line.

   Sample response shown. This sample response is inaccurate but reflects the anticipated prediction that students will make.

   b. Complete the table. Compare the values of x in the table with your graph to check your prediction.

x	−4	−3	−2	−1	0	1	2	3	4
$-2x$	**8**	**6**	**4**	**2**	**0**	**−2**	**−4**	**−6**	**−8**

   c. Write an inequality to represent the solutions to $-2x < 6$.

   $x > -3$

3. How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?

   The solutions to $2x < 6$ are numbers less than 3. The solutions to $-2x < 6$ are numbers greater than −3.

626 Unit 6 Expressions, Equations, and Inequalities

1 Launch

Explain that students will be making and checking predictions about the solutions to inequalities. Suggest that they use strategies discussed in the last activity, such as solving a related equation, to help them predict.

2 Monitor

Look for points of confusion:

- **Predicting the arrows point to the left on both graphs because they have the same inequality sign.** Encourage them to check their predictions with the table.

Look for productive strategies:

- Writing and solving a related equation for each inequality. Note students who use this method.

3 Connect

Have students share their solutions for each inequality and explain how they are different.

Highlight that solving the equation related to the inequality gives the boundary value between solutions and non-solutions. Demonstrate that a table isn't necessary to check values on either side of the boundary value. Instead, test one number greater than the boundary value and one number less than the boundary value. Whichever number makes the inequality true is on the same side of the boundary value as *all* the points that make the inequality true.

Ask, "How can you use a related equation to help you solve an inequality? How would the solutions to these inequalities change if the sign was ≤ instead of <?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide pre-completed tables for students to use for Problems 1b and 2b to check their predictions. This will allow students to spend more time comparing the two inequalities.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the two inequalities and their solutions. Draw students' attention to how the inequalities and their solutions are similar and different. Ask:

- "How are the inequalities $2x < 6$ and $-2x < 6$ similar? How are they different?"
- "How are the solutions $x < 3$ and $x > -3$ similar? How are they different?"
- "Why do you think the solutions to $-2x < 6$ aren't represented by the inequality $x < -3$?"

English Learners

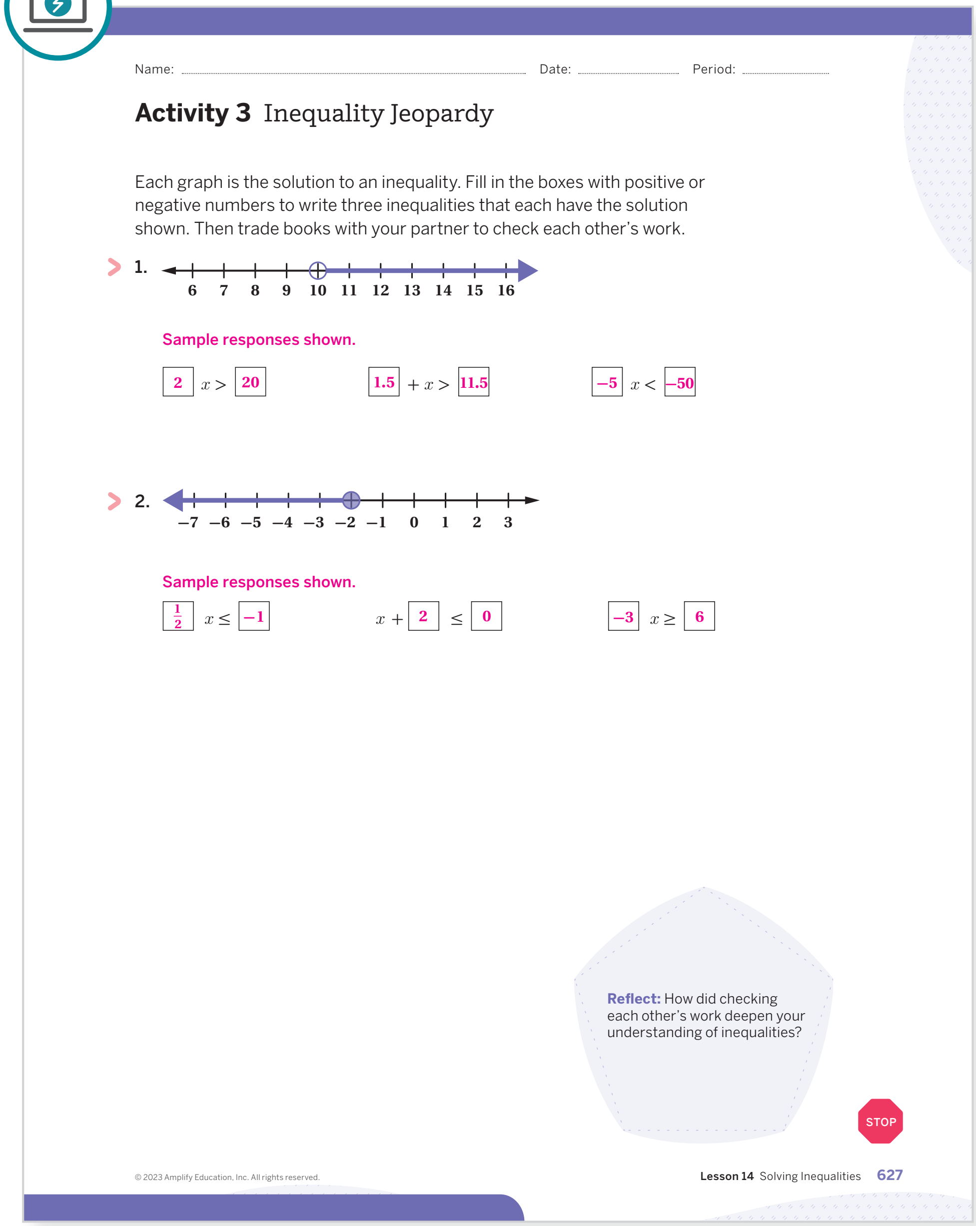
Annotate and/or color code the inequalities with their similarities and differences.

Optional

Pairs | 10 min

Activity 3 Inequality Jeopardy

Students write inequalities that have a given solution and trade with a partner to check their work to practice reasoning about and solving inequalities.



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Activity 3 Inequality Jeopardy

Each graph is the solution to an inequality. Fill in the boxes with positive or negative numbers to write three inequalities that each have the solution shown. Then trade books with your partner to check each other's work.

1.

Sample responses shown.

$\boxed{2}\,x > \boxed{20}$ $\boxed{1.5} + x > \boxed{11.5}$ $\boxed{-5}\,x < \boxed{-50}$

2.

Sample responses shown.

$\boxed{\frac{1}{2}}\,x \leq \boxed{-1}$ $x + \boxed{2} \leq \boxed{0}$ $\boxed{-3}\,x \geq \boxed{6}$

Reflect: How did checking each other's work deepen your understanding of inequalities?

STOP

Lesson 14 Solving Inequalities 627

1 Launch

Have a student read the directions to the class. Explain that students should swap with their partner after Problem 1 and again after Problem 2.

2 Monitor

Help students get started by suggesting they write an inequality to represent the solution. Then tell them to consider how they could use what they know about creating equivalent equations to create a one-step inequality.

Look for points of confusion:

- **Writing incorrect inequalities.** Monitor pairs to make sure they are really checking each other's work.
- **Disagreeing with each other about whether an inequality has the given solution.** Urge students to use mathematical reasoning and precise language to defend their positions.

3 Connect

Display the two number lines.

Have students share an inequality they wrote for each number line. Select a few students to share how they checked that their partner's inequality was correct.

Highlight how to check that the solution of an inequality is correct.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them write just one inequality for each number line. This will provide each student with additional time to reason about the solution before checking their partner's work.

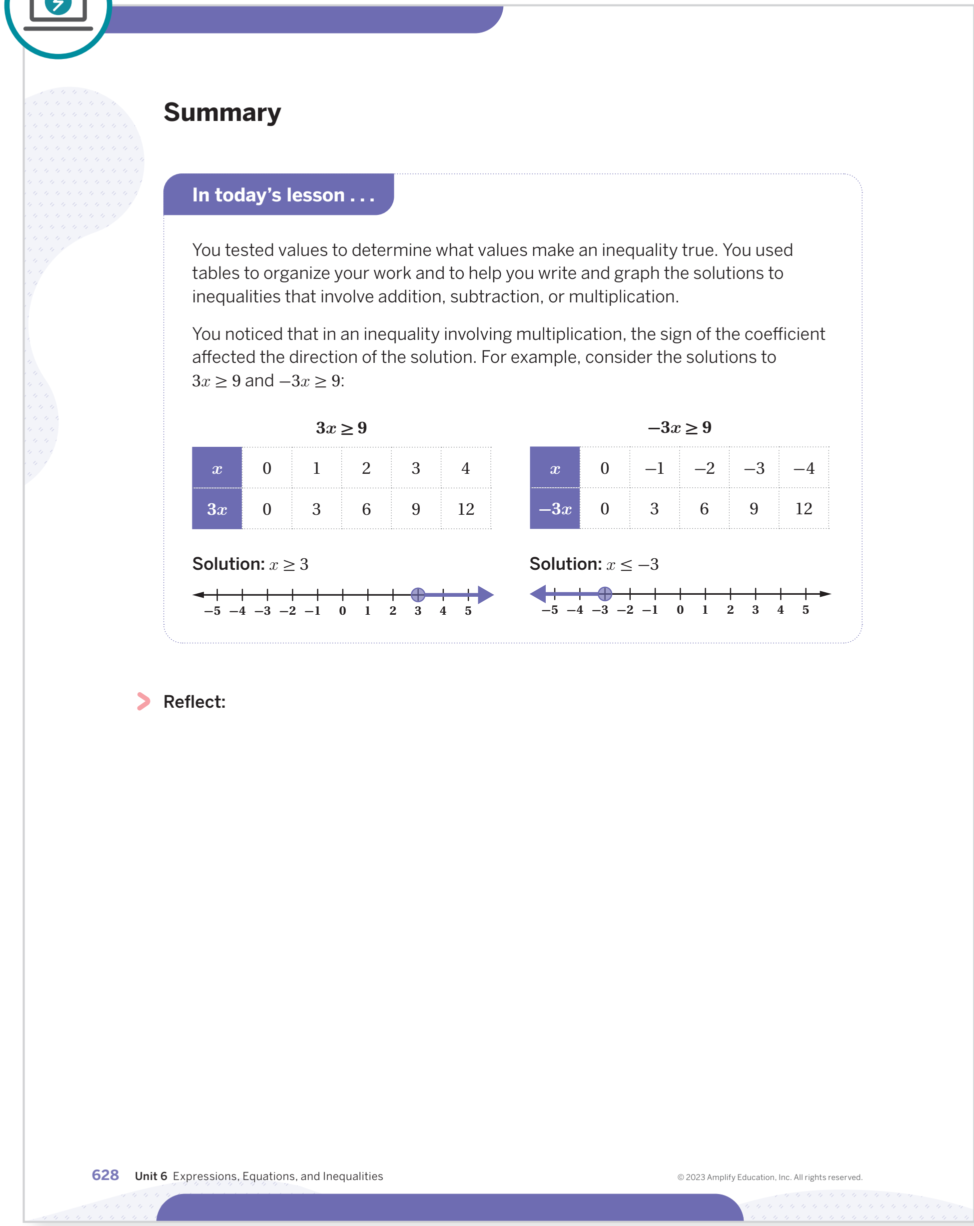
Extension: *Math Enrichment*

Have students explain why the third inequality in each problem must include a negative coefficient on the variable. Sample response: The inequality sign changed directions. In Problem 1, for example, in order for the product of the variable and a number to be less than another number, the number multiplied by the variable must be negative.

Whole Class | 5 min

Summary

Review and synthesize how solving an inequality can be thought of as solving a related equation and then checking values greater and less than the solution.



Summary

In today's lesson . . .

You tested values to determine what values make an inequality true. You used tables to organize your work and to help you write and graph the solutions to inequalities that involve addition, subtraction, or multiplication.

You noticed that in an inequality involving multiplication, the sign of the coefficient affected the direction of the solution. For example, consider the solutions to $3x \geq 9$ and $-3x \geq 9$:

$3x \geq 9$

x	0	1	2	3	4
$3x$	0	3	6	9	12

Solution: $x \geq 3$

$-3x \geq 9$

x	0	–1	–2	–3	–4
$-3x$	0	3	6	9	12

Solution: $x \leq -3$

Reflect:

Synthesize

Display the inequality $5x < -15$ and a blank number line.

Highlight the process for solving an inequality. Write the related equation $5x = -15$. Solve for x. Discuss whether the solution makes the equation true. Then have students choose one value greater than the solution and one value less than the solution. Check which of the values makes the inequality true. Then write the solution and review how to graph it.

Ask, "Why is there an open circle on the graph instead of a closed circle? How do you know that the solution is less than -3, instead of greater than?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

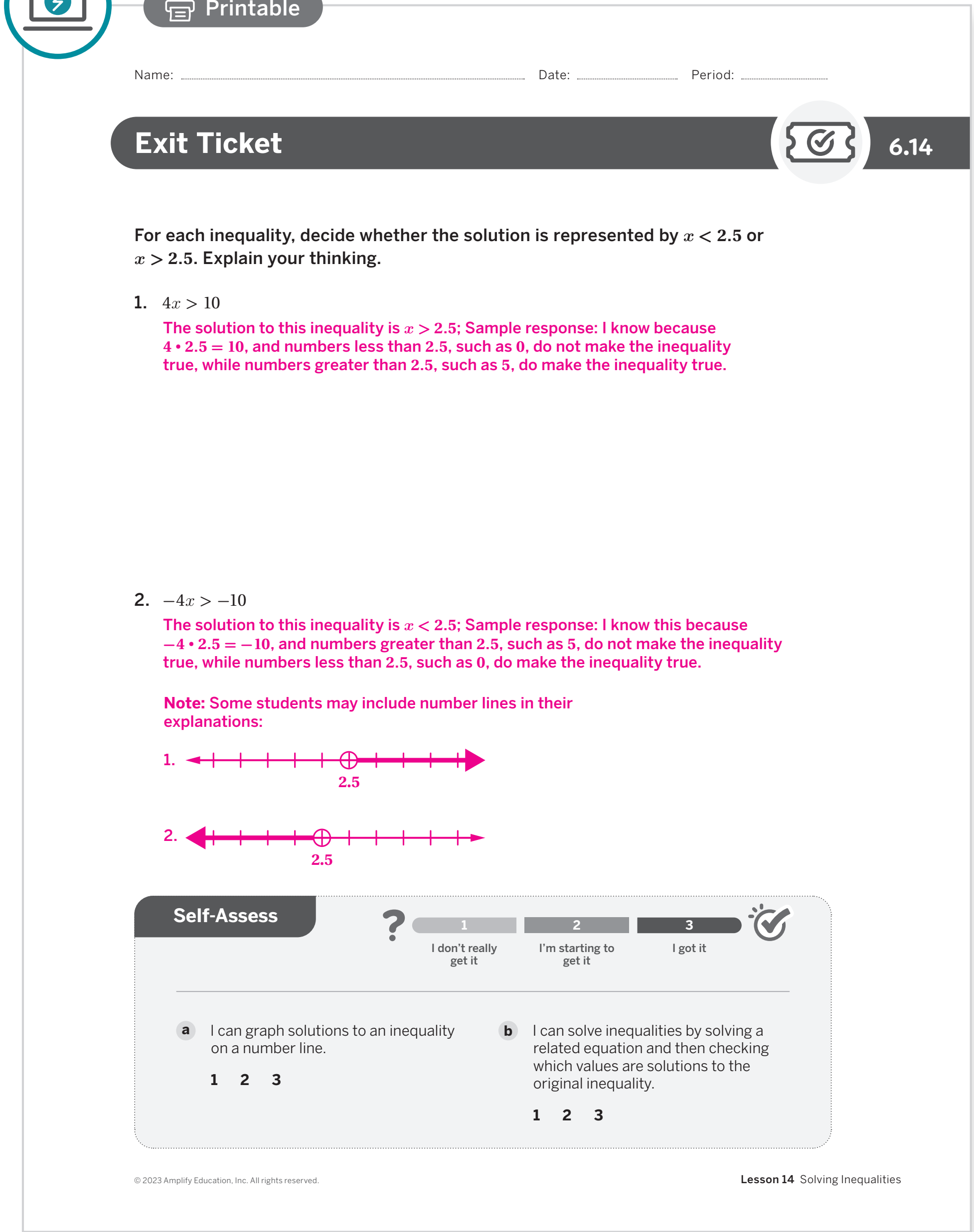
- "Which strategies that worked for solving simple equations or inequalities can be put to use when solving more complex ones?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by matching solutions to inequalities and explaining their choice.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 6.14

For each inequality, decide whether the solution is represented by $x < 2.5$ or $x > 2.5$. Explain your thinking.

1. $4x > 10$

   The solution to this inequality is $x > 2.5$; Sample response: I know because $4 \cdot 2.5 = 10$, and numbers less than 2.5, such as 0, do not make the inequality true, while numbers greater than 2.5, such as 5, do make the inequality true.

2. $-4x > -10$

   The solution to this inequality is $x < 2.5$; Sample response: I know this because $-4 \cdot 2.5 = -10$, and numbers greater than 2.5, such as 5, do not make the inequality true, while numbers less than 2.5, such as 0, do make the inequality true.

Note: Some students may include number lines in their explanations:

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can graph solutions to an inequality on a number line.

1 2 3

b I can solve inequalities by solving a related equation and then checking which values are solutions to the original inequality.

1 2 3

Lesson 14 Solving Inequalities

Success looks like . . .

- **Goal:** Using substitution to determine whether a given value for a variable makes an inequality true.
  - » Substituting values on either side of the boundary value to determine which side makes the inequality true in Problems 1 and 2.
- **Goal:** Generalizing that it is possible to solve an inequality of the form $x + q > r$ or $x + q < r$ by solving the equation $x + q = r$ and then testing a value to determine the direction of the inequality in the solution.
- **Goal:** Generalizing that it is possible to solve an inequality of the form $qx > r$ or $qx < r$ by solving the equation $qx = r$ and then testing a value to determine the direction of the inequality in the solution.
  - » Solving the inequalities $4x > 10$ and $-4x > -10$ in Problems 1 and 2.

Suggested next steps

If students say $x > 2.5$ is the solution for both inequalities because they both use the greater than sign, consider:

- Suggesting they check a value greater than 2.5 to see whether it makes both inequalities true.
- Assigning Practice Problem 2.

If students choose solutions for each inequality without providing any explanation for their choice, consider:

- Reviewing the process of checking values greater than or less than the boundary value to find the solution to an inequality.
- Assigning Practice Problem 3.

Professional Learning

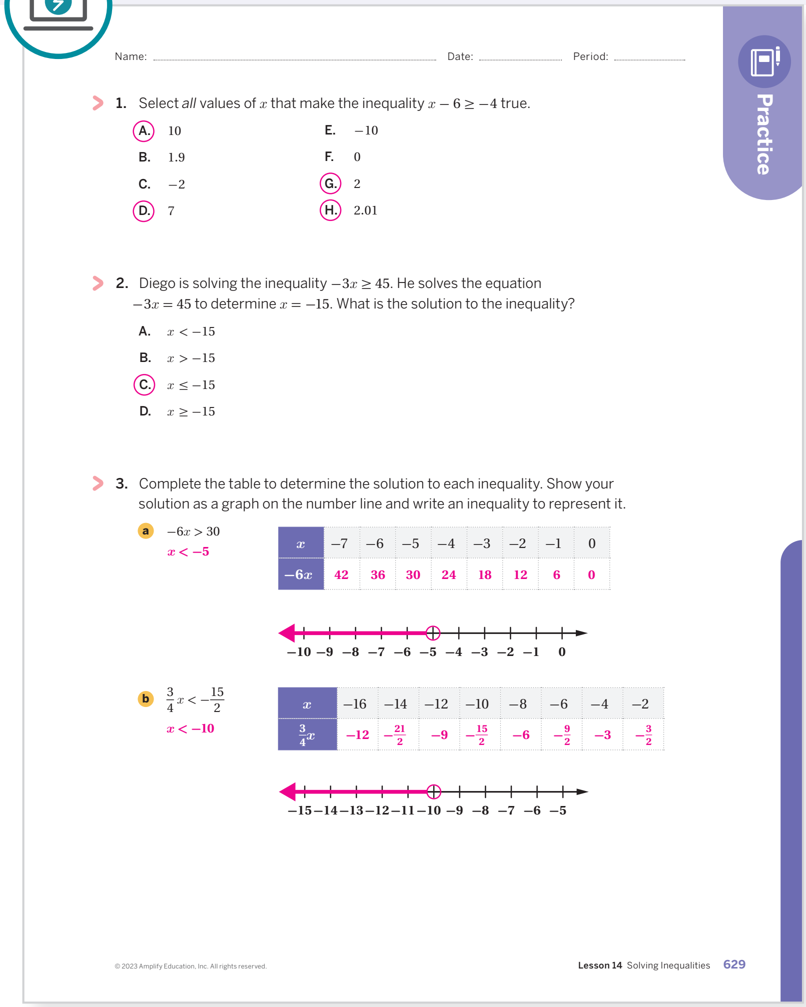
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which students' ideas were you able to highlight during Activity 1?
- Have you changed any ideas you used to have about solving and understanding inequalities as a result of today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Select *all* values of x that make the inequality $x - 6 \geq -4$ true.

[✓ A.] 10
B. 1.9
C. −2
[✓ D.] 7
E. −10
F. 0
[✓ G.] 2
[✓ H.] 2.01

2. Diego is solving the inequality $-3x \geq 45$. He solves the equation $-3x = 45$ to determine $x = -15$. What is the solution to the inequality?

A. $x < -15$
B. $x > -15$
[✓ C.] $x \leq -15$
D. $x \geq -15$

3. Complete the table to determine the solution to each inequality. Show your solution as a graph on the number line and write an inequality to represent it.

a $-6x > 30$

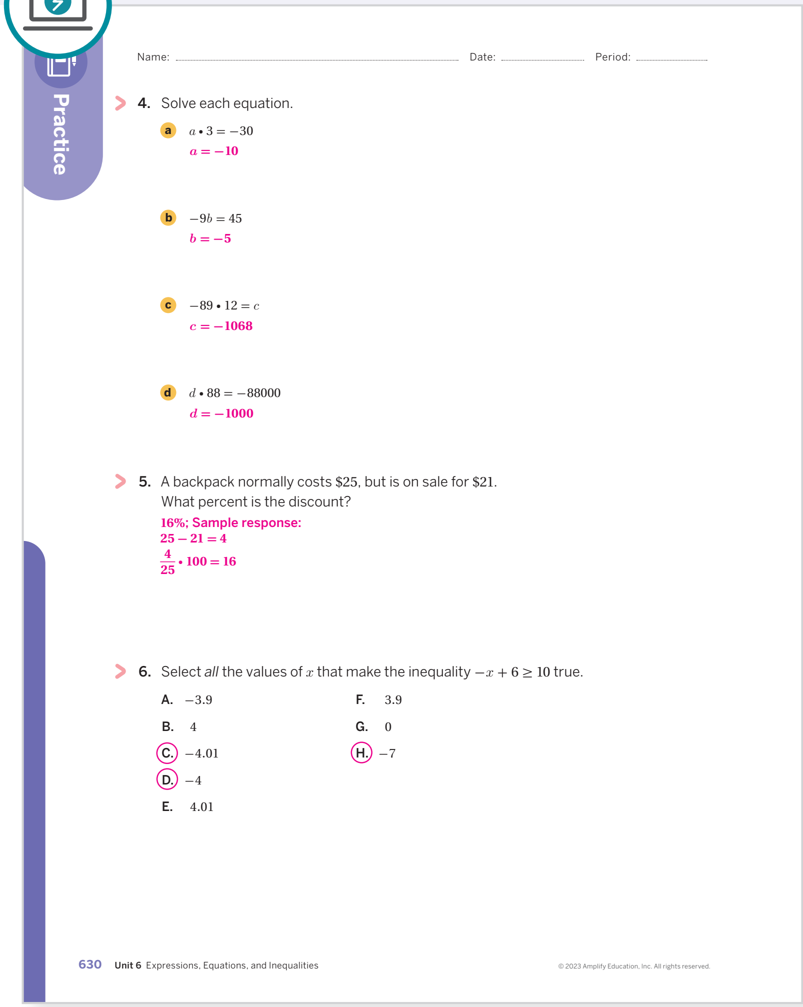
$x < -5$

x	−7	−6	−5	−4	−3	−2	−1	0
$-6x$	42	36	30	24	18	12	6	0

b $\frac{3}{4}x < -\frac{15}{2}$

$x < -10$

x	−16	−14	−12	−10	−8	−6	−4	−2
$\frac{3}{4}x$	−12	$-\frac{21}{2}$	−9	$-\frac{15}{2}$	−6	$-\frac{9}{2}$	−3	$-\frac{3}{2}$



Name: Date: Period:

Practice

4. Solve each equation.

a $a \cdot 3 = -30$

$a = -10$

b $-9b = 45$

$b = -5$

c $-89 \cdot 12 = c$

$c = -1068$

d $d \cdot 88 = -88000$

$d = -1000$

5. A backpack normally costs $25, but is on sale for $21. What percent is the discount?

16%; Sample response:

$25 - 21 = 4$

$\frac{4}{25} \cdot 100 = 16$

6. Select *all* the values of x that make the inequality $-x + 6 \geq 10$ true.

A. −3.9
B. 4
[✓ C.] −4.01
[✓ D.] −4
E. 4.01
F. 3.9
G. 0
[✓ H.] −7

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 18	1
	5	Unit 4 Lesson 5	1
Formative ⚡	6	Unit 6 Lesson 15	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 15**

Finding Solutions to Inequalities in Context

Let's solve more complicated inequalities.

Focus

Goals

1. Interpret inequalities representing situations with a constraint.
2. Solve an equation of the form $px + q = r$ to determine the boundary value for an inequality of the form $px + q > r$ or $px + q < r$.
3. **Language Goal:** Use substitution or reasoning about the context to justify whether the values making an inequality true are greater than or less than the boundary value. **(Speaking and Listening)**

Coherence

Today

Students solve contextual problems involving inequalities using the strategies from previous lessons. After solving for the boundary value, students determine the direction of the inequality. Students reason about the context, substitute values on either side of the boundary value, or reason about the number lines. These techniques exemplify making the problem more concrete and visual by asking, "Does this make sense?".

Previously

Students wrote and solved equations from scenarios in Lessons 9–11. In Lesson 14, students wrote related equations and solved them to help find the solutions to the inequality.

Coming Soon

Students will continue to solve problems involving inequalities in Lessons 16–18.

Rigor

- Students analyze real-world scenarios to develop **procedural fluency** in determining boundary values and direction of inequalities.
- Students **apply** their understanding of writing equations of the form $px + q = y$ to write inequalities of the form $px + q < y$ and $px + q > y$.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	12 min	12 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- number lines (optional)

Math Language Development

Review words

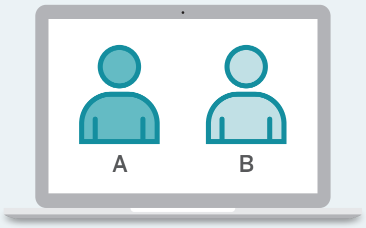
- *at least*
- *at most*
- *inequality*
- *greater than or equal to*
- *less than or equal to*
- *solution to an inequality*

Amps | **Featured Activity**

Activities 1 and 2

See Student Thinking

As students solve equations step by step, see their thinking in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students tend to get concerned when new skills are being applied in real-world situations, but, to alleviate that concern, remind them that they have all of the skills they need to make sense of the problem. Ask students to give examples of self-talk that they use to build their self confidence. Ask students to choose one new way that they will encourage themselves during an internal dialog.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Independent | 10 min

Warm-up One Solution or Many Solutions?

Students see the link between an inequality and its related equation, while recognizing equations with the same solution do not imply the inequalities have the same solutions.

Name: ____________ Date: ________ Period: ________

Unit 6 | Lesson 15

Finding Solutions to Inequalities in Context

Let's solve more complicated inequalities.

Warm-up One Solution or Many Solutions?

1. Solve each equation. Show your thinking.

   a. $-x = 10$

   $-1x = 10$

   $-1x \div (-1) = 10 \div (-1)$

   $x = -10$

   b. $2x = -20$

   $2x \div 2 = -20 \div 2$

   $x = -10$

2. What do you notice about the solutions in Problem 1?

   They are the same.

3. Determine two solutions for each inequality.

   a. $-x > 10$

   Answers may vary, but should satisfy $x < -10$.

   b. $2x > -20$

   Answers may vary, but should satisfy $x > -10$.

4. What do you notice about the solutions in Problem 3?

   Sample response: The solutions for part a does not make the inequality in part b true and vice versa.

Log in to Amplify Math to complete this lesson online.

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Lesson 15 Finding Solutions to Inequalities in Context 631

1 Launch

Set an expectation for the amount of time students will have to work independently on the activity.

2 Monitor

Help students get started by reminding students $-x$ can be written as $-1x$.

Look for points of confusion:

- **Needing a number line to help visualize the boundary value.** Print several copies of a line with unlabeled, evenly-spaced tick marks, and place these in sheet protectors. Students can write on these with dry erase markers and wipe them off.
- **Having difficulty substituting negative values into the inequality.** Remind students that $-x$ can be written as $-(x)$ or $-1 \cdot x$.

3 Connect

Display two number lines centered at -10.

Have students share their responses to Problems 1 and 2, marking them on the number line while gauging the class for agreement. Repeat this with Problems 3 and 4.

Highlight that the equations in Problem 1 have the same solution. The inequalities in Problem 3 have the same structure as the equations, yet they do not have the same solutions. Reinforce that it is important to test values to know the direction of the solutions to an inequality.

Power-up

To power up students' ability to determine whether a value makes an inequality which has a negative coefficient true, have students complete:

Recall that $-x$ is equivalent to $-1x$ or $-1 \cdot x$.
Select *all* values that make the inequality $-x < 6$ true.

[✓ A.] 6
B. -6
[✓ C.] 5
[✓ D.] -5
[✓ E.] 0
[✓ F.] 12

Use: Before the Warm-up

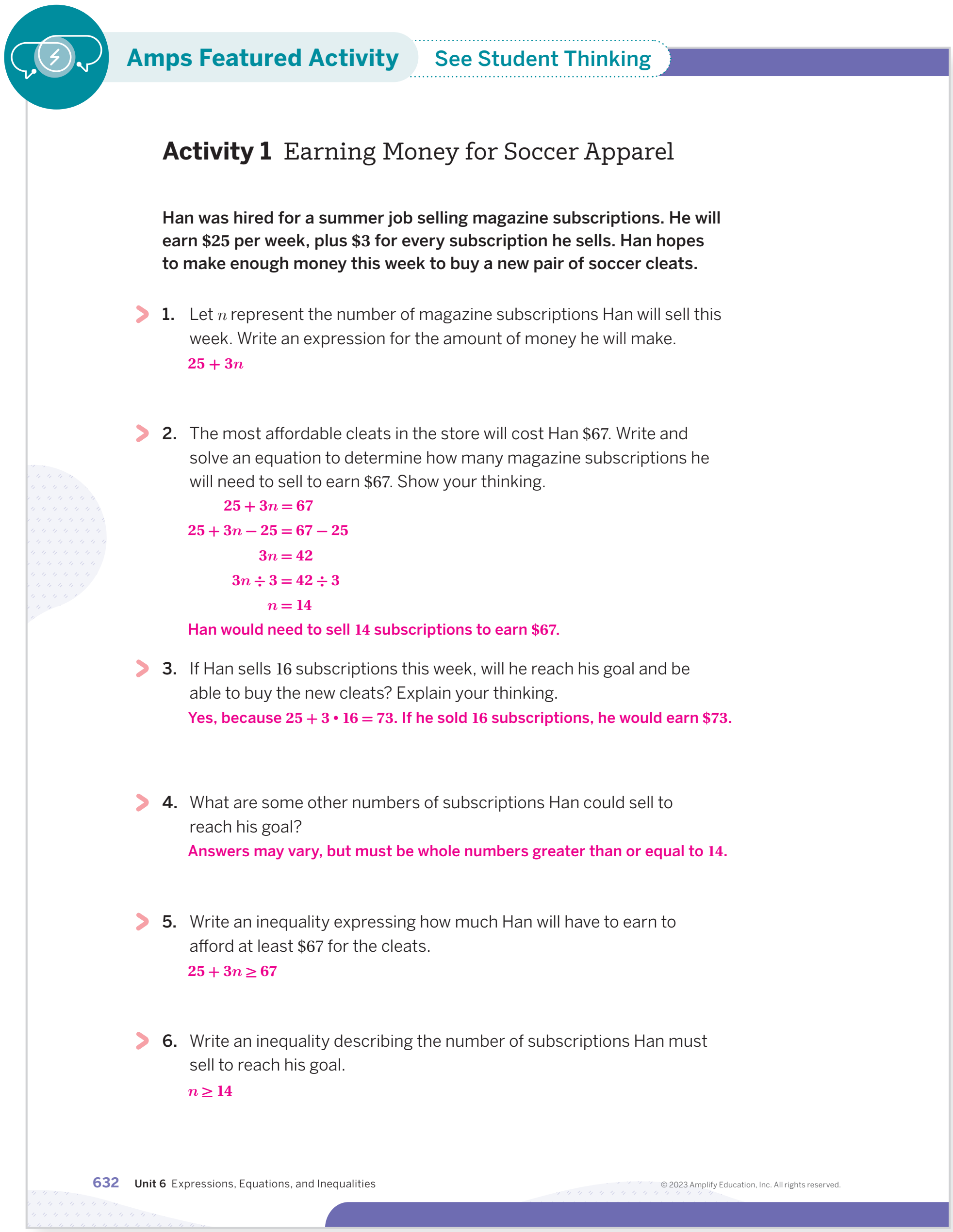
Informed by: Performance on Lesson 14, Practice Problem 6

Pairs | 12 min

Activity 1 Earning Money for Soccer Apparel

Students solve an inequality (whole-number solutions only) by writing a related equation first to answer questions about a real-world scenario.

Amps Featured Activity See Student Thinking



Activity 1 Earning Money for Soccer Apparel

Han was hired for a summer job selling magazine subscriptions. He will earn $25 per week, plus $3 for every subscription he sells. Han hopes to make enough money this week to buy a new pair of soccer cleats.

1. Let n represent the number of magazine subscriptions Han will sell this week. Write an expression for the amount of money he will make.
   $25 + 3n$

2. The most affordable cleats in the store will cost Han $67. Write and solve an equation to determine how many magazine subscriptions he will need to sell to earn $67. Show your thinking.
   $$\begin{aligned} 25 + 3n &= 67 \\ 25 + 3n - 25 &= 67 - 25 \\ 3n &= 42 \\ 3n \div 3 &= 42 \div 3 \\ n &= 14 \end{aligned}$$
   Han would need to sell 14 subscriptions to earn $67.

3. If Han sells 16 subscriptions this week, will he reach his goal and be able to buy the new cleats? Explain your thinking.
   Yes, because $25 + 3 \cdot 16 = 73$. If he sold 16 subscriptions, he would earn $73.

4. What are some other numbers of subscriptions Han could sell to reach his goal?
   Answers may vary, but must be whole numbers greater than or equal to 14.

5. Write an inequality expressing how much Han will have to earn to afford at least $67 for the cleats.
   $25 + 3n \geq 67$

6. Write an inequality describing the number of subscriptions Han must sell to reach his goal.
   $n \geq 14$

632 **Unit 6** Expressions, Equations, and Inequalities

1 Launch

Set an expectation for the amount of time students have to work in pairs, or small groups, on the activity.

2 Monitor

Help students get started by asking, "If Han sells one subscription, how much money will he have? If he sells two subscriptions, how much money will he have?" Asking questions like these helps students develop the expression $3n + 25$.

Look for points of confusion:

- **Thinking *at least* means "less than or equal to."** Give examples of possible amounts Han needs. Ask, "Would Han be able to afford his soccer cleats with $45 or with $70?"

3 Connect

Have students share their solutions and strategies on how to determine which inequality to use.

Highlight that solving the related equation helps find the boundary value, but to determine the solutions to the inequality, students should test values and/or use the context of the scenario to help.

Ask:

- "How does solving the related equation help you solve the inequality?"
- "Are there restrictions to the types of numbers that are solutions?" Han can only sell whole-number subscriptions.
- "Is this always the case or just with some scenarios?" Only some scenarios are restricted to specific values. A common occurence of this is when the scenario requires the counting of a certain item.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their classmate's responses after they submit their own response.

Extension: *Math Enrichment*

Have students complete the following problem:

If Han can sell subscriptions for two weeks, how would the inequality and solution change? The inequality would become $50 + 3n \geq 67$ and the solution would be $n \geq 5\frac{2}{3}$, which means that Han needs to sell at least 6 subscriptions.

Math Language Development

MLR5: Co-craft Questions

During the Launch, reveal the introductory text and ask students to work with their partner and to write 2–3 mathematical questions they could ask about this situation. Have volunteers share their questions with the class. Listen for and amplify questions students write that use the phrase *at least*. Sample questions shown.

- How much do the soccer cleats cost?
- If Han sells 10 subscriptions this week, how much will he earn?
- Does Han need to earn exactly the same amount as the cost of the soccer cleats, at most this amount, or at least this amount?

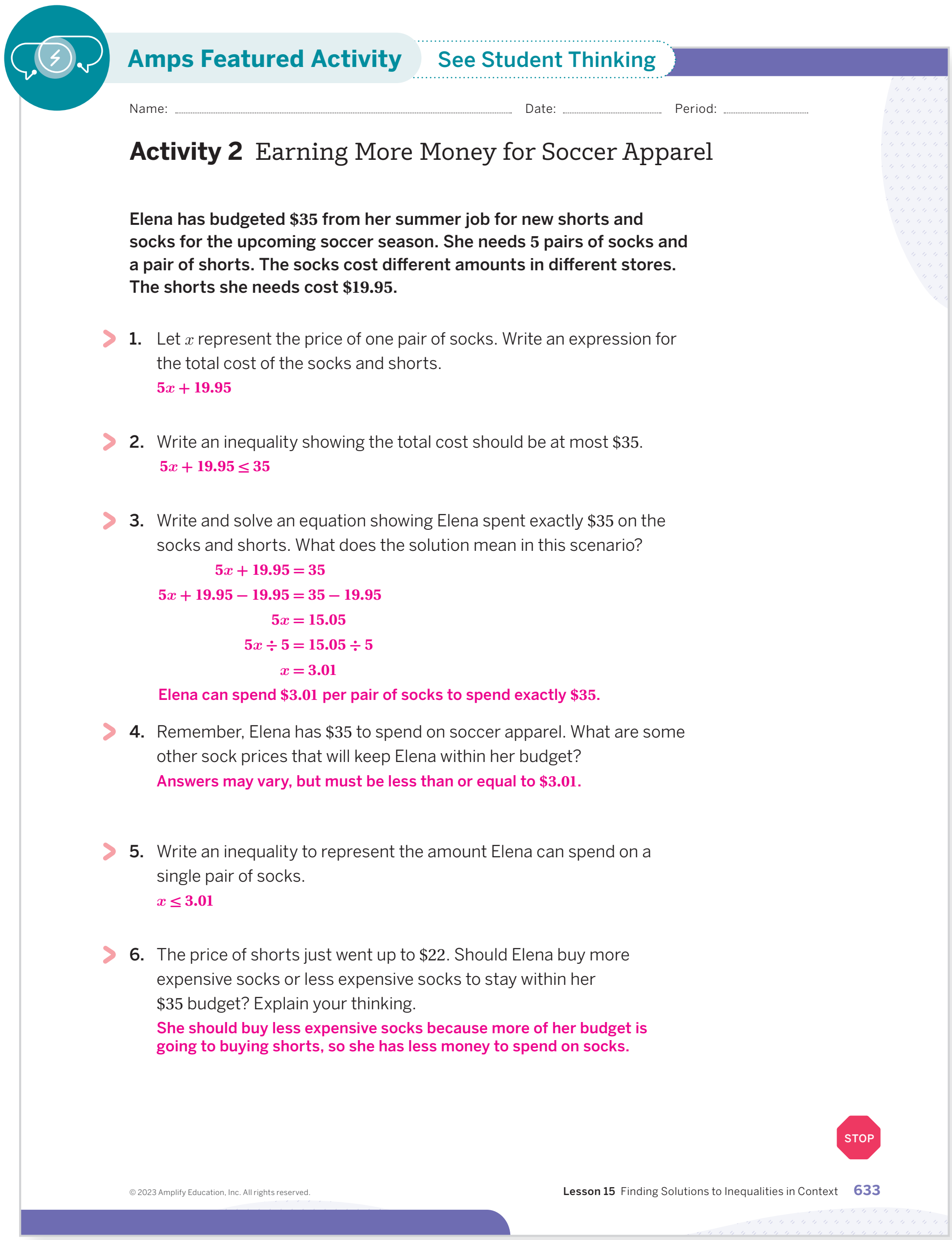
English Learners

Consider showing an image of soccer cleats to help students understand what this term means.

Pairs | 12 min

Activity 2 Earning More Money for Soccer Apparel

Students solve an inequality (rational solutions) by writing a related equation first to answer questions about a real-world scenario.



Amps Featured Activity See Student Thinking

Name: ________ Date: ________ Period: ________

Activity 2 Earning More Money for Soccer Apparel

Elena has budgeted $35 from her summer job for new shorts and socks for the upcoming soccer season. She needs 5 pairs of socks and a pair of shorts. The socks cost different amounts in different stores. The shorts she needs cost $19.95.

1. Let x represent the price of one pair of socks. Write an expression for the total cost of the socks and shorts.
   $5x + 19.95$
2. Write an inequality showing the total cost should be at most $35.
   $5x + 19.95 \le 35$
3. Write and solve an equation showing Elena spent exactly $35 on the socks and shorts. What does the solution mean in this scenario?
   $$\begin{aligned} 5x + 19.95 &= 35 \\ 5x + 19.95 - 19.95 &= 35 - 19.95 \\ 5x &= 15.05 \\ 5x \div 5 &= 15.05 \div 5 \\ x &= 3.01 \end{aligned}$$
   Elena can spend $3.01 per pair of socks to spend exactly $35.
4. Remember, Elena has $35 to spend on soccer apparel. What are some other sock prices that will keep Elena within her budget?
   Answers may vary, but must be less than or equal to $3.01.
5. Write an inequality to represent the amount Elena can spend on a single pair of socks.
   $x \le 3.01$
6. The price of shorts just went up to $22. Should Elena buy more expensive socks or less expensive socks to stay within her $35 budget? Explain your thinking.
   She should buy less expensive socks because more of her budget is going to buying shorts, so she has less money to spend on socks.

STOP

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Lesson 15 Finding Solutions to Inequalities in Context 633

1 Launch

Set an expectation for the amount of time students will have to work in pairs, or small groups, on the activity.

2 Monitor

Help students get started by asking, "What do you *know* about Elena?" and "What do you *need* to know?"

Look for points of confusion:

- **Thinking *at most* means greater than or equal to.** Ask, "Would Elena be within budget if she spent $4 per pair or $2 per pair?"

3 Connect

Have students share their solutions and strategies on which inequality to use.

Highlight similarities and differences among Han's and Elena's scenarios. Testing values will always help determine what the solutions are, but students can also reason about the scenario. If Elena wants to spend less money, she should spend less on each pair of socks.

Ask:

- "Can Elena spend exactly $3.01 on a pair of socks?"
- "Are there restrictions to these values like there were with Han's subscriptions?" No. In this situation, the variable represents money, which can be decimals to the hundredths place. In Han's situation, the variable represented the number of magazine subscriptions, which are restricted to whole numbers.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their classmate's responses after they submit their own response.

Extension: *Math Enrichment*

Have students complete the following problem:

If Elena selects socks that cost $4 per pair, how much can she spend on the pair of shorts, if her budget remains the same? At most $15; Let s represent the cost of the pair of shorts; $5(4) + s \le 35$; $s \le 15$.

Math Language Development

MLR5: Co-craft Questions

During the Launch, reveal the introductory text and ask students to work with their partner and to write 2–3 mathematical questions they could ask about this situation. Have volunteers share their questions with the class. Listen for and amplify questions students write that use the phrase *at most*. Sample questions shown.

- How much do the pairs of socks cost?
- Can Elena spend exactly $35, at least $35, or at most $35?
- How much can Elean spend on each pair of socks?

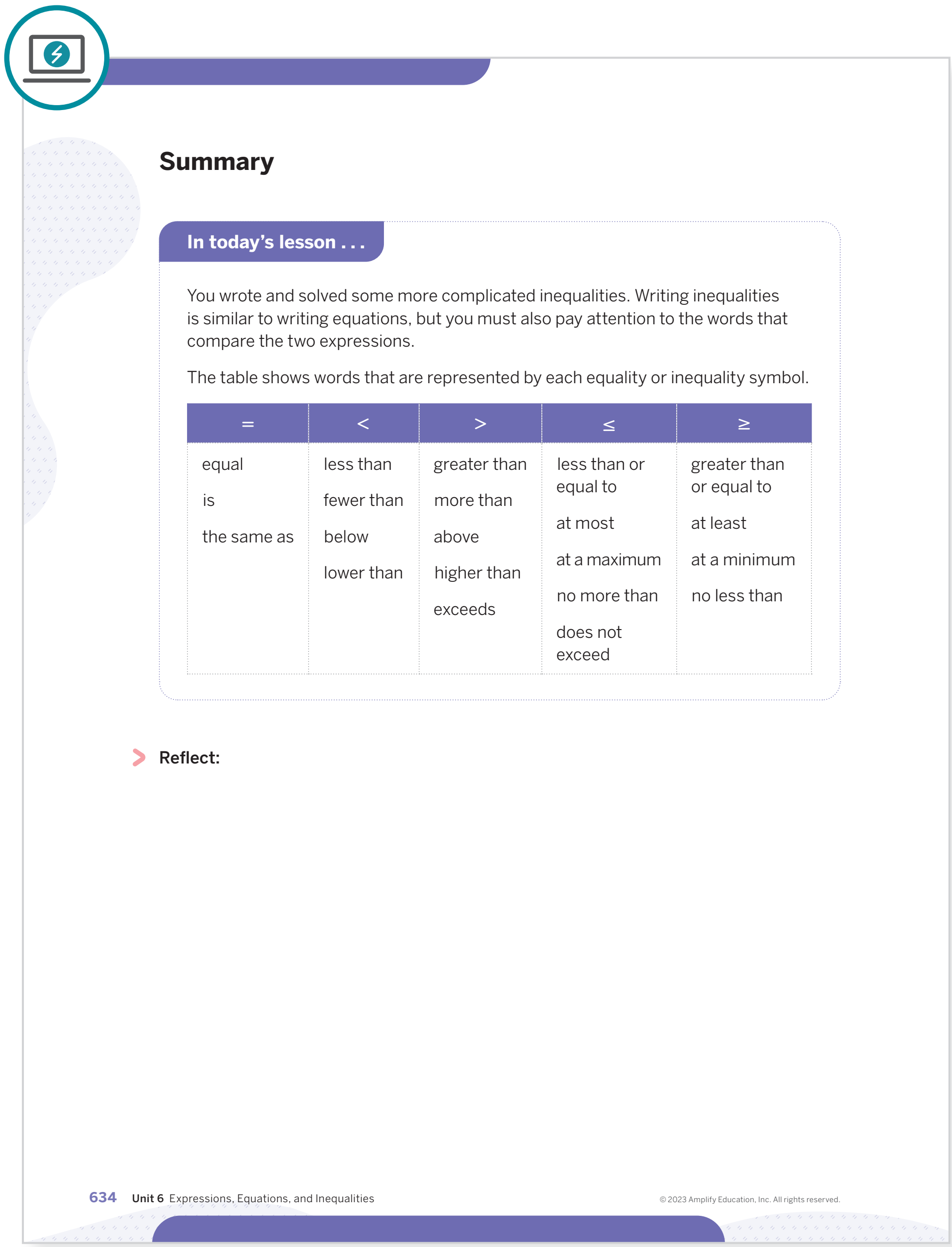
English Learners

Be sure students understand that a "pair of shorts" represents one quantity, not two.

Whole Class | 5 min

Summary

Review and synthesize how to interpret and solve inequalities that represent real-world situations.



Summary

In today's lesson . . .

You wrote and solved some more complicated inequalities. Writing inequalities is similar to writing equations, but you must also pay attention to the words that compare the two expressions.

The table shows words that are represented by each equality or inequality symbol.

=	<	>	≤	≥
equal	less than	greater than	less than or equal to	greater than or equal to
is	fewer than	more than	at most	at least
the same as	below	above	at a maximum	at a minimum
	lower than	higher than	no more than	no less than
		exceeds	does not exceed	

Reflect:

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Synthesize

Display the inequality symbols on the board and write common phrases used for each.

Have students share strategies they use to determine which inequality symbol to use.

Highlight that substituting values into the inequality will always tell students the direction of the solutions to the inequality, and that reasoning through the language of the problem is a way to ensure that the solutions in context make sense.

Ask, "Which phrases do you find most challenging to understand?" Address any concerns presented by the students.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

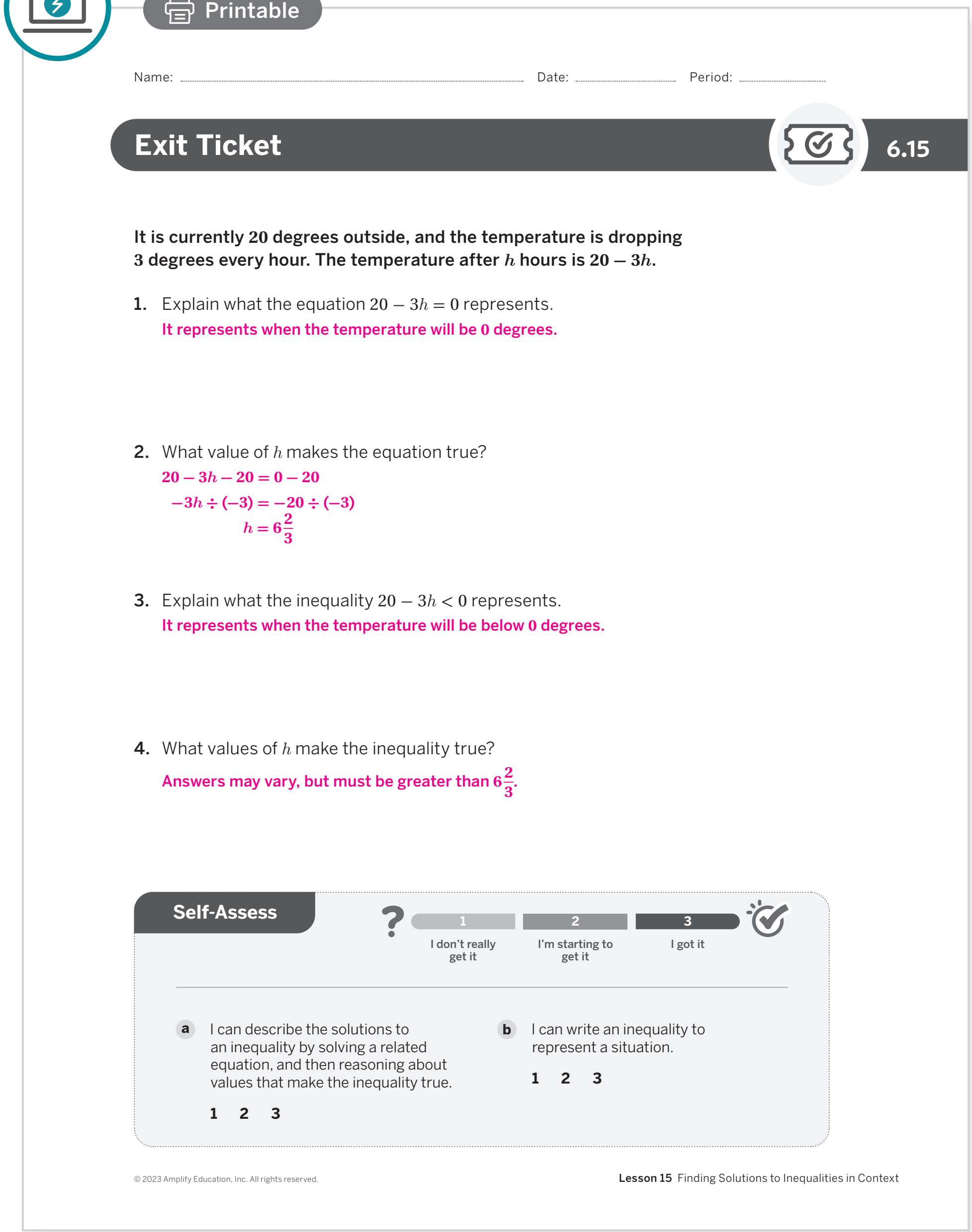
- "Which strategies that worked for solving simple equations or inequalities can be put to use when solving more complex ones?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of solving an inequality by first solving a related equation, and then testing values on either side of the boundary value.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.15

It is currently 20 degrees outside, and the temperature is dropping 3 degrees every hour. The temperature after h hours is $20 - 3h$.

1. Explain what the equation $20 - 3h = 0$ represents.
   It represents when the temperature will be 0 degrees.

2. What value of h makes the equation true?

$$\begin{aligned} 20 - 3h - 20 &= 0 - 20 \\ -3h \div (-3) &= -20 \div (-3) \\ h &= 6\tfrac{2}{3} \end{aligned}$$

3. Explain what the inequality $20 - 3h < 0$ represents.
   It represents when the temperature will be below 0 degrees.

4. What values of h make the inequality true?
   Answers may vary, but must be greater than $6\frac{2}{3}$.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can describe the solutions to an inequality by solving a related equation, and then reasoning about values that make the inequality true.
1 2 3

b. I can write an inequality to represent a situation.
1 2 3

Lesson 15 Finding Solutions to Inequalities in Context

Success looks like . . .

- **Goal:** Interpreting inequalities representing situations with a constraint.
  - » Interpreting the inequality $20 - 3h < 0$ in the context of temperature in Problem 3.
- **Goal:** Solving an equation of the form $px + q = r$ to determine the boundary value for an inequality of the form $px + q > r$ or $px + q < r$.
  - » Determining the boundary value h that makes the equation true in Problem 2.
- **Language Goal:** Using substitution or reasoning about the context to justify whether the values making an inequality true are greater than or less than the boundary value. **(Speaking and Listening)**

Suggested next steps

If students solve the equation correctly but solve the inequality incorrectly, consider:

- Reminding them to test values on either side of the boundary value. The side where the values make the inequality true is the solution.
- Assigning Practice Problems 1 and 2.

If students have difficulty with the process of solving a related equation, consider:

- Assigning Practice Problem 3.

Professional Learning

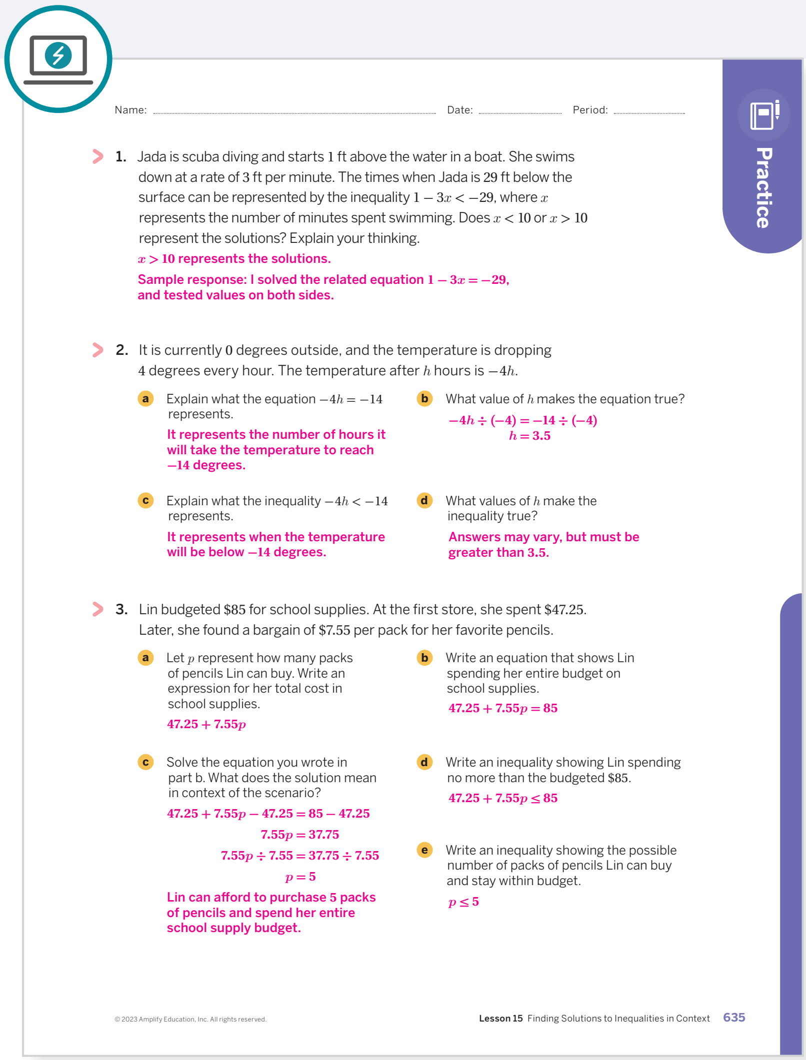
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students wrote and solved inequalities of the form $px + q > r$ and $px + q < r$. How did that build on the earlier work students did with writing and solving equations of the form $px + q = r$?
- **7.EE.B.4.b** asks students to interpret the solution set of an inequality in the context of a problem. Where in your students' work today did you see or hear evidence of them doing this? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Jada is scuba diving and starts 1 ft above the water in a boat. She swims down at a rate of 3 ft per minute. The times when Jada is 29 ft below the surface can be represented by the inequality $1 - 3x < -29$, where x represents the number of minutes spent swimming. Does $x < 10$ or $x > 10$ represent the solutions? Explain your thinking.

   $x > 10$ represents the solutions.

   Sample response: I solved the related equation $1 - 3x = -29$, and tested values on both sides.

2. It is currently 0 degrees outside, and the temperature is dropping 4 degrees every hour. The temperature after h hours is $-4h$.

   a. Explain what the equation $-4h = -14$ represents.

   It represents the number of hours it will take the temperature to reach −14 degrees.

   b. What value of h makes the equation true?

   $$-4h \div (-4) = -14 \div (-4)$$
   $$h = 3.5$$

   c. Explain what the inequality $-4h < -14$ represents.

   It represents when the temperature will be below −14 degrees.

   d. What values of h make the inequality true?

   Answers may vary, but must be greater than 3.5.

3. Lin budgeted $85 for school supplies. At the first store, she spent $47.25. Later, she found a bargain of $7.55 per pack for her favorite pencils.

   a. Let p represent how many packs of pencils Lin can buy. Write an expression for her total cost in school supplies.

   $47.25 + 7.55p$

   b. Write an equation that shows Lin spending her entire budget on school supplies.

   $47.25 + 7.55p = 85$

   c. Solve the equation you wrote in part b. What does the solution mean in context of the scenario?

   $$47.25 + 7.55p - 47.25 = 85 - 47.25$$
   $$7.55p = 37.75$$
   $$7.55p \div 7.55 = 37.75 \div 7.55$$
   $$p = 5$$

   Lin can afford to purchase 5 packs of pencils and spend her entire school supply budget.

   d. Write an inequality showing Lin spending no more than the budgeted $85.

   $47.25 + 7.55p \leq 85$

   e. Write an inequality showing the possible number of packs of pencils Lin can buy and stay within budget.

   $p \leq 5$

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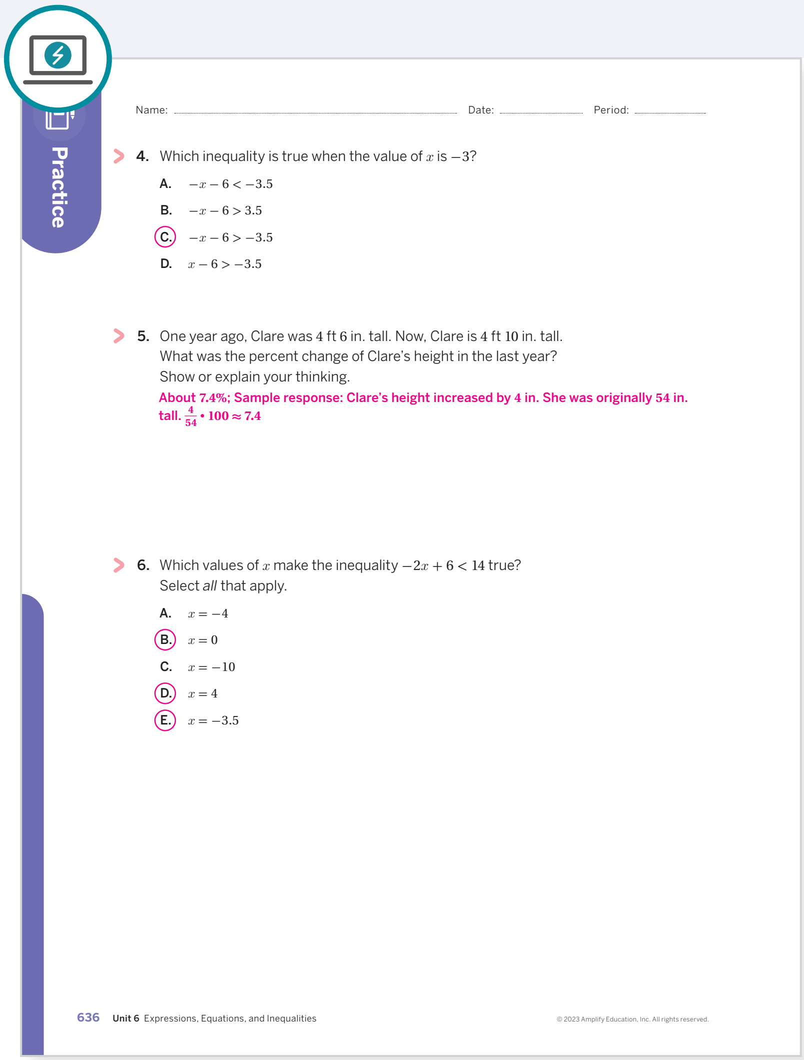
Lesson 15 Finding Solutions to Inequalities in Context 635



Practice

Name: Date: Period:

4. Which inequality is true when the value of x is -3?

   A. $-x - 6 < -3.5$

   B. $-x - 6 > 3.5$

   [✓ C.] $-x - 6 > -3.5$

   D. $x - 6 > -3.5$

5. One year ago, Clare was 4 ft 6 in. tall. Now, Clare is 4 ft 10 in. tall. What was the percent change of Clare's height in the last year? Show or explain your thinking.

   About 7.4%; Sample response: Clare's height increased by 4 in. She was originally 54 in. tall. $\frac{4}{54} \cdot 100 \approx 7.4$

6. Which values of x make the inequality $-2x + 6 < 14$ true? Select *all* that apply.

   A. $x = -4$

   [✓ B.] $x = 0$

   C. $x = -10$

   [✓ D.] $x = 4$

   [✓ E.] $x = -3.5$

636 Unit 6 Expressions, Equations, and Inequalities

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	2
	2	Activities 1 and 2	2
	3	Activities 1 and 2	2
Spiral	4	Unit 6 Lesson 14	1
	5	Unit 4 Lesson 5	1
Formative ⚡	6	Unit 6 Lesson 16	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 16**

Efficiently Solving Inequalities

Let's solve more complicated inequalities.

Focus

Goals

1. **Language Goal:** Compare and contrast solutions to equations and solutions to inequalities. **(Speaking and Listening)**
2. Draw and label a graph on the number line that represents all the solutions to an inequality.
3. **Language Goal:** Generalize the solutions of an inequality of the form $px + q > r$ or $px + q < r$ by solving the equation $px + q = r$ and then testing a value to determine the direction of the inequality in the solution. **(Speaking and Listening)**

Coherence

Today

Students solve inequalities of the forms $px + q < r$ and $p(x + q) < r$ by first writing and solving a related equation. Then they test values to determine the direction of the inequality in the solution.

Previously

In Lesson 14, students solved inequalities of the forms $px < q$ and $x + p < q$ by writing and solving a related equation and testing values to determine the direction of the inequality in the solution.

Coming Soon

In Lesson 17, students will solve word problems by writing inequalities of the forms $px + q < r$ and $p(x + q) < r$ and solving them using the methods addressed in today's lesson.

Rigor

- Students solve inequalities and test solutions to develop their **conceptual understanding** of graphing the solutions of an inequality on a number line.
- Students develop **procedural fluency** in solving and graphing the solutions of an inequality.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Inequalities* (for display, as needed)
- Anchor Chart PDF, *Inequalities* (answers)

Math Language Development

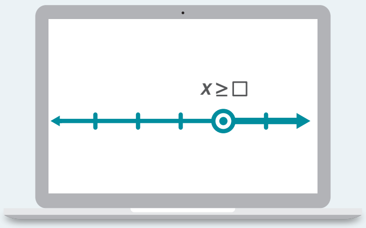
Review words

- *inequality*
- *solution to an equation*
- *solutions to an inequality*

Amps ⋮ **Featured Activity**

Activity 1
Dynamic Number Lines

Students can represent solutions to inequalities on digital numbers lines. You can view their responses in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively solve an inequality just like they would solve an equation but without considering the special cases required of inequalities with signed numbers. Encourage students to write anything extra that they need to remember when solving an inequality at the top of the page. After they have solved all of the inequalities, they need to persevere and go back to look at each case making sure that they did not forget to apply the additional steps.

Modifications to Pacing

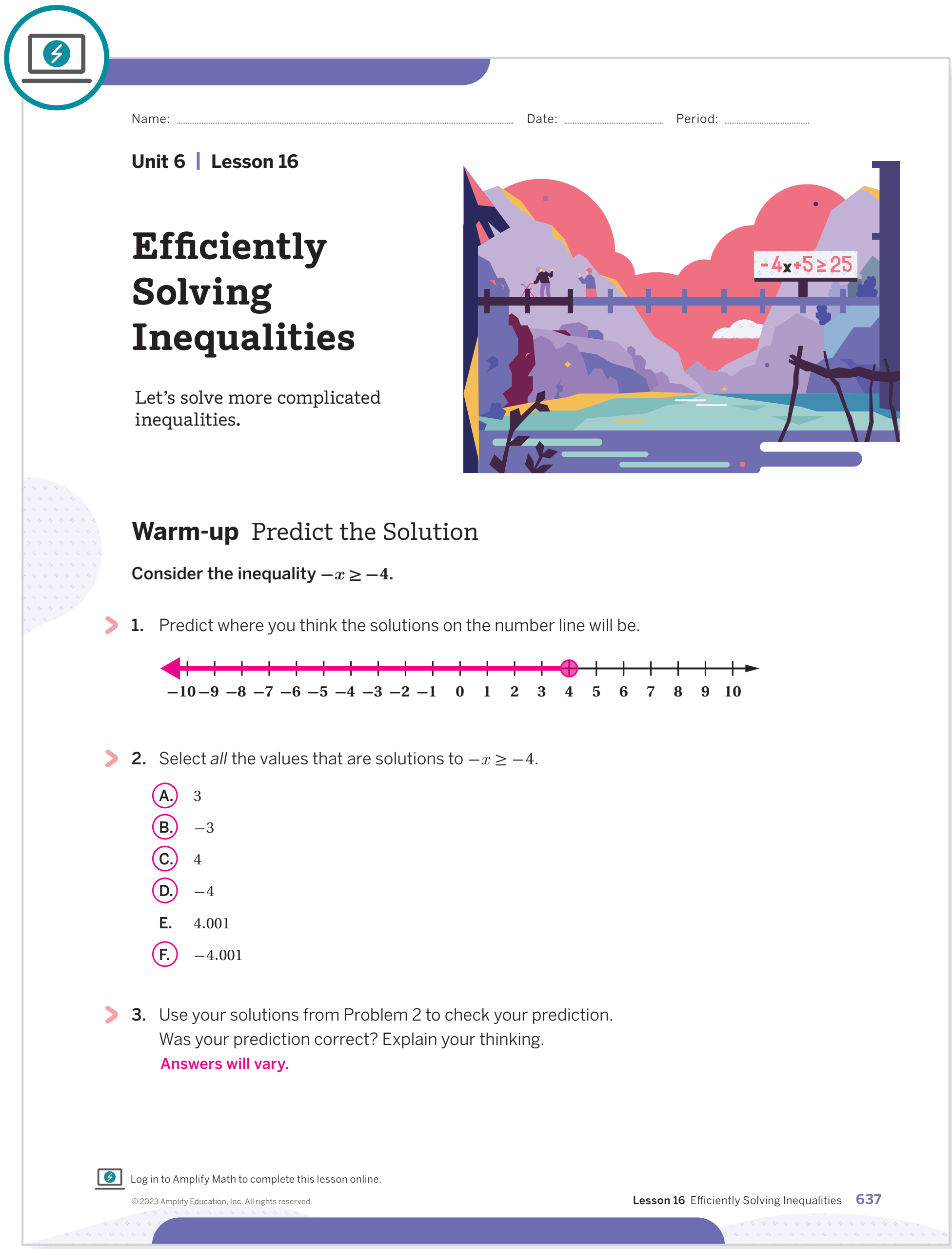
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, Problem 1 may be omitted.

Independent | 5 min

Warm-up Predict the Solution

Students make a prediction about an inequality with a negative coefficient and then test values to check their prediction. This helps prepare them for thinking about solving inequalities with negative coefficients.



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Unit 6 | Lesson 16

Efficiently Solving Inequalities

Let's solve more complicated inequalities.

Warm-up Predict the Solution

Consider the inequality $-x \geq -4$.

1. Predict where you think the solutions on the number line will be.

2. Select *all* the values that are solutions to $-x \geq -4$.

[✓ A.] 3

[✓ B.] -3

[✓ C.] 4

[✓ D.] -4

E. 4.001

[✓ F.] -4.001

3. Use your solutions from Problem 2 to check your prediction. Was your prediction correct? Explain your thinking.

Answers will vary.

Log in to Amplify Math to complete this lesson online.

Lesson 16 Efficiently Solving Inequalities 637

1 Launch

Set an expectation for the amount of time students have to work independently on the activity.

2 Monitor

Help students get started by asking, "How do you represent solutions to inequalities on the number line? and How can you test if a value is a solution to an inequality?"

Look for points of confusion:

- **Ignoring the negative sign on the variable when selecting values that are solutions.** Remind students that $-x$ is the same as $-1 \cdot x$ or $-1x$. Have students substitute the given values of x into the inequality and check if it is still true.

3 Connect

Have students share which of the given values they predicted were solutions. Select some students to explain their thinking for each value and others to share how their predictions differed from their final solutions.

Highlight how negative values in the inequality can make it difficult to predict the solutions. Review the strategy of first determining the value for which both sides are equal, and then testing values to determine the direction of the solutions to the inequality.

Ask, "How are the solutions to $-x \geq -4$ different from the solutions to $x \geq 4$?"

Power-up

To power up students' ability to determine what value make more complex inequalities true, have students complete:

Select *all* values that make the inequality $2x + 8 > 6$ true.

[✓ A.] 1

[✓ B.] 0

[✓ C.] 1.5

[✓ D.] 2

E. -1

[✓ F.] 12

Use: Before Activity 1

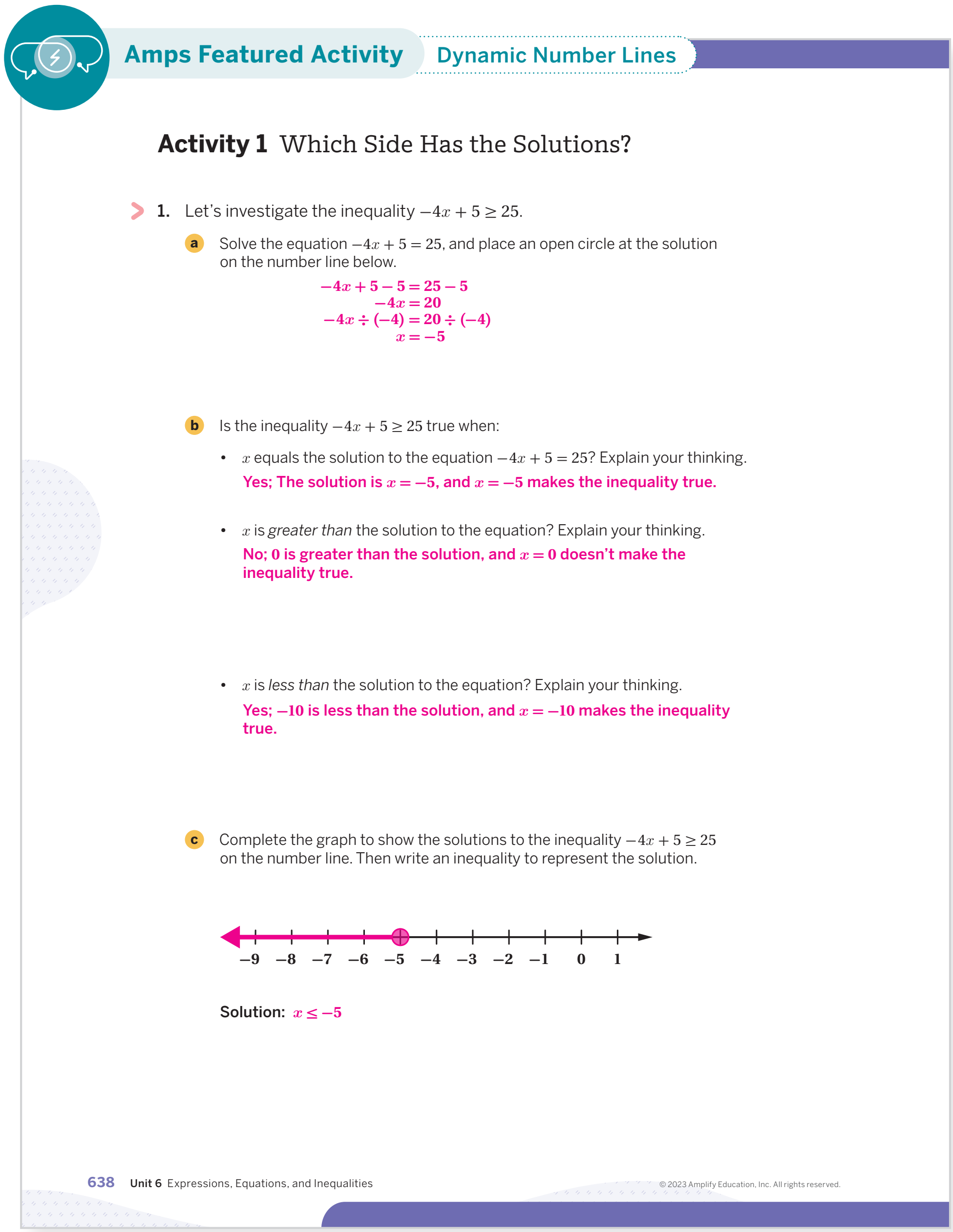
Informed by: Performance on Lesson 15, Practice Problem 6

Pairs | 15 min

Activity 1 Which Side Has the Solutions?

Students solve inequalities by solving related equations and testing solutions and by formalizing a process for solving inequalities.

Amps Featured Activity Dynamic Number Lines



Activity 1 Which Side Has the Solutions?

1. Let's investigate the inequality $-4x + 5 \geq 25$.

   a. Solve the equation $-4x + 5 = 25$, and place an open circle at the solution on the number line below.

   $$\begin{gathered} -4x + 5 - 5 = 25 - 5 \\ -4x = 20 \\ -4x \div (-4) = 20 \div (-4) \\ x = -5 \end{gathered}$$

   b. Is the inequality $-4x + 5 \geq 25$ true when:

   - x equals the solution to the equation $-4x + 5 = 25$? Explain your thinking.

     Yes; The solution is $x = -5$, and $x = -5$ makes the inequality true.

   - x is *greater than* the solution to the equation? Explain your thinking.

     No; 0 is greater than the solution, and $x = 0$ doesn't make the inequality true.

   - x is *less than* the solution to the equation? Explain your thinking.

     Yes; −10 is less than the solution, and $x = -10$ makes the inequality true.

   c. Complete the graph to show the solutions to the inequality $-4x + 5 \geq 25$ on the number line. Then write an inequality to represent the solution.

   Solution: $x \leq -5$

638 **Unit 6** Expressions, Equations, and Inequalities

1 Launch

Conduct the ***Think-Pair-Share*** routine. Give students 5 minutes of independent work time. Then give pairs of students time to share their responses and reasoning with each other.

2 Monitor

Help students get started solving the related equations by reviewing the process of subtracting the constant from both sides and dividing by the coefficient.

Look for points of confusion:

- **Struggling to select values greater than or less than the solutions in Problem 1b.** Rephrase the problem and ask students to select a number to the left and to the right on the number line.
- **Struggling to use their responses to part b to help them complete part c.** Remind students of their work in Lesson 14 and have them consider which values will make the inequality true.
- **Making errors in labeling the number line when graphing the solution.** Suggest they start in the middle with the boundary value and then label the tick marks to the left and right of this value.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can represent solutions to inequalities on digital numbers lines. You can view their responses in real time.

Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on Problem 1 and then review their responses together before they move on to Problem 2.

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as you discuss how students chose the values to test for each inequality, press them for details in their reasoning. For example:

If a student says . . .	Press for detail by asking . . .
"I chose to test the values 1.7 and 1.9." (Problem 2)	"Why did you choose these values? Are there different values you could have chosen? Are some values less challenging to use than others?"

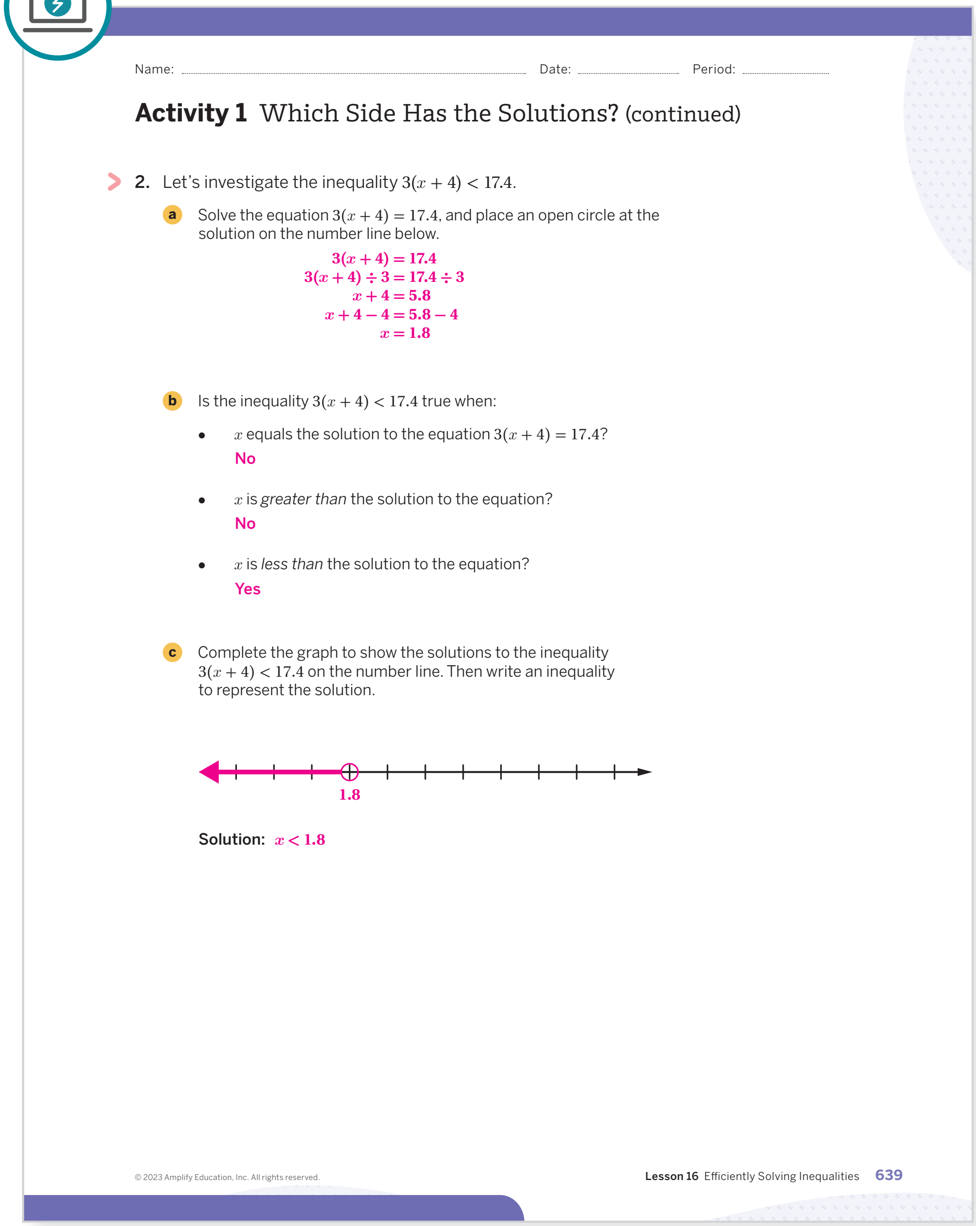
English Learners

Annotate the number lines with how they illustrate whether the boundary value is/is not a solution and whether values on each side of the boundary value are/are not solutions.

Pairs | 15 min

Activity 1 Which Side Has the Solutions? (continued)

Students solve inequalities by solving related equations and testing solutions and by formalizing a process for solving inequalities.



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Activity 1 Which Side Has the Solutions? (continued)

2. Let's investigate the inequality $3(x + 4) < 17.4$.

   a. Solve the equation $3(x + 4) = 17.4$, and place an open circle at the solution on the number line below.

$$\begin{aligned} 3(x+4) &= 17.4 \\ 3(x+4) \div 3 &= 17.4 \div 3 \\ x+4 &= 5.8 \\ x+4-4 &= 5.8-4 \\ x &= 1.8 \end{aligned}$$

   b. Is the inequality $3(x + 4) < 17.4$ true when:

   - x equals the solution to the equation $3(x + 4) = 17.4$?
     No
   - x is *greater than* the solution to the equation?
     No
   - x is *less than* the solution to the equation?
     Yes

   c. Complete the graph to show the solutions to the inequality $3(x + 4) < 17.4$ on the number line. Then write an inequality to represent the solution.

Solution: $x < 1.8$

3 Connect

Have students share how they graphed the solution to each inequality and how they wrote the solution in the form of an inequality, based on the graph.

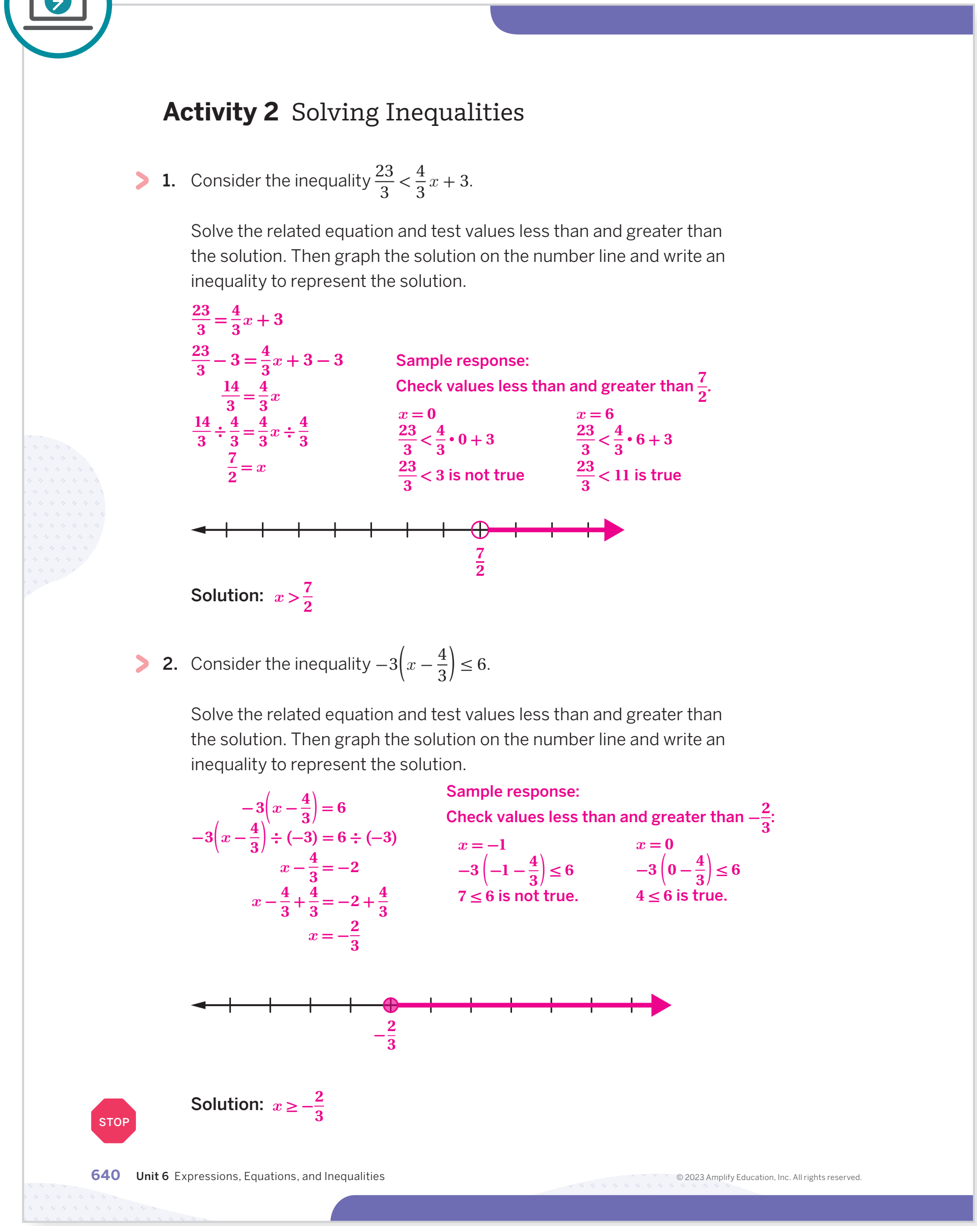
Highlight how students determined the boundary value, their process for testing numbers on either side to determine which side has the values that make the inequality true, and how to represent the solution on the number line. Discuss how students chose the values to test (part b), emphasizing that since they can select any value, they should choose convenient ones.

Ask, If someone asked for your help with how to solve an inequality, what would you tell them?”

Pairs | 15 min

Activity 2 Solving Inequalities

Students practice solving inequalities by solving related equations and testing solutions.



Activity 2 Solving Inequalities

1. Consider the inequality $\frac{23}{3} < \frac{4}{3}x + 3$.

Solve the related equation and test values less than and greater than the solution. Then graph the solution on the number line and write an inequality to represent the solution.

$$\frac{23}{3} = \frac{4}{3}x + 3$$
$$\frac{23}{3} - 3 = \frac{4}{3}x + 3 - 3$$
$$\frac{14}{3} = \frac{4}{3}x$$
$$\frac{14}{3} \div \frac{4}{3} = \frac{4}{3}x \div \frac{4}{3}$$
$$\frac{7}{2} = x$$

Sample response:

Check values less than and greater than $\frac{7}{2}$.

$x = 0$
$\frac{23}{3} < \frac{4}{3} \cdot 0 + 3$
$\frac{23}{3} < 3$ is not true

$x = 6$
$\frac{23}{3} < \frac{4}{3} \cdot 6 + 3$
$\frac{23}{3} < 11$ is true

$\frac{7}{2}$

Solution: $x > \frac{7}{2}$

2. Consider the inequality $-3\left(x - \frac{4}{3}\right) \leq 6$.

Solve the related equation and test values less than and greater than the solution. Then graph the solution on the number line and write an inequality to represent the solution.

$$-3\left(x - \frac{4}{3}\right) = 6$$
$$-3\left(x - \frac{4}{3}\right) \div (-3) = 6 \div (-3)$$
$$x - \frac{4}{3} = -2$$
$$x - \frac{4}{3} + \frac{4}{3} = -2 + \frac{4}{3}$$
$$x = -\frac{2}{3}$$

Sample response:

Check values less than and greater than $-\frac{2}{3}$:

$x = -1$
$-3\left(-1 - \frac{4}{3}\right) \leq 6$
$7 \leq 6$ is not true.

$x = 0$
$-3\left(0 - \frac{4}{3}\right) \leq 6$
$4 \leq 6$ is true.

$-\frac{2}{3}$

Solution: $x \geq -\frac{2}{3}$

STOP

640 **Unit 6** Expressions, Equations, and Inequalities

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1 Launch

Conduct the *Think-Pair-Share* routine. Have students continue working independently for 5 minutes. Then have them share their responses with a partner.

2 Monitor

Help students get started by asking, "What equation can you write to find the boundary value?"

Look for points of confusion:

- **Struggling to complete the problem without the scaffolding given in the previous activity.** Suggest they use their work from Activity 1 as a reference.

3 Connect

Display student work showing the graph of the solution for each inequality.

Have students share how they solved and graphed each inequality.

Highlight strategies for labeling the number line when graphing the solution to an inequality in which the boundary value is not a whole number. Review the process for solving an inequality if students would benefit from it.

Ask, "How would you describe to someone how to solve any inequality?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Help students create a checklist that documents the steps for solving an inequality. A sample checklist is shown. Alternatively, provide students with a copy of the Anchor Chart PDF, *Inequalities*.

☐ Write a related equation and solve the equation. The solution is the boundary value.
☐ Determine if the boundary value is a solution to the inequality. This will tell you whether to use the >/< or ≥/≤.
☐ Test values on either side of the inequality to determine if they are solutions. This will tell you whether to use the symbols >/≥ or </≤.
☐ Write and graph the solution.

Math Language Development

MLR2: Collect and Display

During the Connect, as students respond to the Ask question, "How would you describe to someone how to solve any inequality?," ask them to consider how multiplying or dividing by a negative coefficient affects the solution to an inequality. Collect and display language students use in their response and connect their language to number line diagrams.

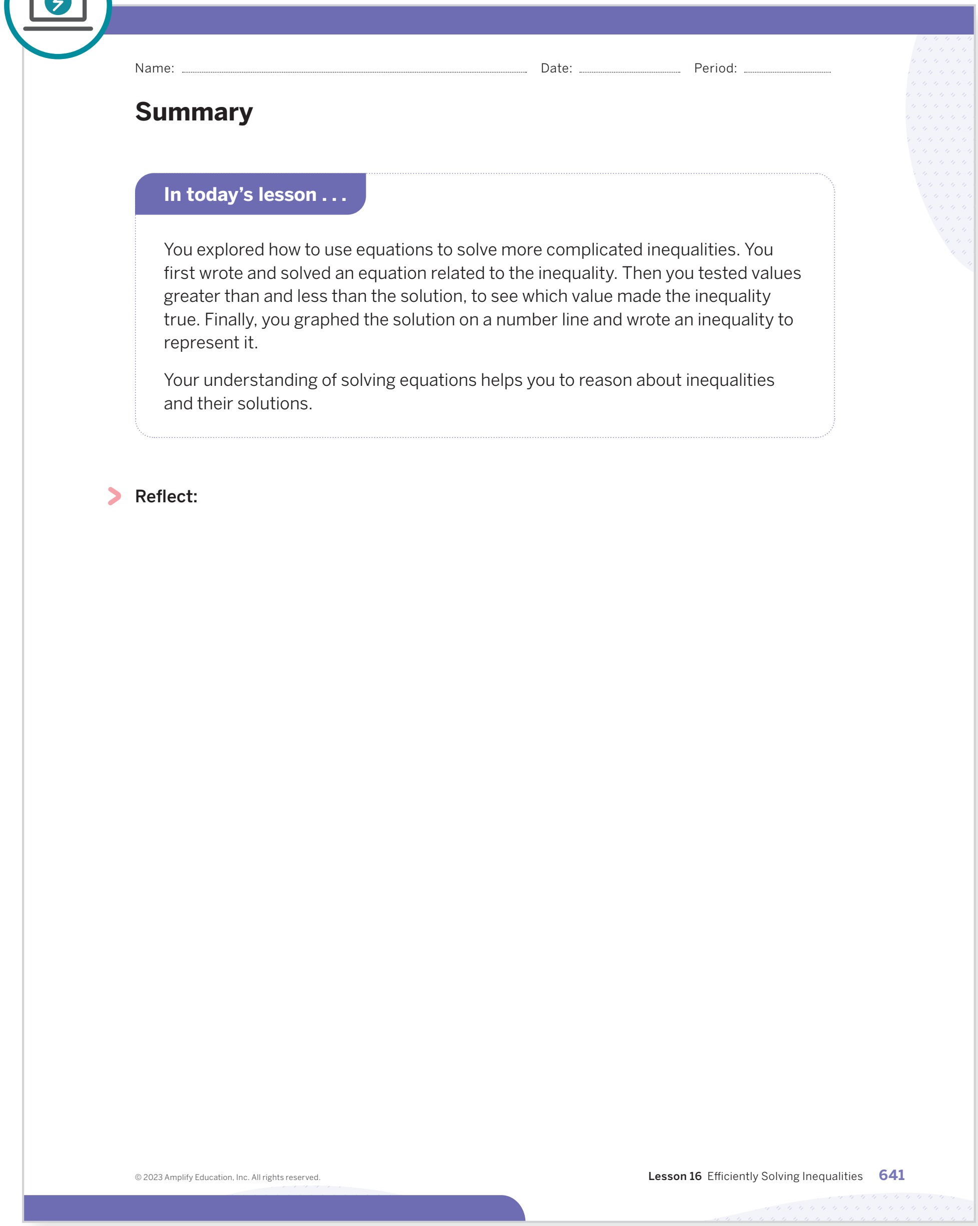
English Learners

Provide students time to record their ideas individually and then share with a partner before sharing with the whole class.

Whole Class | 5 min

Summary

Review and synthesize how to solve a more complicated inequality using the same reasoning used for solving simpler inequalities.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how to use equations to solve more complicated inequalities. You first wrote and solved an equation related to the inequality. Then you tested values greater than and less than the solution, to see which value made the inequality true. Finally, you graphed the solution on a number line and wrote an inequality to represent it.

Your understanding of solving equations helps you to reason about inequalities and their solutions.

Reflect:

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Lesson 16 Efficiently Solving Inequalities 641

Synthesize

Display the Anchor Chart PDF, *Solving Inequalities* and complete the bottom section as a class.

Highlight the steps for solving an inequality by writing and solving a related equation and then checking whether values less than or greater than the equation's solution make the inequality true. Demonstrate how to graph the solution and write an inequality to represent the solution.

Ask:

- "How does the equation relate to the inequality?"
- "How do you use the solution to the equation to help you solve the inequality?
- "What are you looking for when you test values less than and greater than the solution to the equation?"
- "What will the graph of the solution look like?"
- "How do you know whether 7 is included in the solution?"
- "How do you determine the inequality you write for the solution?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

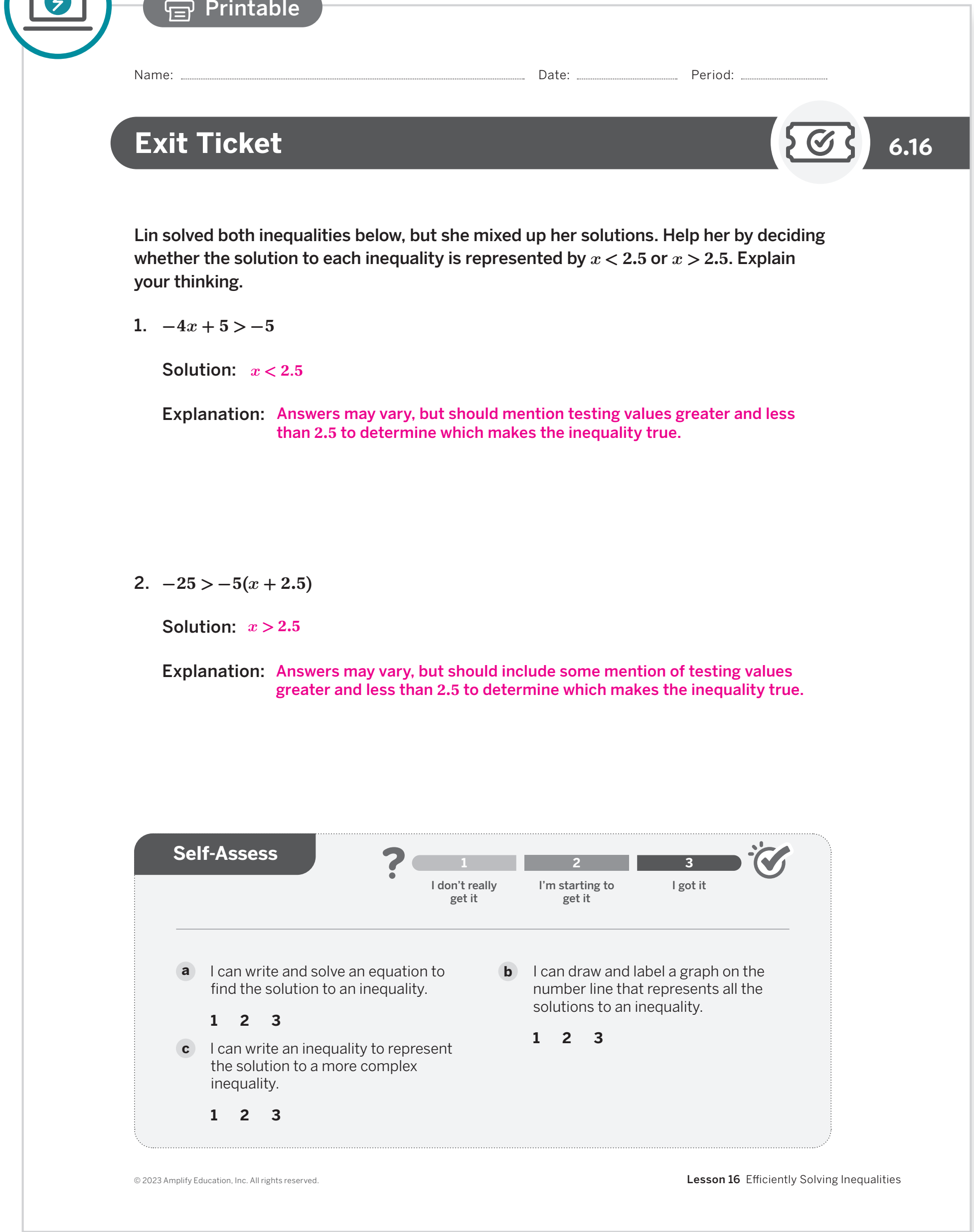
- "Which strategies that worked for solving simple equations or inequalities can be put to use when solving more complex ones?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining which of two similar solutions solves each of two inequalities.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.16

Lin solved both inequalities below, but she mixed up her solutions. Help her by deciding whether the solution to each inequality is represented by $x < 2.5$ or $x > 2.5$. Explain your thinking.

1. $-4x + 5 > -5$

Solution: $x < 2.5$

Explanation: Answers may vary, but should mention testing values greater and less than 2.5 to determine which makes the inequality true.

2. $-25 > -5(x + 2.5)$

Solution: $x > 2.5$

Explanation: Answers may vary, but should include some mention of testing values greater and less than 2.5 to determine which makes the inequality true.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can write and solve an equation to find the solution to an inequality.
1 2 3

b I can draw and label a graph on the number line that represents all the solutions to an inequality.
1 2 3

c I can write an inequality to represent the solution to a more complex inequality.
1 2 3

Lesson 16 Efficiently Solving Inequalities

Success looks like . . .

- **Language Goal:** Comparing and contrasting solutions to equations and solutions to inequalities. **(Speaking and Listening)**
- **Goal:** Drawing and labeling a graph on the number line that represents all the solutions to an inequality.
- **Language Goal:** Generalizing the solutions of an inequality of the form $px + q > r$ or $px + q < r$ by solving the equation $px + q = r$ and then testing a value to determine the direction of the inequality in the solution. **(Speaking and Listening)**
  - » Deciding whether $x < 2.5$ or $x > 2.5$ is a solution to the inequalities by testing values in Problems 1 and 2.

Suggested next steps

If students choose $x > 2.5$ as the solution for both inequalities, consider:

- Double checking that they aren't matching the inequality sign in the solution to the direction of the sign in the given inequality.
- Having them check values to find the correct solutions.
- Assigning Practice Problems 1 and 2.

If students write answers with no work shown or explanation given, consider:

- Having them show work or explain their answers.
- Assigning Practice Problem 3.

Professional Learning

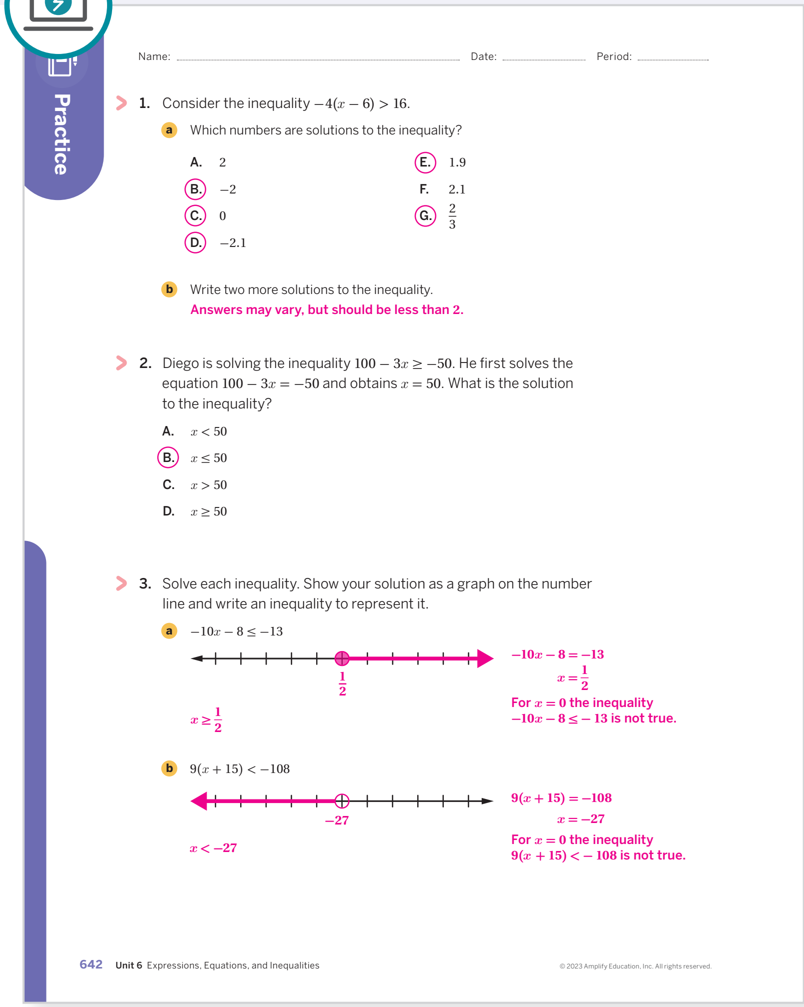
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Who participated and who not participate in Activity 1 today? What trends do you see in participation?
- What did students find frustrating about Activity 1? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Consider the inequality $-4(x-6) > 16$.

   a. Which numbers are solutions to the inequality?

   A. 2
   [✓ B.] -2
   [✓ C.] 0
   [✓ D.] -2.1
   [✓ E.] 1.9
   F. 2.1
   [✓ G.] $\frac{2}{3}$

   b. Write two more solutions to the inequality.

   Answers may vary, but should be less than 2.

2. Diego is solving the inequality $100 - 3x \geq -50$. He first solves the equation $100 - 3x = -50$ and obtains $x = 50$. What is the solution to the inequality?

   A. $x < 50$
   [✓ B.] $x \leq 50$
   C. $x > 50$
   D. $x \geq 50$

3. Solve each inequality. Show your solution as a graph on the number line and write an inequality to represent it.

   a. $-10x - 8 \leq -13$

   $\frac{1}{2}$

   $x \geq \frac{1}{2}$

   $-10x - 8 = -13$
   $x = \frac{1}{2}$
   For $x = 0$ the inequality $-10x - 8 \leq -13$ is not true.

   b. $9(x + 15) < -108$

   -27

   $x < -27$

   $9(x + 15) = -108$
   $x = -27$
   For $x = 0$ the inequality $9(x + 15) < -108$ is not true.

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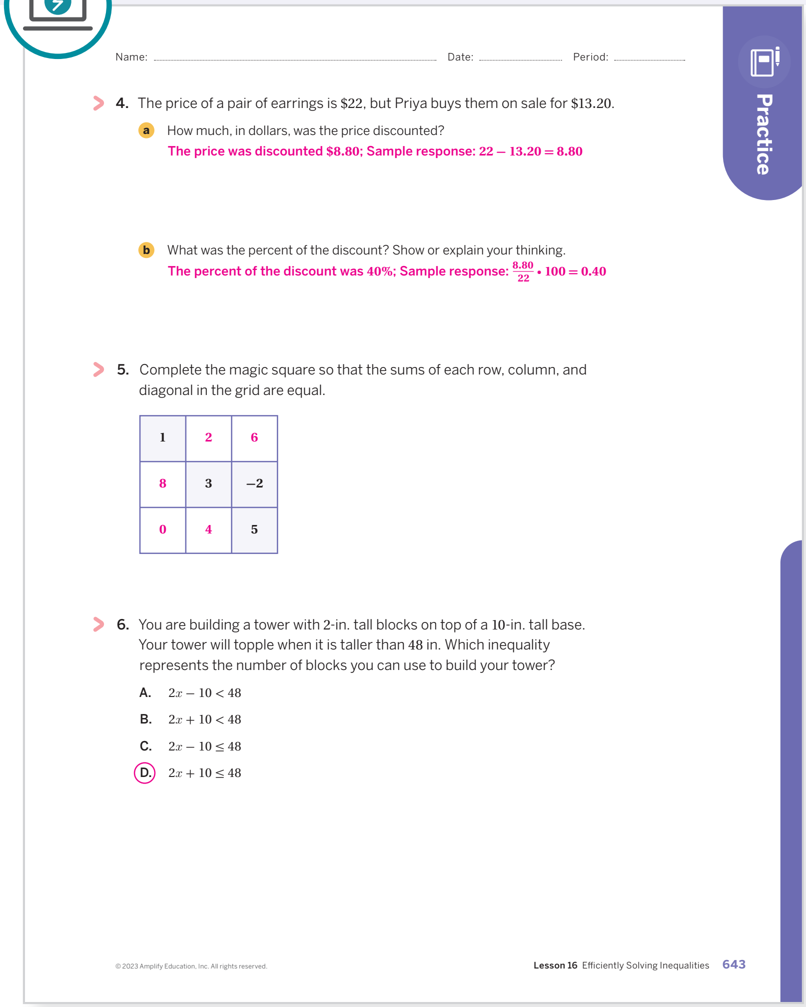

Name: ______ Date: ______ Period: ______

4. The price of a pair of earrings is $22, but Priya buys them on sale for $13.20.

   a. How much, in dollars, was the price discounted?

   The price was discounted $8.80; Sample response: $22 - 13.20 = 8.80$

   b. What was the percent of the discount? Show or explain your thinking.

   The percent of the discount was 40%; Sample response: $\frac{8.80}{22} \cdot 100 = 0.40$

5. Complete the magic square so that the sums of each row, column, and diagonal in the grid are equal.

1	2	6
8	3	−2
0	4	5

6. You are building a tower with 2-in. tall blocks on top of a 10-in. tall base. Your tower will topple when it is taller than 48 in. Which inequality represents the number of blocks you can use to build your tower?

   A. $2x - 10 < 48$
   B. $2x + 10 < 48$
   C. $2x - 10 \leq 48$
   [✓ D.] $2x + 10 \leq 48$

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Lesson 16 Efficiently Solving Inequalities 643

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 12	2
	5	Unit 5 Lesson 4	2
Formative ⚡	6	Unit 6 Lesson 17	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 17**

Interpreting Inequalities

Let’s write some inequalities.

Focus

Goals

1. **Language Goal:** Identify the inequality that represents a situation, and justify the choice. **(Writing)**
2. **Language Goal:** Present (using multiple representations) the solution method for a problem involving an inequality and interpret the solution. **(Speaking and Listening, Writing)**

Coherence

Today

Students interpret and solve inequalities that represent real-world situations, making sense of quantities and their relationships in the problem.

Previously

Students wrote and solved equations from scenarios in Lesson 9–11. In Lesson 14, students wrote related equations and solved them to help find the solutions to the inequality.

Coming Soon

In Lesson 18, students will begin to focus on the modeling process, starting with a question they want to answer and then independently deciding how they will represent the situation mathematically.

Rigor

- Students build their **procedural fluency** in solving and graphing the solutions of inequalities.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	10 min	15 min	5 min	7 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent | Slides x–x

Materials

- Exit Ticket
- Additional Practice

Math Language Development

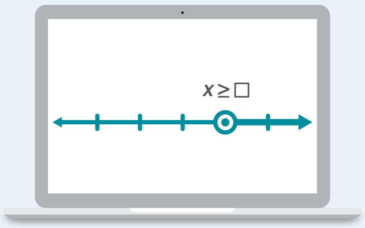
Review words

- *at least*
- *at most*
- *inequality*
- *solution to an inequality*

Amps | **Featured Activity**

Warm-up
Interactive Inequalities

Students drag and drop values to test whether their inequality works and receive instant feedback.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might become distracted as they try to match inequalities and scenarios in Activity 1. They might not put forth the needed focus to approach the problem with both abstract and quantitative reasoning. While working in pairs, have students help each other stay focused. Encourage them to explain their thinking to their partner so that they can make sure their reasoning is sound.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Independent | 8 min

Warm-up Mystery Inequalities

Students create their own inequality following certain rules. This helps students reason about the components of an inequality in an abstract way.

Amps Featured Activity Interactive Inequalities

Unit 6 | Lesson 17

Interpreting Inequalities

Let's write some inequalities.

Warm-up Mystery Inequalities

Using each of the numbers from 1 to 9 at most once, fill in the boxes so that the solution to the inequality is $x \leq 2$. Repeat as many times as possible to determine different sets of numbers.

$$\square x - \square \leq \square$$

Sample responses:

2, 1, 3
2, 3, 1
3, 1, 5
3, 2, 4
3, 4, 2
3, 5, 1
4, 1, 7
4, 2, 6
4, 3, 5
4, 5, 3
4, 6, 2
4, 7, 1
5, 1, 9
5, 2, 8
5, 3, 7
5, 4, 6
5, 6, 4
5, 7, 3
5, 8, 2
5, 9, 1
6, 3, 9
6, 4, 8
6, 5, 7
6, 7, 5
6, 8, 4
6, 9, 3
7, 5, 9
7, 6, 8
7, 8, 6
7, 9, 5
8, 7, 9
8, 9, 7

Log in to Amplify Math to complete this lesson online.

644 Unit 6 Expressions, Equations, and Inequalities

1 Launch

Read the directions aloud, to ensure that students understand the expectations for this problem. It may be helpful to give one example of an inequality that satisfies the prompt, such as $2x - 1 \leq 3$.

2 Monitor

Help students get started by suggesting they try different numbers and check which work.

Look for points of confusion:

- **Using a number more than once.** Ask, "Have you noticed that you've broken one of the rules?"
- **Trying to place the 2 to the right of the ≤ symbol.** Say, "The solution needs to only have an x on the left side. How can you do that?"

Look for productive strategies:

- Finding a pattern in the solutions. Say, "Explain to me what you are noticing. How has it helped you?"

3 Connect

Display the problem and a blank chart to be filled in with possible answers.

Have students share their process for selecting numbers with a partner.

Highlight a student that substituted 2 for x in order to make sense of the problem.

Ask, "Once you found one solution, was there a pattern you could use to find other solutions?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they drag and drop values to test whether their inequality works and receive instant feedback.

Power-up

To power up students' ability to write expressions to represent real-world scenarios, have students complete:

1. It costs $\$0.25$ to play each game at a fair. Admission is $\$4$. Write an expression to show the total cost of playing x games at the fair. $0.25x + 4$
2. It costs $\$0.25$ to play each game at a new arcade. Today, in honor of opening day, the arcade is giving away $\$4$ of free tokens. Write an expression to show the total cost of playing x games at the arcade today. $0.25x - 4$

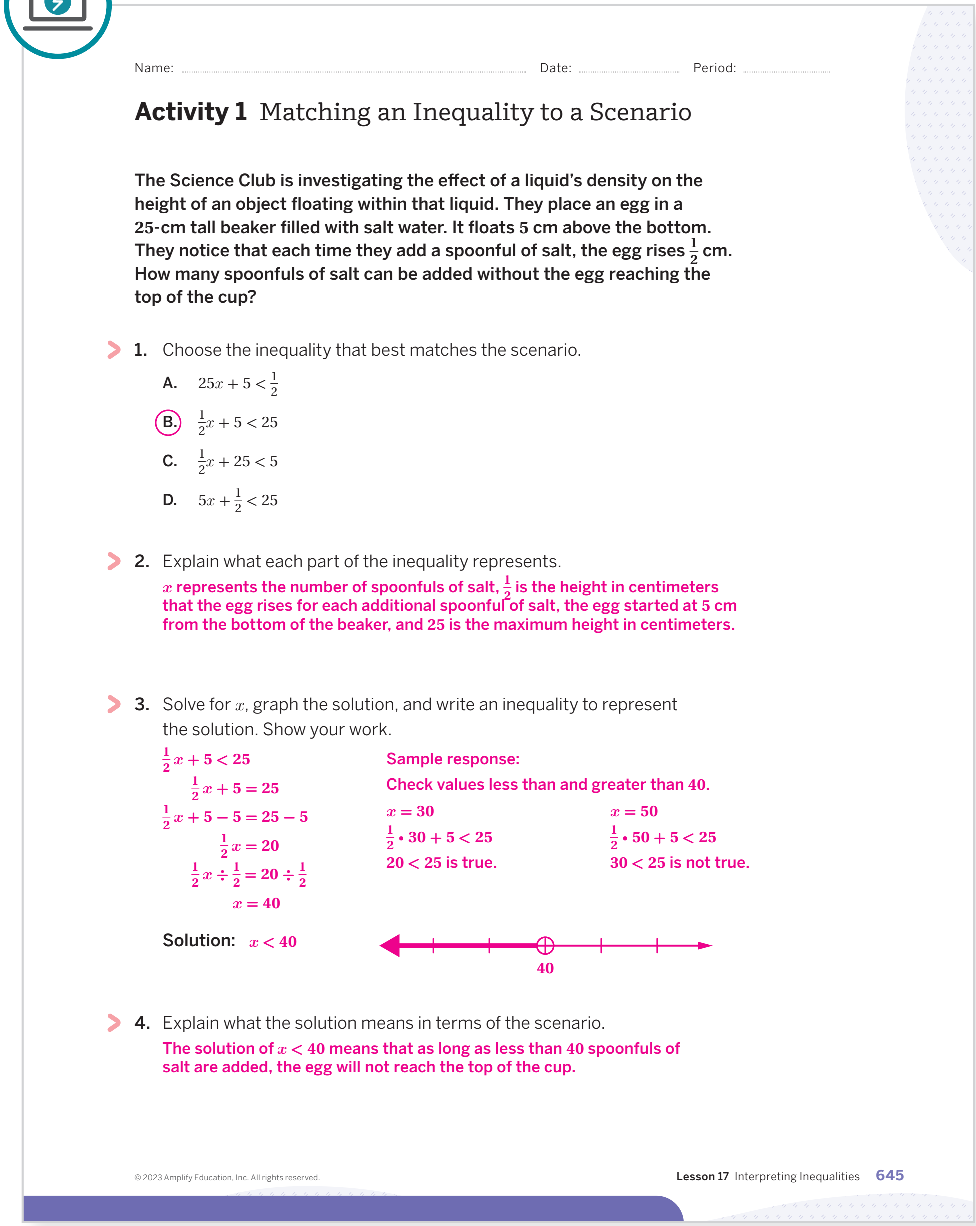
Use: Before Activity 1

Informed by: Performance on Lesson 16, Practice Problem 6

Pairs | 10 min

Activity 1 Matching an Inequality to a Scenario

Students interpret a scenario that leads to an inequality. This activity helps students make sense of the quantities and their relationships.



Name: ______ Date: ______ Period: ______

Activity 1 Matching an Inequality to a Scenario

The Science Club is investigating the effect of a liquid's density on the height of an object floating within that liquid. They place an egg in a 25-cm tall beaker filled with salt water. It floats 5 cm above the bottom. They notice that each time they add a spoonful of salt, the egg rises $\frac{1}{2}$ cm. How many spoonfuls of salt can be added without the egg reaching the top of the cup?

1. Choose the inequality that best matches the scenario.
   - A. $25x + 5 < \frac{1}{2}$
   - [✓ B.] $\frac{1}{2}x + 5 < 25$
   - C. $\frac{1}{2}x + 25 < 5$
   - D. $5x + \frac{1}{2} < 25$

2. Explain what each part of the inequality represents.
   x represents the number of spoonfuls of salt, $\frac{1}{2}$ is the height in centimeters that the egg rises for each additional spoonful of salt, the egg started at 5 cm from the bottom of the beaker, and 25 is the maximum height in centimeters.

3. Solve for x, graph the solution, and write an inequality to represent the solution. Show your work.

$$\frac{1}{2}x + 5 < 25$$
$$\frac{1}{2}x + 5 = 25$$
$$\frac{1}{2}x + 5 - 5 = 25 - 5$$
$$\frac{1}{2}x = 20$$
$$\frac{1}{2}x \div \frac{1}{2} = 20 \div \frac{1}{2}$$
$$x = 40$$

Sample response:

Check values less than and greater than 40.

$x = 30$	$x = 50$
$\frac{1}{2} \cdot 30 + 5 < 25$	$\frac{1}{2} \cdot 50 + 5 < 25$
$20 < 25$ is true.	$30 < 25$ is not true.

Solution: $x < 40$

4. Explain what the solution means in terms of the scenario.
   The solution of $x < 40$ means that as long as less than 40 spoonfuls of salt are added, the egg will not reach the top of the cup.

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Lesson 17 Interpreting Inequalities 645

① Launch

Activate students' background knowledge by asking, "Has anyone noticed that it is easier to float in the ocean than in a pool? Why do you think that is?" The salt in the ocean makes it easier for objects (and people) to float.

② Monitor

Help students get started by asking, "What quantity could be represented by the variable in this scenario?"

Look for points of confusion:

- **Assuming one quantity will always be on the opposite side of the variable.** Allow for this conjecture and ask students to re-evaluate their thinking at the end of the lesson.

Look for productive strategies:

- Expressing the solution in words or by graphing on a number line. Applaud student use of these representations while encouraging them to express the solution using the efficient algebraic notation.

③ Connect

Display one student's solution to Problem 3.

Have students share what each quantity and variable represent in the original inequality. Annotate the inequality as the student explains.

Highlight what the solution to the inequality represents in the scenario.

Ask, "What does it mean for x to be less than 40?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Instead of asking students to select the correct inequality for Problem 1, provide them with the correct inequality and ask them to explain how it matches the scenario. Provide access to colored pencils and suggest students color code key words and phrases from the text and how they are represented in the inequality.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that an egg is floating in a beaker of salt water.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as each time a spoonful of salt is added, the height of the egg in the water increases by $\frac{1}{2}$ cm.
- **Read 3:** Ask students to identify what the unknown amount should represent in this context.

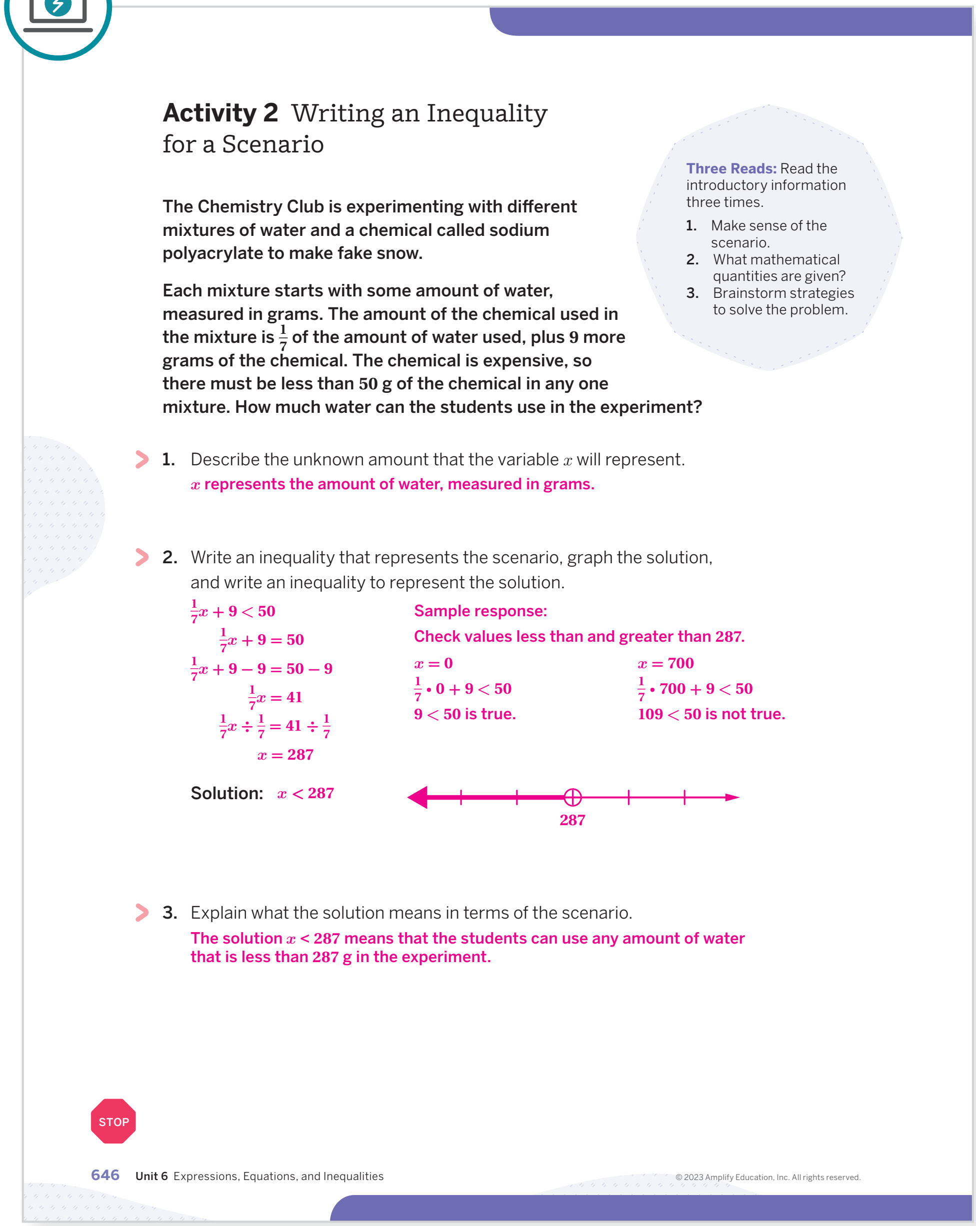
English Learners

Draw a picture representing this context showing an egg floating in a beaker. Then draw a new picture showing as salt is added, the egg rises.

Pairs | 15 min

Activity 2 Writing an Inequality for a Scenario

Students are now asked to write and solve their own inequality to match a scenario. This is a gradual release of support from Activity 1 to prepare students for the Exit Ticket.



Activity 2 Writing an Inequality for a Scenario

Three Reads: Read the introductory information three times.

1. Make sense of the scenario.
2. What mathematical quantities are given?
3. Brainstorm strategies to solve the problem.

The Chemistry Club is experimenting with different mixtures of water and a chemical called sodium polyacrylate to make fake snow.

Each mixture starts with some amount of water, measured in grams. The amount of the chemical used in the mixture is $\frac{1}{7}$ of the amount of water used, plus 9 more grams of the chemical. The chemical is expensive, so there must be less than 50 g of the chemical in any one mixture. How much water can the students use in the experiment?

1. Describe the unknown amount that the variable x will represent.

   x represents the amount of water, measured in grams.

2. Write an inequality that represents the scenario, graph the solution, and write an inequality to represent the solution.

   $\frac{1}{7}x + 9 < 50$

   $\frac{1}{7}x + 9 = 50$

   $\frac{1}{7}x + 9 - 9 = 50 - 9$

   $\frac{1}{7}x = 41$

   $\frac{1}{7}x \div \frac{1}{7} = 41 \div \frac{1}{7}$

   $x = 287$

   Sample response:

   Check values less than and greater than 287.

$x = 0$	$x = 700$
$\frac{1}{7} \cdot 0 + 9 < 50$	$\frac{1}{7} \cdot 700 + 9 < 50$
9 < 50 is true.	109 < 50 is not true.

   Solution: $x < 287$

   287

3. Explain what the solution means in terms of the scenario.

   The solution $x < 287$ means that the students can use any amount of water that is less than 287 g in the experiment.

STOP

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1 Launch

Have the students in each pair take turns reading the scenario to each other. Then ask students to each write their inequality independently before comparing with their partner.

2 Monitor

Help students get started by suggesting they read the scenario backwards, starting with the last sentence and finishing with the first.

Look for points of confusion:

- **Representing the scenario with $\frac{1}{7} + 9x < 50$.** Ask, "What does it mean to have $\frac{1}{7}$ of an amount? Do we know what that amount is?"
- **Using "≤".** Ask, "Can the Chemistry Club use exactly 50 grams of the chemical? How do you know?"
- **Thinking that the solution represents the amount of chemical in the mixture.** Ask, "What did you say the variable represented when you read the scenario?"

3 Connect

Display one student's solution to Problem 2.

Have students share what each quantity and variable represent in their original inequality. Annotate the inequality as the student explains.

Highlight how the inequality and solution relate to the scenario.

Ask:

- "How did you determine what the $\frac{1}{7}$ term represents?"
- "How did you decide on the direction of the inequality for the solution?"
- "What does it mean that x is less than 287?"

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Prior Knowledge*

Before students begin, ask them to explain in their own words what it means that the amount of the chemical is $\frac{1}{7}$ the amount of the water. Connect this relationship to their prior understanding of ratios. Have them complete the following statements.

- For every 1 gram of water, there are ___ grams of the chemical.
- For 7 grams of water, there are ___ grams of the chemical.
- For 14 grams of water, there are ___ grams of the chemical.
- For x grams of water, there are ___ grams of the chemical.

Math Language Development

MLR6: Three Reads

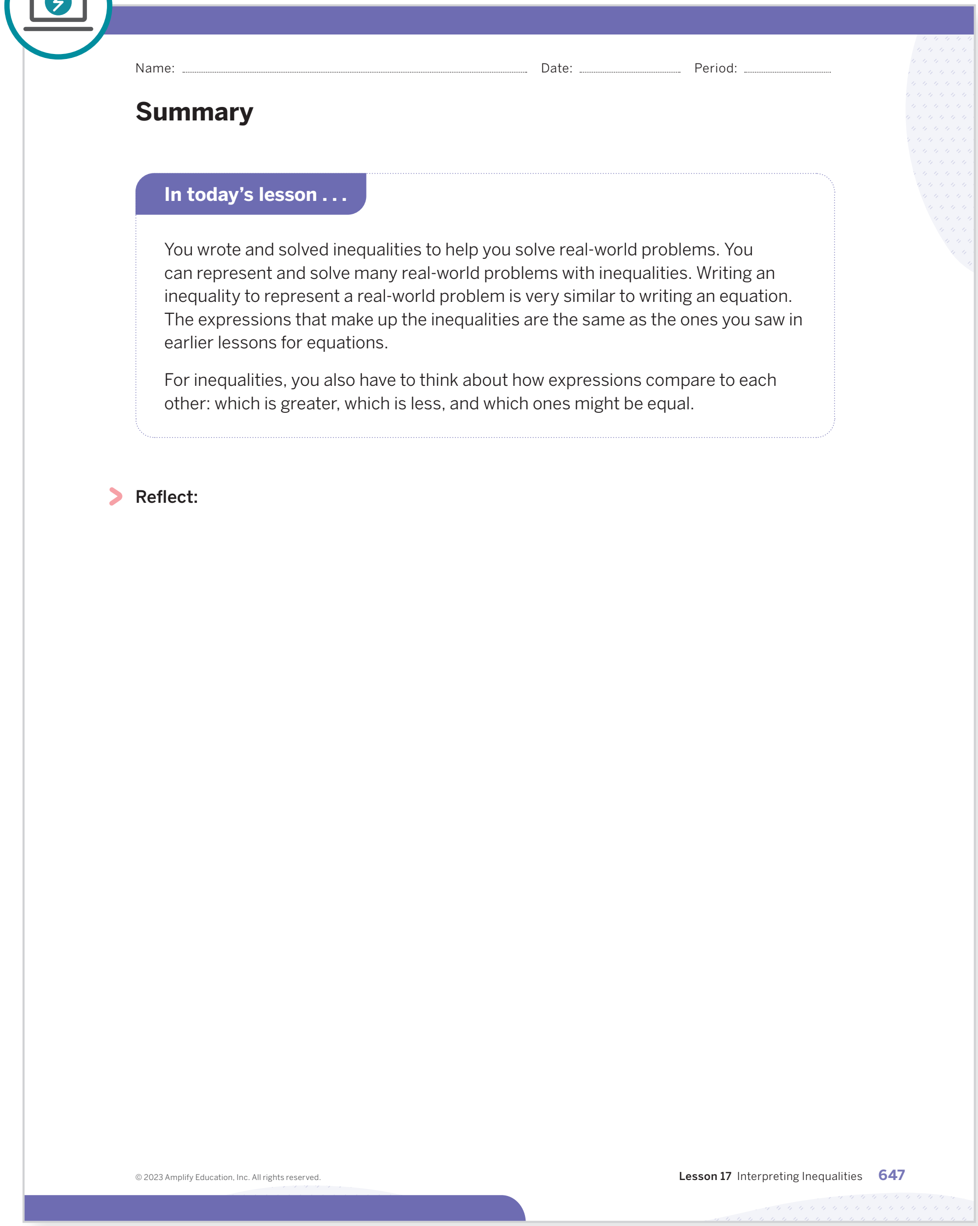
Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that water is mixed with a chemical to make fake snow. Tell them they do not need to worry about how to pronounce the chemical name.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the amount of the chemical used is $\frac{1}{7}$ of the amount water used plus 9 more grams of the chemical.
- **Read 3:** Ask students to identify what the unknown amount should represent in this context.

Whole Class | 5 min

Summary

Review and synthesize how inequalities can represent and help solve real-world problems.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You wrote and solved inequalities to help you solve real-world problems. You can represent and solve many real-world problems with inequalities. Writing an inequality to represent a real-world problem is very similar to writing an equation. The expressions that make up the inequalities are the same as the ones you saw in earlier lessons for equations.

For inequalities, you also have to think about how expressions compare to each other: which is greater, which is less, and which ones might be equal.

Reflect:

Synthesize

Display the following, "Suppose your friend asks you to write some practice problems for solving inequalities. You want to write an inequality that has a solution of $x \leq -8\frac{2}{3}$. Describe how to write such an inequality."

Have students share with a partner how they would write such an inequality. Circulate and note the different strategies students use.

Highlight that there are many approaches to writing such an inequality. As students share different approaches, pause the class and highlight each one.

Ask, "How many different inequalities can be written with this solution?" An infinite number.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

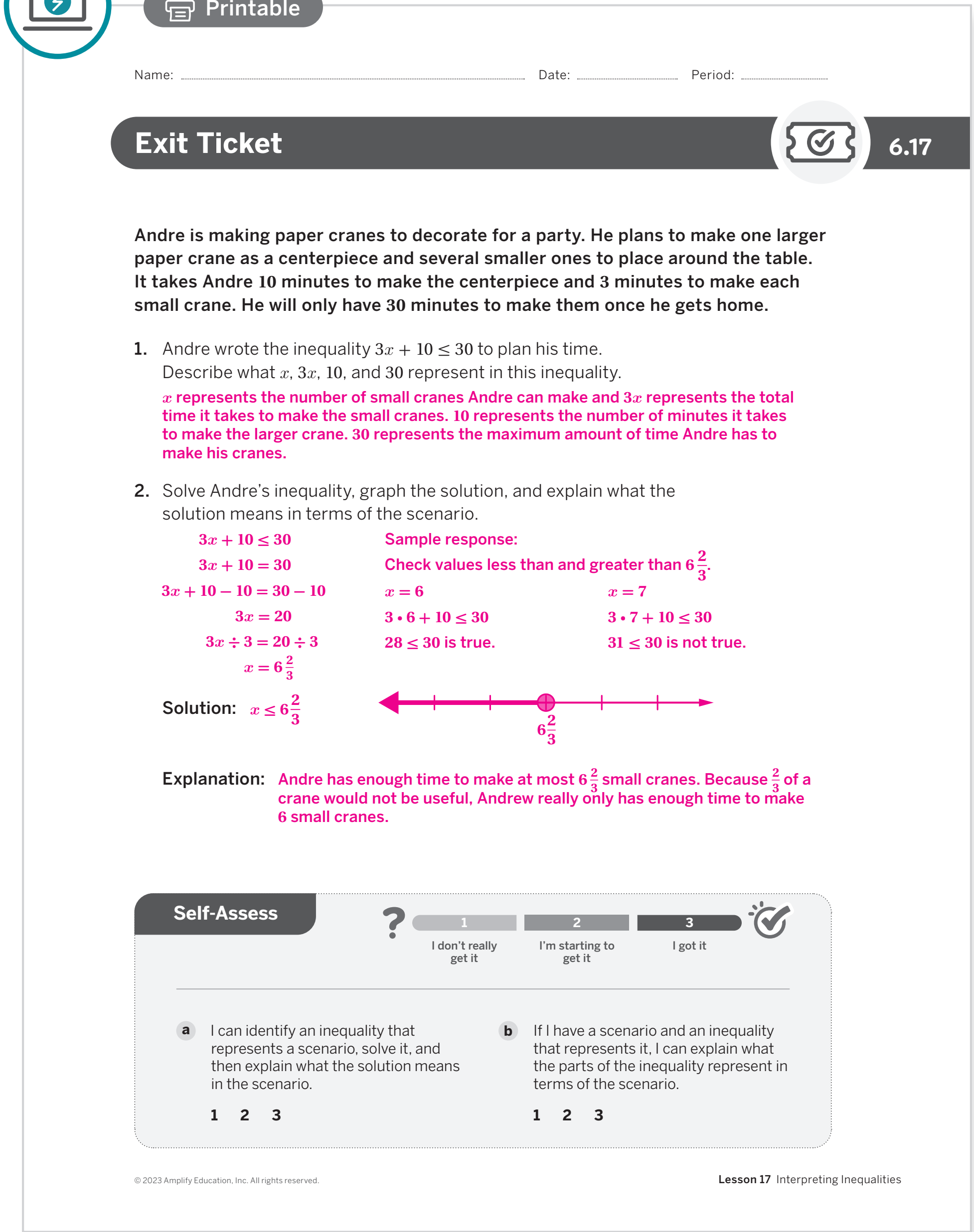
- "How is writing and solving inequalities the same or different from writing and solving equations?"
- "What strategies did you use when determining how to graph the solutions of an inequality?

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by explaining how each of the each of the terms, and the solution to, a given inequality relate to a scenario.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 6.17

Andre is making paper cranes to decorate for a party. He plans to make one larger paper crane as a centerpiece and several smaller ones to place around the table. It takes Andre 10 minutes to make the centerpiece and 3 minutes to make each small crane. He will only have 30 minutes to make them once he gets home.

1. Andre wrote the inequality $3x + 10 \le 30$ to plan his time. Describe what x, $3x$, 10, and 30 represent in this inequality.

   x represents the number of small cranes Andre can make and $3x$ represents the total time it takes to make the small cranes. 10 represents the number of minutes it takes to make the larger crane. 30 represents the maximum amount of time Andre has to make his cranes.

2. Solve Andre's inequality, graph the solution, and explain what the solution means in terms of the scenario.

$$3x + 10 \le 30$$
$$3x + 10 = 30$$
$$3x + 10 - 10 = 30 - 10$$
$$3x = 20$$
$$3x \div 3 = 20 \div 3$$
$$x = 6\tfrac{2}{3}$$

Sample response:

Check values less than and greater than $6\frac{2}{3}$.

$x = 6$	$x = 7$
$3 \cdot 6 + 10 \le 30$	$3 \cdot 7 + 10 \le 30$
$28 \le 30$ is true.	**$31 \le 30$ is not true.**

Solution: $x \le 6\frac{2}{3}$

$6\frac{2}{3}$

Explanation: **Andre has enough time to make at most $6\frac{2}{3}$ small cranes. Because $\frac{2}{3}$ of a crane would not be useful, Andrew really only has enough time to make 6 small cranes.**

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can identify an inequality that represents a scenario, solve it, and then explain what the solution means in the scenario.
1 2 3

b If I have a scenario and an inequality that represents it, I can explain what the parts of the inequality represent in terms of the scenario.
1 2 3

Lesson 17 Interpreting Inequalities

Success looks like . . .

- **Language Goal:** Identifying the inequality that represents a situation, and justifying the choice. **(Writing)**
- **Language Goal:** Presenting (using multiple representations) the solution method for a problem involving an inequality, and interpreting the solution. **(Speaking and Listening, Writing)**
  - » Solving Andre's inequality and graphing and interpreting the solution in Problem 2.

Suggested next steps

If students struggle to describe terms accurately, consider:

- Giving students practice reading scenarios and identifying the inequality with multiple choice answers.
- Assigning Practice Problem 2.

If students struggle to solve the inequality, consider:

- Giving students practice starting with simpler inequalities and working up to two-step inequalities with rational-number values.
- Assigning Practice Problem 3.

If students misinterpret the fractional solution, consider:

- Checking in throughout Lesson 18 to see that they are making sense of the solutions to inequalities.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was Activity 2 similar to or different from the work students did with writing and solving equations previously in this unit?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Math Language Development

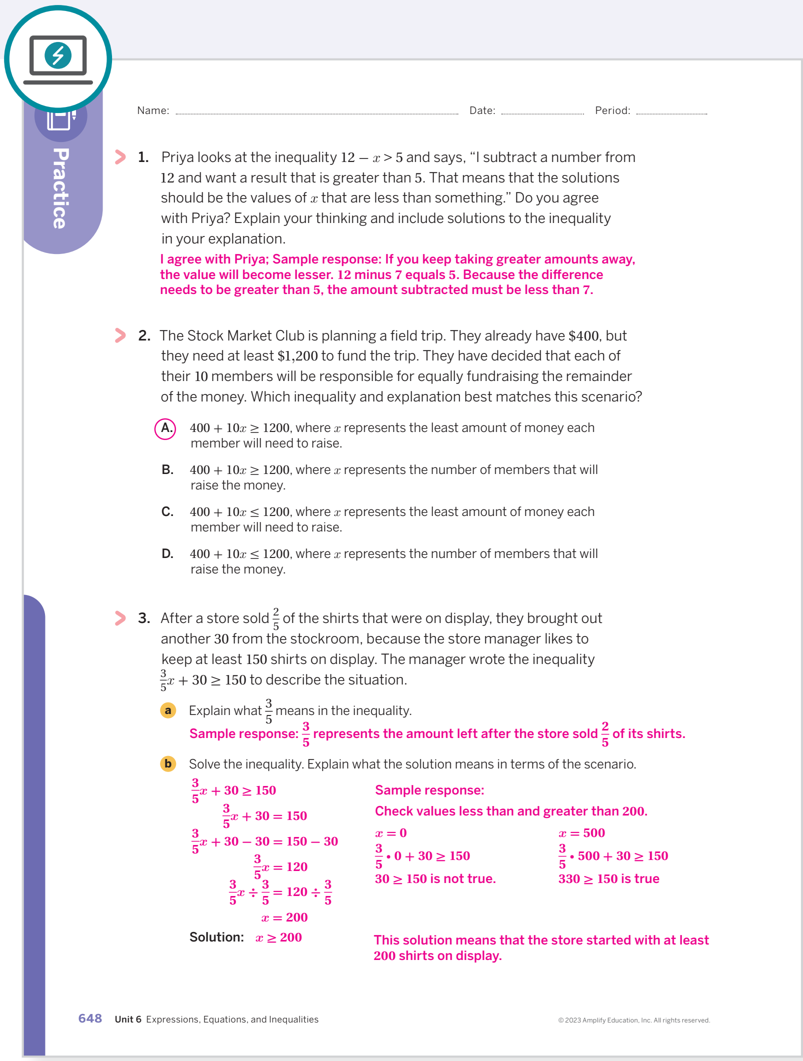
Language Goal: Presenting (using multiple representations) the solution method for a problem involving an inequality, and interpreting the solution.

Reflect on students' language development toward this goal.

- How have students progressed in this unit toward . . .
  - Making sense of real-world problems that involve equations or inequalities?
  - Defining variables to represent the unknown quantities?
  - Interpreting the solutions to their equations or inequalities within the context of the problem?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Priya looks at the inequality $12 - x > 5$ and says, “I subtract a number from 12 and want a result that is greater than 5. That means that the solutions should be the values of x that are less than something.” Do you agree with Priya? Explain your thinking and include solutions to the inequality in your explanation.

I agree with Priya; Sample response: If you keep taking greater amounts away, the value will become lesser. 12 minus 7 equals 5. Because the difference needs to be greater than 5, the amount subtracted must be less than 7.

2. The Stock Market Club is planning a field trip. They already have $400, but they need at least $1,200 to fund the trip. They have decided that each of their 10 members will be responsible for equally fundraising the remainder of the money. Which inequality and explanation best matches this scenario?

[✓ A.] $400 + 10x \geq 1200$, where x represents the least amount of money each member will need to raise.

B. $400 + 10x \geq 1200$, where x represents the number of members that will raise the money.

C. $400 + 10x \leq 1200$, where x represents the least amount of money each member will need to raise.

D. $400 + 10x \leq 1200$, where x represents the number of members that will raise the money.

3. After a store sold $\frac{2}{5}$ of the shirts that were on display, they brought out another 30 from the stockroom, because the store manager likes to keep at least 150 shirts on display. The manager wrote the inequality $\frac{3}{5}x + 30 \geq 150$ to describe the situation.

a. Explain what $\frac{3}{5}$ means in the inequality.

Sample response: $\frac{3}{5}$ represents the amount left after the store sold $\frac{2}{5}$ of its shirts.

b. Solve the inequality. Explain what the solution means in terms of the scenario.

$$\frac{3}{5}x + 30 \geq 150$$
$$\frac{3}{5}x + 30 = 150$$
$$\frac{3}{5}x + 30 - 30 = 150 - 30$$
$$\frac{3}{5}x = 120$$
$$\frac{3}{5}x \div \frac{3}{5} = 120 \div \frac{3}{5}$$
$$x = 200$$

Solution: $x \geq 200$

Sample response:

Check values less than and greater than 200.

$x = 0$

$\frac{3}{5} \cdot 0 + 30 \geq 150$

$30 \geq 150$ is not true.

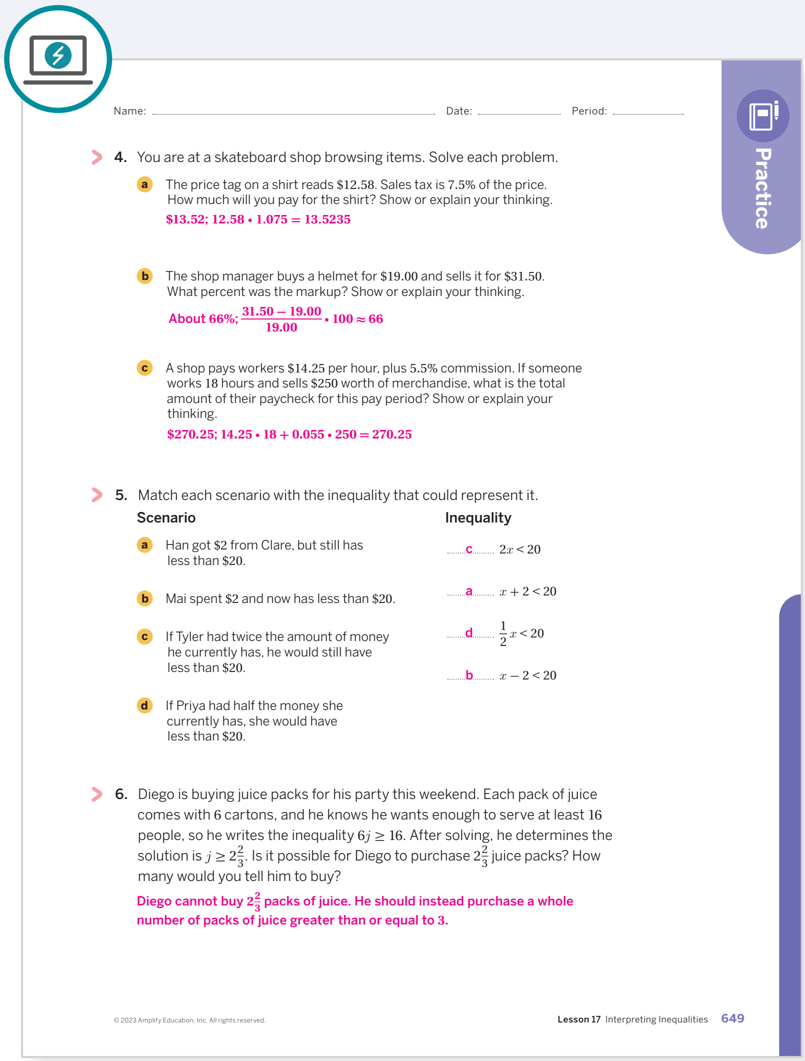
$x = 500$

$\frac{3}{5} \cdot 500 + 30 \geq 150$

$330 \geq 150$ is true

This solution means that the store started with at least 200 shirts on display.

648 **Unit 6** Expressions, Equations, and Inequalities



Practice

Name: ______ Date: ______ Period: ______

4. You are at a skateboard shop browsing items. Solve each problem.

a. The price tag on a shirt reads $12.58. Sales tax is 7.5% of the price. How much will you pay for the shirt? Show or explain your thinking.

$13.52; $12.58 \cdot 1.075 = 13.5235$

b. The shop manager buys a helmet for $19.00 and sells it for $31.50. What percent was the markup? Show or explain your thinking.

About 66%; $\frac{31.50 - 19.00}{19.00} \cdot 100 \approx 66$

c. A shop pays workers $14.25 per hour, plus 5.5% commission. If someone works 18 hours and sells $250 worth of merchandise, what is the total amount of their paycheck for this pay period? Show or explain your thinking.

$270.25; $14.25 \cdot 18 + 0.055 \cdot 250 = 270.25$

5. Match each scenario with the inequality that could represent it.

Scenario	Inequality
a. Han got $2 from Clare, but still has less than $20.	**c** $2x < 20$
b. Mai spent $2 and now has less than $20.	**a** $x + 2 < 20$
c. If Tyler had twice the amount of money he currently has, he would still have less than $20.	**d** $\frac{1}{2}x < 20$
d. If Priya had half the money she currently has, she would have less than $20.	**b** $x - 2 < 20$

6. Diego is buying juice packs for his party this weekend. Each pack of juice comes with 6 cartons, and he knows he wants enough to serve at least 16 people, so he writes the inequality $6j \geq 16$. After solving, he determines the solution is $j \geq 2\frac{2}{3}$. Is it possible for Diego to purchase $2\frac{2}{3}$ juice packs? How many would you tell him to buy?

Diego cannot buy $2\frac{2}{3}$ packs of juice. He should instead purchase a whole number of packs of juice greater than or equal to 3.

Lesson 17 Interpreting Inequalities 649

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 9	1
	5	Unit 6 Lesson 15	2
Formative	6	Unit 6 Lesson 18	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 18**

Modeling With Inequalities

Let's look at solutions to inequalities.

Focus

Goals

1. **Language Goal:** Critique the solution to an inequality, including whether fractional or negative values are reasonable. **(Speaking and Listening)**
2. Determine what information is needed to solve a problem involving a quantity constrained by a maximum or minimum acceptable value.
3. Write and solve an inequality of the form $px + q > r$ or $px + q < r$ to solve a problem about a situation with a constraint.

Coherence

Today

In this lesson, students determine if their solutions are reasonable within context of the scenarios they represent. This lesson focuses on the modeling process, in which students start with a question they want to answer and independently decide how they will represent the situation mathematically.

Previously

In Lesson 16 and 17, students wrote and solved inequalities of the form $px + q > r$ and $p(x + q) < r$.

Coming Soon

Students will continue their work with inequalities in Grade 8 when they solve linear inequalities.

Rigor

- Students continue to build **conceptual understanding** of solutions to inequalities by analyzing real-world scenarios.
- Students develop **procedural fluency** in solving and graphing solutions to inequalities through an Info Gap routine.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 1 PDF (answers)
- Activity 2 PDF, pre-cut cards, one set per pair

Math Language Development

Review words

- *inequality*
- *solution to an inequality*

Amps | **Featured Activity**

Exit Ticket

Real-time Exit Ticket

Check in real time if your students can correct errors in an inequality using a digital Exit Ticket.

Building Math Identity and Community

Connecting to Mathematical Practices

When working with mathematical models, students must make sure that they are appropriate for the scenario, otherwise, the model is completely ineffective. The effectiveness of the model is evaluated after it has been applied by considering whether the solution is discrete or continuous and whether the answer needs to be rounded. Discuss how students evaluate their life decisions and why the reflection process is important.

Modifications to Pacing

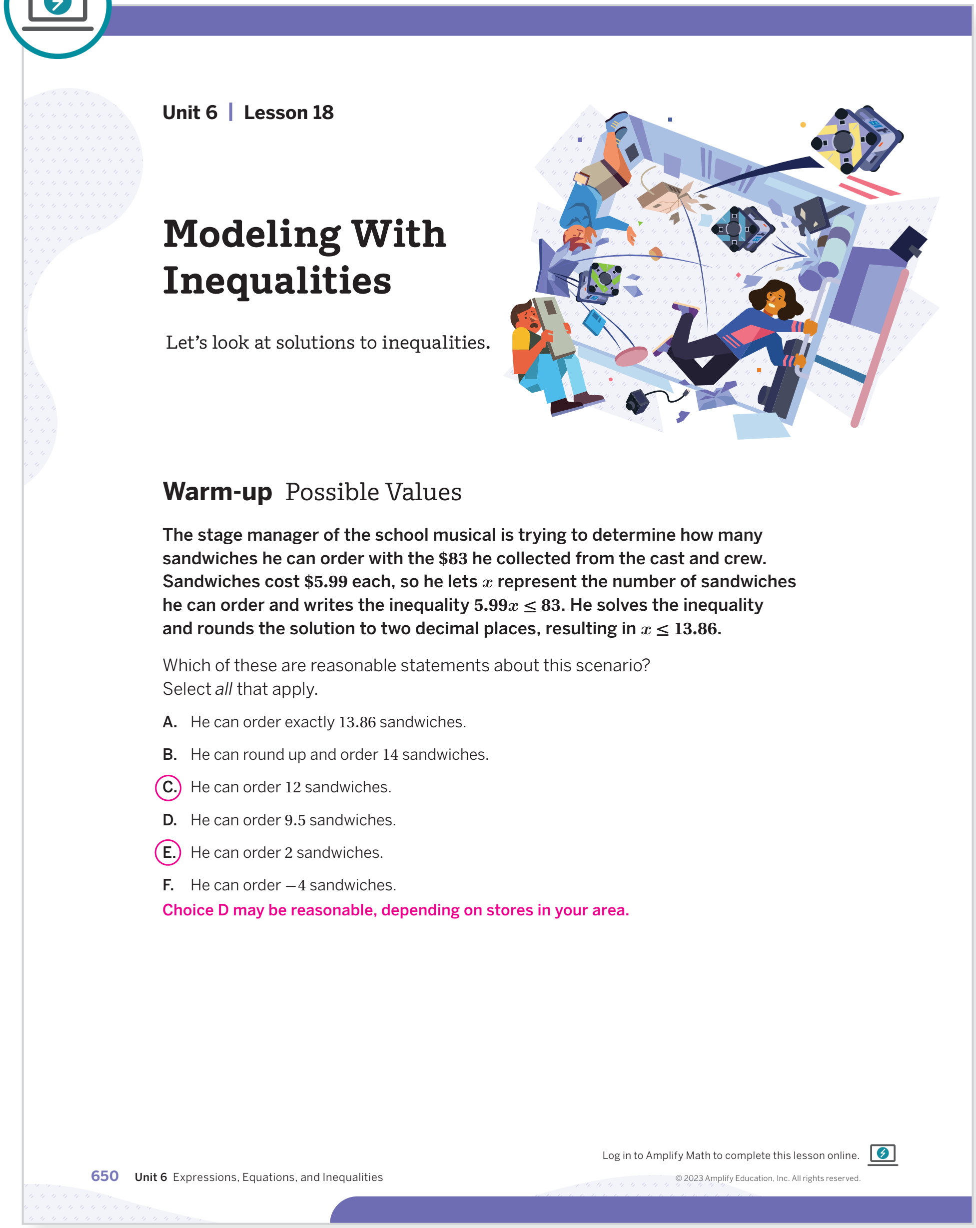
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students only complete Problem 1 from the ***Info Gap***.

Pairs | 5 min

Warm-up Possible Values

Students read a real-world scenario and determine which solutions are possible based on the context. This will begin a discussion about realistic solutions.



Unit 6 | Lesson 18

Modeling With Inequalities

Let's look at solutions to inequalities.

Warm-up Possible Values

The stage manager of the school musical is trying to determine how many sandwiches he can order with the \$83 he collected from the cast and crew. Sandwiches cost \$5.99 each, so he lets x represent the number of sandwiches he can order and writes the inequality $5.99x \leq 83$. He solves the inequality and rounds the solution to two decimal places, resulting in $x \leq 13.86$.

Which of these are reasonable statements about this scenario? Select *all* that apply.

- **A.** He can order exactly 13.86 sandwiches.
- **B.** He can round up and order 14 sandwiches.
- [✓ **C.**] He can order 12 sandwiches.
- **D.** He can order 9.5 sandwiches.
- [✓ **E.**] He can order 2 sandwiches.
- **F.** He can order -4 sandwiches.

Choice D may be reasonable, depending on stores in your area.

Log in to Amplify Math to complete this lesson online.

650 **Unit 6** Expressions, Equations, and Inequalities

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1 Launch

Set an expectation for the amount of time students have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "Can a fraction of a sandwich be ordered? Can the stage manager order a negative number of sandwiches?"

Look for points of confusion:

- **Thinking that the value 14 is a solution because 13.86 rounds up.** Model how to substitute 14 into the inequality showing that it is not a solution.

3 Connect

Have students share their thinking for each value. Conduct the ***Poll the Class*** routine for agreement and discuss any disagreements.

Highlight that the values 13.86 and -4 satisfy the inequality. Emphasize, however, they do not make sense in the context of sandwiches. The value 9.5 also satisfies the inequality, but may or may not be reasonable or possible, depending on what the class decides.

Power-up

To power up students' ability to determine appropriate solutions based on the context of a scenario, have students complete:

Noah needs to buy at least 3 packs of pencils at the store to have enough to last the school year.

Select *all* of the possible numbers of packs of pencils he could buy at the store.

[✓ A.] 3 packs **B.** 3.5 packs [✓ C.] 4 packs **D.** $5\frac{1}{3}$ packs

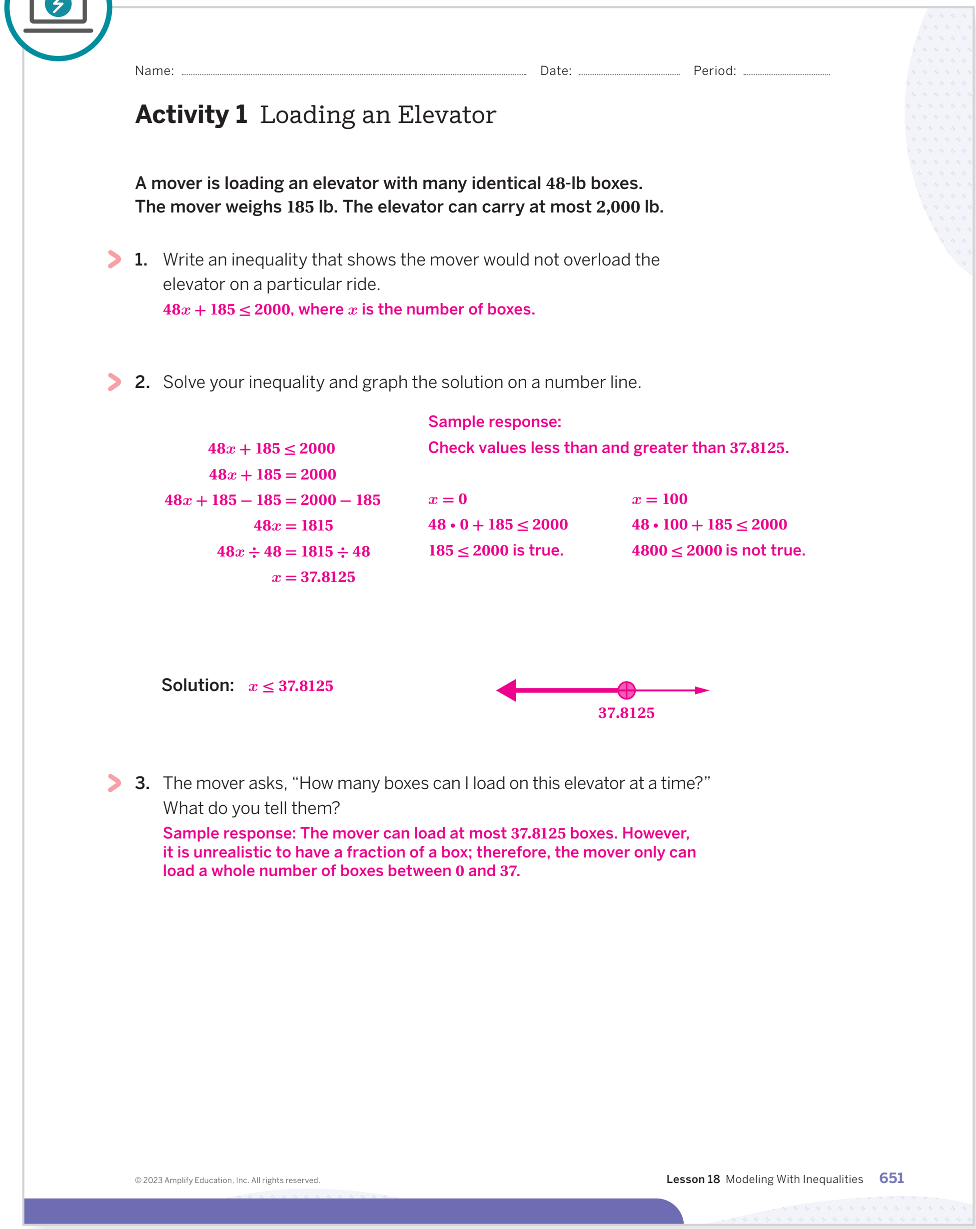
Use: Before the Warm-up

Informed by: Performance on Lesson 17, Practice Problem 6

Pairs | 10 min

Activity 1 Loading an Elevator

Students write and solve an inequality to represent a real-world problem, and consider what solutions are realistic in context.



Name: ________ Date: ________ Period: ________

Activity 1 Loading an Elevator

A mover is loading an elevator with many identical 48-lb boxes. The mover weighs 185 lb. The elevator can carry at most 2,000 lb.

1. Write an inequality that shows the mover would not overload the elevator on a particular ride.
   $48x + 185 \leq 2000$, where x is the number of boxes.

2. Solve your inequality and graph the solution on a number line.

   Sample response:
   Check values less than and greater than 37.8125.

   $$48x + 185 \leq 2000$$
   $$48x + 185 = 2000$$
   $$48x + 185 - 185 = 2000 - 185$$
   $$48x = 1815$$
   $$48x \div 48 = 1815 \div 48$$
   $$x = 37.8125$$

   $x = 0$
   $48 \cdot 0 + 185 \leq 2000$
   $185 \leq 2000$ **is true.**

   $x = 100$
   $48 \cdot 100 + 185 \leq 2000$
   $4800 \leq 2000$ **is not true.**

   Solution: $x \leq 37.8125$

3. The mover asks, “How many boxes can I load on this elevator at a time?” What do you tell them?
   Sample response: The mover can load at most 37.8125 boxes. However, it is unrealistic to have a fraction of a box; therefore, the mover only can load a whole number of boxes between 0 and 37.

Lesson 18 Modeling With Inequalities 651

1 Launch

Ask students to close their books and display the Activity 1 PDF. Give pairs of students a few moments to brainstorm what information they need in order to answer the question. After students share what missing information is needed, have them open their books and read the scenario for the Activity.

2 Monitor

Help students get started by asking, “Can the mover put one box on the elevator? Would that be efficient?”

Look for points of confusion:

- **Thinking it is possible to have 38 boxes in the elevator.** Have students substitute the value into the inequality and determine it is not a solution.

3 Connect

Have students share strategies for solving the inequality.

Highlight modeling the scenario with the inequality and how the related equation helps solve the inequality.

Ask:

- “How can you represent the solution on a number line? Is 5.5 a solution?” Sample response: It is not a solution in the context of this problem because it doesn’t make sense to have half a box.
- “Do you want to change the number line somehow to show this?” Sample response: I could plot points or I could simply leave it as is, but just know that for a problem with this context, I will only use integer solutions.
- “How did you know which way to round?” Sample response: I should round down, otherwise the mover has gone over the weight limit.
- “What other limitations do the contexts place on the solutions?” Sample response: There must be a positive number of boxes.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

To help students make sense of the introductory text, ask these questions before they begin the activity. Then distribute the Activity 1 PDF for students to record all of the possibilities.

- “Can the mover take all 48 boxes in one load? Why or why not?”
- “Can the mover take 10 boxes in one load? More than 10?”
- “Can the mover take 24 boxes in one load? More than 10?”

Extension: *Math Enrichment*

Have students complete the following problem:
If there were 140 boxes to move, how many trips would it take? 4 trips; 140 divided by 37 is about 3.7, which means 4 trips are needed.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight how the inequality models the scenario, display the scenario and its related inequality. Ask the following questions. As students respond, annotate or color code the key words and phrases in the text with how they are represented in the inequality.

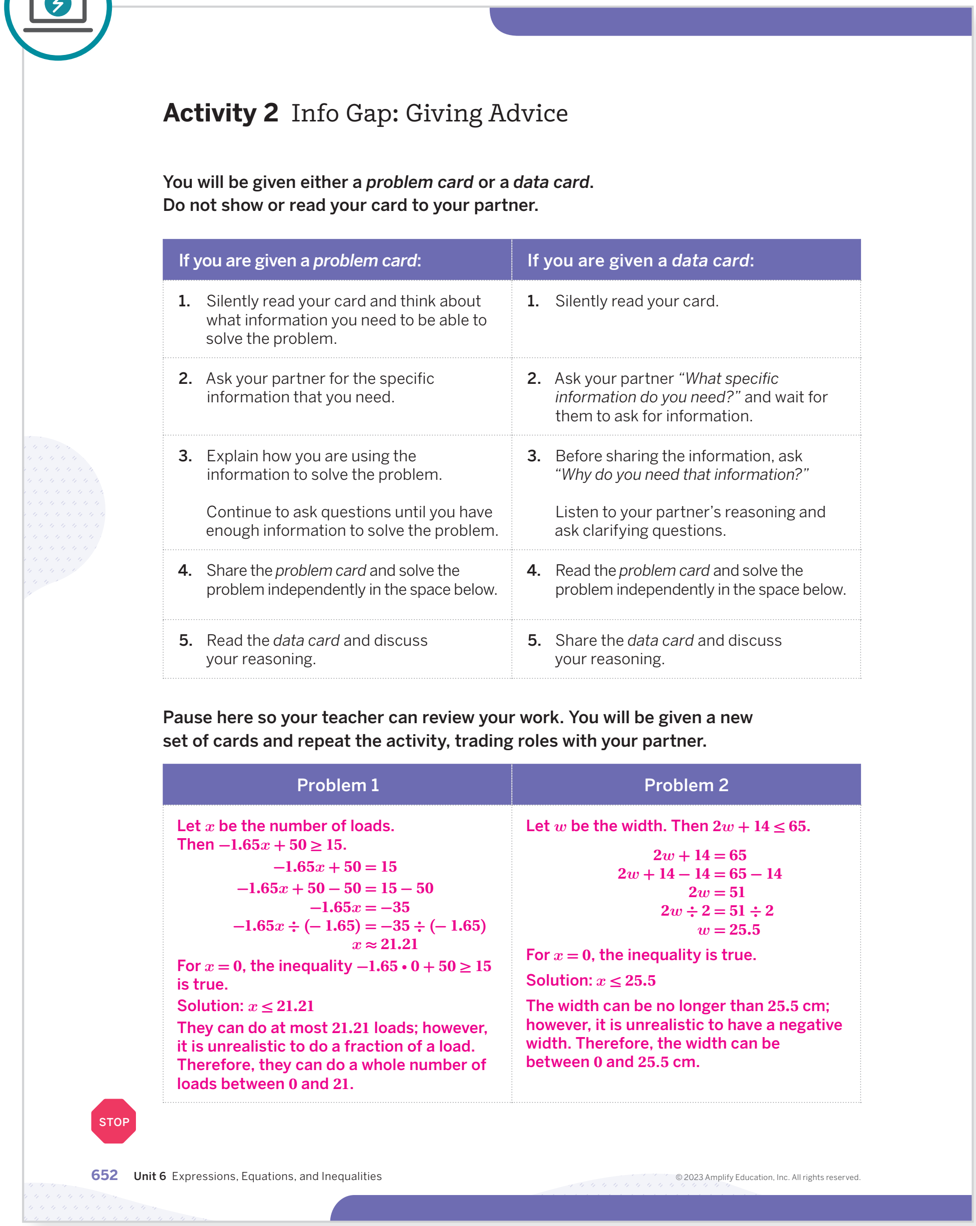
Ask, “Where do you see . . .”

- “The unknown? What does it represent?”
- “The weight constraint of the elevator in the text and in the inequality? Why was this particular inequality symbol used?”
- “The weight of the mover in each representation? Why is it added?”
- “The weight of each box? Why is it multiplied by the unknown?”

Pairs | 20 min

Activity 2 Info Gap: Giving Advice

Students set up and solve inequalities representing real-world scenarios. They use the context of the scenario to interpret the solutions.



Activity 2 Info Gap: Giving Advice

You will be given either a *problem card* or a *data card*.
Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	1. Silently read your card.
2. Ask your partner for the specific information that you need.	2. Ask your partner *"What specific information do you need?"* and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	3. Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space below.	4. Read the *problem card* and solve the problem independently in the space below.
5. Read the *data card* and discuss your reasoning.	5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. You will be given a new set of cards and repeat the activity, trading roles with your partner.

Problem 1	Problem 2
Let x be the number of loads. Then $-1.65x + 50 \geq 15$. $-1.65x + 50 = 15$ $-1.65x + 50 - 50 = 15 - 50$ $-1.65x = -35$ $-1.65x \div (-1.65) = -35 \div (-1.65)$ $x \approx 21.21$ For $x = 0$, the inequality $-1.65 \cdot 0 + 50 \geq 15$ is true. Solution: $x \leq 21.21$ They can do at most 21.21 loads; however, it is unrealistic to do a fraction of a load. Therefore, they can do a whole number of loads between 0 and 21.	Let w be the width. Then $2w + 14 \leq 65$. $2w + 14 = 65$ $2w + 14 - 14 = 65 - 14$ $2w = 51$ $2w \div 2 = 51 \div 2$ $w = 25.5$ For $x = 0$, the inequality is true. Solution: $x \leq 25.5$ The width can be no longer than 25.5 cm; however, it is unrealistic to have a negative width. Therefore, the width can be between 0 and 25.5 cm.

STOP

652 **Unit 6** Expressions, Equations, and Inequalities

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1 Launch

Distribute a set of cards from Activity 2 PDF to each pair of students. Conduct the ***Info Gap*** routine.

2 Monitor

Help students get started by reminding students they can represent their situation using words, an inequality, and a graph. They also need to determine what the variable represents.

Look for points of confusion:

- **Calculating the area instead of perimeter for Problem Card 2.** Remind students that the term *border* implies a distance (length) around the outside.
- **Not remembering how to determine the perimeter or not remembering there are two lengths.** Have them draw a picture of a rectangle and label the length as 7 cm and the width as the unknown quantity.

3 Connect

Highlight that some scenarios can only have discrete solutions. For instance, Noah cannot do 2.5 loads of laundry; he can only do whole numbers of loads. Some scenarios will have continuous solutions. For instance, Elena can make the width any amount between 0 and 25.5 cm.

Ask:

- "In Noah's problem, should you round up or down? Why?" Down; Noah does not have enough money to do 3 loads.
- "Do you need to round for Elena's problem? Why or why not?" No; the width does not have to be a whole-number value.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following during the think-aloud.

- "I know the family wants to keep a minimum balance on the card, but I don't know what that is. I will ask for this amount."
- "I need to determine how many loads of laundry Noah's family can do before needing to add money to the card, but I don't know how much money is already on the card. I will ask for this amount."

Math Language Development

MLR4: Information Gap

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

English Learners

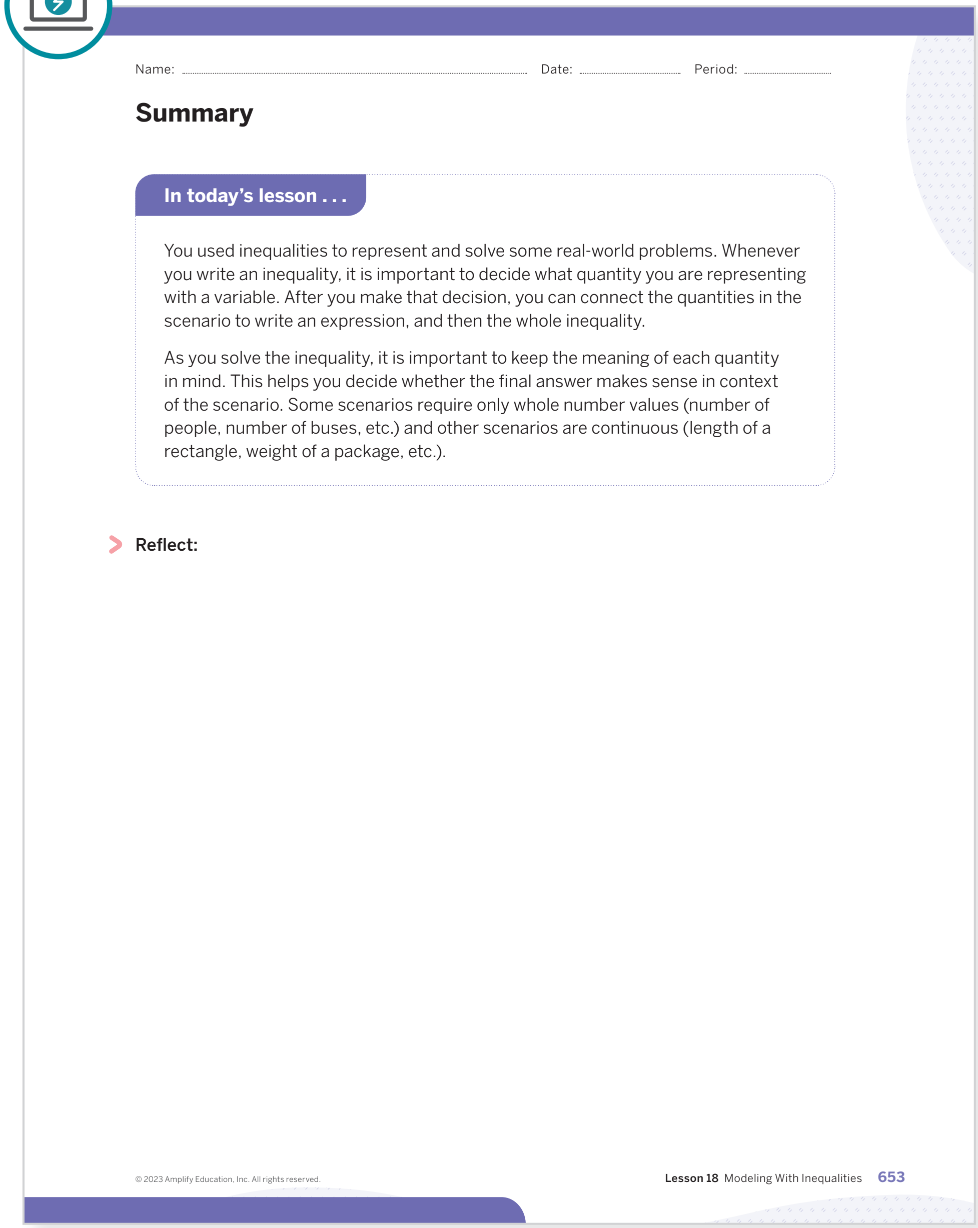
Consider providing sample questions students could ask, such as the following for Problem Card 1:

- "How much does a load of laundry cost?"
- "How much money is currently on the card?"

Whole Class | 5 min

Summary

Review and synthesize how to model real-world situations with inequalities.



Name: Date: Period:

Summary

In today's lesson . . .

You used inequalities to represent and solve some real-world problems. Whenever you write an inequality, it is important to decide what quantity you are representing with a variable. After you make that decision, you can connect the quantities in the scenario to write an expression, and then the whole inequality.

As you solve the inequality, it is important to keep the meaning of each quantity in mind. This helps you decide whether the final answer makes sense in context of the scenario. Some scenarios require only whole number values (number of people, number of buses, etc.) and other scenarios are continuous (length of a rectangle, weight of a package, etc.).

Reflect:

Synthesize

Display the scenario, "Andre is saving money to purchase something and needs at least $100. He already has $20 in his piggy bank and earns $7 each week in allowance."

Ask students what information needs to be decided or what steps need to be completed. For example, students need to define a variable, write an inequality, solve the inequality, and interpret the solution within the context of the scenario.

Highlight that possible solutions to a scenario are different than the mathematical solutions. For instance, some solutions may only be positive whole-number values (*number of people*). Other scenarios may have continuous solutions (*length of a rope*).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "When it was your turn for the "Problem" card, how did you decide which questions to ask?"
- "When it was your turn for the "Data" card, how did you decide what information you should share with your partner?"

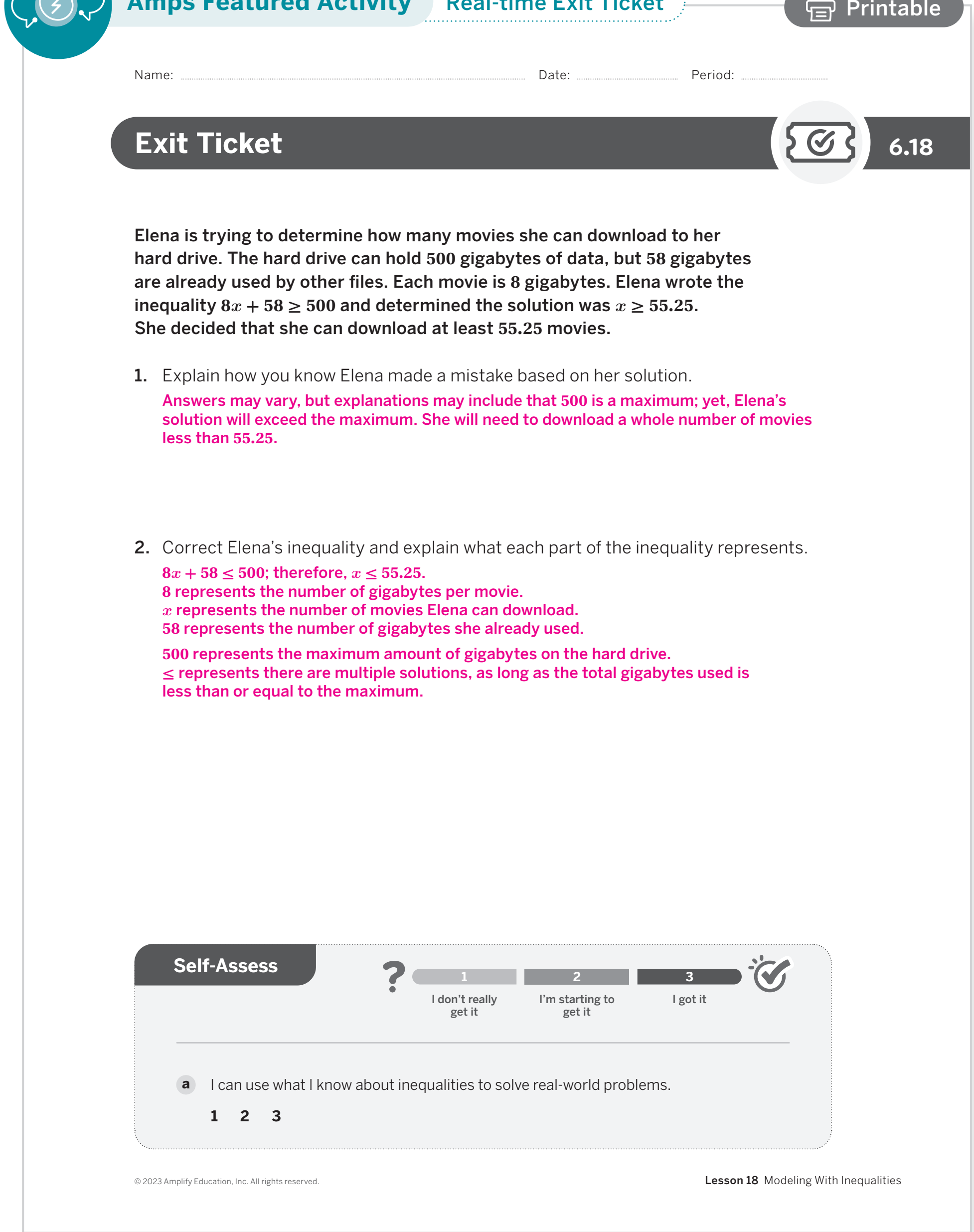
Independent | 5 min

Exit Ticket

Students demonstrate their understanding by critiquing Elena’s inequality and solution to a real-world problem.

Amps Featured Activity Real-time Exit Ticket

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.18

Elena is trying to determine how many movies she can download to her hard drive. The hard drive can hold 500 gigabytes of data, but 58 gigabytes are already used by other files. Each movie is 8 gigabytes. Elena wrote the inequality $8x + 58 \geq 500$ and determined the solution was $x \geq 55.25$. She decided that she can download at least 55.25 movies.

1. Explain how you know Elena made a mistake based on her solution.

   Answers may vary, but explanations may include that 500 is a maximum; yet, Elena’s solution will exceed the maximum. She will need to download a whole number of movies less than 55.25.

2. Correct Elena’s inequality and explain what each part of the inequality represents.

   $8x + 58 \leq 500$; therefore, $x \leq 55.25$.
   8 represents the number of gigabytes per movie.
   x represents the number of movies Elena can download.
   58 represents the number of gigabytes she already used.

   500 represents the maximum amount of gigabytes on the hard drive.
   $\leq$ represents there are multiple solutions, as long as the total gigabytes used is less than or equal to the maximum.

Self-Assess

? 1	2	3 ✓
I don’t really get it	I’m starting to get it	I got it

a I can use what I know about inequalities to solve real-world problems.

1 2 3

Lesson 18 Modeling With Inequalities

Success looks like . . .

- **Language Goal:** Critiquing the solution to an inequality, including whether fractional or negative values are reasonable. **(Speaking and Listening)**
  - » Explaining why Elena’s solution is not correct in Problem 1.
- **Goal:** Determining what information is needed to solve a problem involving a quantity constrained by a maximum or minimum acceptable value.
- **Goal:** Writing and solving an inequality of the form $px + q > r$ or $px + q < r$ to solve a problem about a situation with a constraint.
  - » Correcting Elena’s solution to the inequality in Problem 2.

Suggested next steps

If students do not find Elena’s mistake with the inequality, consider:

- Reviewing the Summary for Lesson 15.
- Discussing examples of inequality phrases.

If students cannot explain the meaning of the parts of the inequality, consider:

- Having students identifying the meaning of the parts of the inequalities in Practice Problems 1, 2, and 3.

Professional Learning

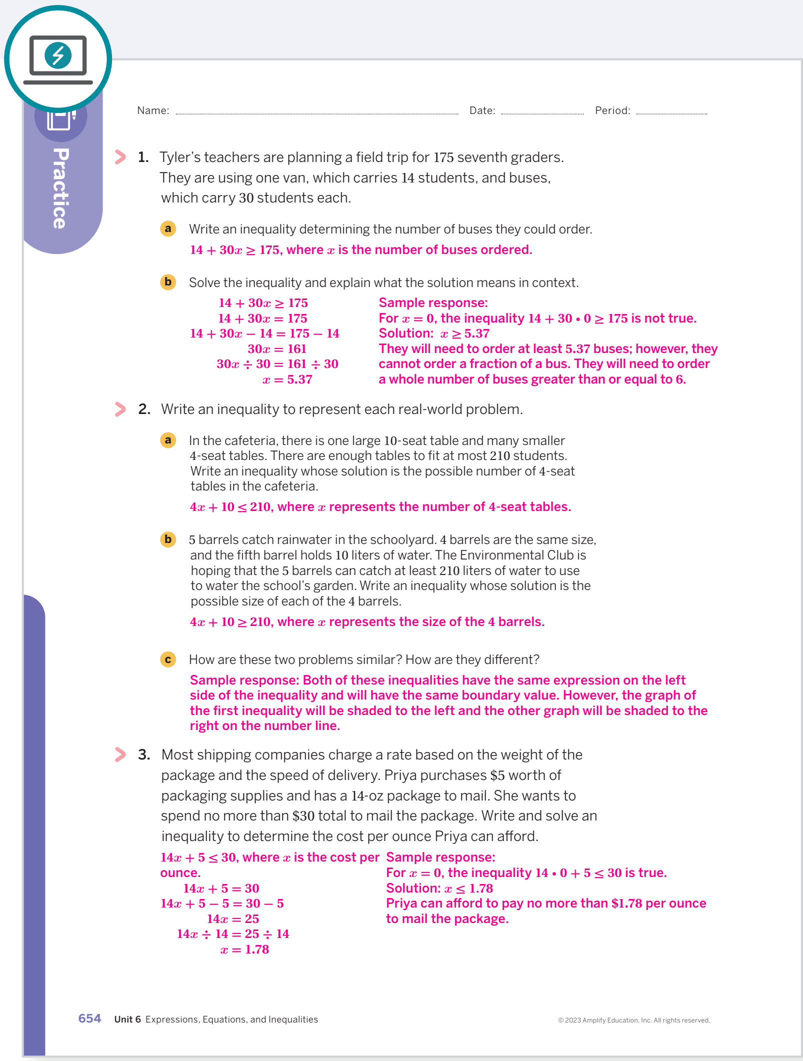
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn’t work today? How did students model with mathematics today? How are you helping students become aware of how they are progressing in this area?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Tyler's teachers are planning a field trip for 175 seventh graders. They are using one van, which carries 14 students, and buses, which carry 30 students each.

   a. Write an inequality determining the number of buses they could order.

   $14 + 30x \geq 175$, where x is the number of buses ordered.

   b. Solve the inequality and explain what the solution means in context.

   $$\begin{aligned} 14 + 30x &\geq 175 \\ 14 + 30x &= 175 \\ 14 + 30x - 14 &= 175 - 14 \\ 30x &= 161 \\ 30x \div 30 &= 161 \div 30 \\ x &= 5.37 \end{aligned}$$

   Sample response:
   For $x = 0$, the inequality $14 + 30 \cdot 0 \geq 175$ is not true.
   Solution: $x \geq 5.37$
   They will need to order at least 5.37 buses; however, they cannot order a fraction of a bus. They will need to order a whole number of buses greater than or equal to 6.

2. Write an inequality to represent each real-world problem.

   a. In the cafeteria, there is one large 10-seat table and many smaller 4-seat tables. There are enough tables to fit at most 210 students. Write an inequality whose solution is the possible number of 4-seat tables in the cafeteria.

   $4x + 10 \leq 210$, where x represents the number of 4-seat tables.

   b. 5 barrels catch rainwater in the schoolyard. 4 barrels are the same size, and the fifth barrel holds 10 liters of water. The Environmental Club is hoping that the 5 barrels can catch at least 210 liters of water to use to water the school's garden. Write an inequality whose solution is the possible size of each of the 4 barrels.

   $4x + 10 \geq 210$, where x represents the size of the 4 barrels.

   c. How are these two problems similar? How are they different?

   Sample response: Both of these inequalities have the same expression on the left side of the inequality and will have the same boundary value. However, the graph of the first inequality will be shaded to the left and the other graph will be shaded to the right on the number line.

3. Most shipping companies charge a rate based on the weight of the package and the speed of delivery. Priya purchases $5 worth of packaging supplies and has a 14-oz package to mail. She wants to spend no more than $30 total to mail the package. Write and solve an inequality to determine the cost per ounce Priya can afford.

   $14x + 5 \leq 30$, where x is the cost per ounce.

   $$\begin{aligned} 14x + 5 &= 30 \\ 14x + 5 - 5 &= 30 - 5 \\ 14x &= 25 \\ 14x \div 14 &= 25 \div 14 \\ x &= 1.78 \end{aligned}$$

   Sample response:
   For $x = 0$, the inequality $14 \cdot 0 + 5 \leq 30$ is true.
   Solution: $x \leq 1.78$
   Priya can afford to pay no more than $1.78 per ounce to mail the package.

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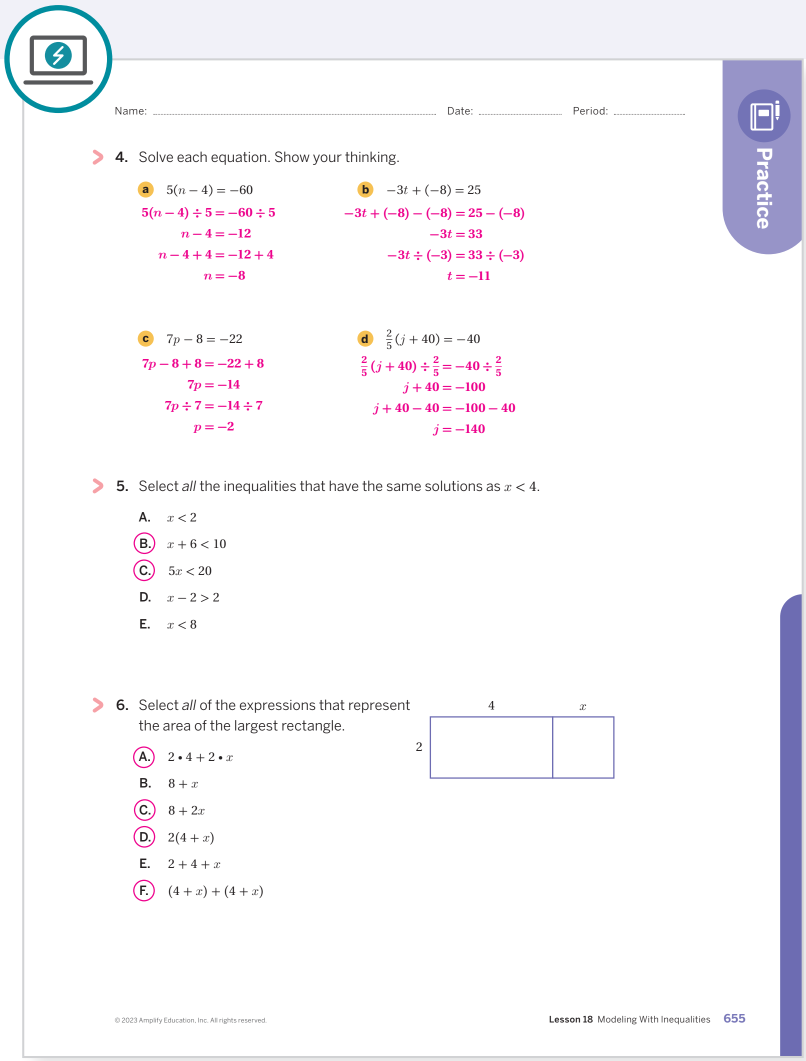

Practice

Name: ______ Date: ______ Period: ______

4. Solve each equation. Show your thinking.

   a. $5(n - 4) = -60$

   $$\begin{aligned} 5(n-4) \div 5 &= -60 \div 5 \\ n - 4 &= -12 \\ n - 4 + 4 &= -12 + 4 \\ n &= -8 \end{aligned}$$

   b. $-3t + (-8) = 25$

   $$\begin{aligned} -3t + (-8) - (-8) &= 25 - (-8) \\ -3t &= 33 \\ -3t \div (-3) &= 33 \div (-3) \\ t &= -11 \end{aligned}$$

   c. $7p - 8 = -22$

   $$\begin{aligned} 7p - 8 + 8 &= -22 + 8 \\ 7p &= -14 \\ 7p \div 7 &= -14 \div 7 \\ p &= -2 \end{aligned}$$

   d. $\frac{2}{5}(j + 40) = -40$

   $$\begin{aligned} \frac{2}{5}(j + 40) \div \frac{2}{5} &= -40 \div \frac{2}{5} \\ j + 40 &= -100 \\ j + 40 - 40 &= -100 - 40 \\ j &= -140 \end{aligned}$$

5. Select *all* the inequalities that have the same solutions as $x < 4$.

   A. $x < 2$
   [✓ B.] $x + 6 < 10$
   [✓ C.] $5x < 20$
   D. $x - 2 > 2$
   E. $x < 8$

6. Select *all* of the expressions that represent the area of the largest rectangle.

   [✓ A.] $2 \cdot 4 + 2 \cdot x$
   B. $8 + x$
   [✓ C.] $8 + 2x$
   [✓ D.] $2(4 + x)$
   E. $2 + 4 + x$
   [✓ F.] $(4 + x) + (4 + x)$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 6	1
	5	Unit 6 Lesson 14	1
Formative	6	Unit 6 Lesson 19	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 4

Equivalent Expressions

Whole Class

In this Sub-Unit, students see that expressions can be rewritten using fewer terms, while taking extra care when dealing with negative terms.

SUB-UNIT

4 Equivalent Expressions

Narrative Connections

Which three blockheads did NASA send into space?

In 2019, NASA launched three robots to the International Space Station. But these were not the clunky bots you would find in a science fiction film. Instead, imagine three cubes, each about a foot wide, and equipped with advanced sensors, touch screens, and a set of electric fans for zipping around.

These were the “Astrobees”: Bumble, Honey, and Queen. NASA hoped these boxy bots would help astronauts with their daily routines. That way the humans onboard would have time to focus on more important things.

The Astrobees were programmed to navigate the space station on their own, dock themselves whenever their batteries ran low, and even put themselves away. They could also track tools, take inventory, help with experiments, and move cargo, making life on the space station easier and more productive.

As a species, we humans have often used technology to do things better. And look how far we have come! From spears and clubs, to the steam engine, to personal computers, and now to robots. By finding more efficient ways to get things done, we can be faster, make fewer mistakes, and arrive at more elegant solutions.

As you’ll see, the same goes for solving equations. By writing expressions into equivalent forms, you can see structure (and solutions) where you might not have before.

Sub-Unit 4 Equivalent Expressions 657

Narrative Connections

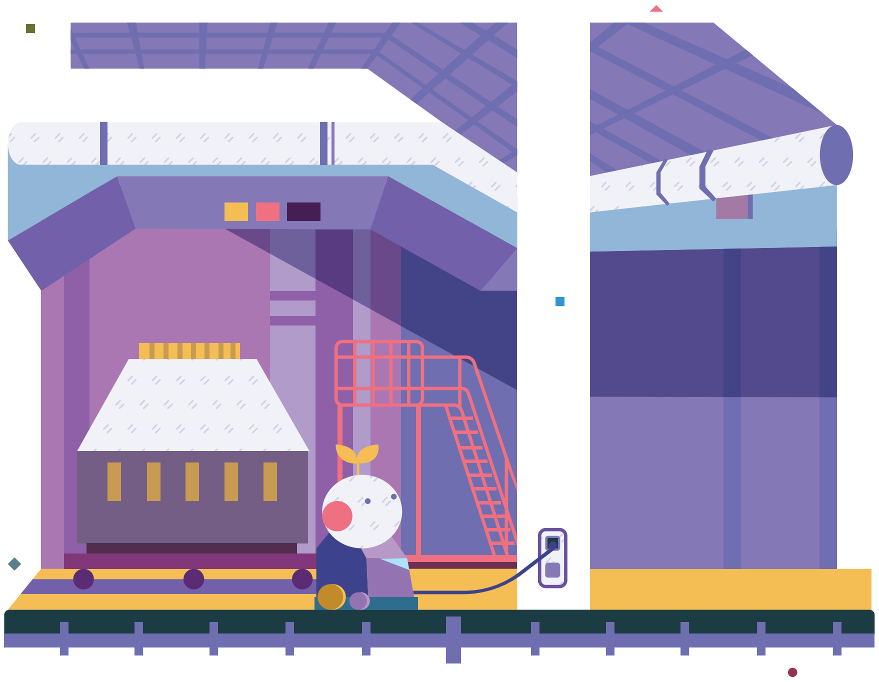
Read the narrative aloud as a class or have students read it individually. Students continue to make use of structure in expressions, and even look at a few more robots, in the following places:

- **Lesson 19, Activity 1:** Robot Recharge
- **Lesson 21, Activity 1:** A’s and B’s
- **Lesson 22, Activity 2:** Grouping Differently

UNIT 6 | **LESSON 19**

Subtraction in Equivalent Expressions

Let's find ways to work with subtraction in expressions.



Focus

Goals

1. **Language Goal:** Explain (using multiple representations) how the Distributive and commutative properties apply to expressions with negative coefficients. **(Speaking and Listening, Writing)**
2. Identify whether expressions are equivalent, including rewriting subtraction as adding the opposite.

Coherence

Today

This lesson prepares students for working with more complicated expressions and rewriting those expressions in a more helpful form. It is meant to guide students against making errors involving subtraction and negative signs when rewriting those expressions.

Previously

Earlier this unit, in Lessons 3–7, students solved equations of the forms $px + q = r$ and $p(x + q) = r$.

Coming Soon

In Lessons 20 and 21, students will work with factoring and expanding expressions that include subtraction and negative terms.

Rigor

- Students build **conceptual understanding** of the Commutative Property of Addition and Distributive Property with negative values by comparing them to similar expressions with positive values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	5 min	10 min	5 min	10 min
Pairs	Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (as needed)

Math Language Development

Review words

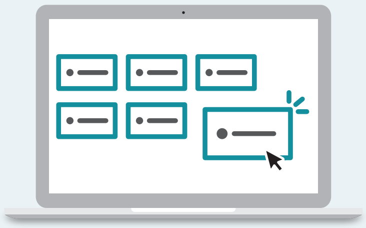
- *additive inverse*
- *Commutative Property of Addition*
- *Distributive Property*
- *equivalent expressions*
- *terms*

Amps | Featured Activity

Activity 3

Digital Card Sort

Students use a digital sorting activity to make sense of area models that include negative values. You can eliminate the prep work needed for this activity.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Because each activity in this lesson reviews a skill students learned in the past, they might impulsively rush through, without putting forth much thought. Prior to beginning each activity, have students explain why the topic is important enough to spend some time reviewing. Explain that each provides a structure in which students can work to be successful.

Modifications to Pacing

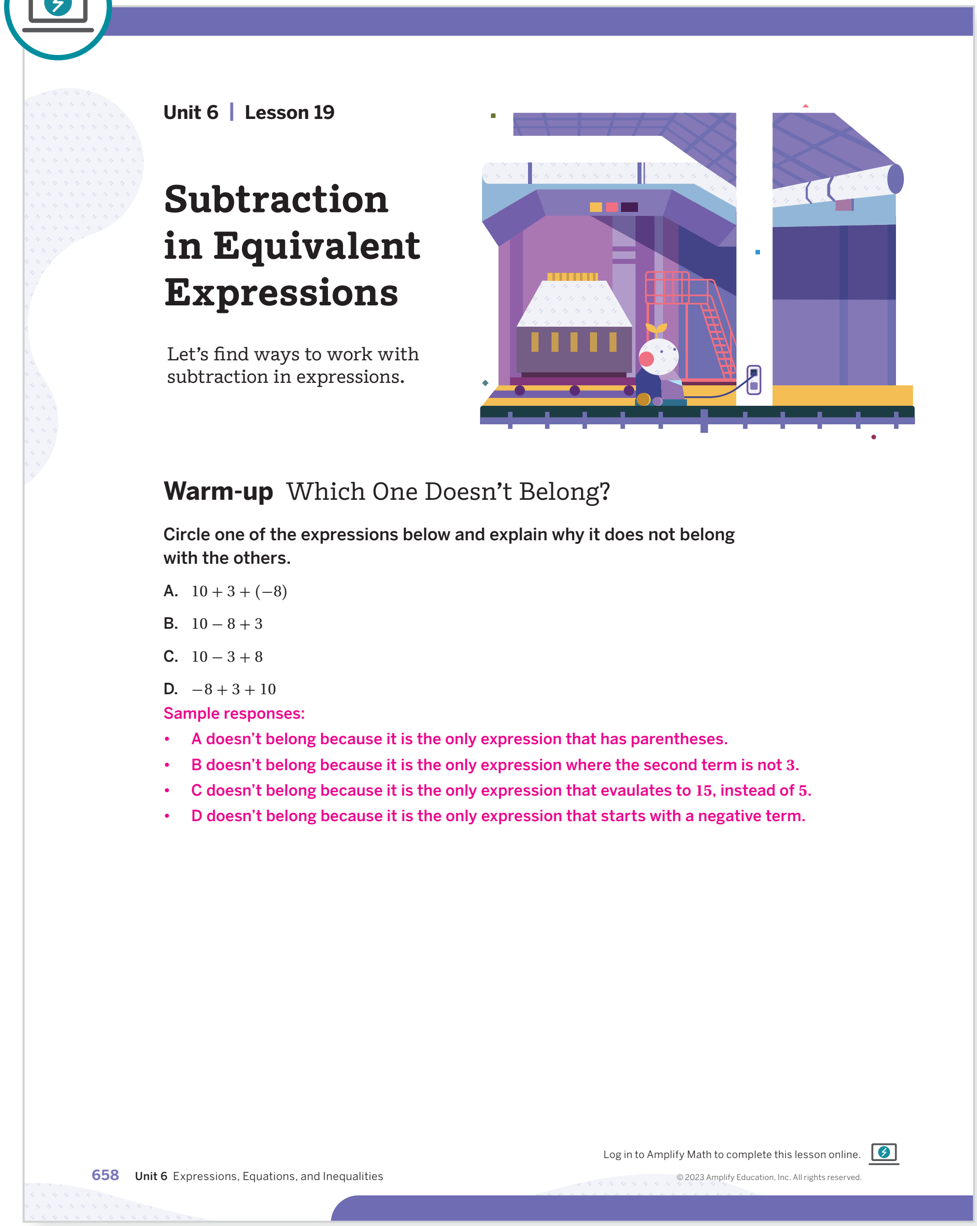
You may want to consider these additional modifications if you are short on time.

- During **Activity 1**, set a time limit and have students write as many number sentences as they can during that time.
- **Activity 3** may be omitted. Consider assigning the Activity as Additional Practice.

Pairs | 5 min

Warm-up Which One Doesn't Belong?

Students compare four expressions involving negative values to prepare students for similar algebraic expressions in this lesson.



Unit 6 | **Lesson 19**

Subtraction in Equivalent Expressions

Let's find ways to work with subtraction in expressions.

Warm-up Which One Doesn't Belong?

Circle one of the expressions below and explain why it does not belong with the others.

A. $10 + 3 + (-8)$

B. $10 - 8 + 3$

C. $10 - 3 + 8$

D. $-8 + 3 + 10$

Sample responses:

- A doesn't belong because it is the only expression that has parentheses.
- B doesn't belong because it is the only expression where the second term is not 3.
- C doesn't belong because it is the only expression that evaulates to 15, instead of 5.
- D doesn't belong because it is the only expression that starts with a negative term.

Log in to Amplify Math to complete this lesson online.

658 **Unit 6** Expressions, Equations, and Inequalities

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Conduct the ***Which One Doesn't Belong?*** routine.

2 Monitor

Help students get started by suggesting they find one difference between two expressions first, and then check if the difference exists in the other expressions.

Look for points of confusion:

- **Stopping after finding one reason.** Encourage students to find as many reasons as they can.

Look for points of confusion:

- Finding these possible reasons: A is the only one with parentheses. B is the only one where 3 is not the middle term. C has a different value than the others. D begins with a negative value.

3 Connect

Display the four expressions.

Have students share at least one reason for each expression as to why it does not belong, starting with A, B, and D. Discuss C last. After each reason, ask the class if they agree or disagree.

Ask students to identify what causes the terms in Expression C to have a different value.

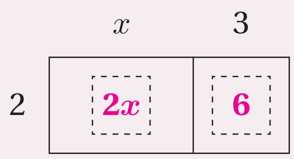
Highlight that Expression C is the only Expression with a different value. Notice that Expressions A, B, and D all preserve the signs of the terms, if not mentioned already.

Power-up

To power up students' ability to recognize equivalent expressions that represent an area model, have students complete:

1. Determine the missing area of each rectangle in the area model shown.
2. Which expressions represent the entire area of the model in Problem 1? Select *all* that apply.

[✓ A.] $2x + 6$ **B.** $2 + x + 3$ [✓ C.] $2(x + 3)$ **D.** $12x$



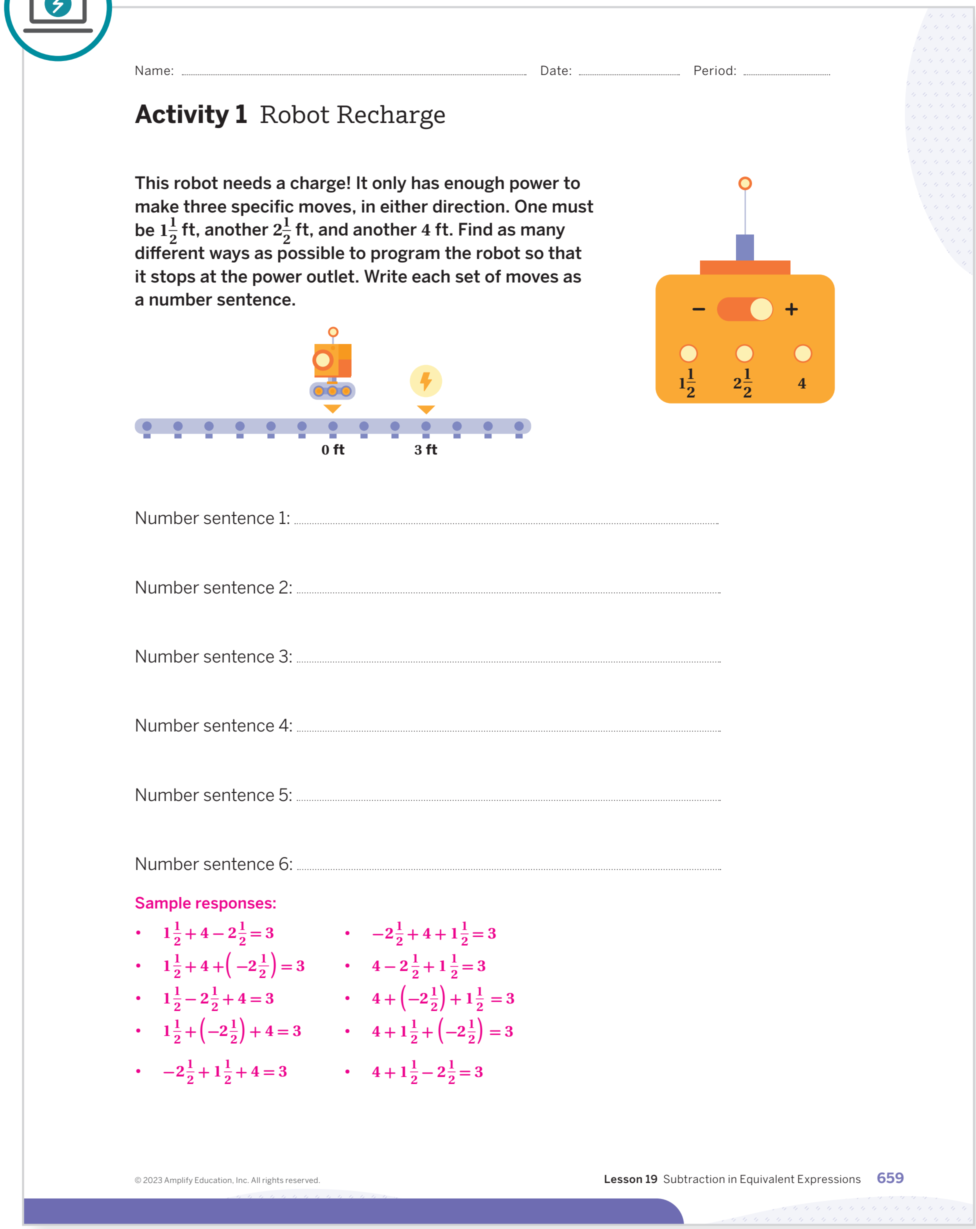
Use: Before Activity 2

Informed by: Performance on Lesson 18, Practice Problem 6

Pairs | 10 min

Activity 1 Robot Recharge

Students recall that subtracting is the same as adding the inverse. This activity reinforces the idea that the subtraction sign must stay with the term that follows it.



Name: Date: Period:

Activity 1 Robot Recharge

This robot needs a charge! It only has enough power to make three specific moves, in either direction. One must be $1\frac{1}{2}$ ft, another $2\frac{1}{2}$ ft, and another 4 ft. Find as many different ways as possible to program the robot so that it stops at the power outlet. Write each set of moves as a number sentence.

Number sentence 1:

Number sentence 2:

Number sentence 3:

Number sentence 4:

Number sentence 5:

Number sentence 6:

Sample responses:

- $1\frac{1}{2} + 4 - 2\frac{1}{2} = 3$
- $1\frac{1}{2} + 4 + \left(-2\frac{1}{2}\right) = 3$
- $1\frac{1}{2} - 2\frac{1}{2} + 4 = 3$
- $1\frac{1}{2} + \left(-2\frac{1}{2}\right) + 4 = 3$
- $-2\frac{1}{2} + 1\frac{1}{2} + 4 = 3$
- $-2\frac{1}{2} + 4 + 1\frac{1}{2} = 3$
- $4 - 2\frac{1}{2} + 1\frac{1}{2} = 3$
- $4 + \left(-2\frac{1}{2}\right) + 1\frac{1}{2} = 3$
- $4 + 1\frac{1}{2} + \left(-2\frac{1}{2}\right) = 3$
- $4 + 1\frac{1}{2} - 2\frac{1}{2} = 3$

Lesson 19 Subtraction in Equivalent Expressions 659

① Launch

Suggest to students they may want to mark important points on the number line before thinking about the moves.

② Monitor

Help students get started by saying, "Try three moves in the same direction. Where does that get you?"

Look for points of confusion:

- **Struggling with the fractional values.** Refer to the Differentiated Support: Accessibility section.
- **Always starting with a move to the right.** Ask, "Is it possible to get to the outlet if the robot moves left first?"
- **Writing a number sentence that doesn't get the robot to the outlet.** Say, "Show me these moves on the number line."

③ Connect

Display the diagram and empty lines to fill in with the expressions that students share.

Have students share each of the number sentences they found to get the robot to the outlet.

Ask:

- "What do these expressions have in common?" All have a value of 3. All have the $2\frac{1}{2}$ subtracted or represented as a negative.
- "Can you explain on the model why the equation $1\frac{1}{2} + 4 - 2\frac{1}{2} = 1\frac{1}{2} + \left(-2\frac{1}{2}\right) + 4$ is true?"

Highlight that we can treat subtracting $2\frac{1}{2}$ as adding $-2\frac{1}{2}$. The Commutative Property of Addition tells that the order of these values can change and still have an equivalent expression. All of the expressions are equivalent.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider simplifying the values in this activity to whole numbers, such as 1, 2, and 4. This will still allow students to access the targeted goal of the activity, which is to reinforce how subtracting is the same as adding the inverse, paying careful attention to negative signs.

Accessibility: *Vary Demands to Optimize Challenge*

Provide access to copies of the Activity 1 PDF for students to use these blank number lines to help them represent the robot's moves as number sentences.

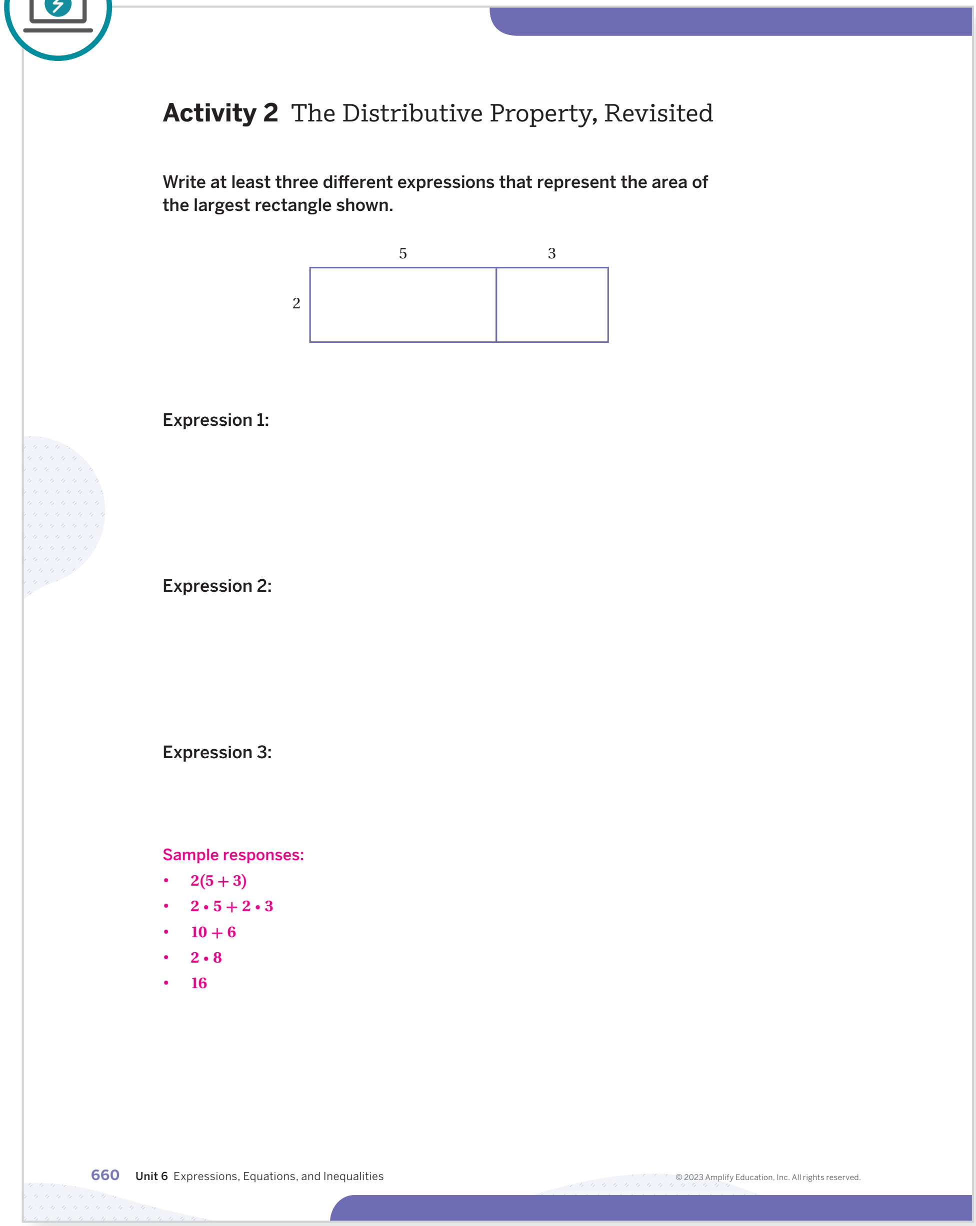
Extension: *Math Enrichment*

Tell students that the robot must move in opposite directions each time. Ask them to determine which number sentence(s) satisfy this new condition. Sample responses: $4 - 2\frac{1}{2} + 1\frac{1}{2} = 3$, $1\frac{1}{2} - 2\frac{1}{2} + 4 = 3$, $4 + \left(-2\frac{1}{2}\right) + 1\frac{1}{2} = 3$, $1\frac{1}{2} + \left(-2\frac{1}{2}\right) + 4 = 3$

Independent | 5 min

Activity 2 The Distributive Property, Revisited

Students write different expressions to represent an area model with whole-number values before moving to negative terms in the following activity.



Activity 2 The Distributive Property, Revisited

Write at least three different expressions that represent the area of the largest rectangle shown.

Expression 1:

Expression 2:

Expression 3:

Sample responses:
- $2(5 + 3)$
- $2 \cdot 5 + 2 \cdot 3$
- $10 + 6$
- $2 \cdot 8$
- 16

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1 Launch

Display the image and give students 2–3 minutes to write an expression for the area of the rectangle in at least three different ways.

2 Monitor

Help students get started by asking, “How can you find the area of the rectangle?”

Look for points of confusion:

- **Providing a response of 16.** Ask students to show how they got to 16.
- **Not seeing $2(5 + 3)$ is modeled by the diagram.** Annotate the length that represents $5 + 3$.

3 Connect

Display the completed area model.

Have students share the expressions, including addition in $2 \cdot 5 + 2 \cdot 3$ and $2(5 + 3)$, and their explanation for how they got them.

Highlight that using the Distributive Property creates equivalent expressions. The area model helps students organize what they need to multiply when using the Distributive Property.

Ask:

- “How does thinking about area give you a way to understand the Distributive Property?” Thinking about the area of a divided rectangle helps to see the different expressions.
- “How do you know that the different expressions are equivalent?” The area model shows that I am thinking about the same area in different ways.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

Remind students they have previously learned and applied the Distributive Property. Ask a student volunteer to provide an example of a numerical expression that illustrates the Distributive Property. Sample response: $4(6 + 1) = 4(6) + 4(1)$

Extension: *Math Enrichment*

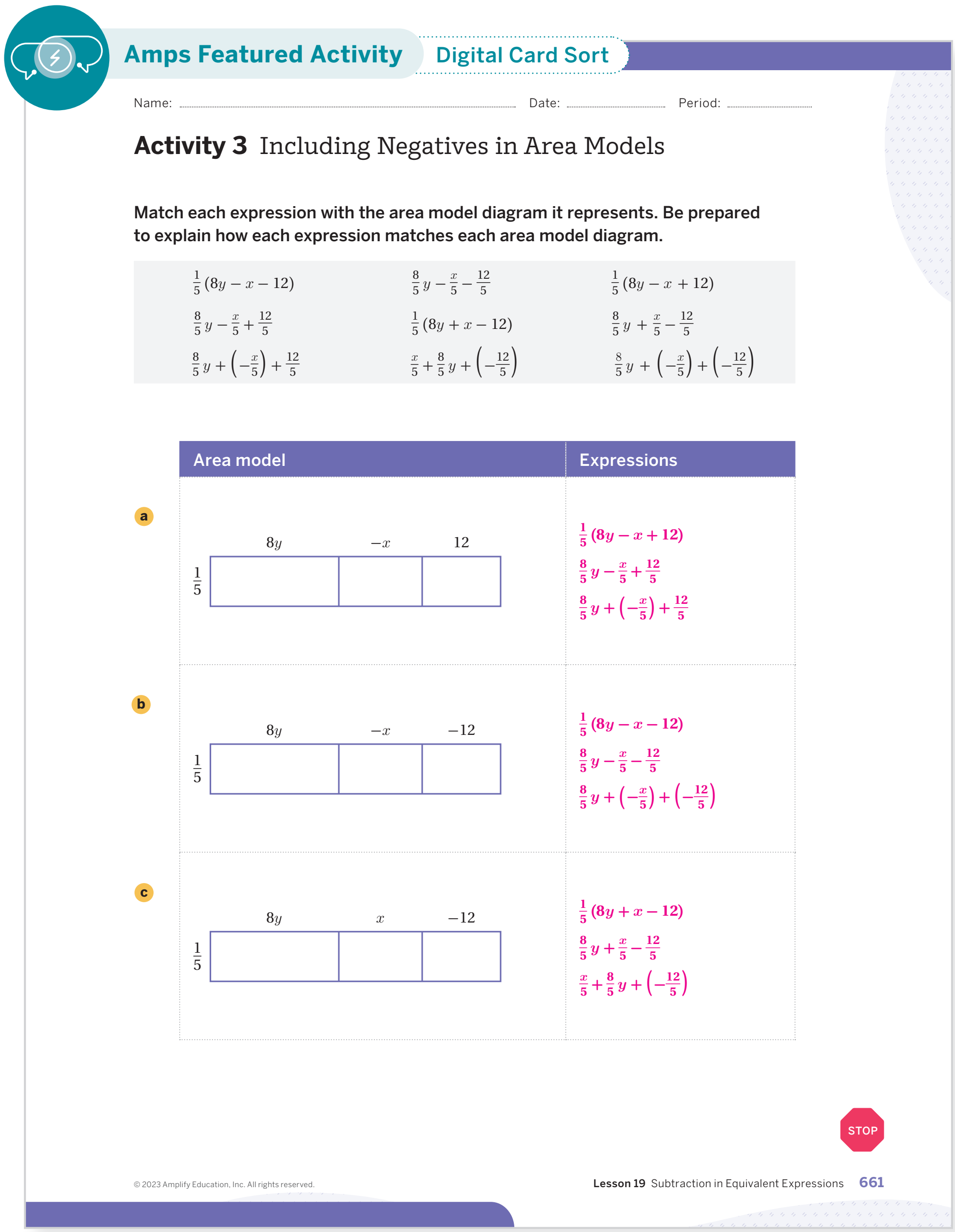
Have students draw an area model that has a total area of $2x + \frac{2}{3}y$. Students’ drawings may vary, but should show how the factor $\frac{2}{3}$ is common to both terms; $\frac{2}{3}(3x + y)$

Pairs | 10 min

Activity 3 Including Negatives in Area Models

Students match area model diagrams with expressions that include negatives and subtraction to help them pay attention to the signs.

Amps Featured Activity Digital Card Sort



Name: ______ Date: ______ Period: ______

Activity 3 Including Negatives in Area Models

Match each expression with the area model diagram it represents. Be prepared to explain how each expression matches each area model diagram.

$\frac{1}{5}(8y - x - 12)$	$\frac{8}{5}y - \frac{x}{5} - \frac{12}{5}$	$\frac{1}{5}(8y - x + 12)$
$\frac{8}{5}y - \frac{x}{5} + \frac{12}{5}$	$\frac{1}{5}(8y + x - 12)$	$\frac{8}{5}y + \frac{x}{5} - \frac{12}{5}$
$\frac{8}{5}y + \left(-\frac{x}{5}\right) + \frac{12}{5}$	$\frac{x}{5} + \frac{8}{5}y + \left(-\frac{12}{5}\right)$	$\frac{8}{5}y + \left(-\frac{x}{5}\right) + \left(-\frac{12}{5}\right)$

	Area model	Expressions
a	$8y$ $-x$ 12 $\frac{1}{5}$	$\frac{1}{5}(8y - x + 12)$ $\frac{8}{5}y - \frac{x}{5} + \frac{12}{5}$ $\frac{8}{5}y + \left(-\frac{x}{5}\right) + \frac{12}{5}$
b	$8y$ $-x$ -12 $\frac{1}{5}$	$\frac{1}{5}(8y - x - 12)$ $\frac{8}{5}y - \frac{x}{5} - \frac{12}{5}$ $\frac{8}{5}y + \left(-\frac{x}{5}\right) + \left(-\frac{12}{5}\right)$
c	$8y$ x -12 $\frac{1}{5}$	$\frac{1}{5}(8y + x - 12)$ $\frac{8}{5}y + \frac{x}{5} - \frac{12}{5}$ $\frac{x}{5} + \frac{8}{5}y + \left(-\frac{12}{5}\right)$

STOP

Lesson 19 Subtraction in Equivalent Expressions 661

1 Launch

Explain to students that they will be matching each expression with one of the area model diagrams. Clarify that students will use all expressions and each diagram will have more than one expression that represents it.

2 Monitor

Help students get started by suggesting they write the area of each of the small rectangles on the models. This should help them find at least one matching expression for each area model.

Look for points of confusion:

- **Getting stuck on the fractional values.** Give students a separate space to work out the multiplication of the coefficients. They should notice these values are consistent in all the expressions.

3 Connect

Display the properly matched area models and expressions. Write or have a student write the expression for the area of each small rectangle on the diagram.

Ask:

- "Is there a pattern you can use to determine which of these expressions are equivalent?" Sample response: I can look at the signs in the expression and match it with the signs in the area model.
- "What is an example of one expression that did not follow the pattern?" Sample response: $\frac{x}{5} + \frac{8}{5}y + \left(-\frac{12}{5}\right)$, because the x term is first.

Highlight that anytime students see subtraction, they know that it can also be represented as adding the additive inverse.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital sorting activity to make sense of area models that include negative values. You can eliminate the prep work needed for this activity.

Accessibility: *Guide Processing and Visualization*

Suggest that students examine the area diagrams before beginning, noticing how they are similar and different. For example, all of the diagrams have $8y$, but the sign of x is positive in the third diagram and the sign of 12 is negative in the second and third diagrams.

Math Language Development

MLR7: Compare and Connect

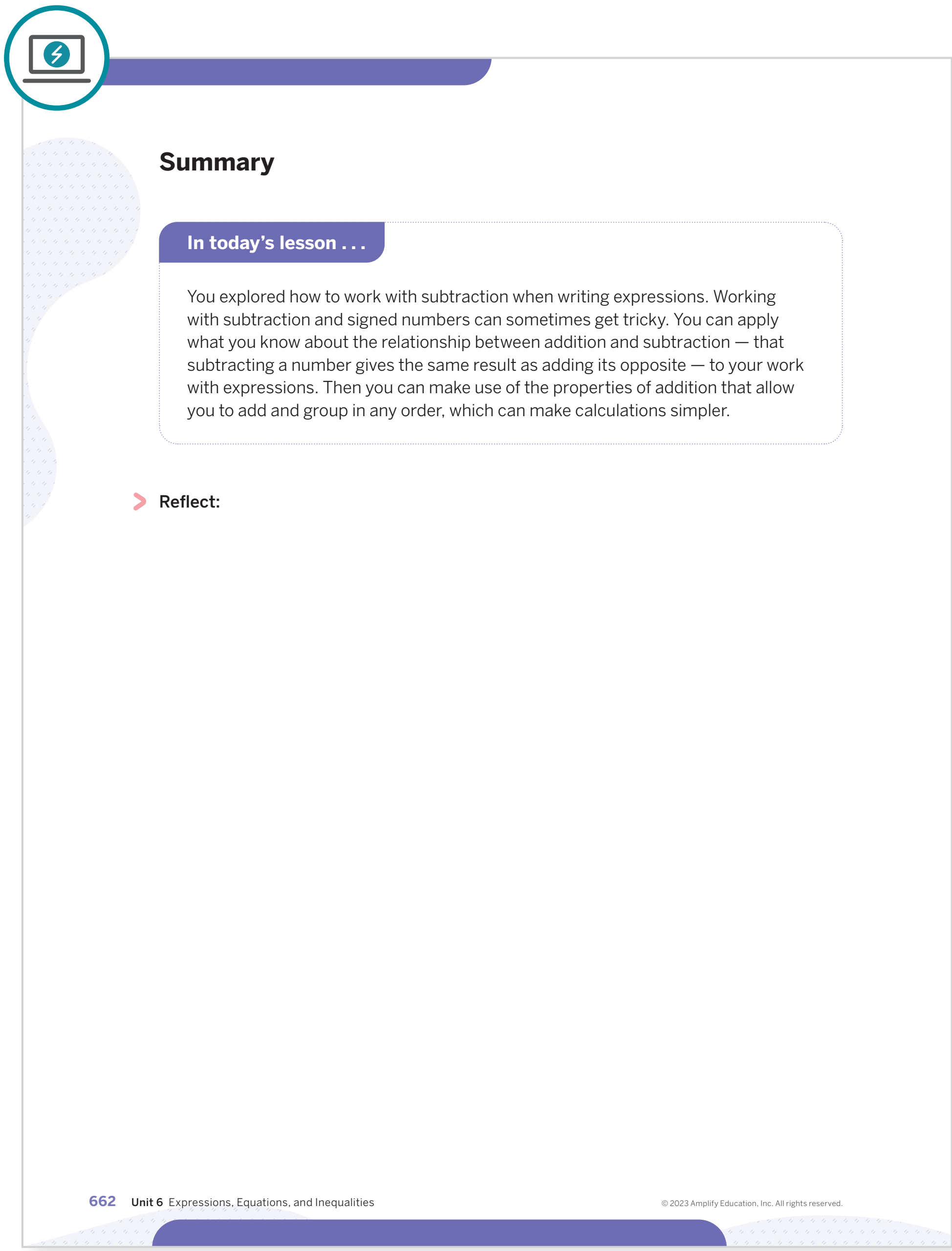
During the Connect, as students respond to the Ask questions, draw their attention to the signs of the terms in the area model and the signs of the terms in the expressions at the beginning of the activity. Ask students to complete these statements, as needed, to drive home the connections.

- "The expressions that include adding a positive x term and either subtracting positive 12 or adding negative 12 match with the area model in part _____, because . . ."
- "The expressions that include adding a positive 12 term and either adding a negative x term or subtracting a positive x term match with the area model in part _____, because . . ."

Whole Class | 5 min

Summary

Review and synthesize how to represent subtraction and signed numbers in area models.



Summary

In today's lesson . . .

You explored how to work with subtraction when writing expressions. Working with subtraction and signed numbers can sometimes get tricky. You can apply what you know about the relationship between addition and subtraction — that subtracting a number gives the same result as adding its opposite — to your work with expressions. Then you can make use of the properties of addition that allow you to add and group in any order, which can make calculations simpler.

Reflect:

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Synthesize

Display the two expressions $x + 2 - 3x - 10$ and $x + 3x - 2 - 10$. Ask students to think about why these expressions are *not* equivalent and explain their reasoning to a partner.

Highlight:

- Subtraction is not commutative. $2 - 3x$ and $3x - 2$ are not equivalent; terms cannot just simply be switched around a subtraction sign.
- Because subtracting $3x$ is the same as adding $(-3x)$, the term $(-3x)$ keeps its negative sign as it is moved around when using the commutative property

Ask, "How can you alter the second expression to make it equivalent to the first?" There are many ways to do this. Recognize students using precise language when describing how they would change signs and/or operations.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

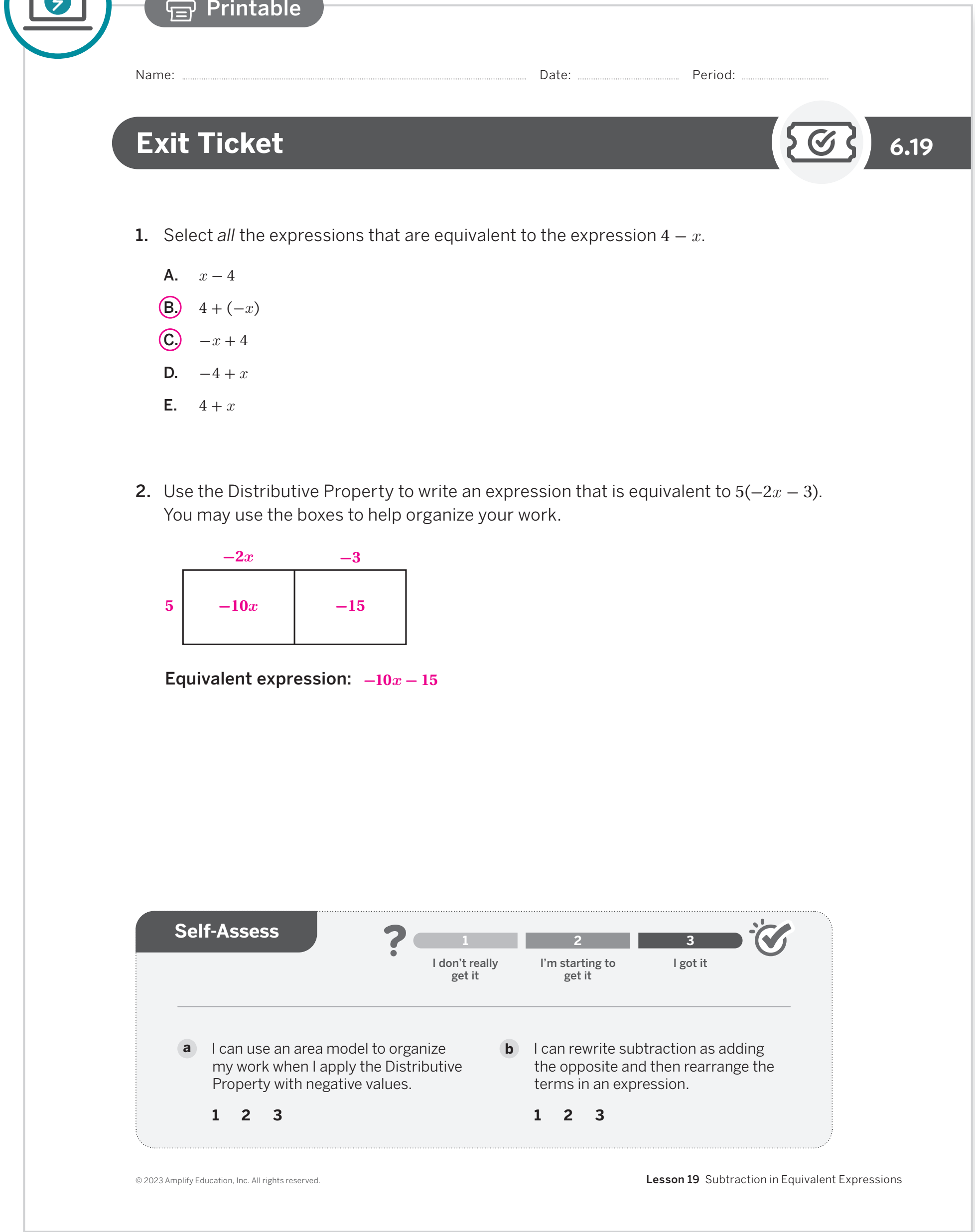
- "Which representations best help you to make sense of certain mathematical scenarios?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by identifying and writing equivalent expressions involving negative terms.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.19

1. Select *all* the expressions that are equivalent to the expression $4 - x$.

A. $x - 4$

[✓ B.] $4 + (-x)$

[✓ C.] $-x + 4$

D. $-4 + x$

E. $4 + x$

2. Use the Distributive Property to write an expression that is equivalent to $5(-2x - 3)$. You may use the boxes to help organize your work.

	$-2x$	-3
5	$-10x$	-15

Equivalent expression: $-10x - 15$

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use an area model to organize my work when I apply the Distributive Property with negative values.

1 2 3

b. I can rewrite subtraction as adding the opposite and then rearrange the terms in an expression.

1 2 3

Lesson 19 Subtraction in Equivalent Expressions

Success looks like . . .

- **Language Goal:** Explaining (using multiple representations) how the Distributive and commutative properties apply to expressions with negative coefficients. **(Speaking and Listening, Writing)**
- **Goal:** Identifying whether expressions are equivalent, including rewriting subtraction as adding the opposite.
  - » Selecting all equivalent expression to $4 - x$ in Problem 1.

Suggested next steps

If students have errors in Problem 1, consider:

- Revisiting appropriate lessons from Unit 5 where students worked with adding and subtracting rational numbers. Help students understand that substituting a variable in place of a number does not change the properties of the operation.
- Reminding students that $-x$ can be rewritten as $-1x$. Encourage them to rewrite each expression adding the coefficient to each x.

If students have errors in Problem 2, consider:

- Giving them opportunities to continue their work with factoring and expanding expressions using the Distributive Property in Lesson 20.
- Assigning Practice Problem 2.

Professional Learning

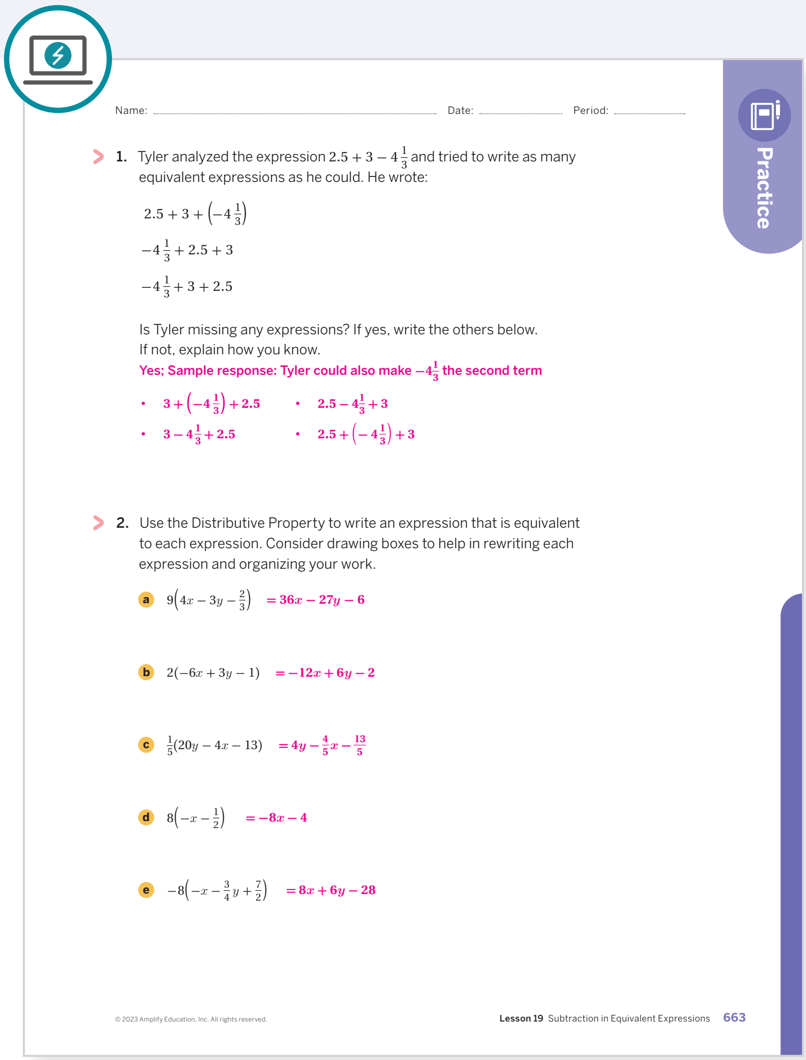
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did you see in the way some students approached Activity 1 that you would like other students to try?
- Knowing where students need to be by the end of this unit, how did using area models to simplify expressions influence that future goal? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Tyler analyzed the expression $2.5 + 3 - 4\frac{1}{3}$ and tried to write as many equivalent expressions as he could. He wrote:

$2.5 + 3 + \left(-4\frac{1}{3}\right)$

$-4\frac{1}{3} + 2.5 + 3$

$-4\frac{1}{3} + 3 + 2.5$

Is Tyler missing any expressions? If yes, write the others below. If not, explain how you know.

Yes; Sample response: Tyler could also make $-4\frac{1}{3}$ the second term

- $3 + \left(-4\frac{1}{3}\right) + 2.5$
- $2.5 - 4\frac{1}{3} + 3$
- $3 - 4\frac{1}{3} + 2.5$
- $2.5 + \left(-4\frac{1}{3}\right) + 3$

2. Use the Distributive Property to write an expression that is equivalent to each expression. Consider drawing boxes to help in rewriting each expression and organizing your work.

a $9\left(4x - 3y - \frac{2}{3}\right)$ $= 36x - 27y - 6$

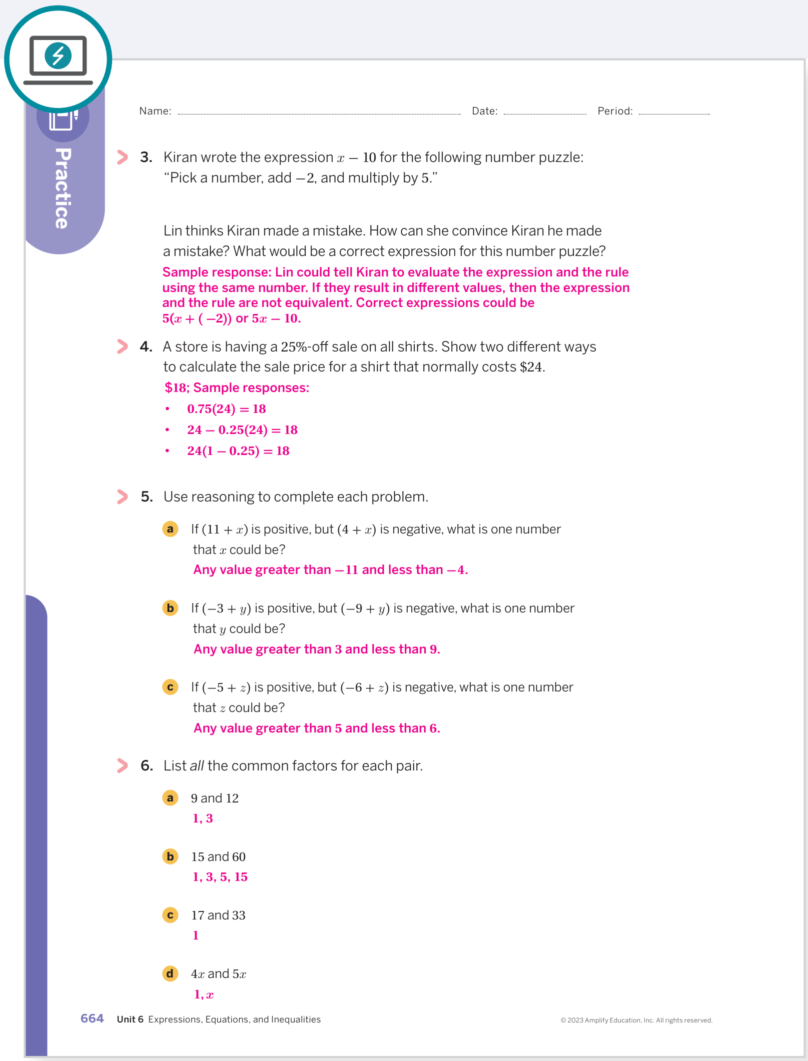
b $2(-6x + 3y - 1)$ $= -12x + 6y - 2$

c $\frac{1}{5}(20y - 4x - 13)$ $= 4y - \frac{4}{5}x - \frac{13}{5}$

d $8\left(-x - \frac{1}{2}\right)$ $= -8x - 4$

e $-8\left(-x - \frac{3}{4}y + \frac{7}{2}\right)$ $= 8x + 6y - 28$

Lesson 19 Subtraction in Equivalent Expressions 663



Name: Date: Period:

3. Kiran wrote the expression $x - 10$ for the following number puzzle: "Pick a number, add -2, and multiply by 5."

Lin thinks Kiran made a mistake. How can she convince Kiran he made a mistake? What would be a correct expression for this number puzzle?

Sample response: Lin could tell Kiran to evaluate the expression and the rule using the same number. If they result in different values, then the expression and the rule are not equivalent. Correct expressions could be $5(x + (-2))$ or $5x - 10$.

4. A store is having a 25%-off sale on all shirts. Show two different ways to calculate the sale price for a shirt that normally costs \$24.

\$18; Sample responses:

- $0.75(24) = 18$
- $24 - 0.25(24) = 18$
- $24(1 - 0.25) = 18$

5. Use reasoning to complete each problem.

a If $(11 + x)$ is positive, but $(4 + x)$ is negative, what is one number that x could be?

Any value greater than -11 and less than -4.

b If $(-3 + y)$ is positive, but $(-9 + y)$ is negative, what is one number that y could be?

Any value greater than 3 and less than 9.

c If $(-5 + z)$ is positive, but $(-6 + z)$ is negative, what is one number that z could be?

Any value greater than 5 and less than 6.

6. List *all* the common factors for each pair.

a 9 and 12

1, 3

b 15 and 60

1, 3, 5, 15

c 17 and 33

1

d $4x$ and $5x$

$1, x$

664 **Unit 6** Expressions, Equations, and Inequalities

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 3	1
	3	Activity 3	2
Spiral	4	Unit 4 Lesson 3	2
	5	Unit 6 Lesson 13	2
Formative ⚡	6	Unit 6 Lesson 20	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Expanding and Factoring

Let's use the Distributive Property to write expressions in different ways.

Focus

Goals

1. **Language Goal:** Comprehend the terms *expand* and *factor* in relation to the Distributive Property. **(Speaking and Listening)**
2. **Language Goal:** Apply the Distributive Property to expand or factor an expression that includes negative coefficients and explain (using multiple representations) the reasoning. **(Speaking and Listening)**

Coherence

Today

Students use the Distributive Property to expand and factor expressions that include subtraction and negative values.

Previously

In Lesson 19, students wrote equivalent expressions with subtraction using the Distributive and commutative properties.

Coming Soon

In Lessons 21 and 22, students will use properties, including the Distributive Property, to write equivalent expressions with fewer terms.

Rigor

- Students use area models to build their **conceptual understanding** of expanding and factoring expressions.
- Students **apply** their understanding of negative values and the Distributive Property to expand and factor expressions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair

Math Language Development

New words

- *expand*
- *factor*

Review words

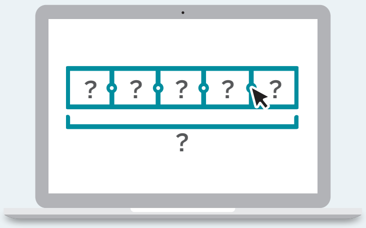
- *common factor*
- *Distributive Property*
- *equivalent expressions*
- *terms*

Amps **Featured Activity**

Activities 1 and 2

Dynamic Area Models

Students can create digital area models to represent the Distributive Property.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Much precision must be taken by students as they write equivalent expressions, and some students might make careless mistakes throughout the process. Remind students that, when working with a partner they have an obligation to help the other person learn from their mistakes. Have students identify approaches that they think will result in effective communication when someone needs to pay more attention.

Modifications to Pacing

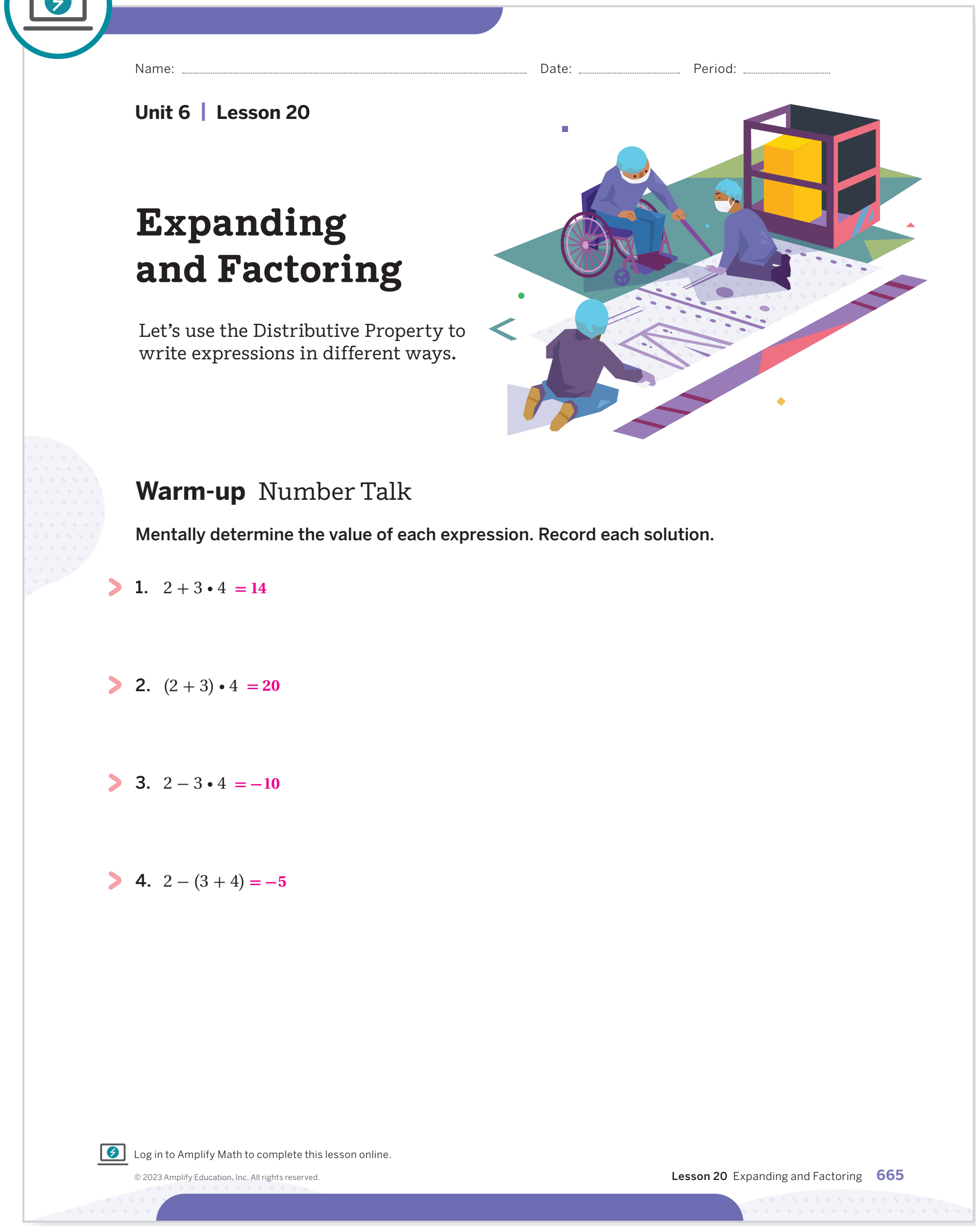
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up**.
- In **Activity 2**, have students complete the card sort and discuss their matches orally.
- In **Activity 3**, may be omitted. Consider assigning the Activity as Additional Practice.

Independent | 5 min

Warm-up Number Talk

Students mentally evaluate expressions in order to review the order of operations and see how the position of parentheses may change the value of the expression.



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Unit 6 | Lesson 20

Expanding and Factoring

Let's use the Distributive Property to write expressions in different ways.

Warm-up Number Talk

Mentally determine the value of each expression. Record each solution.

1. $2 + 3 \cdot 4$ $= 14$
2. $(2 + 3) \cdot 4$ $= 20$
3. $2 - 3 \cdot 4$ $= -10$
4. $2 - (3 + 4)$ $= -5$

Log in to Amplify Math to complete this lesson online.

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Lesson 20 Expanding and Factoring 665

1 Launch

Conduct the *Number Talk* routine. Instruct students to consider one problem at a time. Give students 30 seconds for each problem and ask them to give a signal when they have an answer and a strategy.

2 Monitor

Look for points of confusion:

- **Evaluating the expressions from left to right instead of following the order of operations.** Review the order of operations and remind students that operations are not necessarily performed in order from left to right.

3 Connect

Display the expressions.

Have students share their strategies for evaluating each expression.

Highlight that a number immediately preceding or following a set of parentheses signals multiplication. Discuss how the expressions could be rewritten using the Distributive Property. Bring attention to the effect of a negative on the outside of parenthesis, explaining that $2 - (3 + 4)$ can also be written as $2 - 3 - 4$.

Ask:

- "Who evaluated the expressions a different way?"
- "Do you agree or disagree with ___'s strategy? What are your reasons?"
- "Who can restate ___'s reasoning in a different way?"

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share their responses and strategies for evaluating each expression, press them for detail in their reasoning by asking these probing questions:

- "Why is the value of Problem 2 greater than the value of Problem 1, when they use the same numbers? What do the parentheses indicate?"
- "In Problems 1 and 3, there are no parentheses. How did you know which operation to do first?"

Power-up

To power up students' ability to determine common factors between two values, have students complete:

Recall that a *factor* is a number that divides evenly into a given whole number. A *common factor* is a factor that two or more numbers share. For example 4 is a common factor of 12 and 16. Determine common factors of each pair of values:

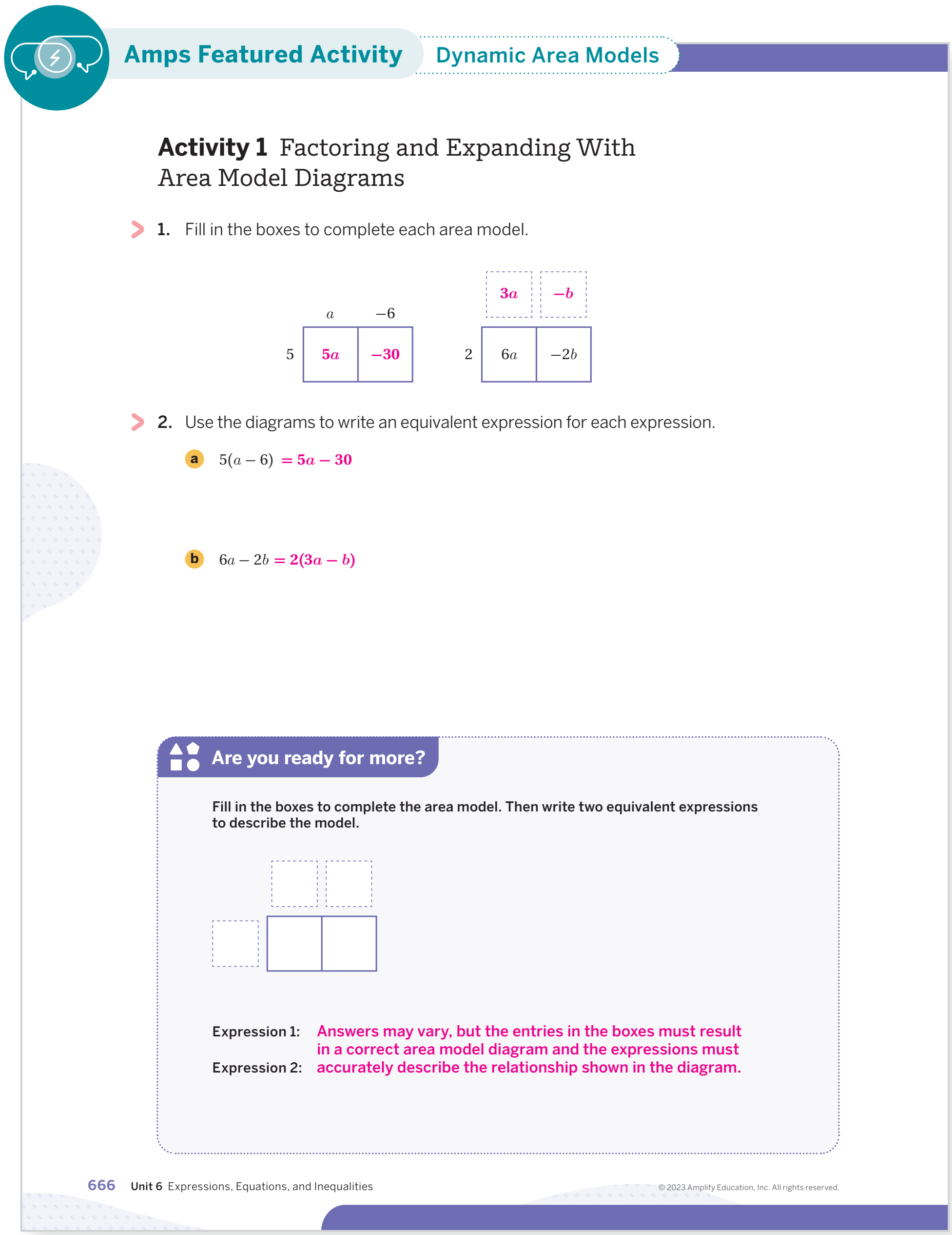
1. 8 and 20 1, 2, 4
2. 12 and 36 1, 2, 3, 4, 6, 12
3. $4y$ and $6y$ $1, 2, y, 2y$

Use: Before Activity 1

Informed by: Performance on Lesson 19, Practice Problem 6

Activity 1 Factoring and Expanding With Area Model Diagrams

Students fill in the missing labels on area model diagrams and use them to write equivalent expressions.



Amps Featured Activity Dynamic Area Models

Activity 1 Factoring and Expanding With Area Model Diagrams

1. Fill in the boxes to complete each area model.

	a	-6
5	$5a$	-30

	$3a$	$-b$
2	$6a$	$-2b$

2. Use the diagrams to write an equivalent expression for each expression.

a. $5(a-6)$ $= 5a - 30$

b. $6a-2b$ $= 2(3a-b)$

Are you ready for more?

Fill in the boxes to complete the area model. Then write two equivalent expressions to describe the model.

Expression 1:

Expression 2:

Answers may vary, but the entries in the boxes must result in a correct area model diagram and the expressions must accurately describe the relationship shown in the diagram.

666 Unit 6 Expressions, Equations, and Inequalities

1 Launch

Display a completed area model diagram modeling the relationship between the expressions $-3(5-2y)$ and $-15+6y$. Discuss how the diagram demonstrates the Distributive Property and shows the expressions are equivalent.

2 Monitor

Help students get started by asking "How do you determine the area of a rectangle? What do you need to multiply 2 by to get $6a$?"

Look for points of confusion:

- **Not making the connection between subtraction and adding the opposite.** Ask, "If you add -6, how can you write that as subtraction?"

3 Connect

Have students share how they completed each diagram and how they used the diagrams to write equivalent expressions.

Define the term ***expand*** as using the Distributive Property to rewrite a product as a sum and the term ***factor*** as using the Distributive Property to rewrite a sum as a product.

Highlight that when *expanding* and *factoring*, the original and resulting expressions are equivalent.

Ask:

- "How does the diagram model the Distributive Property?"
- "Which expressions are in factored form and which are in expanded form?"
- "Where do you see the factored and expanded forms in the area model diagram?"
- "Why is 2 the number outside the box on the second diagram? Could you have 3 or $2a$ there instead?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital area models to represent the Distributive Property.

Extension: *Math Enrichment*

Have students complete the following problems:

- Write the expression $\frac{3}{4}\left(8x-2y+\frac{1}{3}z\right)$ in expanded form. $6x-\frac{3}{2}y+\frac{1}{4}z$
- Write the expression $\frac{3}{5}mn-\frac{2}{5}mp+\frac{1}{5}m$ in factored form. $\frac{1}{5}m(3n-2p+1)$

Math Language Development

MLR8: Discussion Supports—Press for Details

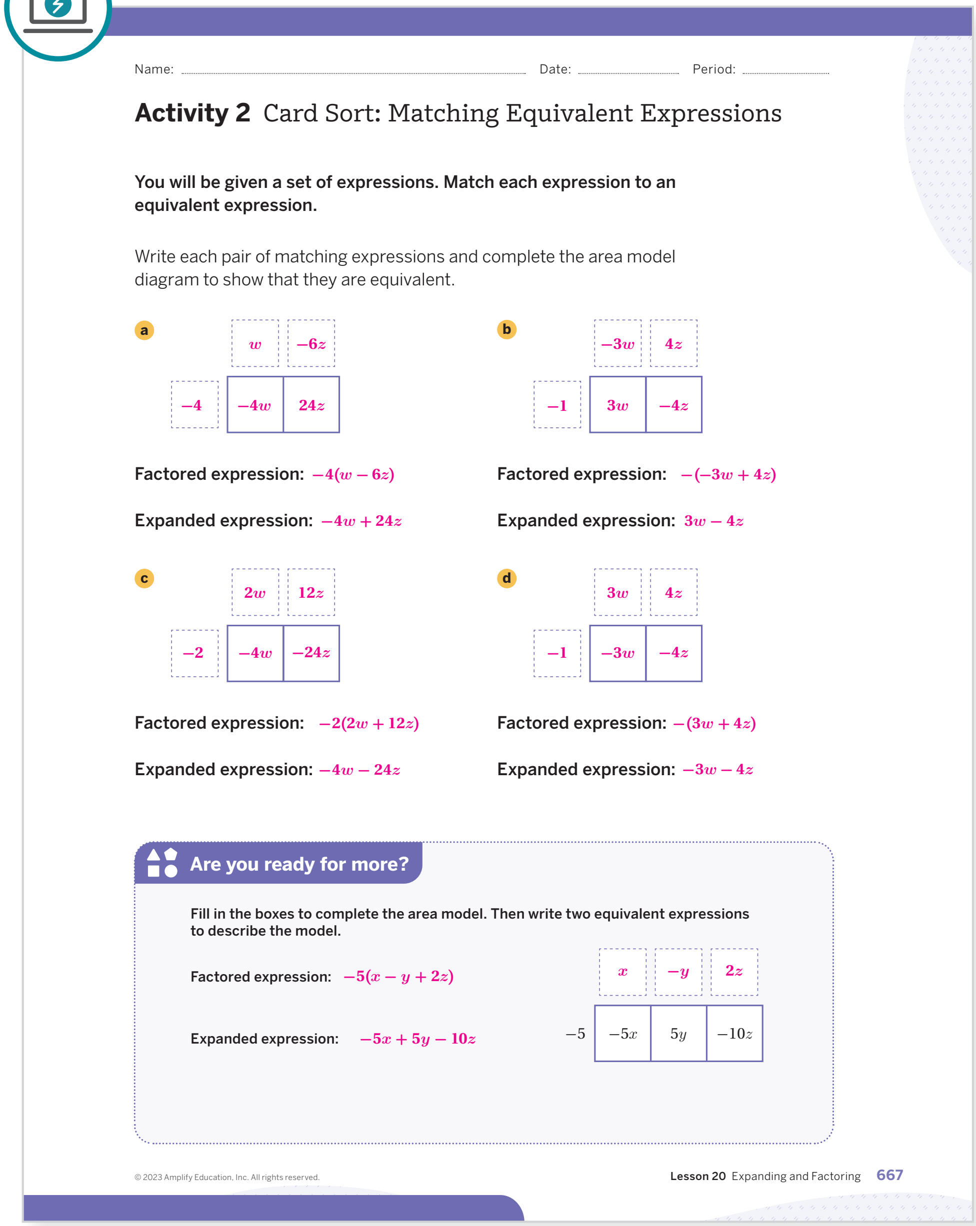
During the Connect, as you define the terms *expand* and *factor*, and as students respond to the Ask questions, press for more detail in their reasoning. For example:

If a student says . . .	Press for detail by asking . . .
"The expression $5(a-6)$ is in factored form."	"Why is the expression $5(a-6)$ in factored form? What does it mean for an expression to be in factored form?"
"The expression $6a-2b$ is in expanded form."	"Why is the expression $6a-2b$ in expanded form? What does it mean for an expression to be in expanded form?"

Pairs | 10 min

Activity 2 Card Sort: Matching Equivalent Expressions

Students match equivalent expressions in factored and expanded form to prepare for writing their own equivalent expressions.



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Activity 2 Card Sort: Matching Equivalent Expressions

You will be given a set of expressions. Match each expression to an equivalent expression.

Write each pair of matching expressions and complete the area model diagram to show that they are equivalent.

a

	w	$-6z$
-4	$-4w$	$24z$

Factored expression: $-4(w - 6z)$

Expanded expression: $-4w + 24z$

b

	$-3w$	$4z$
-1	$3w$	$-4z$

Factored expression: $-(-3w + 4z)$

Expanded expression: $3w - 4z$

c

	$2w$	$12z$
-2	$-4w$	$-24z$

Factored expression: $-2(2w + 12z)$

Expanded expression: $-4w - 24z$

d

	$3w$	$4z$
-1	$-3w$	$-4z$

Factored expression: $-(3w + 4z)$

Expanded expression: $-3w - 4z$

Are you ready for more?

Fill in the boxes to complete the area model. Then write two equivalent expressions to describe the model.

Factored expression: $-5(x - y + 2z)$

Expanded expression: $-5x + 5y - 10z$

	x	$-y$	$2z$
-5	$-5x$	$5y$	$-10z$

Lesson 20 Expanding and Factoring 667

1 Launch

Distribute one set of cards to each pair from the Activity 2 PDF. Conduct the ***Card Sort*** routine. Encourage students to use the area model diagrams to check their matches.

2 Monitor

Help students get started by asking, "How can you represent the expressions in an area model?"

Look for points of confusion:

- **Matching expressions based on coefficients and ignoring signs.** Have students use the area models to prove their matches are correct.
- **Being confused by a negative sign immediately preceding parentheses (e.g., $-(3w + 4z)$).** Explain that the expression is the same as $-1(3w + 4z)$.

3 Connect

Have students share the matches they found and conduct the ***Poll the Class*** routine to reach consensus. Students can show an area model for any disputed matches. Have students identify which expression in each pair is *factored* and which is *expanded.*

Highlight that the expressions in each pair are equivalent. To factor expressions, divide each term by their common factor. Look at each pair of expressions and see if there are any other common factors of the terms. Discuss the other ways the expanded expression could be factored, e.g., 4 is the greatest common factor of the terms in $-4w - 24z$, so $-4(w + 6z)$ is also equivalent. Note that when factoring, it is more efficient to factor out the greatest common factor.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital area models to represent the Distributive Property.

Accessibility: *Guide Processing and Visualization*

Provide students with an example of an area model diagram and corresponding equivalent expressions labeled expanded and factored. For example, provide students with an area model diagram and these corresponding expressions.
Expanded: $-18x + 12y$
Factored: $-6(3x - 2y)$

Math Language Development

MLR7: Compare and Connect

During the Connect, display Cards 2, 4, 7, and 8. Ask students to respond to these questions to encourage the use of their developing vocabulary around factored and expanded form.

- "For which expanded form expression(s) could you factor out a negative sign? Why?" The expression on Card 4, because both terms contain a negative sign.
- "For which factored form expression(s) would result in both terms being negative when written in expanded form? Why?" The expression on Card 7, because the negative sign will be distributed to both positive terms on the inside of the parentheses, making them both now negative.

Pairs | 10 min

Activity 3 Factor and Expand

Students complete a table by writing equivalent expressions to solidify the relationship between factoring and expanding.

Activity 3 Factor and Expand

Complete the table by writing an equivalent expression in each row.

Factored	Expanded
$6(2x - 3a)$	$12x - 18a$
$-2(4a - 5b)$	$-8a + 10b$
$a(10 - 13)$	$10a - 13a$
$4b(a - 1)$	$4ab - 4b$
$\frac{2}{3}(-6a - x)$	$-4a - \frac{2}{3}x$
$-(x - 2b)$	$-x + 2b$

Note: Students are not required to completely factor the expressions. Partially factored expressions are acceptable as long as they are equivalent. For example, in Row 4, $4(ab - b)$ and $b(4a - 4)$ are also valid responses.

Are you ready for more?

Complete the table by writing an equivalent expression in each row.

Factored	Expanded
$b(a - c - 3d)$	$ab - bc - 3bd$
$5(4x - 2a - 3)$	$20x - 10a - 15$

STOP

668 Unit 6 Expressions, Equations, and Inequalities

1 Launch

Conduct the ***Think-Pair-Share*** routine. Have students complete each row independently, and then compare results with a partner.

2 Monitor

Help students get started by suggesting those who are struggling to sketch an area model diagram.

Look for points of confusion:

- **Struggling with factoring the expression $10a - 13a$.** Explain how a variable appearing in both terms can be factored in the same way as a common numerical factor.

Look for productive strategies:

- Drawing an area model diagram for each problem. Note students who use this strategy.
- Factoring a quantity other than the greatest common factor out of the first and fourth rows. Suggest students ask themselves if they have factored out all the common factors.

3 Connect

Display the completed table.

Have students share how they found the equivalent expression in each row.

Highlight the first and fourth rows and note that while factoring out any common factor will result in an equivalent expression, it is more efficient to factor out as much as possible at once. Compare the expressions $7x - 7a$ and $10a - 13a$. Explain that the terms in $7x - 7a$ have the same coefficient (except for the sign) and the terms in $10a - 13a$ have the same variable part. Discuss how $a(10 - 13)$ can be written as $-3a$ and why this isn't the case with $7(x - a)$.

Ask, "What processes can you use to factor an expanded expression, and to expand a factored expression?"

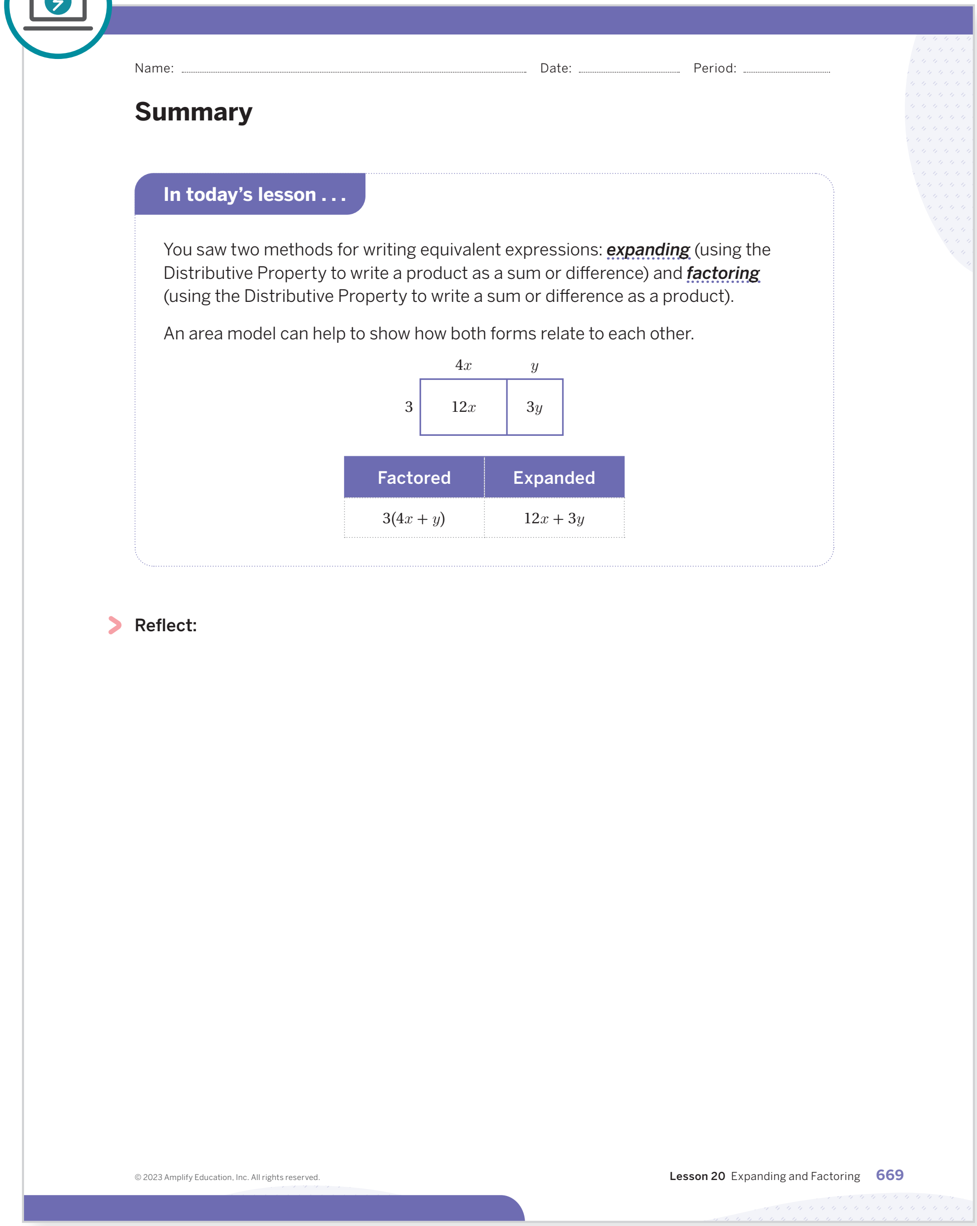
Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

If students need more processing time, allow them to choose three of the six rows to complete and provide them with a template for drawing an area model. Allowing them to choose which rows to complete can increase engagement and ownership of the task.

Summary

Review and synthesize how to use the Distributive Property to factor and expand expressions.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw two methods for writing equivalent expressions: ***expanding*** (using the Distributive Property to write a product as a sum or difference) and ***factoring*** (using the Distributive Property to write a sum or difference as a product).

An area model can help to show how both forms relate to each other.

	$4x$	y
3	$12x$	$3y$

Factored	Expanded
$3(4x + y)$	$12x + 3y$

Reflect:

Lesson 20 Expanding and Factoring 669

Synthesize

Display the expression $12x - 8$ and a partially-completed area model with the expanded expression entered.

Highlight how to factor and expand expressions. Mentally solve the problem aloud while demonstrating how to complete the area model and write the factored expression. Say, "You have seen how to use the Distributive Property to expand an expression. Now you can also use the Distributive Property backwards to factor an expression."

Formalize vocabulary:

- *expand*
- *factor*

Ask:

- "Is $12x - 8$ in factored or expanded form?"
- "What factor do $12x$ and -8 have in common?"
- "4 times what is $12x$?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the *Essential Questions* for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which representations best help you to make sense of certain mathematical scenarios?"

Math Language Development

MLR2: Collect and Display

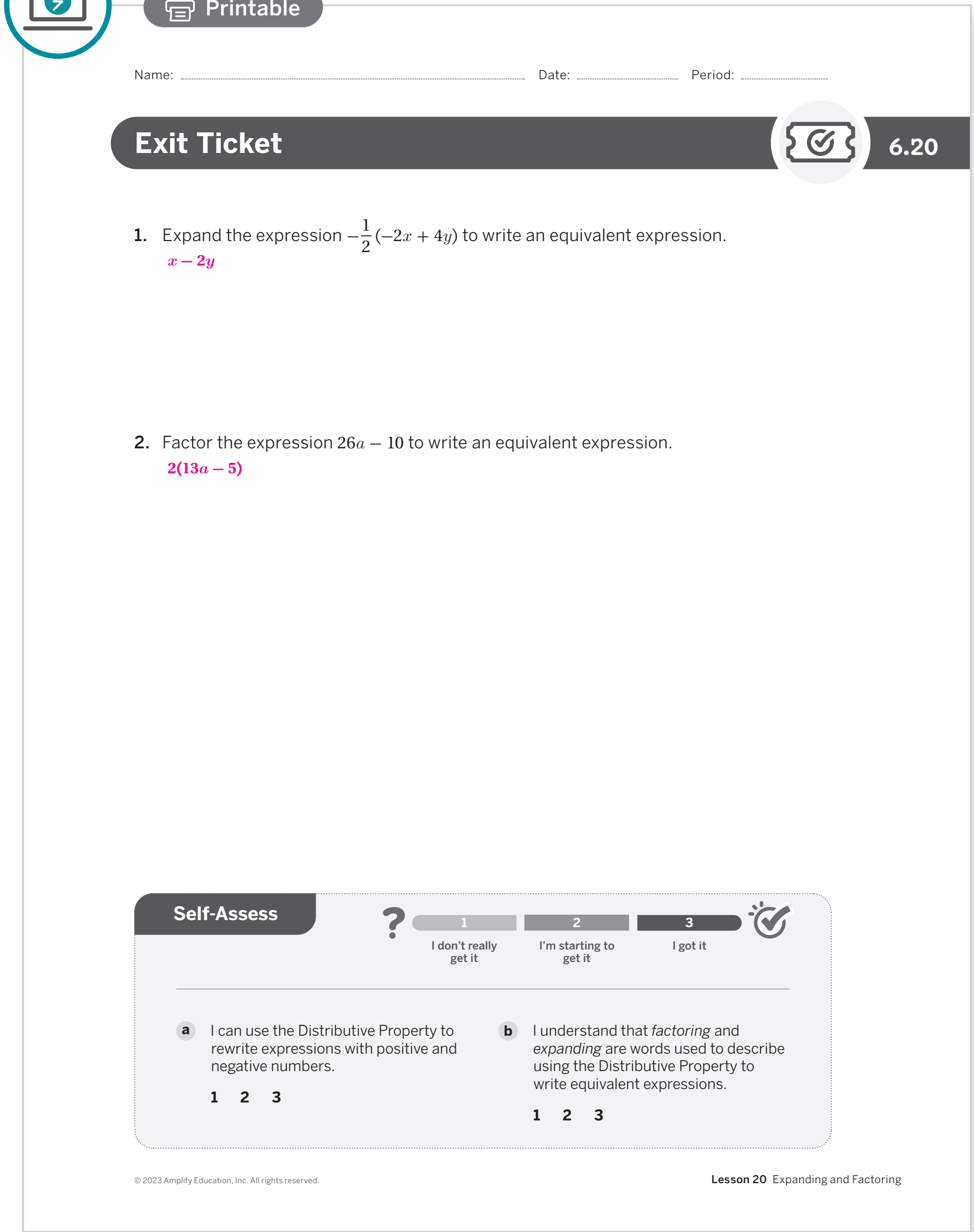
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *expand* and *factor* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of factoring and expanding by writing equivalent expressions.

Printable



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Exit Ticket 6.20

1. Expand the expression $-\frac{1}{2}(-2x + 4y)$ to write an equivalent expression.
   $x - 2y$

2. Factor the expression $26a - 10$ to write an equivalent expression.
   $\mathbf{2(13a - 5)}$

Self-Assess

? **1** I don't really get it | **2** I'm starting to get it | **3** I got it

a I can use the Distributive Property to rewrite expressions with positive and negative numbers.
1 2 3

b I understand that *factoring* and *expanding* are words used to describe using the Distributive Property to write equivalent expressions.
1 2 3

Success looks like . . .

- **Language Goal:** Comprehending the terms *expand* and *factor* in relation to the Distributive Property. **(Speaking and Listening, Writing)**
- **Language Goal:** Applying the Distributive Property to expand or factor an expression that includes negative coefficients and explaining (using multiple representations) the reasoning. **(Speaking and Listening)**
  - » Expanding or factoring an expression using the Distributive Property in Problems 1 and 2.

Suggested next steps

If students arrive at an incorrect answer with a missing or incorrect diagram, consider:

- Reviewing how to draw a diagram for each problem.
- Assigning Practice Problem 1, reviewing the solution, and then having students draw diagrams to complete Practice Problem 2.

If students arrive at an incorrect answer with a correct diagram, consider:

- Reviewing how to use the diagram to write the equivalent expressions.
- Assigning Practice Problem 1, reviewing the solution, and then having students complete Practice Problem 3, part b.

Professional Learning

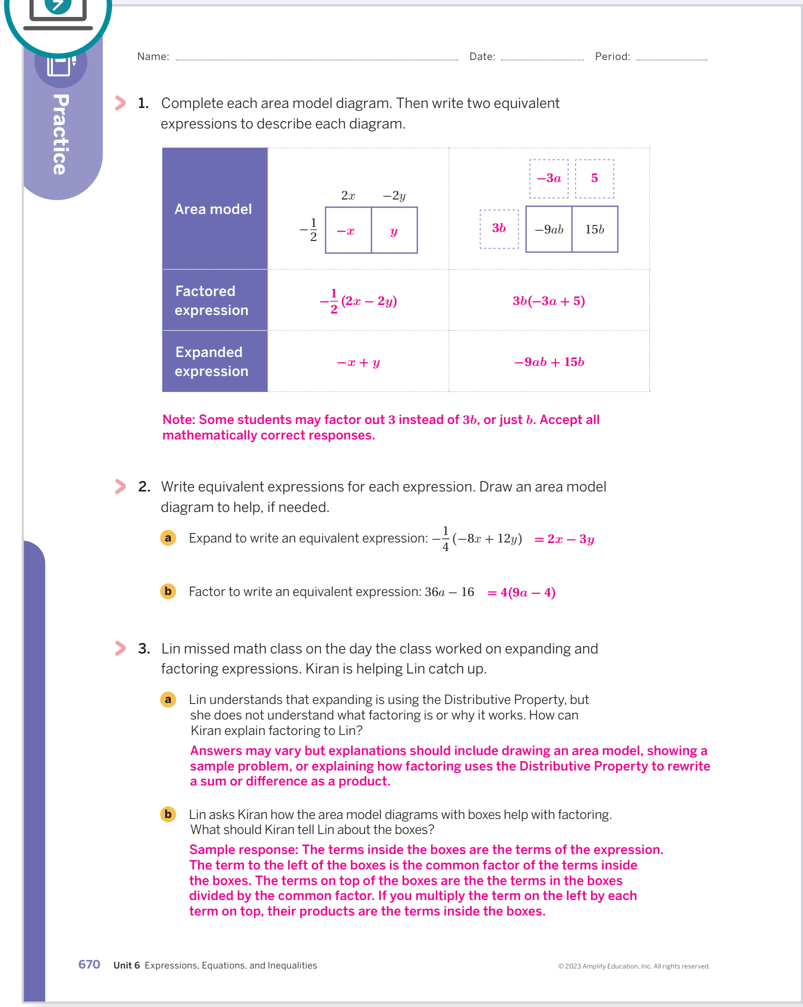
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- The instructional goal for this lesson was for students to comprehend the terms *expand* and *factor* in relation to the Distributive Property. How well did students accomplish this? What did you specifically do to help students accomplish it? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Complete each area model diagram. Then write two equivalent expressions to describe each diagram.

Area model	$2x$ $-2y$ $-\frac{1}{2}$ | $-x$ | y	$-3a$ 5 $3b$ | $-9ab$ | $15b$
Factored expression	$-\frac{1}{2}(2x-2y)$	$3b(-3a+5)$
Expanded expression	$-x+y$	$-9ab+15b$

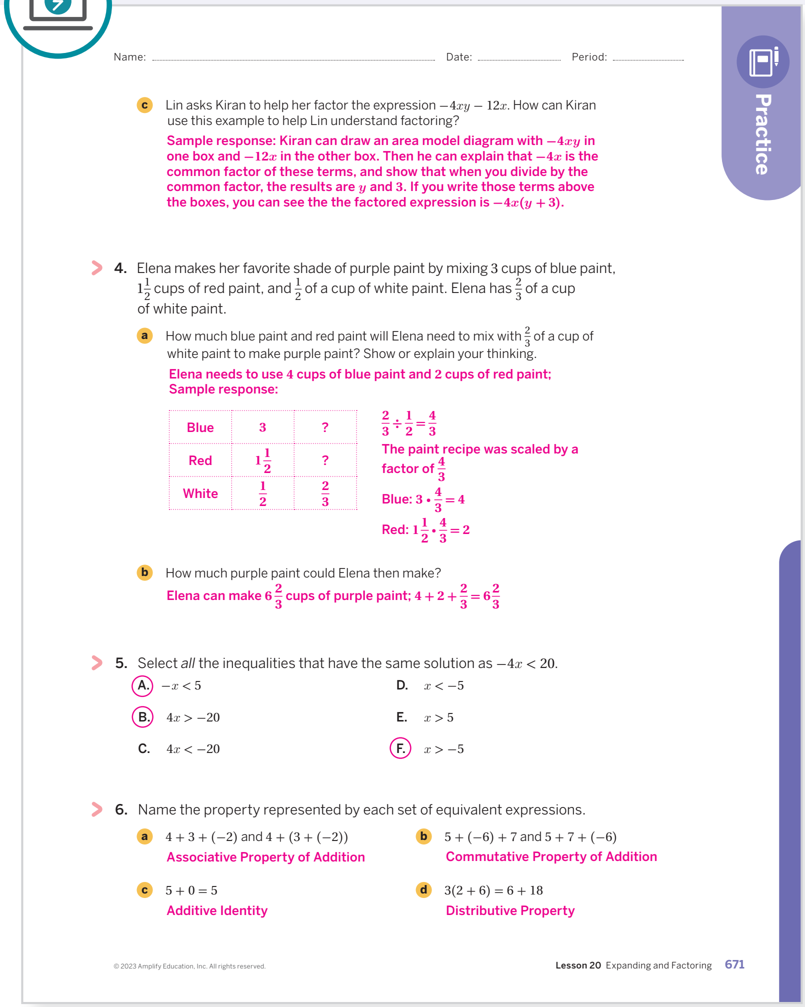
Note: Some students may factor out 3 instead of $3b$, or just b. Accept all mathematically correct responses.

2. Write equivalent expressions for each expression. Draw an area model diagram to help, if needed.
   - a Expand to write an equivalent expression: $-\frac{1}{4}(-8x+12y)$ $= 2x - 3y$
   - b Factor to write an equivalent expression: $36a - 16$ $= 4(9a-4)$

3. Lin missed math class on the day the class worked on expanding and factoring expressions. Kiran is helping Lin catch up.
   - a Lin understands that expanding is using the Distributive Property, but she does not understand what factoring is or why it works. How can Kiran explain factoring to Lin?

     Answers may vary but explanations should include drawing an area model, showing a sample problem, or explaining how factoring uses the Distributive Property to rewrite a sum or difference as a product.
   - b Lin asks Kiran how the area model diagrams with boxes help with factoring. What should Kiran tell Lin about the boxes?

     Sample response: The terms inside the boxes are the terms of the expression. The term to the left of the boxes is the common factor of the terms inside the boxes. The terms on top of the boxes are the the terms in the boxes divided by the common factor. If you multiply the term on the left by each term on top, their products are the terms inside the boxes.



Name: ________ Date: ________ Period: ________

   - c Lin asks Kiran to help her factor the expression $-4xy - 12x$. How can Kiran use this example to help Lin understand factoring?

     Sample response: Kiran can draw an area model diagram with $-4xy$ in one box and $-12x$ in the other box. Then he can explain that $-4x$ is the common factor of these terms, and show that when you divide by the common factor, the results are y and 3. If you write those terms above the boxes, you can see the the factored expression is $-4x(y+3)$.

4. Elena makes her favorite shade of purple paint by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ of a cup of white paint. Elena has $\frac{2}{3}$ of a cup of white paint.
   - a How much blue paint and red paint will Elena need to mix with $\frac{2}{3}$ of a cup of white paint to make purple paint? Show or explain your thinking.

     Elena needs to use 4 cups of blue paint and 2 cups of red paint; Sample response:

Blue	3	?
Red	$1\frac{1}{2}$	?
White	$\frac{1}{2}$	$\frac{2}{3}$

$\frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$

The paint recipe was scaled by a factor of $\frac{4}{3}$

Blue: $3 \cdot \frac{4}{3} = 4$

Red: $1\frac{1}{2} \cdot \frac{4}{3} = 2$

   - b How much purple paint could Elena then make?

     Elena can make $6\frac{2}{3}$ cups of purple paint; $4 + 2 + \frac{2}{3} = 6\frac{2}{3}$

5. Select *all* the inequalities that have the same solution as $-4x < 20$.

[✓ A.] $-x < 5$

[✓ B.] $4x > -20$

C. $4x < -20$

D. $x < -5$

E. $x > 5$

[✓ F.] $x > -5$

6. Name the property represented by each set of equivalent expressions.
   - a $4 + 3 + (-2)$ and $4 + (3 + (-2))$

     Associative Property of Addition
   - b $5 + (-6) + 7$ and $5 + 7 + (-6)$

     Commutative Property of Addition
   - c $5 + 0 = 5$

     Additive Identity
   - d $3(2+6) = 6 + 18$

     Distributive Property

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 3	2
	3	Activity 3	3
Spiral	4	Unit 2 Lesson 2	2
	5	Unit 6 Lesson 14	2
Formative ⚡	6	Unit 6 Lesson 21	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 21**

Combining Like Terms (Part 1)

Let's see how we can tell expressions are equivalent.

Focus

Goals

1. **Language Goal:** Apply properties of operations to justify that expressions are equivalent. **(Speaking and Listening, Writing)**
2. Generate an equivalent expression with fewer terms.
3. **Language Goal:** Interpret different methods for determining whether expressions are equivalent and evaluate their usefulness. **(Speaking and Listening)**

Coherence

Today

Students use the properties of operations they previously studied to understand how to properly write an equivalent expression using fewer terms. Students consider complicated expressions made of several parts.

Previously

In Lesson 20, students wrote equivalent expressions by expanding and factoring expressions.

Coming Soon

In Lesson 22, students will continue their work writing equivalent expressions using properties of operations.

Rigor

- Students build **conceptual understanding** of simplifying expressions with like terms.
- Students develop **procedural fluency** in combining like terms by determining the missing like term in equations.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New word

- *like terms*

Review words

- *associative property*
- *commutative property*
- *Distributive Property*
- *equivalent expressions*
- *identity property*
- *terms*

Amps | Featured Activity

Activity 1
Using Work From Previous Slides

Students can use their responses from previous slides to make a comparison between the values.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Seeing expressions that contain only variables and symbols might cause some students' stress levels to increase. Ask them to brainstorm ways to monitor and manage their stress levels. Explain that the very structure that mathematics provides should help alleviate some of their concerns. The same properties that applied to expressions with numbers apply to expressions with variables.

Modifications to Pacing

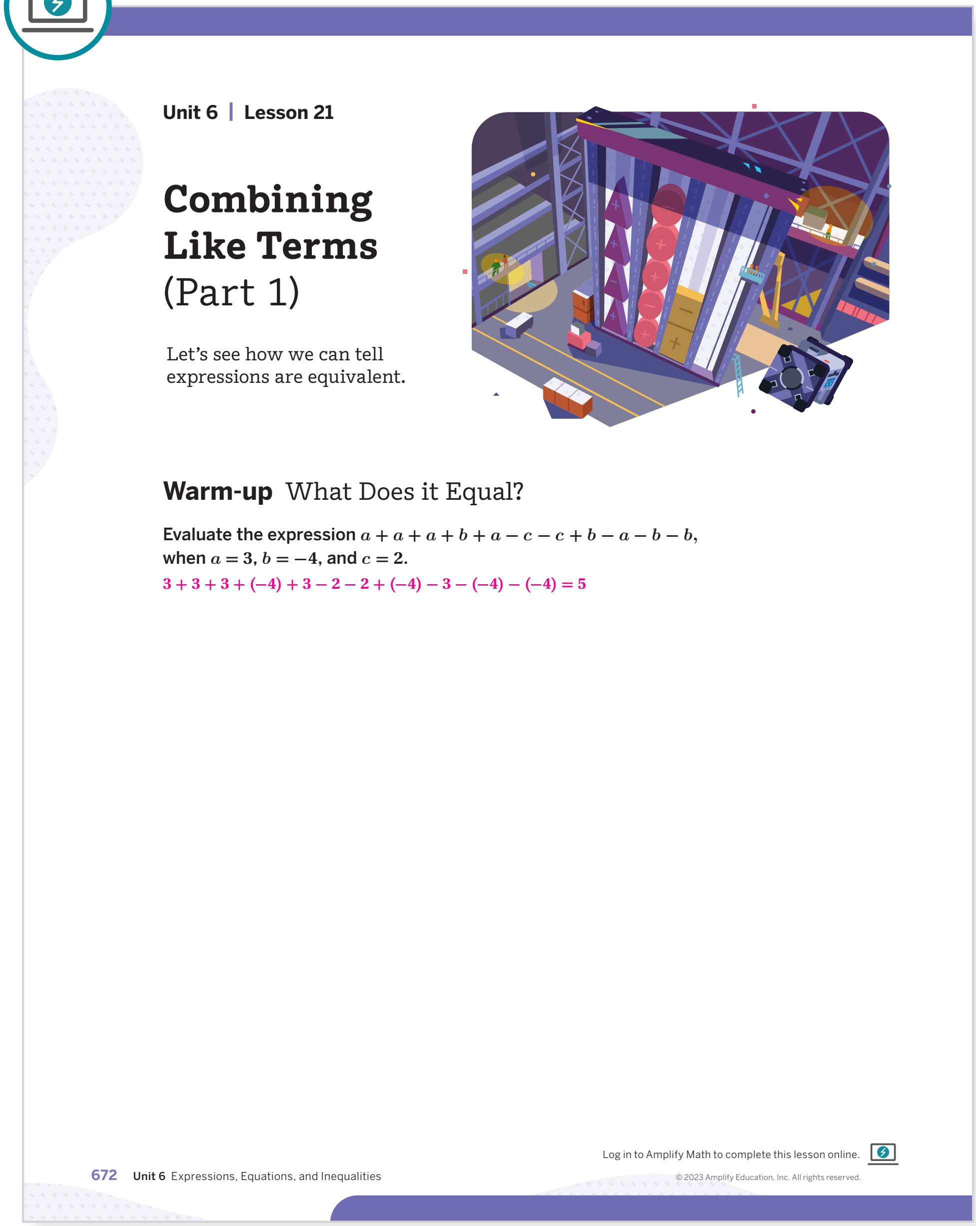
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students only complete either the even or the odd problems.

Pairs | 5 min

Warm-up What Does It Equal?

Students evaluate an expression that they will see later in Activity 1 to begin the discussion of equivalent expressions.



Unit 6 | Lesson 21

Combining Like Terms (Part 1)

Let's see how we can tell expressions are equivalent.

Warm-up What Does it Equal?

Evaluate the expression $a + a + a + b + a - c - c + b - a - b - b$, when $a = 3$, $b = -4$, and $c = 2$.

$3 + 3 + 3 + (-4) + 3 - 2 - 2 + (-4) - 3 - (-4) - (-4) = 5$

672 **Unit 6** Expressions, Equations, and Inequalities

Log in to Amplify Math to complete this lesson online.

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What does *evaluate* mean?" or "What can you replace a with?"

Look for points of confusion:

- **Making computational mistakes.** Remind students to be careful and use the properties of operations to make computations more efficient.

Look for productive strategies:

- Wanting to group like variables or numbers together to assist in the evaluating process. Make note of students who do this and reference their work when discussing Activity 1.

3 Connect

Display the expression.

Have students share strategies for evaluating the expression.

Highlight the correct use of precise, mathematical language and give students opportunities to revise their responses to be more precise. Review the properties of operations if any students use them during their computations. Explain that substituting numbers for the variables several times can be tedious and, in Activity 1, a more efficient method will be used.

Power-up

To power up students' ability to recognize properties of operations, have students complete:

Match each equation to the property it models.

a. $6 + x + 3 = x + 6 + 3$	c	Additive Identity
b. $6x + 12 = 6(x + 2)$	a	Commutative Property of Addition
c. $y + 0 = y$	b	Distributive Property
d. $(4 + x) + 2x = 4 + (x + 2x)$	a	Associative Property of Addition

Use: Before Activity 1

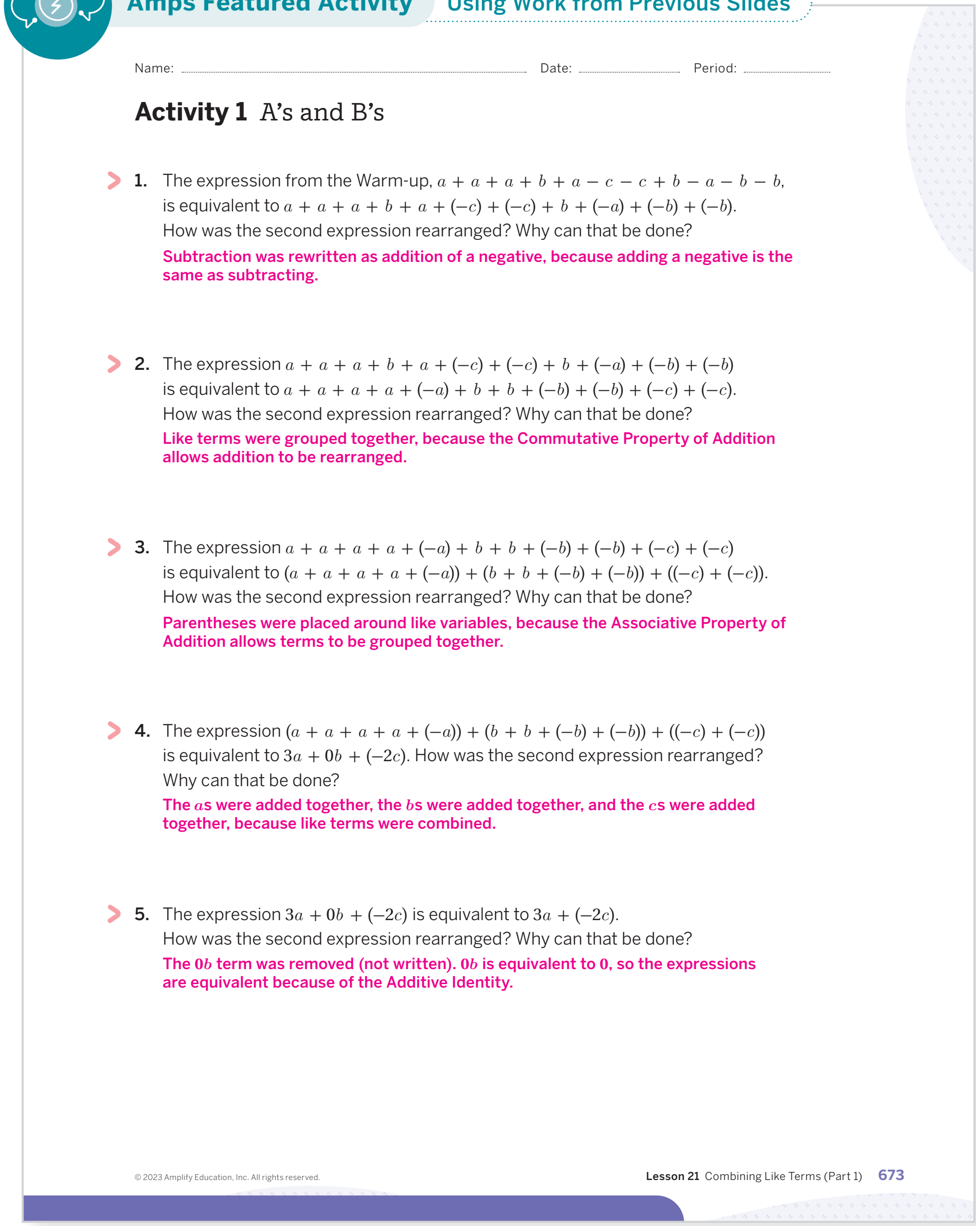
Informed by: Performance on Lesson 20, Practice Problem 6

Pairs | 15 min

Activity 1 A's and B's

Students compare two expressions to determine which property makes them equivalent. They also evaluate expressions and discuss combining like terms to simplify expressions.

Amps Featured Activity Using Work from Previous Slides



Name: .... Date: .... Period: ....

Activity 1 A's and B's

1. The expression from the Warm-up, $a + a + a + b + a - c - c + b - a - b - b$, is equivalent to $a + a + a + b + a + (-c) + (-c) + b + (-a) + (-b) + (-b)$. How was the second expression rearranged? Why can that be done?
   Subtraction was rewritten as addition of a negative, because adding a negative is the same as subtracting.

2. The expression $a + a + a + b + a + (-c) + (-c) + b + (-a) + (-b) + (-b)$ is equivalent to $a + a + a + a + (-a) + b + b + (-b) + (-b) + (-c) + (-c)$. How was the second expression rearranged? Why can that be done?
   Like terms were grouped together, because the Commutative Property of Addition allows addition to be rearranged.

3. The expression $a + a + a + a + (-a) + b + b + (-b) + (-b) + (-c) + (-c)$ is equivalent to $(a + a + a + a + (-a)) + (b + b + (-b) + (-b)) + ((-c) + (-c))$. How was the second expression rearranged? Why can that be done?
   Parentheses were placed around like variables, because the Associative Property of Addition allows terms to be grouped together.

4. The expression $(a + a + a + a + (-a)) + (b + b + (-b) + (-b)) + ((-c) + (-c))$ is equivalent to $3a + 0b + (-2c)$. How was the second expression rearranged? Why can that be done?
   The as were added together, the bs were added together, and the cs were added together, because like terms were combined.

5. The expression $3a + 0b + (-2c)$ is equivalent to $3a + (-2c)$. How was the second expression rearranged? Why can that be done?
   The $0b$ term was removed (not written). $0b$ is equivalent to 0, so the expressions are equivalent because of the Additive Identity.

1 Launch

Keep the Warm-up visible for students, as this expression is simplified during Activity 1. Display a list of the properties of operations with examples of each on the board for students to reference throughout the lesson.

2 Monitor

Help students get started by asking, "What changed between the two expressions?" and "Why can that change be made and still have equivalent expressions?"

Look for points of confusion:

- **Thinking the expressions are equivalent because they evaluated the expressions for $a = 3$, $b = -4$, and $c = 2$.** Remind students that equivalent expressions must be equal for *every* possible value of the variables.
- **Not remembering the formal names of the properties.** The goal is for students to understand the concept of equivalent expressions and explain the process from one expression to another. Encourage the general understanding of the properties by having students reference the example you posted on the board.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use their responses from previous slides to make a comparison between the values.

Accessibility: *Guide Processing and Visualization*

Distribute colored pencils and suggest that students color code the a terms in one color, the b terms in a second color, and the c terms in a third color to help them visualize how the terms in the expressions were rearranged.

Math Language Development

MLR8: Discussion Supports—Press for Details

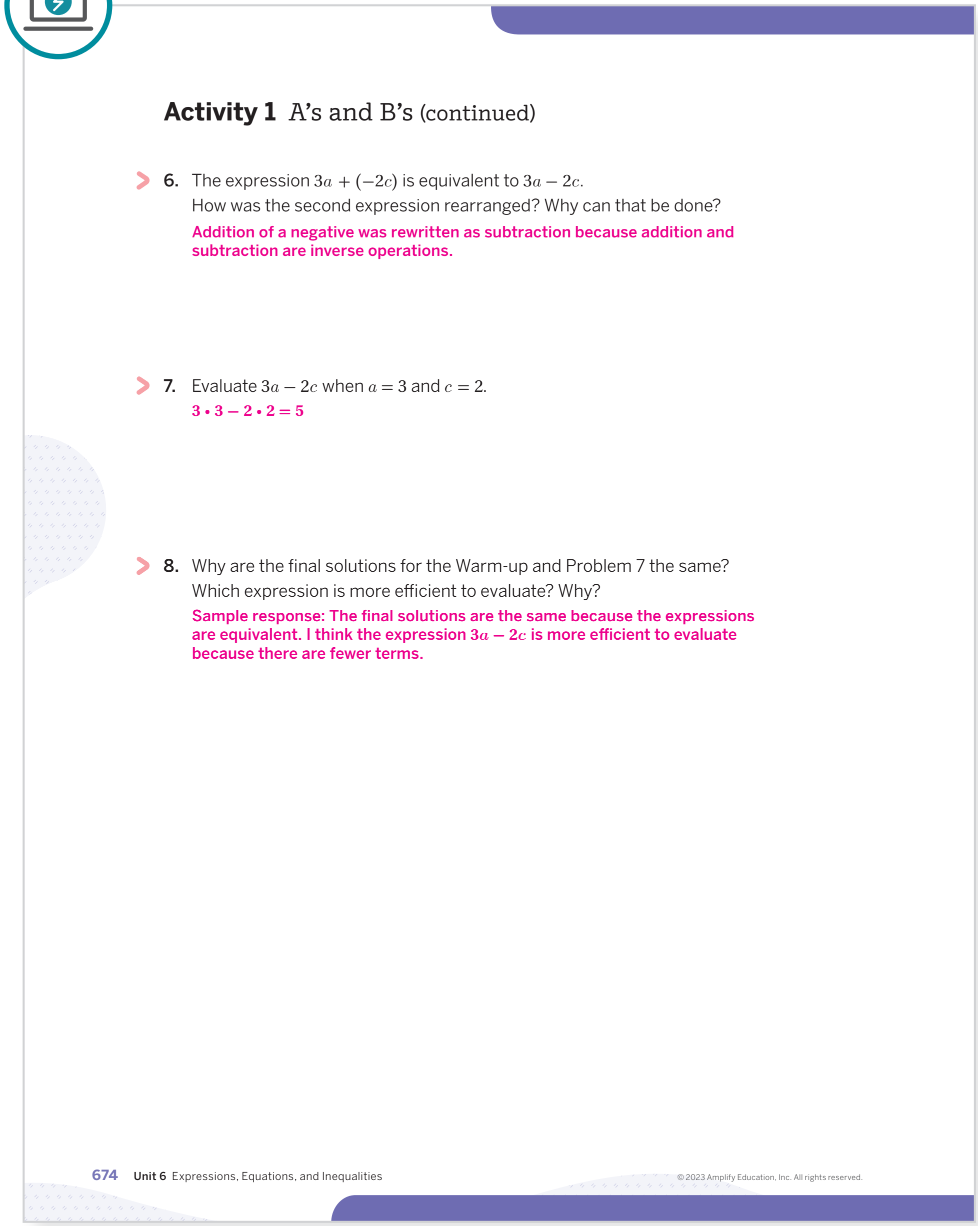
During the Connect, as students share their responses for why the expressions can be rearranged and still be equivalent, press for details in their reasoning. For example:

If a student says . . .	Press for detail by asking . . .
"The terms were rearranged so that all of the terms with the same letter are next to each other." (Problem 2)	"Why would this be beneficial to do? How does it help us further simplify the expression? What property allows us to move the terms around?"
"Term $0b$ is removed." (Problem 5)	"Why was this term removed? What property allows us to do so?"

Pairs | 15 min

Activity 1 A's and B's (continued)

Students compare two expressions to determine which property makes them equivalent. They also evaluate expressions and discuss combining like terms to simplify expressions.



Activity 1 A's and B's (continued)

6. The expression $3a + (-2c)$ is equivalent to $3a - 2c$.
   How was the second expression rearranged? Why can that be done?
   Addition of a negative was rewritten as subtraction because addition and subtraction are inverse operations.

7. Evaluate $3a - 2c$ when $a = 3$ and $c = 2$.
   $3 \cdot 3 - 2 \cdot 2 = 5$

8. Why are the final solutions for the Warm-up and Problem 7 the same? Which expression is more efficient to evaluate? Why?
   Sample response: The final solutions are the same because the expressions are equivalent. I think the expression $3a - 2c$ is more efficient to evaluate because there are fewer terms.

674 **Unit 6** Expressions, Equations, and Inequalities

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3 Connect

Have students share their reasoning of why each statement is true.

Highlight the the correct use of precise, mathematical language. Give students opportunities to revise their responses to be more precise. Review the properties of operations.

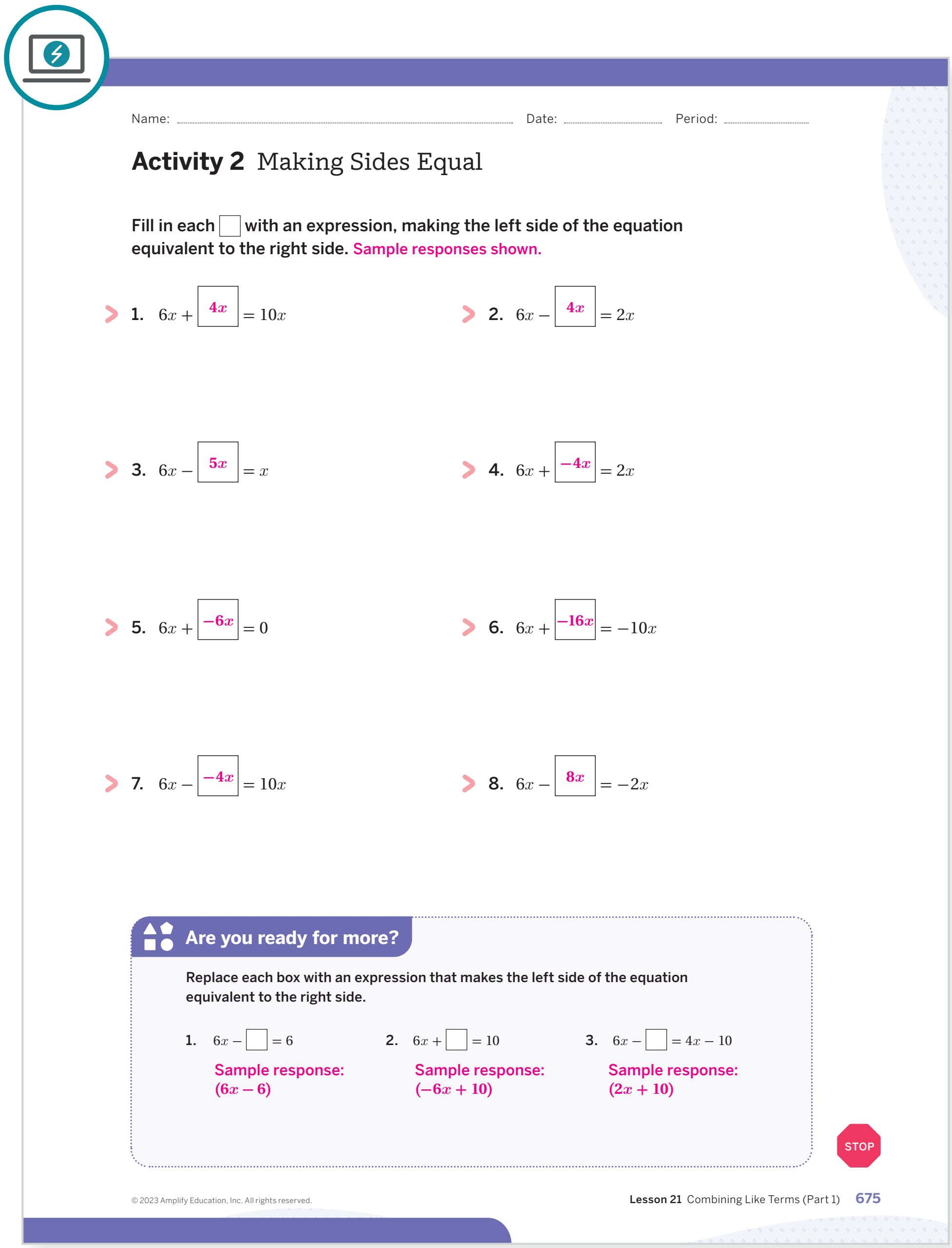
Ask:

- "Why are these expressions called *equivalent expressions*?" Sample response: I used the properties of operations to rewrite each expression so I know they are equal for all possible values of the variables.
- "The expression $3a - 2c$ is known as 'simplest form.' Why?" It is written with the fewest number of terms.

Pairs | 15 min

Activity 2 Making Sides Equal

Students determine which terms can be combined to make an expression with fewer terms in order to solidify the concepts of like terms and equivalent expressions.



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Activity 2 Making Sides Equal

Fill in each ☐ with an expression, making the left side of the equation equivalent to the right side. Sample responses shown.

1. $6x + \boxed{4x} = 10x$
2. $6x - \boxed{4x} = 2x$
3. $6x - \boxed{5x} = x$
4. $6x + \boxed{-4x} = 2x$
5. $6x + \boxed{-6x} = 0$
6. $6x + \boxed{-16x} = -10x$
7. $6x - \boxed{-4x} = 10x$
8. $6x - \boxed{8x} = -2x$

Are you ready for more?

Replace each box with an expression that makes the left side of the equation equivalent to the right side.

1. $6x - \square = 6$
   Sample response: $(6x - 6)$
2. $6x + \square = 10$
   Sample response: $(-6x + 10)$
3. $6x - \square = 4x - 10$
   Sample response: $(2x + 10)$

STOP

Lesson 21 Combining Like Terms (Part 1) 675

1 Launch

Let students know there are multiple correct responses, but that they should check that, when the like terms are combined on the left side of the equation, the resulting expression equals the right side of the equation.

2 Monitor

Help students get started by asking, "What added to 6 will equal 10?" What added to 6 apples will give 10 apples?" To further solidify their understanding, give students the expression $3b + 4c$ and ask if it can be combined to one term. Use the example of 3 bees plus 4 cows does not equal 7 bee-cows.

Look for points of confusion:

- **Needing many examples to understand.** Take time to give as many examples as needed and highlight how all expressions replacing the box in each question are equivalent. For example, in Problem 1, both of the expressions $4x$ and $5x - 1x$ could work.

3 Connect

Have students share their solutions and strategies for each problem. Encourage them to be creative. Conduct the *Poll the Class* routine as students give suggestions to determine if there is agreement.

Highlight alternative strategies and/or expressions. Encourage students to think of additional expressions which will yield the same result. For example, on Problem 1, students may write out the sum of $6x$'s on the left side and $10x$'s on the right side and reason $4x$'s are needed to make the sides equal. Another student might reason with the Distributive Property, and rewrite the question as $x(6 + \square) = 10x$.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them complete the first column of problems (Problems 1, 3, 5, 7).

Accessibility: *Guide Processing and Visualization*

Provide a bank of possible expressions that students can select from for this activity. Include both variable terms and numerical terms to help students discern between them. For example:

$5x$	$4x$	$-4x$	$8x$	$-6x$	$6x$
$-16x$	$10x$	$12x$	$-8x$	4	-4
5	8	-6	-16	10	6

Math Language Development

MLR8: Discussion Supports

During the Connect, display these sentence frames to help students state whether they agree with each other's solutions and strategies.

- "I agree because . . ."
- "I disagree because . . ."

Encourage them to connect the properties with the processes they used to confirm equivalent expressions. For example, Problem 5 illustrates the Additive Identity. Display the property names and have students use the property names in their responses.

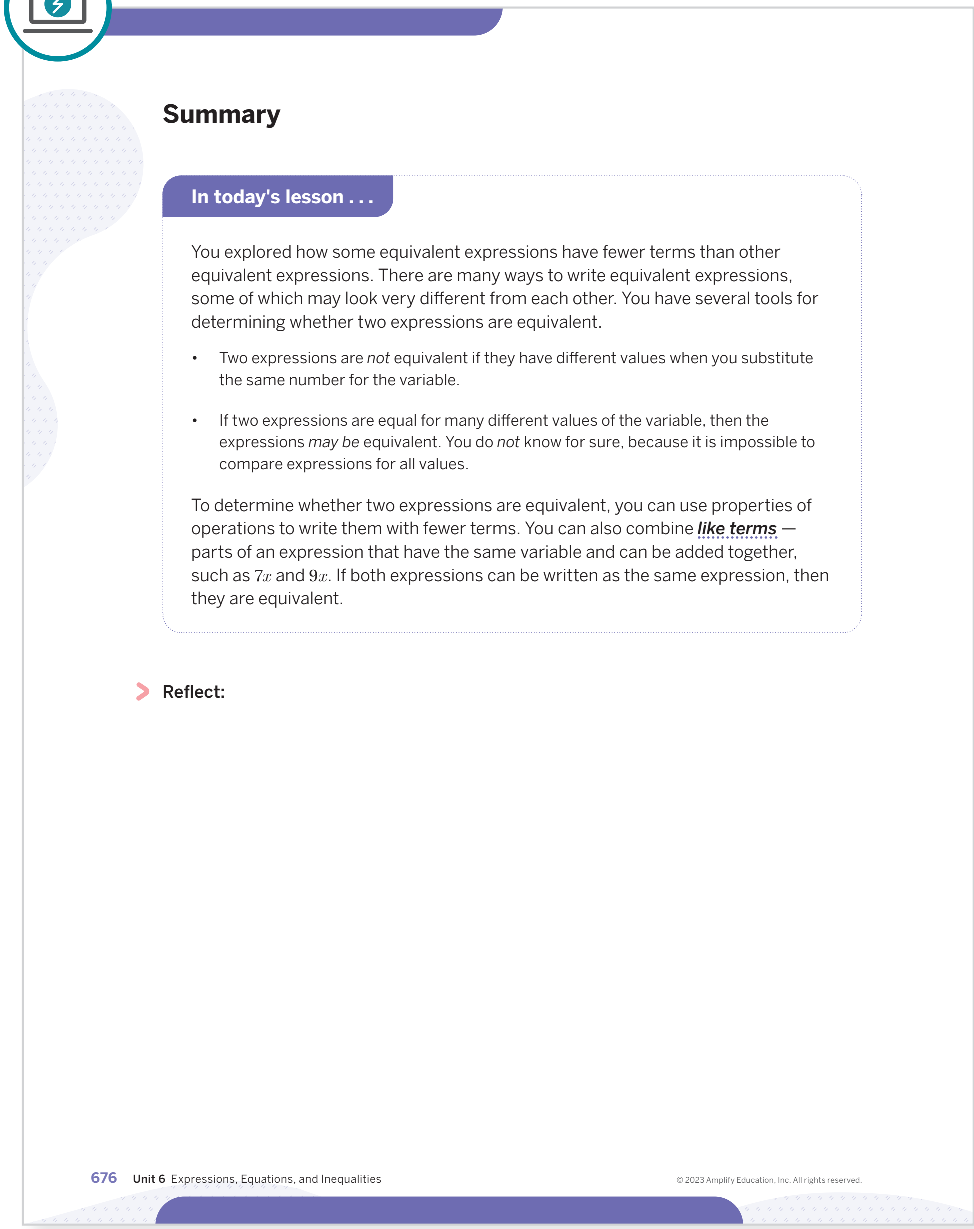
English Learners

Encourage students to draw pictures of shapes and/or algebra tiles to model the equations.

Whole Class | 5 min

Summary

Review and synthesize how to combine like terms.



Summary

In today's lesson . . .

You explored how some equivalent expressions have fewer terms than other equivalent expressions. There are many ways to write equivalent expressions, some of which may look very different from each other. You have several tools for determining whether two expressions are equivalent.

- Two expressions are *not* equivalent if they have different values when you substitute the same number for the variable.
- If two expressions are equal for many different values of the variable, then the expressions *may be* equivalent. You do *not* know for sure, because it is impossible to compare expressions for all values.

To determine whether two expressions are equivalent, you can use properties of operations to write them with fewer terms. You can also combine ***like terms*** — parts of an expression that have the same variable and can be added together, such as $7x$ and $9x$. If both expressions can be written as the same expression, then they are equivalent.

Reflect:

Synthesize

Display the expression $4x + 5x + 8y - 3y$.

Have students share examples of like terms and strategies they use to determine if terms are like terms. Have students demonstrate how to write the above expression in fewer terms.

Highlight that it is impossible to test every number to determine if expressions are equivalent. This is why the properties of operations are used to simplify expressions. Writing expressions in the simplest form makes evaluating them more efficient.

Formalize vocabulary: *like terms*

Highlight that like terms have the same variables raised to the same powers. Provide several examples, and point out that constants are also considered like terms. Have students suggest examples of their own.

Ask, "How do you know whether two expressions are equivalent?" If both expressions simplify or expand to the same expression using properties of operations, the expressions are equivalent.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is simplifying an expression with variables the same as or different from simplifying expressions with only numbers?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit. Ask them to review and reflect on any terms and phrases related to the term *like terms* that were added to the display during the lesson. Add examples and counterexamples of like terms to the display. For example, add the expression $3x + 2 - 4x + 1 + 2xy$ along with the following:

Like terms: $3x$ and $-4x$; 2 and 1

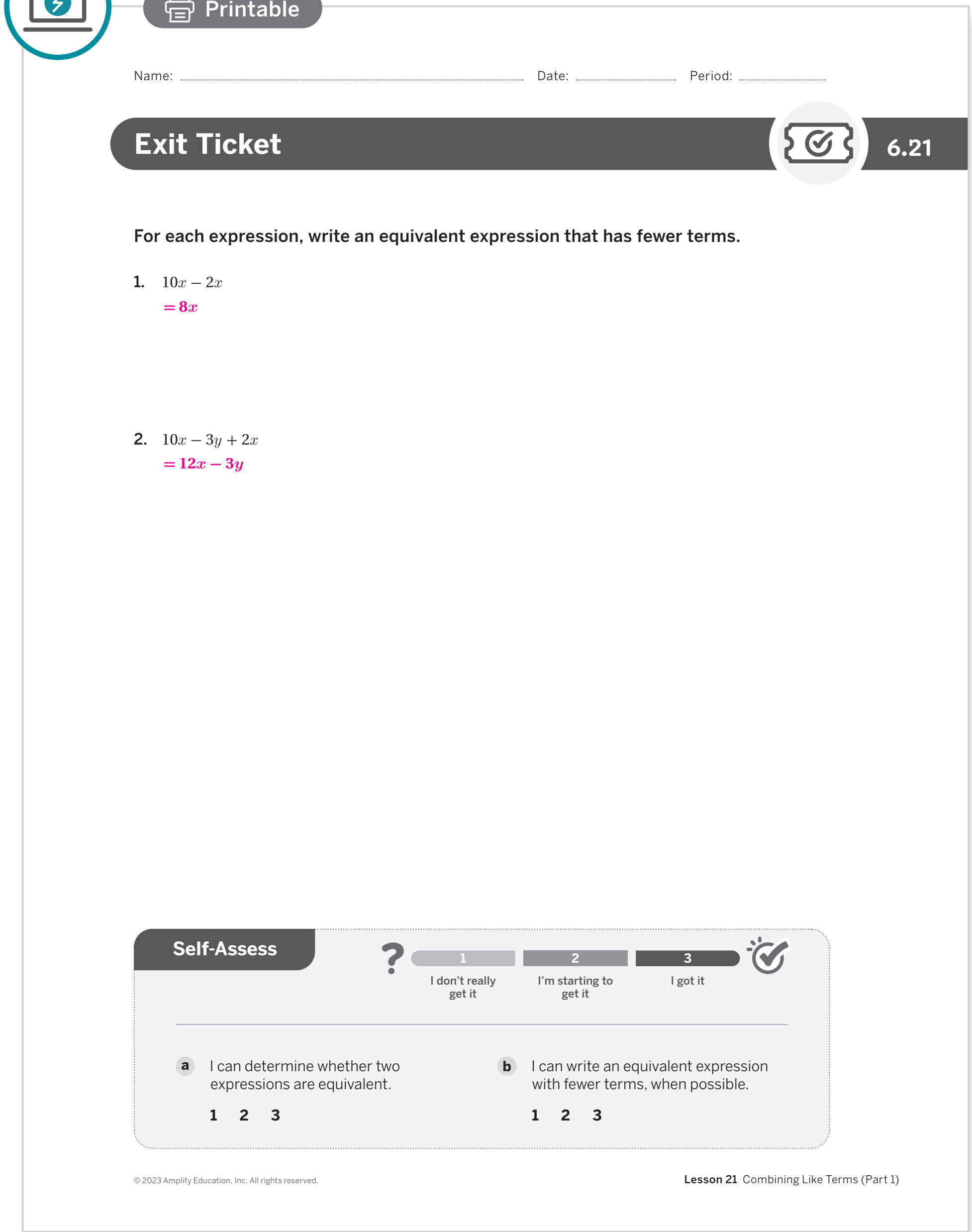
$2xy$ and $3x$ are *not* like terms.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by combining like terms and simplifying expressions.

Printable



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Exit Ticket 6.21

For each expression, write an equivalent expression that has fewer terms.

1. $10x - 2x$
   $= 8x$

2. $10x - 3y + 2x$
   $= 12x - 3y$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine whether two expressions are equivalent.
1 2 3

b. I can write an equivalent expression with fewer terms, when possible.
1 2 3

Lesson 21 Combining Like Terms (Part 1)

Success looks like . . .

- **Language Goal:** Applying properties of operations to justify that expressions are equivalent. **(Speaking and Listening, Writing)**
- **Goal:** Generating an equivalent expression with fewer terms.
  - » Writing equivalent expressions with fewer terms in Problems 1 and 2.
- **Language Goal:** Interpreting different methods for determining whether expressions are equivalent and evaluating their usefulness. **(Speaking and Listening)**

Suggested next steps

If students expand the expression, instead of simplifying, consider:

- Reminding them of the difference between expanding and writing fewer terms.
- Assigning Practice Problem 2.

If students combine the y term with the x terms in Problem 2, consider:

- Reminding them that like terms have the same variables.
- Assigning Practice Problem 1.

Professional Learning

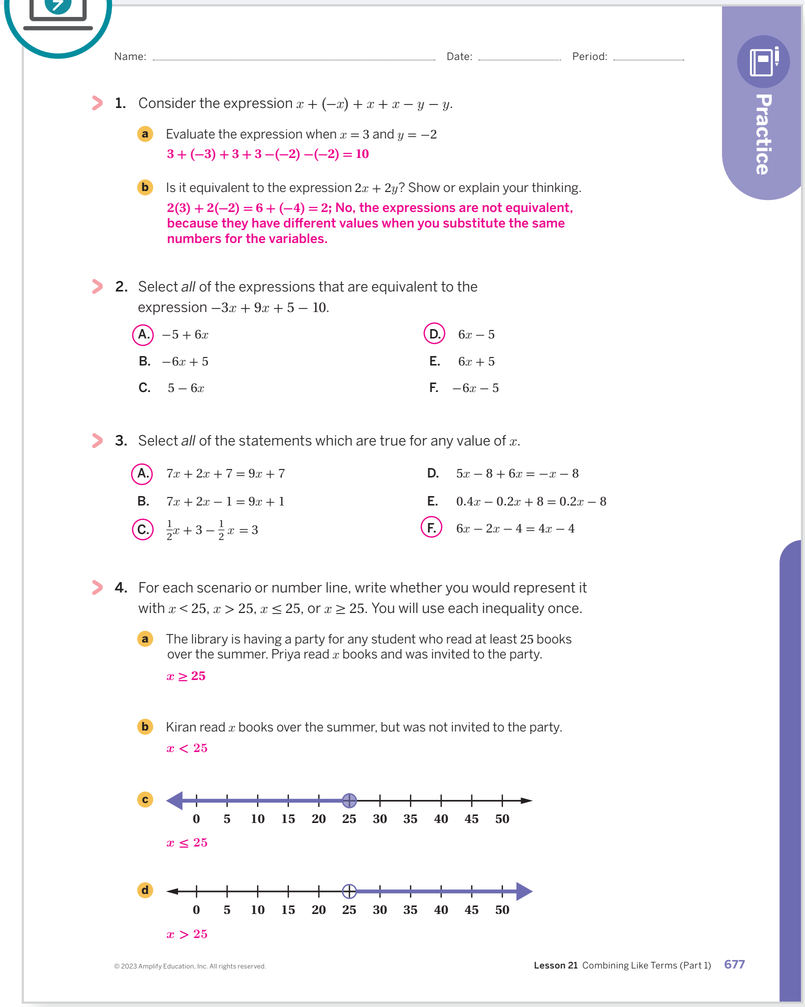
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach the Warm-up? What does that tell you about similarities and differences among your students?
- In what ways in Activity 1 did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

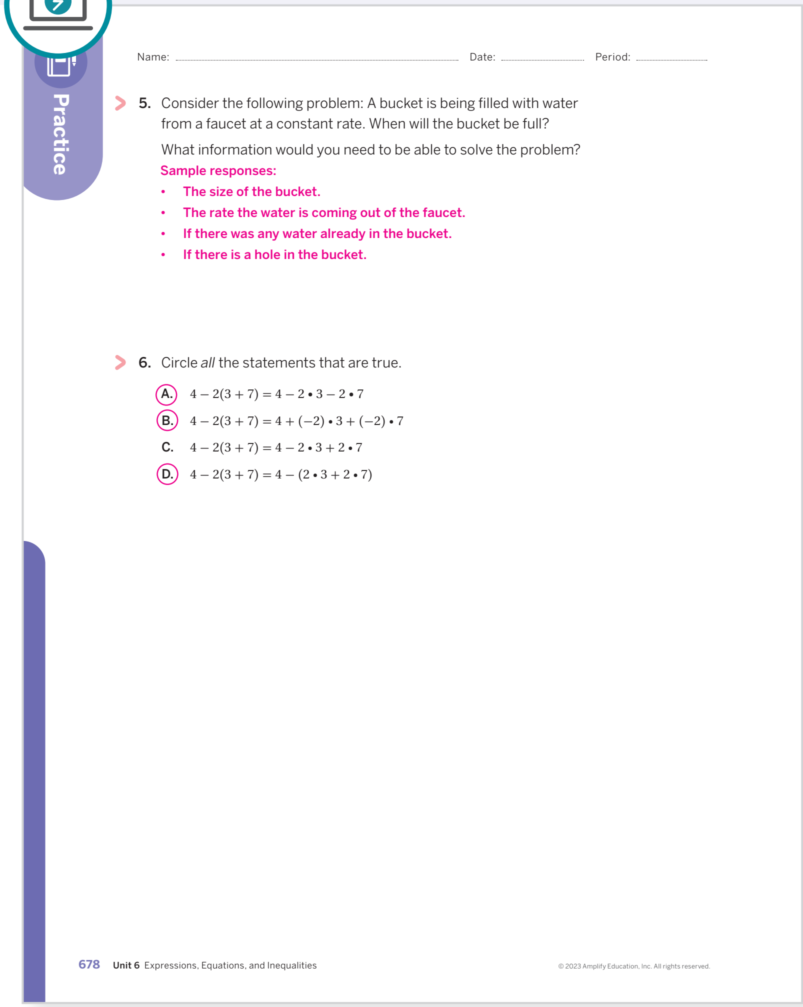
Practice

1. Consider the expression $x + (-x) + x + x - y - y$.
   - **a** Evaluate the expression when $x = 3$ and $y = -2$
     $3 + (-3) + 3 + 3 - (-2) - (-2) = 10$
   - **b** Is it equivalent to the expression $2x + 2y$? Show or explain your thinking.
     $2(3) + 2(-2) = 6 + (-4) = 2$; No, the expressions are not equivalent, because they have different values when you substitute the same numbers for the variables.

2. Select *all* of the expressions that are equivalent to the expression $-3x + 9x + 5 - 10$.
   - [✓ A.] $-5 + 6x$
   - B. $-6x + 5$
   - C. $5 - 6x$
   - [✓ D.] $6x - 5$
   - E. $6x + 5$
   - F. $-6x - 5$

3. Select *all* of the statements which are true for any value of x.
   - [✓ A.] $7x + 2x + 7 = 9x + 7$
   - B. $7x + 2x - 1 = 9x + 1$
   - [✓ C.] $\frac{1}{2}x + 3 - \frac{1}{2}x = 3$
   - D. $5x - 8 + 6x = -x - 8$
   - E. $0.4x - 0.2x + 8 = 0.2x - 8$
   - [✓ F.] $6x - 2x - 4 = 4x - 4$

4. For each scenario or number line, write whether you would represent it with $x < 25$, $x > 25$, $x \leq 25$, or $x \geq 25$. You will use each inequality once.
   - **a** The library is having a party for any student who read at least 25 books over the summer. Priya read x books and was invited to the party.
     $x \geq 25$
   - **b** Kiran read x books over the summer, but was not invited to the party.
     $x < 25$
   - **c** 0 5 10 15 20 25 30 35 40 45 50
     $x \leq 25$
   - **d** 0 5 10 15 20 25 30 35 40 45 50
     $x > 25$



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Practice

5. Consider the following problem: A bucket is being filled with water from a faucet at a constant rate. When will the bucket be full?

   What information would you need to be able to solve the problem?
   Sample responses:
   - The size of the bucket.
   - The rate the water is coming out of the faucet.
   - If there was any water already in the bucket.
   - If there is a hole in the bucket.

6. Circle *all* the statements that are true.
   - [✓ A.] $4 - 2(3 + 7) = 4 - 2 \bullet 3 - 2 \bullet 7$
   - [✓ B.] $4 - 2(3 + 7) = 4 + (-2) \bullet 3 + (-2) \bullet 7$
   - C. $4 - 2(3 + 7) = 4 - 2 \bullet 3 + 2 \bullet 7$
   - [✓ D.] $4 - 2(3 + 7) = 4 - (2 \bullet 3 + 2 \bullet 7)$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 13	1
	5	Unit 2 Lesson 10	2
Formative ⚡	6	Unit 6 Lesson 22	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 22**

Combining Like Terms (Part 2)

Let's see how to write equivalent expressions with parentheses and negative numbers.

Focus

Goals

1. **Language Goal:** Identify expressions that are not equivalent, but differ only in the placement of parentheses, and justify that they are not equivalent. **(Speaking and Listening, Writing)**
2. Write expressions with fewer terms that are equivalent to given expressions that include negative coefficients and parentheses.

Coherence

Today

Students combine like terms to write expressions with fewer terms that are equivalent to expressions with negative coefficients and parentheses. Special attention is given to the effect of negative values on the values within a set of parentheses.

Previously

In Lesson 21, students combined like terms to write equivalent expressions with fewer terms.

Coming Soon

In Lesson 23, the culminating lesson of the unit, students will make connections between equivalent expressions, non-proportional linear relationships, and pattern growth.

Rigor

- Students deepen their **conceptual understanding** of equivalent expressions by comparing and contrasting expressions with and without parentheses.
- Students continue to build **procedural fluency** in simplifying expressions by combining like terms with and without the Distributive Property.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Writing Equivalent Expressions* (for display)
- Anchor Chart PDF, *Writing Equivalent Expressions* (answers)

Math Language Development

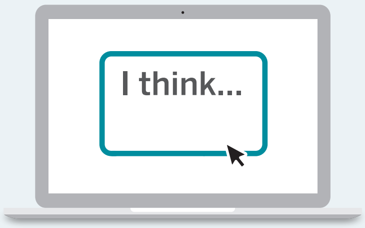
Review words

- *equivalent expressions*
- *like terms*
- *term*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to explain their thinking when determining whether they agree or disagree with strategies for writing equivalent expressions.



Amps
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Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students have to decide whether they agree with each strategy and identify errors that were made. During this debate, students might find themselves in greater conflict than expected with their partner. Encourage students to work through that conflict and resolve it in a way that builds the relationship rather than destroying it.

Modifications to Pacing

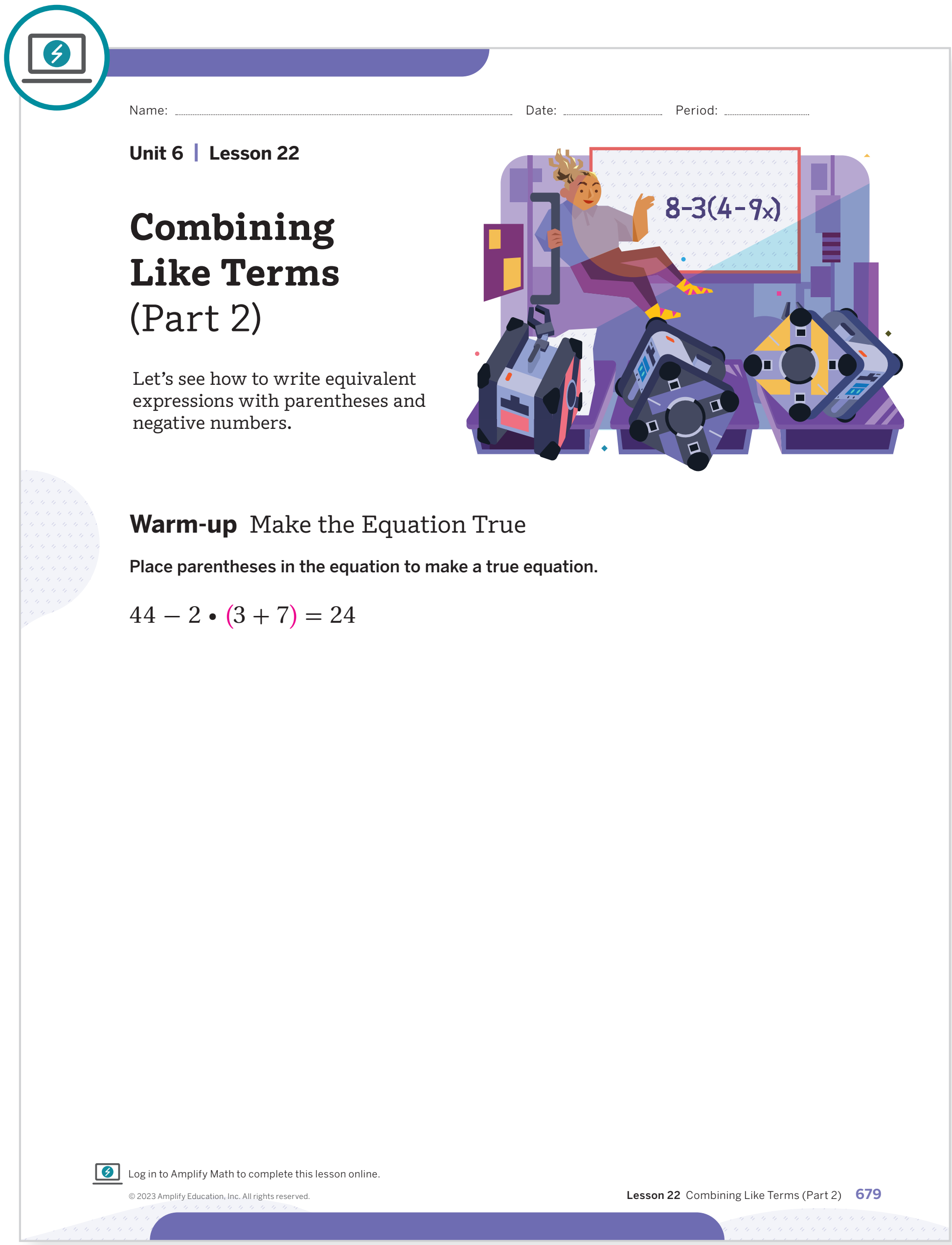
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students discuss the strategies orally without writing down their thinking.
- In **Activity 2**, Problem 1 may be omitted.

Independent | 5 min

Warm-up Make the Equation True

Students place parentheses in an equation to make the equation true.



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Unit 6 | Lesson 22

Combining Like Terms (Part 2)

Let's see how to write equivalent expressions with parentheses and negative numbers.

Warm-up Make the Equation True

Place parentheses in the equation to make a true equation.

$44 - 2 \cdot (3 + 7) = 24$

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students have to work independently on the activity.

2 Monitor

Help students get started by demonstrating how to place the parentheses in an *incorrect* location and evaluating the expression to show the equation is *not* true. Then tell students to try to place the parentheses somewhere else to make it true.

Look for points of confusion:

- **Placing parentheses incorrectly.** Ask them to evaluate the expression to be sure the equation is true.
- **Failing to follow the order of operations when evaluating the expression to check that the equation is true.** Ask students which operation should be done first after they place the parentheses.

3 Connect

Have students share their equations. Choose a few that are *incorrect*. Conduct the ***Poll the Class*** routine to see whether they agree or disagree with their classmates' responses.

Highlight that an equation is true when both sides are equivalent. Review the order of operations by evaluating the given expression on the left side of the equal sign and each student-generated expression.

Power-up

To power up students' ability to identify equivalent number expressions, have students complete:

Recall that subtraction can be rewritten as adding the opposite. For example, $6 - 1$ can be rewritten as $6 + (-1)$. Determine *all* expressions that are equivalent to $6 - 1(3 + 2)$.

[✓ A.] $6 + (-1)(3 + 2)$

[✓ B.] $6 + (-1) \cdot 3 + (-1) \cdot 2$

C. $6 - 1 \cdot 3 + 1 \cdot 2$

[✓ D.] $6 - 1 \cdot 3 - 1 \cdot 2$

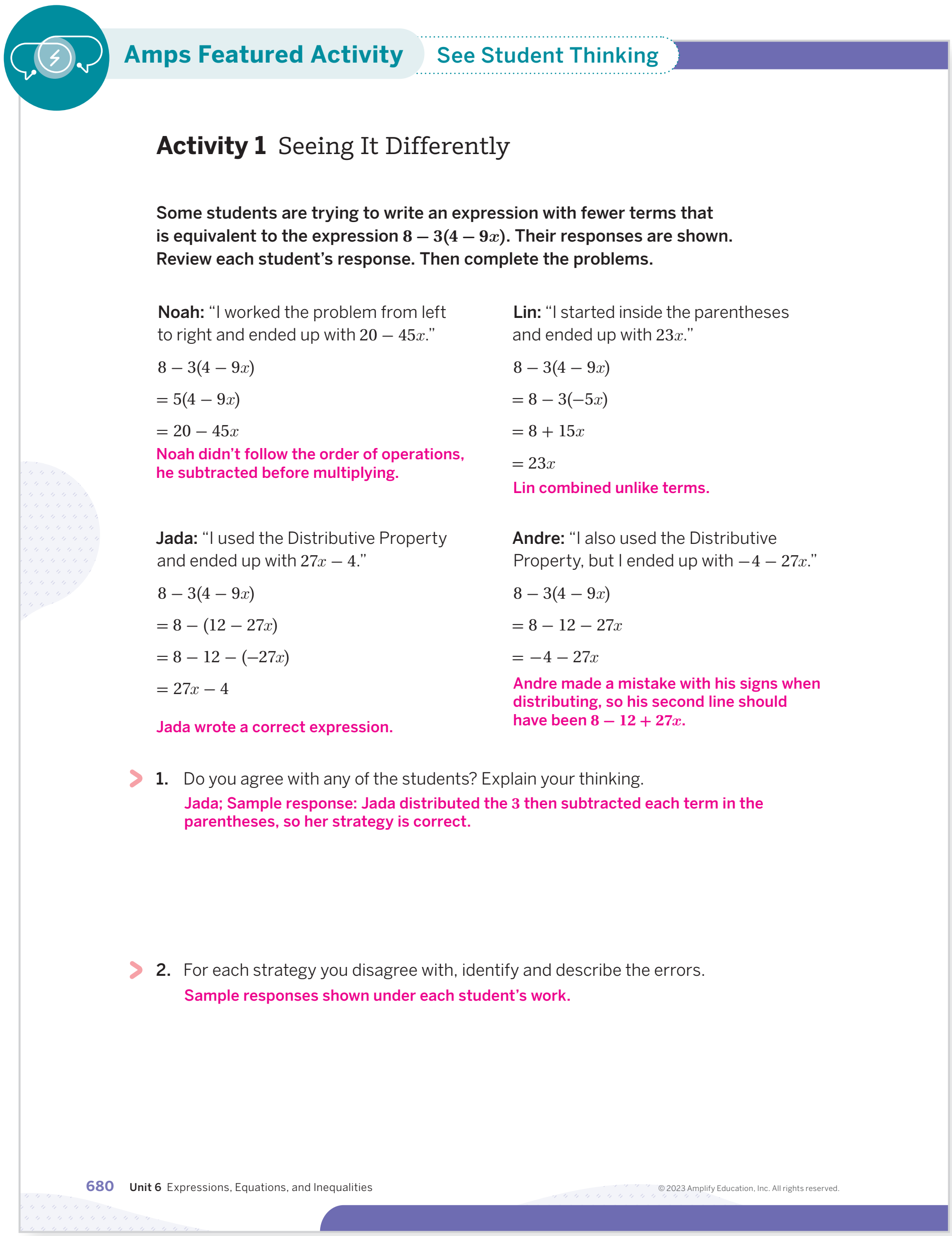
Use: Before Activity 1

Informed by: Performance on Lesson 21, Practice Problem 6

Pairs | 15 min

Activity 1 Seeing It Differently

Students identify typical errors with signed numbers, operations, and properties to help them develop strategies for writing equivalent expressions.



Amps Featured Activity See Student Thinking

Activity 1 Seeing It Differently

Some students are trying to write an expression with fewer terms that is equivalent to the expression $8 - 3(4 - 9x)$. Their responses are shown. Review each student's response. Then complete the problems.

Noah: "I worked the problem from left to right and ended up with $20 - 45x$."

$8 - 3(4 - 9x)$

$= 5(4 - 9x)$

$= 20 - 45x$

Noah didn't follow the order of operations, he subtracted before multiplying.

Lin: "I started inside the parentheses and ended up with $23x$."

$8 - 3(4 - 9x)$

$= 8 - 3(-5x)$

$= 8 + 15x$

$= 23x$

Lin combined unlike terms.

Jada: "I used the Distributive Property and ended up with $27x - 4$."

$8 - 3(4 - 9x)$

$= 8 - (12 - 27x)$

$= 8 - 12 - (-27x)$

$= 27x - 4$

Jada wrote a correct expression.

Andre: "I also used the Distributive Property, but I ended up with $-4 - 27x$."

$8 - 3(4 - 9x)$

$= 8 - 12 - 27x$

$= -4 - 27x$

Andre made a mistake with his signs when distributing, so his second line should have been $8 - 12 + 27x$.

1. Do you agree with any of the students? Explain your thinking.
   Jada; Sample response: Jada distributed the 3 then subtracted each term in the parentheses, so her strategy is correct.

2. For each strategy you disagree with, identify and describe the errors.
   Sample responses shown under each student's work.

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① Launch

Ensure students understand that the task has two parts. First they need to decide whether they agree with each person's strategy, then they need to describe the errors that were made if they disagree.

② Monitor

Help students get started by asking them, "How is the expression changing from one step to the next? Are the expressions on each step equivalent?"

Look for points of confusion:

- **Misidentifying incorrect work as correct.** Suggest substituting the same value for x in each expression and evaluating to see if the results are the same value. This will not prove the expressions are equivalent, but it can prove that they are not equivalent.

③ Connect

Display each student's work given in the problems.

Have students share their thinking. For each work sample, conduct the *Poll the Class* routine to assess classwide agreement with each sample student's work. Have students explain where they see errors in the work with which they disagree.

Highlight the common misconceptions demonstrated by the errors in each work sample; Noah: evaluating from left to right instead of following the order of operations; Lin: combining unlike terms; Andre: forgetting to distribute the negative sign to both terms in the parentheses. Identify each step Jada took to write the expression in fewer terms. Note that Jada's result is the equivalent expression with the fewest terms possible.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can explain their thinking when determining whether they agree or disagree with the given strategies for writing equivalent expressions. Their explanations are available to you digitally, in real time.

Accessibility: *Guide Processing and Visualization*

Have students examine the work sample for one student at a time, pausing for a brief class discussion between work samples.

Math Language Development

MLR3: Critique, Correct, Clarify

The entirety of this activity is designed similarly to the routine *Critique, Correct, Clarify.* During the Connect, as students share where they see errors in the work samples with which they disagree, ask them to work with their partner to either write a few sentences or verbally share an explanation directed to that student as to what error was made and how they could correct their work. Ask student volunteers to share their explanations with the class.

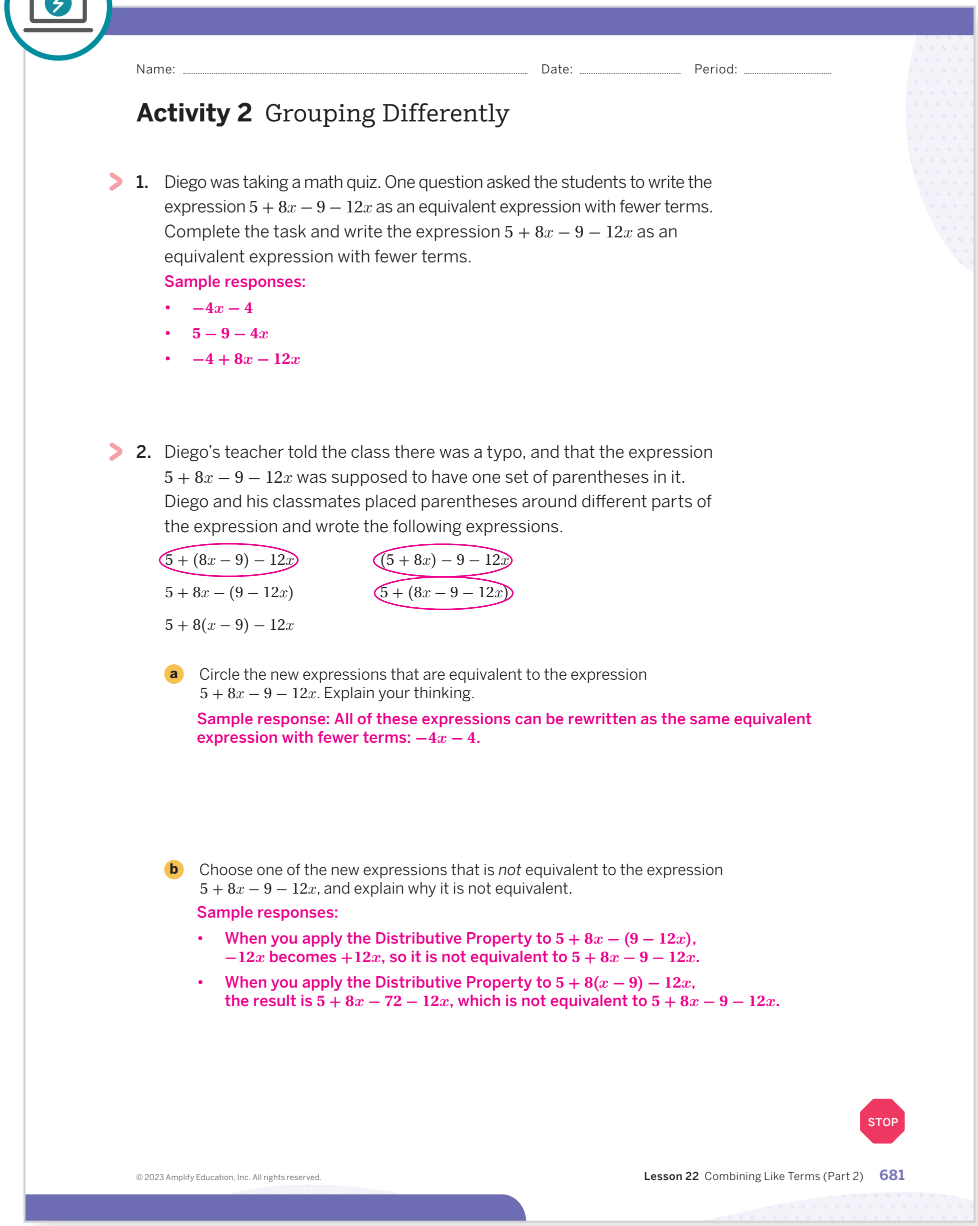
English Learners

Pair students together who speak the same primary language. This will support students in giving and receiving feedback on their explanations.

Pairs | 15 min

Activity 2 Grouping Differently

Students compare expressions that are identical except in the placement of parentheses to explore how the placement of parentheses affects expressions.



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Activity 2 Grouping Differently

1. Diego was taking a math quiz. One question asked the students to write the expression $5 + 8x - 9 - 12x$ as an equivalent expression with fewer terms. Complete the task and write the expression $5 + 8x - 9 - 12x$ as an equivalent expression with fewer terms.

   Sample responses:
   - $-4x - 4$
   - $5 - 9 - 4x$
   - $-4 + 8x - 12x$

2. Diego's teacher told the class there was a typo, and that the expression $5 + 8x - 9 - 12x$ was supposed to have one set of parentheses in it. Diego and his classmates placed parentheses around different parts of the expression and wrote the following expressions.

   $5 + (8x - 9) - 12x$

   $5 + 8x - (9 - 12x)$

   $5 + 8(x - 9) - 12x$

   $(5 + 8x) - 9 - 12x$

   $5 + (8x - 9 - 12x)$

   a. Circle the new expressions that are equivalent to the expression $5 + 8x - 9 - 12x$. Explain your thinking.

   Sample response: All of these expressions can be rewritten as the same equivalent expression with fewer terms: $-4x - 4$.

   b. Choose one of the new expressions that is *not* equivalent to the expression $5 + 8x - 9 - 12x$, and explain why it is not equivalent.

   Sample responses:
   - **When you apply the Distributive Property to $5 + 8x - (9 - 12x)$, $-12x$ becomes $+12x$, so it is not equivalent to $5 + 8x - 9 - 12x$.**
   - **When you apply the Distributive Property to $5 + 8(x - 9) - 12x$, the result is $5 + 8x - 72 - 12x$, which is not equivalent to $5 + 8x - 9 - 12x$.**

STOP

Lesson 22 Combining Like Terms (Part 2) 681

1 Launch

Conduct the *Think-Pair-Share* routine. Have students complete the first problem independently before comparing their equivalent expression with their partner. Then have partners work on the second problem together.

2 Monitor

Help students get started by asking, "Which terms in the expression are *like terms*?"

Look for points of confusion:

- **Making errors writing an expression equivalent to the given expression.** Have students review their work with their partner to see whether they can identify their error.

Look for productive strategies:

- Writing an equivalent expression in factored form $-4(x + 1)$ or $4(-x - 1)$. Note students who do this, and mention it in the class discussion.

3 Connect

Display each expression given in the problem, as it is discussed.

Have students share their equivalent expressions for Problem 1. Conduct the *Poll the Class* routine to see which expressions with parentheses they thought were or were not equivalent to the original. Select students to share their thinking about each expression.

Highlight that, while there are a variety of equivalent expressions with fewer terms students could have written for Problem 1, $-4x - 4$ has the *fewest terms*. Note that equivalent expressions will be identical when written with the fewest terms. Demonstrate this with the expressions in Problem 2.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider removing one or two of the expressions in Problem 2 and have students focus on analyzing the remaining expressions.

Math Language Development

MLR7: Compare and Connect

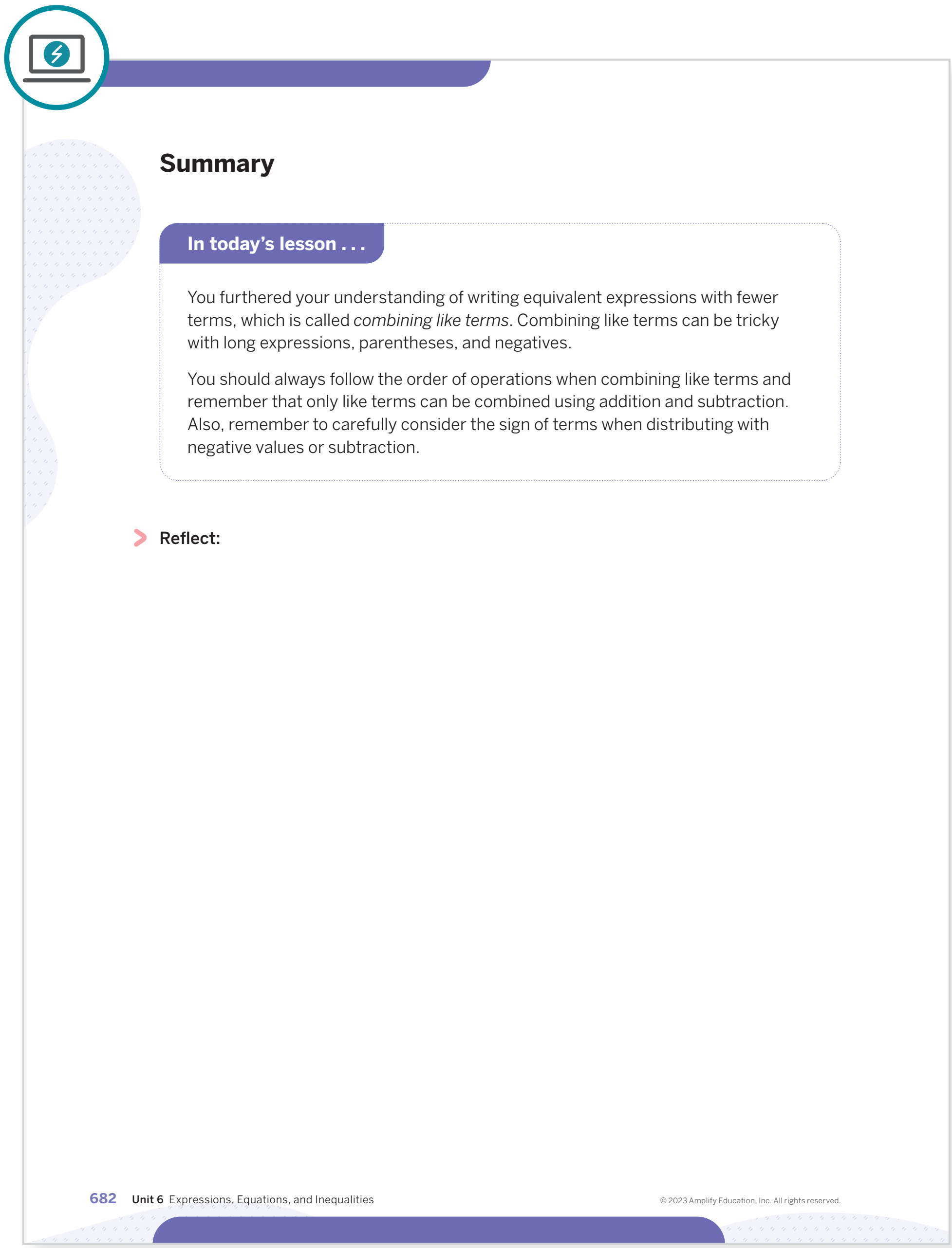
During the Connect, display the expressions from Problem 2. Use color coding to annotate where the parentheses are placed in each expression and whether or not the placement altered the value of the expression. Draw students' attention to the signs or coefficients in front of the parentheses that would result in a different evaluation. Remind them that a negative sign in front of the parentheses indicates a coefficient of –1.

Equivalent to $-4x - 4$	Not equivalent to $-4x - 4$
$5 + (8x - 9) - 12x$ $(5 + 8x) - 9 - 12x$ $5 + (8x - 9 - 12x)$	$5 + 8x - (9 - 12x)$ $5 + 8(x - 9) - 12x$

Whole Class | 5 min

Summary

Review and synthesize how combining like terms and using the properties of operations can help write equivalent expressions with fewer terms.



Summary

In today's lesson . . .

You furthered your understanding of writing equivalent expressions with fewer terms, which is called *combining like terms*. Combining like terms can be tricky with long expressions, parentheses, and negatives.

You should always follow the order of operations when combining like terms and remember that only like terms can be combined using addition and subtraction. Also, remember to carefully consider the sign of terms when distributing with negative values or subtraction.

Reflect:

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Synthesize

Display the Anchor Chart PDF, *Writing Equivalent Expressions* and complete as a class.

Have students share the equivalent expression that should be written in each row of the table that matches the given step.

Ask, "What are some strategies for preventing mistakes while writing expressions in fewer terms?" Rewriting subtraction as adding the opposite; following the order of operations; being careful to only add and subtract like terms.

Highlight how to write the expression as an equivalent expression with the fewest terms.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

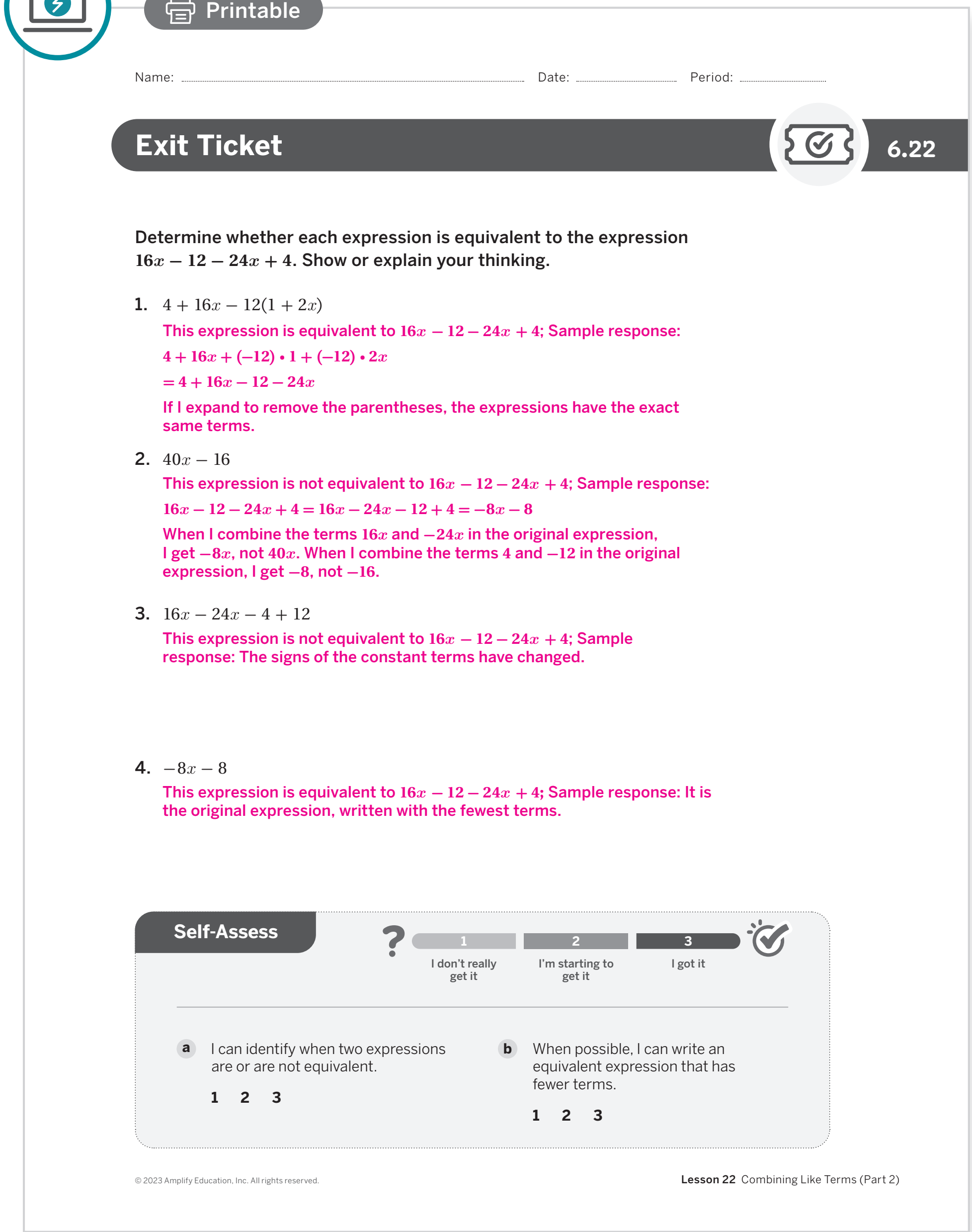
- "What does it mean for two or more expressions to be equivalent?"
- "What strategies did you use to determine if two or more expressions were equivalent?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining which expression in a set of expressions is equivalent to a given expression.

Printable



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Exit Ticket

6.22

Determine whether each expression is equivalent to the expression $16x - 12 - 24x + 4$. Show or explain your thinking.

1. $4 + 16x - 12(1 + 2x)$

   This expression is equivalent to $16x - 12 - 24x + 4$; Sample response:

   $4 + 16x + (-12) \cdot 1 + (-12) \cdot 2x$

   $= 4 + 16x - 12 - 24x$

   If I expand to remove the parentheses, the expressions have the exact same terms.

2. $40x - 16$

   This expression is not equivalent to $16x - 12 - 24x + 4$; Sample response:

   $16x - 12 - 24x + 4 = 16x - 24x - 12 + 4 = -8x - 8$

   When I combine the terms $16x$ and $-24x$ in the original expression, I get $-8x$, not $40x$. When I combine the terms 4 and -12 in the original expression, I get -8, not -16.

3. $16x - 24x - 4 + 12$

   This expression is not equivalent to $16x - 12 - 24x + 4$; Sample response: The signs of the constant terms have changed.

4. $-8x - 8$

   This expression is equivalent to $16x - 12 - 24x + 4$; Sample response: It is the original expression, written with the fewest terms.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can identify when two expressions are or are not equivalent.

1 2 3

b When possible, I can write an equivalent expression that has fewer terms.

1 2 3

Lesson 22 Combining Like Terms (Part 2)

Success looks like . . .

- **Language Goal:** Identifying expressions that are not equivalent, but differ only in the placement of parentheses, and justifying that they are not equivalent. **(Speaking and Listening, Writing)**
- **Goal:** Writing expressions with fewer terms that are equivalent to given expressions that include negative coefficients and parentheses.
  - » Determining that the expression $4 + 16x - 12(1 + 2x)$ is equivalent to the expression $16x - 12 - 24x + 4$ in Problem 1.

Suggested next steps

If students say the first expression is not equivalent because it has parentheses, consider:

- Having them expand the expression to show that it is equivalent to the original expression.
- Assigning Practice Problem 2.

If students say the third expression is equivalent because the numbers are the same as in the original expression, consider:

- Having them write each expression as an equivalent expression with the fewest terms to determine that they are not equivalent.
- Assigning Practice Problem 2.

If students misidentify whether the second and fourth expressions are equivalent, consider:

- Having them rewrite the subtraction in the original expression as adding the opposite. Then have them combine like terms to determine that the fourth expression is equivalent and the second expression is not equivalent.
- Assigning Practice Problem 1.

Professional Learning

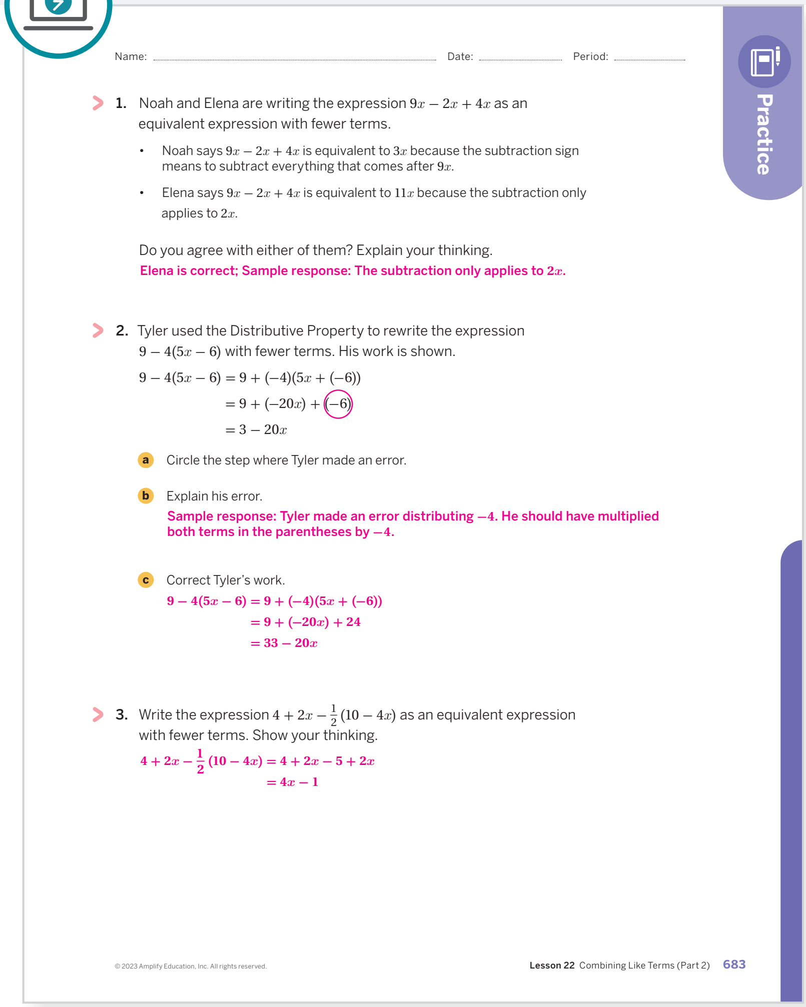
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about Activity 1 how did you encourage each student to share their understandings?
- What challenges did students encounter as they worked on Activity 2? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Noah and Elena are writing the expression $9x - 2x + 4x$ as an equivalent expression with fewer terms.
   - Noah says $9x - 2x + 4x$ is equivalent to $3x$ because the subtraction sign means to subtract everything that comes after $9x$.
   - Elena says $9x - 2x + 4x$ is equivalent to $11x$ because the subtraction only applies to $2x$.

   Do you agree with either of them? Explain your thinking.

   Elena is correct; Sample response: The subtraction only applies to $2x$.

2. Tyler used the Distributive Property to rewrite the expression $9 - 4(5x - 6)$ with fewer terms. His work is shown.

$$\begin{aligned} 9 - 4(5x - 6) &= 9 + (-4)(5x + (-6)) \\ &= 9 + (-20x) + \bigcirc\!\!\!\!\!\!(-6) \\ &= 3 - 20x \end{aligned}$$

   a. Circle the step where Tyler made an error.

   b. Explain his error.
   Sample response: Tyler made an error distributing -4. He should have multiplied both terms in the parentheses by -4.

   c. Correct Tyler's work.

$$\begin{aligned} 9 - 4(5x - 6) &= 9 + (-4)(5x + (-6)) \\ &= 9 + (-20x) + 24 \\ &= 33 - 20x \end{aligned}$$

3. Write the expression $4 + 2x - \frac{1}{2}(10 - 4x)$ as an equivalent expression with fewer terms. Show your thinking.

$$\begin{aligned} 4 + 2x - \frac{1}{2}(10 - 4x) &= 4 + 2x - 5 + 2x \\ &= 4x - 1 \end{aligned}$$

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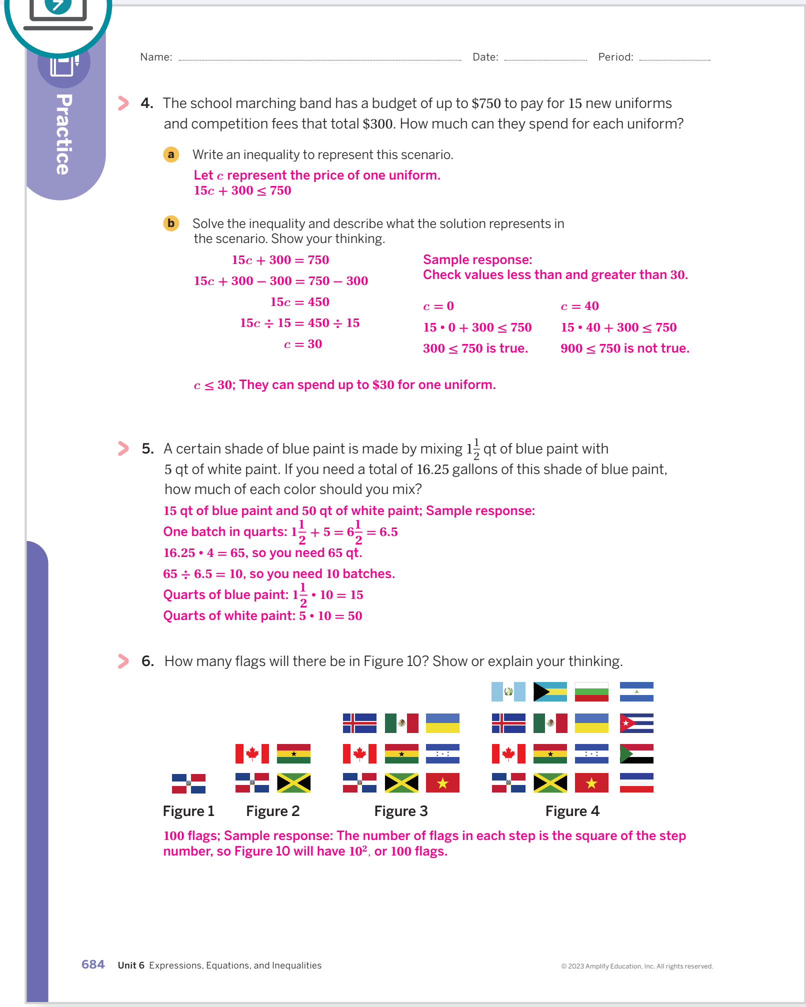
Lesson 22 Combining Like Terms (Part 2) 683



Name: Date: Period:

Practice

4. The school marching band has a budget of up to $750 to pay for 15 new uniforms and competition fees that total $300. How much can they spend for each uniform?

   a. Write an inequality to represent this scenario.
   Let c represent the price of one uniform.
   $15c + 300 \leq 750$

   b. Solve the inequality and describe what the solution represents in the scenario. Show your thinking.

$$\begin{aligned} 15c + 300 &= 750 \\ 15c + 300 - 300 &= 750 - 300 \\ 15c &= 450 \\ 15c \div 15 &= 450 \div 15 \\ c &= 30 \end{aligned}$$

   Sample response:
   Check values less than and greater than 30.

   $c = 0$
   $15 \cdot 0 + 300 \leq 750$
   $300 \leq 750$ **is true.**

   $c = 40$
   $15 \cdot 40 + 300 \leq 750$
   $900 \leq 750$ **is not true.**

   $c \leq 30$; They can spend up to $30 for one uniform.

5. A certain shade of blue paint is made by mixing $1\frac{1}{2}$ qt of blue paint with 5 qt of white paint. If you need a total of 16.25 gallons of this shade of blue paint, how much of each color should you mix?
   15 qt of blue paint and 50 qt of white paint; Sample response:
   One batch in quarts: $1\frac{1}{2} + 5 = 6\frac{1}{2} = 6.5$
   $16.25 \cdot 4 = 65$, so you need 65 qt.
   $65 \div 6.5 = 10$, so you need 10 batches.
   Quarts of blue paint: $1\frac{1}{2} \cdot 10 = 15$
   Quarts of white paint: $5 \cdot 10 = 50$

6. How many flags will there be in Figure 10? Show or explain your thinking.

   Figure 1 Figure 2 Figure 3 Figure 4

   100 flags; Sample response: The number of flags in each step is the square of the step number, so Figure 10 will have 10^2, or 100 flags.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 17	2
	5	Unit 2 Lesson 2	2
Formative ⚡	6	Unit 6 Lesson 23	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 6 | **LESSON 23 – CAPSTONE**

Pattern Thinking

Let's write expressions to describe patterns of growth.

Focus

Goals

1. Write an algebraic expression to describe a linear relationship.
2. **Language Goal:** Justify whether algebraic expressions are equivalent. **(Speaking and Listening, Writing)**

Coherence

Today

This culminating lesson of the unit helps students make connections between equivalent expressions, non-proportional linear relationships, and pattern growth.

Previously

In Unit 2, students became familiar with multiple representations of proportional relationships, using tables, graphs, and equations. Throughout Unit 6, students expanded on their work with proportional relationships to write expressions and equations to represent nonproportional, linear relationships.

Coming Soon

Students will continue to work with solving equations in new contexts, such as angle relationships, in Unit 7.

Rigor

- Students **apply** their understanding of writing and solving equations and expressions to study patterns and predict the values in nonproportional relationships.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
10 min	5 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

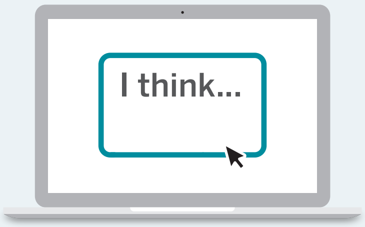
Review words

- *commutative property*
- *Distributive Property*
- *equivalent expressions*

Amps | Featured Activity

Activity 2
Students Test Expressions Instantly

After writing their expression to find the number of tiles for the border, students can see their thinking instantly tested for various sizes of pools and have the opportunity to revise their expression, if needed.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students look for and use the structure in the growth pattern in Activity 1, they might not think that there is a purpose for the task. Have students speculate on why this task is important. Have them explain what they think they will learn and how they will use it in the future. Ask them to explain how it relates to what they have learned before. With a better understanding of this trajectory, students will be able to approach the activity with a spirit of optimism.

Modifications to Pacing

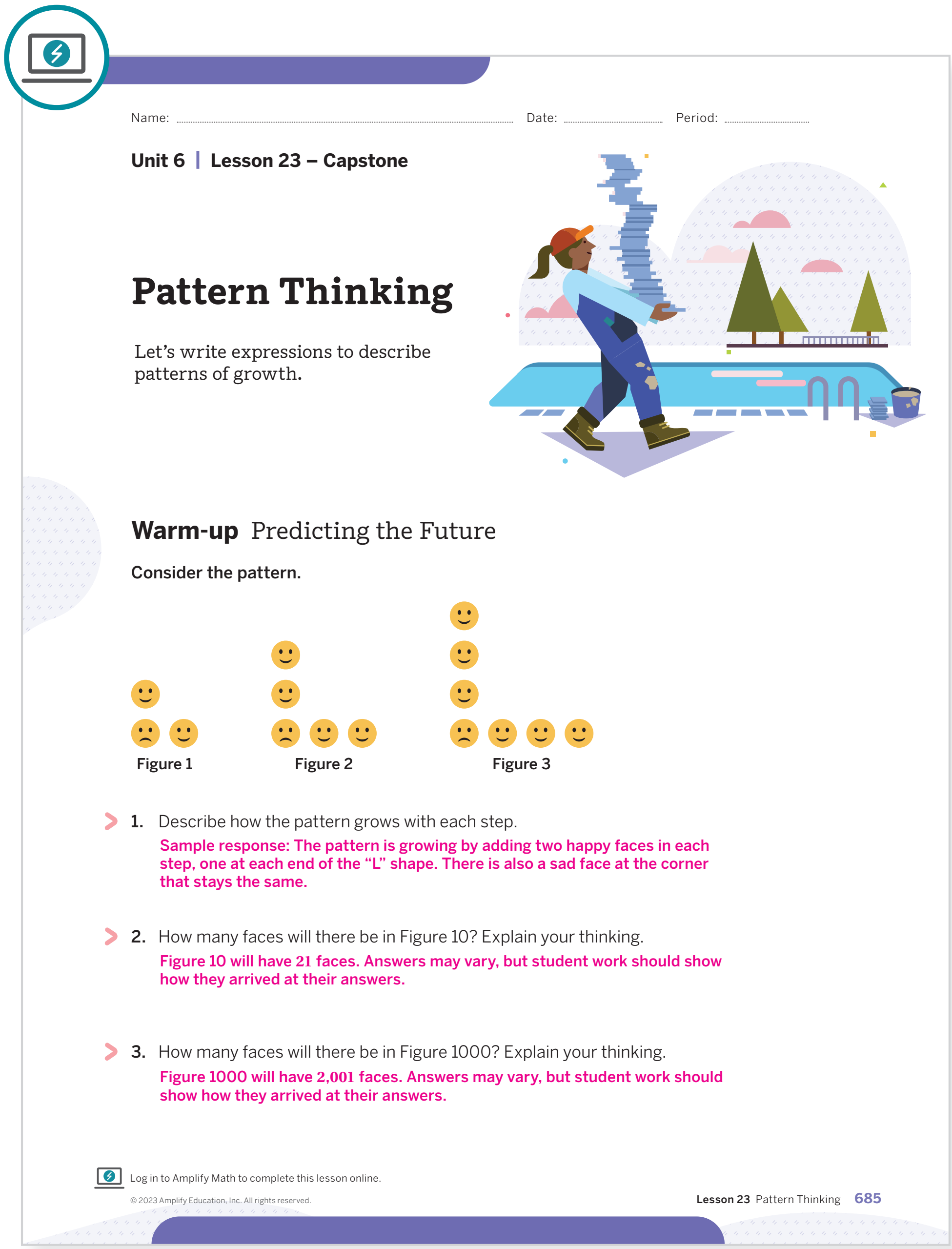
You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, have one partner complete each each problem, and then compare their methods.

Pairs | 10 min

Warm-up Predicting the Future

Students analyze a pattern to prepare for representing pattern growth using expressions in the upcoming activities.



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Unit 6 | Lesson 23 – Capstone

Pattern Thinking

Let's write expressions to describe patterns of growth.

Warm-up Predicting the Future

Consider the pattern.

Figure 1 Figure 2 Figure 3

1. Describe how the pattern grows with each step.
   Sample response: The pattern is growing by adding two happy faces in each step, one at each end of the "L" shape. There is also a sad face at the corner that stays the same.

2. How many faces will there be in Figure 10? Explain your thinking.
   Figure 10 will have 21 faces. Answers may vary, but student work should show how they arrived at their answers.

3. How many faces will there be in Figure 1000? Explain your thinking.
   Figure 1000 will have 2,001 faces. Answers may vary, but student work should show how they arrived at their answers.

Log in to Amplify Math to complete this lesson online.

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Lesson 23 Pattern Thinking 685

① Launch

Conduct the ***Think-Pair-Share*** routine for Problem 1. Have students work together to answer Problems 2 and 3.

② Monitor

Help students get started by asking students to draw what they think Figure 4 will look like.

Look for points of confusion:

- **Multiplying the step number by two.** Ask students to test this strategy by relating the step number with the number of faces.

Look for productive strategies:

- Noticing that the number of happy faces increases by two each step, but that the total number is always one more than double the step number.

③ Connect

Have students share their descriptions of how the pattern is growing.

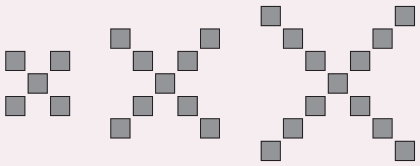
Ask, "Is the number of faces in each figure proportional to the figure number? How do you know?" No; Sample response: I can tell because the ratio of the number of faces to the step number is not equivalent for all steps.

Highlight that even though this relationship is not proportional, it is still a special type of relationship because the amount of change from figure to figure stays constant. This is called a *linear relationship*, and will be explored further in Grade 8.

Power-up

To power up students' ability to use a pattern to determine the number of items in a figure, have students complete:

Determine whether each statement is *true* or *false*.



a. The first figure has 5 blocks and each figure adds 5 additional blocks. False

b. The first figure has 5 blocks and each figure adds 4 additional blocks. True

c. The next figure in the pattern will have 17 blocks. True

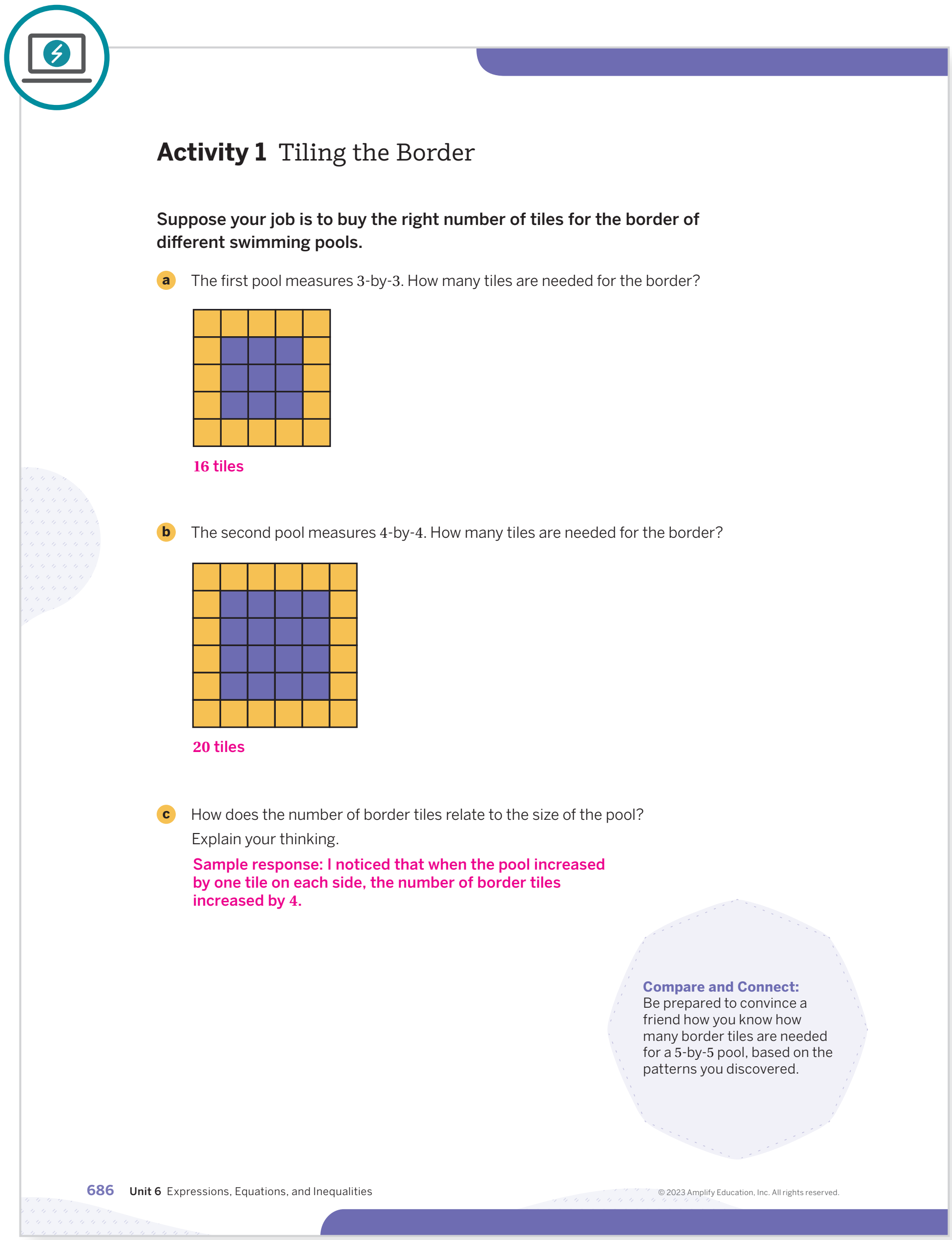
d. The tenth figure in the pattern will have 41 blocks. True

Use: Before the Warm-up
Informed by: Performance on Lesson 22, Practice Problem 6

Pairs | 5 min

Activity 1 Tiling the Border

Students look for and make use of structure in the growth of a pattern. This will prepare them for writing an expression to represent an unknown step in the pattern.



Activity 1 Tiling the Border

Suppose your job is to buy the right number of tiles for the border of different swimming pools.

a The first pool measures 3-by-3. How many tiles are needed for the border?

16 tiles

b The second pool measures 4-by-4. How many tiles are needed for the border?

20 tiles

c How does the number of border tiles relate to the size of the pool? Explain your thinking.

Sample response: I noticed that when the pool increased by one tile on each side, the number of border tiles increased by 4.

Compare and Connect: Be prepared to convince a friend how you know how many border tiles are needed for a 5-by-5 pool, based on the patterns you discovered.

1 Launch

Ask students to try to determine the number of tiles, without counting each one.

2 Monitor

Help students get started by asking, “Do you notice any relationship between the number of tiles in the border and the number of tiles in the inside?” Students may notice that each tile on the sides of the interior square matches with one square tile in the border plus four additional square tiles for the corners.

Look for points of confusion:

- **Counting each tile.** Ask students to try the next problem without counting, or ask about a 5-by-5 pool.

Look for productive strategies:

- Noticing possible relationships. Sample relationships:
  - » 4(side length + 1)
  - » side length + side length + side length + side length + 4
  - » 4 • side length + 4

3 Connect

Have students share different descriptions of the relationship between the pool size and the amount of border tiles.

Display a diagram of the 4-by-4 pool and annotate it as they share.

Highlight that there are different ways to think about the relationship between pool size and border tiles. Each of these ways could be represented by a different expression.

Ask, “How many border tiles will be needed for a 5-by-5 pool?” 24 tiles; You should encourage students to explain their thinking.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their thinking instantly tested for various sizes of pools and have the opportunity to revise their expression, if needed.

Extension: *Math Enrichment*

Have students complete the following problem:
Imagine the image shown in part a is a side view of one side of a cube. Assume the colors of the cubes remain the same throughout the cube as shown in the side view. How many of the smaller cubes are on the outer border? How many are in the interior? 45 outer border cubes; 80 interior cubes

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to share their responses for the task posed to them in their Student Edition, “Be prepared to convince a friend how you know how many border tiles are needed for a 5-by-5 pool, based on the patterns you discovered.” Have volunteers share their arguments with the class and consider these questions as they share:

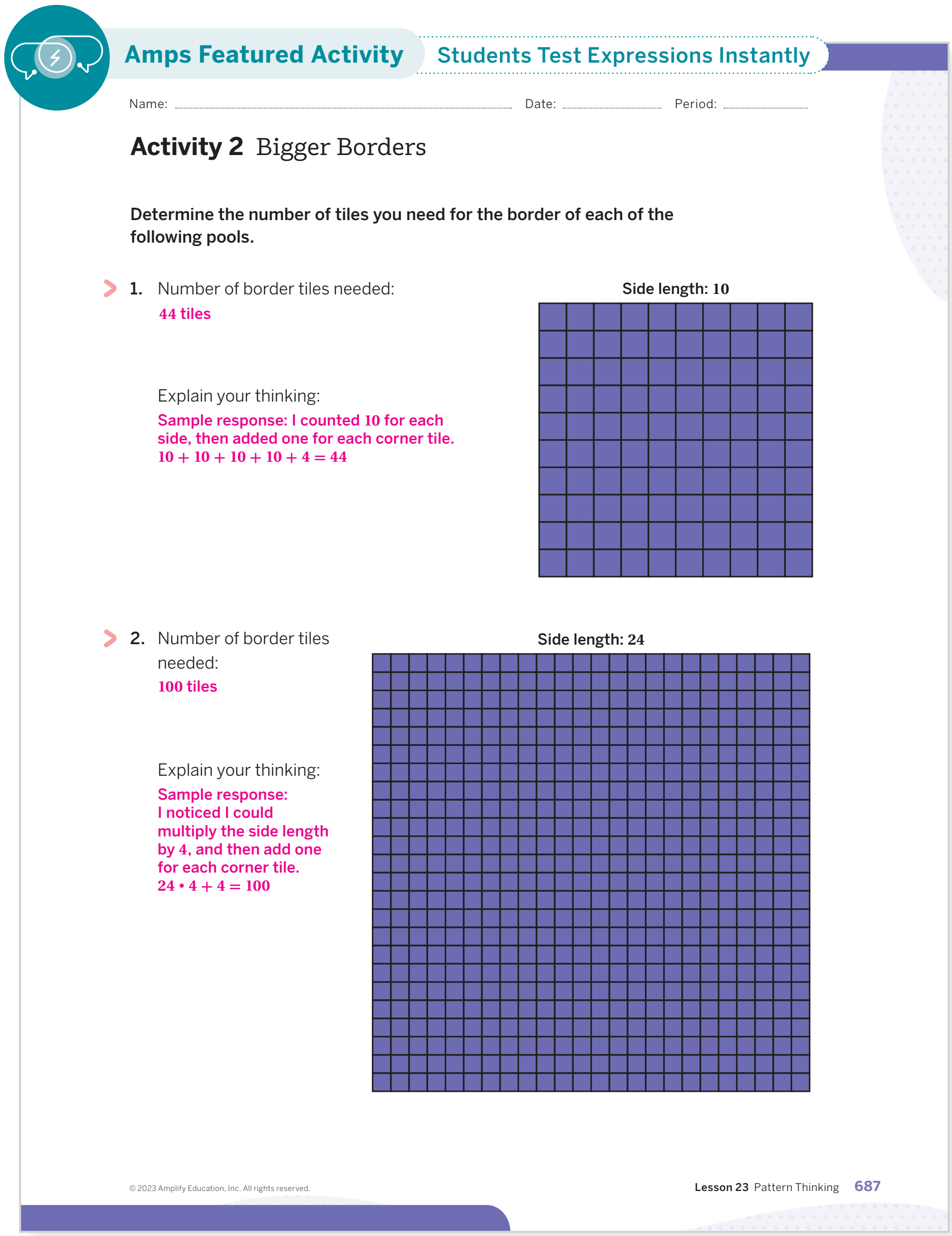
- “Does your argument not include counting the number of tiles?”
- “Does your argument build upon the number of tiles used for a 3-by-3 or 4-by-4 pool?”
- “Can you extend your argument to determine how many border tiles are needed for an 8-by-8 pool?” 36 tiles

Pairs | 10 min

Activity 2 Bigger Borders

Students extend their understanding of the pool border relationship to greater numbers to foster algebraic thinking.

Amps Featured Activity Students Test Expressions Instantly



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Activity 2 Bigger Borders

Determine the number of tiles you need for the border of each of the following pools.

1. Number of border tiles needed:
   44 tiles

   Explain your thinking:
   Sample response: I counted 10 for each side, then added one for each corner tile. $10 + 10 + 10 + 10 + 4 = 44$

2. Number of border tiles needed:
   100 tiles

   Explain your thinking:
   Sample response: I noticed I could multiply the side length by 4, and then add one for each corner tile. $24 \cdot 4 + 4 = 100$

1 Launch

Encourage students to write numerical expressions to find the number of tiles for each border.

2 Monitor

Help students get started by having them represent each grouping of tiles with a number next to the diagram of each pool before writing their expressions.

Look for points of confusion:

- **Counting each tile.** Have students cover the pool, except for the information about the side length of the pool, and ask them to use this information to determine the number of tiles.

Look for productive strategies:

- Generating valid expressions. Possible expressions: $4(10) + 4$, $4(10 + 1)$, $(10 + 2)^2 - 10^2$.

3 Connect

Have students share expressions of the same form for each of the pools. Display the expressions underneath each other.

Highlight that the form of the expressions is exactly the same, and the only difference is the value of the side length, e.g., $4(10) + 4$ and $4(24) + 4$. If time permits, show this for other forms of the expression, e.g., $4(10 + 1)$ and $4(24 + 1)$.

Ask, "What would the expressions look like if the pool had a side length of 1,000?" Substitute the number 1,000 in place of the number 10 or 24.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their thinking instantly tested for various sizes of pools and have the opportunity to revise their expression, if needed.

Accessibility: *Vary Demands to Optimize Challenge*

After students complete Problem 1, have them pause for a brief discussion before moving on to Problem 2. Ask:

- "What expressions can you write to represent the number of border tiles in Problem 1?"
- "What patterns do you notice between Activity 1 and this activity so far?"

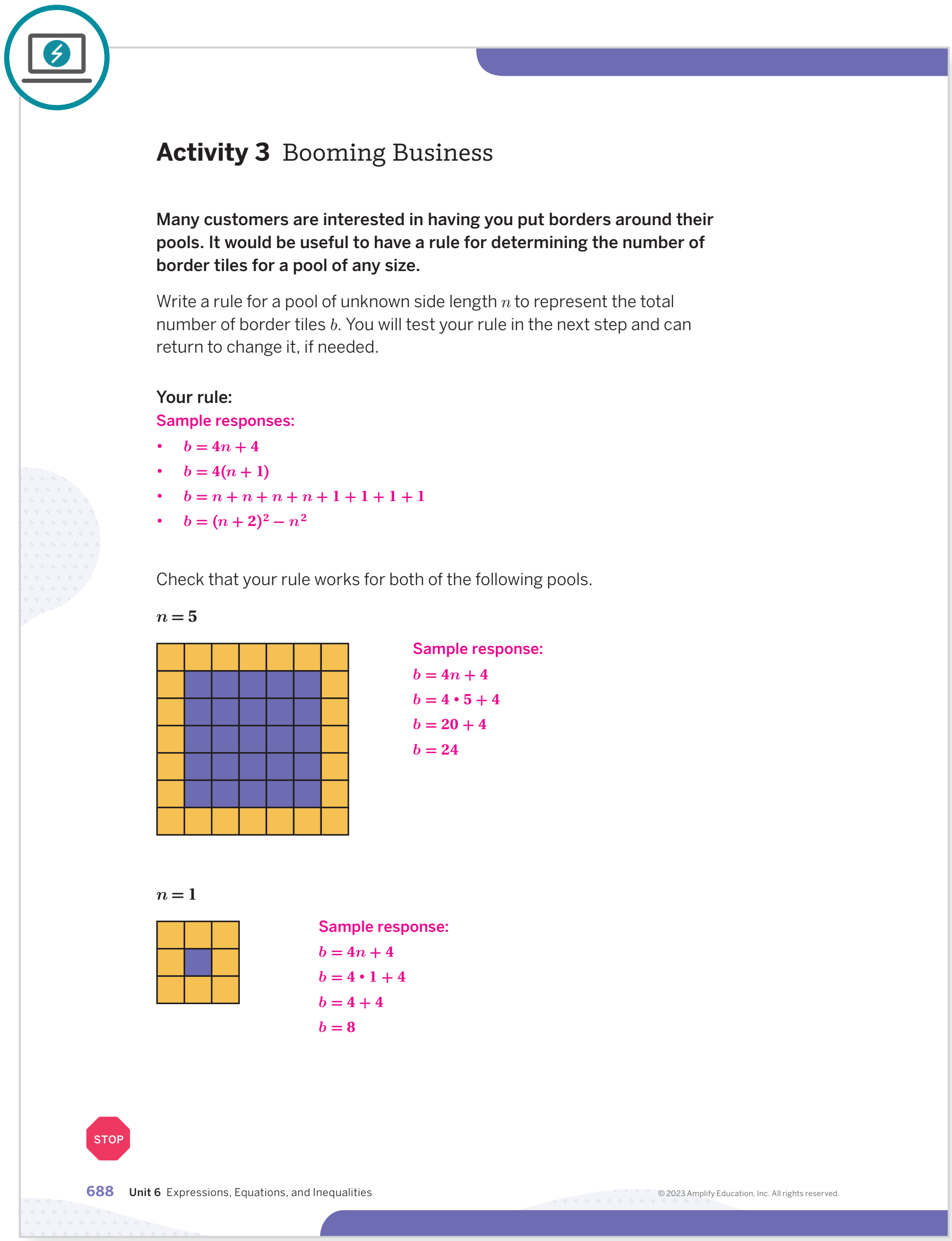
Extension: *Math Enrichment*

Have students complete the following problem:
If the side length of the pool doubles, does the length of the border also double? Why or why not? No; Sample response: When the side length was 3, the total border was 16 tiles. If the side length is now 6, the total border is actually 28 tiles, which is not double 16. There are always 4 corner tiles shared by adjacent sides.

Independent | 10 min

Activity 3 Booming Business

Students write a formula to describe the pool border relationship for any size pool. This helps them reason abstractly about the pattern.



Activity 3 Booming Business

Many customers are interested in having you put borders around their pools. It would be useful to have a rule for determining the number of border tiles for a pool of any size.

Write a rule for a pool of unknown side length n to represent the total number of border tiles b. You will test your rule in the next step and can return to change it, if needed.

Your rule:

Sample responses:

- $b = 4n + 4$
- $b = 4(n + 1)$
- $b = n + n + n + n + 1 + 1 + 1 + 1$
- $b = (n + 2)^2 - n^2$

Check that your rule works for both of the following pools.

$n = 5$

Sample response:

$b = 4n + 4$

$b = 4 \cdot 5 + 4$

$b = 20 + 4$

$b = 24$

$n = 1$

Sample response:

$b = 4n + 4$

$b = 4 \cdot 1 + 4$

$b = 4 + 4$

$b = 8$

STOP

688 Unit 6 Expressions, Equations, and Inequalities

1 Launch

Remind students that a rule is an equation relating two variables. One of the variables will take a value (the side length) and allow you to solve for the other variable (the number of border tiles).

2 Monitor

Help students get started by saying, "Look back at the previous activity. What did your expression look like? Which numbers changed and which stayed the same?"

Look for points of confusion:

- **Writing a rule that works for only one of the pools.** Say, "Your rule needs to work for all sizes of the pool."

Look for productive strategies:

- First counting the border tiles to know what values they are aiming for when writing their equation.

3 Connect

Display a list of all the different student-created rules to the whole class.

Have students share which rules they think are equivalent to each other. Have them first discuss with a partner for one minute and then share their ideas with the class.

Highlight that all the rules are equivalent to each other because they all describe the same relationship between the size of the pool and the number of border tiles.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their thinking instantly tested for various sizes of pools and have the opportunity to revise their expression, if needed.

Extension: *Math Enrichment*

Have students use their rule to determine the length of a square pool if the number of border tiles going around the pool is 132.
The pool has a side length of 32 tiles.

Math Language Development

MLR7: Compare and Connect

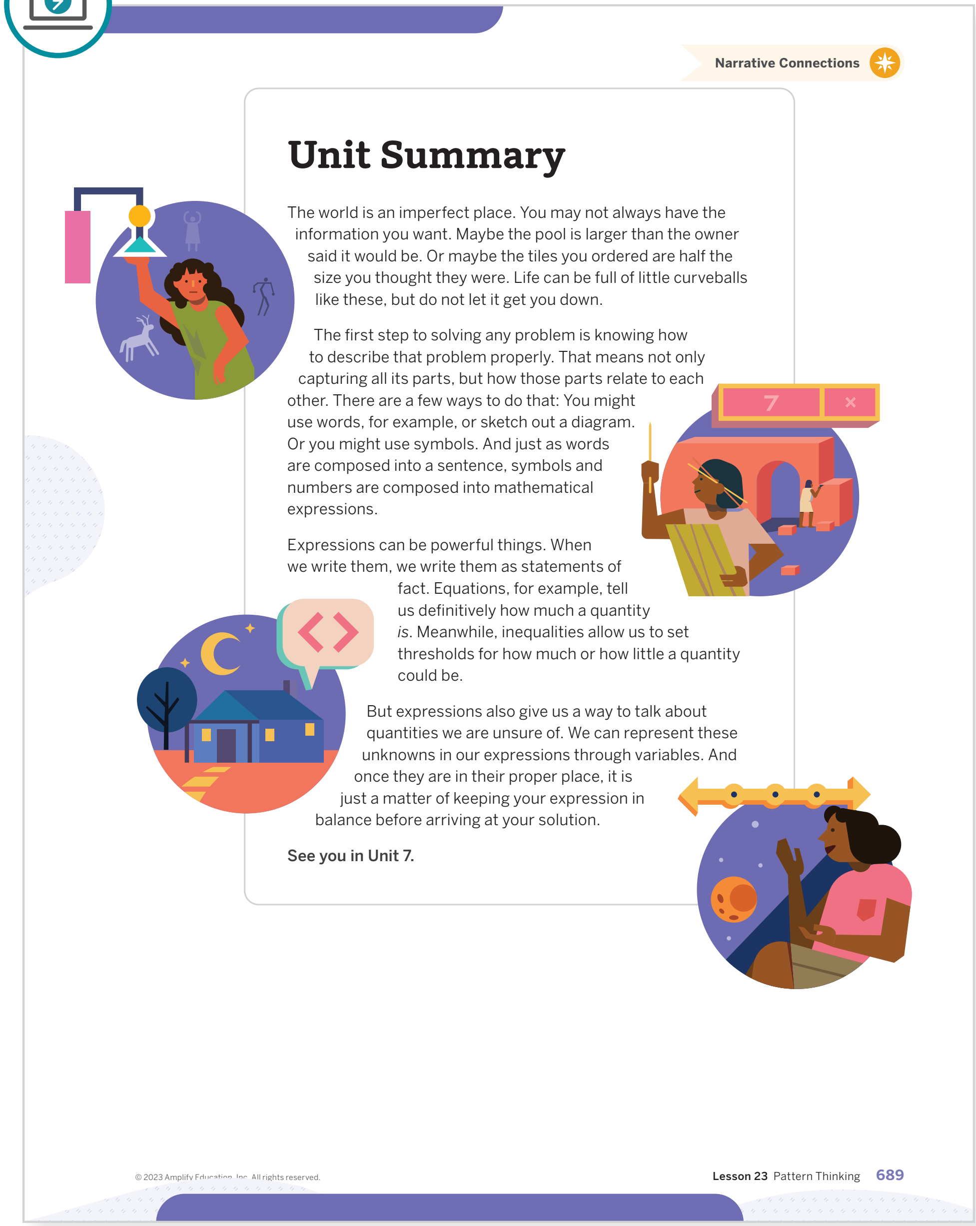
During the Connect, after displaying the different student-created rules to the whole class, ask students to share with a partner what they notice about the rules. Encourage students to compare the different rules and identify which rules they think are equivalent and why.

Highlight connections students make, such as, "they are equivalent because they describe the same relationship between the size and the number of border tiles."

Whole Class | 5 min

Unit Summary

Review and synthesize the main concepts of the unit.



Narrative Connections

Unit Summary

The world is an imperfect place. You may not always have the information you want. Maybe the pool is larger than the owner said it would be. Or maybe the tiles you ordered are half the size you thought they were. Life can be full of little curveballs like these, but do not let it get you down.

The first step to solving any problem is knowing how to describe that problem properly. That means not only capturing all its parts, but how those parts relate to each other. There are a few ways to do that: You might use words, for example, or sketch out a diagram. Or you might use symbols. And just as words are composed into a sentence, symbols and numbers are composed into mathematical expressions.

Expressions can be powerful things. When we write them, we write them as statements of fact. Equations, for example, tell us definitively how much a quantity *is*. Meanwhile, inequalities allow us to set thresholds for how much or how little a quantity could be.

But expressions also give us a way to talk about quantities we are unsure of. We can represent these unknowns in our expressions through variables. And once they are in their proper place, it is just a matter of keeping your expression in balance before arriving at your solution.

See you in Unit 7.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary. Have students read the Summary or have a student volunteer read it aloud.

Highlight that it is very useful to spend some time reflecting on one's own learning at the conclusion of a unit. This helps to retain information and make connections that students may not have been able to make while focusing on each individual lesson.

Ask students to take a few minutes to recall what they have learned about expressions, equations, and inequalities throughout this unit.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

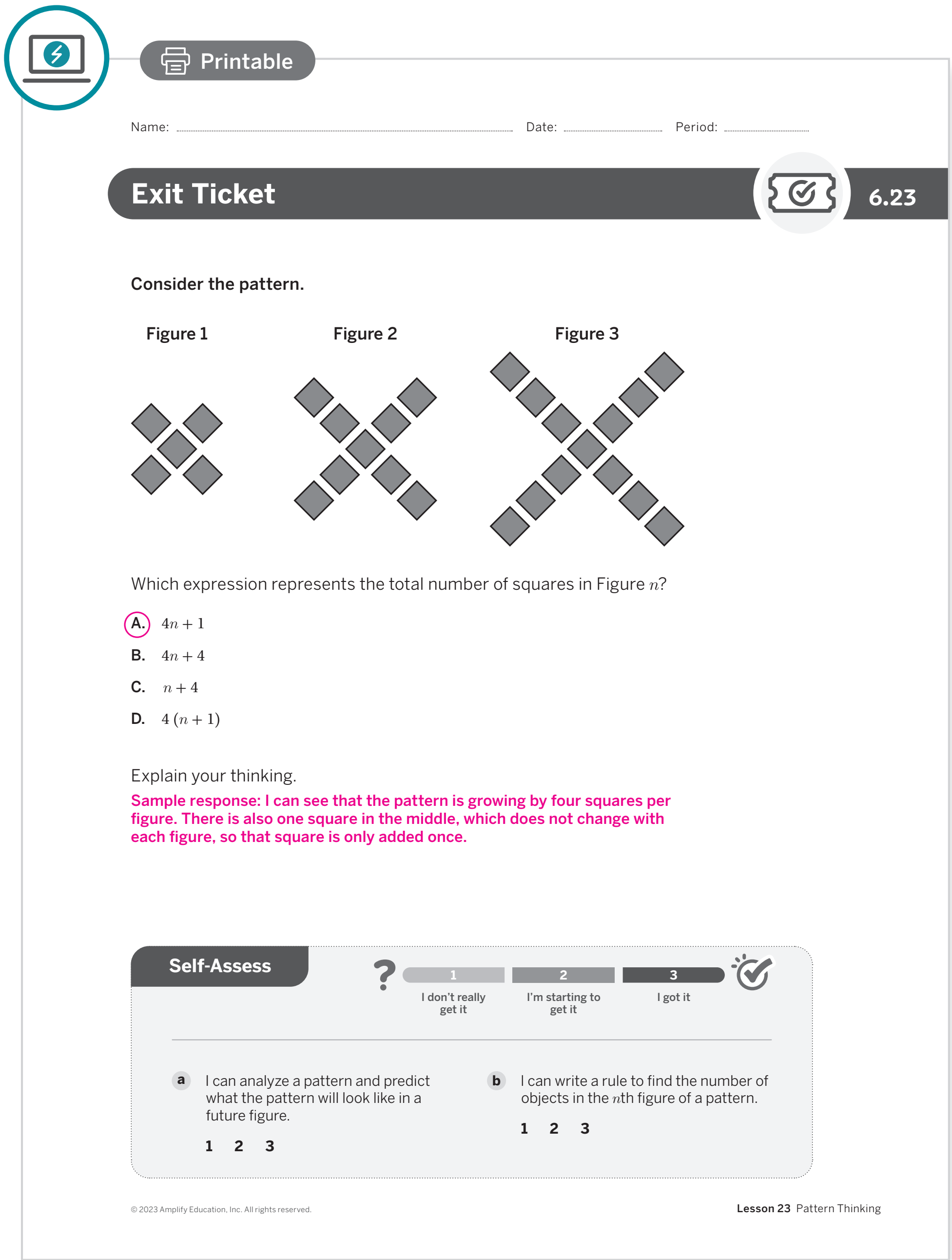
- Did anything surprise you while reading the narratives of this unit?
- Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing a growing pattern and identifying the expression that models the pattern's growth.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.23

Consider the pattern.

Which expression represents the total number of squares in Figure n?

[✓ A.] $4n + 1$

B. $4n + 4$

C. $n + 4$

D. $4(n + 1)$

Explain your thinking.

Sample response: I can see that the pattern is growing by four squares per figure. There is also one square in the middle, which does not change with each figure, so that square is only added once.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can analyze a pattern and predict what the pattern will look like in a future figure. 1 2 3

b I can write a rule to find the number of objects in the nth figure of a pattern. 1 2 3

Lesson 23 Pattern Thinking

Success looks like . . .

- **Goal:** Writing an algebraic expression to describe a linear relationship.
  - » Selecting the correct expression for the total number of squares in Figure n.
- **Language Goal:** Justifying whether algebraic expressions are equivalent **(Speaking and Listening, Writing)**.

Suggested next steps

If students choose all expressions with $4n$, consider:

- Reviewing how to represent constant terms in expressions. These students likely understood how to represent a growth of 4 diamonds in each step, but don't understand how to represent a constant term in the expression.
- Asking, "Are $4n + 1$, $4n + 4$, and $4(n + 1)$ all equivalent expressions?"
- Assigning Practice Problems 1 and 3.

Professional Learning

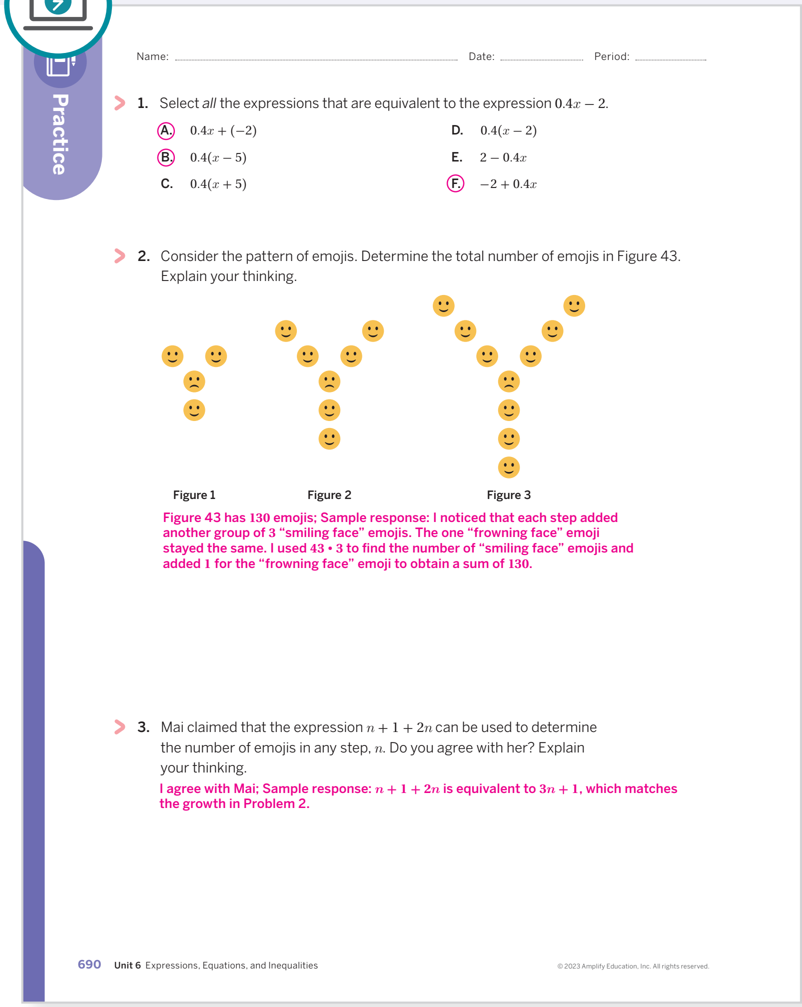
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students gotten better at looking for and making use of structure?
- In earlier lessons, students use proportional reasoning to model and make sense of patterns. How did that support students in writing algebraic expressions to describe patterns? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Select *all* the expressions that are equivalent to the expression $0.4x - 2$.

   [✓ A.] $0.4x + (-2)$
   [✓ B.] $0.4(x - 5)$
   C. $0.4(x + 5)$
   D. $0.4(x - 2)$
   E. $2 - 0.4x$
   [✓ F.] $-2 + 0.4x$

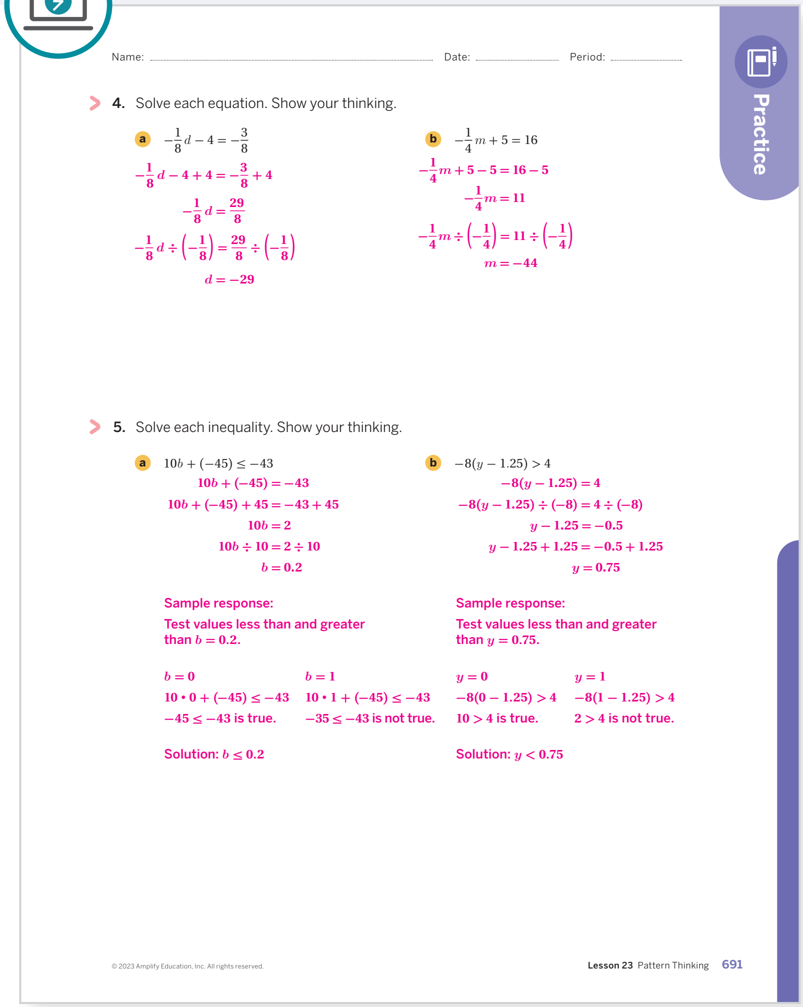
2. Consider the pattern of emojis. Determine the total number of emojis in Figure 43. Explain your thinking.

Figure 1 Figure 2 Figure 3

Figure 43 has 130 emojis; Sample response: I noticed that each step added another group of 3 "smiling face" emojis. The one "frowning face" emoji stayed the same. I used 43 • 3 to find the number of "smiling face" emojis and added 1 for the "frowning face" emoji to obtain a sum of 130.

3. Mai claimed that the expression $n + 1 + 2n$ can be used to determine the number of emojis in any step, n. Do you agree with her? Explain your thinking.

I agree with Mai; Sample response: $n + 1 + 2n$ is equivalent to $3n + 1$, which matches the growth in Problem 2.



Practice

Name: Date: Period:

4. Solve each equation. Show your thinking.

a $-\frac{1}{8}d - 4 = -\frac{3}{8}$

$-\frac{1}{8}d - 4 + 4 = -\frac{3}{8} + 4$

$-\frac{1}{8}d = \frac{29}{8}$

$-\frac{1}{8}d \div \left(-\frac{1}{8}\right) = \frac{29}{8} \div \left(-\frac{1}{8}\right)$

$d = -29$

b $-\frac{1}{4}m + 5 = 16$

$-\frac{1}{4}m + 5 - 5 = 16 - 5$

$-\frac{1}{4}m = 11$

$-\frac{1}{4}m \div \left(-\frac{1}{4}\right) = 11 \div \left(-\frac{1}{4}\right)$

$m = -44$

5. Solve each inequality. Show your thinking.

a $10b + (-45) \le -43$

$10b + (-45) = -43$

$10b + (-45) + 45 = -43 + 45$

$10b = 2$

$10b \div 10 = 2 \div 10$

$b = 0.2$

Sample response:

Test values less than and greater than $b = 0.2$.

$b = 0$	$b = 1$
$10 \cdot 0 + (-45) \le -43$	$10 \cdot 1 + (-45) \le -43$
$-45 \le -43$ is true.	$-35 \le -43$ is not true.

Solution: $b \le 0.2$

b $-8(y - 1.25) > 4$

$-8(y - 1.25) = 4$

$-8(y - 1.25) \div (-8) = 4 \div (-8)$

$y - 1.25 = -0.5$

$y - 1.25 + 1.25 = -0.5 + 1.25$

$y = 0.75$

Sample response:

Test values less than and greater than $y = 0.75$.

$y = 0$	$y = 1$
$-8(0 - 1.25) > 4$	$-8(1 - 1.25) > 4$
$10 > 4$ is true.	$2 > 4$ is not true.

Solution: $y < 0.75$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Warm-up	2
	3	Activity 3	3
Spiral	4	Unit 6 Lesson 10	1
	5	Unit 6 Lesson 10	1

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

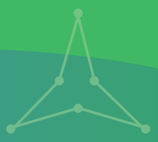
UNIT 7

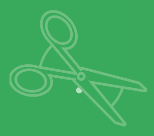
Angles, Triangles, and Prisms

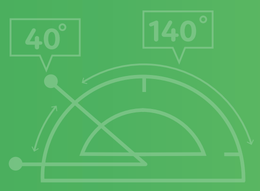
This unit is about the math of what can be seen and what can be held. Through constructing and drawing, students explore relationships among angles, lines, surfaces, and solids.

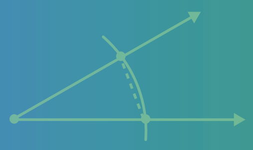
Essential Questions

- When do combinations of angles form special angles?
- Given certain segments and angles, how many unique polygons can be made?
- What shapes can be seen when you slice through solid figures?
- *(By the way, why are triangles stronger than squares?)*









Key Shifts in Mathematics

Focus

In this unit . . .

Students study and apply angle relationships, learning to understand and use the terms *complementary*, *supplementary*, *vertical angles*, and *unique*. This work gives them practice with rational numbers and equations that represent angle relationships. Students also investigate whether sets of angle and side length measurements determine unique triangles, multiple triangles, or fail to form any triangles. Students analyze and describe cross sections of prisms, pyramids, and other polyhedra. They extend their understanding of finding the volume of a right rectangular prism to any prism, and solve problems involving area, surface area, and volume.

Coherence

Previously . . .

Students last worked with angles in Grade 4, where they measured, composed, and decomposed angles. In Grade 5, students classified two-dimensional figures based on their properties. In Grade 6, they found the volume of right rectangular prisms and the surface area of three-dimensional figures.

Coming soon . . .

Students will explore congruence and similarity in Grade 8 by transforming two-dimensional figures. They will continue their work with volumes, discussing volumes of cones and cylinders. Constructions and cross sections will reappear in high school geometry.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students construct various shapes given certain conditions, and build understanding about the properties of those shapes through exploration (Lessons 8 and 9).

Procedural Fluency

Students practice modeling angle relationships with equations, then solve the equations to find unknown angle measurements (Lesson 6).

Application

Students apply their understanding of what surface area and volume measure for a solid and make decisions about which is most appropriate for solving a particular problem (Lesson 17).

Journey to the Third Dimension

SUB-UNIT

1

Lessons 2–7

Angle Relationships

Students notice that some angles can join forces to form complementary, supplementary, or vertical angles — and that these relationships play important roles in certain polygons. Students synthesize their understanding of new angle relationships and solving equations to find the measure of unknown angles.

Narrative: Whether you plan to sail across an ocean or not, using some angles to calculate unknown angles can be a powerful tool.

SUB-UNIT

2

Lessons 8–12

Drawing Polygons With Given Conditions

Euclid, the straightedge, and a compass. Students join the mathematical tradition of constructing geometric figures. Given certain conditions, they notice that sometimes many figures can be constructed, while at other times, no figure can be constructed. And while it may appear that many figures can be constructed, they could just be identical copies of the same figure.

Narrative: Triangles are an important part of construction. So what are the important parts of triangles?

Launch

Lesson 1

Shaping Up

A social and physical activity kicks off the unit. Students reacquaint themselves with a few common two-dimensional figures as they team up to construct the figures using a length of string and use precise language to defend their constructions.

SUB-UNIT

3

Lessons 13–17

Solid Geometry

Students slice, dice, unfold, wrap, and fill three-dimensionsional figures to discover relationships between their sizes and shapes. To conclude, students design and construct an office building, given building specifications and cost constraints.

Narrative: Volume and surface area are important measures of solids. When peering inside, cross sections are another great tool.

Capstone

Lesson 18

Applying Volume and Surface Area

Students design and construct an office building, given certain specifications and cost constraints. As they consider the factors that affect cost, they experience how tradeoffs are often a necessary part of any planning process.

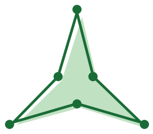
Unit at a Glance

Spoiler Alert: Once you notice the relationship between a prism's base, height, and volume, you'll never need to memorize a formula for its volume ever again.

Assessment

A Pre-Unit Readiness Assessment

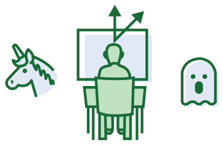
Launch Lesson



1 Shaping Up

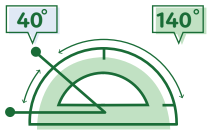
Reintroduce working with polygons and attempt to construct them according to their properties.

Sub-Unit 1: Angle Relationships



2 Relationships of Angles ●

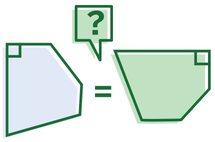
Compose, decompose, and measure angles.



3 Supplementary and Complementary Angles (Part 1)

Determine the measure of an unknown angle, given the measure of a supplementary or complementary angle.

Sub-Unit 2: Drawing Polygons With Given Conditions

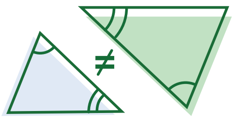


8 Building Polygons (Part 1) ●

Experiment with making polygons with various orders and combinations of side lengths, using linkage strips.

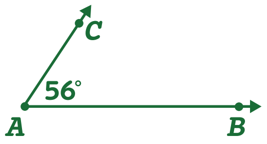
9 Building Polygons (Part 2) ●

Experiment with constructing triangles, given two or three side lengths, and notice that some lengths cannot form a triangle.



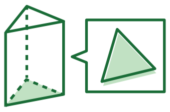
10 Triangles With Three Common Measures

Examine and compare sets of triangles that share three common angle measures or side lengths.



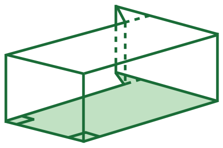
11 Drawing Triangles (Part 1)

Use various tools to draw triangles, noticing that certain conditions determine the number of unique triangles that can be drawn.



15 Decomposing Bases for Area

Determine the volumes of certain right prisms by decomposing the base into triangles and rectangles.



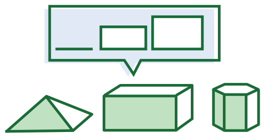
16 Surface Area of Right Prisms

Determine the surface area of prisms and see that a prism's structure lends itself to multiple strategies for finding the surface area.

17 Distinguishing Surface Area and Volume

Distinguish between surface area and volume, and choose which is appropriate for solving different real-world problems.

Capstone Lesson



18 Applying Volume and Surface Area ●

Explore how adjusting the dimensions of a fixed-volume prism changes its surface area.

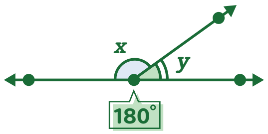
Key Concepts

Lesson 6: Equations can be used to model angle relationships and find unknown angles.
Lesson 11: Determine how many triangles can be constructed given certain conditions.
Lesson 14: Extend understanding of the volume of a right rectangular prism to *any* prism.

Pacing

18 Lessons: 45 min each
3 Assessments: 45 min each
Full Unit: 21 days
● **Modified Unit:** 17 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

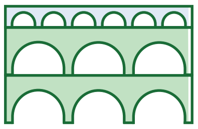


4 Supplementary and Complementary Angles (Part 2)

Determine the measures of non-adjacent supplementary and complementary angles and draw conclusions about the angle relationships of polygons.

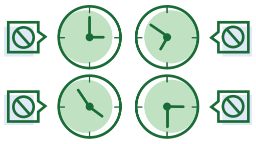
5 Vertical Angles

Notice that vertical angles have equal measures and use this and other angle relationships to solve multi-step problems.



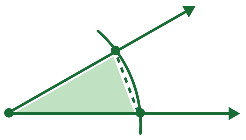
6 Using Equations to Solve for Unknown Angles

Write and solve equations of the form $px + q = r$ and $p(x + q) = r$ to represent angle relationships shown in diagrams.



7 Like Clockwork ●

Explore the close relationship between how time is measured on an analog clock and how rotation is measured using degrees.



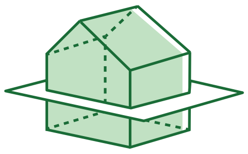
12 Drawing Triangles (Part 2)

Draw triangles given one angle and two side lengths, or three angles.

Assessment

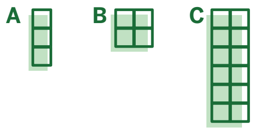
A Mid-Unit Assessment

Sub-Unit 3: Solid Geometry



13 Slicing Solids

Introduce the idea that slicing a three-dimensional figure with a plane results in a two-dimensional cross section.



14 Volume of Right Prisms

Calculate the volume of any right prism by multiplying the area of its base by its height.

Assessment

A End-of-Unit Assessment

● Modifications to Pacing

Lesson 2: This lesson reviews angle relationships from earlier grades, and may be omitted.

Lesson 7: This lesson serves as an exploration into how geometry and equations can work together, but may be omitted.

Lesson 8–9: These two lessons focus on constructing polygons given certain side lengths. You might choose to combine these two lessons, or omit one of them.

Lesson 18: This Capstone lesson has students grapple with the literal costs of maximizing volume or surface area, but may be omitted, as no new standards are introduced.

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	adjacent angles
3	complementary angles supplementary angles
5	vertical angles
13	cross section

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
3, 12, 14, 16	**MLR1:** Stronger and Clearer Each Time
1–3, 5, 8, 10, 13, 17	**MLR2:** Collect and Display
3, 10, 14, 15	**MLR3:** Critique, Correct, Clarify
12, 15	**MLR5:** Co-craft Questions
18	**MLR6:** Three Reads
6, 8, 9, 11, 13, 17	**MLR7:** Compare and Connect
2, 4–6, 8, 11, 14, 16	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket

Additional Practice

Additional required materials include:

Lesson	Materials
2	analog clock (or picture of one)
1, 3–5, 7–12	geometry toolkits
1	lengths of string
1–3, 5, 7–9, 12–18	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
14	rulers marked with centimeters
14	snap cubes
8	sticky notes

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
13	Card Sort
7, 13, 18	Gallery Tour
5	Notice and Wonder
4	Partner Problems
3, 4, 7, 14, 17	Poll the Class
1, 3, 4, 15	Think-Pair-Share
9	True or False?
1, 7	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 12
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 18

Amps

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Social & Collaborative Digital Moments

Featured Activity

Two Sides and One Angle

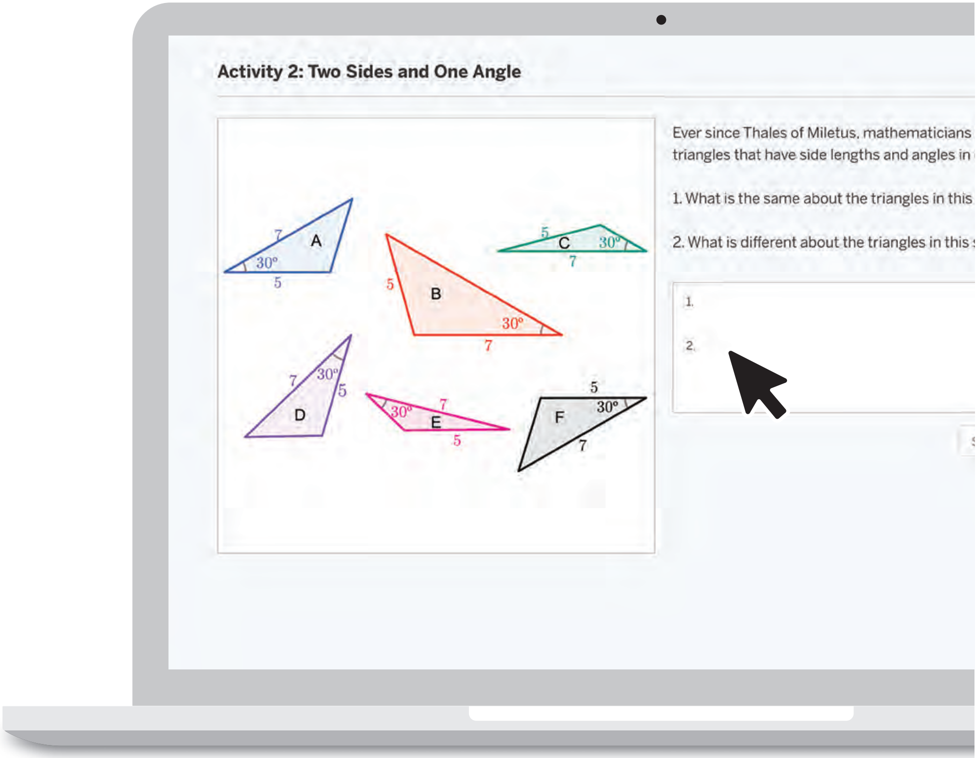
Put on your student hat and work through **Lesson 10, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Precision Timekeeping **(Lesson 7)**
- Is it Identical? **(Lesson 8)**
- What's the Cross Section? **(Lesson 13)**
- Multifaceted **(Lesson 16)**



Activity 2: Two Sides and One Angle

Ever since Thales of Miletus, mathematicians
triangles that have side lengths and angles in

1. What is the same about the triangles in this

2. What is different about the triangles in this

1.

2.

Unit Study
Professional Learning

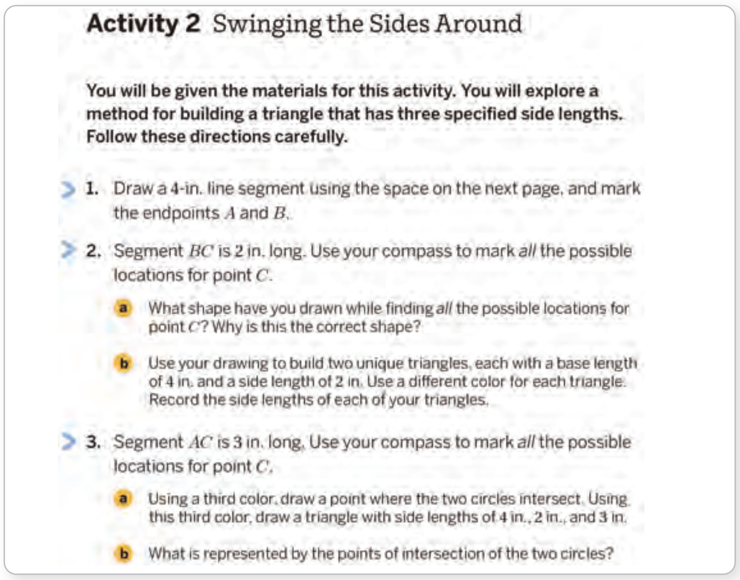
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces the idea of building different polygons with specific side lengths. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 9, Activity 2:**



Activity 2 Swinging the Sides Around

You will be given the materials for this activity. You will explore a method for building a triangle that has three specified side lengths. Follow these directions carefully.

1. Draw a 4-in. line segment using the space on the next page, and mark the endpoints A and B.
2. Segment BC is 2 in. long. Use your compass to mark *all* the possible locations for point C.
   - a. What shape have you drawn while finding *all* the possible locations for point C? Why is this the correct shape?
   - b. Use your drawing to build two unique triangles, each with a base length of 4 in. and a side length of 2 in. Use a different color for each triangle. Record the side lengths of each of your triangles.
3. Segment AC is 3 in. long. Use your compass to mark *all* the possible locations for point C.
   - a. Using a third color, draw a point where the two circles intersect. Using this third color, draw a triangle with side lengths of 4 in., 2 in., and 3 in.
   - b. What is represented by the points of intersection of the two circles?

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

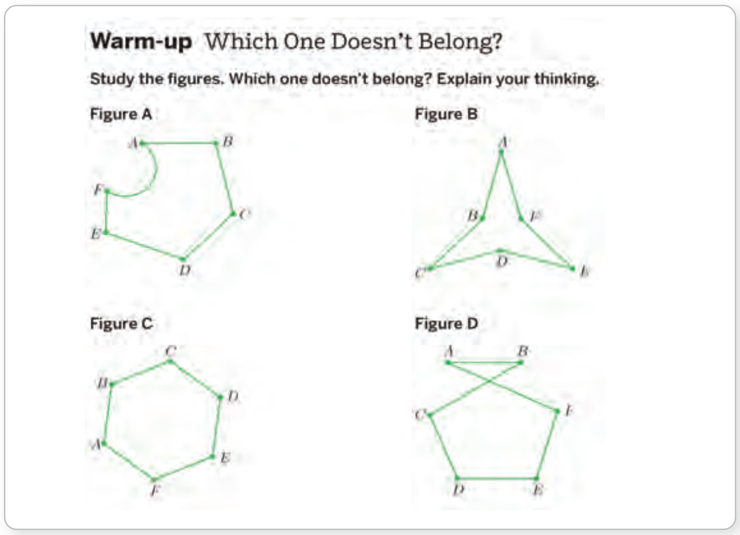
- What was it like to engage in this problem as a learner?
- Do any approaches reveal a misconception or difficulty that might arise for students?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Which One Doesn’t Belong?

Rehearse . . .

How you’ll facilitate the ***Which One Doesn’t Belong?*** instructional routine in **Lesson 1, Warm-up:**



Warm-up Which One Doesn’t Belong?

Study the figures. Which one doesn’t belong? Explain your thinking.

Points to Ponder . . .

- Which answer choice is the low-floor choice that will allow all students to contribute?
- Which answer choice will lead you to the next activity? How could you steer the conversation in that direction if no students suggest it?
- How will you sequence answer choices during the discussion so that everyone has a chance to share ideas, but the discussion still moves in the right direction?

This routine . . .

- Encourages students to think creatively about math.
- Asks students to identify patterns and decide what features or elements stand out from those patterns.
- Engages several learning styles.
- Offers a low-floor, high-ceiling task.

Anticipate . . .

- Students thinking there is only one “correct answer.”
- If you *haven’t* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Support Productive Struggle in Learning Mathematics

This effective teaching practice . . .

- Provides students with the opportunity to wrestle with mathematical concepts and relationships before you intervene, which builds student confidence and perseverance.
- Allows you greater opportunities to monitor student progress, probe for student understanding, and offer differentiated support.

Points to Ponder . . .

- How comfortable are you with allowing students the time to wrestle with mathematical ideas before you intervene?
- When is the right moment to intervene? What can you look for as you monitor student work and student conversations to know when they are engaging in productive struggle vs. unproductive struggle?

Math Language Development

MLR1: Stronger and Clearer Each Time

MLR1 appears in Lessons 3, 12, 14, and 16.

- In these lessons, opportunities are provided to have students first craft an initial draft of their response to a particular problem. Students then share their responses with 2–3 partners to receive feedback and then revise or refine their original response.
- Often, specific suggestions are provided to help reviewing partners look for clarity in the responses. For example:
  - In Lesson 14, display the suggested questions so that reviewers look for whether the responses indicate how students know the given figure is a prism and what each layer must look like when the figure is a prism.
  - In Lesson 16, reviewers are encouraged to ask how students can add more detail to their responses or draw a picture to support their explanations.

Point to Ponder . . .

- How can you help your students grow in both giving and receiving feedback? How will you structure your classroom culture so that there is an expected norm in which your students feel supported, not criticized?

Differentiated Support

Accessibility: *Optimize Access to Tools, Optimize Access to Technology*

Opportunities to provide physical manipulatives, the use of technology (through the Amps slides), or other tools appear in Lessons 2–9, 11–14, 17, and 18.

- In Lessons 11 and 12, students can use an interactive digital tool to create triangles with given conditions.
- In Lesson 13, students can use technology to manipulate a two-dimensional plane in three dimensions to view highlighted cross sections.
- In Lesson 14, students can use digital blocks to build layers of a prism to better understand how volume is related to slices.
- In Lesson 18, pre-assemble the nets from the Warm-up PDF to form three-dimensional solids that students can physically hold and examine.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology, physical manipulatives, or other tools to deepen student understanding?

Unit Assessments

- Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students to better visualize all the major concepts? Do you think your students will generally:
  - Struggle to visualize cross sections or specific faces of three-dimensional figures?
  - Have trouble organizing their work and, as a result, skip steps or fail to finish problems?
  - Not have enough geometric intuition and, as a result, struggle with creating multiple figures matching the same criteria?

Building Math Identity and Community

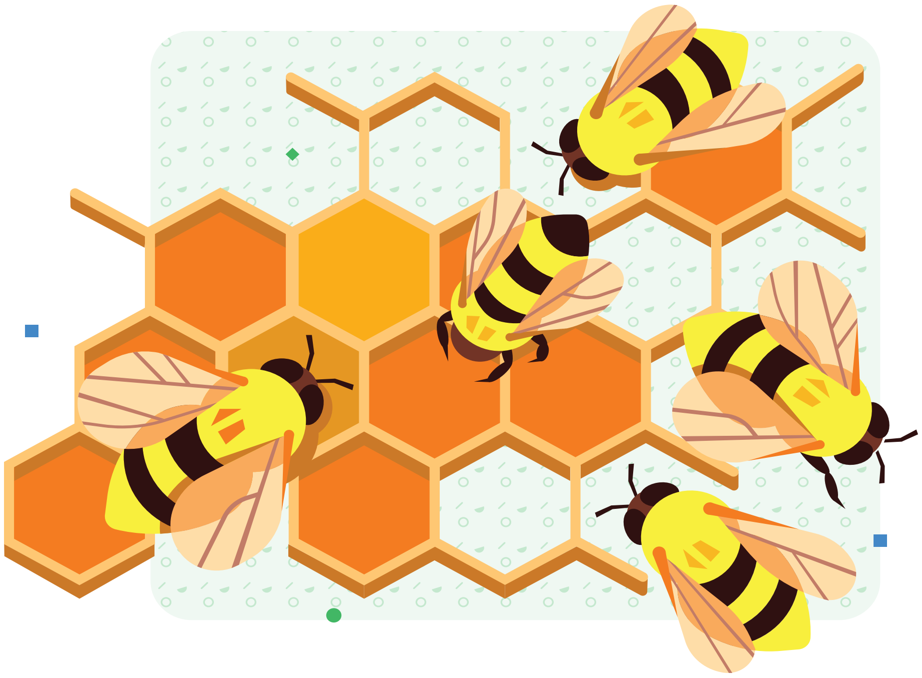
Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their social awareness and self-awareness skills.

Points to Ponder . . .

- Are students able to take on other students' perspectives, recognizing the possible validity of their responses even though their solution process follows a different path?
- Are students able to reflect on their own answers, accurately assessing the effectiveness of their solution strategies well as the reasonableness of their results?

UNIT 7 | **LESSON 1 – LAUNCH**



Shaping Up

Let's reacquaint ourselves with polygons.

Focus

Goals

1. **Language Goal:** Recognize and classify polygons according to their geometric properties. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Reason about geometric properties that are shared by or are unique to certain polygons. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students are reintroduced to working with polygons and informally constructing them according to their properties. Students are exposed to their geometry toolkit and experiment with how to use the tools to measure and construct. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Previously

Students classified polygons in Grade 5 and studied angles in Grade 4.

Coming Soon

Students will study angles more closely. They will review right angles, straight angles, and angles which compose a full circle. Students will use pattern blocks and simple equations to determine the measurements of angles.

Rigor

- Students use visual models to develop **conceptual understanding** of polygons.
- Students use geometry toolkits to build **conceptual understanding** of the properties of polygons.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
7 min	25 min	5 min	10 min
Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut and folded, one set of cards per group
- geometry toolkits: compasses, protractors, rulers
- lengths of string, one per group (about 5 ft each) plus a few short ones for students to work on the *Are you ready for more?*

Math Language Development

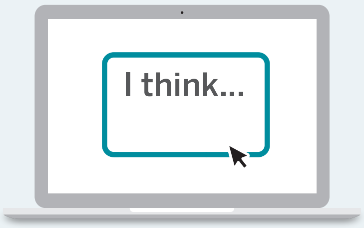
Review words

- *hexagon*
- *polygon*

Amps | **Featured Activity**

Activity 1
See Student Thinking

Students are asked to explain what they already know about several shapes, giving you insight into what vocabulary students recall and what needs to be added or reviewed.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel nervous about defending their construction in Activity 1. Remind students that the purpose of sharing is to seek and offer help when needed, but that respect should be the overarching guide for all discussion. Students should negotiate conflict positively and constructively to increase understanding, not to make another student feel bad because of an error.

Modifications to Pacing

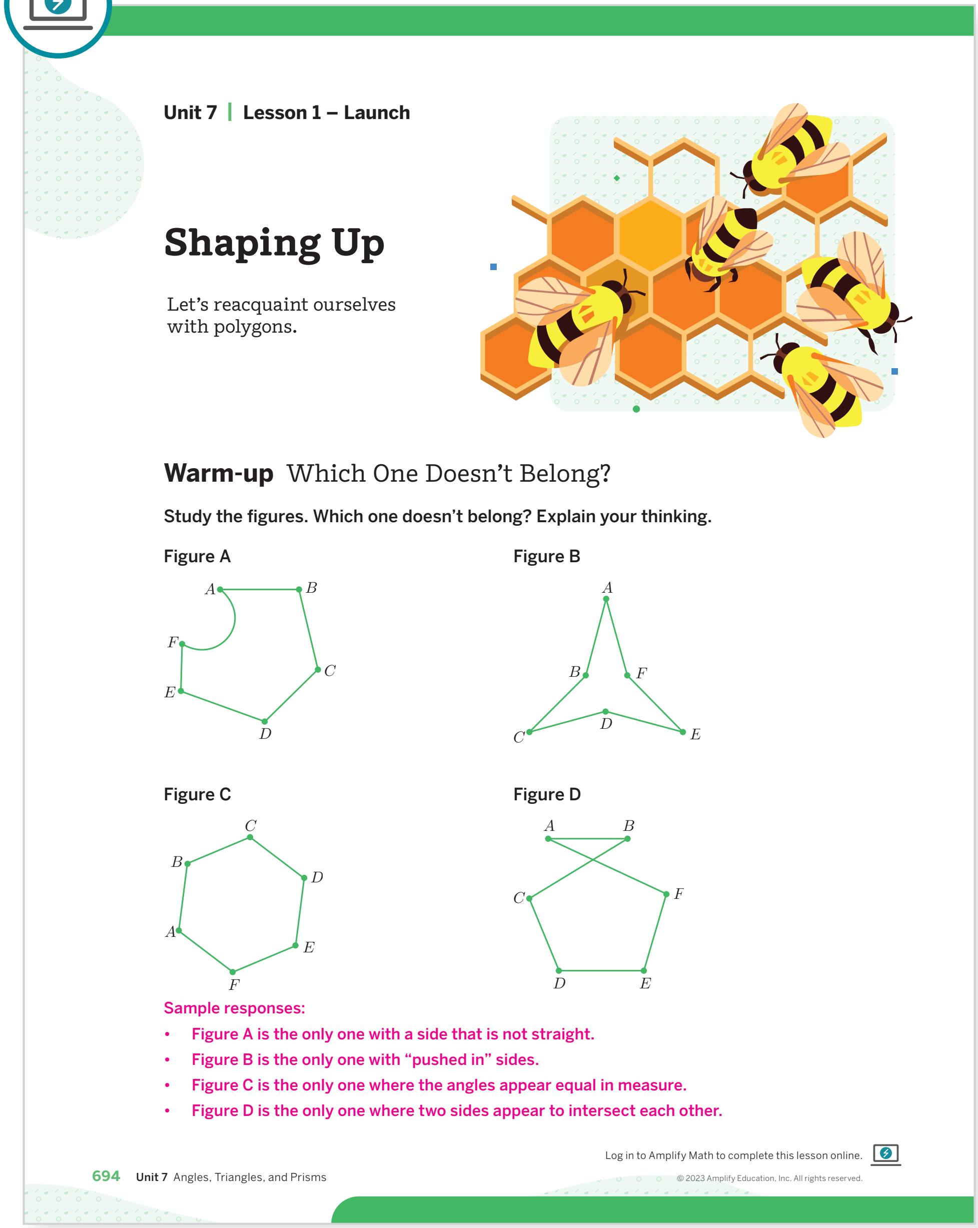
You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Independent | 7 min

Warm-up Which One Doesn't Belong?

Students compare four figures — each with six vertices — to reason about the properties of polygons.



Unit 7 | Lesson 1 – Launch

Shaping Up

Let's reacquaint ourselves with polygons.

Warm-up Which One Doesn't Belong?

Study the figures. Which one doesn't belong? Explain your thinking.

Figure A

Figure B

Figure C

Figure D

Sample responses:

- Figure A is the only one with a side that is not straight.
- Figure B is the only one with "pushed in" sides.
- Figure C is the only one where the angles appear equal in measure.
- Figure D is the only one where two sides appear to intersect each other.

Log in to Amplify Math to complete this lesson online.

694 Unit 7 Angles, Triangles, and Prisms

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1 Launch

Conduct the ***Which One Doesn't Belong*** routine. Have students share their explanation with a partner prior to them recording their response.

2 Monitor

Help students get started by having them find one difference between two shapes first, and then checking whether the same difference exists between other shapes.

Look for points of confusion:

- **Stopping after finding one reason for one shape.** Encourage students to find as many reasons for as many shapes as they can.

Look for productive strategies:

- **Finding these possible reasons**: Figure A is the only one that has a non-straight side. Figure B is the only one with "pushed-in" sides. Figure C is the only one where the angles appear to have equal measure. Figure D is the only one where two sides appear to intersect each other.

3 Connect

Display the four figures to the whole class.

Have students share at least one reason why each figure might not belong, starting with Figures A and D. Discuss Figures B and C last. After each reason, ask the class whether they agree or disagree.

Ask, "You may have called shapes *hexagons* if they have six vertices. Which shapes here would you say are *actually* hexagons?"

Highlight that a *polygon* is a closed, two-dimensional shape with straight sides that do not cross each other. Have students discuss which shapes in the Warm-up are polygons.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

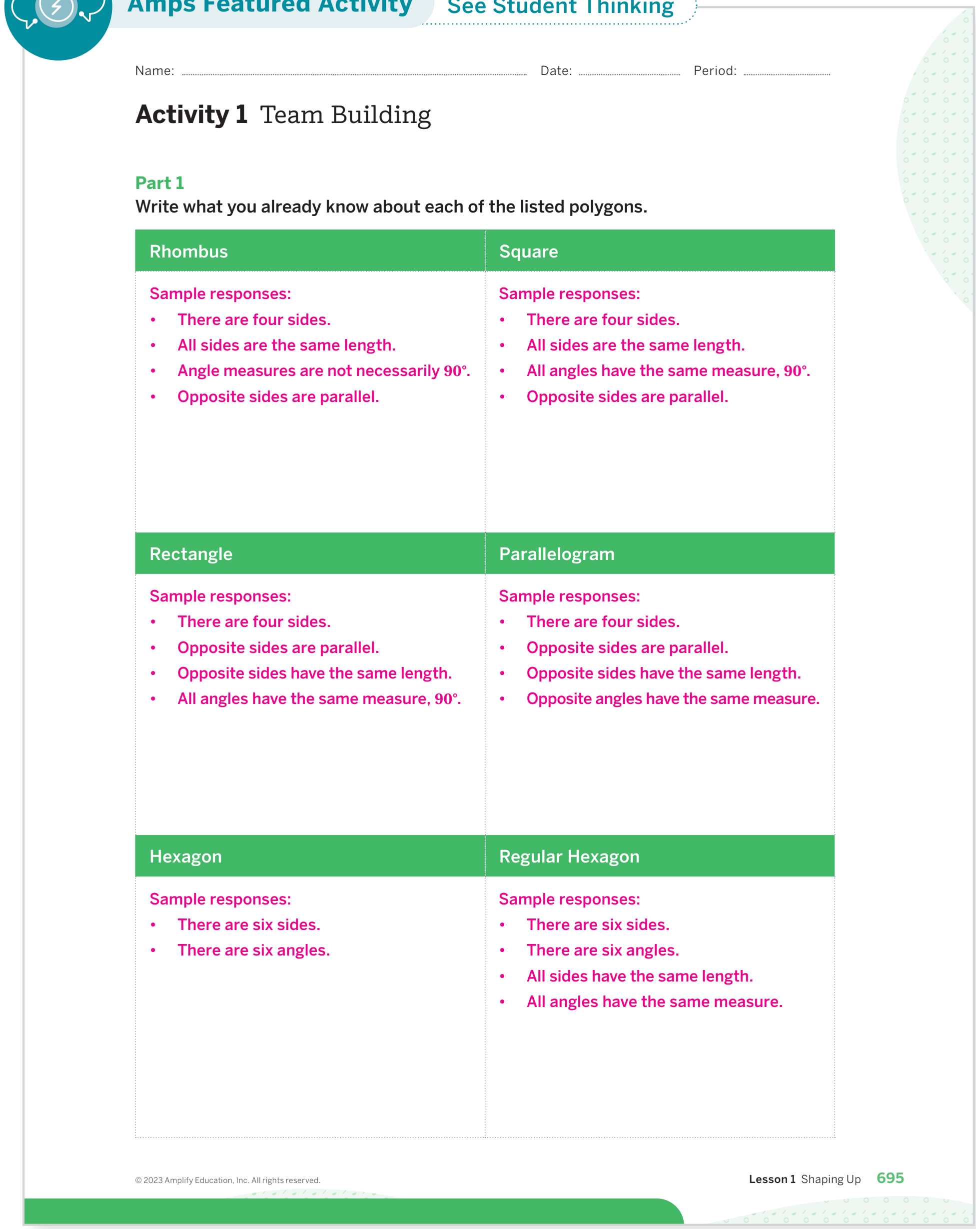
Suggest that students compare two figures at a time to look for similarities and differences. For example, comparing Figures B and C will reveal that they both consist of straight line segments, but Figure B's sides are "pushed in."

Small Groups | 25 min

Activity 1 Team Building

In teams, students use string to build polygons and orally defend their constructions, in order to reacquaint themselves with the properties of polygons.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 1 Team Building

Part 1
Write what you already know about each of the listed polygons.

Rhombus	Square
Sample responses: There are four sides. All sides are the same length. Angle measures are not necessarily 90°. Opposite sides are parallel.	Sample responses: There are four sides. All sides are the same length. All angles have the same measure, 90°. Opposite sides are parallel.
Rectangle	**Parallelogram**
Sample responses: There are four sides. Opposite sides are parallel. Opposite sides have the same length. All angles have the same measure, 90°.	Sample responses: There are four sides. Opposite sides are parallel. Opposite sides have the same length. Opposite angles have the same measure.
Hexagon	**Regular Hexagon**
Sample responses: There are six sides. There are six angles.	Sample responses: There are six sides. There are six angles. All sides have the same length. All angles have the same measure.

Lesson 1 Shaping Up 695

1 Launch

Activate students' prior knowledge by asking them to recall the properties of polygons they learned in prior grades. Conduct the *Think-Pair-Share* routine. Distribute the strings and envelopes with pre-cut, folded cards from the Activity 1 PDF. Be sure all students in each group participate in the activity and can defend their construction.

2 Monitor

Help students get started by reading the shape's properties on the second section of the card together. Stop and correct groups if their polygons are not closed.

Look for points of confusion:

- **Difficulty forming segments of the same length.** Suggest students use folding to match the side lengths.
- **Difficulty forming a right angle.** Have students explore the room to find an object that may help them form a right angle.
- **Difficulty forming a regular hexagon.** Ask students if they can find a way to informally mark the measures of their angles.

Look for productive strategies:

- Using precise vocabulary to discuss aspects of their constructed shape.
- Using a square, rhombus or a rectangle to show a parallelogram.

Activity 1 continued >

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Before students complete Part 1, draw — or ask a student volunteer to draw — examples of each type of polygon listed in the table. Display the following questions that students can ask themselves as they think about the properties of each type of polygon.

- How many sides are there? How many angles are there?
- Are all the sides the same length? Are some of the sides the same length?
- What is true about the angle measures?
- Are any sides parallel?

Math Language Development

MLR2: Collect and Display

While students complete Part 2, circulate and listen to their conversations. Record the words and phrases used that show a developing understanding of the vocabulary needed to describe the shapes. Listen for phrases such as, "All squares are rectangles, but not every rectangle is a square." Display the language collected for the whole class to use as a reference during further discussions in the lesson and unit.

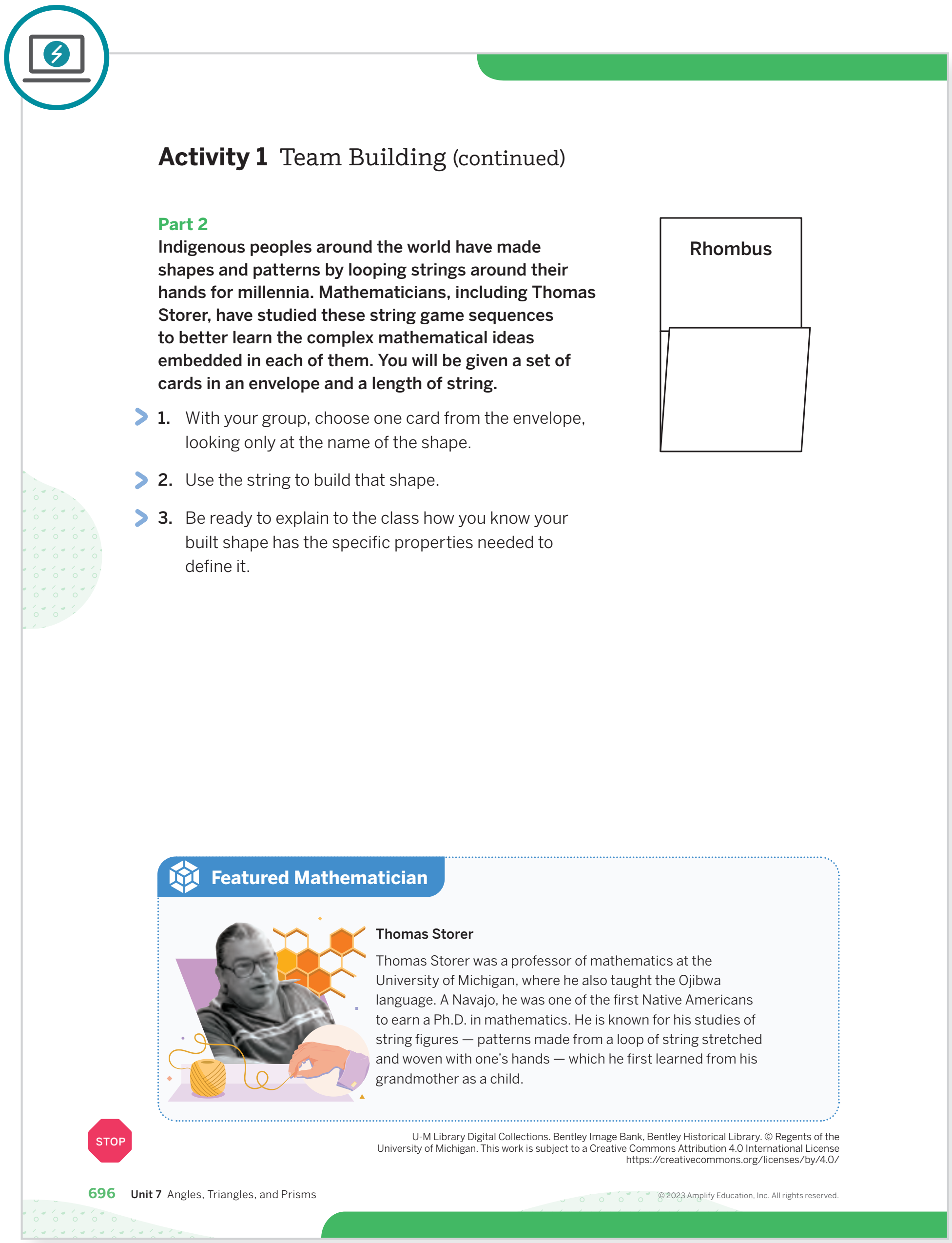
English Learners

Display a visual representation of the shapes next to the collected words and phrases related to the shape.

Small Groups | 25 min

Activity 1 Team Building (continued)

In teams, students use string to build polygons and orally defend their constructions, to reacquaint themselves with the properties of polygons.



Activity 1 Team Building (continued)

Part 2

Indigenous peoples around the world have made shapes and patterns by looping strings around their hands for millennia. Mathematicians, including Thomas Storer, have studied these string game sequences to better learn the complex mathematical ideas embedded in each of them. You will be given a set of cards in an envelope and a length of string.

1. With your group, choose one card from the envelope, looking only at the name of the shape.
2. Use the string to build that shape.
3. Be ready to explain to the class how you know your built shape has the specific properties needed to define it.

Rhombus

Featured Mathematician

Thomas Storer

Thomas Storer was a professor of mathematics at the University of Michigan, where he also taught the Ojibwa language. A Navajo, he was one of the first Native Americans to earn a Ph.D. in mathematics. He is known for his studies of string figures — patterns made from a loop of string stretched and woven with one's hands — which he first learned from his grandmother as a child.

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STOP

696 Unit 7 Angles, Triangles, and Prisms

3 Connect

Have groups of students share what their shape looks like and how they can prove it has the properties that define it. Sequence the shapes from least to most sophisticated. After each group shares, encourage other groups to critique any claims made or ask probing questions, such as "How do you know? Will that always work?"

Highlight that each polygon can be reduced to the set of properties that define it. Since this unit will require students to be precise about angle and side length measurements, it is important that they look carefully at these details.

Featured Mathematician

Thomas Storer

Have students read about featured mathematician Thomas Storer, who studied string figures — handmade patterns woven from a loop of string.

Whole Class | 5 min

Summary Journey to the Third Dimension

Review and synthesize how the properties of polygons play a role when honeybees — and humans — select shapes to use when building.

Narrative Connections

Unit 7 Angles, Triangles, and Prisms

Journey to the Third Dimension

Hooray for the humble honeybee! These pint-sized pollinators might not look like much, but in the whole animal kingdom, who can match the bee's ingenuity?

They are the industrious and dedicated builders of the natural world. They even work weekends!

And their tool of choice is the hexagon. Every honeycomb is a dazzling pattern of interlocking hexagons, each one a sturdy, stackable cubbyhole for storing honey and housing their young.

Using a brain the size of a sesame seed, the honeybee not only understands the shape of hexagons, but can build them in three dimensions. Like architects and engineers, the honeybee takes an idea that once only lived in its brain and, layer-by-layer, brings it to the world.

Why did the bee choose the hexagon to use? Have humans chosen a shape to use when building? What is important about the shapes we choose to use?

Some shapes hold their form better than others. Some shapes fit together more easily. In this unit, you will explore what makes certain shapes special, build those shapes, and think more deeply about three-dimensional shapes.

Welcome to Unit 7.

Lesson 1 Shaping Up 697

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary. Have students read the Summary or have a student volunteer read it aloud. Call attention to the honeycomb pattern behind the text.

Have students share what they notice about the honeycomb.

Ask students whether humans have chosen a common shape, like the honeybee has, when building. There is no right answer, of course, but the discussion may take interesting directions. Some students may suggest that squares, rectangles, and triangles may be most often used by humans. Other students may point out that humans have also used circles for dwellings or other buildings.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful *reflection*, consider asking:

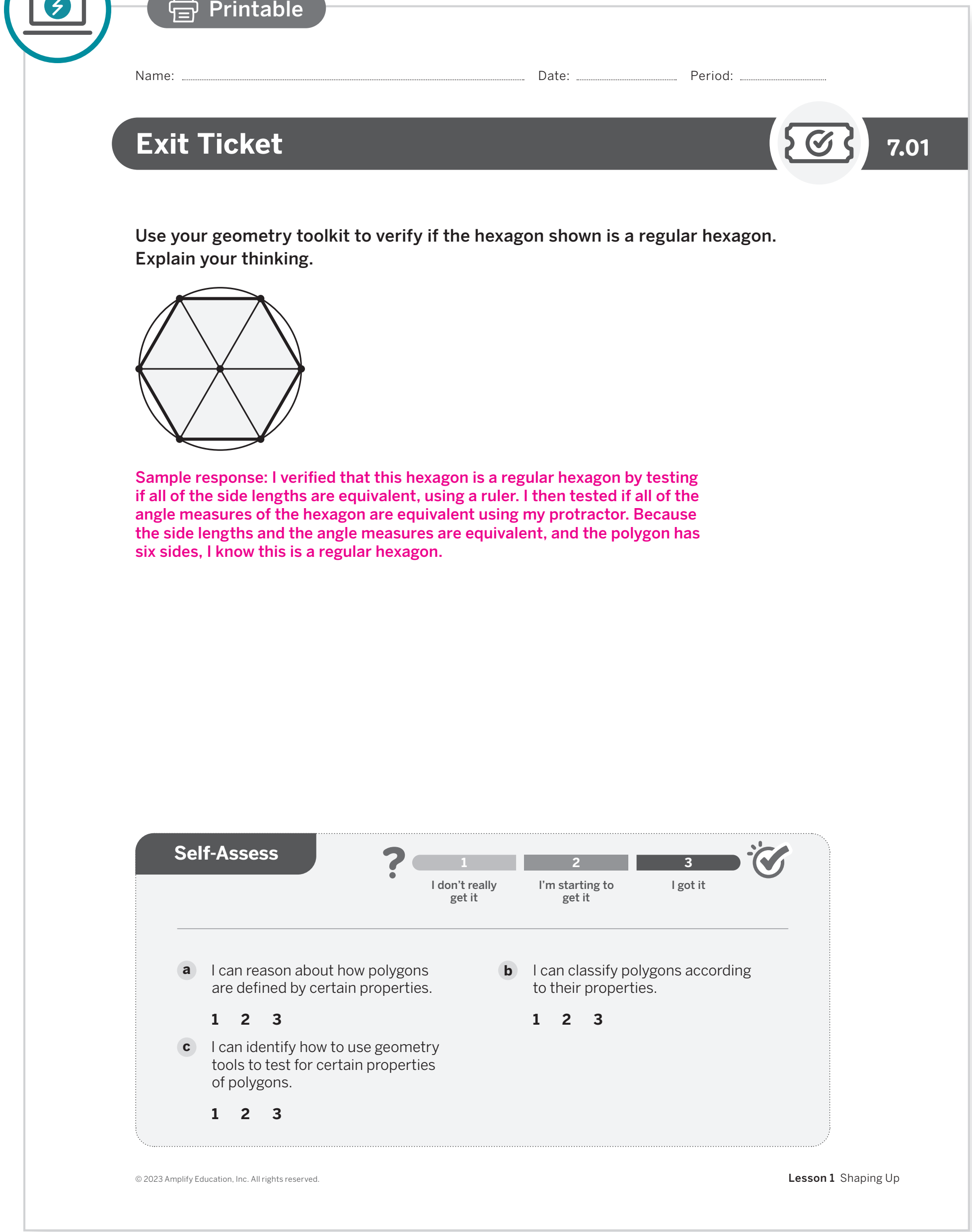
- "What strategies or tools did you find helpful today in classifying polygons? How were they helpful?"
- "What characteristics or properties did you discuss today when describing and building polygons?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by determining whether a certain hexagon is a regular hexagon, using their geometry toolkits.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 7.01

Use your geometry toolkit to verify if the hexagon shown is a regular hexagon. Explain your thinking.

Sample response: I verified that this hexagon is a regular hexagon by testing if all of the side lengths are equivalent, using a ruler. I then tested if all of the angle measures of the hexagon are equivalent using my protractor. Because the side lengths and the angle measures are equivalent, and the polygon has six sides, I know this is a regular hexagon.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can reason about how polygons are defined by certain properties.

1 2 3

b I can classify polygons according to their properties.

1 2 3

c I can identify how to use geometry tools to test for certain properties of polygons.

1 2 3

Lesson 1 Shaping Up

Success looks like . . .

- **Language Goal:** Recognizing and classifying polygons according to their geometric properties. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Reasoning about geometric properties that are shared by or are unique to certain polygons. **(Speaking and Listening, Reading and Writing)**
  - » Verifying that the hexagon is a regular hexagon.

Suggested next steps

If students think they need to measure all segments, including the radii, consider:

- Having them trace the outline of the hexagon and ask them to review the properties of a regular hexagon.
- You might also consider extending this line of thinking by asking them whether, for a regular hexagon, these radii will always have the same measure as the side lengths.

If students do not understand that all the angles must have the same measure, consider:

- Referring them back to Activity 1 to identify what made the regular hexagon special.

Professional Learning

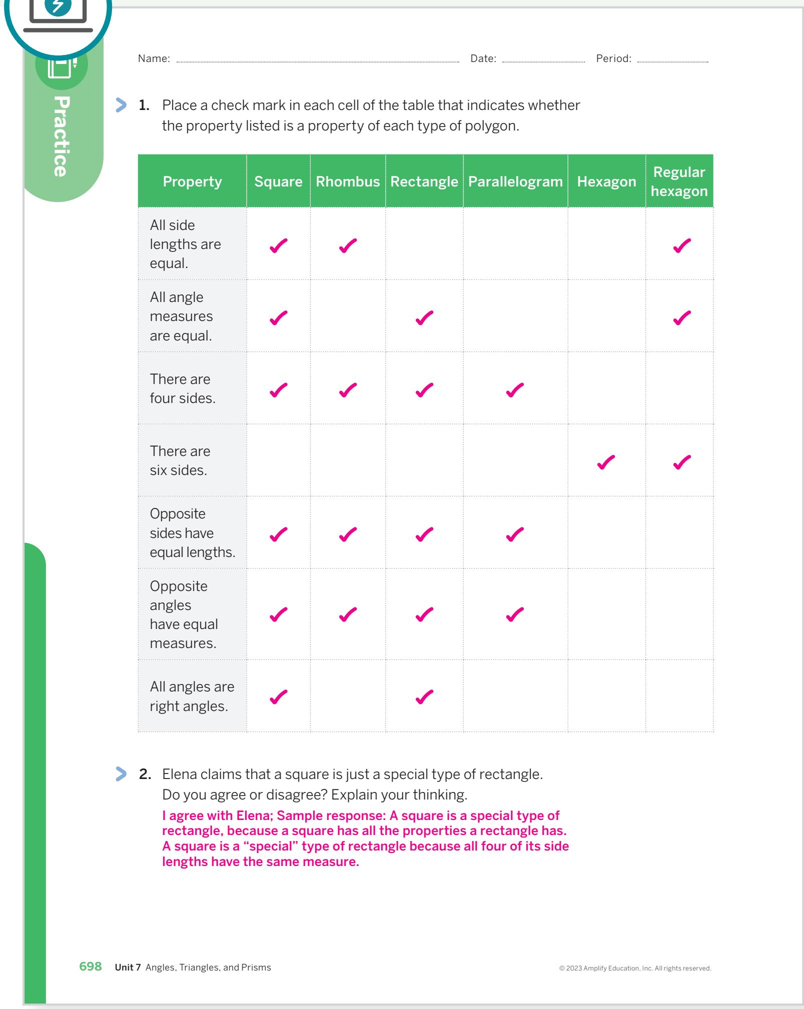
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find frustrating about Activity 1? What helped them work through this frustration?
- During the discussion about the constructed polygons, how did you encourage each student to listen to one another's strategies? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

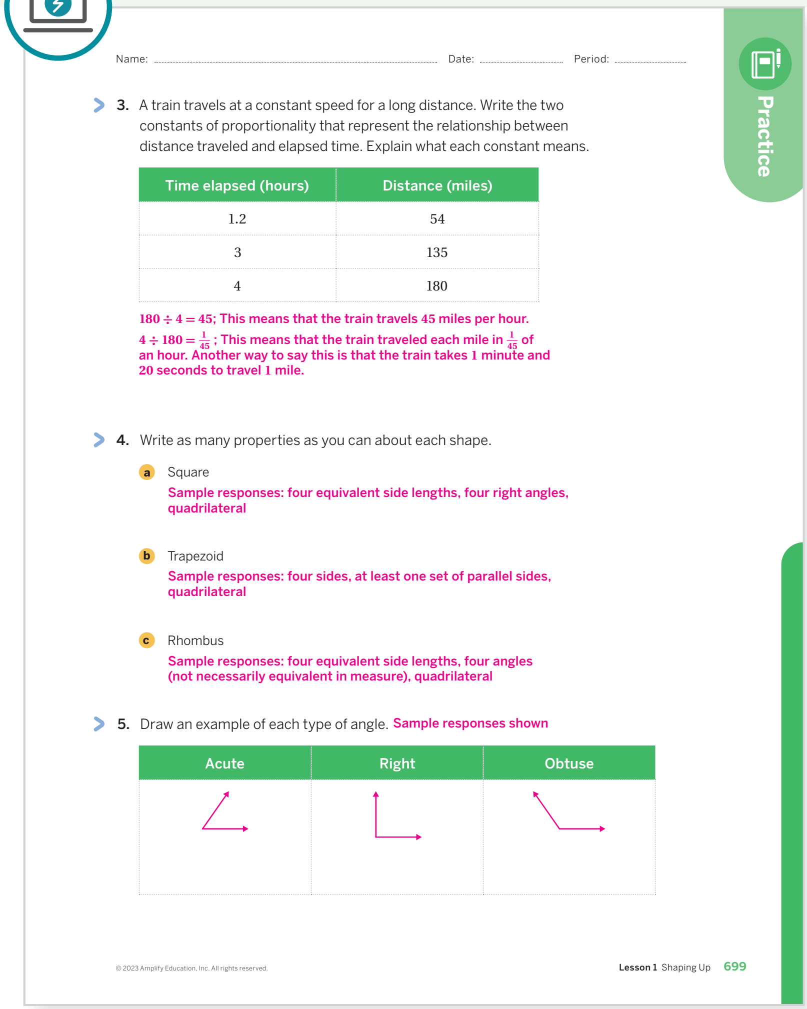
Name: ________ Date: ________ Period: ________

1. Place a check mark in each cell of the table that indicates whether the property listed is a property of each type of polygon.

Property	Square	Rhombus	Rectangle	Parallelogram	Hexagon	Regular hexagon
All side lengths are equal.	✓	✓				✓
All angle measures are equal.	✓		✓			✓
There are four sides.	✓	✓	✓	✓		
There are six sides.					✓	✓
Opposite sides have equal lengths.	✓	✓	✓	✓		
Opposite angles have equal measures.	✓	✓	✓	✓		
All angles are right angles.	✓		✓			

2. Elena claims that a square is just a special type of rectangle. Do you agree or disagree? Explain your thinking.

I agree with Elena; Sample response: A square is a special type of rectangle, because a square has all the properties a rectangle has. A square is a "special" type of rectangle because all four of its side lengths have the same measure.



Practice

Name: ________ Date: ________ Period: ________

3. A train travels at a constant speed for a long distance. Write the two constants of proportionality that represent the relationship between distance traveled and elapsed time. Explain what each constant means.

Time elapsed (hours)	Distance (miles)
1.2	54
3	135
4	180

$180 \div 4 = 45$; This means that the train travels 45 miles per hour.
$4 \div 180 = \frac{1}{45}$; This means that the train traveled each mile in $\frac{1}{45}$ of an hour. Another way to say this is that the train takes 1 minute and 20 seconds to travel 1 mile.

4. Write as many properties as you can about each shape.

a. Square
Sample responses: four equivalent side lengths, four right angles, quadrilateral

b. Trapezoid
Sample responses: four sides, at least one set of parallel sides, quadrilateral

c. Rhombus
Sample responses: four equivalent side lengths, four angles (not necessarily equivalent in measure), quadrilateral

5. Draw an example of each type of angle. **Sample responses shown**

Acute	Right	Obtuse

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	3
Spiral	3	Unit 2 Lesson 2	2
	4	Grade 4	1
Formative ⚡	5	Unit 7 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 1
Angle Relationships

Whole Class

In this Sub-Unit, students notice that some angles can join forces to form complementary, supplementary, or vertical angles — and that these relationships play important roles in certain polygons.

SUB-UNIT

1 Angle Relationships

Narrative Connections

Did radio kill the aviation star?

Few pilots are more famous than Amelia Earhart. She was a national hero and a pioneer. She was the first woman to fly solo across the Atlantic, and the breaker of countless distance, speed, and altitude records.

But tragedy struck in 1937, when she attempted to fly around the world.

For the flight, her plane was outfitted with state-of-the-art radio equipment. This would allow her to communicate with nearby ships. Those ships would then use radio signals to get a bearing on her plane.

The problem was that neither Earhart nor her navigator Fred Noonan, had training experience with radios. This led to a breakdown in communication, which cost the two of them their lives while flying over the South Pacific.

Traditionally, navigators relied on "celestial navigation" to find their way. Using a tool called a sextant, navigators **measured the angle** of a star (usually the Sun) or other celestial body, over the horizon. They would then cross-reference that angle against a nautical chart. With a few calculations they could get a rough idea of their position on a map.

Unfortunately, heavy rain and clouds over the South Pacific made celestial navigation difficult. Had they not put blind trust in their technology, they might have chosen a flight path with clearer skies, where they could have used this tried-and-true method of navigation.

Narrative Connections

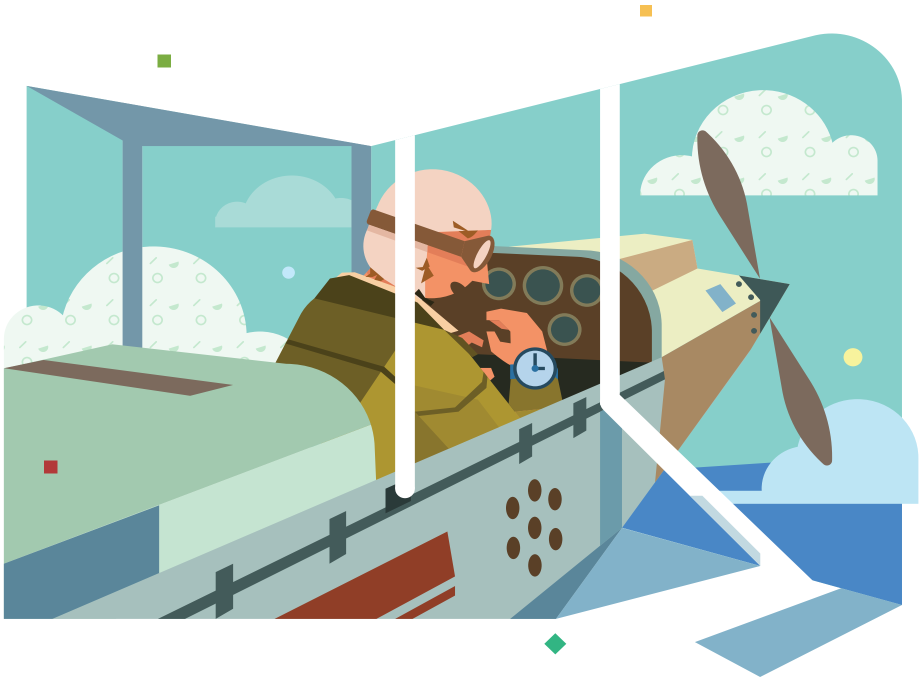
Read the narrative aloud as a class or have students read it individually. Students continue to explore how angles (whether estimated or precisely calculated) can be used for navigation and timekeeping in the following places:

- **Lesson 2, Warm-up:** 3 O'Clock
- **Lesson 6, Warm-up:** It's All Downhill From Here
- **Lesson 7, Activities 2:** Precision Timekeeping

UNIT 7 | **LESSON 2**

Relationships of Angles

Let's investigate some special angles.



Focus

Goals

1. **Language Goal:** Comprehend and use the word *degrees* and the symbol ° to refer to the amount of turn between two different directions. **(Speaking and Listening, Writing)**
2. Recognize $180°$ and $360°$ angles, and identify when adjacent angle measures have these sums.
3. **Language Goal:** Use reasoning about adjacent angles to determine the angle measures of pattern blocks, and justify the reasoning. **(Speaking and Listening)**

Coherence

Today

Students compose, decompose, and measure angles. They review right angles, straight angles, and angles which compose a full circle. They use pattern blocks and simple equations to find the measurements of angles. Students make plausible arguments, justify their conclusions, and communicate them to others.

Previously

Students were introduced to angles in Grade 4. Earlier in Unit 1, they briefly discussed angles during their work with scale drawings.

Coming Soon

In Lessons 3–4, students will continue their work with angles focusing on complementary and supplementary angle pairs.

Rigor

- Students use pattern blocks to develop **conceptual understanding** of composing, decomposing, and measuring angles.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	12 min	15 min	5 min	7 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Angle Relationships*
- Anchor Chart PDF, *Angle Relationships* (answers)
- analog clock or a picture of one
- pattern blocks, or the Activity 1 PDF, pre-cut cards, one set per pair
  - » at least 3 hexagons
  - » at least 6 of each of the other shapes

Math Language Development

New word

- ***adjacent angles***

Review words

- *degree**
- *right angle*
- *straight angle*

*Students may confuse the mathematical meaning of the term *degree* with its meaning in other contexts, such as with temperature or diplomas. Be ready to address the differences.

Amps | Featured Activity

Activity 1

Digital Pattern Blocks

Students use digital pattern blocks to create designs to determine the measures of angles.

Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might assume that there is only one strategy to use to find the correct answers in Activity 1. Help students to take the perspective of others when discussing solution strategies, trying to think like the other person and understand their reasoning. Once students clearly understand the other person's argument, they can respectfully disagree, if needed.

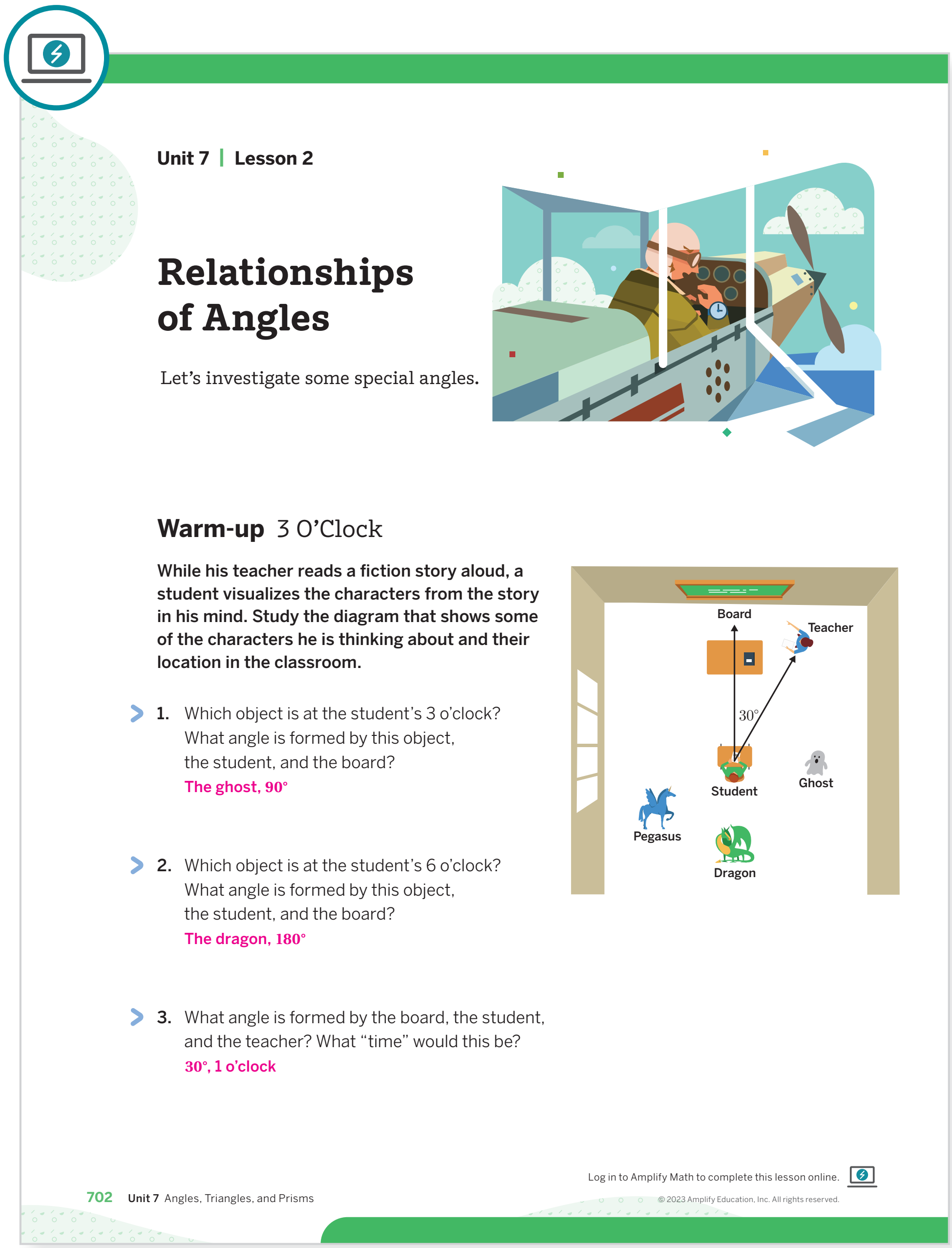
Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Omit **Activity 2**. Instead, during the Connect in Activity 1, highlight how other shapes can also be arranged to create angles that sum to 180° and 360°.

Warm-up 3 O'Clock

Students analyze the relative position of objects based on their relation to the hands on a clock to review the concept of angles.



Unit 7 | Lesson 2

Relationships of Angles

Let's investigate some special angles.

Warm-up 3 O'Clock

While his teacher reads a fiction story aloud, a student visualizes the characters from the story in his mind. Study the diagram that shows some of the characters he is thinking about and their location in the classroom.

1. Which object is at the student's 3 o'clock? What angle is formed by this object, the student, and the board?
   The ghost, 90°

2. Which object is at the student's 6 o'clock? What angle is formed by this object, the student, and the board?
   The dragon, 180°

3. What angle is formed by the board, the student, and the teacher? What "time" would this be?
   30°, 1 o'clock

Log in to Amplify Math to complete this lesson online.

702 Unit 7 Angles, Triangles, and Prisms

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1 Launch

Display an analog clock or an image of one. Activate students' background knowledge by asking them where they have seen analog clocks used and if they are familiar with how to read them.

2 Monitor

Help students get started by having them draw a clock on top of the diagram or write the clock numbers around the diagram.

Look for points of confusion:

- **Struggling to determine the angle representing 6 o'clock.** Remind students that an angle measures the amount of rotation between two rays.

Look for productive strategies:

- Listing "quarter turn" or "half turn" in their responses. Note students who respond in this manner.

3 Connect

Display the diagram from the Warm-up.

Have students share their responses and strategies.

Highlight that a full circle, "all the way around," measures 360°. If any students responded with "quarter turn" or "half turn", have them use these phrases to determine the angle measures. Write student responses using the $\angle$ symbol to represent *angle* and $m\angle$ to represent *the measurement of an angle*.

Ask, "Has anyone used, or heard someone else use, these clock phrases when talking about the location of objects?"

Differentiated Support

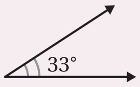
Accessibility: *Guide Processing and Visualization*

Provide students with a copy of an analog clock that they can use as a reference of where 3 o'clock and 6 o'clock are located on the analog clock and what type of angle they may form.

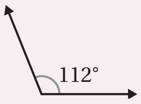
Power-up

To power up students' ability to identify acute, right, and obtuse angles, have students complete:

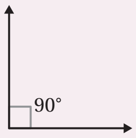
Recall that a *right angle* measures 90° while an *acute angle* is less than 90° and an *obtuse angle* is greater than 90°. For each angle, determine whether it is *acute*, *right*, or *obtuse*.



Acute



Obtuse



Right

Use: Before the Warm-up.

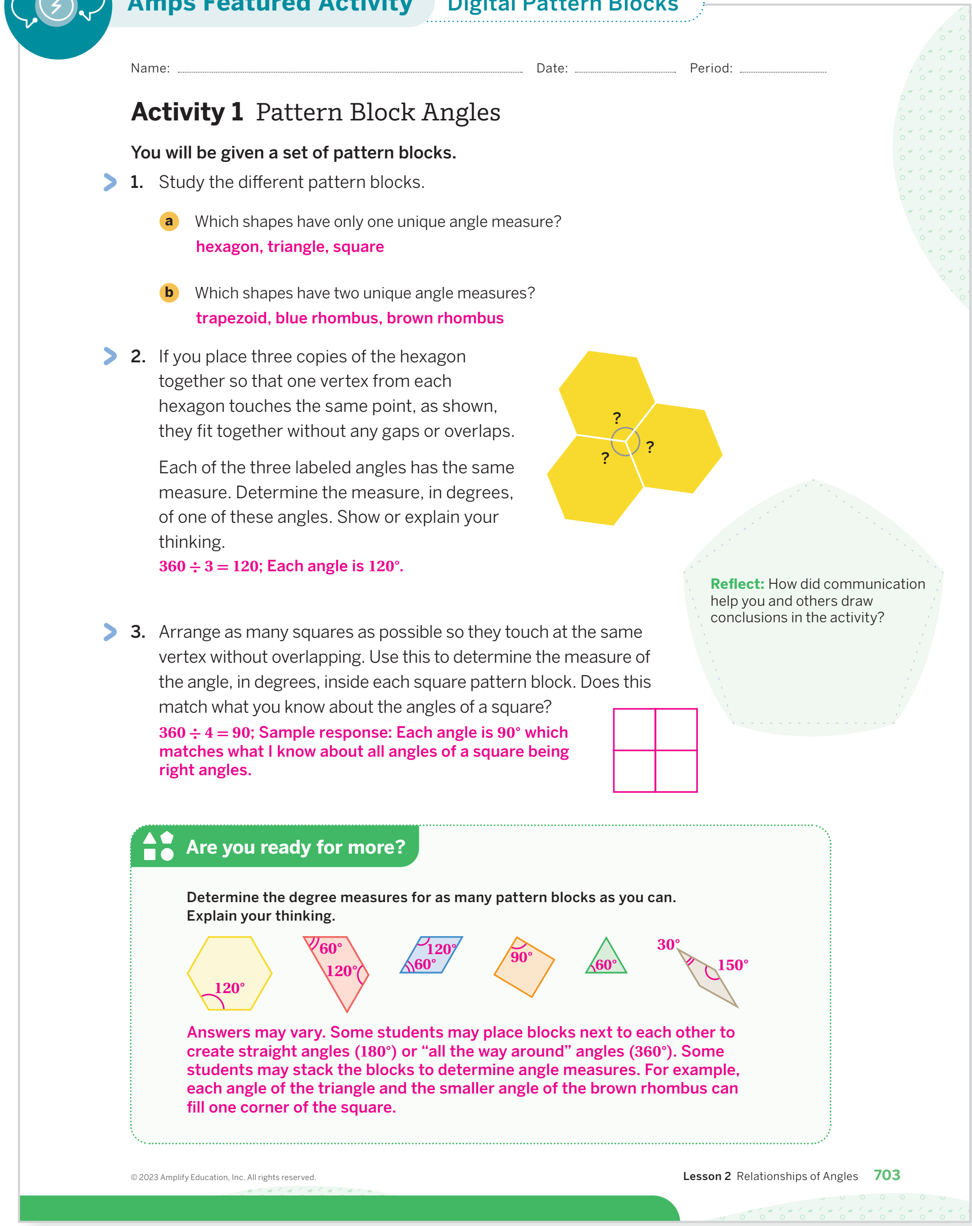
Informed by: Performance on Lesson 1, Practice Problem 5 and Pre-Unit Readiness Assessment, Problems 1 and 5.

Pairs | 12 min

Activity 1 Pattern Block Angles

Students explore how arranging pattern blocks to form angles measuring 360° can help them find the measures of the angles in each pattern block.

Amps Featured Activity Digital Pattern Blocks



Name: Date: Period:

Activity 1 Pattern Block Angles

You will be given a set of pattern blocks.

1. Study the different pattern blocks.
   - a Which shapes have only one unique angle measure?
     hexagon, triangle, square
   - b Which shapes have two unique angle measures?
     trapezoid, blue rhombus, brown rhombus
2. If you place three copies of the hexagon together so that one vertex from each hexagon touches the same point, as shown, they fit together without any gaps or overlaps.

   Each of the three labeled angles has the same measure. Determine the measure, in degrees, of one of these angles. Show or explain your thinking.

   $360 \div 3 = 120$; Each angle is 120°.
3. Arrange as many squares as possible so they touch at the same vertex without overlapping. Use this to determine the measure of the angle, in degrees, inside each square pattern block. Does this match what you know about the angles of a square?

   $360 \div 4 = 90$; Sample response: Each angle is 90° which matches what I know about all angles of a square being right angles.

Reflect: How did communication help you and others draw conclusions in the activity?

Are you ready for more?

Determine the degree measures for as many pattern blocks as you can. Explain your thinking.

Answers may vary. Some students may place blocks next to each other to create straight angles (180°) or "all the way around" angles (360°). Some students may stack the blocks to determine angle measures. For example, each angle of the triangle and the smaller angle of the brown rhombus can fill one corner of the square.

1 Launch

Remind students that 360° represents a full turn around a point. Distribute pattern blocks or one set of cards from the Activity 1 PDF.

2 Monitor

Help students get started by saying, "If you place two of the same blocks on top of each other, do the angles match up? Rotate or flip the pattern blocks to see if this holds true for all the angles."

Look for points of confusion:

- **Misunderstanding what *unique* means in this context.** Explain that *unique* means *one and only one*. Although there are six angles in the hexagon, the angles are the same size, so there is *one and only one* angle measure of 120°.

Look for productive strategies:

- Writing simple equations to determine the angle measures. Note students who use this strategy.

3 Connect

Display digital pattern blocks or use a document camera to display the physical pattern blocks.

Highlight strategies involving the use of equations. If students did not use equations, listen to their descriptions and have them write an equation to match their thinking. Encourage them to explain and critique the reasoning of others.

Define the term ***adjacent angles*** as angles which share a common side and common vertex. Show an example of adjacent angles using the arrangement in Problem 2.

Ask, "How are adjacent angles helpful in determining the angle measures of the pattern blocks?" By using adjacent blocks to form a straight angle or an angle measuring 360°, simple division can be used to determine the angle measures, if the angles have the same measure.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can interact with digital pattern blocks to determine the measures of angles.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 2 and 3.

Math Language Development

MLR2: Collect and Display

While students work, circulate and record words, phrases, drawings, and writings they use to explain their strategies for determining the angle measures. Display collected language and invite students to borrow from, and add to, the display.

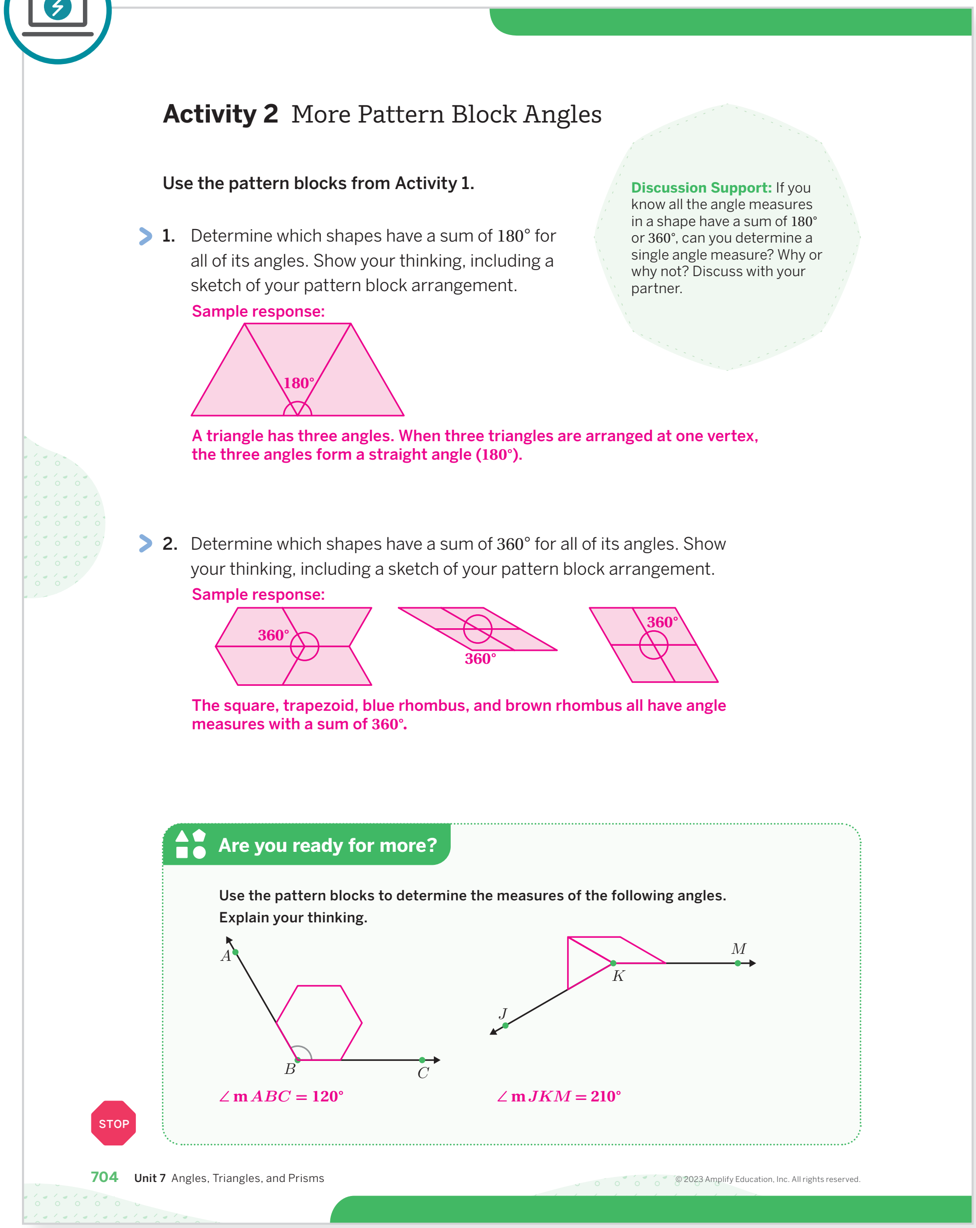
English Learners

Encourage the use of physical and/or digital manipulatives as they explain strategies for determining the angle measures.

Pairs | 15 min

Activity 2 More Pattern Block Angles

Students arrange pattern blocks to see how some shapes can be arranged to form angles measuring 180°, and others can be arranged to form angles measuring 360°.



Activity 2 More Pattern Block Angles

Use the pattern blocks from Activity 1.

Discussion Support: If you know all the angle measures in a shape have a sum of 180° or 360°, can you determine a single angle measure? Why or why not? Discuss with your partner.

1. Determine which shapes have a sum of 180° for all of its angles. Show your thinking, including a sketch of your pattern block arrangement.

   Sample response:

   A triangle has three angles. When three triangles are arranged at one vertex, the three angles form a straight angle (180°).

2. Determine which shapes have a sum of 360° for all of its angles. Show your thinking, including a sketch of your pattern block arrangement.

   Sample response:

   The square, trapezoid, blue rhombus, and brown rhombus all have angle measures with a sum of 360°.

Are you ready for more?

Use the pattern blocks to determine the measures of the following angles. Explain your thinking.

$\angle \mathrm{m}\,ABC = 120°$

$\angle \mathrm{m}\,JKM = 210°$

704 **Unit 7** Angles, Triangles, and Prisms

1 Launch

Explain that the unique angles of the shapes must meet at a common vertex for this activity.

2 Monitor

Help students get started by displaying responses to the *Are you ready for more?* problem from Activity 1.

Look for points of confusion:

- **Thinking only the square has angle measures with a sum of 360°.** Ask students to arrange the unique angles of the trapezoids at a common vertex.
- **Using only some of the angles when arranging the trapezoids and rhombuses.** Remind students that every unique angle needs to be present at the vertex.
- **Interpreting non-congruent angles as congruent.** For example, an arrangement of four copies of the blue rhombus forms a 360° angle, but it does not mean each angle measures one fourth of 360°. Have students place the square block on top of the blue rhombus to show the angles of the rhombus are not 90°.

Look for productive strategies:

- Forming multiple arrangements to show sums of 180° or 360°. Note these patterns for displaying to the class during the Connect.

3 Connect

Display student arrangements of the pattern blocks.

Ask:

- "If you know that all the angle measures in a shape have a sum of 180° or 360°, can you determine a single angle measure?" For shapes that have equal angle measures (equilateral triangle and square), yes. For the shapes that do not have equal angle measures (rhombus and trapezoid), it is not possible.
- "Does anyone notice similarities between the shapes whose angle measures have a sum of 360°?" They are quadrilaterals.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Regardless of whether students completed the Are you ready for more? from Activity 1, display the degree measures of each interior angle for the pattern blocks.

Hexagon	Trapezoid	Blue rhombus	Square	Triangle	Brown rhombus
120°	60° and 120°	120° and 60°	90°	60°	30° and 150°

Math Language Development

MLR8: Discussion Supports

Use a *Think-Write-Pair-Share* to unpack this question from the Connect. Ask, "If you know all the angle measures in a shape have a sum of 180° or 360°, can you determine a single angle measure?"

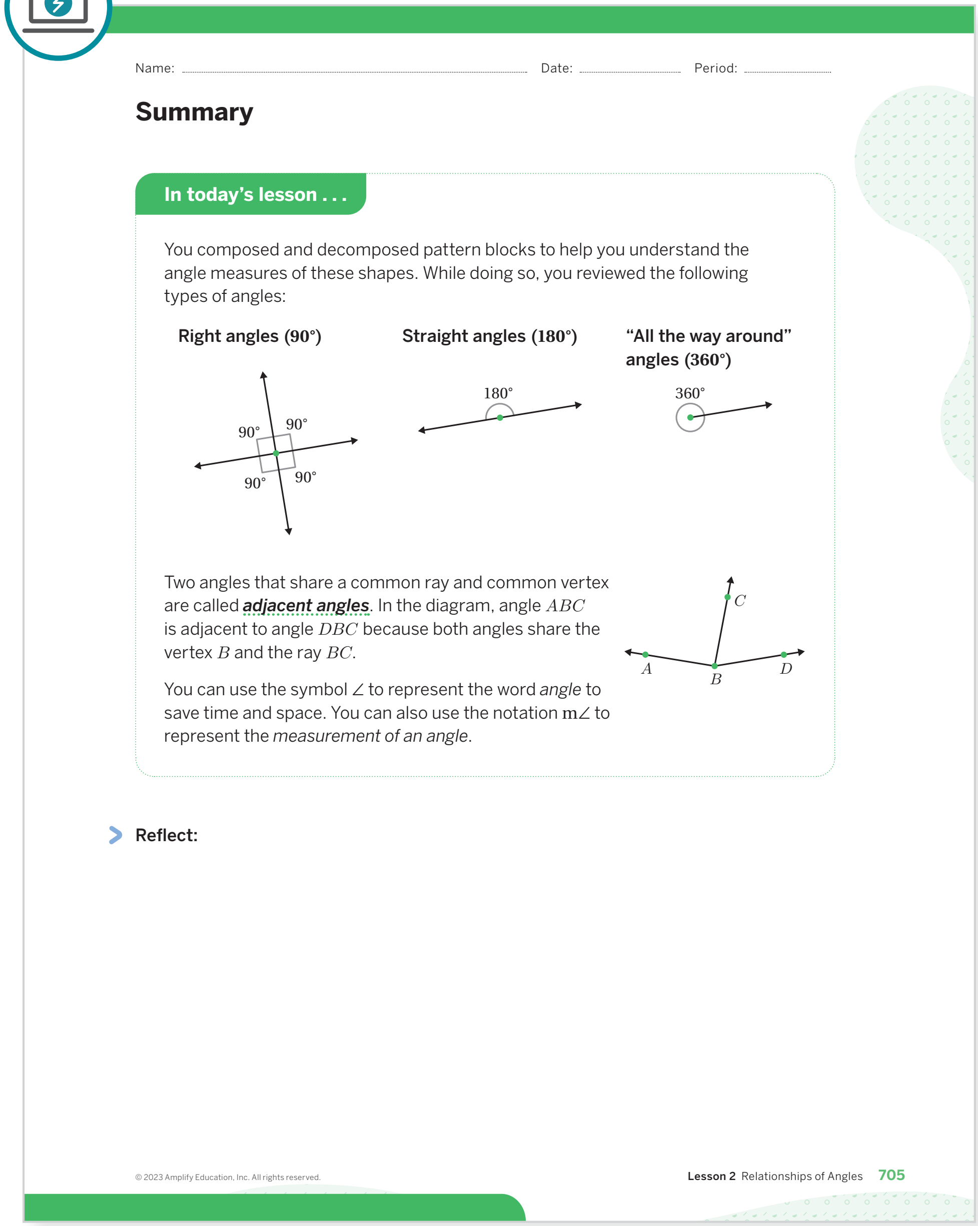
English Learners

Provide a visual representation of an isosceles and a right triangle and ask students whether they can determine a single angle measure in either triangle.

Whole Class | 5 min

Summary

Review and synthesize how adjacent angles can be used to compose angles of 90°, 180°, and 360°. By doing so, some of the unique angle measures of the shapes formed by these angles can be determined.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You composed and decomposed pattern blocks to help you understand the angle measures of these shapes. While doing so, you reviewed the following types of angles:

Right angles (90°) **Straight angles (180°)** **"All the way around" angles (360°)**

Two angles that share a common ray and common vertex are called ***adjacent angles***. In the diagram, angle ABC is adjacent to angle DBC because both angles share the vertex B and the ray BC.

You can use the symbol ∠ to represent the word *angle* to save time and space. You can also use the notation $\mathrm{m}\angle$ to represent the *measurement of an angle*.

Reflect:

Synthesize

Display the Anchor Chart PDF, *Angle Relationships* and complete the information for right, straight, "all the way around", and adjacent angles during the discussion.

Highlight that the measures of adjacent angles can be added to determine the measure of a larger angle. The measure of the larger angle can be used to determine the measures of the smaller angles, if they have equal measures.

Formalize vocabulary: *adjacent angles*

Ask:

- "What are the three main types of angles you studied in this lesson, and what are their measures?" right: 90°, straight: 180°, "all the way around": 360°
- "What does it look like when angles are adjacent, and what can you say about their measures?" Adjacent angles share a common side and vertex. Their angle measures can be added together forming a new angle. If they form a straight angle, the sum of their measures if 180°. If they form a right angle, the sum of their measures is 90°.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies in combining pattern blocks did you try that were successful? What made them successful?"
- "What strategies were not successful? Why not?"

Math Language Development

MLR2: Collect and Display

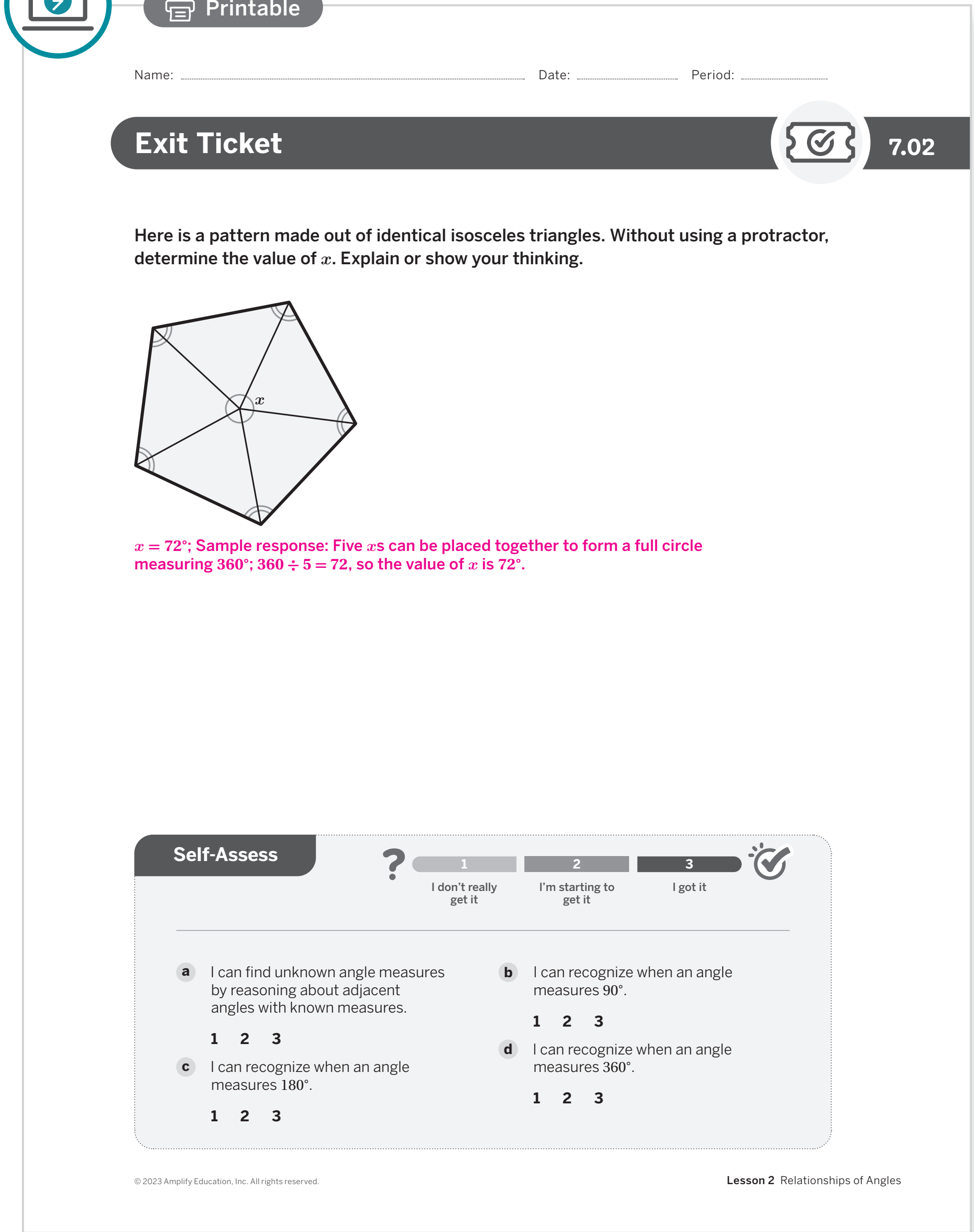
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *adjacent angles* that were added to the display during the lesson.

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by decomposing a 360° angle to determine an unknown angle measure.

Printable



Name: Date: Period:

Exit Ticket 7.02

Here is a pattern made out of identical isosceles triangles. Without using a protractor, determine the value of x. Explain or show your thinking.

$x = 72°$; Sample response: Five xs can be placed together to form a full circle measuring 360°; $360 \div 5 = 72$, so the value of x is 72°.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can find unknown angle measures by reasoning about adjacent angles with known measures.

1 2 3

b I can recognize when an angle measures 90°.

1 2 3

c I can recognize when an angle measures 180°.

1 2 3

d I can recognize when an angle measures 360°.

1 2 3

Lesson 2 Relationships of Angles

Success looks like . . .

- **Language Goal:** Comprehending and using the word *degrees* and the symbol ° to refer to the amount of turn between two different directions. **(Speaking and Listening, Writing)**
- **Goal:** Recognizing 180° and 360° angles, and identifying when adjacent angle measures have these sums.
- **Language Goal:** Using reasoning about adjacent angles to determine the angle measures of pattern blocks, and justifying the reasoning. **(Speaking and Listening)**
  - » Reasoning that the five congruent angles in the center of the pattern give a measure of 360 degrees and that dividing by 5 gives the measure of x

Suggested next steps

If students cannot not determine the value of x, consider:

- Reviewing the angle measure of a full circle is 360° and that simple division can determine the value of one of the angles, assuming the smaller angles have the same measure.
- Assigning Practice Problem 2.

Professional Learning

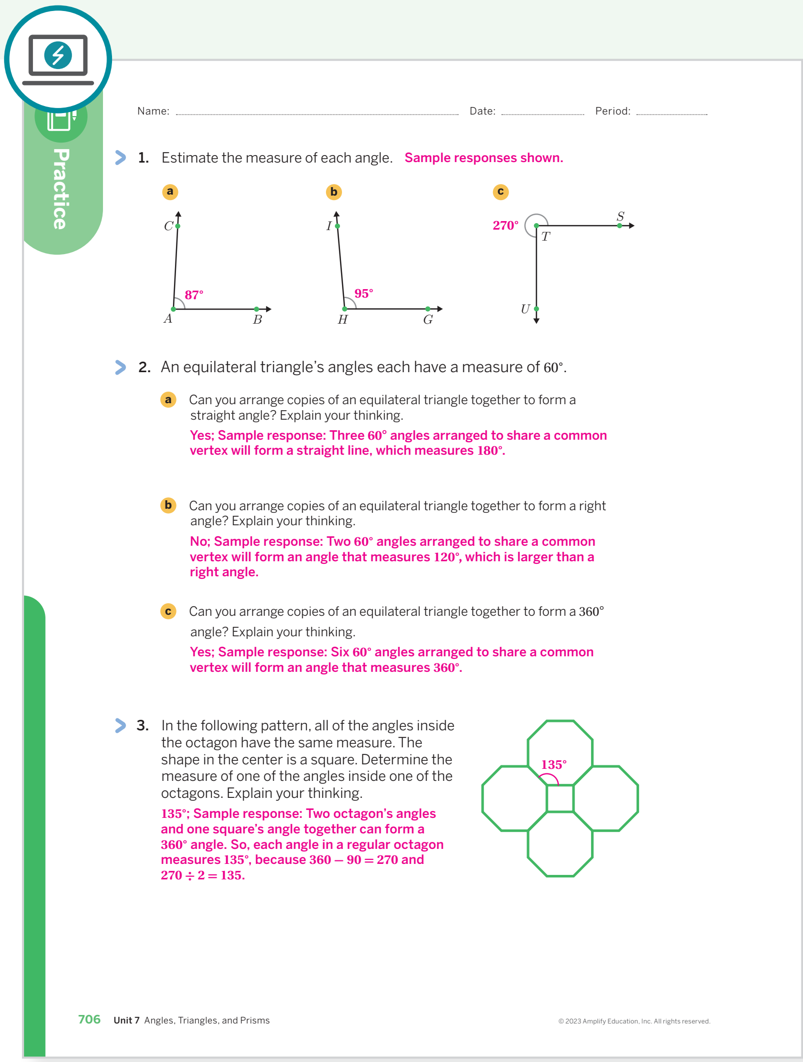
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach Activity 1 today? What does that tell you about similarities and differences among your students?
- In this lesson, students used pattern blocks to compose, decompose, and measure angles. How will that support their work with supplementary, complementary, and vertical angles in the next three lessons? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Estimate the measure of each angle. **Sample responses shown.**

2. An equilateral triangle's angles each have a measure of 60°.

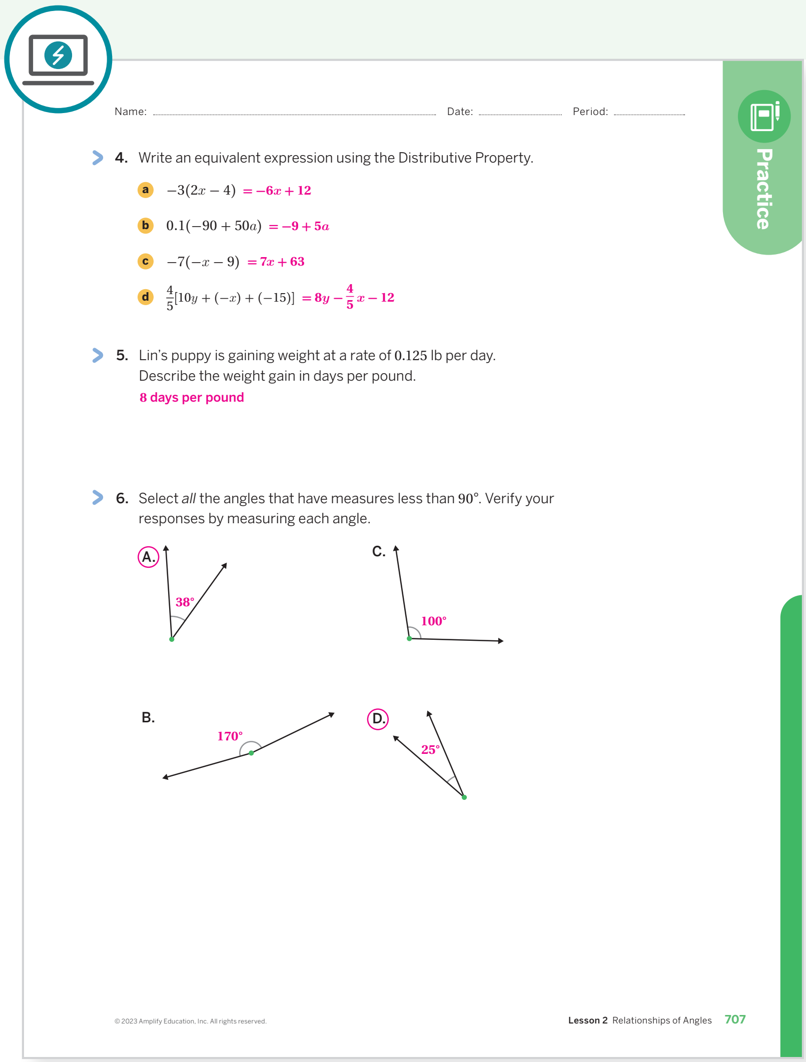
   a. Can you arrange copies of an equilateral triangle together to form a straight angle? Explain your thinking.
   Yes; Sample response: Three 60° angles arranged to share a common vertex will form a straight line, which measures 180°.

   b. Can you arrange copies of an equilateral triangle together to form a right angle? Explain your thinking.
   No; Sample response: Two 60° angles arranged to share a common vertex will form an angle that measures 120°, which is larger than a right angle.

   c. Can you arrange copies of an equilateral triangle together to form a 360° angle? Explain your thinking.
   Yes; Sample response: Six 60° angles arranged to share a common vertex will form an angle that measures 360°.

3. In the following pattern, all of the angles inside the octagon have the same measure. The shape in the center is a square. Determine the measure of one of the angles inside one of the octagons. Explain your thinking.
135°; Sample response: Two octagon's angles and one square's angle together can form a 360° angle. So, each angle in a regular octagon measures 135°, because $360 - 90 = 270$ and $270 \div 2 = 135$.

706 **Unit 7** Angles, Triangles, and Prisms



Practice

Name: Date: Period:

4. Write an equivalent expression using the Distributive Property.

   a. $-3(2x - 4)$ $\mathbf{= -6x + 12}$

   b. $0.1(-90 + 50a)$ $\mathbf{= -9 + 5a}$

   c. $-7(-x - 9)$ $\mathbf{= 7x + 63}$

   d. $\frac{4}{5}[10y + (-x) + (-15)]$ $\mathbf{= 8y - \frac{4}{5}x - 12}$

5. Lin's puppy is gaining weight at a rate of 0.125 lb per day. Describe the weight gain in days per pound.
8 days per pound

6. Select *all* the angles that have measures less than 90°. Verify your responses by measuring each angle.

Lesson 2 Relationships of Angles 707

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 19	1
	5	Unit 2 Lesson 3	2
Formative ⚡	6	Unit 7 Lesson 3	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 3**

Supplementary and Complementary Angles (Part 1)

Let's investigate some special pairs of angles.

Focus

Goals

1. **Language Goal:** Comprehend the terms *complementary* and *supplementary* as they describe pairs of angles. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain how to find an unknown angle measure, given adjacent complementary or supplementary angles. **(Speaking and Listening, Writing)**
3. **Language Goal:** Generalize that, when a straight angle or a right angle is decomposed, the measures of the resulting angles add up to 180° or 90°, respectively. **(Speaking and Listening)**

Coherence

Today

Students are introduced to the terms complementary angles and supplementary angles. They practice finding an unknown angle given the measure of another angle that is complementary or supplementary and justifying their methods. Many of the angles in this lesson share the same vertex as another angle, so students need to be careful when naming each angle in addition to describing the relationship between pairs of angles.

Previously

In Lesson 2, students reviewed angle relationships using pattern blocks and used simple equations to solve for missing angle measures.

Coming Soon

In Lesson 4, students will build on their understanding of complementary and supplementary angles to describe the relationships between angle measures in polygons.

Rigor

- Students use compasses and visual models to build **conceptual understanding** of complementary and supplementary angles.
- Students strengthen their **fluency** in composing, decomposing, and measuring angles.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Small Groups	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Angle Relationships*
- Anchor Chart PDF, *Angle Relationships* (answers)
- geometry toolkits: protractors, scissors, rulers, plain sheets of paper (two sheets per student)

Math Language Development

New words

- ***complementary angles****
- ***supplementary angles***

Review words

- *adjacent angles*
- *protractor*
- *right angle*
- *straight angle*

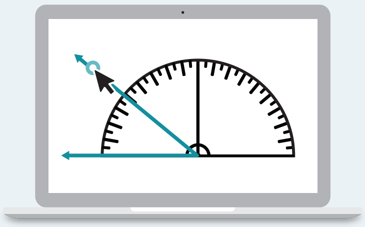
*Students may confuse the term *complementary* with the terms *compliment* or *complimentary*. Be ready to address the differences.

Amps | Featured Activity

Activities 1 and 2

Dynamic Angle Measures

Students can adjust the position of a ray to change the size of two complementary or supplementary angles. They see the resulting angle measurements change in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might resist learning new definitions of angles and rely on the previously-learned skill of measuring angles with a protractor to find the measures in Activity 3. Ask students to make efficiency a goal for this activity. Have them identify what they can do to meet that goal.

Modifications to Pacing

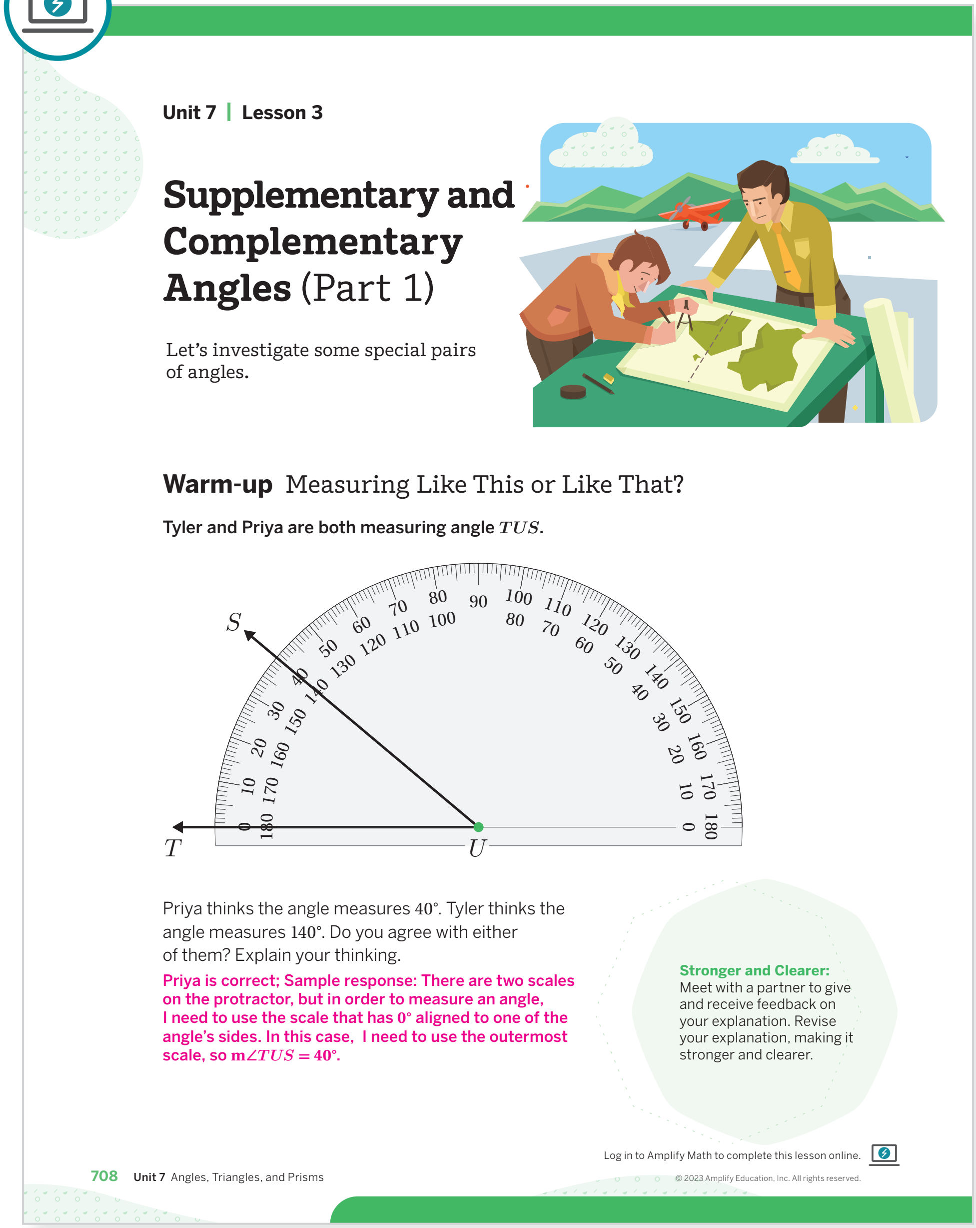
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up**.
- Half the class should complete **Activity 1**, while the other half completes **Activity 2**. Have both groups share their findings with the class.

Independent | 5 min

Warm-up Measuring Like This or Like That?

Students critique another student's reasoning to address the common error of reading a protractor from the wrong end.



Unit 7 | Lesson 3

Supplementary and Complementary Angles (Part 1)

Let's investigate some special pairs of angles.

Warm-up Measuring Like This or Like That?

Tyler and Priya are both measuring angle TUS.

Priya thinks the angle measures 40°. Tyler thinks the angle measures 140°. Do you agree with either of them? Explain your thinking.

Priya is correct; Sample response: There are two scales on the protractor, but in order to measure an angle, I need to use the scale that has 0° aligned to one of the angle's sides. In this case, I need to use the outermost scale, so $m\angle TUS = 40°$.

Stronger and Clearer: Meet with a partner to give and receive feedback on your explanation. Revise your explanation, making it stronger and clearer.

Log in to Amplify Math to complete this lesson online.

708 **Unit 7** Angles, Triangles, and Prisms

① Launch

Distribute the geometry toolkits for students to use throughout this lesson. Activate students' background knowledge by asking them whether they have used a protractor in prior math classes or outside of school. Consider using the *Think-Pair-Share* routine.

② Monitor

Help students get started by suggesting they mark the angle being measured and consider what they think the correct measure is.

Look for points of confusion:

- **Measuring the wrong angle.** Make sure students can correctly identify which angle is being measured.

Look for productive strategies:

- Matching up 0° on the protractor with one ray of the angle and using that to choose the correct scale.
- Considering whether the angle is greater or less than 90° and using that information to choose the correct scale.

③ Connect

Display the image given in the problem. Conduct the ***Poll the Class*** routine to assess student thinking.

Have students share which student they agree with and why.

Highlight how Tyler could know that his answer of 140° is unreasonable. Review how to use a protractor. Note that one segment of the angle should align horizontally with 0° on the protractor. The angle should be measured with the scale that includes this segment.

Ask, "Is angle TUS acute, right, or obtuse?" Acute "Can you see an angle that measures 140° in this figure?" The angle adjacent to angle TUS formed by ray US and the other side of the protractor

Math Language Development

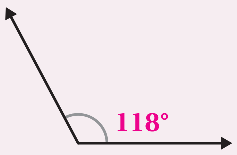
MLR1: Stronger and Clearer Each Time

Before the Connect, have students meet with 1–2 partners to give and receive feedback on their explanations for whether they agree with either Tyler or Priya. Display these prompts that reviewers can use to press for details as they discuss their responses.

- "Does the response include how to read a protractor?"
- "Does the response include making sense of the size of the angle to know whether it is more reasonable to measure 40° or 140°?"

Power-up

To power up students' ability to use a protractor to measure angles, have students complete:



1. Is the given angle greater than or less than 90°? Greater than 90°.
2. Use a protractor to determine the measure of the angle.

Use: Before the Warm-up.

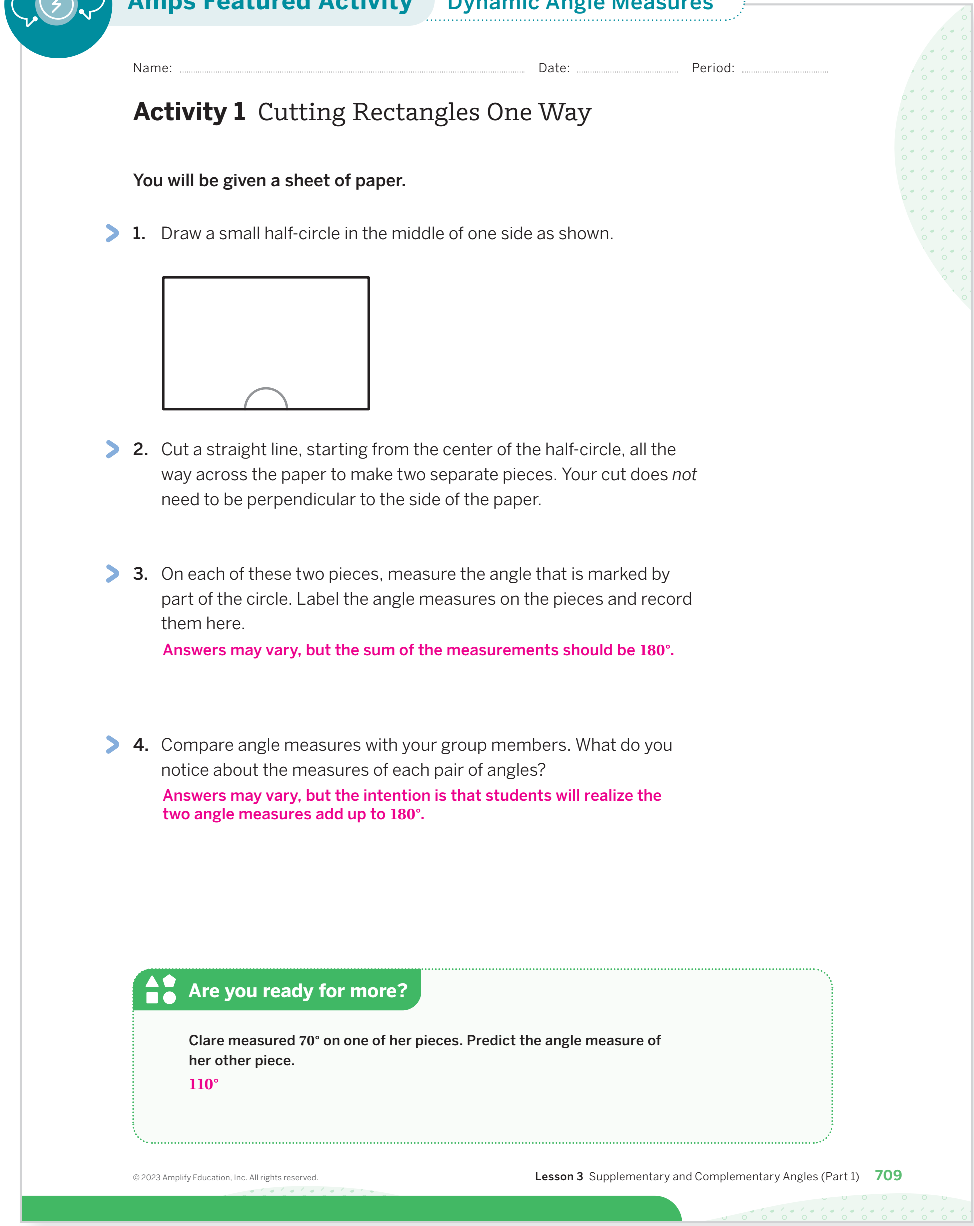
Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 2 and 3.

Small Groups | 10 min

Activity 1 Cutting Rectangles One Way

Students use paper and scissors to create two supplementary angles to see that the measures of any two supplementary angles have a sum of 180°.

Amps Featured Activity Dynamic Angle Measures



Name: ______ Date: ______ Period: ______

Activity 1 Cutting Rectangles One Way

You will be given a sheet of paper.

1. Draw a small half-circle in the middle of one side as shown.

2. Cut a straight line, starting from the center of the half-circle, all the way across the paper to make two separate pieces. Your cut does *not* need to be perpendicular to the side of the paper.

3. On each of these two pieces, measure the angle that is marked by part of the circle. Label the angle measures on the pieces and record them here.
   Answers may vary, but the sum of the measurements should be 180°.

4. Compare angle measures with your group members. What do you notice about the measures of each pair of angles?
   Answers may vary, but the intention is that students will realize the two angle measures add up to 180°.

Are you ready for more?

Clare measured 70° on one of her pieces. Predict the angle measure of her other piece.

110°

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Lesson 3 Supplementary and Complementary Angles (Part 1) 709

1 Launch

Provide each student a sheet of paper, a pair of scissors, and a protractor. Review the instructions and emphasize students should use the straight edge of their protractor to draw the line they will cut, before using the scissors. Instruct students to each make their own cuts and measurements, and then compare their work with their group members.

2 Monitor

Look for points of confusion:

- **Struggling to correctly position the protractor to measure their angles.** Ensure students are positioning the center of the protractor with the vertex of their angle, lining up one side of the angle with 0°, and using the correct scale.
- **Not ending up with two angles whose measures have a sum of 180°.** If the sum is close, allow this for now and discuss during the Connect. If the sum is not close, ask students to show you how they measured their angles.

Look for productive strategies:

- Noticing that the angle measures have a sum of 180°.

3 Connect

Display the pairs of angle measurements students found.

Have students share the measurements for the pair of angles they cut and what they noticed about the angle measures.

Highlight that the angle measures add up to about 180°. This is because they started with a straight angle and cut it, to create two angles. Note that measuring errors or inaccuracies account for any angle pairs whose measures do not have a sum of 180°.

Define the term ***supplementary angles*** as two angles whose measures have a sum of 180°.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can adjust the position of a ray to change the size of two supplementary angles. They can see the resulting angle measurements change in real time.

Accessibility: *Vary Demands to Optimize Challenge*

Provide 2–3 pairs of pre-cut angles. Ask students to measure the angles and describe what they notice. This will still allow them to access the goal of the activity without having to physically cut the materials.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display a potential student misconception illustrating improper placement of the protractor that results in incorrect measures. Ask:

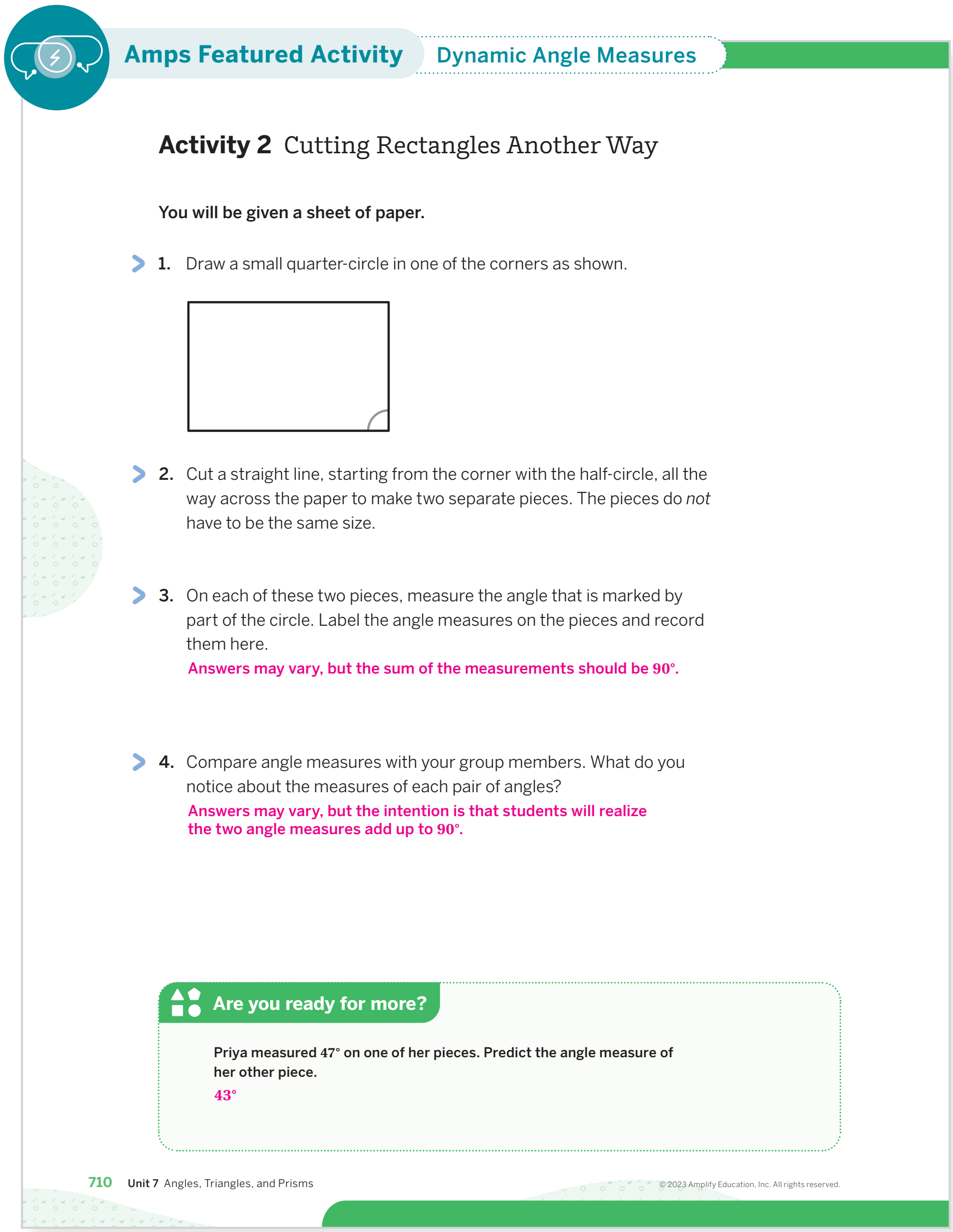
- ***Critique:*** "Do you agree or disagree with these measures? Can you think of how these measures were determined and whether they were measured correctly?"
- ***Correct:*** "Show me how to correctly determine the measures."
- ***Clarify:*** "How do you know that these measures are correct? Other than using the protractor, how else could you confirm your results?"

Small Groups | 10 min

Activity 2 Cutting Rectangles Another Way

Students use paper and scissors to create two complementary angles to see that the measures of any two complementary angles have a sum of 90°.

Amps Featured Activity Dynamic Angle Measures



Activity 2 Cutting Rectangles Another Way

You will be given a sheet of paper.

1. Draw a small quarter-circle in one of the corners as shown.

2. Cut a straight line, starting from the corner with the half-circle, all the way across the paper to make two separate pieces. The pieces do *not* have to be the same size.

3. On each of these two pieces, measure the angle that is marked by part of the circle. Label the angle measures on the pieces and record them here.
   Answers may vary, but the sum of the measurements should be 90°.

4. Compare angle measures with your group members. What do you notice about the measures of each pair of angles?
   Answers may vary, but the intention is that students will realize the two angle measures add up to 90°.

Are you ready for more?

Priya measured 47° on one of her pieces. Predict the angle measure of her other piece.

43°

710 **Unit 7** Angles, Triangles, and Prisms

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1 Launch

Note the difference between this activity and the previous one. Instruct students to each make their own cuts and measurements, and then compare their work with their group members.

2 Monitor

Look for points of confusion:

- **Struggling to correctly position the protractor to measure their angles.** Ensure students are positioning the center of the protractor with the vertex of their angle, lining up one side of the angle with 0°, and using the correct scale.
- **Not ending up with two angles whose measures have a sum of 90°.** If the sum is close, allow this for now and discuss during the Connect. If the sum is not close, ask students to show you how they measured their angles.

Look for productive strategies:

- Noticing that the angle measures have a sum of 90°.

3 Connect

Display the pairs of angle measurements students found.

Have students share the measurements for the pair of angles they cut and what they noticed about the angle measures.

Highlight that the angle measures add up to about 90°. This is because they cut a right angle to create two angles. Note that measuring errors or inaccuracies account for any angle pairs whose measures do not have a sum of 90°.

Define the term ***complementary angles*** as two angles whose measures have a sum of 90°.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can adjust the position of a ray to change the size of two complementary angles. They can see the resulting angle measurements change in real time.

Accessibility: *Vary Demands to Optimize Challenge*

As with Activity 1, provide 2–3 pairs of pre-cut angles. Ask students to measure the angles and describe what they notice. This will still allow them to access the goal of the activity without having to physically cut the materials.

Math Language Development

MLR2: Collect and Display

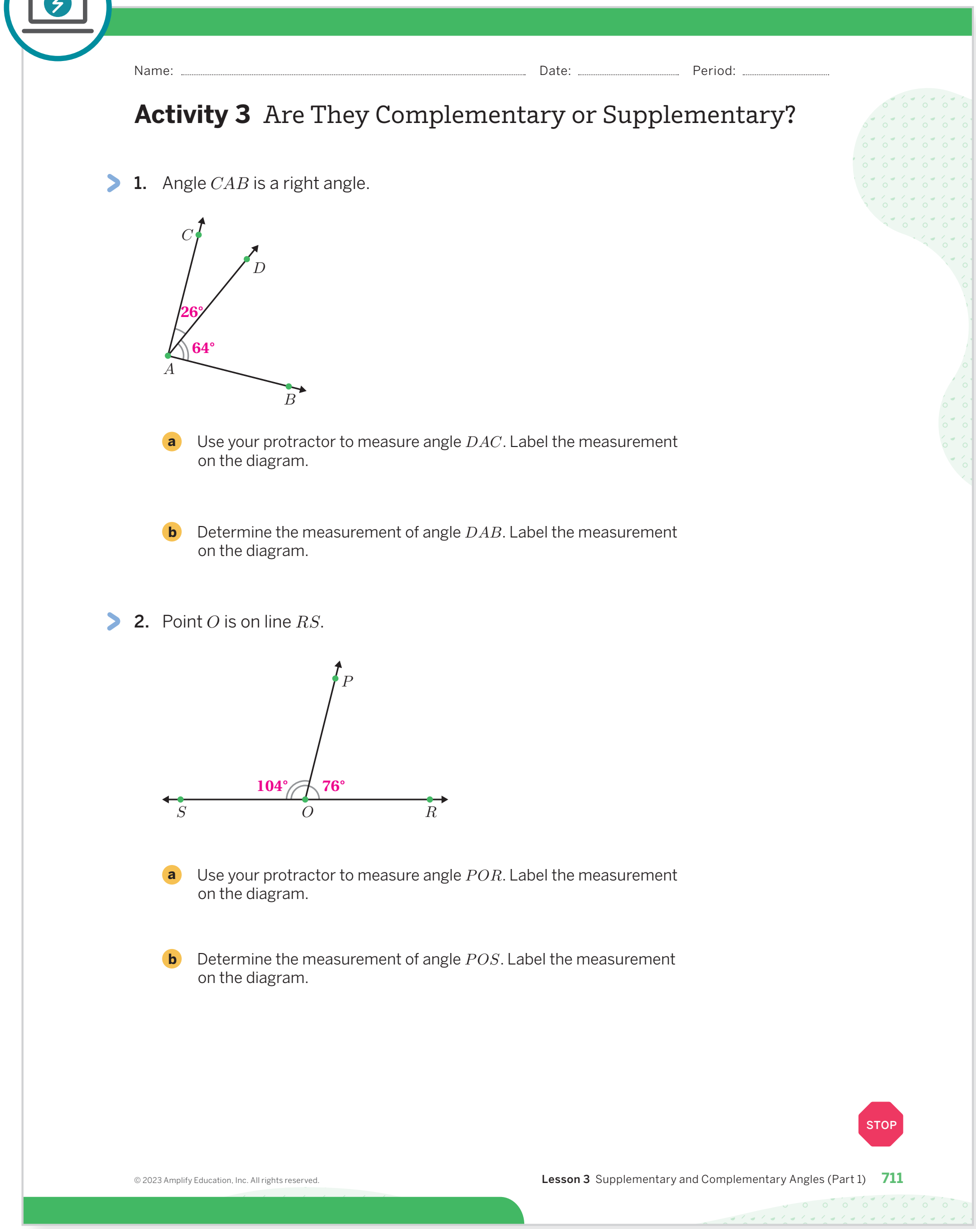
During the Connect, as you define the term *complementary angles*, refer back to Activity 1 where *supplementary angles* were defined. Students may confuse these terms. Add these terms to the class display, along with visual examples of each, illustrating the sum of the angle measures for two complementary or two supplementary angles.

Mention that early mathematicians frequently studied right triangles and used the term complementary to describe two angles that "complete" or "fill up" a right angle.

Pairs | 10 min

Activity 3 Are They Complementary or Supplementary?

Students find the measures of complementary and supplementary angle pairs and begin to formalize a process for calculating angle measures, based on their understanding of these angle relationships.



Name: ______ Date: ______ Period: ______

Activity 3 Are They Complementary or Supplementary?

1. Angle CAB is a right angle.

a. Use your protractor to measure angle DAC. Label the measurement on the diagram.

b. Determine the measurement of angle DAB. Label the measurement on the diagram.

2. Point O is on line RS.

a. Use your protractor to measure angle POR. Label the measurement on the diagram.

b. Determine the measurement of angle POS. Label the measurement on the diagram.

STOP

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Lesson 3 Supplementary and Complementary Angles (Part 1) 711

1 Launch

Ask student to explain what they learned about complementary and supplementary angles in their own words prior to starting this activity.

2 Monitor

Help students get started by suggesting they use the straight edge of the protractor to elongate each ray of the angle. This will make it easier to accurately measure angle DAC.

Look for points of confusion:

- **Thinking they have to use the protractor to measure both angles in each problem.** Ask them to consider how they can use what they learned in Activities 1 and 2.

Look for productive strategies:

- Measuring the first angle with a protractor, and then calculating the measure of the second angle.

3 Connect

Have students share their strategies for finding the angle measures. Select students who calculated the value of the second angle instead of measuring it.

Highlight the relationships between the pair of angles in each problem. Discuss how to tell that the angles in Problem 1 are complementary and that this means their measures add up to $90°$. Discuss the same for the supplementary angles in Problem 2. Explain how to use subtraction to find the measure of an unknown angle.

Ask how students can use the vocabulary of this lesson to describe each diagram. Problem 1: $\angle CAD$ and $\angle DAB$ are adjacent complementary angles; Problem 2: $\angle POR$ and $\angle POS$ are adjacent supplementary angles.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide the angle measures in part a of each problem. Then have students determine the measure of the second angle using what they know about complementary or supplementary angles. This will allow students to focus on the targeted goal of this activity without having to physically measure the angles.

Accessibility: *Guide Processing and Visualization*

Have students annotate each diagram as showing either *complementary* or *supplementary* angles. Then have them annotate the sum of the angle measures for each to help them make connections between the terms and the sums of the angle measures.

Extension: *Math Enrichment*

Challenge students to write equations that describe the relationships between the angle measures for each problem.

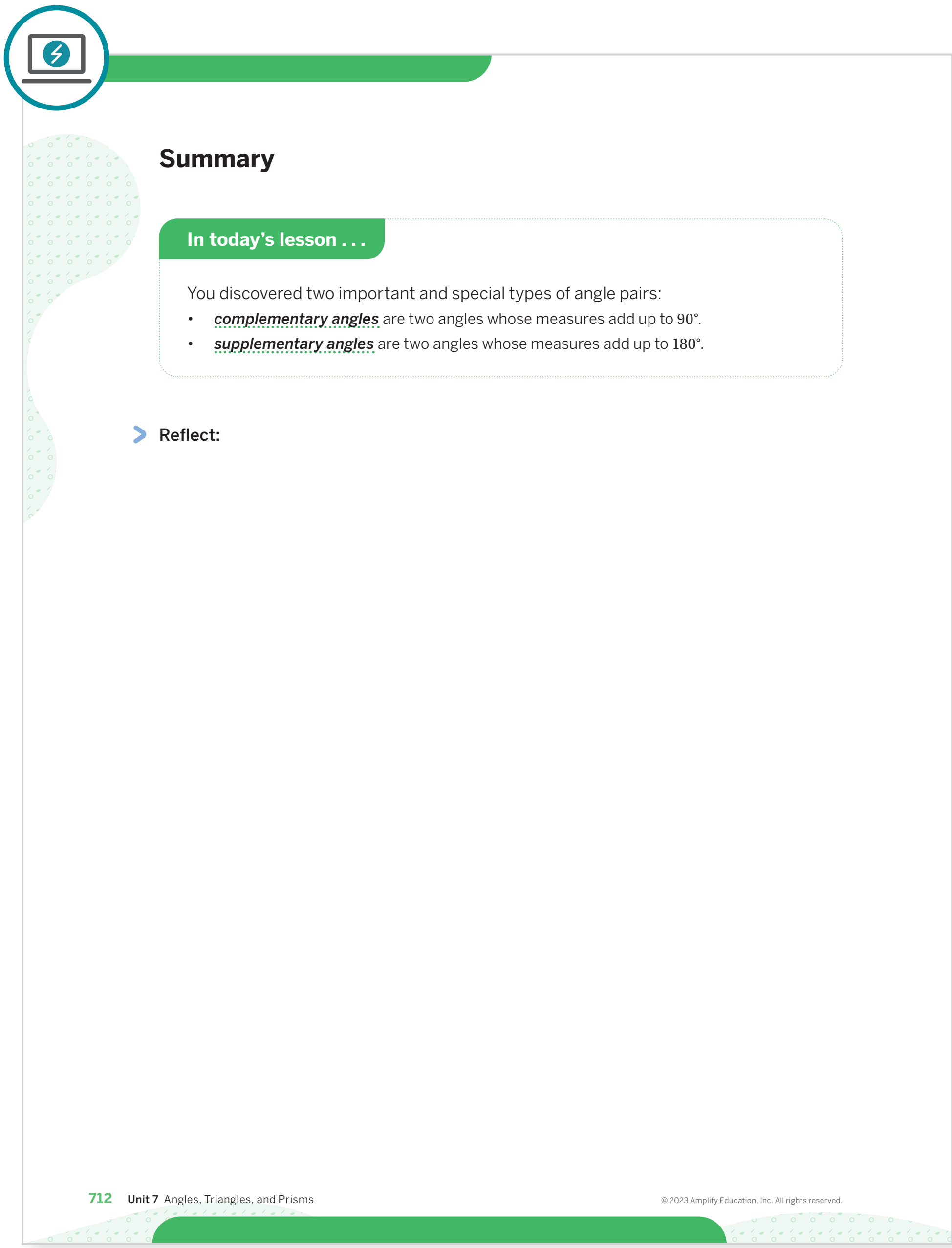
Problem 1: $26 + 64 = 90$

Problem 2: $104 + 76 = 180$

Whole Class | 5 min

Summary

Review and synthesize how complementary and supplementary angles can be used to find unknown angle measures.



Summary

In today's lesson . . .

You discovered two important and special types of angle pairs:

- *complementary angles* are two angles whose measures add up to 90°.
- *supplementary angles* are two angles whose measures add up to 180°.

Reflect:

712 Unit 7 Angles, Triangles, and Prisms

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Synthesize

Display the Anchor Chart PDF, *Angle Relationships* and complete the complementary and supplementary definitions during the discussion.

Formalize vocabulary:

- *complementary angles*
- *supplementary angles*

Highlight how to identify supplementary and complementary angles. Demonstrate how to use a protractor to measure one angle in each pair. Then discuss how to calculate the measure of the other angle in each pair.

Ask:

- What does it mean for two angles to be supplementary?” Their measures have a sum of 180°.
- “What does it mean for two angles to be complementary?” Their measures have a sum of 90°.
- “If you know two angles are supplementary and you know the measure of one angle, how can you find the other?” Subtract the known angle measure from 180°.
- “If you know two angles are complementary and you know the measure of one angle, how can you find the other?” Subtract the known angle measure from 90°.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- “When do certain combinations of angles form special angles?”

Math Language Development

MLR2: Collect and Display

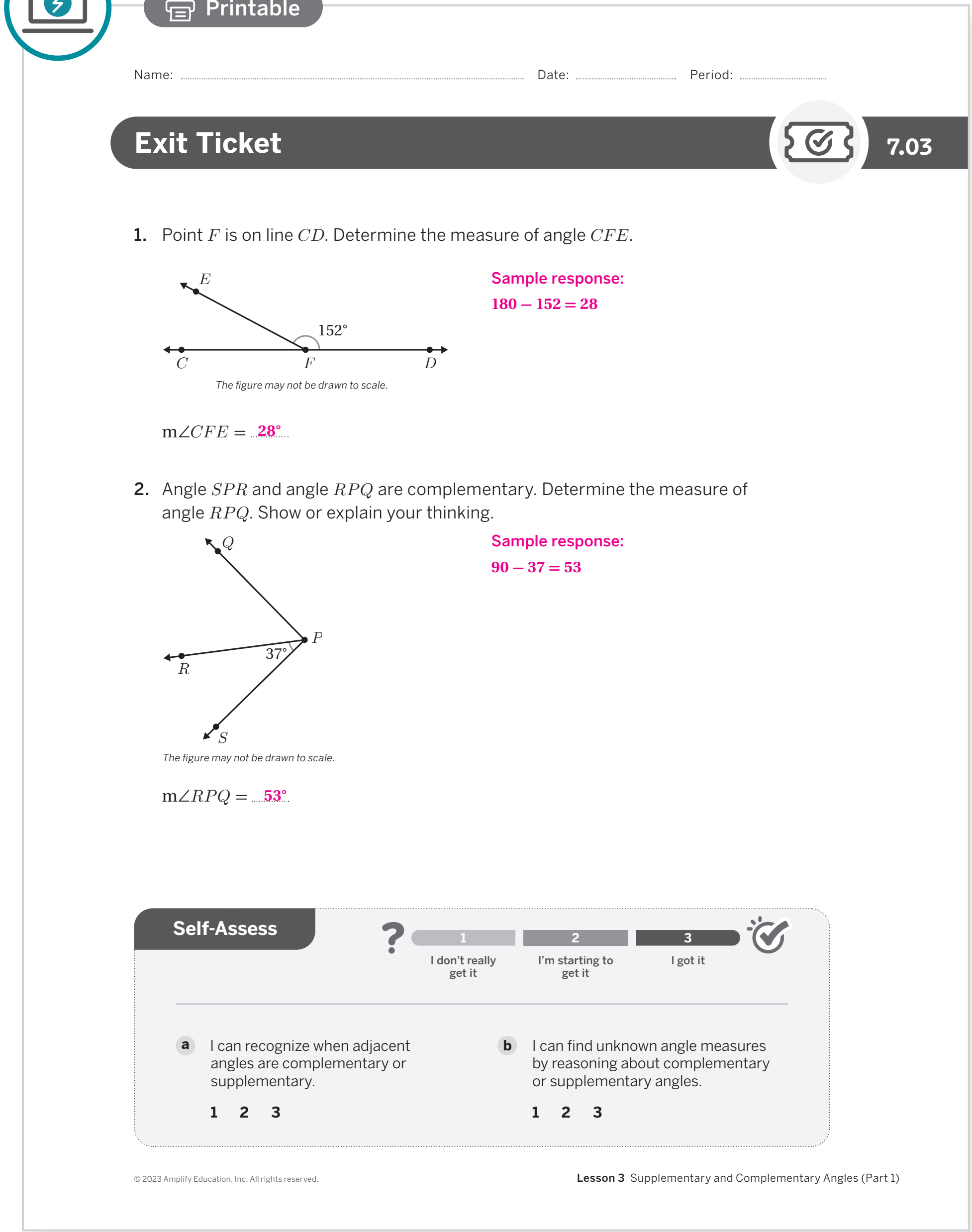
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *complementary angles* and *supplementary angles* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the measure of an unknown angle in a complementary or supplementary angle pair.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 7.03

1. Point F is on line CD. Determine the measure of angle CFE.

The figure may not be drawn to scale.

Sample response:

$180 - 152 = 28$

$m\angle CFE =$ 28°

2. Angle SPR and angle RPQ are complementary. Determine the measure of angle RPQ. Show or explain your thinking.

The figure may not be drawn to scale.

Sample response:

$90 - 37 = 53$

$m\angle RPQ =$ 53°

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can recognize when adjacent angles are complementary or supplementary.

1 2 3

b I can find unknown angle measures by reasoning about complementary or supplementary angles.

1 2 3

Lesson 3 Supplementary and Complementary Angles (Part 1)

Success looks like . . .

- **Language Goal:** Comprehending the terms *complementary* and *supplementary* as they describe pairs of angles. **(Speaking and Listening, Writing)**
  - » Using the information that angles SPR and RPQ are complementary to determine the measure of angle RPQ.
- **Language Goal:** Explaining how to determine an unknown angle measure, given adjacent complementary or supplementary angles. **(Speaking and Listening, Writing)**
  - » Explaining how to determine the measure of angle RPQ in Problem 2.
- **Language Goal:** Generalizing that, when a straight angle or a right angle is decomposed, the measures of the resulting angles add up to 180° or 90°, respectively. **(Speaking and Listening)**

Suggested next steps

If students ask for a protractor, consider:

- Reviewing strategies for determining unknown angles from Activity 3.

If students say the answer to Problem 1 is 62° and/or the answer to Problem 2 is 143°, consider:

- Reviewing the difference between complementary and supplementary angles.
- Assigning Practice Problem 1.

Professional Learning

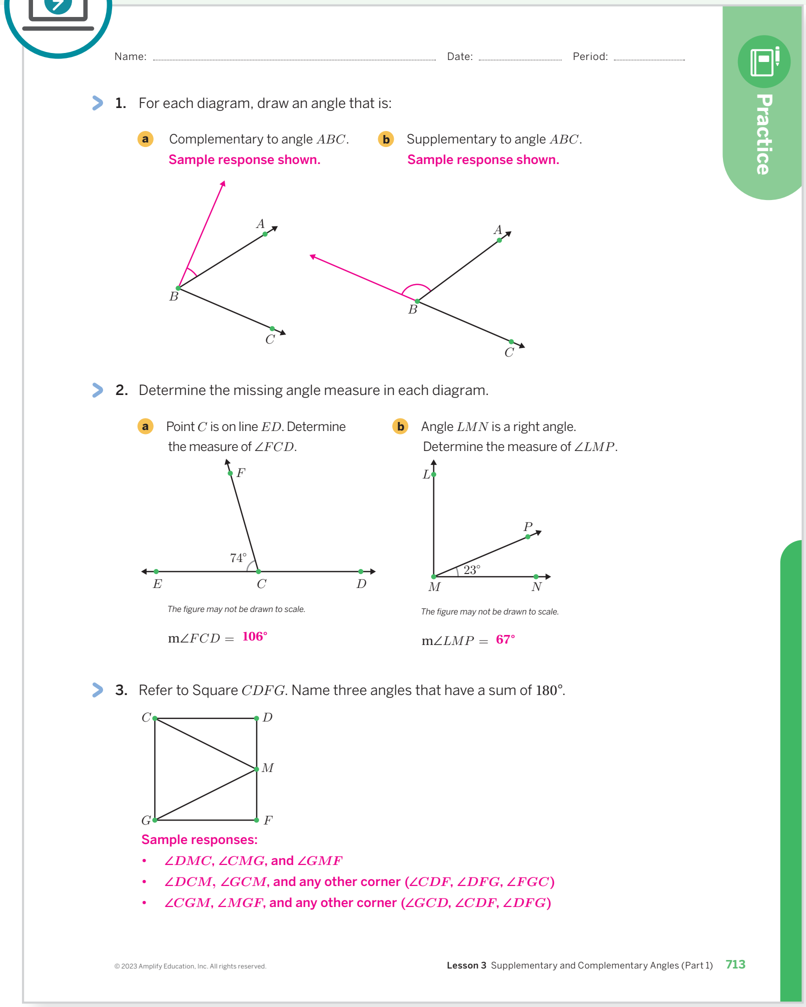
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which students' ideas were you able to highlight during Activity 3?
- How were Activities 1 and 2 from today similar or different from the Pattern Block Angles activity from the previous lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. For each diagram, draw an angle that is:

   a. Complementary to angle ABC.
   Sample response shown.

   b. Supplementary to angle ABC.
   Sample response shown.

2. Determine the missing angle measure in each diagram.

   a. Point C is on line ED. Determine the measure of $\angle FCD$.

   The figure may not be drawn to scale.

   $m\angle FCD =$ **$106°$**

   b. Angle LMN is a right angle. Determine the measure of $\angle LMP$.

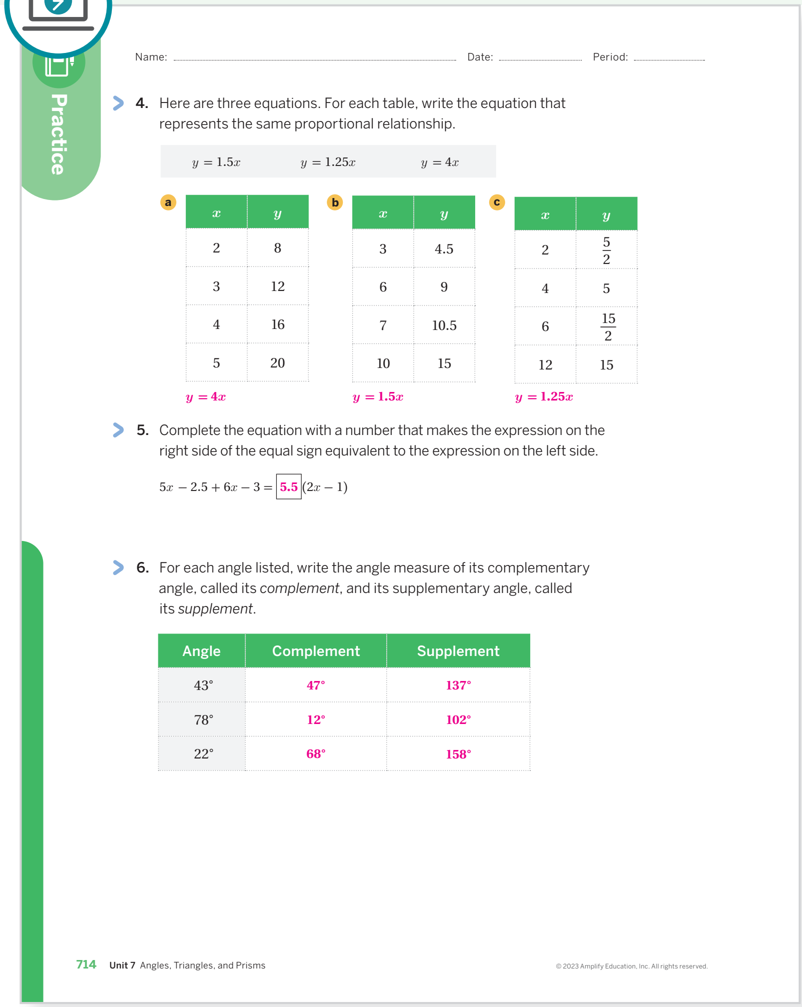
   The figure may not be drawn to scale.

   $m\angle LMP =$ **$67°$**

3. Refer to Square $CDFG$. Name three angles that have a sum of $180°$.

Sample responses:

- $\angle DMC$, $\angle CMG$, and $\angle GMF$
- $\angle DCM$, $\angle GCM$, and any other corner ($\angle CDF$, $\angle DFG$, $\angle FGC$)
- $\angle CGM$, $\angle MGF$, and any other corner ($\angle GCD$, $\angle CDF$, $\angle DFG$)



Name: ______ Date: ______ Period: ______

Practice

4. Here are three equations. For each table, write the equation that represents the same proportional relationship.

   $y = 1.5x$ $y = 1.25x$ $y = 4x$

a.

x	y
2	8
3	12
4	16
5	20

$y = 4x$

b.

x	y
3	4.5
6	9
7	10.5
10	15

$y = 1.5x$

c.

x	y
2	$\frac{5}{2}$
4	5
6	$\frac{15}{2}$
12	15

$y = 1.25x$

5. Complete the equation with a number that makes the expression on the right side of the equal sign equivalent to the expression on the left side.

   $5x - 2.5 + 6x - 3 = \boxed{5.5}(2x - 1)$

6. For each angle listed, write the angle measure of its complementary angle, called its *complement*, and its supplementary angle, called its *supplement*.

Angle	Complement	Supplement
43°	47°	137°
78°	12°	102°
22°	68°	158°

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	1
	2	Activity 3	2
	3	Activity 3	2
Spiral	4	Unit 2 Lesson 5	2
	5	Unit 6 Lesson 22	2
Formative ⚡	6	Unit 7 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 4**

Supplementary and Complementary Angles (Part 2)

Let's investigate angles that are not right next to each other.

Focus

Goals

1. **Language Goal:** Use reasoning about angle measures to identify complementary or supplementary angles. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain how to determine an unknown angle measure, given complementary or supplementary angles. **(Speaking and Listening, Writing)**

Coherence

Today

Students see that angles do not need to be adjacent to be complementary or supplementary. They find supplementary and complementary angles as they measure and draw conclusions about the angle relationships of polygons. Students attend to precision as they use protractors to measure angles. They also make use of structure to determine the measure of unknown angles.

Previously

In Lesson 3, students were introduced to supplementary angles and complementary angles, and used these relationships to determine the measures of unknown angles.

Coming Soon

In Lesson 5, students will be introduced to vertical angles and will use this relationship to determine the measures of unknown angles when two lines intersect.

Rigor

- Students use special polygons to further their **conceptual understanding** of complementary and supplementary angles.
- Students calculate the unknown angle in complementary and supplementary angle pairs to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, protractors, tracing paper

Math Language Development

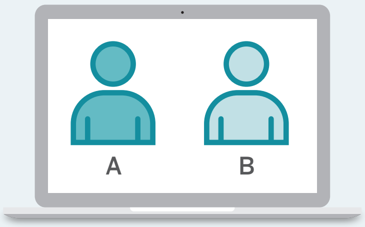
Review words

- *adjacent angles*
- *complementary angles*
- *protractor*
- *right angle*
- *supplementary angles*
- *straight angle*

Amps | Featured Activity

Activity 3
Partner Problems

Pairs of students solve related problems which happen to have the same solution, fostering discussion and encouraging collaboration.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to understand how they can determine angle measures from the given triangles in Activity 3. Ask students what they will do to resist their impulse to use a protractor to determine angle measures. Instead they should rely on the structure of the diagram and the mathematical notations within to find the measures.

Modifications to Pacing

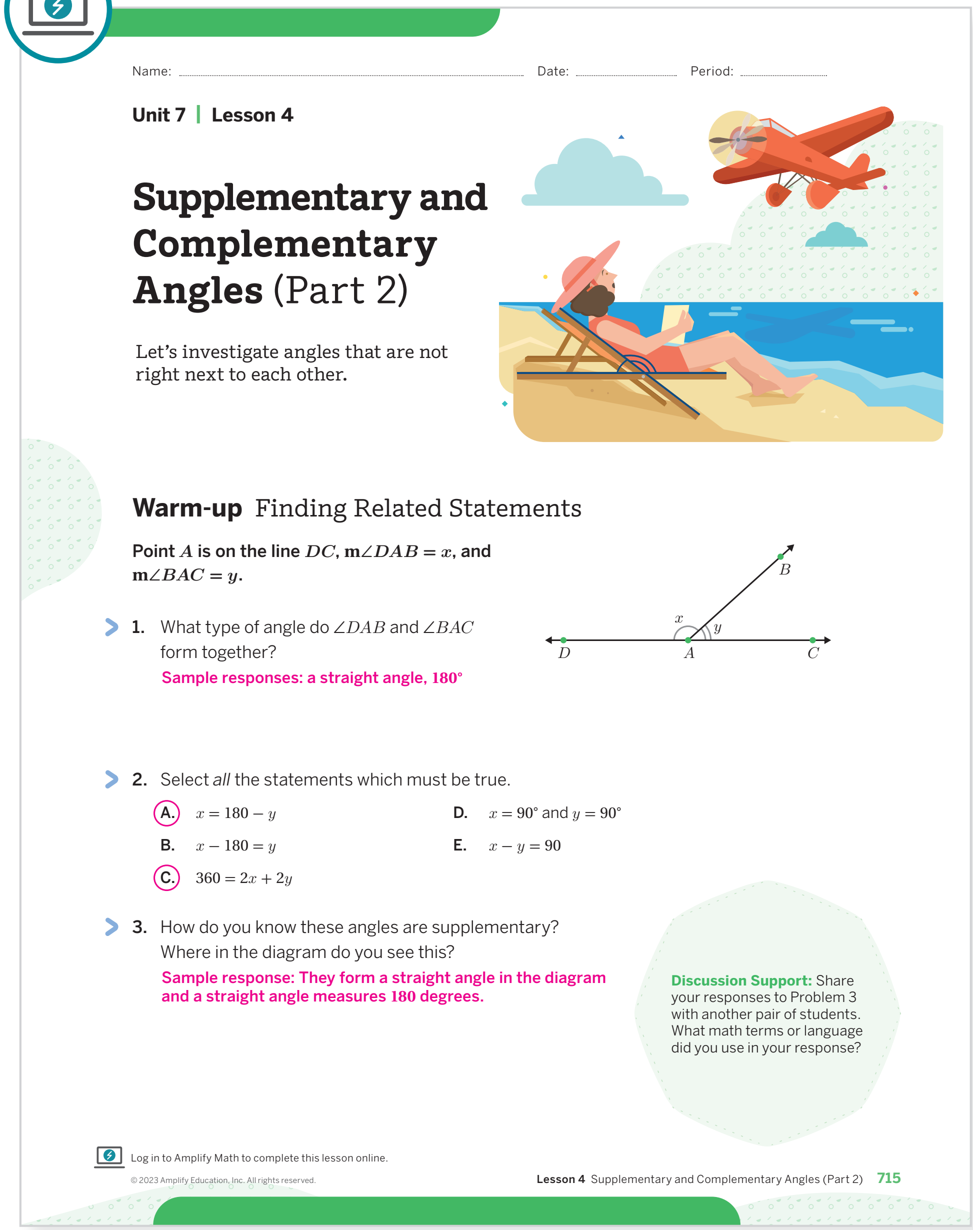
You may want to consider these additional modifications if you are short on time.

- Half the class should complete **Activity 1**, while the other half completes **Activity 2**. Have both groups share their findings with the class.
- Omit part c and Problem 2 from both **Activity 1** and **Activity 2**. During the discussion, have students share what they noticed with the class.

Pairs | 5 min

Warm-up Finding Related Statements

Students examine a pair of supplementary angles and reason about how the relationship can be represented with an equation.



Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 4

Supplementary and Complementary Angles (Part 2)

Let's investigate angles that are not right next to each other.

Warm-up Finding Related Statements

Point A is on the line DC, $m\angle DAB = x$, and $m\angle BAC = y$.

1. What type of angle do $\angle DAB$ and $\angle BAC$ form together?
   Sample responses: a straight angle, 180°

2. Select *all* the statements which must be true.
   [✓ A.] $x = 180 - y$
   B. $x - 180 = y$
   [✓ C.] $360 = 2x + 2y$
   D. $x = 90°$ and $y = 90°$
   E. $x - y = 90$

3. How do you know these angles are supplementary? Where in the diagram do you see this?
   Sample response: They form a straight angle in the diagram and a straight angle measures 180 degrees.

Discussion Support: Share your responses to Problem 3 with another pair of students. What math terms or language did you use in your response?

Log in to Amplify Math to complete this lesson online.

Lesson 4 Supplementary and Complementary Angles (Part 2) 715

1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by asking what they see in the diagram. Ask follow-up questions to help students see the straight angle and ask what they know about straight angles.

Look for points of confusion:

- **Thinking both x and y measure 90°.** Explain this may be true, but it is also possible for x and y to not be equal.
- **Not choosing $360 = 2x + 2y$ as a correct equation.** Draw an identical diagram and discuss the doubling relationship if both diagrams are considered together. (This can be discussed again once vertical angles are introduced in Lesson 5 by extending line BA.)

Look for productive strategies:

- Writing the equation $x + y = 180$ or an equivalent equation. Note students who do this.

3 Connect

Display the diagram.

Have students share their responses for Problem 2. Use the ***Poll the Class*** routine to see if they agree or disagree and allow students to discuss until they come to an agreement.

Highlight all three ways to describe the angle(s) in the diagram: straight, supplementary, and measuring 180°. For Problem 2, writing the equation $x + y = 180$ can help in determining which equations are true.

Ask:

- "For Problem 3, do you have an example that supports this statement?"
- "What other values of x and y might work?"

Math Language Development

MLR8: Discussion Supports

During the Connect, have students share their responses to Problem 3 with another pair of students. Ask them to focus on the math terms and phrases they can use, such as "they form a straight angle," and "a straight angle measures 180 degrees."

English Learners

Provide sentence frames for students to justify their responses, such as:

- "I know the angles are supplementary because _____."
- "I agree/disagree because _____."

Power-up

To power up students' ability to determine the complement and supplement of a given angle measure, have students complete:

Recall that two angles are *complementary* if the sum of their measures is 90°. Two angles are *supplementary* if the sum is 180°.

For each pair of angle measures, determine whether they are *complementary*, *supplementary*, or *neither*.

1. 90° and 20° Neither
2. 95° and 85° Supplementary
3. 30° and 60° Complementary
4. 90° and 90° Supplementary

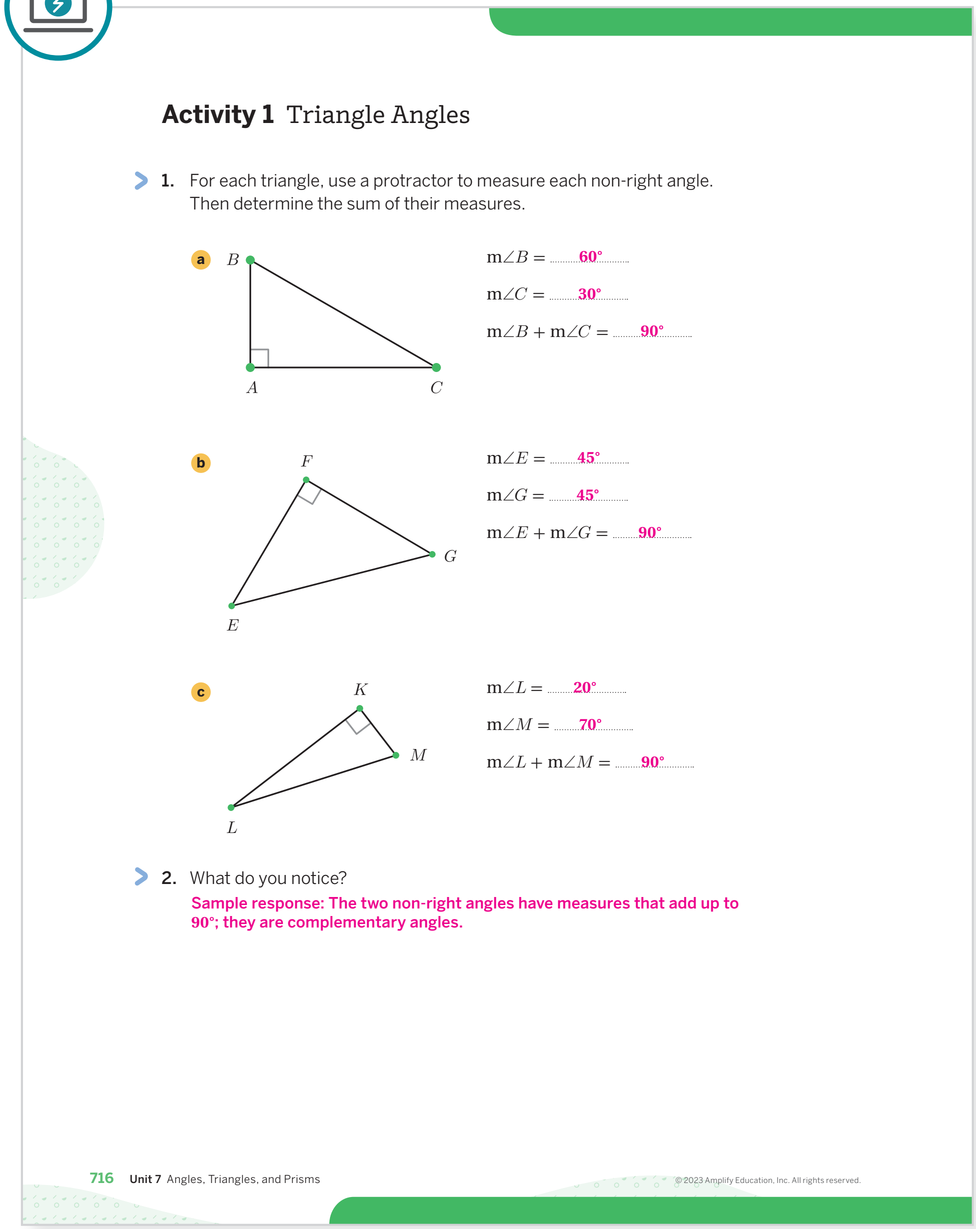
Use: Before the Warm-up.

Informed by: Performance on Lesson 3, Practice Problem 6.

Pairs | 10 min

Activity 1 Triangle Angles

Students measure the non-right angles of a right triangle to conclude that they are complementary. (Students are not expected to know a formal proof of this relationship.)



Activity 1 Triangle Angles

1. For each triangle, use a protractor to measure each non-right angle. Then determine the sum of their measures.

a

$m\angle B =$ 60°

$m\angle C =$ 30°

$m\angle B + m\angle C =$ 90°

b

$m\angle E =$ 45°

$m\angle G =$ 45°

$m\angle E + m\angle G =$ 90°

c

$m\angle L =$ 20°

$m\angle M =$ 70°

$m\angle L + m\angle M =$ 90°

2. What do you notice?
Sample response: The two non-right angles have measures that add up to 90°; they are complementary angles.

1 Launch

Activate prior knowledge by asking students to share the definitions of the terms *complementary* and *supplementary* and consider displaying these definitions for the remainder of class. Distribute geometry toolkits to pairs of students. Tell them accurate measurements are very important in this task.

2 Monitor

Help students get started by asking, "Which angles in Triangle ABC are *not* right angles?" Ask them to trace these angles with tracing paper and then rearrange them to be adjacent angles, showing they are complementary.

Look for points of confusion:

- **Struggling to use a protractor to measure the angles.** Have students extend the sides of the triangles to help make measuring the angle clearer.
- **Thinking complementary angles must be adjacent.** Point out this was true for the examples they have seen so far; however, the angles do not have to be adjacent to be complementary. Their measures just need to have a sum of 90°.

3 Connect

Display the triangles.

Have students share their angle measurements and what they noticed about the sum of the two non-right angles.

Highlight the use of precise mathematical language in describing this conclusion that the two non-right angles of a right triangle are complementary.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view aggregate class data to draw a conclusion about the two non-right angles of a right triangle.

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide students with enlarged versions of each parallelogram to help them measure the angles with greater accuracy. Consider providing the angle measurements for the two non-right angles in each diagram, which will still allow students to access the goal of the activity without having to physically measure the angles.

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share, encourage the use of their developing mathematical vocabulary by pressing for details. For example, if a student says, "The two angles add up to 90°," ask:

- "Which two angles do you mean? How can you be more detailed in your description?"
- "Is there another term for 'add up to 90°' that you can use?"

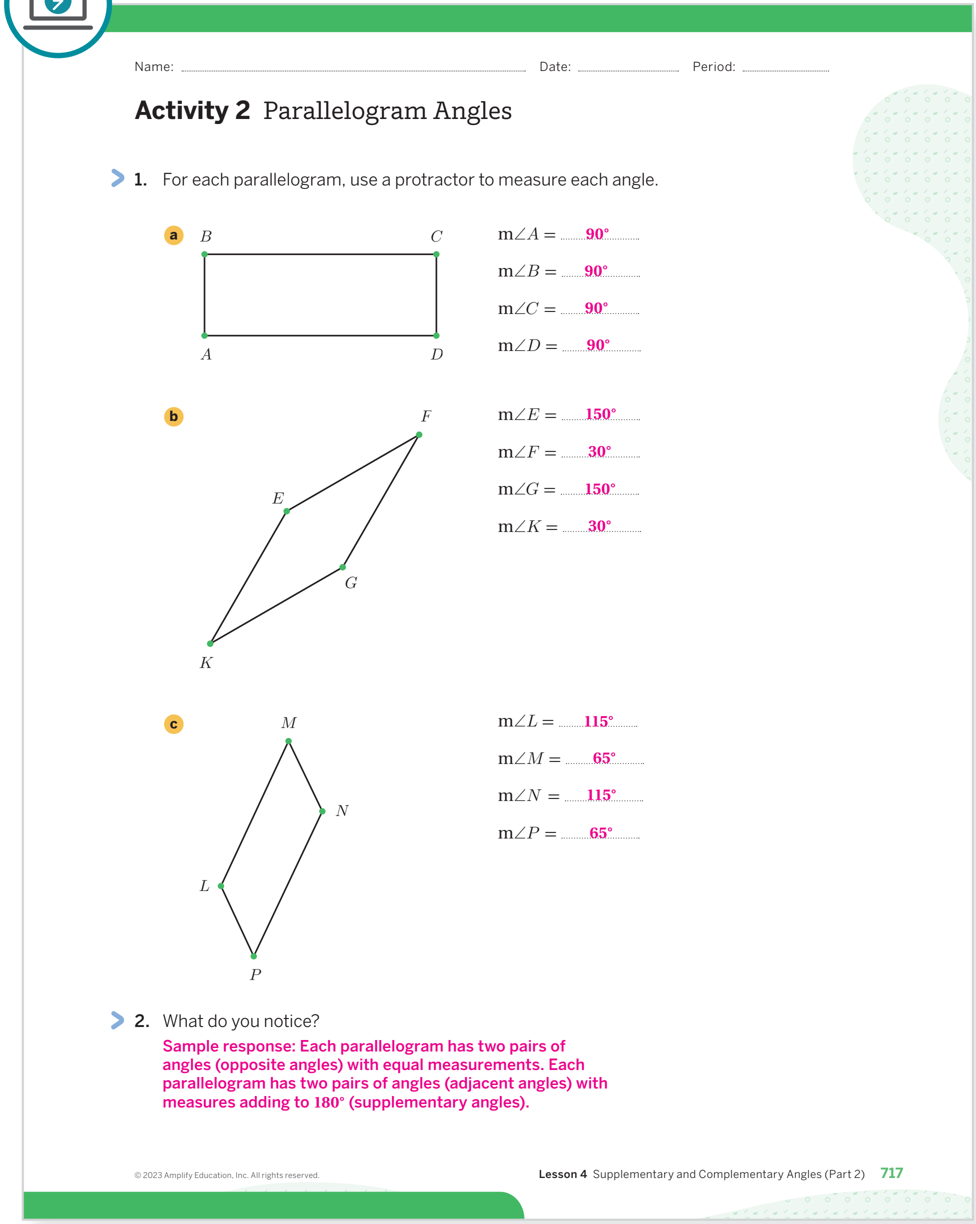
English Learners

Annotate the right angle in each triangle by writing *right angle*. Then annotate the two non-right angles by writing *non-right angles* and drawing arrows to those angles.

Pairs | 10 min

Activity 2 Parallelogram Angles

Students measure the angles of parallelograms to reason about their angle relationships. (Students are not expected to know a formal proof of these relationships.)



Name: ____________ Date: ________ Period: ________

Activity 2 Parallelogram Angles

1. For each parallelogram, use a protractor to measure each angle.

a

$m\angle A =$ 90°

$m\angle B =$ 90°

$m\angle C =$ 90°

$m\angle D =$ 90°

b

$m\angle E =$ 150°

$m\angle F =$ 30°

$m\angle G =$ 150°

$m\angle K =$ 30°

c

$m\angle L =$ 115°

$m\angle M =$ 65°

$m\angle N =$ 115°

$m\angle P =$ 65°

2. What do you notice?

Sample response: Each parallelogram has two pairs of angles (opposite angles) with equal measurements. Each parallelogram has two pairs of angles (adjacent angles) with measures adding to 180° (supplementary angles).

Lesson 4 Supplementary and Complementary Angles (Part 2) 717

1 Launch

Tell students that accurate measurements continue to be important for this task. Activate their background knowledge by asking them what they know about parallelograms and whether they can explain why the figures shown are parallelograms.

2 Monitor

Help students get started by explaining this activity is similar to Activity 1, except they will measure the angles of parallelograms instead of triangles.

Look for points of confusion:

- **Struggling to use a protractor to measure the angles.** Have students extend the sides of the parallelograms to help make measuring the angle clearer.
- **Thinking supplementary angles must be adjacent.** Point out this was true for the examples they have seen so far; however, the angles do not have to be adjacent to be supplementary. Their measures just need to have a sum of 180°.
- **Not seeing a relationship between the angles.** Have students only study the rhombus and parallelogram to determine any angle relationships. Then have them study the rectangle to see if the rectangle also has these same angle relationships.

3 Connect

Display the parallelograms.

Have students share their angle measurements and what they noticed about them.

Highlight that opposite angles of parallelograms have the same measure. This means that there are *two pairs* of supplementary angles in parallelograms. Angles that are "next to each other" (adjacent) are supplementary.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide students with enlarged versions of each parallelogram to help them measure the angles with greater accuracy. Consider providing the angle measurements for each parallelogram, which will still allow students to access the goal of the activity without having to physically measure the angles.

Extension: *Math Enrichment*

Have students make and test a conjecture — by drawing or creating other parallelograms — as to the sum of the angle measures of any parallelogram. 360°

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share, encourage their developing mathematical vocabulary by pressing for details. For example, if a student says, "The angles are the same," "The angles across from each other are the same," or "The angles next to each other add to 180°," ask:

- "Which angles are the same? Are they all the same, or only some of them? How can you be more detailed in your description?"
- "Is there another term for 'across from each other' that you can use?" Is there another term for 'next to each other' you can use?"

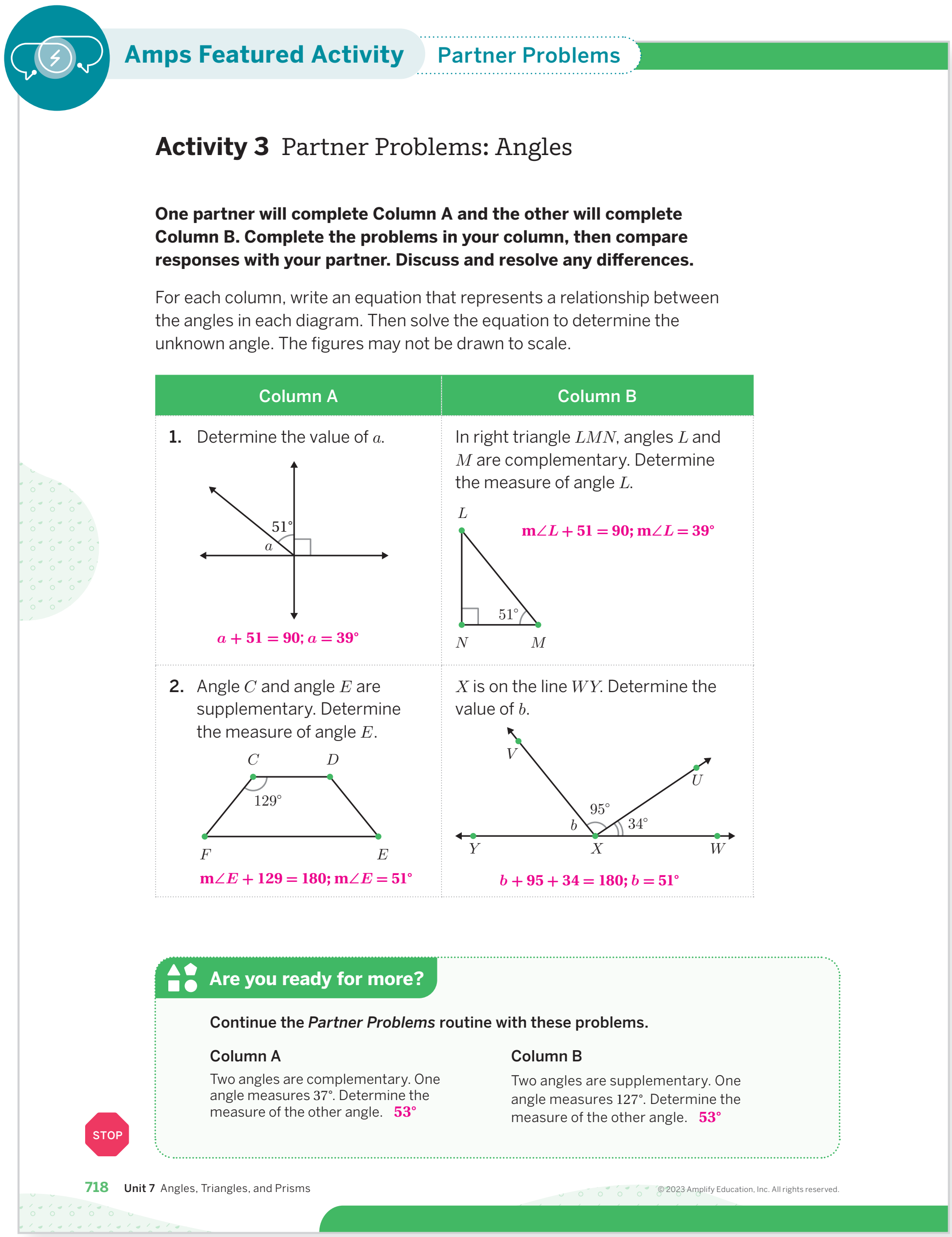
English Learners

Annotate the *adjacent angles* and *opposite angles* in each parallelogram.

Pairs | 10 min

Activity 3 Partner Problems: Angles

Students determine unknown angle measures when given supplementary or complementary angles to build fluency with writing and solving equations that represent these angle relationships.



Amps Featured Activity | Partner Problems

Activity 3 Partner Problems: Angles

One partner will complete Column A and the other will complete Column B. Complete the problems in your column, then compare responses with your partner. Discuss and resolve any differences.

For each column, write an equation that represents a relationship between the angles in each diagram. Then solve the equation to determine the unknown angle. The figures may not be drawn to scale.

Column A	Column B
1. Determine the value of a. $a + 51 = 90; a = 39°$	In right triangle LMN, angles L and M are complementary. Determine the measure of angle L. $m\angle L + 51 = 90; m\angle L = 39°$
2. Angle C and angle E are supplementary. Determine the measure of angle E. $m\angle E + 129 = 180; m\angle E = 51°$	X is on the line WY. Determine the value of b. $b + 95 + 34 = 180; b = 51°$

Are you ready for more?

Continue the *Partner Problems* routine with these problems.

Column A	Column B
Two angles are complementary. One angle measures 37°. Determine the measure of the other angle. 53°	Two angles are supplementary. One angle measures 127°. Determine the measure of the other angle. 53°

718 Unit 7 Angles, Triangles, and Prisms

1 Launch

Do not allow the use of protractors with this activity. Let students know they will need to make use of the structure of the diagrams to determine the unknown measurements. Conduct the *Partner Problems* routine.

2 Monitor

Help students get started by asking, "What do you know about your diagram? Can you find any complementary or supplementary angles?" Have them label their diagrams with the relationships they notice.

Look for points of confusion:

- **Thinking they have to complete each problem.** Remind them that if they both arrive at the same solution, they can move to the next problem. They only have to work on their partner's problem if they have different solutions.

Look for productive strategies:

- Writing equations to represent the angle relationships. Note students who do this.

3 Connect

Display the diagrams.

Have students share any problems where they did not have the same solution as their partner, and how they came to an agreement.

Highlight any equations students created to determine the unknown angle measures.

Ask:

- "Did you and your partner use the same strategy for each row?"
- "Did anyone learn a new strategy from their partner?"
- "Compare the diagrams. What do you notice? What do you wonder?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide 3 or 4 different equations that can be used to represent each diagram and ask students to determine which equation is correct before using the equation to determine the unknown angle measure.

Extension: *Math Enrichment*

Challenge students to draw their own diagram involving complementary or supplementary angles, or angles in triangles and parallelograms. They should include some angle measures and an unknown angle measure. Have them trade diagrams with a partner to determine the unknown angle measure.

Math Language Development

MLR8: Discussion Supports

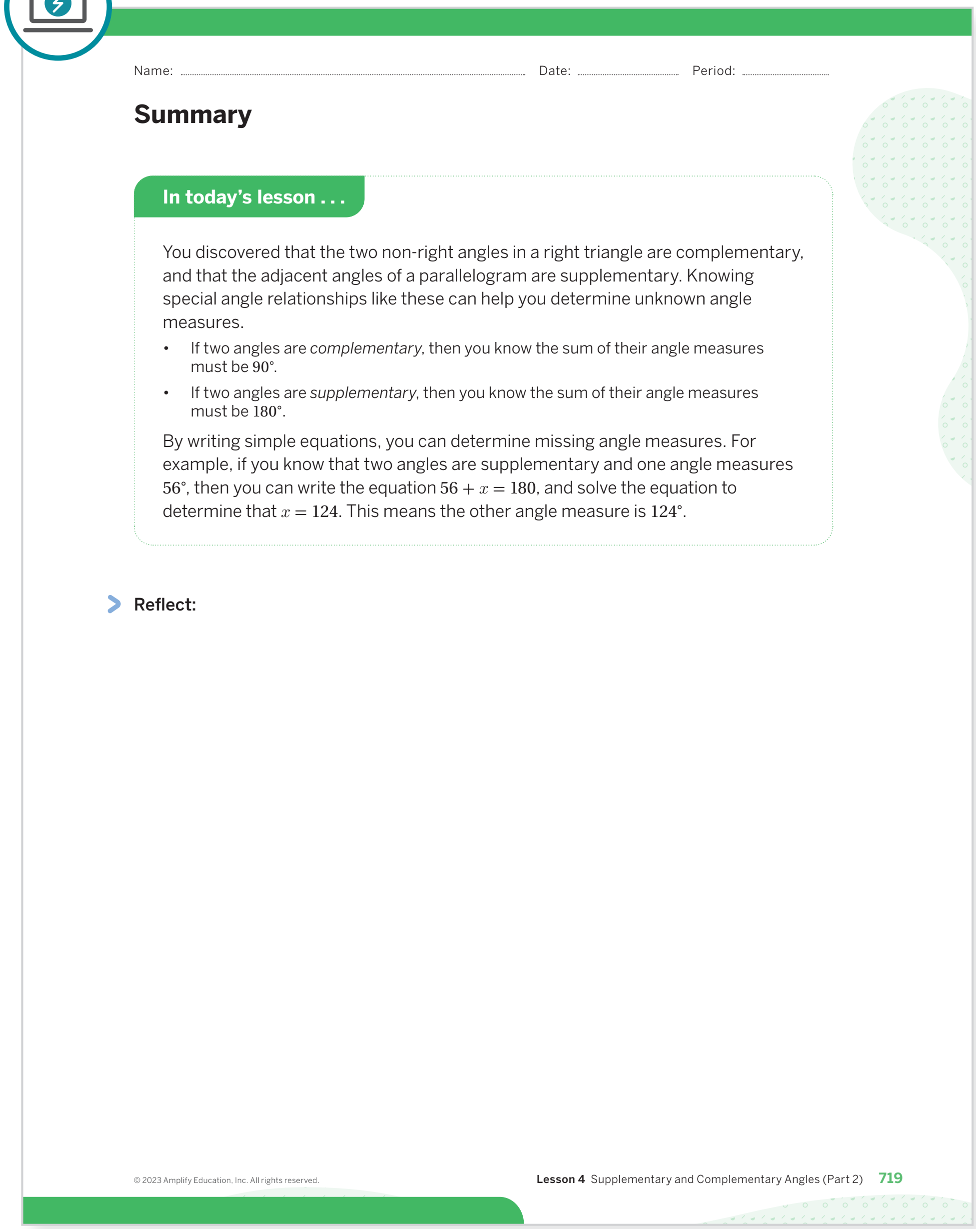
As students share their response with their partners, display these sentence frames to support them as they discuss and resolve any differences.

- "I agree with the solution because . . ."
- "I disagree with the solution because I determined . . ."
- "I wrote the equation ___ because . . ."
- "The diagram shows that angles ___ and ___ are complementary/supplementary because . . ."

Whole Class | 5 min

Summary

Review and synthesize how equations can represent complementary and supplementary angle relationships, and how solving those equations can help find missing angle measures.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You discovered that the two non-right angles in a right triangle are complementary, and that the adjacent angles of a parallelogram are supplementary. Knowing special angle relationships like these can help you determine unknown angle measures.

- If two angles are *complementary*, then you know the sum of their angle measures must be 90°.
- If two angles are *supplementary*, then you know the sum of their angle measures must be 180°.

By writing simple equations, you can determine missing angle measures. For example, if you know that two angles are supplementary and one angle measures 56°, then you can write the equation $56 + x = 180$, and solve the equation to determine that $x = 124$. This means the other angle measure is 124°.

Reflect:

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Lesson 4 Supplementary and Complementary Angles (Part 2) 719

Synthesize

Display the terms *complementary* and *supplementary*, along with respective diagrams. Be sure to include nonadjacent examples.

Ask:

- "Do supplementary or complementary angles need to be next to each other (adjacent)?" No; they just need to have a sum of 180° or 90°, respectively.
- "If you know two angles are complementary/supplementary but only know one angle's measurement, how can you find the measurement of the other angle?" Subtract the known angle's measurement from 90° or 180°, respectively.

Highlight examples of adjacent and nonadjacent complementary and supplementary angle pairs. While the term *nonadjacent* is not a formal vocabulary term, it can be helpful for students to hear and use this term when describing angle relationships. Demonstrate how equations can help determine an unknown angle when one angle measure is known and whether the angle pair is complementary or supplementary.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

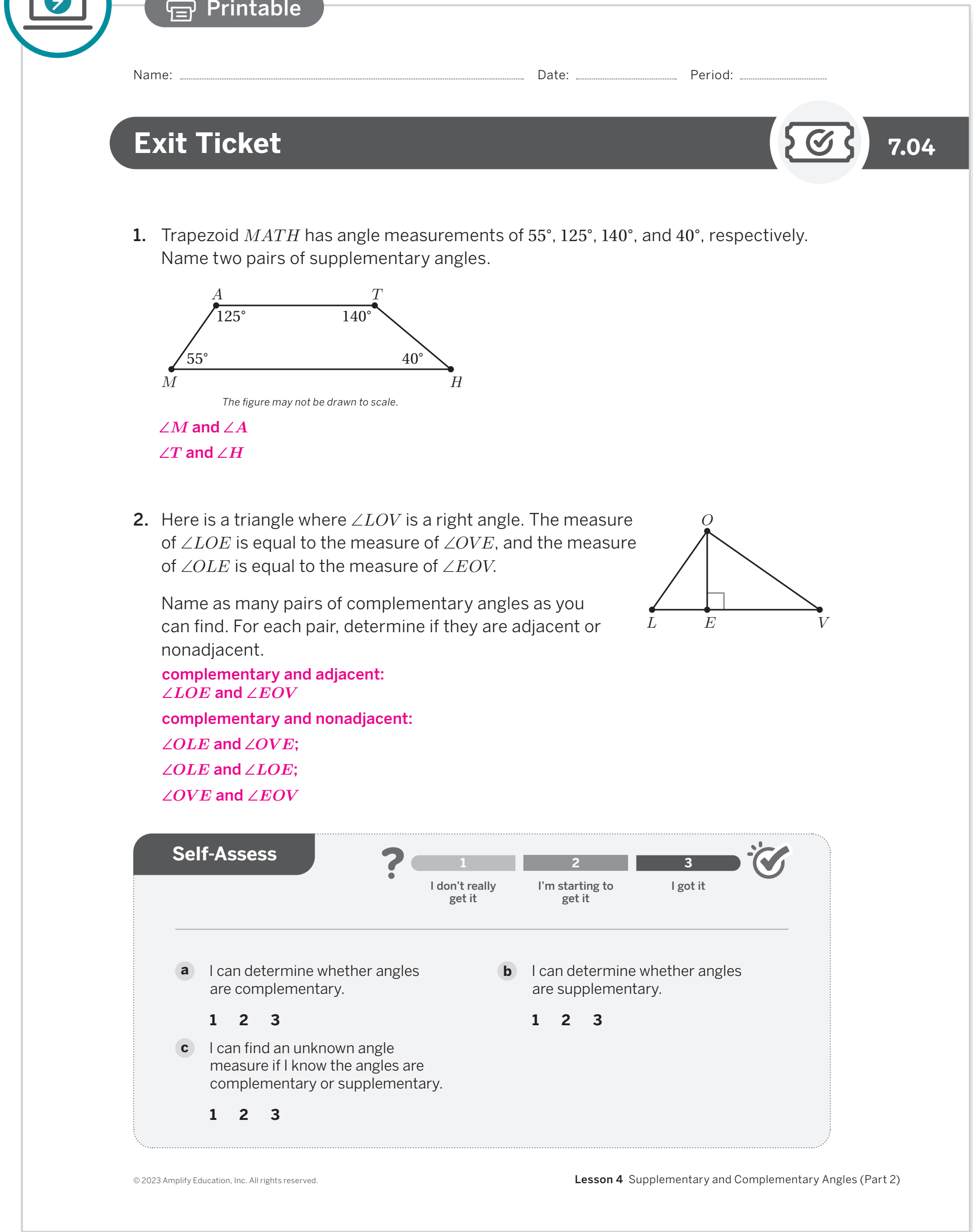
- "What pattern did you notice when measuring angles in right triangles? In parallelograms?"
- "What methods or strategies did you use to determine an angle measure without using a protractor?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by finding pairs of supplementary and complementary angles.

Printable



Name: Date: Period:

Exit Ticket 7.04

1. Trapezoid $MATH$ has angle measurements of $55°$, $125°$, $140°$, and $40°$, respectively. Name two pairs of supplementary angles.

The figure may not be drawn to scale.

$\angle M$ and $\angle A$

$\angle T$ and $\angle H$

2. Here is a triangle where $\angle LOV$ is a right angle. The measure of $\angle LOE$ is equal to the measure of $\angle OVE$, and the measure of $\angle OLE$ is equal to the measure of $\angle EOV$.

Name as many pairs of complementary angles as you can find. For each pair, determine if they are adjacent or nonadjacent.

complementary and adjacent:
$\angle LOE$ and $\angle EOV$

complementary and nonadjacent:
$\angle OLE$ and $\angle OVE$;
$\angle OLE$ and $\angle LOE$;
$\angle OVE$ and $\angle EOV$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can determine whether angles are complementary.
1 2 3

b I can determine whether angles are supplementary.
1 2 3

c I can find an unknown angle measure if I know the angles are complementary or supplementary.
1 2 3

Lesson 4 Supplementary and Complementary Angles (Part 2)

Success looks like . . .

- **Language Goal:** Using reasoning about angle measures to identify complementary or supplementary angles. **(Speaking and Listening, Writing)**
  - » Identifying supplementary angles by finding those with measures that add to 180 degrees in Problem 1.
- **Language Goal:** Explaining how to determine an unknown angle measure, given complementary or supplementary angles. **(Speaking and Listening, Writing)**

Suggested next steps

If students only find one supplementary pair in Problem 1, consider:

- Reminding them the problem asked for two pairs of supplementary angles.
- Asking if they considered the remaining pair of angles.

If students did not find all the pairs of complementary angles in Problem 2, consider:

- Having them label the diagram with the relationships that they know.
- Reviewing what the term *complementary* means.
- Assigning Practice Problems 1 and 2.

Professional Learning

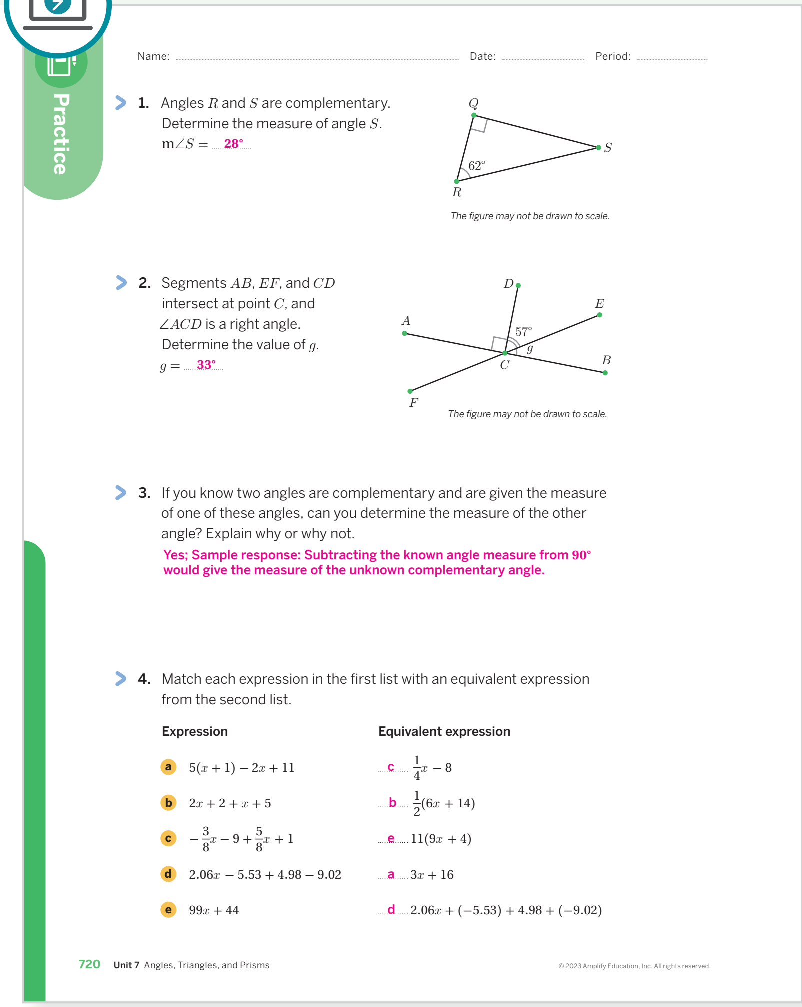
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? One instructional goal for this lesson was for students to use reasoning to find an unknown angle measure, given complementary or supplementary angles. What did you do specifically to help students accomplish this goal?
- How did students attend to precision today? How are you helping students become aware of how they are progressing in this area? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Angles R and S are complementary. Determine the measure of angle S.

$m\angle S =$ 28°

The figure may not be drawn to scale.

2. Segments AB, EF, and CD intersect at point C, and $\angle ACD$ is a right angle. Determine the value of g.

$g =$ 33°

The figure may not be drawn to scale.

3. If you know two angles are complementary and are given the measure of one of these angles, can you determine the measure of the other angle? Explain why or why not.

Yes; Sample response: Subtracting the known angle measure from 90° would give the measure of the unknown complementary angle.

4. Match each expression in the first list with an equivalent expression from the second list.

Expression	Equivalent expression
a $5(x+1)-2x+11$	c $\frac{1}{4}x-8$
b $2x+2+x+5$	b $\frac{1}{2}(6x+14)$
c $-\frac{3}{8}x-9+\frac{5}{8}x+1$	e $11(9x+4)$
d $2.06x-5.53+4.98-9.02$	a $3x+16$
e $99x+44$	d $2.06x+(-5.53)+4.98+(-9.02)$

720 **Unit 7** Angles, Triangles, and Prisms

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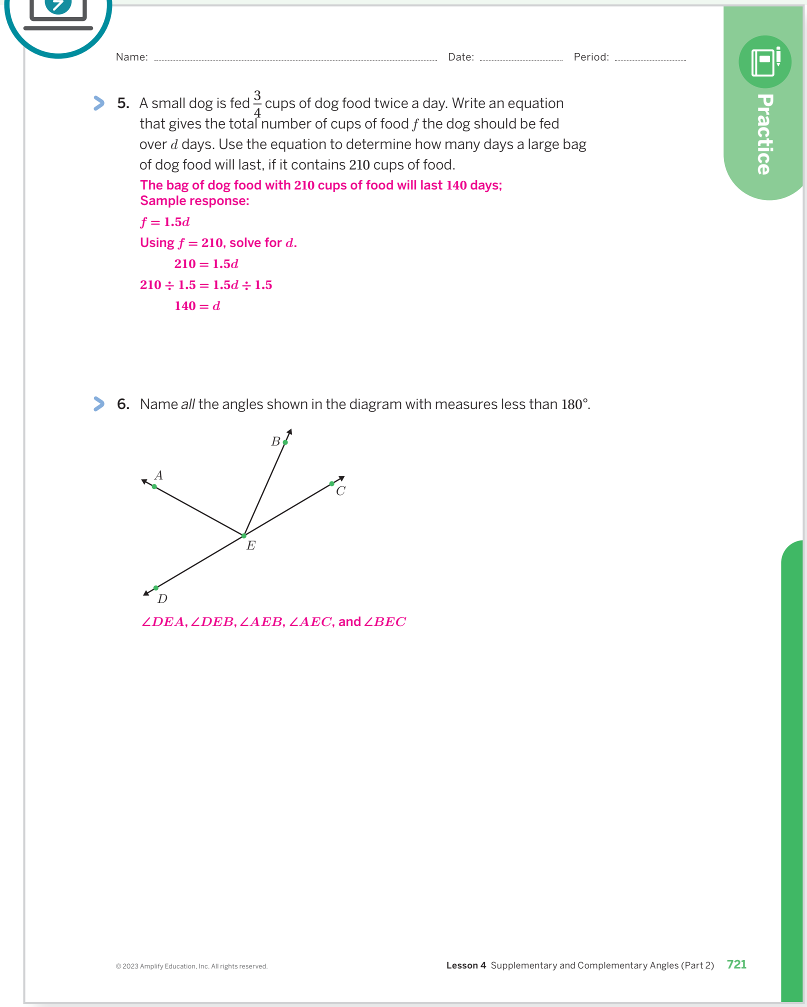

Name: Date: Period:

5. A small dog is fed $\frac{3}{4}$ cups of dog food twice a day. Write an equation that gives the total number of cups of food f the dog should be fed over d days. Use the equation to determine how many days a large bag of dog food will last, if it contains 210 cups of food.

The bag of dog food with 210 cups of food will last 140 days;
Sample response:

$f = 1.5d$

Using $f = 210$, solve for d.

$210 = 1.5d$

$210 \div 1.5 = 1.5d \div 1.5$

$140 = d$

6. Name *all* the angles shown in the diagram with measures less than 180°.

$\angle DEA$, $\angle DEB$, $\angle AEB$, $\angle AEC$, and $\angle BEC$

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Lesson 4 Supplementary and Complementary Angles (Part 2) 721

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 3	1
	3	Activities 1–3	2
Spiral	4	Unit 6 Lesson 22	1
	5	Unit 2 Lesson 8	2
Formative	6	Unit 7 Lesson 5	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 5**

Vertical Angles

Let's investigate angles that are across from each other.

Focus

Goals

1. **Language Goal:** Comprehend that the term *vertical angles* refers to a pair of angles formed by two intersecting lines. **(Speaking and Listening, Writing)**
2. **Language Goal:** Generalize that the opposite angles formed by two intersecting lines have equal angle measures. **(Speaking and Listening, Writing)**
3. **Language Goal:** Solve multi-step problems involving vertical angles, and explain the reasoning used. **(Speaking and Listening, Writing)**

Coherence

Today

In this lesson, students use the term *vertical angles* to describe the opposite angles that are formed when two lines intersect. They see that vertical angles have equal measures and use this relationship, along with what they have learned about other angle relationships, to find unknown angle measures in multi-step problems.

Previously

In Lessons 3 and 4, students learned about supplementary and complementary angles. They used these relationships to find unknown angle measures and to draw conclusions about the angle relationships of triangles and parallelograms.

Coming Soon

In Lesson 6, students will represent angle relationships using equations of the form $px + q = r$ and $p(x + q) = r$. They will solve equations of this form to find unknown angle measures.

Rigor

- Students use visual models to develop **conceptual understanding** of vertical angles.
- Students strengthen their **fluency** in calculating unknown angle measures.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Anchor Chart PDF, *Angle Relationships*
- Anchor Chart PDF, *Angle Relationships* (answers)
- geometry toolkits: protractors, rulers

Math Language Development

New word

- ***vertical angles***

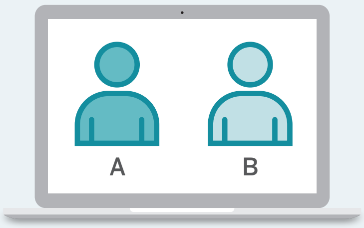
Review words

- *adjacent angles*
- *complementary angles*
- *protractor*
- *right angle*
- *straight angle*
- *supplementary angles*

Amps | **Featured Activity**

Activity 1
Collaborative Measurement

Students share their angle measures with their classmates. Then they use the collective data to make an observation about the measures of vertical angles.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not be successful if they choose not to follow all of the directions from Activity 1. Ask students to identify how their groups could have had better precision by making more constructive choices.

Modifications to Pacing

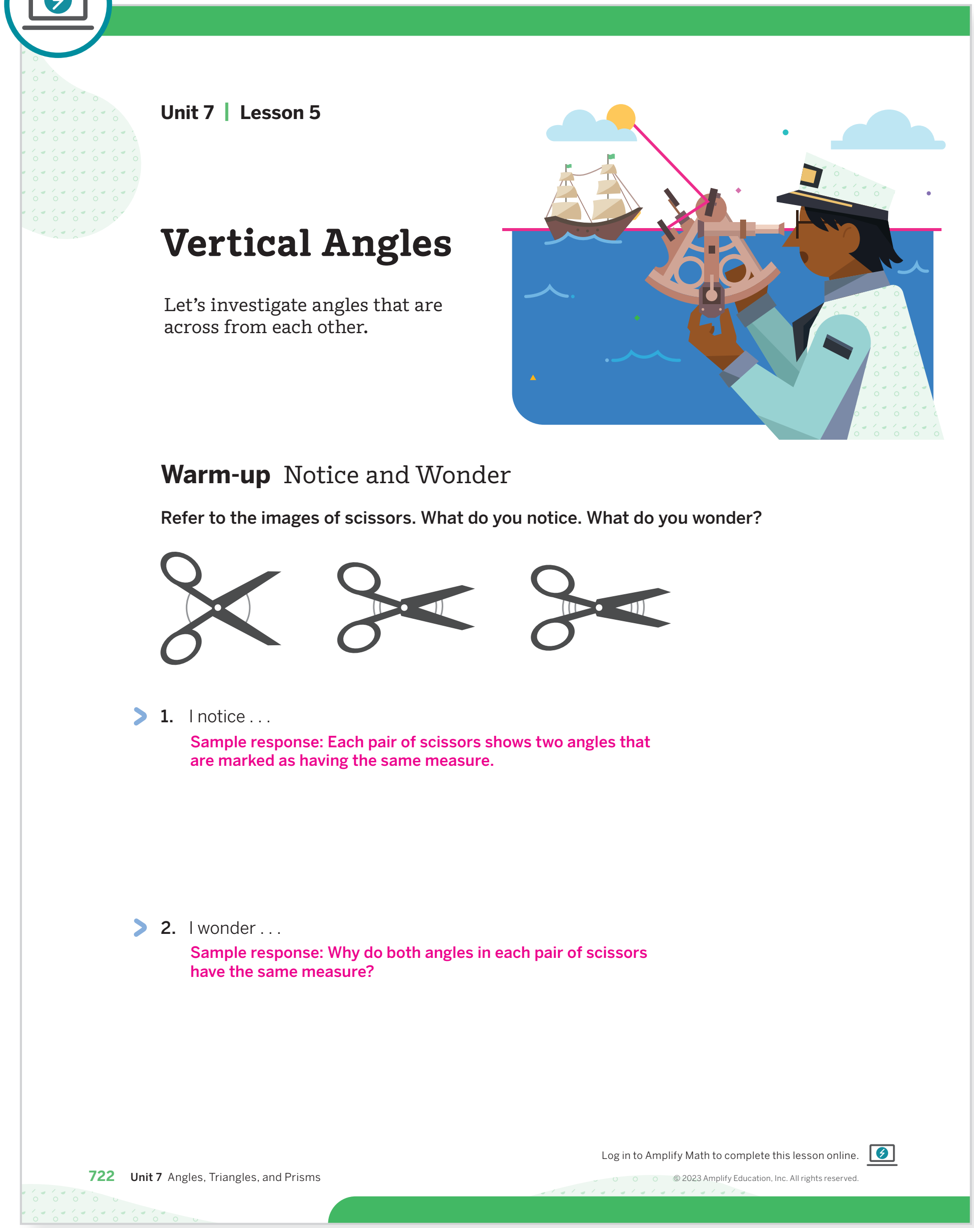
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, provide students with choice by having them complete only 2 of the 4 problems.

Independent | 5 min

Warm-up Notice and Wonder

Students analyze images of scissors with vertical angles marked to introduce the idea that vertical angles have equal measures.



Unit 7 | Lesson 5

Vertical Angles

Let's investigate angles that are across from each other.

Warm-up Notice and Wonder

Refer to the images of scissors. What do you notice. What do you wonder?

1. I notice . . .
   Sample response: Each pair of scissors shows two angles that are marked as having the same measure.

2. I wonder . . .
   Sample response: Why do both angles in each pair of scissors have the same measure?

Log in to Amplify Math to complete this lesson online.

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1 Launch

Conduct the *Notice and Wonder* routine using the images of the scissors.

2 Monitor

Help students get started by asking, "What do you notice about the images?"

Look for points of confusion:

- **Not understanding that the arcs refer to the angles.** Ask students to think about where they have seen those types of marks used in this unit.

Look for productive strategies:

- Noticing the same arc is drawn on both angles in each pair of scissors and wondering if that means the angles have the same measure.

3 Connect

Have students share what they noticed and wondered about the images of the scissors.

Highlight the connections between real-world objects and geometric objects that can be used to model or represent them. Point out that the two parts of a pair of scissors can be thought of as two intersecting lines.

Differentiated Support

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

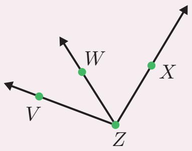
Consider bringing in a pair of scissors to demonstrate opening and closing the scissors. Or allow students to physically manipulate scissors from their geometry toolkits to visualize the angle relationships.

Consider using the internet, or another source, to show images of other everyday objects that illustrate two opposite angles that have the same measure. Examples include: railroad crossing signs, street intersections, and artwork.

Power-up

To power up students' ability to name angles, have students complete:

Recall that angles are named using three points, with the vertex as the middle point. For example, one angle in the given diagram is $\angle XZV$. Name as many other angles as you can.



- $\angle VZX$
- $\angle VZW$
- $\angle WZX$
- $\angle XZW$
- $\angle WZV$

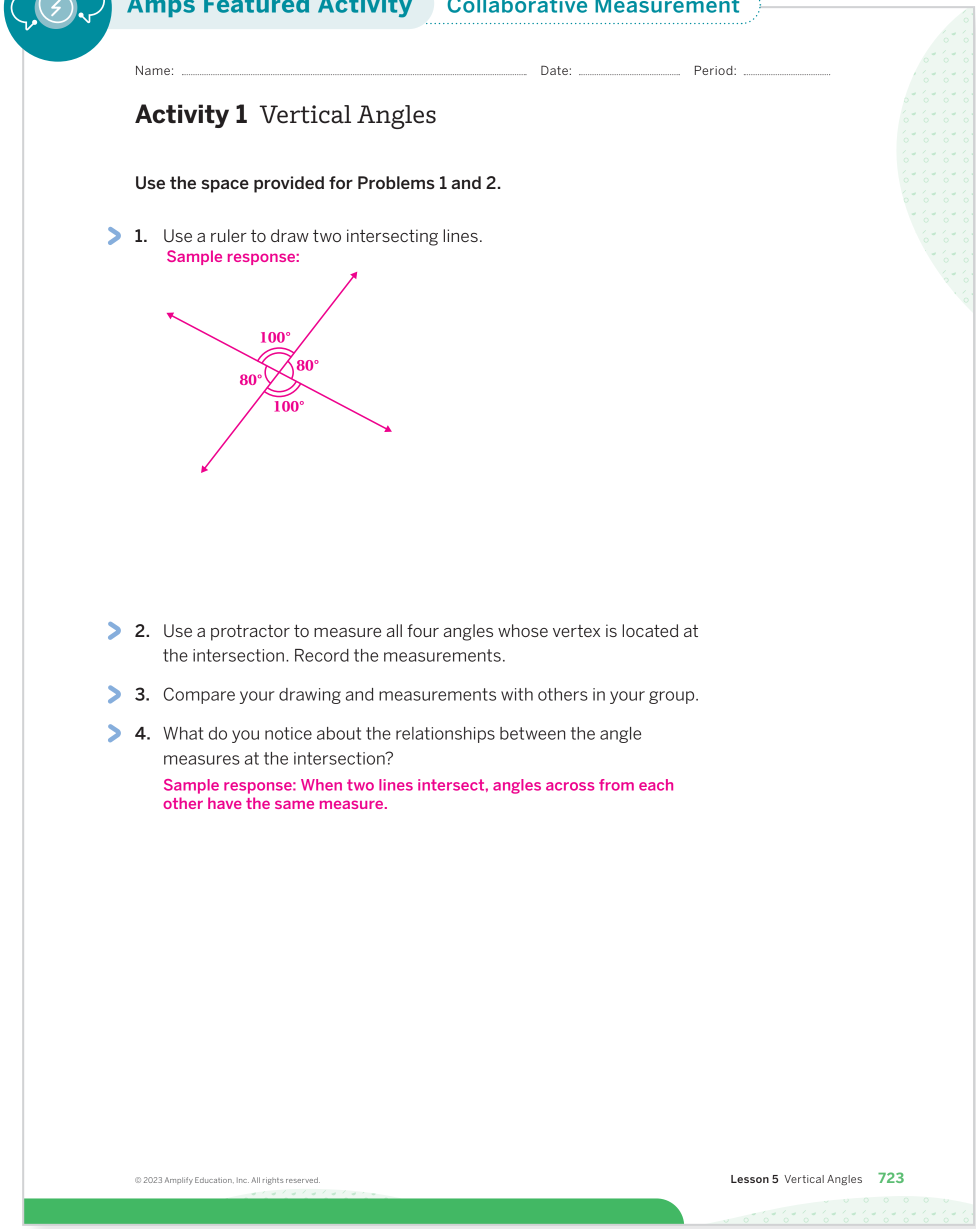
Use: Before Activity 1.
Informed by: Performance on Lesson 4, Practice Problem 6.

Small Groups | 20 min

Activity 1 Vertical Angles

Students measure the angles formed by two intersecting lines to conclude that vertical angles have the same measure. (Students are not expected to know a formal proof of this relationship.)

Amps Featured Activity Collaborative Measurement



Name: ________ Date: ________ Period: ________

Activity 1 Vertical Angles

Use the space provided for Problems 1 and 2.

1. Use a ruler to draw two intersecting lines.
   Sample response:

2. Use a protractor to measure all four angles whose vertex is located at the intersection. Record the measurements.
3. Compare your drawing and measurements with others in your group.
4. What do you notice about the relationships between the angle measures at the intersection?
   Sample response: When two lines intersect, angles across from each other have the same measure.

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Lesson 5 Vertical Angles 723

1 Launch

Provide each student with a protractor and a ruler. Tell students that accurate measurements are important in this activity.

2 Monitor

Help students get started by demonstrating what it means for two lines to *intersect*. Consider holding up two meter sticks to demonstrate several examples.

Look for points of confusion:

- **Thinking that intersecting lines must be perpendicular.** Explain that the term *intersect* is another way of saying *cross* and that intersecting lines do not have to cross at right angles.

Look for productive strategies:

- Using supplementary angles to explain why opposite angles have equal measures.

3 Connect

Have students share what they notice about the angles at the intersection of the two lines.

Display the Activity 1 PDF.

Define the term ***vertical angles*** as opposite angles that share the same vertex. They are formed by two intersecting lines.

Highlight that two intersecting lines form vertical angles. Since each vertical angle is supplementary to the same adjacent angle, the vertical angles must have equal measures.

Ask students to identify four pairs of vertical angles shown in the diagram. Make sure students mention $\angle CEA$ and $\angle DEB$ as vertical angles and stress that here "vertical" doesn't necessarily mean up and down when referring to angles.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide several different examples of pre-drawn intersecting lines for students to use, with angle measurements indicated. Have them complete Problem 4, using the examples provided. This will still allow them to access the goal of the activity, without having to draw their own diagram.

Extension: *Math Enrichment*

Challenge students to construct an argument that explains *why* vertical angles must be congruent.

Math Language Development

MLR2: Collect and Display

As students work, listen for and collect vocabulary, gestures, and phrases that students use to describe the relationships they notice between angles. For example, they may describe vertical angles as "opposite" angles. Add these examples to the class display for this unit.

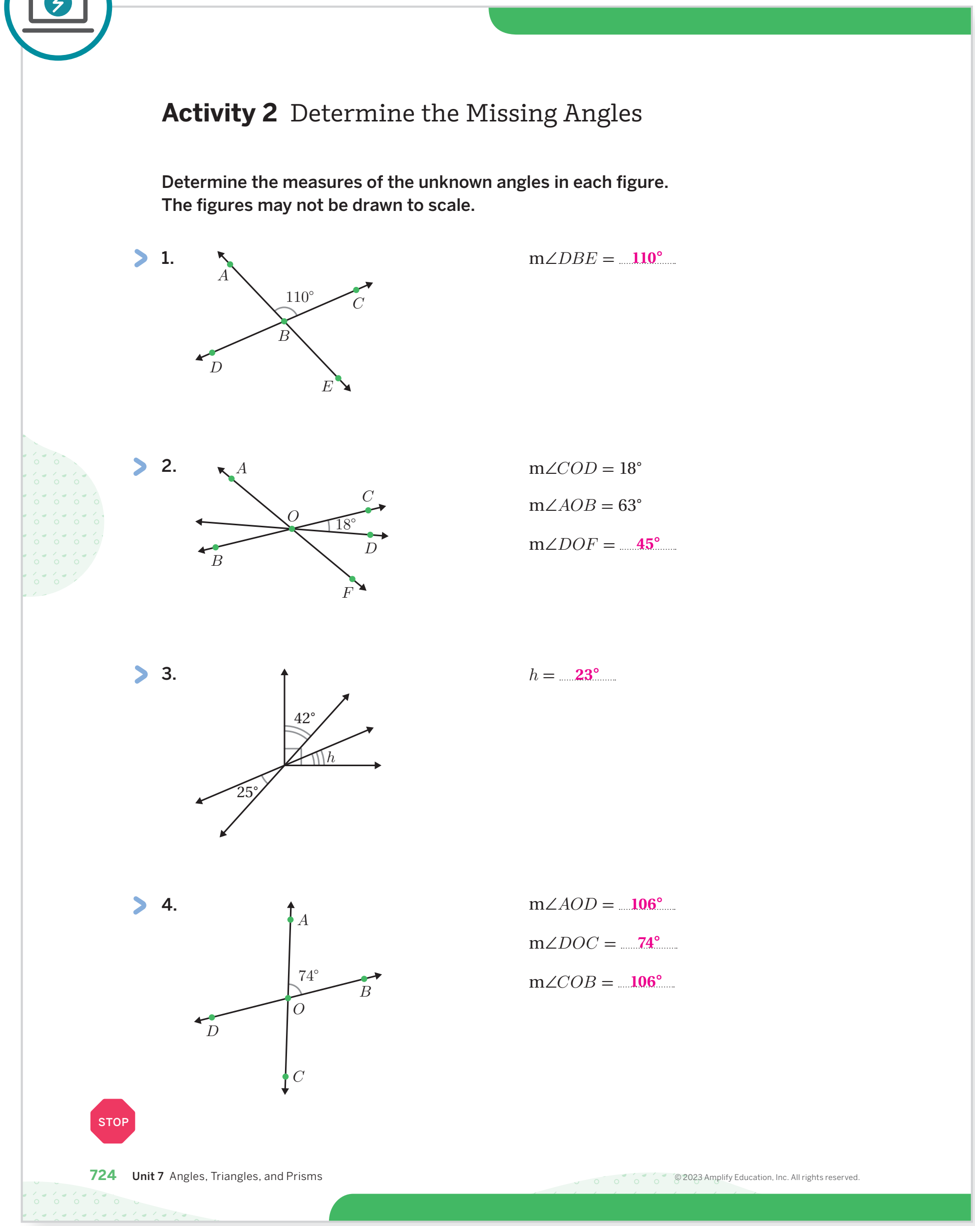
English Learners

Use gestures to model vertical and supplementary angle relationships. For example, cross your arms indicating an "X" to illustrate vertical angles. Keep one arm horizontal and use your other arm to indicate another angle, illustrating supplementary angles.

Pairs | 10 min

Activity 2 Determine the Missing Angles

Students determine the measures of unknown angles by applying their understanding of vertical, complementary, and supplementary angles to reinforce understanding of these relationships.



Activity 2 Determine the Missing Angles

Determine the measures of the unknown angles in each figure. The figures may not be drawn to scale.

1. m∠DBE = 110°

2. m∠COD = 18°

m∠AOB = 63°

m∠DOF = 45°

3. h = 23°

4. m∠AOD = 106°

m∠DOC = 74°

m∠COB = 106°

STOP

724 Unit 7 Angles, Triangles, and Prisms

1 Launch

Instruct students to complete each problem independently, before discussing it with their partner.

2 Monitor

Help students get started by having them identify the vertical angles in the diagram for Problem 1 and then asking them what they know about the measures of vertical angles.

Look for points of confusion:

- **Not recognizing the complementary angle relationship in Problem 3.** Remind them to consider what other angle relationships are provided in each diagram.

Look for productive strategies:

- Annotating the diagrams.
- Using vertical, complementary, and supplementary angle relationships to determine the unknown angle measures in each diagram.

3 Connect

Have students share their strategies for determining the unknown angle measures.

Highlight how angle relationships can be used to determine unknown angle measures. Review vocabulary by naming the angle relationships in each diagram.

Ask, "Can you determine *all* the unknown angle measures in each diagram? Explain your thinking." Sample response: Yes, I can determine the measures of all the vertical angle pairs, then use what I know about complementary and supplementary angles to determine the other unknown angle measures.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1–2.

Extension: *Math Enrichment*

Challenge students to determine the measure of *every* angle for each diagram in Problems 1–3.

Problem 1: 110°, 110°, 70°, 70°

Problem 2: Starting with m∠COD and going clockwise: 18°, 45°, 117°, 18°, 45°, 117°

Problem 3: Starting with 42° and going clockwise: 42°, 25°, 23°, 132°, 25°, 113°

Math Language Development

MLR8: Discussion Supports—Press for Details

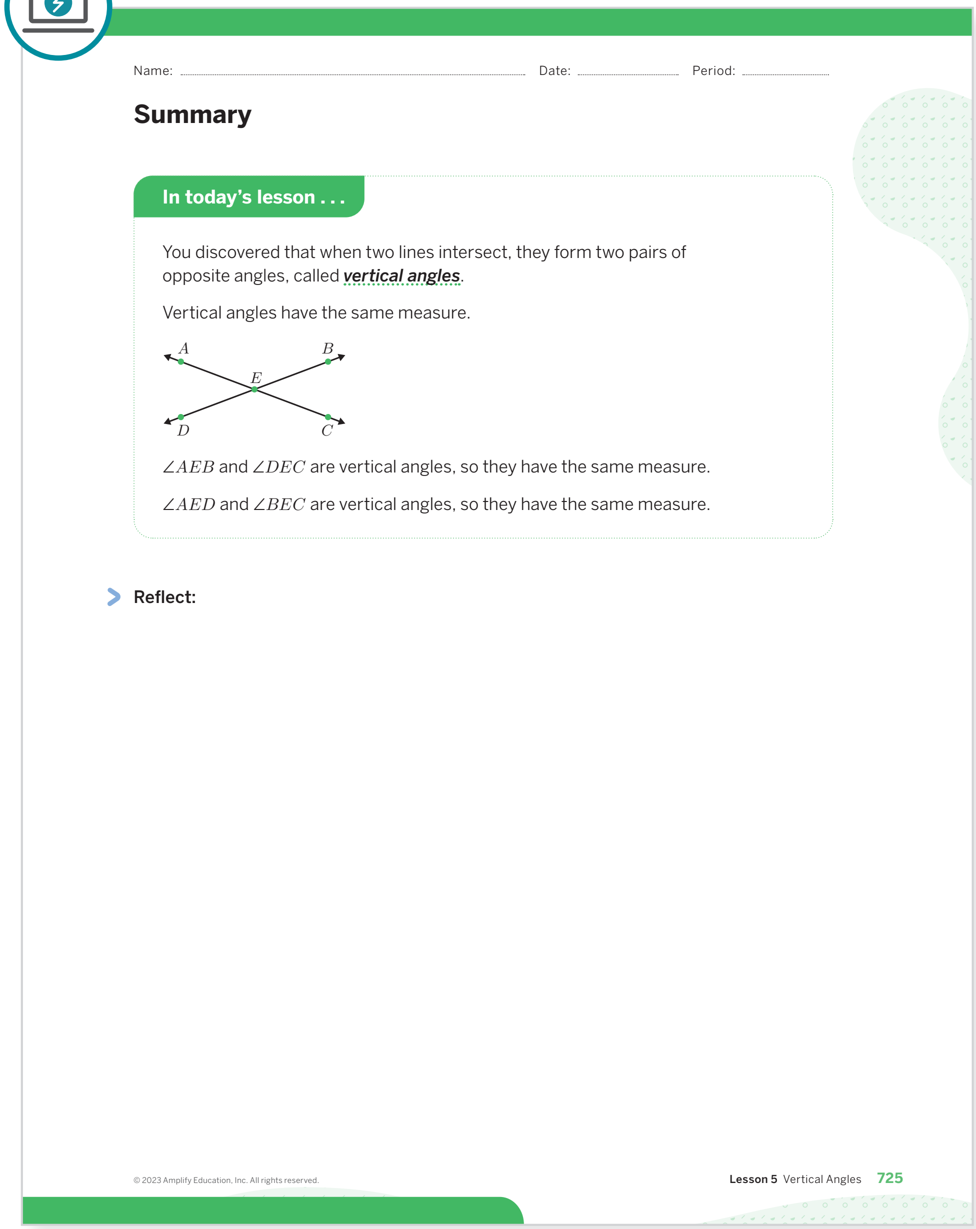
During the Connect, as students share their strategies for determining the unknown angle measures, press for details in their reasoning by asking the following questions:

- "What angle relationships did you use in Problem 1? What math terms can you use to describe these angles?"
- "What did you do first in Problem 2? What math term describes this relationship?"
- "What are some different approaches you could do first in Problem 3? Use math terms to describe these relationships."

Whole Class | 5 min

Summary

Review and synthesize how to use vertical angle relationships to determine unknown angle measures.



Name: Date: Period:

Summary

In today's lesson . . .

You discovered that when two lines intersect, they form two pairs of opposite angles, called ***vertical angles***.

Vertical angles have the same measure.

$\angle AEB$ and $\angle DEC$ are vertical angles, so they have the same measure.

$\angle AED$ and $\angle BEC$ are vertical angles, so they have the same measure.

Reflect:

Synthesize

Display the Anchor Chart PDF, *Angle Relationships* and complete the vertical angles section during the class discussion.

Formalize vocabulary: *vertical angles*

Ask:

- "What angle relationships do you notice?"
- "Which angles are vertical angles?"
- "Which angles, if any, are complementary or supplementary?"
- "What does knowing these angle relationships tell you about the measures of these angles?"
- "If I told you the measure of one angle, how could you find the measures of the other three angles?"

Highlight the angle relationships shown in the diagram on the Anchor Chart PDF. Have students note which angles are vertical, which are supplementary, and which are vertical. You may wish to also review which angles are adjacent or nonadjacent.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What pattern did you notice when measuring angles created by two intersecting lines?"
- "What methods or strategies did you use to determine an angle measure without using a protractor?"

Math Language Development

MLR2: Collect and Display

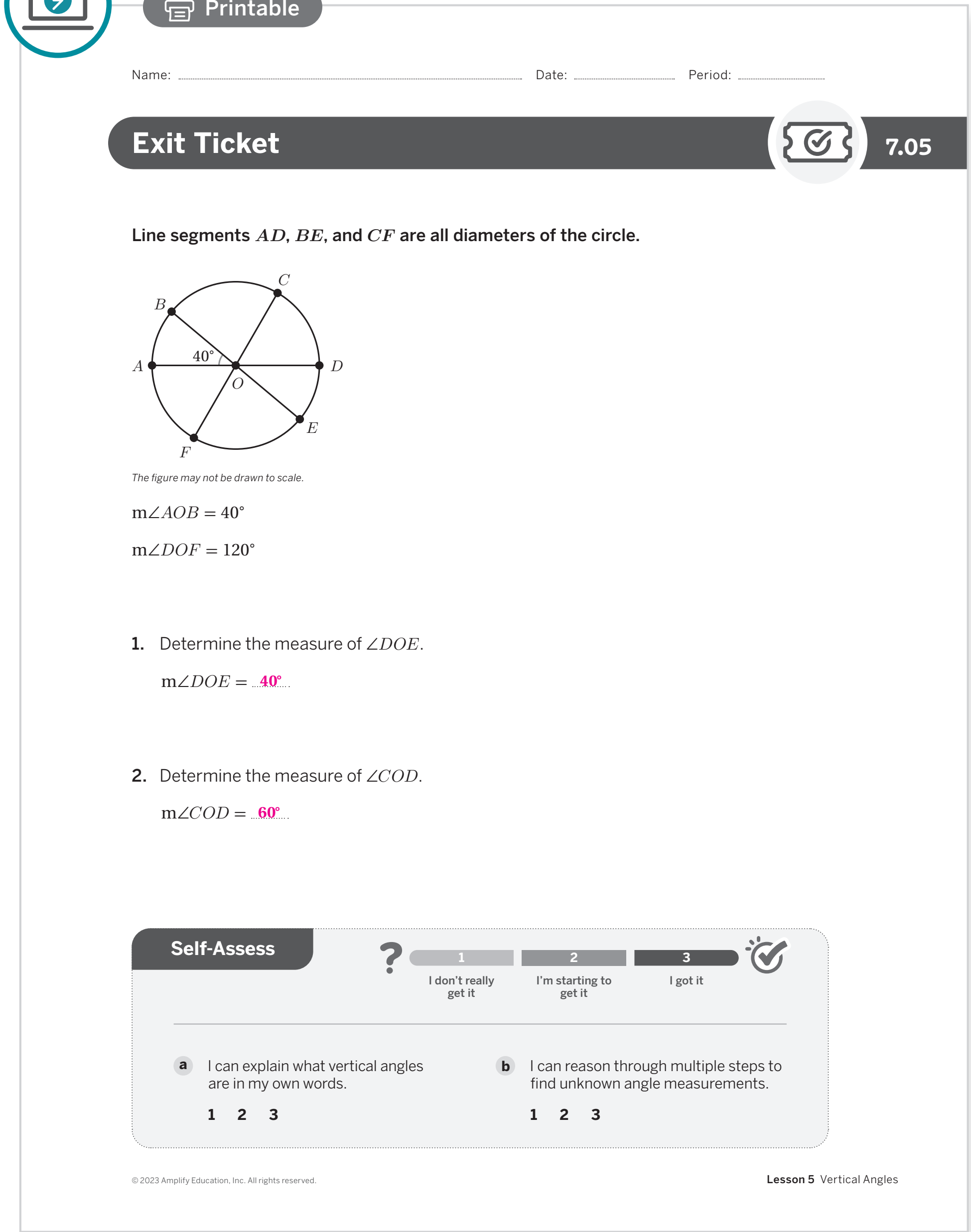
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *vertical angles* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of vertical angles by finding unknown angle measures in a diagram.

Printable



Name: Date: Period:

Exit Ticket 7.05

Line segments AD, BE, and CF are all diameters of the circle.

The figure may not be drawn to scale.

$m\angle AOB = 40°$

$m\angle DOF = 120°$

1. Determine the measure of $\angle DOE$.

   $m\angle DOE =$ 40°

2. Determine the measure of $\angle COD$.

   $m\angle COD =$ 60°

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain what vertical angles are in my own words. 1 2 3

b. I can reason through multiple steps to find unknown angle measurements. 1 2 3

Lesson 5 Vertical Angles

Success looks like . . .

- **Language Goal:** Comprehending that the term *vertical angles* refers to a pair of angles formed by two intersecting lines. **(Speaking and Listening, Writing)**
  - » Understanding that $\angle DOE$ and $\angle AOB$ are vertical angles to determine the measure of $\angle DOE$ in Problem 1.
- **Language Goal:** Generalizing that the opposite angles formed by two intersecting lines have equal angle measures. **(Speaking and Listening, Writing)**
- **Language Goal:** Solving multi-step problems involving vertical angles, and explaining the reasoning used. **(Speaking and Listening, Writing)**
  - » Using the measures of adjacent vertical angles to determine the measure of $\angle COD$ in Problem 2.

Suggested next steps

If students answer Problem 1 correctly and Problem 2 incorrectly, consider:

- Having them annotate the diagram with the given information and their response from Problem 1. Then have them identify the relationship between $\angle COD$ and other angles with known measures.
- Assigning Practice Problem 2.

If students solve for incorrect angles in the diagrams, consider:

- Reviewing how to name angles.
- Assigning Practice Problem 2.

Professional Learning

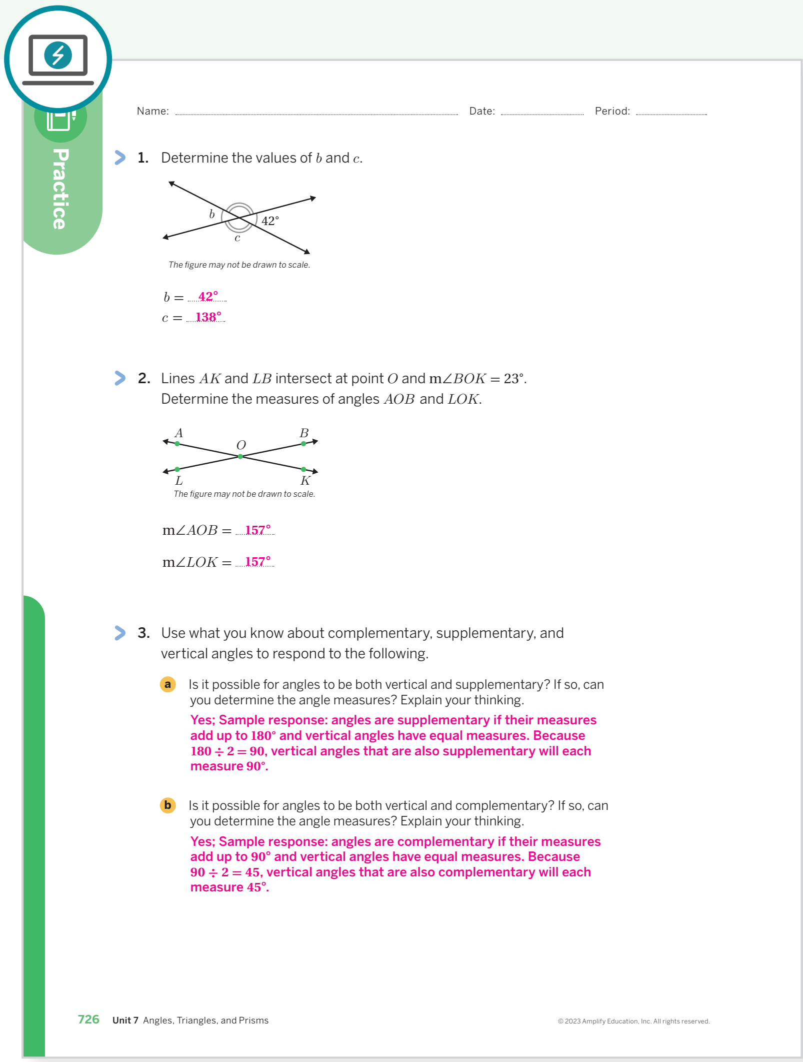
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did the ***Notice and Wonder*** routine support students in generalizing that vertical angles have equal angle measures?
- In earlier lessons, students studied complementary and supplementary angles. How did that support them in building their understanding of vertical angles? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Determine the values of b and c.

The figure may not be drawn to scale.

$b =$ 42°
$c =$ 138°

2. Lines AK and LB intersect at point O and $m\angle BOK = 23°$. Determine the measures of angles AOB and LOK.

The figure may not be drawn to scale.

$m\angle AOB =$ 157°

$m\angle LOK =$ 157°

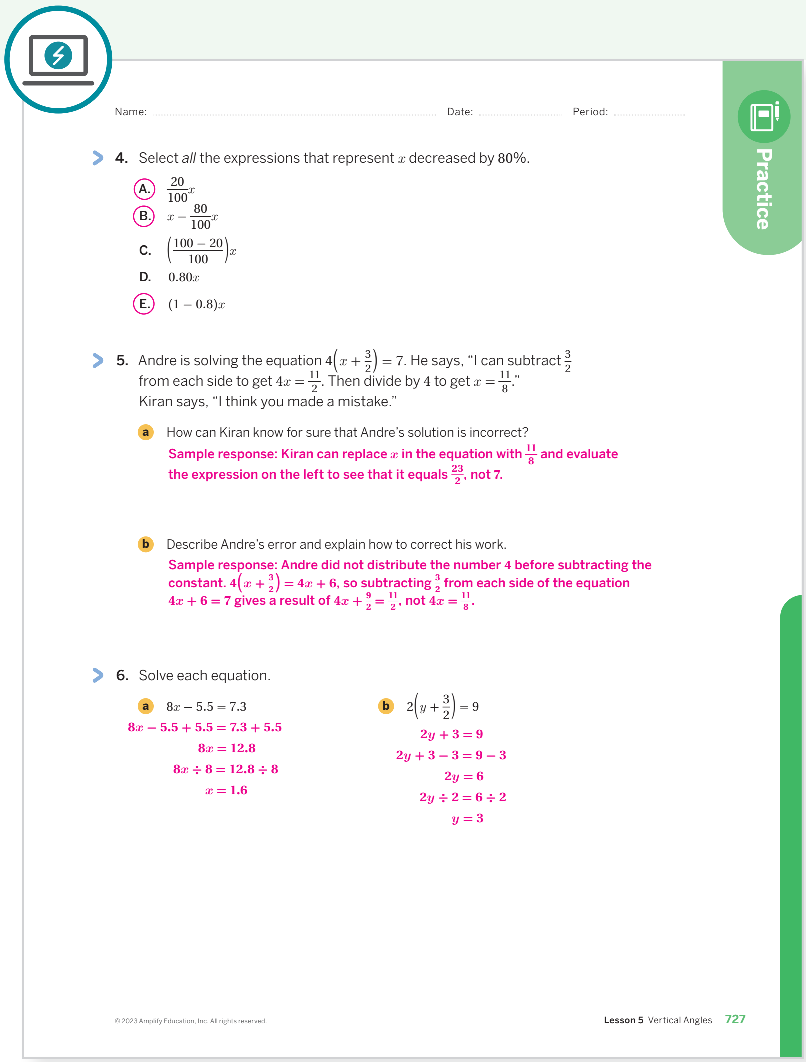
3. Use what you know about complementary, supplementary, and vertical angles to respond to the following.

   a. Is it possible for angles to be both vertical and supplementary? If so, can you determine the angle measures? Explain your thinking.

   Yes; Sample response: angles are supplementary if their measures add up to 180° and vertical angles have equal measures. Because $180 \div 2 = 90$, vertical angles that are also supplementary will each measure 90°.

   b. Is it possible for angles to be both vertical and complementary? If so, can you determine the angle measures? Explain your thinking.

   Yes; Sample response: angles are complementary if their measures add up to 90° and vertical angles have equal measures. Because $90 \div 2 = 45$, vertical angles that are also complementary will each measure 45°.



Practice

Name: Date: Period:

4. Select *all* the expressions that represent x decreased by 80%.

   [✓ A.] $\frac{20}{100}x$
   [✓ B.] $x - \frac{80}{100}x$
   C. $\left(\frac{100-20}{100}\right)x$
   D. $0.80x$
   [✓ E.] $(1 - 0.8)x$

5. Andre is solving the equation $4\left(x + \frac{3}{2}\right) = 7$. He says, "I can subtract $\frac{3}{2}$ from each side to get $4x = \frac{11}{2}$. Then divide by 4 to get $x = \frac{11}{8}$."
   Kiran says, "I think you made a mistake."

   a. How can Kiran know for sure that Andre's solution is incorrect?

   Sample response: Kiran can replace x in the equation with $\frac{11}{8}$ and evaluate the expression on the left to see that it equals $\frac{23}{2}$, not 7.

   b. Describe Andre's error and explain how to correct his work.

   Sample response: Andre did not distribute the number 4 before subtracting the constant. $4\left(x + \frac{3}{2}\right) = 4x + 6$, so subtracting $\frac{3}{2}$ from each side of the equation $4x + 6 = 7$ gives a result of $4x + \frac{9}{2} = \frac{11}{2}$, not $4x = \frac{11}{8}$.

6. Solve each equation.

   a. $8x - 5.5 = 7.3$

   $$8x - 5.5 + 5.5 = 7.3 + 5.5$$
   $$8x = 12.8$$
   $$8x \div 8 = 12.8 \div 8$$
   $$x = 1.6$$

   b. $2\left(y + \frac{3}{2}\right) = 9$

   $$2y + 3 = 9$$
   $$2y + 3 - 3 = 9 - 3$$
   $$2y = 6$$
   $$2y \div 2 = 6 \div 2$$
   $$y = 3$$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 12	2
	5	Unit 6 Lesson 6	2
Formative ⚡	6	Unit 7 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Using Equations to Solve for Unknown Angles

Let’s use equations to determine missing angle measures.

Focus

Goals

1. **Language Goal:** Critique whether a given equation represents the relationship between angles in a diagram. **(Speaking and Listening, Writing)**
2. Write an equation of the form $px + q = r$ or $p(x + q) = r$ to represent the relationship between angles in a given diagram.
3. **Language Goal:** Solve an equation that represents a relationship between angle measures, and explain the reasoning used. **(Writing)**

Coherence

Today

Students write equations of the form $px + q = r$ and $p(x + q) = r$ to accurately represent relationships between angles shown in diagrams. They solve the equations to find unknown angle measures.

Previously

In Lessons 3, 4, and 5, students were introduced to supplementary, complementary, and vertical angle relationships.

Coming Soon

In Lesson 7, students will use what they have learned about equations and angle relationships to describe the angles formed by the hands of a clock.

Rigor

- Students identify special angle relationships to further their **conceptual understanding** of angle relationships.
- Students strengthen their **fluency** in writing and solving equations that represent angle relationships.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *adjacent angles*
- *complementary angles*
- *right angle*
- *straight angle*
- *supplementary angles*
- *vertical angles*

Amps | Featured Activity

Warm-up

Checking Angle Measure Estimates

With the press of a button, angles adjust to show how close the student's estimate is to the actual measure.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might struggle to work together, preventing them from achieving precise solutions. Before starting the activity, have students identify ways that building relationships will make the activity more successful. Lead them to conclude that every member of a group is valuable and has something to contribute to the group's success.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

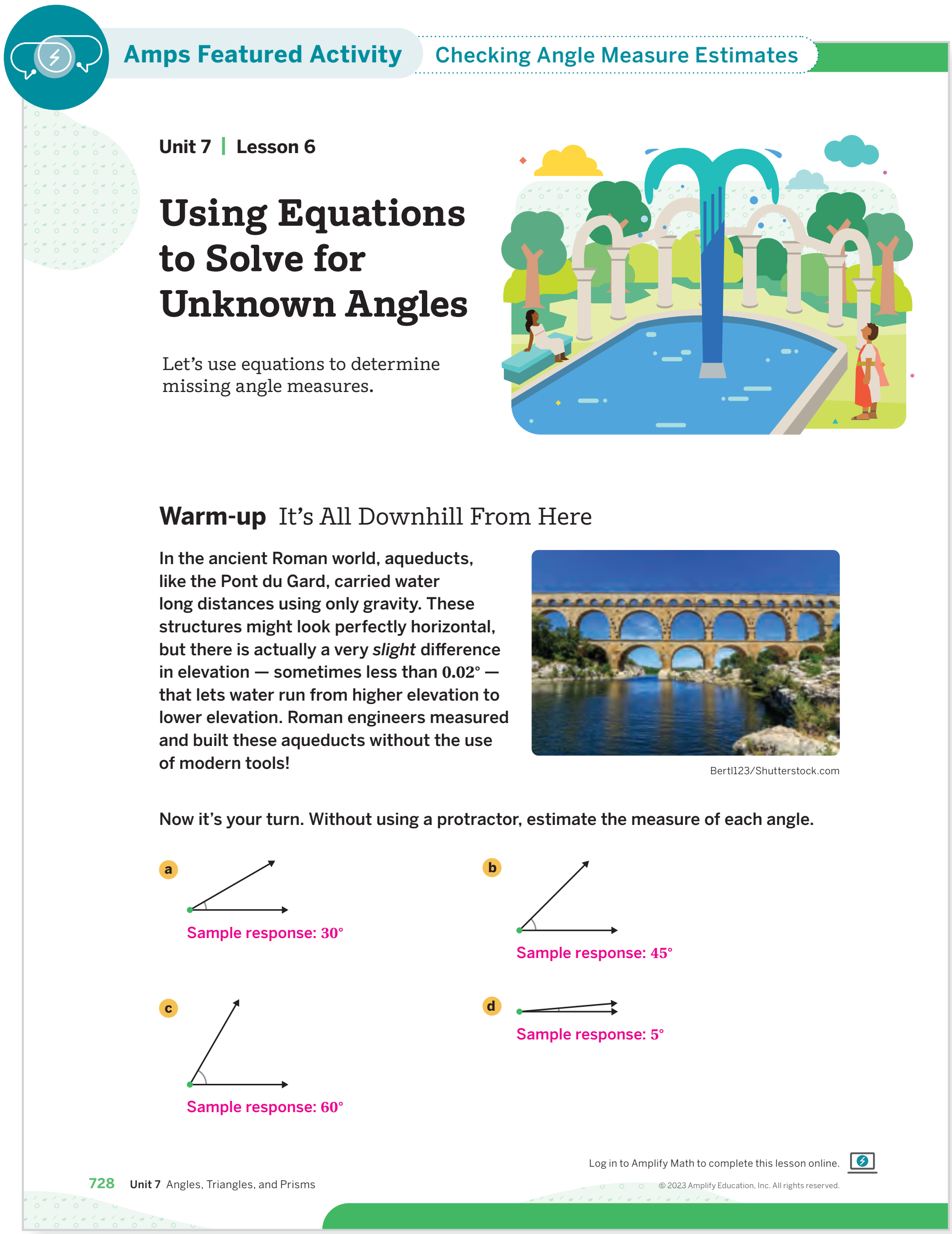
- In **Activity 1**, instead of writing their explanations, have students share with the class during the discussion.
- In **Activity 2**, provide student choice by having them choose any 2 of the 4 problems to complete.

Independent | 5 min

Warm-up It's All Downhill From Here

Students estimate the measures of angles shown in diagrams, which prepares them to check the reasonableness of their solutions when solving for unknown angle measures.

Amps Featured Activity Checking Angle Measure Estimates



Unit 7 | Lesson 6

Using Equations to Solve for Unknown Angles

Let's use equations to determine missing angle measures.

Warm-up It's All Downhill From Here

In the ancient Roman world, aqueducts, like the Pont du Gard, carried water long distances using only gravity. These structures might look perfectly horizontal, but there is actually a very *slight* difference in elevation — sometimes less than 0.02° — that lets water run from higher elevation to lower elevation. Roman engineers measured and built these aqueducts without the use of modern tools!

Bertl123/Shutterstock.com

Now it's your turn. Without using a protractor, estimate the measure of each angle.

a Sample response: 30°

b Sample response: 45°

c Sample response: 60°

d Sample response: 5°

Log in to Amplify Math to complete this lesson online.

728 Unit 7 Angles, Triangles, and Prisms

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① Launch

Read the passage as a class. Activate students' background knowledge by asking them how a waterslide might be similar to or different from an aqueduct.

② Monitor

Help students get started by suggesting they think of angle measures they already know (right angles, straight angles, the 60° angle of an equilateral triangle) and compare them to each given angle.

Look for productive strategies:

- Adding a ray perpendicular to the bottom ray of each angle to compare the given angle with a right angle or half of a right angle.

③ Connect

Display the angles.

Have students share their estimates of each angle measure. Record these estimates for the class to see. For each angle, select one student who chose an average measurement and one student who chose an outlier measurement. Have each student explain their thinking.

Highlight that when finding unknown measures, estimation is a great tool to make sure the measures make sense, given the angle relationships. Diagrams will not always be drawn to scale, but students can use benchmark measurements, such as 90° and 45°, to consider which measures are reasonable and which are not.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which angles automatically resize to match their estimated measures.

Accessibility: *Guide Processing and Visualization*

Consider displaying angles with benchmark measures, such as 90°, 60°, 45°, or 30° to support students in their estimations.

Power-up

To power up students' ability to solve equation with more than one step, have students complete:

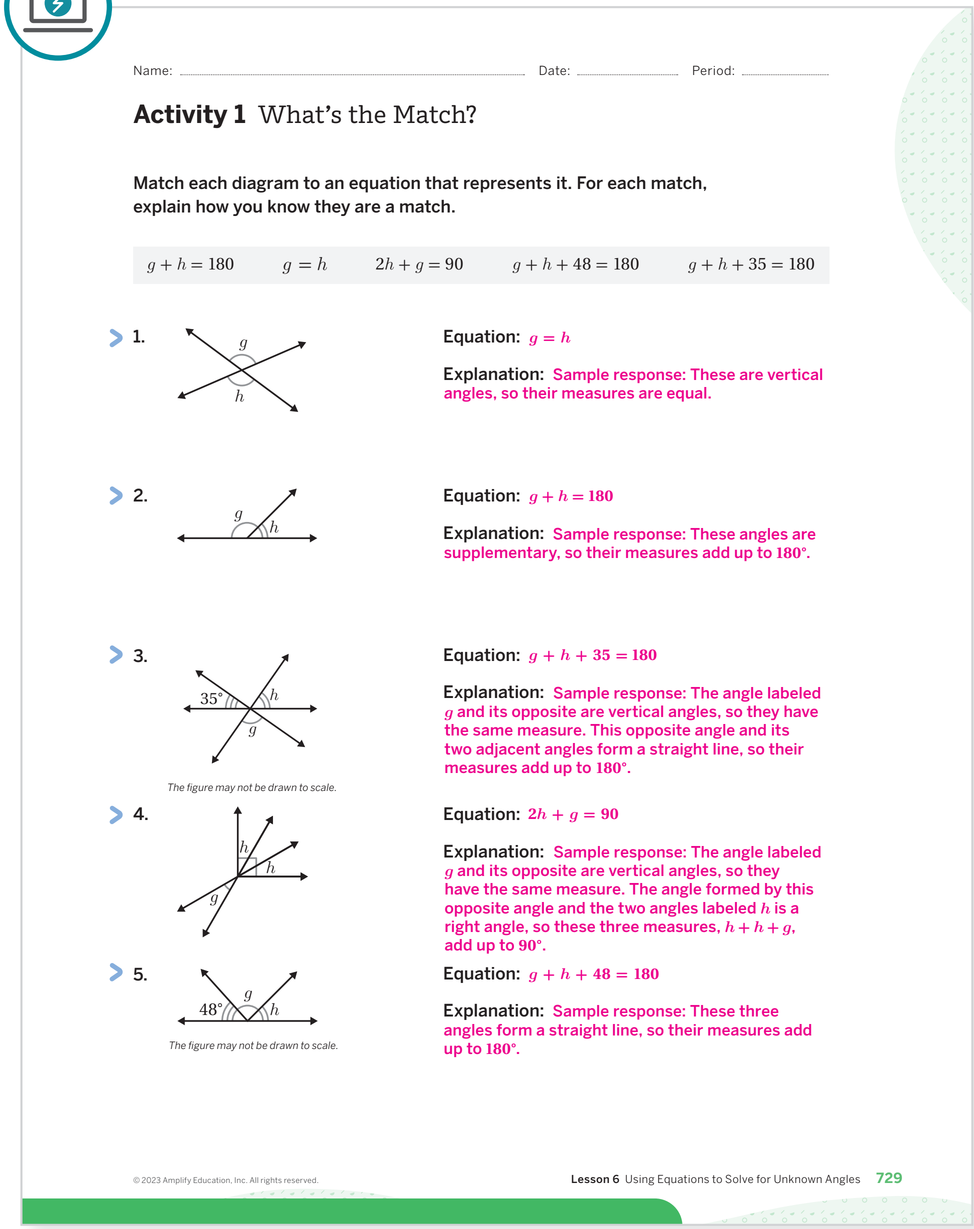
1. For each equation, determine which step would be most efficient to complete first when solving.
   a. $5x + 6 = 20$ $\boxed{\text{Subtract 6 from both sides.}}$ Divide by 5 on both sides.
   b. $5(x + 6) = 20$ Subtract 6 from both sides. $\boxed{\text{Divide by 5 on both sides.}}$
2. Solve each equation. Explain or show your thinking.
   a. $5x + 6 = 20$
      $x = 2.8$
   b. $5(x + 6) = 20$
      $x = -2$

Use: Before Activity 2.
Informed by: Performance on Lesson 5, Practice Problem 6.

Small Groups | 10 min

Activity 1 What's the Match?

Students match diagrams of angle relationships with equations that represent those relationships, which prepares them to write and solve similar equations.



Name: ____________ Date: ________ Period: ________

Activity 1 What's the Match?

Match each diagram to an equation that represents it. For each match, explain how you know they are a match.

$g + h = 180$ $g = h$ $2h + g = 90$ $g + h + 48 = 180$ $g + h + 35 = 180$

1. Equation: $g = h$

Explanation: Sample response: These are vertical angles, so their measures are equal.

2. Equation: $g + h = 180$

Explanation: Sample response: These angles are supplementary, so their measures add up to 180°.

3. Equation: $g + h + 35 = 180$

The figure may not be drawn to scale.

Explanation: Sample response: The angle labeled g and its opposite are vertical angles, so they have the same measure. This opposite angle and its two adjacent angles form a straight line, so their measures add up to 180°.

4. Equation: $2h + g = 90$

Explanation: Sample response: The angle labeled g and its opposite are vertical angles, so they have the same measure. The angle formed by this opposite angle and the two angles labeled h is a right angle, so these three measures, $h + h + g$, add up to 90°.

5. Equation: $g + h + 48 = 180$

The figure may not be drawn to scale.

Explanation: Sample response: These three angles form a straight line, so their measures add up to 180°.

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Lesson 6 Using Equations to Solve for Unknown Angles 729

1 Launch

Activate prior knowledge by asking, "What are the angle relationships you have studied so far in this unit?" Explain that students should keep these relationships in mind throughout the activity.

2 Monitor

Help students get started by suggesting they determine which angles are supplementary, complementary, or vertical in each diagram and then consider what this tells them about the angle relationships.

Look for points of confusion:

- **Trying to solve the equations to find the angle measures.** Direct students to make connections between the relationships shown in the equations and the diagrams.

Look for productive strategies:

- Using appropriate vocabulary and language (i.e., vertical angles have equal measures, supplementary angles have measures with a sum of 180°, etc.) when discussing their thinking with their group.

3 Connect

Display each diagram, one at a time.

Have students share which equation they matched with each diagram and why. Encourage students to use precise mathematical vocabulary, such as *vertical*, *complementary*, and *supplementary*, when explaining their thinking.

Highlight the angle relationships in each diagram and the corresponding equation.

Ask:

- "What angle relationship did you need to recognize so that you could match the equation to the diagram?"
- "Where do you see the relationship expressed in the equation in its corresponding diagram?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display these questions that students can ask themselves to help them determine the matches. Responses are shown for Problem 3.

- Does this diagram show vertical angles? Supplementary angles? Complementary angles?
- What does this information tell you?

The angle labeled g and its opposite are vertical angles; Vertical angles have the same measure. The three angles (35°, g, and h) are supplementary. The sum of their measures is 180°.

Math Language Development

MLR8: Discussion Supports—Pressing for Details

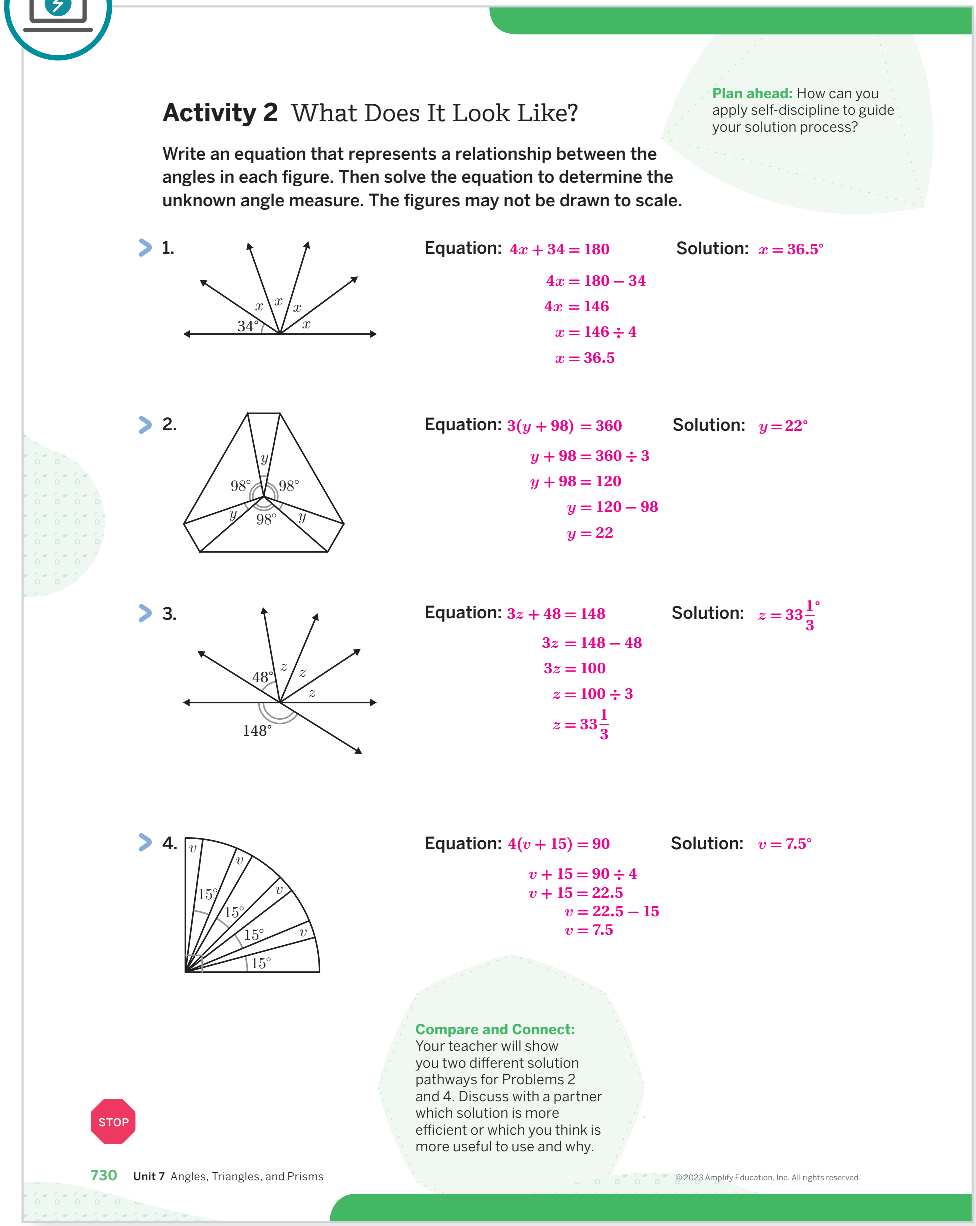
During the Connect, as students share how they determined their matches, encourage their developing mathematical vocabulary by pressing for details. For example, if a student says, "2 hs and g add up to 90°" for Problem 4, ask:

- "How do you know that angle g is part of this sum? What angle relationships do you see?"
- "Did you use the terms *opposite*, *vertical*, or *right angle* in your explanation? If not, how can you revise your explanation to use these terms?"

Small Groups | 20 min

Activity 2 What Does It Look Like?

Students find unknown angle measures by writing and solving equations to reinforce their understanding of angle relationships and build fluency in solving equations.



Activity 2 What Does It Look Like?

Plan ahead: How can you apply self-discipline to guide your solution process?

Write an equation that represents a relationship between the angles in each figure. Then solve the equation to determine the unknown angle measure. The figures may not be drawn to scale.

1. Equation: $4x + 34 = 180$ Solution: $x = 36.5°$

$4x = 180 - 34$
$4x = 146$
$x = 146 \div 4$
$x = 36.5$

2. Equation: $3(y + 98) = 360$ Solution: $y = 22°$

$y + 98 = 360 \div 3$
$y + 98 = 120$
$y = 120 - 98$
$y = 22$

3. Equation: $3z + 48 = 148$ Solution: $z = 33\frac{1}{3}°$

$3z = 148 - 48$
$3z = 100$
$z = 100 \div 3$
$z = 33\frac{1}{3}$

4. Equation: $4(v + 15) = 90$ Solution: $v = 7.5°$

$v + 15 = 90 \div 4$
$v + 15 = 22.5$
$v = 22.5 - 15$
$v = 7.5$

Compare and Connect: Your teacher will show you two different solution pathways for Problems 2 and 4. Discuss with a partner which solution is more efficient or which you think is more useful to use and why.

STOP

730 **Unit 7** Angles, Triangles, and Prisms

1 Launch

Let students know they will be writing and solving their own equations to find unknown angle measures. Note that there may be more than one possible equation to write for each diagram, but there is only one potential angle measure for each angle.

2 Monitor

Help students get started by prompting them to look for any angles that are vertical, complementary, or supplementary.

Look for points of confusion:

- **Finding the unknown angle measure without writing an equation.** Suggest students consider the steps they took to find the angle measure and write an equation that would be solved using the same steps.

Look for productive strategies:

- Labeling the diagrams with angle relationships.

3 Connect

Display each diagram, one at a time.

Have groups of students share the equations they wrote for each diagram. Have them explain their thinking using precise mathematical vocabulary.

Highlight how equations can be used to represent angle relationships.

Ask:

- "Did anyone use a different equation for this diagram? If so, did you get the same solution?"
- "Were any of the problems more challenging than others? Why?"
- "How can you check that the solution to the equation makes sense, given the diagram?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display a series of questions that students can ask themselves to help them write the equation for each diagram. An example is shown.

- Which diagram shows a full circle? Problem 2
- How many degrees are in a full circle? 360°
- How many angles are shown? 6 angles; three labeled y and three labeled 98°
- What must be true about the sum of these measures? The sum is equal to 360°.

Math Language Development

MLR7: Compare and Connect

During the Connect, mention that the diagrams in Problems 2 and 4 both utilize structure. Ask students to compare and connect the solution pathways for each diagram. For example, ask:

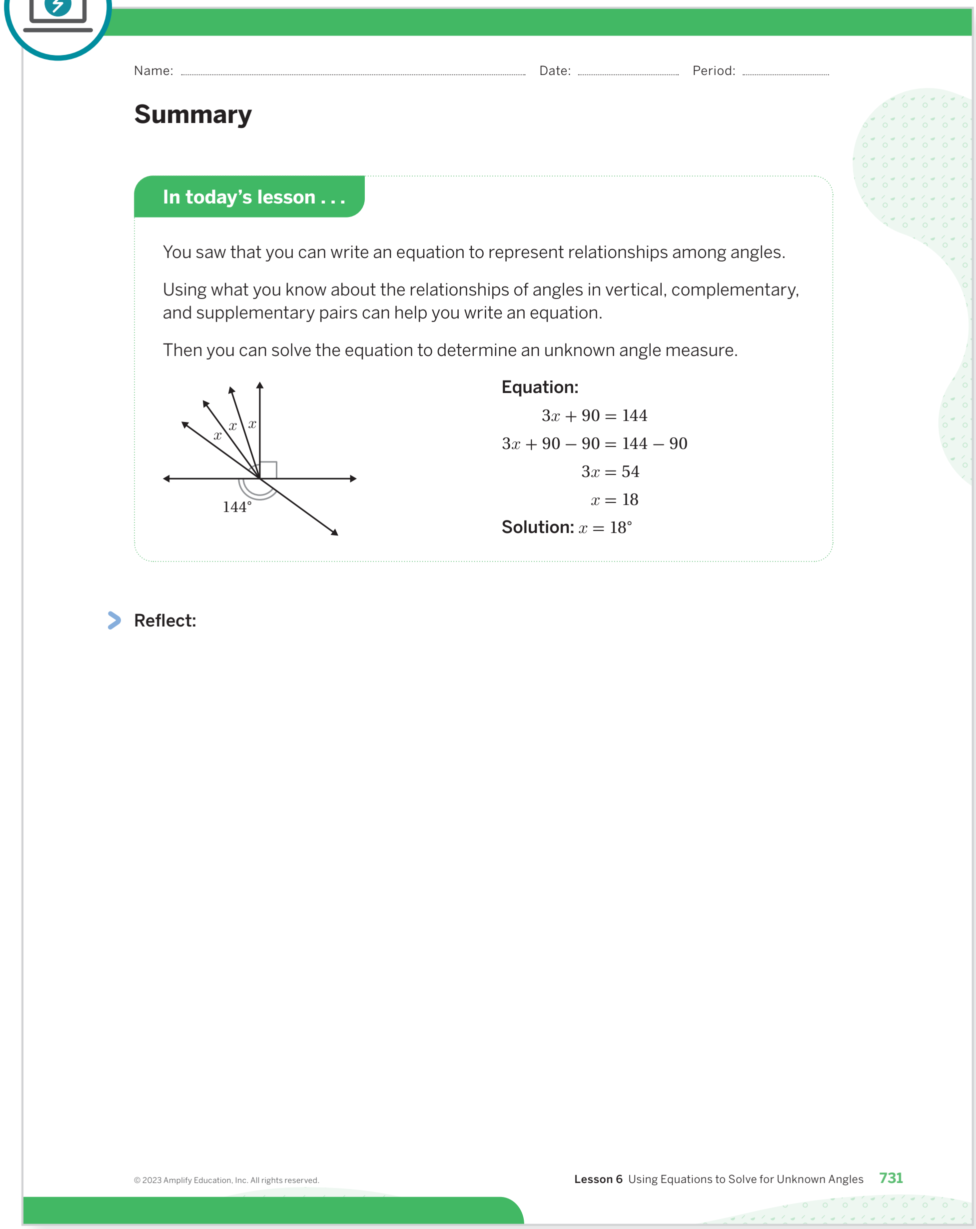
- "In Problem 2, you could begin by writing either of the equations $3(y + 98) = 360$ or $3y + 3 \cdot 98 = 360$. Which solution pathway seems more efficient? Why?"
- "Similarly, in Problem 4, you could write $4(v + 15) = 90$ or $4v + 4 \cdot 15 = 90$. Which solution pathway seems more efficient? Why?"

Encourage students to compare and connect the usefulness and efficiency of each approach.

Whole Class | 5 min

Summary

Review and synthesize how equations can be used to represent angle relationships in order to find unknown angle measures.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that you can write an equation to represent relationships among angles.

Using what you know about the relationships of angles in vertical, complementary, and supplementary pairs can help you write an equation.

Then you can solve the equation to determine an unknown angle measure.

Equation:

$3x + 90 = 144$

$3x + 90 - 90 = 144 - 90$

$3x = 54$

$x = 18$

Solution: $x = 18°$

Reflect:

Lesson 6 Using Equations to Solve for Unknown Angles 731

Synthesize

Display the diagram from the Student Edition.

Have students share the angle relationships they see in the diagram. Students should use precise mathematical language and their vocabulary terms to describe these relationships, such as *adjacent*, *complementary*, *right angle*, *straight angle*, *supplementary*, and *vertical*.

Ask where students see an example of each of these angle relationships or vocabulary terms in the diagram.

Highlight how an equation can be used to represent the angle relationships in the diagram.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

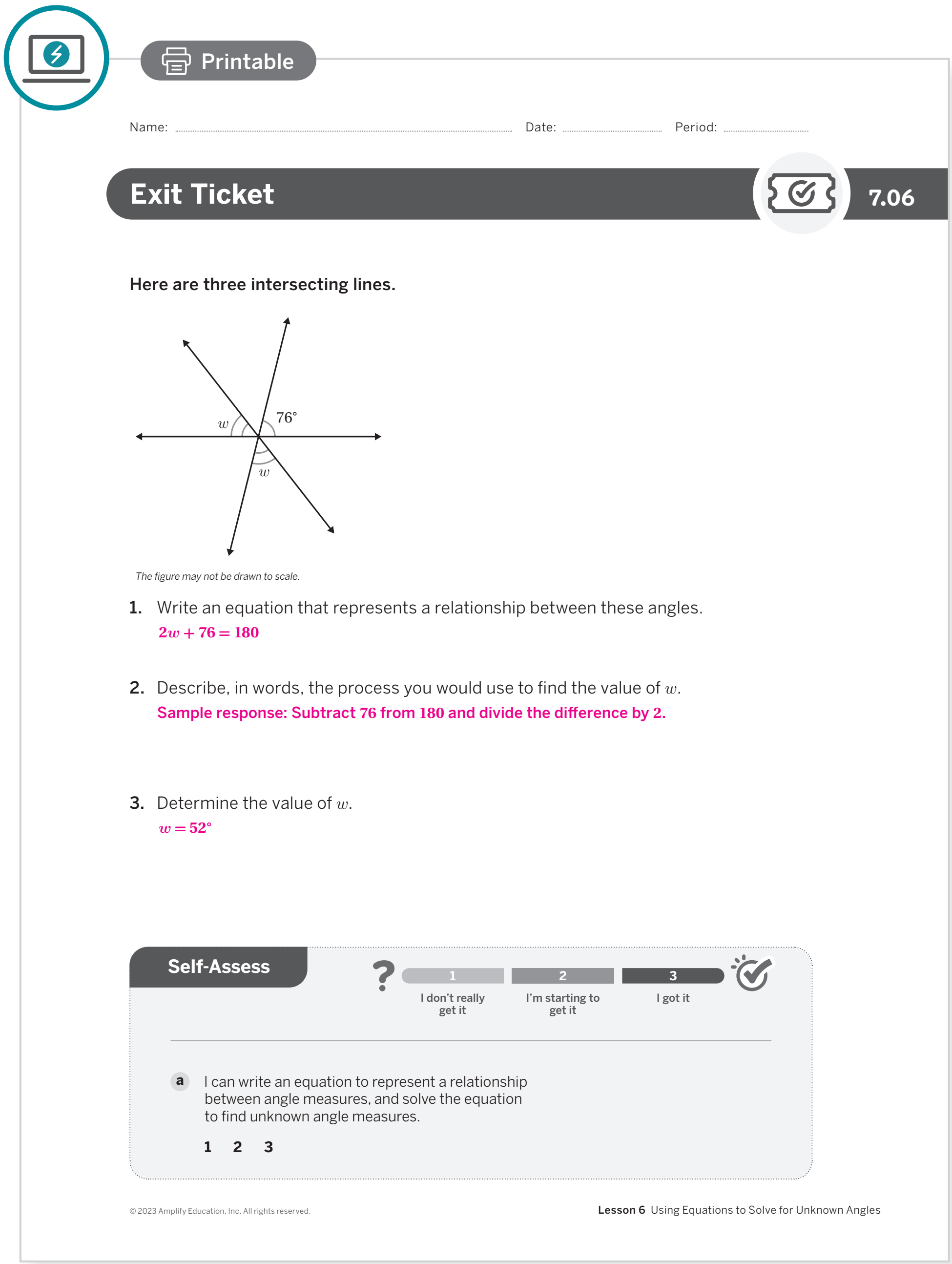
- "What types of special angles or angle relationships did you look for in each diagram?"
- "How did identifying special angles or angle relationships help you to write equations?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing an equation to describe angle relationships in a diagram and describing how to determine the unknown angle measures.

Printable



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Exit Ticket 7.06

Here are three intersecting lines.

The figure may not be drawn to scale.

1. Write an equation that represents a relationship between these angles.
   $2w + 76 = 180$
2. Describe, in words, the process you would use to find the value of w.
   Sample response: Subtract 76 from 180 and divide the difference by 2.
3. Determine the value of w.
   $w = 52°$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can write an equation to represent a relationship between angle measures, and solve the equation to find unknown angle measures.

1 2 3

Lesson 6 Using Equations to Solve for Unknown Angles

Success looks like . . .

- **Language Goal:** Critiquing whether a given equation represents the relationship between angles in a diagram. **(Speaking and Listening, Writing)**
- **Goal:** Writing an equation of the form $px + q = r$ or $p(x + q) = r$ to represent the relationship between angles in a given diagram.
  - » Writing an equation of the form $px + q = r$ to represent the relationship between the angles in Problem 1.
- **Language Goal:** Solving an equation that represents a relationship between angle measures, and explaining the reasoning used. **(Writing)**
  - » Explaining how to determine the unknown angle w and determining the value of w in Problems 2 and 3.

Suggested next steps

If students answer Problem 1 incorrectly, but Problems 2 or 3 correctly, consider:

- Having students work with a partner to help them use their explanation to write an equation.
- Assigning Practice Problem 1.

If students answer Problem 2 incorrectly, but Problems 1 and 3 correctly, consider:

- Asking them to describe in words how they solved their equation.
- Assigning Practice Problem 2.

If students answer Problem 3 iççncorrectly, but Problem 1 correctly, consider:

- Reviewing the process for solving equations.
- Assigning Practice Problem 5.

Professional Learning

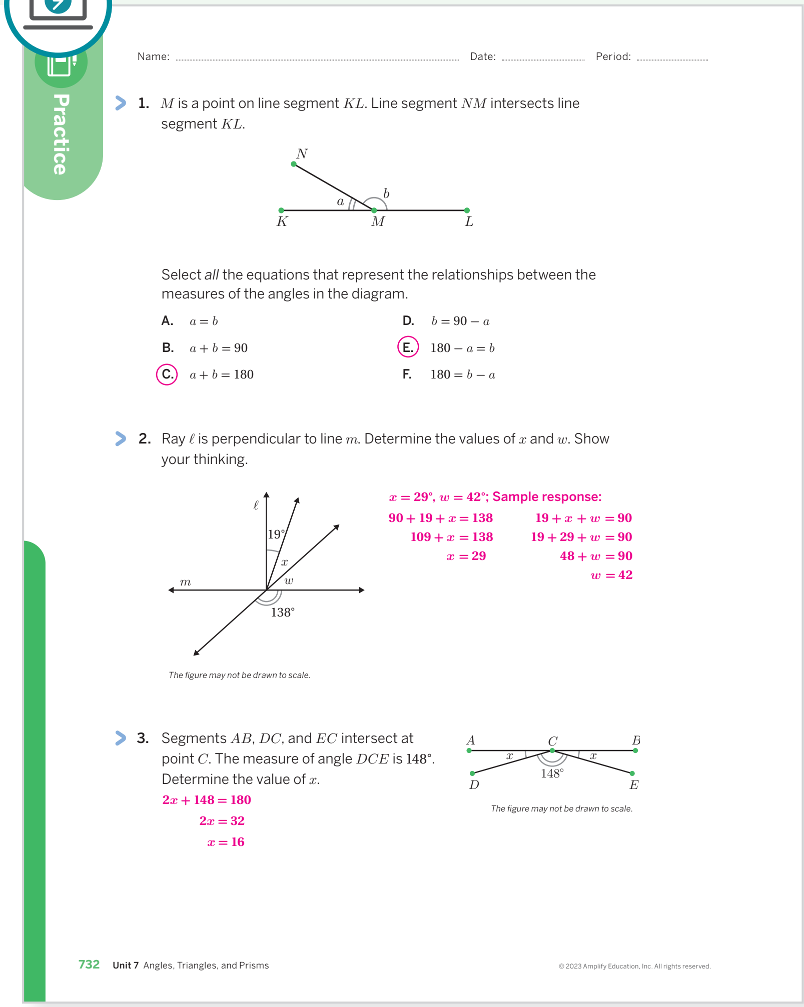
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what students said or did as a result of the questions, which question was the most effective?
- When you compare and contrast today's work with work students did earlier this year on writing equations, what similarities and differences do you see? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. M is a point on line segment KL. Line segment NM intersects line segment KL.

Select *all* the equations that represent the relationships between the measures of the angles in the diagram.

A.	$a = b$	D.	$b = 90 - a$
B.	$a + b = 90$	[✓ E.]	$180 - a = b$
[✓ C.]	$a + b = 180$	F.	$180 = b - a$

2. Ray ℓ is perpendicular to line m. Determine the values of x and w. Show your thinking.

The figure may not be drawn to scale.

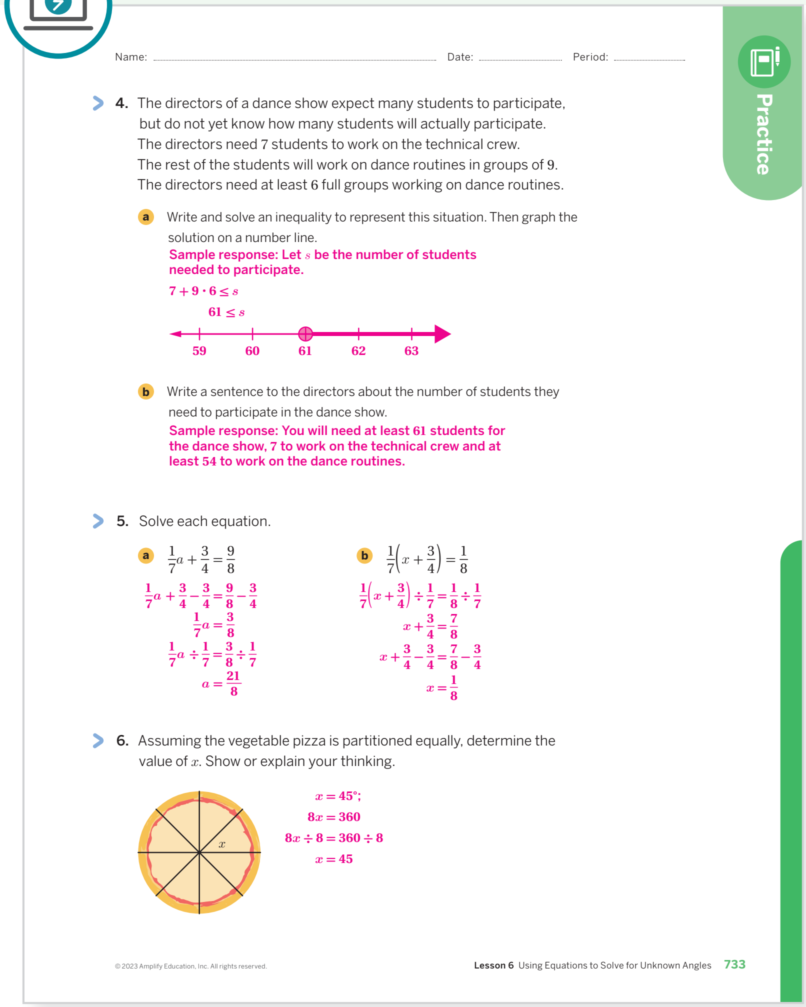
$x = 29°$, $w = 42°$; Sample response:

$$\begin{aligned} 90 + 19 + x &= 138 \\ 109 + x &= 138 \\ x &= 29 \end{aligned} \qquad \begin{aligned} 19 + x + w &= 90 \\ 19 + 29 + w &= 90 \\ 48 + w &= 90 \\ w &= 42 \end{aligned}$$

3. Segments AB, DC, and EC intersect at point C. The measure of angle DCE is $148°$. Determine the value of x.

$$\begin{aligned} 2x + 148 &= 180 \\ 2x &= 32 \\ x &= 16 \end{aligned}$$

The figure may not be drawn to scale.



Name: Date: Period:

Practice

4. The directors of a dance show expect many students to participate, but do not yet know how many students will actually participate. The directors need 7 students to work on the technical crew. The rest of the students will work on dance routines in groups of 9. The directors need at least 6 full groups working on dance routines.

a. Write and solve an inequality to represent this situation. Then graph the solution on a number line.

Sample response: Let s be the number of students needed to participate.

$$\begin{aligned} 7 + 9 \cdot 6 &\le s \\ 61 &\le s \end{aligned}$$

b. Write a sentence to the directors about the number of students they need to participate in the dance show.

Sample response: You will need at least 61 students for the dance show, 7 to work on the technical crew and at least 54 to work on the dance routines.

5. Solve each equation.

a. $\frac{1}{7}a + \frac{3}{4} = \frac{9}{8}$

$$\begin{aligned} \frac{1}{7}a + \frac{3}{4} - \frac{3}{4} &= \frac{9}{8} - \frac{3}{4} \\ \frac{1}{7}a &= \frac{3}{8} \\ \frac{1}{7}a \div \frac{1}{7} &= \frac{3}{8} \div \frac{1}{7} \\ a &= \frac{21}{8} \end{aligned}$$

b. $\frac{1}{7}\left(x + \frac{3}{4}\right) = \frac{1}{8}$

$$\begin{aligned} \frac{1}{7}\left(x + \frac{3}{4}\right) \div \frac{1}{7} &= \frac{1}{8} \div \frac{1}{7} \\ x + \frac{3}{4} &= \frac{7}{8} \\ x + \frac{3}{4} - \frac{3}{4} &= \frac{7}{8} - \frac{3}{4} \\ x &= \frac{1}{8} \end{aligned}$$

6. Assuming the vegetable pizza is partitioned equally, determine the value of x. Show or explain your thinking.

$$\begin{aligned} x &= 45°; \\ 8x &= 360 \\ 8x \div 8 &= 360 \div 8 \\ x &= 45 \end{aligned}$$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 18	2
	5	Unit 6 Lesson 7	2
Formative ⚡	6	Unit 7 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 7**

Like Clockwork

Let's apply our understanding of angles and proportional reasoning to the hands on a clock.

Focus

Goals

1. Use proportional reasoning to calculate the angle of rotation of a hand on a clock.
2. Generalize an equation that determines the angle of rotation of the hands of a clock.

Rigor

- Students **apply** their understanding of angles and proportional reasoning to create equations to model the angles formed by hands of a clock.

Coherence

Today

Students explore the close relationship between how time is measured on an analog clock and how rotation is measured using degrees. The hands of the clock can be represented by the rays of an angle and proportional reasoning is used to both determine angle measures and create equations that represent the relationship.

Previously

Earlier in this grade, students studied proportional relationships and calculated unit rates. Earlier in this unit, students measured and constructed angles.

Coming Soon

In the next section of Unit 7, students will learn about drawing polygons given certain conditions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	12 min	15 min	5 min	8 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, one set of cards per group
- Activity 2 PDF (answers)
- geometry toolkits: protractors, rulers

Math Language Development

Review words

- *adjacent angles*
- *complementary angles*
- *right angle*
- *straight angle*
- *supplementary angles*
- *vertical angles*

Amps | **Featured Activity**

Activity 2
Dynamic Clocks With Instant Feedback

Students write and check their equations immediately with clock hands that rotate automatically.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not continue to persevere as algebra and geometry come together in Activity 2. Have students explain how they can change their thought patterns and relieve their stress. Point out that quantitative algebraic thinking can help make sense of abstract concepts such as time.

Modifications to Pacing

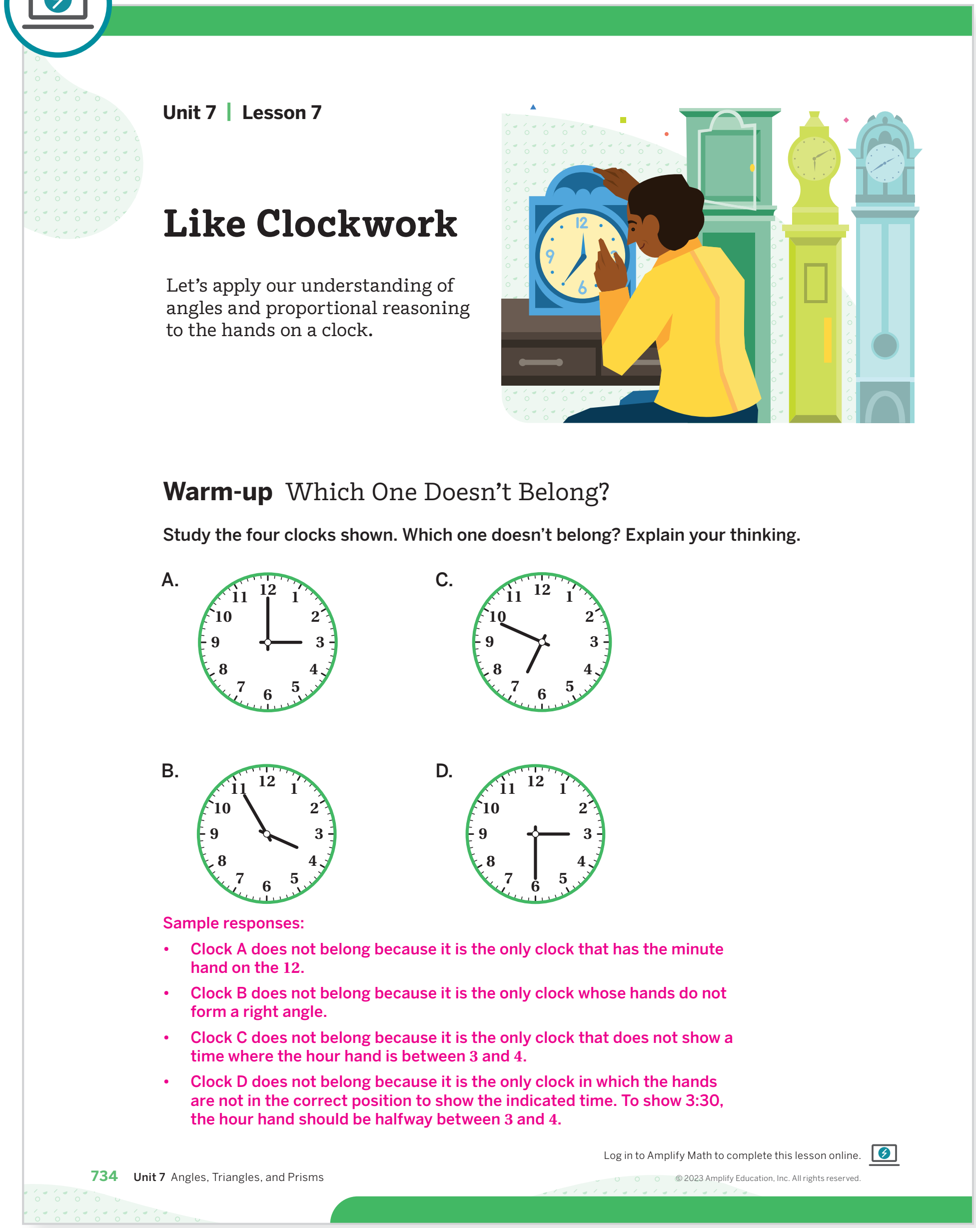
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, have students complete 3 of the 6 cards.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students reacquaint themselves with clocks and the significance of the position of the hands on a clock to prepare for upcoming work related to the angles formed by the hands of a clock.



Unit 7 | Lesson 7

Like Clockwork

Let's apply our understanding of angles and proportional reasoning to the hands on a clock.

Warm-up Which One Doesn't Belong?

Study the four clocks shown. Which one doesn't belong? Explain your thinking.

A.

C.

B.

D.

Sample responses:

- Clock A does not belong because it is the only clock that has the minute hand on the 12.
- Clock B does not belong because it is the only clock whose hands do not form a right angle.
- Clock C does not belong because it is the only clock that does not show a time where the hour hand is between 3 and 4.
- Clock D does not belong because it is the only clock in which the hands are not in the correct position to show the indicated time. To show 3:30, the hour hand should be halfway between 3 and 4.

Log in to Amplify Math to complete this lesson online.

734 Unit 7 Angles, Triangles, and Prisms

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1 Launch

Activate students' background knowledge by asking them how we tell time from analog clocks. If students are unfamiliar with analog clocks, provide an overview of how these clocks measure time. Conduct the ***Which One Doesn't Belong*** routine.

2 Monitor

Help students get started by asking them to consider the position of the hands on the clock.

Look for points of confusion:

- **Not realizing that some hand positions shown are not correct.** Have students consider where the hour hand *should* be in clock D.

Look for productive strategies:

- Analyzing multiple aspects of the clocks. Students may notice the time, the relative positions of the hands to the clock, or the relative positions of the hands to each other.

3 Connect

Display the four clocks and conduct the ***Poll the Class*** routine to assess student thinking.

Highlight that the position of the hour and minute hands on a clock are related to each other. There are interesting questions to be asked about what positions are possible and impossible. This will be the focus of the work today.

Ask, "How do you know that clock D shows an incorrect position for the hands?" If the minute hand shows 30 minutes, then the hour hand needs to be in the middle between 3 and 4.

Differentiated Support

Accessibility: *Bridge Knowledge Gaps*

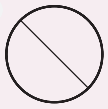
Some students may struggle with reading analog clocks fluently. They will need to access this skill in the upcoming activities. Have students study Clocks A and D and point out that while they show the same angle, 90°, Clock A is read as "three o'clock" and Clock D is read as "three thirty" (when the hour hand is adjusted). Provide other examples of different times that measure 90°. Then ask students to study Clocks B and C and point out why the times shown are *before* four o'clock and seven o'clock, respectively.

Power-up

To power up students' ability to partition the degrees of a circle into equal parts, have students complete:

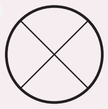
Recall that a circle is 360°. Each circle is partitioned equally. Determine the measure of each angle.

a.



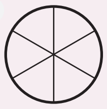
180°

b.



90°

c.



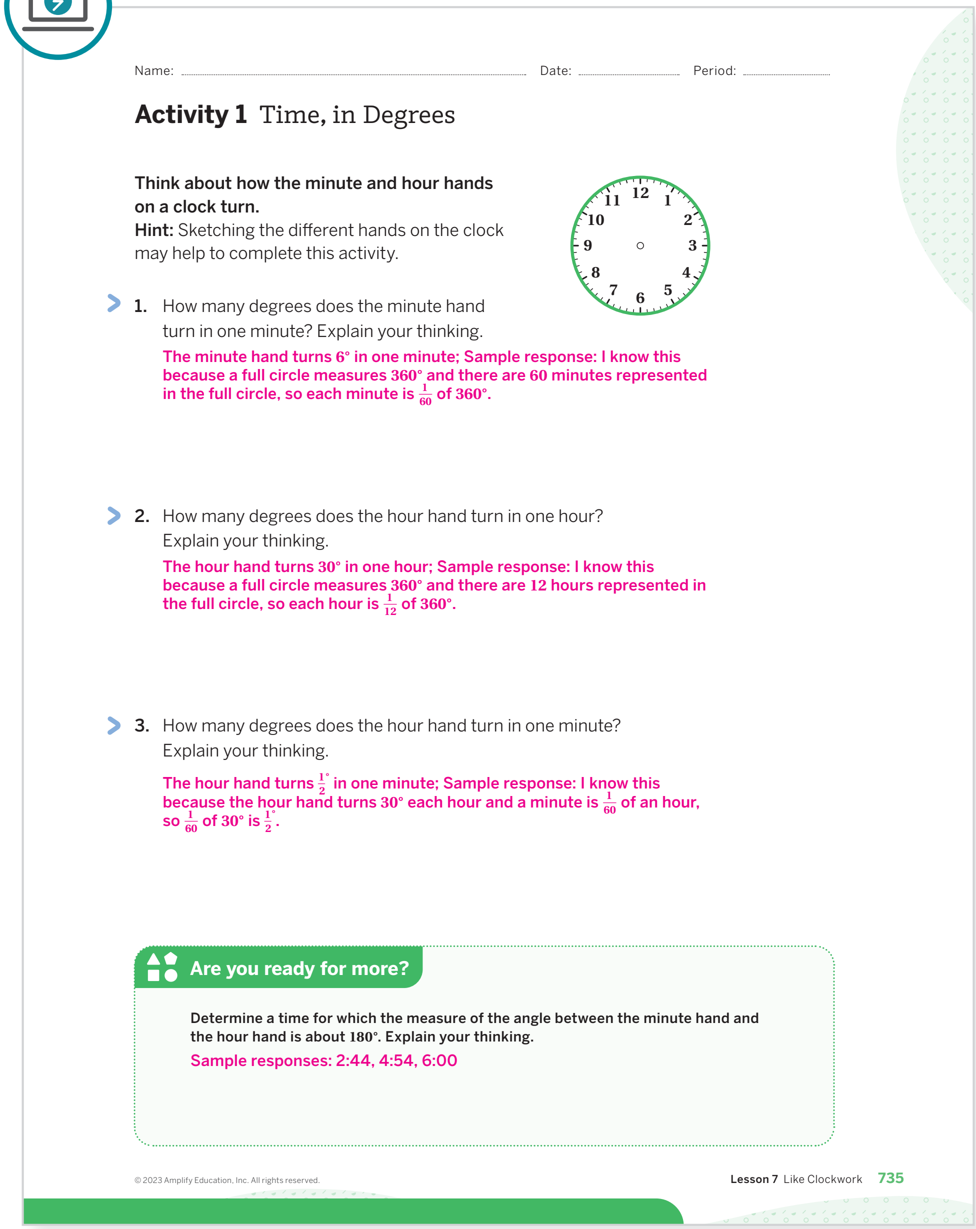
60°

Use: Before Activity 1.
Informed by: Performance on Lesson 6, Practice Problem 6.

Pairs | 12 min

Activity 1 Time, in Degrees

Students find the angle of rotation between the hour and minute hands for different periods of time to understand the proportional relationship between the quantities.



Name: ________ Date: ________ Period: ________

Activity 1 Time, in Degrees

Think about how the minute and hour hands on a clock turn.
Hint: Sketching the different hands on the clock may help to complete this activity.

1. How many degrees does the minute hand turn in one minute? Explain your thinking.
   The minute hand turns 6° in one minute; Sample response: I know this because a full circle measures 360° and there are 60 minutes represented in the full circle, so each minute is $\frac{1}{60}$ of 360°.

2. How many degrees does the hour hand turn in one hour? Explain your thinking.
   The hour hand turns 30° in one hour; Sample response: I know this because a full circle measures 360° and there are 12 hours represented in the full circle, so each hour is $\frac{1}{12}$ of 360°.

3. How many degrees does the hour hand turn in one minute? Explain your thinking.
   The hour hand turns $\frac{1}{2}^{\circ}$ in one minute; Sample response: I know this because the hour hand turns 30° each hour and a minute is $\frac{1}{60}$ of an hour, so $\frac{1}{60}$ of 30° is $\frac{1}{2}^{\circ}$.

Are you ready for more?

Determine a time for which the measure of the angle between the minute hand and the hour hand is about 180°. Explain your thinking.

Sample responses: 2:44, 4:54, 6:00

Lesson 7 Like Clockwork 735

1 Launch

If students are unfamiliar with analog clocks, you may choose to show the online animation.

2 Monitor

Help students get started by having them determine the total number of minutes and total number of degrees that are represented on the entire clock face.

Look for points of confusion:

- **Confusing the minute hand and hour hand.** Ask, "How many degrees does the minute hand turn each minute? Sketch what the minute hand looks like before and after one turn."
- **Struggling to calculate unit rates.** Ask, "How many degrees does the minute hand turn in one hour? How many degrees does a minute hand turn in half an hour (30 minutes)?"

Look for productive strategies:

- Recognizing that a quarter turn measures 90° and a half turn measures 180°.
- Calculating the unit rate for each hand's rotation.

3 Connect

Display the image of the empty clock face. Mark the clock face as students explain their strategies.

Have students share their strategies for finding the degrees of the rotations (turns).

Highlight that finding the unit rate can be very useful when there is a proportional relationship between two quantities. Knowing the unit rate helps to find the number of degrees a hand turns in a certain period of time.

Ask, "What are some different ways in which a clock face can be partitioned?" Sample responses: halves, thirds, quarters, fifths, sixths, tenths, twelfths, fifteenths, twentieths, twenty-fourths, thirtieths.

Differentiated Support

Extension: *Math Around the World, Interdisciplinary Connections*

Ask students if they have ever wondered, "Why are there 24 hours in a day?" Our concept of a 24-hour day comes from the ancient Egyptian civilization. They used shadow clocks (or sundials) to measure the passing of time each day and divided the day into three sections: 10 hours of "day time," 1 hour of "twilight" at the beginning of "day time," and 1 hour of "twilight" at the end of "day time," thus making 12 hours. The night was divided into 12 hours as well. Students may be interested to learn that ancient Egyptians varied the length of the "day time" and "night time" hours, based on the season. In the summer, "day time" hours were longer than "night time" hours. In winter, it was the reverse.

Ask the following questions to facilitate class discussion:

- "Why do you think it was important for ancient Egyptian civilization to create and consequently adopt this '24 hours in a day' segmentation?"
- "Why do you think subsequent civilizations selected to also adopt this segmentation?"

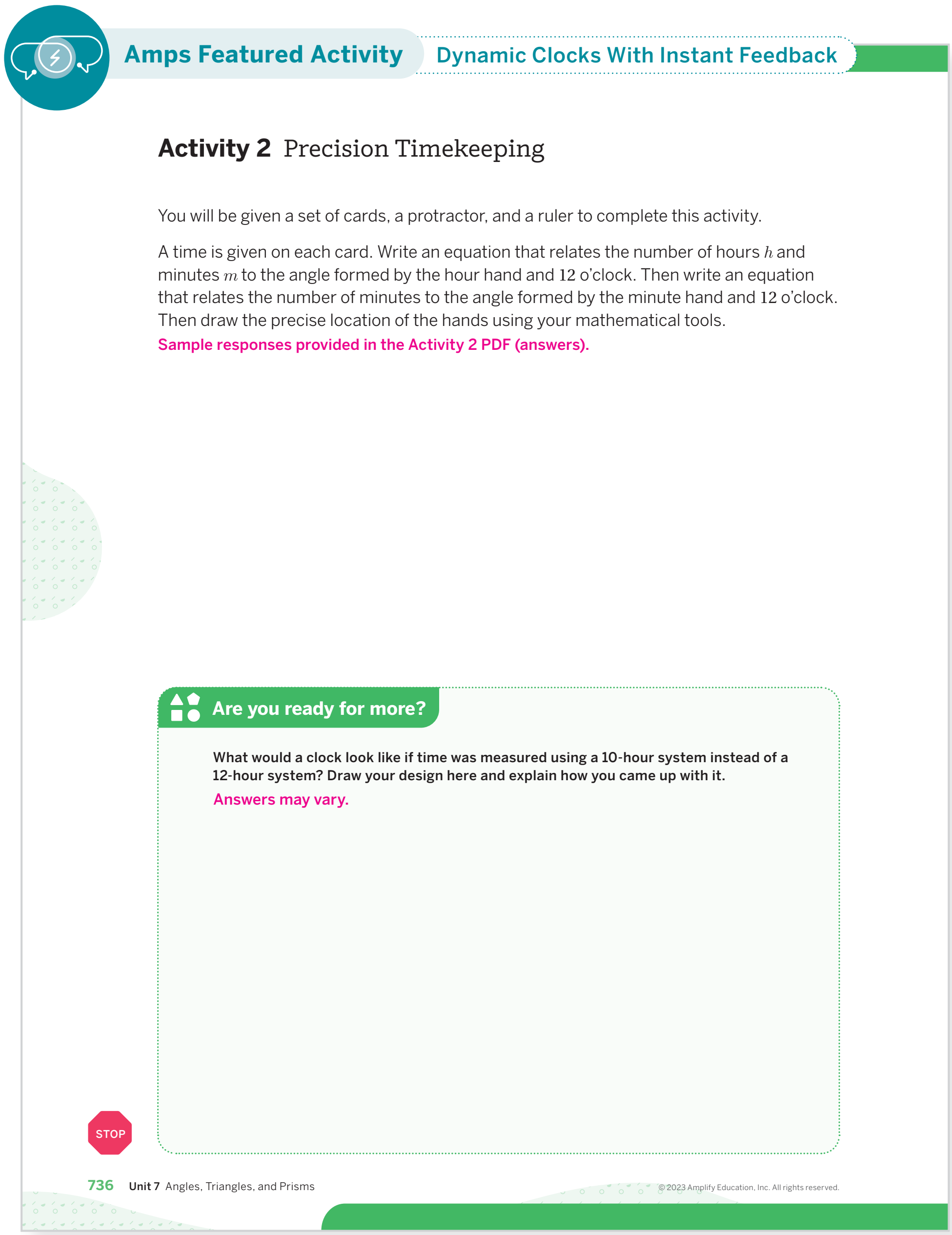
Have students research shadow clocks (sundials) and ask students to compare them to the analog clocks used today. An example of an ancient sundial is shown in the Summary of this lesson. **(History)**

Small Groups | 15 min

Activity 2 Precision Timekeeping

Students create equations that relate time to the measure of the angles formed by the hands of an analog clock to strengthen students' understanding of the connection to proportional relationships.

Amps Featured Activity Dynamic Clocks With Instant Feedback



Activity 2 Precision Timekeeping

You will be given a set of cards, a protractor, and a ruler to complete this activity.

A time is given on each card. Write an equation that relates the number of hours h and minutes m to the angle formed by the hour hand and 12 o'clock. Then write an equation that relates the number of minutes to the angle formed by the minute hand and 12 o'clock. Then draw the precise location of the hands using your mathematical tools.
Sample responses provided in the Activity 2 PDF (answers).

Are you ready for more?

What would a clock look like if time was measured using a 10-hour system instead of a 12-hour system? Draw your design here and explain how you came up with it.
Answers may vary.

STOP

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1 Launch

Distribute copies of the Activity 2 PDF, and provide access to geometry toolkits, in particular, protractors and rulers. Have students first make sense of the activity and its directions in their groups before beginning.

2 Monitor

Help students get started by asking them to find where the minute hand should be first for each card.

Look for points of confusion:

- **Thinking that the hour hand will point exactly at the hour's number.** Have students look back at the Warm-up and Problem 3 in Activity 1 to think about how the hour hand rotates.

Look for productive strategies:

- Recalling work from Unit 6 and writing equations using the unit rate as the coefficient of the variable.
- Converting hours into minutes and combining with the number of minutes, and then using the rate for minutes only.
- Noticing that there are two different rates and using a different variable for hours and minutes.

3 Connect

Have students share all the unique equations they wrote for the activity. Consider using the *Gallery Tour* routine and have students display their equations around the classroom and observe other students' work.

Highlight that this activity illustrates how algebra and geometry share a close relationship with each other. When geometric relationships are proportional, it is often useful to model those relationships using algebra.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can write and check their equations immediately with clock hands that automatically rotate.

Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on Card A or Card E. This will still allow students to access the goal of the activity, which is to relate the angle measures formed by the hands of the clock to proportional relationships.

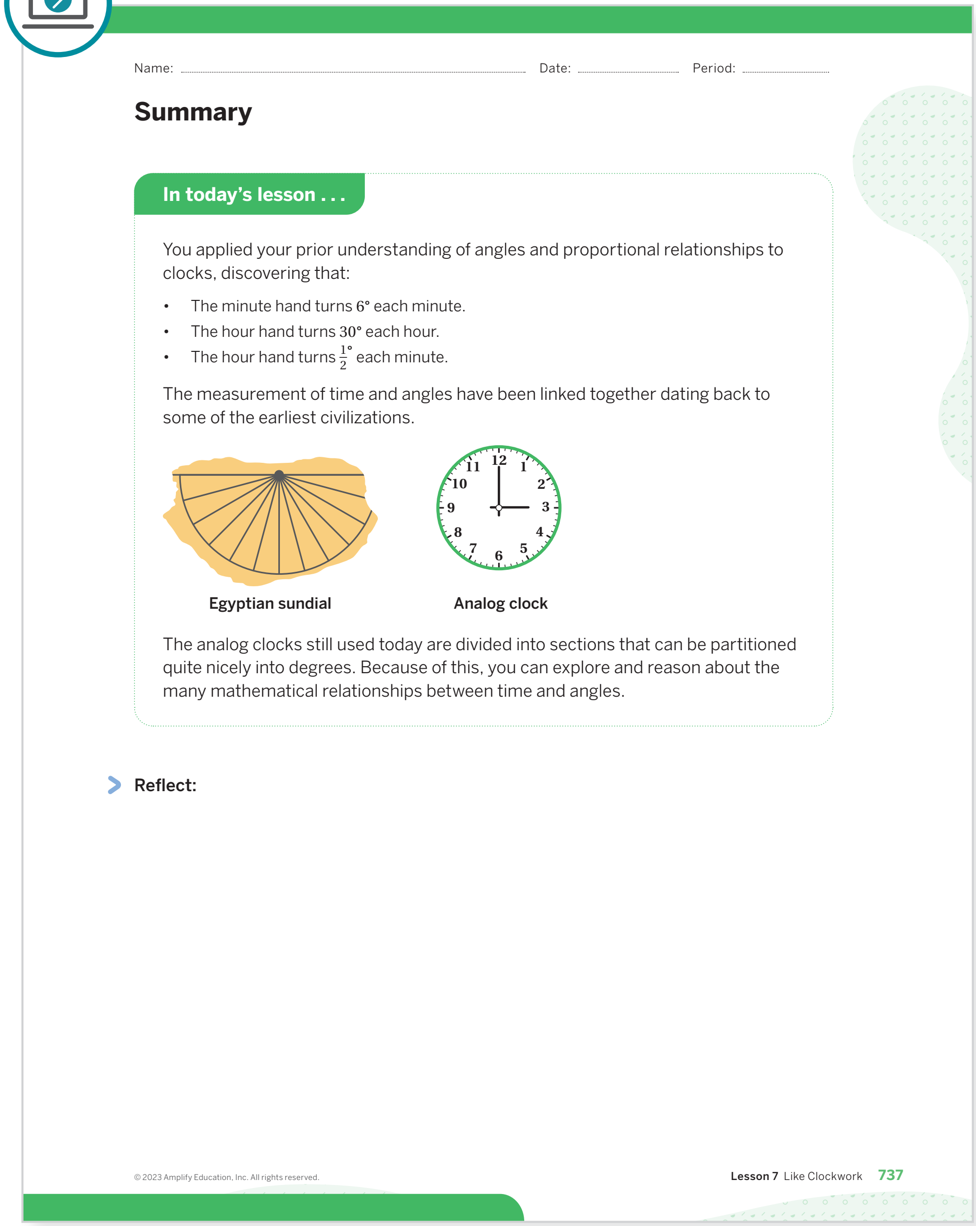
Extension: *Math Around the World, Interdisciplinary Connections*

Ask students if they have ever wondered, "Why are there 60 minutes in 1 hour, and 60 seconds in 1 minute?" This concept comes from the ancient Babylonians who used a base-60 place value system. Interested students can further research the base-60 place value system. Ancient Babylonians also divided the day into 360 partitions, which is different from how we divide the day into 24 partitions (hours). Many historians believe that they divided the day into 360 partitions because they had estimated the number of days in a year to be 360. Ask students, "How do you think the ancient Babylonians were able to estimate with such close accuracy that a year consisted of 360 days?" Have students work in small groups to address this question and to learn about the role of mathematics in Babylonian society. **(History)**

Whole Class | 5 min

Summary

Review and synthesize how proportional reasoning and algebraic representations can be used to represent and analyze geometric relationships.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You applied your prior understanding of angles and proportional relationships to clocks, discovering that:

- The minute hand turns $6°$ each minute.
- The hour hand turns $30°$ each hour.
- The hour hand turns $\frac{1}{2}^{\circ}$ each minute.

The measurement of time and angles have been linked together dating back to some of the earliest civilizations.

Egyptian sundial

Analog clock

The analog clocks still used today are divided into sections that can be partitioned quite nicely into degrees. Because of this, you can explore and reason about the many mathematical relationships between time and angles.

Reflect:

Lesson 7 Like Clockwork 737

Synthesize

Display the images from the Student Edition.

Ask, "How is the ancient sundial that ancient Egyptians used in the 13th century BCE similar to or different from the analog clocks used today?"

Have students share their thoughts on the similarities and differences between the Egyptian sundial and the modern analog clock.

Highlight ideas that use precise vocabulary and reference understandings from the lesson activities. For example, the sundial is only a semi-circle, so it measures $180°$. It is partitioned differently, so perhaps their time was measured differently. Hours may have been longer. Students who are interested in this topic may research this further.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What methods or strategies did you use to determine the angle measures within each clock?"
- "Did you notice any patterns or 'rules' as you determined each angle measure?"

Differentiated Support

Extension: *Math Around the World, Interdisciplinary Connections*

Cultures around the world have used different ways of representing and measuring time. For example, in ancient China, people divided the day into twelve 2-hour periods. Each double hour was called "shi". There is also evidence of a separate system for dividing a day into 100 equal-sized partitions called *ke*. For a while, these two systems did not correlate well with each other because 12 does not evenly divide 100. In 1628 CE, the number of *ke* in a day was changed to be 96 for this reason. Prior to this change, previous dynasties in China utilized different numbers of *ke's* that included 100 (the majority of China's history), 120, 96, and 108.

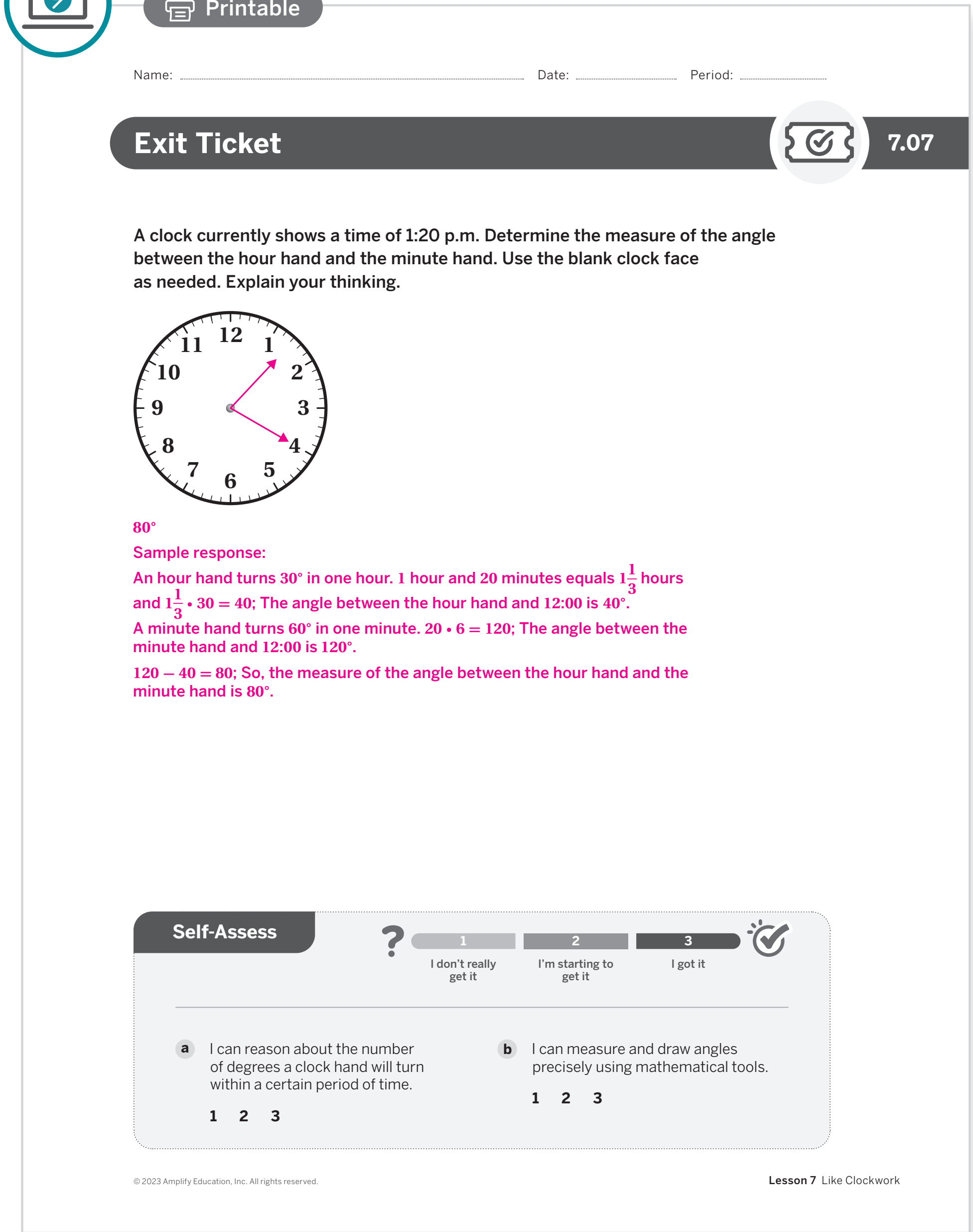
Around 723 CE, during the Tang Dynasty, Chinese inventors developed its first astronomical clock indicating time. This clock used water to function. Have students work in small groups and use the internet or another source to research how water clocks work and how different civilizations — including ancient Egyptian and Babylonian civilizations — constructed and used different types of these water clocks. Have groups create a visual display of their findings and share them with the class. **(History)**

Independent | 8 min

Exit Ticket

Students demonstrate their understanding by determining the measure of the angle between the hands of a clock for a given time.

Printable



Name: __________ Date: __________ Period: __________

Exit Ticket

7.07

A clock currently shows a time of 1:20 p.m. Determine the measure of the angle between the hour hand and the minute hand. Use the blank clock face as needed. Explain your thinking.

80°

Sample response:

An hour hand turns 30° in one hour. 1 hour and 20 minutes equals $1\frac{1}{3}$ hours and $1\frac{1}{3} \cdot 30 = 40$; The angle between the hour hand and 12:00 is 40°.

A minute hand turns 60° in one minute. $20 \cdot 6 = 120$; The angle between the minute hand and 12:00 is 120°.

$120 - 40 = 80$; So, the measure of the angle between the hour hand and the minute hand is 80°.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can reason about the number of degrees a clock hand will turn within a certain period of time.

1 2 3

b I can measure and draw angles precisely using mathematical tools.

1 2 3

Lesson 7 Like Clockwork

Success looks like . . .

- **Goal:** Using proportional reasoning to calculate the angle of rotation of a hand on a clock.
  - » Determining the measure of the angle between the clock hands when the time is 1:20 p.m.
- **Goal:** Generalizing an equation that determines the angle of rotation of the hands of a clock.

Suggested next steps

If students add the measure of the angles from the vertical line at 12:00, consider:

- Asking, "Since you know where each hand is, how can you find the measure of the angle between them?"

If students draw the hour hand pointing directly at 1, consider:

- Asking, "What happens to the hour hand between 1 p.m. and 2 p.m.? How many degrees will it turn?"

If students use an incorrect number of degrees per minute or per hour, consider:

- Having them review their work from Activity 1 and explain how many degrees each hand rotates per minute and per hour.

Professional Learning

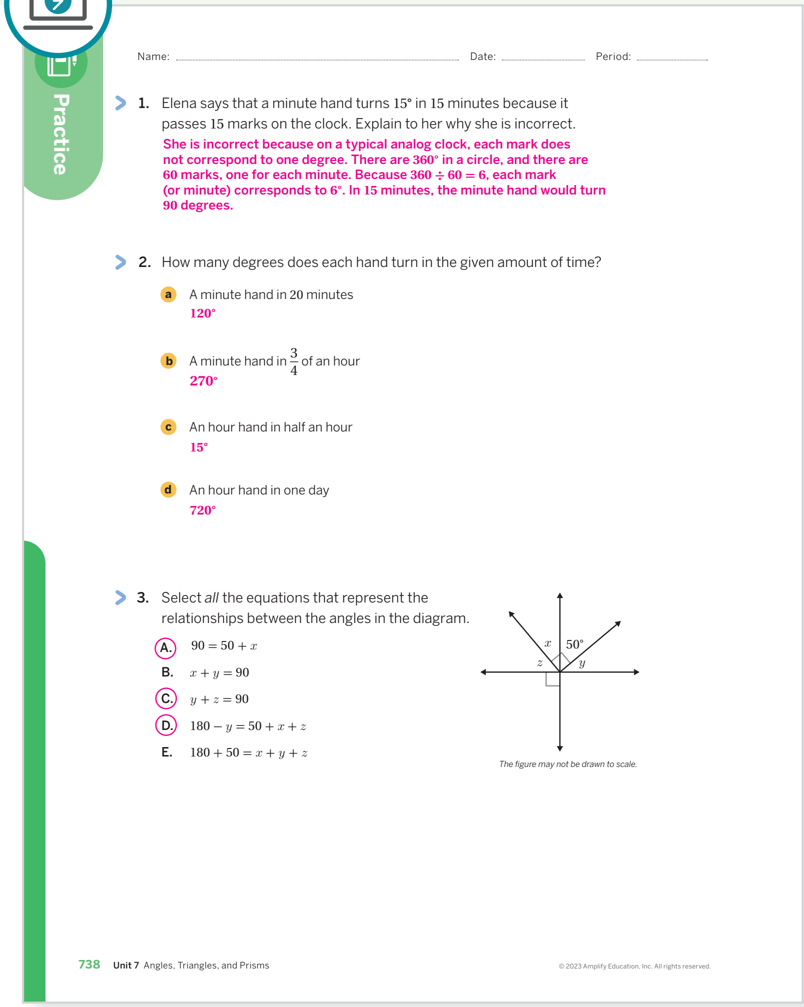
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the Warm-up discussion, how did you encourage each student to share their understanding?
- What challenges did students encounter as they worked through Activity 2? How did they work through it? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ____________ Date: ________ Period: ________

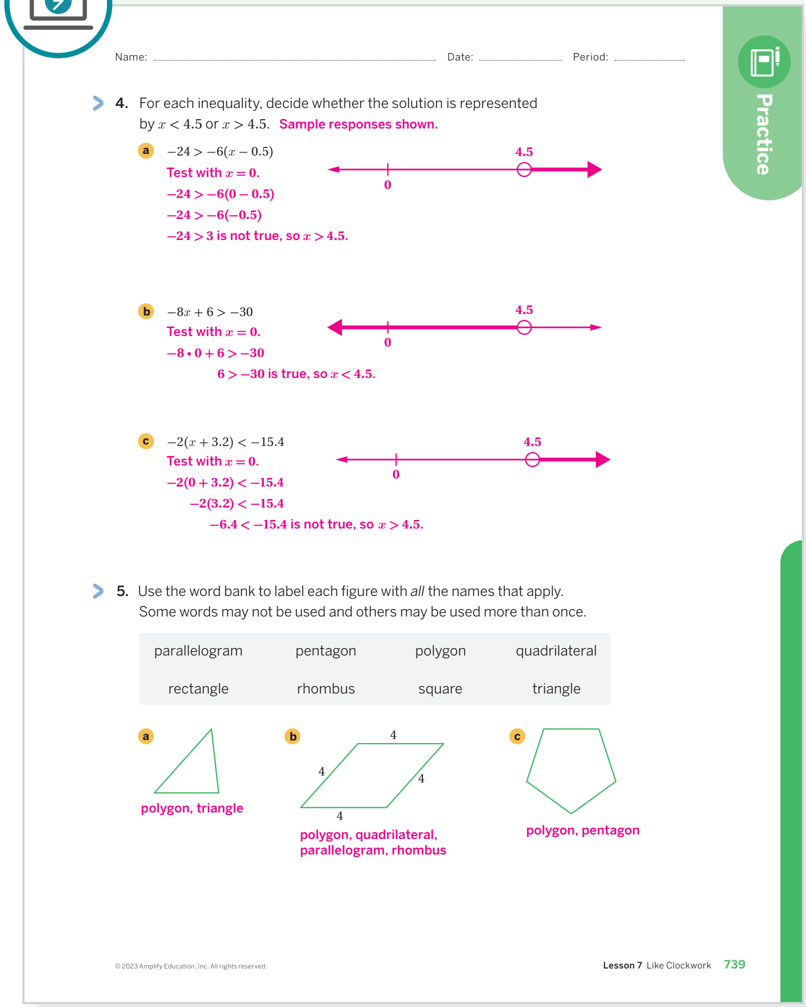
1. Elena says that a minute hand turns 15° in 15 minutes because it passes 15 marks on the clock. Explain to her why she is incorrect.
She is incorrect because on a typical analog clock, each mark does not correspond to one degree. There are 360° in a circle, and there are 60 marks, one for each minute. Because $360 \div 60 = 6$, each mark (or minute) corresponds to 6°. In 15 minutes, the minute hand would turn 90 degrees.

2. How many degrees does each hand turn in the given amount of time?
   - a. A minute hand in 20 minutes
     120°
   - b. A minute hand in $\frac{3}{4}$ of an hour
     270°
   - c. An hour hand in half an hour
     15°
   - d. An hour hand in one day
     720°

3. Select *all* the equations that represent the relationships between the angles in the diagram.
   - [✓ A.] $90 = 50 + x$
   - B. $x + y = 90$
   - [✓ C.] $y + z = 90$
   - [✓ D.] $180 - y = 50 + x + z$
   - E. $180 + 50 = x + y + z$

The figure may not be drawn to scale.

738 **Unit 7** Angles, Triangles, and Prisms



Name: ____________ Date: ________ Period: ________

4. For each inequality, decide whether the solution is represented by $x < 4.5$ or $x > 4.5$. **Sample responses shown.**
   - a. $-24 > -6(x - 0.5)$
     Test with $x = 0$.
     $-24 > -6(0 - 0.5)$
     $-24 > -6(-0.5)$
     $-24 > 3$ is not true, so $x > 4.5$.
   - b. $-8x + 6 > -30$
     Test with $x = 0$.
     $-8 \cdot 0 + 6 > -30$
     $6 > -30$ is true, so $x < 4.5$.
   - c. $-2(x + 3.2) < -15.4$
     Test with $x = 0$.
     $-2(0 + 3.2) < -15.4$
     $-2(3.2) < -15.4$
     $-6.4 < -15.4$ is not true, so $x > 4.5$.

5. Use the word bank to label each figure with *all* the names that apply. Some words may not be used and others may be used more than once.

parallelogram	pentagon	polygon	quadrilateral
rectangle	rhombus	square	triangle

- a. **polygon, triangle**
- b. **polygon, quadrilateral, parallelogram, rhombus**
- c. **polygon, pentagon**

Lesson 7 Like Clockwork 739

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	3
	2	Activity 2	2
Spiral	3	Unit 7 Lesson 6	2
	4	Unit 6 Lesson 19	2
Formative ⚡	5	Unit 7 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Whole Class

Sub-Unit 2

Drawing Polygons With Given Conditions

In this Sub-Unit, students join the mathematical tradition of constructing geometric figures. Given certain conditions, they notice that sometimes many figures can be constructed, while other times no figures can be constructed.

SUB-UNIT

2 Drawing Polygons With Given Conditions

Narrative Connections

How did triangles help win a war?

During the Civil War, one advantage the North had over the South was its railroads. While the South had their own railways, theirs was nowhere near as extensive as the North's. The North had over 22,000 miles worth of track. With this network, soldiers and supplies could travel quickly and efficiently to the frontlines.

So where do triangles come in?

Well, in order for a train to cover these vast distances, it would have to cross many bridges. At the time, most bridges were made of wood. Wood was a strong material, but a far cry from the steel and concrete bridges of today. To make wooden bridges that could support the weight of a locomotive, engineers turned to "trusses". These were structures made of overlapping triangles that made the bridges more stable.

Truss bridges were easy to build and required less material compared to other bridges. Their stability was due to the **rigid** properties of triangles, making these bridges remarkably **strong**. They could easily support the weight of a train carrying a regiment of Union soldiers, without deforming or collapsing.

For centuries, triangles have had a starring role in our architecture. From the simple A-frame house to the Great Pyramids of Giza – if there's a building upright today, chances are that triangles have kept it standing.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to encounter triangles in construction and location in the following places:

- **Lesson 8, Activity 1:** What Can You Build?
- **Lesson 9, Activity 1:** Where Is the Hideout?
- **Lesson 11, Warm-up:** Construct It

Building Polygons
(Part 1)

Let's build some polygons.

Focus

Goals

1. **Language Goal:** Comprehend that two shapes are considered "identical copies" if they can be placed on top of each other and match up exactly. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Recognize that four side lengths do not determine a unique quadrilateral, but that three side lengths can determine a unique triangle. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Use manipulatives to create a polygon with given side lengths, and describe the resulting shape. **(Speaking and Listening, Writing)**

Coherence

Today

This lesson is the first in a series of lessons in which students create shapes with given conditions. Students experiment with building polygons of various orders and combinations of side lengths, using linkage strips and metal paper fasteners.

Previously

These lessons continue the language used in Grade 6, in that two polygons are identical if they match up exactly when placed one on top of the other.

Coming Soon

In the next lesson, students will formulate a rule for which side lengths are possible for triangles. Future work with congruence will continue in Grade 8 and throughout high school.

Rigor

- Students use manipulatives to further their **conceptual understanding** of unique versus copied polygons.
- Students use a variety of tools to build their **conceptual understanding** of constructing polygons.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	8 min	5 min	5 min
Independent	Pairs	Independent	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut and organized, one set per pair
- sticky notes
- geometry toolkits: linkage strips (which may be substituted for Activity 1 PDF), tracing paper, protractors, rulers, metal paper fasteners

Math Language Development

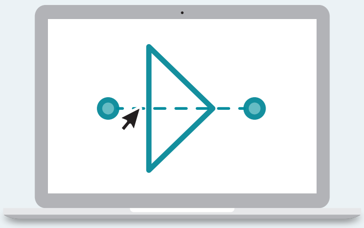
Review word

- *polygon*

Amps | **Featured Activity**

Activity 1
Interactive Geometry

Students can quickly make and manipulate the polygons they build with this tool. Student work is carried over from screen to screen so they can adjust their thinking.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might be insensitive as others present their results from Activity 3. Ask students to identify how they want others to treat them when they share their thinking and then have them set some norms for how the class will treat each other during these discussions.

Modifications to Pacing

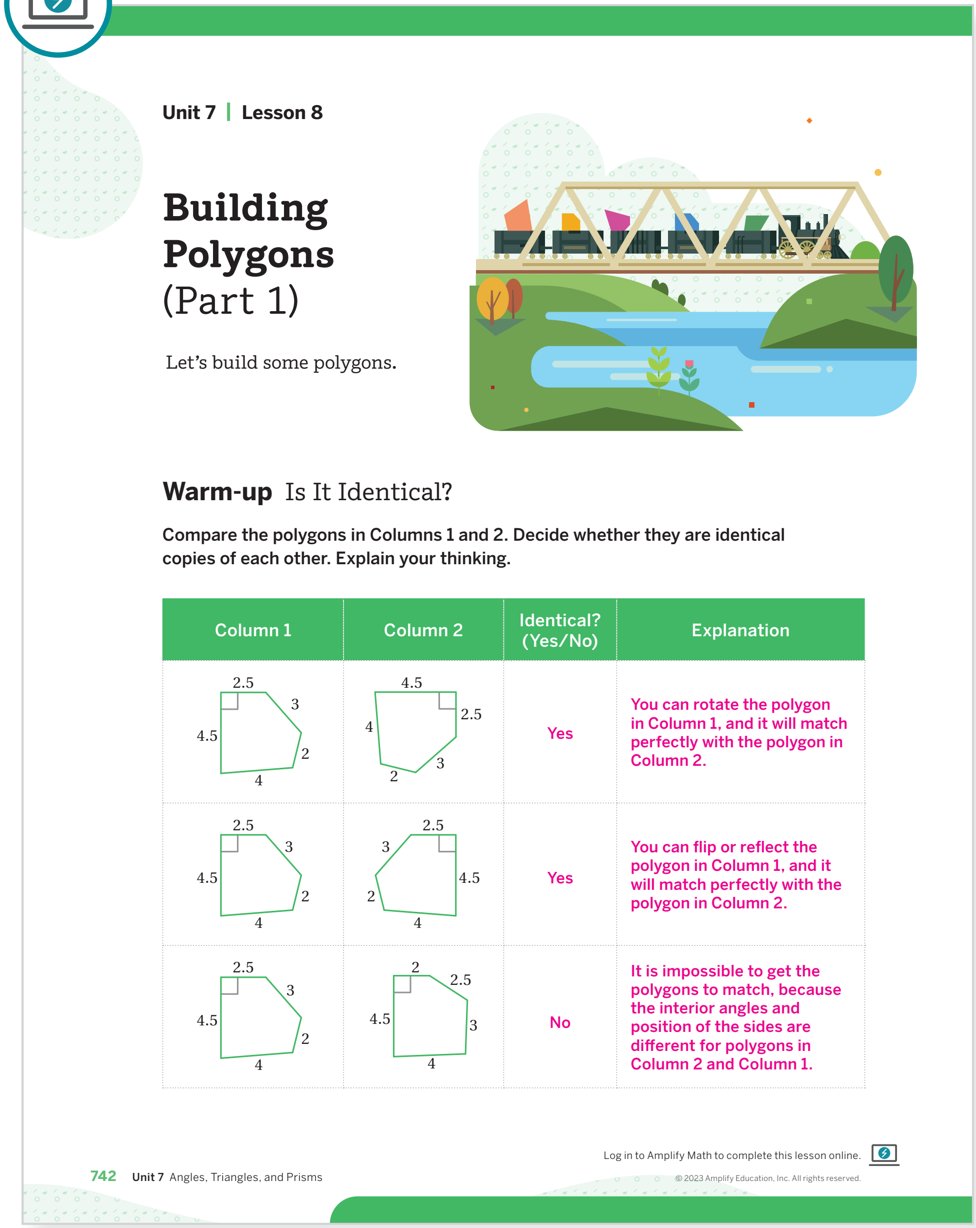
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up**. Then, during the Connect in Activity 2, highlight that the triangle is identical to other copies even when it is rotated or flipped.
- Omit **Activity 1.** Then, during the Connect in Activity 2, highlight that many different quadrilaterals can be built with the same side lengths.

Independent | 5 min

Warm-up Is It Identical?

Students compare polygons to determine if they are identical, which prepares them for determining if two polygons are unique.



Unit 7 | Lesson 8

Building Polygons (Part 1)

Let's build some polygons.

Warm-up Is It Identical?

Compare the polygons in Columns 1 and 2. Decide whether they are identical copies of each other. Explain your thinking.

Column 1	Column 2	Identical? (Yes/No)	Explanation
2.5 3 4.5 2 4	4.5 4 2.5 2 3	Yes	You can rotate the polygon in Column 1, and it will match perfectly with the polygon in Column 2.
2.5 3 4.5 2 4	2.5 3 4.5 2 4	Yes	You can flip or reflect the polygon in Column 1, and it will match perfectly with the polygon in Column 2.
2.5 3 4.5 2 4	2 2.5 4.5 3 4	No	It is impossible to get the polygons to match, because the interior angles and position of the sides are different for polygons in Column 2 and Column 1.

Log in to Amplify Math to complete this lesson online.

742 Unit 7 Angles, Triangles, and Prisms

1 Launch

Provide access to the geometry toolkits, in particular rulers, protractors, and tracing paper. Encourage students to consider which tools will be beneficial in determining if two polygons are the same.

2 Monitor

Help students get started by mentioning that paying attention to side lengths can help them understand how the polygon may have been repositioned.

Look for points of confusion:

- **Difficulty analyzing the positions of the polygons mentally**. Supply students with tracing paper and have them trace the polygons.
- **Not considering that some polygons may be flipped.** Point out that a polygon is a two-dimensional object that can be flipped, just like a piece of paper.

Look for productive strategies:

- Color coding or annotating to match sides or side lengths.
- Using their hands to simulate turning or flipping the polygons.
- Using terms, such as *flip*, *turn*, *reflect*, *slide*, and *rotate*.

3 Connect

Display each pair of polygons so that you can physically manipulate them.

Have students share their explanations for whether the polygons in each set are identical. Listen for and highlight students using relevant vocabulary.

Highlight that the first two rows in the table show identical polygons because the figures were merely rotated or flipped.

Math Language Development

MLR2: Collect and Display

While students work, circulate and look for productive strategies, such as students color coding and annotating the diagrams, as well as the use of terms *flip*, *turn*, and *reflect*. Organize the productive strategies onto a visual display and refer to the display during the discussion.

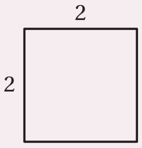
English Learners

Use hand gestures to represent flipping, turning, and reflecting the polygons.

Power-up

To power up students' ability to classify polygons, have students complete:

Which terms could be used to describe the given figure. Select *all* that apply.



[✓ **A.**] Square
[✓ **B.**] Rectangle
[✓ **C.**] Parallelogram
D. Pentagon
[✓ **E.**] Rhombus
[✓ **F.**] Polygon
G. Triangle
[✓ **H.**] Quadrilateral

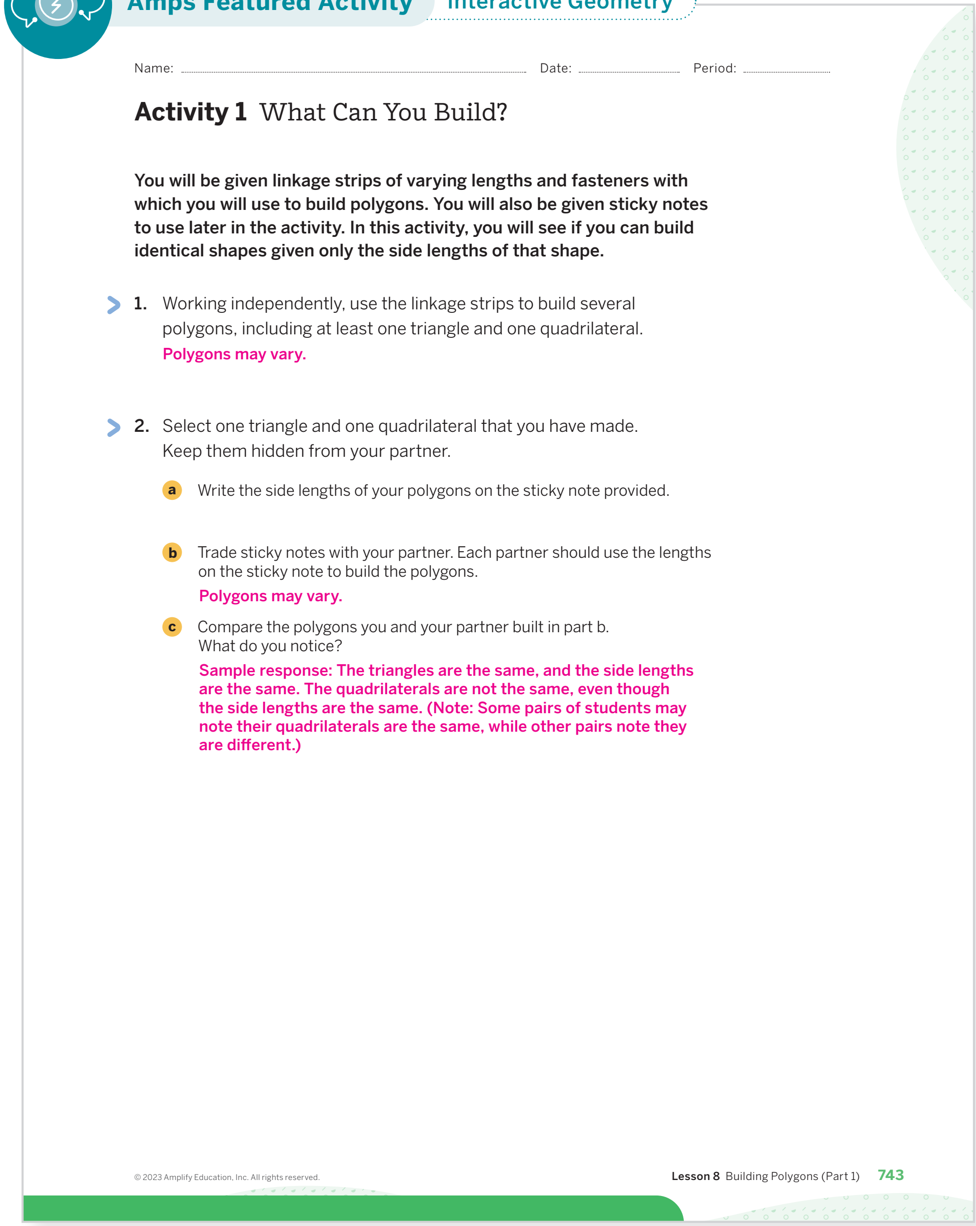
Use: Before Activity 1.
Informed by: Performance on Lesson 7, Practice Problem 5.

Pairs | 15 min

Activity 1 What Can You Build?

Students explore and make observations about a physical representation of polygons to familiarize themselves with the tools and definitions they will use in future activities.

Amps Featured Activity Interactive Geometry



Name: ______ Date: ______ Period: ______

Activity 1 What Can You Build?

You will be given linkage strips of varying lengths and fasteners with which you will use to build polygons. You will also be given sticky notes to use later in the activity. In this activity, you will see if you can build identical shapes given only the side lengths of that shape.

1. Working independently, use the linkage strips to build several polygons, including at least one triangle and one quadrilateral.
   Polygons may vary.

2. Select one triangle and one quadrilateral that you have made. Keep them hidden from your partner.
   - a Write the side lengths of your polygons on the sticky note provided.
   - b Trade sticky notes with your partner. Each partner should use the lengths on the sticky note to build the polygons.
     Polygons may vary.
   - c Compare the polygons you and your partner built in part b. What do you notice?
     Sample response: The triangles are the same, and the side lengths are the same. The quadrilaterals are not the same, even though the side lengths are the same. (Note: Some pairs of students may note their quadrilaterals are the same, while other pairs note they are different.)

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Lesson 8 Building Polygons (Part 1) 743

1 Launch

Distribute the linkage strips from the Activity 1 PDF and at least 12 metal paper fasteners to each pair of students. If necessary, demonstrate how to connect the linkage strips with the fasteners.

2 Monitor

Help students get started by assisting them with connecting their linkage strips.

Look for points of confusion:

- **Taking apart their original shape before comparing with their partner.** Have students try to rebuild their shape, as best they can, in order to make the comparison.
- **Trying to build a triangle with side lengths that do not work.** Let them know they discovered something important, and that it will be discussed later in the lesson.

3 Connect

Have students share the side lengths they chose for their triangle. Create a copy of the triangle with another set of strips and fasteners. Display it for all to see alongside the group's original triangle, but oriented differently. Repeat for another group's quadrilateral.

Ask, "Are the two polygons identical? How can you tell?" Sample responses: Yes, because I can place them on top of each other and they match up exactly; No, there is no way to match them up exactly.

Highlight that the two quadrilaterals are not necessarily identical copies, even though they have the same side lengths. The quadrilateral is "floppy" because its sides can be "pushed" or moved to create a different quadrilateral with the same side lengths. This is not true for triangles, so triangles are called *rigid*.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create and manipulate the polygons they build using an interactive tool.

Accessibility: *Guide Processing and Visualization*

Demonstrate how to use the fasteners to connect the strips to build one polygon students can use for Problem 1. Alternatively, provide pre-built polygons — that you or another class created — for students to use. This will allow them to access the goals of the activity, without having to physically build the polygons.

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Connect, as students share their side lengths, ask others to revoice what they heard, using mathematical language. For example,

If a classmate says:	Another student could say:
"The triangles are the same but the quadrilaterals are different."	"I think you are saying that the triangles have the same side lengths and look exactly the same. The quadrilaterals also have the same side lengths, but they look different from each other. Is that correct?"

Ask the original speaker if their peer was able to restate their thinking.

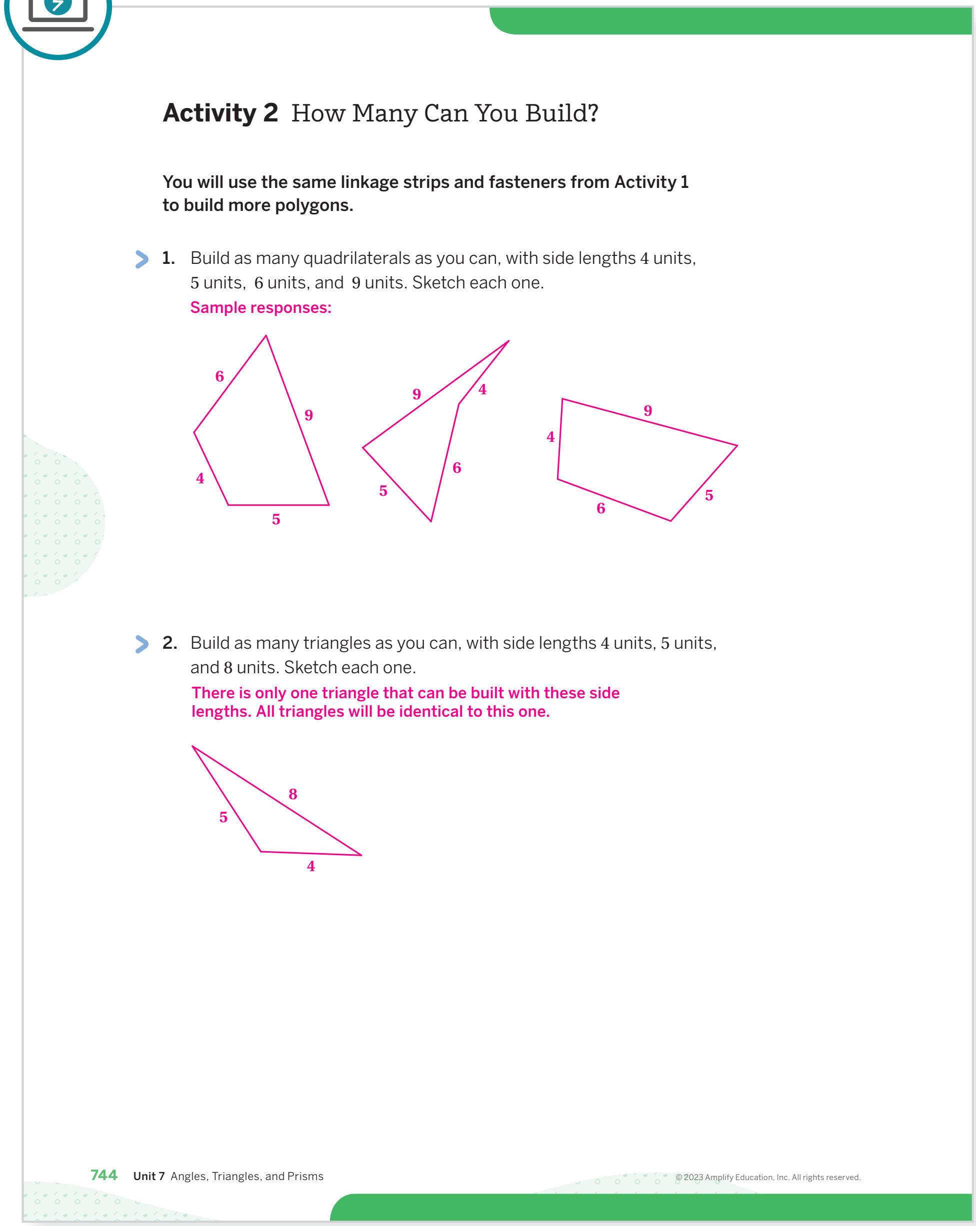
English Learners

The terms *rigid* and *floppy* are likely new terms for students. Be ready to address what these words mean.

Independent | 10 min

Activity 2 How Many Can You Build?

Students build polygons given only a description of their side lengths, to reinforce that certain conditions define a unique polygon.



Activity 2 How Many Can You Build?

You will use the same linkage strips and fasteners from Activity 1 to build more polygons.

1. Build as many quadrilaterals as you can, with side lengths 4 units, 5 units, 6 units, and 9 units. Sketch each one.

   Sample responses:

2. Build as many triangles as you can, with side lengths 4 units, 5 units, and 8 units. Sketch each one.

   There is only one triangle that can be built with these side lengths. All triangles will be identical to this one.

744 **Unit 7** Angles, Triangles, and Prisms

1 Launch

Students will need the same materials from Activity 1. Tell students they will now work independently to build polygons.

2 Monitor

Help students get started by asking, "What does it mean for two polygons to be identical? What does it mean if they are not identical?"

Look for points of confusion:

- **Thinking that there is only one possible quadrilateral in Problem 1.** Suggest students "push" the sides in or out and ask if it still looks like an exact copy of the other.
- **Thinking there is more than one triangle in Problem 2.** Ask students to create two triangles to see there is only one way to form the side lengths.
- **Creating identical polygons.** Ask, "Is there a way to check if this polygon is a copy of any of your others?"

Look for productive strategies:

- Testing uniqueness by manipulating polygons to see if they are an exact copy of what they already created.
- Arranging the side lengths in different orders.

3 Connect

Have students share their constructions and explanations for how many they were able to build.

Display each student's constructed polygons for all to see.

Highlight that the triangle is the only polygon that is rigid. For stability, the internal structures of many buildings (and bridges) will include triangles because they are rigid. Rectangles or other polygons with more than three sides often include triangular supports on the inside, to make the construction more rigid and less floppy. Activate students' background knowledge by asking them whether they have seen triangular supports used in construction.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students work in pairs so they can alternate building polygons and spend more time analyzing them. Alternatively, provide pre-built polygons — that you or another class created — for students to use. This will allow them to access the goals of the activity, without having to physically build the polygons.

Math Language Development

MLR7: Compare and Connect

Ask students to consider what is the same and what is different about their constructed quadrilaterals. Draw students' attention to the conditions that define a unique quadrilateral.

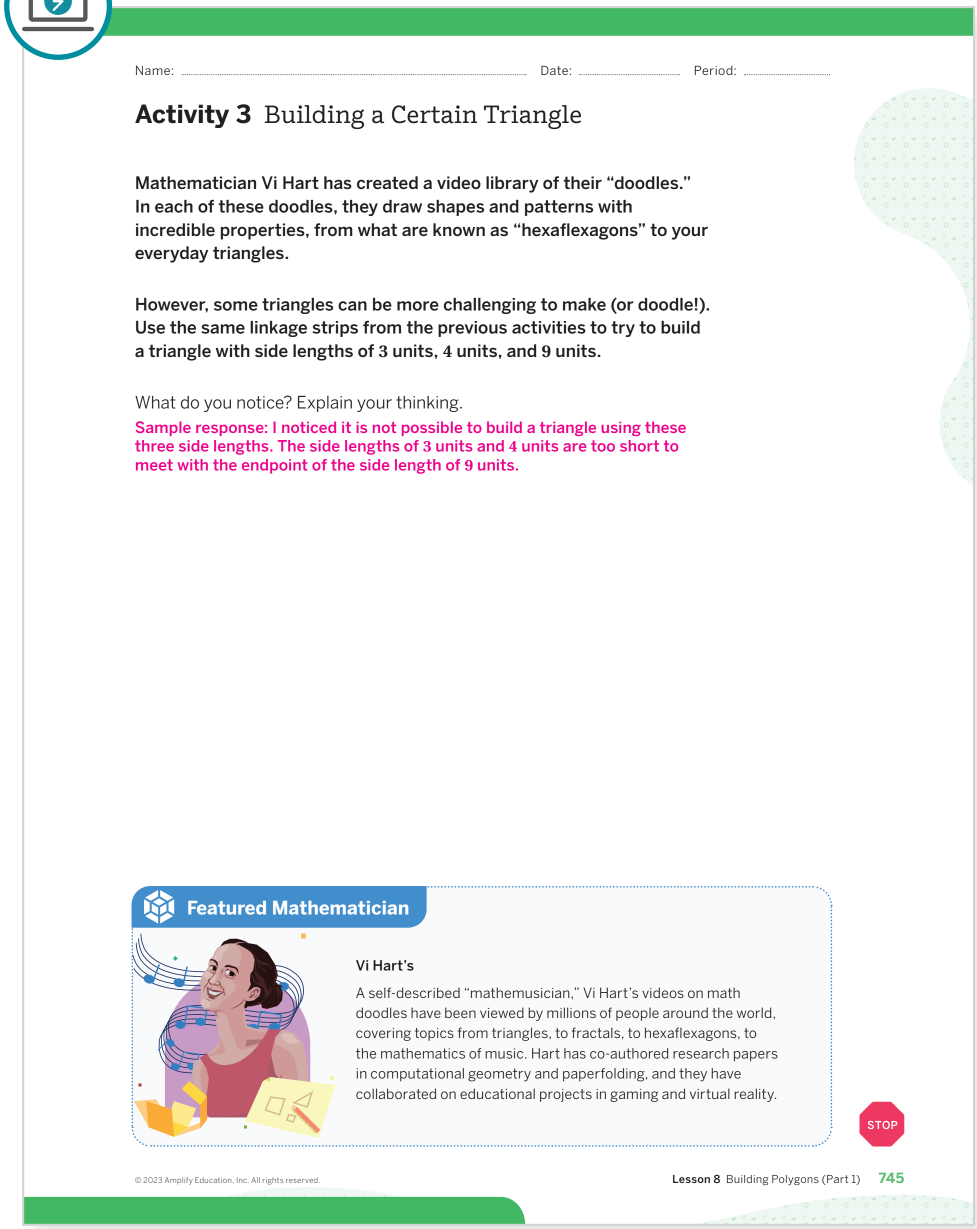
English Learners

Use this time to formalize the term *rigid* and connect the mathematical meaning of *rigid* to the everyday meaning of the term by using physical manipulatives. For example, show a flexible tape measure and say "not rigid." Show a metal or wooden ruler and say "rigid."

Independent | 8 min

Activity 3 Building a Certain Triangle

Students attempt to build a triangle with three lengths that cannot form a triangle, which helps them see that sometimes it is impossible to build a polygon with certain conditions.



Name: ________ Date: ________ Period: ________

Activity 3 Building a Certain Triangle

Mathematician Vi Hart has created a video library of their "doodles." In each of these doodles, they draw shapes and patterns with incredible properties, from what are known as "hexaflexagons" to your everyday triangles.

However, some triangles can be more challenging to make (or doodle!). Use the same linkage strips from the previous activities to try to build a triangle with side lengths of 3 units, 4 units, and 9 units.

What do you notice? Explain your thinking.

Sample response: I noticed it is not possible to build a triangle using these three side lengths. The side lengths of 3 units and 4 units are too short to meet with the endpoint of the side length of 9 units.

Featured Mathematician

Vi Hart's

A self-described "mathemusician," Vi Hart's videos on math doodles have been viewed by millions of people around the world, covering topics from triangles, to fractals, to hexaflexagons, to the mathematics of music. Hart has co-authored research papers in computational geometry and paperfolding, and they have collaborated on educational projects in gaming and virtual reality.

STOP

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Lesson 8 Building Polygons (Part 1) 745

1 Launch

Students will need the same materials as with Activities 1 and 2. Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by making sure they are precise with their unit measurements on the linkage strips.

Look for points of confusion:

- **Bending the strips in order to make the lengths connect.** Remind them that a polygon must contain only straight sides.

Look for productive strategies:

- Reasoning about other potential side lengths that would not form a triangle.

3 Connect

Display a student's incomplete triangle.

Have students share their explanations for why a triangle cannot be formed from these three lengths.

Highlight that as the unit progresses, students will be asked to create or draw shapes that include certain conditions. These conditions can determine whether the shape will be unique or even impossible.

Differentiated Support

Accessibility: *Optimize Access to Tools, Vary Demands to Optimize Challenge*

Instead of having students use linkage strips, allow them to use a ruler to try to draw a triangle with side lengths that are 3 units, 4 units, and 9 units long. They can choose their unit of measure, such as inches or centimeters, as long as they use the same unit for each line segment.

Extension: *Math Enrichment*

Ask students to create other sets of side lengths that would not form triangles. Have them come up with a reasonable explanation for why certain side lengths will not form a triangle. Sample response: The two shortest sides need to be at least a certain length in order to meet with the endpoint of the longest side.

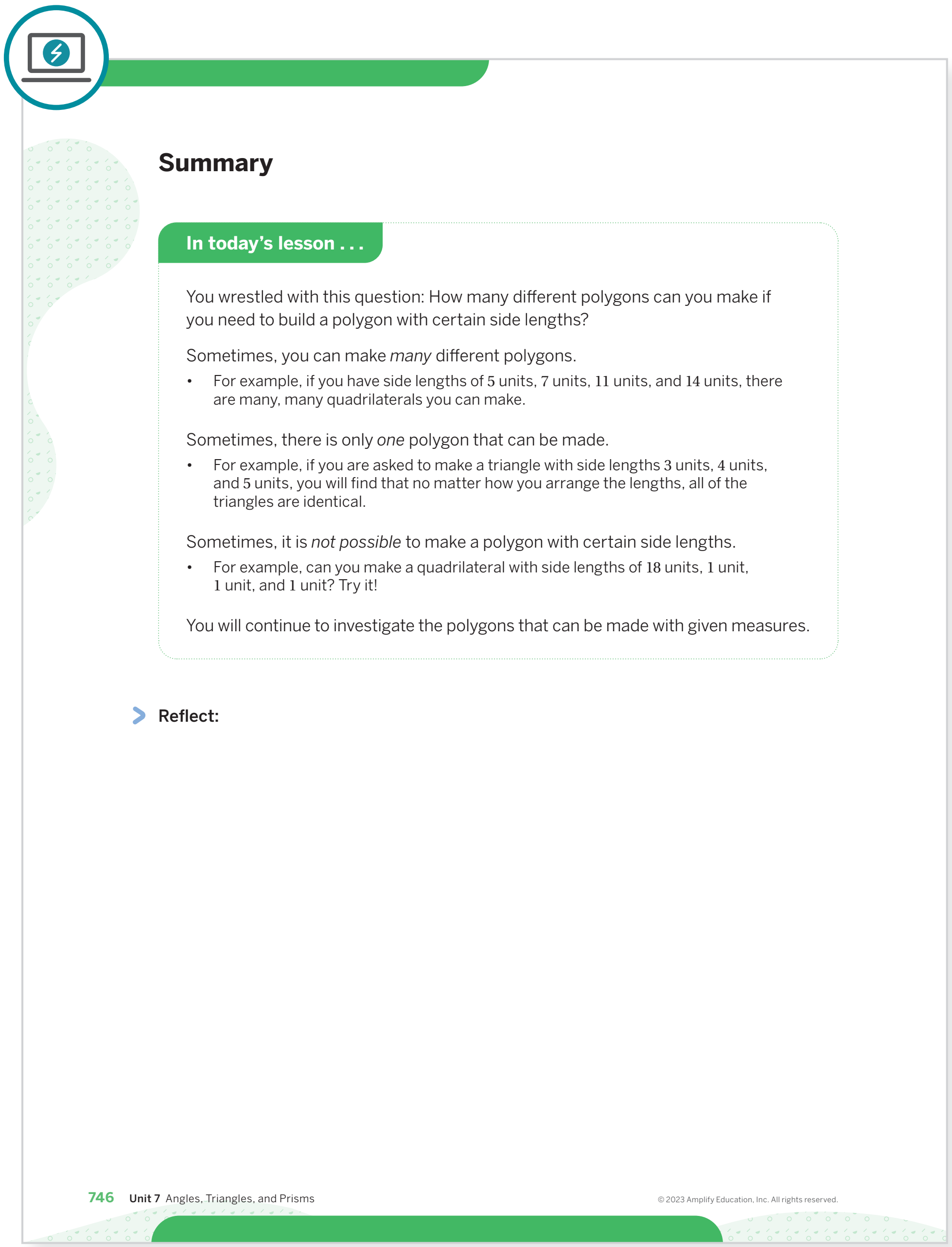
Featured Mathematician

Have students read about Featured Mathematician Vi Hart, whose popular videos show them doodling fascinating mathematical topics, often related to geometry.

Whole Class | 5 min

Summary

Review and synthesize how polygons can be constructed given side lengths, and that sometimes it is not possible to construct a polygon given certain side lengths.



Summary

In today's lesson . . .

You wrestled with this question: How many different polygons can you make if you need to build a polygon with certain side lengths?

Sometimes, you can make *many* different polygons.

- For example, if you have side lengths of 5 units, 7 units, 11 units, and 14 units, there are many, many quadrilaterals you can make.

Sometimes, there is only *one* polygon that can be made.

- For example, if you are asked to make a triangle with side lengths 3 units, 4 units, and 5 units, you will find that no matter how you arrange the lengths, all of the triangles are identical.

Sometimes, it is *not possible* to make a polygon with certain side lengths.

- For example, can you make a quadrilateral with side lengths of 18 units, 1 unit, 1 unit, and 1 unit? Try it!

You will continue to investigate the polygons that can be made with given measures.

> Reflect:

746 Unit 7 Angles, Triangles, and Prisms

Synthesize

Display this set of lengths: 4 units, 4 units, 10 units, and 10 units.

Ask, "What polygons can be made from these side lengths? Be specific about which lengths you are using and how you are describing the polygons."

Have students share with examples and descriptions of the polygons they could make.

Highlight that a triangle can only be made using three of these side lengths, specifically 4 units, 10 units, and 10 units. A rectangle, parallelogram, and a quadrilateral can be made using all four lengths, depending on how they are arranged.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

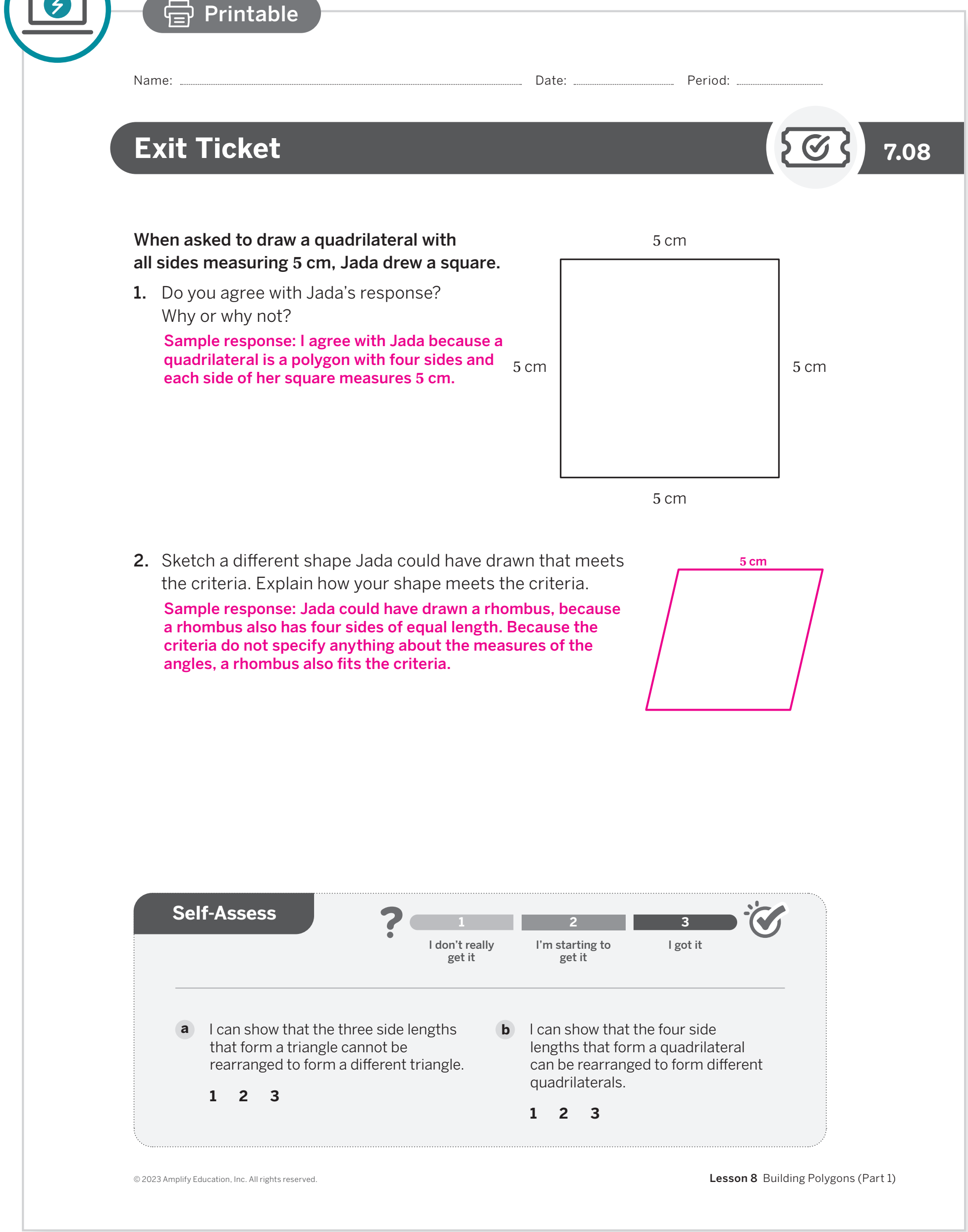
- "Given certain segments and angles, how many unique polygons can be made?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reasoning about the polygons that can be constructed using four side lengths of 5 cm each.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 7.08

When asked to draw a quadrilateral with all sides measuring 5 cm, Jada drew a square.

1. Do you agree with Jada's response? Why or why not?

   Sample response: I agree with Jada because a quadrilateral is a polygon with four sides and each side of her square measures 5 cm.

2. Sketch a different shape Jada could have drawn that meets the criteria. Explain how your shape meets the criteria.

   Sample response: Jada could have drawn a rhombus, because a rhombus also has four sides of equal length. Because the criteria do not specify anything about the measures of the angles, a rhombus also fits the criteria.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can show that the three side lengths that form a triangle cannot be rearranged to form a different triangle.
1 2 3

b I can show that the four side lengths that form a quadrilateral can be rearranged to form different quadrilaterals.
1 2 3

Lesson 8 Building Polygons (Part 1)

Success looks like . . .

- **Language Goal:** Comprehending that two shapes are considered "identical copies" if they can be placed on top of each other and match up exactly. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Recognizing that four side lengths do not determine a unique quadrilateral, but that three side lengths can determine a unique triangle. **(Speaking and Listening, Reading and Writing)**
  - » Explaining that two different quadrilaterals can be drawn with a side length of 5 cm in Problems 1 and 2.
- **Language Goal:** Using manipulatives to create a polygon with given side lengths, and describing the resulting shape. **(Speaking and Listening, Writing)**
  - » Sketching another polygon, such as a rhombus, that also has four sides of length 5 cm.

Suggested next steps

If students disagree with Jada, consider:

- Reviewing the definition and properties of a quadrilateral.

If students do not think there is another polygon Jada could have drawn, consider:

- Having students try to build the given polygon using linkage strips. Ask them to move the linkage strip in a way that creates a different quadrilateral, while the lengths of the sides remain the same.

Professional Learning

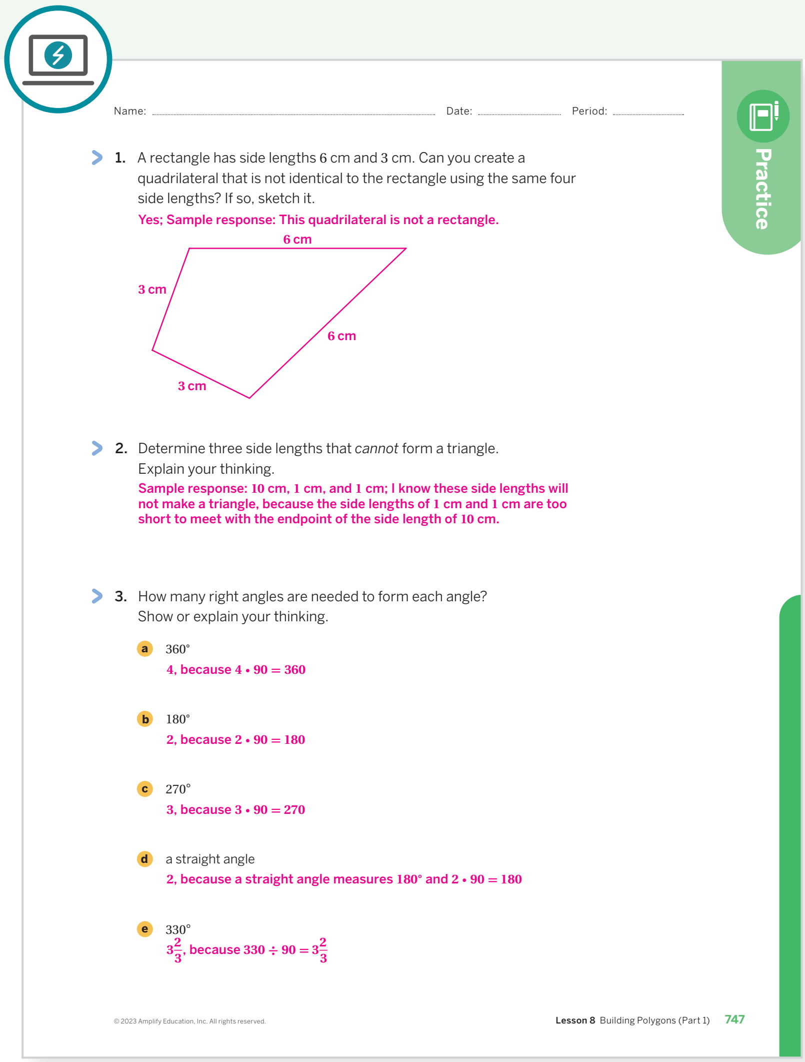
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activities 1 or 2 more engaging today? Why do you think that is?
- In what ways did Activity 3 go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A rectangle has side lengths 6 cm and 3 cm. Can you create a quadrilateral that is not identical to the rectangle using the same four side lengths? If so, sketch it.

Yes; Sample response: This quadrilateral is not a rectangle.

2. Determine three side lengths that *cannot* form a triangle. Explain your thinking.

Sample response: 10 cm, 1 cm, and 1 cm; I know these side lengths will not make a triangle, because the side lengths of 1 cm and 1 cm are too short to meet with the endpoint of the side length of 10 cm.

3. How many right angles are needed to form each angle? Show or explain your thinking.

a. 360°

4, because $4 \cdot 90 = 360$

b. 180°

2, because $2 \cdot 90 = 180$

c. 270°

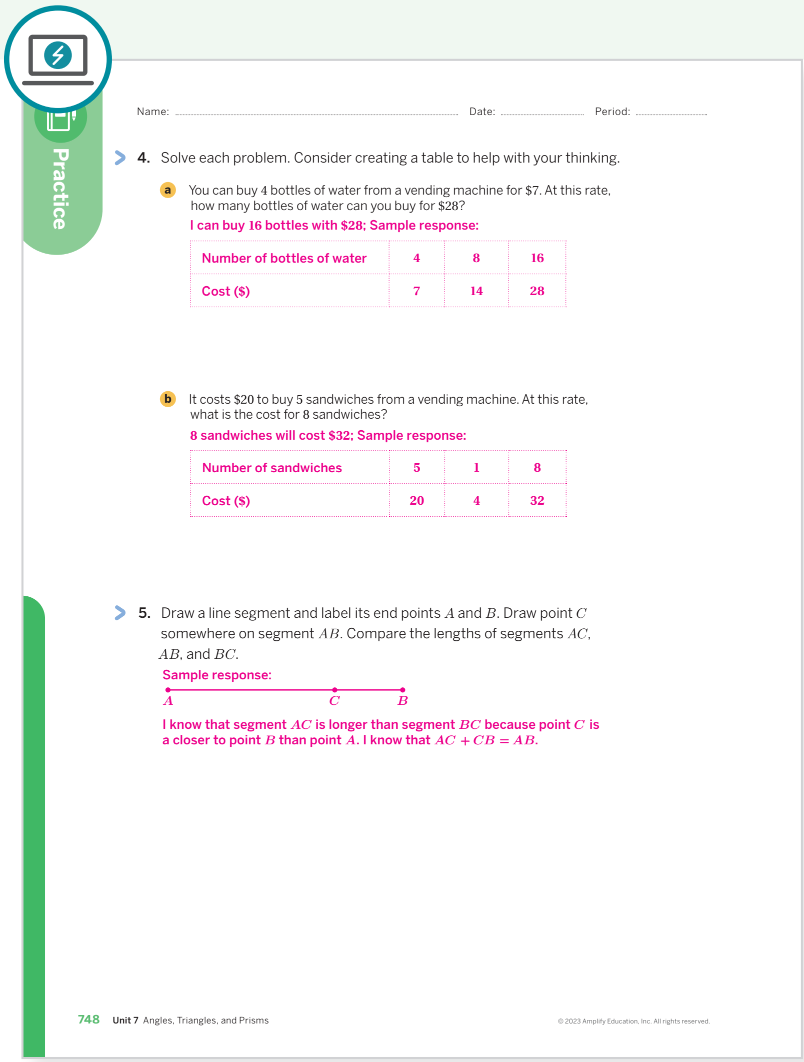
3, because $3 \cdot 90 = 270$

d. a straight angle

2, because a straight angle measures 180° and $2 \cdot 90 = 180$

e. 330°

$3\frac{2}{3}$, because $330 \div 90 = 3\frac{2}{3}$



Name: Date: Period:

4. Solve each problem. Consider creating a table to help with your thinking.

a. You can buy 4 bottles of water from a vending machine for $7. At this rate, how many bottles of water can you buy for $28?

I can buy 16 bottles with $28; Sample response:

Number of bottles of water	4	8	16
Cost ($)	7	14	28

b. It costs $20 to buy 5 sandwiches from a vending machine. At this rate, what is the cost for 8 sandwiches?

8 sandwiches will cost $32; Sample response:

Number of sandwiches	5	1	8
Cost ($)	20	4	32

5. Draw a line segment and label its end points A and B. Draw point C somewhere on segment AB. Compare the lengths of segments AC, AB, and BC.

Sample response:

I know that segment AC is longer than segment BC because point C is a closer to point B than point A. I know that $AC + CB = AB$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 3	2
Spiral	3	Unit 7 Lesson 2	2
	4	Unit 2 Lesson 2	2
Formative	5	Unit 7 Lesson 9	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

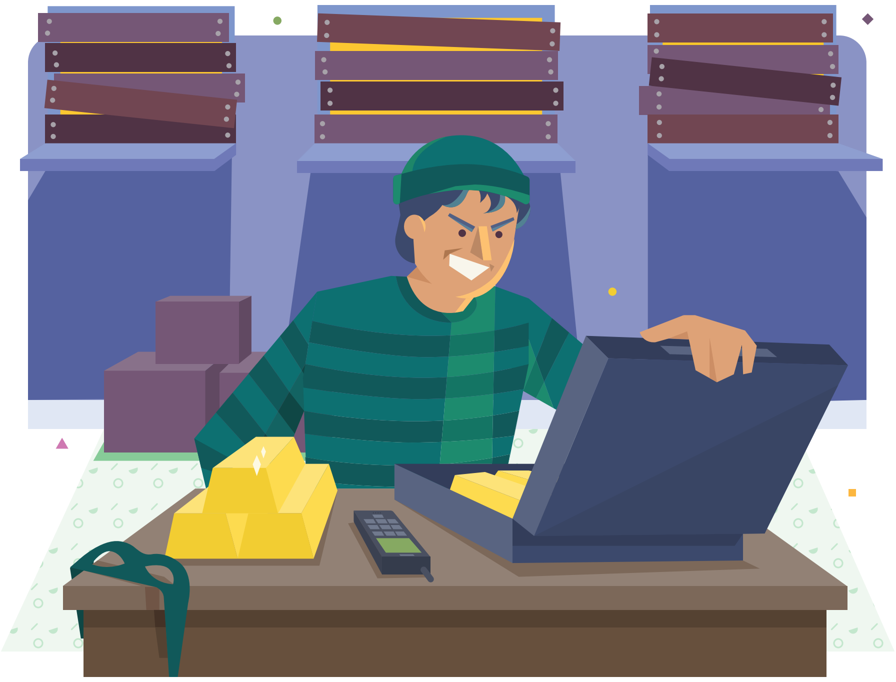
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 9**

Building Polygons (Part 2)

Let's build some more triangles.



Focus

Goals

1. **Language Goal:** Explain how to use circles to locate the point where the sides of a triangle, with known side lengths, should meet. **(Writing)**
2. **Language Goal:** Use manipulatives or tools to justify when it is not possible to make a triangle with three given side lengths. **(Speaking and Listening, Reading and Writing)**
3. Use manipulatives or tools to show that there is a minimum and maximum length for the third side of a triangle, given the other two side lengths.

Coherence

Today

In this lesson, students experiment with constructing triangles given two or three side lengths. They discover that there are some combinations of lengths that do not create a triangle. Students notice that there are certain relationships between the side lengths that cause the formation of a triangle to be possible or not possible.

Previously

Students built triangles and quadrilaterals with varying side lengths, using linkage strips. They reasoned about whether the side lengths of certain polygons created identical polygons or not.

Coming Soon

In the next few lessons, students will construct triangles given certain conditions (angle measures and side lengths).

Rigor

- Students use compasses to further their **conceptual understanding** of triangle construction.
- Students use a variety of tools to construct triangles with given side lengths to develop **fluency**.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut and organized, one set per pair
- geometry toolkits: compasses, rulers, linkage strips (may be substituted for Activity 2 PDF), metal paper fasteners

Math Language Development

Review word

- *compass**

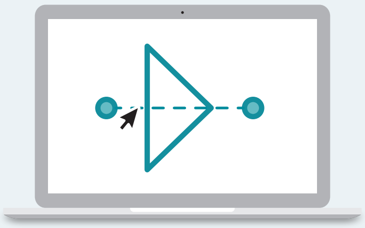
*Students may be familiar with the term *compass*, as it relates to a directional compass. Be ready to address the differences between the everyday use and the mathematical use.

Amps | Featured Activity

Activity 1

Digital Collaboration

Students individually plot potential locations for a call that satisfy mathematical constraints. Then, they can see all the points plotted by their classmates.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not understand how a compass can be helpful in Activity 2. Ask students to start by marking one place where point C may lie. Then ask them to mark a second place, a third place, etc. By applying repeated reasoning with a compass and the definition of a circle, students will be on their way to the correct results.

Modifications to Pacing

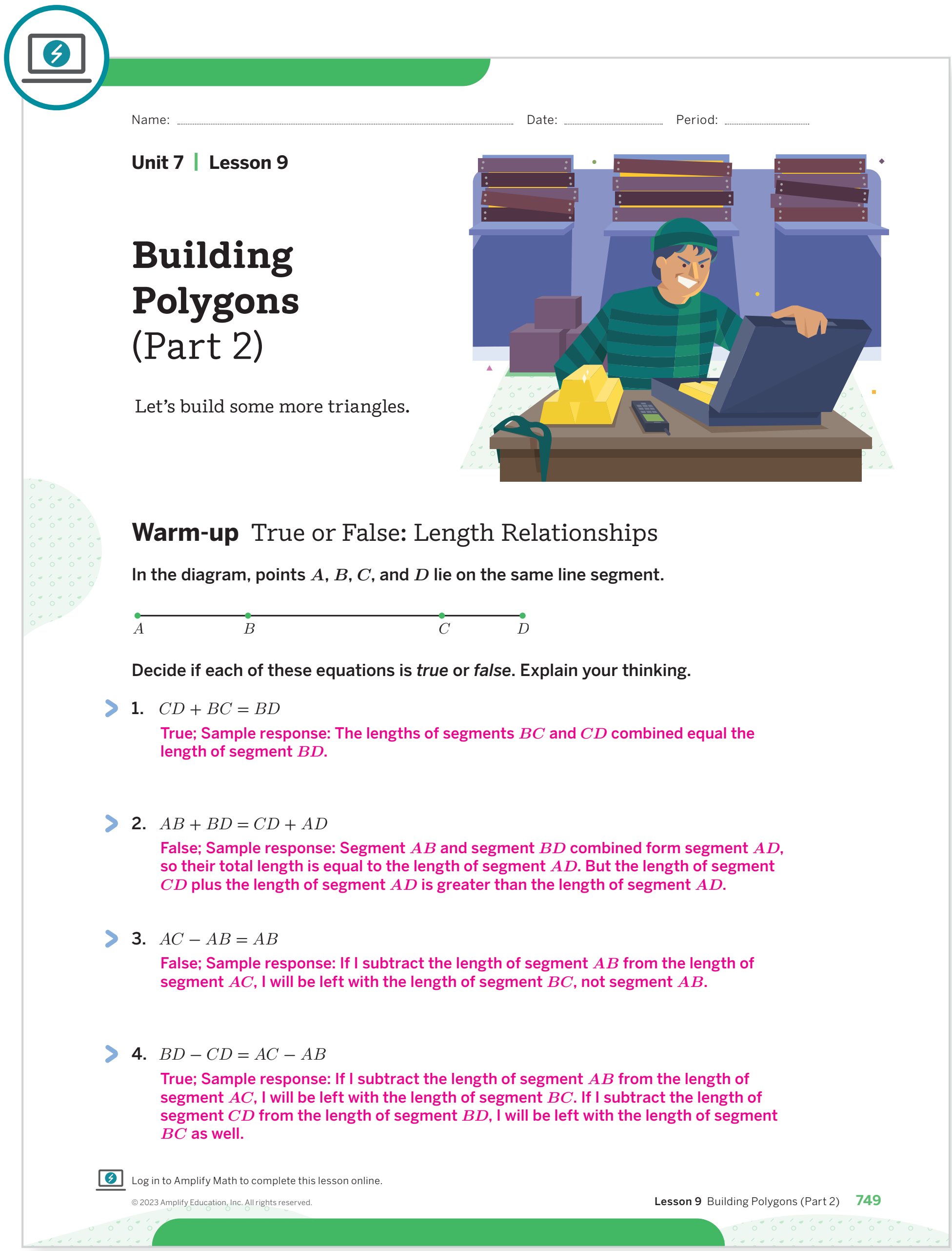
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- **Activity 1** may be omitted. Instead, during the Connect in Activity 2, highlight how the set of all points a certain distance from a starting point form a circle.

Pairs | 10 min

Warm-up True or False: Length Relationships

Students express relationships between length measures with equations, in preparation for working with straight-line distances and radial distances.



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Unit 7 | Lesson 9

Building Polygons (Part 2)

Let's build some more triangles.

Warm-up True or False: Length Relationships

In the diagram, points A, B, C, and D lie on the same line segment.

Decide if each of these equations is *true* or *false*. Explain your thinking.

1. $CD + BC = BD$
   True; Sample response: The lengths of segments BC and CD combined equal the length of segment BD.

2. $AB + BD = CD + AD$
   False; Sample response: Segment AB and segment BD combined form segment AD, so their total length is equal to the length of segment AD. But the length of segment CD plus the length of segment AD is greater than the length of segment AD.

3. $AC - AB = AB$
   False; Sample response: If I subtract the length of segment AB from the length of segment AC, I will be left with the length of segment BC, not segment AB.

4. $BD - CD = AC - AB$
   True; Sample response: If I subtract the length of segment AB from the length of segment AC, I will be left with the length of segment BC. If I subtract the length of segment CD from the length of segment BD, I will be left with the length of segment BC as well.

Log in to Amplify Math to complete this lesson online.

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Lesson 9 Building Polygons (Part 2) 749

1 Launch

Set an expectation for the amount of time students have to work in pairs on the activity.

2 Monitor

Help students get started by reminding them the length of a segment is referred to by naming its endpoints. For example, AB means the length of the line segment from point A to point B.

Look for points of confusion:

- **Thinking they need to know the exact measure of each segment.** Have students use estimation or informal measurement with their hand or fingers.

Look for productive strategies:

- Assigning estimated, reasonable values to the lengths of segments to check the truth of the equations.

3 Connect

Display the equations one at a time.

Have students share their thinking for why each equation is true or false. Record and display their responses for all to see.

Highlight that students can work with distances the same way they work with numbers. Since a distance is a value—even if estimated—students can use operations like addition and subtraction with them.

Ask:

- "Do you agree or disagree with ___'s reasoning? Why?"
- "Who can restate ___'s reasoning in a different way?"
- "Does anyone want to add on to ___'s reasoning?"

Power-up

To power up students' ability to construct and compare the lengths of line segments, have students complete:

1. Name all of the line segments you see in the diagram.

   Segment EG, segment EF, segment GF, segment FG, segment GE, and segment FE.

2. Determine whether each statement about the line segment(s) is *true* or *false*.
   a. Segment EG is the shortest segment. true
   b. Segment GF is the longest segment. false
   c. Segment EF is the same as segment FE. true
   d. Segment EG is the same as segment FG. false

Use: Before the Warm-up.

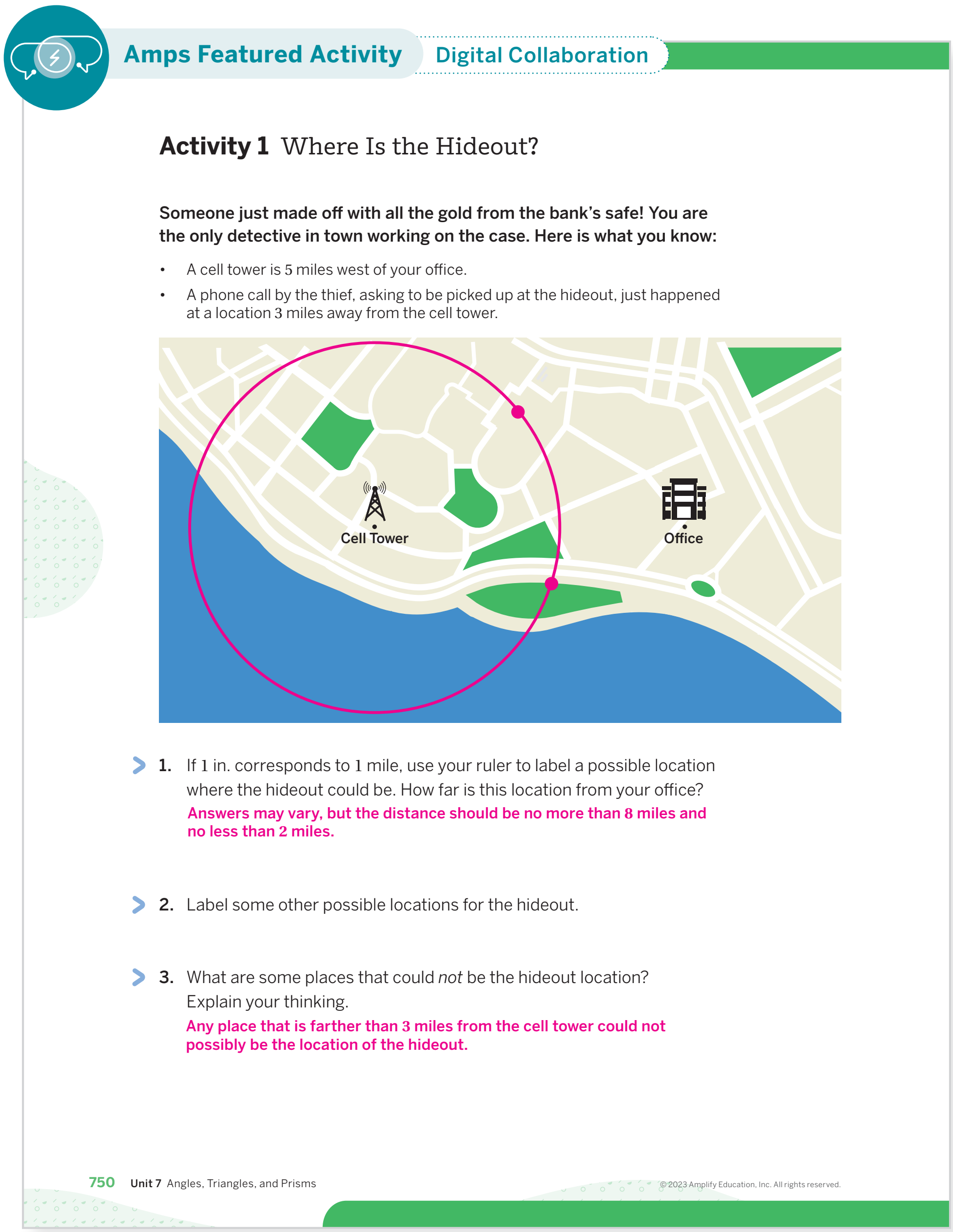
Informed by: Performance on Lesson 8, Practice Problem 5.

Pairs | 10 min

Activity 1 Where Is the Hideout?

Students use geometry tools to remind themselves that all the points that are a certain distance from a starting point form a circle.

Amps Featured Activity | Digital Collaboration



Activity 1 Where Is the Hideout?

Someone just made off with all the gold from the bank's safe! You are the only detective in town working on the case. Here is what you know:

- A cell tower is 5 miles west of your office.
- A phone call by the thief, asking to be picked up at the hideout, just happened at a location 3 miles away from the cell tower.

1. If 1 in. corresponds to 1 mile, use your ruler to label a possible location where the hideout could be. How far is this location from your office?
   Answers may vary, but the distance should be no more than 8 miles and no less than 2 miles.
2. Label some other possible locations for the hideout.
3. What are some places that could *not* be the hideout location? Explain your thinking.
   Any place that is farther than 3 miles from the cell tower could not possibly be the location of the hideout.

750 **Unit 7** Angles, Triangles, and Prisms

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1 Launch

Provide access to the geometry toolkits. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by activating their prior knowledge by asking them how scale helps them to understand a map. Highlight that each inch on the ruler represents one mile.

Look for points of confusion:

- **Assuming that the office, cell tower, and hideout are all on a straight line, and thus the hideout must be 8 miles away.** Ask these students if the problem tells them in which direction the hideout is from the cell tower.

Look for productive strategies:

- **Finding** and labeling as many locations for the hideout as possible.

3 Connect

Have pairs of students share their responses and reasoning.

Ask:

- "What is the closest the hideout could be to the office?" 2 miles
- "What is the farthest the hideout could be away from the office?" 8 miles
- "What shape is made by all the possible locations where the hideout could be?" a circle

Highlight that there is a tool — the compass — in the students' geometry toolkits that can be used to create all the points that are a certain distance from a starting point. As students discovered in Unit 3, this set of points will always form a circle.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the map from the Student Edition with several points plotted on it. Have them use a ruler to measure whether each point could reasonably represent the location of the hideout. Be sure to provide them with the scale used in Problem 1. Plot enough points on the map that both could and could not represent the location of the hideout.

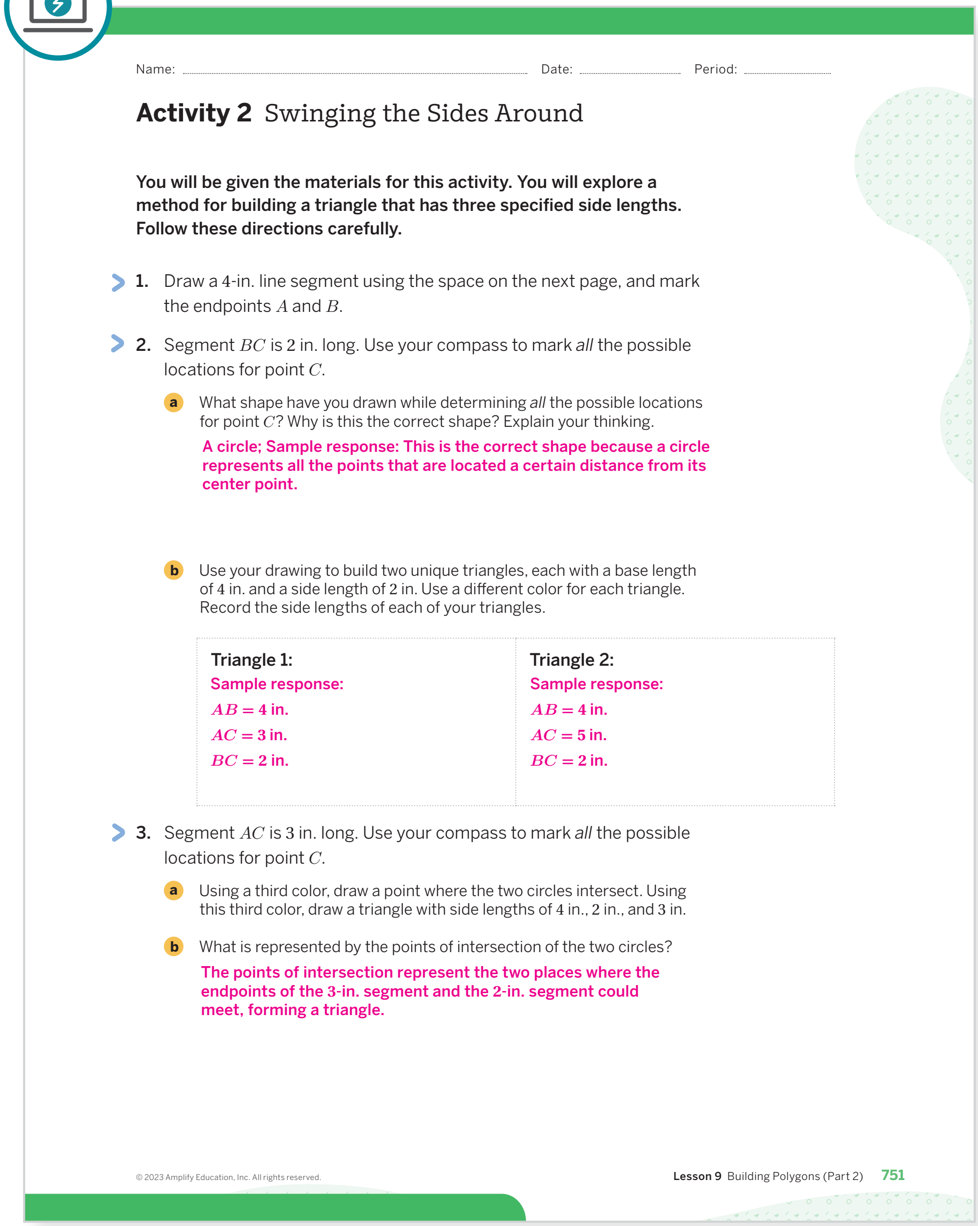
Extension: *Math Enrichment*

After the Connect discussion, ask students if the hideout could be located *in the interior of the circle*, or the hideout must be a point that lies *on the circle's curve*. Have them justify their response. Students' responses will depend upon their interpretation of whether the phone call was located *exactly* 3 miles from the cell tower, or a *maximum* of 3 miles from the cell tower. Accept all reasonable responses, based on their interpretation.

Pairs | 15 min

Activity 2 Swinging the Sides Around

Students use geometry tools to draw all the possible endpoints for given segments, which allows them to practice constructing triangles given the side lengths.



Name: ______ Date: ______ Period: ______

Activity 2 Swinging the Sides Around

You will be given the materials for this activity. You will explore a method for building a triangle that has three specified side lengths. Follow these directions carefully.

1. Draw a 4-in. line segment using the space on the next page, and mark the endpoints A and B.
2. Segment BC is 2 in. long. Use your compass to mark *all* the possible locations for point C.
   - a. What shape have you drawn while determining *all* the possible locations for point C? Why is this the correct shape? Explain your thinking.
     A circle; Sample response: This is the correct shape because a circle represents all the points that are located a certain distance from its center point.
   - b. Use your drawing to build two unique triangles, each with a base length of 4 in. and a side length of 2 in. Use a different color for each triangle. Record the side lengths of each of your triangles.

Triangle 1:	Triangle 2:
Sample response: $AB = 4$ in. $AC = 3$ in. $BC = 2$ in.	**Sample response:** $AB = 4$ in. $AC = 5$ in. $BC = 2$ in.

3. Segment AC is 3 in. long. Use your compass to mark *all* the possible locations for point C.
   - a. Using a third color, draw a point where the two circles intersect. Using this third color, draw a triangle with side lengths of 4 in., 2 in., and 3 in.
   - b. What is represented by the points of intersection of the two circles?
     The points of intersection represent the two places where the endpoints of the 3-in. segment and the 2-in. segment could meet, forming a triangle.

1 Launch

Distribute a set of linkage strips from the Activity 2 PDF and at least 12 metal paper fasteners to each pair of students. Activate prior knowledge by asking a student to model how to use the compass to make a circle and an arc.

2 Monitor

Help students get started by reading the steps aloud and having students demonstrate each step as you read it.

Look for points of confusion:

- **Trying to center their pencil in the center of the hole, creating a wobbly circle.** Have students place their pencil along the outer edge of the hole as they rotate the linkage strip.

Look for productive strategies:

- Noticing that two identical triangles can be created.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create and manipulate the triangles they build using an interactive tool. They can use the interactive tool to rotate and trace points.

Extension: *Math Enrichment*

Have students construct a triangle with side lengths of 2 in., 3 in., and 4 in., given that the 2-in. side is placed horizontally.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their drawings with the class, ask, "What is the same and what is different about the drawings you see?" Draw students' attention to the connections between building and drawing such as opening the hinge between the cardboard strips and drawing the circle using the compass.

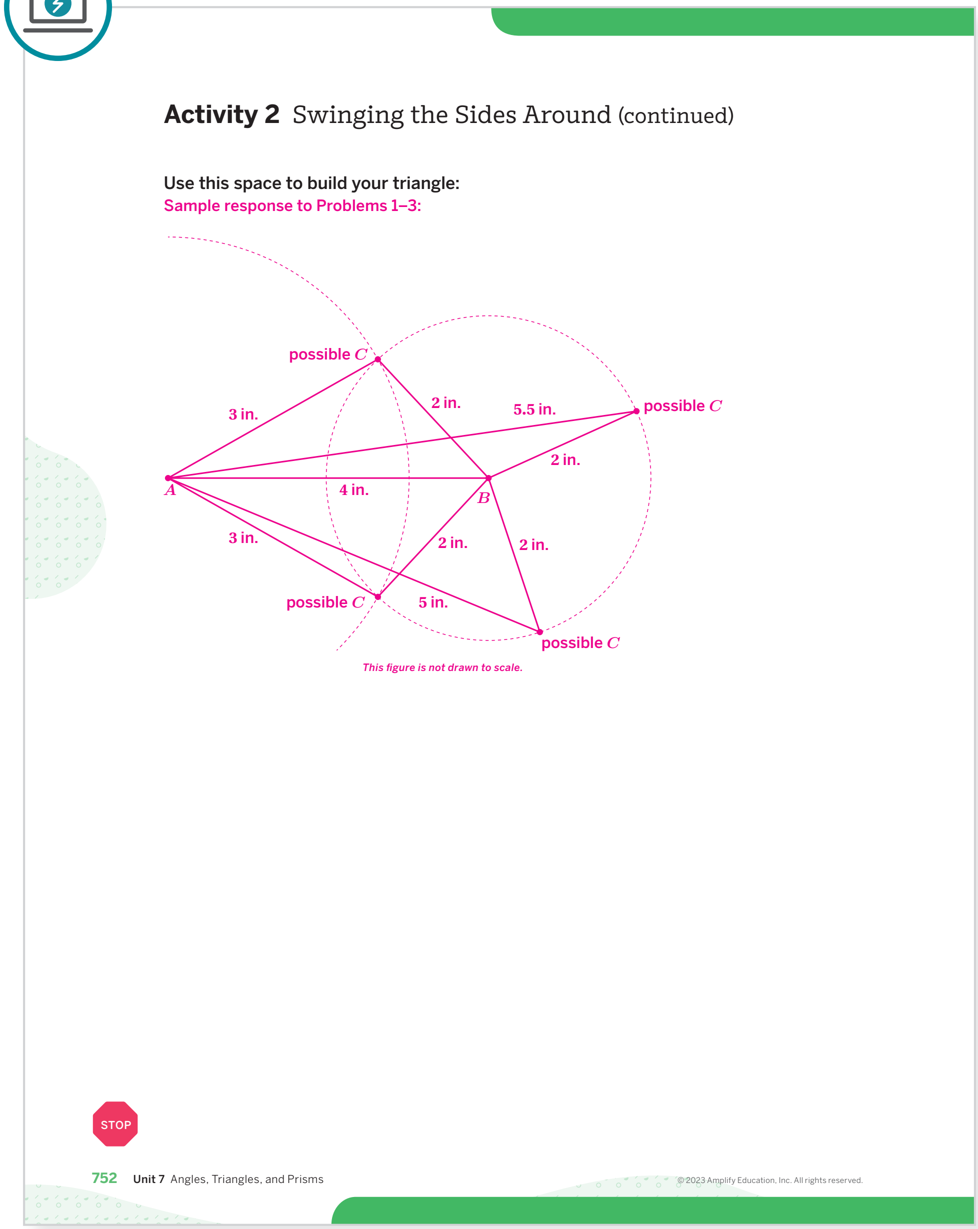
English Learners

As students respond to the Ask questions, have them point to their drawings to help justify their responses.

Pairs | 15 min

Activity 2 Swinging the Sides Around (continued)

Students use geometry tools to draw all the possible endpoints for given segments, which allows them to practice constructing triangles given the side lengths.



Activity 2 Swinging the Sides Around (continued)

Use this space to build your triangle:

Sample response to Problems 1–3:

This figure is not drawn to scale.

STOP

3 Connect

Have students share their drawings with the class.

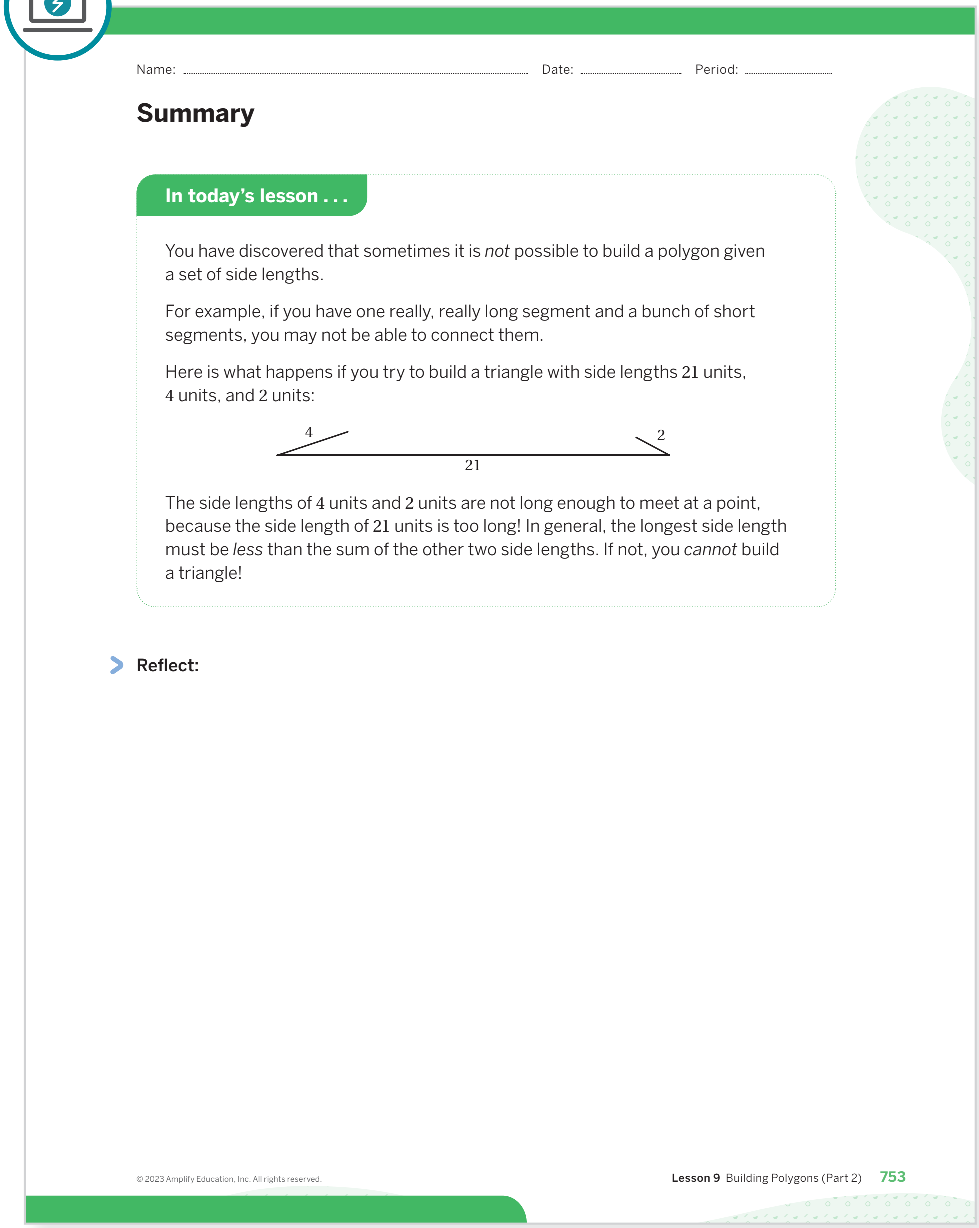
Ask:

- "How many different triangles could you draw when you had only traced a circle on one side? Why?" Sample response: Lots of different triangles, because we only used two of the given side lengths.
- "What is the longest the third side could have been? The shortest?" The length should be between 1 in. and 7 in.
- "How many different triangles could you draw once you traced a circle on each side?" Sample response: It looked like there were two different triangles, but they are identical copies, so there is really only one triangle.

Whole Class | 5 min

Summary

Review and synthesize that it is not always possible to construct a triangle given three side lengths.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You have discovered that sometimes it is *not* possible to build a polygon given a set of side lengths.

For example, if you have one really, really long segment and a bunch of short segments, you may not be able to connect them.

Here is what happens if you try to build a triangle with side lengths 21 units, 4 units, and 2 units:

The side lengths of 4 units and 2 units are not long enough to meet at a point, because the side length of 21 units is too long! In general, the longest side length must be *less* than the sum of the other two side lengths. If not, you *cannot* build a triangle!

Reflect:

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Lesson 9 Building Polygons (Part 2) 753

Synthesize

Ask, "When you are given three side lengths and asked to draw a triangle, what steps should you take?" Sample response: First draw one length. Then, using the endpoints of the first side and a compass, mark the circles representing all the possible endpoints of the other two sides. Use the intersections of those two circles to find the point where the second and third sides should meet.

Display a demonstration of a construction of a triangle with side lengths of 3 in., 4 in., and 5 in.

Ask:

- After the demonstration, ask "What side lengths make a triangle possible?" Any side lengths where the sum of the lengths of the two shorter sides are longer than the length of the longest side.
- "Which conditions make constructing a triangle *impossible*?" When the length of the longest side of the triangles is greater than or equal to the sum of the lengths of the two shorter sides.

Highlight that when students are given three lengths for the possible sides of a triangle, they must think about the side lengths in order to know if it is even possible to build a triangle.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

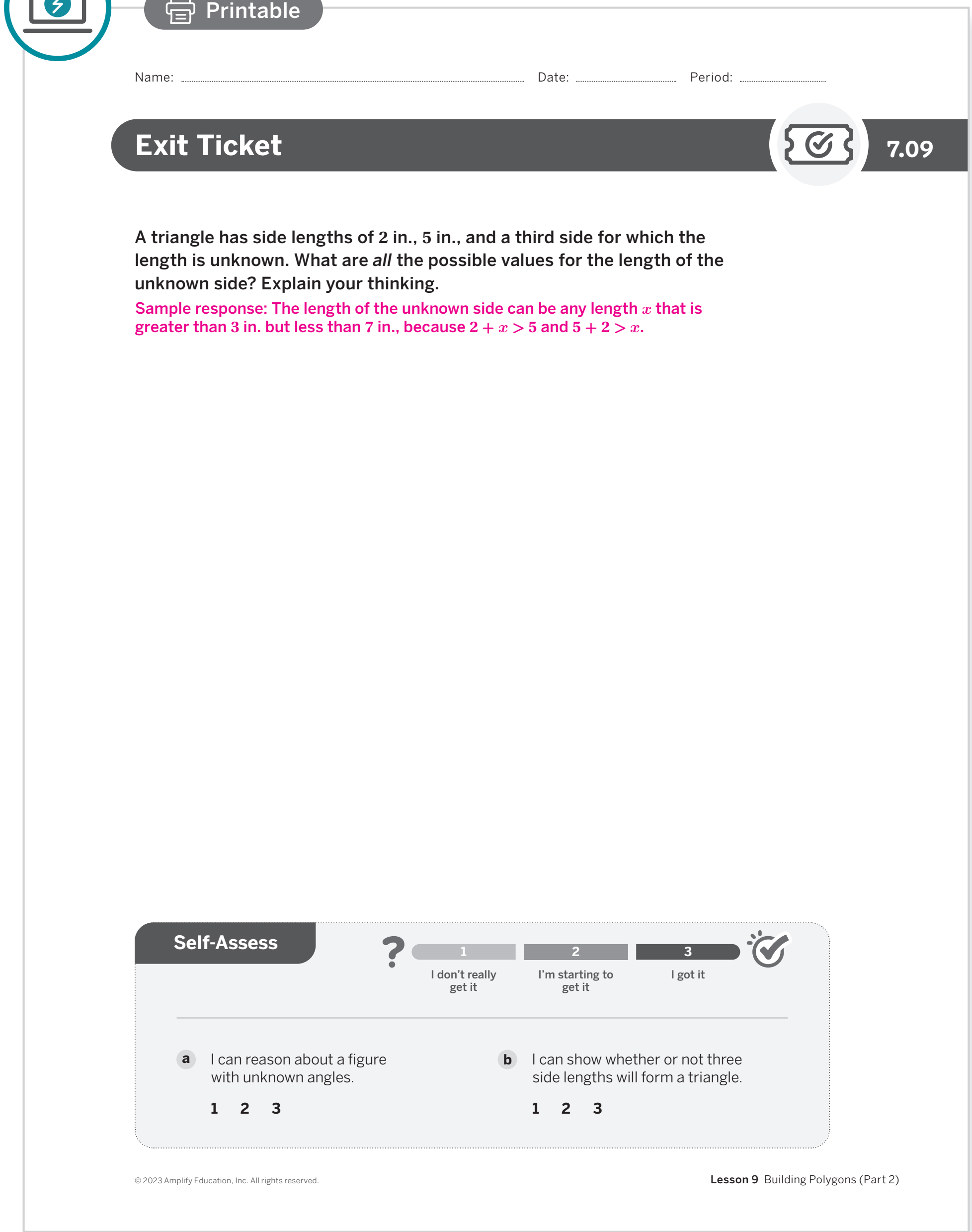
- "Which tools did you use that were most useful in constructing triangles? In what way(s) were they useful?"
- "Which tools did you find most challenging to use? What made them challenging?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reasoning about the possible values for the unknown side length of a triangle.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket

7.09

A triangle has side lengths of 2 in., 5 in., and a third side for which the length is unknown. What are *all* the possible values for the length of the unknown side? Explain your thinking.

Sample response: The length of the unknown side can be any length x that is greater than 3 in. but less than 7 in., because $2 + x > 5$ and $5 + 2 > x$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can reason about a figure with unknown angles.

1 2 3

b I can show whether or not three side lengths will form a triangle.

1 2 3

Lesson 9 Building Polygons (Part 2)

Success looks like . . .

- **Language Goal:** Explaining how to use circles to locate the point where the sides of a triangle, with known side lengths, should meet. **(Writing)**
- **Language Goal:** Using manipulatives or tools to justify when it is not possible to make a triangle with three given side lengths. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Using manipulatives or tools to show that there is a minimum and maximum length for the third side of a triangle, given the other two side lengths.
    - » Determining the possible lengths of the unknown side of a triangle when the two given sides are 2 in. and 5 in.

Suggested next steps

If students include 7 in. and 3 in. as possible side lengths, consider:

- Having them try to draw a triangle with these side lengths and then explain why it is not possible.

If students only give discrete values as the possible lengths, consider:

- Suggesting they try some fractional values. Ask if they can find a way to express all the possible values using an inequality symbol.

Professional Learning

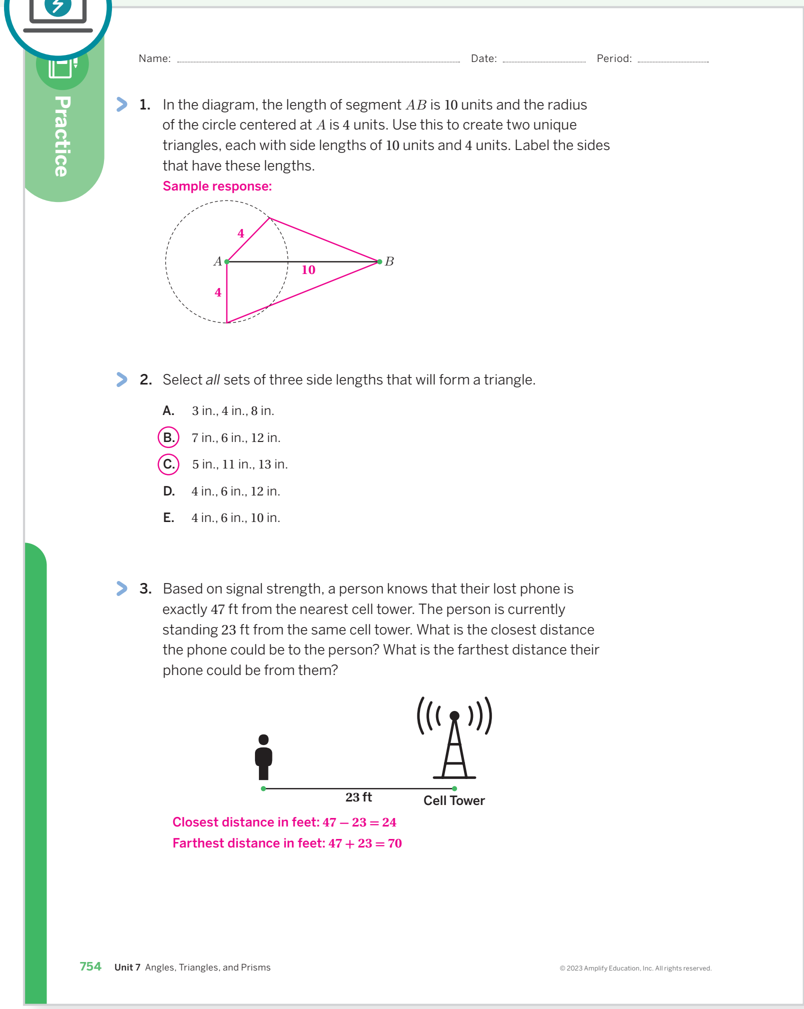
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Who participated and who didn't in Activity 2? What trends do you see in participation?
- How did using a variety of tools to construct triangles set up students to develop the skills they will need to determine what characteristics determine unique triangles? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. In the diagram, the length of segment AB is 10 units and the radius of the circle centered at A is 4 units. Use this to create two unique triangles, each with side lengths of 10 units and 4 units. Label the sides that have these lengths.

Sample response:

2. Select *all* sets of three side lengths that will form a triangle.

A. 3 in., 4 in., 8 in.
[✓ B.] 7 in., 6 in., 12 in.
[✓ C.] 5 in., 11 in., 13 in.
D. 4 in., 6 in., 12 in.
E. 4 in., 6 in., 10 in.

3. Based on signal strength, a person knows that their lost phone is exactly 47 ft from the nearest cell tower. The person is currently standing 23 ft from the same cell tower. What is the closest distance the phone could be to the person? What is the farthest distance their phone could be from them?

Closest distance in feet: $47 - 23 = 24$

Farthest distance in feet: $47 + 23 = 70$

754 Unit 7 Angles, Triangles, and Prisms

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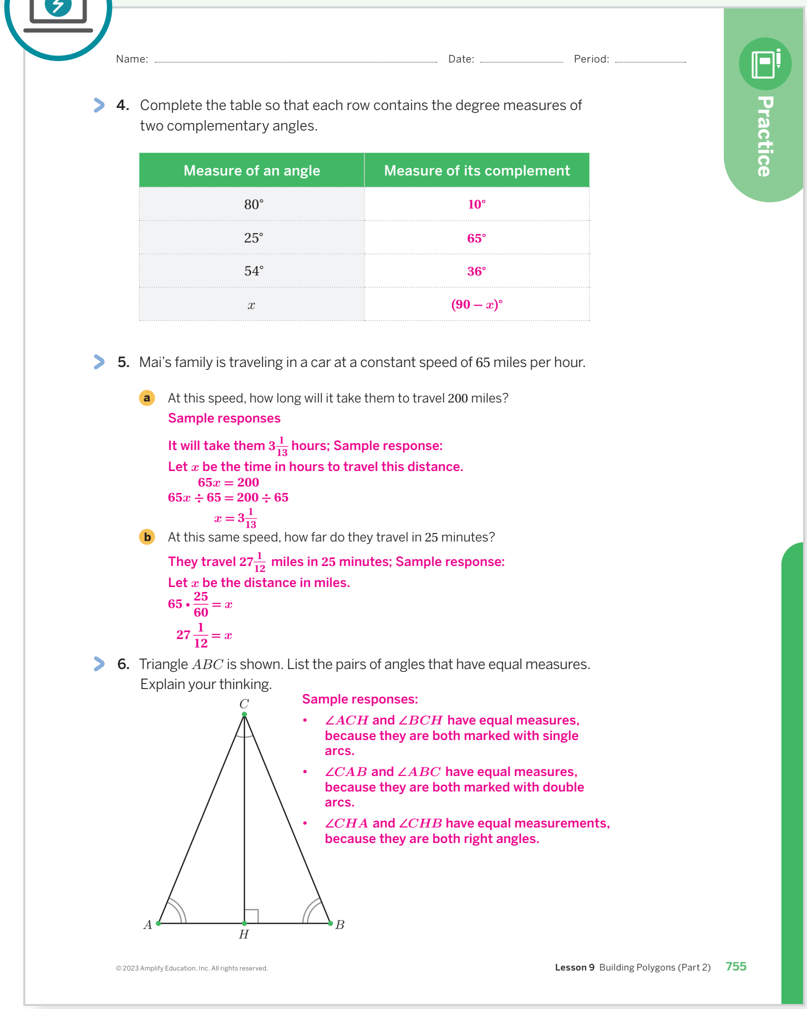

Name: ______ Date: ______ Period: ______

4. Complete the table so that each row contains the degree measures of two complementary angles.

Measure of an angle	Measure of its complement
80°	**10°**
25°	**65°**
54°	**36°**
x	**$(90 - x)°$**

5. Mai's family is traveling in a car at a constant speed of 65 miles per hour.

a. At this speed, how long will it take them to travel 200 miles?

Sample responses

It will take them $3\frac{1}{13}$ hours; Sample response:

Let x be the time in hours to travel this distance.

$$65x = 200$$
$$65x \div 65 = 200 \div 65$$
$$x = 3\frac{1}{13}$$

b. At this same speed, how far do they travel in 25 minutes?

They travel $27\frac{1}{12}$ miles in 25 minutes; Sample response:

Let x be the distance in miles.

$$65 \cdot \frac{25}{60} = x$$
$$27\frac{1}{12} = x$$

6. Triangle ABC is shown. List the pairs of angles that have equal measures. Explain your thinking.

Sample responses:

- **$\angle ACH$ and $\angle BCH$ have equal measures, because they are both marked with single arcs.**
- **$\angle CAB$ and $\angle ABC$ have equal measures, because they are both marked with double arcs.**
- **$\angle CHA$ and $\angle CHB$ have equal measurements, because they are both right angles.**

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Lesson 9 Building Polygons (Part 2) 755

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 7 Lesson 2	2
	5	Unit 2 Lesson 6	2
Formative	6	Unit 7 Lesson 10	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 10**

Triangles With Three Common Measures

Let's compare triangles that have common side lengths or angle measures.

Focus

Goals

1. **Language Goal:** Describe, compare, and contrast triangles that share three common measures of angles or sides. **(Speaking and Listening, Writing)**
2. **Language Goal:** Justify (using multiple representations) whether triangles are identical copies (unique) or are different triangles. **(Speaking and Listening, Writing)**
3. **Language Goal:** Recognize that examining which side lengths and angle measures are adjacent can help determine whether triangles are identical copies. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students examine sets of triangles in which all the triangles share three common measures of angles or sides. Students learn to recognize when triangles are identical copies, even if they are oriented differently, and when triangles are not unique. Students make conjectures and justify their conclusions.

Previously

In Lesson 9, students constructed triangles given various side lengths and determined when it was not possible to construct a triangle.

Coming Soon

In Lesson 11, students will draw triangles given three measurements and determine if the one they created is unique or not.

Rigor

- Students use a variety of tools to construct triangles with given characteristics to develop **fluency**.
- Students **apply** their understanding of triangle constructions to determine and justify if two triangles are copies or are unique.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
7 min	7 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: tracing paper, protractors, rulers

Math Language Development

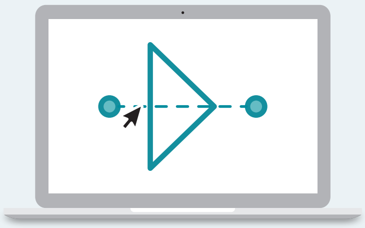
Review word

- *corresponding*

Amps | Featured Activity

Activity 2
Dynamic Triangles

Students can digitally trace, flip, and rotate as they compare whether two triangles are identical copies or not.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack the motivation to make good and purposeful observations of the sets of triangles that they will share in Activity 1. Ask students to explain what they will do to motivate themselves to use their mathematical minds to make their observations and be ready to defend them to the class.

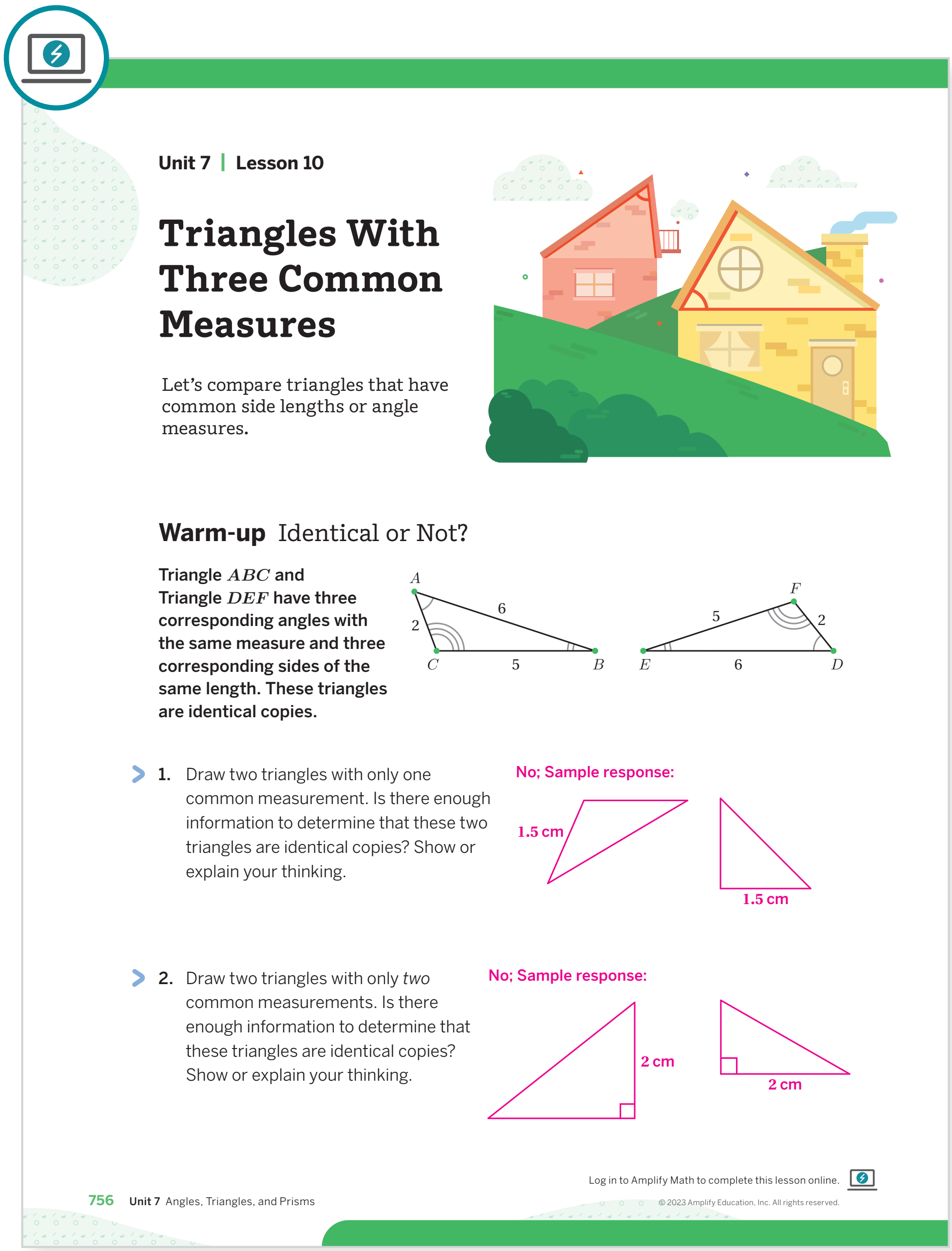
Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Divide the class and have one half complete **Activity 2** and the other half complete **Activity 3**. Have students share what they noticed during a single discussion for both activities.

Warm-up Identical or Not?

Students examine triangles to observe how many measurements are needed to determine if the triangles are identical copies.



Unit 7 | Lesson 10

Triangles With Three Common Measures

Let's compare triangles that have common side lengths or angle measures.

Warm-up Identical or Not?

Triangle ABC and Triangle DEF have three corresponding angles with the same measure and three corresponding sides of the same length. These triangles are identical copies.

1. Draw two triangles with only one common measurement. Is there enough information to determine that these two triangles are identical copies? Show or explain your thinking.

No; Sample response:

2. Draw two triangles with only *two* common measurements. Is there enough information to determine that these triangles are identical copies? Show or explain your thinking.

No; Sample response:

Log in to Amplify Math to complete this lesson online.

756 Unit 7 Angles, Triangles, and Prisms

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1 Launch

Provide access to the geometry toolkits. Activate prior knowledge by asking what it means when two or more angles have the same number of arcs.

2 Monitor

Help students get started by directing partners to each draw a triangle with a specific side length, such as 3 in., and then complete Problem 1.

Look for points of confusion:

- **Saying the first set of triangles are "equal".** Explain that shapes cannot be equal; only values, such as measurements, can be equal. Instead, they should say, "These triangles are identical copies." **Note:** The term *congruent* will be formalized in Grade 8.
- **Thinking they need to draw identical copies.** Ask students to show that one measurement is not enough to determine whether the triangles are identical copies. Have them create triangles that only share one or two common measurements.

3 Connect

Ask, "How many measurements are enough to determine if two triangles are identical copies?"

Display student responses.

Have students share their reasoning behind how they answered the question. Record their reasoning. As students complete Activities 1–3, check with the list to determine whether any opinions have been proven or disproven.

Highlight that knowing all six corresponding measurements are equal will guarantee the triangles are identical copies. However, it is not necessary to know all six measurements, and the next activities will help determine how many, and what type, are needed.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

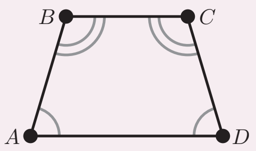
Provide pairs of pre-drawn triangles that each satisfy the conditions given in Problems 1 and 2. Ask students to study the pre-drawn triangles to see if they have enough information to determine if they are identical copies, instead of drawing their own.

Power-up

To power up students' ability to use arc notation to identify angles of equal measure, have students complete:

Recall that when two angles have the same measure, they are marked with the same number of arcs.

Determine all pairs of angles with equal measure in the figure.



$\angle DAB$ and $\angle ADC$

$\angle ABC$ and $\angle BCD$

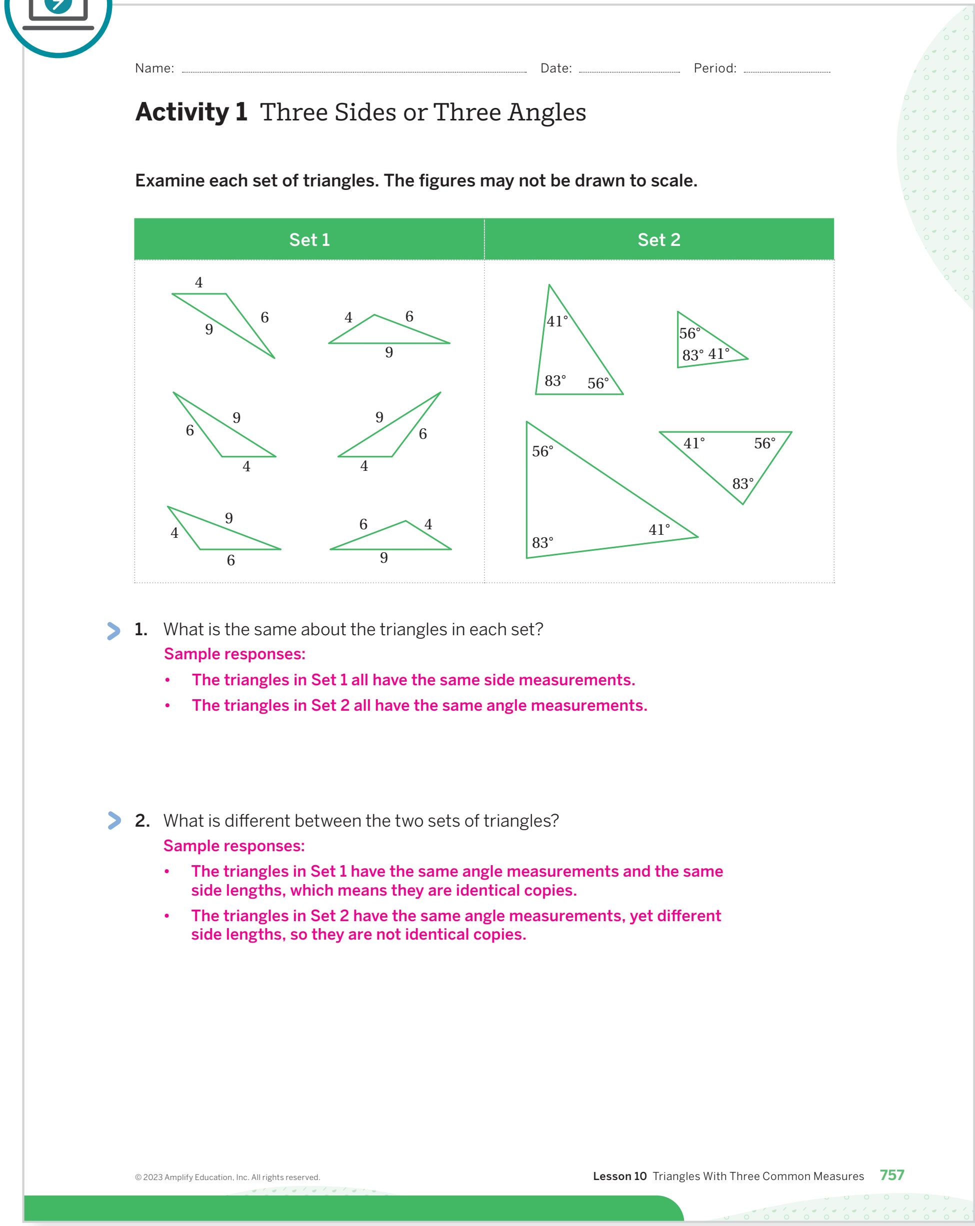
Use: Before the Warm-up.

Informed by: Performance on Lesson 9, Practice Problem 6.

Pairs | 7 min

Activity 1 Three Sides or Three Angles

Students study triangles in two sets — one in which all side lengths are equal and one in which all angle measures are equal — to determine which set contains identical copies.



Name: ______ Date: ______ Period: ______

Activity 1 Three Sides or Three Angles

Examine each set of triangles. The figures may not be drawn to scale.

1. What is the same about the triangles in each set?
   Sample responses:
   - The triangles in Set 1 all have the same side measurements.
   - The triangles in Set 2 all have the same angle measurements.

2. What is different between the two sets of triangles?
   Sample responses:
   - The triangles in Set 1 have the same angle measurements and the same side lengths, which means they are identical copies.
   - The triangles in Set 2 have the same angle measurements, yet different side lengths, so they are not identical copies.

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Lesson 10 Triangles With Three Common Measures 757

1 Launch

Provide access to geometry toolkits. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by tracing the first triangle on tracing paper and using it to determine if the triangles are identical copies.

Look for points of confusion:

- **Thinking the triangles in Set 1 are different.** Explain that the triangles in Set 1 are identical copies, so there really is just one triangle that has been copied many times.
- **Thinking the triangles in Set 2 are identical copies because the angle measures are the same.** Have students use tracing paper to trace one triangle to see if it can be matched exactly with another triangle.

Look for productive strategies:

- Using tracing paper to trace one triangle and transforming it to try to match the other triangles. Note students who do this.
- Extending the sides and measuring angles with a protractor. Note students who do this.

3 Connect

Have students share what they notice is similar and different about the triangles in each set.

Highlight:

- In Set 1, all the triangles are identical copies, just in different orientations. The corresponding side lengths and angle measures are the same.
- In Set 2, the triangles are not identical copies. They are scaled copies. They have the same three angle measures, but different side lengths.

Note: The terms *congruent* and *similar* will be developed in Grade 8.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the triangles in each set for students to cut out and examine individually. Alternatively, provide pre-cut copies of the triangles so students do not need to cut them out themselves.

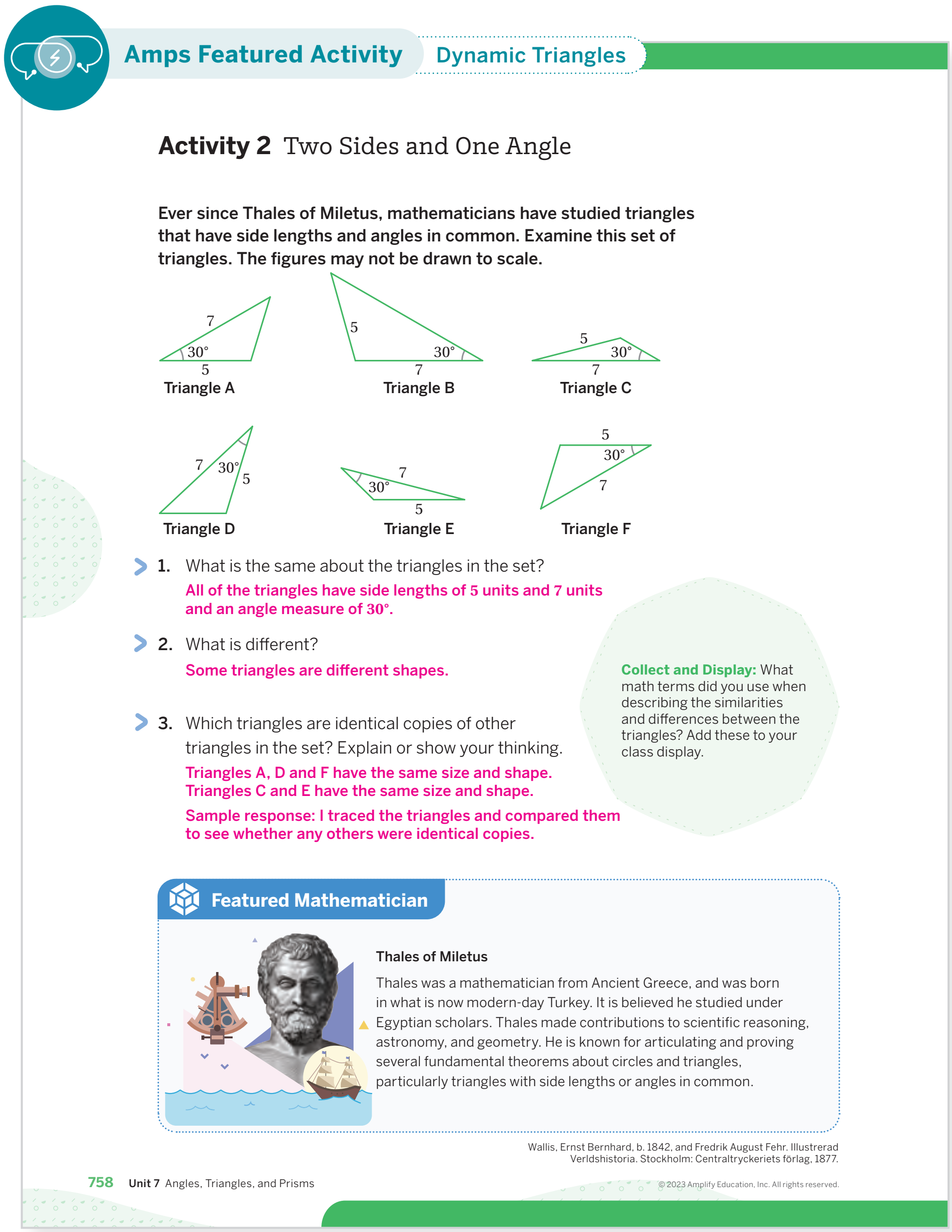
Extension: *Math Enrichment*

Have students complete the following problem:
Draw a parallelogram and draw one diagonal partitioning the parallelogram into two triangles. Determine whether the triangles formed are identical copies and explain your thinking. Yes; Sample response: The diagonal line segment separates the parallelogram into two triangles of equal size. The triangles have the same side lengths and the same angle measures.

Pairs | 10 min

Activity 2 Two Sides and One Angle

Students examine a set of triangles given two side lengths and one angle measurement to determine if any are identical copies, and how many unique triangles exist in the set.



Amps Featured Activity Dynamic Triangles

Activity 2 Two Sides and One Angle

Ever since Thales of Miletus, mathematicians have studied triangles that have side lengths and angles in common. Examine this set of triangles. The figures may not be drawn to scale.

Triangle A Triangle B Triangle C

Triangle D Triangle E Triangle F

1. What is the same about the triangles in the set?
   All of the triangles have side lengths of 5 units and 7 units and an angle measure of 30°.
2. What is different?
   Some triangles are different shapes.
3. Which triangles are identical copies of other triangles in the set? Explain or show your thinking.
   Triangles A, D and F have the same size and shape.
   Triangles C and E have the same size and shape.
   Sample response: I traced the triangles and compared them to see whether any others were identical copies.

Collect and Display: What math terms did you use when describing the similarities and differences between the triangles? Add these to your class display.

Featured Mathematician

Thales of Miletus

Thales was a mathematician from Ancient Greece, and was born in what is now modern-day Turkey. It is believed he studied under Egyptian scholars. Thales made contributions to scientific reasoning, astronomy, and geometry. He is known for articulating and proving several fundamental theorems about circles and triangles, particularly triangles with side lengths or angles in common.

Wallis, Ernst Bernhard, b. 1842, and Fredrik August Fehr. Illustrerad Verldshistoria. Stockholm: Centraltryckeriets förlag, 1877.

758 Unit 7 Angles, Triangles, and Prisms

1 Launch

Provide access to geometry toolkits. Demonstrate, using one of the triangles, how to determine which angle is across, or opposite, from each side.

2 Monitor

Help students get started by having them trace Triangle A to determine whether any other triangles are identical copies of Triangle A.

Look for points of confusion:

- **Thinking if two triangles both have a side length of 5 across from a 30° angle, then they are identical.** Point out that Triangles B and C both satisfy this criteria, but are not identical.

3 Connect

Display the set of triangles.

Highlight that two side lengths and one angle measure can produce many triangles. It is important to know the order of the measurements. In Triangles D and F, the known angle is *between* the two known side lengths and they are identical copies. In Triangles B and E, the known 30° angle is *across* from the side length of 5; however, they are *not* identical copies.

Have students share what they notice about the triangles that are identical copies and those that are not.

Ask:

- "When given one angle measurement and two side lengths, how do you know if they will form a unique triangle?" It will only be unique if the known angle is *between* the two side lengths.
- "When given one angle measurement and two side lengths, how do you know when identical copies will be made?" I need to know where the angle is. Even if the location of the angle is given, I cannot prove the triangles will be identical. (Refer to Triangles B and C for an example.)

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of Triangles A–F for students to cut out and examine individually. In Problem 3, students can group the triangles together that are identical copies of one another.

Math Language Development

MLR2: Collect and Display

During the Connect, collect vocabulary and diagrams students use to describe the similarities and differences between the triangles.

English Learners

Provide a graphic organizer for students to keep track of the different cases of side length and angle placement for the triangles.

Featured Mathematician

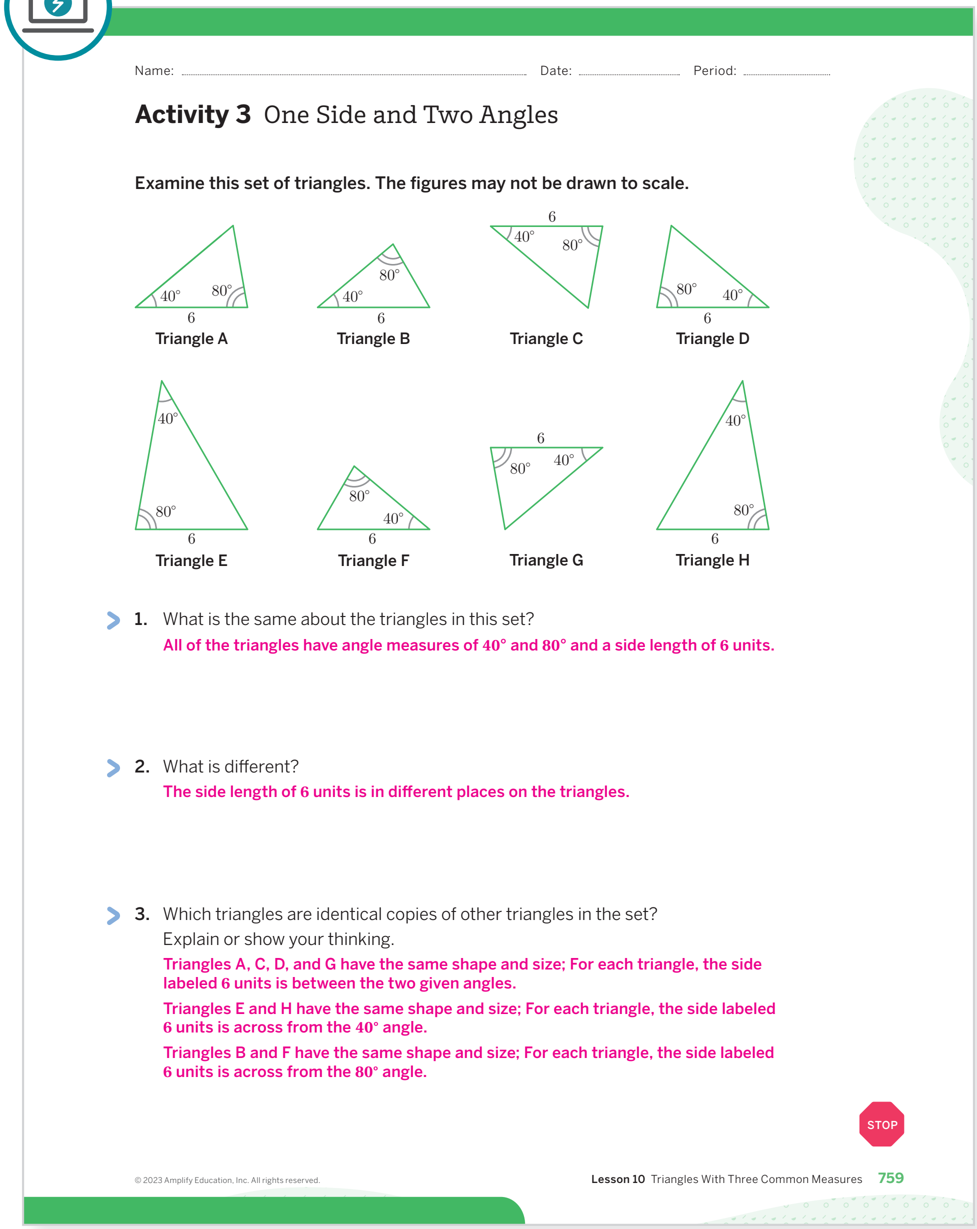
Thales of Miletus

Have students read about featured mathematician Thales of Miletus, a mathematician, astronomer and philosopher from Ancient Greece who used geometry to calculate the heights of pyramids and the distance of ships from the shore.

Pairs | 10 min

Activity 3 One Side and Two Angles

Students examine a set of triangles given one side length and two angle measurements to determine whether any are identical copies, and how many unique triangles exist in the set.



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Activity 3 One Side and Two Angles

Examine this set of triangles. The figures may not be drawn to scale.

1. What is the same about the triangles in this set?
   All of the triangles have angle measures of 40° and 80° and a side length of 6 units.

2. What is different?
   The side length of 6 units is in different places on the triangles.

3. Which triangles are identical copies of other triangles in the set? Explain or show your thinking.
   Triangles A, C, D, and G have the same shape and size; For each triangle, the side labeled 6 units is between the two given angles.
   Triangles E and H have the same shape and size; For each triangle, the side labeled 6 units is across from the 40° angle.
   Triangles B and F have the same shape and size; For each triangle, the side labeled 6 units is across from the 80° angle.

STOP

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Lesson 10 Triangles With Three Common Measures 759

1 Launch

Provide access to geometry toolkits. Tell students this activity is similar to Activity 2; however, different types of measurements are given here.

2 Monitor

Help students get started by having them trace Triangle A to determine whether any other triangles are identical copies of Triangle A.

Look for points of confusion:

- **Saying there are only two unique triangles.** Prompt students to notice where the 80° angle is located in comparison to the side length of 6 on the smaller triangles.

3 Connect

Display the set of triangles.

Have students share their responses to Problems 1–3, the strategies they used, and their thinking.

Highlight there are three unique triangles in the set. In each set, the known side is either (1) between the two known angles, as in Triangles A, C, D, and G, (2) across from the 40° angle, as in Triangles E and H, or (3) across from the 80° angle, as in Triangles B and F.

Ask:

- "What differences do you see between the triangles in Activities 2 and 3?" The given measures in Activity 3 were for two angles and one side. In Activity 2, they were for two sides and one angle.
- "What similarities do you see?" Three measures are known for each triangle in each activity, a combination of side lengths and angle measures.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of Triangles A–H for students to cut out and examine individually. In Problem 3, students can group the triangles together that are identical copies of one another.

Extension: *Math Enrichment*

Have students draw other sets of triangles with the same two angle measurements and the same side length. Ask, "Which ones are always identical copies?" Have students make a conjecture about what they discovered. Alternatively, have students measure the third angle in all the triangles and ask them what they notice.

Math Language Development

MLR3: Critique, Correct, Clarify

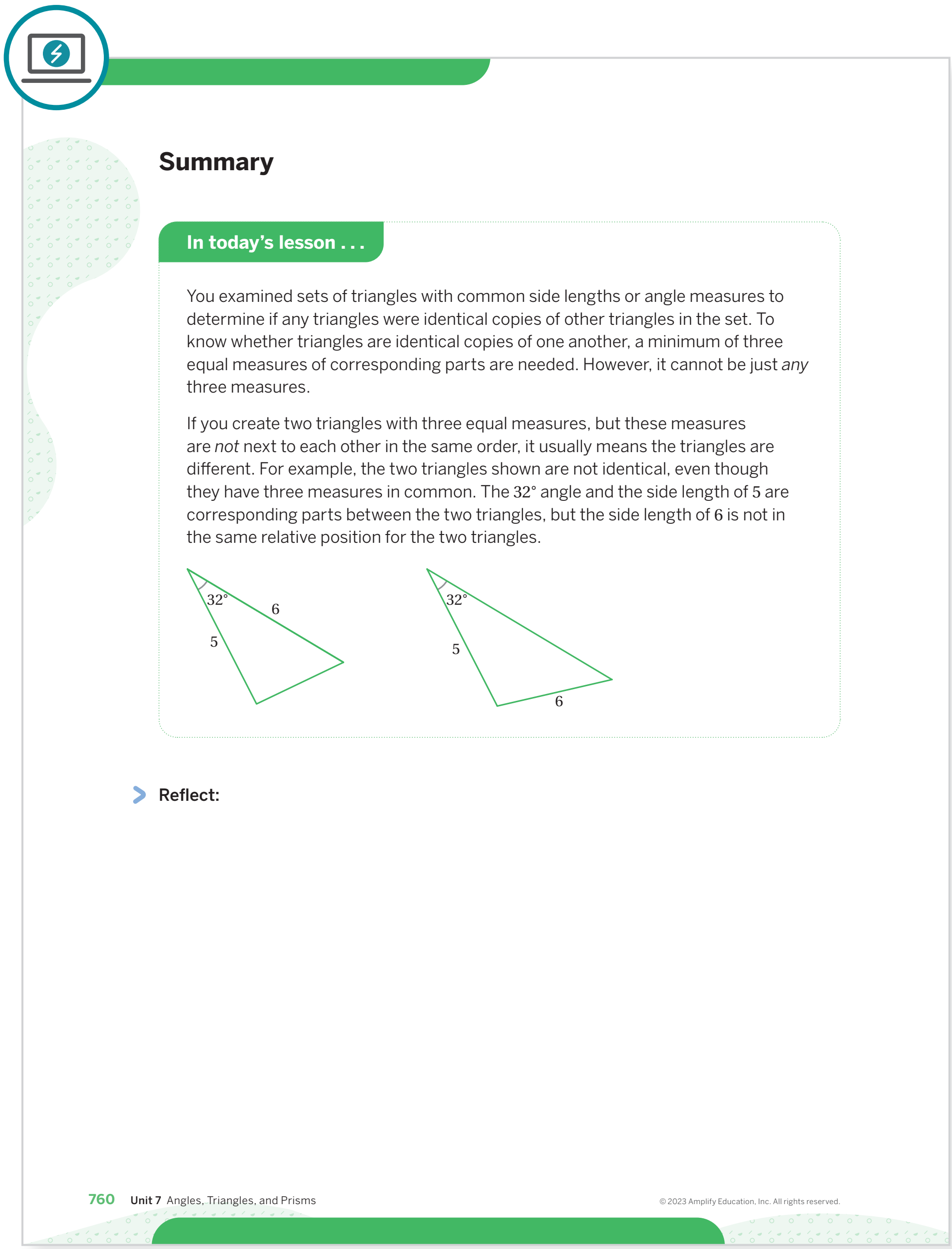
During the Connect, display two triangles that are not identical copies and an incorrect statement, such as "Triangles A and F are identical copies because they have the same measures shown." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Why?"
  Sample response: I disagree. The measures are not always shown in the same location. The side labeled 6 is between the two angles in Triangle A, but not between the two angles in Triangle F.
  Listen for understanding of the term *between* and clarify, as needed.
- ***Correct and Clarify:*** "Write a corrected statement that is now true. How do you know that the statement is now true?"

Whole Class | 5 min

Summary

Review and synthesize how triangles with the same three measurements may still not be identical copies. The order of the measures helps determine whether they are identical.



Summary

In today's lesson . . .

You examined sets of triangles with common side lengths or angle measures to determine if any triangles were identical copies of other triangles in the set. To know whether triangles are identical copies of one another, a minimum of three equal measures of corresponding parts are needed. However, it cannot be just *any* three measures.

If you create two triangles with three equal measures, but these measures are *not* next to each other in the same order, it usually means the triangles are different. For example, the two triangles shown are not identical, even though they have three measures in common. The 32° angle and the side length of 5 are corresponding parts between the two triangles, but the side length of 6 is not in the same relative position for the two triangles.

Reflect:

Synthesize

Display the responses to the question you asked in the Warm-up.

Have students share if they want to keep or change anything about their responses. Make sure students explain their thinking.

Highlight that there are many arrangements of triangle measurements and it is important to pay attention to not only the values themselves, but also the location of the measurements.

Ask:

- "Using your work in this lesson as an example, what does it mean for two triangles to be different?" They are not identical copies.
- "If you have a drawing of two triangles, how can you determine whether they are identical copies?" If by tracing one triangle and overlaying it on top of the other triangle so that it matches perfectly, then they are identical copies.
- "If you know three measurements are the same in a pair of triangles, are the triangles identical copies?" It depends on what measurements are known and if they are corresponding parts.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What tools or strategies did you find most useful in helping you to determine if two triangles were identical? Why?"
- "What characteristics did you consider when deciding if two triangles were identical?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing two triangles to determine whether they are identical copies.

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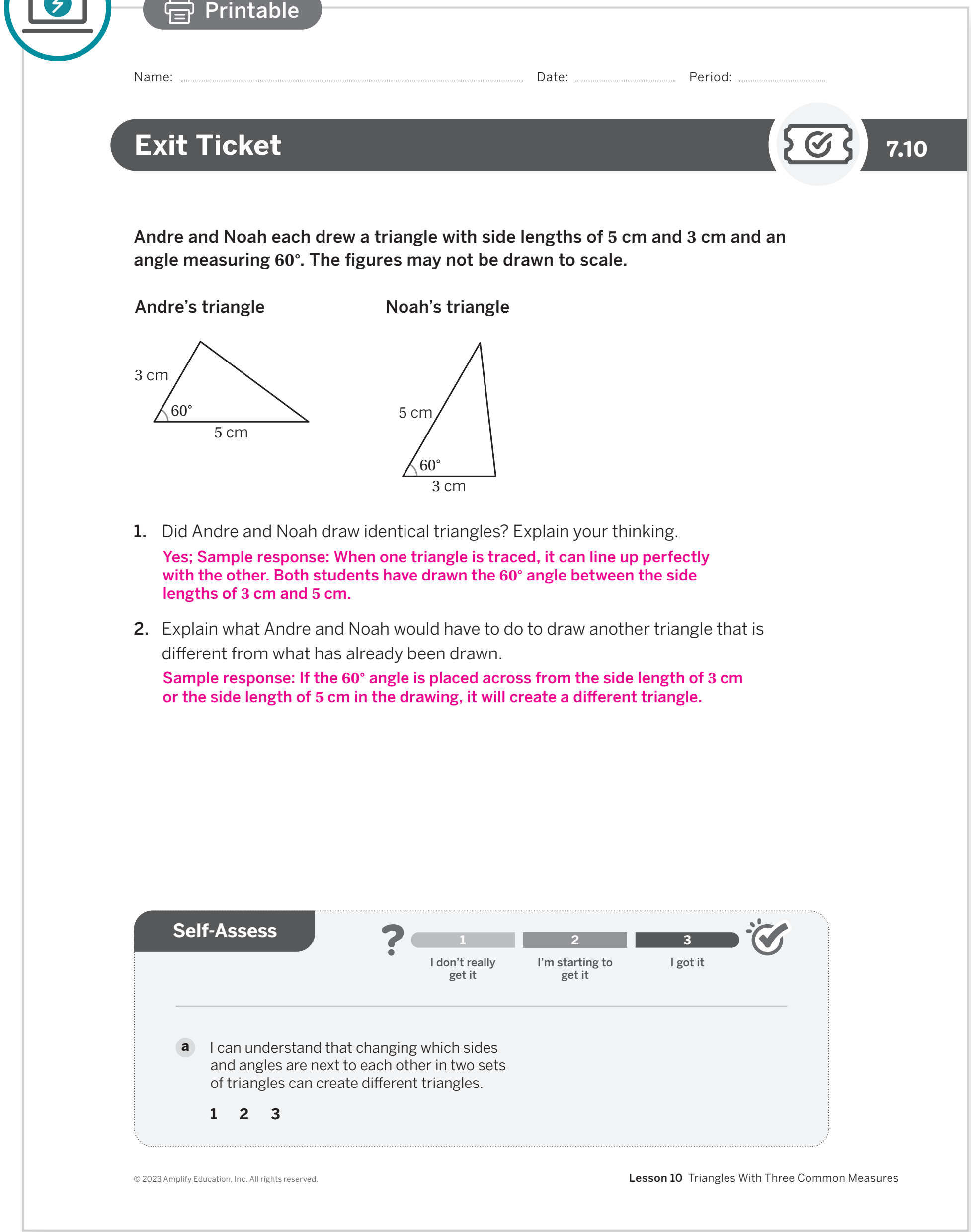

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Exit Ticket 7.10

Andre and Noah each drew a triangle with side lengths of 5 cm and 3 cm and an angle measuring 60°. The figures may not be drawn to scale.

Andre's triangle **Noah's triangle**

1. Did Andre and Noah draw identical triangles? Explain your thinking.
   Yes; Sample response: When one triangle is traced, it can line up perfectly with the other. Both students have drawn the 60° angle between the side lengths of 3 cm and 5 cm.
2. Explain what Andre and Noah would have to do to draw another triangle that is different from what has already been drawn.
   Sample response: If the 60° angle is placed across from the side length of 3 cm or the side length of 5 cm in the drawing, it will create a different triangle.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can understand that changing which sides and angles are next to each other in two sets of triangles can create different triangles.
1 2 3

Lesson 10 Triangles With Three Common Measures

Success looks like . . .

- **Language Goal:** Describing, comparing, and contrasting triangles that share three common measures of angles or sides. **(Speaking and Listening, Writing)**
  - » Explaining how to make a different triangle with the same three common measures in Problem 2.
- **Language Goal:** Justifying (using multiple representations) whether triangles are identical copies (unique) or are different triangles. **(Speaking and Listening, Writing)**
  - » Explaining why Andre's and Noah's triangles are identical in Problem 1.
- **Language Goal:** Recognizing that examining which side lengths and angle measures are adjacent can help determine whether triangles are identical copies. **(Speaking and Listening, Reading and Writing)**
  - » Comparing side lengths and angles of Andre's and Noah's triangles to determine whether the triangles are identical in Problem 1.

Suggested next steps

If students complete Problem 1 incorrectly, consider:

- Reviewing that the term *identical copies* means the triangles are the same shape and size, but do not have to be in the same location or have the same orientation.
- Using tracing paper to trace one triangle, and then overlaying it on top of the other triangle to see how they match up.
- Assigning Practice Problem 1.

If students complete Problem 2 incorrectly, consider:

- Assigning Practice Problems 2 and 3.
- Providing additional support during Lesson 11.

Professional Learning

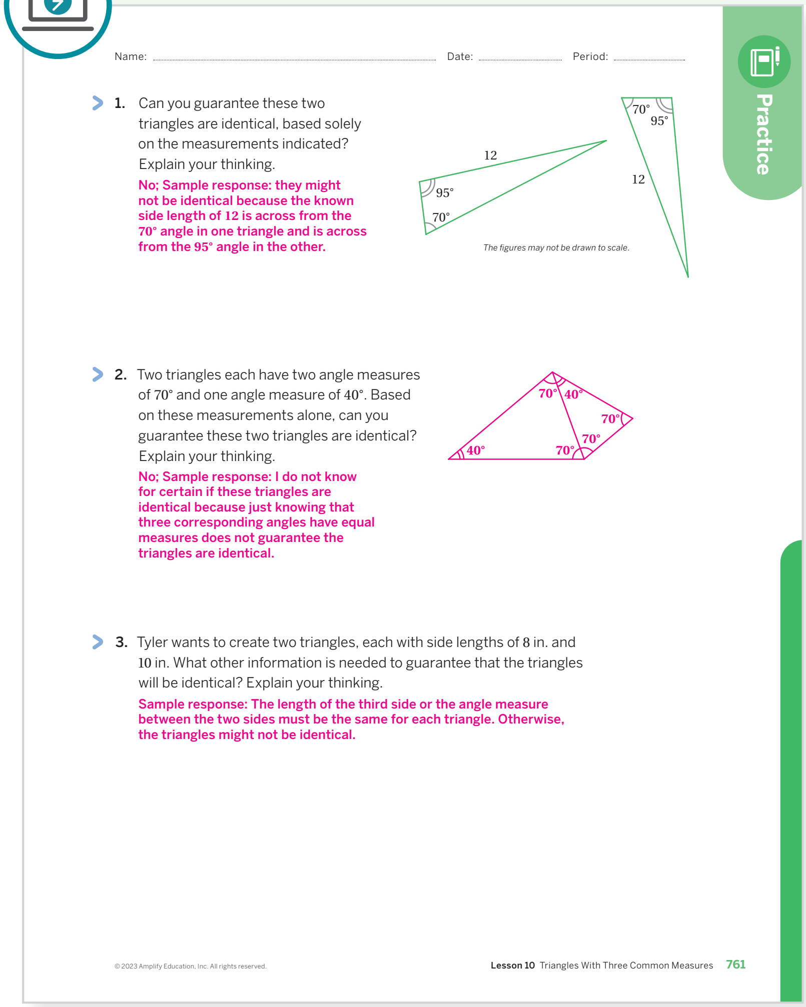
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students gotten better at making conjectures and justifying their conclusions?
- This lesson asked students to notice when conditions determine a unique triangle or more than one triangle. Where in your students' work today did you see or hear evidence of them doing this? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Can you guarantee these two triangles are identical, based solely on the measurements indicated? Explain your thinking.

No; Sample response: they might not be identical because the known side length of 12 is across from the 70° angle in one triangle and is across from the 95° angle in the other.

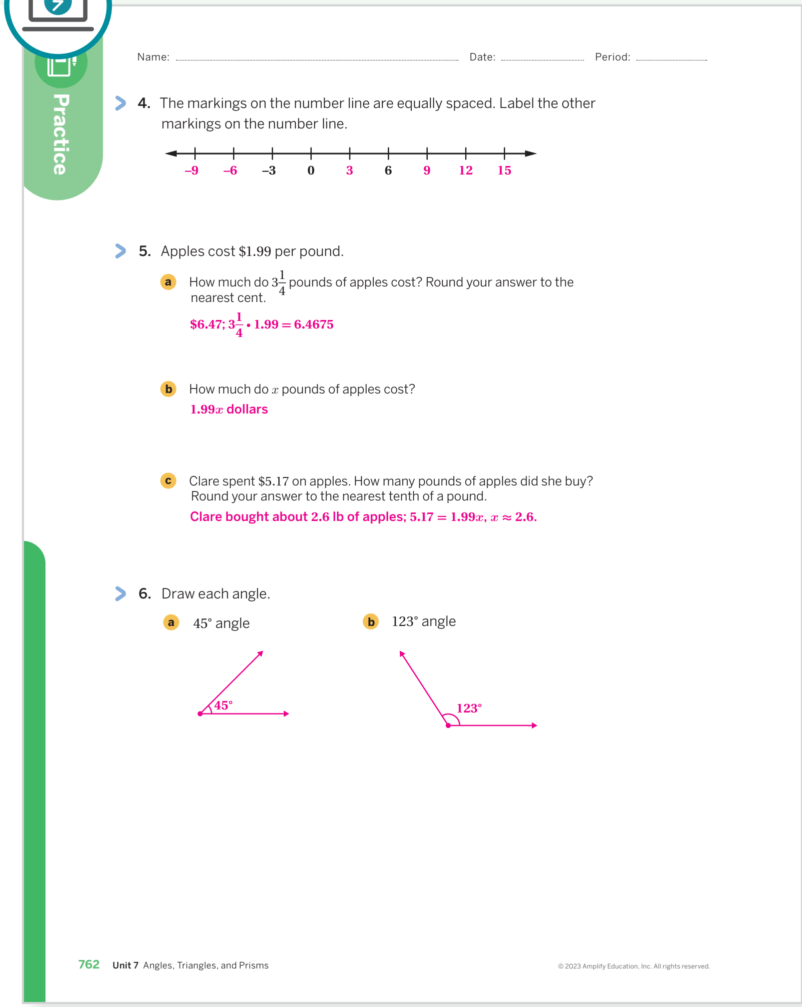
The figures may not be drawn to scale.

2. Two triangles each have two angle measures of 70° and one angle measure of 40°. Based on these measurements alone, can you guarantee these two triangles are identical? Explain your thinking.

No; Sample response: I do not know for certain if these triangles are identical because just knowing that three corresponding angles have equal measures does not guarantee the triangles are identical.

3. Tyler wants to create two triangles, each with side lengths of 8 in. and 10 in. What other information is needed to guarantee that the triangles will be identical? Explain your thinking.

Sample response: The length of the third side or the angle measure between the two sides must be the same for each triangle. Otherwise, the triangles might not be identical.



Practice

Name: Date: Period:

4. The markings on the number line are equally spaced. Label the other markings on the number line.

5. Apples cost $1.99 per pound.

a. How much do $3\frac{1}{4}$ pounds of apples cost? Round your answer to the nearest cent.

$\$6.47; 3\frac{1}{4} \cdot 1.99 = 6.4675$

b. How much do x pounds of apples cost?

$1.99x$ dollars

c. Clare spent $5.17 on apples. How many pounds of apples did she buy? Round your answer to the nearest tenth of a pound.

Clare bought about 2.6 lb of apples; $5.17 = 1.99x$, $x \approx 2.6$.

6. Draw each angle.

a. 45° angle

b. 123° angle

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 10	1
	5	Unit 2 Lesson 8	1
Formative ⚡	6	Unit 7 Lesson 11	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 11**

Drawing Triangles (Part 1)

Let's see how many different triangles we can draw with certain measurements.

Focus

Goals

1. **Language Goal:** Draw triangles with two given angle measures and one side length, and describe how many different triangles could be drawn with the given conditions. **(Speaking and Listening)**
2. **Language Goal:** Use drawings to justify whether two given angle measures and one side length determine one unique triangle. **(Writing)**

Coherence

Today

Students draw their own triangles given certain measurements for two given angles and a side length. They gain experience using various tools drawing triangles with given conditions to help them understand that sometimes there is only one possible triangle, more than one triangle, or no triangle. Students must attend to the structure of each triangle to ensure uniqueness.

Previously

In Lesson 10, students were given sets of triangles and noticed they shared angle and side measures, and that sometimes there was more than one unique triangle which met the same conditions.

Coming Soon

In Lesson 12, students will continue to draw their own triangles, but will focus on given conditions consisting of two side lengths and one angle measure.

Rigor

- Students use a variety of tools to construct triangles with given characteristics to develop **fluency**.
- Students **apply** their understanding of triangle constructions to determine and justify if two triangles are copies or are unique.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	12 min	12 min	5 min	7 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: tracing paper, protractors, rulers

Math Language Development

Review words

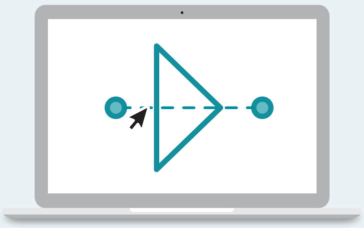
- *compass*
- *corresponding*

Amps | Featured Activity

Activities 1 and 2

Digital Constructions

Students create triangles with given conditions using digital edges and angles.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack the self-discipline to determine additional triangles after drawing one in Activity 2. Ask students to identify how the patterns they have observed in the structure of triangles can help them manage their progress, determining whether or not they can do more.

Modifications to Pacing

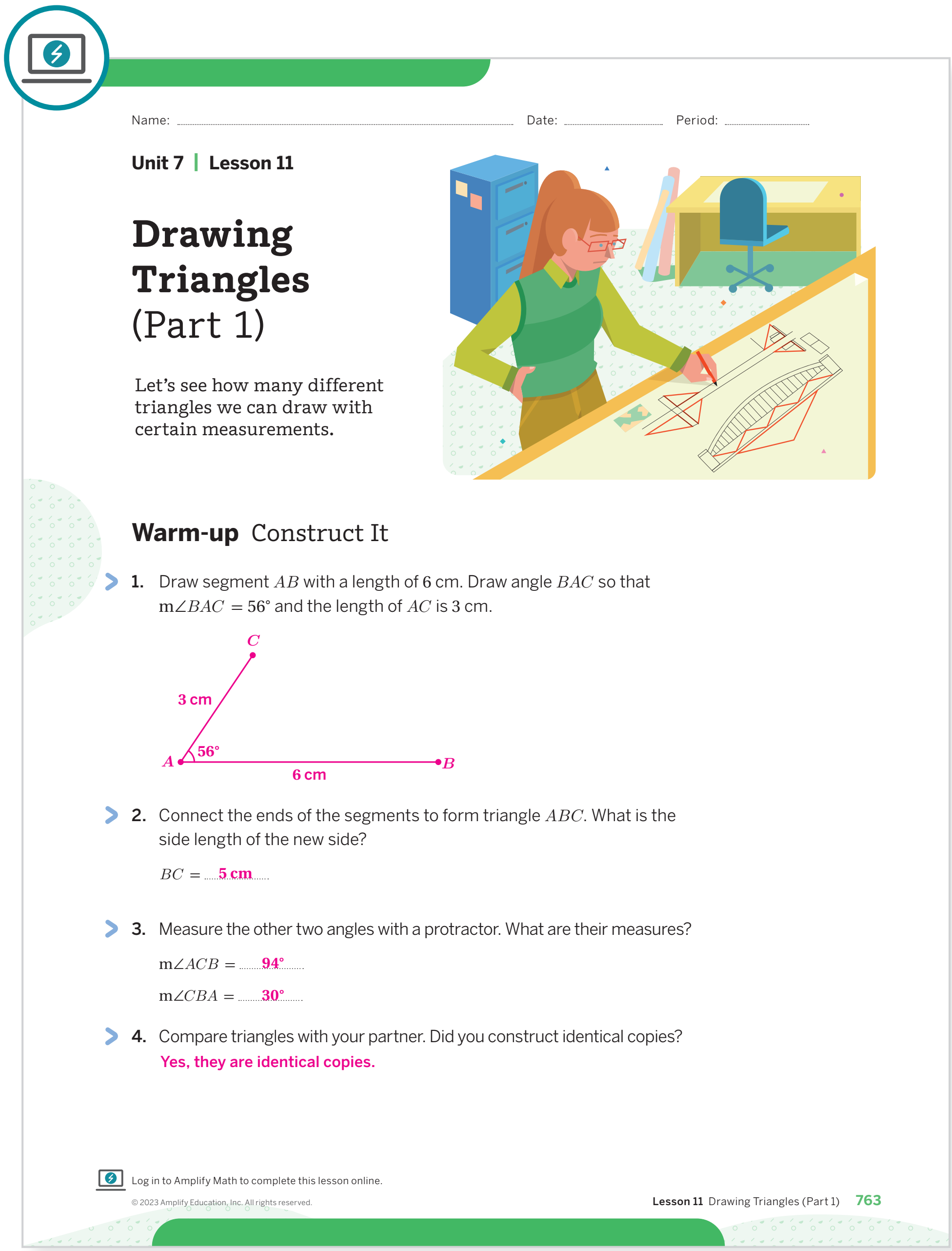
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students work in groups of three. Each student should be responsible for one set in Problem 1, then they should share their findings with their group.

Pairs | 10 min

Warm-up Construct It

Students create a triangle with given conditions and measure the unknown angles and side to determine whether the triangle is unique.



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Unit 7 | Lesson 11

Drawing Triangles
(Part 1)

Let's see how many different triangles we can draw with certain measurements.

Warm-up Construct It

1. Draw segment AB with a length of 6 cm. Draw angle BAC so that $m\angle BAC = 56°$ and the length of AC is 3 cm.

2. Connect the ends of the segments to form triangle ABC. What is the side length of the new side?

$BC =$ 5 cm

3. Measure the other two angles with a protractor. What are their measures?

$m\angle ACB =$ 94°

$m\angle CBA =$ 30°

4. Compare triangles with your partner. Did you construct identical copies?
Yes, they are identical copies.

Log in to Amplify Math to complete this lesson online.

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Lesson 11 Drawing Triangles (Part 1) 763

1 Launch

Provide access to the geometry toolkits.

2 Monitor

Help students get started by explaining or demonstrating how to draw a triangle with the given conditions.

Look for points of confusion:

- **Difficulty drawing an angle.** Remind students, or demonstrate, how to draw one side first, create the angle, and then draw the other side.
- **Labeling the angle incorrectly**. Remind students that the vertex of the angle is the middle letter in the angle's name.
- **Struggling to make the sides the correct length.** After the angle is drawn, have students measure the appropriate lengths and place points at each end. If their line extends beyond, reassure them this is fine, as long they know where the sides are supposed to end.

Look for productive strategies:

- Drawing and measuring accurately. Note students who do this and have them support and help others.

3 Connect

Display students' drawings.

Have pairs of students share whether or not their triangles were identical copies. Have them compare their triangles and measurements with the ones displayed.

Highlight that everyone should have drawn identical copies because the given conditions were in an arrangement that creates a unique triangle.

Ask, "Why do you think everyone drew an identical triangle?" The arrangement of the given conditions were such that only one triangle was possible.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Provide students with the side length of 6 cm already drawn. Have them create the 56° angle and side length of 3 cm.

Extension: ***Math Enrichment***

After posing the Ask question in the Connect section, ask students whether they can draw other given conditions that might result in more than one triangle being possible.

Power-up

To power up students' ability to draw a given angle measurement, have students complete:

In order to construct an angle, first draw a line segment on your paper, then line up the center of your protractor with the endpoint of the line segment. Make a mark on your paper where the indicated angle measurement is, then use your ruler to connect it to the endpoint.

Use your ruler and protractor to construct a 75° angle. Answers may vary.

Use: Before the Warm-up.

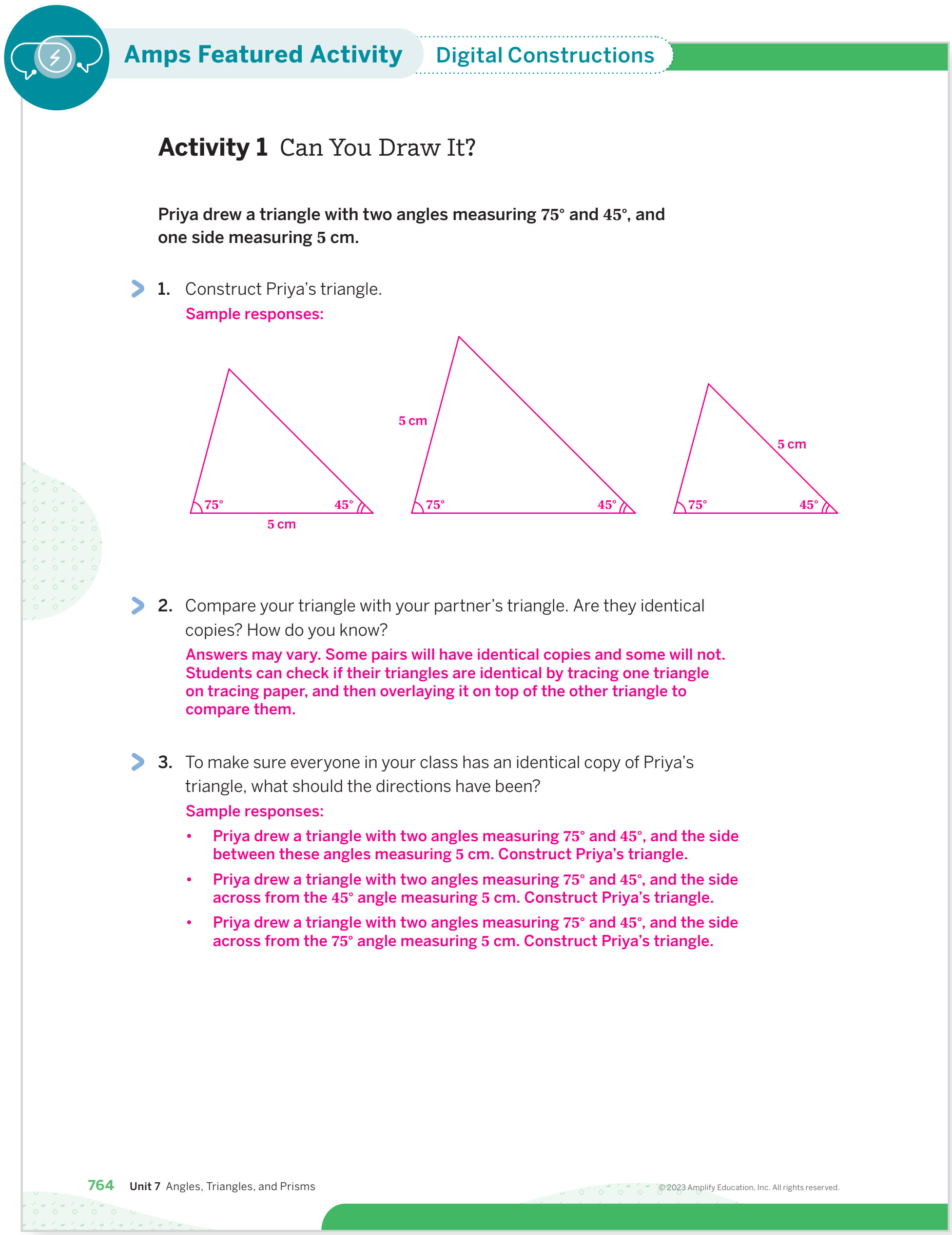
Informed by: Performance on Lesson 10, Practice Problem 6.

Pairs | 12 min

Activity 1 Can You Draw It?

Students reconstruct a triangle with given conditions to determine whether it is unique.

Amps Featured Activity Digital Constructions



Activity 1 Can You Draw It?

Priya drew a triangle with two angles measuring 75° and 45°, and one side measuring 5 cm.

1. Construct Priya's triangle.

Sample responses:

2. Compare your triangle with your partner's triangle. Are they identical copies? How do you know?

Answers may vary. Some pairs will have identical copies and some will not. Students can check if their triangles are identical by tracing one triangle on tracing paper, and then overlaying it on top of the other triangle to compare them.

3. To make sure everyone in your class has an identical copy of Priya's triangle, what should the directions have been?

Sample responses:

- **Priya drew a triangle with two angles measuring 75° and 45°, and the side between these angles measuring 5 cm. Construct Priya's triangle.**
- **Priya drew a triangle with two angles measuring 75° and 45°, and the side across from the 45° angle measuring 5 cm. Construct Priya's triangle.**
- **Priya drew a triangle with two angles measuring 75° and 45°, and the side across from the 75° angle measuring 5 cm. Construct Priya's triangle.**

764 **Unit 7** Angles, Triangles, and Prisms ©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Let students know that although they are working in pairs, each of them should draw their own triangles with supplies from their geometry toolkits.

2 Monitor

Help students get started by having them draw the 5 cm side just above the directions for Problem 2 to provide room for the rest of their triangle.

Look for points of confusion:

- **Difficulty drawing a triangle.** Have them start by drawing the 5 cm side first, followed by the 75° angle.
- **Drawing a triangle that does not fit on the page.** Tell students it is okay if their triangle covers part of the directions. Provide a separate sheet of paper if needed.

3 Connect

Ask, "Did anybody draw a triangle identical to the one drawn by their partner?"

Display students' triangles. Have the three sample responses available in case one of these was not drawn.

Have students share what they notice about the different triangles.

Highlight there are three possible triangles given these conditions. However, if triangles have the same arrangement of these conditions, they will be identical. Use tracing paper to quickly show the identical copies. Review Problem 3, referencing students' triangles or the sample triangles shown in the answer key noted on the Student Edition page here.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create triangles with given conditions using an interactive tool.

Accessibility: *Vary Demands to Optimize Challenge*

Consider one of these alternative approaches to this activity:

- Demonstrating how to construct a possible triangle for Problem 1. Having students describe how to construct other possible triangles.
- Providing copies of triangles that may or may not meet the given conditions. Have students measure sides and angles to determine which triangles meet the given conditions.

Math Language Development

MLR8: Discussion Supports

Before students complete Problem 3, use the ***Think-Pair-Share*** routine to encourage them to discuss with their partner what the directions might be for creating identical triangles. Based on their discussions, have students record the directions for creating identical triangles.

English Learners

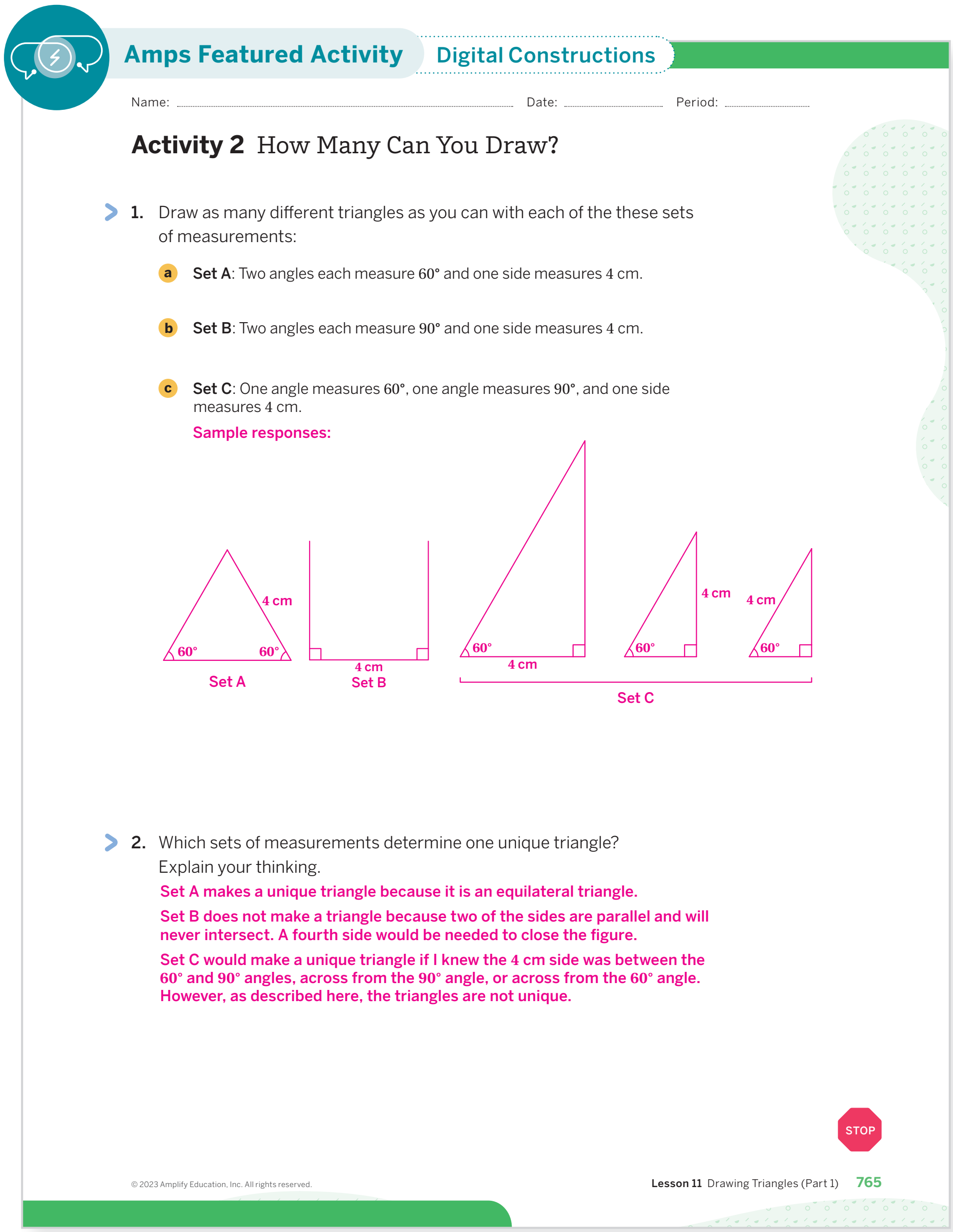
Provide sentence frames, such as:

- "I noticed ___, so I . . ."
- "This triangle is/is not identical because . . ."

Pairs | 12 min

Activity 2 How Many Can You Draw?

Students draw triangles given certain conditions to determine whether they are unique.



Amps Featured Activity Digital Constructions

Name: ______________________ Date: __________ Period: __________

Activity 2 How Many Can You Draw?

1. Draw as many different triangles as you can with each of the these sets of measurements:
   - a **Set A**: Two angles each measure 60° and one side measures 4 cm.
   - b **Set B**: Two angles each measure 90° and one side measures 4 cm.
   - c **Set C**: One angle measures 60°, one angle measures 90°, and one side measures 4 cm.

   Sample responses:

2. Which sets of measurements determine one unique triangle? Explain your thinking.

   Set A makes a unique triangle because it is an equilateral triangle.

   Set B does not make a triangle because two of the sides are parallel and will never intersect. A fourth side would be needed to close the figure.

   Set C would make a unique triangle if I knew the 4 cm side was between the 60° and 90° angles, across from the 90° angle, or across from the 60° angle. However, as described here, the triangles are not unique.

STOP

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Lesson 11 Drawing Triangles (Part 1) 765

1 Launch

Provide access to the geometry toolkits.

2 Monitor

Help students get started by helping them draw the 60° angle first.

Look for points of confusion:

- **Thinking the criteria in Set A produces different triangles because this same arrangement produced a different triangle in Lesson 10, Activity 3.** Have students try to draw a different triangle given these conditions. This is a special case, because it is an equilateral triangle.
- **Confusing which of their drawings belongs to which set of measurements.** Have students draw Set B on the front of a sheet of paper and Set C on the back.
- **Saying the conditions in set C produce a unique triangle.** They may assume the known side length is between the two angles. Remind them of Priya's triangle from Activity 1.

3 Connect

Have students share their drawings and reasoning about the uniqueness of each problem. Discuss strategies students used to think about other triangles that might fit the conditions.

Ask:

- "Which conditions produced a unique triangle? Why?" Set A because it is an equilateral triangle.
- "Which conditions produced more than one triangle?" Set C
- "Which conditions did not produce a triangle? Why?" Set B; because the two sides are parallel and will never intersect.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create triangles with given conditions using an interactive tool.

Accessibility: *Vary Demands to Optimize Challenge*

Consider one of these alternative approaches to this activity:

- Demonstrating how to draw the triangles for each set.
- Providing copies of triangles for each set. Have students determine which triangles meet the given conditions.

Math Language Development

MLR7: Compare and Connect

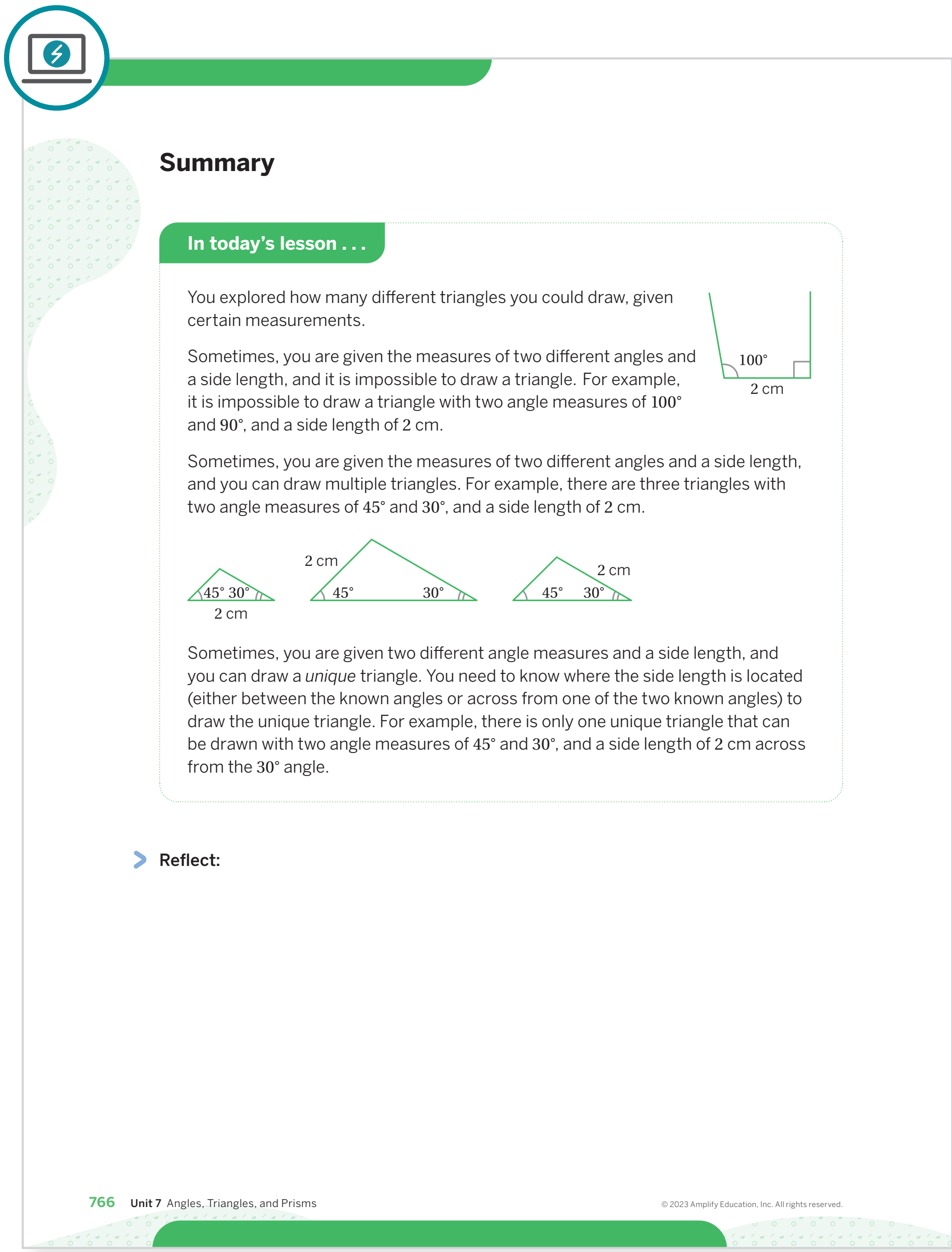
During the Connect, ask students to consider what is the same and what is different about the conditions given in each set and connect this information to whether each set produced a *unique triangle*, *more than one triangle*, or *no triangle*.

English Learners

Encourage students to use their drawings and/or hand gestures to illustrate whether the conditions in each set produced a unique triangle. Connect the term *unique* to "one triangle."

Summary

Review and synthesize why the number of unique triangles that can be formed given two angle measurements and one side length depends on the arrangement of the given conditions.



Summary

In today's lesson . . .

You explored how many different triangles you could draw, given certain measurements.

Sometimes, you are given the measures of two different angles and a side length, and it is impossible to draw a triangle. For example, it is impossible to draw a triangle with two angle measures of 100° and 90°, and a side length of 2 cm.

Sometimes, you are given the measures of two different angles and a side length, and you can draw multiple triangles. For example, there are three triangles with two angle measures of 45° and 30°, and a side length of 2 cm.

Sometimes, you are given two different angle measures and a side length, and you can draw a *unique* triangle. You need to know where the side length is located (either between the known angles or across from one of the two known angles) to draw the unique triangle. For example, there is only one unique triangle that can be drawn with two angle measures of 45° and 30°, and a side length of 2 cm across from the 30° angle.

Reflect:

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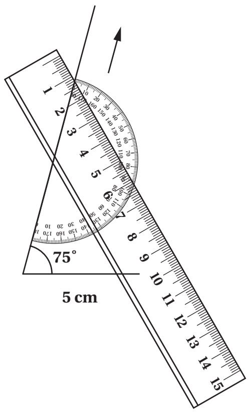
Synthesize

Have students share a set of conditions of two angle measurements and one side length which will produce one unique triangle, many triangles, and no triangle.

Highlight the strategies students use to draw their triangles. Demonstrate the strategy of drawing the side length and one angle, and then using the ruler and protractor to construct the other angle at the other end of the known line segment.



Ask:

- "If given a side length and two angle measures, what are the possible outcomes for the number of triangles?" One unique triangle, many triangles, or no triangle.
- "If you are given a side length and two angles, what would you do to make different triangles?" For one triangle, draw the two angles on either end of the line segment. For another triangle, draw a known angle across from the known side length. For the third triangle, draw the known side length across from the other known angle.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did you find most challenging in constructing triangles? How did you overcome these challenges?"
- "Which tools or strategies did you find most useful when constructing your triangles?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by drawing a triangle with two given angle measurements and one side length, and then determining whether it is unique.

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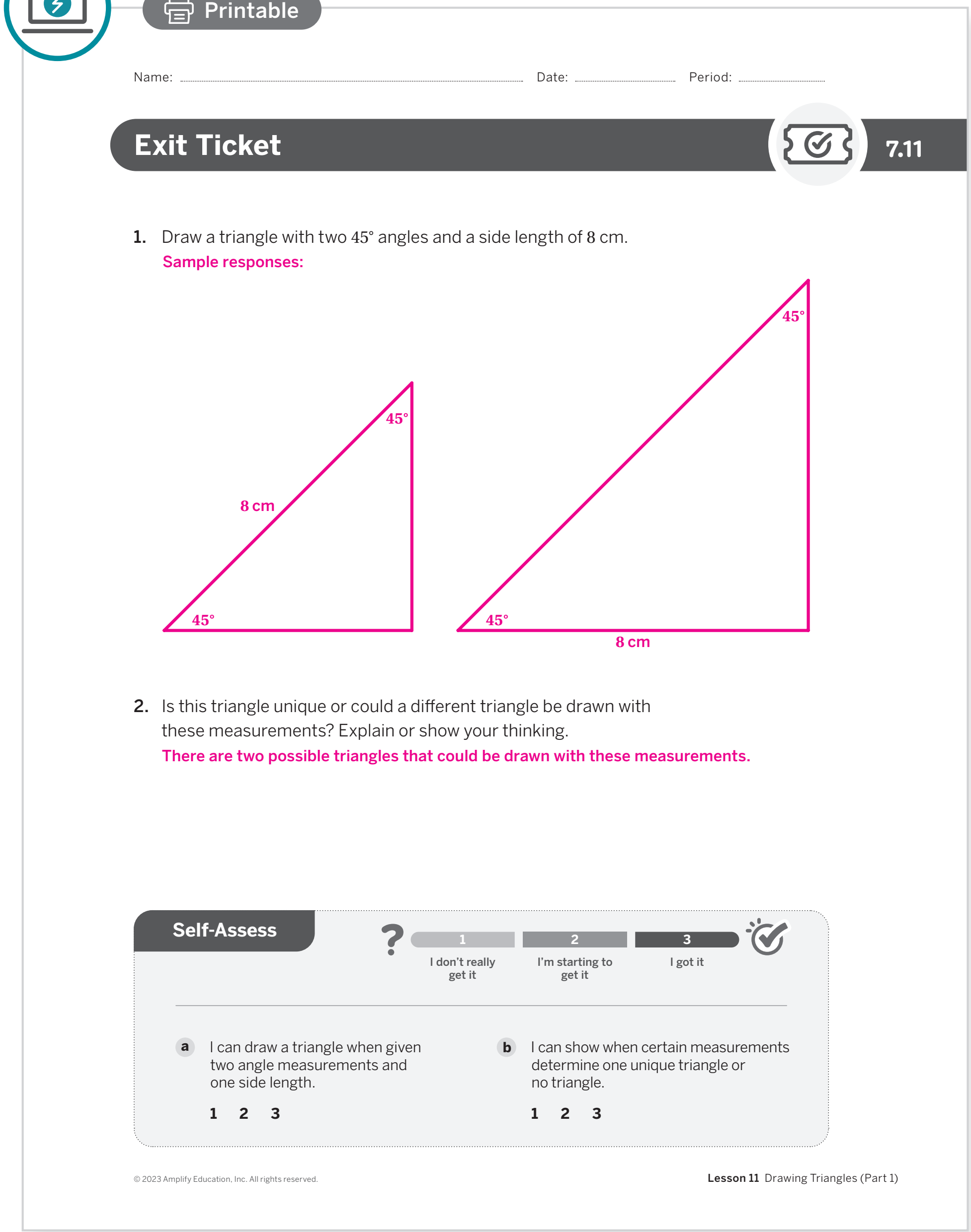


Name: ______________________ Date: __________ Period: __________

Exit Ticket 7.11

1. Draw a triangle with two 45° angles and a side length of 8 cm.
   Sample responses:

2. Is this triangle unique or could a different triangle be drawn with these measurements? Explain or show your thinking.
   There are two possible triangles that could be drawn with these measurements.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can draw a triangle when given two angle measurements and one side length.
1 2 3

b I can show when certain measurements determine one unique triangle or no triangle.
1 2 3

Lesson 11 Drawing Triangles (Part 1)

Success looks like . . .

- **Language Goal:** Drawing triangles with two given angle measures and one side length, and describing how many different triangles could be drawn with the given conditions. **(Speaking and Listening)**
  - » Explaining whether two different triangles could be drawn with two 45° angles and a side length of 8 cm in Problem 2.
- **Language Goal:** Using drawings to justify whether two given angle measures and one side length determine one unique triangle. **(Writing)**

Suggested next steps

If students were unable to draw a triangle in Problem 1, consider:

- Reviewing how to use a protractor and draw a triangle.
- Assigning Practice Problem 2.

If students were unable to determine whether the triangle was unique in Problem 2, consider:

- Reviewing the conditions resulting in a unique triangle.
- Assigning Practice Problem 1.

Professional Learning

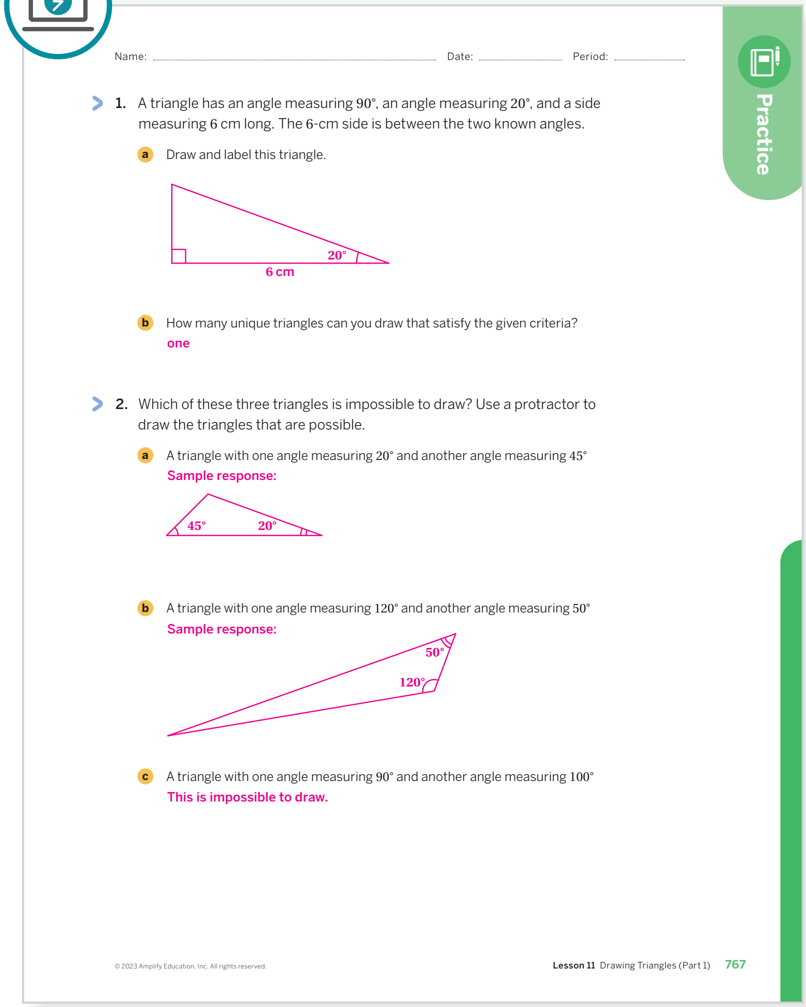
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did you see in the way some students approached constructing triangles that you would like other students to try?
- Did anything unexpected happen during Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. A triangle has an angle measuring 90°, an angle measuring 20°, and a side measuring 6 cm long. The 6-cm side is between the two known angles.

   a. Draw and label this triangle.

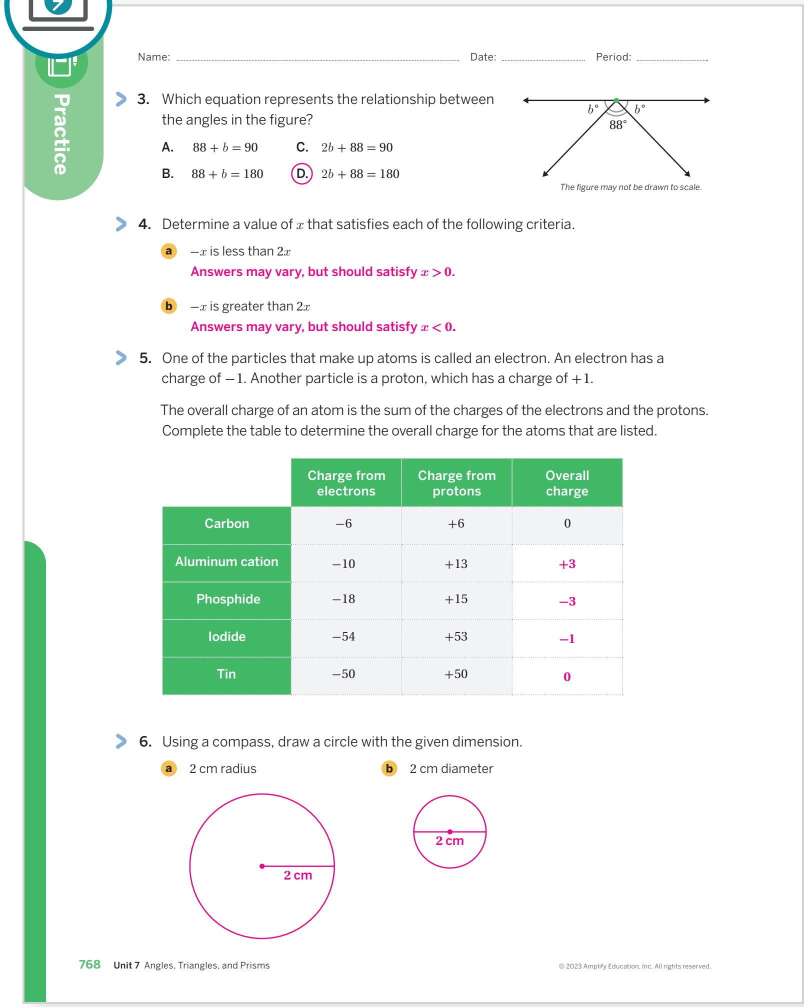
   b. How many unique triangles can you draw that satisfy the given criteria?
   one

2. Which of these three triangles is impossible to draw? Use a protractor to draw the triangles that are possible.

   a. A triangle with one angle measuring 20° and another angle measuring 45°
   Sample response:

   b. A triangle with one angle measuring 120° and another angle measuring 50°
   Sample response:

   c. A triangle with one angle measuring 90° and another angle measuring 100°
   This is impossible to draw.



Name: ________ Date: ________ Period: ________

Practice

3. Which equation represents the relationship between the angles in the figure?

The figure may not be drawn to scale.

   A. $88 + b = 90$
   B. $88 + b = 180$
   C. $2b + 88 = 90$
   [✓ D.] $2b + 88 = 180$

4. Determine a value of x that satisfies each of the following criteria.

   a. $-x$ is less than $2x$
   Answers may vary, but should satisfy $x > 0$.

   b. $-x$ is greater than $2x$
   Answers may vary, but should satisfy $x < 0$.

5. One of the particles that make up atoms is called an electron. An electron has a charge of -1. Another particle is a proton, which has a charge of $+1$.

   The overall charge of an atom is the sum of the charges of the electrons and the protons. Complete the table to determine the overall charge for the atoms that are listed.

	Charge from electrons	Charge from protons	Overall charge
Carbon	-6	$+6$	0
Aluminum cation	-10	$+13$	**+3**
Phosphide	-18	$+15$	**−3**
Iodide	-54	$+53$	**−1**
Tin	-50	$+50$	**0**

6. Using a compass, draw a circle with the given dimension.

   a. 2 cm radius
   b. 2 cm diameter

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 7 Lesson 4	1
	4	Unit 5 Lesson 12	1
	5	Unit 5 Lesson 4	1
Formative ⚡	6	Unit 7 Lesson 12	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

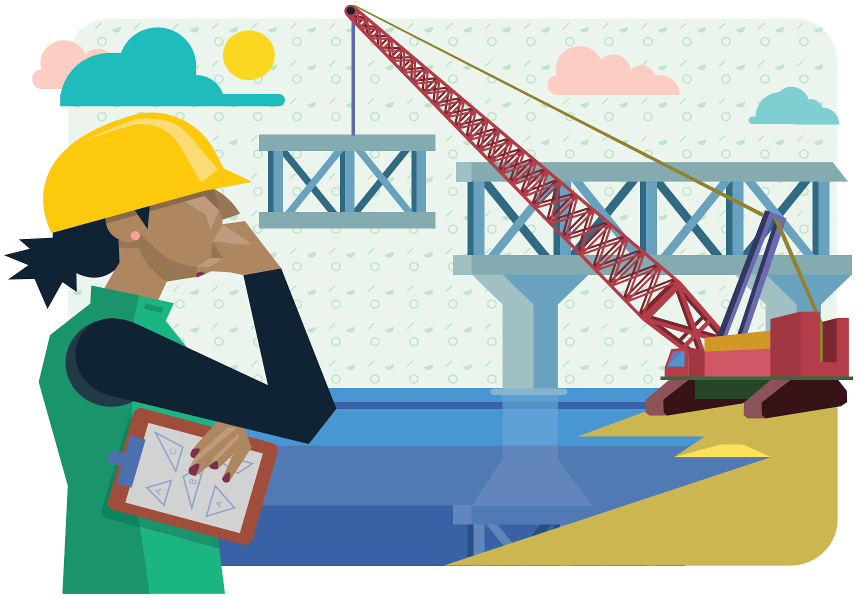
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 12**

Drawing Triangles (Part 2)

Let's draw some more triangles.



Focus

Goals

1. **Language Goal:** Draw triangles with three given angles and triangles with two given side lengths and one angle measure, and describe how many different triangles could be drawn with the given conditions. **(Speaking and Listening)**
2. **Language Goal:** Use drawings to justify whether two given side lengths and one angle measure can determine one unique triangle. **(Writing)**

Coherence

Today

Students draw their own triangles given measurements of one given angle and two side lengths, or three angles. They gain experience using various tools (i.e., compass and protractor) when drawing triangles with given conditions to help them see that sometimes there is only one possible triangle, more than one triangle, or no triangle. Students must pay attention to the structure of each triangle to ensure uniqueness.

Previously

In Lesson 11, students drew triangles given two angle measures and one side length to determine if the conditions resulted in one unique triangle, many triangles, or no triangles.

Coming Soon

In Lesson 13, students will determine the shape of a cross section of a sliced prism. In later lessons, students will find the volume and surface area of right prisms.

Rigor

- Students **apply** their understanding of triangle constructions to determine the number of unique triangles that can be formed given certain conditions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	12 min	15 min	5 min	7 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (as needed)
- geometry toolkits: compasses, tracing paper, protractors, rulers

Math Language Development

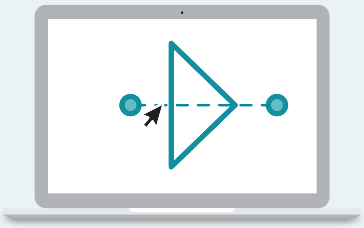
Review words

- *compass*
- *corresponding*

Amps | Featured Activity

Activity 2
Digital Constructions

As students construct triangles with specific conditions, this digital tool allows them to shorten the time between their first, subsequent, and final drafts.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack the self-discipline to determine additional triangles after drawing one in Activity 2. Have students describe how the tools they have can help them determine whether or not other triangles can be drawn or if they have found them all.

Modifications to Pacing

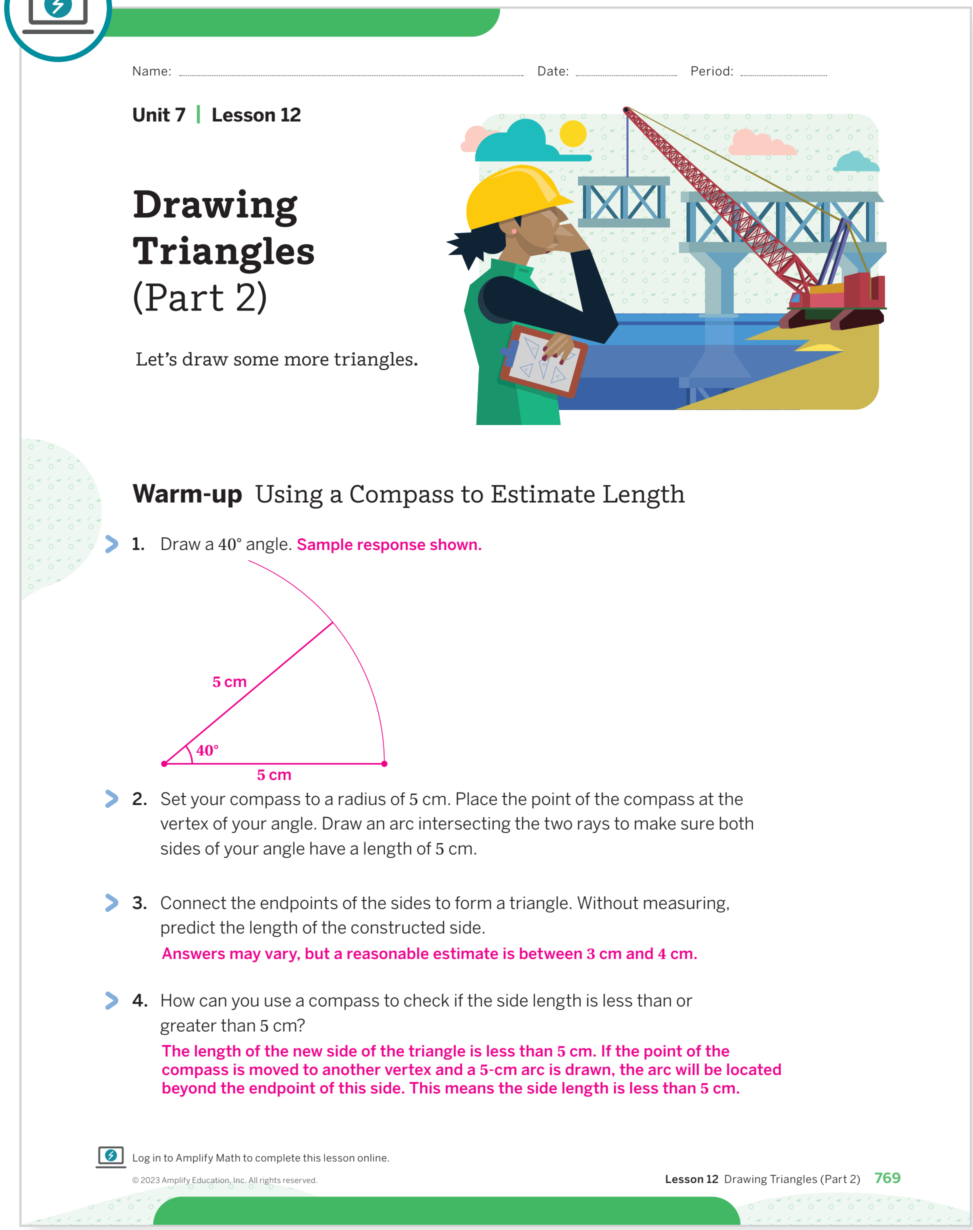
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up**. Instead, demonstrate how to set the compass to a specific length during the Activity 1 Launch.
- In **Activity 2**, assign each pair of students two of the four sets. Be careful to ensure all sets are being completed for the whole-class discussion.

Pairs | 7 min

Warm-up Using a Compass to Estimate Length

Students draw an angle using a compass to realize a compass can be used for more than drawing a circle, such as drawing equivalent side lengths.



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Unit 7 | Lesson 12

Drawing Triangles (Part 2)

Let's draw some more triangles.

Warm-up Using a Compass to Estimate Length

1. Draw a 40° angle. **Sample response shown.**

2. Set your compass to a radius of 5 cm. Place the point of the compass at the vertex of your angle. Draw an arc intersecting the two rays to make sure both sides of your angle have a length of 5 cm.

3. Connect the endpoints of the sides to form a triangle. Without measuring, predict the length of the constructed side.
Answers may vary, but a reasonable estimate is between 3 cm and 4 cm.

4. How can you use a compass to check if the side length is less than or greater than 5 cm?
The length of the new side of the triangle is less than 5 cm. If the point of the compass is moved to another vertex and a 5-cm arc is drawn, the arc will be located beyond the endpoint of this side. This means the side length is less than 5 cm.

Log in to Amplify Math to complete this lesson online.

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Lesson 12 Drawing Triangles (Part 2) 769

1 Launch

Provide access to the geometry toolkits. Activate students' prior knowledge by asking, "How did you use your compass to show distance in Activity 1 from Lesson 9?"

2 Monitor

Help students get started by reminding them how to draw a 40° angle.

Look for points of confusion:

- **Using a ruler to draw the side lengths of 5 cm.** Tell them to mark the side lengths *after* they draw an arc with the compass.
- **Struggling to use the compass to measure length.** Have them read the directions carefully to determine where to place the compass. Some may need to draw the entire circle.

3 Connect

Have students share their diagrams and responses to Problems 3 and 4.

Highlight that compasses can be used for transferring lengths (i.e., making sure that lengths are drawn to be the same measure), not just drawing circles. If students do not mention how to use a compass to estimate the length of the third side, demonstrate this process.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

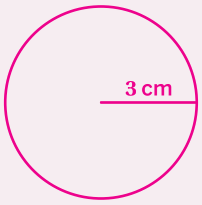
Provide copies of a 40° angle with side lengths greater than 5 cm for students to use, instead of drawing their own angle for Problem 1. Have them begin with the compass directions in Problem 2.

Power-up

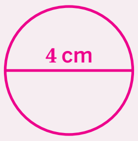
To power up students' ability to construct a circle for a given radius or diameter, have students complete:

Recall that the radius of a circle is the distance from its center to a point on a circle. The *diameter* of a circle is the distance across a circle through its center. Use your ruler and compass to construct each circle.

1. Radius: 3 cm



2. Diameter: 4 cm



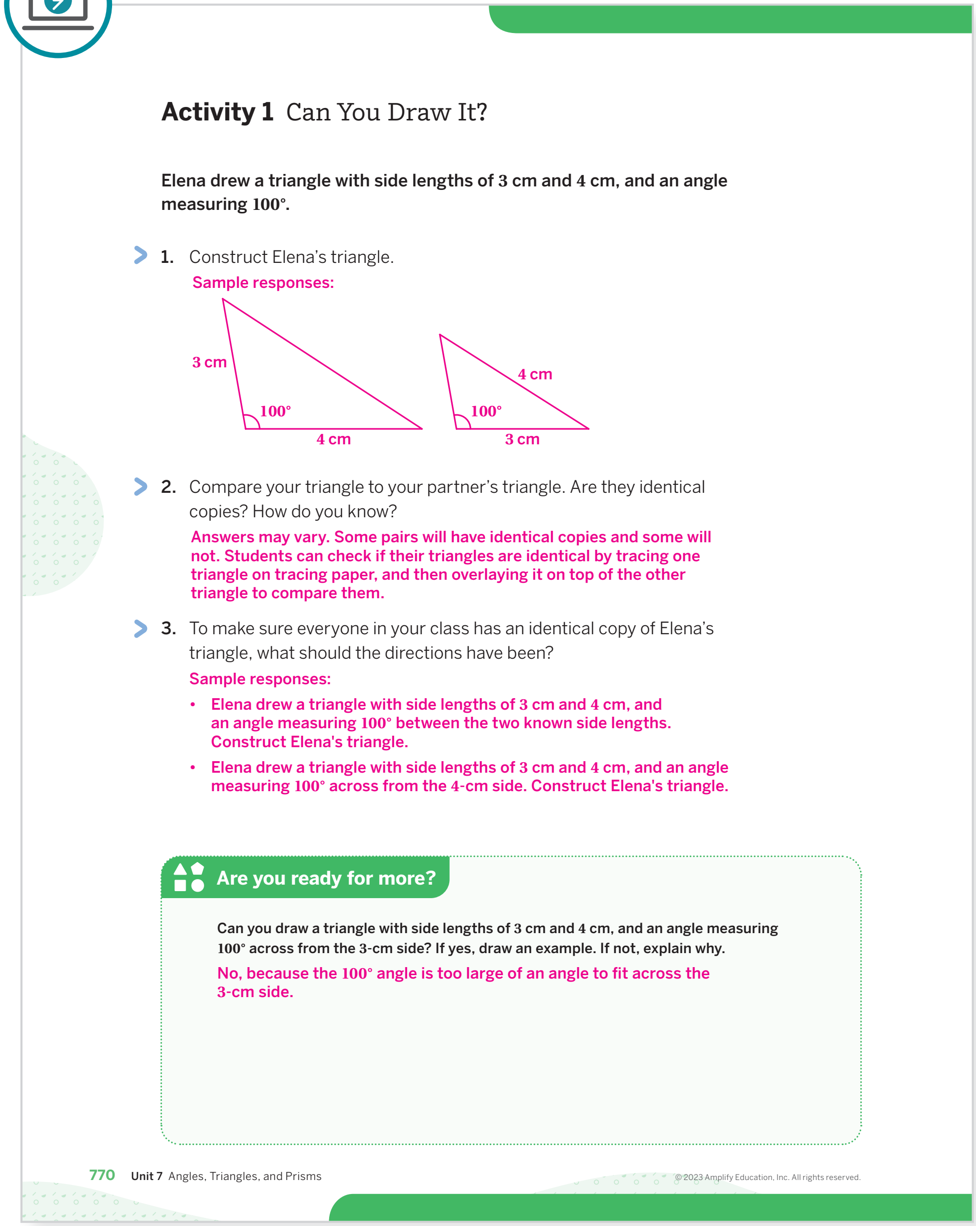
Use: Before the Warm-up.

Informed by: Performance on Lesson 11, Practice Problem 6.

Pairs | 12 min

Activity 1 Can You Draw It?

Students draw a triangle given two side lengths and an obtuse angle measure to determine if these conditions result in one unique triangle.



Activity 1 Can You Draw It?

Elena drew a triangle with side lengths of 3 cm and 4 cm, and an angle measuring 100°.

1. Construct Elena's triangle.

Sample responses:

2. Compare your triangle to your partner's triangle. Are they identical copies? How do you know?

Answers may vary. Some pairs will have identical copies and some will not. Students can check if their triangles are identical by tracing one triangle on tracing paper, and then overlaying it on top of the other triangle to compare them.

3. To make sure everyone in your class has an identical copy of Elena's triangle, what should the directions have been?

Sample responses:

- Elena drew a triangle with side lengths of 3 cm and 4 cm, and an angle measuring 100° between the two known side lengths. Construct Elena's triangle.
- Elena drew a triangle with side lengths of 3 cm and 4 cm, and an angle measuring 100° across from the 4-cm side. Construct Elena's triangle.

Are you ready for more?

Can you draw a triangle with side lengths of 3 cm and 4 cm, and an angle measuring 100° across from the 3-cm side? If yes, draw an example. If not, explain why.

No, because the 100° angle is too large of an angle to fit across the 3-cm side.

770 Unit 7 Angles, Triangles, and Prisms

1 Launch

Provide access to the geometry toolkits. Let students know this activity is similar to Lesson 11, Activity 1.

2 Monitor

Help students get started by drawing the 100° angle first, and then draw the side lengths of 3 cm and 4 cm. Ask them if there is another arrangement of the sides.

Look for points of confusion:

- **Drawing different orientations of the same triangle and thinking this means the triangles are not unique.** Have students trace one triangle on tracing paper and see how it compares to the other. If it matches, then the triangles are identical copies and not unique.
- **Attempting to draw a triangle with the 3-cm side across from the 100° angle.** Have them use their drawing to help respond to the *Are you ready for more?* problem.
- **Drawing a triangle that does not fit on the page.** Tell students their triangle may cover part of the directions.

3 Connect

Ask, "Did anybody draw a triangle identical to the one drawn by their partner?"

Display students' triangles. Have the two sample responses available in case one of these was not drawn.

Highlight there are two possible triangles given these conditions. If triangles have the same arrangement of the conditions, they will be identical. Use tracing paper to show the identical copies. Review Problem 3, referencing students' triangles or the sample triangles shown in the answer key of the Student Edition page here.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a partially-drawn triangle and have them complete it, making sure the conditions of Elena's triangle are met.

Extension: *Math Enrichment*

If students complete the *Are you ready for more?* problem, mention that in their future studies of geometry, they will learn that the longest side of a triangle is always across from the largest angle, and vice versa. Ask students to explain why this makes sense. Sample response: The largest angle will need to have the side opposite it long enough to meet at both endpoints of the angle's rays.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory text about Elena's triangle. Have students work with their partner to write 2–3 mathematical questions they may have about the triangle Elena drew.

Sample questions could be:

- "How many triangles fit this criteria?"
- "Is the 100° angle between the two given sides?"

English Learners

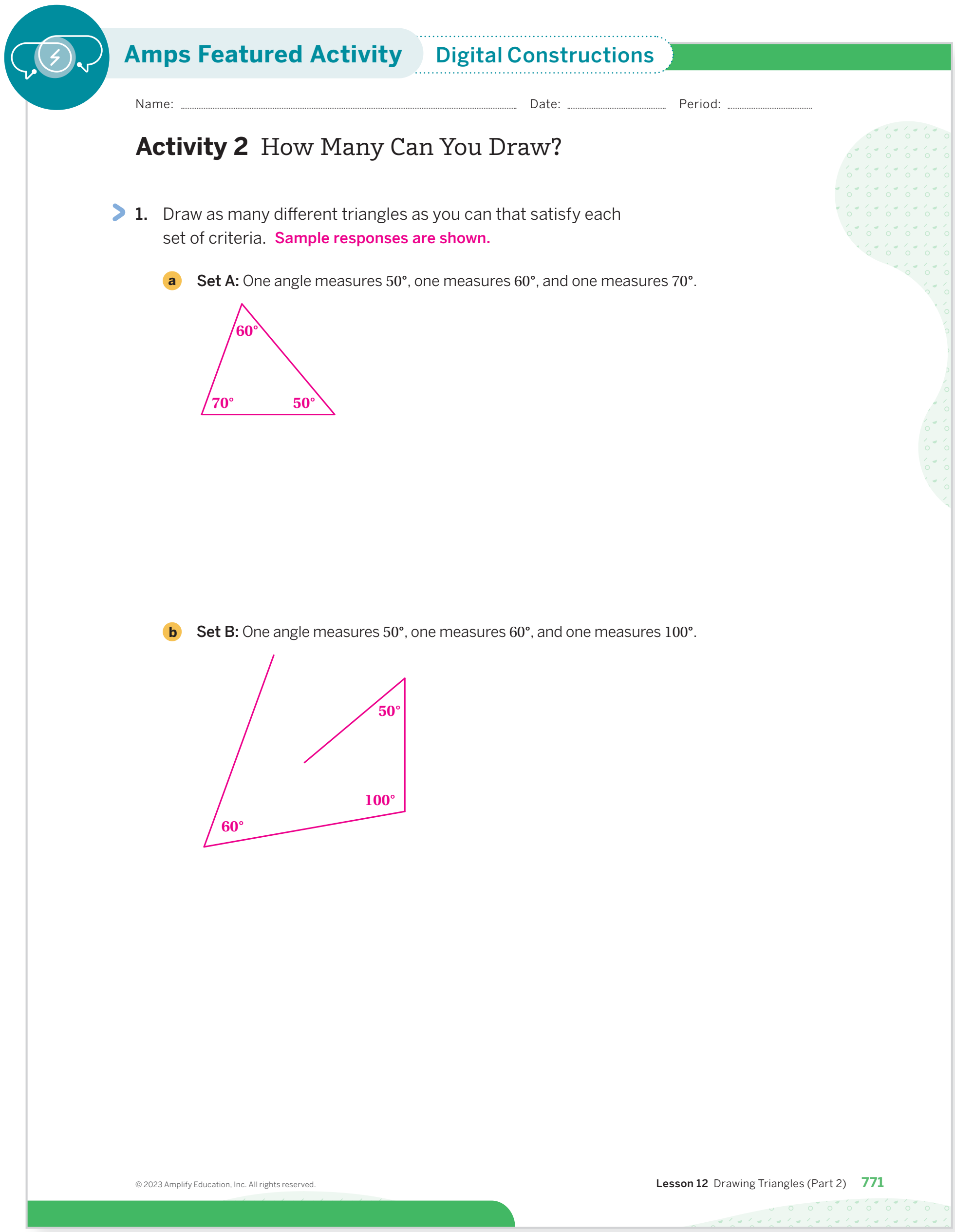
Model for students 1–2 mathematical questions that could be asked about this situation. Then ask them to revise one of your questions and create one of their own.

Pairs | 15 min

Activity 2 How Many Can You Draw?

Students draw triangles for given conditions to determine whether there is one unique triangle, many triangles, or no triangle.

Amps Featured Activity Digital Constructions



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Activity 2 How Many Can You Draw?

1. Draw as many different triangles as you can that satisfy each set of criteria. **Sample responses are shown.**

   a **Set A:** One angle measures 50°, one measures 60°, and one measures 70°.

   b **Set B:** One angle measures 50°, one measures 60°, and one measures 100°.

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Lesson 12 Drawing Triangles (Part 2) 771

1 Launch

Provide access to the geometry toolkits. Tell students they will attempt to create a triangle with the given conditions. They will then try to draw another unique triangle with the same conditions or justify to themselves why it cannot be done. If no triangle is possible, they should show valid attempts.

2 Monitor

Help students get started by helping them organize their work for Set A.

Look for points of confusion:

- **Drawing the same triangle in different orientations.** Have students trace their triangle and see if it compares to another to determine whether they are identical copies.
- **Drawing only one triangle.** Have students write down the order of their known measurements, then rearrange them to see if they can create another triangle.
- **Struggling to draw the triangles with three angles in Sets A and B.** Have them start by drawing the largest angle, then moving to one end of the segment to draw the next largest angle. They can also use the technique with the ruler and protractor mentioned in Lesson 11.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create triangles with given conditions using an interactive tool.

Accessibility

Have students try to build just one triangle for each set of conditions. Provide them with a copy of the Activity 2 PDF that they can use as a guide for the criteria in Set C.

Extension: *Math Enrichment*

Have students make conjectures about what type of conditions will result in one unique triangle, many triangles, or no triangle. Have them refer to Lessons 10 and 11 for more examples for them to study.

Math Language Development

MLR1: Stronger and Clearer Each Time

Before the Connect, have students share their responses for Problem 2 with another pair of students to give and receive feedback. Display these prompts that reviewers can use to press for details.

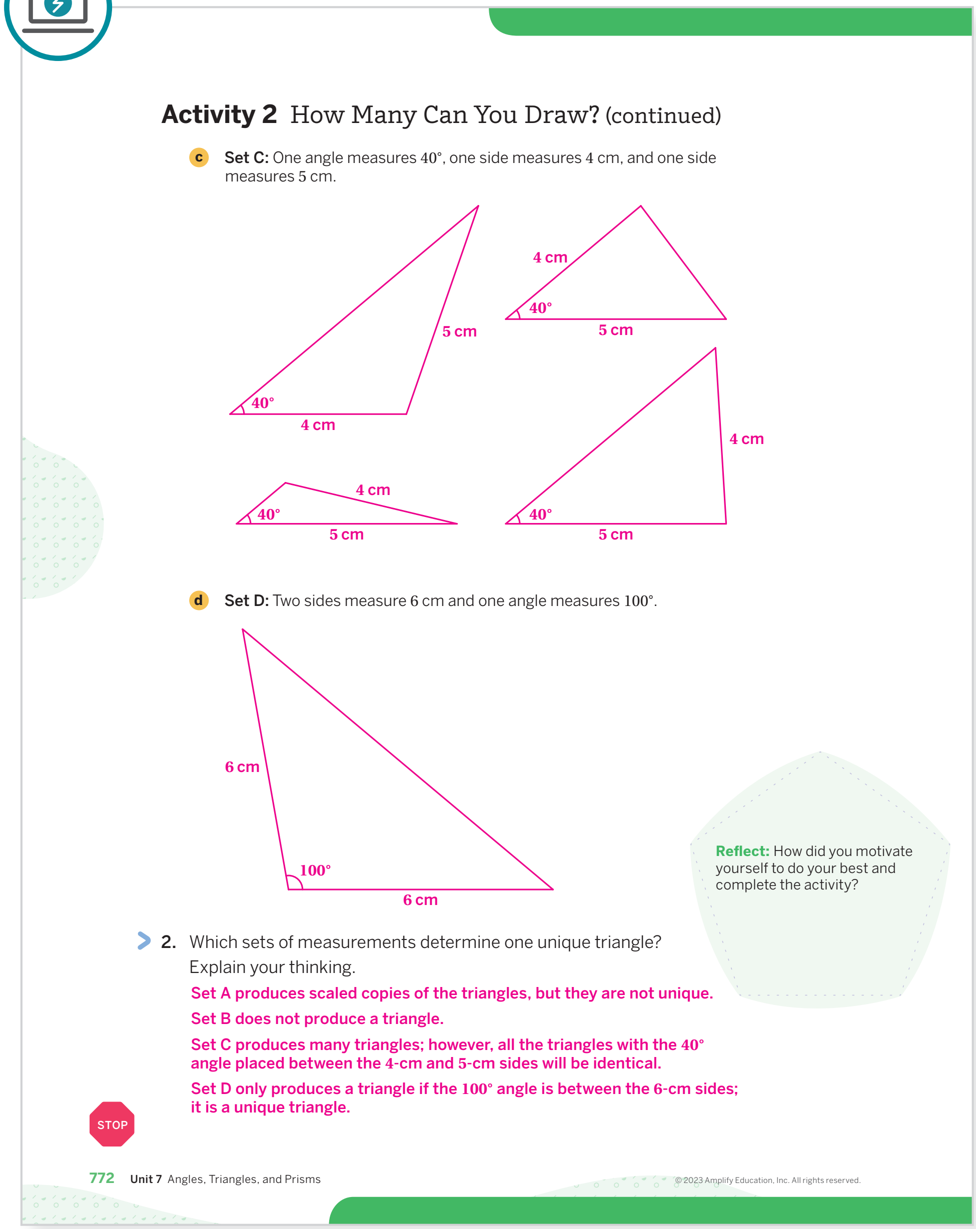
- "How do you know that this set produces scaled copies/many triangles/a unique triangle/no triangle?"
- "Could you explain how you know that _____?"

English Learners

Provide students with a word bank or create an anchor chart with mathematical vocabulary needed to reason about the number of triangles, such as *scaled copies, unique, identical*, etc.

Activity 2 How Many Can You Draw? (continued)

Students draw triangles given conditions to determine whether there is one unique triangle, many triangles, or no triangle.



Activity 2 How Many Can You Draw? (continued)

c **Set C:** One angle measures 40°, one side measures 4 cm, and one side measures 5 cm.

d **Set D:** Two sides measure 6 cm and one angle measures 100°.

Reflect: How did you motivate yourself to do your best and complete the activity?

2. Which sets of measurements determine one unique triangle? Explain your thinking.
   Set A produces scaled copies of the triangles, but they are not unique.
   Set B does not produce a triangle.
   Set C produces many triangles; however, all the triangles with the 40° angle placed between the 4-cm and 5-cm sides will be identical.
   Set D only produces a triangle if the 100° angle is between the 6-cm sides; it is a unique triangle.

772 **Unit 7** Angles, Triangles, and Prisms

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3 Connect

Display students' triangles for each set of conditions.

Have a few students share how many triangles were possible for each set of conditions. Have them share how they knew when one unique triangle, many triangles, or no triangles were possible.

Highlight:

- Set A results in many triangles which are all scaled triangles of each other. Mention that corresponding angles have the same measure and corresponding side lengths are multiplied by a scale factor.
- It is impossible to create a triangle for Set B.
  Note: At this level, students do not need to know that the sum of all the interior angles of a triangle is 180°.
- If not brought up in student explanations, point out that for Set C, one possible order for the measurements (40°, 5 cm, 4 cm) can result in two different triangles. One way to show this is to draw a 5-cm segment. Then use a compass to draw a circle with a radius of 4 cm centered at one of the segment's endpoints. Draw a 40° angle on the segment's other endpoint to demonstrate that this ray will intersect the circle twice. Each intersection could be the third vertex of the triangle.
  Note: Students do not need to learn the rules about the number of possible triangles given different sets of conditions.
- The measurements in Set D will only result in a triangle if the 100° angle is between the two 6-cm sides. Show an attempt with the 100° angle across from a 6-cm side.

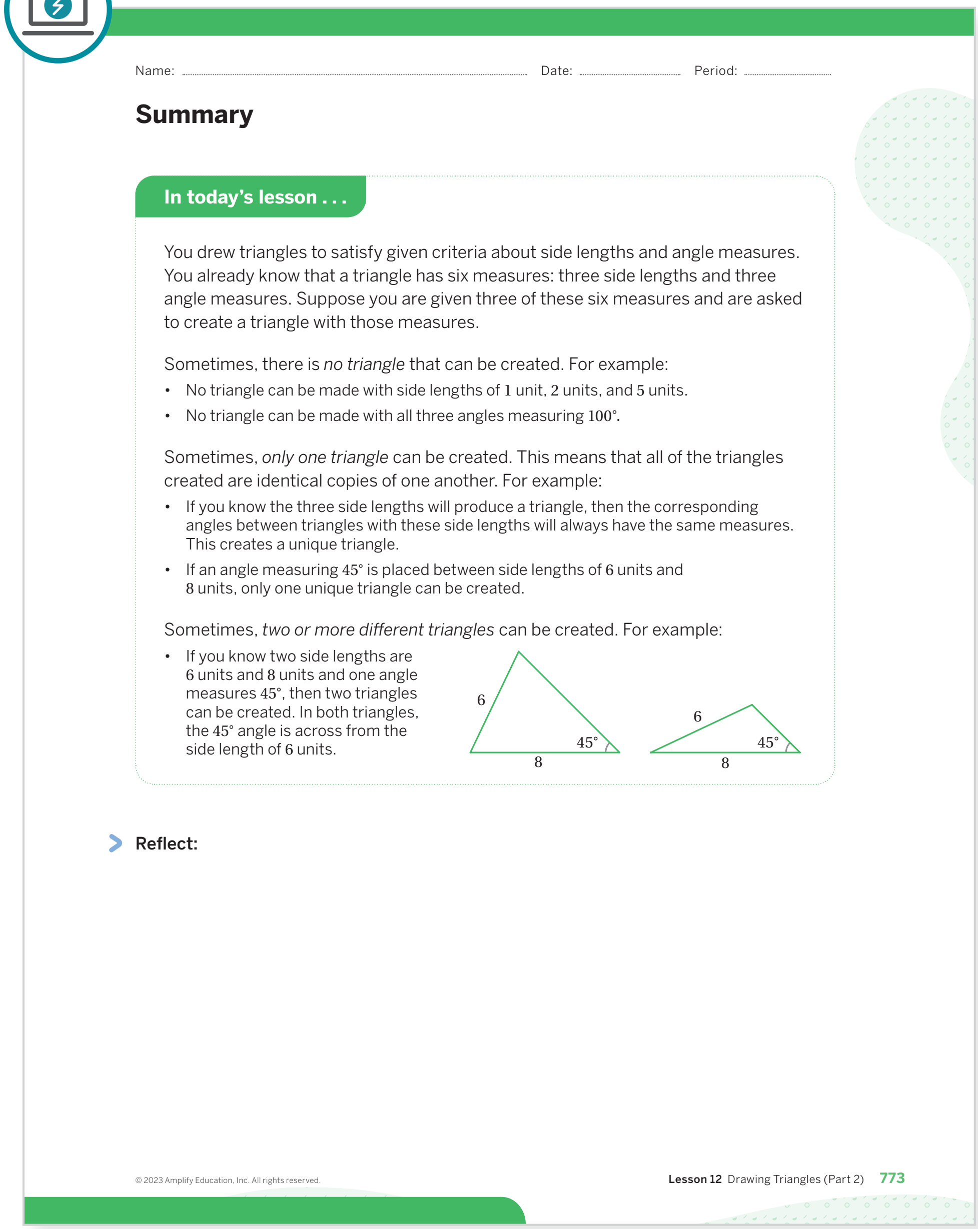
Ask:

- "Did any set of conditions result in a unique triangle?" Set D
- "Which set of conditions could not be drawn?" Set B
- "Why can more than one triangle be made for the conditions in Set A?" There were no side lengths, so the triangles can be made as scaled copies with shorter or longer sides.

Whole Class | 5 min

Summary

Review and synthesize how knowing given conditions of a triangle will help determine whether one unique triangle, many triangles, or no triangle can be drawn to meet those conditions.



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Summary

In today's lesson . . .

You drew triangles to satisfy given criteria about side lengths and angle measures. You already know that a triangle has six measures: three side lengths and three angle measures. Suppose you are given three of these six measures and are asked to create a triangle with those measures.

Sometimes, there is *no triangle* that can be created. For example:

- No triangle can be made with side lengths of 1 unit, 2 units, and 5 units.
- No triangle can be made with all three angles measuring 100°.

Sometimes, *only one triangle* can be created. This means that all of the triangles created are identical copies of one another. For example:

- If you know the three side lengths will produce a triangle, then the corresponding angles between triangles with these side lengths will always have the same measures. This creates a unique triangle.
- If an angle measuring 45° is placed between side lengths of 6 units and 8 units, only one unique triangle can be created.

Sometimes, *two or more different triangles* can be created. For example:

- If you know two side lengths are 6 units and 8 units and one angle measures 45°, then two triangles can be created. In both triangles, the 45° angle is across from the side length of 6 units.

Reflect:

Lesson 12 Drawing Triangles (Part 2) 773

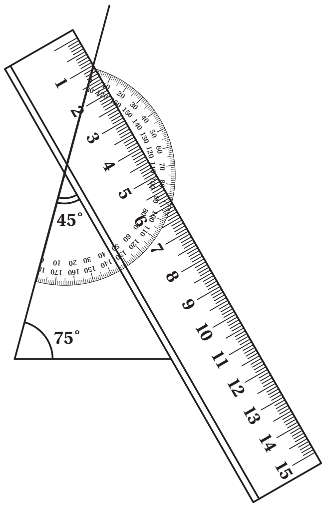
Synthesize

Display different sets of conditions and drawn triangles from the activities of this lesson.

Highlight that a triangle has six measures (three angle measurements and three side lengths). A minimum of three measurements are needed to determine whether there is only one triangle, many triangles, or no possible triangle. It is important to note what types of measurements they are, but also their order, such as the location of the measurements in reference to the other measurements (i.e., adjacent or across).

Ask:

- "How was a compass useful in today's activities?" It helps find the points a certain distance away.
- "What strategies did you use to include two given side lengths and a given angle?" Draw one of the side lengths, use a protractor to draw the angle at one end, and use a compass to finish the triangle by drawing the other side length.
- "What strategies did you use to include three given angles?" Draw one angle and then use a protractor and ruler to slide along one side of the first angle.



Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Given certain segments and angles, how many unique triangles can be made?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by explaining how to complete a triangle with given conditions and determining whether there is another possible triangle that can be drawn.

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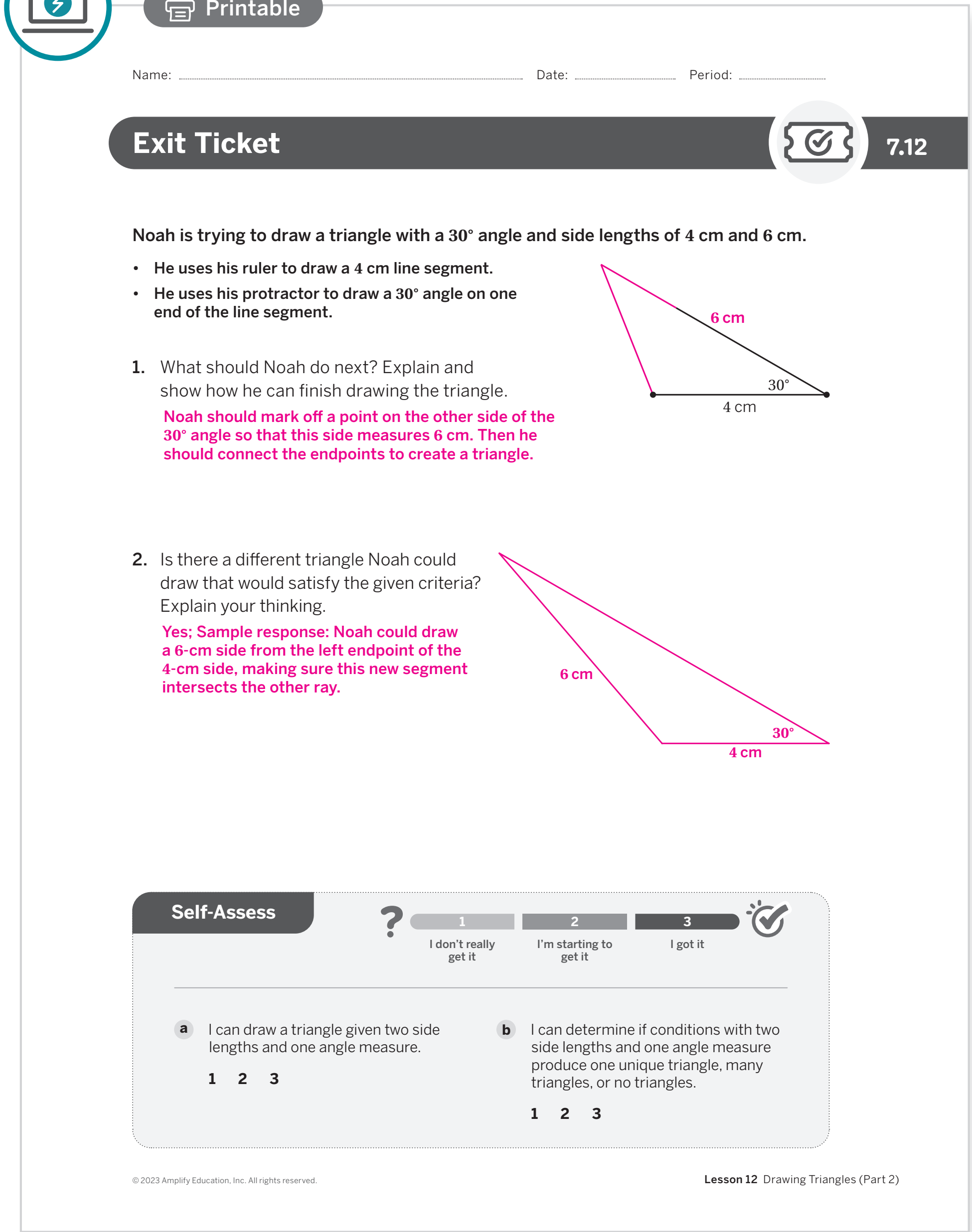


Name: ________ Date: ________ Period: ________

Exit Ticket

7.12

Noah is trying to draw a triangle with a 30° angle and side lengths of 4 cm and 6 cm.

- He uses his ruler to draw a 4 cm line segment.
- He uses his protractor to draw a 30° angle on one end of the line segment.

1. What should Noah do next? Explain and show how he can finish drawing the triangle.

   Noah should mark off a point on the other side of the 30° angle so that this side measures 6 cm. Then he should connect the endpoints to create a triangle.

2. Is there a different triangle Noah could draw that would satisfy the given criteria? Explain your thinking.

   Yes; Sample response: Noah could draw a 6-cm side from the left endpoint of the 4-cm side, making sure this new segment intersects the other ray.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can draw a triangle given two side lengths and one angle measure.

1 2 3

b I can determine if conditions with two side lengths and one angle measure produce one unique triangle, many triangles, or no triangles.

1 2 3

Lesson 12 Drawing Triangles (Part 2)

Success looks like . . .

- **Language Goal:** Drawing triangles with three given anglesand triangles with two given side lengths and one angle measure, and describing how many different triangles could be drawn with the given conditions. **(Speaking and Listening)**
    - » Explaining whether Noah could draw a different triangle with a 30° angle and side lengths of 4 cm and 6 cm in Problem 2.
- **Language Goal:** Using drawings to justify whether two given side lengths and one angle measure can determine one unique triangle. **(Writing)**
    - » Drawing a different triangle with a 30° angle and side lengths of 4 cm and 6 cm in Problem 2.

Suggested next steps

If students cannot think of a possible different triangle that Noah could draw in Problem 2, consider:

- Checking to see whether they drew the triangle instead of explaining how to draw it.
- Assigning Practice Problem 2.

If students could not think of the second possible triangle, consider:

- Assigning Practice Problems 1 and 3.

Professional Learning

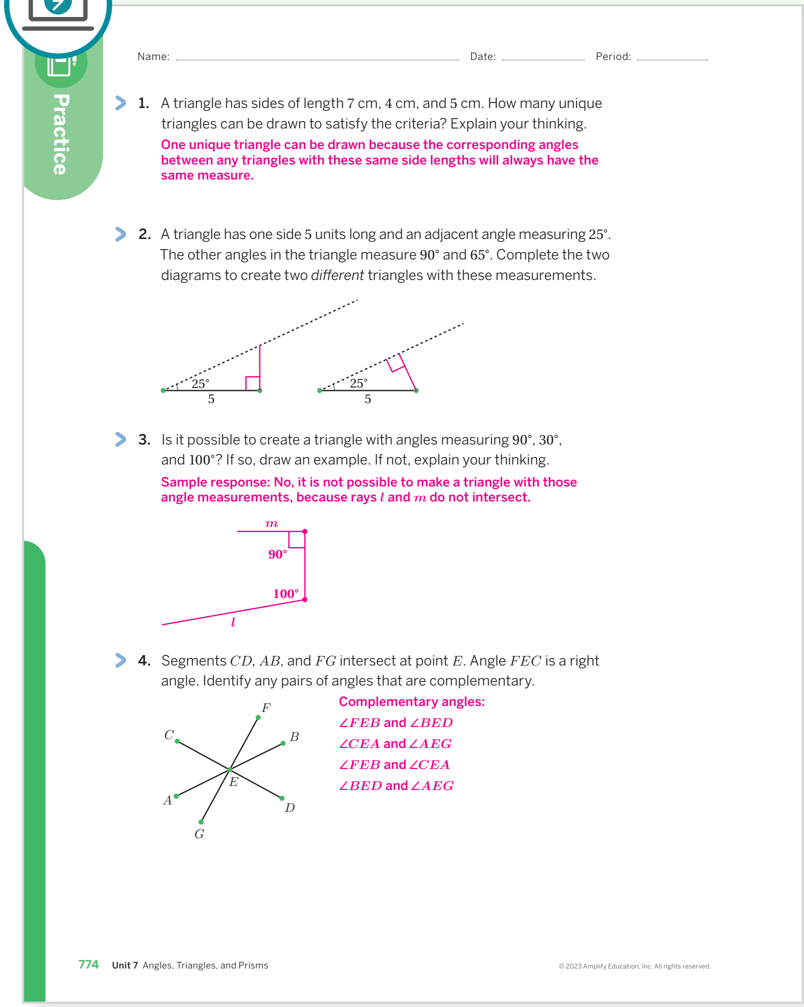
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- What resources did students use as they worked through Activity 2? Which resources were especially helpful? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. A triangle has sides of length 7 cm, 4 cm, and 5 cm. How many unique triangles can be drawn to satisfy the criteria? Explain your thinking.

   One unique triangle can be drawn because the corresponding angles between any triangles with these same side lengths will always have the same measure.

2. A triangle has one side 5 units long and an adjacent angle measuring 25°. The other angles in the triangle measure 90° and 65°. Complete the two diagrams to create two *different* triangles with these measurements.

3. Is it possible to create a triangle with angles measuring 90°, 30°, and 100°? If so, draw an example. If not, explain your thinking.

   Sample response: No, it is not possible to make a triangle with those angle measurements, because rays l and m do not intersect.

4. Segments CD, AB, and FG intersect at point E. Angle FEC is a right angle. Identify any pairs of angles that are complementary.

   Complementary angles:
   $\angle FEB$ and $\angle BED$
   $\angle CEA$ and $\angle AEG$
   $\angle FEB$ and $\angle CEA$
   $\angle BED$ and $\angle AEG$

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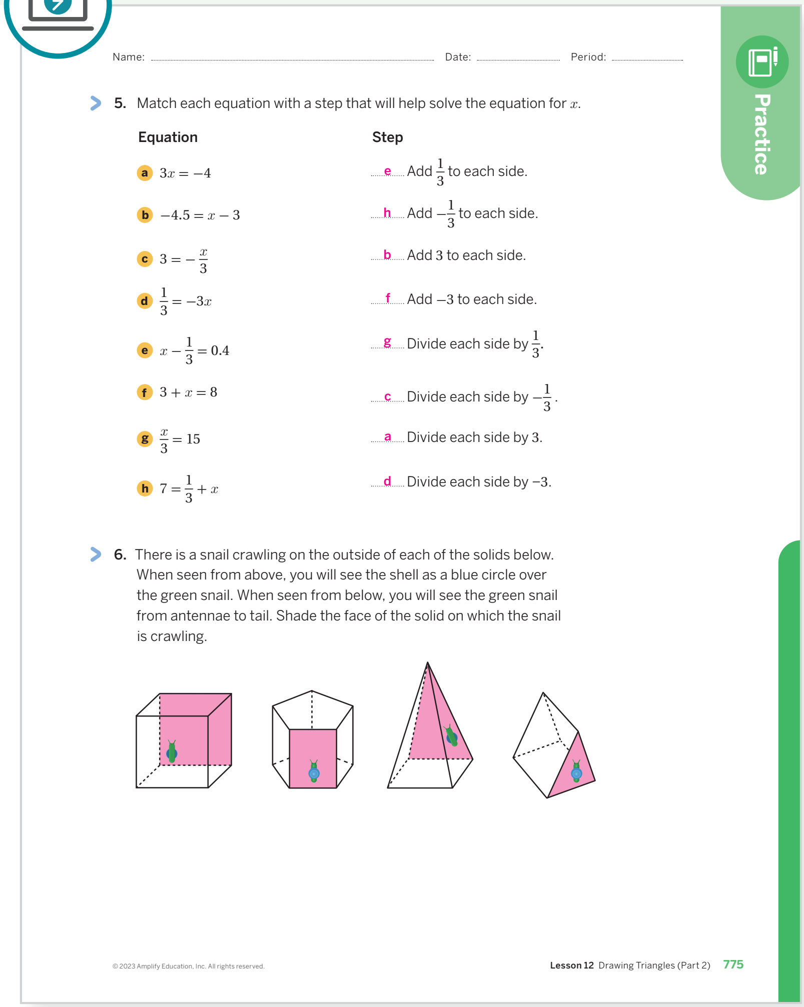

Name: ______ Date: ______ Period: ______

5. Match each equation with a step that will help solve the equation for x.

Equation	Step
a $3x = -4$	**e** Add $\frac{1}{3}$ to each side.
b $-4.5 = x - 3$	**h** Add $-\frac{1}{3}$ to each side.
c $3 = -\frac{x}{3}$	**b** Add 3 to each side.
d $\frac{1}{3} = -3x$	**f** Add –3 to each side.
e $x - \frac{1}{3} = 0.4$	**g** Divide each side by $\frac{1}{3}$.
f $3 + x = 8$	**c** Divide each side by $-\frac{1}{3}$.
g $\frac{x}{3} = 15$	**a** Divide each side by 3.
h $7 = \frac{1}{3} + x$	**d** Divide each side by –3.

6. There is a snail crawling on the outside of each of the solids below. When seen from above, you will see the shell as a blue circle over the green snail. When seen from below, you will see the green snail from antennae to tail. Shade the face of the solid on which the snail is crawling.

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Lesson 12 Drawing Triangles (Part 2) 775

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 3	2
	5	Unit 5 Lesson 18	1
Formative ⚡	6	Unit 7 Lesson 13	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

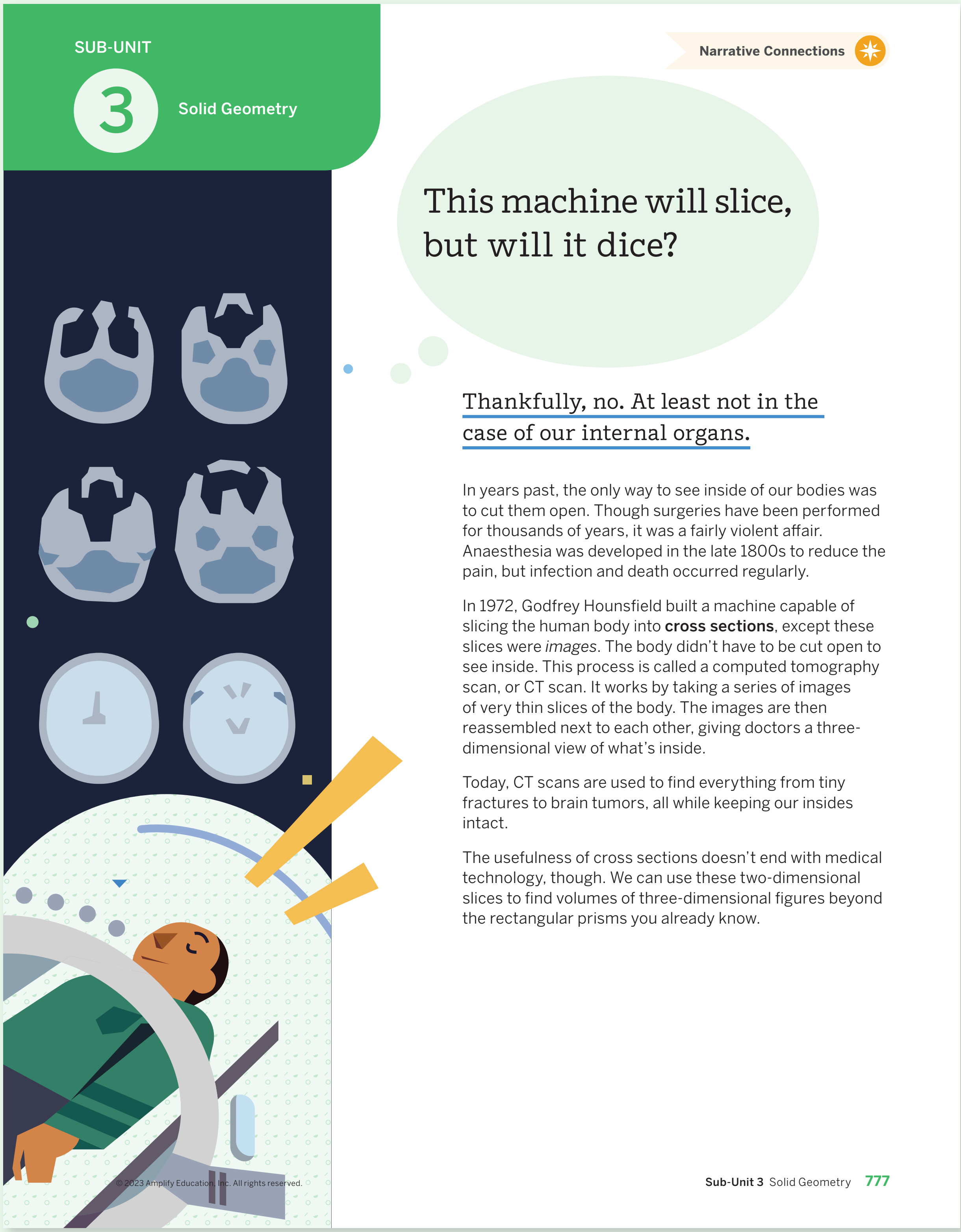
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 3
Solid Geometry

Whole Class

In this Sub-Unit, students slice, dice, unfold, wrap, and fill three-dimensional figures to discover relationships between their sizes and shapes.



SUB-UNIT

3 Solid Geometry

Narrative Connections

This machine will slice, but will it dice?

Thankfully, no. At least not in the case of our internal organs.

In years past, the only way to see inside of our bodies was to cut them open. Though surgeries have been performed for thousands of years, it was a fairly violent affair. Anaesthesia was developed in the late 1800s to reduce the pain, but infection and death occurred regularly.

In 1972, Godfrey Hounsfield built a machine capable of slicing the human body into **cross sections**, except these slices were *images*. The body didn't have to be cut open to see inside. This process is called a computed tomography scan, or CT scan. It works by taking a series of images of very thin slices of the body. The images are then reassembled next to each other, giving doctors a three-dimensional view of what's inside.

Today, CT scans are used to find everything from tiny fractures to brain tumors, all while keeping our insides intact.

The usefulness of cross sections doesn't end with medical technology, though. We can use these two-dimensional slices to find volumes of three-dimensional figures beyond the rectangular prisms you already know.

Narrative Connections

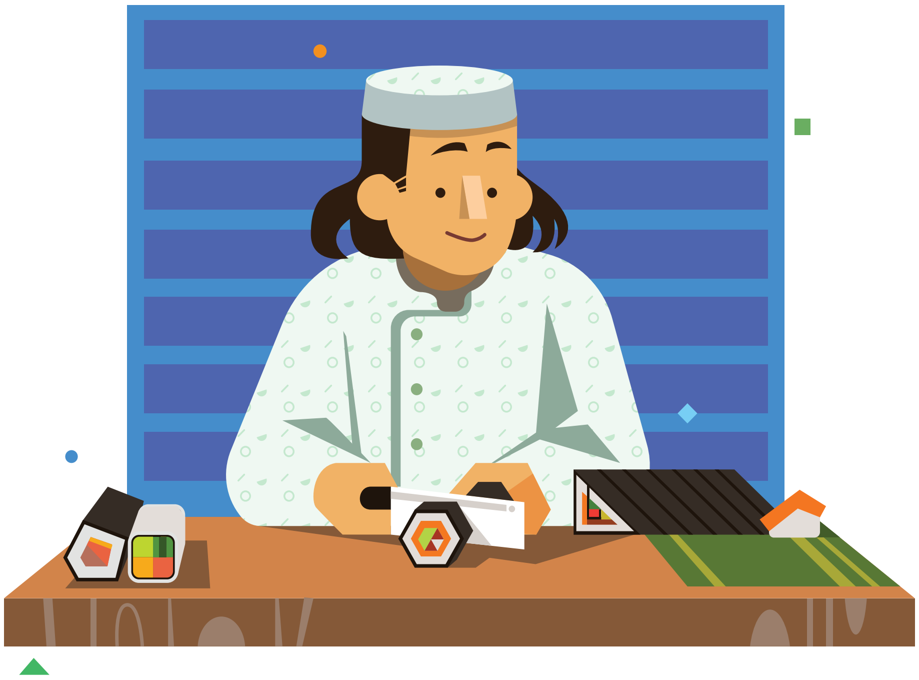
Read the narrative aloud as a class or have students read it individually. Students continue to make sense of three-dimensional solids by slicing or decomposing them in the following places:

- **Lesson 14, Activity 1:** Determining Volume With Cubes
- **Lesson 15, Activity 2:** A Heart-Shaped Box
- **Lesson 17, Warm-up:** The Science Fair

UNIT 7 | **LESSON 13**

Slicing Solids

Let's see what shapes you get when you slice a three-dimensional solid.



Focus

Goals

1. **Language Goal:** Categorize images of planes intersecting pyramids and prisms and describe the categories. **(Speaking and Listening)**
2. **Language Goal:** Comprehend that the term *cross section* refers to the two-dimensional face that results from slicing a three-dimensional figure. **(Speaking and Listening, Writing)**
3. **Language Goal:** Describe, compare, and contrast different cross sections that could result from slicing the same pyramid or prism. **(Speaking and Listening, Writing)**

Coherence

Today

This lesson introduces the idea that slicing a three-dimensional solid with a plane results in a two-dimensional cross section. Given two-dimensional representations of how solids are sliced, students practice visualizing the three-dimensional solids and the resulting cross sections.

Previously

In Grade 6, students learned to find the volume of right rectangular prisms with fractional side lengths. They also learned to represent three-dimensional figures using nets.

Coming Soon

In the next lesson, students will generalize a formula for the volume of a prism. They will build on the ideas of taking slices identical to the base and stacking them to fill the prism.

Rigor

- Students use visual models to build their **conceptual understanding** of cross sections of solids.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair

Math Language Development

New word

- ***cross section***

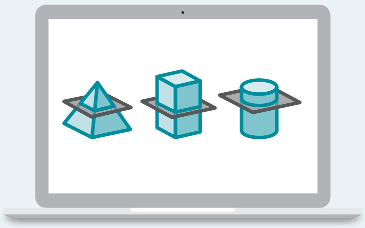
Review words

- *base (of a prism or pyramid)*
- *prism*
- *pyramid*

Amps | Featured Activity

Activity 1
Interactive Cross Sections

Students are able to manipulate a two-dimensional plane in three dimensions to see highlighted cross sections where the plane intersects the solid.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might think that they have found the only way to sort the cards. Explain that there are many ways to sort the cards. They should rely on their own strengths, being prepared to explain their reasoning while accepting and appreciating the valid reasoning of others.

Modifications to Pacing

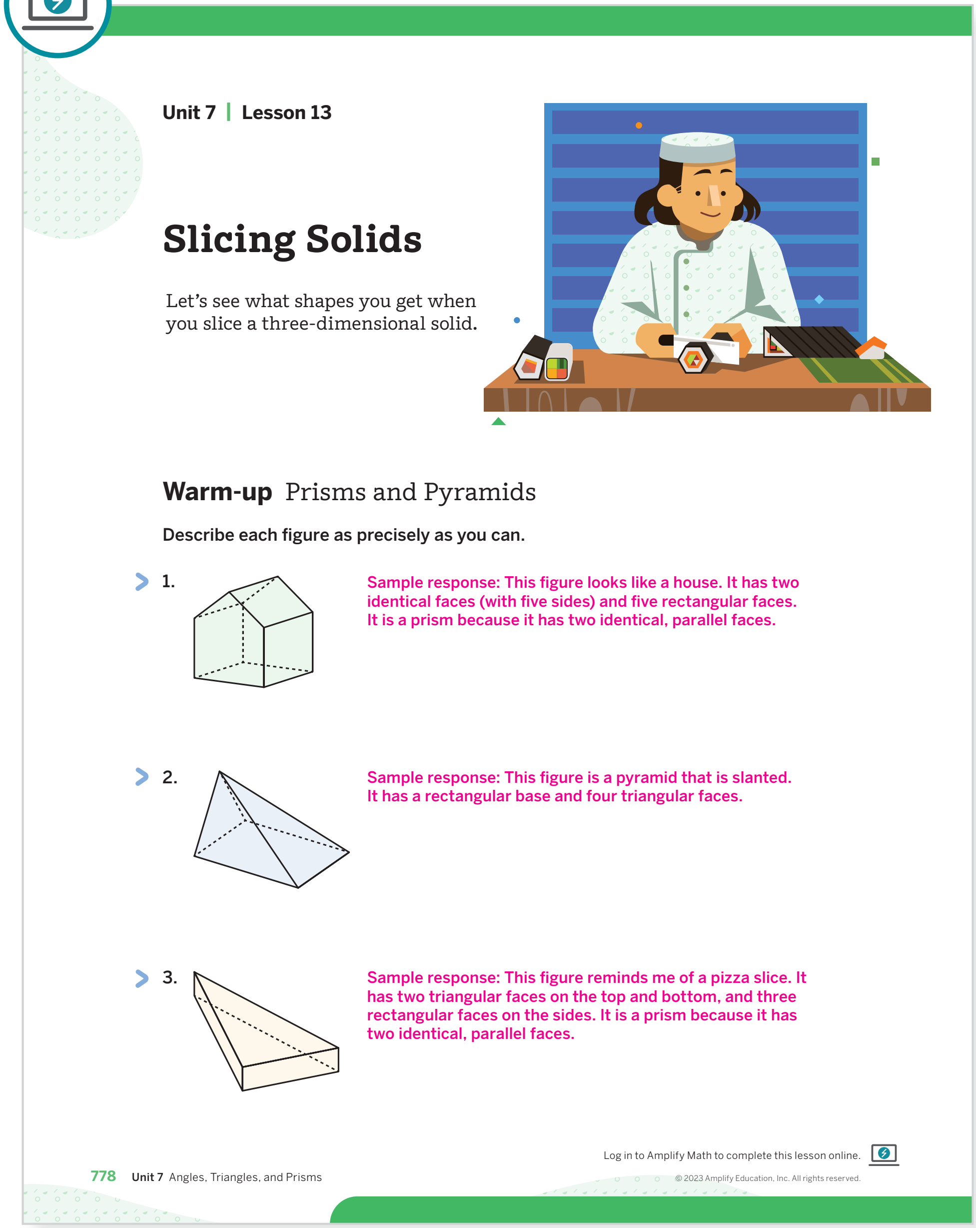
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up Prisms and Pyramids

Students review the characteristics of prisms and pyramids, preparing them for the upcoming activities.



Unit 7 | Lesson 13

Slicing Solids

Let's see what shapes you get when you slice a three-dimensional solid.

Warm-up Prisms and Pyramids

Describe each figure as precisely as you can.

1. Sample response: This figure looks like a house. It has two identical faces (with five sides) and five rectangular faces. It is a prism because it has two identical, parallel faces.

2. Sample response: This figure is a pyramid that is slanted. It has a rectangular base and four triangular faces.

3. Sample response: This figure reminds me of a pizza slice. It has two triangular faces on the top and bottom, and three rectangular faces on the sides. It is a prism because it has two identical, parallel faces.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Ask students, "What do you see? Describe each figure and its parts as precisely as you can."

2 Monitor

Help students get started by having them describe aloud something they notice about the first figure.

Look for points of confusion:

- **Not being sure how to interpret the dashed lines.** Have students focus on a corner in the foreground (where three solid lines meet).
- **Thinking that a prism must have its base on the bottom.** Ask the students if they can find two identical, parallel bases.

Look for productive strategies:

- Using vocabulary that will be important throughout the lesson, such as *prism*, *face*, and *pyramid*.
- Identifying the first and third figures as prisms.

3 Connect

Have students share their descriptions of each figure. Record and display their responses for all to see.

Define a *prism* as a three-dimensional figure with two identical polygonal bases, connected by rectangles. A *pyramid* is a three-dimensional figure with one polygonal base, and all other faces are triangles which meet at a single point. A *plane* is a flat two-dimensional surface, similar to a piece of paper, that students will use in the upcoming activities to slice through solids.

Highlight that students will be thinking about what happens when they slice three-dimensional solids, the different ways those solids can be sliced, and the shapes of the cross sections created by those slices.

Differentiated Support

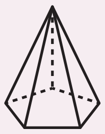
Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide students with a word bank of attributes they can choose from to describe each figure, such as the following.

Shape of the base	Shape of each face	Number of faces	Are the faces identical?
Are the faces parallel?	Is this a prism?	Is this a pyramid?	

Power-up

To power up students' ability to identify faces of three-dimensional figures, have students complete:



1. How many faces does the figure have? 6
2. Identify the polygons that make up the faces of the figure. One pentagon and 5 triangles.

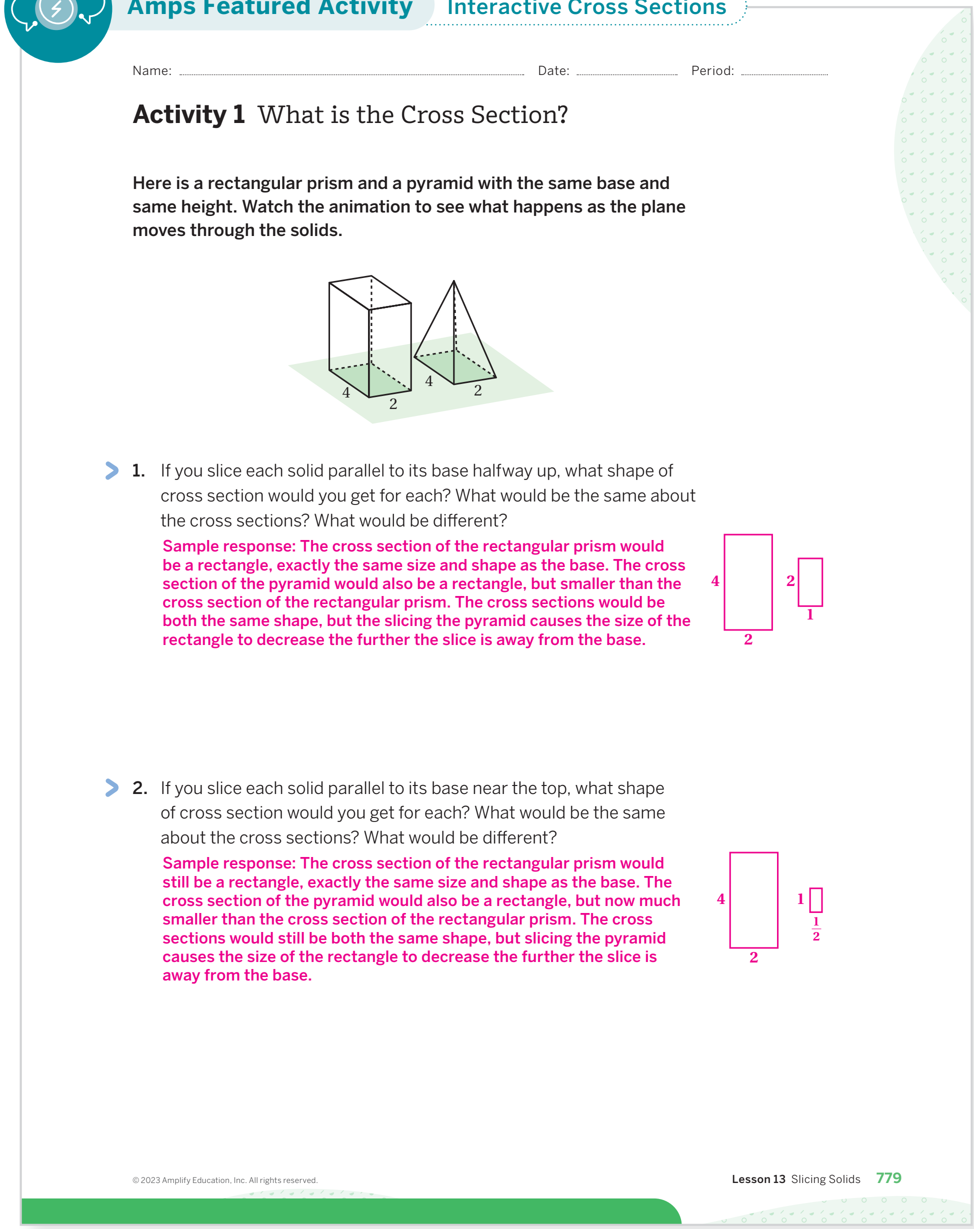
Use: Before the Warm-up.
Informed by: Performance on Lesson 12, Practice Problem 6.

Pairs | 15 min

Activity 1 What Is the Cross Section?

Students view a demonstration of slicing a solid to help them visualize two-dimensional cross sections of three-dimensional solids.

Amps Featured Activity Interactive Cross Sections



Name: ______ Date: ______ Period: ______

Activity 1 What is the Cross Section?

Here is a rectangular prism and a pyramid with the same base and same height. Watch the animation to see what happens as the plane moves through the solids.

1. If you slice each solid parallel to its base halfway up, what shape of cross section would you get for each? What would be the same about the cross sections? What would be different?

Sample response: The cross section of the rectangular prism would be a rectangle, exactly the same size and shape as the base. The cross section of the pyramid would also be a rectangle, but smaller than the cross section of the rectangular prism. The cross sections would be both the same shape, but the slicing the pyramid causes the size of the rectangle to decrease the further the slice is away from the base.

2. If you slice each solid parallel to its base near the top, what shape of cross section would you get for each? What would be the same about the cross sections? What would be different?

Sample response: The cross section of the rectangular prism would still be a rectangle, exactly the same size and shape as the base. The cross section of the pyramid would also be a rectangle, but now much smaller than the cross section of the rectangular prism. The cross sections would still be both the same shape, but slicing the pyramid causes the size of the rectangle to decrease the further the slice is away from the base.

Lesson 13 Slicing Solids 779

1 Launch

Use the digital tool to demonstrate a plane slicing through a rectangular prism and pyramid. Say, "When you slice a three-dimensional solid, you expose new faces that are two-dimensional. Each two-dimensional face is called a ***cross section***."

2 Monitor

Help students get started by leaving the digital tool visible and showing the plane passing through the three-dimensional figures halfway.

Look for points of confusion:

- **Not understanding what *parallel* means for planes.** Have students simulate parallel planes with their hands.
- **Thinking the cross section is a parallelogram.** Remind students that they know the base is a rectangle, even though it looks slanted.

Look for productive strategies:

- Noticing that the size of the pyramid's cross section is a scaled version of the base.

3 Connect

Have students share their descriptions of the cross sections.

Ask, "How do the dimensions of each cross section change as the plane moves upward?" The cross section of the rectangular prism remains the same size, but the cross section of the pyramid gets smaller.

Highlight there are many ways to slice a three-dimensional solid. In this activity, students only sliced horizontally. In the next activity, they will see what the cross sections look like when they slice vertically and diagonally.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate a two-dimensional plane in three dimensions to see highlighted cross sections where the plane intersects the solid.

Accessibility: *Optimize Access to Tools*

If you choose to not use the Amps slides for this activity, consider bringing in three-dimensional models of a rectangular prism and pyramid for students to physically examine.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their descriptions and respond to the Ask question, listen for and amplify the mathematical language they use in their descriptions. Add these words and phrases, such as *cross section*, *same size*, *gets smaller*, *slice*, and *parallel to the base* to the class display.

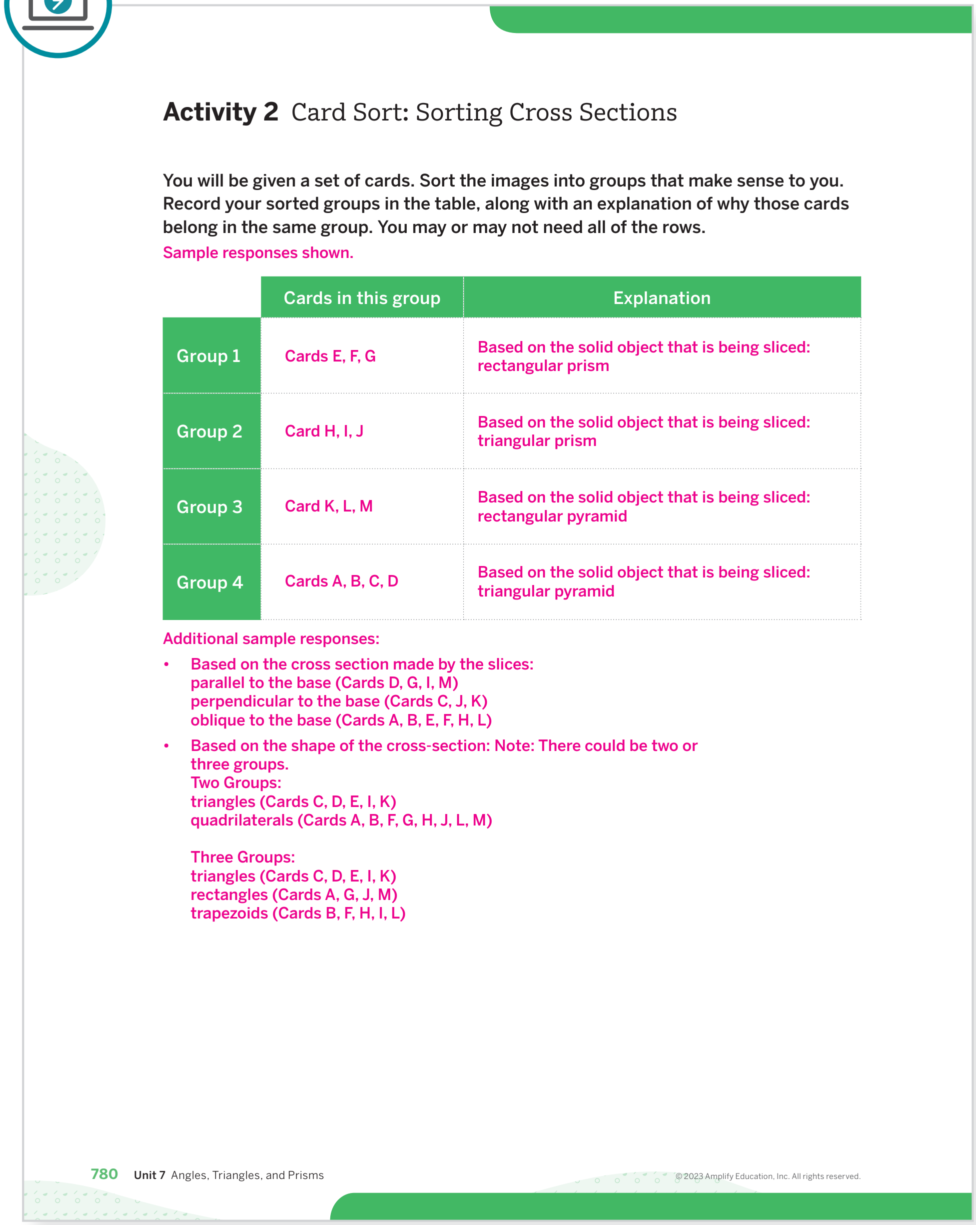
English Learners

Use gestures and pictures to illustrate the slice type and the shapes formed. For example, hold your arm horizontal and demonstrate what it looks like to slice the solid parallel to its base.

Pairs | 15 min

Activity 2 Card Sort: Sorting Cross Sections

Students visualize cross sections in a more abstract way by sorting images of three-dimensional solids sliced by a plane by the cross sections created by the slices.



Activity 2 Card Sort: Sorting Cross Sections

You will be given a set of cards. Sort the images into groups that make sense to you. Record your sorted groups in the table, along with an explanation of why those cards belong in the same group. You may or may not need all of the rows.

Sample responses shown.

	Cards in this group	Explanation
Group 1	Cards E, F, G	Based on the solid object that is being sliced: rectangular prism
Group 2	Card H, I, J	Based on the solid object that is being sliced: triangular prism
Group 3	Card K, L, M	Based on the solid object that is being sliced: rectangular pyramid
Group 4	Cards A, B, C, D	Based on the solid object that is being sliced: triangular pyramid

Additional sample responses:

- Based on the cross section made by the slices:
  parallel to the base (Cards D, G, I, M)
  perpendicular to the base (Cards C, J, K)
  oblique to the base (Cards A, B, E, F, H, L)
- Based on the shape of the cross-section: Note: There could be two or three groups.
  Two Groups:
  triangles (Cards C, D, E, I, K)
  quadrilaterals (Cards A, B, F, G, H, J, L, M)

  Three Groups:
  triangles (Cards C, D, E, I, K)
  rectangles (Cards A, G, J, M)
  trapezoids (Cards B, F, H, I, L)

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1 Launch

Distribute one set of cards from the Activity 2 PDF to each pair of students. Tell students that these cards have several qualities in common, so each pair of students might have different reasons for sorting the images. Conduct the *Card Sort* routine.

2 Monitor

Help students get started by mentioning that they may want to make note of the shapes of each cross section first.

Look for points of confusion:

- **Struggling to identify the cross section.** Remind students how the perspective can make a shape look different from different angles of viewing.

Look for productive strategies:

- Sorting based on the solid being sliced.
- Sorting based on the direction of the slice (vertically, horizontally, diagonally).
- Sorting based on the shape of the cross section.

3 Connect

Display a pair's set of cards that are organized by the shape of the cross section.

Highlight that there are multiple ways to obtain cross sections that are triangles. Note the more obvious ways (such as from a pyramid or triangular prism) and the less obvious (such as slicing a corner of a cube).

Ask, "What kind of slice creates a cross section that is identical to the base of that shape?" When you slice a prism parallel to its base, the cross section is identical to the base.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on sorting Cards A-J. This will still allow them to access the mathematical goal of the activity.

Extension: *Math Enrichment*

Have students compare the horizontal cross sections for the prisms related to the pyramids. Ask them why all horizontal cross sections of a prism will be the same size, regardless of how far up the prism the slice occurs. Then have them explain why this is *not* true for all horizontal cross sections of pyramids.

Math Language Development

MLR7: Compare and Connect

After students have sorted the cards into groups, ask them to investigate each other's work by taking a tour of other group's sorting methods via a *Gallery Tour* routine. Facilitate discussion among students by asking questions, such as "What similarities or differences do you see in how other groups sorted the cards compared to how your group sorted the cards?"

English Learners

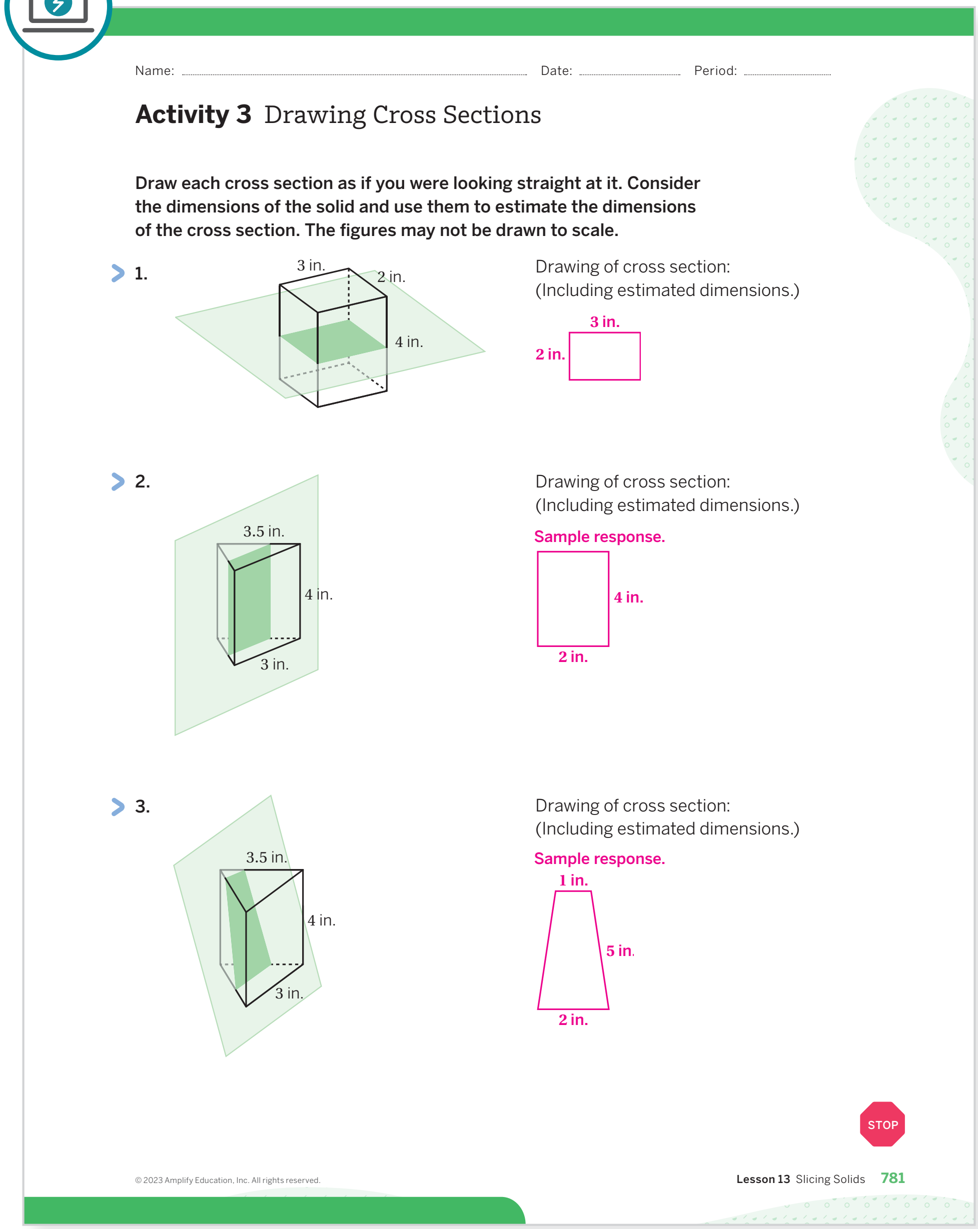
Model using, and encourage students to use, arm gestures when talking about *horizontal*, *vertical*, and *diagonal* cross sections.

Optional

Pairs | 10 min

Activity 3 Drawing Cross Sections

Students draw cross sections of various slices and use them to estimate the dimensions of the cross sections, given the dimensions of the three-dimensional solids.



Name: Date: Period:

Activity 3 Drawing Cross Sections

Draw each cross section as if you were looking straight at it. Consider the dimensions of the solid and use them to estimate the dimensions of the cross section. The figures may not be drawn to scale.

1.

Drawing of cross section:
(Including estimated dimensions.)

2.

Drawing of cross section:
(Including estimated dimensions.)

Sample response.

3.

Drawing of cross section:
(Including estimated dimensions.)

Sample response.

STOP

Lesson 13 Slicing Solids 781

1 Launch

Set an expectation for the amount of time students have to work in pairs on the activity.

2 Monitor

Help students get started by reminding them that they have seen a rectangular prism sliced horizontally in Activity 1.

Look for points of confusion:

- **Not knowing which of the figure's dimensions are dimensions of the cross section.** Have students shade the face of the prism that is parallel to the slice.

Look for productive strategies:

- Drawing the cross sections first and then returning to consider the dimensions.

3 Connect

Have pairs of students share how they reasoned about the dimensions of their cross section.

Ask, "Why is it important to draw a cross section as if you were looking directly at it?" This is the true shape of the two-dimensional shape. The perspective is distorted when viewing it in the three-dimensional solid.

Highlight, "Sometimes a cross section can look just like the face of the shape, and other times it can look quite different. It's important to consider the direction of the slice and what type of three-dimensional solid is being sliced when visualizing what the cross section will look like."

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Consider providing pre-drawn cross sections that may or may not match each figure. Have students determine which cross section is correct for each figure.

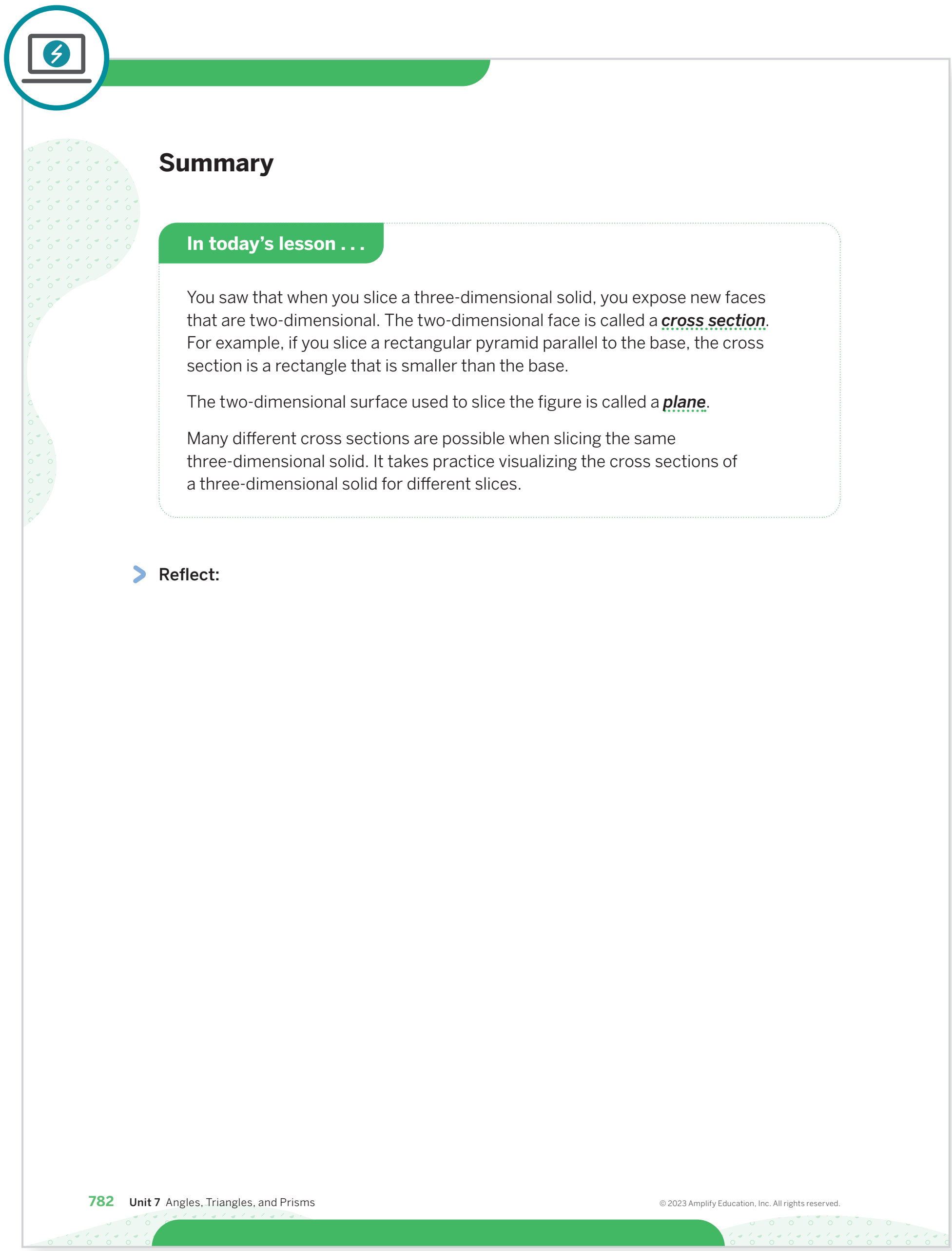
Extension: *Math Enrichment*

Have students complete the following problem:
For any given three-dimensional solid, what is the greatest number of sides you can have for a cross section of that solid? For any solid with n faces, the greatest number of sides for any of its cross sections is n. For example, for a rectangular prism, the greatest number of sides for a cross section is 6 because a rectangular prism has 6 sides.

Whole Class | 5 min

Summary

Review and synthesize why slicing a three-dimensional solid with a plane results in a two-dimensional cross section.



Summary

In today's lesson . . .

You saw that when you slice a three-dimensional solid, you expose new faces that are two-dimensional. The two-dimensional face is called a ***cross section***. For example, if you slice a rectangular pyramid parallel to the base, the cross section is a rectangle that is smaller than the base.

The two-dimensional surface used to slice the figure is called a ***plane***.

Many different cross sections are possible when slicing the same three-dimensional solid. It takes practice visualizing the cross sections of a three-dimensional solid for different slices.

Reflect:

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Synthesize

Display the digital tool that allows you, or students, to manipulate the cross section of a rectangular prism with a square base.

Ask, "What are all the possible cross sections that can be made when slicing this rectangular prism?" triangle, square, rectangle, pentagon, hexagon

Have pairs of students share or draw all the different cross sections they think can be made.

Highlight that there are many possible cross sections when slicing a three-dimensional solid.

Formalize vocabulary:

- *cross section*
- *plane*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What shapes can be seen when you slice through solid figures?"

Math Language Development

MLR2: Collect and Display

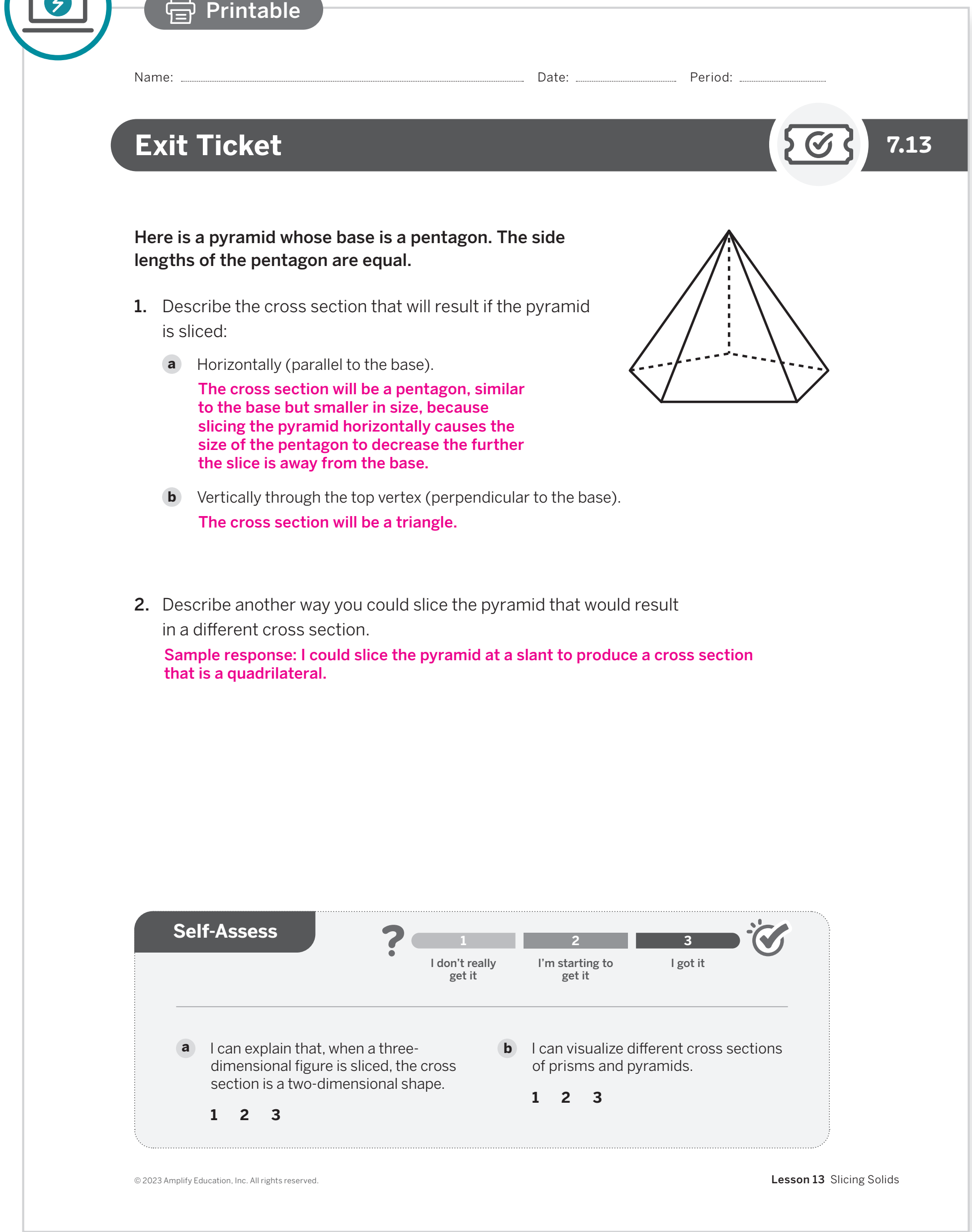
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *cross section* and *plane* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing cross sections that result from slicing a pentagonal pyramid.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 7.13

Here is a pyramid whose base is a pentagon. The side lengths of the pentagon are equal.

1. Describe the cross section that will result if the pyramid is sliced:
   - **a** Horizontally (parallel to the base).
     The cross section will be a pentagon, similar to the base but smaller in size, because slicing the pyramid horizontally causes the size of the pentagon to decrease the further the slice is away from the base.
   - **b** Vertically through the top vertex (perpendicular to the base).
     The cross section will be a triangle.
2. Describe another way you could slice the pyramid that would result in a different cross section.
   Sample response: I could slice the pyramid at a slant to produce a cross section that is a quadrilateral.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can explain that, when a three-dimensional figure is sliced, the cross section is a two-dimensional shape.
1 2 3

b I can visualize different cross sections of prisms and pyramids.
1 2 3

Success looks like . . .

- **Language Goal:** Categorizing images of planes intersecting pyramids and prisms and describing the categories. **(Speaking and Listening)**
  - » Describing the cross sections of the pyramid in Problem 1.
- **Language Goal:** Comprehending that the term *cross section* refers to the two-dimensional face that results from slicing a three-dimensional figure. **(Speaking and Listening, Writing)**
- **Language Goal:** Describing, comparing, and contrasting different cross sections that could result from slicing the same pyramid or prism. **(Speaking and Listening, Writing)**
  - » Describing a different cross section of the pyramid in Problem 2.

Suggested next steps

If students confuse the terms *horizontal* and *vertical*, consider:

- Creating an anchor chart to support them throughout the unit.

If students only think of a triangle and a pentagon and cannot think of another cross section for Problem 2, consider:

- Asking them to consider a vertical slice that does *not* pass through the *apex* (the single point at the top of the figure where the triangular faces meet).

Professional Learning

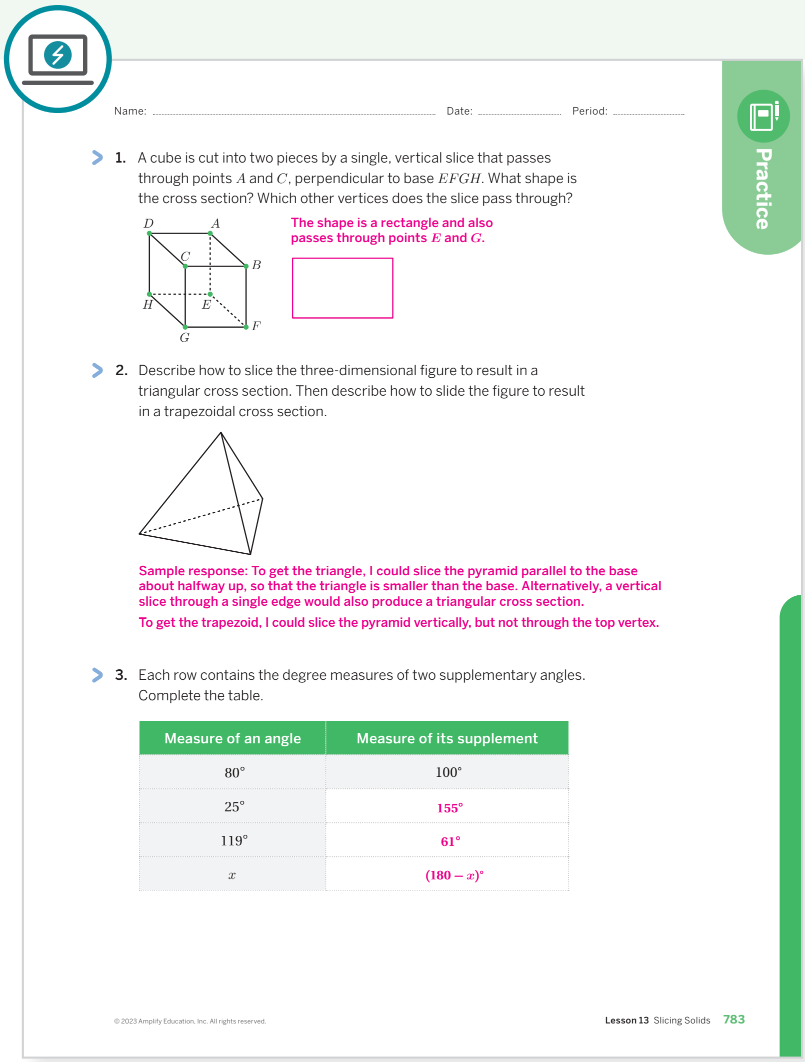
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Have you changed any ideas you used to have about cross sections as a result of today's lesson?
- What surprised you as your students worked through Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. A cube is cut into two pieces by a single, vertical slice that passes through points A and C, perpendicular to base $EFGH$. What shape is the cross section? Which other vertices does the slice pass through?

The shape is a rectangle and also passes through points E and G.

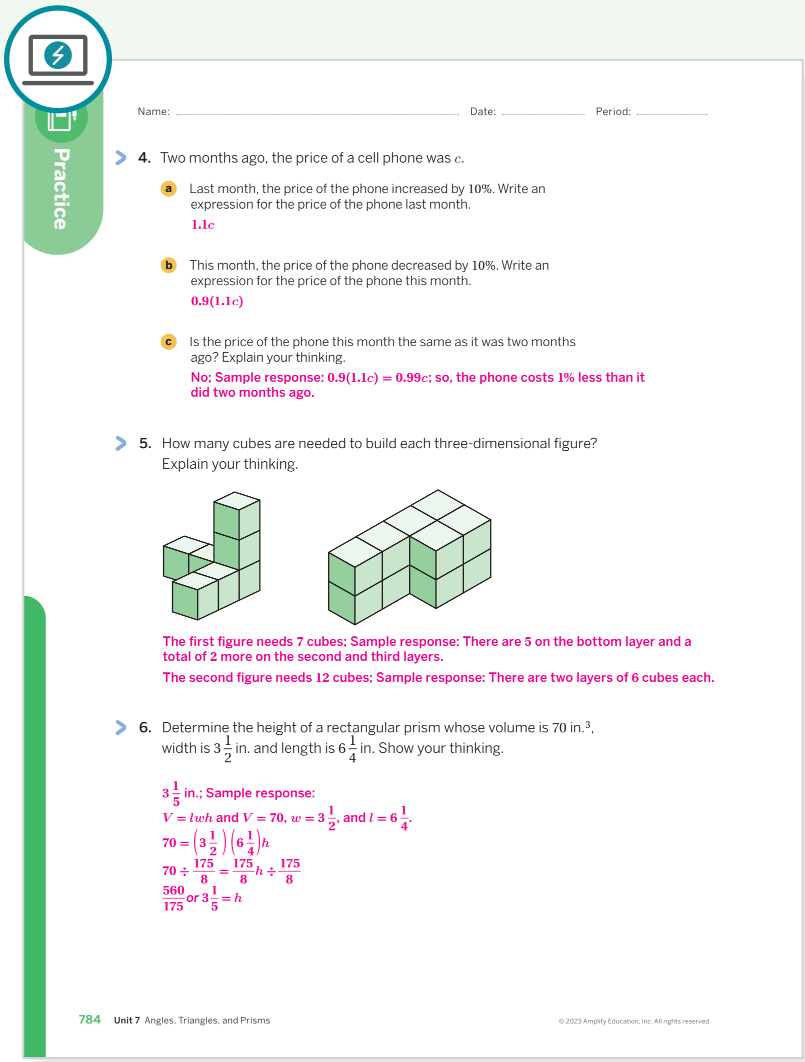
2. Describe how to slice the three-dimensional figure to result in a triangular cross section. Then describe how to slide the figure to result in a trapezoidal cross section.

Sample response: To get the triangle, I could slice the pyramid parallel to the base about halfway up, so that the triangle is smaller than the base. Alternatively, a vertical slice through a single edge would also produce a triangular cross section.

To get the trapezoid, I could slice the pyramid vertically, but not through the top vertex.

3. Each row contains the degree measures of two supplementary angles. Complete the table.

Measure of an angle	Measure of its supplement
80°	100°
25°	**155°**
119°	**61°**
x	**$(180 - x)°$**



Name: Date: Period:

Practice

4. Two months ago, the price of a cell phone was c.

   a. Last month, the price of the phone increased by 10%. Write an expression for the price of the phone last month.

   $1.1c$

   b. This month, the price of the phone decreased by 10%. Write an expression for the price of the phone this month.

   $0.9(1.1c)$

   c. Is the price of the phone this month the same as it was two months ago? Explain your thinking.

   No; Sample response: $0.9(1.1c) = 0.99c$; so, the phone costs 1% less than it did two months ago.

5. How many cubes are needed to build each three-dimensional figure? Explain your thinking.

The first figure needs 7 cubes; Sample response: There are 5 on the bottom layer and a total of 2 more on the second and third layers.

The second figure needs 12 cubes; Sample response: There are two layers of 6 cubes each.

6. Determine the height of a rectangular prism whose volume is 70 in.^{3}, width is $3\frac{1}{2}$ in. and length is $6\frac{1}{4}$ in. Show your thinking.

$3\frac{1}{5}$ in.; Sample response:

$V = lwh$ and $V = 70$, $w = 3\frac{1}{2}$, and $l = 6\frac{1}{4}$.

$$70 = \left(3\frac{1}{2}\right)\left(6\frac{1}{4}\right)h$$

$$70 \div \frac{175}{8} = \frac{175}{8}h \div \frac{175}{8}$$

$$\frac{560}{175} \text{ or } 3\frac{1}{5} = h$$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 3	2
Spiral	3	Unit 7 Lesson 3	1
	4	Unit 4 Lesson 6	2
	5	Grade 6	2
Formative ⚡	6	Unit 7 Lesson 14	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 14**

Volume of Right Prisms

Let’s look at volumes of prisms.

Focus

Goals

1. Determine the volume of a right prism by counting how many unit cubes it takes to build one layer, then multiplying by the number of layers.
2. **Language Goal:** Generalize the relationship between the volume of a prism, the area of its base, and its height. **(Speaking and Listening)**
3. Identify whether a given figure is a prism and, if so, identify its base and height.

Coherence

Today

In this lesson, students learn that they can calculate the volume of any right prism by multiplying the area of the base by the height of the prism. Students make sense of the structure of this formula by visualizing the prism decomposed into identical layers, each layer being 1 unit tall.

Previously

In Grades 5 and 6, students calculated the volume of rectangular prisms. In Lesson 13, students were reacquainted with three-dimensional solids in the form of prisms and pyramids.

Coming Soon

Students will continue working with the volume of right prisms, adding cases where the area of the base is found by decomposing a polygon into triangles and rectangles.

Rigor

- Students use snap cubes to create models to build their **conceptual understanding** of volume of right prisms.
- Students generalize the relationship between a prism’s base area and its height to develop **procedural fluency** in calculating volume.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut figure, one per group
- Activity 2 PDF, pre-cut and assembled, one set per pair
- Anchor Chart PDF, *Solids*
- Anchor Chart PDF, *Solids* (answers)
- rulers marked with centimeters
- snap cubes
  - » 30-60 per group

Math Language Development

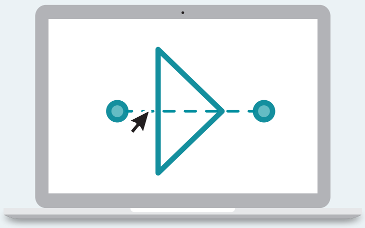
Review words

- *base (of a prism or pyramid)*
- *cross section*
- *plane*
- *prism*
- *pyramid*
- *volume*

Amps | Featured Activity

Activity 1
Building Layers of a Prism

Students use digital block manipulatives to build layers of a prism to better understand determining volume using slices.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students will use snap cubes to build a prism, layer by layer. They use the structure of the layers of the prism to determine a strategy for finding the volume for different numbers of layers. Some students will see this structure more readily than others; ask them to help explain the structure to their classmates who may not see the structure at first.

Modifications to Pacing

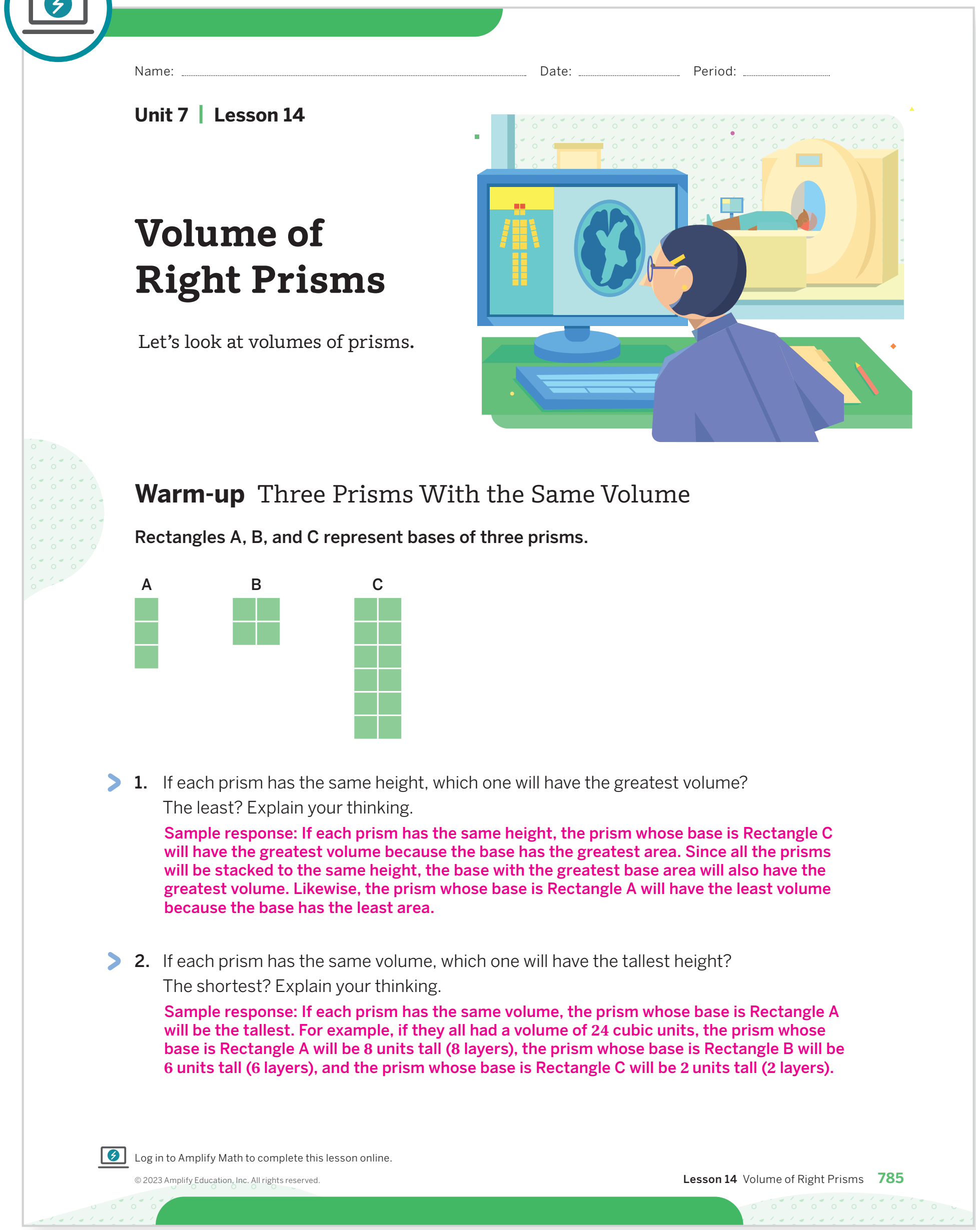
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students only consider bases A and B.
- In **Activity 2**, provide students with choice by allowing them to complete the table for any 2 or 3 figures.

Pairs | 5 min

Warm-up Three Prisms With the Same Volume

Students reason about the relationship between the area of the base of a prism and the prism's volume to activate knowledge from Grade 6 about the volume of a prism.



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Unit 7 | Lesson 14

Volume of Right Prisms

Let's look at volumes of prisms.

Warm-up Three Prisms With the Same Volume

Rectangles A, B, and C represent bases of three prisms.

A B C

1. If each prism has the same height, which one will have the greatest volume? The least? Explain your thinking.
   Sample response: If each prism has the same height, the prism whose base is Rectangle C will have the greatest volume because the base has the greatest area. Since all the prisms will be stacked to the same height, the base with the greatest base area will also have the greatest volume. Likewise, the prism whose base is Rectangle A will have the least volume because the base has the least area.

2. If each prism has the same volume, which one will have the tallest height? The shortest? Explain your thinking.
   Sample response: If each prism has the same volume, the prism whose base is Rectangle A will be the tallest. For example, if they all had a volume of 24 cubic units, the prism whose base is Rectangle A will be 8 units tall (8 layers), the prism whose base is Rectangle B will be 6 units tall (6 layers), and the prism whose base is Rectangle C will be 2 units tall (2 layers).

Log in to Amplify Math to complete this lesson online.

Lesson 14 Volume of Right Prisms 785

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by showing that each square in the base represents where a stack of cubes will be placed on top.

Look for points of confusion:

- **Being unsure how to find the volume.** Remind students that *volume* is the number of cubic units that occupy a three-dimensional figure. To find the volume, multiply the area of the base by the height.

Look for productive strategies:

- Selecting common multiples of 3, 4, and 8 to test their reasoning for Problem 2.

3 Connect

Display Rectangles A, B, and C.

Have students share the prism they found to have the greatest and least volume and the tallest and shortest height. Record and display their responses for all to see. Use the ***Poll the Class*** routine to determine whether students agree or disagree. If students all agree, ask a few students to share their reasoning. If some students do not agree, ask students to explain their reasoning until they reach an agreement.

Ask, "If each prism has the same volume and the prism associated with base B has a height of 6 units, what is the height of the prism associated with base C?" 3 units

Highlight that the area of the base of a prism directly affects the volume of the prism. For prisms with the same height, a prism with a greater base area will have a greater volume. For prisms with the same volume, a prism with a greater base area will be shorter.

Math Language Development

MLR8: Discussion Supports

During the Connect, when discussing how the area of the base of a prism directly affects the volume of a prism, consider providing students with manipulatives, such as two different rectangular prisms with the same height, but different base areas. Encourage students to discuss how changing the area of the base affects the *volume* of the rectangular prisms.

Power-up

To power up students' ability to determine the height of a rectangular prism given its volume, length, and width, have students complete:

Recall that the relationship between the volume of a rectangular prism and its dimension can be represented by $V = B \cdot h$ or $V = l \cdot w \cdot h$.

Determine the height of a rectangular prism whose volume is 50 cm^3, length is 10 cm, and width is 2.5 cm. Show your thinking.

2 cm; $V = l \cdot w \cdot h$ where $V = 50$, $l = 10$, and $w = 2.5$. $50 = (10) \cdot (2.5) \cdot h$ or $50 = 25h$.

$50 \div 25 = 25h \div 25$ or $2 = h$

Use: Before the Warm-up.

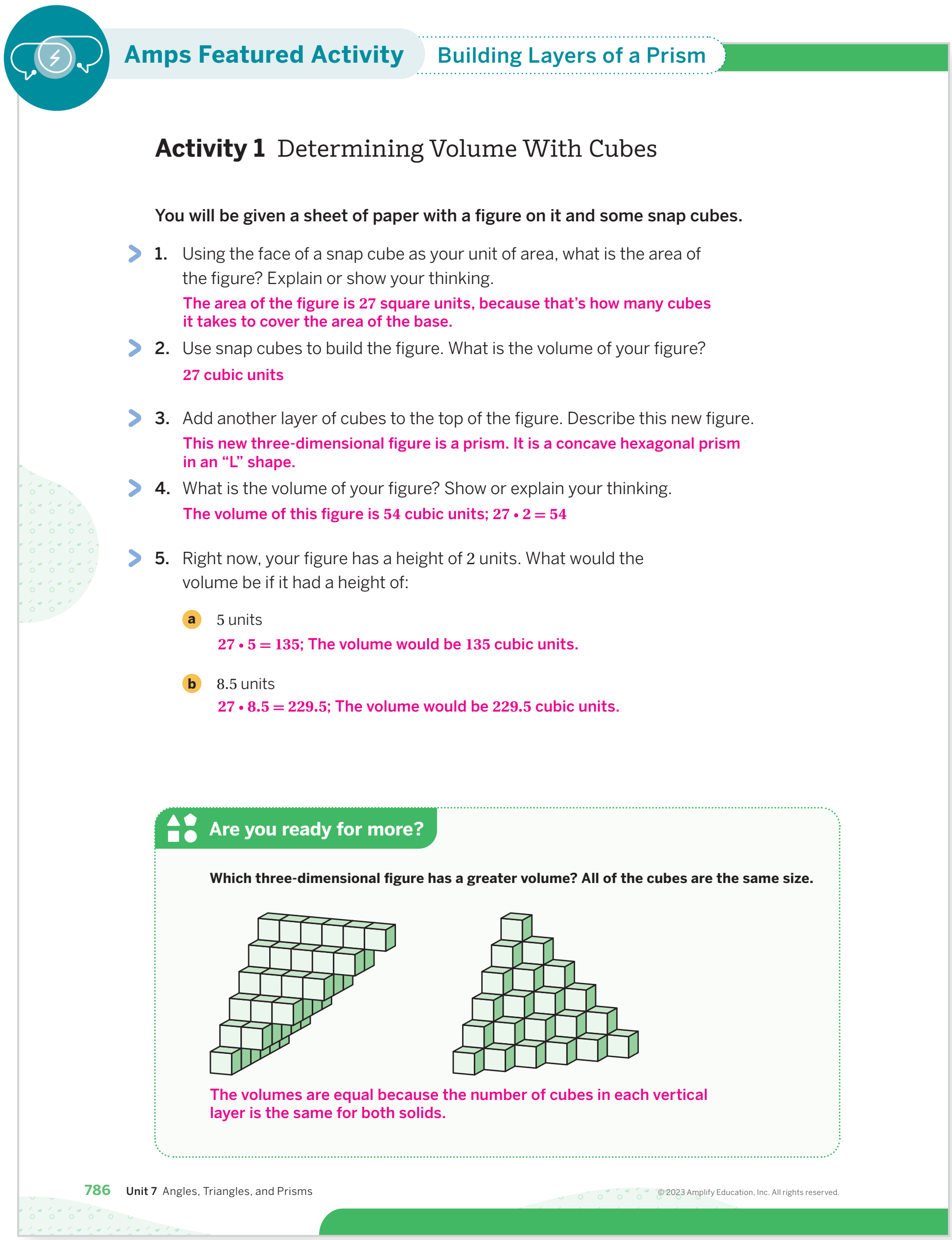
Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 6 and 7.

Small Groups | 15 min

Activity 1 Determine Volume With Cubes

Students use snap cubes to build a prism, helping them to generalize the formula for the volume of a prism.

Amps Featured Activity Building Layers of a Prism



Activity 1 Determining Volume With Cubes

You will be given a sheet of paper with a figure on it and some snap cubes.

1. Using the face of a snap cube as your unit of area, what is the area of the figure? Explain or show your thinking.
   The area of the figure is 27 square units, because that's how many cubes it takes to cover the area of the base.
2. Use snap cubes to build the figure. What is the volume of your figure?
   27 cubic units
3. Add another layer of cubes to the top of the figure. Describe this new figure.
   This new three-dimensional figure is a prism. It is a concave hexagonal prism in an "L" shape.
4. What is the volume of your figure? Show or explain your thinking.
   The volume of this figure is 54 cubic units; $27 \cdot 2 = 54$
5. Right now, your figure has a height of 2 units. What would the volume be if it had a height of:
   - a 5 units
     $27 \cdot 5 = 135$; The volume would be 135 cubic units.
   - b 8.5 units
     $27 \cdot 8.5 = 229.5$; The volume would be 229.5 cubic units.

Are you ready for more?

Which three-dimensional figure has a greater volume? All of the cubes are the same size.

The volumes are equal because the number of cubes in each vertical layer is the same for both solids.

786 **Unit 7** Angles, Triangles, and Prisms

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1 Launch

Distribute the shapes from the Activity 1 PDF to each group. Tell students they will build a prism using snap cubes, so that the base of the prism is the same shape as the figure on the PDF.

2 Monitor

Help students get started by showing them how to make sure the cubes are tight to the corners and edges of the figure on the PDF.

Look for points of confusion:

- **Being unsure how to use the face of a cube as a unit.** Ask, "Anything can be a unit if it is consistent. How many of the cube faces cover the shape?"

Look for productive strategies:

- Using multiplication strategies to find the volume for different heights of the prism.

3 Connect

Ask:

- "How do you know this figure is a prism?"
- "What is the area of the base of this prism?"
- "How do you calculate the total number of cubes to make the prism?"
- "If you find the area of the base, how do you use that information to calculate the volume of the prism?"
- "How would the volume of the prism change if the shape of the base changed, but you still used 27 cubes to build it?" The volume would not change.

Highlight that calculating the total number of cubes to make the prism is the same as calculating the volume of the prism. Students can find the area of the base of the prism and multiply that by the number of layers in the prism, i.e., the height of the prism.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital block manipulatives to build layers of a prism to better understand determining volume using slices.

Accessibility: *Optimize Access to Tools*

If you choose not to use the Amps slides for this activity, consider providing pre-built figures, each with a different number of layers, using snap cubes and asking students to analyze the figures and the number of layers to determine the volume of each.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, invite students to draft a response to the first Ask question, "How do you know this figure is a prism?" Have them meet with 2–3 partners to share and receive feedback on their responses. Display these prompts that reviewers can use to press for details.

- "Does the base have to be a rectangle to be a prism?"
- "What does each layer have to look like to be a prism?"

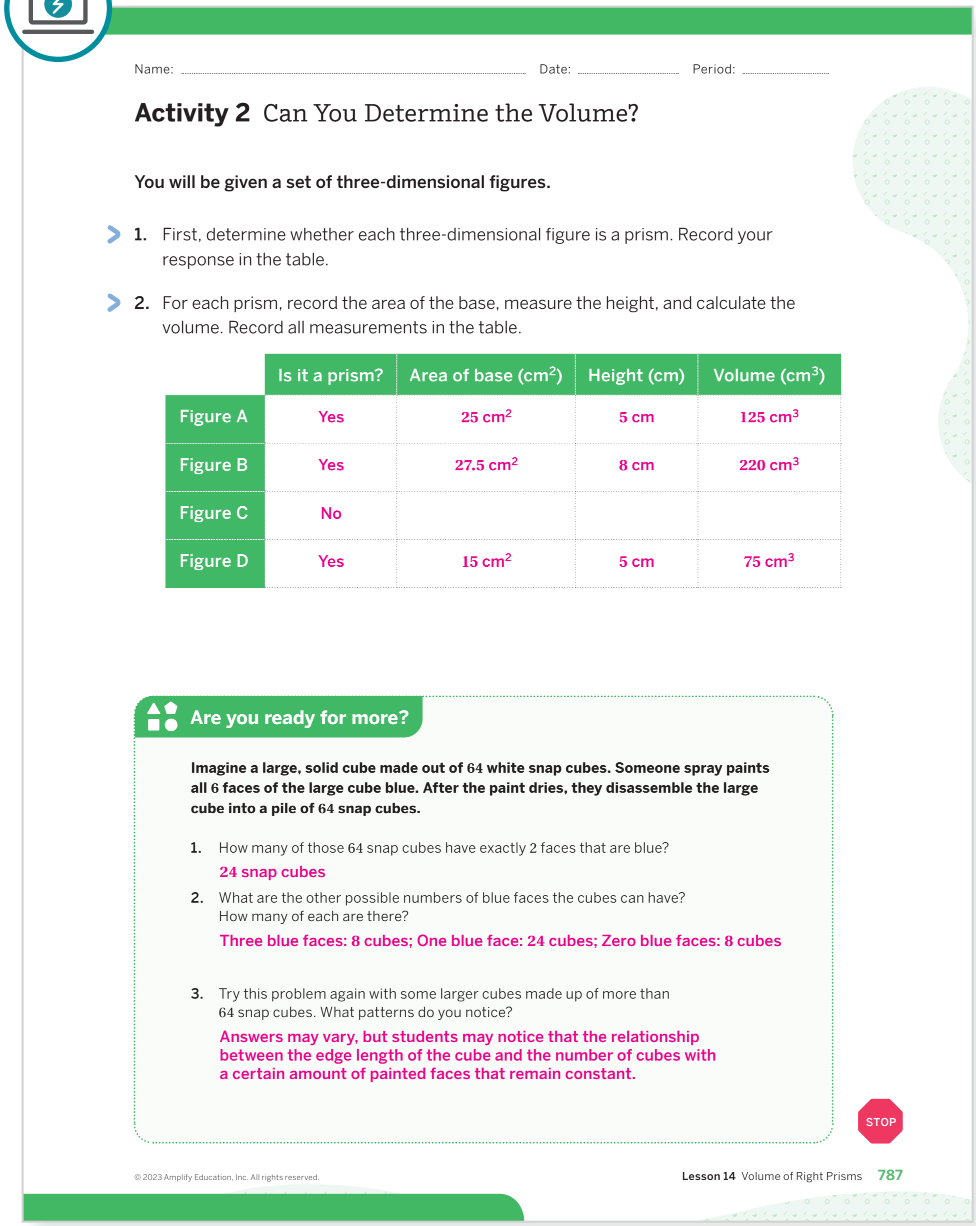
English Learners

Allow students to write their draft response in their primary language. Ask them to translate their final response to English.

Pairs | 15 min

Activity 2 Can You Determine the Volume?

Students engage in a hands-on experience with three-dimensional figures, recognizing whether a figure is a prism and, if so, determining its base area and finding its volume.



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Activity 2 Can You Determine the Volume?

You will be given a set of three-dimensional figures.

1. First, determine whether each three-dimensional figure is a prism. Record your response in the table.
2. For each prism, record the area of the base, measure the height, and calculate the volume. Record all measurements in the table.

	Is it a prism?	Area of base (cm^2)	Height (cm)	Volume (cm^3)
Figure A	Yes	25 cm^2	5 cm	125 cm^3
Figure B	Yes	27.5 cm^2	8 cm	220 cm^3
Figure C	No			
Figure D	Yes	15 cm^2	5 cm	75 cm^3

Are you ready for more?

Imagine a large, solid cube made out of 64 white snap cubes. Someone spray paints all 6 faces of the large cube blue. After the paint dries, they disassemble the large cube into a pile of 64 snap cubes.

1. How many of those 64 snap cubes have exactly 2 faces that are blue?

   24 snap cubes

2. What are the other possible numbers of blue faces the cubes can have? How many of each are there?

   Three blue faces: 8 cubes; One blue face: 24 cubes; Zero blue faces: 8 cubes

3. Try this problem again with some larger cubes made up of more than 64 snap cubes. What patterns do you notice?

   Answers may vary, but students may notice that the relationship between the edge length of the cube and the number of cubes with a certain amount of painted faces that remain constant.

STOP

1 Launch

Distribute one pre-assembled figures from the Activity 2 PDF and a centimeter ruler to each pair of students.

2 Monitor

Help students get started by asking them to explain the differences between prisms and pyramids.

Look for points of confusion:

- **Thinking that a cube is not a prism.** Ask students whether the cross sections would be identical if they made various slices parallel to one side.

Look for productive strategies:

- Looking for identical bases in the figures.
- Using the table to multiply the base area by the height to get the volume.

3 Connect

Have students share their responses to each part of the table and ask 1 or 2 volunteers to explain how they arrived at their responses. Display the responses in the table for all to see.

Ask:

- "What is different about the structure of non-prisms in comparison to prisms?"
- "Why can't you use the formula *area of the base times the height* to calculate the volume of the figures that are not prisms?"

Highlight that students have now found two ways to find the volume of a prism. If they can count the square units of the base, then they can multiply that amount by the height. If they can find the area of the base, then they can also multiply that by the height. Review the formula $V = Bh$, where V is the volume of the prism, B is the area of the prism's base, and h is the prism's height.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Eliminate Figure D from the task and have students focus on Figures A, B, and C. You may also choose to provide the measurements for the area of the base and the height for Figures A and B to allow students to focus on calculating the volume, as opposed to measuring.

Math Language Development

MLR3: Critique, Correct, Clarify

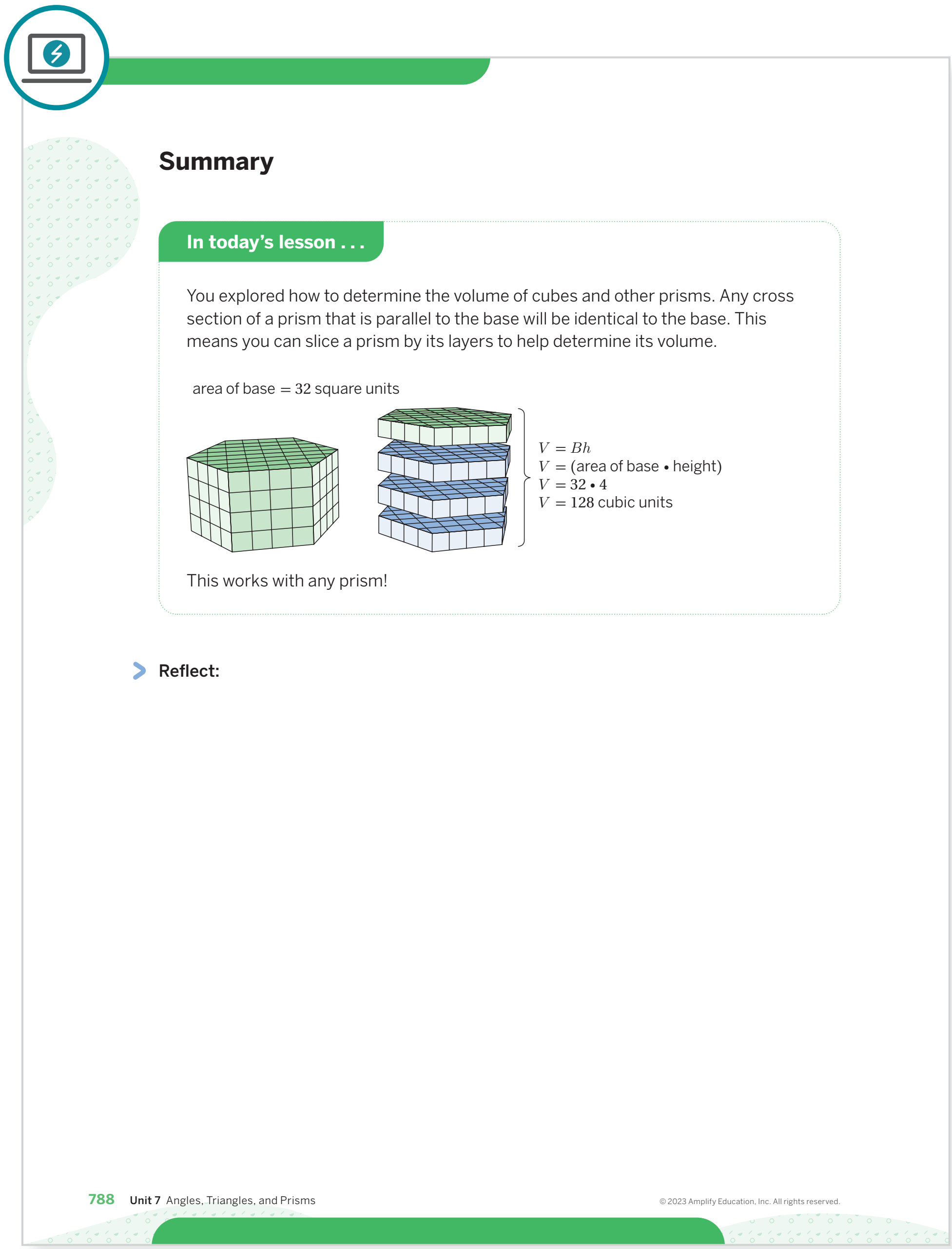
During the Connect and before students share, display an incorrect response such as, "Figure A is not a prism because it is a cube." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? What must be true for a figure to be a prism?" Sample response: I disagree. A cube is a prism because it has two parallel bases and all of the other faces are rectangles. It just happens that in a cube, all of these faces are equal-sized squares.
- ***Correct:*** "Write a corrected statement that is now true." Sample response: Figure A is a prism because it is a cube, and all cubes are prisms.
- ***Clarify:*** "How did you correct the statement? How do you know that the statement is true?"

Whole Class | 5 min

Summary

Review and synthesize how to find the volume of a prism using slices.



Summary

In today's lesson . . .

You explored how to determine the volume of cubes and other prisms. Any cross section of a prism that is parallel to the base will be identical to the base. This means you can slice a prism by its layers to help determine its volume.

area of base = 32 square units

$V = Bh$
$V =$ (area of base • height)
$V = 32 \cdot 4$
$V = 128$ cubic units

This works with any prism!

Reflect:

788 **Unit 7** Angles, Triangles, and Prisms

Synthesize

Display the Anchor Chart PDF, *Solids*, and complete the Volume section as you facilitate a class discussion using the following questions.

Ask:

- "How could you use layers or slices to find the volume of a prism?" If you look at the first layer of a prism, you can find how many cubes are in that layer by finding the area of the base. Once you find the number of cubes on the first layer, you multiply that by the number of layers it takes to stack up to the height of the prism.
- "Two prisms have the same base area and height, but different base shapes. Which prism has a greater volume? Explain." The two prisms have the same volume. The shape of the base does not matter if it is a prism, only the *area* of the base.

Have pairs of students share their responses to one of these questions with each other.

Highlight that any cross section of a prism that is parallel to the base will be identical to the base. This means students can think of slicing prisms to help find their volume. For example, if a rectangular prism has a height of 3 units, with a base measuring 4 units by 5 units, students can think of this as 3 layers, where each layer has 4×5 cubic units.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

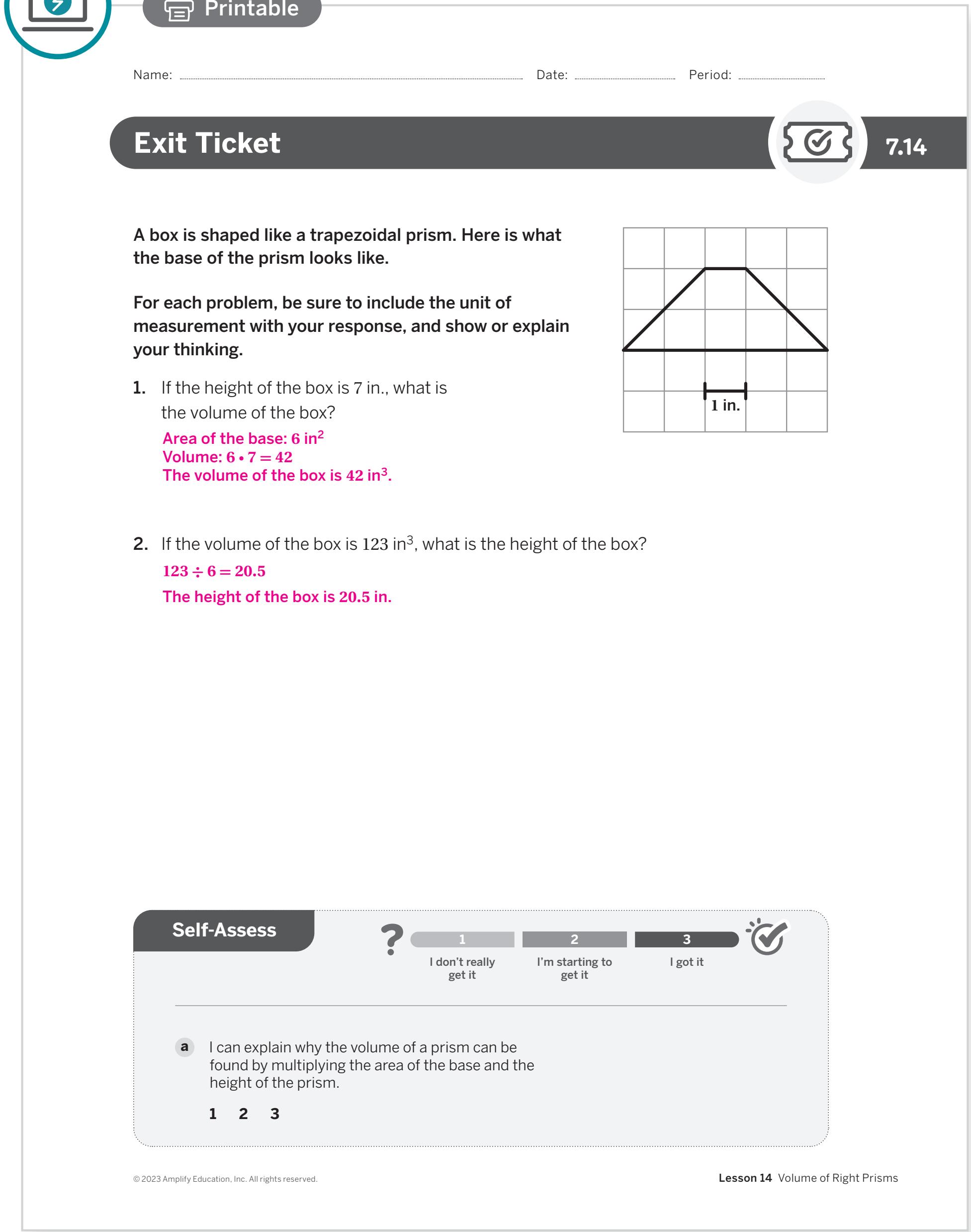
- "What process did you follow when creating your figure out of snap cubes? Do you think there was a more effective process?"
- "What characteristics did you look for when deciding whether a figure was a prism?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by finding the volume of a trapezoidal prism.

Printable



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Exit Ticket 7.14

A box is shaped like a trapezoidal prism. Here is what the base of the prism looks like.

For each problem, be sure to include the unit of measurement with your response, and show or explain your thinking.

1. If the height of the box is 7 in., what is the volume of the box?

   Area of the base: 6 in^2
   Volume: $6 \cdot 7 = 42$
   The volume of the box is 42 in^3.

2. If the volume of the box is 123 in^3, what is the height of the box?

   $123 \div 6 = 20.5$

   The height of the box is 20.5 in.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain why the volume of a prism can be found by multiplying the area of the base and the height of the prism.

1 2 3

Lesson 14 Volume of Right Prisms

Success looks like . . .

- **Goal:** Determining the volume of a right prism by counting how many unit cubes it takes to build one layer, then multiplying by the number of layers.
- **Language Goal:** Generalizing the relationship between the volume of a prism, the area of its base, and its height. **(Speaking and Listening)**
  - » Calculating the volume of the trapezoidal prism using the area of its base and the height in Problem 2.
- Identifying whether a given figure is a prism and, if so, identifying its base and height.

Suggested next steps

If students struggle to find the area of the base, consider:

- Having them count the unit squares.
- Partitioning the trapezoid into smaller shapes.

If students use the incorrect unit, consider:

- Having them think about what they are measuring. Say, "When measuring how many cubes fit in an object, use cubic units. When measuring how many unit squares cover an area, use square units."

If students struggle to find the height of the prism, consider:

- Asking them to imagine how they would find the number of layers in a prism that had a certain base area, if they know the total number of cubes.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What routines enabled all students to do math in today's lesson?
- Which teacher actions made facilitating the students in generalizing the relationship between base area, height, and volume strong? What might you change for the next time you teach this lesson?

Math Language Development

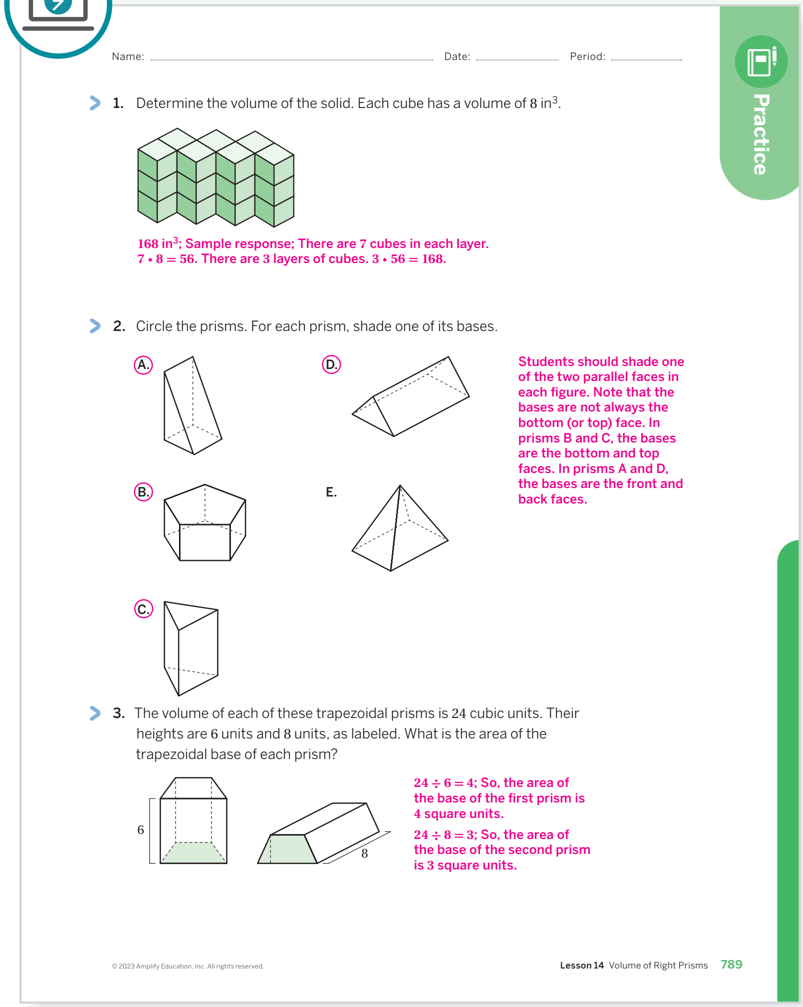
Language Goal: Generalizing the relationship between the volume of a prism, the area of its base, and its height.

Reflect on students' language development toward this goal.

- Are students able to describe two ways to determine the volume of a prism? What math language are they using in their descriptions? How can you help them be more precise in their descriptions?
- Reflect on the language routines used in this lesson? Were there any that were more helpful than others? Why? Would you change anything the next time you use these routines?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. Determine the volume of the solid. Each cube has a volume of 8 in^3.

168 in^3; Sample response; There are 7 cubes in each layer. $7 \cdot 8 = 56$. There are 3 layers of cubes. $3 \cdot 56 = 168$.

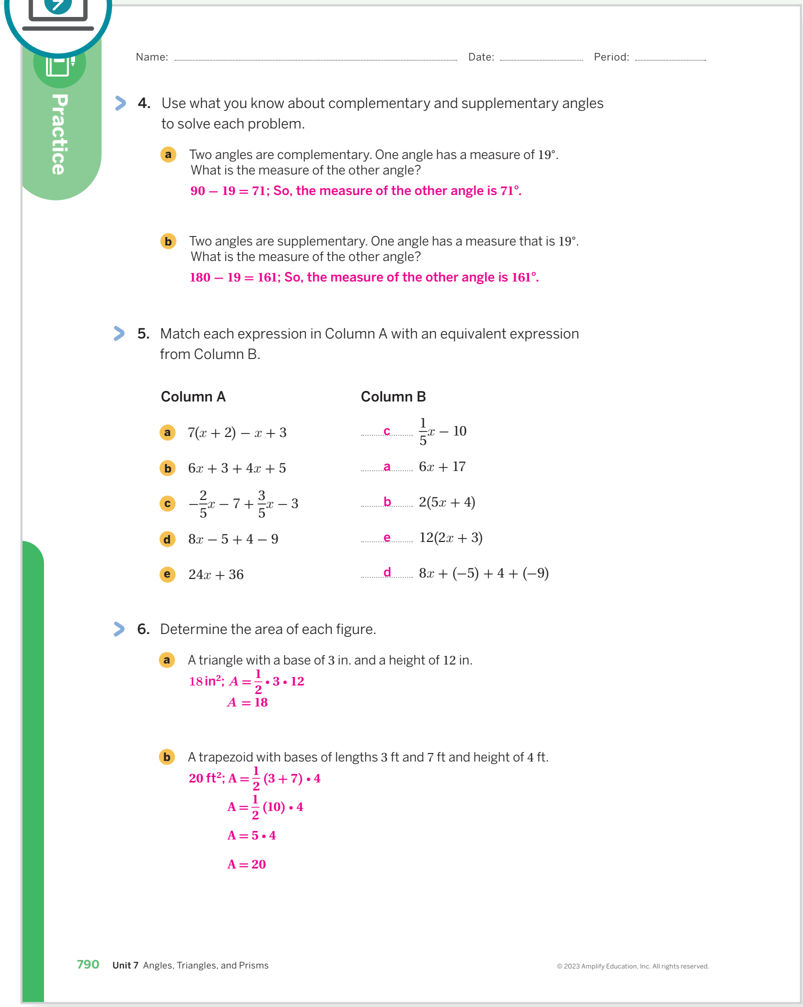
2. Circle the prisms. For each prism, shade one of its bases.

Students should shade one of the two parallel faces in each figure. Note that the bases are not always the bottom (or top) face. In prisms B and C, the bases are the bottom and top faces. In prisms A and D, the bases are the front and back faces.

3. The volume of each of these trapezoidal prisms is 24 cubic units. Their heights are 6 units and 8 units, as labeled. What is the area of the trapezoidal base of each prism?

$24 \div 6 = 4$; So, the area of the base of the first prism is 4 square units.

$24 \div 8 = 3$; So, the area of the base of the second prism is 3 square units.



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Practice

4. Use what you know about complementary and supplementary angles to solve each problem.

a. Two angles are complementary. One angle has a measure of 19°. What is the measure of the other angle?

$90 - 19 = 71$; So, the measure of the other angle is 71°.

b. Two angles are supplementary. One angle has a measure that is 19°. What is the measure of the other angle?

$180 - 19 = 161$; So, the measure of the other angle is 161°.

5. Match each expression in Column A with an equivalent expression from Column B.

Column A		Column B
a. $7(x+2) - x + 3$	c	$\frac{1}{5}x - 10$
b. $6x + 3 + 4x + 5$	a	$6x + 17$
c. $-\frac{2}{5}x - 7 + \frac{3}{5}x - 3$	b	$2(5x + 4)$
d. $8x - 5 + 4 - 9$	e	$12(2x + 3)$
e. $24x + 36$	d	$8x + (-5) + 4 + (-9)$

6. Determine the area of each figure.

a. A triangle with a base of 3 in. and a height of 12 in.

18 in^2; $A = \frac{1}{2} \cdot 3 \cdot 12$

$A = 18$

b. A trapezoid with bases of lengths 3 ft and 7 ft and height of 4 ft.

20 ft^2; $A = \frac{1}{2}(3 + 7) \cdot 4$

$A = \frac{1}{2}(10) \cdot 4$

$A = 5 \cdot 4$

$A = 20$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 3	1
	5	Unit 6 Lesson 22	2
Formative	6	Unit 7 Lesson 15	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 15**

Decomposing Bases for Area

Let's look at the bases
of different solids.

Focus

Goals

1. **Language Goal:** Critique different methods for decomposing and calculating the area of a prism's base. **(Speaking and Listening)**
2. **Language Goal:** Explain how to decompose and calculate the area of a prism's base and then use it to calculate the prism's volume. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue working with the volume of right prisms. They encounter prisms where the base is composed of triangles and rectangles and decompose the base to calculate its area. They also work with solids, such as a heart-shaped box or house-shaped figures, where they have to identify the base in order to view the solid as a prism and calculate its volume. Students look for the prism structure in a shape to solve problems.

Previously

In Lesson 14, students learned that the volume of any right prism is found by multiplying the area of the base and the height of the prism.

Coming Soon

In future lessons, students will find the surface area for right prisms and draw conclusions of when surface area or volume is needed to solve a problem. In Lesson 17, students will revisit the heart-shaped box to find its surface area.

Rigor

- Students strengthen their **fluency** in calculating the volume of right prisms.
- Students **apply** their understanding of area and volume to determine the volume of real-world objects.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	15 min	5 min	10 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- Activity 2 PDF, *Plans*, one per student (as needed)
- Activity 2 PDF, *Lin's Plan* (for display)

Math Language Development

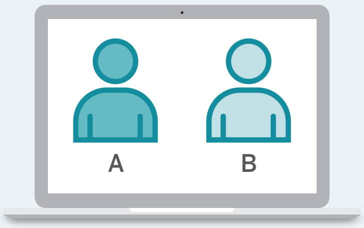
Review words

- *base (of a prism or pyramid)*
- *cross section*
- *plane*
- *prism*
- *pyramid*
- *volume*

Amps | **Featured Activity**

Activity 2
Comparing Methods

Students choose one method and their partners choose another method for finding the area of the base. Afterwards, they compare areas, discussing and resolving any differences.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students might not make sense of problems by focusing on the shape which does not show a traditionally-curved heart in Activity 2. Have students control their impulse to get stuck on something that is not familiar and explore the purpose of it. After completing the activity, ask students to identify why they think the heart was not curved.

Modifications to Pacing

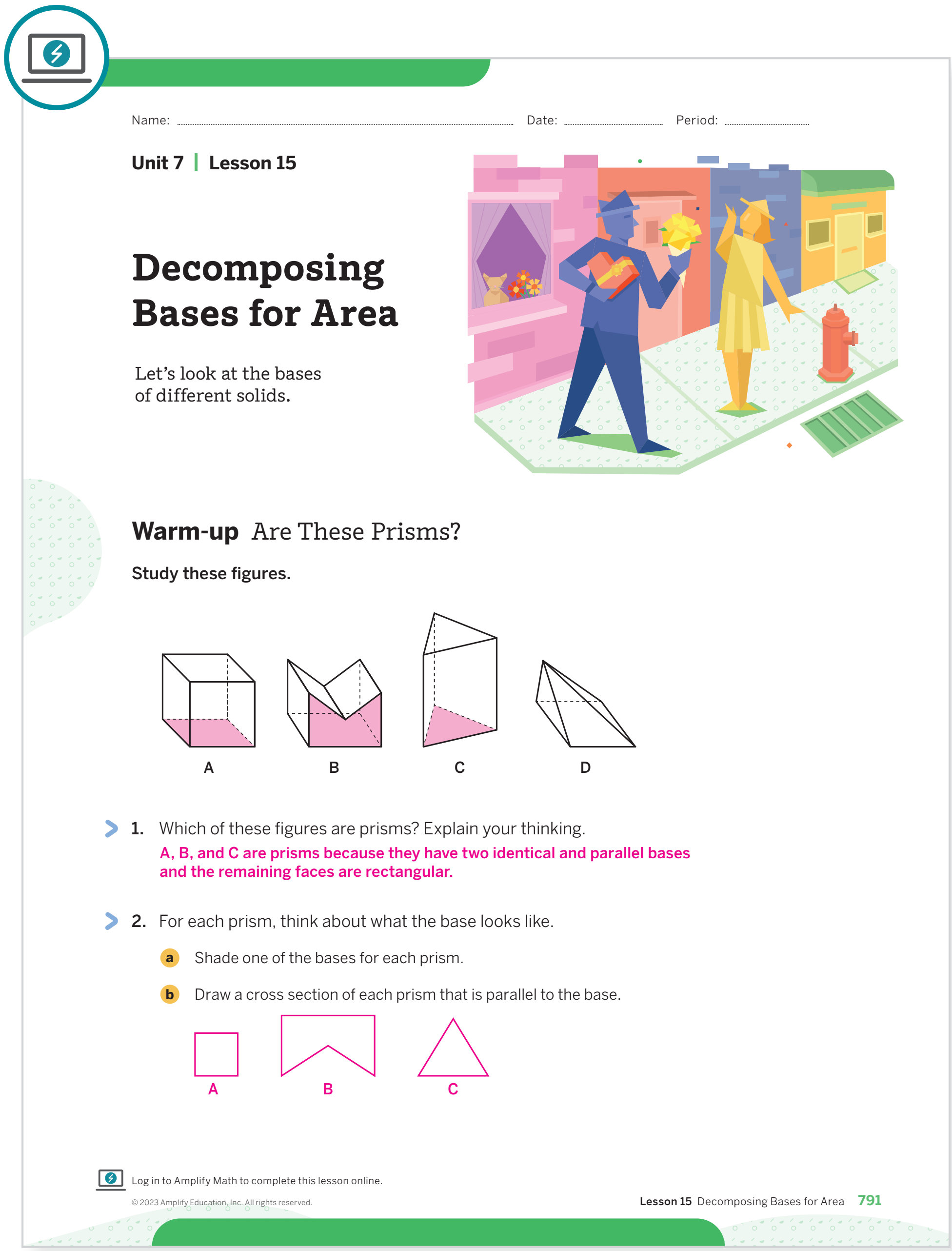
You may want to consider these additional modifications if you are short on time.

- The **Warm-up**, Problem 2 may be omitted.
- You may also choose to have the class complete only **Activity 1** or **Activity 2**. If Activity 1 is omitted, be prepared to support students with decomposing the figure in Activity 2.

Pairs | 5 min

Warm-up Are These Prisms?

Students determine what the bases of different prisms look like to prepare them for decomposing complex base shapes to find their areas.



Name: Date: Period:

Unit 7 | Lesson 15

Decomposing Bases for Area

Let's look at the bases of different solids.

Warm-up Are These Prisms?

Study these figures.

1. Which of these figures are prisms? Explain your thinking.
   A, B, and C are prisms because they have two identical and parallel bases and the remaining faces are rectangular.

2. For each prism, think about what the base looks like.
   - a Shade one of the bases for each prism.
   - b Draw a cross section of each prism that is parallel to the base.

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by using physical or digital models of prisms (not necessarily the ones shown) to demonstrate how to find bases and visualize cross sections.

Look for points of confusion:

- **Not realizing that Figure B is a prism.** Remind students that *base* does not always mean *bottom*. Ask students which face of this prism represents the base, mentioning that a prism has two parallel bases that are the same size and shape.
- **Drawing a parallelogram (but not a rectangle) for the base of Figure A.** Remind students these diagrams are drawn to show multiple perspectives, which distorts the actual shape of the faces.

3 Connect

Display the figures.

Have students share their responses to Problems 1-3.

Highlight that the base and the cross section parallel to the base will be identical if the figure is a prism. Ask students to draw other prisms to verify this is true. Then ask students why a cross section parallel to the base of a pyramid will not be identical to the base.

Differentiated Support

Extension: *Math Enrichment*

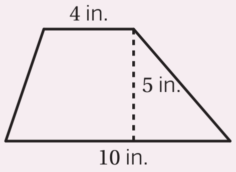
During the Connect, as students discuss why a cross section parallel to the base of a *pyramid* will *not* be identical to the base, have them draw various cross sections to illustrate why this is so. Ask them to explain what happens to the size of the cross section as it gets farther away from the base.

Power-up

To power up students' ability to determine the area of a trapezoid, have students complete:

Recall that the formula for determining the area of a trapezoid is $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the two bases and h is the height.

Determine the area of the given trapezoid. 35 in^2



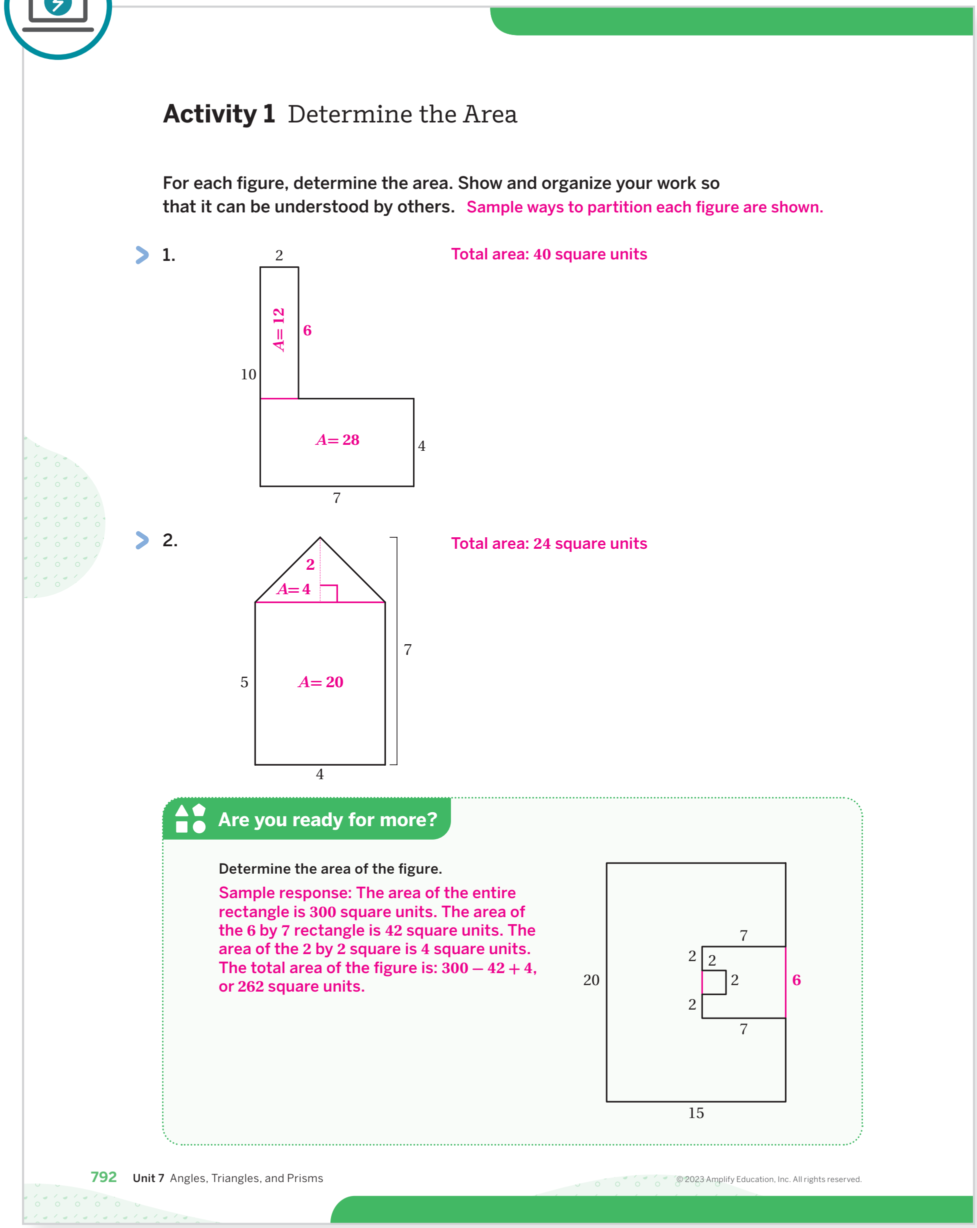
Use: Before the Warm-up.

Informed by: Performance on Lesson 14, Practice Problem 6b and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 10 min

Activity 1 Determine the Area

Students decompose complex shapes into simpler shapes and find their areas to prepare them for decomposing complex shapes that represent the bases of prisms.



Activity 1 Determine the Area

For each figure, determine the area. Show and organize your work so that it can be understood by others. Sample ways to partition each figure are shown.

1\. Total area: 40 square units

2\. Total area: 24 square units

Are you ready for more?

Determine the area of the figure.

Sample response: The area of the entire rectangle is 300 square units. The area of the 6 by 7 rectangle is 42 square units. The area of the 2 by 2 square is 4 square units. The total area of the figure is: 300 – 42 + 4, or 262 square units.

792 Unit 7 Angles, Triangles, and Prisms

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① Launch

Activate background knowledge by asking students to explain how to find the area of a rectangle and triangle. Explain that it will be important for them to organize their work so that others can follow it. Ask, "Once you find the area of the smaller figures, how do you find the total area?"

② Monitor

Help students get started by asking them to decompose the figures into simpler shapes.

Look for points of confusion:

- **Mislabeling the dimensions of the smaller figures.** Have students use a pencil to trace the sides that are parallel and use them to determine the lengths of the unknown dimensions using addition and or subtraction.

Look for productive strategies:

- **Showing their work in an organized manner.** Use their work as examples during the Connect.
- **Finding the area of the larger 7 by 10 rectangle and subtracting the area of the smaller 6 by 5 rectangle in Problem 1.** Use this as an alternative solution during the class discussion.

③ Connect

Display student work that is organized and easy to follow. If possible, have an example of pre-made disorganized work to discuss, being careful not to call out any individual student work that may be disorganized.

Highlight that decomposing complicated shapes into triangles and rectangles helps determine the area of the larger shape. However, the large shapes should be decomposed in ways where students can determine the dimensions of the smaller shapes. Ask, "Are there any other ways to decompose these shapes?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to partition the figure in Problem 1 in two different ways. Illustrate for students how the total area is the same regardless of how the figure is partitioned.

Extension: *Math Enrichment*

Use the internet, or another source, to provide images of real-world objects that are composed of triangles and rectangles. Ask students to find an image — or draw one — that can be decomposed into rectangles and triangles and have them show how to partition the figure.

Math Language Development

MLR3: Critique, Correct, Clarify

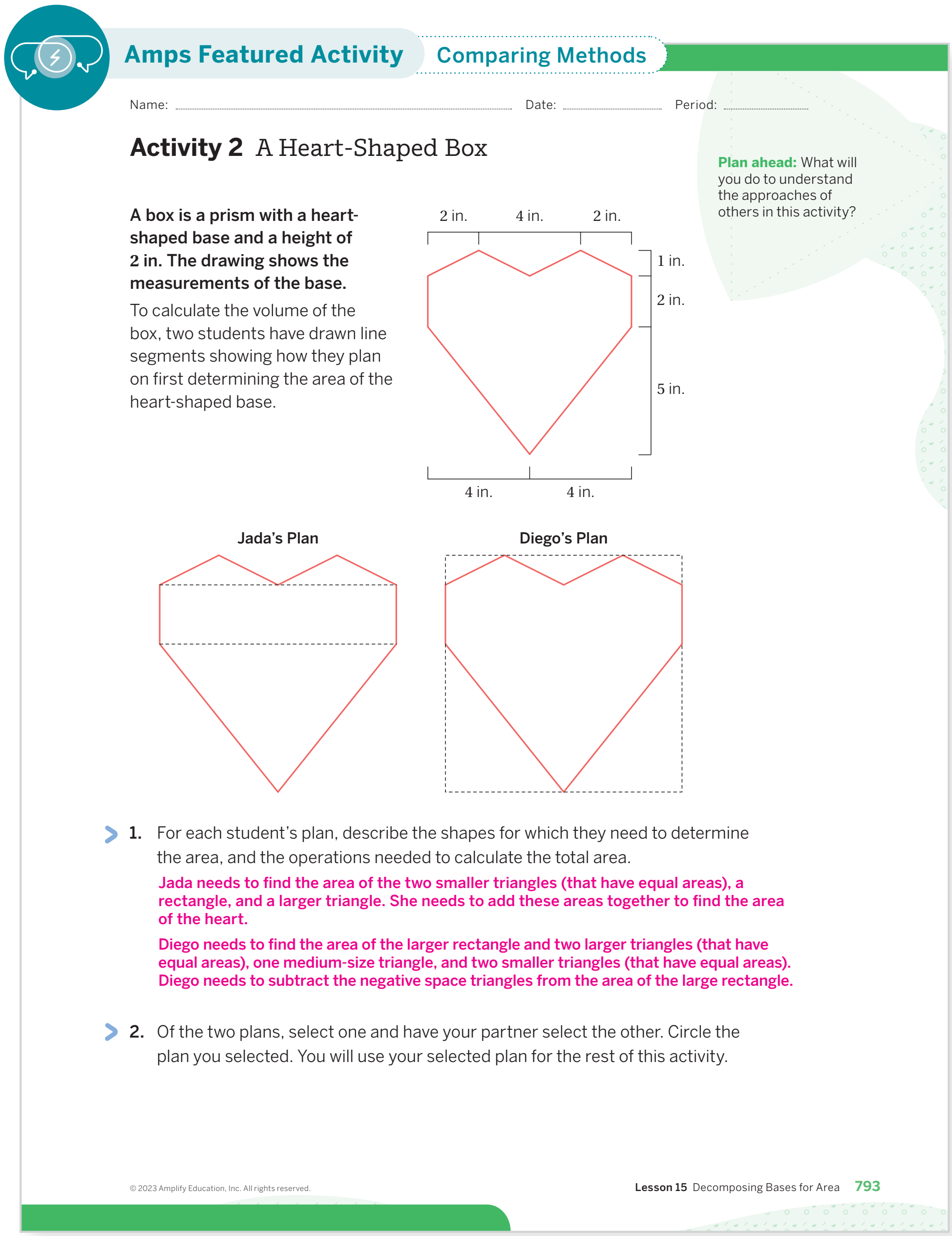
Display or provide copies of a premade example of disorganized decomposition work. Ask:

- ***Critique:*** Have students discuss in pairs what makes the work disorganized. Encourage them to include specific details or examples of why they think the work is unclear.
- ***Correct and Clarify:*** Ask students to write a few sentences describing how the work could be better organized so that others can understand it. Have them assume they are writing to a peer helping them understand how to show their work more clearly.

Pairs | 15 min

Activity 2 A Heart-Shaped Box

Students analyze two decompositions of a heart-shaped box, which helps them consider different strategies for decomposing figures.



Amps Featured Activity Comparing Methods

Name: ______________________ Date: __________ Period: __________

Activity 2 A Heart-Shaped Box

Plan ahead: What will you do to understand the approaches of others in this activity?

A box is a prism with a heart-shaped base and a height of 2 in. The drawing shows the measurements of the base.

To calculate the volume of the box, two students have drawn line segments showing how they plan on first determining the area of the heart-shaped base.

1. For each student's plan, describe the shapes for which they need to determine the area, and the operations needed to calculate the total area.

   Jada needs to find the area of the two smaller triangles (that have equal areas), a rectangle, and a larger triangle. She needs to add these areas together to find the area of the heart.

   Diego needs to find the area of the larger rectangle and two larger triangles (that have equal areas), one medium-size triangle, and two smaller triangles (that have equal areas). Diego needs to subtract the negative space triangles from the area of the large rectangle.

2. Of the two plans, select one and have your partner select the other. Circle the plan you selected. You will use your selected plan for the rest of this activity.

1 Launch

Display the Activity 2 PDF, showing the heart-shaped box. Have students brainstorm about how the shape of the base could be decomposed into simpler shapes, such as rectangles and triangles.

2 Monitor

Help students get started by providing a copy of Activity 2 PDF and having them choose a plan (Jada's or Diego's) and mark known lengths on the plan they choose.

Look for points of confusion:

- **Missing sections.** Have students write and circle their known areas in the sections of the heart. Have them count the number of sections to make sure they know all the smaller areas before finding the sum.
- **Not understanding Diego's plan.** Ask, "What shape is formed by the dotted lines around the figure? If you knew the area of this shape, how could you find the area of the heart?"

Look for productive strategies:

- **Noticing repetitions among sections.** If students notice the sections have the same shape, they can save time by not calculating the same area twice.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide copies of the Activity 2 PDF to students with the dimensions for each plan pre-labeled. They may use these copies of Jada's plan and Diego's plan to help them organize their work.

Extension: *Math Enrichment*

Ask students where they may have seen a heart-shaped box before, or any box that is in the shape of a prism with a unique base shape. Have them research product packaging to learn more about the different shapes of packages that companies use and why they may choose certain shapes over others, depending on cost efficiencies or the shapes of the products they need to ship.

Math Language Development

MLR5: Co-craft Questions

Display the image of the heart-shaped box from the Activity 2 PDF and the introduction to Activity 2 that shows the dimensions of the base. Ask students to work with their partner to write 2–3 mathematical questions about the heart-shaped box. Sample questions could be:

- "How could I decompose this shape to determine its area?"
- "Is the right side of the shape the same as the left side? Can I use that information somehow?"

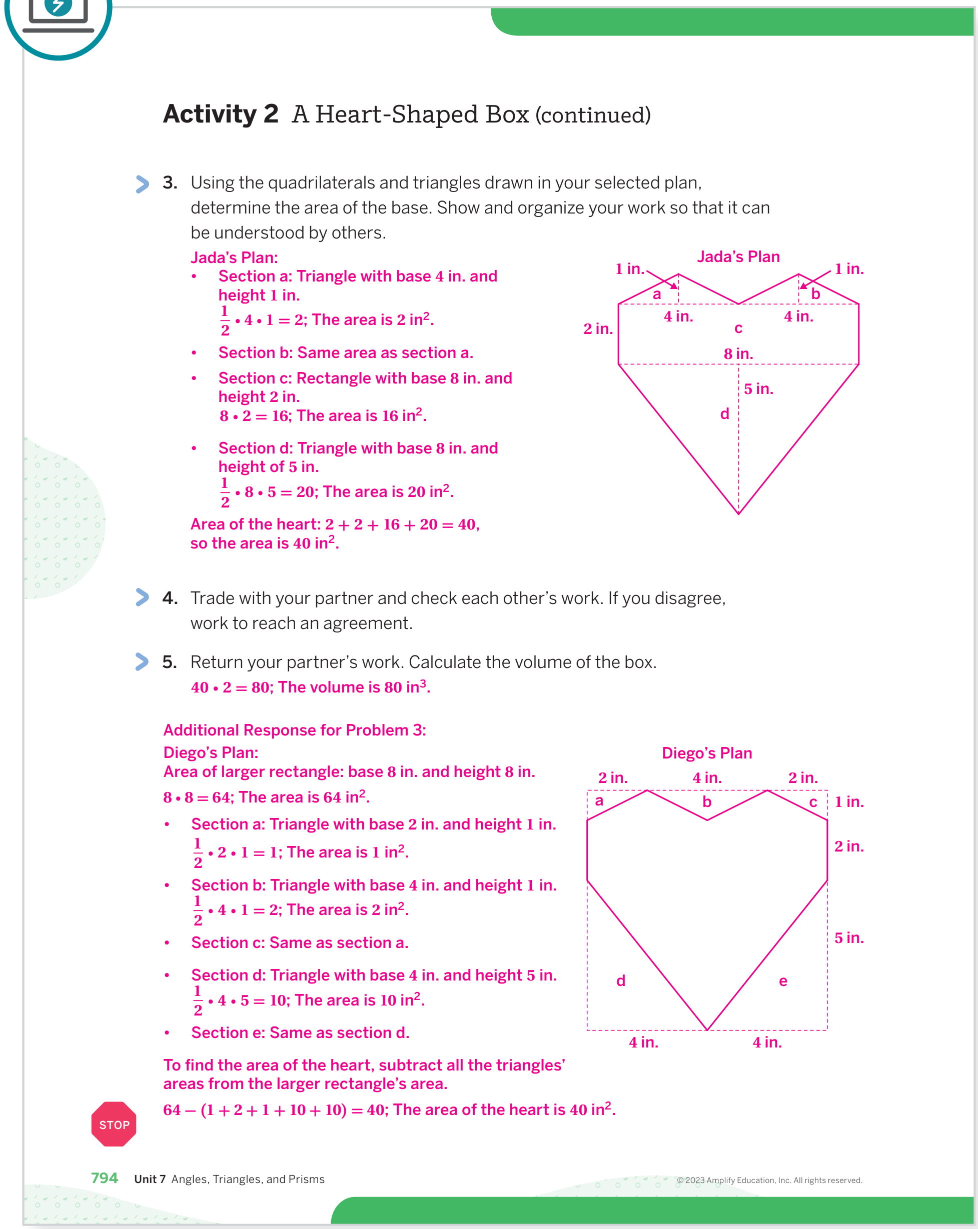
English Learners

Display 1 or 2 sample questions and allow students to say their questions aloud as opposed to writing them, or vice versa.

Pairs | 15 min

Activity 2 A Heart-Shaped Box (continued)

Students analyze two decompositions of the same heart-shaped box, which helps them consider different strategies for decomposing figures.



Activity 2 A Heart-Shaped Box (continued)

3. Using the quadrilaterals and triangles drawn in your selected plan, determine the area of the base. Show and organize your work so that it can be understood by others.

Jada's Plan:

- **Section a: Triangle with base 4 in. and height 1 in.**
  $\frac{1}{2} \cdot 4 \cdot 1 = 2$; The area is 2 in^2.
- **Section b: Same area as section a.**
- **Section c: Rectangle with base 8 in. and height 2 in.**
  $8 \cdot 2 = 16$; The area is 16 in^2.
- **Section d: Triangle with base 8 in. and height of 5 in.**
  $\frac{1}{2} \cdot 8 \cdot 5 = 20$; The area is 20 in^2.

Area of the heart: $2 + 2 + 16 + 20 = 40$, so the area is 40 in^2.

4. Trade with your partner and check each other's work. If you disagree, work to reach an agreement.

5. Return your partner's work. Calculate the volume of the box.
$40 \cdot 2 = 80$; The volume is 80 in^3.

Additional Response for Problem 3:
Diego's Plan:
Area of larger rectangle: base 8 in. and height 8 in.
$8 \cdot 8 = 64$; The area is 64 in^2.

- **Section a: Triangle with base 2 in. and height 1 in.**
  $\frac{1}{2} \cdot 2 \cdot 1 = 1$; The area is 1 in^2.
- **Section b: Triangle with base 4 in. and height 1 in.**
  $\frac{1}{2} \cdot 4 \cdot 1 = 2$; The area is 2 in^2.
- **Section c: Same as section a.**
- **Section d: Triangle with base 4 in. and height 5 in.**
  $\frac{1}{2} \cdot 4 \cdot 5 = 10$; The area is 10 in^2.
- **Section e: Same as section d.**

To find the area of the heart, subtract all the triangles' areas from the larger rectangle's area.
$64 - (1 + 2 + 1 + 10 + 10) = 40$; The area of the heart is 40 in^2.

3 Connect

Have students share their work and how they found the area of the heart.

Display the Activity 2 PDF, *Lin's Plan*.

Ask:

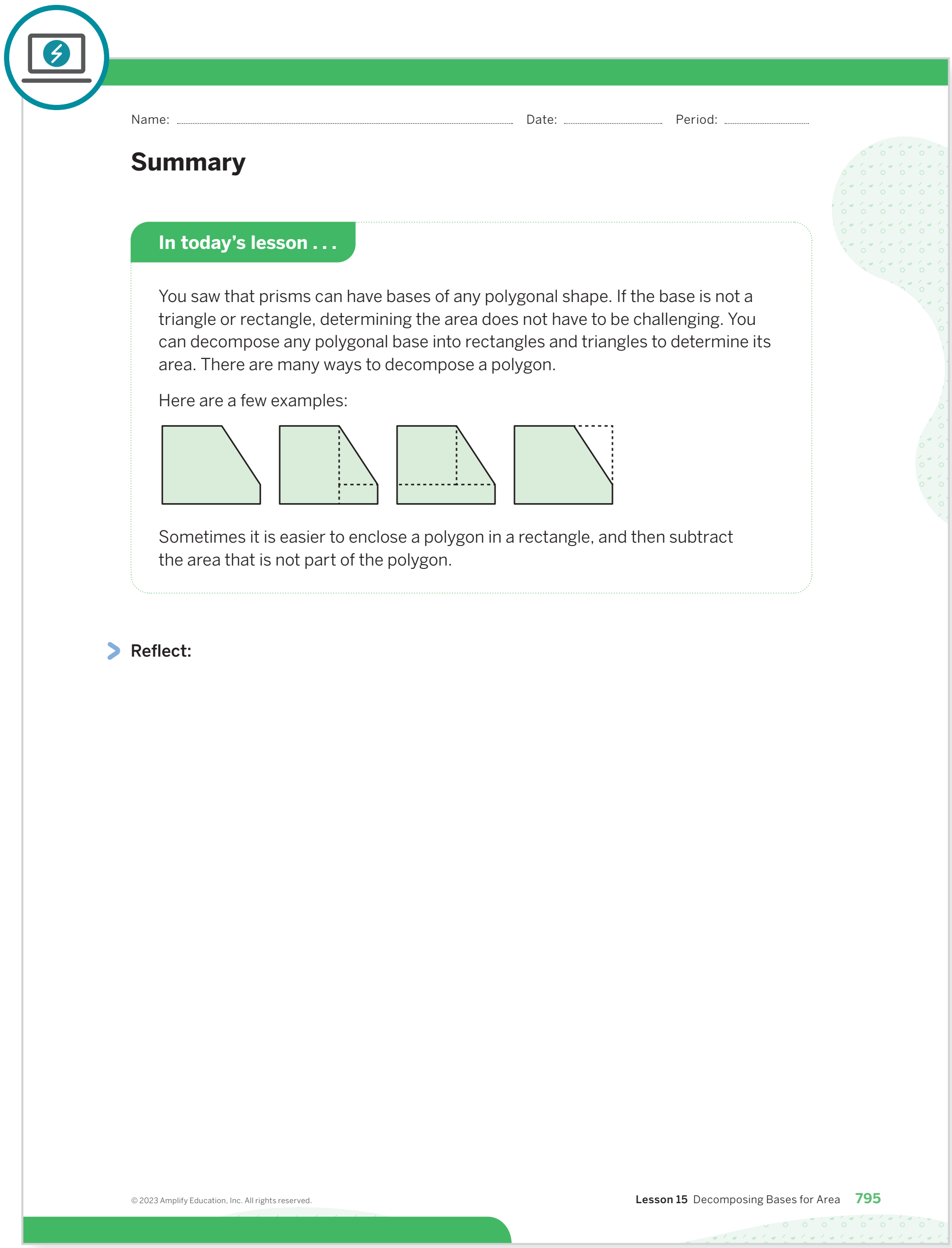
- "What do you think of Lin's plan? Can you use Lin's plan?" Sample response: I don't know how to find the bases of the trapezoid, because they are slanted.
  Note: In Grade 7, students do not know how to find slanted lengths, unless those measures are known. In Grade 8, they will learn about the Pythagorean Theorem, which will allow them to find measures like these.
- "When you have a base that is not a rectangle or triangle, what can you do to find the area?" Decompose it into smaller shapes of rectangles and triangles. Or, draw a larger rectangle around the shape and subtract the negative space.
- "After finding the area of the base, what did you have to do to find the volume?" Multiply the area by the height of the box which is 2 in.

Highlight student work which is organized and easy to follow. Point out that they might encounter figures with non-rectangular bases in future lessons. It will be important for them to think about using different strategies to calculate the area of the base.

Whole Class | 5 min

Summary

Review and synthesize how decomposing bases into rectangles and triangles can help in determining the area of a complex shape.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that prisms can have bases of any polygonal shape. If the base is not a triangle or rectangle, determining the area does not have to be challenging. You can decompose any polygonal base into rectangles and triangles to determine its area. There are many ways to decompose a polygon.

Here are a few examples:

Sometimes it is easier to enclose a polygon in a rectangle, and then subtract the area that is not part of the polygon.

Reflect:

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Lesson 15 Decomposing Bases for Area 795

Synthesize

Display the figures from the Summary.

Ask, "When the base is not a rectangle or a triangle, what are some methods for determining the area?" Decompose the base into rectangles and triangles. Another method is to imagine a larger figure (e.g., rectangle) that can be decomposed into the base and a "missing" figure, and subtract the area of the missing figure from the larger figure.

Have students share how they would decompose the figure into smaller rectangles and triangles.

Highlight:

- The different strategies for decomposing the base to determine its area. Each strategy works because the whole area is equal to the sum of its parts.
- Remind students the volume of a prism is the area of the base multiplied by the height of the prism.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

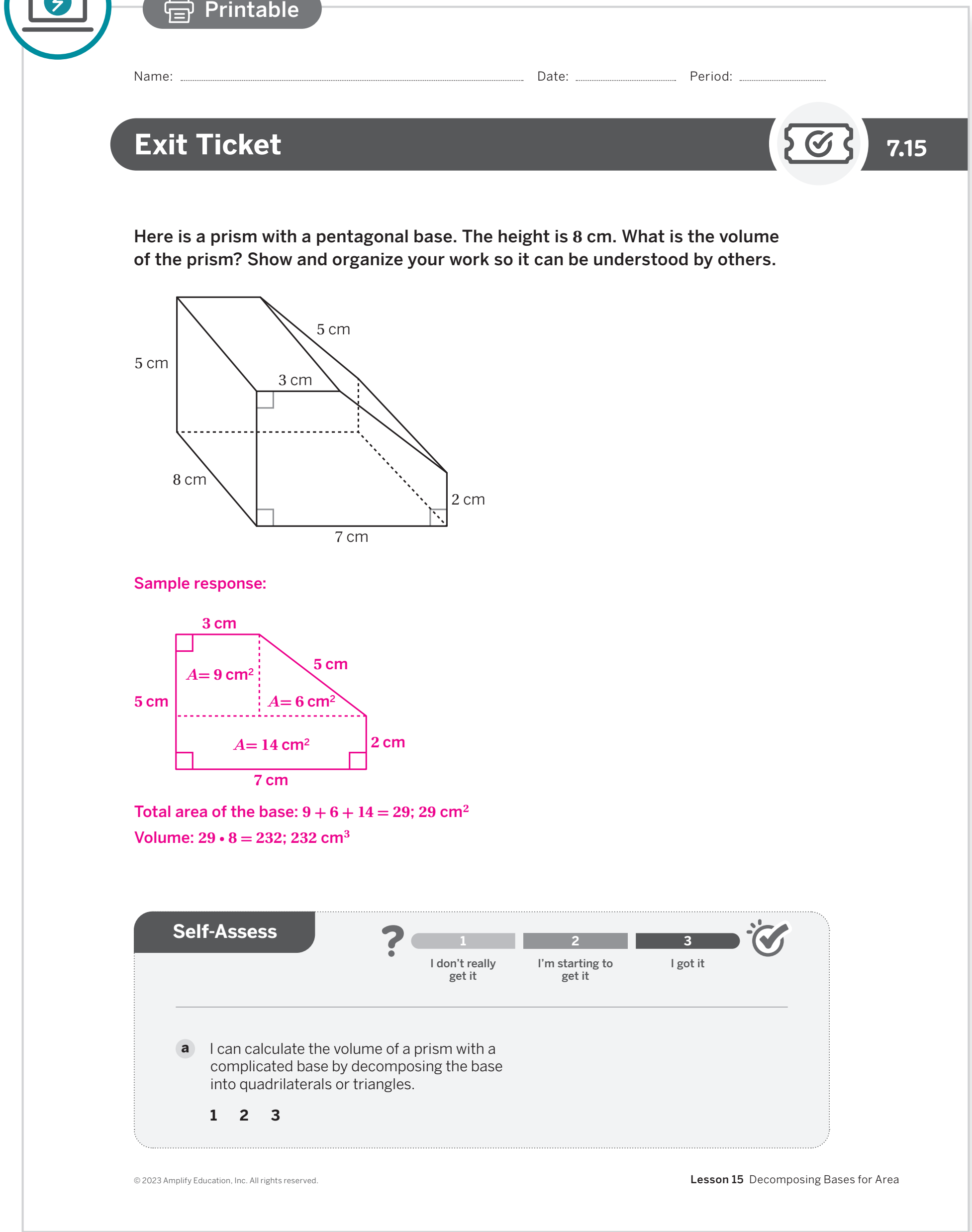
- "What strategies did you find helpful in finding the area of the base of prisms today? How were they helpful?"
- "Were any strategies not helpful? Why?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by explaining how to decompose the base of a complex solid to find its volume.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 7.15

Here is a prism with a pentagonal base. The height is 8 cm. What is the volume of the prism? Show and organize your work so it can be understood by others.

Sample response:

Total area of the base: $9 + 6 + 14 = 29$; 29 cm^2

Volume: $29 \cdot 8 = 232$; 232 cm^3

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can calculate the volume of a prism with a complicated base by decomposing the base into quadrilaterals or triangles.

1 2 3

Lesson 15 Decomposing Bases for Area

Success looks like . . .

- **Language Goal:** Critiquing different methods for decomposing and calculating the area of a prism's base. **(Speaking and Listening)**
- **Language Goal:** Explaining how to decompose and calculate the area of a prism's base and then using it to calculate the prism's volume. **(Speaking and Listening, Writing)**
  - » Decomposing and calculating the volume of the pentagonal prism.

Suggested next steps

If students mistake the base as rectangular, consider:

- Asking, "How do you know this figure is a prism? What must be true about the figure in order to consider it a prism?"
- Reminding them that in a prism, the non-base faces are rectangular, but the base can be any shape.
- Revisiting the Warm-up.

If students decompose the base of the figure incorrectly, consider:

- Reminding them to mark the dimensions on the smaller figures after they decompose the larger figure.
- Assigning Practice Problems 1 and 2.

If students find the area of the base, but do not find the volume, consider:

- Reminding them to carefully read the directions.
- Assigning Practice Problem 1.

Professional Learning

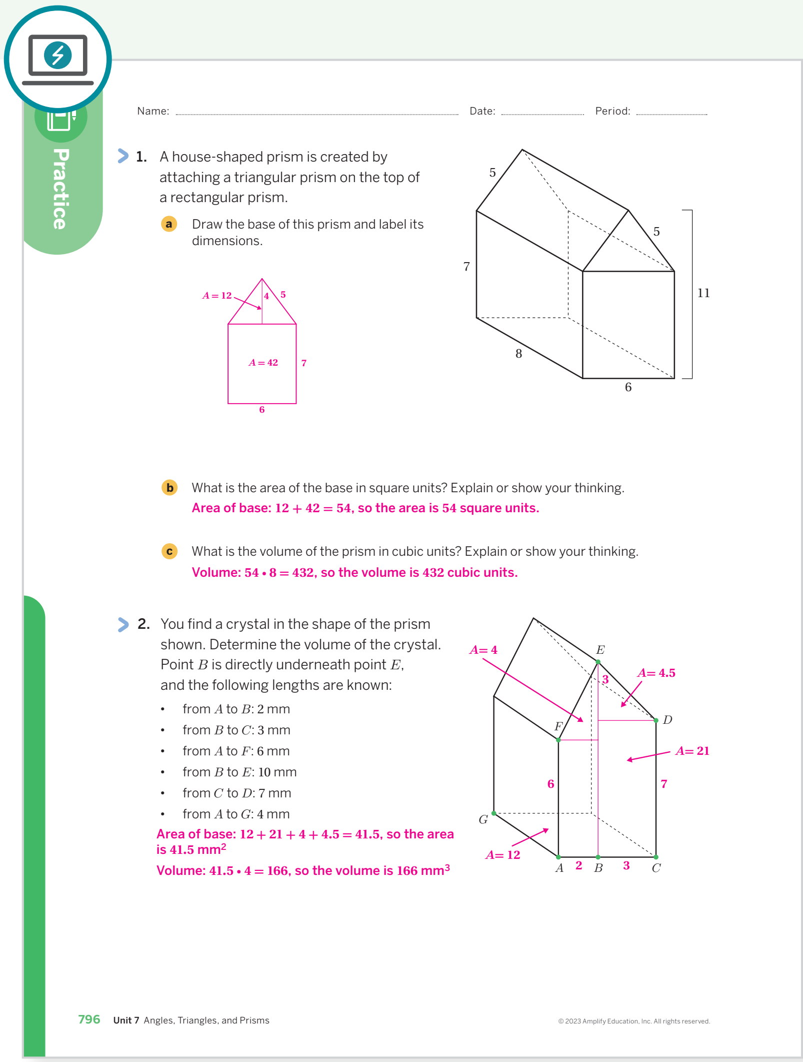
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did the process of decomposing shapes to find the area reveal about your students as learners?
- The focus of this lesson was decomposing and calculating the area of a prism's base to be able to calculate its volume. How did this focus go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. A house-shaped prism is created by attaching a triangular prism on the top of a rectangular prism.

   a. Draw the base of this prism and label its dimensions.

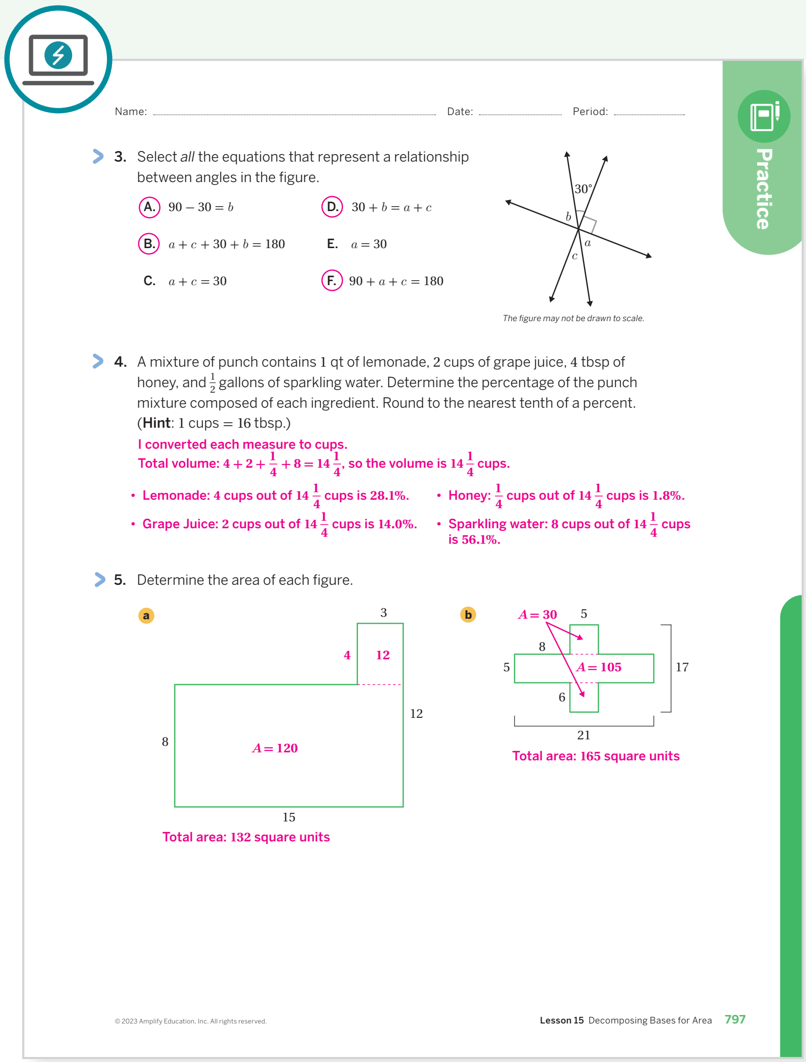
   b. What is the area of the base in square units? Explain or show your thinking.
   Area of base: $12 + 42 = 54$, so the area is 54 square units.

   c. What is the volume of the prism in cubic units? Explain or show your thinking.
   Volume: $54 \cdot 8 = 432$, so the volume is 432 cubic units.

2. You find a crystal in the shape of the prism shown. Determine the volume of the crystal. Point B is directly underneath point E, and the following lengths are known:
   - from A to B: 2 mm
   - from B to C: 3 mm
   - from A to F: 6 mm
   - from B to E: 10 mm
   - from C to D: 7 mm
   - from A to G: 4 mm

   Area of base: $12 + 21 + 4 + 4.5 = 41.5$, so the area is 41.5 mm^2
   Volume: $41.5 \cdot 4 = 166$, so the volume is 166 mm^3

796 **Unit 7** Angles, Triangles, and Prisms © 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

Practice

3. Select *all* the equations that represent a relationship between angles in the figure.

   [✓ A.] $90 - 30 = b$
   [✓ B.] $a + c + 30 + b = 180$
   C. $a + c = 30$
   [✓ D.] $30 + b = a + c$
   E. $a = 30$
   [✓ F.] $90 + a + c = 180$

   The figure may not be drawn to scale.

4. A mixture of punch contains 1 qt of lemonade, 2 cups of grape juice, 4 tbsp of honey, and $\frac{1}{2}$ gallons of sparkling water. Determine the percentage of the punch mixture composed of each ingredient. Round to the nearest tenth of a percent. (**Hint**: 1 cups = 16 tbsp.)

   I converted each measure to cups.
   Total volume: $4 + 2 + \frac{1}{4} + 8 = 14\frac{1}{4}$, so the volume is $14\frac{1}{4}$ cups.
   - **Lemonade: 4 cups out of $14\frac{1}{4}$ cups is 28.1%.**
   - **Grape Juice: 2 cups out of $14\frac{1}{4}$ cups is 14.0%.**
   - **Honey: $\frac{1}{4}$ cups out of $14\frac{1}{4}$ cups is 1.8%.**
   - **Sparkling water: 8 cups out of $14\frac{1}{4}$ cups is 56.1%.**

5. Determine the area of each figure.

   a. **Total area: 132 square units**

   b. **Total area: 165 square units**

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	3
Spiral	3	Unit 7 Lesson 5	2
	4	Unit 4 Lesson 2	2
Formative ⚡	5	Unit 7 Lesson 16	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 16**

Surface Area of Right Prisms

Let's look at the surface area of prisms.

Focus

Goals

1. **Language Goal:** Calculate the surface area of a prism and explain the solution method used. **(Speaking and Listening, Reading and Writing)**
2. Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.
3. **Language Goal:** Interpret different methods for calculating the surface area of a prism and evaluate their usefulness. **(Speaking and Listening, Reading and Writing)**

Rigor

- Students use visual models to develop **conceptual understanding** of surface area of right prisms.
- Students are introduced to multiple methods of calculating surface area to build **procedural skills**.

Coherence

Today

Students critique two methods for finding the surface area of prisms. They see that the structure of a prism allows for shortcuts in adding the areas of the faces or if the prism is sitting on its base, then the vertical sides can be unfolded into a single rectangle whose height is the height of the prism and whose length is the perimeter of the base. The purpose of the lesson is not to come up with a formula for the surface area of a right prism, but to help students see and make use of the structure of the prism to find surface area efficiently.

Previously

In Grade 6, students used nets made up of rectangles and triangles to find the surface area of three-dimensional figures.

Coming Soon

In the next lesson, students will determine if surface area or volume would be the appropriate measure for certain real-world problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	7 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, precut and assembled (for display)
- Activity 2 PDF, pre-cut nets, one per pair (as needed)
- Anchor Chart PDF, *Solids*
- Anchor Chart PDF, *Solids* (answers)

Math Language Development

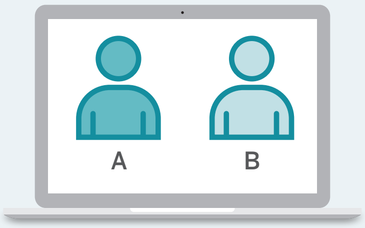
Review words

- *base (of a prism or pyramid)*
- *cross section*
- *plane*
- *prism*
- *pyramid*
- *surface area*
- *volume*

Amps | Featured Activity

Activity 2
Choice and Collaboration

Students select a prism to determine the surface area of, then compare their strategy with a partner who chose the same prism.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students might exhibit a chaotic approach to finding surface area in Activity 1. Ask students how they can use the structure of the figure to organize their work so that they are sure to include all faces when calculating the surface area.

Modifications to Pacing

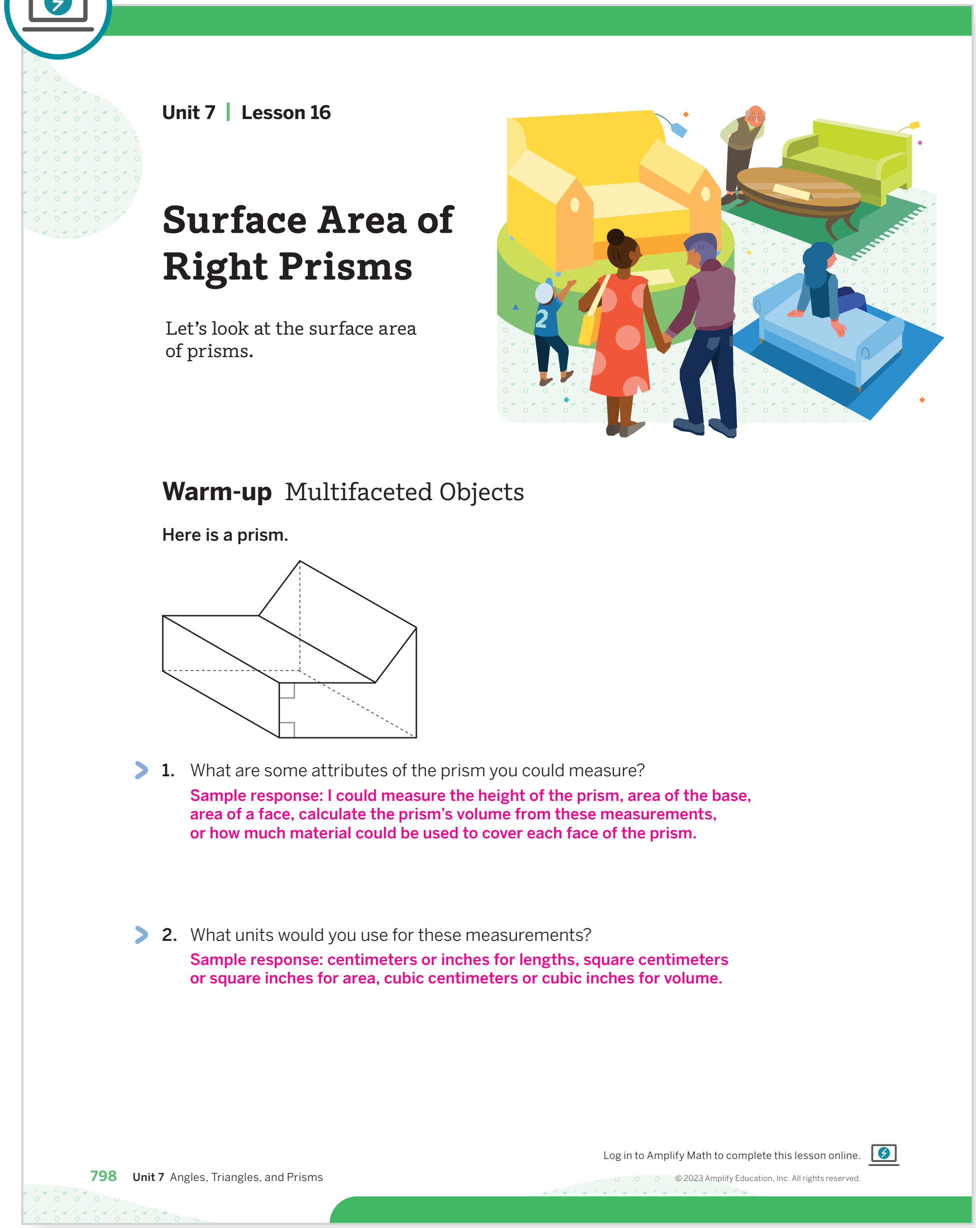
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- **Activity 2** may be omitted. You may consider assigning this Activity as Additional Practice.

Independent | 5 min

Warm-up Multifaceted Objects

Students recognize important attributes of solids in anticipation of computing volume and surface area.



Unit 7 | **Lesson 16**

Surface Area of Right Prisms

Let's look at the surface area of prisms.

Warm-up Multifaceted Objects

Here is a prism.

1. What are some attributes of the prism you could measure?
   Sample response: I could measure the height of the prism, area of the base, area of a face, calculate the prism's volume from these measurements, or how much material could be used to cover each face of the prism.

2. What units would you use for these measurements?
   Sample response: centimeters or inches for lengths, square centimeters or square inches for area, cubic centimeters or cubic inches for volume.

Log in to Amplify Math to complete this lesson online.

798 **Unit 7** Angles, Triangles, and Prisms

1 Launch

Display the prism from the Warm-up PDF, which will be used for both the Warm-up and Activity 1.

2 Monitor

Help students get started by reminding them of the measures they have calculated in previous lessons, such as area of a base and volume of a prism.

Look for points of confusion:

- **Listing only one attribute in Problem 1 and only one unit in Problem 2.** Challenge students to write down as many attributes and their units of measurement as they can.
- **Not knowing what the term *attribute* means.** Explain that an *attribute* is like a characteristic or feature. If students still struggle, provide an example, such as the prism's height.

3 Connect

Display the prism.

Have students share their responses to Problems 1–2.

Ask the class to think of unreasonable units for the prism, such as yards or kilometers. Have students share why these units are unreasonable.

Differentiated Support

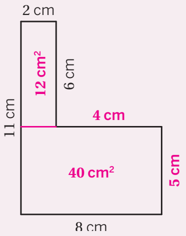
Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

For Problem 1, provide students with a list of attributes from which they could choose ones that are appropriate for this prism.

For Problem 2, provide students with a list of units from which they could choose ones that are appropriate for each attribute they listed in Problem 1. Be sure to include linear units, square units, and cubic units.

Power-up

To power up students' ability to determine the area of polygons by composing and decomposing them into familiar shapes, have students complete:



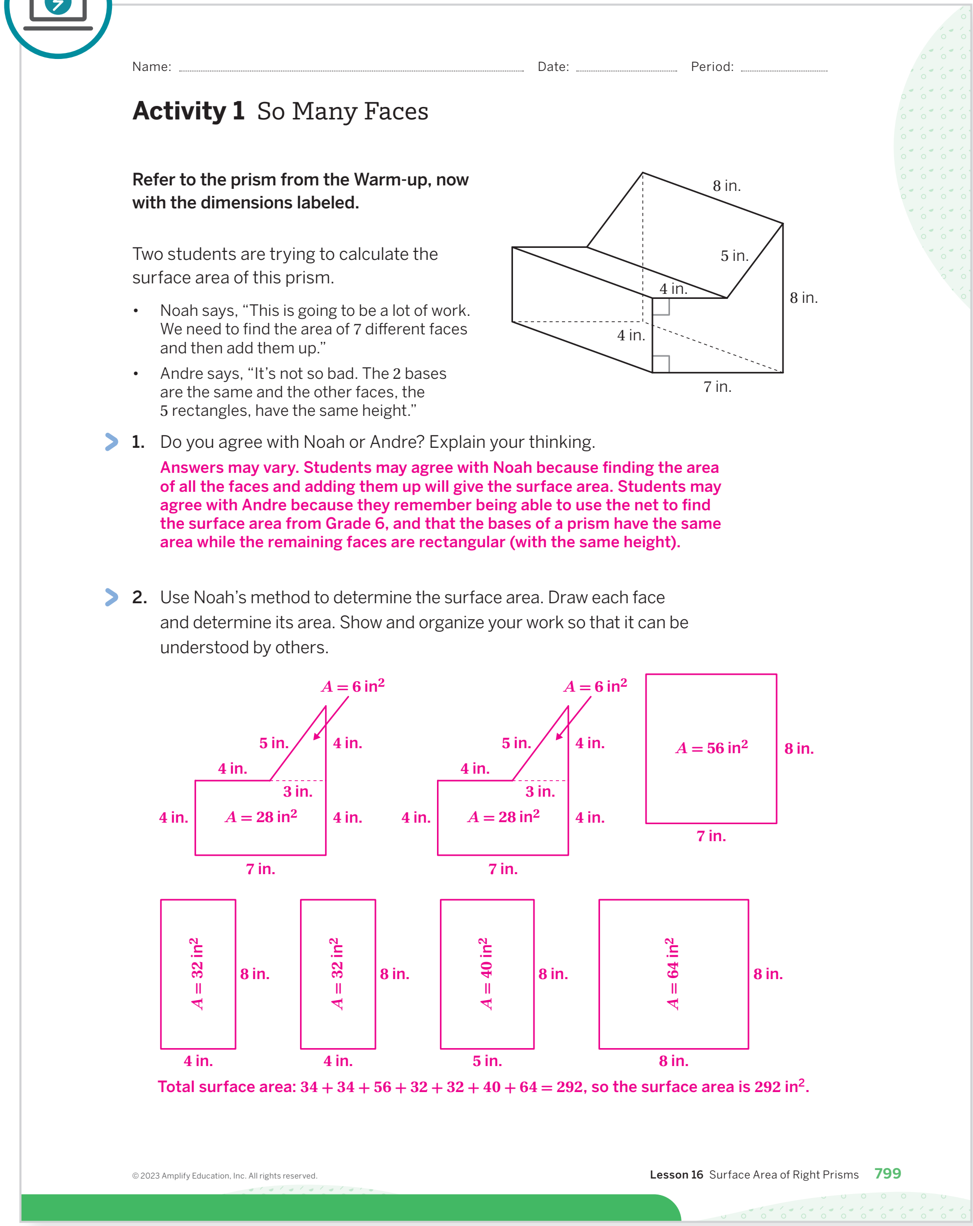
1. Draw one line on the given figure to break it into two rectangular pieces. Sample response shown.
2. Determine the lengths of the sides that are not labeled and add them to the diagram. Sample response shown.
3. Determine the area of each resulting rectangle then determine their sum to calculate the area of the entire figure. 52 cm^2

Use: Before Activity 1.
Informed by: Performance on Lesson 15, Practice Problem 5.

Pairs | 20 min

Activity 1 So Many Faces

Students make sense of different methods for calculating the surface area of a prism to generalize whether they will work for any prism.



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Activity 1 So Many Faces

Refer to the prism from the Warm-up, now with the dimensions labeled.

Two students are trying to calculate the surface area of this prism.

- Noah says, "This is going to be a lot of work. We need to find the area of 7 different faces and then add them up."
- Andre says, "It's not so bad. The 2 bases are the same and the other faces, the 5 rectangles, have the same height."

1. Do you agree with Noah or Andre? Explain your thinking.

   Answers may vary. Students may agree with Noah because finding the area of all the faces and adding them up will give the surface area. Students may agree with Andre because they remember being able to use the net to find the surface area from Grade 6, and that the bases of a prism have the same area while the remaining faces are rectangular (with the same height).

2. Use Noah's method to determine the surface area. Draw each face and determine its area. Show and organize your work so that it can be understood by others.

Total surface area: $34 + 34 + 56 + 32 + 32 + 40 + 64 = 292$, so the surface area is 292 in^2.

1 Launch

Display the prism from the Warm-up and let students know they will be critiquing the reasoning of other students. Activate students' prior knowledge by asking them to how they determined the surface area of rectangular prisms in Grade 6.

2 Monitor

Help students get started by having them restate each method in their own words using the physical model if necessary.

Look for points of confusion:

- **Thinking only one method is correct.** Have students reason through each method to find that both methods work for this prism.
- **Missing the area of some faces in Problem 2.** Have students count the number of faces and check this number against the number of faces mentioned by Noah and Andre.
- **Not understanding that $7 + 4 + 4 + 5 + 8$ is the perimeter of the base.** Have students trace these sides in order.

Activity 1 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Instead of having students draw the shapes in Problems 2 and 3, provide pre-drawn shapes and ask them to label the measurements on each shape and find the total surface area.

Extension: *Math Enrichment*

Have students derive the formula for the surface area $S.A.$ of any prism using Andre's method, defining the variables they use.

$S.A. = ph + 2B$; $p =$ perimeter of the base, $h =$ height of the prism, and $B =$ area of the base.

Math Language Development

MLR1: Stronger and Clearer Each Time

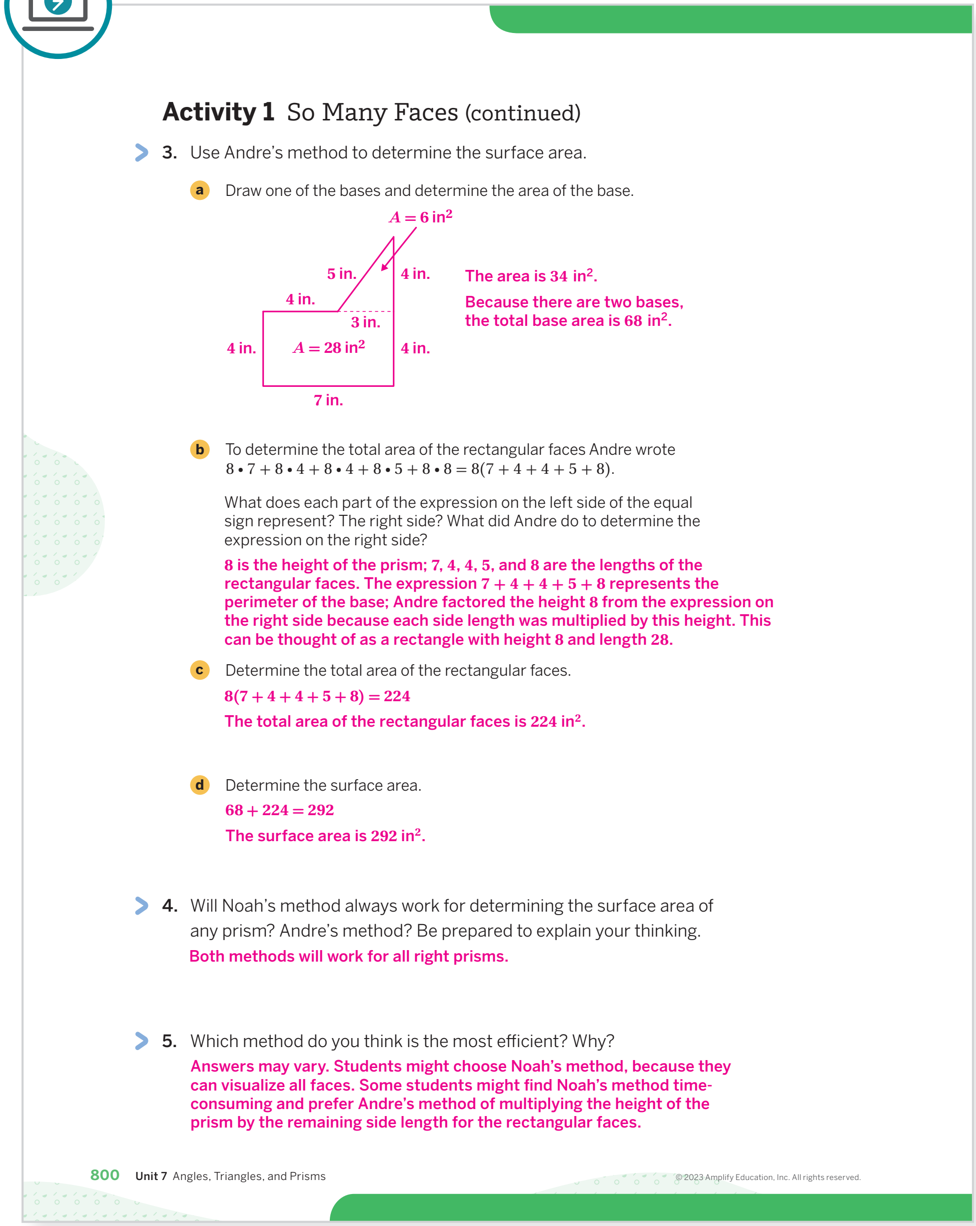
While students study Noah's and Andre's strategies and respond to Problem 1, have them use a ***Think-Write-Pair-Share*** routine. Have them individually make sense of each strategy, write an individual response to Problem 1, and share their response with their partner. Partners should review each other's responses and make suggestions for improvement. Provide prompts that will help partners strengthen ideas and clarify language, such as:

- "How can you draw a picture to support your explanation?"
- "How can you expand on . . .?"

Pairs | 20 min

Activity 1 So Many Faces (continued)

Students make sense of different methods for calculating the surface area of a prism to generalize whether they will work for any prism.



Activity 1 So Many Faces (continued)

3. Use Andre's method to determine the surface area.

a Draw one of the bases and determine the area of the base.

The area is 34 in^2.

Because there are two bases, the total base area is 68 in^2.

b To determine the total area of the rectangular faces Andre wrote $8 \cdot 7 + 8 \cdot 4 + 8 \cdot 4 + 8 \cdot 5 + 8 \cdot 8 = 8(7 + 4 + 4 + 5 + 8)$.

What does each part of the expression on the left side of the equal sign represent? The right side? What did Andre do to determine the expression on the right side?

8 is the height of the prism; 7, 4, 4, 5, and 8 are the lengths of the rectangular faces. The expression $7 + 4 + 4 + 5 + 8$ represents the perimeter of the base; Andre factored the height 8 from the expression on the right side because each side length was multiplied by this height. This can be thought of as a rectangle with height 8 and length 28.

c Determine the total area of the rectangular faces.

$8(7 + 4 + 4 + 5 + 8) = 224$

The total area of the rectangular faces is 224 in^2.

d Determine the surface area.

$68 + 224 = 292$

The surface area is 292 in^2.

4. Will Noah's method always work for determining the surface area of any prism? Andre's method? Be prepared to explain your thinking.

Both methods will work for all right prisms.

5. Which method do you think is the most efficient? Why?

Answers may vary. Students might choose Noah's method, because they can visualize all faces. Some students might find Noah's method time-consuming and prefer Andre's method of multiplying the height of the prism by the remaining side length for the rectangular faces.

800 Unit 7 Angles, Triangles, and Prisms

3 Connect

Display the prism with the measurements labeled, and, if possible, provide or display a net created for Andre's method.

Have previously identified students share their reasoning for each method.

Ask:

- "How did you determine that the triangle forming part of the base was a right triangle?" The right angles indicate the horizontal and vertical lines are perpendicular, so when the base is cut into a rectangle and triangle, the angles are right angles.
- "Which method will work for finding the surface area of *any prism*?" Both methods will work.
- "Which method will work for finding the surface area of *any solid*, not just prisms?" Noah's method will work.

Highlight:

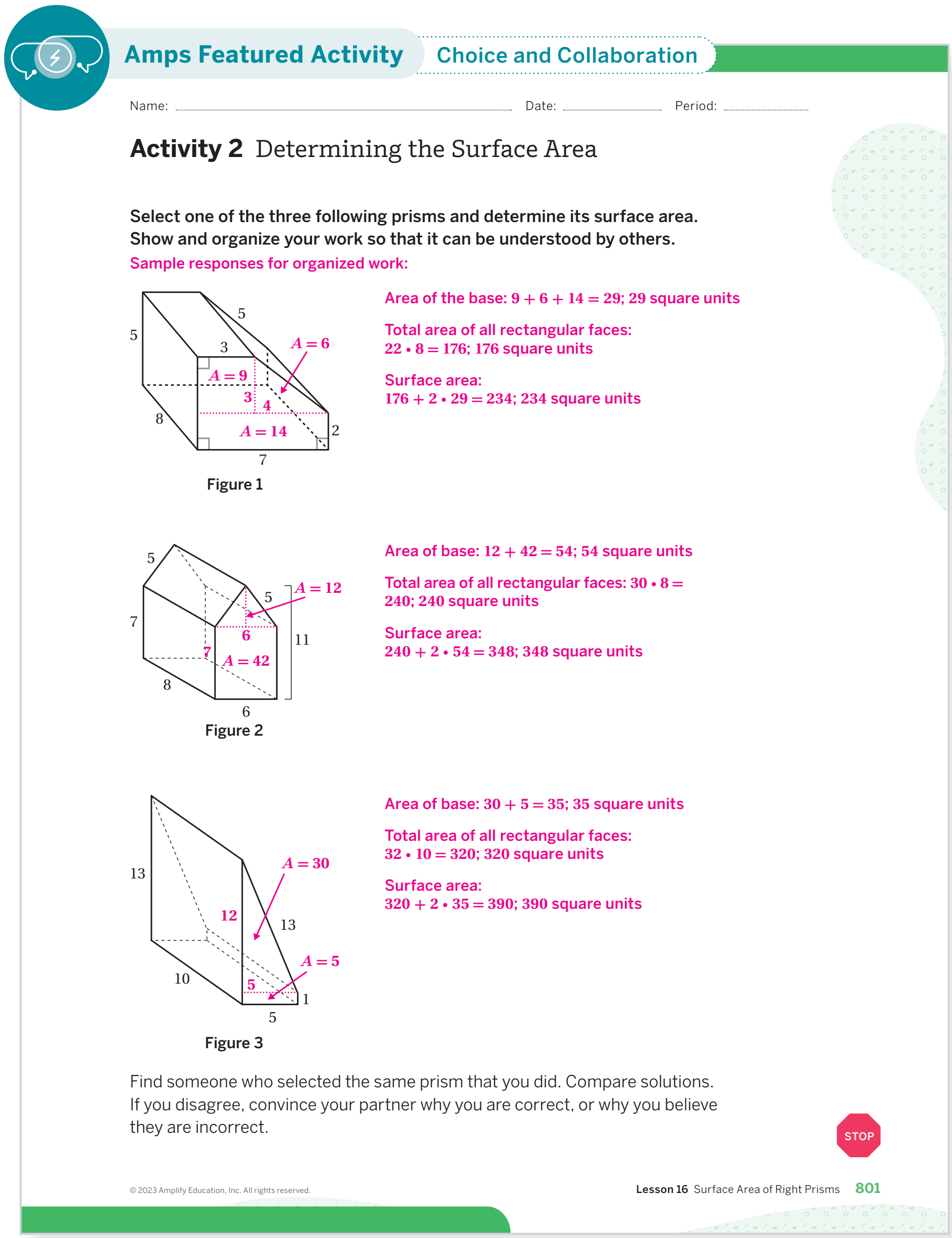
- Students' work that is organized and easy to follow.
- Noah's and Andre's methods work for all prisms. Students may find Andre's method more efficient because the surfaces of prisms can always be thought of in three sections: two bases and one rectangle whose length is the perimeter of the base and whose width is the height of the prism**.** **Note:** It is not necessary for students to know the formula of surface area. However, Andre's method lends itself to sharing that the surface area $S.A.$ of right prisms can be found by multiplying the perimeter p of the base by the height h of the prism and then adding twice the area of the base B: $S.A. = ph + 2B$.
- When discussing Problem 3b, mention factoring and the Distributive Property to spiral concepts from Unit 6.

Pairs | 10 min

Activity 2 Determining the Surface Area

Students explore solids from a previous lesson to build fluency with determining the surface area of prisms.

Amps Featured Activity Choice and Collaboration



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Activity 2 Determining the Surface Area

Select one of the three following prisms and determine its surface area. Show and organize your work so that it can be understood by others.

Sample responses for organized work:

Figure 1

Area of the base: $9 + 6 + 14 = 29$; 29 square units

Total area of all rectangular faces: $22 \cdot 8 = 176$; 176 square units

Surface area: $176 + 2 \cdot 29 = 234$; 234 square units

Figure 2

Area of base: $12 + 42 = 54$; 54 square units

Total area of all rectangular faces: $30 \cdot 8 = 240$; 240 square units

Surface area: $240 + 2 \cdot 54 = 348$; 348 square units

Figure 3

Area of base: $30 + 5 = 35$; 35 square units

Total area of all rectangular faces: $32 \cdot 10 = 320$; 320 square units

Surface area: $320 + 2 \cdot 35 = 390$; 390 square units

Find someone who selected the same prism that you did. Compare solutions. If you disagree, convince your partner why you are correct, or why you believe they are incorrect.

STOP

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Lesson 16 Surface Area of Right Prisms 801

1 Launch

Let students know they may recognize the figures from a previous lesson, but now they will determine the surface area of the figures.

2 Monitor

Help students get started by having them draw the bases of the figures with the known dimensions.

Look for points of confusion:

- **Determining the volume instead of the surface area.** Remind students to attend to the directions.

Look for productive strategies:

- **Remembering these Figures 1 and 2 from Lesson 15.** If students completed the Exit Ticket and Practice Problem 1, they have already calculated the base areas of these prisms, which can save time. If they cannot locate the base areas, use this as an opportunity to reinforce why organizing their work is important.

3 Connect

Have students share the methods they used to determine the surface area.

Ask:

- "How did you determine the area of the base and any other areas you needed to solve the problem?"
- "Which methods are more efficient than others? Why?"
- "Do you think you will prefer this same method for every problem? If not, what would make you change methods?"

Highlight the methods from Activity 1. Determining the area of all the faces (Noah's method) requires calculating the area of many figures. Determining the perimeter of the base and multiplying by the height of the prism (Andre's method) requires visualizing the prism in a different way, but not as many areas need to be calculated. Both methods work and are valuable.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Use copies of the Activity 2 PDF to provide students with nets of Figures 1–3 to help them determine the area.

Extension: *Math Enrichment*

Challenge students to draw their own prisms, whose bases can be decomposed into smaller, known figures. Have them provide and label the dimensions needed to calculate their prism's surface area. Ask them to trade prisms with a partner to see if each student can calculate the surface area.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students describe the methods they used to determine the surface area of each prism, provide sentence frames, such as:

- "First, I _____ because . . . Then I _____, because. . ."
- "I decomposed (or unfolded) _____ to determine . . ."
- "I know the base is a _____ because . . ."

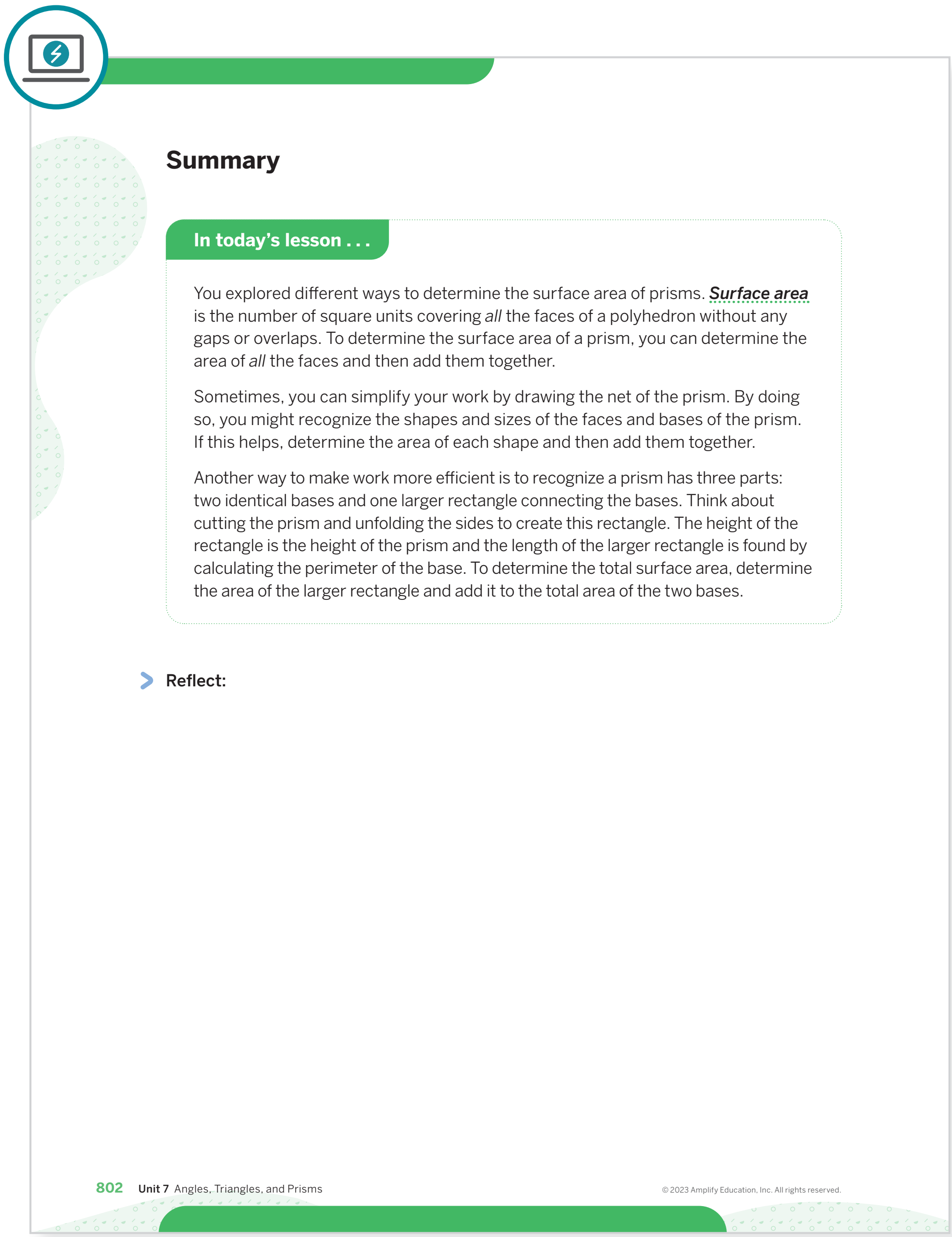
English Learners

Provide a word bank students can use to assist them as they describe their strategy, such as: *area of the base*, *rectangular faces*, *square units*, *trapezoid*, and *triangle*.

Whole Class | 5 min

Summary

Review and synthesize how to determine the surface area of a right prism with a non-rectangular base.



Summary

In today's lesson . . .

You explored different ways to determine the surface area of prisms. ***Surface area*** is the number of square units covering *all* the faces of a polyhedron without any gaps or overlaps. To determine the surface area of a prism, you can determine the area of *all* the faces and then add them together.

Sometimes, you can simplify your work by drawing the net of the prism. By doing so, you might recognize the shapes and sizes of the faces and bases of the prism. If this helps, determine the area of each shape and then add them together.

Another way to make work more efficient is to recognize a prism has three parts: two identical bases and one larger rectangle connecting the bases. Think about cutting the prism and unfolding the sides to create this rectangle. The height of the rectangle is the height of the prism and the length of the larger rectangle is found by calculating the perimeter of the base. To determine the total surface area, determine the area of the larger rectangle and add it to the total area of the two bases.

> **Reflect:**

802 **Unit 7** Angles, Triangles, and Prisms

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Synthesize

Display the Anchor Chart PDF, *Solids*, and complete the Surface Area section as you facilitate a class discussion using the following questions.

Ask:

- "What is surface area?" The total area of all the exposed faces of an object.
- "What are some methods for determining the surface area of prisms?"
  Sample responses:
  - Calculate the area of each face and add them to find the total.
  - Determine groups of faces that have the same area, which saves some calculation.
  - Calculate the area of the bases and then add the area of the long rectangle made up of the sides.

Highlight that determining the area of the base might involve decomposing the figure into rectangles and triangles first. To determine the surface area, the area of the sides must also be included.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

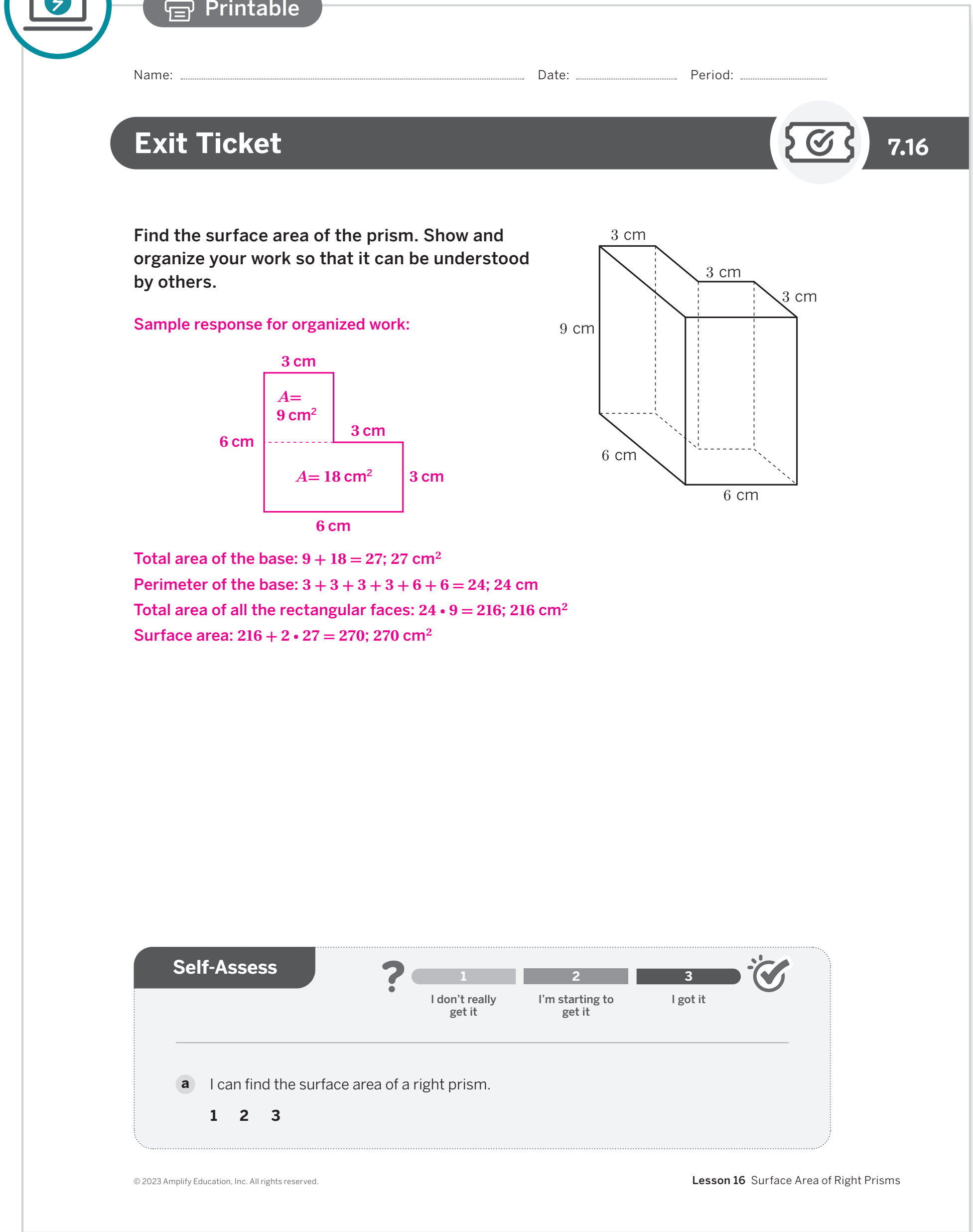
- "What attributes of your prism did you consider when calculating its surface area?"
- "What did you find most challenging about calculating surface area today? How did you work through any challenges?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding of surface area by reasoning about a prism with a non-rectangular base.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 7.16

Find the surface area of the prism. Show and organize your work so that it can be understood by others.

Sample response for organized work:

Total area of the base: $9 + 18 = 27$; 27 cm^2

Perimeter of the base: $3 + 3 + 3 + 3 + 6 + 6 = 24$; 24 cm

Total area of all the rectangular faces: $24 \cdot 9 = 216$; 216 cm^2

Surface area: $216 + 2 \cdot 27 = 270$; 270 cm^2

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can find the surface area of a right prism.

1 2 3

Lesson 16 Surface Area of Right Prisms

Success looks like . . .

- **Language Goal:** Calculating the surface area of a prism and explaining the solution method used. **(Speaking and Listening, Reading and Writing)**
  - » Calculating the surface area of the prism and explaining how it was determined.
- **Goal:** Comprehending that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.
- **Language Goal:** Interpreting different methods for calculating the surface area of a prism and evaluating their usefulness. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students use a rectangle for the base, consider:

- Assigning Practice Problems 1 and 2.

If students do not complete the process of determining the surface area, consider:

- Reminding them of the methods from Activity 1.
- Having them choose Noah's or Andre's method and apply the method to the prism in the Exit Ticket.
- Assigning Practice Problem 2 to practice the chosen method (Noah's or Andre's method).

Professional Learning

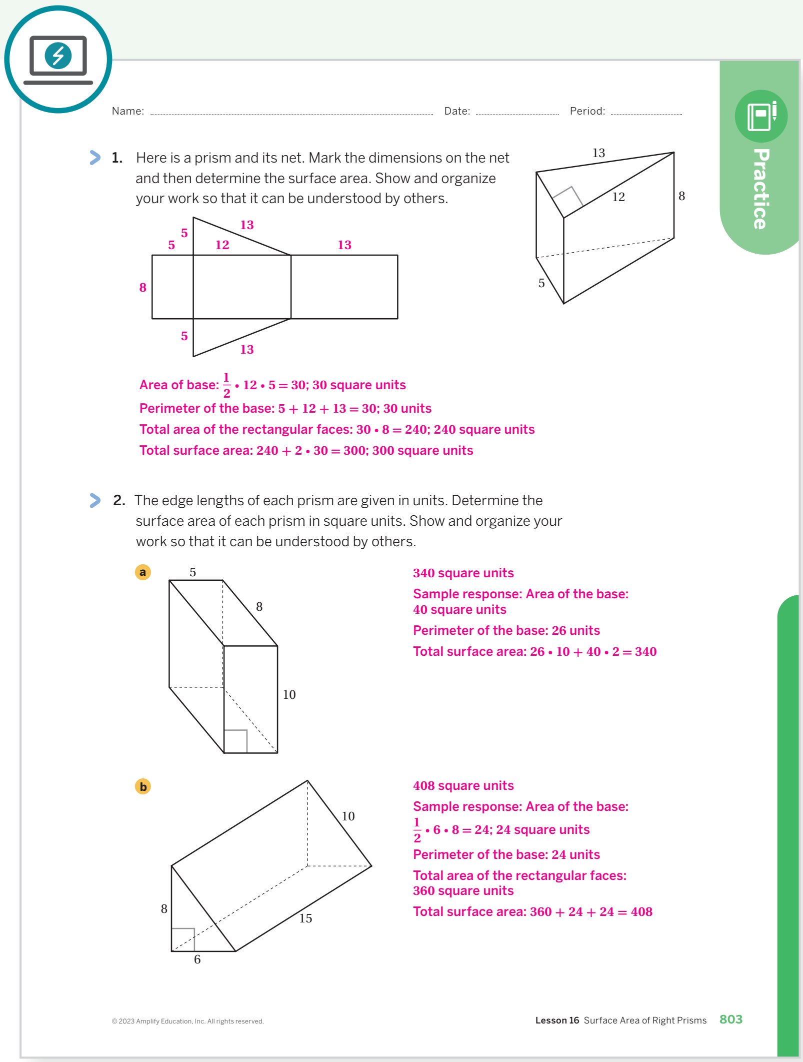
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students see and make use of structure today? How are you helping them become aware of how they are progressing in this area?
- What did students find frustrating, if anything, during Activity 1? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Here is a prism and its net. Mark the dimensions on the net and then determine the surface area. Show and organize your work so that it can be understood by others.

Area of base: $\frac{1}{2} \cdot 12 \cdot 5 = 30$; 30 square units
Perimeter of the base: $5 + 12 + 13 = 30$; 30 units
Total area of the rectangular faces: $30 \cdot 8 = 240$; 240 square units
Total surface area: $240 + 2 \cdot 30 = 300$; 300 square units

2. The edge lengths of each prism are given in units. Determine the surface area of each prism in square units. Show and organize your work so that it can be understood by others.

a

340 square units
Sample response: Area of the base: 40 square units
Perimeter of the base: 26 units
Total surface area: $26 \cdot 10 + 40 \cdot 2 = 340$

b

408 square units
Sample response: Area of the base: $\frac{1}{2} \cdot 6 \cdot 8 = 24$; 24 square units
Perimeter of the base: 24 units
Total area of the rectangular faces: 360 square units
Total surface area: $360 + 24 + 24 = 408$

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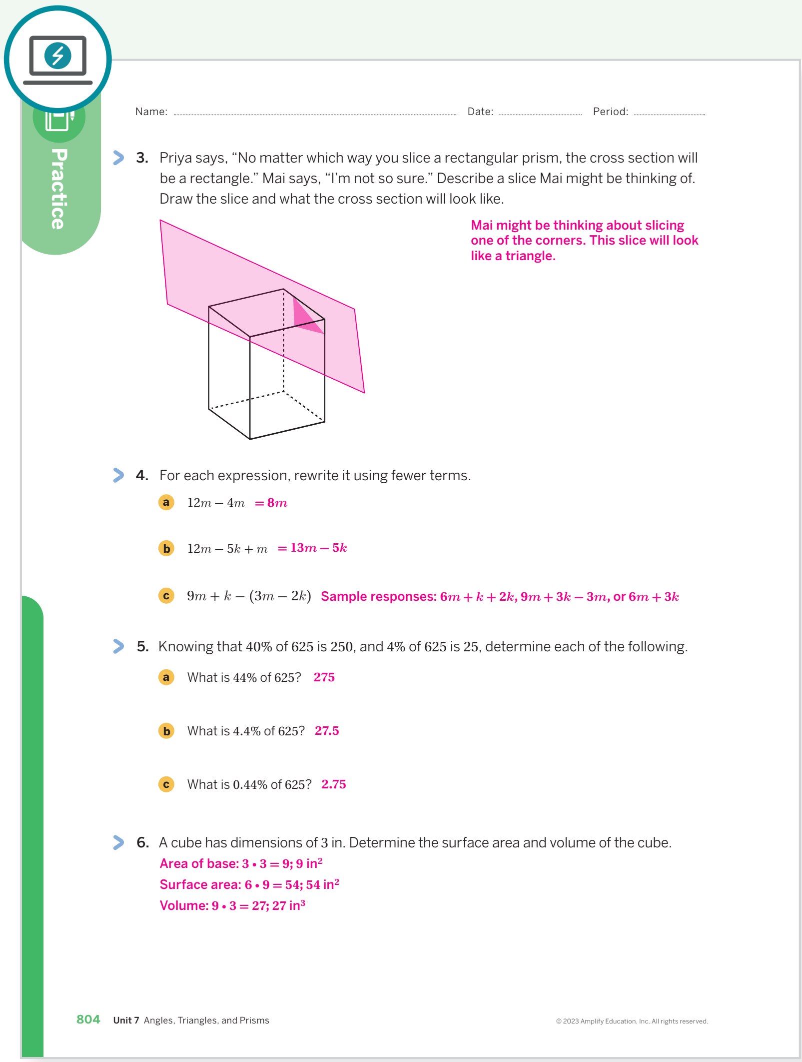
Lesson 16 Surface Area of Right Prisms 803



Name: Date: Period:

Practice

3. Priya says, "No matter which way you slice a rectangular prism, the cross section will be a rectangle." Mai says, "I'm not so sure." Describe a slice Mai might be thinking of. Draw the slice and what the cross section will look like.

Mai might be thinking about slicing one of the corners. This slice will look like a triangle.

4. For each expression, rewrite it using fewer terms.

a $12m - 4m$ $= 8m$

b $12m - 5k + m$ $= 13m - 5k$

c $9m + k - (3m - 2k)$ Sample responses: $6m + k + 2k$, $9m + 3k - 3m$, or $6m + 3k$

5. Knowing that 40% of 625 is 250, and 4% of 625 is 25, determine each of the following.

a What is 44% of 625? 275

b What is 4.4% of 625? 27.5

c What is 0.44% of 625? 2.75

6. A cube has dimensions of 3 in. Determine the surface area and volume of the cube.
Area of base: $3 \cdot 3 = 9$; 9 in^2
Surface area: $6 \cdot 9 = 54$; 54 in^2
Volume: $9 \cdot 3 = 27$; 27 in^3

804 **Unit 7** Angles, Triangles, and Prisms

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
Spiral	3	Unit 7 Lesson 13	1
	4	Unit 6 Lesson 21	1
	5	Unit 4 Lesson 2	1
Formative	6	Unit 7 Lesson 17	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 7 | **LESSON 17**

Distinguishing Surface Area and Volume

Let's work with surface area and volume in real-world situations.

Focus

Goals

1. **Language Goal:** Compare and contrast problems involving surface area and volume of prisms. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Decide whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation and justify the decision. **(Speaking and Listening)**

Rigor

- Students strengthen their **fluency** in calculating surface area and volume of right prisms.
- Students **apply** their knowledge of surface area and volume to solve real-world problems.

Coherence

Today

Students apply their knowledge of surface area and volume to solve real-world problems. The purpose of this lesson is to help students distinguish between surface area and volume and to choose which of the two measures is appropriate for solving the problem.

Previously

In previous lessons, students found the volume and surface area for many prisms, including ones with non-rectangular or non-triangular bases.

Coming Soon

In the final lesson of the unit, students will see how adjusting dimensions can affect the volume and surface area.

Pacing Guide

Suggested Total Lesson Time **~45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	12 min	15 min	7 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)

Math Language Development

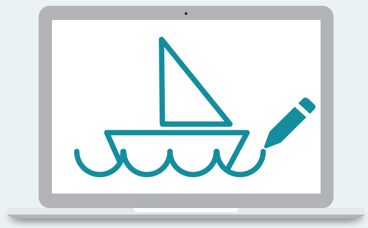
Review words

- *base (of a prism or pyramid)*
- *cross section*
- *plane*
- *prism*
- *pyramid*
- *surface area*
- *volume*

Amps | **Featured Activity**

Activity 1
Digital Sketches

Students decompose the heart-shaped base while drawing directly on the image using the sketch tool.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might find themselves locking onto key words, instead of reading each question completely when choosing a model in Activity 2. Have students present strategies for maintaining focus and thoroughly reading the question before selecting an answer. Acknowledge that this requires self-discipline and impulse control.

Modifications to Pacing

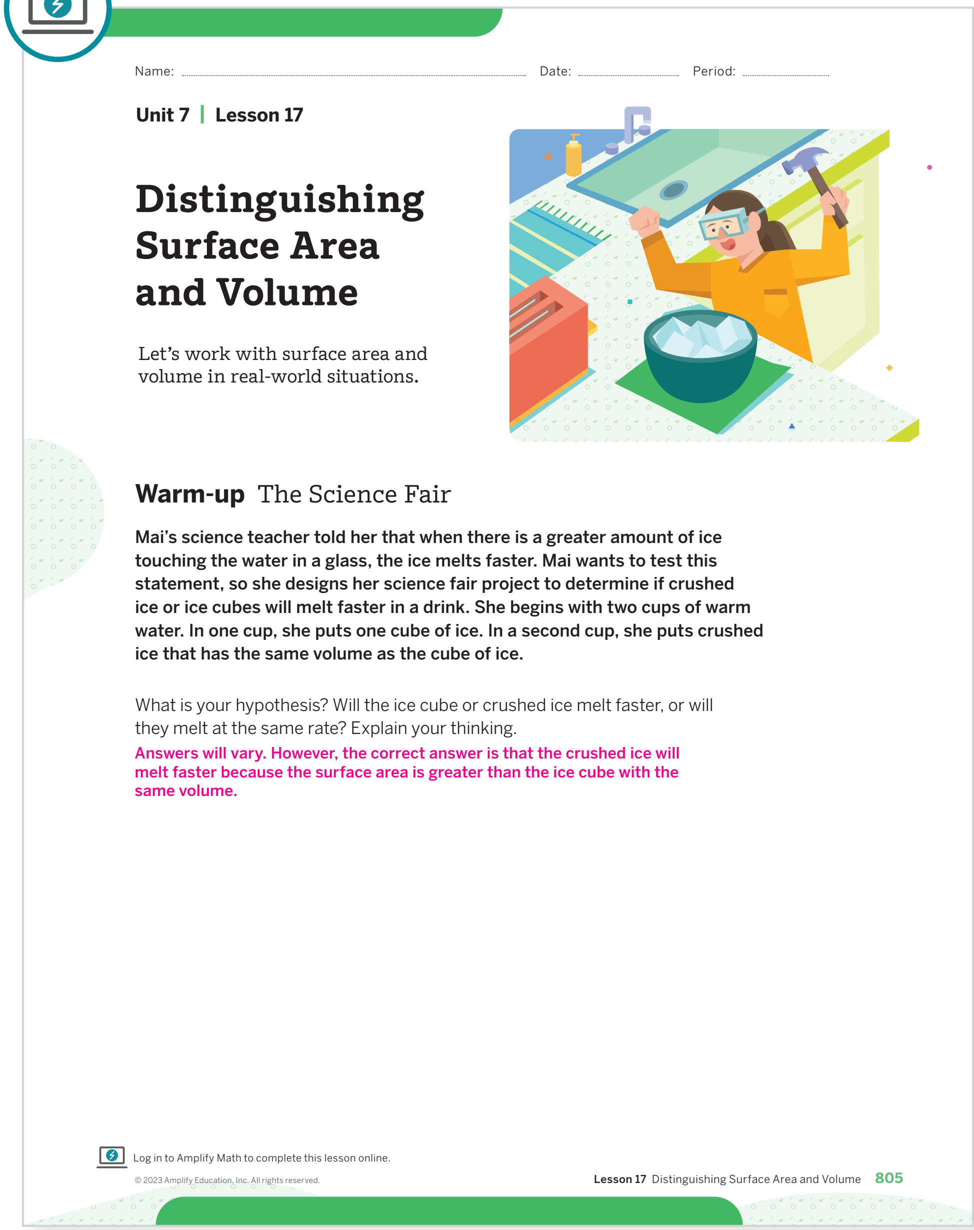
You may want to consider this additional modification if you are short on time.

- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up The Science Fair

Students think about two objects with the same volume to reason about their surface areas.



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Unit 7 | Lesson 17

Distinguishing Surface Area and Volume

Let's work with surface area and volume in real-world situations.

Warm-up The Science Fair

Mai's science teacher told her that when there is a greater amount of ice touching the water in a glass, the ice melts faster. Mai wants to test this statement, so she designs her science fair project to determine if crushed ice or ice cubes will melt faster in a drink. She begins with two cups of warm water. In one cup, she puts one cube of ice. In a second cup, she puts crushed ice that has the same volume as the cube of ice.

What is your hypothesis? Will the ice cube or crushed ice melt faster, or will they melt at the same rate? Explain your thinking.

Answers will vary. However, the correct answer is that the crushed ice will melt faster because the surface area is greater than the ice cube with the same volume.

Log in to Amplify Math to complete this lesson online.

Lesson 17 Distinguishing Surface Area and Volume 805

1 Launch

Activate students' background knowledge by asking them why ice cubes in a glass of water melt over time.

2 Monitor

Help students get started by asking them to visualize or draw one cube of ice versus crushed ice with the same volume.

3 Connect

Display images of crushed ice and cubed ice.

Have students share their hypotheses by using the ***Poll the Class*** routine. Record and display their responses. If all students agree that crushed ice will melt faster, ask them to share their reasoning. If there are different hypotheses, ask students to share their reasoning, and have students ask each other questions about their hypotheses and reasoning. Continue the discussion until the students reach an agreement that the crushed ice will melt faster.

Highlight that, because crushed ice has more surfaces that touch the warm water, it should melt faster. The center of the cube is surrounded by cold ice and will not melt until it is touching the water. The total surface area of the crushed ice is greater, so it will melt faster than the ice cube although both have the same volume.

Note: This Warm-up allows for cross-curricular lessons with science. If possible, ask your science teachers to conduct an experiment measuring the length of time for ice of the same volume to melt.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display images of cubes of ice and crushed ice to help students visualize the differences between them.

Power-up

To power up students' ability to determine the surface area and volume of a cube, have students complete:

Recall that the formula for surface area of a cube is $S = 6s^2$ and the formula for its volume is $V = s^3$ where s is the length of one side.

Determine the surface area and volume of a cube with side lengths of 2 ft.

Surface area: 24 ft^2;

$S = 6(2)2$

$S = 6 \cdot 4$

$S = 24$

Volume: 8 ft^3;

$V = (2)^3$

$V = 8$

Use: Before Activity 1.

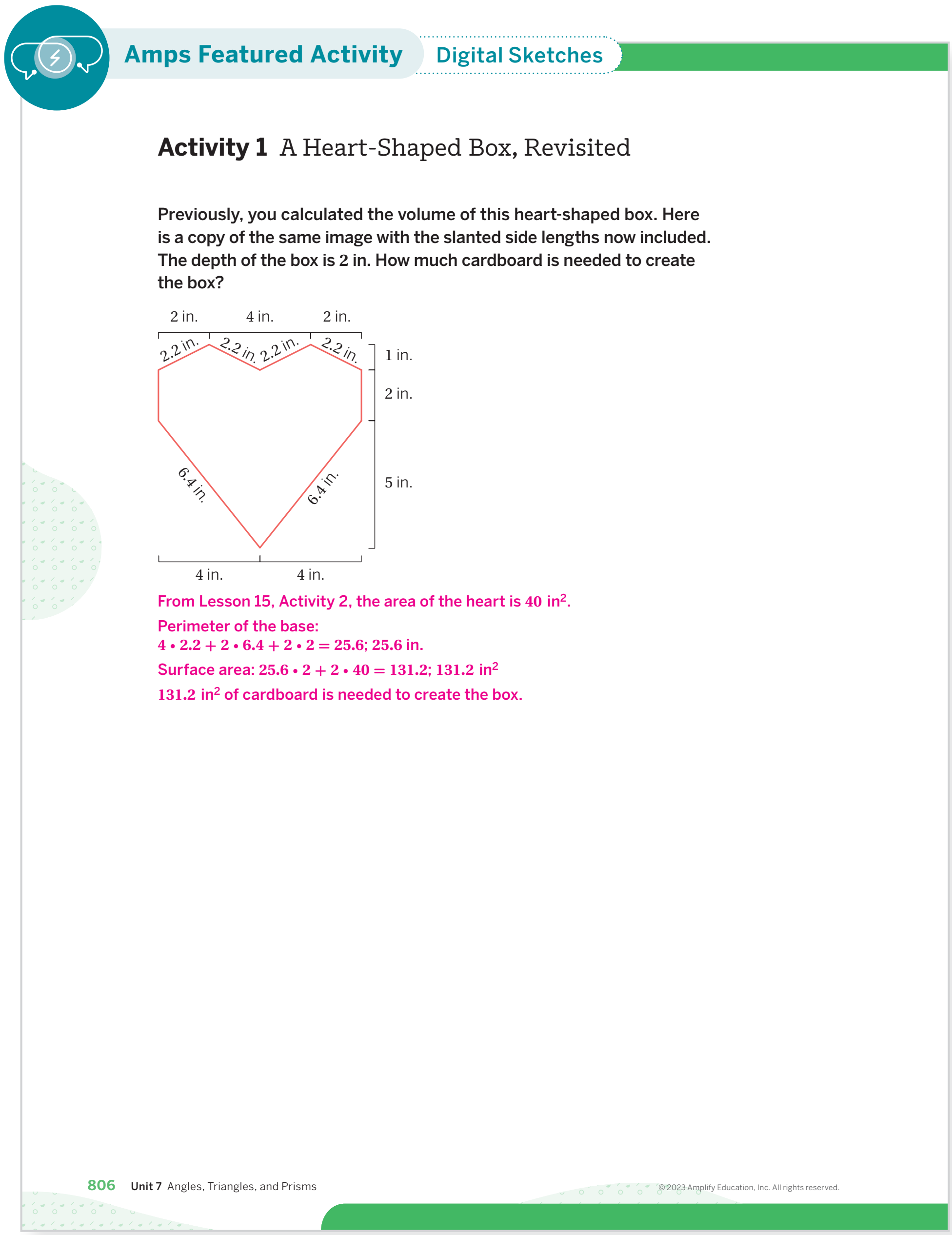
Informed by: Performance on Lesson 16, Practice Problem 6.

Pairs | 15 min

Activity 1 A Heart-Shaped Box, Revisited

Students use the heart-shaped box from a previous lesson to calculate the surface area of complex shapes.

Amps Featured Activity Digital Sketches



Activity 1 A Heart-Shaped Box, Revisited

Previously, you calculated the volume of this heart-shaped box. Here is a copy of the same image with the slanted side lengths now included. The depth of the box is 2 in. How much cardboard is needed to create the box?

From Lesson 15, Activity 2, the area of the heart is 40 in^2.

Perimeter of the base:
$4 \cdot 2.2 + 2 \cdot 6.4 + 2 \cdot 2 = 25.6$; 25.6 in.

Surface area: $25.6 \cdot 2 + 2 \cdot 40 = 131.2$; 131.2 in^2

131.2 in^2 of cardboard is needed to create the box.

806 **Unit 7** Angles, Triangles, and Prisms

1 Launch

Remind students they calculated the volume of this box in a previous lesson. Display the image of the heart with the dimensions labeled.

2 Monitor

Help students get started by having them write down all the dimensions they know and what they need to find to calculate the surface area.

Look for productive strategies:

- **Referencing Lesson 15 for the area of the heart.** Encourage students to work efficiently using known dimensions.
- **Doubling the lateral area.** Students who are familiar with actual heart-shaped boxes may want to double the lateral area to represent the way the top and bottom pieces nest together. Students who do this are making the mathematical aspects of this problem fit the real-world example. This should be developed further with discussion.

3 Connect

Display the image of the heart.

Have students share their solutions and methods for calculating the surface area.

Ask:

- "How did you know you had to calculate the surface area?" I need to find how much cardboard is needed.
- "Suppose you want to make a set of two boxes that are each half of the heart and placed side-by-side to form a full heart, how does the amount of cardboard needed to make two half-heart boxes compare to making one full-heart box?" More cardboard is needed to make two half-heart boxes.
- "How could you reduce the surface area of the one box without reducing the volume?" Sample response: I could change the shape to something with fewer segments, such as a triangle.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can decompose the heart-shaped box while drawing directly on the image using the sketch tool.

Accessibility: *Activate Prior Knowledge*

Review the heart-shaped box calculations from the earlier, related activity in this unit. Consider displaying the various strategies that were used to calculate the area of the heart.

Extension: *Math Enrichment*

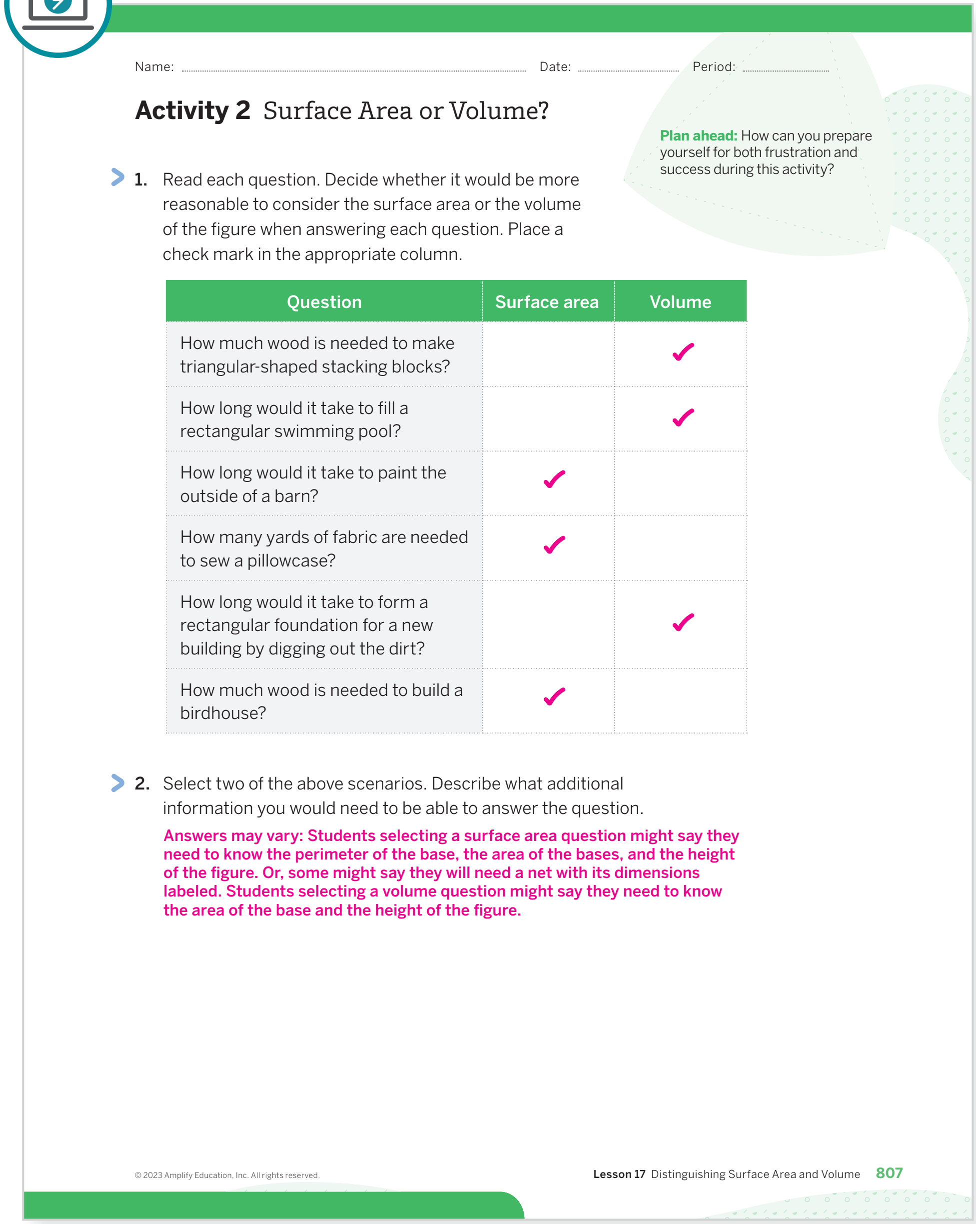
Ask students how the surface area would change if the depth of the box was changed to each of the following lengths.

Depth of box (in.)	3	5	x
Surface area (in^2)	156.8	208	$25.6x + 80$

Pairs | 12 min

Activity 2 Surface Area or Volume?

Students determine whether it is more reasonable to use volume or surface area when answering real-world questions.



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Activity 2 Surface Area or Volume?

Plan ahead: How can you prepare yourself for both frustration and success during this activity?

1. Read each question. Decide whether it would be more reasonable to consider the surface area or the volume of the figure when answering each question. Place a check mark in the appropriate column.

Question	Surface area	Volume
How much wood is needed to make triangular-shaped stacking blocks?		✓
How long would it take to fill a rectangular swimming pool?		✓
How long would it take to paint the outside of a barn?	✓	
How many yards of fabric are needed to sew a pillowcase?	✓	
How long would it take to form a rectangular foundation for a new building by digging out the dirt?		✓
How much wood is needed to build a birdhouse?	✓	

2. Select two of the above scenarios. Describe what additional information you would need to be able to answer the question.
Answers may vary: Students selecting a surface area question might say they need to know the perimeter of the base, the area of the bases, and the height of the figure. Or, some might say they will need a net with its dimensions labeled. Students selecting a volume question might say they need to know the area of the base and the height of the figure.

1 Launch

Once pairs have completed Problem 1, use the ***Poll the Class*** routine to discuss which measurement is needed for the first row. Repeat for the second row.

2 Monitor

Help students get started by asking if the question is referring to what the object can hold (volume) or how much of a material is covering the object (surface area).

3 Connect

Display the results from the poll for each problem.

Ask:

- "How did you determine which measure you needed to answer each question?"
- "What is different about the measures used in each question?"

Have students share their explanations of how they determined whether a question required surface area or volume.

Highlight students' verbal descriptions of their reasoning because they will later be asked to write about similarities and differences in the Exit Ticket. Refer back to the poll responses to help this discussion.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Use the internet, or another source, to provide images, such as a swimming pool, a pillowcase, and a birdhouse, to help students visualize each scenario.

Extension: *Math Enrichment*

Have students brainstorm two questions, one involving surface area and the other volume, that could be applicable to their neighborhood or city. Sample responses:

- Surface area: How much tile is needed to tile the walls in a bathroom?
- Volume: How many cubic feet of air is circulated in an office building?

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they determined whether surface area or volume is the more reasonable measure, call their attention to the words indicated in the text of each question by asking:

- "How does the word *fill* help indicate which measure is more reasonable when filling a swimming pool?"
- "When you think about *painting* something, are you covering a surface or filling a container?"

English Learners

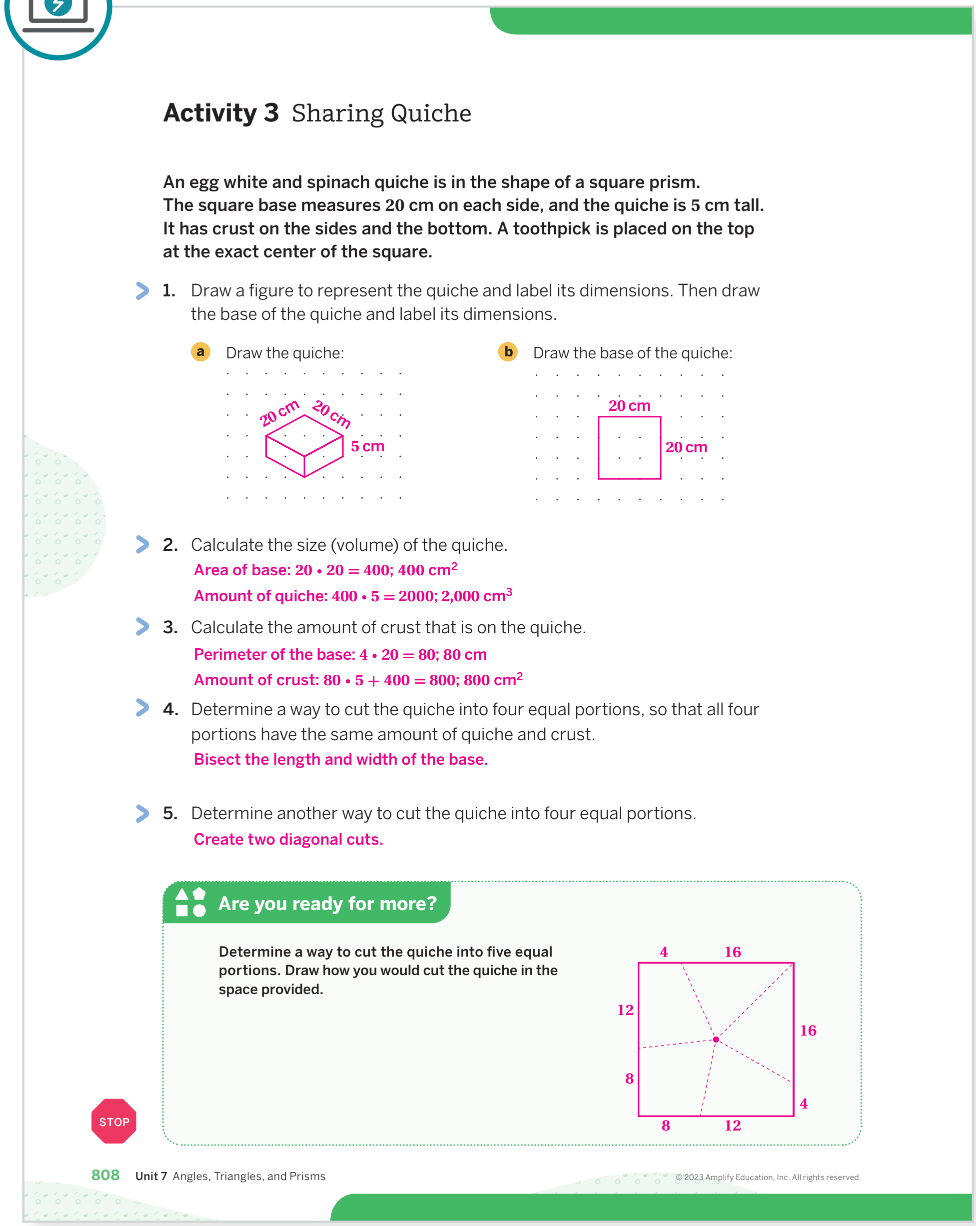
Use pictures or physical objects to help students visualize each scenario.

Optional

Pairs | 15 min

Activity 3 Sharing Quiche

Using a scenario involving a quiche, students decide whether to use surface area or volume to determine the amount of crust needed and the size (volume) of the quiche.



Activity 3 Sharing Quiche

An egg white and spinach quiche is in the shape of a square prism. The square base measures 20 cm on each side, and the quiche is 5 cm tall. It has crust on the sides and the bottom. A toothpick is placed on the top at the exact center of the square.

1. Draw a figure to represent the quiche and label its dimensions. Then draw the base of the quiche and label its dimensions.

   a. Draw the quiche:

   b. Draw the base of the quiche:

2. Calculate the size (volume) of the quiche.

   Area of base: 20 • 20 = 400; 400 cm^2

   Amount of quiche: 400 • 5 = 2000; 2,000 cm^3

3. Calculate the amount of crust that is on the quiche.

   Perimeter of the base: 4 • 20 = 80; 80 cm

   Amount of crust: 80 • 5 + 400 = 800; 800 cm^2

4. Determine a way to cut the quiche into four equal portions, so that all four portions have the same amount of quiche and crust.

   Bisect the length and width of the base.

5. Determine another way to cut the quiche into four equal portions.

   Create two diagonal cuts.

Are you ready for more?

Determine a way to cut the quiche into five equal portions. Draw how you would cut the quiche in the space provided.

808 **Unit 7** Angles, Triangles, and Prisms

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1 Launch

Show students a picture of a quiche in the shape of a square prism, and ask them what mathematical observations they could make or find about the quiche.

2 Monitor

Help students get started by asking, "What measure would you find to know how much quiche there was? What measure would you find to know how much crust there was?"

Look for points of confusion:

- **Having difficulty drawing on the dot paper.** The dot paper is meant to help them draw a prism. If it is distracting, have them draw and label the prism in the margins or on another sheet of paper.
- **Finding the *entire* surface area to represent the crust.** Ask students if all surfaces are visible and covered with crust to get them to realize the top face should not be included. They will need only one base area to find the amount of crust used.

Look for productive strategies:

- Showing work in an organized manner. Use these as models during the whole class discussion.

3 Connect

Display students' drawings and their work.

Ask:

- "How did you determine which measure you needed to answer the question?"
- "What is different about the measures used in each question?"

Have students share how they knew which measure to find for each scenario and what phrases helped them know which measure to use.

Highlight similarities and differences in the measurements for the quiche.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing pre-drawn figures for Problem 1 and have students begin the activity with Problem 2. You may also choose to have students omit Problem 5.

Accessibility: *Guide Processing and Visualization, Clarify Vocabulary and Symbols*

Students may be unfamiliar with the term *quiche*. Display images of quiches and help them pronounce the term. Draw an image of what the quiche might look like with crust on the sides and bottom.

Math Language Development

MLR2: Collect and Display

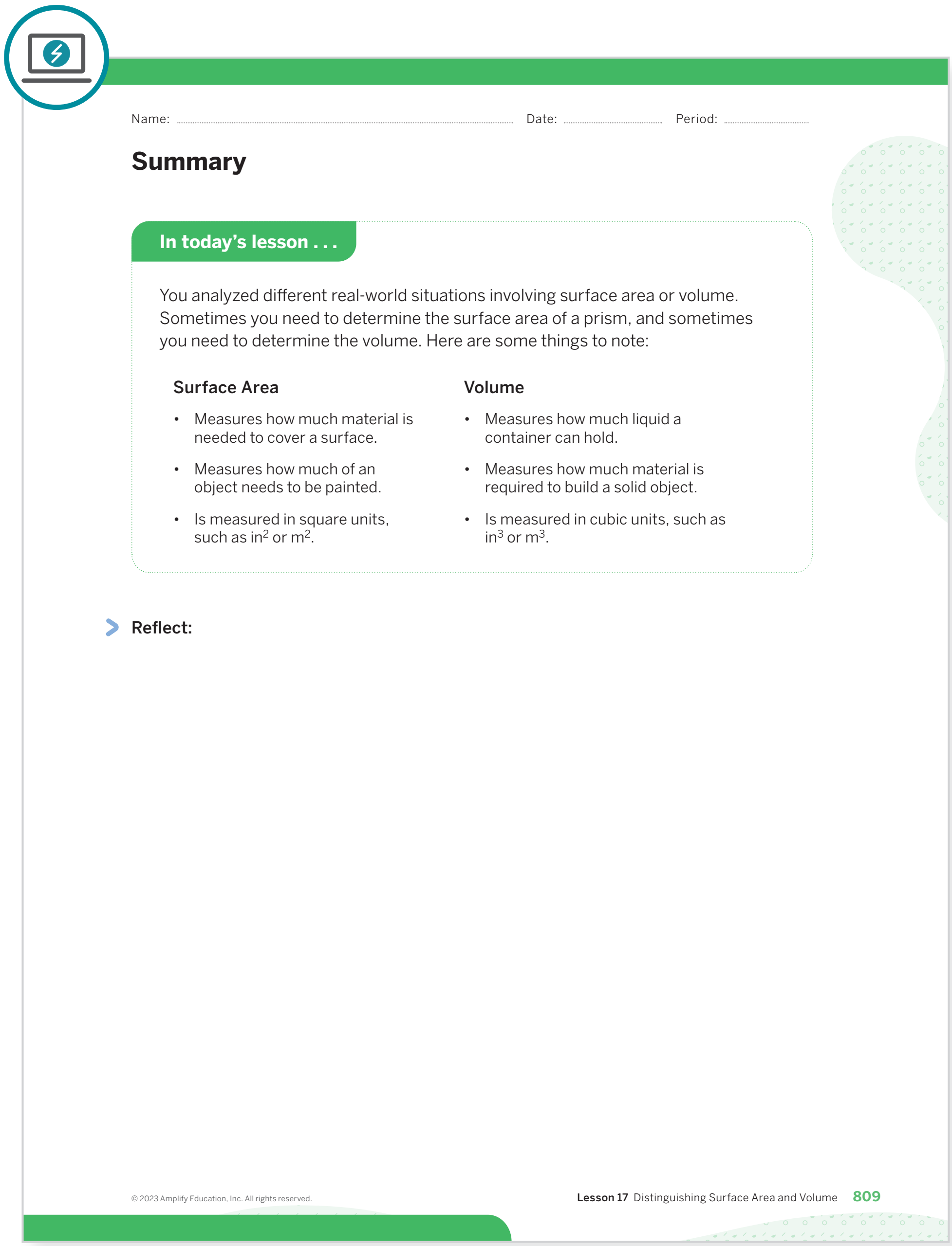
During the Connect, add to the class display any phrases and explanations students use when determining whether to find the quiche's volume or surface area. Have students who need support deciding which measure to use refer to the display.

English Learners

Display images of correctly labeled and organized student work so that students can visualize the work as they participate in the discussions. For example, annotate any work or calculations that provide the area of the base as "area of base."

Summary

Review and synthesize how surface area and volume each measure different attributes of three-dimensional figures, and how to select the appropriate measure to use to solve a real-world problem.



Name: Date: Period:

Summary

In today's lesson . . .

You analyzed different real-world situations involving surface area or volume. Sometimes you need to determine the surface area of a prism, and sometimes you need to determine the volume. Here are some things to note:

Surface Area	Volume
Measures how much material is needed to cover a surface.	Measures how much liquid a container can hold.
Measures how much of an object needs to be painted.	Measures how much material is required to build a solid object.
Is measured in square units, such as in^2 or m^2.	Is measured in cubic units, such as in^3 or m^3.

Reflect:

Synthesize

Have students share strategies they have for determining whether surface area or volume should be used to solve a real-world problem. Have students add these strategies to their notes section and the methods for determining each measurement.

Note: At this time, students do not need to know or use a formula for surface area; however, if they have created one, check to make sure it is accurate.

Highlight examples of each measure, making sure students understand why surface area is used versus volume, and vice versa.

Ask:

- "When is it better to know surface area rather than volume?" Sample responses: when you are covering an object, when you want to know how much is exposed to the environment, etc.
- "When is it better to know volume rather than surface area?" Sample responses: When you are filling up an object, when you need to know how much is already inside, etc.
- "If you cut an object in half and consider the totality of the two halves, how are the surface area and volume affected?" The volume remains unchanged, but the surface area will increase.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

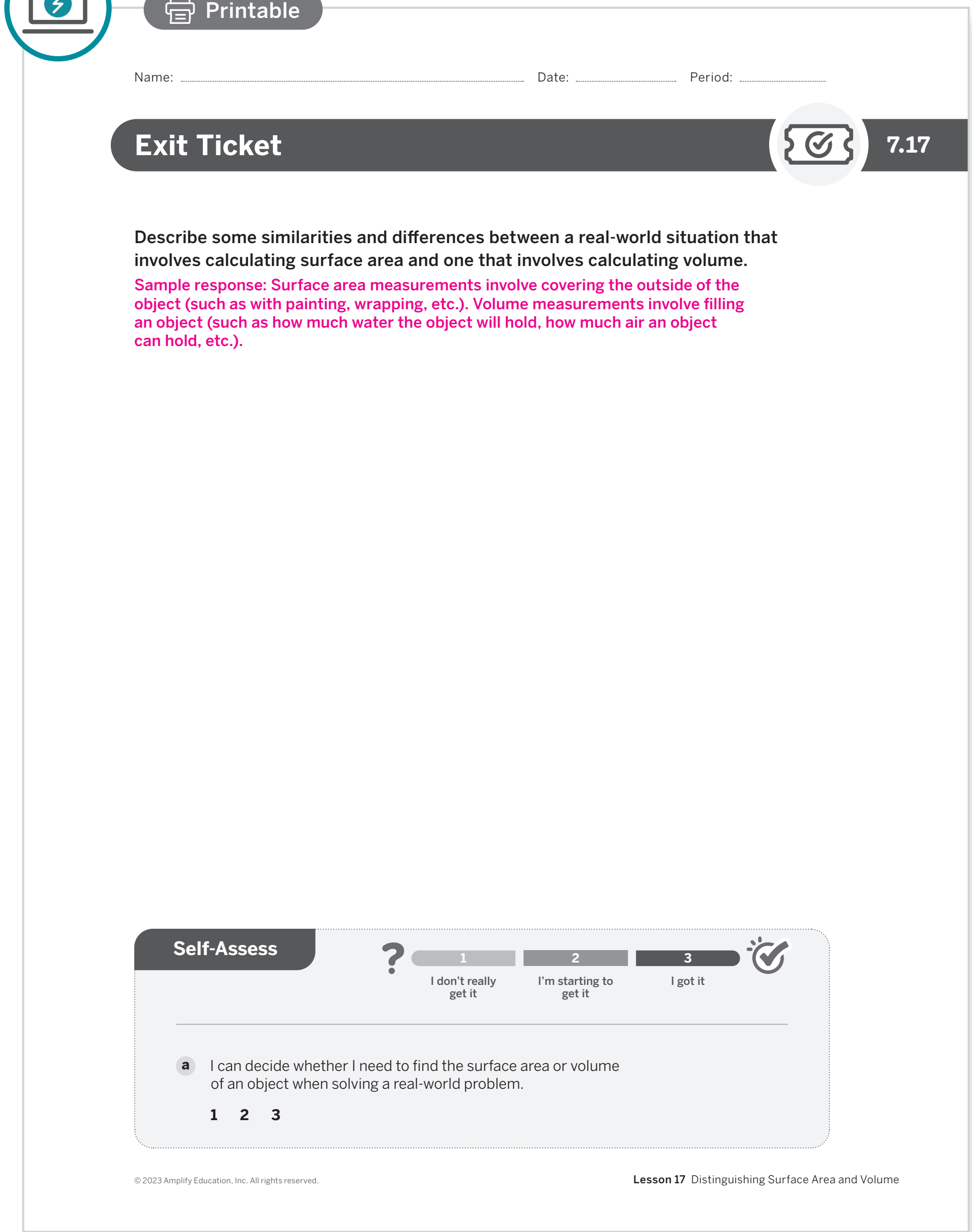
- "In your own words, how would you describe surface area? What about volume?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing similarities and differences between real-world situations in which surface area or volume is the appropriate measure.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 7.17

Describe some similarities and differences between a real-world situation that involves calculating surface area and one that involves calculating volume.

Sample response: Surface area measurements involve covering the outside of the object (such as with painting, wrapping, etc.). Volume measurements involve filling an object (such as how much water the object will hold, how much air an object can hold, etc.).

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can decide whether I need to find the surface area or volume of an object when solving a real-world problem.

1 2 3

Lesson 17 Distinguishing Surface Area and Volume

Success looks like . . .

- **Language Goal:** Comparing and contrasting problems involving surface area and volume of prisms. **(Speaking and Listening, Reading and Writing)**
  - » Explaining the similarities and differences between calculating surface area and volume in a real-world situation.
- **Language Goal:** Deciding whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation and justifying the decision. **(Speaking and Listening)**

Suggested next steps

If students confuse surface area and volume scenarios, consider:

- Reviewing Activity 2.
- Assigning Practice Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion in Activity 2, how did you encourage each student to listen to one another's strategies?
- The instructional goal for this lesson was for students to compare and contrast surface area and volume in order to solve real world problems. How well did students accomplish this goal? What did you specifically do to help students accomplish it? What might you change for the next time you teach this lesson?

Math Language Development

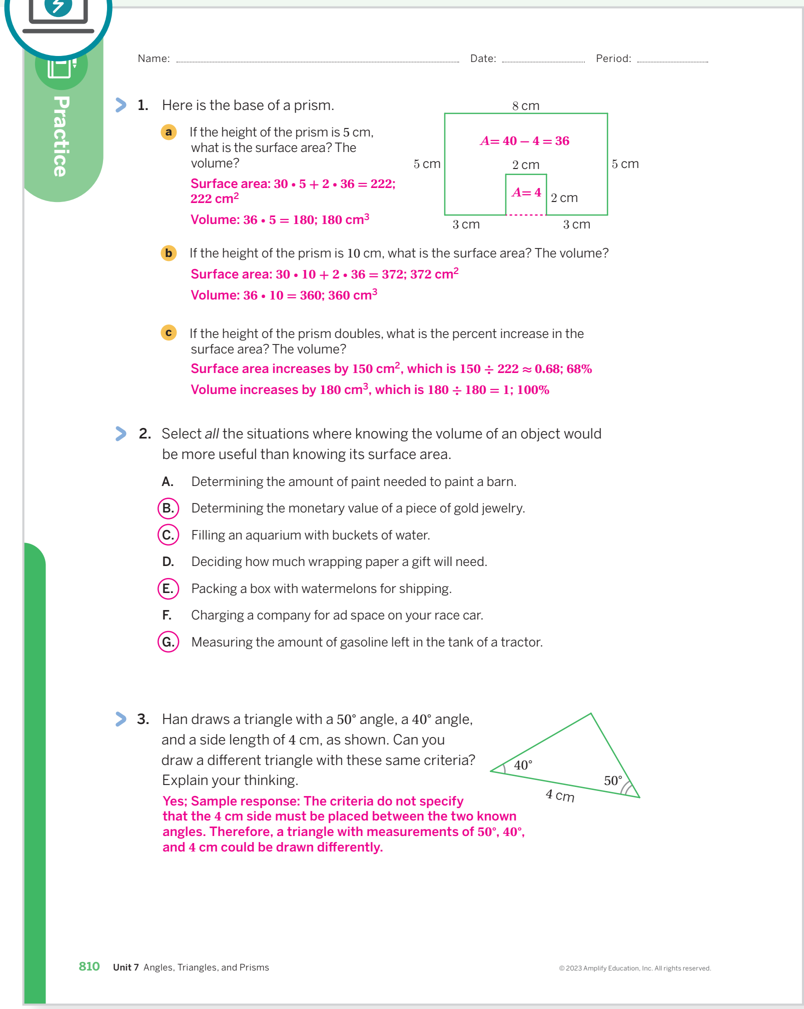
Language Goal: Deciding whether to calculate the surface area of volume of a prism to solve a problem in a real-world situation and justifying the decision.

Reflect on students' language development toward this goal.

- How have students progressed in their justifications for deciding whether to calculate the surface area or volume, given a real-world situation? What key words do they look for?
- How did using the ***Collect and Display*** routine in this lesson help students look for certain key words as they make their decisions? Would you change anything the next time you use this routine?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Here is the base of a prism.

a. If the height of the prism is 5 cm, what is the surface area? The volume?

Surface area: $30 \cdot 5 + 2 \cdot 36 = 222$; 222 cm^2

Volume: $36 \cdot 5 = 180$; 180 cm^3

b. If the height of the prism is 10 cm, what is the surface area? The volume?

Surface area: $30 \cdot 10 + 2 \cdot 36 = 372$; 372 cm^2

Volume: $36 \cdot 10 = 360$; 360 cm^3

c. If the height of the prism doubles, what is the percent increase in the surface area? The volume?

Surface area increases by 150 cm^2, which is $150 \div 222 \approx 0.68$; 68%

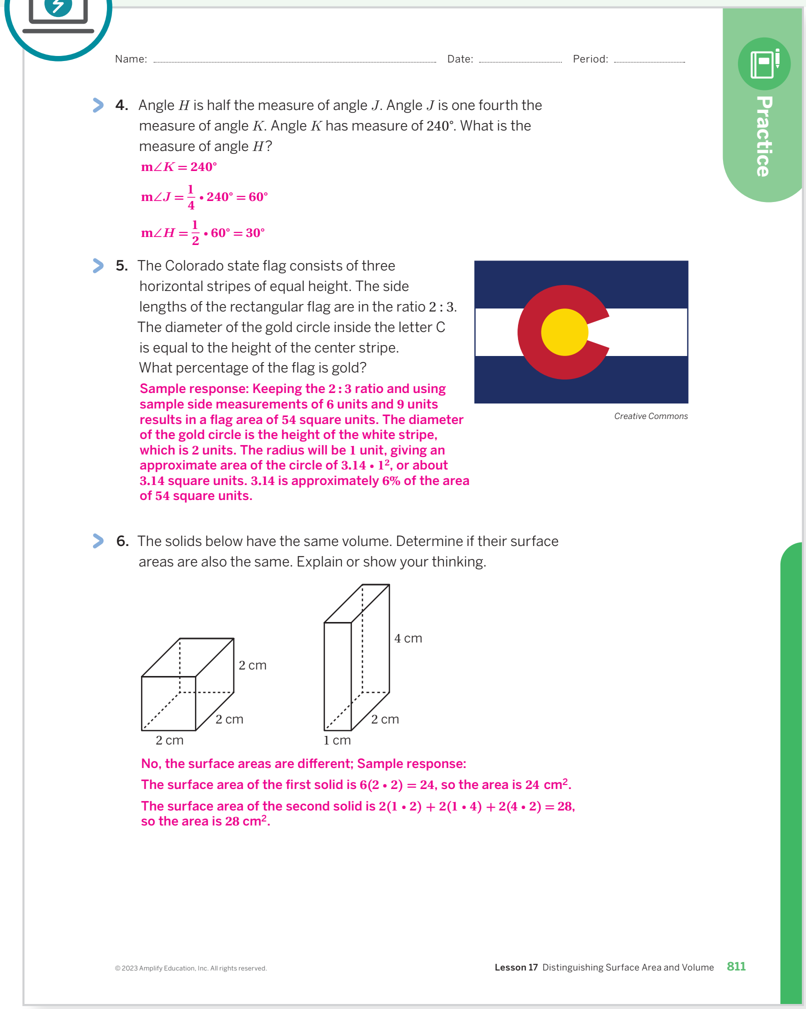
Volume increases by 180 cm^3, which is $180 \div 180 = 1$; 100%

2. Select *all* the situations where knowing the volume of an object would be more useful than knowing its surface area.

A. Determining the amount of paint needed to paint a barn.
[✓ B.] Determining the monetary value of a piece of gold jewelry.
[✓ C.] Filling an aquarium with buckets of water.
D. Deciding how much wrapping paper a gift will need.
[✓ E.] Packing a box with watermelons for shipping.
F. Charging a company for ad space on your race car.
[✓ G.] Measuring the amount of gasoline left in the tank of a tractor.

3. Han draws a triangle with a 50° angle, a 40° angle, and a side length of 4 cm, as shown. Can you draw a different triangle with these same criteria? Explain your thinking.

Yes; Sample response: The criteria do not specify that the 4 cm side must be placed between the two known angles. Therefore, a triangle with measurements of 50°, 40°, and 4 cm could be drawn differently.



Practice

Name: ______ Date: ______ Period: ______

4. Angle H is half the measure of angle J. Angle J is one fourth the measure of angle K. Angle K has measure of 240°. What is the measure of angle H?

$m\angle K = 240°$

$m\angle J = \frac{1}{4} \cdot 240° = 60°$

$m\angle H = \frac{1}{2} \cdot 60° = 30°$

5. The Colorado state flag consists of three horizontal stripes of equal height. The side lengths of the rectangular flag are in the ratio 2 : 3. The diameter of the gold circle inside the letter C is equal to the height of the center stripe. What percentage of the flag is gold?

Creative Commons

Sample response: Keeping the 2 : 3 ratio and using sample side measurements of 6 units and 9 units results in a flag area of 54 square units. The diameter of the gold circle is the height of the white stripe, which is 2 units. The radius will be 1 unit, giving an approximate area of the circle of $3.14 \cdot 1^2$, or about 3.14 square units. 3.14 is approximately 6% of the area of 54 square units.

6. The solids below have the same volume. Determine if their surface areas are also the same. Explain or show your thinking.

No, the surface areas are different; Sample response:

The surface area of the first solid is $6(2 \cdot 2) = 24$, so the area is 24 cm^2.

The surface area of the second solid is $2(1 \cdot 2) + 2(1 \cdot 4) + 2(4 \cdot 2) = 28$, so the area is 28 cm^2.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 2	1
Spiral	3	Unit 7 Lesson 11	2
	4	Unit 7 Lesson 4	2
	5	Unit 3 Lesson 10	3
Formative ⚡	6	Unit 7 Lesson 18	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice.**

UNIT 7 | **LESSON 18 – CAPSTONE**

Applying Volume and Surface Area

Let's explore applications of volume and surface area.

Focus

Goals

1. **Language Goal:** Apply reasoning about volume and surface area of prisms as well as proportional relationships to calculate how much the material to build an office building will cost and explain the solution method. **(Speaking and Listening, Writing)**
2. **Language Goal:** Describe, compare, and contrast the solids from which a given set of cross sections could have originated. **(Speaking and Listening)**

Coherence

Today

Students return to cross sections in the Warm-up to build further understanding of planes that are not parallel or perpendicular to a base of a solid. They explore how adjusting dimensions of a fixed-volume prism impacts the surface area. Further, students model with mathematics when considering how to minimize costs by varying the dimensions of the prism.

Previously

Students worked with cross sections in Lesson 13 and with volume and surface area of prisms in Lessons 14–17.

Coming Soon

Students will continue their work with cross sections in high school when they identify three-dimensional objects generated by rotations of two-dimensional objects.

Rigor

- Students **apply** their understanding of cross sections, surface area, and volume to the creation of office buildings.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	8 min
Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, pre-cut and pre-assembled, one set per pair

Math Language Development

Review words

- *base (of a prism or pyramid)*
- *cross section*
- *plane*
- *prism*
- *pyramid*
- *surface area*
- *volume*

Amps | **Featured Activity**

Activity 1
Dynamic Building Blocks

Students use the building tool to quickly construct an office building of varying dimensions. This allows them to test, analyze, and rebuild quickly. Students compete to find who can design the building with the least cost. Students receive a notification of their current place in the competition.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle with evaluating which design model works best each set of specifications in Activity 1. Encourage students to compare their models and think about what ethical responsibilities might come into play when building a building. As they present their best design, have them reflect on why they made the choice that they made.

Modifications to Pacing

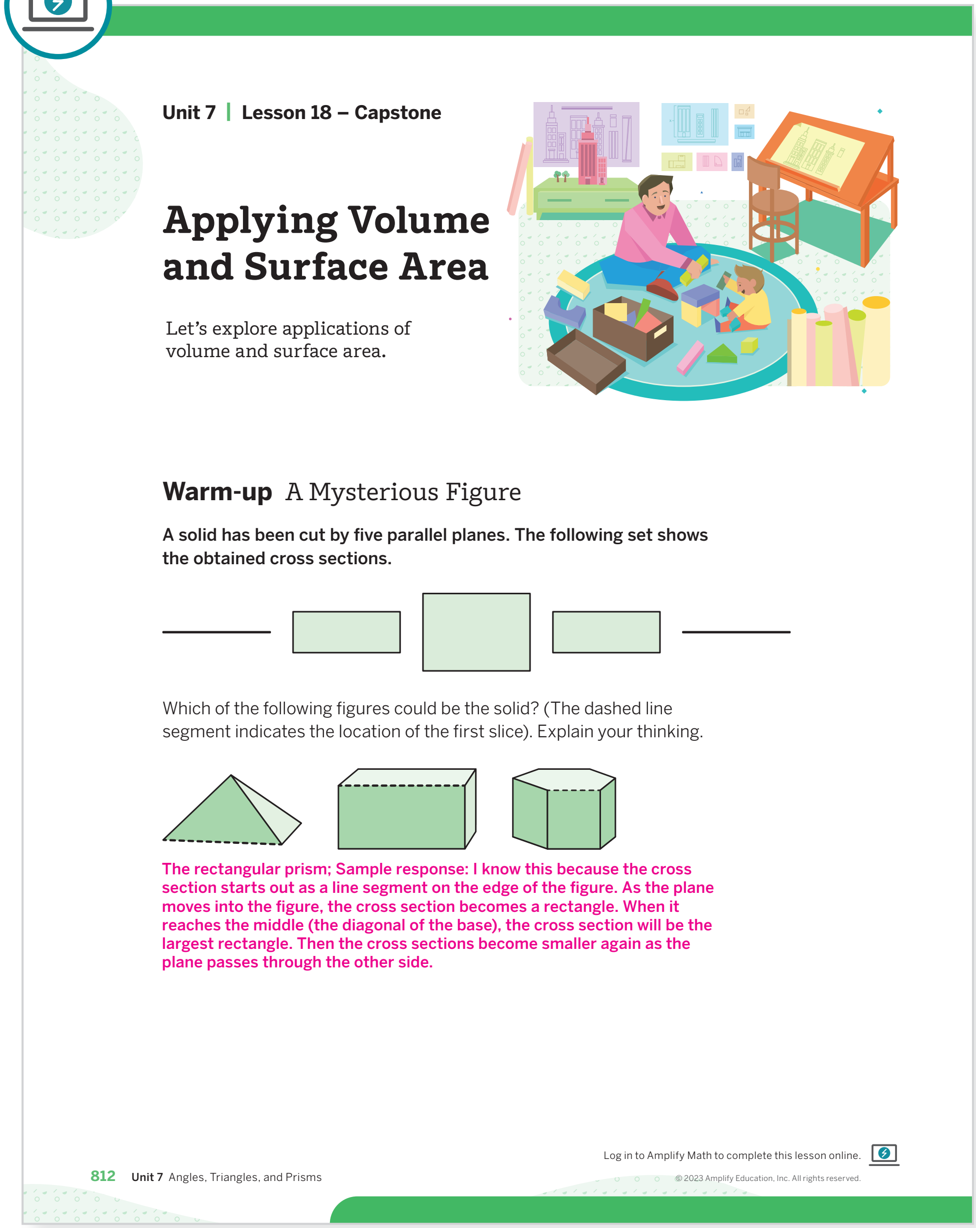
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Problems 2 and 3 may be omitted.

Pairs | 10 min

Warm-up A Mysterious Figure

Students interpret a set of cross sections to determine from which solid each cross section could have originated, preparing them to think about cross sections in a new way.



Unit 7 | Lesson 18 – Capstone

Applying Volume and Surface Area

Let's explore applications of volume and surface area.

Warm-up A Mysterious Figure

A solid has been cut by five parallel planes. The following set shows the obtained cross sections.

Which of the following figures could be the solid? (The dashed line segment indicates the location of the first slice). Explain your thinking.

The rectangular prism; Sample response: I know this because the cross section starts out as a line segment on the edge of the figure. As the plane moves into the figure, the cross section becomes a rectangle. When it reaches the middle (the diagonal of the base), the cross section will be the largest rectangle. Then the cross sections become smaller again as the plane passes through the other side.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Consider demonstrating a set of parallel planes slicing a solid different from those in this Warm-up.

2 Monitor

Help students get started by asking how the cross sections change from the start to the end of the set.

Look for points of confusion:

- **Thinking that, because the cross sections are rectangles, the figure must be the rectangular prism.** Challenge their thinking by asking, "Have you noticed that rectangular pyramids can have rectangular cross sections?"

Look for productive strategies:

- Sketching the planes through the solids to check if given cross sections are possible.

3 Connect

Display the digital tool that shows the cross sections.

Have individual students share what the cross sections of the other two solids would look like if they were also sliced by planes diagonal to the base.

Ask, "When is the cross section of a three-dimensional solid a line segment?" When the plane passes through the edge of the figure.

Highlight that, when considering the shape of the cross section, it is important to think about which edges and faces are included in the slice.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

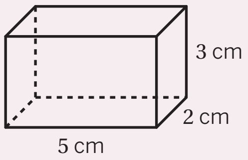
Pre-assemble nets from the Warm-up PDF to form the three-dimensional solids shown in the Warm-up. Allow students to hold these models to support their visualization and reasoning about the possible cross sections.

Power-up

To power up students' ability to determine the surface area of a prism, have students complete:

Recall that one method of determining the surface area of a prism is to add the area of each base B to the area of the rectangular faces. Their area can be determined by multiplying the perimeter of the base by the height of the prism.

Determine the surface area of the prism. 62 cm^2



Use: Before Activity 1.

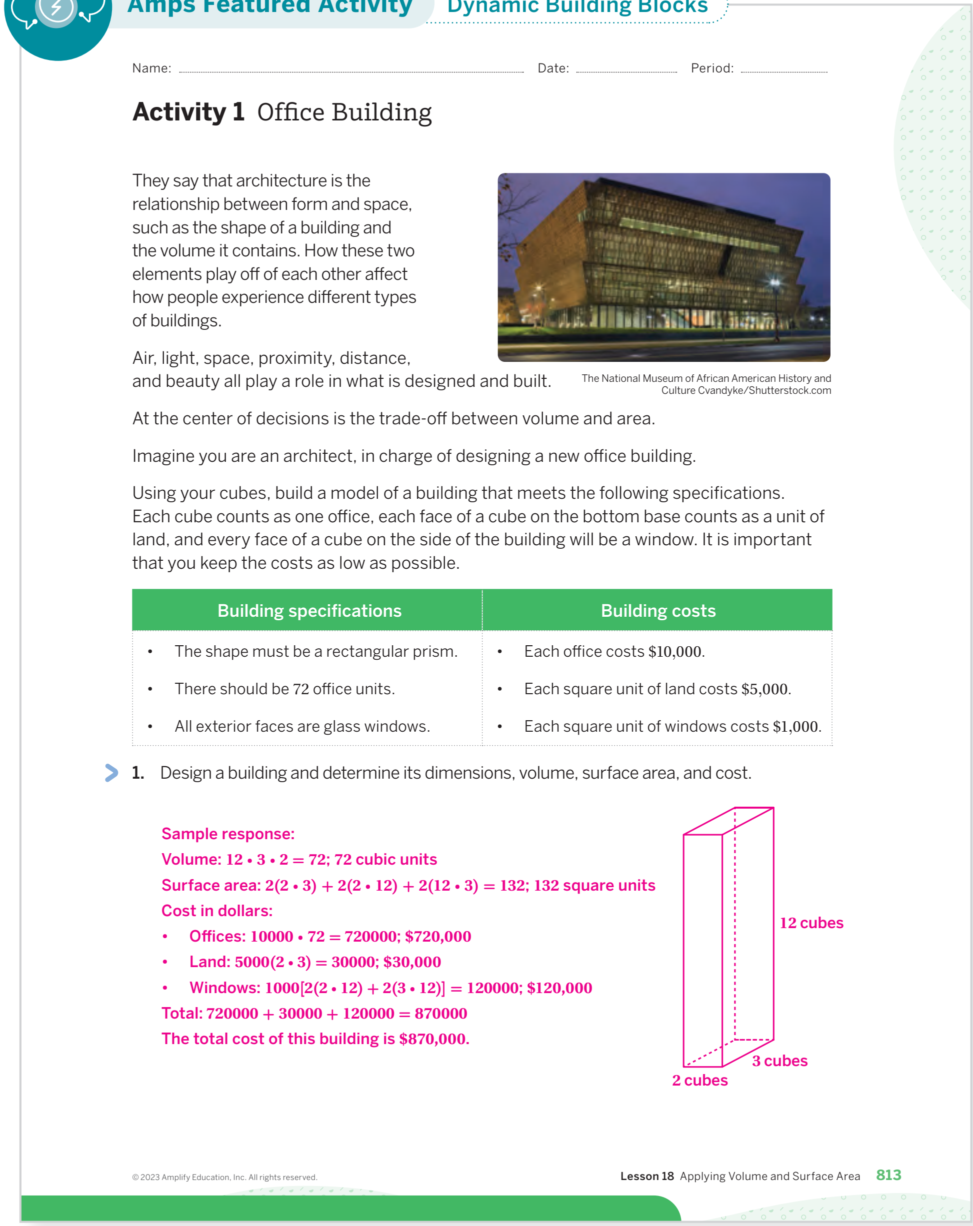
Informed by: Performance on Lesson 17, Practice Problem 6.

Small Groups | 25 min

Activity 1 Office Building

Students design an office building according to certain specifications, while thinking strategically about minimizing costs without adjusting volume (72 office units).

Amps Featured Activity Dynamic Building Blocks



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Activity 1 Office Building

They say that architecture is the relationship between form and space, such as the shape of a building and the volume it contains. How these two elements play off of each other affect how people experience different types of buildings.

Air, light, space, proximity, distance, and beauty all play a role in what is designed and built.

The National Museum of African American History and Culture Cvandyke/Shutterstock.com

At the center of decisions is the trade-off between volume and area.

Imagine you are an architect, in charge of designing a new office building.

Using your cubes, build a model of a building that meets the following specifications. Each cube counts as one office, each face of a cube on the bottom base counts as a unit of land, and every face of a cube on the side of the building will be a window. It is important that you keep the costs as low as possible.

Building specifications	Building costs
The shape must be a rectangular prism.	Each office costs $10,000.
There should be 72 office units.	Each square unit of land costs $5,000.
All exterior faces are glass windows.	Each square unit of windows costs $1,000.

1. Design a building and determine its dimensions, volume, surface area, and cost.

Sample response:

Volume: $12 \cdot 3 \cdot 2 = 72$; 72 cubic units

Surface area: $2(2 \cdot 3) + 2(2 \cdot 12) + 2(12 \cdot 3) = 132$; 132 square units

Cost in dollars:

- Offices: $10000 \cdot 72 = 720000$; $720,000
- Land: $5000(2 \cdot 3) = 30000$; $30,000
- Windows: $1000[2(2 \cdot 12) + 2(3 \cdot 12)] = 120000$; $120,000

Total: $720000 + 30000 + 120000 = 870000$

The total cost of this building is $870,000.

1 Launch

Read the introduction as a class. Tell students they may try out more than two plans and that the plan that costs the least (without adjusting the volume of 72 office units) will be named at the end. Distribute 72 linking cubes to each group.

2 Monitor

Help students get started by activating their background knowledge. Ask, “Have you visited an office building in which all sides of the building are windows?”

Look for points of confusion:

- **Minimizing only the land area.** Ask, “When the base area is small, how does that affect the number of windows in your building?”
- **Minimizing only the surface area.** Have students reread the specifications. Have them verify that their building meets each specification.

Look for productive strategies:

- Using an organizational strategy to track which faces have been accounted for when finding the surface area. Consider organizing results by adding them to a spreadsheet that all students can see. This will encourage groups to try different combinations of dimensions.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use the interactive tool to construct an office building given different dimensions. This allows them to test, analyze, and quickly rebuild.

Extension: *Math Enrichment*

Have students complete the following problem:

If the volume of a rectangular prism is 216 cm^3, what are the dimensions of the prism that minimize the surface area? A cube with a side length of 6 cm; The surface area is 216 cm^2.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the demands of the task.

- ***Read 1:*** Students should understand that they need to design a building that meets the specifications given in the table.
- ***Read 2:*** Ask students to name the given quantities and relationships, such as *each office costs* $10,000.
- ***Read 3:*** Ask students to brainstorm strategies for how to design their building that keeps the cost as low as possible.

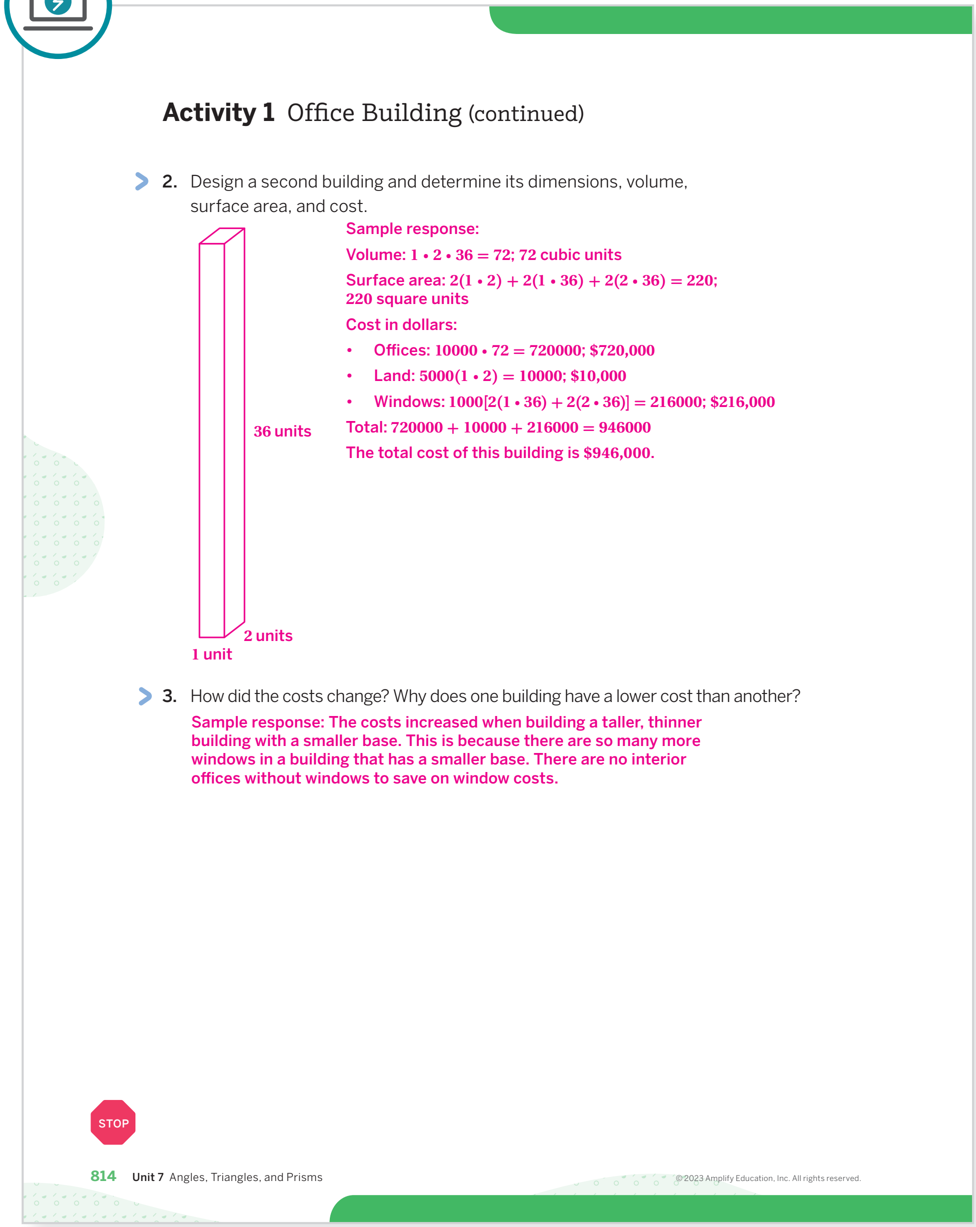
English Learners

Draw a sample rectangular prism, without labels, and illustrate where the office units and windows will be located.

Small Groups | 25 min

Activity 1 Office Building (continued)

Students design an office building according to certain specifications, while thinking strategically about minimizing costs without adjusting volume (72 office units).



Activity 1 Office Building (continued)

2. Design a second building and determine its dimensions, volume, surface area, and cost.

Sample response:

Volume: $1 \cdot 2 \cdot 36 = 72$; 72 cubic units

Surface area: $2(1 \cdot 2) + 2(1 \cdot 36) + 2(2 \cdot 36) = 220$; 220 square units

Cost in dollars:

- Offices: $10000 \cdot 72 = 720000$; \$720,000
- Land: $5000(1 \cdot 2) = 10000$; \$10,000
- Windows: $1000[2(1 \cdot 36) + 2(2 \cdot 36)] = 216000$; \$216,000

Total: $720000 + 10000 + 216000 = 946000$

The total cost of this building is \$946,000.

3. How did the costs change? Why does one building have a lower cost than another?

Sample response: The costs increased when building a taller, thinner building with a smaller base. This is because there are so many more windows in a building that has a smaller base. There are no interior offices without windows to save on window costs.

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3 Connect

Display a few different models of offices that students have built. Include the design with the least cost.

Have groups of students share their best design and the values they found for their design using the ***Gallery Tour*** routine.

Highlight that there seems to be a pattern related to finding the lowest cost for a building. There is a lower cost for buildings that have a smaller base, but yet not too tall, which incurs a greater cost for windows.

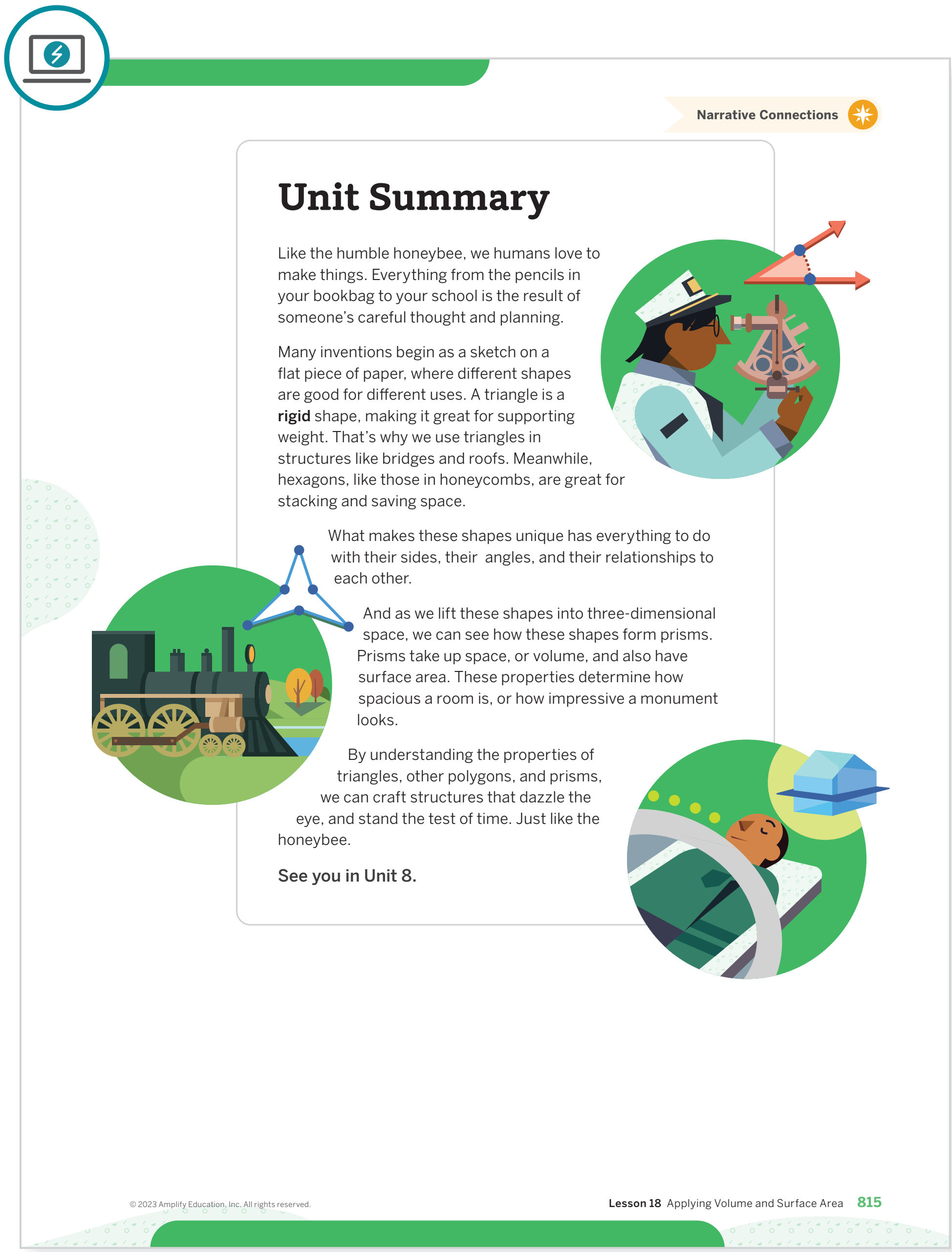
Ask:

- "Which design had the least cost? Why do you think that is?"
- "Which costs stayed the same? Which changed?"
- "What is the tallest building that could be constructed and meet these specifications?"

Whole Class | 5 min

Unit Summary

Review and synthesize what students understand from this unit by having them complete a graphic organizer and compare their notes with others.



Narrative Connections

Unit Summary

Like the humble honeybee, we humans love to make things. Everything from the pencils in your bookbag to your school is the result of someone's careful thought and planning.

Many inventions begin as a sketch on a flat piece of paper, where different shapes are good for different uses. A triangle is a **rigid** shape, making it great for supporting weight. That's why we use triangles in structures like bridges and roofs. Meanwhile, hexagons, like those in honeycombs, are great for stacking and saving space.

What makes these shapes unique has everything to do with their sides, their angles, and their relationships to each other.

And as we lift these shapes into three-dimensional space, we can see how these shapes form prisms. Prisms take up space, or volume, and also have surface area. These properties determine how spacious a room is, or how impressive a monument looks.

By understanding the properties of triangles, other polygons, and prisms, we can craft structures that dazzle the eye, and stand the test of time. Just like the honeybee.

See you in Unit 8.

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Lesson 18 Applying Volume and Surface Area 815

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary. Have students read the Summary or have a student volunteer read it aloud.

Highlight that it is very useful to spend some time reflecting on one's own learning at the conclusion of a unit. This helps to retain information and make connections that may not have been made when focusing on each individual lesson.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the unit narratives. To help them engage in meaningful reflection, consider asking:

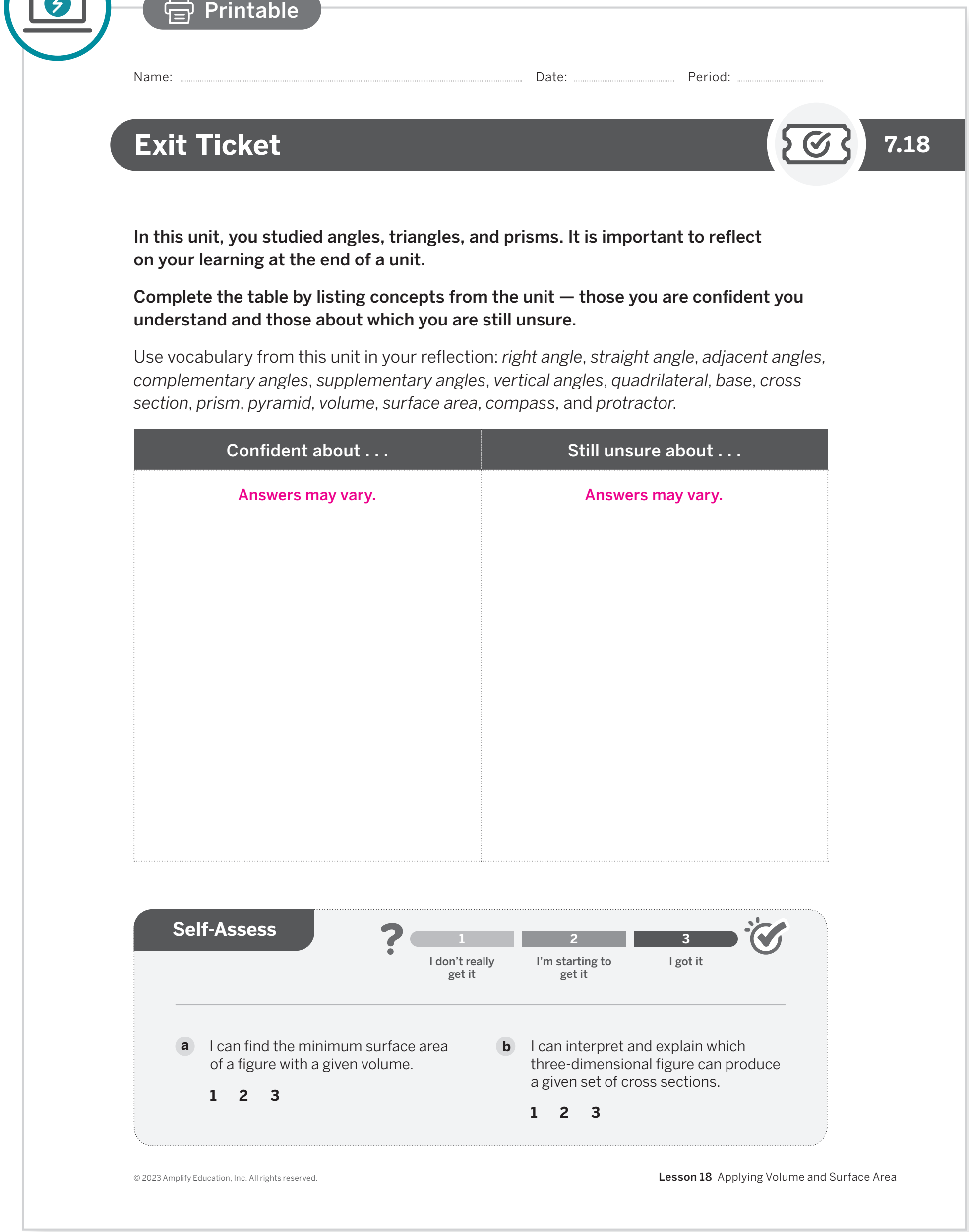
- "Can you think of other places where triangles are found in nature or architecture? In the room you are in right now?"
- "Which objects or spaces around you are shaped like prisms? What things in the wider world are shaped like prisms?"

Independent | 8 min

Exit Ticket

Students demonstrate their understanding of this unit by reflecting on their confidence with the geometrical concepts and terms used throughout the unit.

Printable



Name: ______________________ Date: __________ Period: ________

Exit Ticket 7.18

In this unit, you studied angles, triangles, and prisms. It is important to reflect on your learning at the end of a unit.

Complete the table by listing concepts from the unit — those you are confident you understand and those about which you are still unsure.

Use vocabulary from this unit in your reflection: *right angle*, *straight angle*, *adjacent angles*, *complementary angles*, *supplementary angles*, *vertical angles*, *quadrilateral*, *base*, *cross section*, *prism*, *pyramid*, *volume*, *surface area*, *compass*, and *protractor*.

Confident about . . .	Still unsure about . . .
Answers may vary.	Answers may vary.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can find the minimum surface area of a figure with a given volume.

1 2 3

b. I can interpret and explain which three-dimensional figure can produce a given set of cross sections.

1 2 3

Success looks like . . .

- **Language Goal:** Applying reasoning about volume and surface area of prisms as well as proportional relationships to calculate how much the material to build an office building will cost and explaining the solution method. **(Speaking and Listening, Writing)**
- **Language Goal:** Describing, comparing, and contrasting the solids from which a given set of cross sections could have originated. **(Speaking and Listening)**

Suggested next steps

If students are struggling to access or recall the vocabulary from this unit, consider:

- Having them use the glossary while completing the Exit Ticket.
- **Math Language Development:** Having them create *Frayer Model* graphic organizers for vocabulary with which they are having the most difficulty. This type of graphic organizer includes sections for students to list examples and nonexamples, draw illustrations, or note characteristics of vocabulary terms.

Professional Learning

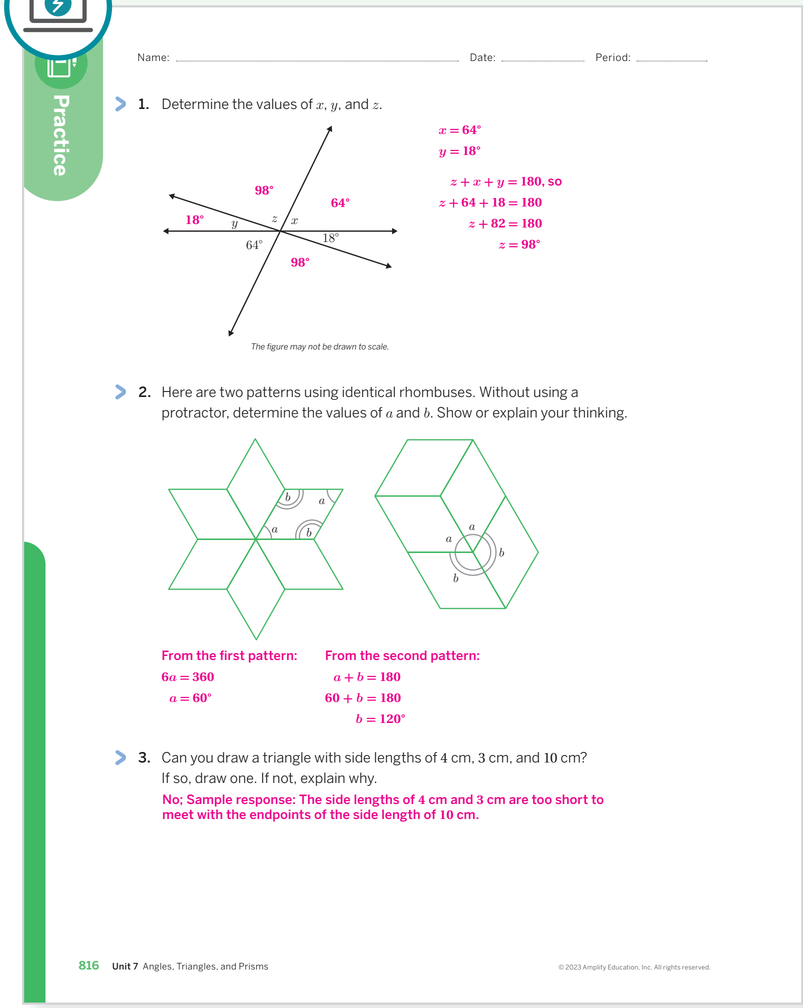
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What was especially satisfying about how students worked in teams to create their building designs?
- What challenges did students encounter as they worked through Activity 1? How did they work through it? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Determine the values of x, y, and z.

The figure may not be drawn to scale.

$x = 64°$

$y = 18°$

$z + x + y = 180$, so

$z + 64 + 18 = 180$

$z + 82 = 180$

$z = 98°$

2. Here are two patterns using identical rhombuses. Without using a protractor, determine the values of a and b. Show or explain your thinking.

From the first pattern:

$6a = 360$

$a = 60°$

From the second pattern:

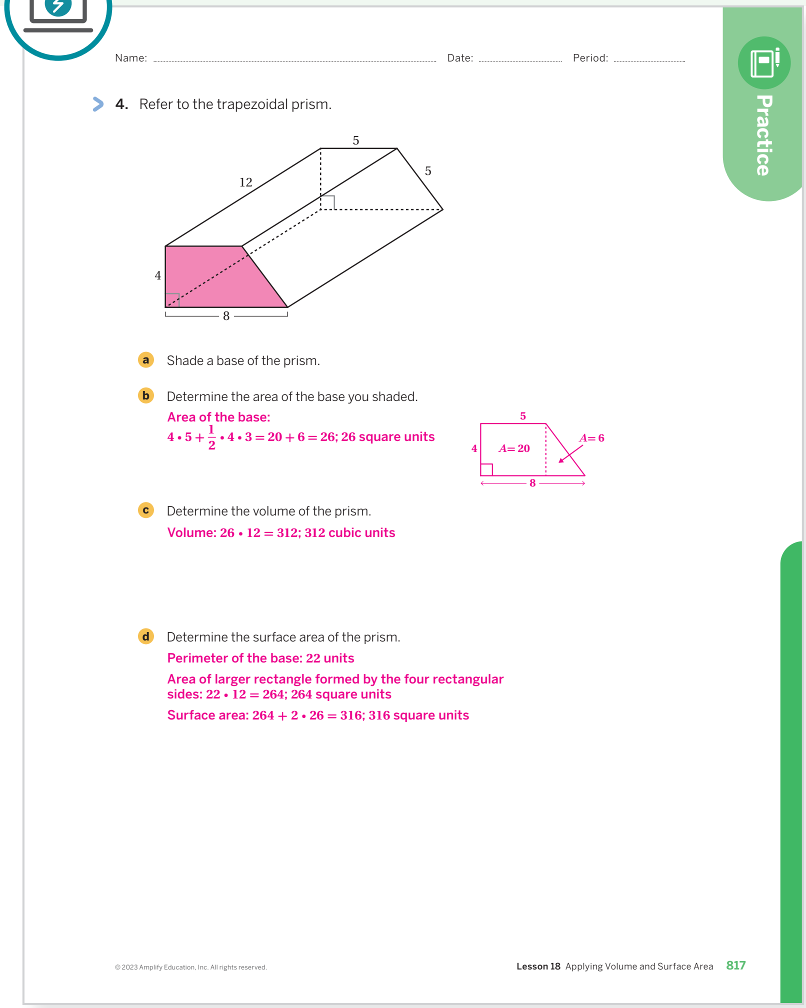
$a + b = 180$

$60 + b = 180$

$b = 120°$

3. Can you draw a triangle with side lengths of 4 cm, 3 cm, and 10 cm? If so, draw one. If not, explain why.

No; Sample response: The side lengths of 4 cm and 3 cm are too short to meet with the endpoints of the side length of 10 cm.



Name: ________ Date: ________ Period: ________

4. Refer to the trapezoidal prism.

a. Shade a base of the prism.

b. Determine the area of the base you shaded.

Area of the base:

$4 \cdot 5 + \frac{1}{2} \cdot 4 \cdot 3 = 20 + 6 = 26$; 26 square units

c. Determine the volume of the prism.

Volume: $26 \cdot 12 = 312$; 312 cubic units

d. Determine the surface area of the prism.

Perimeter of the base: 22 units

Area of larger rectangle formed by the four rectangular sides: $22 \cdot 12 = 264$; 264 square units

Surface area: $264 + 2 \cdot 26 = 316$; 316 square units

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 7 Lesson 4	2
	2	Unit 7 Lesson 2	2
	3	Unit 7 Lesson 8	2
	4	Unit 7 Lesson 17	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

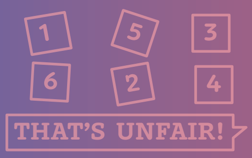
UNIT 8

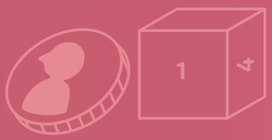
Probability and Sampling

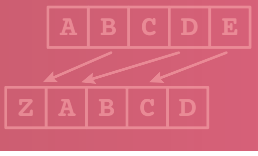
For the first time, students encounter how to quantify the chances of something happening. Though the future is unwritten, probability and statistics help us make better predictions and thus better decisions.

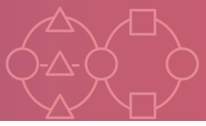
Essential Questions

- When faced with more than one possibility, how can you determine which is more likely to happen?
- Our world is really complicated — how can we simulate parts of it to make better predictions?
- When is a sample *not* representative of a population?
- *(By the way, how do you crack a Caesar cipher-encoded message?)*









Key Shifts in Mathematics

Focus

In this unit . . .

Students develop the knowledge and language to describe the probability of single-step and multi-step chance events. When chance events grow more complex, they learn how to represent sample spaces in tables, tree diagrams, and lists — and to design and use simulations. Samples of populations are generated in hands-on experiments, compared, and analyzed for bias. Finally, students share the results of their own statistical study based on the understandings gained throughout the unit.

Coherence

Previously . . .

Students were introduced to statistics in Grade 6. They reasoned about statistical questions and statistical variability by thinking about the shapes of data distributions and how a single value could represent the measure of center or variation of a numerical data set.

Coming soon . . .

Students will continue to explore the ways statistical data are represented visually on scatter plots. They will look for patterns of association and try to determine linear models that fit the relationship – or decide when an association is not linear.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students see that the best way to estimate the probability of an event is to observe it over the long-run. As students roll dice and collect data from classmates, they notice that many trials are needed (Lesson 4).

Procedural Fluency

As students learn how to represent sample spaces for experiments with multiple parts, they are provided ample opportunity to practice with various scenarios and levels of complexity (Lesson 7).

Application

Students are provided with an opportunity to apply what they have learned about the benefits of different simulation types and design their own simulations (Lesson 10).

Winning Chance

SUB-UNIT

1

Lessons 2–6

Probabilities of Single-step Events

Students begin their formal study of probability and develop the mathematical language necessary for describing the ***probability*** of ***single-step events***. Through games and experiments, they learn that the ***chance*** of an event occurring is related to its ***sample space***.

Narrative: The women of Bletchley Park use probability to decode enemy messages during World War II.

SUB-UNIT

2

Lessons 7–10

Probabilities of Multi-step Events

Students discover that ***multi-step events*** are composed of more than one event. They organize the total number of possible outcomes in the sample space, using ***tree diagrams***, lists, and tables. To estimate the probability of even more complex events, students design and conduct ***simulations***.

Narrative: Discover how to determine the chances of drawing *both* Blazing Shoal and Dragonstorm.

Launch

The Invention of Fairness

Students explore the possibility that fairness can be quantified and measured. They play an unfair game and are given the power to change the rules to make it more fair. As they do so, they notice that they can begin to predict which events in the game are more likely than others.

SUB-UNIT

3

Lessons 11–16

Sampling

Students identify whether a sample is representative of a population, both numerically and visually. They begin to understand the importance of random sampling. As they watch out for sampling bias, they become more data-literate and better shepherds of their own data they collect.

Narrative: Use sampling and statistics to answer the questions that interest *you*.

Capstone

Lesson 17

Presentation of Findings

Students share the research and analysis of a statistical question they have been working on throughout the third Sub-Unit. They study each other's presentations, provide feedback, and take time to reflect on the feedback their presentation received. This lesson simulates the experience of conducting and refining academic research.

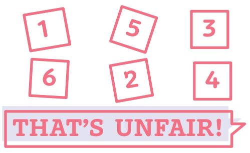
Unit at a Glance

Spoiler Alert: When tree diagrams grow too large and complex, use multiplication to help determine the total number of possible outcomes for a multi-step event.

Assessment

A Pre-Unit Readiness Assessment

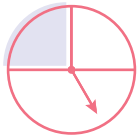
Launch Lesson



1 The Invention of Fairness

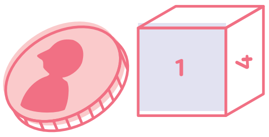
Introduce how fairness can be measured quantitatively and adjust rules to make an unfair game more fair.

Sub-Unit 1: Probabilities of Single-Step Events



2 Chance Experiments

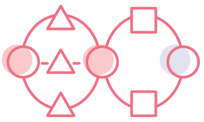
Investigate chance events and begin to describe likelihood using language such as *impossible*, *unlikely*, *likely*, and *certain*.



3 What Are Probabilities?

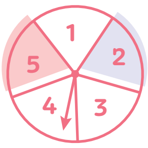
Assign probabilities to chance events and understand that the greater the probability, the more likely it is the event will occur.

Sub-Unit 2: Probabilities of Multi-Step Events



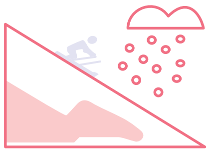
7 Keeping Track of All Possible Outcomes

Use tree diagrams, tables, and organized lists to describe the sample space of a multi-step event.



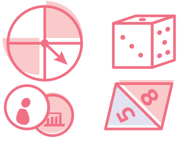
8 Experiments With Multi-step Events

Write sample spaces for multi-step chance experiments and begin using those sample spaces to calculate the probability of a multi-step event.



9 Simulating Multi-step Events

Discover that multi-step events can be simulated by using multi-step chance experiments.



10 Designing Simulations (optional) ●

Estimate probabilities by designing and executing simulations of multi-step events.

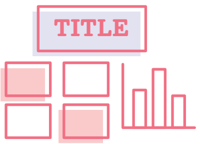
15 Estimating Population Measures of Center

Observe that a population with less variability produces samples that are more representative than populations with greater variability.

16 Estimating Population Proportions

Discover that having a representative sample makes it possible to use proportional reasoning to make predictions about a population.

Capstone



17 Presentation of Findings ●

Share the statistical studies students have worked on throughout the last Sub-Unit.

Assessment

A End-of-Unit Assessment

Key Concepts

Lesson 3: Identifying the sample space helps determine the probability of a chance event occurring.

Lesson 7: Tree diagrams, tables, and organized lists can be used to determine the total number of possible combinations of a multi-step event.

Lesson 14: Some samples from a population can be biased.

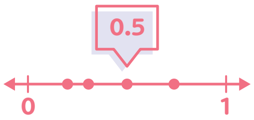
Pacing

17 Lessons: 45 min each
3 Assessments: 45 min each

Full Unit: 20 days
● **Modified Unit:** 17 days

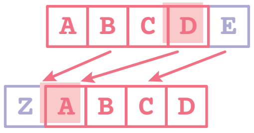
Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Assessment



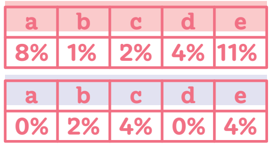
4 Estimating Probabilities Through Repeated Experiments

Perform an experiment and see that in the long run, the relative frequency approaches the theoretical probability of the chance event.



5 **Code Breaking** (Part 1) ●

Build on experience with ratios and experimental probability to perform a frequency analysis of letters in an encrypted message.

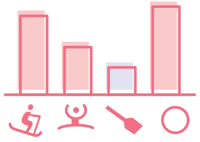


6 **Code Breaking** (Part 2) ●

Decode a message written and encrypted by classmates.

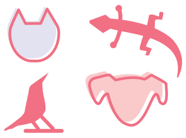
A Mid-Unit Assessment

Sub-Unit 3: Sampling



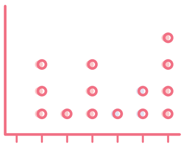
11 Comparing Two Populations

Decide whether two populations are very different from each other by comparing their means and mean absolute deviations.



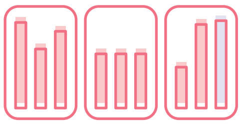
12 Larger Populations

Introduces how data can be gathered from a sample of a population when it is impractical to collect from every individual.



13 What Makes a Good Sample?

Examine different samples of the same population and learn what it means to be representative of the population.



14 Sampling in a Fair Way

Consider different methods for selecting a sample and see that bias may prevent a sample from being representative of the population.

● Modifications to Pacing

Lessons 5 and 6: In these lessons, students study encrypted messages to explore the frequency of letters used. The standards addressed will be re-introduced in the third Sub-Unit, so you may consider omitting Lessons 5 and 6.

Lesson 10: This lesson provides an opportunity for students to select an appropriate tool for a simulation, but does not introduce any new tools or strategies. It may be omitted.

Lesson 17: In the Capstone lesson, students who have completed all the components for their statistical study share and receive feedback on their work with the whole class. Alternatively, you might display students' work in the classroom or hall and omit the lesson.

Unit Supports

Math Language Development

Lesson	New Vocabulary	
2	chance experiment chance experiment equally likely as not event*	impossible likely outcome unlikely
3	probability sample space	
4	relative frequency	
7	multi-step event tree diagram	
9	simulation	
12	population sample	
13	representative sample	
14	random sample	
16	population proportion	

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
4, 9, 15	**MLR1:** Stronger and Clearer Each Time
1–4, 7, 9, 12–16	**MLR2:** Collect and Display
16	**MLR3:** Critique, Correct, Clarify
8	**MLR5:** Co-craft Questions
5, 7	**MLR6:** Three Reads
2, 4, 5, 7–9, 11, 13, 16	**MLR7:** Compare and Connect
8, 12–15	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket   Additional Practice

Lesson(s)	Additional required materials
5	calculators round head fasteners
12	class list of first and last names
10	coins colored blocks or marbles spinners
6	lined paper
5, 17	markers
1, 4, 9, 10	number cubes
2, 8, 9, 10	paper clips
3, 9, 10, 14, 16	bags
2, 3, 5–9, 11, 12, 14–16	PDFs are required for these lessons. Refer to each lesson to see which activities are required.
14	rulers straws
17	sticky notes
13	tracing paper

Instructional Routines

Activities throughout this unit include these routines.

Lesson(s)	Instructional Routine
2, 12	Card Sort
7, 9, 11, 15	Poll the Class
15	Would You Rather?
17	Gallery Tour
2, 13, 15	Think-Pair-Share

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 10
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 17

Amps

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Social & Collaborative Digital Moments

Featured Activity

What's in the Bag?

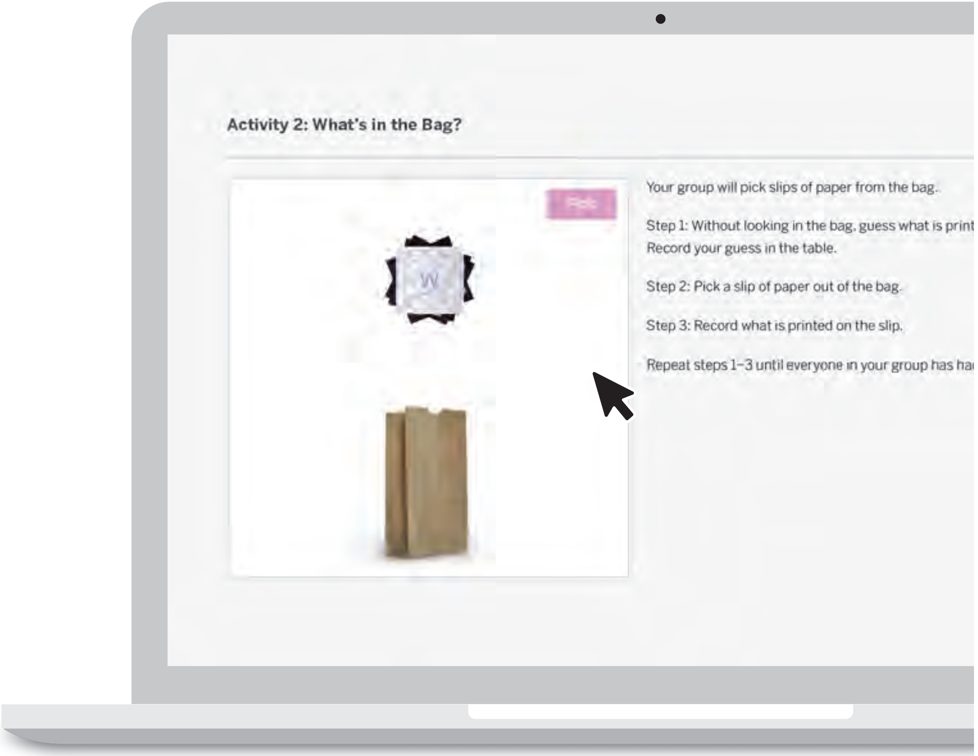
Put on your student hat and work through **Lesson 3, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Cubes and Coins **(Lesson 8)**
- In the Long Run **(Lesson 4)**
- Multi-Step Events **(Lesson 7)**
- Breeding Mice (Part 2) **(Lesson 10)**



Activity 2: What's in the Bag?

Your group will pick slips of paper from the bag.

Step 1: Without looking in the bag, guess what is print
Record your guess in the table.

Step 2: Pick a slip of paper out of the bag.

Step 3: Record what is printed on the slip.

Repeat steps 1–3 until everyone in your group has ha

Unit Study

Professional Learning

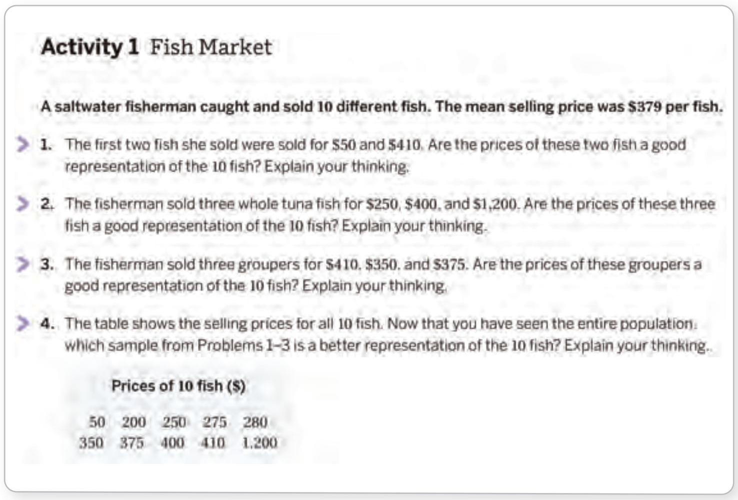
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 3 introduces the idea of larger populations where it's not possible or reasonable to survey everyone, but data can be gathered from a sample of the population. Students learn to look for fair samples that are unbiased to represent the population. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 13, Activity 1:**



Activity 1 Fish Market

A saltwater fisherman caught and sold 10 different fish. The mean selling price was $379 per fish.

1. The first two fish she sold were sold for $50 and $410. Are the prices of these two fish a good representation of the 10 fish? Explain your thinking.
2. The fisherman sold three whole tuna fish for $250, $400, and $1,200. Are the prices of these three fish a good representation of the 10 fish? Explain your thinking.
3. The fisherman sold three groupers for $410, $350, and $375. Are the prices of these groupers a good representation of the 10 fish? Explain your thinking.
4. The table shows the selling prices for all 10 fish. Now that you have seen the entire population, which sample from Problems 1–3 is a better representation of the 10 fish? Explain your thinking.

Prices of 10 fish ($)

50	200	250	275	280
350	375	400	410	1,200

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

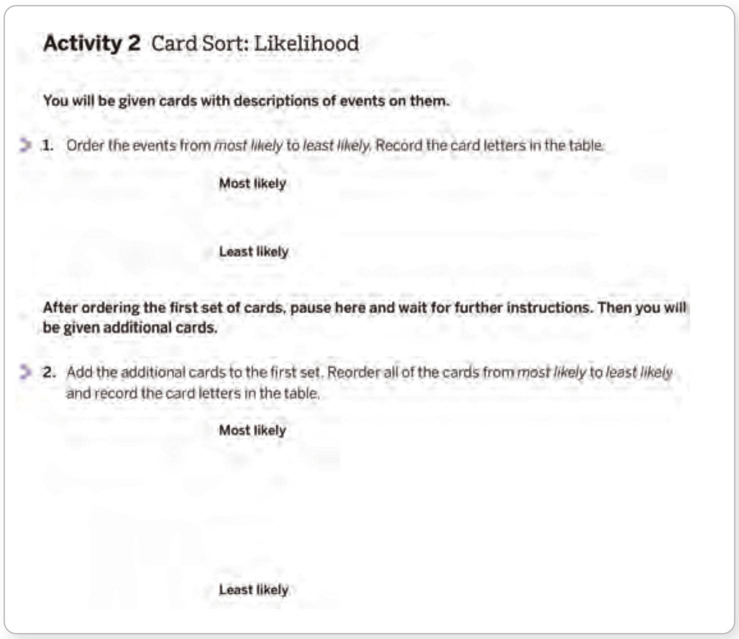
- What was it like to engage in this problem as a learner?
- Other than finding the mean price of each sample, are there other central measures that might be appropriate?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Choose One Instructional Routine to list here

Rehearse . . .

How you'll facilitate the ***Card Sort*** instructional routine in **Lesson 2, Activity 2:**



Activity 2 Card Sort: Likelihood

You will be given cards with descriptions of events on them.

1. Order the events from *most likely* to *least likely*. Record the card letters in the table.

Most likely

Least likely

After ordering the first set of cards, pause here and wait for further instructions. Then you will be given additional cards.

2. Add the additional cards to the first set. Reorder all of the cards from *most likely* to *least likely* and record the card letters in the table.

Most likely

Least likely

Points to Ponder . . .

- In this routine, students organize and classify information according to specific rules or goals.

This routine . . .

- Normalizes error as part of the process of learning.
- Encourages flexible thinking and sense-making.
- Engages visual and tactile learners.
- Allows for students to quickly adjust or correct their previous thinking.

Anticipate . . .

- Students may have difficulty keeping track of all of the cards.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Pose Purposeful Questions

This effective teaching practice . . .

- Helps you assess the reasoning behind student responses. They may arrive at a correct response using flawed reasoning; probing for their reasoning helps you know if they truly understand the concept.
- Helps you advance student reasoning and sense making by asking deeper questions about mathematical ideas and relationships.

Points to Ponder . . .

- How can you probe for student reasoning to ensure they understand a mathematical concept, beyond just providing a correct response?
- What kinds of questions can you ask that will help advance their understanding to a deeper level?

Math Language Development

MLR7: Compare and Connect

MLR7 appears in Lessons 2, 4, 5, 7–9, 11, 13, and 16.

- In Lesson 7, students compare three different representations for the sample space of a multi-step event. Probing questions are provided to help students use math language in their comparisons.
- In Lesson 11, students compare numerical values for mean and MAD to the visual displays of dot plots. Probing questions are provided for you to help students connect these two concepts.
- **English Learners:** In Lesson 5, Spanish letter frequencies are provided for students whose primary language is Spanish. Students can write a code in Spanish, create a cipher, and decode it using the Spanish letter frequencies provided.

Point to Ponder . . .

- How will you help students make greater connections between mathematical concepts and deepen their understanding of comparing two populations, sampling, and probability in this unit?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with using proportional reasoning in the context of probability problems throughout the unit? Do you think your students will generally:
  - » struggle with converting fluently between decimal, fraction, and percent representations?
  - » be able to adapt to different representations of situations involving probability?
  - » understand the concept of probability, but face challenges identifying the relevant information in a problem?

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Opportunities to provide visual support and guidance to help students process new information appear in Lessons 1, 3–8 and 12–17.

- In Lesson 8, provide students with a copy of the Activity 1 PDF (sample spaces) to help them make sense of the sample space for the multi-step event. Suggest that students use colored pencils to mark the favorable events in the sample space.
- Also, in Lesson 8, provide students with a copy of the correct response for Lesson 7, Practice Problem 6 which shows the sample space for rolling two number cubes to help them visualize the multi-step event.
- In Lessons 11, 12, and 15, as students calculate the mean absolute deviation, provide copies of the Graphic Organizer PDF, *MAD Recording Sheet* to help them visualize the calculations needed.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when your students may benefit from visual support or suggested guidance? What clues will you gather from your students?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their social awareness and self-awareness skills.

Points to Ponder . . .

- Students will often be asked to predict what might happen, based on their intuition. How can you prepare students to handle the disappointment or frustration when their intuition is countered by contrary evidence? How can you encourage students to persist in valuing their intuition when it is contradicted?
- What kind of relationships do students have with each other? Do they encourage each other and bring out the best in others? Are they able to cooperate and negotiate conflict? How well do they work in groups?

The Invention of Fairness

Let's figure out how to make complex games fair.

Focus

Goals

1. **Language Goal:** Compare the likelihoods of different outcomes when rolling two number cubes. **(Speaking and Listening)**
2. Create fair rules for a game, based on the likelihood of the outcomes when rolling two number cubes.

Coherence

Today

Students consider fairness from a quantitative perspective. By playing a game with two number cubes, they recognize that not all outcomes are equally likely, and they informally begin to reason about probability. When creating new rules for a game based on their observations during experiments, students reason both abstractly and quantitatively.

Previously

This is a special moment in your students' math careers — the start of work in a new branch of mathematics. Students have not yet been formally introduced to probability in prior grades. Get ready to help them navigate these uncharted waters. While students have not yet formally studied probability, they have developed ratio reasoning in Grade 6 and in earlier units of Grade 7. Ratio reasoning will be particularly helpful as they apply their understanding of ratios to probability.

Coming Soon

Students will formalize their understanding of probability and how to calculate the probability of certain outcomes.

Rigor

- Students build **conceptual understanding** of "fairness" by analyzing the outcomes of games with number cubes.
- Students play games with number cubes to develop **conceptual understanding** of the likelihood of events (probability).

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
5 min	25 min	10 min	5 min
Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

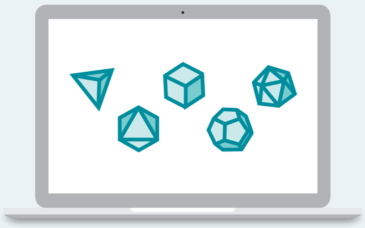
Materials

- Exit Ticket
- Additional Practice
- number cubes, two per group

Amps | **Featured Activity**

Activity 1
Digital Number Cubes

Students quickly roll and record their results, allowing for more representative outcomes.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated that the rules for the game they create in Round 2 of Activity 1 may not seem fair after playing only one time. Mention to students that — as they will see throughout the rest of the unit — some games must be played several times before you can tell whether they are fair.

Modifications to Pacing

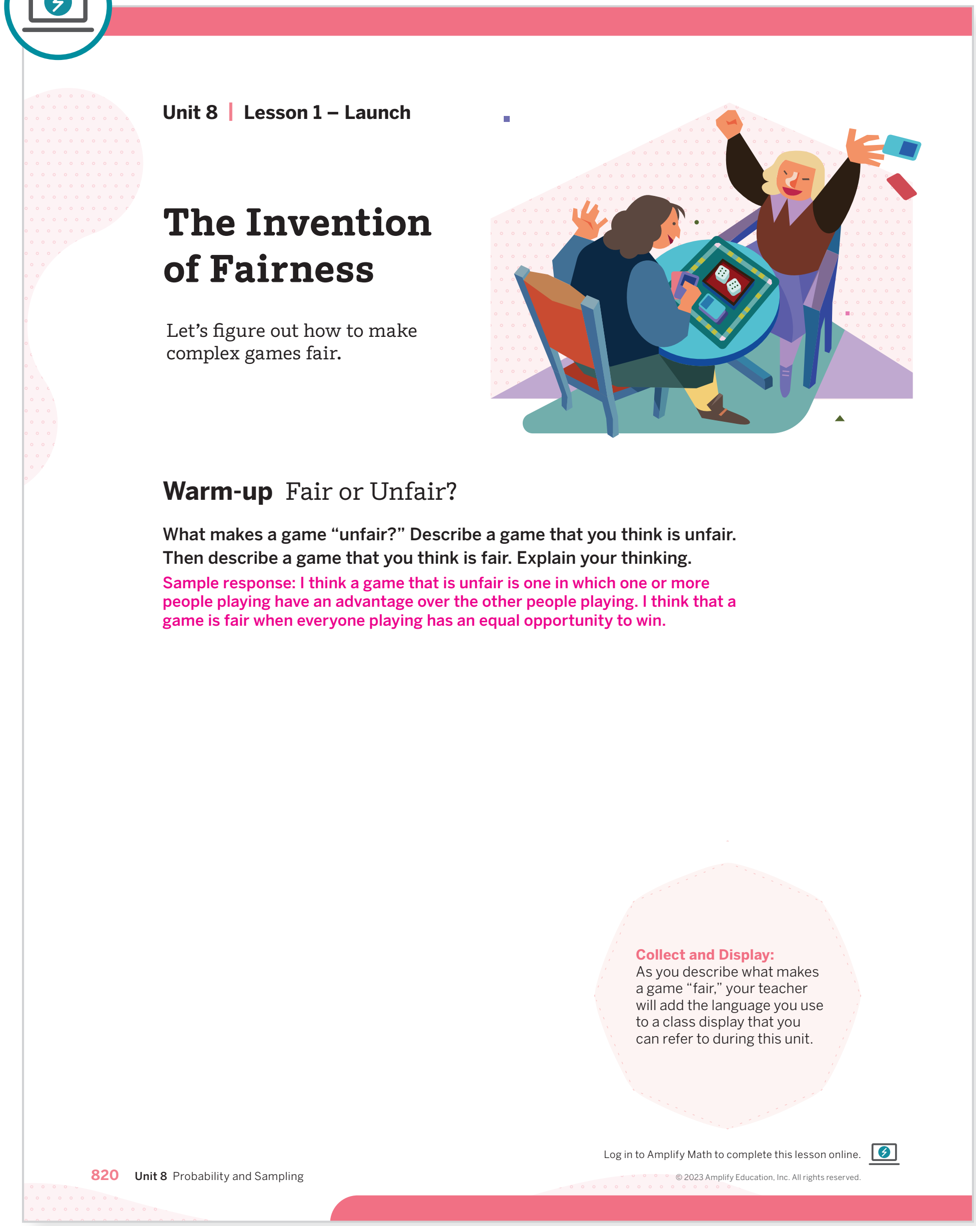
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Problem 3 may be omitted. If additional time is needed, Problems 6 and 7 may also be omitted.

Pairs | 5 min

Warm-up Fair or Unfair?

Students activate their background knowledge by considering how they determine what is "fair" and recall any prior experiences with unfair games.



Unit 8 | Lesson 1 – Launch

The Invention of Fairness

Let's figure out how to make complex games fair.

Warm-up Fair or Unfair?

What makes a game "unfair?" Describe a game that you think is unfair. Then describe a game that you think is fair. Explain your thinking.

Sample response: I think a game that is unfair is one in which one or more people playing have an advantage over the other people playing. I think that a game is fair when everyone playing has an equal opportunity to win.

Collect and Display: As you describe what makes a game "fair," your teacher will add the language you use to a class display that you can refer to during this unit.

Log in to Amplify Math to complete this lesson online.

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820 Unit 8 Probability and Sampling

1 Launch

Tell a story — real or invented — about a game you have played that you thought was unfair. Carnival or arcade games may serve as good examples. Activate students' background knowledge by asking them if they have ever played any games that they thought were fair or unfair.

2 Monitor

Help students get started by suggesting some common games that students may have played, such as board games.

Look for points of confusion:

- **Thinking about games that involve a player's skill rather than a game of chance.** Discussing the fairness of skill games can be tricky — try to steer students back to thinking about games of chance.

Look for productive strategies:

- Mentioning specific rules related to fairness, such as taking turns, receiving the same numbers of cards, or starting with the same amount of money.

3 Connect

Have pairs of students share their descriptions of unfair games with each other. Call on a few students who are willing to share with the whole class.

Highlight some of the commonalities that emerge from the discussions of unfair games, such as earning a prize that was not worth the effort, starting with different numbers of points, or not having an equal opportunity to earn new points.

Math Language Development

MLR2: Collect and Display

While students respond to the Warm-up prompt, circulate and collect any language they use to describe whether a game is "fair" or "unfair." Add this language to a class display for this unit, and invite students to add to and refer to this class display throughout the rest of this unit.

English Learners

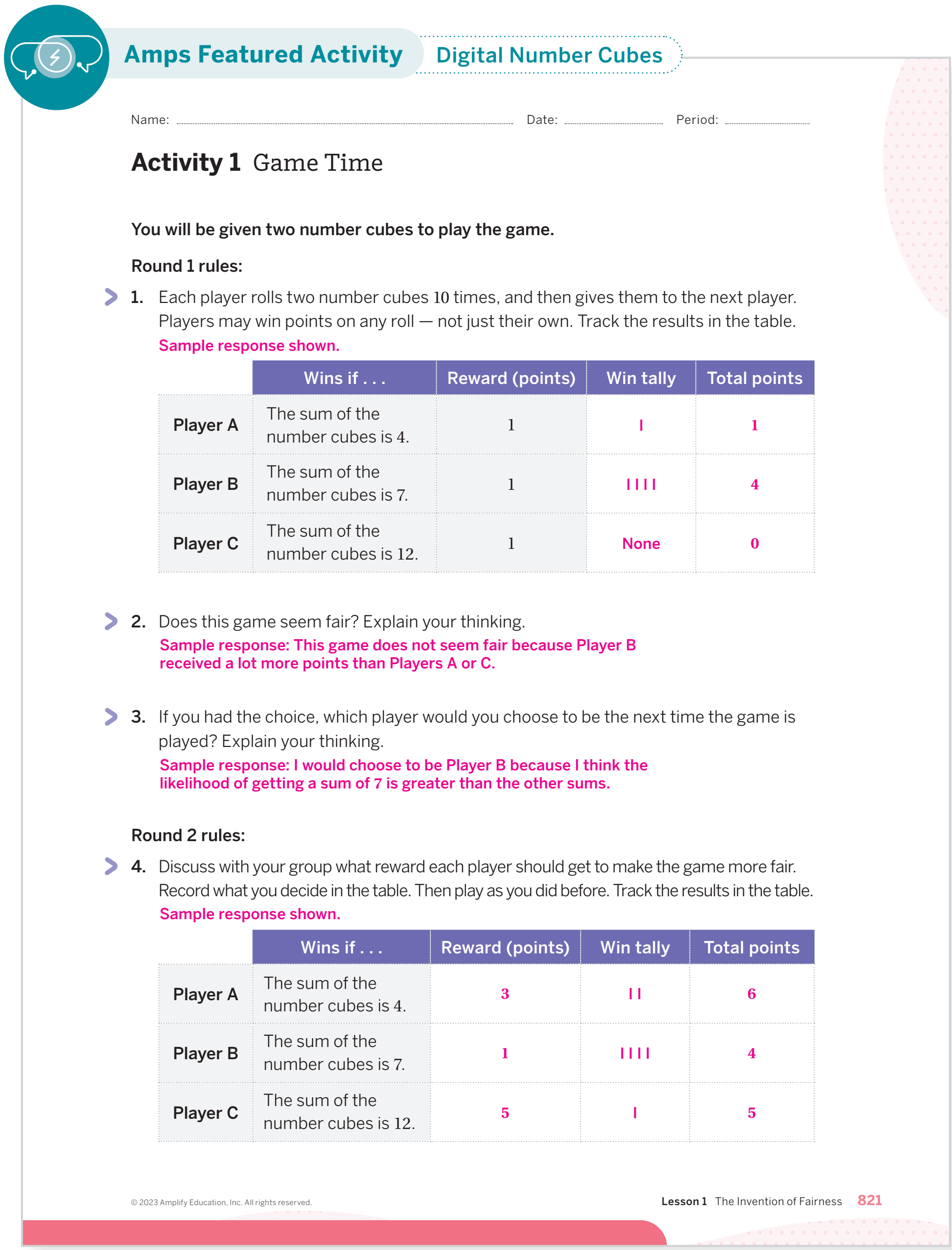
Allow students to work with a partner who shares the same primary language.

Small Groups | 25 min

Activity 1 Game Time

Students play a game with number cubes and craft new rules that are more fair, based on the outcomes they observe from playing the game.

Amps Featured Activity Digital Number Cubes



Name: ______ Date: ______ Period: ______

Activity 1 Game Time

You will be given two number cubes to play the game.

Round 1 rules:

1. Each player rolls two number cubes 10 times, and then gives them to the next player. Players may win points on any roll — not just their own. Track the results in the table.
Sample response shown.

	Wins if . . .	Reward (points)	Win tally	Total points
Player A	The sum of the number cubes is 4.	1	I	1
Player B	The sum of the number cubes is 7.	1	IIII	4
Player C	The sum of the number cubes is 12.	1	None	0

2. Does this game seem fair? Explain your thinking.
Sample response: This game does not seem fair because Player B received a lot more points than Players A or C.

3. If you had the choice, which player would you choose to be the next time the game is played? Explain your thinking.
Sample response: I would choose to be Player B because I think the likelihood of getting a sum of 7 is greater than the other sums.

Round 2 rules:

4. Discuss with your group what reward each player should get to make the game more fair. Record what you decide in the table. Then play as you did before. Track the results in the table.
Sample response shown.

	Wins if . . .	Reward (points)	Win tally	Total points
Player A	The sum of the number cubes is 4.	3	II	6
Player B	The sum of the number cubes is 7.	1	IIII	4
Player C	The sum of the number cubes is 12.	5	I	5

1 Launch

Read the directions for playing the game. Model how to shake and roll the number cubes gently.

2 Monitor

Help students get started by suggesting a neutral characteristic they can use to select which player they will be in the activity. For example, "closest to the door," or "first name by reverse alphabetical order."

Look for points of confusion:

- **Thinking that not receiving any points means the game is unfair.** Ask students to consider whether they had the same opportunity to earn points as the other players, not merely whether they earned points.

Look for productive strategies:

- Noticing that certain outcomes are more likely to occur than others, and using these observations to craft new rules.
- Systematically analyzing the likelihood of the sums when rolling two number cubes, such as using a table or ordered list to record possibilities.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can interact with digital number cubes and roll them more quickly, allowing for more representative outcomes.

Accessibility: *Guide Processing and Visualization*

Demonstrate how points are scored for Round 1 by rolling two number cubes until at least Player A, B, or C would have scored a point. Show how the points are scored based on the sum of the two number cubes.

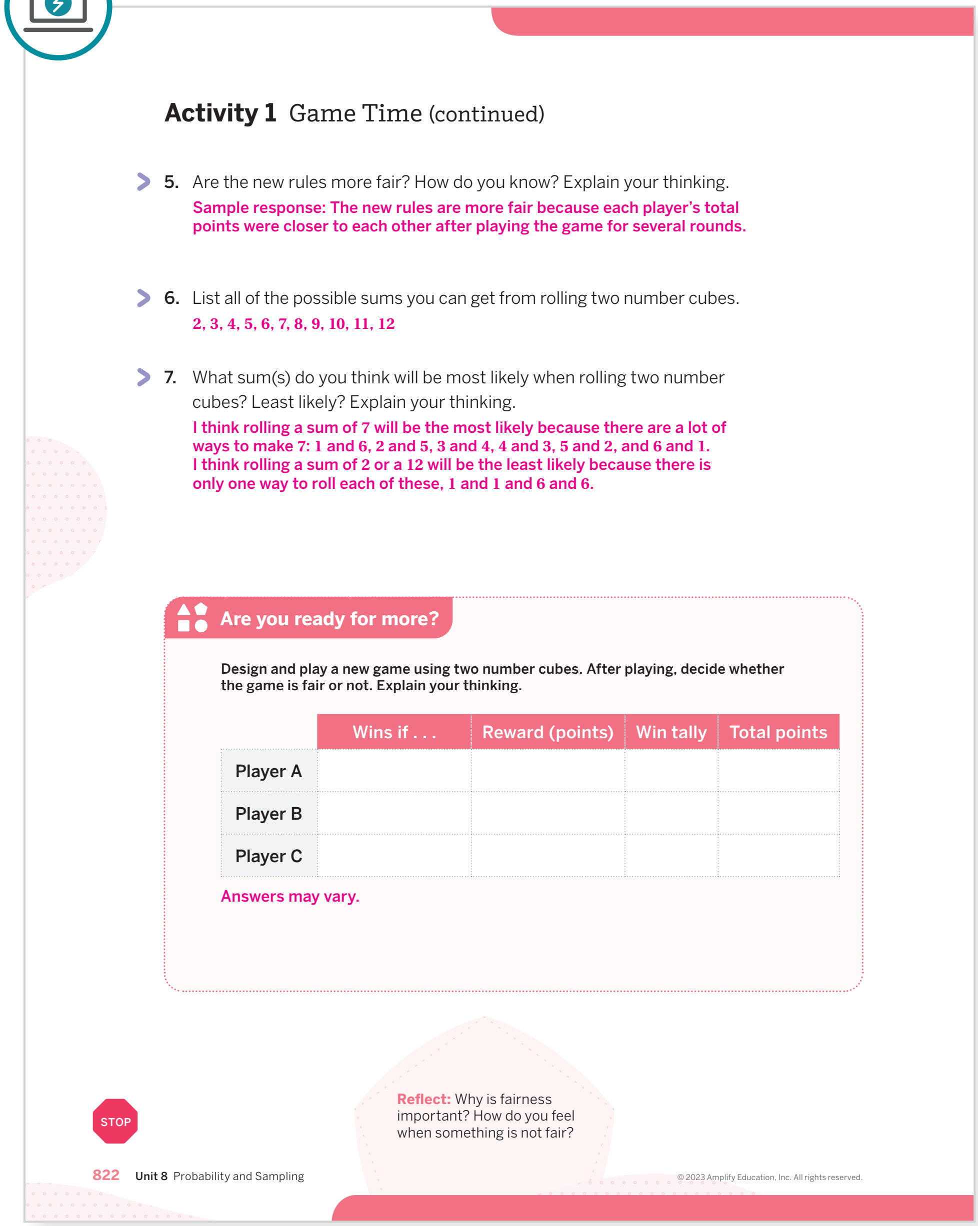
Extension: *Math Enrichment*

Have students make a list to record all of the possible ways to roll a sum, when rolling two number cubes. Then have them revisit their response to Problem 7 to see if the list they created supports their response.

Small Groups | 25 min

Activity 1 Game Time (continued)

Students play a game with number cubes and craft new rules that are more fair, based on the outcomes they observe playing the game.



Activity 1 Game Time (continued)

5. Are the new rules more fair? How do you know? Explain your thinking.
Sample response: The new rules are more fair because each player's total points were closer to each other after playing the game for several rounds.

6. List all of the possible sums you can get from rolling two number cubes.
2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

7. What sum(s) do you think will be most likely when rolling two number cubes? Least likely? Explain your thinking.
I think rolling a sum of 7 will be the most likely because there are a lot of ways to make 7: 1 and 6, 2 and 5, 3 and 4, 4 and 3, 5 and 2, and 6 and 1.
I think rolling a sum of 2 or a 12 will be the least likely because there is only one way to roll each of these, 1 and 1 and 6 and 6.

Are you ready for more?

Design and play a new game using two number cubes. After playing, decide whether the game is fair or not. Explain your thinking.

	Wins if . . .	Reward (points)	Win tally	Total points
Player A				
Player B				
Player C				

Answers may vary.

STOP

Reflect: Why is fairness important? How do you feel when something is not fair?

822 Unit 8 Probability and Sampling

3 Connect

Have groups of students share their responses to Problem 7. Record each group's list in a separate row of a table. Then select a few groups to share their reasoning for how they ordered their list. Allow groups to reconsider the order of their lists following the discussion.

Ask, "How much should each sum be worth? If you assume rolling a sum of 7 is one point, what should be the worth of rolling a sum of 12? Why do you think that?" Answers may vary.

Highlight that even though there is an equal chance of rolling each number on a number cube, when you add the two number-cube values together, the sums do not have the same chances of occurring.

Differentiated Support

Extension: *Math Around the World*

Have students use the internet, or another source, to research games that have been played by other cultures around the world. Have them select one game, describe the rules for play, and then decide whether they think the game is fair. They should explain their reasoning for why they think the game is either fair or unfair. Ask students to include in their research how the games were part of each culture or civilization.

Some games that students can research are provided here:

- Ashbii (Native American)
- Dreidel (a Jewish game that originated in Germany)
- Hubbub (Penobscot Nation, New England)
- Lu-Lu Dice (Hawaii)
- Mancala (ancient Egypt)
- Tomo Todo (Mexico)
- Zambales (Philippines)
- Zara (ancient Italy)

Whole Class | 10 min

Summary Winning Chance

Review and synthesize that analyzing the likelihood of outcomes helps to determine whether games are fair or unfair and can be used to create rules for fair games.

Narrative Connections

Unit 8 Probability and Sampling

Winning Chance

Imagine a game with two players, where the first to win 10 rounds gets a big cash prize. Now imagine you are in the middle of this game, winning and losing rounds here and there. But before anyone can claim that fateful 10th round, the game is forced to stop!

How much prize money should you get?

This is the problem that plagued the mathematician Blaise Pascal in the summer of 1654. He was 31, a year away from publishing his famous treatise on arithmetic triangles.

Pascal wrote to the esteemed Pierre de Fermat for help. Fermat, a lawyer, was already well revered in mathematical circles. He had produced groundbreaking work in number theory and analytic geometry. Over several months, the two corresponded by mail — Pascal in Paris, Fermat from his home in southern France. If you read their letters, it's clear how much the men admired each other. Over the course of working together, and seeing how the other thought, they formed a deep and profound friendship.

Together, they recognized that the fairest way to split the prize was based on the probability of each player winning the game.

But what was the likelihood that a young Pascal would reach out to the older and more esteemed Fermat for help?

And what was the likelihood that Fermat would agree?

Thanks to their friendship, we have better ways to describe moments of uncertainty. From drawing a card to predicting the weather, we can use probability as a check on our intuition, and to help us make more informed decisions.

Welcome to Unit 8.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Highlight that, in this unit, students will engage in a new branch of math called *probability*. Even though it is new, students likely have prior experience understanding the chance of an event happening. Encourage them to share their own experiences in class during this unit.

Ask, "Why do you think Pascal and Fermat decided to use math to make the game more fair? Why were they not satisfied with just using their 'gut instinct' or intuition?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

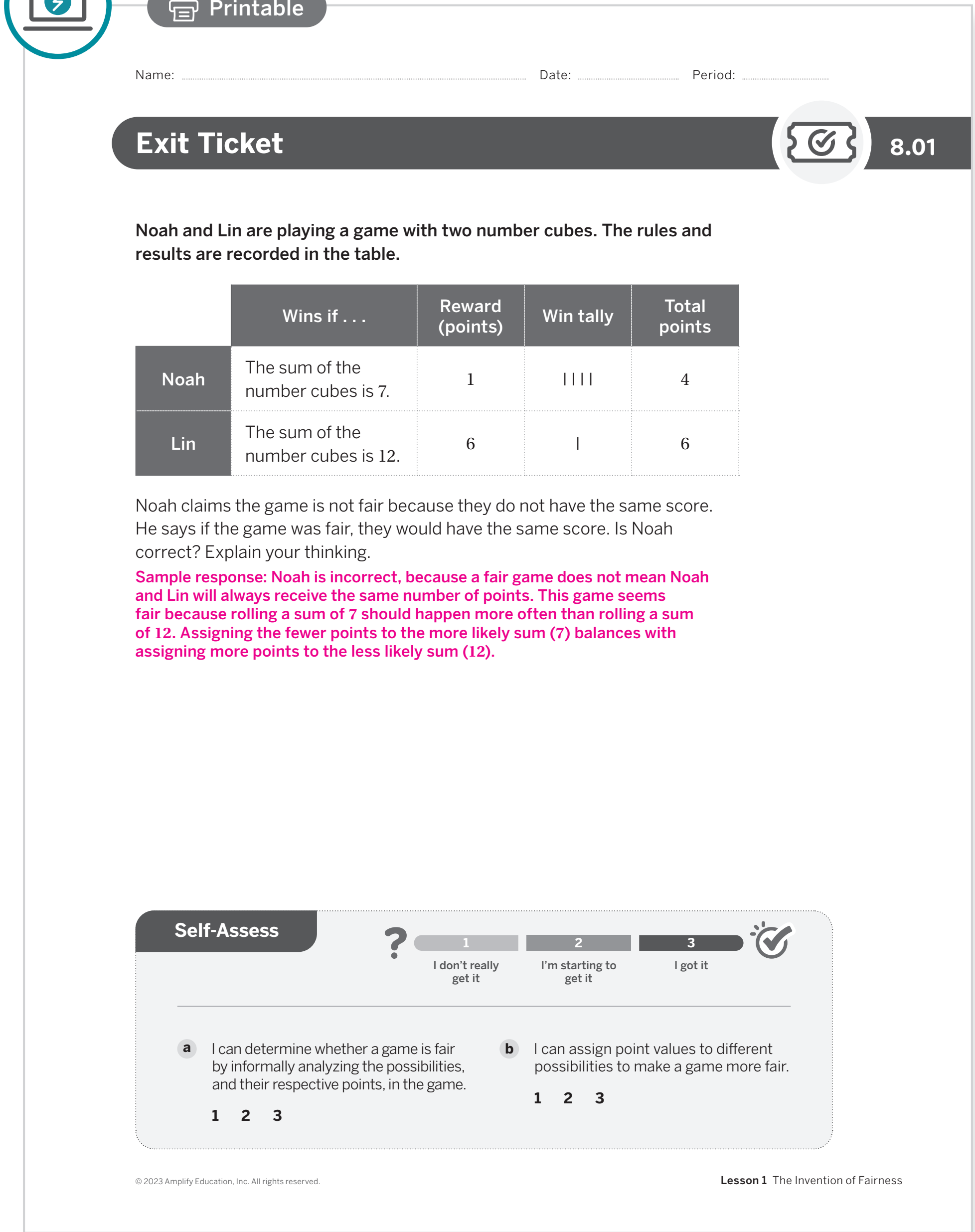
- "What does it mean to be fair?"
- "Which strategies did you use with your group today to make your game fair? Were any successful? Unsuccessful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing the results of a game and determining whether the game is fair or unfair.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

8.01

Noah and Lin are playing a game with two number cubes. The rules and results are recorded in the table.

	Wins if . . .	Reward (points)	Win tally	Total points
Noah	The sum of the number cubes is 7.	1	||||	4
Lin	The sum of the number cubes is 12.	6	|	6

Noah claims the game is not fair because they do not have the same score. He says if the game was fair, they would have the same score. Is Noah correct? Explain your thinking.

Sample response: Noah is incorrect, because a fair game does not mean Noah and Lin will always receive the same number of points. This game seems fair because rolling a sum of 7 should happen more often than rolling a sum of 12. Assigning the fewer points to the more likely sum (7) balances with assigning more points to the less likely sum (12).

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can determine whether a game is fair by informally analyzing the possibilities, and their respective points, in the game.

1 2 3

b. I can assign point values to different possibilities to make a game more fair.

1 2 3

Success looks like . . .

- **Language Goal:** Comparing the likelihoods of different outcomes when rolling two number cubes. **(Speaking and Listening)**
  - » Explaining whether the game with two number cubes is fair by comparing the likelihood of a sum of 7 with the likelihood of a sum of 12.
- **Goal:** Creating fair rules for a game, based on the likelihood of the outcomes when rolling two number cubes.

Suggested next steps

If students claim the game is not fair because each player receives a different reward, consider:

- Having them review Activity 1, Problem 7 and describe why certain sums, such as 2 and 12, are less likely to occur than other sums.

Professional Learning

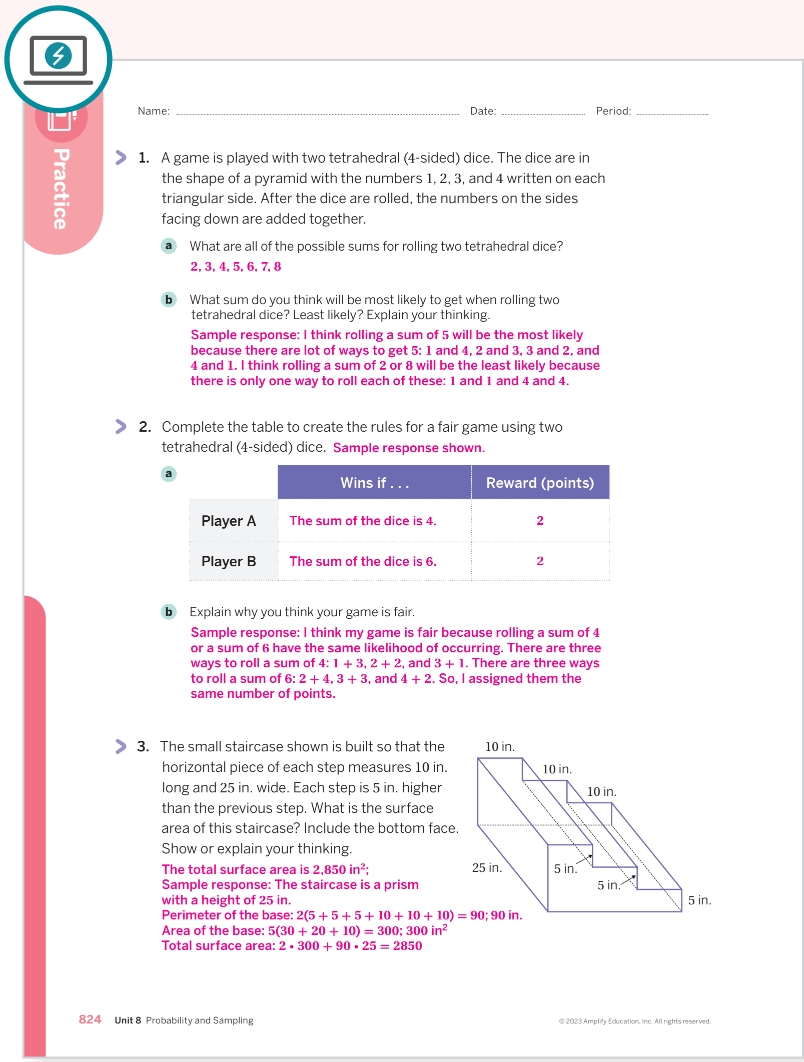
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find frustrating about Activity 1? What helped them work through this frustration?
- Knowing where your students need to be by the end of this unit, how did developing the concept of fairness influence that future goal? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A game is played with two tetrahedral (4-sided) dice. The dice are in the shape of a pyramid with the numbers 1, 2, 3, and 4 written on each triangular side. After the dice are rolled, the numbers on the sides facing down are added together.
   - a What are all of the possible sums for rolling two tetrahedral dice?

     2, 3, 4, 5, 6, 7, 8
   - b What sum do you think will be most likely to get when rolling two tetrahedral dice? Least likely? Explain your thinking.

     Sample response: I think rolling a sum of 5 will be the most likely because there are lot of ways to get 5: 1 and 4, 2 and 3, 3 and 2, and 4 and 1. I think rolling a sum of 2 or 8 will be the least likely because there is only one way to roll each of these: 1 and 1 and 4 and 4.

2. Complete the table to create the rules for a fair game using two tetrahedral (4-sided) dice. **Sample response shown.**
   - a

	Wins if . . .	Reward (points)
Player A	The sum of the dice is 4.	2
Player B	The sum of the dice is 6.	2

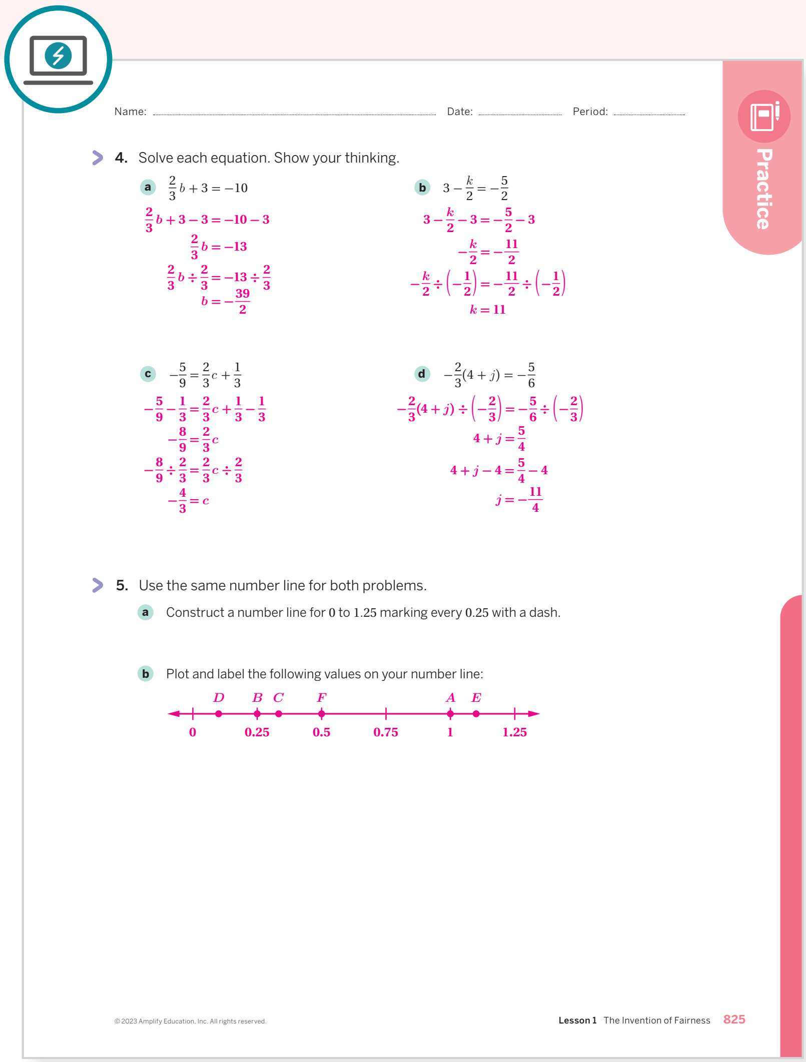
   - b Explain why you think your game is fair.

     Sample response: I think my game is fair because rolling a sum of 4 or a sum of 6 have the same likelihood of occurring. There are three ways to roll a sum of 4: 1 + 3, 2 + 2, and 3 + 1. There are three ways to roll a sum of 6: 2 + 4, 3 + 3, and 4 + 2. So, I assigned them the same number of points.

3. The small staircase shown is built so that the horizontal piece of each step measures 10 in. long and 25 in. wide. Each step is 5 in. higher than the previous step. What is the surface area of this staircase? Include the bottom face. Show or explain your thinking.

The total surface area is 2,850 in^2;
Sample response: The staircase is a prism with a height of 25 in.
Perimeter of the base: $2(5 + 5 + 5 + 10 + 10 + 10) = 90$; 90 in.
Area of the base: $5(30 + 20 + 10) = 300$; 300 in^2
Total surface area: $2 \cdot 300 + 90 \cdot 25 = 2850$

824 **Unit 8** Probability and Sampling



Name: Date: Period:

4. Solve each equation. Show your thinking.
   - a $\frac{2}{3}b + 3 = -10$

$$\frac{2}{3}b + 3 - 3 = -10 - 3$$
$$\frac{2}{3}b = -13$$
$$\frac{2}{3}b \div \frac{2}{3} = -13 \div \frac{2}{3}$$
$$b = -\frac{39}{2}$$

   - b $3 - \frac{k}{2} = -\frac{5}{2}$

$$3 - \frac{k}{2} - 3 = -\frac{5}{2} - 3$$
$$-\frac{k}{2} = -\frac{11}{2}$$
$$-\frac{k}{2} \div \left(-\frac{1}{2}\right) = -\frac{11}{2} \div \left(-\frac{1}{2}\right)$$
$$k = 11$$

   - c $-\frac{5}{9} = \frac{2}{3}c + \frac{1}{3}$

$$-\frac{5}{9} - \frac{1}{3} = \frac{2}{3}c + \frac{1}{3} - \frac{1}{3}$$
$$-\frac{8}{9} = \frac{2}{3}c$$
$$-\frac{8}{9} \div \frac{2}{3} = \frac{2}{3}c \div \frac{2}{3}$$
$$-\frac{4}{3} = c$$

   - d $-\frac{2}{3}(4 + j) = -\frac{5}{6}$

$$-\frac{2}{3}(4 + j) \div \left(-\frac{2}{3}\right) = -\frac{5}{6} \div \left(-\frac{2}{3}\right)$$
$$4 + j = \frac{5}{4}$$
$$4 + j - 4 = \frac{5}{4} - 4$$
$$j = -\frac{11}{4}$$

5. Use the same number line for both problems.
   - a Construct a number line for 0 to 1.25 marking every 0.25 with a dash.
   - b Plot and label the following values on your number line:

Lesson 1 The Invention of Fairness 825

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
Spiral	3	Unit 7 Lesson 16	2
	4	Unit 6 Lesson 7	2
Formative ⚡	5	Unit 8 Lesson 2	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 1

Probabilities of Single-step Events

Whole Class

In this Sub-Unit, students examine the probability of simple events through games and experiments.

SUB-UNIT

1 Probabilities of Single-step Events

Narrative Connections

How did the women of Bletchley Park save the free world?

Before World War II, Bletchley Park was a quaint country estate outside of London. But by 1938, it had become the headquarters for a crack team of British codebreakers. This team, composed mostly of women, was tasked with decoding messages intercepted from the Nazis and their allies.

But how does code breaking work? The key is knowing the language a code is written in. For example, in English, 'e' is the most common letter. So if you were decoding a secret message written in English, chances are good that the symbol that occurred most would represent the letter 'e'. Once you figure out one letter, it becomes easier to deduce the rest.

That is exactly what the codebreakers did. By looking at a message's length and the frequency of different letters, codebreakers such as Mavis Batey, Jane Fawcett, and Joan Clarke decoded enemy messages back into their original language.

For years, their contribution remained a secret. But in 2009 the British government finally acknowledged the teams' work. Thanks to their efforts, the Allies gained valuable information. They learned about enemy troop movements, attack plans, and even spy activity. Historians estimate that the Bletchley codebreakers shortened the war by about three years, saving countless lives.

Sub-Unit 1 Probabilities of Single-step Events 827

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how probability and frequency is connected to decoding messages in the following places:

- **Lesson 5, Activity 1:** Caesar Cipher
- **Lesson 5, Activity 2:** Crack the Code
- **Lesson 6, Activity 1:** Send a Secret Message
- **Lesson 6, Activity 2:** Decoding the Secret Message

UNIT 8 | **LESSON 2**

Chance Experiments

Let's investigate experiments of chance.

Focus

Goals

1. **Language Goal:** Comprehend and use the terms *impossible*, *unlikely*, *equally likely as not*, *likely*, and *certain* to describe the likelihood of an event. **(Speaking and Listening, Writing)**
2. **Language Goal:** Order a given set of events from least likely to occur to most likely, and justify the reasoning. **(Speaking and Listening)**

Coherence

Today

Students investigate chance events. They use language such as *impossible*, *unlikely*, *equally likely as not*, *likely*, or *certain* to describe the likelihood of a chance event. Students make sense of the chance events and sort them into these categories. By comparing informal groupings first and numerical quantities later, students attend to precision as they order the events from least likely to most likely.

Previously

In Lesson 1, students considered the fairness of a game by playing a game using two number cubes. They recognized that not all outcomes are equally likely and informally reasoned about chance events.

Coming Soon

In Lesson 3, students will use a sample space to calculate the probability of a chance event. They will connect the language that describes the likelihood of an event to more precise numerical values.

Rigor

- Students use real-world examples and spinners to build their **conceptual understanding** of the likelihood of an event.
- Students order events from least likely to most likely to develop **procedural fluency** of ordering events by likelihood.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per group
- Activity 3 PDF, pre-cut spinners, one per pair
- paper clips, one per pair

Math Language Development

New words

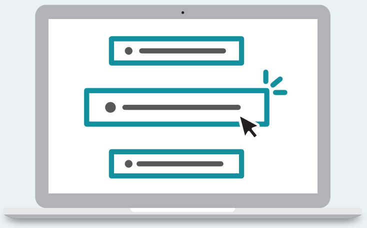
- *certain*
- *chance experiment*
- *equally likely as not*
- *event**
- *impossible*
- *likely*
- *outcome*
- *unlikely*

*Students may be familiar with the term *event* in its everyday use of the term, such as a sporting event. Be ready to address how the everyday meaning relates to the statistical meaning.

Amps | Featured Activity

Activity 2
Digital Ordered Lists

Students can drag and drop scenarios in an ordered list, showing how they order the likelihood of events.



Building Math Identity and Community

Connecting to Mathematical Practices

During the Warm-up, students may doubt their intuition and feel confused at first when thinking about pairs of shoes, especially, thinking that a specific pair needs to be selected. Encourage students to reread the question and discuss with their partner. Have them ask each other follow-up questions such as, “Does this make sense?”

Modifications to Pacing

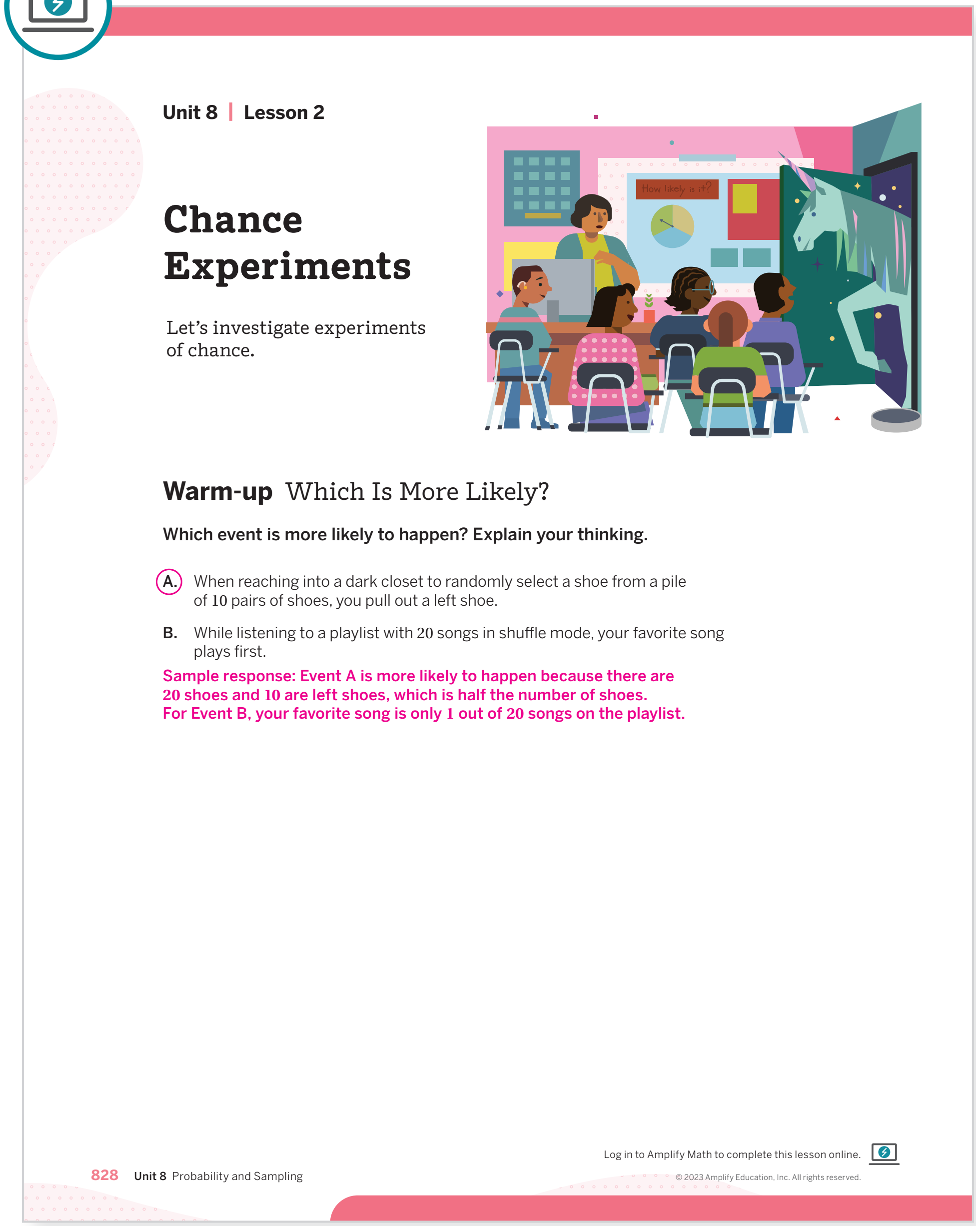
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students only complete 5–6 of the events. Select at least one event for them to complete from each category: *possible*, *impossible*, *certain*.
- In **Activity 3**, Problem 3, have students spin the spinner 5 times and combine their results with another set of partners.

Pairs | 5 min

Warm-up Which Is More Likely?

Students use two real-world scenarios to reason about likelihood, which will help them define probability in future activities.



Unit 8 | Lesson 2

Chance Experiments

Let's investigate experiments of chance.

Warm-up Which Is More Likely?

Which event is more likely to happen? Explain your thinking.

Ⓐ. When reaching into a dark closet to randomly select a shoe from a pile of 10 pairs of shoes, you pull out a left shoe.

B. While listening to a playlist with 20 songs in shuffle mode, your favorite song plays first.

Sample response: Event A is more likely to happen because there are 20 shoes and 10 are left shoes, which is half the number of shoes. For Event B, your favorite song is only 1 out of 20 songs on the playlist.

828 Unit 8 Probability and Sampling

Log in to Amplify Math to complete this lesson online.

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1 Launch

Have students use the ***Think-Pair-Share*** routine. Provide them 1 minute of individual think time. Then have them complete the Warm-up with a partner.

2 Monitor

Help students get started by asking them to consider the total number of shoes in the closet and the total number of songs on the playlist.

Look for points of confusion:

- **Thinking that a specific shoe has to be selected.** Have students visualize a pile of shoes and tell them they can choose any left shoe. To clarify, have them determine how many right shoes are in the closet.

3 Connect

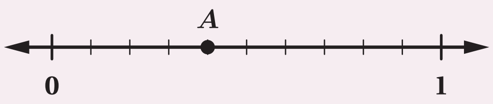
Have students share their thinking with the class.

Highlight that when comparing the two events, it is more likely that you will pull out a left shoe from a closet of 10 pairs of shoes, because there are 10 left shoes and 10 right shoes. If there were only 1 left shoe in the closet — and the remaining 19 shoes were right shoes — then the events would have the same likelihood. When using terms like *possible*, *likely*, or *not very likely* to describe the chances of an event happening, students are describing the *likelihood* of that event.

Power-up

To power up students' ability to identify benchmark fractions and decimals on a number line, have students complete:

Use the number line to complete each problem.



1. What fraction does the first tick mark represent? $\frac{1}{10}$
2. What decimal does the first tick mark represent? 0.1
3. Write the value of point A as both a fraction and a decimal.

   Fraction: $\frac{4}{10}$ or equivalent
   Decimal: 0.4 or equivalent

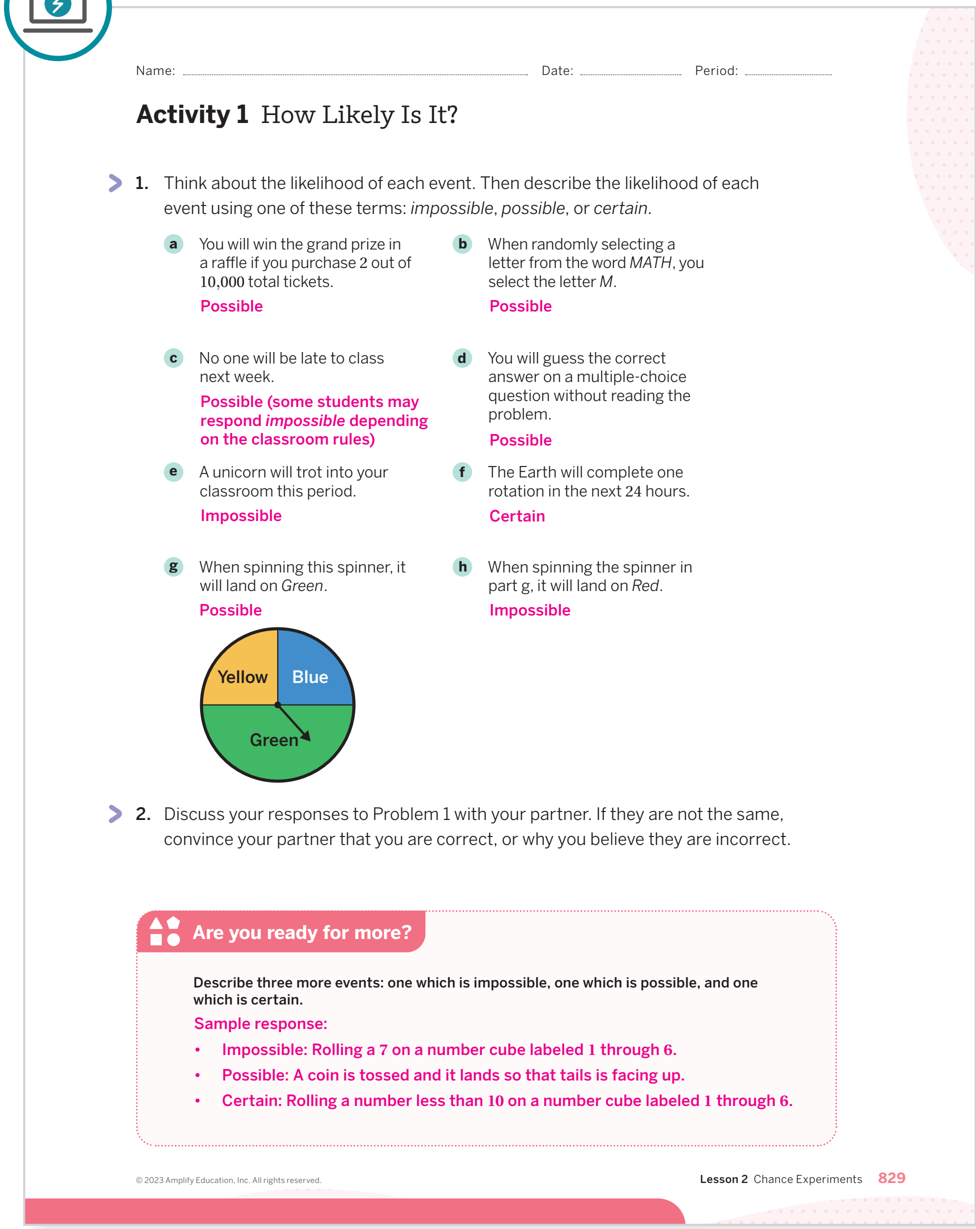
Use: Before Activity 2

Informed by: Performance on Lesson 1, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 10 min

Activity 1 How Likely Is It?

Students engage their intuition about the likelihood of events and informally assign the terms *impossible*, *possible*, and *certain* to describe them.



Name: ______ Date: ______ Period: ______

Activity 1 How Likely Is It?

1. Think about the likelihood of each event. Then describe the likelihood of each event using one of these terms: *impossible*, *possible*, or *certain*.

   a. You will win the grand prize in a raffle if you purchase 2 out of 10,000 total tickets.
   Possible

   b. When randomly selecting a letter from the word *MATH*, you select the letter *M*.
   Possible

   c. No one will be late to class next week.
   Possible (some students may respond *impossible* depending on the classroom rules)

   d. You will guess the correct answer on a multiple-choice question without reading the problem.
   Possible

   e. A unicorn will trot into your classroom this period.
   Impossible

   f. The Earth will complete one rotation in the next 24 hours.
   Certain

   g. When spinning this spinner, it will land on *Green*.
   Possible

   h. When spinning the spinner in part g, it will land on *Red*.
   Impossible

2. Discuss your responses to Problem 1 with your partner. If they are not the same, convince your partner that you are correct, or why you believe they are incorrect.

Are you ready for more?

Describe three more events: one which is impossible, one which is possible, and one which is certain.

Sample response:

- Impossible: Rolling a 7 on a number cube labeled 1 through 6.
- Possible: A coin is tossed and it lands so that tails is facing up.
- Certain: Rolling a number less than 10 on a number cube labeled 1 through 6.

Lesson 2 Chance Experiments 829

1 Launch

Activate students' background knowledge by asking them what the terms *impossible*, *possible*, and *certain* mean to them.

2 Monitor

Help students get started by having them read through the descriptions of all the events, first looking for which events are impossible.

Look for points of confusion:

- **Not understanding the difference between *certain* and *possible*.** Tell students that *certain* means "it will definitely happen." *Possible* means "it might happen, but it also might not happen."

3 Connect

Have pairs share how they defined each term in their own terms with specific examples from the activity.

Highlight how students organized the events in Problem 1 as *impossible*, *possible*, or *certain*.

Ask:

- "Were there any disagreements among you and your partner? How did you resolve them?" Answers may vary.
- "Were any of these events challenging to describe using these terms?" Answers may vary.
- "Which terms are the most strict about what types of events can be described by them?" Certain and impossible
- "What does it mean for an event to be *certain*?" Sample response: The event will definitely happen.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students choose 6 of the events in Problem 1 to complete. Allowing them to choose which parts they would like to complete helps increase their engagement in the activity.

Accessibility: *Clarify Vocabulary and Symbols*

Students are likely to be familiar with the terms *possible*, *impossible (not possible)*, and *certain*. Display the following as a reminder of these terms.

Possible:	Impossible:	Certain:
It might happen or it might not happen.	It definitely will not happen.	It definitely will happen.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask questions, draw their attention to the terms that are the most strict about what types of events they describe. If students do not say *certain* and *impossible*, ask these follow-up questions:

- "If you are sure that the event will either happen or not happen, which term(s) are best to describe these?" Certain or Impossible.
- "If you are unsure as to whether the event will happen, which term(s) are best to describe the event?" Possible.

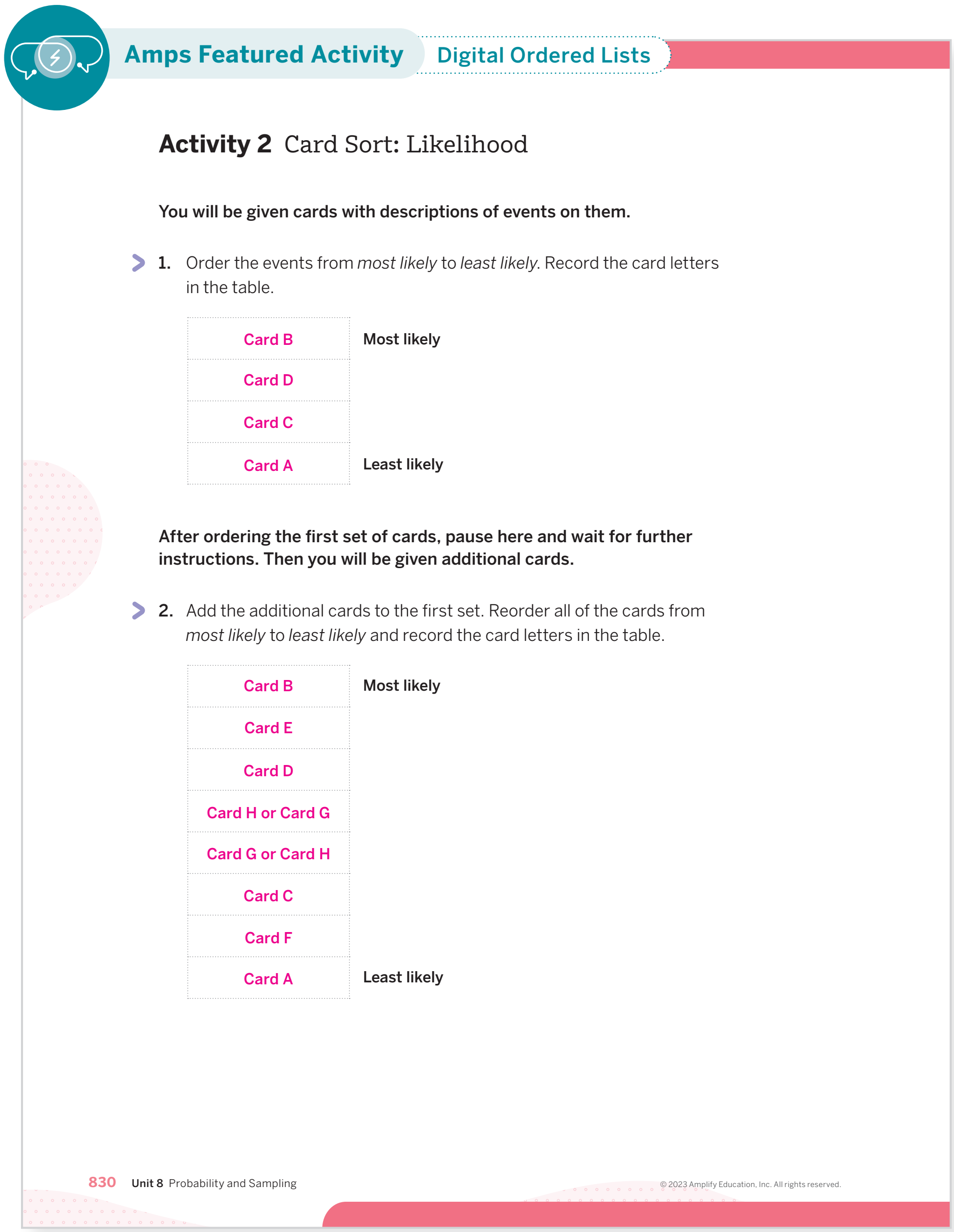
English Learners

Students may be confused by the term *trot* in Problem 1e. Mention this term describes how an animal might move.

Small Groups | 10 min

Activity 2 Card Sort: Likelihood

Students move toward formally describing the likelihood of events as *impossible*, *unlikely*, *equally likely as not*, *likely*, and *certain*.



Amps Featured Activity Digital Ordered Lists

Activity 2 Card Sort: Likelihood

You will be given cards with descriptions of events on them.

1. Order the events from *most likely* to *least likely*. Record the card letters in the table.

Card B	Most likely
Card D	
Card C	
Card A	Least likely

After ordering the first set of cards, pause here and wait for further instructions. Then you will be given additional cards.

2. Add the additional cards to the first set. Reorder all of the cards from *most likely* to *least likely* and record the card letters in the table.

Card B	Most likely
Card E	
Card D	
Card H or Card G	
Card G or Card H	
Card C	
Card F	
Card A	Least likely

830 **Unit 8** Probability and Sampling

1 Launch

Distribute one set of cards from the Activity 2 PDF to each group. Distribute Cards A–D first and then Cards E–H after Problem 1 has been completed. Conduct the *Card Sort* routine.

2 Monitor

Help students get started by choosing two cards first to compare them, and then adding in additional cards one at a time.

Look for points of confusion:

- **Not noticing that Card G and Card H have the same likelihood.** Ask students, “How much of each book consists of even-numbered pages?”

Look for productive strategies:

- Thinking about the likelihood of each event in terms of ratios.

3 Connect

Have groups of students share their reasoning for ordering the cards in the way they did.

Ask, “How can you know that Card F is less likely to happen than Card H?”

Highlight that students can use mathematical reasoning to gain a sense of which kinds of events are more likely to occur than others. Also, it is possible for two events to have the same likelihood of occurring.

Define the terms ***impossible***, ***unlikely***, ***equally likely as not***, ***likely***, and ***certain***.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can drag and drop scenarios into a digital ordered list.

Accessibility: *Clarify Vocabulary and Symbols*

Be sure students understand the phrases *least likely* and *most likely*.

Least likely: Either impossible or probably won’t happen (compared to the others).

Most likely: Either certain or probably will happen (compared to the others).

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the ask question, display Card F and Card H. Follow-up with these questions:

- “How many ways are there for Card F to occur? 1 way Card H?” 50 ways
- “What if a new card, Card I, stated, ‘Opening a 100-page book to a page numbered greater than 35’? Which card would be more likely, Card I or Card H? Why?” Card I because there would be 65 ways for Card I to occur.

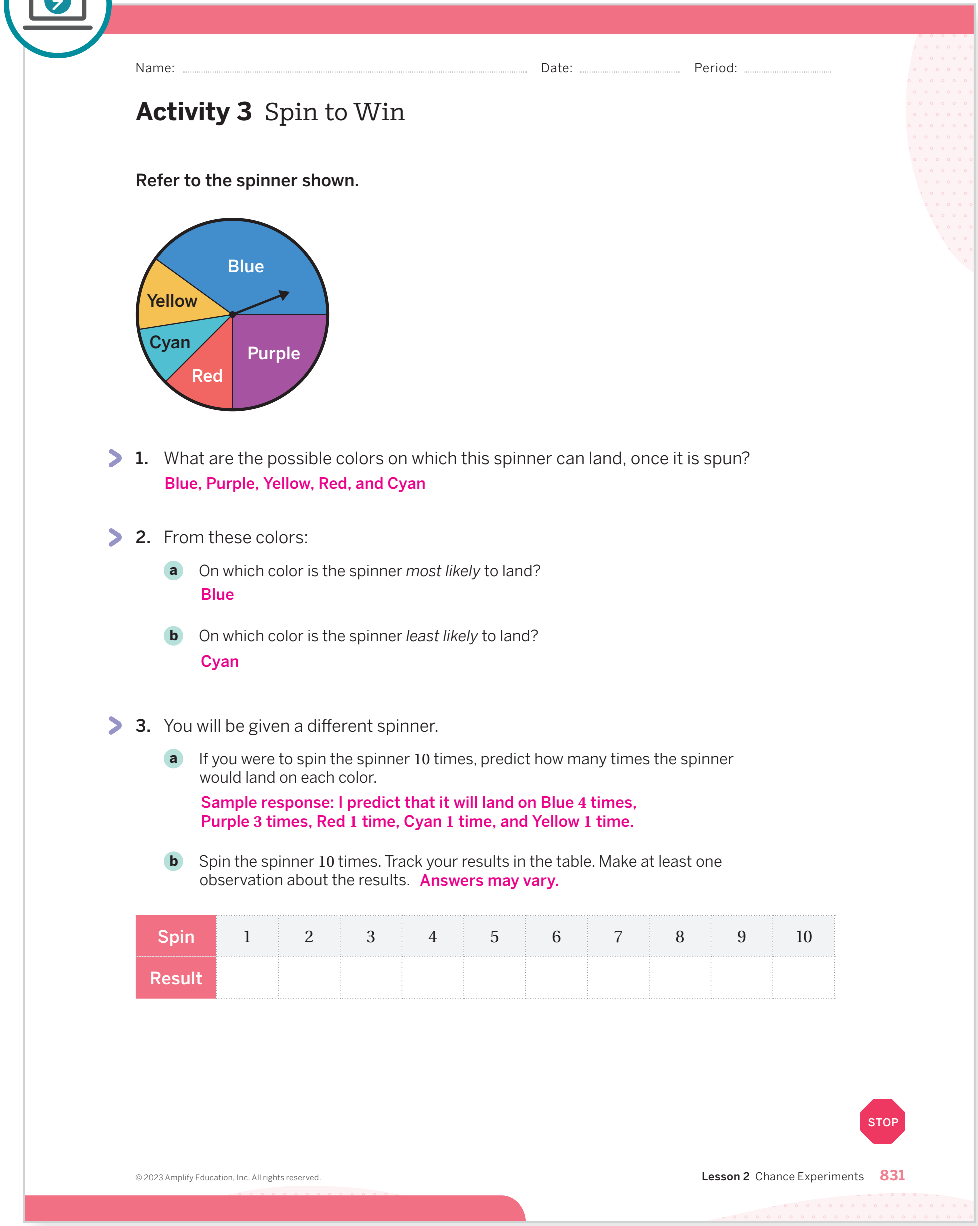
English Learners

Show examples of an even-numbered page and a “double-digit” page to help students comprehend Cards E, G, and H.

Pairs | 10 min

Activity 3 Spin to Win

Students begin to move toward a more quantitative understanding of likelihood by observing a chance experiment that has several outcomes.



Name: Date: Period:

Activity 3 Spin to Win

Refer to the spinner shown.

1. What are the possible colors on which this spinner can land, once it is spun?
   Blue, Purple, Yellow, Red, and Cyan

2. From these colors:
   - a On which color is the spinner *most likely* to land?
     Blue
   - b On which color is the spinner *least likely* to land?
     Cyan

3. You will be given a different spinner.
   - a If you were to spin the spinner 10 times, predict how many times the spinner would land on each color.
     Sample response: I predict that it will land on Blue 4 times, Purple 3 times, Red 1 time, Cyan 1 time, and Yellow 1 time.
   - b Spin the spinner 10 times. Track your results in the table. Make at least one observation about the results. **Answers may vary.**

Spin	1	2	3	4	5	6	7	8	9	10
Result										

STOP

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Lesson 2 Chance Experiments 831

1 Launch

Distribute a paper clip and a spinner from the Activity 3 PDF to each pair of students.

2 Monitor

Help students get started by having them generate three informal observations about the spinner.

Look for points of confusion:

- **Thinking it is impossible to land on cyan.** Ask, "Impossible means it will *never* happen. Is that what you mean?"

3 Connect

Have pairs of students share the reasoning behind their predictions from Problem 3a.

Define the following terms:

- A ***chance experiment*** is an experiment you can perform multiple times, in which the outcome may be different each time. For example, spinning the spinner 10 times to see what the result is each time.
- The ***outcome*** of a chance experiment is something that can happen when you perform the experiment. For example, when you toss a coin, one possible outcome is landing heads facing up.
- An ***event*** is a set of one or more outcomes in a chance experiment.

Highlight that students can reason about the likelihood of an event by comparing the chances for different outcomes.

Ask, "If you performed this chance experiment again, would you get different outcomes?" Yes.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

In Problem 3, have students focus on a particular outcome, such as spinning blue 5 times, because spinning blue is the most likely event to happen. Discuss reasons why blue might not have actually occurred that exact number of times.

Extension: *Math Enrichment*

Have students explain whether the likelihood would change for each color if the spinner was spun 5 times, 10 times, or 15 times. No, the likelihood will not change regardless of how many times the spinner is spun. For example, each time the spinner is spun, landing on blue will always have the greatest likelihood.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their reasoning behind their predictions for Problem 3a, draw their attention to the connections between the relative size of each section and the comparative number of times they predicted the spinner would land on each color. Ask:

- "What do you notice about the largest-size section, compared to your prediction in Problem 3a? The smallest-size section?"
- "What did the sum of your predictions need to be? How did that affect your predictions?"

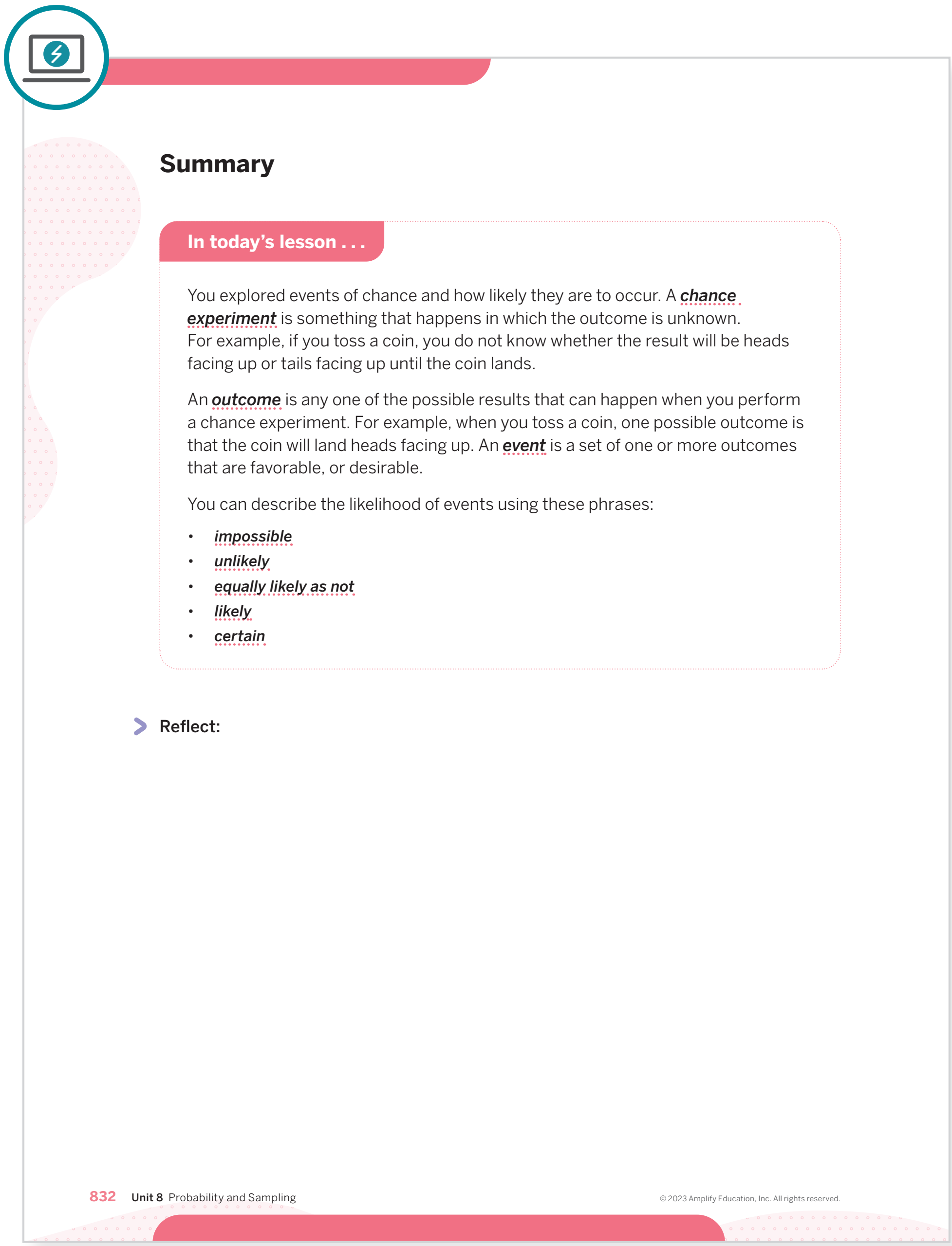
English Learners

Annotate the blue section of the spinner with the phrase *most likely* and the cyan section with the phrase *least likely*.

Whole Class | 5 min

Summary

Review and synthesize how to compare and describe likelihood.



Summary

In today's lesson . . .

You explored events of chance and how likely they are to occur. A ***chance experiment*** is something that happens in which the outcome is unknown. For example, if you toss a coin, you do not know whether the result will be heads facing up or tails facing up until the coin lands.

An ***outcome*** is any one of the possible results that can happen when you perform a chance experiment. For example, when you toss a coin, one possible outcome is that the coin will land heads facing up. An ***event*** is a set of one or more outcomes that are favorable, or desirable.

You can describe the likelihood of events using these phrases:

- *impossible*
- *unlikely*
- *equally likely as not*
- *likely*
- *certain*

Reflect:

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Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *chance experiment*, *outcome*, *event*, *impossible*, *unlikely*, *equally likely as not*, *likely*, and *certain* that were added to the display during the lesson.

Synthesize

Display the Summary from the Student Edition showing the terms used to describe likelihood.

Formalize vocabulary:

- *certain*
- *chance experiment*
- *equally likely as not*
- *event*
- *impossible*
- *likely*
- *outcome*
- *unlikely*

Have pairs of students share examples of each of the types of likelihood using examples from this lesson.

Ask:

- "If the chance experiment is tossing a coin, what are the possible outcomes?" Heads, tails
- "If the chance experiment is rolling a standard number cube, what are the possible outcomes?" 1, 2, 3, 4, 5, 6
- "Suppose you roll a standard number cube and want to describe the likelihood of rolling an even number. What are the favorable (or desirable) events?" Rolling a 2, 4, or 6.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments to reflect on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

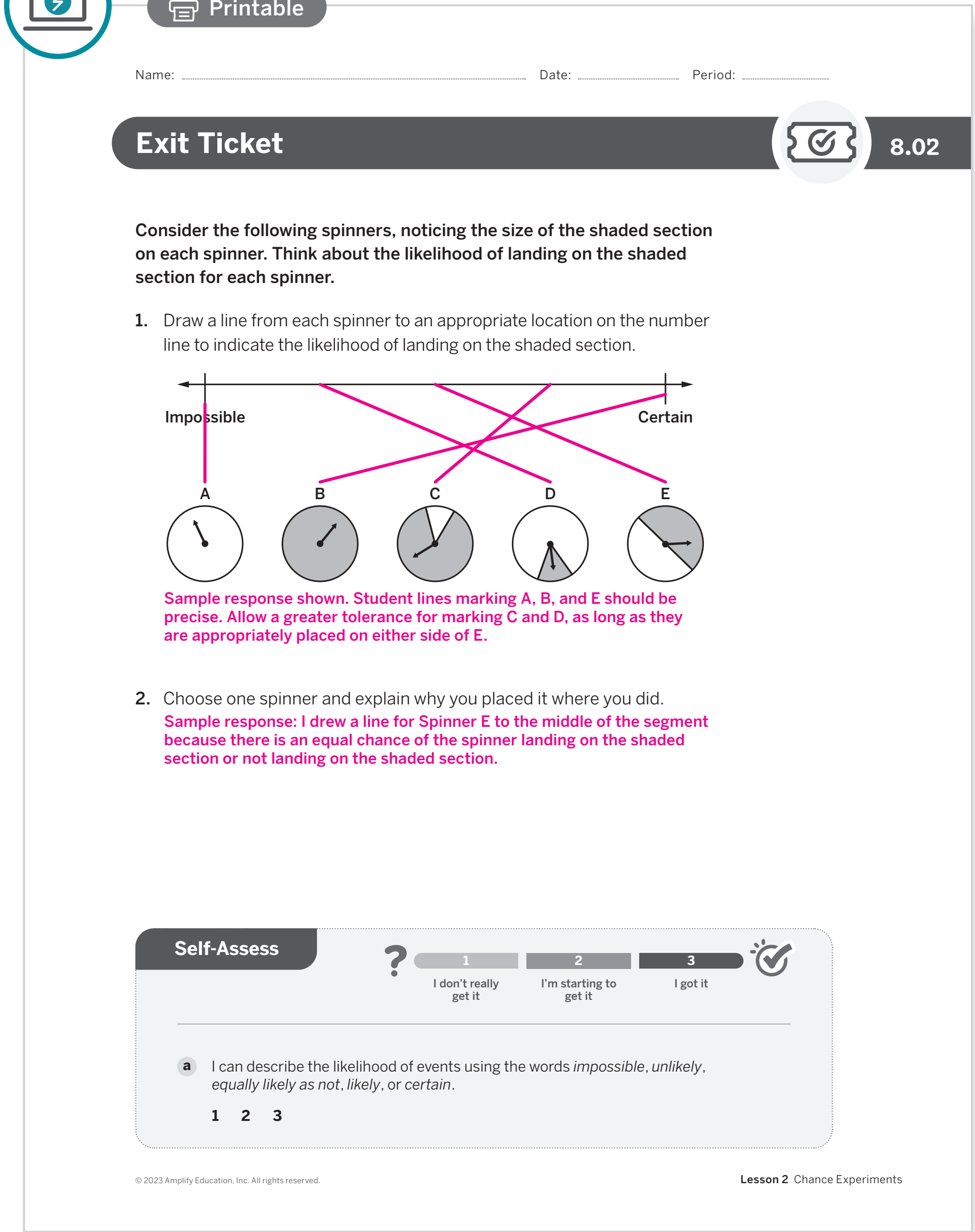
- "When faced with more than one possibility, how can you determine which outcome is more likely to happen?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by comparing the likelihood of events.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 8.02

Consider the following spinners, noticing the size of the shaded section on each spinner. Think about the likelihood of landing on the shaded section for each spinner.

1. Draw a line from each spinner to an appropriate location on the number line to indicate the likelihood of landing on the shaded section.

Sample response shown. Student lines marking A, B, and E should be precise. Allow a greater tolerance for marking C and D, as long as they are appropriately placed on either side of E.

2. Choose one spinner and explain why you placed it where you did.
Sample response: I drew a line for Spinner E to the middle of the segment because there is an equal chance of the spinner landing on the shaded section or not landing on the shaded section.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can describe the likelihood of events using the words *impossible*, *unlikely*, *equally likely as not*, *likely*, or *certain*.

1 2 3

Lesson 2 Chance Experiments

Success looks like . . .

- **Language Goal:** Comprehending and using the terms *impossible*, *unlikely*, *equally likely as not*, *likely*, and *certain* to describe the likelihood of an event. **(Speaking and Listening, Writing)**
- **Language Goal:** Ordering a given set of events from least likely to occur to most likely, and justifying the reasoning. **(Speaking and Listening)**
  - » Ordering the likelihoods of the shaded areas of the spinners in Problem 1.

Suggested next steps

If students order the spinners in a different order, consider:

- Reviewing the terms *impossible* and *certain*.
- Reviewing the events in Activity 2 and applying these terms to those events.
- Asking students to identify which spinner has no shaded section and which spinner is entirely shaded.
- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did you see in the way some students approached Activity 2? What does that tell you about similarities and differences among your students?
- During the discussion about Activity 1, how did you encourage each student to share their understanding? What might you change for the next time you teach this lesson?

MLR Math Language Development

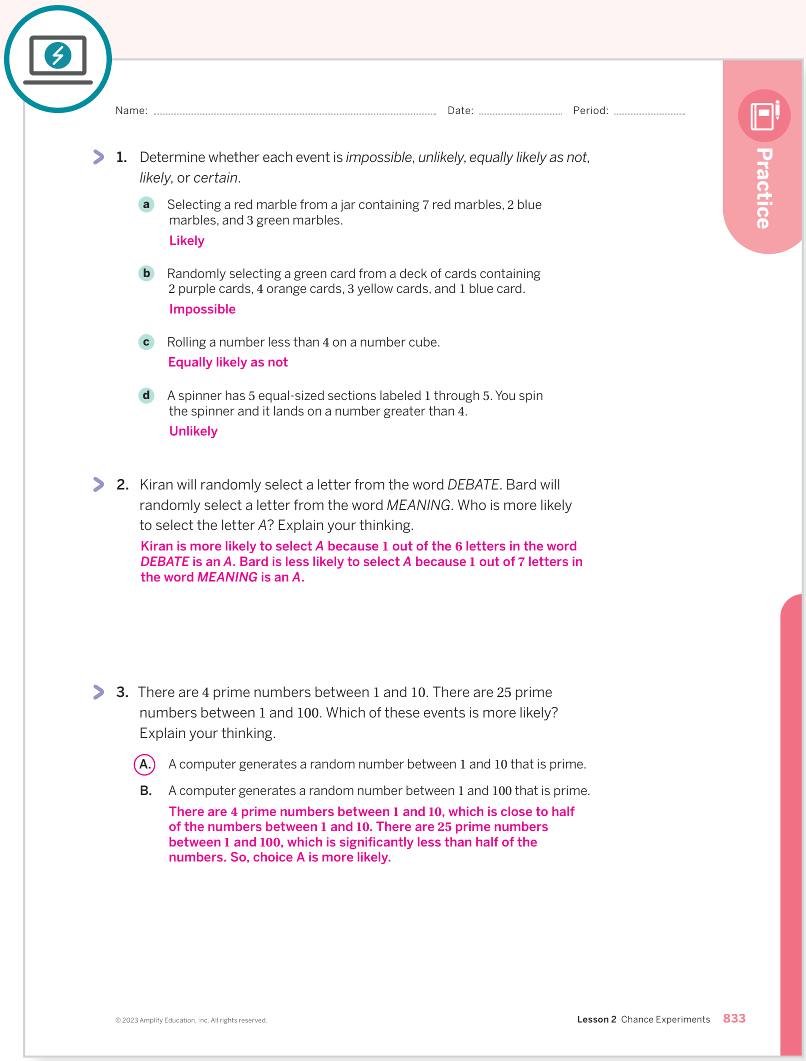
Language Goal: Comprehending and using the terms *impossible, unlikely, equally likely as not, likely*, and *certain* to describe the likelihood of an event.

Reflect on students' language development toward this goal.

- How are students progressing in their use of these terms to describe the likelihood of events? How can you model the use of these terms to support them?
- Reflect on the language routines used in this lesson? Were there any that were more helpful than others? Why? Would you change anything the next time you use these routines?

Practice

Independent



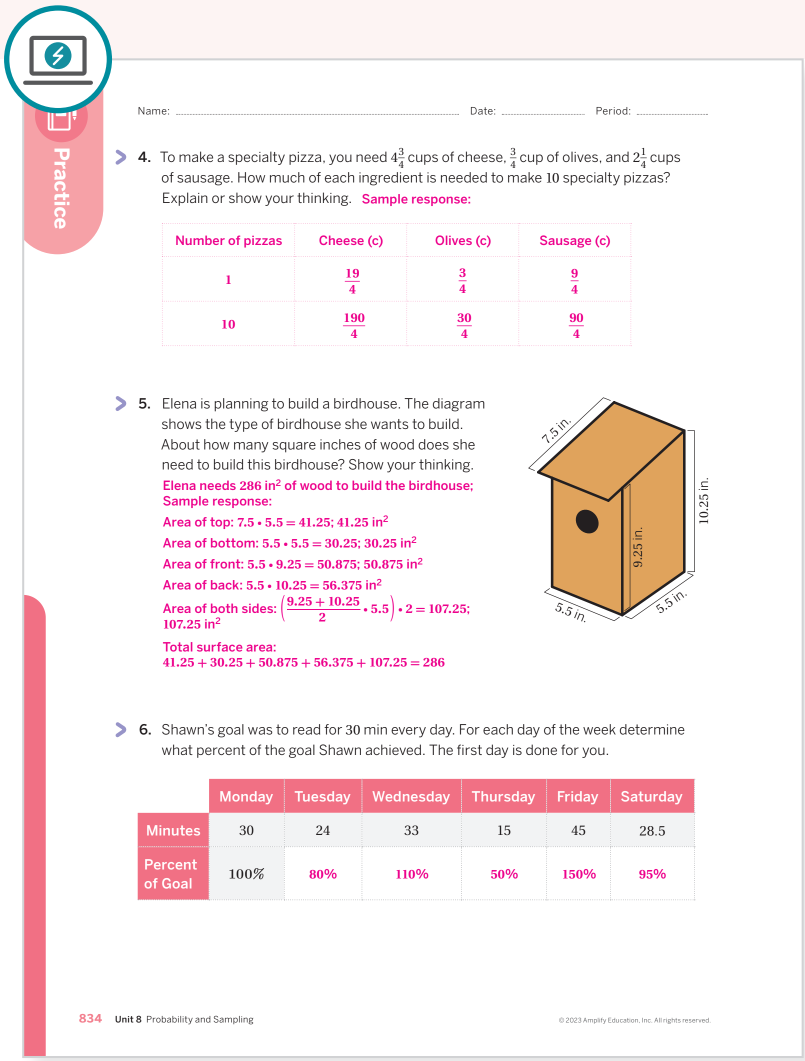
Practice

Name: ______________ Date: ________ Period: ________

1. Determine whether each event is *impossible*, *unlikely*, *equally likely as not*, *likely*, or *certain*.
   - a Selecting a red marble from a jar containing 7 red marbles, 2 blue marbles, and 3 green marbles.
     Likely
   - b Randomly selecting a green card from a deck of cards containing 2 purple cards, 4 orange cards, 3 yellow cards, and 1 blue card.
     Impossible
   - c Rolling a number less than 4 on a number cube.
     Equally likely as not
   - d A spinner has 5 equal-sized sections labeled 1 through 5. You spin the spinner and it lands on a number greater than 4.
     Unlikely

2. Kiran will randomly select a letter from the word *DEBATE*. Bard will randomly select a letter from the word *MEANING*. Who is more likely to select the letter *A*? Explain your thinking.
   Kiran is more likely to select *A* because 1 out of the 6 letters in the word *DEBATE* is an *A*. Bard is less likely to select *A* because 1 out of 7 letters in the word *MEANING* is an *A*.

3. There are 4 prime numbers between 1 and 10. There are 25 prime numbers between 1 and 100. Which of these events is more likely? Explain your thinking.
   [✓ A.] A computer generates a random number between 1 and 10 that is prime.
   B. A computer generates a random number between 1 and 100 that is prime.
   There are 4 prime numbers between 1 and 10, which is close to half of the numbers between 1 and 10. There are 25 prime numbers between 1 and 100, which is significantly less than half of the numbers. So, choice A is more likely.



Practice

Name: ______________ Date: ________ Period: ________

4. To make a specialty pizza, you need $4\frac{3}{4}$ cups of cheese, $\frac{3}{4}$ cup of olives, and $2\frac{1}{4}$ cups of sausage. How much of each ingredient is needed to make 10 specialty pizzas? Explain or show your thinking. **Sample response:**

Number of pizzas	Cheese (c)	Olives (c)	Sausage (c)
1	$\frac{19}{4}$	$\frac{3}{4}$	$\frac{9}{4}$
10	$\frac{190}{4}$	$\frac{30}{4}$	$\frac{90}{4}$

5. Elena is planning to build a birdhouse. The diagram shows the type of birdhouse she wants to build. About how many square inches of wood does she need to build this birdhouse? Show your thinking.
   Elena needs 286 in^2 of wood to build the birdhouse; Sample response:
   Area of top: $7.5 \cdot 5.5 = 41.25$; 41.25 in^2
   Area of bottom: $5.5 \cdot 5.5 = 30.25$; 30.25 in^2
   Area of front: $5.5 \cdot 9.25 = 50.875$; 50.875 in^2
   Area of back: $5.5 \cdot 10.25 = 56.375$ in^2
   Area of both sides: $\left(\frac{9.25 + 10.25}{2} \cdot 5.5\right) \cdot 2 = 107.25$; 107.25 in^2
   Total surface area: $41.25 + 30.25 + 50.875 + 56.375 + 107.25 = 286$

6. Shawn's goal was to read for 30 min every day. For each day of the week determine what percent of the goal Shawn achieved. The first day is done for you.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Minutes	30	24	33	15	45	28.5
Percent of Goal	100%	**80%**	**110%**	**50%**	**150%**	**95%**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 5	1
	5	Unit 7 Lesson 16	2
Formative ⚡	6	Unit 8 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 3**

What Are Probabilities?

Let's find out what's possible.

Focus

Goals

1. **Language Goal:** Generalize the relationship between the probability of an event and the number of possible outcomes in the sample space, for an experiment in which each outcome in the sample space is equally likely. **(Speaking and Listening)**
2. **Language Goal:** List the sample space of a simple chance experiment. **(Writing and Reading)**
3. Use the sample space to determine the probability of an event, and express it as a fraction.

Coherence

Today

Students begin to assign probabilities to chance events. They understand that the greater the probability, the more likely the event will occur. They learn that the sample space is the set of all possible outcomes. Students reason that if there are n equally likely outcomes for a chance experiment, they can construct the argument that the probability of each of these outcomes is $\frac{1}{n}$.

Previously

In Lesson 2, students used the terms *impossible*, *unlikely*, *equally likely as not*, *likely*, and *certain* to describe the likelihood of a chance event.

Coming Soon

Students will conduct experiments and see that in the long run, the relative frequency of a chance event approaches its theoretical probability.

Rigor

- Students build **conceptual understanding** of sample space by creating lists of outcomes.
- Students build **conceptual understanding** of probability by using sample spaces to model possible outcomes.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	15 min	5 min	10 min
Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut slips, one set per group
- bags, one per group

Math Language Development

New words

- *probability*
- *sample space*

Review words

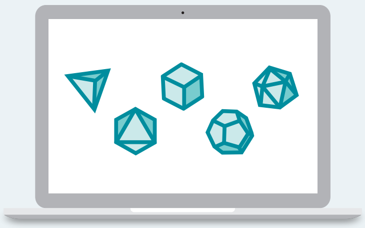
- *chance experiment*
- *outcome*
- *event*

Amps | **Featured Activity**

Activity 2

Mystery Bags

Students will select slips of paper from a virtual bag. They will then try to guess the contents of the bag based on their selection.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may lose focus while they attempt to determine what is in the bag as they select slips of paper out of the bag. Encourage students to discuss their feelings with their group members. Others may be feeling similarly and can empathize. Together they can work on a strategy for staying focused without frustration.

Modifications to Pacing

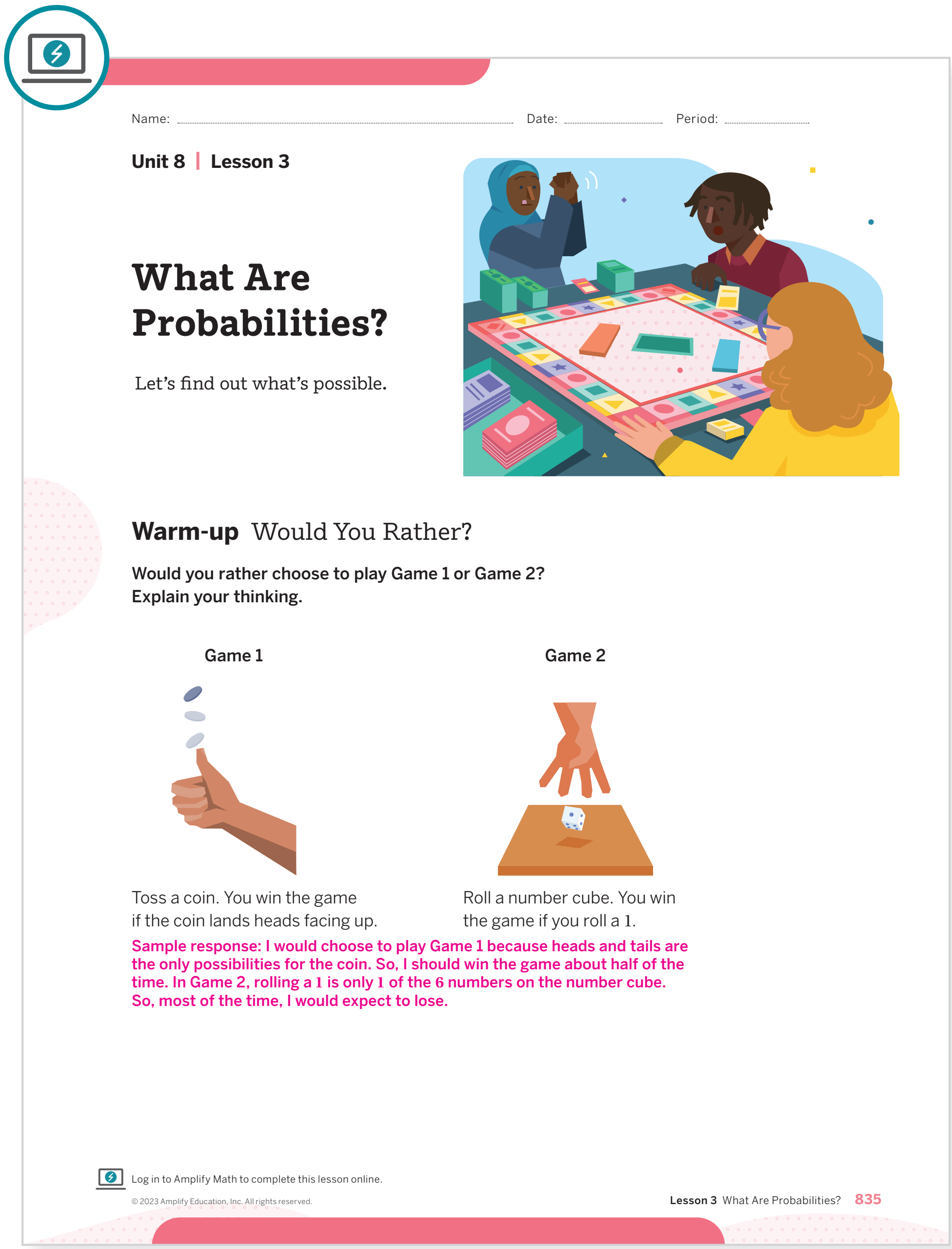
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, Problem 4 may be omitted.

Pairs | 5 min

Warm-up Would You Rather?

Students compare likelihoods of two events to get a better idea of how the size of the sample space affects the likelihood.



Name: ______ Date: ______ Period: ______

Unit 8 | Lesson 3

What Are Probabilities?

Let's find out what's possible.

Warm-up Would You Rather?

Would you rather choose to play Game 1 or Game 2? Explain your thinking.

Game 1	Game 2
Toss a coin. You win the game if the coin lands heads facing up.	Roll a number cube. You win the game if you roll a 1.

Sample response: I would choose to play Game 1 because heads and tails are the only possibilities for the coin. So, I should win the game about half of the time. In Game 2, rolling a 1 is only 1 of the 6 numbers on the number cube. So, most of the time, I would expect to lose.

Log in to Amplify Math to complete this lesson online.

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Lesson 3 What Are Probabilities? 835

1 Launch

Activate students' background knowledge by asking if they have ever played a game that involved tossing a coin or rolling a number cube. Conduct the ***Would You Rather?*** routine.

2 Monitor

Help students get started by thinking about all the possible outcomes for each game.

Look for points of confusion:

- **Thinking that the chances to win are the same for each game because there is one favorable outcome for each game.** Ask students to experiment with a coin and a number cube to observe what happens.

Look for productive strategies:

- Listing the total outcomes for each game and comparing the number of favorable outcomes for each game to the total.

3 Connect

Have pairs of students share which game they chose and their reasoning.

Highlight that the likelihood of winning depends on how the number of favorable outcomes compares to the total number of outcomes.

Define the ***sample space*** as the list of every possible outcome for a chance experiment. For example, when rolling a number cube once, the sample space is 1, 2, 3, 4, 5, and 6.

Ask:

- "What is important to consider when judging how likely you are to win a game of chance?" How many chances you have to win compared to how many chances you have to lose (or how many total outcomes there are).
- "What is the sample space if you were to randomly select one letter from the word *DEBATE*?"

Math Language Development

MLR2: Collect and Display

During the Connect, as students share which game they chose and their reasoning, collect informal language that students use to describe the sample space and connect their language to the formal definition introduced.

English Learners

Use diagrams and organized lists to list the total outcomes of the games played to support students in visualizing the sample space.

Power-up

To power up students' ability to calculate percentages based on real-world data, have students complete:

Recall that determining the percentage from a *part* p and a *whole* w you can use the expression $\frac{p}{w} \cdot 100$

Jada's teacher assigned 30 minutes of reading for homework. Determine the percent of the assigned reading each student completed.

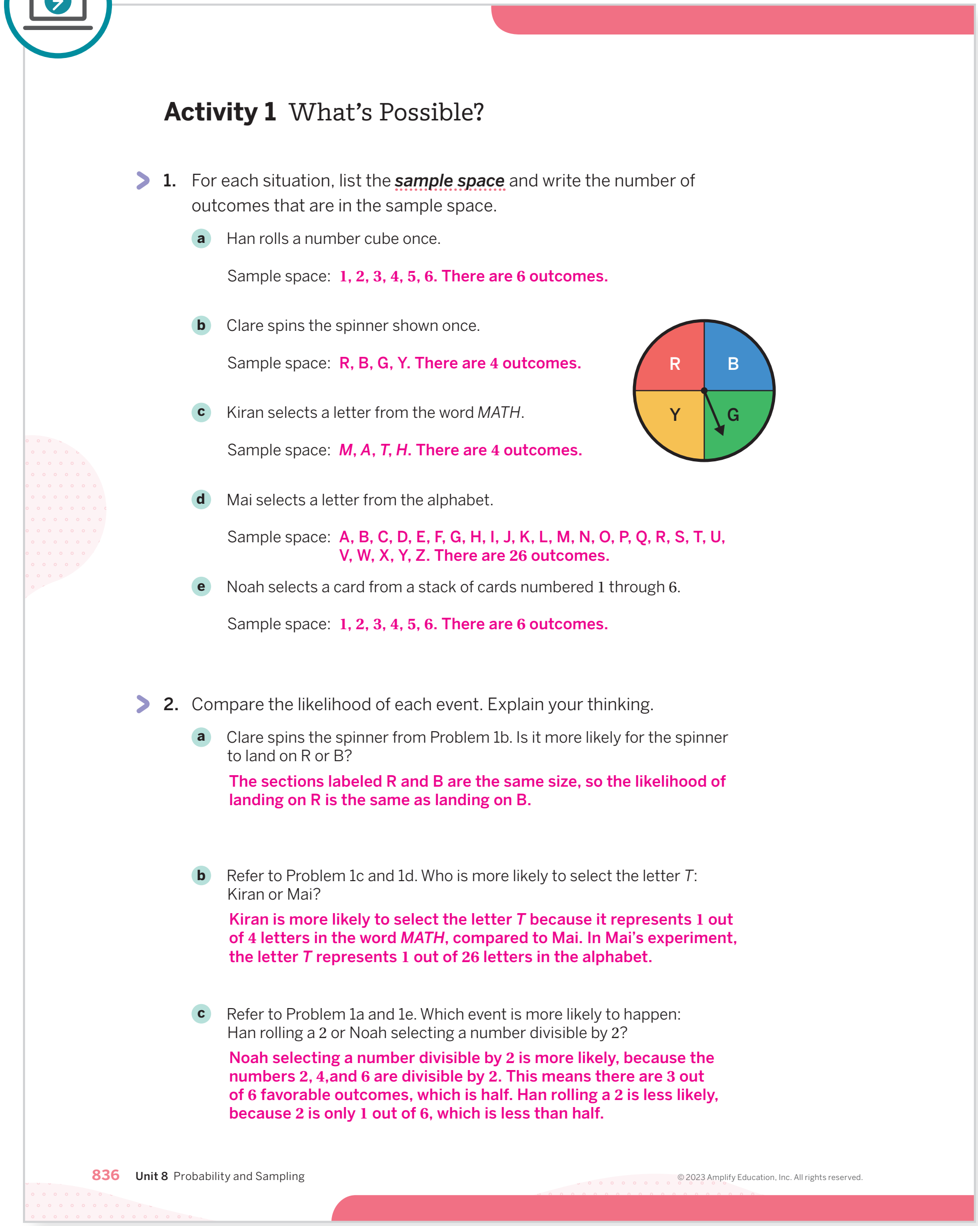
a Diego read for 21 minutes. 70%
b Priya read for 15 minutes. 50%
c Bard read for 36 minutes. 120%

Use: Before Activity 2
Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2

Pairs | 10 min

Activity 1 What's Possible?

Students think about chance situations, determine the sample space, and then consider the likelihood of certain outcomes.



Activity 1 What's Possible?

1. For each situation, list the *sample space* and write the number of outcomes that are in the sample space.
   - a Han rolls a number cube once.

     Sample space: **1, 2, 3, 4, 5, 6. There are 6 outcomes.**
   - b Clare spins the spinner shown once.

     Sample space: **R, B, G, Y. There are 4 outcomes.**
   - c Kiran selects a letter from the word *MATH*.

     Sample space: ***M, A, T, H*. There are 4 outcomes.**
   - d Mai selects a letter from the alphabet.

     Sample space: **A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. There are 26 outcomes.**
   - e Noah selects a card from a stack of cards numbered 1 through 6.

     Sample space: **1, 2, 3, 4, 5, 6. There are 6 outcomes.**

2. Compare the likelihood of each event. Explain your thinking.
   - a Clare spins the spinner from Problem 1b. Is it more likely for the spinner to land on R or B?

     The sections labeled R and B are the same size, so the likelihood of landing on R is the same as landing on B.
   - b Refer to Problem 1c and 1d. Who is more likely to select the letter *T*: Kiran or Mai?

     Kiran is more likely to select the letter *T* because it represents 1 out of 4 letters in the word *MATH*, compared to Mai. In Mai's experiment, the letter *T* represents 1 out of 26 letters in the alphabet.
   - c Refer to Problem 1a and 1e. Which event is more likely to happen: Han rolling a 2 or Noah selecting a number divisible by 2?

     Noah selecting a number divisible by 2 is more likely, because the numbers 2, 4,and 6 are divisible by 2. This means there are 3 out of 6 favorable outcomes, which is half. Han rolling a 2 is less likely, because 2 is only 1 out of 6, which is less than half.

836 **Unit 8** Probability and Sampling

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What are all of the possible numbers you can roll on a number cube?"

Look for points of confusion:

- **Thinking Mai is more likely to select the letter *T* in Problem 2b because there are more possible outcomes.** Have students imagine placing each outcome in the sample space into a bag and selecting one without looking.

3 Connect

Have pairs of students share their responses to Problem 2.

Highlight that when the sample space is greater in one event than another, but the number of favorable outcomes is the same, then the likelihood of the event with the greater sample space is less than the likelihood of the other event, as seen in Problem 2b. Conversely, when the sample space is the same, but the number of favorable outcomes is greater, there is a greater likelihood of the event, as seen in Problem 2c. Point out that there is a ratio that will help them measure likelihood.

Define the term ***probability*** as a number that tells how likely it is for an event to happen. When outcomes are equally likely, the probability is the ratio of the number of favorable outcomes to the total possible outcomes.

Ask:

- "What is the probability of tossing a coin and landing heads facing up?" $\frac{1}{2}$
- "What is the probability of rolling a standard number cube and rolling a 1 or 4?" $\frac{2}{6}$ or $\frac{1}{3}$

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider demonstrating how to determine the sample space for Problem 1a and then have students complete the rest of the problems.

Accessibility: *Clarify Vocabulary and Symbols*

During the Connect, as you define the term *probability*, add the following to the class display:

$$\text{probability} = \text{favorable outcomes} : \text{possible outcomes}$$

$$\text{probability} = \frac{\text{favorable outcomes}}{\text{possible outcomes}}$$

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

During the Connect, as students share their responses to Problem 2, press them to explain their reasoning for Problem 2b. Ask:

- "What do you notice about the sample space for Kiran's experiment compared to Mai's experiment?"
- "How does the sample space affect the likelihood of the event?"

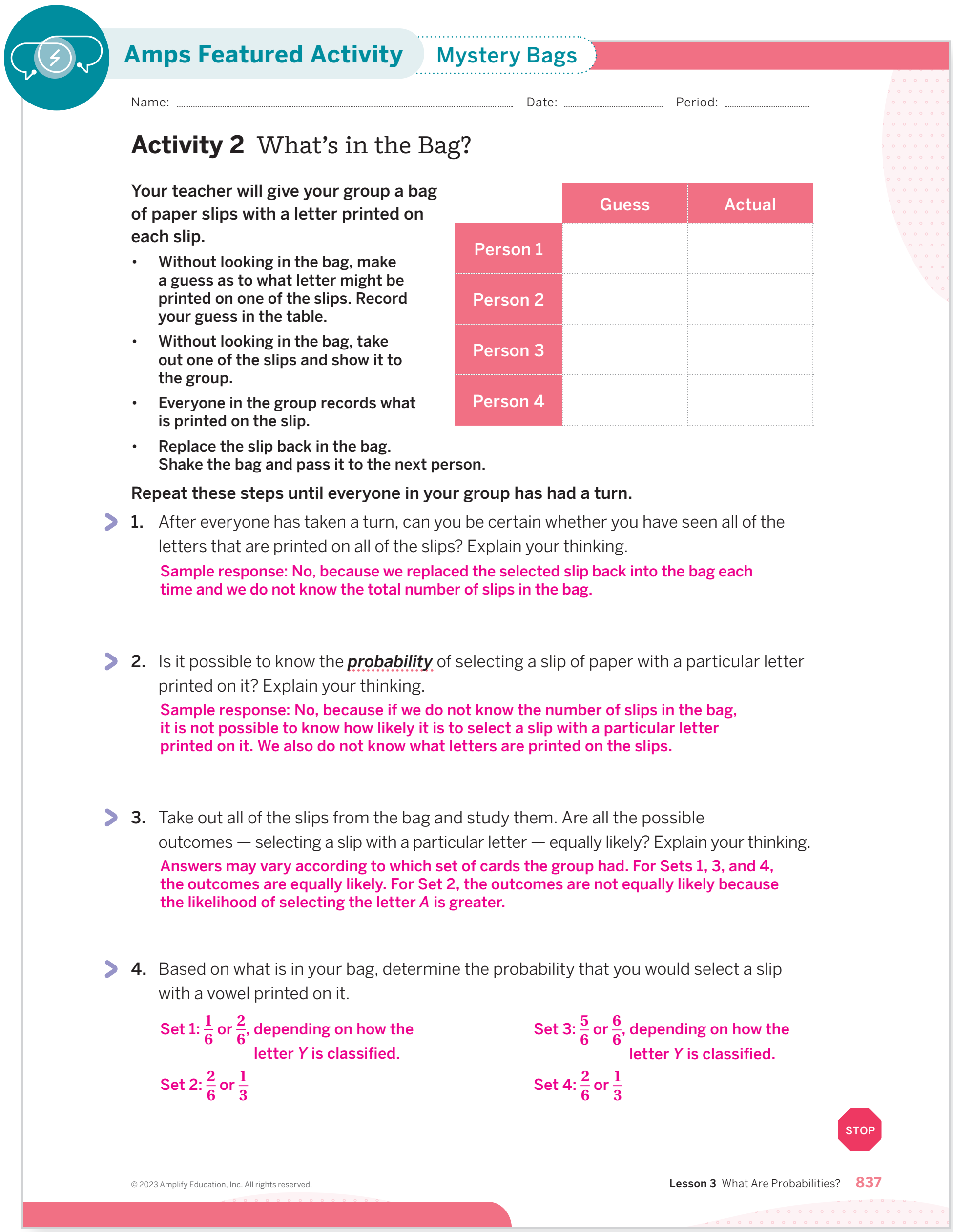
English Learners

Consider providing students with notecards or small pieces of paper to try a few of the experiments. This will help support students' sense making around sample space and favorable outcomes.

Small Groups | 15 min

Activity 2 What's in the Bag?

Students conduct an experiment, seeing that the probability of an event cannot be found without first understanding the sample space.



Amps Featured Activity Mystery Bags

Name: ______________ Date: ________ Period: ________

Activity 2 What's in the Bag?

Your teacher will give your group a bag of paper slips with a letter printed on each slip.

- Without looking in the bag, make a guess as to what letter might be printed on one of the slips. Record your guess in the table.
- Without looking in the bag, take out one of the slips and show it to the group.
- Everyone in the group records what is printed on the slip.
- Replace the slip back in the bag. Shake the bag and pass it to the next person.

	Guess	Actual
Person 1		
Person 2		
Person 3		
Person 4		

Repeat these steps until everyone in your group has had a turn.

1. After everyone has taken a turn, can you be certain whether you have seen all of the letters that are printed on all of the slips? Explain your thinking.
   Sample response: No, because we replaced the selected slip back into the bag each time and we do not know the total number of slips in the bag.

2. Is it possible to know the ***probability*** of selecting a slip of paper with a particular letter printed on it? Explain your thinking.
   Sample response: No, because if we do not know the number of slips in the bag, it is not possible to know how likely it is to select a slip with a particular letter printed on it. We also do not know what letters are printed on the slips.

3. Take out all of the slips from the bag and study them. Are all the possible outcomes — selecting a slip with a particular letter — equally likely? Explain your thinking.
   Answers may vary according to which set of cards the group had. For Sets 1, 3, and 4, the outcomes are equally likely. For Set 2, the outcomes are not equally likely because the likelihood of selecting the letter *A* is greater.

4. Based on what is in your bag, determine the probability that you would select a slip with a vowel printed on it.
   Set 1: $\frac{1}{6}$ or $\frac{2}{6}$, depending on how the letter *Y* is classified.
   Set 2: $\frac{2}{6}$ or $\frac{1}{3}$
   Set 3: $\frac{5}{6}$ or $\frac{6}{6}$, depending on how the letter *Y* is classified.
   Set 4: $\frac{2}{6}$ or $\frac{1}{3}$

STOP

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Lesson 3 What Are Probabilities? 837

1 Launch

Provide each group with a bag containing a set of paper slips from the Activity 2 PDF. Inform students that their task is to determine what is printed on the slips of paper in their bag.

2 Monitor

Help students get started by acting out the steps they will take to perform the experiment.

Look for points of confusion:

- **Thinking it is possible to know the probability of selecting a slip of paper with a particular letter on it in Problem 2.** Ask, "If I asked whether you would like a snack from a mystery bag, wouldn't you want to know what the possibilities are first?"

3 Connect

Ask, "What reasoning did you use to refine your predictions after each person took their turn?"

Highlight that students can think of the contents of the bag as the ***sample space*** of the experiment. Sometimes, in the real world, the sample space is unknown before conducting an experiment. When the sample space is not known, an accurate prediction cannot be made about the probability of outcomes. Once the sample space is better understood, then accurate predictions can be made.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can select slips of paper from a virtual bag and guess the contents of the bag based on their selection.

Accessibility: *Clarify Vocabulary and Symbols*

During the Launch or before students attempt Problem 3, preview the vocabulary term *equally likely* and be sure students understand its meaning. Remind them that outcomes in an experiment are equally likely if the likelihood of selecting each outcome is the same. Consider providing an example, such as rolling a number cube or tossing a coin.

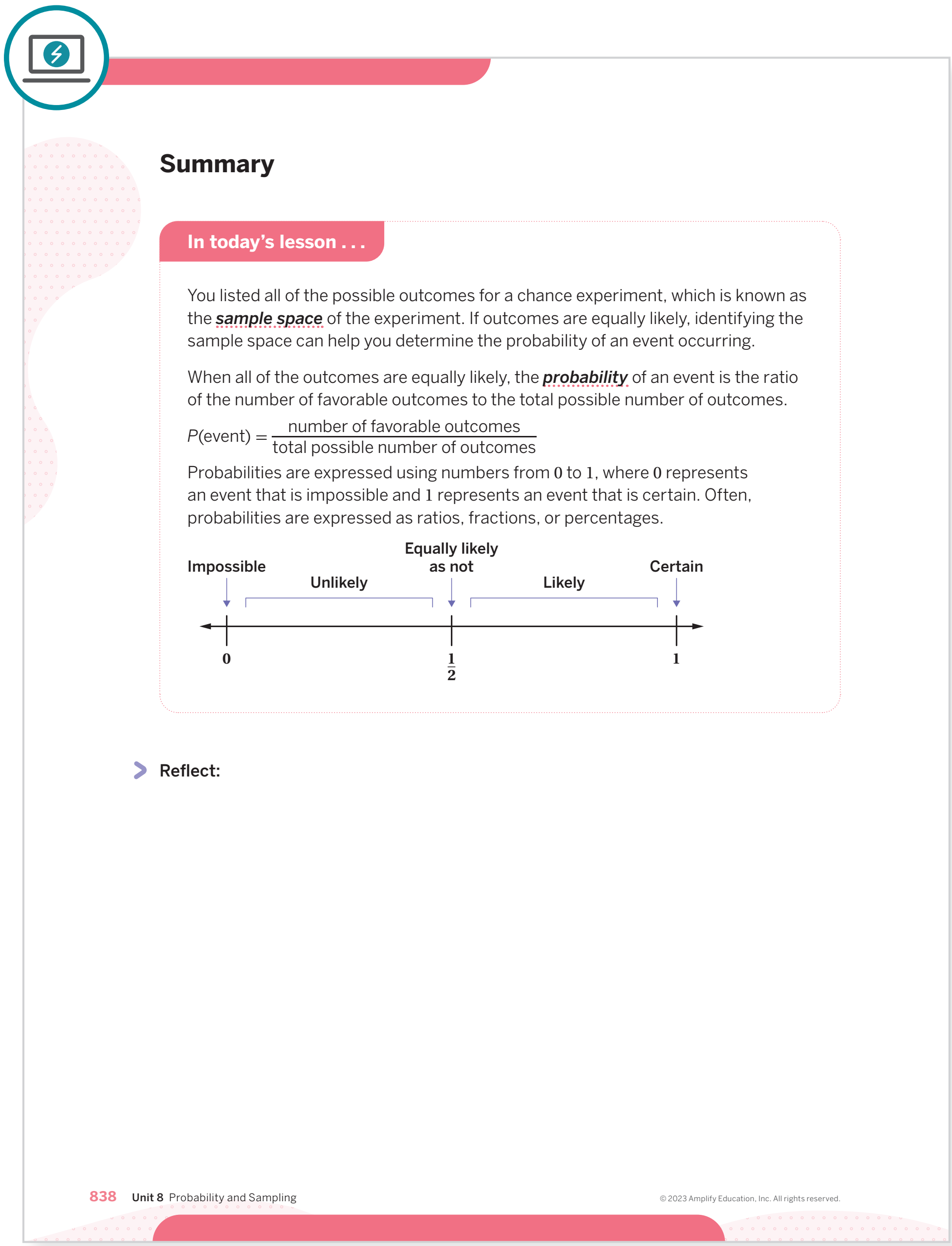
Extension: *Math Enrichment*

Have students consider the following scenario:
You are given a new bag with slips of paper in it. Each slip has a letter printed on it. You are told that the bag contains all the letters of the alphabet. You selected and replaced slips of paper 50 times and never selected the letter Z. Does this mean there is no letter Z in the bag? Explain your thinking.
Not necessarily, but it might make me wonder if it is missing.

Whole Class | 5 min

Summary

Review and synthesize how the sample space of a chance experiment can help determine the probability of an event occurring.



Summary

In today's lesson . . .

You listed all of the possible outcomes for a chance experiment, which is known as the ***sample space*** of the experiment. If outcomes are equally likely, identifying the sample space can help you determine the probability of an event occurring.

When all of the outcomes are equally likely, the ***probability*** of an event is the ratio of the number of favorable outcomes to the total possible number of outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total possible number of outcomes}}$$

Probabilities are expressed using numbers from 0 to 1, where 0 represents an event that is impossible and 1 represents an event that is certain. Often, probabilities are expressed as ratios, fractions, or percentages.

Reflect:

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *sample space* and *probability* that were added to the display during the lesson.

Synthesize

Highlight that the probability of an event occurring is the ratio of the number of favorable outcomes to the total possible number of outcomes in an experiment. This ratio is a number between 0 and 1, where 0 represents an event that is impossible and 1 represents an event that is certain. The probability of an event can be written as a fraction, a decimal, or a percentage.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total possible number of outcomes}}$$

Formalize vocabulary:

- ***sample space***
- ***probability***

Ask:

- "If you randomly select one letter from the English alphabet, how many outcomes are in the sample space? There are 26 outcomes in the sample space.
- Suppose you want to select a vowel (not including *Y*). How many favorable outcomes are there? What is the probability of selecting a vowel?" There are 5 favorable outcomes that are vowels: *A*, *E*, *I*, *O*, and *U*. The probability of selecting a vowel is $\frac{5}{26}$.
- "Suppose, in a different chance experiment, there are 100 different outcomes in the sample space that are equally likely. What is the probability that a specific outcome will occur?" $\frac{1}{100}$
- "Is it possible to have a probability of 3? Why or why not?" No; Sample response: An event that is certain to happen has a probability of 1. Any number greater than 1 does not make sense within the context of probability.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

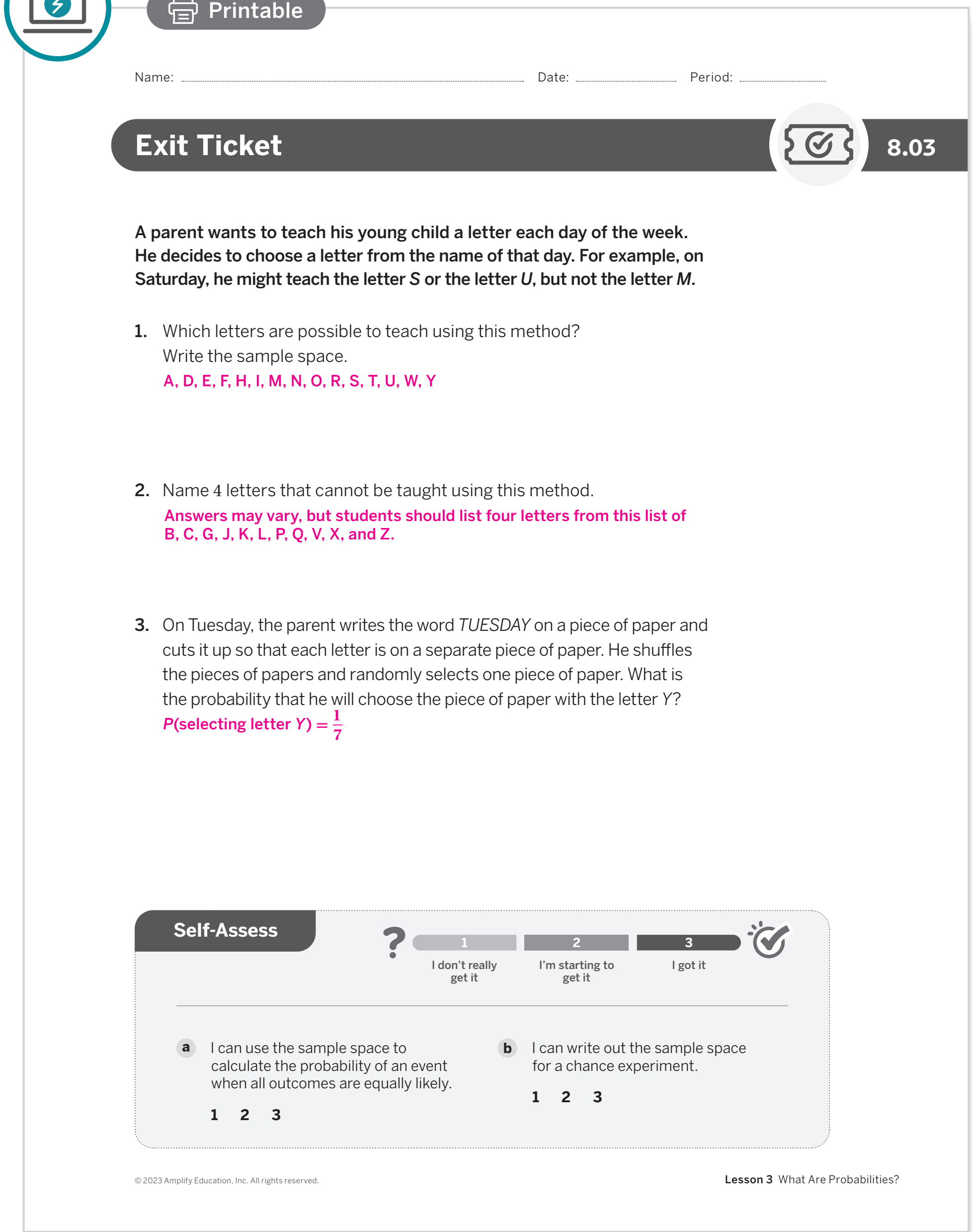
- "What methods did you use to organize your sample spaces in class today? Which methods did you find helpful?"
- "Why is it important to consider all of the outcomes when determining the likelihood or probability of an event occurring?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by listing the sample space of a chance experiment and finding the probability of an event occurring.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 8.03

A parent wants to teach his young child a letter each day of the week. He decides to choose a letter from the name of that day. For example, on Saturday, he might teach the letter *S* or the letter *U*, but not the letter *M*.

1. Which letters are possible to teach using this method? Write the sample space.
   A, D, E, F, H, I, M, N, O, R, S, T, U, W, Y

2. Name 4 letters that cannot be taught using this method.
   Answers may vary, but students should list four letters from this list of B, C, G, J, K, L, P, Q, V, X, and Z.

3. On Tuesday, the parent writes the word *TUESDAY* on a piece of paper and cuts it up so that each letter is on a separate piece of paper. He shuffles the pieces of papers and randomly selects one piece of paper. What is the probability that he will choose the piece of paper with the letter *Y*?
   $P(\text{selecting letter } Y) = \frac{1}{7}$

Self-Assess

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it ✓

a. I can use the sample space to calculate the probability of an event when all outcomes are equally likely.
1 2 3

b. I can write out the sample space for a chance experiment.
1 2 3

Lesson 3 What Are Probabilities?

Success looks like . . .

- **Language Goal:** Generalizing the relationship between the probability of an event and the number of possible outcomes in the sample space, for an experiment in which each outcome in the sample space is equally likely. **(Speaking and Listening)**
- **Language Goal:** Listing the sample space of a simple chance experiment. **(Writing and Reading)**
  - » Listing the sample space of letters that are possible for the parent to teach in Problem 1.
- **Goal:** Using the sample space to determine the probability of an event, and expressing it as a fraction.
  - » Determining the probability of selecting the letter Y as a fraction in Problem 3.

Suggested next steps

If students struggle to spell the days of the week, consider:

- Having a calendar available to aid them.

If students struggle to list the sample space in Problem 1, consider:

- Reviewing Activity 1.

If students struggle to find the probability in Problem 3, consider:

- Assigning Practice Problems 2 and 3.

Professional Learning

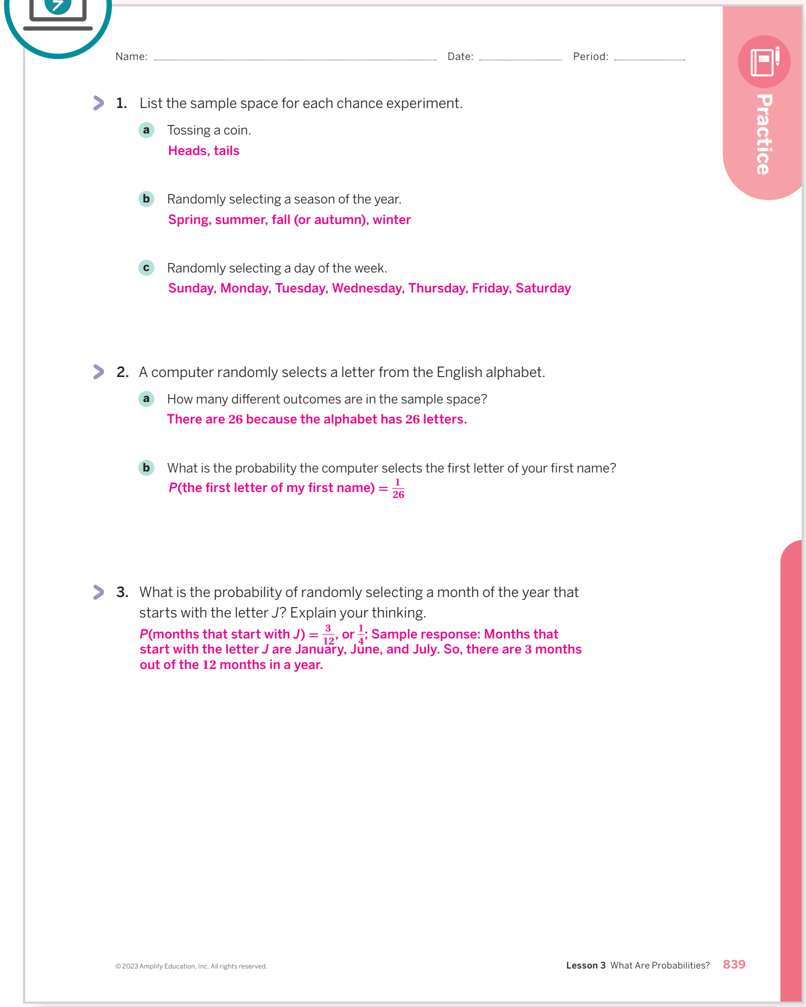
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was for students to create sample space to determine the probability of single events. How well did students accomplish this? What did you specifically do to help students accomplish it?
- In what ways in Activity 2, did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. List the sample space for each chance experiment.
   - a Tossing a coin.
     Heads, tails
   - b Randomly selecting a season of the year.
     Spring, summer, fall (or autumn), winter
   - c Randomly selecting a day of the week.
     Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday

2. A computer randomly selects a letter from the English alphabet.
   - a How many different outcomes are in the sample space?
     There are 26 because the alphabet has 26 letters.
   - b What is the probability the computer selects the first letter of your first name?
     P(the first letter of my first name) $= \frac{1}{26}$

3. What is the probability of randomly selecting a month of the year that starts with the letter *J*? Explain your thinking.
   P(months that start with J) $= \frac{3}{12}$, or $\frac{1}{4}$; Sample response: Months that start with the letter *J* are January, June, and July. So, there are 3 months out of the 12 months in a year.

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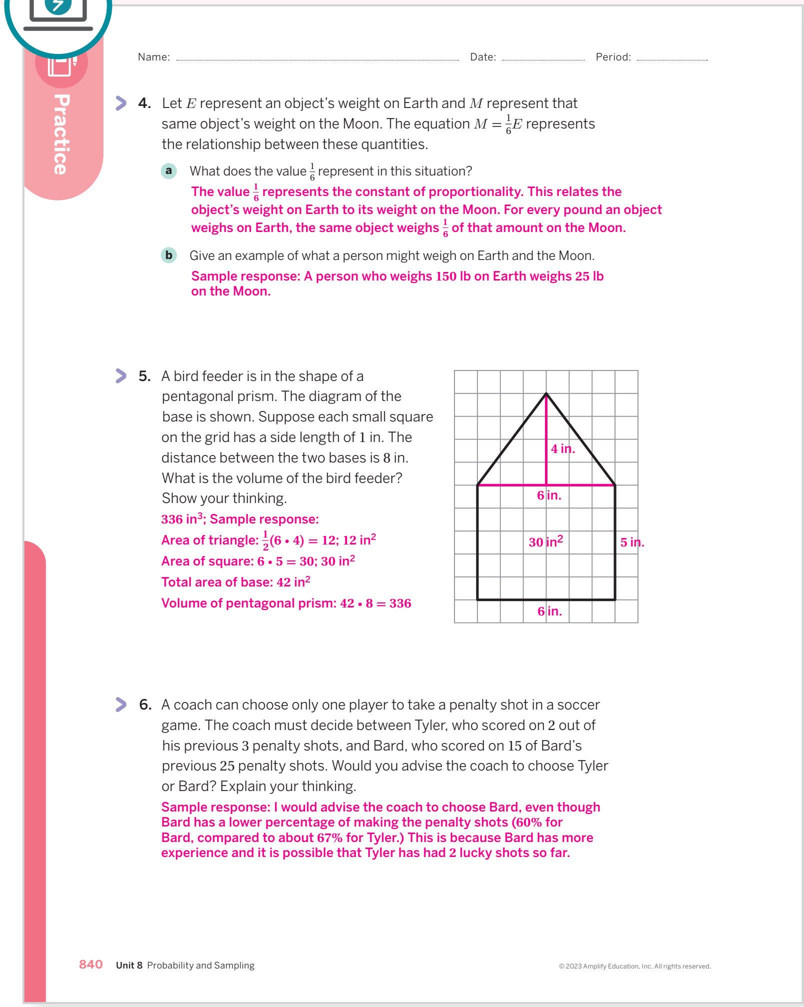
Lesson 3 What Are Probabilities? 839



Name: ______ Date: ______ Period: ______

4. Let E represent an object's weight on Earth and M represent that same object's weight on the Moon. The equation $M = \frac{1}{6}E$ represents the relationship between these quantities.
   - a What does the value $\frac{1}{6}$ represent in this situation?
     The value $\frac{1}{6}$ represents the constant of proportionality. This relates the object's weight on Earth to its weight on the Moon. For every pound an object weighs on Earth, the same object weighs $\frac{1}{6}$ of that amount on the Moon.
   - b Give an example of what a person might weigh on Earth and the Moon.
     Sample response: A person who weighs 150 lb on Earth weighs 25 lb on the Moon.

5. A bird feeder is in the shape of a pentagonal prism. The diagram of the base is shown. Suppose each small square on the grid has a side length of 1 in. The distance between the two bases is 8 in. What is the volume of the bird feeder? Show your thinking.
   336 in^3; Sample response:
   Area of triangle: $\frac{1}{2}(6 \cdot 4) = 12$; 12 in^2
   Area of square: $6 \cdot 5 = 30$; 30 in^2
   Total area of base: 42 in^2
   Volume of pentagonal prism: $42 \cdot 8 = 336$

6. A coach can choose only one player to take a penalty shot in a soccer game. The coach must decide between Tyler, who scored on 2 out of his previous 3 penalty shots, and Bard, who scored on 15 of Bard's previous 25 penalty shots. Would you advise the coach to choose Tyler or Bard? Explain your thinking.
   Sample response: I would advise the coach to choose Bard, even though Bard has a lower percentage of making the penalty shots (60% for Bard, compared to about 67% for Tyler.) This is because Bard has more experience and it is possible that Tyler has had 2 lucky shots so far.

840 **Unit 8** Probability and Sampling

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 5	2
	5	Unit 7 Lesson 15	2
Formative ⚡	6	Unit 8 Lesson 4	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 4**

Estimating Probabilities Through Repeated Experiments

Let's do some experimenting.

Focus

Goals

1. **Language Goal:** Describe patterns, observed in a table or on a graph, that show the relative frequency for a repeated experiment. **(Speaking and Listening, Writing)**
2. **Language Goal:** Generalize that the cumulative relative frequency approaches the probability of the event as an experiment is repeated many times. **(Speaking and Listening)**
3. **Language Goal:** Generate possible results that would or would not be surprising for a repeated experiment, and justify the reasoning. **(Speaking and Listening)**

Coherence

Today

Students roll a number cube many times and see that in the long run, the relative frequency approaches the probability of the chance event. They repeat the experiment and examine the structure of the results. They also see that the relative frequency of a chance event seldom has the exact same value as the expected probability.

Previously

In Lesson 3, students were introduced to assigning probabilities to chance events and learned that the greater the probability, the more likely the event will occur.

Coming Soon

In Lessons 5 and 6, students will engage in a project asking them to compare the relative frequencies of letters in texts in order to crack a code.

Rigor

- Students build **conceptual understanding** of relative frequency by repeatedly rolling number cubes.
- Students build **fluency** in determining the probability of single-step events by comparing them to the cumulative relative frequency of the events in an experiment.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	10 min	5 min	10 min
Pairs	Pairs	Pairs	Whole Class	Independent

desmos Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (optional, for display)
- number cubes, one per pair

Math Language Development

New word

- ***relative frequency***

Review words

- *chance experiment*
- *event*
- *outcome*
- *probability*
- *sample space*

Amps | Featured Activity

Activity 1

Aggregate Class Data

Students see how, over the long term, the experimental (observed) probability approaches the theoretical probability of an event.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel confused when considering the theoretical outcomes of tossing a coin without actually conducting the experiment. Encourage students to use drawings or objects to simulate the experience of tossing a coin when it cannot actually be done.

Modifications to Pacing

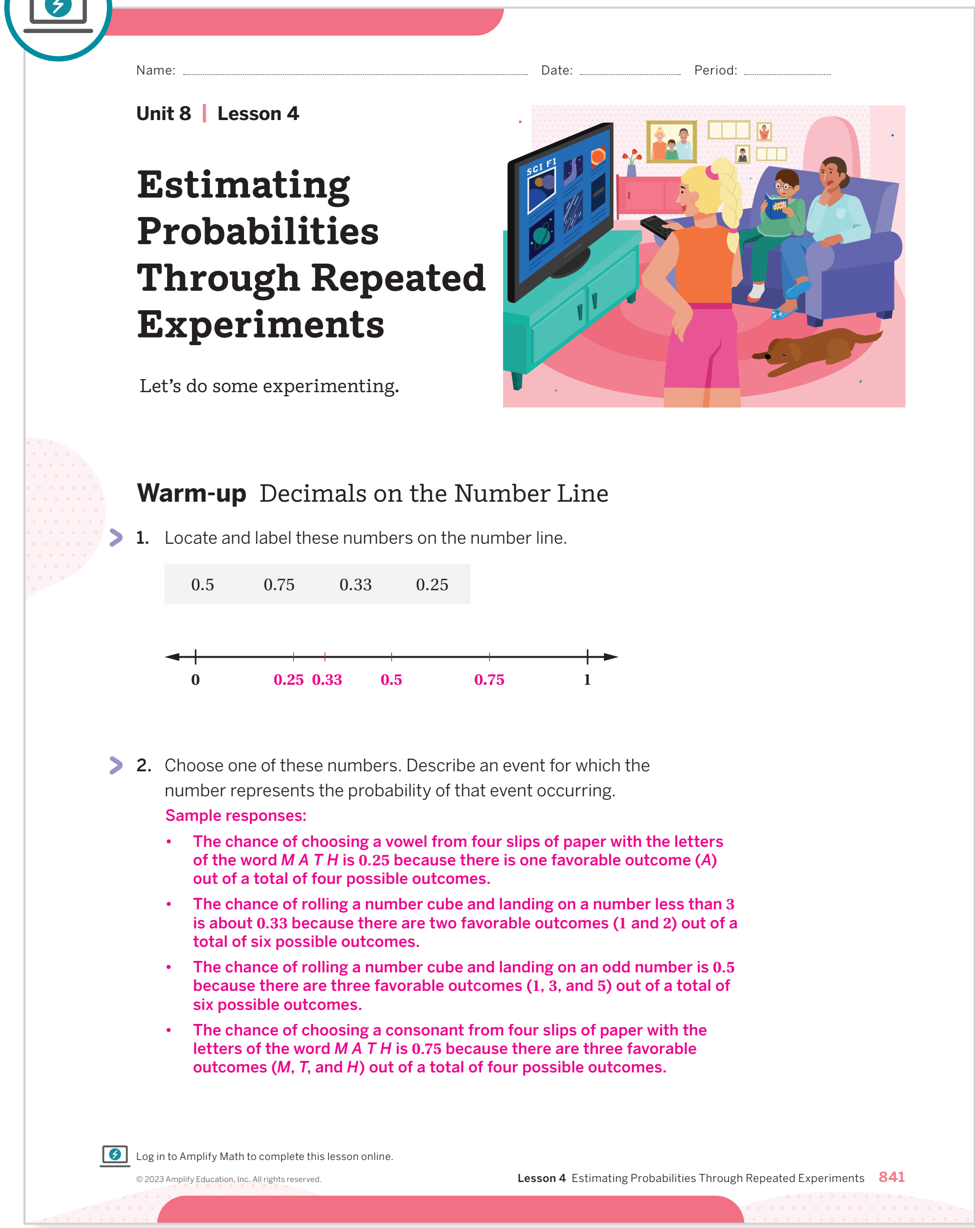
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 2**, Problem 3 may be omitted.

Pairs | 5 min

Warm-up Decimals on the Number Line

Students place decimal values on a number line to prepare them for working with decimals and probability in the upcoming activities.



Name: ______ Date: ______ Period: ______

Unit 8 | Lesson 4

Estimating Probabilities Through Repeated Experiments

Let's do some experimenting.

Warm-up Decimals on the Number Line

1. Locate and label these numbers on the number line.

0.5	0.75	0.33	0.25

2. Choose one of these numbers. Describe an event for which the number represents the probability of that event occurring.

Sample responses:

- **The chance of choosing a vowel from four slips of paper with the letters of the word *M A T H* is 0.25 because there is one favorable outcome (*A*) out of a total of four possible outcomes.**
- **The chance of rolling a number cube and landing on a number less than 3 is about 0.33 because there are two favorable outcomes (1 and 2) out of a total of six possible outcomes.**
- **The chance of rolling a number cube and landing on an odd number is 0.5 because there are three favorable outcomes (1, 3, and 5) out of a total of six possible outcomes.**
- **The chance of choosing a consonant from four slips of paper with the letters of the word *M A T H* is 0.75 because there are three favorable outcomes (*M*, *T*, and *H*) out of a total of four possible outcomes.**

Log in to Amplify Math to complete this lesson online.

1 Launch

Encourage students to discuss their plan, with a partner, for locating the numbers on the number line before they place the values.

2 Monitor

Help students get started by suggesting they partition the number line into smaller units, such as halves, fourths, or tenths.

Look for points of confusion:

- **Placing 0.33 halfway between 0.25 and 0.5 in Problem 1.** Have students mark locations for 0.1, 0.2, 0.3, and 0.4, and ask them how these partitions help them place these values.

Look for productive strategies:

- Partitioning the number line into equal intervals before placing the values in Problem 1.
- Using objects they have seen before, such as number cubes and spinners, to describe events in Problem 2.

3 Connect

Display the number line with the values located.

Have pairs of students share their responses to Problem 2 with the whole class.

Highlight the vocabulary for the likelihood for each event shared and for the values of 0 and 1. Label the number line with the regions that correspond with each term: *impossible*, *unlikely*, *equally likely as not*, *likely*, and *certain*.

Ask, "When calculating the probability of an event, what is the range of possible values? Explain your thinking." All probabilities must be between 0 and 1, 0 and 1 included. 0 represents a 0% chance — an outcome that is impossible. 1 represents a 100% chance — an outcome that is certain. It is not possible for an outcome to have a probability below 0% or above 100%.

Math Language Development

MLR2: Collect and Display

During the Connect, collect and display the mathematical language students used to describe the likelihood for each event shared. Listen for words such as *chance*, *favorable outcomes*, *possible outcomes*, *probability*, etc. Add the number line to the class display and annotate the number line with the corresponding terms: *impossible*, *unlikely*, *equally likely as not*, *likely*, and *certain*. Include numerical examples written as fractions, decimals, and percentages.

Power-up

To power up students' ability to reason about probabilities, have students complete:

In the first two games of the season Elena made 3 out of 4 goals that she attempted. During the entire month, she made 12 out of 20 goals that she attempted.

1. What percent of goals did she make during the first two games? 75%
2. What percent of goals did she make during her first month of games? 60%
3. Which percent better reflects Elena's ability to make a goal during a game? 60%

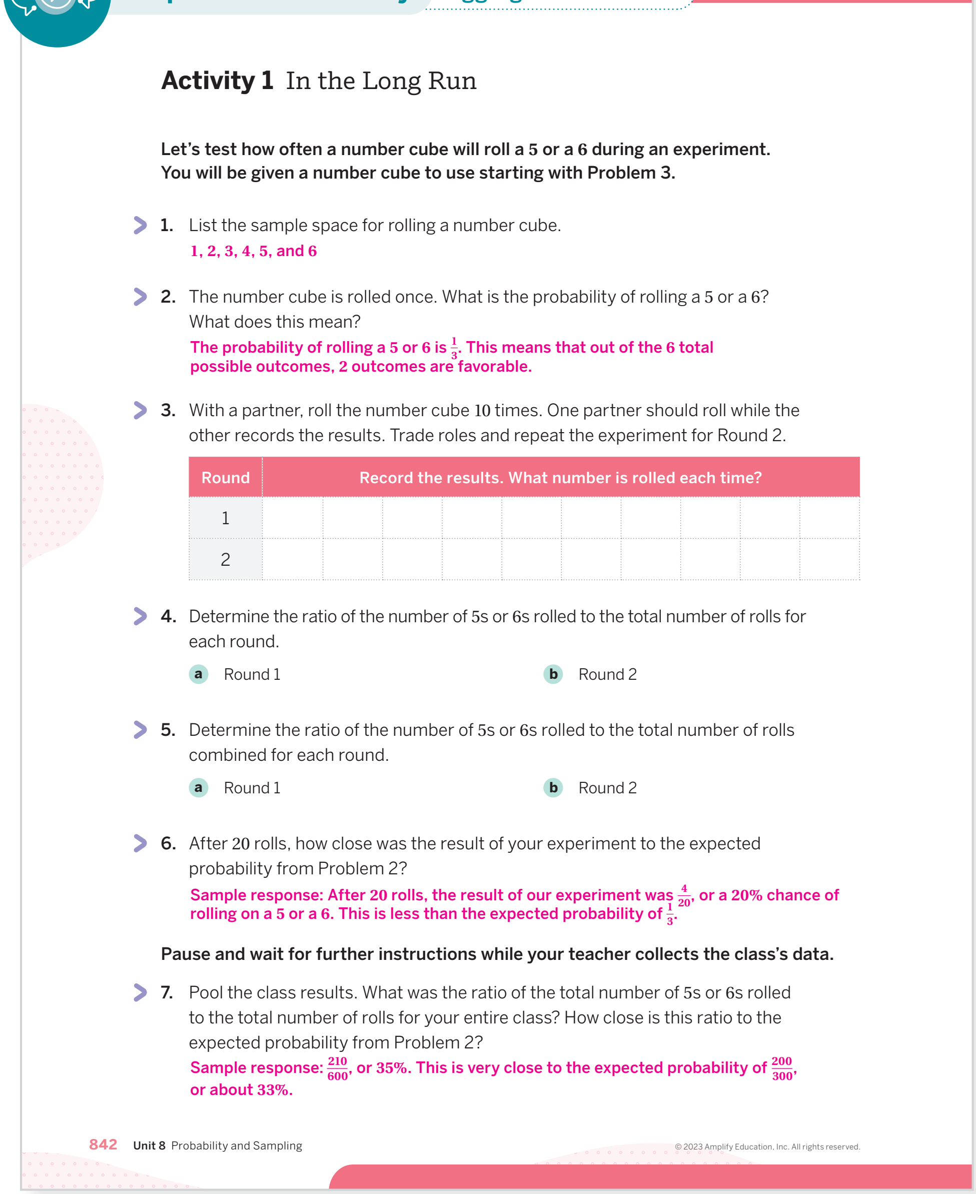
Use: Before Activity 1
Informed by: Performance on Lesson 3, Practice Problem 6

Pairs | 15 min

Activity 1 In the Long Run

Students conduct an experiment to observe the long-run relative frequency of an event and compare it to the event's probability.

Amps Featured Activity Aggregate Class Data



Activity 1 In the Long Run

Let's test how often a number cube will roll a 5 or a 6 during an experiment. You will be given a number cube to use starting with Problem 3.

1. List the sample space for rolling a number cube.
   1, 2, 3, 4, 5, and 6

2. The number cube is rolled once. What is the probability of rolling a 5 or a 6? What does this mean?
   The probability of rolling a 5 or 6 is $\frac{1}{3}$. This means that out of the 6 total possible outcomes, 2 outcomes are favorable.

3. With a partner, roll the number cube 10 times. One partner should roll while the other records the results. Trade roles and repeat the experiment for Round 2.

Round	Record the results. What number is rolled each time?									
1										
2										

4. Determine the ratio of the number of 5s or 6s rolled to the total number of rolls for each round.
   a. Round 1
   b. Round 2

5. Determine the ratio of the number of 5s or 6s rolled to the total number of rolls combined for each round.
   a. Round 1
   b. Round 2

6. After 20 rolls, how close was the result of your experiment to the expected probability from Problem 2?
   Sample response: After 20 rolls, the result of our experiment was $\frac{4}{20}$, or a 20% chance of rolling on a 5 or a 6. This is less than the expected probability of $\frac{1}{3}$.

Pause and wait for further instructions while your teacher collects the class's data.

7. Pool the class results. What was the ratio of the total number of 5s or 6s rolled to the total number of rolls for your entire class? How close is this ratio to the expected probability from Problem 2?
   Sample response: $\frac{210}{600}$, or 35%. This is very close to the expected probability of $\frac{200}{300}$, or about 33%.

842 Unit 8 Probability and Sampling

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1 Launch

Provide one number cube for each pair of students.

2 Monitor

Help students get started by asking them what the sample space of an event means.

Look for points of confusion:

- **Thinking the probability of rolling a 5 or 6 is $\frac{1}{6}$ in Problem 2.** Remind students that probability is a ratio of the number of favorable outcomes, 2, to the total number of possible outcomes, 6.
- **Generalizing from their results that the small sample was very close to the probability.** Share results from other pairs of students to show this is not always the case.

Look for productive strategies:

- Writing an equivalent percentage for each ratio in the table in Problem 3.

3 Connect

Have pairs of students share their results from their experiment, and display them for the class to see, including a graph of the data. To share results, you may create your own spreadsheet to aggregate class data or use the Activity 1 PDF containing a graph of sample data.

Ask:

- "What do you notice about the shape of the graph? Why does the graph jump around more at first?"
- "After how many trials does the graph seem to get closer to the expected (theoretical) probability?"

Highlight that the graph of the class data (or in Activity 1 PDF) appears to flatten in the long run and approach the probability from Problem 2. Mention that the term ***relative frequency*** refers to the observed (experimental) probability.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can see, over the long term, how the observed probability approaches the theoretical probability.

Extension: *Math Enrichment*

Have students complete the following problem: If a computer simulated 7,329,210 rolls of the number cube, about how many 5s or 6s would you expect? About 2,400,000 (about 1,200,000 of each number).

Math Language Development

MLR7: Compare and Connect

During the Connect, as you either aggregate class data or use the sample data from the Activity 1 PDF, draw students' attention to the shape of the graph and the mathematical language that describes what is happening. Highlight the following:

- The graph shows the *observed (experimental) probabilities*. These probabilities are the same as the *relative frequencies*.
- As the *number of trials* (rolls) *increases*, the graph approaches the *expected (theoretical probability)*. This is known as the *long-run relative frequency* because it describes what happens in the long run.

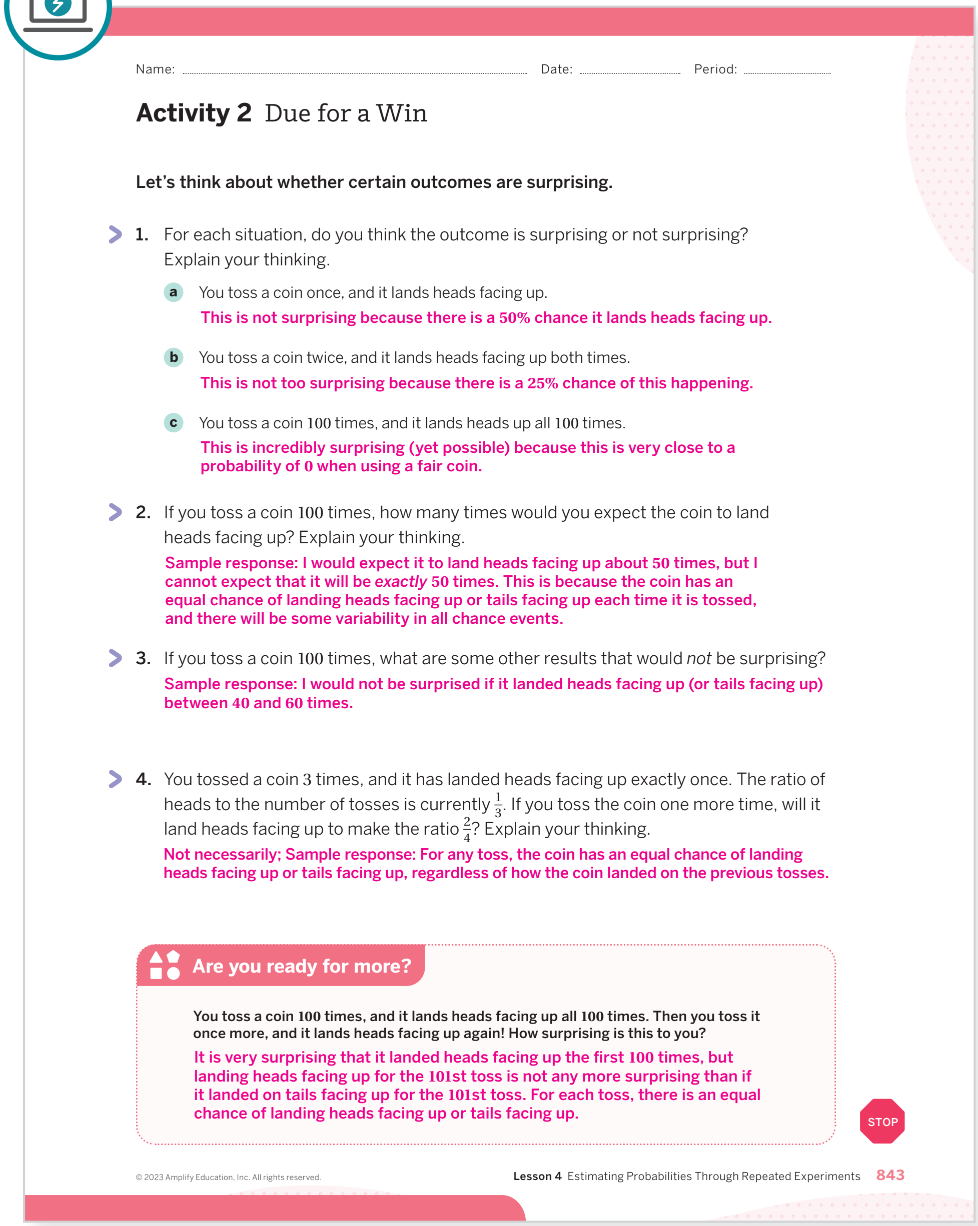
English Learners

Annotate the graph with the term *long-run relative frequency* as the graph approaches the *theoretical probability*, $\frac{1}{3}$.

Pairs | 10 min

Activity 2 Due For a Win

Students reason about chance events that may be surprising to understand that the probability of an event is not dependent upon the results of prior events.



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Activity 2 Due for a Win

Let's think about whether certain outcomes are surprising.

1. For each situation, do you think the outcome is surprising or not surprising? Explain your thinking.
   - a You toss a coin once, and it lands heads facing up.
     This is not surprising because there is a 50% chance it lands heads facing up.
   - b You toss a coin twice, and it lands heads facing up both times.
     This is not too surprising because there is a 25% chance of this happening.
   - c You toss a coin 100 times, and it lands heads up all 100 times.
     This is incredibly surprising (yet possible) because this is very close to a probability of 0 when using a fair coin.
2. If you toss a coin 100 times, how many times would you expect the coin to land heads facing up? Explain your thinking.
   Sample response: I would expect it to land heads facing up about 50 times, but I cannot expect that it will be *exactly* 50 times. This is because the coin has an equal chance of landing heads facing up or tails facing up each time it is tossed, and there will be some variability in all chance events.
3. If you toss a coin 100 times, what are some other results that would *not* be surprising?
   Sample response: I would not be surprised if it landed heads facing up (or tails facing up) between 40 and 60 times.
4. You tossed a coin 3 times, and it has landed heads facing up exactly once. The ratio of heads to the number of tosses is currently $\frac{1}{3}$. If you toss the coin one more time, will it land heads facing up to make the ratio $\frac{2}{4}$? Explain your thinking.
   Not necessarily; Sample response: For any toss, the coin has an equal chance of landing heads facing up or tails facing up, regardless of how the coin landed on the previous tosses.

Are you ready for more?

You toss a coin 100 times, and it lands heads facing up all 100 times. Then you toss it once more, and it lands heads facing up again! How surprising is this to you?

It is very surprising that it landed heads facing up the first 100 times, but landing heads facing up for the 101st toss is not any more surprising than if it landed on tails facing up for the 101st toss. For each toss, there is an equal chance of landing heads facing up or tails facing up.

STOP

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Lesson 4 Estimating Probabilities Through Repeated Experiments 843

1 Launch

Encourage students to discuss any surprising prior experiences they may have had with tossing coins or other probability events.

2 Monitor

Help students get started by telling them the probability of a tossed coin landing heads facing up is $\frac{1}{2}$.

Look for points of confusion:

- **Thinking a tossed coin will land heads facing up half of the time, no matter how many times the coin is tossed.** Give students a coin to experiment with a few times to see what happens.

Look for productive strategies:

- Reasoning or discussing with a partner about the point at which a result becomes "surprising".
- Finding that the probability for Problem 1c is $\left(\frac{1}{2}\right)^{100}$. **Note:** This calculation or expression is not required for this activity.

3 Connect

Have individual students share their responses for Problems 1b and 1c and display for all to see. Then create a range of values that might not be surprising based on student responses.

Ask the class if they agree with this range or to provide a reason for why the range is too large.

Highlight that a theoretical probability represents the expected likelihood of an event occurring for a single trial of an experiment. In the short term, the outcomes from your trials will likely not match the theoretical probability exactly, but should stay close enough that it is not surprising. Regardless of what has come before, each coin flip should be equally as likely to land heads up as tails up.

Differentiated Support

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Provide access to coins and allow students to toss an actual coin during the activity to help them visualize the experiment and ground their reasoning.

Extension: *Math Enrichment*

Tell students that the expected (theoretical) probability of tossing a coin 100 times and having the coin land on heads all 100 times is 1 out of 1,267,650,600,228,229,401,496,703,205,376, an extremely unlikely event, but not impossible.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 4, have them meet with 1–2 other pairs of students to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "Can you include an example in your response?"
- "What mathematical language did you use in your response?"

Have students write a final response, based on the feedback they received.

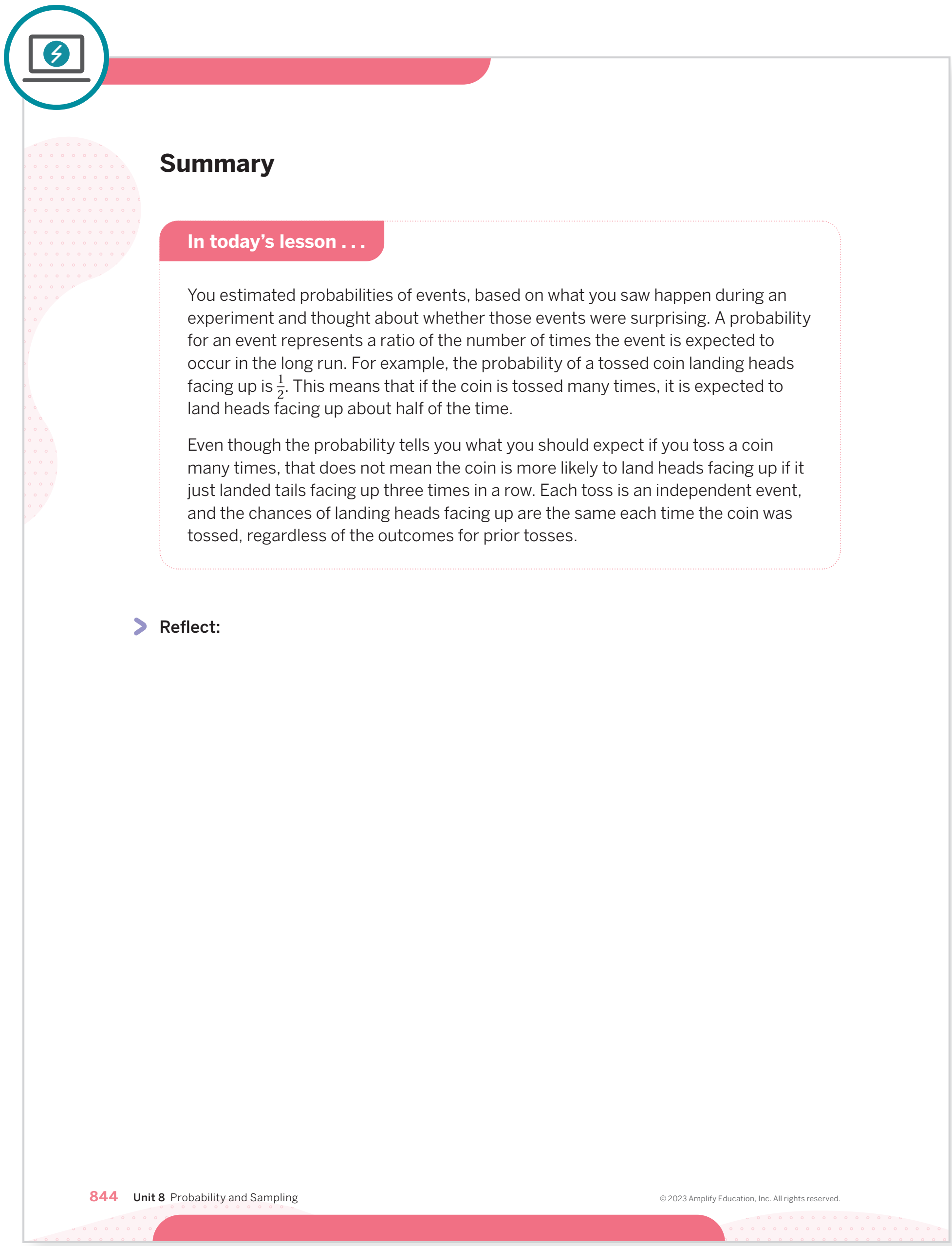
English Learners

Encourage students to use diagrams or illustrations in their response indicating the outcomes of the experiment.

Whole Class | 5 min

Summary

Review and synthesize that in the long run, the relative frequency of a chance event will approach the expected probability — but not necessarily in the short term.



Summary

In today's lesson . . .

You estimated probabilities of events, based on what you saw happen during an experiment and thought about whether those events were surprising. A probability for an event represents a ratio of the number of times the event is expected to occur in the long run. For example, the probability of a tossed coin landing heads facing up is $\frac{1}{2}$. This means that if the coin is tossed many times, it is expected to land heads facing up about half of the time.

Even though the probability tells you what you should expect if you toss a coin many times, that does not mean the coin is more likely to land heads facing up if it just landed tails facing up three times in a row. Each toss is an independent event, and the chances of landing heads facing up are the same each time the coin was tossed, regardless of the outcomes for prior tosses.

Reflect:

844 Unit 8 Probability and Sampling

Synthesize

Formalize vocabulary: ***relative frequency***

Ask:

- "What is the probability of rolling a 2, 3, or 4 on a standard number cube?" $\frac{1}{2}$; There are three favorable outcomes out of six possible outcomes.
- "If you roll the number cube three times and *none* of them result in a 2, 3, or 4, does the chance of rolling one of these numbers increase with the next roll? Why or why not?" No; Sample response: Each roll is considered independent of the results of prior rolls.
- "Suppose the probability of getting the flu during flu season is $\frac{1}{8}$. If a family has 8 people living in the same house, is it guaranteed that one of them will get the flu? Sample response: No, it is possible that none of the people in the family will get the flu and also possible that more than 1 person will get the flu.
- "If a country has 8 million people, about how many do you expect will get the flu? Would you be surprised if the actual number of people that get the flu is a different number?" Sample response: about 1 million people; I would not be surprised if the actual number is different, as long as it is not significantly higher or lower than 1 million.
- "How can you check whether a coin is a fair coin?" Sample response: I could flip the coin many times and check that the relative frequency of landing heads facing up or tails facing up is close to $\frac{1}{2}$.

Highlight that an interesting problem in statistics is trying to define when an outcome should be considered "surprising." Tossing a fair coin 100 times and having it land heads facing up 55 times should not be surprising, but landing heads facing up either 5 or 95 times would be surprising, based on the theoretical probability. It is expected that the coin should land heads facing up about 50 times, so if the number of times it lands heads facing up is significantly less than or greater than 50, it would be surprising.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments to reflect on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

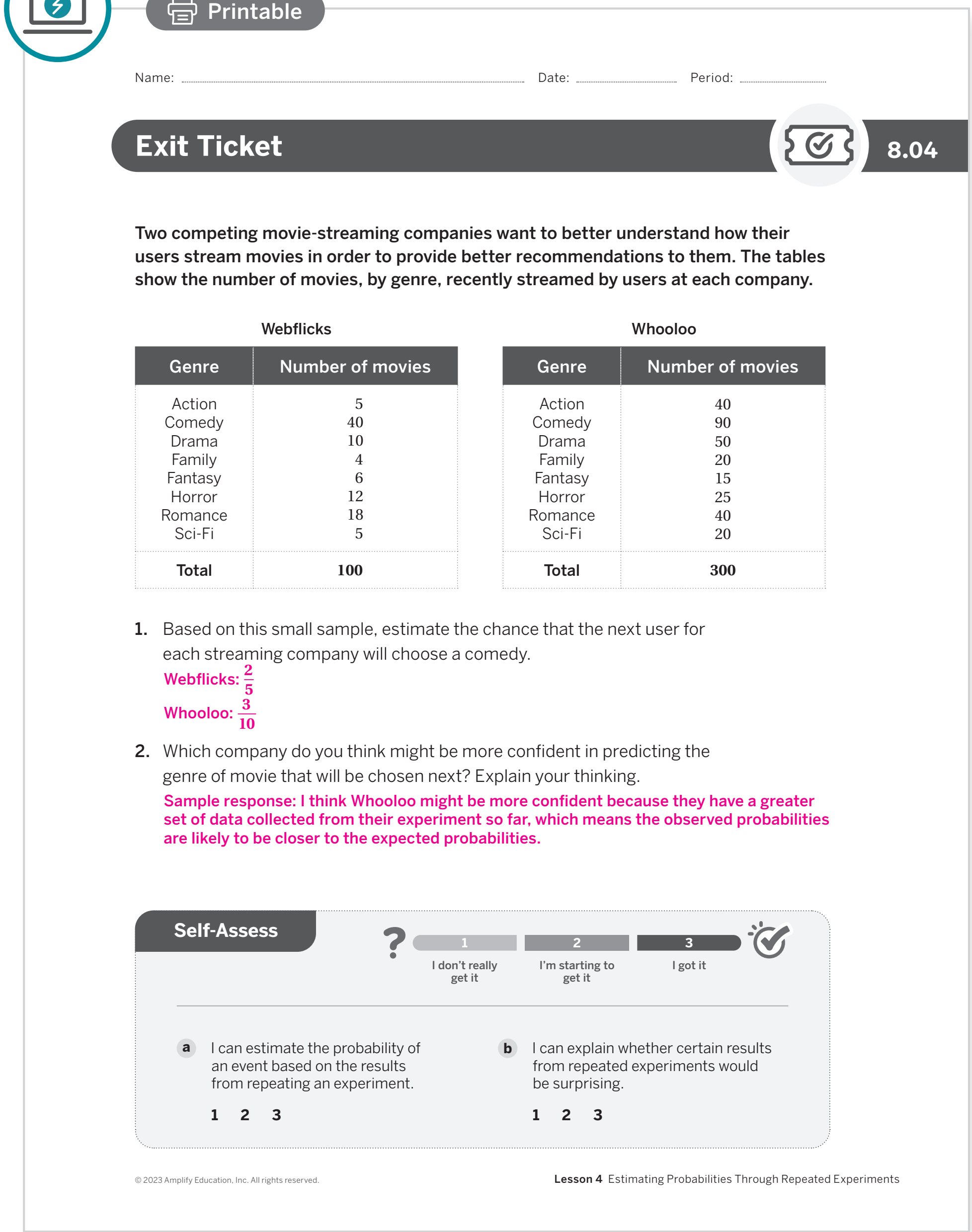
- "Our world is really complex — how can we simulate parts of it to make better predictions?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of long-term relative frequency by comparing surveys with different sample sizes.

Printable



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Exit Ticket 8.04

Two competing movie-streaming companies want to better understand how their users stream movies in order to provide better recommendations to them. The tables show the number of movies, by genre, recently streamed by users at each company.

Webflicks

Genre	Number of movies
Action	5
Comedy	40
Drama	10
Family	4
Fantasy	6
Horror	12
Romance	18
Sci-Fi	5
Total	100

Whooloo

Genre	Number of movies
Action	40
Comedy	90
Drama	50
Family	20
Fantasy	15
Horror	25
Romance	40
Sci-Fi	20
Total	300

1. Based on this small sample, estimate the chance that the next user for each streaming company will choose a comedy.
   Webflicks: $\frac{2}{5}$
   Whooloo: $\frac{3}{10}$

2. Which company do you think might be more confident in predicting the genre of movie that will be chosen next? Explain your thinking.
   Sample response: I think Whooloo might be more confident because they have a greater set of data collected from their experiment so far, which means the observed probabilities are likely to be closer to the expected probabilities.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can estimate the probability of an event based on the results from repeating an experiment.
1 2 3

b I can explain whether certain results from repeated experiments would be surprising.
1 2 3

Lesson 4 Estimating Probabilities Through Repeated Experiments

Success looks like . . .

- Language **Goal:** Describing patterns, observed in a table or on a graph, that show the relative frequency for a repeated experiment. **(Speaking and Listening, Writing)**
  - » Estimating the chance that the ext user will choose a comedy in Problem 1.
- **Language Goal:** Generalizing that the cumulative relative frequency approaches the probability of the event as an experiment is repeated many times. **(Speaking and Listening)**
  - » Explaining that Whooloo might more confident about predicting the next genre in Problem 2.
- **Language Goal:** Generating possible results that would or would not be surprising for a repeated experiment, and justifying the reasoning. **(Speaking and Listening)**

Suggested next steps

If students say the chance that the next Webflicks user will choose a comedy is $\frac{1}{40}$ in Problem 1, consider:

- Asking them to think about how many users had chosen a comedy compared to the total number of users.

If students say that Webflicks might be more confident, consider:

- Reviewing Activity 1.
- Asking them to identify whether a smaller or larger sample size yields more accurate predictions.

Professional Learning

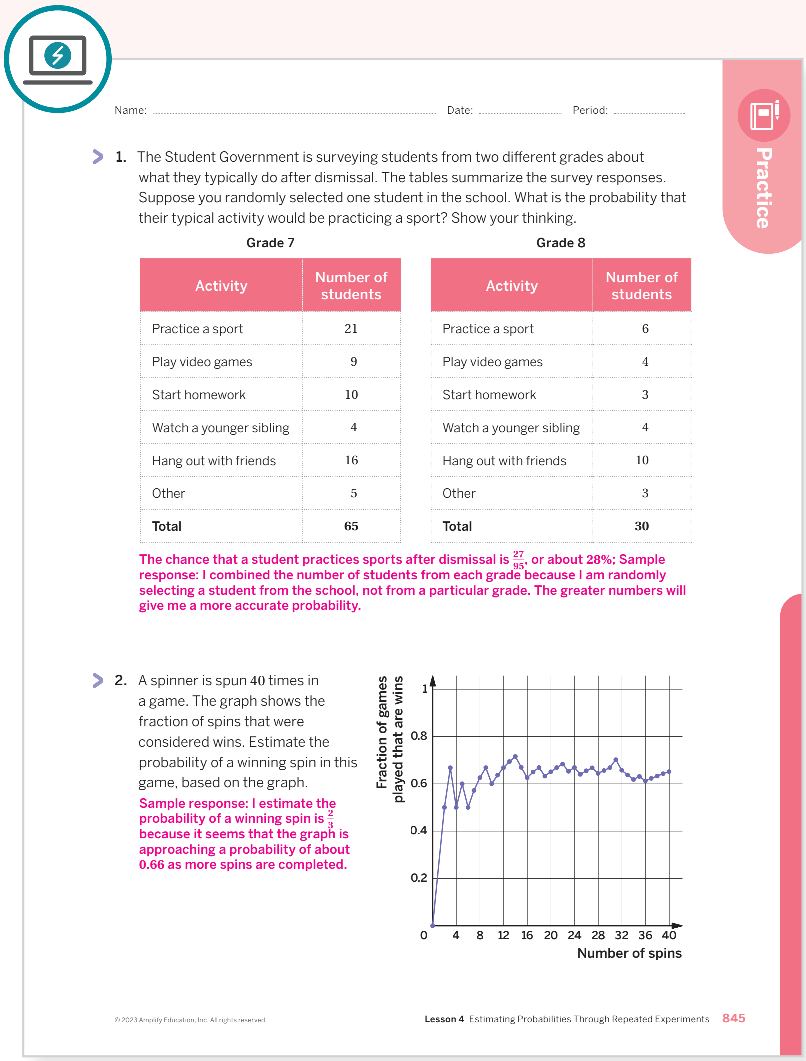
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which students' ideas were you able to highlight during Activity 2?
- In what ways have your students improved at looking for and expressing regularity in repeated reasoning? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

Practice

1. The Student Government is surveying students from two different grades about what they typically do after dismissal. The tables summarize the survey responses. Suppose you randomly selected one student in the school. What is the probability that their typical activity would be practicing a sport? Show your thinking.

Grade 7

Activity	Number of students
Practice a sport	21
Play video games	9
Start homework	10
Watch a younger sibling	4
Hang out with friends	16
Other	5
Total	**65**

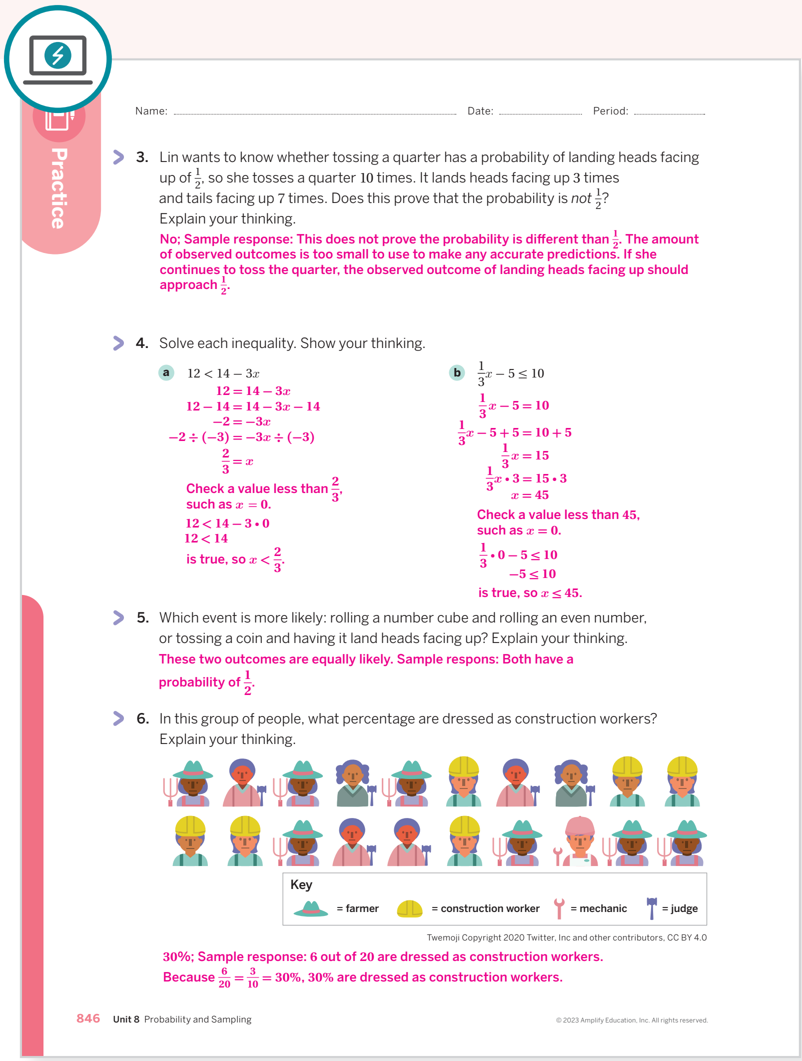
Grade 8

Activity	Number of students
Practice a sport	6
Play video games	4
Start homework	3
Watch a younger sibling	4
Hang out with friends	10
Other	3
Total	**30**

The chance that a student practices sports after dismissal is $\frac{27}{95}$, or about 28%; Sample response: I combined the number of students from each grade because I am randomly selecting a student from the school, not from a particular grade. The greater numbers will give me a more accurate probability.

2. A spinner is spun 40 times in a game. The graph shows the fraction of spins that were considered wins. Estimate the probability of a winning spin in this game, based on the graph.

Sample response: I estimate the probability of a winning spin is $\frac{2}{3}$ because it seems that the graph is approaching a probability of about 0.66 as more spins are completed.



Name: .... Date: .... Period: ....

Practice

3. Lin wants to know whether tossing a quarter has a probability of landing heads facing up of $\frac{1}{2}$, so she tosses a quarter 10 times. It lands heads facing up 3 times and tails facing up 7 times. Does this prove that the probability is *not* $\frac{1}{2}$? Explain your thinking.

No; Sample response: This does not prove the probability is different than $\frac{1}{2}$. The amount of observed outcomes is too small to use to make any accurate predictions. If she continues to toss the quarter, the observed outcome of landing heads facing up should approach $\frac{1}{2}$.

4. Solve each inequality. Show your thinking.

a. $12 < 14 - 3x$

$$12 = 14 - 3x$$
$$12 - 14 = 14 - 3x - 14$$
$$-2 = -3x$$
$$-2 \div (-3) = -3x \div (-3)$$
$$\frac{2}{3} = x$$

Check a value less than $\frac{2}{3}$, such as $x = 0$.

$$12 < 14 - 3 \cdot 0$$
$$12 < 14$$

is true, so $x < \frac{2}{3}$.

b. $\frac{1}{3}x - 5 \leq 10$

$$\frac{1}{3}x - 5 = 10$$
$$\frac{1}{3}x - 5 + 5 = 10 + 5$$
$$\frac{1}{3}x = 15$$
$$\frac{1}{3}x \cdot 3 = 15 \cdot 3$$
$$x = 45$$

Check a value less than 45, such as $x = 0$.

$$\frac{1}{3} \cdot 0 - 5 \leq 10$$
$$-5 \leq 10$$

is true, so $x \leq 45$.

5. Which event is more likely: rolling a number cube and rolling an even number, or tossing a coin and having it land heads facing up? Explain your thinking.

These two outcomes are equally likely. Sample respons: Both have a probability of $\frac{1}{2}$.

6. In this group of people, what percentage are dressed as construction workers? Explain your thinking.

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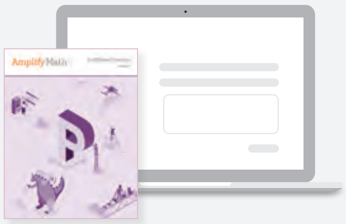
30%; Sample response: 6 out of 20 are dressed as construction workers. Because $\frac{6}{20} = \frac{3}{10} = 30\%$, 30% are dressed as construction workers.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Exit Ticket	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 16	2
	5	Unit 8 Lesson 3	2
Formative	6	Unit 8 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 5**

Code Breaking (Part 1)

Let's use probability to decode encrypted messages.

Focus

Goals

1. **Language Goal:** Describe reasons why the relative frequency from an experiment may not exactly match the expected probability of the event. **(Speaking and Listening, Writing)**
2. Recognize that sometimes the outcomes in a sample space are not equally likely.

Rigor

- Students build **procedural skills** for finding the relative frequencies by comparing the amount of a certain letter to the total letters in a text.
- Students **apply** their understanding of likelihood by comparing relative frequencies to decode a secret message.

Coherence

Today

Students use their knowledge of ratio and relative frequency to conduct a frequency analysis of letters in a message encrypted by a Caesar cipher. They reason about the structure of the words and notice which letter appears at about the same frequency in the encrypted message as in typical English usage. Finally, they compare and contrast the frequency in typical English usage to the frequency in a short speech.

Previously

In Lesson 4, students saw that in the long run, the relative frequency approaches the probability of the chance event, but this will not necessarily be seen using a small sample.

Coming Soon

Students will continue to explore how to use frequency analysis to crack codes. They will try to decode each other's messages using their understanding of the Caesar cipher.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

desmos Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per student
- Activity 2 PDF, pre-cut and assembled, one per pair
- calculators
- markers
- round head fasteners, one per pair

Math Language Development

Review words

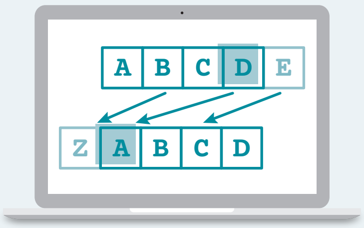
- *chance experiment*
- *event*
- *outcome*
- *probability*
- *relative frequency**
- *sample space*

*Students may confuse the term *relative* with its familial meaning. Be ready to address the similarities and differences between the familial meaning and the mathematical meaning.

Amps | Featured Activity

Activity 2
Digital Decoder Ring

Students use a dynamic decoder to help them quickly shift the letters in the alphabet to decode a message using the Caesar cipher.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may lose confidence if they look at the encoded text in Activity 1 and are not able to decipher it immediately. Let students know that decoding a text can be challenging, and even professional codebreakers sometimes take a really long time. Acknowledge that persistence may pay off.

Modifications to Pacing

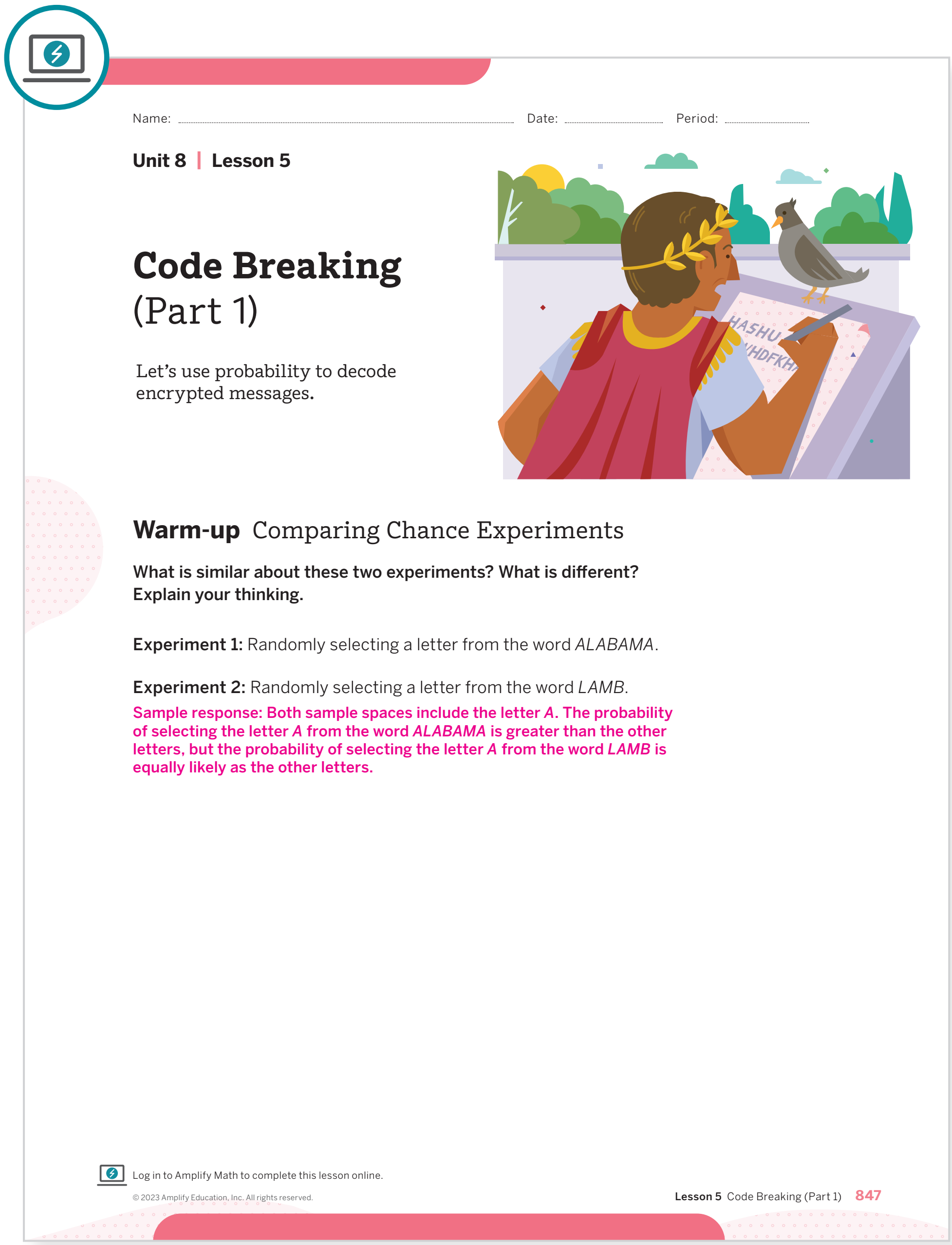
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, students do not need to decode the entire message in Problem 3. Instead, have them decode the first four words.

Independent | 5 min

Warm-up Comparing Chance Experiments

Students compare the relative frequencies of letters in different words, to prepare for finding relative frequencies in larger texts in the upcoming activities.



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Unit 8 | Lesson 5

Code Breaking
(Part 1)

Let's use probability to decode encrypted messages.

Warm-up Comparing Chance Experiments

What is similar about these two experiments? What is different? Explain your thinking.

Experiment 1: Randomly selecting a letter from the word *ALABAMA*.

Experiment 2: Randomly selecting a letter from the word *LAMB*.

Sample response: Both sample spaces include the letter *A*. The probability of selecting the letter *A* from the word *ALABAMA* is greater than the other letters, but the probability of selecting the letter *A* from the word *LAMB* is equally likely as the other letters.

Log in to Amplify Math to complete this lesson online.

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Lesson 5 Code Breaking (Part 1) 847

1 Launch

Remind students that randomly selecting a letter or an object is similar to choosing that letter or object from a bag, without looking.

2 Monitor

Help students get started by asking them how many letters are in each word and what the letters are in each word.

Look for points of confusion:

- **Thinking the experiments are exactly the same because both words only have the letters *A*, *L*, *M*, and *B*.** Ask, "Is the chance of choosing the letter *A* the same for both words?"

Look for productive strategies:

- Noticing that even though the sample spaces are the same, the relative frequency of the letters are different.

3 Connect

Have pairs of students share their responses with each other.

Highlight that the event with the greatest probability is choosing the letter *A* from the word *ALABAMA*.

Ask students if they noticed which letters they use most frequently when writing. Have them share their thinking with a partner before inviting a few to share with the class.

Define the ***relative frequency*** as the ratio of the number of times an an outcome occurs to the total number of trials the activity is performed. It is expressed as a fraction, a decimal, or a percentage of the total number.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students rewrite each word in large, spaced-out letters on a separate sheet of paper. Have them visualize placing each letter from the word *ALABAMA* in a bag and randomly selecting one of the letters. This will help make the abstract idea more concrete.

Power-up

To power up students' ability to determine percentage from analyzing a diagram or phrase, have students complete:

Determine the ratio that compares the number of times the letter *O* occurs in the word *ONOMATOPOEIA* to the total number of letters in the word. Write the ratio as a percentage. $\frac{4}{12}$ or approximately 33%.

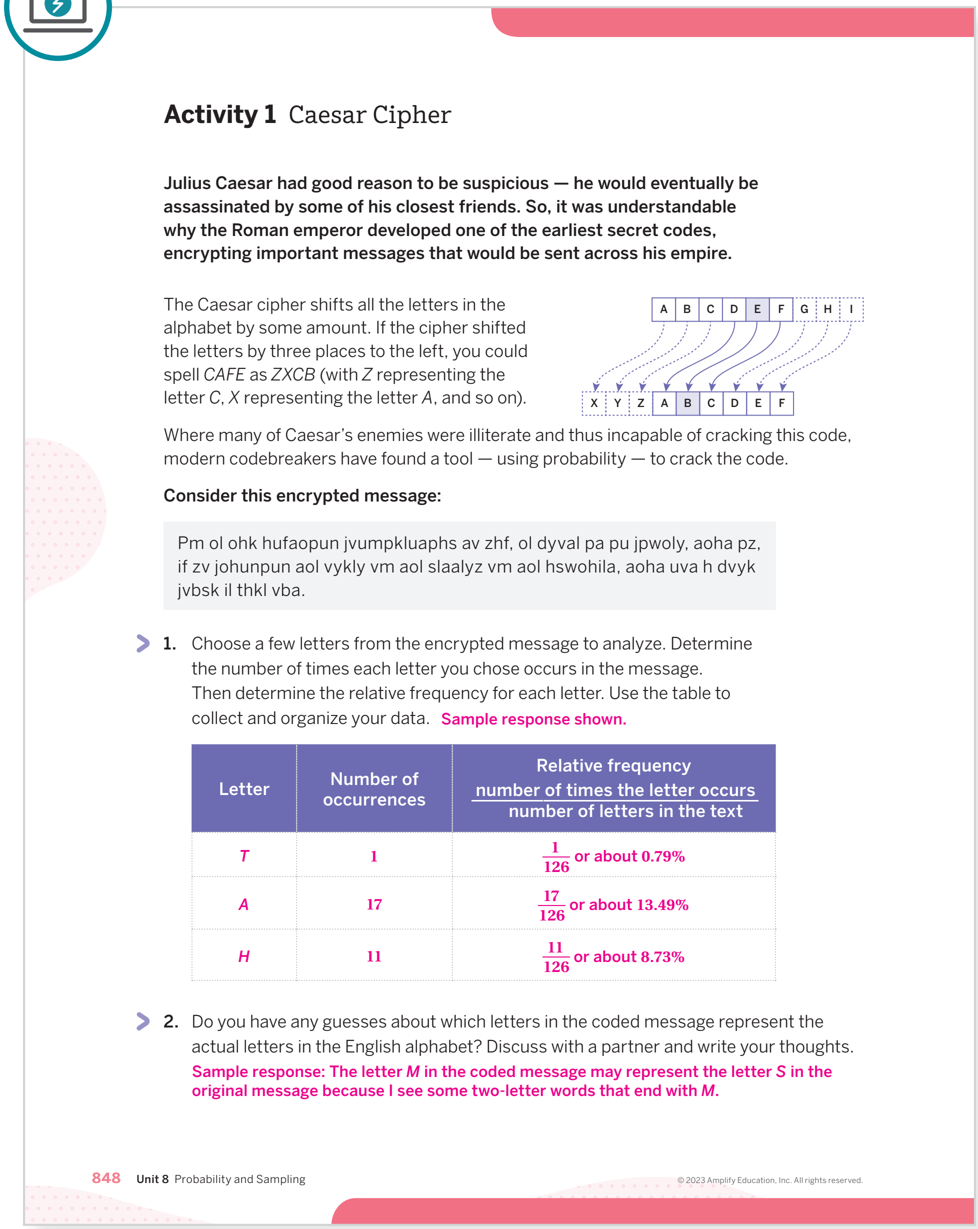
Use: Before the Warm-up

Informed by: Performance on Lesson 4, Practice Problem 6

Pairs | 15 min

Activity 1 Caesar Cipher

Students learn how a Caesar cipher works and perform a frequency analysis for a few letters in a short message to notice that certain letters are used more frequently.



Activity 1 Caesar Cipher

Julius Caesar had good reason to be suspicious — he would eventually be assassinated by some of his closest friends. So, it was understandable why the Roman emperor developed one of the earliest secret codes, encrypting important messages that would be sent across his empire.

The Caesar cipher shifts all the letters in the alphabet by some amount. If the cipher shifted the letters by three places to the left, you could spell *CAFE* as *ZXCB* (with *Z* representing the letter *C*, *X* representing the letter *A*, and so on).

Where many of Caesar's enemies were illiterate and thus incapable of cracking this code, modern codebreakers have found a tool — using probability — to crack the code.

Consider this encrypted message:

Pm ol ohk hufaopun jvumpkluaphs av zhf, ol dyval pa pu jpwoly, aoha pz, if zv johunpun aol vykly vm aol slaalyz vm aol hswohila, aoha uva h dvyk jvbsk il thkl vba.

1. Choose a few letters from the encrypted message to analyze. Determine the number of times each letter you chose occurs in the message. Then determine the relative frequency for each letter. Use the table to collect and organize your data. **Sample response shown.**

Letter	Number of occurrences	Relative frequency $\frac{\text{number of times the letter occurs}}{\text{number of letters in the text}}$
T	1	$\frac{1}{126}$ or about 0.79%
A	17	$\frac{17}{126}$ or about 13.49%
H	11	$\frac{11}{126}$ or about 8.73%

2. Do you have any guesses about which letters in the coded message represent the actual letters in the English alphabet? Discuss with a partner and write your thoughts.
Sample response: The letter *M* in the coded message may represent the letter *S* in the original message because I see some two-letter words that end with *M*.

848 **Unit 8** Probability and Sampling

1 Launch

Read the introduction as a class. Distribute calculators and markers to each pair and assign each pair to find the frequency of a set of three specific letters (be sure to account for the whole alphabet).

2 Monitor

Help students get started by suggesting one student in each pair counts certain letters while the other student counts the total number of letters.

Look for points of confusion:

- **Thinking that the chance of each letter occurring is $\frac{1}{26}$.** Ask students to describe the sample space for randomly selecting a letter from the first three words, and then ask if the frequency of each letter is the same or different.

Look for productive strategies:

- Marking or tallying letters as they are counted.

3 Connect

Note: Distribute the Activity 1 PDF to each student. Students will use this PDF in Activity 2.

Highlight that the table was created to help students first find the frequency of a letter and then to find the relative frequency. Ask a student to describe the difference between the frequency of a letter and its relative frequency.

Display the letter frequency table and complete the rows as students share their work. If student responses do not match the answer key, you may wish to record the answer key now for students, and follow up at another time.

Have pairs of students share their work with the class.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols, Guide Processing and Visualization*

Explain that a *cipher* provides the rules for writing a secret message in code. The message that is written in code is called the *encrypted message*. Illustrate the Caesar cipher by showing how the letter *D* shifts three letters to the left, becoming the letter *A*. Ask students why the letters *A*, *B*, and *C* are shifted to the end of the alphabet.

Accessibility: *Vary Demands to Optimize Challenge*

Provide the total number of letters in the message (126) so that students can concentrate on determining the frequency of their assigned letters.

Math Language Development

MLR6: Three Reads

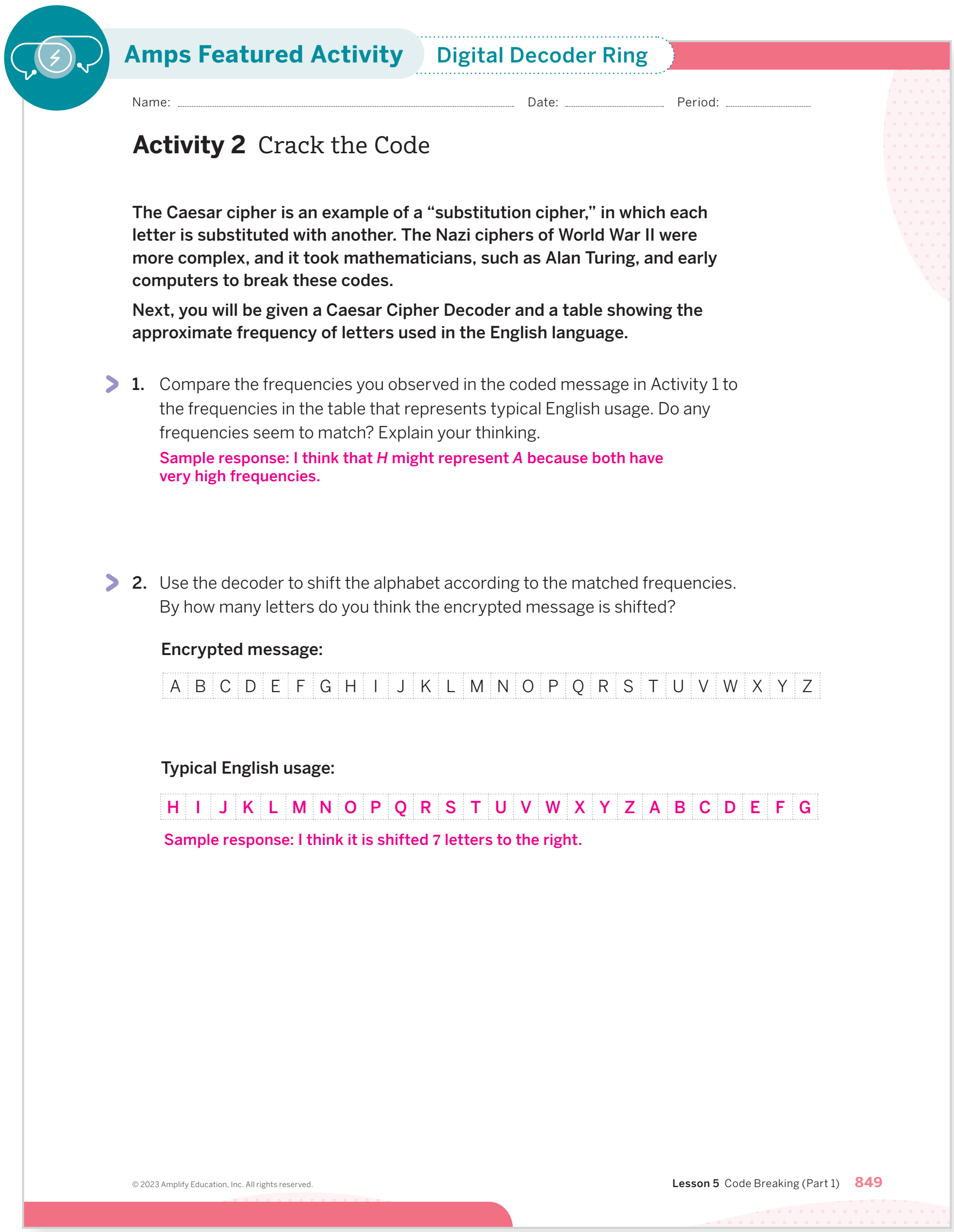
Use this routine to help students make sense of the introductory text:

- **Read 1:** Students should understand that they will be analyzing an encrypted code.
- **Read 2:** Ask students to describe the given relationship between the letters in the word *CAFE* and its encrypted message *ZXCB*.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problem 1.

Pairs | 15 min

Activity 2 Crack the Code

Students analyze and compare the letter frequencies in typical English usage to the frequencies in the coded message from Activity 1, and use this to decode the message.



Amps Featured Activity Digital Decoder Ring

Name: ______ Date: ______ Period: ______

Activity 2 Crack the Code

The Caesar cipher is an example of a “substitution cipher,” in which each letter is substituted with another. The Nazi ciphers of World War II were more complex, and it took mathematicians, such as Alan Turing, and early computers to break these codes.

Next, you will be given a Caesar Cipher Decoder and a table showing the approximate frequency of letters used in the English language.

1. Compare the frequencies you observed in the coded message in Activity 1 to the frequencies in the table that represents typical English usage. Do any frequencies seem to match? Explain your thinking.

Sample response: I think that *H* might represent *A* because both have very high frequencies.

2. Use the decoder to shift the alphabet according to the matched frequencies. By how many letters do you think the encrypted message is shifted?

Encrypted message:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Typical English usage:

H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G

Sample response: I think it is shifted 7 letters to the right.

1 Launch

Distribute the pre-assembled Activity 2 PDF to each student pair. Discuss which letters are the most and least frequently used in the English language.

2 Monitor

Help students get started by having them highlight letters in each table that are high-usage, medium-usage, and low-usage, using different colors. This will help them to see correspondences across the two tables.

Look for points of confusion:

- **Thinking that the frequencies need to match up perfectly.** Have students consider a very simple sentence, such as “Math is amazing.” Then ask them whether the letter frequencies from this small sample size would match those of the typical English usage table.

Look for productive strategies:

- Drawing lines to connect possible letter matches from the typical English usage table to the encrypted message table. This will help students visualize the shift more easily.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital decoder to help them quickly shift the letters in the alphabet to decode the message using the Caesar cipher.

Math Language Development

MLR7: Compare and Connect

During the Connect, as time allows, have pairs of students write sentences or phrases in their primary language and create a cipher to share with their classmates.

English Learners

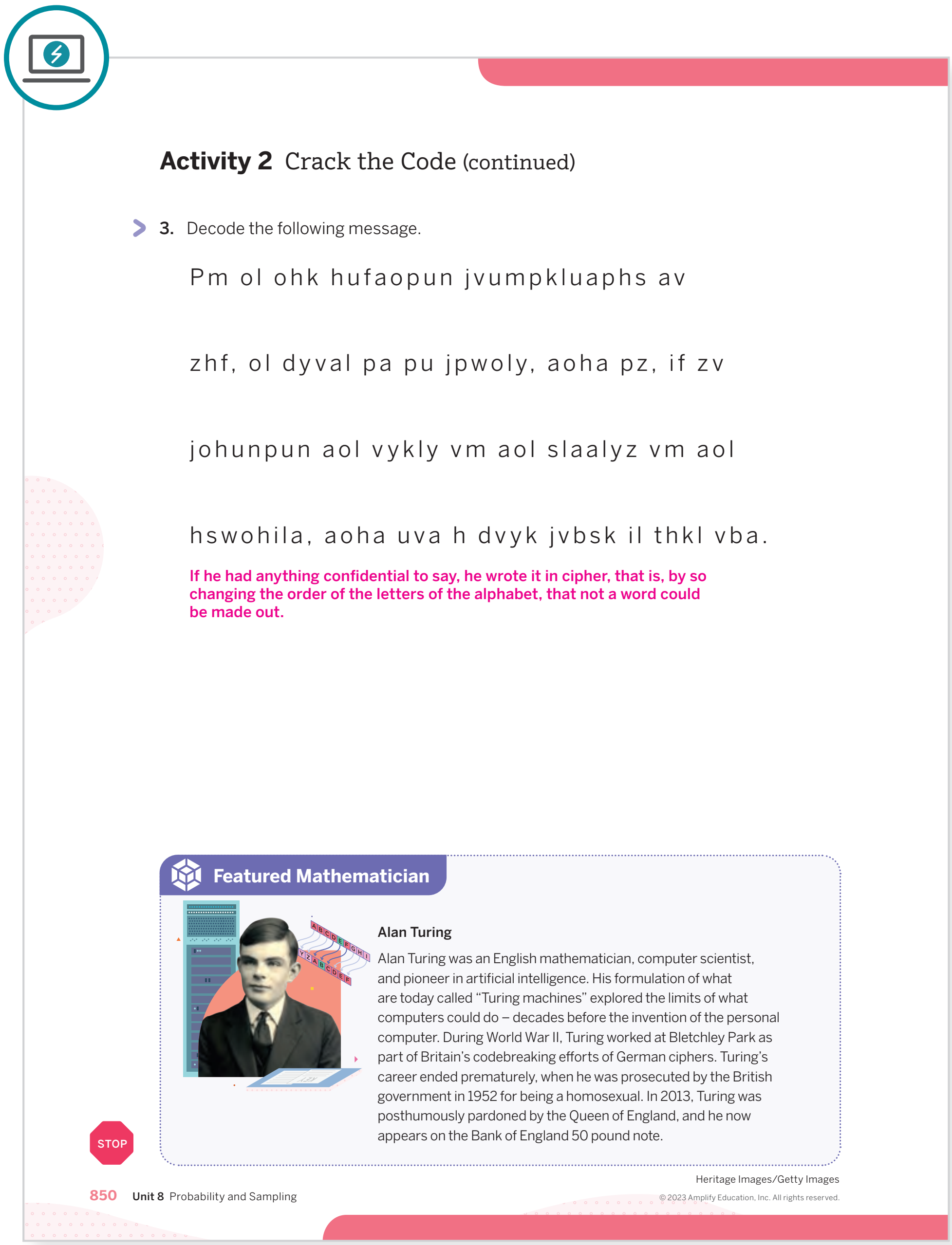
Provide the following Spanish letter frequencies for students whose primary language is Spanish and use the internet to research frequencies for other languages.

A	B	C	D	E	F	G	H	I	J	K	L	M	N
12.5	1.3	4.4	5.1	13.2	0.8	1.2	0.8	6.9	0.5	0.1	5.8	2.6	7.1
Ñ	O	P	Q	R	S	T	U	V	W	X	Y	Z	
0.2	9.0	2.8	0.8	6.6	7.4	4.4	4.0	1.0	0.03	0.2	0.8	0.4	

Pairs | 15 min

Activity 2 Crack the Code (continued)

Students analyze and compare the letter frequencies in typical English usage to the frequencies in the coded message from Activity 1, and use this to decode the message.



Activity 2 Crack the Code (continued)

3. Decode the following message.

Pm ol ohk hufaopun jvumpkluaphs av

zhf, ol dyval pa pu jpwoly, aoha pz, if zv

johunpun aol vykly vm aol slaalyz vm aol

hswohila, aoha uva h dvyk jvbsk il thkl vba.

If he had anything confidential to say, he wrote it in cipher, that is, by so changing the order of the letters of the alphabet, that not a word could be made out.

Featured Mathematician

Alan Turing

Alan Turing was an English mathematician, computer scientist, and pioneer in artificial intelligence. His formulation of what are today called "Turing machines" explored the limits of what computers could do – decades before the invention of the personal computer. During World War II, Turing worked at Bletchley Park as part of Britain's codebreaking efforts of German ciphers. Turing's career ended prematurely, when he was prosecuted by the British government in 1952 for being a homosexual. In 2013, Turing was posthumously pardoned by the Queen of England, and he now appears on the Bank of England 50 pound note.

Heritage Images/Getty Images

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3 Connect

Display the completed tables from the Activity 1 PDF to the class.

Have individual students share their process for matching the letters from one table to the other.

Ask, "Why is it that the frequencies do not match perfectly?"

Highlight that a short message will not necessarily match the letter frequency of typical English for a few reasons:

- The message might not contain all of the letters of the alphabet.
- The message might contain atypical words.
- The message might not be long enough. The actual frequencies should approach the expected frequency as the message grows longer.

Featured Mathematician

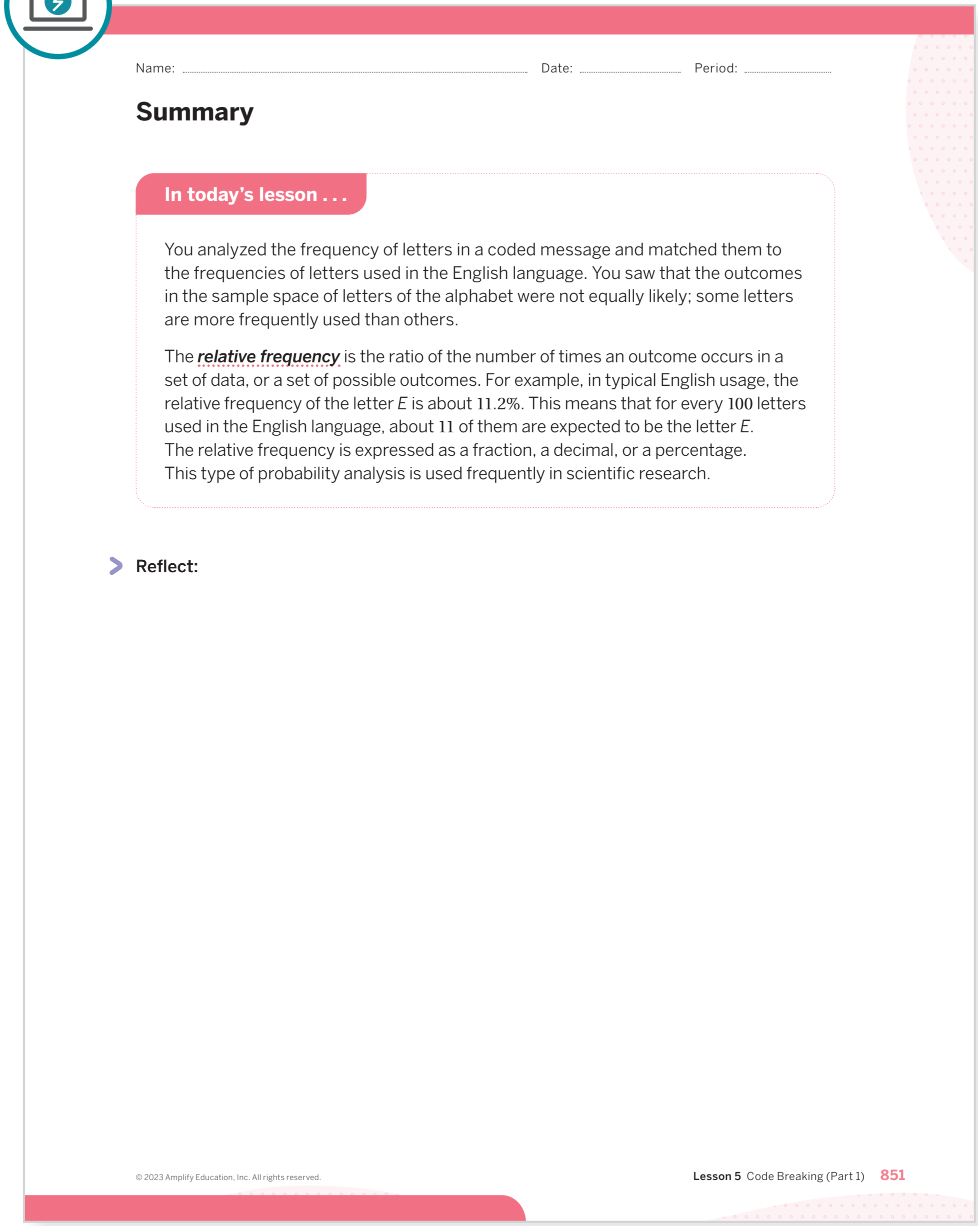
Alan Turing

Have students read about Featured Mathematician Alan Turing, a mathematician and computer scientist from England who used computers to decipher German communications during WWII.

Whole Class | 5 min

Summary

Review and synthesize the reasons why the relative frequency from an experiment may vary from the expected frequency.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You analyzed the frequency of letters in a coded message and matched them to the frequencies of letters used in the English language. You saw that the outcomes in the sample space of letters of the alphabet were not equally likely; some letters are more frequently used than others.

The ***relative frequency*** is the ratio of the number of times an outcome occurs in a set of data, or a set of possible outcomes. For example, in typical English usage, the relative frequency of the letter *E* is about 11.2%. This means that for every 100 letters used in the English language, about 11 of them are expected to be the letter *E*. The relative frequency is expressed as a fraction, a decimal, or a percentage. This type of probability analysis is used frequently in scientific research.

Reflect:

Synthesize

Highlight that finding the frequency of a letter in a text can be considered an experiment.

Ask, "Suppose a chance experiment is repeated many times, but the fraction of outcomes for which a certain event occurs does not match the expected probability of the event. What are some reasons why this may happen?"
Sample responses:

- The experiment may not have been repeated enough times.
- The experiment was not conducted properly.
- Some variance is expected between the estimated probability and the expected probability.

Note: Prior to the next lesson, have students think of a quote or positive message they would like to send to a random classmate. Provide students the opportunity to prepare this quote or message before the next lesson begins.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

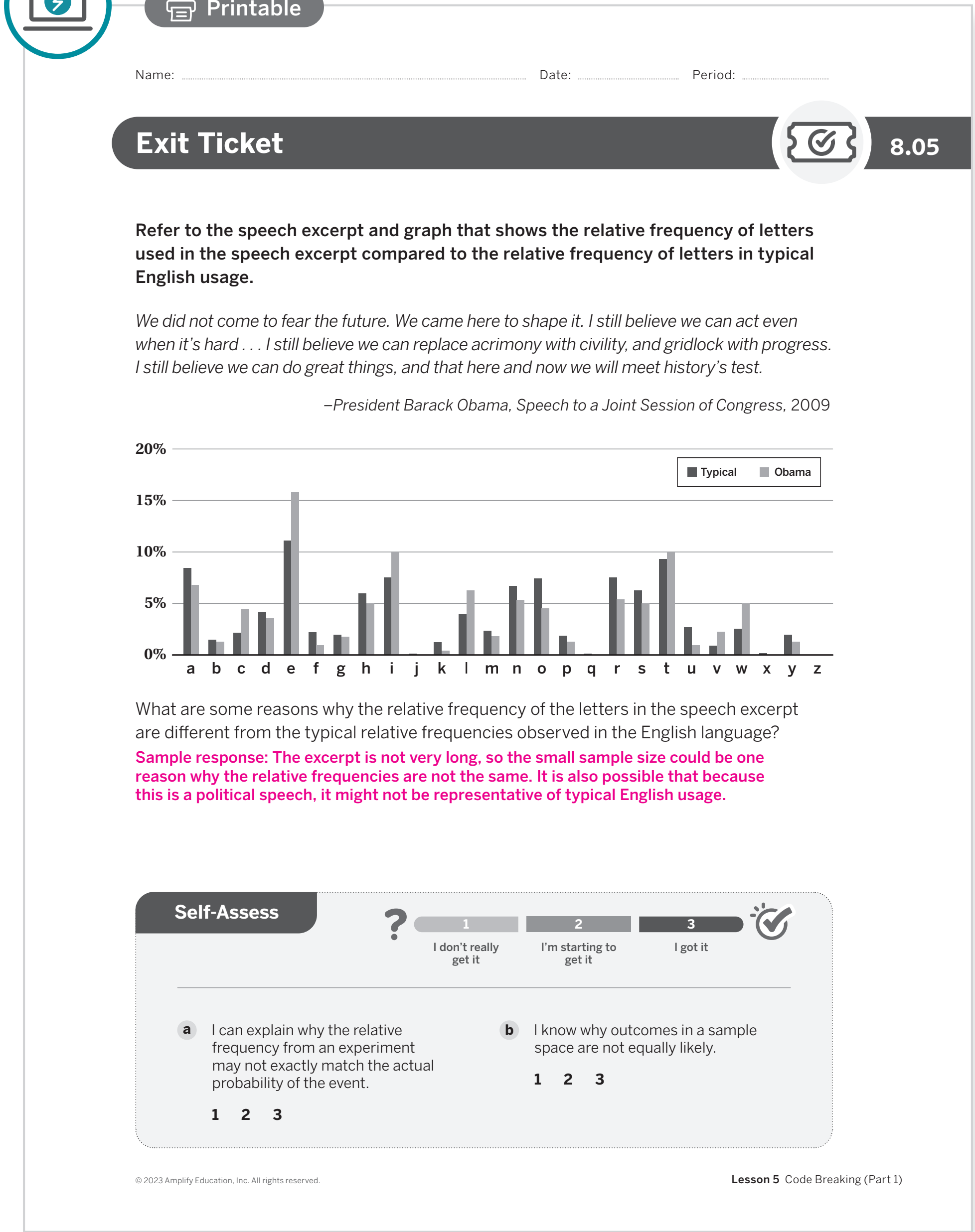
- "What strategies did you use to help you decode the secret message?"
- "Which strategy was the most successful? Which strategy was the least successful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing the discrepancy of letter frequency in a new text to typical English usage.

Printable



Name: Date: Period:

Exit Ticket 8.05

Refer to the speech excerpt and graph that shows the relative frequency of letters used in the speech excerpt compared to the relative frequency of letters in typical English usage.

We did not come to fear the future. We came here to shape it. I still believe we can act even when it's hard . . . I still believe we can replace acrimony with civility, and gridlock with progress. I still believe we can do great things, and that here and now we will meet history's test.

–President Barack Obama, Speech to a Joint Session of Congress, 2009

What are some reasons why the relative frequency of the letters in the speech excerpt are different from the typical relative frequencies observed in the English language?

Sample response: The excerpt is not very long, so the small sample size could be one reason why the relative frequencies are not the same. It is also possible that because this is a political speech, it might not be representative of typical English usage.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain why the relative frequency from an experiment may not exactly match the actual probability of the event.

1 2 3

b I know why outcomes in a sample space are not equally likely.

1 2 3

Lesson 5 Code Breaking (Part 1)

Success looks like . . .

- **Language Goal:** Describing reasons why the relative frequency from an experiment may not exactly match the expected probability of the event. **(Speaking and Listening, Writing)**
    - » Explaining why the relative frequency of the letters in the speech excerpt are different from the typical relative frequencies found in the English language.
- **Goal:** Recognizing that sometimes the outcomes in a sample space are not equally likely.

Suggested next steps

If students struggle to identify valid reasons for the differences among the relative frequencies, consider:

- This is just a starting point for further exploration that will occur later in this unit. Allow students to cite any possible reasons they can think of to explain the discrepancy among the relative frequencies, and accept all reasonable responses.

If students do not mention that the excerpt is a relatively small sample, consider:

- Reviewing Lesson 5, Activity 1.

If students do not mention that this excerpt might not be representative of typical English usage because it is a political speech, consider:

- Making a note to revisit this text after Lesson 13, where students learn more about representative samples.

Professional Learning

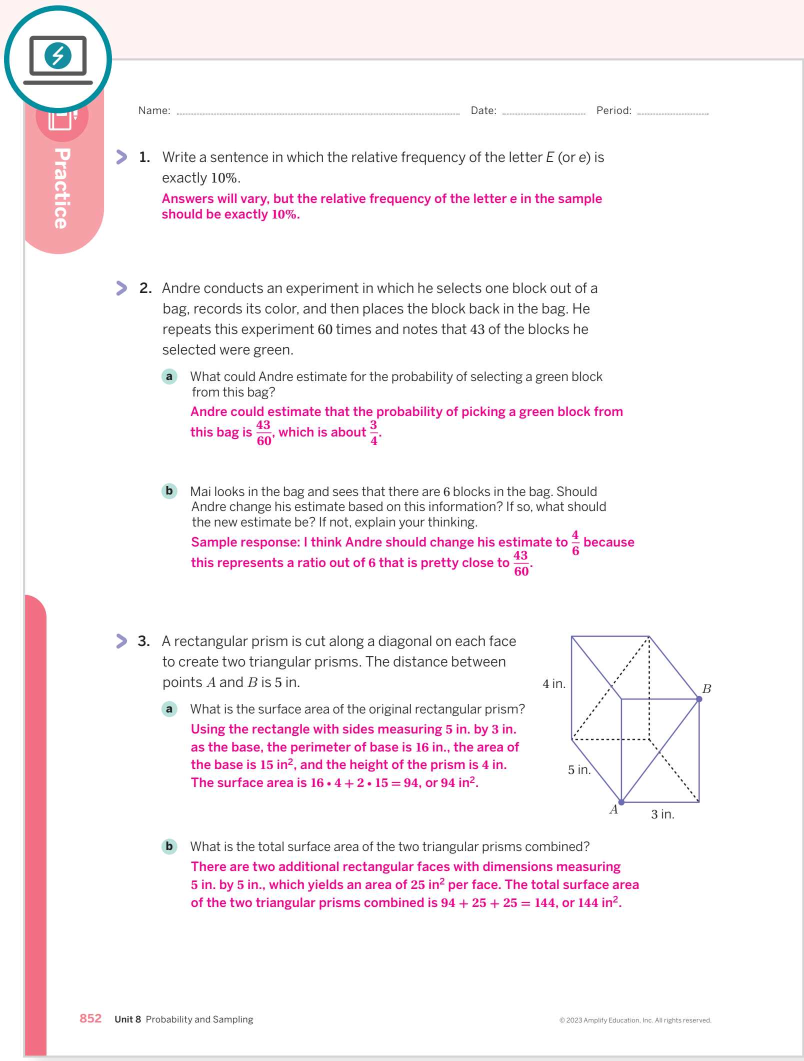
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did or didn't have their ideas seen and heard today?
- What surprised you as your students worked on Activity 1? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Write a sentence in which the relative frequency of the letter *E* (or *e*) is exactly 10%.

   Answers will vary, but the relative frequency of the letter e in the sample should be exactly 10%.

2. Andre conducts an experiment in which he selects one block out of a bag, records its color, and then places the block back in the bag. He repeats this experiment 60 times and notes that 43 of the blocks he selected were green.

   a. What could Andre estimate for the probability of selecting a green block from this bag?

   Andre could estimate that the probability of picking a green block from this bag is $\frac{43}{60}$, which is about $\frac{3}{4}$.

   b. Mai looks in the bag and sees that there are 6 blocks in the bag. Should Andre change his estimate based on this information? If so, what should the new estimate be? If not, explain your thinking.

   Sample response: I think Andre should change his estimate to $\frac{4}{6}$ because this represents a ratio out of 6 that is pretty close to $\frac{43}{60}$.

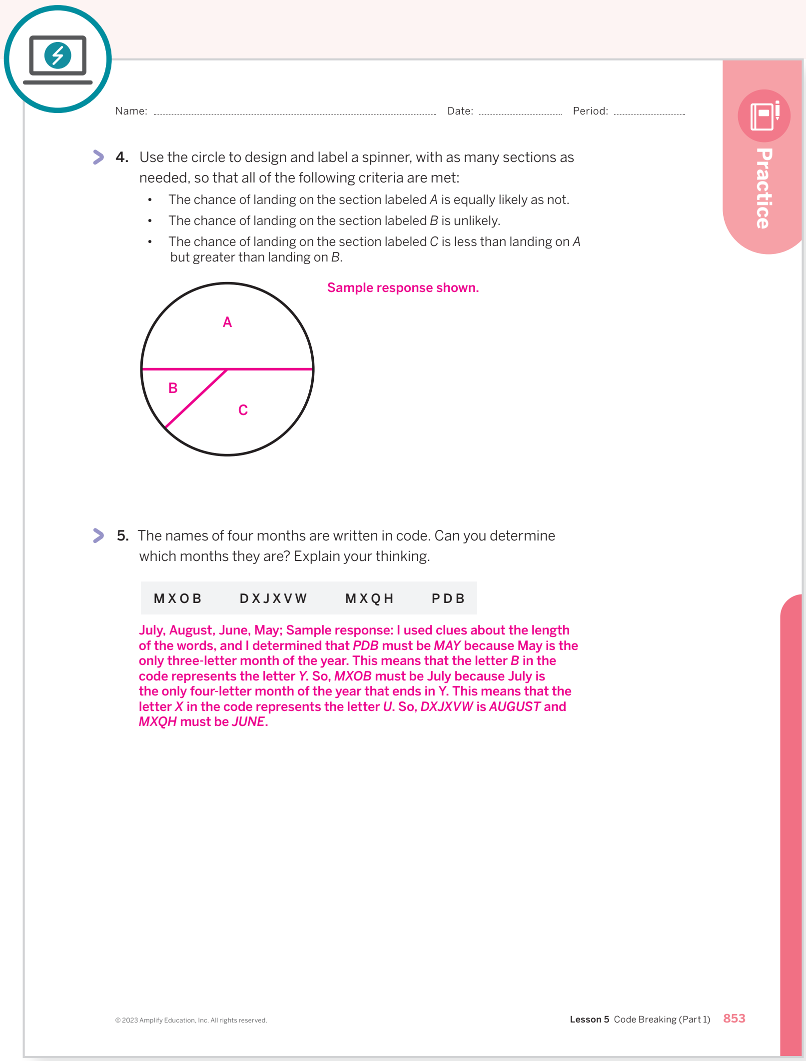
3. A rectangular prism is cut along a diagonal on each face to create two triangular prisms. The distance between points A and B is 5 in.

   a. What is the surface area of the original rectangular prism?

   Using the rectangle with sides measuring 5 in. by 3 in. as the base, the perimeter of base is 16 in., the area of the base is 15 in^2, and the height of the prism is 4 in. The surface area is $16 \cdot 4 + 2 \cdot 15 = 94$, or 94 in^2.

   b. What is the total surface area of the two triangular prisms combined?

   There are two additional rectangular faces with dimensions measuring 5 in. by 5 in., which yields an area of 25 in^2 per face. The total surface area of the two triangular prisms combined is $94 + 25 + 25 = 144$, or 144 in^2.



Name: ______ Date: ______ Period: ______

Practice

4. Use the circle to design and label a spinner, with as many sections as needed, so that all of the following criteria are met:
   - The chance of landing on the section labeled *A* is equally likely as not.
   - The chance of landing on the section labeled *B* is unlikely.
   - The chance of landing on the section labeled *C* is less than landing on *A* but greater than landing on *B*.

   Sample response shown.

5. The names of four months are written in code. Can you determine which months they are? Explain your thinking.

   MXOB DXJXVW MXQH PDB

   July, August, June, May; Sample response: I used clues about the length of the words, and I determined that *PDB* must be *MAY* because May is the only three-letter month of the year. This means that the letter *B* in the code represents the letter *Y*. So, *MXOB* must be July because July is the only four-letter month of the year that ends in Y. This means that the letter *X* in the code represents the letter *U*. So, *DXJXVW* is *AUGUST* and *MXQH* must be *JUNE*.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	3
	2	Activity 2	2
Spiral	3	Unit 7 Lesson 16	2
	4	Unit 8 Lesson 2	2
Formative ⚡	5	Unit 8 Lesson 6	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 6**

Code Breaking (Part 2)

Let's use probability to decode encrypted messages.

Focus

Goal

1. Perform a frequency analysis, and use the results to decipher a coded text.

Coherence

Today

Students continue their work with frequency analysis of coded messages. They create their own message to be sent to a classmate and then look for and make use of patterns while comparing frequencies in two tables.

Previously

In Lessons 1–4, students were introduced to the idea of how analyzing the probability of an event can help shed light on future outcomes.

Coming Soon

Students will learn about multi-step experiments and how to find the probability of multi-step events.

Rigor

- Students strengthen their **fluency** reasoning about fraction and percent equivalents.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Independent	Independent	Whole Class	Independent

desmos Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, *Caesar Cipher Decoder*, one per student (also used in Activity 1)
- Activity 1 PDF, *Sample Text* (optional)
- lined paper, one per student

Math Language Development

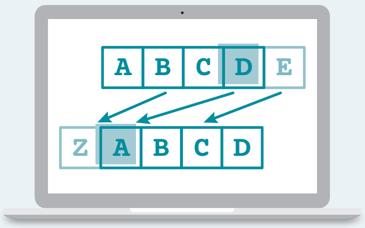
Review words

- *chance experiment*
- *event*
- *outcome*
- *probability*
- *relative frequency*
- *sample space*

Amps Featured Activity

Activity 1
Digital Decoder Ring

Students use a dynamic decoder to help them quickly shift the letters in the alphabet to decode a message using the Caesar cipher.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may lose track while counting letters and feel disorganized. Ask if there are any tools or strategies they can think of that might help them maintain their place while counting.

Modifications to Pacing

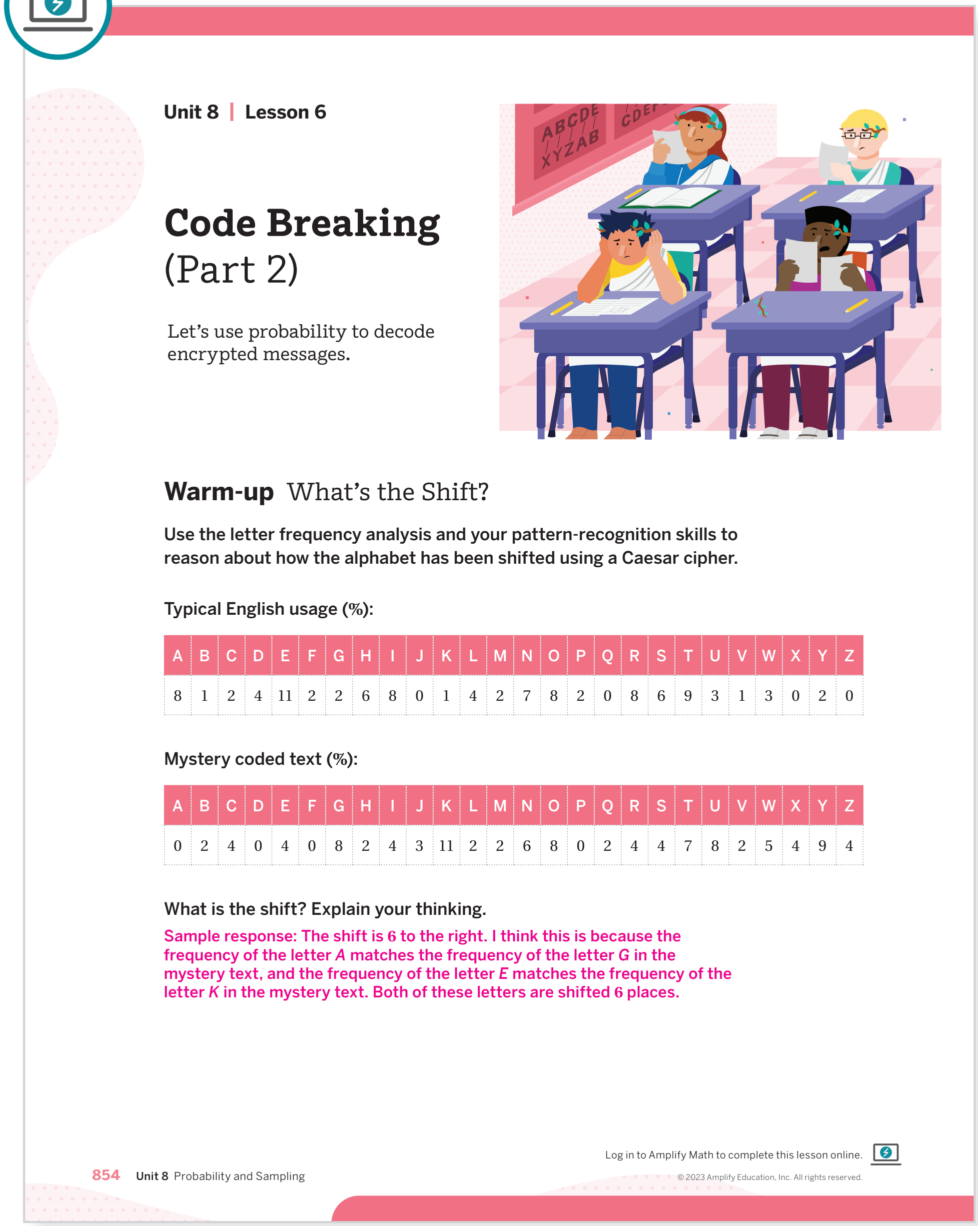
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students keep their message to less than 10 words.
- In **Activity 2**, Problem 3 may be omitted.

Pairs | 5 min

Warm-up What's the Shift?

Students compare two sets of frequency data and notice patterns to practice finding the shift of a Caesar cipher.



Unit 8 | Lesson 6

Code Breaking (Part 2)

Let's use probability to decode encrypted messages.

Warm-up What's the Shift?

Use the letter frequency analysis and your pattern-recognition skills to reason about how the alphabet has been shifted using a Caesar cipher.

Typical English usage (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
8	1	2	4	11	2	2	6	8	0	1	4	2	7	8	2	0	8	6	9	3	1	3	0	2	0

Mystery coded text (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	2	4	0	4	0	8	2	4	3	11	2	2	6	8	0	2	4	4	7	8	2	5	4	9	4

What is the shift? Explain your thinking.

Sample response: The shift is 6 to the right. I think this is because the frequency of the letter *A* matches the frequency of the letter *G* in the mystery text, and the frequency of the letter *E* matches the frequency of the letter *K* in the mystery text. Both of these letters are shifted 6 places.

Log in to Amplify Math to complete this lesson online.

854 Unit 8 Probability and Sampling

1 Launch

Activate students' prior knowledge by asking for a student, or students, to explain how they used the Caesar Cipher in the previous lesson.

Remind students that a letter frequency analysis means comparing the frequencies of one set of letters to another.

2 Monitor

Help students get started by reminding them that data from an experiment will often not exactly match the expected probability.

Look for points of confusion:

- **Thinking that determining any two values that match is the key to the shift.** Ask students to also compare the values to the left and the right to make sure those also match. Provide access to the Warm-up PDF, *Caesar Cipher Decoder* to allow students to better compare these values.

Look for productive strategies:

- Finding the absolute value of differences to find the most likely shift.

3 Connect

Have individual students share their strategy for reasoning about the shift.

Highlight that it may take checking several letters, in various places, to confirm the letter shift.

Ask, "Why is it that the coded text does not match the typical English letter frequency?" It does not match because the sample is not representative of the entire English language.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students circle or highlight the most frequently occurring letter in the Typical English usage table. Then ask, "Which letter in the mystery coded text table has the greatest percentage? What might this tell you?"

Power-up

To power up students' ability to use patterns to decipher phrases, have students complete:

The names of four days of the week are written in code. Can you determine which days they are?

ACVLIG UWVLIG BCMALIG

a Consider the days of the week, what are the three letters that they all end in? DAY

b Using your answer from Part A and your Caesar Cipher, what is the shift in letters. 8

c What are the three days of the week? Sunday, Monday, and Tuesday.

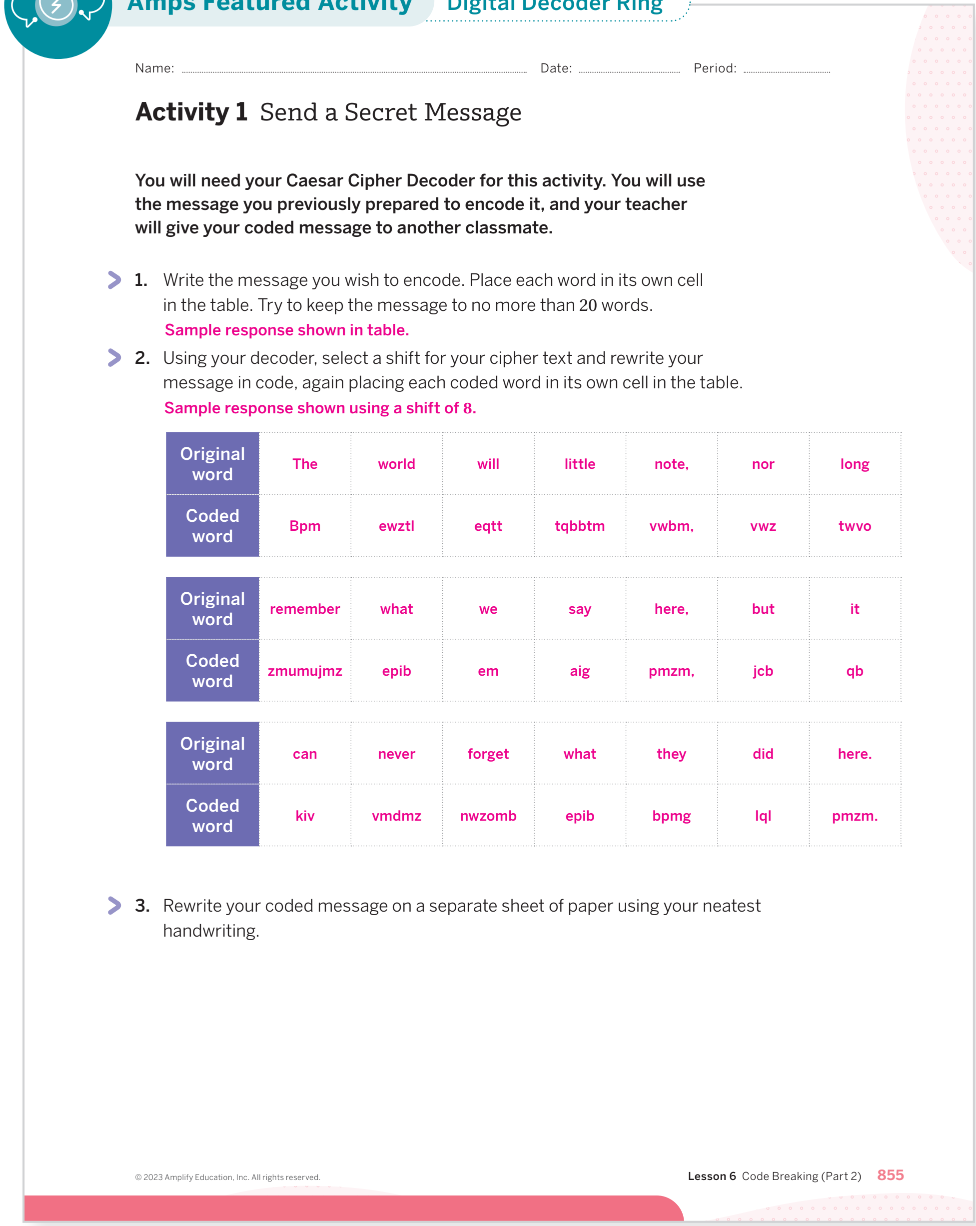
Use: Before Activity 2
Informed by: Performance on Lesson 5, Practice Problem 5

Independent | 15 min

Activity 1 Send a Secret Message

Students select and encode a secret message that will be used in the next activity for other students to decode.

Amps Featured Activity Digital Decoder Ring



Name: ______ Date: ______ Period: ______

Activity 1 Send a Secret Message

You will need your Caesar Cipher Decoder for this activity. You will use the message you previously prepared to encode it, and your teacher will give your coded message to another classmate.

1. Write the message you wish to encode. Place each word in its own cell in the table. Try to keep the message to no more than 20 words.
   Sample response shown in table.
2. Using your decoder, select a shift for your cipher text and rewrite your message in code, again placing each coded word in its own cell in the table.
   Sample response shown using a shift of 8.

Original word	The	world	will	little	note,	nor	long
Coded word	Bpm	ewztl	eqtt	tqbbtm	vwbm,	vwz	twvo
Original word	remember	what	we	say	here,	but	it
Coded word	zmumujmz	epib	em	aig	pmzm,	jcb	qb
Original word	can	never	forget	what	they	did	here.
Coded word	kiv	vmdmz	nwzomb	epib	bpmg	lql	pmzm.

3. Rewrite your coded message on a separate sheet of paper using your neatest handwriting.

1 Launch

Have students take out the quote or positive message they prepared ahead of time. Distribute lined paper and the pre-assembled decoder from the Warm-up PDF, *Caesar Cipher Decoder*. For students who have not prepared a message or would like to choose a message from a text, distribute the sample text from the Activity 1 PDF, *Sample Text*.

2 Monitor

Help students get started by suggesting they encode their text one letter at a time, e.g., first encoding the letter *a* throughout the message.

Look for points of confusion:

- **Sending personal messages to specific people in their class.** Remind them that the message will be delivered to a random classmate, so it is important to keep the message general and positive.

3 Connect

Highlight that some students may have noticed that they used certain letters that are not used as frequently.

Ask, "What may be a cause of a text varying a lot from typical English letter frequencies?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital decoder to help them quickly shift the letters in the alphabet to decode the message using the Caesar cipher.

Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Tools*

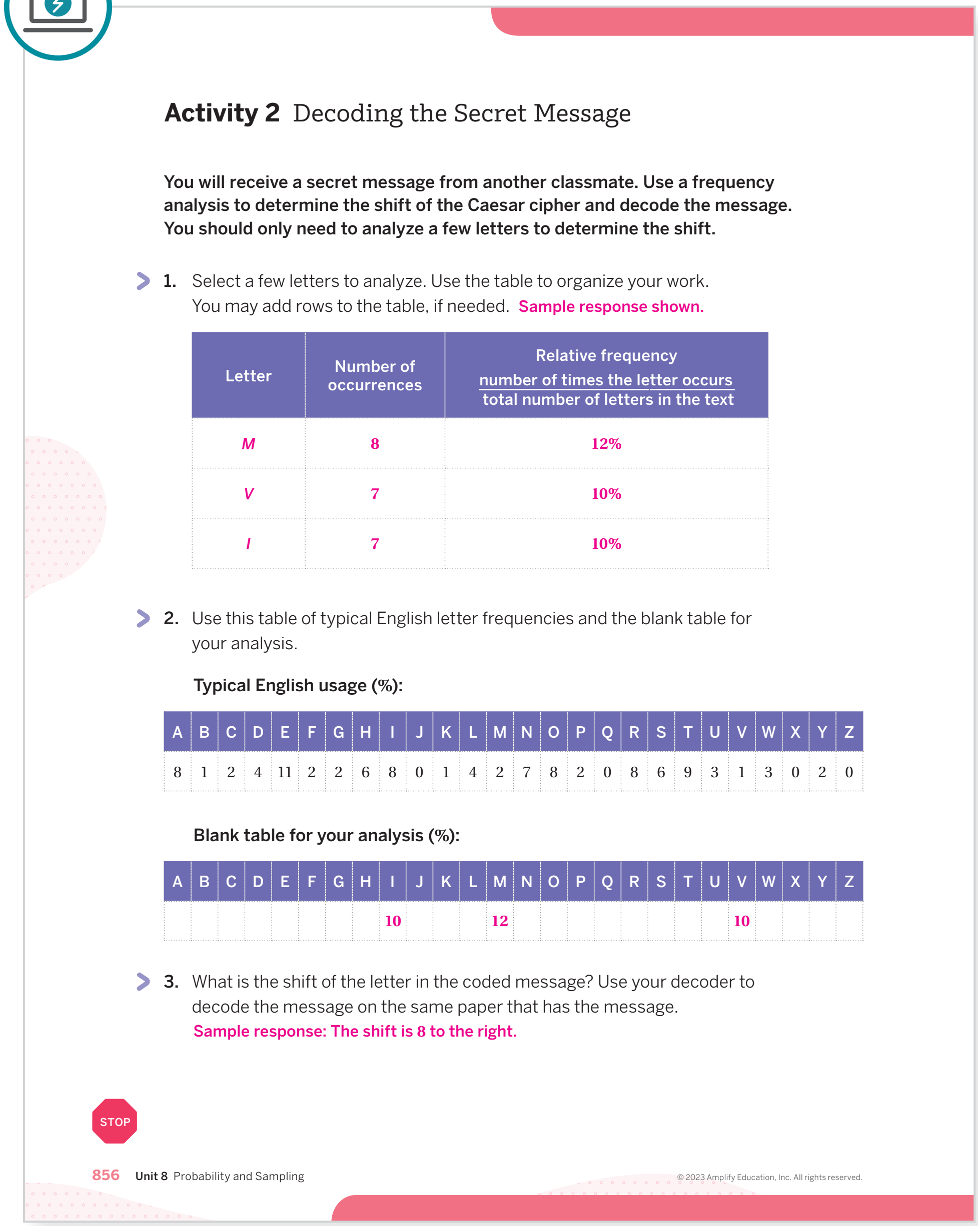
Consider one or more of these additional supports:

- Suggest that students use a shorter message for their original message, limiting them to about 10 words.
- Display sample messages from which to choose a message, instead of having them create their own.
- Provide access to graph paper and suggest they use it to help organize and align the letters in their original and coded messages.

Independent | 15 min

Activity 2 Decoding the Secret Message

Students receive a coded message from a classmate and use a frequency analysis on a sample letter to determine the shift of the Caesar cipher used.



Activity 2 Decoding the Secret Message

You will receive a secret message from another classmate. Use a frequency analysis to determine the shift of the Caesar cipher and decode the message. You should only need to analyze a few letters to determine the shift.

1. Select a few letters to analyze. Use the table to organize your work. You may add rows to the table, if needed. **Sample response shown.**

Letter	Number of occurrences	Relative frequency $\frac{\text{number of times the letter occurs}}{\text{total number of letters in the text}}$
M	8	12%
V	7	10%
I	7	10%

2. Use this table of typical English letter frequencies and the blank table for your analysis.

Typical English usage (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
8	1	2	4	11	2	2	6	8	0	1	4	2	7	8	2	0	8	6	9	3	1	3	0	2	0

Blank table for your analysis (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
								10				12									10				

3. What is the shift of the letter in the coded message? Use your decoder to decode the message on the same paper that has the message.
Sample response: The shift is 8 to the right.

STOP

856 **Unit 8** Probability and Sampling

1 Launch

Collect and re-distribute the coded messages randomly from Activity 1.

2 Monitor

Help students get started by directing them to look over the text they have received and notice any clues or patterns that may help in their analysis.

Look for points of confusion:

- **Assuming that any frequencies that match exactly are indicative of the shift.** Make sure students are comparing the amount of the shift they are noticing for multiple letters to see that they are in alignment. Remind them that a Caesar cipher uses the same shift value for all letters.

Look for productive strategies:

- Testing the possible decoded letters as they go. If students are noticing certain patterns or letter combinations that do not make sense, they should adjust their thinking.

3 Connect

Highlight that the class used the tool of probability to uncover hidden patterns.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital decoder to help them quickly shift the letters in the alphabet to decode the message using the Caesar cipher.

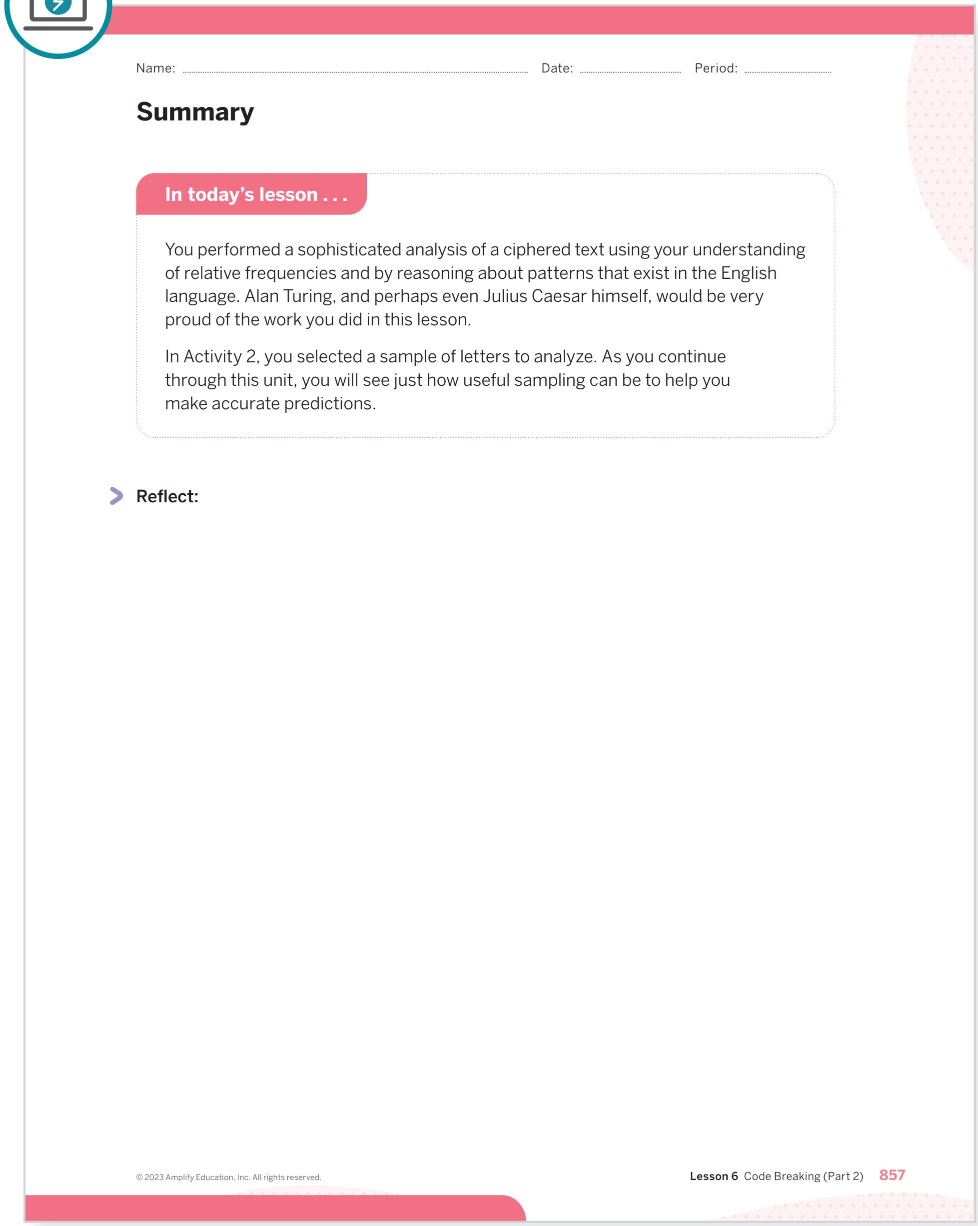
Accessibility: *Optimize Access to Tools*

Provide access to graph paper and suggest they use it to help organize and align the letters in their original and coded messages.

Whole Class | 5 min

Summary

Review and synthesize how a frequency analysis can help crack a code.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You performed a sophisticated analysis of a ciphered text using your understanding of relative frequencies and by reasoning about patterns that exist in the English language. Alan Turing, and perhaps even Julius Caesar himself, would be very proud of the work you did in this lesson.

In Activity 2, you selected a sample of letters to analyze. As you continue through this unit, you will see just how useful sampling can be to help you make accurate predictions.

Reflect:

Synthesize

Display an example of a decoded message, including the coded message from Activity 2.

Ask, "Why is a frequency analysis a good tool to help decipher a coded message?" A frequency analysis reveals information about patterns that exist. It helps us to identify a letter by a property that cannot be hidden by a cipher.

Highlight that the frequency analysis used to decipher the messages are a great example of how students can make inferences about a larger set of information from a smaller set of related information. This will be an important aspect of the statistics work in the next part of the unit.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

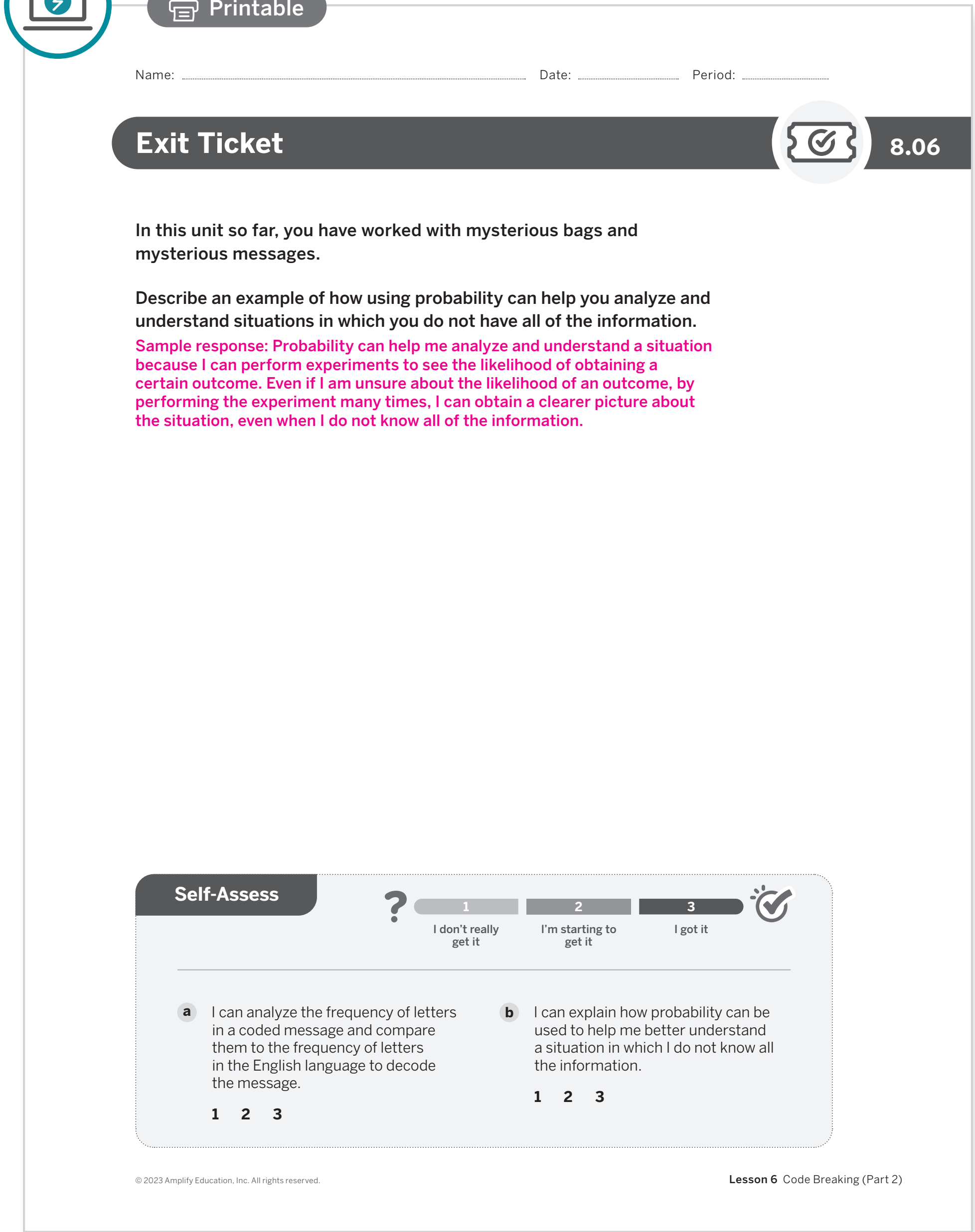
- "Did you reuse any of your strategies from the previous lesson on code breaking? If so, which one(s)"
- "Did you make any changes to your strategies from the previous lesson on code breaking? If so, how?"

Exit Ticket

Independent | 5 min

Students demonstrate their understanding of how using probability can help gain information and make predictions that aren't readily observable.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

8.06

In this unit so far, you have worked with mysterious bags and mysterious messages.

Describe an example of how using probability can help you analyze and understand situations in which you do not have all of the information.

Sample response: Probability can help me analyze and understand a situation because I can perform experiments to see the likelihood of obtaining a certain outcome. Even if I am unsure about the likelihood of an outcome, by performing the experiment many times, I can obtain a clearer picture about the situation, even when I do not know all of the information.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can analyze the frequency of letters in a coded message and compare them to the frequency of letters in the English language to decode the message.

1 2 3

b I can explain how probability can be used to help me better understand a situation in which I do not know all the information.

1 2 3

Lesson 6 Code Breaking (Part 2)

Success looks like . . .

- **Goal:** Performing a frequency analysis, and using the results to decipher a coded text.

Suggested next steps

If students cannot recall the ways in which they used probability in the first lessons of the unit, consider:

- Having them turn back in their textbook or look over previous digital lessons.

Professional Learning

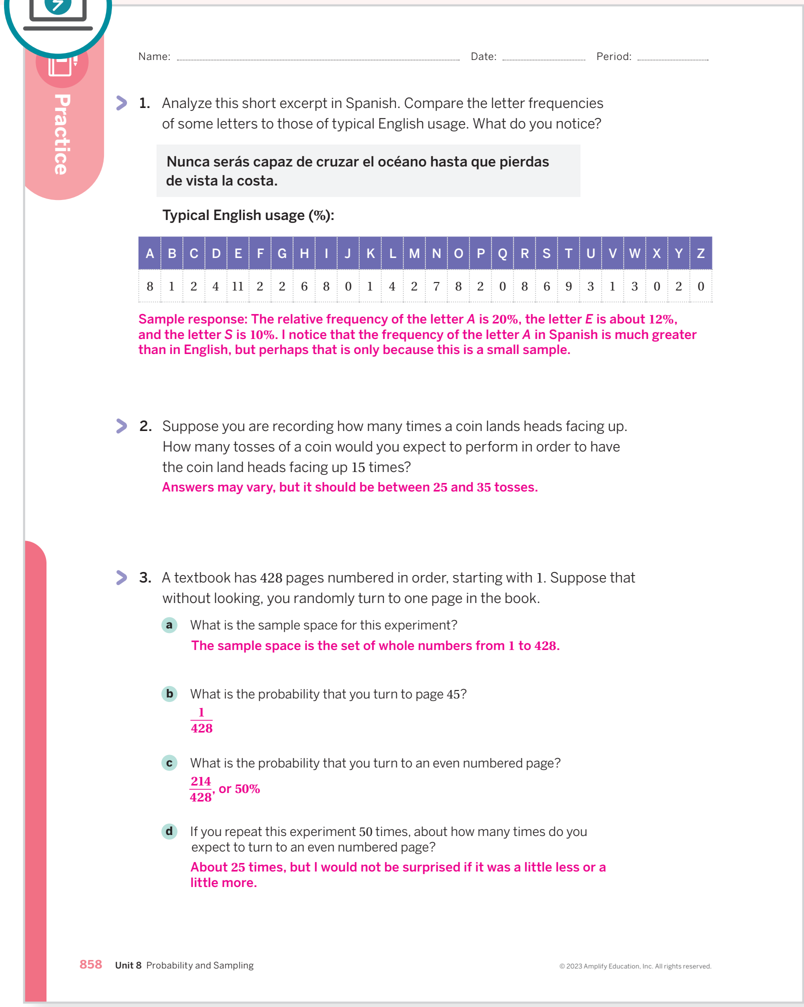
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What challenges did students encounter as they worked on Activity 1? How did they work through them?
- In what ways did Activity 2 go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Analyze this short excerpt in Spanish. Compare the letter frequencies of some letters to those of typical English usage. What do you notice?

   Nunca serás capaz de cruzar el océano hasta que pierdas de vista la costa.

   Typical English usage (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
8	1	2	4	11	2	2	6	8	0	1	4	2	7	8	2	0	8	6	9	3	1	3	0	2	0

Sample response: The relative frequency of the letter *A* is 20%, the letter *E* is about 12%, and the letter *S* is 10%. I notice that the frequency of the letter *A* in Spanish is much greater than in English, but perhaps that is only because this is a small sample.

2. Suppose you are recording how many times a coin lands heads facing up. How many tosses of a coin would you expect to perform in order to have the coin land heads facing up 15 times?
   Answers may vary, but it should be between 25 and 35 tosses.

3. A textbook has 428 pages numbered in order, starting with 1. Suppose that without looking, you randomly turn to one page in the book.
   - a What is the sample space for this experiment?
     The sample space is the set of whole numbers from 1 to 428.
   - b What is the probability that you turn to page 45?
     $\frac{1}{428}$
   - c What is the probability that you turn to an even numbered page?
     $\frac{214}{428}$, **or 50%**
   - d If you repeat this experiment 50 times, about how many times do you expect to turn to an even numbered page?
     About 25 times, but I would not be surprised if it was a little less or a little more.

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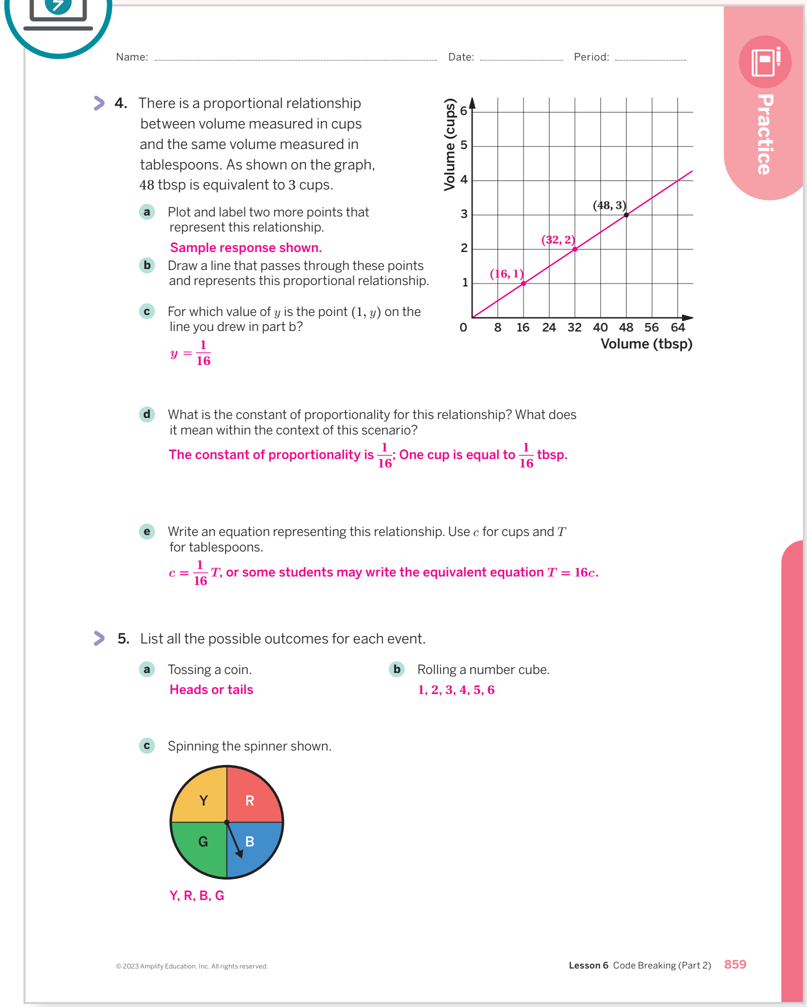


Name: Date: Period:

4. There is a proportional relationship between volume measured in cups and the same volume measured in tablespoons. As shown on the graph, 48 tbsp is equivalent to 3 cups.
   - a Plot and label two more points that represent this relationship.
     Sample response shown.
   - b Draw a line that passes through these points and represents this proportional relationship.
   - c For which value of y is the point $(1, y)$ on the line you drew in part b?
     $y = \frac{1}{16}$
   - d What is the constant of proportionality for this relationship? What does it mean within the context of this scenario?
     The constant of proportionality is $\frac{1}{16}$; **One cup is equal to** $\frac{1}{16}$ **tbsp.**
   - e Write an equation representing this relationship. Use c for cups and T for tablespoons.
     $c = \frac{1}{16}T$, **or some students may write the equivalent equation** $T = 16c$.

5. List all the possible outcomes for each event.
   - a Tossing a coin.
     Heads or tails
   - b Rolling a number cube.
     1, 2, 3, 4, 5, 6
   - c Spinning the spinner shown.

Y, R, B, G

Lesson 6 Code Breaking (Part 2) 859

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	3
Spiral	2	Unit 8 Lesson 4	2
	3	Unit 8 Lesson 3	2
	4	Unit 2 Lesson 12	2
Formative ⚡	5	Unit 8 Lesson 7	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 2

Probabilities of Multi-step Events

Whole Class

In this Sub-Unit, students will explore different methods for finding the probability of a multi-step event and learn how to design and conduct simulations.

SUB-UNIT

2 Probabilities of Multi-step Events

Narrative Connections

How did a Blazing Shoal bring the Philadelphia Convention Center to its feet?

It was a warm September day in the city of brotherly love. But inside the Philadelphia Convention Center, it was an intense competition. That day in 2011, players from all over the world had gathered to compete in a tournament for a popular collectible fantasy card game. Here, players assumed the role of powerful wizards, drawing cards from custom decks that allowed them to summon monsters and cast spells to score points against their opponents.

Facing off were Sam Black and Josh Leyton-Utter. That year, Black had built a deck never seen before in tournament play. How it worked is, well, complicated — but it boiled down to drawing the right cards in the right order.

At the heart of his strategy was to use a card called "Blazing Shoal," which could end the game in a single turn. But for this to work, Black first had to use another card called "Dragonstorm." This strategy was so powerful, it would later be banned from tournament play. But it was also risky. From a deck of 75 cards, Black had to draw *both* Blazing Shoal and Dragonstorm.

In the final round, Black was a turn away from defeat. But with Dragonstorm already in his hand, all he needed was Blazing Shoal in order to win. The crowd was tense as Black reshuffled his deck. With one last draw, Black slammed down the card without even looking.

A roar went through the convention center. He looked down and saw that he had failed. With a grin, Black shook his opponent's hand in defeat.

Drawing *both* Blazing Shoal and Dragonstorm is an example of a *multi-step* event, because more than one event has to occur. You can apply probability concepts to multi-step events to help you win at games of chance.

Sub-Unit 2 Probabilities of Multi-step Events 861

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how understanding the probability of multi-step events can help them solve real-world problems in the following places:

- **Lesson 7, Activity 3:** How Many Sandwiches?
- **Lesson 9, Activity 1:** Graphington Slopes (Part 2)
- **Lesson 10, Activity 1:** Breeding Mice (Part 2)

UNIT 8 | **LESSON 7**

Keeping Track of All Possible Outcomes

Let's explore sample spaces for experiments with multiple events.

Focus

Goals

1. **Language Goal:** Compare and contrast different methods for representing the sample space of a multi-step event, and evaluate their usefulness. **(Speaking and Listening, Writing)**
2. **Language Goal:** Determine the total number of possible outcomes for a multi-step event, and justify the reasoning for using other representations. **(Speaking and Listening, Writing)**
3. Interpret or create a list, table, or tree diagram representing the sample space of a multi-step event.

Coherence

Today

Students practice listing the sample space for a multi-step event. They make use of the structure of tree diagrams, tables, and organized lists as methods of organizing this information. Students notice that the total number of outcomes in the sample space for an experiment consisting of multiple events can be found by multiplying the number of outcomes for each event.

Previously

Students determined the probabilities of single-step events and analyzed coded messages by using the frequency of the letters.

Coming Soon

In the next lesson, students will use sample spaces to calculate the probability of multi-step events.

Rigor

- Students build their **conceptual understanding** of multi-step events.
- Students **apply** their understanding of finding the sample space for single-step events to multi-step events.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (Optional)	Summary	Exit Ticket
5 min	10 min	15 min	15 min	7 min	7 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

desmos Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New words

- ***multi-step event***
- ***tree diagram***

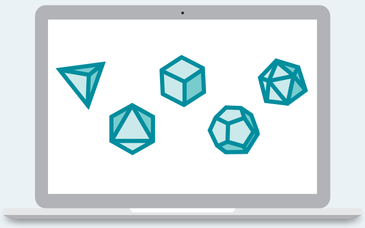
Review words

- *chance experiment*
- *event*
- *outcome*
- *probability*
- *sample space*

Amps | **Featured Activity**

Activity 2
Student Choice

Students choose two of four experiments for which to determine the sample space.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel disorganized as they try to list the sample spaces of multi-step events; they may repeat or forget outcomes as they try to make use of the structure of tree diagrams, tables, or lists. Help them grow their organizational skills as they list and pair the outcomes of one event with the outcomes of the other event, ensuring that all possible outcomes are listed.

Modifications to Pacing

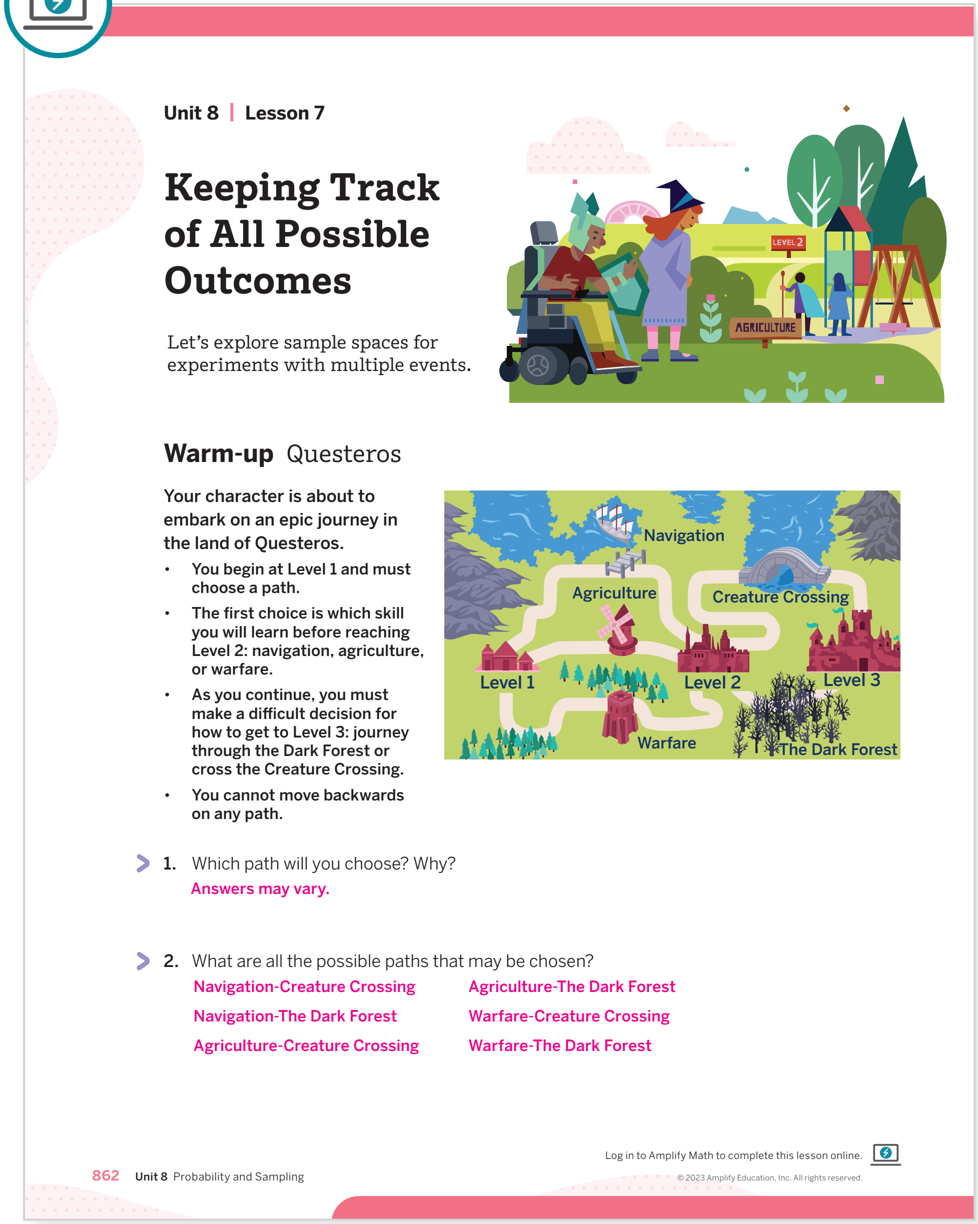
You may want to consider this additional modification if you are short on time.

- Complete optional **Activity 3** as a whole class, or **Activity 3** may be omitted.

Independent | 5 min

Warm-up Questeros

Students use their own methods to organize an inventory of different outcomes to prepare them for keeping track of all of the possible outcomes in a multi-step probability event.



Unit 8 | **Lesson 7**

Keeping Track of All Possible Outcomes

Let's explore sample spaces for experiments with multiple events.

Warm-up Questeros

Your character is about to embark on an epic journey in the land of Questeros.

- You begin at Level 1 and must choose a path.
- The first choice is which skill you will learn before reaching Level 2: navigation, agriculture, or warfare.
- As you continue, you must make a difficult decision for how to get to Level 3: journey through the Dark Forest or cross the Creature Crossing.
- You cannot move backwards on any path.

1. Which path will you choose? Why?
   Answers may vary.

2. What are all the possible paths that may be chosen?
   Navigation-Creature Crossing
   Navigation-The Dark Forest
   Agriculture-Creature Crossing
   Agriculture-The Dark Forest
   Warfare-Creature Crossing
   Warfare-The Dark Forest

Log in to Amplify Math to complete this lesson online.

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1 Launch

Activate students' background knowledge by asking if they have ever played a game where they needed to choose a path.

2 Monitor

Help students get started by asking them how many paths branch out from Level 1.

Look for points of confusion:

- **Thinking they can obtain every skill.** Let students know they cannot move backwards and have them trace the paths with their finger or pencil.

Look for productive strategies:

- Organizing the possible paths using a list or table.

3 Connect

Display the image from the Warm-up.

Have students share their methods for organizing the list of possible paths.

Ask:

- "How do you know that you counted all the possible paths?"
- "How do you know that you did not repeat any path(s)?"

Highlight the importance of creating an organized list so that possible outcomes are neither missed nor repeated.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that they will start at Level 1 and choose one of the three paths shown by selecting a skill to learn.
- **Read 2:** Ask students to name or highlight the given constraint of not being able to move backwards on any path.
- **Read 3:** Ask students to decide which path they will choose.

English Learners

Use gestures, such as pointing, to illustrate the choices that need to be made at each level and the corresponding paths based on those choices.

Power-up

To power up students' ability to list outcomes of an event, have students complete:

Recall that an *outcome* is a possible result of an experiment. For example, the outcomes of tossing a coin are heads and tails.

List the possible outcomes of rolling an 8-sided dice.

1, 2, 3, 4, 5, 6, 7, and 8

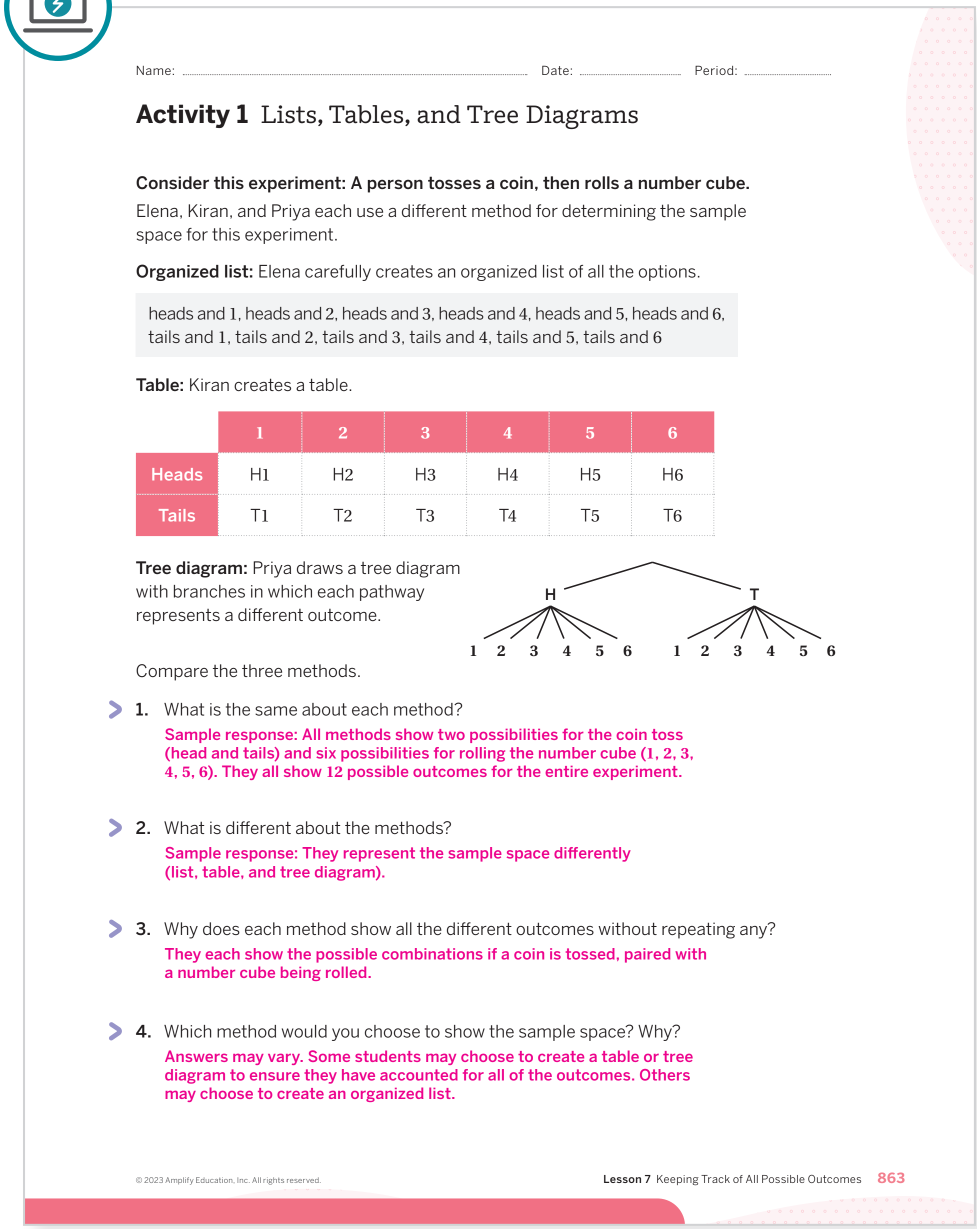
Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 5

Pairs | 10 min

Activity 1 Lists, Tables, and Tree Diagrams

Students are shown three methods for writing sample spaces to understand the possible outcomes of a *multi-step event*.



Name: ______________ Date: __________ Period: __________

Activity 1 Lists, Tables, and Tree Diagrams

Consider this experiment: A person tosses a coin, then rolls a number cube.
Elena, Kiran, and Priya each use a different method for determining the sample space for this experiment.

Organized list: Elena carefully creates an organized list of all the options.

heads and 1, heads and 2, heads and 3, heads and 4, heads and 5, heads and 6, tails and 1, tails and 2, tails and 3, tails and 4, tails and 5, tails and 6

Table: Kiran creates a table.

	1	2	3	4	5	6
Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

Tree diagram: Priya draws a tree diagram with branches in which each pathway represents a different outcome.

Compare the three methods.

1. What is the same about each method?
   Sample response: All methods show two possibilities for the coin toss (head and tails) and six possibilities for rolling the number cube (1, 2, 3, 4, 5, 6). They all show 12 possible outcomes for the entire experiment.

2. What is different about the methods?
   Sample response: They represent the sample space differently (list, table, and tree diagram).

3. Why does each method show all the different outcomes without repeating any?
   They each show the possible combinations if a coin is tossed, paired with a number cube being rolled.

4. Which method would you choose to show the sample space? Why?
   Answers may vary. Some students may choose to create a table or tree diagram to ensure they have accounted for all of the outcomes. Others may choose to create an organized list.

1 Launch

Explain that there are two events: tossing a coin and rolling a number cube. When an experiment consists of two or more events, it is called a *multi-step event.*

2 Monitor

Help students get started by asking, “What are the possible outcomes for flipping a coin? Rolling a number cube?”

Look for points of confusion:

- **Misinterpreting the tree diagram.** Help students see that a single outcome is represented by following the “branches.” Have students highlight the paths using different colors.

Look for productive strategies:

- Checking one sample space against another to ensure each method shows all of the outcomes.

3 Connect

Display the three sample space representations.

Have students share their preferred method. Use the *Poll the Class* routine, and then ask one student for each method to explain why they prefer it.

Highlight that all three methods show the same possible outcomes, but organize them differently.

Define the term ***multi-step event*** as an event that consists of two or more events.

Ask:

- “Why is it important to organize the sample space?”
- “How can you make sure not to repeat any outcomes in the sample space?”
- “Would the sample space change if the number cube was rolled before the coin is tossed?”

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Annotate the tree diagram by writing “coin toss” next to the first row and “number cube” next to the second row to help students make sense of the two events occurring.

Extension: *Math Enrichment*

Have students determine the sample space for tossing two coins and rolling one number cube.

HH1, HH2, HH3, HH4, HH5, HH6
HT1, HT2, HT3, HT4, HT5, HT6
TH1, TH2, TH3, TH4, TH5, TH6
TT1, TT2, TT3, TT4, TT5, TT6

Math Language Development

MLR7: Compare and Connect

During the Connect, have students compare the three methods used to represent the sample space. Connect how the three different representations all show the same information. Ask:

- “How many total possible outcomes are there? How do you see this in the organized list? Table? Tree diagram?”
- “How does the tree diagram show this is a multi-step event?”
  There are two steps: (1) heads versus tails, and (2) each of those has the 6 numbers listed underneath.

English Learners

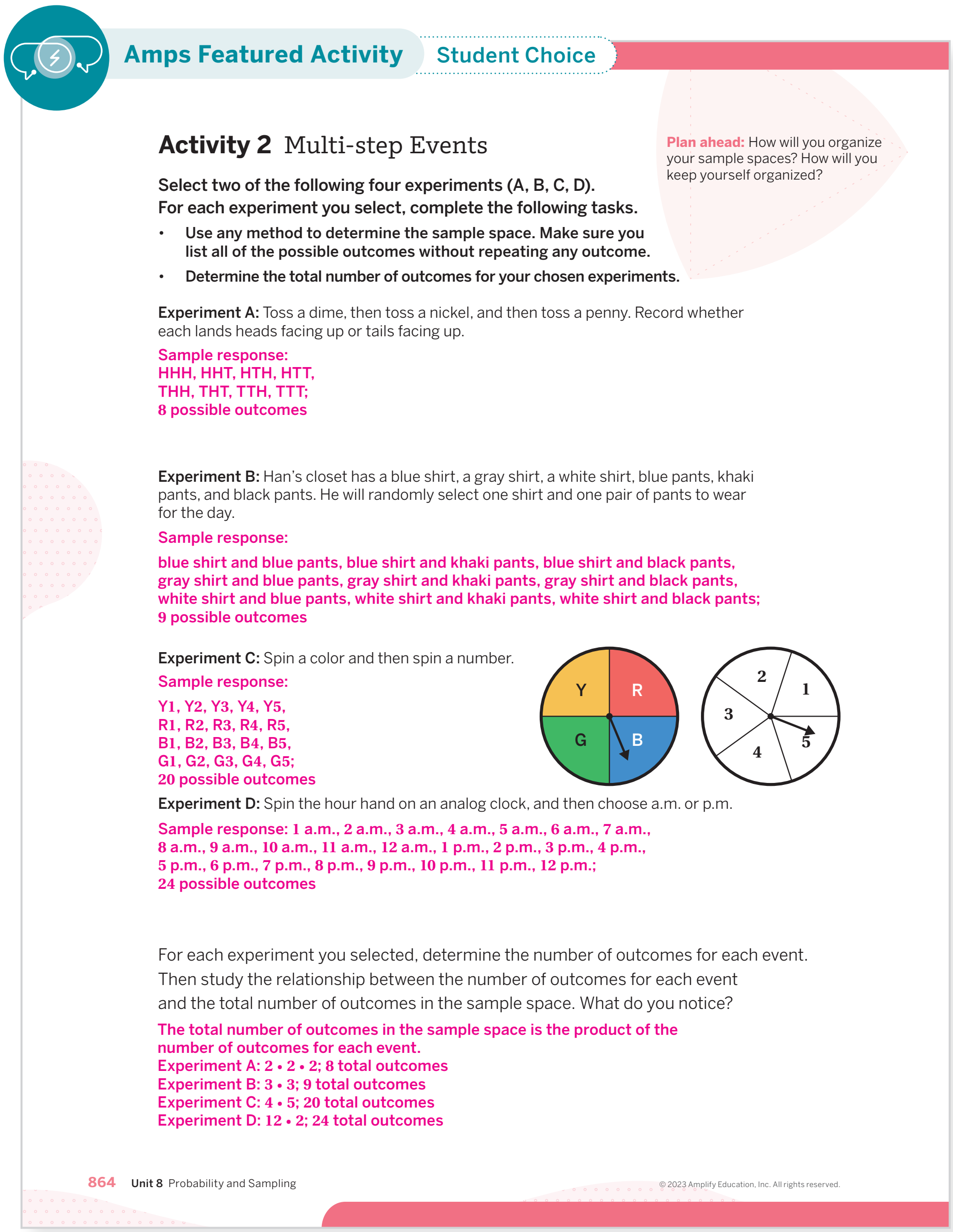
Color code one of the outcomes in each representation to illustrate how they are each displayed.

Pairs | 15 min

Activity 2 Multi-step Events

Students record the sample spaces to learn the total number of outcomes can be found by multiplying the number of outcomes of each experiment.

Amps Featured Activity Student Choice



Activity 2 Multi-step Events

Plan ahead: How will you organize your sample spaces? How will you keep yourself organized?

Select two of the following four experiments (A, B, C, D). For each experiment you select, complete the following tasks.

- **Use any method to determine the sample space. Make sure you list all of the possible outcomes without repeating any outcome.**
- **Determine the total number of outcomes for your chosen experiments.**

Experiment A: Toss a dime, then toss a nickel, and then toss a penny. Record whether each lands heads facing up or tails facing up.

Sample response:
HHH, HHT, HTH, HTT,
THH, THT, TTH, TTT;
8 possible outcomes

Experiment B: Han's closet has a blue shirt, a gray shirt, a white shirt, blue pants, khaki pants, and black pants. He will randomly select one shirt and one pair of pants to wear for the day.

Sample response:

blue shirt and blue pants, blue shirt and khaki pants, blue shirt and black pants, gray shirt and blue pants, gray shirt and khaki pants, gray shirt and black pants, white shirt and blue pants, white shirt and khaki pants, white shirt and black pants; 9 possible outcomes

Experiment C: Spin a color and then spin a number.

Sample response:

Y1, Y2, Y3, Y4, Y5,
R1, R2, R3, R4, R5,
B1, B2, B3, B4, B5,
G1, G2, G3, G4, G5;
20 possible outcomes

Experiment D: Spin the hour hand on an analog clock, and then choose a.m. or p.m.

Sample response: 1 a.m., 2 a.m., 3 a.m., 4 a.m., 5 a.m., 6 a.m., 7 a.m., 8 a.m., 9 a.m., 10 a.m., 11 a.m., 12 a.m., 1 p.m., 2 p.m., 3 p.m., 4 p.m., 5 p.m., 6 p.m., 7 p.m., 8 p.m., 9 p.m., 10 p.m., 11 p.m., 12 p.m.; 24 possible outcomes

For each experiment you selected, determine the number of outcomes for each event. Then study the relationship between the number of outcomes for each event and the total number of outcomes in the sample space. What do you notice?

The total number of outcomes in the sample space is the product of the number of outcomes for each event.
Experiment A: 2 • 2 • 2; 8 total outcomes
Experiment B: 3 • 3; 9 total outcomes
Experiment C: 4 • 5; 20 total outcomes
Experiment D: 12 • 2; 24 total outcomes

864 **Unit 8** Probability and Sampling

1 Launch

Let students know they are to choose two experiments and can represent the sample spaces using whichever method(s) they would prefer.

2 Monitor

Help students get started by asking what the possible outcomes are for each event, and then asking them to decide which method they will use to organize the sample space.

Look for points of confusion:

- **Repeating outcomes or missing outcomes.** Point out any repeated or missing outcomes. Ask students how they can check to see if they have listed all the possible outcomes. Consider having students show the sample space in another method to check their work.

3 Connect

Have students share their sample spaces. Include displays of each method.

Highlight students using different methods for the different experiments. Connect the total possible outcomes for each experiment to the product of the number of outcomes for each event in the experiment. Some students might see this connection better by creating a table or a tree diagram.

Ask:

- "What structure or method did you use for each experiment to make sure all outcomes were included without duplication?"
- "Are all three methods possible for each experiment?" A table would not work for Experiment A because it has three events. A table shows two events, one event along the column headers and one event along the row headers.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students begin by selecting one of the four experiments to complete. As time permits, have them select another experiment.

Extension: *Math Enrichment*

Have students use the relationship they noticed in this activity to determine the number of outcomes for the following multi-step events.

- Toss a coin 5 times and record each outcome.
  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ total outcomes
- Spin each of the spinners shown in Experiment C three times and record each outcome.
  $4 \cdot 4 \cdot 4 \cdot 5 \cdot 5 \cdot 5 = 8{,}000$ total outcomes

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their observations, provide the following sentence frames to help organize their thinking.

- "The number of outcomes in each event are ___ to give the total number of outcomes in the sample space."
- "The total number of outcomes in the sample space is the ___ of the number of outcomes in each event."

English Learners

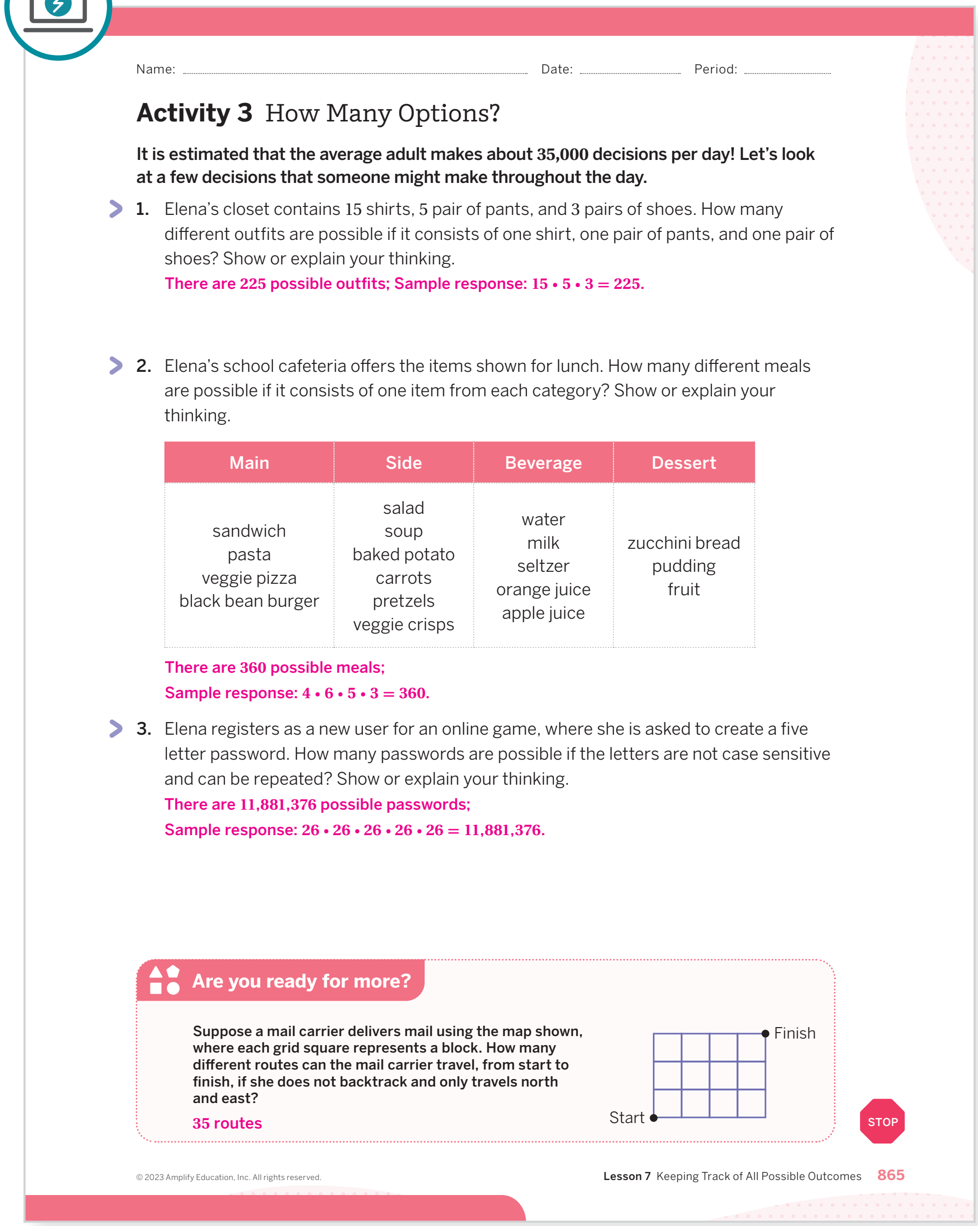
All students, including English Learners, may benefit from organizing the pattern in a table to visualize the relationship more clearly.

Optional

Pairs | 15 min

Activity 3 How Many Options? (optional)

Students apply their understanding of calculating the total number of possible outcomes in a multi-step event without listing all of the possible outcomes in the sample space.



Name: __________ Date: __________ Period: __________

Activity 3 How Many Options?

It is estimated that the average adult makes about 35,000 decisions per day! Let's look at a few decisions that someone might make throughout the day.

1. Elena's closet contains 15 shirts, 5 pair of pants, and 3 pairs of shoes. How many different outfits are possible if it consists of one shirt, one pair of pants, and one pair of shoes? Show or explain your thinking.
   There are 225 possible outfits; Sample response: $15 \cdot 5 \cdot 3 = 225$.

2. Elena's school cafeteria offers the items shown for lunch. How many different meals are possible if it consists of one item from each category? Show or explain your thinking.

Main	Side	Beverage	Dessert
sandwich pasta veggie pizza black bean burger	salad soup baked potato carrots pretzels veggie crisps	water milk seltzer orange juice apple juice	zucchini bread pudding fruit

   There are 360 possible meals;
   Sample response: $4 \cdot 6 \cdot 5 \cdot 3 = 360$.

3. Elena registers as a new user for an online game, where she is asked to create a five letter password. How many passwords are possible if the letters are not case sensitive and can be repeated? Show or explain your thinking.
   There are 11,881,376 possible passwords;
   Sample response: $26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 11{,}881{,}376$.

Are you ready for more?

Suppose a mail carrier delivers mail using the map shown, where each grid square represents a block. How many different routes can the mail carrier travel, from start to finish, if she does not backtrack and only travels north and east?

35 routes

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Lesson 7 Keeping Track of All Possible Outcomes 865

1 Launch

Ask, "What types of decisions does someone make each day?"

2 Monitor

Help students get started by having them underline the possible outcomes for each event.

Look for points of confusion:

- **Creating a sample space to determine the possible outcomes.** Tell students that while this is a valid strategy, it may not be the most efficient because it might take a lot of time and space.
- **Struggling to answer Problem 3.** Explain a password that is not case sensitive means it does not matter whether the password includes capital or lowercase letters. Then remind students that there are 26 letters in the alphabet. Allow students write their answer as an expression using exponents.

Look for productive strategies:

- Noticing the order in which they multiply the outcomes does not matter.

3 Connect

Have students share their responses. For Problem 3, select students who wrote different expressions, such as $26 \cdot 26 \cdot 26 \cdot 26 \cdot 26$ or 26^5 to determine their response.

Highlight that some situations have very large sample spaces and recording every possible outcome is not an efficient method. However, the total number of possible outcomes can be found by multiplying the number of outcomes for each event. **Note:** In Grade 7, students do not need to know this is the Fundamental Counting Principle.

Ask, "For Problem 3, how would the number of total possible outcomes change if the letters were case sensitive?" There would be 52^5, or 380,204,032, possible passwords.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students create an expression using blanks, and then fill each blank with the number of outcomes of each event. For example, for Problem 3, students may write _____ • _____ • _____• _____ •_____ , and then fill in the blanks with the number of outcomes for each event.

Extension: *Math Enrichment*

For Problem 3, have students determine the number of total possible outcomes if the letters *cannot* be repeated. $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$, or 7,893,600 possible passwords.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the incorrect statement for Problem 3, "There are 130 possible passwords because $26 \cdot 5 = 130$."

- ***Critique:*** "Do you agree or disagree with this statement? Why or why not?"
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "What was the most likely misunderstanding of the person who wrote this incorrect statement?" They multiplied the number of events by the possible outcome for one event.

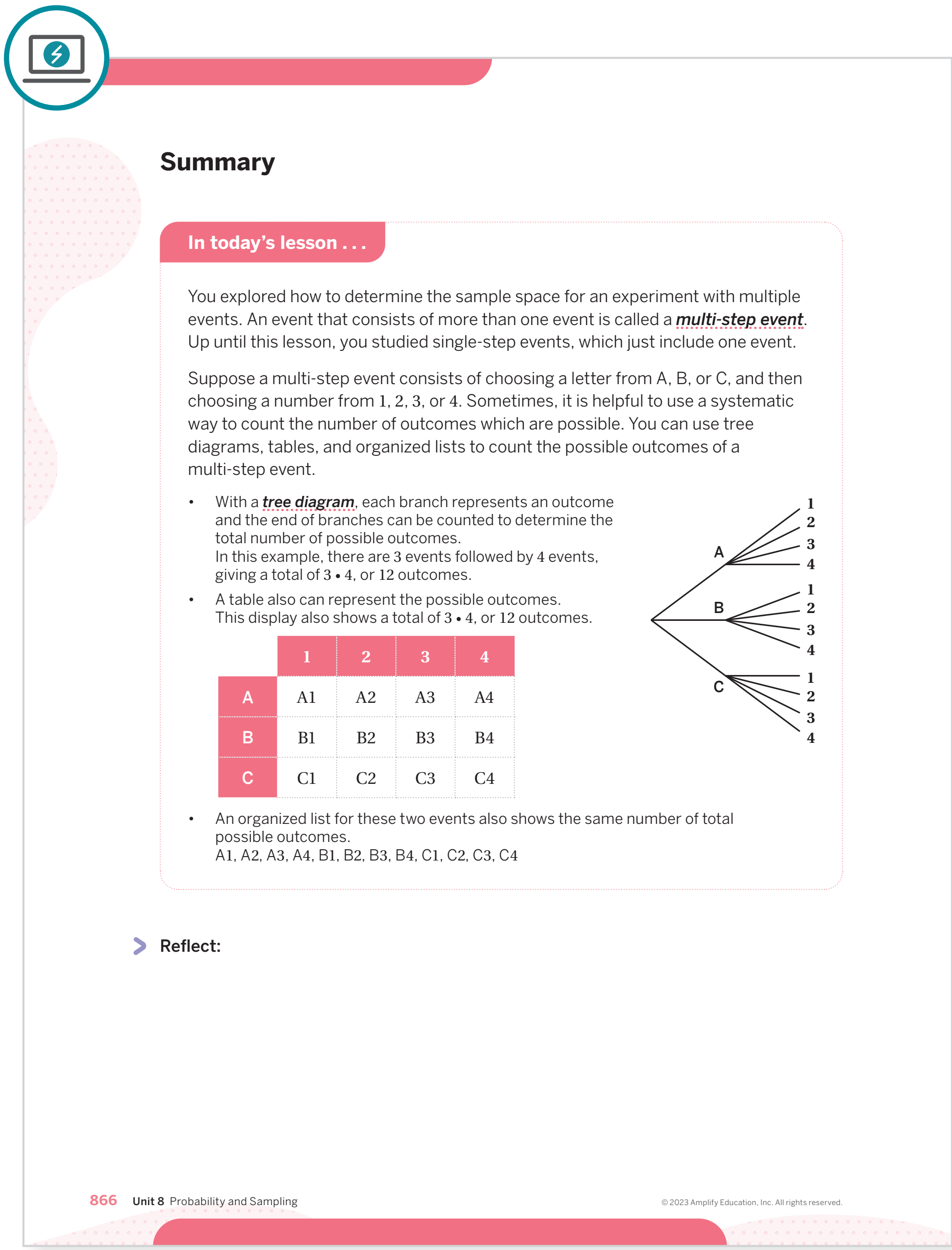
English Learners

Allow students time to rehearse what they will say with a partner before sharing with the whole class.

Whole Class | 7 min

Summary

Review and synthesize the variety of methods that can be used to create and organize the sample space for multi-step events.



Summary

In today's lesson . . .

You explored how to determine the sample space for an experiment with multiple events. An event that consists of more than one event is called a ***multi-step event***. Up until this lesson, you studied single-step events, which just include one event.

Suppose a multi-step event consists of choosing a letter from A, B, or C, and then choosing a number from 1, 2, 3, or 4. Sometimes, it is helpful to use a systematic way to count the number of outcomes which are possible. You can use tree diagrams, tables, and organized lists to count the possible outcomes of a multi-step event.

- With a ***tree diagram***, each branch represents an outcome and the end of branches can be counted to determine the total number of possible outcomes. In this example, there are 3 events followed by 4 events, giving a total of 3 • 4, or 12 outcomes.
- A table also can represent the possible outcomes. This display also shows a total of 3 • 4, or 12 outcomes.

	1	2	3	4
A	A1	A2	A3	A4
B	B1	B2	B3	B4
C	C1	C2	C3	C4

- An organized list for these two events also shows the same number of total possible outcomes. A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4

Reflect:

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Synthesize

Formalize vocabulary:

- *multi-step event*
- *tree diagram*

Have students share which method they would use to find the sample space of a multi-step event, and how they can use the individual events to find the total number of possible outcomes without counting.

Highlight the methods to represent a sample space are an organized list, table, and tree diagram.

Ask:

- "What are some methods for writing out the sample space of a multi-step event?" Tree diagrams, tables, and organized lists.
- "How does using a tree diagram relate to finding the number of outcomes in a sample space?" Each path from the start to the end of the "branches" represents one outcome in the sample space. Counting all the paths gives the number of items in the sample space.
- "Why do you think it will be important to know the total number of possible outcomes in a sample space when finding the probability of an event?" Probability can be found by finding the ratio of the number of favorable outcomes divided by the total number of outcomes in the sample space. I need to know the total number of outcomes in order to find this ratio.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is determining the probability of multi-step events similar to how you determined the probability of single-step events? How is it different?"

Math Language Development

MLR2: Collect and Display

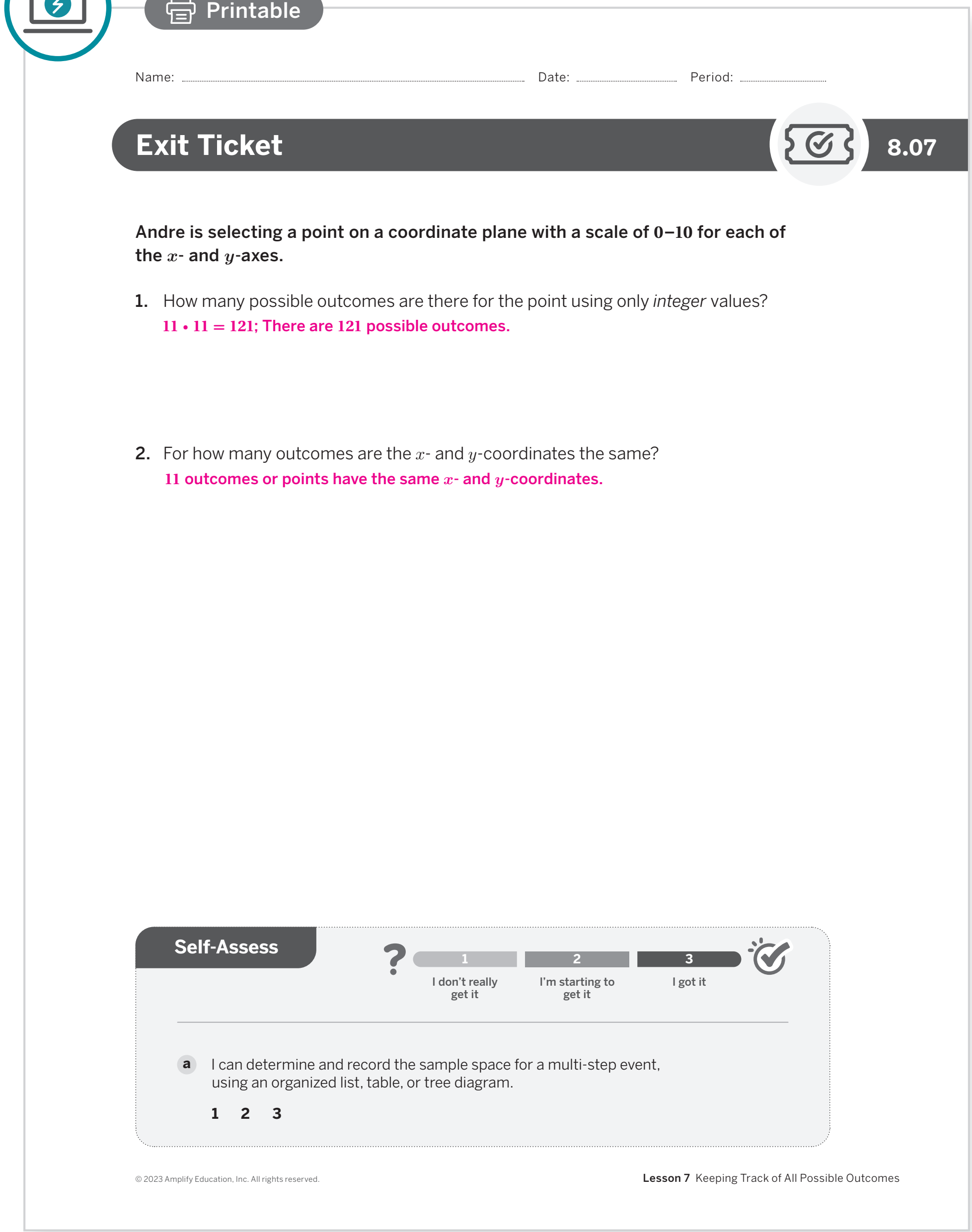
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *multi-step event* and *tree diagram* that were added to the display during the lesson.

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by calculating the total number of possible outcomes for a multi-step event.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 8.07

Andre is selecting a point on a coordinate plane with a scale of 0–10 for each of the x- and y-axes.

1. How many possible outcomes are there for the point using only *integer* values?
   $11 \cdot 11 = 121$; There are 121 possible outcomes.

2. For how many outcomes are the x- and y-coordinates the same?
   11 outcomes or points have the same x- and y-coordinates.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine and record the sample space for a multi-step event, using an organized list, table, or tree diagram.

1 2 3

Lesson 7 Keeping Track of All Possible Outcomes

Success looks like . . .

- **Language Goal:** Comparing and contrasting different methods for representing the sample space of a multi-step event, and evaluating their usefulness. **(Speaking and Listening, Writing)**
- **Language Goal:** Determining the total number of possible outcomes for a multi-step event, and justifying the reasoning for using other representations. **(Speaking and Listening, Writing)**
    - » Determining the number of possible outcomes for the point in Problem 1.
- **Goal:** Interpreting or creating a list, table, or tree diagram representing the sample space of a multi-step event.

Suggested next steps

If students answer Problem 1 with 100, consider:

- Reminding them to include 0 as a possible coordinate.

If students struggle to complete Problem 2, consider:

- Providing them with a piece of graph paper containing quadrant I.
- Assigning Practice Problems 1, 2, and 3.

Professional Learning

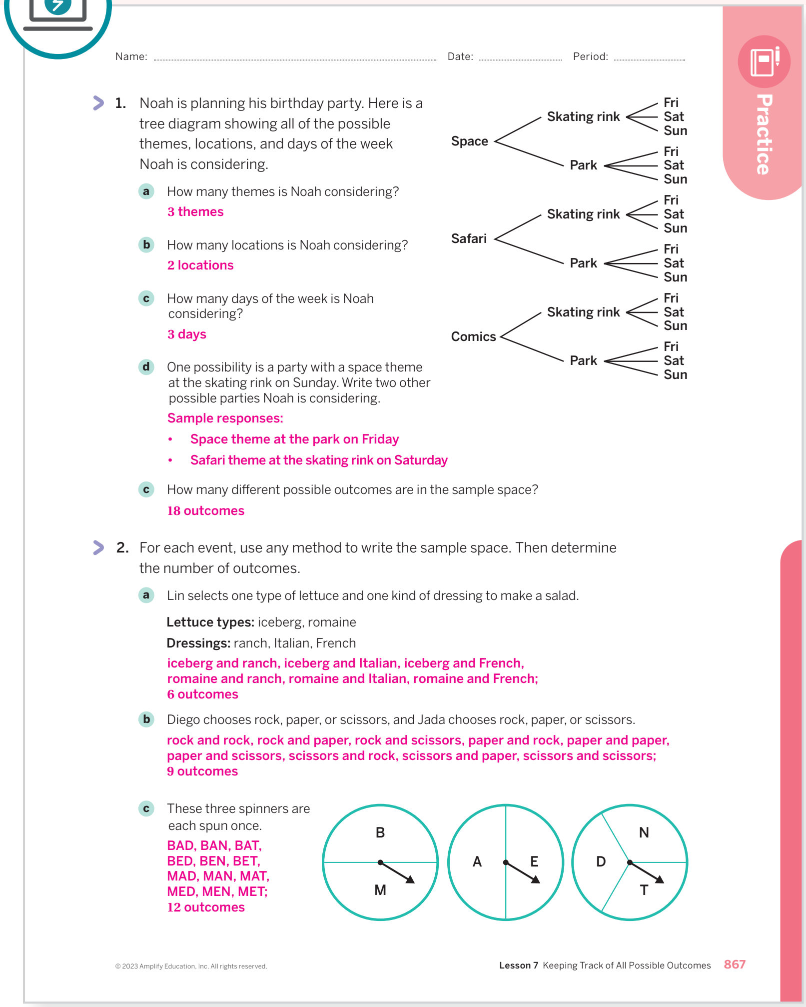
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students used tree diagrams and tables to determine the probability of multi-step events. How did this build on the early work that students did with sample space and simple events?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Noah is planning his birthday party. Here is a tree diagram showing all of the possible themes, locations, and days of the week Noah is considering.

a. How many themes is Noah considering?

3 themes

b. How many locations is Noah considering?

2 locations

c. How many days of the week is Noah considering?

3 days

d. One possibility is a party with a space theme at the skating rink on Sunday. Write two other possible parties Noah is considering.

Sample responses:

- **Space theme at the park on Friday**
- **Safari theme at the skating rink on Saturday**

c. How many different possible outcomes are in the sample space?

18 outcomes

2. For each event, use any method to write the sample space. Then determine the number of outcomes.

a. Lin selects one type of lettuce and one kind of dressing to make a salad.

Lettuce types: iceberg, romaine

Dressings: ranch, Italian, French

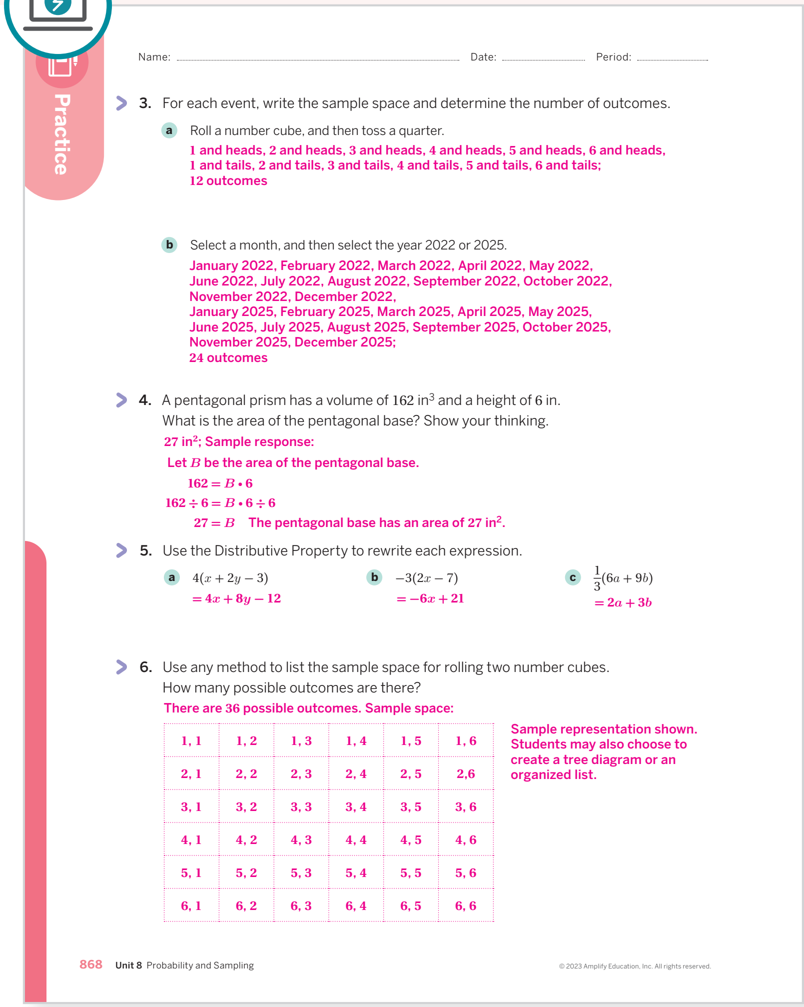
iceberg and ranch, iceberg and Italian, iceberg and French, romaine and ranch, romaine and Italian, romaine and French; 6 outcomes

b. Diego chooses rock, paper, or scissors, and Jada chooses rock, paper, or scissors.

rock and rock, rock and paper, rock and scissors, paper and rock, paper and paper, paper and scissors, scissors and rock, scissors and paper, scissors and scissors; 9 outcomes

c. These three spinners are each spun once.

BAD, BAN, BAT, BED, BEN, BET, MAD, MAN, MAT, MED, MEN, MET; 12 outcomes



Practice

Name: Date: Period:

3. For each event, write the sample space and determine the number of outcomes.

a. Roll a number cube, and then toss a quarter.

1 and heads, 2 and heads, 3 and heads, 4 and heads, 5 and heads, 6 and heads, 1 and tails, 2 and tails, 3 and tails, 4 and tails, 5 and tails, 6 and tails; 12 outcomes

b. Select a month, and then select the year 2022 or 2025.

January 2022, February 2022, March 2022, April 2022, May 2022, June 2022, July 2022, August 2022, September 2022, October 2022, November 2022, December 2022, January 2025, February 2025, March 2025, April 2025, May 2025, June 2025, July 2025, August 2025, September 2025, October 2025, November 2025, December 2025; 24 outcomes

4. A pentagonal prism has a volume of 162 in^3 and a height of 6 in. What is the area of the pentagonal base? Show your thinking.

27 in^2; Sample response:

Let B be the area of the pentagonal base.

$$162 = B \cdot 6$$
$$162 \div 6 = B \cdot 6 \div 6$$
$$27 = B$$

The pentagonal base has an area of 27 in^2.

5. Use the Distributive Property to rewrite each expression.

a. $4(x + 2y - 3)$

$= 4x + 8y - 12$

b. $-3(2x - 7)$

$= -6x + 21$

c. $\frac{1}{3}(6a + 9b)$

$= 2a + 3b$

6. Use any method to list the sample space for rolling two number cubes. How many possible outcomes are there?

There are 36 possible outcomes. Sample space:

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2,6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Sample representation shown. Students may also choose to create a tree diagram or an organized list.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	1
Spiral	4	Unit 7 Lesson 14	2
	5	Unit 6 Lesson 20	1
Formative ⚡	6	Unit 8 Lesson 8	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 8**

Experiments With Multi-step Events

Let's look at probabilities of experiments with multi-step events.

Focus

Goals

1. **Language Goal:** Choose a method for representing the sample space of a multi-step event, and justify the choice. **(Speaking and Listening)**
2. **Language Goal:** Use the sample space to determine the probability of a multi-step event, and explain (using other representations) the reasoning. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue to write sample spaces for chance experiments having multiple steps and begin using those sample spaces to calculate the probability of an event. Students may start listing the sample space using one method and then decide to switch to a different method, or they might recognize certain aspects of the situation leading them to choose a particular method from the beginning. The events mentioned in the lesson use everyday language; therefore, students will need to reason abstractly to create the event outcomes.

Previously

In Lesson 7, students wrote the sample space and reasoned that multiplying the number of outcomes for each event gave the total number of possible outcomes in an experiment.

Coming Soon

In Lessons 9 and 10, students will design and perform a simulation to estimate the probability of a real-world event.

Rigor

- Students build **conceptual understanding** of more efficient methods for finding the total number of possible outcomes for a multi-step event.
- Students **apply** their understanding of the different methods for representing the sample spaces of multi-step events.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Sample Spaces*, one per pair (optional)
- Activity 1 PDF, *Spinners*, one per pair (optional)
- Activity 2 PDF *Sample Spaces*, one per pair (optional)
- Activity 3 PDF (for display)
- paper clips (optional)

Math Language Development

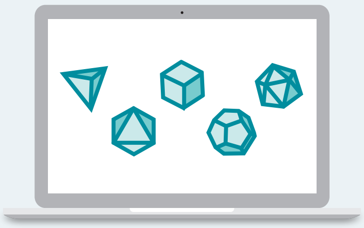
Review words

- *chance experiment*
- *event*
- *multi-step event*
- *outcome*
- *probability*
- *sample space*
- *tree diagram*

Amps | Featured Activity

Activity 2
Digital Number Cubes

Students roll a number cube to determine which sample space method they will use to complete the activity.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated by writing or referencing sample spaces when finding the probability of multi-step experiments; it may be hard to find the favorable outcomes if their work is disorganized. Ask them to use quantitative reasoning to find the total number of outcomes of the sample space by multiplying the number of outcomes for each event. This will allow them to check if they have accounted for all possible outcomes.

Modifications to Pacing

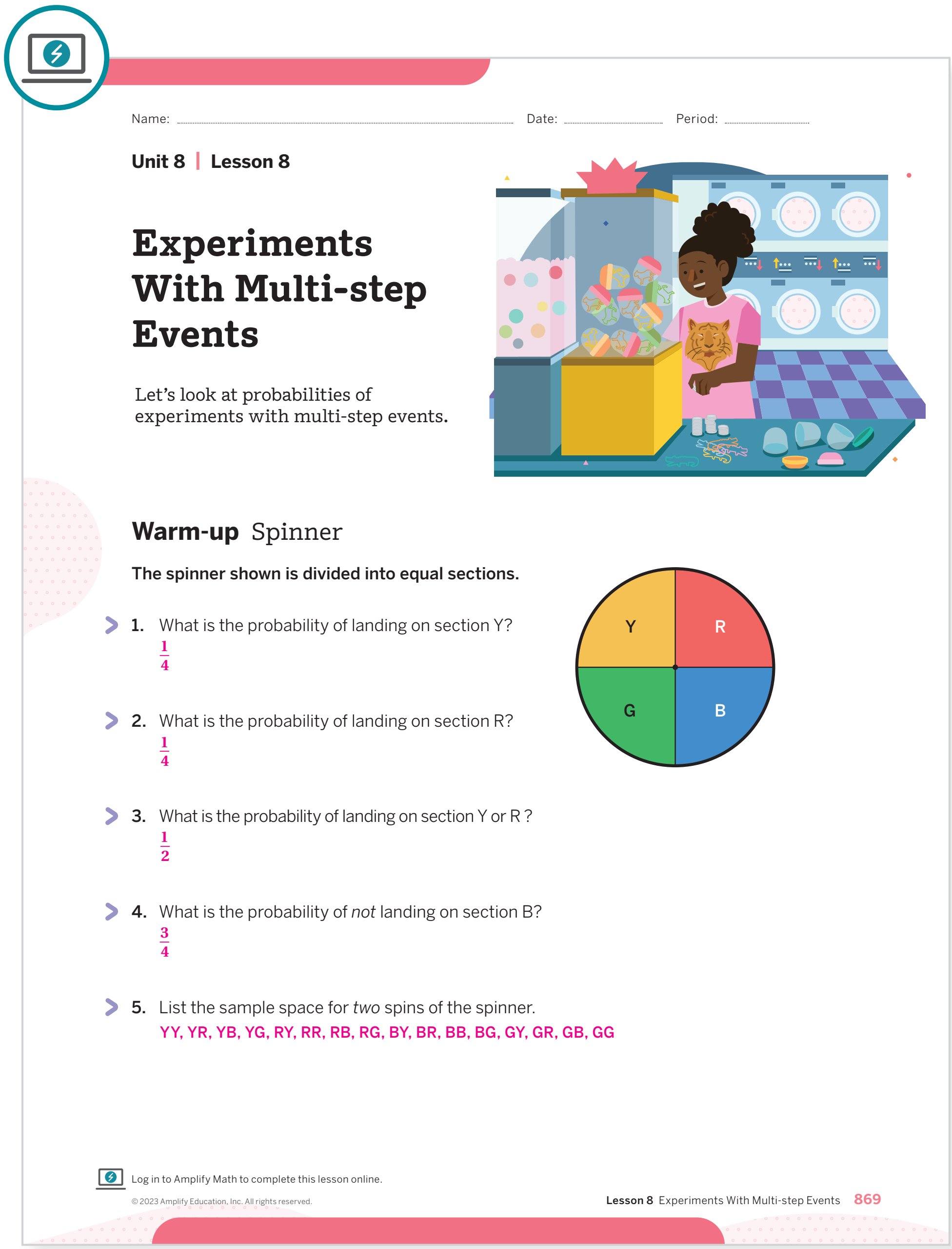
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 4 and 5 may be omitted.
- In **Activity 3**, Problems 3 and 5 may be omitted.

Independent | 5 min

Warm-up Spinner

Students review how to find the probability of single-step events to prepare them for finding the probability of multi-step events.



Name: ____________ Date: ________ Period: ________

Unit 8 | Lesson 8

Experiments With Multi-step Events

Let's look at probabilities of experiments with multi-step events.

Warm-up Spinner

The spinner shown is divided into equal sections.

1. What is the probability of landing on section Y?
$\frac{1}{4}$

2. What is the probability of landing on section R?
$\frac{1}{4}$

3. What is the probability of landing on section Y or R ?
$\frac{1}{2}$

4. What is the probability of *not* landing on section B?
$\frac{3}{4}$

5. List the sample space for *two* spins of the spinner.
YY, YR, YB, YG, RY, RR, RB, RG, BY, BR, BB, BG, GY, GR, GB, GG

Log in to Amplify Math to complete this lesson online.

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Lesson 8 Experiments With Multi-step Events 869

1 Launch

Activate prior knowledge by asking, "What methods did you use previously to determine the sample space of a multi-step event?" Explain that they can use any method for Problem 5.

2 Monitor

Help students get started by asking what fraction of the circle represents each color.

Look for points of confusion:

- Not knowing how to find the probability of not landing on B. Explain to students that "not landing on B" means landing on Y or R or G.

Look for productive strategies:

- Finding the probability for Problem 4 by subtracting the probability of spinning B from 1.

3 Connect

Display the spinner.

Highlight student reasoning and that the probability for each color is $\frac{1}{4}$ because the spinner is segmented into 4 equal sections, not just because there are 4 colors.

Ask, "If you change yellow (Y) to red (R), how does this change the probabilities?" The blue (B) and green (G) probabilities stay the same; however, the probability of spinning red (R) increases and now becomes $\frac{1}{2}$.

Power-up

To power up students' ability to list sample spaces, have students complete:

Recall that some tools you can use to create sample space are a tree diagram, a table, or an organized list.

Choose one tool to determine the sample space for rolling a number cube and flipping a coin, then determine the total number of outcomes.

Answers may vary, but should have 12 total outcomes:

1H, 2H, 3H, 4H, 5H, 6H

1T, 2T, 3T, 4T, 5T, 6T

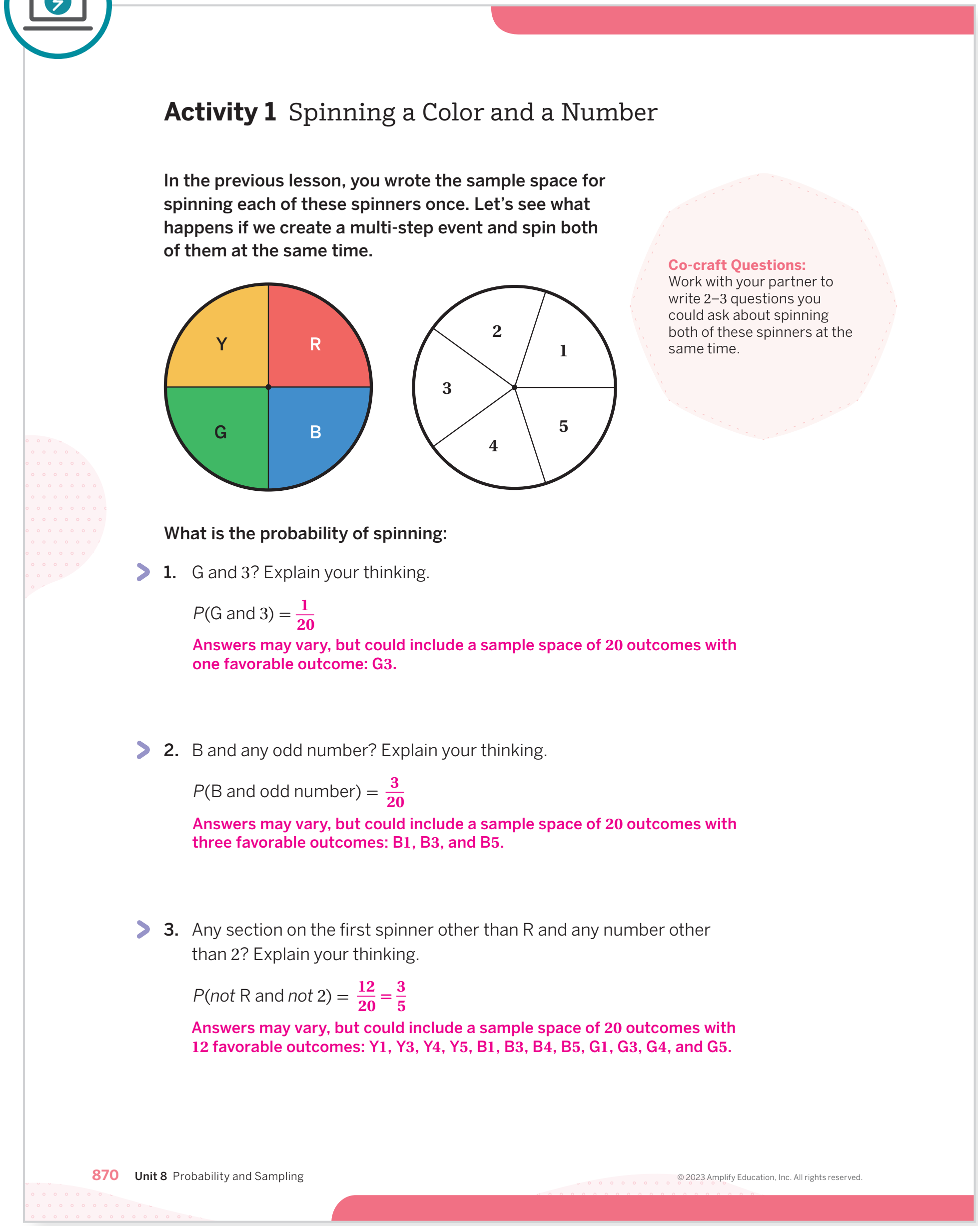
Use: Before Activity 1

Informed by: Performance on Lesson 7, Practice Problem 6

Pairs | 10 min

Activity 1 Spinning a Color and a Number

Students use the sample space of a multi-step experiment to calculate the probability of multi-step events.



Activity 1 Spinning a Color and a Number

In the previous lesson, you wrote the sample space for spinning each of these spinners once. Let's see what happens if we create a multi-step event and spin both of them at the same time.

Co-craft Questions:
Work with your partner to write 2–3 questions you could ask about spinning both of these spinners at the same time.

What is the probability of spinning:

1. G and 3? Explain your thinking.

   $P(\text{G and 3}) = \frac{1}{20}$

   Answers may vary, but could include a sample space of 20 outcomes with one favorable outcome: G3.

2. B and any odd number? Explain your thinking.

   $P(\text{B and odd number}) = \frac{3}{20}$

   Answers may vary, but could include a sample space of 20 outcomes with three favorable outcomes: B1, B3, and B5.

3. Any section on the first spinner other than R and any number other than 2? Explain your thinking.

   $P(\textit{not } \text{R and } \textit{not } 2) = \frac{12}{20} = \frac{3}{5}$

   Answers may vary, but could include a sample space of 20 outcomes with 12 favorable outcomes: Y1, Y3, Y4, Y5, B1, B3, B4, B5, G1, G3, G4, and G5.

870 **Unit 8** Probability and Sampling

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1 Launch

Activate student's prior knowledge by asking, "How do you determine the probability of an event?" The probability of multi-step events is found the same way.

2 Monitor

Help students get started by having them record the sample space.

Look for points of confusion:

- **Counting the outcomes for G or 3.** Have students circle the outcome for G and 3 and ask whether there are any other ways to spin a G and then the number 3.

Look for productive strategies:

- Referencing the sample space from Lesson 7, Activity 2.
- For Problem 3, finding the probability for R or 2 and subtracting it from 1.
- Circling or highlighting favorable outcomes. Note students who do this to share their answers during the discussion.

3 Connect

Display the spinners and sample space.

Have students share their reasoning for determining the number of favorable outcomes and the probability for each event.

Highlight students' use of precise mathematical language to describe their reasoning. As the events are described in everyday language, students must reason abstractly to identify the described outcomes.

Ask, "How did you calculate the number of outcomes in the sample space?" Counting the items in the tree diagram, table, or list, or using the multiplication: $4 \cdot 5 = 20$.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with a copy of the Activity 1 PDF, *Spinners* and a paper clip. Distribute pencils if students do not have their own. Show them how to attach the paper clip to each spinner and spin the paper clips to represent the multi-step event. Students can manipulate the concrete models to help them visualize the multi-step event.

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Activity 1 PDF, *Sample Spaces* to help them make sense of the sample space for the multi-step event. Suggest that students use colored pencils to mark the favorable events in the sample space.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display both spinners and have students work with their partner to write 2–3 mathematical questions they could ask about this multi-step event. Sample questions shown.

- How many total possible outcomes are there?
- How does the likelihood of spinning yellow and 1 relate to the likelihood of spinning red and 3?
- What is the probability of spinning blue and an even number?

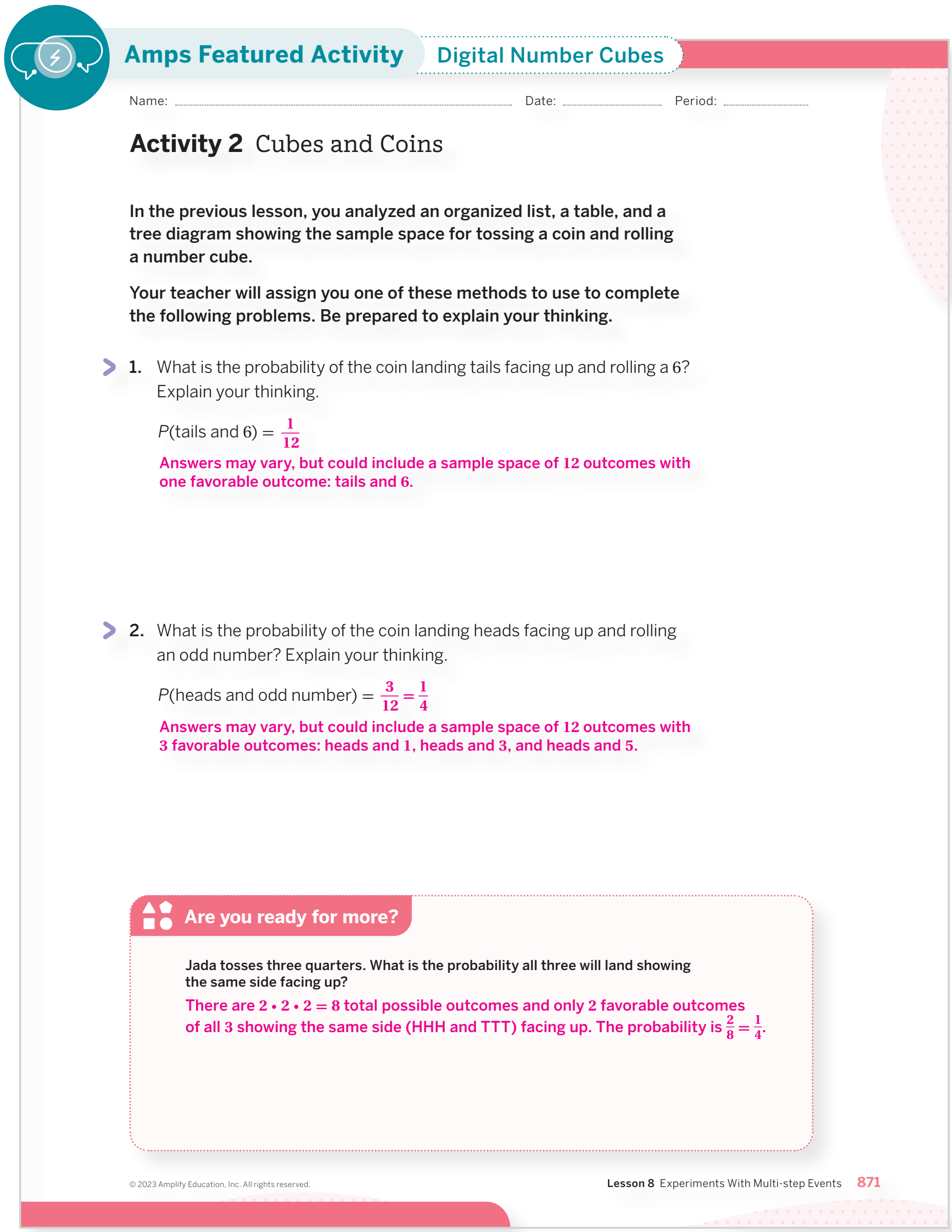
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 10 min

Activity 2 Cubes and Coins

Students continue thinking about multi-step experiments and use different representations of the sample space to compute probabilities.



Amps Featured Activity Digital Number Cubes

Name: ______ Date: ______ Period: ______

Activity 2 Cubes and Coins

In the previous lesson, you analyzed an organized list, a table, and a tree diagram showing the sample space for tossing a coin and rolling a number cube.

Your teacher will assign you one of these methods to use to complete the following problems. Be prepared to explain your thinking.

1. What is the probability of the coin landing tails facing up and rolling a 6? Explain your thinking.

   $P(\text{tails and } 6) = \frac{1}{12}$

   Answers may vary, but could include a sample space of 12 outcomes with one favorable outcome: tails and 6.

2. What is the probability of the coin landing heads facing up and rolling an odd number? Explain your thinking.

   $P(\text{heads and odd number}) = \frac{3}{12} = \frac{1}{4}$

   Answers may vary, but could include a sample space of 12 outcomes with 3 favorable outcomes: heads and 1, heads and 3, and heads and 5.

Are you ready for more?

Jada tosses three quarters. What is the probability all three will land showing the same side facing up?

There are $2 \cdot 2 \cdot 2 = 8$ total possible outcomes and only 2 favorable outcomes of all 3 showing the same side (HHH and TTT) facing up. The probability is $\frac{2}{8} = \frac{1}{4}$.

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Lesson 8 Experiments With Multi-step Events 871

1 Launch

Assign each pair of students a representation for writing out the sample space: a tree diagram, a table, or a list.

2 Monitor

Help students get started by having them list the outcomes for each event independently, then creating the sample space for the multi-step event.

Look for points of confusion:

- **Not finding the outcome in their sample space representation.** Have students use another method to record their sample space.

Look for productive strategies:

- Referencing the sample spaces from Lesson 7, Activity 1.

3 Connect

Have pairs of students share their method of representing the sample space and their reasoning for determining each probability.

Highlight how each method of recording the sample space is useful in determining the probability.

Ask:

- "Does one method of representing the sample space work better for this event?"
- "Do you have a preferred method for representing the sample space? Why?"
- "If you know the sample space and the number of favorable events, how can you determine the probability?" The total number of possible outcomes is the divisor and the number of favorable events is the dividend of the probability ratio.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can roll a digital number cube to determine which sample space method they will use to complete the activity.

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Activity 2 PDF, *Sample Spaces* to help them make sense of the sample space for the multi-step event. Suggest that students use colored pencils to mark the favorable events in the sample space.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share the method they used to represent the sample space for each event, highlight the connections between the representations and how they each show the same information. Ask students where they see the total number of possible outcomes and the number of favorable outcomes in each representation.

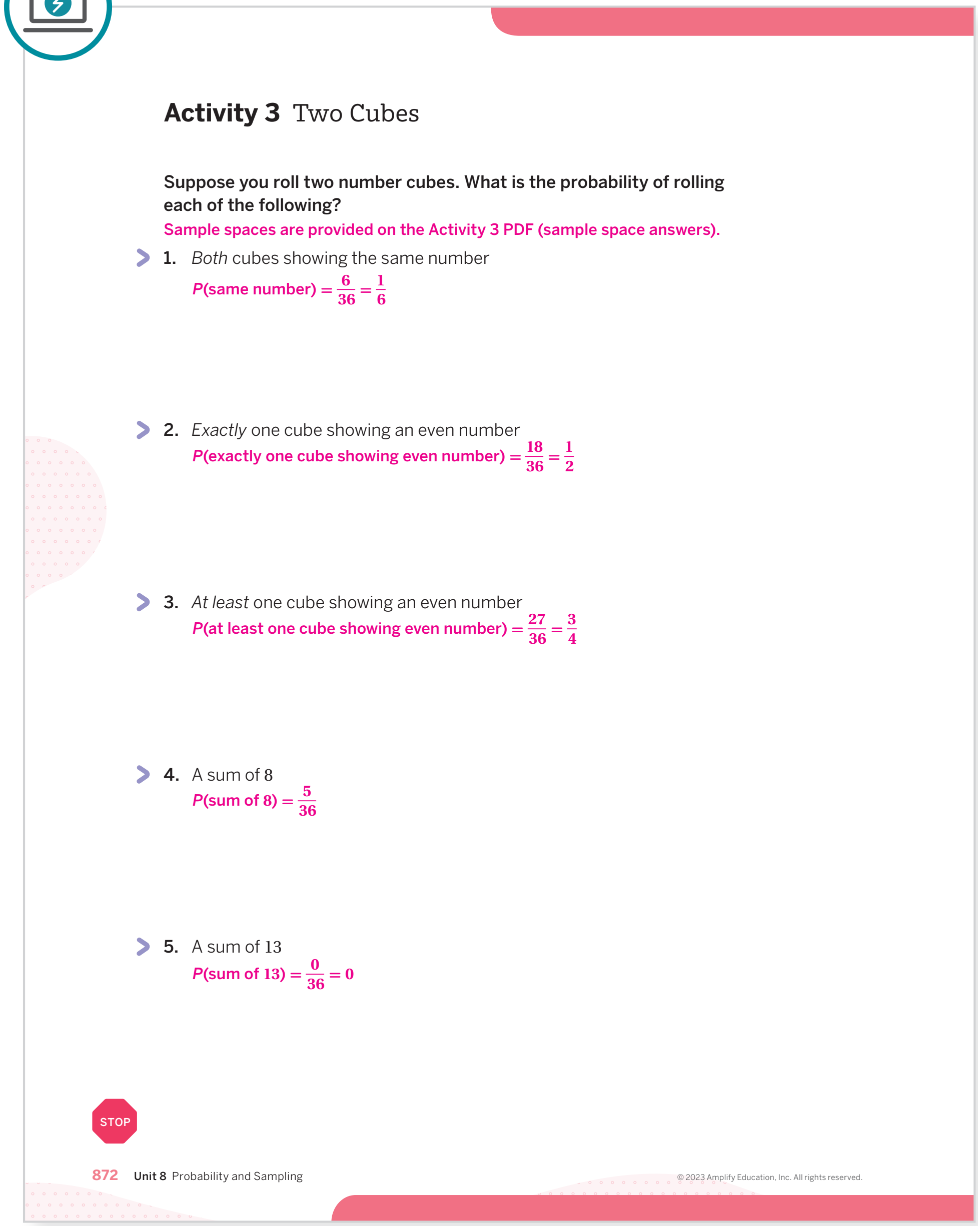
English Learners

Use gestures, such as pointing, or annotations to show where the *number of favorable outcomes* are shown in each representation.

Pairs | 10 min

Activity 3 Two Cubes

Students continue their work with multi-step events using the sample space to find probabilities.



Activity 3 Two Cubes

Suppose you roll two number cubes. What is the probability of rolling each of the following?

Sample spaces are provided on the Activity 3 PDF (sample space answers).

1. *Both* cubes showing the same number

   $P(\text{same number}) = \frac{6}{36} = \frac{1}{6}$

2. *Exactly* one cube showing an even number

   $P(\text{exactly one cube showing even number}) = \frac{18}{36} = \frac{1}{2}$

3. *At least* one cube showing an even number

   $P(\text{at least one cube showing even number}) = \frac{27}{36} = \frac{3}{4}$

4. A sum of 8

   $P(\text{sum of 8}) = \frac{5}{36}$

5. A sum of 13

   $P(\text{sum of 13}) = \frac{0}{36} = 0$

STOP

1 Launch

Explain to students that they can use any of the methods discussed previously to determine the probability for each scenario.

2 Monitor

Help students get started by having them write the sample space for rolling two number cubes. A table might be beneficial for most students.

Look for points of confusion:

- **Not recognizing that rolling a 2 then a 3 is different from rolling a 3 then a 2.** Have students imagine the number cubes are different colors to help them see they are different outcomes.
- **Not realizing a probability of zero is possible.** For Problem 5, have students write the sample space of the sum of two number cubes.

Look for productive strategies:

- Referencing the sample space from Lesson 7, Practice Problem 6.

3 Connect

Display the sample space for rolling two number cubes. When discussing the probabilities for the prompts, display the Activity 3 PDF.

Highlight the different methods students used to determine the probabilities and how listing the sample space aided in their solutions.

Ask:

- "Is there a method for finding the number of outcomes in the sample space that was more efficient than counting them?" The number of outcomes can be found by $6 \cdot 6 = 36$.
- "One of the events had a probability of zero. What does this mean?" It is impossible to occur.
- "What would the probability be of an event if it was certain to occur?" 1
- "Using two number cubes, describe an event that has a probability of 1." Answers may vary, but should describe an event that is certain to occur.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on completing Problems 1, 2, and 4. These problems provide a variety of different types of outcomes to consider for this multi-step event.

Extension: ***Math Enrichment***

Have students determine the sum(s) with the greatest and least probabilities of occurring when two number cubes are rolled. Ask them to explain their thinking. The sum with the greatest probability is a sum of 7 because there are 6 ways for that sum to occur. The sums with the least probability are 2 and 12 because there is only one way for each of these to occur.

Accessibility: ***Guide Processing and Visualization***

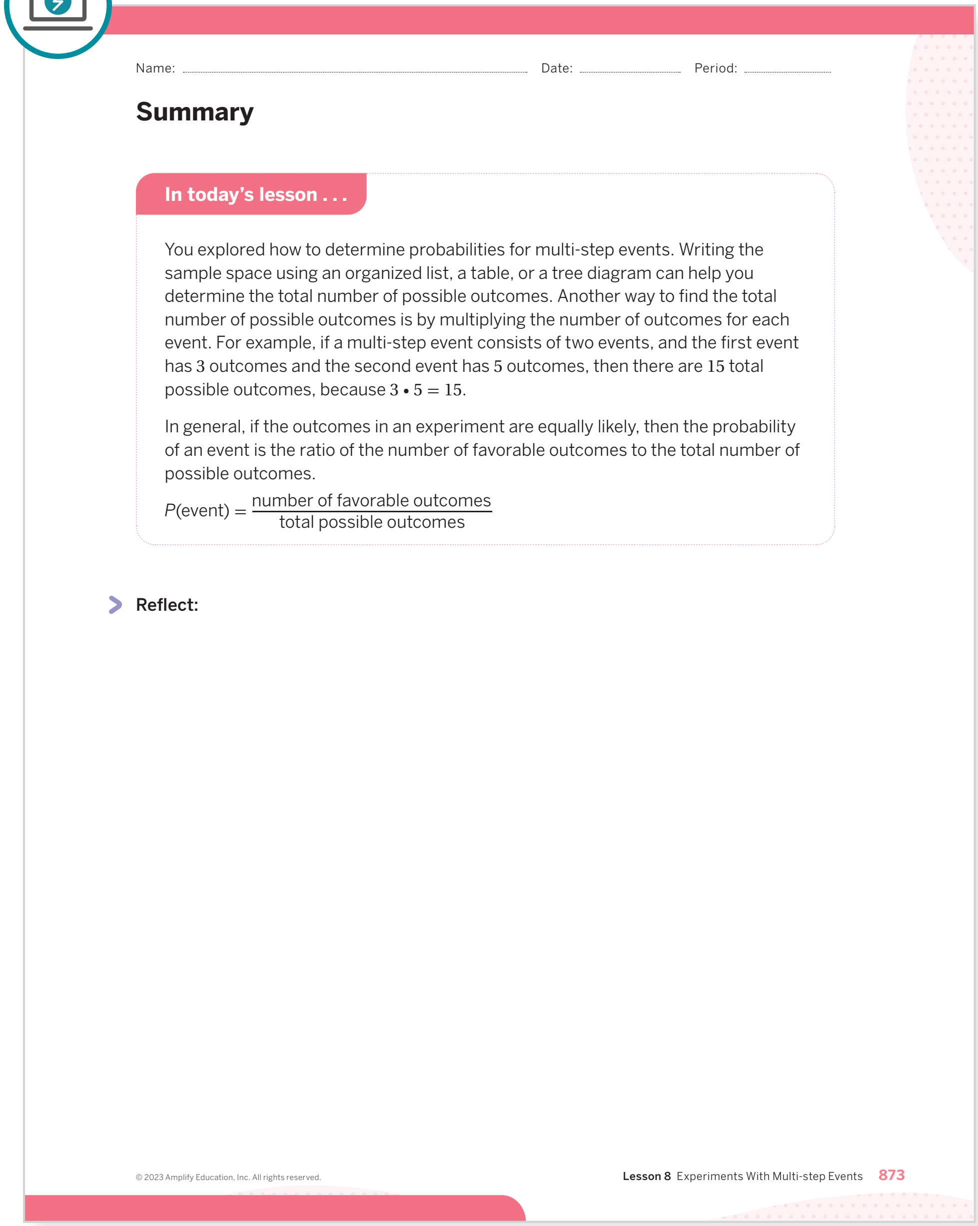
Display or provide students with a copy of the correct response for Lesson 7, Practice Problem 6, which shows the sample space for rolling two number cubes, and is also shown here.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Whole Class | 5 min

Summary

Review and synthesize how to find probabilities of multi-step events using the sample space.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how to determine probabilities for multi-step events. Writing the sample space using an organized list, a table, or a tree diagram can help you determine the total number of possible outcomes. Another way to find the total number of possible outcomes is by multiplying the number of outcomes for each event. For example, if a multi-step event consists of two events, and the first event has 3 outcomes and the second event has 5 outcomes, then there are 15 total possible outcomes, because $3 \cdot 5 = 15$.

In general, if the outcomes in an experiment are equally likely, then the probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total possible outcomes}}$$

Reflect:

Lesson 8 Experiments With Multi-step Events 873

Synthesize

Have students share their methods for determining probabilities of multi-step events using sample spaces.

Ask:

- "When the outcomes in the sample space are equally likely, how do you calculate the probability of the event?"
- "Now that you have plenty of practice, do you have a favorite method for writing the sample space?"
- "Are there times when one strategy for writing the sample space makes more sense than others?"

Highlight writing sample spaces with lists, tables, or tree diagrams can be helpful. Also, knowing that the total possible number of outcomes can be found by multiplying the number of events for each part of the experiment is useful. The probability of an event is found by calculating the ratio of the number of favorable events to the total number of possible events.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

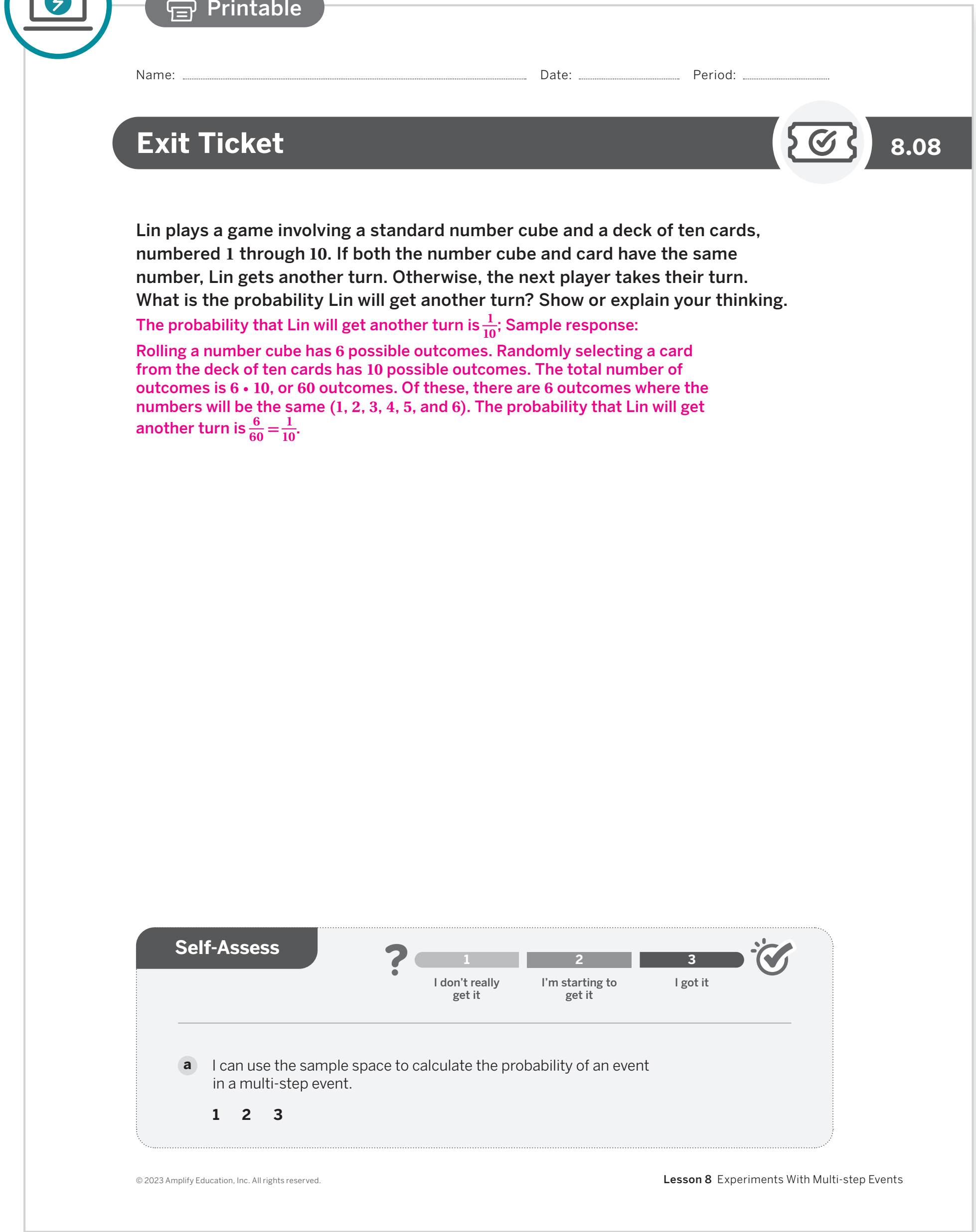
- "How did looking at sample spaces help you in determining the probability of multi-step events?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by finding the probability of a multi-step event.

Printable



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Exit Ticket

8.08

Lin plays a game involving a standard number cube and a deck of ten cards, numbered 1 through 10. If both the number cube and card have the same number, Lin gets another turn. Otherwise, the next player takes their turn. What is the probability Lin will get another turn? Show or explain your thinking.

The probability that Lin will get another turn is $\frac{1}{10}$; Sample response:

Rolling a number cube has 6 possible outcomes. Randomly selecting a card from the deck of ten cards has 10 possible outcomes. The total number of outcomes is $6 \cdot 10$, or 60 outcomes. Of these, there are 6 outcomes where the numbers will be the same (1, 2, 3, 4, 5, and 6). The probability that Lin will get another turn is $\frac{6}{60} = \frac{1}{10}$.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can use the sample space to calculate the probability of an event in a multi-step event.

1 2 3

Lesson 8 Experiments With Multi-step Events

Success looks like . . .

- **Language Goal:** Choosing a method for representing the sample space of a multi-step event, and justifying the choice. **(Speaking and Listening)**
- **Language Goal:** Using the sample space to determine the probability of a multi-step event, and explaining (using other representations) the reasoning. **(Speaking and Listening, Writing)**
  - » Determining and explaining how to find the probability that Lin will get another turn.

Suggested next steps

If students spend time writing out the sample space, consider:

- Reminding them the total number of possible outcomes can be found by multiplying the number of outcomes for each event.
- Giving them a copy of the sample space to determine whether they can find the probability.

Professional Learning

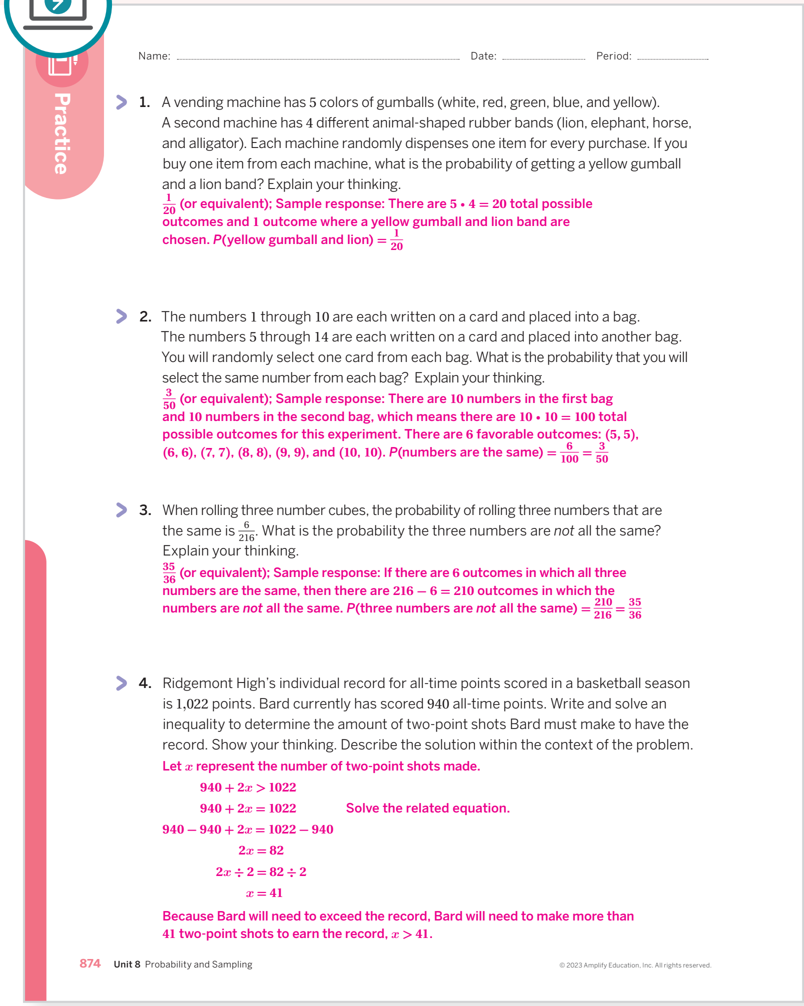
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways in Activity 2 did things happen that you did not expect?
- What did your students' use of lists, tables, and tree diagrams to determine multi-step probability reveal about your students as learners? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. A vending machine has 5 colors of gumballs (white, red, green, blue, and yellow). A second machine has 4 different animal-shaped rubber bands (lion, elephant, horse, and alligator). Each machine randomly dispenses one item for every purchase. If you buy one item from each machine, what is the probability of getting a yellow gumball and a lion band? Explain your thinking.
$\frac{1}{20}$ (or equivalent); Sample response: There are $5 \cdot 4 = 20$ total possible outcomes and 1 outcome where a yellow gumball and lion band are chosen. *P*(yellow gumball and lion) $= \frac{1}{20}$

2. The numbers 1 through 10 are each written on a card and placed into a bag. The numbers 5 through 14 are each written on a card and placed into another bag. You will randomly select one card from each bag. What is the probability that you will select the same number from each bag? Explain your thinking.
$\frac{3}{50}$ (or equivalent); Sample response: There are 10 numbers in the first bag and 10 numbers in the second bag, which means there are $10 \cdot 10 = 100$ total possible outcomes for this experiment. There are 6 favorable outcomes: (5, 5), (6, 6), (7, 7), (8, 8), (9, 9), and (10, 10). *P*(numbers are the same) $= \frac{6}{100} = \frac{3}{50}$

3. When rolling three number cubes, the probability of rolling three numbers that are the same is $\frac{6}{216}$. What is the probability the three numbers are *not* all the same? Explain your thinking.
$\frac{35}{36}$ (or equivalent); Sample response: If there are 6 outcomes in which all three numbers are the same, then there are $216 - 6 = 210$ outcomes in which the numbers are *not* all the same. *P*(three numbers are *not* all the same) $= \frac{210}{216} = \frac{35}{36}$

4. Ridgemont High's individual record for all-time points scored in a basketball season is 1,022 points. Bard currently has scored 940 all-time points. Write and solve an inequality to determine the amount of two-point shots Bard must make to have the record. Show your thinking. Describe the solution within the context of the problem.
Let x represent the number of two-point shots made.

$$940 + 2x > 1022$$

$$940 + 2x = 1022$$

Solve the related equation.

$$940 - 940 + 2x = 1022 - 940$$

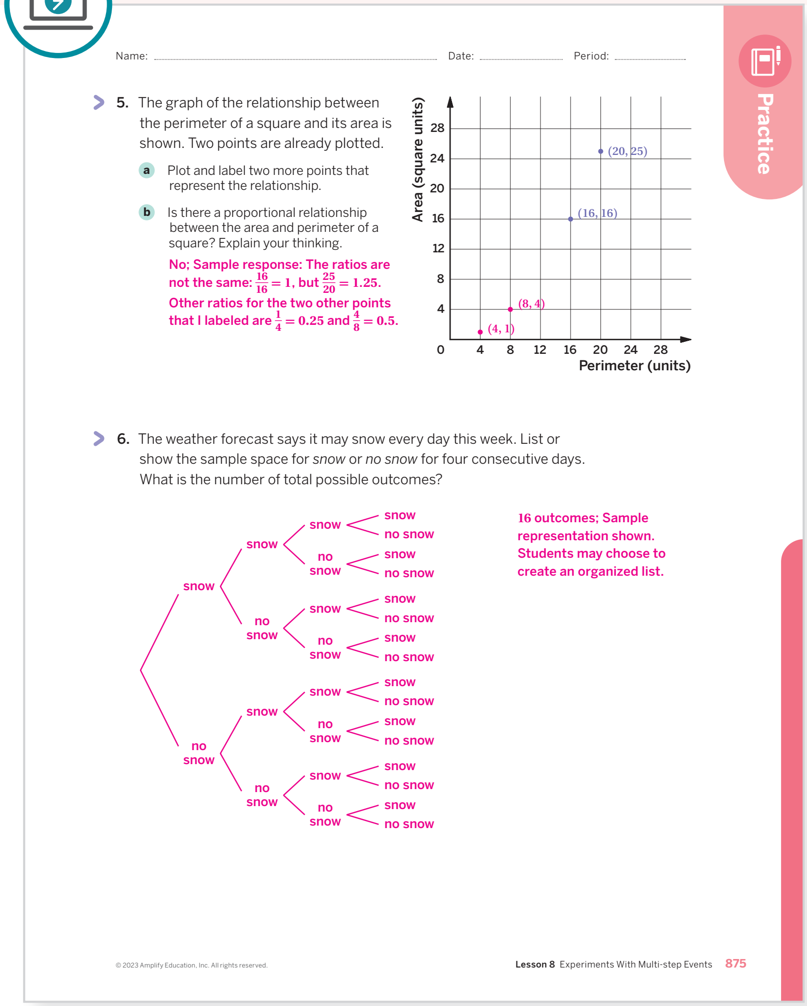
$$2x = 82$$

$$2x \div 2 = 82 \div 2$$

$$x = 41$$

Because Bard will need to exceed the record, Bard will need to make more than 41 two-point shots to earn the record, $x > 41$.

874 **Unit 8** Probability and Sampling



Name: ______ Date: ______ Period: ______

5. The graph of the relationship between the perimeter of a square and its area is shown. Two points are already plotted.
   - **a** Plot and label two more points that represent the relationship.
   - **b** Is there a proportional relationship between the area and perimeter of a square? Explain your thinking.
   No; Sample response: The ratios are not the same: $\frac{16}{16} = 1$, but $\frac{25}{20} = 1.25$. Other ratios for the two other points that I labeled are $\frac{1}{4} = 0.25$ and $\frac{4}{8} = 0.5$.

6. The weather forecast says it may snow every day this week. List or show the sample space for *snow* or *no snow* for four consecutive days. What is the number of total possible outcomes?

16 outcomes; Sample representation shown. Students may choose to create an organized list.

Lesson 8 Experiments With Multi-step Events 875

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1–3	2
	2	Activities 1–3	2
	3	Activities 1–3	2
Spiral	4	Unit 6 Lesson 15	2
	5	Unit 2 Lesson 11	2
Formative ⚡	6	Unit 8 Lesson 9	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 9**

Simulating Multi-step Events

Let’s simulate multi-step events.

Focus

Goals

1. **Language Goal:** Coordinate a real-world situation and a chance event which could be used to simulate that situation. **(Speaking and Listening)**
2. **Language Goal:** Perform a multi-step simulation, and use the results to estimate the probability of a multi-step event in a real-world situation (using other representations). **(Speaking and Listening, Writing)**

Coherence

Today

Students see that multi-step events can be simulated by using multiple chance experiments. In this case, it is important to communicate precisely what represents one outcome of the simulation. Students consider how real-world situations can be represented using simulation.

Previously

In Lesson 4, students simulated a chance experiment and were shown the experimental (observed) probability would approach the theoretical (expected) probability as more simulations were performed. In Lesson 8, students used the sample space to determine probabilities of multi-step events.

Coming Soon

In Lesson 10, students design and perform a simulation to model a real-world event. They use the results of the simulation to estimate probabilities.

Rigor

- Students build **conceptual understanding** of how chance events can be used to simulate real-world situations.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	20 min	10 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers as needed)
- Activity 1 PDF, *Graphington Slopes,* pre-cut spinner, one per group
- Activity 1 PDF, *Graphington Slopes: Theoretical Probability* (mathematical information for the teacher)
- bags with slips of paper
- number cubes
- paper clips

Math Language Development

New word

- ***simulation***

Review words

- *chance experiment*
- *multi-step event*
- *outcome*
- *probability*
- *sample space*

Amps | **Featured Activity**

Activity 1
Aggregate Class Data

Students perform their individual simulations followed by the entire class's data aggregating to create a greater number of trials. Students compare their simulation probability to the class's probability.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may grow disinterested with the repetitive nature of a simulation; they may want to be impulsive and make assumptions instead of modeling the event and completing the simulation. Encourage students to persist and work as a group to finish all trials of the simulation because accurate results are needed for the entire class to learn.

Modifications to Pacing

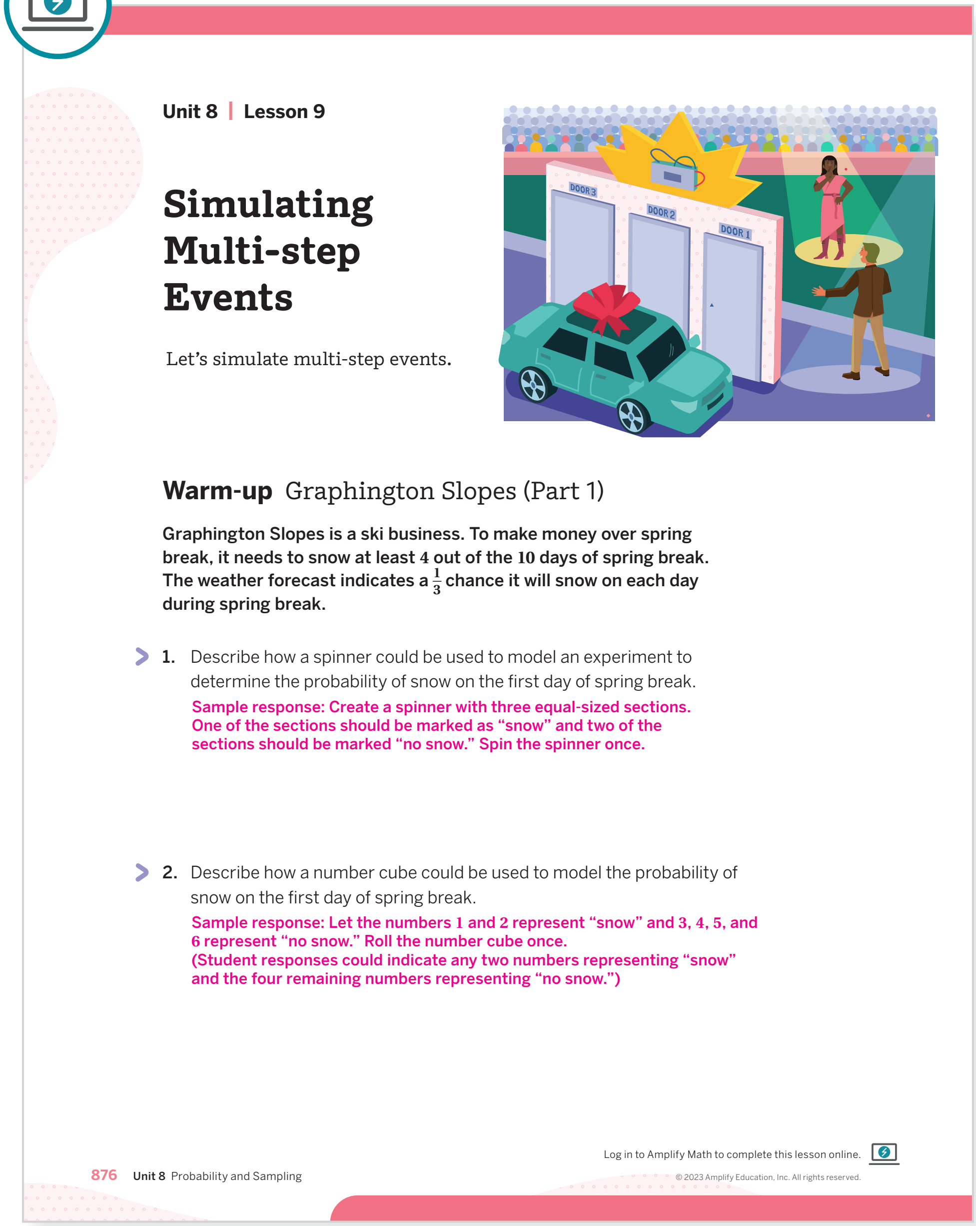
You may want to consider these additional modifications if you are short on time.

- Complete the **Warm-up** as a whole class.
- Reduce the number of simulations in **Activity 1** to three instead of five.
- In **Activity 2**, have students match only the first two situations.

Pairs | 8 min

Warm-up Graphington Slopes (Part 1)

Students design simulations of a real-world event to use in Activity 1.



Unit 8 | **Lesson 9**

Simulating Multi-step Events

Let's simulate multi-step events.

Warm-up Graphington Slopes (Part 1)

Graphington Slopes is a ski business. To make money over spring break, it needs to snow at least 4 out of the 10 days of spring break. The weather forecast indicates a $\frac{1}{3}$ chance it will snow on each day during spring break.

1. Describe how a spinner could be used to model an experiment to determine the probability of snow on the first day of spring break.

   Sample response: Create a spinner with three equal-sized sections. One of the sections should be marked as "snow" and two of the sections should be marked "no snow." Spin the spinner once.

2. Describe how a number cube could be used to model the probability of snow on the first day of spring break.

   Sample response: Let the numbers 1 and 2 represent "snow" and 3, 4, 5, and 6 represent "no snow." Roll the number cube once. (Student responses could indicate any two numbers representing "snow" and the four remaining numbers representing "no snow.")

Log in to Amplify Math to complete this lesson online.

876 **Unit 8** Probability and Sampling

1 Launch

Read the scenario to the class and ask, "What is the probability it will snow on any given day?"

2 Monitor

Help students get started by describing a spinner that is divided into thirds. Ask, "How many sections represent snow and how many sections represent no snow?"

Look for points of confusion:

- **Having difficulty describing a spinner showing a third as snow.** Let students draw a spinner instead of describing it.
- **Not understanding how the number cube could be used.** Have students write the sample space of rolling a number cube then circle one third of it.

Look for productive strategies:

- Describing multiple simulations for the number cube or a simulation using other items such as marbles or slips of paper.

3 Connect

Define the term ***simulation*** as an experiment used to estimate or predict the probability of a real-world event. The chance experiments designed in Problems 1 and 2 are examples of simulations.

Highlight why simulations are used to model a multi-step event using everyday objects like number cubes, marbles, coins, or spinners to help estimate probabilities.

Ask:

- "How can you adjust the simulations from Problems 1 or 2 to find the probability of Graphington Slopes making money?" (Take this conversation and lead into the Launch of Activity 1).
- "Which simulation would you like to perform to help find the probability?" (Based on group responses, provide those materials to the groups for Activity 1.)

Differentiated Support

Accessibility: *Activate Background Knowledge*

Students are likely familiar with weather forecasts. Ask, "When a weather forecast indicates a probability of snow (or other weather), what does that mean to you? Do you think a $\frac{1}{3}$ chance of snow means that snow is likely or unlikely?"

Accessibility: *Optimize Access to Tools*

Provide students with a blank spinner and a number cube that they can hold and physically manipulate to help them visualize how they could use each one to model the probability.

Power-up

To power up students' ability to construct sample spaces to determine the number of outcomes for three or more events:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

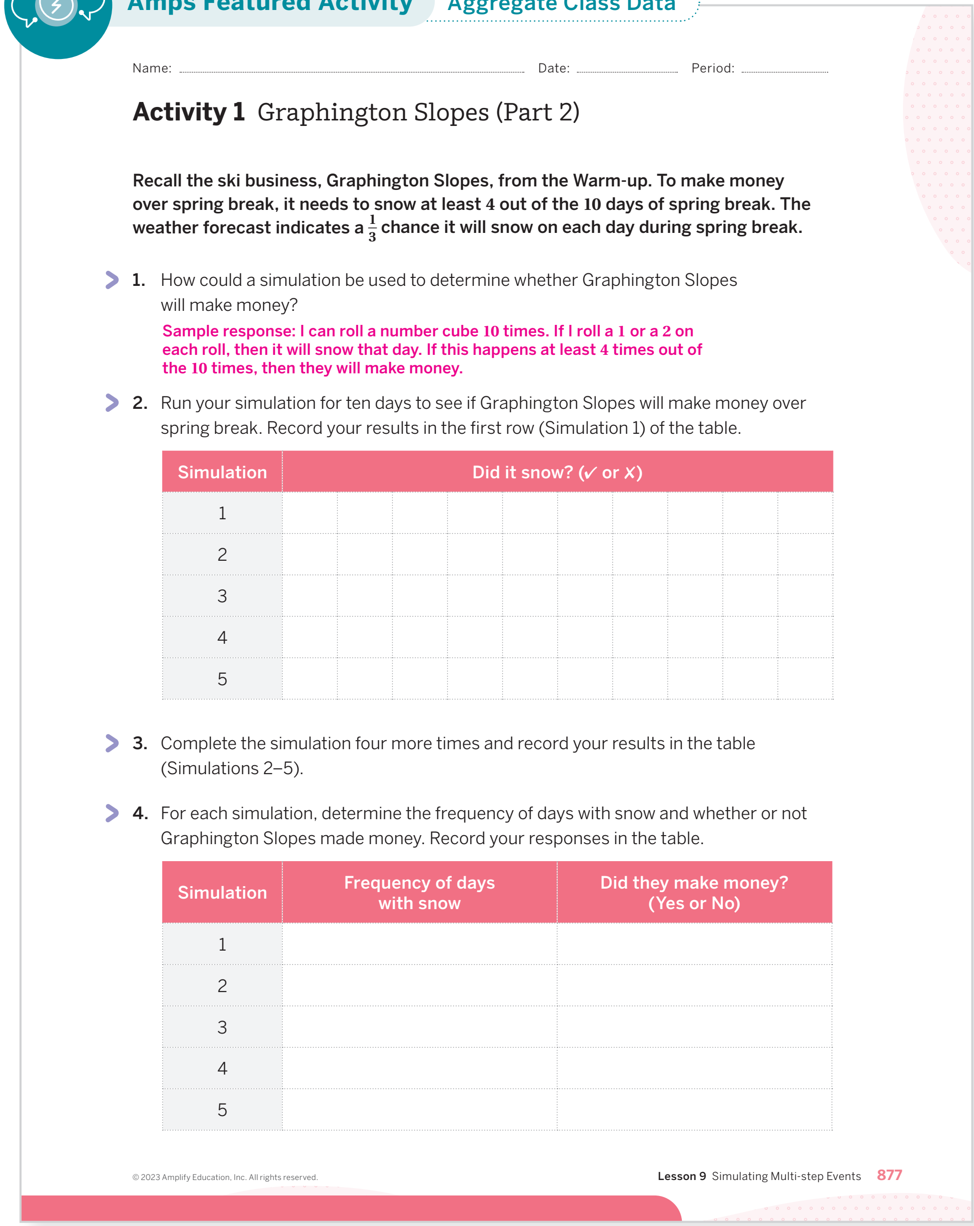
Informed by: Performance on Lesson 8, Practice Problem 6

Small Groups | 20 min

Activity 1 Graphington Slopes (Part 2)

Students perform the simulation created in the Warm-up to determine the relative frequency of a real-world event.

Amps Featured Activity Aggregate Class Data



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Activity 1 Graphington Slopes (Part 2)

Recall the ski business, Graphington Slopes, from the Warm-up. To make money over spring break, it needs to snow at least 4 out of the 10 days of spring break. The weather forecast indicates a $\frac{1}{3}$ chance it will snow on each day during spring break.

1. How could a simulation be used to determine whether Graphington Slopes will make money?

   Sample response: I can roll a number cube 10 times. If I roll a 1 or a 2 on each roll, then it will snow that day. If this happens at least 4 times out of the 10 times, then they will make money.

2. Run your simulation for ten days to see if Graphington Slopes will make money over spring break. Record your results in the first row (Simulation 1) of the table.

Simulation	Did it snow? (✓ or ✗)									
1										
2										
3										
4										
5										

3. Complete the simulation four more times and record your results in the table (Simulations 2–5).

4. For each simulation, determine the frequency of days with snow and whether or not Graphington Slopes made money. Record your responses in the table.

Simulation	Frequency of days with snow	Did they make money? (Yes or No)
1		
2		
3		
4		
5		

Lesson 9 Simulating Multi-step Events 877

1 Launch

Provide groups with the Activity 1 PDF, a pre-cut spinner, paper clips, number cubes, and bags with slips of paper. Give students 2 minutes to complete Problem 1 and have a whole-class discussion regarding any questions they have before starting the simulation.

2 Monitor

Help students get started by helping them analyze and understand their simulation results. Once a simulation is complete (Problem 2), ask, "How do you know if Graphington Slopes will make money?"

Look for points of confusion:

- **Not knowing how to find frequency.** Remind students that *frequency* means the number of occurrences, for example, how many times it snowed.
- **Confusing the phrase "at least 4."** Remind students that this phrase means 4 or more days of snow.
- **Totaling the number of snow days across all simulations.** Remind students one complete simulation represents 10 days, not any more or less.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can perform their own simulations and view the class data aggregated. Students can then compare their individual simulation probability to the class's probability.

Extension: *Math Enrichment*

Ask students how the design of their simulation would change if the weather forecast indicated a 10% chance of snow on each day.

Answers will vary.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 1, have groups meet with another group to share and receive feedback on the design of their simulations. Have reviewers ask these questions:

- "Does the response include a description of the tool to use *and how to use* it (spinner, number cube, slips of paper, etc.)?"
- "Does the response include a description of what the favorable outcome represents when using the tool?"

Have groups revise their designs, based on the feedback, and proceed with the rest of the activity.

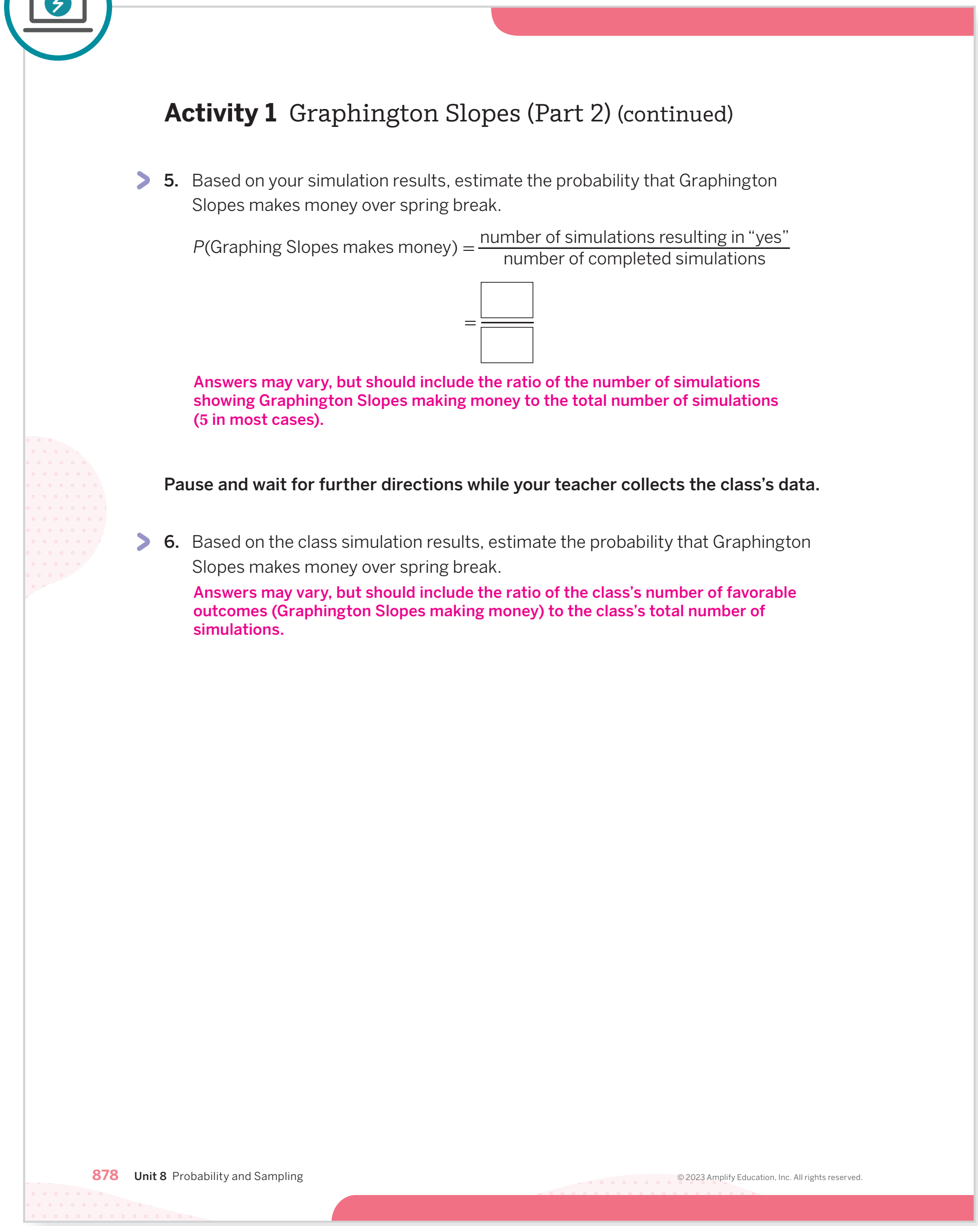
English Learners

Suggest that students draw diagrams or pictures to include in their descriptions.

Small Groups | 20 min

Activity 1 Graphington Slopes (Part 2) (continued)

Students perform the simulation created in the Warm-up to determine the relative frequency of a real-world event.



Activity 1 Graphington Slopes (Part 2) (continued)

5. Based on your simulation results, estimate the probability that Graphington Slopes makes money over spring break.

$$P(\text{Graphing Slopes makes money}) = \frac{\text{number of simulations resulting in “yes”}}{\text{number of completed simulations}}$$

$$= \frac{\square}{\square}$$

Answers may vary, but should include the ratio of the number of simulations showing Graphington Slopes making money to the total number of simulations (5 in most cases).

Pause and wait for further directions while your teacher collects the class's data.

6. Based on the class simulation results, estimate the probability that Graphington Slopes makes money over spring break.

Answers may vary, but should include the ratio of the class's number of favorable outcomes (Graphington Slopes making money) to the class's total number of simulations.

878 **Unit 8** Probability and Sampling

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3 Connect

Have groups of students share their simulation results and display them for the class to see. Have students complete Problem 5 using the class data.

Highlight that the experimental (observed) probability approaches the theoretical (expected) probability when many trials are observed.

Note: At this level, make the assumption that the class's simulation result is the theoretical probability of the event. In later grades, students will learn to calculate the precise theoretical probability of multi-step events. The theoretical probability of it snowing at least 4 days out of the 10 days is 0.44. Refer to the Activity 1 PDF, *Graphington Slopes: Theoretical Probability* for an explanation.

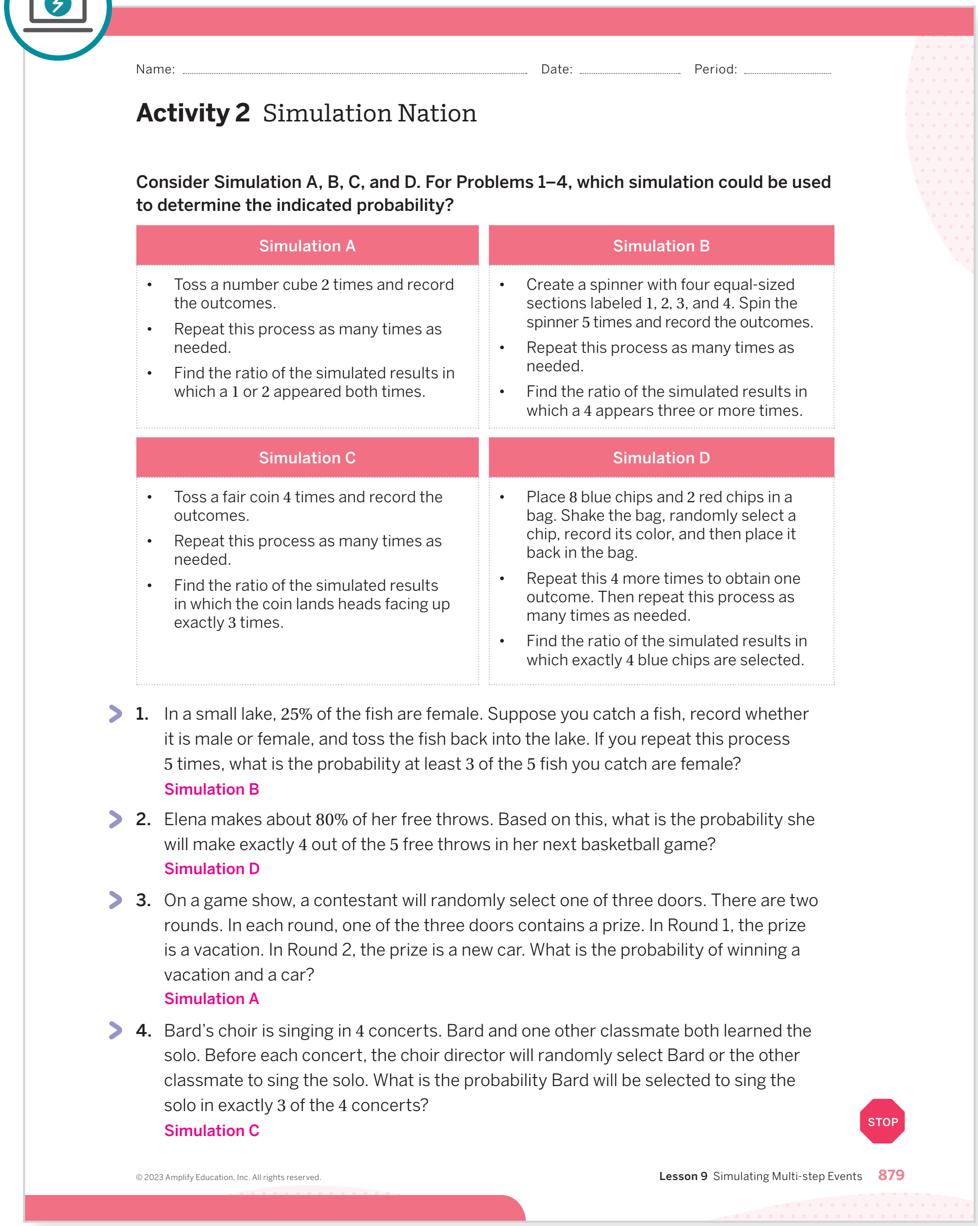
Ask:

- "How close was your group's estimated probability to the class's probability?"
- "Is your group's probability a good representative of the class's probability?"
- "The class performed ___ simulations and calculated the probability of Graphington Slopes making money to be ___. What do you think you could do to make the estimated probability the same as the expected probability?" Perform more simulations.
- "Do you anticipate Graphington Slopes will make money this year?" Answers may vary depending on the class's experimental (observed) probability. If the probability is less than 0.5, perhaps students would not expect to make money.

Small Groups | 10 min

Activity 2 Simulation Nation

Students practice what they learned about simulations by matching them with real-world scenarios.



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Activity 2 Simulation Nation

Consider Simulation A, B, C, and D. For Problems 1–4, which simulation could be used to determine the indicated probability?

Simulation A	Simulation B
• Toss a number cube 2 times and record the outcomes. • Repeat this process as many times as needed. • Find the ratio of the simulated results in which a 1 or 2 appeared both times.	• Create a spinner with four equal-sized sections labeled 1, 2, 3, and 4. Spin the spinner 5 times and record the outcomes. • Repeat this process as many times as needed. • Find the ratio of the simulated results in which a 4 appears three or more times.
Simulation C	**Simulation D**
• Toss a fair coin 4 times and record the outcomes. • Repeat this process as many times as needed. • Find the ratio of the simulated results in which the coin lands heads facing up exactly 3 times.	• Place 8 blue chips and 2 red chips in a bag. Shake the bag, randomly select a chip, record its color, and then place it back in the bag. • Repeat this 4 more times to obtain one outcome. Then repeat this process as many times as needed. • Find the ratio of the simulated results in which exactly 4 blue chips are selected.

1. In a small lake, 25% of the fish are female. Suppose you catch a fish, record whether it is male or female, and toss the fish back into the lake. If you repeat this process 5 times, what is the probability at least 3 of the 5 fish you catch are female?
   Simulation B
2. Elena makes about 80% of her free throws. Based on this, what is the probability she will make exactly 4 out of the 5 free throws in her next basketball game?
   Simulation D
3. On a game show, a contestant will randomly select one of three doors. There are two rounds. In each round, one of the three doors contains a prize. In Round 1, the prize is a vacation. In Round 2, the prize is a new car. What is the probability of winning a vacation and a car?
   Simulation A
4. Bard's choir is singing in 4 concerts. Bard and one other classmate both learned the solo. Before each concert, the choir director will randomly select Bard or the other classmate to sing the solo. What is the probability Bard will be selected to sing the solo in exactly 3 of the 4 concerts?
   Simulation C

STOP

Lesson 9 Simulating Multi-step Events 879

1 Launch

Read the directions and explain to students that each simulation will be used exactly one time.

2 Monitor

Help students get started by having them find the probabilities represented in each simulation, and then reading the problems to see which matches.

Look for points of confusion:

- **Not seeing the connection between Problem 3 and the number cube.** Have students write the sample space for a standard number cube and circle the numbers 1 and 2. Ask them what the probability is of rolling a 1 or 2.

Look for productive strategies:

- Explaining their reasoning with precise mathematical language.

3 Connect

Display the situations and simulations.

Have students share their matches using the *Poll the Class* routine, and explain their thinking.

Highlight how the simulations model the problems using the explanations from the students.

Ask:

- "For Problem 1, why is the spinner spun five times?" Five fish are caught.
- "For Problem 1, why does the same number need to be spun 3 or more times?" We are trying to find the probability of at least 3 being female.
- "For any given problem, could part of it be changed and still result in the simulation working?" Answers may vary, but should include maintaining the probability of the experiment.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with blank spinners, number cubes, coins, slips of paper and a bag or box that they can physically manipulate to help them visualize how they could use each one to simulate the situations.

Accessibility: *Vary Demands to Optimize Challenge*

Consider having students complete only Problems 1–3 and remove Simulation C from the list of simulations to choose.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they determined their matches, listen for and amplify the connection between the numerical quantities in the simulation and the situation. Display sentence frames for students to use, such as:

- "Situation ______ matches with Simulation ______ because . . ."
- "I saw that the text mentions ______, so I . . ."
- "I know that the spinner matches ______, because . . ."

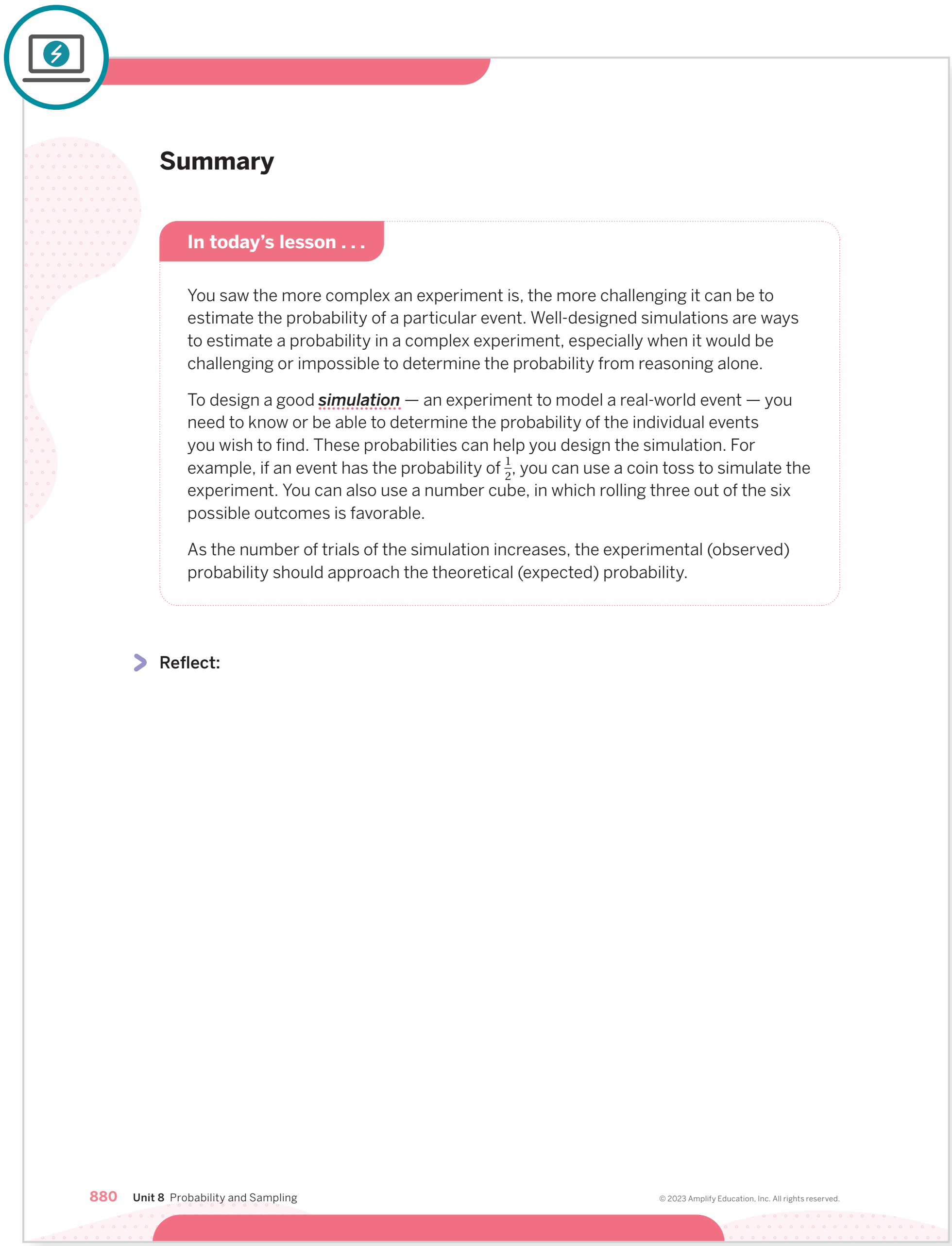
English Learners

Mention that the phrase "simulated results" means the results of the simulation.

Whole Class | 5 min

Summary

Review and synthesize how simulations are used to estimate probabilities.



Summary

In today's lesson . . .

You saw the more complex an experiment is, the more challenging it can be to estimate the probability of a particular event. Well-designed simulations are ways to estimate a probability in a complex experiment, especially when it would be challenging or impossible to determine the probability from reasoning alone.

To design a good ***simulation*** — an experiment to model a real-world event — you need to know or be able to determine the probability of the individual events you wish to find. These probabilities can help you design the simulation. For example, if an event has the probability of $\frac{1}{2}$, you can use a coin toss to simulate the experiment. You can also use a number cube, in which rolling three out of the six possible outcomes is favorable.

As the number of trials of the simulation increases, the experimental (observed) probability should approach the theoretical (expected) probability.

Reflect:

880 **Unit 8** Probability and Sampling

Synthesize

Have students share what they understand about simulations and how they are used to estimate probabilities.

Highlight experimental (observed) probability is calculated as the ratio of the number of observed favorable cases to the number of completed simulations. Performing more simulations should result in an observed probability which is closer to the expected probability. For instance, a simulation using 10,000 trials should have a better observed probability than one using only 100 trials.

Formalize vocabulary: *simulation*

Ask, "Each day, a student randomly reaches into a bowl of fruit and picks one for his lunch. To simulate the situation, he creates a spinner with four equal sections labeled: *apple*, *orange*, *pear*, and *peach*. Why might this simulation not represent the situation very well?" This simulation assumes each fruit is equally likely to be chosen. I do not know if there are the same number of each fruit. Also, as the week progresses, the remaining fruit might not have the same ratio as it did at the beginning of the week.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments to reflect on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Our world is really complicated — how can we simulate parts of it to make better predictions?

Math Language Development

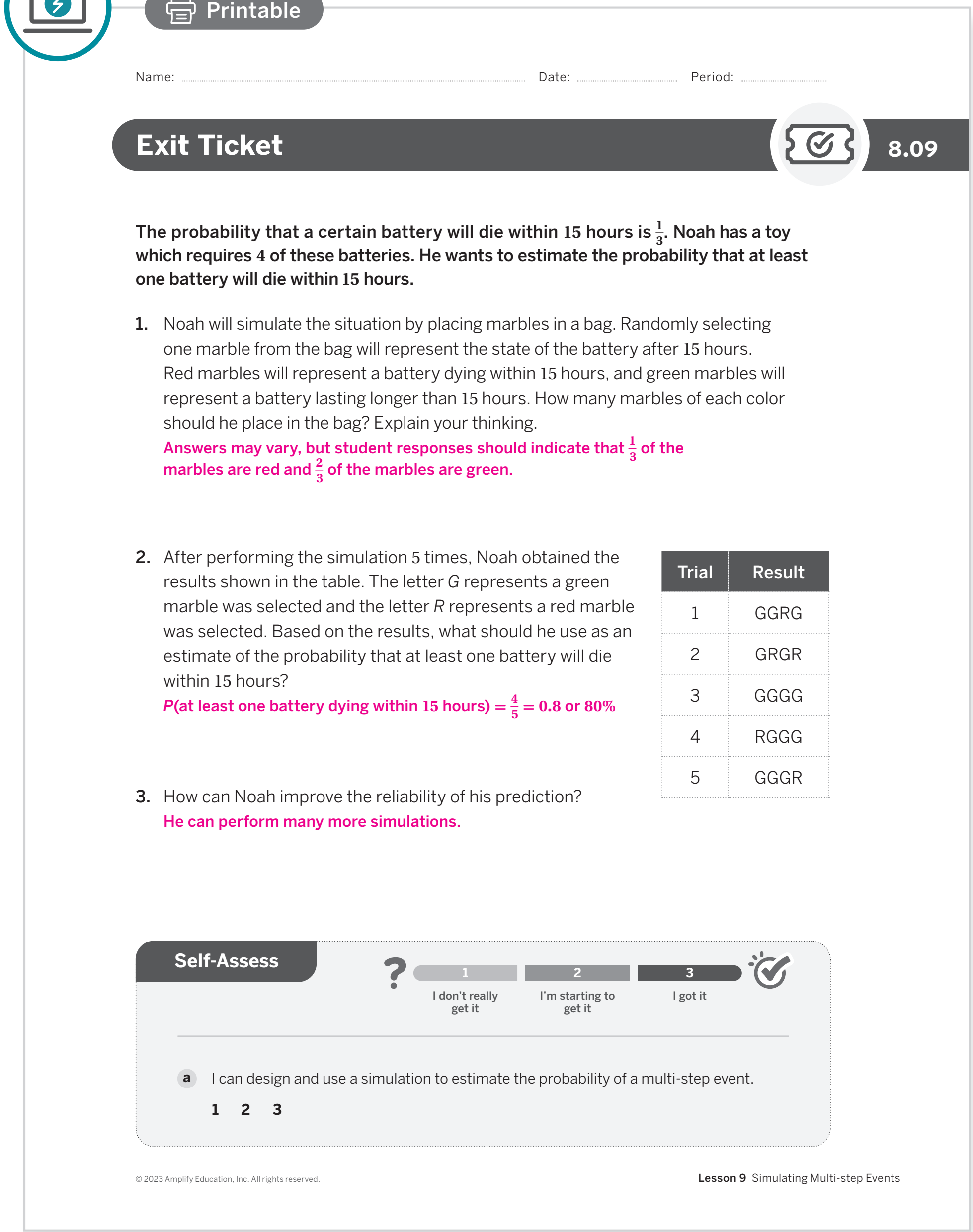
MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *simulation* that were added to the display during the lesson.

Exit Ticket

Students demonstrate their understanding of simulations by designing one and using it to estimate probabilities.

Printable



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Exit Ticket 8.09

The probability that a certain battery will die within 15 hours is $\frac{1}{3}$. Noah has a toy which requires 4 of these batteries. He wants to estimate the probability that at least one battery will die within 15 hours.

1. Noah will simulate the situation by placing marbles in a bag. Randomly selecting one marble from the bag will represent the state of the battery after 15 hours. Red marbles will represent a battery dying within 15 hours, and green marbles will represent a battery lasting longer than 15 hours. How many marbles of each color should he place in the bag? Explain your thinking.
   Answers may vary, but student responses should indicate that $\frac{1}{3}$ of the marbles are red and $\frac{2}{3}$ of the marbles are green.

2. After performing the simulation 5 times, Noah obtained the results shown in the table. The letter *G* represents a green marble was selected and the letter *R* represents a red marble was selected. Based on the results, what should he use as an estimate of the probability that at least one battery will die within 15 hours?
   P(at least one battery dying within 15 hours) $= \frac{4}{5} = 0.8$ or 80%

Trial	Result
1	GGRG
2	GRGR
3	GGGG
4	RGGG
5	GGGR

3. How can Noah improve the reliability of his prediction?
   He can perform many more simulations.

Self-Assess

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it

a I can design and use a simulation to estimate the probability of a multi-step event.
1 2 3

Success looks like . . .

- **Language Goal:** Coordinating a real-world situation and a chance event which could be used to simulate that situation. **(Speaking and Listening)**
  - » Explaining how many marbles of each color Noah should use in his simulation in Problem 1.
- **Language Goal:** Performing a multi-step simulation, and using the results to estimate the probability of a multi-step event in a real-world situation (using other representations). **(Speaking and Listening, Writing)**
  - » Using the results of Noah's five simulations to estimate the probability that at least one battery will die within 15 hours in Problem 2.

Suggested next steps

If students respond to Problem 1 with an incorrect ratio of marbles, consider:

- Reviewing ratios and equivalent fractions.
- Assigning Practice Problem 1.

If students respond to Problem 2 incorrectly, consider:

- Reviewing Activity 1, Problem 4.
- Assigning Practice Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Have you changed any ideas you used to have about simulations as a result of today's lesson?
- How was Activity 1 similar to or different from Activity 1 in Lesson 4? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Based on prior orders, a customer notices that when ordering takeout food from a particular restaurant, napkins were not included in the bag 50% of the time. Design a simulation you can use to estimate the probability of napkins not being included on every one of your next three takeout orders.

   Sample response: I can toss a coin and let heads facing up represent the event that napkins are not included and tails facing up represent the event that napkins are included. I can toss the coin three times to represent the next three takeout orders. This represents one trial. If I perform the simulation many times, I can find the ratio of the number of favorable outcomes (all three tosses landing heads facing up) to the total number of simulated trials.

2. Priya's cat is pregnant with a litter of 5 kittens. Each kitten has a 30% chance of being chocolate brown. Priya wants to know the probability that at least two of the kittens will be chocolate brown. To estimate this probability, Priya designed and conducted the following simulation. The table shows her results. The letter W represents selecting a white cube and the letter G represents selecting a green cube.

   Simulation

   - Place 3 white cubes and 7 green cubes in a bag.
   - Randomly select a cube, record its color, and place it back in the bag.
   - Repeat this 5 times to simulate one trial representing a litter of 5 kittens. Perform 12 trials.

Trial	Outcome
1	GGGGG
2	GGGWG
3	WGWGW
4	GWGGG
5	GGGWG
6	WWGGG
7	GWGGG
8	GGWGW
9	WWWGG
10	GGGGW
11	WGGWG
12	GGGWG

   a. How many successful trials were there? Describe how you determined if a trial was a success.

   There are 5 successful trials because at least two white cubes are present. Selecting a white cube simulates a chocolate brown kitten.

   b. Based on this simulation, estimate the probability that *exactly* two kittens will be chocolate brown.

   P(exactly two chocolate brown kittens) $= \frac{3}{12} = \frac{1}{4}$

   c. Based on this simulation, estimate the probability that *at least* two kittens will be chocolate brown.

   P(at least two chocolate brown kittens) $= \frac{5}{12}$

   d. Write and solve another probability problem Priya could estimate using this simulation.

   Sample response: What is the probability that *none* of the kittens are chocolate brown?
   P(no chocolate brown kittens) $= \frac{1}{12}$

   e. How could Priya increase the accuracy of her predictions?

   She could perform more simulations.

Name: ______ Date: ______ Period: ______

Practice

3. Based on prior games played, a team's record shows they have won 75% of the games they have played so far this season. They will play 3 games this week. To estimate the probability that the team will win at least two of these games, Clare designs the simulation shown in the table. She obtains a result of "win," "win," "lose."

   Simulation

   - Place 4 cards in a bag, 3 of which are labeled "win" and 1 of which is labeled "lose."
   - Randomly select a card, record the result, and place the card back in the bag. This represents one trial of playing a game.
   - Perform three trials to simulate playing three games.

   What is the estimated probability that the team will win at least two games? Explain whether Clare can trust this estimate.

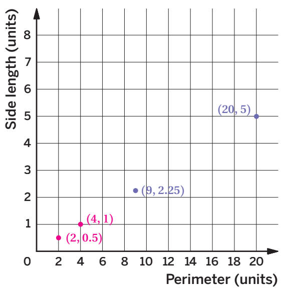
   Sample response: Based on the results, Clare can estimate the probability of winning to be $\frac{1}{1} = 1$. However, this simulation was only performed one time and, to get a more accurate prediction, Clare will need to perform many more trials.

4. The graph of the relationship between the perimeter of a square and its side length are shown. Two points are plotted.

   a. Plot and label two more points that represent the relationship.

   b. Is there a proportional relationship between the perimeter of a square and its side length? Explain your thinking.

   Yes, there is a proportional relationship; Sample response: For every ordered pair, the side length divided by the perimeter is 0.25.



5. Design a simulation of an experiment in which the favorable outcome has a probability of 0.25.

   Sample response: Creating a spinner with four equal-sized sections labeled 1, 2, 3, and 4. Spin the spinner once. A favorable outcome is landing on one of the sections.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 11	2
Formative ⚡	5	Unit 8 Lesson 10	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Optional

UNIT 8 | **LESSON 10**

Designing Simulations

Let's simulate some real-world experiments.

Focus

Goals

1. **Language Goal:** Describe a multi-step experiment to simulate a multi-step event in a real-world situation, and justify that it represents the situation. **(Speaking and Listening, Writing)**
2. **Language Goal:** Perform a simulation to estimate the probability of a multi-step event, and explain how the simulation could be improved. **(Speaking and Listening, Writing)**

Coherence

Today

Students estimate probabilities by designing and performing simulations of multi-step events. This lesson gives the option of every group performing the scaffolded simulation or designing their own simulation using tools (e.g., spinners, number cubes, blocks, etc.). This provides an opportunity for students to practice communicating precisely.

Previously

In earlier lessons, students calculated probabilities of multi-step events and described simulations to model real-world events.

Coming Soon

In the next Sub-Unit, students compare distributions and discuss ways to use sample populations of data. Throughout the last lessons, Practice Problems, labeled as *Capstone project helper*, will aid students in their statistical Capstone project for Lesson 17.

Rigor

- Students gain **procedural skills** by performing simulations and tracking results.
- Students **apply** their understanding of simulations to reason how they can be improved.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
5 min	25 min	5 min	10 min
Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers, as needed)
- Activity 1 PDF, *Are you ready for more?* (optional)
- bags
- coins
- colored blocks or marbles
- number cubes
- paper clips
- spinners

Math Language Development

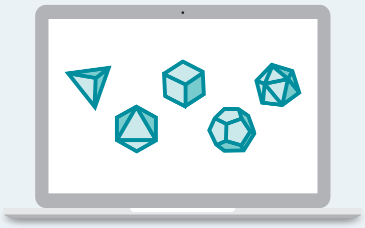
Review words

- *multi-step event*
- *simulation*

Amps | Featured Activity

Activity 1
Digital Simulation

Students are able to perform the simulation digitally which will limit the number of materials required and provide a more realistic idea of how simulations are performed in the real world.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Throughout this lesson, students will select tools to simulate probability experiments. They may be unsure of selecting a tool, especially if they are unfamiliar with the tool. Encourage them to experiment with different tools until they find one that will work for each simulation. You may want to spend extra time during the Warm-up to familiarize everyone with the tools listed.

Modifications to Pacing

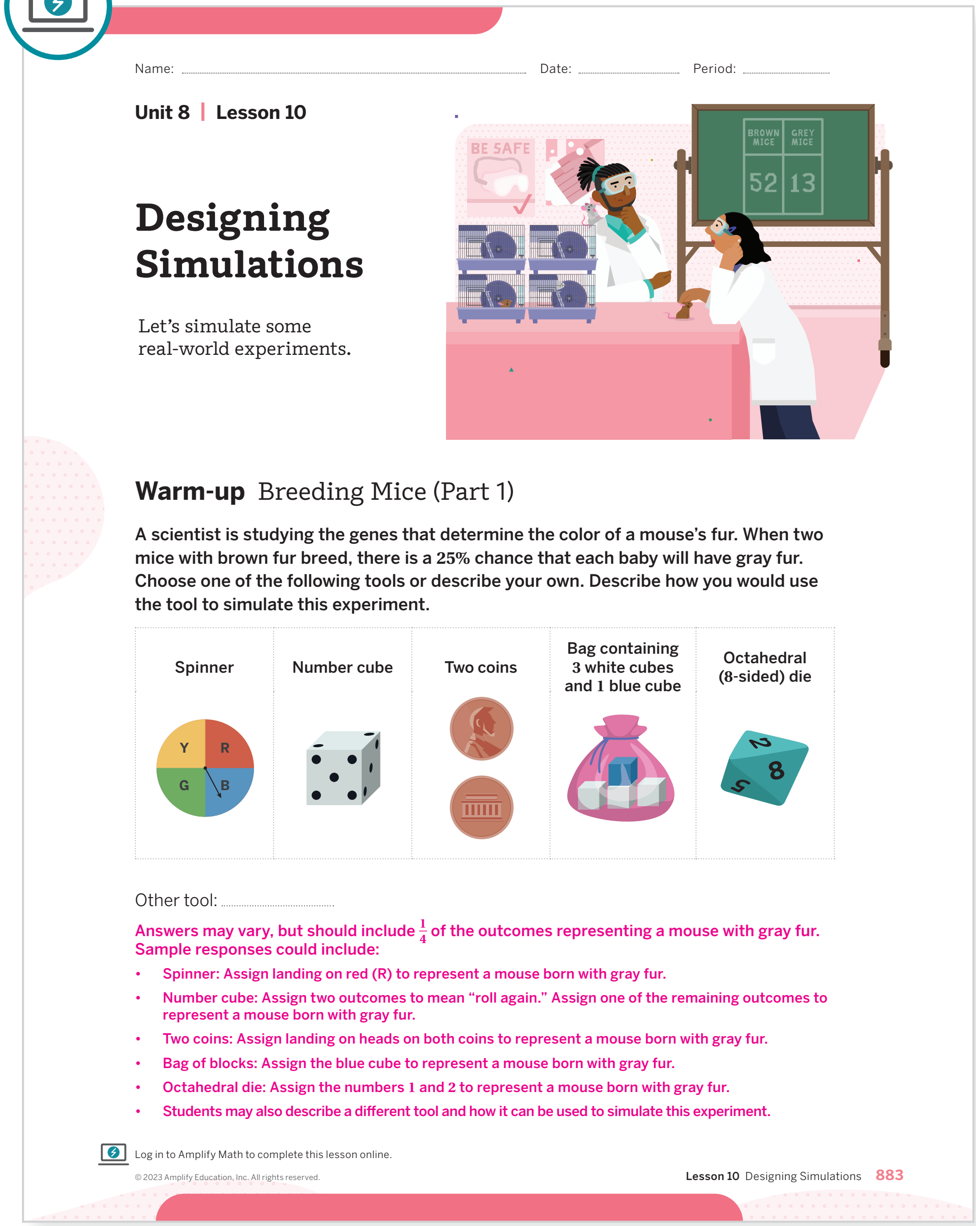
You may want to consider these additional modifications if you are short on time.

- This entire lesson may be omitted, as it is an optional lesson.
- In **Activity 1**, perform the simulations digitally using the Desmos Featured Activity.

Independent | 5 min

Warm-up Breeding Mice (Part 1)

Students select a tool and describe how it could be used to simulate a multi-step event from Activity 1.



Name: .... Date: .... Period: ....

Unit 8 | Lesson 10

Designing Simulations

Let's simulate some real-world experiments.

Warm-up Breeding Mice (Part 1)

A scientist is studying the genes that determine the color of a mouse's fur. When two mice with brown fur breed, there is a 25% chance that each baby will have gray fur. Choose one of the following tools or describe your own. Describe how you would use the tool to simulate this experiment.

Spinner	Number cube	Two coins	Bag containing 3 white cubes and 1 blue cube	Octahedral (8-sided) die

Other tool: ....

Answers may vary, but should include $\frac{1}{4}$ of the outcomes representing a mouse with gray fur. Sample responses could include:

- **Spinner: Assign landing on red (R) to represent a mouse born with gray fur.**
- **Number cube: Assign two outcomes to mean "roll again." Assign one of the remaining outcomes to represent a mouse born with gray fur.**
- **Two coins: Assign landing on heads on both coins to represent a mouse born with gray fur.**
- **Bag of blocks: Assign the blue cube to represent a mouse born with gray fur.**
- **Octahedral die: Assign the numbers 1 and 2 to represent a mouse born with gray fur.**
- **Students may also describe a different tool and how it can be used to simulate this experiment.**

Log in to Amplify Math to complete this lesson online.

Lesson 10 Designing Simulations 883

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking them what the favorable event is and what the probability is of that event occurring.

Look for points of confusion:

- **Thinking they have to describe the simulation for each tool.** Remind students they only have to choose one.
- **Selecting 5 sides of the number cube to represent non-white fur.** Remind students the probability for the simulation must match the probability of the event.

3 Connect

Display the images of the tools from the Warm-up.

Have students share what tool they chose and how they would use it to simulate the real-world experiment.

Ask, "Did anyone choose the same tool, but set up their simulation differently?"

Highlight that no matter which tool the students chose, they are able to simulate the experiment as long as the probability of the favorable outcome matches the probability of the simulated event.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with blank spinners, paper clips, number cubes, coins, slips of paper and a bag or box that they can physically manipulate to help them visualize how they could use each one to simulate the situations. Students may be unfamiliar with an octahedral die. If they are available, allow students to manipulate them. Otherwise, explain that an octahedral die is an eight-sided die labeled with the numbers 1–8.

Power-up

To power up students' ability to simulate an event given a probability have students complete:

Provide students with a copy of the Power-up PDF.

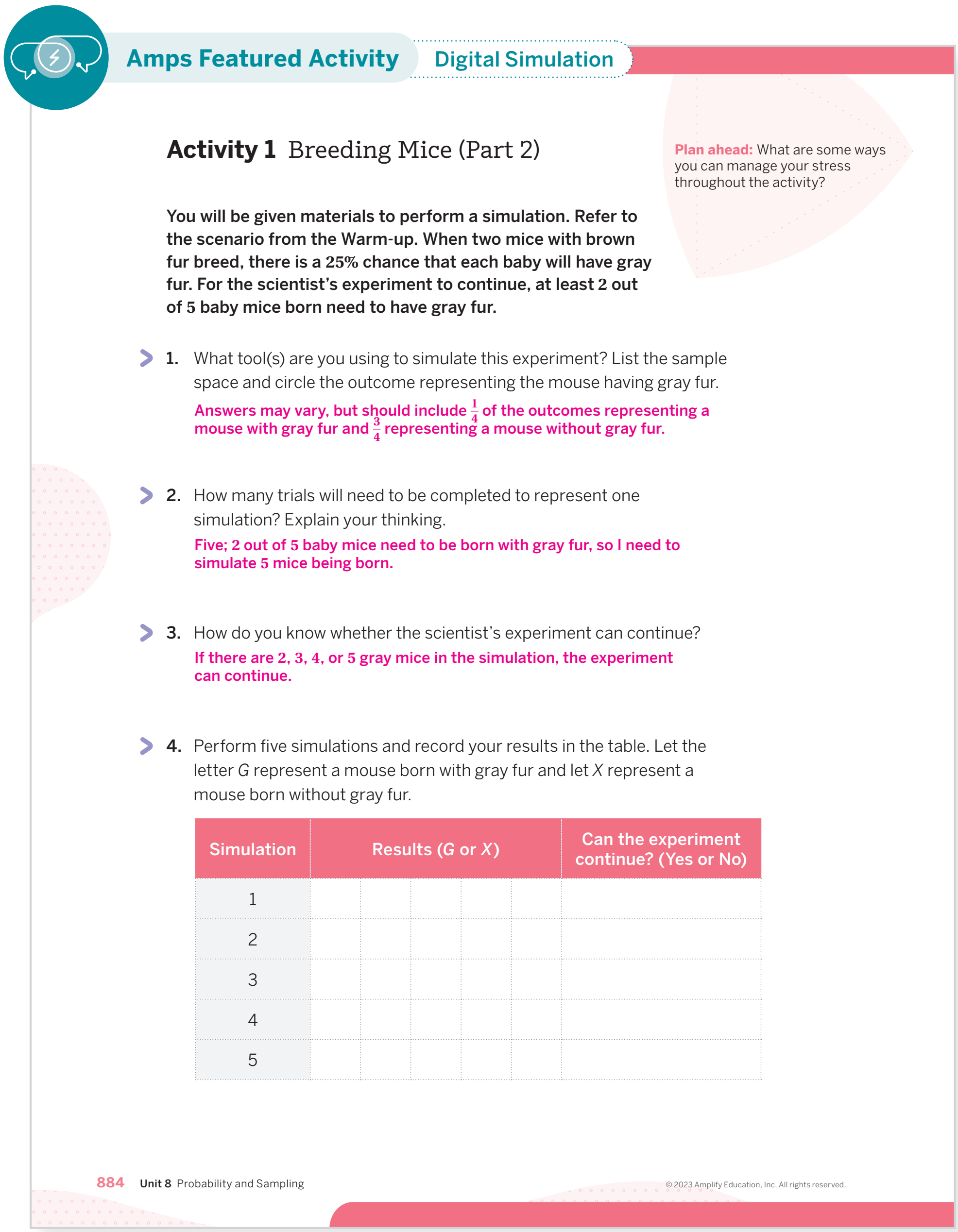
Use: Before the Warm-up

Informed by: Performance on Lesson 9, Practice Problem 5

Small Groups | 25 min

Activity 1 Breeding Mice (Part 2)

Students perform the simulation of their choice to estimate probabilities.



Amps Featured Activity Digital Simulation

Activity 1 Breeding Mice (Part 2)

Plan ahead: What are some ways you can manage your stress throughout the activity?

You will be given materials to perform a simulation. Refer to the scenario from the Warm-up. When two mice with brown fur breed, there is a 25% chance that each baby will have gray fur. For the scientist's experiment to continue, at least 2 out of 5 baby mice born need to have gray fur.

1. What tool(s) are you using to simulate this experiment? List the sample space and circle the outcome representing the mouse having gray fur.

   Answers may vary, but should include $\frac{1}{4}$ of the outcomes representing a mouse with gray fur and $\frac{3}{4}$ representing a mouse without gray fur.

2. How many trials will need to be completed to represent one simulation? Explain your thinking.

   Five; 2 out of 5 baby mice need to be born with gray fur, so I need to simulate 5 mice being born.

3. How do you know whether the scientist's experiment can continue?

   If there are 2, 3, 4, or 5 gray mice in the simulation, the experiment can continue.

4. Perform five simulations and record your results in the table. Let the letter G represent a mouse born with gray fur and let X represent a mouse born without gray fur.

Simulation	Results (G or X)					Can the experiment continue? (Yes or No)
1						
2						
3						
4						
5						

884 **Unit 8** Probability and Sampling

1 Launch

Provide groups of students with the necessary materials (e.g., number cube, coins, bags with colored blocks, spinner, etc.) needed to perform their simulations. If students opt to design their own simulation modeling the problem from the *Are you ready for more?* problems, provide a copy of the Activity 1 PDF, *Are you ready for more?*

2 Monitor

Help students get started by having them record the sample space.

Have students list their sample space based on their chosen simulation tool(s) and helping them complete Problem 1.

Look for points of confusion:

- **Thinking each trial completes a simulation.** Remind students they need 5 trials to complete their simulation and then need to complete the simulation 5 times.
- **Combining all the simulation results to calculate the probability.** Remind students the probability is found by finding the ratio of the number of successful simulations to the total number of completed simulations (most likely 5 in this situation).

Look for productive strategies:

- Using precise mathematical language to describe their simulation and the resulting probabilities of the simulation.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*
Display the same images of the tools from the Warm-up to help students select a tool for their simulation. Allow access to blank spinners, number cubes, coins, slips of paper and a bag or box to help them visualize how they could use each one to simulate the experiment.

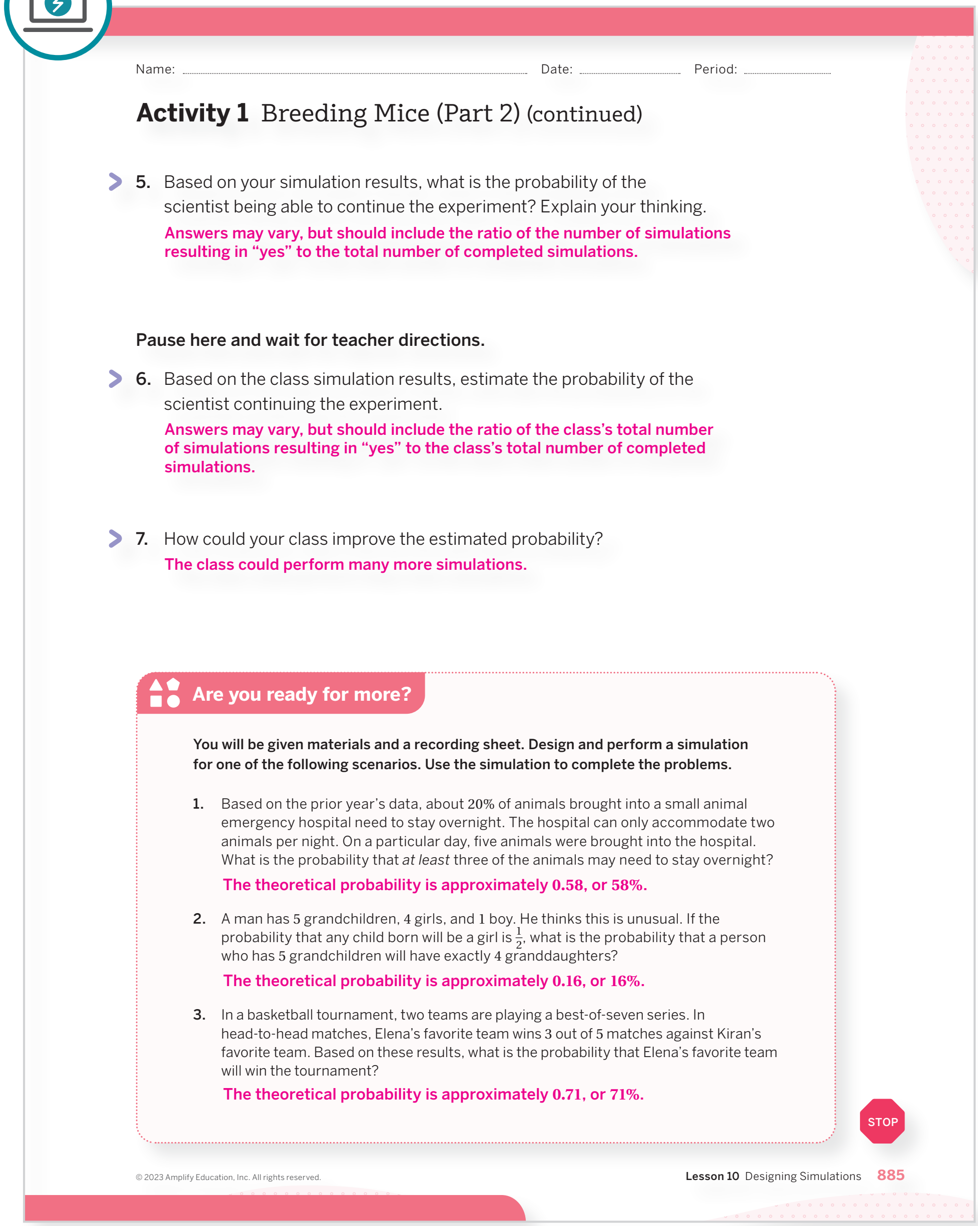
Accessibility: *Optimize Access to Technology*
Have students use the Amps slides for this activity, in which they can perform the simulation in a digital environment which limits the number of materials required. By doing so, students are provided with a more realistic way of how simulations are often performed in the real world.

Extension: *Math Enrichment*
Have students complete the Activity 1 PDF, *Are you ready for more?*, instead of this activity. They will choose a scenario from the *Are you ready for more?* in their Student Edition and design and conduct a simulation.

Small Groups | 25 min

Activity 1 Breeding Mice (Part 2) (continued)

Students perform the simulation of their choice to estimate probabilities.



Name: ______________________ Date: __________ Period: __________

Activity 1 Breeding Mice (Part 2) (continued)

5. Based on your simulation results, what is the probability of the scientist being able to continue the experiment? Explain your thinking.
   Answers may vary, but should include the ratio of the number of simulations resulting in "yes" to the total number of completed simulations.

Pause here and wait for teacher directions.

6. Based on the class simulation results, estimate the probability of the scientist continuing the experiment.
   Answers may vary, but should include the ratio of the class's total number of simulations resulting in "yes" to the class's total number of completed simulations.

7. How could your class improve the estimated probability?
   The class could perform many more simulations.

Are you ready for more?

You will be given materials and a recording sheet. Design and perform a simulation for one of the following scenarios. Use the simulation to complete the problems.

1. Based on the prior year's data, about 20% of animals brought into a small animal emergency hospital need to stay overnight. The hospital can only accommodate two animals per night. On a particular day, five animals were brought into the hospital. What is the probability that *at least* three of the animals may need to stay overnight?
   The theoretical probability is approximately 0.58, or 58%.
2. A man has 5 grandchildren, 4 girls, and 1 boy. He thinks this is unusual. If the probability that any child born will be a girl is $\frac{1}{2}$, what is the probability that a person who has 5 grandchildren will have exactly 4 granddaughters?
   The theoretical probability is approximately 0.16, or 16%.
3. In a basketball tournament, two teams are playing a best-of-seven series. In head-to-head matches, Elena's favorite team wins 3 out of 5 matches against Kiran's favorite team. Based on these results, what is the probability that Elena's favorite team will win the tournament?
   The theoretical probability is approximately 0.71, or 71%.

STOP

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3 Connect

Display the class results for Problem 5. Allow students to complete Problems 6 and 7 before the class discussion.

Have groups of students share their simulation method and results. If students completed the *Are you ready for more?* problems, have them share their prompt and their simulation design, perhaps in a planned presentation.

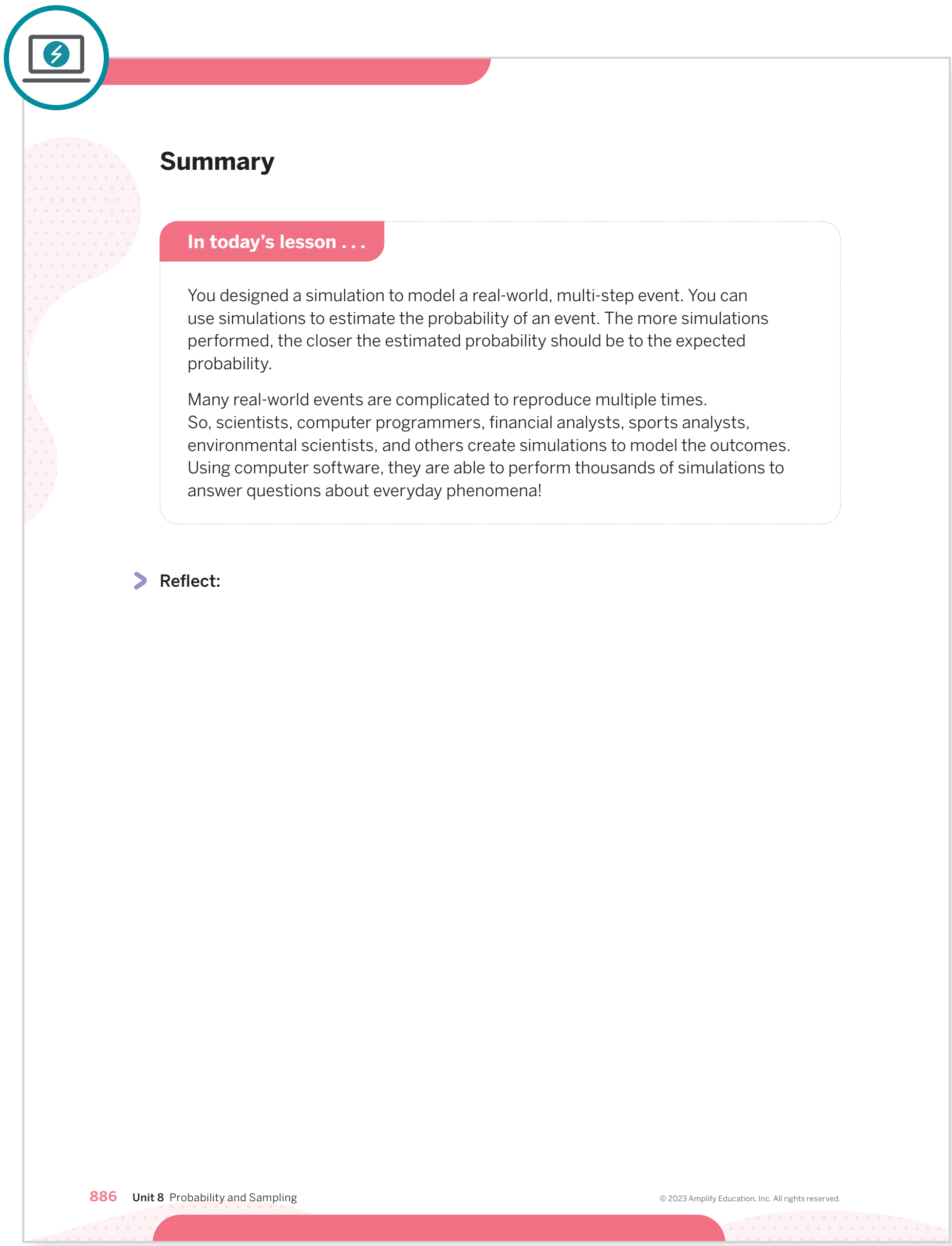
Highlight that good simulations model the real-world event and have matching probabilities for the events.

Ask, "How could you get a better estimate than what you got in your group?" Repeat the experiment many more times and combine the data from the class.

Whole Class | 5 min

Summary

Review and synthesize the characteristics of a good simulation.



Summary

In today's lesson . . .

You designed a simulation to model a real-world, multi-step event. You can use simulations to estimate the probability of an event. The more simulations performed, the closer the estimated probability should be to the expected probability.

Many real-world events are complicated to reproduce multiple times. So, scientists, computer programmers, financial analysts, sports analysts, environmental scientists, and others create simulations to model the outcomes. Using computer software, they are able to perform thousands of simulations to answer questions about everyday phenomena!

Reflect:

886 Unit 8 Probability and Sampling

Synthesize

Highlight that a good simulation must model the event and have the same probability. To make a simulation produce a better estimated probability, perform many trials. This is why computer programs are often used to simulate experiments.

Ask:

- "What are some things you had to consider when designing your simulation?" The simulation needed to have the same theoretical probability as the actual event.
- "What did you learn from the simulations the other groups performed?"
- "Were the results of any of the simulations surprising?"
- "Why would it make sense to design and perform a simulation rather than repeat the actual experiment multiple times?" It would make sense when the actual experiment is costly in time or resources or cannot be controlled or repeated.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments to reflect on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

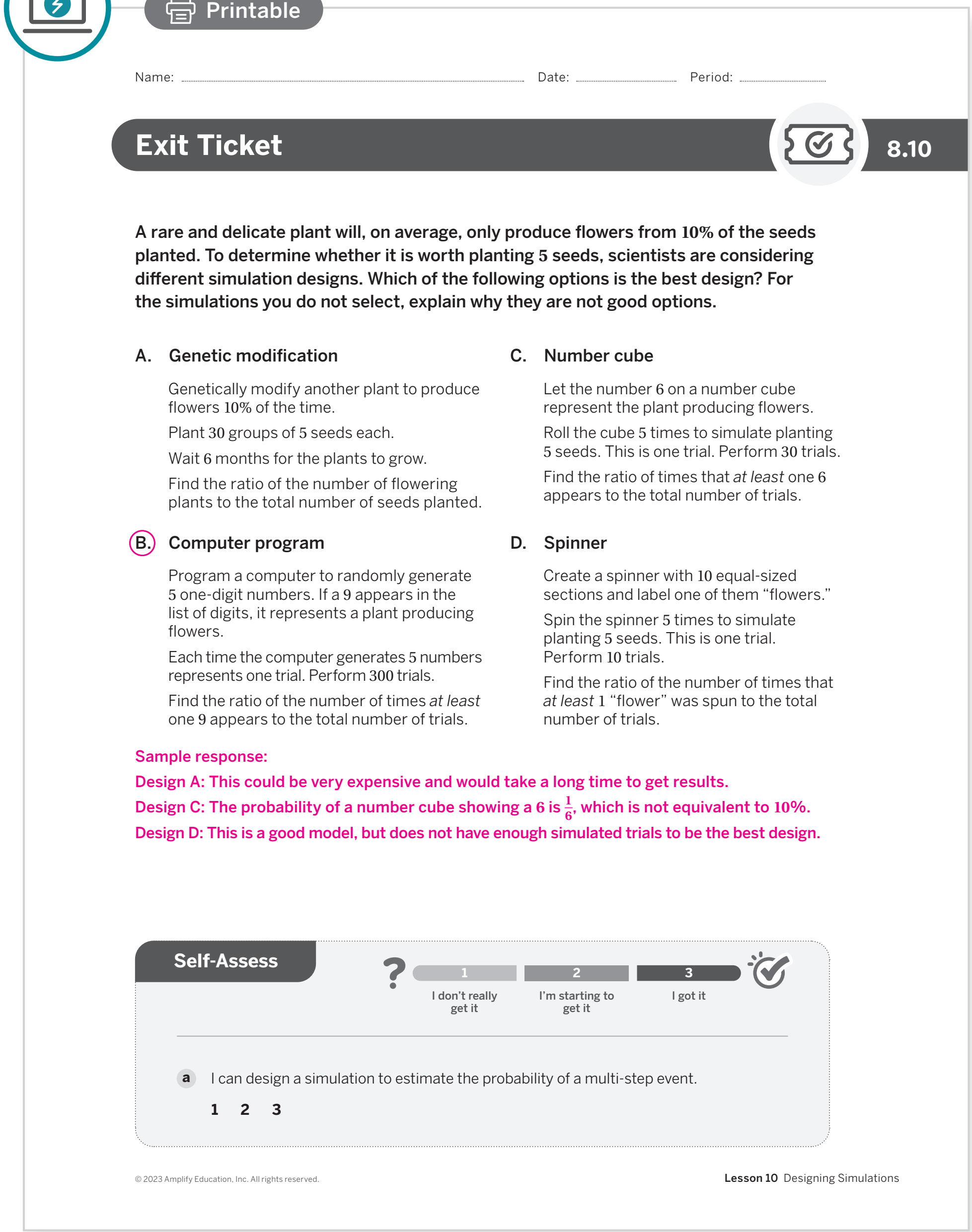
- "Our world is really complicated — how can we simulate parts of it to make better predictions?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by analyzing simulation designs to determine which design is the best and why the others are not good simulations.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 8.10

A rare and delicate plant will, on average, only produce flowers from 10% of the seeds planted. To determine whether it is worth planting 5 seeds, scientists are considering different simulation designs. Which of the following options is the best design? For the simulations you do not select, explain why they are not good options.

A. **Genetic modification**

Genetically modify another plant to produce flowers 10% of the time.

Plant 30 groups of 5 seeds each.

Wait 6 months for the plants to grow.

Find the ratio of the number of flowering plants to the total number of seeds planted.

[✓ B.] **Computer program**

Program a computer to randomly generate 5 one-digit numbers. If a 9 appears in the list of digits, it represents a plant producing flowers.

Each time the computer generates 5 numbers represents one trial. Perform 300 trials.

Find the ratio of the number of times *at least* one 9 appears to the total number of trials.

C. **Number cube**

Let the number 6 on a number cube represent the plant producing flowers.

Roll the cube 5 times to simulate planting 5 seeds. This is one trial. Perform 30 trials.

Find the ratio of times that *at least* one 6 appears to the total number of trials.

D. **Spinner**

Create a spinner with 10 equal-sized sections and label one of them “flowers.”

Spin the spinner 5 times to simulate planting 5 seeds. This is one trial. Perform 10 trials.

Find the ratio of the number of times that *at least* 1 “flower” was spun to the total number of trials.

Sample response:

Design A: This could be very expensive and would take a long time to get results.

Design C: The probability of a number cube showing a 6 is $\frac{1}{6}$, which is not equivalent to 10%.

Design D: This is a good model, but does not have enough simulated trials to be the best design.

Self-Assess

? **1** I don’t really get it | **2** I’m starting to get it | **3** I got it ✓

a I can design a simulation to estimate the probability of a multi-step event.

1 2 3

Success looks like . . .

- **Language Goal:** Describing a multi-step experiment to simulate a multi-step event in a real-world situation, and justifying that it represents the situation. **(Speaking and Listening, Writing)**
  - » Explaining which option is the best design of a simulation for the scientists to perform.
- **Language Goal:** Performing a simulation to estimate the probability of a multi-step event, and explaining how the simulation could be improved. **(Speaking and Listening, Writing)**

Suggested next steps

If students select A as the best simulation, consider:

- Reminding them that a simulation is meant to get results quickly so the data can be analyzed.

If students select B as the best simulation, consider:

- Reminding them that a simulation should have the same theoretical probability as the actual events.

If students select D as the best simulation, consider:

- Reminding them that the more trials a simulation has, the better it will estimate the probability.

Professional Learning

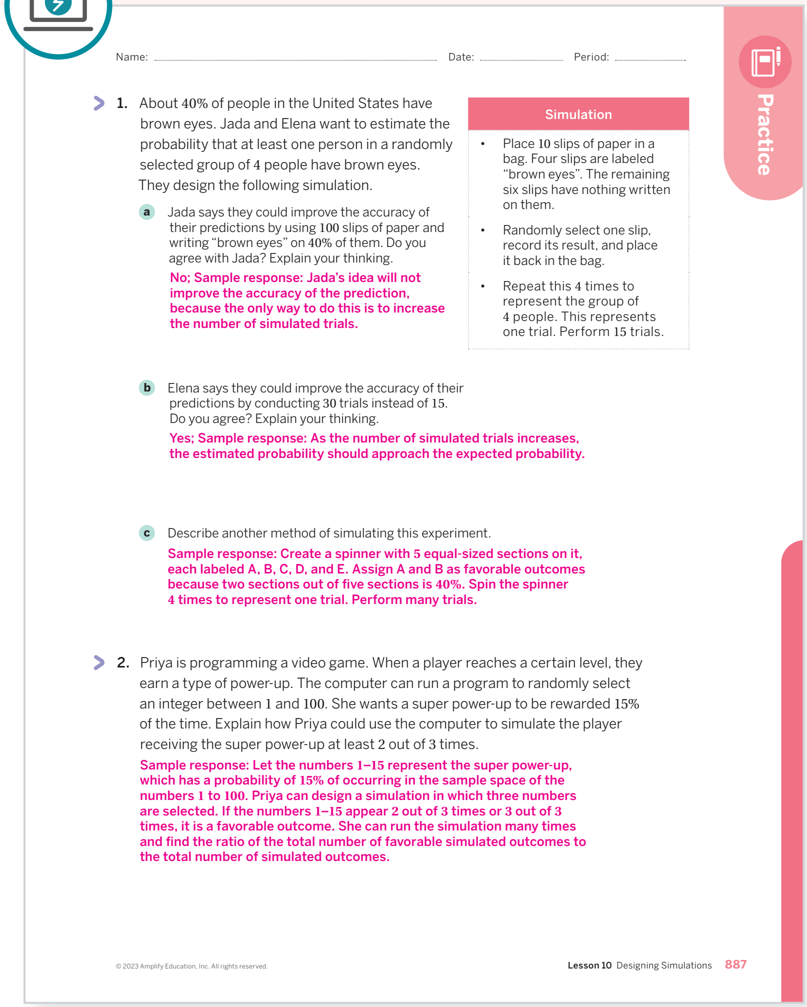
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn’t work today? What did students find frustrating about Activity 1? What helped them work through this frustration?
- The instructional goal for this lesson was for students to design and perform simulations for multi-step events. How well did students accomplish this? What did you specifically do to help students accomplish it? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. About 40% of people in the United States have brown eyes. Jada and Elena want to estimate the probability that at least one person in a randomly selected group of 4 people have brown eyes. They design the following simulation.

Simulation

- Place 10 slips of paper in a bag. Four slips are labeled "brown eyes". The remaining six slips have nothing written on them.
- Randomly select one slip, record its result, and place it back in the bag.
- Repeat this 4 times to represent the group of 4 people. This represents one trial. Perform 15 trials.

a Jada says they could improve the accuracy of their predictions by using 100 slips of paper and writing "brown eyes" on 40% of them. Do you agree with Jada? Explain your thinking.

No; Sample response: Jada's idea will not improve the accuracy of the prediction, because the only way to do this is to increase the number of simulated trials.

b Elena says they could improve the accuracy of their predictions by conducting 30 trials instead of 15. Do you agree? Explain your thinking.

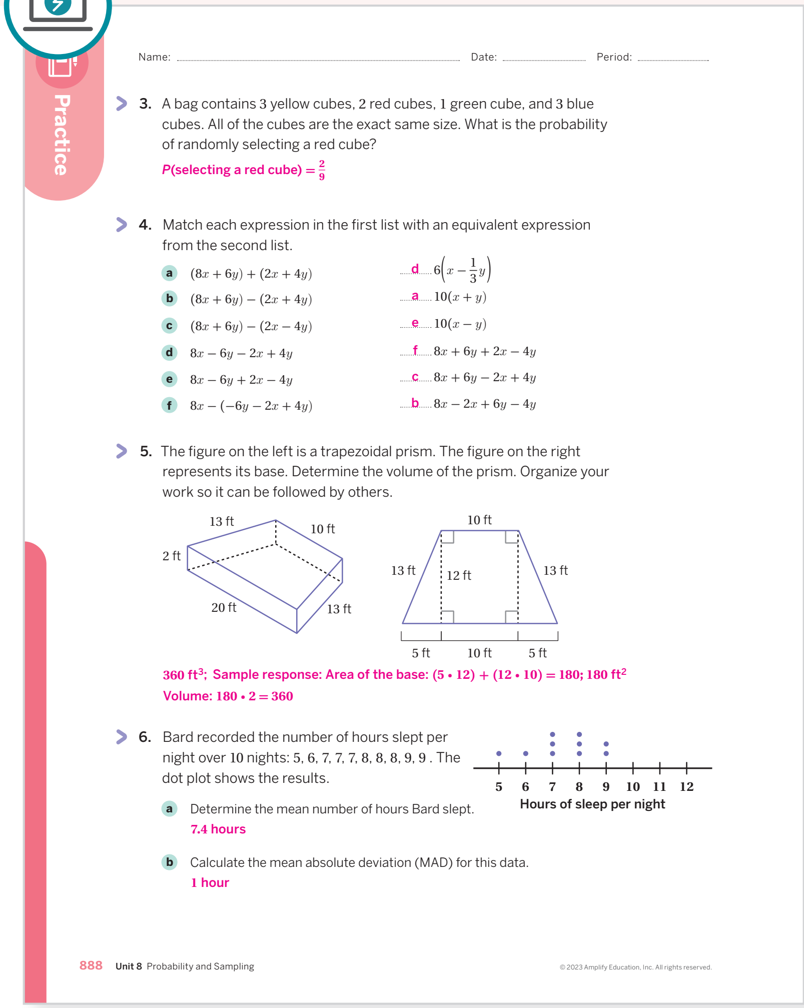
Yes; Sample response: As the number of simulated trials increases, the estimated probability should approach the expected probability.

c Describe another method of simulating this experiment.

Sample response: Create a spinner with 5 equal-sized sections on it, each labeled A, B, C, D, and E. Assign A and B as favorable outcomes because two sections out of five sections is 40%. Spin the spinner 4 times to represent one trial. Perform many trials.

2. Priya is programming a video game. When a player reaches a certain level, they earn a type of power-up. The computer can run a program to randomly select an integer between 1 and 100. She wants a super power-up to be rewarded 15% of the time. Explain how Priya could use the computer to simulate the player receiving the super power-up at least 2 out of 3 times.

Sample response: Let the numbers 1–15 represent the super power-up, which has a probability of 15% of occurring in the sample space of the numbers 1 to 100. Priya can design a simulation in which three numbers are selected. If the numbers 1–15 appear 2 out of 3 times or 3 out of 3 times, it is a favorable outcome. She can run the simulation many times and find the ratio of the total number of favorable simulated outcomes to the total number of simulated outcomes.



Practice

Name: Date: Period:

3. A bag contains 3 yellow cubes, 2 red cubes, 1 green cube, and 3 blue cubes. All of the cubes are the exact same size. What is the probability of randomly selecting a red cube?

P(selecting a red cube) $= \frac{2}{9}$

4. Match each expression in the first list with an equivalent expression from the second list.

a	$(8x + 6y) + (2x + 4y)$	**d**	$6\left(x - \frac{1}{3}y\right)$
b	$(8x + 6y) - (2x + 4y)$	**a**	$10(x + y)$
c	$(8x + 6y) - (2x - 4y)$	**e**	$10(x - y)$
d	$8x - 6y - 2x + 4y$	**f**	$8x + 6y + 2x - 4y$
e	$8x - 6y + 2x - 4y$	**c**	$8x + 6y - 2x + 4y$
f	$8x - (-6y - 2x + 4y)$	**b**	$8x - 2x + 6y - 4y$

5. The figure on the left is a trapezoidal prism. The figure on the right represents its base. Determine the volume of the prism. Organize your work so it can be followed by others.

360 ft^3; Sample response: Area of the base: $(5 \cdot 12) + (12 \cdot 10) = 180$; 180 ft^2

Volume: $180 \cdot 2 = 360$

6. Bard recorded the number of hours slept per night over 10 nights: 5, 6, 7, 7, 7, 8, 8, 8, 9, 9 . The dot plot shows the results.

a Determine the mean number of hours Bard slept.

7.4 hours

b Calculate the mean absolute deviation (MAD) for this data.

1 hour

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	3
Spiral	3	Unit 8 Lesson 3	1
	4	Unit 6 Lesson 22	1
	5	Unit 7 Lesson 15	1
Formative ⚡	6	Unit 8 Lesson 11	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Sub-Unit 3
Sampling

Whole Class

In this Sub-Unit, students will learn how to identify whether a sample is representative of its population and understand the importance of random sampling.

SUB-UNIT

3 Sampling

Narrative Connections

What's on your mind?

Forget about school for a second. Let's talk about you.

What are the things *you* enjoy? What are you passionate about?

Everyone has something that drives them. Maybe it is the environment. Maybe it is community organizing. Maybe it is competitive chicken grooming. As you think about what you want to do when you grow up, do not lose track of the things you care about.

What starts as an interest can turn into a hobby, then a passion, and even a full-fledged career!

The trick is to stay curious about what you love. In fact, that's what researchers do. They look at the things that interest them, and turn these things into research questions.

Is the new traffic light on Main Street keeping people safe?

Does owning a pet help you live longer?

Do video games help with problem-solving?

With the right questions, you can dig deep into the way people think, open doors to new knowledge, and even change the way people behave.

In these next few lessons, it will be up to you to develop your own research question. And whatever that question may be, the key to getting accurate answers will come down to finding the right group of people to ask.

Sub-Unit 3 Sampling 889

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how sampling and statistics can help them answer questions in the following places:

- **Lesson 11, Activities 1-2:** Team Heights, Family Heights
- **Lesson 13, Activities 1-3:** Fish Market, Sampling the Fish Market (Part 1), Sampling the Fish Market (Part 2)
- **Lesson 15, Activities 1-2:** Three Different Shows, Making a Recommendation
- **Lesson 16, Activities 1-2:** Travel Times, A New Comic Book Hero

UNIT 8 | **LESSON 11**

Comparing Two Populations

Let's compare two populations of data.

Focus

Goals

1. **Language Goal:** Calculate the mean and mean absolute deviation (MAD) for a data set, and interpret these measures. **(Speaking and Listening)**
2. **Language Goal:** Compare and contrast populations represented on dot plots in terms of their center, spread, and visual overlap. **(Speaking and Listening, Writing)**
3. **Language Goal:** Justify whether two populations are "very different" based on the difference in their means expressed as a multiple of the mean absolute deviation. **(Writing)**

Coherence

Today

Students work at deciding whether two groups are different from each other. They use a quantifiable method of determining if the two groups are relatively close or relatively different using means and MADs of the two groups. For the problems in this lesson, the populations under study are small and the data for the entire populations are known.

Previously

In Grade 6, students calculated the mean absolute deviation (MAD) of a data set and used it to describe the spread of the data.

Coming Soon

In Lesson 12, students will revisit the method of reasoning used in today's lesson to decide whether there is a meaningful difference between two populations' given data from only a sample of each of the populations. At the end of the unit, students will present their findings concerning their self-selected question. In order to prepare for the Capstone project, students will be given a milestone problem in each of the remaining lessons.

Rigor

- Students build **conceptual understanding** of how the mean and mean absolute deviation can be used to determine whether two sets of data are very different from each other.
- Students further develop **procedural skills** for calculating the mean absolute deviation.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Whole Class	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, *Comparing Two Populations* (for display, as needed)
- Graphic Organizer PDF, *MAD Recording Sheet,* one per student

Math Language Development

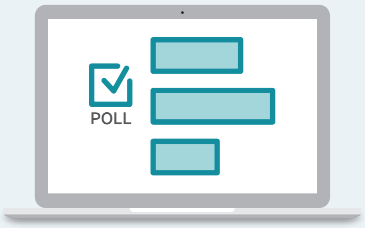
Review words

- *center*
- *mean*
- *mean absolute deviation (MAD)*
- *spread*
- *variability*

Amps | Featured Activity

Warm-up
Take a Poll

Poll the class to determine what sport your students would like to add to the school to facilitate a conversation on collecting data from a sample.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may dislike the tedious process of calculating the mean and mean absolute deviation; they may be lost on how to compare the groups of data. Help them practice taking control of their own impulses by suggesting they seek out support from two to three people, such as other students or you, when they feel frustrated.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

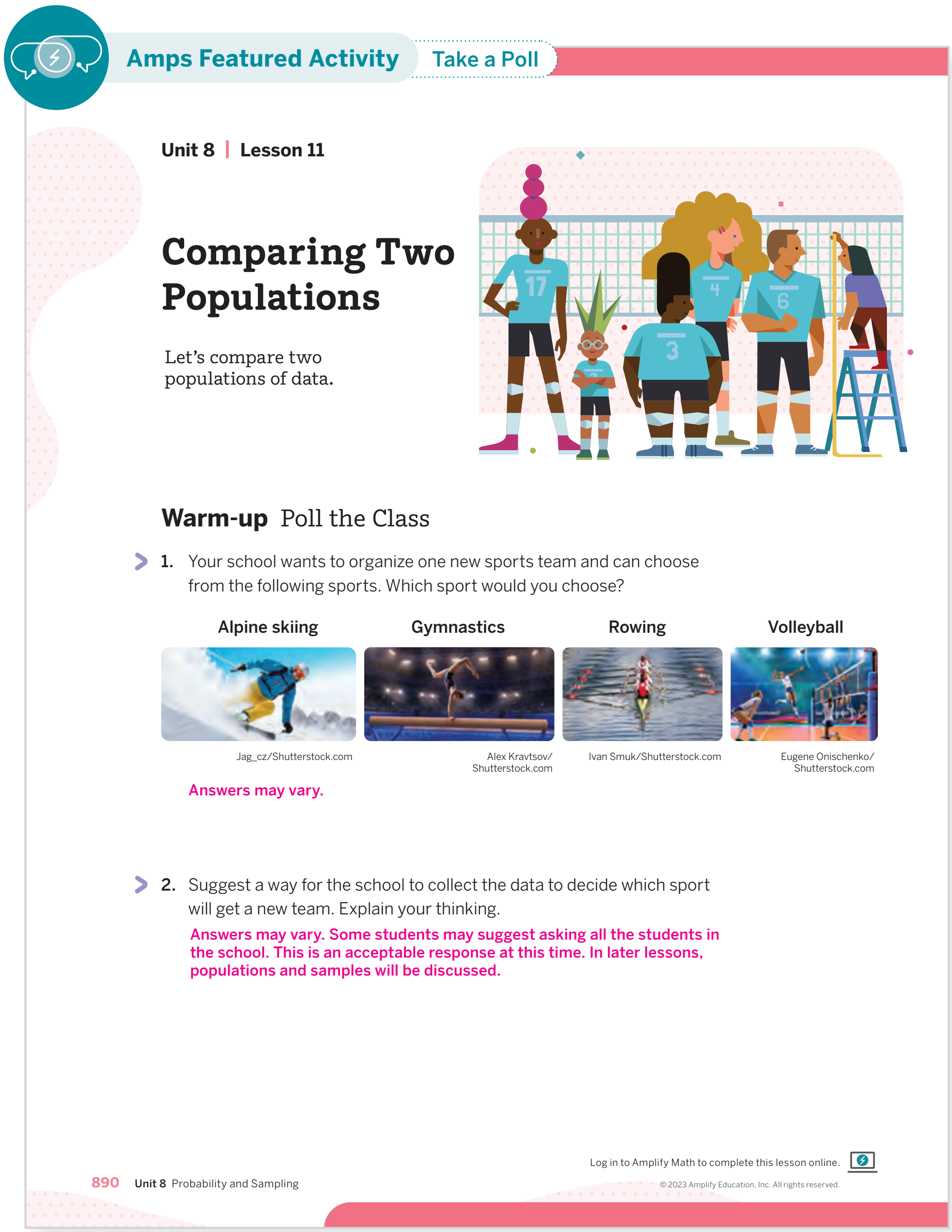
- In **Activity 1**, give students the means for the teams in Problem 3 so they can focus on the comparison of the data.
- In **Activity 3**, have students focus on the smaller data sets to compare.

Whole Class | 5 min

Warm-up Poll the Class

Students respond to a poll and suggest how to collect survey results.

Amps Featured Activity Take a Poll



Unit 8 | **Lesson 11**

Comparing Two Populations

Let's compare two populations of data.

Warm-up Poll the Class

1. Your school wants to organize one new sports team and can choose from the following sports. Which sport would you choose?

Alpine skiing — Jag_cz/Shutterstock.com

Gymnastics — Alex Kravtsov/Shutterstock.com

Rowing — Ivan Smuk/Shutterstock.com

Volleyball — Eugene Onischenko/Shutterstock.com

Answers may vary.

2. Suggest a way for the school to collect the data to decide which sport will get a new team. Explain your thinking.

Answers may vary. Some students may suggest asking all the students in the school. This is an acceptable response at this time. In later lessons, populations and samples will be discussed.

Log in to Amplify Math to complete this lesson online.

890 **Unit 8** Probability and Sampling

1 Launch

Activate students' background knowledge by asking students to share if they play a sport and what team they play for if it is not a school team.

2 Monitor

Help students get started by having them think about who should be included in the survey.

Look for points of confusion:

- **Thinking that collecting responses from all the students in the school is necessary.** Ask students how long they think that would take and then ask, "What if you expanded the survey to the whole school? The whole city? The whole state?"

Look for productive strategies:

- Thinking that collecting responses from groups of students is more efficient.

3 Connect

Display the options from Problem 1 and conduct the *Poll the Class* routine.

Have individual students share which new sports team they prefer and their suggestion for collecting data. Begin with students who want to collect data from all the students in the school and finish with students who devised a plan to collect survey results from groups of students.

Ask:

- "How might you use the previous lessons to help you organize your survey results?"
- "How might you organize, represent, and show the results of the data so that you can present it?"

Differentiated Support

Accessibility: *Activate Background Knowledge*

Use the *Poll the Class* routine to determine what sport your students would choose to add to their school, from the ones shown in the Warm-up. Consider using the digital poll provided in the Amps slides.

Power-up

To power up students' ability to construct dot plots to determine the measures of center and spread from a set of data, have students complete:

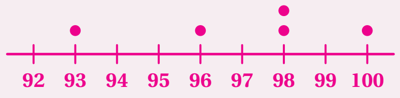
Recall that the *mean* of a data set is the average, while the *MAD* is the average distance each value is from the mean.

Clare's scores for her last five math quizzes were 93, 96, 98, 98, and 100.

a Construct a dot plot representing her scores.

b What is her mean score? 97

c What is the MAD of her scores? 2

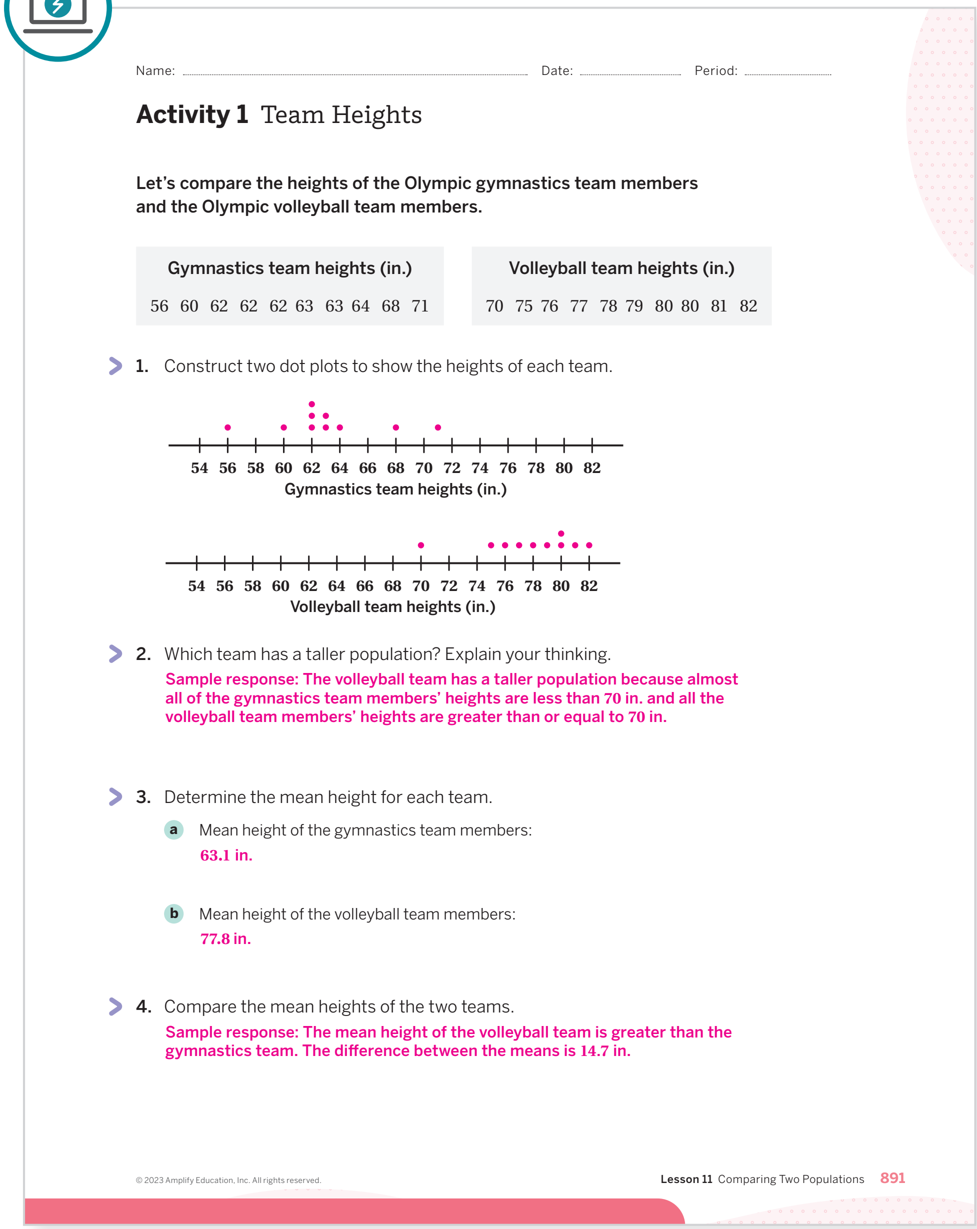


Use: Before Activity 1
Informed by: Performance on Lesson 10, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 5–7

Pairs | 10 min

Activity 1 Team Heights

Students create dot plots and calculate means to compare the heights of two groups to prepare them for using the MAD to compare two populations of data in Activity 3.



Name: .... Date: .... Period: ....

Activity 1 Team Heights

Let's compare the heights of the Olympic gymnastics team members and the Olympic volleyball team members.

Gymnastics team heights (in.)									
56	60	62	62	62	63	63	64	68	71

Volleyball team heights (in.)									
70	75	76	77	78	79	80	80	81	82

1. Construct two dot plots to show the heights of each team.

2. Which team has a taller population? Explain your thinking.
Sample response: The volleyball team has a taller population because almost all of the gymnastics team members' heights are less than 70 in. and all the volleyball team members' heights are greater than or equal to 70 in.

3. Determine the mean height for each team.
   - a Mean height of the gymnastics team members:
     63.1 in.
   - b Mean height of the volleyball team members:
     77.8 in.

4. Compare the mean heights of the two teams.
Sample response: The mean height of the volleyball team is greater than the gymnastics team. The difference between the means is 14.7 in.

Lesson 11 Comparing Two Populations 891

1 Launch

Have students work in pairs and assign each partner a team for Problem 1. Activate students' prior knowledge by asking them how to calculate the mean.

2 Monitor

Help students get started by reviewing how to construct dot plots given the groups of data.

Look for points of confusion:

- **Forgetting to divide after adding the data when finding the mean.** Ask, "Does it make sense that the mean is so much larger than the values in the set?"

3 Connect

Have pairs of students share the dot plots they constructed as well as the mean for each team.

Ask:

- "What do you notice about your dot plots?" Sample response: There is no overlap between the two dot plots. For the gymnastics team, the center of the distribution is about 63 in. For the volleyball team the center is about 78 in., and the dots seem to be closer together.
- "On which team do you think the athletes are generally taller?"

Highlight how dot plots display the distribution and spread of the data.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they constructed and analyzed dot plots in Grade 6 and calculated the means of data sets. Students may need a refresher of these concepts. Demonstrate how to plot 2 or 3 values from the Gymnastics data set and ask students to complete the dot plots.
Remind them that the mean of a data set is a measure of center that describes the data set with a single value and is also called the *average*.

$$\text{mean} = \frac{\text{sum of the data values}}{\text{number of data values}}$$

Accessibility: *Vary Demands to Optimize Challenge*

Consider one of the alternative approaches to this activity:

- Provide pre-completed dot plots for students to use and have them begin the activity with Problem 2.
- Providing the calculations for the mean in Problem 3, so that students can spend more time comparing the means.

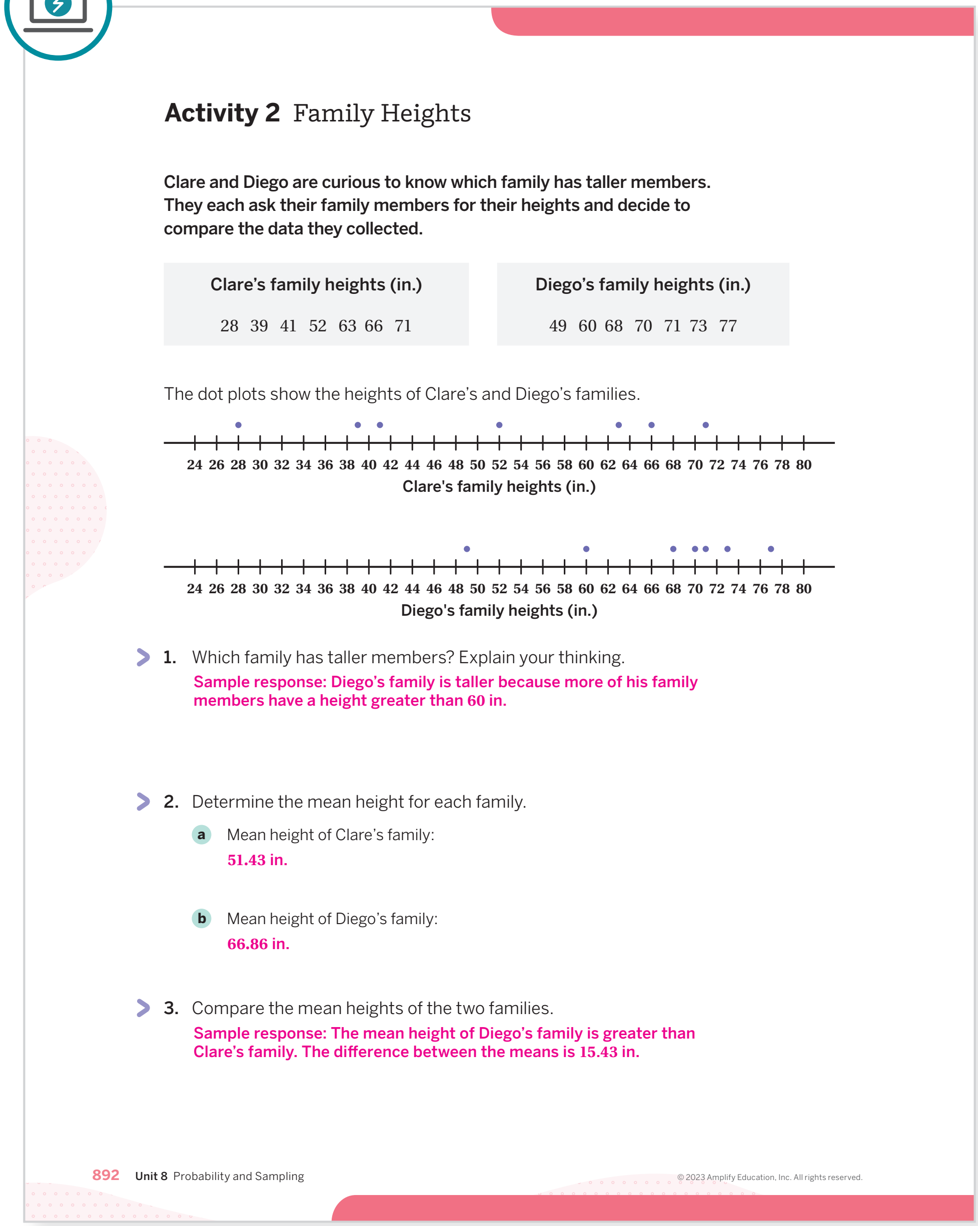
Extension: *Math Enrichment*

Tell students that the coach of each team has added an additional player. Ask them to describe how the means might be affected. If the additional player's height is close to the mean height, the mean will likely not vary much. If it is far away from the mean height, the mean will likely change.

Pairs | 10 min

Activity 2 Family Heights

Students calculate means to compare the heights of two families to prepare them for using the MAD to compare two populations of data in Activity 3.



Activity 2 Family Heights

Clare and Diego are curious to know which family has taller members. They each ask their family members for their heights and decide to compare the data they collected.

Clare's family heights (in.)	Diego's family heights (in.)
28 39 41 52 63 66 71	49 60 68 70 71 73 77

The dot plots show the heights of Clare's and Diego's families.

1. Which family has taller members? Explain your thinking.
   Sample response: Diego's family is taller because more of his family members have a height greater than 60 in.

2. Determine the mean height for each family.
   - a Mean height of Clare's family:
     51.43 in.
   - b Mean height of Diego's family:
     66.86 in.

3. Compare the mean heights of the two families.
   Sample response: The mean height of Diego's family is greater than Clare's family. The difference between the means is 15.43 in.

892 **Unit 8** Probability and Sampling

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1 Launch

Have students work in pairs and assign each partner a different family for Problem 2.

2 Monitor

Look for points of confusion:

- **Having no clear comparison between Clare's and Diego's families.** Have students think about arguments to support the idea that Diego's family is taller and counterarguments why this might not be true.

3 Connect

Have pairs of students share the means they calculated for the two family teams.

Ask:

- "How would you describe the distribution of the data in each dot plot?" Sample response: In each dot plot, the data are spread apart.
- "Do the data for these two teams overlap?" There is an overlap between 49 in. and 71 in.
- "In which family are the family members generally taller?" Diego's family
- "For which sets of data did you find it more straightforward to compare the distributions: the Olympic teams or the families?" Sample response: It was more straightforward to compare the data for the Olympic teams because there was less overlap between the two distributions.

Highlight that the differences between the means are more or less the same, but the difference in the spreads vary. The distributions for the Olympic teams have a smaller difference in spreads, meaning the teams' heights do not differ as much as the families' heights. The spread for the families is much greater.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Provide the calculations for one or both means in Problem 2, so that students can focus on comparing the means in Problem 3.

Extension: ***Math Enrichment***

Have students complete the following problem:
How would the mean heights and their comparison change if the least value from each data set was removed? The new mean heights would be about 55.33 in. (Clare's family) and about 69.83 in. (Diego's family). Diego's family still has the greater mean, but not by as much. The difference between the means is now about 14.5 in.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, draw their attention between the numerical calculations for the mean for each family and the distributions of the dot plots. Ask:

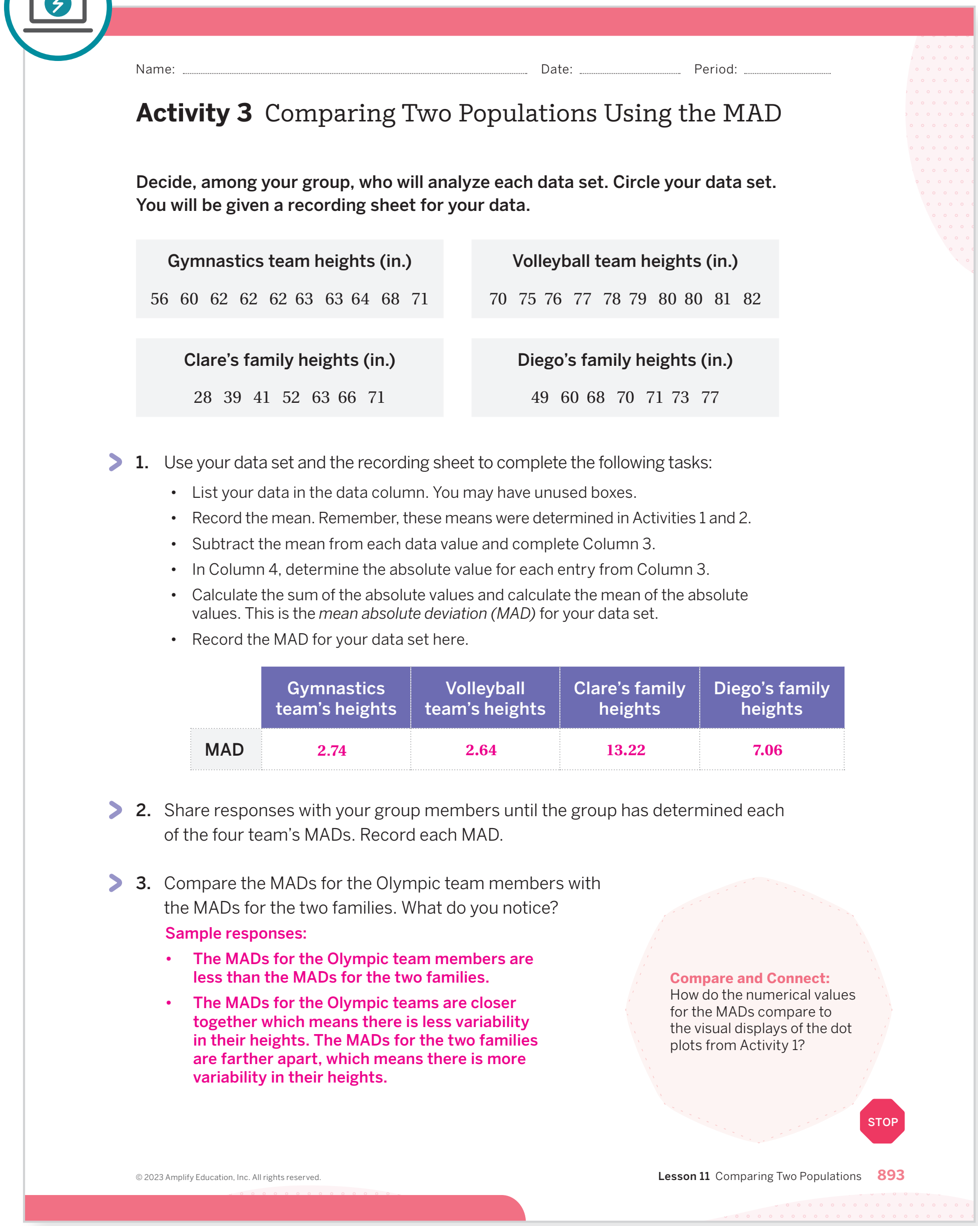
- "Look back at Activity 1. How does the difference between the means in Activity 2 compare to Activity 1?"
- "How do the distributions compare between Activities 1 and 2?"

Listen for, and amplify, student reasoning that recognizes the data are more spread out in Activity 2 than they are in Activity 1. Ask students if the mean is a good measure to indicate this variation and amplify student responses that indicate the mean is not a measure of variation.

Small Groups | 10 min

Activity 3 Comparing Two Populations Using the MAD

Students compare groups of data by calculating the difference between the means as the multiple of the MAD and use this value to determine whether two populations are very different from each other.



Name: ________ Date: ________ Period: ________

Activity 3 Comparing Two Populations Using the MAD

Decide, among your group, who will analyze each data set. Circle your data set. You will be given a recording sheet for your data.

Gymnastics team heights (in.)
56 60 62 62 62 63 63 64 68 71

Volleyball team heights (in.)
70 75 76 77 78 79 80 80 81 82

Clare's family heights (in.)
28 39 41 52 63 66 71

Diego's family heights (in.)
49 60 68 70 71 73 77

1. Use your data set and the recording sheet to complete the following tasks:
   - List your data in the data column. You may have unused boxes.
   - Record the mean. Remember, these means were determined in Activities 1 and 2.
   - Subtract the mean from each data value and complete Column 3.
   - In Column 4, determine the absolute value for each entry from Column 3.
   - Calculate the sum of the absolute values and calculate the mean of the absolute values. This is the *mean absolute deviation (MAD)* for your data set.
   - Record the MAD for your data set here.

	Gymnastics team's heights	Volleyball team's heights	Clare's family heights	Diego's family heights
MAD	2.74	2.64	13.22	7.06

2. Share responses with your group members until the group has determined each of the four team's MADs. Record each MAD.

3. Compare the MADs for the Olympic team members with the MADs for the two families. What do you notice?

   Sample responses:
   - **The MADs for the Olympic team members are less than the MADs for the two families.**
   - **The MADs for the Olympic teams are closer together which means there is less variability in their heights. The MADs for the two families are farther apart, which means there is more variability in their heights.**

Compare and Connect: How do the numerical values for the MADs compare to the visual displays of the dot plots from Activity 1?

STOP

Lesson 11 Comparing Two Populations 893

1 Launch

Have students work in small groups and assign each team to a different group member. Provide students with the Graphic Organizer PDF, *MAD Recording Sheet* to help organize and calculate the MAD.

2 Monitor

Look for points of confusion:

- **Forgetting how to evaluate absolute value.** Remind students the absolute value represents the distance of a value from 0.

3 Connect

Have students share their responses to Problem 3.

Display the Activity 3 PDF, *Comparing Two Populations*, and ask, "Is there a big difference between the variabilities of the Olympic teams and of the family teams?"

Highlight that the differences in the means for both data sets are close to 15. However, the difference between the means is not enough information to know whether the data sets are very different. One way to express the amount of overlap is to divide the difference in means by the larger MAD. For the Olympic teams, the difference in means is about 5 times the measure of variability, and for the family teams it is about 1 time the measure of variability. This indicates that the Olympic teams have a large difference among the two distributions. As a general rule, these materials will consider the difference between the data sets to be significant if the difference in means is *more than twice* the larger MAD.

Note: Although the median and interquartile range (IQR) are not needed in this activity, it may be useful to review how to calculate those values as well.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Assign each group a different data set for which to calculate the MAD, as opposed to each group calculating all four MADs. Display group's calculations and encourage groups to discuss and resolve any differences before they proceed with Problem 3.

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to respond to the question posed in their Student Edition, "How do the numerical values for the MADs compare to the visual displays of the dot plots from Activity 1?" Consider displaying the dot plots from Activities 1 and 2 and annotate them with their respective MADs, so students can see them at a glance. Ask:

- "Does either set of dot plots show a greater visual difference? What do you notice about their corresponding means or MADs?"
- Does either set of dot plots show a greater visual overlap of data values? What do you notice about their corresponding means and MADs?"

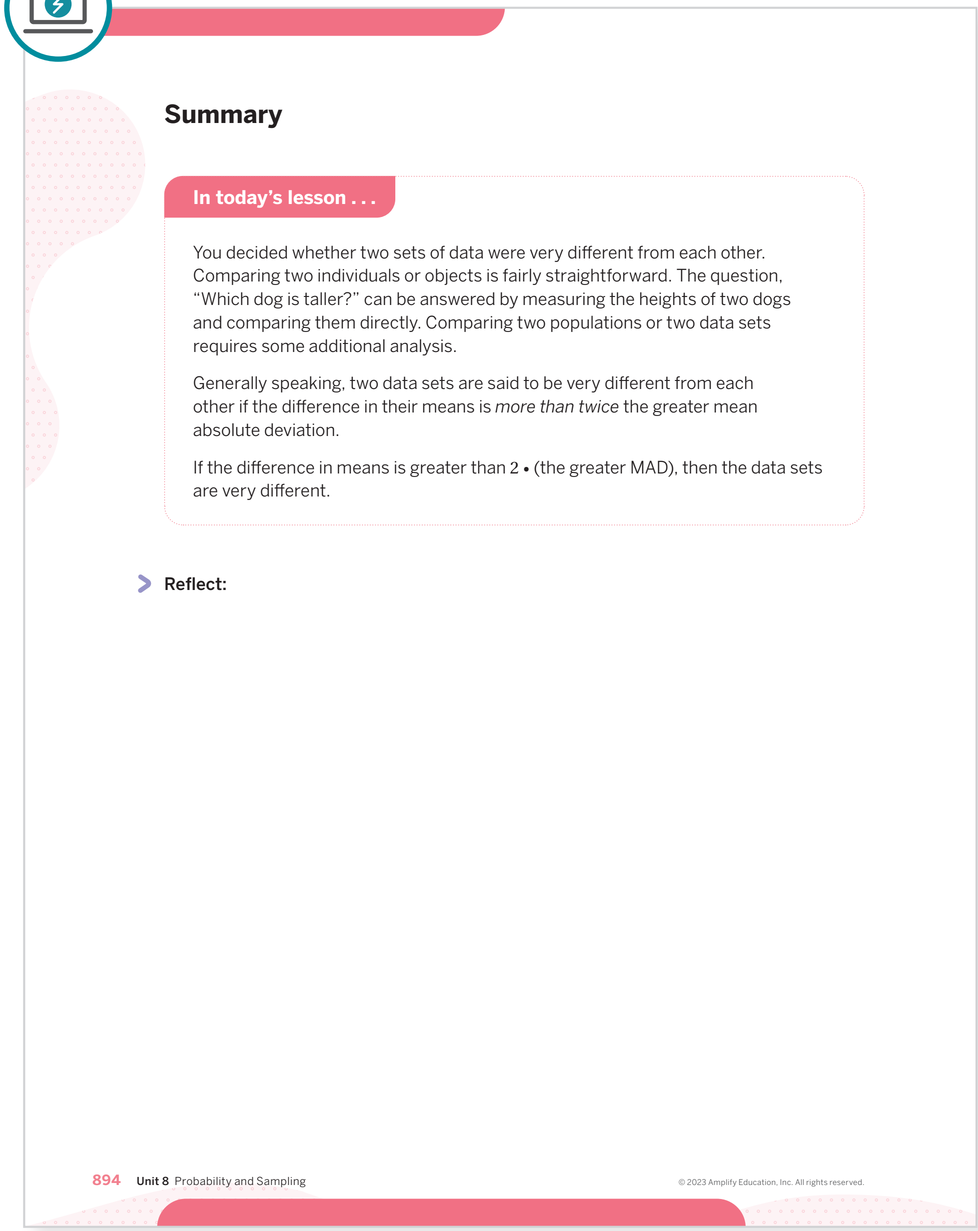
English Learners

Use colored pencils to highlight where the visual overlap appears to be for each set of dot plots.

Whole Class | 5 min

Summary

Review and synthesize how to compare populations using the mean and mean absolute deviation (MAD).



Summary

In today's lesson . . .

You decided whether two sets of data were very different from each other. Comparing two individuals or objects is fairly straightforward. The question, "Which dog is taller?" can be answered by measuring the heights of two dogs and comparing them directly. Comparing two populations or two data sets requires some additional analysis.

Generally speaking, two data sets are said to be very different from each other if the difference in their means is *more than twice* the greater mean absolute deviation.

If the difference in means is greater than 2 • (the greater MAD), then the data sets are very different.

Reflect:

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Synthesize

Highlight that variation or variability is something that can be measured. Students may recall the statistical tool mean absolute deviation (MAD) from Grade 6. While the mean can help understand where the center is, MAD can help understand how much variation (or spread) a data set has. When comparing two populations of data, more than just the mean is needed. Two populations can have the same mean, or similar means, and yet be very different because one population may have a greater spread than the other population.

Ask:

- "What are some measures of center, and how are they calculated?" Two measures are the mean and median. The mean is calculated by finding the sum of the data values and then dividing by the total number of data values. The median is calculated by finding the middle data value (or average of the two middle data values) after placing the data values in order.
- "Why is the mean useful for comparing two populations?" The mean helps describe whether the centers of the populations are similar or different.
- "Why is the MAD also needed when comparing two populations?" The MAD helps describe the spread of each population, or how far away the data values in each population are from the mean.
- "What is the general rule you will use to determine whether two populations have a large difference or not?" If the difference in the means is greater than twice the MAD, then the two populations are considered to be very different from each other.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

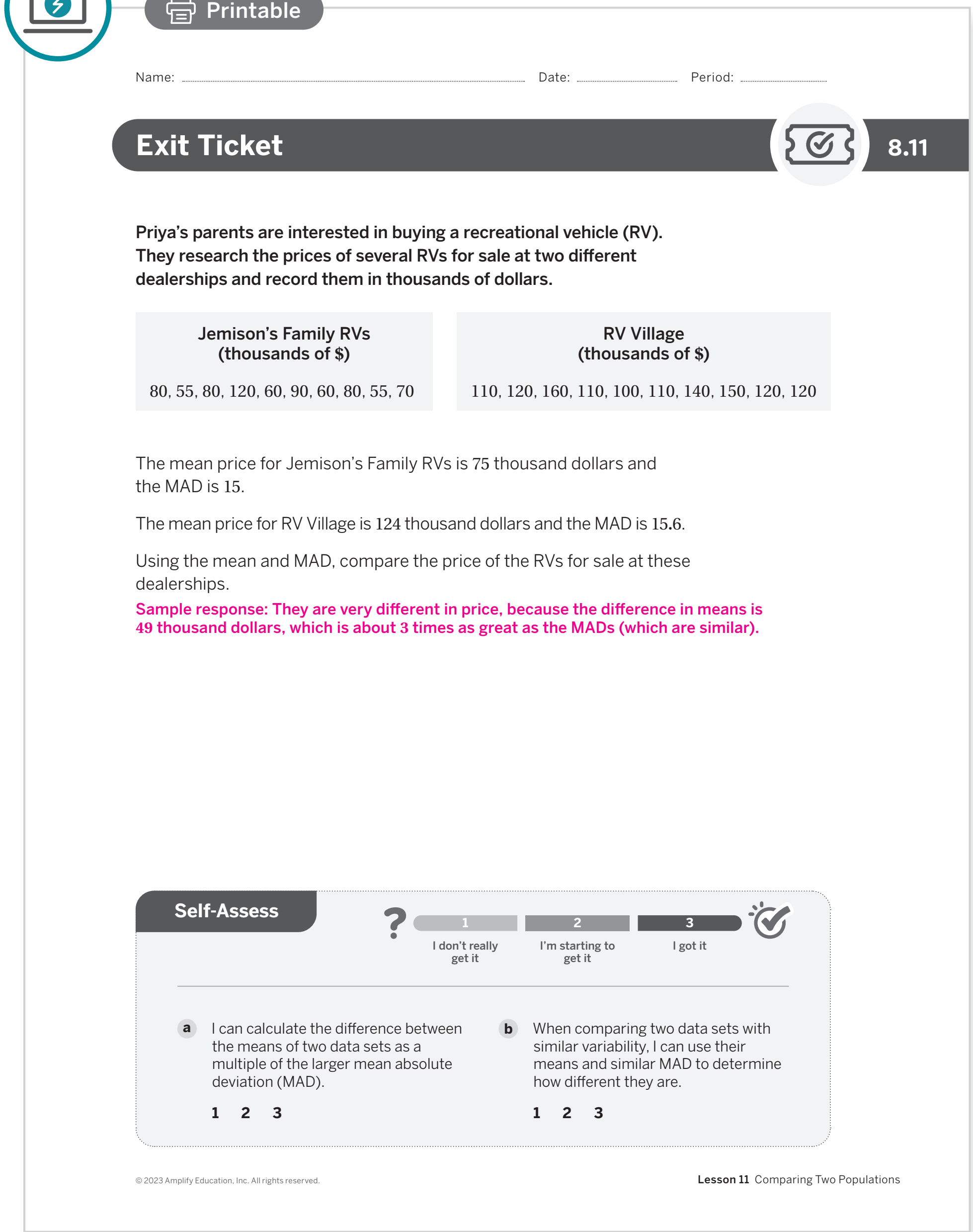
- "In what ways can you compare two populations?"
- "In what way did creating dot plots help you to visualize the similarities and differences between two populations?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to determine whether two populations of data are very different from each other by comparing means and mean absolute deviations.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 8.11

Priya's parents are interested in buying a recreational vehicle (RV). They research the prices of several RVs for sale at two different dealerships and record them in thousands of dollars.

Jemison's Family RVs (thousands of $)	**RV Village (thousands of $)**
80, 55, 80, 120, 60, 90, 60, 80, 55, 70	110, 120, 160, 110, 100, 110, 140, 150, 120, 120

The mean price for Jemison's Family RVs is 75 thousand dollars and the MAD is 15.

The mean price for RV Village is 124 thousand dollars and the MAD is 15.6.

Using the mean and MAD, compare the price of the RVs for sale at these dealerships.

Sample response: They are very different in price, because the difference in means is 49 thousand dollars, which is about 3 times as great as the MADs (which are similar).

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can calculate the difference between the means of two data sets as a multiple of the larger mean absolute deviation (MAD).

1 2 3

b When comparing two data sets with similar variability, I can use their means and similar MAD to determine how different they are.

1 2 3

Lesson 11 Comparing Two Populations

Success looks like . . .

- **Language Goal:** Calculating the mean and mean absolute deviation (MAD) for a data set, and interpreting these measures. **(Speaking and Listening)**
- **Language Goal:** Comparing and contrasting populations represented on dot plots in terms of their center, spread, and visual overlap. **(Speaking and Listening, Writing)**
- **Language Goal:** Justifying whether two populations are "very different" based on the difference in their means expressed as a multiple of the mean absolute deviation. **(Writing)**
    - » Comparing the prices of the RVs by expressing the difference in means of the two sets of prices as a multiple of the MADs.

Suggested next steps

If students provide an incorrect response or explanation, consider:

- Reviewing how to determine whether two populations are different from Activity 3.
- Assigning Practice Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups did and didn't have their ideas seen and heard today?
- In what ways did the Warm-up go as planned? What might you change for the next time you teach this lesson?

Math Language Development

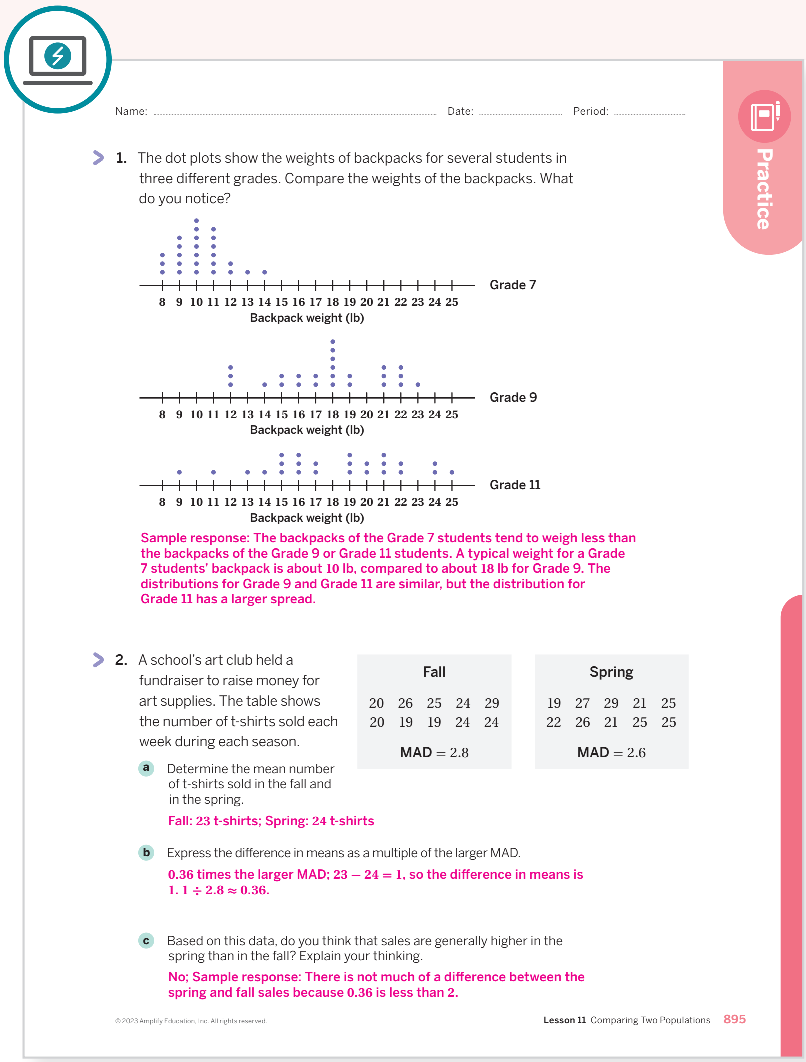
Language Goal: Justifying whether two populations are "very different" based on the difference in their means expressed as a multiple of the mean absolute deviation.

Reflect on students' language development toward this goal.

- Do students' responses to the Exit Ticket problem indicate an understanding that the prices of RVs for sale at the two companies are very different?
- Do their explanations include math language, such as "the difference in means", "three times as great as the MAD", etc? How can you help them be more precise in their explanations?

Practice

Independent



Practice

Name: Date: Period:

1. The dot plots show the weights of backpacks for several students in three different grades. Compare the weights of the backpacks. What do you notice?

Sample response: The backpacks of the Grade 7 students tend to weigh less than the backpacks of the Grade 9 or Grade 11 students. A typical weight for a Grade 7 students' backpack is about 10 lb, compared to about 18 lb for Grade 9. The distributions for Grade 9 and Grade 11 are similar, but the distribution for Grade 11 has a larger spread.

2. A school's art club held a fundraiser to raise money for art supplies. The table shows the number of t-shirts sold each week during each season.

Fall

20	26	25	24	29
20	19	19	24	24

MAD $= 2.8$

Spring

19	27	29	21	25
22	26	21	25	25

MAD $= 2.6$

a. Determine the mean number of t-shirts sold in the fall and in the spring.

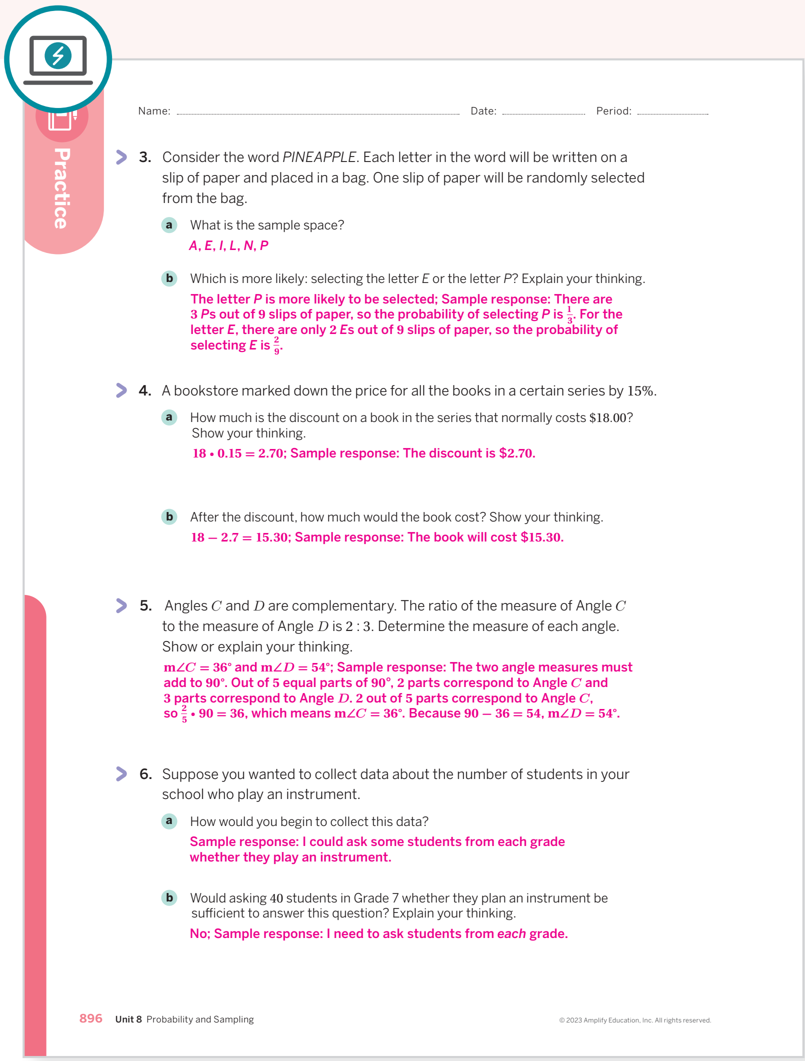
Fall: 23 t-shirts; Spring: 24 t-shirts

b. Express the difference in means as a multiple of the larger MAD.

0.36 times the larger MAD; $23 - 24 = 1$, so the difference in means is 1. $1 \div 2.8 \approx 0.36$.

c. Based on this data, do you think that sales are generally higher in the spring than in the fall? Explain your thinking.

No; Sample response: There is not much of a difference between the spring and fall sales because 0.36 is less than 2.



Practice

Name: Date: Period:

3. Consider the word *PINEAPPLE*. Each letter in the word will be written on a slip of paper and placed in a bag. One slip of paper will be randomly selected from the bag.

a. What is the sample space?

A, E, I, L, N, P

b. Which is more likely: selecting the letter *E* or the letter *P*? Explain your thinking.

The letter *P* is more likely to be selected; Sample response: There are 3 *P*s out of 9 slips of paper, so the probability of selecting *P* is $\frac{1}{3}$. For the letter *E*, there are only 2 *E*s out of 9 slips of paper, so the probability of selecting *E* is $\frac{2}{9}$.

4. A bookstore marked down the price for all the books in a certain series by 15%.

a. How much is the discount on a book in the series that normally costs $18.00? Show your thinking.

$18 \cdot 0.15 = 2.70$; Sample response: The discount is $2.70.

b. After the discount, how much would the book cost? Show your thinking.

$18 - 2.7 = 15.30$; Sample response: The book will cost $15.30.

5. Angles C and D are complementary. The ratio of the measure of Angle C to the measure of Angle D is $2 : 3$. Determine the measure of each angle. Show or explain your thinking.

$m\angle C = 36°$ and $m\angle D = 54°$; Sample response: The two angle measures must add to 90°. Out of 5 equal parts of 90°, 2 parts correspond to Angle C and 3 parts correspond to Angle D. 2 out of 5 parts correspond to Angle C, so $\frac{2}{5} \cdot 90 = 36$, which means $m\angle C = 36°$. Because $90 - 36 = 54$, $m\angle D = 54°$.

6. Suppose you wanted to collect data about the number of students in your school who play an instrument.

a. How would you begin to collect this data?

Sample response: I could ask some students from each grade whether they play an instrument.

b. Would asking 40 students in Grade 7 whether they plan an instrument be sufficient to answer this question? Explain your thinking.

No; Sample response: I need to ask students from *each* grade.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	1
	2	Activity 3	2
Spiral	3	Unit 8 Lesson 3	1
	4	Unit 4 Lesson 3	2
	5	Unit 7 Lesson 5	3
Formative ⚡	6	Unit 8 Lesson 12	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 12**

Larger Populations

Let's compare larger populations of data.

Focus

Goals

1. **Language Goal:** Comprehend that the terms *population* and *sample* refer to the whole group (population) and a part of the group under consideration (sample). **(Speaking and Listening, Writing)**
2. **Language Goal:** Describe a sample for a given population. **(Speaking and Listening, Writing)**
3. **Language Goal:** Explain that a sample may be used when it is unreasonable to gather data about an entire population. **(Speaking and Listening)**

Coherence

Today

Students are introduced to the idea of using data from a sample of a population when it is impractical or impossible to gather data from every individual in the population. Students consider whether the people in their class would be an adequate sample for several different questions and associated populations. Students will use the quantifiable method they learned in the previous lesson. **Note:** This lesson's Practice contains a milestone for the Capstone project.

Previously

In Lesson 11, students compared dot plots and calculated the difference between two means as a multiple of the mean absolute deviation (MAD) to show when two data sets are different.

Coming Soon

Students will learn what makes some samples more representative of a population than others. Students will also explore the best ways to obtain such samples.

Rigor

- Students build **conceptual understanding** of how a population and a sample of that population are related.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per student
- Graphic Organizer PDF, *MAD Recording Sheet*, one per pair
- class list of first and last names

Math Language Development

New words

- ***population***
- ***sample***

Review words

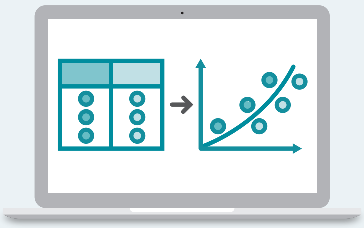
- *mean*
- *mean absolute deviation (MAD)*
- *statistical question*
- *variability*

Amps | **Featured Activity**

Activity 2

Aggregate Class Data

Student-submitted data is quickly collected, aggregated, and shared with fellow classmates, automatically.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may rush to a conclusion that they have enough information about the population they are considering. Encourage students to think about their own thinking process to make sure their conclusions make sense given the context of the data.

Modifications to Pacing

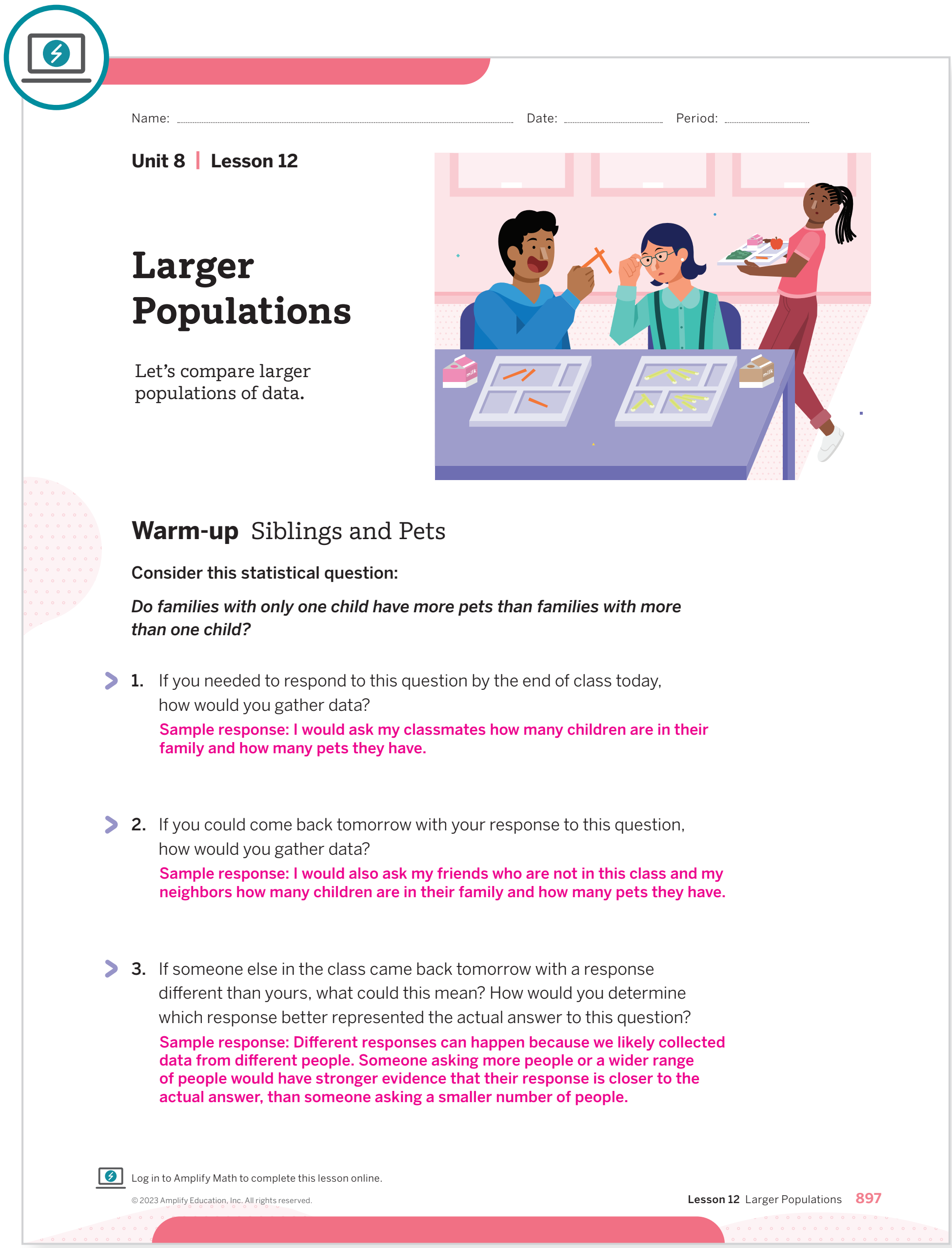
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, omit Problem 3.
- For **Activity 2**, provide a class list with the letters in each name already counted. To save more time, provide the mean for the class data and perhaps the MAD, as well.

Independent | 10 min

Warm-up Siblings and Pets

Students consider ways they gather data to realize it is unreasonable to collect data from everyone addressed by the question.



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Unit 8 | Lesson 12

Larger Populations

Let's compare larger populations of data.

Warm-up Siblings and Pets

Consider this statistical question:

Do families with only one child have more pets than families with more than one child?

1. If you needed to respond to this question by the end of class today, how would you gather data?
   Sample response: I would ask my classmates how many children are in their family and how many pets they have.

2. If you could come back tomorrow with your response to this question, how would you gather data?
   Sample response: I would also ask my friends who are not in this class and my neighbors how many children are in their family and how many pets they have.

3. If someone else in the class came back tomorrow with a response different than yours, what could this mean? How would you determine which response better represented the actual answer to this question?
   Sample response: Different responses can happen because we likely collected data from different people. Someone asking more people or a wider range of people would have stronger evidence that their response is closer to the actual answer, than someone asking a smaller number of people.

Log in to Amplify Math to complete this lesson online.

Lesson 12 Larger Populations 897

1 Launch

Activate students' prior knowledge by asking them to explain the term *statistical question*.

2 Monitor

Help students get started by having them think about who should be included in answering the question.

Look for points of confusion:

- **Thinking that families with only one child are the population.** Remind students that they need to ask all the family units with at least one child.

3 Connect

Have pairs of students share how they would gather data to answer the question.

Highlight the difference between the data students would like to have to answer the question and the data students have available.

Define the term ***population*** as the set of individuals from which the data are taken. A ***sample*** is the part of the population from which the data are actually collected.

Ask:

- "What is the population for the statistical question?" Every family unit in the world with at least one child.
- "What might be a sample you could use to answer the question?" Students in our class.
- "Why is it unreasonable to actually collect all the necessary data to answer the question?" There are too many people to collect data from.

Math Language Development

MLR2: Collect and Display

To demonstrate how a sample is a subset of a population, ask everyone in the class to stand up. Say, "This is the population of our class." Then ask everyone to sit down with the exception of 5 students you select. When those 5 students are the only ones standing, say, "This is the sample."

Add an illustration of this concept to the class display. For example, draw a diagram of a few smaller circles inside a larger circle. Label the larger circle "population" and smaller circles "sample."

Power-up

To power up students' ability to reason about collecting data from a group, have students complete:

Your teacher wants to know what students are planning to do over the summer. Which would be the best way for them to collect this data?

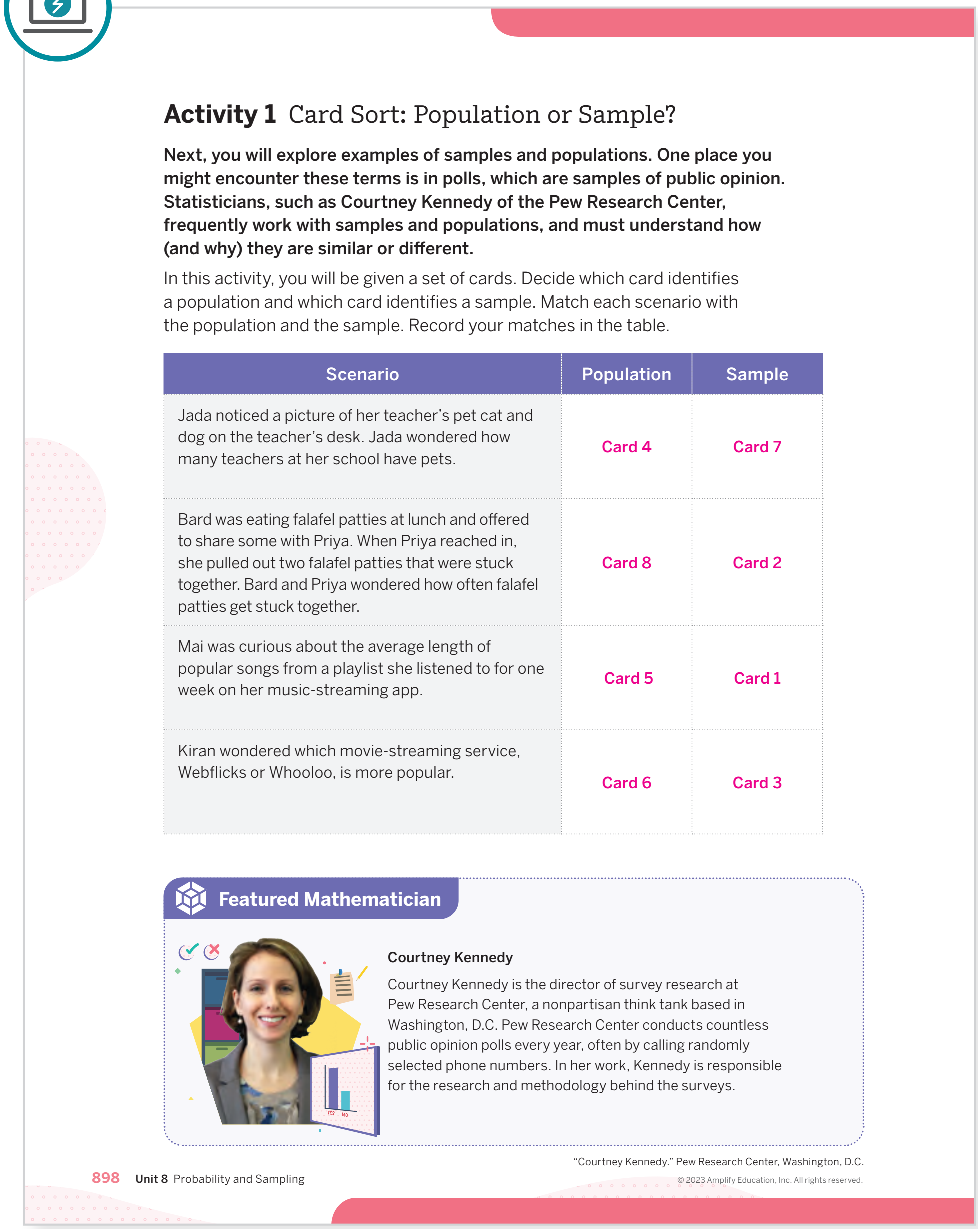
A. Ask other teachers in the school what they have heard.
B. Ask their homeroom students only.
C. Ask all of the students who are involved in Chorus.
[✓ D.] Ask six students from each class that they teach.

Use: Before the Warm-up
Informed by: Performance on Lesson 11, Practice Problem 6

Pairs | 10 min

Activity 1 Card Sort: Population or Sample?

Students practice identifying populations and samples based on several scenarios.



Activity 1 Card Sort: Population or Sample?

Next, you will explore examples of samples and populations. One place you might encounter these terms is in polls, which are samples of public opinion. Statisticians, such as Courtney Kennedy of the Pew Research Center, frequently work with samples and populations, and must understand how (and why) they are similar or different.

In this activity, you will be given a set of cards. Decide which card identifies a population and which card identifies a sample. Match each scenario with the population and the sample. Record your matches in the table.

Scenario	Population	Sample
Jada noticed a picture of her teacher's pet cat and dog on the teacher's desk. Jada wondered how many teachers at her school have pets.	Card 4	Card 7
Bard was eating falafel patties at lunch and offered to share some with Priya. When Priya reached in, she pulled out two falafel patties that were stuck together. Bard and Priya wondered how often falafel patties get stuck together.	Card 8	Card 2
Mai was curious about the average length of popular songs from a playlist she listened to for one week on her music-streaming app.	Card 5	Card 1
Kiran wondered which movie-streaming service, Webflicks or Whooloo, is more popular.	Card 6	Card 3

Featured Mathematician

Courtney Kennedy

Courtney Kennedy is the director of survey research at Pew Research Center, a nonpartisan think tank based in Washington, D.C. Pew Research Center conducts countless public opinion polls every year, often by calling randomly selected phone numbers. In her work, Kennedy is responsible for the research and methodology behind the surveys.

"Courtney Kennedy." Pew Research Center, Washington, D.C.

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1 Launch

Distribute the pre-cut cards from the Activity 1 PDF. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by having them match pairs of cards together before reading the scenarios.

Look for points of confusion:

- **Mixing up population and sample.** Have students think about which card represents the larger group (population) and which card represents the smaller group (sample).

3 Connect

Have pairs of students share the populations and samples for the scenarios.

Ask:

- For each scenario, could there be another population other than the ones given?" No. The scenario should describe the population you are wanting to research.
- "For each scenario, could there be another sample other than the ones given?" Yes. A sample refers to a few of the individuals from the population that will be collected.
- "If you were to answer the question the student wonders about in each scenario, what are some advantages and disadvantages of using the sample?" Some samples are more convenient, but might miss large sections of the population and might not be an accurate representation.

Highlight that well-phrased questions should have known populations. A question that is not well-phrased should be reconsidered so that the purpose of the question is clear. However, there are usually many ways to find samples within the populations.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with Cards 2, 4, 7, and 8. Have them focus on completing Problems 1 and 2. As time permits, distribute the remaining cards.

Extension: *Math Enrichment*

Have students generate their own scenario and write a description of the population and sample for that scenario. Ask them to share their descriptions with the class.

Math Language Development

MLR2: Collect and Display

Create a visual display with two columns labeled *Population* and *Sample*. Tape a class set of the cards from this activity onto the display where each card is taped under its appropriate category. Leave the display up for students to reference throughout the rest of this unit.

Featured Mathematician

Courtney Kennedy

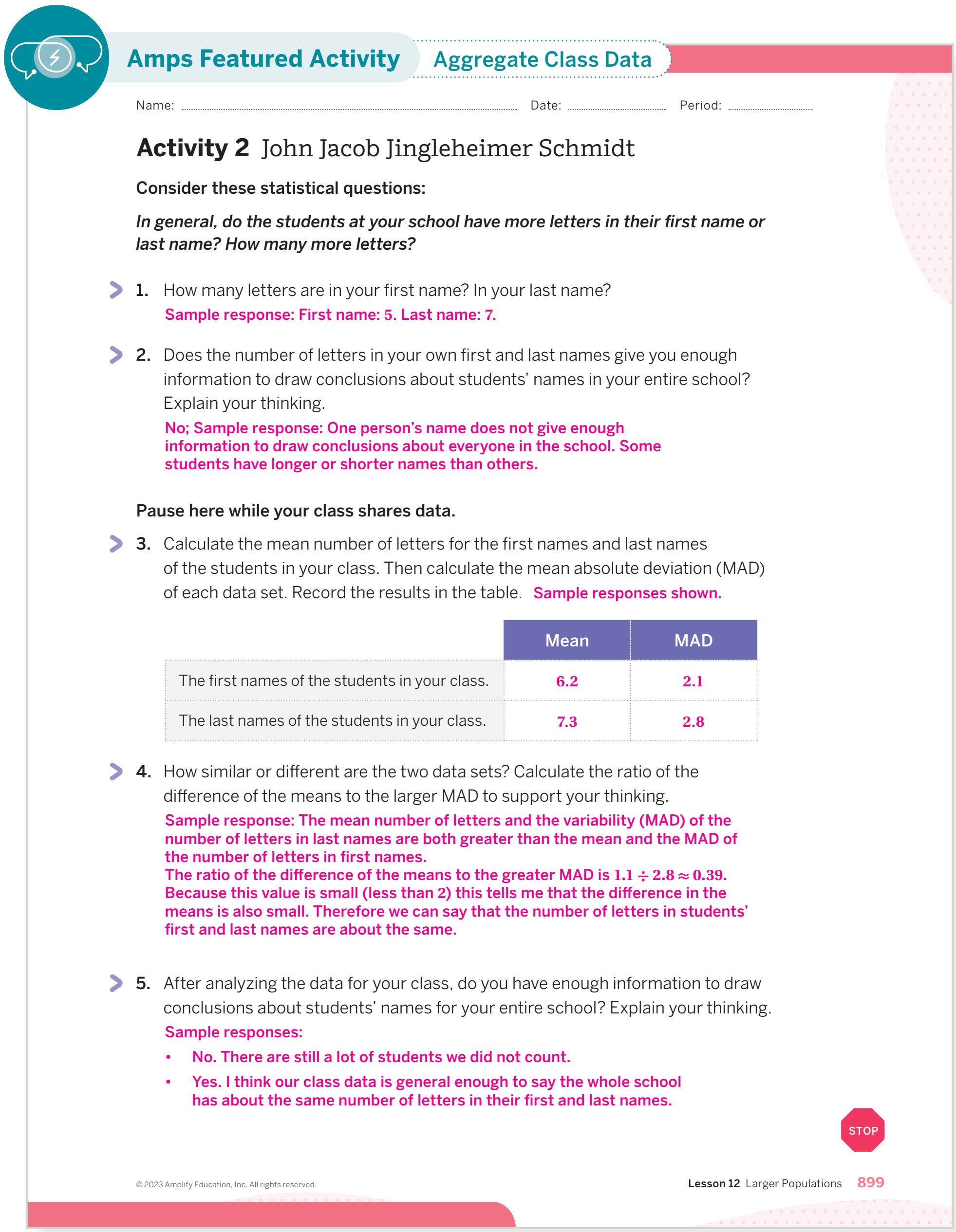
Have students read about featured mathematician Courtney Kennedy, the director of survey research at Pew Research Center, a nonpartisan think tank based in Washington, D.C.

Pairs | 15 min

Activity 2 John Jacob Jingleheimer Schmidt

Students compare two groups by collecting data to draw a conclusion about a larger group.

Amps Featured Activity Aggregate Class Data



Name: Date: Period:

Activity 2 John Jacob Jingleheimer Schmidt

Consider these statistical questions:

In general, do the students at your school have more letters in their first name or last name? How many more letters?

1. How many letters are in your first name? In your last name?
   Sample response: First name: 5. Last name: 7.

2. Does the number of letters in your own first and last names give you enough information to draw conclusions about students' names in your entire school? Explain your thinking.
   No; Sample response: One person's name does not give enough information to draw conclusions about everyone in the school. Some students have longer or shorter names than others.

Pause here while your class shares data.

3. Calculate the mean number of letters for the first names and last names of the students in your class. Then calculate the mean absolute deviation (MAD) of each data set. Record the results in the table. **Sample responses shown.**

	Mean	MAD
The first names of the students in your class.	6.2	2.1
The last names of the students in your class.	7.3	2.8

4. How similar or different are the two data sets? Calculate the ratio of the difference of the means to the larger MAD to support your thinking.
   Sample response: The mean number of letters and the variability (MAD) of the number of letters in last names are both greater than the mean and the MAD of the number of letters in first names.
   The ratio of the difference of the means to the greater MAD is $1.1 \div 2.8 \approx 0.39$. Because this value is small (less than 2) this tells me that the difference in the means is also small. Therefore we can say that the number of letters in students' first and last names are about the same.

5. After analyzing the data for your class, do you have enough information to draw conclusions about students' names for your entire school? Explain your thinking.
   Sample responses:
   - **No. There are still a lot of students we did not count.**
   - **Yes. I think our class data is general enough to say the whole school has about the same number of letters in their first and last names.**

STOP

1 Launch

Ask students why knowing the length of names would be helpful (e.g., printing name cards for an event, diplomas, etc.) Have students complete Problems 1 and 2, then provide students with the class list and the Graphic Organizer PDF, *MAD Recording Sheet*. Have students share the number of letters in their names.

2 Monitor

Help students get started by providing the class list with the letters already counted.

Look for points of confusion:

- **Comparing the mean and MAD directly instead of finding the quotient between the difference of means and the MAD (Problems 4 and 5).** Have students find the difference between the means, and then divide by the MAD. If it is less than 2, there is not much difference.

3 Connect

Have individual students share their conclusions about the entire school's data based on the class data .

Highlight how the data they have might relate to a larger group. A sample might give some estimate of a larger population, but the estimate should not be assumed to be exact.

Ask:

- "Do you expect the mean length of first names for the school to be exactly the same as the mean length for the class?" Not the same, but close.
- "Do you expect the mean length of first names for the school to be larger, smaller, or about the same as the mean length for the class?" Unless there are one or two names which are considerably longer than most names, it should be close to the mean from the class.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which the class data can be aggregated and shared.
If not using the Amps slides, provide the mean and MAD of the class data already calculated for students to record in the table for Problem 3. This will allow them to focus on comparing the means and MADs in Problems 4 and 5.

Accessibility: *Guide Processing and Visualization*

Provide students with copies of the Graphic Organizer PDF, *MAD Recording Sheet*, to help them visualize the calculations needed to determine the mean and MAD.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their conclusions, display these sentence frame to help them structure their thinking:

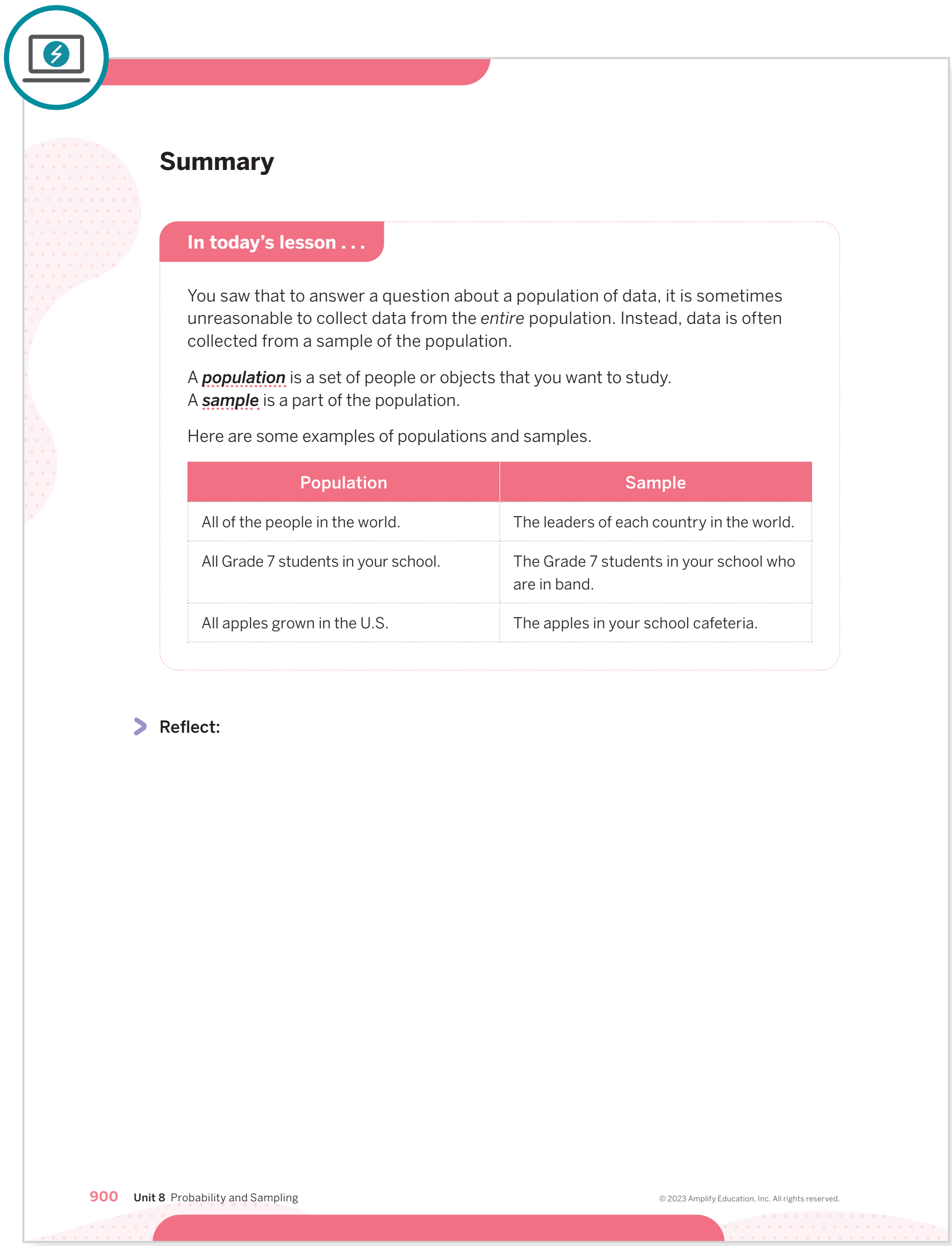
- "The mean length of first names for the school will not be exactly the same as the mean length for the class because . . . ."
- "The mean length of first names for the _____ should be greater than/less than/about the same as the mean length for the _____ because . . . ."

English Learners
Encourage students to use the terms and phrases from the class display to strengthen their use of appropriate mathematical language.

Whole Class | 5 min

Summary

Review and synthesize why collecting data from an entire population is not always reasonable or efficient, and why using samples can help answer statistical questions about the population.



Summary

In today's lesson . . .

You saw that to answer a question about a population of data, it is sometimes unreasonable to collect data from the *entire* population. Instead, data is often collected from a sample of the population.

A ***population*** is a set of people or objects that you want to study.
A ***sample*** is a part of the population.

Here are some examples of populations and samples.

Population	Sample
All of the people in the world.	The leaders of each country in the world.
All Grade 7 students in your school.	The Grade 7 students in your school who are in band.
All apples grown in the U.S.	The apples in your school cafeteria.

Reflect:

900 **Unit 8** Probability and Sampling

Synthesize

Have students share a question exploring something that interests them, and then have them identify the population and sample they could use in their study.

Highlight how a population represents the larger group under study by the statistical question. The sample is a smaller group of the population.

Formalize vocabulary:

- *population*
- *sample*

Ask:

- "When the population is too large, how can you obtain some data to begin answering a question about the population?" By taking a sample.
- "What are some drawbacks of using samples instead of the entire population?" The sample may not be representative of the population. Some groups or individuals may not be included that could affect the results.
- "What are some reasons samples are necessary?" Sample responses: Samples can be more manageable to study or more cost effective. It may be impossible to ask the entire population, take too long, or be too costly.
- "If you wanted to know what breed of dog is most popular as a pet in our state, think about different samples you could use. Would asking all the teachers at our school be a good sample? The people at a dog park? A few dog owners from around the state?" Teachers would be convenient to ask, but they may not have dogs. By asking people at a dog park, I would only be asking people living in one area. A few dog owners from across the state would be more challenging to collect data from, but would be a better sample for studying the most popular dog breed in the state.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What characteristics or key words did you look for when completing your card sort today?"
- "What questions do you still have about the difference between a population and a sample?"

Math Language Development

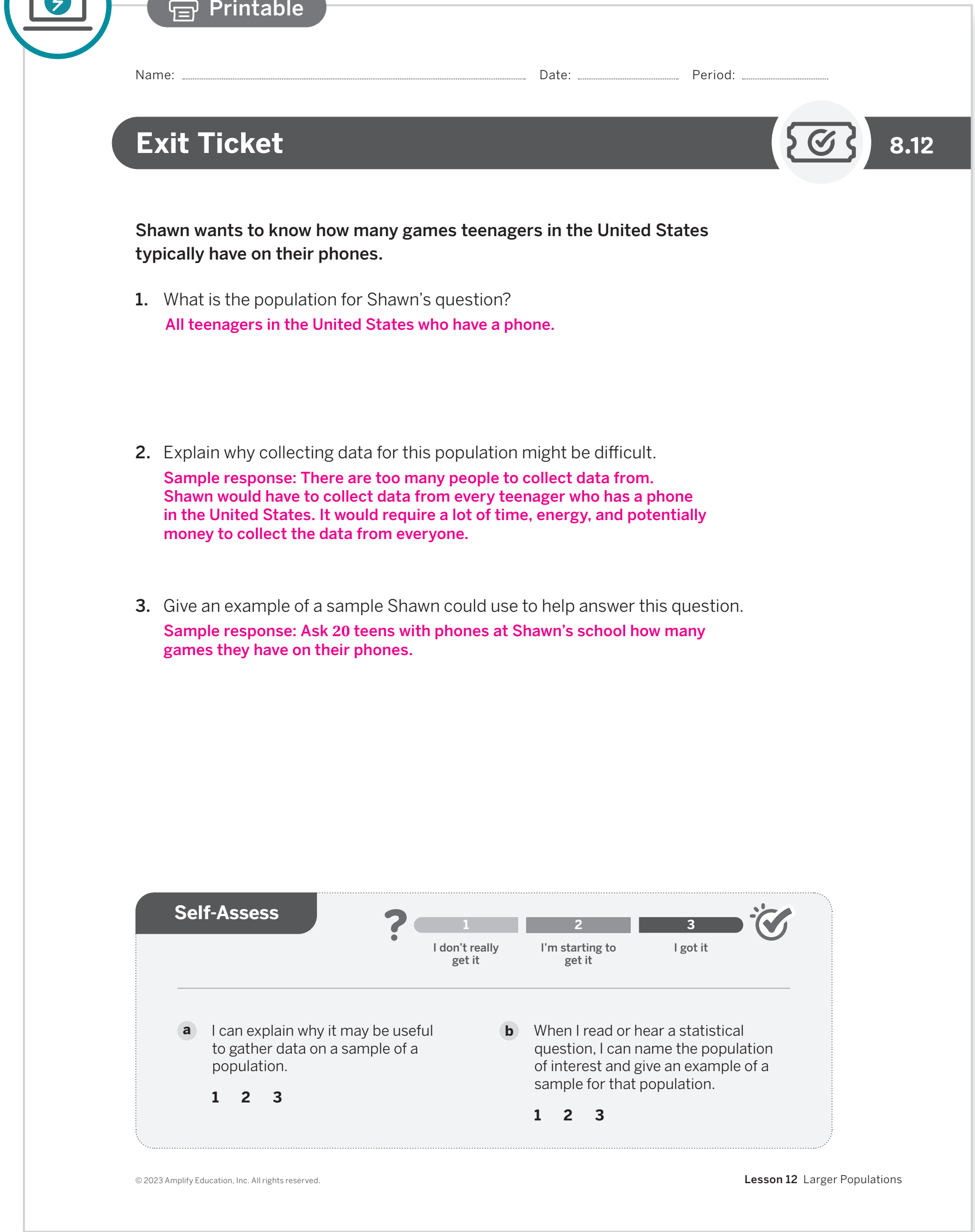
MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *population* and *sample* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of populations and samples by gathering data related to a population and a sample.



Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 8.12

Shawn wants to know how many games teenagers in the United States typically have on their phones.

1. What is the population for Shawn's question?
   All teenagers in the United States who have a phone.

2. Explain why collecting data for this population might be difficult.
   Sample response: There are too many people to collect data from. Shawn would have to collect data from every teenager who has a phone in the United States. It would require a lot of time, energy, and potentially money to collect the data from everyone.

3. Give an example of a sample Shawn could use to help answer this question.
   Sample response: Ask 20 teens with phones at Shawn's school how many games they have on their phones.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can explain why it may be useful to gather data on a sample of a population.
1 2 3

b When I read or hear a statistical question, I can name the population of interest and give an example of a sample for that population.
1 2 3

Success looks like . . .

- **Language Goal:** Comprehending that the terms *population* and *sample* to refer to the whole group (population) and a part of the group under consideration (sample). **(Speaking and Listening, Writing)**
  - » Identifying the population for Shawn's question in Problem 1.
- **Language Goal:** Describing a sample for a given population. **(Speaking and Listening, Writing)**
- **Language Goal:** Explaining that a sample may be used when it is unreasonable to gather data about an entire population. **(Speaking and Listening)**
  - » Identifying a sample that Shawn could use to answer their question in Problem 3.

Suggested next steps

If students struggle to identify the population or sample, consider:

- Reviewing Activity 3.
- Assigning Practice Problems 1 and 2.

If students struggle to explain why collecting data for a population would be difficult, consider:

- Reviewing Activity 2.

Professional Learning

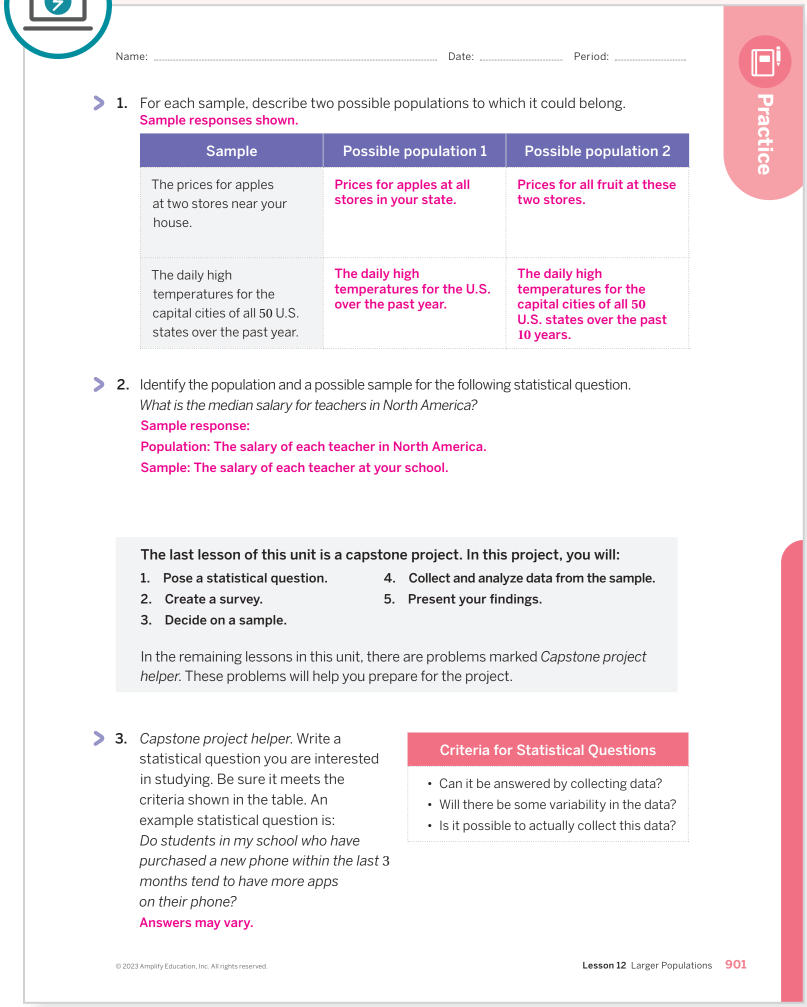
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about Activity 1, how did you encourage each student to listen to one another's strategies?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. For each sample, describe two possible populations to which it could belong.
Sample responses shown.

Sample	Possible population 1	Possible population 2
The prices for apples at two stores near your house.	Prices for apples at all stores in your state.	Prices for all fruit at these two stores.
The daily high temperatures for the capital cities of all 50 U.S. states over the past year.	The daily high temperatures for the U.S. over the past year.	The daily high temperatures for the capital cities of all 50 U.S. states over the past 10 years.

2. Identify the population and a possible sample for the following statistical question.
What is the median salary for teachers in North America?
Sample response:
Population: The salary of each teacher in North America.
Sample: The salary of each teacher at your school.

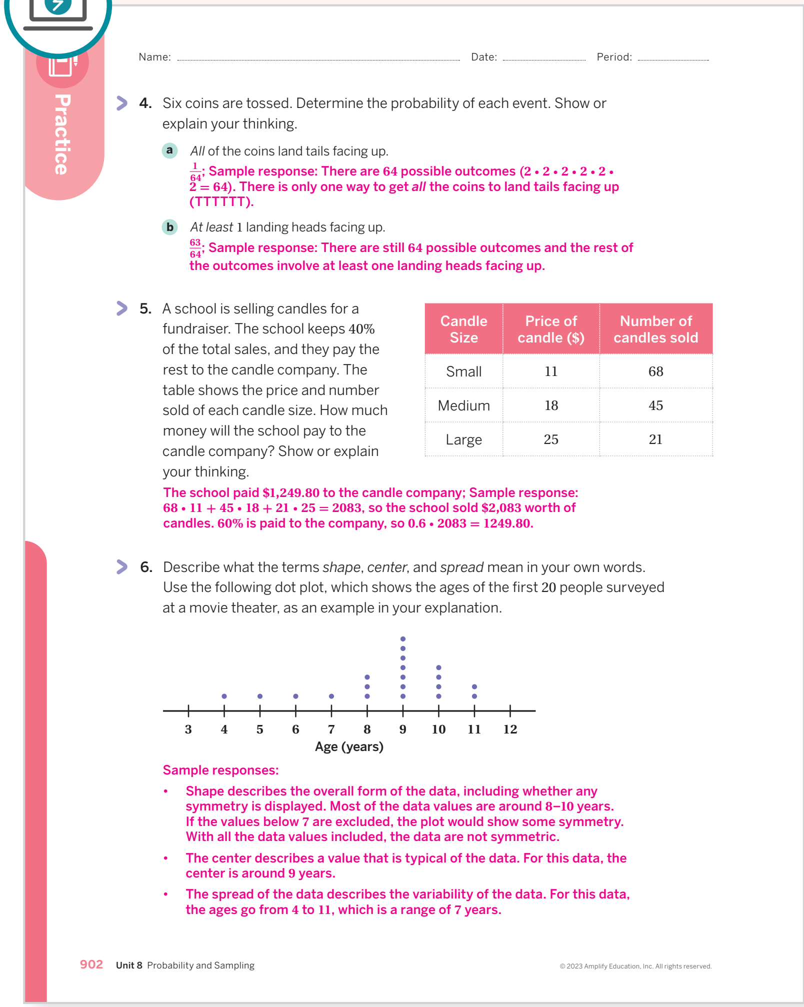
The last lesson of this unit is a capstone project. In this project, you will:

1. **Pose a statistical question.**
2. **Create a survey.**
3. **Decide on a sample.**
4. **Collect and analyze data from the sample.**
5. **Present your findings.**

In the remaining lessons in this unit, there are problems marked *Capstone project helper.* These problems will help you prepare for the project.

3. *Capstone project helper.* Write a statistical question you are interested in studying. Be sure it meets the criteria shown in the table. An example statistical question is: *Do students in my school who have purchased a new phone within the last* 3 *months tend to have more apps on their phone?*
Answers may vary.

Criteria for Statistical Questions
• Can it be answered by collecting data? • Will there be some variability in the data? • Is it possible to actually collect this data?



Name: Date: Period:

Practice

4. Six coins are tossed. Determine the probability of each event. Show or explain your thinking.
   - a *All* of the coins land tails facing up.
   $\frac{1}{64}$; Sample response: There are 64 possible outcomes ($2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$). There is only one way to get *all* the coins to land tails facing up (TTTTTT).
   - b *At least* 1 landing heads facing up.
   $\frac{63}{64}$; Sample response: There are still 64 possible outcomes and the rest of the outcomes involve at least one landing heads facing up.

5. A school is selling candles for a fundraiser. The school keeps 40% of the total sales, and they pay the rest to the candle company. The table shows the price and number sold of each candle size. How much money will the school pay to the candle company? Show or explain your thinking.

Candle Size	Price of candle ($)	Number of candles sold
Small	11	68
Medium	18	45
Large	25	21

The school paid $1,249.80 to the candle company; Sample response: $68 \cdot 11 + 45 \cdot 18 + 21 \cdot 25 = 2083$, so the school sold $2,083 worth of candles. 60% is paid to the company, so $0.6 \cdot 2083 = 1249.80$.

6. Describe what the terms *shape*, *center*, and *spread* mean in your own words. Use the following dot plot, which shows the ages of the first 20 people surveyed at a movie theater, as an example in your explanation.

Sample responses:

- Shape describes the overall form of the data, including whether any symmetry is displayed. Most of the data values are around 8–10 years. If the values below 7 are excluded, the plot would show some symmetry. With all the data values included, the data are not symmetric.
- The center describes a value that is typical of the data. For this data, the center is around 9 years.
- The spread of the data describes the variability of the data. For this data, the ages go from 4 to 11, which is a range of 7 years.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 8 Lesson 9	2
	5	Unit 4 Lesson 9	2
Formative ⚡	6	Unit 8 Lesson 13	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 13**

What Makes a Good Sample?

Let's see what makes a good sample.

Focus

Goals

1. Calculate the mean of various samples, and compare them with the mean of the population.
2. **Language Goal:** Comprehend that the term *representative* refers to a sample with a distribution that closely resembles the population's shape, center, and spread. **(Speaking and Listening, Writing)**
3. **Language Goal:** Given dot plots that represent a population and several samples, determine whether each sample is representative of the population, and explain the reasoning. **(Speaking and Listening, Writing)**

Coherence

Today

Students examine multiple samples of the same population and learn what it means for a sample to be representative of the population. Students study the structure of dot plots, attending to center, shape, and spread, to help them compare the samples and the population. The problems in this lesson use smaller populations so that students can compare each sample against the entire population. **Note:** This lesson's Practice contains a milestone for the Capstone project.

Previously

In Lesson 12, students began to think about the impracticality of studying entire populations and instead saw the usefulness of selecting samples of the population to study.

Coming Soon

In Lesson 14, students will critique different sampling methods to determine which sampling methods are fair which helps to reduce bias.

Rigor

- Students build **conceptual understanding** of samples and representative samples by comparing characteristics of a sample to the characteristics of the population from which it came.
- Students **apply** their understanding of representative samples to sampling at a fish market.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	8 min	8 min	5 min	7 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- tracing paper (optional)

Math Language Development

New word

- ***representative sample***

Review words

- *center*
- *mean*
- *population*
- *sample*
- *shape*
- *spread*
- *variability*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to explain their thinking after deciding if a sample is representative or not, and these explanations are available to you digitally, in real time.

Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may resist thinking deeply of the structure of the data in order to compare the dot plots. Encourage students to consider the perspective of others when writing their descriptions. Have them consider if someone could visualize the shape, center, and spread of the dot plot based solely on their description.

Modifications to Pacing

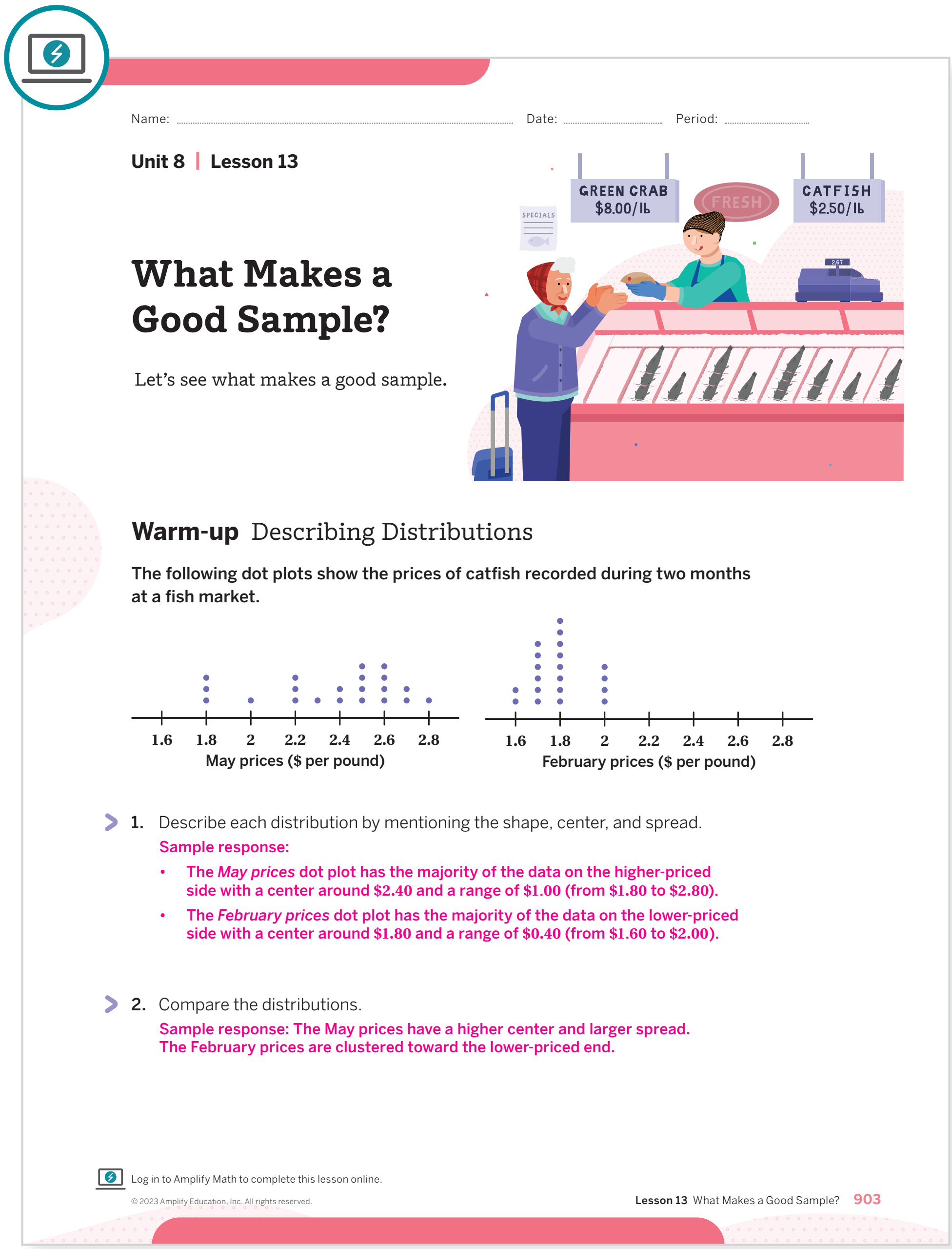
You may want to consider this additional modification if you are short on time.

- **Activity 3** may be omitted or assigned as an *Are you ready for more?* problem for students who finish **Activity 2** early.

Pairs | 5 min

Warm-up Describing Distributions

Students describe dot plots that they will use in Activity 2 to practice comparing the shape, center, and spread of distributions.



Name: ______________________ Date: __________ Period: __________

Unit 8 | Lesson 13

What Makes a Good Sample?

Let's see what makes a good sample.

Warm-up Describing Distributions

The following dot plots show the prices of catfish recorded during two months at a fish market.

1.6 1.8 2 2.2 2.4 2.6 2.8
May prices ($ per pound)

1.6 1.8 2 2.2 2.4 2.6 2.8
February prices ($ per pound)

1. Describe each distribution by mentioning the shape, center, and spread.
   Sample response:
   - **The *May prices* dot plot has the majority of the data on the higher-priced side with a center around $2.40 and a range of $1.00 (from $1.80 to $2.80).**
   - **The *February prices* dot plot has the majority of the data on the lower-priced side with a center around $1.80 and a range of $0.40 (from $1.60 to $2.00).**

2. Compare the distributions.
   Sample response: The May prices have a higher center and larger spread. The February prices are clustered toward the lower-priced end.

Log in to Amplify Math to complete this lesson online.

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Lesson 13 What Makes a Good Sample? 903

1 Launch

Activate students' background knowledge by asking if they have ever been to a market, and if so, what type. If someone is familiar with a fish market, ask them to share what they know about them. If not mentioned, be sure to share that the same type of fish can come in different sizes and that fish are usually sold by weight.

2 Monitor

Help students get started by having them drawing a curve around the data and asking if it looks symmetric.

Look for points of confusion:

- **Thinking the February prices are not symmetric because the data are not centered on the number line at 2.2.** Remind students that the number line could start and end so that the middle of the number line is 1.8. If students can draw a curve around the data that looks symmetric, then the data appear to be symmetric.

3 Connect

Display the dot plots.

Have students share their descriptions.

Ask:

- "When describing the center, did anyone use the mean? Median?"
- "Which data has more variability? How do you know?" The prices in May because they have a larger spread.
- "Which data appear more symmetric?" The prices in February appear more symmetric than the prices in May.

Highlight mathematically precise language and the structure of the dots plots as students share their comparisons.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their descriptions of the distributions, display a three column table similar to the following. Record the words and phrases students use to describe the shape, center, and spread of the two distributions.

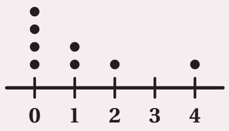
Shape	Center	Spread

Power-up

To power up students' ability to understand shape, center, and spread, have students complete:

Recall that the *spread* of a set of data describes its variability while the center describes its typical value.
Identify which statements are true based on the dot plot. Select *all* that apply.

[✓ A.] The mean is 1.
B. The median is 1.
[✓ C.] The MAD is 1.
[✓ D.] The range is 4.
[✓ E.] The median is 0.5.
F. The data is symmetric.



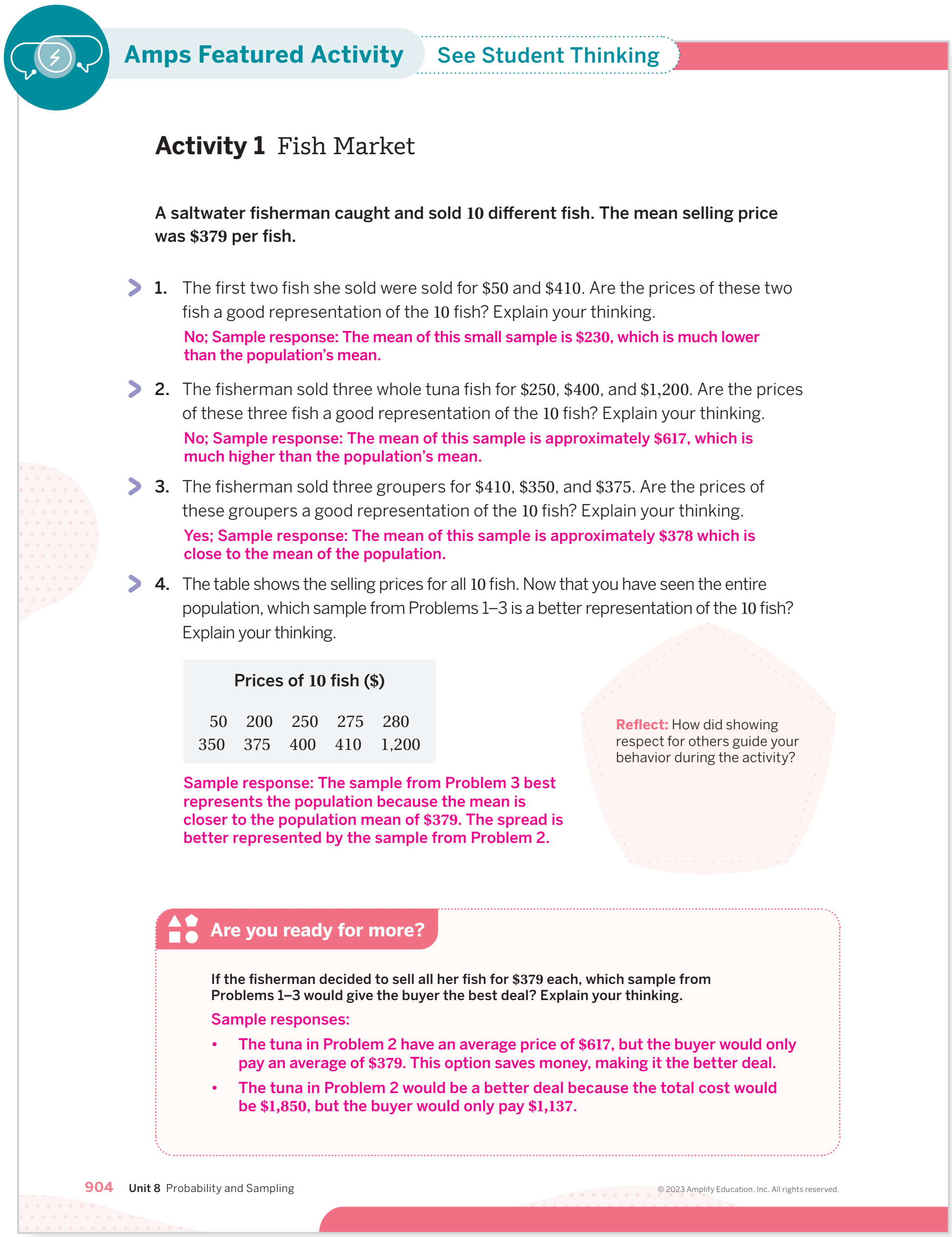
Use: Before the Warm-up
Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Pairs | 12 min

Activity 1 Fish Market

Students find the mean of samples and compare it to the mean of the population to determine which is a more representative sample using numerical evidence.

Amps Featured Activity See Student Thinking



Activity 1 Fish Market

A saltwater fisherman caught and sold 10 different fish. The mean selling price was $379 per fish.

1. The first two fish she sold were sold for $50 and $410. Are the prices of these two fish a good representation of the 10 fish? Explain your thinking.
   No; Sample response: The mean of this small sample is $230, which is much lower than the population's mean.
2. The fisherman sold three whole tuna fish for $250, $400, and $1,200. Are the prices of these three fish a good representation of the 10 fish? Explain your thinking.
   No; Sample response: The mean of this sample is approximately $617, which is much higher than the population's mean.
3. The fisherman sold three groupers for $410, $350, and $375. Are the prices of these groupers a good representation of the 10 fish? Explain your thinking.
   Yes; Sample response: The mean of this sample is approximately $378 which is close to the mean of the population.
4. The table shows the selling prices for all 10 fish. Now that you have seen the entire population, which sample from Problems 1–3 is a better representation of the 10 fish? Explain your thinking.

Prices of 10 fish ($)

50	200	250	275	280
350	375	400	410	1,200

Sample response: The sample from Problem 3 best represents the population because the mean is closer to the population mean of $379. The spread is better represented by the sample from Problem 2.

Reflect: How did showing respect for others guide your behavior during the activity?

Are you ready for more?

If the fisherman decided to sell all her fish for $379 each, which sample from Problems 1–3 would give the buyer the best deal? Explain your thinking.

Sample responses:

- The tuna in Problem 2 have an average price of $617, but the buyer would only pay an average of $379. This option saves money, making it the better deal.
- The tuna in Problem 2 would be a better deal because the total cost would be $1,850, but the buyer would only pay $1,137.

904 Unit 8 Probability and Sampling

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1 Launch

Activate students' background knowledge by asking what the term *representative* means and encourage students to think about situations outside of mathematics.

2 Monitor

Help students get started by saying one way to think about if a sample is representative of the population is to compare the mean of the sample to the mean of the population.

Look for points of confusion:

- **Calculating the mean incorrectly.** Remind students of the process to find the mean.

Look for productive strategies:

- Recognizing the $1,200 tuna is not like the other priced fish.
- Constructing a dot plot or another display of the data.

3 Connect

Have students share their means and reasoning if the sample is representative of the population.

Highlight that the data set for this population is small enough that it is not necessary to use a sample; however, it is helpful to get an idea of how data from a sample compares to the population data.

Ask:

- "What is the population for this situation?" All of the fish sold.
- "What are the samples used in the calculations?" The first two fish sold, the tuna sold, and the grouper sold.
- "Why did the different samples have different means?" Because they used different fish.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to compare the sample mean with the population mean for Problem 1. Annotate the given mean selling price of $379 per fish given in the introduction as the *population mean*. Then show how to calculate the mean of the sample in Problem 1 and annotate it with the term *sample mean*.

Accessibility: *Clarify Vocabulary and Symbols*

Be sure students understand what it means for the prices of the fish to be a "good representation" of the population of 10 fish. Ask, "What does it mean to 'represent' something?" Sample response: To be similar to, or to serve as a sign or symbol of that object.

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Connect, as students share their reasoning, have their classmates restate or revoice what they heard using appropriate mathematical language. Ask the original speaker whether their peer was accurate. Call students' attention to words or phrases that clarified the original statement. This will provide students with an opportunity to produce language as they interpret the reasoning of others.

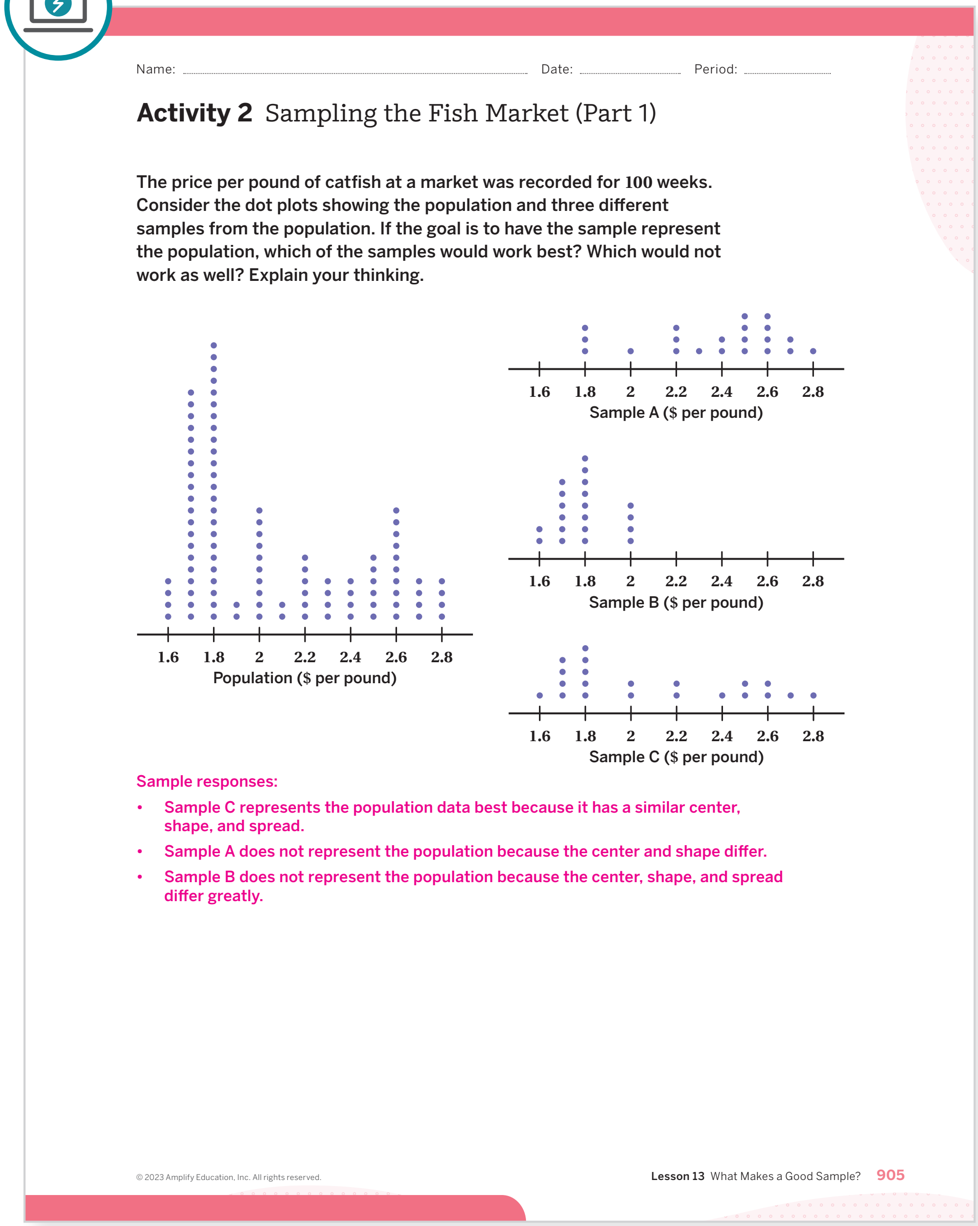
English Learners

Encourage students to refer to the class display to use language to support their use of appropriate mathematical language.

Pairs | 8 min

Activity 2 Sampling the Fish Market (Part 1)

Students compare three samples of population to the population itself to determine which is the most representative sample.



Name: ______ Date: ______ Period: ______

Activity 2 Sampling the Fish Market (Part 1)

The price per pound of catfish at a market was recorded for 100 weeks. Consider the dot plots showing the population and three different samples from the population. If the goal is to have the sample represent the population, which of the samples would work best? Which would not work as well? Explain your thinking.

Sample responses:

- Sample C represents the population data best because it has a similar center, shape, and spread.
- Sample A does not represent the population because the center and shape differ.
- Sample B does not represent the population because the center, shape, and spread differ greatly.

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Lesson 13 What Makes a Good Sample? 905

1 Launch

Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by describing the shape, center, and spread of the population. Then have them refer to their responses from the Warm-up.

Look for points of confusion:

- **Thinking the sample must perfectly match the population.** Let students know that these samples all came from the population, but that one sample is a better match for the population than the others.

3 Connect

Display the dot plots.

Have students share their selection and reasoning.

Ask, "What are some aspects that make for a 'good' sample? A sample that is 'not as good'?" A sample is "good" if it has a similar distribution to the population data. A sample is "not as good" if the data does not have a similar distribution to the population data. For example, Sample B is "not as good" because its center is not similar to the center of the population.

Define the term ***representative sample*** as having a distribution which closely resembles the distribution of the population in shape, center, and spread.

Highlight that a sample that only has the same mean as the population is not necessarily representative. It may miss other important aspects of the population like shape and spread. It is important to look at shape, spread, and center when determining whether a sample is representative of the population.

Differentiated Support

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Provide students with tracing paper and suggest they trace the overall shape of the population. Consider demonstrating this action. Then have them move the tracing paper to the other dot plots to determine which dot plot might be the most similar.

Math Language Development

MLR2: Collect and Display

During the Connect, create a table with column headings "Good sample" and "Not as good." Record the words and phrases students use to describe the samples. For example:

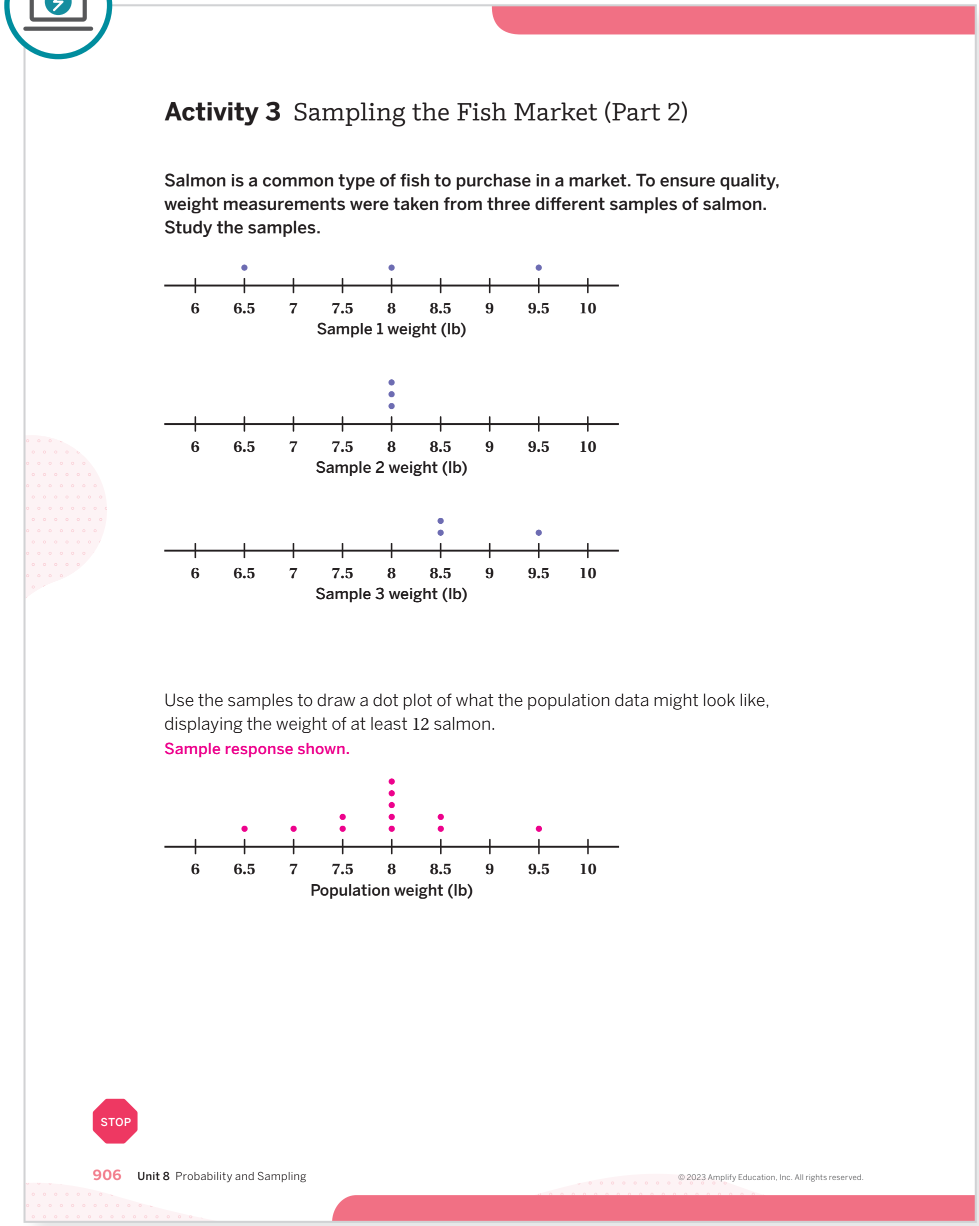
Good sample	Not as good
They have similar shapes. The peaks are in the same places. The centers are close to each other. The variabilities are similar.	The shapes are very different. The peaks are not in the same places. The centers are not close to each other. The data are more/less spread out.

Use the words and phrases that you recorded that describe a good sample to help define the term *representative sample*.

Pairs | 8 min

Activity 3 Sampling the Fish Market (Part 2)

Students use sample distributions to create a possible population distribution.



Activity 3 Sampling the Fish Market (Part 2)

Salmon is a common type of fish to purchase in a market. To ensure quality, weight measurements were taken from three different samples of salmon. Study the samples.

Use the samples to draw a dot plot of what the population data might look like, displaying the weight of at least 12 salmon.

Sample response shown.

906 Unit 8 Probability and Sampling

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "Which sample can help determine the spread of your data? The center? The shape?"

Look for points of confusion:

- **Thinking the sample data can be combined represent the population.** Remind students that different samples might have chosen the same fish, and therefore, there might be overlap in some of the data values.
- **Not including the known data.** Have students start by plotting the known "data values" on their population number line. Then have them add "data values" to it and make sure there are at least 12 "data values."

3 Connect

Display students' dot plots.

Have students share their methods for drawing the population distribution. Ask them to compare the student-generated distributions.

Highlight that gaining an understanding of the population data from a sample can be challenging, especially when it is not known whether samples are or are not representative of the population.

Ask, "What could be done to make the samples more representative of the population data without knowing what the population would be?" Sample response: Include more information in the samples to find how the samples were selected.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Suggest that students list out the fish weights from the three samples and then build their dot plot for the population using these values.

Math Language Development

MLR7: Compare and Connect

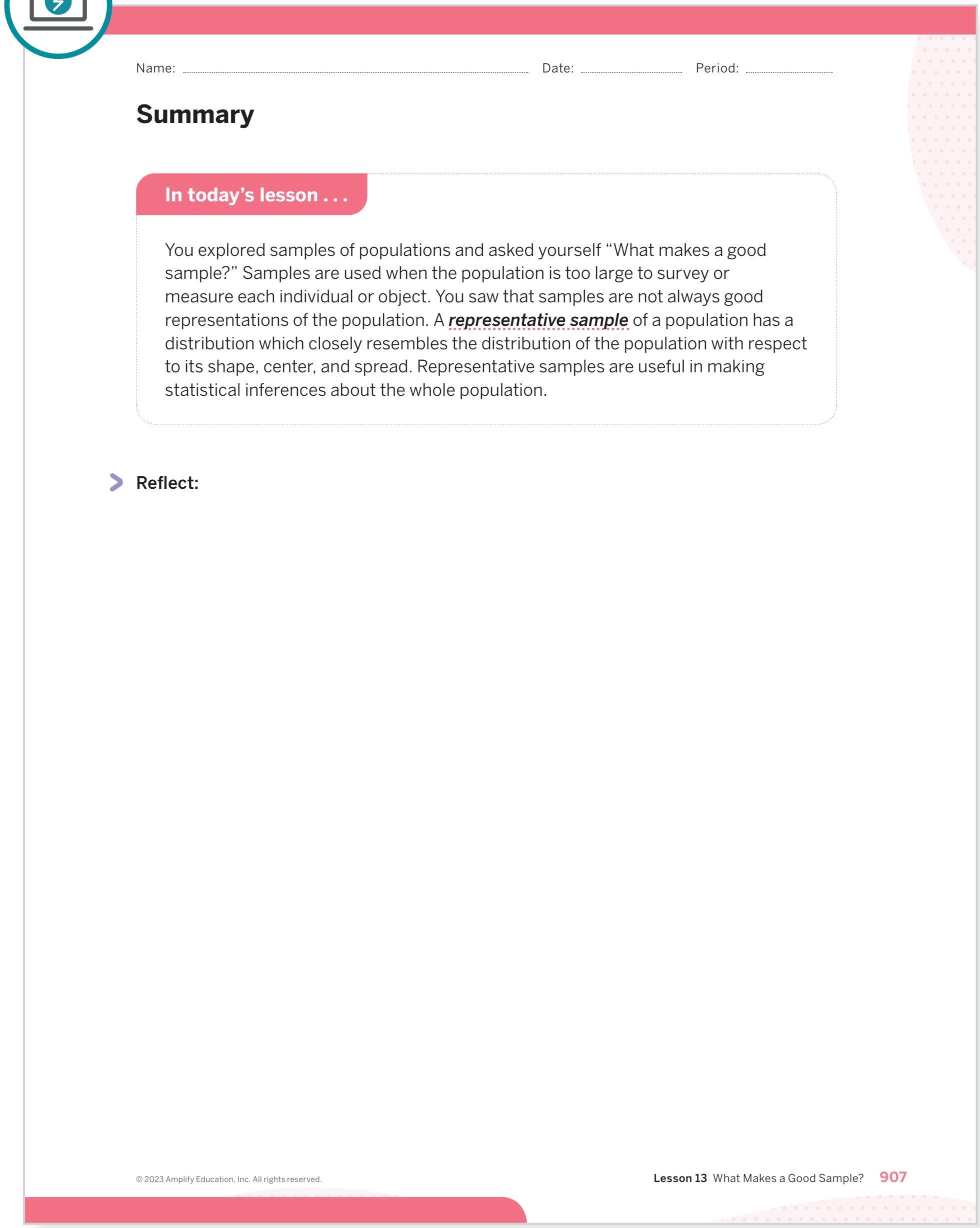
Invite students to prepare a visual display of their dot plots. Then have students investigate each other's work and compare their dot plots. Listen for and amplify the language students use to describe the population, and how they explain why it might be challenging to understand what the population might look like, based on three very different samples. Ask:

- "Are any of the three samples representative of the population dot plot you or others created?"
- "Could there be other population dot plots for which these three dot plots are samples?"

Whole Class | 5 min

Summary

Review and synthesize how a sample that is representative of the population closely resembles the population distribution's shape, center, and spread.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored samples of populations and asked yourself "What makes a good sample?" Samples are used when the population is too large to survey or measure each individual or object. You saw that samples are not always good representations of the population. A ***representative sample*** of a population has a distribution which closely resembles the distribution of the population with respect to its shape, center, and spread. Representative samples are useful in making statistical inferences about the whole population.

Reflect:

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Lesson 13 What Makes a Good Sample? 907

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *representative sample* that were added to the display during the lesson.

Synthesize

Formalize vocabulary: *representative sample*

Ask:

- "What does it mean for a sample to be representative of the population?" The sample has a similar center, shape, and spread as the population data.
- "Why might it be important to get a representative sample, rather than a more convenient sample?" If I am going to answer questions about the entire population, it is useful if the sample looks similar to the population data. If not, I may miss some important information.
- "Usually, a sample is used because it is challenging to obtain data for the entire population. How do you know if the sample is representative of the population if you do not know the population?" **Note:** It is okay for students to wrestle with this question at this point. In the next lesson, they will explore ways to make their best attempt at getting a representative sample.

Highlight that a representative sample is the ideal type of sample they would like to collect, but, if they do not know the data for the population, it will be challenging to know if a sample they collect is representative or not. If they do know the population data, then a sample is probably unnecessary. In future lessons, they will explore methods of collecting samples that are more likely to produce representative samples (although it is still not guaranteed).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

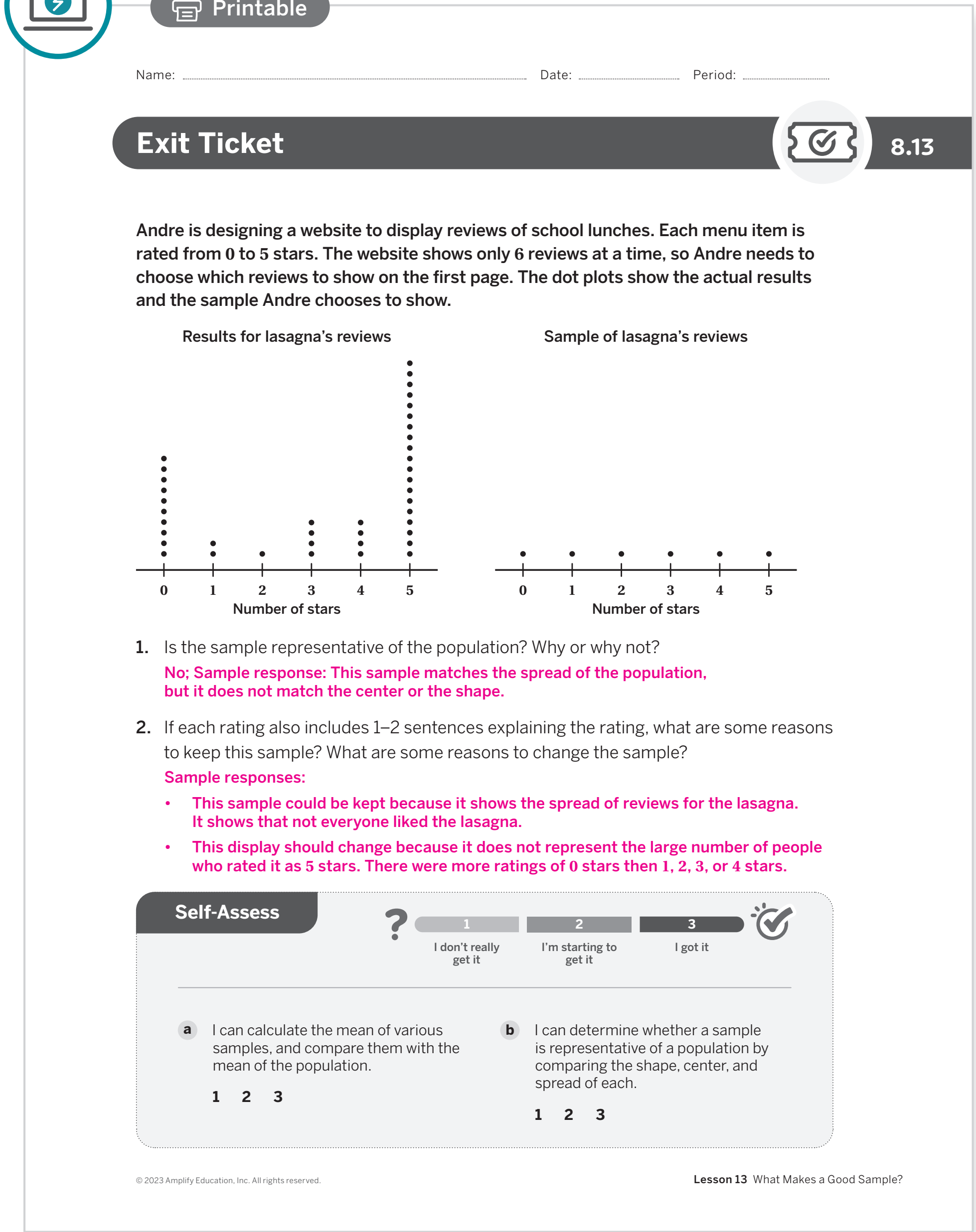
- "When is a sample *not* representative of a population?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by analyzing a sample of reviews compared to the population.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

8.13

Andre is designing a website to display reviews of school lunches. Each menu item is rated from 0 to 5 stars. The website shows only 6 reviews at a time, so Andre needs to choose which reviews to show on the first page. The dot plots show the actual results and the sample Andre chooses to show.

1. Is the sample representative of the population? Why or why not?
   No; Sample response: This sample matches the spread of the population, but it does not match the center or the shape.

2. If each rating also includes 1–2 sentences explaining the rating, what are some reasons to keep this sample? What are some reasons to change the sample?
   Sample responses:
   - **This sample could be kept because it shows the spread of reviews for the lasagna. It shows that not everyone liked the lasagna.**
   - **This display should change because it does not represent the large number of people who rated it as 5 stars. There were more ratings of 0 stars then 1, 2, 3, or 4 stars.**

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can calculate the mean of various samples, and compare them with the mean of the population.
1 2 3

b I can determine whether a sample is representative of a population by comparing the shape, center, and spread of each.
1 2 3

Success looks like . . .

- **Goal:** Calculating the mean of various samples, and comparing them with the mean of the population.
- **Language Goal:** Comprehending that the term *representative* refers to a sample with a distribution that closely resembles the population's shape, center, and spread. **(Speaking and Listening, Writing)**
  - » Determining whether the 6 samples are representative of the population in Problem 1.
- **Language Goal:** Given dot plots that represent a population and several samples, determining whether each sample is representative of the population, and explaining the reasoning. **(Speaking and Listening, Writing)**
  - » Explaining whether the sample of 6 reviews is representative of the population in Problem 1.

Suggested next steps

If students cannot think of a positive or a negative reason for keeping the sample, consider:

- Having them compare the distributions by their shapes, centers, and spreads.
- Assigning Practice Problems 2 and 3.

If students think the sample is representative of the population, consider:

- Having them compare the distributions by their shapes, centers, and spreads.
- Assigning Practice Problems 1, 2, and 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students look for and make use of structure today? How are you helping students become aware of how they are progressing in this area?
- The focus of this lesson was for students to comprehend that the term *representative sample* refers to a sample that closely resembles the population's shape, center, and spread. How did this go? What might you change for the next time you teach this lesson?

Math Language Development

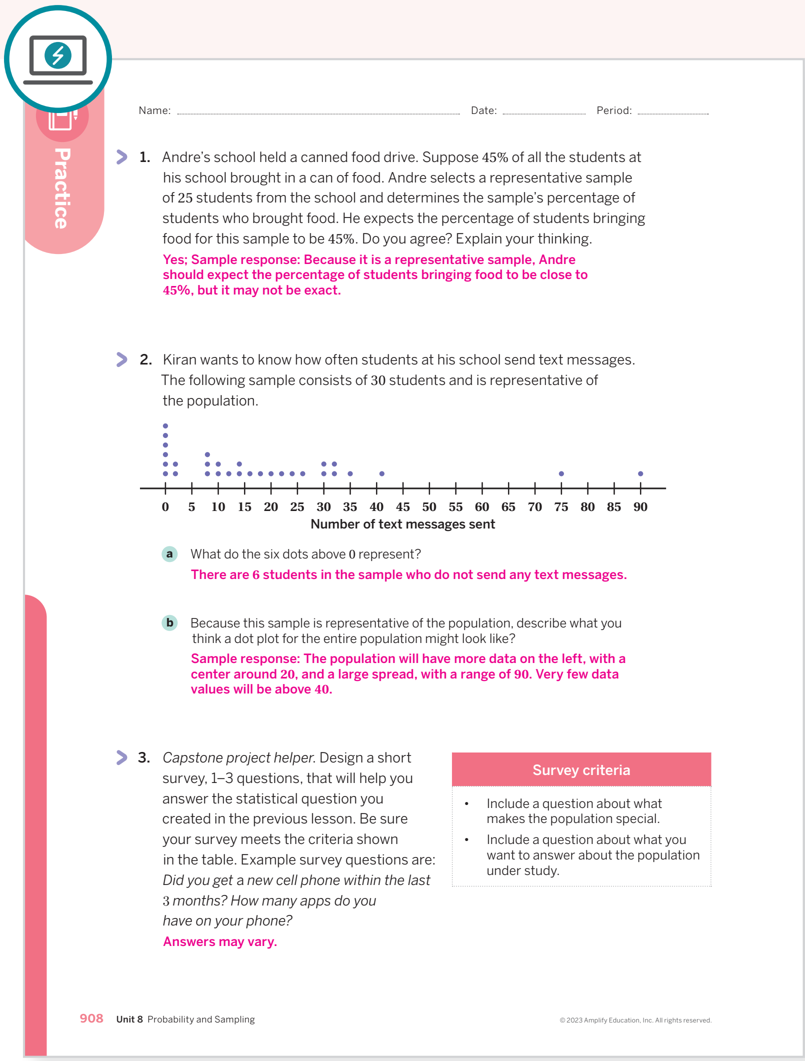
Language Goal: Justifying whether two populations are "very different" based on the difference in their means expressed as a multiple of the mean absolute deviation.

Reflect on students' language development toward this goal.

- Do students' responses to the Exit Ticket problem indicate an understanding that the prices of RVs for sale at the two companies are very different?
- Do their explanations include math language, such as "the difference in means", "three times as great as the MAD", etc? How can you help them be more precise in their explanations?

Practice

Independent



Name: Date: Period:

Practice

1. Andre's school held a canned food drive. Suppose 45% of all the students at his school brought in a can of food. Andre selects a representative sample of 25 students from the school and determines the sample's percentage of students who brought food. He expects the percentage of students bringing food for this sample to be 45%. Do you agree? Explain your thinking.

Yes; Sample response: Because it is a representative sample, Andre should expect the percentage of students bringing food to be close to 45%, but it may not be exact.

2. Kiran wants to know how often students at his school send text messages. The following sample consists of 30 students and is representative of the population.

a. What do the six dots above 0 represent?

There are 6 students in the sample who do not send any text messages.

b. Because this sample is representative of the population, describe what you think a dot plot for the entire population might look like?

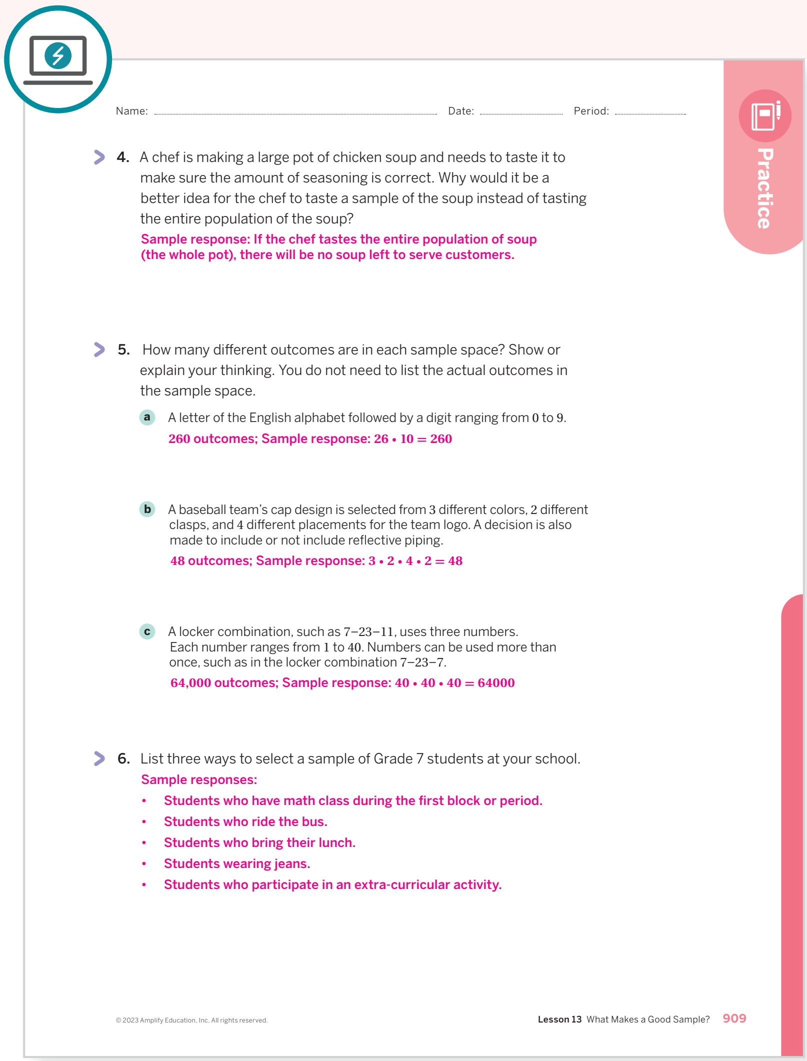
Sample response: The population will have more data on the left, with a center around 20, and a large spread, with a range of 90. Very few data values will be above 40.

3. *Capstone project helper*. Design a short survey, 1–3 questions, that will help you answer the statistical question you created in the previous lesson. Be sure your survey meets the criteria shown in the table. Example survey questions are: *Did you get a new cell phone within the last 3 months? How many apps do you have on your phone?*

Answers may vary.

Survey criteria

- Include a question about what makes the population special.
- Include a question about what you want to answer about the population under study.



Name: Date: Period:

Practice

4. A chef is making a large pot of chicken soup and needs to taste it to make sure the amount of seasoning is correct. Why would it be a better idea for the chef to taste a sample of the soup instead of tasting the entire population of the soup?

Sample response: If the chef tastes the entire population of soup (the whole pot), there will be no soup left to serve customers.

5. How many different outcomes are in each sample space? Show or explain your thinking. You do not need to list the actual outcomes in the sample space.

a. A letter of the English alphabet followed by a digit ranging from 0 to 9.

260 outcomes; Sample response: 26 • 10 = 260

b. A baseball team's cap design is selected from 3 different colors, 2 different clasps, and 4 different placements for the team logo. A decision is also made to include or not include reflective piping.

48 outcomes; Sample response: 3 • 2 • 4 • 2 = 48

c. A locker combination, such as 7–23–11, uses three numbers. Each number ranges from 1 to 40. Numbers can be used more than once, such as in the locker combination 7–23–7.

64,000 outcomes; Sample response: 40 • 40 • 40 = 64000

6. List three ways to select a sample of Grade 7 students at your school.

Sample responses:

- **Students who have math class during the first block or period.**
- **Students who ride the bus.**
- **Students who bring their lunch.**
- **Students wearing jeans.**
- **Students who participate in an extra-curricular activity.**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	1
Spiral	4	Unit 8 Lesson 12	1
	5	Unit 8 Lesson 7	1
Formative ⚡	6	Unit 8 Lesson 14	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 14**

Sampling in a Fair Way

Let's explore ways to obtain representative samples.

Focus

Goals

1. **Language Goal:** Describe methods to obtain a random sample from a population. **(Speaking and Listening, Writing)**
2. **Language Goal:** Justify whether a given sampling method is fair. **(Speaking and Listening)**
3. Recognize that random sampling tends to produce representative samples and support valid inferences.

Coherence

Today

Students consider different methods of selecting a sample and begin by critiquing different sampling methods for their benefits and drawbacks. The Warm-up shows that some methods may seem to be unbiased at first, but have a hidden bias that restricts the sample from being representative of the population. Students practice recognizing when a sampling method is likely to be biased, and they see that selecting a sample at random is more likely to produce a representative sample. **Note:** This lesson's Practice contains a milestone for the Capstone project.

Previously

In Lesson 13, students explored the characteristics of samples that are representative of the population by studying the shapes, centers, and spreads of the samples and the population.

Coming Soon

In Lesson 15, students will estimate the measures of center of a population by using representative samples.

Rigor

- Students improve their **conceptual understanding** of sampling by showing how a random sample is most likely representative of the population.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	12 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- bags
- rulers
- straws, cut to the following lengths: 12 at 1 cm, 8 at 2 cm, 6 at 3 cm, 5 at 4 cm, 4 at 5 cm

Note: If you change the lengths of the straws, you will need to edit the mean for Activity 1, Problem 4.

Math Language Development

New words

- ***random sample***

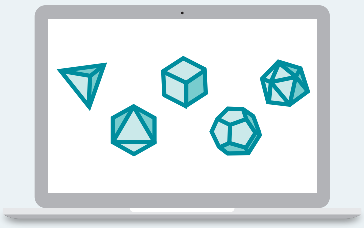
Review words

- *mean*
- *mean absolute deviation (MAD)*
- *population*
- *representative sample*
- *sample*
- *variability*

Amps | Featured Activity

Activity 1
Digital Sampling

Students digitally perform a sampling technique to discuss hidden bias.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may be discouraged to listen to other students' ideas and explanations; they may only think their own reasoning is correct. Encourage students to actively listen to what others say while thinking about the sampling methods. Have them ask a clarifying question to the speaker to positively engage in critiquing the reasoning of others.

Modifications to Pacing

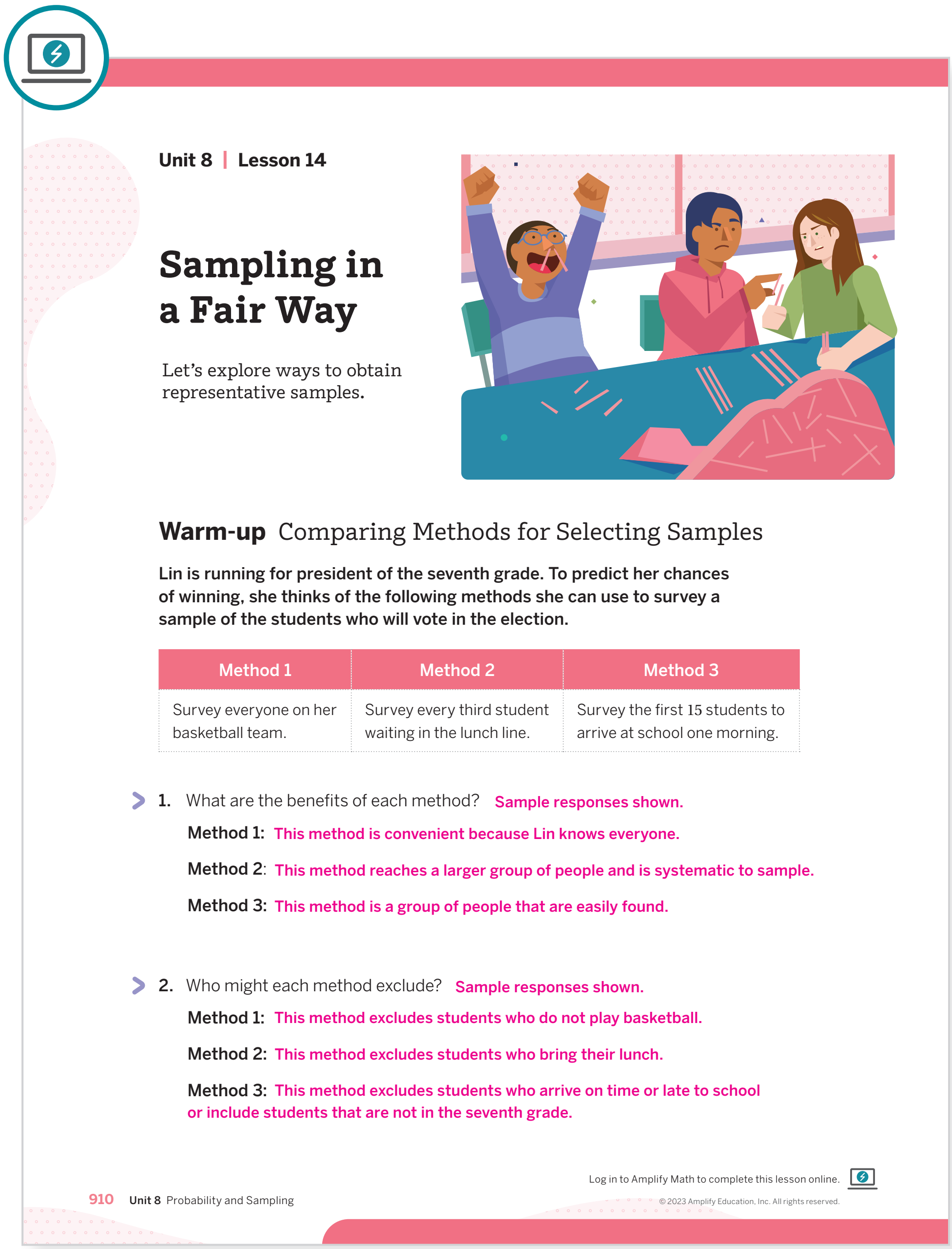
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be completed as a whole class.
- **Activity 1** may be completed as a whole-class discussion.

Pairs | 10 min

Warm-up Comparing Methods for Selecting Samples

Students reason about different sampling methods to determine if they are unbiased.



Unit 8 | Lesson 14

Sampling in a Fair Way

Let's explore ways to obtain representative samples.

Warm-up Comparing Methods for Selecting Samples

Lin is running for president of the seventh grade. To predict her chances of winning, she thinks of the following methods she can use to survey a sample of the students who will vote in the election.

Method 1	Method 2	Method 3
Survey everyone on her basketball team.	Survey every third student waiting in the lunch line.	Survey the first 15 students to arrive at school one morning.

1. What are the benefits of each method? Sample responses shown.

   Method 1: This method is convenient because Lin knows everyone.

   Method 2: This method reaches a larger group of people and is systematic to sample.

   Method 3: This method is a group of people that are easily found.

2. Who might each method exclude? Sample responses shown.

   Method 1: This method excludes students who do not play basketball.

   Method 2: This method excludes students who bring their lunch.

   Method 3: This method excludes students who arrive on time or late to school or include students that are not in the seventh grade.

Log in to Amplify Math to complete this lesson online.

910 Unit 8 Probability and Sampling

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1 Launch

Activate students' background knowledge by asking students to share what they know about elections. Share information regarding the school's election process or how the electoral college works for the United States.

2 Monitor

Help students get started by asking them to explain each method in their own words and clarify any instances of misunderstanding.

Look for productive strategies:

- Not thinking any of the methods are good sampling techniques and can explain the flaws in each method.

3 Connect

Have students share the benefits and drawbacks of each of the sampling methods.

Highlight that people often have biases which may lead to over- or under-representing some groups in their samples whether the biases are obvious or not. Due to the (sometimes hidden) biases, the best method for selecting samples is to remove as much of the personal selection as possible.

Ask:

- "What are the benefits of each method?"
- "What might each method overlook?"
- "What are some important things to consider when selecting a sample?" Is there a group that this method shows preference for or does a group automatically get left out by this method?
- "Can you think of a better way to select a sample for this situation?" Sample response: Obtain an alphabetized list of all the 7th graders and ask every fourth student.

Power-up

To power up students' ability to describe how to obtain a sample from a population, have students complete:

Recall that a *population* is a set of people or objects that are to be studied and that a *sample* is a part of a population.

A researcher wants to know more about how seventh graders use cell phones. Which of the following would be a sample of seventh graders? Select *all* that apply.

A. Asking all students at 50 middle schools.
[✓ **B.**] Sending a survey to a selection of schools and having them give it to seventh grade students.
[✓ **C.**] Polling students at a seventh grade math contest.
D. Asking guardians of seventh graders to report on what they have noticed about their children.

Use: Before the Warm-up

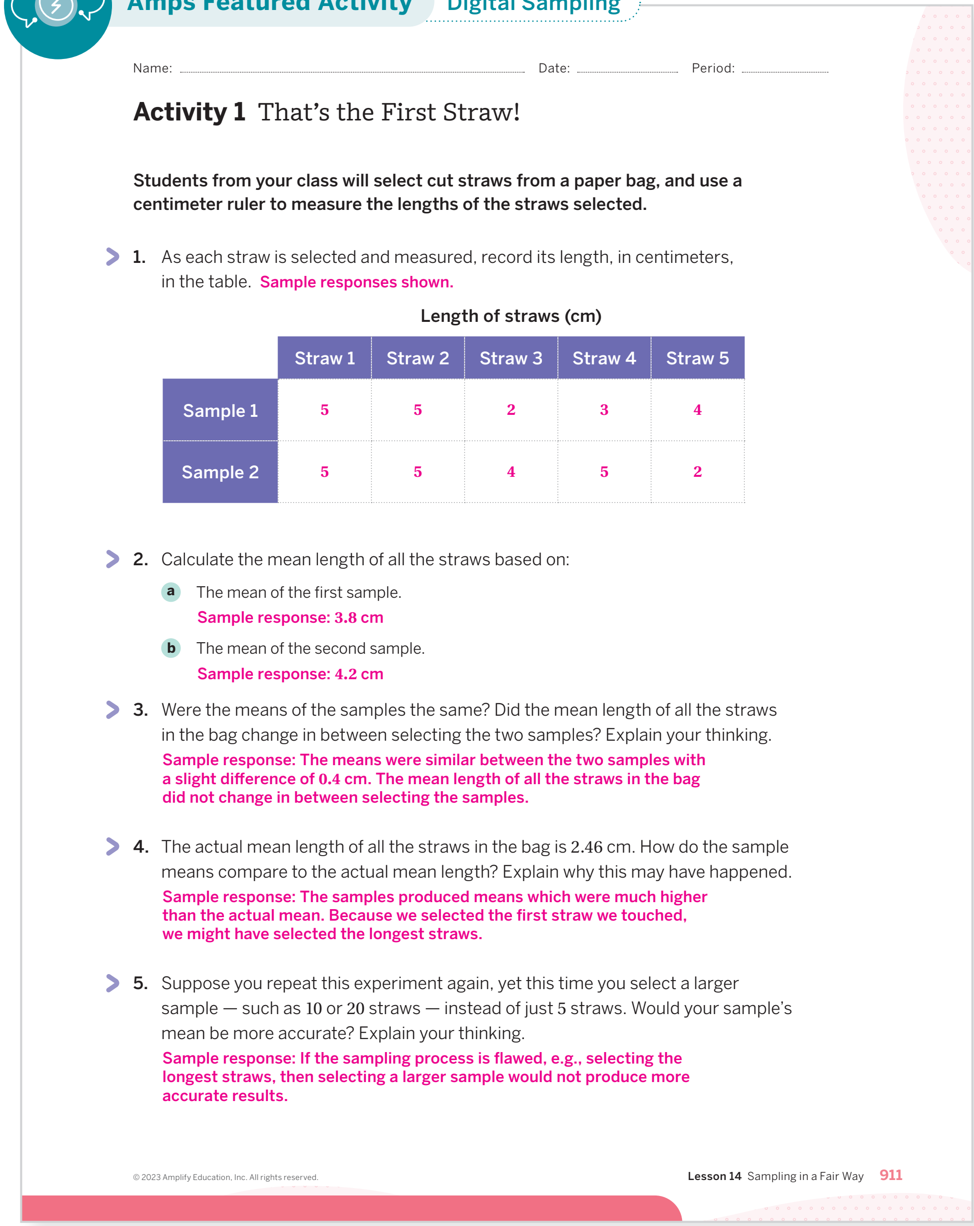
Informed by: Performance on Lesson 13, Practice Problem 6

Pairs | 15 min

Activity 1 That’s the First Straw!

Students experience a hidden bias while collecting data to compare the means of the samples to the mean of the population.

Amps Featured Activity Digital Sampling



Name: Date: Period:

Activity 1 That’s the First Straw!

Students from your class will select cut straws from a paper bag, and use a centimeter ruler to measure the lengths of the straws selected.

1. As each straw is selected and measured, record its length, in centimeters, in the table. **Sample responses shown.**

Length of straws (cm)

	Straw 1	Straw 2	Straw 3	Straw 4	Straw 5
Sample 1	5	5	2	3	4
Sample 2	5	5	4	5	2

2. Calculate the mean length of all the straws based on:
   a. The mean of the first sample.
   Sample response: 3.8 cm
   b. The mean of the second sample.
   Sample response: 4.2 cm

3. Were the means of the samples the same? Did the mean length of all the straws in the bag change in between selecting the two samples? Explain your thinking.
Sample response: The means were similar between the two samples with a slight difference of 0.4 cm. The mean length of all the straws in the bag did not change in between selecting the samples.

4. The actual mean length of all the straws in the bag is 2.46 cm. How do the sample means compare to the actual mean length? Explain why this may have happened.
Sample response: The samples produced means which were much higher than the actual mean. Because we selected the first straw we touched, we might have selected the longest straws.

5. Suppose you repeat this experiment again, yet this time you select a larger sample — such as 10 or 20 straws — instead of just 5 straws. Would your sample’s mean be more accurate? Explain your thinking.
Sample response: If the sampling process is flawed, e.g., selecting the longest straws, then selecting a larger sample would not produce more accurate results.

1 Launch

Cut straws according to measurements in the materials list and place them in a bag. Have five students take turns selecting the first straw they touch. **Note:** Taking out the first straw the student touches, rather than reaching around in the bag is important for this task. They should measure the straw to the nearest centimeter while the class records the data. Repeat this process for a second sample.

2 Monitor

Help students get started by reviewing how to calculate the mean of a data set.

3 Connect

Ask:

- “Why do you think the population’s mean is much smaller than the mean of your sample? What does this suggest about the rest of the data?” Most of the data are smaller than the sample.
- “What would it mean for the process of selecting straws to be fair (unbiased)?” There should be an equal chance for each straw to be selected.
- “Was this selection process fair (unbiased)?”

Display the contents of the bag, perhaps with a document camera.

Have students share their thoughts on whether each straw had an equal chance of being selected. Longer straws are more likely to be touched first, because the smaller ones fell to the bottom of the bag.

Highlight that increasing the sample size will not make the sample more representative. This is because if the sampling method is flawed, it may increase the over-representation of the longer straws and be even more misleading.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital technology to understand sampling techniques and discuss hidden bias.

Accessibility: *Vary Demands to Optimize Challenge*

Have different sets of students calculate the mean for each sample and ask them to share their results with the class.

Math Language Development

MLR8: Discussion Supports

Have pairs use a ***Think-Write-Pair-Share*** routine as they complete Problem 5. Ask them to individually think about how they will respond and craft an individual response before sharing their responses with their partner. Allow time for partners to discuss and agree on a final response and then ask pairs of students to share their responses with the class during the Connect.

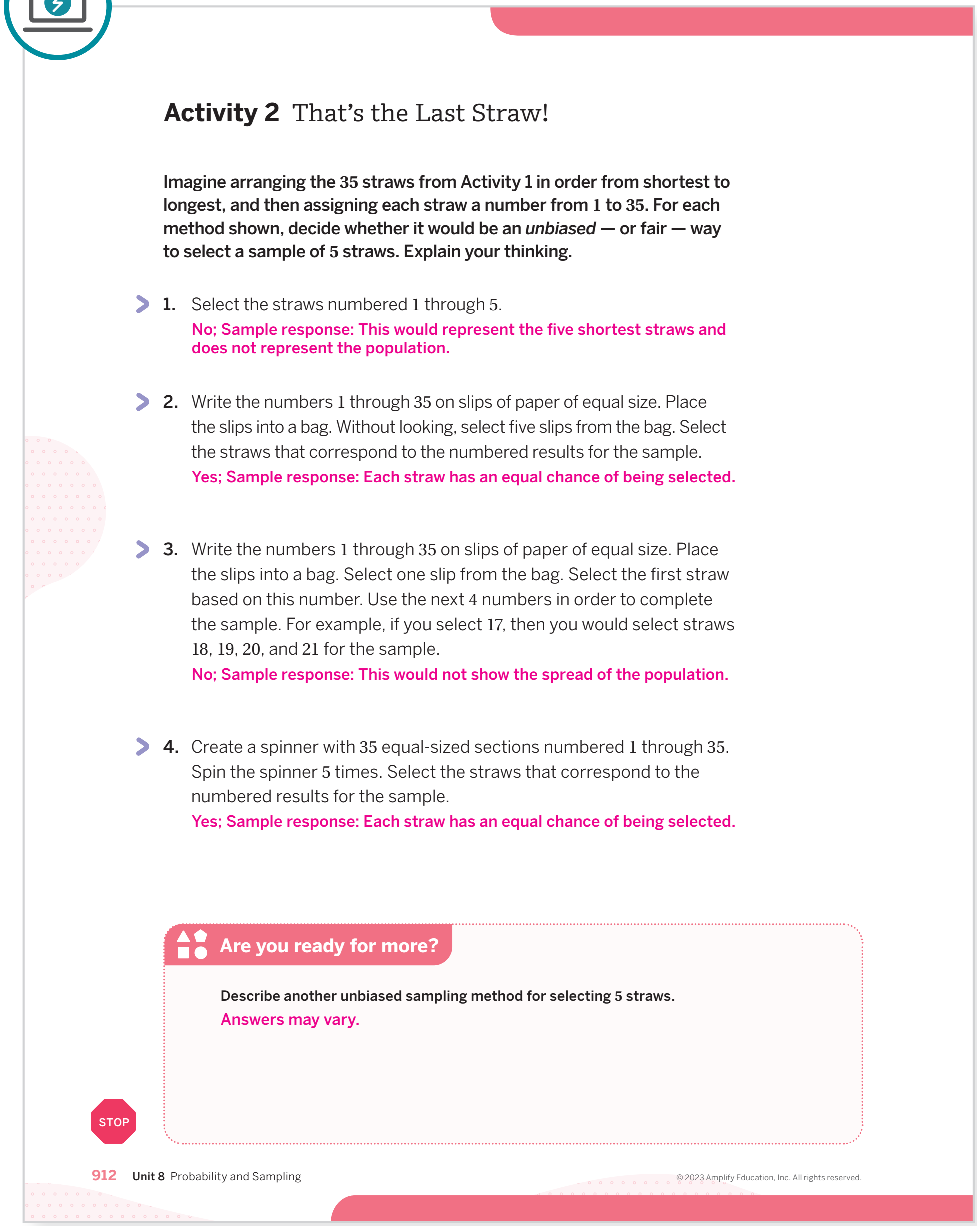
English Learners

Provide ample wait time for students to formulate a response in writing and orally before sharing with a partner and the class.

Pairs | 12 min

Activity 2 That's the Last Straw!

Students analyze alternate methods of sampling to determine whether they will eliminate bias.



Activity 2 That's the Last Straw!

Imagine arranging the 35 straws from Activity 1 in order from shortest to longest, and then assigning each straw a number from 1 to 35. For each method shown, decide whether it would be an *unbiased* — or fair — way to select a sample of 5 straws. Explain your thinking.

1. Select the straws numbered 1 through 5.
   No; Sample response: This would represent the five shortest straws and does not represent the population.
2. Write the numbers 1 through 35 on slips of paper of equal size. Place the slips into a bag. Without looking, select five slips from the bag. Select the straws that correspond to the numbered results for the sample.
   Yes; Sample response: Each straw has an equal chance of being selected.
3. Write the numbers 1 through 35 on slips of paper of equal size. Place the slips into a bag. Select one slip from the bag. Select the first straw based on this number. Use the next 4 numbers in order to complete the sample. For example, if you select 17, then you would select straws 18, 19, 20, and 21 for the sample.
   No; Sample response: This would not show the spread of the population.
4. Create a spinner with 35 equal-sized sections numbered 1 through 35. Spin the spinner 5 times. Select the straws that correspond to the numbered results for the sample.
   Yes; Sample response: Each straw has an equal chance of being selected.

Are you ready for more?

Describe another unbiased sampling method for selecting 5 straws.
Answers may vary.

STOP

1 Launch

Display the Activity 2 PDF and let students know the straws from the previous activity are represented with 1 being the shortest straw and 35 being the longest straw. Ask students, "What would it mean for the sampling method to be fair?"

2 Monitor

Help students get started by referring to the display and asking if the straws numbered 1 through 5 represent the population of the data.

Look for points of confusion:

- **Confusing a sample with a simulation.** Let students know a sample is a group of data from a population of data; whereas, a simulation models a real-world event.

Look for productive strategies:

- Finding the bias (hidden or not) in the experiments and being able to precisely describe the biases.

3 Connect

Define the term ***random sample*** as a sample that has an equal chance of being selected from the population as any sample of the same size.

Ask:

- "Which of the four methods proposed would be a random sample?"
- "What is the most common straw in the bag?"
- "When selecting one of the straws at random, what is the probability of selecting the most common straw?" $\frac{12}{35}$

Highlight that the goal is to have a sample representative of the population. One way to get close to a representative sample is to find a random sample. A random sample does not guarantee a representative sample, but it avoids methods that might over- or under-represent items of the population.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider drawing, or showing, 5 straws that are very short and are arranged in order from shortest to longest. Demonstrate how selecting these 5 straws out of the entire population of 35 straws that are much longer would not represent the population. Have students continue with the rest of the activity.

Math Language Development

MLR2: Collect and Display

During the Connect, ask students to describe situations in which they may have heard or used the term random. Consider displaying these phrases, which students may have heard or used in everyday life. Ask students what they think these phrases mean.

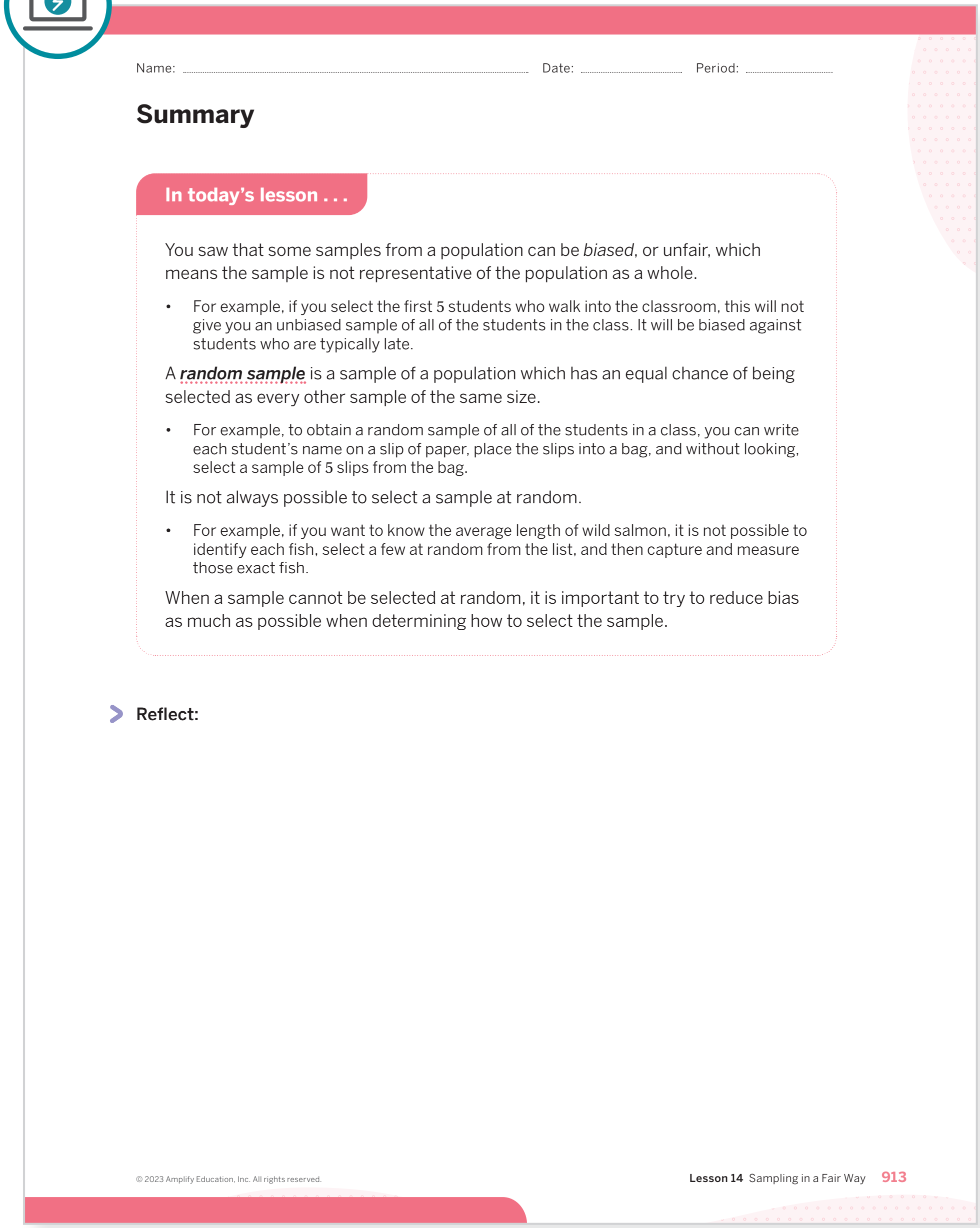
- "Some random dog greeted me on my way home."
- "Wow, that was random!"
- "I just randomly chose an appetizer from the menu."

Help students connect these everyday meanings of the term *random* to the definition of a random sample as "every sample (of the same size) has an equal chance of being selected."

Whole Class | 5 min

Summary

Review and synthesize why random sampling is a good way to eliminate bias.



Name: ______________ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that some samples from a population can be *biased*, or unfair, which means the sample is not representative of the population as a whole.

- For example, if you select the first 5 students who walk into the classroom, this will not give you an unbiased sample of all of the students in the class. It will be biased against students who are typically late.

A ***random sample*** is a sample of a population which has an equal chance of being selected as every other sample of the same size.

- For example, to obtain a random sample of all of the students in a class, you can write each student's name on a slip of paper, place the slips into a bag, and without looking, select a sample of 5 slips from the bag.

It is not always possible to select a sample at random.

- For example, if you want to know the average length of wild salmon, it is not possible to identify each fish, select a few at random from the list, and then capture and measure those exact fish.

When a sample cannot be selected at random, it is important to try to reduce bias as much as possible when determining how to select the sample.

Reflect:

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Synthesize

Ask:

- "What makes a sample selected at random the best way to select individuals for a sample?" It avoids biases that might be introduced using other methods.
- "As part of an English project, you want to look at the length of lines in Shakespeare's plays. What are some methods of selecting a random sample of lines from these plays?" Assign each line in the plays a number and use a computer to select several random numbers that correspond to the lines.
- "Would the sampling techniques in Activity 2 work for other situations? For example, to select 50 people in a large city to represent the views of the city residents?" Although they would work in theory for large populations, it would be too time consuming to write over a million numbers (or names) on pieces of paper and put them in a bag. Similarly, a spinner that is divided into a million sections would be difficult to manage.

Highlight that representative samples are better for measuring the populations. Random samples are more likely to eliminate bias (over- or under-representing certain groups). Random samples are not always representative samples and can be challenging to obtain. For example, getting a random sample of ants would be difficult for scientists to gather.

Formalize vocabulary: ***random sample***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "When is a sample *not* representative of a population?"

Math Language Development

MLR2: Collect and Display

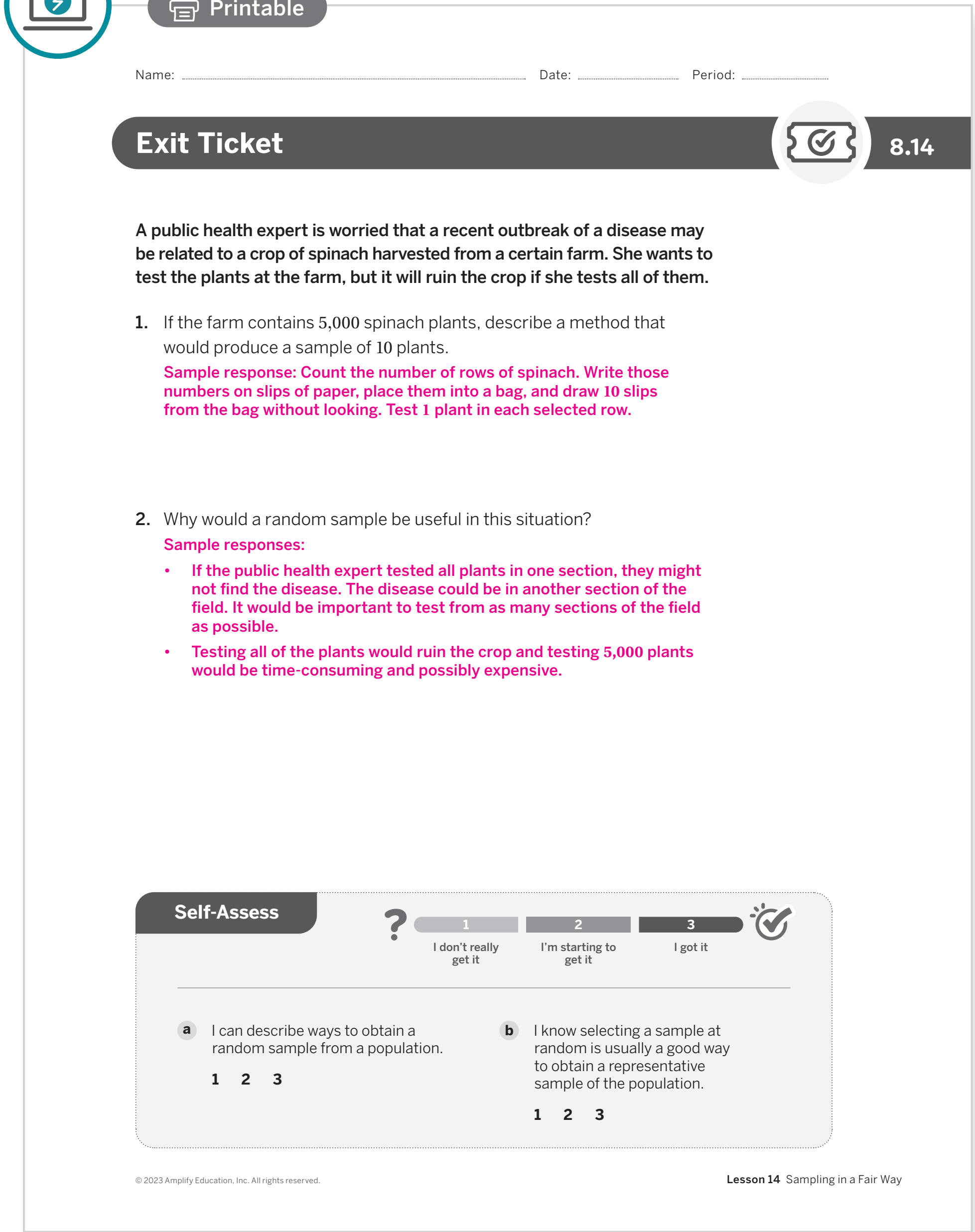
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *random sample* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of sampling by describing a sampling method and describing why random samples are beneficial.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

8.14

A public health expert is worried that a recent outbreak of a disease may be related to a crop of spinach harvested from a certain farm. She wants to test the plants at the farm, but it will ruin the crop if she tests all of them.

1. If the farm contains 5,000 spinach plants, describe a method that would produce a sample of 10 plants.
   Sample response: Count the number of rows of spinach. Write those numbers on slips of paper, place them into a bag, and draw 10 slips from the bag without looking. Test 1 plant in each selected row.

2. Why would a random sample be useful in this situation?
   Sample responses:
   - **If the public health expert tested all plants in one section, they might not find the disease. The disease could be in another section of the field. It would be important to test from as many sections of the field as possible.**
   - **Testing all of the plants would ruin the crop and testing 5,000 plants would be time-consuming and possibly expensive.**

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a) I can describe ways to obtain a random sample from a population.
1 2 3

b) I know selecting a sample at random is usually a good way to obtain a representative sample of the population.
1 2 3

Lesson 14 Sampling in a Fair Way

Success looks like . . .

- **Language Goal:** Describing methods to obtain a random sample from a population. **(Speaking and Listening, Writing)**
  - » Explaining how to produce a sample of 10 plants from 5,000 plants in Problem 1.
- **Language Goal:** Justifying whether a given sampling method is fair. **(Speaking and Listening)**
- **Goal:** Recognizing that random sampling tends to produce representative samples and support valid inferences.
  - » Explaining why a random sample would be useful in the context of testing the plants at the farm in Problem 2.

Suggested next steps

If students do not write a sampling technique which produces a random sample, consider:

- Reviewing Activity 2.
- Reviewing the Warm-up.
- Assigning Practice Problem 1.

If students are unable to describe why a random sample is beneficial, consider:

- Assigning Practice Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What other ways are there to perform the experiment in Activity 1?
- The instructional goal for this lesson was for students to understand that statistics can be used to gain information about a population by examining a sample. Where in your students' work today did you see or hear evidence of them doing this? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ________ Date: ________ Period: ________

1. Select *all* the reasons why random samples are preferred over other methods of collecting a sample.
   - A. A random sample is the only way to determine how many people or objects you want in the sample.
   - B. A random sample is always the most convenient way to select a sample from a population.
   - [✓ C.] A random sample is likely to give you a representative sample of the population.
   - [✓ D.] A random sample is a fair way to select a sample, because each person in the population has an equal chance of being selected.
   - E. If you use a random sample, the sample mean will always be the same as the population mean.

2. Jada wants to create a representative sample of 6 whole numbers between 1 and 100. She uses a computer's random number generator to produce these numbers: 1, 2, 3, 4, 5, and 6. Should she use these numbers or have the computer generate a new set of numbers? Explain your thinking.

   Sample response: This is a random sample, but is not representative of the population. If she uses this sample, the results might not represent the entire population she is studying.

   Note: This problem is well-suited for a rich class discussion.

3. *Capstone project helper*. Collect data to answer your statistical question by giving the survey you created in the previous lesson to a representative sample. Example method: *I will survey every 10th student who arrives at school in the morning*. Describe your method here:

   Answers may vary.

Important items to consider

- A representative sample that is randomly selected.
- The process you will use to select your sample.
- A realistic sample size.

Name: ________ Date: ________ Period: ________

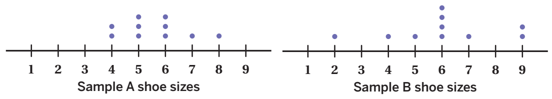
Practice

4. At an office party, 100 people are divided into 5 groups with 20 people in each group. One person's name is randomly chosen, and everyone in their group wins a prize. Noah simulates this event by writing 100 different names on slips of papers, placing them in a bag, and then selecting one slip of paper without looking. Explain a way to simulate this event using fewer than 100 slips of paper.

   Sample response: Number each of the 5 groups 1 through 5 and write these numbers on slips of paper. Place the slips into a bag and select one slip of paper without looking.

5. Data collected from a survey of American teenagers, ages 13 to 17, was used to estimate that 29% of teens believe in ghosts. This estimate was based on data from 510 American teenagers. Which of the following represents the population that this survey was studying?
   - A. All American teenagers.
   - B. The 510 teens that were surveyed.
   - [✓ C.] All American teens who are between the ages of 13 and 17.
   - D. The 29% of the teens surveyed who said they believe in ghosts.

6. Suppose the average shoe size of a Grade 7 student in a school is 5.5. Each dot plot shows the shoe size for a sample of ten Grade 7 students. Which sample is more representative of the population of Grade 7 students at the school? Explain your thinking.



   Sample A; Sample response: The values of Sample A are clustered closer to the population mean of 5.5.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 1	1
Spiral	4	Unit 8 Lesson 9	2
	5	Unit 8 Lesson 12	1
Formative	6	Unit 8 Lesson 15	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 15**

Estimating Population Measures of Center

Let's think about when the mean is an appropriate measure of center, and when it is not.

Focus

Goals

1. Generalize that an estimate for the center of a population distribution is more likely to be accurate when it is based on a random sample with less variability.
2. **Language Goal:** Use the mean of a random sample to make inferences about the population, and explain the reasoning. **(Speaking and Listening, Writing)**

Coherence

Today

Students calculate the mean and MAD for samples from different populations and consider the meaning of these quantities in terms of the situation. Students see that when there is less variability in the data, they can assume that the mean of that sample is a better estimate for the mean of a population than when a sample has greater variability. **Note:** This lesson's Practice contains a milestone for the Capstone project.

Previously

In Lesson 14, students selected a sample and critiqued different sampling methods as to their benefits and drawbacks.

Coming Soon

In Lesson 16, students will see that if a sample is representative of the population, then they can use proportional reasoning to make predictions about the population.

Rigor

- Students **apply** their understanding of mean, MAD, and samples to make predictions and recommendations for a streaming media company.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	10 min	5 min	10 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- Graphic Organizer PDF, *MAD Recording Sheet*

Math Language Development

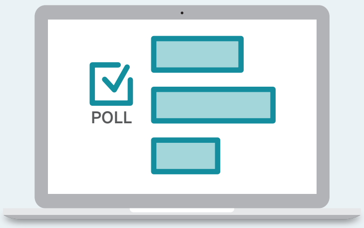
Review words

- *center*
- *mean*
- *mean absolute deviation (MAD)*
- *population*
- *random sample*
- *representative sample*
- *sample*
- *shape*
- *spread*
- *variability*

Amps | Featured Activity

Warm-up
Take a Poll

Get a sense of your students' thinking and encourage a discourse by quickly polling the class.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel disorganized while calculating the mean and MAD of the data set in Activity 1. Ask students if they can find ways to keep track of which numbers they have used already and which numbers they have not used. Have students share their strategies in small groups so that they can seek ways to improve their own organization.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

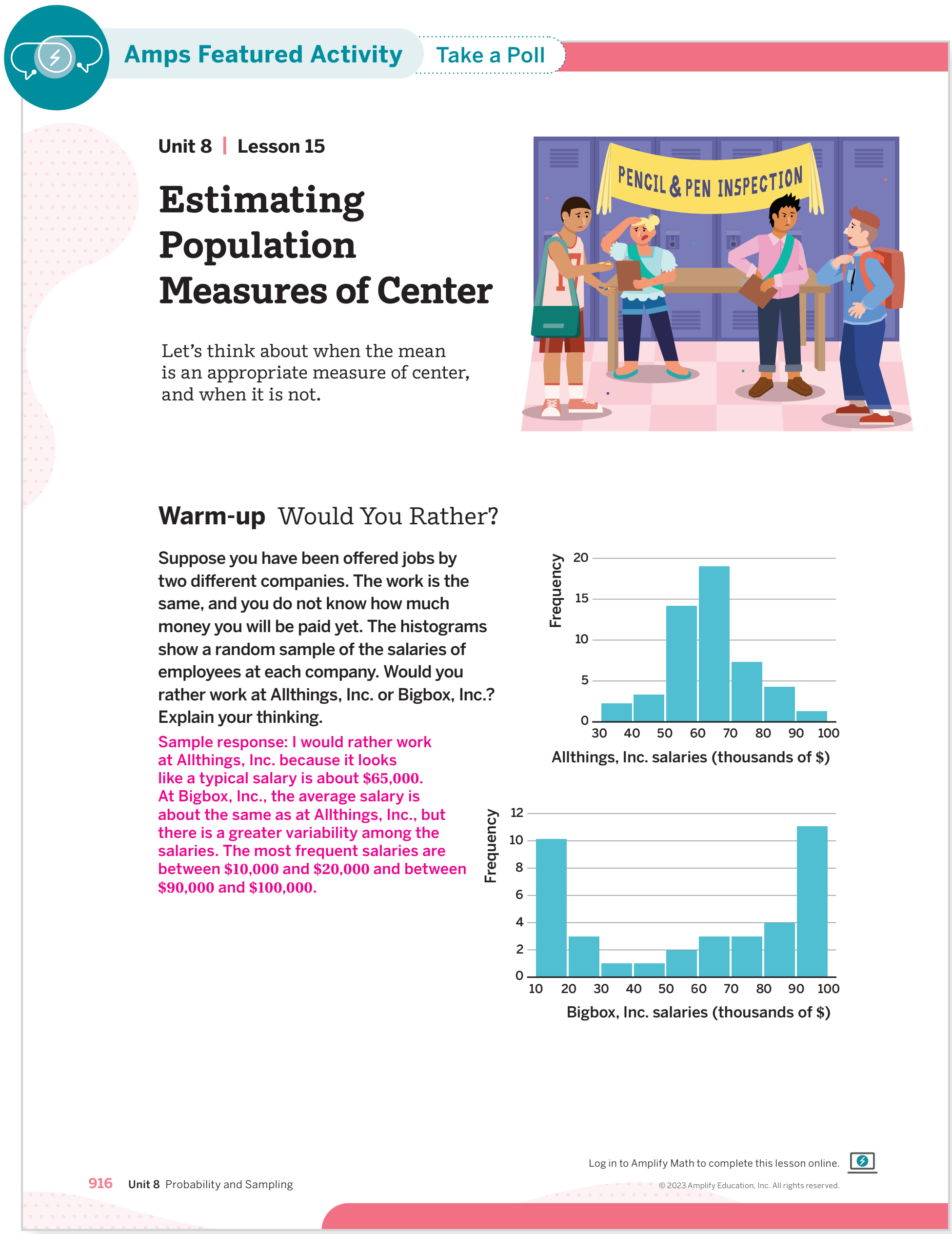
- In **Activity 1**, provide students with the mean and MAD to save them calculation time.

Pairs | 5 min

Warm-up Would You Rather?

Students compare the distributions of a random sample of salaries at two different companies to spur discussion about average and typical values.

Amps Featured Activity Take a Poll



Unit 8 | Lesson 15

Estimating Population Measures of Center

Let's think about when the mean is an appropriate measure of center, and when it is not.

Warm-up Would You Rather?

Suppose you have been offered jobs by two different companies. The work is the same, and you do not know how much money you will be paid yet. The histograms show a random sample of the salaries of employees at each company. Would you rather work at Allthings, Inc. or Bigbox, Inc.? Explain your thinking.

Sample response: I would rather work at Allthings, Inc. because it looks like a typical salary is about $65,000. At Bigbox, Inc., the average salary is about the same as at Allthings, Inc., but there is a greater variability among the salaries. The most frequent salaries are between $10,000 and $20,000 and between $90,000 and $100,000.

Log in to Amplify Math to complete this lesson online.

916 Unit 8 Probability and Sampling

1 Launch

Activate students' background knowledge by asking students what they know about the term *salary*. Describe how a salary differs from an hourly wage. Have students use the *Think-Pair-Share* routine. Provide them 1 minute of individual think time. Then have them complete the Warm-up with a partner. Conduct the *Poll the Class* routine to assess student thinking.

2 Monitor

Help students get started by sharing that the median salary at both companies is about $65,000.

Look for points of confusion:

- **Thinking that the average salary is the middle value on the horizontal axis.** Have students explain what the height of each bar represents.

Look for productive strategies:

- Finding a value that is typical for the data.
- Finding the means of the salaries at both companies.

3 Connect

Display the class results for which company they would choose to work.

Have students share observations they made about the data — particularly focusing on the center, shape, and spread of the data.

Highlight that the shape of the data can tell students as much — and sometimes more — about a data set as the measures of center.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their observations, collect the language they use that describe the center, shape, and spread of the data. For example, they may use these words and phrases: *typical salary, average salary, greater variability,* and *most frequent salaries*. Invite students to add to the display during the lesson and encourage students to refer back to the display during class discussions.

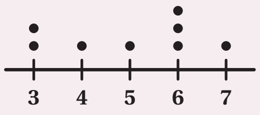
English Learners

Annotate the distributions with the terms and phrases students use to describe them. For example, annotate the peaks of each distribution with the phrase *most frequent*.

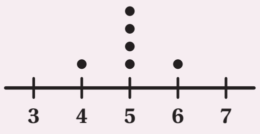
Power-up

To power up students' ability to compare data displayed by dot plots, have students complete:

A set of data has a mean of 5 and a range of 4. Which dot plot represents the set of data? Be prepared to explain your thinking.



Data Set A



Data Set B

Data Set A

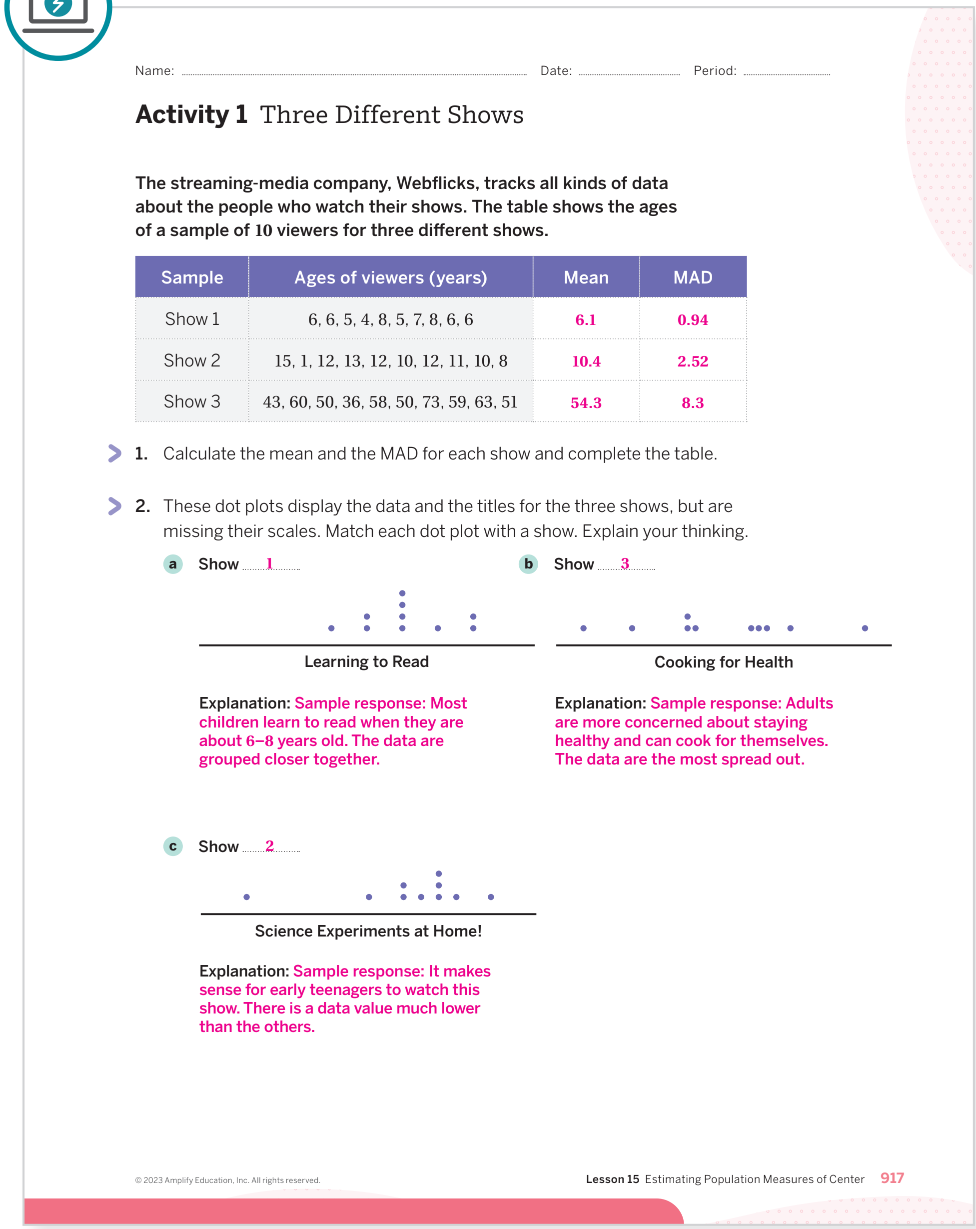
Use: Before the Warm-up

Informed by: Performance on Lesson 14, Practice Problem 6

Small Groups | 15 min

Activity 1 Three Different Shows

Students analyze data from samples of viewers for different media streaming shows to better understand the population of viewers.



Name: .... Date: .... Period: ....

Activity 1 Three Different Shows

The streaming-media company, Webflicks, tracks all kinds of data about the people who watch their shows. The table shows the ages of a sample of 10 viewers for three different shows.

Sample	Ages of viewers (years)	Mean	MAD
Show 1	6, 6, 5, 4, 8, 5, 7, 8, 6, 6	6.1	0.94
Show 2	15, 1, 12, 13, 12, 10, 12, 11, 10, 8	10.4	2.52
Show 3	43, 60, 50, 36, 58, 50, 73, 59, 63, 51	54.3	8.3

1. Calculate the mean and the MAD for each show and complete the table.

2. These dot plots display the data and the titles for the three shows, but are missing their scales. Match each dot plot with a show. Explain your thinking.

a Show 1

Explanation: **Sample response: Most children learn to read when they are about 6–8 years old. The data are grouped closer together.**

b Show 3

Explanation: **Sample response: Adults are more concerned about staying healthy and can cook for themselves. The data are the most spread out.**

c Show 2

Explanation: **Sample response: It makes sense for early teenagers to watch this show. There is a data value much lower than the others.**

1 Launch

Tell students that each person in the group should analyze a different sample, then share their results with their group. Distribute copies of the Graphic Organizer, *MAD Recording Sheet*, to students who would benefit from using it.

2 Monitor

Help students get started by demonstrating how to work through one set of show data using the Graphic Organizer PDF, *MAD Recording Sheet*.

Look for points of confusion:

- **Not using the absolute value when finding the MAD.** Because this will cause their MAD to be 0, ask if it makes sense that the MAD is 0 when all of the data are not the same value as the mean.

Look for productive strategies:

- Relating the ages of the viewers to the titles of the shows.
- Relating the MAD of the shows to the spread of the data on the dot plots.

3 Connect

Have students share their thinking for which dot plot represents the data from each show, beginning with students who reasoned using the titles, then moving to students who reasoned using the mean or MAD.

Display the dot plots, annotated with the mean and MAD associated with each.

Ask:

- "Describe the shape of the data on the dot plot for the show with the highest MAD."
- "Describe the shape of the data on the dot plot for the show with the lowest MAD."

Highlight that on a dot plot without labels for the number line, much cannot be said about the mean, but a lot can be learned from the MAD. The MAD corresponds directly to the spread of the data.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with the calculations for the mean and MAD for each show so that they can focus more time thinking about which show corresponds to each dot plot, as opposed to spending a lot of time on calculations.

Accessibility: *Guide Processing and Visualization*

Provide students with copies of the Graphic Organizer PDF, *MAD Recording Sheet*, to help them visualize the calculations needed to determine the mean and MAD.

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Connect, as students share their matches and their thinking, demonstrate mathematical language use by using revoicing to restate a student statement as a question in order to clarify, apply appropriate language, and involve more students. For example, if a student says, "The cooking for health dot plot shows dots that are farther apart," revoice this by asking, "What do you mean by farther apart? Is there a measure that can describe this? Is it a measure of center or variation?"

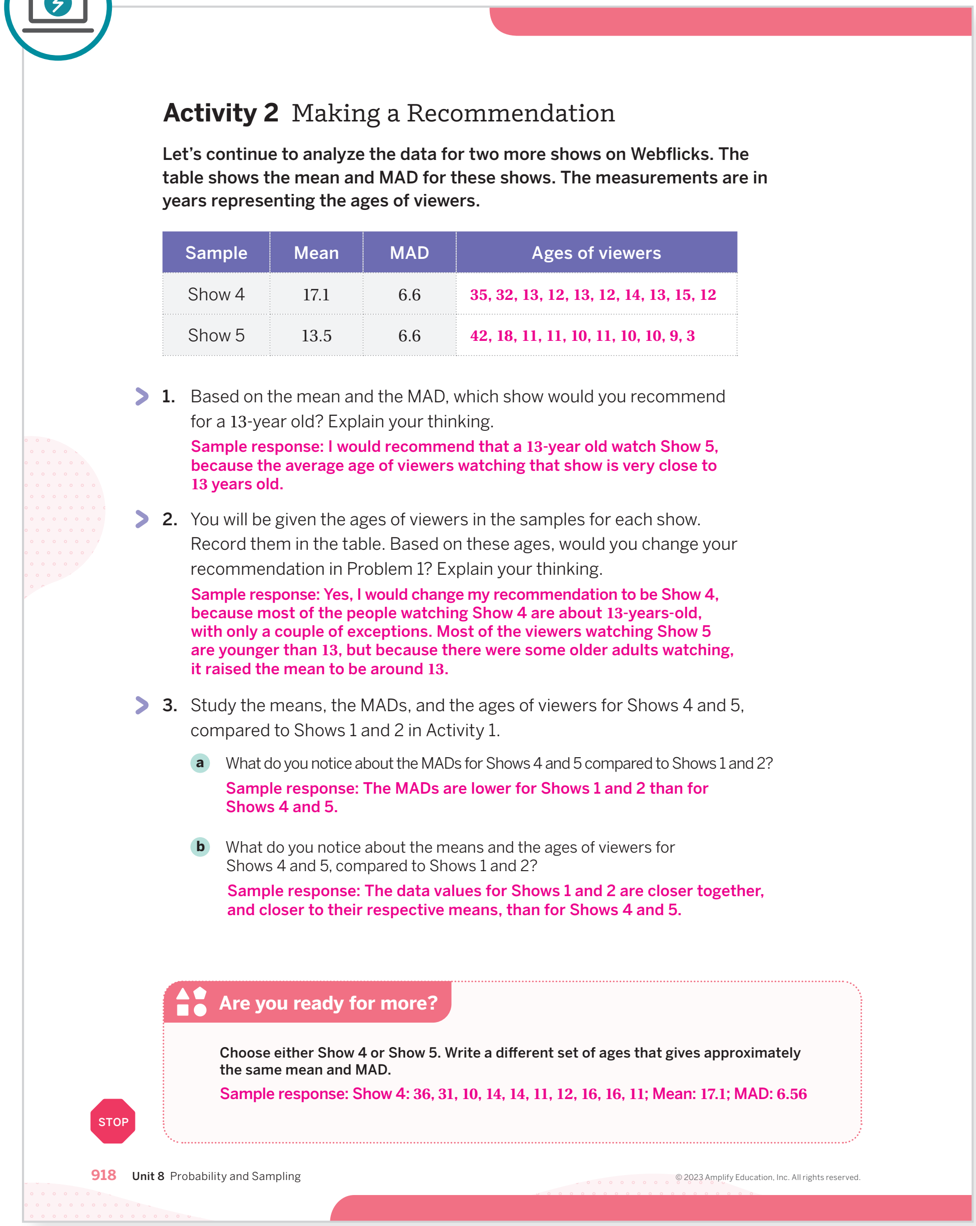
English Learners

Model how you use the class display to apply appropriate mathematical language to the discussion.

Pairs | 10 min

Activity 2 Making a Recommendation

Students find that a sample with a relatively large MAD can make the mean less reliable as a measure of center for that population.



Activity 2 Making a Recommendation

Let's continue to analyze the data for two more shows on Webflicks. The table shows the mean and MAD for these shows. The measurements are in years representing the ages of viewers.

Sample	Mean	MAD	Ages of viewers
Show 4	17.1	6.6	35, 32, 13, 12, 13, 12, 14, 13, 15, 12
Show 5	13.5	6.6	42, 18, 11, 11, 10, 11, 10, 10, 9, 3

1. Based on the mean and the MAD, which show would you recommend for a 13-year old? Explain your thinking.
   Sample response: I would recommend that a 13-year old watch Show 5, because the average age of viewers watching that show is very close to 13 years old.

2. You will be given the ages of viewers in the samples for each show. Record them in the table. Based on these ages, would you change your recommendation in Problem 1? Explain your thinking.
   Sample response: Yes, I would change my recommendation to be Show 4, because most of the people watching Show 4 are about 13-years-old, with only a couple of exceptions. Most of the viewers watching Show 5 are younger than 13, but because there were some older adults watching, it raised the mean to be around 13.

3. Study the means, the MADs, and the ages of viewers for Shows 4 and 5, compared to Shows 1 and 2 in Activity 1.
   - a. What do you notice about the MADs for Shows 4 and 5 compared to Shows 1 and 2?
     Sample response: The MADs are lower for Shows 1 and 2 than for Shows 4 and 5.
   - b. What do you notice about the means and the ages of viewers for Shows 4 and 5, compared to Shows 1 and 2?
     Sample response: The data values for Shows 1 and 2 are closer together, and closer to their respective means, than for Shows 4 and 5.

Are you ready for more?

Choose either Show 4 or Show 5. Write a different set of ages that gives approximately the same mean and MAD.

Sample response: Show 4: 36, 31, 10, 14, 14, 11, 12, 16, 16, 11; Mean: 17.1; MAD: 6.56

STOP

918 **Unit 8** Probability and Sampling

1 Launch

Activate students' background knowledge by asking students why media companies might want to know the ages of the viewers watching their shows. Give groups a few minutes to finish Problem 1, then display the Activity 2 PDF to show the ages of viewers for Shows 4 and 5.

2 Monitor

Help students get started by prompting them to think about which number, the mean or the MAD, tells them about the ages of viewers.

Look for points of confusion:

- **Thinking that Show 5 is a better recommendation because the mean is closer to 13, even after seeing the ages of viewers.** Ask students to explain why the mean is 13, even though the typical age is 10 or 11.

Look for productive strategies:

- Being skeptical about using either mean for the recommendation, given that the MADs are relatively large.

3 Connect

Have students share with a partner whether they would change their recommendation. Then have them record their response to Problem 2.

Highlight that because the MADs were relatively large, the means were not very reliable as a measure of center.

Ask:

- "Do certain values in the set of numbers seem non-typical for the set?"
- "Why do you think some people of this age might be viewing the show?"
- "How do these non-typical values affect the MAD of the set?" These non-typical values cause the MAD to increase.
- "How do these non-typical values affect the mean?" Outliers do not necessarily affect the mean if they are balanced on either side, but they could skew the data if they are unbalanced on one side.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

After sharing the ages of viewers for Shows 4 and 5, suggest that students arrange the values in order from least to greatest. Then suggest they circle the values they think are close to the age of 13 to help them better make sense of the data set.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, after students record their response for Problem 2, invite them to meet with another pair of students to give and receive feedback on their responses to both Problems 2 and 3. Provide these prompts for feedback to help strengthen ideas and clarify language.

- "How would you describe the ages of the viewers for Shows 4 and 5?"
- "Do you think the means and MADs of these data sets describe the ages of the viewers? Why or why not?"
- "What mathematical language can you use in your responses?"

Allow time to complete a final draft based on feedback.

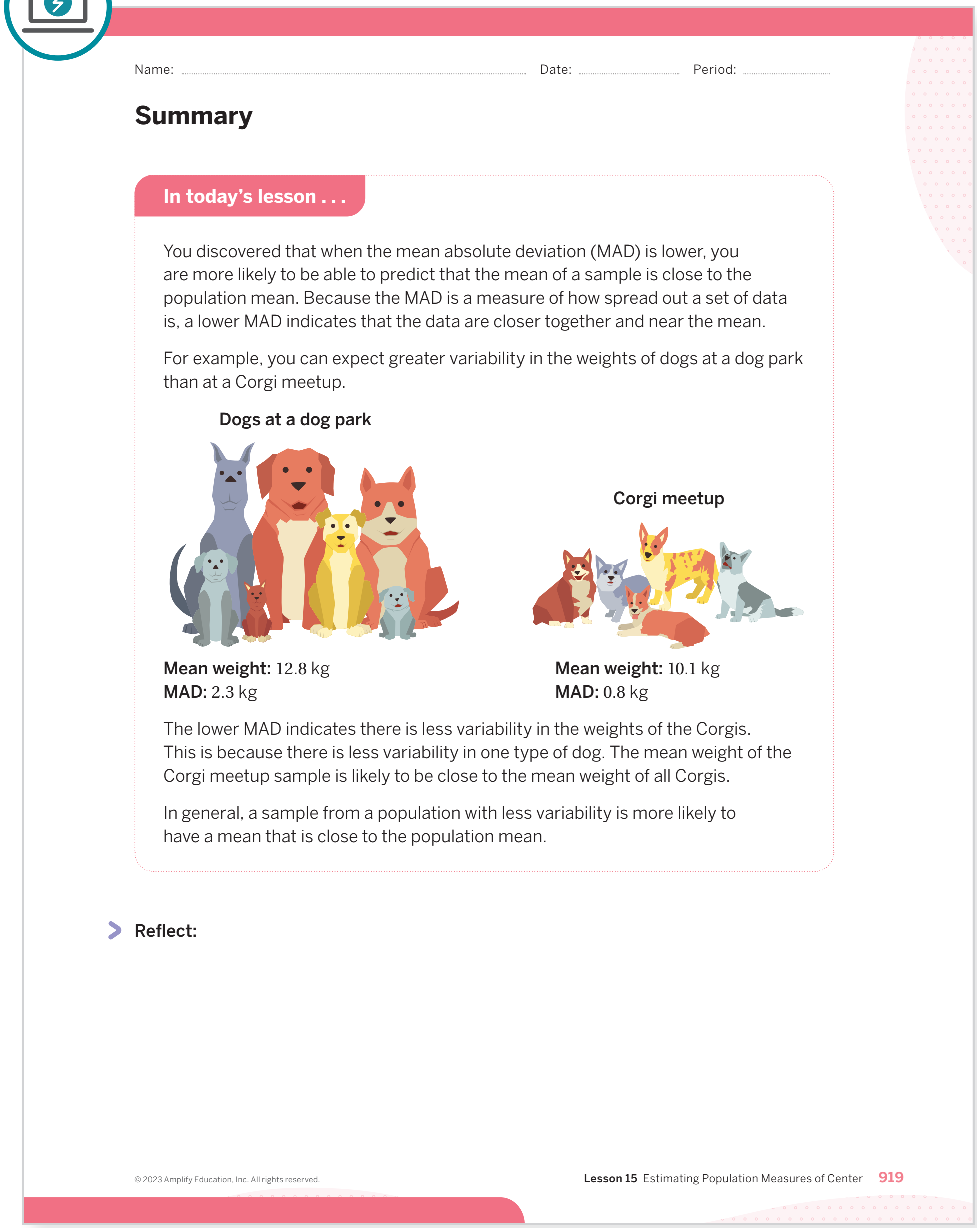
English Learners

Allow students time to formulate with their partner how they will improve their final draft before proceeding with the Connect discussion.

Whole Class | 5 min

Summary

Review and synthesize how the mean of a sample is more likely to be close to the mean of the population when the MAD is a lower value.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You discovered that when the mean absolute deviation (MAD) is lower, you are more likely to be able to predict that the mean of a sample is close to the population mean. Because the MAD is a measure of how spread out a set of data is, a lower MAD indicates that the data are closer together and near the mean.

For example, you can expect greater variability in the weights of dogs at a dog park than at a Corgi meetup.

Dogs at a dog park

Mean weight: 12.8 kg
MAD: 2.3 kg

Corgi meetup

Mean weight: 10.1 kg
MAD: 0.8 kg

The lower MAD indicates there is less variability in the weights of the Corgis. This is because there is less variability in one type of dog. The mean weight of the Corgi meetup sample is likely to be close to the mean weight of all Corgis.

In general, a sample from a population with less variability is more likely to have a mean that is close to the population mean.

Reflect:

Synthesize

Display the image of the two groups of dogs.

Ask, "Which sample of dogs is more likely to have a mean closer to the mean of the population?"

Have students share which set of dogs will have a greater MAD for their weights.

Highlight that if each set of dogs was selected from a population of dogs, students can make some inferences based on these samples.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

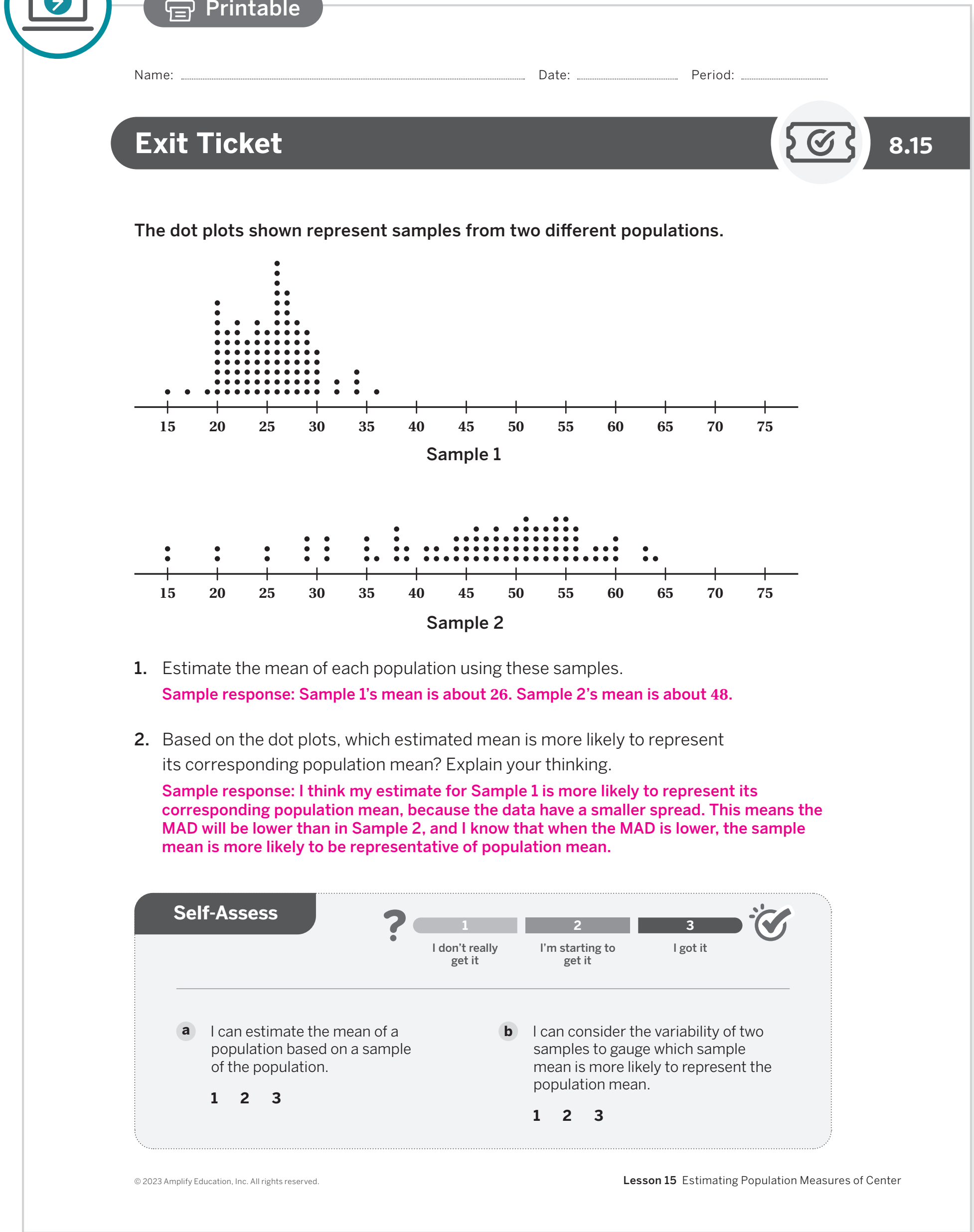
- "When is a sample *not* representative of a population?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of how representative the mean of a sample is by comparing the dot plots of samples from separate populations.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 8.15

The dot plots shown represent samples from two different populations.

1. Estimate the mean of each population using these samples.
   Sample response: Sample 1's mean is about 26. Sample 2's mean is about 48.

2. Based on the dot plots, which estimated mean is more likely to represent its corresponding population mean? Explain your thinking.
   Sample response: I think my estimate for Sample 1 is more likely to represent its corresponding population mean, because the data have a smaller spread. This means the MAD will be lower than in Sample 2, and I know that when the MAD is lower, the sample mean is more likely to be representative of population mean.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can estimate the mean of a population based on a sample of the population.
1 2 3

b. I can consider the variability of two samples to gauge which sample mean is more likely to represent the population mean.
1 2 3

Lesson 15 Estimating Population Measures of Center

Success looks like . . .

- **Goal:** Generalizing that an estimate for the center of a population distribution is more likely to be accurate when it is based on a random sample with less variability.
  - » Explaining that the mean of Sample 1 is more representative of the population mean because that data have a smaller spread in Problem 2.
- **Language Goal:** Using the mean of a random sample to make inferences about the population, and explaining the reasoning. **(Speaking and Listening, Writing)**

Suggested next steps

If students say that both samples are likely to be accurate, consider:

- Asking them about which sample would have a greater MAD.

If students say that Sample 1 has some outliers, consider:

- Asking them to compare the spread of each sample.

Professional Learning

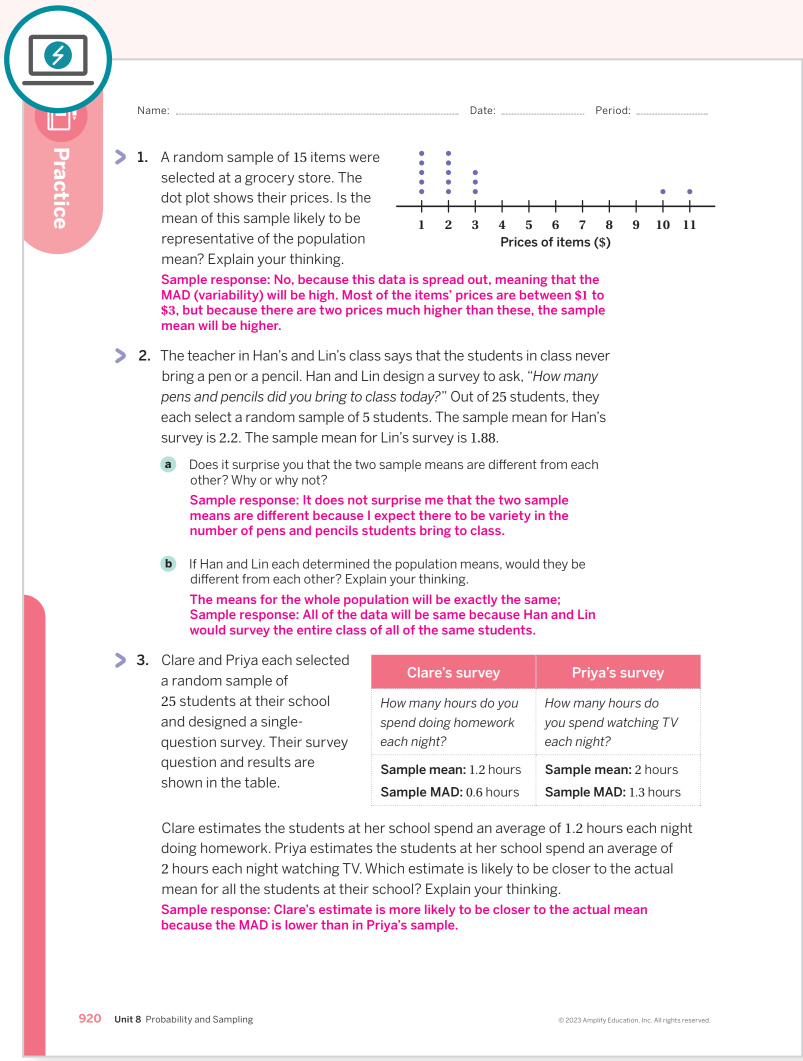
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students calculated the mean absolute deviation. How did that support determining if samples were better predictors of the population?
- Who participated and who didn't participate in calculating the mean and mean absolute deviation today? What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. A random sample of 15 items were selected at a grocery store. The dot plot shows their prices. Is the mean of this sample likely to be representative of the population mean? Explain your thinking.

Sample response: No, because this data is spread out, meaning that the MAD (variability) will be high. Most of the items' prices are between $1 to $3, but because there are two prices much higher than these, the sample mean will be higher.

2. The teacher in Han's and Lin's class says that the students in class never bring a pen or a pencil. Han and Lin design a survey to ask, "*How many pens and pencils did you bring to class today?*" Out of 25 students, they each select a random sample of 5 students. The sample mean for Han's survey is 2.2. The sample mean for Lin's survey is 1.88.

   a Does it surprise you that the two sample means are different from each other? Why or why not?

   Sample response: It does not surprise me that the two sample means are different because I expect there to be variety in the number of pens and pencils students bring to class.

   b If Han and Lin each determined the population means, would they be different from each other? Explain your thinking.

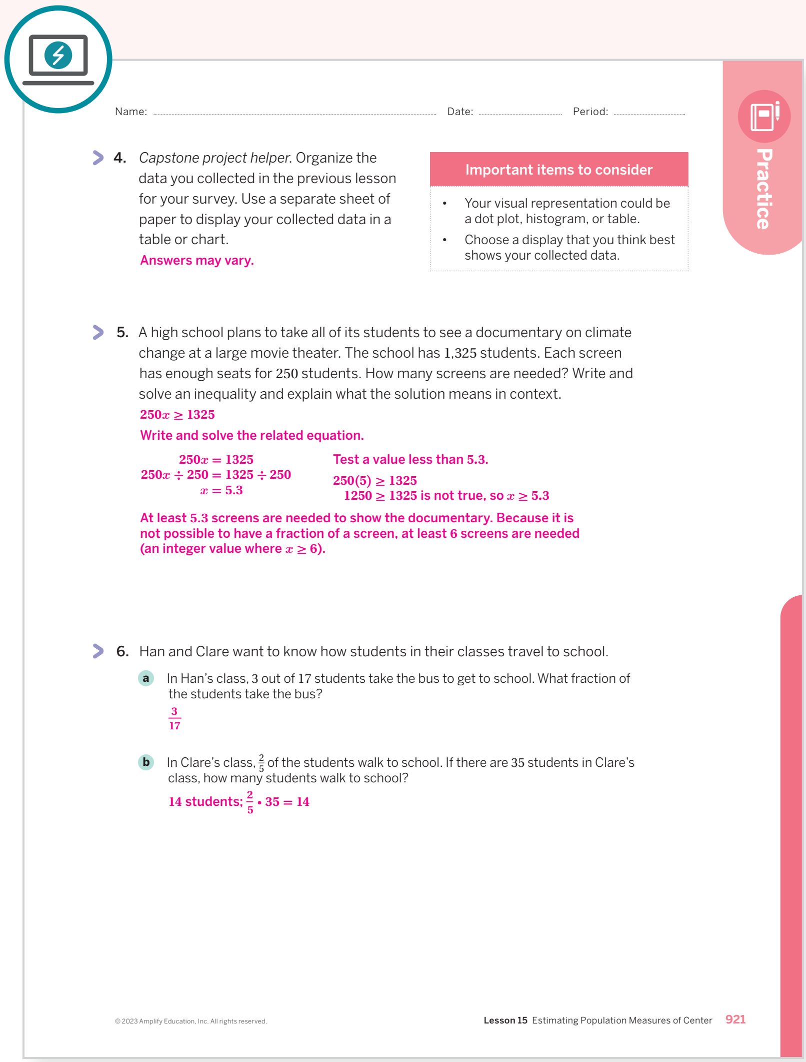
   The means for the whole population will be exactly the same; Sample response: All of the data will be same because Han and Lin would survey the entire class of all of the same students.

3. Clare and Priya each selected a random sample of 25 students at their school and designed a single-question survey. Their survey question and results are shown in the table.

Clare's survey	Priya's survey
How many hours do you spend doing homework each night?	*How many hours do you spend watching TV each night?*
Sample mean: 1.2 hours **Sample MAD:** 0.6 hours	**Sample mean:** 2 hours **Sample MAD:** 1.3 hours

Clare estimates the students at her school spend an average of 1.2 hours each night doing homework. Priya estimates the students at her school spend an average of 2 hours each night watching TV. Which estimate is likely to be closer to the actual mean for all the students at their school? Explain your thinking.

Sample response: Clare's estimate is more likely to be closer to the actual mean because the MAD is lower than in Priya's sample.



Practice

Name: Date: Period:

4. *Capstone project helper*. Organize the data you collected in the previous lesson for your survey. Use a separate sheet of paper to display your collected data in a table or chart.

   Answers may vary.

Important items to consider

- Your visual representation could be a dot plot, histogram, or table.
- Choose a display that you think best shows your collected data.

5. A high school plans to take all of its students to see a documentary on climate change at a large movie theater. The school has 1,325 students. Each screen has enough seats for 250 students. How many screens are needed? Write and solve an inequality and explain what the solution means in context.

   $250x \geq 1325$

   Write and solve the related equation.

   $$250x = 1325$$
   $$250x \div 250 = 1325 \div 250$$
   $$x = 5.3$$

   Test a value less than 5.3.

   $250(5) \geq 1325$

   $1250 \geq 1325$ **is not true, so** $x \geq 5.3$

   At least 5.3 screens are needed to show the documentary. Because it is not possible to have a fraction of a screen, at least 6 screens are needed (an integer value where $x \geq 6$).

6. Han and Clare want to know how students in their classes travel to school.

   a In Han's class, 3 out of 17 students take the bus to get to school. What fraction of the students take the bus?

   $\frac{3}{17}$

   b In Clare's class, $\frac{2}{5}$ of the students walk to school. If there are 35 students in Clare's class, how many students walk to school?

   14 students; $\frac{2}{5} \cdot 35 = 14$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	3
	3	Activity 2	2
Spiral	4	Grade 6	3
	5	Unit 6 Lesson 15	2
Formative ⚡	6	Unit 8 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 16**

Estimating Population Proportions

Let's use samples to estimate population proportions.

Focus

Goals

1. Comprehend that the term *population proportion* refers to a number between 0 and 1 that represents the fraction of the data within a certain category.
2. **Language Goal:** Compare population proportions for the same category from different samples of a population. **(Speaking and Listening)**
3. **Language Goal:** Use the population proportion of a random sample that is within a certain category to make inferences about the population, and explain the reasoning. **(Speaking and Listening, Writing)**

Coherence

Today

Students see that having a representative sample makes it possible to use proportional reasoning to make predictions about the population. Students understand that these predictions are only estimates.
Note: This lesson's Practice contains a milestone for the Capstone project. Remind students that they should bring their project's visual representation and analysis for the next lesson. You may want to distribute the *Capstone Project Rubric* PDF at the end of this lesson.

Previously

In Lesson 15, students used samples to estimate measures of center of a population.

Coming Soon

Lesson 17 is the Capstone lesson for this unit. Students should bring the presentation of their statistical study to class for this lesson.

Rigor

- Students **apply** their understanding of sampling and probability to estimate population proportions in a variety of real-world contexts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	10 min	5 min	10 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- *Capstone Project Rubric* PDF, one per student
- bags, one per pair
- number of Grade 7 students in the school

Math Language Development

New words

- ***population proportion***

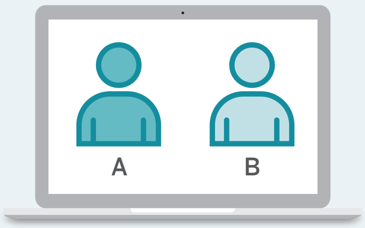
Review words

- *population*
- *random sample*
- *sample*

Amps **Featured Activity**

Activity 1
Random Times

Students are randomly given travel times, making facilitation easier.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Communicating one's opinion can be frightening at times. Remind students to show respect as others speak. Discussions can lead to a better understanding of where one stands on a particular issue. Encourage students to actively listen to what classmates are saying and try to understand their perspectives. Explain that all that is presented can be used to shape and reshape their own thinking.

Modifications to Pacing

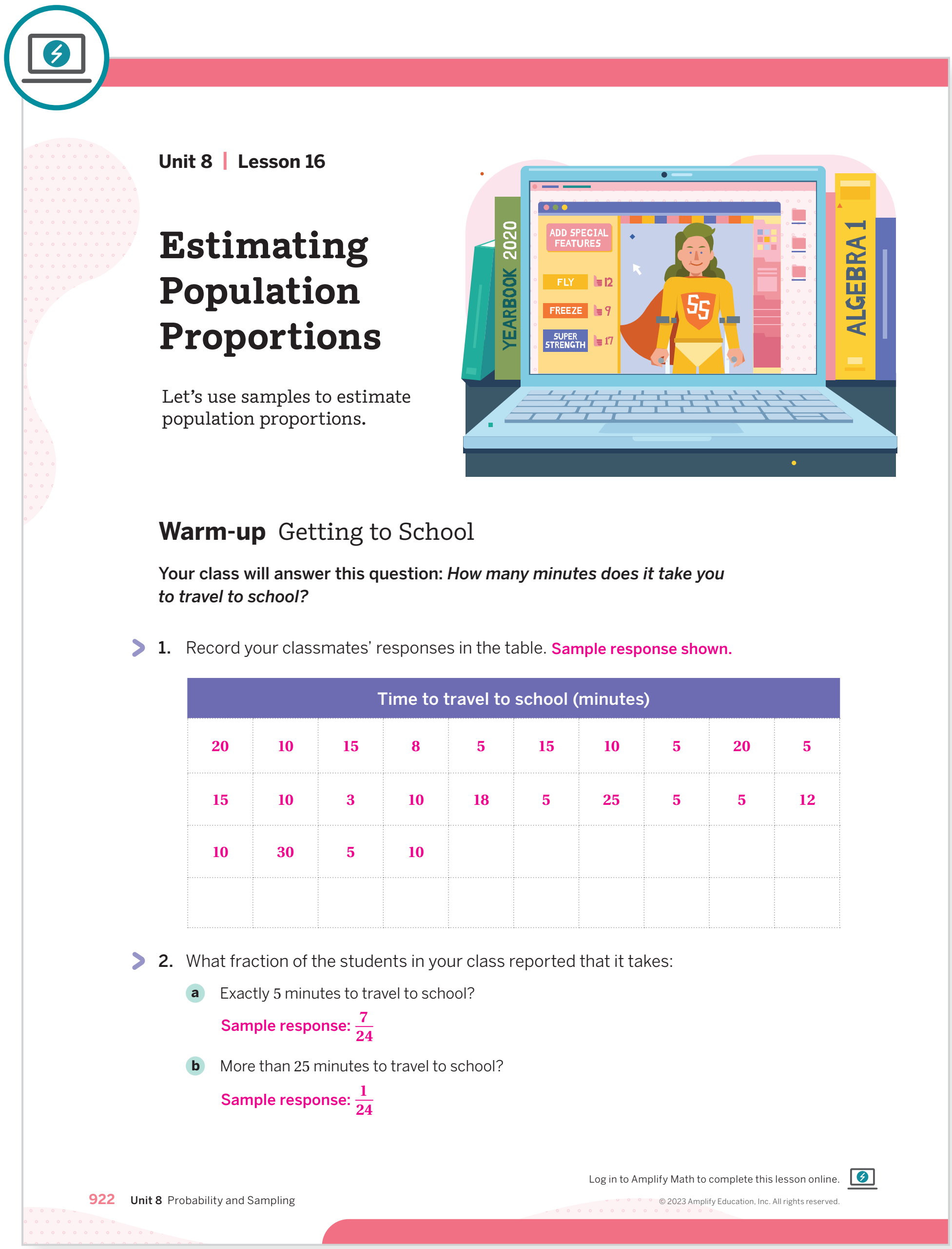
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 4 may be omitted.
- In **Activity 2**, Problems 1 and 2 may be omitted.

Independent | 5 min

Warm-up Getting to School

Students collect data and write corresponding ratios to prepare for the next activity.



Unit 8 | **Lesson 16**

Estimating Population Proportions

Let's use samples to estimate population proportions.

Warm-up Getting to School

Your class will answer this question: ***How many minutes does it take you to travel to school?***

1. Record your classmates' responses in the table. **Sample response shown.**

Time to travel to school (minutes)									
20	10	15	8	5	15	10	5	20	5
15	10	3	10	18	5	25	5	5	12
10	30	5	10						

2. What fraction of the students in your class reported that it takes:

   a. Exactly 5 minutes to travel to school?

      Sample response: $\frac{7}{24}$

   b. More than 25 minutes to travel to school?

      Sample response: $\frac{1}{24}$

Log in to Amplify Math to complete this lesson online.

922 **Unit 8** Probability and Sampling

1 Launch

Display the table from the Warm-up. Have students share their travel times to school, complete the class chart, and then complete Problem 2 independently.

2 Monitor

Help students get started by pointing out that the total number of students in the class is the denominator for the fractions in Problem 2.

Look for points of confusion:

- **Interpreting "more than 25 minutes" as "25 minutes or more" in Problem 2b.** Help students realize that "more than" does not include 25 minutes and they should include only times greater than 25 minutes.

3 Connect

Have individual students share their fractions of the students that fit each category.

Ask:

- "Do more or fewer than half the students travel to school in exactly 5 minutes?"
- "Can you make a generalization based on the fraction of students who take more than 25 minutes to travel to school?" Because only a small fraction of the class takes more than 25 minutes to travel to school, this doesn't seem to be a large problem for the community.
- "If a student from high school said that 10 of his classmates spent more than 25 minutes to travel to school, can you make a generalization about his school?" No, because I do not know the total number of students that represent his classmates.

Highlight that, in order to understand how relatively large a certain category is, students need to find what fraction of the whole group it comprises.

Power-up

To power up students' ability to use proportional relationships to solve problems, have students complete:

On Noah's rugby team, $\frac{2}{3}$ of the players play at least one other sport. Use the ratio table to determine the number of players that play more than one sport if there are 18 players on his team.

Total Players	3	18
Plays more than 1 sport	2	12

Use: Before Activity 2

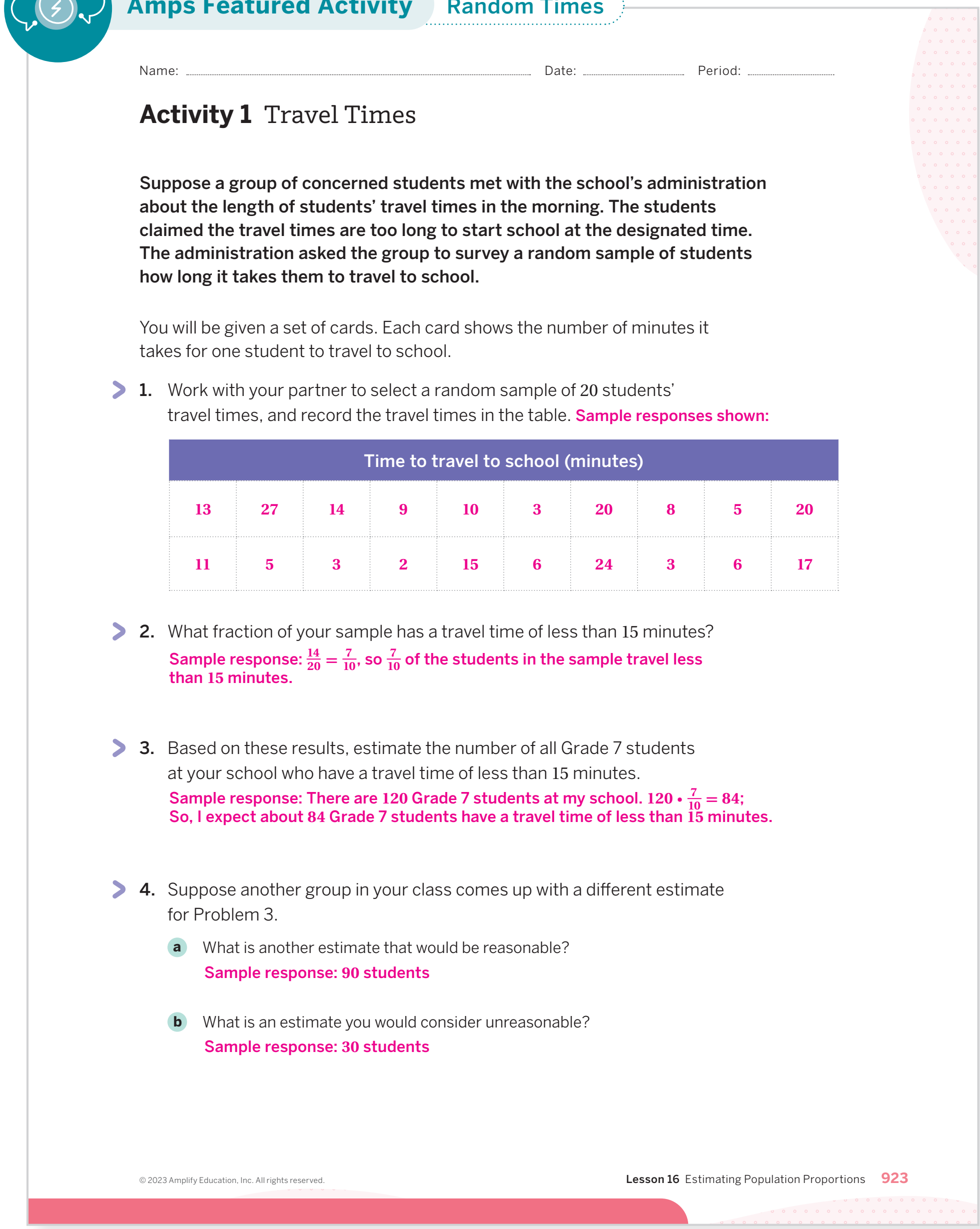
Informed by: Performance on Lesson 15, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

Pairs | 15 min

Activity 1 Travel Times

Students collect a random sample and use the fractions of responses for certain categories to make estimates about the population.

Amps Featured Activity Random Times



Name: ______ Date: ______ Period: ______

Activity 1 Travel Times

Suppose a group of concerned students met with the school's administration about the length of students' travel times in the morning. The students claimed the travel times are too long to start school at the designated time. The administration asked the group to survey a random sample of students how long it takes them to travel to school.

You will be given a set of cards. Each card shows the number of minutes it takes for one student to travel to school.

1. Work with your partner to select a random sample of 20 students' travel times, and record the travel times in the table. **Sample responses shown:**

Time to travel to school (minutes)									
13	27	14	9	10	3	20	8	5	20
11	5	3	2	15	6	24	3	6	17

2. What fraction of your sample has a travel time of less than 15 minutes?
   Sample response: $\frac{14}{20} = \frac{7}{10}$, so $\frac{7}{10}$ of the students in the sample travel less than 15 minutes.

3. Based on these results, estimate the number of all Grade 7 students at your school who have a travel time of less than 15 minutes.
   Sample response: There are 120 Grade 7 students at my school. $120 \cdot \frac{7}{10} = 84$; So, I expect about 84 Grade 7 students have a travel time of less than 15 minutes.

4. Suppose another group in your class comes up with a different estimate for Problem 3.
   - a. What is another estimate that would be reasonable?
     Sample response: 90 students
   - b. What is an estimate you would consider unreasonable?
     Sample response: 30 students

1 Launch

Distribute an opaque bag containing the pre-cut cards from the Activity 1 PDF to pairs of students. Be prepared to give students the total number of 7th grade students in your school for Problem 3.

2 Monitor

Look for points of confusion:

- **Placing the slip back in the bag after it is selected.** Have students think about what they are simulating by selecting the numbers out of the bag. Ask if it would make sense to survey the same student again.
- **Thinking the probability is greater than 1 in Problem 2.** Remind students they are looking for the fraction of a desired category out of the whole group, which will be between 0 and 1.

Look for productive strategies:

- Organizing the data cards into two piles with one containing the times less than 15 minutes.

3 Connect

Have pairs of students share their responses for Problem 3.

Highlight how multiple samples can help revise their estimates in Problem 4 and give an idea of how accurate the individual estimates from the samples might be.

Define the term ***population proportion*** in statistics as a number between 0 and 1 that represents the fraction of the data that fits into the desired category.

Ask:

- "Using the class's data, how accurate do you think your group's estimate is?"
- "The actual population proportion for this population is 0.68. How close was your estimate? Explain why your estimate was not exactly the same."
- "If each group had 40 travel times in their samples instead of 20, do you think the estimate would be more or less accurate?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students select a random sample of 10 travel times, instead of 20.

Extension: *Math Enrichment*

Have students include 20 additional travel times from the random sample. Ask them to explain whether (and how) their responses to Problems 2 and 3 will change by including these additional travel times. Answers will vary, based on the additional values included.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses to Problem 3, display the fractions that different pairs of students calculated based on their responses. Tell them that these fractions are the *sample proportions*. As you define the term *population proportion*, have students compare their sample proportions with the actual population proportion, 0.68.

English Learners

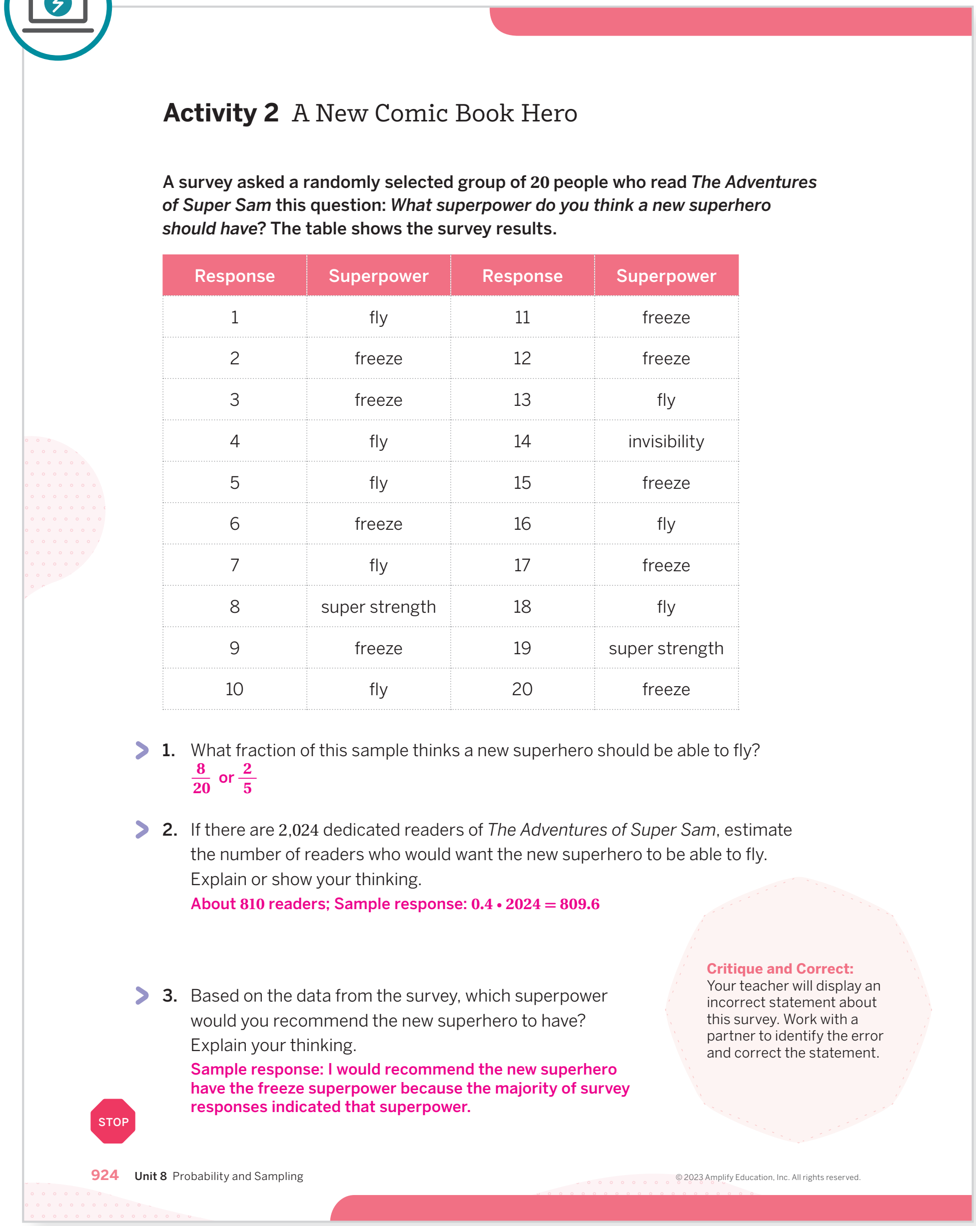
Consider displaying a table, similar to the one shown, to illustrate the difference between sample proportions and the population proportion. Sample values are provided for the sample proportions.

Sample proportions	Population proportion
0.7, 0.4, 0.65, 0.8	0.68

Pairs | 10 min

Activity 2 A New Comic Book Hero

Students use a different context to practice exploring population proportions and making predictions about populations.



Activity 2 A New Comic Book Hero

A survey asked a randomly selected group of 20 people who read *The Adventures of Super Sam* this question: *What superpower do you think a new superhero should have?* The table shows the survey results.

Response	Superpower	Response	Superpower
1	fly	11	freeze
2	freeze	12	freeze
3	freeze	13	fly
4	fly	14	invisibility
5	fly	15	freeze
6	freeze	16	fly
7	fly	17	freeze
8	super strength	18	fly
9	freeze	19	super strength
10	fly	20	freeze

1. What fraction of this sample thinks a new superhero should be able to fly?
   $\frac{8}{20}$ or $\frac{2}{5}$

2. If there are 2,024 dedicated readers of *The Adventures of Super Sam*, estimate the number of readers who would want the new superhero to be able to fly. Explain or show your thinking.
   About 810 readers; Sample response: $0.4 \cdot 2024 = 809.6$

3. Based on the data from the survey, which superpower would you recommend the new superhero to have? Explain your thinking.
   Sample response: I would recommend the new superhero have the freeze superpower because the majority of survey responses indicated that superpower.

Critique and Correct: Your teacher will display an incorrect statement about this survey. Work with a partner to identify the error and correct the statement.

STOP

924 Unit 8 Probability and Sampling

1 Launch

Display the table. Activate students' background knowledge by asking about their favorite comic book or what superpower they would want to have if they are not familiar with comic books.

2 Monitor

Look for points of confusion:

- **Struggling to multiply numbers in Problem 2.** Have students rewrite their response for Problem 1 as a decimal.

3 Connect

Have pairs of students share which superpower they would recommend the new superhero to have.

Highlight that the population of middle school students is too large to collect the data from every one of the readers. However, the sample of 20 students can be used to make estimates about the population.

Ask, "Would your opinion change of which superpower to recommend to the new superhero if there were more results added to the survey?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students use colored pencils to color code the responses by superpower. Alternatively, suggest they create a table, similar to the following, to list the number of responses for each superpower.

	Number of responses
Fly	
Freeze	
Super strength	
Invisibility	

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement about this survey, such as "I would recommend the superhero have the flying superpower because the majority of responses indicated that superpower." Ask:

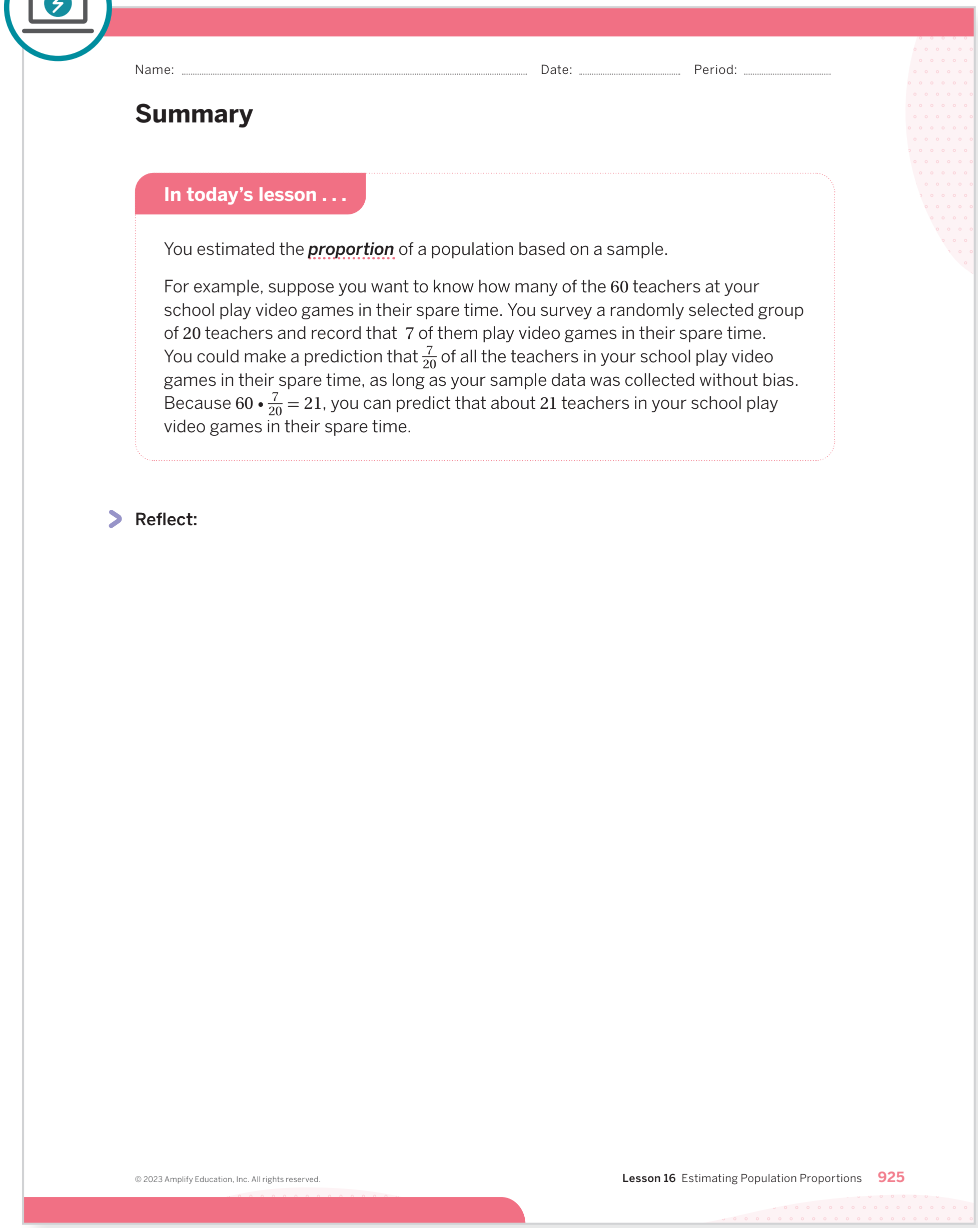
- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking using the results of the survey."
- ***Correct:*** "Write a corrected statement, based on the survey results."
- ***Clarify:*** "How did you correct the statement? Would you change your recommendation if there were more responses? Why or why not?"

Listen for students who recognize that the sample size is relatively small, so the recommendation may change if more data are available.

Whole Class | 5 min

Summary

Review and synthesize how to estimate the population proportion based on a sample.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You estimated the ***proportion*** of a population based on a sample.

For example, suppose you want to know how many of the 60 teachers at your school play video games in their spare time. You survey a randomly selected group of 20 teachers and record that 7 of them play video games in their spare time. You could make a prediction that $\frac{7}{20}$ of all the teachers in your school play video games in their spare time, as long as your sample data was collected without bias. Because $60 \cdot \frac{7}{20} = 21$, you can predict that about 21 teachers in your school play video games in their spare time.

Reflect:

Synthesize

Highlight that samples are used when the population is too large or it would be too time consuming to survey every person. The proportion of the sample is used to estimate the population proportion.

Formalize vocabulary: *population proportion*

Ask:

- "How does a sample change the proportion if more data are added to the sample?" The sample should make the population proportion more accurate.
- "In order to say that more than half of the people in a sample responded with a certain answer, what would the population proportion for that answer be?" Any value greater than 0.5.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What is a population proportion and how is it calculated?"

Math Language Development

MLR2: Collect and Display

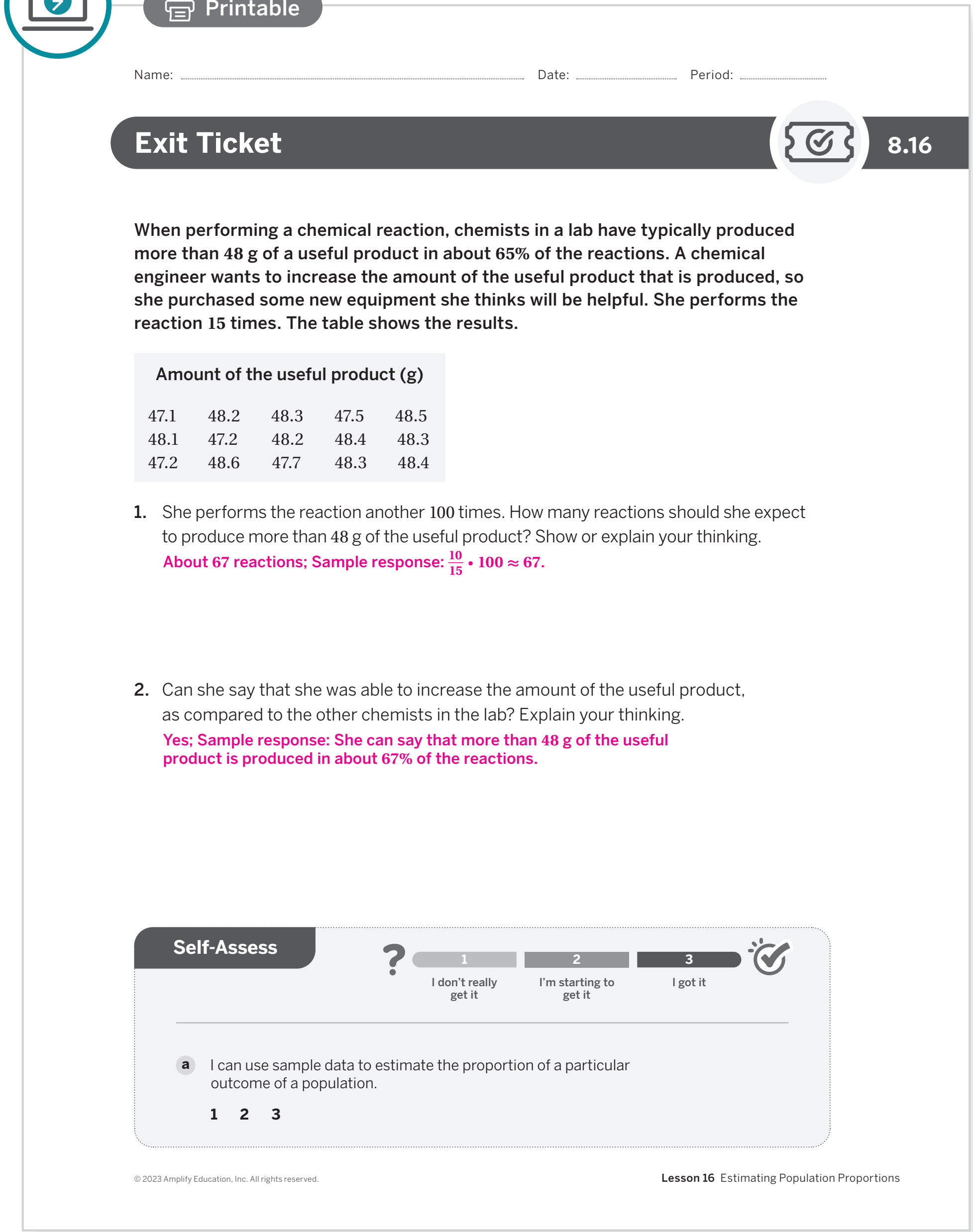
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *population proportion* that were added to the display during the lesson.

Independent | 10 min

Exit Ticket

Students demonstrate their understanding of estimating population proportions for a population based on a random sample.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 8.16

When performing a chemical reaction, chemists in a lab have typically produced more than 48 g of a useful product in about 65% of the reactions. A chemical engineer wants to increase the amount of the useful product that is produced, so she purchased some new equipment she thinks will be helpful. She performs the reaction 15 times. The table shows the results.

Amount of the useful product (g)				
47.1	48.2	48.3	47.5	48.5
48.1	47.2	48.2	48.4	48.3
47.2	48.6	47.7	48.3	48.4

1. She performs the reaction another 100 times. How many reactions should she expect to produce more than 48 g of the useful product? Show or explain your thinking.
   About 67 reactions; Sample response: $\frac{10}{15} \cdot 100 \approx 67$.

2. Can she say that she was able to increase the amount of the useful product, as compared to the other chemists in the lab? Explain your thinking.
   Yes; Sample response: She can say that more than 48 g of the useful product is produced in about 67% of the reactions.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can use sample data to estimate the proportion of a particular outcome of a population.

1 2 3

Lesson 16 Estimating Population Proportions

Success looks like . . .

- **Goal:** Comprehending that the term *population proportion* refers to a number between 0 and 1 that represents the fraction of the data within a certain category.
- **Language Goal:** Comparing population proportions for the same category from different samples of a population. **(Speaking and Listening)**
- **Language Goal:** Using the population proportion of a random sample that is within a certain category to make inferences about the population, and explaining the reasoning. **(Speaking and Listening, Writing)**
  - » Explaining why the chemical engineer was able to increase the amount of useful product in relation to the other chemists in Problem 2.

Suggested next steps

If students incorrectly find the proportion, consider:

- Reviewing converting fractions to percentages.
- Assigning Practice Problem 2.

Professional Learning

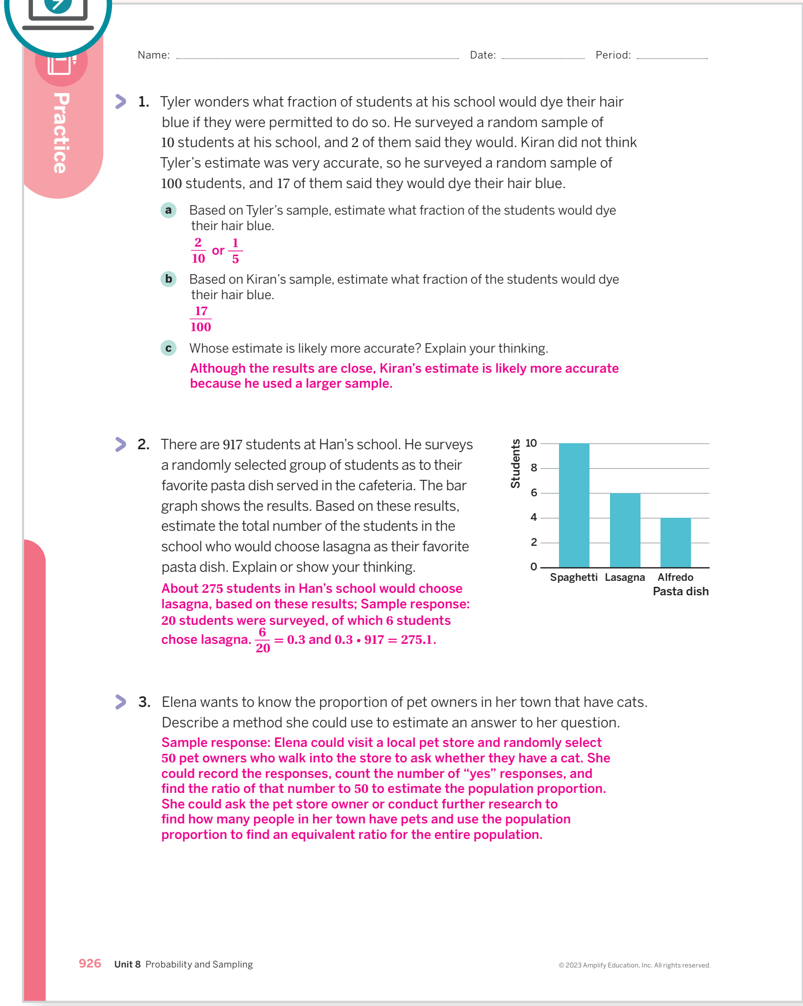
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students used population proportions. How did that build on the earlier work students did with probability?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Tyler wonders what fraction of students at his school would dye their hair blue if they were permitted to do so. He surveyed a random sample of 10 students at his school, and 2 of them said they would. Kiran did not think Tyler's estimate was very accurate, so he surveyed a random sample of 100 students, and 17 of them said they would dye their hair blue.
   - a Based on Tyler's sample, estimate what fraction of the students would dye their hair blue.
     $\frac{2}{10}$ or $\frac{1}{5}$
   - b Based on Kiran's sample, estimate what fraction of the students would dye their hair blue.
     $\frac{17}{100}$
   - c Whose estimate is likely more accurate? Explain your thinking.
     Although the results are close, Kiran's estimate is likely more accurate because he used a larger sample.

2. There are 917 students at Han's school. He surveys a randomly selected group of students as to their favorite pasta dish served in the cafeteria. The bar graph shows the results. Based on these results, estimate the total number of the students in the school who would choose lasagna as their favorite pasta dish. Explain or show your thinking.
   About 275 students in Han's school would choose lasagna, based on these results; Sample response: 20 students were surveyed, of which 6 students chose lasagna. $\frac{6}{20} = 0.3$ and $0.3 \cdot 917 = 275.1$.

3. Elena wants to know the proportion of pet owners in her town that have cats. Describe a method she could use to estimate an answer to her question.
   Sample response: Elena could visit a local pet store and randomly select 50 pet owners who walk into the store to ask whether they have a cat. She could record the responses, count the number of "yes" responses, and find the ratio of that number to 50 to estimate the population proportion. She could ask the pet store owner or conduct further research to find how many people in her town have pets and use the population proportion to find an equivalent ratio for the entire population.

926 **Unit 8** Probability and Sampling

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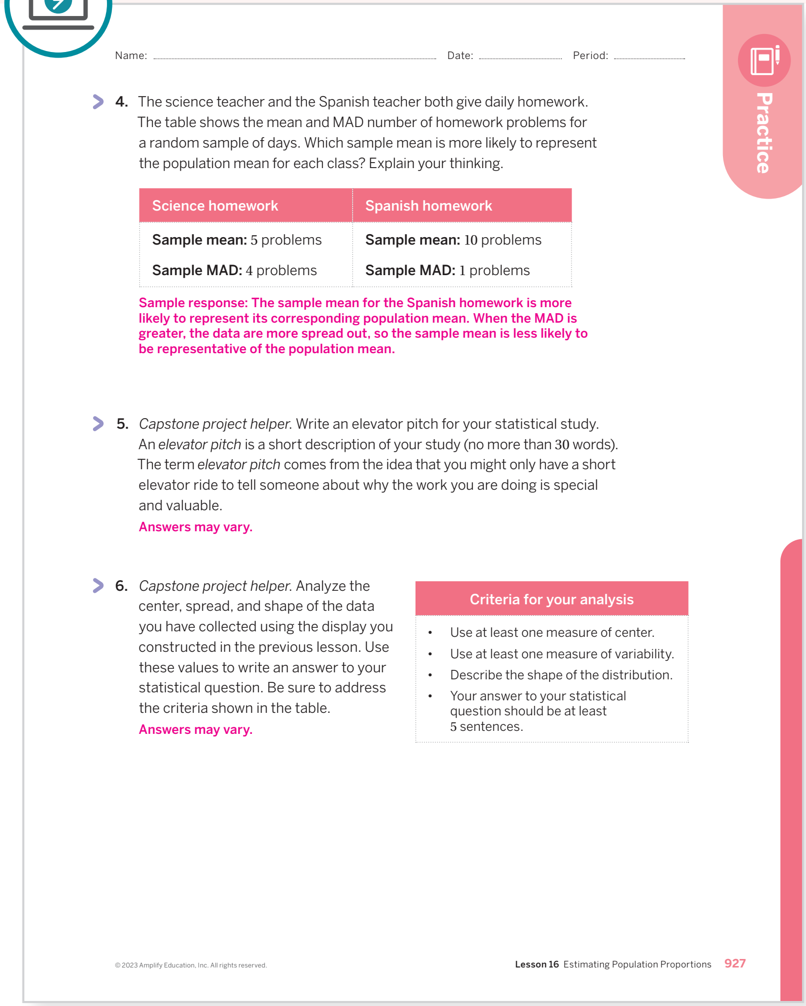

Practice

Name: Date: Period:

4. The science teacher and the Spanish teacher both give daily homework. The table shows the mean and MAD number of homework problems for a random sample of days. Which sample mean is more likely to represent the population mean for each class? Explain your thinking.

Science homework	Spanish homework
Sample mean: 5 problems **Sample MAD:** 4 problems	**Sample mean:** 10 problems **Sample MAD:** 1 problems

Sample response: The sample mean for the Spanish homework is more likely to represent its corresponding population mean. When the MAD is greater, the data are more spread out, so the sample mean is less likely to be representative of the population mean.

5. *Capstone project helper.* Write an elevator pitch for your statistical study. An *elevator pitch* is a short description of your study (no more than 30 words). The term *elevator pitch* comes from the idea that you might only have a short elevator ride to tell someone about why the work you are doing is special and valuable.
   Answers may vary.

6. *Capstone project helper.* Analyze the center, spread, and shape of the data you have collected using the display you constructed in the previous lesson. Use these values to write an answer to your statistical question. Be sure to address the criteria shown in the table.
   Answers may vary.

Criteria for your analysis

- Use at least one measure of center.
- Use at least one measure of variability.
- Describe the shape of the distribution.
- Your answer to your statistical question should be at least 5 sentences.

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Lesson 16 Estimating Population Proportions 927

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 1	2
Spiral	4	Unit 8 Lesson 12	2
	5	Unit 8 Lesson 11	3
Formative ⚡	6	Unit 8 Lesson 17	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

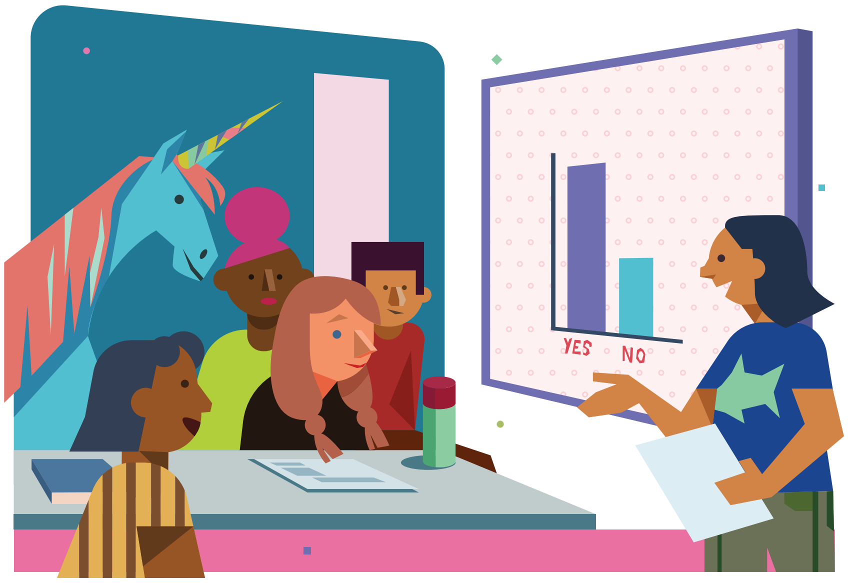
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

UNIT 8 | **LESSON 17 – CAPSTONE**

Presentation of Findings

Let's share some new ideas, based on what we've learned during this unit.



Focus

Goals

1. **Language Goal:** Create a title for a statistical study that is appropriate, engaging, and informative. **(Writing)**
2. **Language Goal:** Analyze visual and written presentations of data from a sample. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

During this Capstone lesson, students share their statistical study from this last Sub-Unit. They learn how to write an engaging title and then participate in a ***Gallery Tour*** while leaving constructive feedback for their peers.

Previously

To prepare for this Capstone lesson, students selected a statistical question, drew conclusions about their data, used math to model and solve real-world problems, and selected tools strategically.

Coming Soon

In Grade 8, students will investigate patterns of association in more complex data sets, including modeling associations with linear relationships.

Rigor

- Students **apply** their knowledge of probability and sampling through their Capstone project presentation.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	20 min	10 min	5 min	5 min
Pairs	Whole Class	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

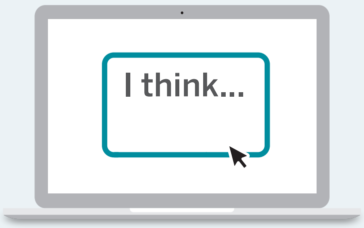
Materials

- Exit Ticket
- Additional Practice
- markers
- sticky notes, about 10 per student

Amps | **Featured Activity**

Activity 2
Share Your Reflections

Students are able to share their reflections of their own work with the rest of the class.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

It may be challenging to receive feedback on something students have worked hard on, especially from their peers. Empathize with students about how difficult receiving feedback can be, while communicating how important it is to hear and incorporate others' perspectives into our work. Stress the importance to all students of being fair, constructive, and kind when providing feedback.

Modifications to Pacing

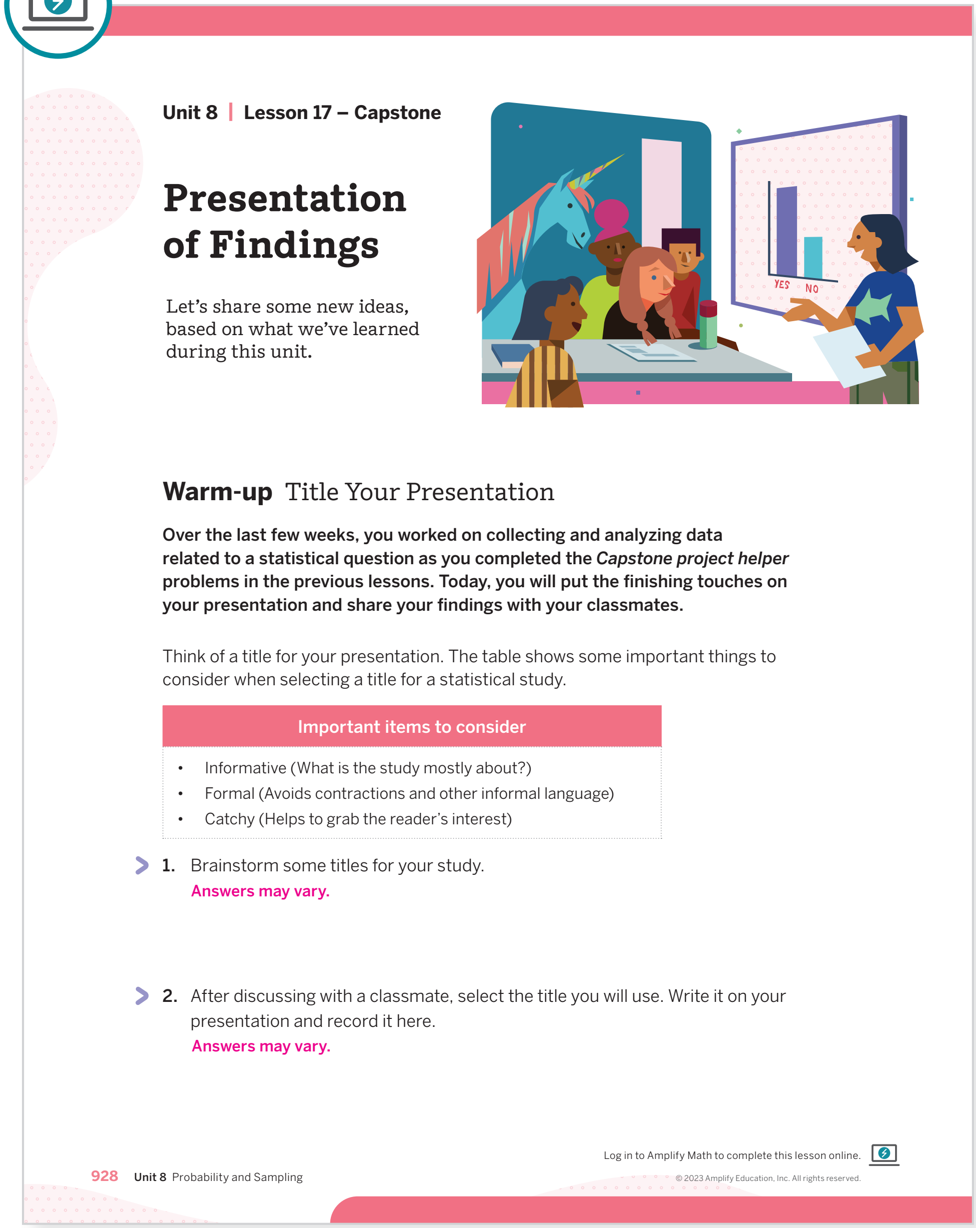
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, you may adjust the number of presentations for which students are expected to provide feedback.

Pairs | 7 min

Warm-up Title Your Presentation

Students craft an engaging and appropriate title for their study by workshopping it with a classmate.



Unit 8 | Lesson 17 – Capstone

Presentation of Findings

Let's share some new ideas, based on what we've learned during this unit.

Warm-up Title Your Presentation

Over the last few weeks, you worked on collecting and analyzing data related to a statistical question as you completed the *Capstone project helper* problems in the previous lessons. Today, you will put the finishing touches on your presentation and share your findings with your classmates.

Think of a title for your presentation. The table shows some important things to consider when selecting a title for a statistical study.

Important items to consider
• Informative (What is the study mostly about?) • Formal (Avoids contractions and other informal language) • Catchy (Helps to grab the reader's interest)

1. Brainstorm some titles for your study.
   Answers may vary.

2. After discussing with a classmate, select the title you will use. Write it on your presentation and record it here.
   Answers may vary.

Log in to Amplify Math to complete this lesson online.

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928 **Unit 8** Probability and Sampling

1 Launch

Share example titles from other statistical studies (real or invented). Provide students with some independent work time before having them work with a partner. Distribute markers so that students can add the title to their presentation.

2 Monitor

Help students get started by asking them to tell you about their statistical study.

Look for points of confusion:

- **Thinking that more words is better.** Have students aim for 5–15 words in their title.
- **Getting stuck while brainstorming.** Model exemplar titles, such as "Do Students Who Study More Get Better Grades?", "The Effect of Screen Time on Memory", and "Do Athletes Perform Better in Class?"

Look for productive strategies:

- Using colons to separate the main title and subtitles.
- Phrasing the title as a question.
- Students giving useful feedback to each other.

3 Connect

Display a list of the titles the class has created.

Highlight a particular piece of feedback from one student to another that helped them title their presentation.

Ask:

- "Do any of the titles stand out to you? Why?"
- "What is one title that makes you want to learn more?"

Power-up

To power up students' ability to analyze a set of data, have students complete:

Match each term with the part of the distribution that it describes — *center*, *shape*, or *variability*.

a cluster
b IQR
c mean
d mean absolute deviation
e median
f peak
g range
h symmetry

c,e center

a, f, h shape

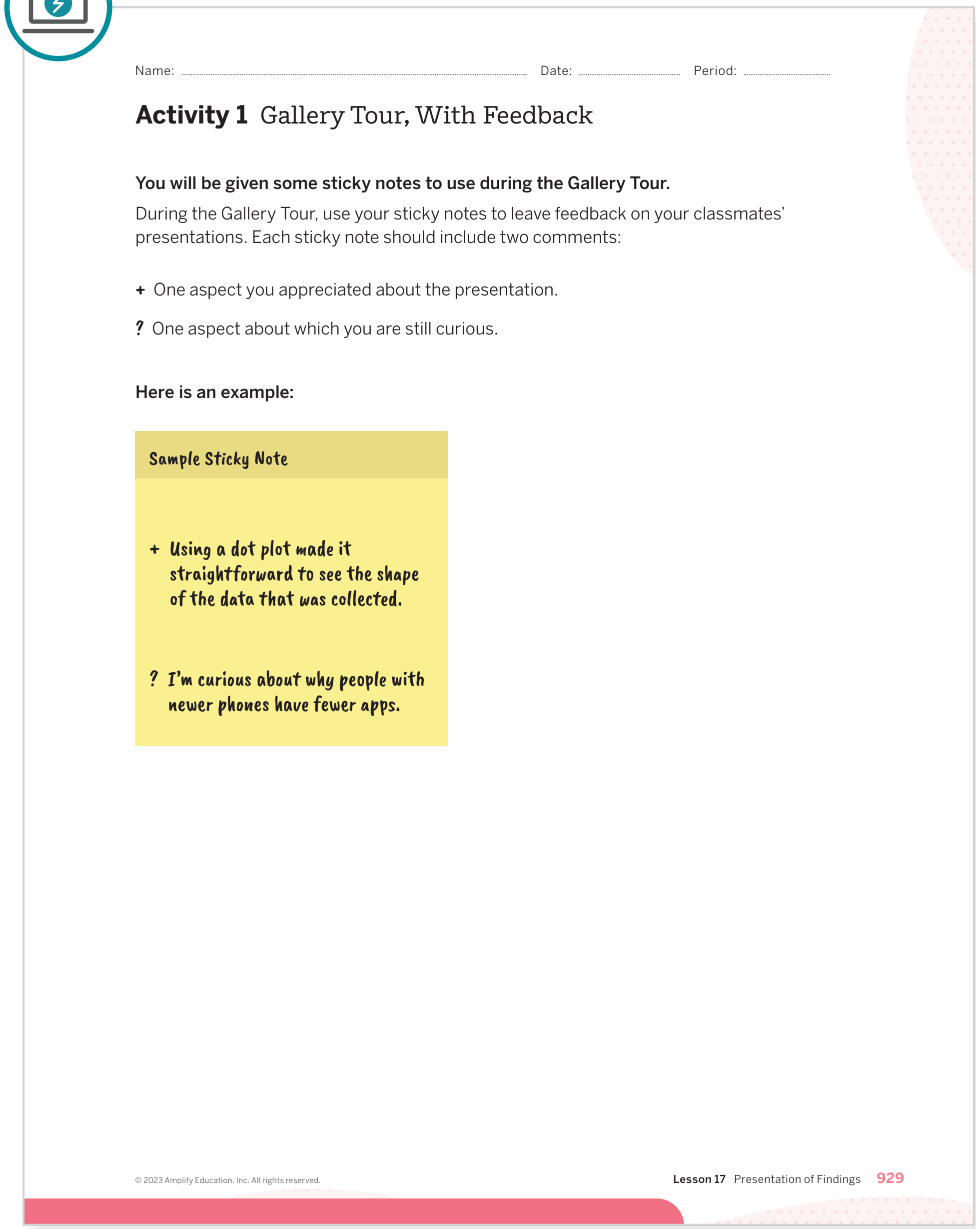
b, d, g variability

Use: Before Activity 1
Informed by: Performance on Lesson 16, Practice Problem 6

Whole Class | 20 min

Activity 1 Gallery Tour, With Feedback

Students learn about their classmates' statistical studies by observing their data and inference presentations. They give constructive feedback to improve their critical abilities.



Name: ______________ Date: ______ Period: ______

Activity 1 Gallery Tour, With Feedback

You will be given some sticky notes to use during the Gallery Tour.

During the Gallery Tour, use your sticky notes to leave feedback on your classmates' presentations. Each sticky note should include two comments:

\+ One aspect you appreciated about the presentation.

? One aspect about which you are still curious.

Here is an example:

Sample Sticky Note

\+ Using a dot plot made it straightforward to see the shape of the data that was collected.

? I'm curious about why people with newer phones have fewer apps.

1 Launch

Use the ***Gallery Tour*** routine; set up areas around the classroom to display student presentations. Demonstrate the type of feedback students might give each other before distributing the sticky notes to students.

2 Monitor

Help students get started by reading the feedback other students have left before writing their own.

Look for points of confusion:

- **Repeating feedback already given.** Have students aim to be additive with their feedback, building on others instead of echoing.

Look for productive strategies:

- Spending some time reading the presentation carefully before giving feedback.
- Placing a check mark to signal agreement with existing feedback.

3 Connect

Have individual students share pieces of specific feedback they received that they think will make their presentation better.

Highlight that every piece of academic research always goes through phases of editing and revising. This is an essential part of the process of studying something unique.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

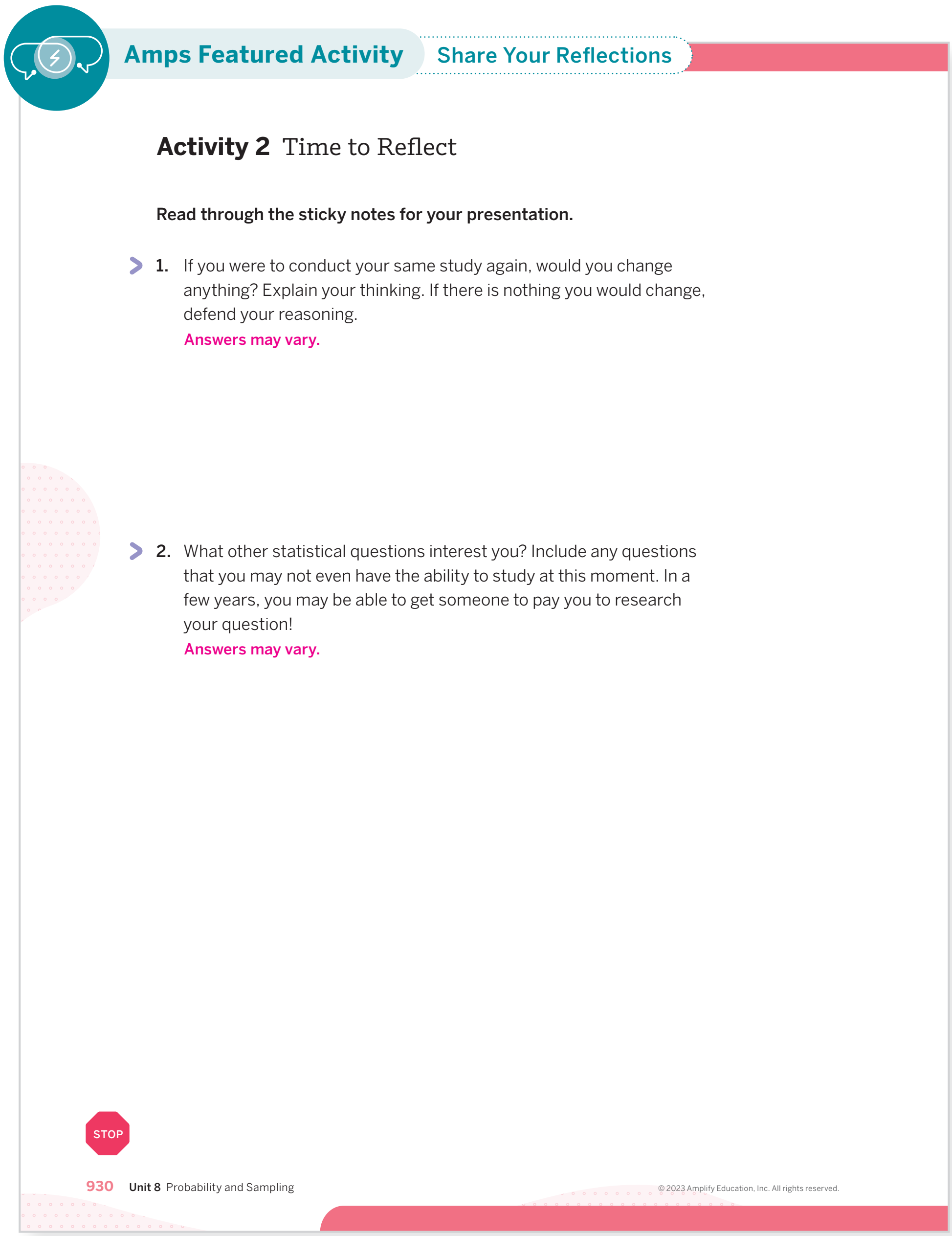
Demonstrate what additive feedback looks like, to illustrate how to build on the feedback of others, instead of mirroring the same feedback that others have given. Suggest that if they agree with the feedback from another person, they place a check mark on that sticky note indicating their agreement.

Independent | 10 min

Activity 2 Time to Reflect

Students take time to read through their feedback carefully and answer questions that prompt them to think about revising and extending their study in meaningful ways.

Amps Featured Activity Share Your Reflections



Activity 2 Time to Reflect

Read through the sticky notes for your presentation.

1. If you were to conduct your same study again, would you change anything? Explain your thinking. If there is nothing you would change, defend your reasoning.
   Answers may vary.

2. What other statistical questions interest you? Include any questions that you may not even have the ability to study at this moment. In a few years, you may be able to get someone to pay you to research your question!
   Answers may vary.

STOP

930 Unit 8 Probability and Sampling

1 Launch

Create an environment conducive to reflection, perhaps using sound or lighting to produce this effect.

2 Monitor

Help students get started by having them organize their feedback into useful categories. For example, they can place the sticky notes that gave feedback on the title into a category.

Look for points of confusion:

- **Copying the feedback verbatim.** Have students describe orally what the feedback means to them before writing.

Look for productive strategies:

- Thinking about things that interest them outside of school — perhaps things that particularly impact themselves, their families, or their communities.

3 Connect

Have pairs of students share what they wrote with each other.

Display a list of the statistical questions students are interested in studying further.

Highlight real-world scenarios that are research-based and emphasize that this process is one that involves thought and persistence. Explain that universities are good for learning, but they are also good at connecting students with avenues of research, peer and professional support, and funding for studies. Remind students that it is okay to not know exactly what they are interested in yet.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Allow students to orally respond to Problems 1 and 2 by sharing them with a partner, you, or the class.

Extension: ***Math Enrichment***

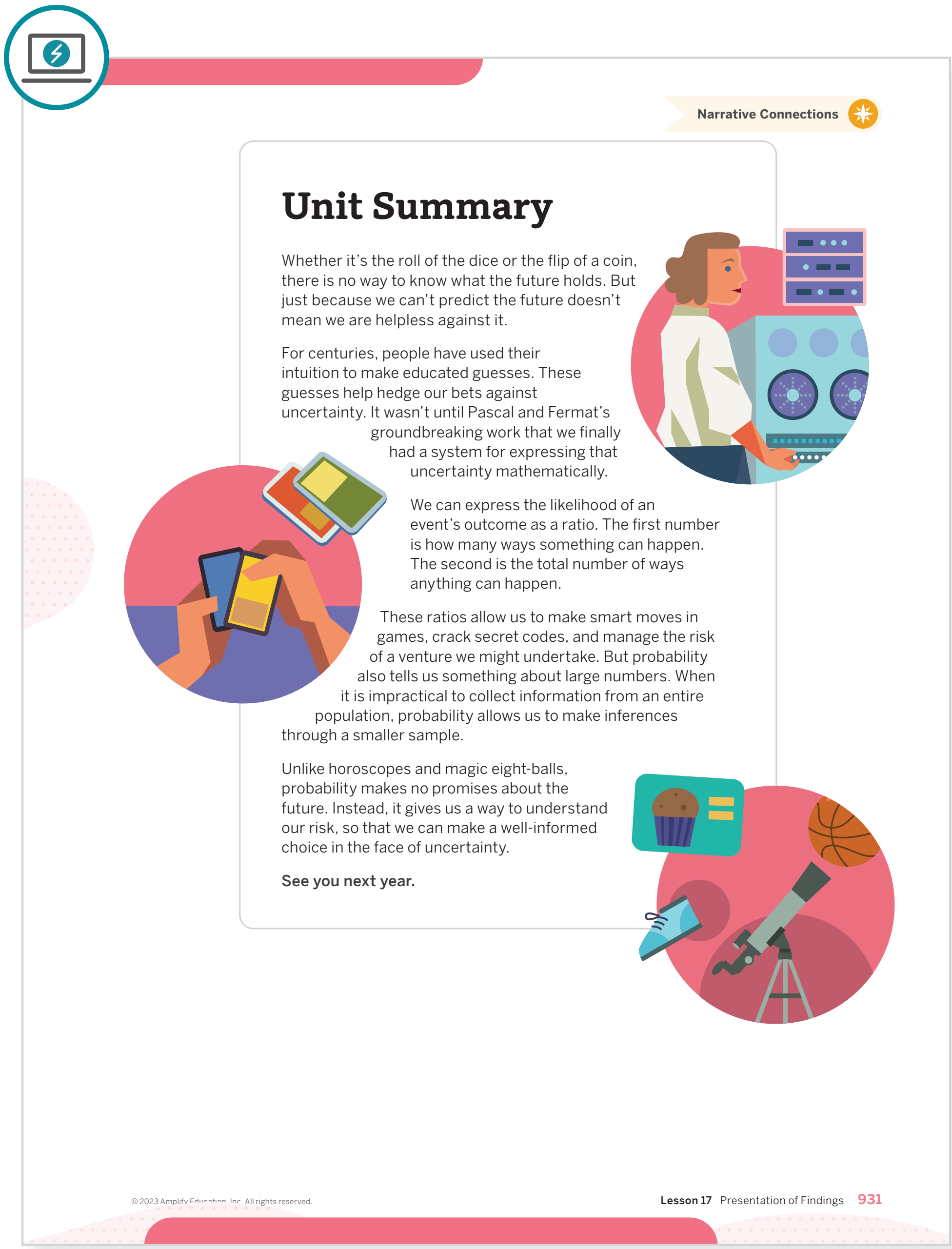
If there is time left in the school year, have students select one of the statistical questions they are interested in studying further and repeat the following steps that were outlined in the following practice problems designated as Capstone project helpers.

Capstone project steps:	Refer to:
1. Pose a statistical question.	Lesson 12, Practice Problem 3
2. Create a survey.	Lesson 13, Practice Problem 3
3. Decide on a sample.	Lesson 14, Practice Problem 3
4. Collect and analyze data from the sample.	Lesson 15, Practice Problem 4, Lesson 16, Practice Problem 5
5. Present your findings.	Lesson 16, Practice Problem 6, Lesson 17, Warm-up and Activities

Whole Class | 5 min

Unit Summary

Review and synthesize the major concepts of this unit.



Narrative Connections

Unit Summary

Whether it's the roll of the dice or the flip of a coin, there is no way to know what the future holds. But just because we can't predict the future doesn't mean we are helpless against it.

For centuries, people have used their intuition to make educated guesses. These guesses help hedge our bets against uncertainty. It wasn't until Pascal and Fermat's groundbreaking work that we finally had a system for expressing that uncertainty mathematically.

We can express the likelihood of an event's outcome as a ratio. The first number is how many ways something can happen. The second is the total number of ways anything can happen.

These ratios allow us to make smart moves in games, crack secret codes, and manage the risk of a venture we might undertake. But probability also tells us something about large numbers. When it is impractical to collect information from an entire population, probability allows us to make inferences through a smaller sample.

Unlike horoscopes and magic eight-balls, probability makes no promises about the future. Instead, it gives us a way to understand our risk, so that we can make a well-informed choice in the face of uncertainty.

See you next year.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Highlight that it is very useful to spend some time reflecting on one's own learning at the conclusion of a unit. This helps to retain information and make connections that one may not have been able to while focusing on each individual lesson.

Ask students to take a few minutes to recall what they've learned about probability and sampling throughout this unit.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "How are probabilities used to help people make predictions and decisions about the future?"
- "When have you seen or used probability in your own life?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the unit concepts by reflecting on what stood out to them and sharing any unresolved questions they may have.

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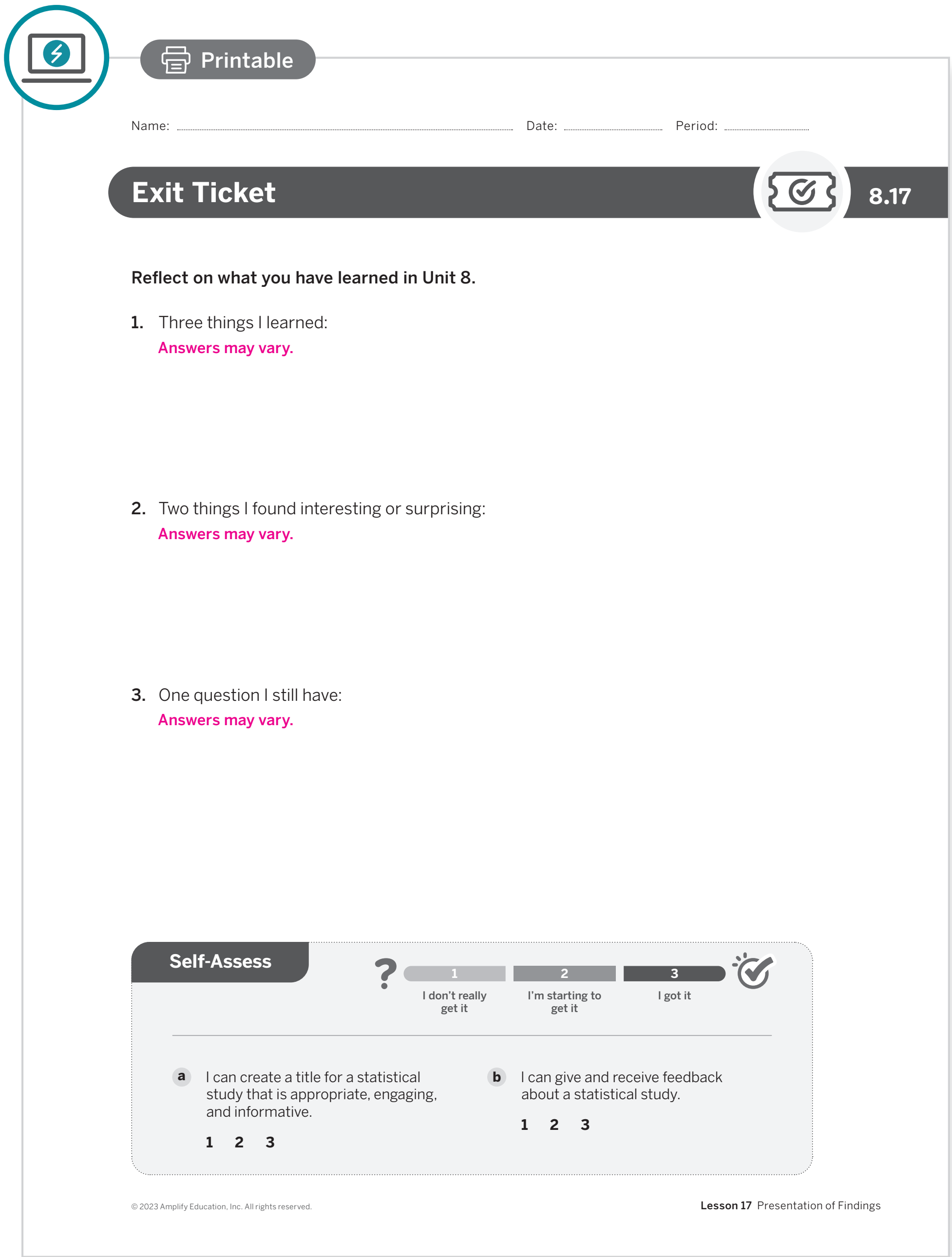


Name: ______ Date: ______ Period: ______

Exit Ticket 8.17

Reflect on what you have learned in Unit 8.

1. Three things I learned:
   Answers may vary.

2. Two things I found interesting or surprising:
   Answers may vary.

3. One question I still have:
   Answers may vary.

Self-Assess

a. I can create a title for a statistical study that is appropriate, engaging, and informative.
1 2 3

b. I can give and receive feedback about a statistical study.
1 2 3

Lesson 17 Presentation of Findings

Success looks like . . .

- **Language Goal:** Creating a title for a statistical study that is appropriate, engaging, and informative. **(Writing)**
- **Language Goal:** Analyzing visual and written presentations of data from a sample. **(Speaking and Listening, Reading and Writing)**
  - » Explaining how sample data were analyzed in Problems 1 and 2.

Suggested next steps

If students cannot think of three things they learned, consider:

- Allowing them to review what they have written in their Summary notes throughout the unit.

If students cannot think of two things they found interesting or surprising, consider:

- Changing the question to "two things that were different than usual during this unit" and have them respond.

Professional Learning

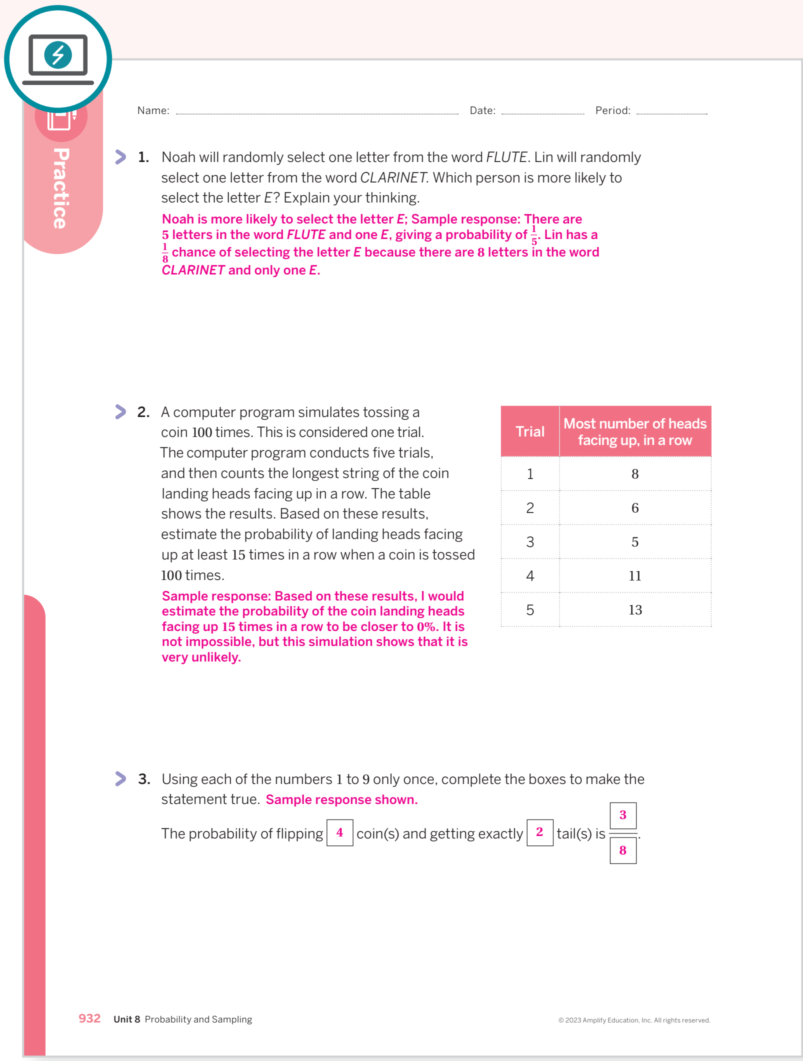
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What was especially satisfying about the Capstone project?
- When you compare and contrast today's work with work students did earlier this year, what similarities and differences do you see? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. Noah will randomly select one letter from the word *FLUTE*. Lin will randomly select one letter from the word *CLARINET*. Which person is more likely to select the letter *E*? Explain your thinking.
   Noah is more likely to select the letter *E*; Sample response: There are 5 letters in the word *FLUTE* and one *E*, giving a probability of $\frac{1}{5}$. Lin has a $\frac{1}{8}$ chance of selecting the letter *E* because there are 8 letters in the word *CLARINET* and only one *E*.

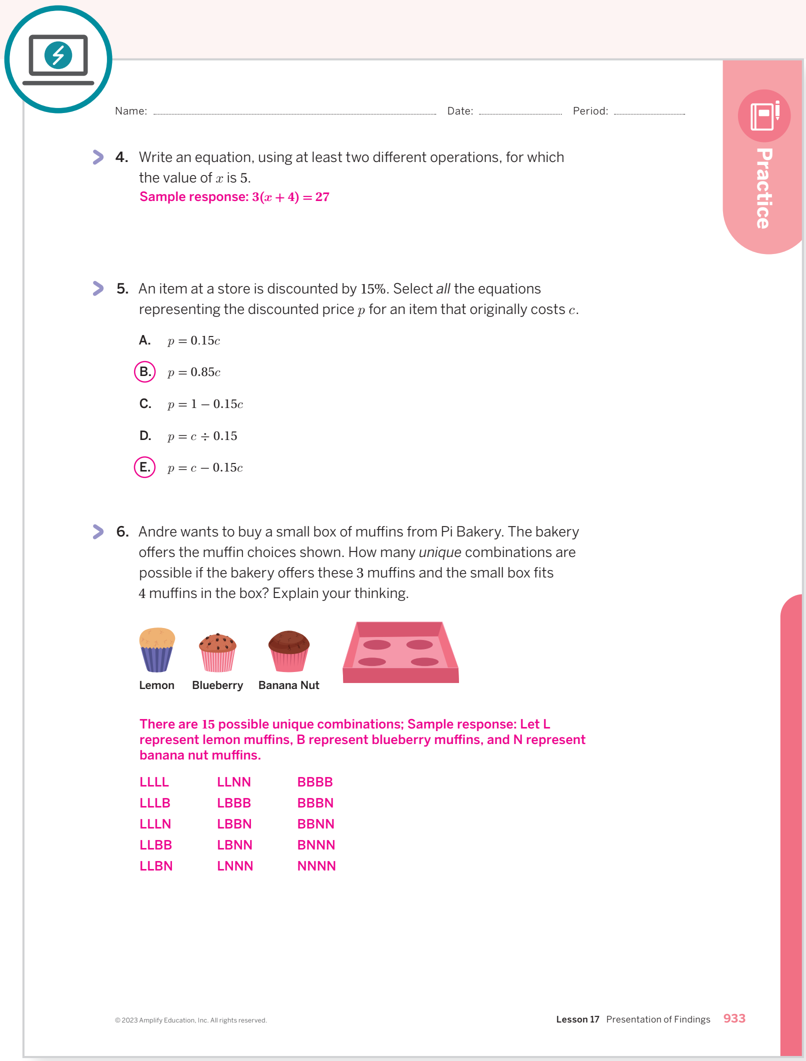
2. A computer program simulates tossing a coin 100 times. This is considered one trial. The computer program conducts five trials, and then counts the longest string of the coin landing heads facing up in a row. The table shows the results. Based on these results, estimate the probability of landing heads facing up at least 15 times in a row when a coin is tossed 100 times.
   Sample response: Based on these results, I would estimate the probability of the coin landing heads facing up 15 times in a row to be closer to 0%. It is not impossible, but this simulation shows that it is very unlikely.

Trial	Most number of heads facing up, in a row
1	8
2	6
3	5
4	11
5	13

3. Using each of the numbers 1 to 9 only once, complete the boxes to make the statement true. **Sample response shown.**

   The probability of flipping $\boxed{4}$ coin(s) and getting exactly $\boxed{2}$ tail(s) is $\frac{\boxed{3}}{\boxed{8}}$.

932 **Unit 8** Probability and Sampling



Name: .... Date: .... Period: ....

4. Write an equation, using at least two different operations, for which the value of x is 5.
   Sample response: $3(x + 4) = 27$

5. An item at a store is discounted by 15%. Select *all* the equations representing the discounted price p for an item that originally costs c.
   - A. $p = 0.15c$
   - [✓ B.] $p = 0.85c$
   - C. $p = 1 - 0.15c$
   - D. $p = c \div 0.15$
   - [✓ E.] $p = c - 0.15c$

6. Andre wants to buy a small box of muffins from Pi Bakery. The bakery offers the muffin choices shown. How many *unique* combinations are possible if the bakery offers these 3 muffins and the small box fits 4 muffins in the box? Explain your thinking.

Lemon Blueberry Banana Nut

There are 15 possible unique combinations; Sample response: Let L represent lemon muffins, B represent blueberry muffins, and N represent banana nut muffins.

LLLL	LLNN	BBBB
LLLB	LBBB	BBBN
LLLN	LBBN	BBNN
LLBB	LBNN	BNNN
LLBN	LNNN	NNNN

Lesson 17 Presentation of Findings 933

Practice

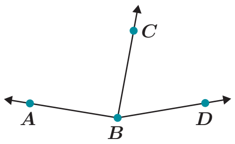
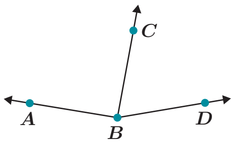
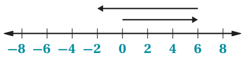
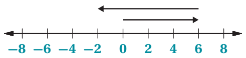
Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 8 Lesson 3	2
	2	Unit 8 Lesson 10	2
	3	Unit 8 Lesson 8	3
	4	Unit 6 Lesson 7	3
	5	Unit 4 Lesson 6	2
	6	Unit 8 Lesson 8	3

Additional Practice Available

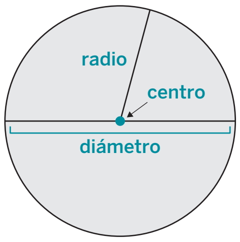
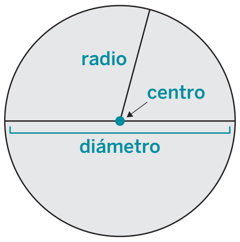
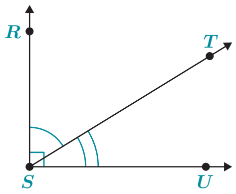
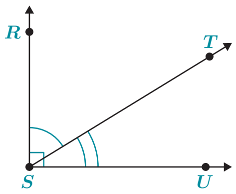
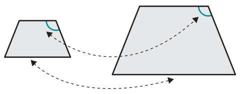
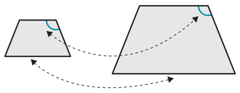
For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

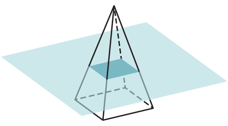
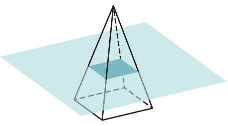
Glossary/Glosario

English	Español
A	**A**
absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $\|-3\| = 3$.	**valor absoluto** Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $\|-3\| = 3$.
Addition Property of Equality A property stating that, if $a = b$, then $a + c = b + c$.	**Propiedad de igualdad en la suma** Propiedad que establece que si $a = b$, entonces $a + c = b + c$.
additive inverse The additive inverse of a number a is the number that, when added to a, gives a sum of zero. It is the number's opposite.	**inverso aditivo** El inverso aditivo de un número a es el número que, cuando se suma a a, resulta en cero. Es el opuesto del número.
adjacent angles Angles that share a common side and vertex. For example, $\angle ABC$ and $\angle CBD$ are adjacent angles. 	**ángulos adyacentes** Ángulos que comparten un lado y un vértice. Por ejemplo, $\angle ABC$ y $\angle CBD$ son ángulos adyacentes. 
area The number of unit squares needed to fill a two-dimensional figure without gaps or overlaps.	**área** Número de unidades cuadradadas necesario para llenar una figura bidimensional sin dejar espacios vacíos ni superposiciones.
arrow diagram A model used in combination with a number line to show positive and negative numbers and operations on them. 	**diagrama de flechas** Modelo que se utiliza en combinación con una línea numérica para mostrar números positivos y negativos, y operaciones sobre estos. 
Associative Property of Addition A property stating that how addends are grouped does not change the result. For example, $(a + b) + c = a + (b + c)$.	**Propiedad asociativa de la suma** Propiedad que establece que la forma en que se agrupan los sumandos en una suma no cambia el resultado. Por ejemplo, $(a + b) + c = a + (b + c)$.
Associative Property of Multiplication A property stating that how factors are grouped in multiplication does not change the product. For example, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$.	**Propiedad asociativa de la multiplicación** Propiedad que establece que la forma en que se agrupan los factores en una multiplicación no cambia el producto. Por ejemplo, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$.
B	**B**
balance The amount that represents the difference between positive and negative amounts of money in an account.	**balance** Cantidad que representa la diferencia entre cantidades positivas y negativas de dinero en una cuenta bancaria.
bar notation Notation that indicates the repeated part of a repeating decimal. For example, $0.\overline{6} = 0.66666\ldots.$	**notación de barras** Notación que indica la parte repetida de un número decimal periódico. Por ejemplo, $0.\overline{6} = 0.66666\ldots.$
base (of a prism) Either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.	**base (de un prisma)** Cualquiera de las dos caras idénticas y paralelas de un prisma que están conectadas por un conjunto de caras rectangulares.
base (of a pyramid) The face of a pyramid that is opposite from the vertex, where all the triangular faces connect.	**base (de una pirámide)** La cara de una pirámide que se opone al vértice, donde todas las caras triangulares se conectan.

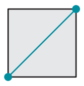
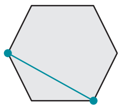
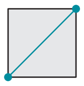
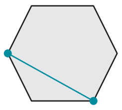
Glossary/Glosario

C

English	Español
center of a circle The point that is the same distance from all points on the circle.	**centro de un círculo** Punto que está a la misma distancia de todos los puntos del círculo.
certain A certain event is an event that is sure to happen. (The probability of the event happening is 1.)	**seguro** Un evento seguro es un evento que ocurrirá con certeza. (La probabilidad de que el evento ocurra es 1.)
chance experiment An experiment that can be performed multiple times, in which the outcome may be different each time.	**experimento aleatorio** Experimento que puede ser llevado a cabo muchas veces, en cada una de las cuales el resultado será diferente.
circle A shape that is made up of all of the points that are the same distance from a given point. 	**círculo** Forma compuesta de todos los puntos que están a la misma distancia de un punto dado. 
circumference The distance around a circle.	**circunferencia** Distancia alrededor de un círculo.
coefficient A number that is multiplied by a variable, typically written in front of or "next to" the variable, often without a multiplication symbol.	**coeficiente** Número por el cual una variable es multiplicada, escrito comúnmente frente o junto a la variable.
commission A fee paid for services, usually as a percentage of the total cost.	**comisión** Pago realizado a cambio de algún servicio, usualmente como porcentaje del costo total.
common factor A number that divides evenly into each of two or more given numbers.	**factor común** Número que divide en partes iguales cada número de entre dos o más números dados.
commutative property Changing the order in which numbers are either added or multiplied does not change the value of the sum or product.	**propiedad conmutativa** Cambiar el orden de los operandos en una suma o multiplicación no cambia el valor final de la suma o el producto.
complementary angles Two angles whose measures add up to 90 degrees. For example, $\angle RST$ and $\angle TSU$ are complementary angles. 	**ángulos complementarios** Dos ángulos cuyas medidas suman 90 grados. Por ejemplo, $\angle RST$ y $\angle TSU$ son ángulos complementarios. 
constant of proportionality The number in a proportional relationship by which the value of one quantity is multiplied to get the value of the other quantity.	**constante de proporcionalidad** En una relación proporcional, el número por el cual el valor de una cantidad es multiplicado para obtener el valor de otra cantidad.
coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both represent values that are positive, negative, or zero.	**plano de coordenadas** Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.
corresponding parts Parts of two scaled copies that match up, or "correspond" with each other. These corresponding parts could be points, segments, angles, or lengths. 	**partes correspondientes** Partes de dos copias a escala que coinciden, o "se corresponden" entre sí. Estas partes correspondientes pueden ser puntos, segmentos, ángulos o longitudes. 

English	Español
cross section A cross section is the new face seen when slicing through a three-dimensional figure. For example, a rectangular pyramid that is sliced parallel to the base has a smaller rectangle as the cross section. 	**corte transversal** Un corte transversal es la nueva cara que aparece cuando una figura tridimensional es rebanada. Por ejemplo, una pirámide rectangular que es rebanada en forma paralela a la base tiene un rectángulo más pequeño como corte transversal. 

D

debt Amount of money that has been borrowed and owed to the person or bank from which it was borrowed.	**deuda** Cantidad de dinero que ha sido pedida prestada y se le debe a la persona o al banco que la prestó.
deposit Money put into an account.	**depósito** Dinero colocado en una cuenta.
diagonal A line segment connecting two vertices on different sides of a polygon. The *diagonal* of a square connects opposite vertices.  	**diagonal** Segmento de una línea que conecta dos vértices en lados diferentes de un polígono. La *diagonal* de un cuadrado conecta vértices opuestos.  
diameter The distance across a circle through its center. The line segment with endpoints on the circle, that passes through its center. (See also *circle*.)	**diámetro** Distancia a través de un círculo que atraviesa su centro. Segmento de línea cuyos extremos limitan con el círculo y que pasa por su centro. (Ver también *círculo*.)
discount A reduction in the price of an item, typically due to a sale.	**descuento** Reducción del precio de un artículo, usualmente debido a una venta de rebaja.
Distributive Property A property that states the product of a number and a sum of numbers is equal to the sum of two products: $a\,(b + c) = ab + ac$.	**Propiedad distributiva** Propiedad que establece que el producto de un número y una suma de números es igual a la suma de dos productos: $a\,(b + c) = ab + ac$.

E

equally likely as not An event that has equal chances of occurring and not occurring. (The probability of the event happening is exactly $\frac{1}{2}$.)	**tan probable como improbable** Evento que tiene las mismas posibilidades de ocurrir que de no ocurrir. (La probabilidad de que ocurra es exactamente $\frac{1}{2}$.)
equation Two expressions with an equal sign between them. When the two expressions are equal, the equation is true. An equation can also be false, when the values of the two expressions are not equal.	**ecuación** Dos expresiones con un signo igual entre sí. Cuando las dos expresiones son iguales, la ecuación es verdadera. Una ecuación también puede ser falsa, cuando los valores de las dos expresiones no son iguales.
equivalent equations Equations that have the same solution.	**ecuaciones equivalentes** Ecuaciones que tienen la misma solución.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
equivalent ratios Any two ratios in which the values for one quantity in each ratio can be multiplied or divided by the same number to get the values for the other quantity in each ratio.	**razones equivalentes** Dos razones entre las cuales los valores de una cantidad en cada razón pueden ser multiplicados o divididos por el mismo número para obtener así los valores de la otra cantidad en cada razón.
equivalent scales Different scales (relating scaled and actual measurements) that have the same scale factor.	**escalas equivalentes** Diferentes escalas (que relacionan medidas a escala y reales) que tienen el mismo factor de escala.
error interval A range of values above and below an exact value, expressed as a percentage.	**intervalo de error** Rango de valores por sobre y por debajo de un valor exacto, expresado como porcentaje.

Glossary/Glosario

English	Español
event A set of one or more outcomes in a chance experiment.	**evento** Conjunto de uno o más resultados de un experimento aleatorio.
expand To expand an expression means to use the Distributive Property to rewrite a product as a sum. The new expression is equivalent to the original expression.	**expandir** Expandir una expresión significa usar la Propiedad distributiva para volver a escribir un producto como una suma. La nueva expresión es equivalente a la expresión original.
F	
factor To factor an expression means to use the Distributive Property to rewrite a sum as a product. The new expression is equivalent to the original expression.	**factorizar** Factorizar una expresión significa usar la Propiedad distributiva para volver a escribir una suma como un producto. La nueva expresión es equivalente a la expresión original.
G	
gratuity See the definition for *tip*.	**gratificación** Ver *propina*.
greater than or equal to $x \geq a$, x is greater than a or x is equal to a.	**mayor o igual a** $x \geq a$, x es mayor que a o x es igual a a.
H	
hanger diagram A model in which quantities are represented as weights attached to either side of a hanger. When the hanger is balanced, the sum of the quantities on either side must be equal.	**diagrama de colgador** Modelo en el cual ciertas cantidades son representadas como pesos sujetos a cada lado de un colgador. Cuando el colgador está en equilibrio, la suma de las cantidades en cualquiera de los lados debe ser igual.
I	
impossible An impossible event is an event that has no chance of occurring. The probability of the event happening is 0.	**imposible** Un evento imposible es un evento que no tiene posibilidad de que ocurra. La probabilidad de que ocurra es 0.
inequality A statement relating two numbers or expressions that are not equal. The phrases *less than*, *less than or equal to*, *greater than*, and *greater than* or *equal to* describe inequalities.	**desigualdad** Enunciado que relaciona dos números o expresiones que no son iguales. Las expresiones "menor que", "menor o igual a", "mayor que" o "mayor o igual a" describen desigualdades.
integers Whole numbers and their opposites.	**enteros** Números completos y sus opuestos.
inverse operations Operations that "undo" each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.	**operaciones inversas** Operaciones que se cancelan entre sí. La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

L

English	Español
less than or equal to $x \leq a$, x is less than a or x is equal to a.	**menor o igual a** $x \leq a$, x es menor que a o x es igual a a.
like terms Terms in an expression that have the same variables and can be combined, such as $7x$ and $9x$.	**términos semejantes** Partes de una expresión que tiene la misma variable y que pueden ser sumadas, tales como $7x$ and $9x$.
likely A likely event is an event that has a greater chance of occurring than not occurring. (The probability of happening is more than $\frac{1}{2}$.)	**probable** Un evento probable es un evento que tiene más posibilidad de ocurrir que de no ocurrir. (La probabilidad de que ocurra es mayor que $\frac{1}{2}$.)
long division A method that shows the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right. $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\;4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$	**división larga** Método que muestra los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha. $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\;4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$

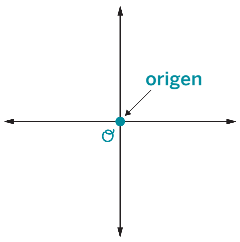
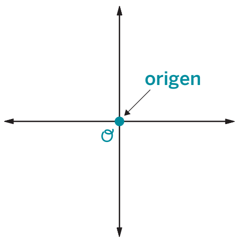
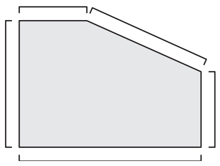
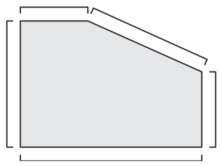
M

magnitude The absolute value of a number, or the distance of a number from 0 on the number line.	**magnitud** Valor absoluto de un número, o distancia de un número con respecto a 0 en la línea numérica.
markdown An amount, expressed as a percentage, subtracted from the cost of an item.	**descuento** Monto, expresado como porcentaje, que se resta al costo de un producto.
markup An amount, expressed as a percentage, added to the cost of an item.	**sobreprecio** Monto, expresado como porcentaje, que se agrega al costo de un producto.
multi-step event When an experiment consists of two or more events, it is called a multi-step event.	**evento de varios pasos** Cuando un experimento consiste en dos o más eventos, es llamado un evento de varios pasos.
multiplicative inverse Another name for the reciprocal of a number; The multiplicative inverse of a number a is the number that, when multiplied by a, gives a product of 1.	**inverso multiplicativo** Otro nombre para el recíproco de un número. El inverso multiplicativo de un número a es el número que, cuando se multiplica por a, tiene como producto 1.

N

negative numbers Numbers whose values are less than zero.	**números negativos** Números cuyos valores son menores que cero.
nonproportional relationship A relationship between two quantities in which the corresponding values do not have a constant ratio. (In other words, a relationship that is *not* a proportional relationship.)	**relación no proporcional** Relación entre dos cantidades, en la cual los valores correspondientes no tienen una razón constante. (En otras palabras, una relación que *no* es una relación proporcional.)

Glossary/Glosario

English	Español
O	**O**
opposites Two numbers that are the same distance from 0, but are on different sides of the number line.	**opuestos** Dos números que están a la misma distancia de 0, pero que están en lados diferentes de la línea numérica.
order of operations When an expression has multiple operations, they are applied in a consistent order (the "order of operations") so that the expression is evaluated the same way by everyone.	**orden de las operaciones** Cuando una expresión contiene múltiples operaciones, estas se aplican en cierto orden consistente (el "orden de las operaciones") de forma que la expresión sea evaluada de la misma manera por todas las personas.
ordered pair Two values, written as (x, y), that represent a point on the coordinate plane.	**par ordenado** Dos valores, escritos como (x, y), que representan un punto en el plano de coordenadas.
origin The point represented by the ordered pair $(0, 0)$ on the coordinate plane. The *origin* is where the x- and y-axes intersect. 	**origen** Punto representado por el par ordenado $(0, 0)$ en el plano de coordenadas. El *origen* es donde los ejes x y y se intersecan. 
outcome One of the possible results that can happen when an experiment is performed. For example, the possible outcomes of tossing a coin are heads and tails.	**resultado** El resultado de un experimento aleatorio es una de las cosas que pueden ocurrir cuando se realiza el experimento. Por ejemplo, los posibles resultados de tirar una moneda al aire son cara o cruz.
P	**P**
percent change How much a quantity changed (increased or decreased), expressed as a percentage of the original amount.	**cambio porcentual** Cuánto ha cambiado una cantidad (aumentado o disminuido), expresado en un porcentaje del monto original.
percent decrease The amount a value has gone down, expressed as a percentage of the original amount.	**disminución porcentual** Cantidad en que un valor ha disminuido, expresada como porcentaje del monto original.
percent error The difference between approximate and exact values, as a percentage of the exact value.	**error porcentual** Diferencia entre valores aproximados y valores exactos, expresada como porcentaje del valor exacto.
percent increase The amount a value has gone up, expressed as a percentage of the original amount.	**aumento porcentual** Monto en que un valor ha incrementado, expresado como porcentaje del monto original.
percentage A rate per 100. (A specific *percentage* is also called a *percent*, such as "70 percent.")	**porcentaje** Tasa por cada 100. (Un *porcentaje* específico también es llamado *por ciento*, como por ejemplo "70 por ciento.")
perimeter The total distance around the sides of a two-dimensional figure. 	**perímetro** Distancia total alrededor de los lados de una forma bidimensional. 

English

pi, or π The ratio between the circumference and the diameter of a circle.

polygon A closed, two-dimensional shape with straight sides that do not cross each other.

population A set of people or objects that are to be studied. For example, if the heights of people on different sports teams are studied, the population would be all the people on the teams.

population proportion A number in statistics, between 0 and 1 that represents the fraction of the data that fits into the desired category.

positive numbers Numbers whose values are greater than zero.

prism A three-dimensional figure with two parallel, identical faces (called *bases*) that are connected by a set of rectangular faces.

probability The ratio of the number of favorable outcomes to the total possible number of outcomes. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen.

profit The amount of money earned, minus expenses.

properties of equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that, if an equation is true, then applying the same operation to both sides will give a new equation that is also true.

proportional relationship A relationship in which the values for one quantity are each multiplied by the same number (the *constant of proprtionality*) to get the values for the other quantity.

pyramid A three-dimensional figure with one base and a set of triangular faces that meet at a singular vertex.

Español

pi, o π Razón entre la circunferencia y el diámetro de un círculo.

porcentaje Tasa por cada 100. (Un porcentaje específico también es llamado “por ciento”, como por ejemplo “70 por ciento”.)

población Una población es un conjunto de personas o cosas por estudiar. Por ejemplo, si se estudia la altura de las personas en diferentes equipos deportivos, la población constaría de todas las personas que conforman los equipos.

proporción de la población En estadística, número entre 0 y 1 que representa la fracción de los datos que cabe en la categoría deseada.

números positivos Números cuyos valores son mayores que cero.

prisma Forma tridimensional con dos caras iguales y paralelas (llamadas *bases*) que se conectan entre sí a través de un conjunto de caras rectangulares.

probabilidad La razón entre el número de resultados favorables y el número total posible de resultados. Una probabilidad de 1 significa que el evento siempre ocurrirá. Una probabilidad de 0 significa que el evento nunca va a ocurrir.

ganancia Monto del dinero obtenido, menos los gastos.

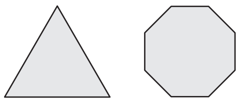
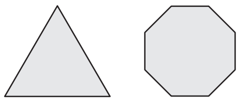
propiedades de igualdad Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al aplicar la misma operación a ambos lados se obtendrá una nueva ecuación que también es verdadera.

relación proporcional Relación en la que los valores de una cantidad se multiplican cada uno por el mismo número (la *constante de proporcionalidad*) para encontrar los valores de la otra cantidad.

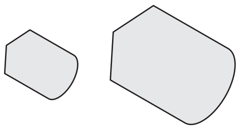
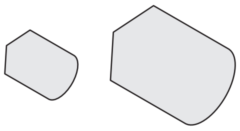
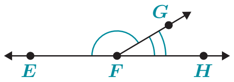
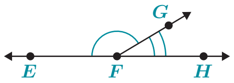
pirámide Forma tridimensional con una base y un conjunto de caras triangulares que se intersecan en un solo vértice.

Glossary/Glosario

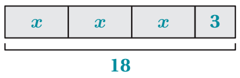
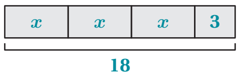
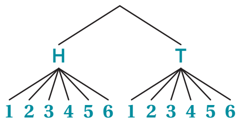
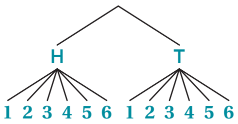
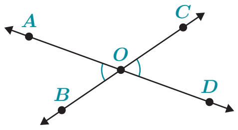
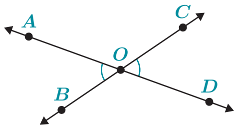
R

English	Español
radius A line segment that connects the center of a circle with a point on the circle. The term *radius* can also refer to the length of this segment. (See also *circle*.)	**radio** Segmento de una línea que conecta el centro de un círculo con un punto del círculo. *Radio* también puede referirse a la longitud de este segmento. (Ver también *círculo*.)
random sample A sample that has an equal chance of being selected from the population as any sample of the same size	**muestra al azar** Muestra que tiene la misma posibilidad de ser seleccionada de entre la población que cualquier otra muestra del mismo tamaño.
rate A comparison of how two quantities change together.	**tasa** Comparación de cuánto cambian dos cantidades en conjunto.
ratio A comparison of two quantities by multiplication or division.	**razón** Comparación de dos cantidades a través de la multiplicación o la división.
rational numbers The set of all numbers, positive and negative, that can be written as fractions. For example, any whole number is a rational number.	**números racionales** Conjunto de todos los números positivos y negativos que pueden ser escritos como fracciones. Por ejemplo, todo número entero es un número racional.
reciprocal Two numbers whose product is 1 are *reciprocals* of each other. (For example, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals.)	**recíproco/a** Dos números cuyo producto es 1 son *recíprocos* entre sí. (Por ejemplo, $\frac{3}{5}$ y $\frac{5}{3}$ son recíprocos.)
regular polygon A polygon whose sides all have the same length and whose angles all have the same measure. 	**polígono regular** Polígono cuyos lados tienen todos la misma longitud y cuyos ángulos tienen todos la misma medida. 
relative frequency The relative frequency is the ratio of the number of times an outcome occurs in a set of data. The relative frequency can be written as a fraction, a decimal, or a percentage.	**frecuencia relativa** La frecuencia relativa es la razón del número de veces que ocurre un resultado en un conjunto de datos. Se puede escribir como una fracción, un decimal o un porcentaje.
repeating decimal A decimal in which there is a sequence of non-zero digits that repeat indefinitely.	**número decimal periódico** Decimal que tiene una secuencia de dígitos distintos de cero que se repite de manera indefinida.
representative sample A sample is representative of a population if its distribution resembles the population's distribution in center, shape, and spread.	**muestra representativa** Una muestra es representativa de una población si su distribución asemeja la distribución de la población en centro, forma y extensión.
retail price The price a store typically charges for an item	**precio de venta al público** Precio que una tienda comercial usualmente cobra por un producto.
right angle An angle whose measure is 90 degrees.	**ángulo recto** Ángulo cuya medida es de 90 grados.

S

English	Español
sales tax An additional cost, as a rate to the cost of certain goods and services, applied by the government.	**impuesto de venta** Costo adicional, como una tasa del costo de ciertos bienes y servicios, aplicado por el gobierno.
sample Part of a population. For example, a population could be all the seventh graders at one school. One sample of that population is all the seventh graders who are in band.	**interés simple** Monto de dinero que se agrega a un monto original, usualmente pagado al titular o a la titular de una cuenta bancaria de ahorros.
sample space A list of every possible outcome for a chance experiment.	**espacio de muestra** Lista de cada resultado posible de un experimento aleatorio.
scale A ratio, sometimes shown as a segment, that indicates how the measurements in a scale drawing represent the actual measurements of the object shown.	**escala** Razón, a veces mostrada como segmento, que indica de qué forma las medidas de un dibujo a escala representan las verdaderas medidas del objeto mostrado.
scale drawing A drawing that represents an actual place, object, or person. All of the measurements in the scale drawing correspond to the measurements of the actual object by the same scale.	**dibujo a escala** Dibujo que representa un lugar, objeto o persona real. Todas las medidas en el dibujo a escala corresponden en la misma escala a las medidas del objeto real.
scale factor The value that side lengths are multiplied by to produce a certain scaled copy.	**factor de escala** Valor por el cual las longitudes de cada lado se multiplican para producir cierta copia a escala.
scaled copy A copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy. 	**copia a escala** Copia de una figura donde cada longitud de la figura original es multiplicada por el mismo valor, para producir longitudes correspondientes en la copia. 
simple interest An amount of money that is added on to an original amount, usually paid to the holder of a bank savings account.	**muestra** Una muestra es una parte de la población. Por ejemplo, una población podría ser todos/as los/as estudiantes de séptimo grado en una escuela. Una muestra de esa población son todos/as los/as estudiantes de séptimo grado que están en una banda.
simulation An experiment that is used to estimate the probability of a real-world event.	**simulación** Un experimento que es utilizado para estimar la probabilidad de un evento en el mundo real.
solution to an equation A value that will make an equation true when substituted into the equation.	**solución a una ecuación** Número que puede sustituir una variable para volver verdadera una ecuación.
solution to an inequality A value that will make an inequality a true statement when substituted into the inequality.	**solución a una desigualdad** Cualquier número que puede sustituir una variable para volver verdadera una desigualdad.
straight angle An angle whose measure is 180 degrees. For example, $\angle EFH$ is a straight angle. 	**ángulo llano** Ángulo cuya medida es de 180 grados.Por ejemplo, $\angle EFH$ es un ángulo llano. 
supplementary angles Two angles whose measures add up to 180 degrees. For example, $\angle EFG$ and $\angle GFH$ are supplementary angles. 	**ángulos suplementarios** Dos ángulos cuyas medidas suman 180 grados. Por ejemplo, $\angle EFG$ y $\angle GFH$ son ángulos suplementarios. 
surface area The number of unit squares needed to cover all of the faces of a three-dimensional figure without gaps or overlaps.	**área de superficie** Número de unidades cuadradas necesarias para cubrir todas las caras de una figura tridimensional sin dejar espacios vacíos ni superposiciones.

Glossary/Glosario

English	Español
T	
tape diagram A model in which quantities are represented as lengths (of tape) placed end-to-end, and which can be used to show addition, subtraction, multiplication, and division. 	**diagrama de cinta** Modelo en el cual las cantidades están representadas como longitudes (de cinta) colocadas de forma continua, y que pueden ser usadas para mostrar suma, resta, multiplicación y división. 
term A term is a part of an expression. It can be a number, a variable, or a product of a number and a variable.	**término** Un término es una parte de una expresión. Puede ser un número individual, una variable o el producto de un número y una variable.
terminating decimal A decimal that ends at a specific place value.	**decimal exacto** Un decimal que termina en un valor posicional específico.
tip An amount given to a server at a restaurant (or other service provider) that is calculated as a percentage of the bill.	**propina** Cantidad dada a un mesero o mesera en un restaurante (o a una persona que presta cualquier otro servicio) que se calcula como porcentaje de la cuenta.
tree diagram A diagram that represent all the possible outcomes in an experiment. 	**diagrama de árbol** Diagrama que representa todos los resultados posibles. 
U	
unit rate How much one quantity changes when the other changes by 1.	**tasa unitaria** Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.
unlikely An unlikely event is an event that has small chance of occurring. (The probablity of the event happening is less than $\frac{1}{2}$.)	**improbable** Un evento improbable es un evento que tiene pocas posibilidades de ocurrir. (La probabilidad de que ocurra es menor que $\frac{1}{2}$.)
V	
variable A letter that represents an unknown number in an expression or equation.	**variable** Letra que representa un número desconocido en una expresión o ecuación.
velocity A quantity that represents the speed and the direction of motion. In general, speed, like distance, is always positive, but velocity can be either positive or negative.	**velocidad** Cantidad que representa la rapidez y la dirección de un movimiento. En general, la rapidez, como la distancia, es siempre positiva, pero la velocidad puede ser tanto positiva como negativa.
vertical angles Opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal. For example, $\angle AOB$ and $\angle COD$ are vertical angles. 	**ángulos verticales** Ángulos opuestos que comparten el mismo vértice. Están compuestos de un par de líneas que se intersecan. Sus medidas de ángulo son iguales. Por ejemplo, $\angle AOB$ y $\angle COD$ son ángulos verticales. 
volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.	**volumen** Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.
W	
withdrawal Money taken out of an account.	**retiro** Dinero que es extraído de una cuenta.

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