Amplify Math
TENNESSEE

Teacher Edition
Grade 8 | Volume 2

FEATURING Amps POWERED BY desmos

Amplify Math

Grade 8

Volume 2: Units 5–8

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify Math is based on the Illustrative Mathematics (IM) curriculum. IM 6–8 Math™ was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is © 2017–2019 Open Up Resources. Additional adaptations and updates to IM 6–8 Math™ are © 2019 Illustrative Mathematics. IM 9–12 Math™ is © 2019 Illustrative Mathematics. IM 6–8 Math™ and IM 9–12 Math™ are licensed under the Creative Commons Attribution 4.0 International license (CC BY 4.0). Additional modifications contained in Amplify Math are © 2020 Amplify Education, Inc. and its licensors. Amplify is not affiliated with the Illustrative Mathematics organization.

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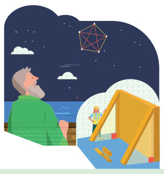
Program Scope and Sequence

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 160 days total	Area and Surface Area 20 Instructional Days 3 Assessment Days 23 days total	Introducing Ratios 20 Instructional Days 2 Assessment Days 22 days total	Rates and Percentages 15 Instructional Days 2 Assessment Days 17 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 153 days total	Scale Drawings 13 Instructional Days 2 Assessment Days 15 days total	Introducing Proportional Relationships 17 Instructional Days 2 Assessment Days 19 days total	Measuring Circles 12 Instructional Days 2 Assessment Days 14 days total	Percentages 13 Instructional Days 2 Assessment Days 15 days total
Grade 8 145 days total	 Rigid Transformation and Congruence 18 Instructional Days 3 Assessment Days 21 days total	 Dilations and Similarity 12 Instructional Days 2 Assessment Days 14 days total	 Linear Relationships 19 Instructional Days 2 Assessment Days 21 days total	 Linear Equations and Systems of Linear Equations 17 Instructional Days 2 Assessment Days 19 days total
Algebra 1 157 days total	Linear Equations, Inequalities, and Systems 26 Instructional Days 3 Assessment Days 29 days total	Data Analysis and Statistics 22 Instructional Days 3 Assessment Days 25 days total	Functions and Their Graphs 22 Instructional Days 3 Assessment Days 25 days total	Introducing Exponential Functions 22 Instructional Days 3 Assessment Days 25 days total

Volume 2

Unit 5	Unit 6	Unit 7	Unit 8
Arithmetic in Base Ten 14 Instructional Days 2 Assessment Days 16 days total	Expressions and Equations 19 Instructional Days 2 Assessment Days 21 days total	Rational Numbers 19 Instructional Days 2 Assessment Days 21 days total	Data Sets and Distributions 17 Instructional Days 3 Assessment Days 20 days total
Rational Number Arithmetic 20 Instructional Days 3 Assessment Days 23 days total	Expressions, Equations, and Inequalities 23 Instructional Days 3 Assessment Days 26 days total	Angles, Triangles, and Prisms 18 Instructional Days 3 Assessment Days 21 days total	Probability and Sampling 17 Instructional Days 3 Assessment Days 20 days total
 **Functions and Volume** 21 Instructional Days 3 Assessment Days 24 days total	 **Exponents and Scientific Notation** 15 Instructional Days 2 Assessment Days 17 days total	 **Irrationals and the Pythagorean Theorem** 16 Instructional Days 2 Assessment Days 18 days total	 **Associations in Data** 9 Instructional Days 2 Assessment Days 11 days total
Introducing Quadratic Functions 23 Instructional Days 3 Assessment Days 26 days total	Quadratic Equations 24 Instructional Days 3 Assessment Days 27 days total		

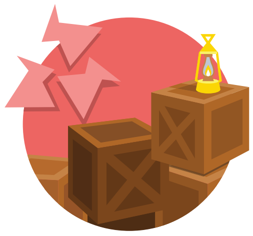
Volume 1

Unit 1 Rigid Transformations and Congruence

Students begin Grade 8 by joining talented architects, artists, and mathematicians in the study of two-dimensional figures. Equipped with their geometry toolkits, students manipulate familiar figures with new methods, and make key discoveries along the way.

Unit Narrative:
The Art of Transformation

Sub-Unit Narrative:
How do you make a piece of cardboard come alive?
Pack your geometry toolkits for a transformational journey into the movement of figures.



Sub-Unit Narrative:
How can a crack make a piece of art priceless?
Something special happens when you perform rigid transformations on a figure.

Sub-Unit Narrative:
What's got 10 billion galaxies and goes great with maple syrup?
Construct a triangle from a straight angle and cut two parallel lines to see what angle relationships you notice.

Unit 2 Dilations and Similarity

Students explore a new type of transformation, dilations, and practice using dilations to create and recognize similar figures. Students' understanding of the characteristics of these similar figures, of similar triangles specifically, will serve as the foundation for their study of the slope of a line.

Unit Narrative:
More Than
Meets the Eye

Sub-Unit Narrative:
Would you put poison in your eye?

Shrink and stretch objects on and off the plane and study the characteristics of the figures you dilate.

Sub-Unit Narrative:
Do you really get what you pay for?

Learn how some companies use dilations to create similar, and slightly smaller, sized packaging, in a process called "shrinkflation."

Unit 3 Linear Relationships

Students make connections between the rate of change, slope, and the constant of proportionality, drawing on previous knowledge to explore an exciting new relationship: the linear relationship.

Unit Narrative:
A Straight Change

Sub-Unit Narrative:
How fast is a geography teacher?
On your mark, get set, go! Use your understanding of slope to show how a geography teacher shocked the world with her record setting speed.

Sub-Unit Narrative:
How did a coal mine help build America's most famous amusement park?
Use linear relationships to collect as many coins as you can at Honest Carl's Funtime World amusement park.

Sub-Unit Narrative:
How did a 16-year-old take down a Chicago Bull?
Create equations from linear relationships and find how a 16-year-old was able to beat Michael Jordan in a game of basketball.

Unit 4 Linear Equations and Systems of Linear Equations

Students begin this unit by developing algebraic methods for solving linear equations with variables on both sides of the equation. They then use these algebraic methods, along with graphs and tables, to solve systems of linear equations.

Unit Narrative:
The Path the Mind Takes

Sub-Unit Narrative:
Who was the Father of Algebra?
When traders in 9th century Baghdad needed a better system for solving problems, a mathematician developed a new method he called “al-jabr” or algebra.

Sub-Unit Narrative:
How is anesthesia like buying live lobsters?
Now that you have practiced solving equations, take a closer look at how you can use linear equations to solve everyday problems.

Volume 2

Unit 5 Functions and Volume

By the end of this unit, students will have derived the formulas for the volumes of cylinders, cones, and spheres. But it all starts with a deep dive into the concept of what makes a relationship a function at the beginning of the unit.

Unit Narrative:
Pumping up the Volume on Functions

Sub-Unit Narrative:
Who has the better camera: you or your grandparents?
Learn how functions can help you tell stories.

Sub-Unit Narrative:
Who invented the waffle cone?
Use your prior knowledge about finding the volume of rectangular prisms to derive formulas for finding the volumes of cylinders, cones, and spheres.

Unit 6 Exponents and Scientific Notation

This unit is about the numbers so large and so small that students must develop new ways of working with them. Students deepen their knowledge of exponents before exploring how powers of 10 and scientific notation can be used to write and work with numbers as small as the mass of a bacterium or as large as the number of atoms in the Universe.

Unit Narrative:
From Teeny-Tiny to
Downright Titanic

Sub-Unit Narrative:
How many carbs are in a game of chess?
You probably already know a thing or two about exponents, but what happens when you multiply or divide expressions with exponents?

Sub-Unit Narrative:
Who should we call when we run out of numbers?
You'll work with numbers that are super small and incredibly large. But you won't waste your time writing pesky zeros!

Unit 7 Irrationals and the Pythagorean Theorem

Students study rational and irrational numbers using geometry and expressions before exploring a proof of the Pythagorean Theorem.

Unit Narrative:
The Mystery of the Pythagoreans

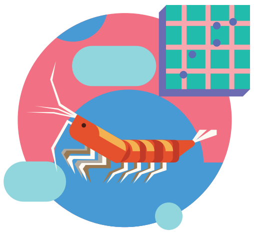
Sub-Unit Narrative:
How rational were the Pythagoreans?
Find out if every number can be represented by a fraction.

Sub-Unit Narrative:
What do the President of the United States and Albert Einstein have in common?
Uncover a special property of right triangles when you explore one of the nearly 500 proofs of the Pythagorean Theorem.

Unit 8 Associations in Data

What makes a cat logo consumer friendly? Is there a relationship between eye distance and height for a species of krill? Are adults just as likely to ride a bike as kids? Did the hole in the ozone layer have an association with skin cancer rates in Australia? In this unit, students will grapple with these questions and more, as they discover new ways to represent associations in data and build their data literacy.

Unit Narrative:
Data and the
Ozone Layer



Sub-Unit 1 Associations in Data

Sub-Unit Narrative:
Who is the biggest mover and shaker in the Antarctic Ocean?

Explore the ozone hole using scatter plots, while learning about the different kinds of associations data can have.

UNIT 5

Functions and Volume

By the end of this unit, students will have derived the formulas for the volumes of cylinders, cones, and spheres. But it all starts with a deep dive into the concept of what makes a relationship a function at the beginning of the unit.

Essential Questions

- What makes a relationship a function?
- How can you compare multiple representations of linear functions to determine which is changing at a faster rate, or which is slower?
- How are the volumes of a cylinder, cone, and sphere related if their dimensions are the same?
- *(By the way, can you create your own music just by manipulating a graph?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students are introduced to the concept of a function as a relationship between *inputs* and *outputs*. Students learn formulas for volumes of cylinders, cones, and spheres. Students express functional relationships described by these formulas as equations. They use these relationships to reason about how the volume of a figure changes as one of its dimensions changes, transforming algebraic expressions to get the information they need.

Coherence

Previously . . .

Students studied how to find the volume of a right rectangular prism in Grade 7. Also in Grade 7, students worked with proportional relationships. In earlier units in Grade 8, students explored linear relationships with an emphasis on finding rates of change or slopes.

Coming soon . . .

In Unit 8, students will continue exploring how to use linear models to describe nonlinear data sets. In Algebra 1, students will deepen their understanding of functions by looking at other types of functions, such as quadratic and exponential functions.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students ground their work in the unit by developing an understanding of what makes a relationship a function (Lesson 2). Later, they explore how to represent functions, including piecewise functions, as graphs (Lessons 4 and 10). At the end of the unit, students derive formulas for the volume of a cylinder, cone, and sphere (Lessons 12, 14, and 17).

Procedural Fluency

Students identify and analyze the qualitative aspects of a graph of a function (Lessons 5 and 6). They practice finding the unknown dimension of a cylinder, cone, or sphere using the volume equations of each (Lessons 13, 15, and 18).

Application

Students compare linear functions in different representations (Lesson 8). After learning how the volume of a cylinder is related to the volume of a cone, students use their knowledge about the volume to determine a close approximation for the volume of a hemisphere (Lesson 16).

Pumping up the Volume on Functions

SUB-UNIT

1

Lessons 2–10

Representing and Interpreting Functions

Students are introduced to the concept of a ***function*** as a special relationship between input and output values. By connecting equations to functions and interpreting graphs of functions, they begin to see how real-world data can be modeled with ***linear functions***.

Narrative: Similar to a function, a camera takes an input—light—and provides an output—a photo.

SUB-UNIT

2

Lessons 11–20

Cylinders, Cones, and Spheres

Students view the volume of a cylinder, cone, or sphere as a function of its radius. They use the formulas for the volume to determine unknown dimensions, given the volume and reason about how the volume of a figure changes as one of its dimensions changes.

Narrative: Discover how much ice cream a cone can hold.

Launch

Lesson 1

Pick a Pitch

By exploring how music can be represented as a graph, students develop the foundation for understanding what makes a relationship a function before being formally introduced to the term.

Capstone

Lesson 21

Packing Spheres

What's the best way to ship bowling balls? Students consider a classic challenge: how to pack spheres.

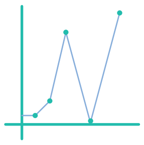
Unit at a Glance

Spoiler Alert: The volume of a cone is one third the volume of a cylinder, if the cone and cylinder have the same radius and height.

Assessment

A Pre-Unit Readiness Assessment

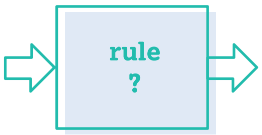
Launch Lesson



1 Pick a Pitch

Explore connections between math and music.

Sub-Unit 1: Representing and



2 Introduction to Functions ●

Understand the structure of a function as having one and only one output for each allowable input.

6 Graphs of Functions (Part 3) ●

Correlate graphs of a function with the contexts they represent.

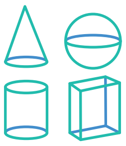
7 Connecting Representations of Functions

Connect tables, equations, graphs, and stories of functions

8 Comparing Linear Functions

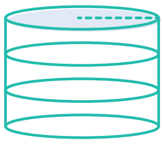
Compare properties of linear functions represented in different forms.

Sub-Unit 2: Cylinders, Cones, and Spheres



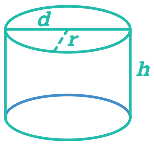
11 Filling Containers

Fill different-sized cylinders to see how their heights are a function of their volumes.



12 The Volume of a Cylinder

Discover the formula for finding the volume of a cylinder.



13 Determining Dimensions of Cylinders

Given the volume and one dimension of a cylinder, use the equation of the formula to find the unknown dimension.

Key Concepts

Lesson 2: A function is a rule that assigns exactly one output value to each possible input value.
Lesson 13: If given the volume and either the height or the radius of a cylinder, the other dimension can be determined by using the structure of the formula for the volume of a cylinder.

Pacing

21 Lessons: 45 min each
3 Assessments: 45 min each

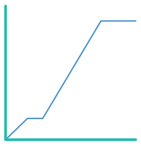
Full Unit: 24 days
● **Modified Unit:** 22 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Interpreting Functions

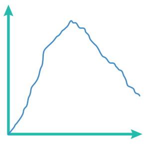
3 Equations for Functions

Connect equations with functions.



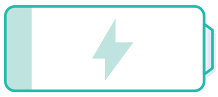
4 Graphs of Functions (Part 1)

Interpret graphs of functions.



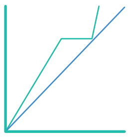
5 Graphs of Functions (Part 2) ●

Interpret intervals of graphs.



9 Modeling With Linear Functions

Model nonlinear data with linear functions.

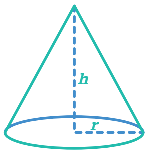


10 Piecewise Functions

Examine linear sections of piecewise functions in context.

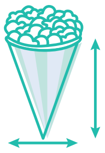
Assessment

A Mid-Unit Assessment



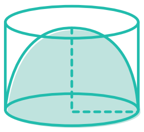
14 The Volume of a Cone

Consider how many cones fill a cylinder of equal dimensions to discover the formula for finding the volume of a cone.



15 Determining Dimensions of Cones

Solve for the unknown dimension of a cone.



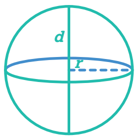
16 Estimating a Hemisphere

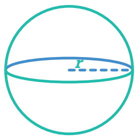
Estimate the volume of a hemisphere as greater than the volume of a cone, but less than the volume of a cylinder of equal radius.

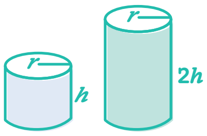
Unit at a Glance

Spoiler Alert: The volume of a cone is one third the volume of a cylinder, if the cone and cylinder have the same radius and height.

‹ continued







17 The Volume of a Sphere

Create an equation to represent the volume of a sphere as a function of its radius.

18 Cylinders, Cones, and Spheres

Build fluency determining the volumes of circular solids.

19 Scaling One Dimension (optional) ●

Discover how changing one dimension changes the volume of a shape.

Key Concepts

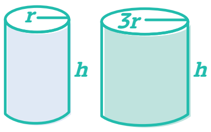
Lesson 2: A function is a rule that assigns exactly one output value to each possible input value.

Lesson 13: If given the volume and either the height or the radius of a cylinder, the other dimension can be determined by using the structure of the formula for the volume of a cylinder.

Pacing

21 Lessons: 45 min each	**Full Unit:** 24 days
3 Assessments: 45 min each	● **Modified Unit:** 22 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



20 Scaling Two Dimensions (optional) ●

Explore how changes to the radius affect the volume of a cone by a nonconstant amount.

Capstone Lesson

21 Packing Spheres

Determine the empty space of a container after packing spheres.

Assessment

A End-of-Unit Assessment

● Modifications to Pacing

Lesson 2: Depending on the needs of your students, you may choose to spend two days on Lesson 2 and consider omitting one of the optional lessons at the end of the unit.

Lessons 5–6: Lessons 4–6 help students build connections between functions and graphs, but Lessons 5–6 can be combined into one lesson, if students show proficiency in Lesson 4.

Lessons 19–20: These optional lessons can be omitted.

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	function
3	dependent variable independent variable
8	linear function
10	piecewise function
16	hemisphere

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
1, 8, 11, 16, 19	**MLR1:** Stronger and Clearer Each Time
2, 3, 8, 10, 11, 16, 17, 21	**MLR2:** Collect and Display
6, 14, 20	**MLR3:** Critique, Correct, Clarify
1, 7, 8, 10, 12, 15	**MLR5:** Co-craft Questions
3, 9, 10	**MLR6:** Three Reads
2, 4, 7, 11, 14, 17, 19	**MLR7:** Compare and Connect
3–6, 8, 9, 12, 13, 15, 18	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket Additional Practice

Lesson(s)	Additional required materials
14–21	calculators
21	cardboard or cardstock glue or tape spheres, 5 of the same type for each group
1	computer/digital headphones
16	globe
11	graduated cylinders water
4	graph paper
20	graphing technology
2–19, 21	PDFs are required for these lessons. Refer to each lesson to see which activities require PDFs.
9, 10, 21	rulers
17	3D models of cylinders, cones, and spheres

Instructional Routines

Activities throughout this unit include these instructional routines:

Lesson(s)	Instructional Routines
1, 4, 5, 10, 14, 17	Notice and Wonder
1, 21	Gallery Tour
2, 10, 13, 15, 18, 20	Poll the Class
3, 7, 10, 13, 15, 20	Think-Pair-Share
4, 11	Card Sort
11	Which One Doesn't Belong?
15	Number Talk

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 10
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 21

Amps POWERED BY **desmos**

Social & Collaborative Digital Moments

Featured Activity

Exploring Height and Volume

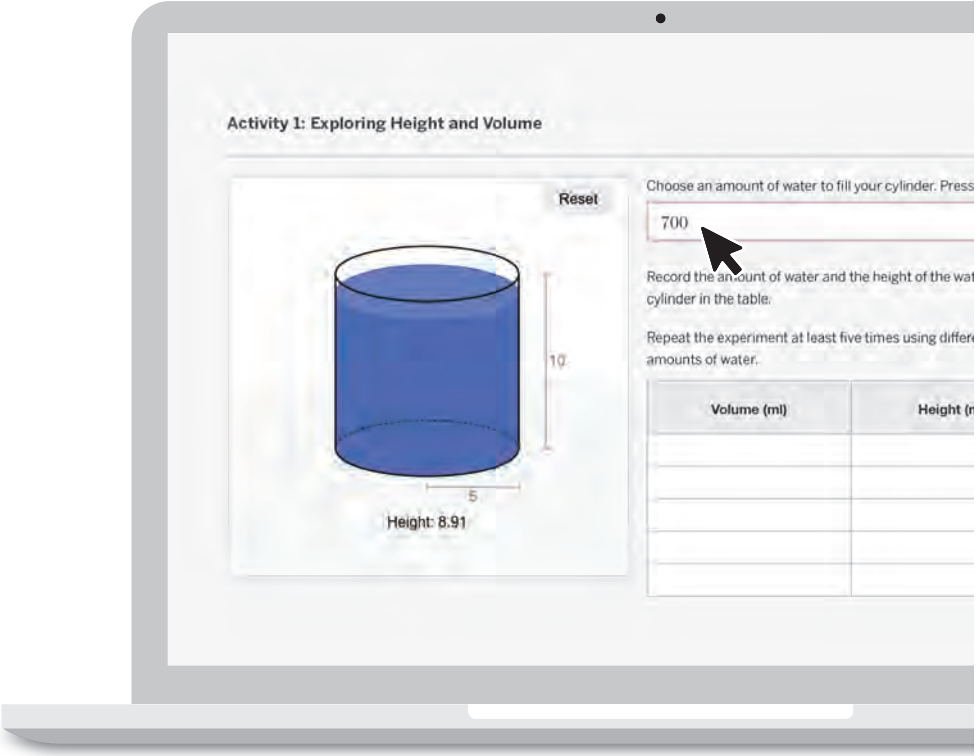
Put on your student hat and work through **Lesson 11, Activity 1:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Digital DJ **(Lesson 1)**
- Sketching a Story **(Lesson 6)**
- The Tortoise and the Hare . . . and the Fox **(Lesson 10)**
- A Sphere in a Cylinder **(Lesson 17)**



Activity 1: Exploring Height and Volume

Choose an amount of water to fill your cylinder. Press

700

Record the amount of water and the height of the wat
cylinder in the table.

Repeat the experiment at least five times using differ
amounts of water.

Volume (ml)	Height (

Unit Study
Professional Learning

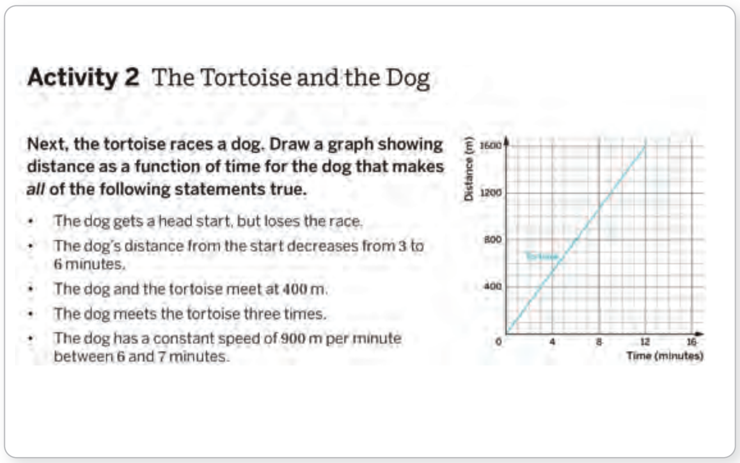
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 1 introduces students to the definition of a function, using a function "machine" that takes in an input, applies a rule to it, and produces an output. Students learn to distinguish a function from a non-function by examining input/output tables, graphs, and real-world scenarios. Sub-Unit 2 takes students through calculations of volume for cylinders, cones, and spheres. They also examine how scaling one dimension of a solid affects its volume. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 10, Activity 2:**



Activity 2 The Tortoise and the Dog

Next, the tortoise races a dog. Draw a graph showing distance as a function of time for the dog that makes *all* of the following statements true.

- The dog gets a head start, but loses the race.
- The dog's distance from the start decreases from 3 to 6 minutes.
- The dog and the tortoise meet at 400 m.
- The dog meets the tortoise three times.
- The dog has a constant speed of 900 m per minute between 6 and 7 minutes.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

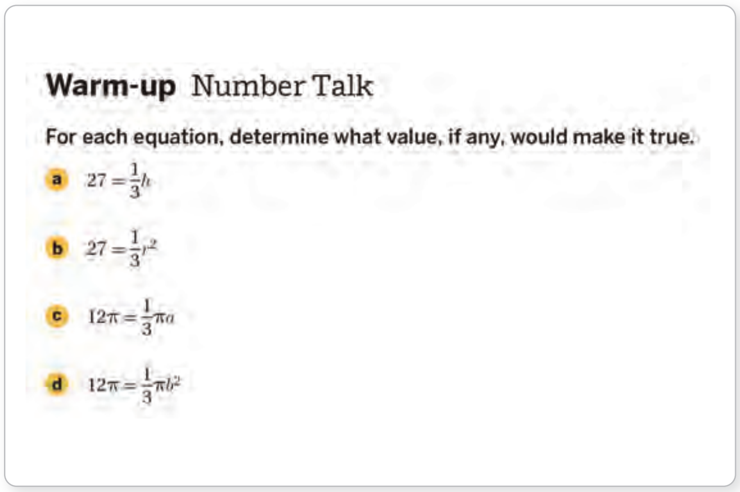
- What was it like to engage in this problem as a learner?
- Which of the five statements about the dog do you think your students may struggle with?
- Some students may interpret the last statement about the dog having a "constant speed" as a horizontal line between 6 and 7 minutes. What question(s) can you ask the student for them to think more about this?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Number Talk

Rehearse . . .

How you'll facilitate the ***Number Talk*** instructional routine in **Lesson 15, Warm-up:**



Warm-up Number Talk

For each equation, determine what value, if any, would make it true.

a $27 = \frac{1}{3}h$

b $27 = \frac{1}{3}r^2$

c $12\pi = \frac{1}{3}\pi a$

d $12\pi = \frac{1}{3}\pi b^2$

Points to Ponder . . .

- What is the purpose of revealing each problem one at a time? How does each problem build off of the previous one?

This routine . . .

- Is structured so that one problem is displayed at a time, and students are given a few moments to quietly think and give a signal when they have a response and a strategy.
- Builds fluency by encouraging students to think about the equations and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem.
- Challenges students to be precise in their word choice and use of language.
- Provides opportunities to notice and make use of structure.

Anticipate . . .

- Your students will be working at different paces — how can you best manage the range of learners and working speeds in your class?
- Some students may want to work with pencil and paper and will be reluctant to use mental math. How can you frame the routine at the beginning to encourage students to use mental math so they can best make use of structure?
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Use and connect mathematical representations.

This effective teaching practice . . .

- Deepens student understanding of mathematical concepts and procedures and builds a toolkit for problem solving.
- Facilitates meaningful mathematical discourse by drawing connections between the different representations used and how each one illustrates the same mathematical ideas.

Points to Ponder . . .

- What representations will be presented in this unit?
- Where do you see opportunities to make connections among the different representations used, and when should students be allowed choice of the representation used?

Math Language Development

MLR5: Co-craft Questions

MLR5 appears in Lessons 1, 7, 8, 10, 12, and 15.

- In Lesson 7, after you display the Activity 1 PDF, ask students to work with their small groups to co-craft questions they have about the multiple representations shown. Sample questions are provided.
- In Lesson 10, ask students to examine the graph before revealing the problems of the activity. Generating their own questions about the graph will help them make sense of the scenario before diving in.
- **English Learners:** Display 1 or 2 sample questions and allow students to say their questions aloud as opposed to writing them, or vice versa.

Point to Ponder . . .

- As students work with each other to create mathematical questions they have about graphs or scenarios, how can you model for them how to use their developing mathematical vocabulary?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with understanding how volume can be considered a function? Do you think your students will generally:
  - Struggle to understand certain representations of functions more than others?
  - Have difficulty working with the volume formulas for three-dimensional circular solids?
  - Be able to apply the concepts of a function in different contexts?

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Opportunities to provide visual support and guidance to help students process new information appear in Lessons 2, 3, 5, 7–9, 11, 13–21.

- In Lesson 3, suggest students color code the independent variable/input in one color and the dependent variable/output in another color.
- In Lesson 9, annotate the graph intervals as increasing or decreasing and suggest that students use index cards to cover up other parts of the graph while they examine one of the intervals.
- In Lesson 15, work with students to brainstorm a checklist to help students think about how to approach the problem. A sample checklist is provided.
- In selected lessons, display the Anchor Chart PDFs, *Representations of Linear Relationships* (from an earlier unit) and *Volumes of Circular Solids* for students to use as references.

Point to Ponder . . .

- When are the best times in a lesson to leverage this differentiated support?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-awareness and social awareness.

Points to Ponder . . .

- Do students identify their strengths and use them as a foundation for self-confidence? Do students have a growth mindset, not feeling defeated by new ideas that are difficult to understand?
- Do students recognize the value in diversity, both in people but also in their approaches to and solutions for problems? Are students able to show respect for their classmates because they have the ability to take on each other's viewpoints?

Pick a Pitch

Let's make connections between music and math.

Focus

Goals

1. **Language Goal:** Describe how music can be represented as a graph. **(Speaking and Listening, Writing)**
2. **Language Goal:** Create and modify audio samples by manipulating a graph that represents the pitch over time. **(Speaking and Listening)**

Coherence

Today

Students begin Unit 5 by exploring how music can be represented as a graph showing the pitch over time. By playing with the graphs, students explore concepts of how a relationship can be represented as a function, with each input represented by exactly one output. They build out these concepts throughout the first Sub-Unit.

Previously

In Unit 3, students studied linear relationships. They learned how to find and describe the slope of a line as it relates to a real-world context.

Coming Soon

In Lesson 2, students will identify rules that produce different input-output pairs. They will be formally introduced to the term *function*.

Rigor

- Students build **conceptual understanding** for how a function represents a unique output for every input before being formally introduced to the term later in the unit.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- computer/digital
- headphones (optional)

Math Language Development

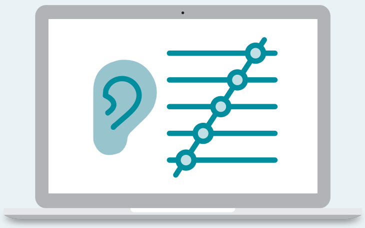
Review words

- *input*
- *linear relationship*
- *output*
- *slope*

Amps | Featured Activity

Activity 2
Produce Your Own Track

Students manipulate a graph to change how a piece of music sounds over time. View student thinking in real-time and play the role of DJ Teacher as you select student-created tracks to play for the whole class.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel uncertain about their ability to complete Activity 3, especially if they have no music training. Remind students to build on their own strengths, but also that they can learn from mistakes. They can use their strengths as well as their new mathematical understandings, to refine their audio samples before the *Gallery Tour*.

Modifications to Pacing

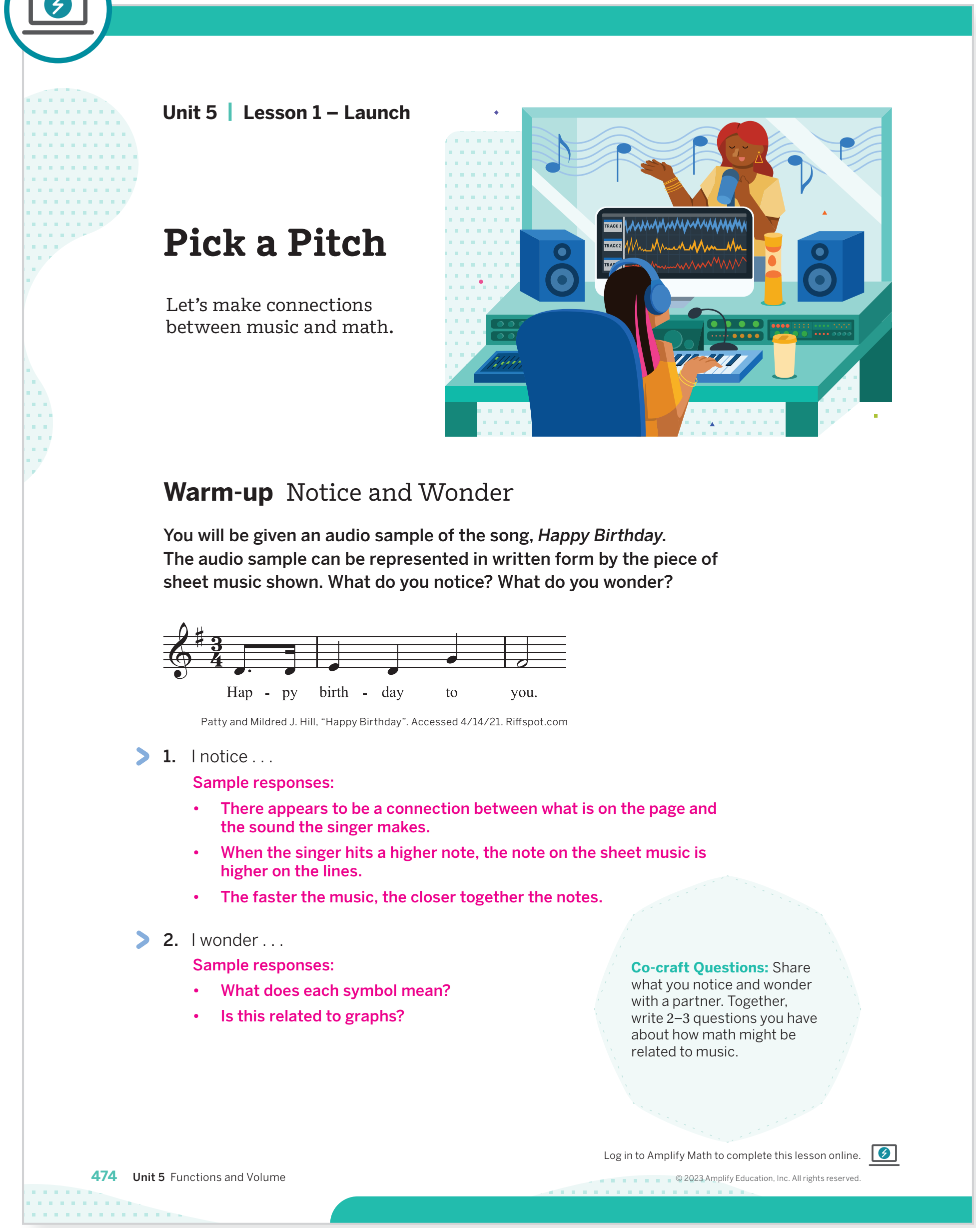
You may want to consider these additional modifications if you are short on time.

- **Activity 2** may be omitted.
- **Activity 3** may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students observe how a song can be represented as sheet music to make a connection between the different ways music can be represented.



Unit 5 | **Lesson 1 – Launch**

Pick a Pitch

Let's make connections between music and math.

Warm-up Notice and Wonder

You will be given an audio sample of the song, *Happy Birthday*. The audio sample can be represented in written form by the piece of sheet music shown. What do you notice? What do you wonder?

Patty and Mildred J. Hill, "Happy Birthday". Accessed 4/14/21. Riffspot.com

1. I notice . . .

   Sample responses:
   - There appears to be a connection between what is on the page and the sound the singer makes.
   - When the singer hits a higher note, the note on the sheet music is higher on the lines.
   - The faster the music, the closer together the notes.

2. I wonder . . .

   Sample responses:
   - What does each symbol mean?
   - Is this related to graphs?

Co-craft Questions: Share what you notice and wonder with a partner. Together, write 2–3 questions you have about how math might be related to music.

Log in to Amplify Math to complete this lesson online.

474 **Unit 5** Functions and Volume

1 Launch

Display the Amps Warm-up. Conduct the ***Notice and Wonder*** routine. Distribute headphones and have students use the Amps slides for the duration of the lesson.

2 Monitor

Help students get started by asking them to look for changes in the sheet music as it relates to changes in the sound.

Look for points of confusion:

- **Being unsure of the connection between the audio and the sheet music.** Play the audio again for students to notice when the notes on the sheet music are highlighted. Pause the audio to help students see a connection.

Look for productive strategies:

- Recognizing the sheet music as similar to a graph on the coordinate plane with vertical and horizontal axes.

3 Connect

Display the sheet music from the Student Edition and activate students' background knowledge by asking whether anyone recognizes the sheet music.

Have students share what they noticed and wondered.

Highlight that the sheet music is like a program, telling the singer or musician exactly what to do for a given point in time. Each note represents a different piece of music to be played at a given time. The higher the note is on the lines, the higher pitch of the sound played.

Ask, "Do you think it is possible to represent this piece of music using standard math notation?"

Math Language Development

MLR5: Co-craft Questions

After students complete Problems 1 and 2 independently, have them share their observations and questions with a partner. Have them work together to write 2–3 questions they might have about how math is related to music.

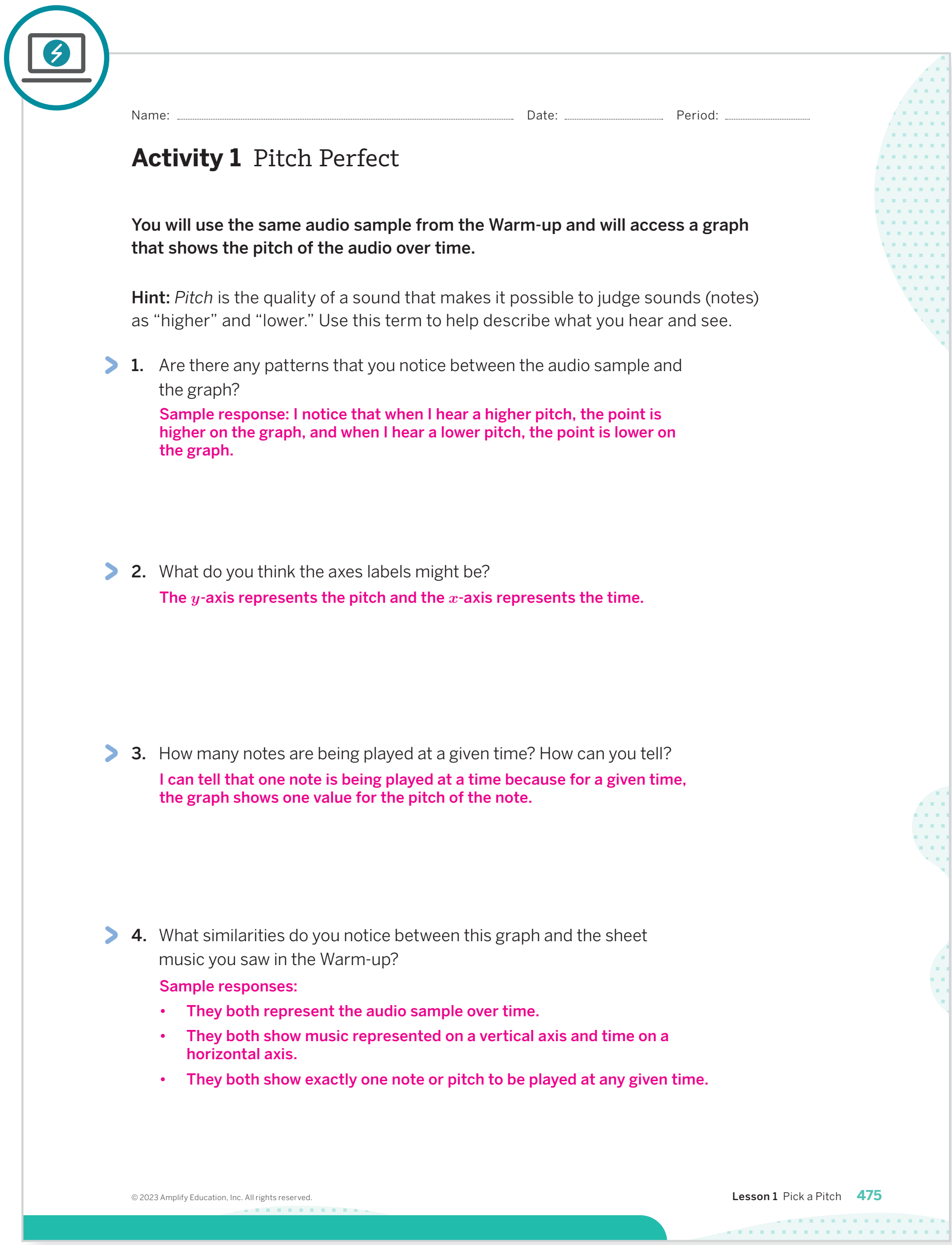
English Learners

Model crafting a question, such as "How is the sheet music similar to and different from a graph?"

Pairs | 10 min

Activity 1 Pitch Perfect?

Students observe how music and pitch can be represented on a graph and describe the graph as a relationship using input and output.



Name: ______ Date: ______ Period: ______

Activity 1 Pitch Perfect

You will use the same audio sample from the Warm-up and will access a graph that shows the pitch of the audio over time.

Hint: *Pitch* is the quality of a sound that makes it possible to judge sounds (notes) as "higher" and "lower." Use this term to help describe what you hear and see.

1. Are there any patterns that you notice between the audio sample and the graph?
   Sample response: I notice that when I hear a higher pitch, the point is higher on the graph, and when I hear a lower pitch, the point is lower on the graph.

2. What do you think the axes labels might be?
   The y-axis represents the pitch and the x-axis represents the time.

3. How many notes are being played at a given time? How can you tell?
   I can tell that one note is being played at a time because for a given time, the graph shows one value for the pitch of the note.

4. What similarities do you notice between this graph and the sheet music you saw in the Warm-up?
   Sample responses:
   - **They both represent the audio sample over time.**
   - **They both show music represented on a vertical axis and time on a horizontal axis.**
   - **They both show exactly one note or pitch to be played at any given time.**

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Lesson 1 Pick a Pitch 475

1 Launch

Say, "The graph shown is another way to represent the sounds you are hearing." Activate students' prior knowledge by discussing what they know about pitch.

2 Monitor

Help students get started by having them focus on the changes in the graph, one piece at a time.

Look for points of confusion:

- **Not knowing what the axes labels are.** Point to the highest point on the graph and ask students to describe what that sounded like to help them see that the y-axis represents the pitch.

Look for productive strategies:

- Making connections between the music represented as a graph and the music represented on sheet music.

3 Connect

Have students share their responses to Problems 1–4.

Ask:

- " How many sounds are being played at a given time? How can you tell?"
- "What do you think you would hear for a point with zero pitch?"
- "How is the graph similar to the sheet music you saw in the Warm-up?" Sample response: Like sheet music, this graph shows sound over time using a vertical and horizontal axis.

Highlight the similarities and differences between the music represented by the graph and by the sheet music. Have students describe the relationship between pitch and time using the terms input and output. Discuss how when the pitch goes to zero hertz, there is no sound.

Differentiated Support

Accessibility: *Activate Background Knowledge*

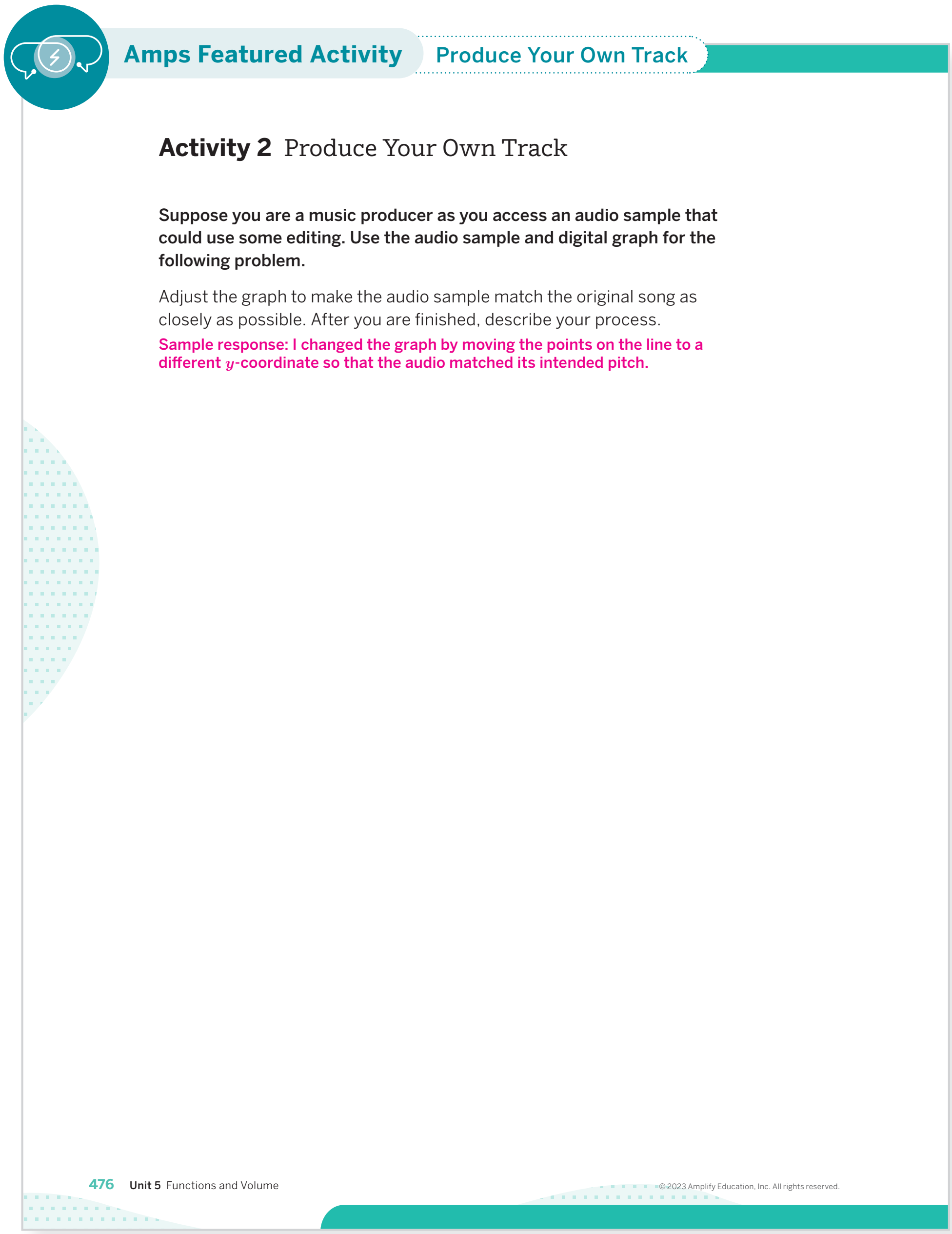
Some students may be more familiar with the term *pitch* than others. Have volunteers describe the term pitch in their own words. Consider having a volunteer demonstrate how a higher pitch compares to a lower pitch.

Pairs | 10 min

Activity 2 Produce Your Own Track

Students manipulate a graph to see how changes in the graph can coordinate with changes in the audio.

Amps Featured Activity Produce Your Own Track



Activity 2 Produce Your Own Track

Suppose you are a music producer as you access an audio sample that could use some editing. Use the audio sample and digital graph for the following problem.

Adjust the graph to make the audio sample match the original song as closely as possible. After you are finished, describe your process.

Sample response: I changed the graph by moving the points on the line to a different y-coordinate so that the audio matched its intended pitch.

476 **Unit 5** Functions and Volume

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1 Launch

Display the Amps slides for this activity. Have students describe what they hear.

2 Monitor

Help students get started by helping them identify the section of the graph that does not match the desired audio.

Look for points of confusion:

- **Being unsure how to coordinate the graph with the audio.** Help students see the difference in sound between a higher point and a lower point.

3 Connect

Have students share their process for manipulating the graph to change the audio.

Ask:

- "How did changing the graph change the music?"
- "Can you think of a faster way to produce tracks besides changing the graph of each sound yourself?"

Highlight that audio can be changed by adjusting the levels of pitch over time. If no student offers the idea, suggest a proposal for creating a computer program to automatically change the tone. Discuss how writing a computer function that takes audio input and produces a desired output automatically could be efficient. Explain how Auto-Tune®, invented in 1997, is a technology that takes the notes sung by an unprocessed human voice and applies a mathematical rule to correct the pitch. This technology replaced slow studio techniques with a real-time process that could also be used in live performance.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate a graph to change how a piece of music sounds over time.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write a draft response describing their process, give them time to meet with 2–3 other pairs of students to give and receive feedback. Students should improve and refine their response based on the feedback they received.

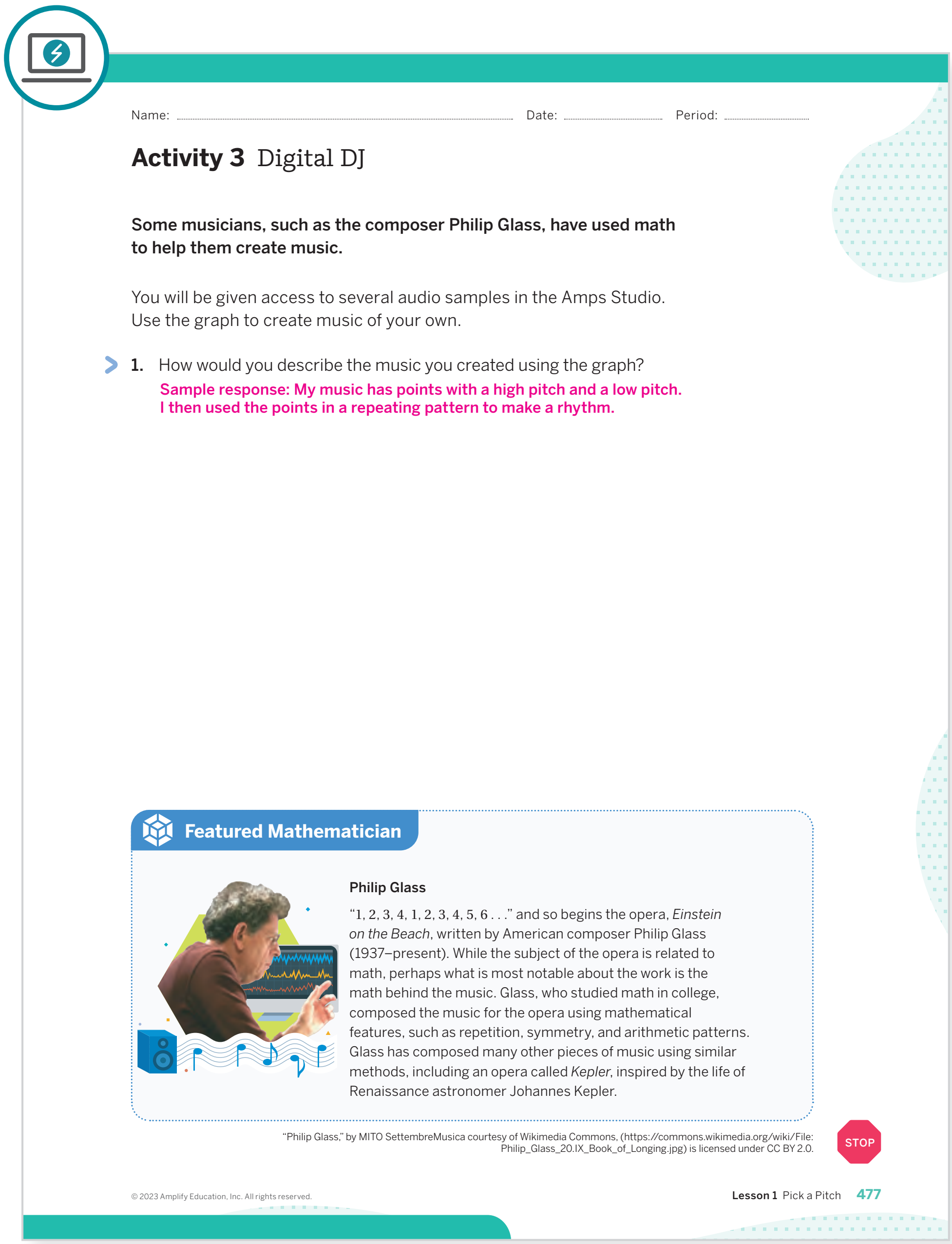
English Learners

Provide clarifying prompts, such as, "Why did you move the points on the graph?" to support students in asking questions while they give feedback.

Pairs | 10 min

Activity 3 Digital DJ

Students create their own track from scratch to apply their understanding of how to coordinate a graph with music.



Name: ______ Date: ______ Period: ______

Activity 3 Digital DJ

Some musicians, such as the composer Philip Glass, have used math to help them create music.

You will be given access to several audio samples in the Amps Studio. Use the graph to create music of your own.

1. How would you describe the music you created using the graph?
   Sample response: My music has points with a high pitch and a low pitch. I then used the points in a repeating pattern to make a rhythm.

Featured Mathematician

Philip Glass

"1, 2, 3, 4, 1, 2, 3, 4, 5, 6 . . ." and so begins the opera, *Einstein on the Beach*, written by American composer Philip Glass (1937–present). While the subject of the opera is related to math, perhaps what is most notable about the work is the math behind the music. Glass, who studied math in college, composed the music for the opera using mathematical features, such as repetition, symmetry, and arithmetic patterns. Glass has composed many other pieces of music using similar methods, including an opera called *Kepler*, inspired by the life of Renaissance astronomer Johannes Kepler.

"Philip Glass," by MITO SettembreMusica courtesy of Wikimedia Commons, (https://commons.wikimedia.org/wiki/File:Philip_Glass_20.IX_Book_of_Longing.jpg) is licensed under CC BY 2.0.

STOP

Lesson 1 Pick a Pitch 477

1 Launch

Activate students' background knowledge about how a DJ might create music from audio samples.

2 Monitor

Help students get started by showing them how to create an audio sample for a section of the graph.

Look for points of confusion:

- **Being unsure how to create a desired sound.** Ask students what it might sound like if there were symmetry in their graph or repeated patterns. Ask students what it would sound like if the pitch goes to zero.

Look for productive strategies:

- Using patterns in the graph to create an audio track.

3 Connect

Have students share how they created their audio sample with a graph. Conduct the ***Gallery Tour*** routine.

Ask:

- "What similarities do you see in the graphs of audio tracks you liked the best? What differences?"
- "Why do you think music is not best represented by the graph of one line?"

Highlight different strategies students used for creating audio samples.

Featured Mathematician

Philip Glass

Have students read about featured mathematician Philip Glass, who composed music inspired by mathematics.

Whole Class | 5 min

Summary Pumping up the Volume on Functions

Review and synthesize connections between the math and the music explored in the lesson.

Narrative Connections

Unit 5 Functions and Volume

Pumping up the Volume on Functions

When a song is in tune, somehow we just know it — no analysis necessary.

But what makes a song sound "in tune" or "out of tune?" Most of the music we listen to is built using musical scales, which themselves follow mathematical patterns. Over the years, we have become so used to these patterns that we understand them intuitively. When someone hits a wrong note, you can *feel* it without having to break out pencil and paper.

But, even if we don't notice the math when we are listening to music, math can help us describe and even modify music. Take pitch correction, for example. Used in technologies such as Auto-Tune®, pitch correction takes the notes sung by an unprocessed human voice and applies a mathematical rule to it. Each note is then adjusted to fit somewhere along a musical scale. What you end up with is a melody that is perfectly in tune every time!

But it's not just music. Much of what many people find pleasing is driven by hidden mathematical rules. Functions allow us to take what we have and change it into something new. Although we might not always notice these functions, understanding the part they play gives us options for experimenting and seeing what new outcomes might result.

Welcome to Unit 5.

478 **Unit 5** Functions and Volume

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Have students share how math and music are connected.

Ask, "What math did you see in graphs of the audio tracks that sounded the best to you?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

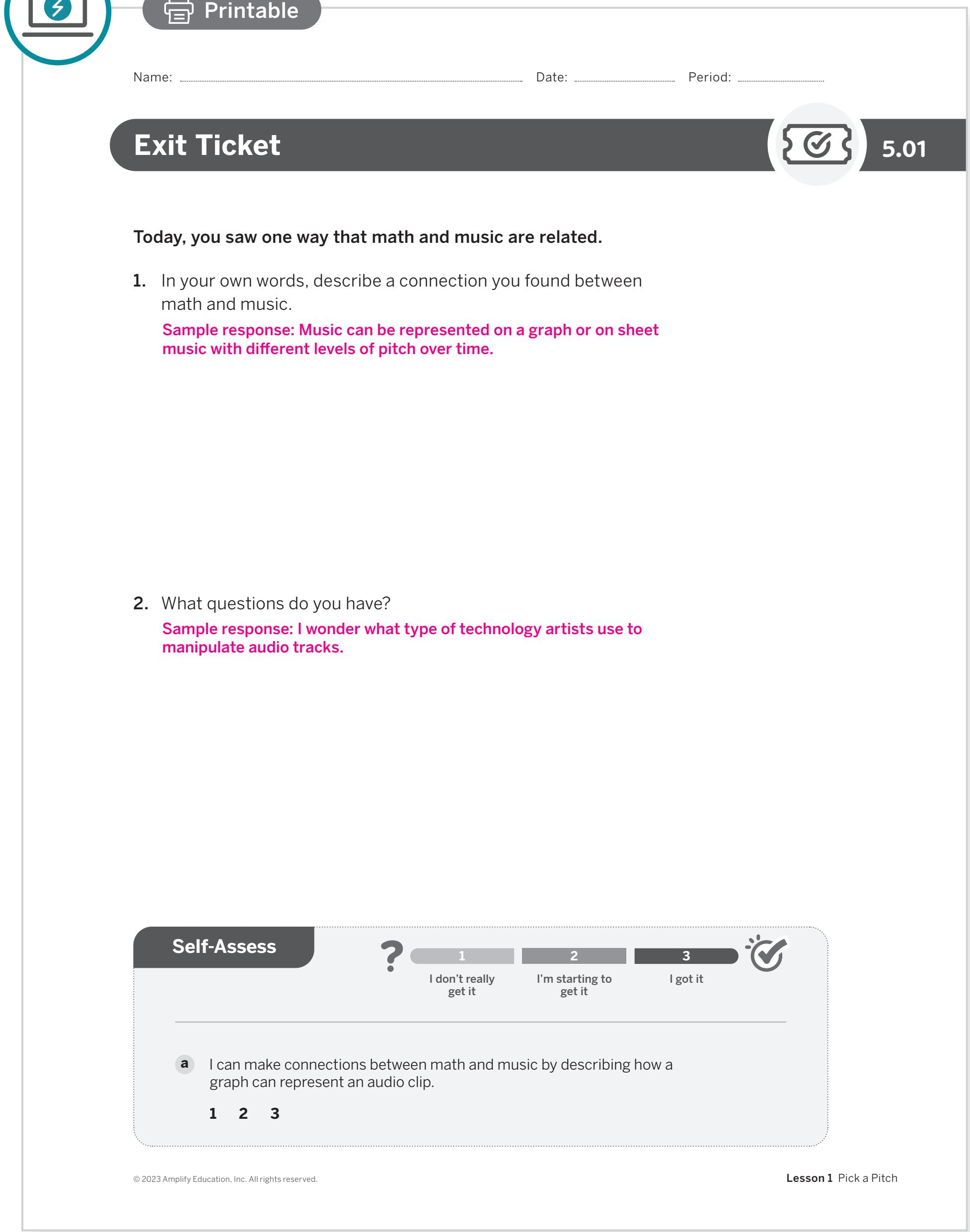
- "Were you surprised to see a connection between music and math? Why or why not?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reflecting on the math behind music.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.01

Today, you saw one way that math and music are related.

1. In your own words, describe a connection you found between math and music.
   Sample response: Music can be represented on a graph or on sheet music with different levels of pitch over time.

2. What questions do you have?
   Sample response: I wonder what type of technology artists use to manipulate audio tracks.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can make connections between math and music by describing how a graph can represent an audio clip.

1 2 3

Lesson 1 Pick a Pitch

Success looks like . . .

- **Language Goal:** Describing how music can be represented as a graph. **(Speaking and Listening, Writing)**
  - » Connecting math and music by explaining that music can be represented on a graph in Problem 1.
- **Language Goal:** Creating and modifying audio samples by manipulating a graph that represents the pitch over time. **(Speaking and Listening)**

Suggested next steps

If students are unsure what to say, consider:

- Reviewing Activity 1 and asking students what connections they noticed between the graph and the music.

If students are too vague in their responses, consider:

- Encouraging them to be more specific about what they learned in Activities 1, 2, or 3.

Professional Learning

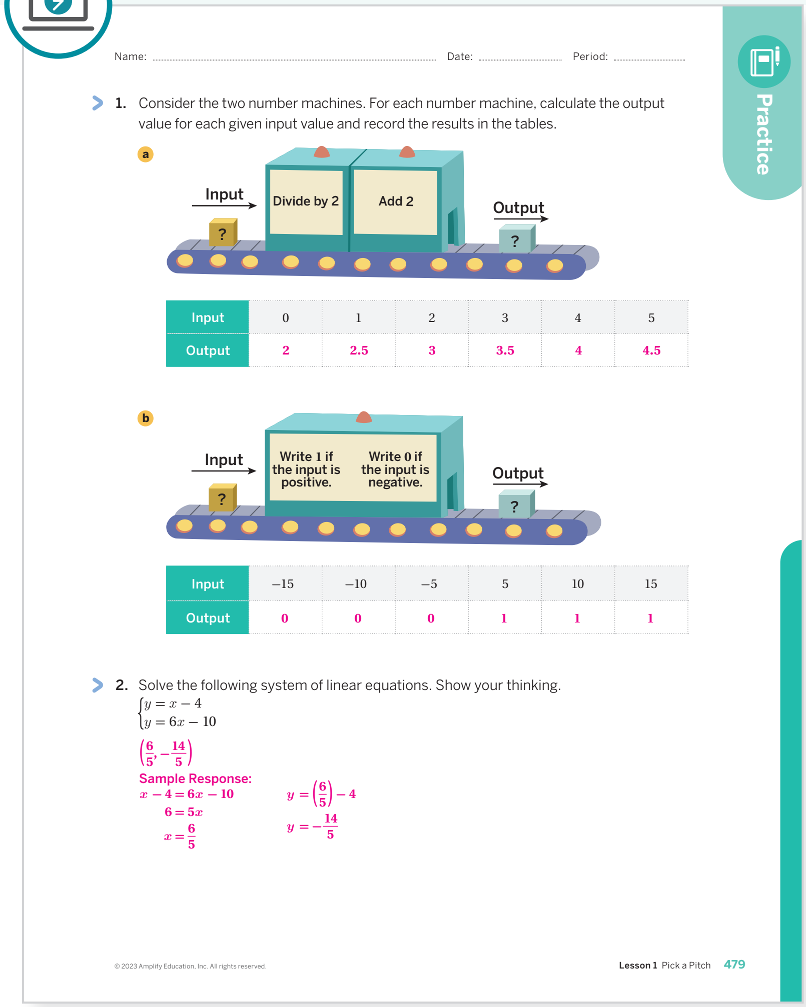
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Knowing where students need to be by the end of this unit, how did today's lesson influence that future goal?
- In what ways did these activities go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



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1. Consider the two number machines. For each number machine, calculate the output value for each given input value and record the results in the tables.

a

Input	0	1	2	3	4	5
Output	2	2.5	3	3.5	4	4.5

b

Input	-15	-10	-5	5	10	15
Output	0	0	0	1	1	1

2. Solve the following system of linear equations. Show your thinking.

$$\begin{cases} y = x - 4 \\ y = 6x - 10 \end{cases}$$

$\left(\frac{6}{5}, -\frac{14}{5}\right)$

Sample Response:

$x - 4 = 6x - 10$

$6 = 5x$

$x = \frac{6}{5}$

$y = \left(\frac{6}{5}\right) - 4$

$y = -\frac{14}{5}$

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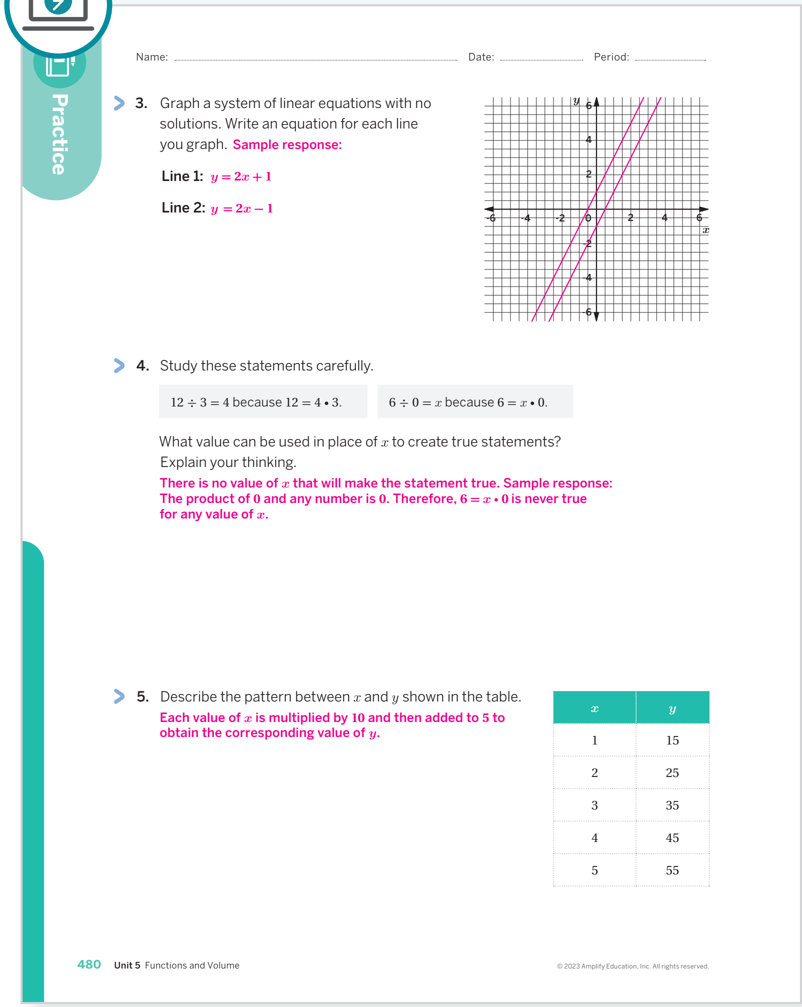
Lesson 1 Pick a Pitch 479



Name: ______ Date: ______ Period: ______

3. Graph a system of linear equations with no solutions. Write an equation for each line you graph. Sample response:

Line 1: $y = 2x + 1$

Line 2: $y = 2x - 1$

4. Study these statements carefully.

$12 \div 3 = 4$ because $12 = 4 \cdot 3$.

$6 \div 0 = x$ because $6 = x \cdot 0$.

What value can be used in place of x to create true statements? Explain your thinking.

There is no value of x that will make the statement true. Sample response: The product of 0 and any number is 0. Therefore, $6 = x \cdot 0$ is never true for any value of x.

5. Describe the pattern between x and y shown in the table.

Each value of x is multiplied by 10 and then added to 5 to obtain the corresponding value of y.

x	y
1	15
2	25
3	35
4	45
5	55

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Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 4 Lesson 1	2
	2	Unit 4 Lesson 15	1
	3	Unit 4 Lesson 12	2
	4	Grade 3	2
Formative ⚡	5	Unit 5 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Sub-Unit 1

Representing and Interpreting Functions

Whole Class

In this Sub-Unit, students are introduced to the term *function*. They build their understanding of functions by studying functions expressed through multiple representations.

SUB-UNIT

1 Representing and Interpreting Functions

Narrative Connections

Who has the better camera: you or your grandparents?

These days, most people can pop out their phone and snap a selfie. But thirty years ago, things weren't so simple. To capture that perfect moment, you needed an actual, honest-to-goodness analog camera. Digital cameras (like those built into smartphones) and analog cameras (which use film) might produce similar images, but differences in *how they work* affect the kind of image you end up with.

Let's start with an analog camera. When you take a picture, light bounces off what you are looking at. The light then travels through the camera's lens, and hits tiny crystals of silver halide in your film. These crystals undergo a chemical reaction, creating a "latent image," which is later treated with chemicals to produce the photograph.

Digital cameras work differently. Rather than hitting film, the light from your subject hits a tiny grid of millions of photosites. A sensor measures the color and brightness of each photosite and stores that information as a number. This process is called "sampling." These numbers are later recombined to create the image.

There are pros and cons to both methods. Sampling is less precise than film. You end up with an image that is not as sharp, and that does not have the same range of colors. But compared to analog photography, digital photography is more efficient. By turning a picture into numbers, the information can be squeezed down to a tiny file size, making it easier to share.

Whatever your preference is, both analog and digital photography are examples of functions. They take raw information — the light bouncing off your subject — and convert it into something you can keep and carry with you.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore relationships in real-world contexts that are examples of functions in the following places:

- **Lesson 2, Activity 2:** Is It a Function?
- **Lesson 3, Activity 2:** Apples and Oranges
- **Lesson 4, Activity 1:** Turtle Crossing
- **Lesson 5, Activities 1-2:** Time and Temperature, Highs and Lows
- **Lesson 7, Activities 1-2:** Junior Olympics, Comparing Volumes
- **Lesson 8, Activities 1-2:** Which Is Growing Faster?, Is It Charging or Losing Charge?
- **Lesson 9, Activities 1-2:** Charging a Phone, Charging a Laptop
- **Lesson 10, Activities 1-2:** The Tortoise and the Hare . . . and the Fox, The Tortoise and the Dog

UNIT 5 | **LESSON 2**

Introduction to Functions

Let's explore the concept of a function.

Focus

Goals

1. **Language Goal:** Identify rules that produce exactly one output for each allowable input and rules that do not. **(Speaking and Listening)**
2. Comprehend the structure of a function as having one and only one output for each allowable input.
3. **Language Goal:** Describe a context using function language. For example, "the [output] is a function of the [input]," or "the [output] depends on the [input]." **(Speaking and Listening, Writing)**

Coherence

Today

Students look for and make use of structure to identify a rule that describes the relationship between an input-output pair. They learn that a *function* is a rule that assigns exactly one output to each possible input and start to use function language to describe a context.

Previously

In Lesson 1, students began exploring inputs and outputs using the context of music.

Coming Soon

In Lesson 3, students will transition from input-output tables to equations of functions. They will be reintroduced to *independent* and *dependent variables* and explore how it is sometimes possible to write either variable as a function of the other.

Rigor

- Students build **conceptual understanding** of what it means for a relationship to be a function.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	20 min	12 min	5 min	5 min
Whole Class	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per group

Math Language Development

New word

- ***function***

Review words

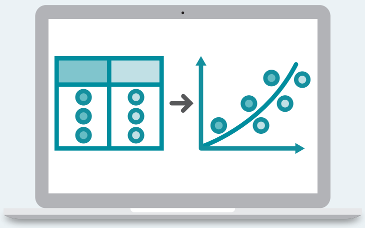
- *input*
- *output*

Amps | **Featured Activity**

Warm-up

Testing Inputs

Students test input values by entering them into a table and use the output values that are automatically generated to guess the rule.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may be confused as to why their rules in Activity 1 are not the same as their classmates'. Encourage students to step back and take a different view of each pattern. While they might immediately see one pattern, their classmates might view the structure differently. By sharing their results, students can take on multiple different and correct views of the same problem.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

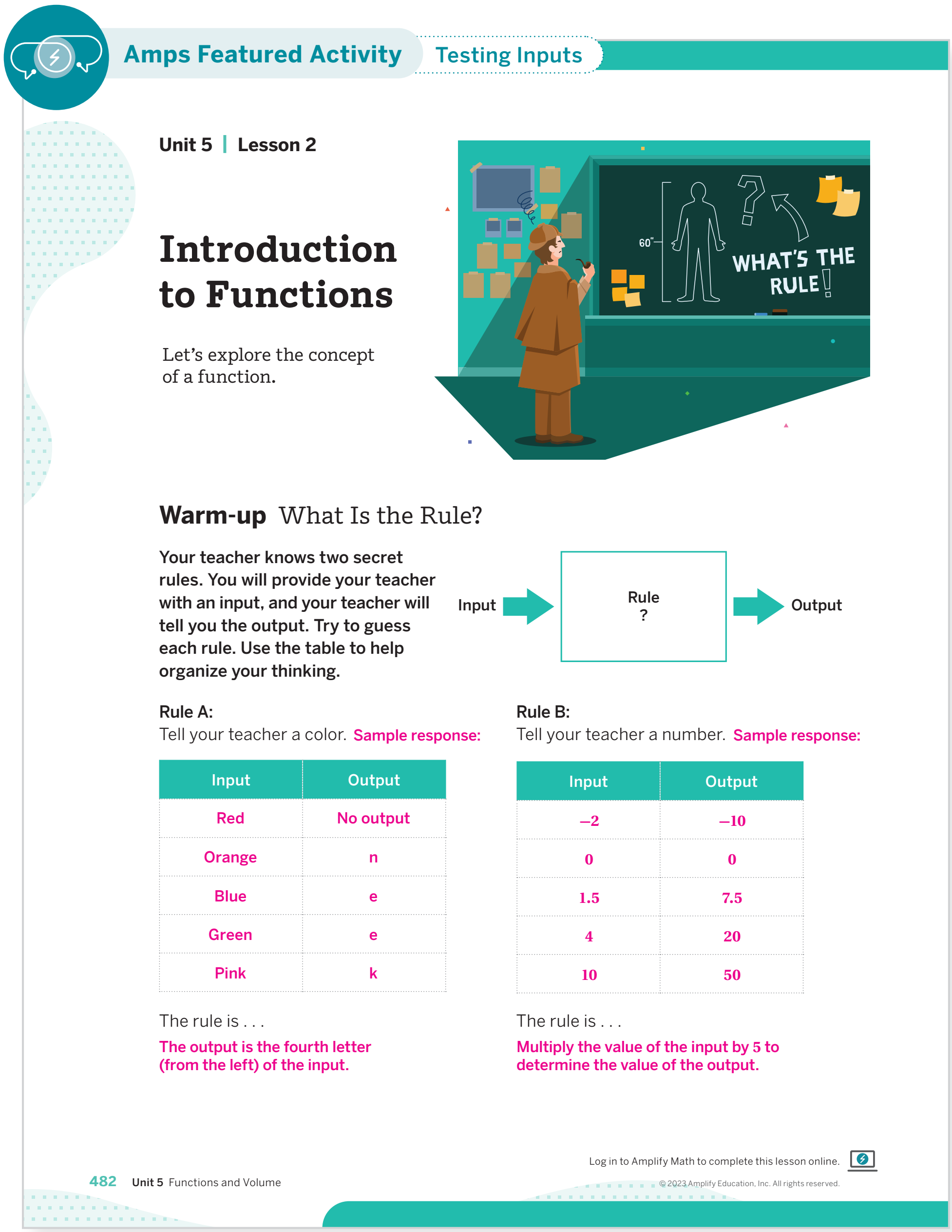
- In the **Warm-up**, complete only Rule B.
- In **Activity 2**, omit Problems 3 and 4.

Whole Class | 8 min

Warm-up What Is the Rule?

Students guess a rule as an introduction to the idea of *input-output* rules.

Amps Featured Activity Testing Inputs



Unit 5 | Lesson 2

Introduction to Functions

Let's explore the concept of a function.

Warm-up What Is the Rule?

Your teacher knows two secret rules. You will provide your teacher with an input, and your teacher will tell you the output. Try to guess each rule. Use the table to help organize your thinking.

Input → Rule ? → Output

Rule A:
Tell your teacher a color. Sample response:

Input	Output
Red	No output
Orange	n
Blue	e
Green	e
Pink	k

The rule is . . .
The output is the fourth letter (from the left) of the input.

Rule B:
Tell your teacher a number. Sample response:

Input	Output
-2	-10
0	0
1.5	7.5
4	20
10	50

The rule is . . .
Multiply the value of the input by 5 to determine the value of the output.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Display the Warm-up from the Student Edition. For each rule, ask students to provide an input. Apply the rule described below and respond to students with the output. Then have students record the given input and output in the table. Repeat the process until students can correctly guess the rule.

- **Rule A:** Write the fourth letter of the input word. **Note:** Provide the output "no output" if a color is less than four letters.
- **Rule B:** Multiply the input by 5.

2 Monitor

Help students get started by having them look for a pattern between each input-output pair.

Look for points of confusion:

- **Not understanding when there is no output for Rule A.** Tell students that the rule does not work for this color.

3 Connect

Have students share their strategies for guessing the rule. Highlight any strategies that may help students in Activity 1.

Ask:

- "For each rule, can the rule be applied for every input?" Rule A: No, Rule B: Yes
- "For each rule, can there be a different output for the same input?" Rule A: No, Rule B: No

Highlight that for an input-output rule, students start with a number or word, called the *input*, and apply a rule to the input which results in a number or word, called the *output*. The output *corresponds* to the input. For some inputs, there might not be an output.

Power-up

To power up students' ability to describe a pattern in a table of values, have students complete:

1. Which statement describes the pattern in the table?

x	1	2	3
y	20	25	30

   A. Add 5 to x.
   B. Add 5 to x then multiply by 15.
   [✓ C.] Multiply x by 5 then add 15.
   D. Multiply x by 15 then add 5.

2. Determine the next value in the table.
   When $x = 4$, $y =$ 35

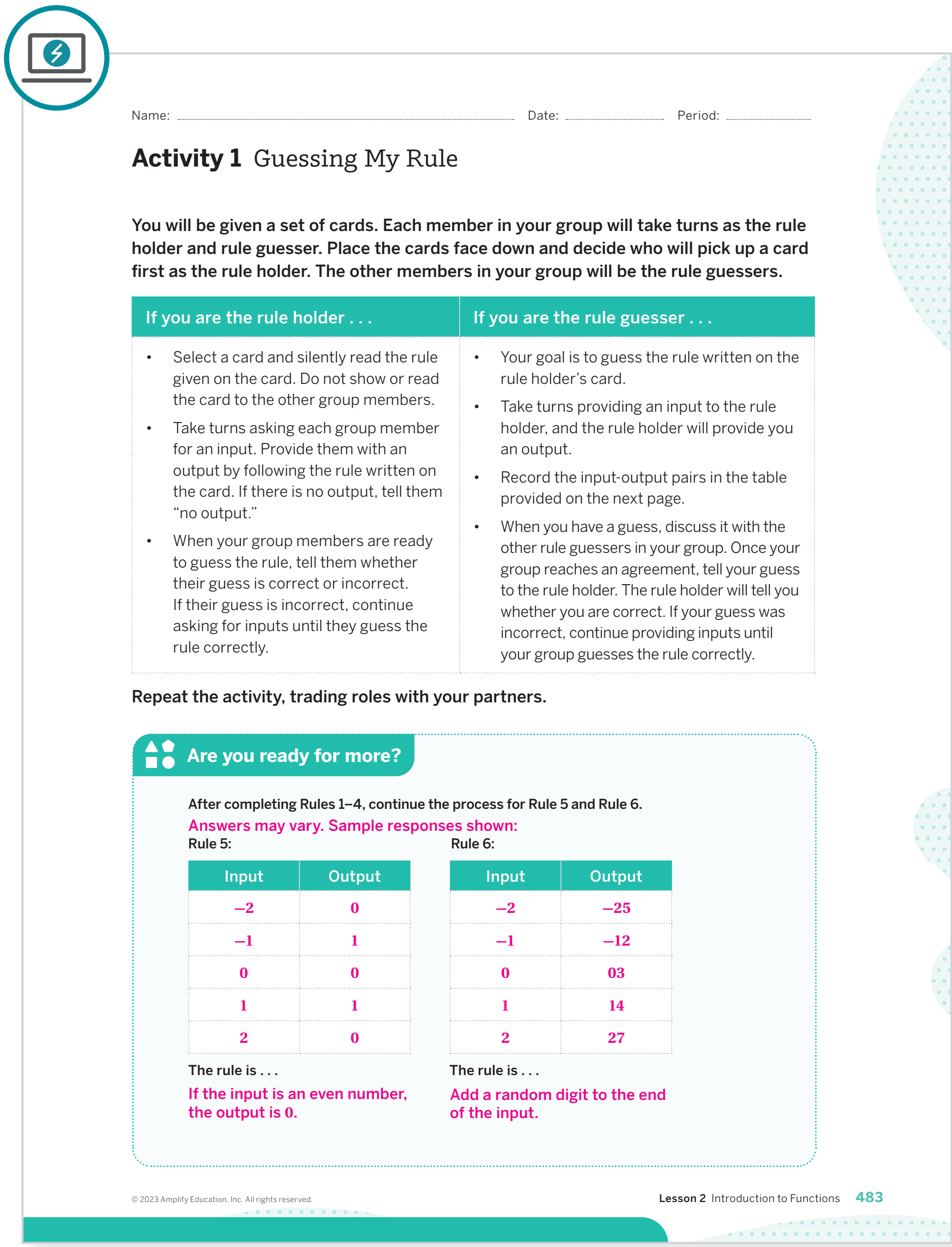
Use: Before the Warm-up

Informed by: Performance on Lesson 1, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 2

Small Groups | 20 min

Activity 1 Guessing My Rule

Students take turns guessing input and output rules to explore the concept of a function.



Name: Date: Period:

Activity 1 Guessing My Rule

You will be given a set of cards. Each member in your group will take turns as the rule holder and rule guesser. Place the cards face down and decide who will pick up a card first as the rule holder. The other members in your group will be the rule guessers.

If you are the rule holder . . .	If you are the rule guesser . . .
• Select a card and silently read the rule given on the card. Do not show or read the card to the other group members. • Take turns asking each group member for an input. Provide them with an output by following the rule written on the card. If there is no output, tell them "no output." • When your group members are ready to guess the rule, tell them whether their guess is correct or incorrect. If their guess is incorrect, continue asking for inputs until they guess the rule correctly.	• Your goal is to guess the rule written on the rule holder's card. • Take turns providing an input to the rule holder, and the rule holder will provide you an output. • Record the input-output pairs in the table provided on the next page. • When you have a guess, discuss it with the other rule guessers in your group. Once your group reaches an agreement, tell your guess to the rule holder. The rule holder will tell you whether you are correct. If your guess was incorrect, continue providing inputs until your group guesses the rule correctly.

Repeat the activity, trading roles with your partners.

Are you ready for more?

After completing Rules 1–4, continue the process for Rule 5 and Rule 6.

Answers may vary. Sample responses shown:

Rule 5:

Input	Output
–2	0
–1	1
0	0
1	1
2	0

The rule is . . .

If the input is an even number, the output is 0.

Rule 6:

Input	Output
–2	–25
–1	–12
0	03
1	14
2	27

The rule is . . .

Add a random digit to the end of the input.

Lesson 2 Introduction to Functions 483

1 Launch

Distribute the pre-cut cards from the Activity 1 PDF to each group. Have students place the cards face down, and then take turns as the rule holder and rule guesser. Remind students to record their input-output pairs in the tables. Have students add more rows in the tables if needed.

2 Monitor

Help students get started by encouraging them to provide strategic inputs to guess the rule.

Look for points of confusion:

- **Struggling to guess the rule.** Provide students with a specific strategy. For example, have students start with 0 then choose a sequence of consecutive whole numbers for the input.
- **Guessing a different, but correct rule. For example, for Rule 3, students may guess the rule "adding the opposite."** Tell the students that although the rule is correct, it is not the rule written on the card. Revisit these students during the whole-class discussion.

Look for productive strategies:

- Noticing that the output may be the same for different inputs.
- Noticing that the same input may produce a different output.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider demonstrating the activity by using the rule described on the card for Rule 1. Ask three student volunteers to provide a letter for the input. After you have provided the output for each input, ask the class if anyone can guess the rule.

Extension: *Math Enrichment*

Ask students to describe their own rule that satisfies each condition.

- A rule in which there are different outputs for the same input.
- A rule in which there are different inputs that have the same output.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you define the term *function*, draw students' attention to the connections between the functions (Rules 2, 3, 4, and 5) and the tables that do not represent functions (Rules 1 and 6). Ask:

- "Why is Rule 3 a function when every single output is the same?"
- "Give an example of two input-output pairs that show why Rule 1 is not a function."

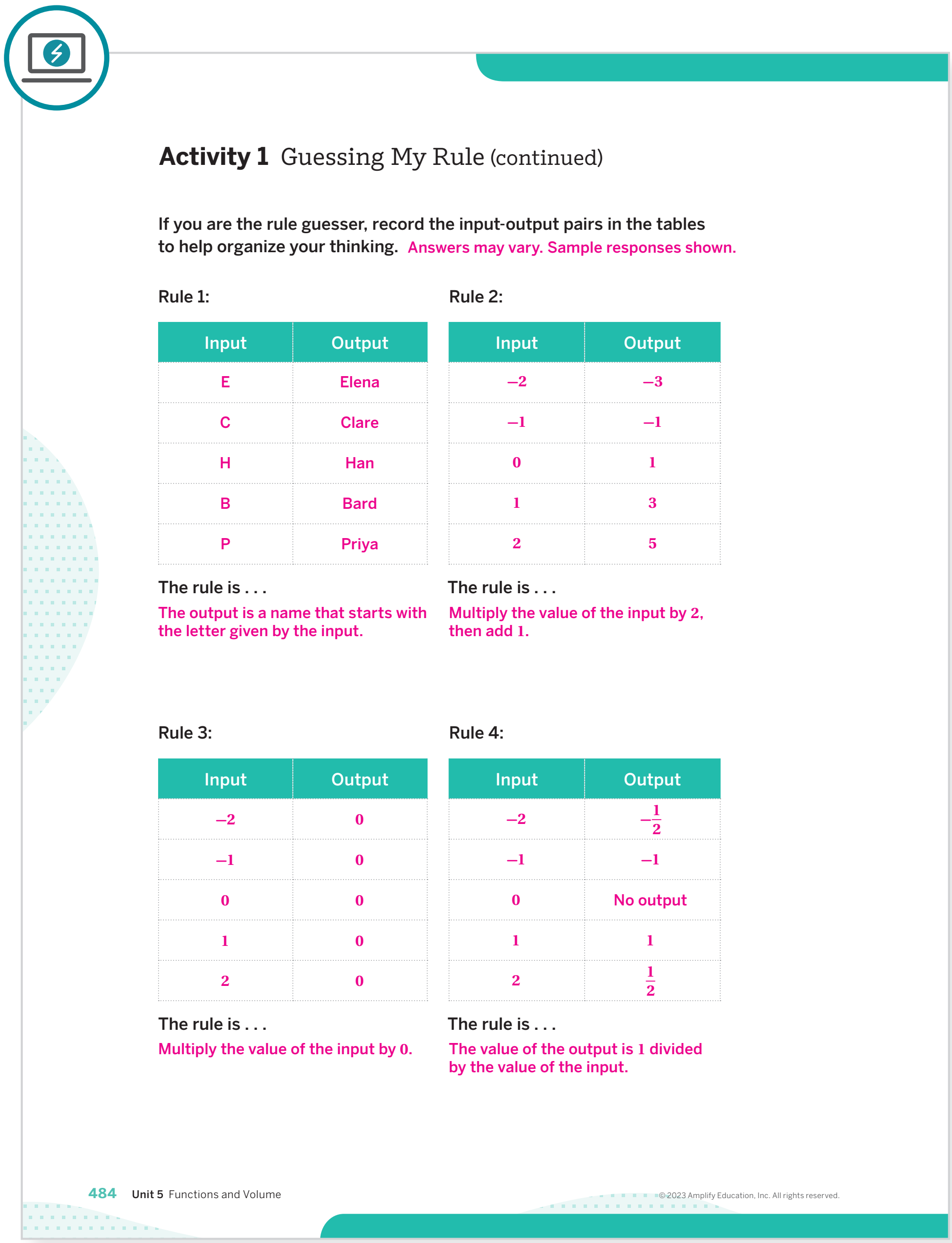
English Learners

Annotate the tables by writing *function* or *not a function* and highlight any input-output pairs that indicate the rule is not a function.

Small Groups | 20 min

Activity 1 Guessing My Rule (continued)

Students take turns guessing input and output rules to explore the concept of a function.



Activity 1 Guessing My Rule (continued)

If you are the rule guesser, record the input-output pairs in the tables to help organize your thinking. Answers may vary. Sample responses shown.

Rule 1:

Input	Output
E	Elena
C	Clare
H	Han
B	Bard
P	Priya

The rule is . . .

The output is a name that starts with the letter given by the input.

Rule 2:

Input	Output
-2	-3
-1	-1
0	1
1	3
2	5

The rule is . . .

Multiply the value of the input by 2, then add 1.

Rule 3:

Input	Output
-2	0
-1	0
0	0
1	0
2	0

The rule is . . .

Multiply the value of the input by 0.

Rule 4:

Input	Output
-2	$-\frac{1}{2}$
-1	-1
0	No output
1	1
2	$\frac{1}{2}$

The rule is . . .

The value of the output is 1 divided by the value of the input.

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3 Connect

Display the cards from the Activity 1 PDF.

Have groups of students share their strategies for guessing each rule. Select previously identified students who appeared to have a specific strategy for determining a rule. Sequence students starting with the most common strategies to the least. Make connections between the successful aspects of each strategy.

Ask:

- "Can an input-output table be represented by different rules?" Yes. Point out Rule 3. Tell students that the value of the input could be multiplied by 0 or added to its opposite to produce the value of the output.
- "Does a rule always provide different outputs?" No. Point out Rule 3 and Rule 5.
- "Can a rule have the same input, but provide a different output?" Yes. Point out Rule 1 and Rule 6.
- "Does an input always provide an output?" No. Point out Rule 4 and Rule 5.

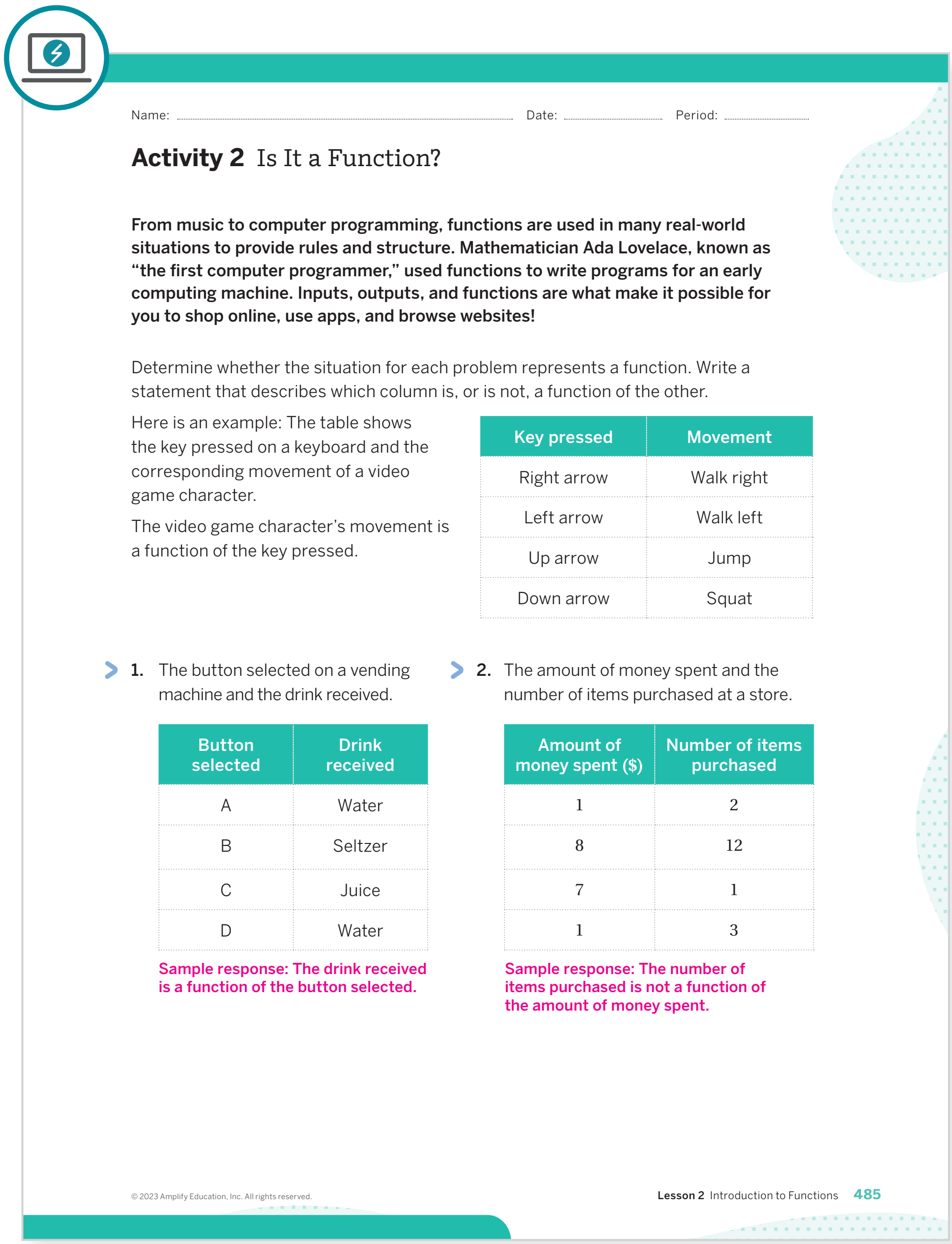
Define the term ***function*** as a rule that assigns exactly one output to each possible input.

Highlight that functions are special types of rules in which each input has only one possible output. Highlight that Rules 1 and 6 do not represent a function because the same input can produce more than one output. For Rule 1, the input "A" could produce the output "Andre" or "Ashley." For Rule 6, the input 1 could produce the output 13 or 15. Highlight that the remaining rules represent a function because there is exactly one output for each possible input. Check to make sure that students understand why Rules 3 and 5 represent a function.

Pairs | 12 min

Activity 2 Is It a Function?

Students determine whether a statement describes a function to develop their understanding of the structure of a function and to use the language of functions.



Name: ______________ Date: ________ Period: ________

Activity 2 Is It a Function?

From music to computer programming, functions are used in many real-world situations to provide rules and structure. Mathematician Ada Lovelace, known as "the first computer programmer," used functions to write programs for an early computing machine. Inputs, outputs, and functions are what make it possible for you to shop online, use apps, and browse websites!

Determine whether the situation for each problem represents a function. Write a statement that describes which column is, or is not, a function of the other.

Here is an example: The table shows the key pressed on a keyboard and the corresponding movement of a video game character.

The video game character's movement is a function of the key pressed.

Key pressed	Movement
Right arrow	Walk right
Left arrow	Walk left
Up arrow	Jump
Down arrow	Squat

1. The button selected on a vending machine and the drink received.

Button selected	Drink received
A	Water
B	Seltzer
C	Juice
D	Water

Sample response: The drink received is a function of the button selected.

2. The amount of money spent and the number of items purchased at a store.

Amount of money spent ($)	Number of items purchased
1	2
8	12
7	1
1	3

Sample response: The number of items purchased is not a function of the amount of money spent.

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Lesson 2 Introduction to Functions 485

1 Launch

Review the prompt with the class. Activate students' background knowledge by asking, "What do you know about coding and computer programming?" Sample response: Computer coding is the use of computer programming languages to give computers and machines a set of instructions on what actions to perform. Use the example to ensure they understand how to describe if a variable is, or is not, a function of the other.

2 Monitor

Help students get started by telling students that the input is given on the left side of the table and the output is given on the right side of the table.

Look for points of confusion:

- **Thinking that Problem 1 or 4 does not represent a function because the same output is given twice.** Ask students if they can determine the output if the input is given. Remind students that the output can be listed more than once.
- **Thinking that Problem 2 represents a function because the number of items purchased *depends* on the amount of money spent.** Tell students that, although they may be able to make generalizations about a situation, because it is not true for every input, it does not represent a function.

Look for productive strategies:

- Looking for the same values under the input column.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Display the following sentence frames for students to use as they complete the activity. If students need more processing time, have them focus on completing Problems 1 and 2.

- "_____ is a function of _____."
- "_____ is not a function of _____."

Extension: *Math Enrichment*

Ask students to select one table that is a function and add an input-output pair as a new row so that the table is no longer a function. Have them explain their thinking.

Math Language Development

MLR2: Collect and Display

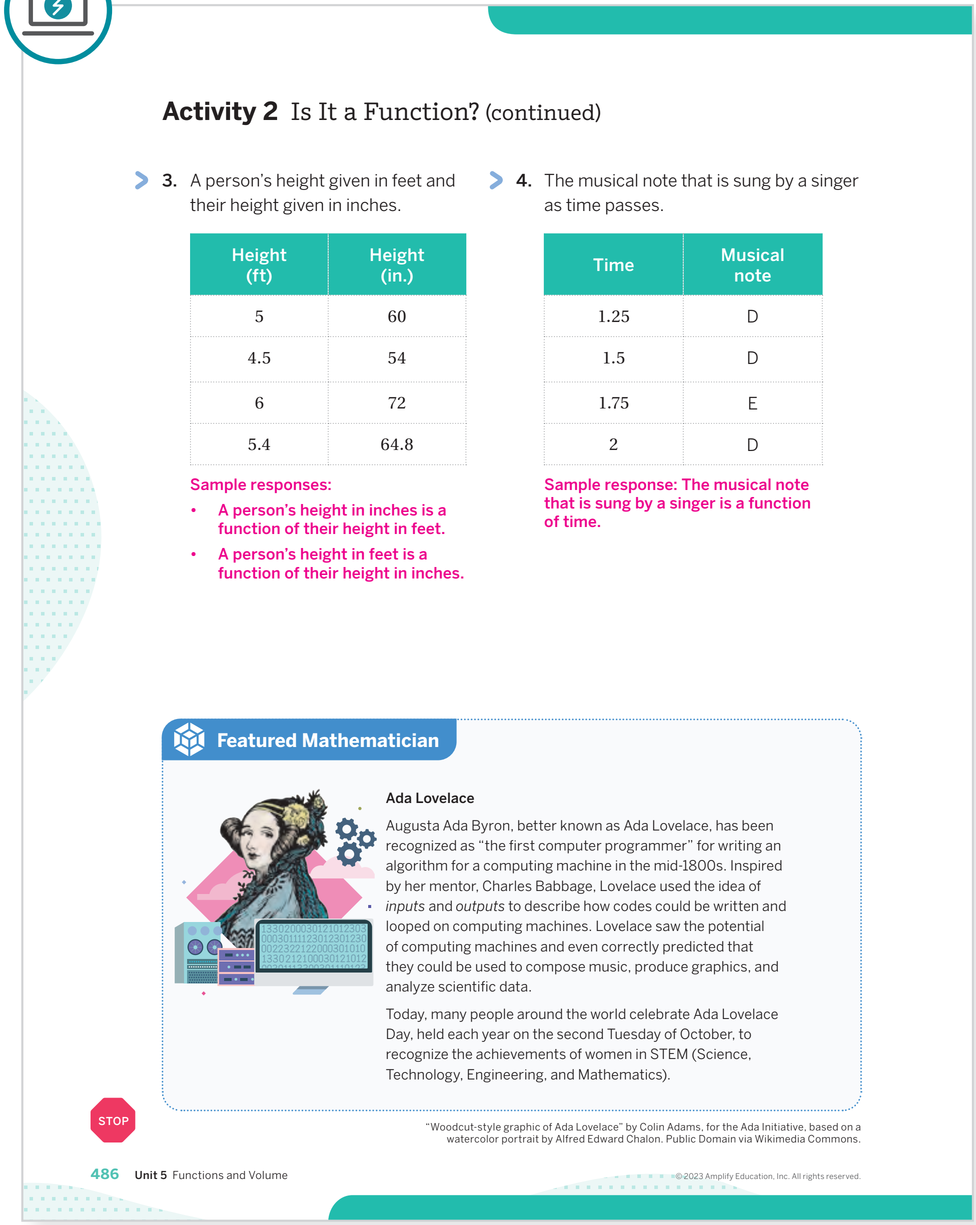
While students work, circulate and listen for the language they use to Listen and record the language students use to determine whether one quantity is, or is not, a function of the other quantity. For example, they may say, "The output can be listed more than once as long as it has different inputs" or "The same input can't have more than one output."

Display the language collected for the whole class to use as a reference throughout the lesson and unit. Invite students to suggest revisions, updates, and connections to the display as they develop new ideas about functions.

Pairs | 12 min

Activity 2 Is It a Function? (continued)

Students determine whether a statement describes a function to develop their understanding of the structure of a function and to use the language of functions.



Activity 2 Is It a Function? (continued)

3. A person's height given in feet and their height given in inches.

Height (ft)	Height (in.)
5	60
4.5	54
6	72
5.4	64.8

Sample responses:

- A person's height in inches is a function of their height in feet.
- A person's height in feet is a function of their height in inches.

4. The musical note that is sung by a singer as time passes.

Time	Musical note
1.25	D
1.5	D
1.75	E
2	D

Sample response: The musical note that is sung by a singer is a function of time.

Featured Mathematician

Ada Lovelace

Augusta Ada Byron, better known as Ada Lovelace, has been recognized as "the first computer programmer" for writing an algorithm for a computing machine in the mid-1800s. Inspired by her mentor, Charles Babbage, Lovelace used the idea of *inputs* and *outputs* to describe how codes could be written and looped on computing machines. Lovelace saw the potential of computing machines and even correctly predicted that they could be used to compose music, produce graphics, and analyze scientific data.

Today, many people around the world celebrate Ada Lovelace Day, held each year on the second Tuesday of October, to recognize the achievements of women in STEM (Science, Technology, Engineering, and Mathematics).

STOP

"Woodcut-style graphic of Ada Lovelace" by Colin Adams, for the Ada Initiative, based on a watercolor portrait by Alfred Edward Chalon. Public Domain via Wikimedia Commons.

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3 Connect

Have pairs of students share their responses. Use the ***Poll the Class*** routine to determine which students thought each problem represented a function. Have students share their thinking and discuss any disagreements. Encourage students to use language such as, "The input does not determine the output because . . ."

Highlight that functions help provide rules and structure to people's lives.

Ask, "Can you think of other real-world examples that represent a function?"
Sample response: A function that has a person's Social Security number as input and the name of the person as output.

Featured Mathematician

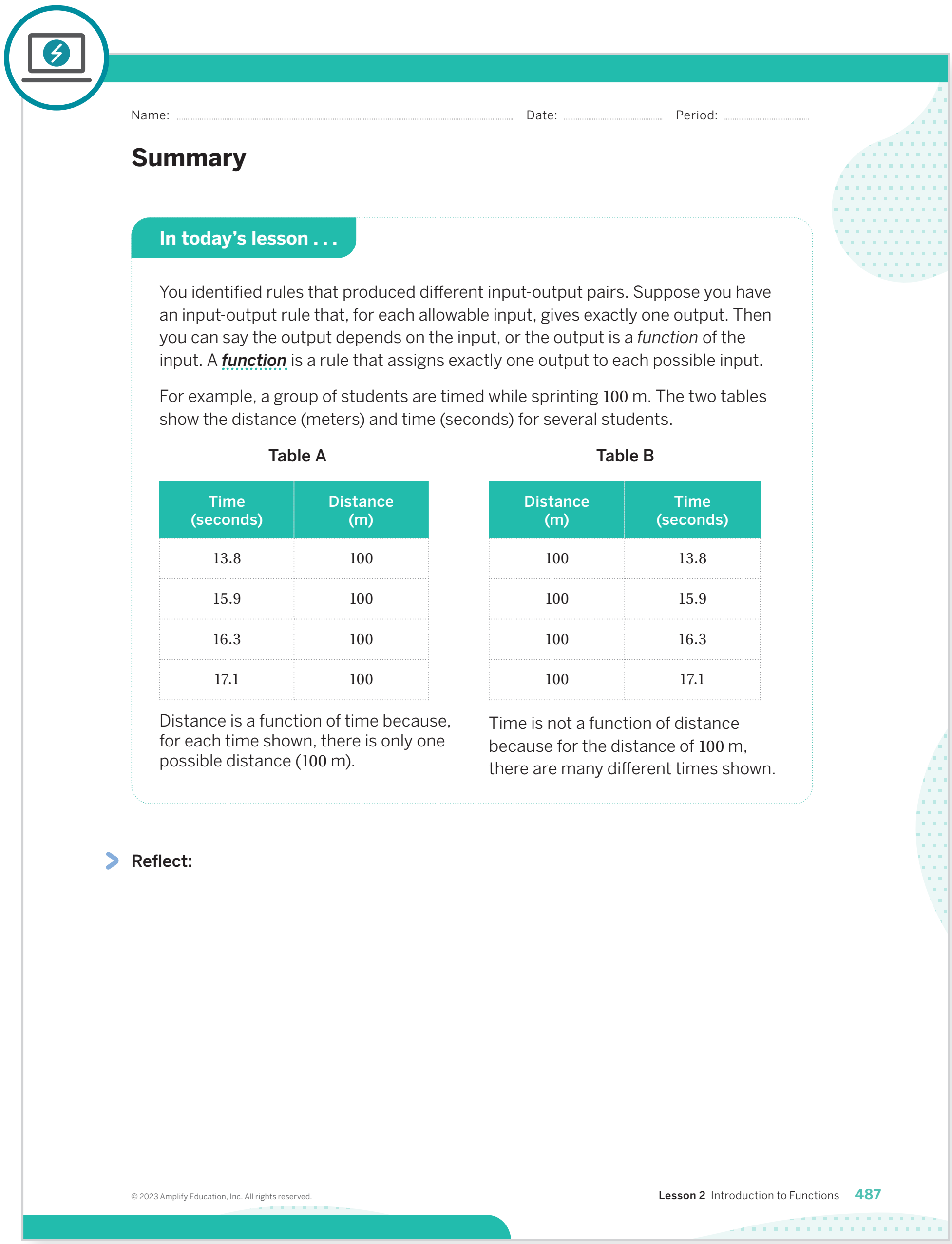
Ada Lovelace

Have students read about featured mathematician Ada Lovelace, who is recognized by many as "the first computer programmer."

Whole Class | 5 min

Summary

Review and synthesize the concept and definition of a function.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You identified rules that produced different input-output pairs. Suppose you have an input-output rule that, for each allowable input, gives exactly one output. Then you can say the output depends on the input, or the output is a *function* of the input. A ***function*** is a rule that assigns exactly one output to each possible input.

For example, a group of students are timed while sprinting 100 m. The two tables show the distance (meters) and time (seconds) for several students.

Table A

Time (seconds)	Distance (m)
13.8	100
15.9	100
16.3	100
17.1	100

Distance is a function of time because, for each time shown, there is only one possible distance (100 m).

Table B

Distance (m)	Time (seconds)
100	13.8
100	15.9
100	16.3
100	17.1

Time is not a function of distance because for the distance of 100 m, there are many different times shown.

Reflect:

Synthesize

Have students share how they can identify a function from a table or description.

Highlight that a function is a rule that assigns exactly one output to each allowable input. Students may use phrases, such as "the output is a function of the input," or "the output depends on the input," when talking about the relationship between inputs and outputs of functions.

Formalize vocabulary: *function*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What makes a relationship a function?"

Math Language Development

MLR2: Collect and Display

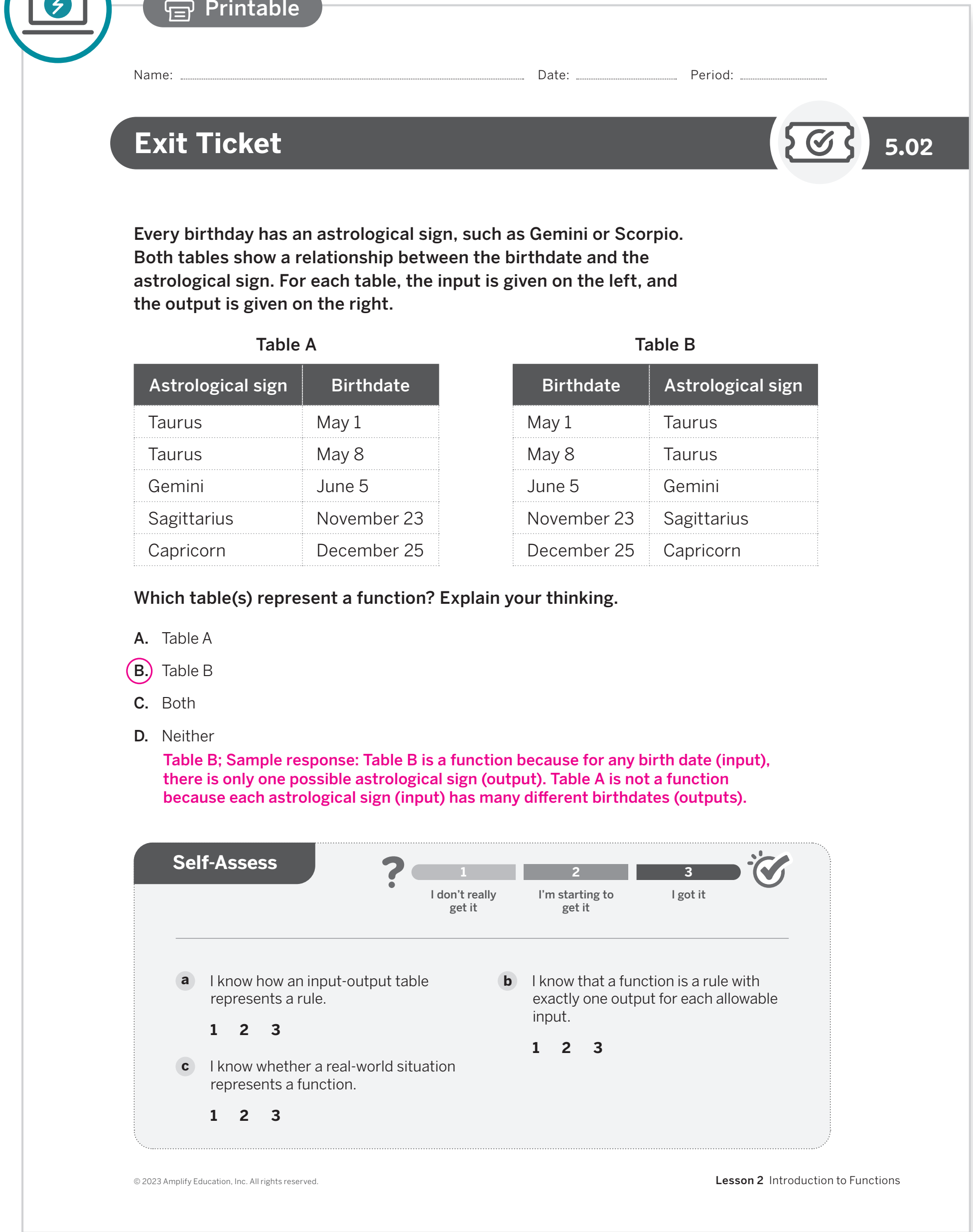
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *function* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying a function given a table.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket

5.02

Every birthday has an astrological sign, such as Gemini or Scorpio. Both tables show a relationship between the birthdate and the astrological sign. For each table, the input is given on the left, and the output is given on the right.

Table A

Astrological sign	Birthdate
Taurus	May 1
Taurus	May 8
Gemini	June 5
Sagittarius	November 23
Capricorn	December 25

Table B

Birthdate	Astrological sign
May 1	Taurus
May 8	Taurus
June 5	Gemini
November 23	Sagittarius
December 25	Capricorn

Which table(s) represent a function? Explain your thinking.

A. Table A

[✓ B.] Table B

C. Both

D. Neither

Table B; Sample response: Table B is a function because for any birth date (input), there is only one possible astrological sign (output). Table A is not a function because each astrological sign (input) has many different birthdates (outputs).

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I know how an input-output table represents a rule.

1 2 3

b I know that a function is a rule with exactly one output for each allowable input.

1 2 3

c I know whether a real-world situation represents a function.

1 2 3

Lesson 2 Introduction to Functions

Success looks like . . .

- **Language Goal:** Identifying rules that produce exactly one output for each allowable input and rules that do not. **(Speaking and Listening)**
  - » Selecting the Table B as representing a function.
- **Goal:** Comprehending the structure of a function as having one and only one output for each allowable input.
  - » Understanding that each birthdate has only one astrological sign.
- **Language Goal:** Describing a context using function language. For example, "the [output] is a function of the [input]," or "the [output] depends on the [input]." **(Speaking and Listening, Writing)**
  - » Using function language to explain their thinking and defend their selection as to which relationship is a function.

Suggested next steps

If students choose A, C, or D consider:

- Reviewing how students can identify a function from a table.
- Assigning Practice Problem 1.
- Asking, "Are there two or more of the same input?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What was especially satisfying about Activity 1?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?

Math Language Development

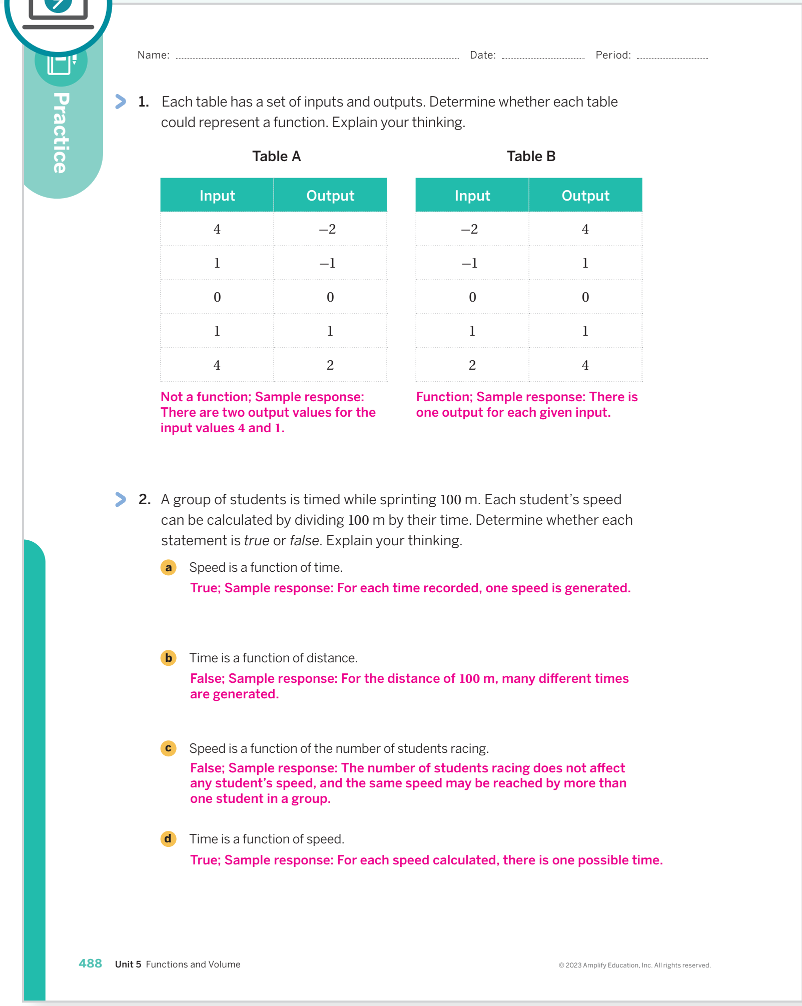
Language Goal: Describing the movement of figures informally and formally using the terms *clockwise*, *counterclockwise*, *translation*, and *rotation*.

Reflect on students' language development toward this goal.

- How did students begin to informally describe the movement of figures in this lesson? What language did they use?
- How has their use of language progressed after being introduced to the terms *clockwise*, *counterclockwise*, *translation*, and *rotation*? How can you support them in using their developing math language?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Each table has a set of inputs and outputs. Determine whether each table could represent a function. Explain your thinking.

Table A

Input	Output
4	−2
1	−1
0	0
1	1
4	2

Not a function; Sample response: There are two output values for the input values 4 and 1.

Table B

Input	Output
−2	4
−1	1
0	0
1	1
2	4

Function; Sample response: There is one output for each given input.

2. A group of students is timed while sprinting 100 m. Each student's speed can be calculated by dividing 100 m by their time. Determine whether each statement is *true* or *false*. Explain your thinking.

   a. Speed is a function of time.
   True; Sample response: For each time recorded, one speed is generated.

   b. Time is a function of distance.
   False; Sample response: For the distance of 100 m, many different times are generated.

   c. Speed is a function of the number of students racing.
   False; Sample response: The number of students racing does not affect any student's speed, and the same speed may be reached by more than one student in a group.

   d. Time is a function of speed.
   True; Sample response: For each speed calculated, there is one possible time.

488 **Unit 5** Functions and Volume

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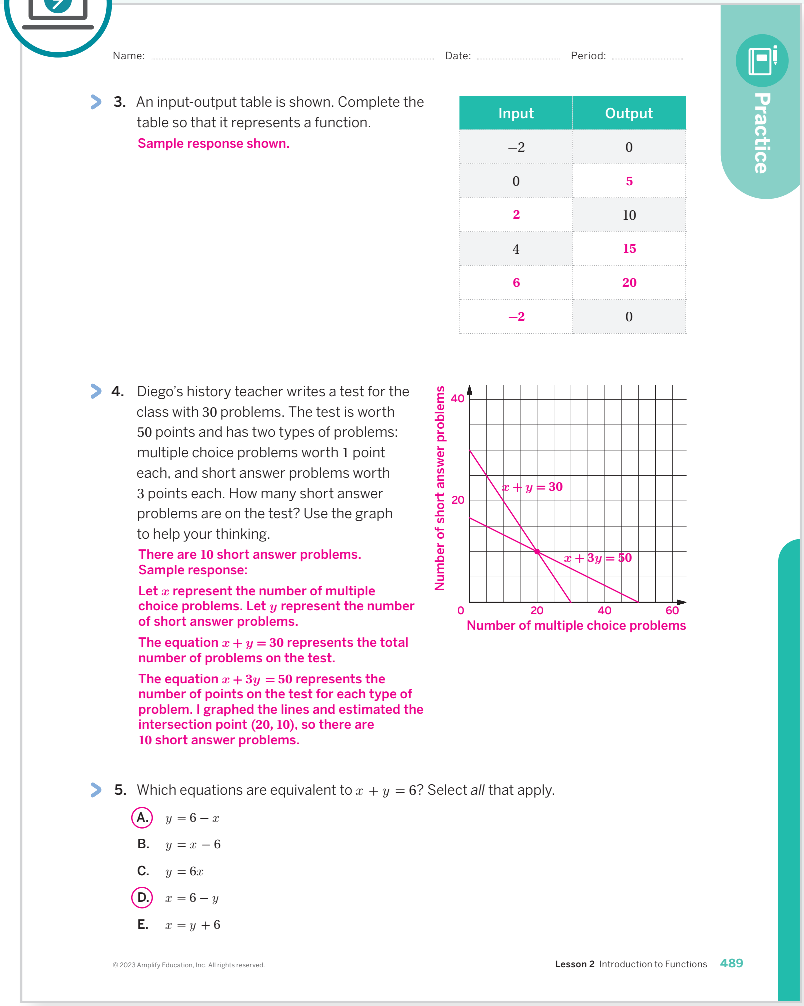

Name: ______ Date: ______ Period: ______

3. An input-output table is shown. Complete the table so that it represents a function.
Sample response shown.

Input	Output
−2	0
0	5
2	10
4	15
6	20
−2	0

4. Diego's history teacher writes a test for the class with 30 problems. The test is worth 50 points and has two types of problems: multiple choice problems worth 1 point each, and short answer problems worth 3 points each. How many short answer problems are on the test? Use the graph to help your thinking.
There are 10 short answer problems.
Sample response:
Let x represent the number of multiple choice problems. Let y represent the number of short answer problems.
The equation $x + y = 30$ represents the total number of problems on the test.
The equation $x + 3y = 50$ represents the number of points on the test for each type of problem. I graphed the lines and estimated the intersection point (20, 10), so there are 10 short answer problems.

5. Which equations are equivalent to $x + y = 6$? Select *all* that apply.
   [✓ A.] $y = 6 - x$
   B. $y = x - 6$
   C. $y = 6x$
   [✓ D.] $x = 6 - y$
   E. $x = y + 6$

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Lesson 2 Introduction to Functions 489

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 11	2
Formative ⚡	5	Unit 5 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 3**

Equations for Functions

Let's connect equations and graphs of functions.

Focus

Goals

1. **Language Goal:** Calculate the output of a function for a given input using an equation in two variables, and interpret the output in context. **(Speaking and Listening, Writing)**
2. Create an equation that represents a function rule.
3. **Language Goal:** Determine the independent and dependent variables of a function, and explain the reasoning. **(Speaking and Listening, Writing)**

Coherence

Today

Students transition from input-output tables to equations of functions. They look for structure between inputs and outputs and are introduced to *independent* and *dependent variables*. In Activity 2, students reason abstractly and quantitatively as they explore an equation where it is possible to write either variable as a function of the other.

Previously

In Grade 6, students analyzed relationships between dependent and independent variables. In Lesson 2, students were introduced to the term *function*. They used input-output diagrams and tables to help them investigate the characteristics of a function.

Coming Soon

In Lesson 4, students will explore graphs of functions. They will determine whether a graph represents a function, and explain the reasoning.

Rigor

- Students further their **conceptual understanding** of functions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Properties of Equality*

Math Language Development

New words

- ***dependent variable***
- ***independent variable***

Review words

- *function*
- *input*
- *output*

Amps | **Featured Activity**

Activity 2
Formative Feedback for Students

Instead of just being told if they are correct or incorrect, students see the consequences of their response and resolve any errors on their own.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be so eager to obtain the answer to a problem that they forget to think about the overarching process involved. Explain that while the correct answer is important, their thought process is what will help them solve future problems. Encourage students to record their thinking by showing all of their work.

Modifications to Pacing

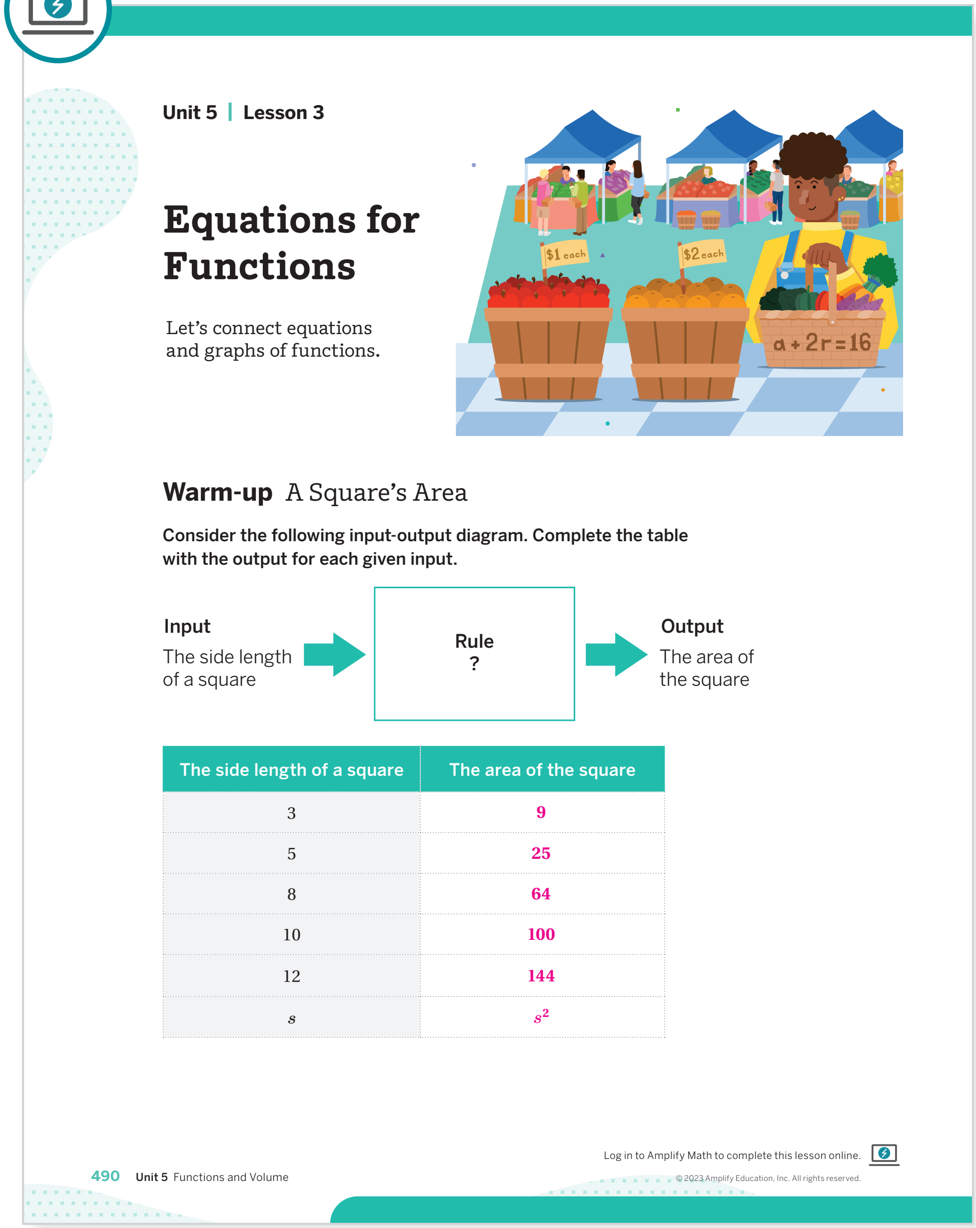
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, have students complete only the first two rows of the table.

Pairs | 5 min

Warm-up A Square's Area

Students use repeated reasoning to write an algebraic expression to represent a rule of a function which introduces them identify the *independent* and *dependent variables*.



Unit 5 | Lesson 3

Equations for Functions

Let's connect equations and graphs of functions.

Warm-up A Square's Area

Consider the following input-output diagram. Complete the table with the output for each given input.

Input The side length of a square → **Rule** ? → **Output** The area of the square

The side length of a square	The area of the square
3	9
5	25
8	64
10	100
12	144
s	s^2

Log in to Amplify Math to complete this lesson online.

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490 Unit 5 Functions and Volume

1 Launch

Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by completing the first row of the table together.

Look for productive strategies:

- Writing s^2 or $s \cdot s$ for the area of a square with the side length s.
- Understanding that negative numbers cannot be inputs for this context.

3 Connect

Have students share their responses.

Define the terms ***independent variable*** and ***dependent variable***. Say, "The *independent variable* represents the input of a function, while the *dependent variable* represents the output of a function."

Ask:

- "What are the independent and dependent variables in this situation?" The area of the square is the dependent variable. The side length of the square is the independent variable.
- "Is -5 a possible input value for this context? Why or why not?" No, a square cannot have a negative side length.

Highlight that, for non-negative numbers, students can represent this rule in two different ways: with a description and with an equation.

Math Language Development

MLR2: Collect and Display

During the Connect, add the terms *independent variable* and *dependent variable* to the class display. As students share what the independent and dependent variables are in the Warm-up diagram, collect terms related to the definitions, such as *input* and *output*, and add these to the display.

English Learners

Annotate the Warm-up diagram by writing *independent variable* next to *input* and *dependent variable* next to *output* to reinforce the meanings of these terms.

Power-up

To power up students' ability to identify equivalent equations, have students complete:

Determine all equations that are equivalent to $3 + 5 = 8$. Select *all* that apply.

[✓ **A.**] $5 + 3 = 8$
D. $5 = 8 + 3$
B. $5 = 3 - 8$
[✓ **E.**] $5 = 8 - 3$
[✓ **C.**] $3 = 8 - 5$

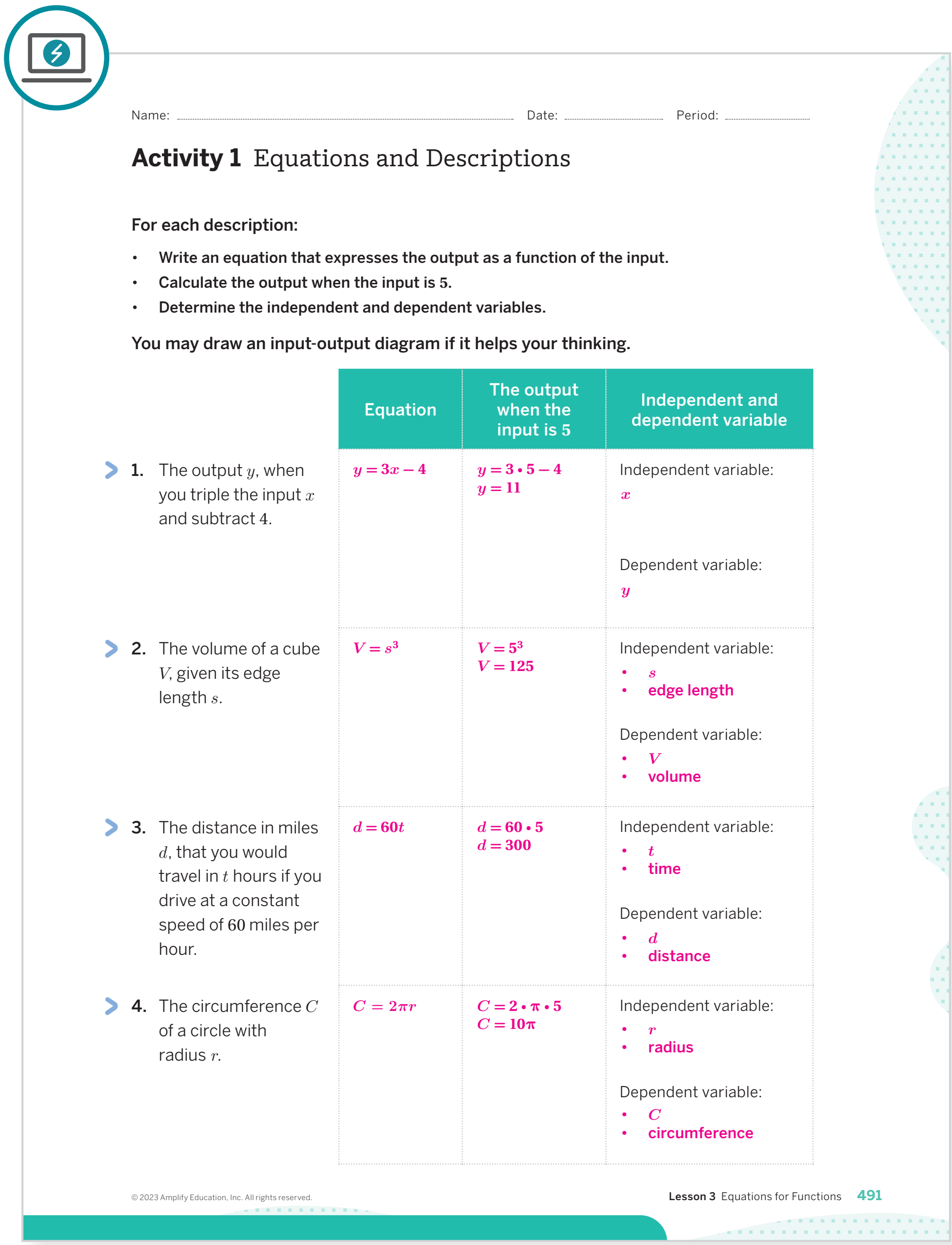
Use: Before Activity 2

Informed by: Performance on Lesson 2, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 15 min

Activity 1 Equations and Descriptions

Students write equations of functions and determine the independent and dependent variables to connect different representations of functions.



Name: ______ Date: ______ Period: ______

Activity 1 Equations and Descriptions

For each description:

- **Write an equation that expresses the output as a function of the input.**
- **Calculate the output when the input is 5.**
- **Determine the independent and dependent variables.**

You may draw an input-output diagram if it helps your thinking.

	Equation	The output when the input is 5	Independent and dependent variable
1. The output y, when you triple the input x and subtract 4.	$y = 3x - 4$	$y = 3 \cdot 5 - 4$ $y = 11$	Independent variable: x Dependent variable: y
2. The volume of a cube V, given its edge length s.	$V = s^3$	$V = 5^3$ $V = 125$	Independent variable: • s • edge length Dependent variable: • V • volume
3. The distance in miles d, that you would travel in t hours if you drive at a constant speed of 60 miles per hour.	$d = 60t$	$d = 60 \cdot 5$ $d = 300$	Independent variable: • t • time Dependent variable: • d • distance
4. The circumference C of a circle with radius r.	$C = 2\pi r$	$C = 2 \cdot \pi \cdot 5$ $C = 10\pi$	Independent variable: • r • radius Dependent variable: • C • circumference

1 Launch

Activate students' prior knowledge by asking them to describe how to determine the volume of a cube and the circumference of a circle.

2 Monitor

Help students get started by having them draw an input-output diagram to help their thinking.

Look for points of confusion:

- **Switching the independent and dependent variables in their equations.** Encourage students to draw an input-output diagram, along with a table with several input-output pairs, to help them determine the variables. Additionally, you may have students revisit the Warm-up.

Look for productive strategies:

- Identifying the independent and dependent variables from the description, input, output, or equation.

3 Connect

Display student work showing the completed table.

Have students share their strategies for writing each equation and identifying the independent and dependent variables for each problem.

Highlight that an equation can be written to represent a rule expressed by a function. The equations can be used to determine different input-output pairs.

Ask, "How can you determine the independent and dependent variables if you are only given an equation?" Collect and display students' responses for all to see and have them revisit the responses as they work on Activity 2.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

If students need more processing time, have them focus on completing the first two rows of the table. Provide access to colored pencils and suggest that students highlight the independent variable/input in one color and the dependent variable/output in another color.

Extension: *Math Enrichment*

Have students write an equation that gives the width P of a rectangle, given the length is 3 units and the perimeter is P units. Sample response: $w = \frac{P-6}{2}$

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students describe the connections they noticed, press for details in their explanations by requesting that other students challenge an idea, elaborate on an idea, or provide an example. For example, if a student says, "The dependent variable is the one that's on the left side of the equation," ask the class, "Can you write an equation so that the dependent variable is on the right side? What about the equation $3x - 4 = y$? Which variable is the dependent variable?"

English Learners

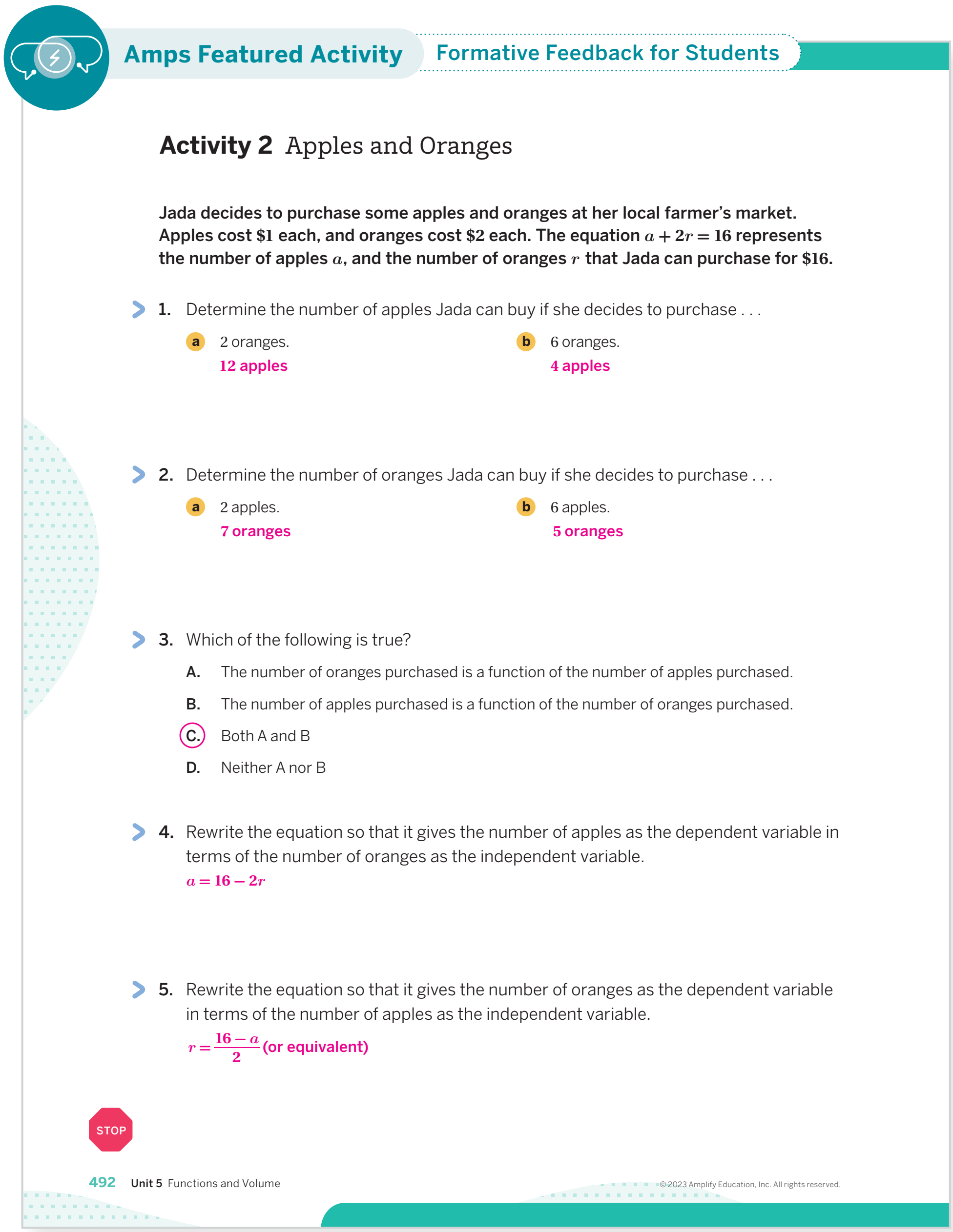
Encourage students to refer to and use the class display to support their use of appropriate mathematical language.

Pairs | 15 min

Activity 2 Apples and Oranges

Students work with an equation to discover that, in some situations, either variable can be the independent variable.

Amps Featured Activity Formative Feedback for Students



Activity 2 Apples and Oranges

Jada decides to purchase some apples and oranges at her local farmer's market. Apples cost $1 each, and oranges cost $2 each. The equation $a + 2r = 16$ represents the number of apples a, and the number of oranges r that Jada can purchase for $16.

1. Determine the number of apples Jada can buy if she decides to purchase . . .
   - a. 2 oranges. **12 apples**
   - b. 6 oranges. **4 apples**

2. Determine the number of oranges Jada can buy if she decides to purchase . . .
   - a. 2 apples. **7 oranges**
   - b. 6 apples. **5 oranges**

3. Which of the following is true?
   - A. The number of oranges purchased is a function of the number of apples purchased.
   - B. The number of apples purchased is a function of the number of oranges purchased.
   - [✓ C.] Both A and B
   - D. Neither A nor B

4. Rewrite the equation so that it gives the number of apples as the dependent variable in terms of the number of oranges as the independent variable.

   $a = 16 - 2r$

5. Rewrite the equation so that it gives the number of oranges as the dependent variable in terms of the number of apples as the independent variable.

   $r = \frac{16 - a}{2}$ **(or equivalent)**

STOP

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. **Note:** The goal of this activity is for students to explore writing equations so that either variable is the independent or dependent variable. Students will further explore this topic in high school.

2 Monitor

Help students get started by reviewing steps to substituting a value and solving an equation.

Look for points of confusion:

- **Having difficulty rearranging the equations in Problems 4 and 5.** Prompt students to use the provided equation and their knowledge about balancing equations to create the new equations. Consider having students replace a variable with a number, asking them to solve the equation, and record their steps without making any calculations.

3 Connect

Display the equations students wrote for Problems 4 and 5 and have them share their strategies for rewriting each equation.

Ask:

- "In this situation, why could either variable be the independent variable?" The number of oranges purchased depends on the number of apples purchased, and the number of apples purchased depends on the number of oranges purchased.
- "In what situation is it helpful to use the equation you wrote for Problem 4? Problem 5?" The equation for Problem 4 could be helpful when the number of oranges purchased is known, while the equation for Problem 5 could be helpful when the number of apples purchased is known.

Highlight that, in this situation, either variable could be the independent variable. Ensure students understand that this is not always possible when working with functions.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see the consequences of their response and resolve any errors on their own.

Accessibility: *Guide Processing and Visualization*

For Problems 4 and 5, provide partially-completed equations, such as the following, to help students remember which variable is the dependent variable.

Problem 4: $a =$ _____ (the number of apples is the dependent variable)

Problem 5: $r =$ _____ (the number of oranges is the dependent variable)

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that Jada purchased an unknown number of apples and oranges.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as apples cost $1 each.
- **Read 3:** Ask students to brainstorm strategies for how they will use the equation to complete Problems 1 and 2.

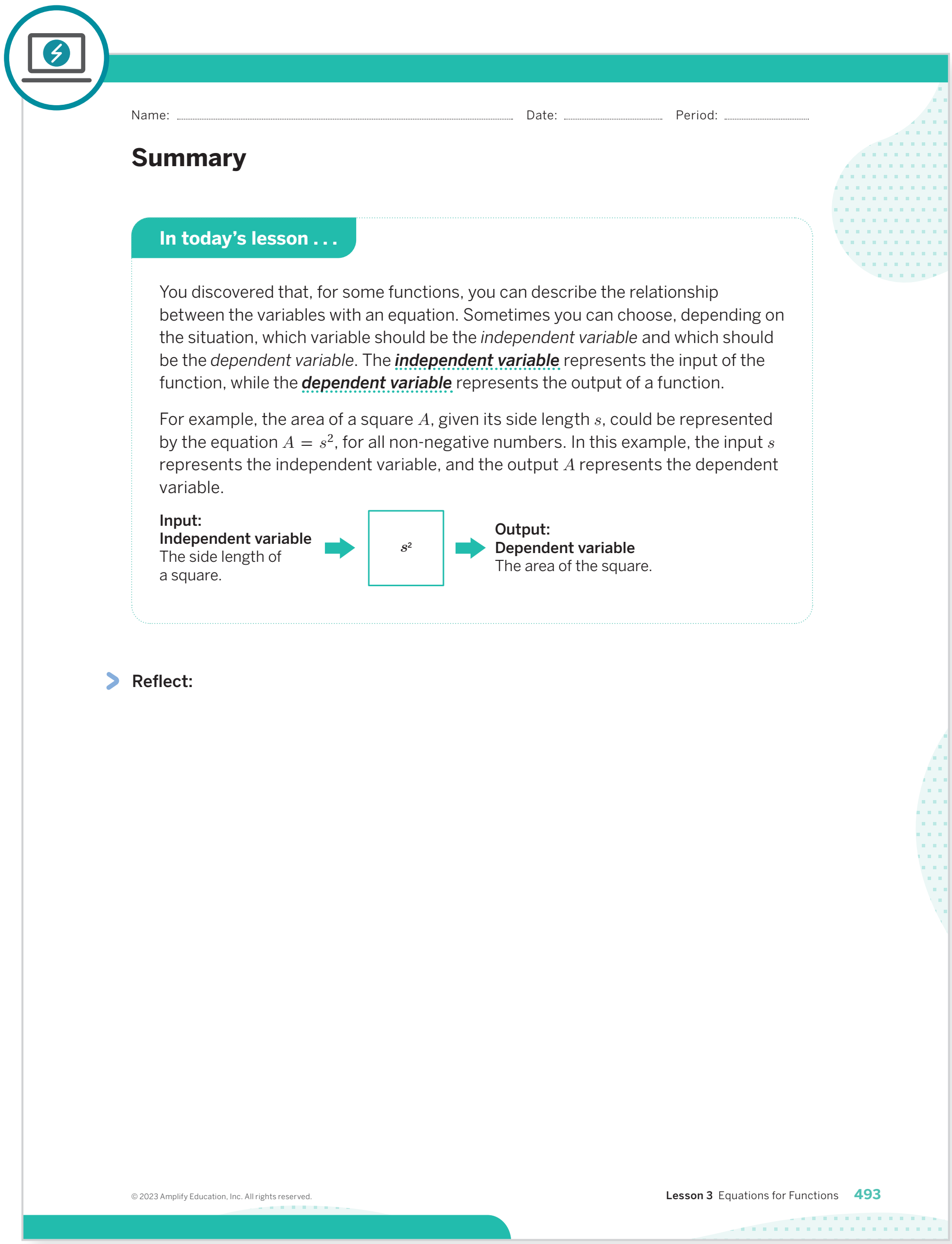
English Learners

Annotate the number of apples and oranges in Problems 1 and 2 with the variables a and r that represent them.

Whole Class | 5 min

Summary

Review and synthesize how to determine the independent and dependent variables of functions.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You discovered that, for some functions, you can describe the relationship between the variables with an equation. Sometimes you can choose, depending on the situation, which variable should be the *independent variable* and which should be the *dependent variable*. The ***independent variable*** represents the input of the function, while the ***dependent variable*** represents the output of a function.

For example, the area of a square A, given its side length s, could be represented by the equation $A = s^2$, for all non-negative numbers. In this example, the input s represents the independent variable, and the output A represents the dependent variable.

Input:
Independent variable
The side length of a square.

s^2

Output:
Dependent variable
The area of the square.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Have students share how they can identify the independent and dependent variables and how they relate to the input and output of a function.

Highlight that the *independent variables* and *dependent variables* represent the inputs and outputs of functions. For some functions, students can describe the relationship between the variables with an equation. Sometimes students can select, depending on the situation, which variable should be the independent and which should be the dependent variable.

Formalize vocabulary:

- *independent variable*
- *dependent variable*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when determining the independent and dependent variables of a function? How were they helpful?"

Math Language Development

MLR2: Collect and Display

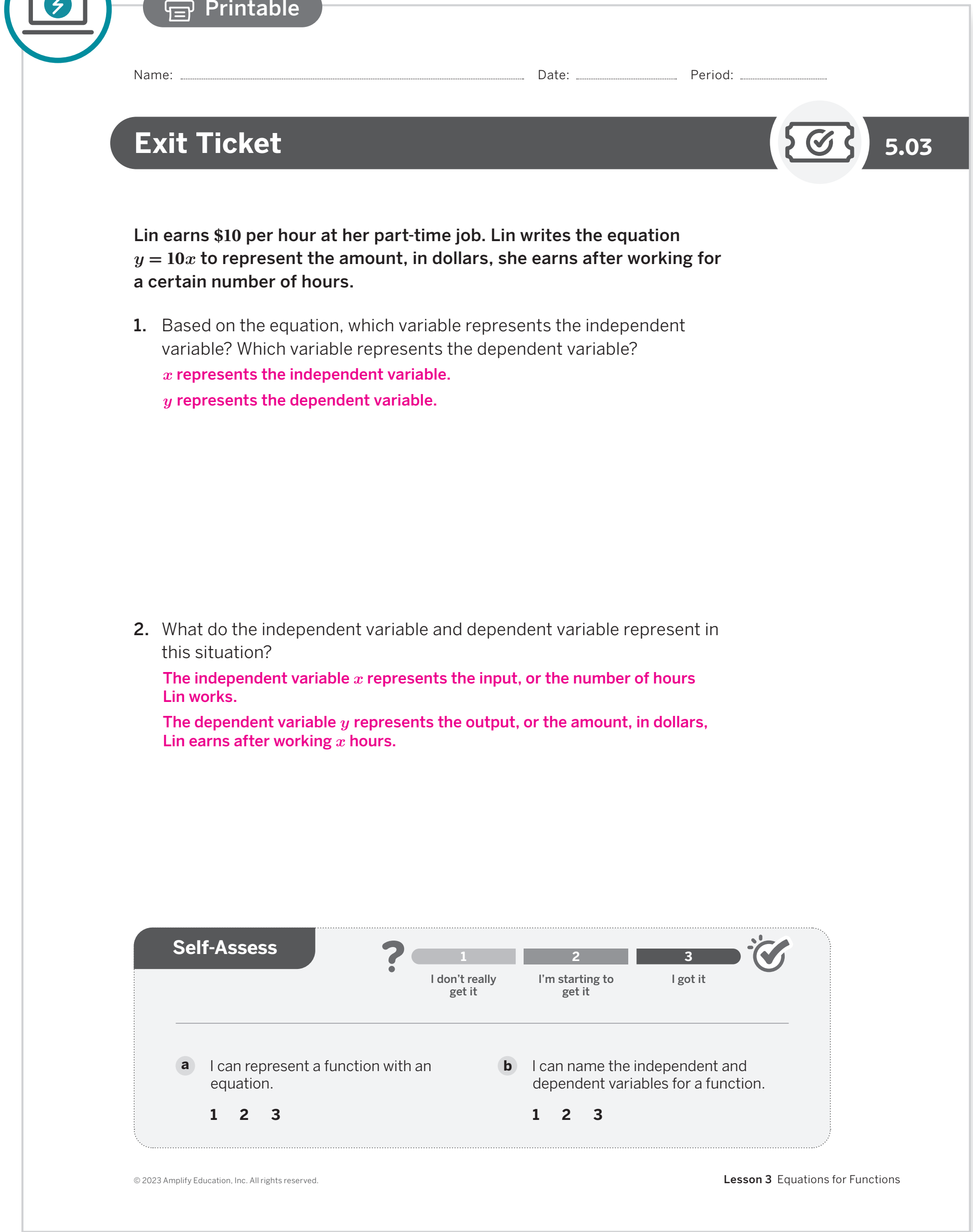
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *independent variable* and *dependent variable* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying the independent and dependent variables, given an equation and describing what each variable represents.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.03

Lin earns $\$10$ per hour at her part-time job. Lin writes the equation $y = 10x$ to represent the amount, in dollars, she earns after working for a certain number of hours.

1. Based on the equation, which variable represents the independent variable? Which variable represents the dependent variable?

   x represents the independent variable.

   y represents the dependent variable.

2. What do the independent variable and dependent variable represent in this situation?

   The independent variable x represents the input, or the number of hours Lin works.

   The dependent variable y represents the output, or the amount, in dollars, Lin earns after working x hours.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can represent a function with an equation.

1 2 3

b I can name the independent and dependent variables for a function.

1 2 3

Lesson 3 Equations for Functions

Success looks like . . .

- **Language Goal:** Calculating the output of a function for a given input using an equation in two variables, and interpreting the output in context. **(Speaking and Listening, Writing)**
- **Goal:** Creating an equation that represents a function rule.
- **Language Goal:** Determining the independent and dependent variables of a function, and explaining the reasoning. **(Speaking and Listening, Writing)**
  - » Determining x to be the independent variable and y to be the dependent variable and what they represent in Problems 1 and 2.

Suggested next steps

If students incorrectly identify the independent and dependent variables, consider:

- Having them create an input-output diagram or a table that represents the context.

If students are unsure of what the variables represent, consider:

- Asking students to reread the description, and identify how much Lin earns if she works 5 hours, 10 hours, and 20 hours. Then ask students what each variable represents.

Professional Learning

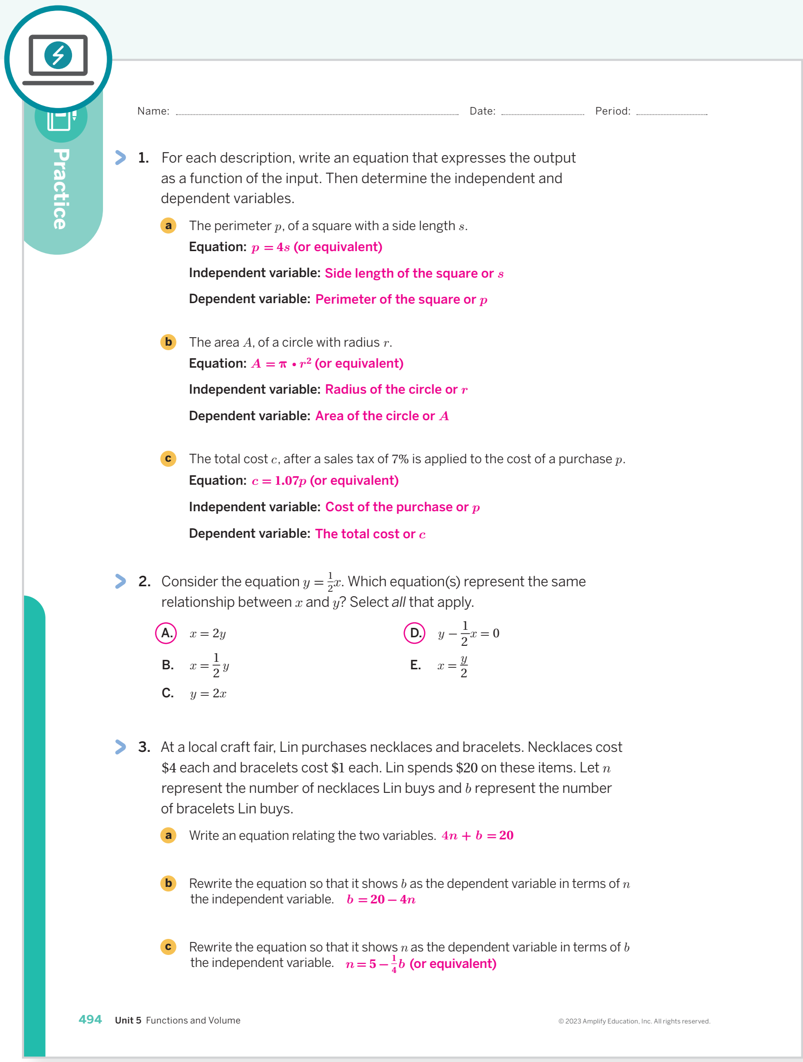
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What resources did students use as they worked on Activity 1? Which resources were especially helpful?
- What challenges did students encounter as they worked on Activity 2? How did they work through them?

Practice

Independent



Name: Date: Period:

1. For each description, write an equation that expresses the output as a function of the input. Then determine the independent and dependent variables.

a The perimeter p, of a square with a side length s.

Equation: $p = 4s$ (or equivalent)

Independent variable: Side length of the square or s

Dependent variable: Perimeter of the square or p

b The area A, of a circle with radius r.

Equation: $A = \pi \cdot r^2$ (or equivalent)

Independent variable: Radius of the circle or r

Dependent variable: Area of the circle or A

c The total cost c, after a sales tax of 7% is applied to the cost of a purchase p.

Equation: $c = 1.07p$ (or equivalent)

Independent variable: Cost of the purchase or p

Dependent variable: The total cost or c

2. Consider the equation $y = \frac{1}{2}x$. Which equation(s) represent the same relationship between x and y? Select *all* that apply.

[✓ A.] $x = 2y$

B. $x = \frac{1}{2}y$

C. $y = 2x$

[✓ D.] $y - \frac{1}{2}x = 0$

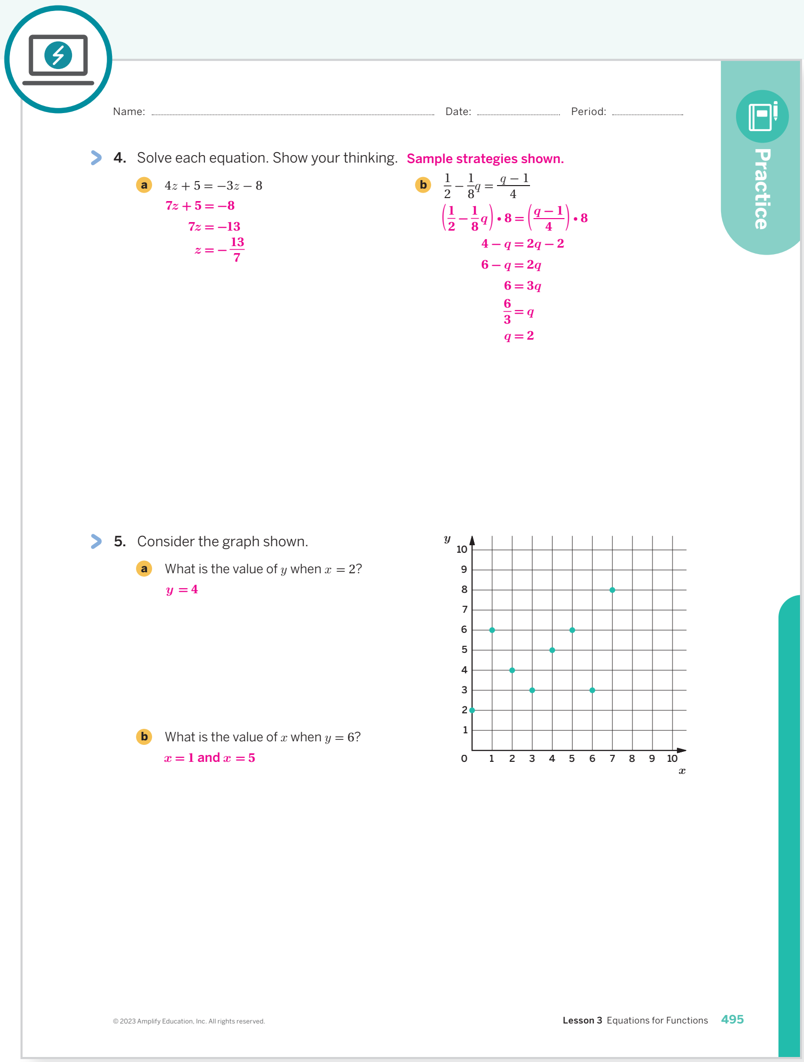
E. $x = \frac{y}{2}$

3. At a local craft fair, Lin purchases necklaces and bracelets. Necklaces cost \$4 each and bracelets cost \$1 each. Lin spends \$20 on these items. Let n represent the number of necklaces Lin buys and b represent the number of bracelets Lin buys.

a Write an equation relating the two variables. $4n + b = 20$

b Rewrite the equation so that it shows b as the dependent variable in terms of n the independent variable. $b = 20 - 4n$

c Rewrite the equation so that it shows n as the dependent variable in terms of b the independent variable. $n = 5 - \frac{1}{4}b$ (or equivalent)



Name: Date: Period:

4. Solve each equation. Show your thinking. Sample strategies shown.

a $4z + 5 = -3z - 8$

$7z + 5 = -8$

$7z = -13$

$z = -\frac{13}{7}$

b $\frac{1}{2} - \frac{1}{8}q = \frac{q-1}{4}$

$\left(\frac{1}{2} - \frac{1}{8}q\right) \cdot 8 = \left(\frac{q-1}{4}\right) \cdot 8$

$4 - q = 2q - 2$

$6 - q = 2q$

$6 = 3q$

$\frac{6}{3} = q$

$q = 2$

5. Consider the graph shown.

a What is the value of y when $x = 2$?

$y = 4$

b What is the value of x when $y = 6$?

$x = 1$ and $x = 5$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 5	2
Formative	5	Unit 5 Lesson 4	1

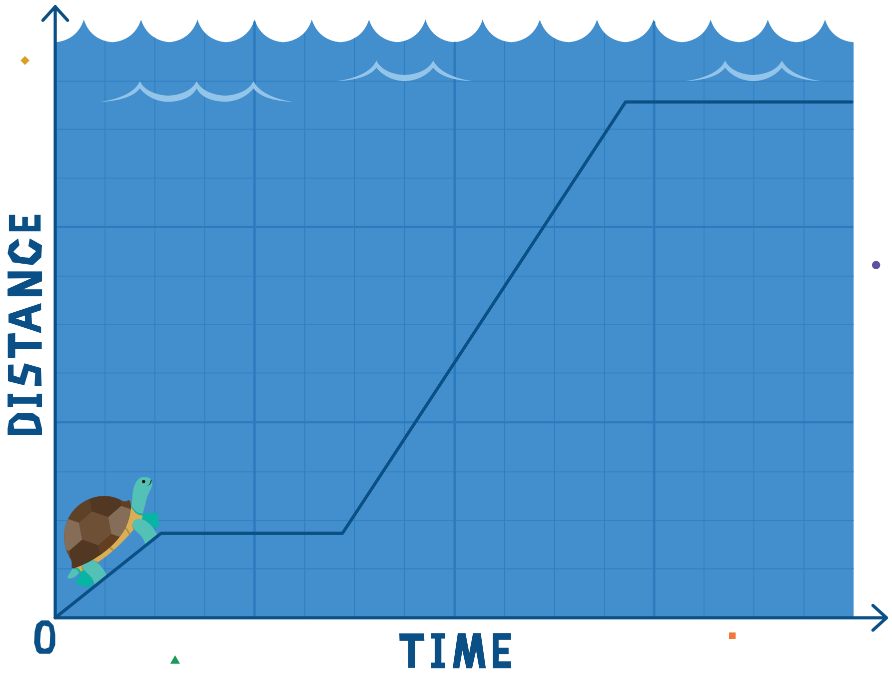
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Graphs of Functions (Part 1)

Let's interpret graphs of functions.



Focus

Goals

1. **Language Goal:** Determine whether a graph represents a function, and explain the reasoning. **(Speaking and Listening)**
2. **Language Goal:** Interpret points on a graph, including a graph of a function and a graph that does not represent a function. **(Speaking and Listening, Writing)**
3. Comprehend that the graph of a function is a set of ordered pairs consisting of an input and corresponding output.

Coherence

Today

Students begin to make connections between scenarios and graphs that represent them. They compare two graphs representing the same context to determine whether or not the graph represents a function. Students attend to precision while determining the characteristics of functions and make use of structure as they sort graphs of functions.

Previously

In Lessons 2 and 3, students explored tables and equations of functions. Students looked for structure between input and outputs and were introduced to the terms independent and dependent variables.

Coming Soon

In Lesson 5, students will interpret specific points on a graph of a function, as well as specific intervals and the overall shape of a graph. Students will make sense of the graph of a function in context and determine what the graph says about the relationship between two variables.

Rigor

- Students build **conceptual understanding** of graphs of functions.
- Students develop **procedural fluency** by identifying graphs of functions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
6 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- graph paper (as needed)

Math Language Development

Review words

- *function*
- *dependent variable*
- *independent variable*
- *input*
- *output*

Amps | Featured Activity

Activity 1
Turtle Crossing Animation

Students watch an animation of a turtle crossing while the graph of the turtle's distance and time is shown at the same time.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students discuss their interpretations of the graphs, they might forget to listen to others. Remind students to show respect by listening attentively and respectfully critiquing the arguments their classmates. By building on each other's experiences, they might be able to create better and more creative stories that could be mathematically represented by the graphs.

Modifications to Pacing

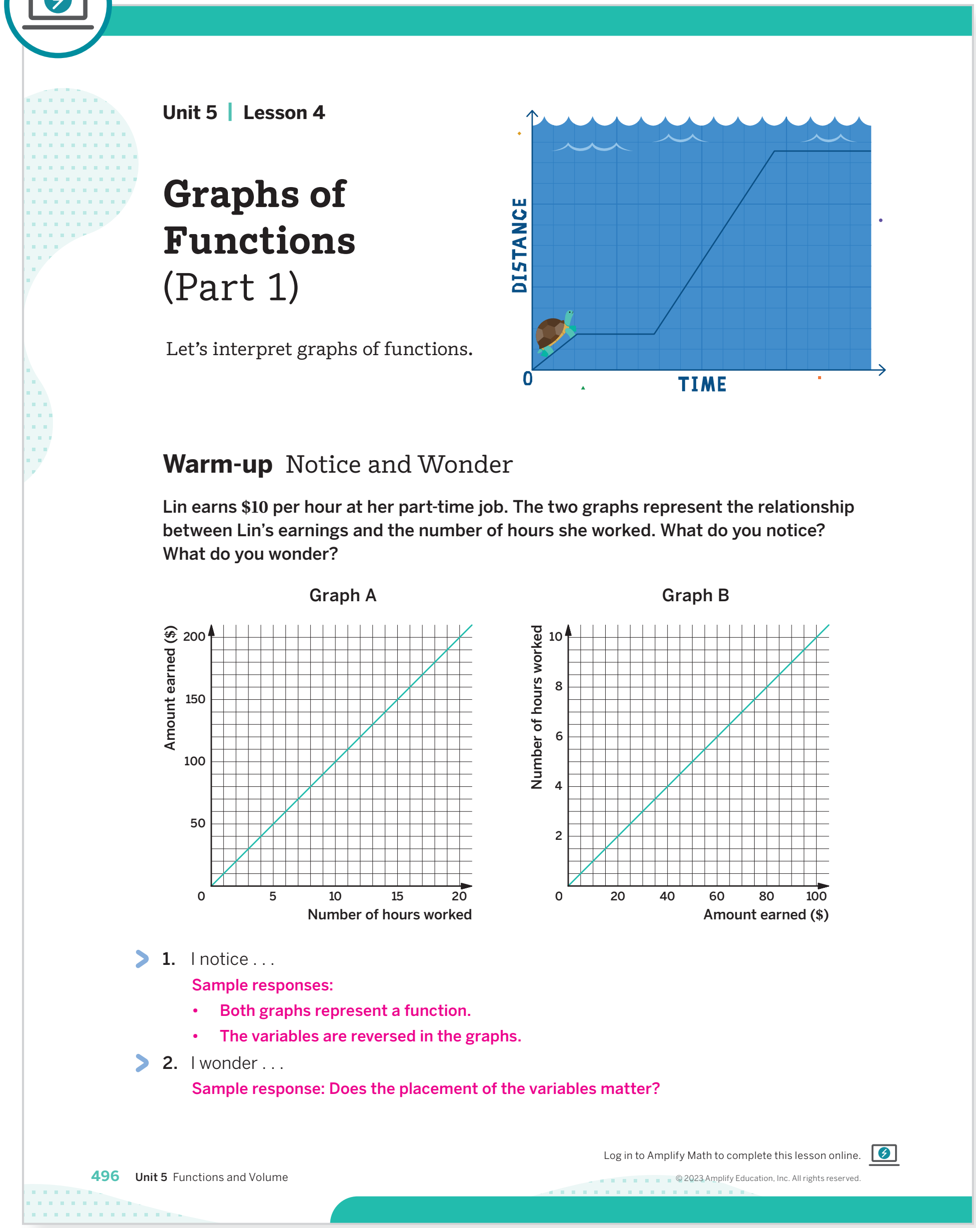
You may want to consider this additional modification if you are short on time.

- Assign **Activity 2** as additional practice.

Pairs | 6 min

Warm-up Notice and Wonder

Students compare two graphs that represent the same context to explore the conventions used to label a graph of a function.



Unit 5 | Lesson 4

Graphs of Functions (Part 1)

Let's interpret graphs of functions.

Warm-up Notice and Wonder

Lin earns $10 per hour at her part-time job. The two graphs represent the relationship between Lin's earnings and the number of hours she worked. What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - Both graphs represent a function.
   - The variables are reversed in the graphs.
2. I wonder . . .
   Sample response: Does the placement of the variables matter?

Log in to Amplify Math to complete this lesson online.

496 Unit 5 Functions and Volume

1 Launch

Conduct the *Notice and Wonder* routine.

2 Monitor

Help students get started by having them study the axes labels for each graph carefully before writing their response.

Look for points of confusion:

- **Thinking that the graphs are identical.** Point out that the scales and labels on the graphs are different.

3 Connect

Have students share what they noticed and wondered with a partner before sharing with the class.

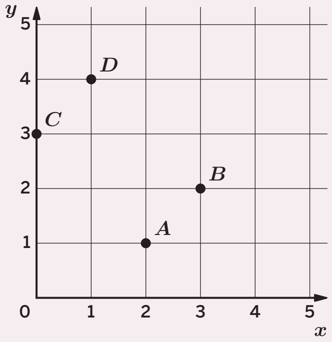
Ask, "Do both graphs represent a function? How do you know?" Yes, for every input, there is only one output.

Highlight that one way to represent a function is by using a graph. While both graphs represent the same relationship, they are different because they use different independent and dependent variables. Then highlight the conventions of labeling a graph. The input, or independent variable, is labeled along the x-axis, while the output, or dependent variable, is labeled along the y-axis. For Graph A, have students label the x-axis with the term *independent variable* and the y-axis with the term *dependent variable*.

Power-up

To power up students' ability to determine inputs and outputs from a graph, have students complete:

1. Write the coordinates for each point:
   A: (2, 1)
   B: (3, 2)
   C: (0, 3)
   D: (1, 4)
2. What is the value of y when $x = 2$? $y = 1$
3. What is the value of x when $y = 3$? $x = 0$



Use: Before Activity 1

Informed by: Performance on Lesson 3, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 15 min

Activity 1 Turtle Crossing

Students make connections between a scenario and two different graphs that represent the scenario to determine whether or not a graph represents a function.

Amps Featured Activity Turtle Crossing Animation

Name: Date: Period:

Activity 1 Turtle Crossing

The graph shows a turtle's distance from the water, in feet, over time, in seconds.

1. Determine the turtle's distance from the water after:
   - a 4 seconds. 2 ft
   - b 6 seconds. 6 ft
   - c 8 seconds. 10 ft

2. For this situation, is the turtle's distance from the water a function of time? If yes, determine the independent and dependent variables and explain why it is a function. If no, explain why it is not a function.
   Yes; Sample response: For every input x, there is exactly one output y. The independent variable is time, in seconds. The dependent variable is the turtle's distance, in feet, from the water.

Clare drew the graph shown to represent the turtle's journey.

3. Using Clare's graph, determine the time when the turtle's distance from the water was:
   - a 1 ft. 1 second
   - b 8 ft. 7 seconds
   - c 11 ft. Answers may vary, but should range from 9 to 12 seconds.

4. For this situation, is the time a function of the turtle's distance from the water? If yes, determine the independent and dependent variables and explain why it is a function. If no, explain why it is not a function.
   No; Sample response: There are several outputs for the same input value, 2. Therefore, for this situation, time is not a function of distance.

Reflect: How did you keep control over your impulses?

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Lesson 4 Graphs of Functions (Part 1) 497

1 Launch

Display the animation, *Turtle Crossing*, from the Activity 1 Amps slides.

2 Monitor

Help students get started by having them carefully study the axes labels on the graph before responding to each problem.

Look for points of confusion:

- **Struggling with Problems 1 and 3.** For each graph, have students create an input-output table using the points on the graph and use the table to determine whether each situation represents a function.
- **Struggling to respond to Problem 3c.** Ask students to highlight the points that correspond to the turtle's distance of 11 ft. Tell students that they may give a single value or a range of values.

Look for productive strategies:

- Noticing Clare's graph does not represent a function because there are multiple outputs for the inputs of 2 and 11.

3 Connect

Have students share how they used each graph to determine the turtle's distance or time and how they used this information to determine whether the graph represented a function.

Ask, "How can you determine whether a graph represents a function?" Sample response: If each input, or x-value, has only one output, or y-value, the graph represents a function.

Highlight that the graph of a function is a set of ordered pairs consisting of an input and a corresponding output, and, for each input, there is exactly one output.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view an animation of a turtle crossing while the graph of the turtle's time and distance is simultaneously shown.

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing the responses to Problems 1 and 3, so that students can focus on responding to Problems 2 and 4.

Math Language Development

MLR7: Compare and Connect

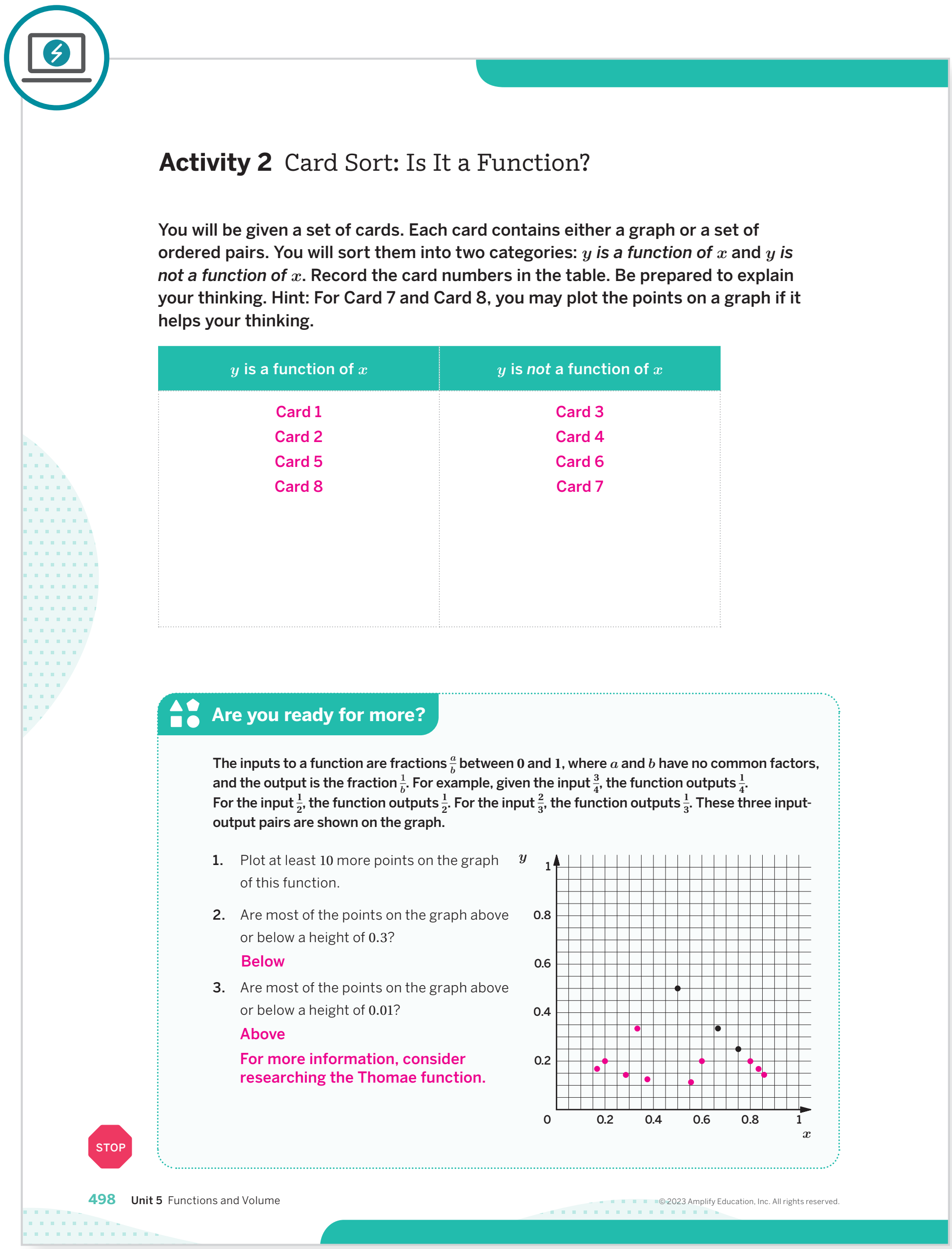
Before the Connect, invite pairs to discuss, "What is the same and what is different about the graphs?" During the Connect, as students share their responses, connect the graphical representations back to previous activities where students analyzed input-output tables. Ask:

- "What would an input-output table look like for the second graph?" Sample response: There would be different outputs for the same input.
- "Why would a vertical line represent a relationship that is not a function?" Sample response: There are different outputs for the same input.

Pairs | 15 min

Activity 2 Card Sort: Is It a Function?

Students sort cards to build fluency in identifying functions, given a graph or set of ordered pairs.



Activity 2 Card Sort: Is It a Function?

You will be given a set of cards. Each card contains either a graph or a set of ordered pairs. You will sort them into two categories: *y is a function of x* and *y is not a function of x*. Record the card numbers in the table. Be prepared to explain your thinking. Hint: For Card 7 and Card 8, you may plot the points on a graph if it helps your thinking.

y is a function of x	y is *not* a function of x
Card 1 Card 2 Card 5 Card 8	Card 3 Card 4 Card 6 Card 7

Are you ready for more?

The inputs to a function are fractions $\frac{a}{b}$ between 0 and 1, where a and b have no common factors, and the output is the fraction $\frac{1}{b}$. For example, given the input $\frac{3}{4}$, the function outputs $\frac{1}{4}$. For the input $\frac{1}{2}$, the function outputs $\frac{1}{2}$. For the input $\frac{2}{3}$, the function outputs $\frac{1}{3}$. These three input-output pairs are shown on the graph.

1. Plot at least 10 more points on the graph of this function.
2. Are most of the points on the graph above or below a height of 0.3?

   Below
3. Are most of the points on the graph above or below a height of 0.01?

   Above

   For more information, consider researching the Thomae function.

498 Unit 5 Functions and Volume

1 Launch

Distribute the pre-cut cards from the Activity 2 PDF to each student pair. Tell students that for Cards 7 and 8, they will be given a set of ordered pairs, and they may plot the points on graph paper if it helps their thinking. Conduct the ***Card Sort*** routine. Provide access to graph paper.

2 Monitor

Help students get started by choosing one graph and looking at several ordered pairs to determine if there are multiple outputs for a single input.

Look for points of confusion:

- **Sorting a graph incorrectly.** Suggest that students read the graph from left to right and stop at each point that is plotted. For each point, ask students to check if there is another output, or y-value, for that specific input.

Look for productive strategies:

- Creating a table for the values to determine if the graph or set of ordered pairs represents a function.

3 Connect

Display student work showing the correctly sorted cards.

Have students share the strategies they used to determine which cards represented functions. Have students share any cards they struggled to sort and what they did to overcome the challenge.

Highlight that, for a graph, students can visually check whether y is a function of x. They can draw a vertical line through an input value, parallel to the y-axis. If the vertical line only intersects that one input value, the graph does represent a function.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Tools*

If students need more processing time, have them focus on sorting Cards 1-6. If you have students sort Cards 7 and 8, provide access to graph paper or graphing technology for students to use if they choose.

Extension: *Math Enrichment*

If students completed the *Are you ready for more*? problems and are interested in learning more, have them research the Thomae function, which is also known as the popcorn function because of its resemblance to what popping corn might look like.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share how they determined which cards represented functions, provide these sentence frames for students to use to organize their thinking.

- "For Card ____, y is a function of x because . . ."
- "For Card ____, y is not a function of x because . . ."

If any disagreements arose during the card sort, ask for an explanation as to how partners came to a consensus.

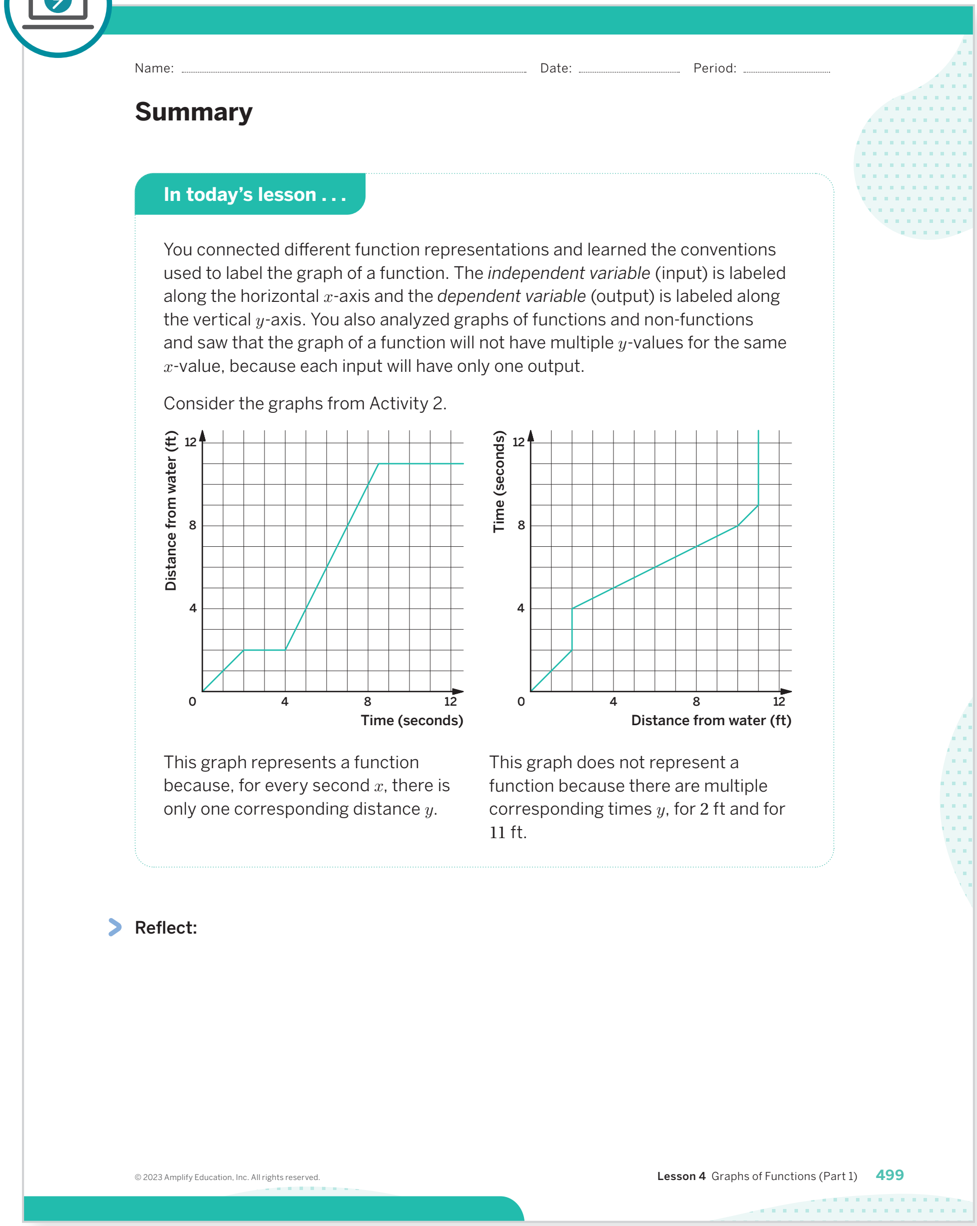
English Learners

Provide language to help complete the sentence frames, such as ". . . because there is more than one output for the same input."

Whole Class | 5 min

Summary

Review and synthesize what the graph of a function looks like and how to determine whether a relationship is a function by studying the structure of the graph.



Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You connected different function representations and learned the conventions used to label the graph of a function. The *independent variable* (input) is labeled along the horizontal x-axis and the *dependent variable* (output) is labeled along the vertical y-axis. You also analyzed graphs of functions and non-functions and saw that the graph of a function will not have multiple y-values for the same x-value, because each input will have only one output.

Consider the graphs from Activity 2.

This graph represents a function because, for every second x, there is only one corresponding distance y.

This graph does not represent a function because there are multiple corresponding times y, for 2 ft and for 11 ft.

Reflect:

Synthesize

Have students share how they would describe the graph of a function to their friend who was absent from class that day. Encourage students to include the terms *independent variable*, *dependent variable*, *input*, and *output* in their description.

Highlight that, for a graph of a function, the independent variable is labeled along the x-axis and the dependent variable is labeled along the y-axis. Also highlight that the graph of a function will not have multiple y-values for the same x-value.

Reflect

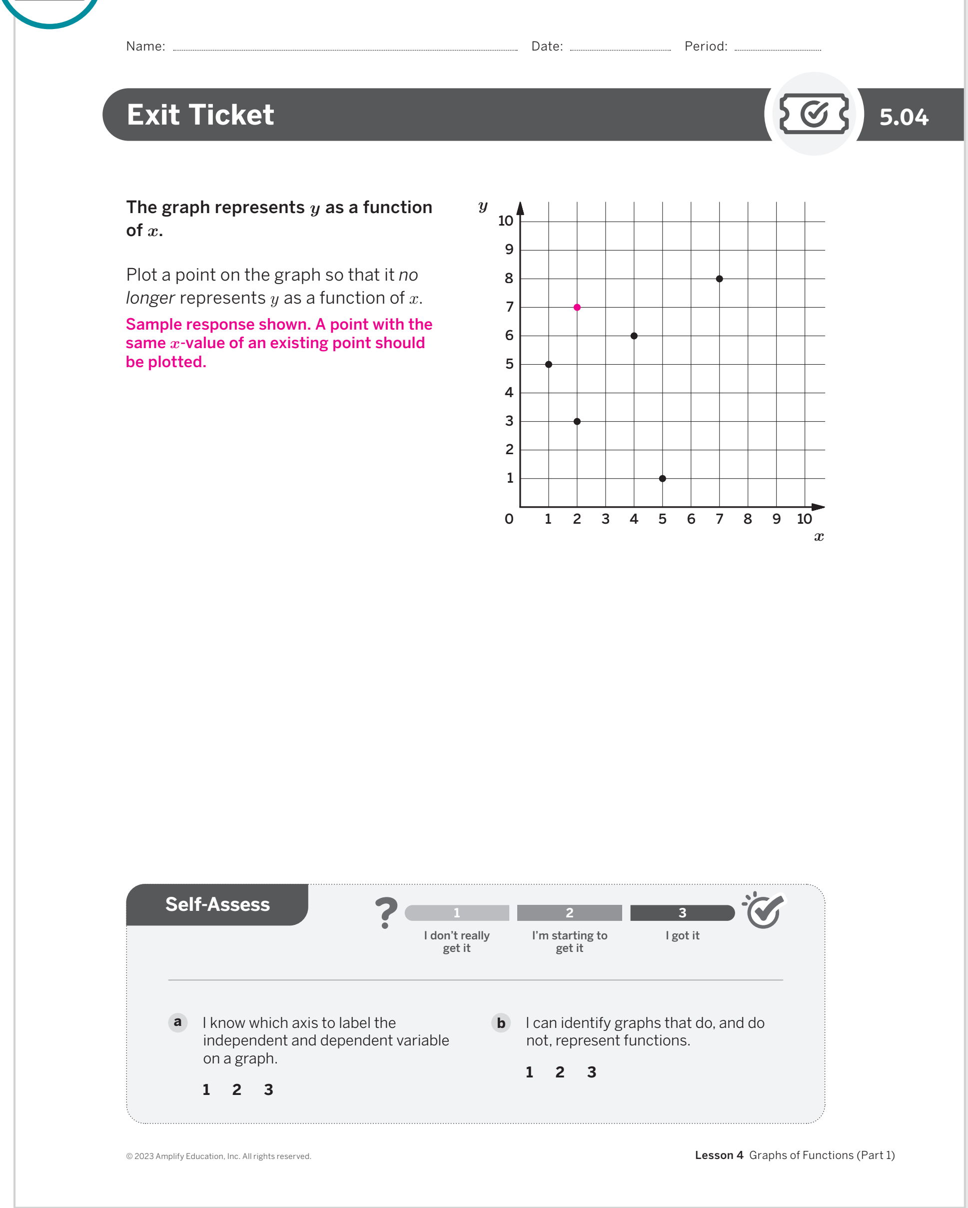
After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What makes a relationship a function?"

Exit Ticket

Students demonstrate their understanding by plotting a point on a graph so that y is no longer a function of x.

Printable



Name: Date: Period:

Exit Ticket

5.04

The graph represents y as a function of x.

Plot a point on the graph so that it *no longer* represents y as a function of x.

Sample response shown. A point with the same x-value of an existing point should be plotted.

Self-Assess

? 1 I don't really get it 2 I'm starting to get it 3 I got it

a I know which axis to label the independent and dependent variable on a graph.

1 2 3

b I can identify graphs that do, and do not, represent functions.

1 2 3

Lesson 4 Graphs of Functions (Part 1)

Success looks like . . .

- **Language Goal:** Determining whether a graph represents a function, and explaining the reasoning. **(Speaking and Listening)**
- **Language Goal:** Interpreting points on a graph, including a graph of a function and a graph that does not represent a function. **(Speaking and Listening, Writing)**
    - » Changing the graph so that y is no longer a function of x.
- **Goal:** Comprehending that the graph of a function is a set of ordered pairs consisting of an input and corresponding output.

Suggested next steps

If students do not plot a point correctly, consider:

- Reviewing Activity 2.
- Having them create an input-output table using the plotted points.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Which students' ideas were you able to highlight during Activity 1?
- In this lesson, students sorted graphs of functions. How did that build on the earlier work students did with identifying functions given a table?

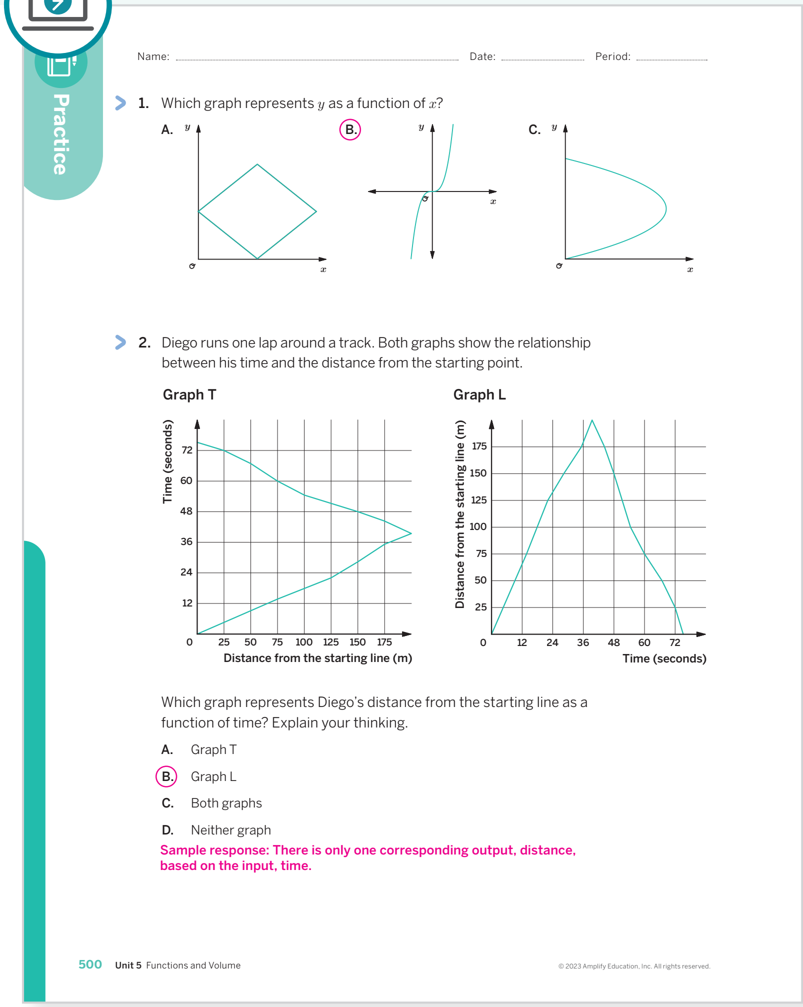
Math Language Development

Language Goal: Determining whether a graph represents a function, and explaining the reasoning.

- Reflect on students' language development toward this goal. How did using the sentence frames provided in the ***Discussion Supports*** routine in Activity 2 help students develop more precise language to explain why given relationships are or are not functions?
- Do students' responses to the Exit Ticket problem indicate that they understand what must be true about an additional point plotted on the graph so that the graph is no longer a function?

Practice

Independent



Name: Date: Period:

1. Which graph represents y as a function of x?

2. Diego runs one lap around a track. Both graphs show the relationship between his time and the distance from the starting point.

Which graph represents Diego's distance from the starting line as a function of time? Explain your thinking.

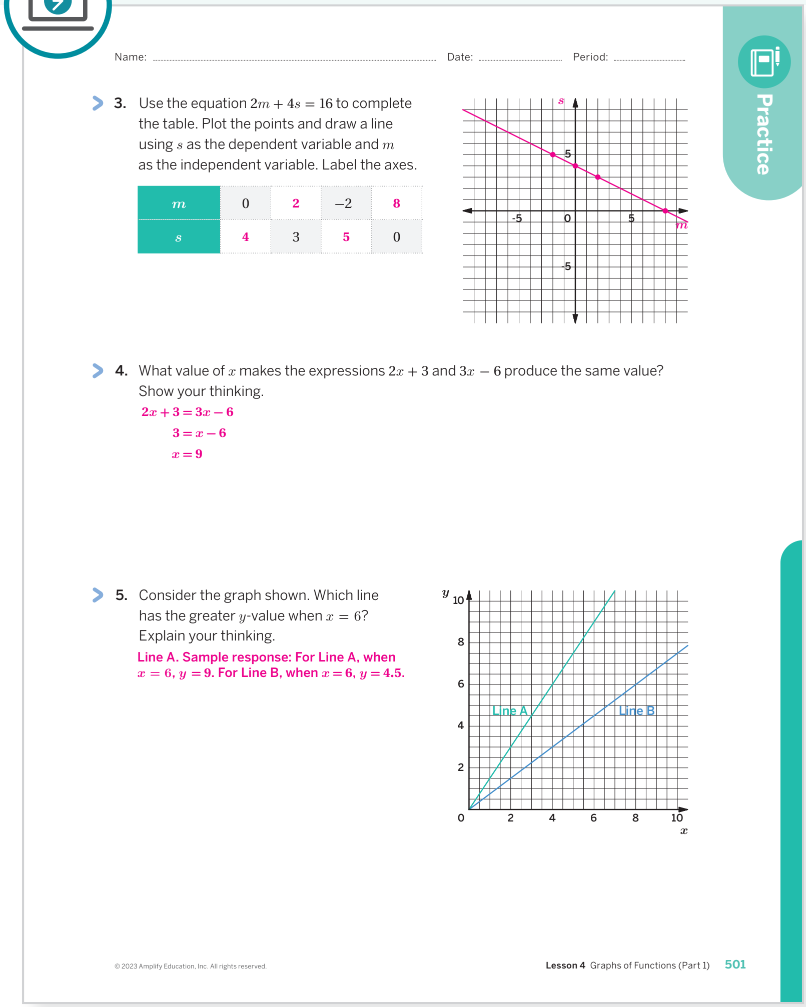
A. Graph T

[✓ B.] Graph L

C. Both graphs

D. Neither graph

Sample response: There is only one corresponding output, distance, based on the input, time.



Name: Date: Period:

3. Use the equation $2m + 4s = 16$ to complete the table. Plot the points and draw a line using s as the dependent variable and m as the independent variable. Label the axes.

m	0	**2**	–2	**8**
s	**4**	3	**5**	0

4. What value of x makes the expressions $2x + 3$ and $3x - 6$ produce the same value? Show your thinking.

$$\mathbf{2x + 3 = 3x - 6}$$
$$\mathbf{3 = x - 6}$$
$$\mathbf{x = 9}$$

5. Consider the graph shown. Which line has the greater y-value when $x = 6$? Explain your thinking.

Line A. Sample response: For Line A, when $x = 6$, $y = 9$. For Line B, when $x = 6$, $y = 4.5$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 1	1
Spiral	4	Unit 4 Lesson 10	2
Formative ⚡	5	Unit 5 Lesson 5	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Graphs of Functions (Part 2)

Let’s interpret graphs of functions.

Focus

Goals

1. **Language Goal:** Describe a graph of a function as *increasing* or *decreasing* over an interval, and explain the reasoning. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret a graph of temperature as a function of time, using language such as *input* and *output*. **(Speaking and Listening, Writing)**

Coherence

Today

Students analyze graphs of functions and use them to respond to problems about a context. Students observe what happens over intervals of input values and learn that graphs can be viewed as dynamic objects that tell stories.

Previously

In Lesson 4, students began making connections between scenarios and graphs that represent them. Students identified the features of graphs that represent functions and graphs that did not represent a function.

Coming Soon

In Lesson 6, students will continue to explore the qualitative aspect of a function relationship by matching graphs to contexts and sketching a graph, given a context.

Rigor

- Students build **conceptual understanding** of graphs of functions.
- Students develop **procedural fluency** by interpreting graphs of functions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	12 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per group
- Activity 2 PDF (answers)

Math Language Development

Review words

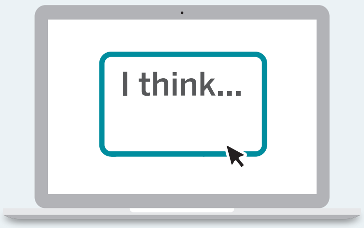
- *decreasing*
- *dependent variable*
- *function*
- *increasing*
- *independent variable*
- *input*
- *output*

Amps | Featured Activity

Activity 2

See Student Thinking

Review students' responses, in real time, to assess their understanding of interpreting time and temperature graphs before they report their findings in small groups.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might consider interpretation of the graph to be a single task in Activity 1 and become overwhelmed about persevering though the activity. Encourage students to break the graph into pieces, and look at smaller pieces that sum to the total graph. By focusing in on a smaller tasks, they can alleviate much of their stress.

Modifications to Pacing

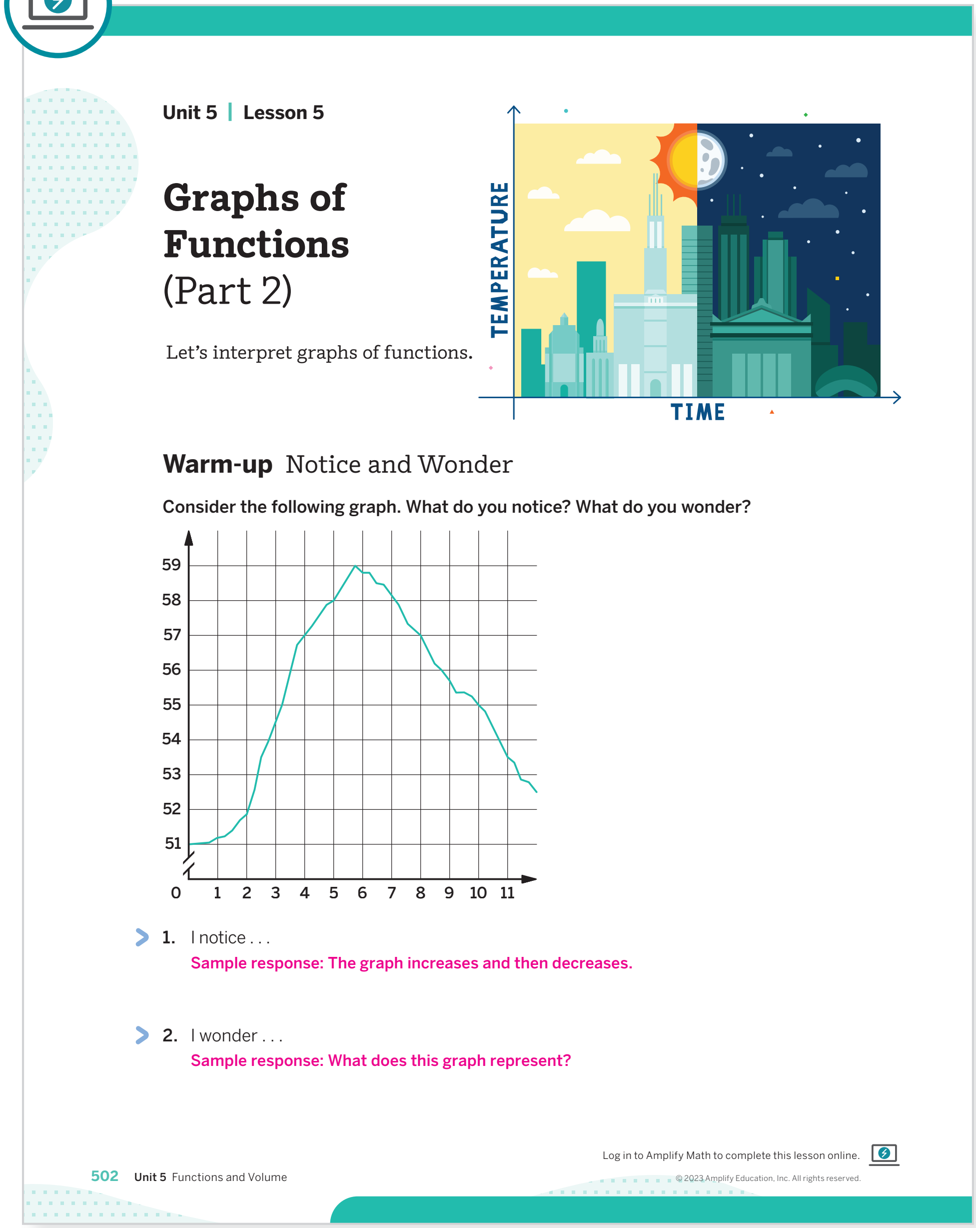
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, Problem 2 may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students analyze a graph of a function to notice and describe the features of the graph using their own language.



Unit 5 | Lesson 5

Graphs of Functions (Part 2)

Let's interpret graphs of functions.

Warm-up Notice and Wonder

Consider the following graph. What do you notice? What do you wonder?

1. I notice . . .
   Sample response: The graph increases and then decreases.

2. I wonder . . .
   Sample response: What does this graph represent?

Log in to Amplify Math to complete this lesson online.

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502 Unit 5 Functions and Volume

1 Launch

Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking them to look for any trends of how the points are plotted on the graph.

Look for productive strategies:

- Noticing the graph increases and then decreases.
- Noticing there are no variables labeled on the axes of the graph.

3 Connect

Display the graph from the Student Edition.

Have students share what they noticed and wondered. Record and display their responses for all to see.

Highlight that students can use the shape of a graph to determine the relationship between two variables. Say, "A function is *increasing* if the output gets larger as the input gets larger. A function is *decreasing* if the output gets smaller as the input gets larger."

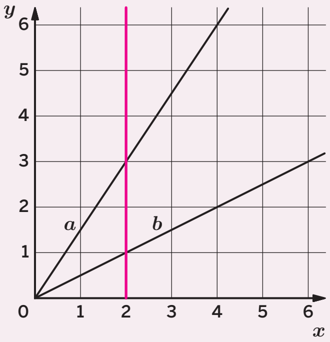
Ask:

- "Does this graph represent a function? How do you know?" Yes. There is one output for each input.
- "What do you think this graph could represent?"

Power-up

To power up students' ability to compare outputs for a given input, have students complete:

1. Draw a vertical line passing through the point $(2, 0)$.
2. Use the line you added in Problem 1 to determine the value of y on line a when $x = 2$. $y = 3$
3. Use the line you added in Problem 1 to determine the value of y on line b when $x = 2$. $y = 1$



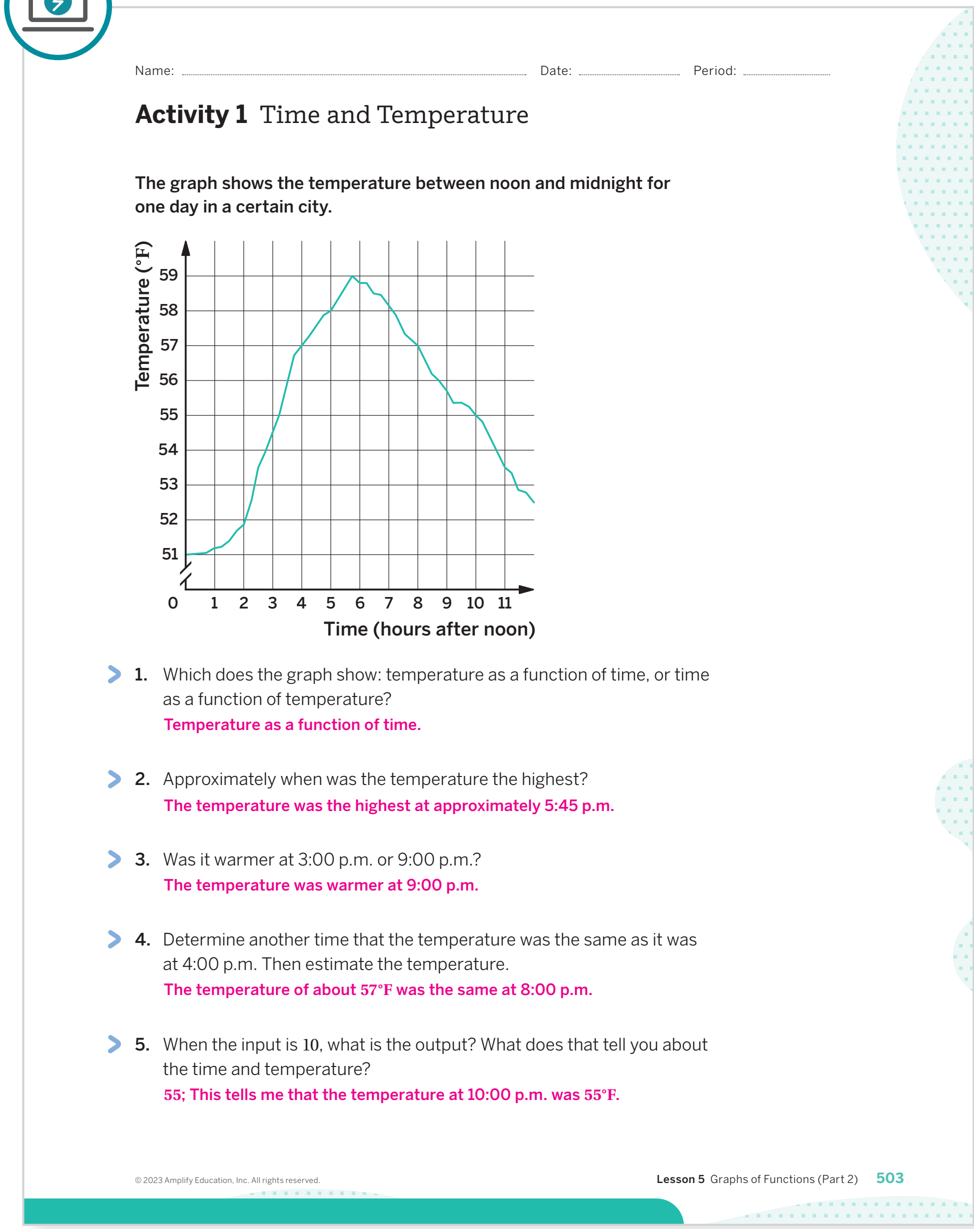
Use: Before Activity 1

Informed by: Performance on Lesson 4, Practice Problem 5

Pairs | 12 min

Activity 1 Time and Temperature

Students analyze a graph of a function to make qualitative observations between two variables.



Name: Date: Period:

Activity 1 Time and Temperature

The graph shows the temperature between noon and midnight for one day in a certain city.

1. Which does the graph show: temperature as a function of time, or time as a function of temperature?
   Temperature as a function of time.

2. Approximately when was the temperature the highest?
   The temperature was the highest at approximately 5:45 p.m.

3. Was it warmer at 3:00 p.m. or 9:00 p.m.?
   The temperature was warmer at 9:00 p.m.

4. Determine another time that the temperature was the same as it was at 4:00 p.m. Then estimate the temperature.
   The temperature of about 57°F was the same at 8:00 p.m.

5. When the input is 10, what is the output? What does that tell you about the time and temperature?
   55; This tells me that the temperature at 10:00 p.m. was 55°F.

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Lesson 5 Graphs of Functions (Part 2) 503

1 Launch

Say, "This graph is the same graph that you saw in the Warm-up, but now with the independent and dependent variables labeled on the axes." Activate students' background knowledge by asking them when they might see this type of weather graph.

2 Monitor

Help students get started by reviewing how to read points on the graph. Consider choosing a single point on the graph and asking students to interpret the context of the point. Tell students they should approximate the values of the points on the graph.

Look for points of confusion:

- **Struggling to interpret points on the graph.** On the graph. have students label *time* as the input and *temperature* as the output. Remind students that each point on the graph can be interpreted as (*time*, *temperature*).

Look for productive strategies:

- Using the shape of the graph rather than focusing on the actual values to respond to certain problems.

3 Connect

Have students share their strategies for Problem 3. Select students who calculated the difference between the two temperatures, and students who compared the vertical change for each input. Have students plot each point on the graph, if they did not do so already.

Highlight that the shape of the graph can help students interpret the context of a graph.

Ask, "When does the temperature get hotter? colder?" Say, "The temperature *increased* from noon to 5:45 p.m. and *decreased* from 5:45 p.m. to midnight."

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can click on specific points of the graph to help them interpret the context of each point selected.

Extension: *Math Enrichment*

Ask students if a graph displaying time and temperature would ever not be a function and have them explain their thinking. Sample response: No, for any specific point in time, there is always one temperature, so temperature will always be a function of time.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

While students work, display the following sentence frame to support their thinking as they respond to Problem 1. Ask them to share during the Connect how they would complete the sentence, focusing on the reasoning.

"_____ is a function of _____ because . . ."

English Learners

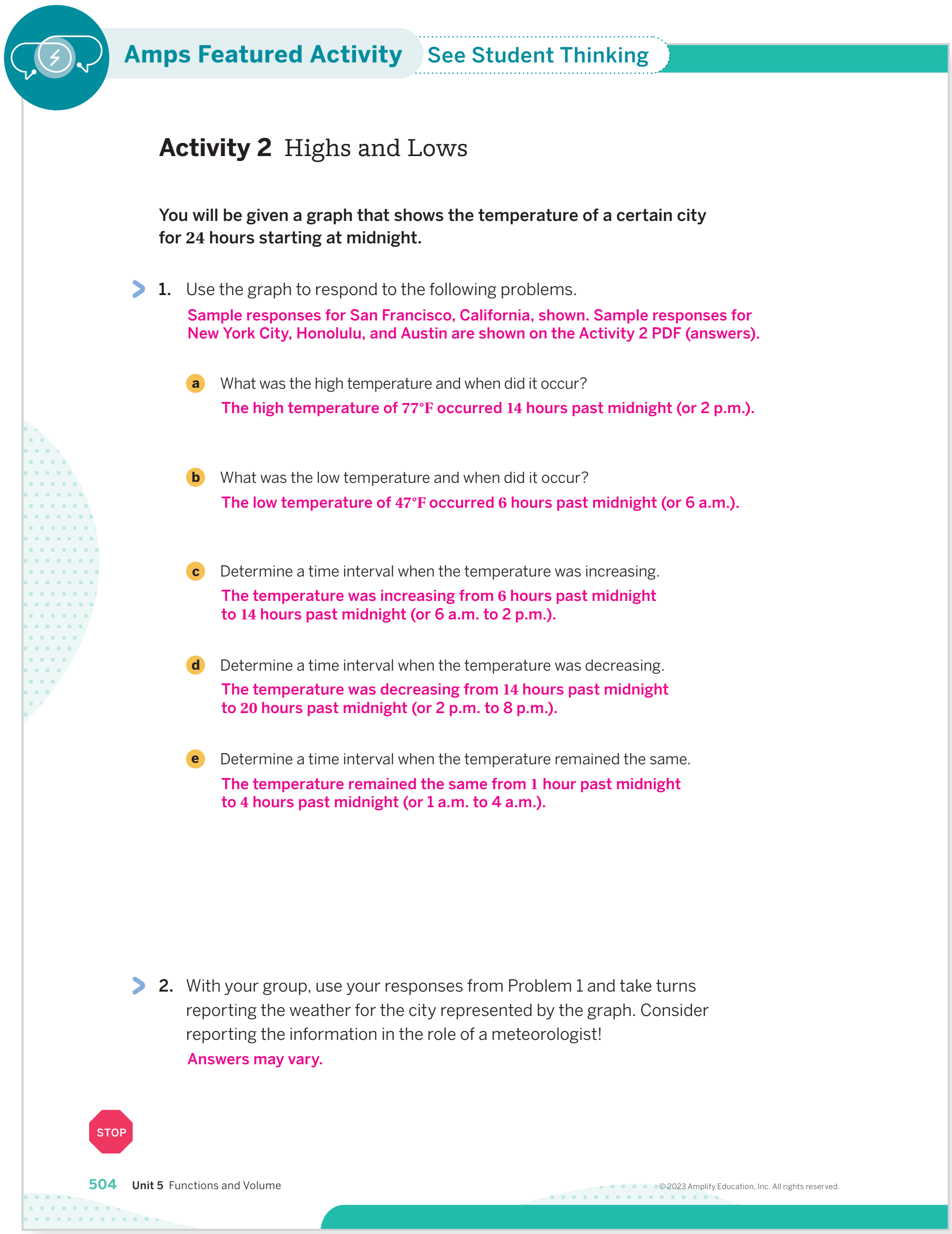
Annotate the increasing sections of the graph as "warmer," "increasing," and "hotter." Annotate the decreasing sections of the graph as "colder" and "decreasing."

Small Groups | 15 min

Activity 2 Highs and Lows

Students interpret more time and temperature graphs to model with mathematics and build procedural fluency.

Amps Featured Activity See Student Thinking



Activity 2 Highs and Lows

You will be given a graph that shows the temperature of a certain city for 24 hours starting at midnight.

1. Use the graph to respond to the following problems.
Sample responses for San Francisco, California, shown. Sample responses for New York City, Honolulu, and Austin are shown on the Activity 2 PDF (answers).

a. What was the high temperature and when did it occur?
The high temperature of 77°F occurred 14 hours past midnight (or 2 p.m.).

b. What was the low temperature and when did it occur?
The low temperature of 47°F occurred 6 hours past midnight (or 6 a.m.).

c. Determine a time interval when the temperature was increasing.
The temperature was increasing from 6 hours past midnight to 14 hours past midnight (or 6 a.m. to 2 p.m.).

d. Determine a time interval when the temperature was decreasing.
The temperature was decreasing from 14 hours past midnight to 20 hours past midnight (or 2 p.m. to 8 p.m.).

e. Determine a time interval when the temperature remained the same.
The temperature remained the same from 1 hour past midnight to 4 hours past midnight (or 1 a.m. to 4 a.m.).

2. With your group, use your responses from Problem 1 and take turns reporting the weather for the city represented by the graph. Consider reporting the information in the role of a meteorologist!
Answers may vary.

STOP

504 **Unit 5** Functions and Volume

1 Launch

Distribute the pre-cut cards from the Activity 2 PDF to each group so that each student in the group receives a different card. Tell students that they will see a *break* on the axes of their graph. Say, "Axis breaks are often used so that two distinct ranges can be displayed in the same graph. Using axis breaks can help you to identify data values more efficiently."

2 Monitor

Help students get started by having them locate the highest point on their graph and estimating the time and temperature.

Look for points of confusion:

- **Not considering the endpoints when determining a high or low temperature.** Remind students that they should look at all of the data starting at 0 hours and ending at 24 hours.
- **Only providing one value for an interval.** Remind students that an interval should have a starting and ended value.
- **Writing a shorter range of an interval that is increasing/decreasing. For example, For San Francisco, students might write the interval from 6 to 9 hours as increasing, rather than 6 to 14 hours**. At this time, allow for these responses, but tell students that the interval 6 to 14 hours is also acceptable as it covers a larger interval where the temperature is generally increasing.

3 Connect

Display the graphs from the Activity 2 PDF.

Ask, "How are the graphs similar? How are they different?"

Highlight that sometimes graphs can have multiple intervals that increase or decrease. The horizontal segments identify where the temperature is constant — it remains the same.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider displaying the graph from Activity 1 and annotate the high temperature, low temperature, where the graph is increasing, and where the graph is decreasing. Keep this graph displayed throughout Activity 2 for students to use as a reference, if needed.

Extension: *Math Enrichment*

Have students research a time-temperature graph for their city that shows how the temperature changes throughout a 24-hour period or other time interval. Have them describe the graph using the language from this activity: high/low temperature, increasing/decreasing/remains the same.

Math Language Development

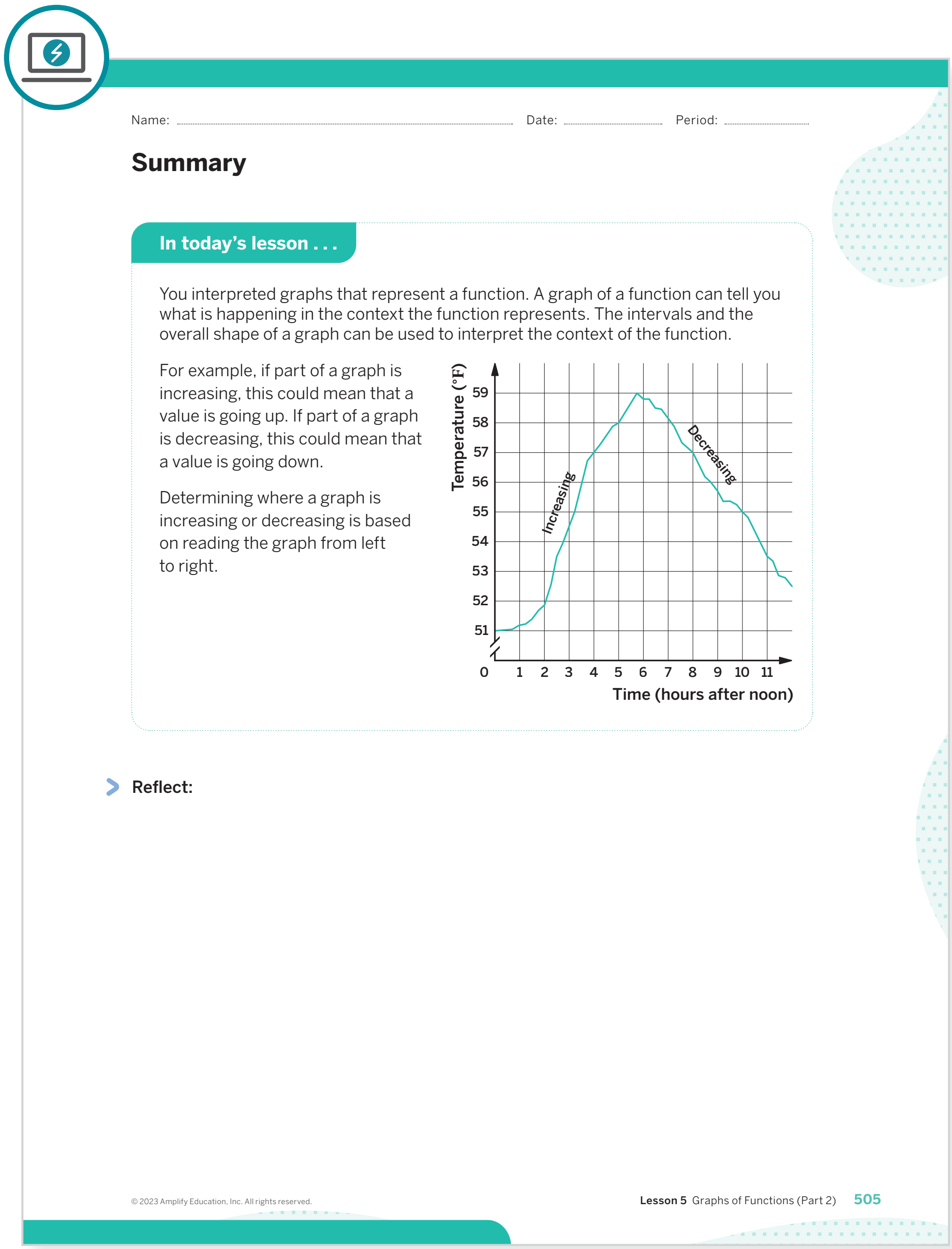
MLR8: Discussion Supports

Display sentence frames to support students as they organize their thoughts and respond to Problem 1. For example,

- "The high/low temperature of _____ degrees occurred at _____."
- "The temperature was increasing/decreasing from _____ to _____."
- "The temperature remained the same from _____ to _____."

Summary

Review and synthesize how to interpret the graphs of functions within real-world contexts, including increasing and decreasing intervals.



Name: ______________________ Date: ____________ Period: ____________

Summary

In today's lesson . . .

You interpreted graphs that represent a function. A graph of a function can tell you what is happening in the context the function represents. The intervals and the overall shape of a graph can be used to interpret the context of the function.

For example, if part of a graph is increasing, this could mean that a value is going up. If part of a graph is decreasing, this could mean that a value is going down.

Determining where a graph is increasing or decreasing is based on reading the graph from left to right.

Reflect:

Synthesize

Have students share their strategies for interpreting points on a graph.

Ask, "How can you tell if the graph of a function is increasing or decreasing?" For a function that is increasing, as the x-value increases, the y-values tend to increase. For a function that is decreasing, as the x-value increases, the y-values tend to decrease.

Highlight that graphs are often used to visually display information. Students can interpret the information on a graph by looking at the shape of the graph.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

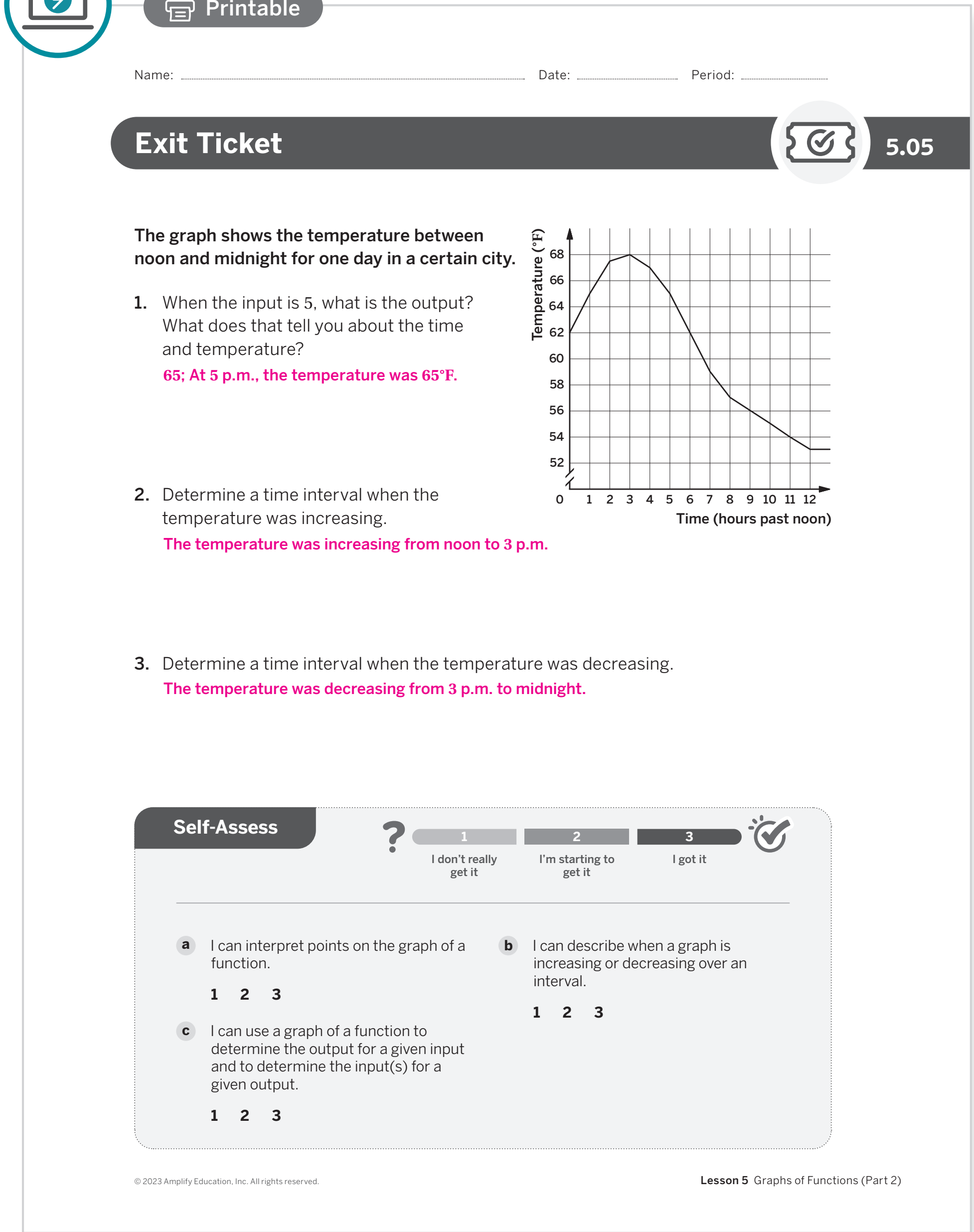
- "What strategies did you find helpful today when interpreting a graph? How were they helpful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by interpreting an input-output pair and naming an interval that is increasing and decreasing.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket

5.05

The graph shows the temperature between noon and midnight for one day in a certain city.

1. When the input is 5, what is the output? What does that tell you about the time and temperature?
   65; At 5 p.m., the temperature was 65°F.
2. Determine a time interval when the temperature was increasing.
   The temperature was increasing from noon to 3 p.m.
3. Determine a time interval when the temperature was decreasing.
   The temperature was decreasing from 3 p.m. to midnight.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can interpret points on the graph of a function.
1 2 3

b. I can describe when a graph is increasing or decreasing over an interval.
1 2 3

c. I can use a graph of a function to determine the output for a given input and to determine the input(s) for a given output.
1 2 3

Lesson 5 Graphs of Functions (Part 2)

Success looks like . . .

- **Language Goal:** Describing a graph of a function as *increasing* or *decreasing* over an interval, and explaining the reasoning. **(Speaking and Listening, Writing)**
  - » Determining time intervals where the temperature was increasing and decreasing in Problems 2 and 3.
- **Language Goal:** Interpreting a graph of temperature as a function of time, using language such as *input* and *output*. **(Speaking and Listening, Writing)**
  - » Determining the output for an input of 5 in Problem 1.

Suggested next steps

If students do not correctly interpret the input-output pair, consider:

- Highlighting the labels on the axes using two different colors. Then writing (hours past noon, temperature) using the corresponding colors.
- Reassessing after Lesson 6.

If students do not correctly determine an interval that is increasing/decreasing or only write part of an interval, consider:

- Reviewing Activity 1.
- Asking, "Between which two times was the temperature going up? Going down?"

Professional Learning

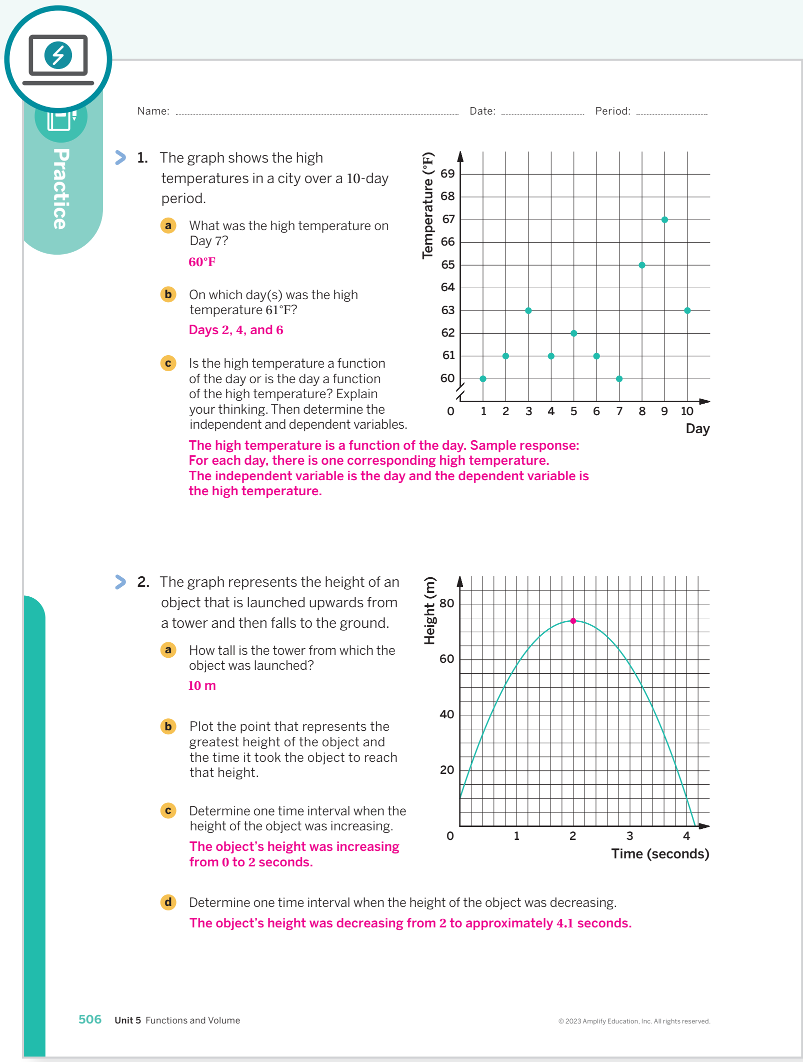
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- This lesson asked students to describe qualitatively the functional relationship between two variables by analyzing a graph. Where in your students' work today did you see or hear evidence of them doing this?
- What surprised you as your students reported the weather cycle in Activity 2, Problem 2?

Practice

Independent



Name: Date: Period:

1. The graph shows the high temperatures in a city over a 10-day period.

a. What was the high temperature on Day 7?

60°F

b. On which day(s) was the high temperature 61°F?

Days 2, 4, and 6

c. Is the high temperature a function of the day or is the day a function of the high temperature? Explain your thinking. Then determine the independent and dependent variables.

The high temperature is a function of the day. Sample response: For each day, there is one corresponding high temperature. The independent variable is the day and the dependent variable is the high temperature.

2. The graph represents the height of an object that is launched upwards from a tower and then falls to the ground.

a. How tall is the tower from which the object was launched?

10 m

b. Plot the point that represents the greatest height of the object and the time it took the object to reach that height.

c. Determine one time interval when the height of the object was increasing.

The object's height was increasing from 0 to 2 seconds.

d. Determine one time interval when the height of the object was decreasing.

The object's height was decreasing from 2 to approximately 4.1 seconds.

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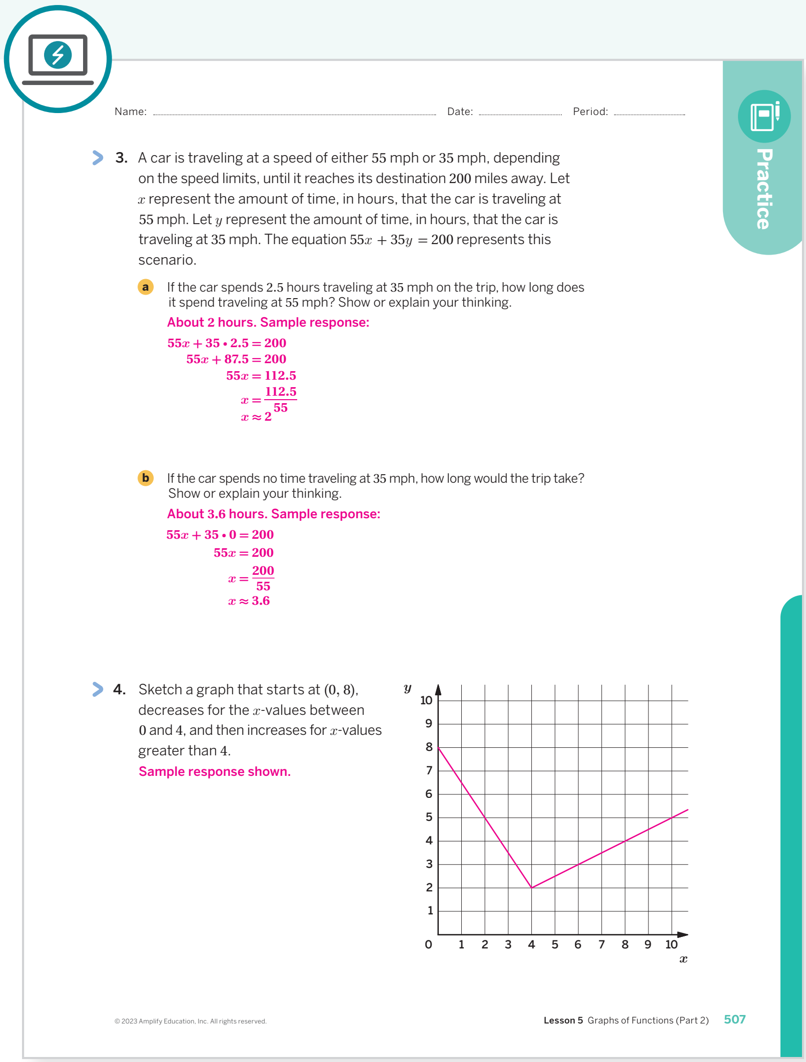

Name: Date: Period:

3. A car is traveling at a speed of either 55 mph or 35 mph, depending on the speed limits, until it reaches its destination 200 miles away. Let x represent the amount of time, in hours, that the car is traveling at 55 mph. Let y represent the amount of time, in hours, that the car is traveling at 35 mph. The equation $55x + 35y = 200$ represents this scenario.

a. If the car spends 2.5 hours traveling at 35 mph on the trip, how long does it spend traveling at 55 mph? Show or explain your thinking.

About 2 hours. Sample response:

$$\begin{aligned} 55x + 35 \cdot 2.5 &= 200 \\ 55x + 87.5 &= 200 \\ 55x &= 112.5 \\ x &= \frac{112.5}{55} \\ x &\approx 2 \end{aligned}$$

b. If the car spends no time traveling at 35 mph, how long would the trip take? Show or explain your thinking.

About 3.6 hours. Sample response:

$$\begin{aligned} 55x + 35 \cdot 0 &= 200 \\ 55x &= 200 \\ x &= \frac{200}{55} \\ x &\approx 3.6 \end{aligned}$$

4. Sketch a graph that starts at $(0, 8)$, decreases for the x-values between 0 and 4, and then increases for x-values greater than 4.

Sample response shown.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	**1**	Activity 1	2
	2	Activity 1	2
Spiral	**3**	Unit 3 Lesson 16	2
Formative	**4**	Unit 5 Lesson 6	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

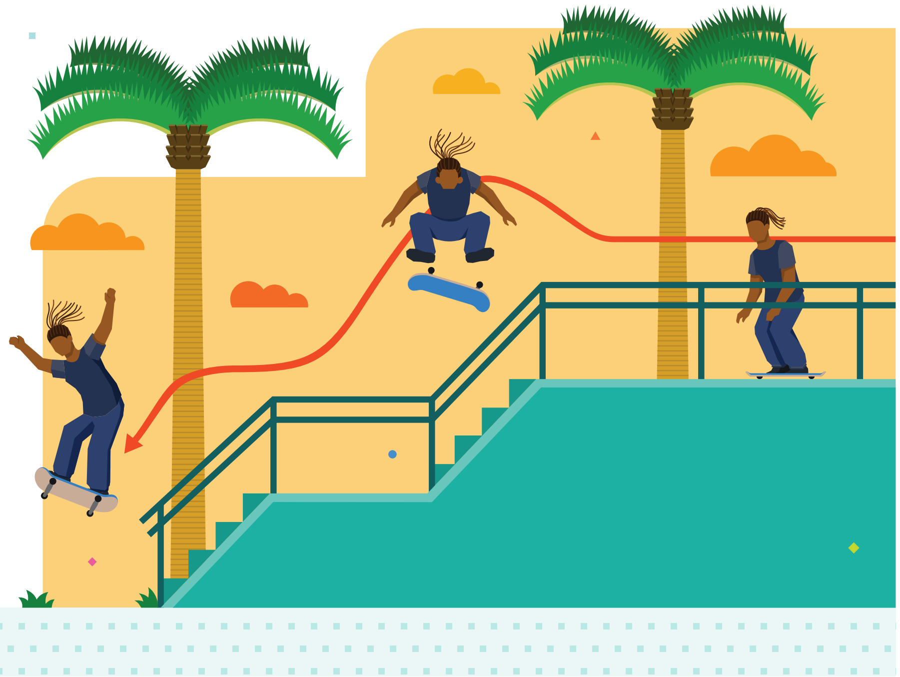
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 6**

Graphs of Functions (Part 3)

Let's make connections between scenarios and the graphs that represent them.



Focus

Goals

1. Comprehend that graphs representing the same context can appear different, depending on the variables chosen.
2. Connect a graph showing the qualitative features of the function described.
3. **Language Goal:** Draw the graph of a function that represents a context, and explain which variable is a function of the other. **(Speaking and Listening)**

Coherence

Today

Students analyze two different graphs that represent the same situation and see that both graphs could represent different aspects of the same scenario, depending on the variables chosen. Students focus on the qualitative aspects of a graph when they match given contexts with their graph and create a sketch after they watch a short video clip of a skateboarding trick.

Previously

In Lesson 5, students analyzed graphs of functions and used them to respond to problems about a context. Students also looked at what happens over intervals of input values and learned that graphs can be viewed as dynamic objects that tell stories.

Coming Soon

In Lesson 7, students will compare tables, equations, graphs, and stories of functions.

Rigor

- Students analyze the qualitative aspects of the graphs of functions to build **fluency**.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, one copy per pair

Math Language Development

Review words

- *decreasing*
- *dependent variable*
- *function*
- *Increasing*
- *independent variable*

Amps | Featured Activity

Activity 2

Repeated Playback

Students watch a video of a skateboarding trick and have the ability to control the playback while they sketch their graph.

Building Math Identity and Community

Connecting to Mathematical Practices

Students' may inadvertently, or even intentionally, show disrespect while critiquing someone else's graph in Activity 2. Challenge students to find appreciation for the differences of interpretation. While errors might be made, by finding similarities and differences in all of the graphs, students can show respect for each other as they determine how to correctly represent the scenario.

Modifications to Pacing

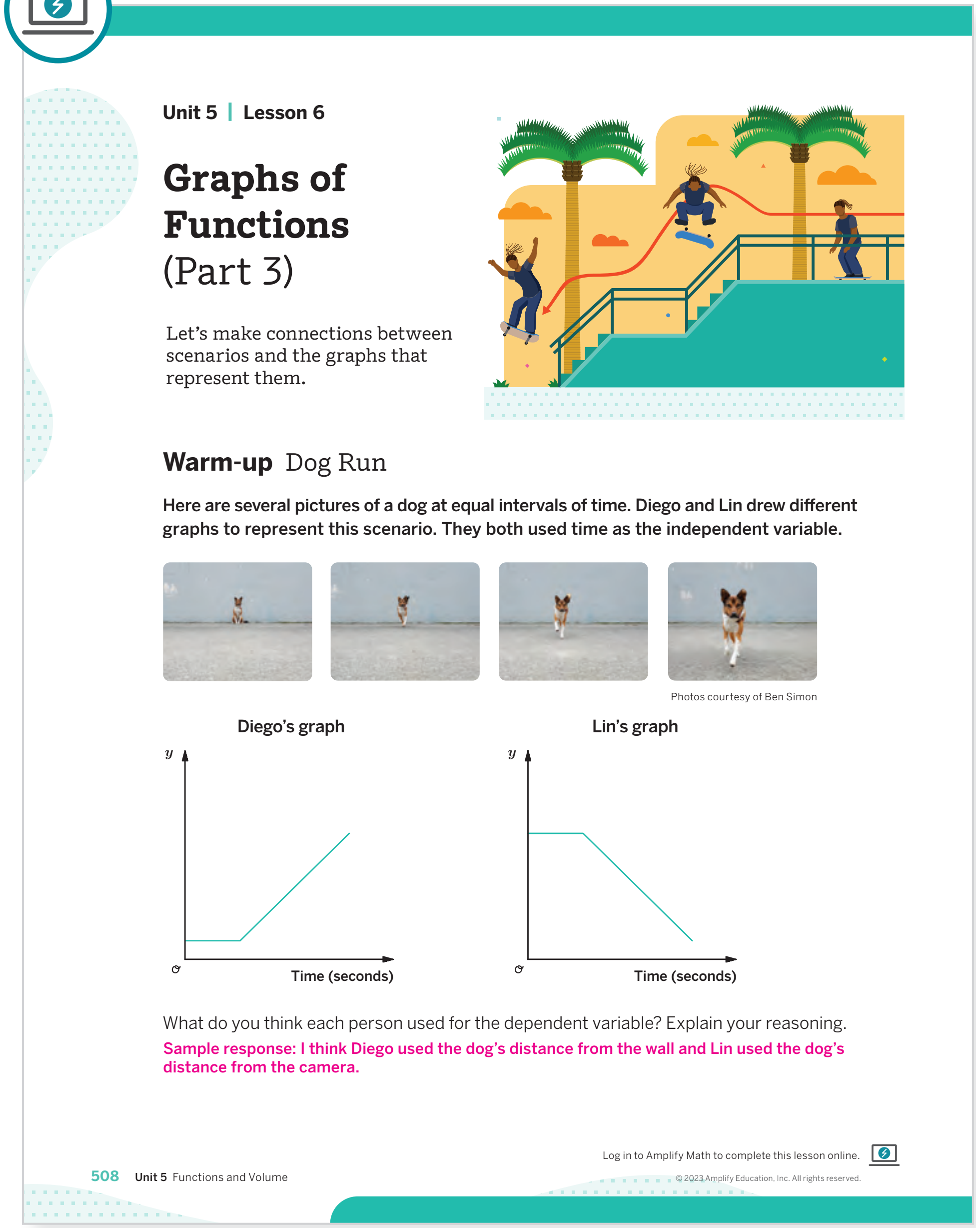
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students only complete Problems 2 and 3.

Pairs | 5 min

Warm-up Dog Run

Students compare two graphs to understand that there are different variables that could be used to describe the same scenario.



Unit 5 | Lesson 6

Graphs of Functions (Part 3)

Let's make connections between scenarios and the graphs that represent them.

Warm-up Dog Run

Here are several pictures of a dog at equal intervals of time. Diego and Lin drew different graphs to represent this scenario. They both used time as the independent variable.

Photos courtesy of Ben Simon

What do you think each person used for the dependent variable? Explain your reasoning.

Sample response: I think Diego used the dog's distance from the wall and Lin used the dog's distance from the camera.

Log in to Amplify Math to complete this lesson online.

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① Launch

Draw students' attention to the different positions of the dog and the dog's distance from the wall and the camera.

② Monitor

Help students get started by asking them if they notice whether any parts of the graph are increasing or decreasing and what it might relate to in the images.

Look for points of confusion:

- **Struggling to determine a dependent variable for either graph.** Ask, "What do you notice about the dog's location in each picture? Is the dog moving closer or further from the wall/camera?"

Look for productive strategies:

- Relating the horizontal part of each graph to no change.

③ Connect

Have students share the different variables they wrote for each graph and their strategies for how they determined the variables.

Ask, "How can labeling the axes of a graph help interpret a scenario?"

Highlight that the same scenario can be represented in different ways depending on what variables are used to describe the scenario.

Power-up

To power up students' ability to sketch parts of a graph given a description:

Provide students with a copy of the Power-up PDF.

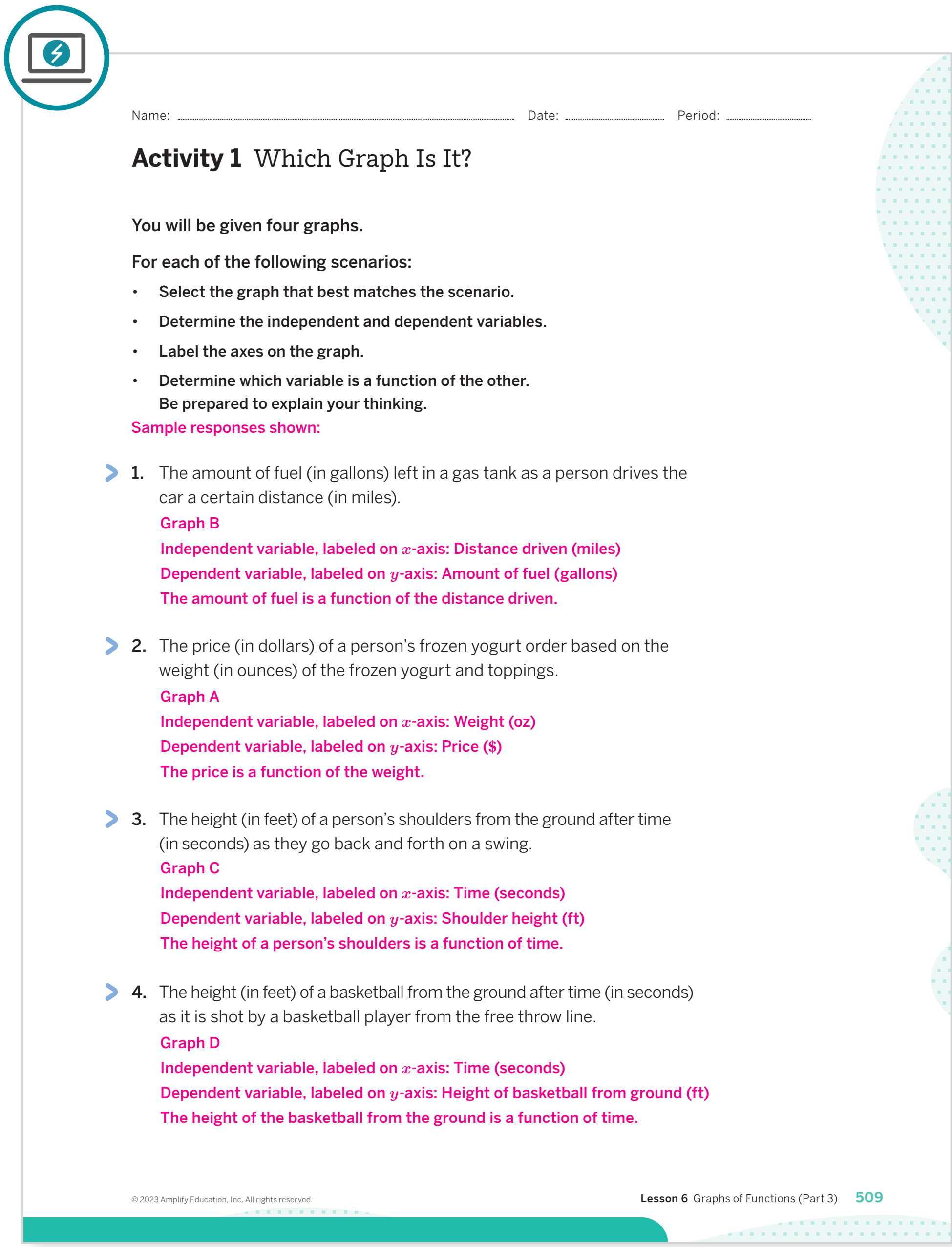
Use: Before the Warm-up

Informed by: Performance on Lesson 5, Practice Problem 4

Pairs | 20 min

Activity 1 Which Graph Is It?

Students match graphs to relate the qualitative features of a function, given a description, and build fluency in naming the independent and dependent variables.



Name: ______ Date: ______ Period: ______

Activity 1 Which Graph Is It?

You will be given four graphs.

For each of the following scenarios:

- Select the graph that best matches the scenario.
- Determine the independent and dependent variables.
- Label the axes on the graph.
- Determine which variable is a function of the other. Be prepared to explain your thinking.

Sample responses shown:

1. The amount of fuel (in gallons) left in a gas tank as a person drives the car a certain distance (in miles).
   Graph B
   Independent variable, labeled on x-axis: Distance driven (miles)
   Dependent variable, labeled on y-axis: Amount of fuel (gallons)
   The amount of fuel is a function of the distance driven.
2. The price (in dollars) of a person's frozen yogurt order based on the weight (in ounces) of the frozen yogurt and toppings.
   Graph A
   Independent variable, labeled on x-axis: Weight (oz)
   Dependent variable, labeled on y-axis: Price ($)
   The price is a function of the weight.
3. The height (in feet) of a person's shoulders from the ground after time (in seconds) as they go back and forth on a swing.
   Graph C
   Independent variable, labeled on x-axis: Time (seconds)
   Dependent variable, labeled on y-axis: Shoulder height (ft)
   The height of a person's shoulders is a function of time.
4. The height (in feet) of a basketball from the ground after time (in seconds) as it is shot by a basketball player from the free throw line.
   Graph D
   Independent variable, labeled on x-axis: Time (seconds)
   Dependent variable, labeled on y-axis: Height of basketball from ground (ft)
   The height of the basketball from the ground is a function of time.

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1 Launch

Distribute the Activity 1 PDF to each student pair and review the directions for the activity.

2 Monitor

Help students get started by asking, "As the independent variable increases, does the dependent variable increase or decrease?"

Look for points of confusion:

- **Struggling to match a graph with its description.** Consider providing sentence frames to support student thinking. For example, "The further a person drives, the amount of gas in the gas tank ____." decreases Then ask students which graph best matches their response.

Look for productive strategies:

- Remembering that the independent variable is labeled along the x-axis and the dependent variable is labeled along the y-axis.

3 Connect

Have students share their responses and strategies for matching each graph to its given context.

Highlight the features of a graph that could help students interpret a scenario.

Ask, "What parts of the graph are important to pay attention to when matching the graph from context?" Sample responses:

- The variables and units.
- Whether the graph increases or decreases.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 2 and 3.

Extension: *Math Enrichment*

Have students sketch a graph that would represent the following scenario.

The height of a bouncing ball (in inches) after it is dropped from a person's hand and rebounds several times over a period of time (in seconds).

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Connect, as students share their matches and strategies, revoice their ideas to demonstrate mathematical language use. For example, if a student says, "Graph B is a function that goes with Problem 1," revoice their idea by saying, "I think I hear you saying that Graph B matches the scenario in Problem 1 and that Graph B is a function. Is that correct? Which variable in this scenario is a function of the other variable?"

English Learners

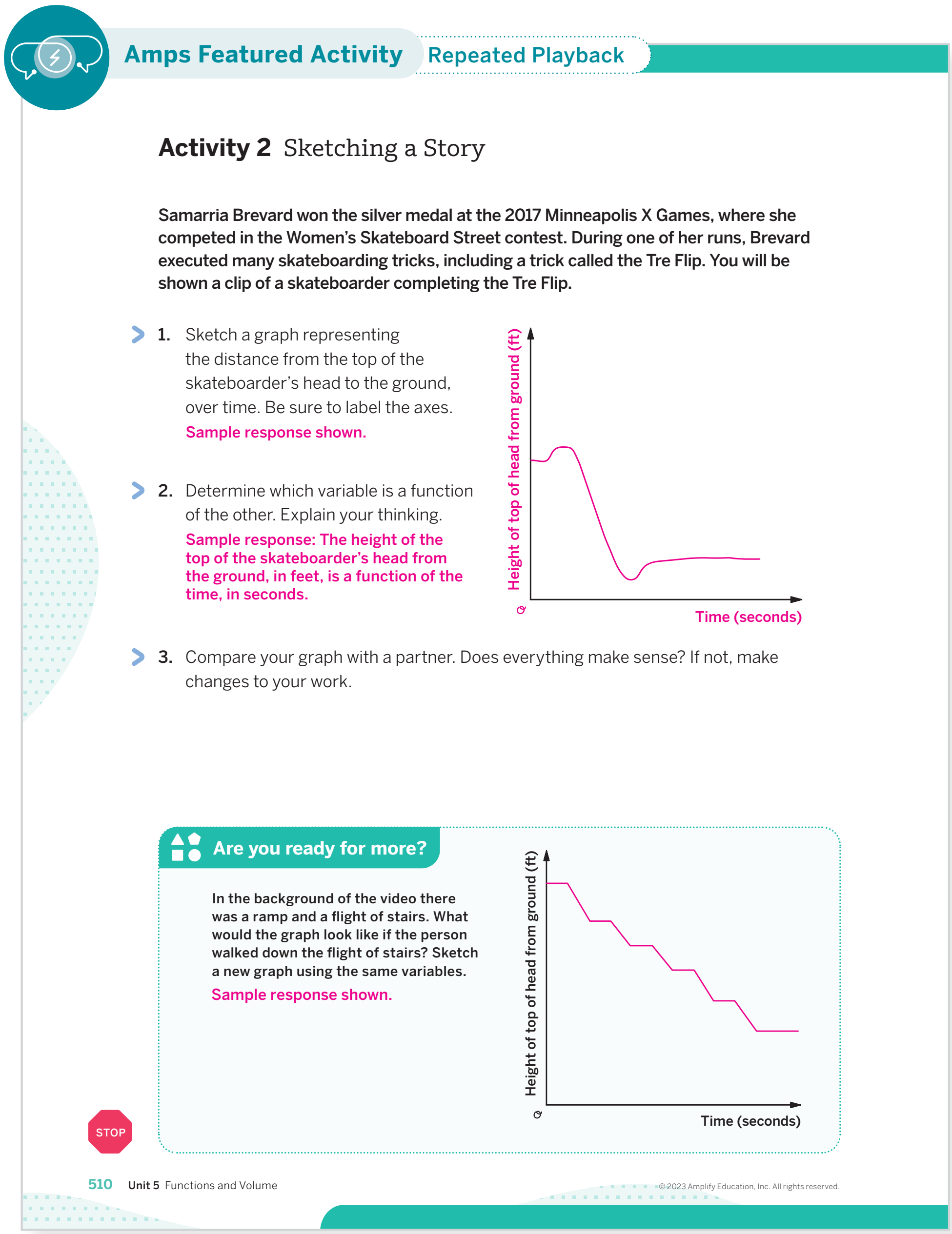
Provide sentence frames to help students organize their thinking, such as:

- "The scenario in Problem ___ matches Graph ___ because . . ."
- "The independent/dependent variable is ___."
- "___ is a function of ___ because . . ."

Pairs | 15 min

Activity 2 Sketching a Story

Students sketch a graph of a function that represents a context to focus on the qualitative aspects of a graph.



Amps Featured Activity Repeated Playback

Activity 2 Sketching a Story

Samarria Brevard won the silver medal at the 2017 Minneapolis X Games, where she competed in the Women's Skateboard Street contest. During one of her runs, Brevard executed many skateboarding tricks, including a trick called the Tre Flip. You will be shown a clip of a skateboarder completing the Tre Flip.

1. Sketch a graph representing the distance from the top of the skateboarder's head to the ground, over time. Be sure to label the axes.
   Sample response shown.

2. Determine which variable is a function of the other. Explain your thinking.
   Sample response: The height of the top of the skateboarder's head from the ground, in feet, is a function of the time, in seconds.

3. Compare your graph with a partner. Does everything make sense? If not, make changes to your work.

Are you ready for more?

In the background of the video there was a ramp and a flight of stairs. What would the graph look like if the person walked down the flight of stairs? Sketch a new graph using the same variables.
Sample response shown.

1 Launch

Display the video, *Completing the TreFlip*, from the Activity 2 Amps slides. Let students know that a "sketch" is a rough drawing and does not need to be an exact representation. Consider replaying the video three times, so that students can adjust and improve their sketches each time.

2 Monitor

Help students get started by activating their prior knowledge about independent and dependent variables.

Look for points of confusion:

- **Switching the independent and dependent variables.** Remind students of input-output diagrams and ask them to identify whether the time would represent the input or the output.
- **Starting or ending the graph at $(0, 0)$.** Ask, "When the video starts/ends, what is the distance from the top of the skateboarder's head to the ground?"

3 Connect

Display student work showing different sketches. Have students identify the similarities and differences in the sketches. Facilitate class discussion by asking students to interpret the minimum and maximum points.

Highlight that when sketching a graph from a context, it is important to pay attention to the unit(s) being measured. For example, the distance from the top of the skateboarder's head to the ground (in feet) versus the distance of the skateboard from the ground.

Ask, "How would the graph change if the dependent variable was the distance (in feet) of the skateboard from the ground?" Sample response: If the same scales were used, the overall graph would be closer to the x-axis.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can watch a video of a skateboarding trick and are able to control the playback while they sketch their graph.

Accessibility: *Vary Demands to Optimize Challenge*

Provide a partially-completed graph in Problem 1, such as the very beginning, and have students watch the video to complete the remaining parts of the graph.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect sketch that does not represent the scenario. Ask:

- ***Critique:*** "Could this graph represent the scenario? Which parts are correct, if any, and which parts are incorrect?"
- ***Correct and Clarify:*** "If a classmate sketched this graph, how could you convince them as to how they should make corrections? What math language can you use?"

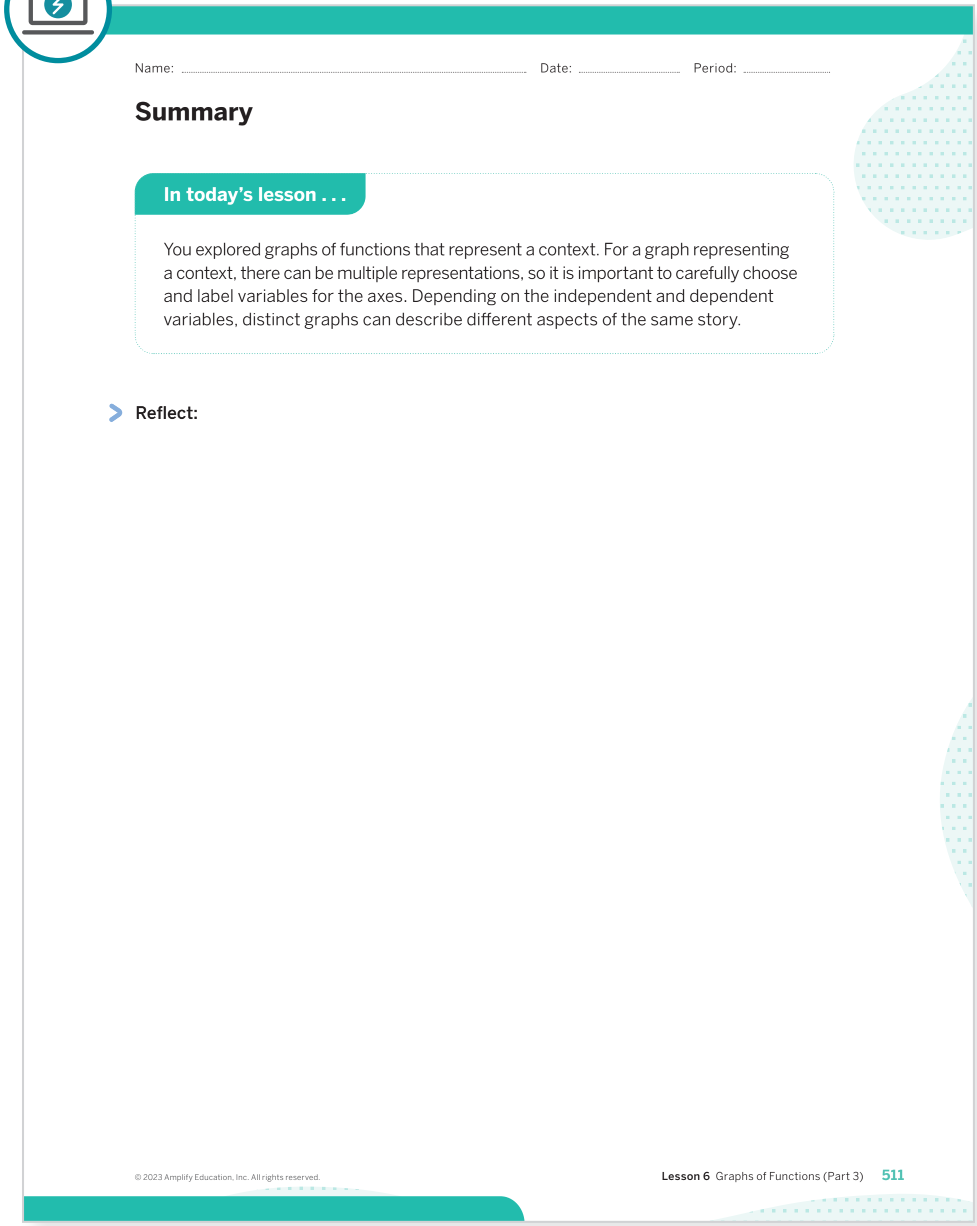
English Learners

Use gestures to trace along the graph to illustrate how it represents the distance from the top of the skateboarder's head over time.

Whole Class | 5 min

Summary

Review and synthesize the importance of selecting and labeling the variables on the axes of a graph that represents a real-world context.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored graphs of functions that represent a context. For a graph representing a context, there can be multiple representations, so it is important to carefully choose and label variables for the axes. Depending on the independent and dependent variables, distinct graphs can describe different aspects of the same story.

Reflect:

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Synthesize

Have students share their strategies for matching or drawing a graph given a context.

Highlight that, for a graph representing a context, there can be multiple representations, so it is important to choose and label variables for the axes.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

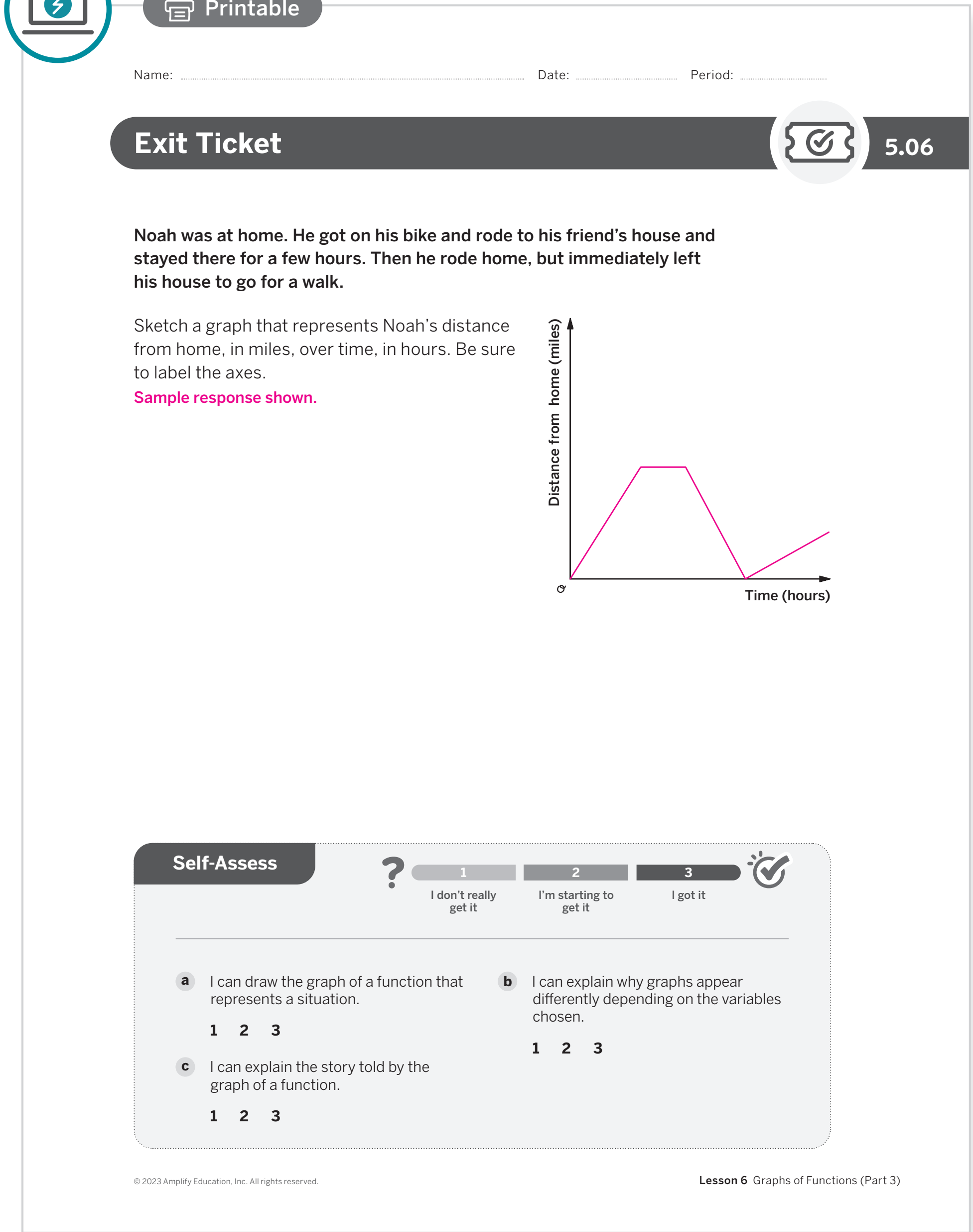
- "What strategies or tools did you find helpful today when sketching the graph of a function from a context? How were they helpful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the qualitative features of graphs by sketching a graph that represents a real-world context, without quantitative values.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.06

Noah was at home. He got on his bike and rode to his friend's house and stayed there for a few hours. Then he rode home, but immediately left his house to go for a walk.

Sketch a graph that represents Noah's distance from home, in miles, over time, in hours. Be sure to label the axes.

Sample response shown.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can draw the graph of a function that represents a situation.
1 2 3

b I can explain why graphs appear differently depending on the variables chosen.
1 2 3

c I can explain the story told by the graph of a function.
1 2 3

Lesson 6 Graphs of Functions (Part 3)

Success looks like . . .

- **Goal:** Comprehending that graphs representing the same context can appear different, depending on the variables chosen.
- **Goal:** Connecting a graph showing the qualitative features of the function described.
- **Language Goal:** Drawing the graph of a function that represents a context, and explaining which variable is a function of the other. **(Speaking and Listening)**
  - » Sketching the graph representing Noah's distance.

Suggested next steps

If students struggle to create an accurate graph, consider:

- Providing the axes labels on the graphs.
- Reviewing Activity 1.
- Reviewing how to identify whether a graph is increasing or decreasing.

Professional Learning

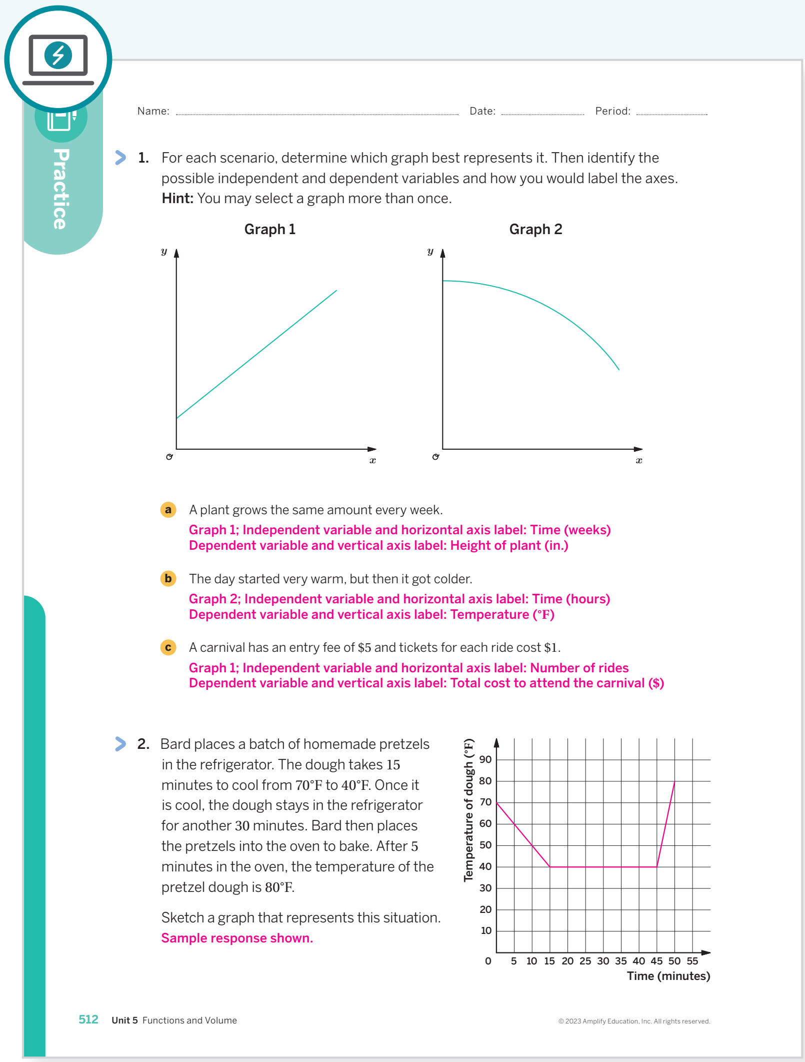
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Who participated and who didn't participate in today's lesson? What trends do you see in participation?
- What different ways did students approach sketching a graph from a context? What does that tell you about similarities and differences among your students?

Practice

Independent



Name: Date: Period:

1. For each scenario, determine which graph best represents it. Then identify the possible independent and dependent variables and how you would label the axes. **Hint:** You may select a graph more than once.

   a. A plant grows the same amount every week.
   Graph 1; Independent variable and horizontal axis label: Time (weeks)
   Dependent variable and vertical axis label: Height of plant (in.)

   b. The day started very warm, but then it got colder.
   Graph 2; Independent variable and horizontal axis label: Time (hours)
   Dependent variable and vertical axis label: Temperature (°F)

   c. A carnival has an entry fee of $5 and tickets for each ride cost $1.
   Graph 1; Independent variable and horizontal axis label: Number of rides
   Dependent variable and vertical axis label: Total cost to attend the carnival ($)

2. Bard places a batch of homemade pretzels in the refrigerator. The dough takes 15 minutes to cool from 70°F to 40°F. Once it is cool, the dough stays in the refrigerator for another 30 minutes. Bard then places the pretzels into the oven to bake. After 5 minutes in the oven, the temperature of the pretzel dough is 80°F.

   Sketch a graph that represents this situation.
   Sample response shown.

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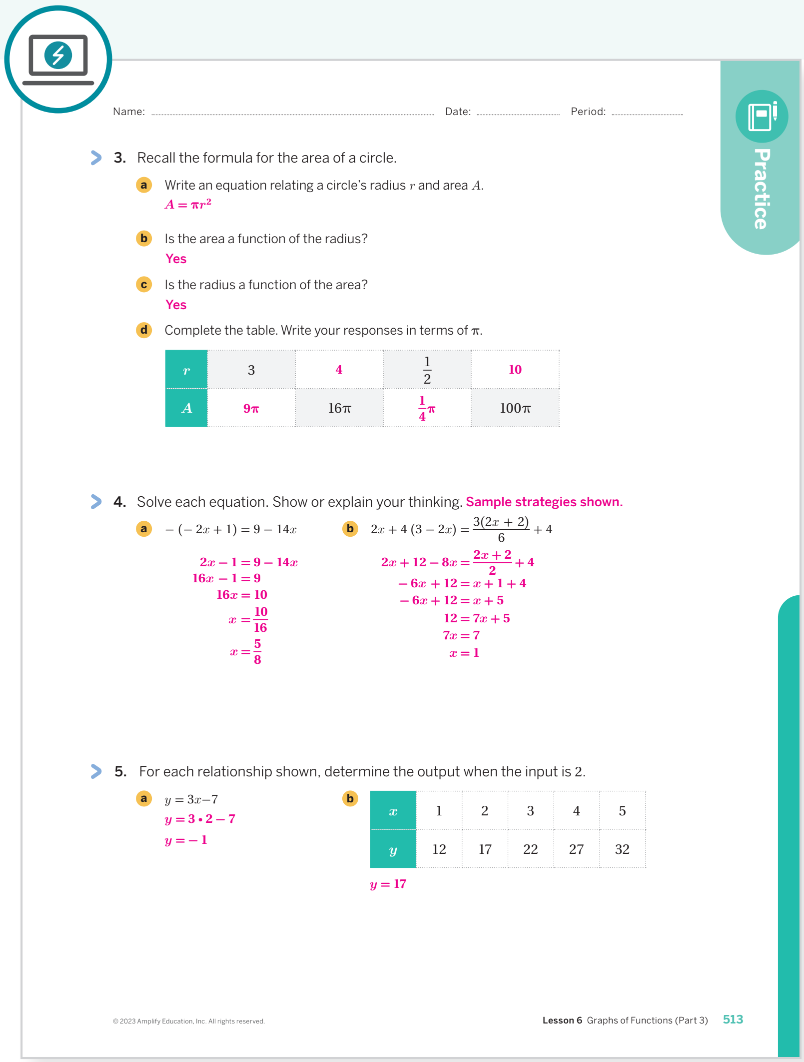

Name: Date: Period:

3. Recall the formula for the area of a circle.

   a. Write an equation relating a circle's radius r and area A.
   $A = \pi r^2$

   b. Is the area a function of the radius?
   Yes

   c. Is the radius a function of the area?
   Yes

   d. Complete the table. Write your responses in terms of π.

r	3	4	$\frac{1}{2}$	10
A	9π	16π	$\frac{1}{4}\pi$	100π

4. Solve each equation. Show or explain your thinking. **Sample strategies shown.**

   a. $-(-2x+1) = 9 - 14x$

$$2x - 1 = 9 - 14x$$
$$16x - 1 = 9$$
$$16x = 10$$
$$x = \frac{10}{16}$$
$$x = \frac{5}{8}$$

   b. $2x + 4\,(3 - 2x) = \frac{3(2x+2)}{6} + 4$

$$2x + 12 - 8x = \frac{2x+2}{2} + 4$$
$$-6x + 12 = x + 1 + 4$$
$$-6x + 12 = x + 5$$
$$12 = 7x + 5$$
$$7x = 7$$
$$x = 1$$

5. For each relationship shown, determine the output when the input is 2.

   a. $y = 3x{-}7$
   $y = 3 \cdot 2 - 7$
   $y = -1$

   b.

x	1	2	3	4	5
y	12	17	22	27	32

   $y = 17$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
Spiral	3	Grade 7	1
	4	Unit 4 Lesson 5	2
Formative ⚡	5	Unit 5 Lesson 7	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 7**

Connecting Representations of Functions

Let's connect tables, equations, graphs, and stories of functions.

Focus

Goals

1. **Language Goal:** Compare and contrast representations of functions, and describe the strengths and weaknesses of each type of representation. **(Speaking and Listening)**
2. **Language Goal:** Interpret multiple representations of functions, including graphs, tables, and equations, and explain how to find information in each type of representation. **(Speaking and Listening)**

Coherence

Today

Students apply their understanding of functions to interpret and compare data given by a table, graph, and equation simultaneously. Students explore the strengths and weaknesses of interpreting data from a table, graph, and equation as they extract different information provided by each representation.

Previously

In prior lessons, students interpreted functions represented either by a table, equation, or graph. They used each representation to determine input-output pairs and interpreted what the input-output pair represented in context.

Coming Soon

In Lesson 8, students will investigate and make connections between linear functions as represented by graphs, descriptions, and equations.

Rigor

- Students interpret different representations of functions to build **procedural skills**.
- Students **apply** their understanding of functions when they interpret different representations of functions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, one copy per group

Math Language Development

Review words

- *function*
- *input*
- *linear relationship*
- *output*

Amps | Featured Activity

Exit Ticket
Real-Time Exit Ticket

Check in real time if your students can interpret different representations of functions using a digital Exit Ticket that is automatically scored.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not persevere with comparing volumes in Activity 2 but stop with determining the volumes. Point out that there is a lot to be gained by listening to others and how they approached the task. By taking on others' perspectives, they can better understand the how to go one step further with comparing.

Modifications to Pacing

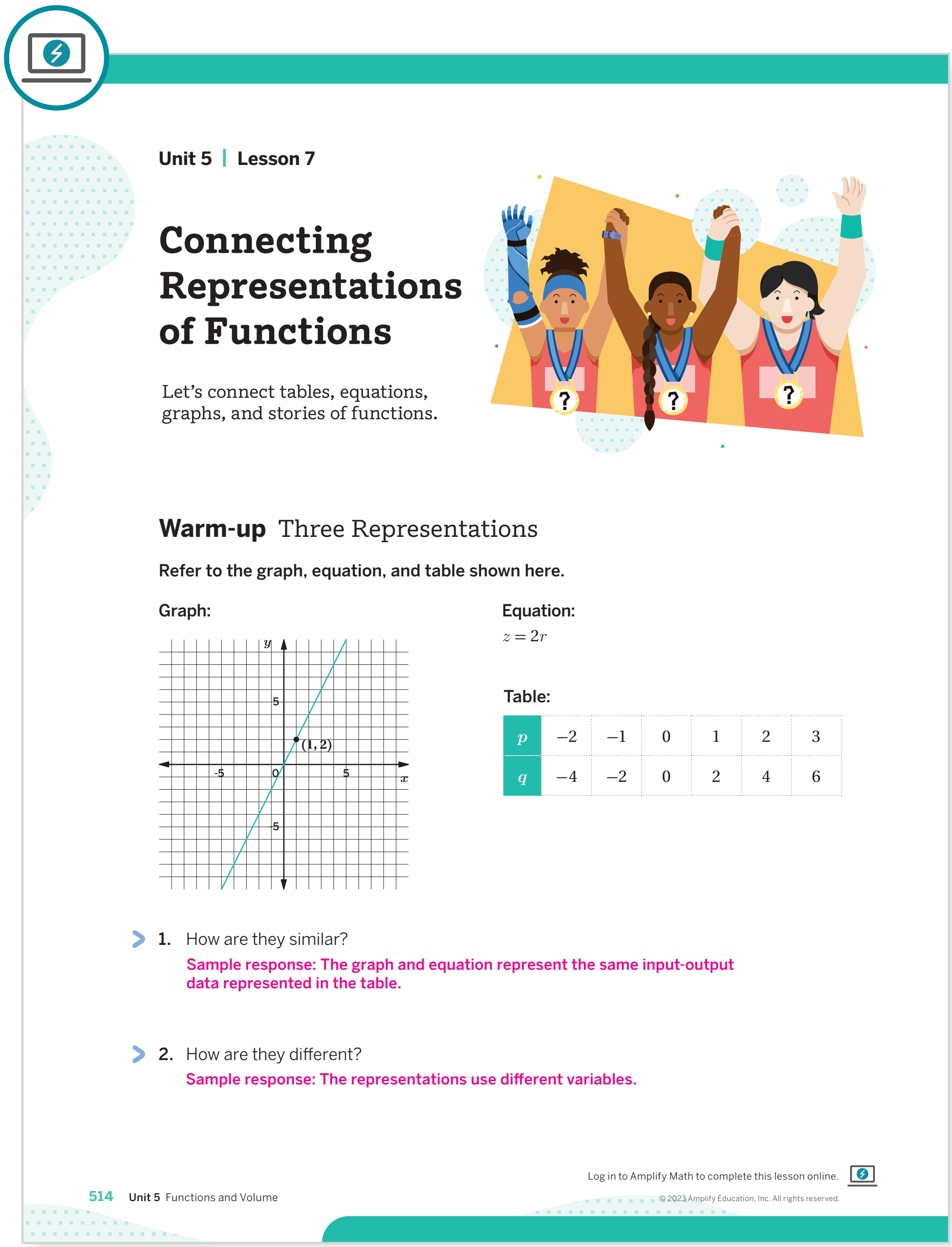
You may want to consider this additional modification if you are short on time.

- **Activity 2** may be assigned as practice, as needed.

Pairs | 5 min

Warm-up Three Representations

Students compare different representations of a linear function to look for similarities and differences of data represented in a table, a graph, and an equation.



Unit 5 | Lesson 7

Connecting Representations of Functions

Let's connect tables, equations, graphs, and stories of functions.

Warm-up Three Representations

Refer to the graph, equation, and table shown here.

Graph:

Equation:

$z = 2r$

Table:

p	−2	−1	0	1	2	3
q	−4	−2	0	2	4	6

1. How are they similar?

Sample response: The graph and equation represent the same input-output data represented in the table.

2. How are they different?

Sample response: The representations use different variables.

Log in to Amplify Math to complete this lesson online.

514 Unit 5 Functions and Volume

1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by asking them about the characteristics of a linear relationship and how to identify these characteristics in a table, graph and equation.

Look for productive strategies:

- Noticing that each representation gives the same output for each input.
- Recognizing that all three representations are linear.
- Identifying the slope and vertical intercept for each representation.

3 Connect

Have students share how the representations are similar and different. Record and display the responses for all to see.

Ask:

- "What type of relationship do the graph, equation, and table represent?" Sample response: Proportional or linear relationships.
- "How could you use each representation to determine the output if the input is 1?"
- "Do the graph, equation, and table each represent a function? How do you know?" Yes, there is only one output for each input.

Highlight that students can use each representation to determine input-output pairs. While each representation uses different letters as their variables, each representation represents the same function.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their observations, revoice the connections they see using mathematical vocabulary they have previously learned (*proportional, linear, slope, rate of change*) and their developing mathematical language from this unit (*function, input, output, independent variable, dependent variable*). Press them to describe how these features can be "seen" in the other representations. For example, ask, "Where do you see the input-output data from the table in the graph?"

English Learners

Use annotations to make visual connections between the representations clear to students.

Power-up

To power up students' ability to determine the output from table, graph, or equation:

Provide students with a copy of the Power-up PDF.

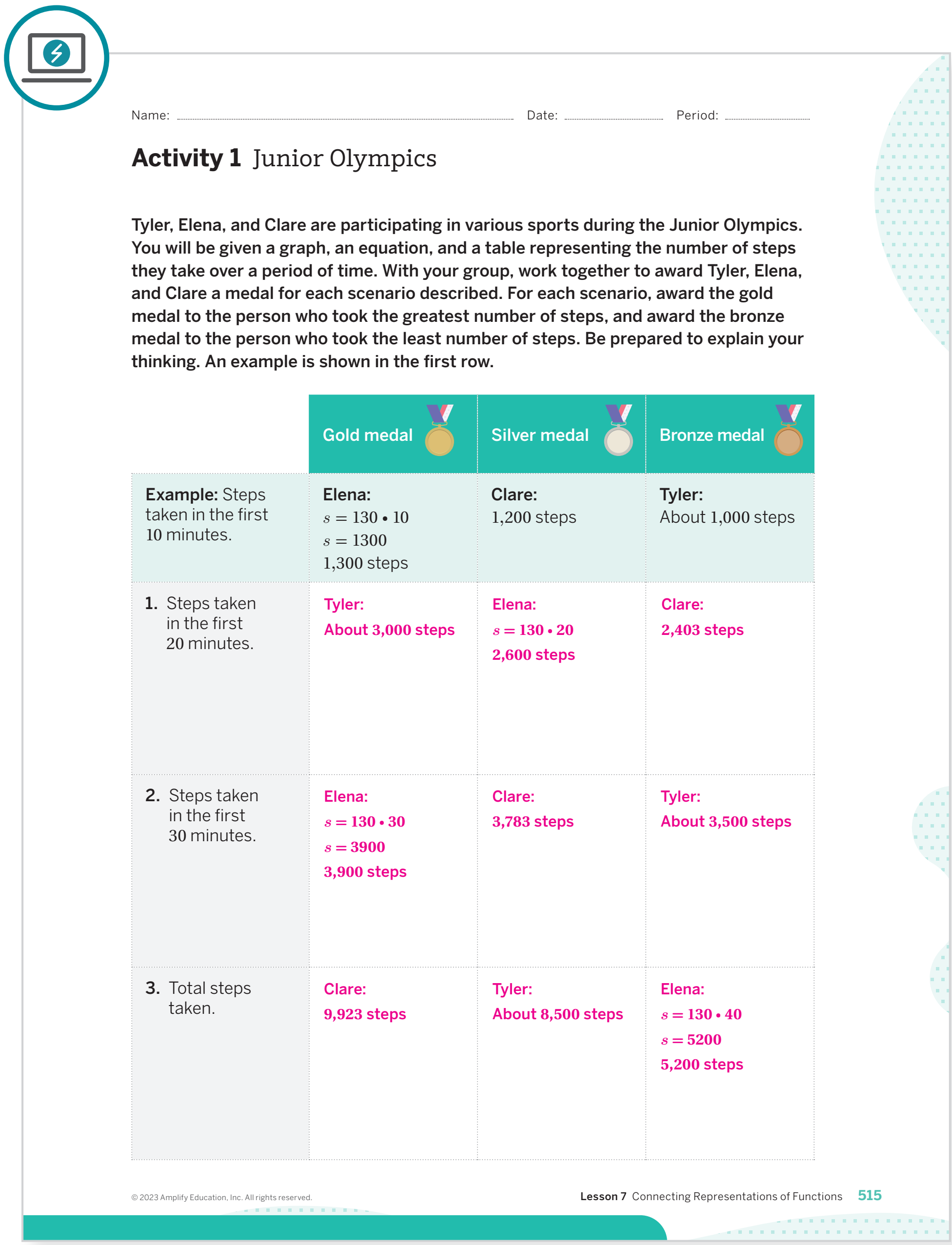
Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 5

Small Groups | 15 min

Activity 1 Junior Olympics

Students compare different functions represented in different ways to make sense of each representation and describe the strengths and weaknesses of each representation.



Name: Date: Period:

Activity 1 Junior Olympics

Tyler, Elena, and Clare are participating in various sports during the Junior Olympics. You will be given a graph, an equation, and a table representing the number of steps they take over a period of time. With your group, work together to award Tyler, Elena, and Clare a medal for each scenario described. For each scenario, award the gold medal to the person who took the greatest number of steps, and award the bronze medal to the person who took the least number of steps. Be prepared to explain your thinking. An example is shown in the first row.

	Gold medal	Silver medal	Bronze medal
Example: Steps taken in the first 10 minutes.	**Elena:** $s = 130 \cdot 10$ $s = 1300$ 1,300 steps	**Clare:** 1,200 steps	**Tyler:** About 1,000 steps
1. Steps taken in the first 20 minutes.	**Tyler:** **About 3,000 steps**	**Elena:** $s = 130 \cdot 20$ **2,600 steps**	**Clare:** **2,403 steps**
2. Steps taken in the first 30 minutes.	**Elena:** $s = 130 \cdot 30$ $s = 3900$ **3,900 steps**	**Clare:** **3,783 steps**	**Tyler:** **About 3,500 steps**
3. Total steps taken.	**Clare:** **9,923 steps**	**Tyler:** **About 8,500 steps**	**Elena:** $s = 130 \cdot 40$ $s = 5200$ **5,200 steps**

1 Launch

Distribute the Activity 1 PDF to each group. Activate students' background knowledge by asking if they have ever used a fit tracker to track the number of steps they take each day. Review the prompt, answering any questions students may have.

2 Monitor

Help students get started by reviewing the example provided in the table and how students can award the medals to Clare, Elena, and Tyler.

Look for points of confusion:

- **Not knowing how to determine the number of steps taken from a certain representation.**
  - » For the graph, remind students to read the labels carefully, and estimate their response as closely as possible.
  - » For the equation, have students annotate what is known and unknown.
  - » For the table, have students label each variable in the table.
- **Using the same time to determine each person's number of steps taken.** Remind students that Tyler competed for 60 minutes, Elena competed for 40 minutes, and Clare competed for 70 minutes. Have students use these corresponding values to determine their response.

3 Connect

Have students share their responses and how they used each representation to determine the information needed to award a placement to each person.

Ask:

- "What was challenging (or not challenging) about using each representation?"
- "What type of question do you prefer to answer with each representation?"

Highlight the benefits and drawbacks of each representation.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to graph paper should students choose to graph Elena's and Clare's relationship. Provide access to colored pencils and suggest that students color code the independent variables in one color and the dependent variables in another color, for each person's representation.

Extension: *Math Enrichment*

Challenge students to estimate who took steps at the fastest average rate overall, and then estimate who took steps at the fastest average rate for the first 40 minutes. Overall: Clare, about 142 steps per minute. First 40 minutes: Elena, 130 steps per minute.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the Activity 1 PDF. Prior to students beginning the activity, ask them to work with their small groups to write 2–3 questions they could ask about the three representations shown. Sample questions shown.

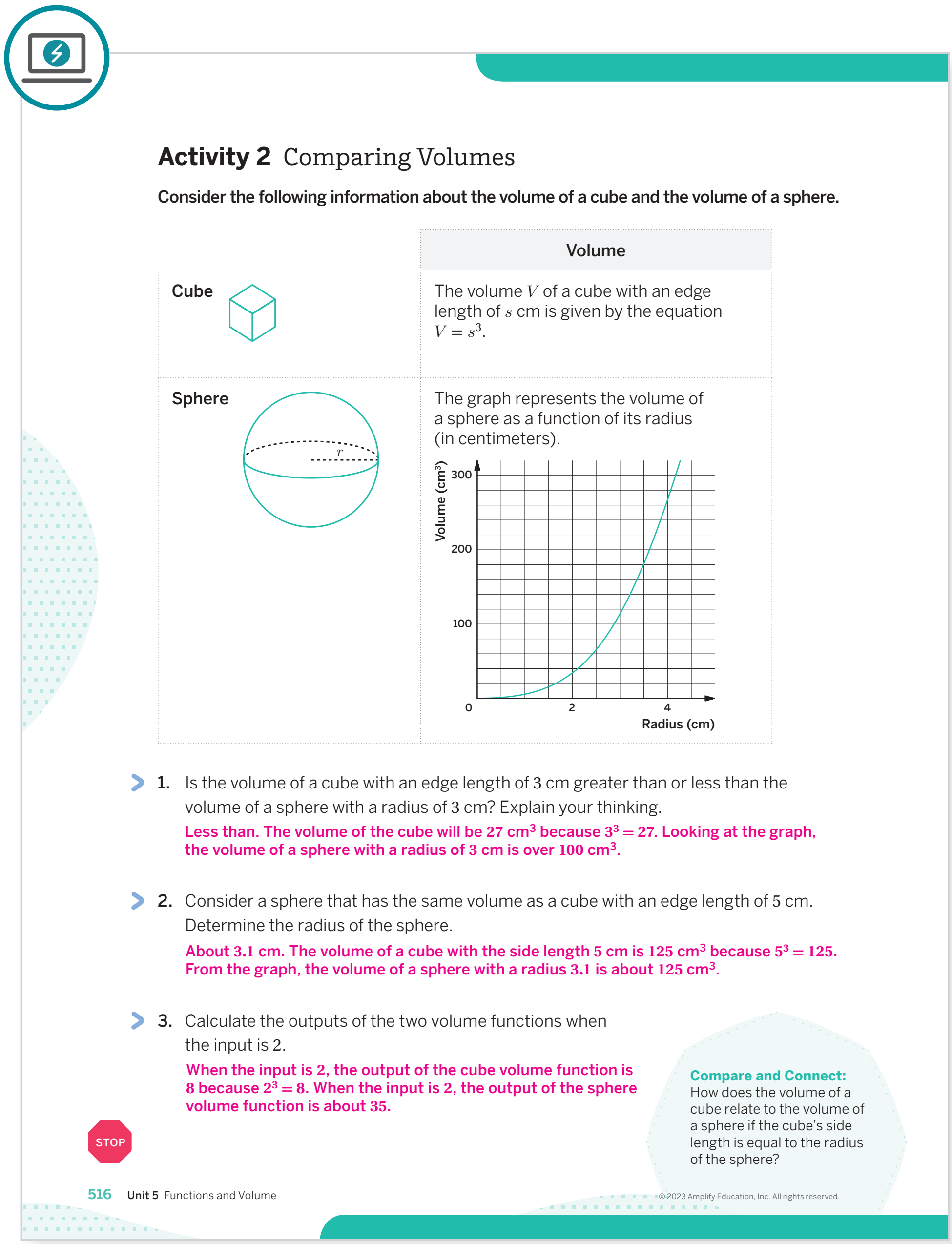
- Who took the most number of steps?
- Who took steps at the fastest rate?

Select 2–3 groups to share their questions with the class. Highlight questions that invite comparisons between each representation.

Pairs | 15 min

Activity 2 Comparing Volumes

Students are given an equation and a graph of the volumes of two different solids to compare inputs and outputs of both functions and interpret the values in context.



Activity 2 Comparing Volumes

Consider the following information about the volume of a cube and the volume of a sphere.

	Volume
Cube	The volume V of a cube with an edge length of s cm is given by the equation $V = s^3$.
Sphere	The graph represents the volume of a sphere as a function of its radius (in centimeters).

1. Is the volume of a cube with an edge length of 3 cm greater than or less than the volume of a sphere with a radius of 3 cm? Explain your thinking.
   Less than. The volume of the cube will be 27 cm^3 because $3^3 = 27$. Looking at the graph, the volume of a sphere with a radius of 3 cm is over 100 cm^3.

2. Consider a sphere that has the same volume as a cube with an edge length of 5 cm. Determine the radius of the sphere.
   About 3.1 cm. The volume of a cube with the side length 5 cm is 125 cm^3 because $5^3 = 125$. From the graph, the volume of a sphere with a radius 3.1 is about 125 cm^3.

3. Calculate the outputs of the two volume functions when the input is 2.
   When the input is 2, the output of the cube volume function is 8 because $2^3 = 8$. When the input is 2, the output of the sphere volume function is about 35.

Compare and Connect: How does the volume of a cube relate to the volume of a sphere if the cube's side length is equal to the radius of the sphere?

1 Launch

Activate students' prior knowledge by asking what it means to find the volume of an object. Ask students to name different objects that are the shape of a cube and a sphere. **Note:** Students will officially explore the volume of a sphere in Lesson 17.

2 Monitor

Help students get started by reviewing how to determine the volume for each solid given the formula or graph.

Look for points of confusion:

- Struggling to understand the problem. Have students reread the problem and then ask, "What information is given to you and what do you need to find?"

Look for productive strategies:

- Labeling the inputs and outputs on the graph for each problem.
- Using the placement of the variables in the equation and the position of the labels in the graph to determine the input and output.

3 Connect

Display the equation and graph.

Have students share how they used each representation to compare the volumes of a cube and a sphere.

Highlight that students can use each representation to determine the same information, but for each representation, they may perform different actions.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students create a table of values for the volume of a cube to help them complete the activity.

Extension: *Math Enrichment*

Show students a graph of the volume of a cube and the volume of a sphere where the side length equals the radius and have them describe what they notice. **Note:** Students will learn the volume of a cube formula in an upcoming lesson.

Math Language Development

MLR7: Compare and Connect

During the Connect, have students respond to the question posed in the Student Edition, "How does the volume of a cube relate to the volume of a sphere if the cube's side length is equal to the radius of the sphere?" Listen for and amplify student reasoning who used the graph of the sphere's volume for a particular radius and compared that value with the volume given by the cube's equation for the same side length (radius). Sample response: When the radius/side length is 2 cm, the sphere's volume is about 35 cm, while the cube's volume is 8 cm. For this value and other values, the sphere's volume is a little over 4 times the cube's volume.

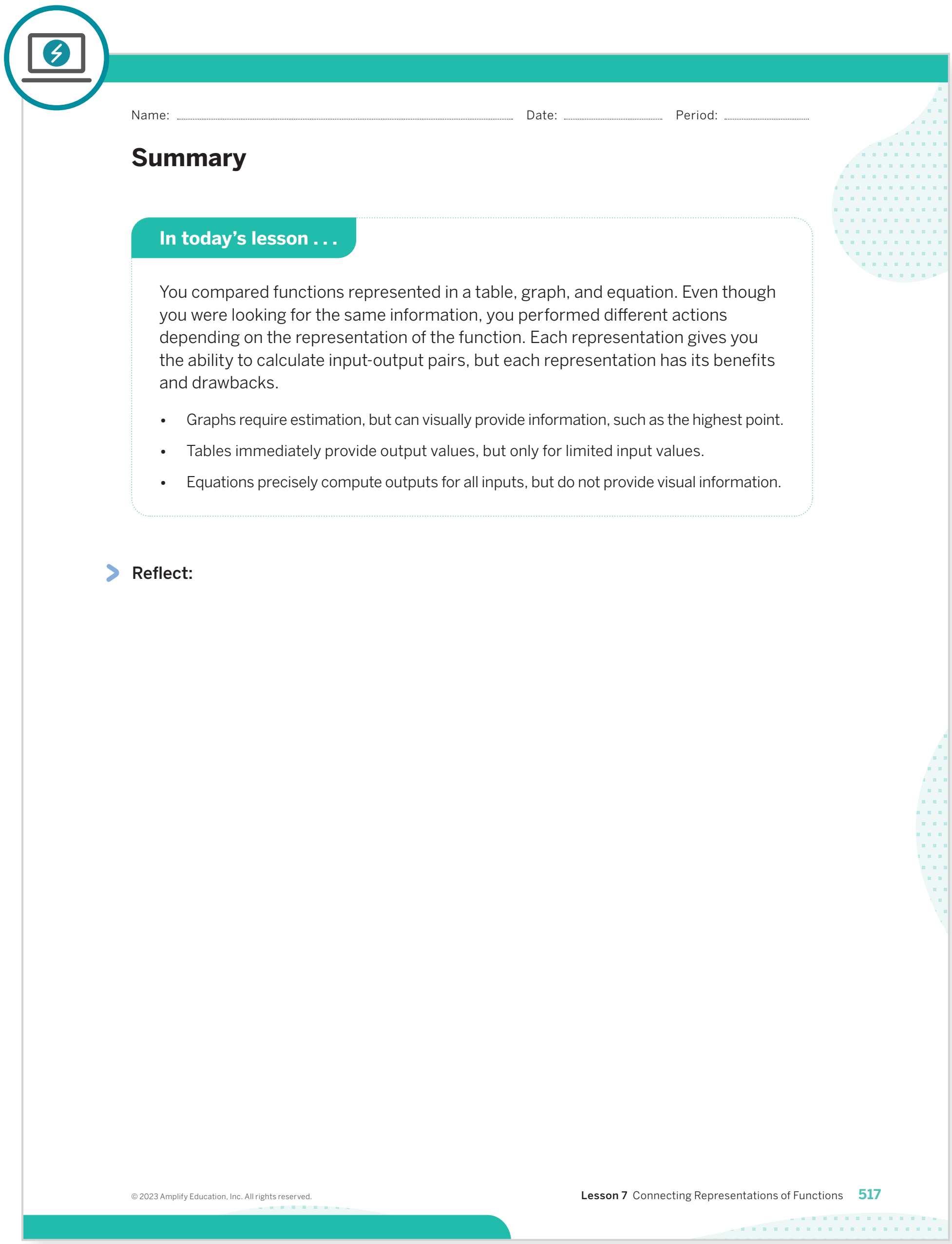
English Learners

Draw a cube and a sphere where the cube's side length is equal to the radius of the sphere. Annotate the cube's side length as r and the sphere's radius as r.

Whole Class | 5 min

Summary

Review and synthesize the interpretation and the benefits and drawbacks of each function representation.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You compared functions represented in a table, graph, and equation. Even though you were looking for the same information, you performed different actions depending on the representation of the function. Each representation gives you the ability to calculate input-output pairs, but each representation has its benefits and drawbacks.

- Graphs require estimation, but can visually provide information, such as the highest point.
- Tables immediately provide output values, but only for limited input values.
- Equations precisely compute outputs for all inputs, but do not provide visual information.

Reflect:

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Lesson 7 Connecting Representations of Functions 517

Synthesize

Have students share how they can interpret information given by a table, a graph, and an equation.

Highlight the ways students can use different representations of a function to interpret information. For example, if students are provided a graph, they can look at the (x, y) coordinates. If students are provided an equation, they can substitute the input to determine output. If students are provided a table, they can find a corresponding input-output pair.

Ask, "What are the benefits and drawbacks of a function represented by a table? A graph? An equation?"

Sample responses:

- Graphs require estimation, but can visually give information, such as the highest point.
- Tables immediately provide output values, but only for limited input values.
- Equations precisely compute outputs for all inputs, but do not provide any visual information.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you compare multiple representations of functions to determine which is changing at a faster rate, or which is slower?"

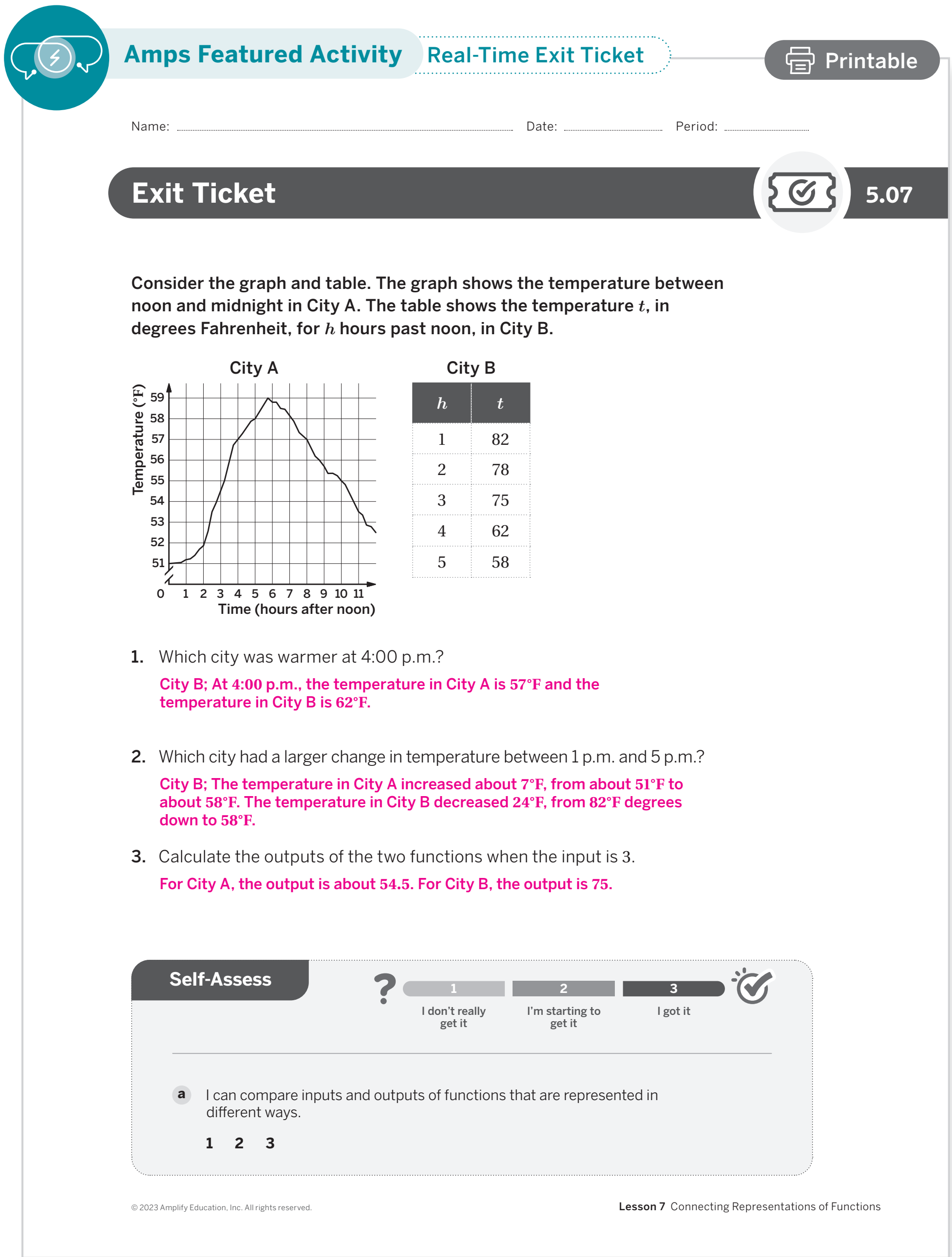
Independent | 5 min

Exit Ticket

Students demonstrate their understanding of multiple representations of functions by calculating input-output pairs given two different representations of functions.

Amps Featured Activity Real-Time Exit Ticket

Printable



Name: Date: Period:

Exit Ticket

5.07

Consider the graph and table. The graph shows the temperature between noon and midnight in City A. The table shows the temperature t, in degrees Fahrenheit, for h hours past noon, in City B.

City B

h	t
1	82
2	78
3	75
4	62
5	58

1. Which city was warmer at 4:00 p.m.?

   City B; At 4:00 p.m., the temperature in City A is 57°F and the temperature in City B is 62°F.

2. Which city had a larger change in temperature between 1 p.m. and 5 p.m.?

   City B; The temperature in City A increased about 7°F, from about 51°F to about 58°F. The temperature in City B decreased 24°F, from 82°F degrees down to 58°F.

3. Calculate the outputs of the two functions when the input is 3.

   For City A, the output is about 54.5. For City B, the output is 75.

Self-Assess

a I can compare inputs and outputs of functions that are represented in different ways.

1 2 3

Lesson 7 Connecting Representations of Functions

Success looks like . . .

- **Language Goal:** Comparing and contrasting representations of functions, and describing the strengths and weaknesses of each type of representation. **(Speaking and Listening)**
- **Language Goal:** Interpreting multiple representations of functions, including graphs, tables, and equations, and explaining how to find information in each type of representation. **(Speaking and Listening)**
  - » Determining information about the temperature of the city using the graph and table in Problems 1–3.

Suggested next steps

If students do not know how to interpret information from the graph or table, consider:

- Reviewing how to calculate the outputs for different inputs using the Activity 1 PDF.
- Having students use two different colors to highlight the input and outputs for each representation.

Professional Learning

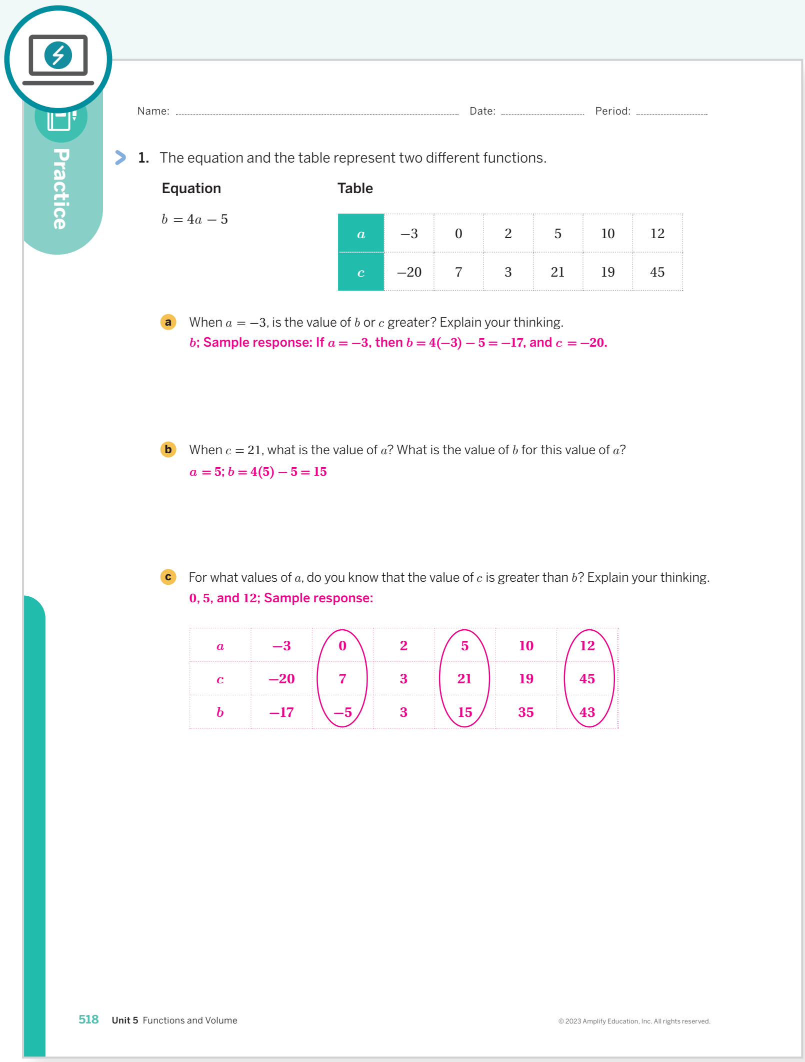
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- During the discussion in Activity 1, how did you encourage each student to share their understandings?
- What challenges did students encounter as they worked on Activity 2? How did they work through them?

Practice

Independent



Practice

Name: Date: Period:

1. The equation and the table represent two different functions.

Equation

$b = 4a - 5$

Table

a	-3	0	2	5	10	12
c	-20	7	3	21	19	45

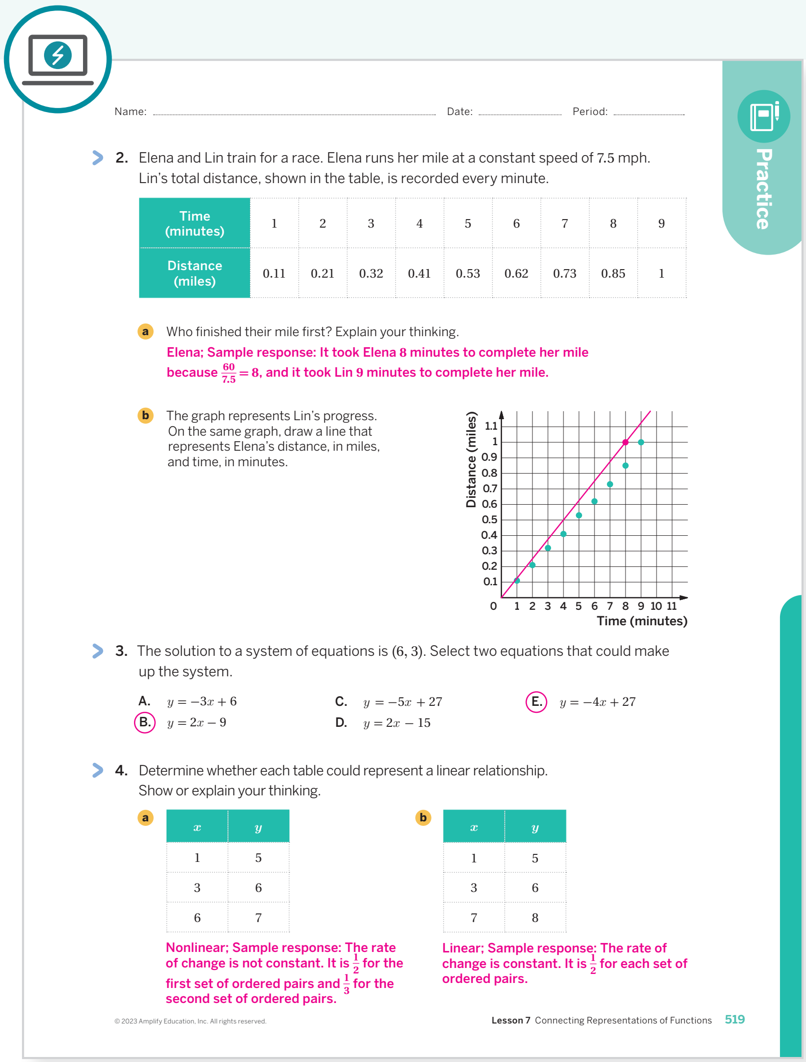
a. When $a = -3$, is the value of b or c greater? Explain your thinking.
b; Sample response: If $a = -3$, then $b = 4(-3) - 5 = -17$, and $c = -20$.

b. When $c = 21$, what is the value of a? What is the value of b for this value of a?
$a = 5$; $b = 4(5) - 5 = 15$

c. For what values of a, do you know that the value of c is greater than b? Explain your thinking.
0, 5, and 12; Sample response:

a	-3	0	2	5	10	12
c	-20	7	3	21	19	45
b	-17	-5	3	15	35	43

518 **Unit 5** Functions and Volume



Practice

Name: Date: Period:

2. Elena and Lin train for a race. Elena runs her mile at a constant speed of 7.5 mph. Lin's total distance, shown in the table, is recorded every minute.

Time (minutes)	1	2	3	4	5	6	7	8	9
Distance (miles)	0.11	0.21	0.32	0.41	0.53	0.62	0.73	0.85	1

a. Who finished their mile first? Explain your thinking.
Elena; Sample response: It took Elena 8 minutes to complete her mile because $\frac{60}{7.5} = 8$, and it took Lin 9 minutes to complete her mile.

b. The graph represents Lin's progress. On the same graph, draw a line that represents Elena's distance, in miles, and time, in minutes.

3. The solution to a system of equations is $(6, 3)$. Select two equations that could make up the system.

A. $y = -3x + 6$
[✓ B.] $y = 2x - 9$
C. $y = -5x + 27$
D. $y = 2x - 15$
[✓ E.] $y = -4x + 27$

4. Determine whether each table could represent a linear relationship. Show or explain your thinking.

a.

x	y
1	5
3	6
6	7

Nonlinear; Sample response: The rate of change is not constant. It is $\frac{1}{2}$ for the first set of ordered pairs and $\frac{1}{3}$ for the second set of ordered pairs.

b.

x	y
1	5
3	6
7	8

Linear; Sample response: The rate of change is constant. It is $\frac{1}{2}$ for each set of ordered pairs.

Lesson 7 Connecting Representations of Functions 519

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 4 Lesson 15	2
Formative	4	Unit 5 Lesson 8	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 8**

Comparing Linear Functions

Let's compare linear functions.

Focus

Goals

1. Comprehend that any linear function can be represented by an equation of the form $y = mx + b$, where m and b are the rate of change and the initial value of the function, respectively.
2. **Language Goal:** Make sense of the graph of a linear function and its rate of change and initial value. **(Speaking and Listening, Writing)**
3. **Language Goal:** Compare properties of linear functions represented in different forms. **(Speaking and Listening, Writing)**

Coherence

Today

Students investigate and make connections between linear functions as represented by graphs, descriptions, tables, and equations of the form $y = mx + b$. They interpret the slope of the line as the rate of change m of the dependent variable with respect to the independent variable, and the vertical intercept of the line as the initial value b. Students also compare properties of linear functions represented in different ways to determine, for example, which function has the greater rate of change.

Previously

In Units 3 and 4, students worked with linear equations and their graphs. In Lesson 7 of this unit, students made connections between the multiple representations of functions.

Coming Soon

In Lesson 9, students will model real-world scenarios with linear functions.

Rigor

- Students show **fluency** in determining the slopes and y-intercepts of linear functions in multiple representations.
- Students **apply** their knowledge of functions and linear relationships to compare linear functions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Representations of Linear Relationships* (from Unit 3)

Math Language Development

New word

- ***linear function***

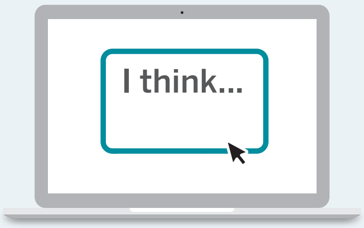
Review words

- *function*
- *linear relationship*
- *slope*
- y*-intercept*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to explain their thinking behind which linear function is changing by a faster rate, and these explanations are available to you digitally, in real time.



Amps POWERED BY desmos

Building Math Identity and Community
Connecting to Mathematical Practices

In both Activities, students may not immediately recognize the relationship between the graphs and the information requested. Encourage students to set a goal of using the structure of a linear equation to find a pattern that applies when solving their problems. Such a goal provides motivation to stay on task while targeting the purpose of the lesson.

Modifications to Pacing

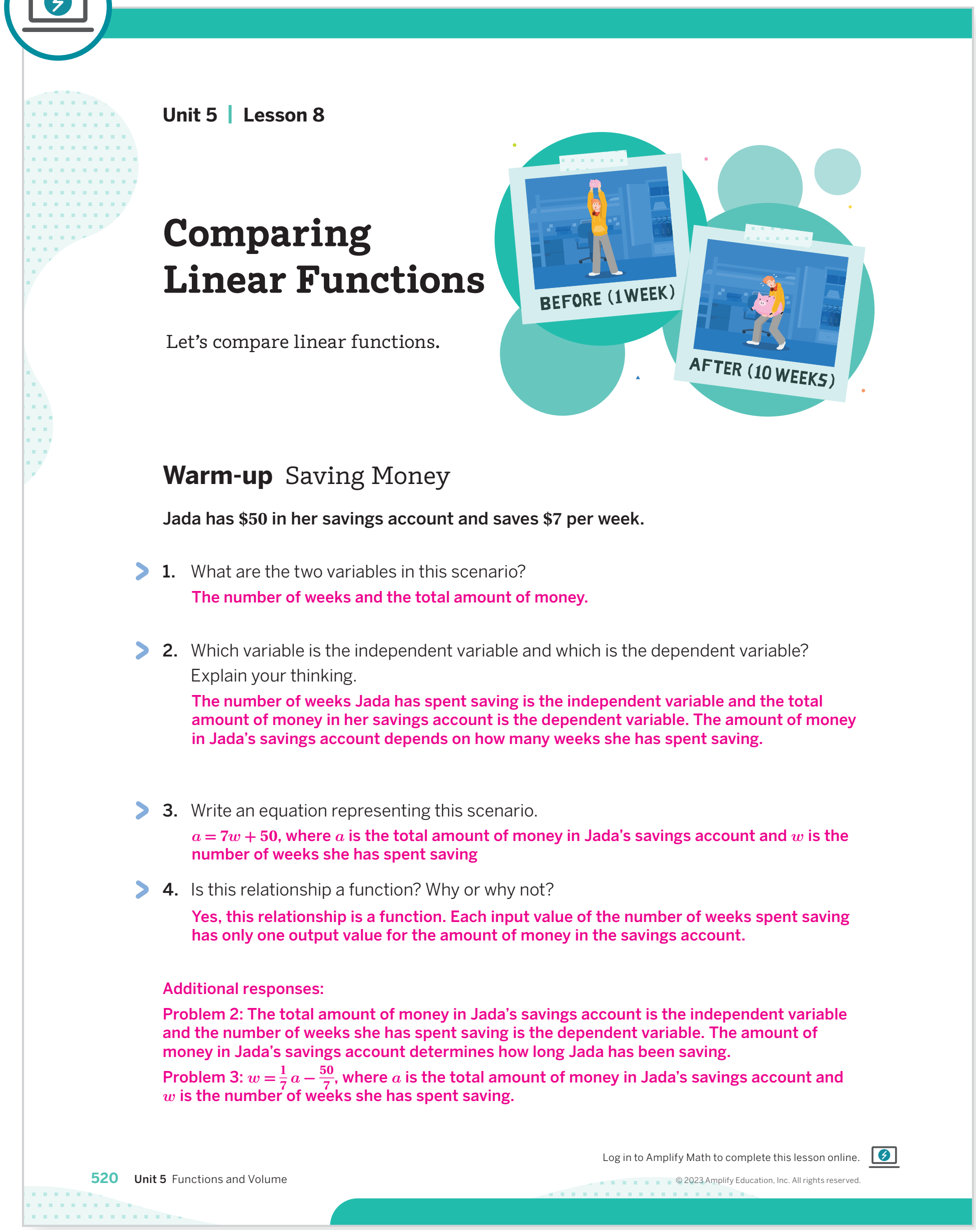
You may want to consider these additional modifications if you are short on time.

- Problem 3 in **Activity 1** may be omitted.
- In **Activity 2**, have students only compare 2 of the 4 batteries.

Pairs | 5 min

Warm-up Saving Money

Students connect what they know about a linear relationship to determine whether a given relationship is a function.



Unit 5 | Lesson 8

Comparing Linear Functions

Let's compare linear functions.

Warm-up Saving Money

Jada has $50 in her savings account and saves $7 per week.

1. What are the two variables in this scenario?
   The number of weeks and the total amount of money.

2. Which variable is the independent variable and which is the dependent variable? Explain your thinking.
   The number of weeks Jada has spent saving is the independent variable and the total amount of money in her savings account is the dependent variable. The amount of money in Jada's savings account depends on how many weeks she has spent saving.

3. Write an equation representing this scenario.
   $a = 7w + 50$, where a is the total amount of money in Jada's savings account and w is the number of weeks she has spent saving

4. Is this relationship a function? Why or why not?
   Yes, this relationship is a function. Each input value of the number of weeks spent saving has only one output value for the amount of money in the savings account.

Additional responses:

Problem 2: The total amount of money in Jada's savings account is the independent variable and the number of weeks she has spent saving is the dependent variable. The amount of money in Jada's savings account determines how long Jada has been saving.

Problem 3: $w = \frac{1}{7}a - \frac{50}{7}$, where a is the total amount of money in Jada's savings account and w is the number of weeks she has spent saving.

Log in to Amplify Math to complete this lesson online.

520 Unit 5 Functions and Volume

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1 Launch

Activate prior knowledge and discuss the characteristics of a function.

2 Monitor

Help students get started by asking what is being measured in the scenario.

Look for points of confusion:

- **Not being able to identify the independent and dependent variable.** Have students create a table of values to represent the scenario. Ask, "Which variable is your input and which is your output?".

3 Connect

Have students share their solutions and reasoning, particularly to Problem 4.

Highlight that this scenario is a function because each input of a number of weeks Jada has saved has exactly one output for the total amount in her savings account. **Note:** If students switch the independent and dependent variables, they need to write the appropriate equation. Encourage students to identify the number of weeks as the independent variable and the total amount of money as the dependent variable.

Ask, "Is this relationship linear or nonlinear? How do you know?"

Define a ***linear function*** as a linear relationship which assigns exactly one output for every possible input. Ask students whether they think every line is a linear function, and then provide an example of a vertical line that is an example of a linear relationship that is not a function.

Math Language Development

MLR8: Discussion Supports

During the Connect, display the following sentence frames to support students as they justify the independent and dependent variables they selected.

"______ is the independent variable, because . . ."

"______ depends on ______, because. . ."

As you define the term linear function, remind students they previously learned about *linear relationships*. Have them compare and contrast *linear relationships* with linear functions by asking:

- "Are all linear functions linear relationships? Why or why not?"
- "Are all linear relationships linear functions? Why or why not?"

Power-up

To power up students' ability to determine whether a table of values represents a linear relationship, have students complete:

Recall that a *linear relationship* is a relationship between quantities where there is a constant rate of change (called slope).

Determine whether the relationship in the table is *linear*. If so, write the equation that represents it.

x	0	1	2	3
y	2	5	8	11

Yes; $y = 3x + 2$

Use: Before Activity 2

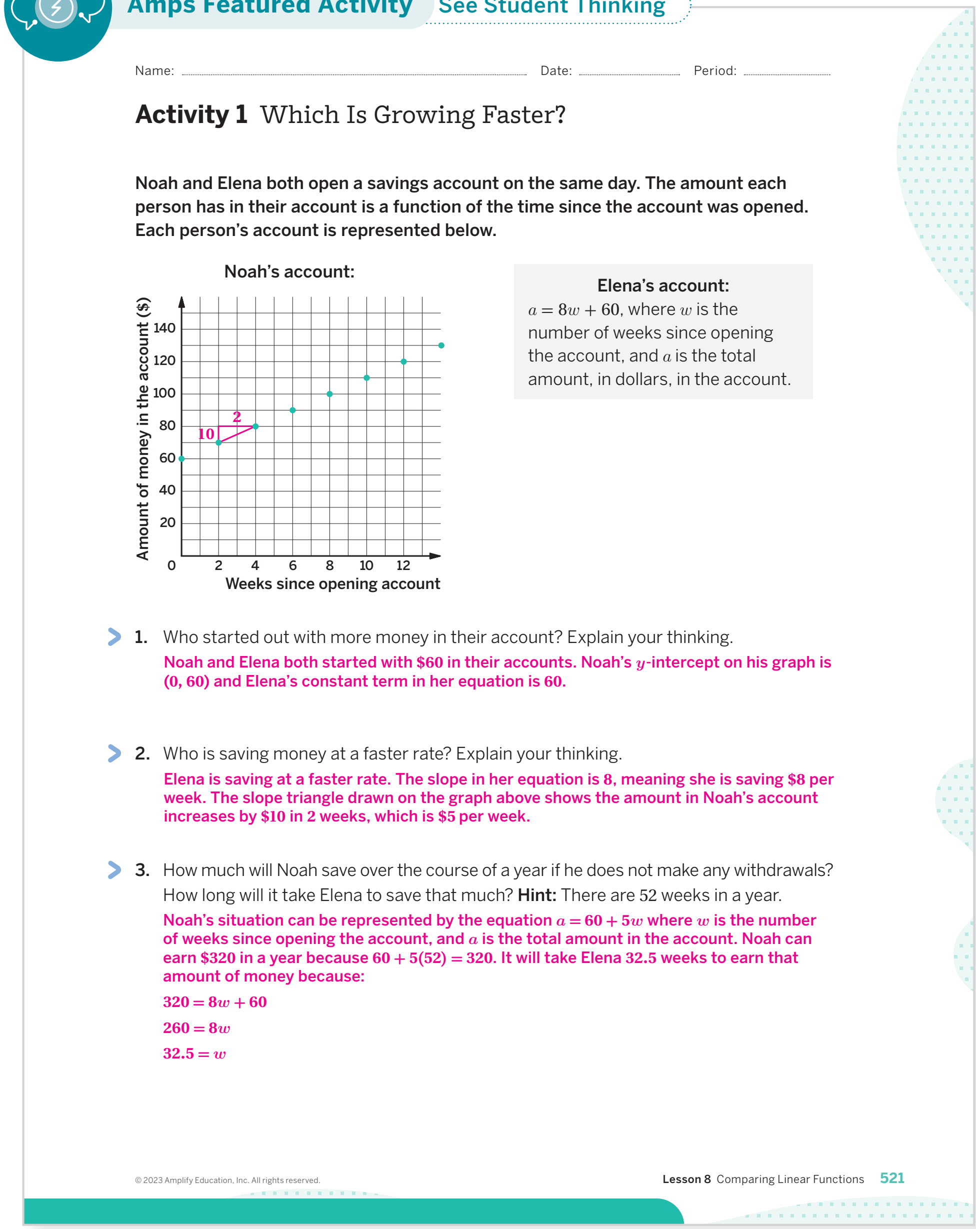
Informed by: Performance on Lesson 7, Practice Problem 4 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 15 min

Activity 1 Which Is Growing Faster?

Students analyze multiple representations of two linear functions to compare their properties, specifically the slopes and y-intercepts.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 1 Which Is Growing Faster?

Noah and Elena both open a savings account on the same day. The amount each person has in their account is a function of the time since the account was opened. Each person's account is represented below.

Noah's account:

Elena's account:
$a = 8w + 60$, where w is the number of weeks since opening the account, and a is the total amount, in dollars, in the account.

1. Who started out with more money in their account? Explain your thinking.
Noah and Elena both started with \$60 in their accounts. Noah's y-intercept on his graph is (0, 60) and Elena's constant term in her equation is 60.

2. Who is saving money at a faster rate? Explain your thinking.
Elena is saving at a faster rate. The slope in her equation is 8, meaning she is saving \$8 per week. The slope triangle drawn on the graph above shows the amount in Noah's account increases by \$10 in 2 weeks, which is \$5 per week.

3. How much will Noah save over the course of a year if he does not make any withdrawals? How long will it take Elena to save that much? **Hint:** There are 52 weeks in a year.
Noah's situation can be represented by the equation $a = 60 + 5w$ where w is the number of weeks since opening the account, and a is the total amount in the account. Noah can earn \$320 in a year because $60 + 5(52) = 320$. It will take Elena 32.5 weeks to earn that amount of money because:

$320 = 8w + 60$

$260 = 8w$

$32.5 = w$

Lesson 8 Comparing Linear Functions 521

1 Launch

Activate prior knowledge and review the slope and y-intercept of linear relationships. Consider displaying the Anchor Chart PDF, *Representations of Linear Relationships*.

2 Monitor

Help students get started by asking what characteristic of a linear relationship represents the starting amount or initial value.

Look for points of confusion:

- **Not finding the slope from the graph.** Have students write in missing values on the scale and draw the slope triangle.

Look for productive strategies:

- Representing Noah's account as an equation or graphing Elena's account on the same coordinate plane as Noah's account.

3 Connect

Display any account information that will help the class discussion.

Have students share their answers and reasoning for the problems.

Highlight that both accounts started at \$60 which is seen as y-intercepts. Both accounts are increasing which is seen in the positive slopes of both functions.

Ask:

- "Are these linear functions? How do you know?"
- "How did you determine the amount Noah saved in a year?"
- "How did you determine how long it would take Elena to save the same amount?"
- "How could you solve the last question without using an equation? If you included Jada from the Warm-up, how would that change your responses for Problems 1 and 2?"

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Display the Anchor Chart PDF, *Representations of Linear Relationships* (from Unit 3) throughout this activity for students to use as a reference. If students need more processing time, have them focus on Problems 1 and 2.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problems 1–3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "What did you do first? Why did you take that approach?"
- "How did you determine how much money Noah and Elena each had at the start?"

Have students revise their responses, as needed.

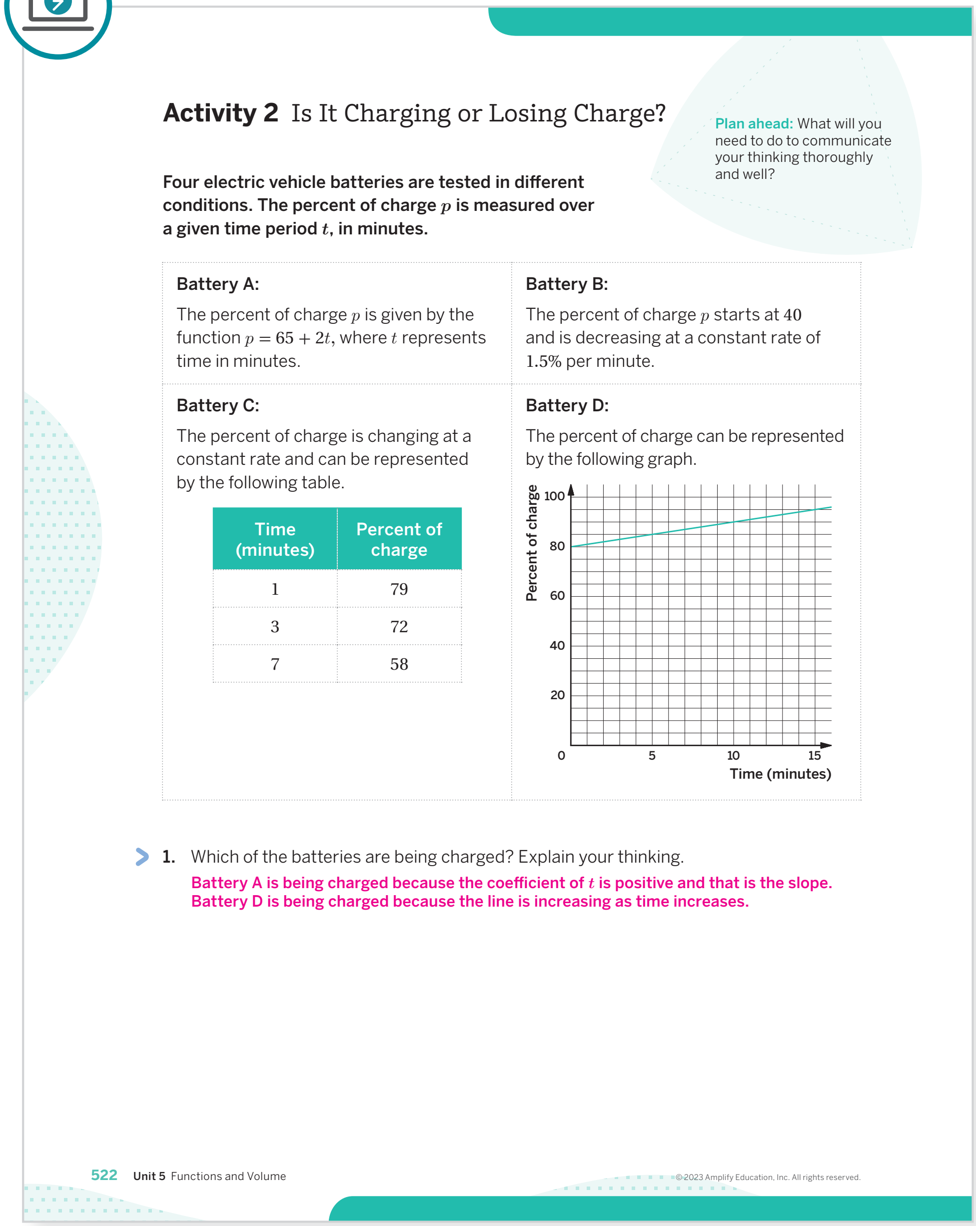
English Learners

Where possible, pair students with different levels of English language proficiency together. This will provide a structured opportunity for English learners to interact with and receive feedback from their peers with varied language backgrounds.

Pairs | 15 min

Activity 2 Is It Charging or Losing Charge?

Students revisit multiple representations of four linear functions to compare their y-intercepts and slopes.



Activity 2 Is It Charging or Losing Charge?

Plan ahead: What will you need to do to communicate your thinking thoroughly and well?

Four electric vehicle batteries are tested in different conditions. The percent of charge p is measured over a given time period t, in minutes.

Battery A:

The percent of charge p is given by the function $p = 65 + 2t$, where t represents time in minutes.

Battery B:

The percent of charge p starts at 40 and is decreasing at a constant rate of 1.5% per minute.

Battery C:

The percent of charge is changing at a constant rate and can be represented by the following table.

Time (minutes)	Percent of charge
1	79
3	72
7	58

Battery D:

The percent of charge can be represented by the following graph.

1. Which of the batteries are being charged? Explain your thinking.
Battery A is being charged because the coefficient of t is positive and that is the slope. Battery D is being charged because the line is increasing as time increases.

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking what should happen if the battery is charging or losing charge and which piece of the linear function determines this.

Look for points of confusion:

- **Not remembering how to identify the slope or y-intercept from a linear relationship.** Provide students with the Anchor Chart PDF, *Representations of Linear Relationships*.
- **Thinking that the greatest rate of change must be positive.** Let students know they are not finding the slope with the largest value but they are finding the steepest line, regardless of whether it increases or decreases.
- **Thinking Battery D has the greatest initial value.** Remind students that the initial value is paired with an x-value of 0 and that Battery C has 79% charge after 1 minute, not at 0 minutes.

Look for productive strategies:

- Finding the slope and y-intercept of each representation.

Activity 2 continued ›

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on comparing two of the four batteries. To leverage the power of choice and support student engagement, consider allowing them to choose which two batteries they will analyze.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the representations for the four batteries. Have students work with their partner to write 2–3 questions they could ask about the representations shown. Ask a few student pairs to share their questions with the class. Amplify questions that ask whether a battery's charge is increasing or decreasing. Sample questions shown.

- Which batteries are increasing in charge? Which are losing charge?
- Which battery is increasing their charge at the fastest rate?
- Which battery is losing their charge at the fastest rate?
- When will the battery be fully charged? Have no charge?

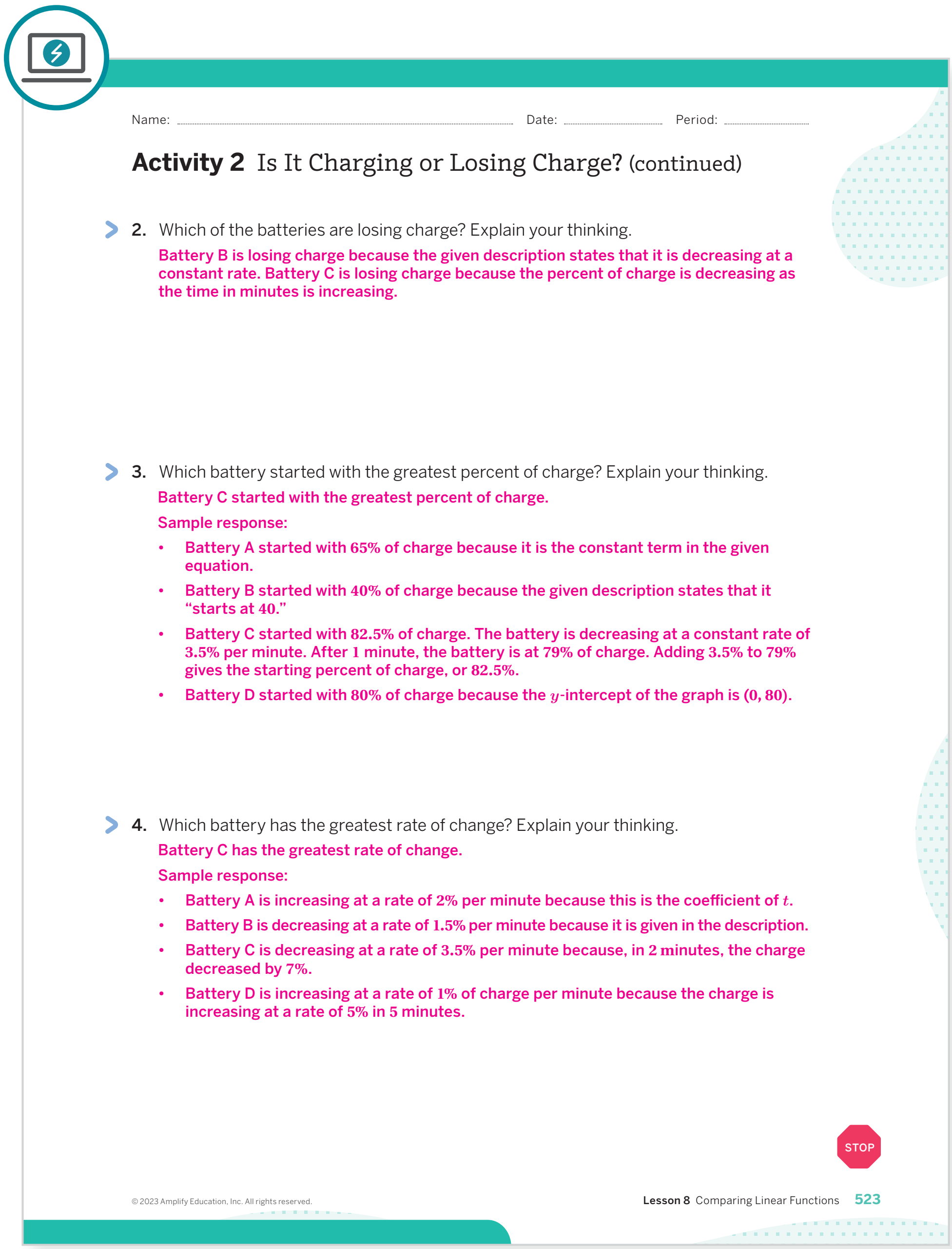
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 15 min

Activity 2 Is It Charging or Losing Charge? (continued)

Students revisit multiple representations of four linear functions to compare their y-intercepts and slopes.



Name: ________ Date: ________ Period: ________

Activity 2 Is It Charging or Losing Charge? (continued)

2. Which of the batteries are losing charge? Explain your thinking.

Battery B is losing charge because the given description states that it is decreasing at a constant rate. Battery C is losing charge because the percent of charge is decreasing as the time in minutes is increasing.

3. Which battery started with the greatest percent of charge? Explain your thinking.

Battery C started with the greatest percent of charge.

Sample response:

- **Battery A started with 65% of charge because it is the constant term in the given equation.**
- **Battery B started with 40% of charge because the given description states that it "starts at 40."**
- **Battery C started with 82.5% of charge. The battery is decreasing at a constant rate of 3.5% per minute. After 1 minute, the battery is at 79% of charge. Adding 3.5% to 79% gives the starting percent of charge, or 82.5%.**
- **Battery D started with 80% of charge because the y-intercept of the graph is (0, 80).**

4. Which battery has the greatest rate of change? Explain your thinking.

Battery C has the greatest rate of change.

Sample response:

- **Battery A is increasing at a rate of 2% per minute because this is the coefficient of t.**
- **Battery B is decreasing at a rate of 1.5% per minute because it is given in the description.**
- **Battery C is decreasing at a rate of 3.5% per minute because, in 2 minutes, the charge decreased by 7%.**
- **Battery D is increasing at a rate of 1% of charge per minute because the charge is increasing at a rate of 5% in 5 minutes.**

STOP

Lesson 8 Comparing Linear Functions 523

3 Connect

Display the table showing each representation for the four batteries

Have students share how they determined which batteries were being charged and which were losing charge.

Highlight that the y-intercepts are used to determine which battery started with the greatest percent of charge, and the slopes are used to determine which rate of change is the greatest.

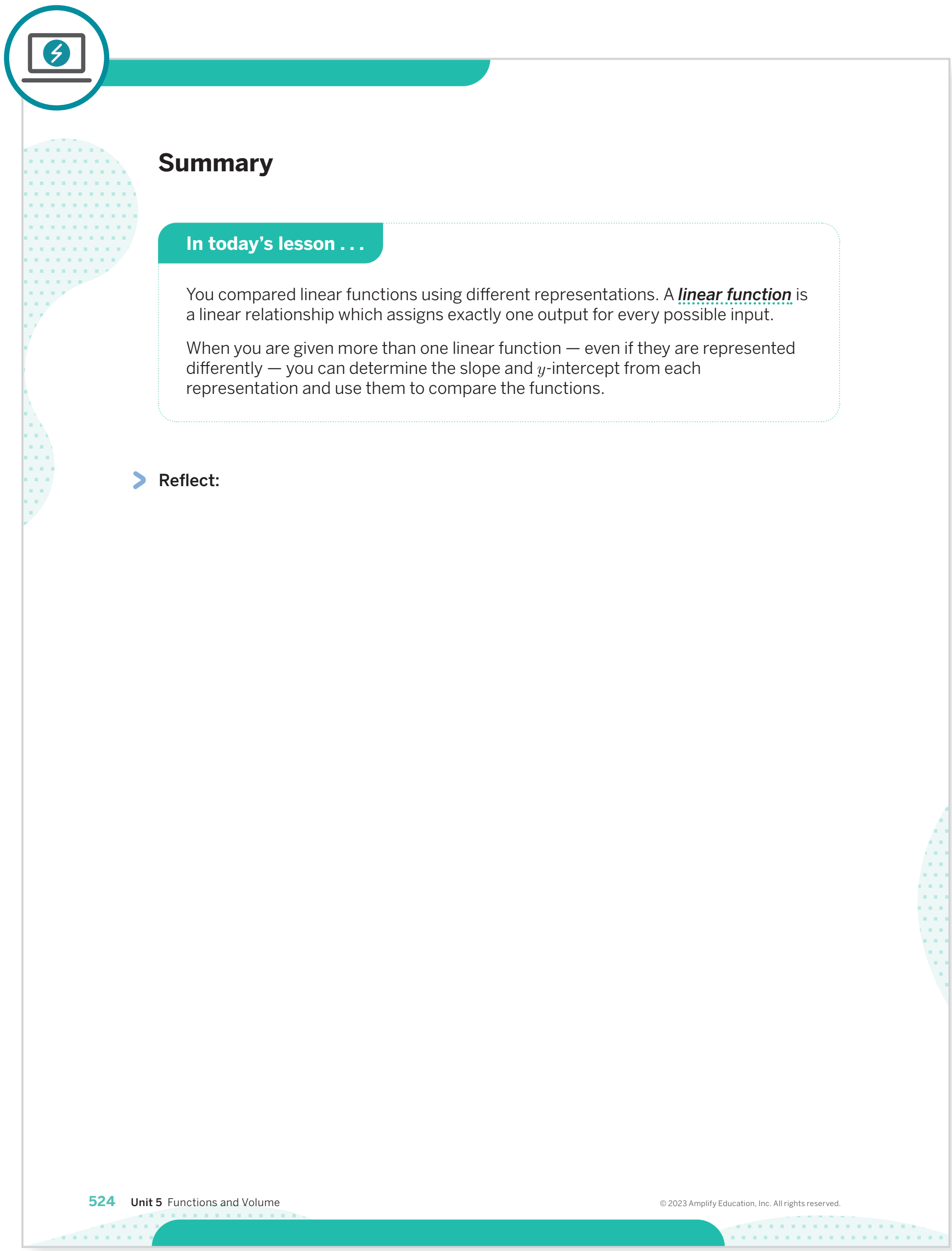
Ask:

- "Are the percent of charge of each battery represented by linear functions?? How do you know?"
- "What would the graph look like if Battery D was losing charge?"
- "What would the equation look like if Battery A was losing charge?"

Whole Class | 5 min

Summary

Review and synthesize comparing multiple representations of linear functions.



Summary

In today's lesson . . .

You compared linear functions using different representations. A ***linear function*** is a linear relationship which assigns exactly one output for every possible input.

When you are given more than one linear function — even if they are represented differently — you can determine the slope and y-intercept from each representation and use them to compare the functions.

Reflect:

524 **Unit 5** Functions and Volume

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Synthesize

Display the Anchor Chart PDF, *Representations of Linear Relationships*.

Ask:

- "Which piece of a linear function do you need to know to compare how the linear functions are changing? And how do you find it from a story, an equation, a table, and a graph?"
- "Which piece of a linear function do you need to know to compare the initial values?" How do you find it from a story, equation, table and graph?"

Highlight that the faster rate of change is the larger value regardless of it being positive or negative.

Formalize vocabulary: ***linear function***

Reflect

After to synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you compare multiple representations of functions to determine which is changing at a faster rate, or which is slower?"

Math Language Development

MLR2: Collect and Display

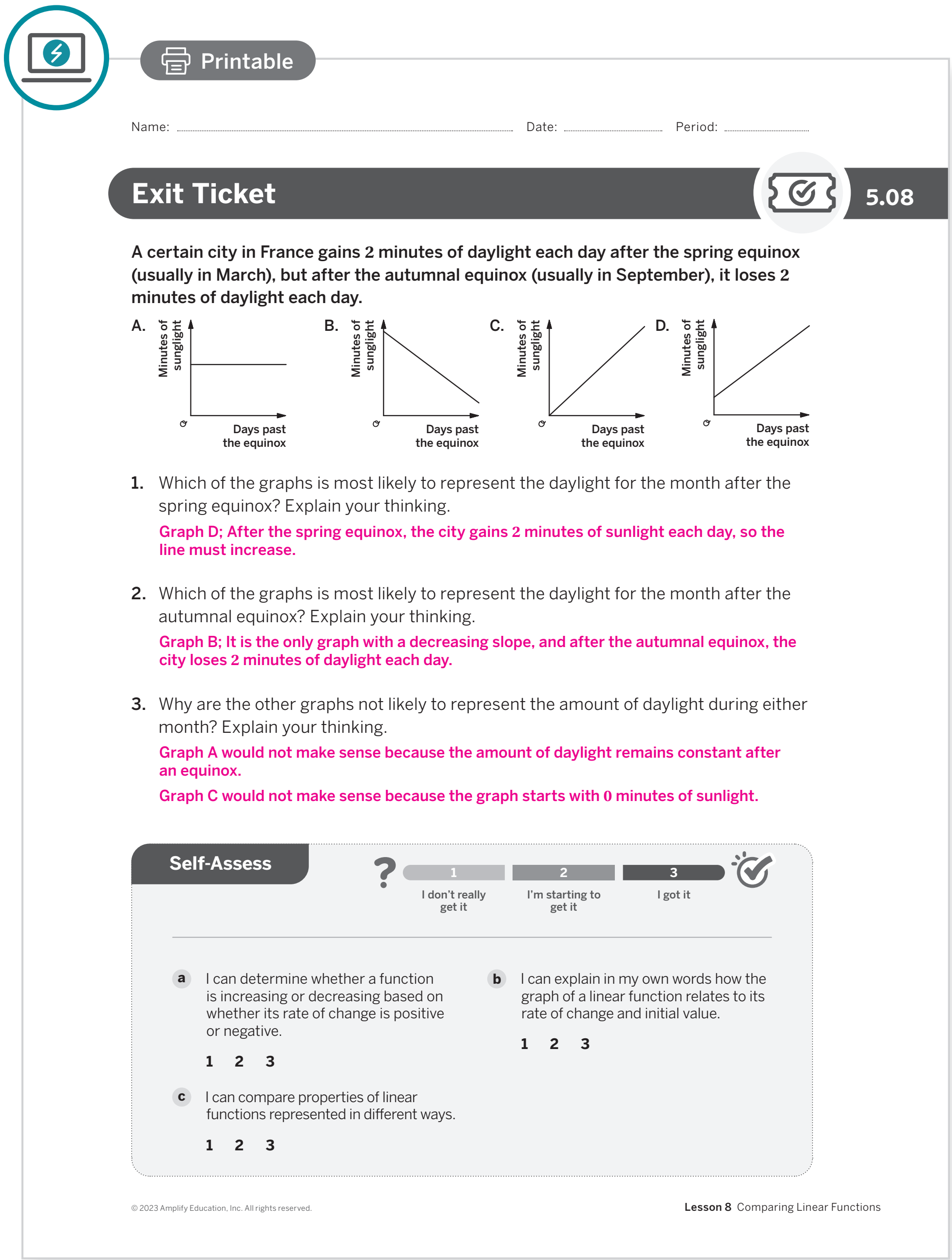
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *linear function* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by matching each graph to the correct increase or decrease in number of minutes of daylight.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 5.08

A certain city in France gains 2 minutes of daylight each day after the spring equinox (usually in March), but after the autumnal equinox (usually in September), it loses 2 minutes of daylight each day.

1. Which of the graphs is most likely to represent the daylight for the month after the spring equinox? Explain your thinking.
   Graph D; After the spring equinox, the city gains 2 minutes of sunlight each day, so the line must increase.

2. Which of the graphs is most likely to represent the daylight for the month after the autumnal equinox? Explain your thinking.
   Graph B; It is the only graph with a decreasing slope, and after the autumnal equinox, the city loses 2 minutes of daylight each day.

3. Why are the other graphs not likely to represent the amount of daylight during either month? Explain your thinking.
   Graph A would not make sense because the amount of daylight remains constant after an equinox.
   Graph C would not make sense because the graph starts with 0 minutes of sunlight.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine whether a function is increasing or decreasing based on whether its rate of change is positive or negative.
1 2 3

b I can explain in my own words how the graph of a linear function relates to its rate of change and initial value.
1 2 3

c I can compare properties of linear functions represented in different ways.
1 2 3

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Lesson 8 Comparing Linear Functions

Success looks like . . .

- **Goal:** Comprehending that any linear function can be represented by an equation of the form $y = mx + b$, where m and b are the rate of change and the initial value of the function, respectively.
- **Language Goal:** Making sense of the graph of a linear function and its rate of change and initial value. **(Speaking and Listening, Writing)**
  - » Determining the graph for each given representation in Problems 1–3.
- **Language Goal:** Comparing properties of linear functions represented in different forms. **(Speaking and Listening, Writing)**

Suggested next steps

If students select Graph C for Problem 1, consider:

- Reminding students what a y-intercept at the origin signifies.

If students correctly identify the graphs but do not explain their thinking, consider:

- Asking them to verbally explain how they knew.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier this year on linear equations, what similarities and differences do you see?
- In Activity 1, you used structured pairing with MLR1 to group students with different levels of English language proficiency. What effect did this grouping strategy have on student conversations and revisions? Would you change anything the next time you use MLR1?

Math Language Development

Language Goal: Identifying what information is needed to transform a polygon. Asking questions to elicit that information.

Reflect on students' language development toward this goal.

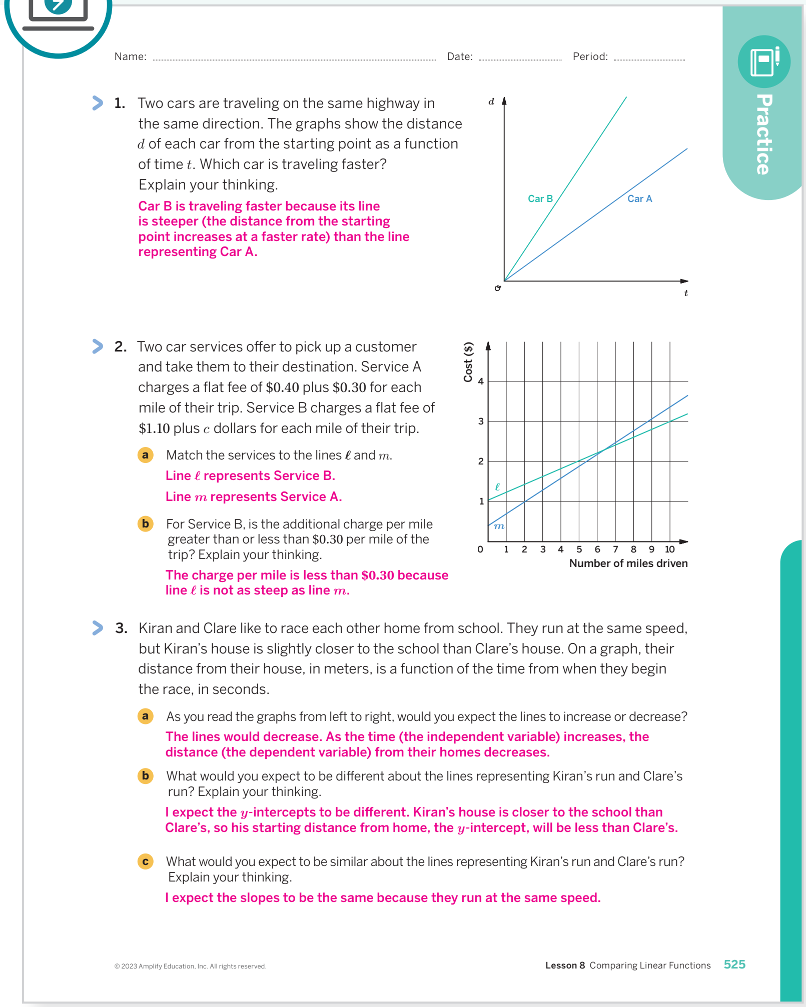
- What are some examples of developing questions and how can you help students be more precise in the questions they ask?

Sample questions for the Exit Ticket problem:

Emerging	Expanding
How far did the polygon move?	What are the horizontal and vertical distances for the translation?
How was it reflected?	What is the line of reflection?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Two cars are traveling on the same highway in the same direction. The graphs show the distance d of each car from the starting point as a function of time t. Which car is traveling faster? Explain your thinking.

   Car B is traveling faster because its line is steeper (the distance from the starting point increases at a faster rate) than the line representing Car A.

2. Two car services offer to pick up a customer and take them to their destination. Service A charges a flat fee of $\$0.40$ plus $\$0.30$ for each mile of their trip. Service B charges a flat fee of $\$1.10$ plus c dollars for each mile of their trip.

   a. Match the services to the lines ℓ and m.

   Line ℓ represents Service B.

   Line m represents Service A.

   b. For Service B, is the additional charge per mile greater than or less than $\$0.30$ per mile of the trip? Explain your thinking.

   The charge per mile is less than $\$0.30$ because line ℓ is not as steep as line m.

3. Kiran and Clare like to race each other home from school. They run at the same speed, but Kiran's house is slightly closer to the school than Clare's house. On a graph, their distance from their house, in meters, is a function of the time from when they begin the race, in seconds.

   a. As you read the graphs from left to right, would you expect the lines to increase or decrease?

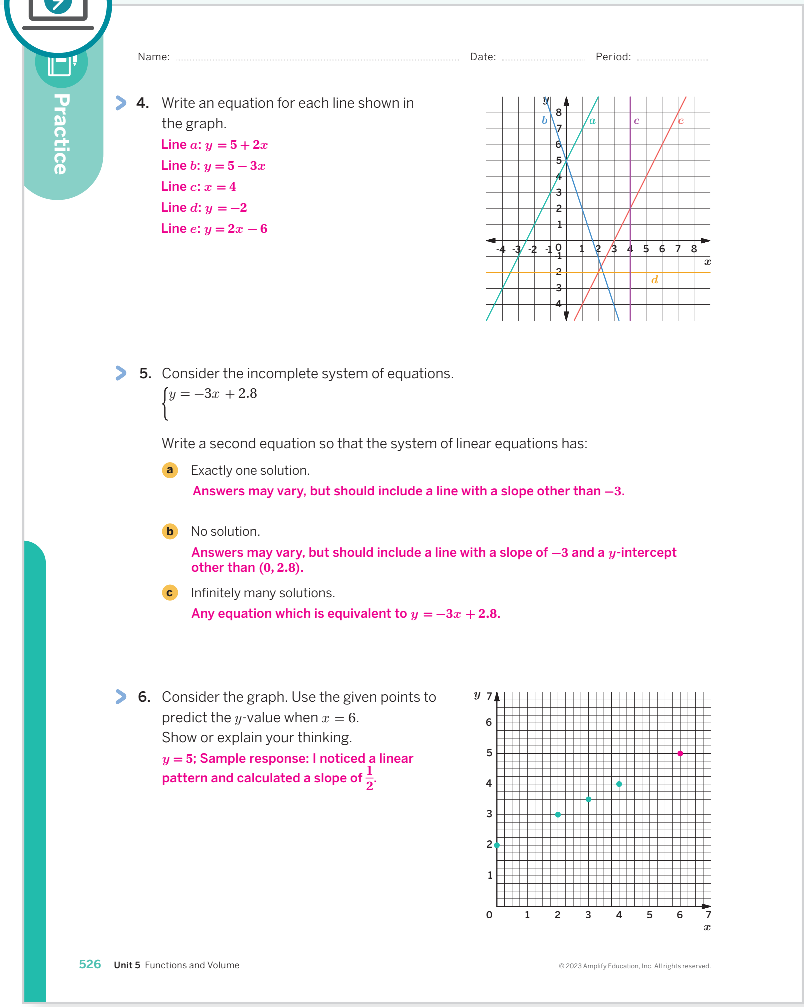
   The lines would decrease. As the time (the independent variable) increases, the distance (the dependent variable) from their homes decreases.

   b. What would you expect to be different about the lines representing Kiran's run and Clare's run? Explain your thinking.

   I expect the y-intercepts to be different. Kiran's house is closer to the school than Clare's, so his starting distance from home, the y-intercept, will be less than Clare's.

   c. What would you expect to be similar about the lines representing Kiran's run and Clare's run? Explain your thinking.

   I expect the slopes to be the same because they run at the same speed.



Name: ______ Date: ______ Period: ______

Practice

4. Write an equation for each line shown in the graph.

   Line a: $y = 5 + 2x$

   Line b: $y = 5 - 3x$

   Line c: $x = 4$

   Line d: $y = -2$

   Line e: $y = 2x - 6$

5. Consider the incomplete system of equations.

$$\begin{cases} y = -3x + 2.8 \\ \end{cases}$$

   Write a second equation so that the system of linear equations has:

   a. Exactly one solution.

   Answers may vary, but should include a line with a slope other than −3.

   b. No solution.

   Answers may vary, but should include a line with a slope of −3 and a y-intercept other than $(0, 2.8)$.

   c. Infinitely many solutions.

   Any equation which is equivalent to $y = -3x + 2.8$.

6. Consider the graph. Use the given points to predict the y-value when $x = 6$. Show or explain your thinking.

   $y = 5$; Sample response: I noticed a linear pattern and calculated a slope of $\frac{1}{2}$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 15	1
	5	Unit 4 Lesson 14	2
Formative ⚡	6	Unit 5 Lesson 9	2

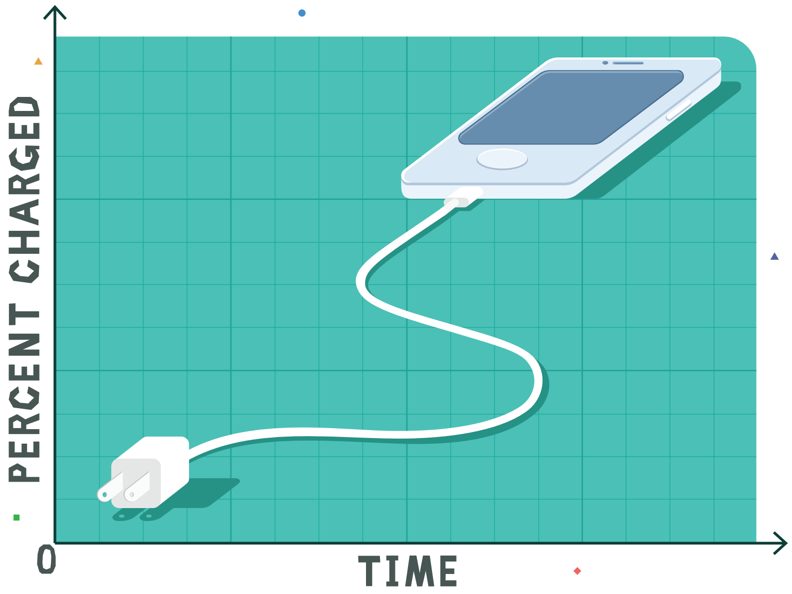
⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Modeling With Linear Functions

Let's model situations with linear functions.



Focus

Goals

1. Use data points to model a linear function.
2. **Language Goal:** Compare and contrast different models of the same data, and determine the range of values for which the model is a good fit for the data. **(Speaking and Listening, Writing)**
3. **Language Goal:** Model nonlinear data using a linear function, and justify whether the model is a good fit for the data. **(Speaking and Listening, Writing)**

Coherence

Today

Students use linear functions to model real-world situations. They create a linear function to make predictions and determine when a linear function is appropriate to model data. **Note:** Students will be introduced formally to lines of fit and modeling data using linear functions in Unit 8.

Previously

In Lesson 8, students compared properties of linear functions represented in different forms.

Coming Soon

In Lesson 10, students will be introduced to piecewise functions. Students will compute and compare the different rates of change given a graph.

Rigor

- Students continue to build their **conceptual understanding** of graphs of linear functions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	12 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Anchor Chart PDF, *Writing Linear Equations in* $y = mx + b$ *Form* (from Unit 3)
- rulers

Math Language Development

Review words

- *decreasing*
- *dependent variable*
- *function*
- *increasing*
- *independent variable*
- *linear function*

Amps | Featured Activity

Activity 1
Interactive Graph

Students plot points and drag a line to determine which points appear to be part of a linear relationship.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may impulsively identify a model without consideration to its accuracy. Ask students to suggest ways that they can control their impulses throughout the activity. At the end of the activity, ask them to self-reflect on how their behavior helped or hindered their work on the mathematical model.

Modifications to Pacing

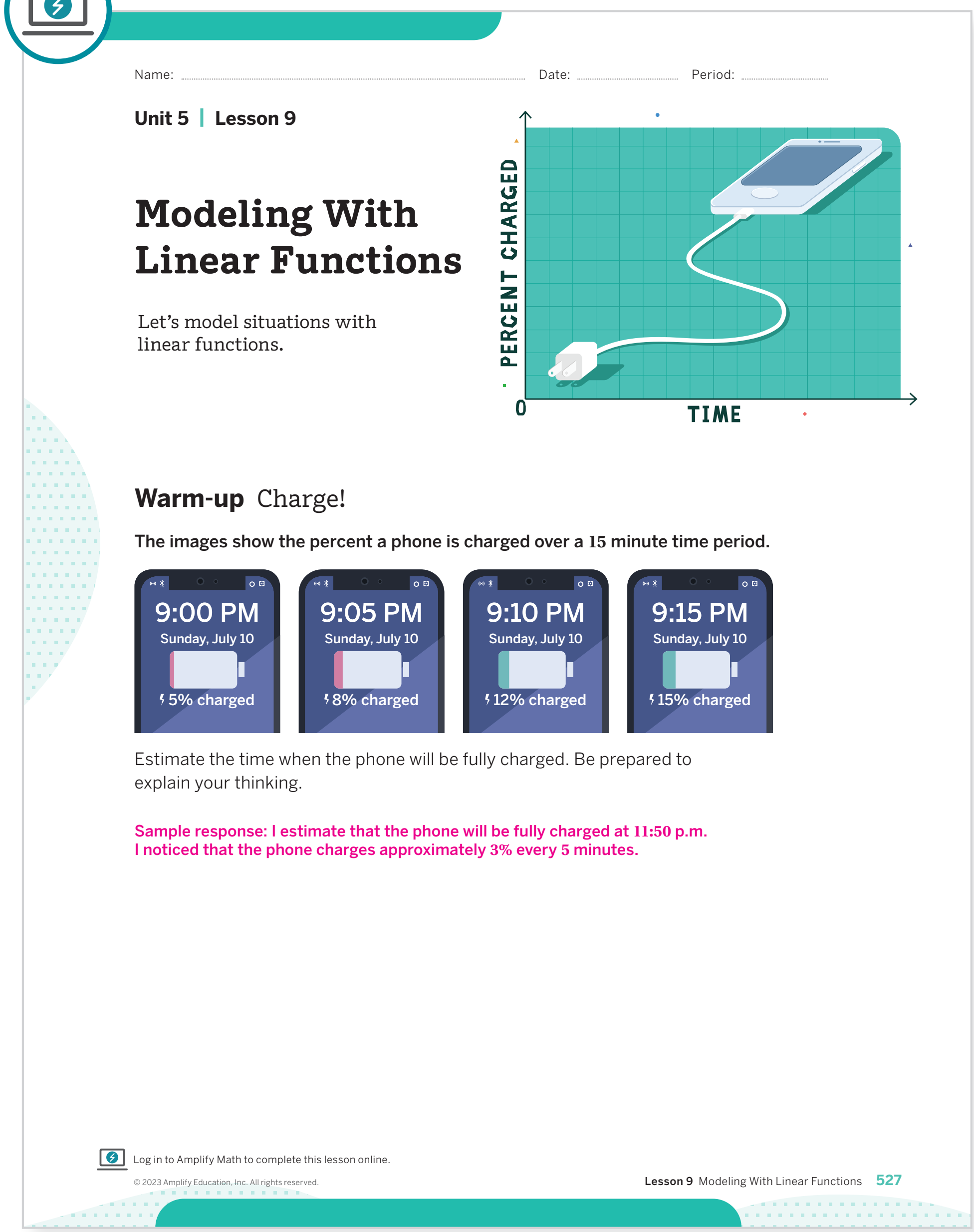
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, display the Activity 1 PDF and have students complete Problems 1, 3, and 4.

Independent | 5 min

Warm-up Charge!

Students analyze images to make sense of a problem and model with mathematics.



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Unit 5 | Lesson 9

Modeling With Linear Functions

Let's model situations with linear functions.

Warm-up Charge!

The images show the percent a phone is charged over a 15 minute time period.

9:00 PM Sunday, July 10 5% charged

9:05 PM Sunday, July 10 8% charged

9:10 PM Sunday, July 10 12% charged

9:15 PM Sunday, July 10 15% charged

Estimate the time when the phone will be fully charged. Be prepared to explain your thinking.

Sample response: I estimate that the phone will be fully charged at 11:50 p.m. I noticed that the phone charges approximately 3% every 5 minutes.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate students' background knowledge by asking, "What happens to the percent of charge of a battery as it charges?" It increases.

2 Monitor

Help students get started by asking them what information they could use to determine when the phone will be fully charged.

Look for productive strategies:

- Calculating the difference in time and the difference in the percent of charge and then comparing the two differences.
- Applying linear reasoning to determine their response.
- Creating a table or equation to determine their response.

3 Connect

Have students share their responses and strategies. Highlight the different strategies that students used to determine their response. At this point, allow for different responses, even if students' estimates and strategies may not be correct.

Ask, "Do you think the percent of charge will be a function of time? Why or why not?" Sample responses: Yes, I think there will be only one corresponding percent with a time. No, I think there could be multiple corresponding percentages with a time.

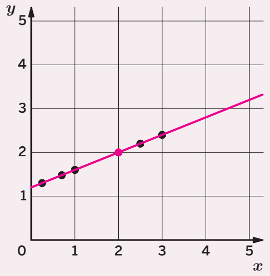
Highlight that creating a graph of the data may help students identify more patterns.

Power-up

To power up students' ability to use the pattern in plotted points to predict a value on the graph, have students complete:

Use the graph to complete each problem.

1. Does the data appear to be increasing or decreasing? Increasing
2. Add a line to the graph connecting the data.
3. Use your line to predict the value of y when $x = 2$ $y = 2$



Use: Before Activity 1

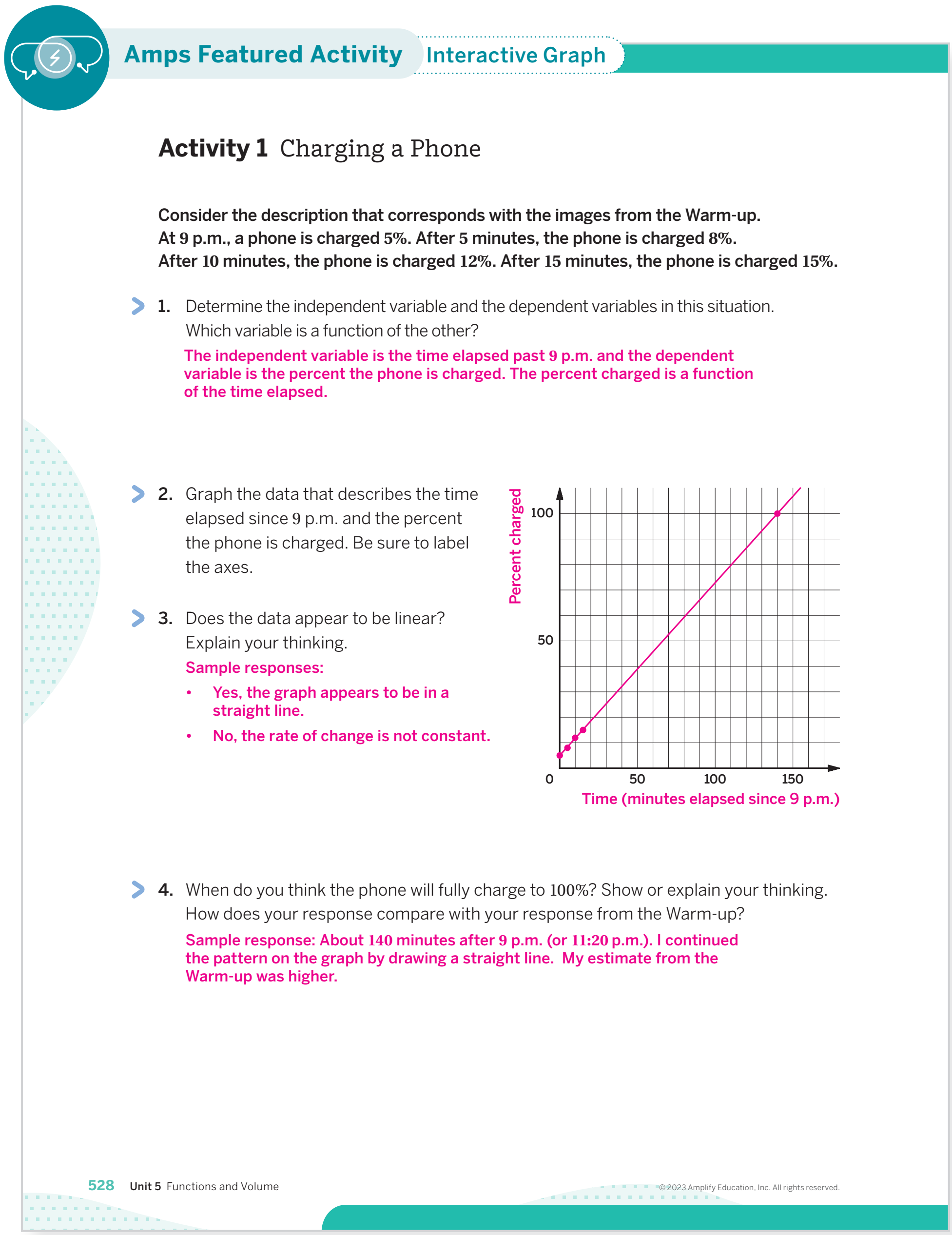
Informed by: Performance on Lesson 8, Practice Problem 6

Pairs | 15 min

Activity 1 Charging a Phone

Students use data points to develop a linear model and then assess the reasonableness of their model.

Amps Featured Activity Interactive Graph



Activity 1 Charging a Phone

Consider the description that corresponds with the images from the Warm-up.
At 9 p.m., a phone is charged 5%. After 5 minutes, the phone is charged 8%.
After 10 minutes, the phone is charged 12%. After 15 minutes, the phone is charged 15%.

1. Determine the independent variable and the dependent variables in this situation. Which variable is a function of the other?
   The independent variable is the time elapsed past 9 p.m. and the dependent variable is the percent the phone is charged. The percent charged is a function of the time elapsed.
2. Graph the data that describes the time elapsed since 9 p.m. and the percent the phone is charged. Be sure to label the axes.
3. Does the data appear to be linear? Explain your thinking.
   Sample responses:
   - **Yes, the graph appears to be in a straight line.**
   - **No, the rate of change is not constant.**

4. When do you think the phone will fully charge to 100%? Show or explain your thinking. How does your response compare with your response from the Warm-up?
   Sample response: About 140 minutes after 9 p.m. (or 11:20 p.m.). I continued the pattern on the graph by drawing a straight line. My estimate from the Warm-up was higher.

528 **Unit 5** Functions and Volume

1 Launch

Tell students that they will investigate the Warm-up further by creating a graph and looking for additional patterns. Provide access to rulers. **Note:** Students will be formally introduced to lines of fit and modeling data using linear functions in Unit 8.

2 Monitor

Help students get started by activating their prior knowledge about functions, independent variables, and dependent variables.

Look for points of confusion:

- **Struggling to determine when the phone will fully charge.** If students think the pattern is linear, have them continue the pattern by drawing a line to make their prediction. Otherwise, encourage students to make an educated guess.

3 Connect

Have students share when they think the phone will fully charge and how they determined their response.

Display page 1 of the Activity 1 PDF. Tell students that, at first, the data appears linear, but it is not linear.

Ask, "Based on the graph, what happens as the phone charges?"

Highlight that, although the data is not precisely linear, students can *model* the data with a linear function because the data is approximately linear. Tell students that to make a more accurate prediction, multiple lines may be drawn to model different parts of the data. Display page 2 of the Activity 1 PDF. Students can use the linear function to predict and estimate when the phone will fully charge.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can predict when a phone will be fully charged and view an animation of the phone's charge while the graph of the data is simultaneously shown.

Accessibility: *Optimize Access to Tools*

Provide access to rulers or other straightedges that students can use to connect the points if they choose to do so.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- Read 1: Students should understand that a phone is charging over time.
- Read 2: Ask students to name or highlight the given quantities and relationships, such as "at 9 p.m., the phone is charged 5%."
- Read 3: Ask students to plan their solution strategy as to how they will determine the independent and dependent variables and which variable is a function of the other.

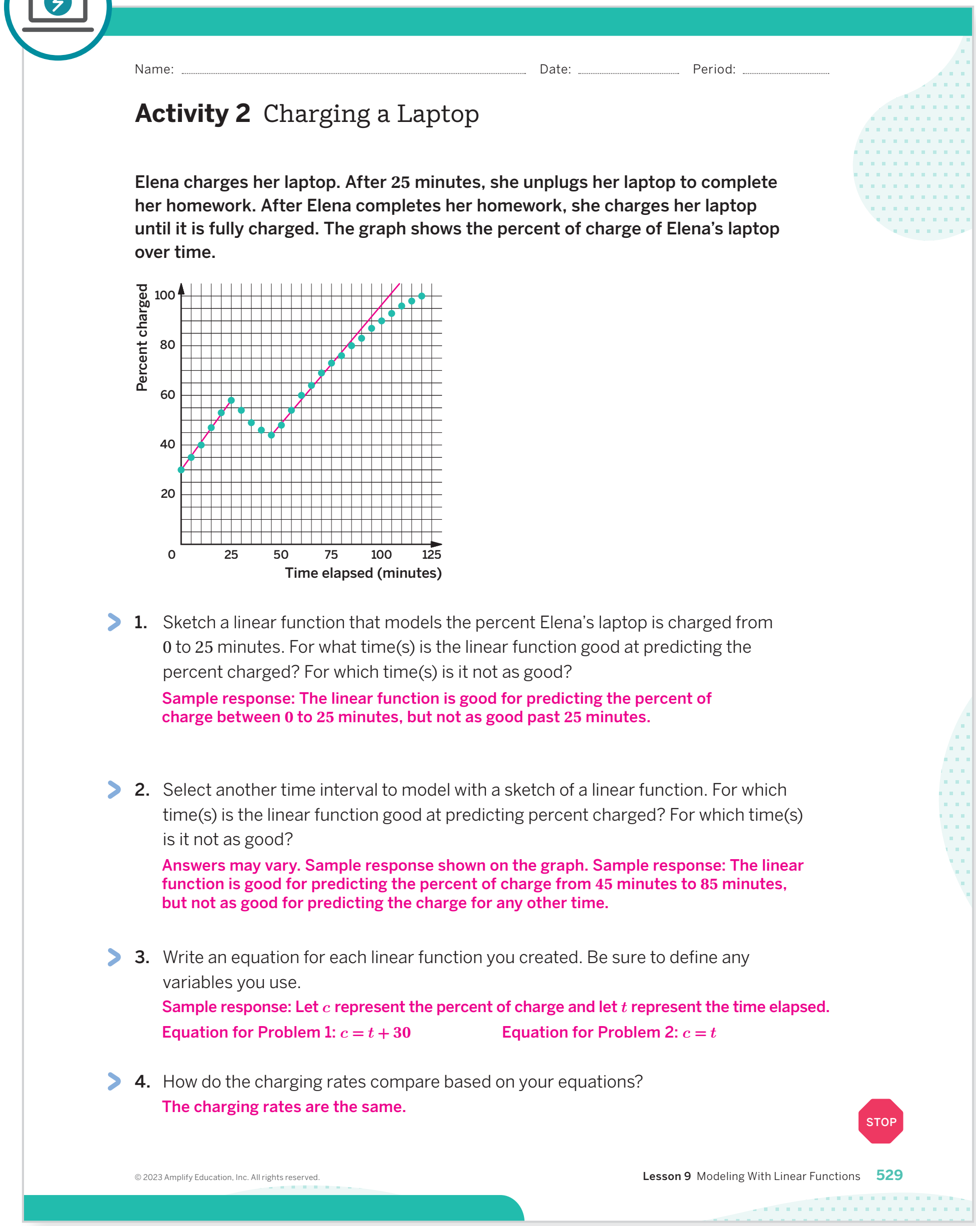
English Learners

Consider providing the information as a table of values, without defining variables, showing the relationship between time and percentage charged.

Small Groups | 12 min

Activity 2 Charging a Laptop

Students sketch multiple linear functions to see that different linear models can apply to different parts of a set of data.



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Activity 2 Charging a Laptop

Elena charges her laptop. After 25 minutes, she unplugs her laptop to complete her homework. After Elena completes her homework, she charges her laptop until it is fully charged. The graph shows the percent of charge of Elena's laptop over time.

1. Sketch a linear function that models the percent Elena's laptop is charged from 0 to 25 minutes. For what time(s) is the linear function good at predicting the percent charged? For which time(s) is it not as good?
   Sample response: The linear function is good for predicting the percent of charge between 0 to 25 minutes, but not as good past 25 minutes.

2. Select another time interval to model with a sketch of a linear function. For which time(s) is the linear function good at predicting percent charged? For which time(s) is it not as good?
   Answers may vary. Sample response shown on the graph. Sample response: The linear function is good for predicting the percent of charge from 45 minutes to 85 minutes, but not as good for predicting the charge for any other time.

3. Write an equation for each linear function you created. Be sure to define any variables you use.
   Sample response: Let c represent the percent of charge and let t represent the time elapsed.
   Equation for Problem 1: $c = t + 30$ **Equation for Problem 2: $c = t$**

4. How do the charging rates compare based on your equations?
   The charging rates are the same.

STOP

1 Launch

Ask students how the laptop charge changes as the time elapses. Check to make sure that students correctly interpret the increasing and decreasing intervals of the graph. Provide access to rulers.

2 Monitor

Help students get started by asking, "Do you think the data is best modeled by a single linear function or multiple linear functions?"

Look for points of confusion:

- **Having trouble with Problem 2.** Consider providing students with a specific time range, such as 25 to 45 minutes, to draw a linear function.
- **Struggling to write an equation.** Have students refer to the Anchor Chart PDF, *Writing Linear Equations in* $y = mx + b$ *Form*.

Look for productive strategies:

- Comparing work with a partner, and then adjusting their line to model the data more closely.

3 Connect

Display the varying linear functions students sketched.

Ask:

- "Why was a linear function used to model the data when the data was not linear?" A linear function could be created to make predictions for certain intervals of the data that are approximately linear.
- "If you drew a single line to model the data from 0 to 120 minutes, what would that line predict well? What would that line predict poorly?"

Highlight that, although a linear function may model and predict data well for one part of a graph, the same linear function may not be good at modeling and predicting other parts of the graph.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

During the Launch, as you discuss the increasing and decreasing intervals of the graph, annotate the graph intervals with *increasing* and *decreasing* and keep it displayed throughout the activity. Provide access to rulers or straightedges and index cards. Suggest that students use the index cards to cover up other parts of the graph while they examine one of the intervals.

Extension: *Math Enrichment*

Ask students to interpret why the graph seems to "level off" the closer the percentage charged gets to 100%. Sample response: The closer the phone gets to fully charged, the slower its rate of increase.

Math Language Development

MLR8: Discussion Supports—Restate It!

During the Connect, as students respond to the Ask questions, pause and ask their classmates to restate what they heard in their own words, ask any clarifying questions, or respectfully challenge an idea. For example:

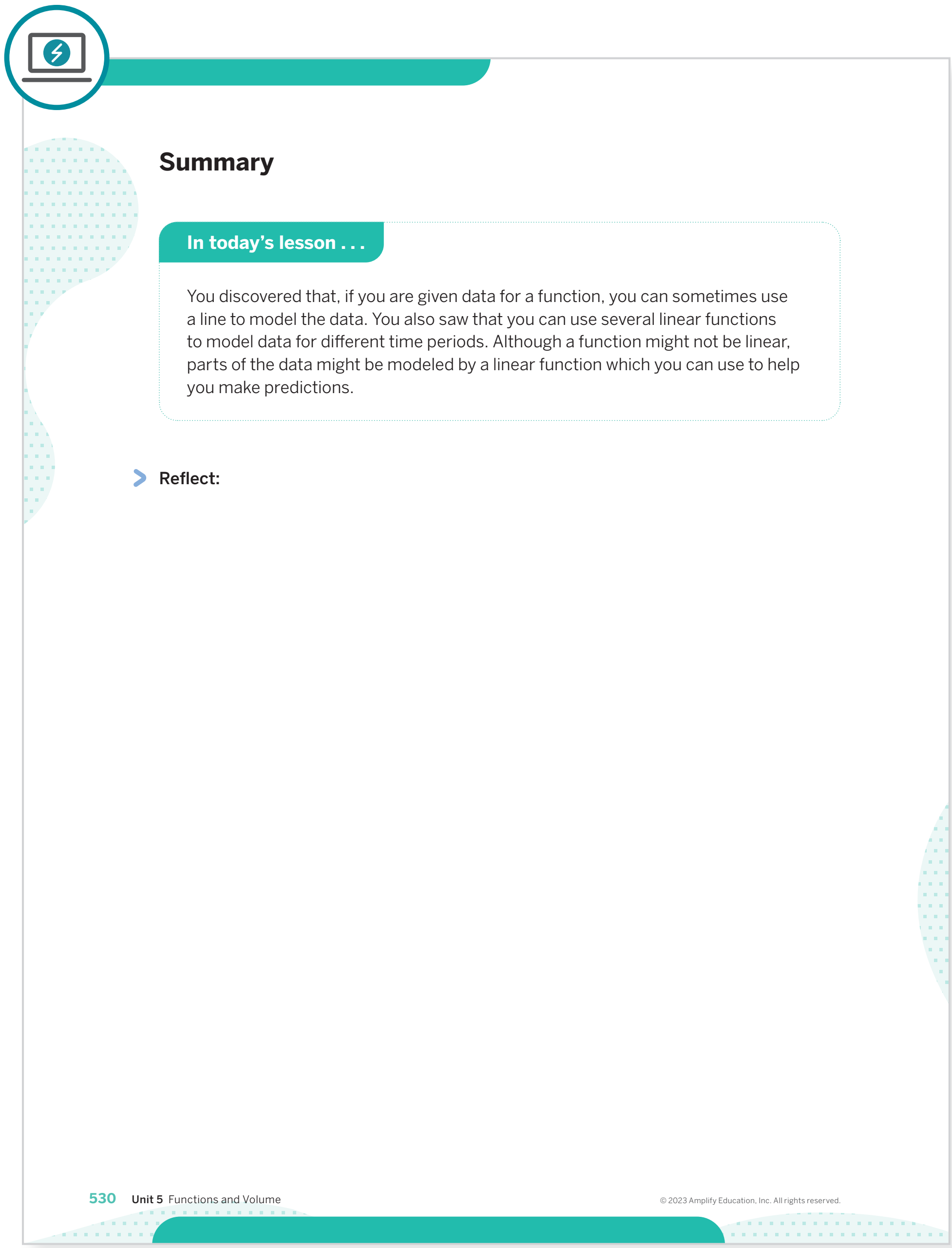
If a student says . . .	A classmate could ask . . .
"A linear function was used because the points mostly fall on a straight line."	"When you say the points mostly fall on a straight line, do you mean all the points, or points within each increasing/decreasing interval?"

Consider modeling for students how to ask a clarifying question.

Whole Class | 5 min

Summary

Review and synthesize how some real-world data sets can be modeled with linear functions.



Summary

In today's lesson . . .

You discovered that, if you are given data for a function, you can sometimes use a line to model the data. You also saw that you can use several linear functions to model data for different time periods. Although a function might not be linear, parts of the data might be modeled by a linear function which you can use to help you make predictions.

Reflect:

530 Unit 5 Functions and Volume

Synthesize

Have students share when a linear function might be used to model data.

Highlight that a linear function can be used to model data that is approximately linear to help students make a prediction.

Ask, "What are some advantages of using a linear function to model data? What are some disadvantages?" Sample response: Linear functions can help to make a quick prediction, but sometimes a linear function does not fit all parts of the data well and may not make a good prediction for a certain interval.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

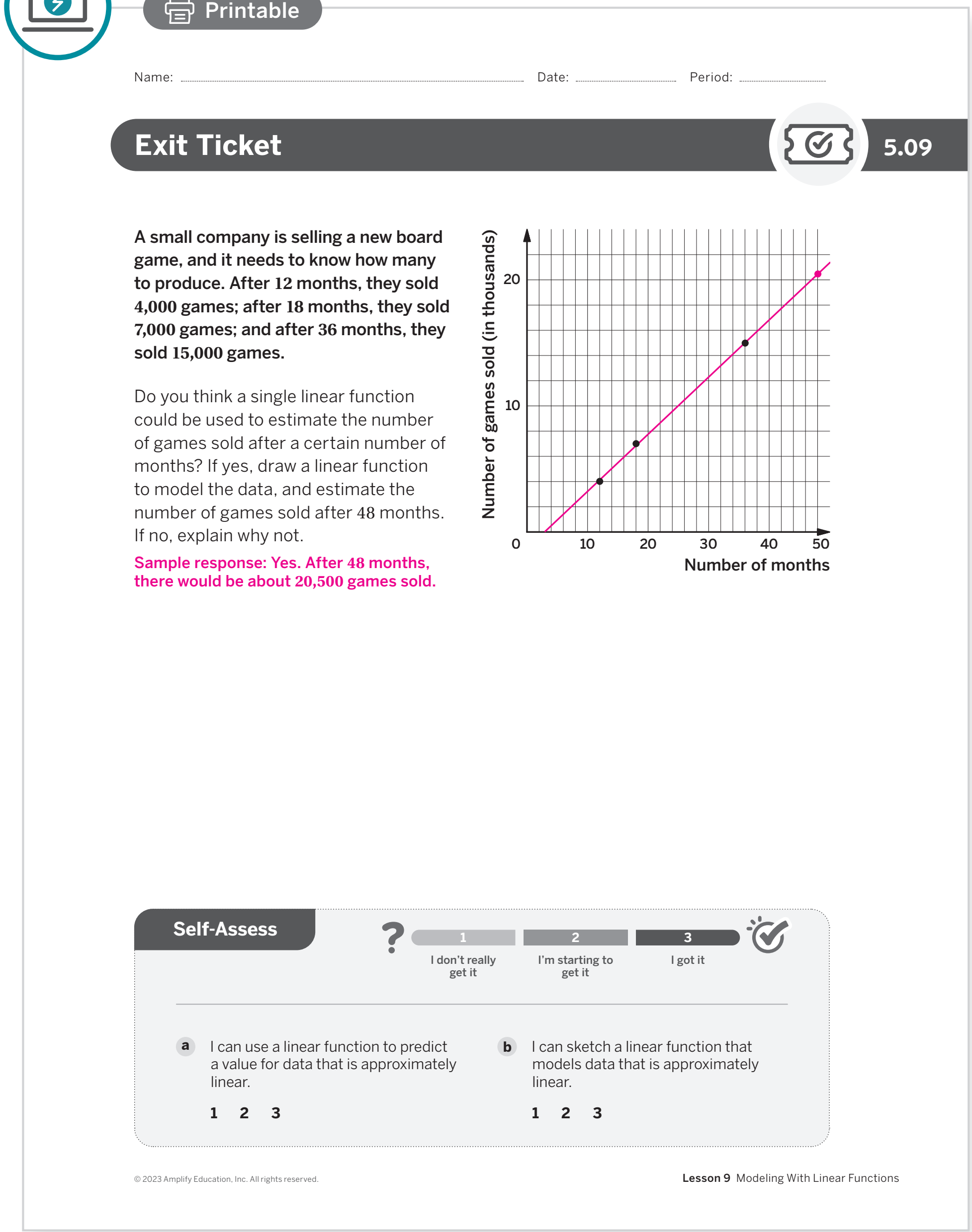
- "When might it be appropriate to model data with a linear function? When might it not be appropriate?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining whether a linear function could model data given by a graph.

Printable



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Exit Ticket 5.09

A small company is selling a new board game, and it needs to know how many to produce. After 12 months, they sold 4,000 games; after 18 months, they sold 7,000 games; and after 36 months, they sold 15,000 games.

Do you think a single linear function could be used to estimate the number of games sold after a certain number of months? If yes, draw a linear function to model the data, and estimate the number of games sold after 48 months. If no, explain why not.

Sample response: Yes. After 48 months, there would be about 20,500 games sold.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use a linear function to predict a value for data that is approximately linear.

1 2 3

b I can sketch a linear function that models data that is approximately linear.

1 2 3

Success looks like . . .

- **Goal:** Using data points to model a linear function.
  - » Modeling the sales of a board game with a linear function.
- **Language Goal:** Comparing and contrasting different linear models of the same data, and determining the range of values for which a given model is a good fit for the data. **(Speaking and Listening, Writing)**
- **Language Goal:** Modeling nonlinear data using a linear function, and justifying whether the model is a good fit for the data. **(Speaking and Listening, Writing)**

Suggested next steps

If students do not think a linear function could model the data, consider:

- Assessing after linear models are formally introduced.

If students do not correctly estimate the number of games sold after 48 months, consider:

- Reviewing Lesson 8.

Professional Learning

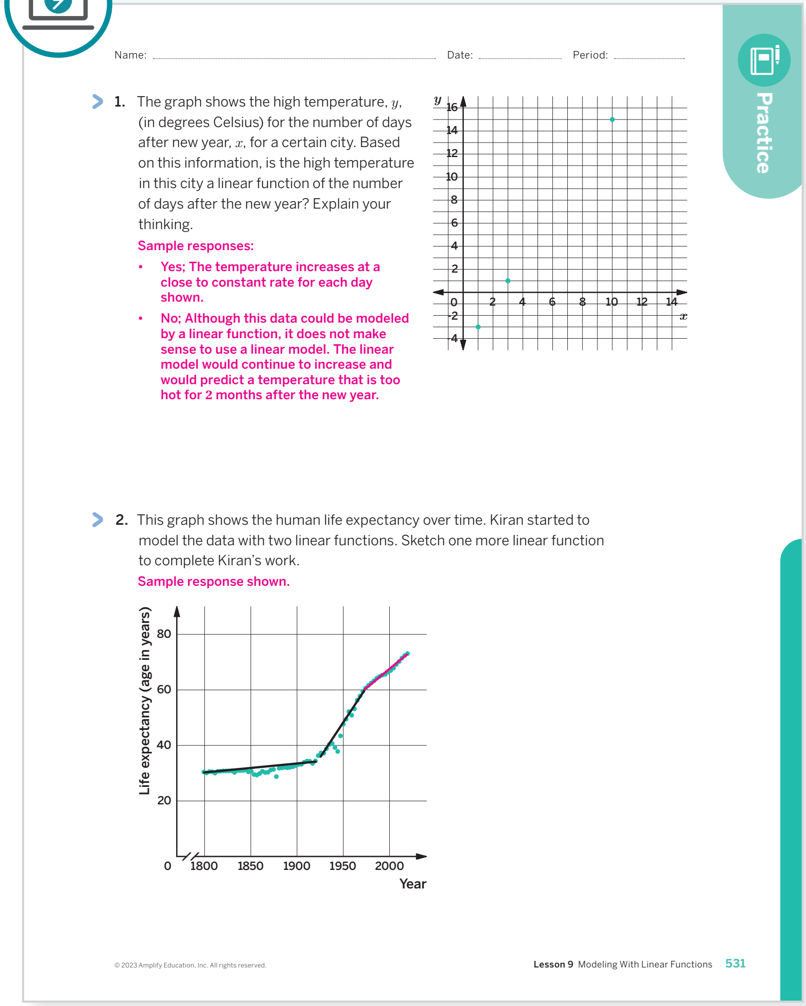
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In a prior unit, students explored linear relationships. How did that help them model data that was approximately linear?
- In this lesson, students used linear functions to model real-world situations. How will that support their understanding when they are formally introduced to linear models in an upcoming unit?

Practice

Independent



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Practice

1. The graph shows the high temperature, y, (in degrees Celsius) for the number of days after new year, x, for a certain city. Based on this information, is the high temperature in this city a linear function of the number of days after the new year? Explain your thinking.

   Sample responses:

   - Yes; The temperature increases at a close to constant rate for each day shown.
   - No; Although this data could be modeled by a linear function, it does not make sense to use a linear model. The linear model would continue to increase and would predict a temperature that is too hot for 2 months after the new year.

2. This graph shows the human life expectancy over time. Kiran started to model the data with two linear functions. Sketch one more linear function to complete Kiran's work.

   Sample response shown.

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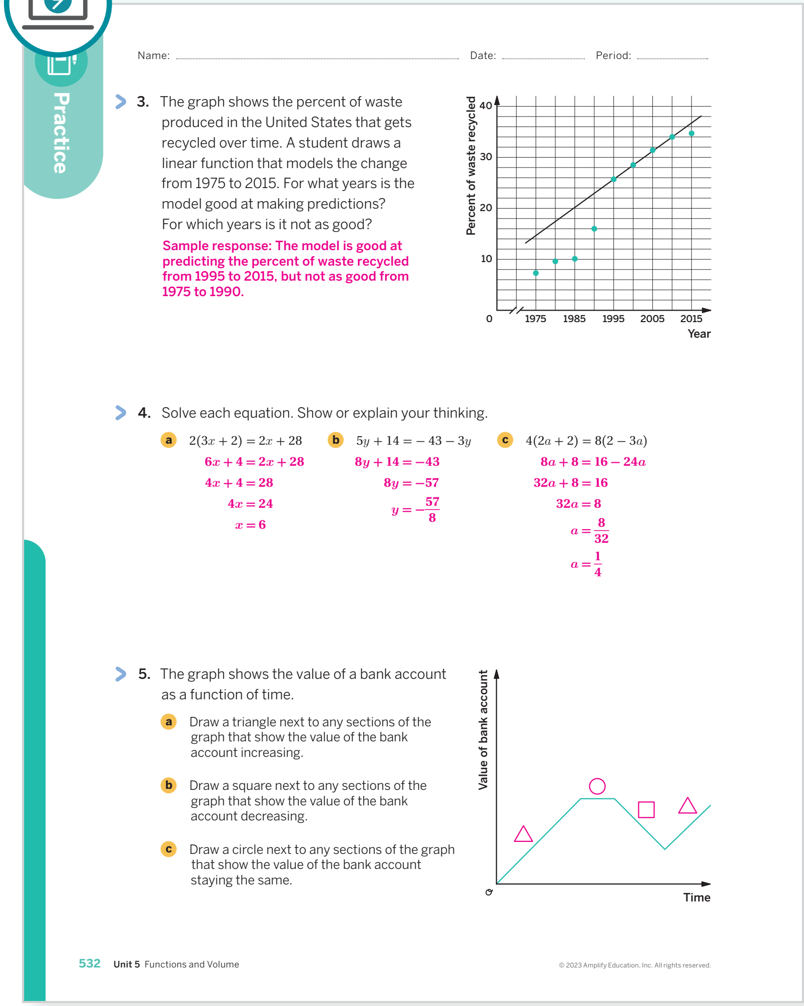
Lesson 9 Modeling With Linear Functions 531



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Practice

3. The graph shows the percent of waste produced in the United States that gets recycled over time. A student draws a linear function that models the change from 1975 to 2015. For what years is the model good at making predictions? For which years is it not as good?

   Sample response: The model is good at predicting the percent of waste recycled from 1995 to 2015, but not as good from 1975 to 1990.

4. Solve each equation. Show or explain your thinking.

   a. $2(3x+2)=2x+28$

   $6x+4=2x+28$

   $4x+4=28$

   $4x=24$

   $x=6$

   b. $5y+14=-43-3y$

   $8y+14=-43$

   $8y=-57$

   $y=-\frac{57}{8}$

   c. $4(2a+2)=8(2-3a)$

   $8a+8=16-24a$

   $32a+8=16$

   $32a=8$

   $a=\frac{8}{32}$

   $a=\frac{1}{4}$

5. The graph shows the value of a bank account as a function of time.

   a. Draw a triangle next to any sections of the graph that show the value of the bank account increasing.

   b. Draw a square next to any sections of the graph that show the value of the bank account decreasing.

   c. Draw a circle next to any sections of the graph that show the value of the bank account staying the same.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 5	2
Formative ⚡	5	Unit 5 Lesson 10	1

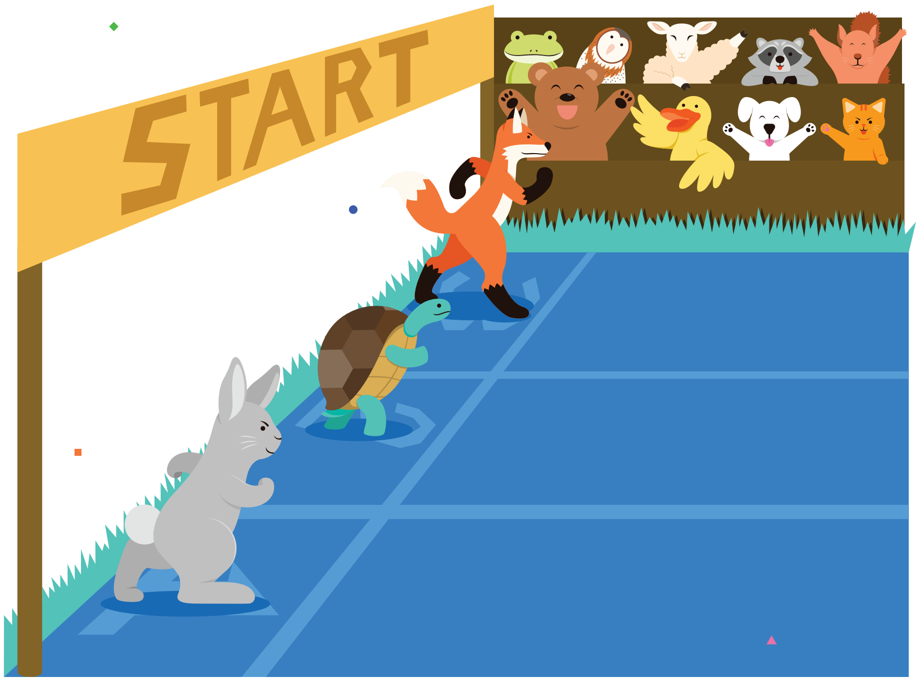
⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Piecewise Functions

Let's explore functions built from pieces of linear functions.



Focus

Goals

1. **Language Goal:** Calculate the different rates of change of a piecewise function using a graph, and interpret the rates of change in context. **(Speaking and Listening, Writing)**
2. **Language Goal:** Create a graph to model a situation using a piecewise function made up of linear segments. **(Speaking and Listening, Writing)**

Coherence

Today

Students use piecewise graphs to determine information about the real-world situations represented. The focus of this lesson is to study the graphs qualitatively and to compute and compare the different rates of change.

Previously

In Lesson 9, students used linear functions to model real-world scenarios.

Coming Soon

In Lesson 11, students will begin the second Sub-Unit by studying the volume of cylinders. Later in the unit, they will explore the relationship between the volumes of a cylinder, cone, and sphere.

Rigor

- Students build **conceptual understanding** of how a piecewise function can represent real-world scenarios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Representations of Linear Relationships* (from Unit 3)
- rulers

Math Language Development

New words

- ***piecewise function***

Review words

- *linear function*
- *slope*
- *y-intercept*

Amps | **Featured Activity**

Activity 2
Formative Feedback for Students

Students sketch a graph representing a dog's position in a race. Then, they can see an animation of the dog based on their graph.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may lack self-motivation to apply reasoning to solve the problems, as they consider what the piecewise function represents in the scenario. Ask students to think of themselves as the tortoise as they approach this new topic. Slow and steady progress by considering only one problem at a time will get them to the finish line.

Modifications to Pacing

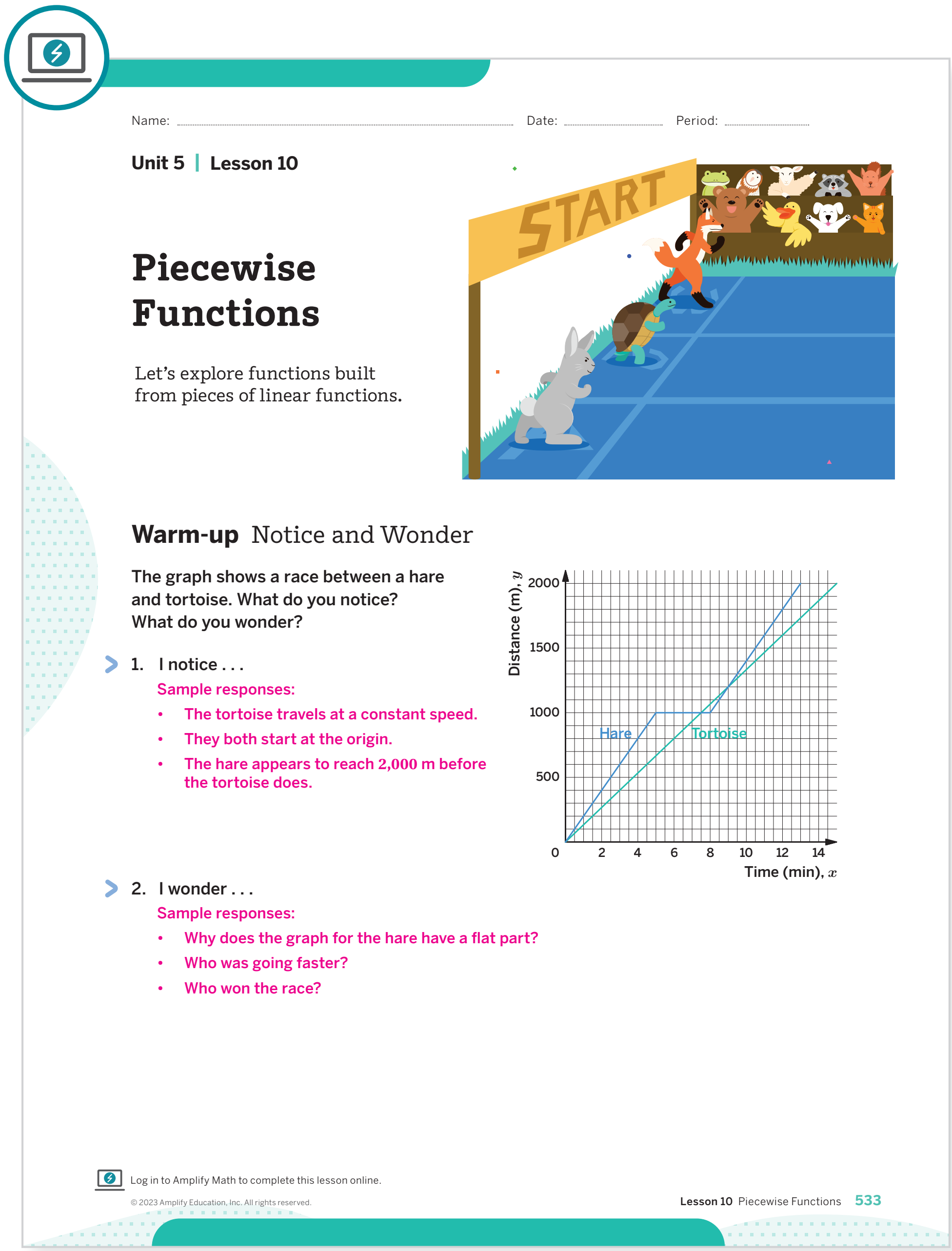
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. Instead, use the ***Notice and Wonder*** routine to introduce Activity 1.
- **Activity 2** may be omitted.

Pairs | 5 min

Warm-up Notice and Wonder

Students explore a graph composed of linear sections as an introduction to piecewise functions.



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Unit 5 | Lesson 10

Piecewise Functions

Let's explore functions built from pieces of linear functions.

Warm-up Notice and Wonder

The graph shows a race between a hare and tortoise. What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - The tortoise travels at a constant speed.
   - They both start at the origin.
   - The hare appears to reach 2,000 m before the tortoise does.

2. I wonder . . .
   Sample responses:
   - Why does the graph for the hare have a flat part?
   - Who was going faster?
   - Who won the race?

Log in to Amplify Math to complete this lesson online.

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Lesson 10 Piecewise Functions 533

1 Launch

Activate students' background knowledge by asking students what they remember about Aesop's fable of the race between the tortoise and the hare. Explain that, they are going to explore races between a hare, a tortoise, a fox, and a dog. Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking them what they notice is similar and what is different about the shapes of the two graphs.

Look for productive strategies:

- Calculating the slope for each section of the hare's graph to compare to the tortoise's graph.
- Describing the changes in the graph for the hare in context.

3 Connect

Have students share what they noticed and wondered about the graphs.

Ask:

- "What do the slopes of the different lines mean?"
- "Which animal was going faster at 4 minutes? How can you tell?"
- "What happened at 6 minutes? At 8 minutes?"
- "Is the graph of the tortoise linear? The hare? Why or why not?"
- "What happened with the hare during the race?"

Highlight the story of the hare during the race.

Define the term ***piecewise function***. Say, "Sometimes you can model a function with multiple line segments. This function is called a piecewise function." Have students describe why the graph of the hare is an example of a piecewise function.

Math Language Development

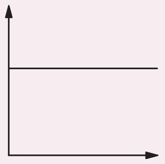
MLR2: Collect and Display

Collect and display language students use to describe the graph that can be connected to the definition of *piecewise function*. To help students make sense of the definition, build on the collected student language when defining *piecewise*. For example, students may describe the graph as "containing multiple pieces," "made of multiple parts," "different line segments," etc. **Note:** Students are not expected to know at this point that a piecewise function does not have to consist of all straight line segments.

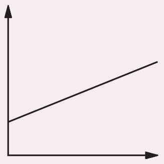
Power-up

To power up students' ability to identify sections of a graph that are increasing, decreasing, and constant, have students complete:

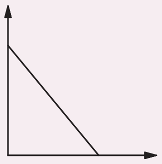
Label each graph as *increasing*, *decreasing*, or *constant*.



constant



increasing



decreasing

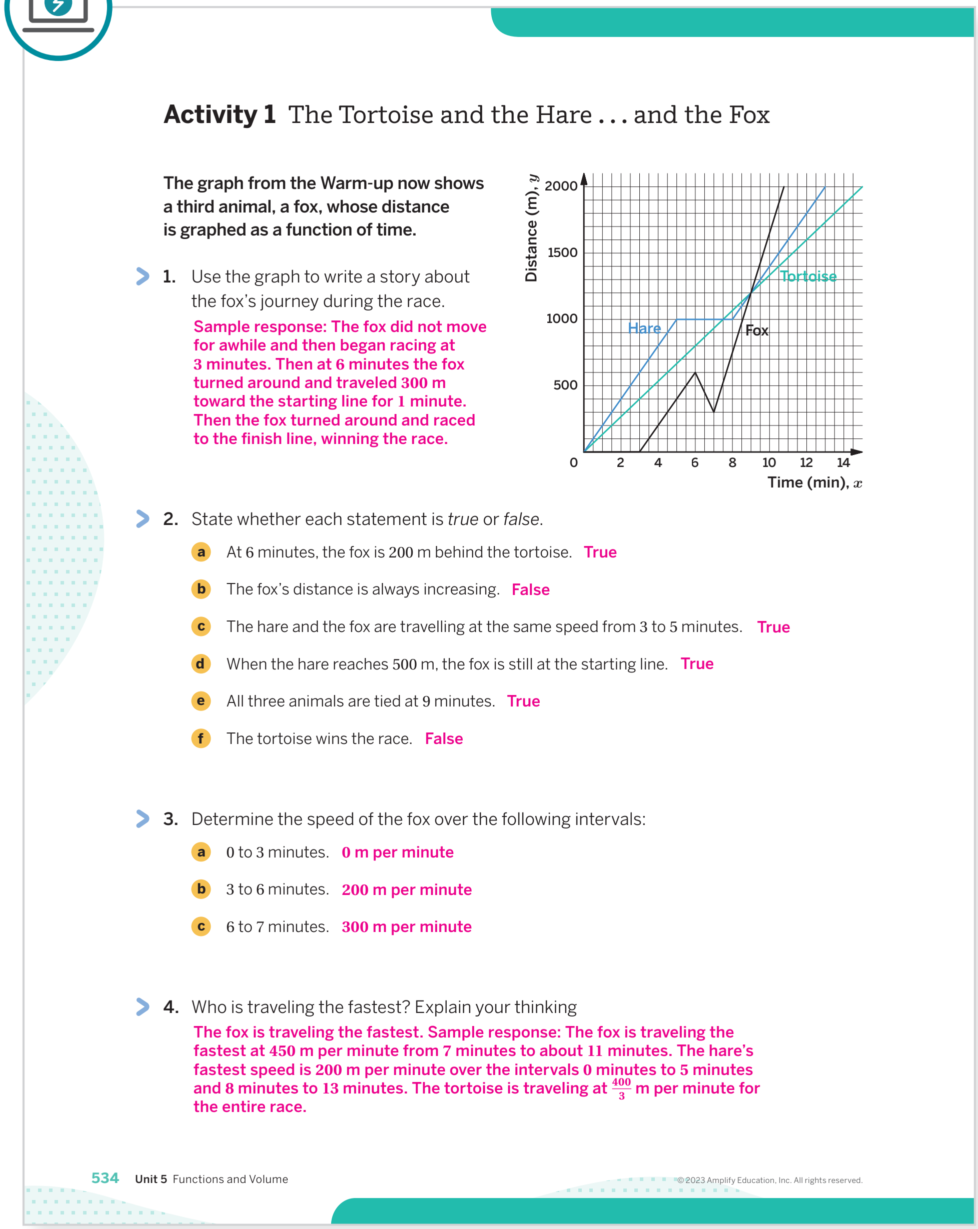
Use: Before the Warm-up

Informed by: Performance on Lesson 9, Practice Problem 5

Pairs | 15 min

Activity 1 The Tortoise and the Hare . . . and the Fox

Students make connections between an animal space race and the graphs for each animal to explore how piecewise functions can represent scenarios.



Activity 1 The Tortoise and the Hare . . . and the Fox

The graph from the Warm-up now shows a third animal, a fox, whose distance is graphed as a function of time.

1. Use the graph to write a story about the fox's journey during the race.
   Sample response: The fox did not move for awhile and then began racing at 3 minutes. Then at 6 minutes the fox turned around and traveled 300 m toward the starting line for 1 minute. Then the fox turned around and raced to the finish line, winning the race.

2. State whether each statement is *true* or *false*.
   - a At 6 minutes, the fox is 200 m behind the tortoise. **True**
   - b The fox's distance is always increasing. **False**
   - c The hare and the fox are travelling at the same speed from 3 to 5 minutes. **True**
   - d When the hare reaches 500 m, the fox is still at the starting line. **True**
   - e All three animals are tied at 9 minutes. **True**
   - f The tortoise wins the race. **False**

3. Determine the speed of the fox over the following intervals:
   - a 0 to 3 minutes. **0 m per minute**
   - b 3 to 6 minutes. **200 m per minute**
   - c 6 to 7 minutes. **300 m per minute**

4. Who is traveling the fastest? Explain your thinking
   The fox is traveling the fastest. Sample response: The fox is traveling the fastest at 450 m per minute from 7 minutes to about 11 minutes. The hare's fastest speed is 200 m per minute over the intervals 0 minutes to 5 minutes and 8 minutes to 13 minutes. The tortoise is traveling at $\frac{400}{3}$ m per minute for the entire race.

534 Unit 5 Functions and Volume

1 Launch

Conduct the ***Think-Pair-Share*** routine for Problem 1 before providing pairs of students work time to complete the rest of the activity.

2 Monitor

Help students get started by asking them to describe which linear section they are examining, using the scale on the x-axis to identify the interval of the section.

Look for points of confusion:

- **Being unsure how to determine the speed represented by a linear section.** Have students reference the Anchor Chart PDF, *Representations of Linear Relationships*.
- **Thinking that the fox's distance is always increasing.** Ask, "During the first 3 minutes of the race, does the fox's distance increase, decrease, or neither? What about from 6 to 7 minutes?"

3 Connect

Have students share their responses for Problem 2 using the ***Poll the Class*** routine. Then have students share how they determined the speed represented by each interval of the graph in Problem 3 before discussing Problem 4.

Ask:

- "What do the slopes of the different lines mean?"
- "Who wins the race? How can you tell?"
- "Is there any information that you would add to your story about the fox's journey in Problem 1?"

Highlight how the speeds (slopes) can be determined for linear sections of a piecewise function. Clarify that piecewise functions do not have one constant speed, or rate of change, and are, therefore, nonlinear as a whole.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students choose four of the six statements to analyze in Problem 2 and two intervals in Problem 3. Allowing them to choose which parts to complete can help support their engagement in the activity.

Extension: *Math Enrichment*

Have students use the graph to estimate what minimum distance the tortoise would need as a head start in order to win the race. Have students draw a graph to represent their conclusion, assuming the tortoise races at the same speed as in the activity. Students should sketch a linear graph with the same slope of $\frac{400}{3}$ and a y-intercept of about 300 (about a 300-m head start).

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the graph without revealing the problems from the activity. Have students work with their partner to write 2–3 questions that could be answered by examining the graph. Ask a few student pairs to share their questions with the class. Highlight questions that make sense of the graphs in context. Sample questions shown.

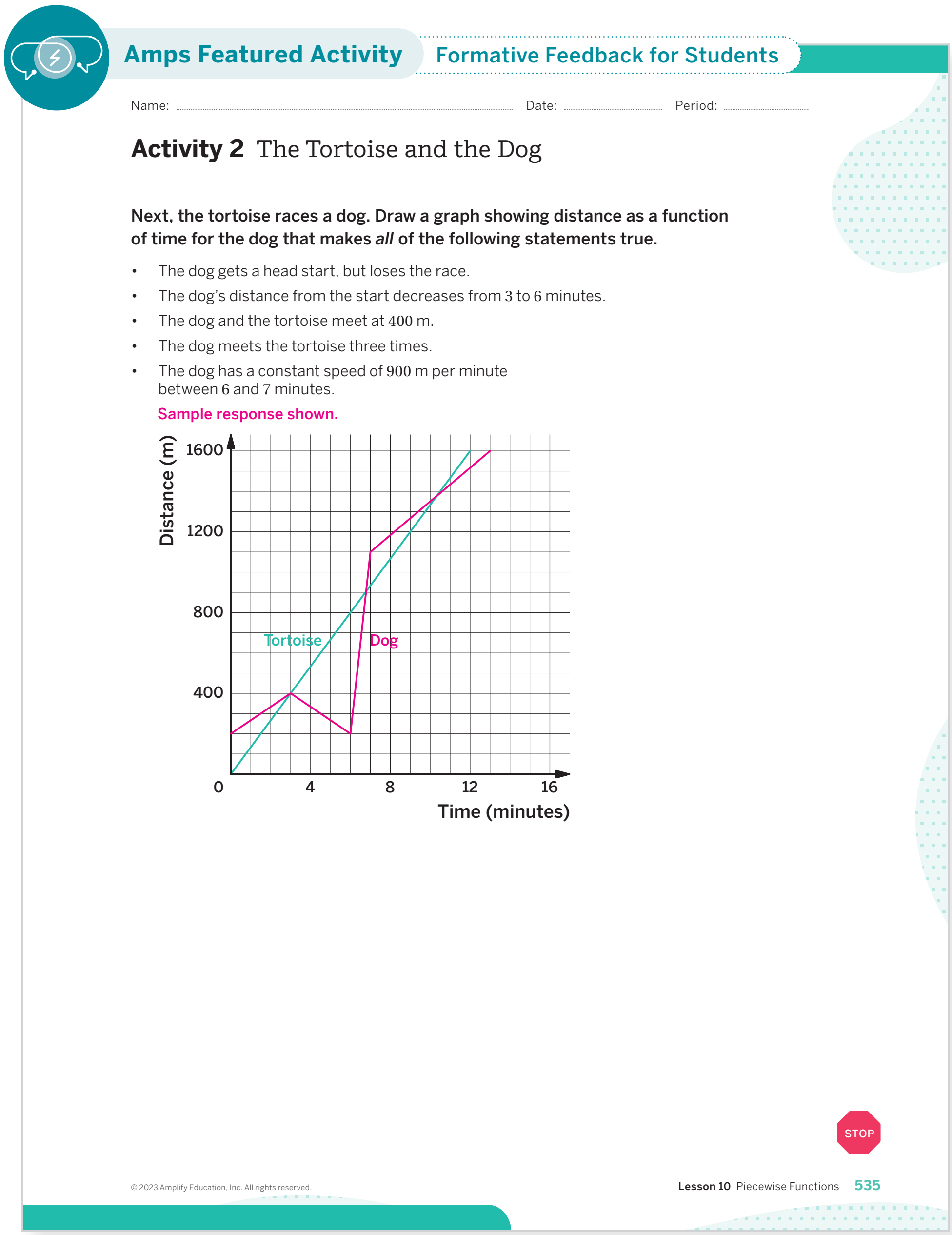
- Which animal appears to have stopped to rest during the race? After doing so, did they increase their rate or keep it the same?
- Which animal appears to have backtracked? After doing so, did they increase their rate or keep it the same?

Pairs | 15 min

Activity 2 The Tortoise and the Dog

Students create their own graph to represent the distance versus time relationship for a new animal, a dog.

Amps Featured Activity Formative Feedback for Students



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Activity 2 The Tortoise and the Dog

Next, the tortoise races a dog. Draw a graph showing distance as a function of time for the dog that makes *all* of the following statements true.

- The dog gets a head start, but loses the race.
- The dog's distance from the start decreases from 3 to 6 minutes.
- The dog and the tortoise meet at 400 m.
- The dog meets the tortoise three times.
- The dog has a constant speed of 900 m per minute between 6 and 7 minutes.

Sample response shown.

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Lesson 10 Piecewise Functions 535

1 Launch

Distribute rulers for the duration of the activity.

2 Monitor

Help students get started by helping them identify how they can show the dog has a head start on the graph.

Look for points of confusion:

- **Being unsure how to show the dog's distance decreasing.** Ask students to identify which section of the graph in Activity 1 shows the fox's distance decreasing.
- **Not accurately showing the dog with a constant speed of 900 m per minute from 6 to 7 minutes.** Have students calculate the slope for their graph segment from 6 to 7 minutes and ask them how they can revise their graph to demonstrate a speed of 900 m per minute.

3 Connect

Display two student graphs that meet all five criteria.

Have students share the similarities and differences between the two graphs.

Ask:

- "Do the two graphs satisfy the first constraint? How do you know? The second? Third? Fourth? Fifth?"
- "Which sections of the graph needed to be a straight line? How do you know?"

Highlight the ways students can use the graph to coordinate with the story. Have students elaborate on what it means for the dog's distance to be decreasing. Emphasize that the graph is nonlinear as a whole, but that there are linear sections. Discuss that piecewise functions can also be made up of nonlinear sections, which students will study in later grades.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally create their own graph of a piecewise function to match a situation.

Extension: *Math Enrichment*

Have students modify their sketch — or explain how to do so — if the dog's speed after 7 minutes slows down, while the dog's distance still increases.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that the tortoise and a dog are racing.
- **Read 2:** Ask students to choose 1 or 2 of the statements and describe them in their own words.
- **Read 3:** Ask students to plan their solution strategy as to how they will sketch the graph for the dog. Consider asking, "Which statement will help you know where the dog starts the race?"

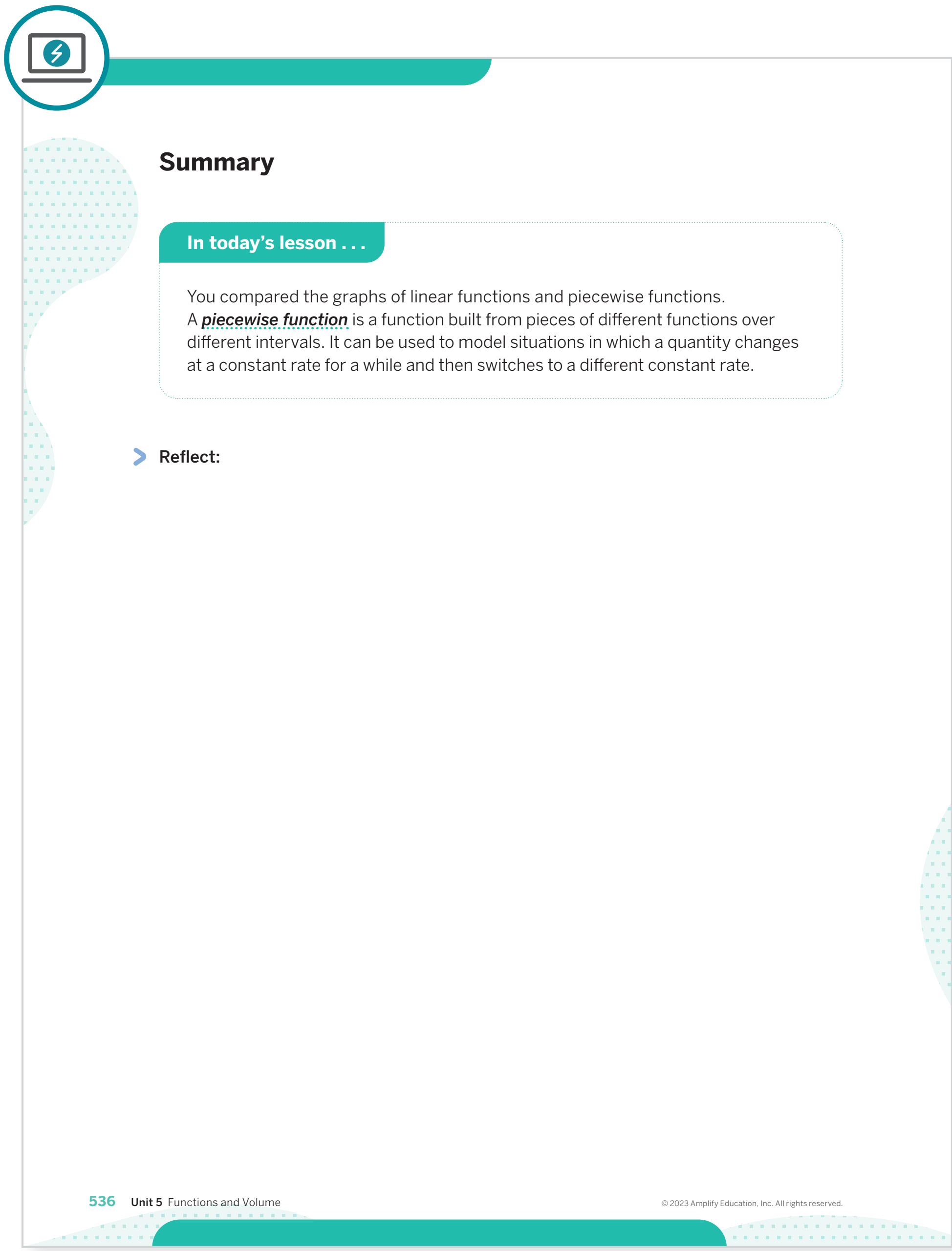
English Learners

Students may be unfamiliar with the phrase *head start*. Consider demonstrating this concept using student volunteers, asking both to walk around the classroom, but giving one of them a head start.

Whole Class | 5 min

Summary

Review and synthesize how piecewise functions built from linear pieces can be used to model some real-world situations.



Summary

In today's lesson . . .

You compared the graphs of linear functions and piecewise functions. A ***piecewise function*** is a function built from pieces of different functions over different intervals. It can be used to model situations in which a quantity changes at a constant rate for a while and then switches to a different constant rate.

Reflect:

536 **Unit 5** Functions and Volume

Synthesize

Ask, "How would you describe a piecewise function to someone who has never seen one?"

Formalize vocabulary: ***piecewise function***

Highlight that a *piecewise function* is a function whose graph is made up of different functions over different intervals. Specifically, for linear piecewise function, there are different intervals for the inputs at which the output changes at different constant rates. A different line is used for each interval.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did your knowledge of linear functions inform your thinking about piecewise functions with linear pieces?"

Math Language Development

MLR2: Collect and Display

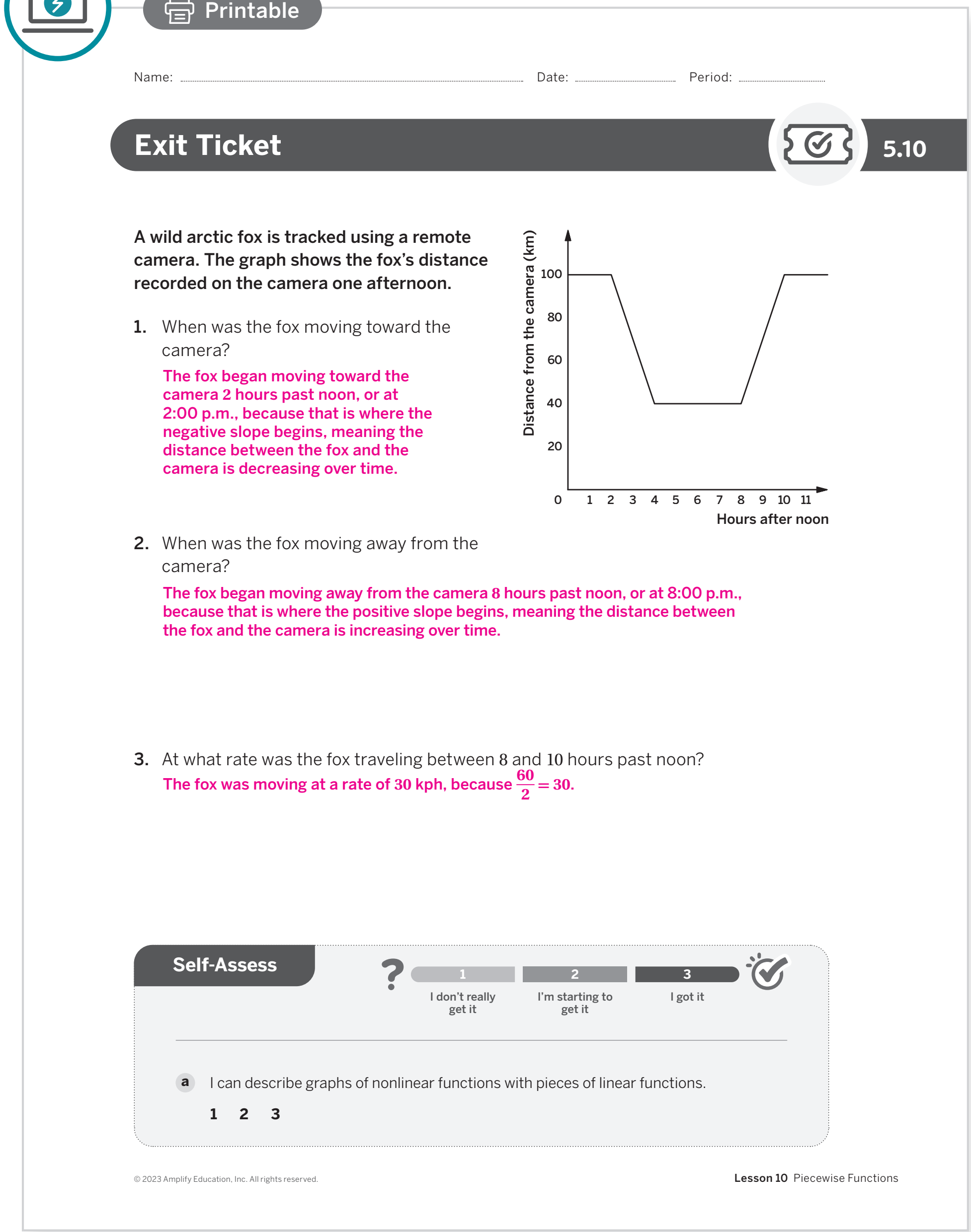
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *piecewise functions* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing a real-world scenario modeled by a piecewise function.

Printable



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Exit Ticket 5.10

A wild arctic fox is tracked using a remote camera. The graph shows the fox's distance recorded on the camera one afternoon.

1. When was the fox moving toward the camera?

   The fox began moving toward the camera 2 hours past noon, or at 2:00 p.m., because that is where the negative slope begins, meaning the distance between the fox and the camera is decreasing over time.

2. When was the fox moving away from the camera?

   The fox began moving away from the camera 8 hours past noon, or at 8:00 p.m., because that is where the positive slope begins, meaning the distance between the fox and the camera is increasing over time.

3. At what rate was the fox traveling between 8 and 10 hours past noon?

   The fox was moving at a rate of 30 kph, because $\frac{60}{2} = 30$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can describe graphs of nonlinear functions with pieces of linear functions.

1 2 3

Lesson 10 Piecewise Functions

Success looks like . . .

- **Language Goal:** Calculating the different rates of change of a piecewise function using a graph, and interpreting the rates of change in context. **(Speaking and Listening, Writing)**
  - » Interpreting the slope of the graph to determine when the fox was moving toward and away from the camera in Problems 1 and 2.
- **Language Goal:** Creating a graph to model a situation using a piecewise function made up of linear segments. **(Speaking and Listening, Writing)**

Suggested next steps

If students have difficulty responding to Problem 1, consider:

- Asking for the initial distance from the camera of the fox.
- Reviewing strategies for identifying changes in the piecewise graph from Activity 1.

If students have difficulty determining the rate in Problem 3, consider:

- Referencing the Anchor Chart PDF, *Representations of Linear Relationships*, to help students determine the slope.
- Reviewing strategies for determining rates of different pieces of piecewise functions from Activity 1.

Professional Learning

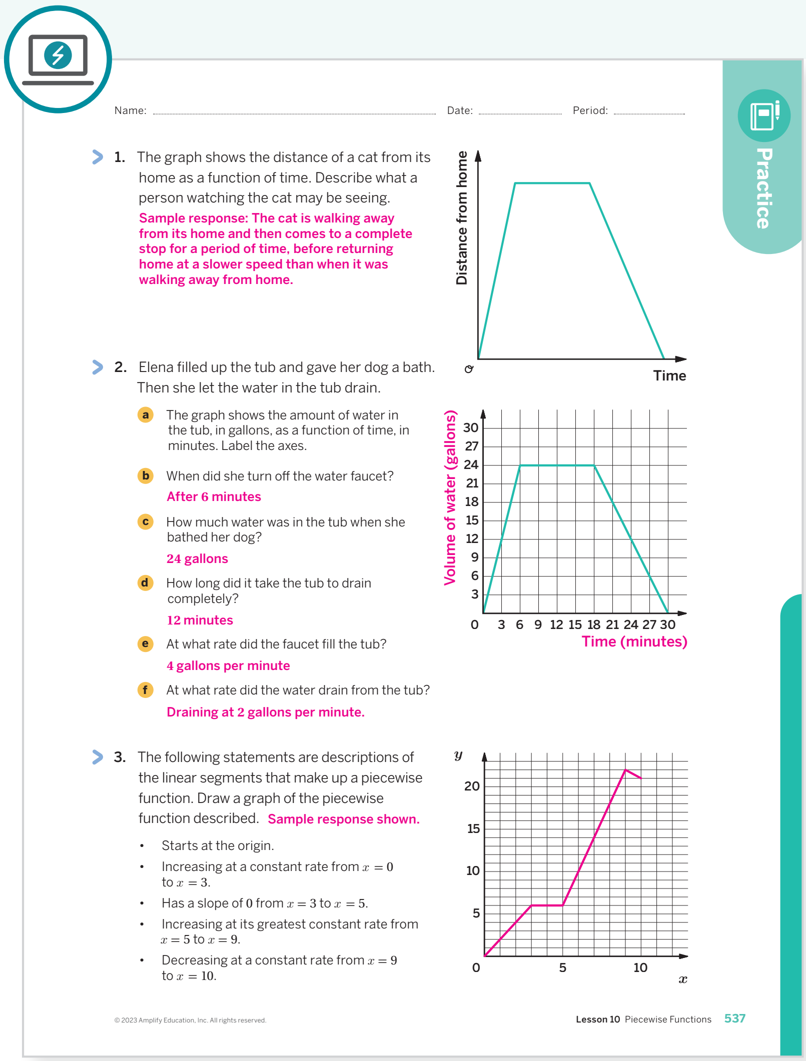
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did the ***Notice and Wonder*** routine support students in learning about piecewise functions?
- What was especially satisfying about seeing students connect what learned about linear functions with what they learned today about piecewise functions?

Practice

Independent



Name: Date: Period:

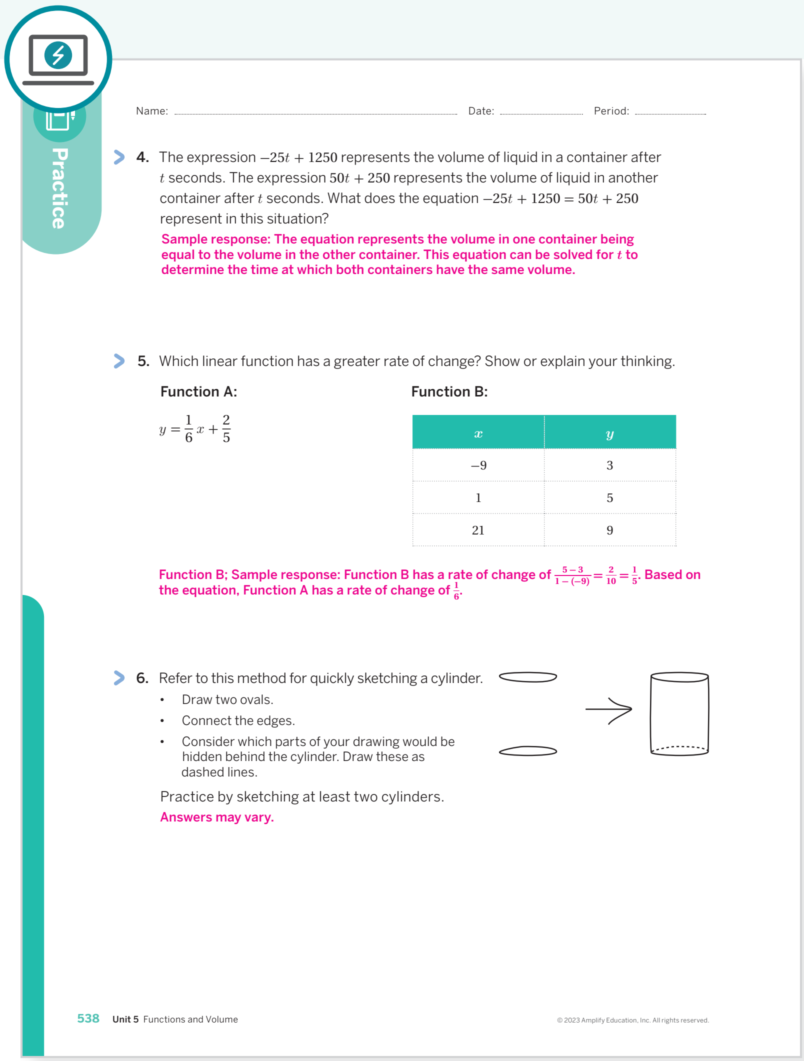
Practice

1. The graph shows the distance of a cat from its home as a function of time. Describe what a person watching the cat may be seeing.
Sample response: The cat is walking away from its home and then comes to a complete stop for a period of time, before returning home at a slower speed than when it was walking away from home.

2. Elena filled up the tub and gave her dog a bath. Then she let the water in the tub drain.
   - a. The graph shows the amount of water in the tub, in gallons, as a function of time, in minutes. Label the axes.
   - b. When did she turn off the water faucet?
   After 6 minutes
   - c. How much water was in the tub when she bathed her dog?
   24 gallons
   - d. How long did it take the tub to drain completely?
   12 minutes
   - e. At what rate did the faucet fill the tub?
   4 gallons per minute
   - f. At what rate did the water drain from the tub?
   Draining at 2 gallons per minute.

3. The following statements are descriptions of the linear segments that make up a piecewise function. Draw a graph of the piecewise function described. **Sample response shown.**
   - Starts at the origin.
   - Increasing at a constant rate from $x = 0$ to $x = 3$.
   - Has a slope of 0 from $x = 3$ to $x = 5$.
   - Increasing at its greatest constant rate from $x = 5$ to $x = 9$.
   - Decreasing at a constant rate from $x = 9$ to $x = 10$.

Lesson 10 Piecewise Functions 537



Name: Date: Period:

Practice

4. The expression $-25t + 1250$ represents the volume of liquid in a container after t seconds. The expression $50t + 250$ represents the volume of liquid in another container after t seconds. What does the equation $-25t + 1250 = 50t + 250$ represent in this situation?
Sample response: The equation represents the volume in one container being equal to the volume in the other container. This equation can be solved for t to determine the time at which both containers have the same volume.

5. Which linear function has a greater rate of change? Show or explain your thinking.

Function A:

$y = \frac{1}{6}x + \frac{2}{5}$

Function B:

x	y
-9	3
1	5
21	9

Function B; Sample response: Function B has a rate of change of $\frac{5-3}{1-(-9)} = \frac{2}{10} = \frac{1}{5}$. Based on the equation, Function A has a rate of change of $\frac{1}{6}$.

6. Refer to this method for quickly sketching a cylinder.
   - Draw two ovals.
   - Connect the edges.
   - Consider which parts of your drawing would be hidden behind the cylinder. Draw these as dashed lines.

   Practice by sketching at least two cylinders.
   Answers may vary.

538 Unit 5 Functions and Volume

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 11	2
	5	Unit 5 Lesson 7	2
Formative	6	Unit 5 Lesson 11	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Sub-Unit 2
Cylinders, Cones, and Spheres

Whole Class

In this Sub-Unit, students explore the relationship between the volumes of a cylinder, cone, and sphere. Students reason about how the volume of a figure changes as another measurement changes.

SUB-UNIT

2 Cylinders, Cones, and Spheres

Narrative Connections

Who invented the waffle cone?

Nobody knows for sure! But over the years, many candidates have tried to claim the title.

First was Italo Marchiony, an Italian immigrant from New Jersey. In the late 1800s, Marchiony sold iced treats out of a pushcart. These ices were originally served out of glass cups. But these cups were impractical. They had to constantly be cleaned. Customers also had the nasty habit of walking off with them or breaking them. This inspired Marchiony to invent a more practical container that his customers could eat.

Then there was the case of David Avayou. Avayou was a Turkish immigrant who worked at the St. Louis Fair in 1904. He noticed that fairgoers were avoiding buying ice cream because of the hassle of eating from a plate. Having seen ice cream in paper cones during his travels in France, he set to work creating an edible version for the fair.

There was also the story of Ernest Hamwi. Hamwi was a Syrian immigrant who was also working at the very same St. Louis Fair. Legend has it, Hamwi was selling a Middle Eastern waffle treat, called zalabia, next to an ice cream vendor. When the vendor ran out of plates, Hamwi helped out by rolling the zalabia into a cone. This created a crisp edible container for the vendor's customers.

These are just a few of the people who have claimed to be the waffle cone's inventor. But just because we don't know its origins for certain, doesn't mean we can't appreciate the cone's design. A well-made waffle cone is easy to grab, reduces waste, and keeps your hands clean.

Let's take a closer look at what makes a cone a cone, and how this shape's dimensions relate to its volume through functions.

Sub-Unit 2 Cylinders, Cones, and Spheres 539

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how a cone's dimensions relates to its volume in the following places:

- **Lesson 14, Activities 1–2:** From Cylinders to Cones, Calculating the Volume of a Cone
- **Lesson 15, Activities 1–2:** Determining Unknown Dimensions, Which Is the Better Deal?
- **Lesson 17, Activity 3:** How Are the Volumes Related?
- **Lesson 18, Activity 2:** Melted Frozen Yogurt
- **Lesson 20, Activities 1–2:** Playing With Cones, Which One Has a Greater Volume?

UNIT 5 | **LESSON 11**

Filling Containers

Let's explore how functions can model the volume of a cylinder.

Focus

Goals

1. **Language Goal:** Create a graph of a function from collected data, and interpret a point on the graph. **(Speaking and Listening, Writing)**
2. **Language Goal:** Analyze a container for which the height of water, as a function of volume, would be represented as a piecewise linear function, and explain the reasoning. **(Speaking and Listening, Writing)**
3. **Language Goal:** Interpret a graph of heights of certain cylinders as a function of volume, and compare the rates of change of the functions. **(Speaking and Listening, Writing)**

Coherence

Today

In this lesson, students fill a graduated cylinder with different amounts of water and draw the graph of the height as a function of the volume. The following activity turns the situation around: when given a graph showing the height of water in a container as a function of the volume of water in the container, students determine the matching image of the container.

Previously

In Lessons 9 and 10, students learned about piecewise functions. In the first Sub-Unit, students learned about concepts related to functions.

Coming Soon

This lesson is the beginning of a sequence of lessons that interweaves the development of the function concept with the development of formulas for volumes of cylinders and cones. In Lesson 12, students will learn a formula for the relation between the height and the radius and the volume of a cylinder.

Rigor

- Students build **conceptual understanding** of how the volume of a cylinder is related to its height and diameter.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	20 min or 40 min*	10 min	5 min	5 min
Pairs	Small Groups	Pairs	Whole Class	Independent

If using the digital version of Activity 1, the suggested pacing is 20 minutes. If using the print version, the suggested pacing is 40 minutes.

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- one 100 ml graduated cylinder per group (optional)
- one container of a 100 ml of water per group (optional)
- Activity 1 PDF, *Height as a Function of Volume* (for display)
- Activity 2 PDF, pre-cut cards, one set per pair

Math Language Development

Review words

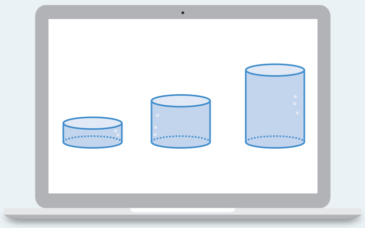
- *cylinder*
- *function*
- *height*
- *piecewise function*
- *radius*
- *slope*
- *volume*

Amps | Featured Activity

Activity 1

Digital Measurements

Students use digital tools to fill cylinders of different sizes with water and record observations about the height, radius, and volume of the water in each cylinder.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not feel comfortable with relating height and volume of a container. Encourage students to have a growth mindset. While it might not make sense to them yet, persistence will help them gain the self-confidence that they need to determine the relationship between the two quantities.

● Modifications to Pacing

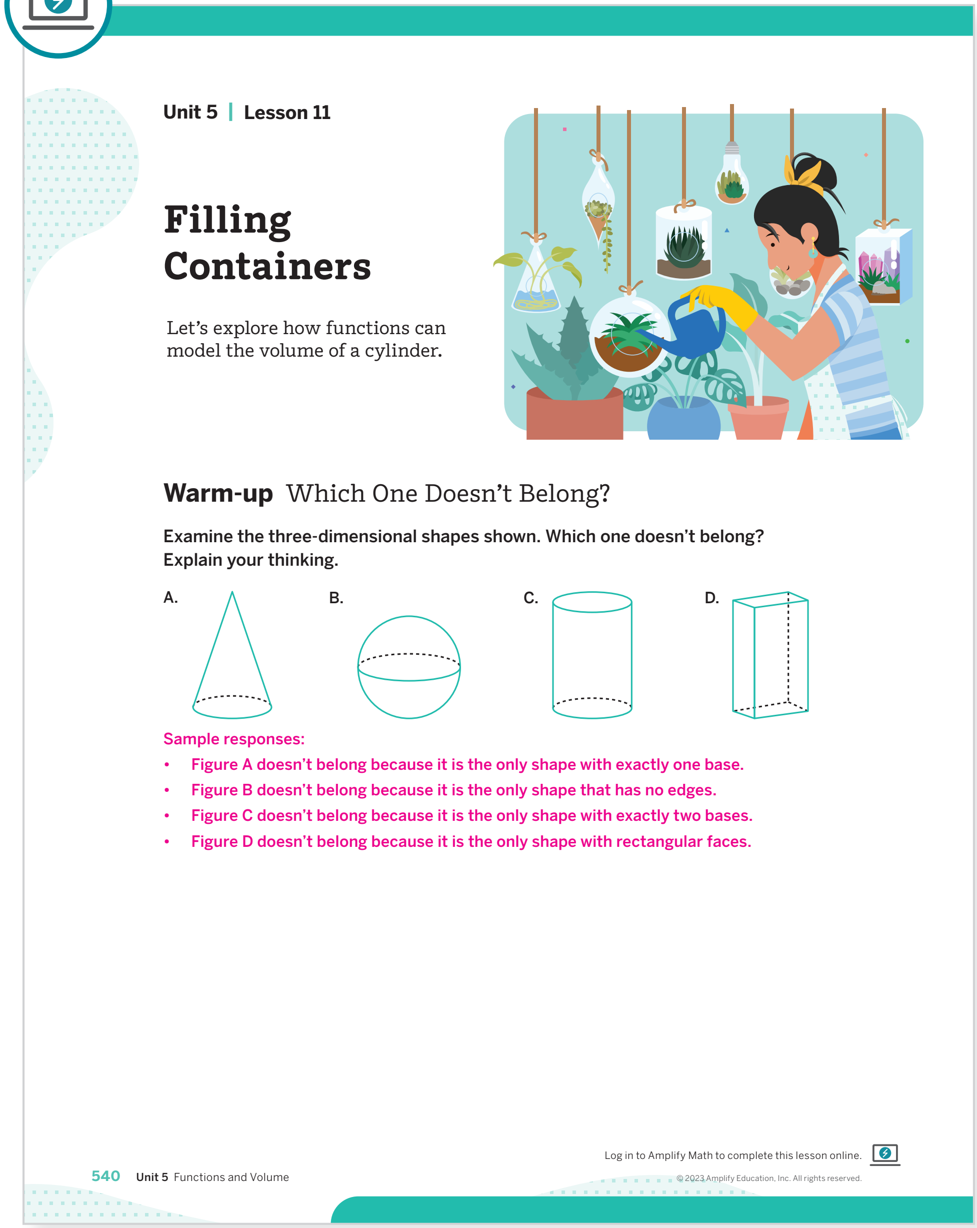
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, Problems 2 and 3 may be omitted.

Pairs | 7 min

Warm-up Which One Doesn't Belong?

Students compare different objects to illicit terms and ideas that will be useful for the duration of the subunit.



Unit 5 | Lesson 11

Filling Containers

Let's explore how functions can model the volume of a cylinder.

Warm-up Which One Doesn't Belong?

Examine the three-dimensional shapes shown. Which one doesn't belong? Explain your thinking.

Sample responses:

- Figure A doesn't belong because it is the only shape with exactly one base.
- Figure B doesn't belong because it is the only shape that has no edges.
- Figure C doesn't belong because it is the only shape with exactly two bases.
- Figure D doesn't belong because it is the only shape with rectangular faces.

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the ***Which One Doesn't Belong?*** routine.

2 Monitor

Help students get started by asking, "What do you notice about the bases of the figures? About the edges?"

Look for points of confusion:

- **Using imprecise or incorrect vocabulary to describe the figures.** Make note of which figures prove most difficult for students to describe and prepare to display correct terms for all to see during the lesson.

3 Connect

Have students share one reason why a particular shape might not belong. Record and display any terminology students use, such as *diameter*, *radius*, *vertex*, *edge*, *face*, or specific names of the figures, such as *sphere*, *cylinder*, *cone*, *rectangular prism*. Have students record the names of each figure.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their reasons for why a particular shape does not belong with the others, listen for and amplify the mathematical terminology they use to describe the shapes. Add these terms and phrases to the class display.

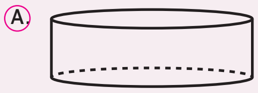
English Learners

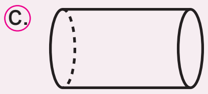
Add the shapes from the Warm-up to the class display and annotate them with their features, such as *base*, *edge*, and *face*.

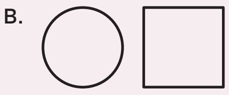
Power-up

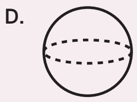
To power up students' ability to sketch a cylinder, have students complete:

Which of the following figures are cylinders? Select *all* that apply.









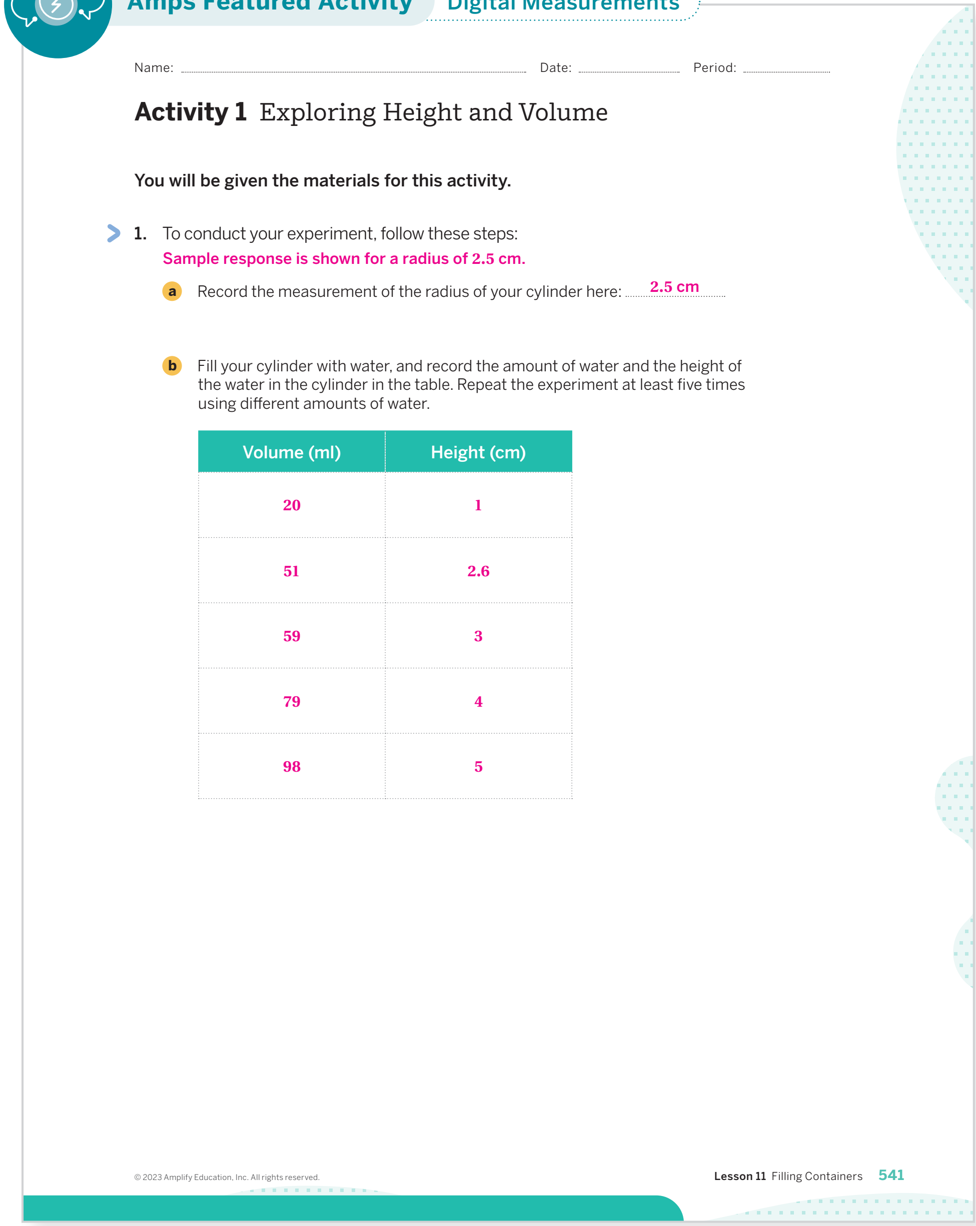
Use: Before the Warm-up
Informed by: Performance on Lesson 10, Practice Problem 6

Small Groups | 20 min or 40 min

Activity 1 Exploring Height and Volume

Students investigate the height of water in different-sized graduated cylinders to explore how the height of a cylinder with a fixed radius is a linear function of cylinder's volume.

Amps Featured Activity Digital Measurements



Name: ______ Date: ______ Period: ______

Activity 1 Exploring Height and Volume

You will be given the materials for this activity.

1. To conduct your experiment, follow these steps:
   Sample response is shown for a radius of 2.5 cm.
   a. Record the measurement of the radius of your cylinder here: 2.5 cm
   b. Fill your cylinder with water, and record the amount of water and the height of the water in the cylinder in the table. Repeat the experiment at least five times using different amounts of water.

Volume (ml)	Height (cm)
20	1
51	2.6
59	3
79	4
98	5

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Lesson 11 Filling Containers 541

1 Launch

Distribute one graduated cylinder and one container of 100 ml of water to each group. For a shorter 20-minute activity, use the Activity 1 Amps slides.

Display a graduated cylinder filled to a specific measurement for all to see and demonstrate to students how to read and interpret the measurement for the volume of the water. Explain that the task is to create a graph that relates the volume of the water and the height of it in the cylinder. Ask students to predict what they think the graph will look like.

2 Monitor

Help students get started by having them determine the label for each axis of the graph.

Look for points of confusion:

- **Getting inaccurate data that does not show a height as a linear function of volume.** Ask students what pattern they see in the data and whether there are any data points they might want to measure again that do not fit the pattern.
- **Being unsure about how to describe a point on the graph.** Have students consider the x- and y-axis labels.
- **Thinking the volume of the water is a function of height.** Help students identify which quantity represents the input and output, and remind students that the output (the height of the water) depends on, or is a function of, the input (the volume of the water).

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital tools to fill cylinders of different sizes with water and record observations about the height, radius, and volume of the water in each cylinder.

Math Language Development

MLR7: Compare and Connect

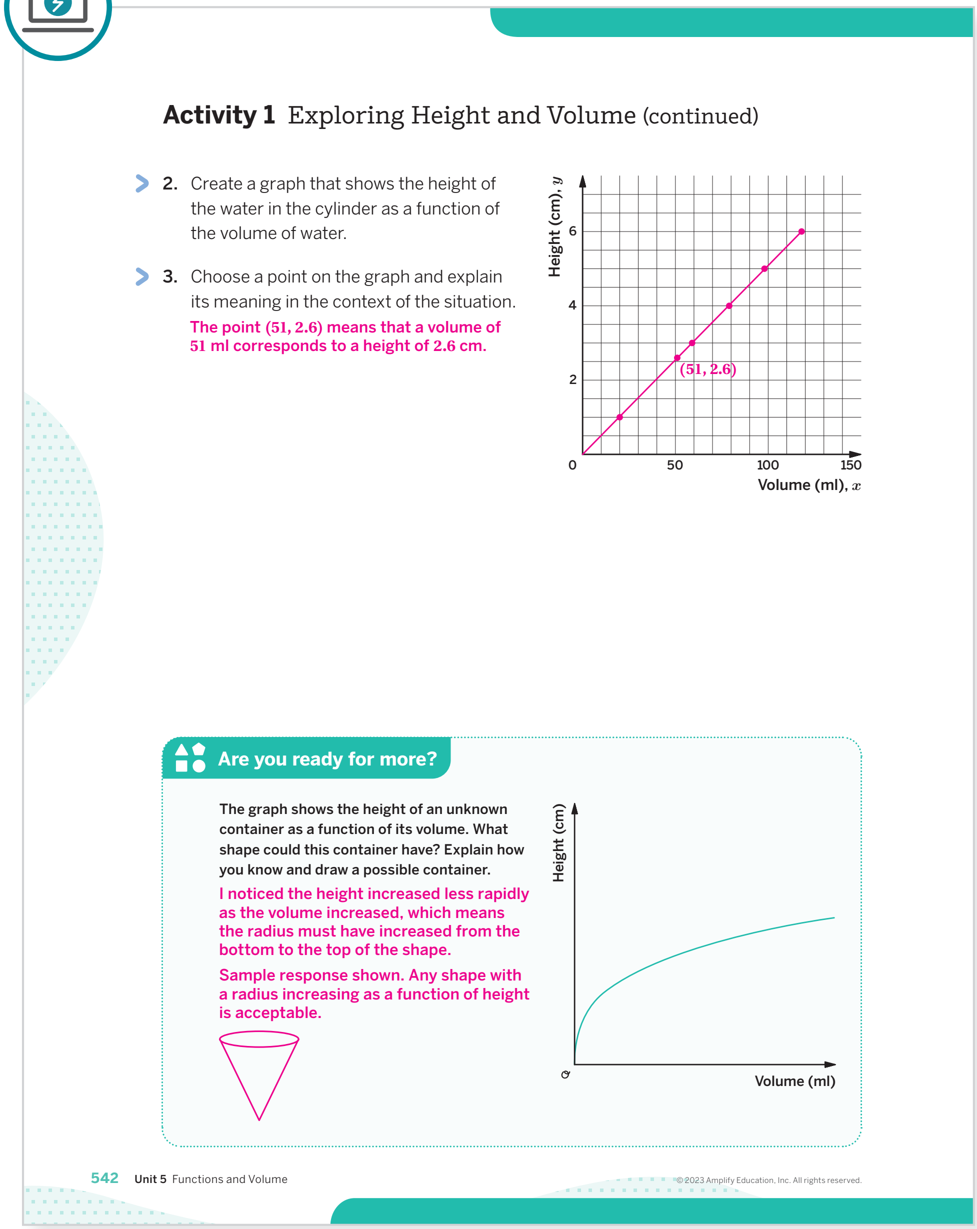
During the Connect, as you display the Activity 1 PDF, *Height as a Function of Volume*, draw students' attention to the connections between the three lines shown. Consider drawing a visual of three cylinders with the different radii of 1 cm, 2 cm, and 3 cm. Have students determine which cylinder will have the *greatest* height of water as it is filled. The cylinder with the *least* radius. Connect this idea to the slope of the line. Display this sentence frame to help students organize their thinking.

"The cylinder with the ___ radius will have the ___ height of water, which means its graph will have the ___ slope."

Small Groups | 20 min or 40 min

Activity 1 Exploring Height and Volume (continued)

Students investigate the height of water in different-sized graduated cylinders to explore how the height of a cylinder with a fixed radius is a linear function of cylinder's volume.



Activity 1 Exploring Height and Volume (continued)

2. Create a graph that shows the height of the water in the cylinder as a function of the volume of water.

3. Choose a point on the graph and explain its meaning in the context of the situation.
The point (51, 2.6) means that a volume of 51 ml corresponds to a height of 2.6 cm.

Are you ready for more?

The graph shows the height of an unknown container as a function of its volume. What shape could this container have? Explain how you know and draw a possible container.

I noticed the height increased less rapidly as the volume increased, which means the radius must have increased from the bottom to the top of the shape.

Sample response shown. Any shape with a radius increasing as a function of height is acceptable.

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3 Connect

Have groups of students share the patterns they noticed and explain what a point represents on the graph.

Ask:

- "What is the independent variable of your graph? Dependent variable?"
- "Why is the height of the water level a linear function of the volume of the water?"
- "Can you say that the volume is a function of the water's height?"

Display the Activity 1 PDF, *Height as a Function of Volume*. Ask students what is similar and what is different about the graphs. Explain that each line represents the graph of a cylinder with a different radius. The three radii given in the PDF are 1 cm, 2 cm, and 3 cm. Have students consider which line must represent which cylinder. Ask, "How did the slope of each graph change as the radius increased?"

Highlight that when the radius is greater, the slope is less steep. This is, because for a cylinder with a greater base area, the same volume of water will not fill as high inside the cylinder.

Differentiated Support

Extension: *Math Around the World, Interdisciplinary Connections*

Tell students that measurement practices across different cultures or civilizations have been diverse and context dependent. For example, some cultures developed their units of measurement based on the size of a seed or a grain because sowing and harvesting the land was essential for survival. In ancient China, length and volume were based on the size of a millet seed.

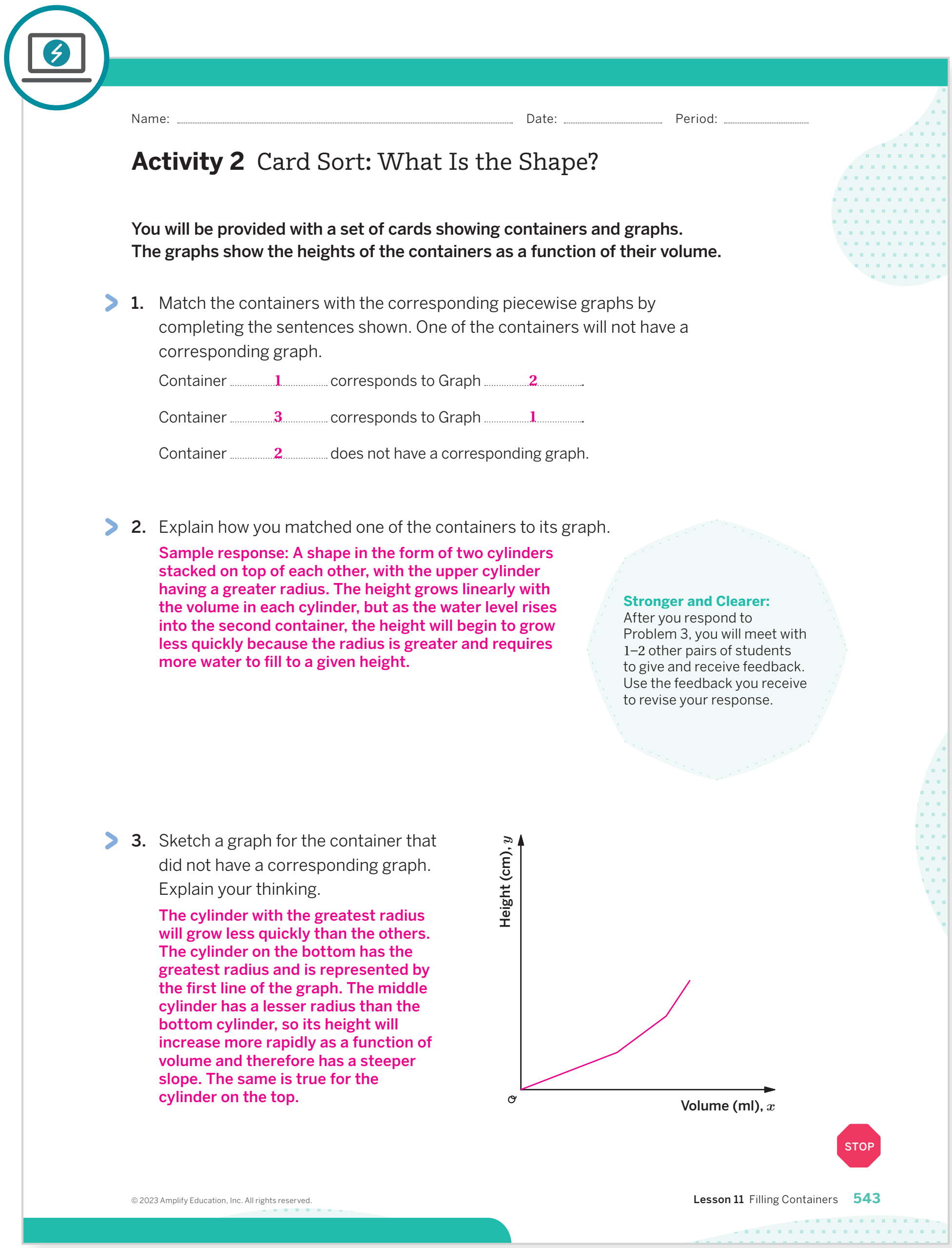
Other types of measurements were developed using the human body. In ancient India, the base unit was the *angula*, which was the average width of a human finger. In ancient Egypt, the *royal cubit* was used to approximate the length of the forearm.

Mention that units of measure can also evolve over time. Even as recently as 2018, the kilogram was redefined so that it no longer is based on an actual object, known as Le Grand K, a small metal cylinder that resides in Paris, France. Le Grand K used to be how all other kilograms on Earth were measured. Yet Le Grand K was losing small amounts of mass over time, due to cleaning or scratches. The kilogram is now based on a constant, which cannot change. **(Science)**

Pairs | 10 min

Activity 2 Card Sort: What Is the Shape?

Students match the sketch of a container to its graph to further investigate how the height of the water is a function of its volume.



Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: What Is the Shape?

You will be provided with a set of cards showing containers and graphs. The graphs show the heights of the containers as a function of their volume.

1. Match the containers with the corresponding piecewise graphs by completing the sentences shown. One of the containers will not have a corresponding graph.

Container 1 corresponds to Graph 2.

Container 3 corresponds to Graph 1.

Container 2 does not have a corresponding graph.

2. Explain how you matched one of the containers to its graph.

Sample response: A shape in the form of two cylinders stacked on top of each other, with the upper cylinder having a greater radius. The height grows linearly with the volume in each cylinder, but as the water level rises into the second container, the height will begin to grow less quickly because the radius is greater and requires more water to fill to a given height.

Stronger and Clearer: After you respond to Problem 3, you will meet with 1–2 other pairs of students to give and receive feedback. Use the feedback you receive to revise your response.

3. Sketch a graph for the container that did not have a corresponding graph. Explain your thinking.

The cylinder with the greatest radius will grow less quickly than the others. The cylinder on the bottom has the greatest radius and is represented by the first line of the graph. The middle cylinder has a lesser radius than the bottom cylinder, so its height will increase more rapidly as a function of volume and therefore has a steeper slope. The same is true for the cylinder on the top.

1 Launch

Ask students what they notice about the graphs in the activity as they relate to the graph they drew in Activity 1. Activate students' prior knowledge about piecewise functions. Then distribute pre-cut cards from the Activity 2 PDF and conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by asking, "Which cylinder will fill to a height faster — one with smaller or larger radius?"

Look for points of confusion:

- **Not being sure how to draw the graph in Problem 3.** Help students start their graph at the origin and make sure students understand that the value of the slope does not matter.

3 Connect

Have students share their matches.

Ask:

- "What does each vertex of the piecewise function in the graphs represent?
- "Suppose the container was filled with water. What does the end point on the line of a graph represent?" The maximum amount of water that could fill the container before overflowing.

Display several correct graphs from Problem 3, and ask what is similar and what is different. Discuss how each graph goes through the origin, and the slope of each linear piece is greater than the previous linear piece in each graph.

Highlight that, as the radius decreases, the slope becomes steeper for the cylinder with a given height. Discuss how the graph represents a piecewise function, and have students share how each section of the graph can be said to represent a cylinder with the same height.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students annotate the images of the containers with phrases such as "greatest radius" and "least radius."

Extension: *Math Enrichment*

Ask students if a graph showing two increasing line segments separated by a horizontal line segment between them could represent a container of three cylinders. No, a horizontal line segment would mean that the volume is increasing with no height increase, which does not make sense in this context.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "Did you look at the radii of each cylinder in the stack? Why or why not?"
- "Which part of the graph — representing which cylinder — will have the greatest slope? The least slope?"

Have students revise their responses, as needed.

English Learners

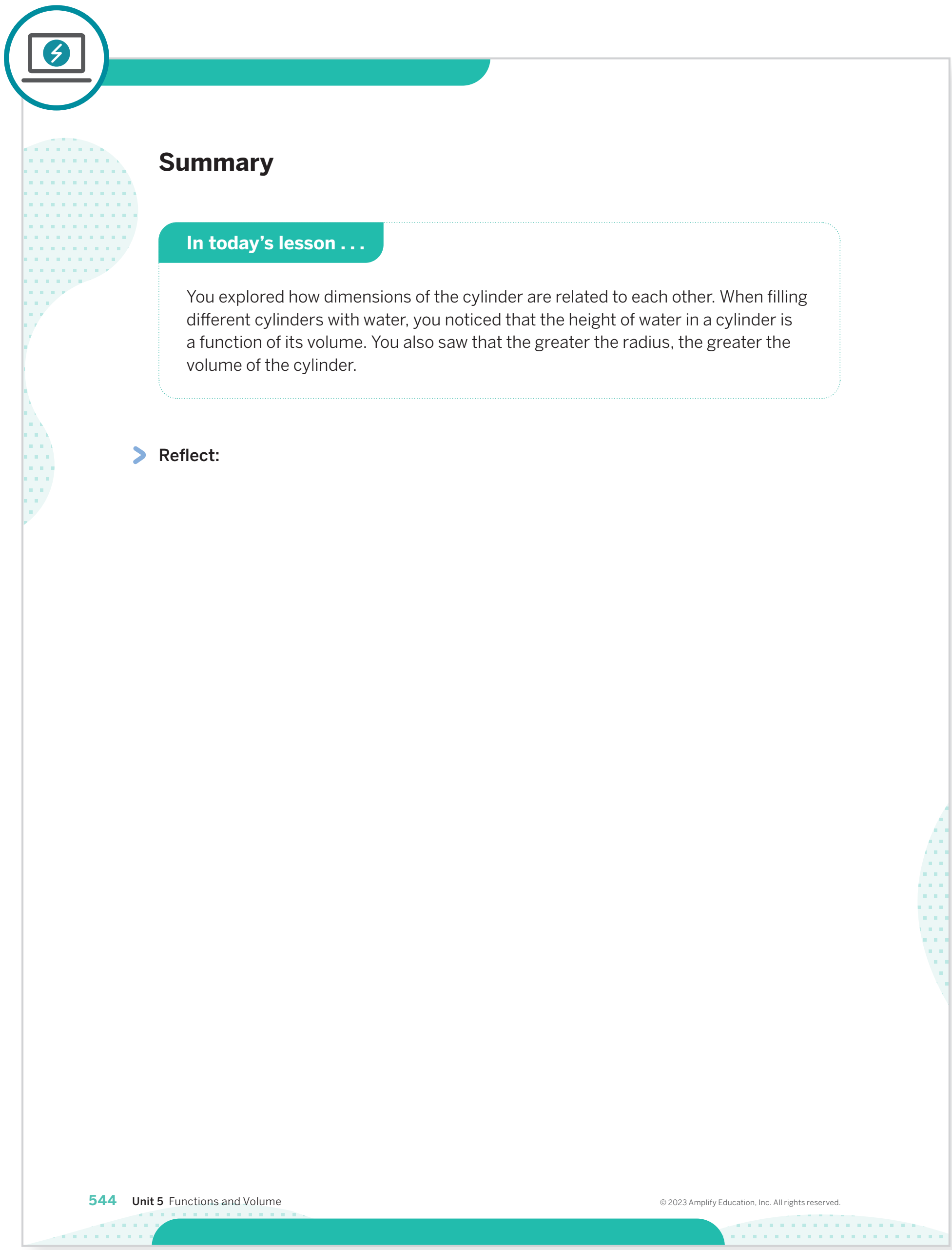
Display this sentence frame to help students organize their thinking.

"The cylinder with the ___ radius will have the ___ height of water, which means its graph will have the ___ slope."

Whole Class | 5 min

Summary

Review and synthesize the relationships between the radius, height, and volume of a cylinder.



Summary

In today's lesson . . .

You explored how dimensions of the cylinder are related to each other. When filling different cylinders with water, you noticed that the height of water in a cylinder is a function of its volume. You also saw that the greater the radius, the greater the volume of the cylinder.

Reflect:

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Synthesize

Ask:

- "How do the volume and radius affect the height of a cylinder?"
- "Suppose two cylinders are being filled with water, one with a lesser radius than the other. Which cylinder will fill faster? How can this be represented on a graph?"
- "Do you think the volume of a cylinder can be represented by a formula, just as the volume of a prism can be represented by a formula? Why or why not?"

Highlight how the height of a figure is a function of its volume.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

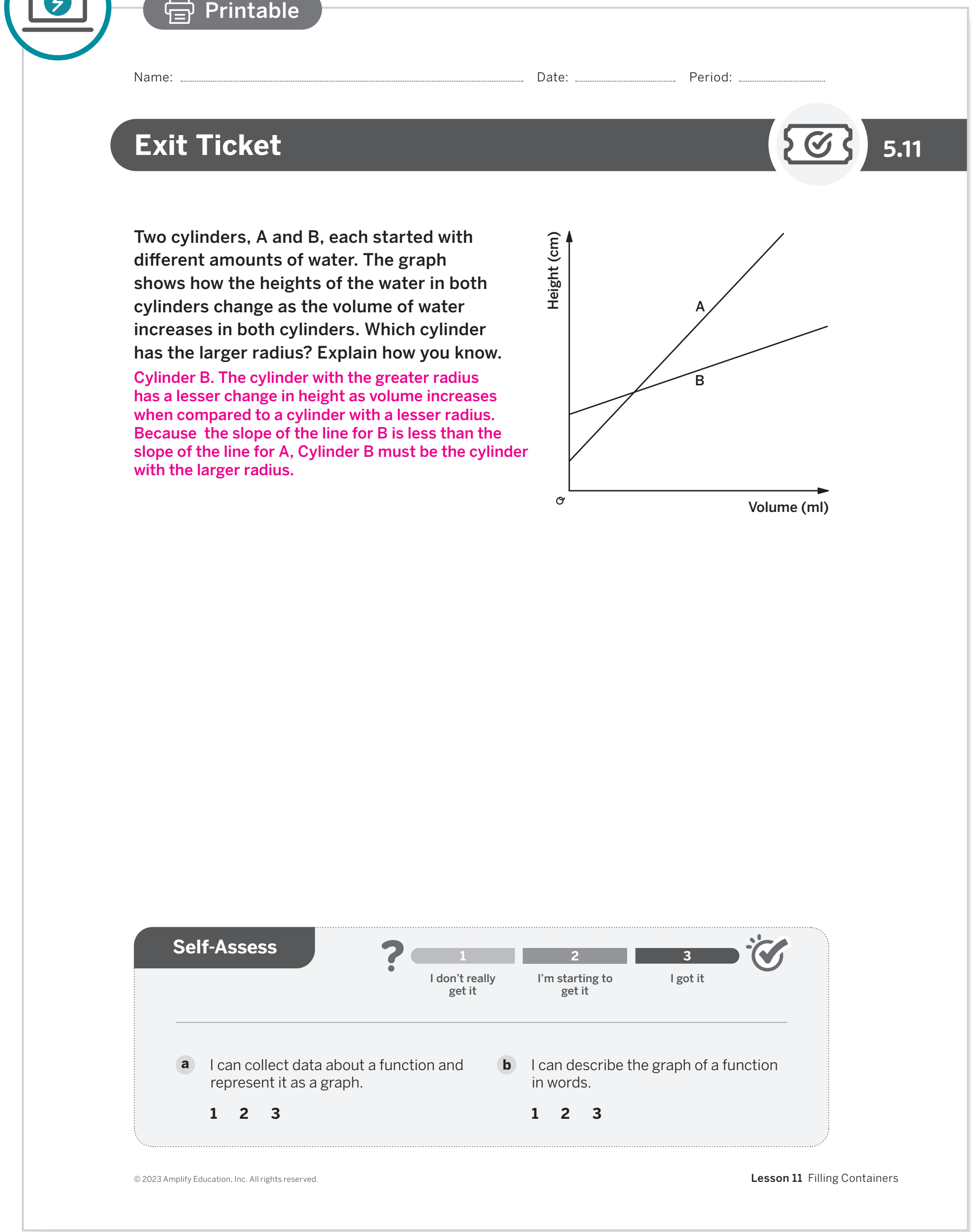
- "How did what you learned today connect to what you learned so far in this unit?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing a graph that shows how the height of a cylinder can be represented as a function of its volume.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 5.11

Two cylinders, A and B, each started with different amounts of water. The graph shows how the heights of the water in both cylinders change as the volume of water increases in both cylinders. Which cylinder has the larger radius? Explain how you know.

Cylinder B. The cylinder with the greater radius has a lesser change in height as volume increases when compared to a cylinder with a lesser radius. Because the slope of the line for B is less than the slope of the line for A, Cylinder B must be the cylinder with the larger radius.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can collect data about a function and represent it as a graph.

1 2 3

b I can describe the graph of a function in words.

1 2 3

Lesson 11 Filling Containers

Success looks like . . .

- **Language Goal:** Creating a graph of a function from collected data, and interpreting a point on the graph. **(Speaking and Listening, Writing)**
- **Language Goal:** Analyzing a container for which the height of water, as a function of volume, would be represented as a piecewise linear function, and explaining the reasoning. **(Speaking and Listening, Writing)**
- **Language Goal:** Interpreting a graph of heights of certain cylinders as a function of volume, and comparing the rates of change of the functions. **(Speaking and Listening, Writing)**
  - » Comparing the slopes of the lines that represent the volumes and heights of the two cylinders A and B.

Suggested next steps

If students think the graph of Cylinder B has a smaller radius because it has a lesser slope, consider:

- Reviewing how the slope relates to the radius of the cylinders in Activity 2.
- Assigning Practice Problem 2.
- Asking, "Which line has a greater slope? What does the slope tell you about how the water level is changing in the cylinder?"

If students are unsure how to get started, consider:

- Having students draw (or provide students with) two cylinders with different radii. Ask students which cylinder's height will increase faster for a given amount of added water.

Professional Learning

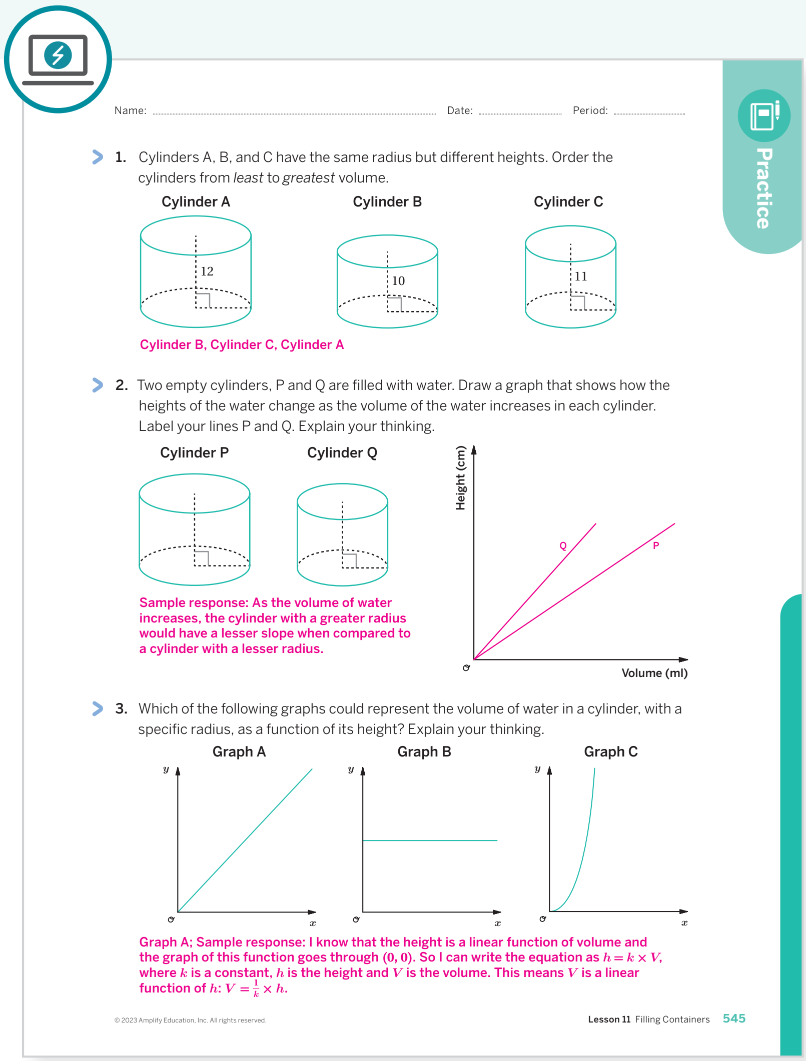
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What did the work in Activity 1 reveal about your students as learners?
- How did Activity 1 set students up to develop key concepts about finding the volume of cylinders, cones, and spheres?

Practice

Independent



Name: Date: Period:

1. Cylinders A, B, and C have the same radius but different heights. Order the cylinders from *least* to *greatest* volume.

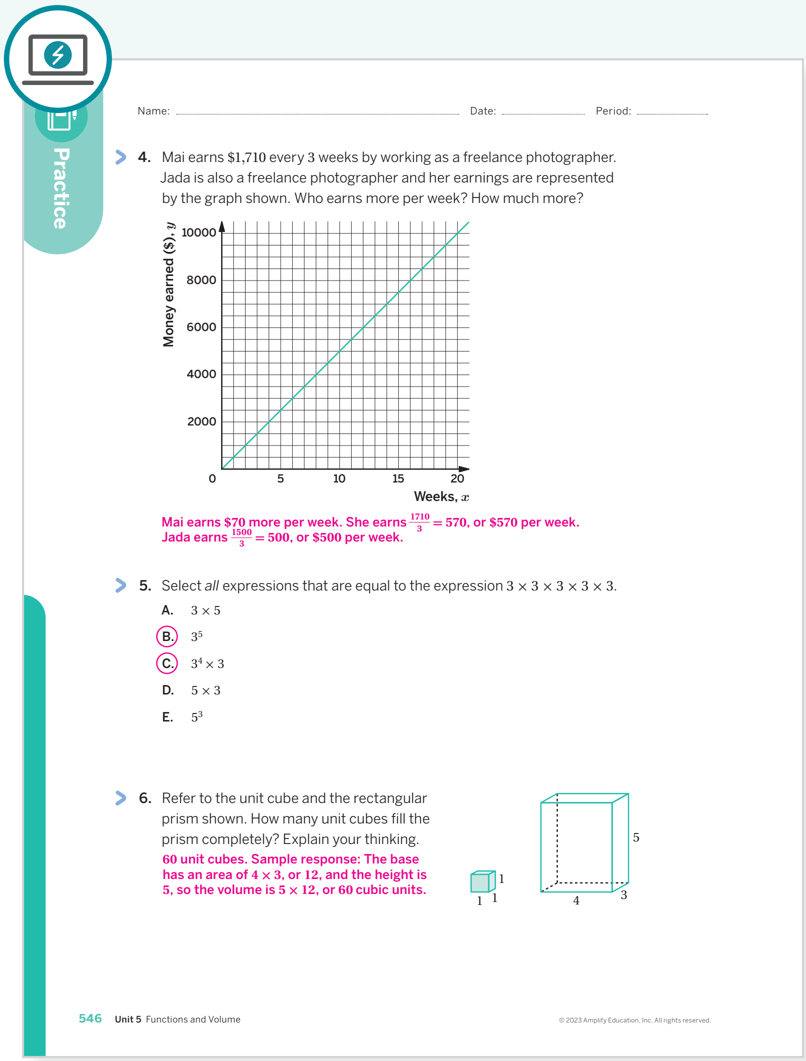
Cylinder B, Cylinder C, Cylinder A

2. Two empty cylinders, P and Q are filled with water. Draw a graph that shows how the heights of the water change as the volume of the water increases in each cylinder. Label your lines P and Q. Explain your thinking.

Sample response: As the volume of water increases, the cylinder with a greater radius would have a lesser slope when compared to a cylinder with a lesser radius.

3. Which of the following graphs could represent the volume of water in a cylinder, with a specific radius, as a function of its height? Explain your thinking.

Graph A; Sample response: I know that the height is a linear function of volume and the graph of this function goes through $(0, 0)$. So I can write the equation as $h = k \times V$, where k is a constant, h is the height and V is the volume. This means V is a linear function of h: $V = \frac{1}{k} \times h$.



Name: Date: Period:

4. Mai earns $\$1{,}710$ every 3 weeks by working as a freelance photographer. Jada is also a freelance photographer and her earnings are represented by the graph shown. Who earns more per week? How much more?

Mai earns \$70 more per week. She earns $\frac{1710}{3} = 570$, or \$570 per week. Jada earns $\frac{1500}{3} = 500$, or \$500 per week.

5. Select *all* expressions that are equal to the expression $3 \times 3 \times 3 \times 3 \times 3$.

A. 3×5
[✓ B.] 3^5
[✓ C.] $3^4 \times 3$
D. 5×3
E. 5^3

6. Refer to the unit cube and the rectangular prism shown. How many unit cubes fill the prism completely? Explain your thinking.

60 unit cubes. Sample response: The base has an area of 4×3, or 12, and the height is 5, so the volume is 5×12, or 60 cubic units.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 1	3
Spiral	4	Unit 5 Lesson 8	2
	5	Grade 6	1
Formative	6	Unit 5 Lesson 12	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 12**

The Volume of a Cylinder

Let's explore cylinders and their volumes.

Focus

Goals

1. **Language Goal:** Calculate the volume of a cylinder, and compare and contrast the formula for the volume of a cylinder with the formula for the volume of a prism. **(Speaking and Listening)**
2. **Language Goal:** Explain how to calculate the volume of a cylinder using the area of the base and height of the cylinder. **(Speaking and Listening)**

Coherence

Today

In this lesson, students learn that the volume of a cylinder is the area of the base times the height, just like a prism. This is accomplished by considering 1-unit layers of a rectangular prism side by side with 1-unit layers of a cylinder. After thinking about how to compute the volume of specific cylinders, students learn the general formulas $V = Bh$ and $V = \pi r^2 h$.

Previously

In Lesson 11, students explored how the volume of a cylinder is a function of its height and radius. In Grade 7, students learned to compute the area of a circle, both in terms of pi and by using an approximation for pi. In Grade 7, students also explored how to find the volume of any right prism.

Coming Soon

In Lesson 13, students will apply formulas for finding the volume of a cylinder to mathematical problems, including ones that require students to find missing dimensions when given the volume. In Lesson 14, students will discover how the volume of a cylinder is related to the volume of a cone.

Rigor

- Students build **conceptual understanding** for how to find the volume of a cylinder.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	15 min	15 min	5 min	5 min
Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Volumes of Circular Solids*
- Anchor Chart PDF, *Volumes of Circular Solids* (answers)
- Anchor Chart PDF, *Circles* (from Grade 7)

Math Language Development

Review words

- *cylinder*
- *height*
- *pi*
- *radius*
- *volume*

Amps | **Featured Activity**

Exit Ticket

Real-Time Exit Ticket

Check, in real time, whether your students can find the volume of a cylinder by using a digital Exit Ticket.

Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might not be able to connect previous lessons to the content of this lesson. Prior to considering the similarities and differences of the figures in the activity itself, encourage students to have the self-discipline to extend this comparison beyond this task to include what they have done in previous lessons.

Modifications to Pacing

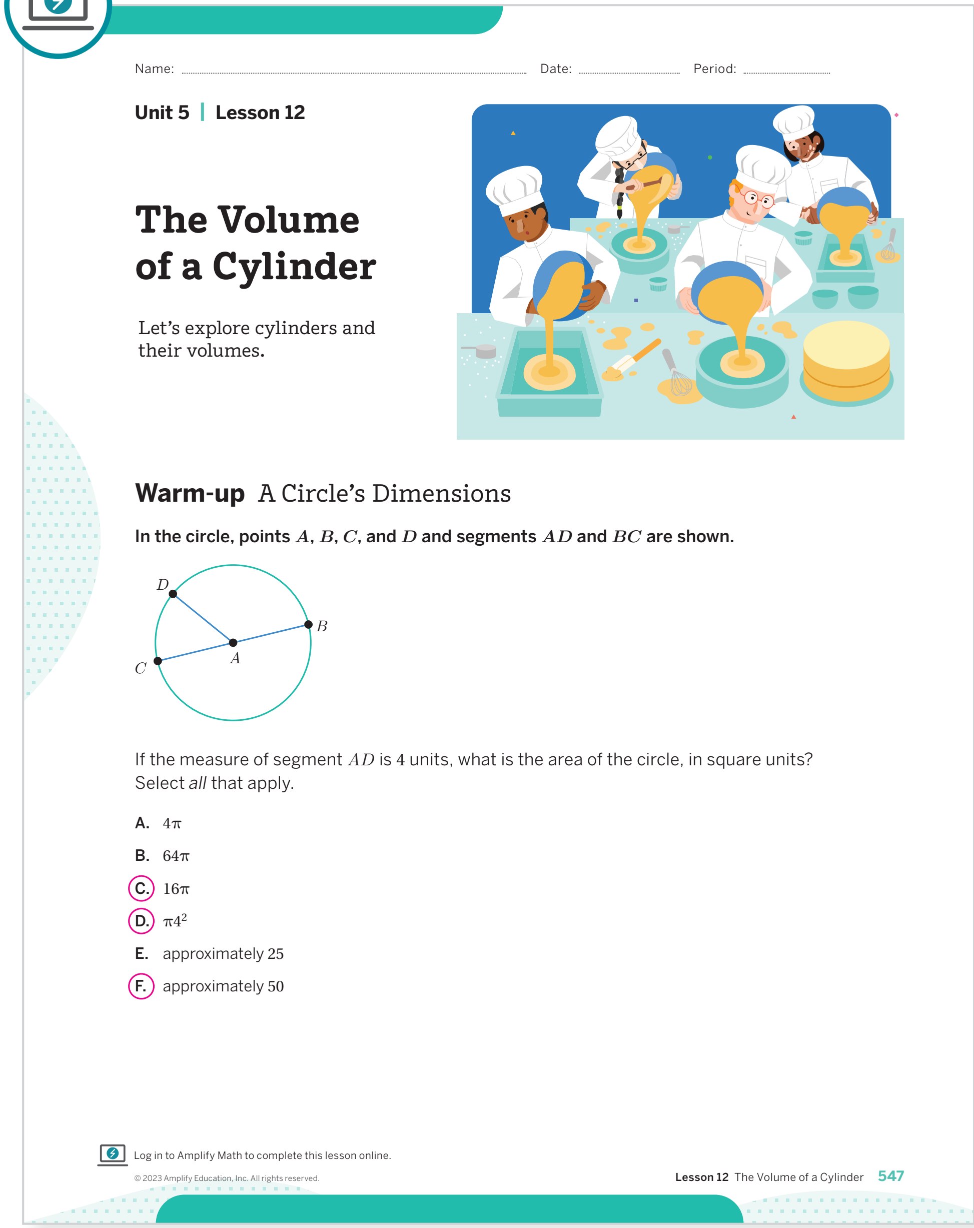
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted and discussed beforehand as a spiral review problem in the previous lesson.
- In **Activity 2,** Problem 1 may be omitted.

Independent | 8 min

Warm-up A Circle's Dimensions

Students review how to compute the area of a circle to prepare for finding the volume of a cylinder.



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Unit 5 | **Lesson 12**

The Volume of a Cylinder

Let's explore cylinders and their volumes.

Warm-up A Circle's Dimensions

In the circle, points A, B, C, and D and segments AD and BC are shown.

If the measure of segment AD is 4 units, what is the area of the circle, in square units? Select *all* that apply.

A. 4π

B. 64π

[✓ C.] 16π

[✓ D.] $\pi 4^2$

E. approximately 25

[✓ F.] approximately 50

Log in to Amplify Math to complete this lesson online.

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Lesson 12 The Volume of a Cylinder 547

1 Launch

Display the diagram and ask:

- "Name a segment that is a radius of Circle A."
- "What do you call a segment, such as Segment BC, with endpoints on the circle that contains the center of the circle?"

Review the meanings of the *radius* and *diameter* of a circle.

2 Monitor

Help students get started by referencing the Anchor Chart PDF, *Circles*, to help them determine the area of a circle.

Look for points of confusion:

- **Not being able to work with π to determine all the correct responses.** Ask students to recall an approximation for π and provide 3.14, if they are unable to offer one.

3 Connect

Have students share why each of the solutions represents the area of the circle

Ask, "When is it better to leave a measurement in terms of π? When is it better to approximate?"

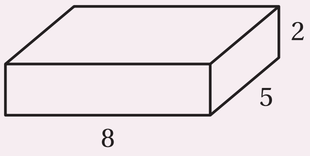
Highlight that, in this unit, students will often write their responses in terms of π.

Power-up

To power up students' ability to determine the volume of a prism, have students complete:

Recall that the volume of a prism can be determined using the formula $V = B \cdot h$ where B is the area of the base.

Determine the volume of the prism shown. Be prepared to explain your thinking.

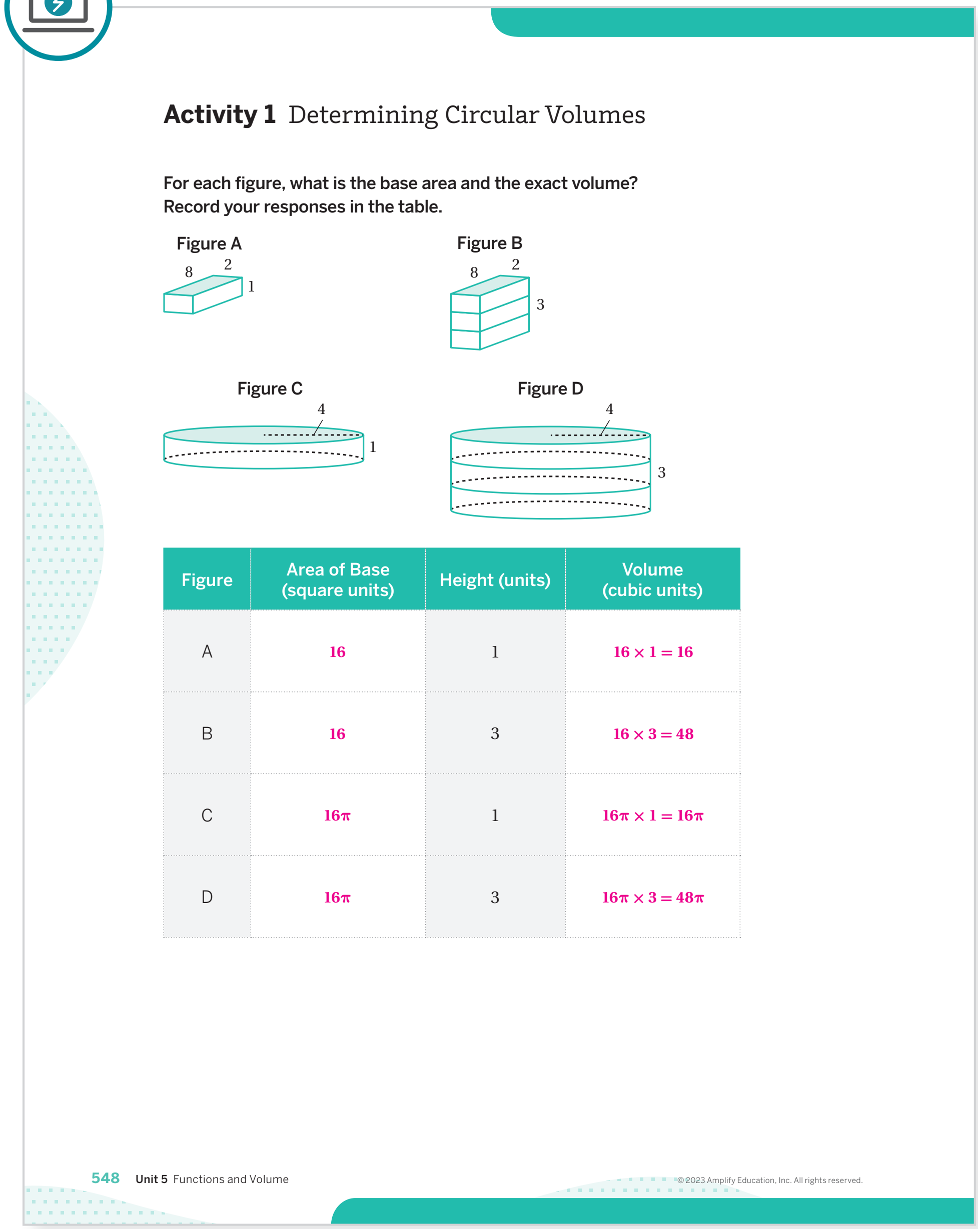


200 cubic units; $V = (8 \cdot 5) \cdot 2$
$V = 40 \cdot 5$
$V = 200$

Use: Before Activity 1
Informed by: Performance on Lesson 11, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6

Activity 1 Determining Circular Volumes

Students activate their prior knowledge of the volume of rectangular prisms to conjecture how to find the volume of cylinders.



Activity 1 Determining Circular Volumes

For each figure, what is the base area and the exact volume? Record your responses in the table.

Figure A

Figure B

Figure C

Figure D

Figure	Area of Base (square units)	Height (units)	Volume (cubic units)
A	16	1	$16 \times 1 = 16$
B	16	3	$16 \times 3 = 48$
C	16π	1	$16\pi \times 1 = 16\pi$
D	16π	3	$16\pi \times 3 = 48\pi$

548 Unit 5 Functions and Volume

1 Launch

Ask students what is the same and what is different about the figures shown. Conduct the *Co-craft Questions* routine as described in the Math Language Development section.

2 Monitor

Help students get started by asking, "How many layers are there in Figure B?"

Look for points of confusion:

- **Not finding the correct volume for Figure D.** Ask students to estimate the volume of Figure C and have them use that value as an estimate of Figure D.

3 Connect

Display student work showing correct responses for the table.

Ask:

- "How are determining the volume of prisms and determining the volume of cylinders similar?"
- "How do you determine the area of the base of a cylinder?"

Highlight the important features of cylinders and their definitions:

- The radius of the cylinder is the radius of the circle that forms its base.
- The height of a cylinder is the length between its circular top and bottom.
- A cylinder of height 1 can be thought of as a "layer" in a cylinder with height h.

Update the Anchor Chart PDF, *Volumes of Circular Solids*, with the formula of the volume of a cylinder, $V = \pi \cdot r^2 \cdot h$.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously learned how to determine the volume of rectangular prisms. Review what the base area of a rectangular prism represents and how to calculate it. Ask, "What do you think the base area of a cylinder represents? What formula can you use to calculate it?"

Extension: *Math Enrichment*

After you introduce the volume formula for a cylinder, let students know that some mathematicians use the formula $V = BH$ to represent the volume of prisms and cylinders, where B represents the area of the base and H represents the height of the figure. In the case of cylinders, $B = \pi \cdot r^2$ because the base is a circle.

Math Language Development

MLR5: Co-craft Questions

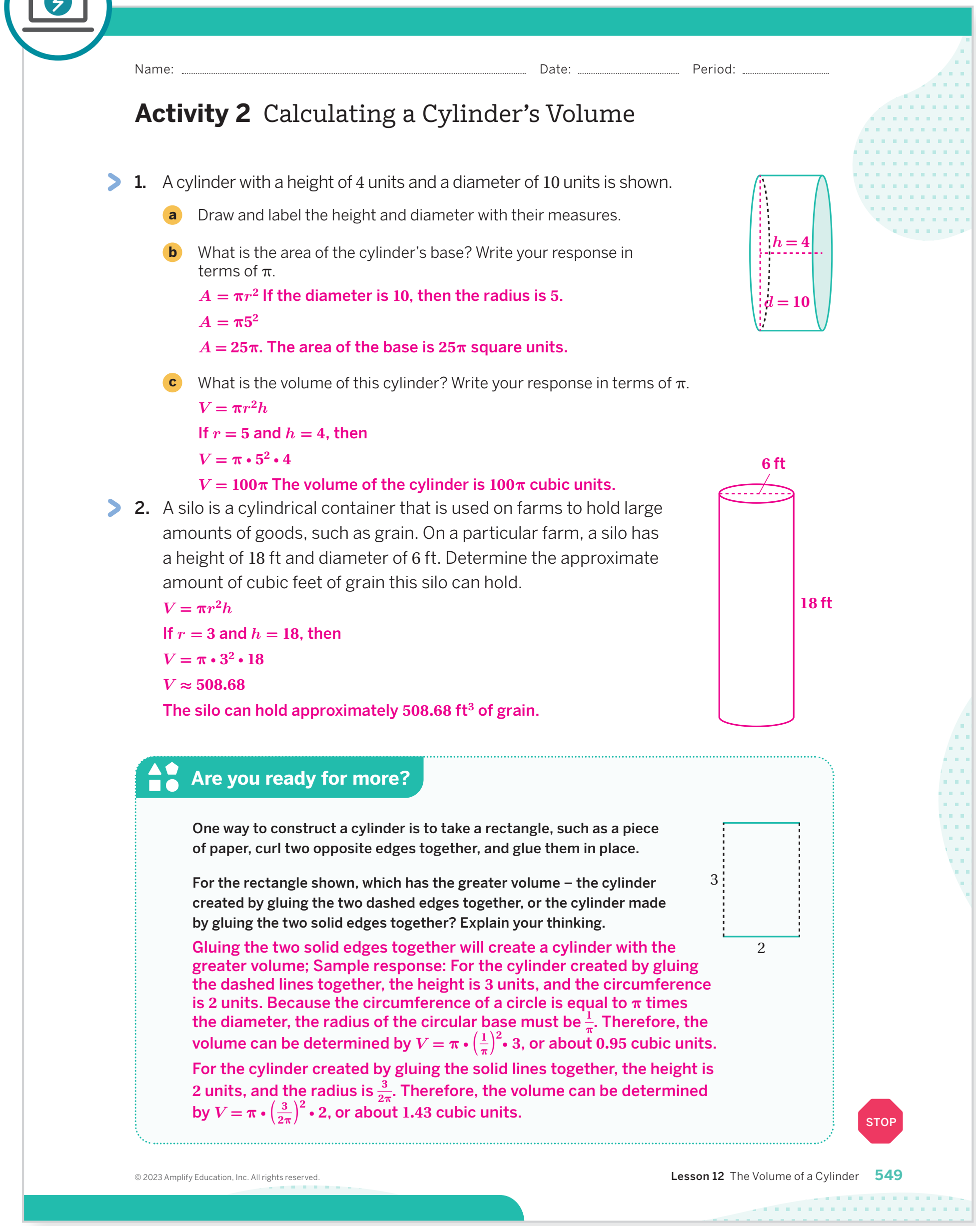
During the Launch, display Figures A-D without revealing the table. Have students work with their partner to write 2–3 mathematical questions that they have about the figures, other than the question asked in the introduction. Ask a few student pairs to share their questions with the class. Highlight questions that compare Figures A and B or Figures C and D. Sample questions shown.

- Is the volume of Figure B three times the volume of Figure A?
- Is the volume of Figure D three times the volume of Figure C?
- If the height of Figure B was 5 units, how would its volume compare to Figure A? (similar question for Figures D and C)

Independent | 15 min

Activity 2 Calculating a Cylinder's Volume

Students solve real-world and mathematical problems involving the volume of cylinders to build fluency using the area of the cylinder's base to determine its volume.



Name: ______ Date: ______ Period: ______

Activity 2 Calculating a Cylinder's Volume

1. A cylinder with a height of 4 units and a diameter of 10 units is shown.

   a. Draw and label the height and diameter with their measures.

   b. What is the area of the cylinder's base? Write your response in terms of π.

   $A = \pi r^2$ If the diameter is 10, then the radius is 5.

   $A = \pi 5^2$

   $A = 25\pi$. The area of the base is 25π square units.

   c. What is the volume of this cylinder? Write your response in terms of π.

   $V = \pi r^2 h$

   If $r = 5$ and $h = 4$, then

   $V = \pi \cdot 5^2 \cdot 4$

   $V = 100\pi$ The volume of the cylinder is 100π cubic units.

2. A silo is a cylindrical container that is used on farms to hold large amounts of goods, such as grain. On a particular farm, a silo has a height of 18 ft and diameter of 6 ft. Determine the approximate amount of cubic feet of grain this silo can hold.

   $V = \pi r^2 h$

   If $r = 3$ and $h = 18$, then

   $V = \pi \cdot 3^2 \cdot 18$

   $V \approx 508.68$

   The silo can hold approximately 508.68 ft^3 of grain.

Are you ready for more?

One way to construct a cylinder is to take a rectangle, such as a piece of paper, curl two opposite edges together, and glue them in place.

For the rectangle shown, which has the greater volume – the cylinder created by gluing the two dashed edges together, or the cylinder made by gluing the two solid edges together? Explain your thinking.

Gluing the two solid edges together will create a cylinder with the greater volume; Sample response: For the cylinder created by gluing the dashed lines together, the height is 3 units, and the circumference is 2 units. Because the circumference of a circle is equal to π times the diameter, the radius of the circular base must be $\frac{1}{\pi}$. Therefore, the volume can be determined by $V = \pi \cdot \left(\frac{1}{\pi}\right)^2 \cdot 3$, or about 0.95 cubic units.

For the cylinder created by gluing the solid lines together, the height is 2 units, and the radius is $\frac{3}{2\pi}$. Therefore, the volume can be determined by $V = \pi \cdot \left(\frac{3}{2\pi}\right)^2 \cdot 2$, or about 1.43 cubic units.

STOP

Lesson 12 The Volume of a Cylinder 549

1 Launch

Have students identify the base of the cylinder with a partner.

2 Monitor

Help students get started by helping them label the dimensions.

Look for points of confusion:

- **Using the diameter, instead of the radius, to find the volume.** Remind students that the radius is half of the diameter and have students label the length of the radius.

Look for productive strategies:

- Drawing a sketch of the silo and labeling the dimensions.
- Finding the volume by multiplying the area of the base and the height.
- Using the volume formula to solve the problem.
- Waiting until the last step of the calculation in Problem 2 to approximate for π.

3 Connect

Have students share how they found the approximate volume for the silo in Problem 2.

Ask:

- "How does knowing the area of a circular base help determine the volume of a cylinder?"
- "If the cylinder were on its side, how do you know which measurements to use for the volume?"
- "Suppose you did not have access to a calculator. What approximation could you use for π?"

Highlight how students can multiply the area of the base by the height to get the volume or they can use the cylinder formula directly. Discuss how it is better to wait until the end before rounding to allow for the most accurate approximation with π.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Prior to students completing Problem 2, consider showing images of silos that are used on farms. Some students may or may not be familiar with what these cylindrical containers look like.

Accessibility: *Clarify Vocabulary and Symbols*

Have students preview Problems 1 and 2. Ask, "For which problem are you asked to determine the exact volume? How do you know? For which problem are you asked to *approximate* the volume?" Clarify, as needed, the differences between exact and *approximate*, and how exact volumes are given in terms of π.

Math Language Development

MLR8: Discussion Supports — Revoicing

During the Connect, as students share how they determined the approximate volume in Problem 2, ask other students to restate and/or revoice what they heard using mathematical language. Ask the original speaker whether their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. Listen for and amplify language students use to describe how the diameter was given, but the formula uses the radius.

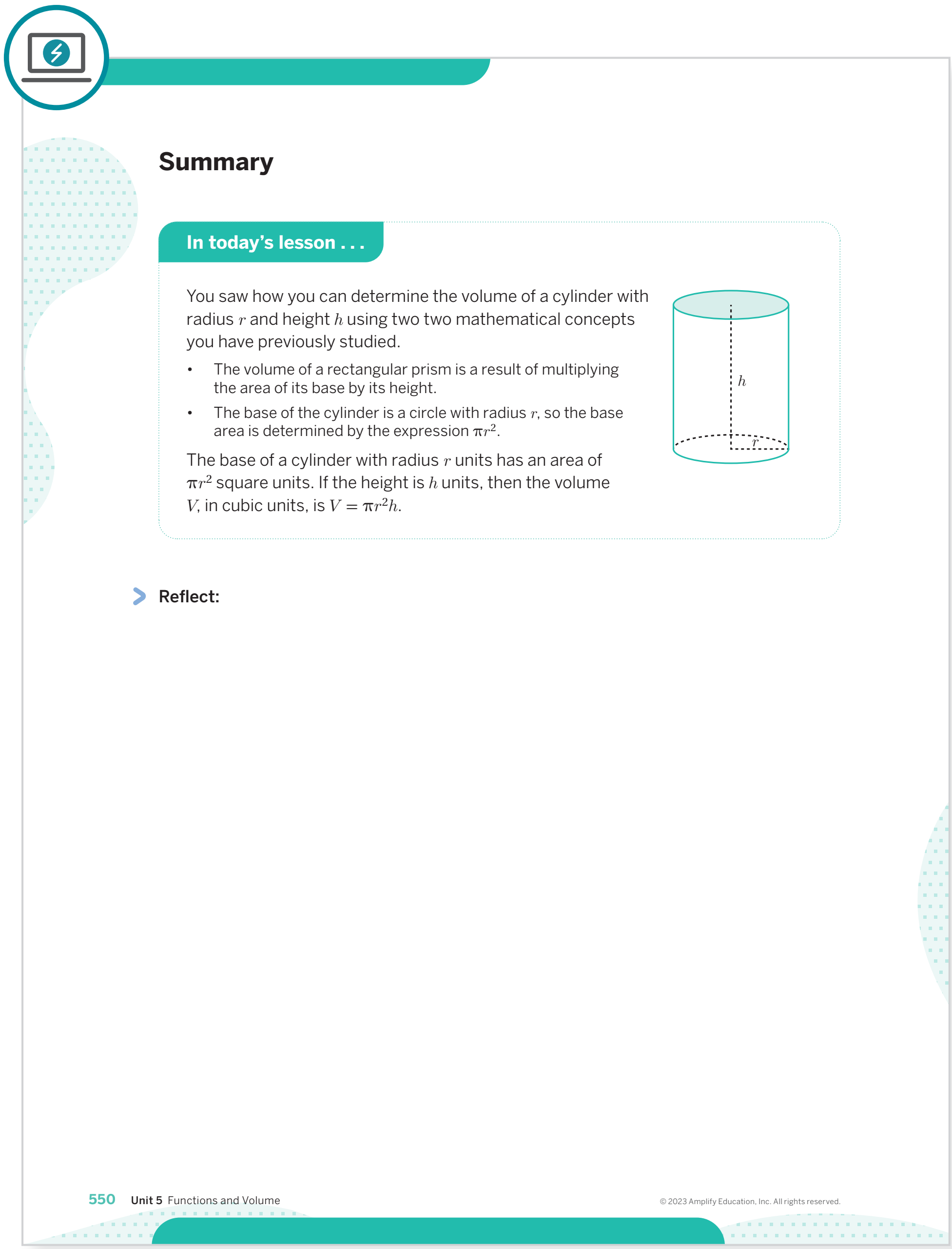
English Learners

Encourage students to refer to and use language from the class display to support their use of appropriate mathematical language.

Whole Class | 5 min

Summary

Review and synthesize the formula for the volume of a cylinder and how the volume of a cylinder is related to its radius and height.



Summary

In today's lesson . . .

You saw how you can determine the volume of a cylinder with radius r and height h using two two mathematical concepts you have previously studied.

- The volume of a rectangular prism is a result of multiplying the area of its base by its height.
- The base of the cylinder is a circle with radius r, so the base area is determined by the expression πr^2.

The base of a cylinder with radius r units has an area of πr^2 square units. If the height is h units, then the volume V, in cubic units, is $V = \pi r^2 h$.

Reflect:

Synthesize

Display and complete the cylinder portion of the Anchor Chart PDF, *Volumes of Circular Solids*.

Ask, "How is finding the volume of a cylinder similar to finding the volume of a prism?"

Highlight that, in order to find the volume of a cylinder, students must know the area of the base or the radius of the base, and the height.

Have students share real-world examples of cylinders and when they might need to know the volumes of those cylinders.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "When finding the volume of a cylinder, did you prefer to find the area of the base and multiply that by the height, or use the formula directly?"

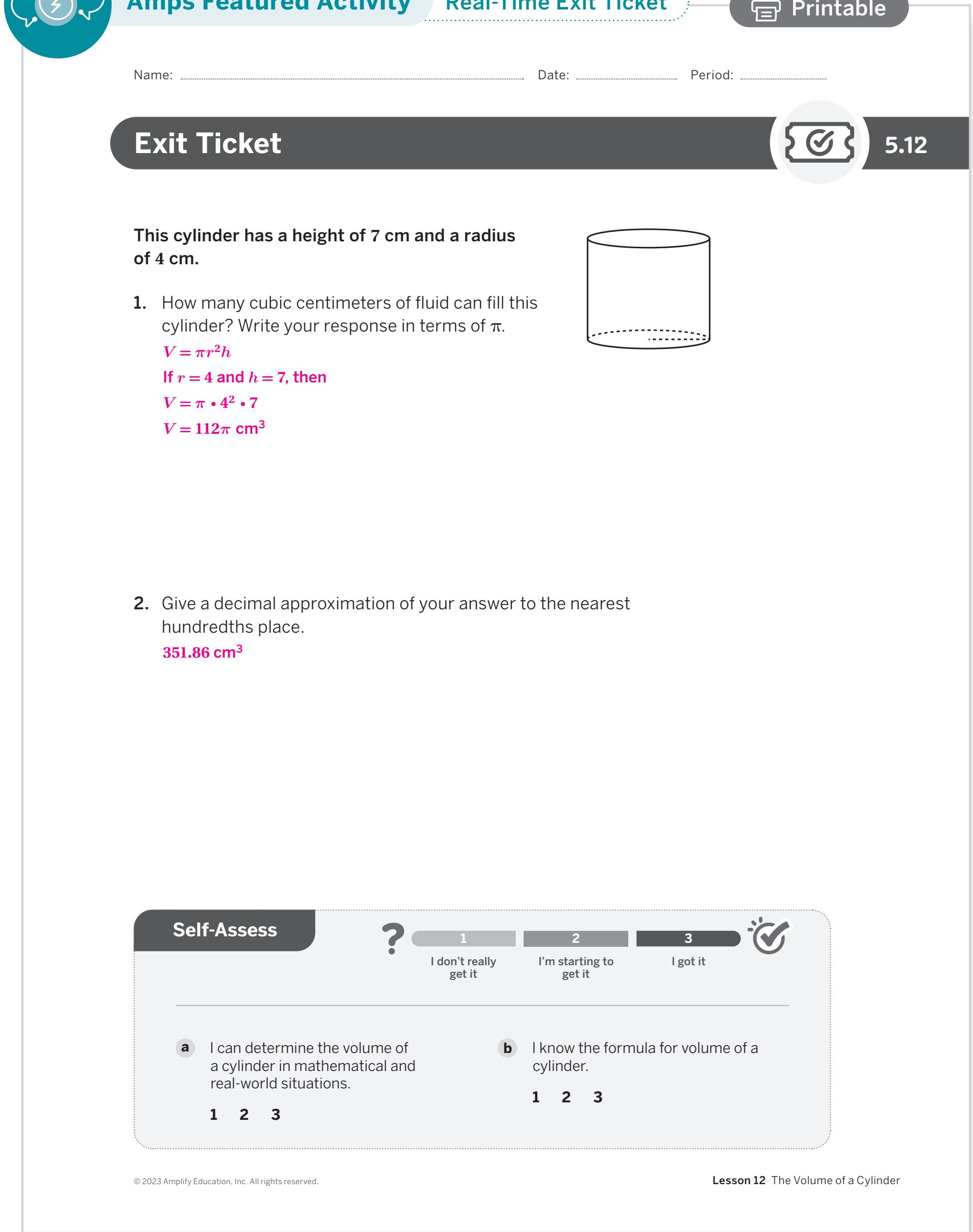
Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the volume of a cylinder, given the radius and the height of the cylinder.

Amps Featured Activity Real-Time Exit Ticket

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 5.12

This cylinder has a height of 7 cm and a radius of 4 cm.

1. How many cubic centimeters of fluid can fill this cylinder? Write your response in terms of π.
   $V = \pi r^2 h$
   If $r = 4$ and $h = 7$, then
   $V = \pi \cdot 4^2 \cdot 7$
   $V = 112\pi$ cm^3

2. Give a decimal approximation of your answer to the nearest hundredths place.
   351.86 cm^3

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine the volume of a cylinder in mathematical and real-world situations.
1 2 3

b. I know the formula for volume of a cylinder.
1 2 3

Lesson 12 The Volume of a Cylinder

Success looks like . . .

- **Language Goal:** Calculating the volume of a cylinder, and comparing and contrasting the formula for the volume of a cylinder with the formula for the volume of a prism. **(Speaking and Listening)**
  - » Determining the amount of cubic centimeters of fluid that can fill the cylinder.
- **Language Goal:** Explaining how to calculate the volume of a cylinder using the area of the base and height of the cylinder. **(Speaking and Listening)**

Suggested next steps

If students do not correctly determine the area of the base, consider:

- Reviewing how to determine the area of a circle from the Warm-up.
- Referencing the Anchor Chart PDF, *Volumes of Circular Solids*.
- Having students label the radius of the base and the height of the cylinder before calculating.

If students have difficulty approximating the value of π, consider:

- Reviewing strategies for approximation in Activity 2, Problem 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was determining the volume of a cylinder. How well do you think your students met this lesson goal? What would you change the next time you teach this lesson?
- What routines enabled all students to engage and participate in the mathematics of this lesson?

Math Language Development

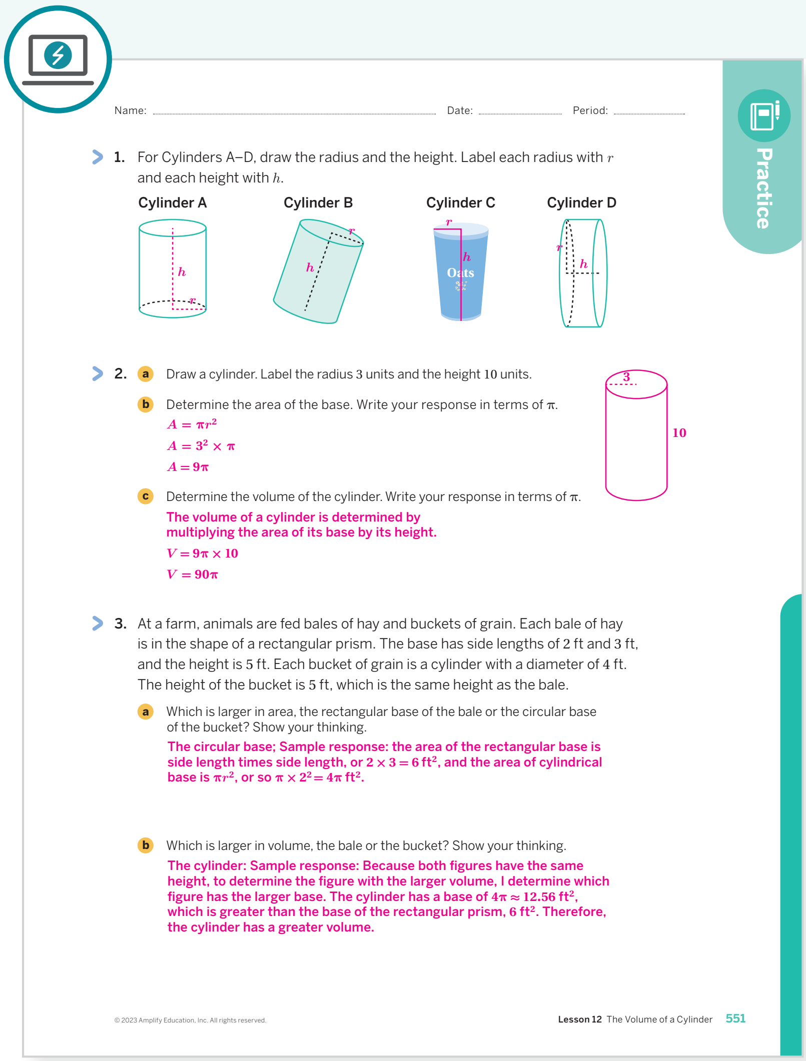
Language Goal: Calculating the volume of a cylinder and comparing and contrasting the formula for the volume of a prism with the formula for the volume of a cylinder with the formula for the volume of a prism.

Reflect on students' language development toward this goal.

- How did using the *Co-craft Questions* routine in Activity 2 help students begin to informally compare prisms with cylinders?
- During the Summary, as students responded to the Ask questions, did you see evidence of their developing math language, such as *area of the base, radius,* or *height*?

Practice

Independent



Name: Date: Period:

1. For Cylinders A–D, draw the radius and the height. Label each radius with r and each height with h.

2. **a** Draw a cylinder. Label the radius 3 units and the height 10 units.

b Determine the area of the base. Write your response in terms of π.

$A = \pi r^2$

$A = 3^2 \times \pi$

$A = 9\pi$

c Determine the volume of the cylinder. Write your response in terms of π.

The volume of a cylinder is determined by multiplying the area of its base by its height.

$V = 9\pi \times 10$

$V = 90\pi$

3. At a farm, animals are fed bales of hay and buckets of grain. Each bale of hay is in the shape of a rectangular prism. The base has side lengths of 2 ft and 3 ft, and the height is 5 ft. Each bucket of grain is a cylinder with a diameter of 4 ft. The height of the bucket is 5 ft, which is the same height as the bale.

a Which is larger in area, the rectangular base of the bale or the circular base of the bucket? Show your thinking.

The circular base; Sample response: the area of the rectangular base is side length times side length, or $2 \times 3 = 6$ ft^2, and the area of cylindrical base is πr^2, or so $\pi \times 2^2 = 4\pi$ ft^2.

b Which is larger in volume, the bale or the bucket? Show your thinking.

The cylinder: Sample response: Because both figures have the same height, to determine the figure with the larger volume, I determine which figure has the larger base. The cylinder has a base of $4\pi \approx 12.56$ ft^2, which is greater than the base of the rectangular prism, 6 ft^2. Therefore, the cylinder has a greater volume.

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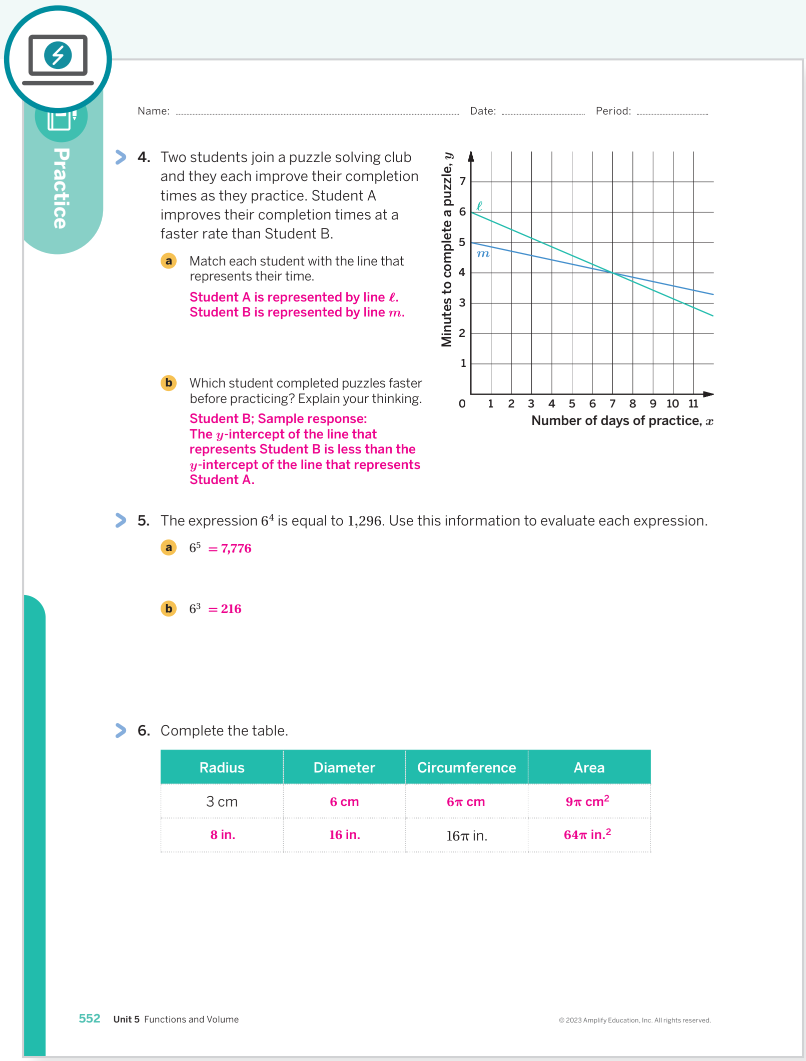
Lesson 12 The Volume of a Cylinder 551



Name: Date: Period:

4. Two students join a puzzle solving club and they each improve their completion times as they practice. Student A improves their completion times at a faster rate than Student B.

a Match each student with the line that represents their time.

Student A is represented by line ℓ.
Student B is represented by line m.

b Which student completed puzzles faster before practicing? Explain your thinking.

Student B; Sample response: The y-intercept of the line that represents Student B is less than the y-intercept of the line that represents Student A.

5. The expression 6^4 is equal to 1,296. Use this information to evaluate each expression.

a $6^5 = 7{,}776$

b $6^3 = 216$

6. Complete the table.

Radius	Diameter	Circumference	Area
3 cm	6 cm	6π cm	9π cm^2
8 in.	16 in.	16π in.	64π in.^{2}

552 Unit 5 Functions and Volume

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 8	2
	5	Grade 6	1
Formative	6	Unit 5 Lesson 13	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 13**

Determining Dimensions of Cylinders

Let's apply the volume formula for a cylinder to determine missing dimensions.

Focus

Goals

1. **Language Goal:** Calculate the value of one dimension of a cylinder, and explain the reasoning. **(Speaking and Listening, Writing)**
2. **Language Goal:** Create a table of dimensions of cylinders, and describe patterns that arise. **(Speaking and Listening, Writing)**

Coherence

Today

Students use the formula $V = \pi \cdot r^2 \cdot h$ for the volume of a cylinder to solve a variety of problems. They compute volumes, given the radius and height, and determine radius or height, given a cylinder's volume and the other dimension, by reasoning about the structure of the volume formula.

Previously

In Lesson 12, students discovered the volume formula of a cylinder, $V = \pi \cdot r^2 \cdot h$.

Coming Soon

In Lesson 14, students will derive the formula for finding the volume of a cone by relating the volume of a cone to the volume of a cylinder.

Rigor

- Students build **procedural skills** working with the volume formula of a cylinder.
- Students **apply** the formula for the volume of a cylinder to scenarios where they must find a missing dimension given the measure of the volume.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	8 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Volumes of Circular Solids*
- Anchor Chart PDF, *Volumes of Circular Solids* (answers)
- Anchor Chart PDF, *Circles* (from Grade 7)

Math Language Development

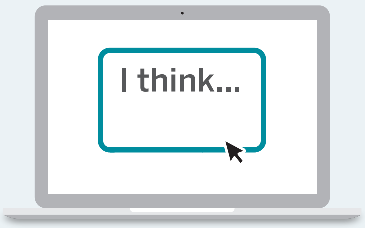
Review words

- *circumference*
- *cylinder*
- *diameter*
- *height*
- *pi* (π)
- *radius*
- *volume*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to explain their thinking behind finding an unknown dimension of a cylinder, and these explanations are available to you digitally, in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

While faced with a new task in Activity 1, students might not have a plan for success. Remind students that goals are not achieved without a plan. Have them develop some intermediate goals, behavioral and academic, to assure their best performance.

Modifications to Pacing

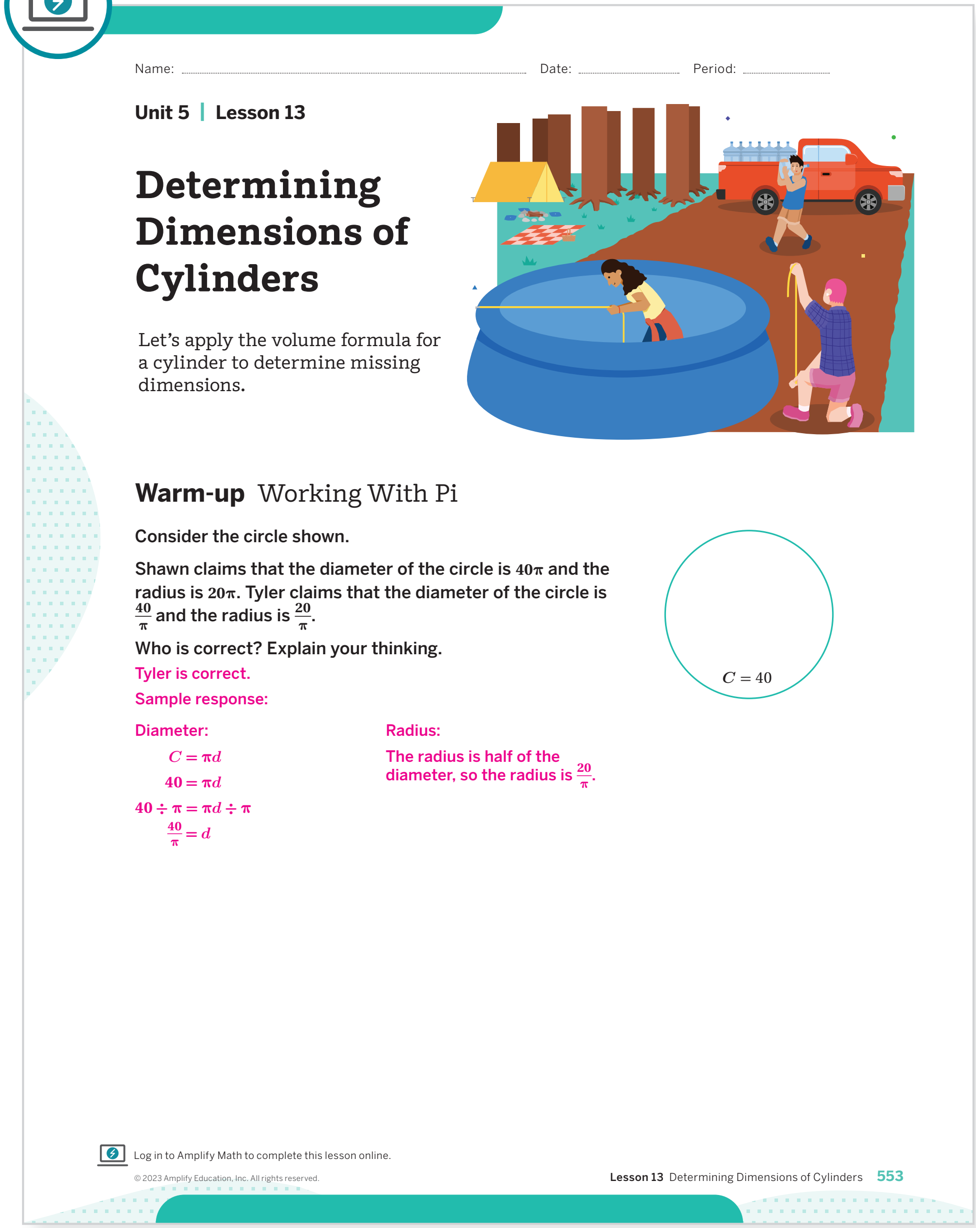
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted if students are proficient working with π.
- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, have students only complete the first three rows of the table.

Pairs | 5 min

Warm-up Working With Pi

Students evaluate two students' responses to activate prior knowledge of π and how to reason and calculate with π.



Name: ____________________ Date: __________ Period: __________

Unit 5 | Lesson 13

Determining Dimensions of Cylinders

Let's apply the volume formula for a cylinder to determine missing dimensions.

Warm-up Working With Pi

Consider the circle shown.

Shawn claims that the diameter of the circle is 40π and the radius is 20π. Tyler claims that the diameter of the circle is $\frac{40}{\pi}$ and the radius is $\frac{20}{\pi}$.

Who is correct? Explain your thinking.

Tyler is correct.

Sample response:

Diameter:

$$C = \pi d$$
$$40 = \pi d$$
$$40 \div \pi = \pi d \div \pi$$
$$\frac{40}{\pi} = d$$

Radius:

The radius is half of the diameter, so the radius is $\frac{20}{\pi}$.

$C = 40$

Log in to Amplify Math to complete this lesson online.

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Lesson 13 Determining Dimensions of Cylinders 553

1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by referencing the Anchor Chart PDF, *Circles*, to help them determine the circumference of a circle.

Look for points of confusion:

- **Thinking that Shawn is correct.** Have students substitute values for the diameter in the formula for the circumference of a circle.

Look for productive strategies:

- Solving for the diameter of the circle by dividing both sides of the circumference formula by π.

3 Connect

Have students share some who they think is correct by conducting the ***Poll the Class*** routine.

Ask, "What would be the circumference of the circle if Shawn was correct?" $40\pi^2$

Highlight that to solve for the diameter measure, students can use the equation $C = \pi \cdot D$ and then divide by π on both sides. Discuss how π should be treated as a regular number, and demonstrate how students can divide and multiply by π, if needed.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

Remind students they previously learned how to determine the circumference of circles in Grade 7. Using the Anchor Chart PDF, *Circles* (from Grade 7), ask a student volunteer to describe the meanings of the variables and the relationship between the circumference of a circle and its diameter or radius.

Power-up

To power up students' ability to relate the radius of a circle to its other measures, have students complete:

Recall that the formula for the circumference of a circle is $C = \pi d$ or $C = 2\pi r$, while the formula for the area of a circle is $A = \pi r^2$.

If the circumference of a circle is 12π ft, determine each measurement.

a. Diameter 12 ft

b. Radius 6 ft

c. Area 36π ft^2

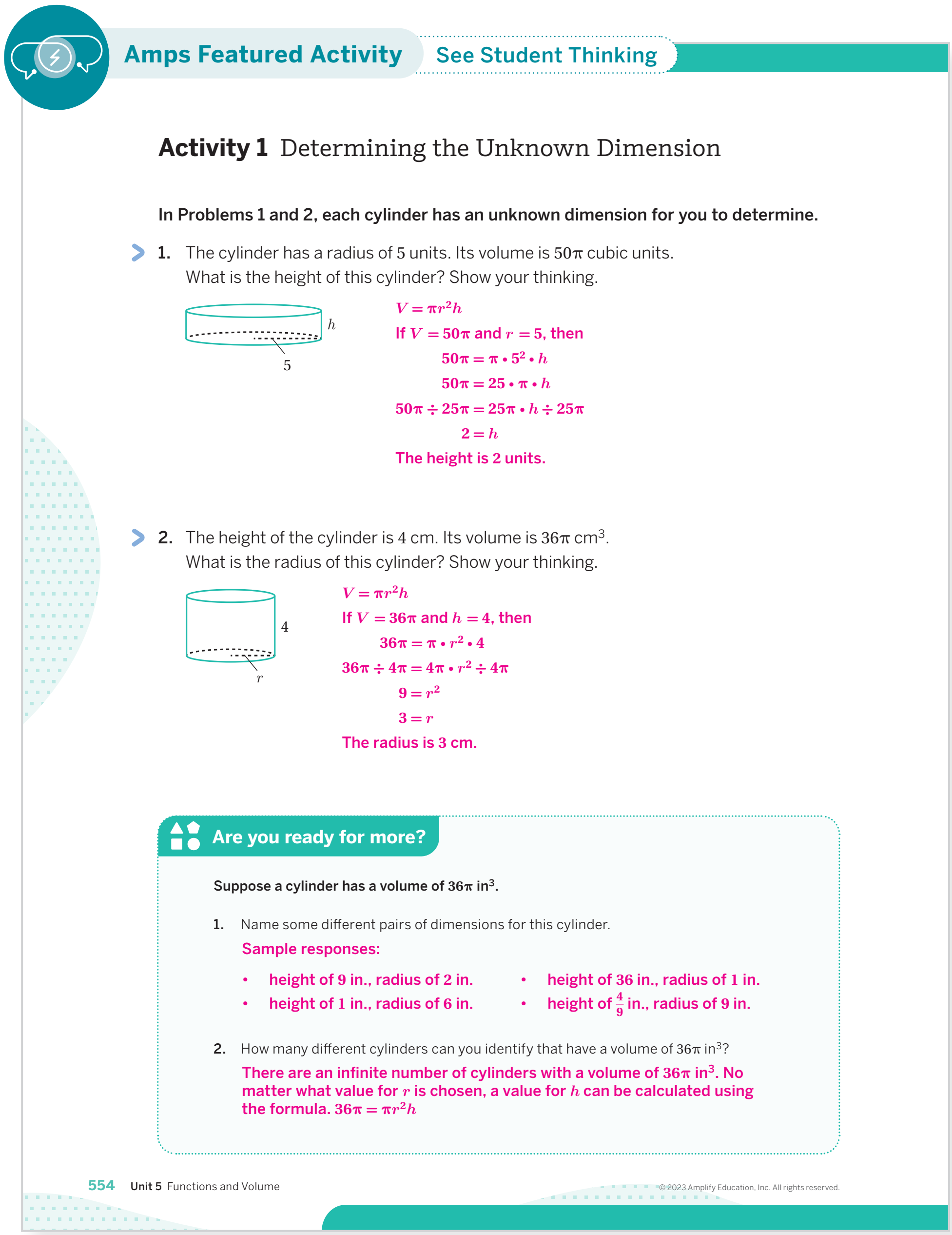
Use: Before the Warm-up

Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 7 and 8

Pairs | 15 min

Activity 1 Determining the Unknown Dimension

Students determine the missing dimensions of cylinders, when given the volume and one dimension, to develop strategies that will help them determine missing dimensions.


Amps Featured Activity See Student Thinking

Activity 1 Determining the Unknown Dimension

In Problems 1 and 2, each cylinder has an unknown dimension for you to determine.

1. The cylinder has a radius of 5 units. Its volume is 50π cubic units. What is the height of this cylinder? Show your thinking.

$V = \pi r^2 h$

If $V = 50\pi$ and $r = 5$, then

$50\pi = \pi \cdot 5^2 \cdot h$

$50\pi = 25 \cdot \pi \cdot h$

$50\pi \div 25\pi = 25\pi \cdot h \div 25\pi$

$2 = h$

The height is 2 units.

2. The height of the cylinder is 4 cm. Its volume is 36π cm^3. What is the radius of this cylinder? Show your thinking.

$V = \pi r^2 h$

If $V = 36\pi$ and $h = 4$, then

$36\pi = \pi \cdot r^2 \cdot 4$

$36\pi \div 4\pi = 4\pi \cdot r^2 \div 4\pi$

$9 = r^2$

$3 = r$

The radius is 3 cm.

Are you ready for more?

Suppose a cylinder has a volume of 36π in^3.

1. Name some different pairs of dimensions for this cylinder.

   Sample responses:

   - height of 9 in., radius of 2 in.
   - height of 1 in., radius of 6 in.
   - height of 36 in., radius of 1 in.
   - height of $\frac{4}{9}$ in., radius of 9 in.

2. How many different cylinders can you identify that have a volume of 36π in^3?

   There are an infinite number of cylinders with a volume of 36π in^3. No matter what value for r is chosen, a value for h can be calculated using the formula. $36\pi = \pi r^2 h$

554 **Unit 5** Functions and Volume

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① Launch

Ask students to identify what is different about the information they are given in Problem 1 compared to the problems they solved in Lesson 12.

② Monitor

Help students get started by referencing the Anchor Chart PDF, *Volumes of Circular Solids* to help them set up an equation.

Look for points of confusion:

- Thinking the height is 10 in Problem 1. Have students substitute 5 for the radius and 10 for the height to check their calculations.

③ Connect

Have students share their strategies in sequence, starting with students who used guess and check and ending with students who used the structure of the equation to find missing dimensions.

Ask:

- "In Problem 1, suppose you change 50π to a different value. Can you determine the new height? How?"
- "Is height a function of volume? Why or why not?"
- "In Problem 2, suppose you change 36π to a different value. Can you determine the new radius? How?"
- "Is the radius a function of volume? Why or why not? What about the diameter?"

Highlight that students can use the structure of the equation to determine the missing values if they are given the volume and either the height or the radius/diameter. **Note:** Students may suggest writing a formula for the height ($h =$ ___) or radius ($r =$ ___) as a function of volume, but this will not be expected in this unit.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students annotate each problem by underlining or circling the measure they are asked to determine. Display the volume formula for a cylinder and ask, "If you know the volume and the radius, how can you use the formula to determine the height?" (for Problem 1)

Extension: *Math Enrichment*

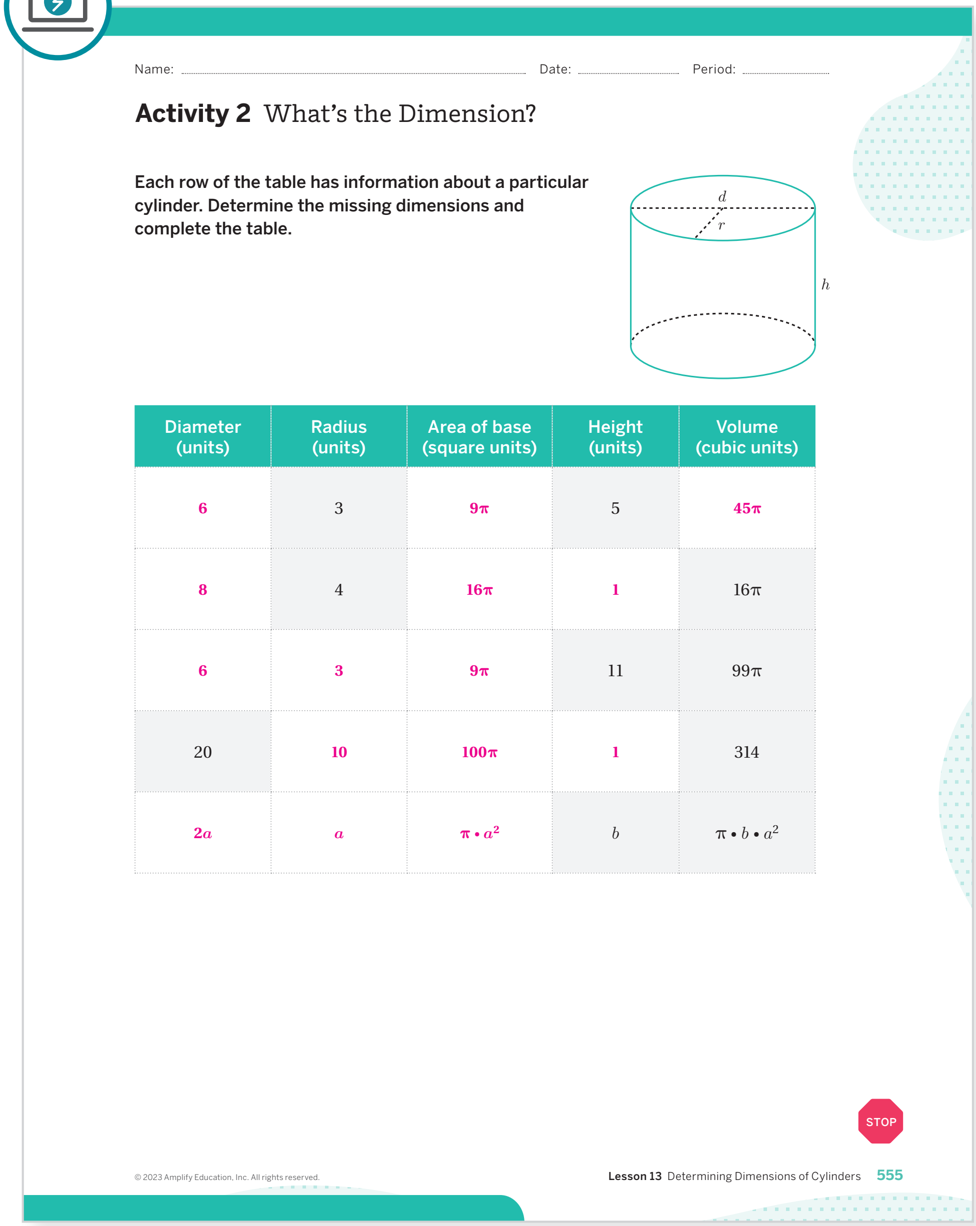
Have students complete the following problem:

A cylinder has a diameter of $\frac{x}{2}$ units and a volume of $2\pi x^2$ cubic units. What is the height of the cylinder? 32 units

Pairs | 15 min

Activity 2 What's the Dimension?

Students use the structure of the volume formula for cylinders to build fluency in determining missing dimensions of a cylinder, given other dimensions.



Name: ______________ Date: ________ Period: ________

Activity 2 What's the Dimension?

Each row of the table has information about a particular cylinder. Determine the missing dimensions and complete the table.

Diameter (units)	Radius (units)	Area of base (square units)	Height (units)	Volume (cubic units)
6	3	9π	5	45π
8	4	16π	1	16π
6	3	9π	11	99π
20	10	100π	1	314
$2a$	a	$\pi \cdot a^2$	b	$\pi \cdot b \cdot a^2$

STOP

Lesson 13 Determining Dimensions of Cylinders 555

1 Launch

Set an expectation for the amount of time students should work on the activity.

2 Monitor

Help students get started by having them write down the formula for the volume of cylinder and asking, "What information were you given in each row? What are you being asked to determine?"

Look for points of confusion:

- **Quickly recording the missing dimensions without the proper calculations.** Encourage students to use the equation for the volume of a cylinder and the given dimensions to determine the unknown dimensions.

Look for productive strategies:

- Manipulating the volume equation using variables to find each dimension in the last row.
- Writing a formula for finding the height.

3 Connect

Display student work showing the correct table.

Have students share the strategies they used for determining the missing dimensions.

Ask:

- "Look at Rows 1 and 3 in the table. How did having one row completed help you complete the other more efficiently?" If the base areas were the same, then the radius and diameter must be the same also.
- "How did you reason about the last row?"

Highlight that, in order to avoid mistakes, students should start from the formula and carefully check what is known and what they need to determine.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose three of the five rows of the table to complete. Allowing them to choose which rows to complete can help support their engagement in the activity.

Extension: *Math Enrichment*

Have students extend the table to one more row and determine the missing dimensions if the volume is $\pi \cdot b \cdot \frac{1}{4}a^2$ and the height is b. The radius is $\frac{1}{2}a$, the diameter is a, and the area of the base is $\pi \cdot \frac{1}{4}a^2$.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their strategies for completing the table, provide these sentence frames to help them organize their thinking.

- "I noticed ______ in the rows/columns, which made me think that . . ."
- "I noticed ______, and it tells me that . . ."
- "If I am given ______, I can ______ to determine the ______."

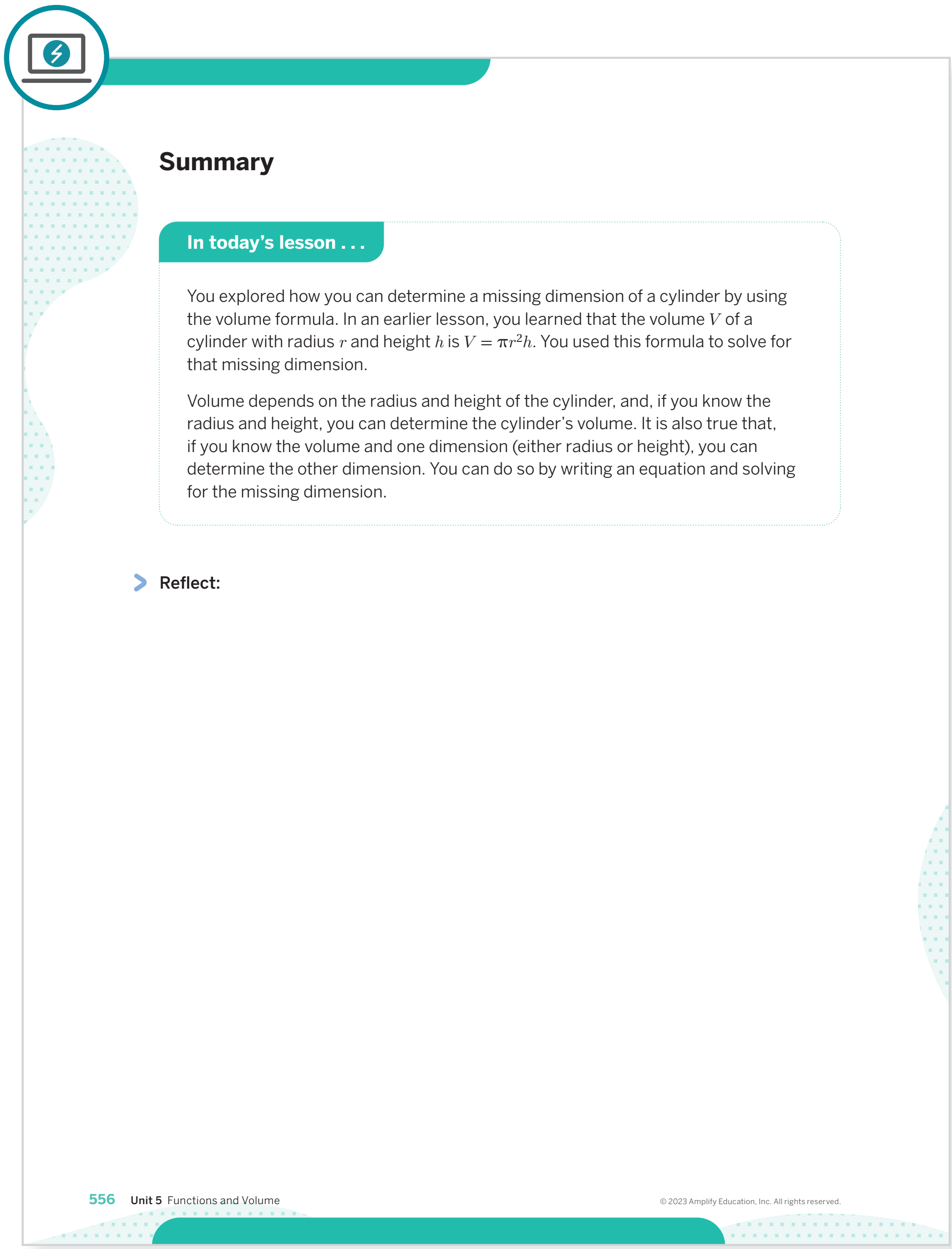
English Learners

Consider drawing cylinders labeled with the given information and use hand gestures, such as pointing to the dimension of the cylinder that is being described.

Whole Class | 8 min

Summary

Review and synthesize how the structure of the volume formula for a cylinder can be used to determine missing dimensions.



Summary

In today's lesson . . .

You explored how you can determine a missing dimension of a cylinder by using the volume formula. In an earlier lesson, you learned that the volume V of a cylinder with radius r and height h is $V = \pi r^2 h$. You used this formula to solve for that missing dimension.

Volume depends on the radius and height of the cylinder, and, if you know the radius and height, you can determine the cylinder's volume. It is also true that, if you know the volume and one dimension (either radius or height), you can determine the other dimension. You can do so by writing an equation and solving for the missing dimension.

Reflect:

Synthesize

Have students work in pairs, and ask them to choose one partner to name a value for the radius and one partner to name a value for the volume of a cylinder. Together, partners create a drawing and determine the height of their cylinder.

Display drawings and have students share their strategies for determining the height.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

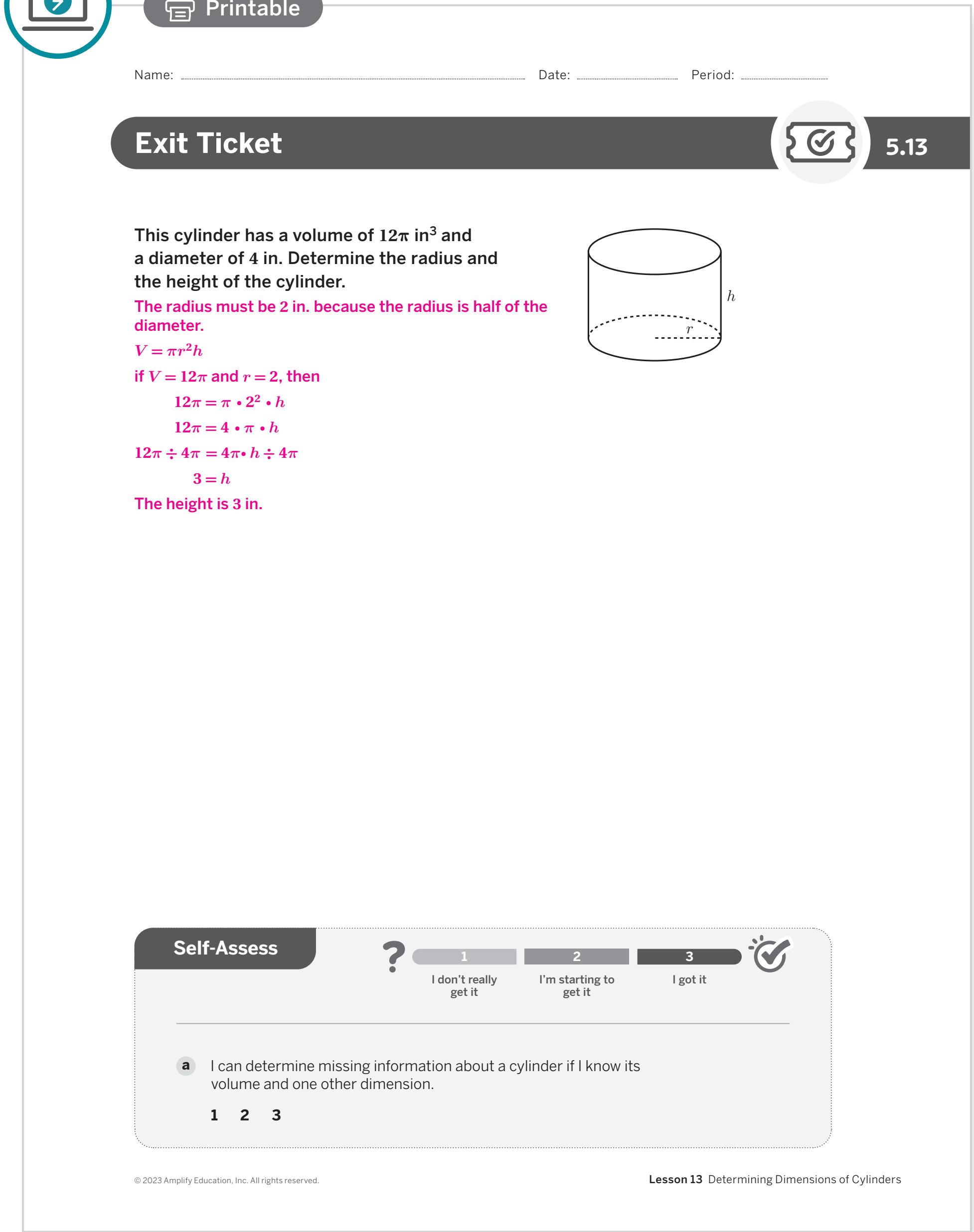
- "Which dimensions were the most efficient to determine? Which were the most challenging?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using the structure of the volume formula to determine the height and radius of a cylinder, given its volume and diameter.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 5.13

This cylinder has a volume of 12π in^3 and a diameter of 4 in. Determine the radius and the height of the cylinder.

The radius must be 2 in. because the radius is half of the diameter.

$V = \pi r^2 h$

if $V = 12\pi$ and $r = 2$, then

$12\pi = \pi \cdot 2^2 \cdot h$

$12\pi = 4 \cdot \pi \cdot h$

$12\pi \div 4\pi = 4\pi \cdot h \div 4\pi$

$3 = h$

The height is 3 in.

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a. I can determine missing information about a cylinder if I know its volume and one other dimension.

1 2 3

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Lesson 13 Determining Dimensions of Cylinders

Success looks like . . .

- **Language Goal:** Calculating the value of one dimension of a cylinder, and explaining the reasoning. **(Speaking and Listening, Writing)**
  - » Determining the height of the cylinder given a volume of 12π in^3 and a diameter of 4 in.
- **Language Goal:** Creating a table of dimensions of cylinders, and describing patterns that arise. **(Speaking and Listening, Writing)**

Suggested next steps

If students have difficulty setting up an equation, consider:

- Having students reference the Anchor Chart PDF, *Volumes of Circular Solids*.
- Reviewing Activity 1 for how to set up an equation.
- Assigning Practice Problem 1.

If students use 4 as the radius, instead of 2, consider:

- Having them label the diameter and radius on the cylinder.

If students are unable to solve for the correct height, consider:

- Having students check their solution using the volume formula for a cylinder.
- Reviewing how to solve for an unknown height in Activity 2.
- Assigning Practice Problem 2.

If students do not include correct units, consider:

- Reminding them to check their units and referencing Activity 1, Problem 2 as an example.

Professional Learning

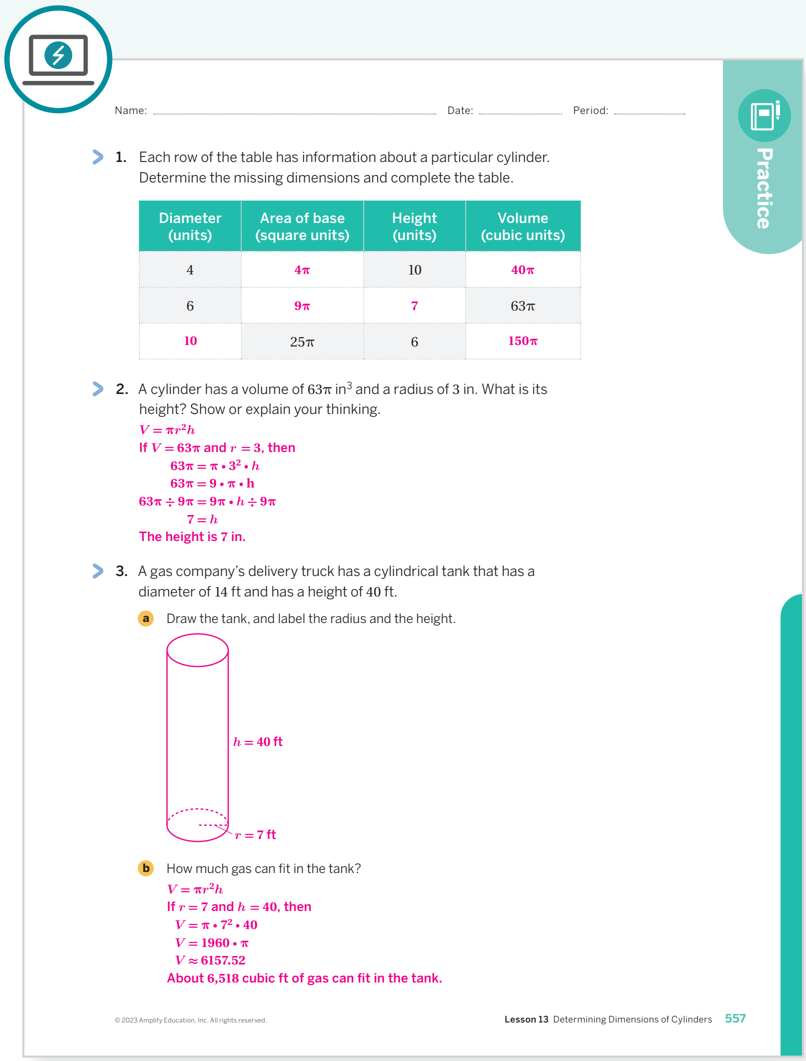
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What did students find frustrating about finding missing dimensions in Activity 2? What helped them work through this frustration?
- In what ways have your students gotten better at working with the formula for the volume of a cylinder?

Practice

Independent



Practice

Name: Date: Period:

1. Each row of the table has information about a particular cylinder. Determine the missing dimensions and complete the table.

Diameter (units)	Area of base (square units)	Height (units)	Volume (cubic units)
4	**4π**	10	**40π**
6	**9π**	**7**	63π
10	25π	6	**150π**

2. A cylinder has a volume of 63π in^3 and a radius of 3 in. What is its height? Show or explain your thinking.

$V = \pi r^2 h$

If $V = 63\pi$ and $r = 3$, then

$63\pi = \pi \cdot 3^2 \cdot h$

$63\pi = 9 \cdot \pi \cdot \mathrm{h}$

$63\pi \div 9\pi = 9\pi \cdot h \div 9\pi$

$7 = h$

The height is 7 in.

3. A gas company's delivery truck has a cylindrical tank that has a diameter of 14 ft and has a height of 40 ft.

a Draw the tank, and label the radius and the height.

b How much gas can fit in the tank?

$V = \pi r^2 h$

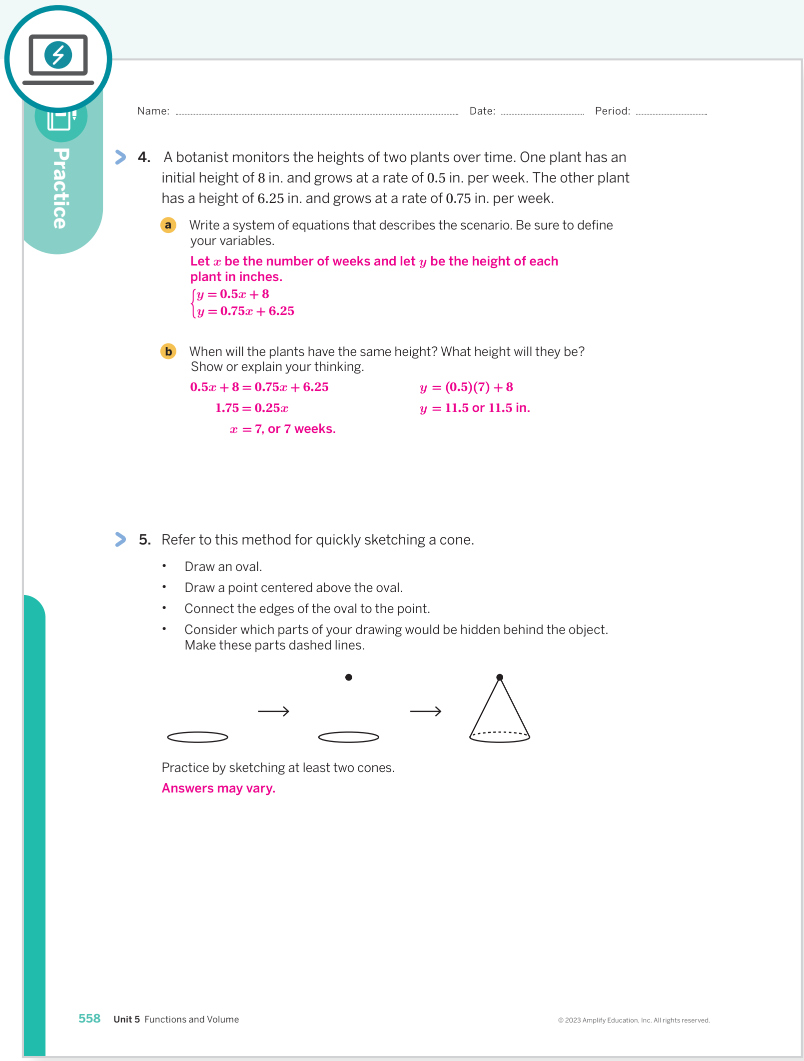
If $r = 7$ and $h = 40$, then

$V = \pi \cdot 7^2 \cdot 40$

$V = 1960 \cdot \pi$

$V \approx 6157.52$

About 6,518 cubic ft of gas can fit in the tank.



Practice

Name: Date: Period:

4. A botanist monitors the heights of two plants over time. One plant has an initial height of 8 in. and grows at a rate of 0.5 in. per week. The other plant has a height of 6.25 in. and grows at a rate of 0.75 in. per week.

a Write a system of equations that describes the scenario. Be sure to define your variables.

Let x be the number of weeks and let y be the height of each plant in inches.

$$\begin{cases} y = 0.5x + 8 \\ y = 0.75x + 6.25 \end{cases}$$

b When will the plants have the same height? What height will they be? Show or explain your thinking.

$0.5x + 8 = 0.75x + 6.25$

$1.75 = 0.25x$

$x = 7$, **or 7 weeks.**

$y = (0.5)(7) + 8$

$y = 11.5$ **or 11.5 in.**

5. Refer to this method for quickly sketching a cone.

- Draw an oval.
- Draw a point centered above the oval.
- Connect the edges of the oval to the point.
- Consider which parts of your drawing would be hidden behind the object. Make these parts dashed lines.

Practice by sketching at least two cones.

Answers may vary.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 4 Lesson 16	2
Formative	5	Unit 5 Lesson 14	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 14**

The Volume of a Cone

Let's explore cones and their volumes.

Focus

Goals

1. **Language Goal:** Calculate the volume of a cone and cylinder given the height and radius, and explain the solution method. **(Speaking and Listening, Writing)**
2. **Language Goal:** Compare the volumes of a cone and a cylinder with the same base and height, and explain the relationship between the volumes. **(Speaking and Listening, Writing)**

Coherence

Today

In this lesson students start working with cones, and learn that the volume of a cone is one third the volume of a cylinder with a congruent base and the same height. They watch a video (or if possible, a live demonstration) showing that it takes three cones of water to fill a cylinder with the same radius and height. At this point, it is taken as a mysterious and beautiful fact that the volume of a cone is one third the volume of the associated cylinder. A proof of this fact requires mathematics beyond this grade level.

Previously

In Lesson 12, students learned that the volume of a cylinder can be represented by the formula $V = \pi \cdot r^2 \cdot h$. In Lesson 13, students had opportunities to practice determining measures of the radius, diameter, and height of a cylinder when given the measure of the volume.

Coming Soon

In Lesson 15, students will have a chance to practice determining the dimensions of a cone, given the volume of a cone, similar to what they practiced in Lesson 13 for the dimensions of a cylinder.

Rigor

- Students build **conceptual understanding** about the relationship between the volume of a cone and the volume of a cylinder.
- Students build **fluency** determining the volume of a cone.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Volumes of Circular Solids*
- Anchor Chart PDF, *Volumes of Circular Solids* (answers)
- calculators

Math Language Development

Review words

- *circumference*
- *cone*
- *cylinder*
- *height*
- *pi*
- *radius*
- *volume*

Amps | Featured Activity

Activity 1

Digital Demonstration

Students watch a video of a cone filling a cylinder to help develop an understanding of the relationship between the volume of a cylinder and the volume of a cone.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might focus on the results during Activity 1 without reflecting on the repeated reasoning used to achieve the results. Encourage students to consider the process of looking for repeated calculations can lead to success in the activity, building confidence. The resulting optimism can influence their behaviors for the rest of the lesson and beyond.

Modifications to Pacing

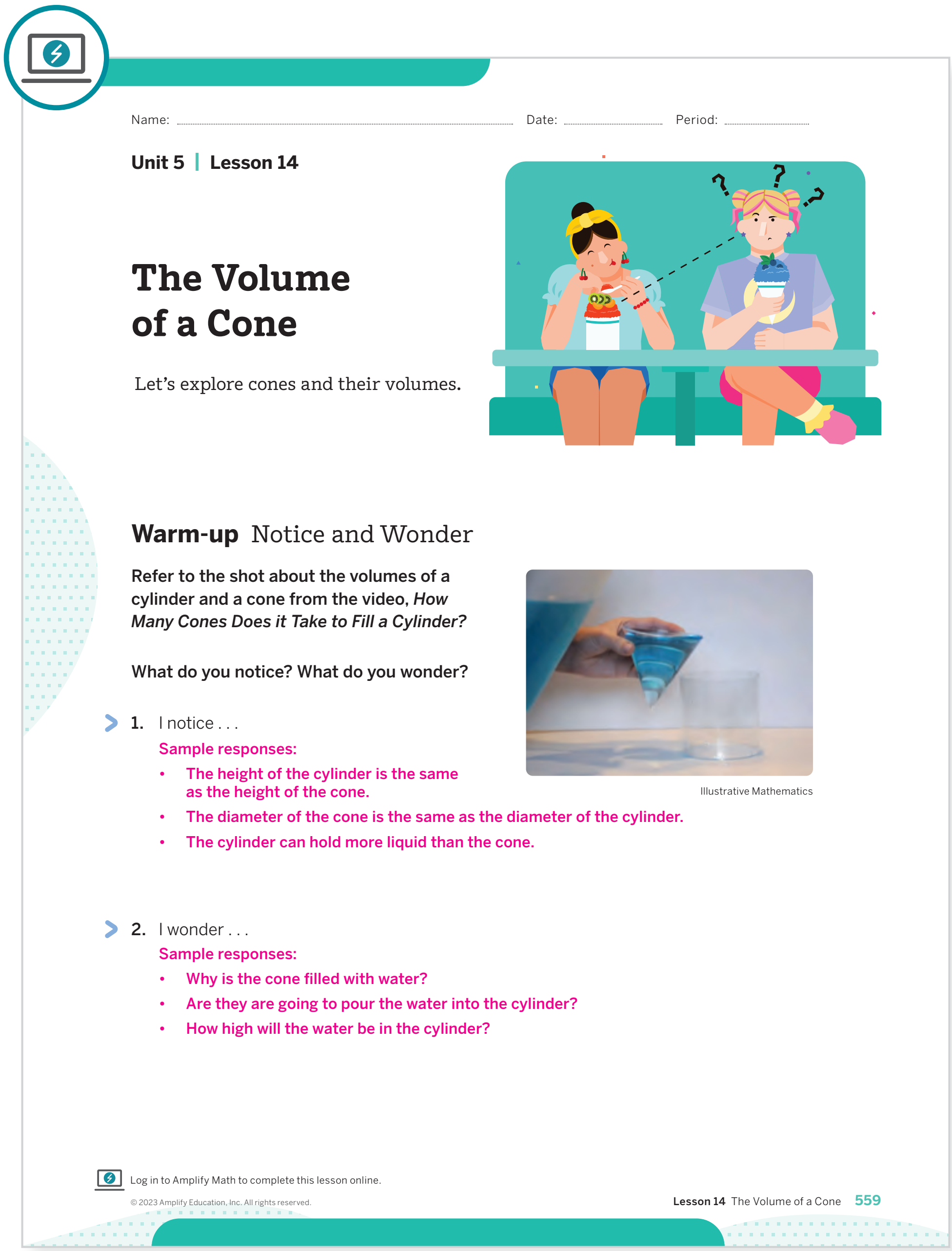
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, have students complete one of the problems.

Pairs | 5 min

Warm-up Notice and Wonder

Students explore how to find the volume of a cone by comparing water in a cone and a cylinder.



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Unit 5 | Lesson 14

The Volume of a Cone

Let's explore cones and their volumes.

Warm-up Notice and Wonder

Refer to the shot about the volumes of a cylinder and a cone from the video, *How Many Cones Does it Take to Fill a Cylinder?*

What do you notice? What do you wonder?

Illustrative Mathematics

1. I notice . . .
   Sample responses:
   - The height of the cylinder is the same as the height of the cone.
   - The diameter of the cone is the same as the diameter of the cylinder.
   - The cylinder can hold more liquid than the cone.
2. I wonder . . .
   Sample responses:
   - Why is the cone filled with water?
   - Are they are going to pour the water into the cylinder?
   - How high will the water be in the cylinder?

Log in to Amplify Math to complete this lesson online.

Lesson 14 The Volume of a Cone 559

1 Launch

Play the first 23 seconds of the online video, *How Many Cones Does it Take to Fill a Cylinder?*, from the Warm-up Amps. Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking what they notice about the circumference and height of the two containers in the video.

Look for points of confusion:

- **Using imprecise language to describe the two containers.** Review the terms *radius*, *circumference* and *height* in relation to a cone and a cylinder.

3 Connect

Have pairs of students share what they noticed and wondered about the video.

Highlight that the cone and cylinder each have the same height and same circumference. Emphasize how the water from the filled cone represents the volume of the cone. This water is then poured into the cylinder.

Ask:

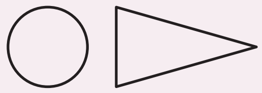
- "If their circumferences are the same, what must be true about their diameters and radii?"
- "What do you think will happen next?"

Power-up

To power up students' ability to sketch a cone, have students complete:

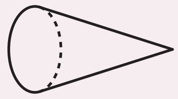
Which of the following figures are cones? Select *all* that apply.

A.

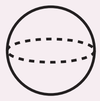


C.

[✓ B.]



D.



Use: Before the Warm-up

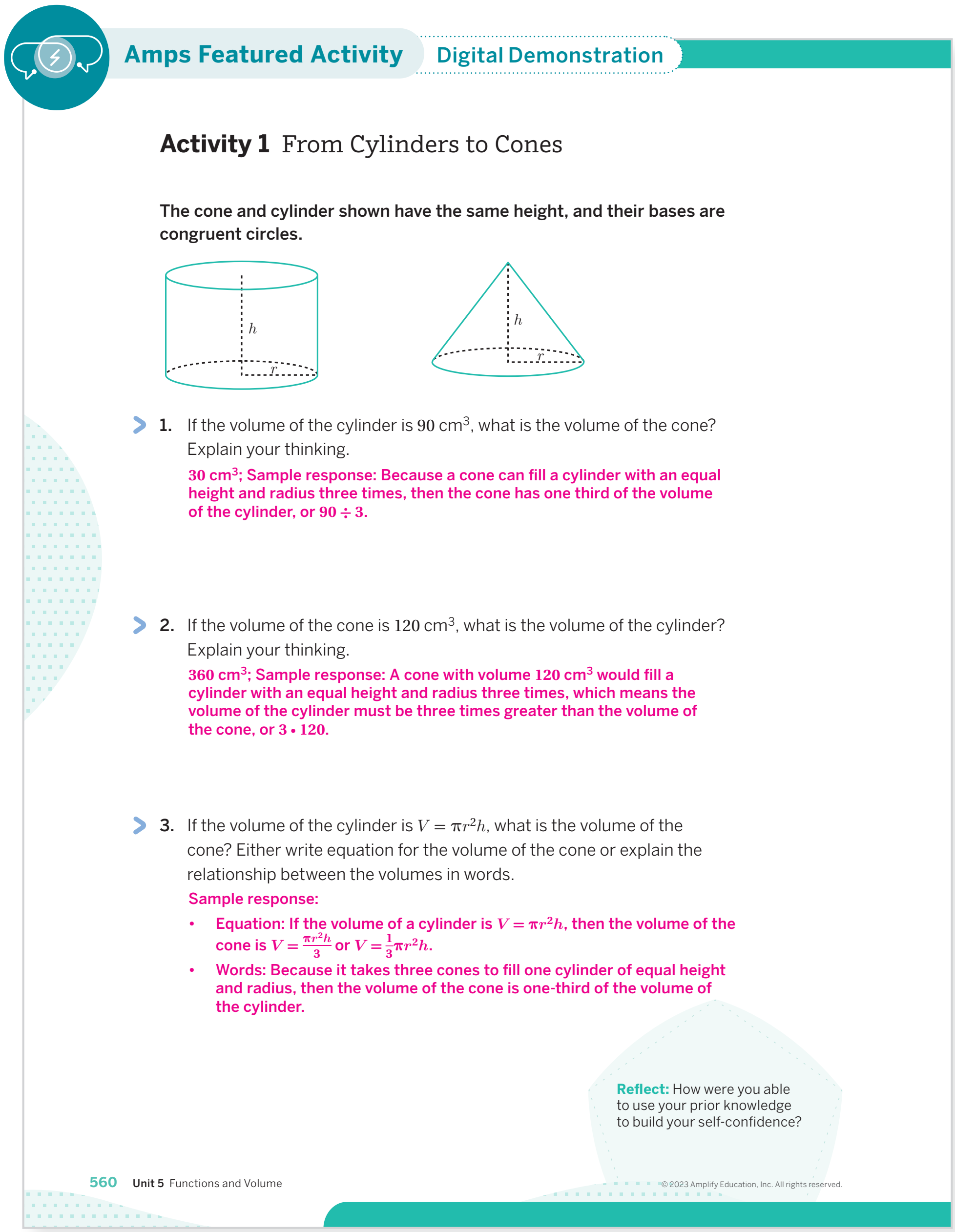
Informed by: Performance on Lesson 13, Practice Problem 5

Pairs | 15 min

Activity 1 From Cylinders to Cones

Students connect the volumes of a cone and a cylinder with the same dimensions to arrive at the formula for the volume of a cone.

Amps Featured Activity Digital Demonstration



Activity 1 From Cylinders to Cones

The cone and cylinder shown have the same height, and their bases are congruent circles.

1. If the volume of the cylinder is 90 cm^3, what is the volume of the cone? Explain your thinking.
   30 cm^3; Sample response: Because a cone can fill a cylinder with an equal height and radius three times, then the cone has one third of the volume of the cylinder, or $90 \div 3$.

2. If the volume of the cone is 120 cm^3, what is the volume of the cylinder? Explain your thinking.
   360 cm^3; Sample response: A cone with volume 120 cm^3 would fill a cylinder with an equal height and radius three times, which means the volume of the cylinder must be three times greater than the volume of the cone, or $3 \cdot 120$.

3. If the volume of the cylinder is $V = \pi r^2 h$, what is the volume of the cone? Either write equation for the volume of the cone or explain the relationship between the volumes in words.
   Sample response:
   - **Equation: If the volume of a cylinder is $V = \pi r^2 h$, then the volume of the cone is $V = \frac{\pi r^2 h}{3}$ or $V = \frac{1}{3}\pi r^2 h$.**
   - **Words: Because it takes three cones to fill one cylinder of equal height and radius, then the volume of the cone is one-third of the volume of the cylinder.**

Reflect: How were you able to use your prior knowledge to build your self-confidence?

560 **Unit 5** Functions and Volume

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1 Launch

Play the remainder of the online video, *How Many Cones Does it Take to Fill a Cylinder?* Discuss how the volume of the cone appears to be one third the volume of the cylinder.

2 Monitor

Help students get started by asking how many times greater the volume of the cylinder was than the volume of the cone in the video.

Look for points of confusion:

- **Having difficulty writing the volume formula in Problem 3.** Ask students how they found the volume of the cone in Problem 1. Ask, "If you know the volume of any cylinder is $\pi r^2 h$, how can you find the volume of any cone?"

3 Connect

Ask, "If you know the volume of a cylinder, how can you find the volume of a cone?"

Display a student response for Problem 3 showing the volume formula of a cone by multiplying the volume formula of a cylinder by $\frac{1}{3}$ and a response dividing the volume formula of a cylinder by 3.

Highlight that the volume of a cone is $\frac{1}{3}V$, where V represents the volume of a cylinder with the same base and height as the cone, which means the volume of the cone is $\frac{1}{3}\pi r^2 h$. Emphasize that this formula is the same as dividing the volume of a cylinder by 3.

Update the Anchor Chart PDF, *Volumes of Circular Solids* with the formula of the volume of a cone, $V = \frac{1}{3}\pi r^2 h$.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can watch an animation of a cone filling a cylinder to help develop an understanding of the relationship between the volume of a cylinder and the volume of a cone.

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

During the Launch, remind students they have previously worked with and explored congruent figures. Ask them to describe in their own words what it means for the bases to be congruent circles.

Math Language Development

MLR7: Compare and Connect

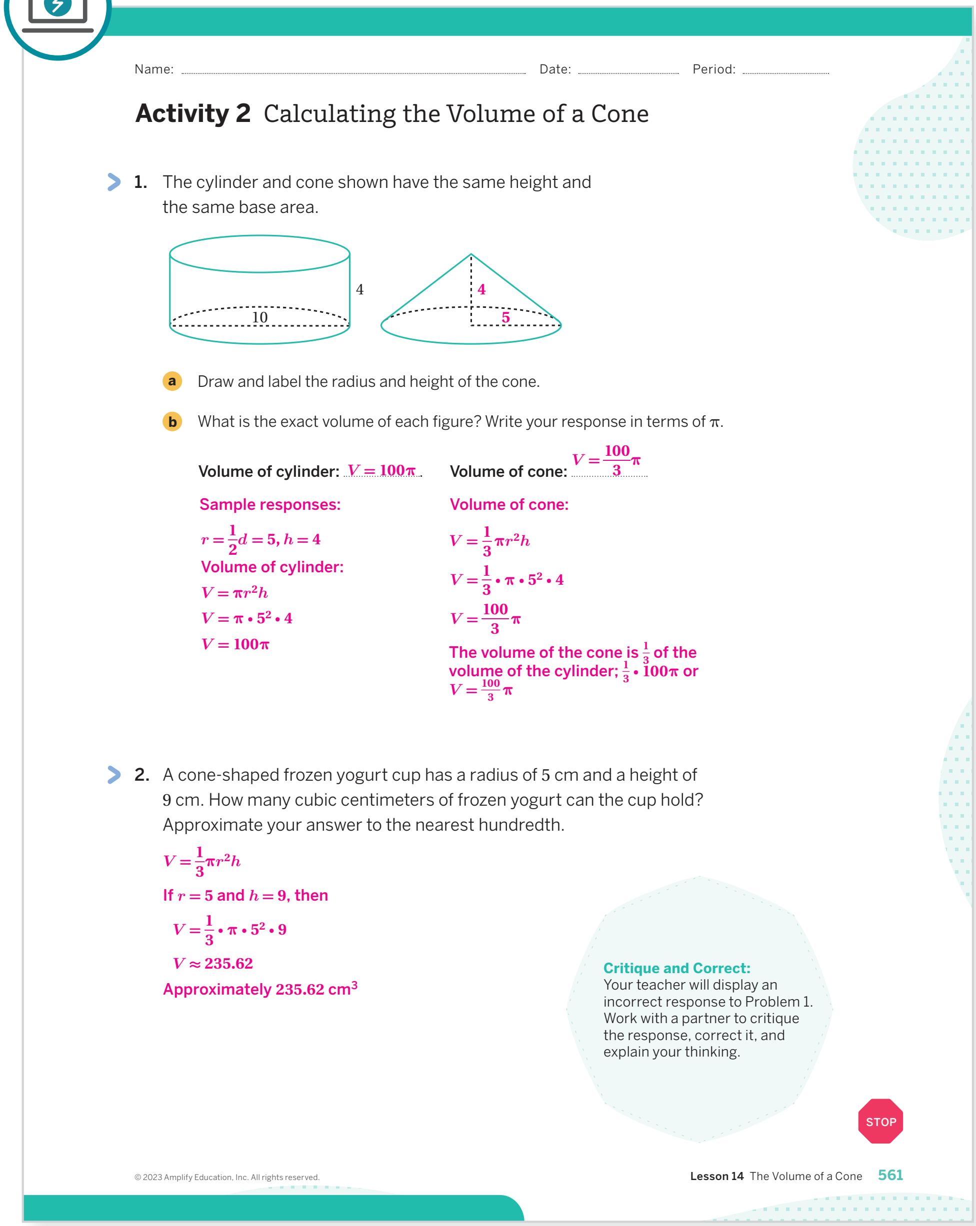
During the Connect, as you display student responses for Problem 3, draw students' attention to the connections between the two different ways to write the formula and describe the relationship in words. Add the following to the class display.

Formulas		Words
$V = \frac{\pi r^2 h}{3}$	$V = \frac{1}{3}\pi r^2 h$	The volume of a cone is one-third the volume of a cylinder, with the same radius.

Independent | 15 min

Activity 2 Calculating the Volume of a Cone

Students calculate the volume of cones to apply the volume relationship they learned in the previous activity.



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Activity 2 Calculating the Volume of a Cone

1. The cylinder and cone shown have the same height and the same base area.

   a Draw and label the radius and height of the cone.

   b What is the exact volume of each figure? Write your response in terms of π.

   Volume of cylinder: $V = 100\pi$ Volume of cone: $V = \frac{100}{3}\pi$

   Sample responses:

   $r = \frac{1}{2}d = 5, h = 4$

   Volume of cylinder:

   $V = \pi r^2 h$

   $V = \pi \cdot 5^2 \cdot 4$

   $V = 100\pi$

   Volume of cone:

   $V = \frac{1}{3}\pi r^2 h$

   $V = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot 4$

   $V = \frac{100}{3}\pi$

   The volume of the cone is $\frac{1}{3}$ of the volume of the cylinder; $\frac{1}{3} \cdot 100\pi$ or $V = \frac{100}{3}\pi$

2. A cone-shaped frozen yogurt cup has a radius of 5 cm and a height of 9 cm. How many cubic centimeters of frozen yogurt can the cup hold? Approximate your answer to the nearest hundredth.

   $V = \frac{1}{3}\pi r^2 h$

   If $r = 5$ and $h = 9$, then

   $V = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot 9$

   $V \approx 235.62$

   Approximately 235.62 cm^3

Critique and Correct:
Your teacher will display an incorrect response to Problem 1. Work with a partner to critique the response, correct it, and explain your thinking.

1 Launch

Distribute calculators for the duration of the activity.

2 Monitor

Help students get started by having them write the formula for the volume of a cone by referencing the Anchor Chart PDF, *Volumes of Circular Solids*.

Look for points of confusion:

- **Thinking that 10 is the radius in Problem 1.** Point out that 10 is the diameter and ask students to label the radius measure on the diagram for each figure.
- **Using the volume of a cylinder formula or not dividing the cylinder formula by 3 in Problem 2.** Have students sketch the shape of the frozen yogurt cup and ask what the relationship is between the volume of a cone and the volume of a cylinder.

Look for productive strategies:

- Using the relationships between the volume of the cylinder and the cone in Problem 1.

3 Connect

Have students share how they calculated the volume of both figures in Problem 1 and have them share the different strategies they used.

Highlight the different ways of using the relationship between the volume of a cone and the volume of a cylinder to find the volume of both figures in Problem 1.

Display student work showing correct work for finding the volume of the cone in Problem 2. If a student used a sketch in their work, discuss how making a quick drawing can help to make sense of a problem such as this.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity.

Extension: *Math Enrichment*

Have students complete the following problem: A cylinder has a volume of 150π. What is the volume of a cone with the same radius, but twice the height? Sample response: If the cone had the same height, the volume would be 50π. Because the cone has twice the height, the volume of the cone is 100π.

Math Language Development

MLR3: Critique, Correct, Clarify

After students have completed Problem 1, present one or both of the following incorrect statements about the volume of the cylinder.

"The volume of the cylinder is 400π because $\pi * 4 * 10^2 = 400\pi$."	"The volume of the cylinder is 100π because $10^2 = 100$."

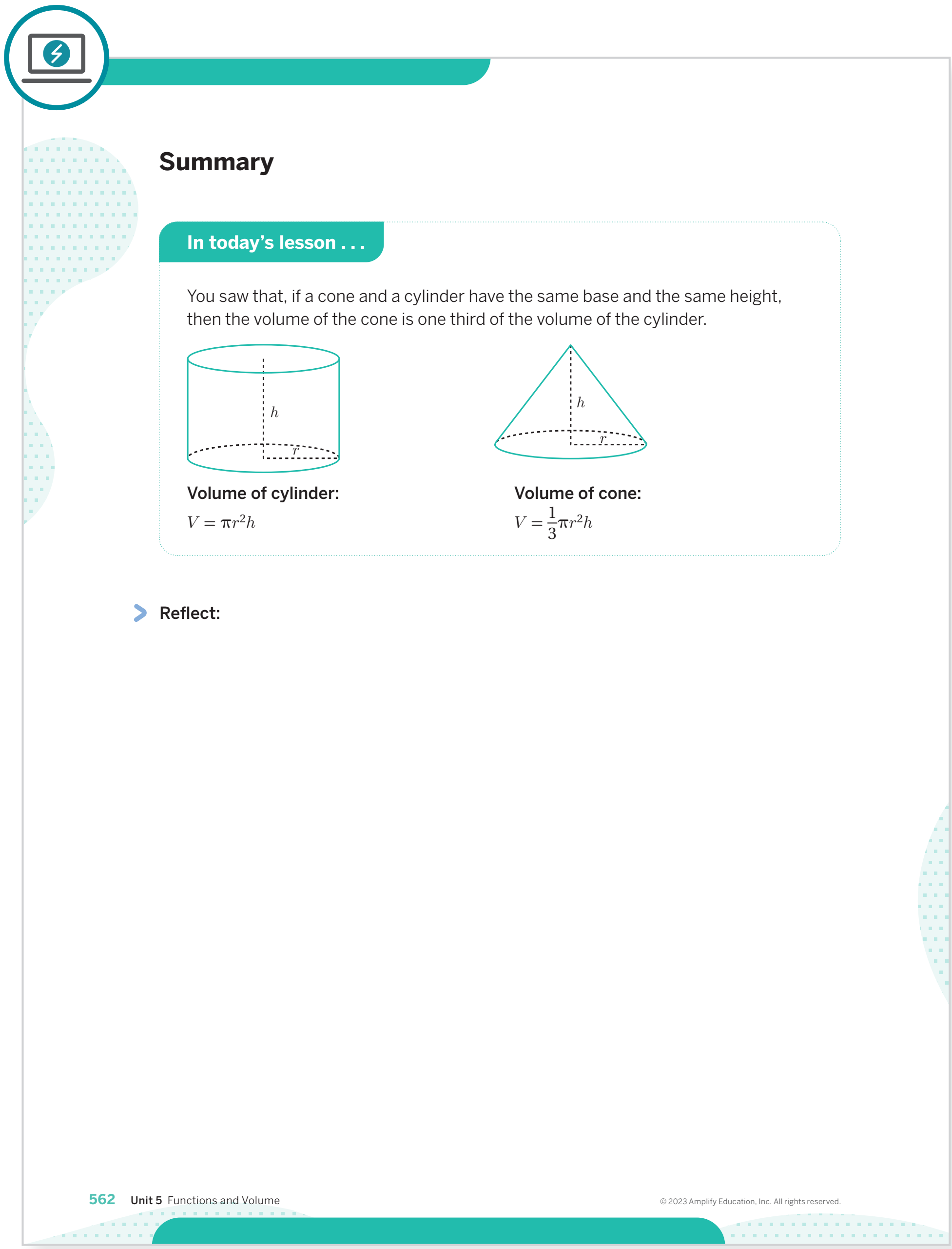
Note: The volume in the second statement is correct, 100π, but was determined using incorrect reasoning. Have students work with a partner to critique this response. Ask:

- ***Critique:*** "Do you agree or disagree with this response and reasoning? Explain your thinking."
- ***Correct:*** "Write a corrected response."
- ***Clarify:*** "How would you convince a classmate that your statement is correct?"

Whole Class | 5 min

Summary

Review and synthesize the relationship between the volume of a cone and the volume of a cylinder, when the figures have the same radius and height.



Summary

In today's lesson . . .

You saw that, if a cone and a cylinder have the same base and the same height, then the volume of the cone is one third of the volume of the cylinder.

Volume of cylinder:
$V = \pi r^2 h$

Volume of cone:
$V = \frac{1}{3}\pi r^2 h$

Reflect:

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Synthesize

Have students share a recollection of how to demonstrate the relationship between the volume of a cone and the volume of a cylinder using liquid.

Ask:

- "If you know the volume of a cone, how do you calculate the volume of a cylinder that has the same height and base area?"
- "If you know the volume of a cylinder, how do you calculate the volume of a cone that has the same height and base area?"
- "If a cylinder and a cone have the same base area, how tall does the cone have to be relative to the cylinder so that they both have the same volume?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

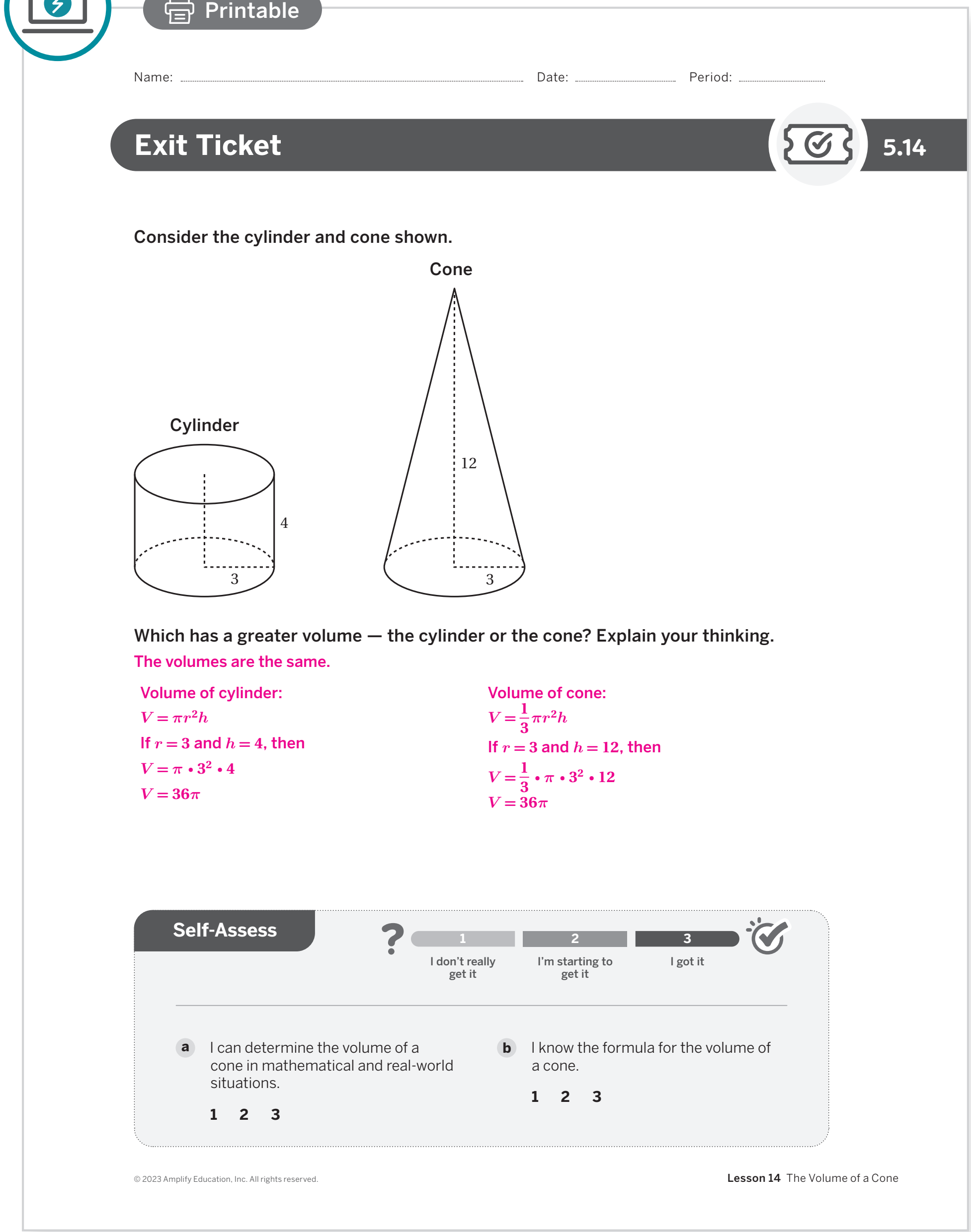
- "How is the volume of a cone related to the volume of a cylinder?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by comparing the volumes of a cone and cylinder, given the same radius but different heights.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.14

Consider the cylinder and cone shown.

Which has a greater volume — the cylinder or the cone? Explain your thinking.

The volumes are the same.

Volume of cylinder:

$V = \pi r^2 h$

If $r = 3$ and $h = 4$, then

$V = \pi \cdot 3^2 \cdot 4$

$V = 36\pi$

Volume of cone:

$V = \frac{1}{3}\pi r^2 h$

If $r = 3$ and $h = 12$, then

$V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 12$

$V = 36\pi$

Self-Assess

? 1 I don't really get it 2 I'm starting to get it 3 I got it

a I can determine the volume of a cone in mathematical and real-world situations.
1 2 3

b I know the formula for the volume of a cone.
1 2 3

Lesson 14 The Volume of a Cone

Success looks like . . .

- **Language Goal:** Calculating the volume of a cone and cylinder given the height and radius, and explaining the solution method. **(Speaking and Listening, Writing)**
- **Language Goal:** Comparing the volumes of a cone and a cylinder with the same base and height, and explaining the relationship between the volumes. **(Speaking and Listening, Writing)**
    - » Determining whether the cone or the cylinder has the greater volume.

Suggested next steps

If students use the wrong formula for either figure in Problem 2, consider:

- Referencing the Anchor Chart PDF, *Volumes of Circular Solids*.
- Having students label the dimensions of each figure.
- Reviewing Activity 2.

Professional Learning

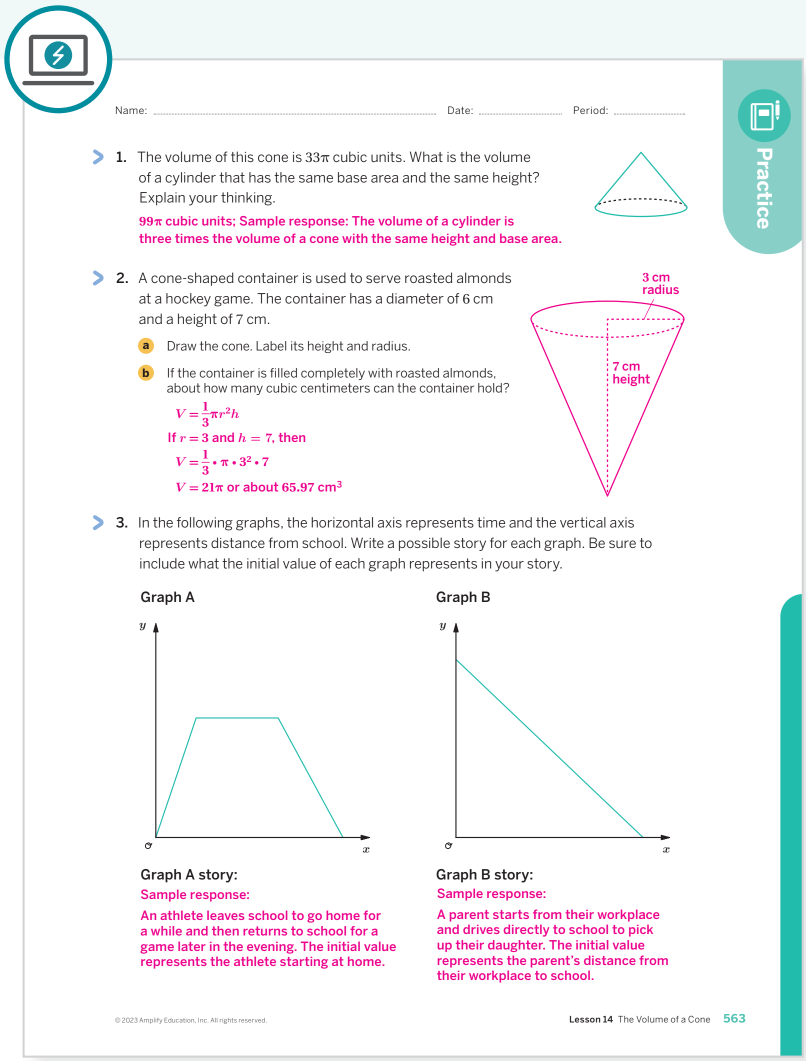
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- During the discussion about the volume of a cone, how did you encourage each student to listen to one another's strategies?
- What other ways are there to try making the connection between the volume of a cone and the volume of a cylinder?

Practice

Independent



Practice

Name: Date: Period:

1. The volume of this cone is 33π cubic units. What is the volume of a cylinder that has the same base area and the same height? Explain your thinking.

99π cubic units; Sample response: The volume of a cylinder is three times the volume of a cone with the same height and base area.

2. A cone-shaped container is used to serve roasted almonds at a hockey game. The container has a diameter of 6 cm and a height of 7 cm.

   a. Draw the cone. Label its height and radius.

   b. If the container is filled completely with roasted almonds, about how many cubic centimeters can the container hold?

$V = \frac{1}{3}\pi r^2 h$

If $r = 3$ and $h = 7$, then

$V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 7$

$V = 21\pi$ or about 65.97 cm^3

3. In the following graphs, the horizontal axis represents time and the vertical axis represents distance from school. Write a possible story for each graph. Be sure to include what the initial value of each graph represents in your story.

Graph A

Graph B

Graph A story:

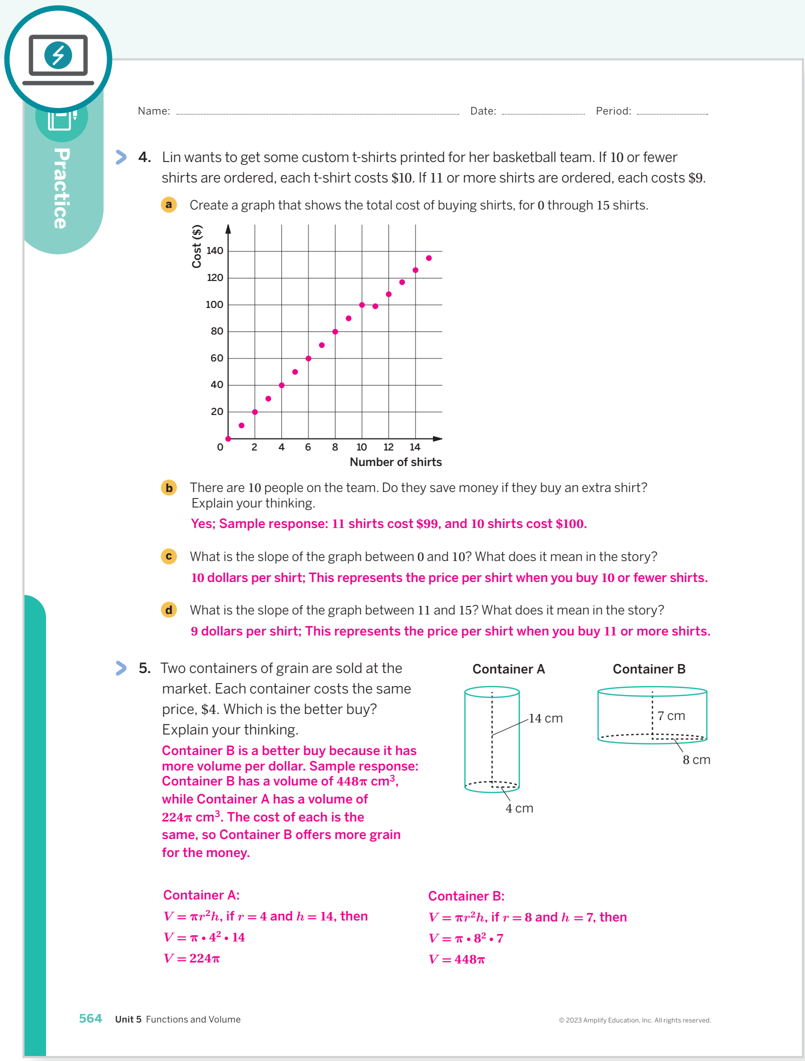
Sample response:

An athlete leaves school to go home for a while and then returns to school for a game later in the evening. The initial value represents the athlete starting at home.

Graph B story:

Sample response:

A parent starts from their workplace and drives directly to school to pick up their daughter. The initial value represents the parent's distance from their workplace to school.



Practice

Name: Date: Period:

4. Lin wants to get some custom t-shirts printed for her basketball team. If 10 or fewer shirts are ordered, each t-shirt costs $10. If 11 or more shirts are ordered, each costs $9.

   a. Create a graph that shows the total cost of buying shirts, for 0 through 15 shirts.

   b. There are 10 people on the team. Do they save money if they buy an extra shirt? Explain your thinking.

Yes; Sample response: 11 shirts cost $99, and 10 shirts cost $100.

   c. What is the slope of the graph between 0 and 10? What does it mean in the story?

10 dollars per shirt; This represents the price per shirt when you buy 10 or fewer shirts.

   d. What is the slope of the graph between 11 and 15? What does it mean in the story?

9 dollars per shirt; This represents the price per shirt when you buy 11 or more shirts.

5. Two containers of grain are sold at the market. Each container costs the same price, $4. Which is the better buy? Explain your thinking.

Container B is a better buy because it has more volume per dollar. Sample response: Container B has a volume of 448π cm^3, while Container A has a volume of 224π cm^3. The cost of each is the same, so Container B offers more grain for the money.

Container A:

$V = \pi r^2 h$, if $r = 4$ and $h = 14$, then

$V = \pi \cdot 4^2 \cdot 14$

$V = 224\pi$

Container B:

$V = \pi r^2 h$, if $r = 8$ and $h = 7$, then

$V = \pi \cdot 8^2 \cdot 7$

$V = 448\pi$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
Spiral	3	Unit 5 Lesson 6	2
	4	Unit 5 Lesson 9	2
Formative ⚡	5	Unit 5 Lesson 15	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 15**

Determining Dimensions of Cones

Let's apply the volume formula for a cone to determine missing dimensions.

Focus

Goals

1. **Language Goal:** Calculate the value of one dimension of a cone, and explain the reasoning. **(Speaking and Listening, Writing)**
2. **Language Goal:** Create a table of dimensions of cones, and describe patterns that arise. **(Speaking and Listening, Writing)**
3. **Language Goal:** Compare volumes of a cone and cylinder in context, and justify which volume is a better value for a given price. **(Speaking and Listening, Writing)**

Rigor

- Students build **procedural skills** working with the volume formula of a cone.
- Students **apply** the formula for the volume of a cone to scenarios where they must determine a missing dimension given the measure of the volume.

Coherence

Today

Students use the formula $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$ to determine the radius or height of a cone given its volume and the other dimension. They also compare the volume of a cone and a cylinder to determine, in context, the better deal.

Previously

In Lesson 14, students discovered that a cone has one third of the volume of a cylinder with the same radius and height as the cone. In Lesson 13, students determined missing dimensions of cylinders using the structure of the volume formula.

Coming Soon

In Lesson 16, students estimate the volume of a hemisphere by fitting a hemisphere inside a cylinder. In Lesson 17, students explore the relationship between the volume of a sphere and the volumes of a cylinder and a cone, all with the same radii and heights.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Volumes of Circular Solids*
- Anchor Chart PDF, *Volumes of Circular Solids* (answers)
- calculators

Math Language Development

Review words

- *cone*
- *cylinder*
- *height*
- *pi*
- *radius*
- *volume*

Amps | **Featured Activity**

Exit Ticket

Real-Time Exit Ticket

Check in real time whether your students can determine possible dimensions of a cone, given the cone's volume, using a digital Exit Ticket.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might feel so confident in their own responses that they forget to consider the thinking of others. Remind them that clearly communicating their own thoughts is important, but equally as important is actively listening to others. Challenge them to learn from others' critiques and viewpoints.

Modifications to Pacing

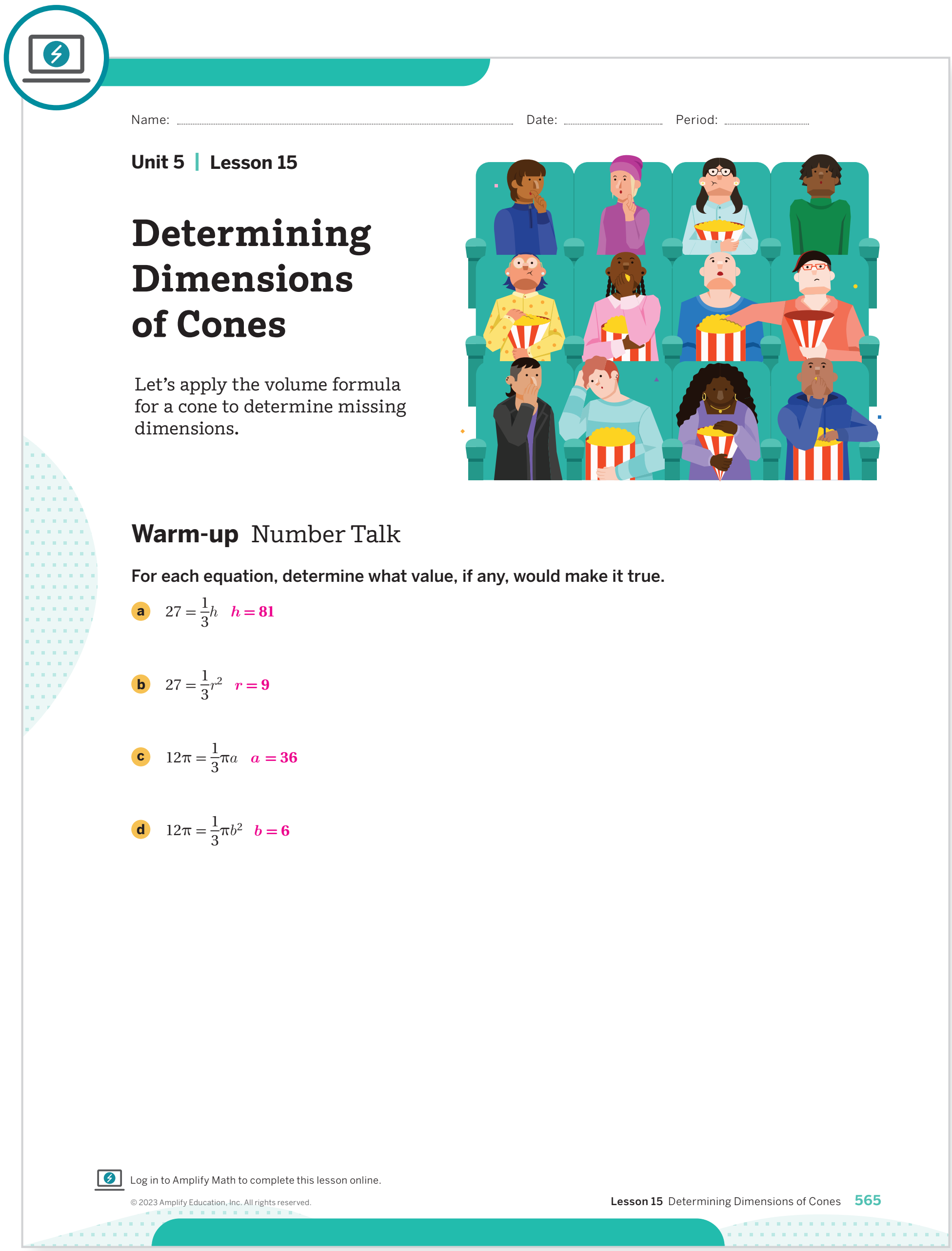
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted and given as a formative practice problem in the previous lesson.
- In **Activity 1**, have students only complete the first three rows of the table.

Independent | 5 min

Warm-up Number Talk

Students build fluency using the structure of an equation to find missing values.



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Unit 5 | Lesson 15

Determining Dimensions of Cones

Let's apply the volume formula for a cone to determine missing dimensions.

Warm-up Number Talk

For each equation, determine what value, if any, would make it true.

a. $27 = \frac{1}{3}h$ $h = 81$

b. $27 = \frac{1}{3}r^2$ $r = 9$

c. $12\pi = \frac{1}{3}\pi a$ $a = 36$

d. $12\pi = \frac{1}{3}\pi b^2$ $b = 6$

Log in to Amplify Math to complete this lesson online.

Lesson 15 Determining Dimensions of Cones 565

1 Launch

Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by asking them whether h is greater than or less than 27 in the first equation.

Look for points of confusion:

- **Dividing 27 by 3 instead of $\frac{1}{3}$.** Ask students what number they can multiply by $\frac{1}{3}$ to get 27.
- **Being unsure how to solve the equations in parts b and d.** Help students rewrite each equation to be a number equal to a variable squared and have them use the guess-and-check strategy.

3 Connect

Display one equation at a time and have students share two or three strategies per equation.

Ask:

- "What is similar between the equations in parts a and b? Between the equations in parts c and d?" The equations in parts a and b have a similar structure and use the values 27 and $\frac{1}{3}$. The equations in parts c and d have a similar structure and use the values 12π and $\frac{1}{3}\pi$.
- "What is the difference between the equations in parts a and c? Between the equations in parts b and d?" The equation in part b replaces h from the equation in part a with r^2. Similarly, the equation in part d replaces a from the equation in part c with b^2.

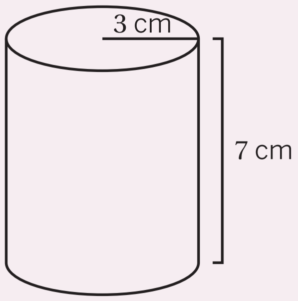
Highlight the ways students worked with $\frac{1}{3}$ when solving for the unknown in each equation. Some students may have multiplied both sides of the equation by 3, while other students may have divided both sides of the equation by $\frac{1}{3}$.

Power-up

To power up students' ability to use the volume of cylinders to compare unit price, have students complete:

Recall that the formula for the volume of a cylinder is $V = \pi r^2 h$ where r is the radius and h is the height.

1. Determine the volume of the container. Show your thinking.
   $V = 63\pi$ cm^3; $V = \pi\,(3)^2(7)$
   $V = \pi\,(9)(7)$
   $V = 63\pi$
2. The container is filled with premium soup. If the cost of the soup is \$2, determine its unit price (cost per cubic cm). About \$0.01 per square inch.



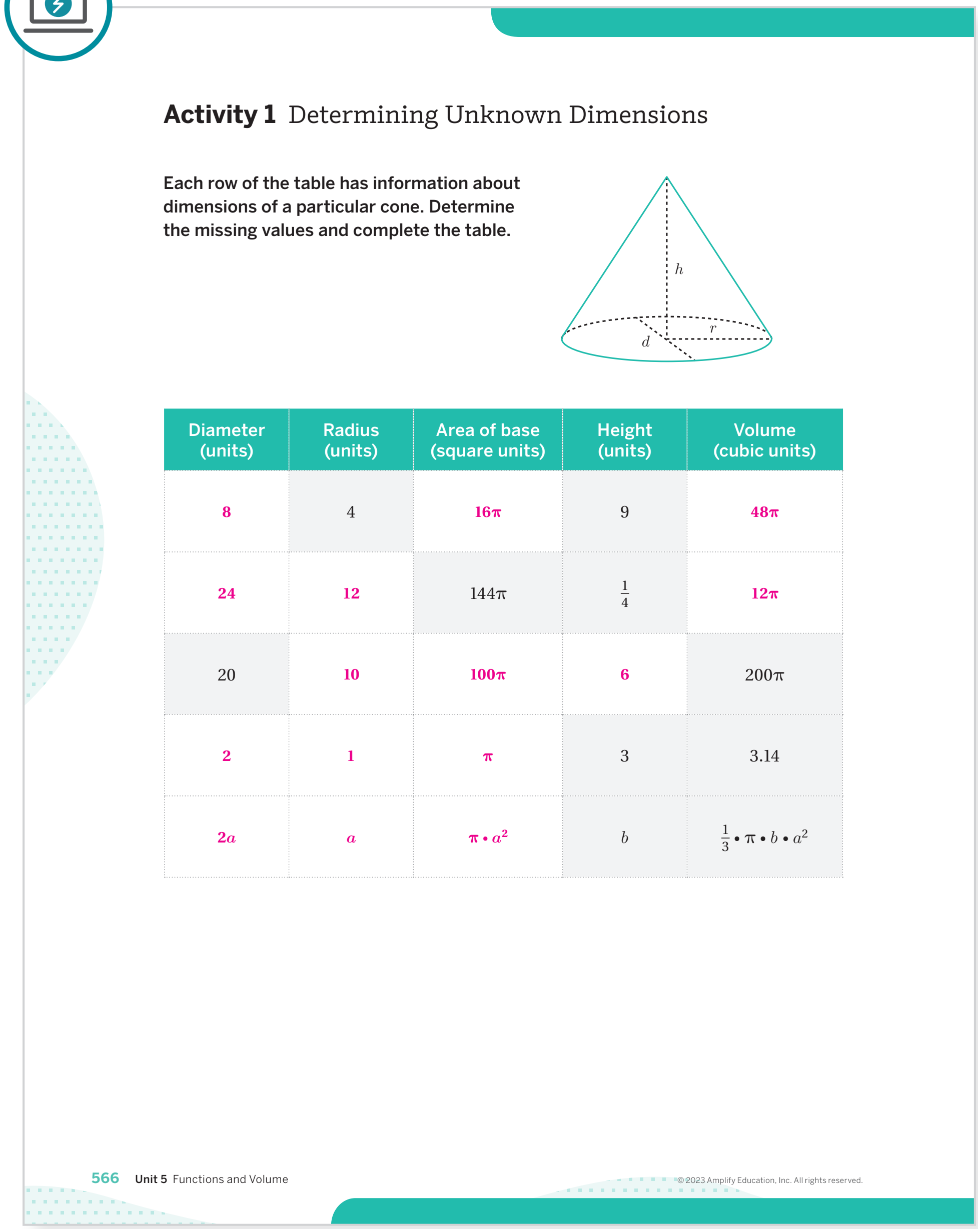
Use: Before Activity 2

Informed by: Performance on Lesson 14, Practice Problem 5

Pairs | 15 min

Activity 1 Determining Unknown Dimensions

Students use the structure of the volume formula for cones to calculate missing dimensions of a cone.



Activity 1 Determining Unknown Dimensions

Each row of the table has information about dimensions of a particular cone. Determine the missing values and complete the table.

Diameter (units)	Radius (units)	Area of base (square units)	Height (units)	Volume (cubic units)
8	4	**16π**	9	**48π**
24	**12**	144π	$\frac{1}{4}$	**12π**
20	**10**	**100π**	**6**	200π
2	**1**	**π**	3	3.14
$2a$	**a**	**$\pi \cdot a^2$**	b	$\frac{1}{3} \cdot \pi \cdot b \cdot a^2$

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1 Launch

Activate students' prior knowledge about strategies they used in Lesson 13 for finding the missing dimensions of a cylinder.

2 Monitor

Help students get started by writing the volume formula for a cone and substituting the values they are given in Row 1.

Look for points of confusion:

- **Having difficulty finding the missing values in a row.** Discuss an order students can use to fill in the row. Ask, "If you know ___, can you find ___? What can you find?"

3 Connect

Display student work showing a correct table.

Have students share their strategies for finding the unknown dimensions. Select a few rows of the table, and ask students how they might find the volume of a cylinder with the same radius and height as the cone.

Ask:

- "Which dimension or measure, in your opinion, was the most challenging to calculate?"
- "If you had to pick two pieces of information given in the table, which information would you choose? Why?"

Highlight that, when working with the volume formula for either a cylinder or cone, if students know two of the three measures for radius, height and volume, they can always calculate the third.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them choose three of the five rows of the table to complete. Allowing them to choose which rows to complete can help support their engagement in the activity.

Extension: *Math Enrichment*

Have students extend the table to one more row and determine the missing dimensions if the diameter is a and the height is b. The radius is $\frac{1}{2}a$, the area of the base is $\pi \cdot \frac{1}{4}a^2$, and the volume is $\frac{1}{12}\pi \cdot a^2 \cdot$ b.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their strategies for completing the table, provide these sentence frames to help them organize their thinking.

- "I noticed ______ in the rows/columns, which made me think that . . ."
- "I noticed ______, and it tells me that . . ."
- "If I am given ______, I can ______ to determine the______."

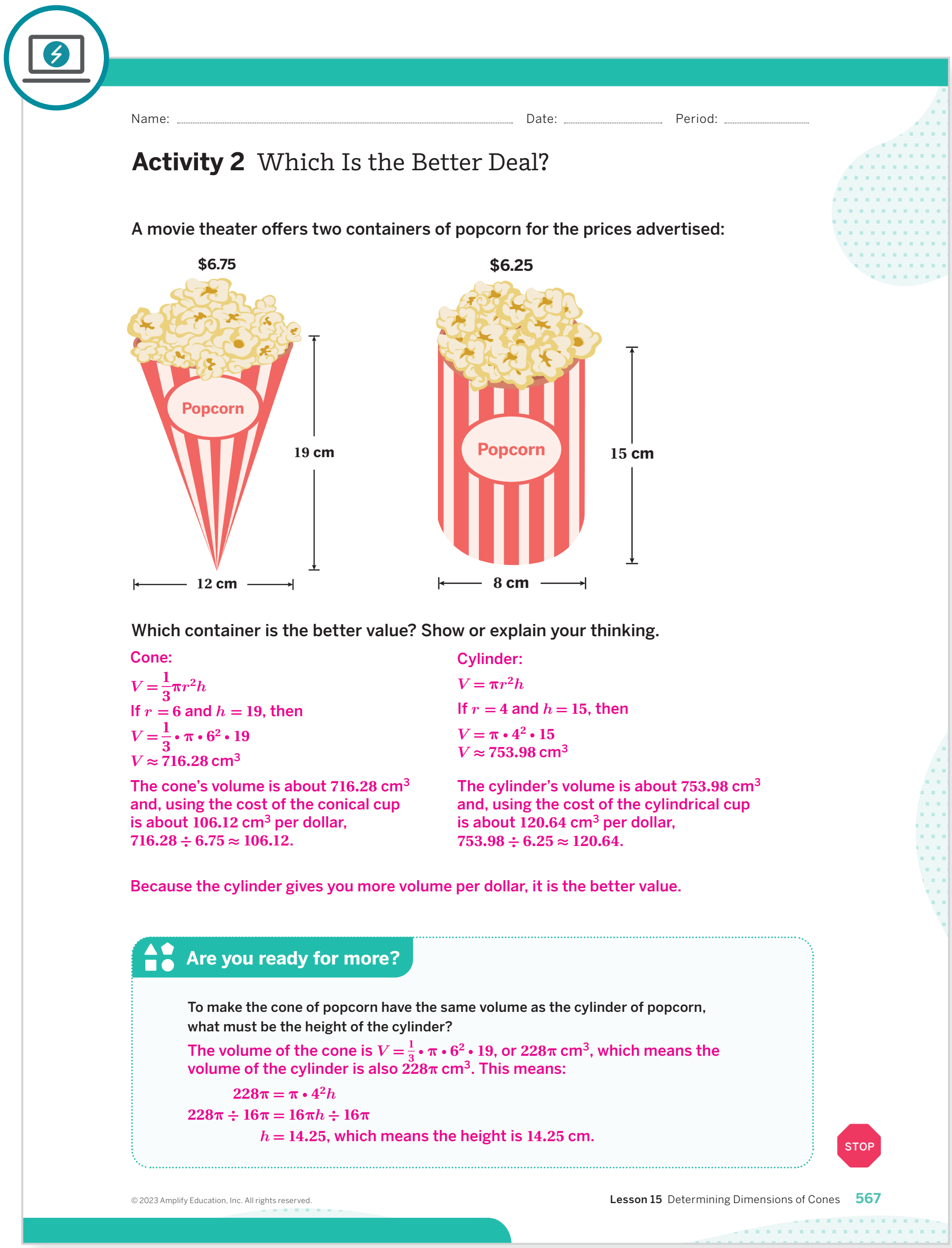
English Learners

Consider drawing cones labeled with the given information and use hand gestures, such as pointing to the dimension of the cone that is being described.

Pairs | 10 min

Activity 2 Which Is the Better Deal?

Students solve a real-world problem relating the volume of a cone to the volume of a cylinder to determine the better buy.



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Activity 2 Which Is the Better Deal?

A movie theater offers two containers of popcorn for the prices advertised:

$6.75

Popcorn

19 cm

12 cm

$6.25

Popcorn

15 cm

8 cm

Which container is the better value? Show or explain your thinking.

Cone:

$V = \frac{1}{3}\pi r^2 h$

If $r = 6$ and $h = 19$, then

$V = \frac{1}{3} \cdot \pi \cdot 6^2 \cdot 19$

$V \approx 716.28 \text{ cm}^3$

The cone's volume is about 716.28 cm^3 and, using the cost of the conical cup is about 106.12 cm^3 per dollar, $716.28 \div 6.75 \approx 106.12$.

Cylinder:

$V = \pi r^2 h$

If $r = 4$ and $h = 15$, then

$V = \pi \cdot 4^2 \cdot 15$

$V \approx 753.98 \text{ cm}^3$

The cylinder's volume is about 753.98 cm^3 and, using the cost of the cylindrical cup is about 120.64 cm^3 per dollar, $753.98 \div 6.25 \approx 120.64$.

Because the cylinder gives you more volume per dollar, it is the better value.

Are you ready for more?

To make the cone of popcorn have the same volume as the cylinder of popcorn, what must be the height of the cylinder?

The volume of the cone is $V = \frac{1}{3} \cdot \pi \cdot 6^2 \cdot 19$, or $228\pi \text{ cm}^3$, which means the volume of the cylinder is also $228\pi \text{ cm}^3$. This means:

$228\pi = \pi \cdot 4^2 h$

$228\pi \div 16\pi = 16\pi h \div 16\pi$

$h = 14.25$, which means the height is 14.25 cm.

STOP

Lesson 15 Determining Dimensions of Cones 567

1 Launch

Activate students' background knowledge by asking if they have ever needed to decide which deal is a better deal. Conduct the ***Think-Pair-Share*** routine to have students decide which popcorn container they would purchase, without doing any calculations. Poll the class and display the results. Distribute calculators for the duration of the activity.

2 Monitor

Help students get started by asking what helps them decide which container is a better deal.

Look for points of confusion:

- **Reasoning about which is a better deal based on the image alone.** Make sure students understand that the image of the two containers is not to scale.
- **Finding only volumes and not taking into account the prices.** Provide students with an example of how something with greater volume may not always be a better deal and ask them what they can do next to find the better deal.

3 Connect

Display the results of the original poll and conduct the ***Poll the Class*** routine to see what the class thinks now.

Have students share their arguments for which container has a better value.

Ask:

- "What steps did you take to find which container was a better deal?"
- "Do you think more people would buy the cone than the cylinder? Why or why not?"

Highlight that the better buy is the container with the cheaper price per cubic centimeter of volume (or the most volume per dollar).

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity. Consider brainstorming a checklist, like the one shown, to help students think about how to approach the problem.

- Determine the volume of each container.
- Determine the unit cost for each container, the cost per cm^3.
- Compare the unit costs to determine which is the better deal.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory text and the two containers of popcorn, without revealing the question posed. Have students work with their partner to write 2–3 mathematical questions that can be answered about the two containers. Ask a few student pairs to share their questions with the class. Highlight questions that compare the volumes of the two containers. Sample questions shown.

- Which container has a greater amount of popcorn?
- Is it worth the extra $0.50 to purchase the container that is a cone?
- These two containers don't have the same radius or height. Can I still compare their volumes?

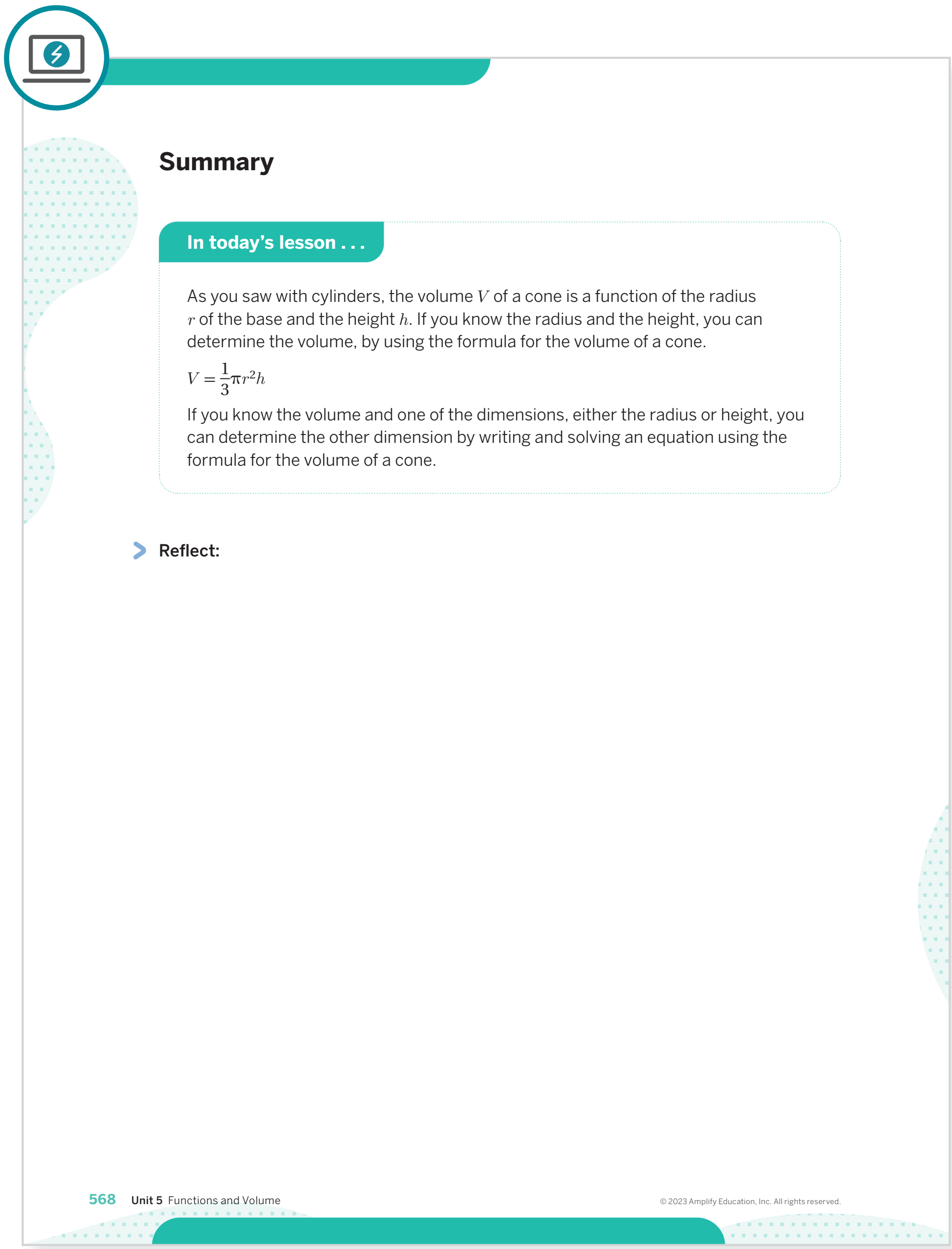
English Learners

Allow students to write their questions in their primary language.

Whole Class | 5 min

Summary

Review and synthesize how to determine unknown dimensions of a cone, given the volume and one dimension of the cone.



Summary

In today's lesson . . .

As you saw with cylinders, the volume V of a cone is a function of the radius r of the base and the height h. If you know the radius and the height, you can determine the volume, by using the formula for the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$

If you know the volume and one of the dimensions, either the radius or height, you can determine the other dimension by writing and solving an equation using the formula for the volume of a cone.

Reflect:

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Synthesize

Ask:

- "Suppose you wanted to determine the height of a cylinder using the volume formula. What other information would you need?"
- "What information would you need to determine which of two containers is a better buy?"

Highlight students' thinking about determining the volume of a cone compared to determining the volume of a cylinder.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What real-world applications can you think of that would use the volume formula of a cone?"

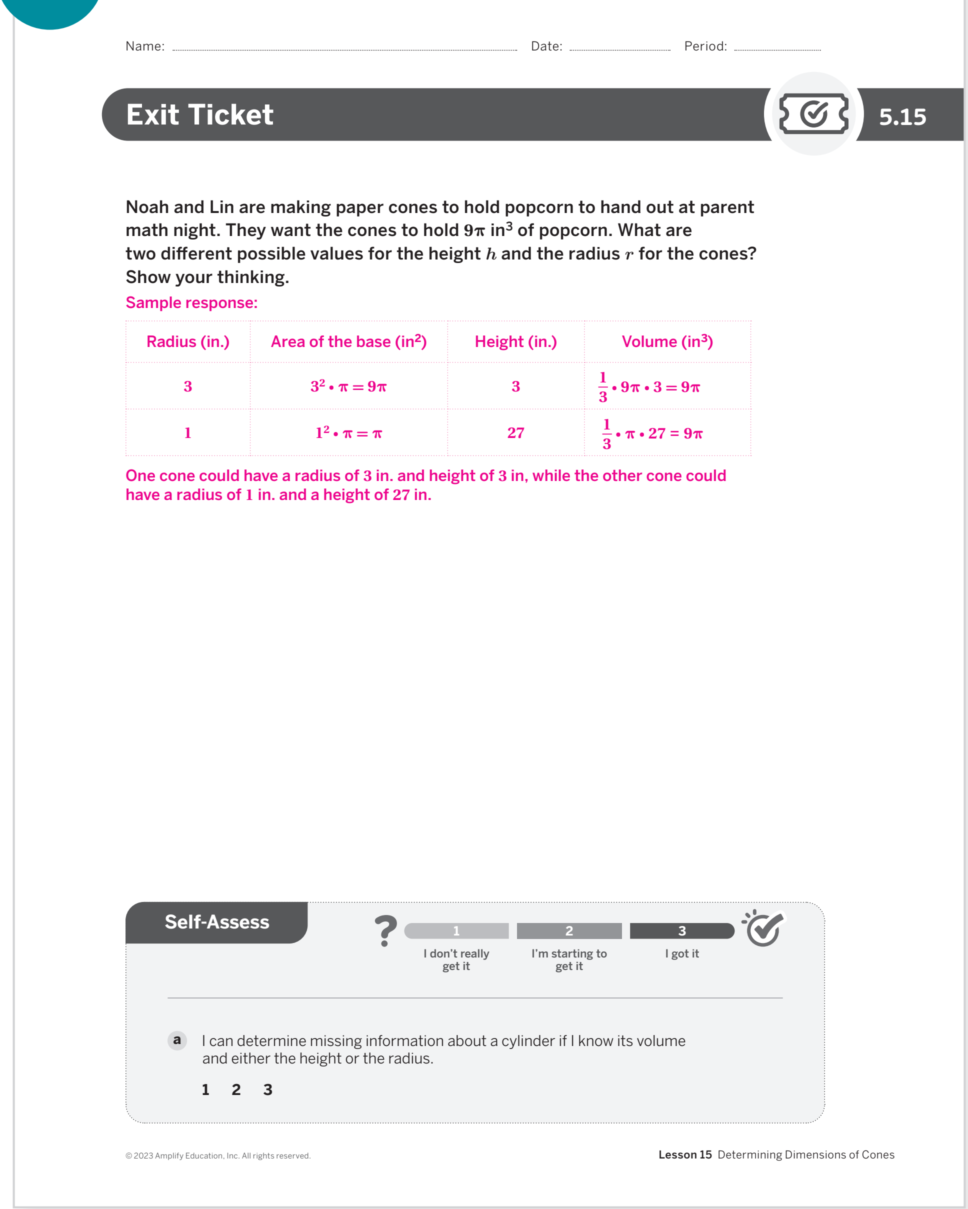
Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining a possible radius and height for a given volume of a cone.

Amps Featured Activity Real-Time Exit Ticket

Printable



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Exit Ticket 5.15

Noah and Lin are making paper cones to hold popcorn to hand out at parent math night. They want the cones to hold 9π in^3 of popcorn. What are two different possible values for the height h and the radius r for the cones? Show your thinking.

Sample response:

Radius (in.)	Area of the base (in^2)	Height (in.)	Volume (in^3)
3	$3^2 \cdot \pi = 9\pi$	3	$\frac{1}{3} \cdot 9\pi \cdot 3 = 9\pi$
1	$1^2 \cdot \pi = \pi$	27	$\frac{1}{3} \cdot \pi \cdot 27 = 9\pi$

One cone could have a radius of 3 in. and height of 3 in, while the other cone could have a radius of 1 in. and a height of 27 in.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine missing information about a cylinder if I know its volume and either the height or the radius.

1 2 3

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Lesson 15 Determining Dimensions of Cones

Success looks like . . .

- **Language Goal:** Calculating the value of one dimension of a cone, and explaining the reasoning. **(Speaking and Listening, Writing)**
- **Language Goal:** Creating a table of dimensions of cones, and describing patterns that arise. **(Speaking and Listening, Writing)**
  - » Creating a table to determine the unknown dimensions of the cone.
- **Language Goal:** Comparing volumes of a cone and cylinder in context, and justifying which volume is a better value for a given price. **(Speaking and Listening, Writing)**

Suggested next steps

If students have difficulty getting started, consider:

- Reviewing Activity 1.
- Having students record the formula of a cone.
- Assigning Practice Problem 3.

If students use the incorrect formula for the volume of a cone, consider:

- Having students reference the Anchor Chart PDF, *Volumes of Circular Solids*.

If students arrive at incorrect values for the radius or height, consider:

- Having students substitute their values into the formula for the volume of a cone to check their solutions.
- Reviewing Activity 1.

Professional Learning

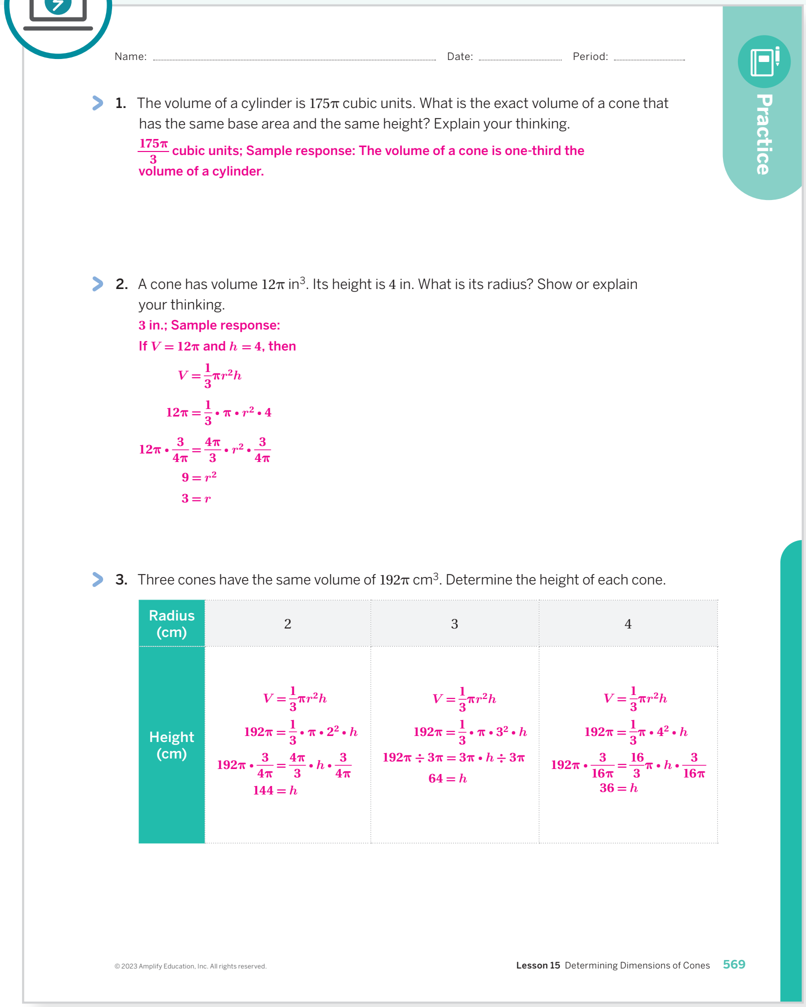
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students determined missing dimensions for a cone. How did that build on the earlier work students did with determining missing dimensions of cylinders?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? Would you change anything the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. The volume of a cylinder is 175π cubic units. What is the exact volume of a cone that has the same base area and the same height? Explain your thinking.

   $\frac{175\pi}{3}$ cubic units; Sample response: The volume of a cone is one-third the volume of a cylinder.

2. A cone has volume 12π in^3. Its height is 4 in. What is its radius? Show or explain your thinking.

   3 in.; Sample response:

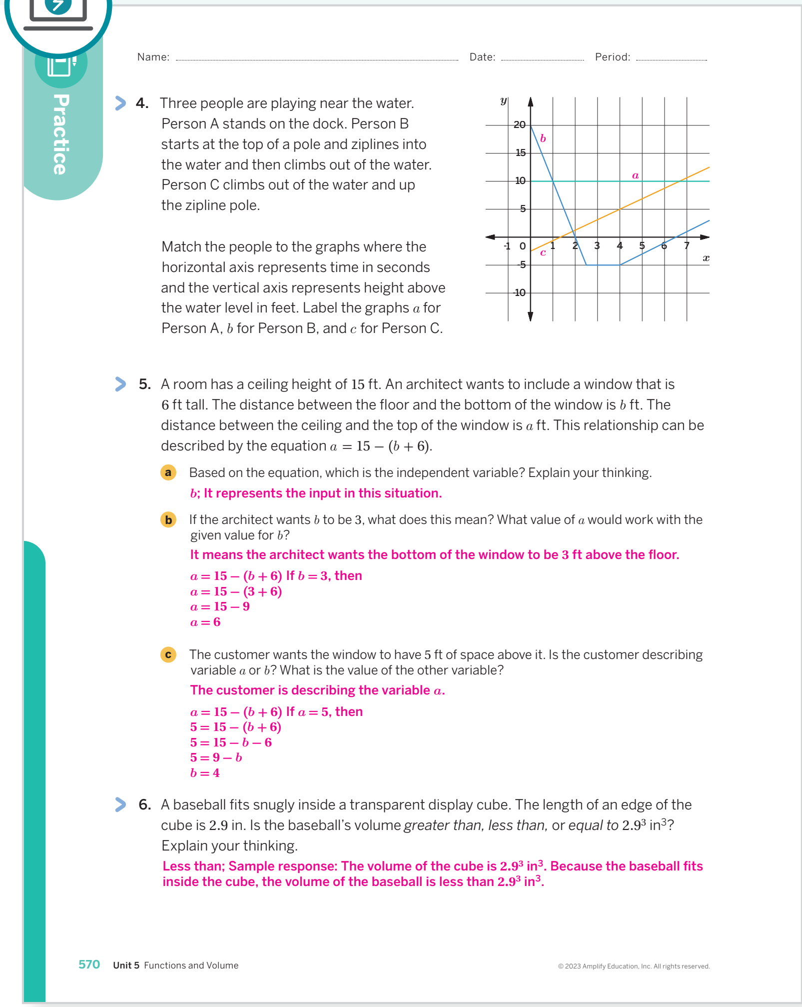
   If $V = 12\pi$ and $h = 4$, then

$$V = \frac{1}{3}\pi r^2 h$$
$$12\pi = \frac{1}{3} \cdot \pi \cdot r^2 \cdot 4$$
$$12\pi \cdot \frac{3}{4\pi} = \frac{4\pi}{3} \cdot r^2 \cdot \frac{3}{4\pi}$$
$$9 = r^2$$
$$3 = r$$

3. Three cones have the same volume of 192π cm^3. Determine the height of each cone.

Radius (cm)	2	3	4
Height (cm)	$V = \frac{1}{3}\pi r^2 h$ $192\pi = \frac{1}{3} \cdot \pi \cdot 2^2 \cdot h$ $192\pi \cdot \frac{3}{4\pi} = \frac{4\pi}{3} \cdot h \cdot \frac{3}{4\pi}$ $144 = h$	$V = \frac{1}{3}\pi r^2 h$ $192\pi = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot h$ $192\pi \div 3\pi = 3\pi \cdot h \div 3\pi$ $64 = h$	$V = \frac{1}{3}\pi r^2 h$ $192\pi = \frac{1}{3}\pi \cdot 4^2 \cdot h$ $192\pi \cdot \frac{3}{16\pi} = \frac{16}{3}\pi \cdot h \cdot \frac{3}{16\pi}$ $36 = h$

Lesson 15 Determining Dimensions of Cones 569



Name: ______ Date: ______ Period: ______

Practice

4. Three people are playing near the water. Person A stands on the dock. Person B starts at the top of a pole and ziplines into the water and then climbs out of the water. Person C climbs out of the water and up the zipline pole.

   Match the people to the graphs where the horizontal axis represents time in seconds and the vertical axis represents height above the water level in feet. Label the graphs a for Person A, b for Person B, and c for Person C.

5. A room has a ceiling height of 15 ft. An architect wants to include a window that is 6 ft tall. The distance between the floor and the bottom of the window is b ft. The distance between the ceiling and the top of the window is a ft. This relationship can be described by the equation $a = 15 - (b + 6)$.

   a. Based on the equation, which is the independent variable? Explain your thinking.

      b; It represents the input in this situation.

   b. If the architect wants b to be 3, what does this mean? What value of a would work with the given value for b?

      It means the architect wants the bottom of the window to be 3 ft above the floor.

      $a = 15 - (b + 6)$ If $b = 3$, then
      $a = 15 - (3 + 6)$
      $a = 15 - 9$
      $a = 6$

   c. The customer wants the window to have 5 ft of space above it. Is the customer describing variable a or b? What is the value of the other variable?

      The customer is describing the variable a.

      $a = 15 - (b + 6)$ If $a = 5$, then
      $5 = 15 - (b + 6)$
      $5 = 15 - b - 6$
      $5 = 9 - b$
      $b = 4$

6. A baseball fits snugly inside a transparent display cube. The length of an edge of the cube is 2.9 in. Is the baseball's volume *greater than, less than,* or *equal to* 2.9^3 in^3? Explain your thinking.

   Less than; Sample response: The volume of the cube is 2.9^3 in^3. Because the baseball fits inside the cube, the volume of the baseball is less than 2.9^3 in^3.

570 **Unit 5** Functions and Volume

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 6	1
	5	Unit 5 Lesson 3	2
Formative	6	Unit 5 Lesson 16	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 16**

Estimating a Hemisphere

Let’s estimate the volume of hemispheres using figures we know.

Focus

Goals

1. **Language Goal:** Estimate the volume of a hemisphere using the formulas for volume of a cone and cylinder, and explain the estimation strategy. **(Speaking and Listening, Writing)**

Coherence

Today

Students estimate the volume of a hemisphere by fitting it inside a cylinder and using the volume of the cylinder to estimate of the volume of the hemisphere. Then they repeat the process with a cone that fits inside the hemisphere. Students discover the volume of the hemisphere has to be between the volume of the cone and the volume of the cylinder, both of which they can calculate from work in previous lessons.

Previously

In Lessons 10–15, students calculated the volumes of cylinders and cones and discovered that, if a cone and cylinder have the same radii and heights, the volume of the cone is a third of the volume of the cylinder.

Coming Soon

In Lesson 17, students will determine the relationship between the volumes of a cylinder, cone, and sphere with the same radius and heights equivalent to the diameter of the sphere.

Rigor

- Students’ **conceptual understanding** of volume is strengthened by their work with upper and lower bounds to estimate the volume of a hemisphere.
- Students **apply** their knowledge about the volume of cylinders and cones to determine a close approximation for the volume of a hemisphere.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Volumes of Circular Solids*
- Anchor Chart PDF, *Volumes of Circular Solids* (answers)
- calculators
- globe (optional)

Math Language Development

New word

- ***hemisphere***

Review words

- *cone*
- *cylinder*
- *height*
- *pi*
- *radius*
- *sphere*
- *volume*

Amps | Featured Activity

Activity 2
Using Work From Previous Slides

Students are shown their estimated from Activity 1 as they refine their estimation of the volume of a hemisphere.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students compare their estimates for the volume of the hemisphere placed snugly inside a cylinder and the volume of the cone placed snugly inside a hemisphere. Remind them to pay attention to what their calculations, estimates, and expressions mean within context. Ask them how their calculations show how the volume of the hemisphere compares to the volume of the cone and cylinder.

Modifications to Pacing

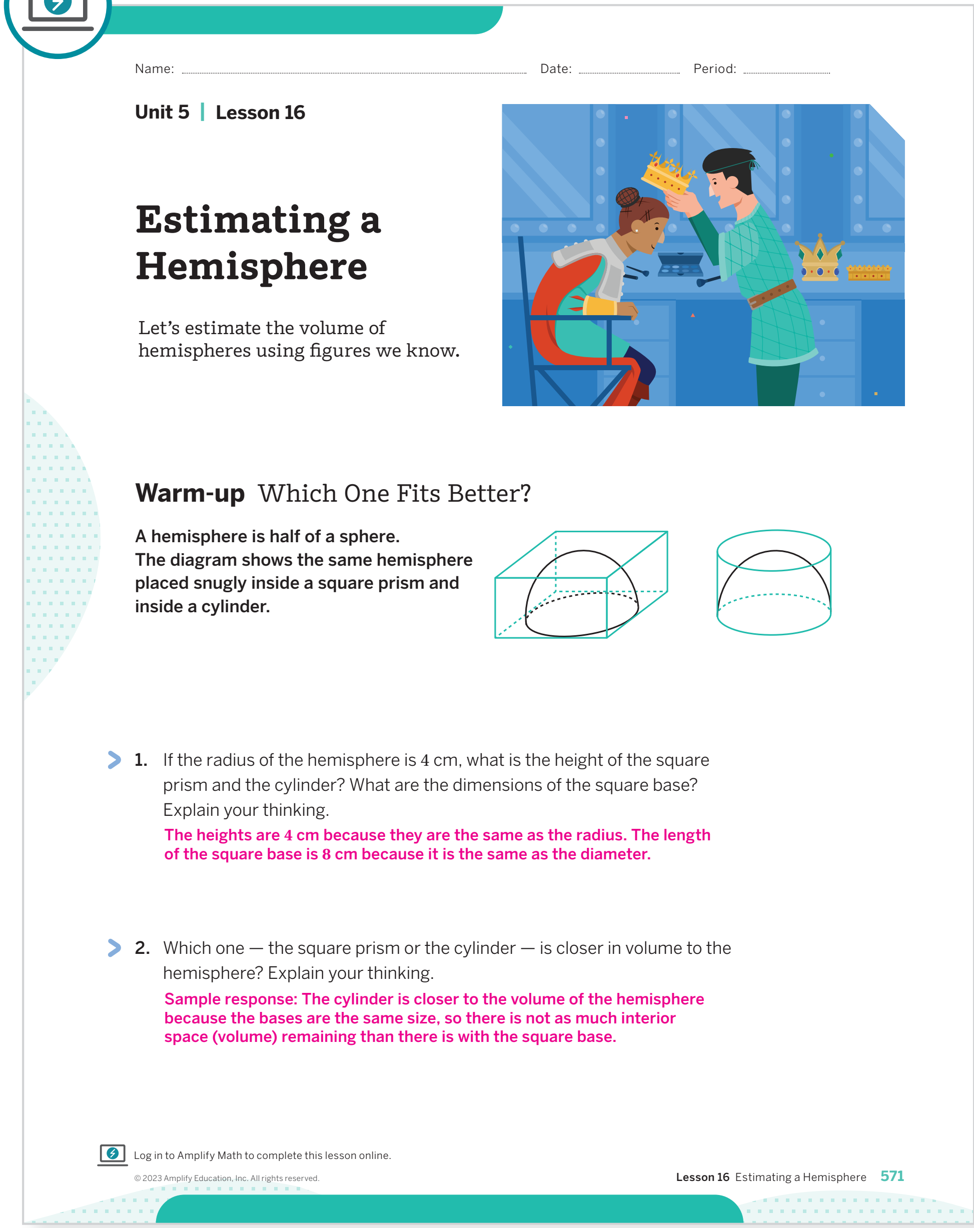
You may want to consider these additional modifications if you are short on time.

- Combine **Activities 1 and 2** into one activity to save time during the whole-class discussion portion.
- **Activity 3** may be omitted.

Independent | 5 min

Warm-up Which One Fits Better?

Students analyze which figure – a rectangular prism or a cylinder – better estimates the volume of a hemisphere.



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Unit 5 | Lesson 16

Estimating a Hemisphere

Let's estimate the volume of hemispheres using figures we know.

Warm-up Which One Fits Better?

A hemisphere is half of a sphere. The diagram shows the same hemisphere placed snugly inside a square prism and inside a cylinder.

1. If the radius of the hemisphere is 4 cm, what is the height of the square prism and the cylinder? What are the dimensions of the square base? Explain your thinking.
   The heights are 4 cm because they are the same as the radius. The length of the square base is 8 cm because it is the same as the diameter.

2. Which one — the square prism or the cylinder — is closer in volume to the hemisphere? Explain your thinking.
   Sample response: The cylinder is closer to the volume of the hemisphere because the bases are the same size, so there is not as much interior space (volume) remaining than there is with the square base.

Log in to Amplify Math to complete this lesson online.

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Lesson 16 Estimating a Hemisphere 571

1 Launch

Display a globe and ask students whether they are familiar with the words *sphere* and *hemisphere*, such as used in *Northern Hemisphere* and *Southern Hemisphere*.

2 Monitor

Help students get started by having them draw the radius of the hemisphere in each image, including the vertical one to show the height.

Look for points of confusion:

- **Thinking that the side length of the square base is 4 cm.** Remind students 4 represents the radius of the circle base of the hemisphere, which comes from the center.

Look for productive strategies:

- Writing the dimensions on the images.

3 Connect

Define a ***hemisphere*** as half of a sphere.

Display the images and have students share their responses and reasoning to Problems 1 and 2.

Highlight that the radius of the hemisphere is the height of the square prism and cylinder and that the diameter is the side length of the square base.

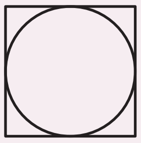
Ask:

- "Are these shapes an overestimate or underestimate of the volume of the hemisphere? How do you know?"
- "What could you do to get an estimate of the volume of the sphere?"

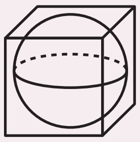
Power-up

To power up students' ability to compare the volume of figures where one figure is contained within the other, have students complete:

1. Which figure has the greater area, the square or the circle?
   Square



2. Which figure has the greater volume, the sphere or the cube?
   Cube



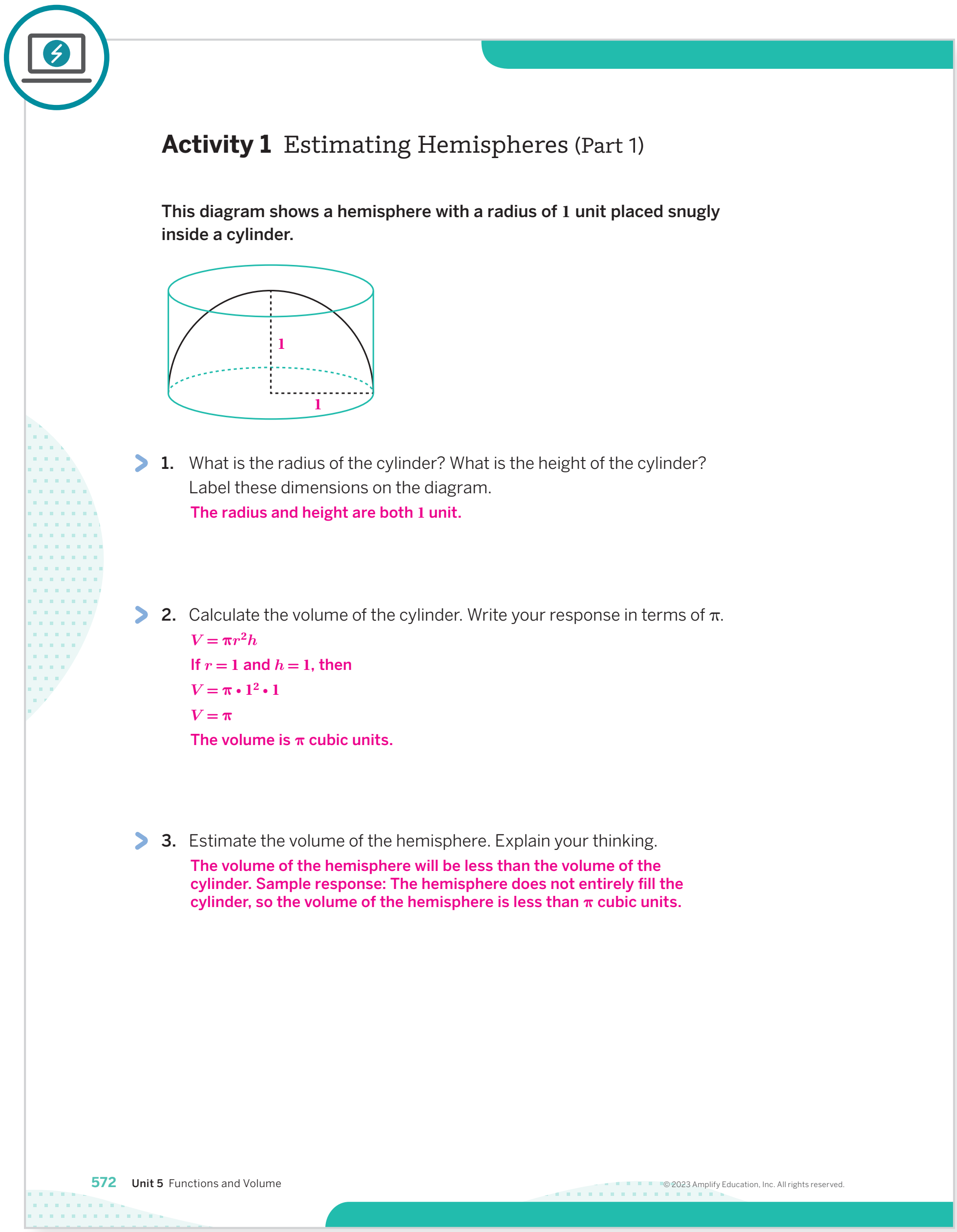
Use: Before the Warm-up

Informed by: Performance on Lesson 15, Practice Problem 6

Pairs | 10 min

Activity 1 Estimating Hemispheres (Part 1)

Students use the volume of a cylinder to estimate the volume of a hemisphere.



Activity 1 Estimating Hemispheres (Part 1)

This diagram shows a hemisphere with a radius of 1 unit placed snugly inside a cylinder.

1. What is the radius of the cylinder? What is the height of the cylinder? Label these dimensions on the diagram.
   The radius and height are both 1 unit.

2. Calculate the volume of the cylinder. Write your response in terms of π.
   $V = \pi r^2 h$
   If $r = 1$ and $h = 1$, then
   $V = \pi \cdot 1^2 \cdot 1$
   $V = \pi$
   The volume is π cubic units.

3. Estimate the volume of the hemisphere. Explain your thinking.
   The volume of the hemisphere will be less than the volume of the cylinder. Sample response: The hemisphere does not entirely fill the cylinder, so the volume of the hemisphere is less than π cubic units.

572 **Unit 5** Functions and Volume

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1 Launch

Let students know that, because they know how to determine the volume of a cylinder, they will use the volume of a cylinder to estimate the volume of the hemisphere.

2 Monitor

Help students get started by asking "What do you need to know to determine the volume of the cylinder?"

Look for points of confusion:

- **Not remembering how to find the volume of a cylinder.** Have students reference the Anchor Chart PDF, *Volumes of Circular Solids.*
- **Not realizing that the radius of the hemisphere determines the height of the cylinder.** Remind students of the Warm-up in which the radius of the hemisphere was also the height of the square prism and cylinder.

Look for productive strategies:

- Making reasonable and specific estimates, such as the volume is less than π but greater than $\frac{1}{2}\pi$.

3 Connect

Display the diagram from the Student Edition.

Have students share their estimates for the volume of the hemisphere. Sequence responses by starting with students who have less than π as an estimate followed by more precise estimates. Ask whether students think $\frac{1}{2}\pi$ would be a good estimate. Consider recording responses on the board to discuss them further during the Connect of Activity 2.

Highlight how to use a cylinder with the radius and height equivalent to the radius of the hemisphere to estimate an upper bound of the volume of the hemisphere. The volume of the cylinder is an overestimate of the volume of the hemisphere.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity.

Extension: *Math Enrichment*

Have students estimate the volume of a hemisphere with a radius of 2 units fitted snugly into the bottom of a cylinder, where the height of the cylinder is 4 units. Sample response: Less than 8π cubic units.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problems 3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions, such as:

- "What would be an unreasonable estimate, one that is too large, for the volume of the hemisphere? Why?"
- "What would be an unreasonable estimate, one that is too small, for the volume of the hemisphere? Why?"
- "How does your response to Problems 1–2 help you respond to Problem 3?"

Have students revise their responses, as needed.

English Learners

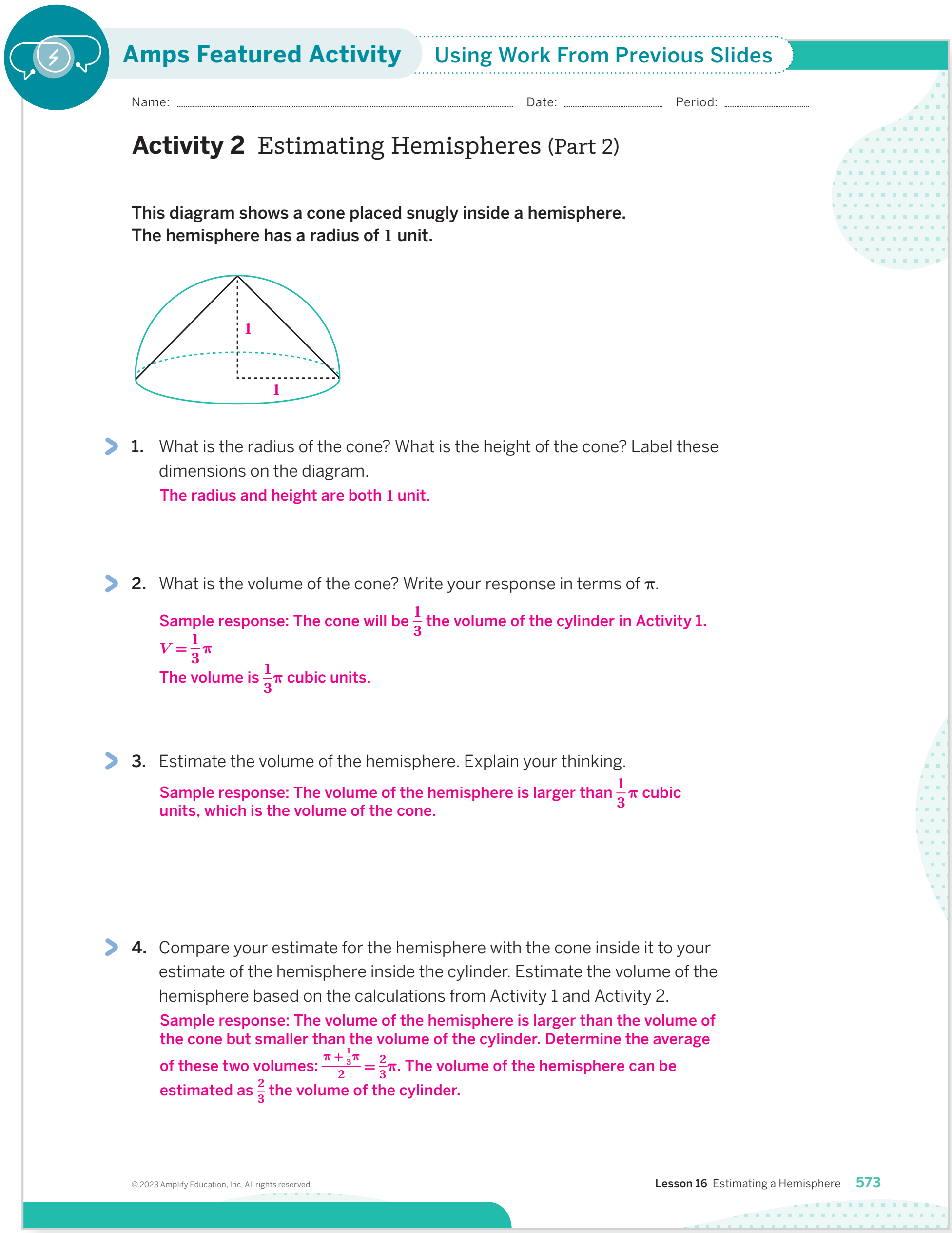
Demonstrate what the phrase *fitted snugly* means in this context by illustrating that the hemisphere touches the walls of the cylinder.

Pairs | 10 min

Activity 2 Estimating Hemispheres (Part 2)

Students use the volume of a cone to estimate the volume of a hemisphere.

Amps Featured Activity Using Work From Previous Slides



Name: ______ Date: ______ Period: ______

Activity 2 Estimating Hemispheres (Part 2)

This diagram shows a cone placed snugly inside a hemisphere.
The hemisphere has a radius of 1 unit.

1. What is the radius of the cone? What is the height of the cone? Label these dimensions on the diagram.
   The radius and height are both 1 unit.

2. What is the volume of the cone? Write your response in terms of π.
   Sample response: The cone will be $\frac{1}{3}$ the volume of the cylinder in Activity 1.
   $V = \frac{1}{3}\pi$
   The volume is $\frac{1}{3}\pi$ cubic units.

3. Estimate the volume of the hemisphere. Explain your thinking.
   Sample response: The volume of the hemisphere is larger than $\frac{1}{3}\pi$ cubic units, which is the volume of the cone.

4. Compare your estimate for the hemisphere with the cone inside it to your estimate of the hemisphere inside the cylinder. Estimate the volume of the hemisphere based on the calculations from Activity 1 and Activity 2.
   Sample response: The volume of the hemisphere is larger than the volume of the cone but smaller than the volume of the cylinder. Determine the average of these two volumes: $\frac{\pi + \frac{1}{3}\pi}{2} = \frac{2}{3}\pi$. The volume of the hemisphere can be estimated as $\frac{2}{3}$ the volume of the cylinder.

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Lesson 16 Estimating a Hemisphere 573

1 Launch

Let students know they will use their familiarity with determining the volume of a cone to estimate a hemisphere's volume.

2 Monitor

Help students get started by asking "What do you need to know to determine the volume of a cone?"

Look for points of confusion:

- **Not remembering how to determine the volume of a cone.** Have students reference the Anchor Chart PDF, *Volume of Circular Solids*.
- **Not realizing that the radius of the hemisphere determines the height of the cone.** Remind students of the Warm-up, where the radius of the hemisphere was also the height of the square prism and the cylinder.

3 Connect

Have students share their estimates for the volume of the hemisphere. Revisit the estimates from Activity 1 and consider drawing a number line indicating $\frac{1}{3}\pi$ and π. Ask students which value would be a reasonable estimate for the volume of the hemisphere.

Highlight how using a cone — with a radius and height equivalent to the radius of a hemisphere — provides a lower-bound estimate, or underestimate, of the volume of the hemisphere.

Ask:

- "What do the volumes of the cone and cylinder tell you about the volume of the hemisphere?"
- "Compare the equations for volume of a cylinder and cone where radius and height are equal. If the volume of the hemisphere has to be between these two, what might an equation for the volume of a hemisphere look like?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problems 3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions, such as:

- "Which figure has the greatest volume? The least volume?"
- "Why does it make sense that the volume of the cone is the smallest of the three figures?"
- "Why must the volume of the hemisphere be greater than $\frac{1}{3}\pi$ cubic units?"

Have students revise their responses, as needed.

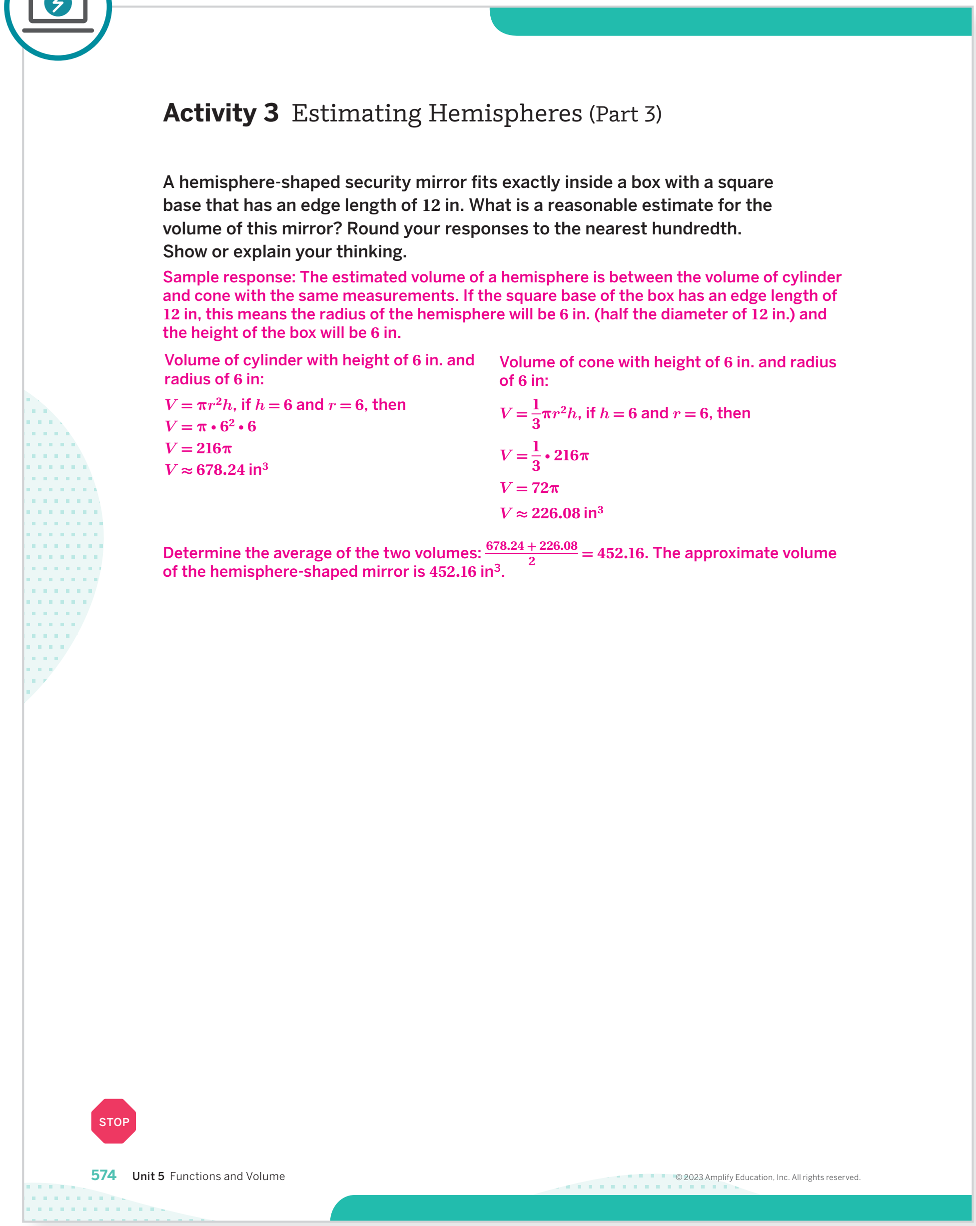
English Learners

Encourage students to use visual diagrams in their responses. Consider drawing a cylinder around the diagram in this activity to reinforce the relative sizes of the volumes of the three figures.

Pairs | 10 min

Activity 3 Estimating Hemispheres (Part 3)

Students use what they learned in Activities 1 and 2 to practice estimating the volume of a hemisphere.



Activity 3 Estimating Hemispheres (Part 3)

A hemisphere-shaped security mirror fits exactly inside a box with a square base that has an edge length of 12 in. What is a reasonable estimate for the volume of this mirror? Round your responses to the nearest hundredth. Show or explain your thinking.

Sample response: The estimated volume of a hemisphere is between the volume of cylinder and cone with the same measurements. If the square base of the box has an edge length of 12 in, this means the radius of the hemisphere will be 6 in. (half the diameter of 12 in.) and the height of the box will be 6 in.

Volume of cylinder with height of 6 in. and radius of 6 in:

$V = \pi r^2 h$, if $h = 6$ and $r = 6$, then

$V = \pi \cdot 6^2 \cdot 6$

$V = 216\pi$

$V \approx 678.24$ in^3

Volume of cone with height of 6 in. and radius of 6 in:

$V = \frac{1}{3}\pi r^2 h$, if $h = 6$ and $r = 6$, then

$V = \frac{1}{3} \cdot 216\pi$

$V = 72\pi$

$V \approx 226.08$ in^3

Determine the average of the two volumes: $\frac{678.24 + 226.08}{2} = 452.16$. The approximate volume of the hemisphere-shaped mirror is 452.16 in^3.

STOP

574 Unit 5 Functions and Volume

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Provide access to calculators.

2 Monitor

Help students get started by having them label the image from the Warm-up to show the dimensions of the box.

Look for points of confusion:

- **Thinking that the radius is 12 in.** Remind students this is the edge length of the square.

Look for productive strategies:

- Drawing a picture to help show their thinking.

3 Connect

Display the image of a hemisphere in a prism from the Warm-up and mark known measurements.

Have students share their responses and reasoning.

Highlight students' methods for determining the upper and lower bounds of the volume of the hemisphere by determining the volumes of a cylinder and a cone with radii and heights of 6 in.

Ask:

- "How can you use the volumes of cylinders and cones to estimate the volume of a hemisphere?"
- "How do you think you can estimate the volume of a sphere?"

Differentiated Support

Accessibility: *Active Prior Knowledge*

Remind students they explored how the volume of a hemisphere can be estimated by using the volume of a cone and a cylinder with the same radius, and where the height is equal to the radius. Display the following sentence frames and ask students to complete them.

- The estimated volume of a hemisphere is between the volume of a _____ and the volume of a _____.
- The estimated volume of a hemisphere is greater than the volume of a _____ and less than the volume of a _____.

Accessibility: *Guide Processing and Visualization*

Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity.

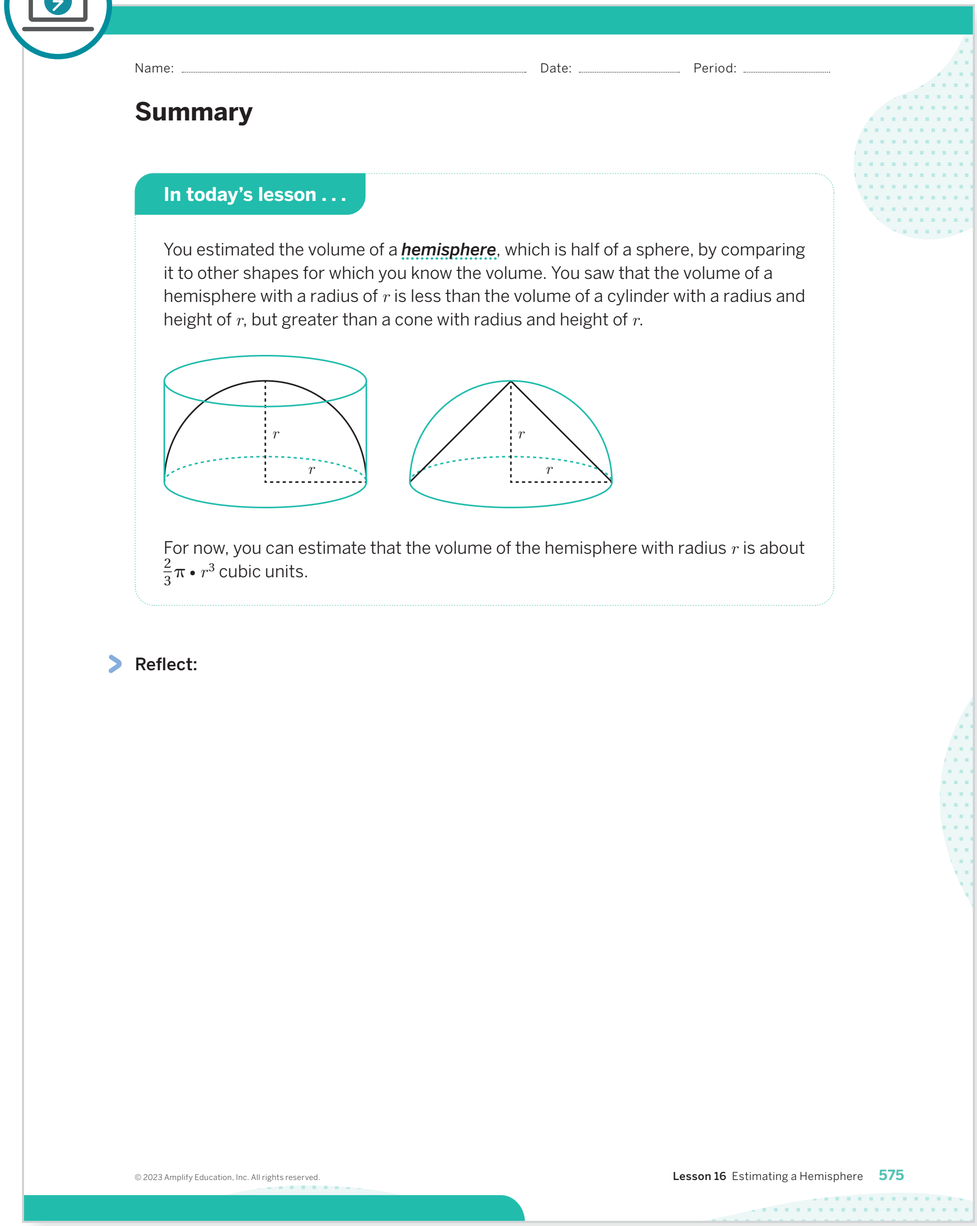
Extension: *Math Enrichment*

Ask students to estimate the volume of the security mirror if it was in the shape of a whole sphere with the same radius. Sample response: About 904.32 in^3.

Whole Class | 5 min

Summary

Review and synthesize how to estimate the volume of a hemisphere by comparing hemispheres to other shapes, such as cylinders and cones.



Name: ____________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You estimated the volume of a ***hemisphere***, which is half of a sphere, by comparing it to other shapes for which you know the volume. You saw that the volume of a hemisphere with a radius of r is less than the volume of a cylinder with a radius and height of r, but greater than a cone with radius and height of r.

For now, you can estimate that the volume of the hemisphere with radius r is about $\frac{2}{3}\pi \cdot r^3$ cubic units.

Reflect:

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Lesson 16 Estimating a Hemisphere 575

Synthesize

Formalize vocabulary: ***hemisphere***

Ask:

- "How did you use a cylinder to estimate a volume of a hemisphere that is an overestimate?"
- "How did you use a cone to estimate a volume of a hemisphere that is an underestimate?"
- "How did you get a closer estimate for the volume of a hemisphere?"
- "How can you use today's work to estimate the volume of a sphere?"

Highlight that using figures for which they know how to determine the volume of (a cone and cylinder) can help students estimate the volume of a figure for which they do not know how to determine (a sphere). In the next lesson, they will learn how to determine the volume of a sphere and see how close their reasoning in this lesson was to the actual calculation.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you use the volume of a cylinder and cone to estimate the volume of a hemisphere?"

Math Language Development

MLR2: Collect and Display

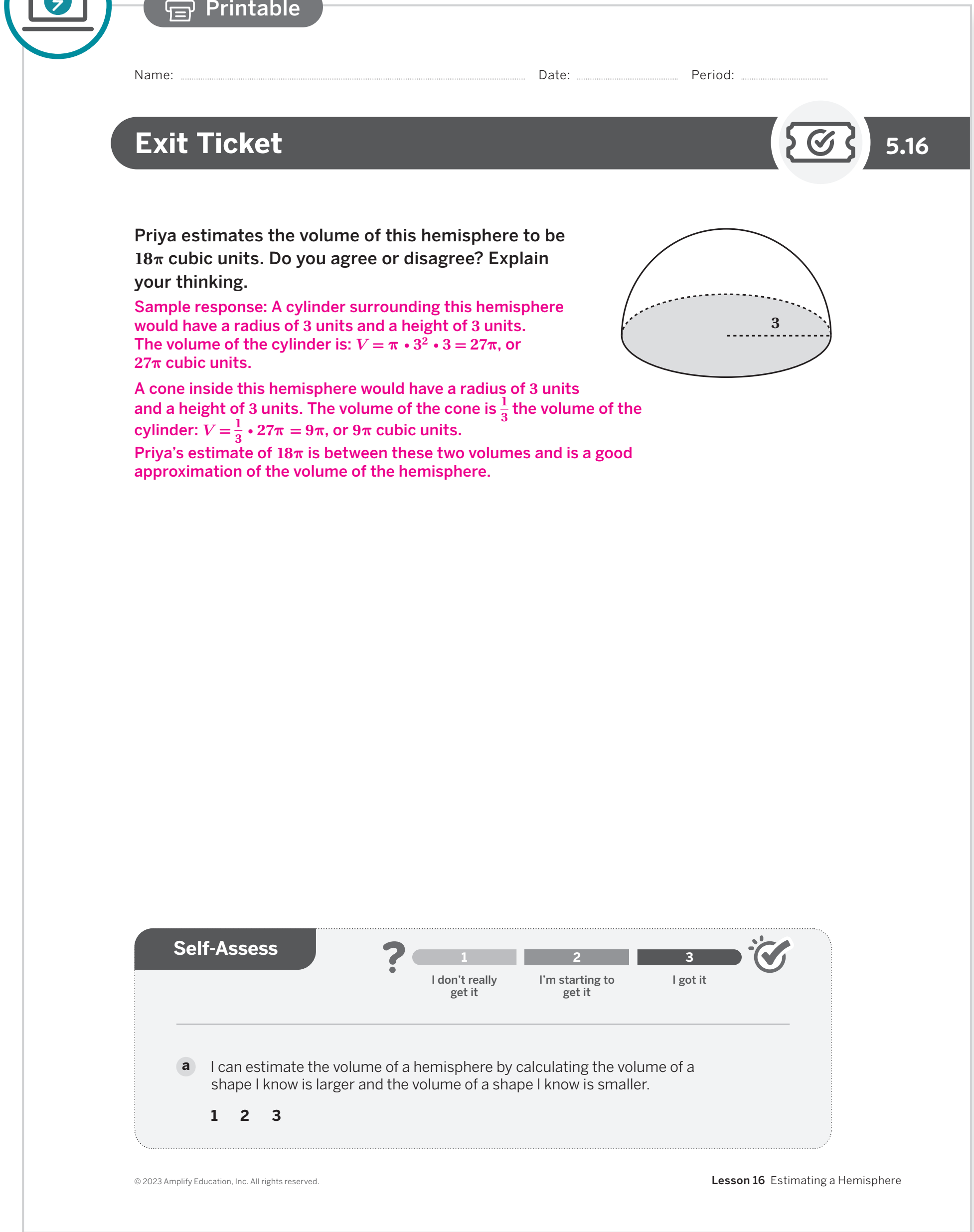
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *hemisphere* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by estimating the volume of a hemisphere.

Printable



Name: Date: Period:

Exit Ticket 5.16

Priya estimates the volume of this hemisphere to be 18π cubic units. Do you agree or disagree? Explain your thinking.

Sample response: A cylinder surrounding this hemisphere would have a radius of 3 units and a height of 3 units. The volume of the cylinder is: $V = \pi \cdot 3^2 \cdot 3 = 27\pi$, or 27π cubic units.

A cone inside this hemisphere would have a radius of 3 units and a height of 3 units. The volume of the cone is $\frac{1}{3}$ the volume of the cylinder: $V = \frac{1}{3} \cdot 27\pi = 9\pi$, or 9π cubic units.
Priya's estimate of 18π is between these two volumes and is a good approximation of the volume of the hemisphere.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can estimate the volume of a hemisphere by calculating the volume of a shape I know is larger and the volume of a shape I know is smaller.

1 2 3

Success looks like . . .

- **Language Goal:** Estimating the volume of a hemisphere using the formulas for volume of a cone and cylinder, and explaining the estimation strategy. **(Speaking and Listening, Writing)**
  - » Explaining whether Priya's estimate of the volume is reasonable.

Suggested next steps

If students do not mention the volume of the cylinder or the cone with radius of 3 and height of 3, consider:

- Reviewing Activities 1 and 2.
- Assigning Practice Problem 2.

Professional Learning

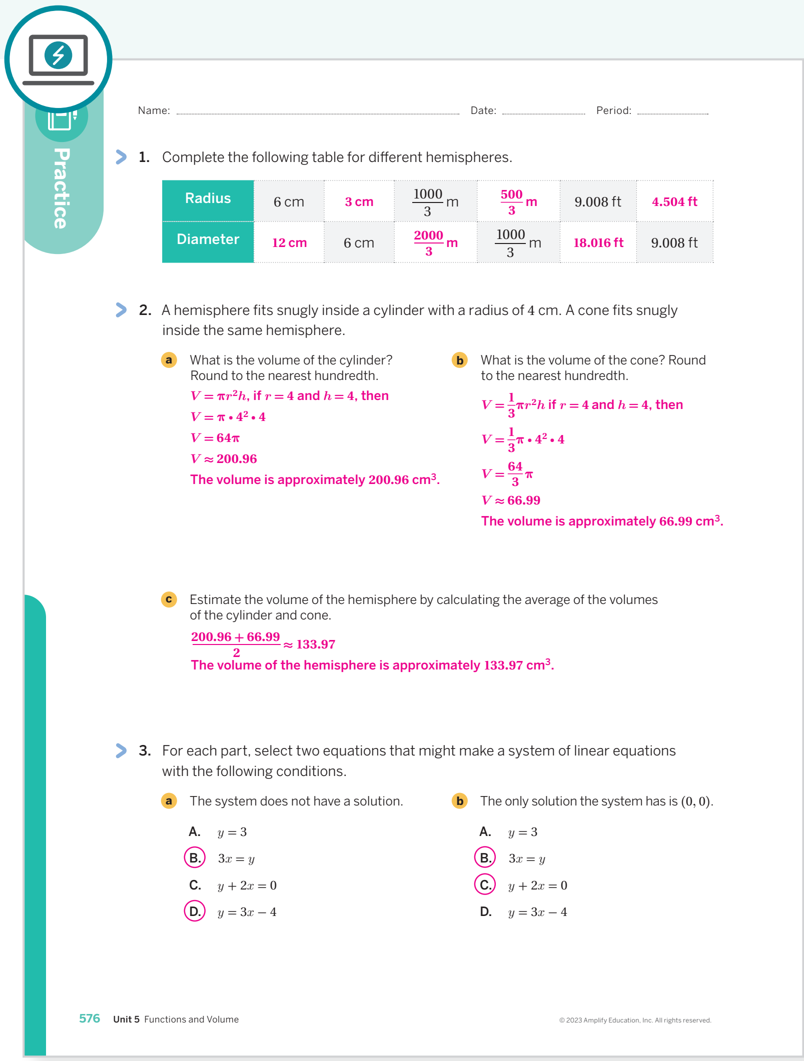
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students estimated the volume of a hemisphere using a cone and cylinder. How will that support the connection between the volumes of all three circular solids?
- How did using the volume of a cylinder and cone set students up to develop the formula for the volume of a sphere? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Complete the following table for different hemispheres.

Radius	6 cm	3 cm	$\frac{1000}{3}$ m	$\frac{500}{3}$ m	9.008 ft	4.504 ft
Diameter	12 cm	6 cm	$\frac{2000}{3}$ m	$\frac{1000}{3}$ m	18.016 ft	9.008 ft

2. A hemisphere fits snugly inside a cylinder with a radius of 4 cm. A cone fits snugly inside the same hemisphere.

a What is the volume of the cylinder? Round to the nearest hundredth.

$V = \pi r^2 h$, if $r = 4$ and $h = 4$, then

$V = \pi \cdot 4^2 \cdot 4$

$V = 64\pi$

$V \approx 200.96$

The volume is approximately 200.96 cm^3.

b What is the volume of the cone? Round to the nearest hundredth.

$V = \frac{1}{3}\pi r^2 h$ if $r = 4$ and $h = 4$, then

$V = \frac{1}{3}\pi \cdot 4^2 \cdot 4$

$V = \frac{64}{3}\pi$

$V \approx 66.99$

The volume is approximately 66.99 cm^3.

c Estimate the volume of the hemisphere by calculating the average of the volumes of the cylinder and cone.

$\frac{200.96 + 66.99}{2} \approx 133.97$

The volume of the hemisphere is approximately 133.97 cm^3.

3. For each part, select two equations that might make a system of linear equations with the following conditions.

a The system does not have a solution.

A. $y = 3$

[✓ B.] $3x = y$

C. $y + 2x = 0$

[✓ D.] $y = 3x - 4$

b The only solution the system has is $(0, 0)$.

A. $y = 3$

[✓ B.] $3x = y$

[✓ C.] $y + 2x = 0$

D. $y = 3x - 4$

576 **Unit 5** Functions and Volume

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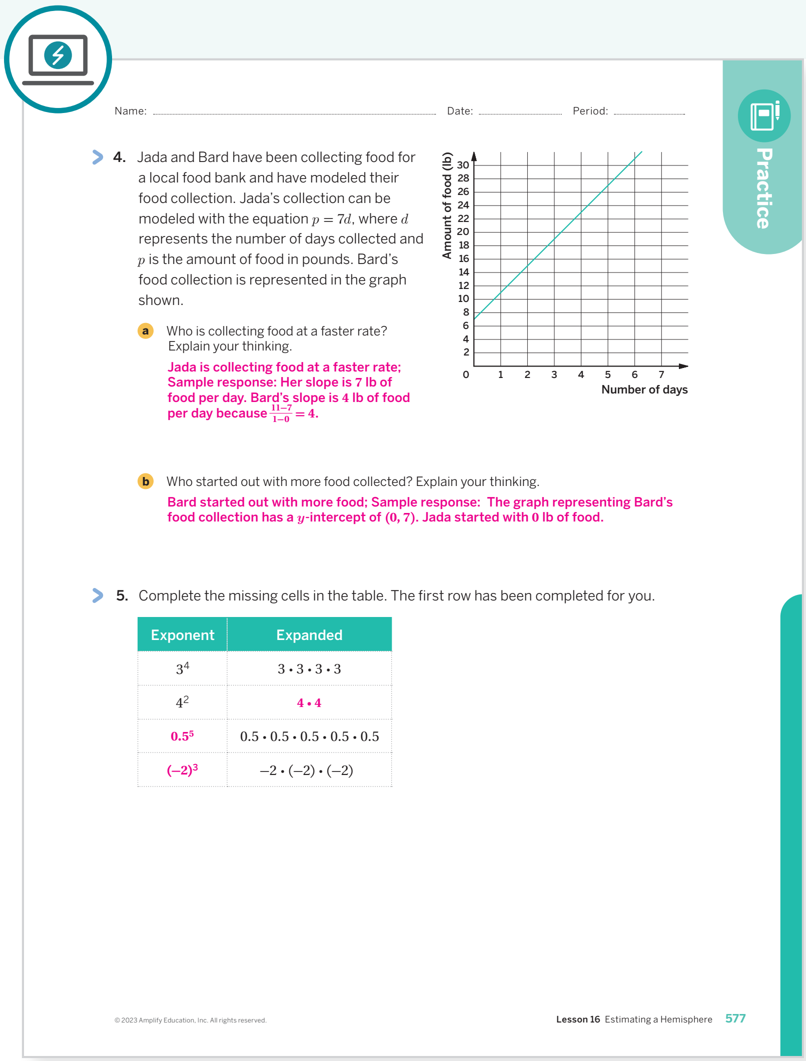

Name: Date: Period:

4. Jada and Bard have been collecting food for a local food bank and have modeled their food collection. Jada's collection can be modeled with the equation $p = 7d$, where d represents the number of days collected and p is the amount of food in pounds. Bard's food collection is represented in the graph shown.

a Who is collecting food at a faster rate? Explain your thinking.

Jada is collecting food at a faster rate; Sample response: Her slope is 7 lb of food per day. Bard's slope is 4 lb of food per day because $\frac{11-7}{1-0} = 4$.

b Who started out with more food collected? Explain your thinking.

Bard started out with more food; Sample response: The graph representing Bard's food collection has a y-intercept of $(0, 7)$. Jada started with 0 lb of food.

5. Complete the missing cells in the table. The first row has been completed for you.

Exponent	Expanded
3^4	$3 \cdot 3 \cdot 3 \cdot 3$
4^2	$4 \cdot 4$
0.5^5	$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5$
$(-2)^3$	$-2 \cdot (-2) \cdot (-2)$

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Lesson 16 Estimating a Hemisphere 577

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activities 1 and 2	2
Spiral	3	Unit 4 Lesson 14	2
	4	Unit 5 Lesson 8	2
Formative ⚡	5	Unit 5 Lesson 17	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

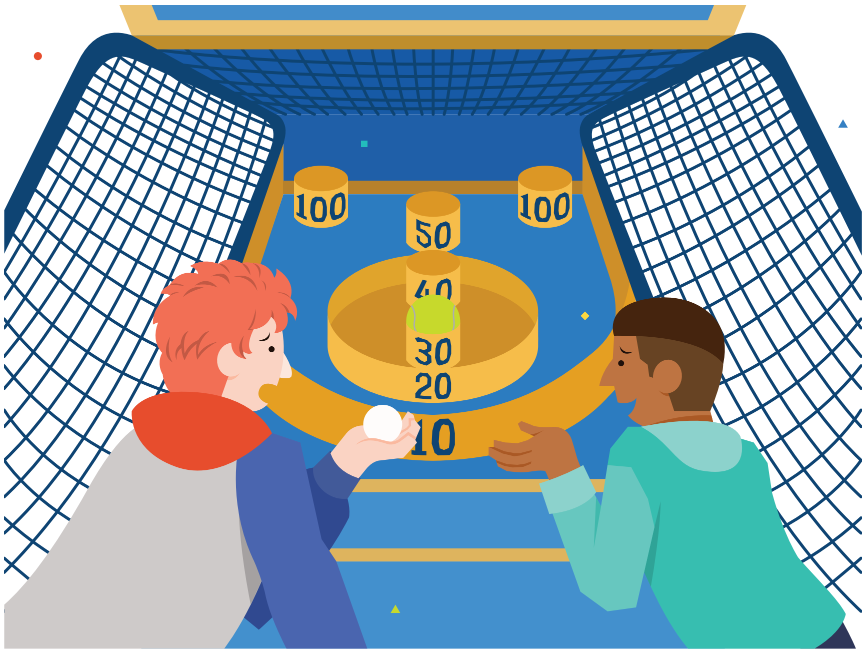
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 17**

The Volume of a Sphere

Let's explore spheres and their volumes.



Focus

Goals

1. **Language Goal:** Calculate the volume of a sphere, cylinder, and cone, which have a radius of r and height (diameter) of $2r$, and explain the relationship between their volumes. **(Speaking and Listening)**
2. **Language Goal:** Create an equation to represent the volume of a sphere as a function of its radius, and explain the reasoning. **(Speaking and Listening, Writing)**

Coherence

Today

Students inspect a sphere that snugly fits inside a cylinder and watch the contents of a cone being poured into the cylinder. The figures each have the same radius, and the heights of the cylinder and cone are equal to the diameter of the sphere. The demonstration shows that for these figures, the cylinder contains the volumes of the sphere and cone together. From this observation, the volume of a specific sphere is computed. Then, the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere is derived.

Previously

In Lesson 16, students estimated the volume of a hemisphere by estimating the upper and lower bounds from the volumes of a cylinder and cone with heights the same as the radius.

Coming Soon

Students will determine the volumes of cylinders, cones, and spheres where the radii are the same and the heights are equivalent to the diameter of the sphere.

Rigor

- Students build **conceptual understanding** of the relationship between the volumes of a cylinder, cone, and sphere with the same radii and in which the heights of the cone and cylinder are equivalent to the diameter of the sphere.
- Students develop a **conceptual understanding** of how the formula for the volume of a sphere can be derived.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Volumes of Circular Solids*
- Anchor Chart PDF, *Volumes of Circular Solids* (answers)
- calculators
- three-dimensional models of a cylinder, cone, and sphere (optional)

Math Language Development

Review words

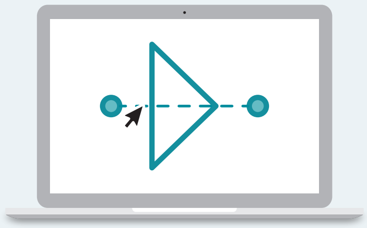
- *cone*
- *cylinder*
- *height*
- *hemisphere*
- *pi*
- *radius*
- *sphere*
- *volume*

Amps | Featured Activity

Warm-up

Digital Demonstration

Using water, students see a demonstration of how the volume of a cone is related to the volume of a cylinder and a sphere with similar dimensions.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might be stressed by feeling unable to compare the different figures. Remind students to pause and consider steps they can take to make this comparison. They need to reason how the meanings of the symbols and the formulas are similar. By probing the meaning of each symbol, students will better be able to understand the relationships.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

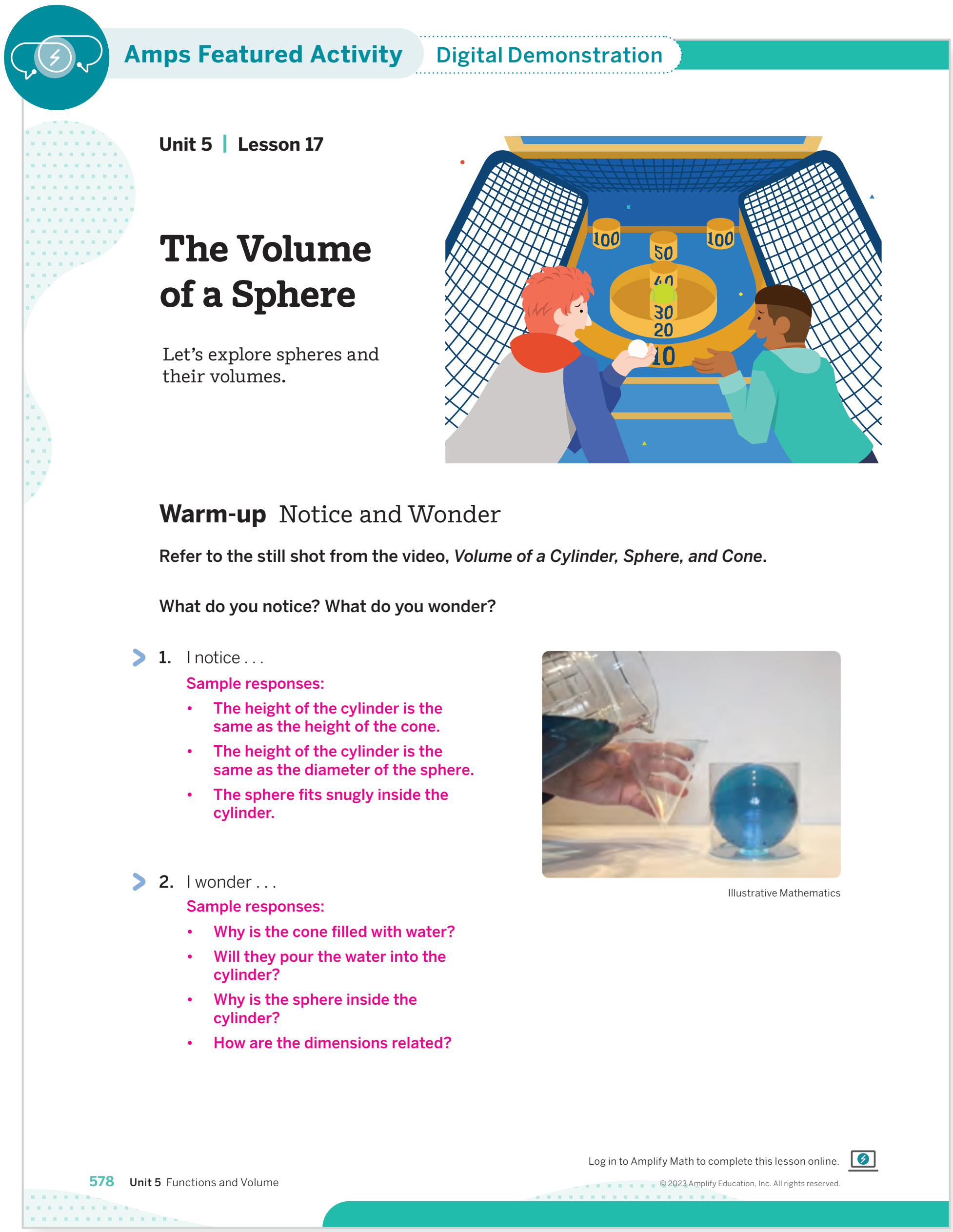
- The **Warm-up** may be omitted, but begin **Activity 1** by showing the video, *The Volume of a Cylinder, Sphere, and Cone*, from the Warm-up Amps slide.
- Complete **Activity 1** in pairs and **Activity 2** as a whole class, or omit **Activity 1** and complete **Activity 2**.

Pairs | 5 min

Warm-up Notice and Wonder

Students analyze three circular solids with the same dimensions to predict how the volumes are related.

Amps Featured Activity Digital Demonstration



Unit 5 | Lesson 17

The Volume of a Sphere

Let's explore spheres and their volumes.

Warm-up Notice and Wonder

Refer to the still shot from the video, *Volume of a Cylinder, Sphere, and Cone*.

What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - The height of the cylinder is the same as the height of the cone.
   - The height of the cylinder is the same as the diameter of the sphere.
   - The sphere fits snugly inside the cylinder.

2. I wonder . . .
   Sample responses:
   - Why is the cone filled with water?
   - Will they pour the water into the cylinder?
   - Why is the sphere inside the cylinder?
   - How are the dimensions related?

Illustrative Mathematics

Log in to Amplify Math to complete this lesson online.

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1 Launch

Display the image from the Student Edition which is the first shot of the video, *The Volume of a Cylinder, Sphere, and Cone*, from the Warm-up Amps slide. Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by activating prior knowledge and asking what they see and what they are curious about.

Look for points of confusion:

- **Not using precise language and saying the solids are the same size**. Encourage students to use precise language, such as *volume, length, width, radius*, etc.

Look for productive strategies:

- Remembering a cone is one third of the volume of a cylinder with matching radius and height.

3 Connect

Have students share what they notice and wonder.

Ask, "In the previous lesson you thought about hemispheres in cylinders. Here is a sphere in a cylinder. Which is bigger, the volume of the cylinder or the volume of the sphere? Do you think the bigger one is twice as big, more than twice as big, or less than twice as big?"

Highlight that, in the next activity, they will see how the volumes of a cylinder, cone, and sphere are related to each other.

Display the rest of the video, *The Volume of a Cylinder, Sphere, and Cone*, from the Warm-up Amps slide. Ask students, "Does this give you any answers to the list of wonders?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this Warm-up, in which they can watch an animation of a sphere inside a cylinder with a cone full of water to begin the discussion of how the volumes are related if the radii are the same and the heights are equivalent to the diameter of the sphere.

Power-up

To power up students' ability to expand and condense expressions using exponents, have students complete:

1. Rewrite each expression using an exponent.

a $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

b $5 \cdot 5 \cdot 5 = 5^3$

2. Expand each expression.

a $4^6 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

b $8^7 = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

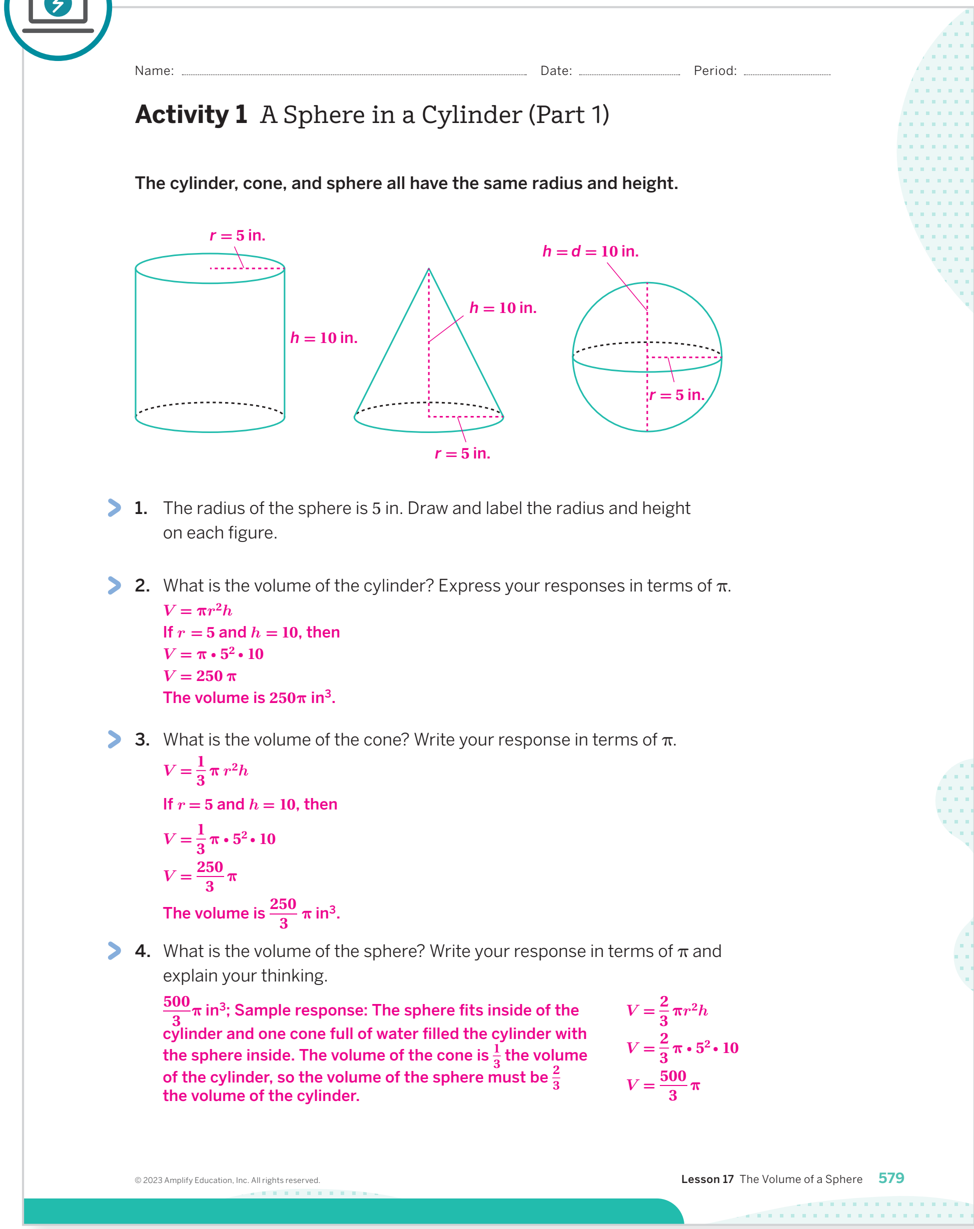
Use: Before the Warm-up

Informed by: Performance on Lesson 16, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 3

Pairs | 10 min

Activity 1 A Sphere in a Cylinder (Part 1)

Students use the connection between the volumes of circular solids with the same dimensions to determine the volume of a sphere.



Name: ________ Date: ________ Period: ________

Activity 1 A Sphere in a Cylinder (Part 1)

The cylinder, cone, and sphere all have the same radius and height.

1. The radius of the sphere is 5 in. Draw and label the radius and height on each figure.

2. What is the volume of the cylinder? Express your responses in terms of π.
   $V = \pi r^2 h$
   If $r = 5$ and $h = 10$, then
   $V = \pi \cdot 5^2 \cdot 10$
   $V = 250\,\pi$
   The volume is 250π in^3.

3. What is the volume of the cone? Write your response in terms of π.
   $V = \frac{1}{3}\pi r^2 h$
   If $r = 5$ and $h = 10$, then
   $V = \frac{1}{3}\pi \cdot 5^2 \cdot 10$
   $V = \frac{250}{3}\pi$
   The volume is $\frac{250}{3}\pi$ in^3.

4. What is the volume of the sphere? Write your response in terms of π and explain your thinking.
   $\frac{500}{3}\pi$ in^3; Sample response: The sphere fits inside of the cylinder and one cone full of water filled the cylinder with the sphere inside. The volume of the cone is $\frac{1}{3}$ the volume of the cylinder, so the volume of the sphere must be $\frac{2}{3}$ the volume of the cylinder.
   $V = \frac{2}{3}\pi r^2 h$
   $V = \frac{2}{3}\pi \cdot 5^2 \cdot 10$
   $V = \frac{500}{3}\pi$

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Lesson 17 The Volume of a Sphere 579

① Launch

If needed, show the video, *The Volume of a Cylinder, Sphere, and Cone*, from the Warm-up again and ask students to think about how they might calculate the volume of the sphere if they know the radius. Provide access to calculators for the duration of the lesson.

② Monitor

Help students get started by activating prior knowledge of the relationship between the volumes of a cone and cylinder with the same radius and height.

Look for points of confusion:

- **Not knowing the heights of the cylinder and cone.** Have students start with finding the diameter of the sphere and using this to help determine the heights of the other figures.

Look for productive strategies:

- Subtracting the volume of the cone from the volume of the cylinder.
- Making the connection that a cone is $\frac{1}{3}$ of the cylinder, so the sphere must be the $\frac{2}{3}$ that fills up the rest of the cylinder.

③ Connect

Have students share their volumes and reasoning and ask, "What does the expression $\pi \cdot 5^2 \cdot 10 - \frac{1}{3}\pi \cdot 5^2 \cdot 10$ represent?"

Highlight any connections the students make between the volumes and remind students that the diameter of the sphere is the height of the cone and cylinder. If a student has not mentioned it, discuss how the sum of the volume of the sphere and the volume of the cone is equivalent to the volume of the cylinder. **Note:** If students do not make the connection that the sphere's volume is $\frac{2}{3}$ the volume of the cylinder, they will have another chance to look at the relationship in the next activity.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity. Consider providing the figures with their dimensions pre-labeled and have students begin the activity with Problem 2.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the connections between the volumes of the figures. Consider displaying the following and add it to the class display.

The volume of a sphere plus the volume of a cone equals the volume of a cylinder, where the figures have the same radius and height (diameter).

$$\underbrace{\frac{2}{3} \cdot \text{Cylinder}}_{\text{Sphere}} + \underbrace{\frac{1}{3} \cdot \text{Cylinder}}_{\text{Cone}} = \underbrace{\text{Cylinder}}_{\text{Cylinder}}$$

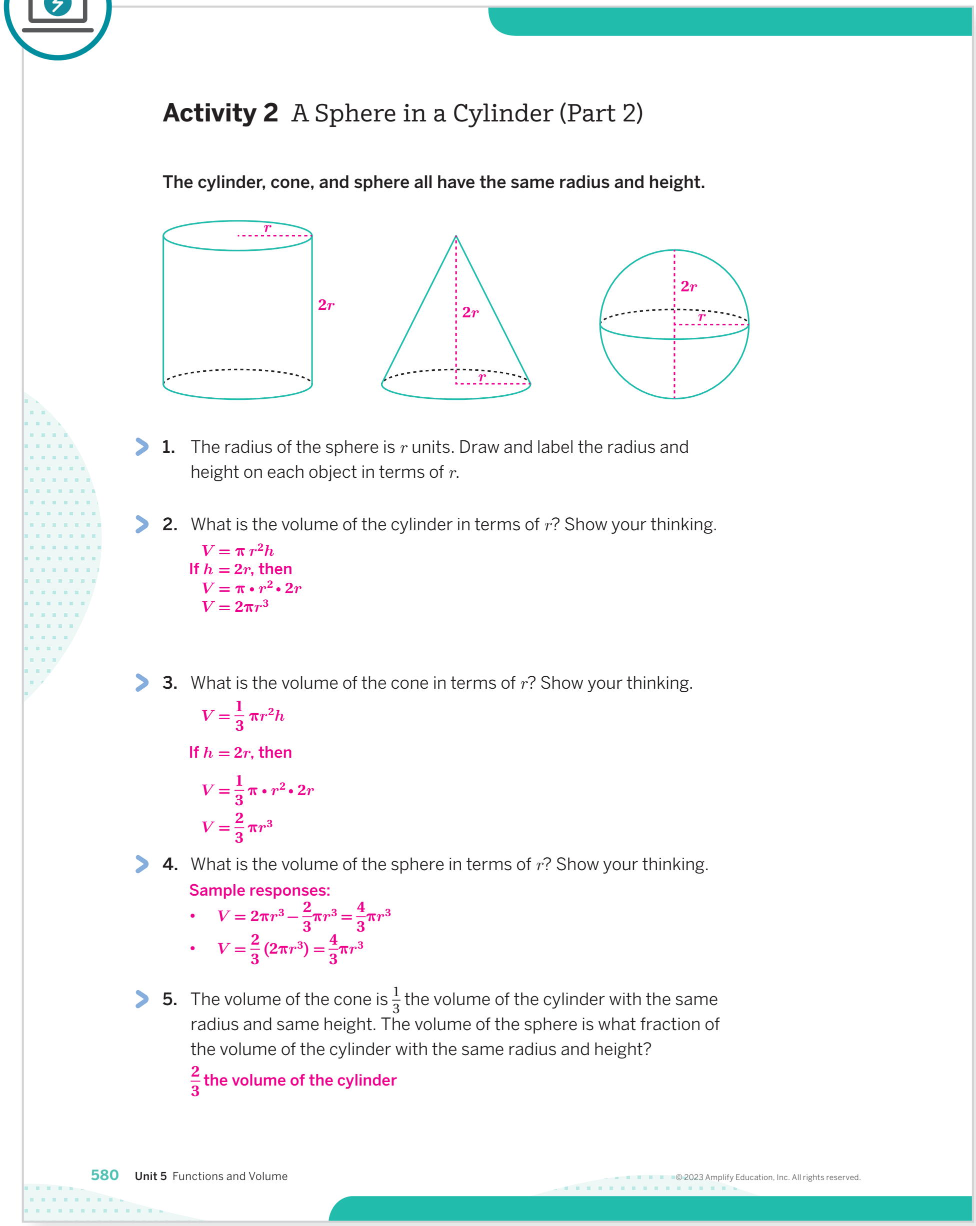
English Learners

Include a diagram where a sphere is fitted snugly inside a cylinder. Shade the remaining space and annotate it with "volume of a cone."

Pairs | 10 min

Activity 2 A Sphere in a Cylinder (Part 2)

Students generalize the connection between circular solids with the same dimensions to discover the formula for the volume of a sphere.



Activity 2 A Sphere in a Cylinder (Part 2)

The cylinder, cone, and sphere all have the same radius and height.

1. The radius of the sphere is r units. Draw and label the radius and height on each object in terms of r.

2. What is the volume of the cylinder in terms of r? Show your thinking.

   $V = \pi r^2 h$
   If $h = 2r$, then
   $V = \pi \cdot r^2 \cdot 2r$
   $V = 2\pi r^3$

3. What is the volume of the cone in terms of r? Show your thinking.

   $V = \frac{1}{3}\pi r^2 h$

   If $h = 2r$, then

   $V = \frac{1}{3}\pi \cdot r^2 \cdot 2r$

   $V = \frac{2}{3}\pi r^3$

4. What is the volume of the sphere in terms of r? Show your thinking.

   Sample responses:
   - $V = 2\pi r^3 - \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$
   - $V = \frac{2}{3}(2\pi r^3) = \frac{4}{3}\pi r^3$

5. The volume of the cone is $\frac{1}{3}$ the volume of the cylinder with the same radius and same height. The volume of the sphere is what fraction of the volume of the cylinder with the same radius and height?

   $\frac{2}{3}$ the volume of the cylinder

580 **Unit 5** Functions and Volume

1 Launch

Let students know this activity is very similar to Activity 1 but is the general case where the radius is r. Ask students what the diameter and heights will be in terms of r.

2 Monitor

Help students get started by ensuring they label the radii r and heights $2r$ correctly.

Look for points of confusion

- **Being unsure of how to write equivalent expressions with $\pi \cdot r^2 \cdot 2r$.** Expand r^2 and use the Commutative Property of Multiplication to rewrite the expression as $2 \cdot \pi \cdot r \cdot r \cdot r$ to help students condense the expression to $2\pi \cdot r^3$.

Look for productive strategies:

- Recognizing that the volume of the sphere is $\frac{2}{3}$ the volume of the cylinder and using that to easily write the general formula for volume of a sphere.
- Using the subtraction method discussed in the previous activity.

3 Connect

Display the figures and have students share how they substituted and manipulated the formulas.

Highlight the equation from Problem 4 is the formula for the volume of a sphere. **Note:** A general proof of the formula for the volume of a sphere would require mathematics beyond this grade level.

Ask:

- "How does the volume of a sphere compare to the volume of a cone with the same dimensions?"
- "Which method did you use to calculate the volume of the sphere?"
- "Examine the method that you did not use. Explain to a partner why that method works."
- "Which method do you think is the most efficient? Why?"

Differentiated Support

Accessibility: *Guide Processing and Visualizatio*

Keep the Anchor Chart PDF, Volumes of Circular Solids displayed throughout the activity. Be sure students understand that this time, the height (diameter) of the figures are labeled as $2r$.

Ask:

- "What do you notice about the dimensions of these figures, compared to the figures from Activity 1?"
- "How does the height/diameter of the figures compare to the radius?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight how the equation from Problem 4 represents the formula for the volume of a sphere in terms of the radius, draw students' attention to the connections between the previous activity and this activity. Ask:

- "In Activity 1, you saw that the expression $\frac{2}{3} \cdot \pi \cdot r^2 \cdot h$ represents the volume of a sphere, where h is the diameter. Compare $\frac{2}{3} \cdot \pi \cdot r^2 \cdot h$ with $\frac{4}{3} \cdot \pi \cdot r^3$. What do you notice?"
- "How can both of these expressions represent the volume of a sphere?" Listen for and amplify responses that connect the height of a sphere with its diameter (twice the radius). Students may benefit from a demonstration of the calculations needed to simplify $\frac{2}{3} \cdot \pi \cdot r^2 \cdot 2r$ or $\frac{2}{3} \cdot 2\pi \cdot r^2 \cdot r$ to $\frac{4}{3} \cdot \pi \cdot r^3$.

Pairs | 10 min

Activity 3 How Are the Volumes Related?

Students use the relationship between the volumes of circular solids with the same dimensions to determine the volume of unknown solids.

Name: ________ Date: ________ Period: ________

Activity 3 How Are the Volumes Related?

Use the relationship between the volumes of a cylinder, cone, and sphere to solve the following problems. Write your responses in terms of π.

1. A cylinder and a sphere have the same radii and the height of the cylinder is the same as the diameter of the sphere.

   a. If the cylinder has a volume of 144π cm^3, what is the volume of the sphere?

   Because the two figures have the same radii, the height of the cylinder is the same as the diameter of the sphere. The sphere's volume is $\frac{2}{3}$ the cylinder's volume: $144\pi \cdot \frac{2}{3} = 96\pi$, or 96π cm^3.

   b. If the sphere has a volume of 18π cm^3, what is the volume of the cylinder?

   Because the two figures have the same radii, the height of the cylinder is the same as the diameter of the sphere. The cylinder's volume is $\frac{3}{2}$ the sphere's volume: $18\pi \cdot \frac{3}{2} = 27\pi$, or 27π cm^3.

2. A cone and a sphere have the same dimensions.

   a. If the cone has a volume of 144π cm^3, what is the volume of the sphere?

   Because the two figures have the same radii, the height of the cone is the same as the diameter of the sphere. The sphere's volume is double the cone's volume: $144\pi \cdot 2 = 288\pi$, or 288π cm^3.

   b. If the sphere has a volume of 18π cm^3, what is the volume of the cone?

   Because the two figures have the same radii, the height of the cone is the same as the diameter of the sphere. The cone's volume is half the sphere's volume: $18\pi \div 2 = 9\pi$, or 9π cm^3.

Historical Moment

Archimedes

The Greek mathematician Archimedes (287–212 BCE) was the first person to discover that the volume of a sphere is $\frac{2}{3}$ the volume of a cylinder with the same dimensions. This was his favorite mathematical discovery, so much that it was engraved on his tombstone.

STOP

Lesson 17 The Volume of a Sphere 581

1 Launch

Activate prior knowledge by asking, "If the circular solids have the same radius and the height is the same as the diameter of the sphere, then what fraction of the cylinder is the volume of a sphere?" $\frac{2}{3}$ "The volume of a cone is what fraction of the sphere?" $\frac{1}{2}$

2 Monitor

Help students get started by asking, "If a sphere's volume is $\frac{2}{3}$ the volume of a cylinder with the same dimensions, this means the volume of the cylinder is what fraction compared to the sphere?" $\frac{3}{2}$

Look for points of confusion:

- **Switching the relationship between the volumes of the circular solids.** Have students reference Activity 1 for the relationship between the cylinder and sphere and the Anchor Chart PDF, *Volumes of Circular Solids*, to remind themselves of the relationship between a cylinder and cone.

Look for productive strategies:

- Determining the dimensions for the figure with the given volume and then determining the unknown volume using the formulas.

3 Connect

Have students share their responses and reasoning to the problems. Sequence responses by starting with students who determined the unknown dimensions followed by students who used proportional reasoning.

Highlight that, if the radii and heights are known to be the same, students can use the relationship between the volumes of the solids to find the unknown volume.

Ask, "In order to guarantee the volume of a sphere is $\frac{2}{3}$ the volume a cylinder, what should be true?" The radii must be equivalent and the height of the cylinder must be equivalent to the diameter of the sphere.

Differentiated Support

***Accessibility:** Guide Processing and Visualization*

If possible, allow students to hold and manipulate physical three-dimensional models of cylinders, cones, and spheres as they complete this activity.

Math Language Development

MLR2: Collect and Display

During the Connect, add the following language to the class display.

- The volume of a sphere is $\frac{2}{3}$ the volume of a cylinder, *only* when they have the same radius and the cylinder's height is equivalent to the sphere's diameter.
- The volume of a cone is $\frac{1}{3}$ the volume of a cylinder, *only* when they have the same radius and heights.
- The sum of the volumes of a sphere and cone is the same as the volume of a cylinder, only when they have the same radius and height - where the height is the diameter of the sphere.

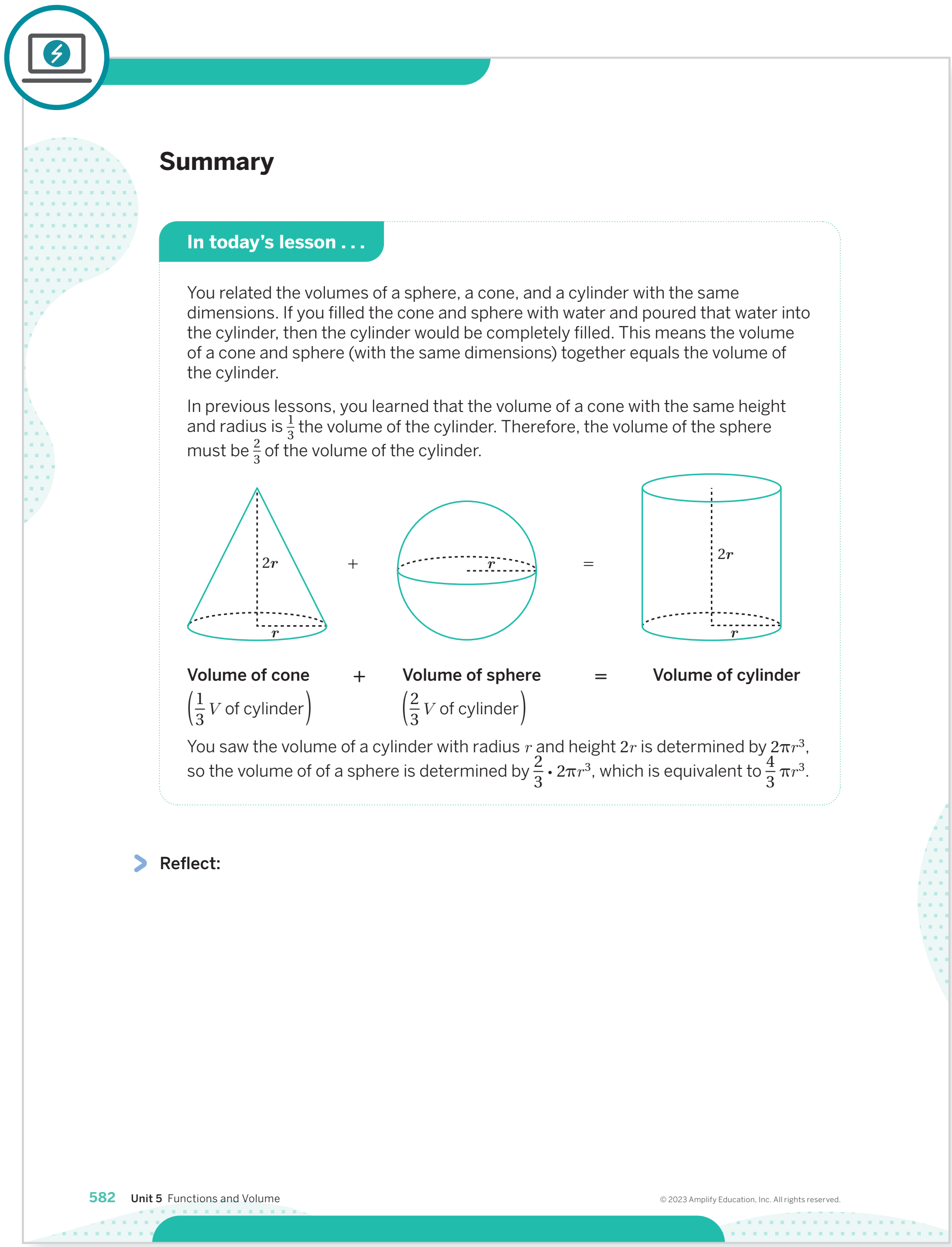
Historical Moment

Archimedes

Have students read about Archimedes and ask them to research how he calculated volumes of irregular shapes.

Summary

Review and synthesize the connection between the volume of a cylinder, cone, and sphere with the same radii and heights (where the heights are equal to the diameter of the sphere).



Summary

In today's lesson . . .

You related the volumes of a sphere, a cone, and a cylinder with the same dimensions. If you filled the cone and sphere with water and poured that water into the cylinder, then the cylinder would be completely filled. This means the volume of a cone and sphere (with the same dimensions) together equals the volume of the cylinder.

In previous lessons, you learned that the volume of a cone with the same height and radius is $\frac{1}{3}$ the volume of the cylinder. Therefore, the volume of the sphere must be $\frac{2}{3}$ of the volume of the cylinder.

Volume of cone + **Volume of sphere** = **Volume of cylinder**

$\left(\frac{1}{3}V \text{ of cylinder}\right)$ $\left(\frac{2}{3}V \text{ of cylinder}\right)$

You saw the volume of a cylinder with radius r and height $2r$ is determined by $2\pi r^3$, so the volume of of a sphere is determined by $\frac{2}{3} \cdot 2\pi r^3$, which is equivalent to $\frac{4}{3}\pi r^3$.

Reflect:

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Synthesize

Have students share the connection between the volume of a sphere and the other circular solids.

Highlight the methods the students discuss to find the volume of a sphere. Have the class choose the method(s) to write on the volume of a sphere section of the Anchor Chart PDF, *Volumes of Circular Solids*.

Ask:

- "If the radii are the same, how tall must a cone be to have the same volume of a sphere?" The height will need to be double the diameter of the sphere.
- "If the radii are the same, how tall must a cylinder be to have the same volume of a sphere?" The height will need to be $\frac{2}{3}$ the diameter of the sphere.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

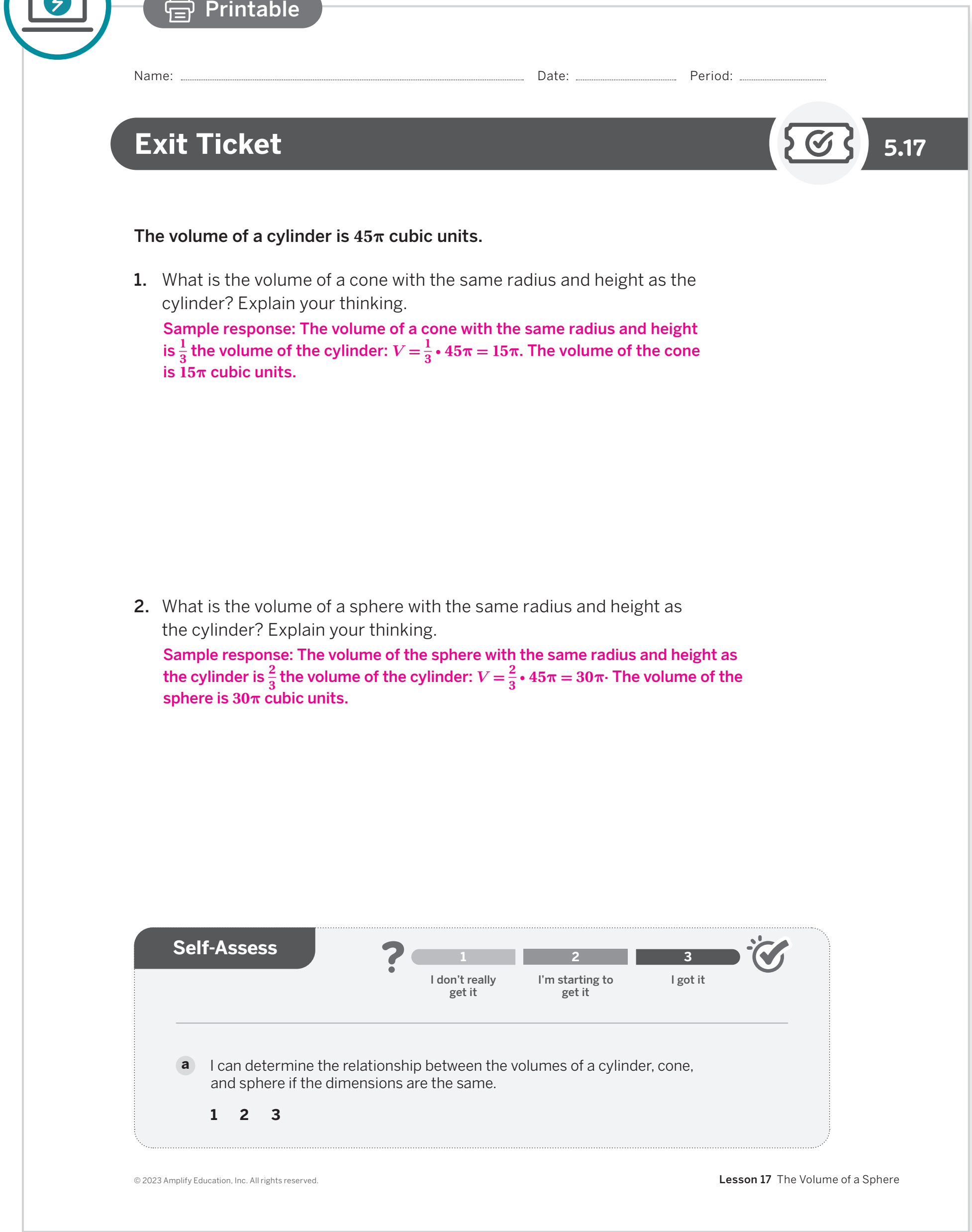
- "How are the volumes of a cylinder, cone, and sphere related if their dimensions are the same?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the volume of a cone and sphere based on the volume of a cylinder with the same dimensions.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 5.17

The volume of a cylinder is 45π cubic units.

1. What is the volume of a cone with the same radius and height as the cylinder? Explain your thinking.
   Sample response: The volume of a cone with the same radius and height is $\frac{1}{3}$ the volume of the cylinder: $V = \frac{1}{3} \cdot 45\pi = 15\pi$. The volume of the cone is 15π cubic units.

2. What is the volume of a sphere with the same radius and height as the cylinder? Explain your thinking.
   Sample response: The volume of the sphere with the same radius and height as the cylinder is $\frac{2}{3}$ the volume of the cylinder: $V = \frac{2}{3} \cdot 45\pi = 30\pi$· The volume of the sphere is 30π cubic units.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine the relationship between the volumes of a cylinder, cone, and sphere if the dimensions are the same.

1 2 3

Success looks like . . .

- **Language Goal:** Calculating the volume of a sphere, cylinder, and cone which have a radius of r and height (diameter) of $2r$, and explaining the relationship between their volumes. **(Speaking and Listening)**
  - » Calculating the volumes of a cone and a sphere with the same radius and height as the given cylinder.
- **Language Goal:** Creating an equation to represent the volume of a sphere as a function of its radius, and explaining the reasoning. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly answer Problem 1, consider:

- Displaying the Anchor Chart PDF, *Volumes of Circular Solids*.
- Reviewing Activities 1 and 2.
- Reviewing Lesson 14.

If students incorrectly answer Problem 2, consider:

- Reviewing Activities 1, 2, and 3.
- Assigning Practice Problems 1, 2, and 3.

Professional Learning

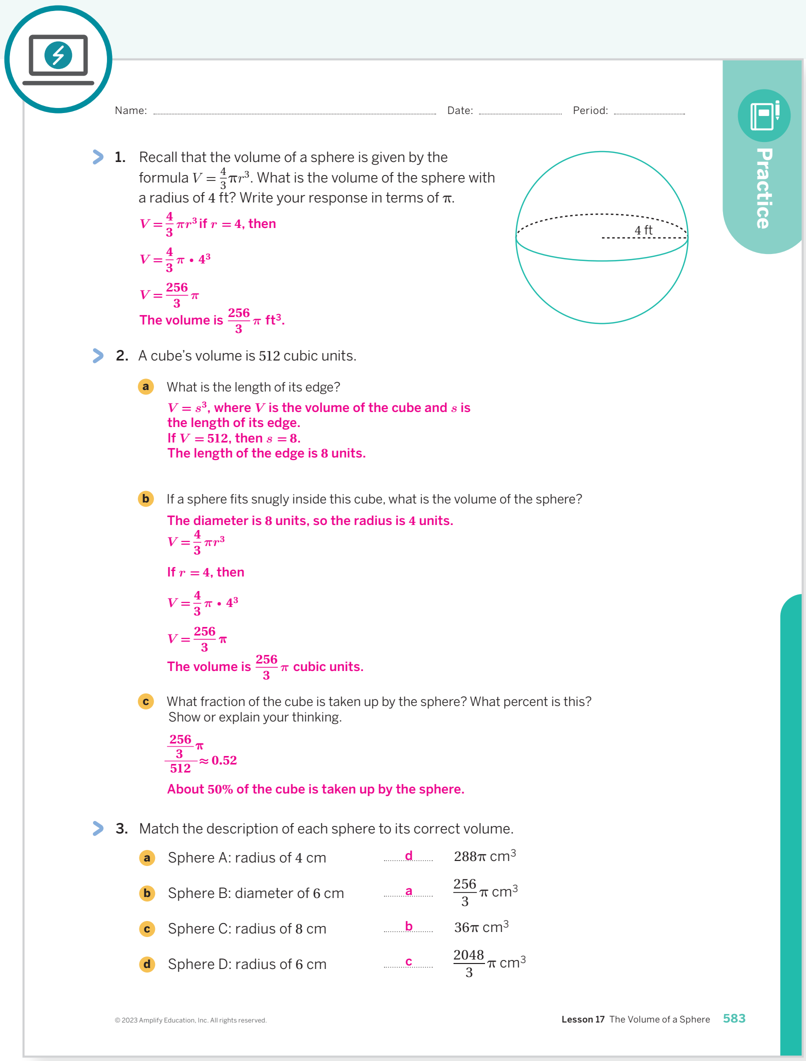
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on discovering the formula for the volume of a sphere?
- In earlier lessons, students found the volume of cylinders and cones. How did that support finding the volume of a sphere? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Recall that the volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. What is the volume of the sphere with a radius of 4 ft? Write your response in terms of π.

   $V = \frac{4}{3}\pi r^3$ if $r = 4$, then

   $V = \frac{4}{3}\pi \cdot 4^3$

   $V = \frac{256}{3}\pi$

   The volume is $\frac{256}{3}\pi$ ft^3.

2. A cube's volume is 512 cubic units.

   a. What is the length of its edge?

   $V = s^3$, where V is the volume of the cube and s is the length of its edge.
   If $V = 512$, then $s = 8$.
   The length of the edge is 8 units.

   b. If a sphere fits snugly inside this cube, what is the volume of the sphere?

   The diameter is 8 units, so the radius is 4 units.

   $V = \frac{4}{3}\pi r^3$

   If $r = 4$, then

   $V = \frac{4}{3}\pi \cdot 4^3$

   $V = \frac{256}{3}\pi$

   The volume is $\frac{256}{3}\pi$ cubic units.

   c. What fraction of the cube is taken up by the sphere? What percent is this? Show or explain your thinking.

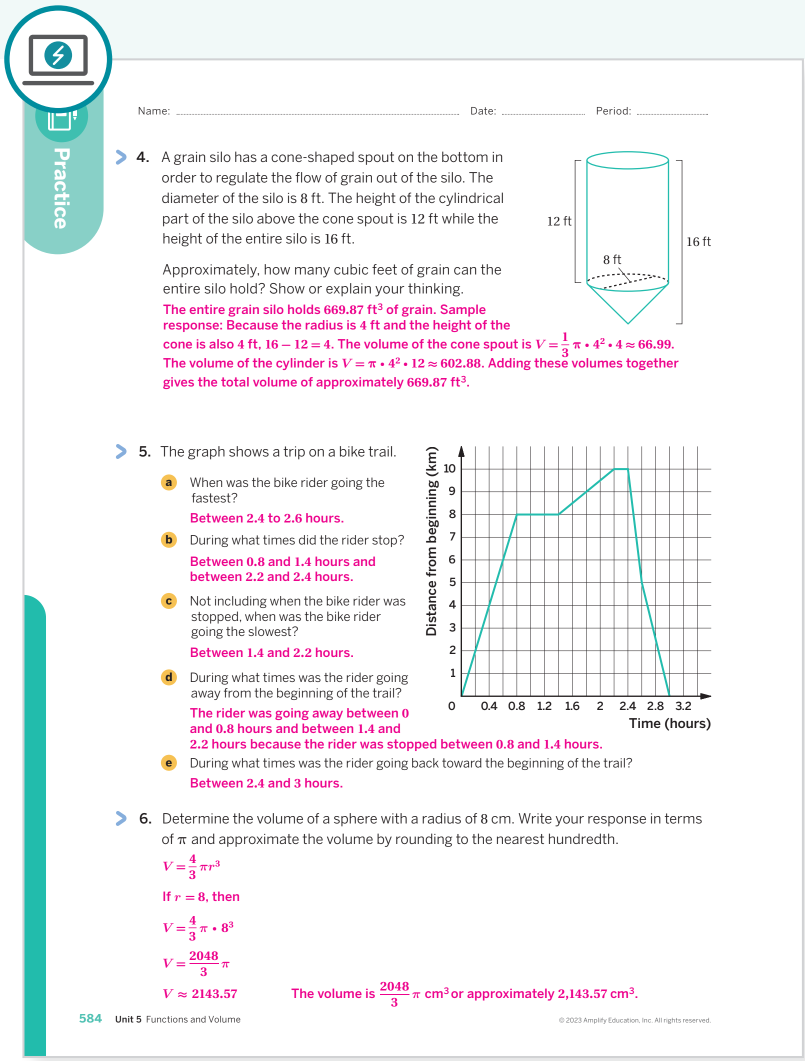
   $\frac{\frac{256}{3}\pi}{512} \approx 0.52$

   About 50% of the cube is taken up by the sphere.

3. Match the description of each sphere to its correct volume.

a. Sphere A: radius of 4 cm	d	288π cm^3
b. Sphere B: diameter of 6 cm	a	$\frac{256}{3}\pi$ cm^3
c. Sphere C: radius of 8 cm	b	36π cm^3
d. Sphere D: radius of 6 cm	c	$\frac{2048}{3}\pi$ cm^3

Lesson 17 The Volume of a Sphere 583



Practice

Name: Date: Period:

4. A grain silo has a cone-shaped spout on the bottom in order to regulate the flow of grain out of the silo. The diameter of the silo is 8 ft. The height of the cylindrical part of the silo above the cone spout is 12 ft while the height of the entire silo is 16 ft.

   Approximately, how many cubic feet of grain can the entire silo hold? Show or explain your thinking.

   The entire grain silo holds 669.87 ft^3 of grain. Sample response: Because the radius is 4 ft and the height of the cone is also 4 ft, $16 - 12 = 4$. The volume of the cone spout is $V = \frac{1}{3}\pi \cdot 4^2 \cdot 4 \approx 66.99$. The volume of the cylinder is $V = \pi \cdot 4^2 \cdot 12 \approx 602.88$. Adding these volumes together gives the total volume of approximately 669.87 ft^3.

5. The graph shows a trip on a bike trail.

   a. When was the bike rider going the fastest?

   Between 2.4 to 2.6 hours.

   b. During what times did the rider stop?

   Between 0.8 and 1.4 hours and between 2.2 and 2.4 hours.

   c. Not including when the bike rider was stopped, when was the bike rider going the slowest?

   Between 1.4 and 2.2 hours.

   d. During what times was the rider going away from the beginning of the trail?

   The rider was going away between 0 and 0.8 hours and between 1.4 and 2.2 hours because the rider was stopped between 0.8 and 1.4 hours.

   e. During what times was the rider going back toward the beginning of the trail?

   Between 2.4 and 3 hours.

6. Determine the volume of a sphere with a radius of 8 cm. Write your response in terms of π and approximate the volume by rounding to the nearest hundredth.

   $V = \frac{4}{3}\pi r^3$

   If $r = 8$, then

   $V = \frac{4}{3}\pi \cdot 8^3$

   $V = \frac{2048}{3}\pi$

   $V \approx 2143.57$ The volume is $\frac{2048}{3}\pi$ cm^3 or approximately 2,143.57 cm^3.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 15	2
	5	Unit 5 Lesson 6	1
Formative ⚡	6	Unit 5 Lesson 18	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

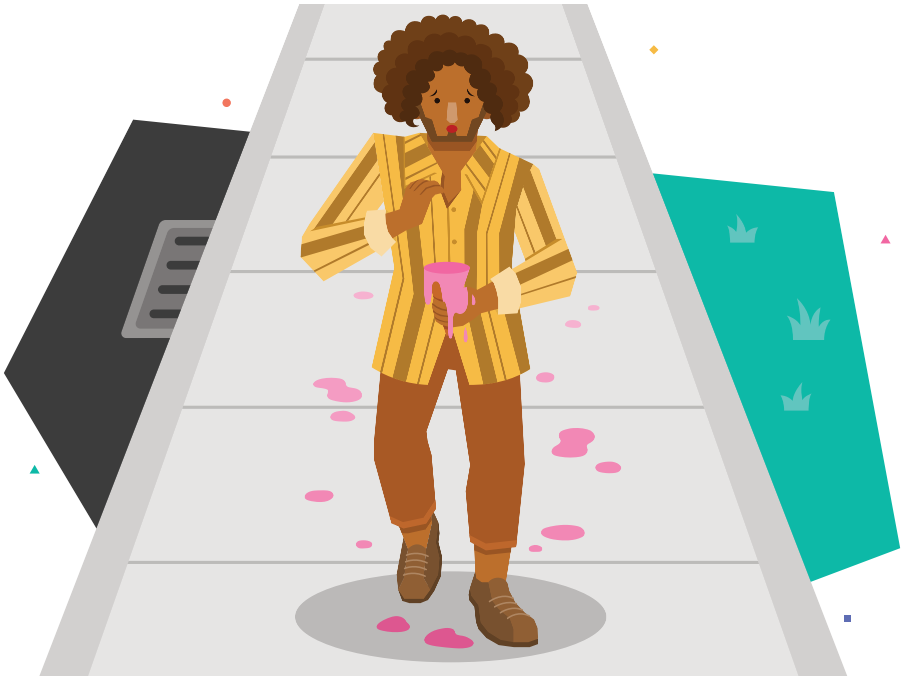
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 18**

Cylinders, Cones, and Spheres

Let's determine the volumes of circular solids.



Focus

Goals

1. **Language Goal:** Calculate the value of the radius of a sphere with a given volume using the structure of the equation, and explain the solution method. **(Speaking and Listening)**
2. **Language Goal:** Describe how a change in the radius affects the volume. **(Speaking and Listening)**

Coherence

Today

In this lesson, students use the formula for the volume of a sphere to solve various problems. They have opportunities to analyze common errors that people make when using this formula. They use the structure of an equation to find the radius of a sphere when they know its volume. Finally, they have opportunities to practice using all of the new volume formulas they have learned in this unit to solve mixed problems with spheres, cylinders, and cones, reasoning about the effect of different dimensions on the volume of different figures.

Previously

In Lesson 17, students discovered the formula for the volume of a sphere.

Coming Soon

In Lessons 19 and 20, students will determine how changing the dimensions affects the volume.

Rigor

- Students develop **fluency** as they use the formulas for volumes of circular solids to solve problems.
- Students **apply** their knowledge of volume to solve real-world problems using cylinders, cones, and spheres.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Volumes of Circular Solids*
- Anchor Chart PDF, *Volumes of Circular Solids* (answers)
- Anchor Chart PDF, *Properties of Equality*
- calculators

Math Language Development

Review words

- *cone*
- *cylinder*
- *height*
- *hemisphere*
- *pi*
- *radius*
- *sphere*
- *volume*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to explain their thinking behind finding the radius of a sphere given its volume, and these explanations are available to you digitally, in real time.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not have any ideas about how to use mathematical properties with the structure of the equation in Activity 1. Have students identify the individual parts that compose the formula. Guide students to step back for an overview before diving into the structure and details of the formula. Help them not lose sight of the connections between the entire equation and the variable.

Modifications to Pacing

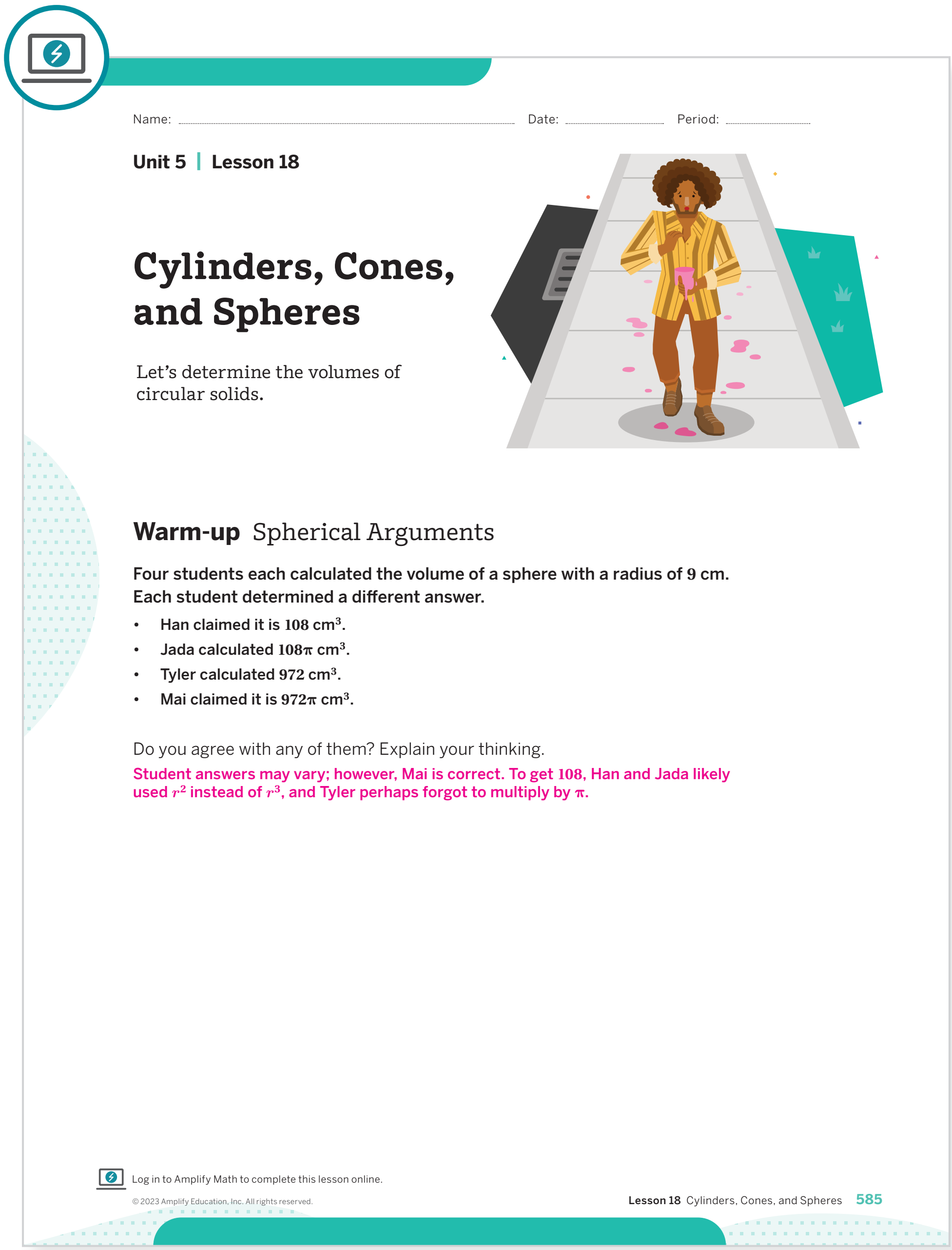
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problem 2 may be used as additional practice.
- In **Activity 3**, assign 1 or 2 problems.

Pairs | 5 min

Warm-up Spherical Arguments

Students reason about possible volumes to critique errors made while calculating the volume of a sphere.



Name: ________ Date: ________ Period: ________

Unit 5 | Lesson 18

Cylinders, Cones, and Spheres

Let's determine the volumes of circular solids.

Warm-up Spherical Arguments

Four students each calculated the volume of a sphere with a radius of 9 cm. Each student determined a different answer.

- **Han claimed it is 108 cm^3.**
- **Jada calculated 108π cm^3.**
- **Tyler calculated 972 cm^3.**
- **Mai claimed it is 972π cm^3.**

Do you agree with any of them? Explain your thinking.

Student answers may vary; however, Mai is correct. To get 108, Han and Jada likely used r^2 instead of r^3, and Tyler perhaps forgot to multiply by π.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Provide access to calculators for the duration of the lesson.

2 Monitor

Help students get started by having them write the formula for the volume of a sphere and substitute the known information into the formula.

Look for points of confusion:

- **Not being able to determine the mistakes that were made.** Have students determine the volume of the sphere and work backwards to see how Tyler could have made his mistake and then work on Han and Jada's reasoning.

Look for productive strategies:

- Providing suggestions for possible mistakes.

3 Connect

Display the prompt and use the ***Poll the Class*** routine to have students indicate whether they agree or disagree with the answer. If they disagree with a student's answer, ask for possible mistakes that student may have made.

Highlight that the mistakes that were made are common and students can sketch and label pictures to help ensure accuracy.

Ask, "Can you think of any other common mistakes people may make?" Using the diameter as the radius.

Power-up

To power up students' ability to determine the volume of a sphere from its radius, have students complete:

Recall that the volume of a sphere is $V = \frac{4}{3}\pi \cdot r^3$ where r is the length of the radius.

Determine the volume of a sphere with a radius of 9 cm.

972π cm^3 or 3,052 cm^3;

$V = \frac{4}{3}\pi \cdot r^3$

$V = \frac{4}{3}\pi \cdot 9^3$

$V = 972\pi$

$V \approx 3052$

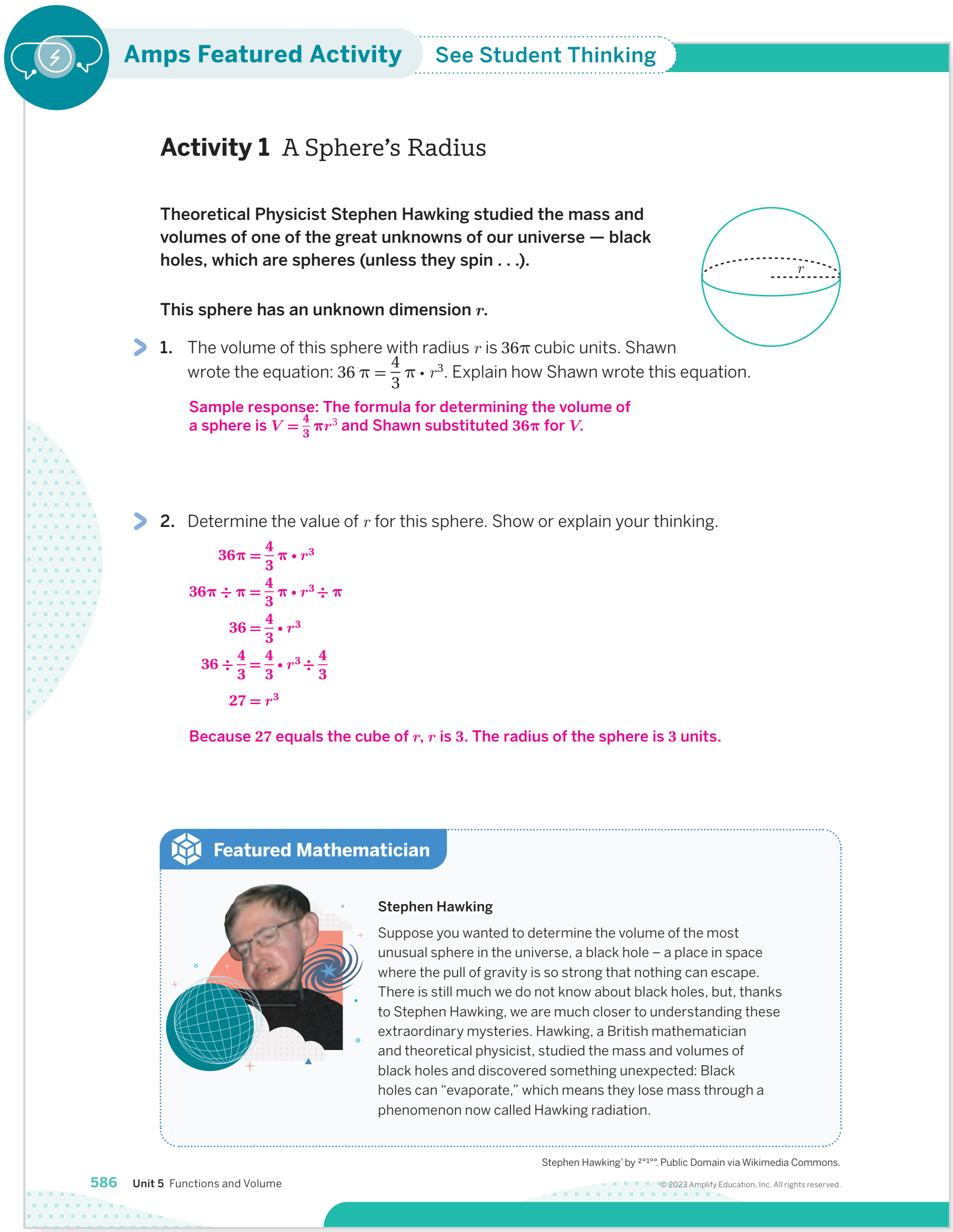
Use: Before the Warm-up

Informed by: Performance on Lesson 17, Practice Problem 6

Pairs | 15 min

Activity 1 A Sphere's Radius

Students use the volume of a sphere to determine its radius.



Amps Featured Activity See Student Thinking

Activity 1 A Sphere's Radius

Theoretical Physicist Stephen Hawking studied the mass and volumes of one of the great unknowns of our universe — black holes, which are spheres (unless they spin . . .).

This sphere has an unknown dimension r.

1. The volume of this sphere with radius r is 36π cubic units. Shawn wrote the equation: $36\,\pi = \frac{4}{3}\pi \cdot r^3$. Explain how Shawn wrote this equation.

   Sample response: The formula for determining the volume of a sphere is $V = \frac{4}{3}\pi r^3$ and Shawn substituted 36π for V.

2. Determine the value of r for this sphere. Show or explain your thinking.

$$36\pi = \frac{4}{3}\pi \cdot r^3$$
$$36\pi \div \pi = \frac{4}{3}\pi \cdot r^3 \div \pi$$
$$36 = \frac{4}{3} \cdot r^3$$
$$36 \div \frac{4}{3} = \frac{4}{3} \cdot r^3 \div \frac{4}{3}$$
$$27 = r^3$$

   Because 27 equals the cube of r, r is 3. The radius of the sphere is 3 units.

Featured Mathematician

Stephen Hawking

Suppose you wanted to determine the volume of the most unusual sphere in the universe, a black hole – a place in space where the pull of gravity is so strong that nothing can escape. There is still much we do not know about black holes, but, thanks to Stephen Hawking, we are much closer to understanding these extraordinary mysteries. Hawking, a British mathematician and theoretical physicist, studied the mass and volumes of black holes and discovered something unexpected: Black holes can "evaporate," which means they lose mass through a phenomenon now called Hawking radiation.

Stephen Hawking' by $^{2\circ 1\circ\circ}$. Public Domain via Wikimedia Commons.

586 **Unit 5** Functions and Volume

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them write down the volume formula for a sphere and substituting the known values into the formula.

Look for points of confusion:

- Getting a value which is not exactly 3. Students may have substituted a value for π and solved the resulting equation to arrive at a value for the radius slightly less than 3, while the actual value is exactly 3. This is a good opportunity to talk about the effects of rounding and how to minimize the error that rounding introduces.

Look for productive strategies:

- Examining the structure of the equation for the volume and reasoning about a number that makes the equation true.
- Noticing that π is a factor on each side of the equation and dividing each side by π.

3 Connect

Have students share their responses and reasoning.

Highlight that students have the skills necessary to solve for the radius and reference the Anchor Chart PDF, *Properties of Equality*, but now they will also need to think about perfect squares and perfect cubes.

Ask:

- "π appears on both sides of the volume equation. Did you deal with this as a first step or later in the solution process?"
- "How did you deal with the fraction in the equation?"
- "If the final step in your solution was solving for r when $r^3 = 27$, how did you solve it?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the formula for the volume of a sphere and provide access to colored pencils. Suggest that students color code V with 36π to help make the connection to how Shawn wrote the equation in Problem 1.

$$V = \frac{4}{3}\pi \cdot r^3$$
$$36\pi = \frac{4}{3}\pi \cdot r^3$$

Extension: *Interdisciplinary Connections*

Have students explore the NASA site, "Imagine the Universe: Black Holes" to study the science behind black holes. They can examine the structure of a black hole, noting the singularity, event horizon, and radius — called the *Schwarzschild radius*. This is the radius at which an object has an escape velocity equivalent to the speed of light. If the object has a radius smaller than the Schwarzschild radius, then it is a black hole. (Science)

Featured Mathematician

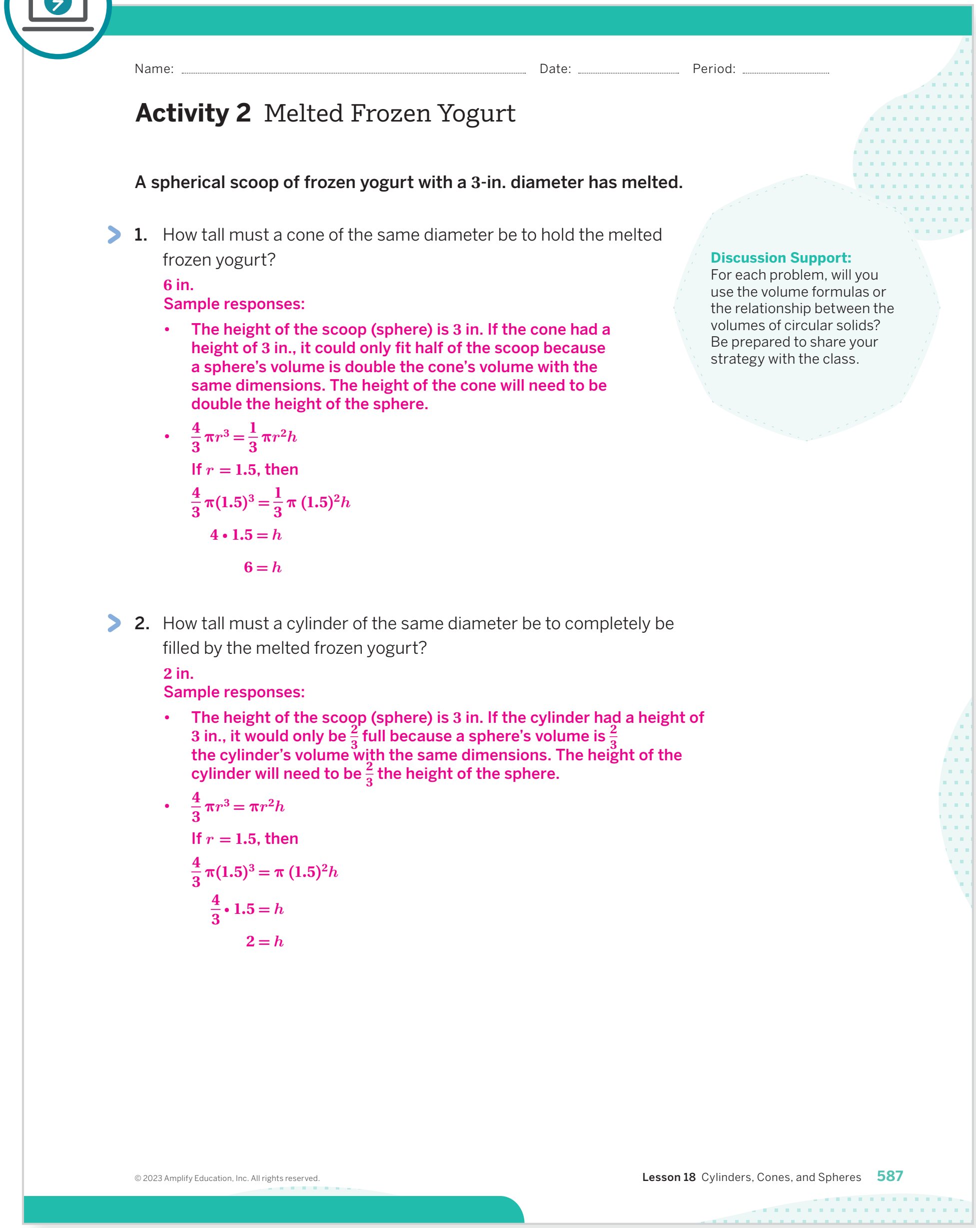
Stephen Hawking

Have students read about featured mathematician, Stephen Hawking, who studied the mass and volumes of black holes.

Pairs | 15 min

Activity 2 Melted Frozen Yogurt

Students use the volume of a sphere to determine the height of a cone and cylinder which have the same volume.



Name: ____________ Date: ____________ Period: ____________

Activity 2 Melted Frozen Yogurt

A spherical scoop of frozen yogurt with a 3-in. diameter has melted.

1. How tall must a cone of the same diameter be to hold the melted frozen yogurt?

   6 in.
   Sample responses:
   - **The height of the scoop (sphere) is 3 in. If the cone had a height of 3 in., it could only fit half of the scoop because a sphere's volume is double the cone's volume with the same dimensions. The height of the cone will need to be double the height of the sphere.**
   - $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$

     If $r = 1.5$, then

     $\frac{4}{3}\pi(1.5)^3 = \frac{1}{3}\pi\,(1.5)^2 h$

     $4 \cdot 1.5 = h$

     $6 = h$

2. How tall must a cylinder of the same diameter be to completely be filled by the melted frozen yogurt?

   2 in.
   Sample responses:
   - **The height of the scoop (sphere) is 3 in. If the cylinder had a height of 3 in., it would only be $\frac{2}{3}$ full because a sphere's volume is $\frac{2}{3}$ the cylinder's volume with the same dimensions. The height of the cylinder will need to be $\frac{2}{3}$ the height of the sphere.**
   - $\frac{4}{3}\pi r^3 = \pi r^2 h$

     If $r = 1.5$, then

     $\frac{4}{3}\pi(1.5)^3 = \pi\,(1.5)^2 h$

     $\frac{4}{3} \cdot 1.5 = h$

     $2 = h$

Discussion Support:
For each problem, will you use the volume formulas or the relationship between the volumes of circular solids? Be prepared to share your strategy with the class.

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Lesson 18 Cylinders, Cones, and Spheres 587

1 Launch

Activate prior knowledge about the relationships between the volume of a cone and sphere or by reviewing the formulas for those volumes.

2 Monitor

Help students get started by encouraging students to sketch and label pictures to determine which pieces are known and which unknown.

Look for points of confusion:

- **Thinking the height of the cone is the same as the diameter of a sphere.** Remind students that, if the radii are the same and the height of the cone is the same as the diameter of the sphere, the sphere's volume is double the cone's volume so the frozen yogurt will not fit.

Look for productive strategies:

- Reasoning about the dimensions and relationship between the volume of a cone and sphere.
- Setting the volumes equal to each other and solving for the height of the cone or cylinder.

3 Connect

Have students share their solutions and reasoning. Sequence student responses by starting with any formula manipulation and ending with any verbal reasoning. Use conclusions drawn in Activity 1 about the solving process to enhance this discussion.

Ask:

- "If you used the relationship between the volumes of circular solids with the same dimensions to reason through the problem, how did you know to scale up or down your height?"
- If you used the formulas, how did you handle the value of π on both sides? The fractions?"

Highlight the many methods for determining the unknown dimensions. Some students may use the connection between the volumes of circular solids and others may set the equations equal to each other and solve.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing sketches of a spherical scoop of frozen yogurt, a cone, and a cylindrical container and have students label them with known dimensions for each problem. Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity.

Extension: *Math Enrichment*

Challenge students to determine the height of a cylindrical container needed to be completely filled, with no overflow, by the melted frozen yogurt if the diameter of the container is 2 in. 4.5 in. tall

Math Language Development

MLR8: Discussion Supports—Press for Details

During the Connect, as students share their solutions and reasoning, press them for details as to whether they used the volume formulas or the relationships between the volumes of circular solids. Let students know that either method is appropriate, yet the relationships between the volumes of circular solids *only holds true* when they have the same dimensions.

English Learners

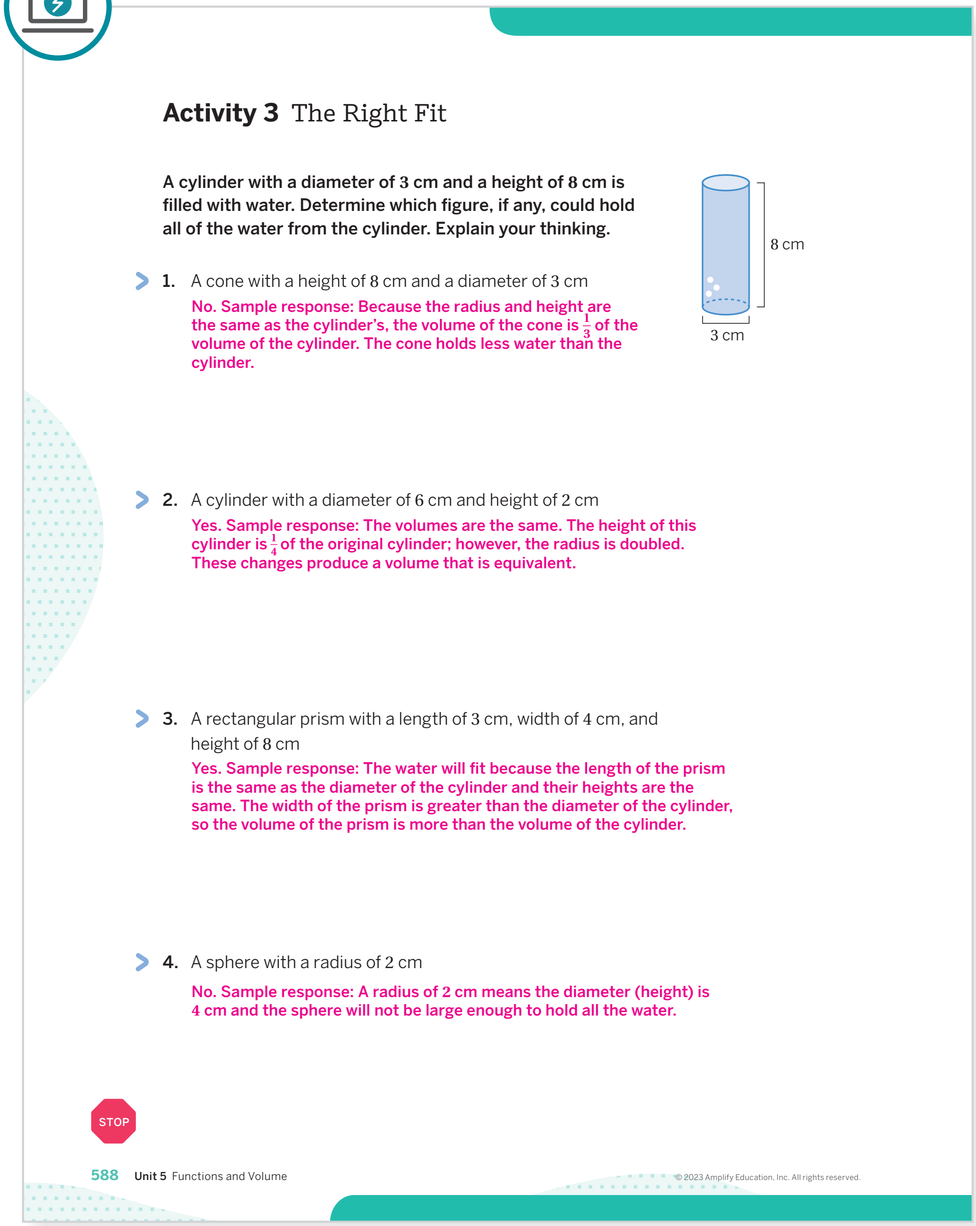
Provide time for students to rehearse and formulate a response with a partner before sharing with the class.

Optional

Pairs | 10 min

Activity 3 The Right Fit

Students reason about the changes to the dimensions to determine if the volume of water will fit into the container.



Activity 3 The Right Fit

A cylinder with a diameter of 3 cm and a height of 8 cm is filled with water. Determine which figure, if any, could hold all of the water from the cylinder. Explain your thinking.

1. A cone with a height of 8 cm and a diameter of 3 cm

   No. Sample response: Because the radius and height are the same as the cylinder's, the volume of the cone is $\frac{1}{3}$ of the volume of the cylinder. The cone holds less water than the cylinder.

2. A cylinder with a diameter of 6 cm and height of 2 cm

   Yes. Sample response: The volumes are the same. The height of this cylinder is $\frac{1}{4}$ of the original cylinder; however, the radius is doubled. These changes produce a volume that is equivalent.

3. A rectangular prism with a length of 3 cm, width of 4 cm, and height of 8 cm

   Yes. Sample response: The water will fit because the length of the prism is the same as the diameter of the cylinder and their heights are the same. The width of the prism is greater than the diameter of the cylinder, so the volume of the prism is more than the volume of the cylinder.

4. A sphere with a radius of 2 cm

   No. Sample response: A radius of 2 cm means the diameter (height) is 4 cm and the sphere will not be large enough to hold all the water.

STOP

588 **Unit 5** Functions and Volume

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1 Launch

Have students compare the dimensions of the figures and consider how those measurements affect the volume of the figures. Encourage students to make predictions before they reason through the tasks.

2 Monitor

Help students get started by encouraging them to draw and label a picture.

Look for points of confusion:

- **Thinking that doubling a dimension doubles the volume.** Help students reason that changes in the radius, because it is squared or cubed when calculating volume, have a greater effect.

Look for productive strategies:

- Thinking about the effects of the dimension changes without computing the actual volume.

3 Connect

Have students share their solutions and reasoning. Sequence student responses by starting with those who used verbal reasoning followed by those who used algebraic reasoning.

Highlight comparing the volumes of different figures by computation and also by considering the effect that different dimensions have on volume.

Ask:

- "How does the volume of a cylinder, cone, or sphere change if the radius is doubled?" The volume of the cylinder and cone each will be 4 times greater, but the volume of the sphere will be 8 times greater.
- "How does the volume of a cylinder, cone, or sphere change if the radius is halved?" The volume of the cylinder and cone will each be one fourth the volume, but the volume of the sphere will be one eighth the volume.

Note: If students struggle with this activity, consider assigning optional Lessons 19 and 20 to further explore the effects of changing dimensions on volume.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity. Suggest that students draw sketches of cones, cylinders, rectangular prisms, and spheres and annotate them with their dimensions to help them visualize the problems.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

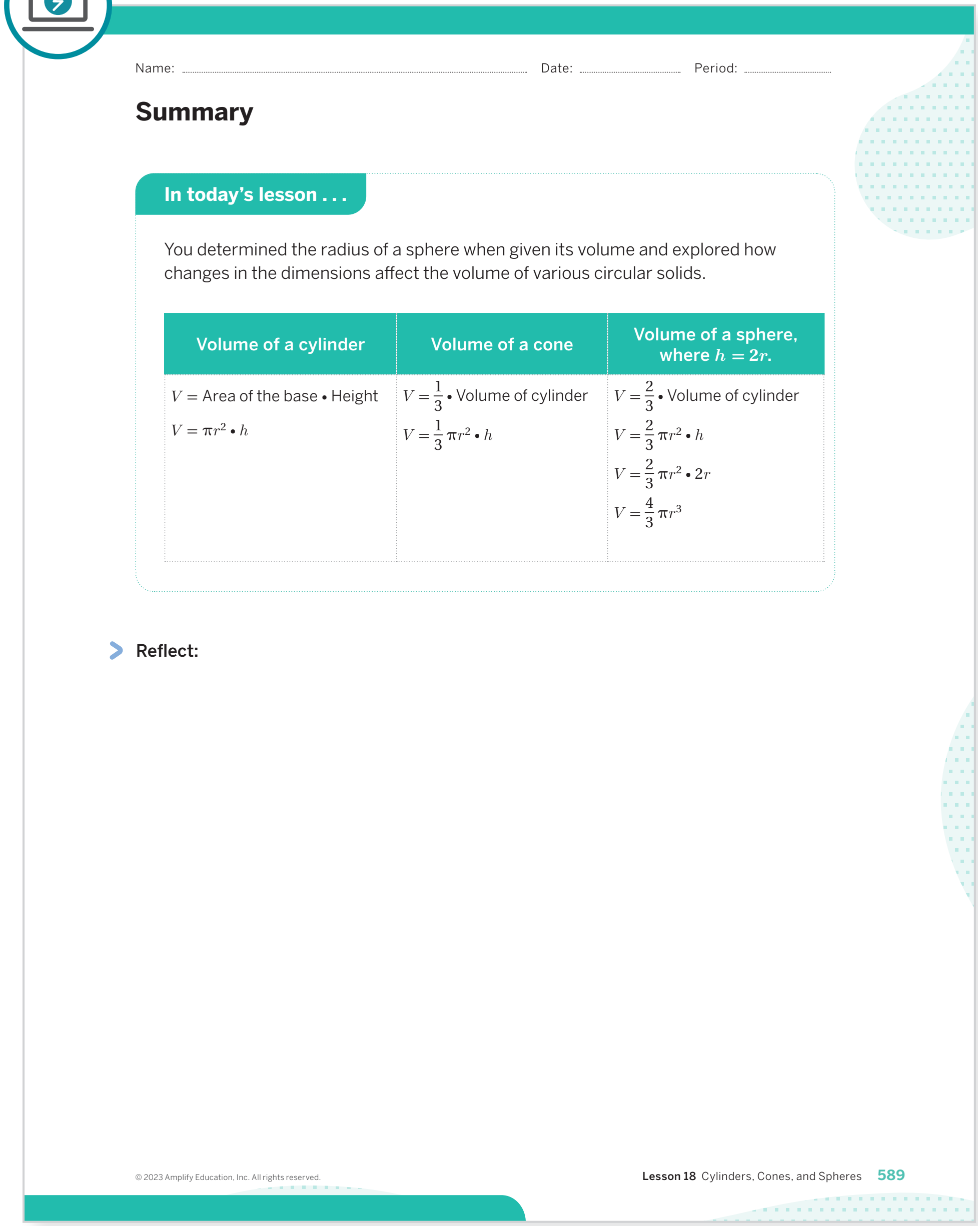
During the Connect, as students share their solutions and reasoning, provide the following sentence frames to help them organize their thinking.

- "The _____ could/could not hold all the water from the cylinder because . . ."
- "The _____ holds less/more water than the cylinder because . . ."
- "I noticed that _____, which means . . ."
- "I agree/disagree because . . ."

Whole Class | 5 min

Summary

Review and synthesize the formulas for the volumes of circular solids and how changing one dimension affects the volumes.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You determined the radius of a sphere when given its volume and explored how changes in the dimensions affect the volume of various circular solids.

Volume of a cylinder	Volume of a cone	Volume of a sphere, where $h = 2r$.
$V =$ Area of the base • Height $V = \pi r^2 \bullet h$	$V = \frac{1}{3} \bullet$ Volume of cylinder $V = \frac{1}{3}\pi r^2 \bullet h$	$V = \frac{2}{3} \bullet$ Volume of cylinder $V = \frac{2}{3}\pi r^2 \bullet h$ $V = \frac{2}{3}\pi r^2 \bullet 2r$ $V = \frac{4}{3}\pi r^3$

Reflect:

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Lesson 18 Cylinders, Cones, and Spheres 589

Synthesize

Display the Anchor Chart PDF, *Volumes of Circular Solids* and complete any remaining sections.

Highlight students have learned how to determine the volumes of cylinders, cones, and spheres and how to determine an unknown dimension when the volume and another dimension are known.

Ask:

- "Describe some relationships between the volumes of cylinders, cones, and spheres."
- "How do you determine a missing dimension when you know the volume and another dimension of a cylinder, cone, or sphere (or just the volume in the case of the sphere)?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

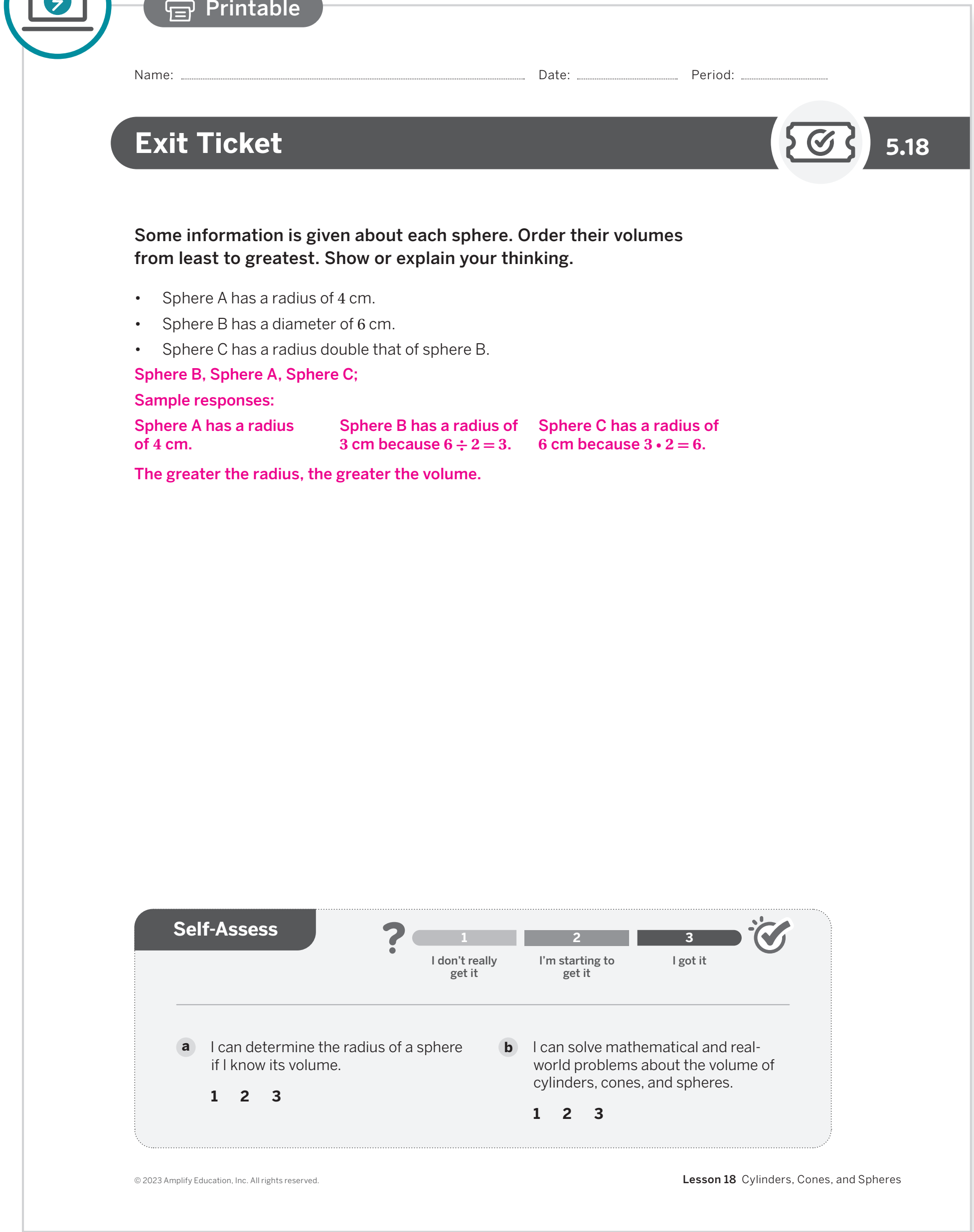
- "How are the volumes of a cylinder, cone, and sphere related if their dimensions are the same?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of volume by demonstrating how changes in the radius of a sphere affects its volume.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.18

Some information is given about each sphere. Order their volumes from least to greatest. Show or explain your thinking.

- Sphere A has a radius of 4 cm.
- Sphere B has a diameter of 6 cm.
- Sphere C has a radius double that of sphere B.

Sphere B, Sphere A, Sphere C;

Sample responses:

Sphere A has a radius of 4 cm.

Sphere B has a radius of 3 cm because $6 \div 2 = 3$.

Sphere C has a radius of 6 cm because $3 \cdot 2 = 6$.

The greater the radius, the greater the volume.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine the radius of a sphere if I know its volume.

1 2 3

b I can solve mathematical and real-world problems about the volume of cylinders, cones, and spheres.

1 2 3

Lesson 18 Cylinders, Cones, and Spheres

Success looks like . . .

- **Language Goal:** Calculating the value of the radius of a sphere with a given volume using the structure of the equation, and explaining the solution method. **(Speaking and Listening)**
- **Language Goal:** Describing how a change in the radius affects the volume. **(Speaking and Listening)**
  - » Explaining how the change in radius changes the volume of each sphere.

Suggested next steps

If students use a radius of 6 cm for Sphere B, consider:

- Reminding students to read carefully and perhaps sketch and label a sphere with the given information.

If students order the spheres incorrectly, consider:

- Ordering the spheres by the lengths of their radii.
- Reviewing Activity 1.
- Assigning Practice Problems 2 and 3.

If students order Sphere C incorrectly, consider

- Having students determine the approximate value of the volume.

Professional Learning

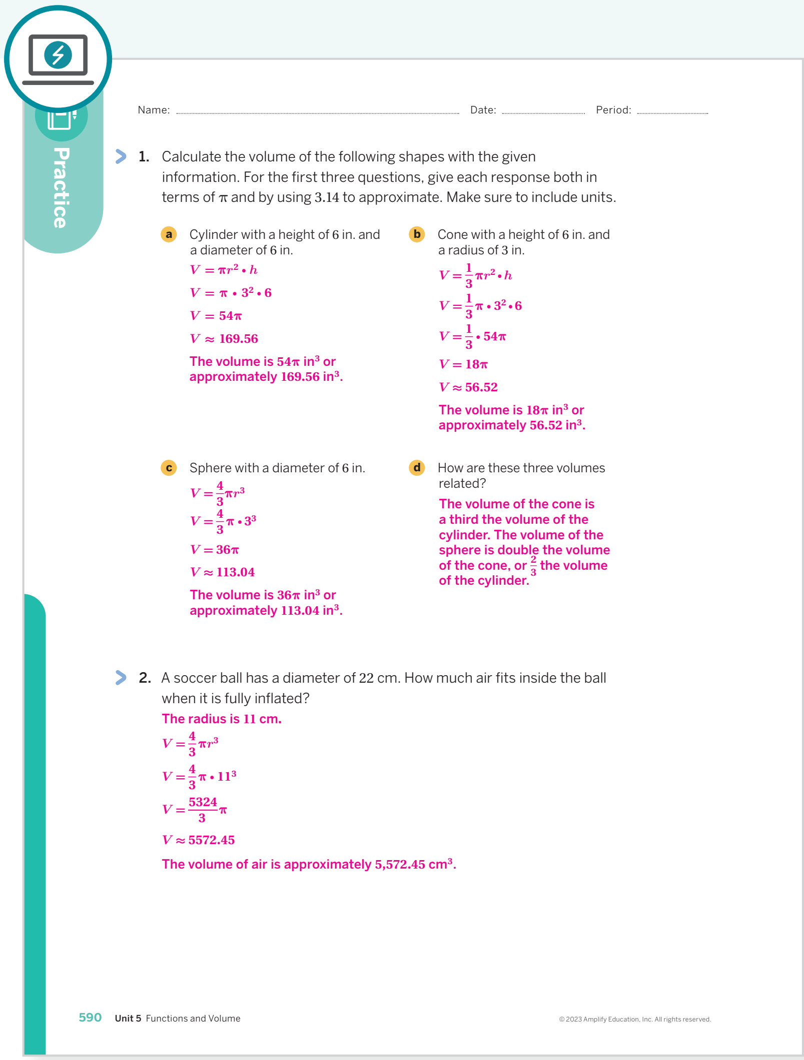
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was to calculate the radius of a sphere given its volume. How well did students accomplish this? What did you specifically do to help students accomplish it?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Calculate the volume of the following shapes with the given information. For the first three questions, give each response both in terms of π and by using 3.14 to approximate. Make sure to include units.

   a. Cylinder with a height of 6 in. and a diameter of 6 in.

   $V = \pi r^2 \cdot h$

   $V = \pi \cdot 3^2 \cdot 6$

   $V = 54\pi$

   $V \approx 169.56$

   The volume is 54π in^3 or approximately 169.56 in^3.

   b. Cone with a height of 6 in. and a radius of 3 in.

   $V = \frac{1}{3}\pi r^2 \cdot h$

   $V = \frac{1}{3}\pi \cdot 3^2 \cdot 6$

   $V = \frac{1}{3} \cdot 54\pi$

   $V = 18\pi$

   $V \approx 56.52$

   The volume is 18π in^3 or approximately 56.52 in^3.

   c. Sphere with a diameter of 6 in.

   $V = \frac{4}{3}\pi r^3$

   $V = \frac{4}{3}\pi \cdot 3^3$

   $V = 36\pi$

   $V \approx 113.04$

   The volume is 36π in^3 or approximately 113.04 in^3.

   d. How are these three volumes related?

   The volume of the cone is a third the volume of the cylinder. The volume of the sphere is double the volume of the cone, or $\frac{2}{3}$ the volume of the cylinder.

2. A soccer ball has a diameter of 22 cm. How much air fits inside the ball when it is fully inflated?

   The radius is 11 cm.

   $V = \frac{4}{3}\pi r^3$

   $V = \frac{4}{3}\pi \cdot 11^3$

   $V = \frac{5324}{3}\pi$

   $V \approx 5572.45$

   The volume of air is approximately 5,572.45 cm^3.

590 **Unit 5** Functions and Volume

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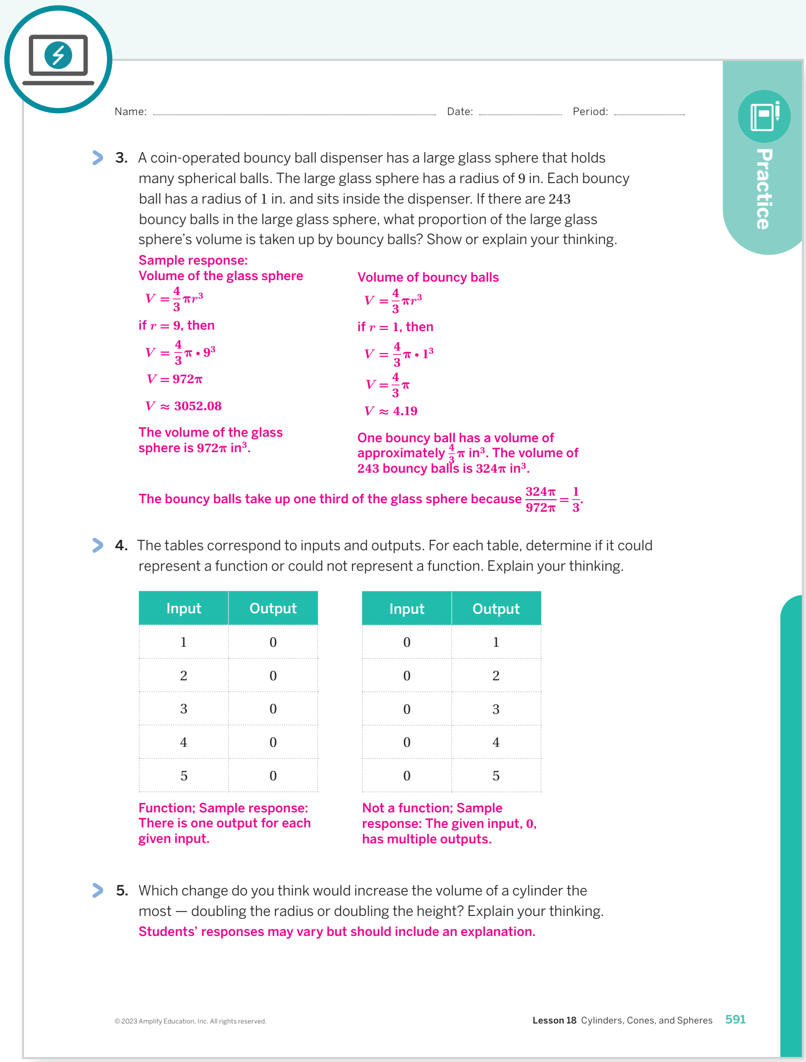

Name: Date: Period:

3. A coin-operated bouncy ball dispenser has a large glass sphere that holds many spherical balls. The large glass sphere has a radius of 9 in. Each bouncy ball has a radius of 1 in. and sits inside the dispenser. If there are 243 bouncy balls in the large glass sphere, what proportion of the large glass sphere's volume is taken up by bouncy balls? Show or explain your thinking.

   Sample response:

   Volume of the glass sphere

   $V = \frac{4}{3}\pi r^3$

   if $r = 9$, then

   $V = \frac{4}{3}\pi \cdot 9^3$

   $V = 972\pi$

   $V \approx 3052.08$

   The volume of the glass sphere is 972π in^3.

   Volume of bouncy balls

   $V = \frac{4}{3}\pi r^3$

   if $r = 1$, then

   $V = \frac{4}{3}\pi \cdot 1^3$

   $V = \frac{4}{3}\pi$

   $V \approx 4.19$

   One bouncy ball has a volume of approximately $\frac{4}{3}\pi$ in^3. The volume of 243 bouncy balls is 324π in^3.

   The bouncy balls take up one third of the glass sphere because $\frac{324\pi}{972\pi} = \frac{1}{3}$.

4. The tables correspond to inputs and outputs. For each table, determine if it could represent a function or could not represent a function. Explain your thinking.

Input	Output
1	0
2	0
3	0
4	0
5	0

Function; Sample response: There is one output for each given input.

Input	Output
0	1
0	2
0	3
0	4
0	5

Not a function; Sample response: The given input, 0, has multiple outputs.

5. Which change do you think would increase the volume of a cylinder the most — doubling the radius or doubling the height? Explain your thinking.

   Students' responses may vary but should include an explanation.

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Lesson 18 Cylinders, Cones, and Spheres 591

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 5 Lesson 2	2
Formative ⚡	5	Unit 5 Lesson 19	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Optional

UNIT 5 | **LESSON 19**

Scaling One Dimension

Let's see how changing one dimension changes the volume of a shape.

Focus

Goals

1. **Language Goal:** Create a graph and an equation to represent the function relationship between the volume of a right rectangular prism or a cylinder and its height, and justify that the relationship is linear. **(Speaking and Listening)**
2. **Language Goal:** Interpret a point on a graph representing the volume of a cylinder as a function of its height, and explain how changing one dimension affects the other. **(Speaking and Listening, Writing)**

Coherence

Today

Students see how the volume of a three-dimensional figure changes when one or more of its dimensions (length, width, height, radius) are scaled. In this lesson, they consider just one of the dimensions. The main purpose of the lesson is to understand that when one of the dimensions of a three-dimensional figure is scaled by a factor, the volume is scaled by the same factor. A secondary purpose is to see some examples of linear functions arising out of geometry.

Previously

Students determined the volumes of cylinders, cones, and spheres, particularly in relation to each other.

Coming Soon

In Lesson 20, students will see the effects of scaling two dimensions on the volume, creating a nonlinear model.

Rigor

- Students **apply** their understanding of proportional relationships and functions to the effects of scaling one dimension and volume.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Volumes of Circular Solids*
- Anchor Chart PDF, *Volumes of Circular Solids* (answers)
- Anchor Chart PDF, *Representations of Proportional Relationships* (from Unit 3)
- calculators

Math Language Development

Review words

- *cylinder*
- *dependent variable*
- *function*
- *independent variable*
- *pi*
- *proportional relationship*
- *radius*
- *volume*

Amps | Featured Activity

Activities 1 and 2

Interactive Graphs

Students can see their equation graphed immediately to save time, allowing them to focus on analyzing the effects of doubling or halving the height.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may not display constructive behavior during Activities 1 and 2, keeping them from making possible conjectures about volume. Pair students up to hold each other accountable for their behaviors. At the end of Activity 1, have students reflect on their choices, concluding with ways on how to successfully complete Activity 2.

Modifications to Pacing

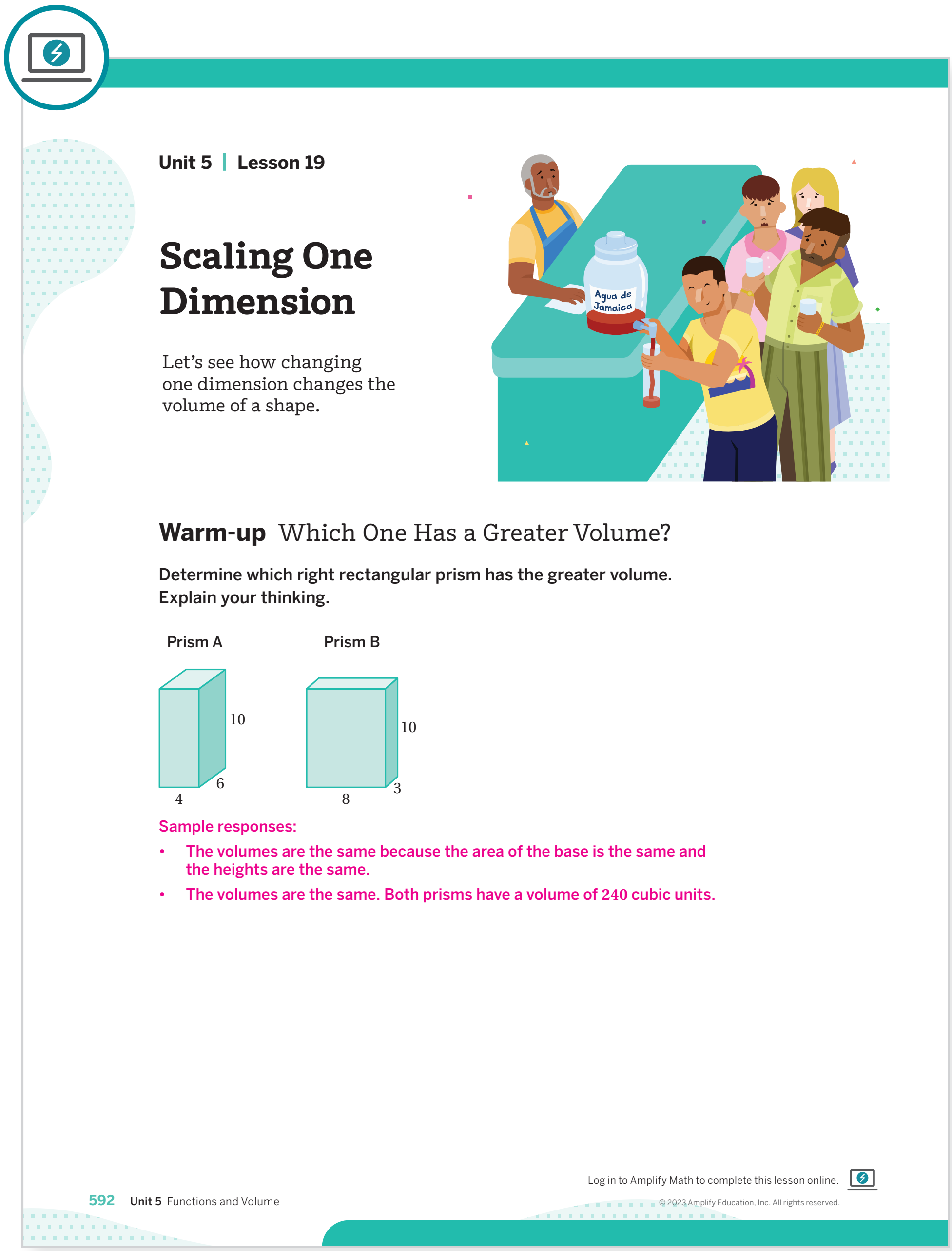
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Consider assigning half the class to **Activity 1** and the other half of the class to **Activity 2** and then share results to the entire class.

Independent | 5 min

Warm-up Which One Has a Greater Volume?

Students are reminded that the volume of a rectangular prism is unchanged when one dimension is halved, one dimension is doubled, and the other is the same.



Unit 5 | Lesson 19

Scaling One Dimension

Let's see how changing one dimension changes the volume of a shape.

Warm-up Which One Has a Greater Volume?

Determine which right rectangular prism has the greater volume. Explain your thinking.

Sample responses:

- The volumes are the same because the area of the base is the same and the heights are the same.
- The volumes are the same. Both prisms have a volume of 240 cubic units.

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by reviewing how to calculate the volume of a rectangular prism.

Look for productive strategies:

- Realizing the volumes will be the same without performing the calculations.
- Not choosing 4 by 6 or 8 by 3 as the base, but still seeing the connection of why the volumes are the same.

3 Connect

Display the prisms from the Student Edition.

Have students share their solutions and reasoning.

Highlight that the area of the bases are the same and the heights of the prisms are the same.

Ask:

- "How did the 4 by 6 edge lengths change to make 8 by 3 and how did this affect volume? One side was doubled, the other side was halved, and the volumes are the same.
- "Do you think those effects always have the same outcome with the volume? Why or why not?"
- "Can you think of another set of dimensions for this rectangular prism which produce the same volume?"

Power-up

To power up students' ability to understand how scaling dimensions affects volume, have students complete:

Determine the exact volume of a cylinder with each of the following dimensions.

1. A radius of 3 units and a height of 3 units. 27π cubic units
2. A radius of 6 units and a height of 3 units. 108π cubic units
3. A radius of 3 units and a height of 6 units. 54π cubic units

Use: Before the Warm-up

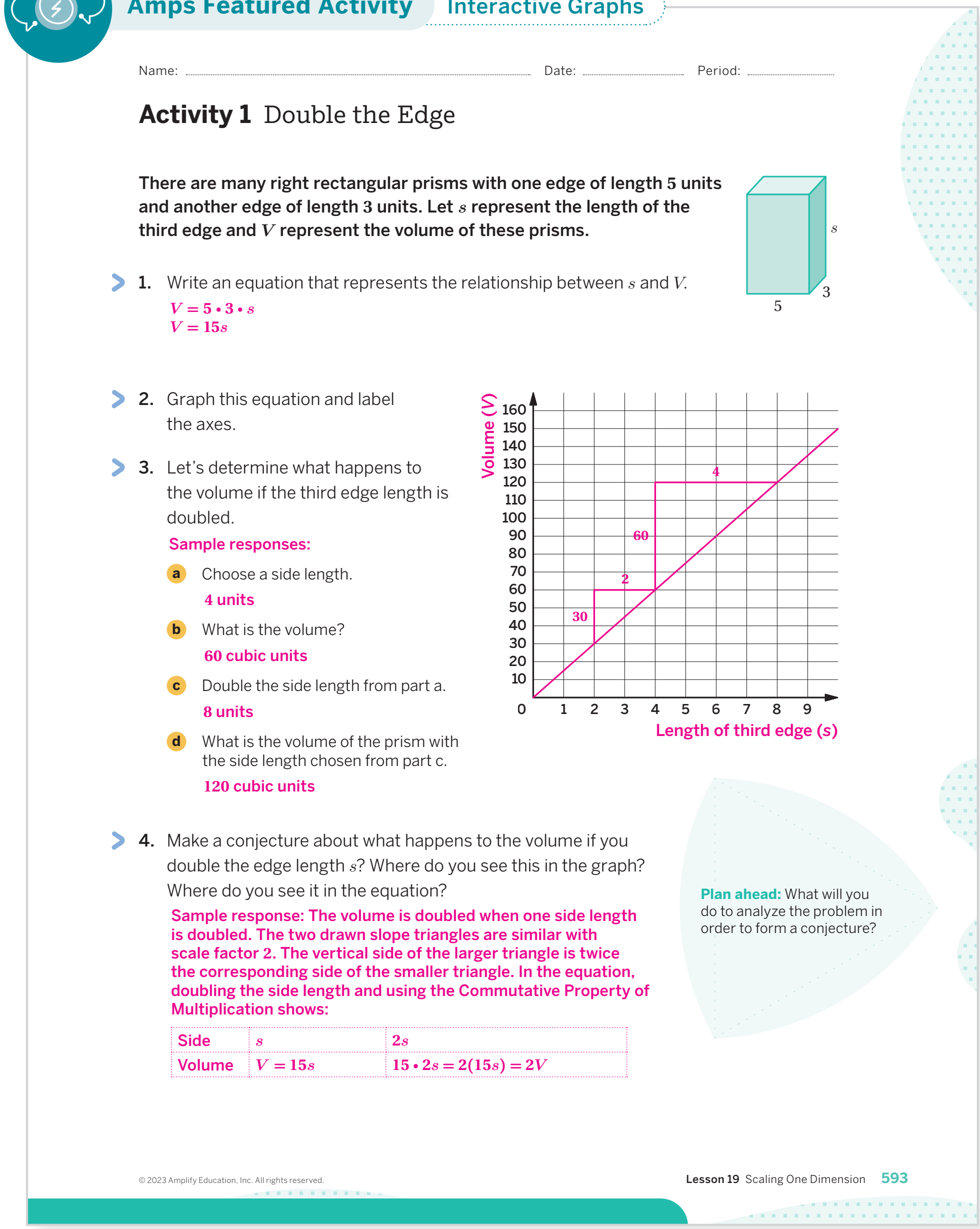
Informed by: Performance on Lesson 18, Practice Problem 5

Pairs | 15 min

Activity 1 Double the Edge

Students apply their knowledge of functions to investigate the effect of a change in one dimension on the volume of a rectangular prism.

Amps Featured Activity Interactive Graphs



Name: ________ Date: ________ Period: ________

Activity 1 Double the Edge

There are many right rectangular prisms with one edge of length 5 units and another edge of length 3 units. Let s represent the length of the third edge and V represent the volume of these prisms.

1. Write an equation that represents the relationship between s and V.

   $V = 5 \cdot 3 \cdot s$
   $V = 15s$

2. Graph this equation and label the axes.

3. Let's determine what happens to the volume if the third edge length is doubled.

   Sample responses:

   a. Choose a side length.
   4 units

   b. What is the volume?
   60 cubic units

   c. Double the side length from part a.
   8 units

   d. What is the volume of the prism with the side length chosen from part c.
   120 cubic units

4. Make a conjecture about what happens to the volume if you double the edge length s? Where do you see this in the graph? Where do you see it in the equation?

   Sample response: The volume is doubled when one side length is doubled. The two drawn slope triangles are similar with scale factor 2. The vertical side of the larger triangle is twice the corresponding side of the smaller triangle. In the equation, doubling the side length and using the Commutative Property of Multiplication shows:

Side	s	$2s$
Volume	$V = 15s$	$15 \cdot 2s = 2(15s) = 2V$

Plan ahead: What will you do to analyze the problem in order to form a conjecture?

1 Launch

Activate students' prior knowledge of functions and functional representation as they prepare to write equations and draw graphs.

2 Monitor

Help students get started by having them create a table with various side lengths and corresponding volume. Ask them to write a side length of a in the last row of the table for students to fill in the volume.

Look for points of confusion:

- **Struggling to see how the change in volume is reflected in the equation.** Have them start with values from the graph and substitute those into the equation to see how the volume changes.

Look for productive strategies:

- Using the function representations to support the idea that the volume doubles when s doubles.

3 Connect

Display student-created graphs and equations**.**

Ask, "Which of your variables is independent? "Dependent?" Which variable is a function of which?"

Have students share where they see the effect of doubling s in the graphs. Sequence student responses by starting with those using specific values and then following with those using a general representation.

Highlight the connections between the different representations by pointing out how the graph and the equation reflect an edge length that is doubled. If it has not been brought up in students' explanations, ask what the volume equation looks like when the edge length s is doubled. Display the volume equation $V = 15(2s)$. Ask, "How can this equation be rewritten to show that the volume doubled when s doubled?".

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their equation graphed immediately to save time, allowing them to focus on analyzing the effects of doubling the height.

Accessibility: *Guide Processing and Visualization*

Display the Anchor Chart PDF, Representations of Proportional *Relationships* (from Unit 3) throughout the activity for students to use as a reference. Remind them they previously learned about proportional relationships and functions and how they can be represented by graphs and equations.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "What part of the graph supports your reasoning?"
- "Can algebraic calculations be applied to tripling the side length?"

Have students revise their responses, as needed.

English Learners

Provide sentence frames for students to use as they craft or revise their responses, such as:

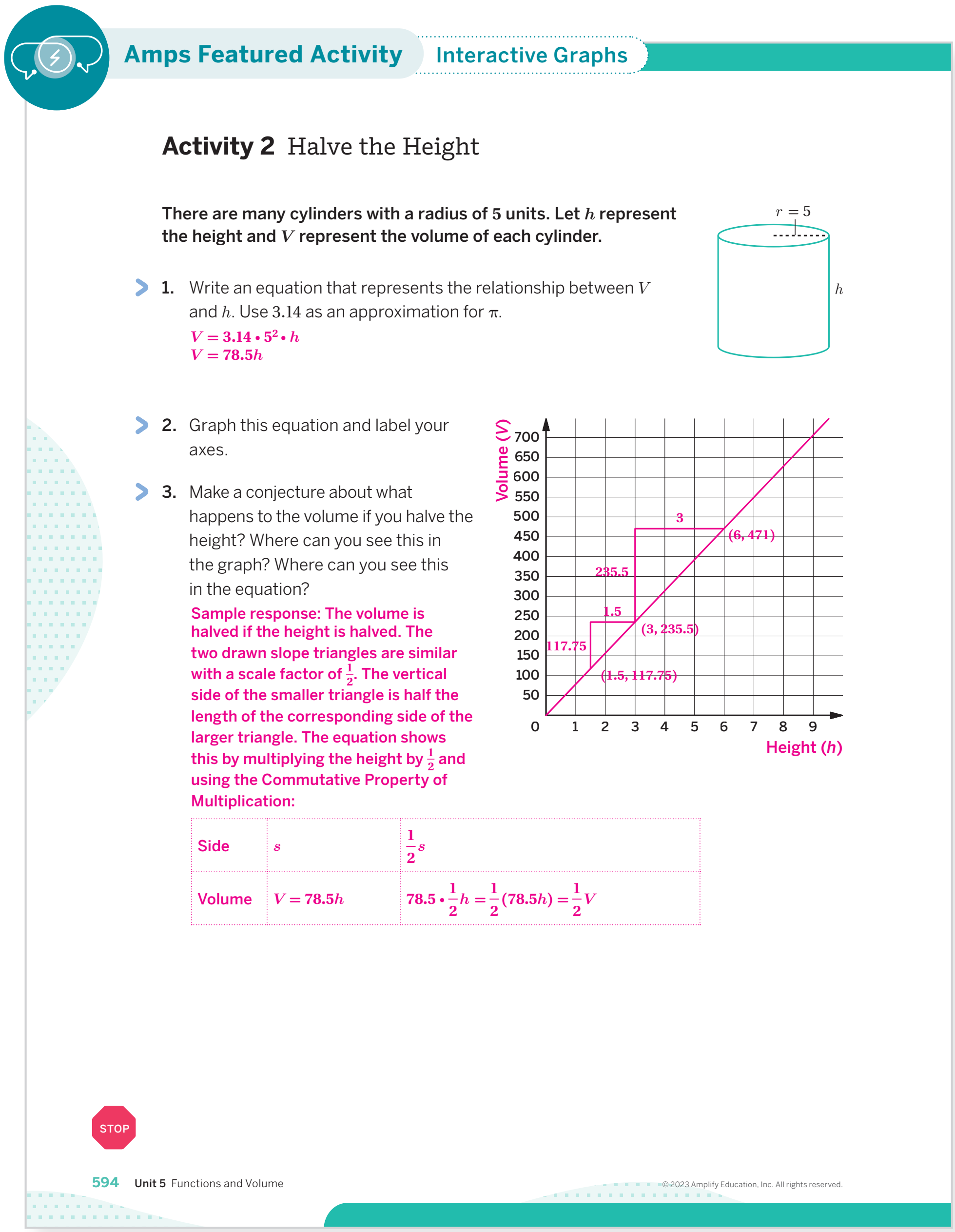
- "The volume is _____ when one side length is doubled."
- "The graph/equation shows that . . ."

Pairs | 15 min

Activity 2 Halve the Height

Students continue working with functions to investigate what happens to the volume of a cylinder when the height is halved.

Amps Featured Activity Interactive Graphs



Activity 2 Halve the Height

There are many cylinders with a radius of 5 units. Let h represent the height and V represent the volume of each cylinder.

1. Write an equation that represents the relationship between V and h. Use 3.14 as an approximation for π.
   $V = 3.14 \cdot 5^2 \cdot h$
   $V = 78.5h$
2. Graph this equation and label your axes.
3. Make a conjecture about what happens to the volume if you halve the height? Where can you see this in the graph? Where can you see this in the equation?
   Sample response: The volume is halved if the height is halved. The two drawn slope triangles are similar with a scale factor of $\frac{1}{2}$. The vertical side of the smaller triangle is half the length of the corresponding side of the larger triangle. The equation shows this by multiplying the height by $\frac{1}{2}$ and using the Commutative Property of Multiplication:

Side	s	$\frac{1}{2}s$
Volume	$V = 78.5h$	$78.5 \cdot \frac{1}{2}h = \frac{1}{2}(78.5h) = \frac{1}{2}V$

594 Unit 5 Functions and Volume

1 Launch

Let students know this activity is similar to Activity 1, but it involves a cylinder. Provide access to calculators.

2 Monitor

Help students get started by activating prior knowledge of how to find volume of a cylinder and reference the Anchor Chart PDF, *Volumes of Circular Solids*.

Look for points of confusion:

- **Halving the radius instead of the height.** Have students read the directions carefully and highlight all the places where height appears including on the figure and on the graph.

Look for productive strategies:

- Making connections between the equation and graphical representation of the function.
- Noticing similarities and differences among this activity and Activity 1.

3 Connect

Display student-created graphs and equations and have students share where they see the effect of halving h in the graphs.

Ask:

- "Compare the graph in this activity to the graph in the last activity. How are they alike? How are they different?"
- "Compare the equation in this activity to the equation in the last activity. How are they alike? How are they different?"
- "How can you tell that this is a linear function?"

Highlight that "it looks like a line" is insufficient evidence for saying a relationship is linear. It is important students connect that the equation is linear (of the form $y = mx + b$). If it has not been brought up in students' explanations, ask what the volume equation looks like when the height h is halved.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their equation graphed immediately to save time, allowing them to focus on analyzing the effects of halving the height.

Extension: *Math Enrichment*

Have students examine the structure of the volume formulas for a cylinder and a cone and explain why volume and height are in a proportional relationship, for a known radius. Sample response: For a known radius, such as 5 units, both volume formulas can be written in the form of a proportional equation, $y = kx$.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask questions, draw their attention to how the multiple representations of the volume function illustrate the effect on the volume when the height is halved (or doubled). Consider adding the following to the class display.

Height	Volume
When the height is multiplied by a scale factor . . .	. . . the volume is multiplied by the same scale factor.

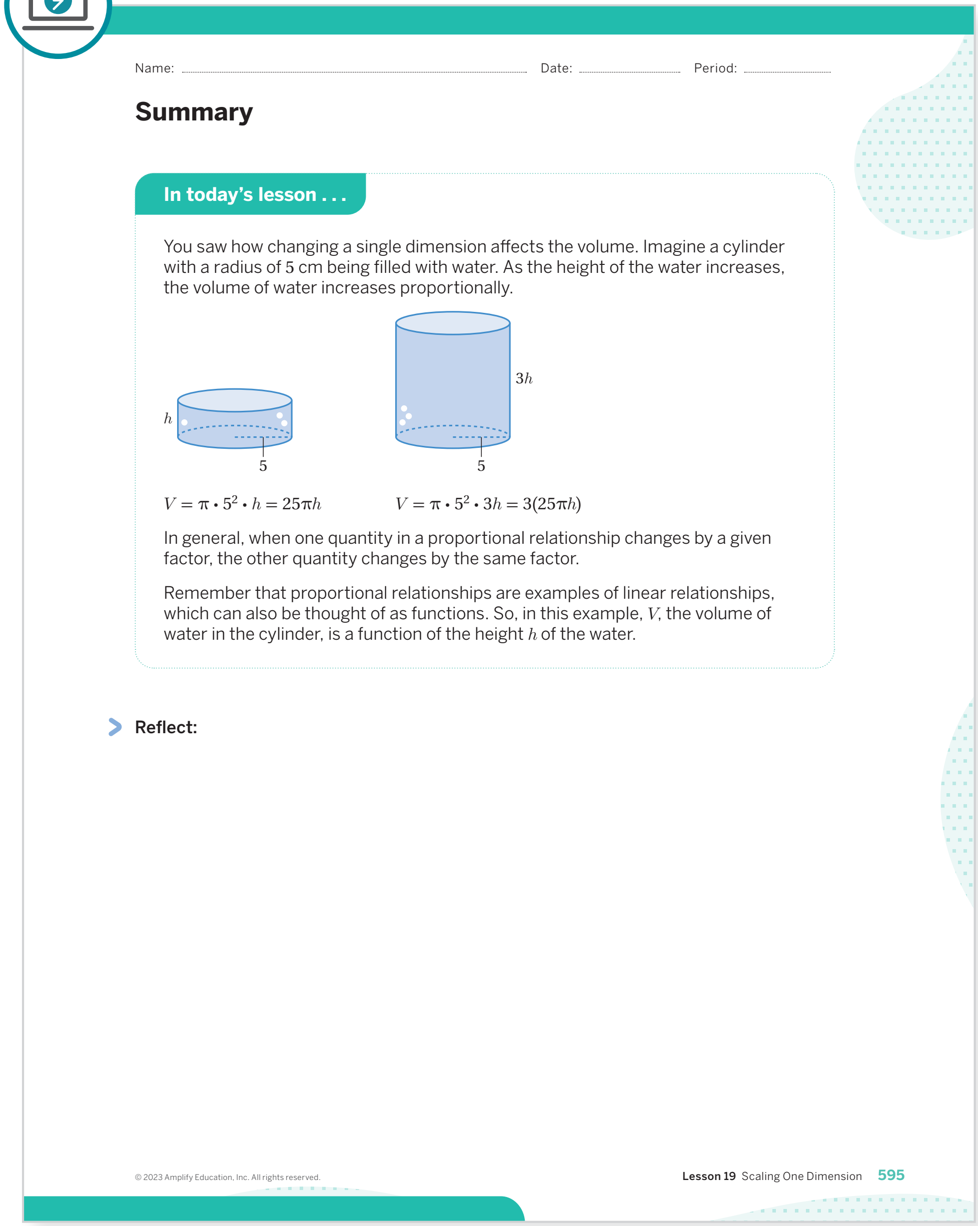
English Learners

Demonstrate that "halving a quantity" means to "take half of the quantity" or to "divide the quantity by 2."

Whole Class | 5 min

Summary

Review and synthesize that changing the height of a cylinder by a certain factor changes the volume of the cylinder by the same factor because height and volume are in a proportional relationship.



Name: Date: Period:

Summary

In today's lesson . . .

You saw how changing a single dimension affects the volume. Imagine a cylinder with a radius of 5 cm being filled with water. As the height of the water increases, the volume of water increases proportionally.

$V = \pi \cdot 5^2 \cdot h = 25\pi h$

$V = \pi \cdot 5^2 \cdot 3h = 3(25\pi h)$

In general, when one quantity in a proportional relationship changes by a given factor, the other quantity changes by the same factor.

Remember that proportional relationships are examples of linear relationships, which can also be thought of as functions. So, in this example, V, the volume of water in the cylinder, is a function of the height h of the water.

Reflect:

Synthesize

Display the Summary from the Student Edition.

Highlight that changing the height by a factor changes the volume by that same factor. This is because the height of a cylinder and its volume are in a proportional relationship.

Ask:

- "How can you change one dimension of the water tank so that the volume of the tank increases by a factor of 2?" Change the height of the tank by a factor of 2.
- "How can you change one dimension of the water tank so that the volume of the tank increases by a factor of a?" Change the height of the tank by a factor of a.
- "Does changing the radius by a certain factor have the same effect on the cylinder's volume? Explain your thinking." No, the radius is squared. Changing the radius by a certain factor actually means the volume of the cylinder is now multiplied by the square of that factor. The radius and volume of a cylinder are not in a proportional relationship.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

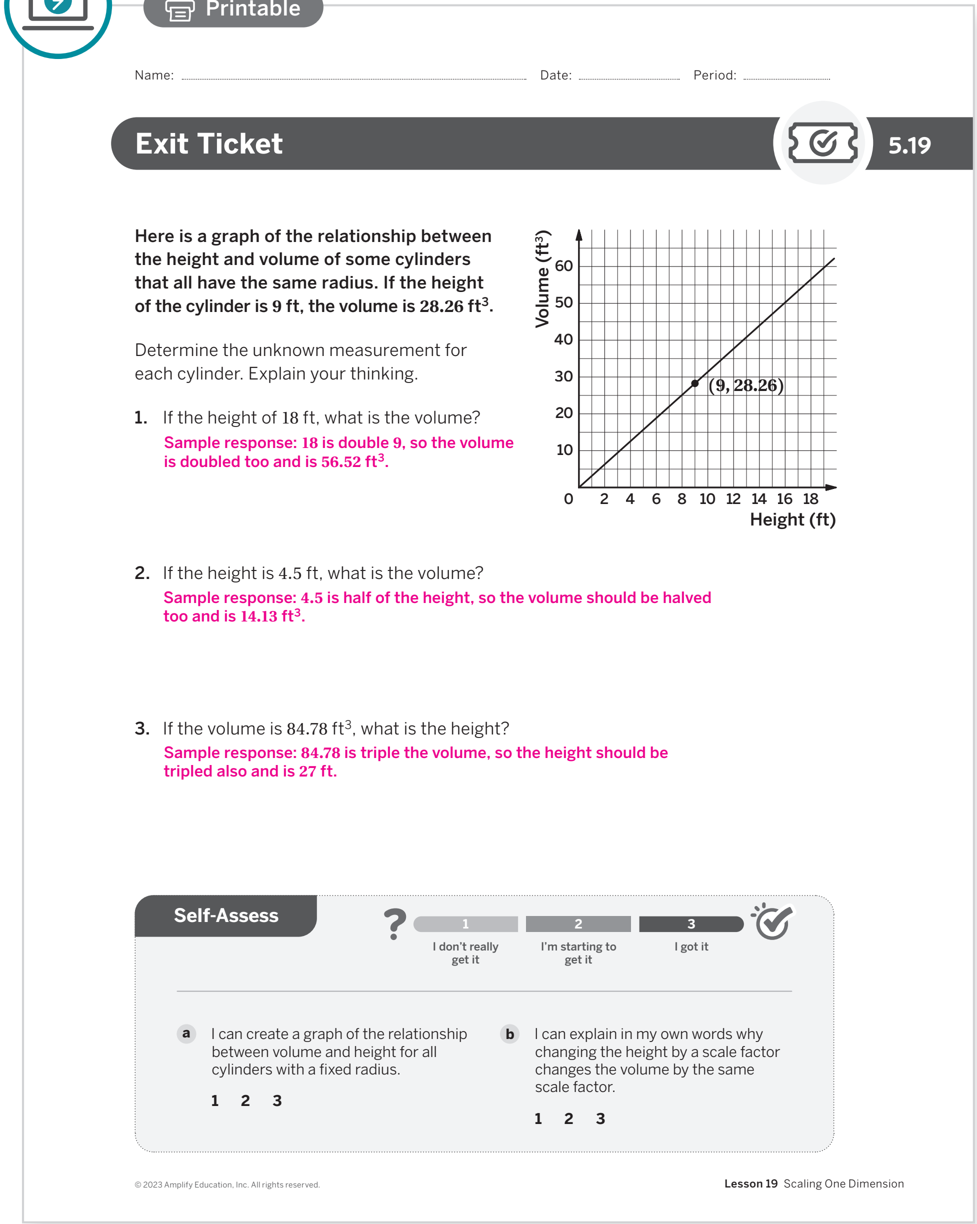
- "How does a change in height affect the volume of a cylinder or cone?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the volume or height after the other dimension is scaled.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.19

Here is a graph of the relationship between the height and volume of some cylinders that all have the same radius. If the height of the cylinder is 9 ft, the volume is 28.26 ft^3.

Determine the unknown measurement for each cylinder. Explain your thinking.

1. If the height of 18 ft, what is the volume?

   Sample response: 18 is double 9, so the volume is doubled too and is 56.52 ft^3.

2. If the height is 4.5 ft, what is the volume?

   Sample response: 4.5 is half of the height, so the volume should be halved too and is 14.13 ft^3.

3. If the volume is 84.78 ft^3, what is the height?

   Sample response: 84.78 is triple the volume, so the height should be tripled also and is 27 ft.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can create a graph of the relationship between volume and height for all cylinders with a fixed radius.

   1 2 3

b. I can explain in my own words why changing the height by a scale factor changes the volume by the same scale factor.

   1 2 3

Success looks like . . .

- **Language Goal:** Creating a graph and an equation to represent the function relationship between the volume of a right rectangular prism or a cylinder and its height, and justifying that the relationship is linear. **(Speaking and Listening)**
- **Language Goal:** Interpreting a point on a graph representing the volume of a cylinder as a function of its height, and explaining how changing one dimension affects the other. **(Speaking and Listening, Writing)**
  - » Explaining the change in volume in terms of the point (9, 28.26) in Problems 1–3.

Suggested next steps

If students incorrectly identify the corresponding values, consider:

- Encouraging students to check the points on the graph.
- Reviewing Activities 1 and 2.
- Assigning Practice Problems 1, 2, and 3.

Professional Learning

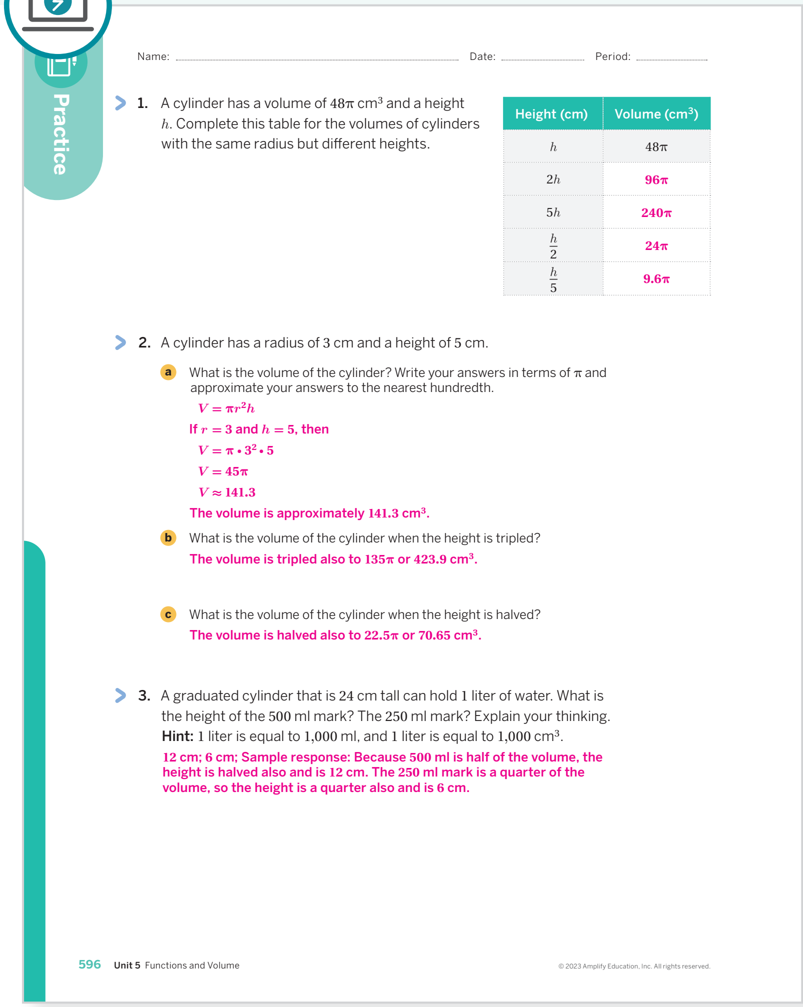
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did representing the proportional relationship go as planned?
- In what ways did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

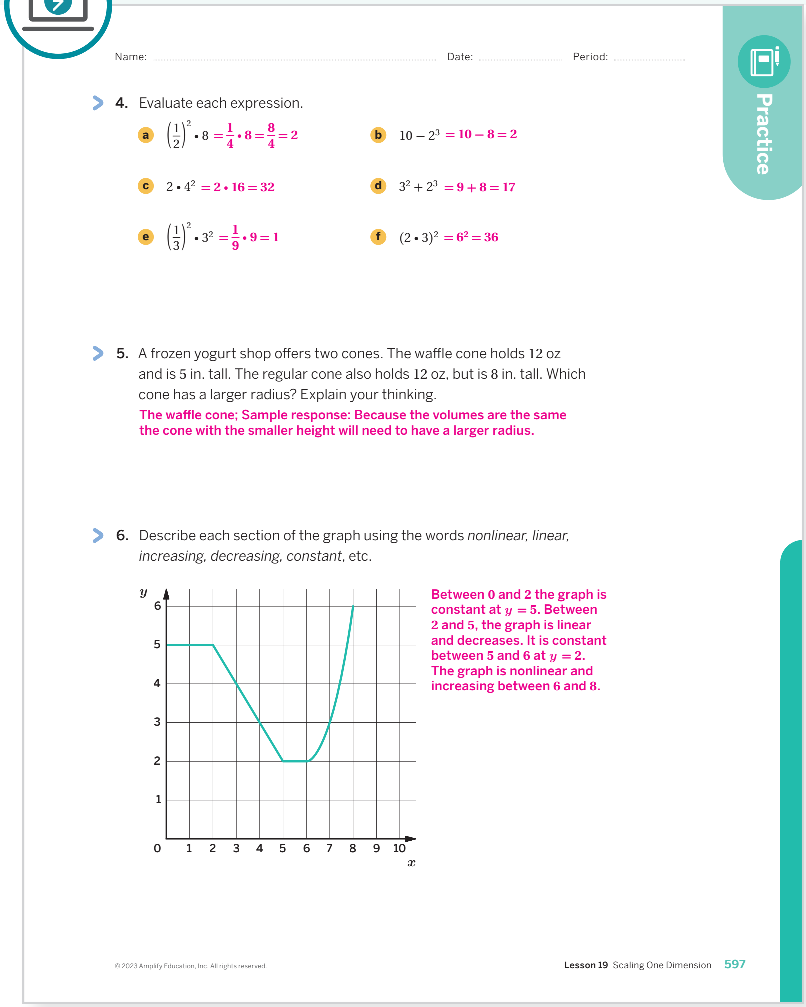
1. A cylinder has a volume of 48π cm^3 and a height h. Complete this table for the volumes of cylinders with the same radius but different heights.

Height (cm)	Volume (cm^3)
h	48π
$2h$	96π
$5h$	240π
$\frac{h}{2}$	24π
$\frac{h}{5}$	9.6π

2. A cylinder has a radius of 3 cm and a height of 5 cm.
   - a. What is the volume of the cylinder? Write your answers in terms of π and approximate your answers to the nearest hundredth.
     $V = \pi r^2 h$
     If $r = 3$ and $h = 5$, then
     $V = \pi \cdot 3^2 \cdot 5$
     $V = 45\pi$
     $V \approx 141.3$
     The volume is approximately 141.3 cm^3.
   - b. What is the volume of the cylinder when the height is tripled?
     The volume is tripled also to 135π or 423.9 cm^3.
   - c. What is the volume of the cylinder when the height is halved?
     The volume is halved also to 22.5π or 70.65 cm^3.

3. A graduated cylinder that is 24 cm tall can hold 1 liter of water. What is the height of the 500 ml mark? The 250 ml mark? Explain your thinking. **Hint:** 1 liter is equal to 1,000 ml, and 1 liter is equal to 1,000 cm^3.
   12 cm; 6 cm; Sample response: Because 500 ml is half of the volume, the height is halved also and is 12 cm. The 250 ml mark is a quarter of the volume, so the height is a quarter also and is 6 cm.

596 **Unit 5** Functions and Volume



Name: Date: Period:

4. Evaluate each expression.
   - a. $\left(\frac{1}{2}\right)^2 \cdot 8 = \frac{1}{4} \cdot 8 = \frac{8}{4} = 2$
   - b. $10 - 2^3 = 10 - 8 = 2$
   - c. $2 \cdot 4^2 = 2 \cdot 16 = 32$
   - d. $3^2 + 2^3 = 9 + 8 = 17$
   - e. $\left(\frac{1}{3}\right)^2 \cdot 3^2 = \frac{1}{9} \cdot 9 = 1$
   - f. $(2 \cdot 3)^2 = 6^2 = 36$

5. A frozen yogurt shop offers two cones. The waffle cone holds 12 oz and is 5 in. tall. The regular cone also holds 12 oz, but is 8 in. tall. Which cone has a larger radius? Explain your thinking.
   The waffle cone; Sample response: Because the volumes are the same the cone with the smaller height will need to have a larger radius.

6. Describe each section of the graph using the words *nonlinear, linear, increasing, decreasing, constant*, etc.
   Between 0 and 2 the graph is constant at $y = 5$. Between 2 and 5, the graph is linear and decreases. It is constant between 5 and 6 at $y = 2$. The graph is nonlinear and increasing between 6 and 8.

Lesson 19 Scaling One Dimension 597

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	3
Spiral	4	Grade 6	1
	5	Unit 5 Lesson 15	2
Formative ⚡	6	Unit 5 Lesson 20	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Optional

UNIT 5 | **LESSON 20**

Scaling Two Dimensions

Let's change more dimensions of shapes.

Focus

Goals

1. **Language Goal:** Compare and contrast graphs of linear and nonlinear functions. **(Speaking and Listening)**
2. **Language Goal:** Create an equation and a graph representing the volume of a cone as a function of its radius, and describe how a change in radius affects the volume. **(Speaking and Listening, Writing)**
3. **Language Goal:** Describe how changing the input of a certain nonlinear function affects the output. **(Speaking and Listening, Writing)**

Coherence

Today

Students see what happens to the volume when two of the dimensions are scaled. They consider the effects on the volume of a cone where the height is kept constant and the radius of the base is varied. They discover that the volume scales by the square of the factor and see examples of nonlinear functions arising from geometry. In general, if two dimensions are scaled by a, the volume is scaled by a^2.

Previously

In Lesson 19, students explored some proportional relationships that arise when the volume of a rectangular prism or cone is considered a function of one of its dimensions, such as side length or height. Students studied what happens to the volume of the figure when that dimension is scaled.

Coming Soon

In the Capstone Lesson, students will determine the amount of unused space when packing spheres or will determine the best container to ship spheres.

Rigor

- Students **apply** their understanding of nonlinear functions to the effects of scaling two dimensions and volume.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- calculators
- graphing technology (optional)

Math Language Development

Review words

- *cone*
- *cylinder*
- *function*
- *nonlinear function*
- *pi*
- *sphere*
- *volume*

Amps | Featured Activity

Activity 1
Interactive Graphs

Students can plot the solutions to an equation on a graph. You can overlay student answers to provide immediate feedback.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not see how their internal self-talk prevents them from modeling the relationship between the radius and volume during Activity 1. Include positive self-talk for students during the activity, and give them a minute to give themselves their own pep talk. Then have them approach the task with a new level of optimism.

Modifications to Pacing

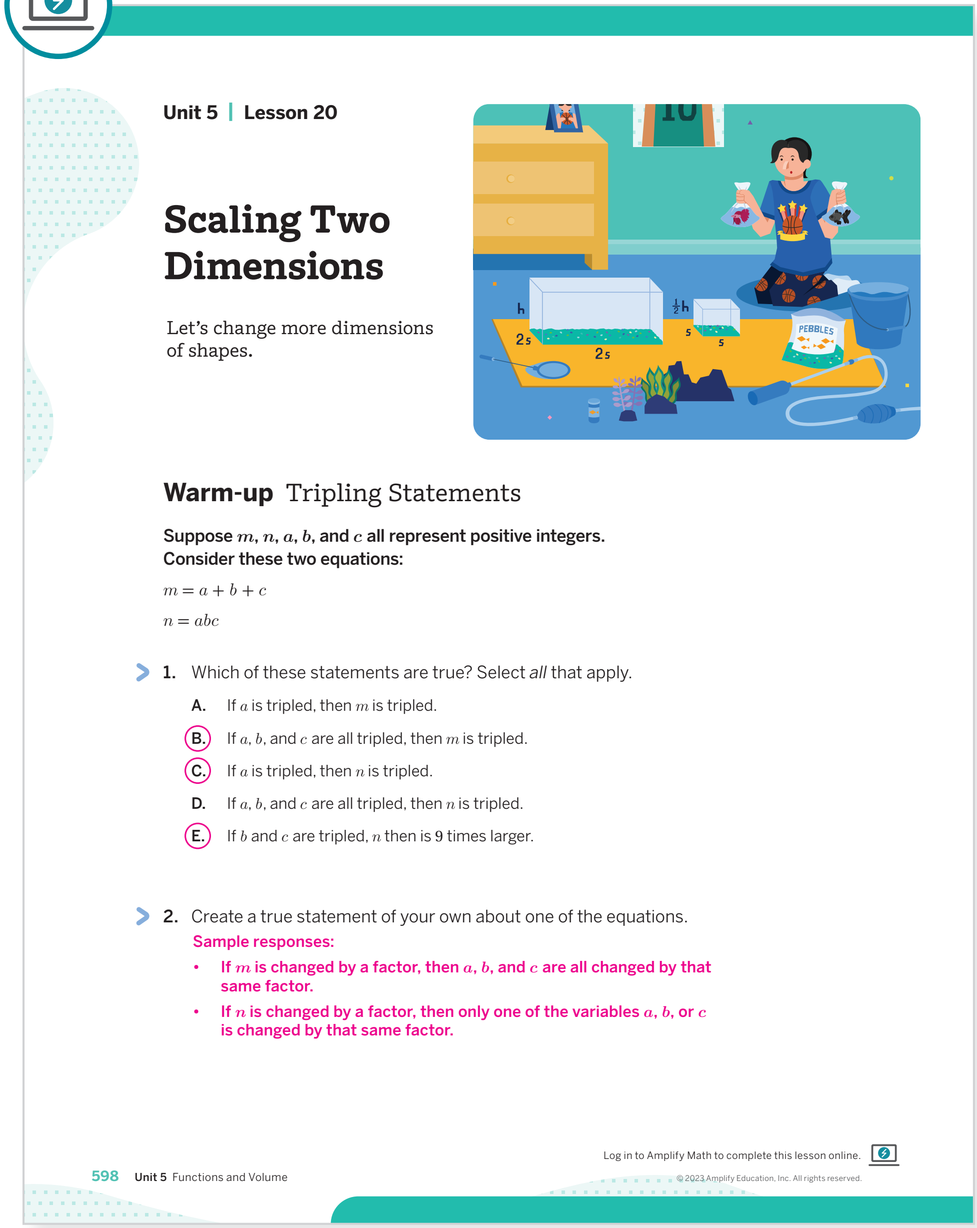
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Consider assigning each problem from **Activity 2** to a different group of students or assigning **Activity 2** after students have completed the Exponent and Scientific Notation unit.

Pairs | 5 min

Warm-up Tripling Statements

Students explore how scaling the addends or factors in an expression affects a sum or product.



Unit 5 | Lesson 20

Scaling Two Dimensions

Let's change more dimensions of shapes.

Warm-up Tripling Statements

Suppose m, n, a, b, and c all represent positive integers. Consider these two equations:

$m = a + b + c$

$n = abc$

1. Which of these statements are true? Select *all* that apply.
   - A. If a is tripled, then m is tripled.
   - [✓ B.] If a, b, and c are all tripled, then m is tripled.
   - [✓ C.] If a is tripled, then n is tripled.
   - D. If a, b, and c are all tripled, then n is tripled.
   - [✓ E.] If b and c are tripled, n then is 9 times larger.

2. Create a true statement of your own about one of the equations.
   Sample responses:
   - **If m is changed by a factor, then a, b, and c are all changed by that same factor.**
   - **If n is changed by a factor, then only one of the variables a, b, or c is changed by that same factor.**

Log in to Amplify Math to complete this lesson online.

598 Unit 5 Functions and Volume

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1 Launch

Activate prior knowledge by asking what it means to triple a number. Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by having them substitute numerical values into the equations to check which statements are true.

Look for points of confusion:

- **Thinking that if a is tripled, then m is tripled.** Have students substitute values to determine which statements are true.

Look for productive strategies:

- Testing numerical values to check the validity of the statements.
- Using the algebraic structure to show the statements are true for any values of the dimensions.

3 Connect

Display the problem and conduct the *Poll the Class* routine to determine which statements are true.

Have students share the reasoning for determining why statements were true or not true, including their examples or counterexamples.

Highlight the following concepts for Problem 1.

- Because $3a + 3b + 3c = 3\,(a + b + c)$, the statement in Choice B is true.
- Because $(3a)bc = 3\,(abc)$, the statement in Choice C is true.
- Because $a(3b)(3c) = 9\,(abc)$, the statement in Choice E is true.

Power-up

To power up students' ability to describe functions:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

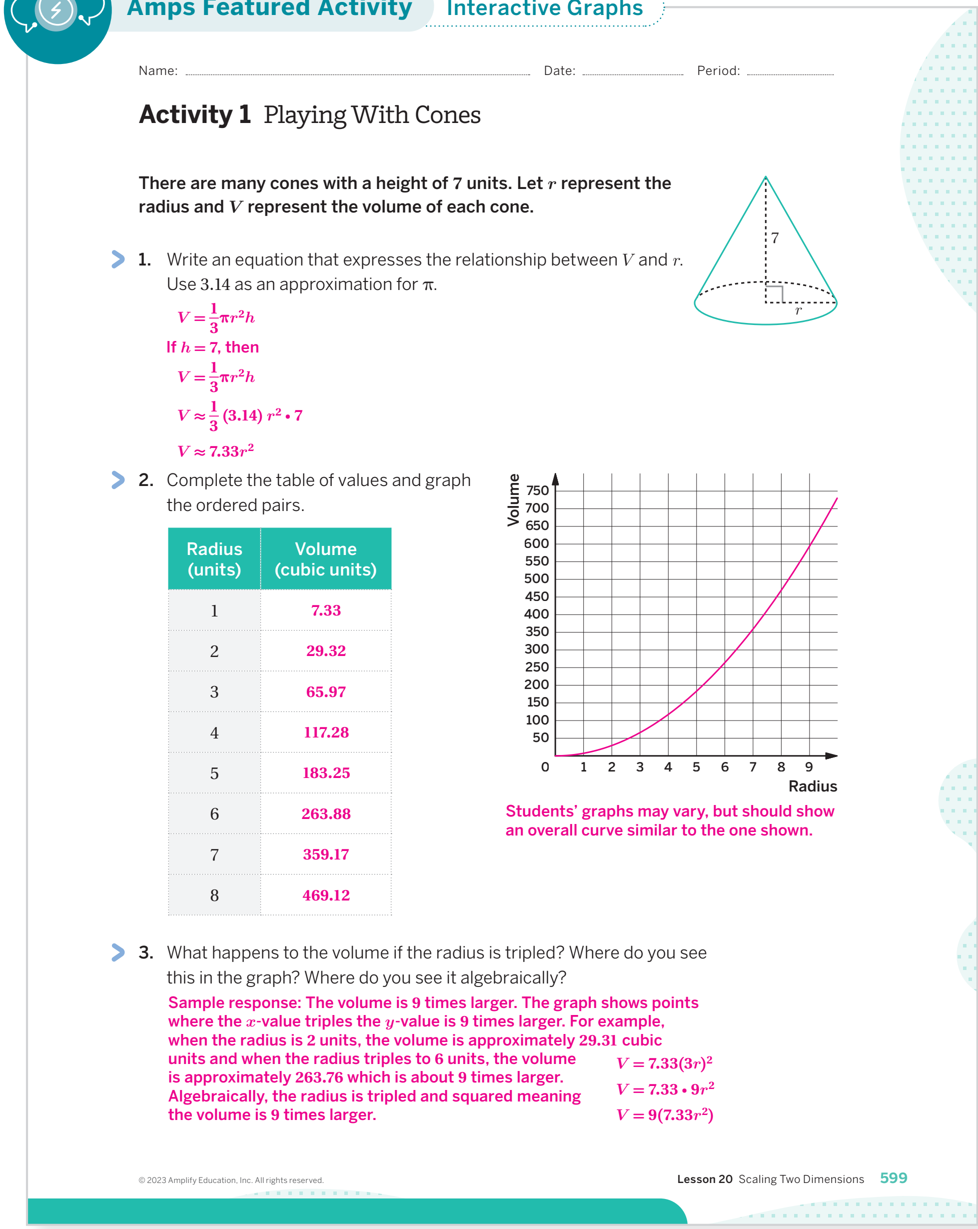
Informed by: Performance on Lesson 19, Practice Problem 6

Pairs | 15 min

Activity 1 Playing With Cones

Students analyze multiple representations of functions to investigate how changing the radius affects the volume of a cone with a fixed height.

Amps Featured Activity Interactive Graphs



Name: Date: Period:

Activity 1 Playing With Cones

There are many cones with a height of 7 units. Let r represent the radius and V represent the volume of each cone.

1. Write an equation that expresses the relationship between V and r. Use 3.14 as an approximation for π.

$V = \frac{1}{3}\pi r^2 h$

If $h = 7$, then

$V = \frac{1}{3}\pi r^2 h$

$V \approx \frac{1}{3}(3.14)\ r^2 \cdot 7$

$V \approx 7.33r^2$

2. Complete the table of values and graph the ordered pairs.

Radius (units)	Volume (cubic units)
1	7.33
2	29.32
3	65.97
4	117.28
5	183.25
6	263.88
7	359.17
8	469.12

Students' graphs may vary, but should show an overall curve similar to the one shown.

3. What happens to the volume if the radius is tripled? Where do you see this in the graph? Where do you see it algebraically?

Sample response: The volume is 9 times larger. The graph shows points where the x-value triples the y-value is 9 times larger. For example, when the radius is 2 units, the volume is approximately 29.31 cubic units and when the radius triples to 6 units, the volume is approximately 263.76 which is about 9 times larger. Algebraically, the radius is tripled and squared meaning the volume is 9 times larger.

$V = 7.33(3r)^2$

$V = 7.33 \cdot 9r^2$

$V = 9(7.33r^2)$

Lesson 20 Scaling Two Dimensions 599

1 Launch

Ask students to make a conjecture about what would happen to the volume of the cone if the radius was tripled. Consider writing predictions on the board to reference during the Connect. Distribute calculators and consider providing access to graphing technology for Problem 2.

2 Monitor

Help students get started by reminding them they examined a similar relationship in Lesson 19. Encourage students to review their work.

Look for points of confusion:

- **Calculating $(3r)^2$ to mean $2 \cdot 3r$ or $3r^2$.** Remind students what squaring a term involves and encourage them to expand the term to $3r \cdot 3r$.

Look for productive strategies:

- Recognizing the relationship is of a nonlinear function.

3 Connect

Display student responses to each problem.

Have students share whether their predictions from the Launch were correct, based on their solutions to the problems.

Highlight how the graph and equation show that when the radius is tripled, the volume is 9 times greater.

Ask:

- If the radius was quadrupled, how would the volume change?" By a factor of 4^2, or 16.
- "If the radius was halved, how would the volume change?" By a factor of $\left(\frac{1}{2}\right)^2$, or $\frac{1}{4}$.
- "If the radius was scaled by an unknown factor a, how would the volume change?" The new volume would be a^2 times the original volume.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their equation graphed immediately to save time, allowing them to focus on analyzing the effects of tripling the radius.

Accessibility: *Guide Processing and Visualization*

Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity.

Math Language Development

MLR3: Critique, Correct, Clarify

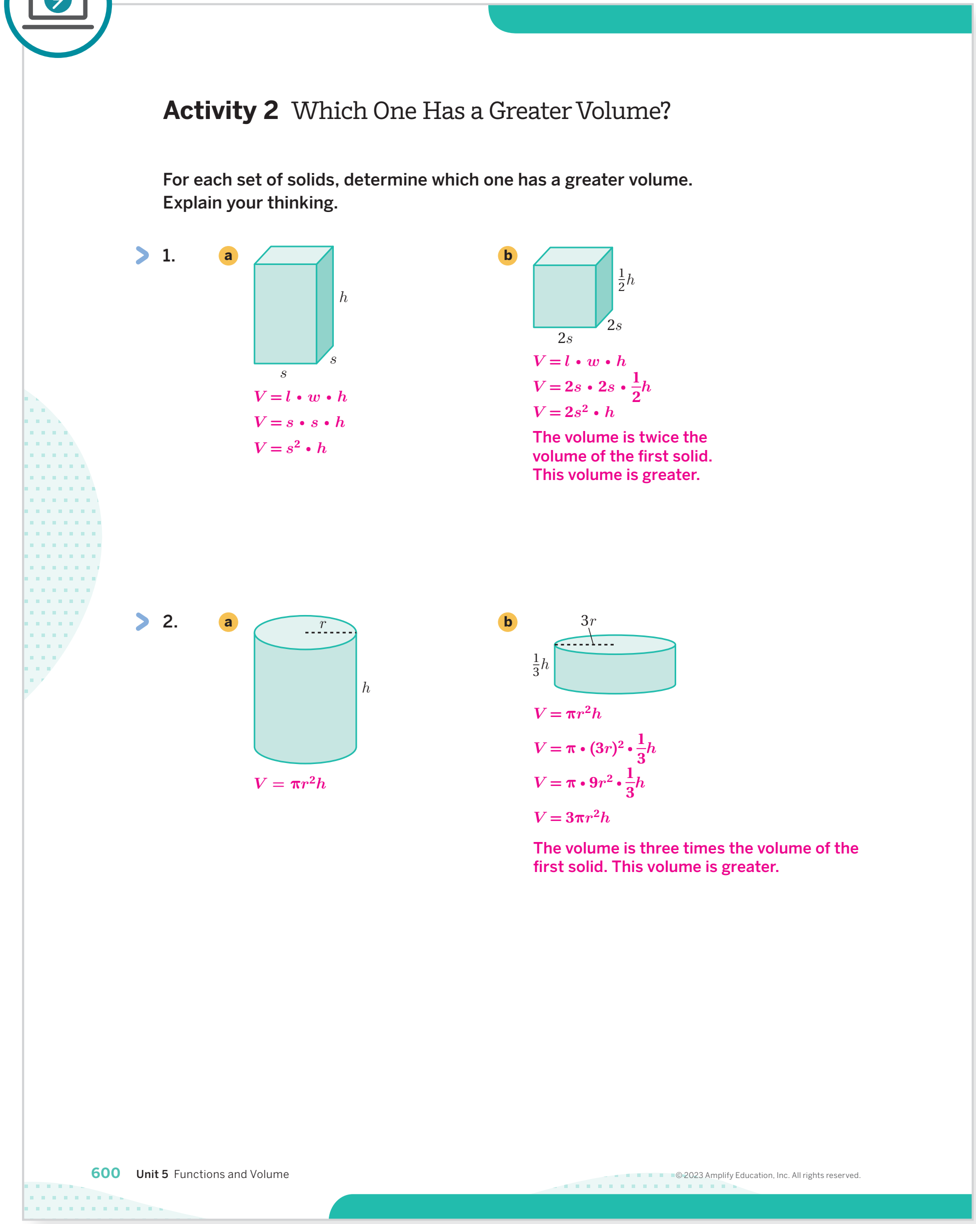
During the Connect, present an incorrect response to Problem 3, such as, "The volume will be 3 times greater because that's what *triple* means and volume and radius are in a proportional relationship." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Does the graph show a proportional relationship? Does the equation show a proportional relationship?"
- ***Correct and Clarify:*** "How would you correct this statement? Why is the volume tripled when the height is tripled, but the volume is not tripled when the radius is tripled?"

Pairs | 15 min

Activity 2 Which One Has a Greater Volume?

Students compare solids to determine which solid has a greater volume, to help solidify their understanding of the effect of changing one dimension has on the volume of the solid.



Activity 2 Which One Has a Greater Volume?

For each set of solids, determine which one has a greater volume. Explain your thinking.

1\. a

$V = l \cdot w \cdot h$

$V = s \cdot s \cdot h$

$V = s^2 \cdot h$

b

$V = l \cdot w \cdot h$

$V = 2s \cdot 2s \cdot \frac{1}{2}h$

$V = 2s^2 \cdot h$

The volume is twice the volume of the first solid. This volume is greater.

2\. a

$V = \pi r^2 h$

b

$V = \pi r^2 h$

$V = \pi \cdot (3r)^2 \cdot \frac{1}{3}h$

$V = \pi \cdot 9r^2 \cdot \frac{1}{3}h$

$V = 3\pi r^2 h$

The volume is three times the volume of the first solid. This volume is greater.

600 Unit 5 Functions and Volume

1 Launch

Activate prior knowledge of what it means to square a term and revisit expanding squaring a term.

2 Monitor

Help students get started by having them substitute numerical values into the appropriate volume formula.

Look for points of confusion:

- **Incorrectly squaring a term.** Have students expand the term before performing the multiplication.

Look for productive strategies:

- Knowing the effect on the volume without manipulating the volume formula. Encourage these students to explain their thinking.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2.

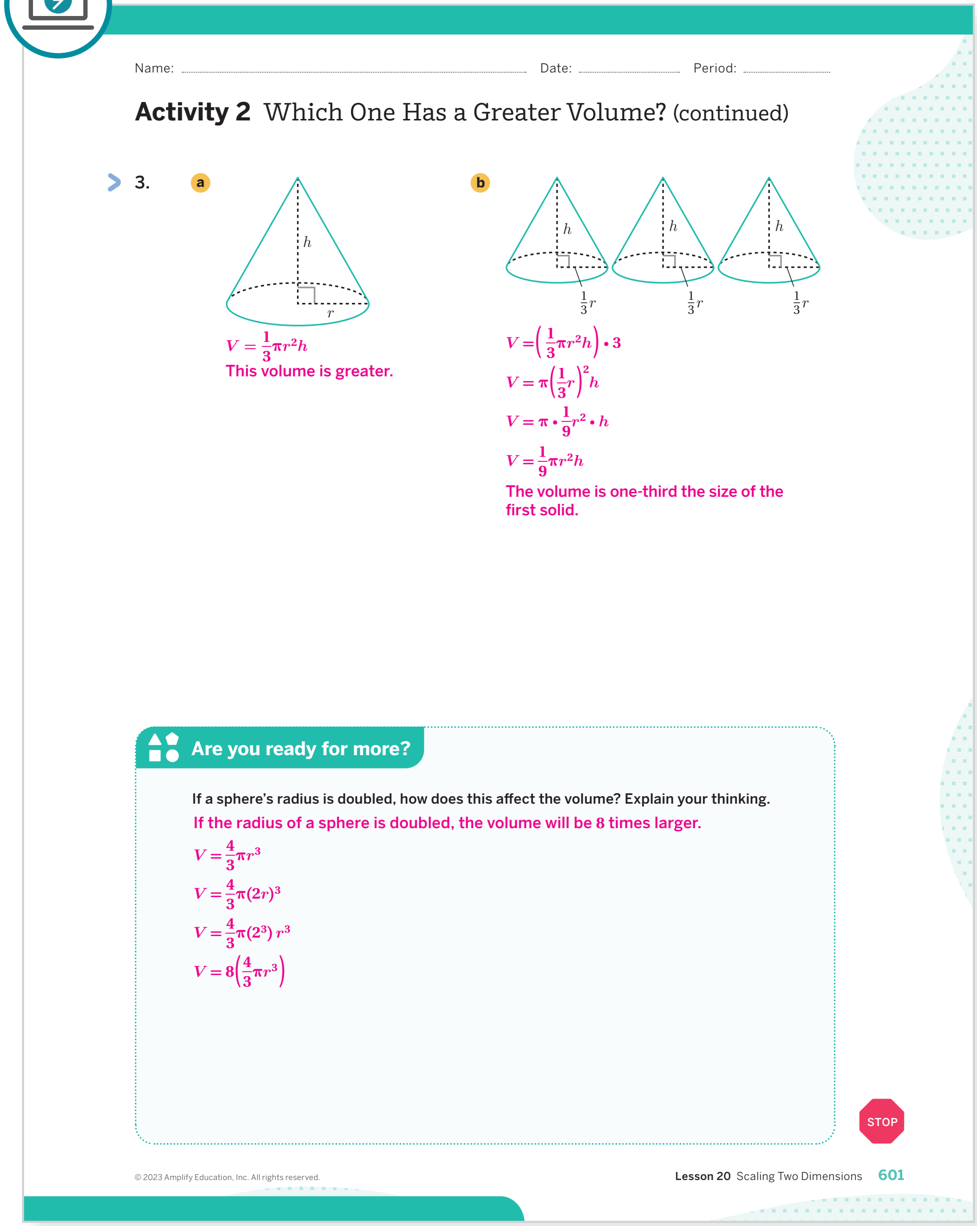
Accessibility: *Guide Processing and Visualization*

Suggest that students begin by annotating how the second figure compares to the first figure for each problem. For example, for Problem 1, they could note that the base side lengths were doubled and the height was halved. This will help them make sense of the figures.

Pairs | 15 min

Activity 2 Which One Has a Greater Volume? (continued)

Students compare solids to determine which solid has a greater volume, to help solidify their understanding of the effect of changing one dimension has on the volume of the solid.



Name: Date: Period:

Activity 2 Which One Has a Greater Volume? (continued)

3. a

$V = \frac{1}{3}\pi r^2 h$

This volume is greater.

b

$V = \left(\frac{1}{3}\pi r^2 h\right) \cdot 3$

$V = \pi\left(\frac{1}{3}r\right)^2 h$

$V = \pi \cdot \frac{1}{9}r^2 \cdot h$

$V = \frac{1}{9}\pi r^2 h$

The volume is one-third the size of the first solid.

Are you ready for more?

If a sphere's radius is doubled, how does this affect the volume? Explain your thinking.

If the radius of a sphere is doubled, the volume will be 8 times larger.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (2r)^3$

$V = \frac{4}{3}\pi (2^3)\, r^3$

$V = 8\left(\frac{4}{3}\pi r^3\right)$

STOP

Lesson 20 Scaling Two Dimensions 601

3 Connect

Display any figures necessary to help with discussion.

Have students share their solutions and reasoning on determining which volume was greater.

Highlight that, in Problem 1, two dimensions are doubled and one dimension is halved and that is why the volume of part b is greater. In Problem 2, the radius is tripled, but the height is a third of the original height and that is why the volume of part b is greater. In Problem 3, the radius is a third of the original radius, but the number of cones is tripled.

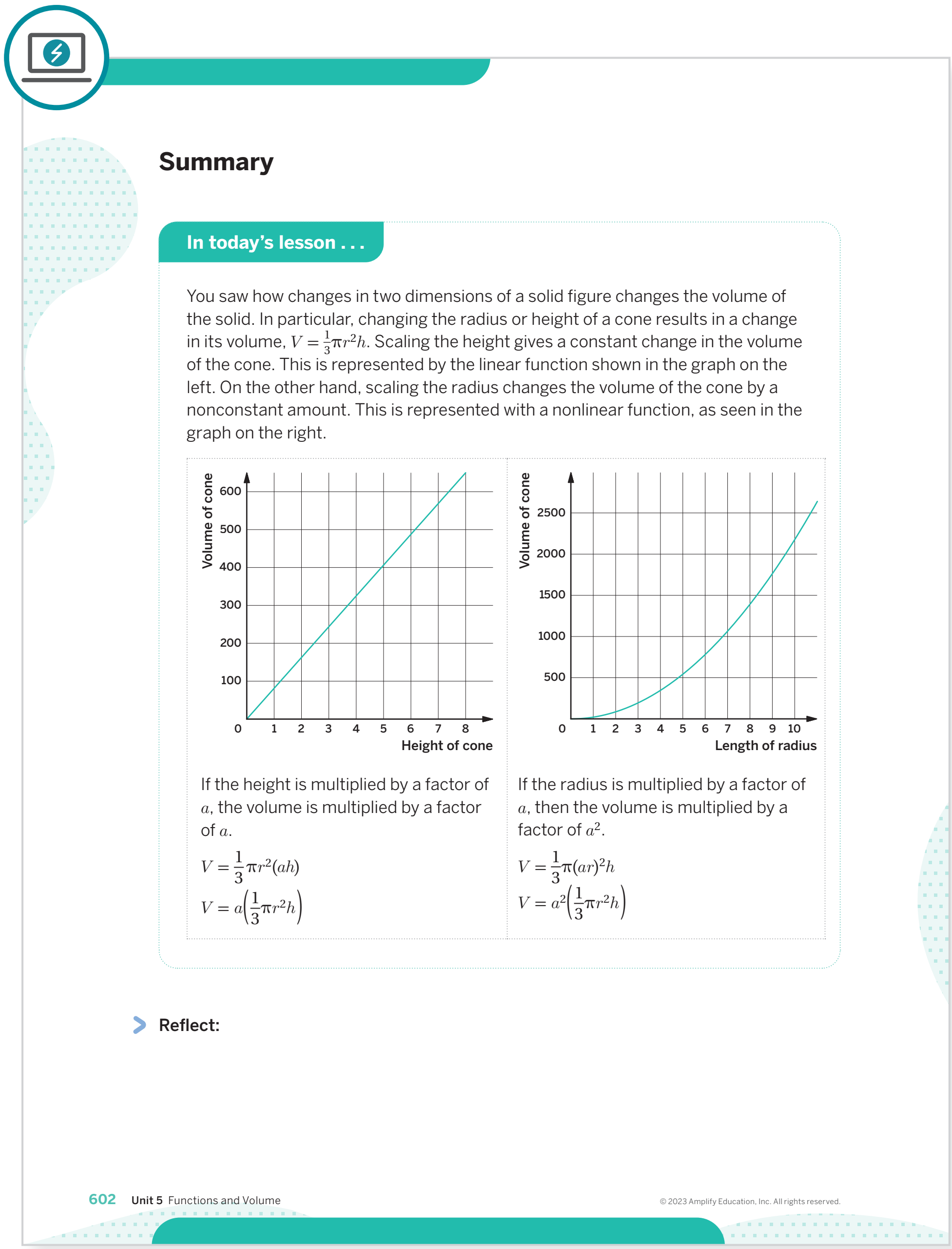
Ask, "Imagine a cone where $r = s$, a cylinder with $h = s$, and a cube with a side of s. Changing s will have the greater effect on which figure's volume?"

Note: Students will have more opportunities to practice with rules of exponents during Unit 6.

Whole Class | 5 min

Summary

Review and synthesize how changing the dimensions of a solid figure affects the volume of the figure.



Summary

In today's lesson . . .

You saw how changes in two dimensions of a solid figure changes the volume of the solid. In particular, changing the radius or height of a cone results in a change in its volume, $V = \frac{1}{3}\pi r^2 h$. Scaling the height gives a constant change in the volume of the cone. This is represented by the linear function shown in the graph on the left. On the other hand, scaling the radius changes the volume of the cone by a nonconstant amount. This is represented with a nonlinear function, as seen in the graph on the right.

If the height is multiplied by a factor of a, the volume is multiplied by a factor of a.

$V = \frac{1}{3}\pi r^2(ah)$

$V = a\left(\frac{1}{3}\pi r^2 h\right)$

If the radius is multiplied by a factor of a, then the volume is multiplied by a factor of a^2.

$V = \frac{1}{3}\pi(ar)^2 h$

$V = a^2\left(\frac{1}{3}\pi r^2 h\right)$

Reflect:

602 Unit 5 Functions and Volume

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Synthesize

Display the graphs from the Student Edition and ask, "What do these graphs represent? How are they similar? How are they different?"

Highlight how changing a single dimension affects the volume by that same factor and how changing two dimensions affects the volume by the square of that factor.

Ask, "If all three dimensions were changed by the same factor, how do you expect the volume to change?" The volume will change by the cube of the factor.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

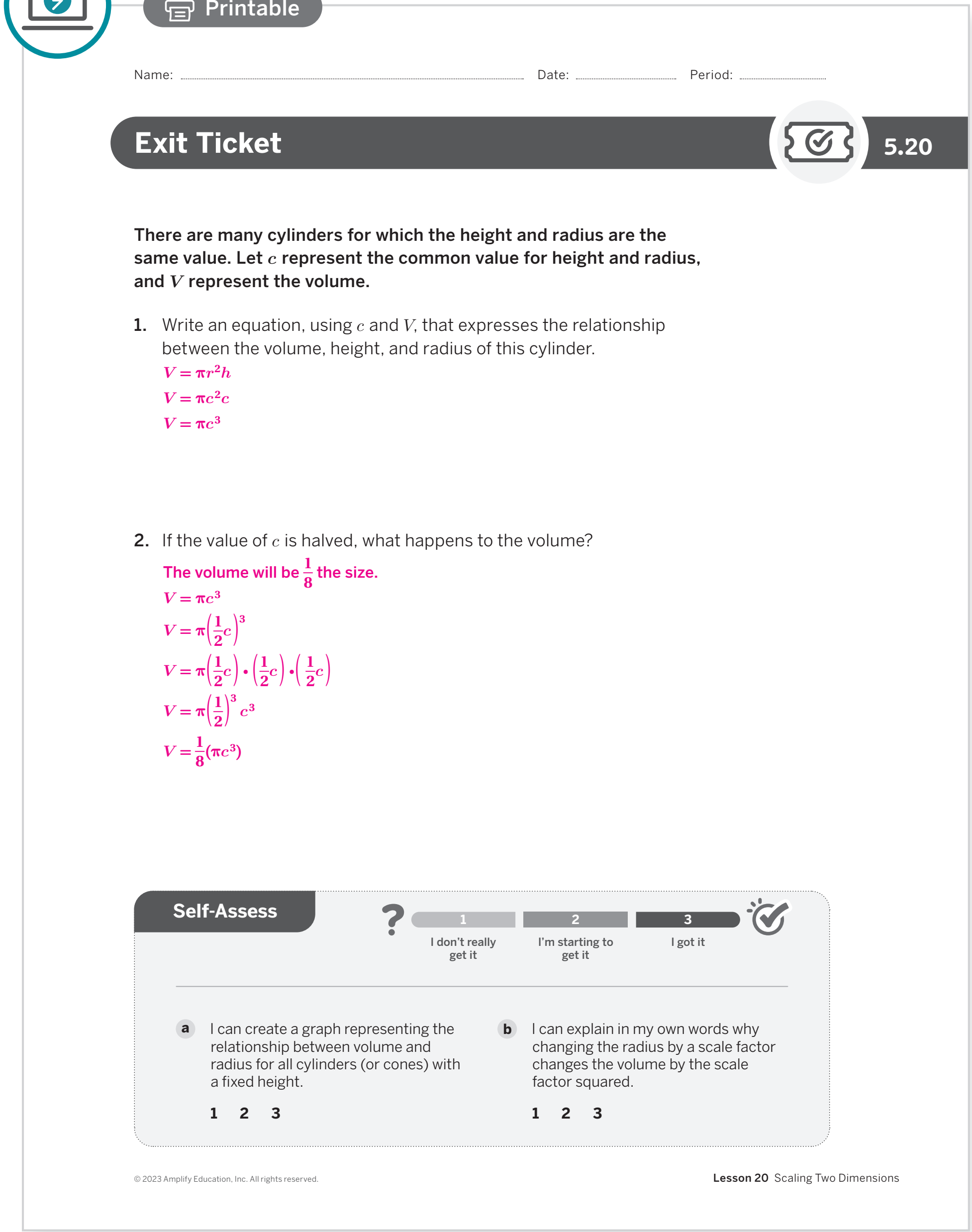
- "How does changing two dimensions affect the volume?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining how halving the radius of a cylinder affects its volume.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 5.20

There are many cylinders for which the height and radius are the same value. Let c represent the common value for height and radius, and V represent the the volume.

1. Write an equation, using c and V, that expresses the relationship between the volume, height, and radius of this cylinder.

   $V = \pi r^2 h$

   $V = \pi c^2 c$

   $V = \pi c^3$

2. If the value of c is halved, what happens to the volume?

   The volume will be $\frac{1}{8}$ the size.

   $V = \pi c^3$

   $V = \pi\left(\frac{1}{2}c\right)^3$

   $V = \pi\left(\frac{1}{2}c\right) \cdot \left(\frac{1}{2}c\right) \cdot \left(\frac{1}{2}c\right)$

   $V = \pi\left(\frac{1}{2}\right)^3 c^3$

   $V = \frac{1}{8}(\pi c^3)$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can create a graph representing the relationship between volume and radius for all cylinders (or cones) with a fixed height.

1 2 3

b I can explain in my own words why changing the radius by a scale factor changes the volume by the scale factor squared.

1 2 3

Success looks like . . .

- **Language Goal:** Comparing and contrasting graphs of linear and nonlinear functions. **(Speaking and Listening)**
- **Language Goal:** Creating an equation and a graph representing the volume of a cone as a function of its radius, and describing how a change in radius affects the volume. **(Speaking and Listening, Writing)**
  - » Writing an equation relating c and V in Problem 1.
- **Language Goal:** Describing how changing the input of a certain nonlinear function affects the output. **(Speaking and Listening, Writing)**
  - » Explaining that if c is halved, V reduces to $\frac{1}{8}$ of the original volume in Problem 2.

Suggested next steps

If students represent the radius being halved correctly but incorrectly determine the value of the volume, consider:

- Checking in with these students during Unit 6.
- Reassigning this Exit Ticket during Unit 6.
- Assigning Practice Problems 1 and 2.

Professional Learning

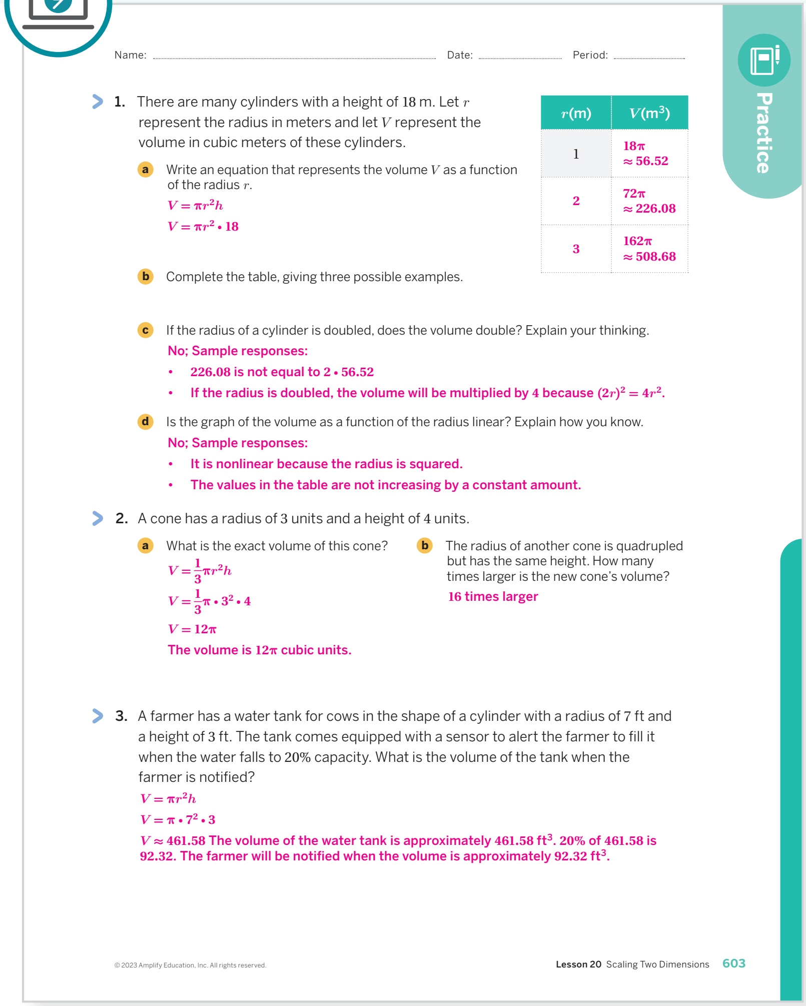
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- What did you see in the way some students approached creating an equation and a graph representing the volume of a cone as a function of its radius that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. There are many cylinders with a height of 18 m. Let r represent the radius in meters and let V represent the volume in cubic meters of these cylinders.

   a. Write an equation that represents the volume V as a function of the radius r.

   $V = \pi r^2 h$

   $V = \pi r^2 \cdot 18$

   b. Complete the table, giving three possible examples.

r(m)	V(m^3)
1	18π ≈ 56.52
2	72π ≈ 226.08
3	162π ≈ 508.68

   c. If the radius of a cylinder is doubled, does the volume double? Explain your thinking.

   No; Sample responses:
   - 226.08 is not equal to $2 \cdot 56.52$
   - If the radius is doubled, the volume will be multiplied by 4 because $(2r)^2 = 4r^2$.

   d. Is the graph of the volume as a function of the radius linear? Explain how you know.

   No; Sample responses:
   - It is nonlinear because the radius is squared.
   - The values in the table are not increasing by a constant amount.

2. A cone has a radius of 3 units and a height of 4 units.

   a. What is the exact volume of this cone?

   $V = \frac{1}{3}\pi r^2 h$

   $V = \frac{1}{3}\pi \cdot 3^2 \cdot 4$

   $V = 12\pi$

   The volume is 12π cubic units.

   b. The radius of another cone is quadrupled but has the same height. How many times larger is the new cone's volume?

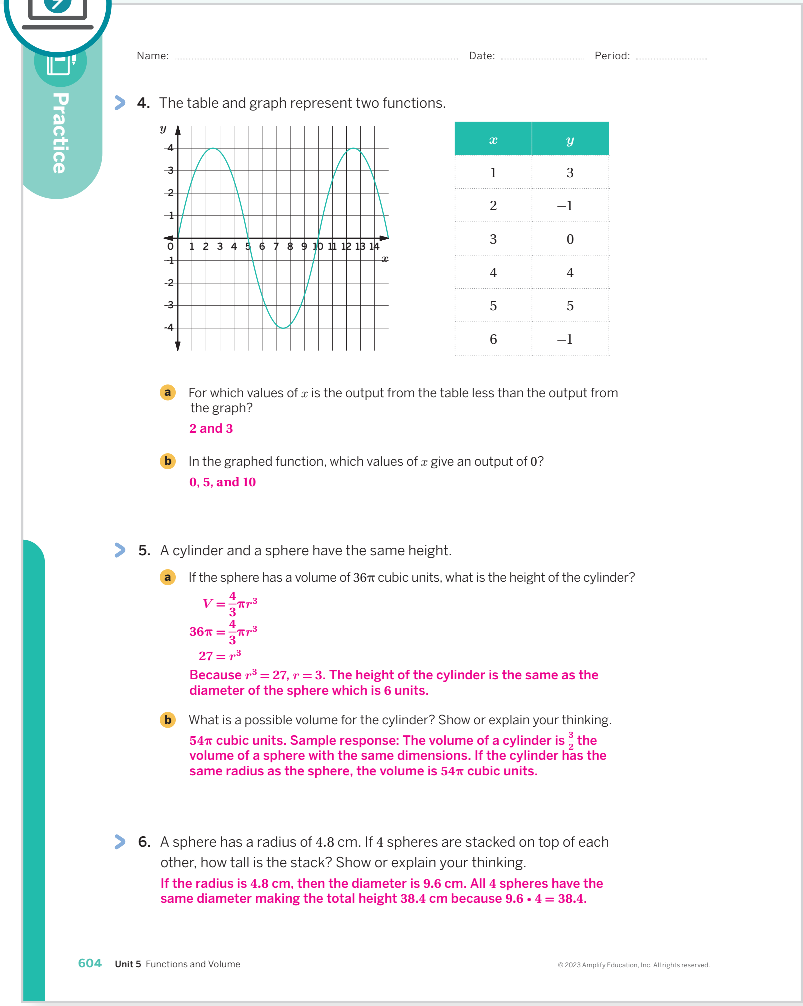
   16 times larger

3. A farmer has a water tank for cows in the shape of a cylinder with a radius of 7 ft and a height of 3 ft. The tank comes equipped with a sensor to alert the farmer to fill it when the water falls to 20% capacity. What is the volume of the tank when the farmer is notified?

   $V = \pi r^2 h$

   $V = \pi \cdot 7^2 \cdot 3$

   $V \approx 461.58$ The volume of the water tank is approximately 461.58 ft^3. 20% of 461.58 is 92.32. The farmer will be notified when the volume is approximately 92.32 ft^3.



Name: Date: Period:

4. The table and graph represent two functions.

x	y
1	3
2	–1
3	0
4	4
5	5
6	–1

   a. For which values of x is the output from the table less than the output from the graph?

   2 and 3

   b. In the graphed function, which values of x give an output of 0?

   0, 5, and 10

5. A cylinder and a sphere have the same height.

   a. If the sphere has a volume of 36π cubic units, what is the height of the cylinder?

   $V = \frac{4}{3}\pi r^3$

   $36\pi = \frac{4}{3}\pi r^3$

   $27 = r^3$

   Because $r^3 = 27$, $r = 3$. The height of the cylinder is the same as the diameter of the sphere which is 6 units.

   b. What is a possible volume for the cylinder? Show or explain your thinking.

   54π cubic units. Sample response: The volume of a cylinder is $\frac{3}{2}$ the volume of a sphere with the same dimensions. If the cylinder has the same radius as the sphere, the volume is 54π cubic units.

6. A sphere has a radius of 4.8 cm. If 4 spheres are stacked on top of each other, how tall is the stack? Show or explain your thinking.

   If the radius is 4.8 cm, then the diameter is 9.6 cm. All 4 spheres have the same diameter making the total height 38.4 cm because $9.6 \cdot 4 = 38.4$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
Spiral	3	Unit 5 Lesson 13	2
	4	Unit 5 Lesson 7	2
	5	Unit 5 Lesson 17	2
Formative ⚡	6	Unit 5 Lesson 21	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 5 | **LESSON 21 – CAPSTONE**

Packing Spheres

Let's apply our understanding of the volume of spheres to packaging problems.

Focus

Goal

1. Determine the volume of empty space in a container of spheres.

Rigor

- Students **apply** their understanding of calculating volumes of circular solids to find the volume of empty space when packing spheres.

Coherence

Today

Students engage with mathematics as they determine the amount of empty space in containers used to ship spheres. **Note:** Consider assigning Activity 1 or Activity 2, or allowing the students to choose which activity to complete.

Previously

In Lessons 19 and 20, students determined the effects of scaling dimensions.

Coming Soon

In their high school Geometry course, students will study Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	30 min	30 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (as needed)
- Activity 2 PDF (answers)
- Anchor Chart PDF, *Volumes of Circular Solids*
- calculators
- cardboard or cardstock (optional)
- glue or tape (optional)
- rulers
- spheres, 5 of the same type for each group (golf balls, tennis balls, bouncy balls, basketballs, etc.)

Math Language Development

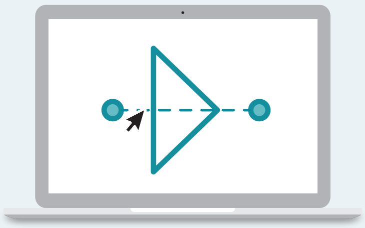
Review words

- *cone*
- *cylinder*
- *height*
- *pi* (π)
- *radius*
- *sphere*
- *volume*

Amps | **Featured Activity**

Activity 1
Interactive Geometry

Students are able to digitally measure a tennis ball and determine the amount of empty space within a cylindrical can of tennis balls.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might rush through the ***Gallery Tour*** without making an effort to understand the work of others. Prior to the walk, have students identify what can be gained by looking at a task through someone else's eyes. Ask then to seek variations in the way students approached the task and appreciate the diversity because it reflects the diversity among students.

Modifications to Pacing

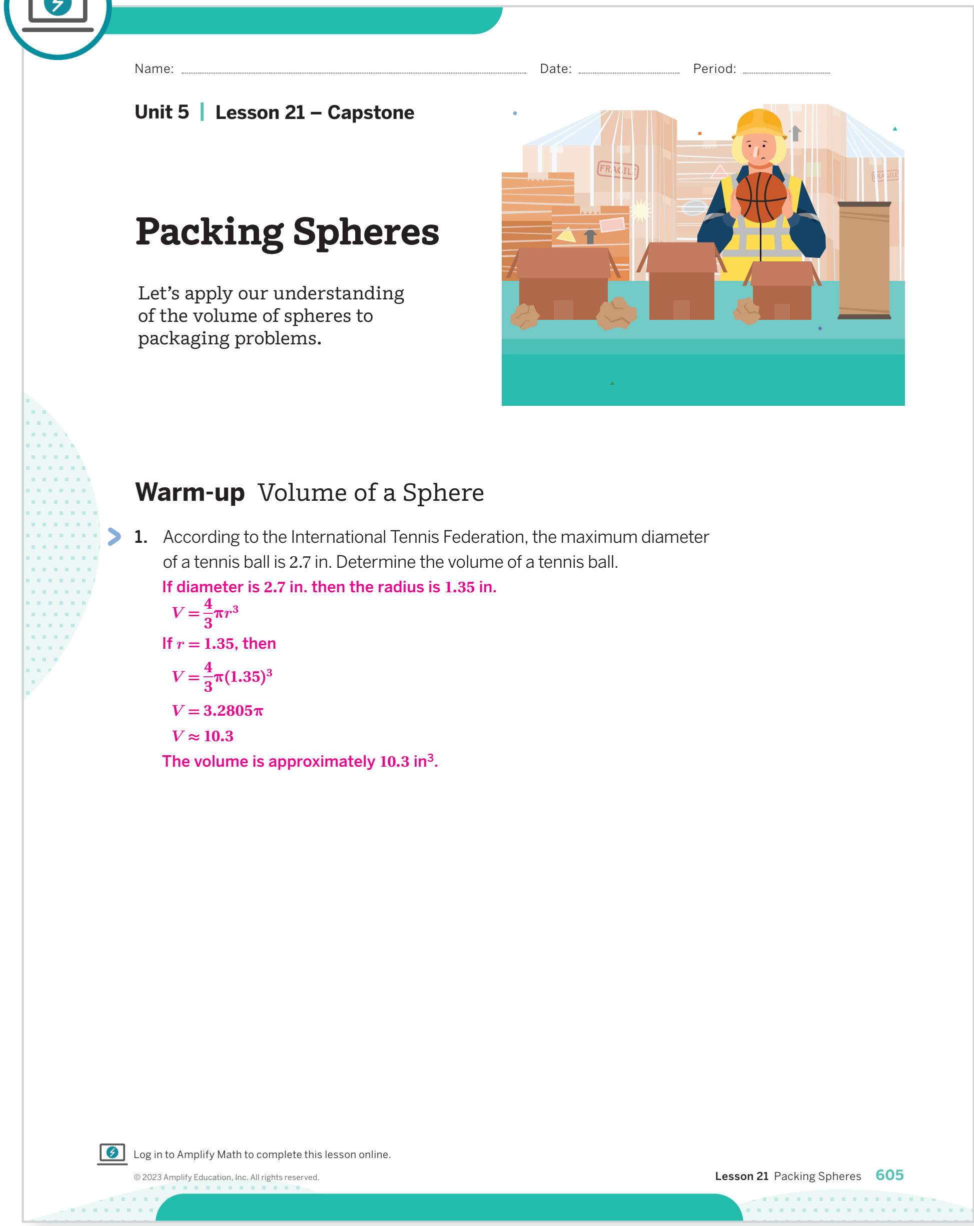
You may want to consider this additional modification if you are short on time.

- Assign **Activity 1** or **Activity 2**, but not both. If assigning **Activity 1**, be sure to assign the **Warm-up**.

Independent | 5 min

Warm-up Volume of a Sphere

Students determine the volume of a tennis ball to prepare them for Activity 1, in which they will apply the volume of a tennis ball to a packaging problem.



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Unit 5 | Lesson 21 – Capstone

Packing Spheres

Let's apply our understanding of the volume of spheres to packaging problems.

Warm-up Volume of a Sphere

1. According to the International Tennis Federation, the maximum diameter of a tennis ball is 2.7 in. Determine the volume of a tennis ball.

If diameter is 2.7 in. then the radius is 1.35 in.

$V = \frac{4}{3}\pi r^3$

If $r = 1.35$, then

$V = \frac{4}{3}\pi(1.35)^3$

$V = 3.2805\pi$

$V \approx 10.3$

The volume is approximately 10.3 in^3.

Log in to Amplify Math to complete this lesson online.

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Lesson 21 Packing Spheres 605

1 Launch

Activate background knowledge and ask students what type of sphere might have a diameter of 2.7 in. Provide access to calculators.

2 Monitor

Help students get started by having them reference the Anchor Chart, *Volumes of Circular Solids* to locate the formula needed.

Look for points of confusion:

- **Thinking the radius is 2.7 in.** Have students read the problem carefully and sketch and label a diagram of a sphere to help them make sense of the given dimension.

3 Connect

Have students share their work for determining the volume.

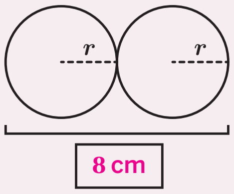
Highlight that this sphere represents a tennis ball and will be used in the next activity. Packing and shipping spherical objects requires some creativity and mathematical calculations because companies want to limit wasted space and materials. In the activities, students will determine the amount of empty space in a container of spheres.

Power-up

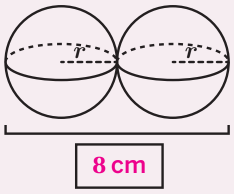
To power up students' ability to visualize the relationship between the radius and height of stacked spheres, have students complete:

Have students determine the length of each row of circles or spheres described.

1. The radius of each circle is 2 cm. What is the missing length?



2. The radius of each sphere is 2 cm. What is the missing length?



Use: Before Activity 1

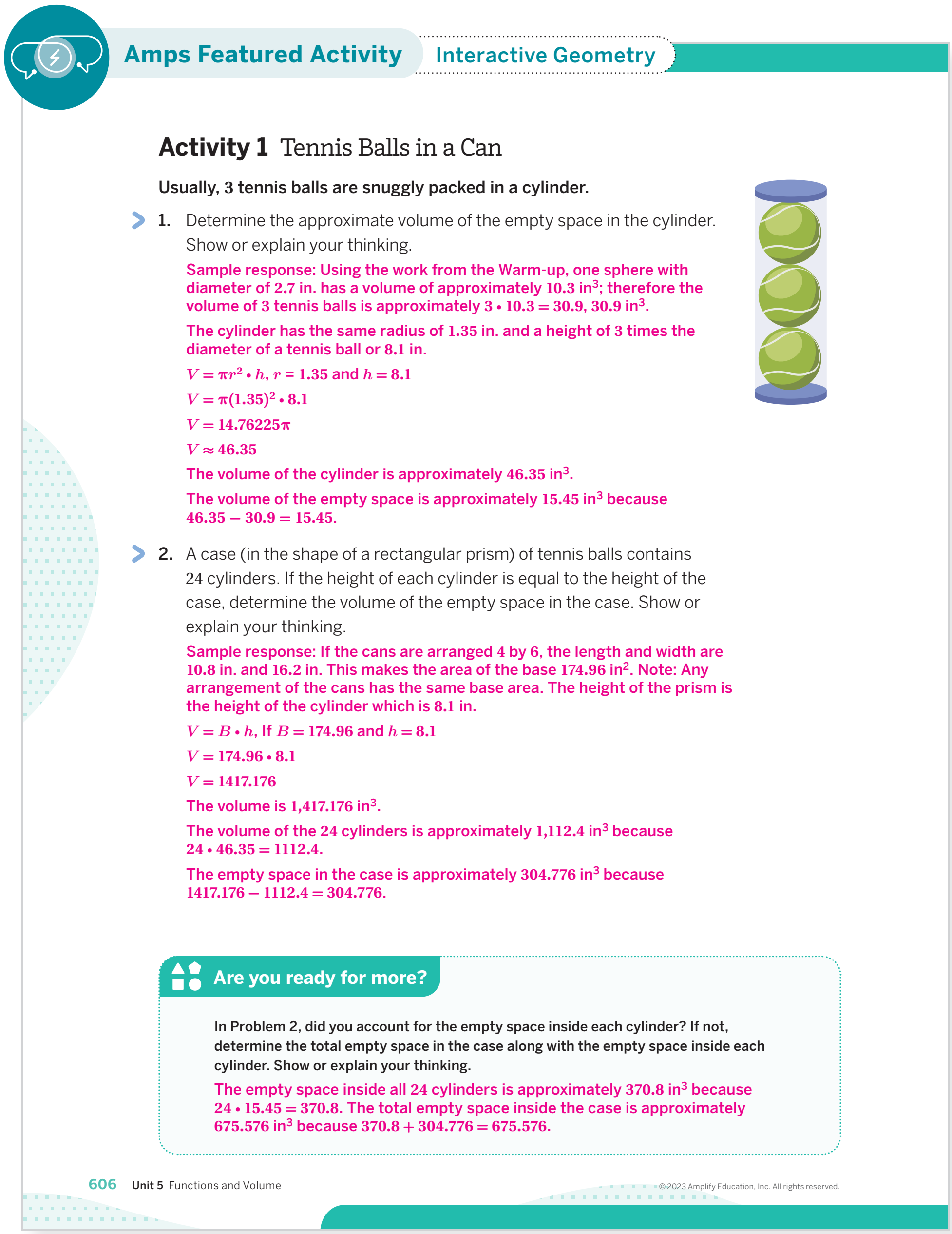
Informed by: Performance on Lesson 20, Practice Problem 6

Small Groups | 30 min

Activity 1 Tennis Balls in a Can

Students determine the volume of the empty space in a can of tennis balls to determine the volume of empty space in an entire case of tennis balls.

Amps Featured Activity Interactive Geometry



Activity 1 Tennis Balls in a Can

Usually, 3 tennis balls are snuggly packed in a cylinder.

1. Determine the approximate volume of the empty space in the cylinder. Show or explain your thinking.

   Sample response: Using the work from the Warm-up, one sphere with diameter of 2.7 in. has a volume of approximately 10.3 in^3; therefore the volume of 3 tennis balls is approximately $3 \cdot 10.3 = 30.9$, 30.9 in^3.

   The cylinder has the same radius of 1.35 in. and a height of 3 times the diameter of a tennis ball or 8.1 in.

   $V = \pi r^2 \cdot h$, $r = 1.35$ and $h = 8.1$

   $V = \pi(1.35)^2 \cdot 8.1$

   $V = 14.76225\pi$

   $V \approx 46.35$

   The volume of the cylinder is approximately 46.35 in^3.

   The volume of the empty space is approximately 15.45 in^3 because $46.35 - 30.9 = 15.45$.

2. A case (in the shape of a rectangular prism) of tennis balls contains 24 cylinders. If the height of each cylinder is equal to the height of the case, determine the volume of the empty space in the case. Show or explain your thinking.

   Sample response: If the cans are arranged 4 by 6, the length and width are 10.8 in. and 16.2 in. This makes the area of the base 174.96 in^2. Note: Any arrangement of the cans has the same base area. The height of the prism is the height of the cylinder which is 8.1 in.

   $V = B \cdot h$, If $B = 174.96$ and $h = 8.1$

   $V = 174.96 \cdot 8.1$

   $V = 1417.176$

   The volume is 1,417.176 in^3.

   The volume of the 24 cylinders is approximately 1,112.4 in^3 because $24 \cdot 46.35 = 1112.4$.

   The empty space in the case is approximately 304.776 in^3 because $1417.176 - 1112.4 = 304.776$.

Are you ready for more?

In Problem 2, did you account for the empty space inside each cylinder? If not, determine the total empty space in the case along with the empty space inside each cylinder. Show or explain your thinking.

The empty space inside all 24 cylinders is approximately 370.8 in^3 because $24 \cdot 15.45 = 370.8$. The total empty space inside the case is approximately 675.576 in^3 because $370.8 + 304.776 = 675.576$.

606 **Unit 5** Functions and Volume

1 Launch

Remind students the diameter of the sphere from the Warm-up was 2.7 in. Provide access to calculators.

2 Monitor

Help students get started by asking how they could determine the volume of all three tennis balls.

Look for points of confusion:

- **Thinking the height of the can is three times the radius instead of three times the diameter.** Encourage students to label the image of the tennis ball can.
- **Not knowing which configuration to make for the cans in the box.** Have students draw a picture of how they can arrange the 24 cans in the box. **Note:** Regardless of their arrangement, the area of the base of the box will be the same: 174 in^2.

Look for productive strategies:

- Organizing their work so that others can understand it.
- Drawing and labeling diagrams to support their work.

3 Connect

Have groups of students share their work and solutions.

Highlight the different strategies and methods students used and amplify student work that is well organized.

Ask, "Does it matter how you arrange the case of 24 cans of tennis balls, such as 4 by 6 or 3 by 8?" For the volume of the empty space, it does not, but, for the surface area of the case, it does.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they are able to digitally measure a tennis ball and determine the amount of empty space within a cylindrical can of tennis balls.

Accessibility: *Guide Processing and Visualization*

Use these scaffolds and supports for this activity.

- Provide students with a copy of the Activity 1 PDF that they can use as a scaffolded guide as they complete this activity.
- Keep the Anchor Chart PDF, *Volumes of Circular Solids* displayed throughout the activity.

Math Language Development

MLR2: Collect and Display

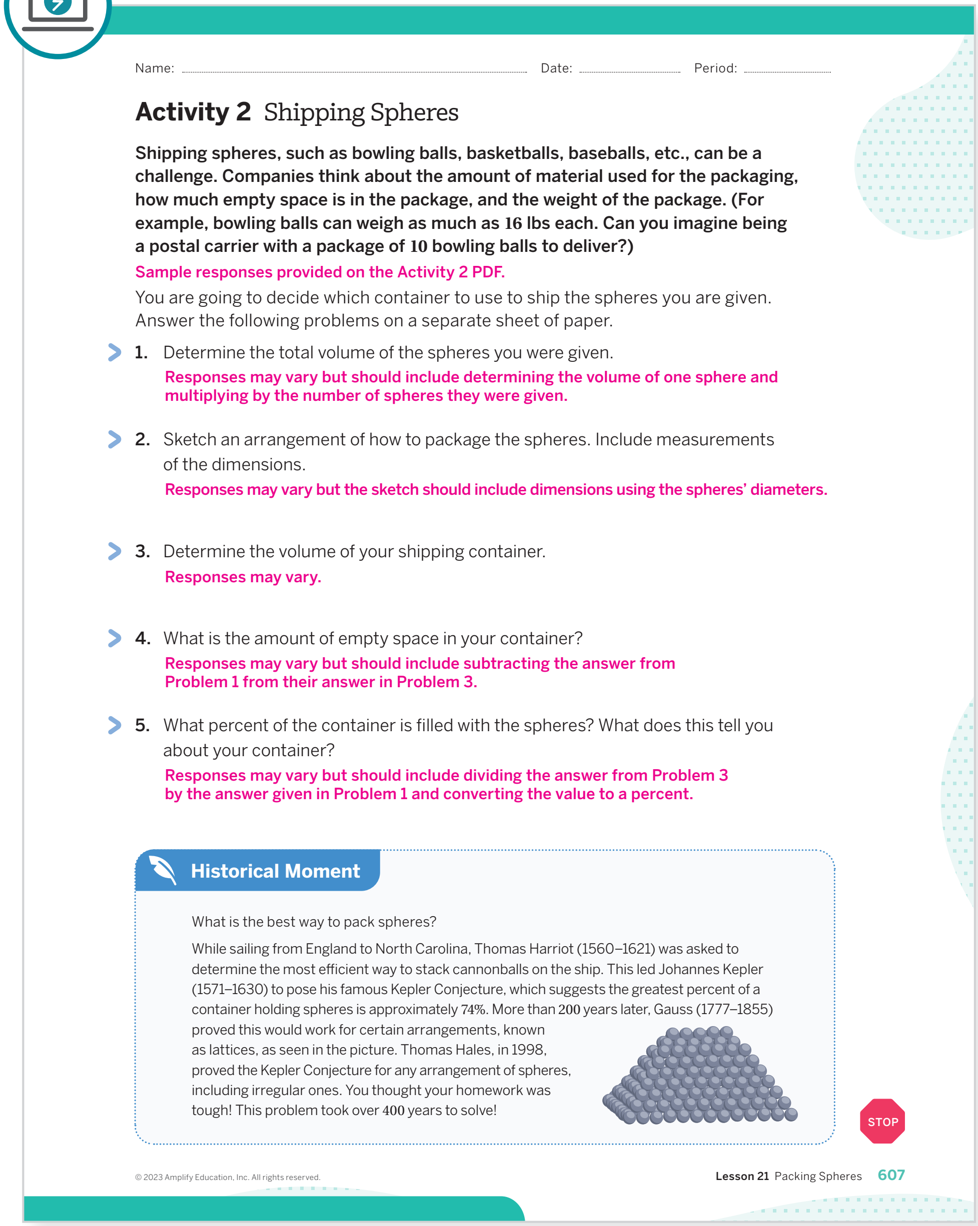
Collect and display the various strategies and methods students used. Highlight organizational strategies, such as flow charts or diagrams, in addition to algebraic strategies. Have students compare the strategies. Ask:

- "Did anyone draw a diagram in Problem 2?"
- "How many different arrangements were considered in Problem 2 for the base layer? What do you notice about the final result?"
- "Why is the height of the cylinder in Problem 1 three times the height of each sphere, but the height of the case in Problem 2 is the height of each sphere?"

Small Groups | 30 min

Activity 2 Shipping Spheres

Students determine the container which will minimize empty space for shipping spheres.



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Activity 2 Shipping Spheres

Shipping spheres, such as bowling balls, basketballs, baseballs, etc., can be a challenge. Companies think about the amount of material used for the packaging, how much empty space is in the package, and the weight of the package. (For example, bowling balls can weigh as much as 16 lbs each. Can you imagine being a postal carrier with a package of 10 bowling balls to deliver?)

Sample responses provided on the Activity 2 PDF.

You are going to decide which container to use to ship the spheres you are given. Answer the following problems on a separate sheet of paper.

1. Determine the total volume of the spheres you were given.
   Responses may vary but should include determining the volume of one sphere and multiplying by the number of spheres they were given.
2. Sketch an arrangement of how to package the spheres. Include measurements of the dimensions.
   Responses may vary but the sketch should include dimensions using the spheres' diameters.
3. Determine the volume of your shipping container.
   Responses may vary.
4. What is the amount of empty space in your container?
   Responses may vary but should include subtracting the answer from Problem 1 from their answer in Problem 3.
5. What percent of the container is filled with the spheres? What does this tell you about your container?
   Responses may vary but should include dividing the answer from Problem 3 by the answer given in Problem 1 and converting the value to a percent.

Historical Moment

What is the best way to pack spheres?

While sailing from England to North Carolina, Thomas Harriot (1560–1621) was asked to determine the most efficient way to stack cannonballs on the ship. This led Johannes Kepler (1571–1630) to pose his famous Kepler Conjecture, which suggests the greatest percent of a container holding spheres is approximately 74%. More than 200 years later, Gauss (1777–1855) proved this would work for certain arrangements, known as lattices, as seen in the picture. Thomas Hales, in 1998, proved the Kepler Conjecture for any arrangement of spheres, including irregular ones. You thought your homework was tough! This problem took over 400 years to solve!

STOP

1 Launch

Give groups of students 5 spheres of the same type (i.e. all are tennis balls), rulers, and other materials deemed necessary.

2 Monitor

Help students get started by asking them to arrange the spheres and tracing the base of the container on a piece of paper to measure the dimensions.

Look for points of confusion:

- **Having difficulty determining the diameter of their sphere.** Encourage students to place the ball on a sheet of a paper and place a pencil tangent to the ball and perpendicular to the paper to draw a line. Do this on both sides and remove the ball to measure the distances marked on the paper.

Look for productive strategies:

- Being creative with the shape of their containers.

3 Connect

Display any sketches students made of their container or any student-made containers and consider conducting a ***Gallery Tour***.

Have students share the shape and volume of their container with the volume of empty space.

Ask, "What percent of your container's volume is filled with the spheres?"

Highlight that determining an efficient way to stack cannonballs is a problem which plagued mathematicians for centuries and led to further exploration in studies of close-packing spheres and the Kepler Conjecture which was proven in 1998 by Thomas Hales.

Differentiated Support

Extension: *Math Enrichment*

Tell students that sphere packing is a mathematical concept that has been studied — and continues to be studied — by mathematicians. Sphere packing is the act of arranging identical spheres within a certain amount of space, such that the spheres do not overlap, but can touch each other. The goal in sphere packing is to determine an arrangement in which the spheres fill up as much of the space as possible. Have interested students research different types of arrangements that have been explored for sphere packing, such as two-dimensional (similar to honeycombs) and three-dimensional (similar to the one shown in the Student Edition).

Historical Moment

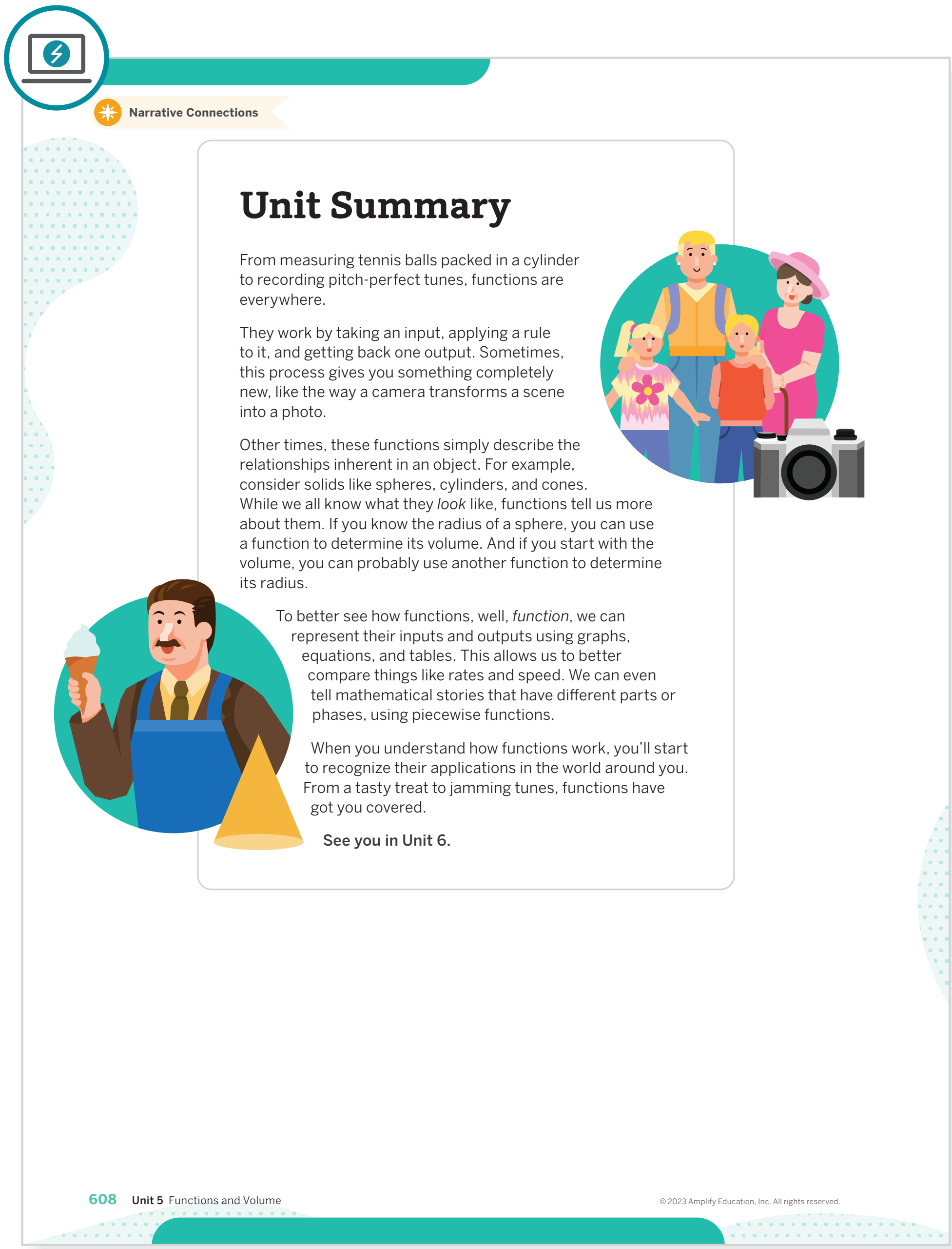
What is the best way to pack spheres?

Have students read the historical moment about the problem of stacking cannonballs started by Thomas Harriot and solved by Hales over 400 years later. Suggest that students research the Kepler Conjecture.

Whole Class | 5 min

Unit Summary

Review and synthesize the concepts of the unit.



Narrative Connections

Unit Summary

From measuring tennis balls packed in a cylinder to recording pitch-perfect tunes, functions are everywhere.

They work by taking an input, applying a rule to it, and getting back one output. Sometimes, this process gives you something completely new, like the way a camera transforms a scene into a photo.

Other times, these functions simply describe the relationships inherent in an object. For example, consider solids like spheres, cylinders, and cones. While we all know what they *look* like, functions tell us more about them. If you know the radius of a sphere, you can use a function to determine its volume. And if you start with the volume, you can probably use another function to determine its radius.

To better see how functions, well, *function*, we can represent their inputs and outputs using graphs, equations, and tables. This allows us to better compare things like rates and speed. We can even tell mathematical stories that have different parts or phases, using piecewise functions.

When you understand how functions work, you'll start to recognize their applications in the world around you. From a tasty treat to jamming tunes, functions have got you covered.

See you in Unit 6.

608 Unit 5 Functions and Volume

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Have students share their reflections from their work in this unit.

Ask:

- "What are your biggest takeaways from this unit?"
- "What are your biggest questions about this unit?"

Highlight that students will continue to study functions and volume in high school. In particular, they will study and explore Cavalieri's principle, which states that, if two three-dimensional solids with the same height also have the same cross-sectional area at every point along that height, then they have the same volume.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

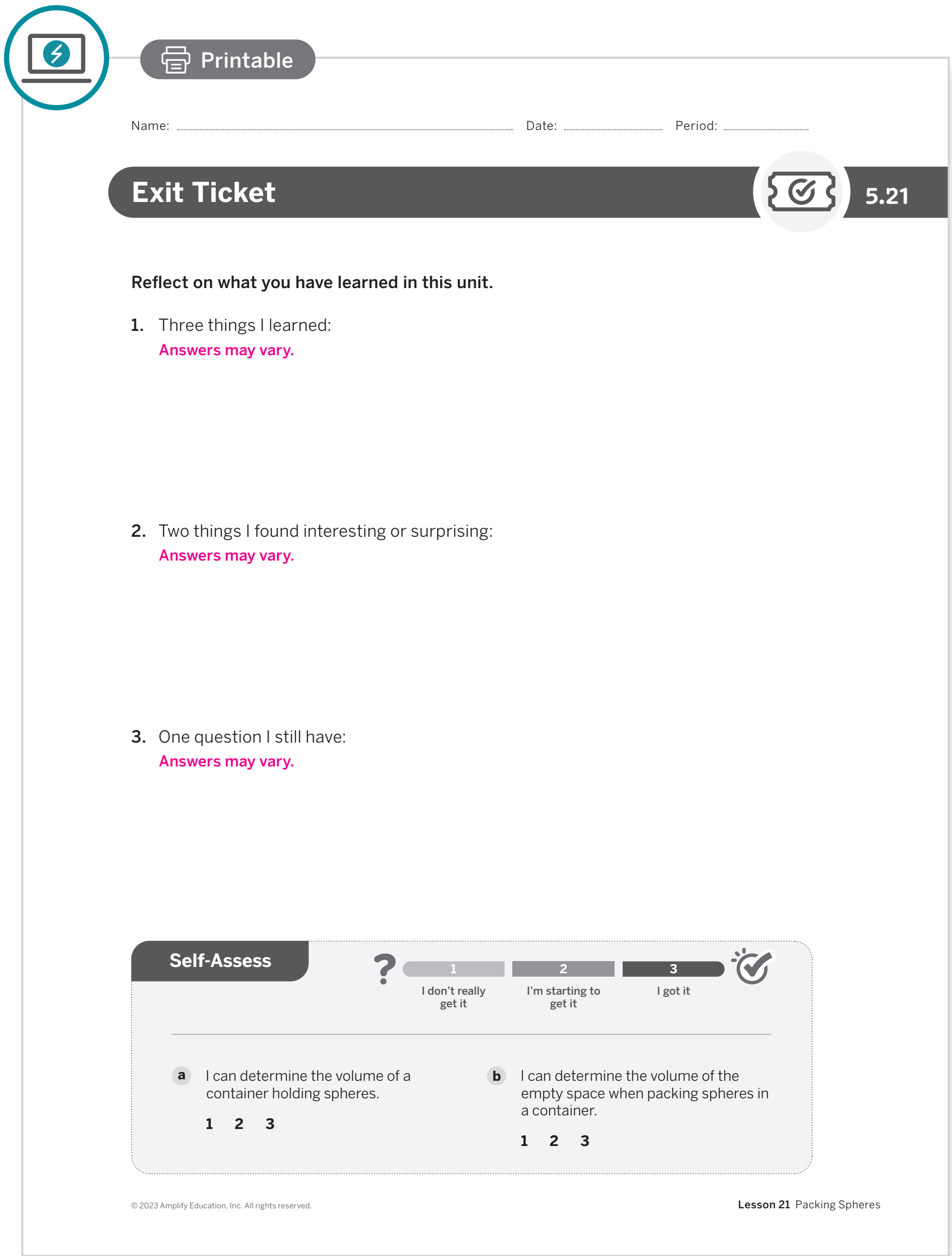
- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reflecting on their work in this unit.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 5.21

Reflect on what you have learned in this unit.

1. Three things I learned:
   Answers may vary.

2. Two things I found interesting or surprising:
   Answers may vary.

3. One question I still have:
   Answers may vary.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can determine the volume of a container holding spheres.
1 2 3

b I can determine the volume of the empty space when packing spheres in a container.
1 2 3

Lesson 21 Packing Spheres

Success looks like . . .

- Determining the volume of empty space in a container of spheres.

Suggested next steps

If students are unsure what to write, consider:

- Activating students' prior knowledge by having them refer to the Anchor Charts from this unit.
- Encourage students to write more than one question if they are having trouble responding to Problems 1 and 2.

Professional Learning

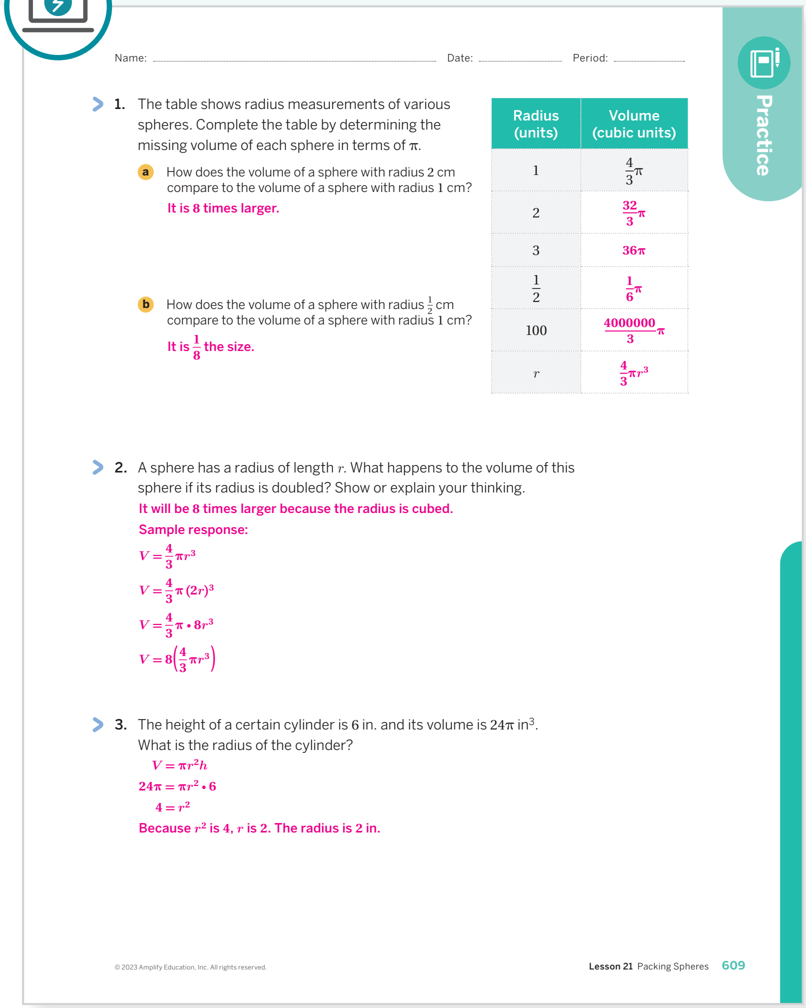
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did you see in the way some students approached Activity 1 or Activity 2 that you would like other students to try?
- Which teacher actions made today's lesson strong? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. The table shows radius measurements of various spheres. Complete the table by determining the missing volume of each sphere in terms of π.

Radius (units)	Volume (cubic units)
1	$\frac{4}{3}\pi$
2	$\frac{32}{3}\pi$
3	36π
$\frac{1}{2}$	$\frac{1}{6}\pi$
100	$\frac{4000000}{3}\pi$
r	$\frac{4}{3}\pi r^3$

a. How does the volume of a sphere with radius 2 cm compare to the volume of a sphere with radius 1 cm?

It is 8 times larger.

b. How does the volume of a sphere with radius $\frac{1}{2}$ cm compare to the volume of a sphere with radius 1 cm?

It is $\frac{1}{8}$ the size.

2. A sphere has a radius of length r. What happens to the volume of this sphere if its radius is doubled? Show or explain your thinking.

It will be 8 times larger because the radius is cubed.

Sample response:

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (2r)^3$

$V = \frac{4}{3}\pi \cdot 8r^3$

$V = 8\left(\frac{4}{3}\pi r^3\right)$

3. The height of a certain cylinder is 6 in. and its volume is 24π in^3. What is the radius of the cylinder?

$V = \pi r^2 h$

$24\pi = \pi r^2 \cdot 6$

$4 = r^2$

Because r^2 is 4, r is 2. The radius is 2 in.

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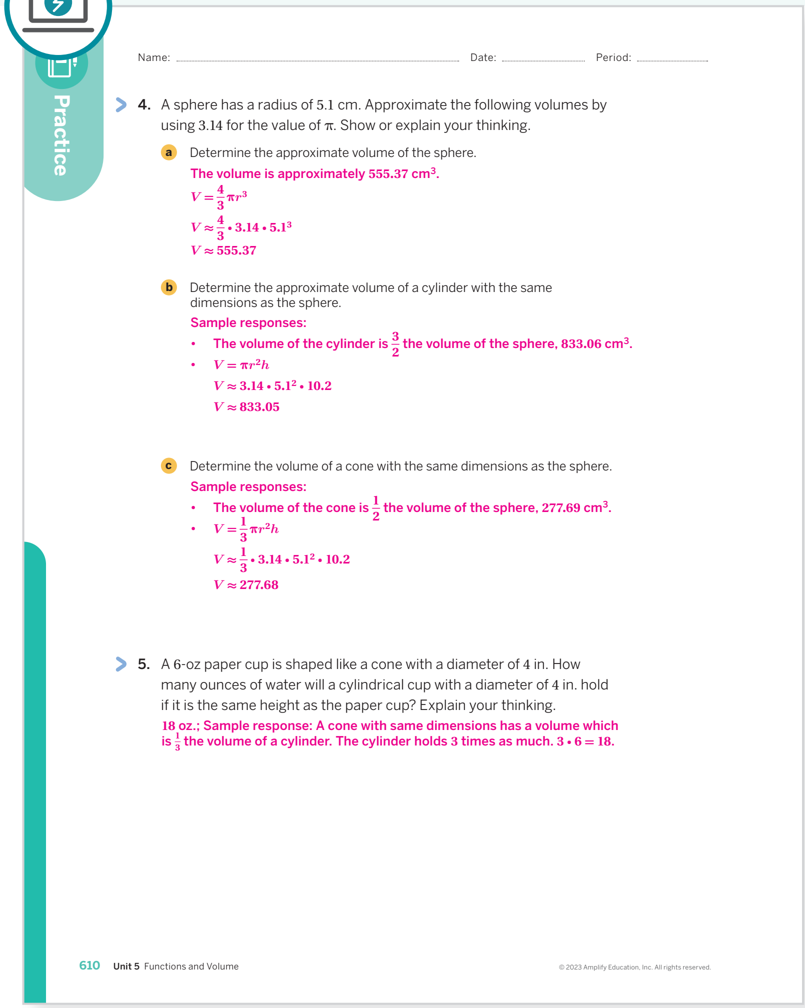
Lesson 21 Packing Spheres 609



Name: Date: Period:

4. A sphere has a radius of 5.1 cm. Approximate the following volumes by using 3.14 for the value of π. Show or explain your thinking.

a. Determine the approximate volume of the sphere.

The volume is approximately 555.37 cm^3.

$V = \frac{4}{3}\pi r^3$

$V \approx \frac{4}{3} \cdot 3.14 \cdot 5.1^3$

$V \approx 555.37$

b. Determine the approximate volume of a cylinder with the same dimensions as the sphere.

Sample responses:

- **The volume of the cylinder is $\frac{3}{2}$ the volume of the sphere, 833.06 cm^3.**
- $V = \pi r^2 h$
  $V \approx 3.14 \cdot 5.1^2 \cdot 10.2$
  $V \approx 833.05$

c. Determine the volume of a cone with the same dimensions as the sphere.

Sample responses:

- **The volume of the cone is $\frac{1}{2}$ the volume of the sphere, 277.69 cm^3.**
- $V = \frac{1}{3}\pi r^2 h$
  $V \approx \frac{1}{3} \cdot 3.14 \cdot 5.1^2 \cdot 10.2$
  $V \approx 277.68$

5. A 6-oz paper cup is shaped like a cone with a diameter of 4 in. How many ounces of water will a cylindrical cup with a diameter of 4 in. hold if it is the same height as the paper cup? Explain your thinking.

18 oz.; Sample response: A cone with same dimensions has a volume which is $\frac{1}{3}$ the volume of a cylinder. The cylinder holds 3 times as much. $3 \cdot 6 = 18$.

610 Unit 5 Functions and Volume

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Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 5 Lesson 20	2
	2	Unit 5 Lesson 20	2
	3	Unit 5 Lesson 13	2
	4	Unit 5 Lesson 17	2
	5	Unit 5 Lesson 14	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

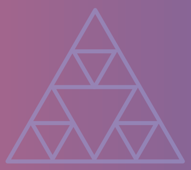
UNIT 6

Exponents and Scientific Notation

This unit is about the numbers so large and so small that students must develop new ways of working with them. Students deepen their knowledge of exponents before exploring how powers of 10 and scientific notation can be used to write and work with numbers as small as the mass of a bacterium or as large as the number of atoms in the Universe.

Essential Questions

- What happens when expressions containing exponents are multiplied or divided?
- Is there a more efficient way to write really small and really large numbers?
- What strategies can be used when working with very large and very small numbers?
- *(By the way, which weighs more: the Burj Khalifa or all the pennies it cost to build the Burj Khalifa?)*



2^2

$2^3 \cdot 2^4$

$(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$

2^{-6} OR $\frac{1}{2^6}$

Key Shifts in Mathematics

Focus

In this unit . . .

Students start by building a strong foundation with expressions involving exponents, learning to multiply and divide these expressions when they have the same base. They develop an understanding of zero as an exponent and generalize a process for working with negative exponents and expressions with the same base, yet different exponents. With this foundation, students then encounter problems with vast quantities, large and small. They will see how powers of 10 can be helpful when working with such numbers, ultimately defining and using scientific notation.

Coherence

Previously . . .

Students were introduced to exponential notation in Grade 6. They worked with expressions that included parentheses and positive whole number exponents with whole number, fractional, decimal, or variable bases, using properties of exponents strategically, but they did not formulate rules for the use of exponents.

Coming soon . . .

Students will continue to use scientific notation and powers of 10 to describe and calculate large and small numbers in future grades and courses. They will explore exponential expressions in greater depth in Algebra 1 when they look at how to model exponential growth using functions and graphs. A solid understanding of exponents will help students better understand how to work with more complex exponential expressions with variables.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students discover and generalize a process for writing exponential expressions, building an understanding of exponential behavior they can carry with them throughout the unit and in future grades (Lessons 3–7). They do not just learn the definition of scientific notation — they learn *why* we have a need for writing numbers in powers of 10 (Lesson 11) and when powers of 10 are most helpful (Lesson 15).

Procedural Fluency

After generalizing how exponents behave in different expressions, students will have the opportunity to practice their fluency with exponential rules and expressions (Lesson 8). In later lessons, students develop their scientific notation skills by finding products and quotients (Lesson 13) and sums and differences (Lesson 14) with large and small numbers.

Application

Students use exponents and powers of 10 to describe and calculate large and small quantities, from the mass of a single bacterium to the number of organisms on Earth. Is a smartphone smart enough to go to the Moon? Students tackle this question and others with their new skills and knowledge in the culminating lesson of the unit (Lesson 15).

From Teeny-Tiny to Downright Titanic

SUB-UNIT

1

Lessons 2–8

Exponent Rules

Students revisit exponents — this time to formulate the rules for multiplying and dividing powers and raising a power to a power. They study patterns to determine what it means when an exponent is zero or negative. At the end of the Sub-Unit, students are provided with ample opportunity to practice using and applying these ***exponent rules***.

Narrative: A chess board can *reveal* the power of exponents.

SUB-UNIT

2

Lessons 9–14

Scientific Notation

Students begin by estimating quantities in terms of multiples of powers of 10. In doing so, they discover the need for a new way to work with these quantities — ***scientific notation***. They practice writing quantities in scientific notation and performing operations on quantities using scientific notation.

Narrative: Discover a more efficient way to talk about the distance across the Universe.

Launch

Lesson 1

Create a Sierpiński Triangle

Students construct a Sierpiński triangle pattern to illustrate the power of exponential growth and decay (although they will not learn these terms until Algebra 1). Will the number of triangles exceed the number of students in a school? Does the size of each triangle in the pattern approach zero? Students will stretch their imaginations as they prepare to explore exponents and scientific notation in greater depth.

Capstone

Lesson 15

Is a Smartphone Smart Enough to Go to the Moon?

Dealing with extremely large numbers can be mind-boggling, but scientific notation can help to de-boggle them a bit. Explore and calculate with numbers so large, they go on for days.

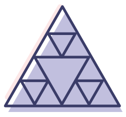
Unit at a Glance

Spoiler Alert: Powers of 10 and scientific notation can be used to estimate calculations with very large and very small numbers.

Assessment

A **Pre-Unit Readiness Assessment**

Launch Lesson



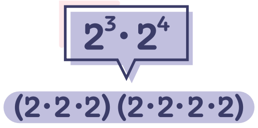
1 **Create a Sierpiński Triangle** ●

Study and create patterns with the Sierpiński triangle to explore repeated multiplication.

Sub-Unit 1: Exponent Rules

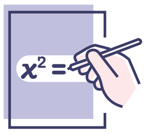
2 **Reviewing Exponents**

Revisit exponents from earlier grades, noting that the structure of powers helps to compare values.



3 **Multiplying Powers**

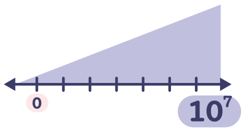
Generalize a process for multiplying exponential expressions with the same base and justify that $a^m \cdot a^n = a^{m+n}$, where $a \neq 0$.



8 **Practice With Rational Bases**

Put it all together and practice all of the exponent rules learned so far, while engaging in a friendly class competition.

Sub-Unit 2: Scientific Notation



9 **Representing Large Numbers on the Number Line** ●

Use number lines to represent and compare large numbers as multiples of powers of 10.

10 **Representing Small Numbers on the Number Line** ●

The sequel to Lesson 9, but this time with small numbers.

11 **Applications of Arithmetic With Powers of 10** ●

Use exponent rules and powers of 10 to solve problems in context and explain the steps used, justifying the need for scientific notation.

Assessment

A **End-of-Unit Assessment**

Key Concepts

Lesson 3: Justify patterns found when multiplying expressions involving exponents.

Lesson 12: Define and identify scientific notation.

Lesson 14: Calculate and estimate sums and differences with numbers written in scientific notation.

Pacing

15 Lessons: 45 min each

2 Assessments: 45 min each

Full Unit: 17 days

● **Modified Unit:** 14 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

$\frac{10^3}{10^2}$ $\frac{10\cdot10\cdot10}{10\cdot10}$

4 Dividing Powers

Generalize a process for dividing exponential expressions with the same base and justify that $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$.

2^{-6} OR $\frac{1}{2^6}$

5 Negative Exponents

Explore how exponent rules extend to expressions involving negative exponents.

$(10^2)^4$

6 Powers of Powers

Generalize a process for finding a power raised to a power and justify that $(a^m)^n = a^{m \cdot n}$, where $a \neq 0$.

$3^3 \cdot 5^3 = 15^3$

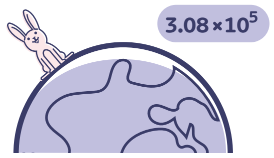
7 Different Bases, Same Exponent

Generalize a process for multiplying expressions with different bases having the same exponent and justify that $a^m \cdot b^m = (a \cdot b)^m$, where $a \neq 0$ and $b \neq 0$.

10^{-4} 10^4

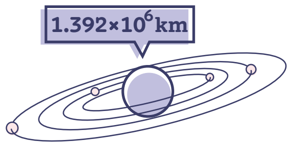
12 Definition of Scientific Notation ●

Put a name to a face! Now that there's a need for efficiently writing large and small numbers, formally define scientific notation.



13 Multiplying, Dividing, and Estimating With Scientific Notation

Multiplying and dividing with large or small numbers? Have no fear, scientific notation is here!



14 Adding and Subtracting With Scientific Notation

The gift that keeps on giving. Adding and subtracting large or small numbers is more efficient, thanks to scientific notation.

Capstone Lesson

10^{googol}

15 Is a Smartphone Smart Enough to Go to the Moon? ●

Apply scientific notation to compare the computing power of smartphones and older devices, and see how scientific notation can help estimate how long it takes to count to 1 million.

● Modifications to Pacing

Lesson 1: To begin with a strong foundation, consider spending two days on this lesson.

Lessons 9–10: Combine these two lessons so that the first part explores large numbers on a number line and the second part explores small numbers on a number line.

Lessons 11–12: Lesson 11 helps students see the need for using scientific notation. It can be omitted and instead used as a modified Warm-up to Lesson 12, which formally introduces scientific notation.

Lesson 15: In this culminating lesson, students apply what they learned throughout the unit to computing power and counting to 1 million. No new standards are addressed, and thus this lesson can be omitted, if needed.

Unit Supports

Math Language Development

Lesson	New Vocabulary
12	**scientific notation**

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
5, 14	**MLR1:** Stronger and Clearer Each Time
1–3, 6, 12	**MLR2:** Collect and Display
6, 7, 10, 12, 14	**MLR3:** Critique, Correct, Clarify
11, 13, 14	**MLR5:** Co-craft Questions
1, 3, 4, 7–9	**MLR7:** Compare and Connect
2, 5, 8, 10, 11, 14, 15	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson	Materials
1, 11, 12	**calculators**
1	**colored pencils** **rulers**
1, 3, 8, 11, 12, 15	**PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.**

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
3, 4, 6, 12	**Card Sort**
1, 5, 14	**Notice and Wonder**
8	**Partner Problems**
1, 2, 9, 10, 13	**Think-Pair-Share**
7	**True or False?**
3	**Which One Doesn't Belong?**

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 15

Amps

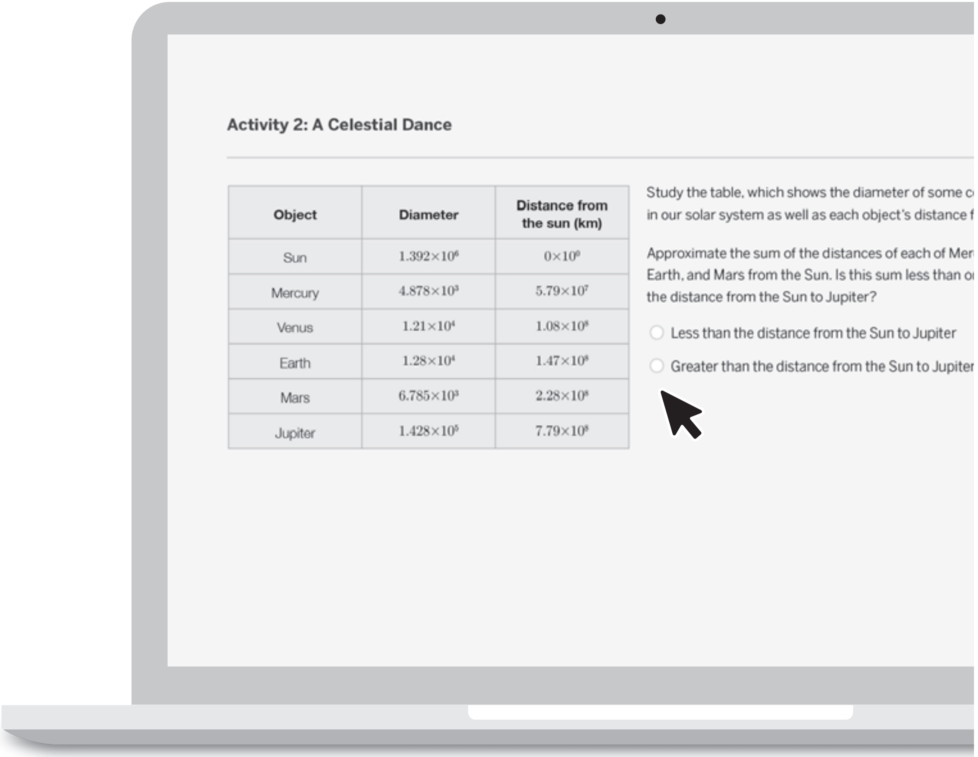
POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

A Celestial Dance

Put on your student hat and work through **Lesson 14, Activity 2:**



Activity 2: A Celestial Dance

Object	Diameter	Distance from the sun (km)
Sun	1.392×10^6	0×10^0
Mercury	4.878×10^3	5.79×10^7
Venus	1.21×10^4	1.08×10^8
Earth	1.28×10^4	1.47×10^8
Mars	6.785×10^3	2.28×10^8
Jupiter	1.428×10^5	7.79×10^8

Study the table, which shows the diameter of some c
in our solar system as well as each object's distance f

Approximate the sum of the distances of each of Mer
Earth, and Mars from the Sun. Is this sum less than o
the distance from the Sun to Jupiter?

○ Less than the distance from the Sun to Jupiter

○ Greater than the distance from the Sun to Jupiter

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities

- The Sierpiński Triangle **(Lesson 1)**
- Covering All Your Bases **(Lesson 8)**
- Even More Pennies **(Lesson 11)**
- Counting to a Million and Beyond **(Lesson 15)**

Unit Study
Professional Learning

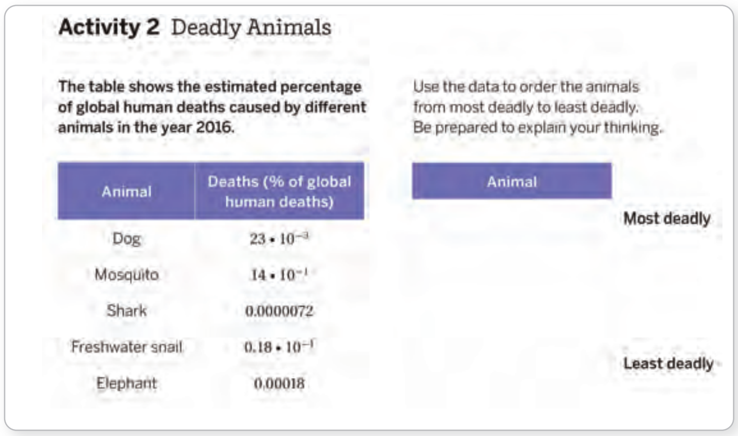
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 1 introduces students to the exponent rules. They learn to see the pattern by examining the expanded form of an expression to its single power form. Students continue to see this pattern as it applies to negative exponents. Then, in Sub-Unit 2, they begin to see the need for using scientific notation for very large and very small numbers. Students learn to calculate large numbers using powers of 10. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 10, Activity 2:**



Activity 2 Deadly Animals

The table shows the estimated percentage of global human deaths caused by different animals in the year 2016.

Animal	Deaths (% of global human deaths)
Dog	$23 \cdot 10^{-3}$
Mosquito	$14 \cdot 10^{-1}$
Shark	0.0000072
Freshwater snail	$0.18 \cdot 10^{-1}$
Elephant	0.00018

Use the data to order the animals from most deadly to least deadly. Be prepared to explain your thinking.

Animal

Most deadly

Least deadly

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

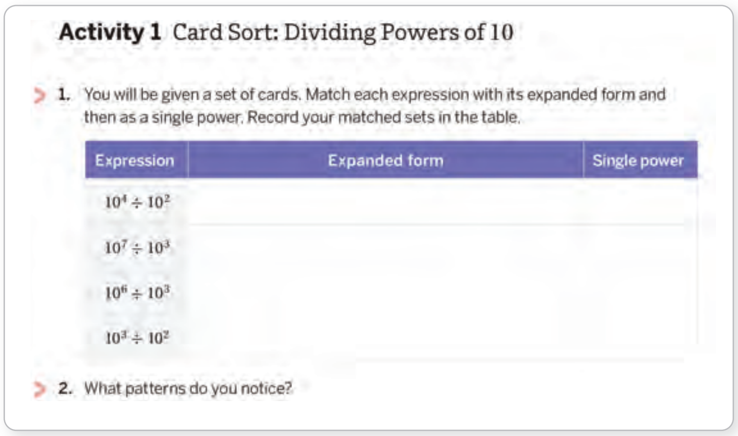
- What was it like to engage in this problem as a learner?
- What strategy did you use to compare the numbers more easily?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Card Sort

Rehearse . . .

How you'll facilitate the ***Card Sort*** instructional routine in **Lesson 4, Activity 1:**



Activity 1 Card Sort: Dividing Powers of 10

1. You will be given a set of cards. Match each expression with its expanded form and then as a single power. Record your matched sets in the table.

Expression	Expanded form	Single power
$10^4 \div 10^2$		
$10^7 \div 10^3$		
$10^6 \div 10^3$		
$10^3 \div 10^2$		

2. What patterns do you notice?

Point to Ponder . . .

- Students might match cards correctly without fully understanding the meaning of the expanded expression. What will you say or do to help them see what is happening within each expression? Why is this important for your students to understand?

This routine . . .

- Enables students to efficiently see patterns in the structures of expanded expressions and their simplified exponential form.
- Provides students with opportunities to analyze expressions closely and make connections.
- Allows students to revise their thinking by recreating groups as new ideas form, or as they are persuaded by a partner's thinking.

Anticipate . . .

- What questions can you ask to help students generalize their thinking?
- How can you tell the difference between student thinking that shows memorizing a procedure or rule and thinking that shows generalizing the concepts behind the process?
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Build procedural fluency from conceptual understanding.

This effective teaching practice . . .

- Begins with a foundation of deep understanding so that students develop sense-making skills, before procedural skills are introduced.
- Provides students with the opportunity to connect procedural skills with contextual or mathematical problems, strengthening their problem solving abilities.

Points to Ponder . . .

- Before introducing a formula or procedure, how will you ensure that your students have a solid understanding of the mathematical concepts?
- Do your students connect procedures to concepts, or are they reliant on memorization of formulas or procedural steps? How can you be sure they understand the "why behind the what"?

Math Language Development

MLR3: Critique, Correct, Clarify

MLR3 appears in Lessons 2, 5, 8, 10, 11, 14, and 15.

- In Lesson 4, students critique an incorrect equation involving an exponent rule and describe how they can convince a classmate as to why their corrected equation is true.
- In Lesson 10, students may have misconceptions about negative powers of 10 placed on a number line. This is a good opportunity to present the misconception as a statement and have students critique it.
- **English Learners:** Provide students time to formulate their responses and allow them to rehearse what they will say with a partner before sharing with the class.

Point to Ponder . . .

- In this routine, students analyze incorrect statements and work to correct them. How can you model what an effective and respectful critique looks like?

Unit Assessments

- Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with evaluating exponential expressions throughout the unit? Do you think your students will generally:
  - Mix up the exponential rules?
  - Be intimidated by large numbers with many digits?
  - Rush through calculations and add or subtract without paying attention to magnitude?

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Opportunities to vary the demands of a task or activity appear in Lessons 1–4, 6–10, 12, 14, and 15.

- Chunking a complex task into smaller, more manageable parts allows students to focus on one part at a time.
- In Lesson 9, questions are provided that you can display to have students ask themselves, which will help them think about how to approach the task.
- Some students may benefit from more processing time. When restricting the number of tasks or problems students need to complete, consider allowing them to choose which problem(s) to complete. Students are often more engaged when they have a choice.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to vary the demands of a particular task or activity? What clues will you gather from your students?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

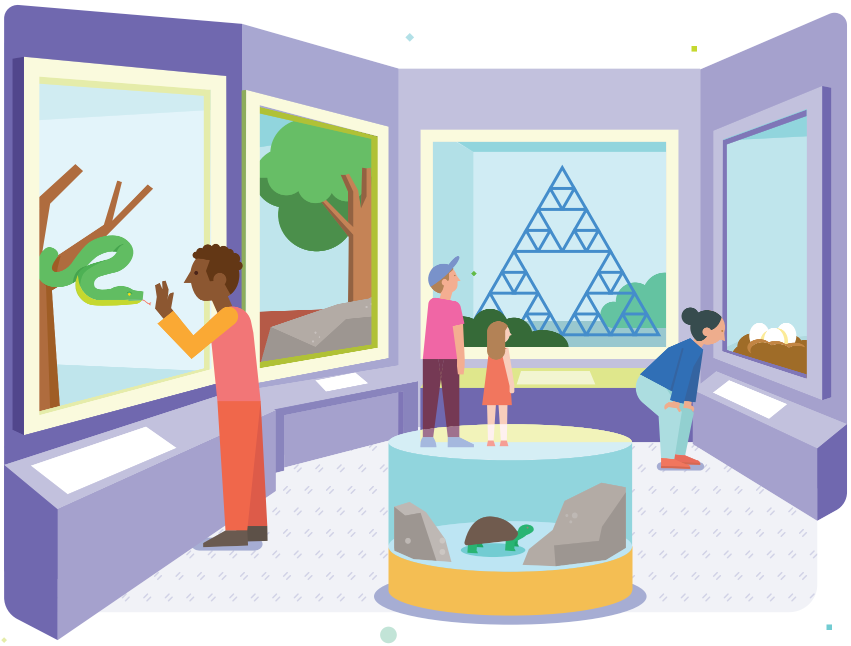
In this unit, pay particular attention to supporting students in building their self-management and relationship skills.

Points to Ponder . . .

- How do students exhibit self-discipline? Are they able to stay focused? What do they do to control their impulses? Are they able to regulate their emotions?
- Do students communicate well with others? Do they engage with others in a manner that builds healthy relationships? Do they seek opportunities to help others understand and to receive help, when needed? How well do they work with others?

Create a Sierpiński Triangle

Let's draw some triangles.



Focus

Goals

1. Create an expression that represents repeated multiplication, and explain how the structure of the expression helps predict quantities.
2. **Language Goal:** Describe a pattern that could be expressed using repeated multiplication. **(Speaking and Listening, Writing)**

Coherence

Today

This lesson uses the context of the Sierpiński triangle to remind students about the need for exponents in thinking about problems involving repeated multiplication. Students create their own Sierpiński triangle to look for patterns of repeated multiplication of both a whole number and a fraction.

Previously

In Grade 6, students used whole number exponents to represent repeated multiplication.

Coming Soon

In Lesson 2, students will analyze the structure of powers and apply their understanding of exponents.

Rigor

- Students build **conceptual understanding** of exponents.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	20 min	20 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per student
- calculators
- colored pencils (optional)
- rulers

Math Language Development

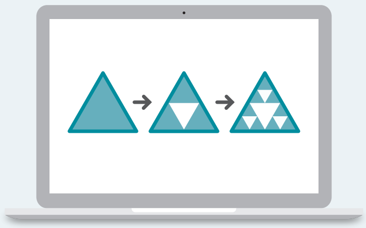
Review words

- *base*
- *exponent*
- *power*

Amps | Featured Activity

Warm-up
Chaos Game

Launch your lesson with an animation of the Chaos game and introduce students to the Sierpiński triangle.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might feel uneasy about having to elaborate on the Sierpiński triangle in Activity 1, especially as they work independently. Remind students that they have a ruler to help them find the midpoints and that after finding the midpoints, they can connect the midpoints to make the new triangles. Encourage students to approach the task with confidence, relying on their strengths, but also understanding that during *Think-Pair-Share*, they will also get the input of another student.

Modifications to Pacing

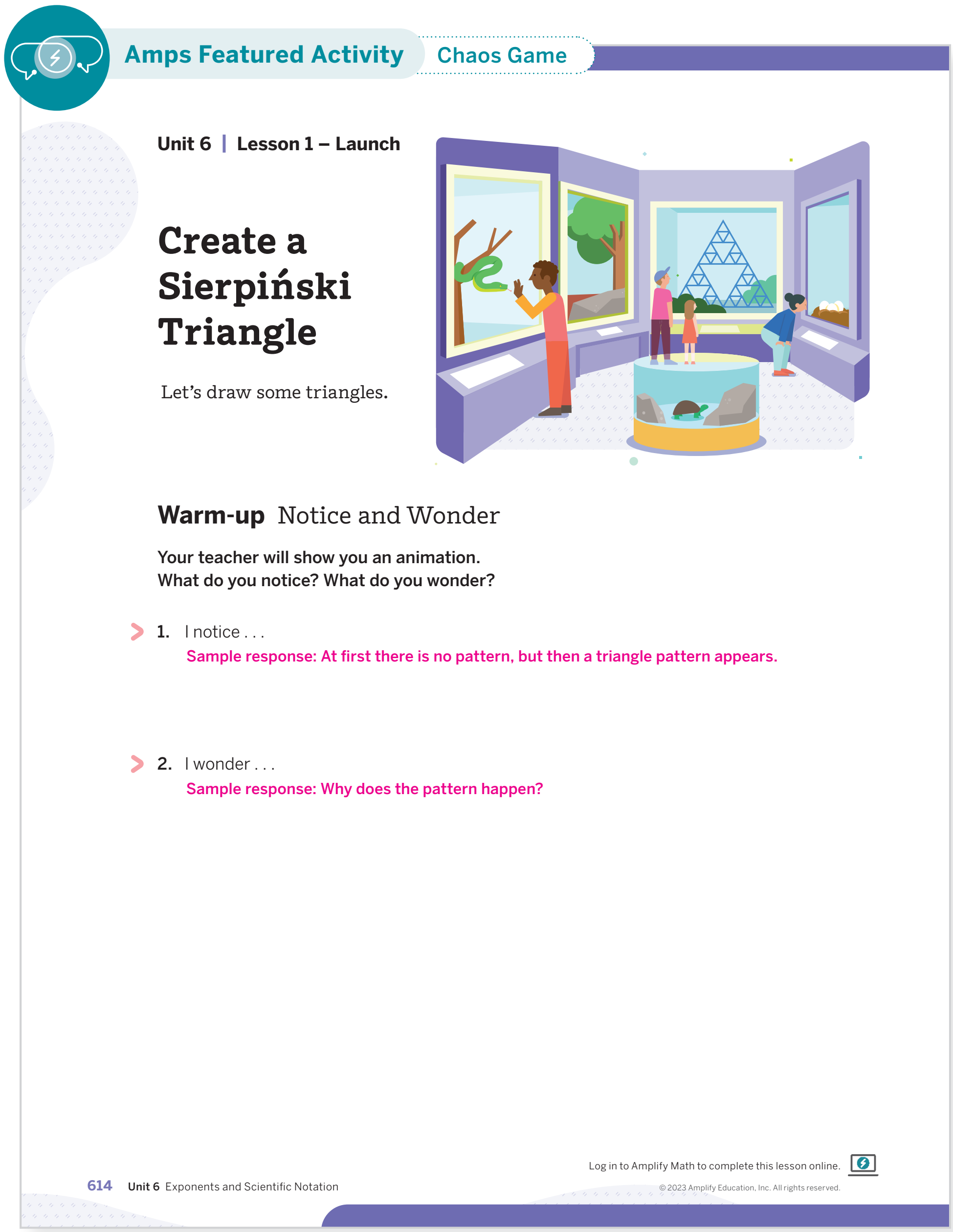
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 5 may be omitted.
- In **Activity 2**, Problems 2 and 3 may be omitted.
- **Activity 1** and **Activity 2** may be completed over two days.

Warm-up Notice and Wonder

Students play the Chaos game, and then watch an animation to introduce them to the Sierpiński triangle and patterns of repeated multiplication. This will prepare them for the mathematics of this unit: exponents.

Amps Featured Activity Chaos Game



Unit 6 | Lesson 1 – Launch

Create a Sierpiński Triangle

Let's draw some triangles.

Warm-up Notice and Wonder

Your teacher will show you an animation.
What do you notice? What do you wonder?

1. I notice . . .
   Sample response: At first there is no pattern, but then a triangle pattern appears.
2. I wonder . . .
   Sample response: Why does the pattern happen?

Log in to Amplify Math to complete this lesson online.

614 **Unit 6** Exponents and Scientific Notation

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1 Launch

Introduce the Chaos game from the digital lesson. Have students choose the number 1, 2, or 3. Start with the first student, and call on every student in order around the room. As each student calls out a number, click the corresponding point on the screen. Continue for about 5 minutes, or until time allows.

Show the Chaos Game animation which shows the pattern after 200 points, 500 points, and 1,000 points. Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking them to look for patterns that appear in the animation as it continues to play.

Look for points of confusion:

- **Struggling to see a pattern.** You may wish to pause the animation at various points and have students jot down what they notice. Some students may benefit from playing the animation more than once.

Look for productive strategies:

- Noticing a repeating triangle pattern.

3 Connect

Have students share what they notice and wonder. Record responses for all to see.

Ask students to describe the pattern in their own words.

Highlight that it seemed as if there was no pattern in the beginning. As more points were added, the pattern became clearer.

Differentiated Support

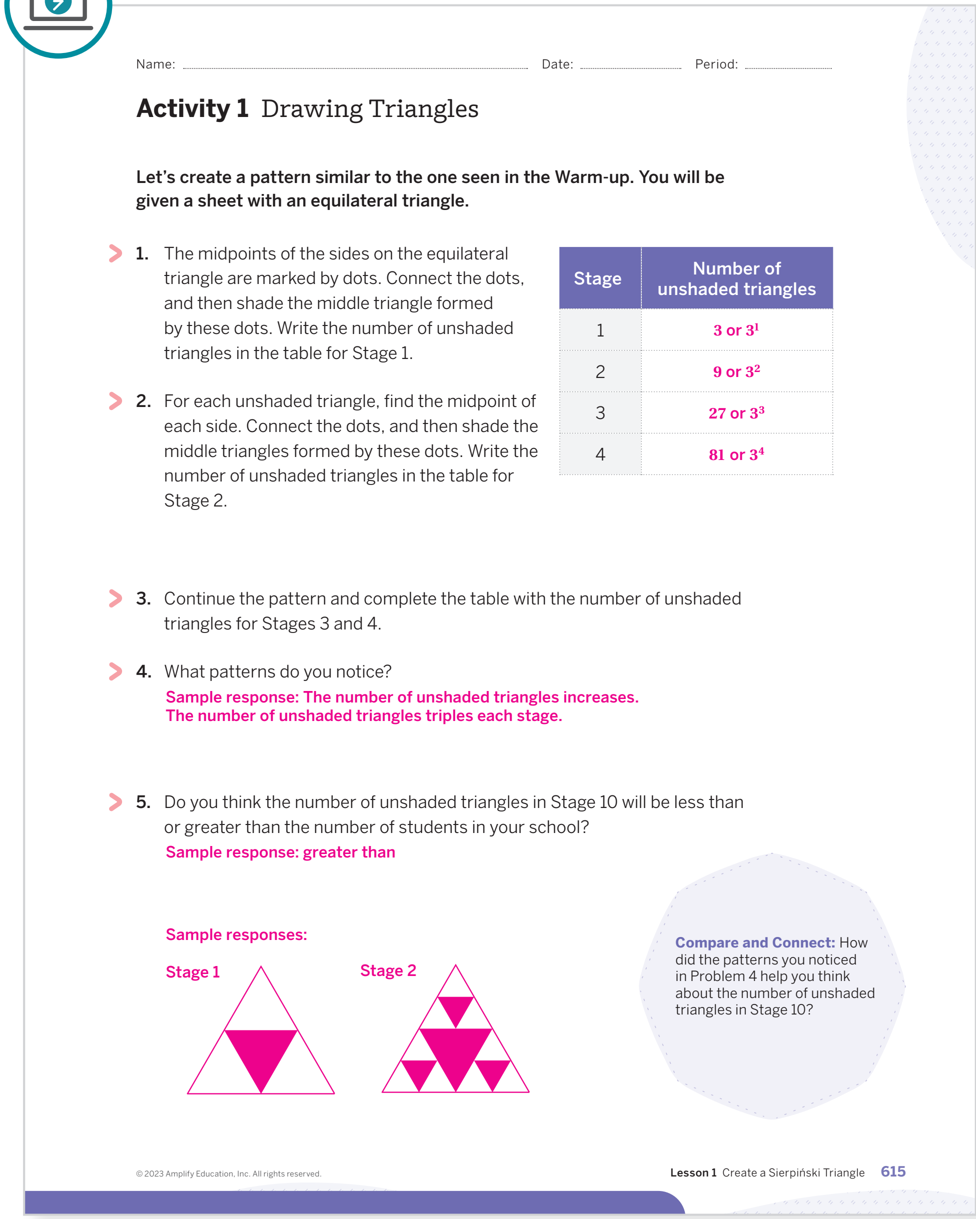
Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can watch an animation of the Chaos game and be introduced to the Sierpiński triangle.

Independent | 20 min

Activity 1 Drawing Triangles

Students draw the first four stages of a Sierpiński triangle pattern to look for repeated multiplication by a whole number.



Name: Date: Period:

Activity 1 Drawing Triangles

Let's create a pattern similar to the one seen in the Warm-up. You will be given a sheet with an equilateral triangle.

1. The midpoints of the sides on the equilateral triangle are marked by dots. Connect the dots, and then shade the middle triangle formed by these dots. Write the number of unshaded triangles in the table for Stage 1.

2. For each unshaded triangle, find the midpoint of each side. Connect the dots, and then shade the middle triangles formed by these dots. Write the number of unshaded triangles in the table for Stage 2.

Stage	Number of unshaded triangles
1	3 or 3^1
2	9 or 3^2
3	27 or 3^3
4	81 or 3^4

3. Continue the pattern and complete the table with the number of unshaded triangles for Stages 3 and 4.

4. What patterns do you notice?
   Sample response: The number of unshaded triangles increases. The number of unshaded triangles triples each stage.

5. Do you think the number of unshaded triangles in Stage 10 will be less than or greater than the number of students in your school?
   Sample response: greater than

Sample responses:

Stage 1 Stage 2

Compare and Connect: How did the patterns you noticed in Problem 4 help you think about the number of unshaded triangles in Stage 10?

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Lesson 1 Create a Sierpiński Triangle 615

1 Launch

Review the instructions provided in Problems 1 and 2 and distribute the Activity 1 PDF, rulers, and colored pencils to each student. Have students use the ***Think-Pair-Share*** routine. Give them 10 minutes to complete the activity independently before sharing responses with a partner.

2 Monitor

Help students get started by reminding them to use their ruler to measure the midpoints and draw the new triangle.

Look for points of confusion:

- **Struggling to find the number of unshaded triangles in Stage 10.** Emphasize the repeated multiplication by 3 and allow access to a calculator.
- **Not knowing the number of students in their school.** Provide this number or suggest a way for students to estimate it.

Look for productive strategies:

- Noticing that the number of unshaded triangles triples for each stage.

3 Connect

Have students share any patterns they noticed.

Ask:

- "How can you write the number of unshaded triangles in each stage as a single power?" 3^1, 3^2, 3^3, 3^4
- "How can you use a single power to write and evaluate an expression that represents the number of triangles in Stage 10?"

Highlight that because the number of unshaded triangles multiplies by 3 for each stage, students can use powers of 3 to represent this number.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with 4 stages of the Sierpiński triangle found in Activity 2. Have students complete the table using the shaded Sierpiński triangle and ask, "What do you notice about how the number of unshaded triangles increases?" This will allow students to focus on analyzing the triangle patterns and identifying the repeated multiplication pattern.

Extension: *Math Enrichment*

Have students write an expression that represents the number of unshaded triangles in Stage 100 and Stage n. 3^{100} and 3^n

Math Language Development

MLR7: Compare and Connect

After completing Problems 1–5 independently, have students compare any patterns they notice with a partner. Ask them to connect the patterns they described in Problem 4 to how they used reasoning in Problem 5 by having them respond to the question posed in the Student Edition, "How did the patterns you noticed in Problem 4 help you think about the number of unshaded triangles in Stage 10?"

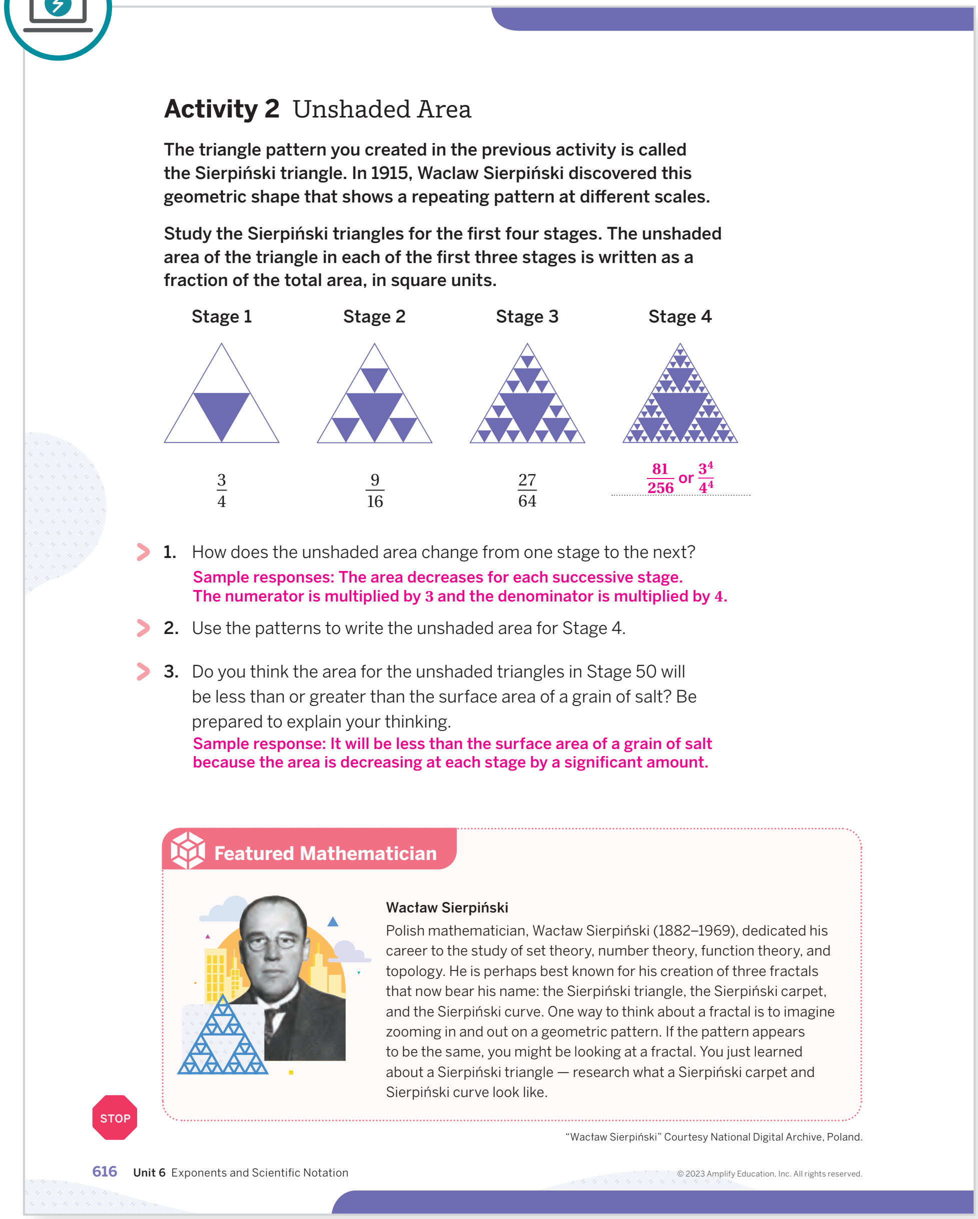
English Learners

Provide students time to rehearse and formulate what they will say independently before sharing with a partner.

Pairs | 20 min

Activity 2 Unshaded Area

Students determine the unshaded area of the Sierpiński triangle, noticing repeated multiplication by a fraction, to understand that exponents can represent repeated multiplication by non-whole number values.



Activity 2 Unshaded Area

The triangle pattern you created in the previous activity is called the Sierpiński triangle. In 1915, Waclaw Sierpiński discovered this geometric shape that shows a repeating pattern at different scales.

Study the Sierpiński triangles for the first four stages. The unshaded area of the triangle in each of the first three stages is written as a fraction of the total area, in square units.

Stage 1	Stage 2	Stage 3	Stage 4
$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$	$\frac{81}{256}$ or $\frac{3^4}{4^4}$

1. How does the unshaded area change from one stage to the next?
   Sample responses: The area decreases for each successive stage. The numerator is multiplied by 3 and the denominator is multiplied by 4.
2. Use the patterns to write the unshaded area for Stage 4.
3. Do you think the area for the unshaded triangles in Stage 50 will be less than or greater than the surface area of a grain of salt? Be prepared to explain your thinking.
   Sample response: It will be less than the surface area of a grain of salt because the area is decreasing at each stage by a significant amount.

Featured Mathematician

Wacław Sierpiński

Polish mathematician, Wacław Sierpiński (1882–1969), dedicated his career to the study of set theory, number theory, function theory, and topology. He is perhaps best known for his creation of three fractals that now bear his name: the Sierpiński triangle, the Sierpiński carpet, and the Sierpiński curve. One way to think about a fractal is to imagine zooming in and out on a geometric pattern. If the pattern appears to be the same, you might be looking at a fractal. You just learned about a Sierpiński triangle — research what a Sierpiński carpet and Sierpiński curve look like.

STOP

"Wacław Sierpiński" Courtesy National Digital Archive, Poland.

616 Unit 6 Exponents and Scientific Notation

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Provide access to calculators.

2 Monitor

Help students get started by asking them by what numbers the numerator and denominator are each multiplied, from each stage to the next stage.

Look for points of confusion:

- **Struggling to attempt Problem 3.** Ask students to represent each number as a power of $\frac{3}{4}$, similar to Activity 1.

Look for productive strategies:

- Noticing the unshaded area is multiplied by $\frac{3}{4}$ for each successive stage.

3 Connect

Have students share any patterns they noticed.

Ask:

- "How can you write the unshaded area in each stage as a single power?" The unshaded areas are $\left(\frac{3}{4}\right)^1$, $\left(\frac{3}{4}\right)^2$, $\left(\frac{3}{4}\right)^3$, and $\left(\frac{3}{4}\right)^4$.
- "Do you think the area of the unshaded triangles in Stage 50 will be less than or greater than the surface area of a grain of salt?" Less than; The unshaded area in Stage 50 is 0.000000566 ft^2, while the surface area of a typical grain of salt is about 0.000005812 ft^2.

Highlight that the unshaded area is multiplied by $\frac{3}{4}$ for each successive stage; therefore, students can use exponents to represent this number. Exponents can be used to show repeated multiplication even when the repeated factor is a fraction.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students of their previous work with multiplying fractions by modeling a simple fraction multiplication problem, such as $\frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12}$, or $\frac{1}{2}$. Ask students what they notice and what they remember about multiplying fractions and then reveal the prompt for the activity.

Math Language Development

MLR2: Collect and Display

During the Connect, add terms that students use to describe writing the unshaded areas as a single power, such as *repeated multiplication* and *exponents*, to the class display. Invite students to continue adding to and using language from the display throughout the unit.

English Learners

Use a table to show the growth pattern and add annotations to the table that highlight the connection between repeated multiplication and exponents.

Featured Mathematician

Wacław Sierpiński

Have students read about Featured Mathematician, Wacław Sierpiński, who is well known for his work with fractals.

Summary From Teeny-Tiny to Downright Titanic

Review and synthesize how the patterns of the Sierpiński triangle show repeated multiplication, which can be indicated using exponents.

Narrative Connections

Unit 6 Exponents and Scientific Notation

From Teeny-Tiny to Downright Titanic

We call this the Sierpiński triangle. Pretty, right? We imagine shapes as things that are settled, static. You wouldn't expect a triangle to grow a pair of legs and hop around the page. But this guy is different! In some ways, it's like a living thing. It moves. It changes. You can see it growing versions of itself, within itself. And those versions grow their own versions. On, and on, and on.

With enough rounds, you'll have a quilt-work of nested triangles. And in each version, those triangles become smaller and smaller — smaller than a pen point, smaller than even the space between atoms.

And yet with each round, the number of triangles keeps increasing — growing into values so colossal, it would cramp the hand, circle the equator, and stagger even the most advanced computer.

These are the numbers that govern how we talk about the vast expanse between planets, or the width of a bacterium, or the chances of getting attacked by an elephant.

Welcome to the Land of Colossal and Infinitesimal Numbers. Your guide? Our old friend, the *exponent*.

Welcome to Unit 6.

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Lesson 1 Create a Sierpiński Triangle 617

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Have students share how exponents can be used to show patterns in the Sierpiński triangle.

Ask:

- "What are some patterns shown by the Sierpiński triangle?" Answers may vary.
- "How are the patterns from Activity 1 and Activity 2 similar? How are they different?" Both patterns show repeated multiplication. One increases, while the other decreases.

Highlight that exponents are an efficient way to show repeated multiplication. For example, expressing repeated multiplication by a factor of 3, a total of ten times, can be written as $3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3$, which can be time consuming to write or understand. Using exponents, this is equal to 3^{10}, which is a more efficient way of writing the same value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. To help them engage in meaningful reflection, consider asking:

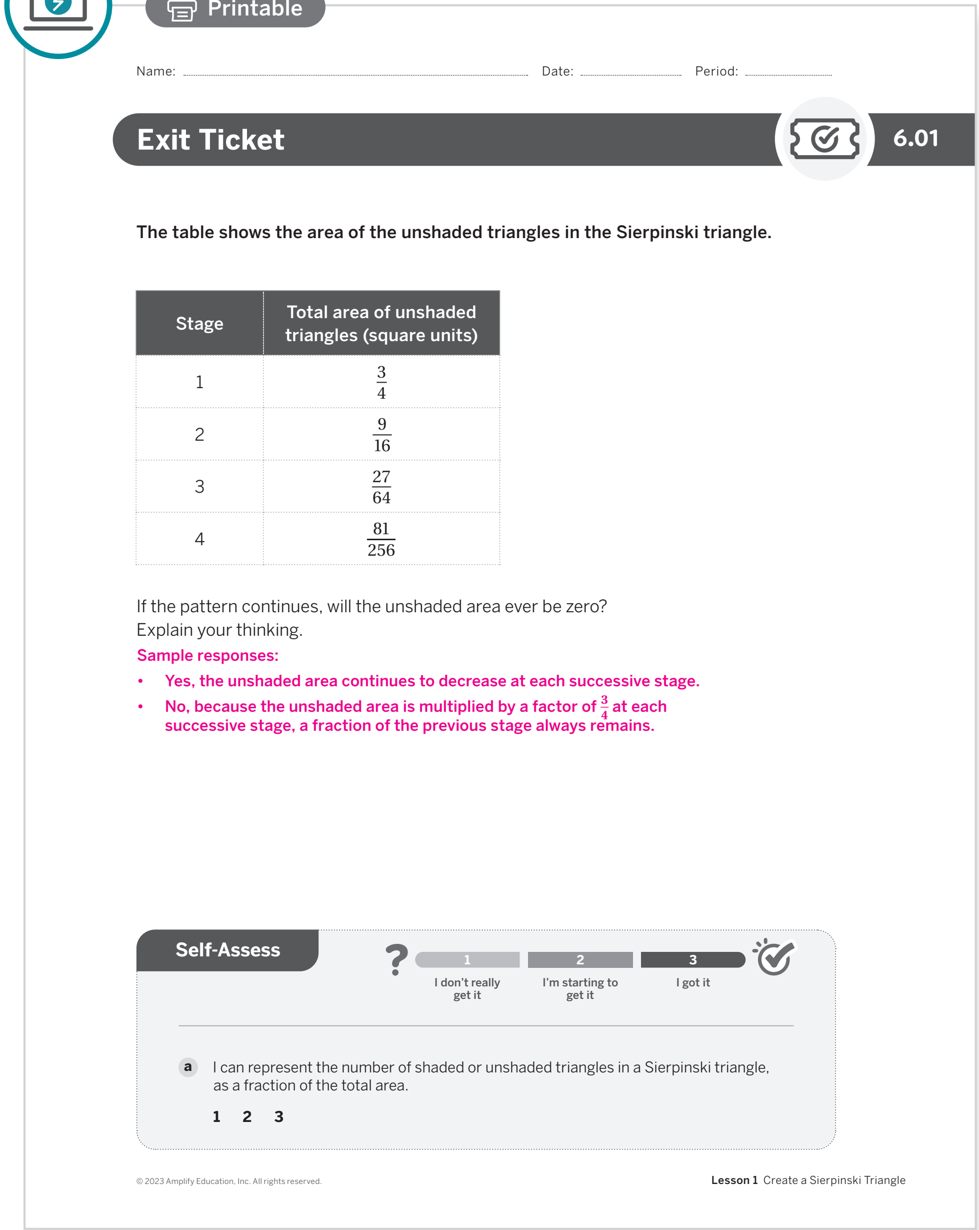
- "Is there a more efficient way to write really small and really large numbers?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using the pattern of the Sierpiński triangle to make a prediction.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 6.01

The table shows the area of the unshaded triangles in the Sierpinski triangle.

Stage	Total area of unshaded triangles (square units)
1	$\frac{3}{4}$
2	$\frac{9}{16}$
3	$\frac{27}{64}$
4	$\frac{81}{256}$

If the pattern continues, will the unshaded area ever be zero? Explain your thinking.

Sample responses:

- **Yes, the unshaded area continues to decrease at each successive stage.**
- **No, because the unshaded area is multiplied by a factor of $\frac{3}{4}$ at each successive stage, a fraction of the previous stage always remains.**

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can represent the number of shaded or unshaded triangles in a Sierpinski triangle, as a fraction of the total area.

1 2 3

Success looks like . . .

- **Goal:** Creating an expression that represents repeated multiplication, and explaining how the structure of the expression helps predict quantities.
- **Language Goal:** Describing a pattern that could be expressed using repeated multiplication. **(Speaking and Listening, Writing)**
  - » Describing the area of the unshaded triangles as a pattern using multiplication.

Suggested next steps

Because this Exit Ticket is designed to help students reason and think abstractly, either response is acceptable at this point in the unit, provided they support their thinking. While the mathematically correct response is *no*, students are not expected to know at this point that the unshaded area will never reach zero.

Professional Learning

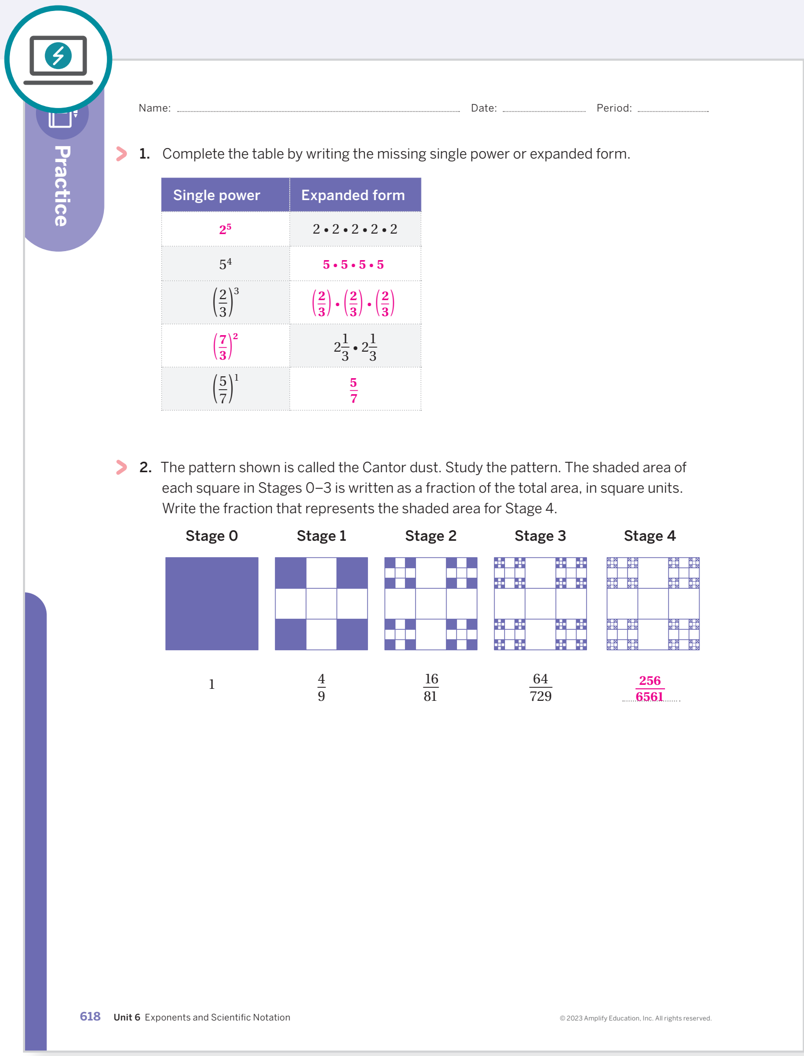
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on Activity 1?
- What did you see in the way how some students looked for patterns that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Complete the table by writing the missing single power or expanded form.

Single power	Expanded form
2^5	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
5^4	$5 \cdot 5 \cdot 5 \cdot 5$
$\left(\frac{2}{3}\right)^3$	$\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)$
$\left(\frac{7}{3}\right)^2$	$2\frac{1}{3} \cdot 2\frac{1}{3}$
$\left(\frac{5}{7}\right)^1$	$\frac{5}{7}$

2. The pattern shown is called the Cantor dust. Study the pattern. The shaded area of each square in Stages 0–3 is written as a fraction of the total area, in square units. Write the fraction that represents the shaded area for Stage 4.

Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
1	$\frac{4}{9}$	$\frac{16}{81}$	$\frac{64}{729}$	$\frac{256}{6561}$

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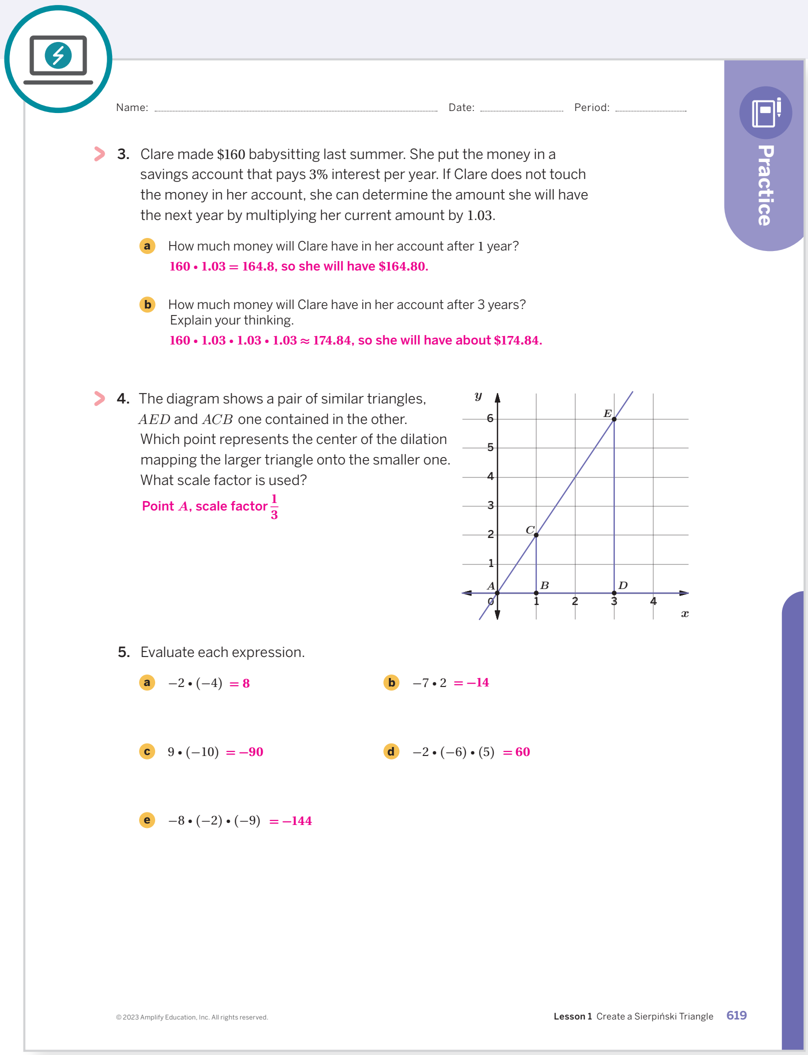

Practice

Name: ________ Date: ________ Period: ________

3. Clare made \$160 babysitting last summer. She put the money in a savings account that pays 3% interest per year. If Clare does not touch the money in her account, she can determine the amount she will have the next year by multiplying her current amount by 1.03.

   a. How much money will Clare have in her account after 1 year?
   $160 \cdot 1.03 = 164.8$, so she will have \$164.80.

   b. How much money will Clare have in her account after 3 years? Explain your thinking.
   $160 \cdot 1.03 \cdot 1.03 \cdot 1.03 \approx 174.84$, so she will have about \$174.84.

4. The diagram shows a pair of similar triangles, AED and ACB one contained in the other. Which point represents the center of the dilation mapping the larger triangle onto the smaller one. What scale factor is used?
   Point A, scale factor $\frac{1}{3}$

5. Evaluate each expression.

   a. $-2 \cdot (-4) = 8$

   b. $-7 \cdot 2 = -14$

   c. $9 \cdot (-10) = -90$

   d. $-2 \cdot (-6) \cdot (5) = 60$

   e. $-8 \cdot (-2) \cdot (-9) = -144$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 2	3
Spiral	4	Unit 2 Lesson 5	1
Formative ⚡	5	Unit 6 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

*Because this is a unit launch, no specific on-grade content standards are addressed in this lesson. Practice Problems 1–3 prepare students for work with integer exponents **(8.EE.A.1)**.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Sub-Unit 1

Exponent Rules

Whole Class

In this Sub-Unit, students build on their work with exponents and develop the rules for exponents.

SUB-UNIT

1 Exponent Rules

Narrative Connections

How many carbs are in a game of chess?

The story goes that there once was a King Shirham of India, whose Grand Vizier, Sissa ben Dahir, invented the game of chess.

The king loved the game so much, he asked ben Dahir to name his reward for having invented it.

The vizier answered, “Your majesty, instead of riches, I ask for wheat. Award me with 2 grain on the first square of the chessboard, 4 grains on the second square, 8 grains on the third, and so on, doubling the number with each square until all 64 squares have been accounted for.”

King Shirham agreed. So they started: 2 grain on the first square; 4 grains on the second square; 8 grains on the third, 16 grains on the fourth . . .

What King Shirham didn’t realize was that the number of grains was growing a lot faster than he’d previously thought. They were growing ***exponentially***. By the time the King reached the last square, he would’ve had to place 36,893,488,147,419,103,231 grains (or just about 2^{65}) — enough to deplete all the wheat in the entire kingdom many times over!

You may remember powers and exponents from prior grades. But what happens when you multiply two powers? Divide two powers? How many times more grains of wheat are on the 30th square than the 12th square? If you recall that exponents represent repeated multiplication, you can answer these questions.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore what it means to multiply or divide powers in the following places:

- **Lesson 3, Activities 1–2:** Card Sort: Multiplying Powers of 10, Multiplying Powers With Bases Other Than 10
- **Lesson 4, Activities 1–2:** Card Sort: Dividing Powers of 10, Dividing Powers With Bases Other Than 10
- **Lesson 5, Activity 2:** Follow the Exponent Rules
- **Lesson 7, Activity 1:** Powers of Products

UNIT 6 | **LESSON 2**

Reviewing Exponents

Let's review exponents.

Focus

Goal

1. **Language Goal:** Create an expression that represents repeated multiplication, and explain how the structure of the expression helps compare quantities. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students analyze the structure of powers to apply their understanding of exponents as repeated multiplication. Students come to realize that they do not always have to calculate the value of expressions involving exponents, but instead can look for and make use of the structure of the powers to understand and compare the values of expressions.

Previously

In Grade 6, students generated and evaluated numerical expressions involving whole number exponents.

Coming Soon

In subsequent lessons, students will make use of repeated reasoning to discover exponent rules when multiplying and dividing with powers.

Rigor

- Students strengthen their **fluency** in evaluating and comparing values written as a single power.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
10 min	10 min	10 min	5 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

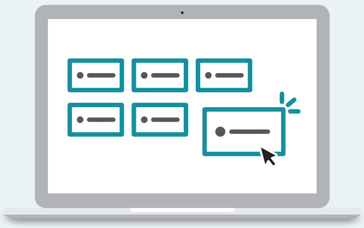
- *base*
- *equivalent expressions*
- *exponent*
- *power*

Amps | Featured Activity

Activity 2

Digitally Sort Expressions

Students match equivalent expressions by dragging and connecting them on screen.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not be focused enough to discern the differences in the expressions in Activity 2. Work with them to set a goal that involves the structure of the expression and the sign of its value. Have students work together to set steps that they will take to reach their goal and achieve success.

Modifications to Pacing

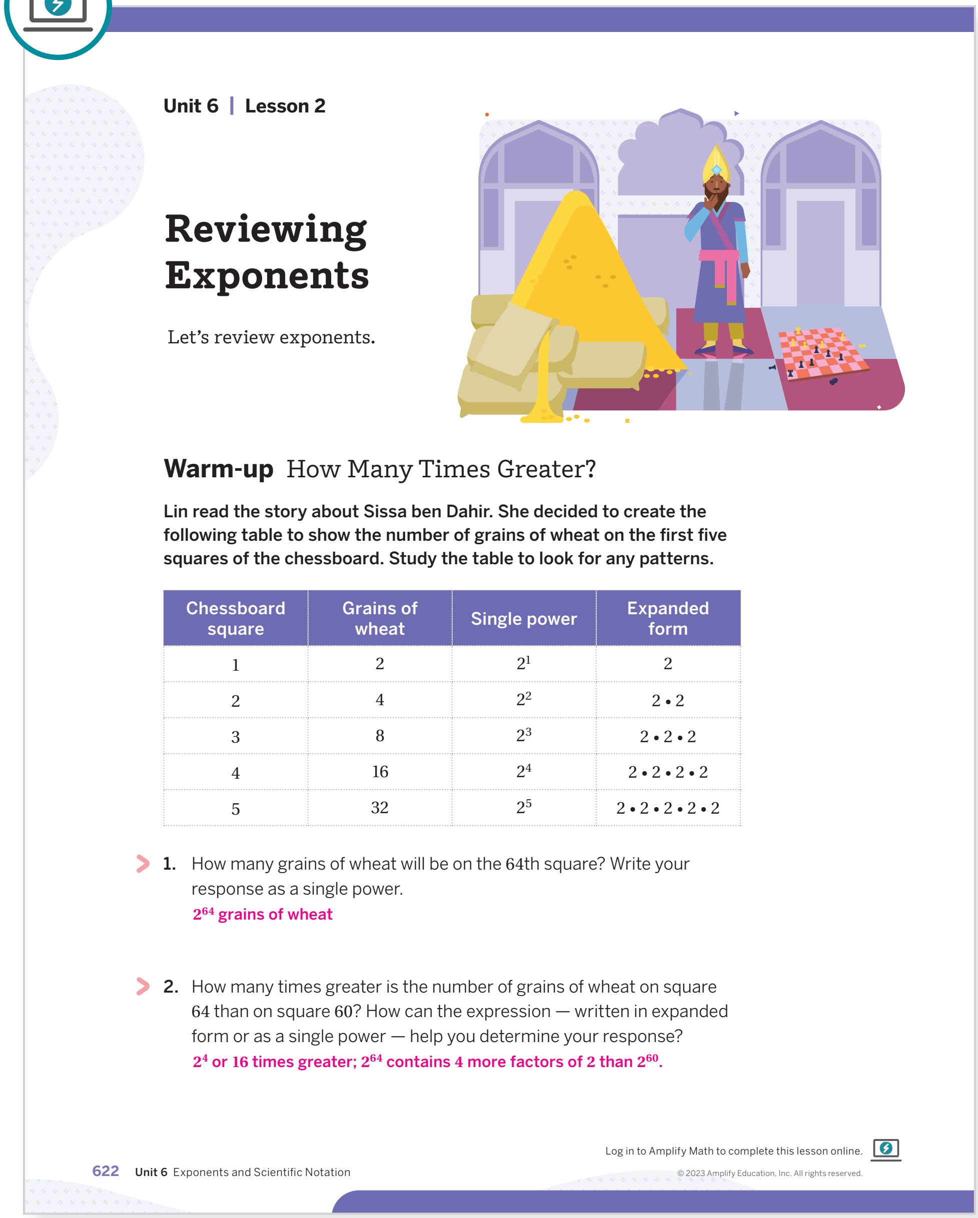
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 1, parts e–h may be omitted.
- **Activity 3** may be assigned as additional practice.

Independent | 10 min

Warm-up How Many Times Greater?

Students compare the grains of wheat on chessboard squares to look for and make use of the structure of expressions written in different forms.



Unit 6 | Lesson 2

Reviewing Exponents

Let's review exponents.

Warm-up How Many Times Greater?

Lin read the story about Sissa ben Dahir. She decided to create the following table to show the number of grains of wheat on the first five squares of the chessboard. Study the table to look for any patterns.

Chessboard square	Grains of wheat	Single power	Expanded form
1	2	2^1	2
2	4	2^2	$2 \cdot 2$
3	8	2^3	$2 \cdot 2 \cdot 2$
4	16	2^4	$2 \cdot 2 \cdot 2 \cdot 2$
5	32	2^5	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

1. How many grains of wheat will be on the 64th square? Write your response as a single power.
   2^{64} grains of wheat

2. How many times greater is the number of grains of wheat on square 64 than on square 60? How can the expression — written in expanded form or as a single power — help you determine your response?
   2^4 or 16 times greater; 2^{64} contains 4 more factors of 2 than 2^{60}.

Log in to Amplify Math to complete this lesson online.

622 Unit 6 Exponents and Scientific Notation

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1 Launch

Tell students that the table shows the first five chessboard squares and the number of grains of wheat written in different forms.

2 Monitor

Help students get started by telling them to look at the *Single power* column. Then ask, "How does the exponent relate to the square number?"

Look for points of confusion:

- **Trying to calculate the grains of wheat on square 64 or square 60**. Tell students that since the number of grains of wheat on these squares is very large, they should look at and use the structure of expressions written as a single power and expanded form.
- **Not knowing how to solve Problem 2.** Have students compare how many times greater the grains of wheat is on square 5 than square 2, and then have them revisit the problem.

Look for productive strategies:

- Noticing 2^{64} has 4 more factors than 2^{60}.

3 Connect

Have students share their strategies and how they used the different expressions to solve the problems.

Highlight the terminology *base* and *exponent*. Point out that there are different ways to say 2^{64}, such as "two raised to the power of sixty-four," or "two to the sixty-fourth power," or "two to the sixty-fourth."

Math Language Development

MLR2: Collect and Display

As students share their responses, add phrases to the class display, such as, "two raised to the power of six" and "two to the sixth power" for the expression 2^6. For exponents of 2 and 3, add "squared" and "cubed" to the class display also, as in "four squared," "four to the second power," etc.

English Learners

Use these sentence frames to help students interpret the base and the exponent.

- "2^6 means _____ to the _____ power."
- "4^2 means _____ squared."
- "5^3 means _____ cubed."

Power-up

To power up students' ability to determine the product of two or more integers, have students complete:

Recall that when a pair of numbers have the same sign, their product is positive. If their signs are different, their product is negative.

Determine the sign of each product.

1. $-6 \cdot (-3)$
   Positive
2. $4 \cdot (-2)$
   Negative
3. $-2 \cdot 3 \cdot (-1)$
   Positive
4. $-1 \cdot (-1) \cdot (-3)$
   Negative

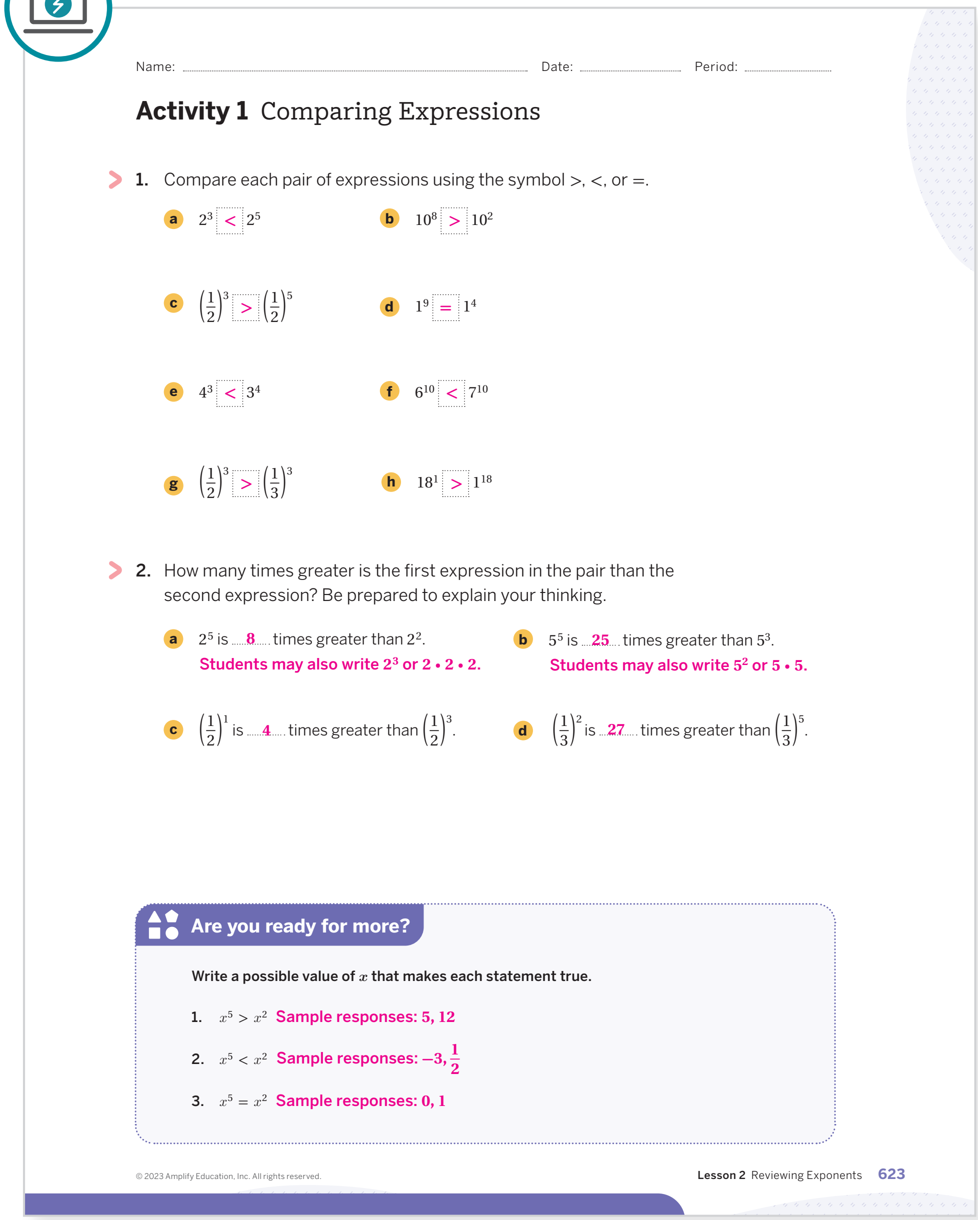
Use: Before Activity 2

Informed by: Performance on Lesson 1, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 10 min

Activity 1 Comparing Expressions

Students compare expressions written as a single power to see how the structure of each expression helps them compare the quantities.



Name: Date: Period:

Activity 1 Comparing Expressions

1. Compare each pair of expressions using the symbol >, <, or =.

a $2^3 \boxed{<} 2^5$

b $10^8 \boxed{>} 10^2$

c $\left(\frac{1}{2}\right)^3 \boxed{>} \left(\frac{1}{2}\right)^5$

d $1^9 \boxed{=} 1^4$

e $4^3 \boxed{<} 3^4$

f $6^{10} \boxed{<} 7^{10}$

g $\left(\frac{1}{2}\right)^3 \boxed{>} \left(\frac{1}{3}\right)^3$

h $18^1 \boxed{>} 1^{18}$

2. How many times greater is the first expression in the pair than the second expression? Be prepared to explain your thinking.

a 2^5 is 8 times greater than 2^2.
Students may also write 2^3 or $2 \cdot 2 \cdot 2$.

b 5^5 is 25 times greater than 5^3.
Students may also write 5^2 or $5 \cdot 5$.

c $\left(\frac{1}{2}\right)^1$ is 4 times greater than $\left(\frac{1}{2}\right)^3$.

d $\left(\frac{1}{3}\right)^2$ is 27 times greater than $\left(\frac{1}{3}\right)^5$.

Are you ready for more?

Write a possible value of x that makes each statement true.

1. $x^5 > x^2$ **Sample responses: 5, 12**
2. $x^5 < x^2$ **Sample responses: $-3, \frac{1}{2}$**
3. $x^5 = x^2$ **Sample responses: 0, 1**

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by reviewing the inequality symbols < and >. Use the examples $5 < 10$ and $15 > 10$ to remind students how to read, write, and interpret the symbols.

Look for points of confusion:

- **Thinking $\left(\frac{1}{2}\right)^5$ is greater than $\left(\frac{1}{2}\right)^3$ for Problem 1c because 5 is greater than 3.** Show the values as $\frac{1}{32}$ and $\frac{1}{8}$ or as 0.125 and 0.03125, and then have students re-evaluate their response.
- **Struggling to compare the fractions in Problems 2c and 2d.** Have students write the expressions in expanded form and compare these expressions to the ones from the Warm-up.

Look for productive strategies:

- Writing expressions in expanded form or evaluating the expressions to compare them.
- Using the exponents to help them see how many times greater an expression is than the other.

3 Connect

Ask, "How can you determine the greater value without computing the value of each expression?"

Have students share how they can use the structure of the expression to determine the greater value.

Highlight that students don't always have to compute the values of expressions with powers to compare them. Sometimes, they can look at the structure of the expression and use the structure to help compare the values.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by having students focus on Problems 1a–1d and Problems 2a and 2b. These problems will still allow them to access bases that are both whole numbers and fractions, yet the comparisons are made when both expressions use the same base.

Accessibility: *Activate Prior Knowledge*

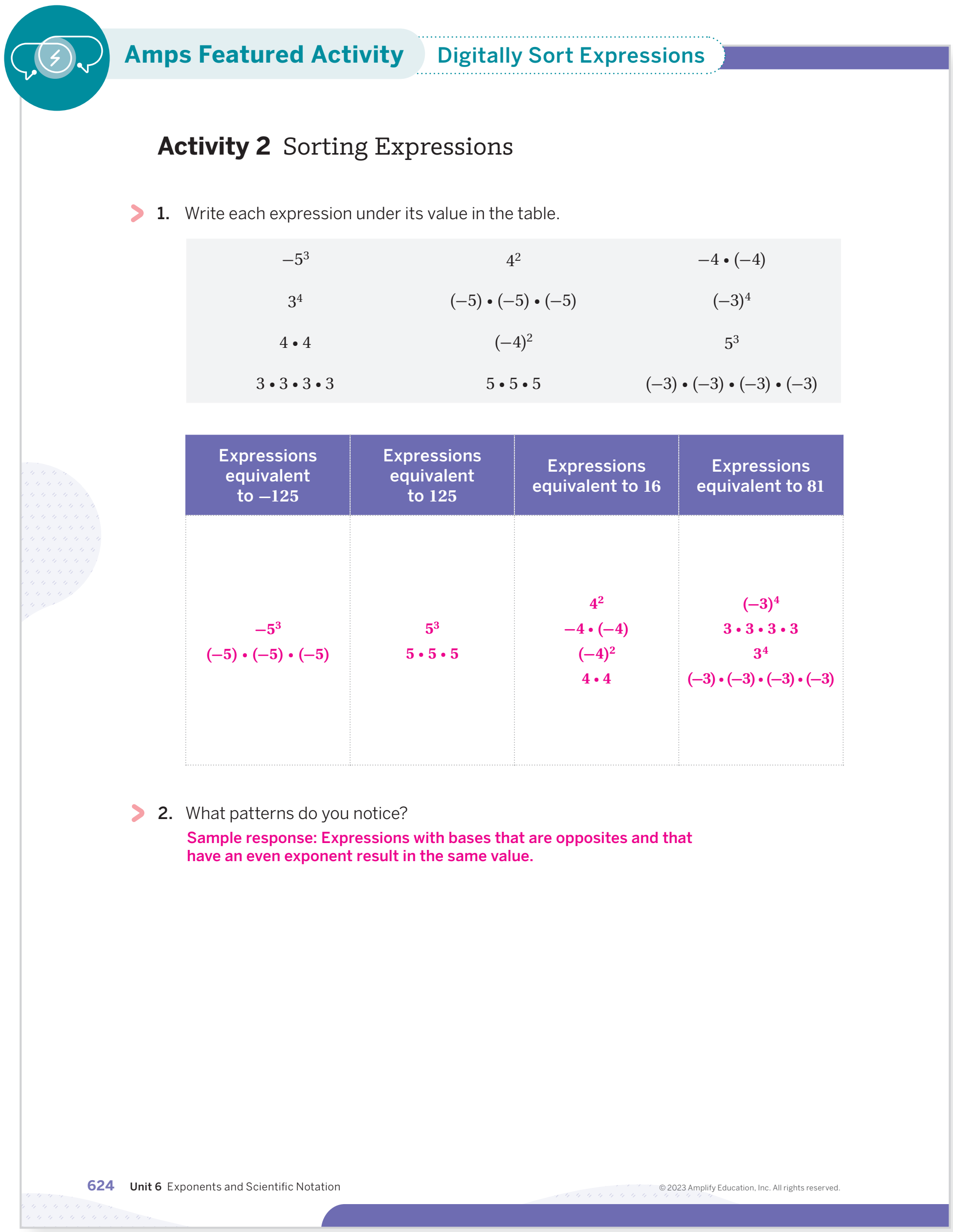
Remind students that they previously learned about inequality symbols in prior grades. Review the language of inequality symbols by displaying $5 < 10$ and writing in words, "five is less than ten." Then display $15 > 10$ and write in words, "fifteen is greater than ten."

Pairs | 10 min

Activity 2 Sorting Expressions

Students evaluate expressions represented in different forms to compare and contrast the structure of expressions involving negative bases.

Amps Featured Activity Digitally Sort Expressions



Activity 2 Sorting Expressions

1. Write each expression under its value in the table.

-5^3	4^2	$-4 \cdot (-4)$
3^4	$(-5) \cdot (-5) \cdot (-5)$	$(-3)^4$
$4 \cdot 4$	$(-4)^2$	5^3
$3 \cdot 3 \cdot 3 \cdot 3$	$5 \cdot 5 \cdot 5$	$(-3) \cdot (-3) \cdot (-3) \cdot (-3)$

Expressions equivalent to -125	Expressions equivalent to 125	Expressions equivalent to 16	Expressions equivalent to 81
-5^3 $(-5) \cdot (-5) \cdot (-5)$	5^3 $5 \cdot 5 \cdot 5$	4^2 $-4 \cdot (-4)$ $(-4)^2$ $4 \cdot 4$	$(-3)^4$ $3 \cdot 3 \cdot 3 \cdot 3$ 3^4 $(-3) \cdot (-3) \cdot (-3) \cdot (-3)$

2. What patterns do you notice?

Sample response: Expressions with bases that are opposites and that have an even exponent result in the same value.

1 Launch

Tell students they will be looking at expressions written in different forms. Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by collectively determining the value of $(-5)^3$.

Look for points of confusion:

- **Thinking that multiplying three factors of a negative number will result in a positive value.** Have students write each step. For example, $-5 \cdot (-5) = 25$, and then $25 \cdot (-5) = -125$.

Look for productive strategies:

- Recognizing the groups of equivalent expressions. Have students compare the base and exponent and come up with a general rule for when the two expressions will be equivalent or not equivalent.

3 Connect

Have pairs of students share their responses for Problem 2. Have students share if they disagree with any of the responses.

Highlight that if powers have an opposite base and the same odd exponent, they will have opposite values. If the powers have an opposite base and the same even exponent, they will have equivalent values.

Ask:

- "Without evaluating, will 10^7 and $(-10)^7$ be equivalent? Explain your thinking."
- "Without evaluating, will the value of $(-10)^3$ be positive or negative? What about $(-10)^2$?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can match equivalent expressions by dragging and connecting them on screen.

Accessibility: *Vary Demands to Optimize Challenge*

Have students circle the expressions that have a base of 4 and examine those first. Then have them examine the expressions that have a base of 3, and, finally, expressions that have a base of 5.

Math Language Development

MLR8: Discussion Supports

During the Connect, to help students explain why two or more expressions in each list are equivalent, provide sentence frames such as:

- "The expressions _____ and _____ are equivalent because . . ."
- "The expressions _____ and _____ have opposite bases and the same even/odd exponent, so they are equivalent."
- "The expressions _____ and _____ have opposite bases and the same even/odd exponent, so they have opposite values."

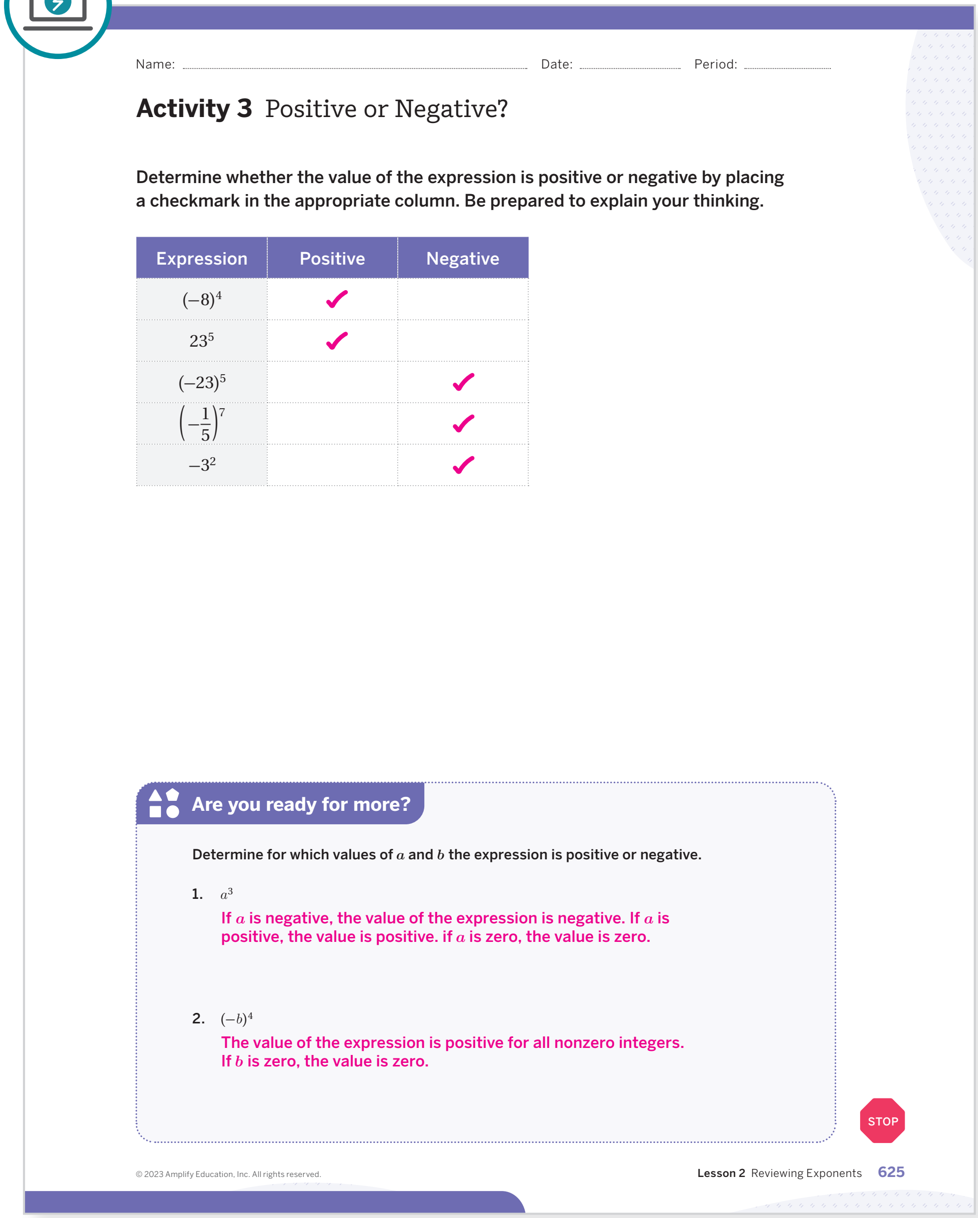
English Learners

Be sure students understand the meanings of the terms *even* and *odd*. Consider adding examples to the class display.

Pairs | 5 min

Activity 3 Positive or Negative?

Students use structure and the order of operations to determine the effect that parentheses around a negative base has on the value of the expression.



Name: ______ Date: ______ Period: ______

Activity 3 Positive or Negative?

Determine whether the value of the expression is positive or negative by placing a checkmark in the appropriate column. Be prepared to explain your thinking.

Expression	Positive	Negative
$(-8)^4$	✓	
23^5	✓	
$(-23)^5$		✓
$\left(-\frac{1}{5}\right)^7$		✓
-3^2		✓

Are you ready for more?

Determine for which values of a and b the expression is positive or negative.

1. a^3

   If a is negative, the value of the expression is negative. If a is positive, the value is positive. if a is zero, the value is zero.

2. $(-b)^4$

   The value of the expression is positive for all nonzero integers. If b is zero, the value is zero.

STOP

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Lesson 2 Reviewing Exponents 625

1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by having them identify whether each exponent is an odd or even number.

Look for points of confusion:

- **Thinking the value of -3^2 is positive**. Revisit this expression during the Connect.

Look for productive strategies:

- Evaluating each expression to determine if its value is positive or negative. Encourage students to use the base, exponent, and category (odd or even) to help them determine their response.

3 Connect

Have students share their responses for each expression by using the ***Poll the Class*** routine.

Ask, "Do you think -3^2 and $(-3)^2$ will result in the same value? Why or why not?"

Highlight that the use of parentheses around a negative base affects its value. For example, the expressions -3^2 and $(-3)^2$ have different values; $-3^2 = -(3) \cdot (3) = -9$ while $(-3) \cdot (-3) = 9$. Have students compare and contrast this with the use of parentheses around a positive base. Ask them whether 3^2 and $(-3)^2$ have the same value. Yes, they have the same value. Ask students why this is different from when the base is negative. The order of operations tells me to evaluate the exponent first and then take the opposite.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate or suggest that students add another column to the table with the header, "Even or Odd?" Ask students to identify whether the exponent is even or odd. This will help students make connections about the effect that an even or odd exponent has on the value of the expression.

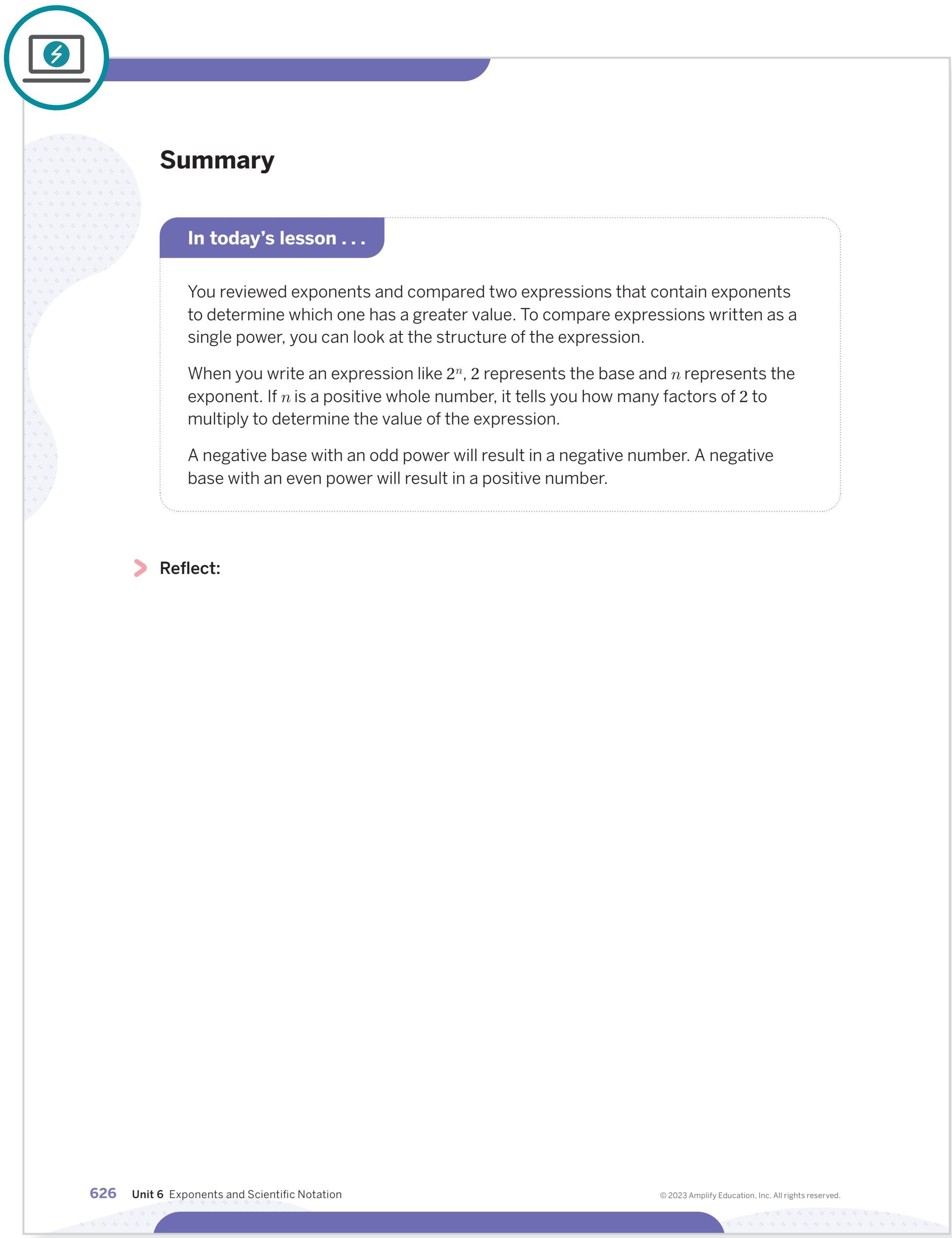
Extension: *Math Enrichment*

Display the statement, "The value of 2^{-x} is always negative because the exponent is negative." Have students decide whether they agree or disagree with the statement and explain their thinking. Sample response: I disagree. If $x = -4$, then $-x = 4$ and 2^4 is positive.

Whole Class | 5 min

Summary

Review and synthesize how understanding the structure of expressions involving exponents can be used to compare expressions without evaluating them.



Summary

In today's lesson . . .

You reviewed exponents and compared two expressions that contain exponents to determine which one has a greater value. To compare expressions written as a single power, you can look at the structure of the expression.

When you write an expression like 2^n, 2 represents the base and n represents the exponent. If n is a positive whole number, it tells you how many factors of 2 to multiply to determine the value of the expression.

A negative base with an odd power will result in a negative number. A negative base with an even power will result in a positive number.

Reflect:

626 Unit 6 Exponents and Scientific Notation

Synthesize

Ask, "What are some different ways you can compare powers that have the same base?"

Have students share how the structure of an expression containing exponents helps them compare quantities.

Highlight that students don't always have to evaluate expressions with powers to compare them. Sometimes, they can look at the structure of the expression and use the structure to help compare. Emphasize the following for expressions containing negative bases:

- A negative base raised to an even power results in a positive value.
- A negative base raised to an odd power results in a negative value.
- Placing parentheses around a negative base affects the value of the expression. For example, -2^4 is negative (-16), while $(-2)^4$ is positive (16).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

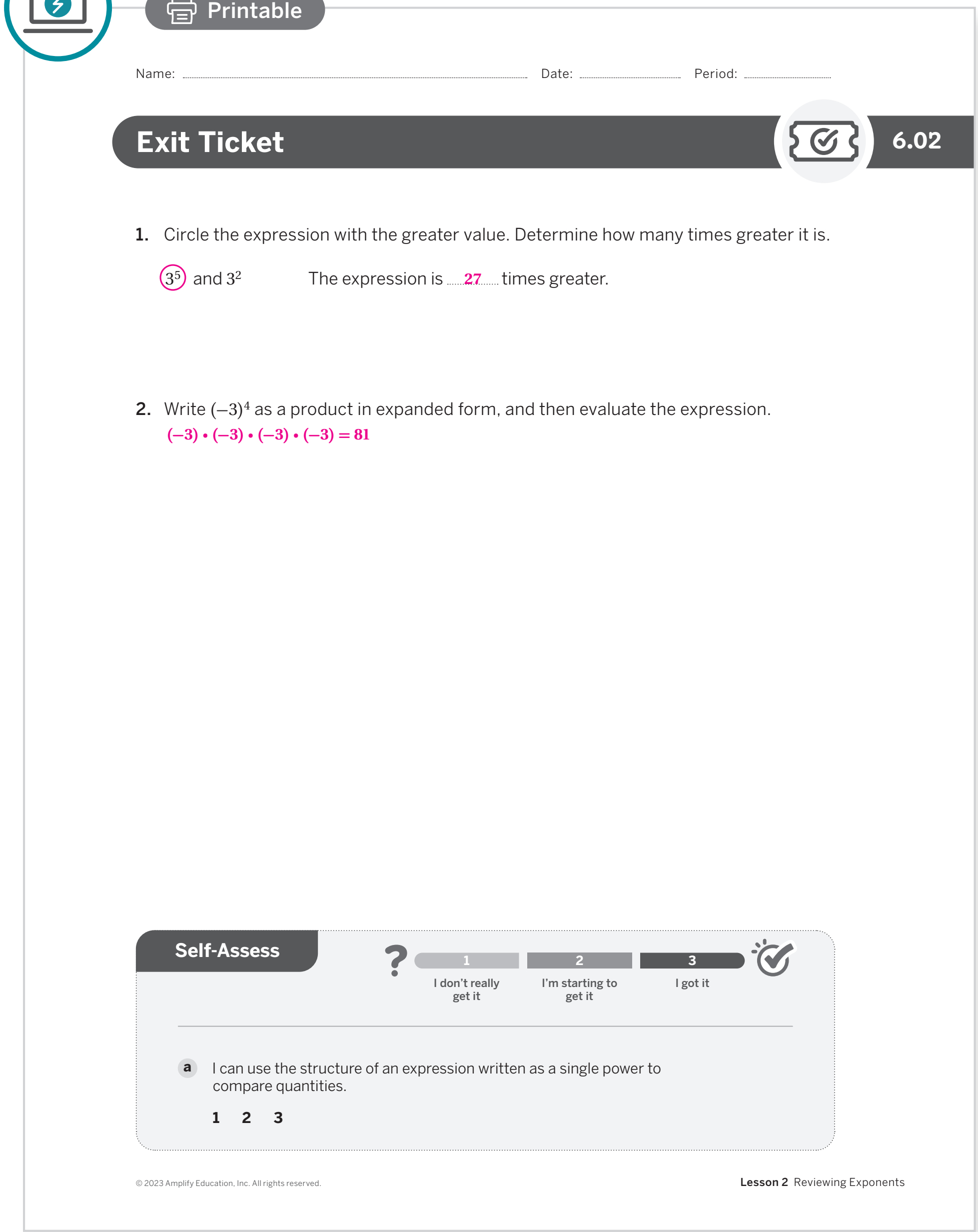
- "How can you use the structure of expressions to compare the value of single powers?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by evaluating powers and determining how many times greater one value is than the other.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 6.02

1. Circle the expression with the greater value. Determine how many times greater it is.

[✓ 3^5] and 3^2 The expression is 27 times greater.

2. Write $(-3)^4$ as a product in expanded form, and then evaluate the expression.

$(-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use the structure of an expression written as a single power to compare quantities.

1 2 3

Lesson 2 Reviewing Exponents

Success looks like . . .

- **Language Goal:** Creating an expression that represents repeated multiplication, and explaining how the structure of the expression helps compare quantities **(Speaking and Listening, Reading and Writing)**.
  - » Selecting the correct expression and determining that it is 27 times greater in Problem 1.

Suggested next steps

If students incorrectly respond to Problem 1, consider:

- Having students write 3^5 and 3^2 in expanded form, and then compare the two values.
- Reviewing Activity 1.
- Assigning Practice Problem 3.

If students do not write the product as $(-3) \cdot (-3) \cdot (-3) \cdot (-3)$ in Problem 2, consider:

- Reviewing different ways to write powers from Activity 2.
- Assigning Practice Problem 1.
- Asking, "How does the exponent help you write an expression in expanded form?"

Professional Learning

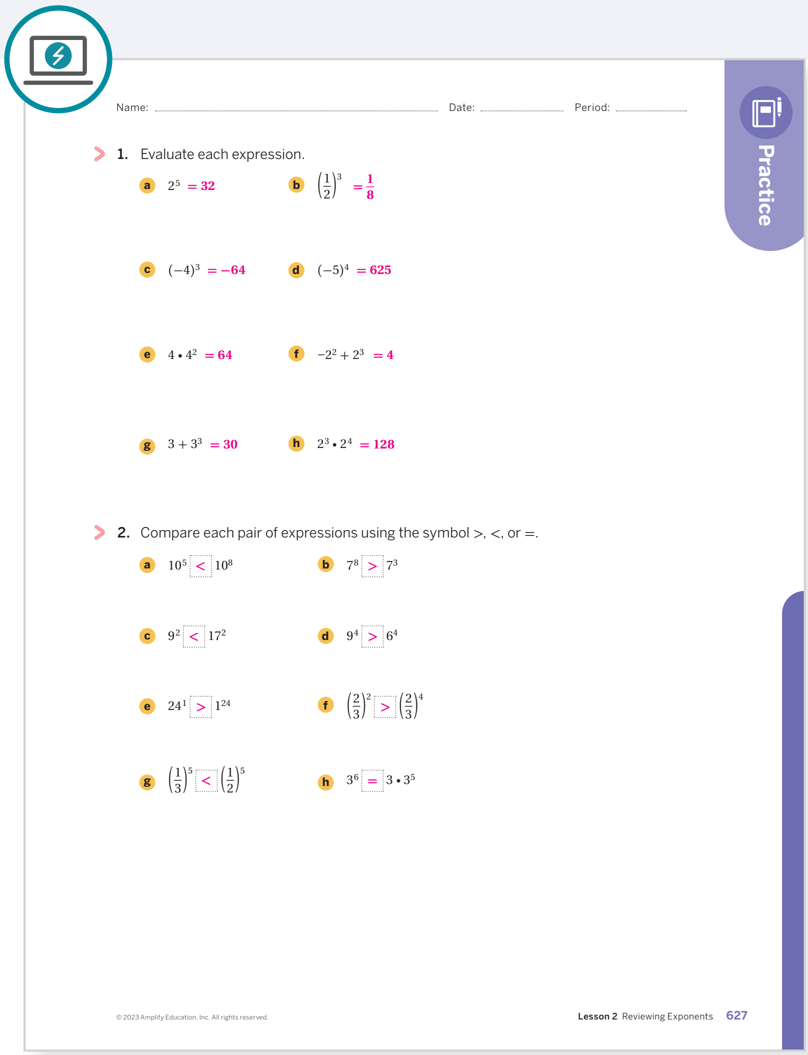
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which groups of students did and didn't have their ideas seen and heard today?
- In this lesson, students compared expressions with exponents. How will that support their understanding when they perform operations involving exponents?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Evaluate each expression.

   a. 2^5 $= 32$
   b. $\left(\frac{1}{2}\right)^3$ $= \frac{1}{8}$
   c. $(-4)^3$ $= -64$
   d. $(-5)^4$ $= 625$
   e. $4 \cdot 4^2$ $= 64$
   f. $-2^2 + 2^3$ $= 4$
   g. $3 + 3^3$ $= 30$
   h. $2^3 \cdot 2^4$ $= 128$

2. Compare each pair of expressions using the symbol >, <, or =.

   a. $10^5 \boxed{<} 10^8$
   b. $7^8 \boxed{>} 7^3$
   c. $9^2 \boxed{<} 17^2$
   d. $9^4 \boxed{>} 6^4$
   e. $24^1 \boxed{>} 1^{24}$
   f. $\left(\frac{2}{3}\right)^2 \boxed{>} \left(\frac{2}{3}\right)^4$
   g. $\left(\frac{1}{3}\right)^5 \boxed{<} \left(\frac{1}{2}\right)^5$
   h. $3^6 \boxed{=} 3 \cdot 3^5$

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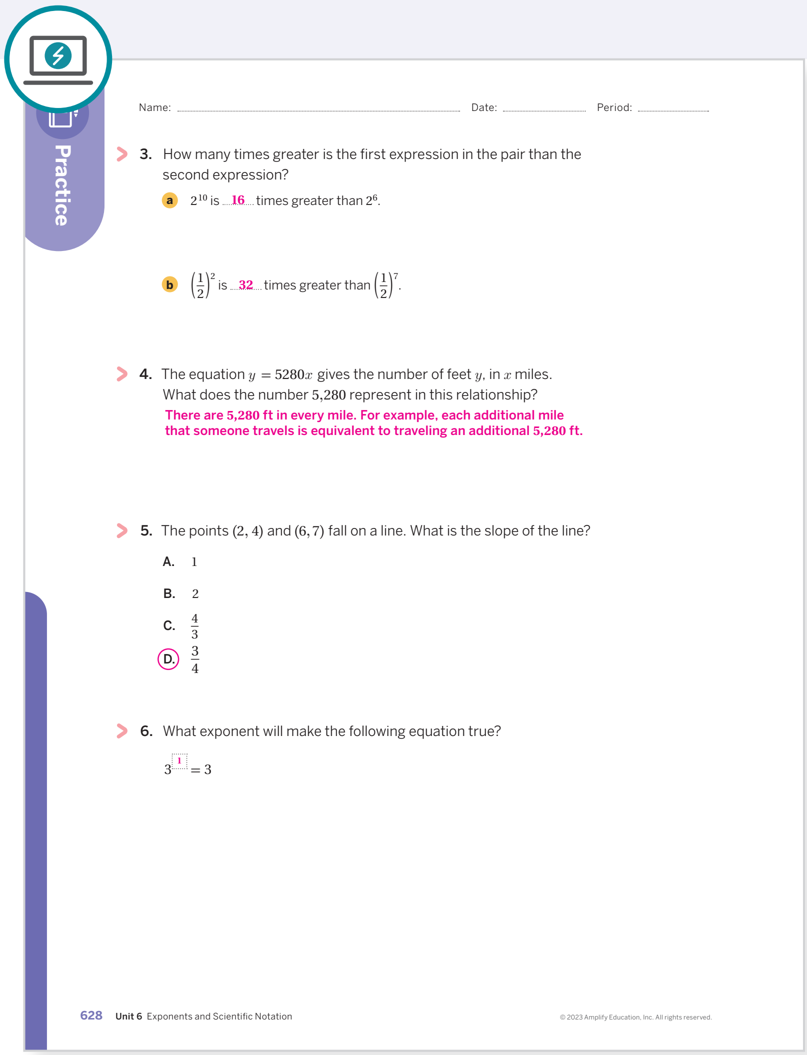

Name: ______ Date: ______ Period: ______

Practice

3. How many times greater is the first expression in the pair than the second expression?

   a. 2^{10} is 16 times greater than 2^6.
   b. $\left(\frac{1}{2}\right)^2$ is 32 times greater than $\left(\frac{1}{2}\right)^7$.

4. The equation $y = 5280x$ gives the number of feet y, in x miles. What does the number 5,280 represent in this relationship?

   There are 5,280 ft in every mile. For example, each additional mile that someone travels is equivalent to traveling an additional 5,280 ft.

5. The points $(2, 4)$ and $(6, 7)$ fall on a line. What is the slope of the line?

   A. 1
   B. 2
   C. $\frac{4}{3}$
   [✓ D.] $\frac{3}{4}$

6. What exponent will make the following equation true?

   $3^{\boxed{1}} = 3$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 2	2
	5	Unit 3 Lesson 11	2
Formative	6	Unit 6 Lesson 3	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 6 | **LESSON 3**

Multiplying Powers

Let's explore patterns when we multiply powers with the same base.

Focus

Goal

1. **Language Goal:** Generalize a process for multiplying exponential expressions with the same base, and justify that $a^m \cdot a^n = a^{m+n}$, where $a \neq 0$. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students make use of repeated reasoning to discover the exponent rule $a^m \cdot a^n = a^{m+n}$, where $a \neq 0$. Students reason abstractly and quantitatively when applying exponent rules, pausing to consider the meaning of quantities, not just how to compute them.
Note: At this grade level, all exponents are understood to be integers. Rational exponents will be addressed in high school.

Previously

In Lesson 2, students used the structure of expressions to evaluate and compare expressions involving exponents.

Coming Soon

In subsequent lessons, students will extend the exponent rule to cases where the exponents are zero or negative.

Rigor

- Students build **conceptual understanding** of multiplying powers that have the same base, but different exponents.
- Students multiply powers that have the same base, but different exponents to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Exponent Rules*

Math Language Development

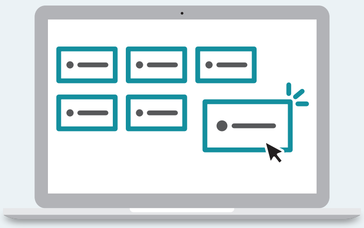
Review words

- *base*
- *equivalent expressions*
- *expanded form*
- *exponent*
- *power*

Amps | **Featured Activity**

Activity 1
Digital Card Sort

Students match equivalent expressions represented in different forms by dragging and connecting them on screen.



Amps
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Building Math Identity and Community

Connecting to Mathematical Practices

The stress of having to use repeated reasoning to determine a pattern and then apply it in a new situation might be paralyzing to some students. Remind students that as they learn something new, just as in science, it is ok to make a hypothesis and then revise it upon more evidence because this process can lend itself to better conceptual understanding. This freedom to make mistakes, and then fix them, should help regulate students' stress levels.

Modifications to Pacing

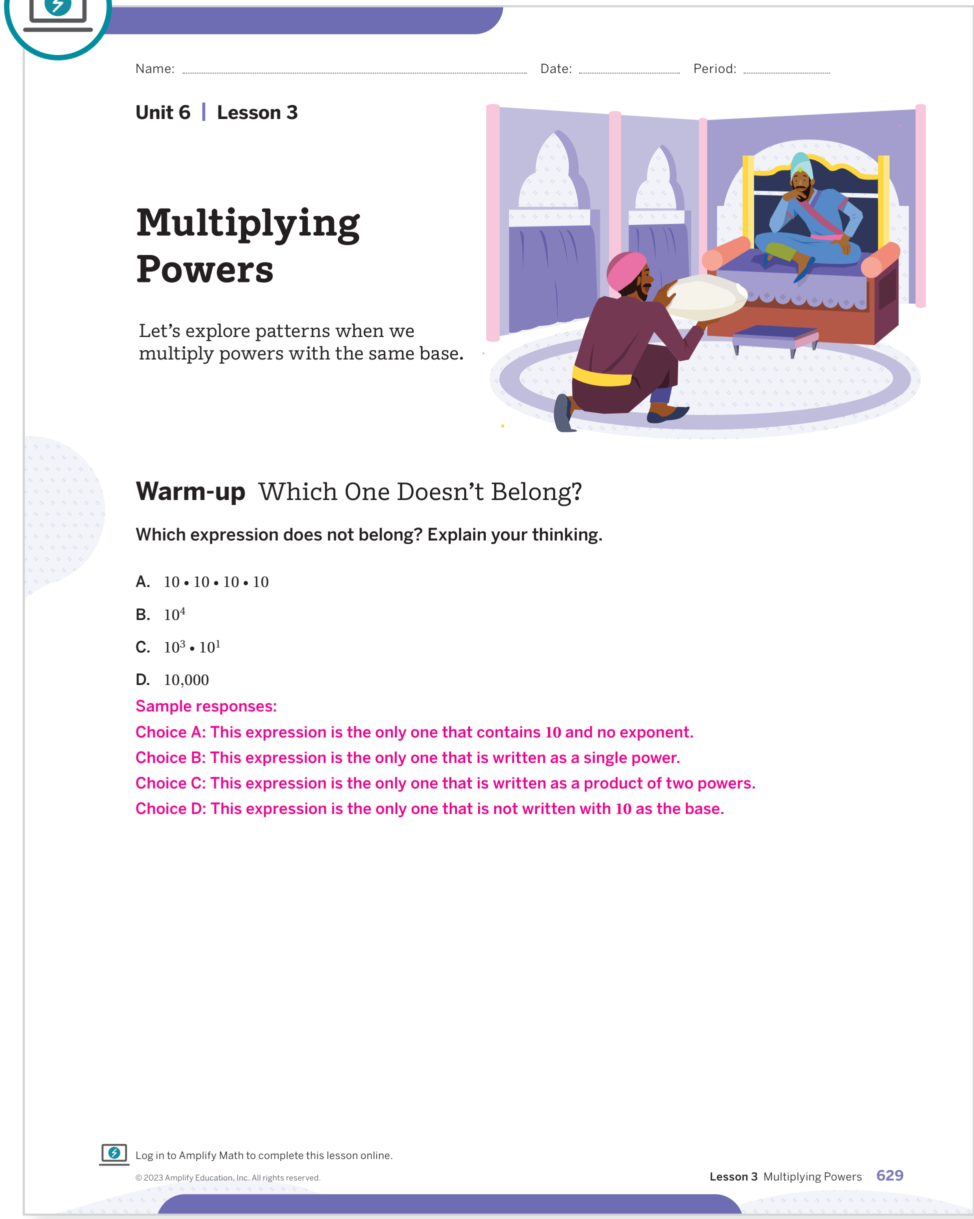
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, the last two rows may be omitted.
- Optional **Activity 3** may be omitted or assigned as additional practice.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students compare four expressions with exponents to generate ideas and terminology (e.g., product, single power) that will be helpful for the upcoming work in this lesson.



Name: ______________ Date: ________ Period: ________

Unit 6 | Lesson 3

Multiplying Powers

Let's explore patterns when we multiply powers with the same base.

Warm-up Which One Doesn't Belong?

Which expression does not belong? Explain your thinking.

A. $10 \cdot 10 \cdot 10 \cdot 10$

B. 10^4

C. $10^3 \cdot 10^1$

D. $10{,}000$

Sample responses:

Choice A: This expression is the only one that contains 10 and no exponent.

Choice B: This expression is the only one that is written as a single power.

Choice C: This expression is the only one that is written as a product of two powers.

Choice D: This expression is the only one that is not written with 10 as the base.

Log in to Amplify Math to complete this lesson online.

Lesson 3 Multiplying Powers 629

1 Launch

Conduct the ***Which One Doesn't Belong?*** routine.

2 Monitor

Help students get started by asking students to choose any expression and identify what makes it different from the other expressions.

Look for points of confusion:

- **Thinking 10^4 represents $10 \cdot 4$.** Write the terms *base* and *exponent* with a diagram, labeling each part of the expression. Show 10^4 as $10 \cdot 10 \cdot 10 \cdot 10$.
- **Thinking $10^3 \cdot 10^1$ doesn't belong because it is not equivalent to the other expressions.** Ask students to write 10^3 and 10^1 in expanded form. This type of problem will be addressed in today's lesson and can be used to transition into Activity 1.

Look for productive strategies:

- Identifying any one expression that is different. Each expression has a feature that makes it different from the others. Challenge students to find the feature in each expression that makes it different from the others.
- Noticing all the expressions are equivalent. Challenge students to write their own equivalent expression using powers.

3 Connect

Have students share their responses. For each expression, select one student to explain their thinking. Draw out reasons for each, attending to appropriate vocabulary and use of language.

Ask students which expression they think has the greatest value. Students may notice that expressions are all equivalent.

Highlight the terms *power*, *exponent*, and *base*.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their responses, press them to explain the particular feature of each expression that makes it different from the others. Capture language used in their responses on the class display. Highlight the terms *power*, *exponent*, *base*, *product*, and *single power*.

English Learners

Consider providing sentence frames to help students explain their thinking, such as: "Choice ___ doesn't belong because . . ."

Power-up

To power up students' ability to understand expressions that have exponents of 1, have students complete:

Determine the exponent that makes each equation true.

1. $3^{\boxed{4}} = 3 \cdot 3 \cdot 3 \cdot 3$
2. $3^{\boxed{3}} = 3 \cdot 3 \cdot 3$
3. $3^{\boxed{2}} = 3 \cdot 3$
4. $3^{\boxed{1}} = 3$

Use: Before the Warm-up

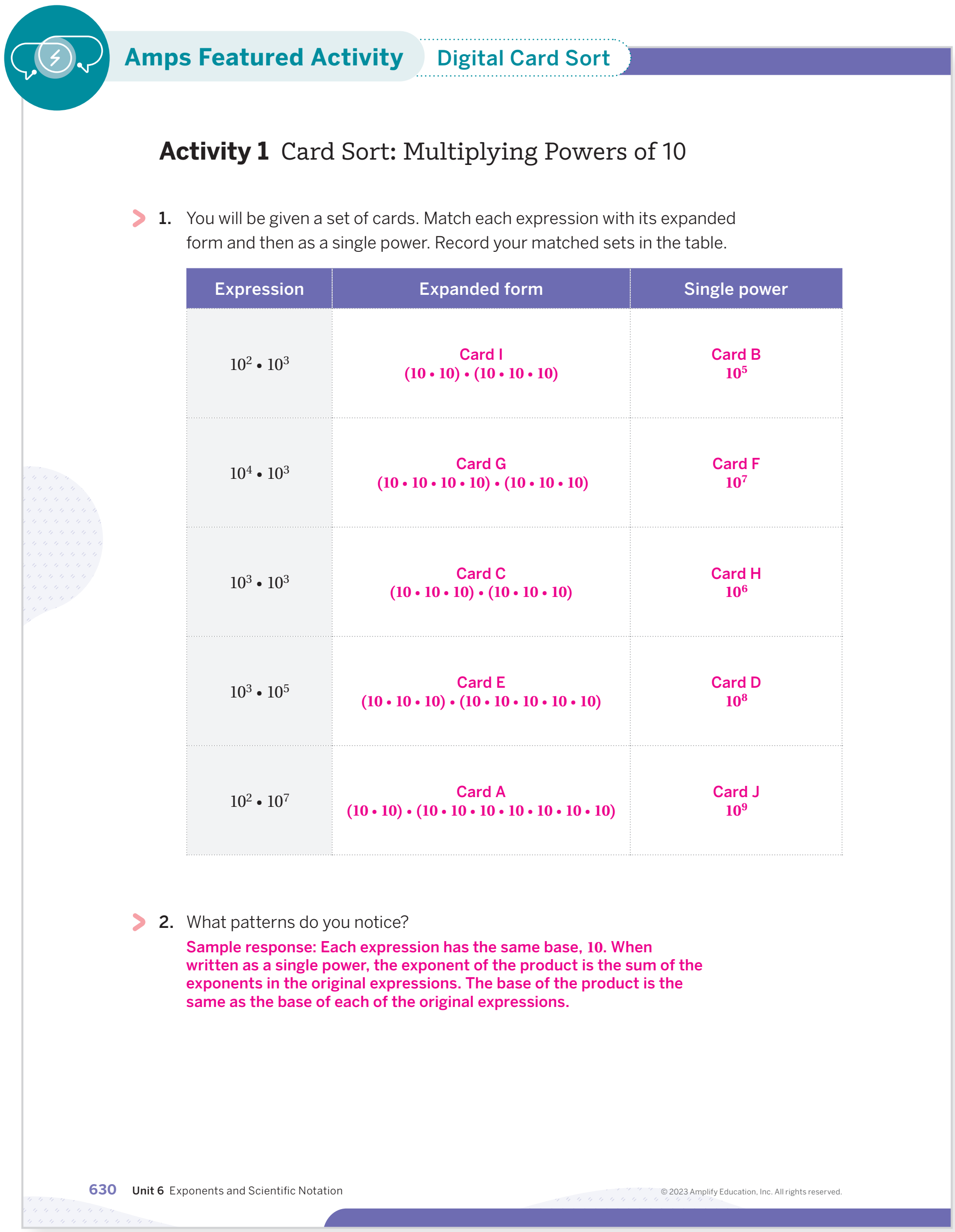
Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

Pairs | 20 min

Activity 1 Card Sort: Multiplying Powers of 10

Students match cards containing products of powers of 10 written in different forms to discover a pattern among the exponents.

Amps Featured Activity Digital Card Sort



Activity 1 Card Sort: Multiplying Powers of 10

1. You will be given a set of cards. Match each expression with its expanded form and then as a single power. Record your matched sets in the table.

Expression	Expanded form	Single power
$10^2 \cdot 10^3$	Card I $(10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$	Card B 10^5
$10^4 \cdot 10^3$	Card G $(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$	Card F 10^7
$10^3 \cdot 10^3$	Card C $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$	Card H 10^6
$10^3 \cdot 10^5$	Card E $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	Card D 10^8
$10^2 \cdot 10^7$	Card A $(10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	Card J 10^9

2. What patterns do you notice?
Sample response: Each expression has the same base, 10. When written as a single power, the exponent of the product is the sum of the exponents in the original expressions. The base of the product is the same as the base of each of the original expressions.

630 **Unit 6** Exponents and Scientific Notation

1 Launch

Distribute one set of cards from the Activity 1 PDF to each pair of students. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by asking which card shows $10^2 \cdot 10^3$ in expanded form.

Look for points of confusion:

- **Thinking that $10^2 \cdot 10^3$ is equivalent to 10^6.** Have students find the matching card written in expanded form before matching the card with a single power.

Look for productive strategies:

- Noticing a pattern where the base remains the same and the exponents are added. Ask students to use the patterns to rewrite $10^m \cdot 10^n$ as a single power of the form $10^{\square}$.

3 Connect

Display student work showing the correct responses.

Have students share any patterns they found. Record responses for all to see.

Highlight that each expression has the same base, 10. When written as a single power, the exponent is the sum of the exponents in the original expression, while the base remains the same.

Ask, "How can you write $10^4 \cdot 10^5$ as a single power without writing it in expanded form first?" Keep the base 10, add the exponents; $10^{4+5} = 10^9$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the first expression and use a think-aloud to model how to determine which card has its matching expanded form and which card has its matching single power. Consider using the following during the think-aloud.

- "I have 'ten to the second power' multiplied by 'ten to the third power.'"
- "When I write these as factors without using exponents, I have 5 factors of 10."
- "I will match this card with the expanded form card that has 5 factors of 10 and the single power card that has an exponent of 5."

Math Language Development

MLR7: Compare and Connect

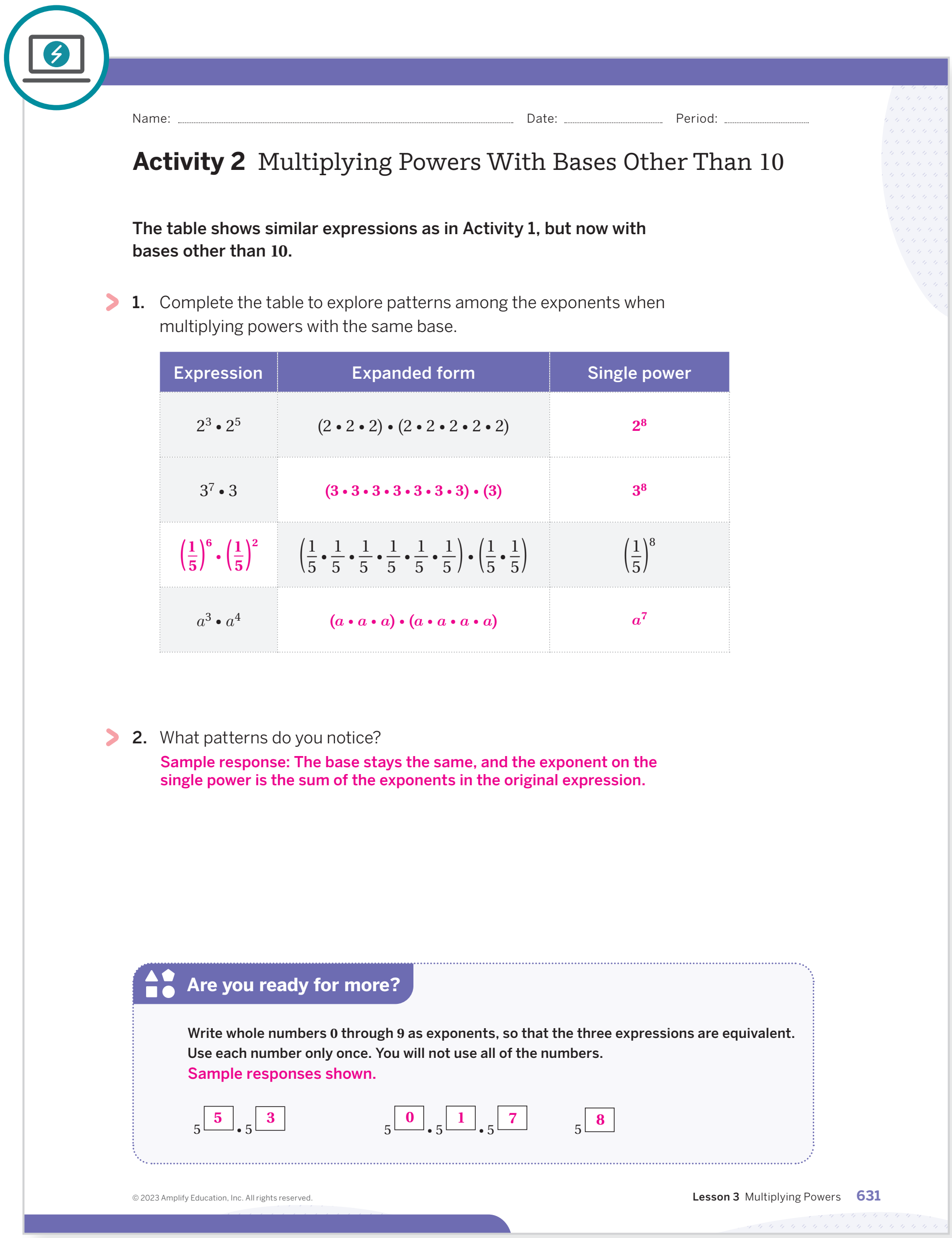
During the Connect, as students share the patterns they noticed, press for details to probe for understanding and to demonstrate the use of precise mathematical language. For example:

If a student says . . .	Press for details by asking . . .
"The exponents are added."	"Are the exponents always added? What did you notice about the bases? What did you notice about the operation(s) that were used in the original expressions?"

Pairs | 10 min

Activity 2 Multiplying Powers With Bases Other Than 10

Students continue exploring patterns of products of powers to understand the pattern they saw in Activity 1 applies to powers with bases other than 10.



Name: ______ Date: ______ Period: ______

Activity 2 Multiplying Powers With Bases Other Than 10

The table shows similar expressions as in Activity 1, but now with bases other than 10.

1. Complete the table to explore patterns among the exponents when multiplying powers with the same base.

Expression	Expanded form	Single power
$2^3 \cdot 2^5$	$(2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$	2^8
$3^7 \cdot 3$	$(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (3)$	3^8
$\left(\frac{1}{5}\right)^6 \cdot \left(\frac{1}{5}\right)^2$	$\left(\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}\right) \cdot \left(\frac{1}{5} \cdot \frac{1}{5}\right)$	$\left(\frac{1}{5}\right)^8$
$a^3 \cdot a^4$	$(a \cdot a \cdot a) \cdot (a \cdot a \cdot a \cdot a)$	a^7

2. What patterns do you notice?
   Sample response: The base stays the same, and the exponent on the single power is the sum of the exponents in the original expression.

Are you ready for more?

Write whole numbers 0 through 9 as exponents, so that the three expressions are equivalent. Use each number only once. You will not use all of the numbers.
Sample responses shown.

$5^{\boxed{5}} \cdot 5^{\boxed{3}}$ $\quad 5^{\boxed{0}} \cdot 5^{\boxed{1}} \cdot 5^{\boxed{7}}$ $\quad 5^{\boxed{8}}$

Lesson 3 Multiplying Powers 631

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking how $2^3 \cdot 2^5$ and $(2 \cdot 2 \cdot 2)$ $(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$ are related.

Look for points of confusion:

- **Thinking 3 is equivalent to 3^0 in the expression $3^7 \cdot 3$.** Ask, "How many factors of 3 do you see in the expression 3?" Have them write a 3 as 3^1 to strengthen this connection.
- **Struggling to write a^3 or a^4 in expanded form.** Show students a^3 as $a \cdot a \cdot a$. Then, have them write a^4 in expanded form.
- **Thinking $3^7 \cdot 3 = 9^8$.** Have students write this expression in expanded form before writing it as a single power.

Look for productive strategies:

- Noticing the patterns from Activity 1 are the same even when the base is not 10. Ask students to use the patterns to rewrite $a^m \cdot a^n$ as a single power of the form $a^{\square}$.

3 Connect

Have students share any patterns they found. Record responses for all to see.

Ask, "Can you rewrite $2^3 \cdot 3^4$ as a single power? Why or why not?" No, the bases are not the same.

Define the exponent rule $a^m \cdot a^n = a^{m+n}$, where $a \neq 0$. This means that when multiplying powers that have the same base, keep the base and add the exponents.

Highlight that the rule $a^m \cdot a^n = a^{m+n}$ is true for *any* base including fractions, decimals, and negative numbers.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by having students focus on the problems in the first three rows of the table in Problem 1. Then have them complete Problem 2.

Accessibility: *Guide Processing and Visualization*

If students appear hesitant to tackle the expression in the third row where the base is a fraction, ask them to think of the fraction $\frac{1}{5}$ as a single entity, such as the variable x. Have them write the expression when the base is x and then replace x with $\frac{1}{5}$ in their final result.

Math Language Development

MLR7: Compare and Connect

During the Connect, press students to represent the patterns they notice as a single rule. Consider having the class collectively write the exponent rule in their own words before you define and display the rule. Consider using the following sentence frame to help organize their thinking.

When multiplying powers that have the same ___, the ___ stays the same, and the ___ are added.

English Learners

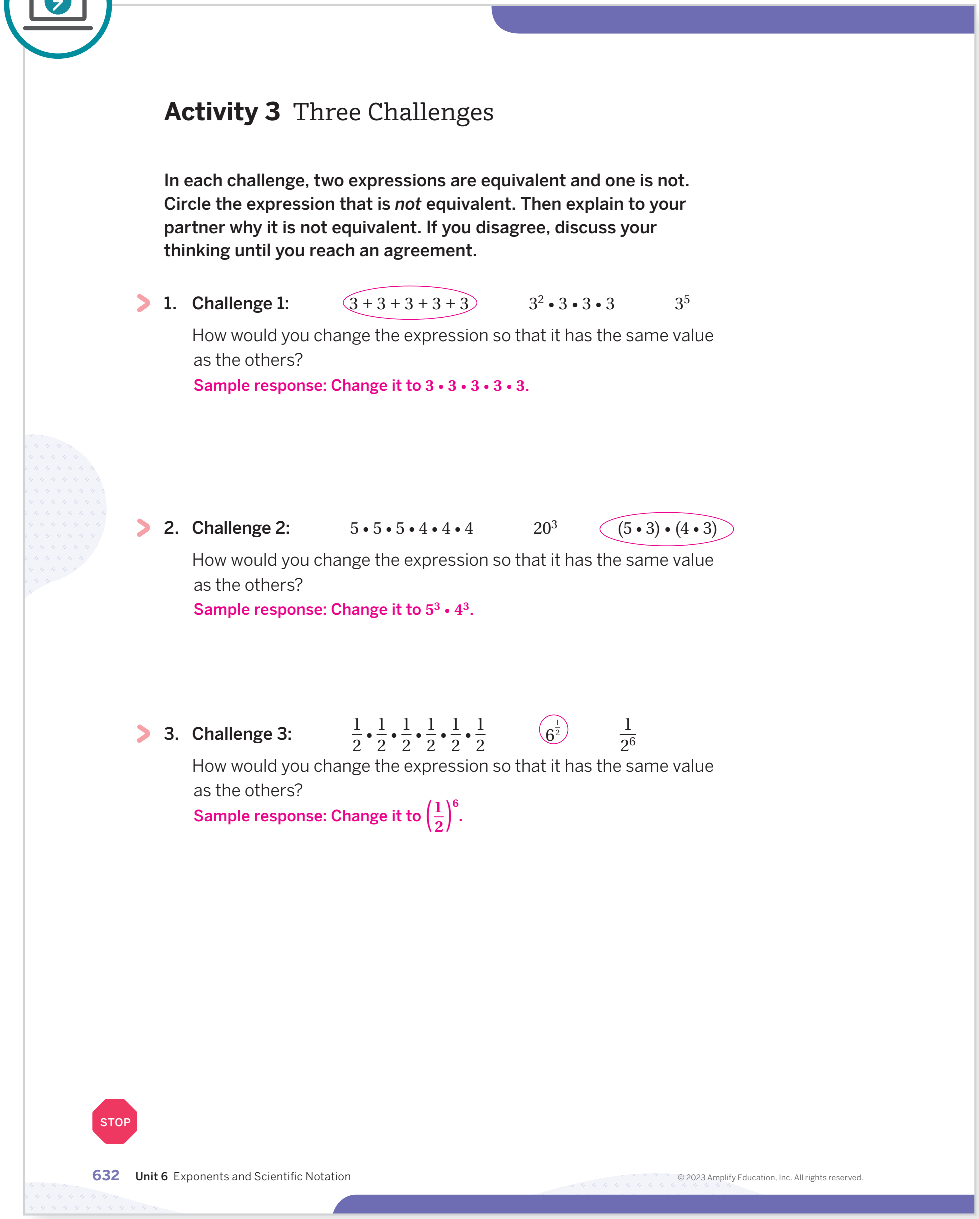
Use gestures, such as pointing to the base and exponent of the expression, expanded form, and single power, as you collectively craft the class rule.

Optional

Pairs | 15 min

Activity 3 Three Challenges

Students compare expressions in different forms to determine which expression in each set is *not* equivalent to the others.



Activity 3 Three Challenges

In each challenge, two expressions are equivalent and one is not. Circle the expression that is *not* equivalent. Then explain to your partner why it is not equivalent. If you disagree, discuss your thinking until you reach an agreement.

1. **Challenge 1:** $\boxed{3 + 3 + 3 + 3 + 3}$ $3^2 \cdot 3 \cdot 3 \cdot 3$ 3^5

   How would you change the expression so that it has the same value as the others?

   Sample response: Change it to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

2. **Challenge 2:** $5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ 20^3 $\boxed{(5 \cdot 3) \cdot (4 \cdot 3)}$

   How would you change the expression so that it has the same value as the others?

   Sample response: Change it to $5^3 \cdot 4^3$.

3. **Challenge 3:** $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ $\boxed{6^{\frac{1}{2}}}$ $\frac{1}{2^6}$

   How would you change the expression so that it has the same value as the others?

   Sample response: Change it to $\left(\frac{1}{2}\right)^6$.

STOP

632 **Unit 6** Exponents and Scientific Notation

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them find the value, or analyze the structure, of each expression in Challenge 1.

Look for points of confusion:

- **Not following the order of operations when evaluating expressions.** Write the order of operations for all to see, and demonstrate them by evaluating $(2 \cdot 3)^4$ using the order of operations.
- **Writing 3^4 for $3 + 3 + 3 + 3$.** Remind students 3^4 is a way to write factors so that it is equivalent to $3 \cdot 3 \cdot 3 \cdot 3$ (repeated multiplication, not repeated addition).

Look for productive strategies:

- Determining the numeric value of each expression. Ask students to compare the structure of each expression to see if they can identify which expression is not equivalent.

3 Connect

Have students share their responses. Use the *Poll the Class* routine to see which expression they chose for each challenge. Ask students to explain their thinking.

Ask students how they can determine which expression is not equivalent without evaluating each expression.

Highlight that examining the structure of an expression can sometimes be a more efficient method than finding the value of an expression when comparing expressions or determining equivalence.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Anchor Chart PDF, *Exponent Rules* for them to reference during this activity. Allow them to keep this anchor chart throughout the rest of the unit. Suggest they cover up all but the Product rule for use in this activity.

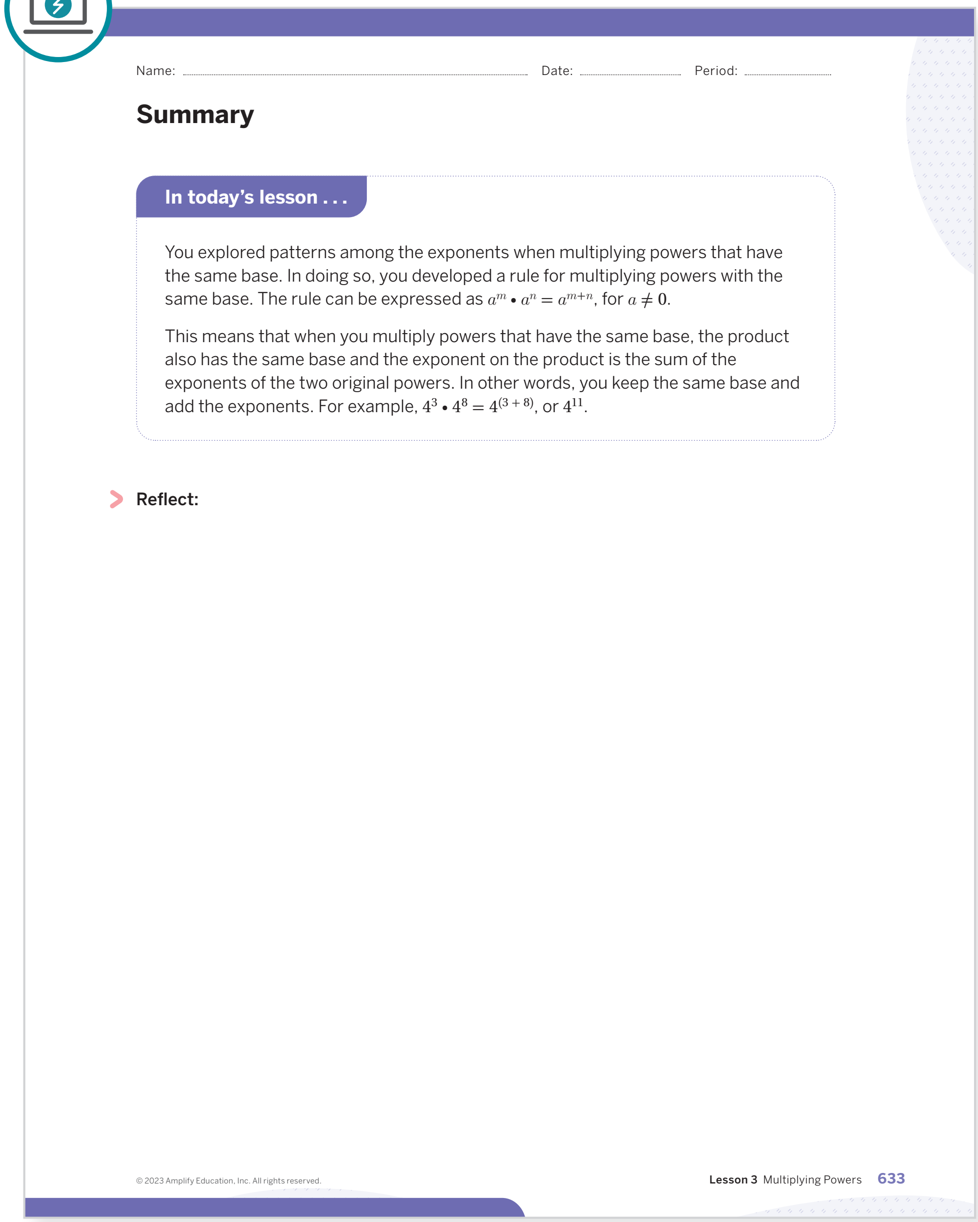
Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by having students focus on either Challenge 1 or Challenge 2. Allow them to choose which Challenge they would like to complete. Offering them the power of choice can result in greater engagement in the task.

Whole Class | 5 min

Summary

Review and synthesize the exponent rule for multiplying powers with the same base.



Name: __________ Date: __________ Period: __________

Summary

In today's lesson . . .

You explored patterns among the exponents when multiplying powers that have the same base. In doing so, you developed a rule for multiplying powers with the same base. The rule can be expressed as $a^m \cdot a^n = a^{m+n}$, for $a \neq 0$.

This means that when you multiply powers that have the same base, the product also has the same base and the exponent on the product is the sum of the exponents of the two original powers. In other words, you keep the same base and add the exponents. For example, $4^3 \cdot 4^8 = 4^{(3+8)}$, or 4^{11}.

Reflect:

Synthesize

Ask:

- "How can you verify that $3^4 \cdot 3^2 = 3^6$?" Sample response: Write each expression as repeated multiplication. There are four factors of 3 multiplied by two factors of 3. This means there are a total of six factors of 3 in the product, or 3^6. Also, the exponent rule for multiplying powers with the same base tells me to add the exponents, $4 + 2 = 6$, and keep the same base, 3.
- "How can you write $10^5 \cdot 10^2 \cdot 10$ as a single power of 10?" Keep the same base, 10. Add the exponents, $5 + 2 + 1 = 8$. The single power of 10 is 10^8.

Display the Anchor Chart PDF, *Exponent Rules*. Show only the Product rule and cover the remaining rules. **Note:** The other rules will be uncovered throughout the unit.

Highlight that $a^m \cdot a^n = a^{m+n}$ for $a \neq 0$ shows the exponent rule for multiplying powers with the same base. This means that when multiplying powers that have the same base, keep the same base and add the exponents.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

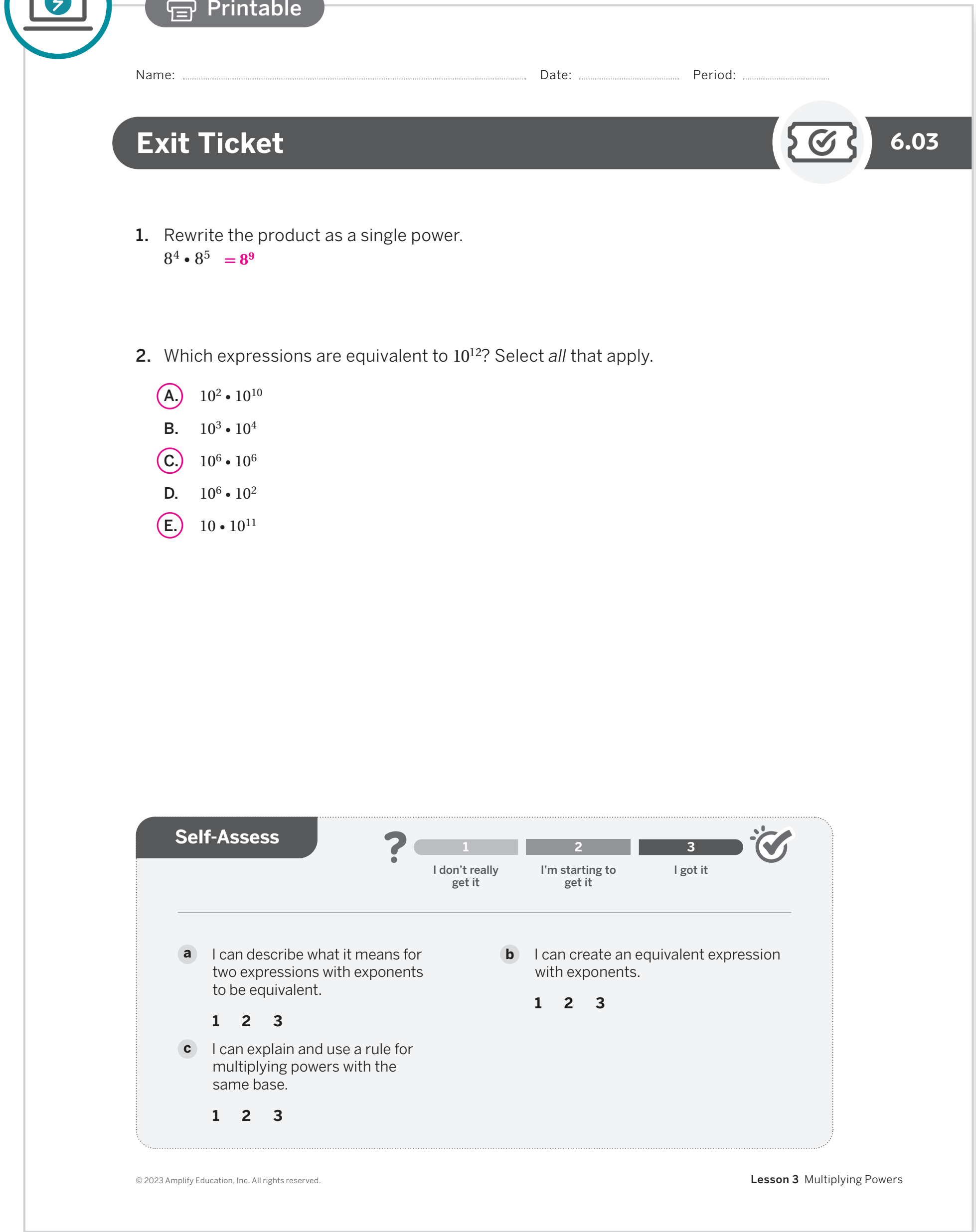
- "What happens when expressions containing exponents are multiplied or divided?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of multiplying powers (that have the same base) by writing an equivalent expression as a single power.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.03

1. Rewrite the product as a single power.
   $8^4 \cdot 8^5$ $= 8^9$

2. Which expressions are equivalent to 10^{12}? Select *all* that apply.
   [✓ A.] $10^2 \cdot 10^{10}$
   B. $10^3 \cdot 10^4$
   [✓ C.] $10^6 \cdot 10^6$
   D. $10^6 \cdot 10^2$
   [✓ E.] $10 \cdot 10^{11}$

Self-Assess

?			
	1	2	3
	I don't really get it	I'm starting to get it	I got it

a. I can describe what it means for two expressions with exponents to be equivalent.
1 2 3

b. I can create an equivalent expression with exponents.
1 2 3

c. I can explain and use a rule for multiplying powers with the same base.
1 2 3

Lesson 3 Multiplying Powers

Success looks like . . .

- **Language Goal:** Generalizing a process for multiplying exponential expressions with the same base, and justifying that $a^m \cdot a^n = a^{m+n}$, where $a \neq 0$. **(Speaking and Listening, Reading and Writing)**
  - » Simplifying the exponential expression in Problem 1.

Suggested next steps

If students do not write 8^9 for Problem 1, consider:

- Having students write each factor in expanded form.
- Asking, "How many factors of eight does 8^4 represent? What about 8^5?"
- Reviewing different ways to write an expression from Activity 2.

If students do not choose A or C for Problem 2, consider:

- Reviewing Activity 1.
- Showing students different ways to write 10^{12} in expanded form, such as . . . $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$ or $(10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$. For each expression students write, ask them to rewrite an equivalent expression showing multiplication of powers with a base of 10 and then apply the exponent rule to rewrite as a single power of 10.

Professional Learning

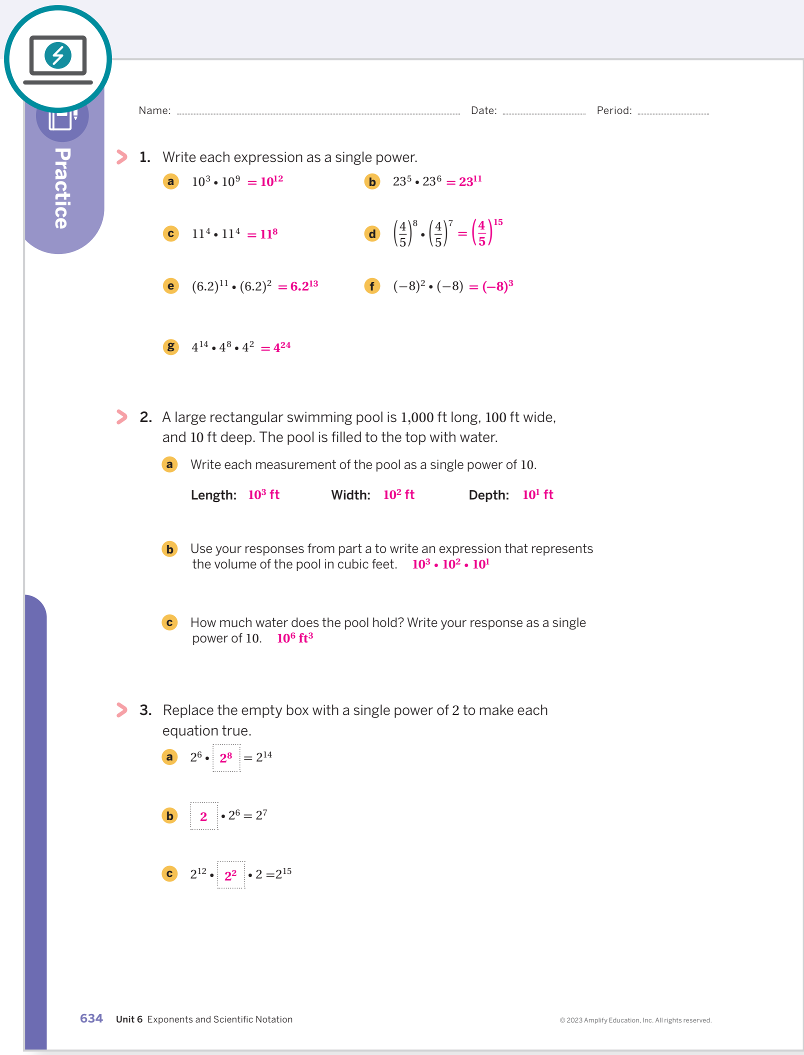
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion in Activity 1, how did you encourage each student to share their understanding?
- During the discussion in Activity 3, how did you encourage each student to listen to one another's strategies? What might you change for the next time you teach this lesson?

Practice

Independent



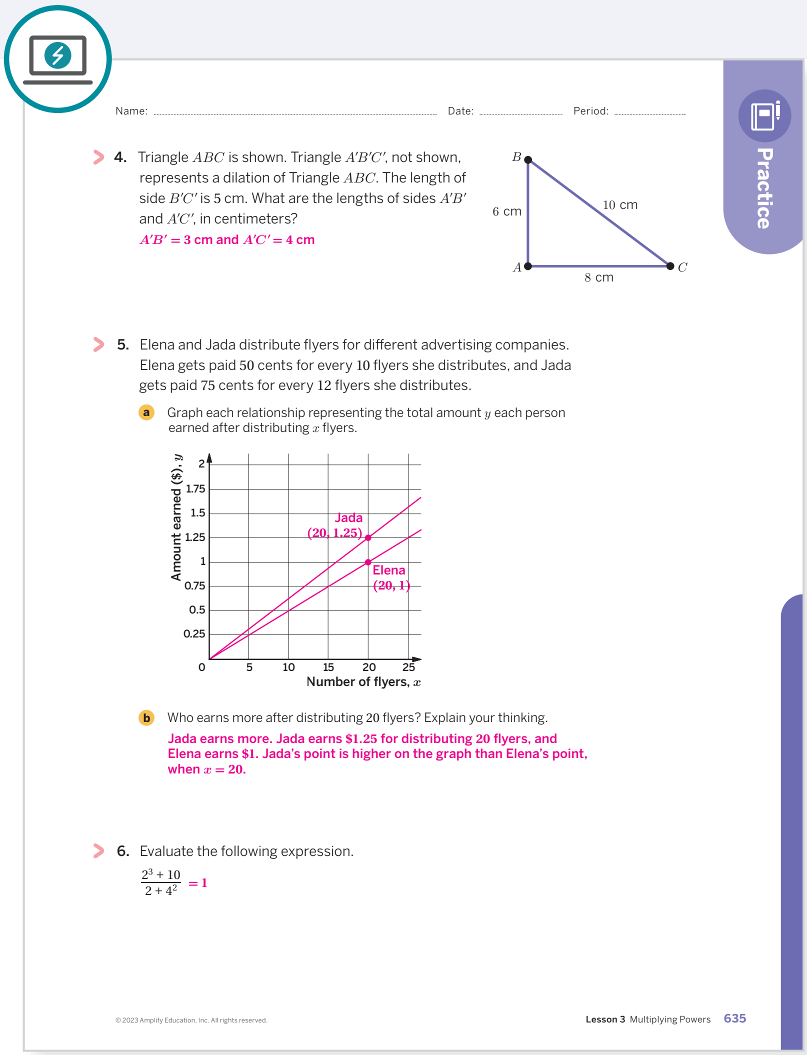
Name: Date: Period:

1. Write each expression as a single power.
   - a $10^3 \cdot 10^9$ $= 10^{12}$
   - b $23^5 \cdot 23^6$ $= 23^{11}$
   - c $11^4 \cdot 11^4$ $= 11^8$
   - d $\left(\frac{4}{5}\right)^8 \cdot \left(\frac{4}{5}\right)^7 = \left(\frac{4}{5}\right)^{15}$
   - e $(6.2)^{11} \cdot (6.2)^2$ $= 6.2^{13}$
   - f $(-8)^2 \cdot (-8)$ $= (-8)^3$
   - g $4^{14} \cdot 4^8 \cdot 4^2$ $= 4^{24}$

2. A large rectangular swimming pool is 1,000 ft long, 100 ft wide, and 10 ft deep. The pool is filled to the top with water.
   - a Write each measurement of the pool as a single power of 10.

     Length: 10^3 ft **Width:** 10^2 ft **Depth:** 10^1 ft
   - b Use your responses from part a to write an expression that represents the volume of the pool in cubic feet. $10^3 \cdot 10^2 \cdot 10^1$
   - c How much water does the pool hold? Write your response as a single power of 10. 10^6 ft^3

3. Replace the empty box with a single power of 2 to make each equation true.
   - a $2^6 \cdot \boxed{2^8} = 2^{14}$
   - b $\boxed{2} \cdot 2^6 = 2^7$
   - c $2^{12} \cdot \boxed{2^2} \cdot 2 = 2^{15}$



Name: Date: Period:

4. Triangle ABC is shown. Triangle $A'B'C'$, not shown, represents a dilation of Triangle ABC. The length of side $B'C'$ is 5 cm. What are the lengths of sides $A'B'$ and $A'C'$, in centimeters?

   $A'B' = 3$ cm and $A'C' = 4$ cm

5. Elena and Jada distribute flyers for different advertising companies. Elena gets paid 50 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.
   - a Graph each relationship representing the total amount y each person earned after distributing x flyers.

   - b Who earns more after distributing 20 flyers? Explain your thinking.

     Jada earns more. Jada earns \$1.25 for distributing 20 flyers, and Elena earns \$1. Jada's point is higher on the graph than Elena's point, when $x = 20$.

6. Evaluate the following expression.

   $\frac{2^3 + 10}{2 + 4^2} = 1$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 9	3
	5	Unit 3 Lesson 6	2
Formative ⚡	6	Unit 6 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 6 | **LESSON 4**

Dividing Powers

Let's explore patterns with exponents when we divide powers with the same base.

Focus

Goals

1. **Language Goal:** Generalize a process for dividing exponential expressions with the same base, and justify that $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Use exponent rules to justify that a^0 is 1. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students use repeated reasoning to discover the quotient of powers exponent rule $\frac{a^m}{a^n} = a^{m-n}$, when $a \neq 0$. For now, students work with expressions where m and n are positive integers and $m > n$. In Activity 2, expressions extend to the case where $m = n$ to make sense of why $a^0 = 1$, for when $a \neq 0$.

Previously

In Lesson 3, students made use of repeated reasoning and discovered the product of powers exponent rule $a^m \cdot a^n = a^{m+n}$.

Coming Soon

In Lesson 5, students will extend the exponent rule $\frac{a^m}{a^n} = a^{m-n}$ to include situations where $m < n$.

Rigor

- Students build **conceptual understanding** of dividing powers that have the same base, but different exponents.
- Students divide powers that have the same base, but different exponents to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Exponent Rules*

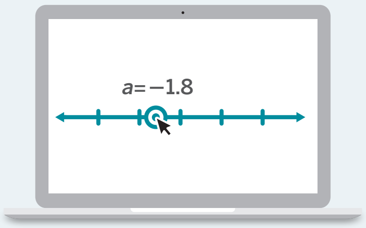
Math Language Development

- *base*
- *equivalent expressions*
- *expanded form*
- *exponent*
- *power*

Amps | **Featured Activity**

Activity 2
Manipulating an Expression

Students drag points, manipulating the numerator and denominator, seeing how the expressions written in expanded form and as single powers are related.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively want to apply the same exponent rule from the previous lesson to this one. Have students compare the activity to the one from the previous lesson and state their similarities and differences. Explain how they will extend the regularity of repeated reasoning from the previous lesson to this one. At the end of the lesson, ask them to explain the relationship between multiplication and division and then correlate it to the relationship between products of powers and quotients of powers.

Modifications to Pacing

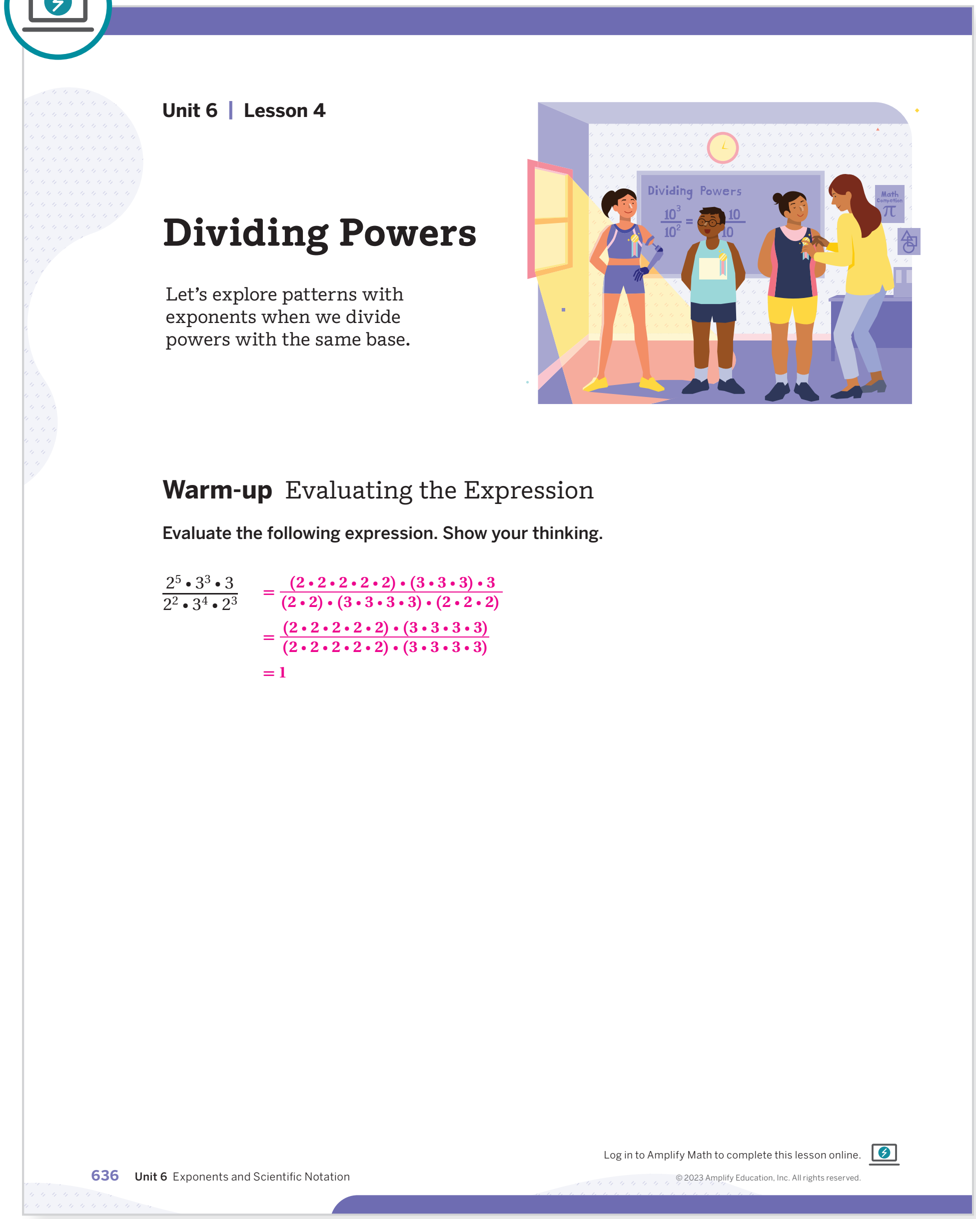
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, the last two rows may be omitted.
- Optional **Activity 3** may be omitted or assigned as additional practice.

Independent | 5 min

Warm-up Evaluating the Expression

Students evaluate a fraction involving powers to see the need for a more efficient method.



Unit 6 | **Lesson 4**

Dividing Powers

Let's explore patterns with exponents when we divide powers with the same base.

Warm-up Evaluating the Expression

Evaluate the following expression. Show your thinking.

$$\frac{2^5 \cdot 3^3 \cdot 3}{2^2 \cdot 3^4 \cdot 2^3} = \frac{(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3) \cdot 3}{(2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3) \cdot (2 \cdot 2 \cdot 2)}$$

$$= \frac{(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3)}{(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3)}$$

$$= 1$$

Log in to Amplify Math to complete this lesson online.

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1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by reminding them to follow the order of operations.

Look for points of confusion:

- **Thinking 2^5 represents $2 \cdot 5$.** Write the terms *base* and *exponent* with a diagram, labeling each part of the expression. Show 2^5 as $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.
- **Writing $2^5 \cdot 3^3 \cdot 3$ as 18^{15}.** Ask students to rewrite each part in expanded form before evaluating the expression.

Look for productive strategies:

- Evaluating each power first. Ask students to use the product of powers rule from Lesson 3 to write the numerator and denominator each as a single power of 2 multiplied by a single power of 3. Have them look at the structure of the expression to give another reason why the value is 1.

3 Connect

Have students share their responses and methods. Use the Poll the Class routine to see if they arrived at the same value and used the same method.

Ask, "What has to be true about a fraction for it to equal 1?" The numerator and denominator must be the same value, except 0.

Highlight that students can find the value of the expression, *without* evaluating it, by looking at the structure of the expression.

Power-up

To power up students' ability to identify fractions that are equivalent to 1, have students complete:

Recall that the quotient of a number and itself is always equal to 1.

Determine which expressions are equal to 1. Select *all* that apply.

[✓ A.] $10 \div 10$

[✓ B.] $\frac{3}{3}$

C. $\frac{(-8+2)}{(-10)}$

[✓ D.] $\frac{30+6}{6^2}$

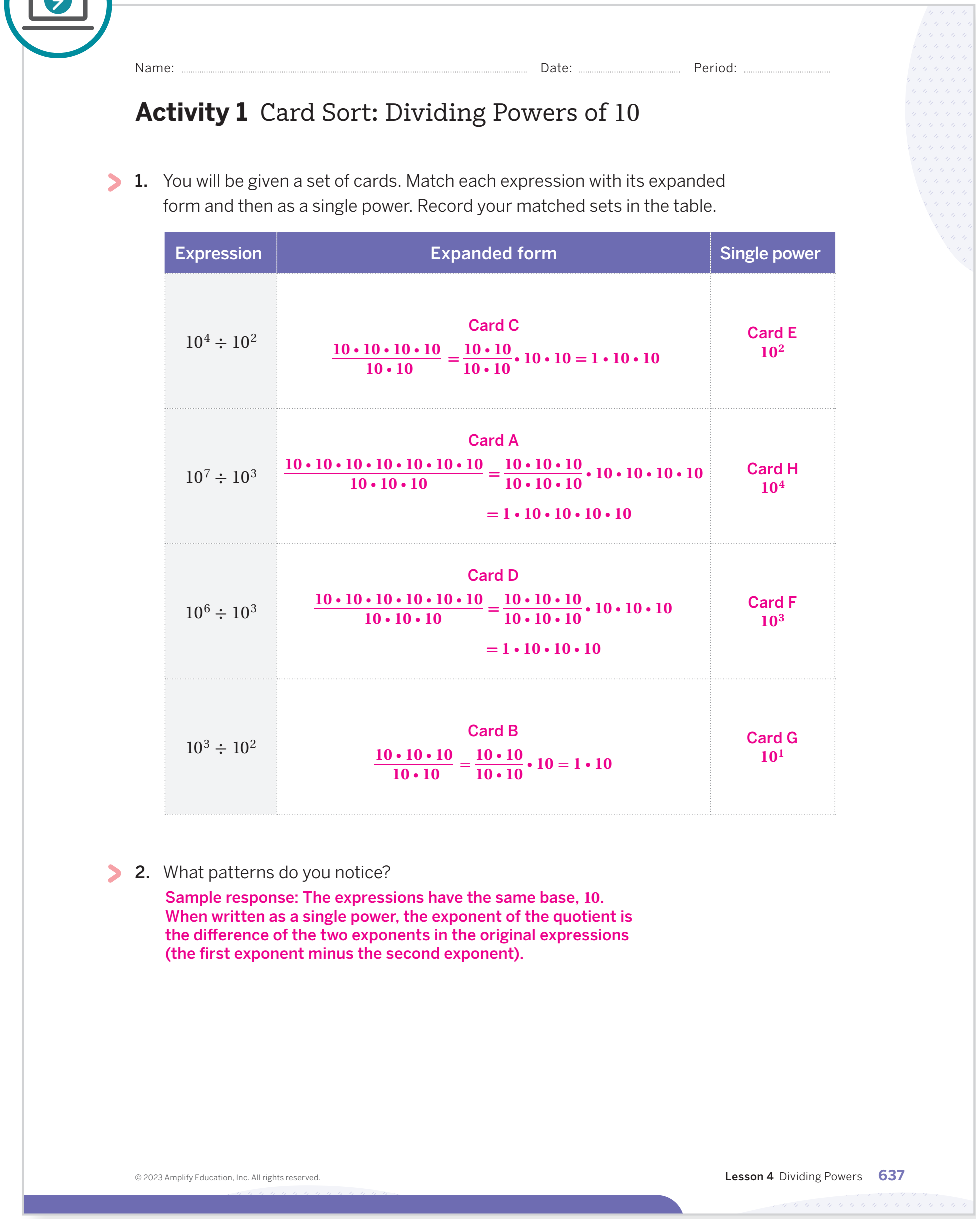
Use: Before the Warm-up

Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7

Pairs | 15 min

Activity 1 Card Sort: Dividing Powers of 10

Students match cards containing quotients of powers of 10 written in different forms to discover a pattern among the exponents.



Name: ______ Date: ______ Period: ______

Activity 1 Card Sort: Dividing Powers of 10

1. You will be given a set of cards. Match each expression with its expanded form and then as a single power. Record your matched sets in the table.

Expression	Expanded form	Single power
$10^4 \div 10^2$	Card C $\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	Card E 10^2
$10^7 \div 10^3$	Card A $\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} = \frac{10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 \cdot 10$ $= 1 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	Card H 10^4
$10^6 \div 10^3$	Card D $\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} = \frac{10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} \cdot 10 \cdot 10 \cdot 10$ $= 1 \cdot 10 \cdot 10 \cdot 10$	Card F 10^3
$10^3 \div 10^2$	Card B $\frac{10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 = 1 \cdot 10$	Card G 10^1

2. What patterns do you notice?

Sample response: The expressions have the same base, 10. When written as a single power, the exponent of the quotient is the difference of the two exponents in the original expressions (the first exponent minus the second exponent).

1 Launch

Distribute one set of cards from the Activity 1 PDF to each pair of students. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by activating prior knowledge about the fraction bar and the division symbol representing the same operation and by asking which card shows $10^4 \div 10^2$ in expanded form.

Look for points of confusion:

- **Thinking that $10^6 \div 10^3$ is equivalent to 10^2 because $6 \div 3 = 2$.** Have students find the matching card written in expanded form before matching the card with a single power.

Look for productive strategies:

- Noticing a pattern where the base stays the same and the exponents are subtracted. Ask students to use the patterns to rewrite $10^m \div 10^n$ as a single power of the form 10^{m-n}.

3 Connect

Have students share any patterns they found. Record responses for all to see.

Highlight that each expression has the same base 10. When written as a single power, the exponent is the difference of the exponents in the original expression, while the base remains the same.

Ask, "How can you write $10^8 \div 10^2$ as a single power without writing it in expanded form?" Keep the base 10, and subtract the exponents; $10^{8-2} = 10^6$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the first expression and use a think-aloud to model how to determine the matching expressions. Consider using the following during the think-aloud.

- "I know that 10^4 will be in the numerator and 10^2 will be in the denominator when I write the expression as a fraction."
- "I can expand 10^4 to $10 \cdot 10 \cdot 10 \cdot 10$ and I can expand 10^2 to $10 \cdot 10$, which matches with Card C."
- "I notice two of the 10s in my numerator cancel with two of the 10s in my denominator."
- "Because I have two 10s left over, I can write it as 10^2, which matches with Card E."

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share the patterns they noticed, press for details to probe for understanding and to demonstrate the use of precise mathematical language. For example:

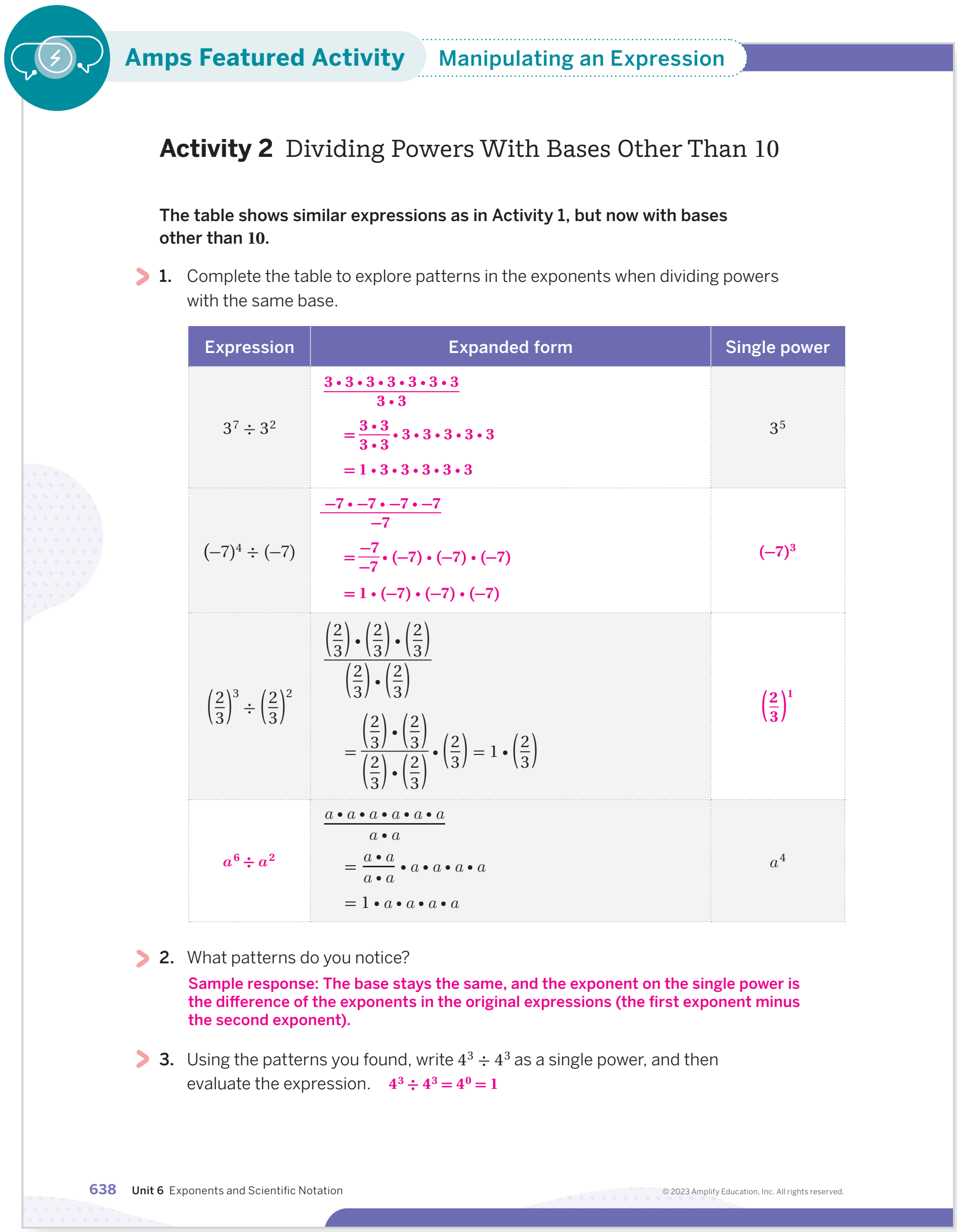
If a student says . . .	Press for details by asking . . .
"The exponents are subtracted."	"Are the exponents always subtracted? What did you notice about the bases? What did you notice about the operation(s) that were used in the original expressions?"

Pairs | 15 min

Activity 2 Dividing Powers With Bases Other Than 10

Students continue exploring patterns of quotients of powers to understand the pattern they saw in Activity 1 applies to powers with bases other than 10.

Amps Featured Activity Manipulating an Expression



Activity 2 Dividing Powers With Bases Other Than 10

The table shows similar expressions as in Activity 1, but now with bases other than 10.

1. Complete the table to explore patterns in the exponents when dividing powers with the same base.

Expression	Expanded form	Single power
$3^7 \div 3^2$	$\frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3}$ $= \frac{3 \cdot 3}{3 \cdot 3} \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ $= 1 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	3^5
$(-7)^4 \div (-7)$	$\frac{-7 \cdot -7 \cdot -7 \cdot -7}{-7}$ $= \frac{-7}{-7} \cdot (-7) \cdot (-7) \cdot (-7)$ $= 1 \cdot (-7) \cdot (-7) \cdot (-7)$	$(-7)^3$
$\left(\frac{2}{3}\right)^3 \div \left(\frac{2}{3}\right)^2$	$\frac{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)}$ $= \frac{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)} \cdot \left(\frac{2}{3}\right) = 1 \cdot \left(\frac{2}{3}\right)$	$\left(\frac{2}{3}\right)^1$
$a^6 \div a^2$	$\frac{a \cdot a \cdot a \cdot a \cdot a \cdot a}{a \cdot a}$ $= \frac{a \cdot a}{a \cdot a} \cdot a \cdot a \cdot a \cdot a$ $= 1 \cdot a \cdot a \cdot a \cdot a$	a^4

2. What patterns do you notice?

   Sample response: The base stays the same, and the exponent on the single power is the difference of the exponents in the original expressions (the first exponent minus the second exponent).

3. Using the patterns you found, write $4^3 \div 4^3$ as a single power, and then evaluate the expression. $4^3 \div 4^3 = 4^0 = 1$

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking them to write 3^7 in expanded form.

Look for points of confusion:

- **Thinking $3^7 \div 3^2 = 1^5$.** Ask students to write the expanded form. Tell them 1^5 represents five factors of 1.
- **Thinking $a^6 \div a^2 = a^3$.** Have students look at the corresponding expanded form. Review each step, emphasizing the final expression showing the four factors of a.
- **Thinking $4^3 \div 4^3 = 0$.** Ask students to evaluate $5 \div 5$ and $8 \div 8$. Ask students to notice the structure of the expressions and relate it to $4^3 \div 4^3$.

3 Connect

Have students share any patterns they found. Record responses for all to see.

Ask, "How can you write $x^{10} \div x^2$ as a single power without writing it in expanded form first? What rule describes the patterns you found? What is the value of x^0?"

Define:

- The exponent rule $a^m \div a^n = a^{m-n}$, where $a \neq 0$.
- Any nonzero *base* with an *exponent* of zero is equal to 1.

Highlight that when dividing powers with the same base, keep the base and subtract the exponent in the denominator from the exponent in the numerator. This is true for *any* base, including fractions, decimals, and negative numbers.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can drag points, manipulating the numerator and denominator, seeing how the expressions written in expanded form and as a single power are related.

Accessibility: *Guide Processing and Visualization*

Students may be intimidated at first by the expression in the third row where the base is a fraction, ask them to think of the fraction $\frac{2}{3}$ as a single entity, such as the variable y. Have them write the expression when the base is y and then replace y with $\frac{2}{3}$ in their final result.

Math Language Development

MLR7: Compare and Connect

During the Connect, press students to represent the patterns they notice as a single rule. Consider having the class collectively write the exponent rule in their own words before you define and display the rule. Consider using the following sentence frame to help organize their thinking.

When dividing powers that have the same ___, the ___ stays the same, and the ___ are subtracted.

English Learners

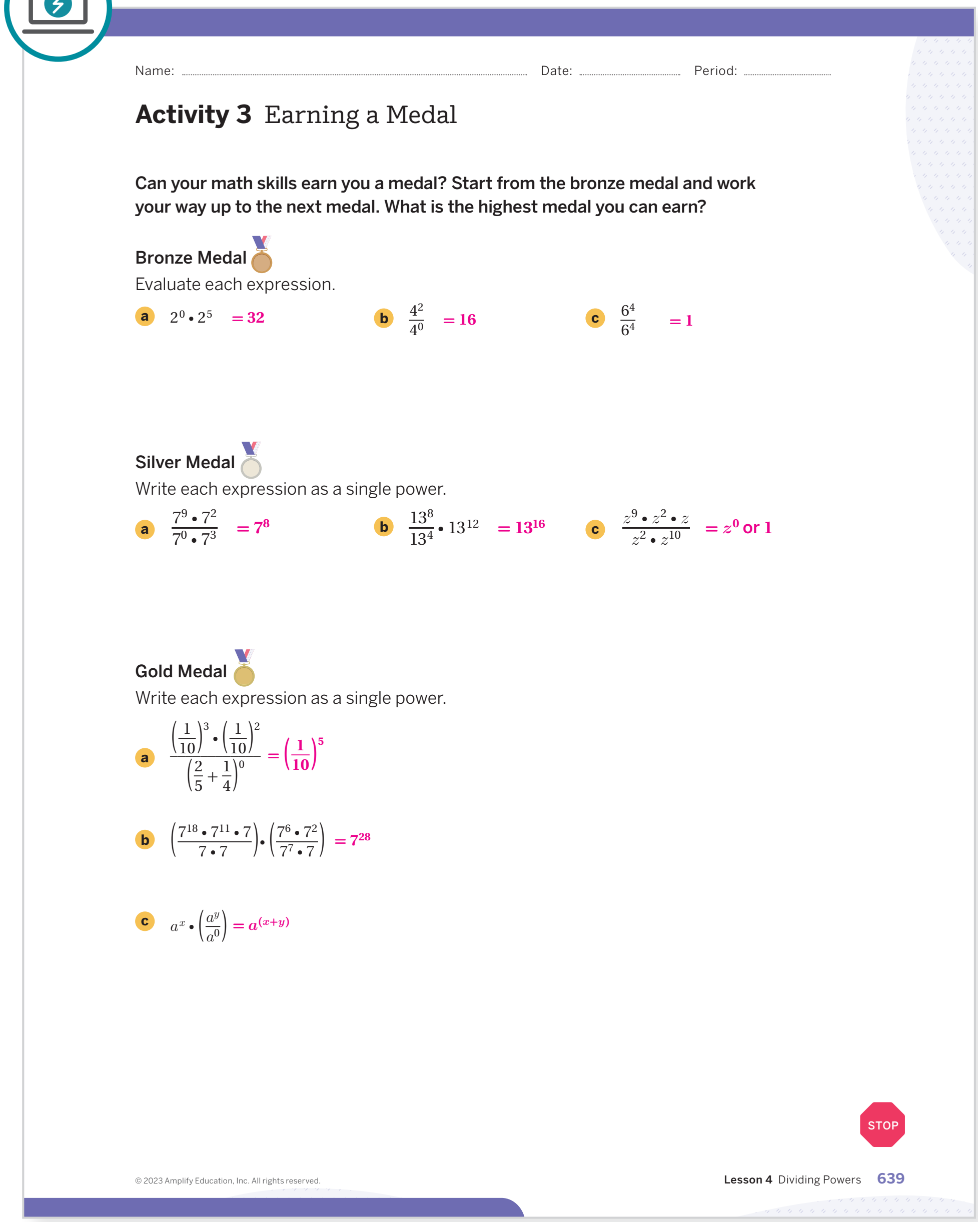
Use gestures, such as pointing to the base and exponent of the expression, expanded form, and single power, as you collectively craft the class rule.

Optional

Independent | 20 min

Activity 3 Earning a Medal

Students evaluate multi-step exponential expressions to build fluency with the product of powers and quotients exponent rules they have learned so far.



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Activity 3 Earning a Medal

Can your math skills earn you a medal? Start from the bronze medal and work your way up to the next medal. What is the highest medal you can earn?

Bronze Medal

Evaluate each expression.

a $2^0 \cdot 2^5$ $= 32$

b $\frac{4^2}{4^0}$ $= 16$

c $\frac{6^4}{6^4}$ $= 1$

Silver Medal

Write each expression as a single power.

a $\frac{7^9 \cdot 7^2}{7^0 \cdot 7^3}$ $= 7^8$

b $\frac{13^8}{13^4} \cdot 13^{12}$ $= 13^{16}$

c $\frac{z^9 \cdot z^2 \cdot z}{z^2 \cdot z^{10}}$ $= z^0$ or 1

Gold Medal

Write each expression as a single power.

a $\frac{\left(\frac{1}{10}\right)^3 \cdot \left(\frac{1}{10}\right)^2}{\left(\frac{2}{5} + \frac{1}{4}\right)^0} = \left(\frac{1}{10}\right)^5$

b $\left(\frac{7^{18} \cdot 7^{11} \cdot 7}{7 \cdot 7}\right) \cdot \left(\frac{7^6 \cdot 7^2}{7^7 \cdot 7}\right) = 7^{28}$

c $a^x \cdot \left(\frac{a^y}{a^0}\right) = a^{(x+y)}$

STOP

Lesson 4 Dividing Powers 639

1 Launch

Tell students they will apply the exponent rules they have learned so far in this unit. All students should start by evaluating the Bronze Medal expressions. After checking their responses, have them proceed through the next two medals at their own pace. After 15 minutes, have students share which expression they found the most challenging.

2 Monitor

Help students get started by evaluating $\frac{5^0 \cdot 5^9}{5^2}$ together, showing each step.

Look for points of confusion:

- **Not following the order of operations.** Write the order of operations for all to see. Tell students to check the order of operations each time they evaluate a new expression.
- **Thinking that 2^0 and 4^0 equal 0 for the Bronze Medal expressions**. Have students write the expression in part c as four repeated factors of 6 in both the numerator and denominator and show how dividing a number by itself equals 1, not 0.

Look for productive strategies:

- Writing their responses in exponential notation for the bronze level. Remind students they should be evaluating the expression, not writing it as a single power for this activity.

3 Connect

Highlight that using exponent rules allows shorter and more efficient ways to write and evaluate expressions involving exponents.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Anchor Chart PDF, *Exponent Rules*, for them to reference during this activity. Allow them to keep this anchor chart throughout the rest of the unit. Suggest they cover up all but the Product rule and Quotient rule for use in this activity.

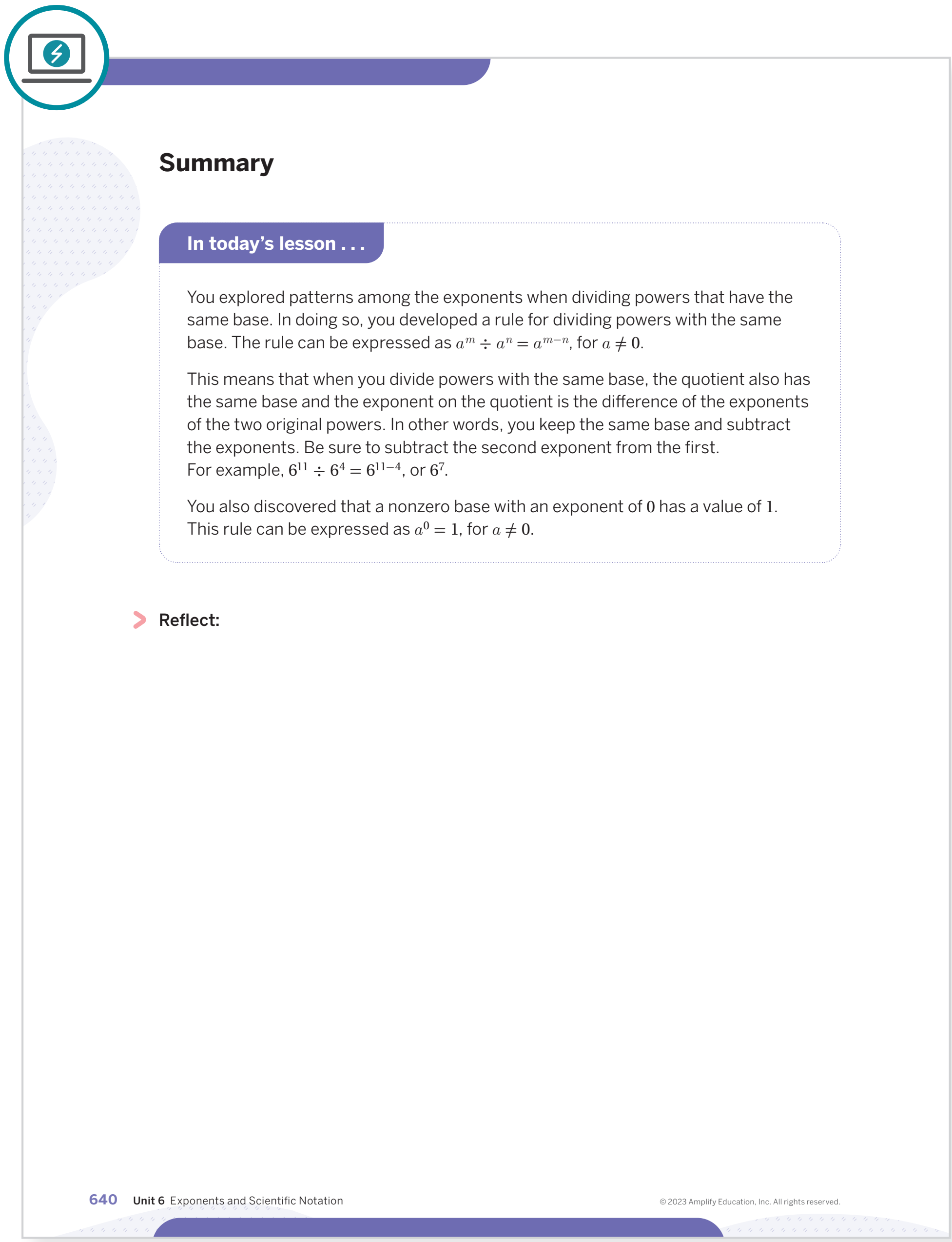
Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by having students complete parts two out of three parts for each medal. Consider allowing them to choose which parts to complete. Offering them the power of choice can result in greater engagement in the task.

Whole Class | 5 min

Summary

Review and synthesize the exponent rule for dividing powers that have the same base.



Summary

In today's lesson . . .

You explored patterns among the exponents when dividing powers that have the same base. In doing so, you developed a rule for dividing powers with the same base. The rule can be expressed as $a^m \div a^n = a^{m-n}$, for $a \neq 0$.

This means that when you divide powers with the same base, the quotient also has the same base and the exponent on the quotient is the difference of the exponents of the two original powers. In other words, you keep the same base and subtract the exponents. Be sure to subtract the second exponent from the first.
For example, $6^{11} \div 6^4 = 6^{11-4}$, or 6^7.

You also discovered that a nonzero base with an exponent of 0 has a value of 1. This rule can be expressed as $a^0 = 1$, for $a \neq 0$.

Reflect:

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Synthesize

Ask:

- "How can you verify that $\frac{9^7}{9^2} = 9^5$?" Sample response: Write each expression as repeated multiplication. There are seven factors of 9 divided by two factors of 9. This means there are a total of five factors of 9 in the quotient, or 9^5. Also, the exponent rule for dividing powers with the same base tells me to subtract the exponents, $7 - 2 = 5$, and keep the same base, 9.
- "How can you show the value of 10^0 using exponent rules?" Sample response: Write the division expressions $10^3 \div 10^3$. Then keep the same base, 10, and subtract the exponents, $3 - 3 = 0$. The single power is 10^0. But because any non-zero number divided by itself equals 1, I know that 10^0 must equal 1.

Display the Anchor Chart PDF, *Exponent Rules*. Uncover the Quotient rule and Zero rule. **Note:** The other rules will be uncovered throughout the unit.

Highlight that $a^m \div a^n = a^{m-n}$, for $a \neq 0$ shows the rule for dividing powers with the same base. This means that when dividing powers with the same base, keep the base and subtract the exponent in the denominator from the exponent in the numerator. Follow up with the rule that $a^0 = 1$. Connect this with the Quotient rule and division of any number by itself is equal to 1.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

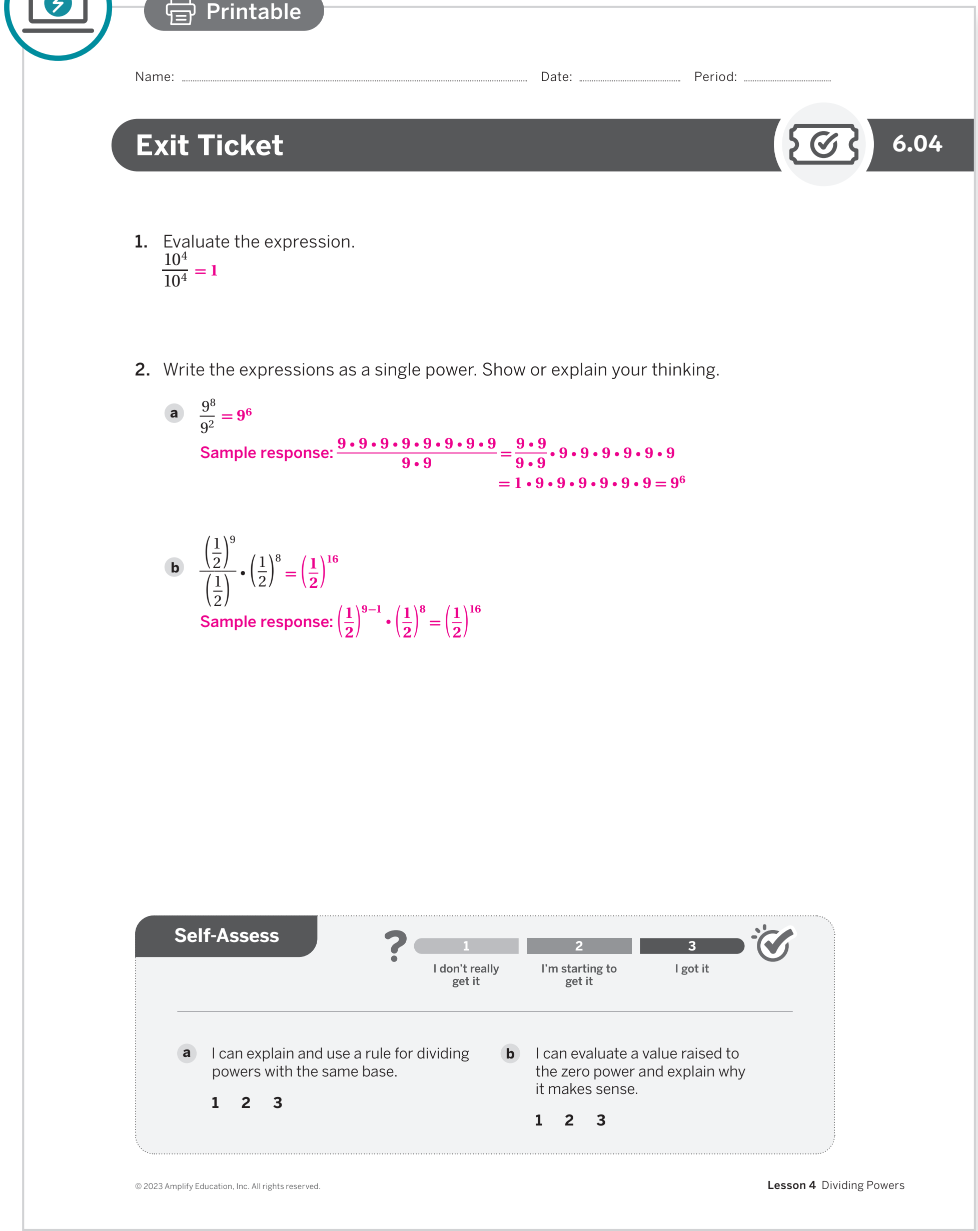
- "What happens when expressions containing exponents are multiplied or divided?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of dividing powers (that have the same base) by writing an equivalent as a single power.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.04

1. Evaluate the expression.
$\frac{10^4}{10^4} = 1$

2. Write the expressions as a single power. Show or explain your thinking.

a $\frac{9^8}{9^2} = 9^6$

Sample response: $\frac{9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9}{9 \cdot 9} = \frac{9 \cdot 9}{9 \cdot 9} \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
$= 1 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 9^6$

b $\frac{\left(\frac{1}{2}\right)^9}{\left(\frac{1}{2}\right)} \cdot \left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{16}$

Sample response: $\left(\frac{1}{2}\right)^{9-1} \cdot \left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{16}$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can explain and use a rule for dividing powers with the same base.
1 2 3

b I can evaluate a value raised to the zero power and explain why it makes sense.
1 2 3

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Lesson 4 Dividing Powers

Success looks like . . .

- **Language Goal:** Generalizing a process for dividing exponential expressions with the same base, and justifying that $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$. **(Speaking and Listening, Reading and Writing)**
  - » Evaluating and justifying the division in Problem 2.
- **Language Goal:** Using exponent rules to justify that a^0 is 1. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students write 0 instead of 1 or 10^0 for Problem 1, consider:

- Reviewing fractions that have the same numerator and denominator or reviewing that any non-zero number divided by itself equals 1.
- Having students write the expression in expanded form.

If students write 1^6 or 9^4 for Problem 2a, consider:

- Reviewing Activity 2.
- Writing the expression in expanded form.

If students do not know how to complete Problem 2b or complete it incorrectly, consider:

- Asking students to evaluate $\frac{\left(\frac{1}{2}\right)^9}{\left(\frac{1}{2}\right)}$ first.
- Having them write $\left(\frac{1}{2}\right)$ as $\left(\frac{1}{2}\right)^1$.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What challenges did students encounter as they worked on Activity 3? How did they work through them?
- What resources did students use as they worked on Activity 2? Which resources were especially helpful? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Evaluate each expression.
   - a $10^0 = 1$
   - b $\frac{8^3}{8^3} = 1$
   - c $\frac{2^4}{11^0} = 16$
   - d $(-9)^0 + 7 = 8$
   - e $\left(\frac{2}{3}\right)^0 + \frac{1}{4} = 1\frac{1}{4}$
   - f $\frac{3^3}{3^0} \bullet 3 = 81$

2. Write each expression as a single power.
   - a $\frac{10^7}{10^2} = 10^5$
   - b $\frac{11^8}{11^2} = 11^6$
   - c $\frac{7^3 \bullet 7^4}{7^5} = 7^2$
   - d $\left(\frac{8^{12}}{8^7}\right) \bullet 8^4 = 8^9$
   - e $\frac{10^{13} \bullet 10^5 \bullet 10^0}{10^3 \bullet 10^7} = 10^8$
   - f $\frac{a^3 \bullet a^5}{a^2 \bullet a^6} = a^0$ or 1

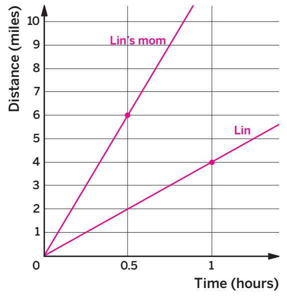
3. The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? Explain your thinking.
   KW Sagittarii is 10^3 or 1,000 times as wide as the Sun, because $10^5 \div 10^2 = 10^3$.

4. Bananas cost $1.50 per pound, and guavas cost $3.00 per pound. Kiran spends $12 on fruit for a breakfast his family is hosting. Let b represent the number of pounds of bananas Kiran buys and g represent the number of pounds of guavas he buys.
   - a Write an equation relating the two variables.
     $1.50b + 3.00g = 12$
   - b Rewrite the equation so that g is the independent variable.
     $b = 8 - 2g$
   - c Rewrite the equation so that b is the independent variable.
     $g = 4 - 0.5b$

Practice

Name: ______ Date: ______ Period: ______

5. Lin's mom bikes at a constant speed of 12 mph. Lin walks at a constant speed that is $\frac{1}{3}$ of the speed at which her mom bikes. Sketch a graph of each relationship.



6. Write each expression under an equivalent value in the table.

$-5 - 7$	$-5 + 7$	$-5 - (-7)$	$5 + (-7)$	$5 - 7$

Equivalent to −2	Equivalent to 2	Equivalent to −12
$5 + (-7)$ $5 - 7$	$-5 + 7$ $-5 - (-7)$	$-5 - 7$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 1	2
Spiral	4	Unit 5 Lesson 3	2
	5	Unit 3 Lesson 4	2
Formative ⚡	6	Unit 6 Lesson 5	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice.**

UNIT 6 | **LESSON 5**

Negative Exponents

Let’s see what happens when exponents are negative.

Focus

Goals

1. **Language Goal:** Describe how exponent rules extend to expressions involving negative exponents. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Describe patterns in repeated multiplication and division, and justify that $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students extend their understanding of exponents to include negative exponents. This lesson starts by presenting negative exponents with the base of 2 as repeated multiplication of $\frac{1}{2}$, leading to the rule $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$. In Activity 2, students continue to analyze the structure of different expressions to strengthen their understanding of negative exponents.

Previously

In Lessons 3 and 4, students discovered rules for multiplying and dividing powers with the same base.

Coming Soon

In subsequent lessons, students will use exponent rules to rewrite exponential expressions involving negative exponents so that they have a single positive exponent, and explain their strategy.

Rigor

- Students build **conceptual understanding** of negative exponents.
- Students strengthen their **fluency** in multiplying and dividing single powers with the same base.

Pacing Guide

Suggested Total Lesson Time | ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	12 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Exponent Rules*

Math Language Development

Review words

- *base*
- *equivalent expressions*
- *expanded form*
- *exponent*
- *power*

Amps | **Featured Activity**

Activity 2

See Student Thinking

Students enter what they think is an equivalent expression and explain their thinking, which you can see in real time.

Building Math Identity and Community

Connecting to Mathematical Practices

As students work in pairs in Activity 2, they might not make constructive choices about their social interactions. Their understanding of negative exponents might influence their behavioral choices. If they understand, they might try to take over, but if they don't, they might sit back and not work with their partner. Ask pairs to rely on and discuss the familiar structures of exponents as they match the equivalent expressions.

Modifications to Pacing

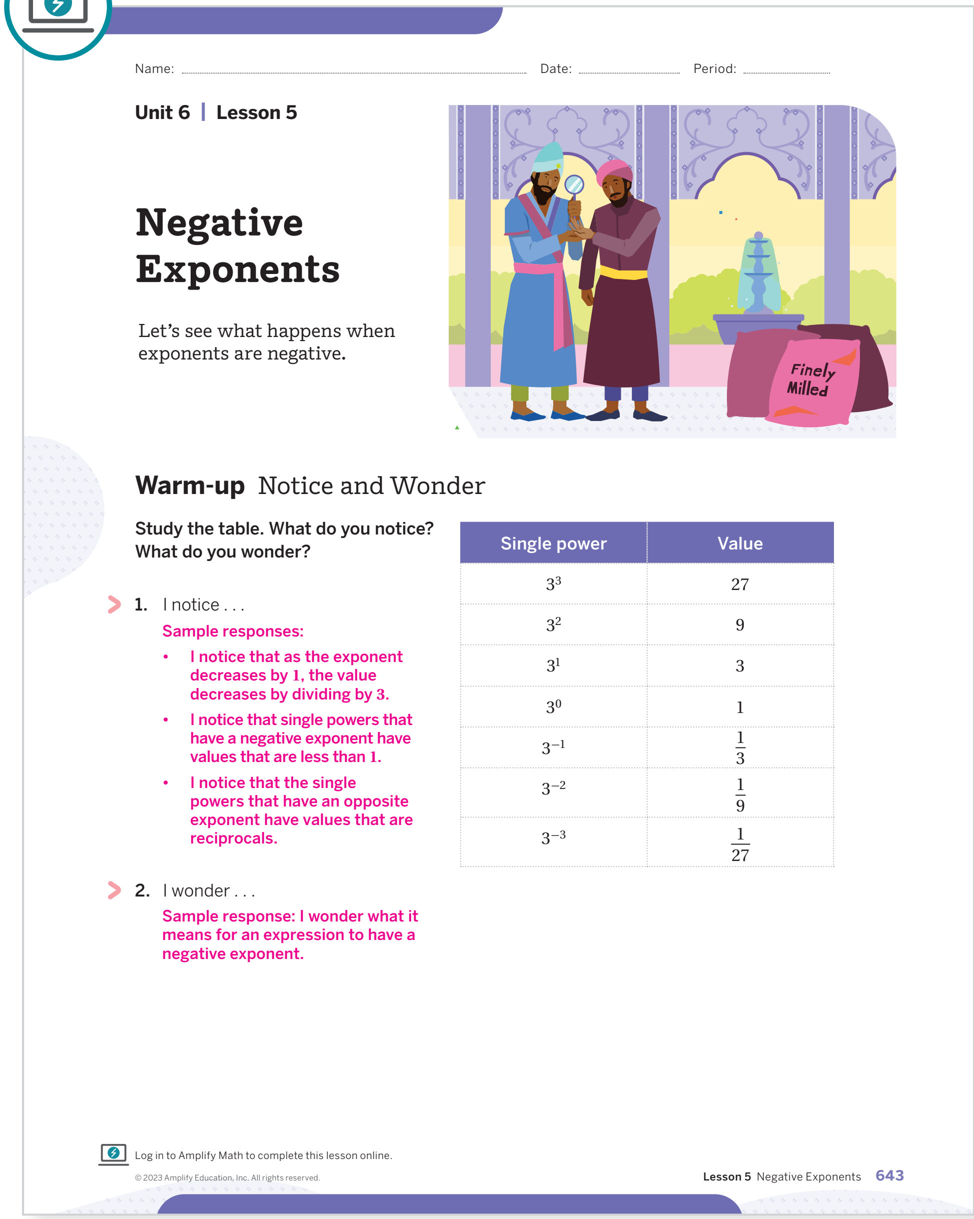
You may want to consider these additional modifications if you are short on time.

- Replace the **Warm-up** with the table in **Activity 1**.
- In **Activity 2**, Problem 5 may be assigned as additional practice.

Independent | 5 min

Warm-up Notice and Wonder

Students use patterns to complete missing values in a table, which introduces them to a negative exponent.



Name: ______________________ Date: __________ Period: __________

Unit 6 | Lesson 5

Negative Exponents

Let's see what happens when exponents are negative.

Warm-up Notice and Wonder

Study the table. What do you notice? What do you wonder?

Single power	Value
3^3	27
3^2	9
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{9}$
3^{-3}	$\frac{1}{27}$

1. I notice . . .

   Sample responses:
   - I notice that as the exponent decreases by 1, the value decreases by dividing by 3.
   - I notice that single powers that have a negative exponent have values that are less than 1.
   - I notice that the single powers that have an opposite exponent have values that are reciprocals.

2. I wonder . . .

   Sample response: I wonder what it means for an expression to have a negative exponent.

Log in to Amplify Math to complete this lesson online.

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Lesson 5 Negative Exponents 643

1 Launch

Conduct the *Notice and Wonder* routine.

2 Monitor

Help students get started by asking, "As the exponent decreases by 1, how does the value change?"

Look for points of confusion:

- **Thinking that a negative exponent will result in a negative value.** Remind students that the fractions $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{27}$ are positive values between 0 and 1.

Look for productive strategies:

- Noticing that as the exponent decreases by 1, the value decreases by dividing by 3.
- Noticing that single powers that have a negative exponent have values that are less than 1.
- Noticing that single powers that have opposite exponents have values that are reciprocals.

3 Connect

Have students share what they notice and wonder.

Highlight that exponents can be positive or negative.

Ask, "What do you think the value of 3^{-4} is?" $\frac{1}{81}$

Power-up

To power up students' ability to add and subtract integers, have students complete:

Recall that when subtracting two integers, you can rewrite subtraction as adding the opposite: $a - b = a + (-b)$.
Determine each sum or difference.

a. $4 + (-9) = -5$

b. $-9 + (-6) = -15$

c. $12 - 15 = -3$

d. $5 - (-3) = 8$

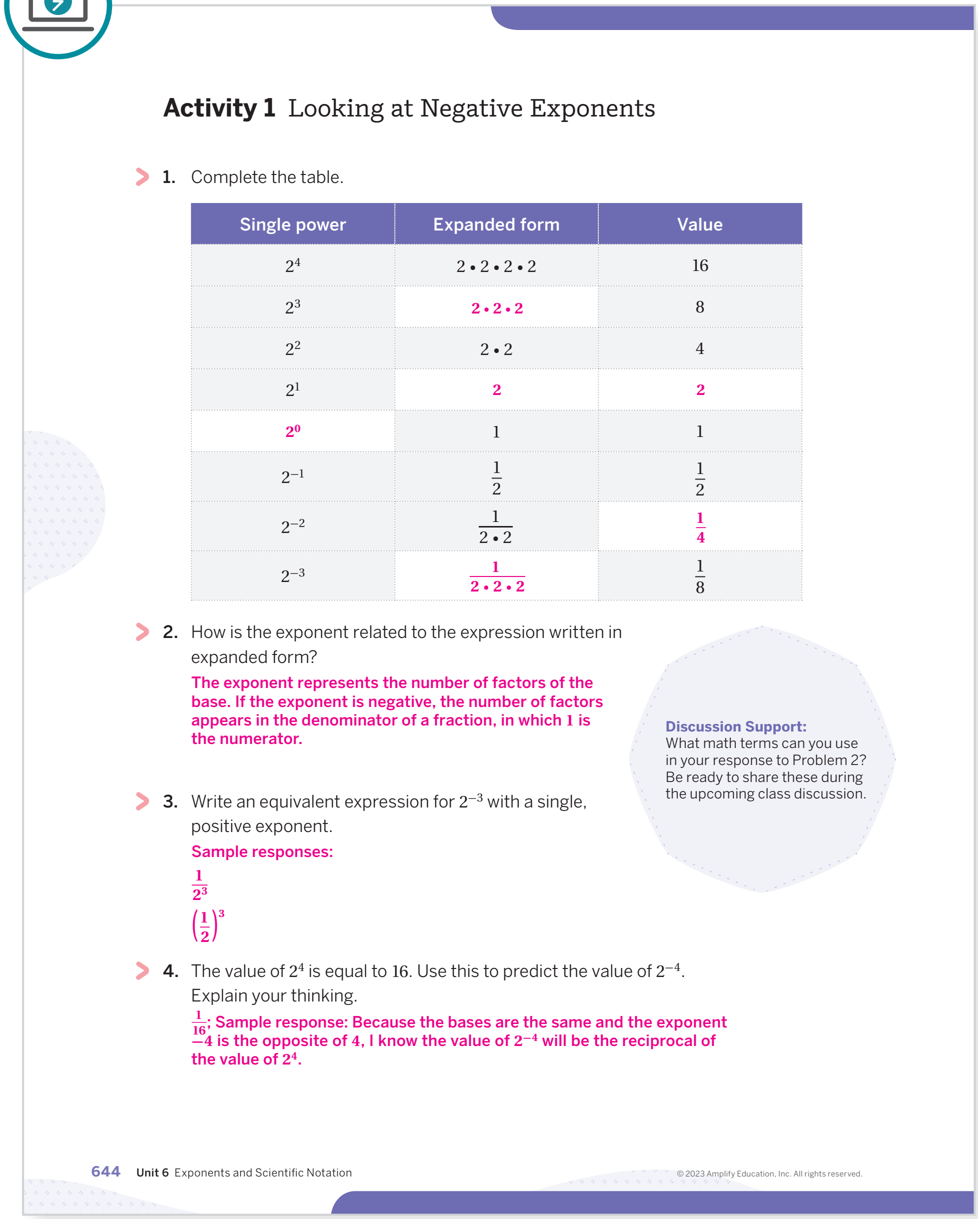
Use: Before Activity 2

Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 12 min

Activity 1 Looking at Negative Exponents

Students use repeated reasoning to recognize that a power with a negative exponent and the base 2 represents repeated multiplication of $\frac{1}{2}$, generalizing the rule $2^{-n} = \frac{1}{2^n}$.



Activity 1 Looking at Negative Exponents

1. Complete the table.

Single power	Expanded form	Value
2^4	$2 \cdot 2 \cdot 2 \cdot 2$	16
2^3	$\mathbf{2 \cdot 2 \cdot 2}$	8
2^2	$2 \cdot 2$	4
2^1	**2**	**2**
$\mathbf{2^0}$	1	1
2^{-1}	$\frac{1}{2}$	$\frac{1}{2}$
2^{-2}	$\frac{1}{2 \cdot 2}$	$\mathbf{\frac{1}{4}}$
2^{-3}	$\mathbf{\frac{1}{2 \cdot 2 \cdot 2}}$	$\frac{1}{8}$

2. How is the exponent related to the expression written in expanded form?
The exponent represents the number of factors of the base. If the exponent is negative, the number of factors appears in the denominator of a fraction, in which 1 is the numerator.

Discussion Support: What math terms can you use in your response to Problem 2? Be ready to share these during the upcoming class discussion.

3. Write an equivalent expression for 2^{-3} with a single, positive exponent.
Sample responses:
$\mathbf{\frac{1}{2^3}}$
$\mathbf{\left(\frac{1}{2}\right)^3}$

4. The value of 2^4 is equal to 16. Use this to predict the value of 2^{-4}. Explain your thinking.
$\mathbf{\frac{1}{16}}$**; Sample response: Because the bases are the same and the exponent -4 is the opposite of 4, I know the value of 2^{-4} will be the reciprocal of the value of 2^4.**

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1 Launch

Tell students that the table shows three different ways an expression can be written.

2 Monitor

Help students get started by looking for a pattern between the columns before completing the table.

Look for points of confusion:

- **Struggling to complete Problem 3.** Have students look at the equivalent expression written in expanded form, and ask them to write the denominator as a single power, then have them write the fraction using the single power.

Look for productive strategies:

- Writing $\frac{1}{2^3}$ or $\left(\frac{1}{2}\right)^3$ for Problem 3.

3 Connect

Have students share what patterns they noticed that allowed them to complete the table.

Highlight that negative exponents with a base of 2 show repeated multiplication of $\frac{1}{2}$.

Define a^{-n} as $\frac{1}{a^n}$ when $a \neq 0$.

Ask, "Are 2^{-3} and $\left(\frac{1}{2}\right)^3$ equivalent?" Yes, $2^{-3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ and $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students study each column at a time before making connections across columns. Ask, "Look at the *Single power* column. What is happening to the exponents?"

Extension: *Math Enrichment*

Have students complete the following problem:

Determine whether $\left(\frac{2}{3}\right)^{-3}$ is greater than 1 or between 0 and 1. Explain your thinking. Greater than 1; Sample response: $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3}$, which is equal to $\frac{1}{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)}$, or $\frac{1}{\frac{8}{27}}$ or $\frac{27}{8}$.

Math Language Development

MLR8: Discussion Supports—Revoicing

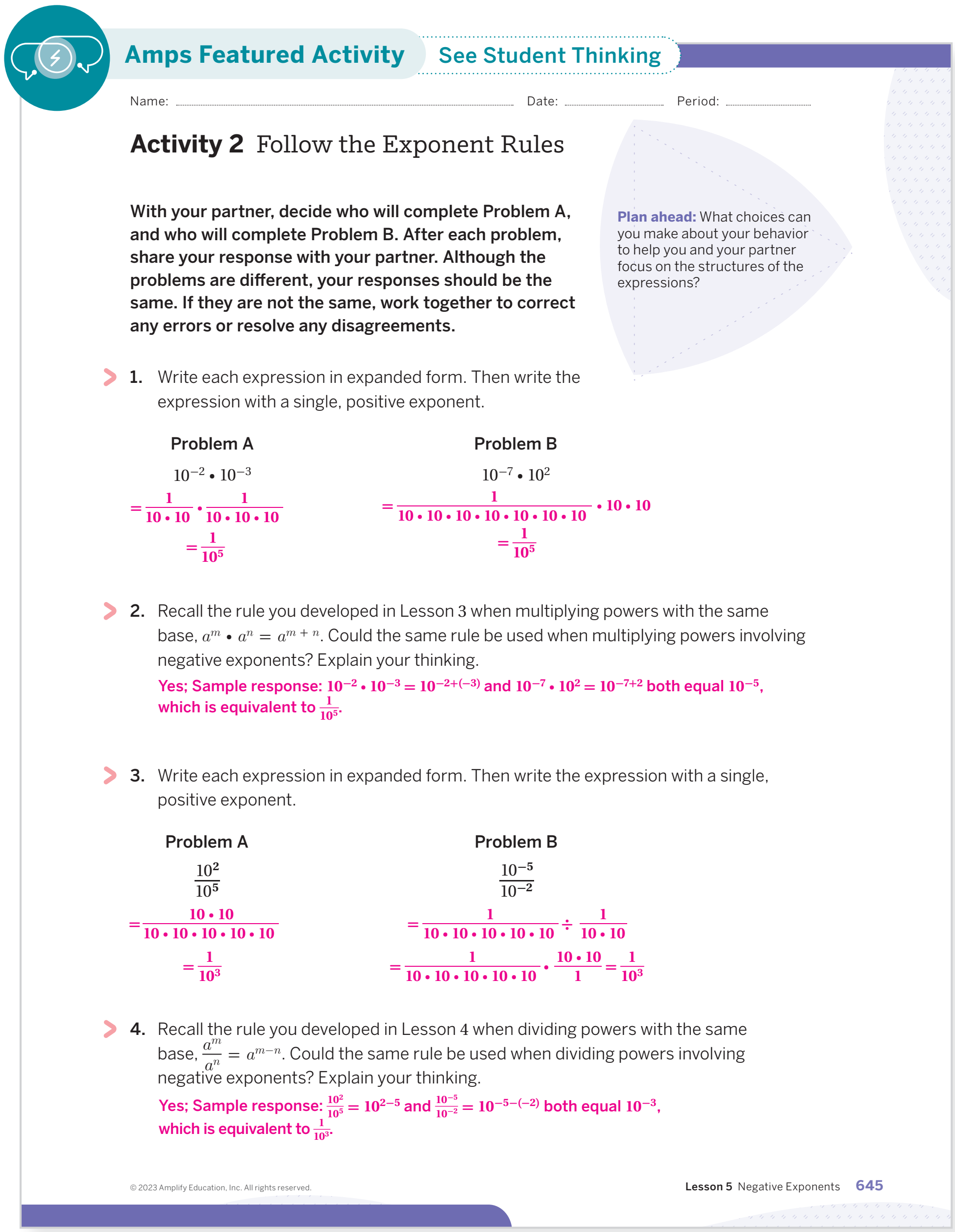
During the Connect, have students share their responses to Problems 2 and ask them what math terms they can use in their responses. Revoice their ideas by restating them in the form of a question using precise mathematical language. This can be a way to invite more students to participate in the discussion. For example:

If a student says . . .	Press for details by asking . . .
"A negative exponent is a fraction."	"I hear you connecting negative exponents to fractions. Can you be more specific in how they relate to each other?"

Pairs | 20 min

Activity 2 Follow the Exponent Rules

Students multiply and divide powers that have the same base, noticing that the exponent rules can still be applied when the exponents are negative.



Amps Featured Activity See Student Thinking

Name: ______ Date: ______ Period: ______

Activity 2 Follow the Exponent Rules

With your partner, decide who will complete Problem A, and who will complete Problem B. After each problem, share your response with your partner. Although the problems are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

Plan ahead: What choices can you make about your behavior to help you and your partner focus on the structures of the expressions?

1. Write each expression in expanded form. Then write the expression with a single, positive exponent.

Problem A

$10^{-2} \cdot 10^{-3}$

$= \frac{1}{10 \cdot 10} \cdot \frac{1}{10 \cdot 10 \cdot 10}$

$= \frac{1}{10^5}$

Problem B

$10^{-7} \cdot 10^2$

$= \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} \cdot 10 \cdot 10$

$= \frac{1}{10^5}$

2. Recall the rule you developed in Lesson 3 when multiplying powers with the same base, $a^m \cdot a^n = a^{m+n}$. Could the same rule be used when multiplying powers involving negative exponents? Explain your thinking.

Yes; Sample response: $10^{-2} \cdot 10^{-3} = 10^{-2+(-3)}$ and $10^{-7} \cdot 10^2 = 10^{-7+2}$ both equal 10^{-5}, which is equivalent to $\frac{1}{10^5}$.

3. Write each expression in expanded form. Then write the expression with a single, positive exponent.

Problem A

$\frac{10^2}{10^5}$

$= \frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

$= \frac{1}{10^3}$

Problem B

$\frac{10^{-5}}{10^{-2}}$

$= \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} \div \frac{1}{10 \cdot 10}$

$= \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} \cdot \frac{10 \cdot 10}{1} = \frac{1}{10^3}$

4. Recall the rule you developed in Lesson 4 when dividing powers with the same base, $\frac{a^m}{a^n} = a^{m-n}$. Could the same rule be used when dividing powers involving negative exponents? Explain your thinking.

Yes; Sample response: $\frac{10^2}{10^5} = 10^{2-5}$ and $\frac{10^{-5}}{10^{-2}} = 10^{-5-(-2)}$ both equal 10^{-3}, which is equivalent to $\frac{1}{10^3}$.

1 Launch

Activate students' prior knowledge about multiplying and dividing fractions and adding and subtracting integers. As a whole class, discuss Problems 1–4, before having students complete Problem 5.

2 Monitor

Help students get started by having them write any powers with a negative exponent as a fraction.

Look for points of confusion:

- **Struggling to write $\frac{10^{-5}}{10^{-2}}$ in expanded form.** Have students write the problem using a division symbol, $10^{-5} \div 10^{-2}$, before writing the expression in expanded form. Then remind students that when dividing fractions, they can multiply the reciprocal of the fraction.
- **For Problem 5, writing the expressions in expanded form.** Encourage students to review the Anchor Chart PDF, *Exponent Rules* and then use the rules for multiplying and dividing powers with the same base.
- **When dividing powers with the same base, subtracting the exponent in the numerator from the exponent in the denominator.** Have students write these expressions in expanded form to compare the factors in the numerator with the factors in the denominator.

Look for productive strategies:

- Noticing that the exponent rules for multiplying and dividing powers with the same base still apply, even when the exponent is negative.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Anchor Chart PDF, *Exponent Rules* for them to reference during this activity. Allow them to keep this anchor chart throughout the rest of the unit.

Extension: *Math Enrichment*

Have students determine whether the equation $a^{-1} \cdot a = 0$ is true or false and explain their thinking. False; Sample response: $a^{-1} \cdot a = a^0$, which means the base is a and the exponent is 0. The actual value is 1, not 0, because any number to the power of zero is 1.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 4 and before moving on to Problem 5, have pairs meet with 1–2 other pairs of students to share their responses to Problems 2 and 4. Encourage reviewers to ask clarifying questions, such as:

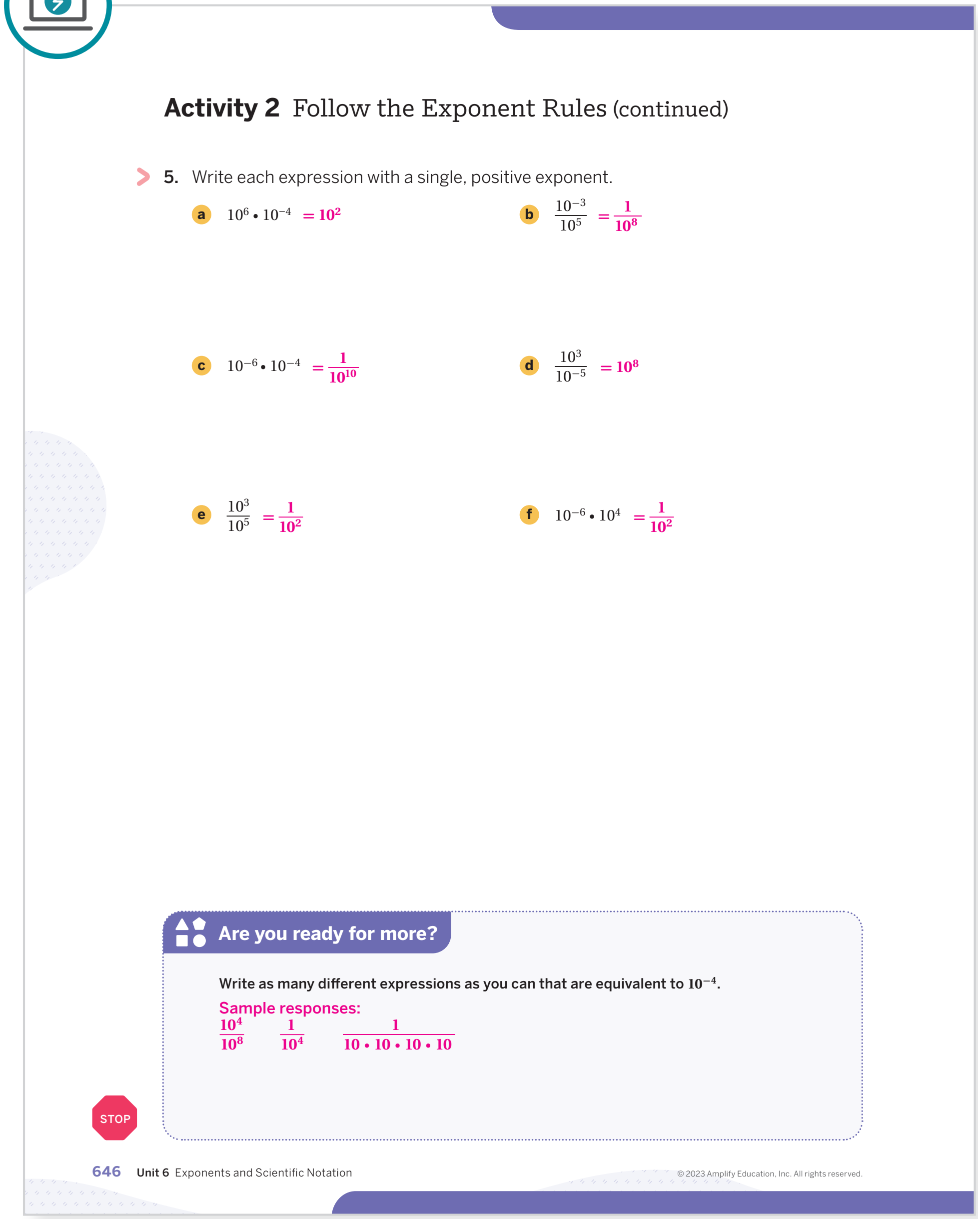
- "Did you include examples in your response? How could examples help illustrate your thoughts?"
- "What math language did you use in your response?"

Have students revise their responses, as needed.

Pairs | 20 min

Activity 2 Follow the Exponent Rules (continued)

Students multiply and divide powers that have the same base, noticing that the exponent rules can still be applied when the exponents are negative.



Activity 2 Follow the Exponent Rules (continued)

5. Write each expression with a single, positive exponent.

a. $10^6 \cdot 10^{-4} = 10^2$

b. $\frac{10^{-3}}{10^5} = \frac{1}{10^8}$

c. $10^{-6} \cdot 10^{-4} = \frac{1}{10^{10}}$

d. $\frac{10^3}{10^{-5}} = 10^8$

e. $\frac{10^3}{10^5} = \frac{1}{10^2}$

f. $10^{-6} \cdot 10^4 = \frac{1}{10^2}$

Are you ready for more?

Write as many different expressions as you can that are equivalent to 10^{-4}.

Sample responses:

$\frac{10^4}{10^8}$ $\frac{1}{10^4}$ $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10}$

STOP

646 **Unit 6** Exponents and Scientific Notation

3 Connect

Have students share any problems in which they did not have the same response as their partner, and how they came to an agreement of their final response.

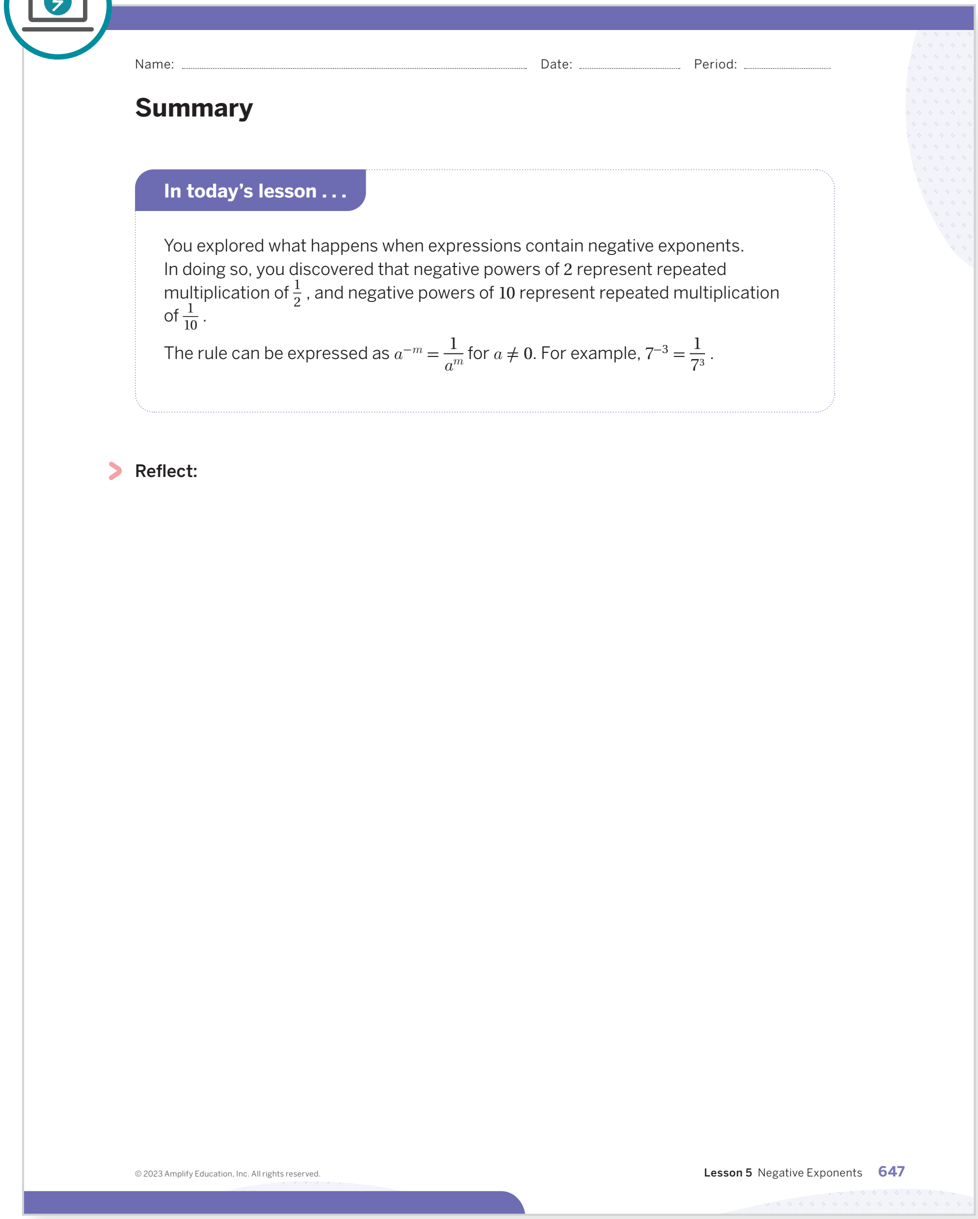
Ask, "How did you look for and make use of structure when writing each expression as a single power?"

Highlight that when multiplying or dividing powers that have the same base and a negative exponent, students can deduce the answer by writing the expression in expanded form or by using the corresponding exponent rule.

Whole Class | 5 min

Summary

Review and synthesize how negative exponents represent repeated multiplication of a fractional base.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored what happens when expressions contain negative exponents. In doing so, you discovered that negative powers of 2 represent repeated multiplication of $\frac{1}{2}$, and negative powers of 10 represent repeated multiplication of $\frac{1}{10}$.

The rule can be expressed as $a^{-m} = \frac{1}{a^m}$ for $a \neq 0$. For example, $7^{-3} = \frac{1}{7^3}$.

Reflect:

Synthesize

Have students share how they can write an expression with negative exponents as an expression with a single positive exponent.

Ask:

- "How is 10^3 related to 10^{-3}?" Sample response: 10^3 represents repeated multiplication of the base 10 (a total of 3 times). 10^{-3} represents repeated multiplication of the base $\frac{1}{10}$ (a total of 3 times). $10^3 = 1{,}000$ and $10^{-3} = \frac{1}{1{,}000}$.
- "How can you convince someone that $10^{-3} = \frac{1}{10^3}$?" Sample response: A negative exponent indicates repeated multiplication of a fractional base, in this case the base is $\frac{1}{10}$. Because $\frac{1}{10}$ multiplied by itself 3 times equals $\frac{1}{10^3}$, this means that $10^{-3} = \frac{1}{10^3}$.

Display the Anchor Chart PDF, *Exponent Rules*. Uncover the Negative exponent rule. **Note:** The other rules will be uncovered throughout the unit.

Highlight that the rule $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$ for $a \neq 0$ helps students evaluate expressions with a negative exponent.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

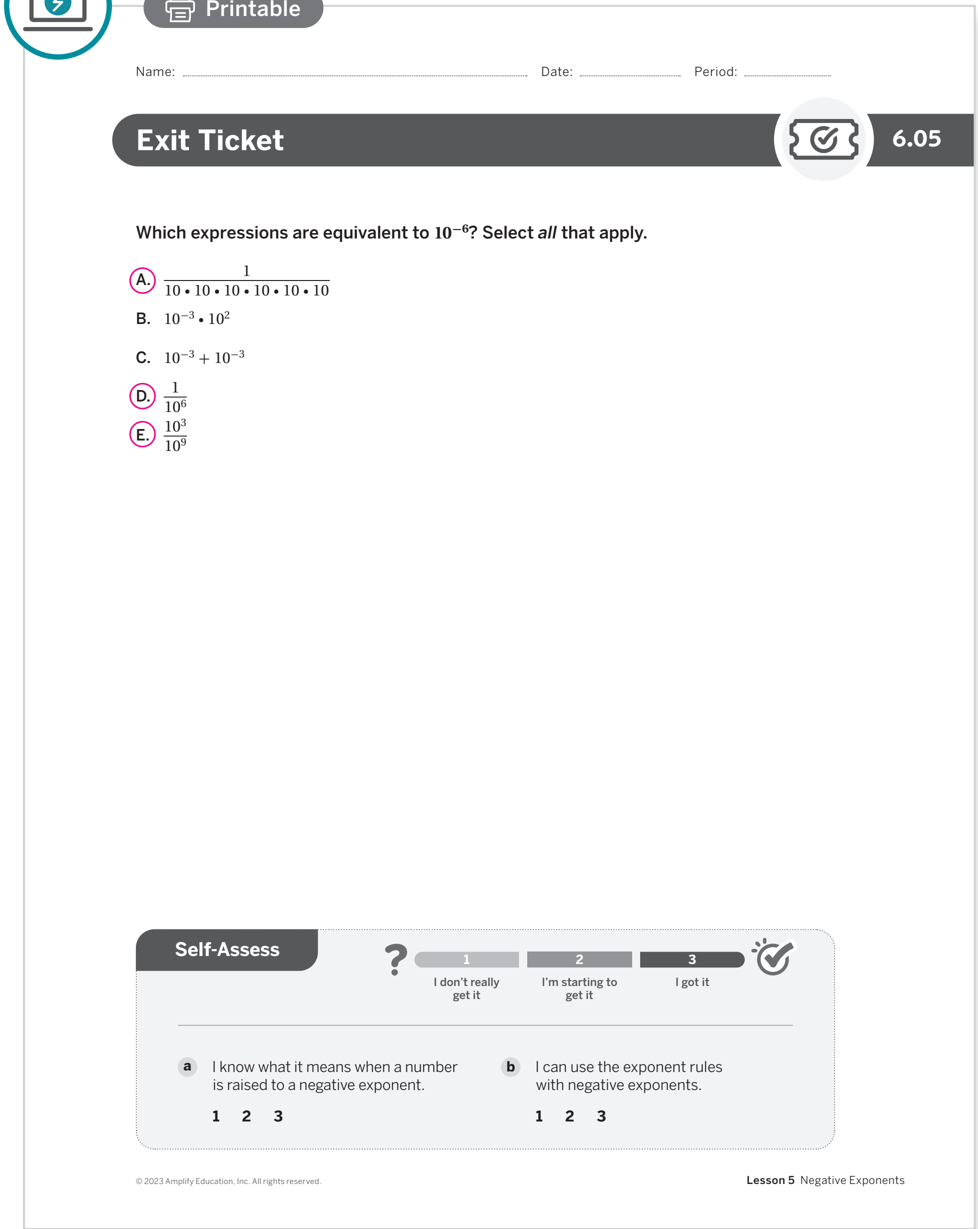
- "What happens when expressions containing exponents are multiplied or divided?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of negative exponents and the exponent rules they have learned so far by identifying equivalent expressions.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.05

Which expressions are equivalent to 10^{-6}? Select *all* that apply.

[✓ A.] $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

B. $10^{-3} \cdot 10^{2}$

C. $10^{-3} + 10^{-3}$

[✓ D.] $\frac{1}{10^{6}}$

[✓ E.] $\frac{10^{3}}{10^{9}}$

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I know what it means when a number is raised to a negative exponent.

1 2 3

b I can use the exponent rules with negative exponents.

1 2 3

Lesson 5 Negative Exponents

Success looks like . . .

- **Language Goal:** Describing how exponent rules extend to expressions involving negative exponents. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Describing patterns in repeated multiplication and division, and justifying that $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$. **(Speaking and Listening, Reading and Writing)**
  - » Show how 10^{-6} can be written using repeated multiplication and division.

Suggested next steps

If students select B or D, consider:

- Having students write each part of the expression in expanded form.
- Reviewing the meaning of negative exponents and the exponent rules students have learned so far.
- Reviewing Lesson 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What different ways did students approach multiplying and dividing powers with negative exponents? What does that tell you about similarities and differences among your students?
- What did students find frustrating about Activity 2? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Math Language Development

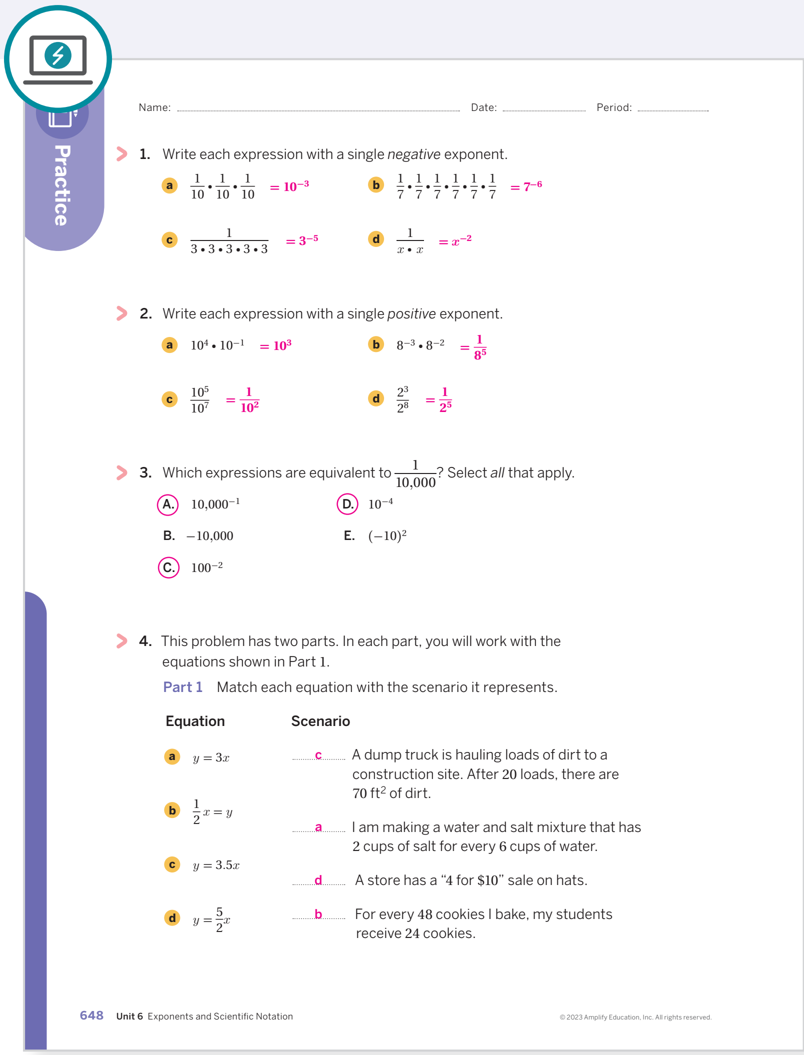
Language Goal: Describing how exponent rules extend to expressions involving negative exponents.

Reflect on students' language development toward this goal.

- How did using the ***Stronger and Clearer Each Time*** routine in Activity 2 help students be more precise in their explanations for how the exponent rules they previously learned can now be applied to expressions involving negative exponents?
- What other strategies might you choose to use to help them be more precise in their descriptions?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. Write each expression with a single *negative* exponent.
   - a $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$ $= 10^{-3}$
   - b $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7}$ $= 7^{-6}$
   - c $\frac{1}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$ $= 3^{-5}$
   - d $\frac{1}{x \cdot x}$ $= x^{-2}$

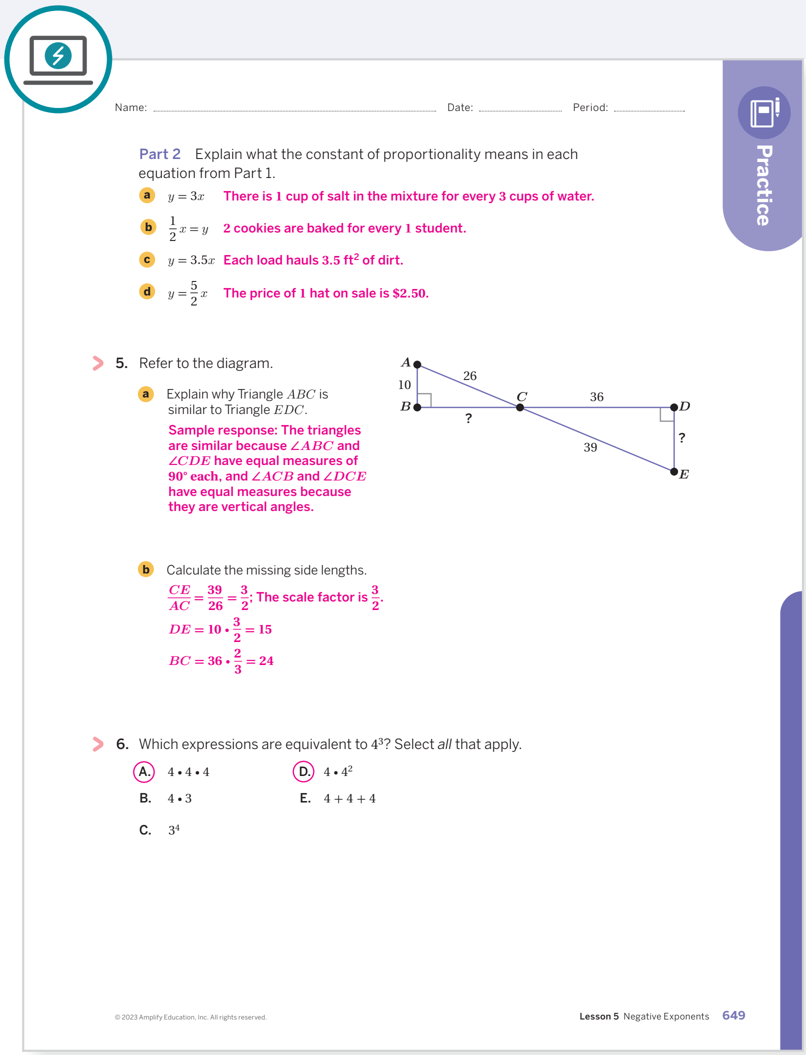
2. Write each expression with a single *positive* exponent.
   - a $10^4 \cdot 10^{-1}$ $= 10^3$
   - b $8^{-3} \cdot 8^{-2}$ $= \frac{1}{8^5}$
   - c $\frac{10^5}{10^7}$ $= \frac{1}{10^2}$
   - d $\frac{2^3}{2^8}$ $= \frac{1}{2^5}$

3. Which expressions are equivalent to $\frac{1}{10{,}000}$? Select *all* that apply.
   - [✓ A.] $10{,}000^{-1}$
   - B. $-10{,}000$
   - [✓ C.] 100^{-2}
   - [✓ D.] 10^{-4}
   - E. $(-10)^2$

4. This problem has two parts. In each part, you will work with the equations shown in Part 1.

   Part 1 Match each equation with the scenario it represents.

Equation	Scenario
a $y = 3x$	c A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 ft² of dirt.
b $\frac{1}{2}x = y$	a I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
c $y = 3.5x$	d A store has a "4 for $10" sale on hats.
d $y = \frac{5}{2}x$	b For every 48 cookies I bake, my students receive 24 cookies.



Name: .... Date: .... Period: ....

Practice

Part 2 Explain what the constant of proportionality means in each equation from Part 1.

- a $y = 3x$ There is 1 cup of salt in the mixture for every 3 cups of water.
- b $\frac{1}{2}x = y$ 2 cookies are baked for every 1 student.
- c $y = 3.5x$ Each load hauls 3.5 ft² of dirt.
- d $y = \frac{5}{2}x$ The price of 1 hat on sale is $2.50.

5. Refer to the diagram.

   - a Explain why Triangle ABC is similar to Triangle EDC.

     Sample response: The triangles are similar because $\angle ABC$ and $\angle CDE$ have equal measures of 90° each, and $\angle ACB$ and $\angle DCE$ have equal measures because they are vertical angles.

   - b Calculate the missing side lengths.

     $\frac{CE}{AC} = \frac{39}{26} = \frac{3}{2}$; The scale factor is $\frac{3}{2}$.

     $DE = 10 \cdot \frac{3}{2} = 15$

     $BC = 36 \cdot \frac{2}{3} = 24$

6. Which expressions are equivalent to 4^3? Select *all* that apply.
   - [✓ A.] $4 \cdot 4 \cdot 4$
   - B. $4 \cdot 3$
   - C. 3^4
   - [✓ D.] $4 \cdot 4^2$
   - E. $4 + 4 + 4$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 3	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 4	2
	5	Unit 2 Lesson 8	3
Formative ⚡	6	Unit 6 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

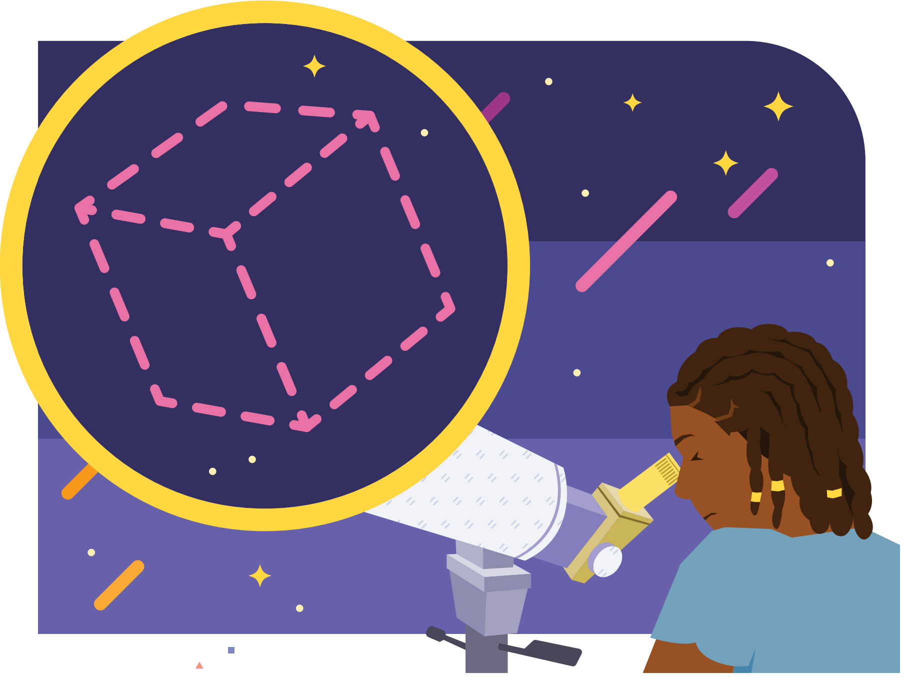
Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 6 | **LESSON 6**

Powers of Powers

Let’s look at powers of powers.



Focus

Goal

1. **Language Goal:** Generalize a process for finding a single power raised to an exponent, and justify that $(a^m)^n = a^{m \cdot n}$, where $a \neq 0$. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students make use of repeated reasoning to discover the exponent rule $(a^m)^n = a^{m \cdot n}$, where $a \neq 0$. and then extend the rule to cases where the exponents are negative. Students reason abstractly and quantitatively when applying exponent rules, pausing to consider the meaning of quantities, not just how to compute them.

Previously

In Lessons 3 and 4, students discovered a rule for multiplying and dividing powers with the same base.

Coming Soon

In Lesson 7, students will continue to explore exponent patterns and develop a rule for multiplying powers that have different bases, yet the same exponent.

Rigor

- Students build **conceptual understanding** of raising powers to another power.
- Students strengthen their **fluency** in raising a power to another power.

Pacing Guide

Suggested Total Lesson Time | ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	8 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Exponent Rules*

Math Language Development

Review words

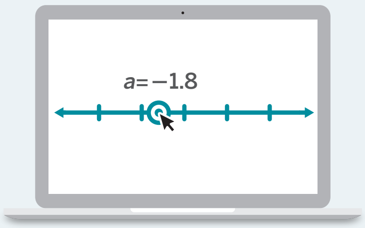
- *base*
- *equivalent expressions*
- *expanded form*
- *exponent*
- *power*

Amps | Featured Activity

Activity 1

Digital Card Sort

Students match expressions and make observations in patterns between expressions written in expanded form and as a single power.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students might begin to lose their motivation to stay focused and discover another rule of exponents using regular and repeated reasoning. Before the activity, have each student think of a small thought reward they can give themselves when they do finish this activity. After students are successful, give them one minute to live in their own thoughts.

Modifications to Pacing

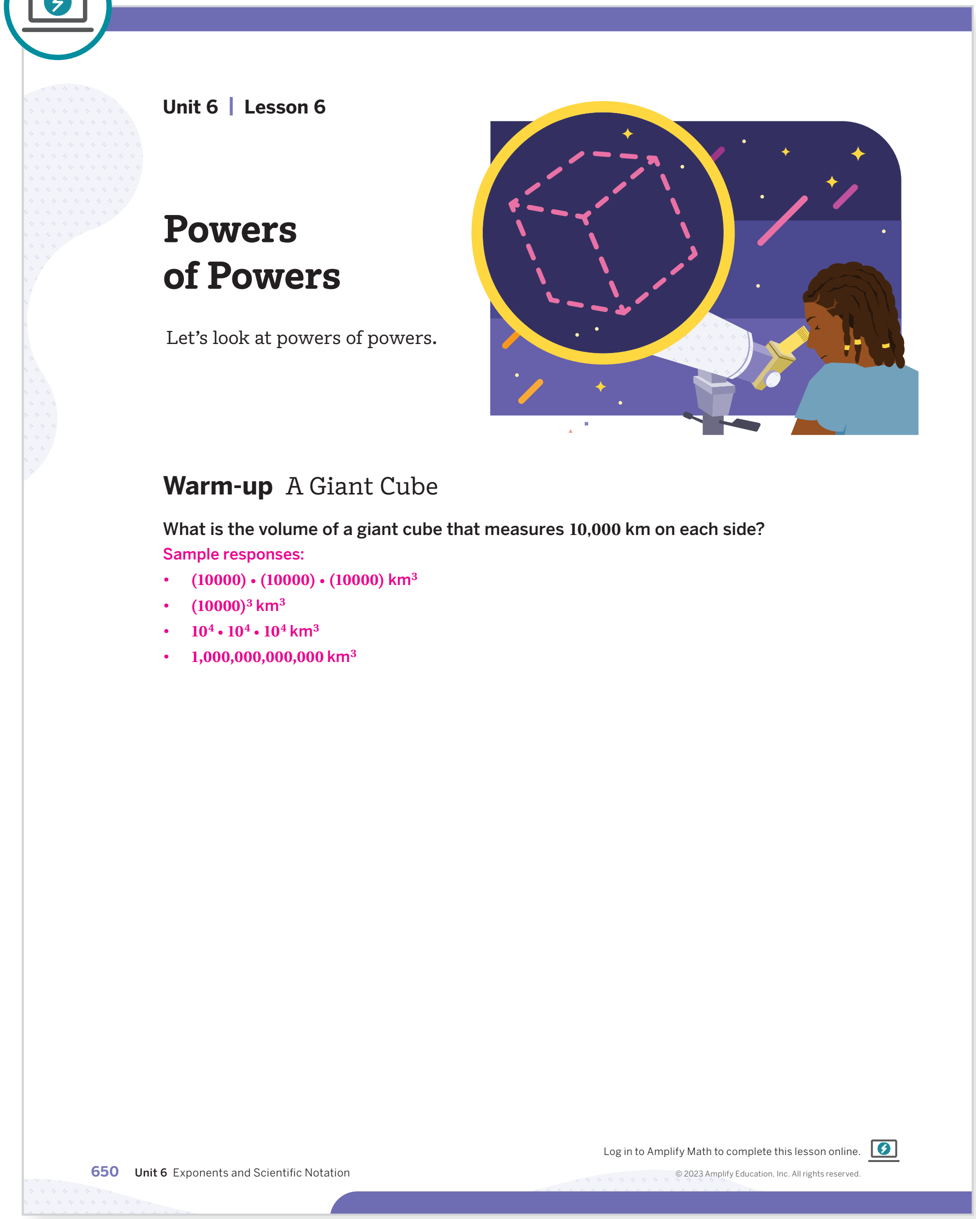
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, the last two rows may be omitted.
- **Activity 3** may be assigned as additional practice.

Independent | 5 min

Warm-up A Giant Cube

Students find the volume of a giant cube to reinforce how exponents show repeated multiplication.



Unit 6 | **Lesson 6**

Powers of Powers

Let's look at powers of powers.

Warm-up A Giant Cube

What is the volume of a giant cube that measures 10,000 km on each side?

Sample responses:

- $(10000) \cdot (10000) \cdot (10000)$ km^3
- $(10000)^3$ km^3
- $10^4 \cdot 10^4 \cdot 10^4$ km^3
- 1,000,000,000,000 km^3

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by activating their background knowledge. Ask, "How do you find the volume of a cube?"

Look for productive strategies:

- Computing the volume. Ask students if they can use powers of 10 and an exponent rule to make their calculations easier.
- Using exponents to write an expression for the volume.

3 Connect

Have students share their responses. Record different strategies to compute $(10{,}000)^3$.

Ask, "How can you rewrite the volume as a single power?"

Highlight that because the volume of the cube is very large, students can express the volume using exponents or repeated multiplication to help make calculations more efficient. Some possible ways to write the volume are $(10{,}000)^3$, $10^4 \cdot 10^4 \cdot 10^4$, $(10^4)^3$.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Ask students how to find the volume of a cube. As students share, display a variety of their strategies. Highlight strategies that use repeated multiplication, as well as those that use exponents.

Extension: *Math Enrichment*

Have students complete the following problem:

The volume of a giant cube is 1,000,000,000 km^3. Write an expression for the volume using repeated multiplication and write an expression using exponents. Sample response:
Repeated multiplication: $1000 \cdot 1000 \cdot 1000$
Exponents: 1000^3

Power-up

To power up students' ability to identify equivalent powers, have students complete:

Expand and evaluate each expression. The first row has been completed for you.

Power	Expand	Evaluate
4^3	$4 \cdot 4 \cdot 4$	64
5^4	$5 \cdot 5 \cdot 5 \cdot 5$	625
2^5	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	32

Use: Before Activity 1

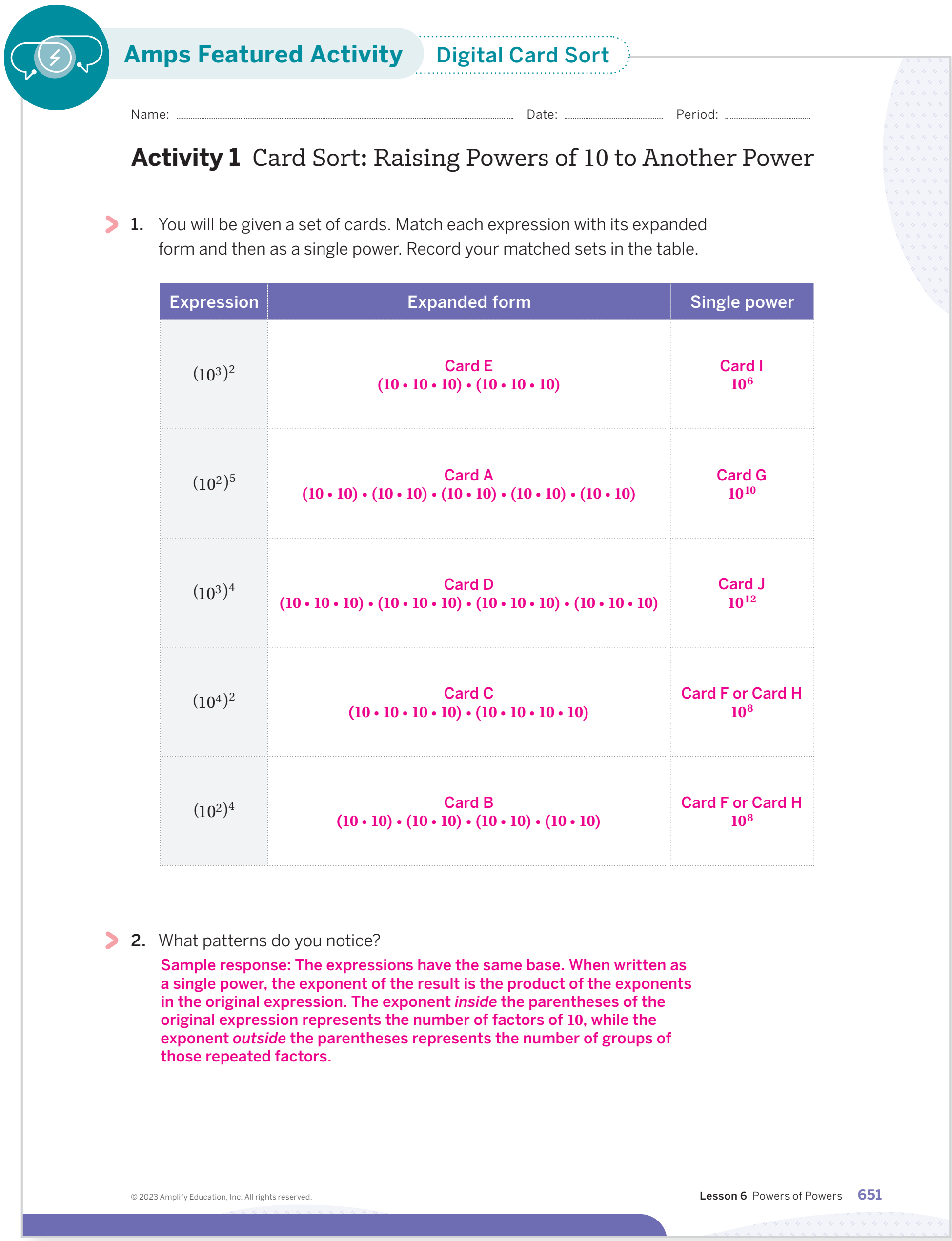
Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

Pairs | 15 min

Activity 1 Card Sort: Raising Powers of 10 to Another Power

Students match cards containing powers of 10 raised to other powers, written in different forms to discover a pattern among the exponents.

Amps Featured Activity Digital Card Sort



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Activity 1 Card Sort: Raising Powers of 10 to Another Power

1. You will be given a set of cards. Match each expression with its expanded form and then as a single power. Record your matched sets in the table.

Expression	Expanded form	Single power
$(10^3)^2$	Card E $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$	Card I 10^6
$(10^2)^5$	Card A $(10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10)$	Card G 10^{10}
$(10^3)^4$	Card D $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$	Card J 10^{12}
$(10^4)^2$	Card C $(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$	Card F or Card H 10^8
$(10^2)^4$	Card B $(10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10)$	Card F or Card H 10^8

2. What patterns do you notice?

   Sample response: The expressions have the same base. When written as a single power, the exponent of the result is the product of the exponents in the original expression. The exponent *inside* the parentheses of the original expression represents the number of factors of 10, while the exponent *outside* the parentheses represents the number of groups of those repeated factors.

Lesson 6 Powers of Powers 651

1 Launch

Distribute one set of cards from the Activity 1 PDF to each pair of students. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by showing $(10^3)^2$ as $(10^3) \cdot (10^3)$ and then as $(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$.

Look for points of confusion:

- **Struggling to organize their work.** Use two different colors to show how each part in Problem 1 corresponds to its expanded form: $(10^3)^2 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) = 10^6$
- **Thinking that $(10^3)^2 = 10^5$.** Have students examine the expanded form and count the number of factors.

Look for productive strategies:

- Noticing a pattern where the base stays the same and the exponents are multiplied. Ask students to use the patterns to rewrite $(10^m)^n$ as an equivalent expression, such as $10^{\square}$.

3 Connect

Display student work showing correct responses.

Have students share any patterns they found. Record responses for all to see.

Highlight that when raising a power of 10 to another power, students can keep the base and multiply the exponents.

Ask, “Why are Cards B and C both expressed as 10^8?”

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Anchor Chart PDF, *Exponent Rules* for them to reference during this activity. Allow them to keep this anchor chart throughout the rest of the unit.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can drag points to manipulate an expression and make observations in patterns between expressions written in expanded form and as a single power.

Accessibility: *Vary Demands to Optimize Challenge*

Consider using one of these alternative approaches to this activity.

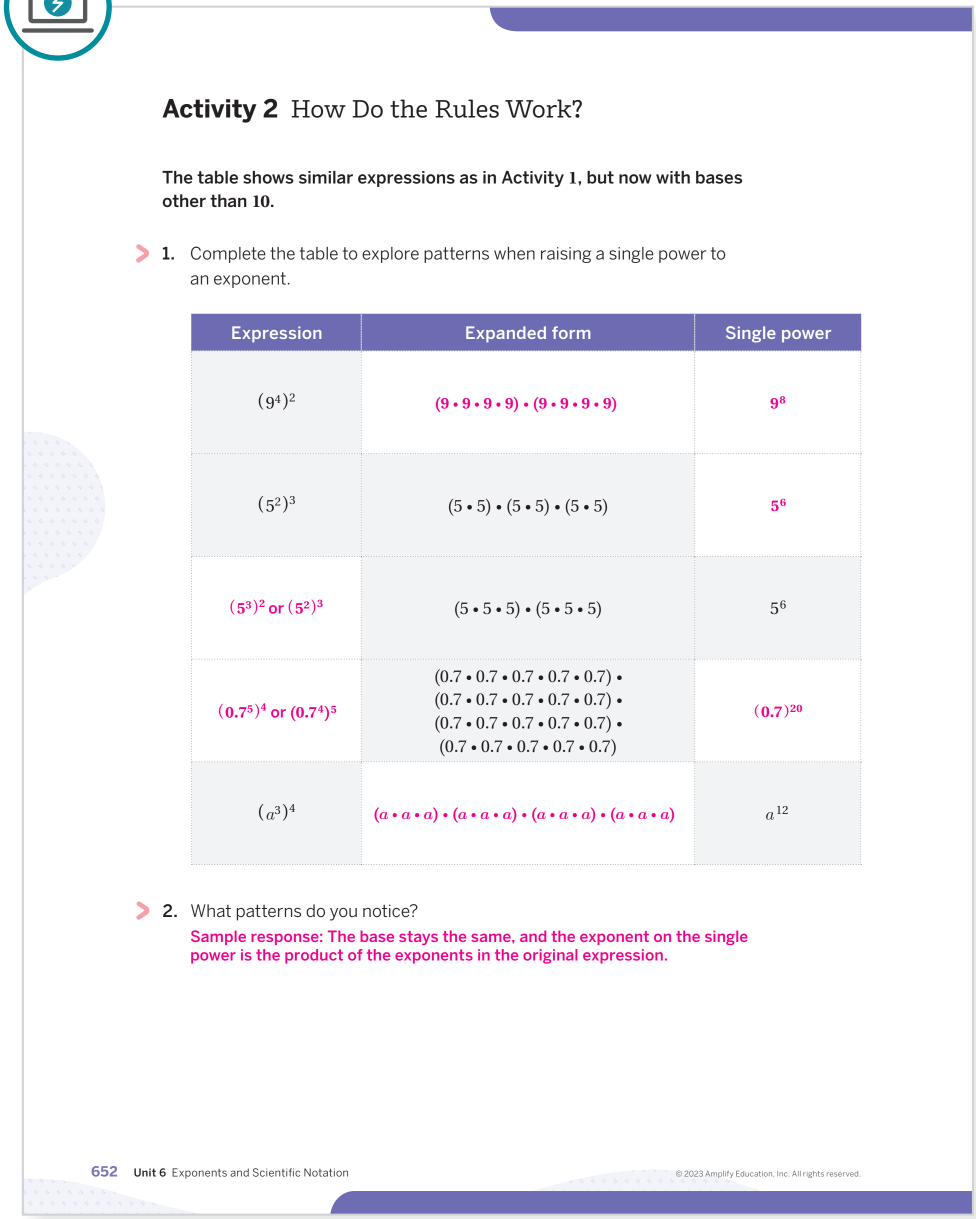
- Assign students the first three expressions and Cards A, D, E, G, I, and J.
- Allow students to choose three of the five expressions to match.

Then have students complete Problem 2.

Pairs | 8 min

Activity 2 How do the Rules Work?

Students continue exploring patterns of powers raised to powers to understand the pattern they saw in Activity 1 applies to powers with bases other than 10.



Activity 2 How Do the Rules Work?

The table shows similar expressions as in Activity 1, but now with bases other than 10.

1. Complete the table to explore patterns when raising a single power to an exponent.

Expression	Expanded form	Single power
$(9^4)^2$	$(9 \bullet 9 \bullet 9 \bullet 9) \bullet (9 \bullet 9 \bullet 9 \bullet 9)$	9^8
$(5^2)^3$	$(5 \bullet 5) \bullet (5 \bullet 5) \bullet (5 \bullet 5)$	5^6
$(5^3)^2$ or $(5^2)^3$	$(5 \bullet 5 \bullet 5) \bullet (5 \bullet 5 \bullet 5)$	5^6
$(0.7^5)^4$ or $(0.7^4)^5$	$(0.7 \bullet 0.7 \bullet 0.7 \bullet 0.7 \bullet 0.7) \bullet$ $(0.7 \bullet 0.7 \bullet 0.7 \bullet 0.7 \bullet 0.7) \bullet$ $(0.7 \bullet 0.7 \bullet 0.7 \bullet 0.7 \bullet 0.7) \bullet$ $(0.7 \bullet 0.7 \bullet 0.7 \bullet 0.7 \bullet 0.7)$	$(0.7)^{20}$
$(a^3)^4$	$(a \bullet a \bullet a) \bullet (a \bullet a \bullet a) \bullet (a \bullet a \bullet a) \bullet (a \bullet a \bullet a)$	a^{12}

2. What patterns do you notice?
Sample response: The base stays the same, and the exponent on the single power is the product of the exponents in the original expression.

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1 Launch

Have students use the patterns from Activity 1 to complete the table.

2 Monitor

Help students get started by asking them to rewrite the expression $(9^4)^2$ as $(9 \bullet 9 \bullet 9 \bullet 9)^2$.

Look for points of confusion:

- **Not knowing how to write the expression in expanded form.** Have students refer to their responses from Activity 1.

Look for productive strategies:

- Writing $(5^2)^3$ as $5^2 \bullet 5^2 \bullet 5^2$.
- Noticing the patterns from Activity 1 are the same even when the base is different than 10. Ask students to use the patterns to rewrite $(a^m)^n$ as an equivalent expression, such as $a^{\square}$.

3 Connect

Have students share any patterns they found. Record responses for all to see.

Ask, "How can you write $(x^{10})^5$ with a single power without writing it in expanded form? What rule describes the patterns you found?"

Define the exponent rule $(a^m)^n = a^{m \bullet n}$, where $a \neq 0$. This means that when raising a single power to another power, keep the same base and multiply the exponents.

Highlight that when raising a single power to another power, students can keep the base and multiply the exponents. This is true for any base, including fractions and decimals.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts to support students' use of structure. For example, give students time to complete the first row and then ask them to share with the class how they made sense of the expanded form and single power expressions. As students complete each row of the table, highlighting sense-making strategies and connections made between each of the three forms of the expressions.

Extension: *Math Enrichment*

Have students rewrite the expression $((5^3)^4)^2$ as a single power.
Sample response: 5^{24}

Math Language Development

MLR3: Critique, Correct, Clarify

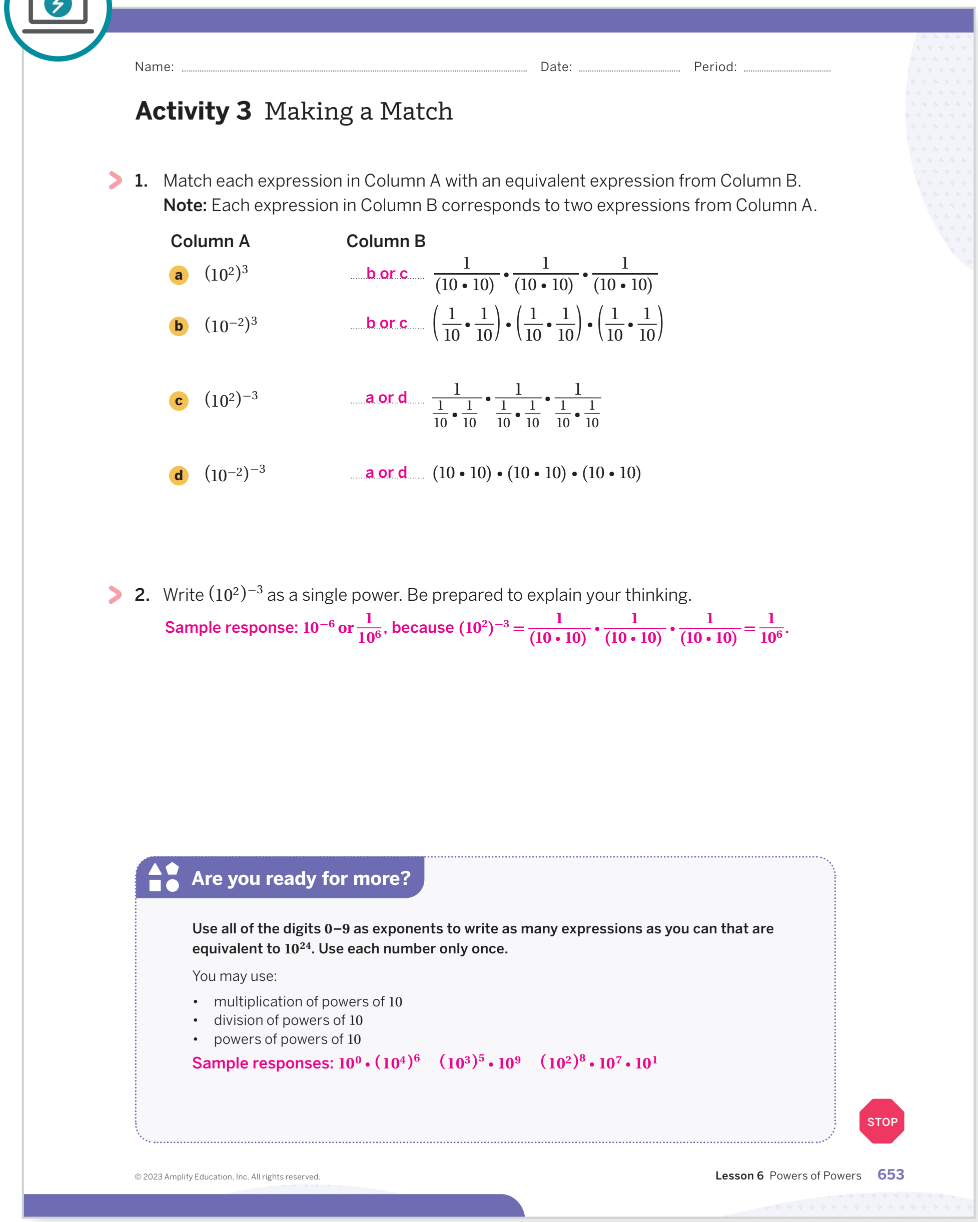
After students complete Problem 2, display an incorrect equation, such as $(5^2)^3 = 5^{(2+3)}$. Ask:

- ***Critique:*** "Is this equation true or false? Explain your thinking."
- ***Correct:*** "Write a corrected equation that is now true."
- ***Clarify:*** "How can you convince someone that your equation is true? What mathematical language or reasoning can you use?"

Pairs | 10 min

Activity 3 Making a Match

Students match equivalent expressions to understand the exponent rule they discovered in Activities 1 and 2 still applies when expressions contain negative exponents.



Name: ________ Date: ________ Period: ________

Activity 3 Making a Match

1. Match each expression in Column A with an equivalent expression from Column B.
Note: Each expression in Column B corresponds to two expressions from Column A.

Column A	Column B	
a $(10^2)^3$	b or c	$\frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)}$
b $(10^{-2})^3$	b or c	$\left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right)$
c $(10^2)^{-3}$	a or d	$\frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}}$
d $(10^{-2})^{-3}$	a or d	$(10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10)$

2. Write $(10^2)^{-3}$ as a single power. Be prepared to explain your thinking.
Sample response: 10^{-6} or $\frac{1}{10^6}$, because $(10^2)^{-3} = \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} = \frac{1}{10^6}$.

Are you ready for more?

Use all of the digits $0–9$ as exponents to write as many expressions as you can that are equivalent to 10^{24}. Use each number only once.

You may use:

- multiplication of powers of 10
- division of powers of 10
- powers of powers of 10

Sample responses: $10^0 \cdot (10^4)^6$ $(10^3)^5 \cdot 10^9$ $(10^2)^8 \cdot 10^7 \cdot 10^1$

STOP

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by reminding them that 10^{-2} is equivalent to $\frac{1}{10} \cdot \frac{1}{10}$.

Look for productive strategies:

- Recognizing negative exponents represent repeated multiplication by the fraction $\frac{1}{10}$.
- Using exponent rules to write the expression as a single power, and then finding the match.
- Recognizing that $(10^{-2})^3$ and $(10^2)^{-3}$ are equivalent expression of 10^{-6}.

3 Connect

Have students share which expressions they matched. For each problem, use the ***Poll the Class*** routine to see which expression they chose.

Ask, "How did you look for and make use of structure when matching equivalent expressions?"

Highlight that the exponent rule for raising a single power to another power can be applied to expressions containing both positive and negative exponents.

Differentiated Support

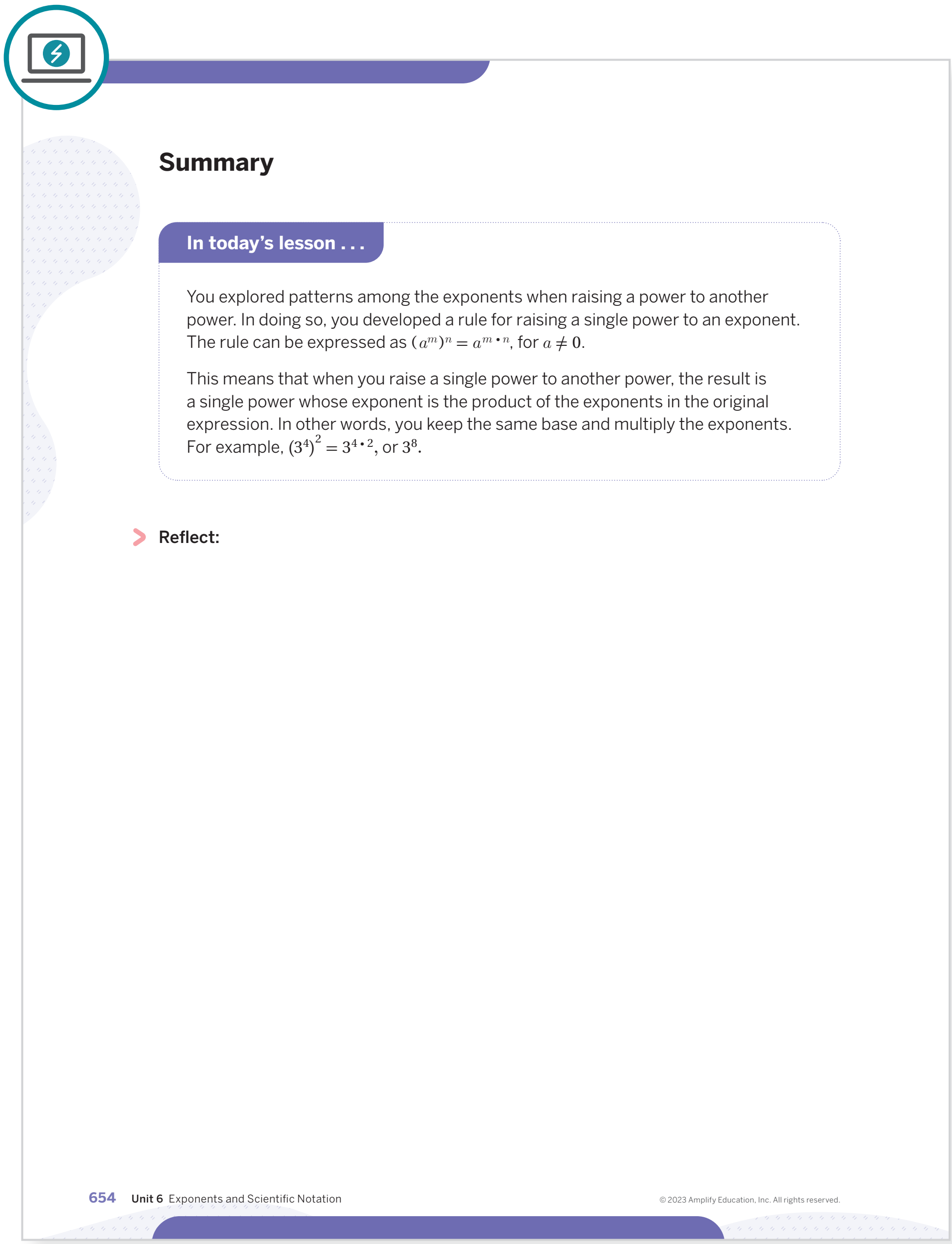
Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing two parts from Problem 1 before moving on to complete Problem 2. Consider allowing them to choose which parts to complete in Problem 1. By doing so, they may have greater engagement and ownership in the activity.

Whole Class | 5 min

Summary

Review and synthesize how a single power raised to another power can be written as a single power.



Summary

In today's lesson . . .

You explored patterns among the exponents when raising a power to another power. In doing so, you developed a rule for raising a single power to an exponent. The rule can be expressed as $(a^m)^n = a^{m \cdot n}$, for $a \neq 0$.

This means that when you raise a single power to another power, the result is a single power whose exponent is the product of the exponents in the original expression. In other words, you keep the same base and multiply the exponents. For example, $(3^4)^2 = 3^{4 \cdot 2}$, or 3^8.

Reflect:

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Synthesize

Ask:

- "How can you write $(1.5 \cdot 1.5) \cdot (1.5 \cdot 1.5) \cdot (1.5 \cdot 1.5)$. as a single power raised to another power?" $(1.5^2)^3$
- "How can you write $(10^{-4})^3$ as a single power?" 10^{-12}
- "In general, why do you multiply the exponents when you raise a power to another power? Give an example to show your thinking." Sample response: The exponent inside the parentheses can be thought of as a single quantity of repeated factors. The exponent outside the parentheses indicates the number of groups of repeated factors. For example, the expression $(2^3)^5$ represents 5 repeated factors of the expression 2^3. The expression 2^3 represents 3 repeated factors of 2. So, there are 5 groups, with 3 repeated factors of 2 in each group. In order words, there are 15 repeated factors of 2, or 2^{15}.

Display the Anchor Chart PDF, *Exponent Rules*. Uncover the Powers of power rule. **Note:** The other rules will be uncovered throughout the unit.

Highlight that the exponent rule $(a^{mn}) = a^{m \cdot n}$ for $a \neq 0$ shows the rule for raising a single power to another power. This means that when raising a single power to another power, keep the same base and multiply the exponents.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

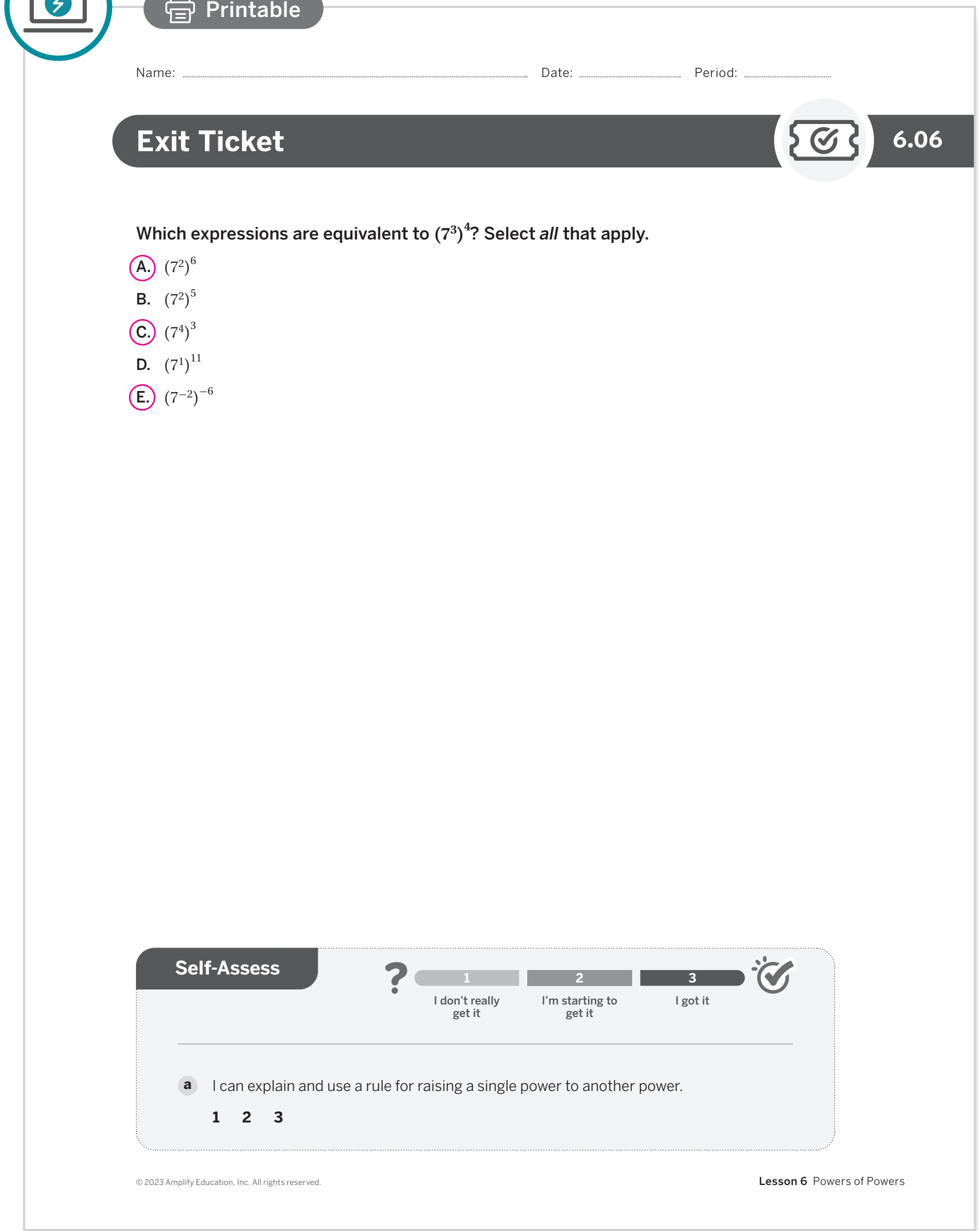
- "What happens when a power is raised to another power?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying equivalent expressions for an expression involving a single power raised to another power.

Printable



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Exit Ticket 6.06

Which expressions are equivalent to $(7^3)^4$? Select *all* that apply.

[✓ A.] $(7^2)^6$

B. $(7^2)^5$

[✓ C.] $(7^4)^3$

D. $(7^1)^{11}$

[✓ E.] $(7^{-2})^{-6}$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can explain and use a rule for raising a single power to another power.

1 2 3

Lesson 6 Powers of Powers

Success looks like . . .

- **Language Goal:** Generalizing a process for finding a single power raised to an exponent, and justifying that $(a^m)^n = a^{m \cdot n}$, where $a \neq 0$. **(Speaking and Listening, Reading and Writing)**
  - » Choosing all equivalent expressions for $(7^3)^4$.

Suggested next steps

If students do not select A, C, or E, consider:

- Having students write each expression in expanded form.
- Reviewing Activity 2.

Professional Learning

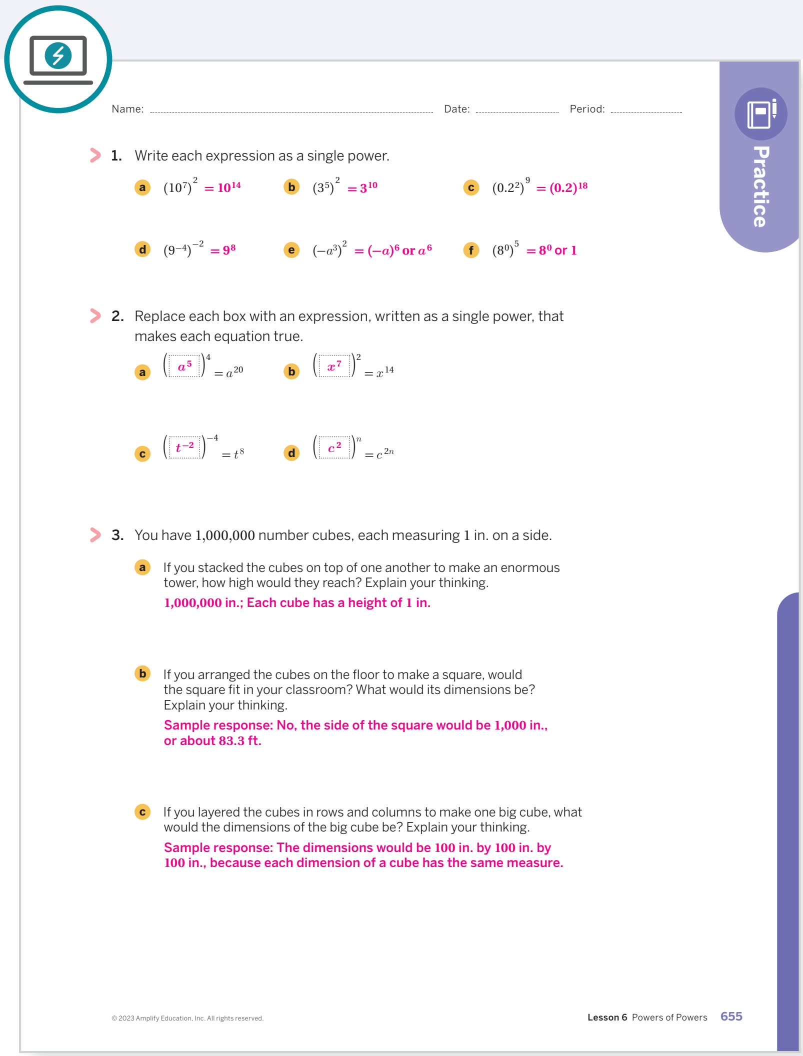
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students gotten better at writing different forms of expressions involving exponents?
- Which students' ideas were you able to highlight during Activity 1? What might you change for the next time you teach this lesson?

Practice

Independent



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Practice

1. Write each expression as a single power.

a $(10^7)^2$ **$= 10^{14}$**

b $(3^5)^2$ **$= 3^{10}$**

c $(0.2^2)^9$ **$= (0.2)^{18}$**

d $(9^{-4})^{-2}$ **$= 9^8$**

e $(-a^3)^2$ **$= (-a)^6$ or a^6**

f $(8^0)^5$ **$= 8^0$ or 1**

2. Replace each box with an expression, written as a single power, that makes each equation true.

a $\left(\boxed{a^5}\right)^4 = a^{20}$

b $\left(\boxed{x^7}\right)^2 = x^{14}$

c $\left(\boxed{t^{-2}}\right)^{-4} = t^8$

d $\left(\boxed{c^2}\right)^n = c^{2n}$

3. You have 1,000,000 number cubes, each measuring 1 in. on a side.

a If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your thinking.

1,000,000 in.; Each cube has a height of 1 in.

b If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your thinking.

Sample response: No, the side of the square would be 1,000 in., or about 83.3 ft.

c If you layered the cubes in rows and columns to make one big cube, what would the dimensions of the big cube be? Explain your thinking.

Sample response: The dimensions would be 100 in. by 100 in. by 100 in., because each dimension of a cube has the same measure.

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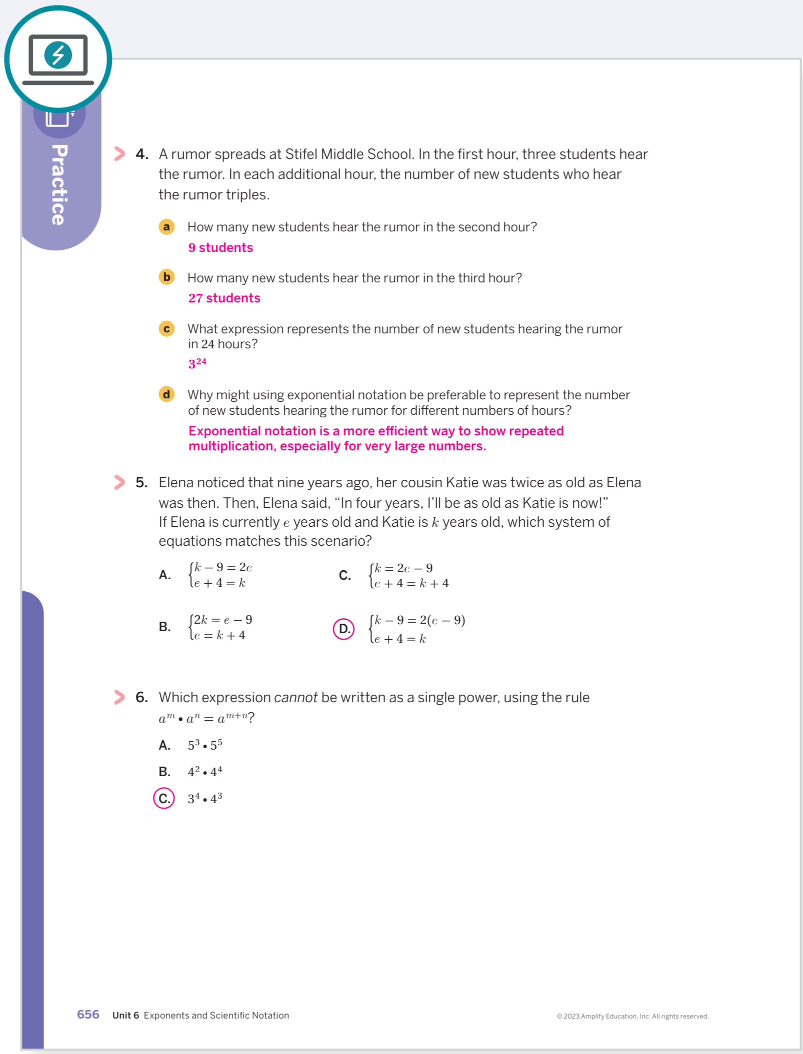
Lesson 6 Powers of Powers 655



Practice

4. A rumor spreads at Stifel Middle School. In the first hour, three students hear the rumor. In each additional hour, the number of new students who hear the rumor triples.

a How many new students hear the rumor in the second hour?

9 students

b How many new students hear the rumor in the third hour?

27 students

c What expression represents the number of new students hearing the rumor in 24 hours?

3^{24}

d Why might using exponential notation be preferable to represent the number of new students hearing the rumor for different numbers of hours?

Exponential notation is a more efficient way to show repeated multiplication, especially for very large numbers.

5. Elena noticed that nine years ago, her cousin Katie was twice as old as Elena was then. Then, Elena said, "In four years, I'll be as old as Katie is now!" If Elena is currently e years old and Katie is k years old, which system of equations matches this scenario?

A. $\begin{cases} k - 9 = 2e \\ e + 4 = k \end{cases}$

B. $\begin{cases} 2k = e - 9 \\ e = k + 4 \end{cases}$

C. $\begin{cases} k = 2e - 9 \\ e + 4 = k + 4 \end{cases}$

[✓ D.] $\begin{cases} k - 9 = 2(e - 9) \\ e + 4 = k \end{cases}$

6. Which expression *cannot* be written as a single power, using the rule $a^m \bullet a^n = a^{m+n}$?

A. $5^3 \bullet 5^5$

B. $4^2 \bullet 4^4$

[✓ C.] $3^4 \bullet 4^3$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 1	3
Spiral	4	Unit 6 Lesson 1	2
	5	Unit 4 Lesson 16	3
Formative ⚡	6	Unit 6 Lesson 7	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice.**

UNIT 6 | **LESSON 7**

Different Bases, Same Exponent

Let's multiply expressions with different bases, yet the same exponent.

Focus

Goal

1. **Language Goal:** Generalize a process for multiplying expressions with different bases, yet the same exponent, and justify that $a^m \cdot b^m = (a \cdot b)^m$ and $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$, where $a \neq 0$ and $b \neq 0$. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students make use of repeated reasoning to discover the exponent rule $a^m \cdot b^m = (a \cdot b)^m$ and $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$, where $a \neq 0$ and $b \neq 0$. Students reason abstractly and quantitatively when applying exponent rules, pausing to consider the meaning of quantities, not just how to compute them.

Previously

In Lesson 3, students applied exponent rules when multiplying powers with the same base.

Coming Soon

In Lesson 8, students will reflect on their conceptual understanding and procedural fluency with the exponent rules they have learned thus far in this unit.

Rigor

- Students build **conceptual understanding** of multiplying powers that have different bases, but the same exponent.
- Students strengthen their **fluency** in multiplying powers that have different bases, but the same exponent.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Exponent Rules*

Math Language Development

Review words

- *base*
- *equivalent expressions*
- *expanded form*
- *exponent*
- *power*

Amps | **Featured Activity**

Activity 1
Manipulating an Expression

Students drag a point to manipulate an expression and make observations in patterns between expressions written in expanded form and as a single power.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to regulate their emotions as working with powers gets more difficult by having different bases. The quantitative reasoning might agitate them if they cannot easily follow the process. Encourage pairs to seek ways to encourage each other. Have them strive to say or do something that positively affects their partner at least twice during the activity.

Modifications to Pacing

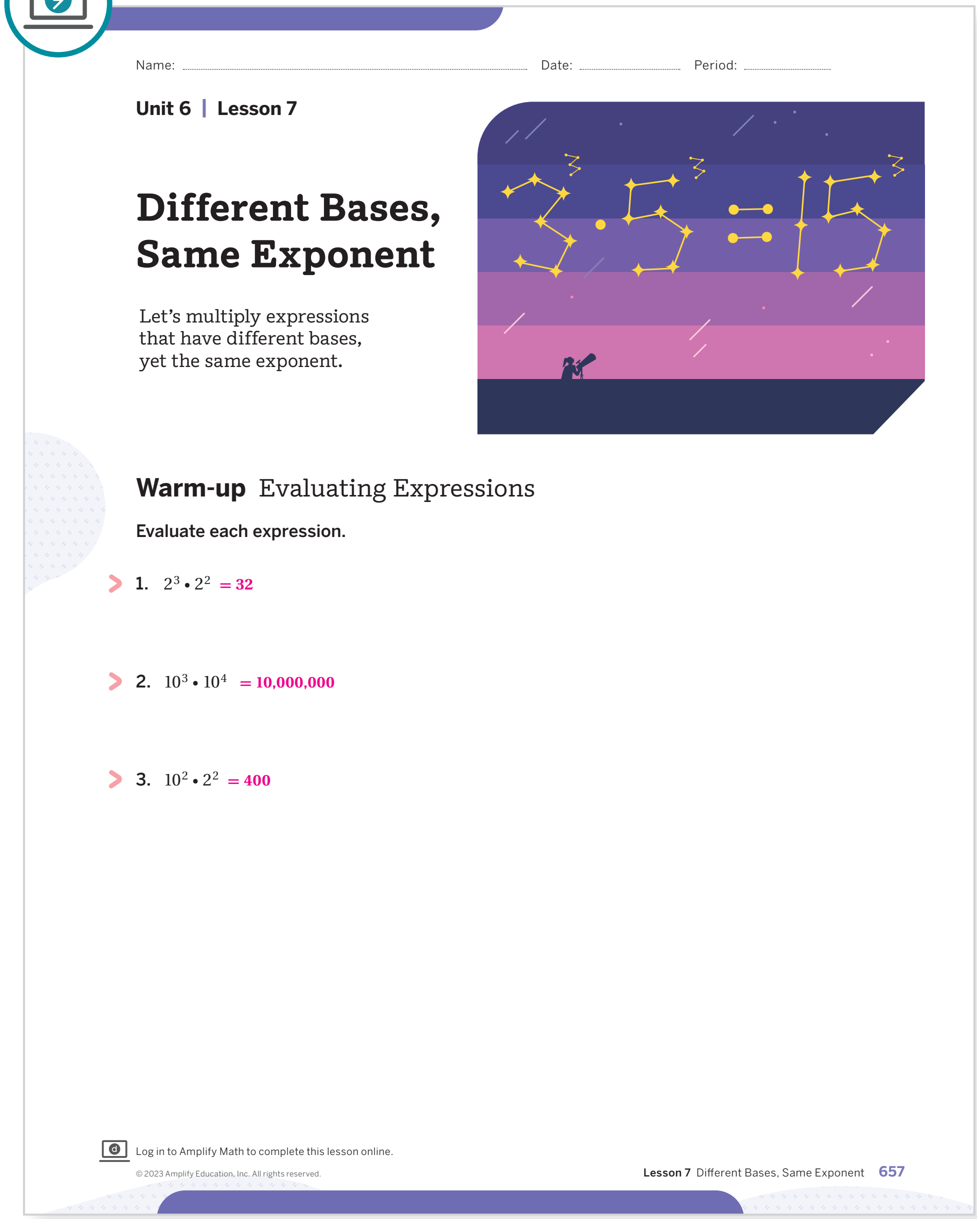
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, the last two rows may be omitted.
- In **Activity 2**, Problems 6 and 7 may be omitted.

Independent | 5 min

Warm-up Evaluating Expressions

Students evaluate expressions as an introduction to multiplying expressions with different bases, yet the same exponent.



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Unit 6 | Lesson 7

Different Bases, Same Exponent

Let's multiply expressions
that have different bases,
yet the same exponent.

Warm-up Evaluating Expressions

Evaluate each expression.

1. $2^3 \cdot 2^2$ = 32

2. $10^3 \cdot 10^4$ = 10,000,000

3. $10^2 \cdot 2^2$ = 400

Log in to Amplify Math to complete this lesson online.

Lesson 7 Different Bases, Same Exponent 657

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by giving them options on how to evaluate the expressions, such as writing the expression as a single power or using the order of operations.

Look for points of confusion:

- **Not remembering the exponent rules.** Tell students to write each expression in expanded form.

Look for productive strategies:

- Noticing the bases in Problem 3 are different, yet the exponents are the same.

3 Connect

Have students share the different methods they used to evaluate each expression.

Highlight that Problem 3 is different from Problems 1 and 2 because the bases are different and the exponents are the same.

Ask, "Can you write 400 from Problem 3 as a single power? What connections do you see between the left side and right side of the equal sign?" Sample response: 400 as a single power is 20^2. The exponent is the same as the exponents of each factor in Problem 3. The base is the product of the bases 10 and 2 in Problem 3.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Anchor Chart PDF, *Exponent Rules* for them to reference during this activity. Allow them to keep this anchor chart throughout the rest of the unit.

Accessibility: *Guide Processing and Visualization*

Encourage students to use exponent rules, yet ask them to first make sure the bases are the same. Ask, "Look at Problem 3. What do you notice? What strategies can you use to evaluate this expression?"

Power-up

To power up students' ability to identify the structure of expressions involving powers, have students complete:

Recall that $a^m \cdot a^n = a^{m+n}$. For example, $2^3 \cdot 2^4 = 2^{3+4} = 2^7$. Determine which expressions can be simplified using the given rule. Select *all* that apply.

A. $5^3 \cdot 3^5$

[✓ B.] $5^2 \cdot 5^4$

[✓ C.] $2^5 \cdot 2^4$

D. $(2^4)^3$

Use: Before the Warm-up

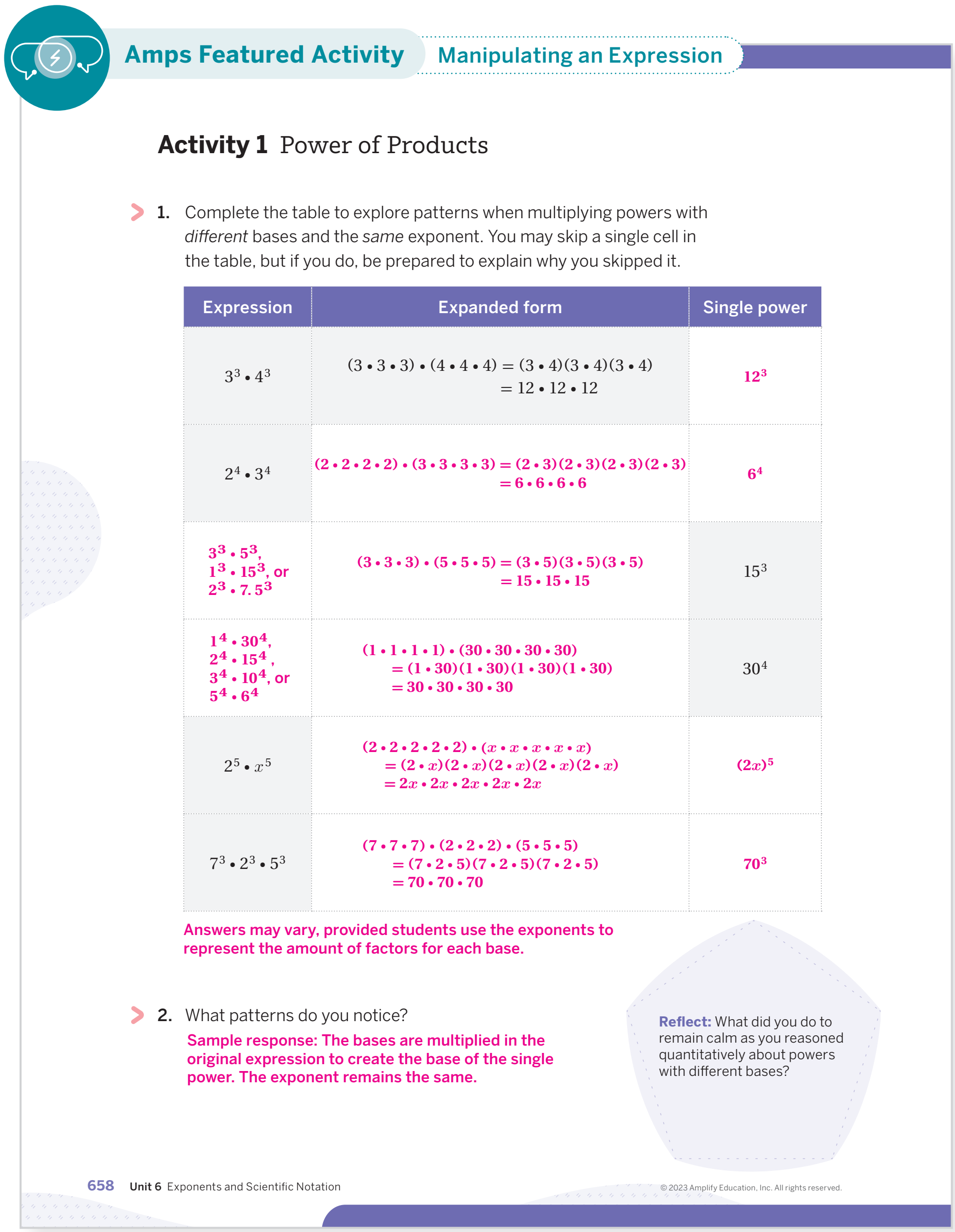
Informed by: Performance on Lesson 6, Practice Problem 6

Pairs | 15 min

Activity 1 Power of Products

Students study expressions that have different bases, yet the same exponent, to develop an exponent rule that can be used to write the expression as a single power.

Amps Featured Activity Manipulating an Expression



Activity 1 Power of Products

1. Complete the table to explore patterns when multiplying powers with *different* bases and the *same* exponent. You may skip a single cell in the table, but if you do, be prepared to explain why you skipped it.

Expression	Expanded form	Single power
$3^3 \cdot 4^3$	$(3 \cdot 3 \cdot 3) \cdot (4 \cdot 4 \cdot 4) = (3 \cdot 4)(3 \cdot 4)(3 \cdot 4)$ $= 12 \cdot 12 \cdot 12$	12^3
$2^4 \cdot 3^4$	$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3) = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3)(2 \cdot 3)$ $= 6 \cdot 6 \cdot 6 \cdot 6$	6^4
$3^3 \cdot 5^3$, $1^3 \cdot 15^3$, or $2^3 \cdot 7.5^3$	$(3 \cdot 3 \cdot 3) \cdot (5 \cdot 5 \cdot 5) = (3 \cdot 5)(3 \cdot 5)(3 \cdot 5)$ $= 15 \cdot 15 \cdot 15$	15^3
$1^4 \cdot 30^4$, $2^4 \cdot 15^4$, $3^4 \cdot 10^4$, or $5^4 \cdot 6^4$	$(1 \cdot 1 \cdot 1 \cdot 1) \cdot (30 \cdot 30 \cdot 30 \cdot 30)$ $= (1 \cdot 30)(1 \cdot 30)(1 \cdot 30)(1 \cdot 30)$ $= 30 \cdot 30 \cdot 30 \cdot 30$	30^4
$2^5 \cdot x^5$	$(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (x \cdot x \cdot x \cdot x \cdot x)$ $= (2 \cdot x)(2 \cdot x)(2 \cdot x)(2 \cdot x)(2 \cdot x)$ $= 2x \cdot 2x \cdot 2x \cdot 2x \cdot 2x$	$(2x)^5$
$7^3 \cdot 2^3 \cdot 5^3$	$(7 \cdot 7 \cdot 7) \cdot (2 \cdot 2 \cdot 2) \cdot (5 \cdot 5 \cdot 5)$ $= (7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)$ $= 70 \cdot 70 \cdot 70$	70^3

Answers may vary, provided students use the exponents to represent the amount of factors for each base.

2. What patterns do you notice?

Sample response: The bases are multiplied in the original expression to create the base of the single power. The exponent remains the same.

Reflect: What did you do to remain calm as you reasoned quantitatively about powers with different bases?

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by studying the first row in the table in Problem 1 and verbally describing each step to write the expression in expanded form.

Look for points of confusion:

- **Having trouble seeing the patterns.** Reorganize the expanded form to show the factors vertically, and emphasize that the exponent shows the same number of factors for each power.
- **Thinking the exponents will simply add or multiply.** Tell them to look at the expanded form before developing a rule.

Look for productive strategies:

- Noticing each power has the same number of factors.
- Noticing the bases are multiplied and the exponent stays the same.

3 Connect

Ask, "What expressions did you write for third and fourth rows of the table? Can there be more than one answer?"

Have students share any patterns they found. Record the answers for all to see.

Define the exponent rule $a^m \cdot b^m = (a \cdot b)^m$, where $a \neq 0$ and $b \neq 0$. This shows that when multiplying powers with different bases, yet the same exponent, multiply the bases and keep the exponent.

Highlight that the rule $a^m \cdot b^m = (a \cdot b)^m$ is true for *any* base except 0, including fractions and decimals.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can drag points to manipulate an expression and make observations in patterns between expressions written in expanded form and as a single power.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them complete four of the six rows of the table. Consider allowing them to choose which rows to complete. Offering them the power of choice can result in greater engagement in the task.

Math Language Development

MLR7: Compare and Connect

During the Connect, press students to represent the patterns they notice as a single rule. Consider having the class collectively write the exponent rule in their own words before you define and display the rule. Consider using the following sentence frame to help organize their thinking.

When multiplying powers with different _____, yet the same _____, the _____ stays the same, and the _____ are multiplied.

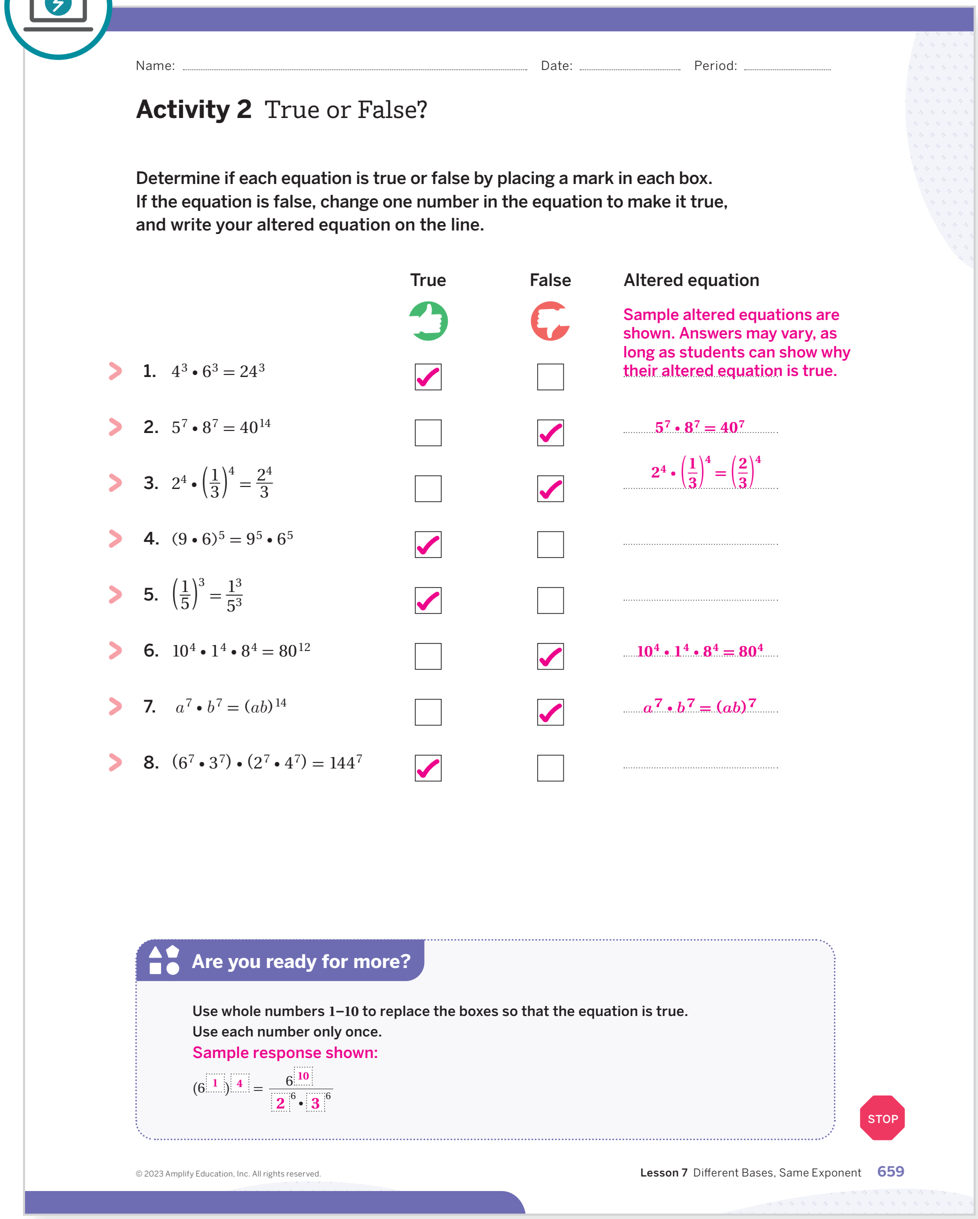
English Learners

Include an annotated example, illustrating how the *bases are different* and the *exponents are the same*.

Pairs | 15 min

Activity 2 True or False?

Students recognize that bases are multiplied and exponents stay the same only when expressions are written in the form $a^m \cdot b^m$.



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Activity 2 True or False?

Determine if each equation is true or false by placing a mark in each box. If the equation is false, change one number in the equation to make it true, and write your altered equation on the line.

	True	False	Altered equation
			Sample altered equations are shown. Answers may vary, as long as students can show why their altered equation is true.
1. $4^3 \cdot 6^3 = 24^3$	☑	☐	
2. $5^7 \cdot 8^7 = 40^{14}$	☐	☑	$5^7 \cdot 8^7 = 40^7$
3. $2^4 \cdot \left(\frac{1}{3}\right)^4 = \frac{2^4}{3}$	☐	☑	$2^4 \cdot \left(\frac{1}{3}\right)^4 = \left(\frac{2}{3}\right)^4$
4. $(9 \cdot 6)^5 = 9^5 \cdot 6^5$	☑	☐	
5. $\left(\frac{1}{5}\right)^3 = \frac{1^3}{5^3}$	☑	☐	
6. $10^4 \cdot 1^4 \cdot 8^4 = 80^{12}$	☐	☑	$10^4 \cdot 1^4 \cdot 8^4 = 80^4$
7. $a^7 \cdot b^7 = (ab)^{14}$	☐	☑	$a^7 \cdot b^7 = (ab)^7$
8. $(6^7 \cdot 3^7) \cdot (2^7 \cdot 4^7) = 144^7$	☑	☐	

Are you ready for more?

Use whole numbers 1–10 to replace the boxes so that the equation is true. Use each number only once.

Sample response shown:

$$(6^{\boxed{1}})^{\boxed{4}} = \frac{6^{\boxed{10}}}{\boxed{2}^6 \cdot \boxed{3}^6}$$

STOP

Lesson 7 Different Bases, Same Exponent 659

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by writing $4^3 \cdot 6^3$ as $(4 \cdot 4 \cdot 4) \cdot (6 \cdot 6 \cdot 6)$.

Look for points of confusion:

- **Confusing exponent rules by thinking the equation in Problem 2 is true.** Have students write $5^7 \cdot 8^7$ in expanded form, and then ask them to count the number of factors of 40.
- **Being unsure about Problems 3 and 5.** Have students evaluate the expressions individually before determining whether they are equal.
- **Thinking the equation in Problem 4 is false.** Show students another way to write the expression as $(9 \cdot 6) \cdot (9 \cdot 6) \cdot (9 \cdot 6) \cdot (9 \cdot 6) \cdot (9 \cdot 6)$. Ask students to count the amount of factors of 9 and 6.
- **Being unsure how to approach Problem 8.** Tell students to write $(6^7 \cdot 3^7)$ as a single power, and then $(2^7 \cdot 4^7)$.

Look for productive strategies:

- Writing expressions in expanded form.
- Applying the exponent rules they have learned so far.

3 Connect

Have students share their responses by reading each problem and having students put their thumbs up to show they thought the equation was true, and put their thumbs down to show they thought the equation was false. Review any problems where the class disagrees.

Highlight that if $a^m \cdot \left(\frac{1}{b}\right)^m = \left(\frac{a}{b}\right)^m$ is true, then the rule is also true for division: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $a \neq 0$ and $b \neq 0$. Also highlight that even if students forget the rules, they can write expressions in expanded form to see whether expressions are equivalent.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the expressions that have different bases *and* the same exponent before beginning the activity.

Math Language Development

MLR3: Critique, Correct, Clarify

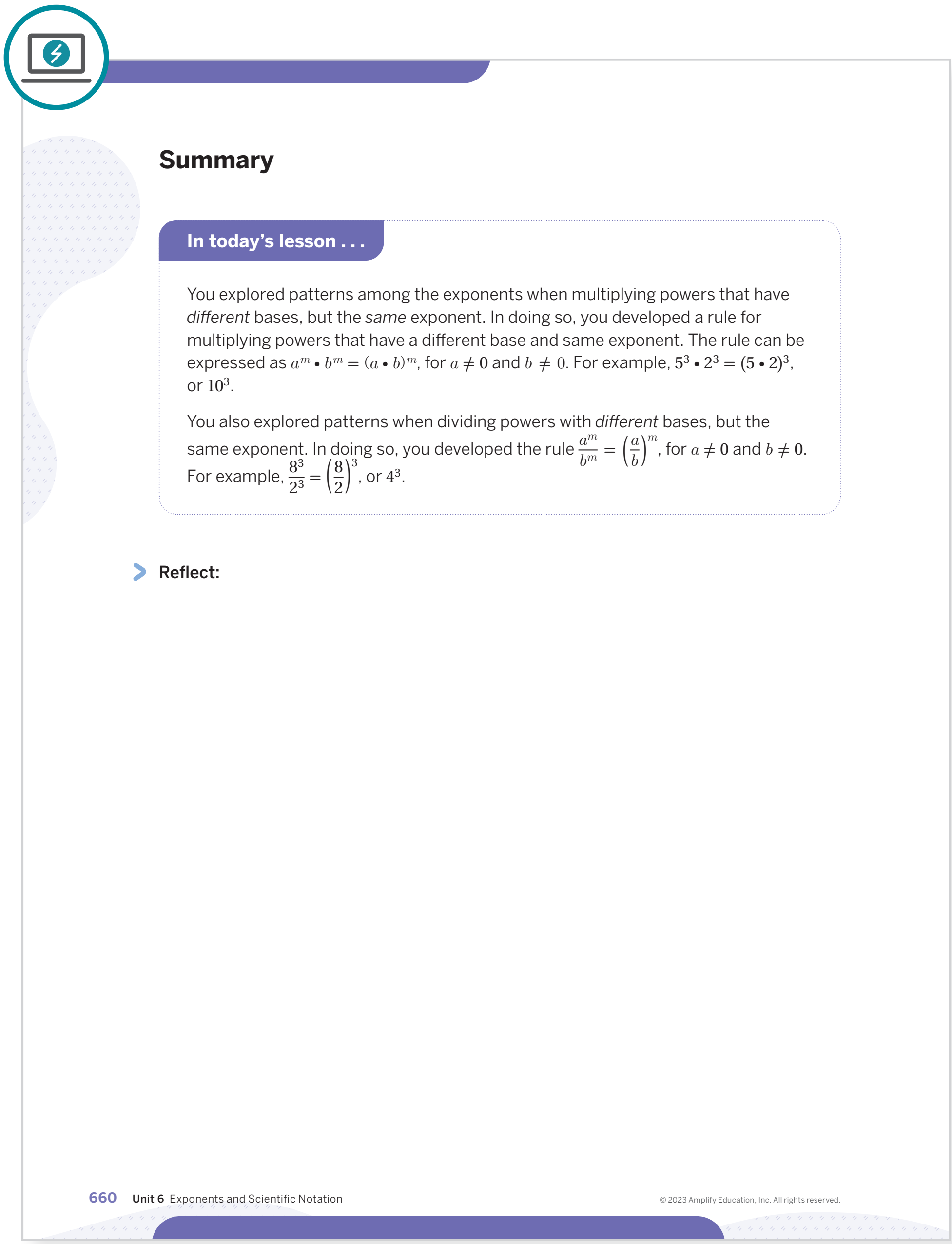
During the Connect, display an incorrect equation, such as, $(4^2) \cdot (6^3) = 24^5$. Tell students the equation is false. Ask:

- ***Critique:*** "What do you think the person who wrote this equation most likely misunderstands?"
- ***Correct:*** "Write a corrected equation."
- ***Clarify:*** "How would you convince the person who wrote this equation of their misunderstanding? What mathematical language or reasoning can you use?"

Whole Class | 5 min

Summary

Review and synthesize how exponent rules can be applied when multiplying powers with different bases, yet the same exponent.



Summary

In today's lesson . . .

You explored patterns among the exponents when multiplying powers that have *different* bases, but the *same* exponent. In doing so, you developed a rule for multiplying powers that have a different base and same exponent. The rule can be expressed as $a^m \bullet b^m = (a \bullet b)^m$, for $a \neq 0$ and $b \neq 0$. For example, $5^3 \bullet 2^3 = (5 \bullet 2)^3$, or 10^3.

You also explored patterns when dividing powers with *different* bases, but the same exponent. In doing so, you developed the rule $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$, for $a \neq 0$ and $b \neq 0$. For example, $\frac{8^3}{2^3} = \left(\frac{8}{2}\right)^3$, or 4^3.

Reflect:

660 Unit 6 Exponents and Scientific Notation

Synthesize

Ask:

- "Is it possible to write $4^5 \bullet 5^5$ as a single power? Explain your thinking." Sample response: Yes, the powers have different bases, but they have the same exponent. By writing the expressions in expanded form, there are 5 repeated factors of 4 and 5 repeated factors of 5. This means there are actually 5 repeated factors of 20. So, as a single power, the expression can be written as 20^5.
- "Is it possible to write $4^3 \bullet 5^5$ as a single power? Explain your thinking." No, this is a product of two powers. But the powers do not have the same base. They also do not satisfy the requirement of having different bases, yet the same exponent. Even by writing the expressions in expanded form, I cannot group or rearrange them so that there is a single power.
- "When is it possible to multiply bases when rewriting expressions as a single power?" Only when a single power is raised to another power, or when there is a product of two powers that have different bases, yet the same exponent.
- "When is it possible to divide bases when rewriting expressions as a single power?" When the base of the powers are the same.

Display the Anchor Chart PDF, *Exponent Rules*. Uncover the Product of powers rule and the Quotient of powers rule.

Highlight that the exponent rule $a^m \bullet b^m = (a \bullet b)^m$ for $a \neq 0$ and shows the rule for multiplying powers with different bases, yet the same exponent. This means that when multiplying powers with different bases, yet the same exponent, multiply the bases and keep the same exponent. This rule is true for *any* bases, which means $a^m \bullet \left(\frac{1}{b}\right)^m = \left(\frac{a}{b}\right)^m$ is also true. The last expression gives the rule for dividing powers with different bases and the same exponent. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$, where $a \neq 0$ and $b \neq 0$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

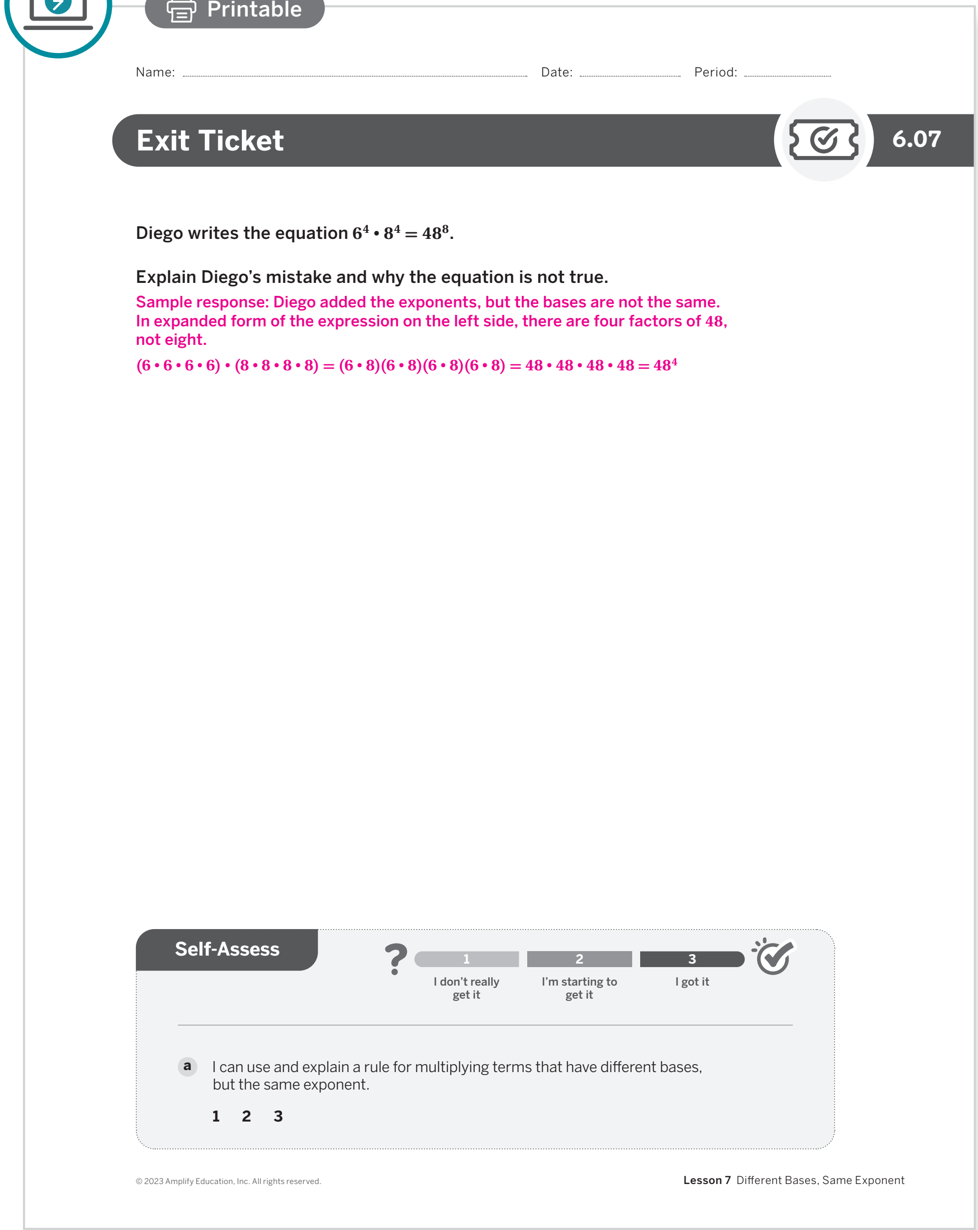
- "What happens when expressions containing exponents are multiplied or divided?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of multiplying two powers that have different bases, yet the same exponent, by critiquing the reasoning of others.

Printable



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Exit Ticket 6.07

Diego writes the equation $6^4 \cdot 8^4 = 48^8$.

Explain Diego's mistake and why the equation is not true.

Sample response: Diego added the exponents, but the bases are not the same. In expanded form of the expression on the left side, there are four factors of 48, not eight.

$(6 \cdot 6 \cdot 6 \cdot 6) \cdot (8 \cdot 8 \cdot 8 \cdot 8) = (6 \cdot 8)(6 \cdot 8)(6 \cdot 8)(6 \cdot 8) = 48 \cdot 48 \cdot 48 \cdot 48 = 48^4$

Self-Assess

? 1 I don't really get it · 2 I'm starting to get it · 3 I got it

a I can use and explain a rule for multiplying terms that have different bases, but the same exponent.

1 2 3

Success looks like . . .

- **Language Goal:** Generalizing a process for multiplying expressions with different bases, yet the same exponent, and justifying that $a^m \cdot b^m = (a \cdot b)^m$ and $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$, where $a \neq 0$ and $b \neq 0$. **(Speaking and Listening, Reading and Writing)**
  - » Correcting Diego's mistake in multiplying two numbers with different bases.

Suggested next steps

If students cannot identify Diego's mistake, consider:

- Having students first write each power in expanded form.
- Reviewing different ways to write an expression with powers from Activity 2.

Professional Learning

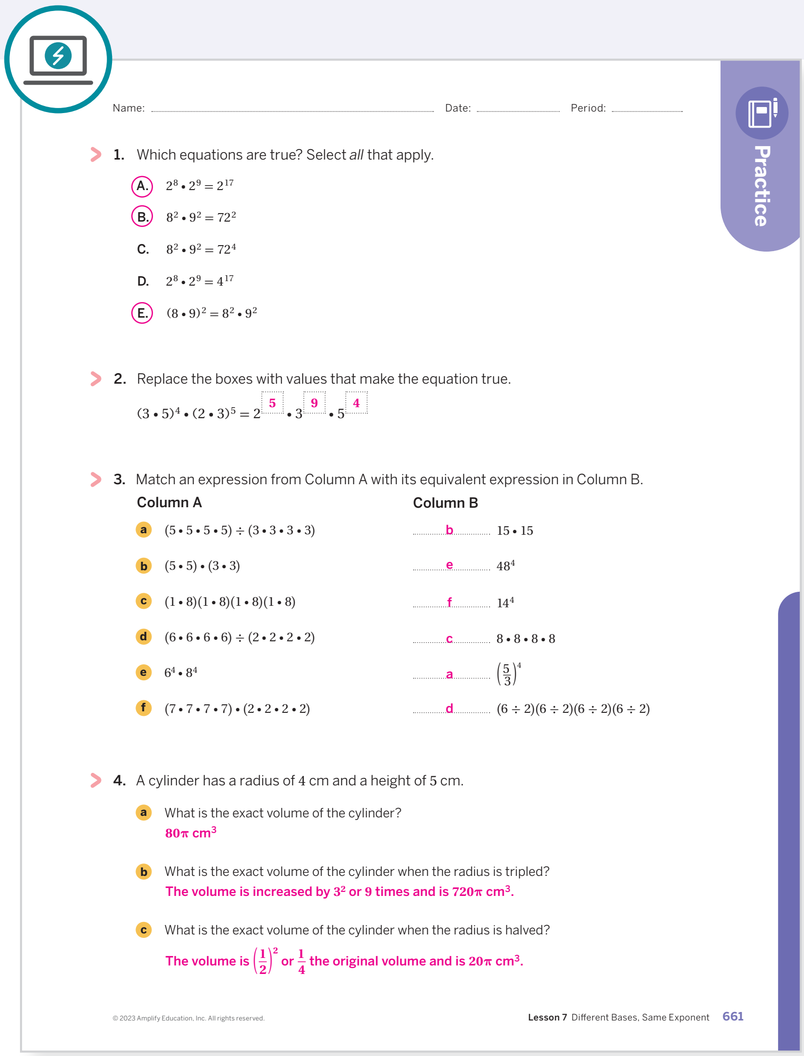
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- In this lesson, students multiplied powers that have different bases, but the same exponent. How did that build on the earlier work students did when they multiplied powers that have the same bases, but different exponents?

Practice

Independent



Name: Date: Period:

Practice

1. Which equations are true? Select *all* that apply.
   - [✓ A.] $2^8 \cdot 2^9 = 2^{17}$
   - [✓ B.] $8^2 \cdot 9^2 = 72^2$
   - C. $8^2 \cdot 9^2 = 72^4$
   - D. $2^8 \cdot 2^9 = 4^{17}$
   - [✓ E.] $(8 \cdot 9)^2 = 8^2 \cdot 9^2$

2. Replace the boxes with values that make the equation true.

   $(3 \cdot 5)^4 \cdot (2 \cdot 3)^5 = 2^{\boxed{5}} \cdot 3^{\boxed{9}} \cdot 5^{\boxed{4}}$

3. Match an expression from Column A with its equivalent expression in Column B.

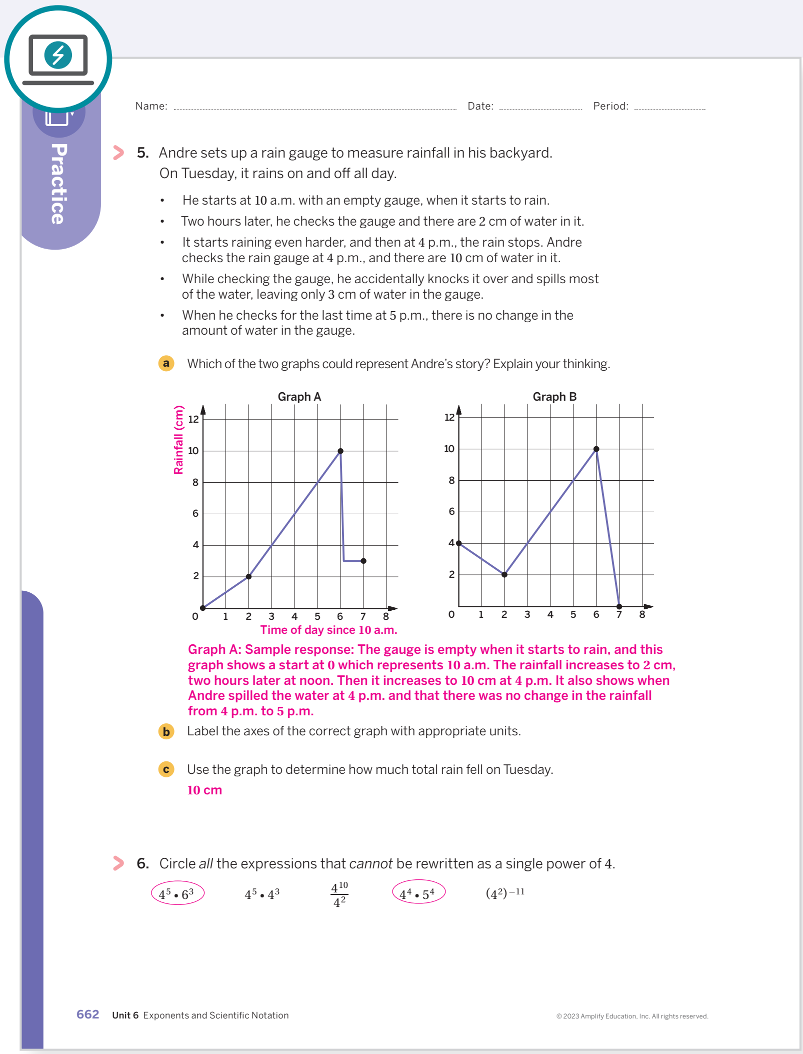
Column A		Column B
a $(5 \cdot 5 \cdot 5 \cdot 5) \div (3 \cdot 3 \cdot 3 \cdot 3)$	b	$15 \cdot 15$
b $(5 \cdot 5) \cdot (3 \cdot 3)$	e	48^4
c $(1 \cdot 8)(1 \cdot 8)(1 \cdot 8)(1 \cdot 8)$	f	14^4
d $(6 \cdot 6 \cdot 6 \cdot 6) \div (2 \cdot 2 \cdot 2 \cdot 2)$	c	$8 \cdot 8 \cdot 8 \cdot 8$
e $6^4 \cdot 8^4$	a	$\left(\frac{5}{3}\right)^4$
f $(7 \cdot 7 \cdot 7 \cdot 7) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$	d	$(6 \div 2)(6 \div 2)(6 \div 2)(6 \div 2)$

4. A cylinder has a radius of 4 cm and a height of 5 cm.
   - a What is the exact volume of the cylinder?

     80π cm^3
   - b What is the exact volume of the cylinder when the radius is tripled?

     The volume is increased by 3^2 or 9 times and is 720π cm^3.
   - c What is the exact volume of the cylinder when the radius is halved?

     The volume is $\left(\frac{1}{2}\right)^2$ or $\frac{1}{4}$ the original volume and is 20π cm^3.



Name: Date: Period:

Practice

5. Andre sets up a rain gauge to measure rainfall in his backyard. On Tuesday, it rains on and off all day.
   - He starts at 10 a.m. with an empty gauge, when it starts to rain.
   - Two hours later, he checks the gauge and there are 2 cm of water in it.
   - It starts raining even harder, and then at 4 p.m., the rain stops. Andre checks the rain gauge at 4 p.m., and there are 10 cm of water in it.
   - While checking the gauge, he accidentally knocks it over and spills most of the water, leaving only 3 cm of water in the gauge.
   - When he checks for the last time at 5 p.m., there is no change in the amount of water in the gauge.

   a Which of the two graphs could represent Andre's story? Explain your thinking.

   Graph A: Sample response: The gauge is empty when it starts to rain, and this graph shows a start at 0 which represents 10 a.m. The rainfall increases to 2 cm, two hours later at noon. Then it increases to 10 cm at 4 p.m. It also shows when Andre spilled the water at 4 p.m. and that there was no change in the rainfall from 4 p.m. to 5 p.m.

   b Label the axes of the correct graph with appropriate units.

   c Use the graph to determine how much total rain fell on Tuesday.

   10 cm

6. Circle *all* the expressions that *cannot* be rewritten as a single power of 4.

   [✓ $4^5 \cdot 6^3$] $4^5 \cdot 4^3$ $\frac{4^{10}}{4^2}$ [✓ $4^4 \cdot 5^4$] $(4^2)^{-11}$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 1	1
Spiral	4	Unit 5 Lesson 20	2
	5	Unit 5 Lesson 5	2
Formative ⚡	6	Unit 6 Lesson 8	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Practice With Rational Bases

Let's practice with exponents.

Focus

Goal

1. **Language Goal:** Use exponent rules to rewrite exponential expressions, including those containing negative exponents, as a single power, and explain the strategy. **(Speaking and Listening)**

Coherence

Today

Students practice all of the exponent rules they have learned so far. They make use of structure when decomposing numbers, create viable arguments, and critique the reasoning of others as they work on problems with a partner and compete in a friendly class competition.

Previously

Students made use of repeated reasoning to discover exponent rules when multiplying and dividing powers with the same base, raising single powers to other powers, and multiplying powers with different bases and the same exponent.

Coming Soon

In the following Sub-Unit, students will apply their knowledge of exponents to solve problems with numbers expressed in scientific notation.

Rigor

- Students strengthen their **fluency** in performing operations with exponents.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- Anchor Chart PDF, *Exponent Rules*

Math Language Development

Review words

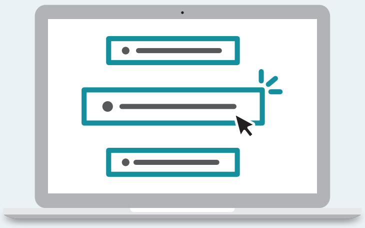
- *base*
- *equivalent expressions*
- *expanded form*
- *exponent*
- *power*

Amps | Featured Activity

Warm-up

Digitally Order Expressions

Students can drag and drop expressions in a list, ordering them from greatest to least.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might become too competitive and create conflict among pairs of students. Before beginning, have students explain the learning goal about powers for the activity. Have each pair explain how they will show respect to each other and other pairs to assure that the competition goes smoothly.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

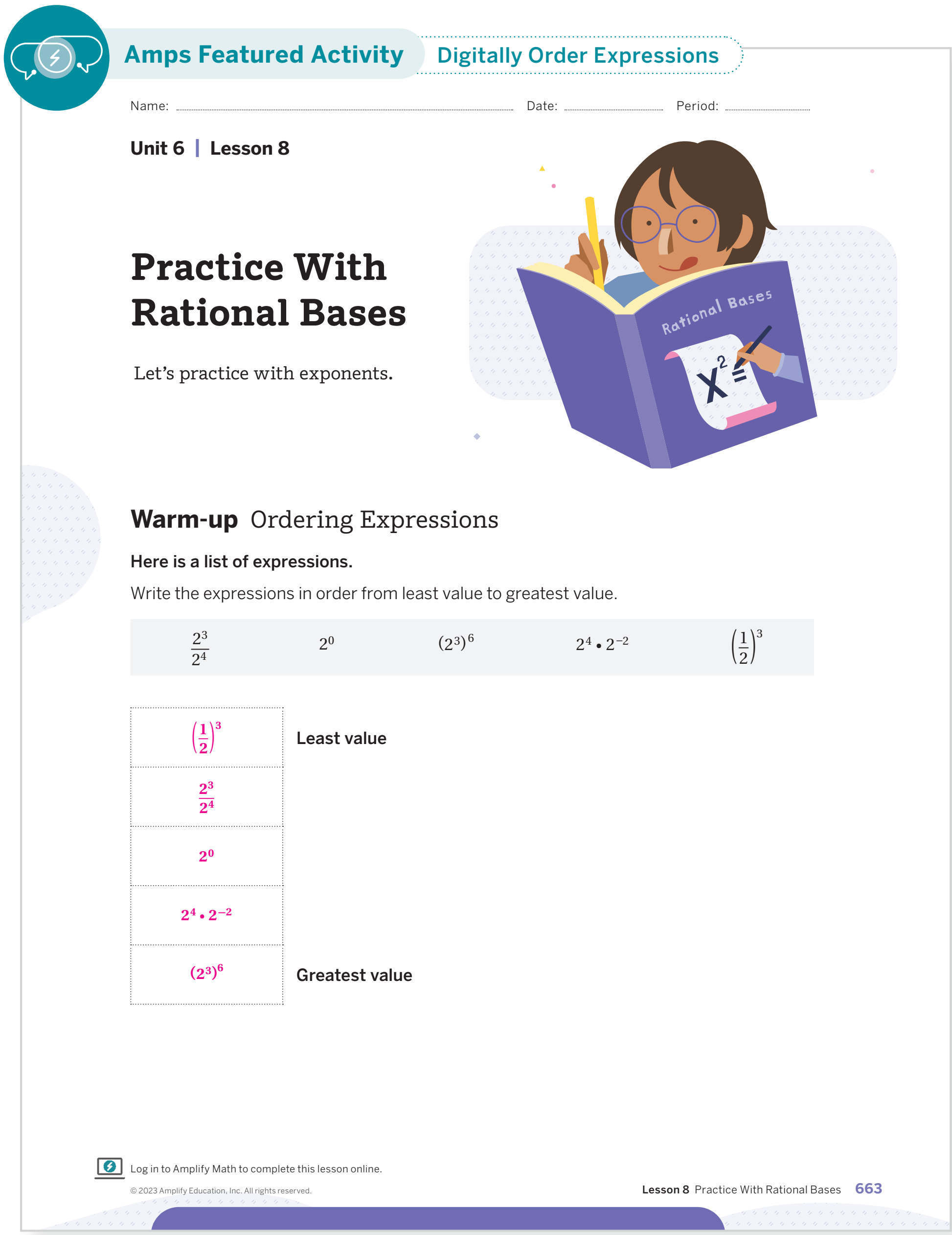
- **Activity 2** may be omitted.

Independent | 5 min

Warm-up Ordering Expressions

Students order a list of expressions to review the exponent rules they have learned in this unit.

Amps Featured Activity Digitally Order Expressions



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Unit 6 | Lesson 8

Practice With Rational Bases

Let's practice with exponents.

Warm-up Ordering Expressions

Here is a list of expressions.

Write the expressions in order from least value to greatest value.

$\frac{2^3}{2^4}$	2^0	$(2^3)^6$	$2^4 \cdot 2^{-2}$	$\left(\frac{1}{2}\right)^3$

$\left(\frac{1}{2}\right)^3$	**Least value**
$\frac{2^3}{2^4}$	
2^0	
$2^4 \cdot 2^{-2}$	
$(2^3)^6$	**Greatest value**

Log in to Amplify Math to complete this lesson online.

Lesson 8 Practice With Rational Bases 663

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking them to find the value of each expression or use an exponent rule to write each expression as a single power, before ordering them.

Look for points of confusion:

- **Not remembering the exponent rules.** Have students write each expression in expanded form or display the Anchor Chart PDF, *Exponent Rules*, for students to refer to.

Look for productive strategies:

- Writing all the expressions as a single power of 2, and then comparing the exponents.

3 Connect

Have students share their ordered list. Discuss any differences, and have students correct any mistakes.

Ask, "What are some strategies for ordering the list? How can you order the list without finding the value of each expression?"

Highlight that there are different methods to ordering the expressions, such as finding each value or rewriting each expression as a single power of 2.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Anchor Chart PDF, *Exponent Rules*, for them to reference during this activity. Allow them to keep this anchor chart throughout the rest of the unit.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can drag and drop expressions in a list, ordering them from greatest to least.

Power-up

To power up students' ability to rewrite expressions as a single power, have students complete:

Recall that you can use the relationships $a^m \cdot a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, and $(a^m)^n = a^{m \cdot n}$ to simplify expressions involving powers with the same base.

Determine which expressions can be simplified using the given rules. Select *all* that apply.

A. $6^3 \cdot 3^6$

[✓ B.] $12^{-4} \div 12^4$

[✓ C.] $8^{20} \cdot 8^{-4}$

[✓ D.] $(4^4)^3$

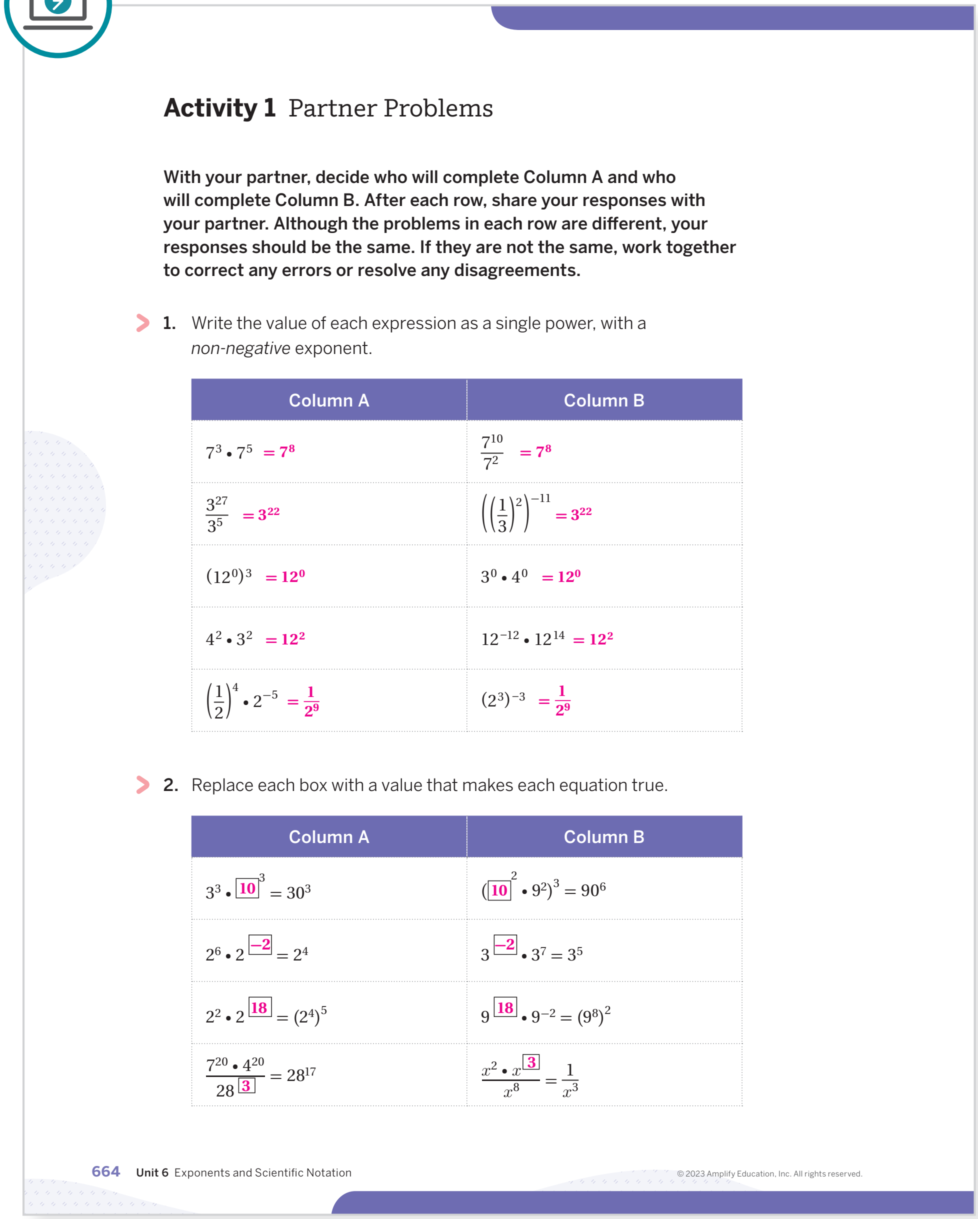
Use: Before the Warm-up

Informed by: Performance on Lesson 7, Practice Problem 6

Pairs | 15 min

Activity 1 Partner Problems

Students work in pairs, but on separate problems, to practice the exponent rules they have learned in this unit.



Activity 1 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with your partner. Although the problems in each row are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

1. Write the value of each expression as a single power, with a *non-negative* exponent.

Column A	Column B
$7^3 \cdot 7^5 = 7^8$	$\frac{7^{10}}{7^2} = 7^8$
$\frac{3^{27}}{3^5} = 3^{22}$	$\left(\left(\frac{1}{3}\right)^2\right)^{-11} = 3^{22}$
$(12^0)^3 = 12^0$	$3^0 \cdot 4^0 = 12^0$
$4^2 \cdot 3^2 = 12^2$	$12^{-12} \cdot 12^{14} = 12^2$
$\left(\frac{1}{2}\right)^4 \cdot 2^{-5} = \frac{1}{2^9}$	$(2^3)^{-3} = \frac{1}{2^9}$

2. Replace each box with a value that makes each equation true.

Column A	Column B
$3^3 \cdot \boxed{10}^3 = 30^3$	$(\boxed{10}^2 \cdot 9^2)^3 = 90^6$
$2^6 \cdot 2^{\boxed{-2}} = 2^4$	$3^{\boxed{-2}} \cdot 3^7 = 3^5$
$2^2 \cdot 2^{\boxed{18}} = (2^4)^5$	$9^{\boxed{18}} \cdot 9^{-2} = (9^8)^2$
$\frac{7^{20} \cdot 4^{20}}{28^{\boxed{3}}} = 28^{17}$	$\frac{x^2 \cdot x^{\boxed{3}}}{x^8} = \frac{1}{x^3}$

664 Unit 6 Exponents and Scientific Notation

1 Launch

Conduct the ***Partner Problems*** routine. Consider assigning Column B to the student who feels more comfortable with operations with integers.

2 Monitor

Help students get started by using the example $\frac{7^{10}}{7^{15}}$ to rewrite as a single, non-negative power.

Look for points of confusion:

- **Not remembering the exponent rules.** Have students rewrite the expressions in expanded form first, or display the Anchor Chart PDF, *Exponent Rules* for students to refer to.

Look for productive strategies:

- Using exponent rules to write each expression as a single power.
- Writing expressions in expanded form before writing them as a single power.

3 Connect

Have students share any rows where they did not have the same response as their partner, and what they did to come to an agreement for their final response.

Ask:

- "Did you and your partner use the same strategy for each row?"
- "Did anyone learn a new strategy from their partner?"

Highlight the different types of strategies used to write expressions as a single power.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code the expressions that have:

- The *same bases* in one color.
- *Different bases* and the same exponent in another color.

Suggest that students annotate each row with "multiplying powers," "dividing powers," and "powers of powers."

Math Language Development

MLR8: Discussion Supports

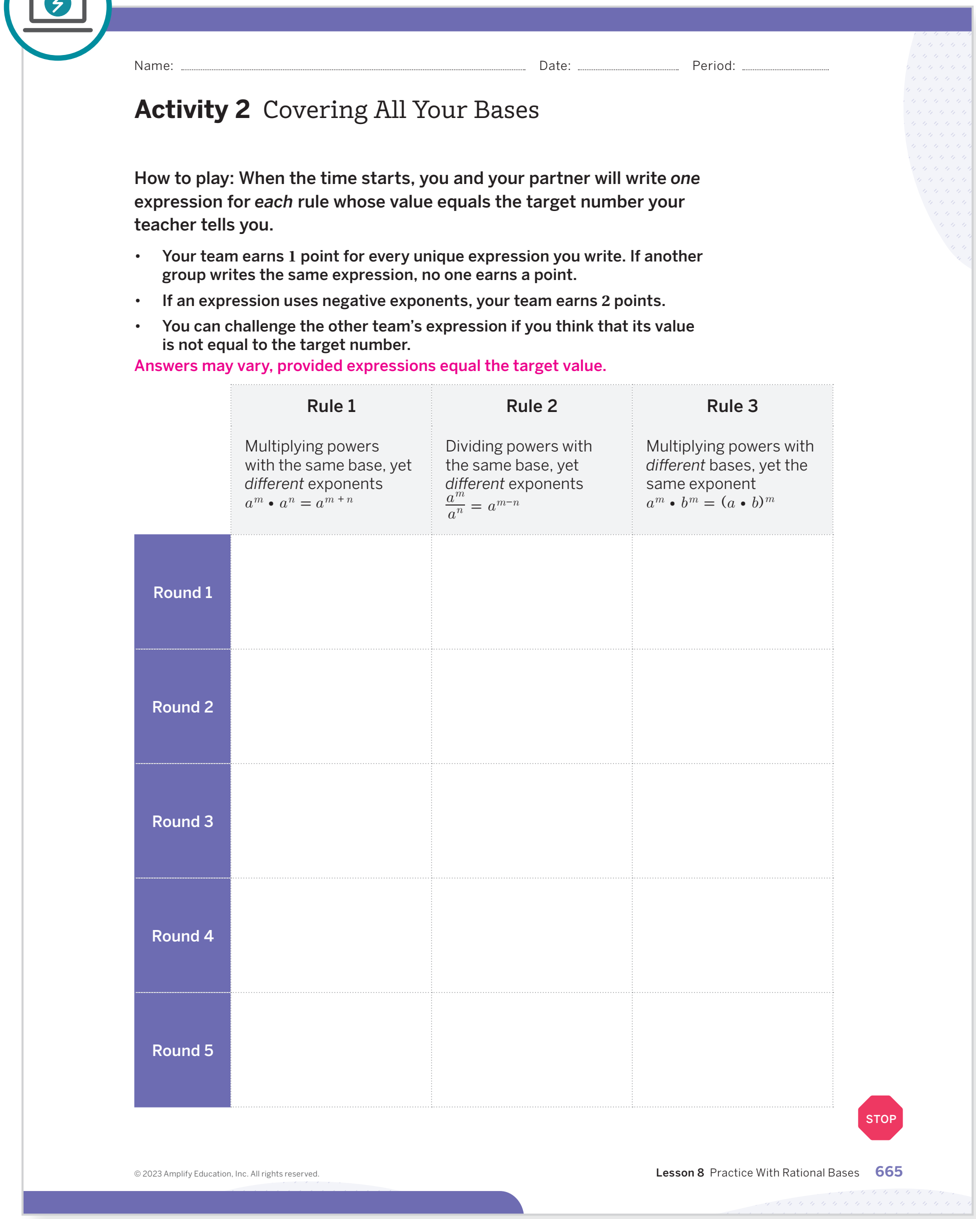
While students work together, display the following sentence frames to support their discussion.

- "I noticed that . . ."
- "First, I _____, because . . ."
- "I disagree because . . ."
- "The expression shows a product/quotient, so I . . ."
- "The expression shows a power raised to another power, so I . . ."
- "The bases were the same, so I . . ."
- "The bases were different, but the exponents were the same, so I . . ."

Small Groups | 15 min

Activity 2 Covering All Your Bases

Students build fluency by generating different equivalent expressions using the rules of exponents.



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Activity 2 Covering All Your Bases

How to play: When the time starts, you and your partner will write *one* expression for *each* rule whose value equals the target number your teacher tells you.

- **Your team earns 1 point for every unique expression you write. If another group writes the same expression, no one earns a point.**
- **If an expression uses negative exponents, your team earns 2 points.**
- **You can challenge the other team's expression if you think that its value is not equal to the target number.**

Answers may vary, provided expressions equal the target value.

	Rule 1	Rule 2	Rule 3
	Multiplying powers with the same base, yet *different* exponents $a^m \cdot a^n = a^{m+n}$	Dividing powers with the same base, yet *different* exponents $\frac{a^m}{a^n} = a^{m-n}$	Multiplying powers with *different* bases, yet the same exponent $a^m \cdot b^m = (a \cdot b)^m$
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			

STOP

Lesson 8 Practice With Rational Bases 665

1 Launch

Have each pair of students play against another pair. Explain the rules of the game using the Activity 2 PDF as an example. Teams can earn a maximum of six points in each round.

For each round, set a timer for 1 minute. Use the following numbers to play additional rounds: $144, 2500, \frac{1}{64}, 4900, \frac{1}{400}$. In subsequent rounds, consider grouping pairs with a different opponent.

2 Monitor

Help students get started by completing a practice round with the target number 100.

Look for points of confusion:

- **Thinking any negative number earns them an extra point.** Point out that an extra point is only earned when a negative number is used as an exponent.

Look for productive strategies:

- Writing a single power to obtain the target value before writing an equivalent expression.
- Challenging teams who write an expression that is not the same as the target value.

3 Connect

Have each pair of students share their strategies to write an equivalent expression to any target value.

Ask, "For which rule did you not have any difficulty generating expressions? Which rule was the most challenging?"

Highlight that numbers can represented as a product of their factors in many ways, and the exponent rules can be used to express the same value in many ways.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Display the Activity 2 PDF and use a think-aloud to demonstrate the steps of the game with a partner. Model one complete round with a student volunteer and then invite the class to ask any clarifying questions they have before playing the game.

Extension: *Math Enrichment*

Challenge students to write an expression that uses more than one exponent rule. Then have them trade expressions with a partner and have each person evaluate their partner's expression. Sample response: $\frac{3^2 \cdot 5^2}{5^{-4}} = 3^2 \cdot 5^6 = 140,625$

Math Language Development

MLR7: Compare and Connect

During the game, pause after each round and ask students to compare their strategies for writing the expressions for each rule with other students' strategies. Press students to critique each other's work and reasoning if they think the value does not equal the target number.

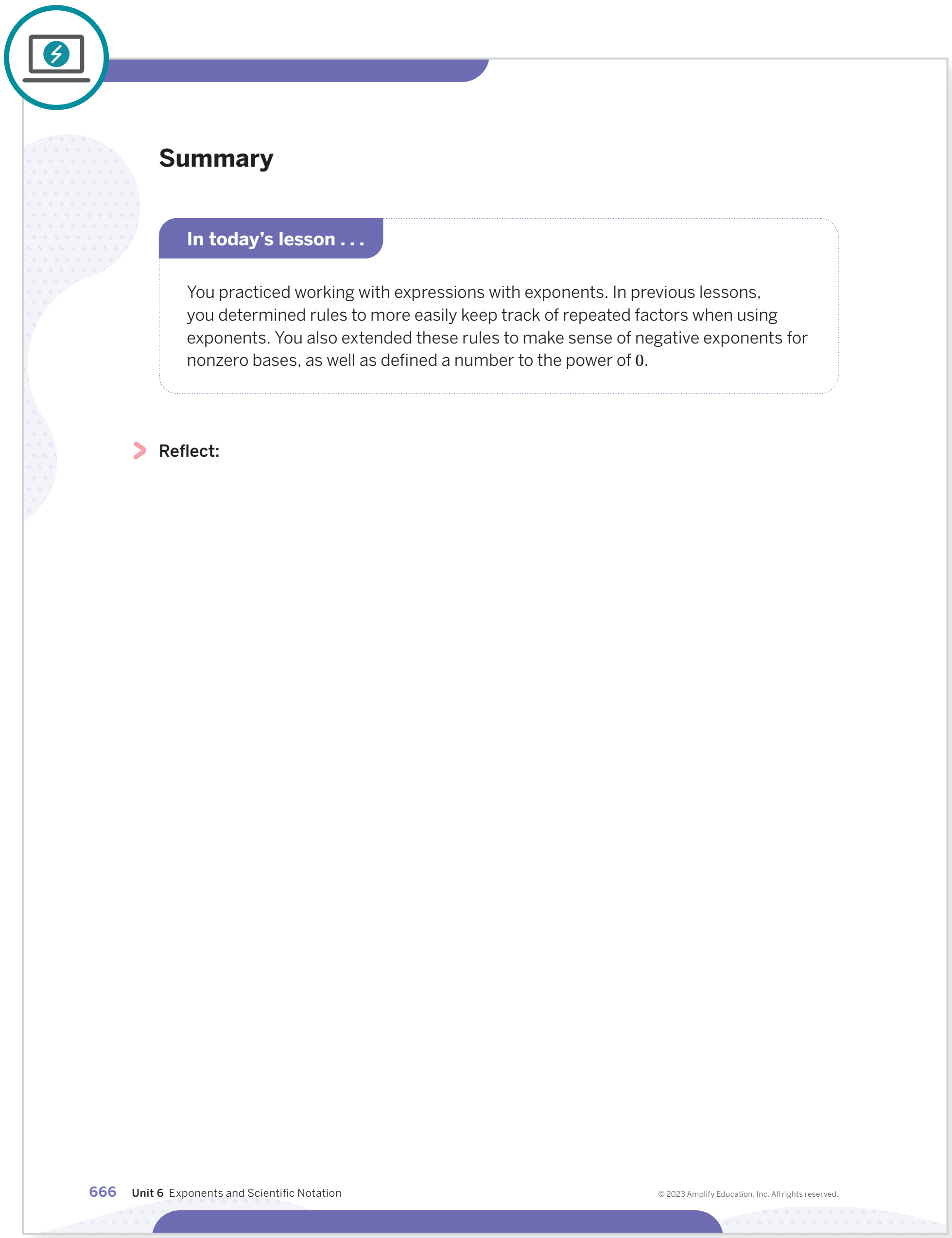
English Learners

- "I noticed that . . ."
- "First, I _____, because . . ."
- "I disagree because . . ."
- "The expression does/does not equal the target value because . . ."

Whole Class | 5 min

Summary

Review and synthesize the exponent rules students have learned in this unit and how they help write exponential expressions in more efficient, equivalent ways.



Summary

In today's lesson . . .

You practiced working with expressions with exponents. In previous lessons, you determined rules to more easily keep track of repeated factors when using exponents. You also extended these rules to make sense of negative exponents for nonzero bases, as well as defined a number to the power of 0.

> **Reflect:**

666 Unit 6 Exponents and Scientific Notation

Synthesize

Display the Anchor Chart PDF, *Exponent Rules*, which illustrates the exponent rules students have learned in this unit.

Ask students to describe a strategy for rewriting an expression as a single power. Tell them to use one of the expressions from the Anchor Chart PDF if it helps their thinking.

Highlight that the exponent rules work because they use patterns of repeated multiplication of a single base.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

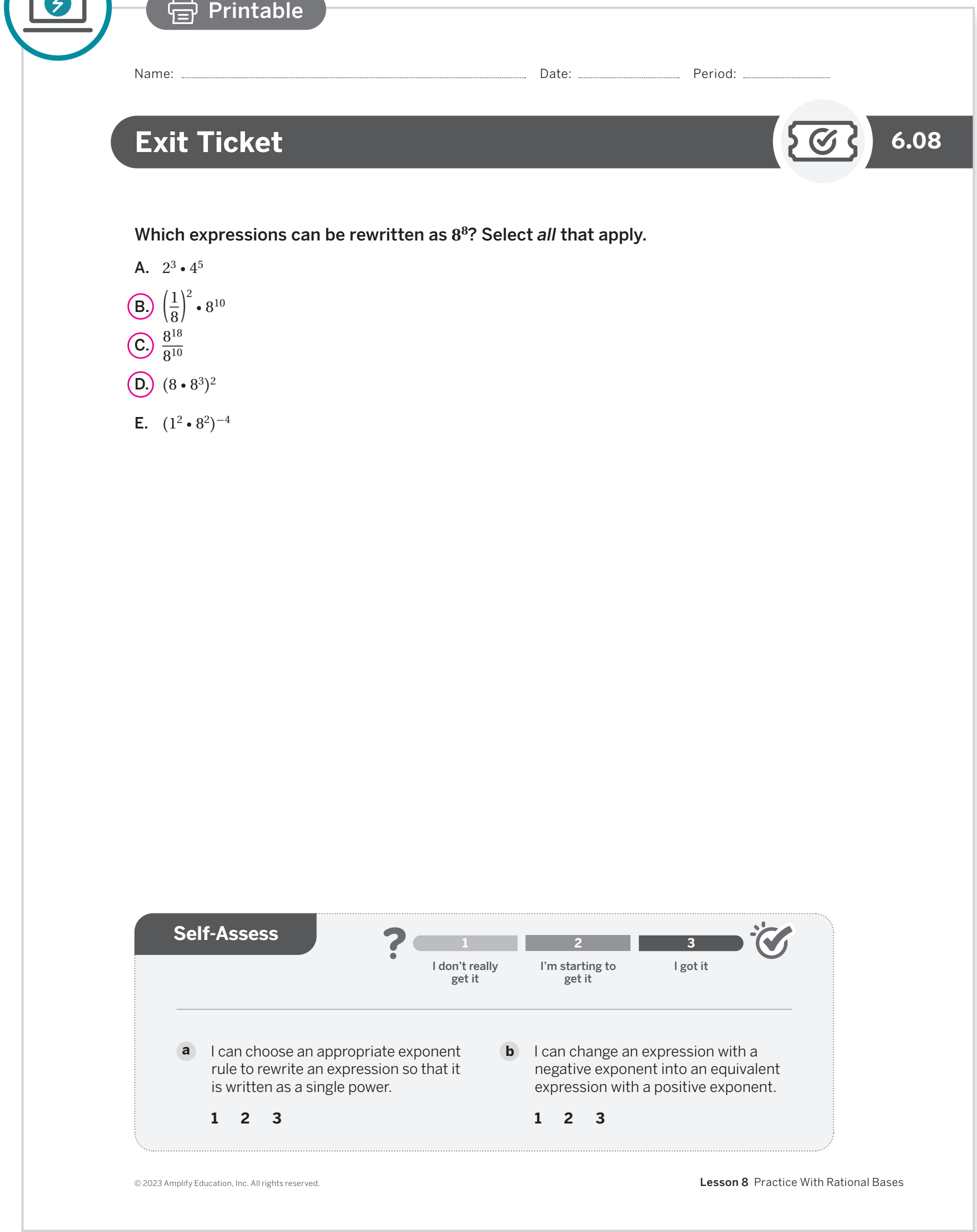
- "What happens when expressions containing exponents are multiplied or divided?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the exponent rules they have learned in this unit by identifying equivalent expressions.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 6.08

Which expressions can be rewritten as 8^8? Select *all* that apply.

A. $2^3 \cdot 4^5$

[✓ B.] $\left(\frac{1}{8}\right)^2 \cdot 8^{10}$

[✓ C.] $\frac{8^{18}}{8^{10}}$

[✓ D.] $(8 \cdot 8^3)^2$

E. $(1^2 \cdot 8^2)^{-4}$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can choose an appropriate exponent rule to rewrite an expression so that it is written as a single power. 1 2 3

b. I can change an expression with a negative exponent into an equivalent expression with a positive exponent. 1 2 3

Lesson 8 Practice With Rational Bases

Success looks like . . .

- **Language Goal:** Using exponent rules to rewrite exponential expressions, including those containing negative exponents, as a single power, and explaining the strategy. **(Speaking and Listening)**
  - » Selecting the choices that are equivalent to 8^8.

Suggested next steps

If students select choice A or E, consider:

- Writing the expression in expanded form to show all the factors.

If students do not select choice D, consider:

- Writing 8 as 8^1.

Professional Learning

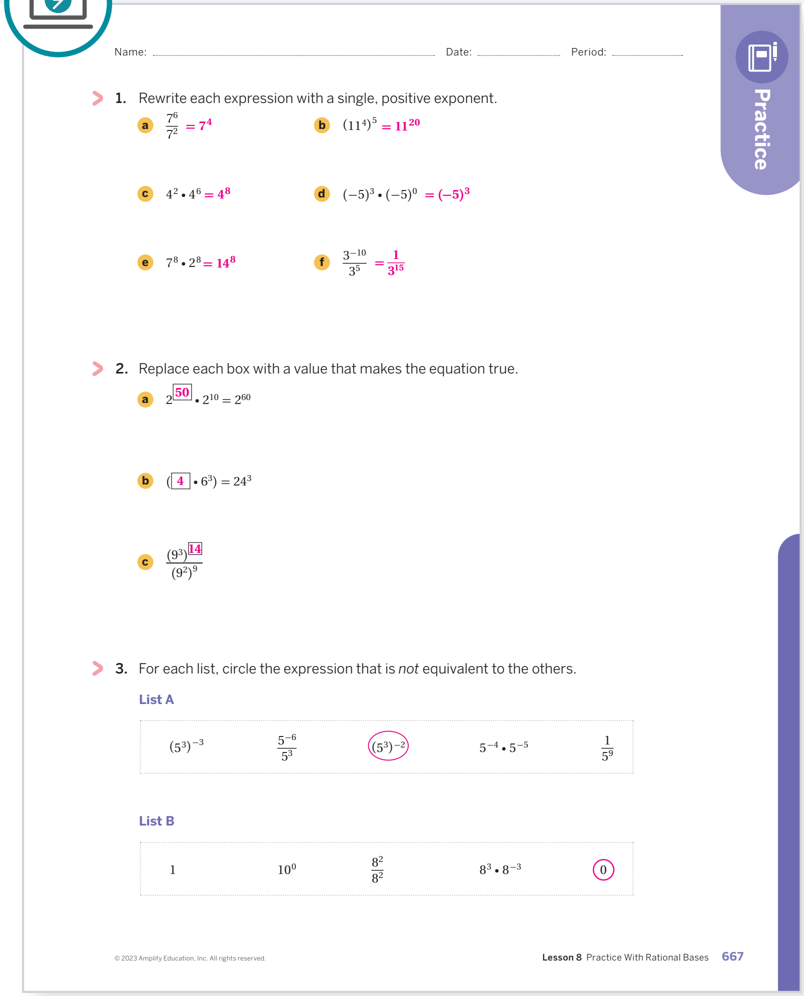
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did Activity 2 go as planned?
- In what ways in Activity 2 did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Rewrite each expression with a single, positive exponent.

   a $\frac{7^6}{7^2}$ $= 7^4$
   b $(11^4)^5$ $= 11^{20}$
   c $4^2 \cdot 4^6 = 4^8$
   d $(-5)^3 \cdot (-5)^0$ $= (-5)^3$
   e $7^8 \cdot 2^8 = 14^8$
   f $\frac{3^{-10}}{3^5}$ $= \frac{1}{3^{15}}$

2. Replace each box with a value that makes the equation true.

   a $2^{\boxed{50}} \cdot 2^{10} = 2^{60}$
   b $(\boxed{4} \cdot 6^3) = 24^3$
   c $\frac{(9^3)^{\boxed{14}}}{(9^2)^9}$

3. For each list, circle the expression that is *not* equivalent to the others.

List A

$(5^3)^{-3}$	$\frac{5^{-6}}{5^3}$	$(5^3)^{-2}$	$5^{-4} \cdot 5^{-5}$	$\frac{1}{5^9}$

List B

1	10^0	$\frac{8^2}{8^2}$	$8^3 \cdot 8^{-3}$	⓪

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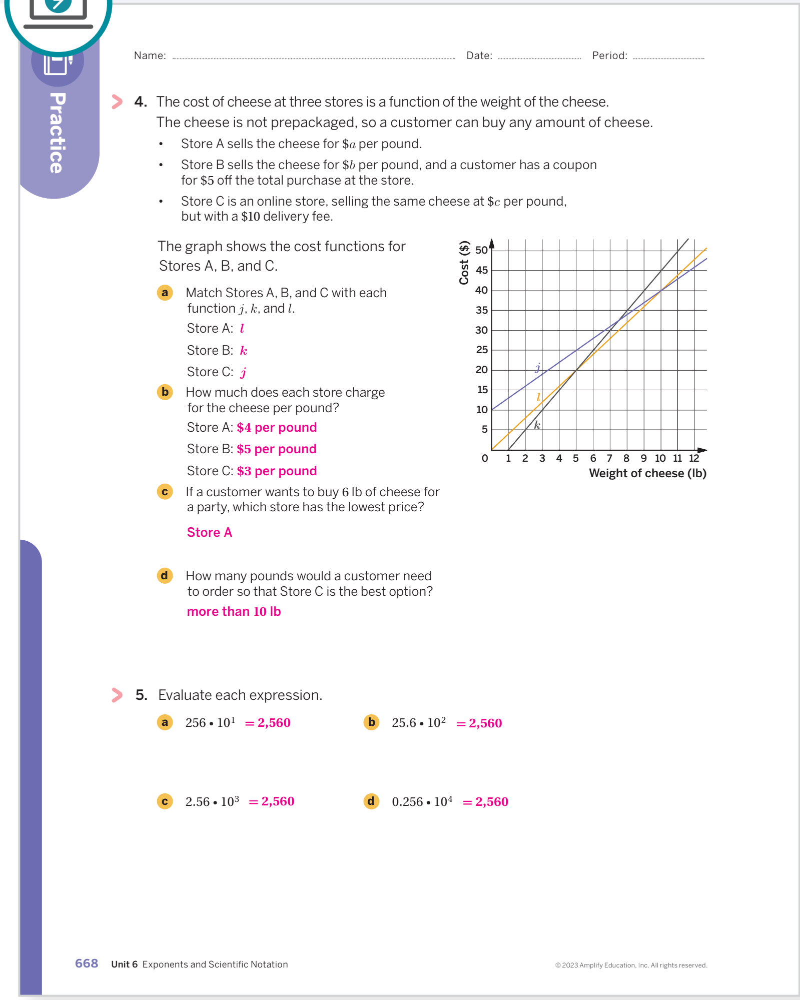
Lesson 8 Practice With Rational Bases 667



Practice

Name: Date: Period:

4. The cost of cheese at three stores is a function of the weight of the cheese. The cheese is not prepackaged, so a customer can buy any amount of cheese.
   - Store A sells the cheese for $\$a$ per pound.
   - Store B sells the cheese for $\$b$ per pound, and a customer has a coupon for $5 off the total purchase at the store.
   - Store C is an online store, selling the same cheese at $\$c$ per pound, but with a $10 delivery fee.

   The graph shows the cost functions for Stores A, B, and C.

   a Match Stores A, B, and C with each function j, k, and l.

   Store A: l

   Store B: k

   Store C: j

   b How much does each store charge for the cheese per pound?

   Store A: **$4 per pound**

   Store B: **$5 per pound**

   Store C: **$3 per pound**

   c If a customer wants to buy 6 lb of cheese for a party, which store has the lowest price?

   Store A

   d How many pounds would a customer need to order so that Store C is the best option?

   more than 10 lb

5. Evaluate each expression.

   a $256 \cdot 10^1$ $= 2{,}560$
   b $25.6 \cdot 10^2$ $= 2{,}560$
   c $2.56 \cdot 10^3$ $= 2{,}560$
   d $0.256 \cdot 10^4$ $= 2{,}560$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	1
	3	Activity 1	2
Spiral	4	Unit 5 Lesson 8	2
Formative ⚡	5	Unit 6 Lesson 9	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

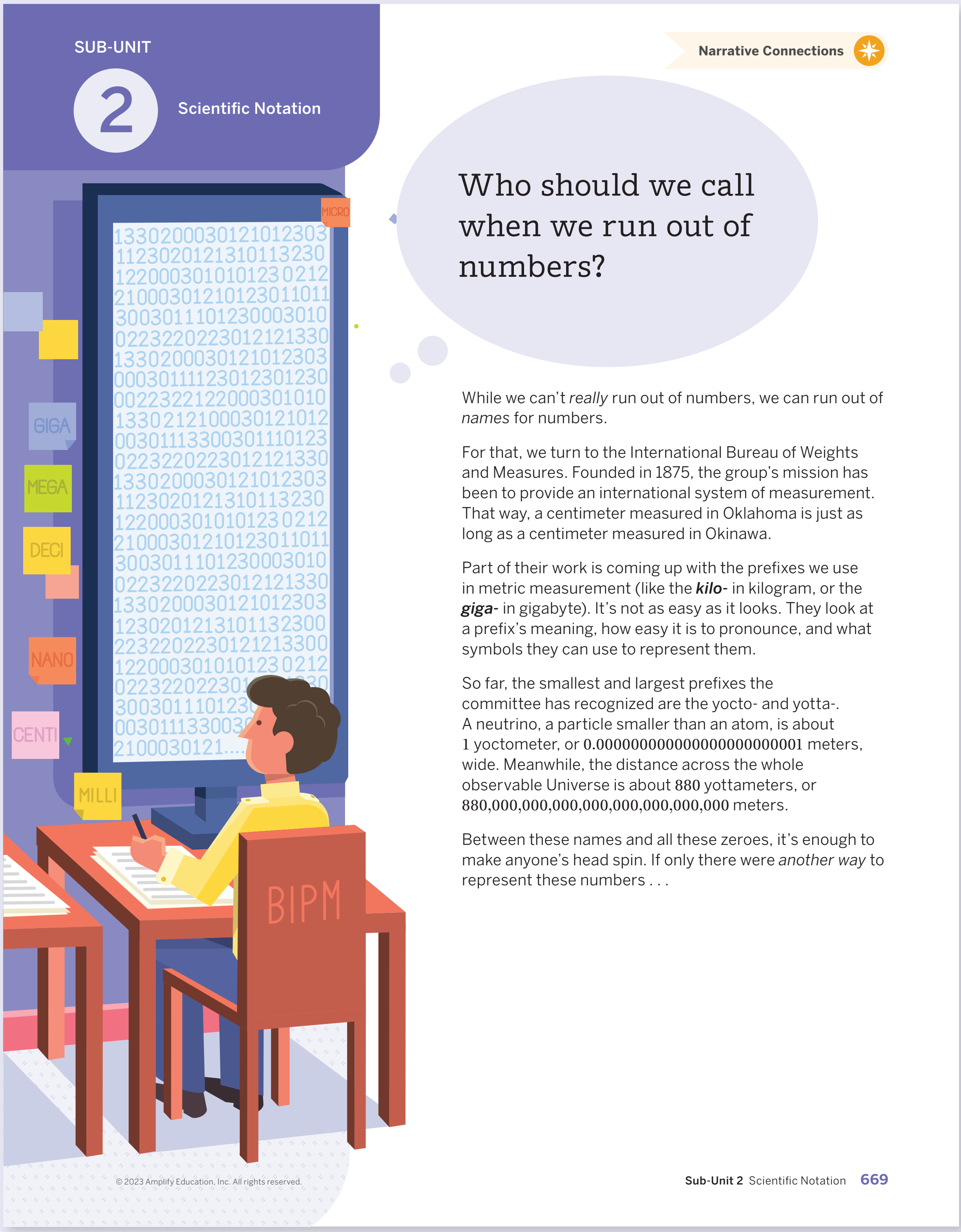
For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Sub-Unit 2

Scientific Notation

Whole Class

In this Sub-Unit, students apply their knowledge of exponents to work with numbers expressed in scientific notation.



SUB-UNIT

2 Scientific Notation

Narrative Connections

Who should we call when we run out of numbers?

While we can't *really* run out of numbers, we can run out of *names* for numbers.

For that, we turn to the International Bureau of Weights and Measures. Founded in 1875, the group's mission has been to provide an international system of measurement. That way, a centimeter measured in Oklahoma is just as long as a centimeter measured in Okinawa.

Part of their work is coming up with the prefixes we use in metric measurement (like the ***kilo-*** in kilogram, or the ***giga-*** in gigabyte). It's not as easy as it looks. They look at a prefix's meaning, how easy it is to pronounce, and what symbols they can use to represent them.

So far, the smallest and largest prefixes the committee has recognized are the yocto- and yotta-. A neutrino, a particle smaller than an atom, is about 1 yoctometer, or 0.000000000000000000000001 meters, wide. Meanwhile, the distance across the whole observable Universe is about 880 yottameters, or 880,000,000,000,000,000,000,000,000 meters.

Between these names and all these zeroes, it's enough to make anyone's head spin. If only there were *another way* to represent these numbers . . .

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore another way to represent very large and very small numbers in the following places:

- **Lesson 9, Activities 1–3:** Labeling a Number Line, Comparing Large Numbers Using a Number Line, The Speeds of Light
- **Lesson 10, Activities 1–2:** Comparing Small Numbers Using a Number Line, Deadly Animals
- **Lesson 11, Activities 1–2:** How Many Pennies?, Even More Pennies
- **Lesson 12, Activities 1–3:** Card Sort: Identifying Scientific Notation, Writing Scientific Notation, Writing Small Numbers in Scientific Notation

UNIT 6 | **LESSON 9**

Representing Large Numbers on the Number Line

Let's visualize large numbers on the number line using powers of 10.

Focus

Goals

1. **Language Goal:** Compare large numbers using powers of 10, and explain the solution method. **(Speaking and Listening)**
2. **Language Goal:** Use number lines to represent large numbers as multiples of powers of 10. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students use number lines to visualize powers of 10, compare very large numbers, and make sense of orders of magnitude. Students attend to precision and construct viable arguments when plotting and comparing values on a number line. This lesson serves as a prelude to scientific notation.

Previously

In Grade 5, students explored patterns in the number of zeros of the product when multiplying a number by powers of 10 and used whole number exponents to denote powers of 10.

Coming Soon

In Lesson 12, students will be officially introduced to the definition of scientific notation.

Rigor

- Students build **conceptual understanding** of large numbers using powers of 10.
- Students **apply** their understanding of small numbers to compare how fast light waves can travel through different materials.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	8 min	12 min	12 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

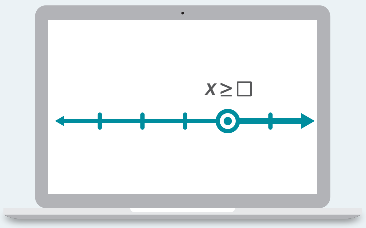
Review words

- *base*
- *equivalent expressions*
- *exponent*
- *power*

Amps | Featured Activity

Activity 2
Collaborative Number Line

Students can plot values on a digital number line. You can overlay students' number lines to see the accuracy and precision of their placements.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might be overly critical or even rude if their partner labeled the number line incorrectly. Remind students that this is an opportunity to learn and that they might be the one who makes the error. Prior to starting the activity, have students role play ways that they can negotiate a conflict with good communication, both seeking and offering help when needed.

Modifications to Pacing

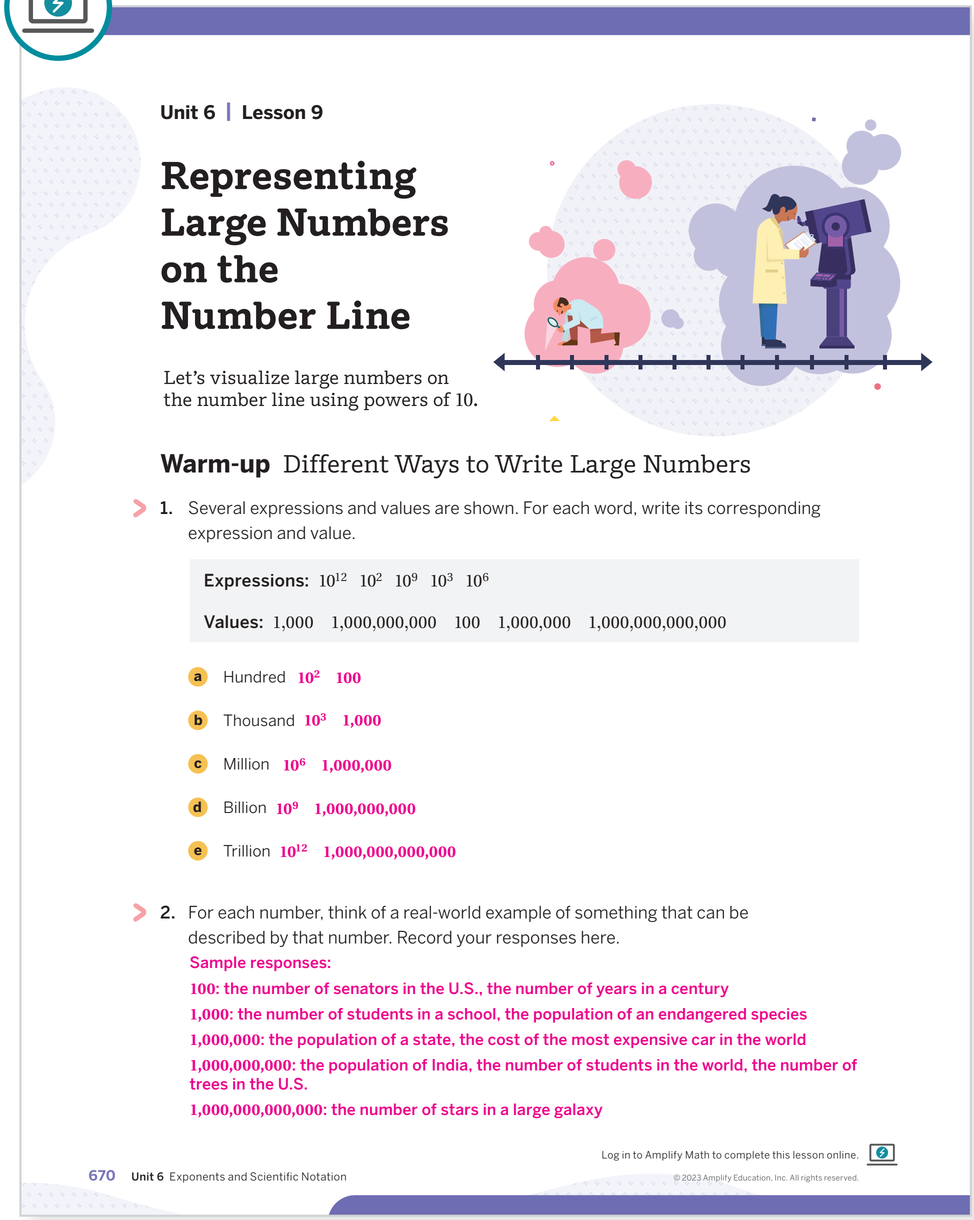
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, have students select three values to plot and label in Problem 1.
- In **Activity 2**, Problem 2 may be omitted.

Pairs | 5 min

Warm-up Matching Large Numbers

Students match words and values to their respective powers of 10 to prepare them for reasoning about the size of large numbers.



Unit 6 | Lesson 9

Representing Large Numbers on the Number Line

Let's visualize large numbers on the number line using powers of 10.

Warm-up Different Ways to Write Large Numbers

1. Several expressions and values are shown. For each word, write its corresponding expression and value.

Expressions: 10^{12} 10^2 10^9 10^3 10^6

Values: 1,000 1,000,000,000 100 1,000,000 1,000,000,000,000

- a. Hundred 10^2 100
- b. Thousand 10^3 1,000
- c. Million 10^6 1,000,000
- d. Billion 10^9 1,000,000,000
- e. Trillion 10^{12} 1,000,000,000,000

2. For each number, think of a real-world example of something that can be described by that number. Record your responses here.

Sample responses:

100: the number of senators in the U.S., the number of years in a century

1,000: the number of students in a school, the population of an endangered species

1,000,000: the population of a state, the cost of the most expensive car in the world

1,000,000,000: the population of India, the number of students in the world, the number of trees in the U.S.

1,000,000,000,000: the number of stars in a large galaxy

Log in to Amplify Math to complete this lesson online.

670 **Unit 6** Exponents and Scientific Notation

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1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by asking them to evaluate 10^2, and then having them find its matching word.

Look for points of confusion:

- **Thinking $10^3 = 100$ because the exponent matches the number of digits.** Have them evaluate $10 \cdot 10 \cdot 10$ and compare the values.
- **Struggling to think of real-world contexts for Problem 2.** Have each pair of students brainstorm with another pair of students.

Look for productive strategies:

- Noticing the number of zeros for each power of 10.

3 Connect

Have students share the corresponding expressions, words, and values. Record responses for all to see. Invite students to share their examples for Problem 2.

Ask if students have heard of the company Google, to help activate background knowledge. Tell them that 10^{100} is called a *googol*. Then ask, "Why do you think the company Google used this word as their name? How many zeros do you think are in this number?" Sample response: Maybe Google used this word to represent their large list of search results. There are 100 zeros in the number googol.

Highlight the patterns between the exponent and number of zeros in the value.

Power-up

To power up students' ability to multiply values by powers of 10, have students complete:

Recall that when simplifying expressions, exponents are evaluated before multiplication.

Evaluate each expression.

1. $175 \cdot 10^2$ $= 17{,}500$
2. $17.5 \cdot 10^3$ $= 17{,}500$
3. $0.175 \cdot 10^5$ $= 17{,}500$

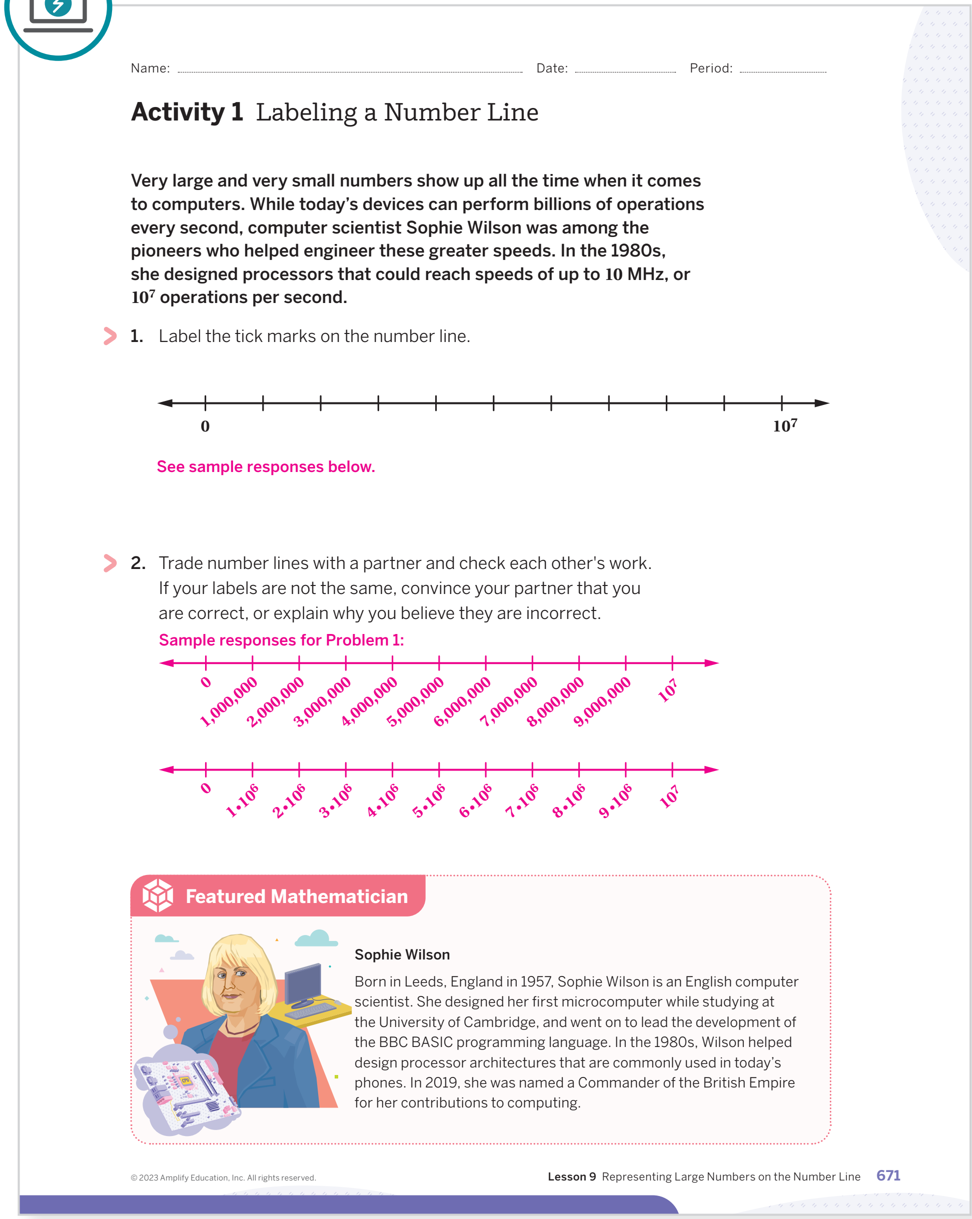
Use: Before the Warm-up

Informed by: Performance on Lesson 8, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 8 min

Activity 1 Labeling a Number Line

Students label a number line given an endpoint of a power of 10 to reason about the placement of powers of 10 on a number line.



Name: ______ Date: ______ Period: ______

Activity 1 Labeling a Number Line

Very large and very small numbers show up all the time when it comes to computers. While today's devices can perform billions of operations every second, computer scientist Sophie Wilson was among the pioneers who helped engineer these greater speeds. In the 1980s, she designed processors that could reach speeds of up to 10 MHz, or 10^7 operations per second.

1. Label the tick marks on the number line.

0 … 10^7

See sample responses below.

2. Trade number lines with a partner and check each other's work. If your labels are not the same, convince your partner that you are correct, or explain why you believe they are incorrect.

Sample responses for Problem 1:

0, 1,000,000, 2,000,000, 3,000,000, 4,000,000, 5,000,000, 6,000,000, 7,000,000, 8,000,000, 9,000,000, 10^7

0, $1 \cdot 10^6$, $2 \cdot 10^6$, $3 \cdot 10^6$, $4 \cdot 10^6$, $5 \cdot 10^6$, $6 \cdot 10^6$, $7 \cdot 10^6$, $8 \cdot 10^6$, $9 \cdot 10^6$, 10^7

Featured Mathematician

Sophie Wilson

Born in Leeds, England in 1957, Sophie Wilson is an English computer scientist. She designed her first microcomputer while studying at the University of Cambridge, and went on to lead the development of the BBC BASIC programming language. In the 1980s, Wilson helped design processor architectures that are commonly used in today's phones. In 2019, she was named a Commander of the British Empire for her contributions to computing.

① Launch

Conduct the *Think-Pair-Share* routine giving time for comparing after each problem.

② Monitor

Help students get started by asking them to identify how many equal segments are marked between 0 and 10^7 on the number line.

Look for points of confusion:

- **Labeling the number line incorrectly.** Student responses will include a variety of incorrect or partially correct ideas. It is not important that students understand the correct notation at this point, but encourage partners to discuss why their labeling does or does not make sense.
- **Labeling the tick marks as 10^1, 10^2, 10^3, etc.** Ask students if this labeling strategy accounts for the 9 unlabeled tick marks between 0 and 10^7.

Look for productive strategies:

- Evaluating 10^7 as $10{,}000{,}000$, then dividing by 10.
- Labeling the tick marks as $1 \cdot 10^6$, $2 \cdot 10^6$, $3 \cdot 10^6$, etc.
- Using the exponent rule, $10^7 \div 10 = 10^6$, to divide the number line into equal parts.

③ Connect

Display the different ways students labeled the number line.

Have pairs of students share any disagreements they had and how they came to an agreement.

Ask, "How can you label the number line using powers of 10? Why would this be helpful?"

Highlight that large numbers can be represented using powers of 10.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Display these questions that students can ask themselves, to help them think about how to approach Problem 1.

- "How many increments are there from 0 to 10^7?"
- "What is another way you can write or think about 10^7? How does this help you think about the label for each tick mark?"

Featured Mathematician

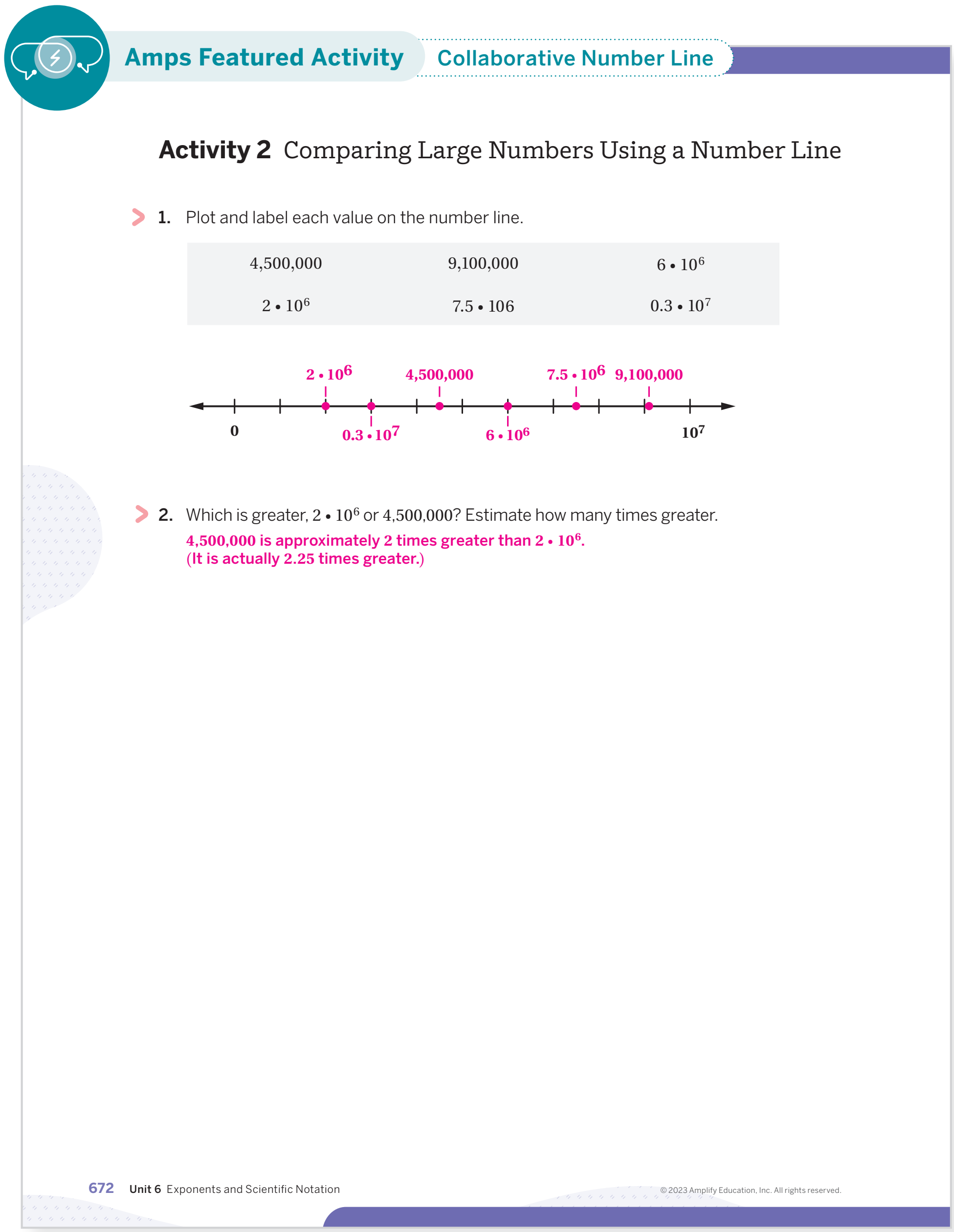
Sophie Wilson

Have students read about Featured Mathematician Sophie Wilson, a computer science pioneer whose processor designs are an important technology powering today's phones.

Pairs | 12 min

Activity 2 Comparing Large Numbers Using a Number Line

Students plot numbers on a number line to make sense of numbers expressed as a multiple of a power of 10.



Amps Featured Activity Collaborative Number Line

Activity 2 Comparing Large Numbers Using a Number Line

1. Plot and label each value on the number line.

4,500,000	9,100,000	$6 \cdot 10^6$
$2 \cdot 10^6$	$7.5 \cdot 106$	$0.3 \cdot 10^7$

2. Which is greater, $2 \cdot 10^6$ or 4,500,000? Estimate how many times greater.
4,500,000 is approximately 2 times greater than $2 \cdot 10^6$.
(It is actually 2.25 times greater.)

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1 Launch

Conduct the ***Think-Pair-Share*** routine giving time for comparing after each problem.

2 Monitor

Help students get started by showing how to write 4,500,000 as a multiple of a power of 10 a few different ways. For example: $4.5 \cdot 10^6$, $45 \cdot 10^5$, $0.45 \cdot 10^7$.

Look for points of confusion:

- **Not knowing where to plot $0.3 \cdot 10^7$.** Have students evaluate it or write it as $3 \cdot 10^6$.
- **Thinking that $7.5 \cdot 10^6$ is written as 75,000,000 because the exponent matches the number of zeros.** Have students evaluate $7.5 \cdot 10^6$ using paper and pencil or a calculator to show that this is not true.

Look for productive strategies:

- Writing the numbers using powers of 10^6.
- Evaluating the expressions written as a multiple of a power of 10.
- Using the number line to estimate their response for Problem 2.

3 Connect

Display student responses.

Have pairs of students share their strategies on how they plotted each value. Have students give each other feedback to determine the precise placement of each point.

Ask, "How can you rewrite 1,500,000, $0.3 \cdot 10^7$, and 9,100,000 so that they have the same power of 10?"

Highlight that writing large values using the same form helps to compare the values.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can plot numbers on a digital number line. You can overlay the number lines to see the accuracy and precision of students' placement of values.

Extension: *Math Enrichment*

Have students generate two values — one that is written as the product of a power of 10 and one that is not — and then trade values with a partner. Each partner should plot the values on the number line and then write a short description comparing the values. Students' responses will vary.

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

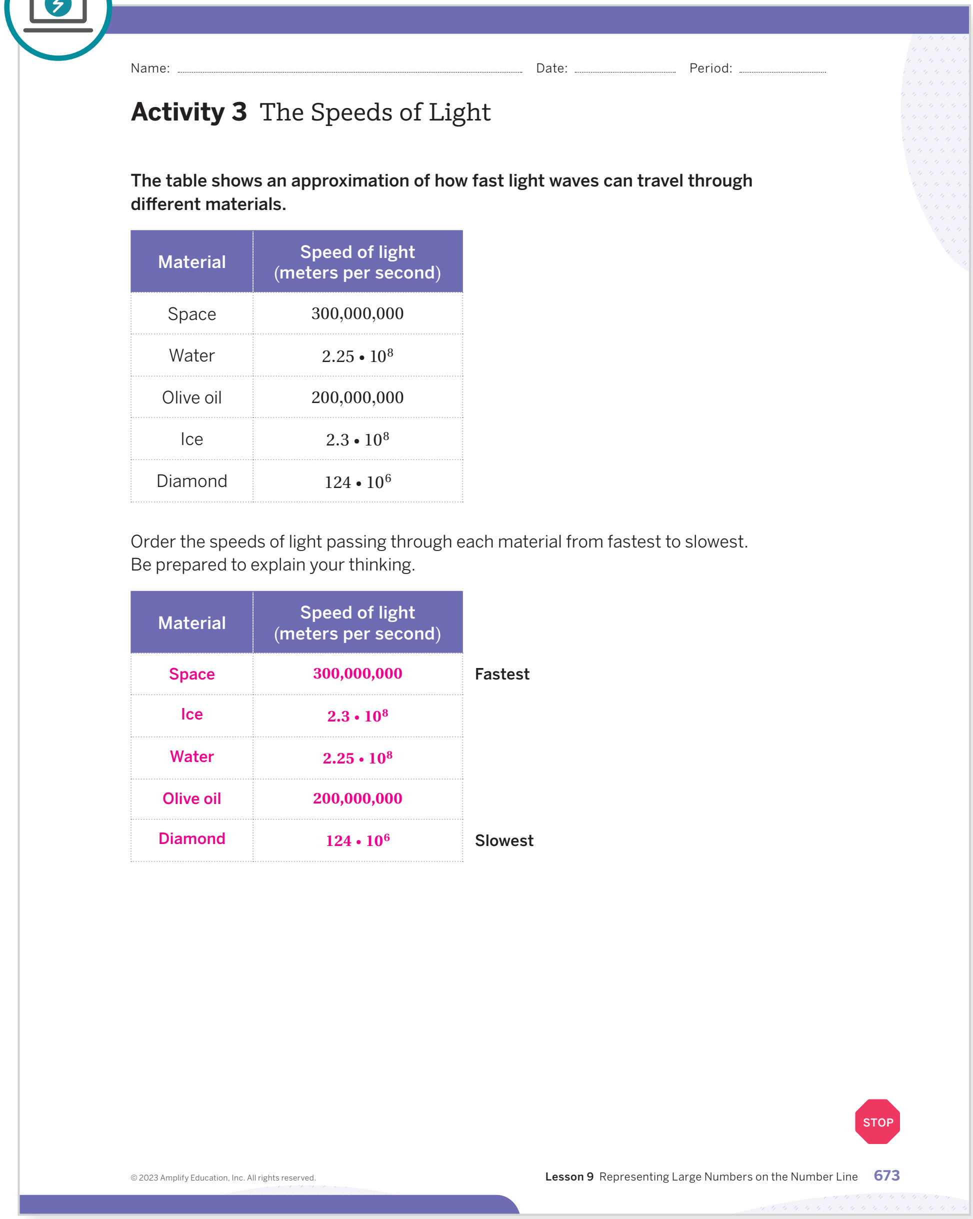
Display these questions that students can ask themselves, to help them think about how to approach Problems 1 and 2.

- "How can you use the number line you labeled in Activity 1 to help you in this activity?"
- "How can you compare 2.10^6 and 4,500,000 when they are not written in the same form? How does the number line help you?"

Pairs | 12 min

Activity 3 The Speeds of Light

Students order numbers, some written as a multiple of a power of 10, to create a need for a unified notation to compare large numbers.



Name: ______________________ Date: __________ Period: __________

Activity 3 The Speeds of Light

The table shows an approximation of how fast light waves can travel through different materials.

Material	Speed of light (meters per second)
Space	300,000,000
Water	$2.25 \cdot 10^8$
Olive oil	200,000,000
Ice	$2.3 \cdot 10^8$
Diamond	$124 \cdot 10^6$

Order the speeds of light passing through each material from fastest to slowest. Be prepared to explain your thinking.

Material	Speed of light (meters per second)	
Space	300,000,000	Fastest
Ice	$2.3 \cdot 10^8$	
Water	$2.25 \cdot 10^8$	
Olive oil	200,000,000	
Diamond	$124 \cdot 10^6$	Slowest

STOP

Lesson 9 Representing Large Numbers on the Number Line 673

1 Launch

Activate prior knowledge by asking students to share what they know about how fast light can travel. As light moves through different materials, it slows down. The speed of light through empty space is roughly 300,000,000 m per second. The speed of light through olive oil is much slower at roughly 200,000,000 m per second.

2 Monitor

Help students get started by having them to write the numbers in the same form to compare them. A greater number represents a faster speed.

Look for points of confusion:

- **Thinking that $2.25 \cdot 10^8$ is written as 22,500,000,000 because the exponent matches the number of zeros.** Have students evaluate this number using a calculator and compare the values.

Look for productive strategies:

- Rewriting the values as a multiple of a power of 10^8.
- Evaluating the expressions written with a power.

3 Connect

Display different student work. Ask students if they agree or disagree with their peers' ordering.

Have pairs share their strategies on how they compared the speeds of light.

Ask, "How can writing the numbers in the same way help to compare numbers?"

Highlight that the speed of light through ice can be written as $2.3 \cdot 10^8$ m per second, and the speed of light through olive oil can be written as $2 \cdot 10^8$ m per second. Writing the numbers in the same form allows for a more efficient way to compare them.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

To help students get started with ordering the speeds of light, write $124 \cdot 10^6$ as $1.24 \cdot 10^8$ so that all multiples of powers of 10 are written as a multiple of the power 10^8.

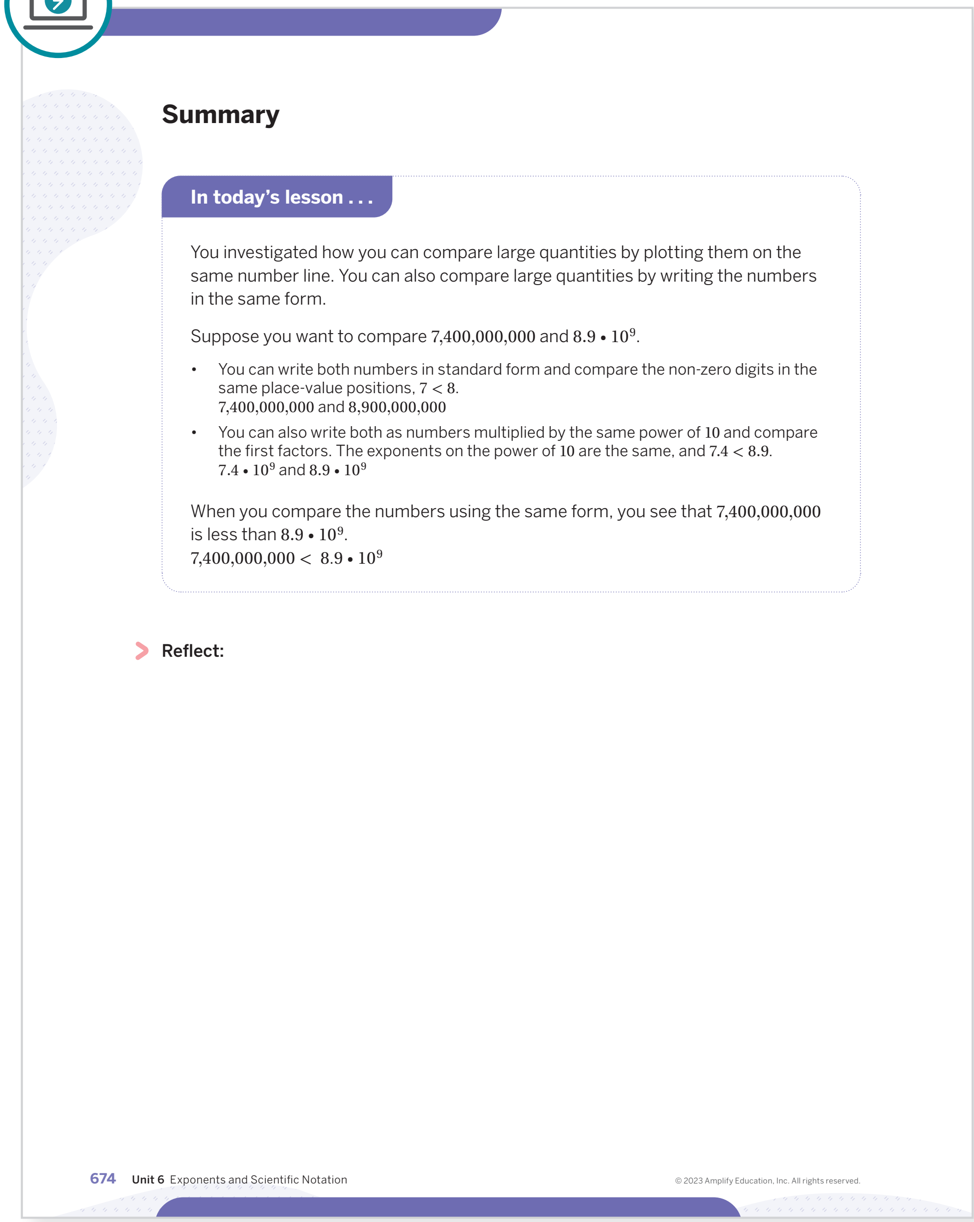
Extension: *Interdisciplinary Connections, Math Enrichment*

Students may have learned about the speed of light in their science classes. Be sure they understand that the speed of light is only the constant 300,000 km per second (186,000 miles per second) in a *vacuum*. This term refers to a space without matter or air. If there are any particles in that space, including dust, it can cause light to slow down as it comes into contact with those particles. Have students determine the percent decrease of the speed of light in a vacuum for each of the materials shown in the activity. **(Science)**
Water: 25% decrease; Olive oil: $33\frac{1}{3}$% decrease; Ice: $23\frac{1}{3}$% decrease; Diamond: $58\frac{2}{3}$% decrease.

Whole Class | 5 min

Summary

Review and synthesize how expressing large numbers as multiples of a power of 10 can help to compare them.



Summary

In today's lesson . . .

You investigated how you can compare large quantities by plotting them on the same number line. You can also compare large quantities by writing the numbers in the same form.

Suppose you want to compare $7{,}400{,}000{,}000$ and $8.9 \cdot 10^9$.

- You can write both numbers in standard form and compare the non-zero digits in the same place-value positions, $7 < 8$.
  $7{,}400{,}000{,}000$ and $8{,}900{,}000{,}000$
- You can also write both as numbers multiplied by the same power of 10 and compare the first factors. The exponents on the power of 10 are the same, and $7.4 < 8.9$.
  $7.4 \cdot 10^9$ and $8.9 \cdot 10^9$

When you compare the numbers using the same form, you see that $7{,}400{,}000{,}000$ is less than $8.9 \cdot 10^9$.

$7{,}400{,}000{,}000 < 8.9 \cdot 10^9$

Reflect:

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Synthesize

Have students share their strategies on how to compare large quantities written in different forms.

Highlight that writing large numbers using powers of 10 can help make comparing their values a more efficient process.

Ask:

- "What are some ways you can write $650{,}000{,}000$ using powers of 10?" $650 \cdot 10^6$, $65 \cdot 10^7$, $6.5 \cdot 10^8$
- "How can you find the value of $5.4 \cdot 10^5$?" $5.4 \cdot 100{,}000 = 540{,}000$
- "Which value is greater: $650{,}000{,}000$ or $5.4 \cdot 10^5$?" How can you tell without writing the numbers in expanded form? $650{,}000{,}000$; When written as multiples of a power of 10, the exponent on the power on 10 for $650{,}000{,}000$ is greater than the exponent on the power of 10 for $540{,}000$.
- There are many ways to write a number as a multiple of a power of 10. Do you see any advantages to having a common, unified way of writing numbers as powers of 10? Answers may vary.

Reflect

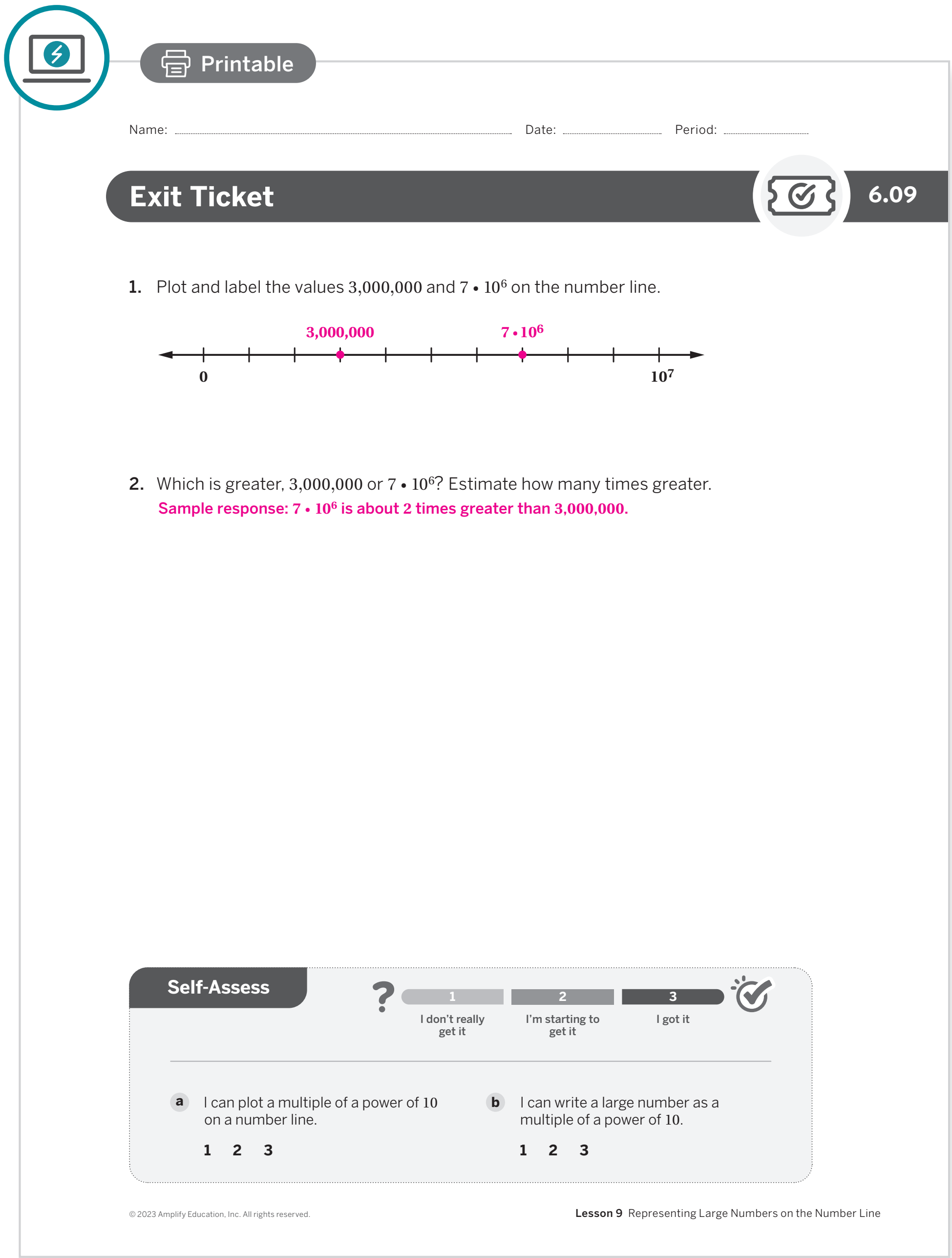
After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies can be used when working with very large numbers?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using a number line to plot and compare large numbers written in different forms.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Name: ______ Date: ______ Period: ______

Exit Ticket 6.09

1. Plot and label the values 3,000,000 and $7 \cdot 10^6$ on the number line.

2. Which is greater, 3,000,000 or $7 \cdot 10^6$? Estimate how many times greater.
Sample response: $7 \cdot 10^6$ is about 2 times greater than 3,000,000.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can plot a multiple of a power of 10 on a number line. 1 2 3

b I can write a large number as a multiple of a power of 10. 1 2 3

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Lesson 9 Representing Large Numbers on the Number Line

Success looks like . . .

- **Language Goal:** Comparing large numbers using powers of 10, and explaining the solution method. **(Speaking and Listening)**
  - » Comparing 3,000,000 and $7 \cdot 10^6$ in Problem 2.
- **Language Goal:** Using number lines to represent large numbers as multiples of powers of 10. **(Speaking and Listening, Reading and Writing)**
  - » Representing 3,000,000 and $7 \cdot 10^6$ on the number line in Problem 1.

Suggested next steps

If students write $7 \cdot 10^6$ as 7,000,000 or 3,000,000 as $3 \cdot 10^6$, but do not know how to plot them on the number line, consider:

- Marking and labeling the tick marks on the number line, and then asking them to plot the points.
- Reviewing Activity 2.
- Revisiting this problem after Lesson 12 where students will be introduced to scientific notation.

If students do not know how many times greater $7 \cdot 10^6$ is than 3,000,000, consider:

- Having them evaluate $7 \cdot 10^6$, and then divide the two values to compare them.
- Using the number line to compare the line segments and distances.

Professional Learning

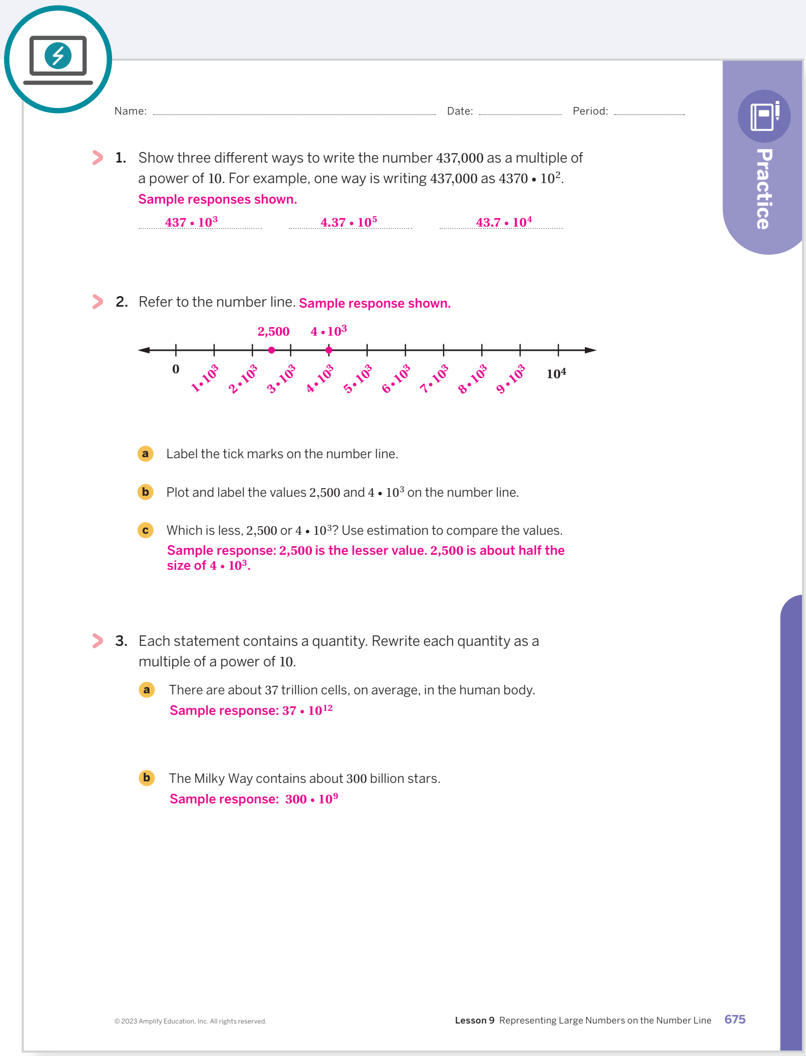
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did Activity 1 go as planned?
- In what ways in Activity 1 did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Show three different ways to write the number 437,000 as a multiple of a power of 10. For example, one way is writing 437,000 as $4370 \cdot 10^2$.
   Sample responses shown.
   $437 \cdot 10^3$ $4.37 \cdot 10^5$ $43.7 \cdot 10^4$

2. Refer to the number line. **Sample response shown.**

   a. Label the tick marks on the number line.
   b. Plot and label the values 2,500 and $4 \cdot 10^3$ on the number line.
   c. Which is less, 2,500 or $4 \cdot 10^3$? Use estimation to compare the values.
   Sample response: 2,500 is the lesser value. 2,500 is about half the size of $4 \cdot 10^3$.

3. Each statement contains a quantity. Rewrite each quantity as a multiple of a power of 10.
   a. There are about 37 trillion cells, on average, in the human body.
   Sample response: $37 \cdot 10^{12}$
   b. The Milky Way contains about 300 billion stars.
   Sample response: $300 \cdot 10^9$

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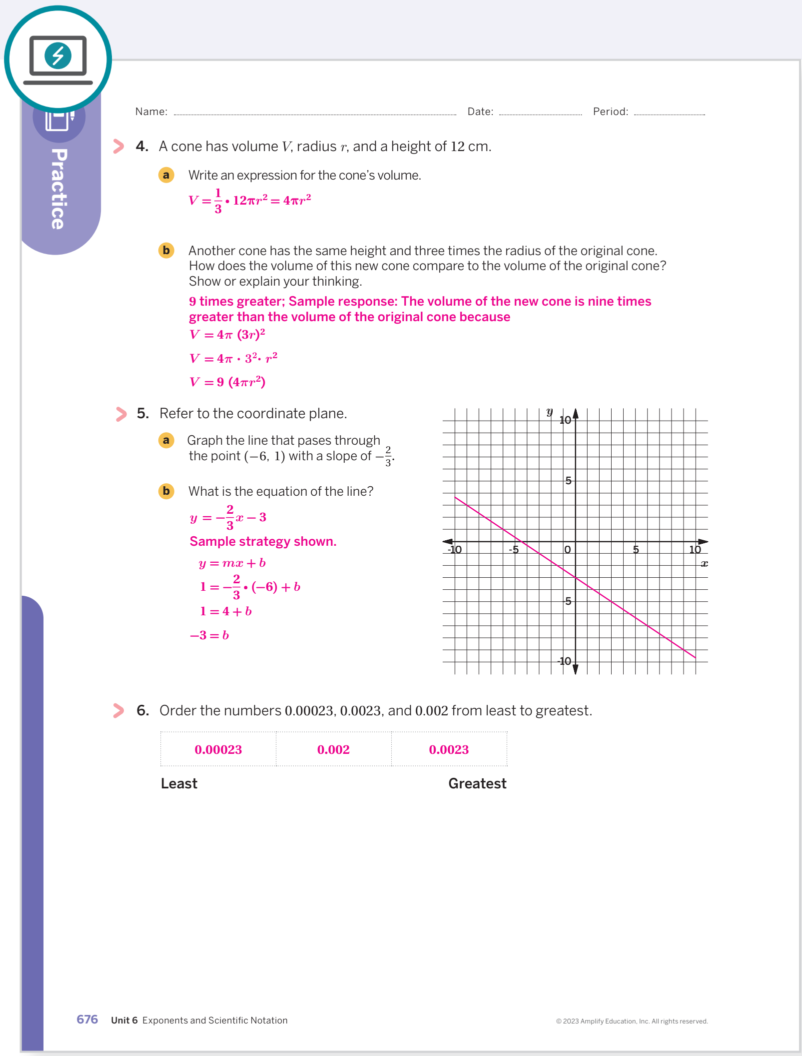
Lesson 9 Representing Large Numbers on the Number Line 675



Practice

Name: Date: Period:

4. A cone has volume V, radius r, and a height of 12 cm.
   a. Write an expression for the cone's volume.
   $V = \frac{1}{3} \cdot 12\pi r^2 = 4\pi r^2$
   b. Another cone has the same height and three times the radius of the original cone. How does the volume of this new cone compare to the volume of the original cone? Show or explain your thinking.
   9 times greater; Sample response: The volume of the new cone is nine times greater than the volume of the original cone because
   $V = 4\pi\ (3r)^2$
   $V = 4\pi \cdot 3^2 \cdot r^2$
   $V = 9\ (4\pi r^2)$

5. Refer to the coordinate plane.
   a. Graph the line that pases through the point $(-6, 1)$ with a slope of $-\frac{2}{3}$.
   b. What is the equation of the line?
   $y = -\frac{2}{3}x - 3$
   Sample strategy shown.
   $y = mx + b$
   $1 = -\frac{2}{3} \cdot (-6) + b$
   $1 = 4 + b$
   $-3 = b$

6. Order the numbers 0.00023, 0.0023, and 0.002 from least to greatest.

0.00023	0.002	0.0023

Least **Greatest**

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 2	1
Spiral	4	Unit 5 Lesson 20	2
	5	Unit 3 Lesson 13	1
Formative ⚡	6	Unit 6 Lesson 10	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 6 | **LESSON 10**

Representing Small Numbers on the Number Line

Let's visualize small numbers on the number line using powers of 10.

Focus

Goals

1. **Language Goal:** Compare small numbers using powers of 10, and explain the solution method. **(Speaking and Listening)**
2. **Language Goal:** Use number lines to represent small numbers as multiples of powers of 10 with negative exponents. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students create viable arguments and critique the reasoning of others when discussing how to represent powers of 10 with negative exponents on a number line. They attend to precision when deciding how to plot numbers correctly on a number line.

Previously

In Lesson 9, students used the number line and positive exponents to explore very large numbers.

Coming Soon

In Lesson 11, students will apply their knowledge of exponents and powers of 10 to reason and model with mathematics.

Rigor

- Students build **conceptual understanding** of small numbers using powers of 10.
- Students **apply** their understanding of small numbers to compare deadly animals.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *base*
- *equivalent expressions*
- *exponent*
- *power*

Amps | Featured Activity

Activity 1
Collaborative Number Line

Students place numbers on a number line. You can overlay the number lines, seeing the accuracy and precision of students' placement of values.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might rush through their work, without concern for the precision of the placement of the numbers on a number line. As students discuss the correct placement, encourage them to seek to be as precise as possible, both with where they locate the number and with their language. By taking more time to be precise, students will also slow down, resulting in a better final product.

Modifications to Pacing

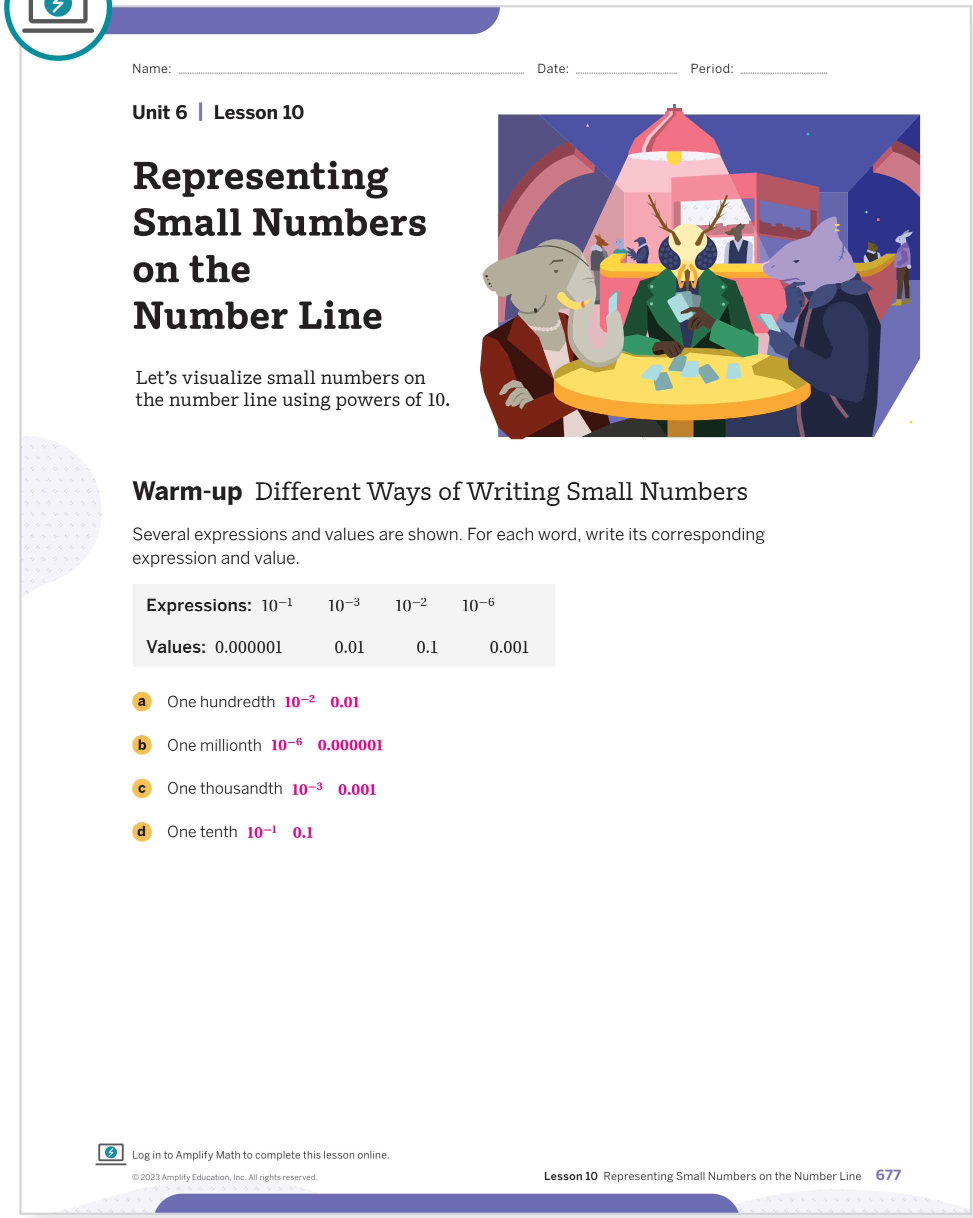
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, omit Problem 1 and have students complete Problem 2 using Andre's number line.

Pairs | 5 min

Warm-up Matching Small Numbers

Students match words and values to their respective powers of 10 to prepare them for reasoning about the size of small numbers.



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Unit 6 | **Lesson 10**

Representing Small Numbers on the Number Line

Let's visualize small numbers on the number line using powers of 10.

Warm-up Different Ways of Writing Small Numbers

Several expressions and values are shown. For each word, write its corresponding expression and value.

Expressions: 10^{-1} 10^{-3} 10^{-2} 10^{-6}

Values: 0.000001 0.01 0.1 0.001

a. One hundredth 10^{-2} 0.01

b. One millionth 10^{-6} 0.000001

c. One thousandth 10^{-3} 0.001

d. One tenth 10^{-1} 0.1

Log in to Amplify Math to complete this lesson online.

Lesson 10 Representing Small Numbers on the Number Line 677

1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by asking them how to write 10^{-1} as a single power with a positive exponent.

Look for points of confusion:

- **Thinking 0.001 is equivalent to one hundredth.** Display a place-value chart showing words matching the digit placement.
- **Writing each expression as a fraction, but being unsure of its decimal equivalent.** Ask students to say the fraction aloud. For example, $\frac{1}{100}$ means *one hundredth*, which is 0.01.

Look for productive strategies:

- Writing the expression as a fraction or single, positive power.
- Noticing that the exponent affects how many digits there are after the decimal point in the value.

3 Connect

Have students share the corresponding expressions, words, and values. Record responses for all to see.

Ask, "How does the exponent affect the value?"

Highlight that the exponent represents the amount of factors of $\frac{1}{10}$. The expression 10^{-3} represents a lesser value than 10^{-2} because it has three factors of $\frac{1}{10}$, while 10^{-2} has two factors of $\frac{1}{10}$.

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Provide students with a two-column table, such as the one shown, that highlights how to write each expression as a fraction. As students complete the table, encourage them to say the fraction aloud. This will help them connect the expression to the decimal form.

Expression	Fraction
10^{-1}	$\frac{1}{10}$
10^{-2}	
10^{-3}	
10^{-6}	

Power-up

To power up students' ability to order decimal values, ask:

Complete each statement with <, >, or =.

1. $0.25 \boxed{>} 0.20$

2. $0.25 \boxed{>} 0.025$

3. $0.25 \boxed{=} 0.250$

4. $0.025 \boxed{<} 0.20$

Use: Before Activity 1

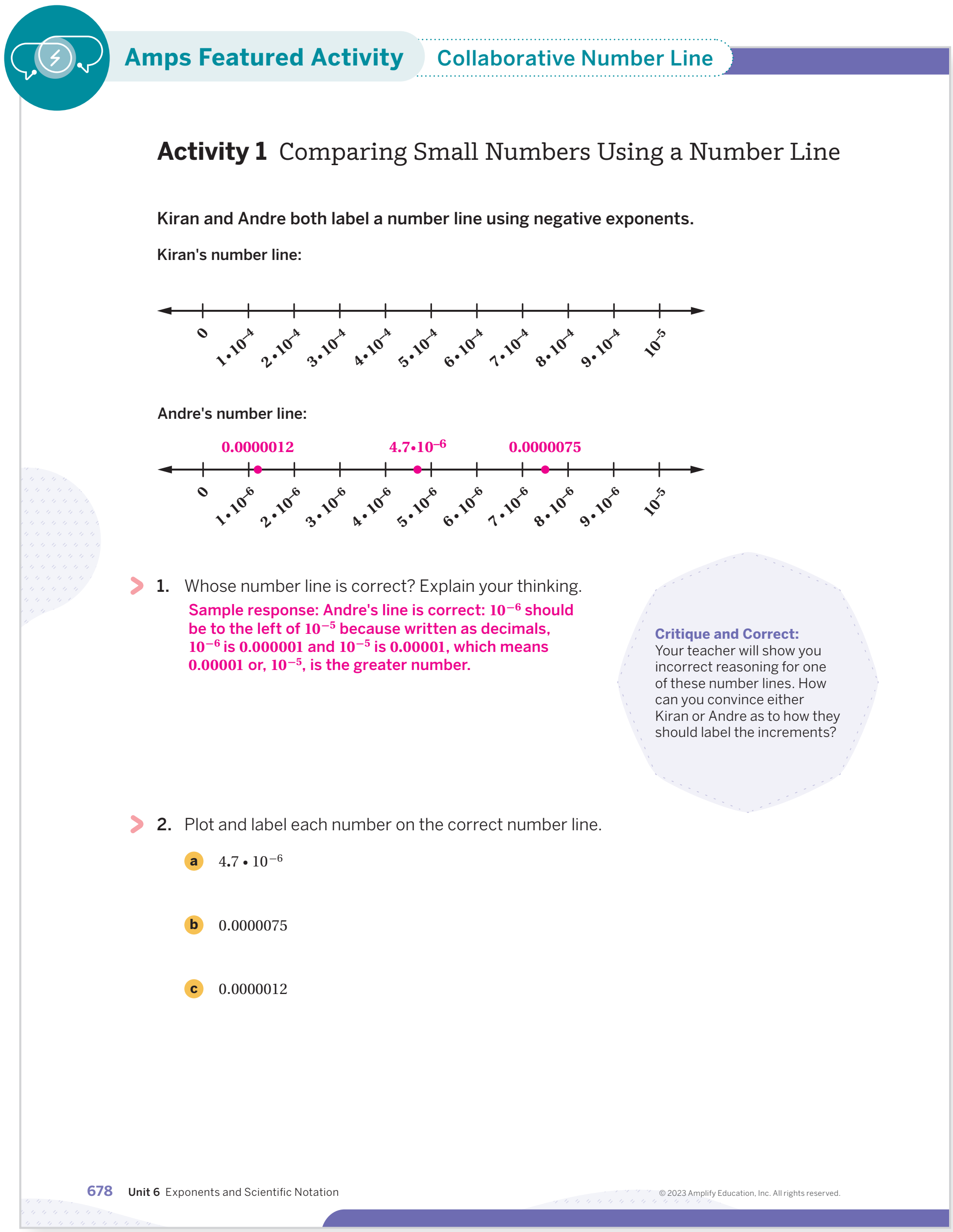
Informed by: Performance on Lesson 9, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 15 min

Activity 1 Comparing Small Numbers Using a Number Line

Students critique the reasoning of others to reason about the locations of the values of expressions with negative exponents on a number line.

Amps Featured Activity Collaborative Number Line



Activity 1 Comparing Small Numbers Using a Number Line

Kiran and Andre both label a number line using negative exponents.

Kiran's number line:

Andre's number line:

1. Whose number line is correct? Explain your thinking.

Sample response: Andre's line is correct: 10^{-6} should be to the left of 10^{-5} because written as decimals, 10^{-6} is 0.000001 and 10^{-5} is 0.00001, which means 0.00001 or, 10^{-5}, is the greater number.

Critique and Correct: Your teacher will show you incorrect reasoning for one of these number lines. How can you convince either Kiran or Andre as to how they should label the increments?

2. Plot and label each number on the correct number line.

a. $4.7 \cdot 10^{-6}$

b. 0.0000075

c. 0.0000012

678 **Unit 6** Exponents and Scientific Notation

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking them if the numbers to the left are less than or greater than 10^{-5}. Have students check their response by evaluating $2 \cdot 10^{-4}$ and $2 \cdot 10^{-6}$ and comparing them to 10^{-5}.

Look for points of confusion:

- **Thinking that Kiran's line is correct**. Have them evaluate $5 \cdot 10^{-4}$ and 10^{-4}, and then compare the values.
- **Being unsure of where to plot the values in Problems 2b and 2c**. Have students write the number as a multiple of a power of 10. If they still struggle, ask them to find the missing value in the expression $? \cdot 10^{-6}$.

Look for productive strategies:

- Comparing the exponents and noticing that $-5 > -6$, which means $10^{-5} > 10^{-6}$.
- Evaluating the expressions.
- Writing Problems 2b and 2c as multiples of a power of 10^{-6}.

3 Connect

Have pairs of students share their answers. Use the ***Poll the Class*** routine to see who agreed with Kiran or Andre. Have students who chose Andre's number line convince the other students that this number line is correct.

Ask, "How did you determine where to plot 0.0000075 and 0.0000012?" Listen and point out ways in which students determined the precise placements of the values.

Highlight that if powers have the same base and different negative exponents, students can compare the absolute value of the exponents to determine which will be the greater value.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can plot numbers on a digital number line. You can overlay the number lines to see the accuracy and precision of students' placement of values.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present an incorrect reasoning for Kiran's number line, such as, "Numbers to the left of 10^{-5} are less than 10^{-5}, so I wrote 10^{-5} as $10 \cdot 10^{-4}$ because $4 < 5$. Then the factors in the other labels should start at 1 and go to 9." Ask:

- ***Critique:*** "Do you agree or disagree with Kiran's reasoning? Explain your thinking. Are any parts of his reasoning correct?"
- ***Correct and Clarify:*** "How can you convince Kiran as to how he should label the increments?"

English Learners

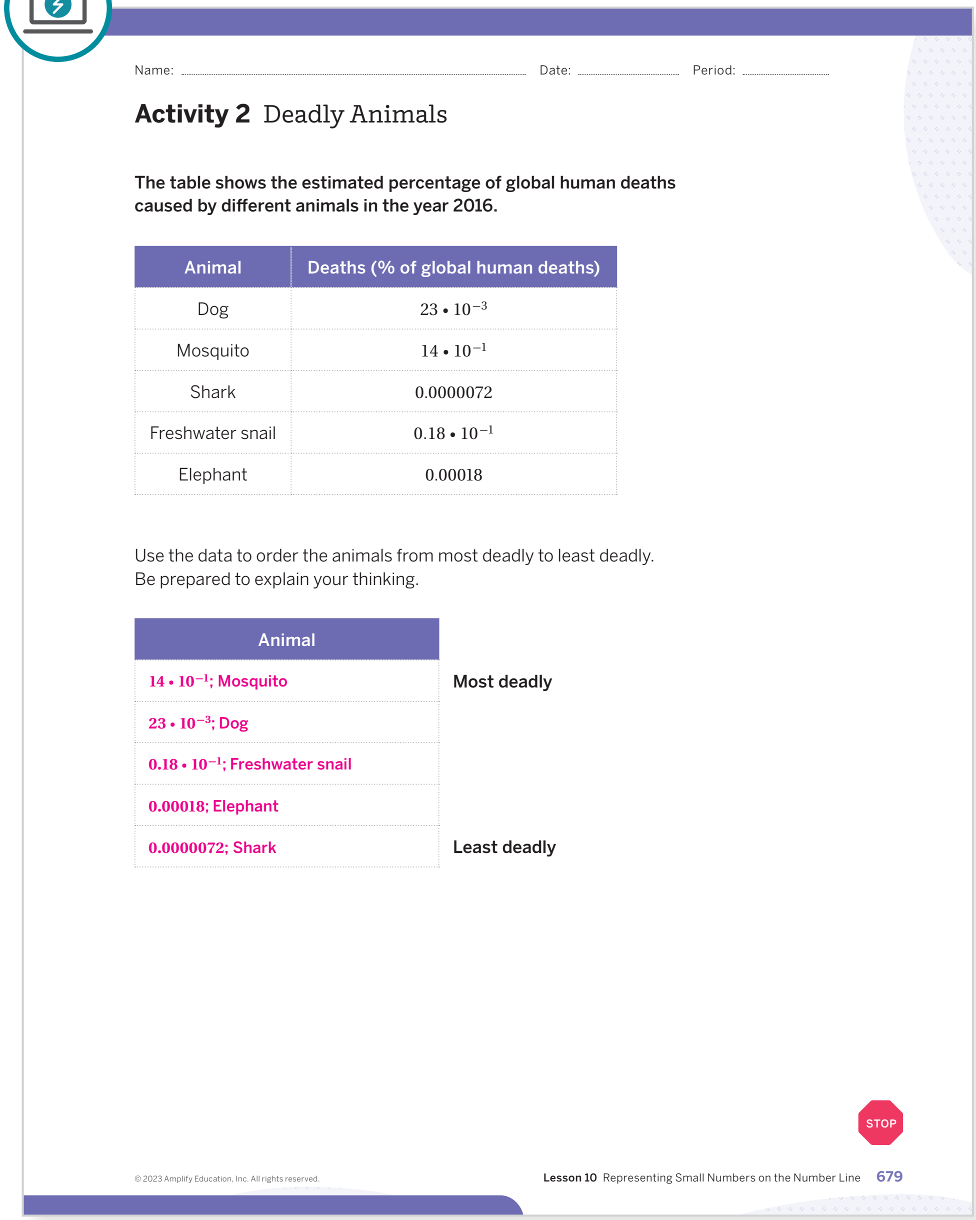
Provide students with sentence frames to help organize their thinking, such as:

- "I agree with Kiran when he says . . ."
- "I disagree with Kiran when he says . . ."

Pairs | 15 min

Activity 2 Deadly Animals

Students order numbers, some written as a multiple of a power of 10, to create a need for a unified notation to compare small numbers.



Name: Date: Period:

Activity 2 Deadly Animals

The table shows the estimated percentage of global human deaths caused by different animals in the year 2016.

Animal	Deaths (% of global human deaths)
Dog	$23 \bullet 10^{-3}$
Mosquito	$14 \bullet 10^{-1}$
Shark	0.0000072
Freshwater snail	$0.18 \bullet 10^{-1}$
Elephant	0.00018

Use the data to order the animals from most deadly to least deadly. Be prepared to explain your thinking.

Animal	
$14 \bullet 10^{-1}$; Mosquito	**Most deadly**
$23 \bullet 10^{-3}$; Dog	
$0.18 \bullet 10^{-1}$; Freshwater snail	
0.00018; Elephant	
0.0000072; Shark	**Least deadly**

STOP

Lesson 10 Representing Small Numbers on the Number Line 679

1 Launch

Activate background knowledge by asking students, "Which animal from the list do you think is the most dangerous to humans?"

Say, "To order the animals, you will compare the percentage of deaths caused by the animal either from direct contact or transmission of disease. The most deadly animal will be the one with the most cases, or the greatest number of deaths."

2 Monitor

Help students get started by having them write the numbers in the same form to compare them.

Look for points of confusion:

- **Only comparing the first factor of the numbers written as a multiple of a power of 10.** Have students evaluate $14 \bullet 10^{-1}$ and $23 \bullet 10^{-3}$ to see that this method does not work.
- **Not knowing how to evaluate $23 \bullet 10^{-3}$.** Remind students that $10^{-3} = \frac{1}{10^3}$ and multiplying by $\frac{1}{10}$ affects the placement of the decimal point.

Look for productive strategies:

- Writing the numbers using the same power of 10.
- Writing the first factor in the numbers written as a multiple of a power of 10 to have the same decimal value.

3 Connect

Have pairs of students share the most and least deadliest animal. Have them share their strategies on how they compared the values.

Highlight that writing small numbers as a multiple of a power of 10 makes the values easier to compare.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Before students begin ordering the animals, remind them to rewrite the numbers in the same form. Ask, "Will you write the numbers all as powers of 10? Or will you choose to write the numbers as decimals?" Consider demonstrating how to write one of the numbers in the other form.

Extension: *Interdisciplinary Connections*

Have students predict the animal they think causes the most number of human deaths on Earth. Ask, "Do you think that animal is one of the ones listed in this activity?" Then tell them that mosquitoes cause the most number of human deaths, more than 1 million per year, according to the World Health Organization. **(Science)**

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share the strategies they used to compare the values, display these sentence frames to help them organize their thinking, such as:

- "First, I ___, then I . . ."
- "I noticed that ___, so I . . ."

If no one mentions how to compare the values for the mosquito and the freshwater snail, ask, "What did you notice about the powers of 10? How did that help you compare the values?"

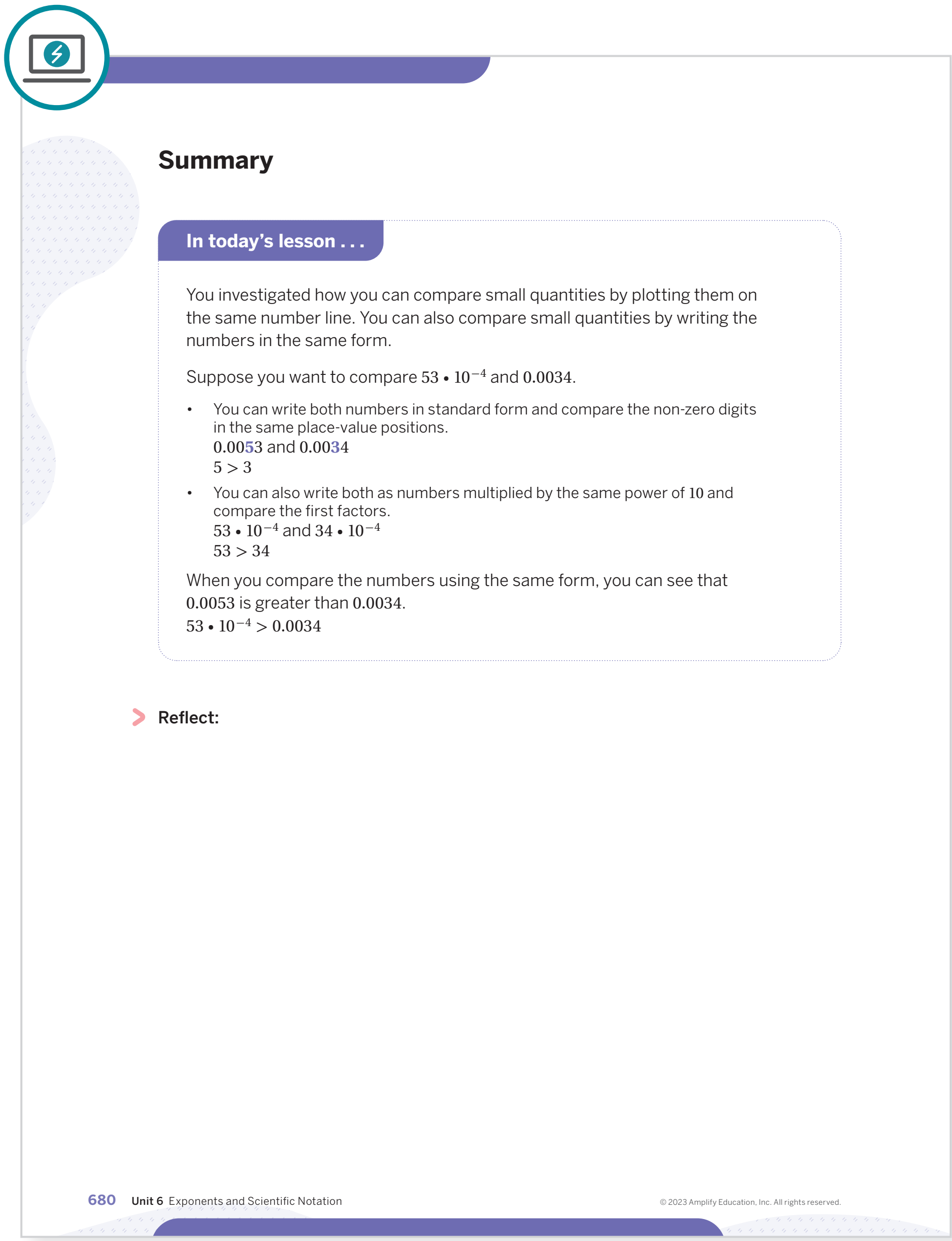
English Learners

Encourage students to refer to and use language from the class display as they describe their strategies for comparing the values.

Whole Class | 5 min

Summary

Review and synthesize how expressing small numbers as multiples of a power of 10 can help to compare them.



Summary

In today's lesson . . .

You investigated how you can compare small quantities by plotting them on the same number line. You can also compare small quantities by writing the numbers in the same form.

Suppose you want to compare $53 \bullet 10^{-4}$ and 0.0034.

- You can write both numbers in standard form and compare the non-zero digits in the same place-value positions.
  0.0053 and 0.0034
  $5 > 3$
- You can also write both as numbers multiplied by the same power of 10 and compare the first factors.
  $53 \bullet 10^{-4}$ and $34 \bullet 10^{-4}$
  $53 > 34$

When you compare the numbers using the same form, you can see that 0.0053 is greater than 0.0034.
$53 \bullet 10^{-4} > 0.0034$

Reflect:

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Synthesize

Display expressions $53 \bullet 10^4$ and 0.0034.

Ask, "What strategies can you use to compare the expressions?"

Highlight that writing small numbers using powers of 10 can make the process of comparing values more efficient.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

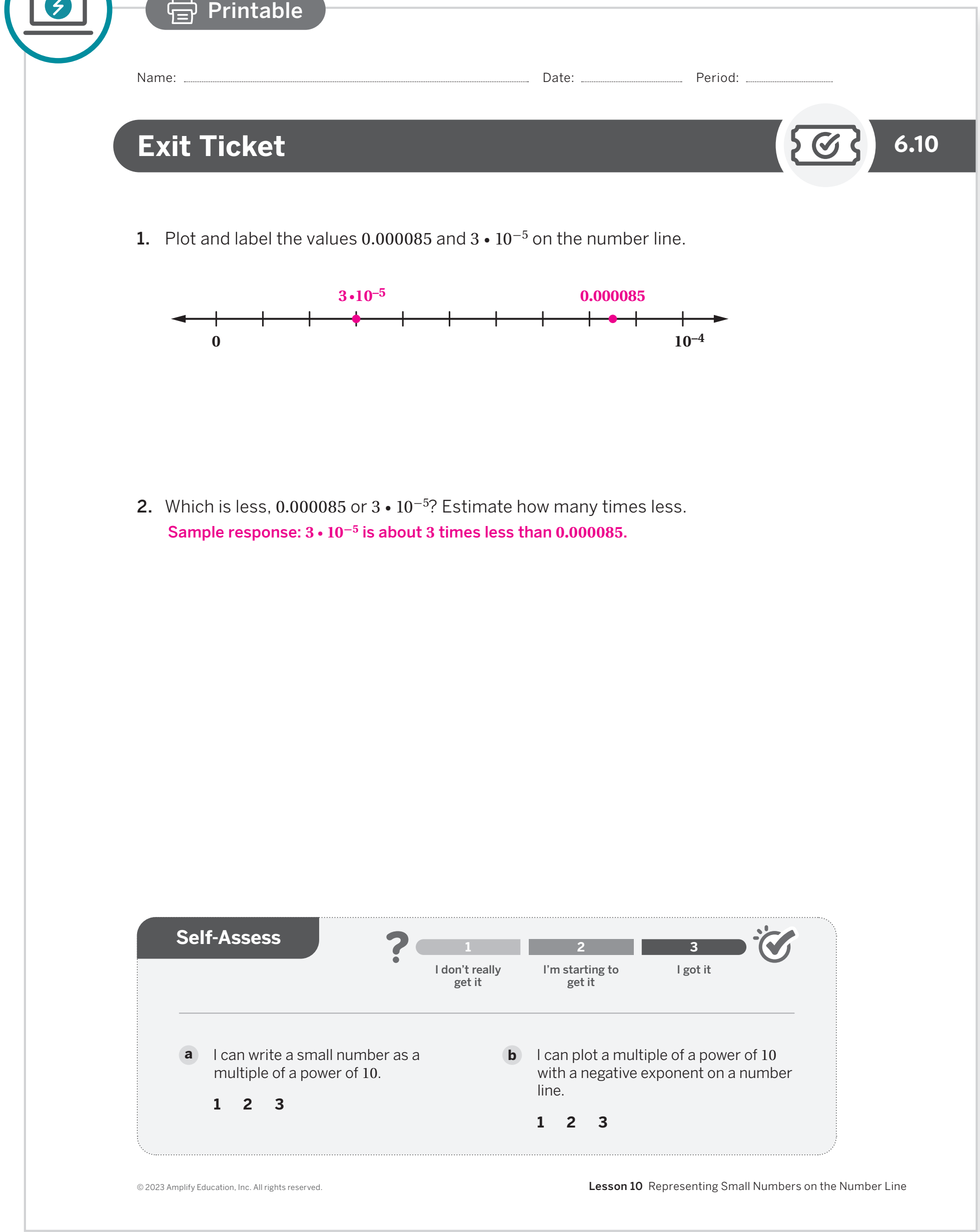
- "What strategies can be used when working with very small numbers?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using a number line to plot and compare small numbers written in different forms.

Printable



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Exit Ticket 6.10

1. Plot and label the values 0.000085 and $3 \cdot 10^{-5}$ on the number line.

2. Which is less, 0.000085 or $3 \cdot 10^{-5}$? Estimate how many times less.

   Sample response: $3 \cdot 10^{-5}$ is about 3 times less than 0.000085.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write a small number as a multiple of a power of 10.

1 2 3

b. I can plot a multiple of a power of 10 with a negative exponent on a number line.

1 2 3

Lesson 10 Representing Small Numbers on the Number Line

Success looks like . . .

- **Language Goal:** Comparing small numbers using powers of 10, and explaining the solution method. **(Speaking and Listening)**
  - » Comparing 0.000085 and $3 \cdot 10^{-5}$ in Problem 2.
- **Language Goal:** Using number lines to represent small numbers as multiples of powers of 10 with negative exponents. **(Speaking and Listening, Reading and Writing)**
  - » Representing 0.000085 and $3 \cdot 10^{-5}$ on the number line in Problem 1.

Suggested next steps

If students do not plot the values on the number line correctly or identify the smaller value, consider:

- Marking and labeling the tick marks on the number line, and then asking them to plot the points and identify the smaller value.
- Reviewing Activity 1.

If students do not know how many times smaller $3 \cdot 10^{-5}$ is than 0.000085, consider:

- Having them evaluate $3 \cdot 10^{-5}$, and then divide the values.
- Using the number line to compare the line segments and distances.

Professional Learning

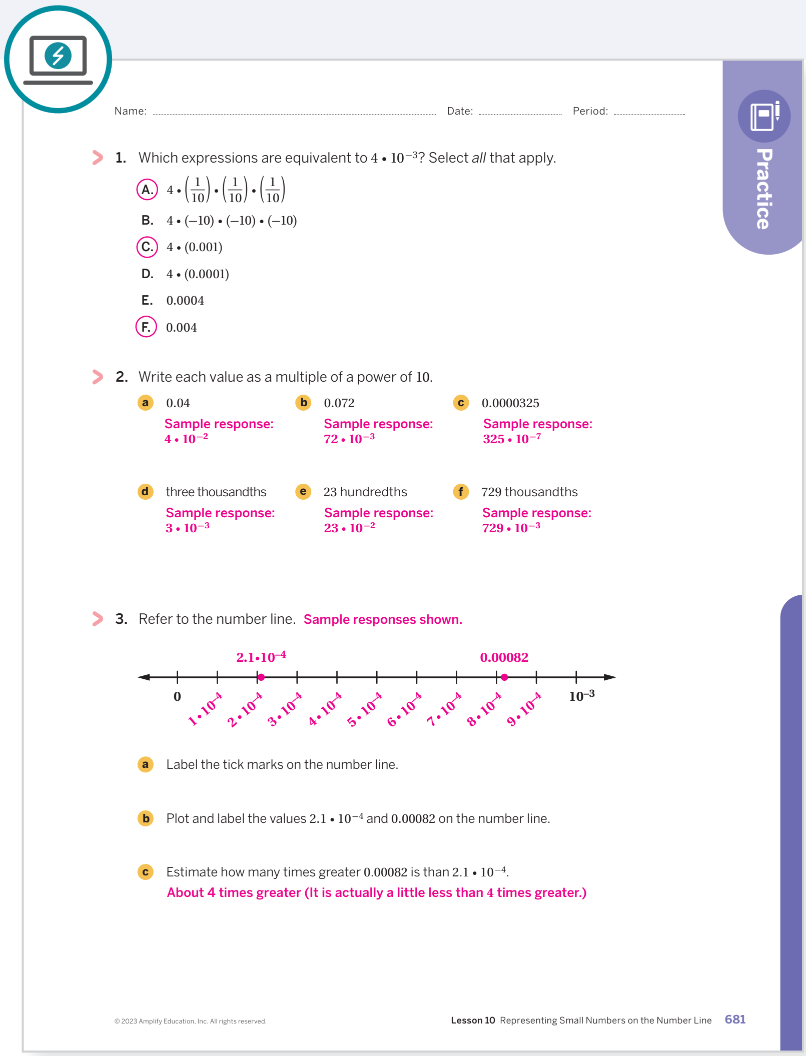
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Who participated and who didn't participate in Activity 2 today? What trends do you see in participation?
- The focus of this lesson was to compare small numbers. How did comparing small numbers go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

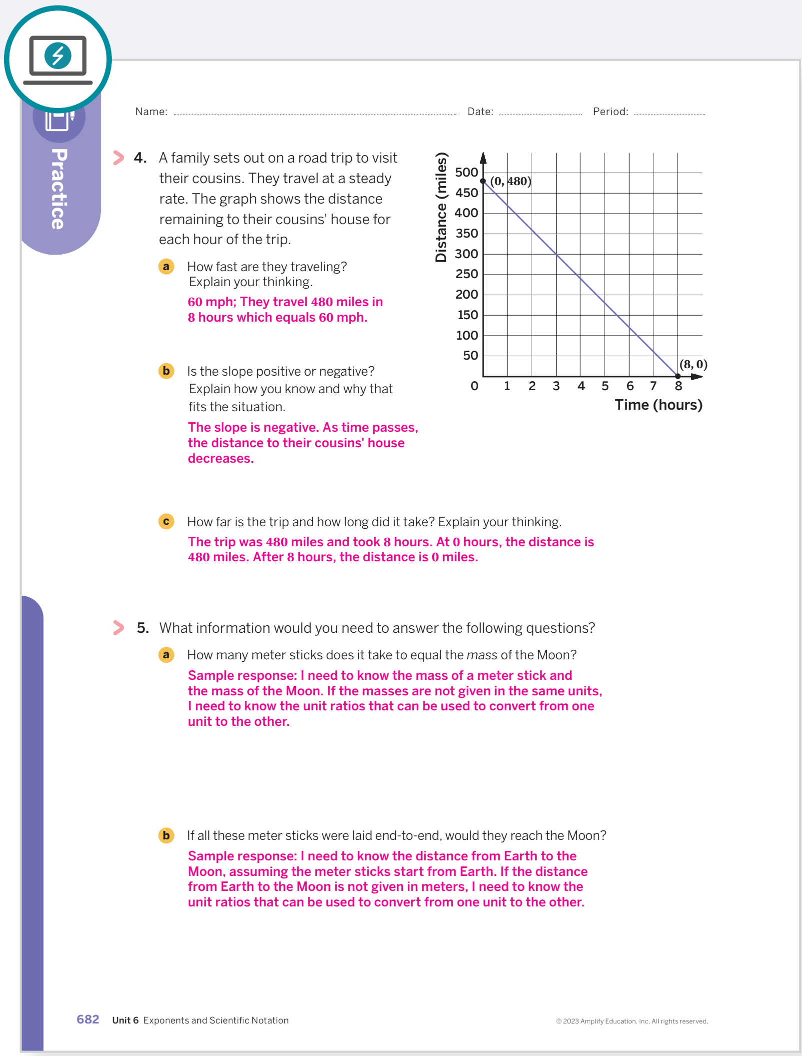
1. Which expressions are equivalent to $4 \cdot 10^{-3}$? Select *all* that apply.
   - [✓ A.] $4 \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right)$
   - B. $4 \cdot (-10) \cdot (-10) \cdot (-10)$
   - [✓ C.] $4 \cdot (0.001)$
   - D. $4 \cdot (0.0001)$
   - E. 0.0004
   - [✓ F.] 0.004

2. Write each value as a multiple of a power of 10.
   - a 0.04 Sample response: $4 \cdot 10^{-2}$
   - b 0.072 Sample response: $72 \cdot 10^{-3}$
   - c 0.0000325 Sample response: $325 \cdot 10^{-7}$
   - d three thousandths Sample response: $3 \cdot 10^{-3}$
   - e 23 hundredths Sample response: $23 \cdot 10^{-2}$
   - f 729 thousandths Sample response: $729 \cdot 10^{-3}$

3. Refer to the number line. **Sample responses shown.**

   - a Label the tick marks on the number line.
   - b Plot and label the values $2.1 \cdot 10^{-4}$ and 0.00082 on the number line.
   - c Estimate how many times greater 0.00082 is than $2.1 \cdot 10^{-4}$.
     About 4 times greater (It is actually a little less than 4 times greater.)

Lesson 10 Representing Small Numbers on the Number Line 681



Name: Date: Period:

Practice

4. A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance remaining to their cousins' house for each hour of the trip.

   - a How fast are they traveling? Explain your thinking.
     60 mph; They travel 480 miles in 8 hours which equals 60 mph.
   - b Is the slope positive or negative? Explain how you know and why that fits the situation.
     The slope is negative. As time passes, the distance to their cousins' house decreases.
   - c How far is the trip and how long did it take? Explain your thinking.
     The trip was 480 miles and took 8 hours. At 0 hours, the distance is 480 miles. After 8 hours, the distance is 0 miles.

5. What information would you need to answer the following questions?
   - a How many meter sticks does it take to equal the *mass* of the Moon?
     Sample response: I need to know the mass of a meter stick and the mass of the Moon. If the masses are not given in the same units, I need to know the unit ratios that can be used to convert from one unit to the other.
   - b If all these meter sticks were laid end-to-end, would they reach the Moon?
     Sample response: I need to know the distance from Earth to the Moon, assuming the meter sticks start from Earth. If the distance from Earth to the Moon is not given in meters, I need to know the unit ratios that can be used to convert from one unit to the other.

682 **Unit 6** Exponents and Scientific Notation

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 13	2
Formative ⚡	5	Unit 6 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 6 | **LESSON 11**

Applications of Arithmetic With Powers of 10

Let’s use powers of 10 to help us make calculations with large numbers.

Focus

Goals

1. **Language Goal:** Determine what information is needed to solve problems involving large numbers, and explain how that information would help solve the problem. **(Speaking and Listening, Writing)**
2. **Language Goal:** Use exponent rules and powers of 10 to solve problems in context, and explain the steps used to organize thinking. **(Speaking and Listening)**

Coherence

Today

Students apply what they have learned about working with exponents (especially powers of 10) to solve rich problems in context. The style of questioning requires students to identify essential features of the problem and persevere to calculate and interpret the solutions in context.

Previously

Students learned the rules of exponents and practiced using powers of 10 when working with large and small quantities.

Coming Soon

In Lesson 12, students will learn how to write numbers in scientific notation using powers of 10. In Lessons 13 and 14, students will learn techniques for multiplying, dividing, adding, and subtracting numbers written in scientific notation to determine exact and approximate values.

Rigor

- Students **apply** their understanding of exponents when they make sense of information to solve a problem in context.

Pacing Guide

Suggested Total Lesson Time | ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	12 min	20 min	7 min	3 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per group
- Activity 2 PDF, one per group
- calculators

Math Language Development

Review words

- *base*
- *equivalent expressions*
- *exponent*
- *power*

Amps | Featured Activity

Activity 1
Interactive Scale

Students test their answers in real time to see if they can determine the number of pennies to balance the scale with the Burj Khalifa.

Building Math Identity and Community

Connecting to Mathematical Practices

While the task might be simple for some to envision, others may have trouble making sense of the task because they are not familiar with the Burj Khalifa or Empire State Building. In a large group, have students discuss any experience or knowledge that they have about the two buildings. Encourage them to consider other students' perspectives and use language that will help others "experience" the buildings with them.

Modifications to Pacing

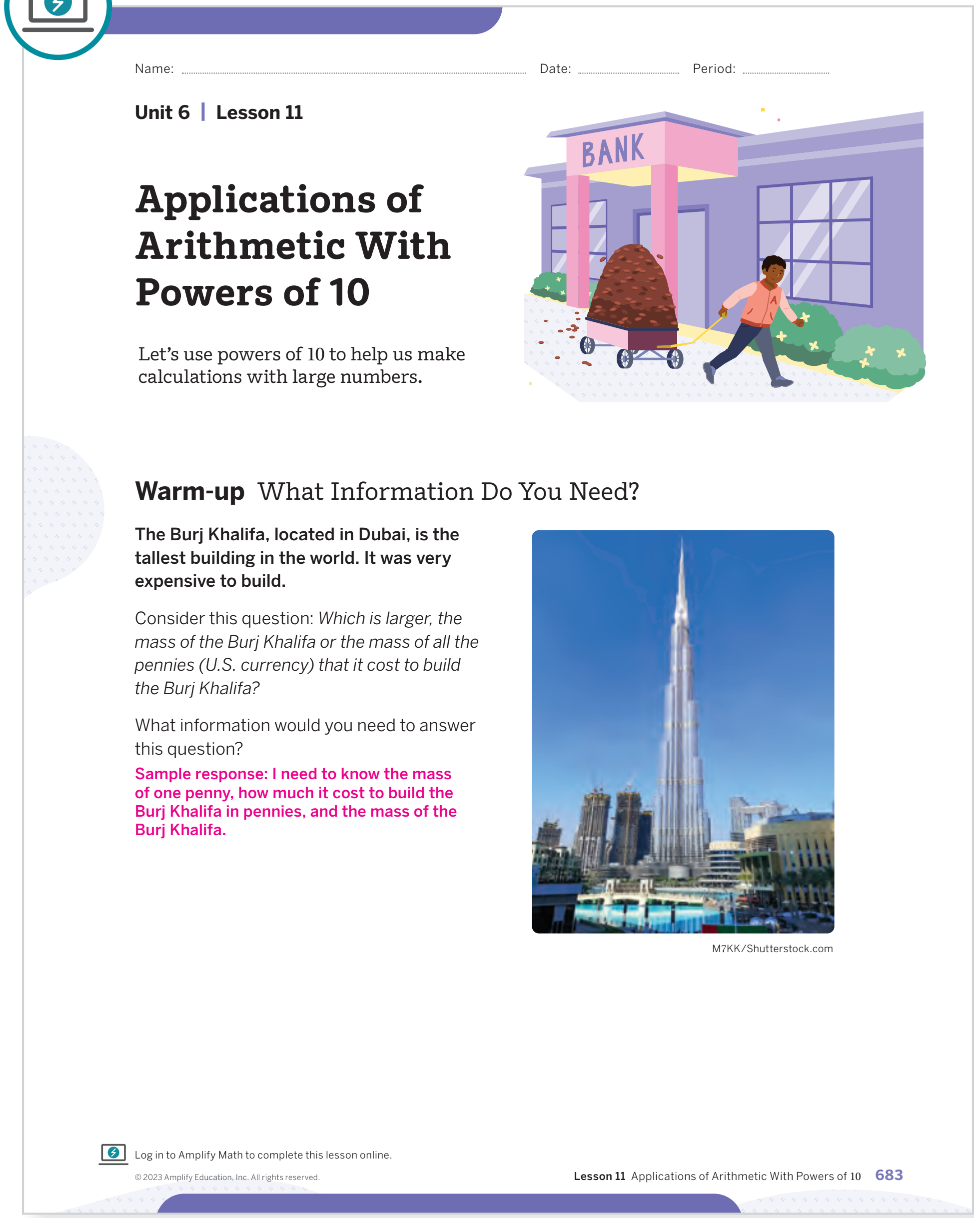
You may want to consider this additional modification if you are short on time:

- Select either **Activity 1** or **Activity 2** to complete.

Pairs | 5 min

Warm-up What Information Do You Need?

Students reason about a real-world problem and consider the essential information required to solve it.



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Unit 6 | Lesson 11

Applications of Arithmetic With Powers of 10

Let's use powers of 10 to help us make calculations with large numbers.

Warm-up What Information Do You Need?

The Burj Khalifa, located in Dubai, is the tallest building in the world. It was very expensive to build.

Consider this question: *Which is larger, the mass of the Burj Khalifa or the mass of all the pennies (U.S. currency) that it cost to build the Burj Khalifa?*

What information would you need to answer this question?

Sample response: I need to know the mass of one penny, how much it cost to build the Burj Khalifa in pennies, and the mass of the Burj Khalifa.

M7KK/Shutterstock.com

Log in to Amplify Math to complete this lesson online.

Lesson 11 Applications of Arithmetic With Powers of 10 683

1 Launch

Give students 1 minute of independent think time, followed by 2 minutes to share their responses with a partner.

2 Monitor

Help students get started by activating background knowledge and asking what they know, if anything, about the Burj Khalifa. Ask if they know how much it weighs and why that would be important information to know.

Look for points of confusion:

- **Not thinking of all the information they might need.** Validate what information they have already identified they will need and then ask them what else they might need to know (based on what is missing from their list).

3 Connect

Have students share their responses with the class.

Ask, "Why do you need that information? How would you use that information in finding the solution? Where would you look to find that information?"

Highlight that it is important to consider what information is required to solve a problem. Point out that students will work to solve this problem in the next activity.

MLR Math Language Development

MLR5: Co-craft Questions

Ask pairs of students to work together to co-craft questions about the information they need to answer the question in the Warm-up. Before the Connect, have them meet with another pair of students to compare questions and decide if they would like to revise their questions before sharing with the class. A sample question could be, "How much does one penny weigh?"

English Learners

Provide students time to rehearse and formulate a question before sharing with a partner.

Power-up

To power up students' ability to determine the necessary information to solve a problem, have students complete:

You are hoping to answer the question, "Will the number of school buses stacked end to end between Earth's surface and the Moon have a mass greater than or less than the Moon?" Determine what information you would need to know in order to answer this question. Select *all* that apply.

[✓ **A.**] The mass of the Moon.
[✓ **B.**] The mass of one bus.
C. The mass of Earth.
[✓ **D.**] The length of one bus.
E. The distance around the Moon.
[✓ **F.**] The distance between Earth and the Moon.

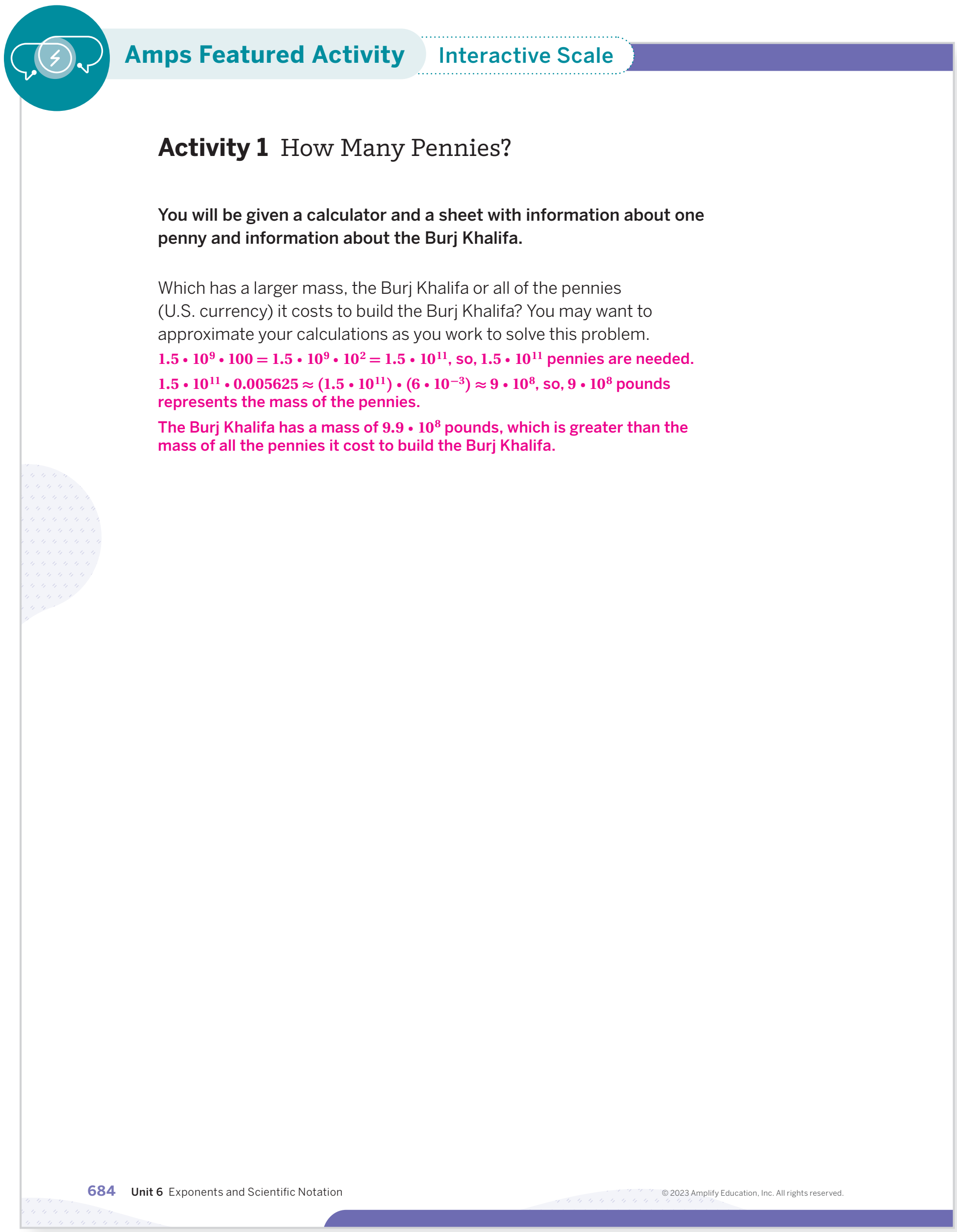
Use: Before the Warm-up
Informed by: Performance on Lesson 10, Practice Problem 5

Small Groups | 12 min

Activity 1 How Many Pennies?

Students make sense of given information and their knowledge of powers of 10 to solve a problem in context.

Amps Featured Activity Interactive Scale



Activity 1 How Many Pennies?

You will be given a calculator and a sheet with information about one penny and information about the Burj Khalifa.

Which has a larger mass, the Burj Khalifa or all of the pennies (U.S. currency) it costs to build the Burj Khalifa? You may want to approximate your calculations as you work to solve this problem.

$1.5 \cdot 10^9 \cdot 100 = 1.5 \cdot 10^9 \cdot 10^2 = 1.5 \cdot 10^{11}$, so, $1.5 \cdot 10^{11}$ pennies are needed.

$1.5 \cdot 10^{11} \cdot 0.005625 \approx (1.5 \cdot 10^{11}) \cdot (6 \cdot 10^{-3}) \approx 9 \cdot 10^8$, so, $9 \cdot 10^8$ pounds represents the mass of the pennies.

The Burj Khalifa has a mass of $9.9 \cdot 10^8$ pounds, which is greater than the mass of all the pennies it cost to build the Burj Khalifa.

684 **Unit 6** Exponents and Scientific Notation

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① Launch

Ask the class to predict which has a greater mass, the Burj Khalifa or all of the pennies it took to build the Burj Khalifa. Record these predictions. Distribute the Activity 1 PDF to each group.

② Monitor

Help students get started by displaying the calculations $1.5 \cdot 10^9 \cdot 100 = 1.5 \cdot 10^9 \cdot 10^2$ to help find the total number of pennies. Have students simplify the result.

Look for points of confusion:

- **Not being able to estimate without a calculator.** Have students round first, then rewrite any values using powers of 10 before calculating.

③ Connect

Ask, "Describe your thinking as you planned a solution path for this problem. Did you ask for information first and then decide what to do with it, or did you decide what needed to be done first and then ask for certain information?"

Display student work showing calculations using powers of 10.

Highlight strategies that included rounding first and using powers of 10 to make calculations more efficient. Point out that the problem indicates that approximations are acceptable.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can test their answers in real time to see if they can determine the number of pennies to balance the scale with the Burj Khalifa.

Accessibility: *Activate Background Knowledge*

Have students think of a more familiar scenario involving the comparison of an object's mass to the mass of the pennies it cost to buy the object. For example, hold up a bag of 100 pennies (or display an image) and hold up a standard pencil. Ask, "Which has more mass, the bag of 100 pennies or the pencil?" Once students have had a chance to visualize and consider the mass of the pennies, reveal the task.

Math Language Development

MLR8: Discussion Supports

While students work, let them know that everyone in their group should be able to explain the strategy they used to solve the problem. Display these questions and let students know they will need to be able to respond to them during the Connect.

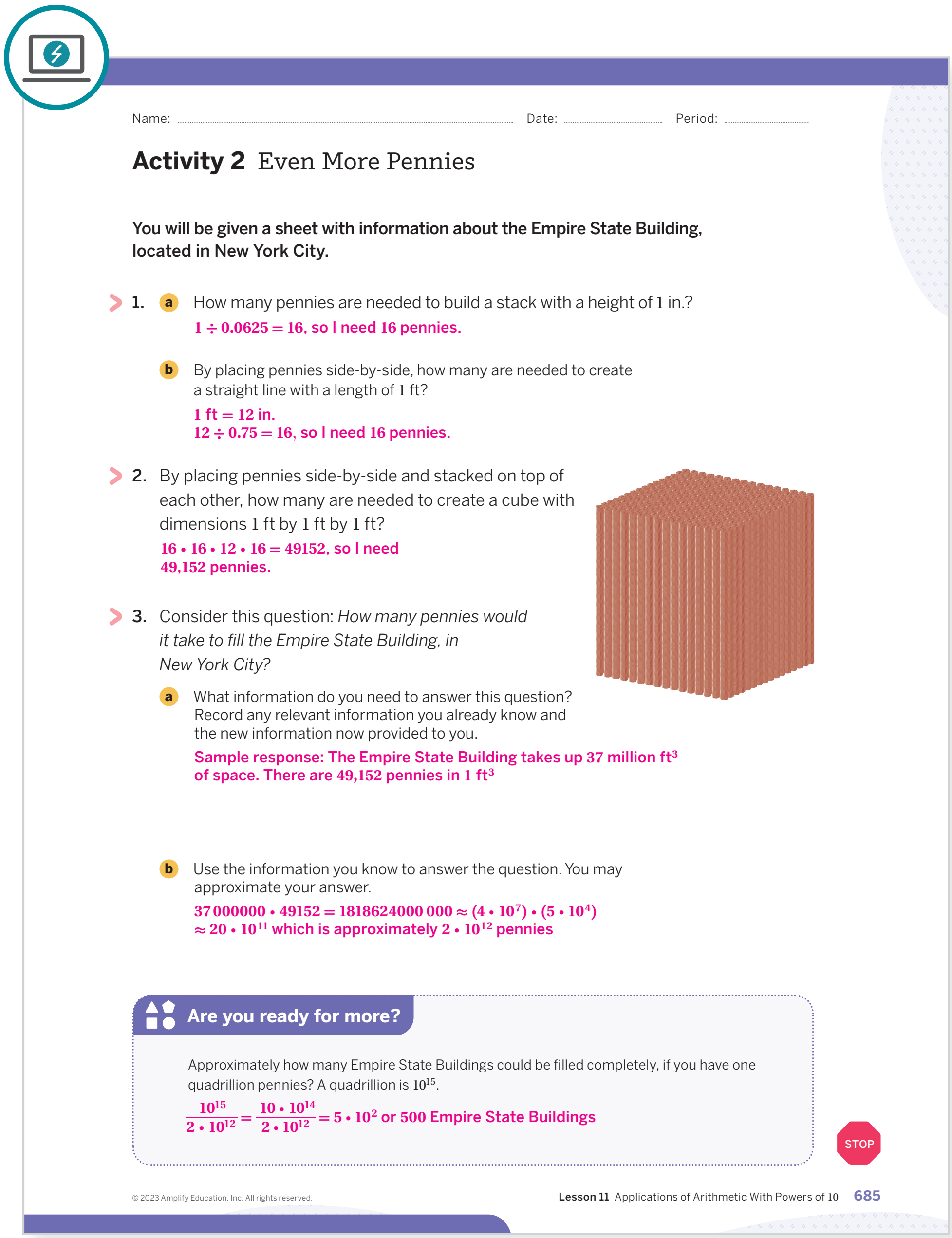
- "What was your first approach to this task? Why did you choose this approach?"
- "How did you use exponent rules to help you?"

During the Connect, vary who is called on to represent the ideas and strategies of each group.

Small Groups | 20 min

Activity 2 Even More Pennies

Students work with large quantities that lend themselves to arithmetic with powers of 10, giving them the opportunity to make use of scientific notation before it is formally introduced.



Name: ______ Date: ______ Period: ______

Activity 2 Even More Pennies

You will be given a sheet with information about the Empire State Building, located in New York City.

1. a. How many pennies are needed to build a stack with a height of 1 in.?

   $1 \div 0.0625 = 16$, so I need 16 pennies.

   b. By placing pennies side-by-side, how many are needed to create a straight line with a length of 1 ft?

   1 ft = 12 in.
   $12 \div 0.75 = 16$, so I need 16 pennies.

2. By placing pennies side-by-side and stacked on top of each other, how many are needed to create a cube with dimensions 1 ft by 1 ft by 1 ft?

   $16 \cdot 16 \cdot 12 \cdot 16 = 49152$, so I need 49,152 pennies.

3. Consider this question: *How many pennies would it take to fill the Empire State Building, in New York City?*

   a. What information do you need to answer this question? Record any relevant information you already know and the new information now provided to you.

   Sample response: The Empire State Building takes up 37 million ft^3 of space. There are 49,152 pennies in 1 ft^3

   b. Use the information you know to answer the question. You may approximate your answer.

   $37\,000000 \cdot 49152 = 1818624000\,000 \approx (4 \cdot 10^7) \cdot (5 \cdot 10^4)$ $\approx 20 \cdot 10^{11}$ which is approximately $2 \cdot 10^{12}$ pennies

Are you ready for more?

Approximately how many Empire State Buildings could be filled completely, if you have one quadrillion pennies? A quadrillion is 10^{15}.

$\frac{10^{15}}{2 \cdot 10^{12}} = \frac{10 \cdot 10^{14}}{2 \cdot 10^{12}} = 5 \cdot 10^2$ or 500 Empire State Buildings

STOP

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Lesson 11 Applications of Arithmetic With Powers of 10 685

1 Launch

Distribute calculators to each group. For Problem 3, distribute the Activity 2 PDF to each group and have students record *only* the information they will need to answer the question.

2 Monitor

Help students get started by asking, "Based on Activity 1, what do you know about the thickness of one penny? What expression represents how many pennies are needed to build a stack with a height of 1 in.?"

Look for points of confusion:

- **Struggling to find the number of pennies in a cubic foot.** Have students visualize or model a stack of 16 pennies. Then ask them how many stacks are needed to build 1 ft^3 of pennies. Make sure students know the diameter of a penny is the same width as the stack of pennies.

Look for productive strategies:

- Using powers of 10 to make calculations more efficient.

3 Connect

Display the animation from the digital lesson that shows pennies increasing by powers of 10.

Ask, "Once you had the information you needed, what were some difficulties you encountered? How did you work through them?"

Highlight the steps students took to plan their solution paths and arrive at their responses for Problem 3.

Differentiated Support

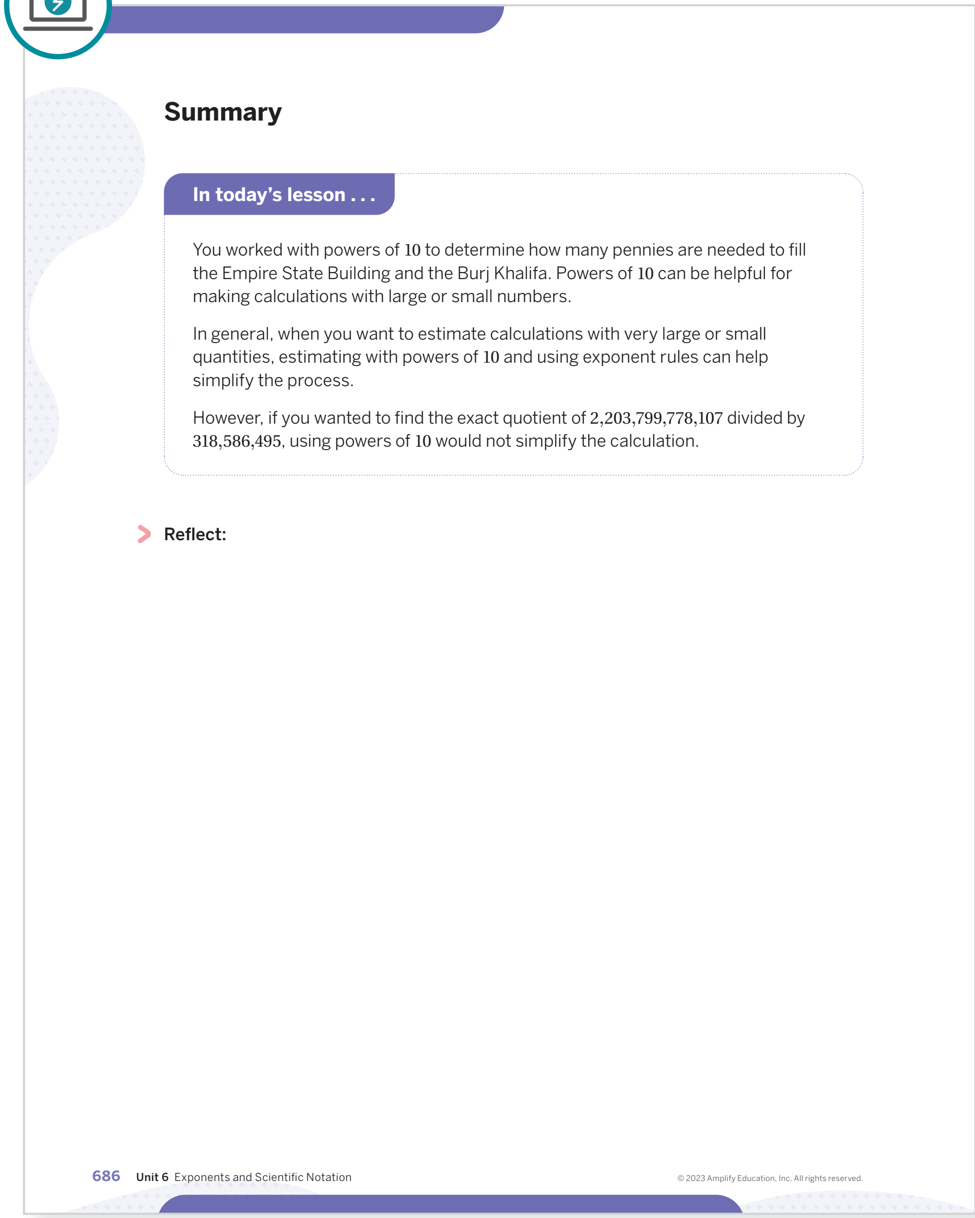
Accessibility: *Guide Processing and Visualization*

Before students complete Problem 3, distribute the Activity 2 PDF and conduct a think-aloud regarding the information needed to solve the problem. Provide access to highlighters or colored pencils and suggest that students highlight the necessary information. Ask students to explain why the highlighted information is the only information needed to complete Problem 3.

Whole Class | 7 min

Summary

Review and synthesize students' problem-solving strategies. Discuss how powers of 10 can be useful when working with large numbers.



Summary

In today's lesson . . .

You worked with powers of 10 to determine how many pennies are needed to fill the Empire State Building and the Burj Khalifa. Powers of 10 can be helpful for making calculations with large or small numbers.

In general, when you want to estimate calculations with very large or small quantities, estimating with powers of 10 and using exponent rules can help simplify the process.

However, if you wanted to find the exact quotient of 2,203,799,778,107 divided by 318,586,495, using powers of 10 would not simplify the calculation.

Reflect:

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Synthesize

Ask:

- "To solve the problems in this lesson, you had to determine what information was needed. Did you find that to be fairly straightforward or challenging? What made it straightforward or challenging?" Answers may vary.
- "How did exponent rules and powers of 10 make the calculations more efficient?" Using powers of 10 allows for a more straightforward way to express and interpret large or small numbers. The rules of exponents were handy for comparing numbers.
- "Would an estimate be an acceptable answer for problems like these? Why or why not? When might you need more precise solutions?" It depends on the questions and how the answers would be used. For example, if we were planning a space exploration, we would likely need a high level of precision to ensure that we hit our targets. But if the answers are for comparison or general information, estimates are likely adequate.

Highlight strong examples of strategies students identified during discussion.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

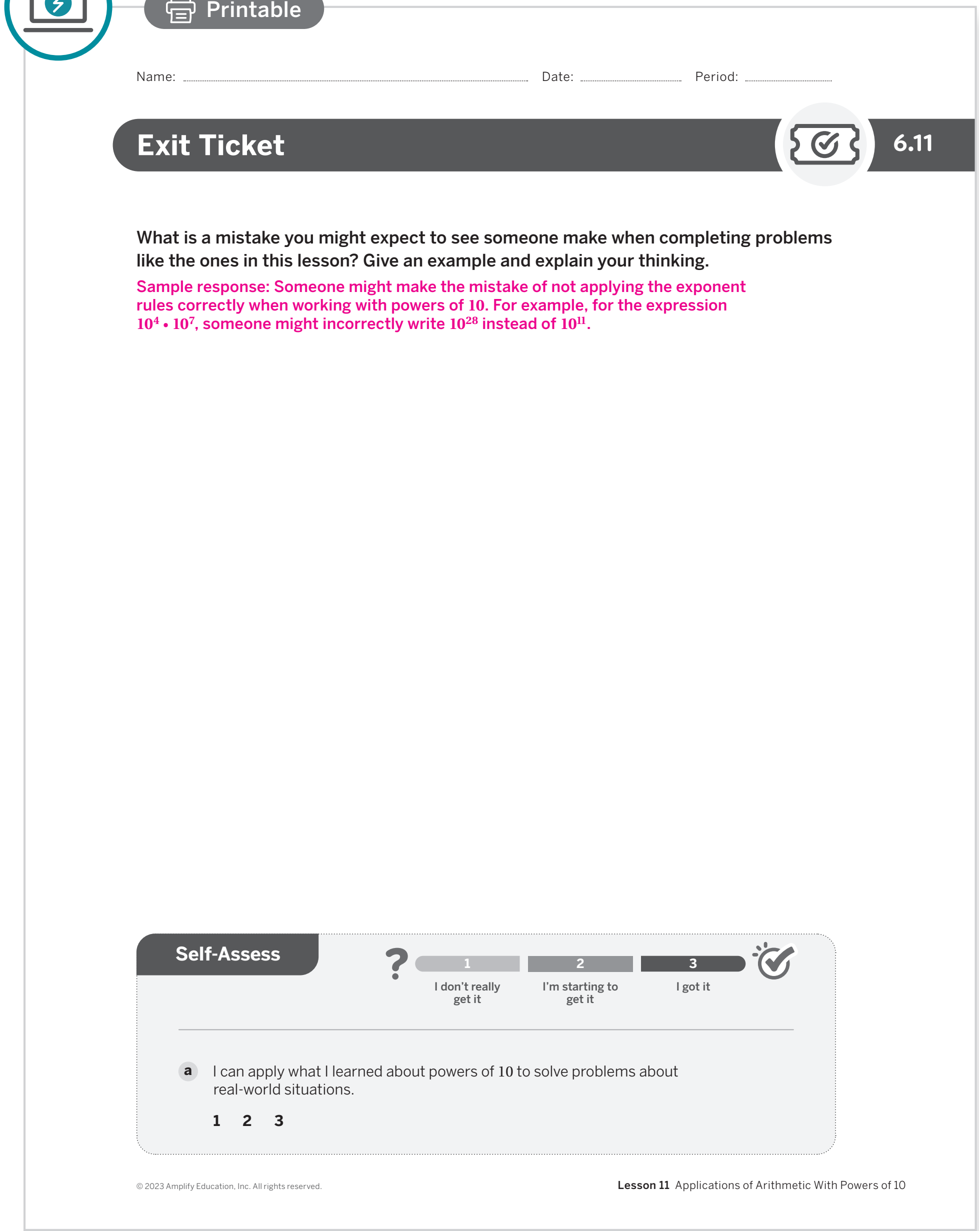
- "What strategies can be used when working with very large and very small numbers?"

Independent | 3 min

Exit Ticket

Students demonstrate their understanding of using powers of 10 to solve problems in context by identifying potential misconceptions.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 6.11

What is a mistake you might expect to see someone make when completing problems like the ones in this lesson? Give an example and explain your thinking.

Sample response: Someone might make the mistake of not applying the exponent rules correctly when working with powers of 10. For example, for the expression $10^4 \cdot 10^7$, someone might incorrectly write 10^{28} instead of 10^{11}.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can apply what I learned about powers of 10 to solve problems about real-world situations.

1 2 3

Lesson 11 Applications of Arithmetic With Powers of 10

Success looks like . . .

- **Language Goal:** Determining what information is needed to solve problems involving large numbers, and explaining how that information would help solve the problem. **(Speaking and Listening, Writing)**
- **Language Goal:** Using exponent rules and powers of 10 to solve problems in context, and explaining the steps used to organize thinking. **(Speaking and Listening)**
    - » Explaining a possible mistake with simplifying exponential expressions.

Suggested next steps

If students are unable to identify any common errors, consider:

- Asking them to recall what was challenging about the problems they encountered today, and where they remember identifying any misconceptions with their small groups.

Professional Learning

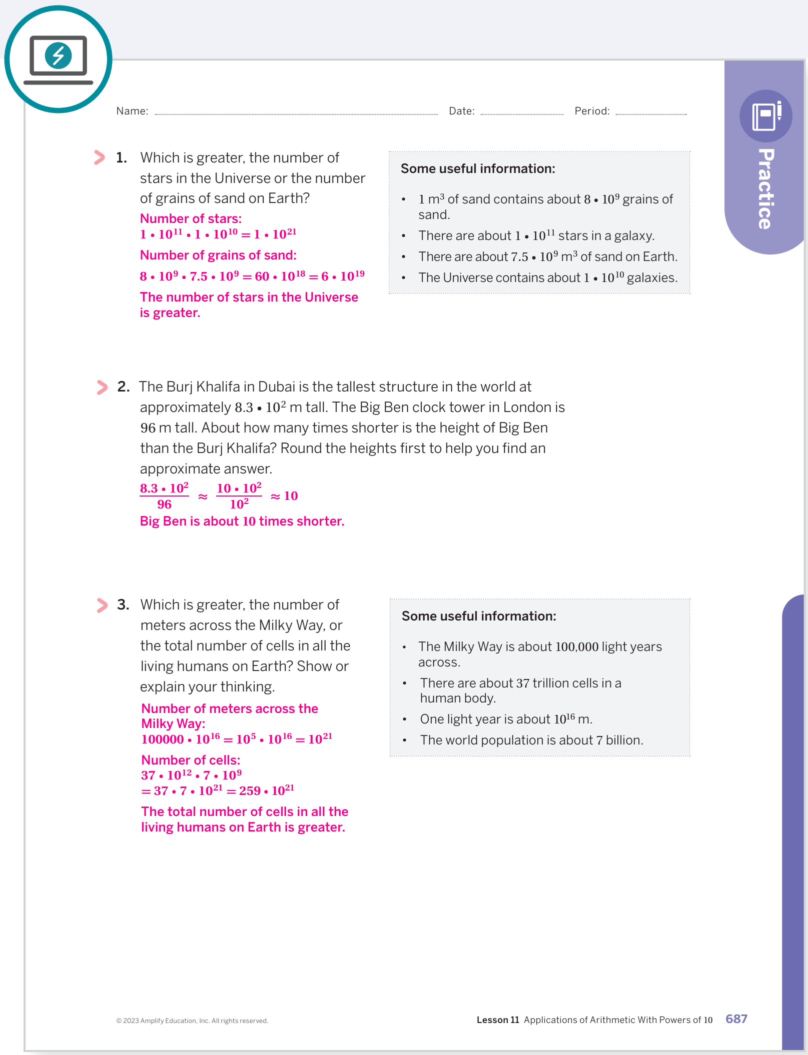
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What other ways are there to use powers of 10 to reason and solve a problem in context?
- What did students find frustrating about Activity 1? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Which is greater, the number of stars in the Universe or the number of grains of sand on Earth?

   Number of stars:
   $1 \cdot 10^{11} \cdot 1 \cdot 10^{10} = 1 \cdot 10^{21}$
   Number of grains of sand:
   $8 \cdot 10^9 \cdot 7.5 \cdot 10^9 = 60 \cdot 10^{18} = 6 \cdot 10^{19}$
   The number of stars in the Universe is greater.

Some useful information:

- 1 m^3 of sand contains about $8 \cdot 10^9$ grains of sand.
- There are about $1 \cdot 10^{11}$ stars in a galaxy.
- There are about $7.5 \cdot 10^9$ m^3 of sand on Earth.
- The Universe contains about $1 \cdot 10^{10}$ galaxies.

2. The Burj Khalifa in Dubai is the tallest structure in the world at approximately $8.3 \cdot 10^2$ m tall. The Big Ben clock tower in London is 96 m tall. About how many times shorter is the height of Big Ben than the Burj Khalifa? Round the heights first to help you find an approximate answer.

   $\frac{8.3 \cdot 10^2}{96} \approx \frac{10 \cdot 10^2}{10^2} \approx 10$
   Big Ben is about 10 times shorter.

3. Which is greater, the number of meters across the Milky Way, or the total number of cells in all the living humans on Earth? Show or explain your thinking.

   Number of meters across the Milky Way:
   $100000 \cdot 10^{16} = 10^5 \cdot 10^{16} = 10^{21}$
   Number of cells:
   $37 \cdot 10^{12} \cdot 7 \cdot 10^9$
   $= 37 \cdot 7 \cdot 10^{21} = 259 \cdot 10^{21}$
   The total number of cells in all the living humans on Earth is greater.

Some useful information:

- The Milky Way is about 100,000 light years across.
- There are about 37 trillion cells in a human body.
- One light year is about 10^{16} m.
- The world population is about 7 billion.

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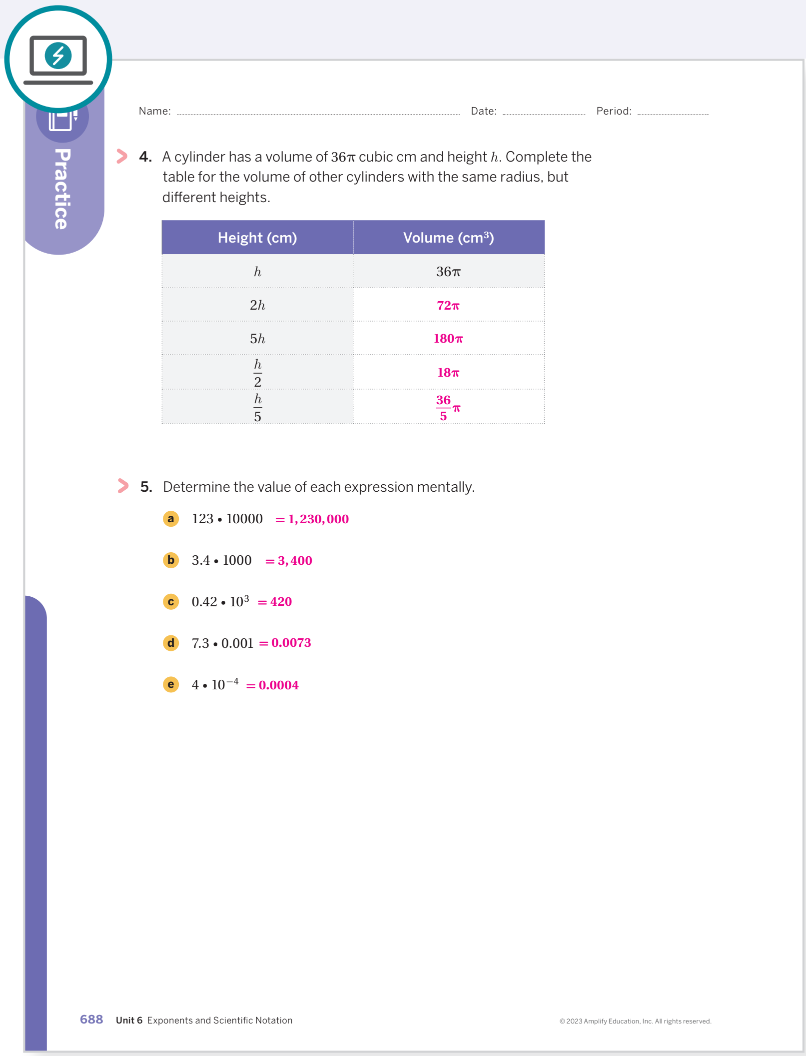
Lesson 11 Applications of Arithmetic With Powers of 10 687



Name: Date: Period:

Practice

4. A cylinder has a volume of 36π cubic cm and height h. Complete the table for the volume of other cylinders with the same radius, but different heights.

Height (cm)	Volume (cm^3)
h	36π
$2h$	72π
$5h$	180π
$\frac{h}{2}$	18π
$\frac{h}{5}$	$\frac{36}{5}\pi$

5. Determine the value of each expression mentally.

   a. $123 \cdot 10000$ $= 1,230,000$
   b. $3.4 \cdot 1000$ $= 3,400$
   c. $0.42 \cdot 10^3$ $= 420$
   d. $7.3 \cdot 0.001$ $= 0.0073$
   e. $4 \cdot 10^{-4}$ $= 0.0004$

688 **Unit 6** Exponents and Scientific Notation

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	1
Spiral	4	Unit 5 Lesson 19	2
Formative ⚡	5	Unit 6 Lesson 12	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 6 | **LESSON 12**

Definition of Scientific Notation

Let's use scientific notation to describe large and small numbers.

Focus

Goals

1. **Language Goal:** Identify numbers written in scientific notation, and describe the features of an expression in scientific notation. **(Speaking and Listening, Reading and Writing)**
2. Rewrite numbers in scientific notation.

Rigor

- Students write numbers in scientific notation to develop **procedural fluency**.

Coherence

Today

This lesson introduces students to the definition of *scientific notation*. Students must attend to precision as they decide whether numbers are in scientific notation and if not, convert them to scientific notation.

Previously

In previous lessons, students built familiarity with arithmetic involving powers of 10 to solve problems with very large and very small quantities.

Coming Soon

In the following lessons, students will perform calculations with numbers written in scientific notation. In Lesson 13, students will learn to multiply, divide, and estimate comparisons with numbers written in scientific notation. In Lesson 14, students will add and subtract with numbers written in scientific notation.

Pacing Guide

Suggested Total Lesson Time | ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
7 min	10 min	8 min	7 min	5 min	8 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- calculators

Math Language Development

New words

- *scientific notation*

Review words

- *power*

Amps | Featured Activity

Activity 2

Place Planets in Orbit

Students use scientific notation to place planets in orbit around the sun.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not cooperate well with their partner as they sort cards in Activity 1. Point out that there is a very precise definition of *scientific notation*, and that they should use the same level of precision when discussing with their partner whether each card shows scientific notation. Explain that precision of language aids the ability to communicate clearly, which helps establish healthy relationships.

Modifications to Pacing

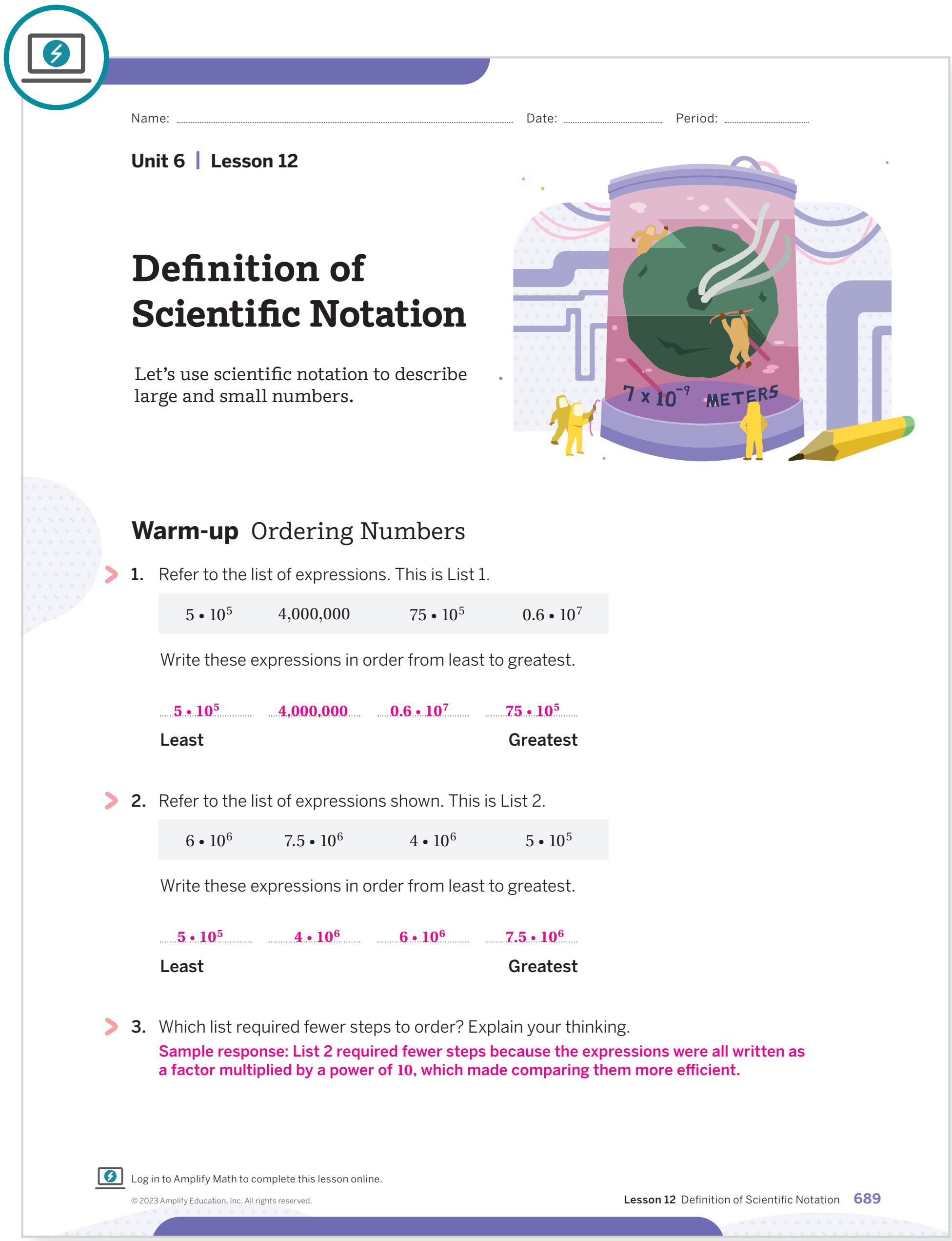
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Cards C, F, and I may be omitted.

Independent | 7 min

Warm-up Ordering Numbers

Students order two lists of numbers to see the usefulness of scientific notation and be introduced to the term *scientific notation*.



Name: .... Date: .... Period: ....

Unit 6 | Lesson 12

Definition of Scientific Notation

Let's use scientific notation to describe large and small numbers.

Warm-up Ordering Numbers

1. Refer to the list of expressions. This is List 1.

$5 \cdot 10^5$	$4{,}000{,}000$	$75 \cdot 10^5$	$0.6 \cdot 10^7$

Write these expressions in order from least to greatest.

$5 \cdot 10^5$ ..... $4{,}000{,}000$ ..... $0.6 \cdot 10^7$ ..... $75 \cdot 10^5$

Least **Greatest**

2. Refer to the list of expressions shown. This is List 2.

$6 \cdot 10^6$	$7.5 \cdot 10^6$	$4 \cdot 10^6$	$5 \cdot 10^5$

Write these expressions in order from least to greatest.

$5 \cdot 10^5$ ..... $4 \cdot 10^6$ ..... $6 \cdot 10^6$ ..... $7.5 \cdot 10^6$

Least **Greatest**

3. Which list required fewer steps to order? Explain your thinking.

Sample response: List 2 required fewer steps because the expressions were all written as a factor multiplied by a power of 10, which made comparing them more efficient.

Log in to Amplify Math to complete this lesson online.

Lesson 12 Definition of Scientific Notation 689

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by having them evaluate $5 \cdot 10^5$. Ask, "What does that equal? How does that compare with $4{,}000{,}000$?"

Look for points of confusion:

- **Thinking the value $6 \cdot 10^7$ is least.** Have students evaluate the expression to find its value in standard form before comparing.
- **Thinking both lists required the same amount of steps to order because they used a calculator.** Prompt students to see how they could order List 2 without using a calculator.

Look for productive strategies:

- Rewriting List 1 using powers of 10 with the same exponents to more easily compare.

3 Connect

Display student work with correctly ordered lists for Problems 1 and 2.

Highlight the similarities in the expressions in List 2 and why it required fewer steps to place them in order.

Define the term ***scientific notation*** as a number that is written as a product of two factors. The first factor is a number greater than or equal to one, but less than ten. The second factor is an integer power of 10. Scientific notation is typically written with multiplication represented using the $\times$ symbol instead of a dot.

Ask, "Is $75 \cdot 10^5$ in List 1 written in scientific notation? How do you know?"

Math Language Development

MLR2: Collect and Display

As students share their reasoning for Problem 3, collect and display language students use to describe and make sense of scientific notation. Add their terms and the formal definition of *scientific notation* to the class display.

English Learners

Provide examples and non-examples of scientific notation to the class display. For example, consider including the following table:

Number	Scientific Notation?
$1.2 \cdot 10^3$	Yes
$0.5 \cdot 10^6$	No
$1.35 \cdot 10^{\frac{1}{2}}$	No

Power-up

To power up students' ability to multiply by a power of ten, have students complete:

Match each expression with its product.

a. $0.03 \cdot 10^3$	b	300
b. $3 \cdot 100$	d	0.3
c. $30 \cdot 10^{-1}$	c	3
d. $0.3 \cdot 10^0$	a	30

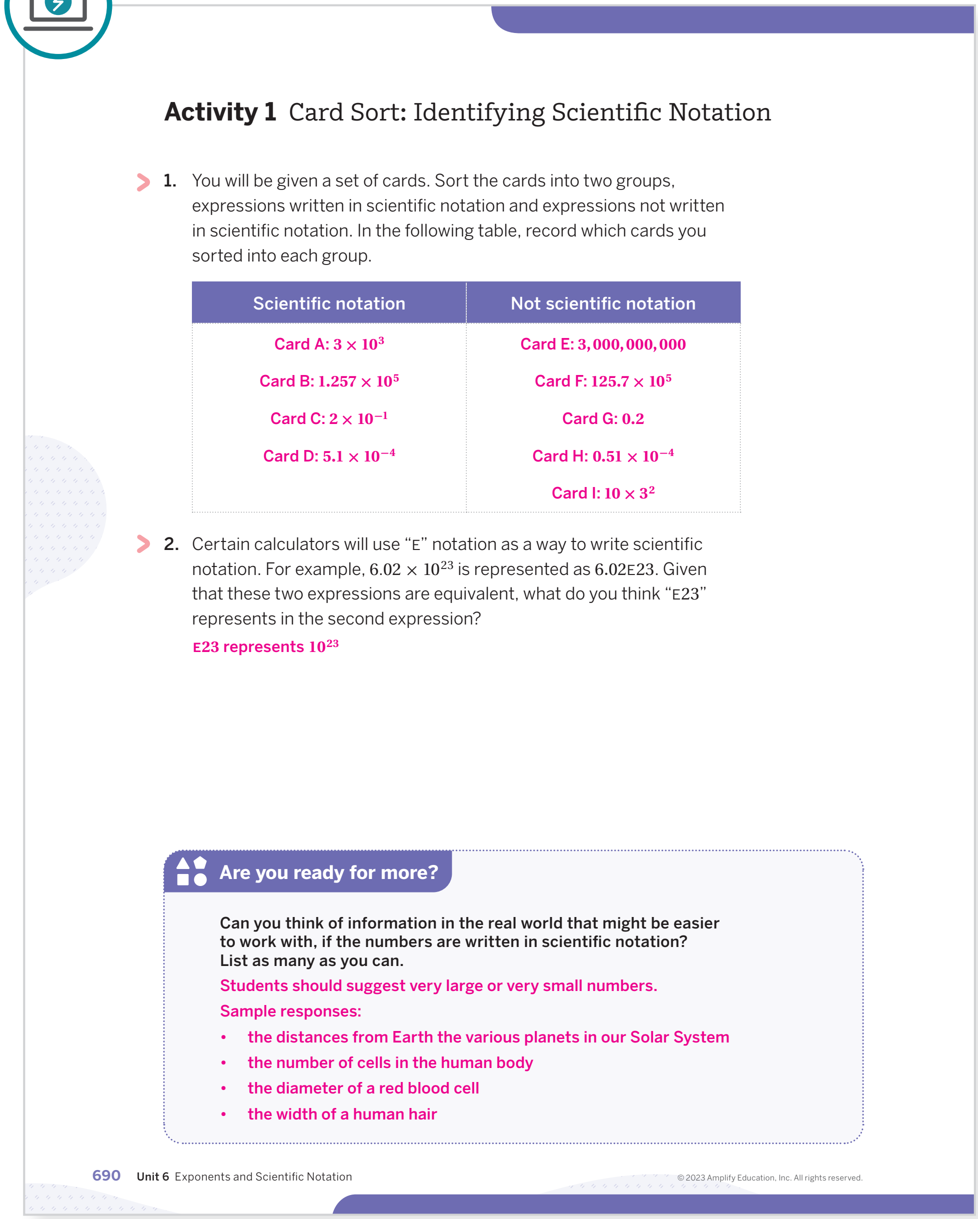
Use: Before the Warm-up

Informed by: Performance on Lesson 11, Practice Problem 5

Pairs | 10 min

Activity 1 Card Sort: Identifying Scientific Notation

Students sort cards using the definition of scientific notation to further understanding of this conventional way of writing large and small numbers.



Activity 1 Card Sort: Identifying Scientific Notation

1. You will be given a set of cards. Sort the cards into two groups, expressions written in scientific notation and expressions not written in scientific notation. In the following table, record which cards you sorted into each group.

Scientific notation	Not scientific notation
Card A: 3×10^3	Card E: $3,000,000,000$
Card B: 1.257×10^5	Card F: 125.7×10^5
Card C: 2×10^{-1}	Card G: 0.2
Card D: 5.1×10^{-4}	Card H: 0.51×10^{-4}
	Card I: 10×3^2

2. Certain calculators will use "E" notation as a way to write scientific notation. For example, 6.02×10^{23} is represented as 6.02E23. Given that these two expressions are equivalent, what do you think "E23" represents in the second expression?

E23 represents 10^{23}

Are you ready for more?

Can you think of information in the real world that might be easier to work with, if the numbers are written in scientific notation? List as many as you can.

Students should suggest very large or very small numbers.

Sample responses:

- **the distances from Earth the various planets in our Solar System**
- **the number of cells in the human body**
- **the diameter of a red blood cell**
- **the width of a human hair**

690 **Unit 6** Exponents and Scientific Notation

1 **Launch**

Say, "Scientific notation can help us compare very large and very small numbers. "

Distribute one set of cards from the Activity 1 PDF to each pair of students. Conduct the ***Card Sort*** routine. Consider providing calculators for Problem 2 especially if they display scientific notation with "E" notation.

2 **Monitor**

Help students get started by asking, "For a number to be in scientific notation, what needs to be true about the first factor?"

Look for points of confusion:

- **Thinking 0.2 is in scientific notation.** Remind students that the first factor must be a number greater than or equal to one, but less than ten. Ask, "What power of 10 can you multiply 0.2 by so that it fits this criteria, but doesn't change its value?"

3 **Connect**

Display and discuss the correct card sort for Problem 1. Then display and discuss an example of "E" notation on a calculator for Problem 2.

Have students share how they determined which numbers were in scientific notation.

Highlight that numbers in scientific notation must have a first factor greater than or equal to one, but less than ten. They must also have a second factor written as an integer power of 10.

Ask, "How could you write 5 using scientific notation?" 5×10^0

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Distribute the Activity 1 PDF and demonstrate sorting a card containing an expression written in scientific notation and a card containing an expression not written in scientific notation. After sorting, pause and invite students to explain why you have sorted the cards the way you did. Have pairs of students finish sorting the rest of the cards.

Extension: *Math Enrichment, Interdisciplinary Connections*

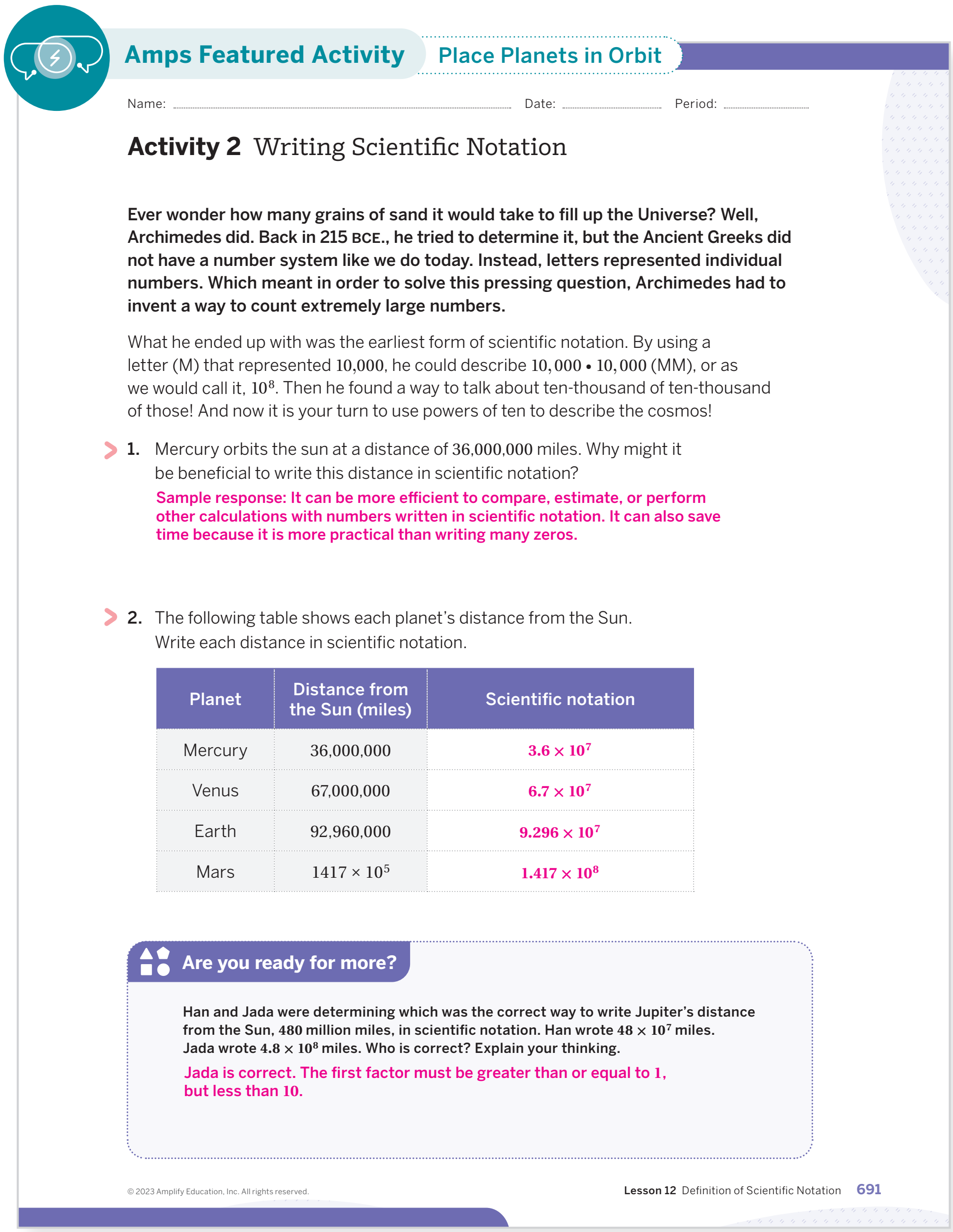
Provide students with the following information and ask them to explain why writing those values in scientific notation is more efficient. (Science)

- The mass of the Earth is estimated to be about 5.972×10^{24} kg.
- Other than the Sun, the closest star to our solar system is Proxima Centauri, which is about $40,208,000,000,000$ km away.
- There are about $25,000,000,000,000$ red blood cells in the typical adult human body.

Pairs | 8 min

Activity 2 Writing Scientific Notation

Students further their understanding by now writing large numbers — orbital distances of planets — in scientific notation.



Amps Featured Activity Place Planets in Orbit

Name: ________ Date: ________ Period: ________

Activity 2 Writing Scientific Notation

Ever wonder how many grains of sand it would take to fill up the Universe? Well, Archimedes did. Back in 215 BCE., he tried to determine it, but the Ancient Greeks did not have a number system like we do today. Instead, letters represented individual numbers. Which meant in order to solve this pressing question, Archimedes had to invent a way to count extremely large numbers.

What he ended up with was the earliest form of scientific notation. By using a letter (M) that represented $10{,}000$, he could describe $10{,}000 \bullet 10{,}000$ (MM), or as we would call it, 10^8. Then he found a way to talk about ten-thousand of ten-thousand of those! And now it is your turn to use powers of ten to describe the cosmos!

1. Mercury orbits the sun at a distance of 36,000,000 miles. Why might it be beneficial to write this distance in scientific notation?

   Sample response: It can be more efficient to compare, estimate, or perform other calculations with numbers written in scientific notation. It can also save time because it is more practical than writing many zeros.

2. The following table shows each planet's distance from the Sun. Write each distance in scientific notation.

Planet	Distance from the Sun (miles)	Scientific notation
Mercury	36,000,000	3.6×10^7
Venus	67,000,000	6.7×10^7
Earth	92,960,000	9.296×10^7
Mars	1417×10^5	1.417×10^8

Are you ready for more?

Han and Jada were determining which was the correct way to write Jupiter's distance from the Sun, 480 million miles, in scientific notation. Han wrote 48×10^7 miles. Jada wrote 4.8×10^8 miles. Who is correct? Explain your thinking.

Jada is correct. The first factor must be greater than or equal to 1, but less than 10.

1 Launch

Give students 1 minute to complete Problem 1 independently, followed by a whole-class discussion. Then have students complete Problem 2 in pairs.

2 Monitor

Help students get started by asking, "How should you write the first factor of 36,000,000 so that the first factor is greater than or equal to one, but less than ten?"

Look for points of confusion:

- **Inaccurately writing the powers of ten.** Have students recall how they can mentally determine the number of place values the decimal point moves if a number is multiplied by a power of 10.

3 Connect

Display student work showing a correctly completed table for Problem 2.

Ask, "What strategies did you use to rewrite 36,000,000 in scientific notation?"

Have students share their strategies for writing large numbers in scientific notation. Select students to share who made the connection between multiplying by powers of 10 and moving the decimal point.

Have students share their strategies for writing a number in scientific notation. Point out that the place-value system we use is based on powers of 10. By multiplying a number by a power of 10, the resulting product has the same non-zero digits but the decimal point has moved to the right the same number of places as indicated by the exponent on the power of 10.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use scientific notation to place planets in orbit around the Sun.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present a hypothetical statement that represents a misunderstanding about how to write values in scientific notation, such as: "Written in scientific notation, the distance of Mars from the Sun is 36×10^6 miles." Ask:

- ***Critique:*** "Do you agree or disagree that this distance is written in scientific notation? Explain your thinking."
- ***Correct and Clarify:*** "How would you correct this distance? What must be true about the first factor?"

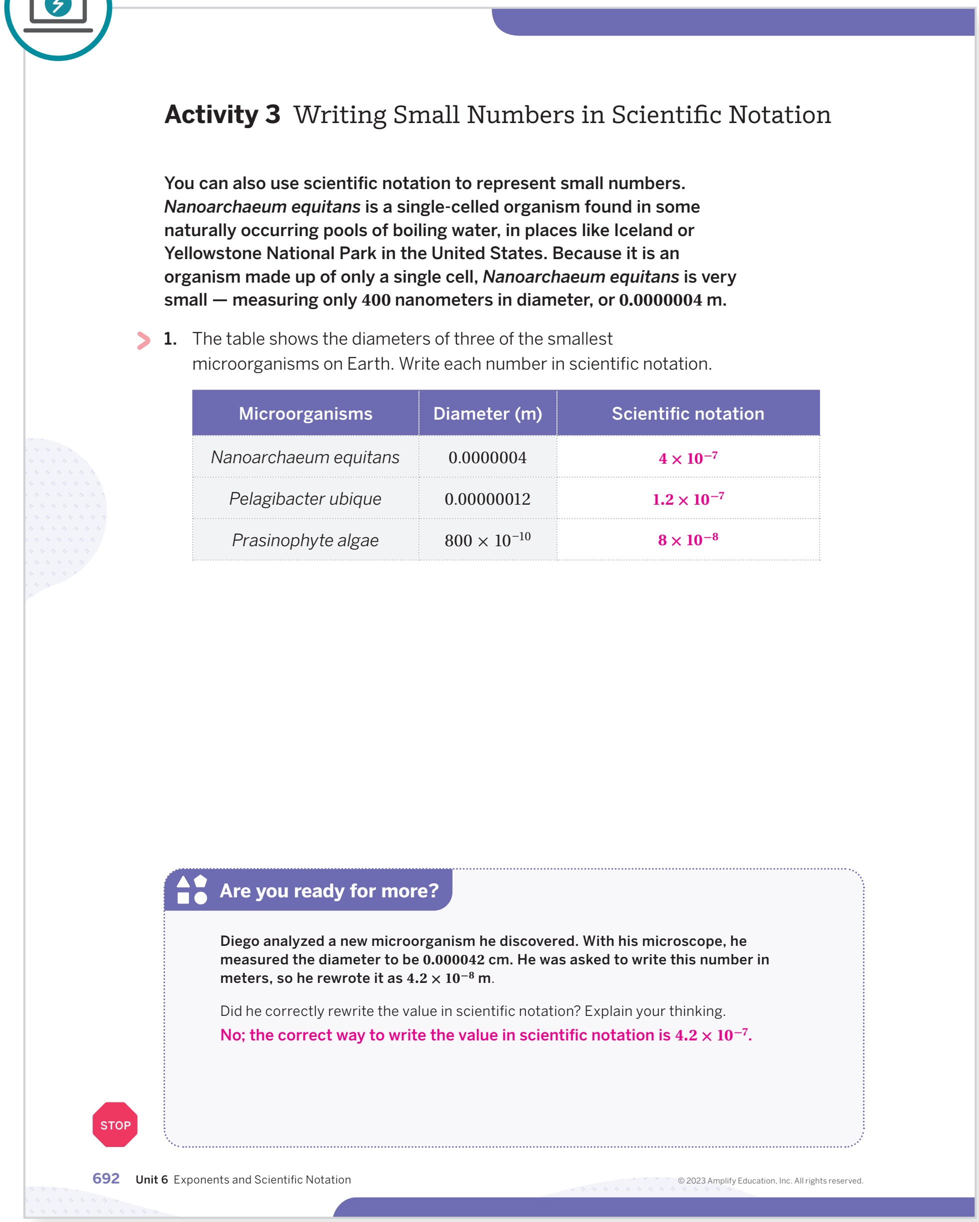
English Learners

Encourage students to refer to the examples and non-examples of scientific notation on the class display as they critique the statement.

Pairs | 7 min

Activity 3 Writing Small Numbers in Scientific Notation

Students further their understanding by now writing small numbers — the diameters of microorganisms — in scientific notation.



Activity 3 Writing Small Numbers in Scientific Notation

You can also use scientific notation to represent small numbers. *Nanoarchaeum equitans* is a single-celled organism found in some naturally occurring pools of boiling water, in places like Iceland or Yellowstone National Park in the United States. Because it is an organism made up of only a single cell, *Nanoarchaeum equitans* is very small — measuring only 400 nanometers in diameter, or 0.0000004 m.

1. The table shows the diameters of three of the smallest microorganisms on Earth. Write each number in scientific notation.

Microorganisms	Diameter (m)	Scientific notation
Nanoarchaeum equitans	0.0000004	4×10^{-7}
Pelagibacter ubique	0.00000012	1.2×10^{-7}
Prasinophyte algae	800×10^{-10}	8×10^{-8}

Are you ready for more?

Diego analyzed a new microorganism he discovered. With his microscope, he measured the diameter to be 0.000042 cm. He was asked to write this number in meters, so he rewrote it as 4.2×10^{-8} m.

Did he correctly rewrite the value in scientific notation? Explain your thinking.

No; the correct way to write the value in scientific notation is 4.2×10^{-7}.

692 **Unit 6** Exponents and Scientific Notation

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by writing 0.0000004 as 4×0.0000001 and asking, "How can you represent the decimal value as a power of 10?"

Look for points of confusion:

- **Writing 4×10^{-6} for 0.0000004 because there are six zeros after the decimal point.** Have students study the expressions 4×10^{-1}, $4 \times \frac{1}{10}$, and 0.4 to notice that when the exponent is -1, there is not 1 zero after the decimal point. Have students study equivalent expressions for when the exponent is -2 and -3 to determine a pattern.

3 Connect

Ask, "How could you write really small numbers in scientific notation?"

Have students share the connection between multiplying by powers of 10 and moving the decimal point.

Highlight that the place-value system is based on powers of 10. By multiplying a number by a negative power of 10, the resulting product has the same non-zero digits but the decimal point has moved. For example, 800×10^{-10} is the same as multiplying 800 by the fraction $\frac{1}{10}$ a total of 10 times. Multiplying 800 by $\frac{1}{10}$ gives a product of 80. Multiplying 80 by $\frac{1}{10}$ gives a product of 8. The decimal point, while not written, is moving to the left each time the number is multiplied by $\frac{1}{10}$.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, omit the second row from the table and have students focus on the first and last row of the table.

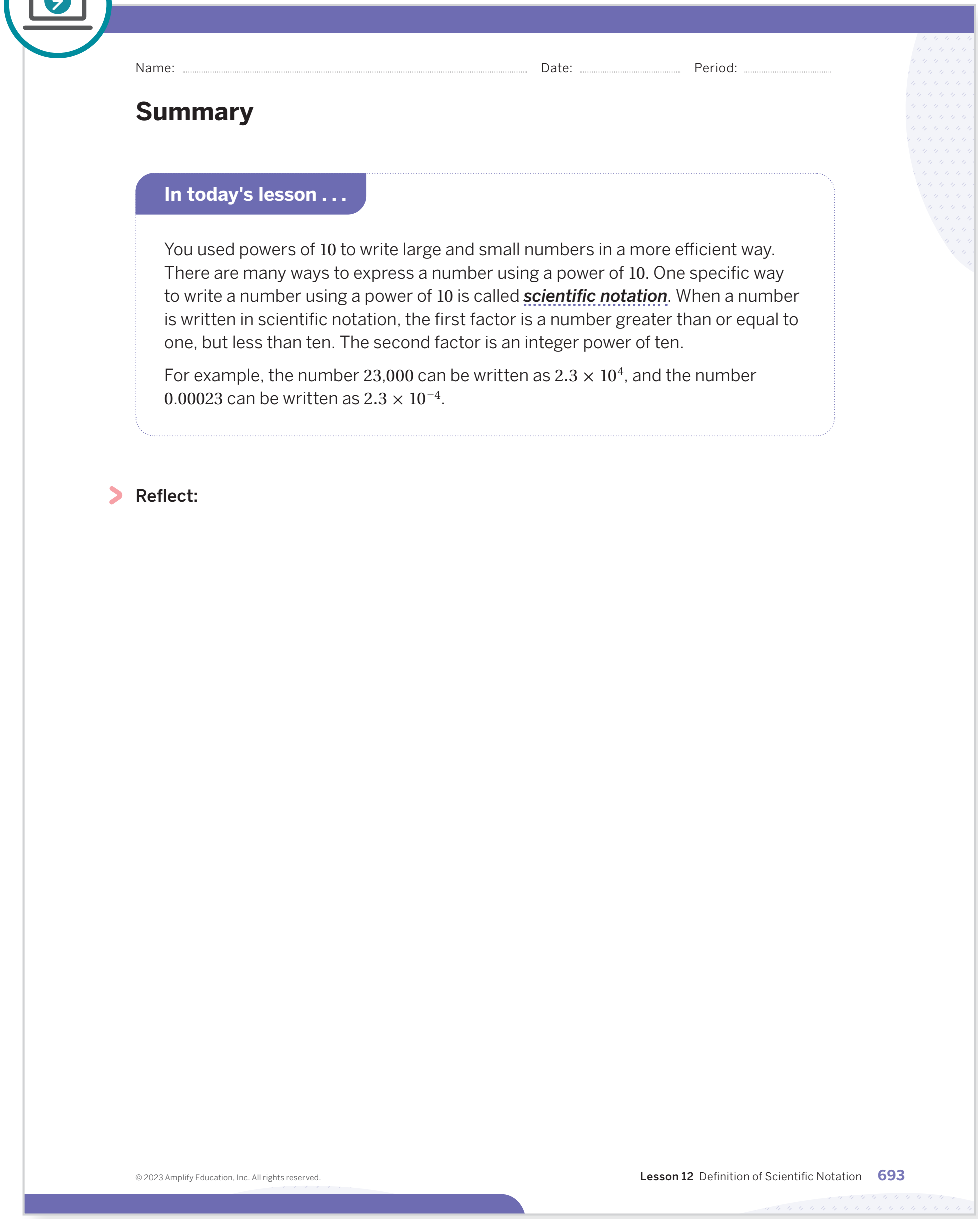
Extension: *Math Around the World*

Tell students that ancient cultures devised their own systems for counting, calculating, writing, and working with really large or small numbers. For example, ancient Chinese mathematicians used a number rod system and to multiply by powers of 10, the rods were moved to the left. To multiply by 10, move the rods one square to the left. To multiply by 100, move the rods two squares to the left. Similarly, to divide by powers of 10, move the rods to the right the corresponding number of squares. The Chinese writing system was developed to fulfill their civilization's needs for counting, representing one of the earliest writing symbols discovered, dating back to about 5000 BCE.

Whole Class | 5 min

Summary

Review and synthesize the definition of scientific notation and its usefulness in working with large and small numbers.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used powers of 10 to write large and small numbers in a more efficient way. There are many ways to express a number using a power of 10. One specific way to write a number using a power of 10 is called ***scientific notation***. When a number is written in scientific notation, the first factor is a number greater than or equal to one, but less than ten. The second factor is an integer power of ten.

For example, the number $23{,}000$ can be written as 2.3×10^4, and the number 0.00023 can be written as 2.3×10^{-4}.

Reflect:

Lesson 12 Definition of Scientific Notation 693

Synthesize

Formalize vocabulary: ***scientific notation***

Ask, "What are some examples of expressions that are in scientific notation? How can you tell they are in scientific notation? Why might scientific notation be useful?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Is there a more efficient way to write really small and really large numbers?"

Math Language Development

MLR2: Collect and Display

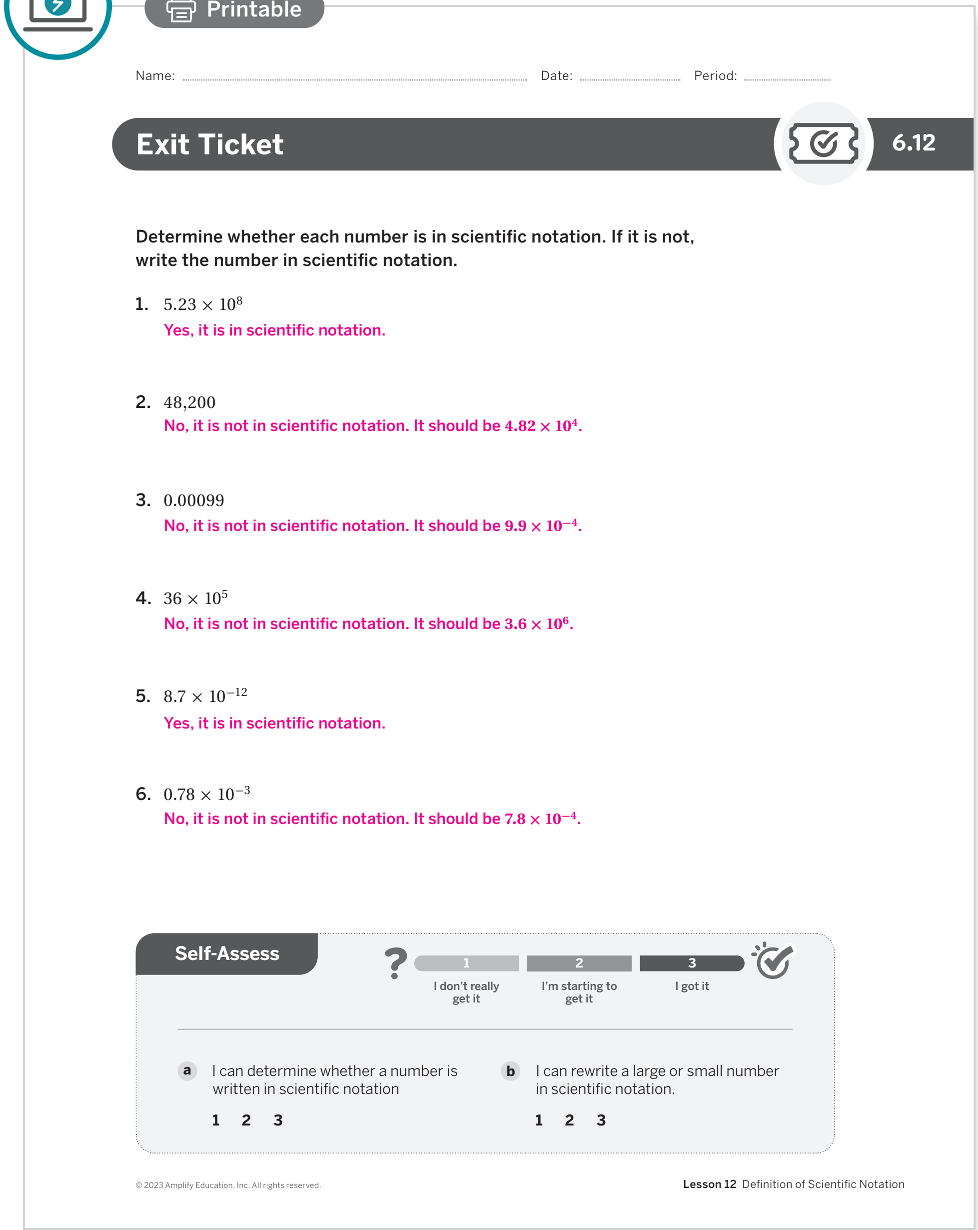
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started for this unit. Ask them to review and reflect on any terms and phrases related to the term *scientific notation* that were added to the display during the lesson.

Independent | 8 min

Exit Ticket

Students demonstrate their understanding by stating whether a number is in scientific notation and, if necessary, rewriting a number in scientific notation.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 6.12

Determine whether each number is in scientific notation. If it is not, write the number in scientific notation.

1. 5.23×10^8
   Yes, it is in scientific notation.
2. 48,200
   No, it is not in scientific notation. It should be 4.82×10^4.
3. 0.00099
   No, it is not in scientific notation. It should be 9.9×10^{-4}.
4. 36×10^5
   No, it is not in scientific notation. It should be 3.6×10^6.
5. 8.7×10^{-12}
   Yes, it is in scientific notation.
6. 0.78×10^{-3}
   No, it is not in scientific notation. It should be 7.8×10^{-4}.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine whether a number is written in scientific notation
1 2 3

b. I can rewrite a large or small number in scientific notation.
1 2 3

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Lesson 12 Definition of Scientific Notation

Success looks like . . .

- **Language Goal:** Identifying numbers written in scientific notation, and describing the features of an expression in scientific notation. **(Speaking and Listening, Reading and Writing)**
  - » Identifying whether a number is written in scientific notation in Problems 1–6.
- **Goal:** Rewriting numbers in scientific notation.
  - » Rewriting numbers in scientific notation in Problems 2–4 and 6.

Suggested next steps

If students misidentify which numbers are in scientific notation, consider:

- Reviewing the definition of scientific notation.
- Reviewing Activity 1.
- Assigning Practice Problem 1.
- Asking, "What needs to be true of the first factor in order for it to be written in scientific notation?"

If students incorrectly write numbers in scientific notation, consider:

- Asking, "Have you tried evaluating your expression using your calculator to see if it is equivalent?"
- Reviewing Activity 2.

Professional Learning

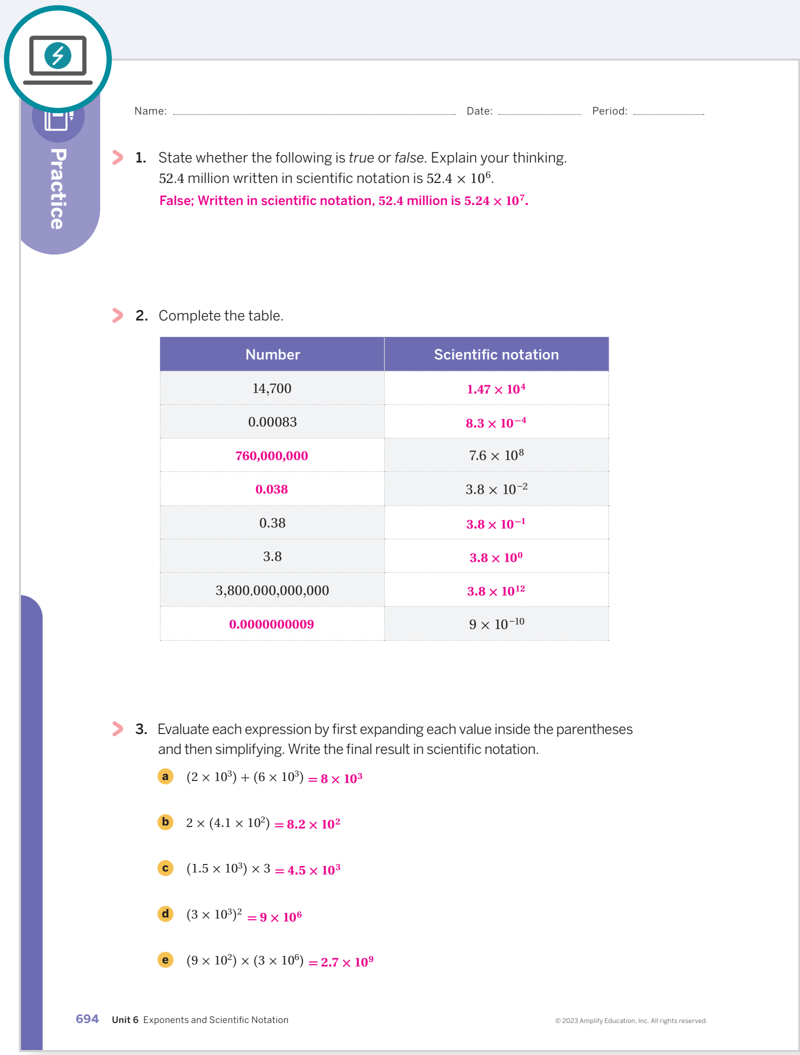
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 2 or Activity 3 more engaging today? Why do you think that is?
- Which teacher actions made students' understanding of scientific notation strong? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

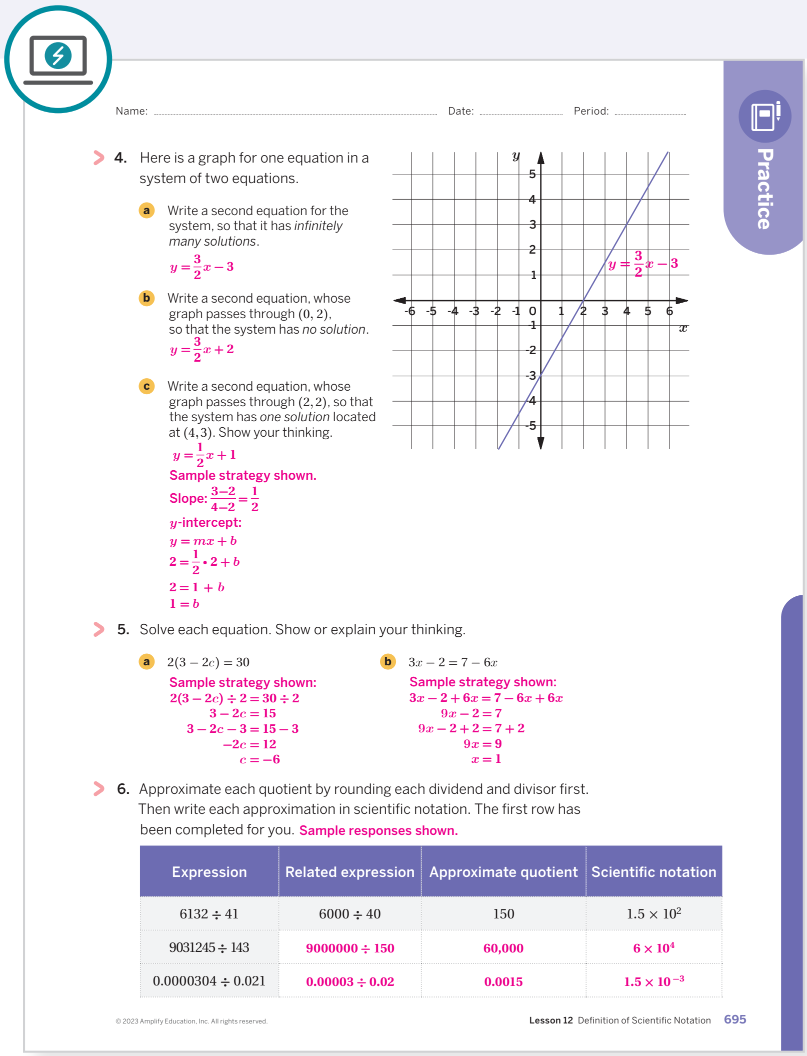
Name: ____________ Date: ____________ Period: ____________

1. State whether the following is *true* or *false*. Explain your thinking.
52.4 million written in scientific notation is 52.4×10^6.
False; Written in scientific notation, 52.4 million is 5.24×10^7.

2. Complete the table.

Number	Scientific notation
14,700	**1.47×10^4**
0.00083	**8.3×10^{-4}**
760,000,000	7.6×10^8
0.038	3.8×10^{-2}
0.38	**3.8×10^{-1}**
3.8	**3.8×10^0**
3,800,000,000,000	**3.8×10^{12}**
0.0000000009	9×10^{-10}

3. Evaluate each expression by first expanding each value inside the parentheses and then simplifying. Write the final result in scientific notation.
   - a $(2 \times 10^3) + (6 \times 10^3)$ **$= 8 \times 10^3$**
   - b $2 \times (4.1 \times 10^2)$ **$= 8.2 \times 10^2$**
   - c $(1.5 \times 10^3) \times 3$ **$= 4.5 \times 10^3$**
   - d $(3 \times 10^3)^2$ **$= 9 \times 10^6$**
   - e $(9 \times 10^2) \times (3 \times 10^6)$ **$= 2.7 \times 10^9$**



Practice

Name: ____________ Date: ____________ Period: ____________

4. Here is a graph for one equation in a system of two equations.
   - a Write a second equation for the system, so that it has *infinitely many solutions*.
     $y = \frac{3}{2}x - 3$
   - b Write a second equation, whose graph passes through $(0, 2)$, so that the system has *no solution*.
     $y = \frac{3}{2}x + 2$
   - c Write a second equation, whose graph passes through $(2, 2)$, so that the system has *one solution* located at $(4, 3)$. Show your thinking.
     $y = \frac{1}{2}x + 1$
     Sample strategy shown.
     Slope: $\frac{3-2}{4-2} = \frac{1}{2}$
     y-intercept:
     $y = mx + b$
     $2 = \frac{1}{2} \cdot 2 + b$
     $2 = 1 + b$
     $1 = b$

5. Solve each equation. Show or explain your thinking.
   - a $2(3 - 2c) = 30$
     Sample strategy shown:
     $2(3 - 2c) \div 2 = 30 \div 2$
     $3 - 2c = 15$
     $3 - 2c - 3 = 15 - 3$
     $-2c = 12$
     $c = -6$
   - b $3x - 2 = 7 - 6x$
     Sample strategy shown:
     $3x - 2 + 6x = 7 - 6x + 6x$
     $9x - 2 = 7$
     $9x - 2 + 2 = 7 + 2$
     $9x = 9$
     $x = 1$

6. Approximate each quotient by rounding each dividend and divisor first. Then write each approximation in scientific notation. The first row has been completed for you. **Sample responses shown.**

Expression	Related expression	Approximate quotient	Scientific notation
$6132 \div 41$	$6000 \div 40$	150	1.5×10^2
$9031245 \div 143$	**$9000000 \div 150$**	**60,000**	**6×10^4**
$0.0000304 \div 0.021$	**$0.00003 \div 0.02$**	**0.0015**	**1.5×10^{-3}**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	1
Spiral	4	Unit 4 Lesson 14	2
	5	Unit 4 Lesson 5	1
Formative	6	Unit 6 Lesson 13	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 6 | **LESSON 13**

Multiplying, Dividing, and Estimating With Scientific Notation

Let's solve problems by multiplying and dividing numbers in scientific notation.

Focus

Goals

1. **Language Goal:** Generalize a process of multiplying and dividing numbers in scientific notation. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Use scientific notation and estimation to compare quantities and interpret results in context. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students perform operations with numbers expressed in scientific notation, use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times as great one quantity is than the other. Students interpret their results in context.

Previously

In Lesson 12, students learned the definition of scientific notation and practiced writing numbers in scientific notation. Earlier, in Lesson 3, students developed an understanding for multiplying numbers with exponents.

Coming Soon

Students will learn how to add and subtract using numbers written in scientific notation in Lesson 14.

Rigor

- Students build **conceptual understanding** of multiplying and dividing numbers expressed in scientific notation.
- Students **apply** their understanding to compare the population and mass of creatures on Earth.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
7 min	8 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *equivalent expressions*
- *scientific notation*

Amps | Featured Activity

Activity 2

Interactive Scale

Students test their estimating skills with scientific notation by using an interactive scale to balance large quantities of different animals, humans, and even bacteria.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack the self-discipline to reason quantitatively about the facts in the table in order to apply them. Prior to the activity, have pairs work together to write their own question using the facts in the table. This process will familiarize students with the data, and hopefully excite and prepare them to work to complete the activity.

Modifications to Pacing

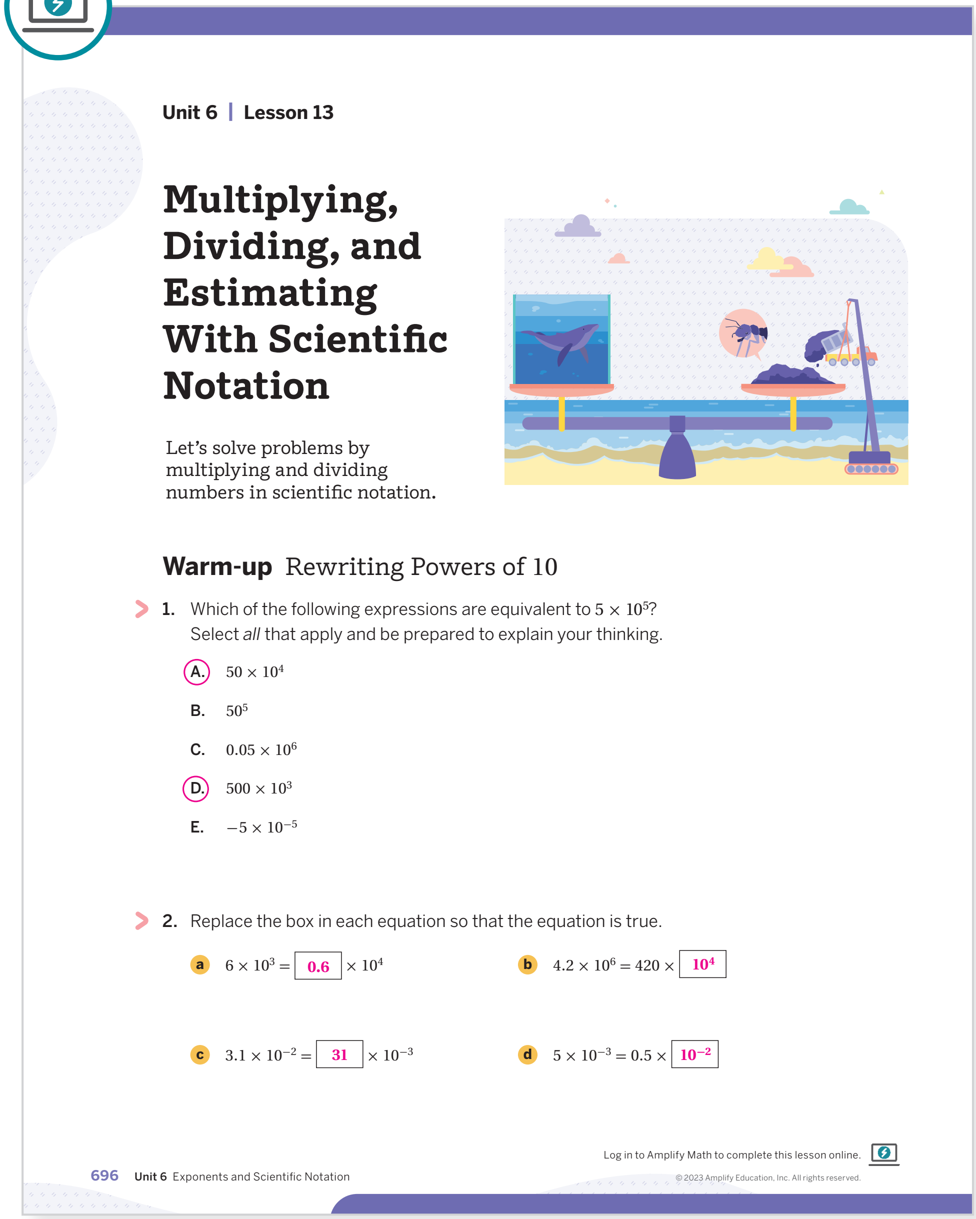
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, Problem 5 may be omitted.

Pairs | 7 min

Warm-up Rewriting Powers of 10

Students practice rewriting numbers in scientific notation using different powers of 10 to reinforce understanding and prepare them for estimation in an upcoming activity.



Unit 6 | **Lesson 13**

Multiplying, Dividing, and Estimating With Scientific Notation

Let's solve problems by multiplying and dividing numbers in scientific notation.

Warm-up Rewriting Powers of 10

1. Which of the following expressions are equivalent to 5×10^5? Select *all* that apply and be prepared to explain your thinking.
   - [✓ A.] 50×10^4
   - B. 50^5
   - C. 0.05×10^6
   - [✓ D.] 500×10^3
   - E. -5×10^{-5}

2. Replace the box in each equation so that the equation is true.
   - a $6 \times 10^3 = \boxed{0.6} \times 10^4$
   - b $4.2 \times 10^6 = 420 \times \boxed{10^4}$
   - c $3.1 \times 10^{-2} = \boxed{31} \times 10^{-3}$
   - d $5 \times 10^{-3} = 0.5 \times \boxed{10^{-2}}$

Log in to Amplify Math to complete this lesson online.

696 **Unit 6** Exponents and Scientific Notation

1 Launch

Tell students that they should not use a calculator during today's lesson.

2 Monitor

Help students get started by evaluating 5×10^5 and 50×10^4 and asking students what they notice.

Look for points of confusion:

- **Thinking that 50^5 is equivalent to 5×10^5.** Remind students to evaluate the power of 10 first before multiplying bases to follow the order of operations.
- **Increasing both the factor and the power of 10 to obtain 60×10^4 in Problem 2a.** Have students evaluate their answer to check their work. Tell them that if they increase the value of a power of 10 by a factor of 10, they must decrease the leading factor by a factor of 10 to maintain equivalence.
- **Incorrectly increasing a negative power, such as writing 10^{-4} in Problem 2d.** Remind students when they increase 10^{-3} by a factor of 10, the result is 10^{-2}.

Look for productive strategies:

- Identifying patterns to find equivalent expressions without evaluating.

3 Connect

Display the correct answers to Problems 1 and 2.

Have students share how they were able to rewrite Problem 2a.

Highlight that if students increase the power of ten by a factor of 10, they need to decrease the first factor by a power of 10 to maintain equivalence.

Ask, "Looking at Problem 2c, what would you need to do to the first factor if you decreased the power of 10 by a factor of 10?" Multiply 3.1 by 10.

Power-up

To power up students' ability to approximate quotients, have students complete:

Approximate each quotient by rounding first dividend and divisor. The first row has been completed for you. Sample answers are shown.

Expression	Related expression	Approximate quotient
$3.7 \div 0.041$	$4 \div 0.04$	100
$603 \div 14$	$600 \div 15$	40
$2434 \div 0.0102$	$2400 \div 0.01$	240,000

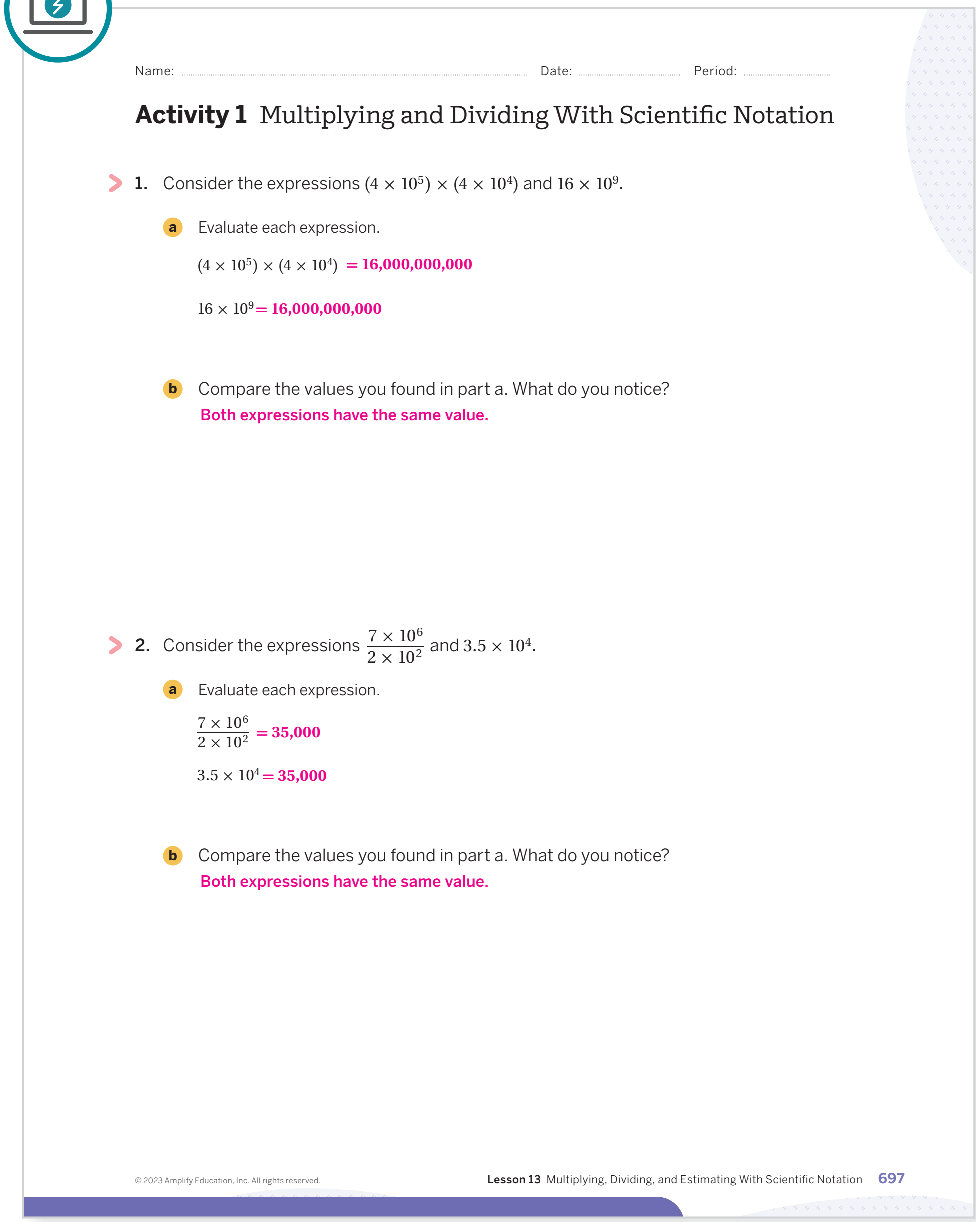
Use: Before Activity 2

Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 2 and 6

Pairs | 8 min

Activity 1 Multiplying and Dividing With Scientific Notation

Students evaluate expressions written in scientific notation to determine strategies for finding products and quotients of large and small numbers.



Name: ________ Date: ________ Period: ________

Activity 1 Multiplying and Dividing With Scientific Notation

1. Consider the expressions $(4 \times 10^5) \times (4 \times 10^4)$ and 16×10^9.

   a. Evaluate each expression.

   $(4 \times 10^5) \times (4 \times 10^4) = 16{,}000{,}000{,}000$

   $16 \times 10^9 = 16{,}000{,}000{,}000$

   b. Compare the values you found in part a. What do you notice?

   Both expressions have the same value.

2. Consider the expressions $\frac{7 \times 10^6}{2 \times 10^2}$ and 3.5×10^4.

   a. Evaluate each expression.

   $\frac{7 \times 10^6}{2 \times 10^2} = 35{,}000$

   $3.5 \times 10^4 = 35{,}000$

   b. Compare the values you found in part a. What do you notice?

   Both expressions have the same value.

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Lesson 13 Multiplying, Dividing, and Estimating With Scientific Notation 697

1 Launch

Conduct the ***Think-Pair-Share*** routine pausing for discussion after each problem.

2 Monitor

Help students get started by referring to Problem 1 and saying, “Let’s use the commutative property to change the order of the factors. We know 4×4 is 16. Now, determine the resulting power of 10.”

Look for points of confusion:

- **Making errors with exponents.** Remind students of exponent rules they derived from earlier lessons.

Look for productive strategies:

- Writing each expression in standard form.
- Using the commutative property to rewrite the expressions in Problem 1.
- Dividing the first factors by each other and the powers of 10 by each other in Problem 2.

3 Connect

Display student work showing efficient strategies for Problem 1, and then again for Problem 2.

Have students share how they were able to find that the expressions were equivalent in each problem.

Highlight using standard form to find equivalence. Then, highlight more sophisticated methods for finding equivalence, first for multiplication in Problem 1, then for division in Problem 2.

Ask, “What should you keep in mind when multiplying and dividing numbers written in scientific notation?”

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students of the exponent rules they previously learned and demonstrate using a similar problem how to rearrange the factors in an expression to simplify the evaluation. For example, $(2 \times 10^4) \times (2 \times 10^3) = (2 \times 2) \times (10^4 \times 10^3)$
Then have students discuss how they would complete the evaluation of the expression before starting Problem 1.

Extension: *Math Enrichment*

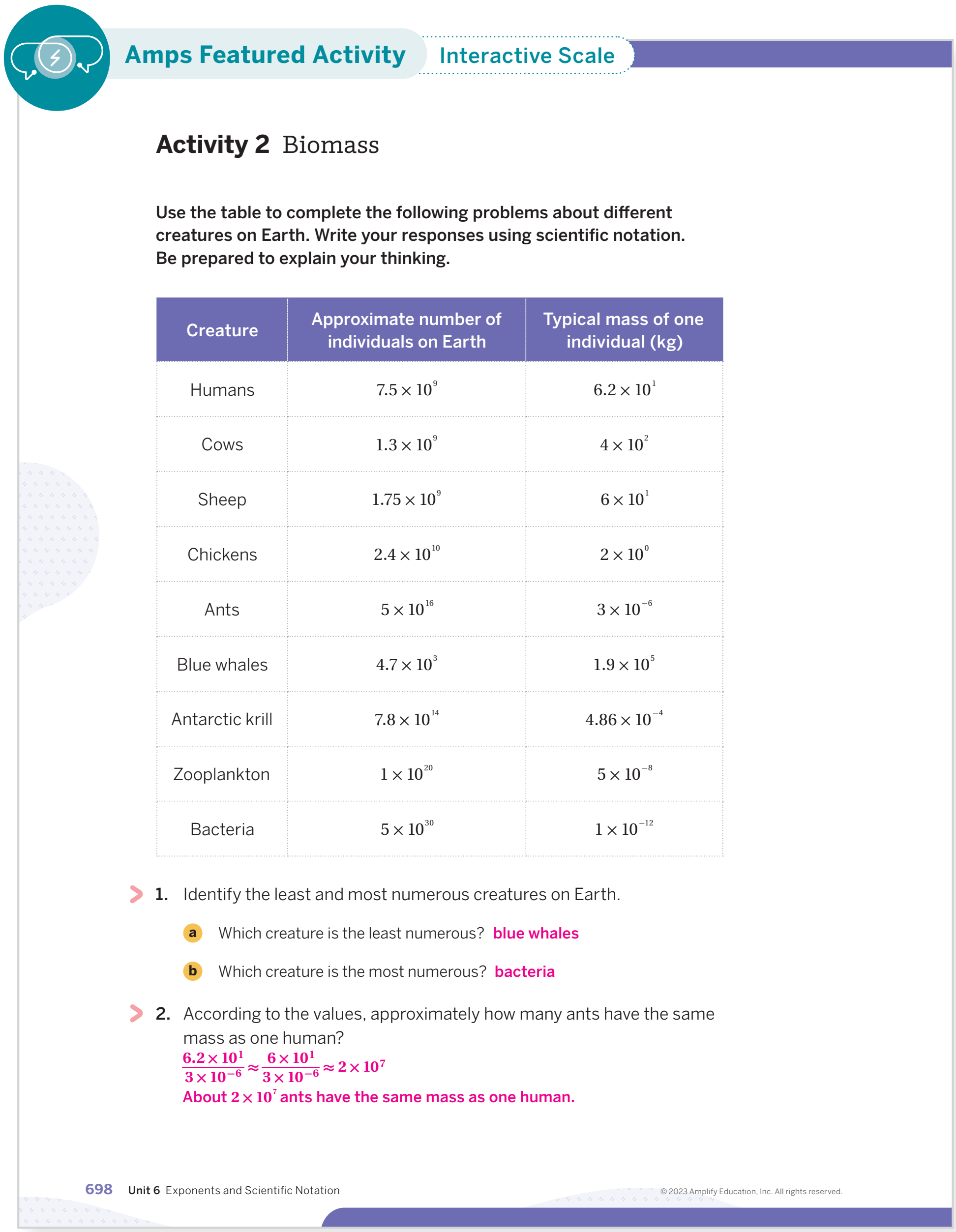
Have students create an expression written in scientific notation for each value. Sample responses are shown.

1. 18,000,000,000 $(3 \times 10^4) \times (6 \times 10^5)$
2. 45,000 $\frac{9 \times 10^6}{2 \times 10^2}$

Pairs | 20 min

Activity 2 Biomass

Students solve problems about quantities in context, using scientific notation as a tool for working with small and large numbers.



Amps Featured Activity Interactive Scale

Activity 2 Biomass

Use the table to complete the following problems about different creatures on Earth. Write your responses using scientific notation. Be prepared to explain your thinking.

Creature	Approximate number of individuals on Earth	Typical mass of one individual (kg)
Humans	7.5×10^9	6.2×10^1
Cows	1.3×10^9	4×10^2
Sheep	1.75×10^9	6×10^1
Chickens	2.4×10^{10}	2×10^0
Ants	5×10^{16}	3×10^{-6}
Blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
Zooplankton	1×10^{20}	5×10^{-8}
Bacteria	5×10^{30}	1×10^{-12}

> **1.** Identify the least and most numerous creatures on Earth.

  a. Which creature is the least numerous? **blue whales**

  b. Which creature is the most numerous? **bacteria**

> **2.** According to the values, approximately how many ants have the same mass as one human?

$$\frac{6.2 \times 10^1}{3 \times 10^{-6}} \approx \frac{6 \times 10^1}{3 \times 10^{-6}} \approx 2 \times 10^7$$

About 2×10^7 ants have the same mass as one human.

698 **Unit 6** Exponents and Scientific Notation

1 Launch

Explain that large numbers, such as populations, are often estimated using scientific notation. Consider checking-in with pairs after they complete Problems 1–3 before continuing.

2 Monitor

Help students get started by asking, "What factor in scientific notation most determines the size of the number? What power of 10 shows the least value(s)?"

Look for points of confusion:

- **Thinking zooplankton is the least numerous.** Ask, "Which factor most determines the size of the number?" Remind students to compare powers of 10 first.
- **Struggling to complete Problem 2.** Ask, "What operation can you use to determine how many of one quantity compare to another quantity?" Remind students they can write division using a fraction and to round before solving to help with estimation calculations.
- **Not rewriting the mass of the blue whale in Problem 4.** Ask students if they can see a relationship between a number with digits close to 1.9 and 6.2. Tell students that 1.9 could be rewritten as approximately 18, as long as they modify the power of 10 in the second factor.

Look for productive strategies:

- Rounding values before multiplying or dividing .
- Rewriting values using powers of 10 to better estimate quotients.

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can test their estimating skills with scientific notation by using an interactive scale to balance large quantities of different animals, humans, and even bacteria.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display only the table, and invite pairs of students to work together to write a list of mathematical questions that could be answered using the information in the table. Have pairs of students share their questions with the class. Sample questions shown.

- Which is greater, the mass of all the humans on Earth or the mass of all the blue whales on Earth?
- How many times more chickens are there on Earth than humans?

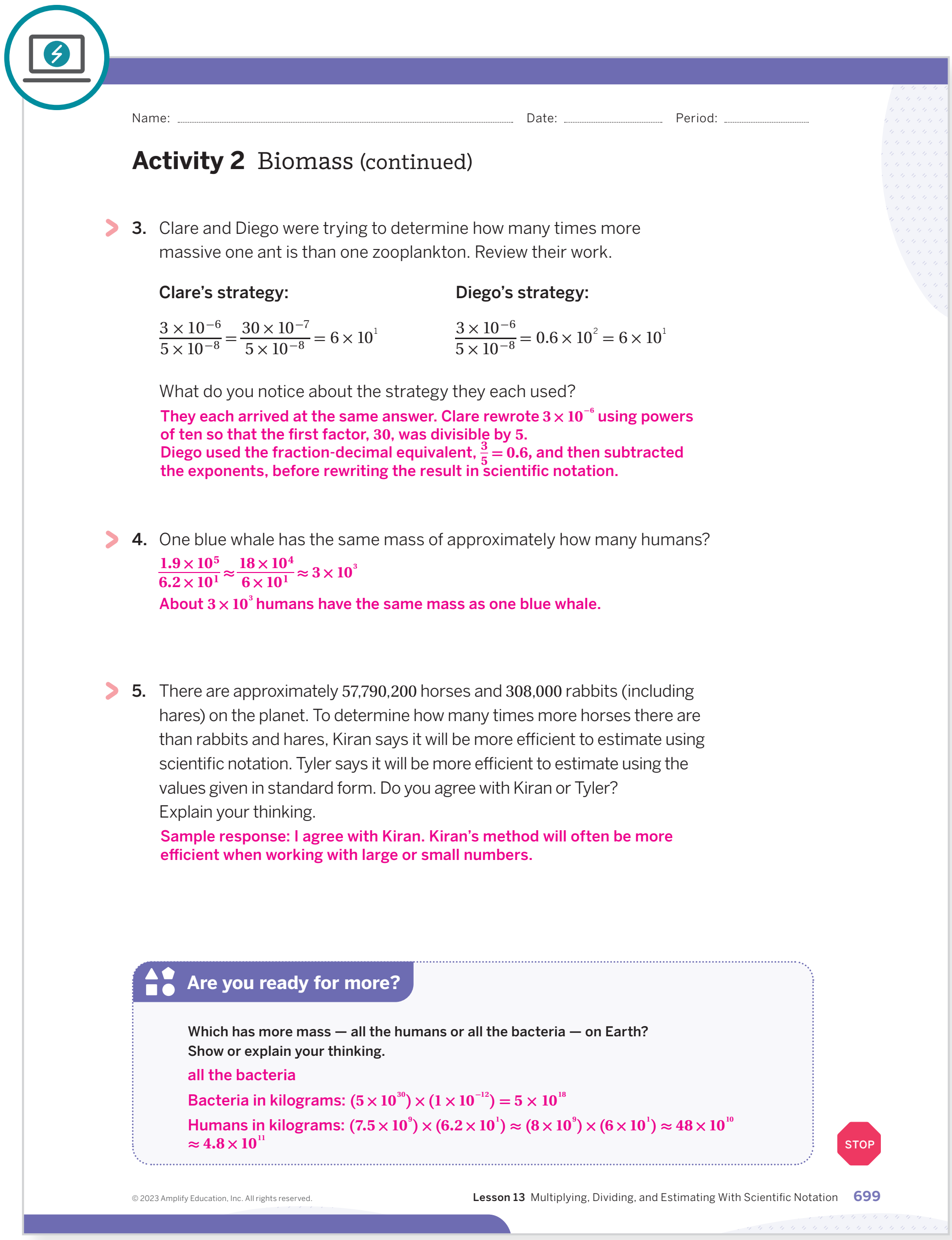
English Learners

Considering modeling how to write one mathematical question that asks students to compare quantities. Demonstrate your process for crafting the question by conducting a think-aloud so that students have access to your metacognitive processes.

Pairs | 20 min

Activity 2 Biomass (continued)

Students solve problems about quantities in context, using scientific notation as a tool for working with small and large numbers.



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Activity 2 Biomass (continued)

3. Clare and Diego were trying to determine how many times more massive one ant is than one zooplankton. Review their work.

Clare's strategy:	Diego's strategy:
$\frac{3 \times 10^{-6}}{5 \times 10^{-8}} = \frac{30 \times 10^{-7}}{5 \times 10^{-8}} = 6 \times 10^{1}$	$\frac{3 \times 10^{-6}}{5 \times 10^{-8}} = 0.6 \times 10^{2} = 6 \times 10^{1}$

What do you notice about the strategy they each used?

They each arrived at the same answer. Clare rewrote 3×10^{-6} using powers of ten so that the first factor, 30, was divisible by 5.
Diego used the fraction-decimal equivalent, $\frac{3}{5} = 0.6$, and then subtracted the exponents, before rewriting the result in scientific notation.

4. One blue whale has the same mass of approximately how many humans?

$\frac{1.9 \times 10^{5}}{6.2 \times 10^{1}} \approx \frac{18 \times 10^{4}}{6 \times 10^{1}} \approx 3 \times 10^{3}$

About 3×10^{3} humans have the same mass as one blue whale.

5. There are approximately 57,790,200 horses and 308,000 rabbits (including hares) on the planet. To determine how many times more horses there are than rabbits and hares, Kiran says it will be more efficient to estimate using scientific notation. Tyler says it will be more efficient to estimate using the values given in standard form. Do you agree with Kiran or Tyler? Explain your thinking.

Sample response: I agree with Kiran. Kiran's method will often be more efficient when working with large or small numbers.

Are you ready for more?

Which has more mass — all the humans or all the bacteria — on Earth? Show or explain your thinking.

all the bacteria

Bacteria in kilograms: $(5 \times 10^{30}) \times (1 \times 10^{-12}) = 5 \times 10^{18}$

Humans in kilograms: $(7.5 \times 10^{9}) \times (6.2 \times 10^{1}) \approx (8 \times 10^{9}) \times (6 \times 10^{1}) \approx 48 \times 10^{10}$ $\approx 4.8 \times 10^{11}$

STOP

Lesson 13 Multiplying, Dividing, and Estimating With Scientific Notation 699

3 Connect

Display the table from the Student Edition.

Have students share how they arrived at their answer for Problem 2 and what they noticed about Clare's strategy in Problem 3. Then, have students share how they estimated the quotient in Problem 4.

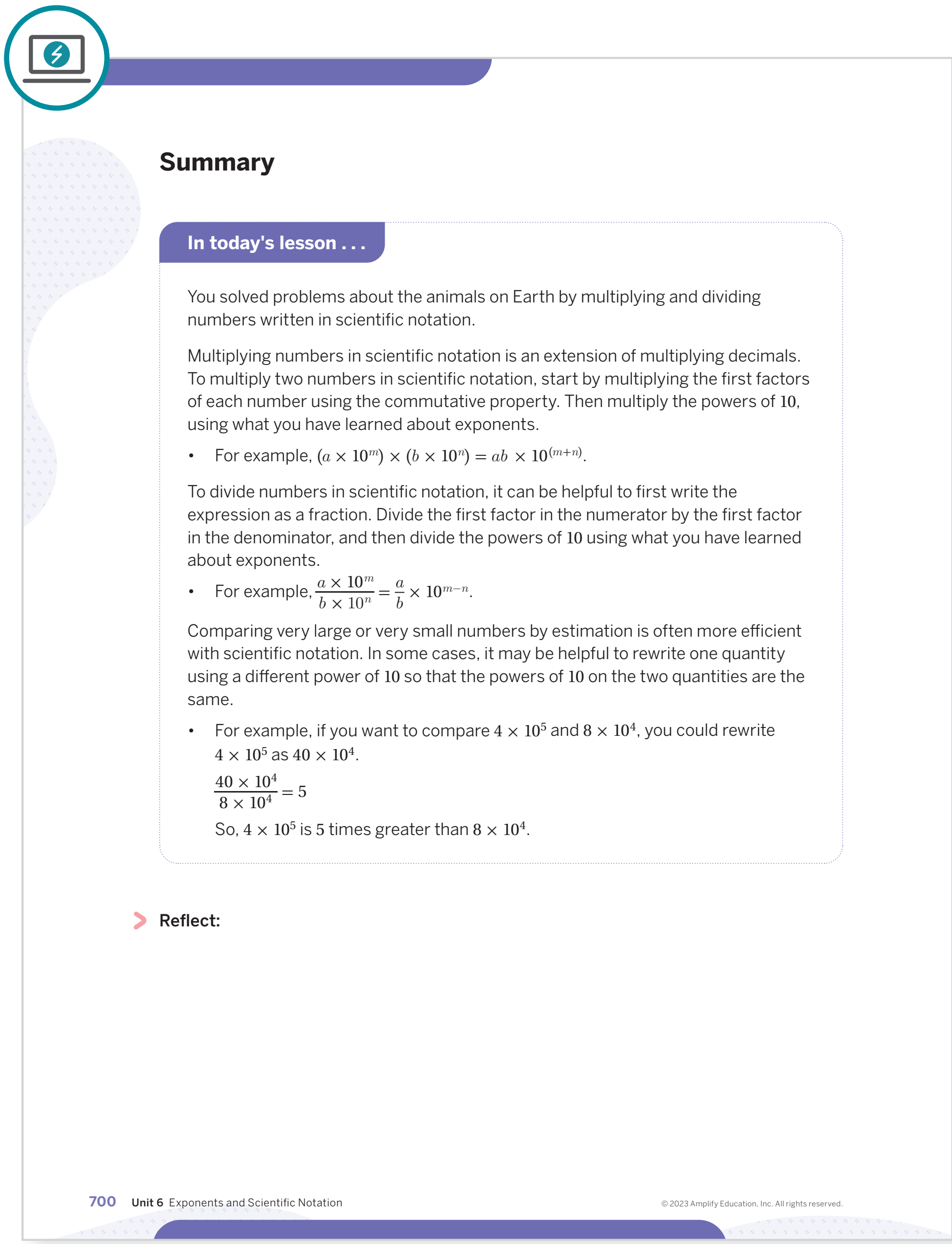
Ask, "What do you notice about Clare's strategy in Problem 3? How were you able to estimate how many humans weigh as much as one blue whale in Problem 4?"

Highlight that it is possible to estimate quotients without rewriting powers of 10. It is often not necessary when the factors are close to values that are easy to divide mentally. However, sometimes the values can be difficult to estimate, as in Problem 4, in which case rewriting numbers using different powers of 10 can help with estimation.

Whole Class | 5 min

Summary

Review and synthesize how scientific notation is useful when making multiplicative comparisons of numbers.



Summary

In today's lesson . . .

You solved problems about the animals on Earth by multiplying and dividing numbers written in scientific notation.

Multiplying numbers in scientific notation is an extension of multiplying decimals. To multiply two numbers in scientific notation, start by multiplying the first factors of each number using the commutative property. Then multiply the powers of 10, using what you have learned about exponents.

- For example, $(a \times 10^m) \times (b \times 10^n) = ab \times 10^{(m+n)}$.

To divide numbers in scientific notation, it can be helpful to first write the expression as a fraction. Divide the first factor in the numerator by the first factor in the denominator, and then divide the powers of 10 using what you have learned about exponents.

- For example, $\frac{a \times 10^m}{b \times 10^n} = \frac{a}{b} \times 10^{m-n}$.

Comparing very large or very small numbers by estimation is often more efficient with scientific notation. In some cases, it may be helpful to rewrite one quantity using a different power of 10 so that the powers of 10 on the two quantities are the same.

- For example, if you want to compare 4×10^5 and 8×10^4, you could rewrite 4×10^5 as 40×10^4.

  $\frac{40 \times 10^4}{8 \times 10^4} = 5$

  So, 4×10^5 is 5 times greater than 8×10^4.

Reflect:

Synthesize

Display the table from Activity 2.

Ask:

- "How can you tell that numbers you saw today in the table were written in scientific notation?" The first factor is greater than one, but less than ten. The second factor is an integer power of 10.
- "How does the convention about the numeric factor help you quickly get an idea about the size of a number?" Sample response: Because the first factors are all greater than one, but less than ten, I can quickly gauge the size by looking at the size of the exponent on the power of 10.
- "How does using scientific notation help you compare numbers?" Sample response: I can compare the exponents on the powers of 10 first. Then look at the first factors.
- "What were some strategies you used to better approximate products and quotients?" Answers may vary.
- "Suppose you had a table where the numbers were not in scientific notation. Would you be able to take a quick look at the table and have a feel for the relative sizes of the numbers?" Answers may vary, but students should note that it would be more efficient if the numbers were written in scientific notation.

Highlight that because the leading factor is always less than 10 in numbers written with scientific notation, the exponent gives most of the information about the size of the number. Numbers can be compared more quickly by examining the exponents and rounding the numerical factor.

If numbers are written using different powers of 10, students need to look at both the first factor *and* the power of 10 to gauge the size of each number.

If numbers are written as decimals, it would be even more challenging, as it would require counting zeros or decimal places.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

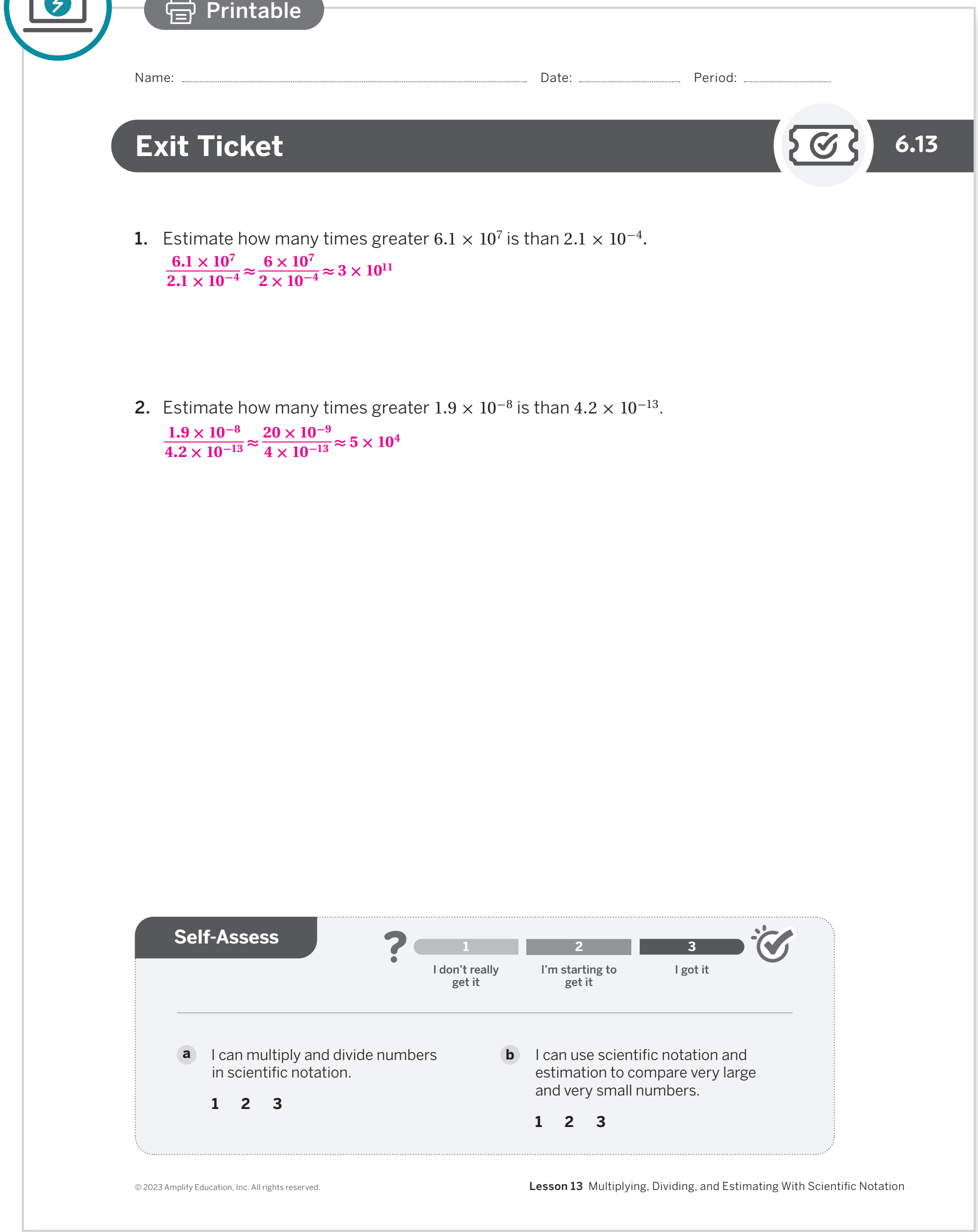
- "What strategies can be used when multiplying and dividing numbers expressed in scientific notation?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by estimating quotients with numbers written in scientific notation.

Printable



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Exit Ticket 6.13

1. Estimate how many times greater 6.1×10^7 is than 2.1×10^{-4}.

$$\frac{6.1 \times 10^7}{2.1 \times 10^{-4}} \approx \frac{6 \times 10^7}{2 \times 10^{-4}} \approx 3 \times 10^{11}$$

2. Estimate how many times greater 1.9×10^{-8} is than 4.2×10^{-13}.

$$\frac{1.9 \times 10^{-8}}{4.2 \times 10^{-13}} \approx \frac{20 \times 10^{-9}}{4 \times 10^{-13}} \approx 5 \times 10^4$$

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I can multiply and divide numbers in scientific notation.

1 2 3

b I can use scientific notation and estimation to compare very large and very small numbers.

1 2 3

Lesson 13 Multiplying, Dividing, and Estimating With Scientific Notation

Success looks like . . .

- **Language Goal:** Generalizing a process of multiplying and dividing numbers in scientific notation. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Using scientific notation and estimation to compare quantities and interpreting results in context. **(Speaking and Listening, Reading and Writing)**
  - » Comparing two numbers written in scientific notation in Problems 1 and 2.

Suggested next steps

If students are unable to complete Problem 1, consider:

- Reviewing estimating strategies from Activity 1.
- Assigning Practice Problem 1b.
- Asking, "How can you round 6.1 and 2.1 so that you can easily recognize a quotient mentally?"

If students are unable to complete Problem 2, consider:

- Reviewing estimating strategies from Activity 2.
- Assigning Practice Problem 2.
- Asking, "How can you rewrite 1.9×10^{-9} using estimation and powers of 10?"

Professional Learning

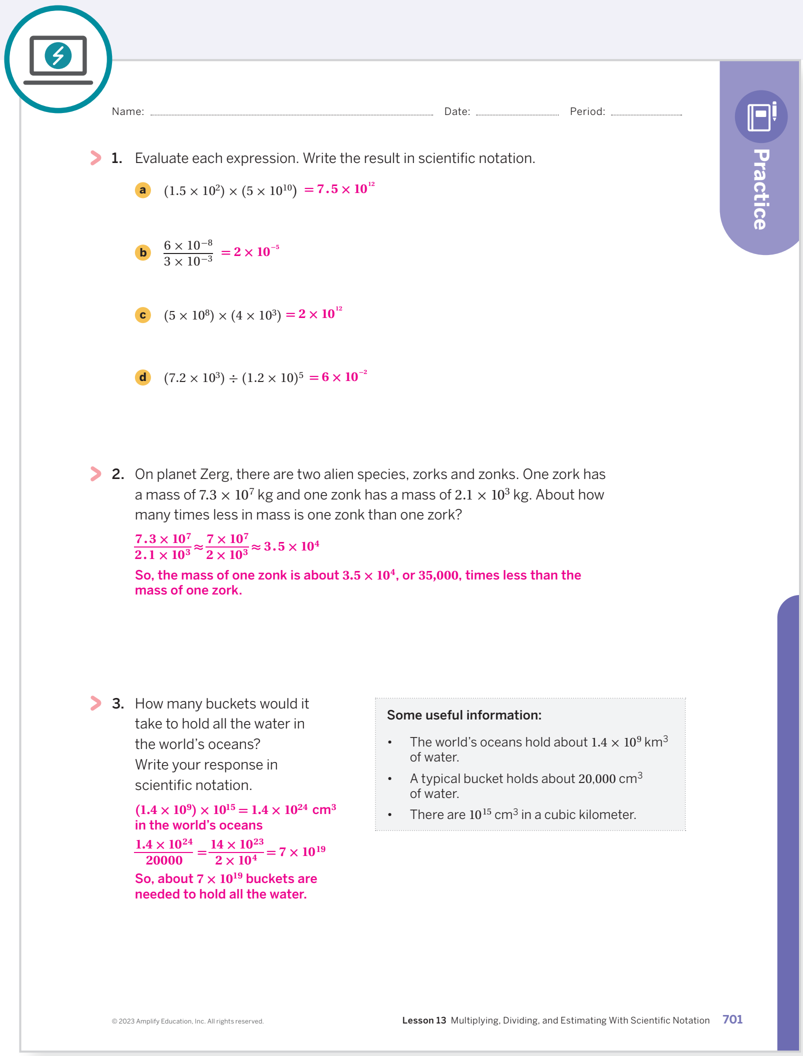
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to solve problems by multiplying and dividing numbers in scientific notation. How well did students accomplish this? What did you specifically do to help students accomplish it?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?

Practice

Independent



Name: Date: Period:

1. Evaluate each expression. Write the result in scientific notation.

a $(1.5 \times 10^2) \times (5 \times 10^{10})$ $= 7.5 \times 10^{12}$

b $\frac{6 \times 10^{-8}}{3 \times 10^{-3}}$ $= 2 \times 10^{-5}$

c $(5 \times 10^8) \times (4 \times 10^3)$ $= 2 \times 10^{12}$

d $(7.2 \times 10^3) \div (1.2 \times 10)^5$ $= 6 \times 10^{-2}$

2. On planet Zerg, there are two alien species, zorks and zonks. One zork has a mass of 7.3×10^7 kg and one zonk has a mass of 2.1×10^3 kg. About how many times less in mass is one zonk than one zork?

$\frac{7.3 \times 10^7}{2.1 \times 10^3} \approx \frac{7 \times 10^7}{2 \times 10^3} \approx 3.5 \times 10^4$

So, the mass of one zonk is about 3.5×10^4, or 35,000, times less than the mass of one zork.

3. How many buckets would it take to hold all the water in the world's oceans? Write your response in scientific notation.

$(1.4 \times 10^9) \times 10^{15} = 1.4 \times 10^{24}$ cm^3 in the world's oceans

$\frac{1.4 \times 10^{24}}{20000} = \frac{14 \times 10^{23}}{2 \times 10^4} = 7 \times 10^{19}$

So, about 7×10^{19} buckets are needed to hold all the water.

Some useful information:

- The world's oceans hold about 1.4×10^9 km^3 of water.
- A typical bucket holds about 20,000 cm^3 of water.
- There are 10^{15} cm^3 in a cubic kilometer.

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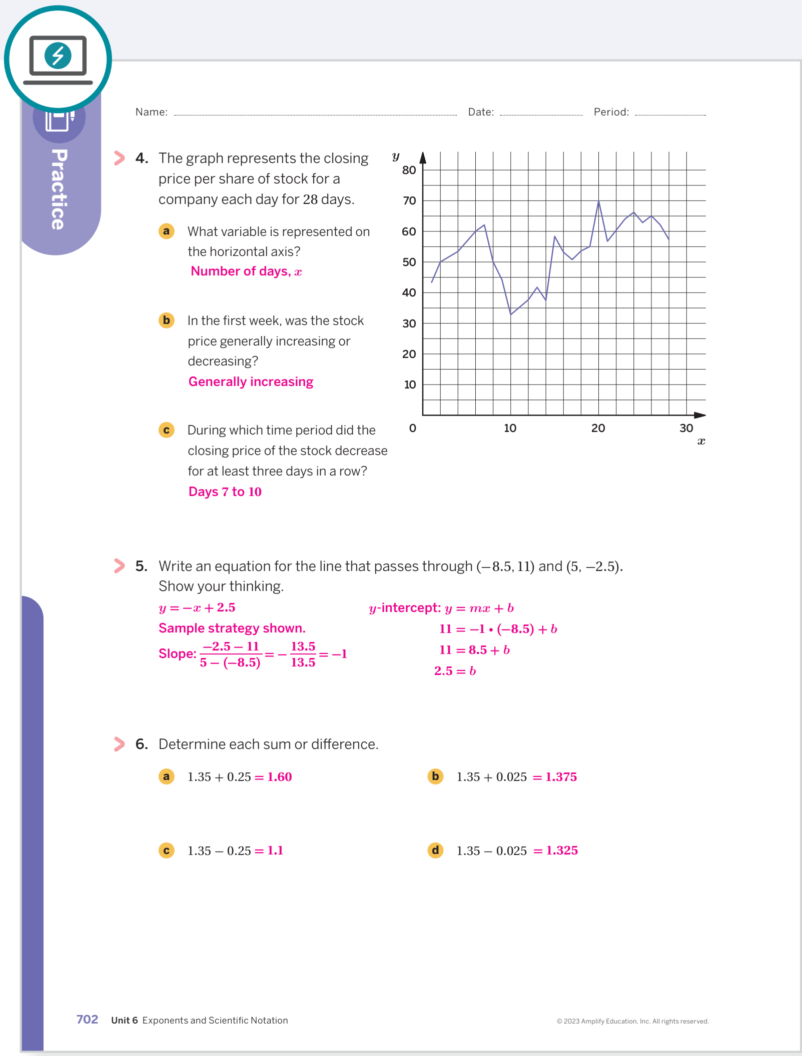

Name: Date: Period:

4. The graph represents the closing price per share of stock for a company each day for 28 days.

a What variable is represented on the horizontal axis?

Number of days, x

b In the first week, was the stock price generally increasing or decreasing?

Generally increasing

c During which time period did the closing price of the stock decrease for at least three days in a row?

Days 7 to 10

5. Write an equation for the line that passes through $(-8.5, 11)$ and $(5, -2.5)$. Show your thinking.

$y = -x + 2.5$

Sample strategy shown.

Slope: $\frac{-2.5 - 11}{5 - (-8.5)} = -\frac{13.5}{13.5} = -1$

y-intercept: $y = mx + b$

$11 = -1 \cdot (-8.5) + b$

$11 = 8.5 + b$

$2.5 = b$

6. Determine each sum or difference.

a $1.35 + 0.25 = 1.60$

b $1.35 + 0.025 = 1.375$

c $1.35 - 0.25 = 1.1$

d $1.35 - 0.025 = 1.325$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	3
Spiral	4	Unit 5 Lesson 5	1
	5	Unit 3 Lesson 13	1
Formative ⚡	6	Unit 6 Lesson 14	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Adding and Subtracting With Scientific Notation

Let's solve problems by adding and subtracting numbers in scientific notation.

Focus

Goal

1. **Language Goal:** Generalize a process of adding and subtracting numbers in scientific notation and interpret results in context. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students add and subtract numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students must make sense of the quantities and use quantitative reasoning to make comparisons, e.g., when comparing whether five planets placed side-by-side are wider than the Sun.

Previously

Students learned how to write numbers with scientific notation in Lesson 12. In Lesson 13, students learned strategies for multiplying, dividing, and estimating with scientific notation, including strategies that involved rewriting numbers using powers of 10.

Coming Soon

In Lesson 15, students will take what they have learned about adding, subtracting, multiplying, and dividing with scientific notation and apply their understanding to numbers in a new context.

Rigor

- Students build **conceptual understanding** of adding and subtracting numbers expressed in scientific notation.
- Students **apply** their understanding as they study a planet's distance from the Sun.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
7 min	7 min	12 min	8 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *equivalent expressions*
- *scientific notation*

Amps | Featured Activity

Activity 2
Interactive Scale

Students test their addition skills with scientific notation by using an interactive scale to balance large quantities of different celestial bodies.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might try to complete the activity with no regard for their partner. Point out that because adding and subtracting numbers in scientific notation requires great precision with regard to place value and decimal placement. Ask students to brainstorm how having a partner can benefit them and how they can help their partner. Encourage them to see this opportunity as a win-win.

Modifications to Pacing

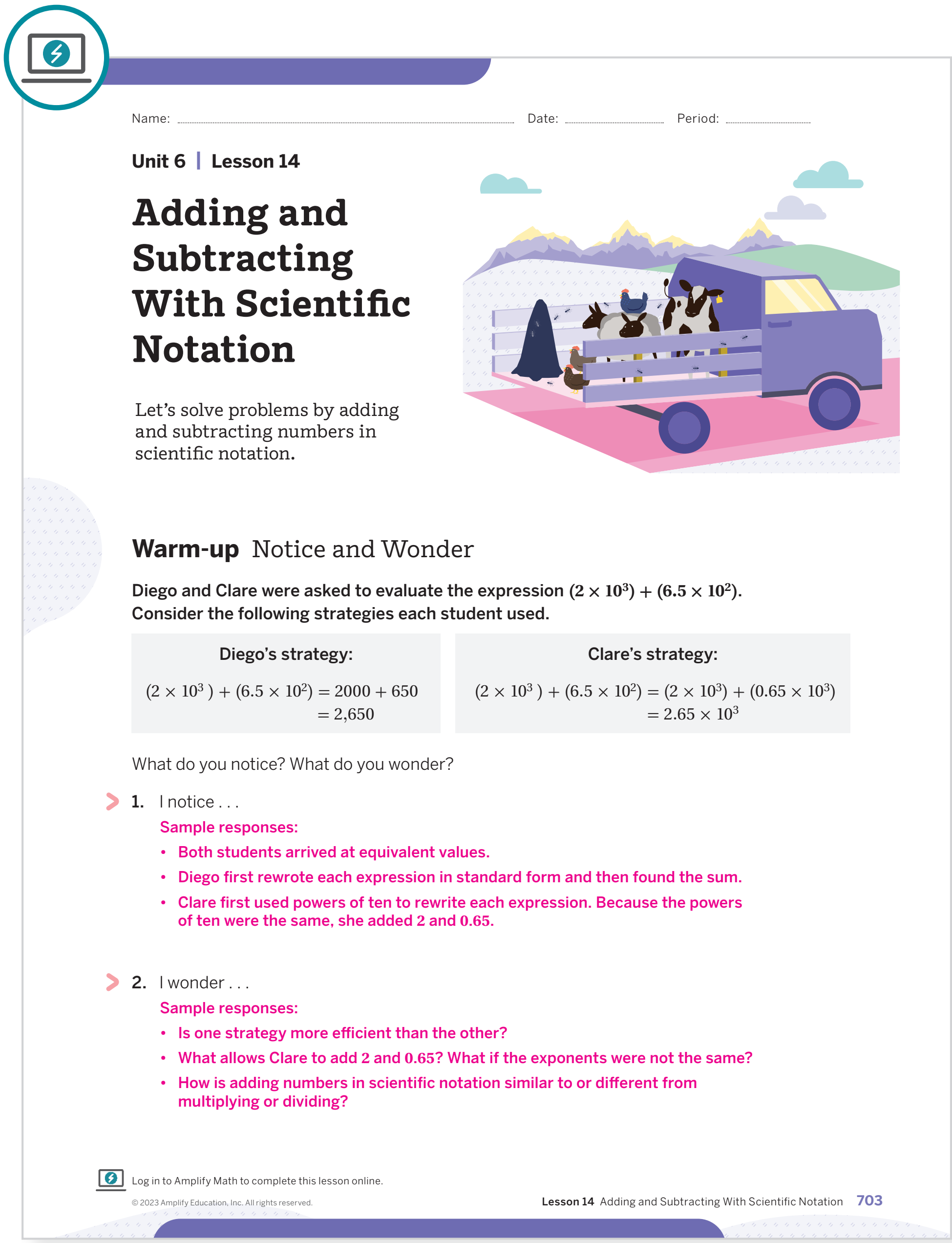
You may want to consider this additional modification if you are short on time.

- **Activity 3** may be omitted.

Independent | 7 min

Warm-up Notice and Wonder

Students study two strategies for adding numbers written in scientific notation to understand that one approach may be more efficient than another.



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Unit 6 | Lesson 14

Adding and Subtracting With Scientific Notation

Let's solve problems by adding and subtracting numbers in scientific notation.

Warm-up Notice and Wonder

Diego and Clare were asked to evaluate the expression $(2 \times 10^3) + (6.5 \times 10^2)$. Consider the following strategies each student used.

Diego's strategy:	**Clare's strategy:**
$(2 \times 10^3) + (6.5 \times 10^2) = 2000 + 650$ $= 2{,}650$	$(2 \times 10^3) + (6.5 \times 10^2) = (2 \times 10^3) + (0.65 \times 10^3)$ $= 2.65 \times 10^3$

What do you notice? What do you wonder?

1. I notice . . .

   Sample responses:
   - Both students arrived at equivalent values.
   - Diego first rewrote each expression in standard form and then found the sum.
   - Clare first used powers of ten to rewrite each expression. Because the powers of ten were the same, she added 2 and 0.65.

2. I wonder . . .

   Sample responses:
   - Is one strategy more efficient than the other?
   - What allows Clare to add 2 and 0.65? What if the exponents were not the same?
   - How is adding numbers in scientific notation similar to or different from multiplying or dividing?

Log in to Amplify Math to complete this lesson online.

1 Launch

Tell students that they should not use a calculator during today's lesson. Conduct the *Notice and Wonder* routine.

2 Monitor

Help students get started by asking, "What do you notice about Diego's first step when he wrote 2×10^3 as $2{,}000$? What did he do next? Why do you think he did that?"

Look for points of confusion:

- **Thinking one of the strategies was incorrect.** Review each strategy to ensure students see mathematically valid steps and equivalent values.

3 Connect

Have students share what they noticed that was similar about both strategies.

Highlight that while both strategies are valid, for numbers that are much larger, using scientific notation (Clare's strategy) will often be more efficient.

Ask, "Why do you think Clare rewrote 6.5×10^2 as 0.65×10^3? Do you think she could have chosen to rewrite 2×10^3 instead? What would that have looked like?"

Math Language Development

MLR5: Co-craft Questions

During the Connect, have students meet with a partner and share what they noticed and wondered about Diego's and Clare's strategies. Invite them to work together to co-craft 2–3 mathematical questions they could ask about the two strategies shown. Have pairs of students share their questions with the class.

English Learners

Model for students an example of one question they might ask, such as: "How is adding numbers in scientific notation similar to or different from multiplying or dividing?"

Power-up

To power up students' ability to add and subtract decimals, have students complete:

Recall that when determining the sum or difference place values must be aligned. Rewrite each horizontal expression as a vertical expression, and then evaluate.

1. $6.05 + 12.3 = 18.35$

$$\begin{array}{r} 6.05 \\ +\ 12.30 \\ \hline 18.35 \end{array}$$

2. $12.35 - 1.025 = 11.325$

$$\begin{array}{r} 12.350 \\ -\ 1.025 \\ \hline 11.325 \end{array}$$

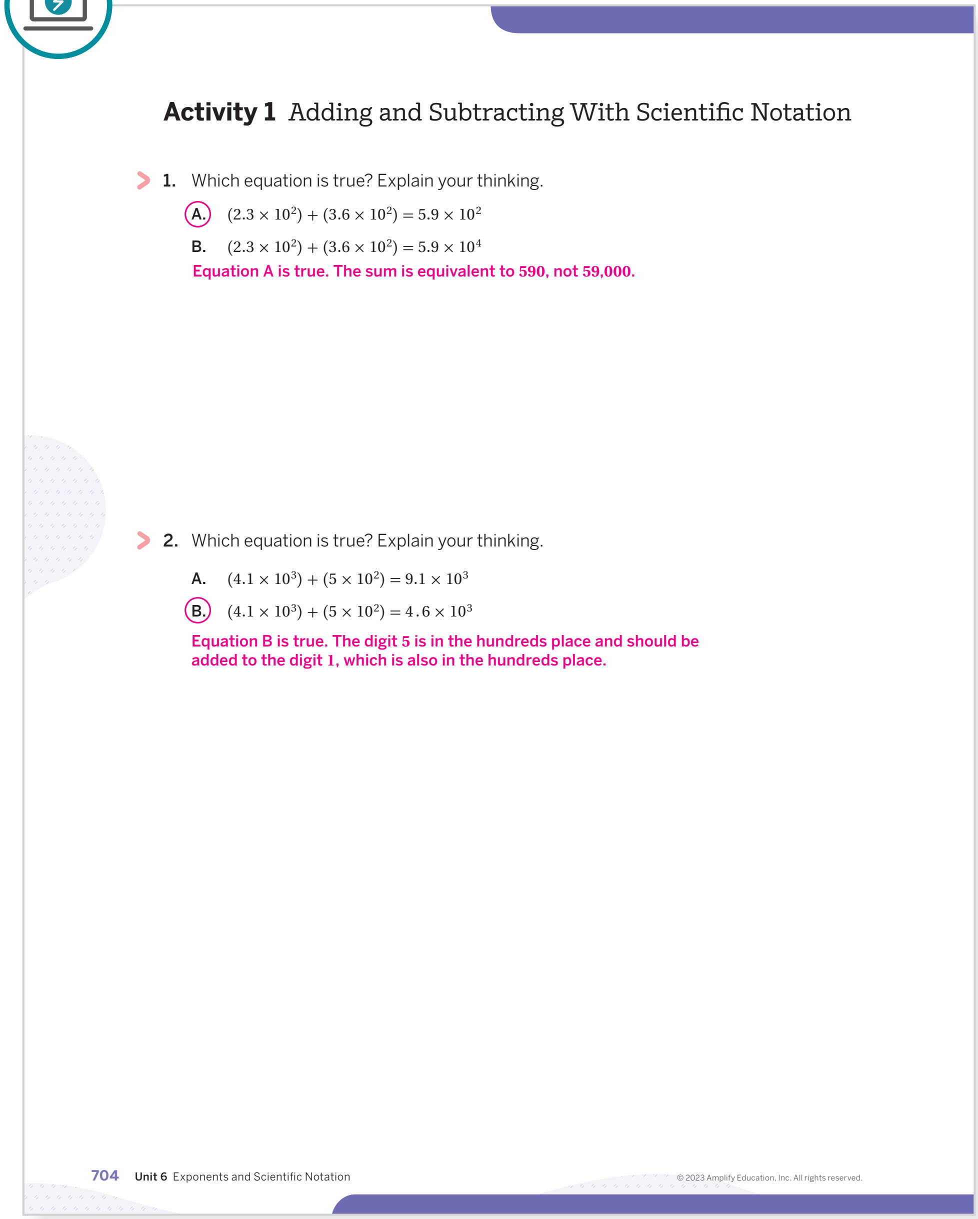
Use: Before Activity 1

Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Pairs | 7 min

Activity 1 Adding and Subtracting With Scientific Notation

Students determine the truth of equations involving scientific notation to better understand the importance of place value when adding and subtracting numbers with scientific notation.



Activity 1 Adding and Subtracting With Scientific Notation

1. Which equation is true? Explain your thinking.

   [✓ A.] $(2.3 \times 10^2) + (3.6 \times 10^2) = 5.9 \times 10^2$

   B. $(2.3 \times 10^2) + (3.6 \times 10^2) = 5.9 \times 10^4$

   Equation A is true. The sum is equivalent to 590, not 59,000.

2. Which equation is true? Explain your thinking.

   A. $(4.1 \times 10^3) + (5 \times 10^2) = 9.1 \times 10^3$

   [✓ B.] $(4.1 \times 10^3) + (5 \times 10^2) = 4.6 \times 10^3$

   Equation B is true. The digit 5 is in the hundreds place and should be added to the digit 1, which is also in the hundreds place.

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by showing students how to write each number in standard form.

Look for points of confusion:

- **Thinking that $10^2 + 10^2 = 10^4$.** Have students write each number in standard form before adding and then ask, "How would you write the sum in scientific notation?"
- **Thinking Equation A in Problem 2 is true.** Ask, "What place value does the 5 represent in 5×10^2? What digit represents the same place value in 4.1×10^3?"

Look for productive strategies:

- Students using scientific notation to add values, rewriting with powers of 10 if needed.

3 Connect

Have students share how they know each equation is true. Have students who chose Diego's strategy from the Warm-up to share their responses first. Then have students who chose Clare's strategy share next.

Highlight: If two numbers written in scientific notation have different powers of 10, then the digits in their first factors represent different place values.

Ask, "Why would you prefer the terms have the same power of 10 before adding? Does this also apply to subtraction?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing students with information for which equations are true and which are false beforehand. This will provide them with more time to think about how they can validate each claim and explain their thinking.

Extension: *Math Enrichment*

Have students complete the missing terms from the equations below to make them true.

$(3.6 \times 10^6) + (?) = (6.4 \times 10^2)$

$(?) - (2 \times 10^2) = (8.6 \times 10^3)$

Math Language Development

MLR8: Discussion Supports — Restate It!

During the Connect, as students share, ask their classmates to restate what they heard using mathematical language. Ask the original speaker if their peer was able to accurately restate their thinking. For example:

If a student says . . .	Their classmates could say . . .
"In Problem 1, Equation B is not true because the exponent should be 2."	"So, you looked at the exponents on the powers of 10 and determined that they should not be added. Is that correct?"

English Learners

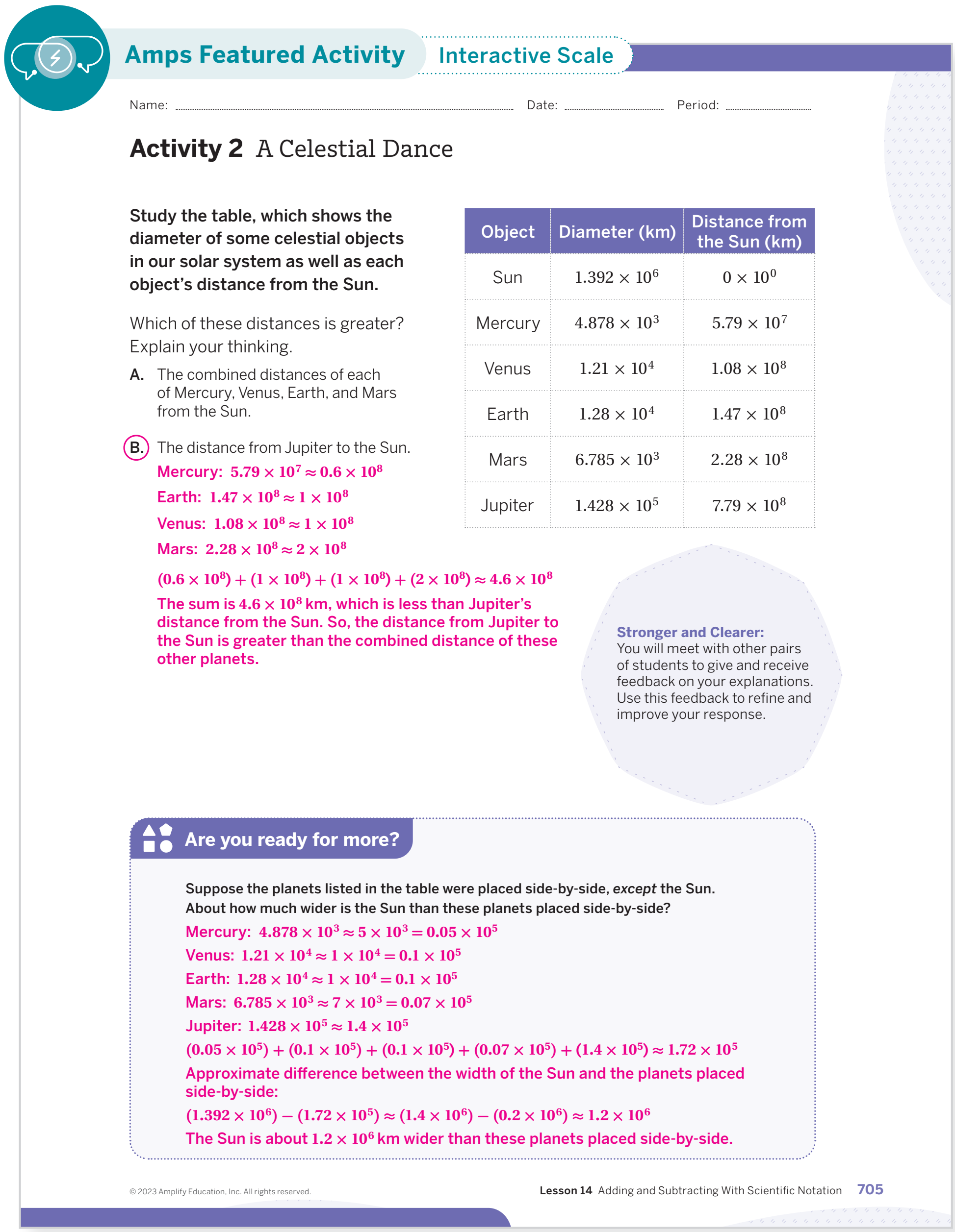
Give students time to rehearse what they will say with a partner before sharing with the class.

Pairs | 12 min

Activity 2 A Celestial Dance

Students add quantities written in scientific notation in order to solve problems in context. Students realize they need to attend to precision by aligning place value.

Amps Featured Activity Interactive Scale



Name: Date: Period:

Activity 2 A Celestial Dance

Study the table, which shows the diameter of some celestial objects in our solar system as well as each object's distance from the Sun.

Object	Diameter (km)	Distance from the Sun (km)
Sun	1.392×10^6	0×10^0
Mercury	4.878×10^3	5.79×10^7
Venus	1.21×10^4	1.08×10^8
Earth	1.28×10^4	1.47×10^8
Mars	6.785×10^3	2.28×10^8
Jupiter	1.428×10^5	7.79×10^8

Which of these distances is greater? Explain your thinking.

A. The combined distances of each of Mercury, Venus, Earth, and Mars from the Sun.

[✓ B.] The distance from Jupiter to the Sun.

Mercury: $5.79 \times 10^7 \approx 0.6 \times 10^8$

Earth: $1.47 \times 10^8 \approx 1 \times 10^8$

Venus: $1.08 \times 10^8 \approx 1 \times 10^8$

Mars: $2.28 \times 10^8 \approx 2 \times 10^8$

$(0.6 \times 10^8) + (1 \times 10^8) + (1 \times 10^8) + (2 \times 10^8) \approx 4.6 \times 10^8$

The sum is 4.6×10^8 km, which is less than Jupiter's distance from the Sun. So, the distance from Jupiter to the Sun is greater than the combined distance of these other planets.

Stronger and Clearer: You will meet with other pairs of students to give and receive feedback on your explanations. Use this feedback to refine and improve your response.

Are you ready for more?

Suppose the planets listed in the table were placed side-by-side, *except* the Sun. About how much wider is the Sun than these planets placed side-by-side?

Mercury: $4.878 \times 10^3 \approx 5 \times 10^3 = 0.05 \times 10^5$

Venus: $1.21 \times 10^4 \approx 1 \times 10^4 = 0.1 \times 10^5$

Earth: $1.28 \times 10^4 \approx 1 \times 10^4 = 0.1 \times 10^5$

Mars: $6.785 \times 10^3 \approx 7 \times 10^3 = 0.07 \times 10^5$

Jupiter: $1.428 \times 10^5 \approx 1.4 \times 10^5$

$(0.05 \times 10^5) + (0.1 \times 10^5) + (0.1 \times 10^5) + (0.07 \times 10^5) + (1.4 \times 10^5) \approx 1.72 \times 10^5$

Approximate difference between the width of the Sun and the planets placed side-by-side:

$(1.392 \times 10^6) - (1.72 \times 10^5) \approx (1.4 \times 10^6) - (0.2 \times 10^6) \approx 1.2 \times 10^6$

The Sun is about 1.2×10^6 km wider than these planets placed side-by-side.

1 Launch

Read the task with students, and ask, "What do you notice about the powers of 10 in the planets listed in the table?" Encourage students to first rewrite the powers of 10 before adding, reminding them that they can review Clare's strategy from the Warm-up.

2 Monitor

Help students get started by saying, "To approximate a sum, first round each value. Then, look to see if you need to rewrite any powers of 10 so that it is more efficient for you to add."

Look for points of confusion:

- **Not being able to determine to which power of 10 they should align the place values.** Students can choose to align their place values to any power of 10 of the addends.
- **Struggling to rewrite each power of 10.** Review for students how to rewrite each factor by the appropriate factor of 10.

3 Connect

Display student work showing different ways of correctly aligning place value.

Have students share their process for choosing which place value to align to and how they rewrote the equivalent expressions using powers of 10.

Highlight that students must pay close attention to place values and powers of 10 when adding or subtracting. Multiple ways of aligning place value are valid as long as each of the powers of 10 are the same and the answer is rewritten in scientific notation.

Differentiated Support

Extension: *Interdisciplinary Connections*

Provide students with the following table and have them determine how the mass of the Sun compares with the combined masses of the planets in our solar system. Tell them the mass of the Sun is 1.989×10^{30} kg. **(Science)**

Planet	Mass (kg)	Planet	Mass (kg)
Mercury	3.285×10^{23}	**Jupiter**	1.898×10^{27}
Venus	4.867×10^{24}	**Saturn**	5.683×10^{26}
Earth	5.972×10^{24}	**Uranus**	8.681×10^{25}
Mars	6.39×10^{23}	**Neptune**	1.024×10^{26}

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write their explanation, hair pairs meet with 1–2 other pairs of students to give and receive feedback. Encourage reviewers to ask clarifying questions such as:

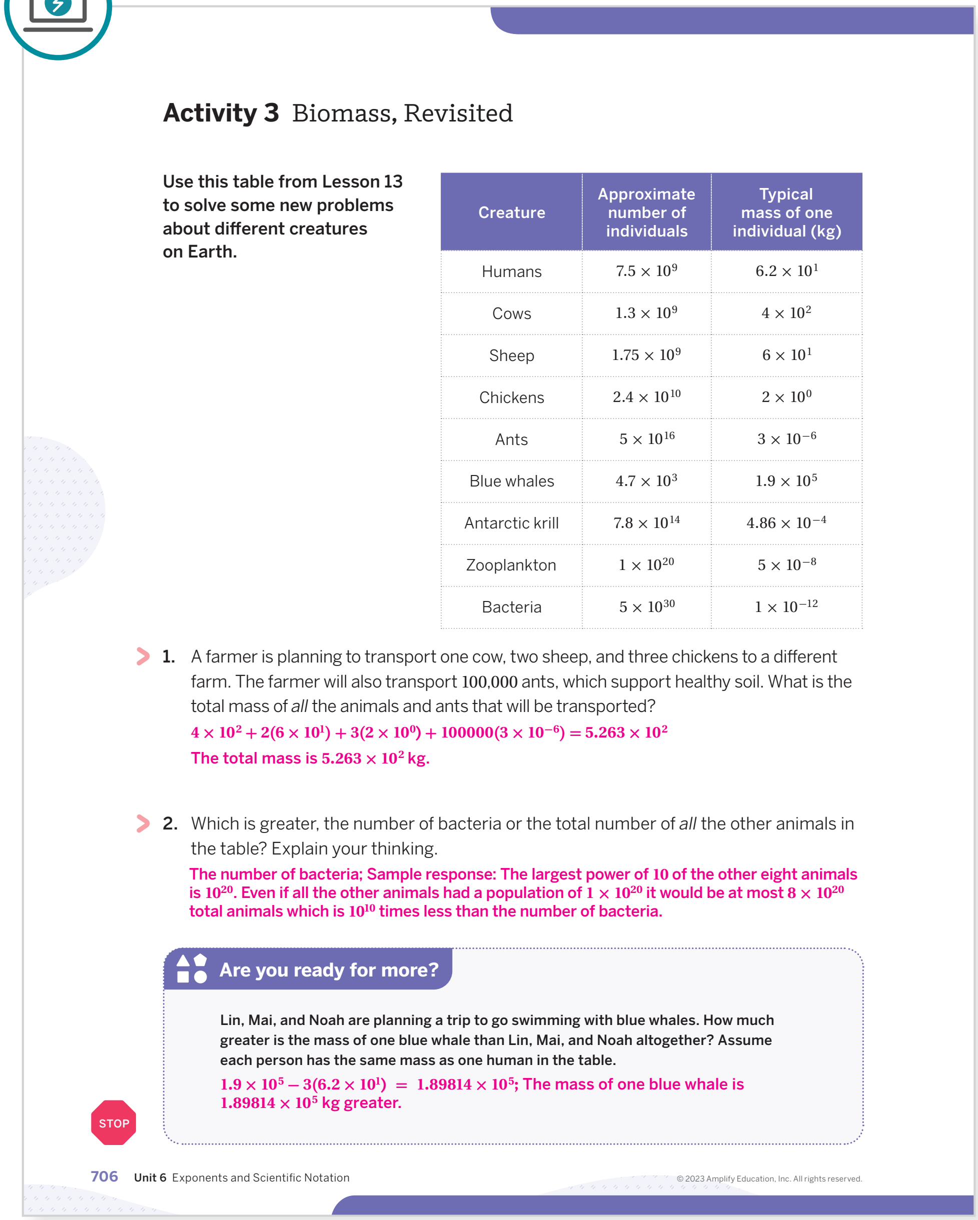
- "What calculations did you use in your response?"
- "Why did you choose these calculations?"
- "What words did you use as text clues?"

Have students revise their responses, as needed.

Pairs | 8 min

Activity 3 Biomass, Revisited

Students answer questions about the same creatures from Lesson 13, using scientific notation as a tool for working with small and large numbers.



Activity 3 Biomass, Revisited

Use this table from Lesson 13 to solve some new problems about different creatures on Earth.

Creature	Approximate number of individuals	Typical mass of one individual (kg)
Humans	7.5×10^9	6.2×10^1
Cows	1.3×10^9	4×10^2
Sheep	1.75×10^9	6×10^1
Chickens	2.4×10^{10}	2×10^0
Ants	5×10^{16}	3×10^{-6}
Blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
Zooplankton	1×10^{20}	5×10^{-8}
Bacteria	5×10^{30}	1×10^{-12}

1. A farmer is planning to transport one cow, two sheep, and three chickens to a different farm. The farmer will also transport 100,000 ants, which support healthy soil. What is the total mass of *all* the animals and ants that will be transported?

   $4 \times 10^2 + 2(6 \times 10^1) + 3(2 \times 10^0) + 100000(3 \times 10^{-6}) = 5.263 \times 10^2$

   The total mass is 5.263×10^2 kg.

2. Which is greater, the number of bacteria or the total number of *all* the other animals in the table? Explain your thinking.

   The number of bacteria; Sample response: The largest power of 10 of the other eight animals is 10^{20}. Even if all the other animals had a population of 1×10^{20} it would be at most 8×10^{20} total animals which is 10^{10} times less than the number of bacteria.

Are you ready for more?

Lin, Mai, and Noah are planning a trip to go swimming with blue whales. How much greater is the mass of one blue whale than Lin, Mai, and Noah altogether? Assume each person has the same mass as one human in the table.

$1.9 \times 10^5 - 3(6.2 \times 10^1) = 1.89814 \times 10^5$; The mass of one blue whale is 1.89814×10^5 kg greater.

STOP

706 **Unit 6** Exponents and Scientific Notation

1 Launch

Tell students they should write their final responses using scientific notation.

2 Monitor

Help students get started by asking, "How can you find the mass of two sheep?"

Look for points of confusion:

- **Not recognizing they need to rewrite their final response in scientific notation.** Say, "Keep in mind the directions. Is your response in scientific notation?"
- **Having difficulty with powers of 10 with negative exponents in Problem 1.** Remind students the process is the same and guide them in rewriting the mass of one ant in order to align the place values.

Look for productive strategies:

- Aligning to larger powers of 10 so that they don't need to write as many digits for their addends.

3 Connect

Display correct work for Problems 1 and 2.

Have students share what strategies they found most efficient and how they were able to approximate their responses.

Highlight strategies that used estimation to complete Problem 2. Highlight that the power of 10 tells them more about the size of a number, when written in scientific notation, than the leading factor does. This is why the total number of bacteria is much greater than the sum of the other animals.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can test their skills related to adding values in scientific notation by using an interaction scale to balance large quantities of different animals.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect response for Problem 2, such as "The total number of all other animals is greater because the sum of the exponents on the powers of 10 for the other animals is 120, which is greater than the exponent on the power of 10 for the number of bacteria." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

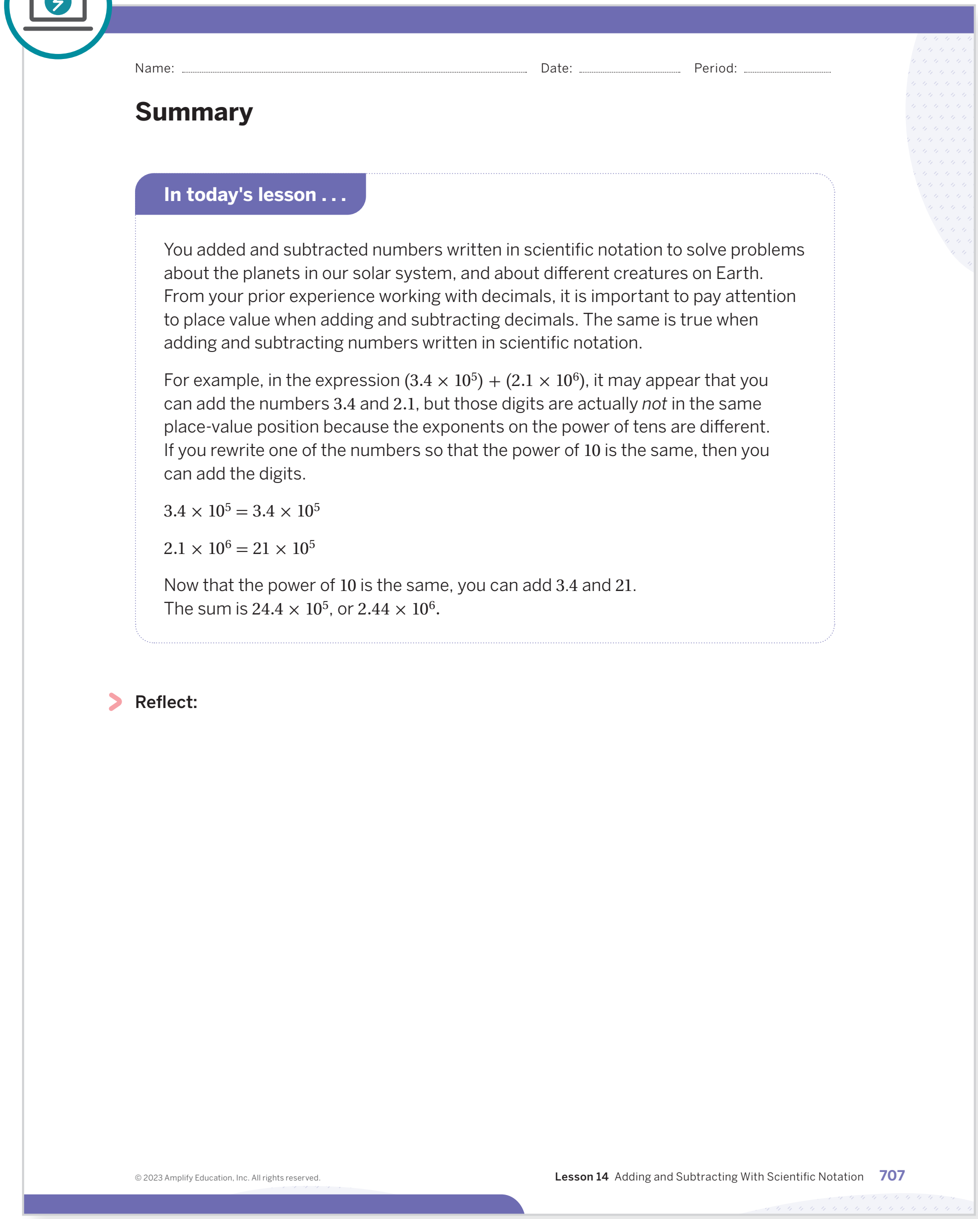
English Learners

Give students time to rehearse what they will say with a partner before sharing with the class.

Whole Class | 5 min

Summary

Review and synthesize the importance of attending to place value when adding and subtracting with numbers written in scientific notation.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You added and subtracted numbers written in scientific notation to solve problems about the planets in our solar system, and about different creatures on Earth. From your prior experience working with decimals, it is important to pay attention to place value when adding and subtracting decimals. The same is true when adding and subtracting numbers written in scientific notation.

For example, in the expression $(3.4 \times 10^5) + (2.1 \times 10^6)$, it may appear that you can add the numbers 3.4 and 2.1, but those digits are actually *not* in the same place-value position because the exponents on the power of tens are different. If you rewrite one of the numbers so that the power of 10 is the same, then you can add the digits.

$3.4 \times 10^5 = 3.4 \times 10^5$

$2.1 \times 10^6 = 21 \times 10^5$

Now that the power of 10 is the same, you can add 3.4 and 21. The sum is 24.4×10^5, or 2.44×10^6.

Reflect:

Synthesize

Display the Warm-up from the beginning of the lesson.

Ask:

- "Thinking back to the Warm-up, which method do you prefer to add two numbers in scientific notation?"
- "How is adding and subtracting with scientific notation different from multiplying and dividing? Which is less challenging? Why do you think that is?"
- "Is there anything you found surprising or interesting in the problems you completed?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

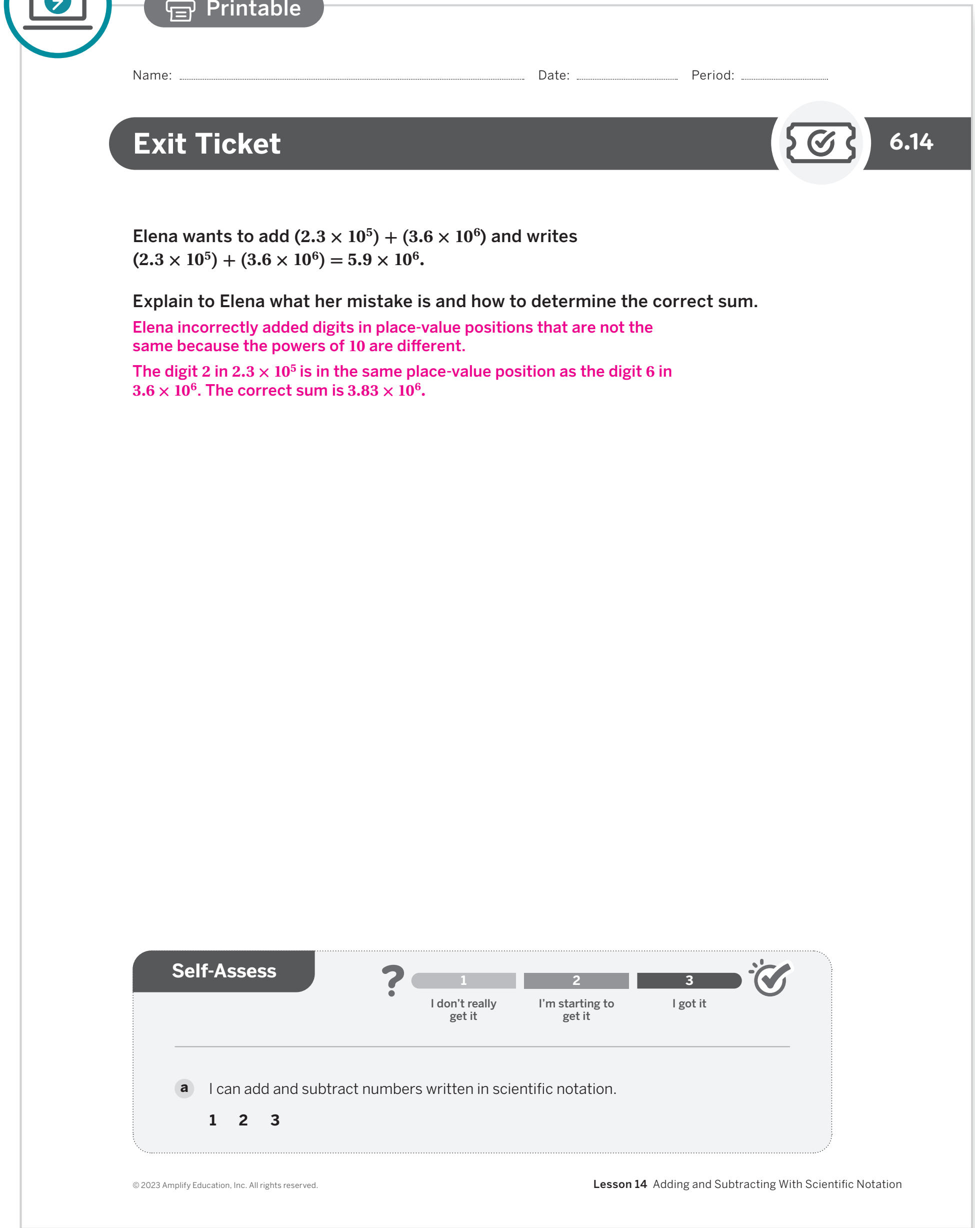
- "What strategies can be used when adding and subtracting numbers expressed in scientific notation?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by evaluating incorrect student work and correctly adding numbers expressed in scientific notation.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 6.14

Elena wants to add $(2.3 \times 10^5) + (3.6 \times 10^6)$ and writes $(2.3 \times 10^5) + (3.6 \times 10^6) = 5.9 \times 10^6$.

Explain to Elena what her mistake is and how to determine the correct sum.

Elena incorrectly added digits in place-value positions that are not the same because the powers of 10 are different.

The digit 2 in 2.3×10^5 is in the same place-value position as the digit 6 in 3.6×10^6. The correct sum is 3.83×10^6.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can add and subtract numbers written in scientific notation.

1 2 3

Lesson 14 Adding and Subtracting With Scientific Notation

Success looks like . . .

- **Language Goal:** Generalizing a process of adding and subtracting numbers in scientific notation and interpreting results in context. **(Speaking and Listening, Reading and Writing)**
  - » Correcting Elena's mistake in adding two numbers written in scientific notation.

Suggested next steps

If students cannot correctly rewrite the powers of 10 to align place values, consider:

- Reviewing strategies from the Warm-up.
- Assigning Practice Problem 1.

If students cannot correctly identify Elena's mistake, consider:

- Asking, "What do you notice about the powers of 10 for each addend? What does that tell you about the place values of each digit in our addends?"
- Reviewing takeaways from Activity 1.
- Assigning Practice Problems 1 and 2.

Professional Learning

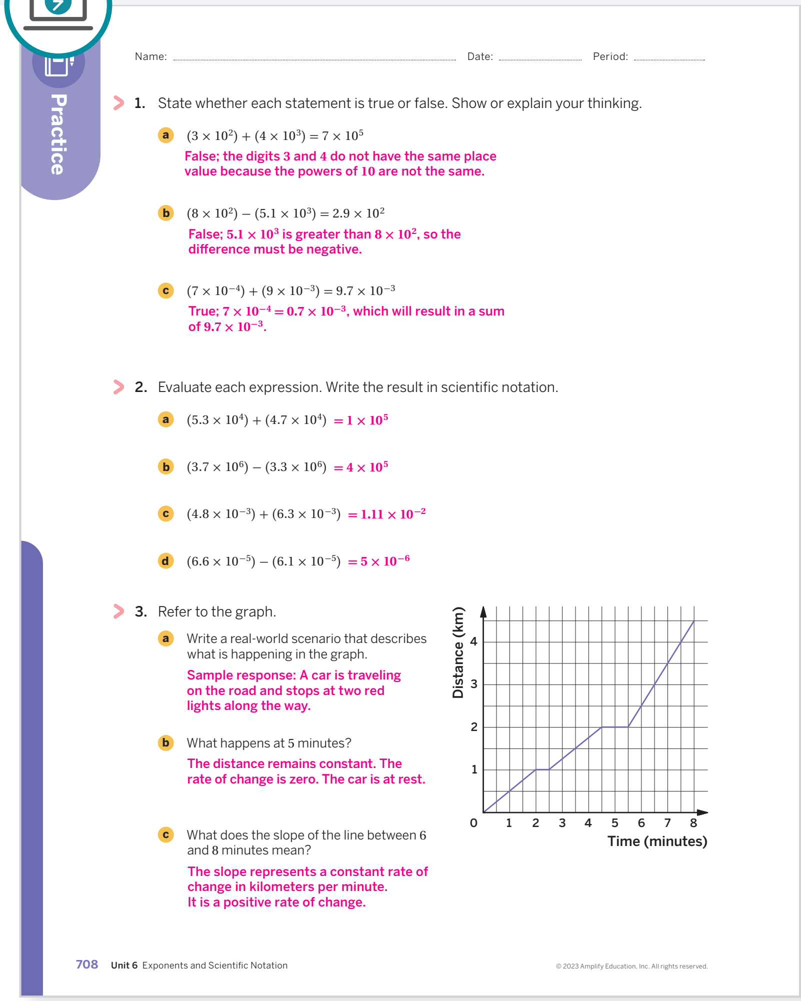
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion in Activity 1 how did you encourage each student to share their understandings?
- Which students' ideas were you able to highlight during Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent


Practice

Name: Date: Period:

1. State whether each statement is true or false. Show or explain your thinking.
   - a $(3 \times 10^2) + (4 \times 10^3) = 7 \times 10^5$
     False; the digits 3 and 4 do not have the same place value because the powers of 10 are not the same.
   - b $(8 \times 10^2) - (5.1 \times 10^3) = 2.9 \times 10^2$
     False; 5.1×10^3 is greater than 8×10^2, so the difference must be negative.
   - c $(7 \times 10^{-4}) + (9 \times 10^{-3}) = 9.7 \times 10^{-3}$
     True; $7 \times 10^{-4} = 0.7 \times 10^{-3}$, which will result in a sum of 9.7×10^{-3}.

2. Evaluate each expression. Write the result in scientific notation.
   - a $(5.3 \times 10^4) + (4.7 \times 10^4)$ **$= 1 \times 10^5$**
   - b $(3.7 \times 10^6) - (3.3 \times 10^6)$ **$= 4 \times 10^5$**
   - c $(4.8 \times 10^{-3}) + (6.3 \times 10^{-3})$ **$= 1.11 \times 10^{-2}$**
   - d $(6.6 \times 10^{-5}) - (6.1 \times 10^{-5})$ **$= 5 \times 10^{-6}$**

3. Refer to the graph.
   - a Write a real-world scenario that describes what is happening in the graph.
     Sample response: A car is traveling on the road and stops at two red lights along the way.
   - b What happens at 5 minutes?
     The distance remains constant. The rate of change is zero. The car is at rest.
   - c What does the slope of the line between 6 and 8 minutes mean?
     The slope represents a constant rate of change in kilometers per minute. It is a positive rate of change.

708 **Unit 6** Exponents and Scientific Notation

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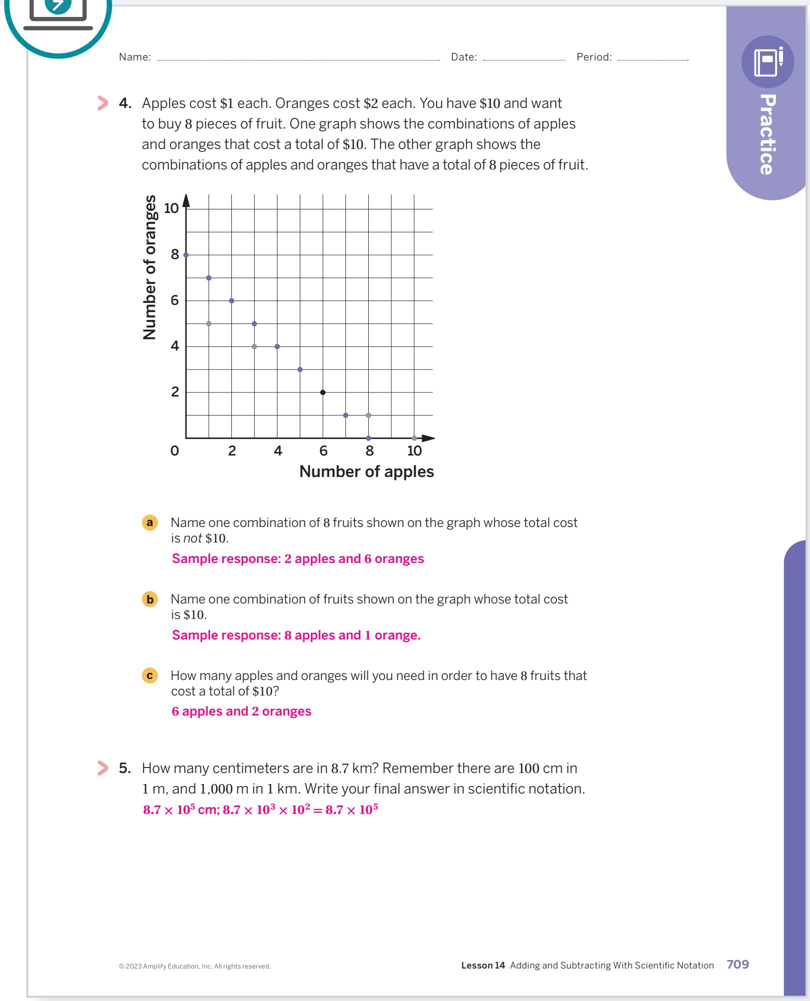
Practice

Name: Date: Period:

4. Apples cost \$1 each. Oranges cost \$2 each. You have \$10 and want to buy 8 pieces of fruit. One graph shows the combinations of apples and oranges that cost a total of \$10. The other graph shows the combinations of apples and oranges that have a total of 8 pieces of fruit.
   - a Name one combination of 8 fruits shown on the graph whose total cost is *not* \$10.
     Sample response: 2 apples and 6 oranges
   - b Name one combination of fruits shown on the graph whose total cost is \$10.
     Sample response: 8 apples and 1 orange.
   - c How many apples and oranges will you need in order to have 8 fruits that cost a total of \$10?
     6 apples and 2 oranges

5. How many centimeters are in 8.7 km? Remember there are 100 cm in 1 m, and 1,000 m in 1 km. Write your final answer in scientific notation.
   8.7×10^5 cm; $8.7 \times 10^3 \times 10^2 = 8.7 \times 10^5$

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Lesson 14 Adding and Subtracting With Scientific Notation 709

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
Spiral	3	Unit 5 Lesson 5	2
	4	Unit 4 Lesson 13	2
Formative ⚡	5	Unit 6 Lesson 15	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 6 | **LESSON 15 – CAPSTONE**

Is a Smartphone Smart Enough to Go to the Moon?

Let's answer some big questions about even larger numbers!

Focus

Goal

1. **Language Goal:** Use scientific notation to compare quantities in context, and describe how using scientific notation helps with making comparisons between very large and very small quantities. **(Speaking and Listening)**

Coherence

Today

In this culminating lesson, students use scientific notation as a tool for making comparisons. Students compare old hardware to new hardware using various digital media as a form of measurement. Students must reason qualitatively and abstractly in order to use scientific notation in context.

Previously

In previous lessons, students learned how to use scientific notation to add, subtract, multiply, and divide large and small numbers.

Coming Soon

In future grades, students will continue to use scientific notation and powers of 10 to describe and calculate large and small numbers.

Rigor

- Students **apply** their understanding as they perform operations with large numbers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	20 min	15 min	3 min	3 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per pair
- Activity 1 PDF (answers)

Math Language Development

Review words

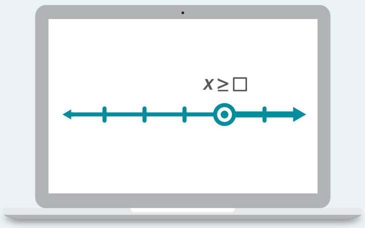
- *scientific notation*

Amps | **Featured Activity**

Activity 2

Interactive Zoom

Students use an interactive zoom tool to see large quantities on a number line.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students investigate the scale of large numbers and may wrestle with understanding the size of one million or one trillion. By contextualizing this concept and thinking about how long it would take to count to each of these numbers, they can have a greater appreciation for how large they really are.

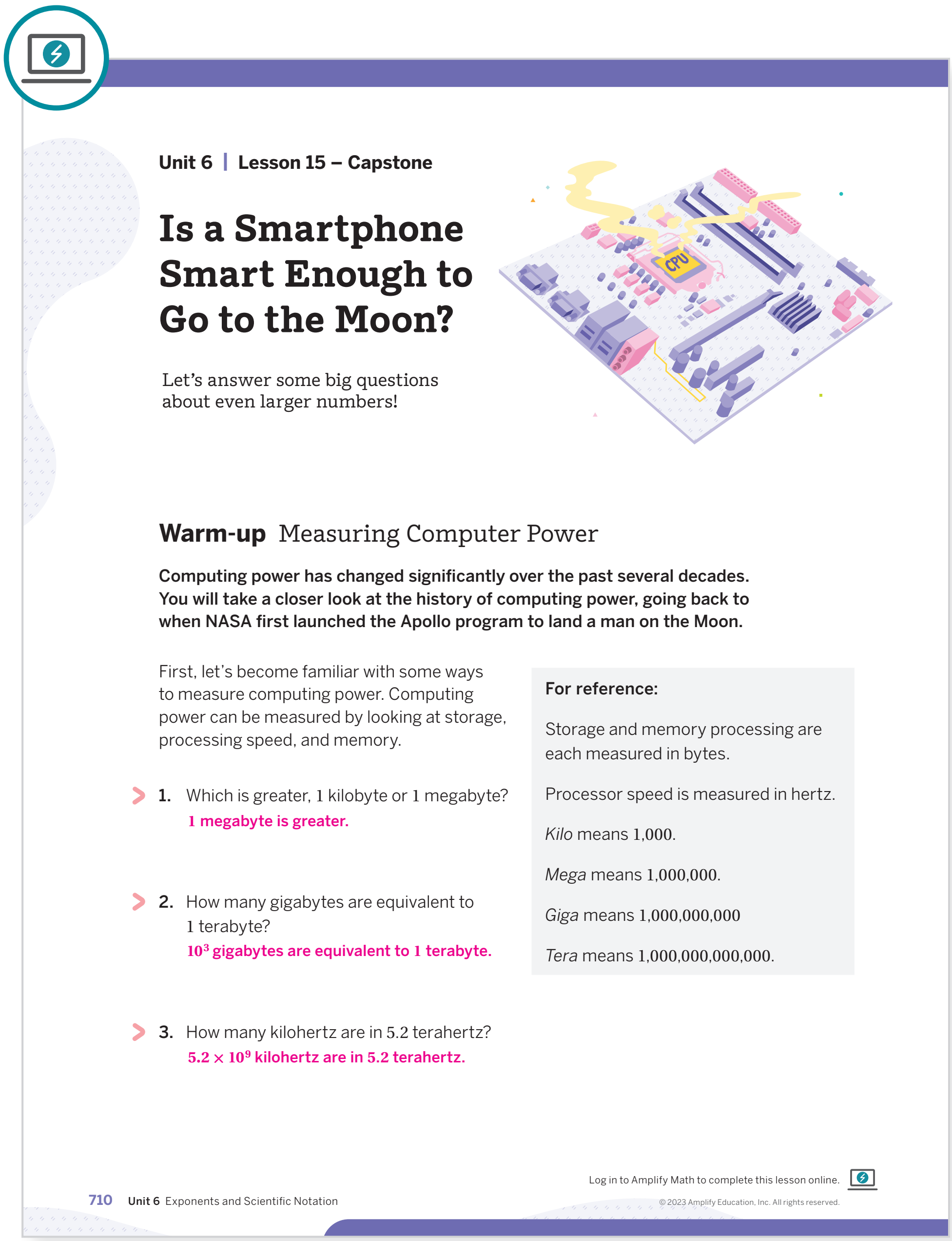
Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Select either **Activity 1** or **Activity 2** to complete.

Warm-up Old Hardware, New Hardware

Students are introduced to different ways to measure computing power as they prepare to solve problems in the next activity about the computing power of different devices.



Unit 6 | Lesson 15 – Capstone

Is a Smartphone Smart Enough to Go to the Moon?

Let's answer some big questions about even larger numbers!

Warm-up Measuring Computer Power

Computing power has changed significantly over the past several decades. You will take a closer look at the history of computing power, going back to when NASA first launched the Apollo program to land a man on the Moon.

First, let's become familiar with some ways to measure computing power. Computing power can be measured by looking at storage, processing speed, and memory.

For reference:

Storage and memory processing are each measured in bytes.

Processor speed is measured in hertz.

Kilo means 1,000.

Mega means 1,000,000.

Giga means 1,000,000,000

Tera means 1,000,000,000,000.

1. Which is greater, 1 kilobyte or 1 megabyte?
   1 megabyte is greater.
2. How many gigabytes are equivalent to 1 terabyte?
   10^3 gigabytes are equivalent to 1 terabyte.
3. How many kilohertz are in 5.2 terahertz?
   5.2×10^9 kilohertz are in 5.2 terahertz.

Log in to Amplify Math to complete this lesson online.

710 Unit 6 Exponents and Scientific Notation

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1 Launch

Frame for students that in this final lesson, they will be asked to consider what they have learned about using scientific notation and apply it to problems in a new context.

2 Monitor

Help students get started by showing that 1 kilobyte is equivalent to 10^3 bytes.

Look for points of confusion:

- **Not knowing how to complete Problem 2.** Have students write both values using powers of 10 and ask, "How many times greater is 10^{12} than 10^9?"

Look for productive strategies:

- Students using powers of 10 to make their calculations more efficient.

3 Connect

Display student work showing correct responses to Problems 1–3.

Have individual students share how they used powers of 10 to make their calculations more efficient.

Highlight that it will often be more efficient to use scientific notation when comparing numbers in the next activity.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they own a smartphone or know someone who owns a smartphone. Ask them to recall or estimate how much cellular data they, or someone they know, use on average each month. Let students discuss and compare data usage and storage of data on smartphones before launching the activity.

Power-up

To power up students' ability to write number using powers of 10, have students complete:

Recall that there are 100 cm in one meter and 1,000 m in one kilometer. Which expressions can be used to determine the number of kilometers in 12 cm. Select *all* that apply.

A. $12 \div 100 \cdot 1000$
[✓ B.] $12 \cdot 100 \cdot 1000$
[✓ C.] $12 \cdot 10^5$
D. $12 \div 10^2 \cdot 10^3$
[✓ E.] $12 \cdot 10^2 \cdot 10^3$
F. $12 \div 10^5$

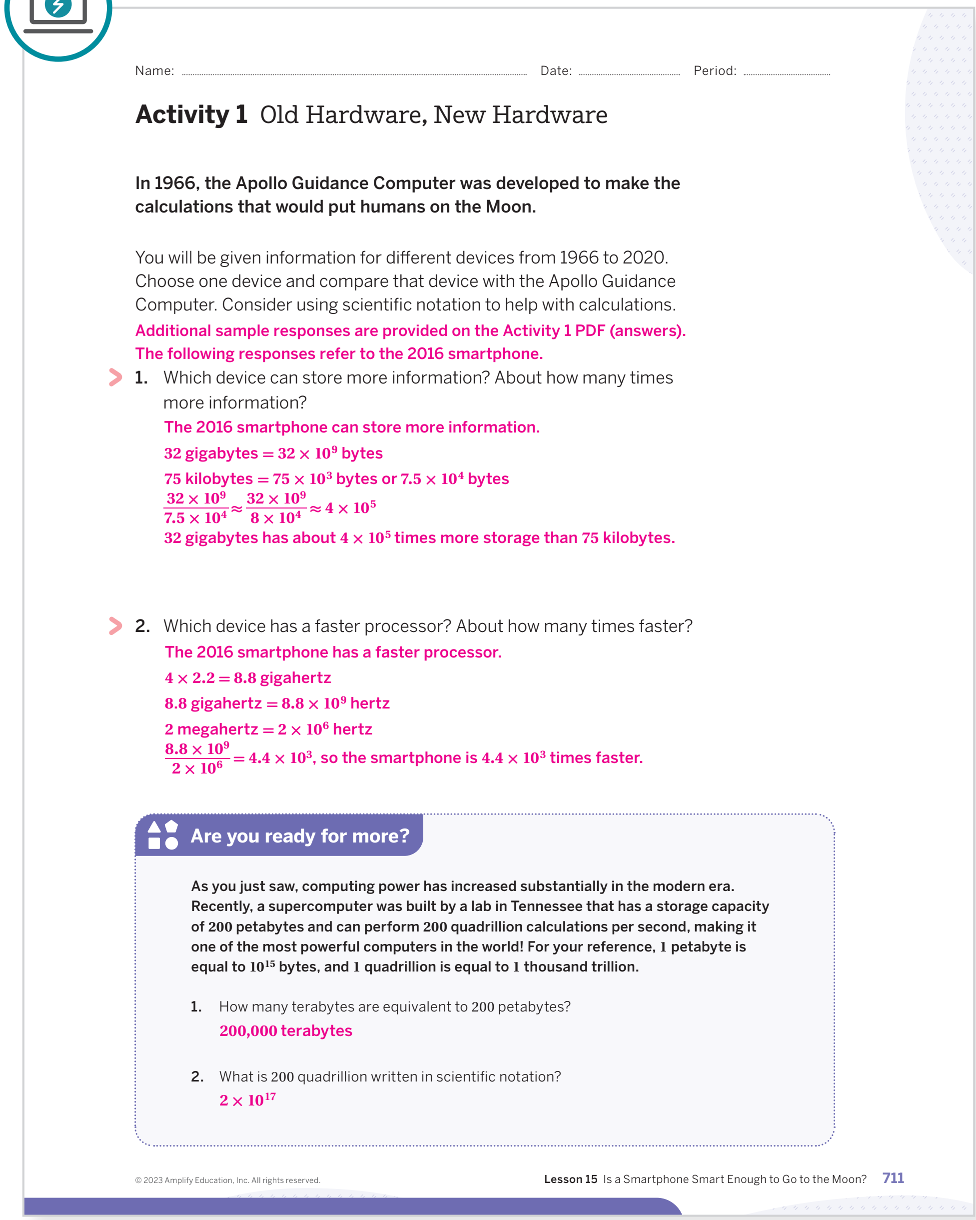
Use: Before the Warm-up

Informed by: Performance on Lesson 14, Practice Problem 5

Pairs | 20 min

Activity 1 Old Hardware, New Hardware

Students apply their understanding of scientific notation to solve problems about computing speed of different devices.



Name: ________ Date: ________ Period: ________

Activity 1 Old Hardware, New Hardware

In 1966, the Apollo Guidance Computer was developed to make the calculations that would put humans on the Moon.

You will be given information for different devices from 1966 to 2020. Choose one device and compare that device with the Apollo Guidance Computer. Consider using scientific notation to help with calculations.

Additional sample responses are provided on the Activity 1 PDF (answers).
The following responses refer to the 2016 smartphone.

1. Which device can store more information? About how many times more information?

   The 2016 smartphone can store more information.

   32 gigabytes $= 32 \times 10^9$ **bytes**

   75 kilobytes $= 75 \times 10^3$ **bytes or** 7.5×10^4 **bytes**

   $\frac{32 \times 10^9}{7.5 \times 10^4} \approx \frac{32 \times 10^9}{8 \times 10^4} \approx 4 \times 10^5$

   32 gigabytes has about 4×10^5 **times more storage than 75 kilobytes.**

2. Which device has a faster processor? About how many times faster?

   The 2016 smartphone has a faster processor.

   $4 \times 2.2 = 8.8$ **gigahertz**

   8.8 gigahertz $= 8.8 \times 10^9$ **hertz**

   2 megahertz $= 2 \times 10^6$ **hertz**

   $\frac{8.8 \times 10^9}{2 \times 10^6} = 4.4 \times 10^3$**, so the smartphone is** 4.4×10^3 **times faster.**

Are you ready for more?

As you just saw, computing power has increased substantially in the modern era. Recently, a supercomputer was built by a lab in Tennessee that has a storage capacity of 200 petabytes and can perform 200 quadrillion calculations per second, making it one of the most powerful computers in the world! For your reference, 1 petabyte is equal to 10^{15} bytes, and 1 quadrillion is equal to 1 thousand trillion.

1. How many terabytes are equivalent to 200 petabytes?

   200,000 terabytes

2. What is 200 quadrillion written in scientific notation?

   2×10^{17}

1 Launch

Distribute the Activity 1 PDF to each pair of students. **Note:** If students compare to a 1977 desktop computer, have students complete a second comparison that encourages the use of scientific notation.

2 Monitor

Help students get started by asking about the storage capacity for the device they have chosen to compare. Have them demonstrate how to convert the values to the same unit and write that capacity in scientific notation.

Look for points of confusion:

- **Not knowing how best to estimate.** Remind students to round first before estimating and to use powers of 10.
- **Being unable to find "how many times more information" in Problem 1.** To help them see what operation they can use, ask, "How many times more is 1,000 than 10? What operation did you use to determine your response?"

Look for productive strategies:

- Using scientific notation to make their calculations more efficient.

3 Connect

Have pairs of students share their findings for different computing devices. Have at least one pair share how using scientific notation made their calculations more efficient.

Ask, "What did you find interesting or surprising when comparing computing power?"

Highlight how technology improves rapidly, and how modern smartphones are much, much more sophisticated than the computer that put people on the Moon.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus only on completing Problem 1.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their findings and how using scientific notation aided their calculations, display these sentence frames to help them organize their thinking.

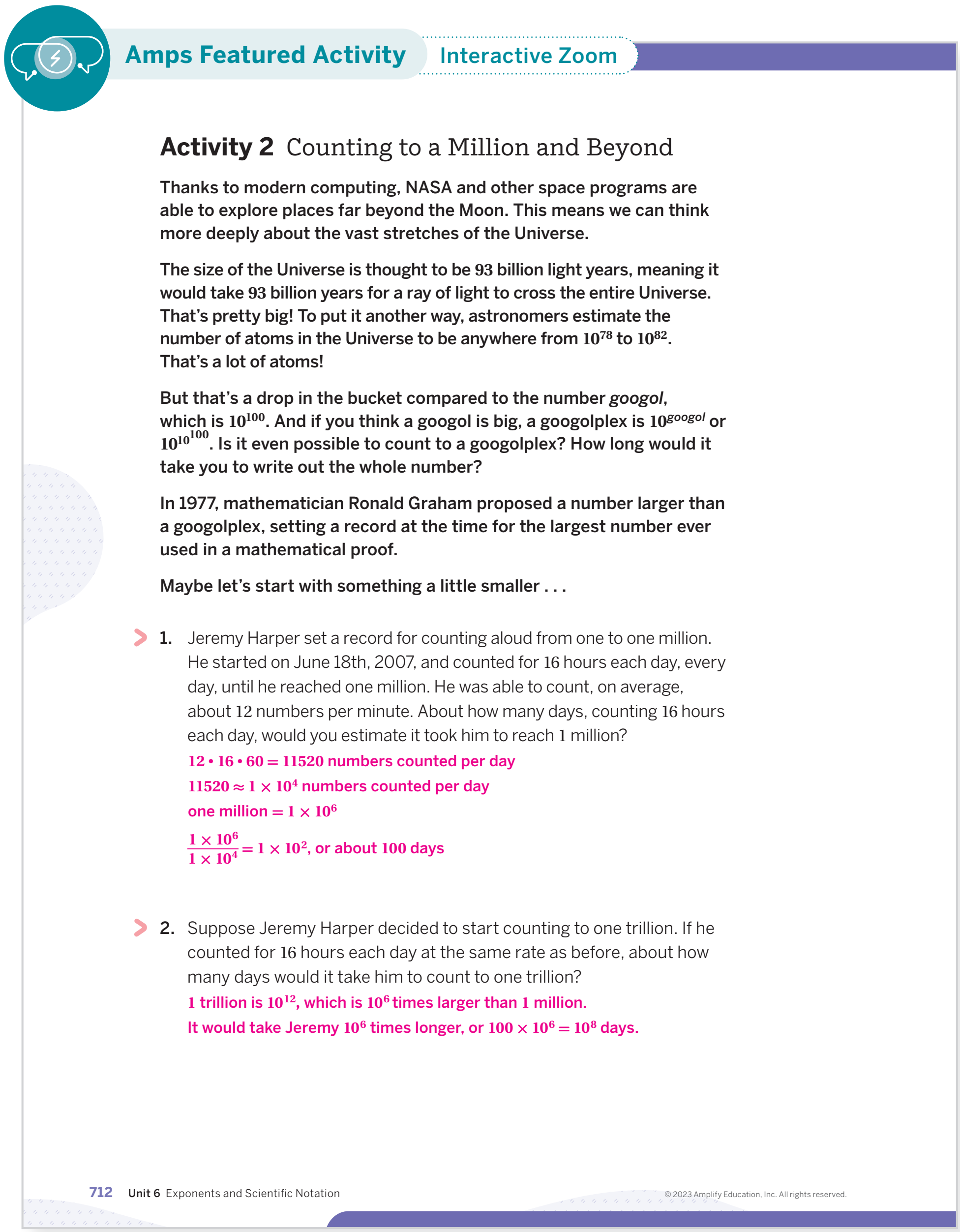
- "The ___ can store more information, about ___ more storage, than the ___."
- "The ___ has a faster processor, about ___ times faster, than the ___."
- "Using scientific notation helped me to . . ."

Pairs | 15 min

Activity 2 Counting to a Million and Beyond

Students use scientific notation as a tool to understand the scale of large numbers.

Amps Featured Activity Interactive Zoom



Activity 2 Counting to a Million and Beyond

Thanks to modern computing, NASA and other space programs are able to explore places far beyond the Moon. This means we can think more deeply about the vast stretches of the Universe.

The size of the Universe is thought to be 93 billion light years, meaning it would take 93 billion years for a ray of light to cross the entire Universe. That's pretty big! To put it another way, astronomers estimate the number of atoms in the Universe to be anywhere from 10^{78} to 10^{82}. That's a lot of atoms!

But that's a drop in the bucket compared to the number *googol*, which is 10^{100}. And if you think a googol is big, a googolplex is 10^{googol} or $10^{10^{100}}$. Is it even possible to count to a googolplex? How long would it take you to write out the whole number?

In 1977, mathematician Ronald Graham proposed a number larger than a googolplex, setting a record at the time for the largest number ever used in a mathematical proof.

Maybe let's start with something a little smaller . . .

1. Jeremy Harper set a record for counting aloud from one to one million. He started on June 18th, 2007, and counted for 16 hours each day, every day, until he reached one million. He was able to count, on average, about 12 numbers per minute. About how many days, counting 16 hours each day, would you estimate it took him to reach 1 million?

   $12 \cdot 16 \cdot 60 = 11520$ numbers counted per day

   $11520 \approx 1 \times 10^4$ numbers counted per day

   one million $= 1 \times 10^6$

   $\frac{1 \times 10^6}{1 \times 10^4} = 1 \times 10^2$, or about 100 days

2. Suppose Jeremy Harper decided to start counting to one trillion. If he counted for 16 hours each day at the same rate as before, about how many days would it take him to count to one trillion?

   1 trillion is 10^{12}, which is 10^6 times larger than 1 million.

   It would take Jeremy 10^6 times longer, or $100 \times 10^6 = 10^8$ days.

712 **Unit 6** Exponents and Scientific Notation

1 Launch

Read the introductory paragraphs aloud with students and ask them if they have ever thought how long it would take to count to one million.

2 Monitor

Help students get started by determining the value of the expression $16 \times 60 \times 12$ to calculate how many numbers Jeremy can say in one day.

Look for points of confusion:

- **Not being able to estimate in Problem 1.** Have students round $11,520$ to 1×10^{4}.
- **Unsure of how to find how long it would take to count to one trillion in Problem 2.** Have students use their response from Problem 1. Ask, "How many times greater is one trillion than one million? How can you use that to find how many days it would take Jeremy to count to one trillion?"
- **Not knowing how to write a googol as a number in Problem 3a.** Remind students that a googol is 10^{100} and ask how many zeros need to be listed.
- **Being unsure how to order the list in Problem 3b.** Have students provide their best estimate based on their own knowledge of each item.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a checklist for Problem 1 to help them break up the problem into smaller, more manageable parts.

☐ How many numbers did Jeremy count per hour?
☐ How many numbers did he count per day, at 16 hours per day?
☐ How many days did it take to count one million numbers?

Extension: *Math Enrichment*

Have students complete the following problem:
How many times more zeros does a googolplex have than a googol?

Sample response: $\frac{10^{100}}{10^2} = 10^{98}$ times more zeros.

Featured Mathematician

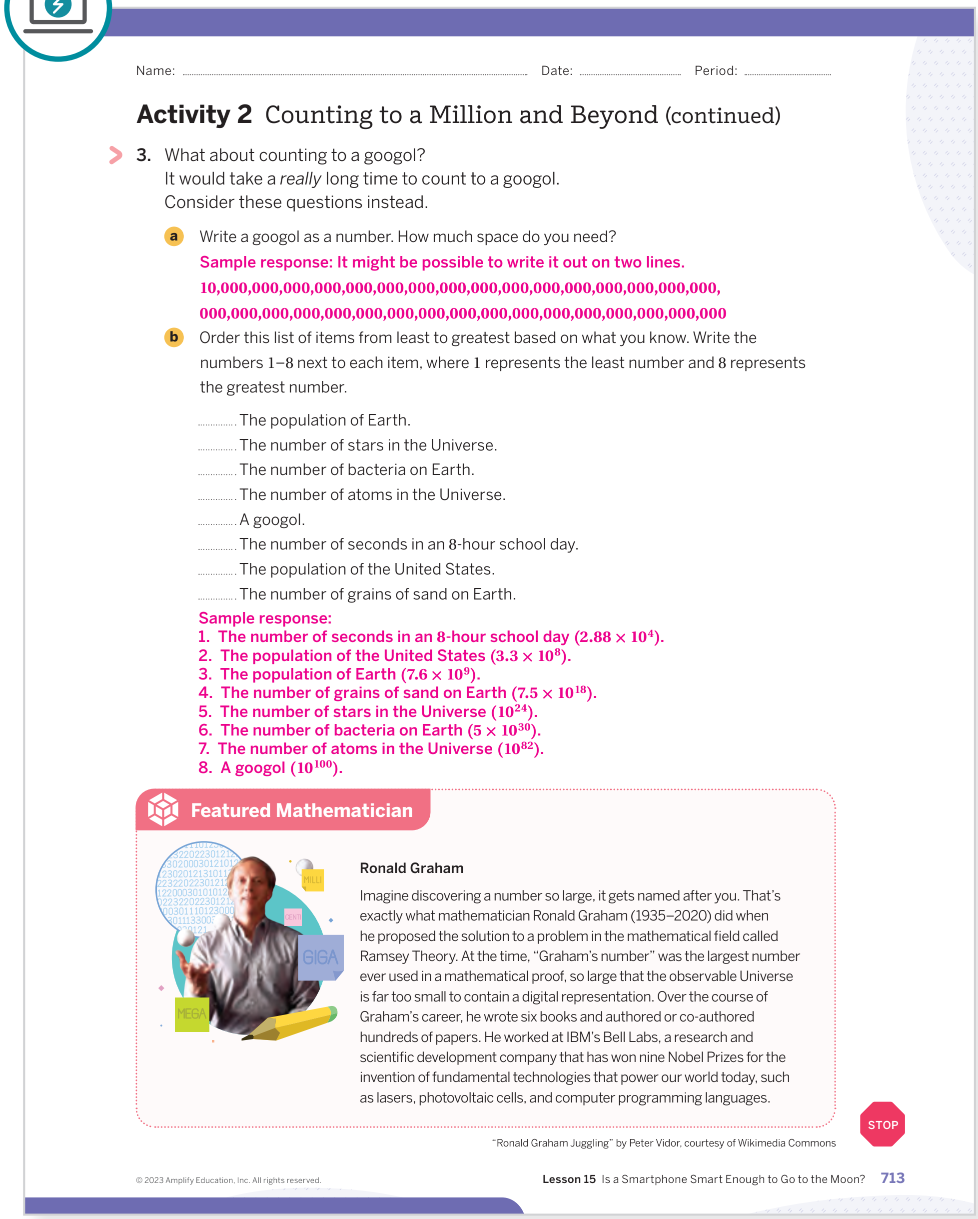
Ron Graham

On the next page, have students read about Featured Mathematician, Ron Graham, one of the preeminent mathematicians of the 20th century.

Pairs | 15 min

Activity 2 Counting to a Million and Beyond (continued)

Students use scientific notation as a tool to understand the scale of large numbers.



Name: Date: Period:

Activity 2 Counting to a Million and Beyond (continued)

3. What about counting to a googol?
It would take a *really* long time to count to a googol.
Consider these questions instead.

a. Write a googol as a number. How much space do you need?
Sample response: It might be possible to write it out on two lines.
10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,
000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000

b. Order this list of items from least to greatest based on what you know. Write the numbers 1–8 next to each item, where 1 represents the least number and 8 represents the greatest number.

.... The population of Earth.
.... The number of stars in the Universe.
.... The number of bacteria on Earth.
.... The number of atoms in the Universe.
.... A googol.
.... The number of seconds in an 8-hour school day.
.... The population of the United States.
.... The number of grains of sand on Earth.

Sample response:
1. **The number of seconds in an 8-hour school day (2.88×10^4).**
2. **The population of the United States (3.3×10^8).**
3. **The population of Earth (7.6×10^9).**
4. **The number of grains of sand on Earth (7.5×10^{18}).**
5. **The number of stars in the Universe (10^{24}).**
6. **The number of bacteria on Earth (5×10^{30}).**
7. **The number of atoms in the Universe (10^{82}).**
8. **A googol (10^{100}).**

Featured Mathematician

Ronald Graham

Imagine discovering a number so large, it gets named after you. That's exactly what mathematician Ronald Graham (1935–2020) did when he proposed the solution to a problem in the mathematical field called Ramsey Theory. At the time, "Graham's number" was the largest number ever used in a mathematical proof, so large that the observable Universe is far too small to contain a digital representation. Over the course of Graham's career, he wrote six books and authored or co-authored hundreds of papers. He worked at IBM's Bell Labs, a research and scientific development company that has won nine Nobel Prizes for the invention of fundamental technologies that power our world today, such as lasers, photovoltaic cells, and computer programming languages.

STOP

"Ronald Graham Juggling" by Peter Vidor, courtesy of Wikimedia Commons

3 Connect

Display student work showing accurate estimates using scientific notation for Problem 1. For Problem 3a, display a work sample showing the number of zeros in a googol before discussing their lists for Problem 3b.

Have pairs of students share how they arrived at their responses for Problems 1 and 2. Then, show the interactive number line from the digital lesson and ask students to share their reactions to seeing the size of a googol compared to the other quantities.

Highlight productive strategies for completing Problems 1 and 2. After showing the interactive number line, highlight how much greater a googol is than the other items.

Promoting Equity

Mary W. Jackson

Have students read the online article, "NASA Names Headquarters After 'Hidden Figure' Mary W. Jackson" by NASA to learn more about Mary W. Jackson, who was the first African American female engineer at NASA. She was a mathematician and aerospace engineer in NASA's segregated Computing Unit and was influential in paving the way for NASA to expand its hiring practices to include more women in its STEM careers. Mary was not alone in her efforts to challenge the racially segregated divisions within NASA where most of the decision makers were men. Mary and the group of women she worked with in the West Area Computing Unit received national media attention in the 2016 book and subsequent film, *Hidden Figures*.

Ask these questions to facilitate class discussion:

- "Despite segregated schooling, and eventually a segregated working environment, Mary W. Jackson persevered to become an influential STEM leader and civil rights advocate. How you do think Mary's work for NASA influenced the types of careers that were available to women, particularly women of color, before and after the 1960s?"
- "Are you interested in pursuing a career in STEM? What steps can you take to learn more about STEM careers?"

Whole Class | 3 min

Unit Summary

Review and synthesize what students may have found interesting or surprising about large numbers they encountered in the lesson.

Narrative Connections

Unit Summary

Scientists estimate there are more than 10 million, million, million individual insects alive on the planet right now. Meanwhile, the average human being houses 39,000,000,000,000 bacteria in their gut. And with every shuffle of a deck of 52 cards, you are ordering the cards into one of approximately 80 unvigintillion (that’s an 8 followed by 67 zeros!) possible permutations.

Whether we notice it or not, we are surrounded by numbers that are both astronomically large and infinitesimally small. From the number of atoms in the Universe to the weight of a microorganism, it is important to find a way to handle these numbers. And for that, there are no better tools than exponents and scientific notation.

Scientific notation allows us to rewrite incredibly large or incredibly small numbers in a way that is friendly to the eye. Using powers of 10, these numbers can be compared much more efficiently. So rather than reach for the dictionary, we can write 80 unvigintillion as 8×10^{67}, and 39,000,000,000,000 as 3.9×10^{13}.

And when numbers share a common base, multiplying and dividing become a snap! When you are multiplying, just add the exponents. And when you are dividing, subtract. When one power is raised to another, you can multiply the exponents to find the new resulting exponent.

With an assist from exponents and scientific notation, you can tackle any number, great or small.

See you in Unit 7.

714 Unit 6 Exponents and Scientific Notation

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Ask, “What did you find interesting or surprising? In what ways was using scientific notation helpful in your work today?”

Highlight that scientific notation can often make students’ calculations easier when working with large or small numbers.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

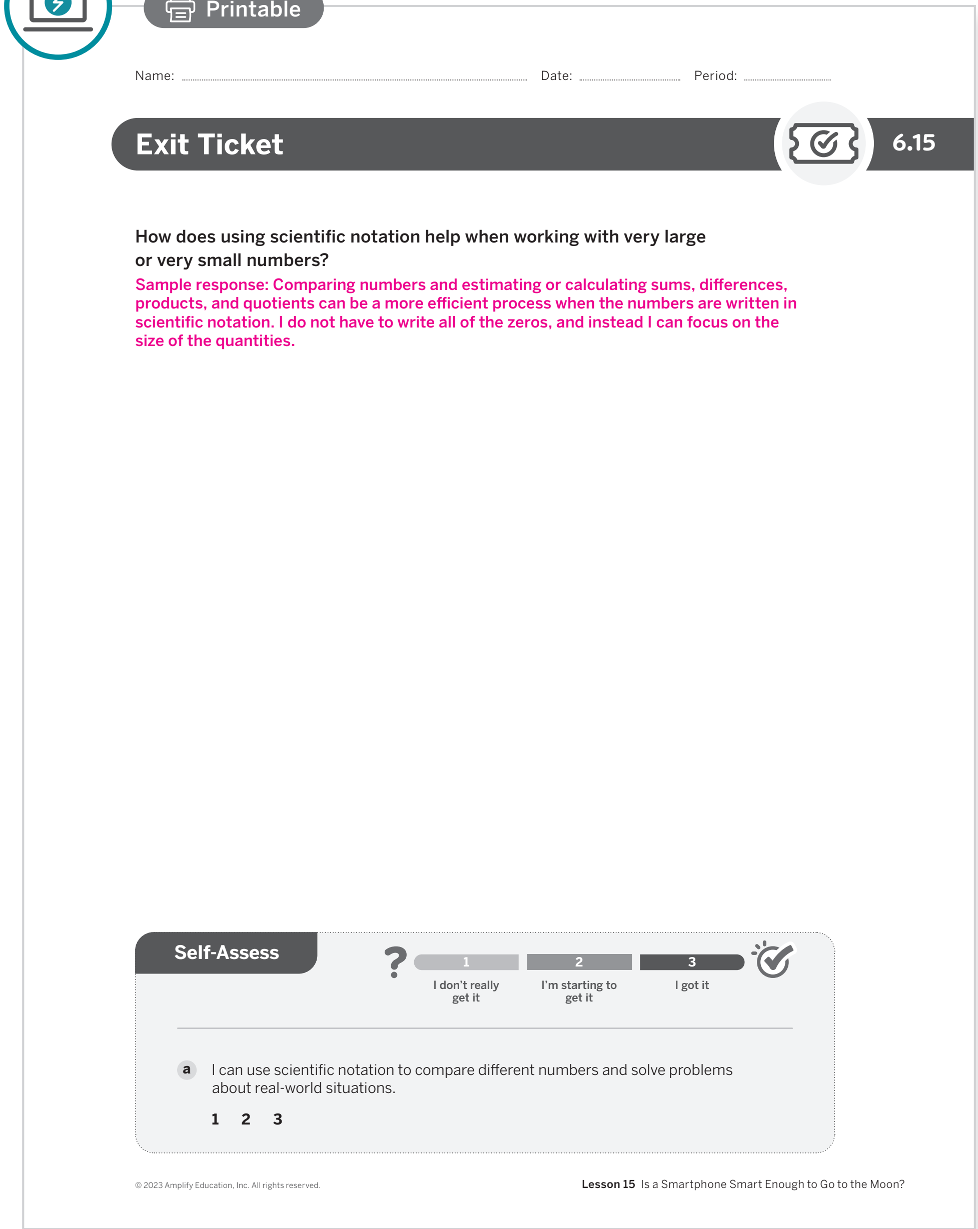
- “Did anything surprise you while reading the narratives of this unit?”
- “Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?”

Independent | 3 min

Exit Ticket

Students demonstrate their understanding of scientific notation by reflecting on its usefulness when working with really large or really small numbers.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 6.15

How does using scientific notation help when working with very large or very small numbers?

Sample response: Comparing numbers and estimating or calculating sums, differences, products, and quotients can be a more efficient process when the numbers are written in scientific notation. I do not have to write all of the zeros, and instead I can focus on the size of the quantities.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use scientific notation to compare different numbers and solve problems about real-world situations.

1 2 3

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Lesson 15 Is a Smartphone Smart Enough to Go to the Moon?

Success looks like . . .

- **Language Goal:** Using scientific notation to compare quantities in context, and describing how using scientific notation helps with making comparisons between very large and very small quantities. **(Speaking and Listening)**
  - » Explaining how to use scientific notation to compare two numbers.

Suggested next steps

If students are unable to come up with a reason why scientific notation is helpful, consider:

- Reviewing strategies that used scientific notation for Activity 1.
- Asking, "Why do we often decide to write a number, such as 10^{15}, using powers of 10?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In earlier lessons, students performed operations with numbers expressed in scientific notation. How did that support their understanding to solve problems about computing speed of different devices?
- What was especially satisfying as students completed Activity 2? What might you change for the next time you teach this lesson?

Math Language Development

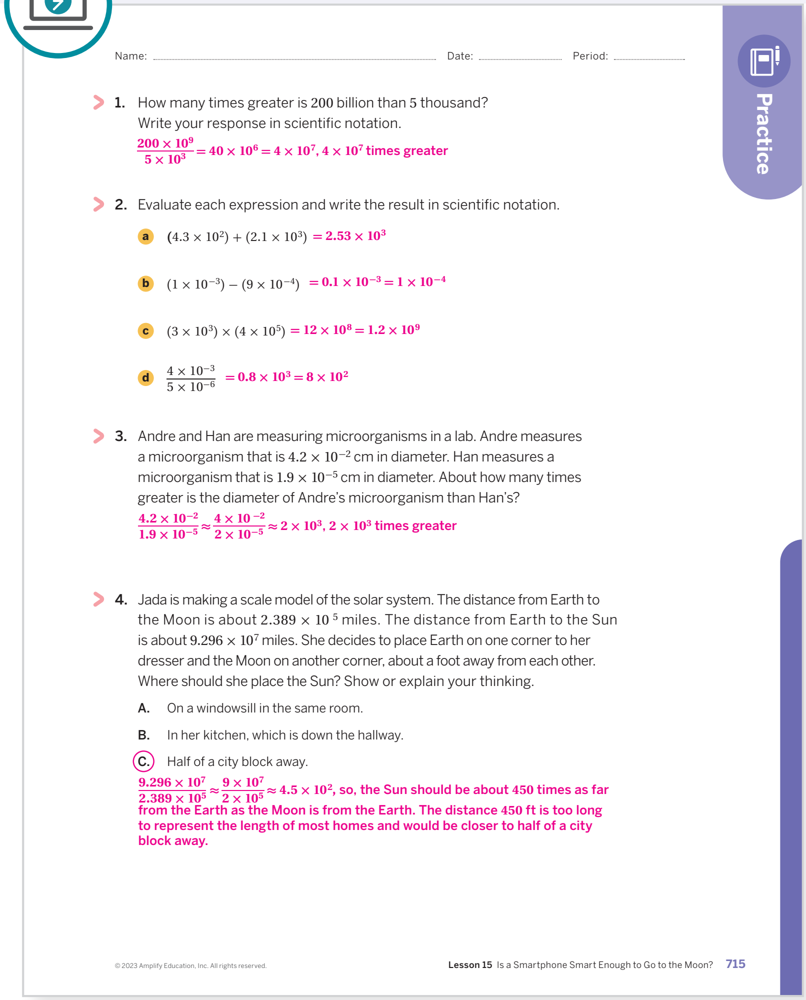
Language Goal: Using scientific notation to compare quantities in context, and describing how using scientific notation helps with making comparisons between very large and very small quantities.

Reflect on students' language development toward this goal.

- Do students' responses to the Exit Ticket problem indicate why using scientific notation is helpful when working with very large or very small numbers? Do they use language such as *comparing, estimating*, or *calculating sums, differences, products, or quotients*?
- How have the language routines used in this unit help students understand the benefits for working with exponents, using exponent rules, and scientific notation?

Practice

Independent



Name: ______ Date: ______ Period: ______

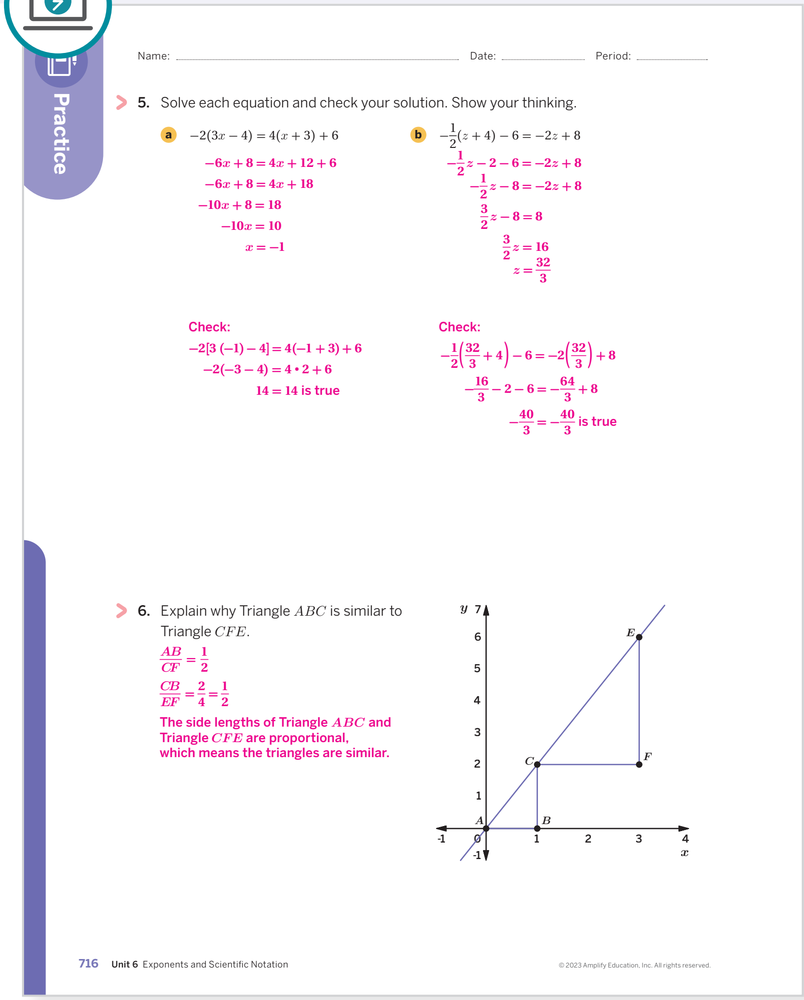
1. How many times greater is 200 billion than 5 thousand? Write your response in scientific notation.
$\frac{200 \times 10^9}{5 \times 10^3} = 40 \times 10^6 = 4 \times 10^7$, 4×10^7 times greater

2. Evaluate each expression and write the result in scientific notation.
   - a $(4.3 \times 10^2) + (2.1 \times 10^3)$ $= 2.53 \times 10^3$
   - b $(1 \times 10^{-3}) - (9 \times 10^{-4})$ $= 0.1 \times 10^{-3} = 1 \times 10^{-4}$
   - c $(3 \times 10^3) \times (4 \times 10^5)$ $= 12 \times 10^8 = 1.2 \times 10^9$
   - d $\frac{4 \times 10^{-3}}{5 \times 10^{-6}}$ $= 0.8 \times 10^3 = 8 \times 10^2$

3. Andre and Han are measuring microorganisms in a lab. Andre measures a microorganism that is 4.2×10^{-2} cm in diameter. Han measures a microorganism that is 1.9×10^{-5} cm in diameter. About how many times greater is the diameter of Andre's microorganism than Han's?
$\frac{4.2 \times 10^{-2}}{1.9 \times 10^{-5}} \approx \frac{4 \times 10^{-2}}{2 \times 10^{-5}} \approx 2 \times 10^3$, 2×10^3 times greater

4. Jada is making a scale model of the solar system. The distance from Earth to the Moon is about 2.389×10^5 miles. The distance from Earth to the Sun is about 9.296×10^7 miles. She decides to place Earth on one corner to her dresser and the Moon on another corner, about a foot away from each other. Where should she place the Sun? Show or explain your thinking.
   - A. On a windowsill in the same room.
   - B. In her kitchen, which is down the hallway.
   - [✓ C.] Half of a city block away.

$\frac{9.296 \times 10^7}{2.389 \times 10^5} \approx \frac{9 \times 10^7}{2 \times 10^5} \approx 4.5 \times 10^2$, so, the Sun should be about 450 times as far from the Earth as the Moon is from the Earth. The distance 450 ft is too long to represent the length of most homes and would be closer to half of a city block away.



Name: ______ Date: ______ Period: ______

5. Solve each equation and check your solution. Show your thinking.

a $-2(3x - 4) = 4(x + 3) + 6$

$$-6x + 8 = 4x + 12 + 6$$
$$-6x + 8 = 4x + 18$$
$$-10x + 8 = 18$$
$$-10x = 10$$
$$x = -1$$

Check:

$$-2[3(-1) - 4] = 4(-1 + 3) + 6$$
$$-2(-3 - 4) = 4 \cdot 2 + 6$$
$$14 = 14 \text{ is true}$$

b $-\frac{1}{2}(z + 4) - 6 = -2z + 8$

$$-\frac{1}{2}z - 2 - 6 = -2z + 8$$
$$-\frac{1}{2}z - 8 = -2z + 8$$
$$\frac{3}{2}z - 8 = 8$$
$$\frac{3}{2}z = 16$$
$$z = \frac{32}{3}$$

Check:

$$-\frac{1}{2}\left(\frac{32}{3} + 4\right) - 6 = -2\left(\frac{32}{3}\right) + 8$$
$$-\frac{16}{3} - 2 - 6 = -\frac{64}{3} + 8$$
$$-\frac{40}{3} = -\frac{40}{3} \text{ is true}$$

6. Explain why Triangle ABC is similar to Triangle CFE.

$$\frac{AB}{CF} = \frac{1}{2}$$
$$\frac{CB}{EF} = \frac{2}{4} = \frac{1}{2}$$

The side lengths of Triangle ABC and Triangle CFE are proportional, which means the triangles are similar.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
Spiral	2	Unit 6 Lesson 14	1
	3	Unit 6 Lesson 13	1
	4	Unit 6 Lesson 13	3
	5	Unit 4 Lesson 5	1
	6	Unit 2 Lesson 9	2

Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

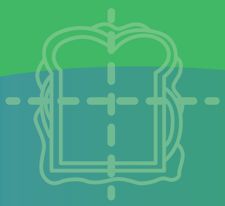
UNIT 7

Irrationals and the Pythagorean Theorem

Students study rational and irrational numbers using geometry and expressions before exploring a proof of the Pythagorean Theorem.

Essential Questions

- What is the difference between a rational number and an irrational number?
- How can you estimate the square root of a number? And what does it represent?
- Is it true that $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ for all right triangles? If so, can you prove it?
- *(By the way, what is the longest cut you can make in a sandwich?)*



$x^2 = \sqrt{x}$

$0.\overline{2}$
$=$
$0.2222...$

$\sqrt{2} = \frac{a}{b}$?

Key Shifts in Mathematics

Focus

In this unit . . .

Students work with geometric and symbolic representations of square and cube roots. They understand the terms *rational number* and *irrational number*, using long division to express fractions as decimals. They use their understanding of fractions to plot rational numbers on the number line and their understanding of approximation of irrationals by rationals to approximate the location of a given irrational on the number line. They understand a geometric proof of the Pythagorean Theorem that involves decomposing and rearranging two squares. They apply the Pythagorean Theorem in two and three dimensions.

Coherence

Previously . . .

In Grade 5, students began classifying shapes based on sides and angles. Also in Grade 5, students learned to square a number by multiplying that number by itself. Students discovered negative numbers in Grade 6, which allowed them to solve any linear equation. In Grade 7, students studied triangles and learned that the longest side of a triangle must be less than the sum of the other two sides. Also in Grade 7, students used long division in order to write fractions as decimals and learned that such decimals either repeat or terminate.

Coming soon . . .

Students will continue working with rational and irrational numbers, square and cube roots, and the Pythagorean Theorem in high school. These topics will serve as a backbone to their exploration in advanced topics in algebra, geometry and trigonometry.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students explore side lengths and areas of squares to build a conceptual understanding of square roots (Lesson 3), and later edge lengths and volume of cubes to understand cube roots (Lesson 5). Next, students discover that numbers which are not rational are called irrational numbers (Lesson 6).

Procedural Fluency

Students practice estimating square roots (Lesson 4) and cube roots (Lesson 5). Later, students use bar notation to represent repeating, non-terminating decimals (Lesson 7). In the second Sub-Unit, students use the Pythagorean Theorem to find an unknown side of a right triangle (Lesson 11).

Application

Students apply the Pythagorean Theorem first to find distances on the coordinate plane (Lessons 13 and 14), and next to solve real-world problems involving right triangles (Lesson 15).

The Mystery of the Pythagoreans

SUB-UNIT

1

Lessons 2–8

Rational and Irrational Numbers

Students revisit ***rational numbers*** to learn about ***irrational numbers***. They use the geometric measurement related to squares and cubes ***square roots*** and ***cube roots***. Students use bar notation to represent the decimal expansion of repeating rational numbers.

Narrative: Beliefs were challenged with the idea that not every number can be expressed as a ratio.

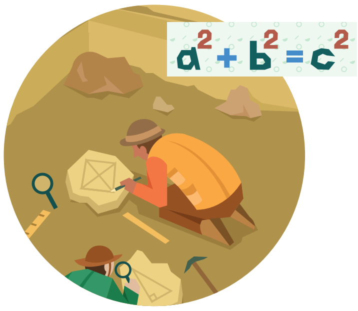
SUB-UNIT

2

Lessons 9–15

The Pythagorean Theorem

Students explore a proof of the ***Pythagorean Theorem***, before applying the theorem to solve real-world and mathematical problems, including determining the distance between two points on the coordinate plane. They use the converse of the Pythagorean Theorem to determine whether a triangle is a right triangle.



Narrative: Discover and use “the most proven theorem of all time”.

Launch

Lesson 1

Sliced Bread

Students draw cuts on sandwiches to explore the relationship between the diagonal of a rectangle and the sides of the rectangle.

Capstone

Lesson 16

Pythagorean Triples

Students look for patterns among a series of ***Pythagorean triples*** and they explore Fermat's Last Theorem.

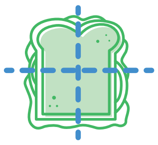
Unit at a Glance

Spoiler Alert: The squared length of the hypotenuse of a right triangle is equal to the sum of the squared lengths of the two legs.

Assessment

A **Pre-Unit Readiness Assessment**

Launch Lesson



1 **Sliced Bread**

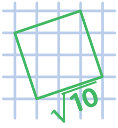
Discover the longest cut of a rectangular figure.

Sub-Unit 1: Rational and Irrational Numbers

$x^2 = \sqrt{x}$

2 **The Square Root**

Understand the term *square root* and evaluate perfect squares using square root notation.



3 **The Areas of Squares and Their Side Lengths**

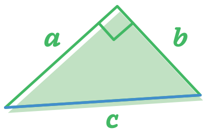
Use the areas and side lengths of different squares to explore square roots.

$0.\overline{2}$
$=$
$0.2222...$

8 **Converting Repeating Decimals Into Fractions**

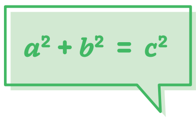
Coordinate repeating decimals and rational numbers that represent the same number.

Sub-Unit 2: The Pythagorean Theorem



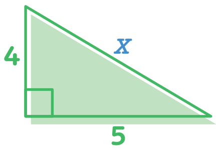
9 **Observing the Pythagorean Theorem**

Discover that the relationship between side lengths in a right triangle is known as the Pythagorean Theorem



10 **Proving the Pythagorean Theorem**

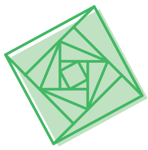
Work through a proof of the Pythagorean Theorem using a cut-out area model.



11 **Determining Unknown Side Lengths**

Apply the Pythagorean Theorem to determine unknown side lengths in 2D and 3D figures.

Capstone Lesson



16 **Pythagorean Triples**

Determine patterns in Pythagorean triples and explore Fermat's Last Theorem.

Assessment

A **End-of-Unit Assessment**

Key Concepts

Lesson 3: The side length of a square is the square root of its area.

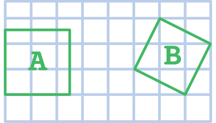
Lesson 6: The $\sqrt{2}$ is irrational, as are all numbers that cannot be written as fractions (ratios of two integers).

Lesson 11: If the lengths of two sides of a right triangle are known, the Pythagorean Theorem can be used to determine the length of the third side.

Pacing

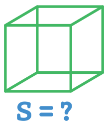
16 Lessons: 45 min each	**Full Unit:** 18 days
2 Assessments: 45 min each	● **Modified Unit:** 15 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



4 Estimating Square Roots

Approximate the value of a square root to the nearest tenth and place square roots on a number line.



5 The Cube Root

Understand the term *cube root* by exploring the volumes and edge lengths of cubes.

$$\sqrt{2} = \frac{a}{b}\ ?$$

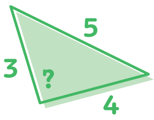
6 Rational and Irrational Numbers

Comprehend that numbers that can be written as a fraction are called *rational numbers*, and numbers that cannot are called *irrational*.

$$\frac{1}{7} = 0.142857\ldots$$

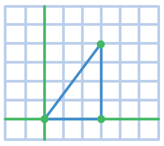
7 Decimal Representations of Rational Numbers ●

Use long division to write fractions as decimals, with a particular focus on using bar notation to represent repeating decimals.



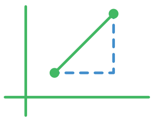
12 Converse of the Pythagorean Theorem

Use the converse of the Pythagorean Theorem to determine whether a triangle is acute, right, or obtuse.



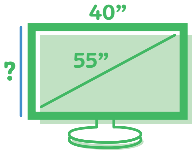
13 Distances on the Coordinate Plane (Part 1)

Apply the Pythagorean Theorem to determine the length of a segment on the coordinate plane.



14 Distances on the Coordinate Plane (Part 2)

Apply the Pythagorean Theorem to determine the distance between coordinate pairs.



15 Applications of the Pythagorean Theorem ●

Solve real-world problems using the Pythagorean Theorem.

● Modifications to Pacing

Lessons 7 and 8: Students typically need at least two days to build fluency skills working with decimal expansions of fractional numbers, but these two lessons can be combined for pacing purposes if needed.

Lessons 11 and 15: In Lesson 11, students learn to apply the Pythagorean Theorem to determine unknown side lengths. Replace Activity 2 with Activity 1 from Lesson 15, where students apply the Pythagorean Theorem to determine unknown side lengths in context, and omit the rest of Lesson 15 for pacing purposes.

Lesson 16: Lesson 16 introduces students to the fascinating concept of Pythagorean triples, but because it does not introduce any new content related to grade level standards, it can be omitted for pacing.

Unit Supports

Math Language Development

Lesson	New vocabulary
2	perfect square square root
5	cube root perfect cube
6	irrational number rational number
7	bar notation repeating decimal terminating decimal
9	hypotenuse legs Pythagorean Theorem
16	Pythagorean triple

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
7, 12, 15	**MLR1:** Stronger and Clearer Each Time
1, 2, 5–7, 16	**MLR2:** Collect and Display
3, 6, 11	**MLR3:** Critique, Correct, Clarify
2, 10	**MLR5:** Co-craft Questions
15	**MLR6:** Three Reads
2, 4, 7, 8	**MLR7:** Compare and Connect
3–7, 9, 10, 12	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Additional required materials
2, 4–8, 11, 13–16	calculators
1	cardboard boxes
11	colored pencils
2	dot grid paper
14	graph paper
13	index cards
1–6, 8–16	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
12	plain sheets of paper
1, 12, 15	rulers
10	scissors
4	sticky notes
1	string

Instructional Routines

Activities throughout this unit include these instructional routines:

Lesson(s)	Instructional Routines
2	Algebra Talk
6, 7	Number Talk
7, 10, 16	Notice and Wonder
6, 7, 9, 15	Poll the Class
1, 4, 6, 9, 12–14	Think-Pair-Share
11	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 16

Amps
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Social & Collaborative Digital Moments

Featured Activity

Arranging Shapes

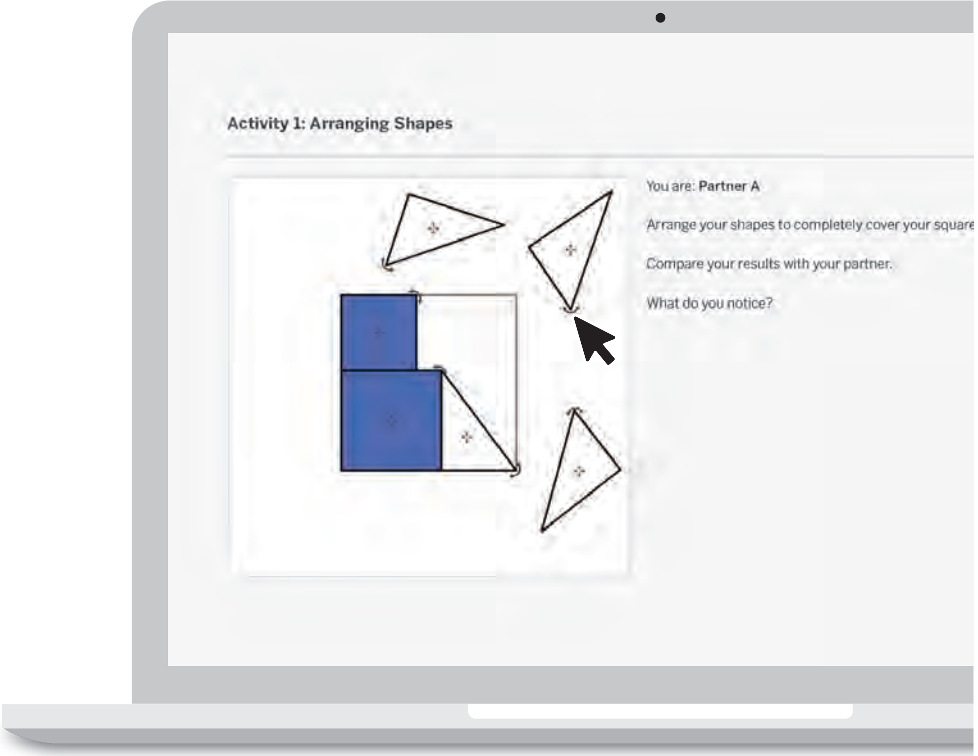
Put on your student hat and work through **Lesson 10, Activity 1:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- The Longest Cut **(Lesson 1)**
- Comparing Squares **(Lesson 3)**
- Determining the Values of x and x^3 **(Lesson 5)**
- Fastest Route **(Lesson 15)**



Activity 1: Arranging Shapes

You are: **Partner A**

Arrange your shapes to completely cover your square

Compare your results with your partner.

What do you notice?

Unit Study
Professional Learning

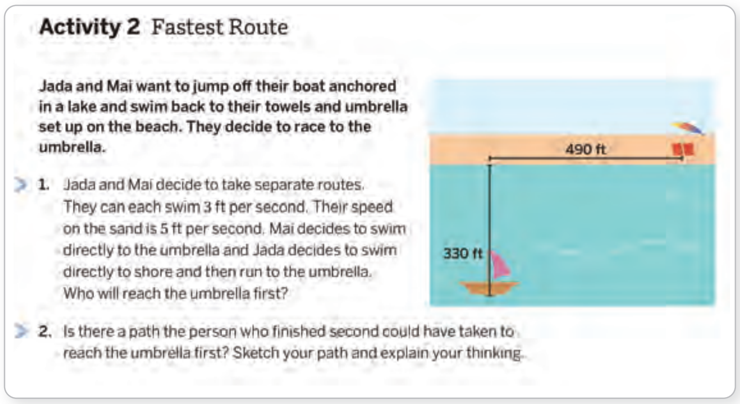
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces students to the Pythagorean Theorem after they have learned the difference between rational and irrational numbers. Students learn how to find the missing side of a right triangle given the other two sides. Likewise, they can determine if a triangle is a right triangle given the lengths of its 3 sides. Students then use the theorem to determine the distance between two points on the coordinate plane. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 15, Activity 2:**



Activity 2 Fastest Route

Jada and Mai want to jump off their boat anchored in a lake and swim back to their towels and umbrella set up on the beach. They decide to race to the umbrella.

1. Jada and Mai decide to take separate routes. They can each swim 3 ft per second. Their speed on the sand is 5 ft per second. Mai decides to swim directly to the umbrella and Jada decides to swim directly to shore and then run to the umbrella. Who will reach the umbrella first?

2. Is there a path the person who finished second could have taken to reach the umbrella first? Sketch your path and explain your thinking.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

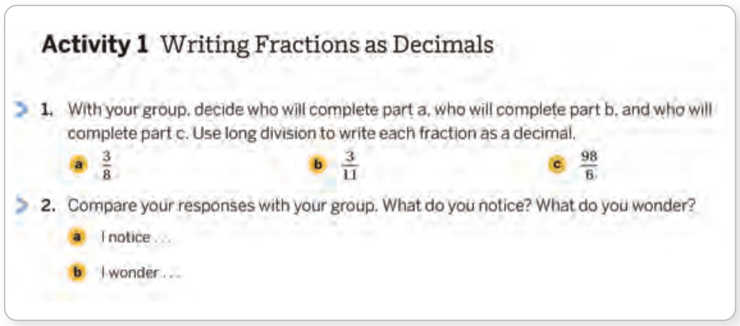
- What was it like to engage in this problem as a learner?
- Other than applying the Pythagorean Theorem, is there another strategy to evaluating Problem 1?
- How does your solution to Problem 2 compare with your colleague's? What is another faster route?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Notice and Wonder

Rehearse . . .

How you'll facilitate the ***Notice and Wonder*** instructional routine in **Lesson 7, Activity 1:**



Activity 1 Writing Fractions as Decimals

1. With your group, decide who will complete part a, who will complete part b, and who will complete part c. Use long division to write each fraction as a decimal.
   - a $\frac{3}{8}$
   - b $\frac{3}{11}$
   - c $\frac{98}{6}$
2. Compare your responses with your group. What do you notice? What do you wonder?
   - a I notice . . .
   - b I wonder . . .

Point to Ponder . . .

- How have students shown progress with this routine over time this year? How have you improved your facilitation of this routine since it was last highlighted? And what can you do to take this routine to the next level for your students?

This routine . . .

- Makes a mathematical task accessible to all students with these two approachable questions.
- Provides students with an entry point into the mathematics and/or context of a problem.
- Piques student curiosity about the mathematics and/or context of a problem.
- Helps students build their sense-making and observation skills.

Anticipate . . .

- What student statements will you be looking for as you monitor student progress during the Warm-up? How will you determine how to sequence those statements during the discussion?
- How can you help a student who does not know what to write for the "I notice . . ." or "I wonder . . . prompts?"
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Pose purposeful questions.

This effective teaching practice . . .

- Helps you assess the reasoning behind student responses. They may arrive at a correct response using flawed reasoning; probing for their reasoning helps you know if they truly understand the concept.
- Helps you advance student reasoning and sense making by asking deeper questions about mathematical ideas and relationships.

Points to Ponder . . .

- How can you probe for student reasoning to ensure they understand a mathematical concept, beyond just providing a correct response?
- What kinds of questions can you ask that will help advance their understanding to a deeper level?

Math Language Development

MLR8: Discussion Supports

MLR8 appears in Lessons 3–7, 9, 10, 12.

- In Lessons 4–7, 8, 19, and 12, sentence frames are provided for you to display to your students. Students can use these prompts to help frame their responses and add structure and organization to their thinking.
- In Lesson 9, further probing questions are provided so that you can ask your students for further clarification or to press for details in their reasoning.
- **English Learners:** Provide wait time to allow students to formulate a response before sharing with others and allow students to rehearse what they will say with a partner before sharing with the class.

Point to Ponder . . .

- During class discussions, how will you know when to probe further to assess student understanding, provide sentence frames, and encourage your students to use their developing mathematical vocabulary?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed in it.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving one-step equations with a variable throughout the unit? Do you think your students will generally:
  - Have difficulty working with square or cube roots?
  - Struggle to identify the differences between rational and irrational numbers?
  - Be unable to successfully apply the Pythagorean Theorem in various contexts?

Differentiated Support

Accessibility: *Optimize Access to Technology*

Opportunities to provide visual support, guide student processing, or provide the use of technology appear in Lessons 1–6, 9–11, 14, and 16.

- In Lesson 9, Activity 1, students can use digital geometry tools to determine the squares of side lengths of triangles, allowing them to make observations about the relationships between their values.
- In Lesson 10, Activity 1, students can digitally arrange right triangles as they work through a proof of the Pythagorean Theorem.
- In Lesson 11, Activity 2, students can explore the dimensions of a 3D prism digitally, as they try to determine the measure of its diagonal.
- In Lesson 16, Warm-up, students can view an animation of nested squares to discover the connection between specific Pythagorean triples.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology to deepen student understanding?

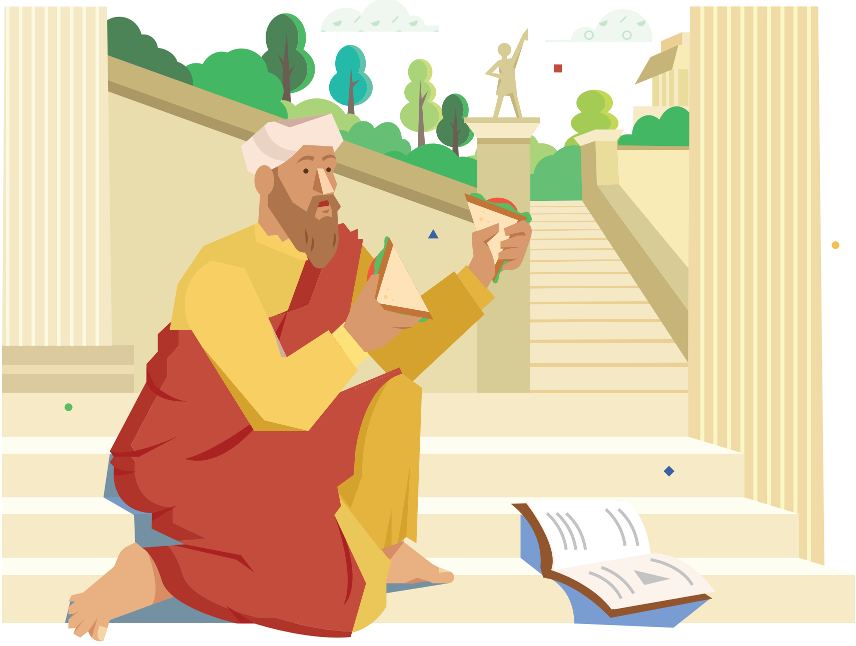
Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-management and self-awareness.

Points to Ponder . . .

- Do students exhibit self-discipline? Are they organized? Are they able to control their impulses? How well do students manage stress? In what ways do they motivate themselves? Can they set goals and accomplish them?
- How would students rank their self-efficacy? Are they able to approach assignments with confidence? Can they use their emotions to their advantage? Do they recognize their strengths and maintain a growth mindset?



Sliced Bread

Let's cut some sandwiches.

Focus

Goals

1. **Language Goal:** Understand and explain that the longest length inside a rectangle and rectangular prism is a diagonal. **(Speaking and Listening)**
2. Develop an intuition that the exact diagonal measure of a rectangle or rectangular prism may be impossible to determine using numbers students know.

Coherence

Today

In this lesson, students explore how to determine the longest length inside a rectangle. They consider the measures of side lengths in relation to the measure of a diagonal to prime their thinking for later lessons when they learn the Pythagorean Theorem. Students may be surprised to discover that the lengths of the diagonals of some rectangles, such as the hypotenuse of a right triangle, have values that are difficult to determine. This prepares students to learn about rational and irrational numbers.

Previously

In Grade 7, students studied triangles and learned that the longest side of a triangle must be less than the sum of the other two sides.

Coming Soon

In the first Sub-Unit, students learn about square roots, including how to estimate them, before exploring the differences between rational and irrational numbers. In the second Sub-Unit, students will learn that the longest side of a right triangle has a special relationship to the measures of its sides, and will go on to prove why the Pythagorean Theorem is true for any right triangle.

Rigor

- Students build **conceptual understanding** about the length of the diagonal of a rectangle and its relation to its sides.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one set per group
- cardboard boxes of different sizes, one per group
- rulers
- string

Math Language Development

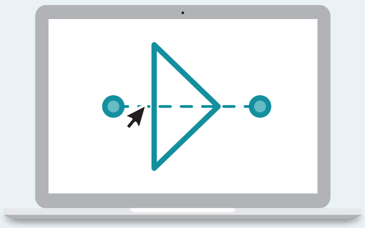
Review words

- *diagonal*
- *rectangular prism*

Amps | Featured Activity

Activity 1
Digital Geometry

Students digitally explore how to determine the longest cut for a rectangular sandwich. You can view their responses in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not feel confident that they have found the longest cut of all possible cuts in Activity 1. Encourage students to rely on their strengths as well as the structures within the task, such as the shape of the sandwich and the ways a sandwich can be cut, to feel self-assured that their answer is correct.

Modifications to Pacing

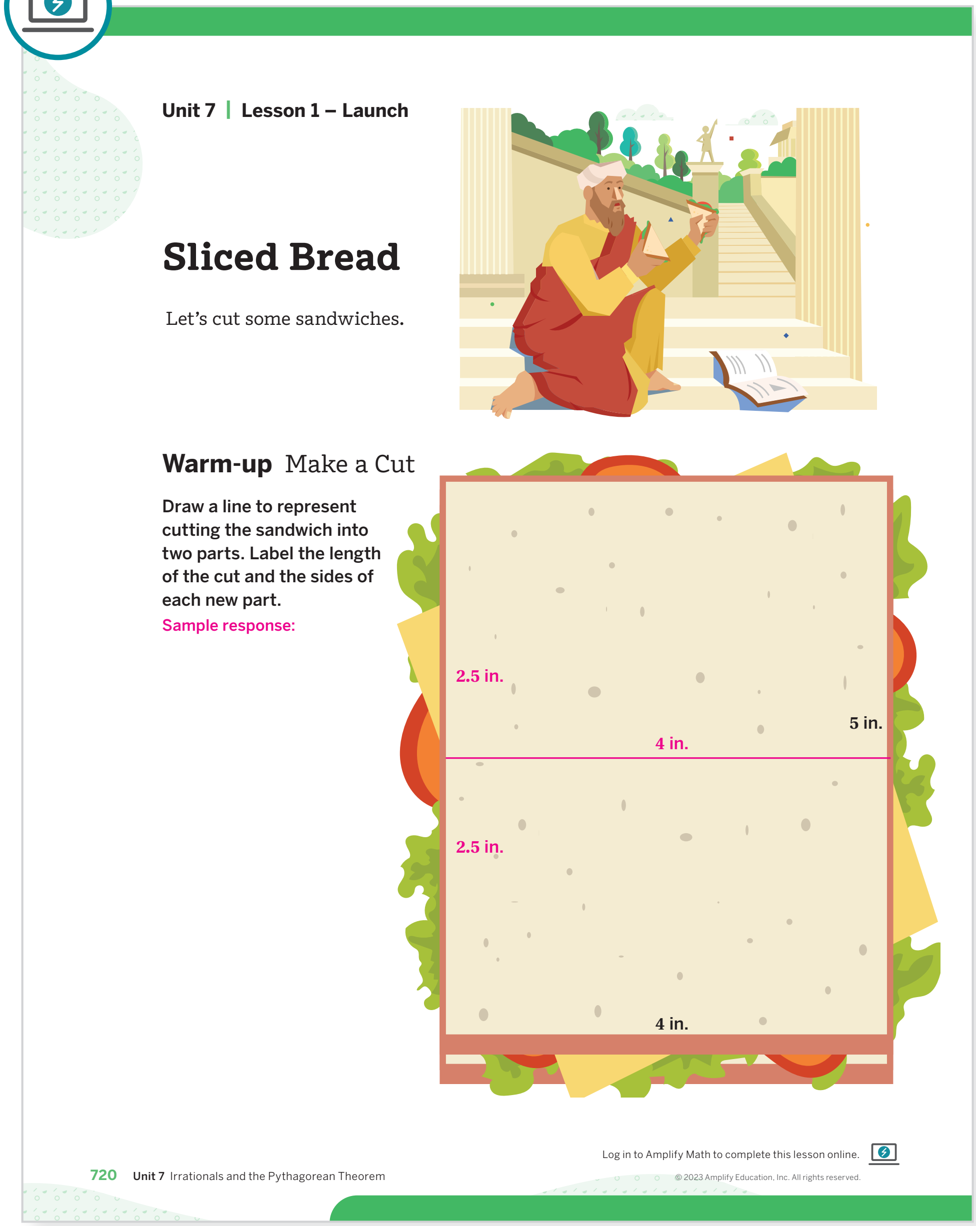
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- For a shorter version of **Activity 1**, give students the measurements for the side lengths.
- **Activity 2** may be omitted.

Pairs | 5 min

Warm-up Make a Cut

Students cut a rectangle to explore the different measures that can be made of the two parts.



Unit 7 | Lesson 1 – Launch

Sliced Bread

Let's cut some sandwiches.

Warm-up Make a Cut

Draw a line to represent cutting the sandwich into two parts. Label the length of the cut and the sides of each new part.

Sample response:

Log in to Amplify Math to complete this lesson online.

720 Unit 7 Irrationals and the Pythagorean Theorem

1 Launch

Activate prior knowledge by asking, "In your opinion, what do you think is the best way to cut a sandwich and why?" Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by asking them what would happen if they cut the sandwich vertically or horizontally. Provide access to rulers, should students choose to use them.

Look for points of confusion:

- **Making more than one cut or a cut that is not straight.** Remind students that they should make only one complete, straight cut.
- **Mislabeling a side measure.** Ensure students have cut the sandwich in a way that allows them to determine the side lengths (horizontally or vertically). Demonstrate for students how to use the side lengths of the original sandwich to determine the new lengths.

3 Connect

Display different examples of how students made their cuts and the measurements they determined.

Ask:

- "How did you decide to make your chosen cut?"
- "How can you be sure you know the exact lengths of your cut and sides?"
- "Did anybody try a cut besides horizontal or vertical? Were you able to find the measure of your cut?"

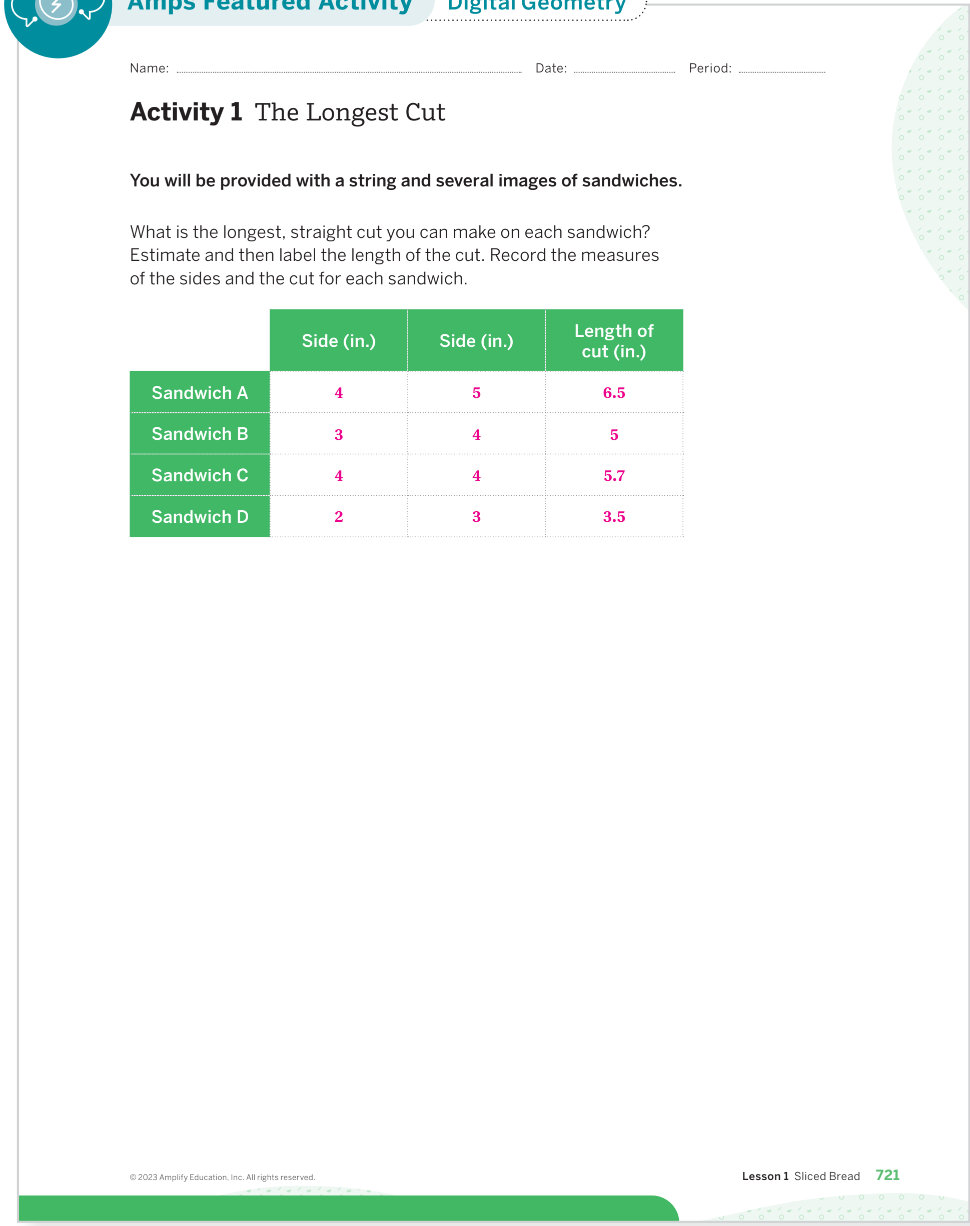
Highlight that to measure cuts that are not vertical or horizontal, students will need different tools or strategies.

Small Groups | 20 min

Activity 1 The Longest Cut

Students explore how to make the longest cut for a rectangular sandwich to gain insight into its measure in relation to the sides.

Amps Featured Activity Digital Geometry



Name: ______________ Date: ________ Period: ________

Activity 1 The Longest Cut

You will be provided with a string and several images of sandwiches.

What is the longest, straight cut you can make on each sandwich? Estimate and then label the length of the cut. Record the measures of the sides and the cut for each sandwich.

	Side (in.)	Side (in.)	Length of cut (in.)
Sandwich A	4	5	6.5
Sandwich B	3	4	5
Sandwich C	4	4	5.7
Sandwich D	2	3	3.5

Lesson 1 Sliced Bread 721

1 Launch

Distribute one set of the Activity 1 PDF and string to each group. Have students estimate measures independently before sharing with a partner.

2 Monitor

Help students get started by having them try three different cuts and measure them to see which type of cut will result in the longest length.

Look for points of confusion:

- **Not being sure how to estimate the length using the string.** Have students hold the length of the cut as measured by the string. Then have students place the string next to the known sides to help them determine an appropriate estimate.

3 Connect

Have students share how they made their cuts and what predictions they made about measurements of the longest cut. After establishing the fact that the longest cut is a diagonal cut, record several measurements for the diagonal of each sandwich. Discuss which measurements are more reasonable and which are less reasonable.

Display the animation from the Amps slide showing that the precise measurement of Sandwich A cannot be determined. Then play an animation for Sandwich B showing the exact length of 5 can be determined.

Ask, "What do these animations tell you about the measure of the longest cut?"

Highlight that the diagonal of a rectangle will always be the longest cut. It is always possible to estimate the length of this diagonal. However, there appears to be something unknowable about the exact measurement of some of the diagonals. Tell students they will explore these mysteries in this unit.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally explore how to determine the longest cut for a rectangular sandwich. You can view their responses in real time.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Sandwiches A and B. If time permits, they can move on to Sandwiches C and D. Consider also providing the measurements for the first two columns of the table so that students can focus on determining the length of the cut.

Math Language Development

MLR2: Collect and Display

While students work, circulate and collect the language they use to describe the type of cut that will result in the longest length. Listen for language used to describe the *diagonal*. Start a class display for this unit and add this language to the class display. During the Connect, press for details in their reasoning by asking:

- "How do you know the diagonal is the longest cut?"
- "Will it always be the longest cut?"

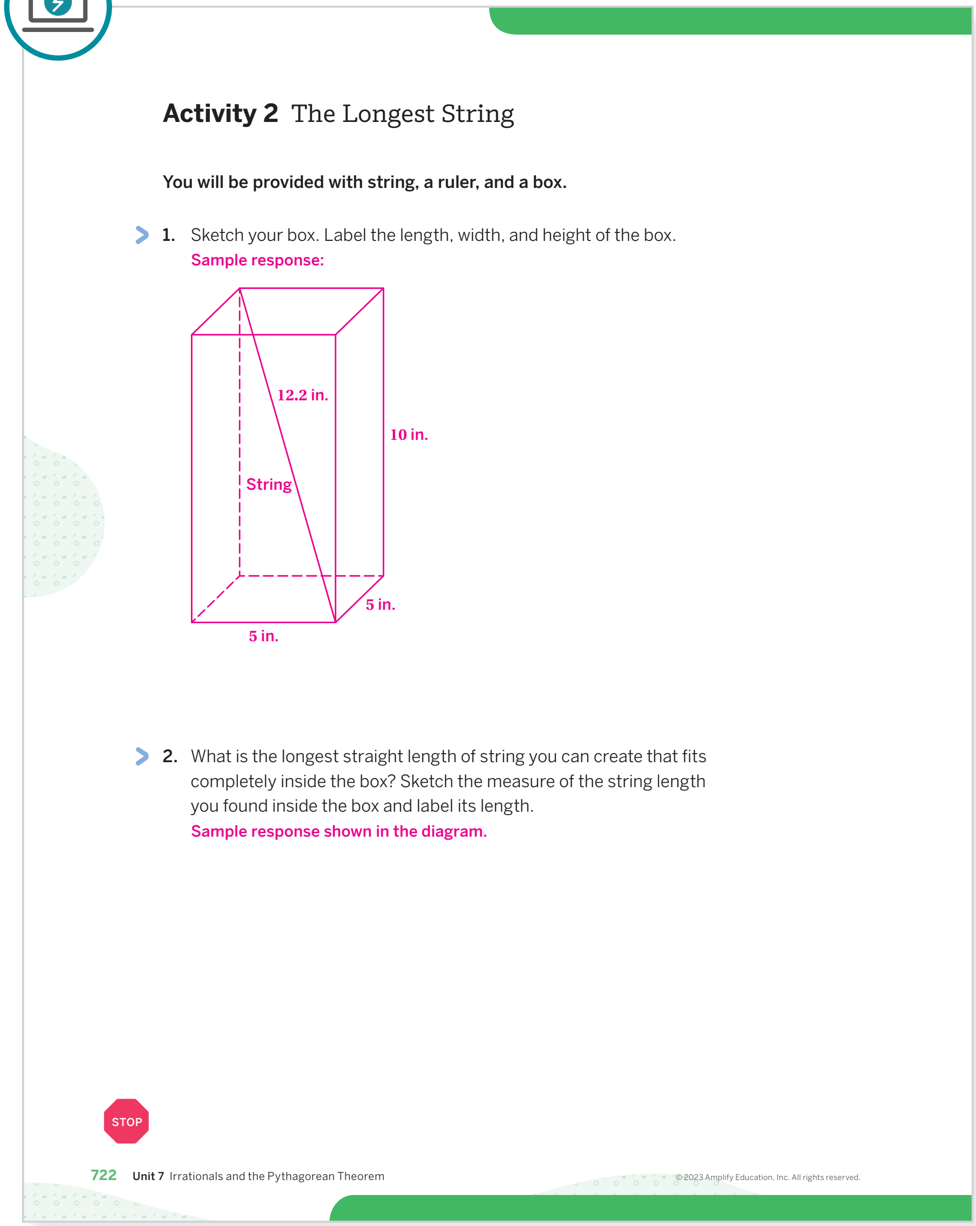
English Learners

If the term *diagonal* is unfamiliar to students, be sure to include visual examples of rectangles with *side* and *diagonal* labeled. Point out there are two diagonals for every rectangle.

Small Groups | 15 min

Activity 2 The Longest String

Students fit a string in a box to estimate the longest diagonal length of a rectangular prism.



Activity 2 The Longest String

You will be provided with string, a ruler, and a box.

1. Sketch your box. Label the length, width, and height of the box.
   Sample response:

2. What is the longest straight length of string you can create that fits completely inside the box? Sketch the measure of the string length you found inside the box and label its length.
   Sample response shown in the diagram.

STOP

722 **Unit 7** Irrationals and the Pythagorean Theorem

1 Launch

Distribute boxes, rulers, and string to groups of 2–4 students. Activate students' prior knowledge by asking them to recall the geometric figure that takes the shape of the box. Rectangular prism Then have students identify its dimensions with their group.

2 Monitor

Help students get started by having them try three different string lengths and have them compare their measurements.

Look for points of confusion:

- **Thinking a diagonal along a face of the prism will be the longest.** Tell students this is the longest length on a 2D surface, and ask them to see if they can find a longer length using more of the space in the 3D prism.

3 Connect

Display different examples of student sketches showing the longest length.

Have students share what they notice is the same about the longest string length for each box.

Ask:

- "What is true about the longest length you can make in a prism?" It is a big diagonal.
- "Do you think the diagonal measures will have the same phenomenon as the diagonals you saw with your sandwiches?"
- "Can you think of a real-world application for this problem?"

Highlight that the longest length of a prism will be the diagonal that connects opposite vertices. Similar to a diagonal drawn on a 2D rectangular figure, students will learn how to determine the exact measure of this length in this unit.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Conduct a class demonstration using one box and ask for student volunteers to describe where you should place the string so that it is the longest. After you cut and place the string, ask a student to measure the string length. Repeat for a different-sized box, as time permits.

Extension: *Math Enrichment*

Ask students if the diagonal from the top corner of a box to the opposite bottom corner will always be longer than the height of the box. Have them explain their thinking. Sample response: Yes, if I use pieces of string to represent each measure, the diagonal is longer than the height.

Whole Class | 5 min

Summary The Mystery of the Pythagoreans

Review and synthesize how to determine the longest length of a cut section of a rectangular figure or a rectangular prism.

Narrative Connections

Unit 7 Irrationals and the Pythagorean Theorem

The Mystery of the Pythagoreans

What makes a perfect sandwich? Fresh ingredients? Toasty bread?

No! It all comes down to presentation. With one diagonal cut, a boring square is transformed into two perfect right triangles of sandwichy goodness.

Diagonals might not seem like a big deal. We use them everyday — taking a shortcut across a yard, moving a bishop along a chessboard, or folding down a corner for the perfect paper airplane. But would you believe that behind these simple, everyday occurrences, there lies a dark and twisted story, as bewildering as any mystery thriller?

It's a story centuries in the making involving cults, mystics, mysterious deaths, and baffling clues in notebook margins. So, let's roll up our sleeves, get off the grid, and peer into the infinite with the Pythagorean Theorem and irrational numbers.

Welcome to Unit 7.

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Lesson 1 Sliced Bread 723

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary. Have students read the Summary or have a student volunteer read it aloud.

Ask:

- "What did you learn about the diagonal of a rectangular figure or rectangular prism?"
- "What do you still not know?"

Highlight that students will learn more about how to determine the exact length of the diagonals they drew today.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

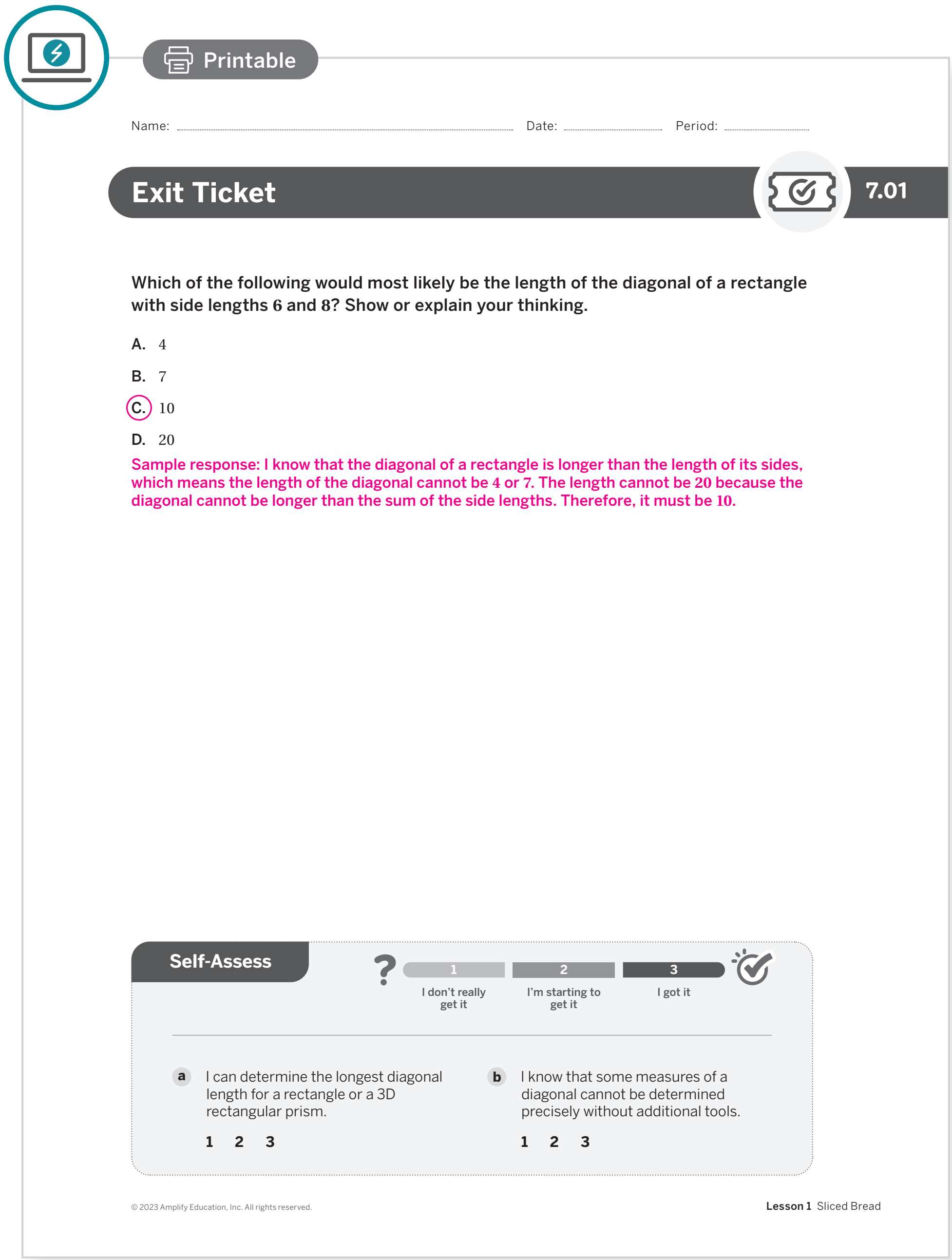
- "What questions do you still have after today's lesson?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reasoning about the length of the diagonal of a rectangle.

Printable



Name: __________ Date: ________ Period: ________

Exit Ticket 7.01

Which of the following would most likely be the length of the diagonal of a rectangle with side lengths 6 and 8? Show or explain your thinking.

A. 4

B. 7

[✓ C.] 10

D. 20

Sample response: I know that the diagonal of a rectangle is longer than the length of its sides, which means the length of the diagonal cannot be 4 or 7. The length cannot be 20 because the diagonal cannot be longer than the sum of the side lengths. Therefore, it must be 10.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine the longest diagonal length for a rectangle or a 3D rectangular prism.

1 2 3

b. I know that some measures of a diagonal cannot be determined precisely without additional tools.

1 2 3

Lesson 1 Sliced Bread

Success looks like . . .

- **Language Goal:** Understanding and explaining that the longest length inside a rectangle and rectangular prism is a diagonal. **(Speaking and Listening)**
  - » Explaining why the diagonal of the rectangle would most likely be a length of 10.
- **Goal:** Developing an intuition that the exact diagonal measure of a rectangle or rectangular prism may be impossible to determine using numbers students know.

Suggested next steps

If students think the correct response is A or B, consider:

- Reviewing what was true about the measures of the diagonals in Activity 1 in relation to the sides.
- Assigning Practice Problem 1.
- Asking, "What is the longest — the diagonal or the side length measures?"

If students think the correct response is D, consider:

- Having students sketch the figure.
- Assigning Practice Problem 1.
- Asking, "Can the diagonal be longer than the sum of the side lengths? Why or why not?"

Professional Learning

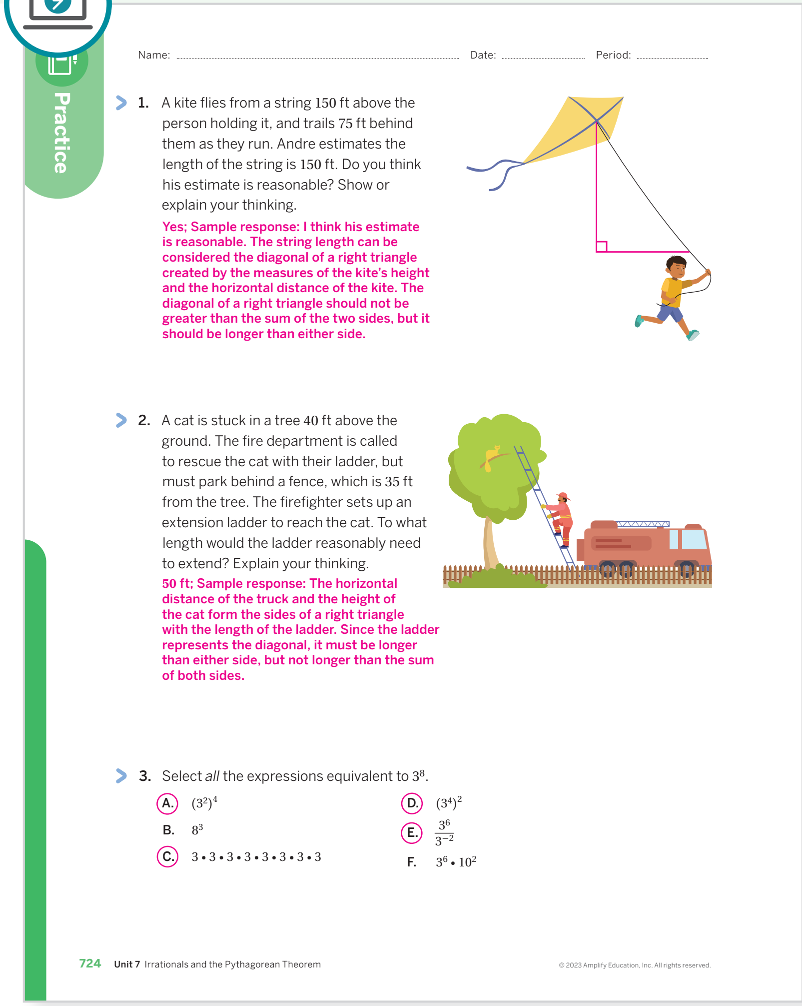
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today?
- What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. A kite flies from a string 150 ft above the person holding it, and trails 75 ft behind them as they run. Andre estimates the length of the string is 150 ft. Do you think his estimate is reasonable? Show or explain your thinking.

Yes; Sample response: I think his estimate is reasonable. The string length can be considered the diagonal of a right triangle created by the measures of the kite's height and the horizontal distance of the kite. The diagonal of a right triangle should not be greater than the sum of the two sides, but it should be longer than either side.

2. A cat is stuck in a tree 40 ft above the ground. The fire department is called to rescue the cat with their ladder, but must park behind a fence, which is 35 ft from the tree. The firefighter sets up an extension ladder to reach the cat. To what length would the ladder reasonably need to extend? Explain your thinking.

50 ft; Sample response: The horizontal distance of the truck and the height of the cat form the sides of a right triangle with the length of the ladder. Since the ladder represents the diagonal, it must be longer than either side, but not longer than the sum of both sides.

3. Select *all* the expressions equivalent to 3^8.

Ⓐ $(3^2)^4$
B. 8^3
Ⓒ $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
Ⓓ $(3^4)^2$
Ⓔ $\frac{3^6}{3^{-2}}$
F. $3^6 \cdot 10^2$

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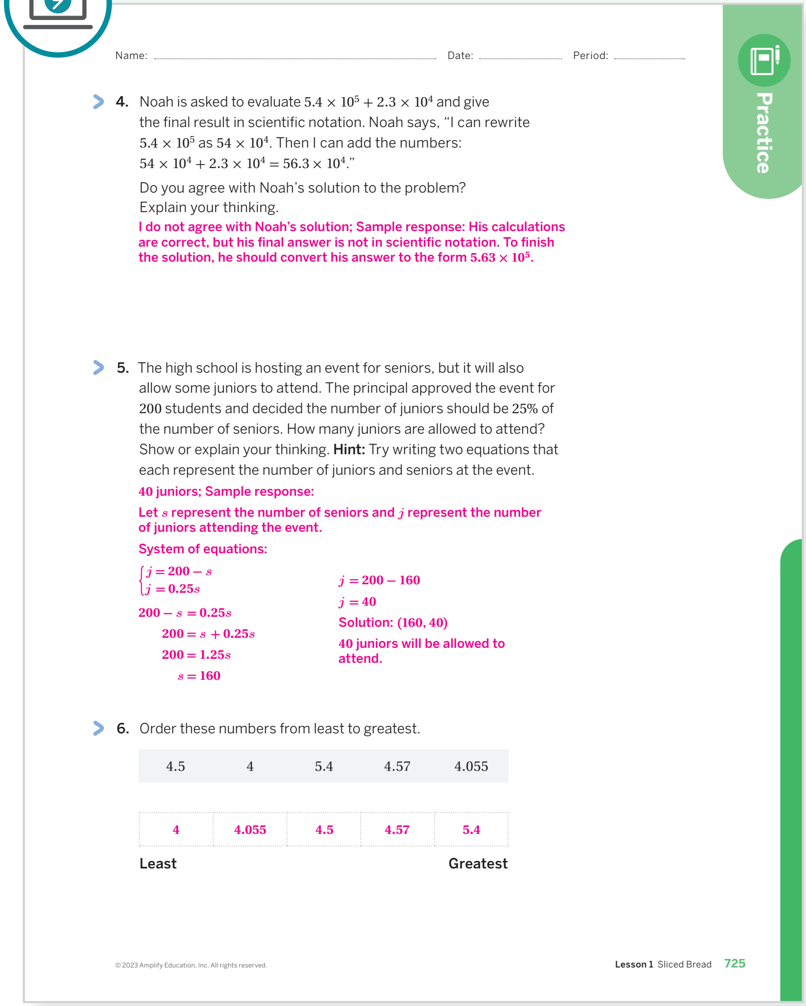

Practice

Name: Date: Period:

4. Noah is asked to evaluate $5.4 \times 10^5 + 2.3 \times 10^4$ and give the final result in scientific notation. Noah says, "I can rewrite 5.4×10^5 as 54×10^4. Then I can add the numbers: $54 \times 10^4 + 2.3 \times 10^4 = 56.3 \times 10^4$."

Do you agree with Noah's solution to the problem? Explain your thinking.

I do not agree with Noah's solution; Sample response: His calculations are correct, but his final answer is not in scientific notation. To finish the solution, he should convert his answer to the form 5.63×10^5.

5. The high school is hosting an event for seniors, but it will also allow some juniors to attend. The principal approved the event for 200 students and decided the number of juniors should be 25% of the number of seniors. How many juniors are allowed to attend? Show or explain your thinking. **Hint:** Try writing two equations that each represent the number of juniors and seniors at the event.

40 juniors; Sample response:

Let s represent the number of seniors and j represent the number of juniors attending the event.

System of equations:

$$\begin{cases} j = 200 - s \\ j = 0.25s \end{cases}$$

$$200 - s = 0.25s$$
$$200 = s + 0.25s$$
$$200 = 1.25s$$
$$s = 160$$

$$j = 200 - 160$$
$$j = 40$$

Solution: (160, 40)

40 juniors will be allowed to attend.

6. Order these numbers from least to greatest.

4.5	4	5.4	4.57	4.055

4	**4.055**	**4.5**	**4.57**	**5.4**

Least **Greatest**

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Lesson 1 Sliced Bread 725

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
Spiral	3	Unit 6 Lesson 7	1
	4	Unit 6 Lesson 14	2
	5	Unit 4 Lesson 16	2
Formative ⚡	6	Unit 7 Lesson 2	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

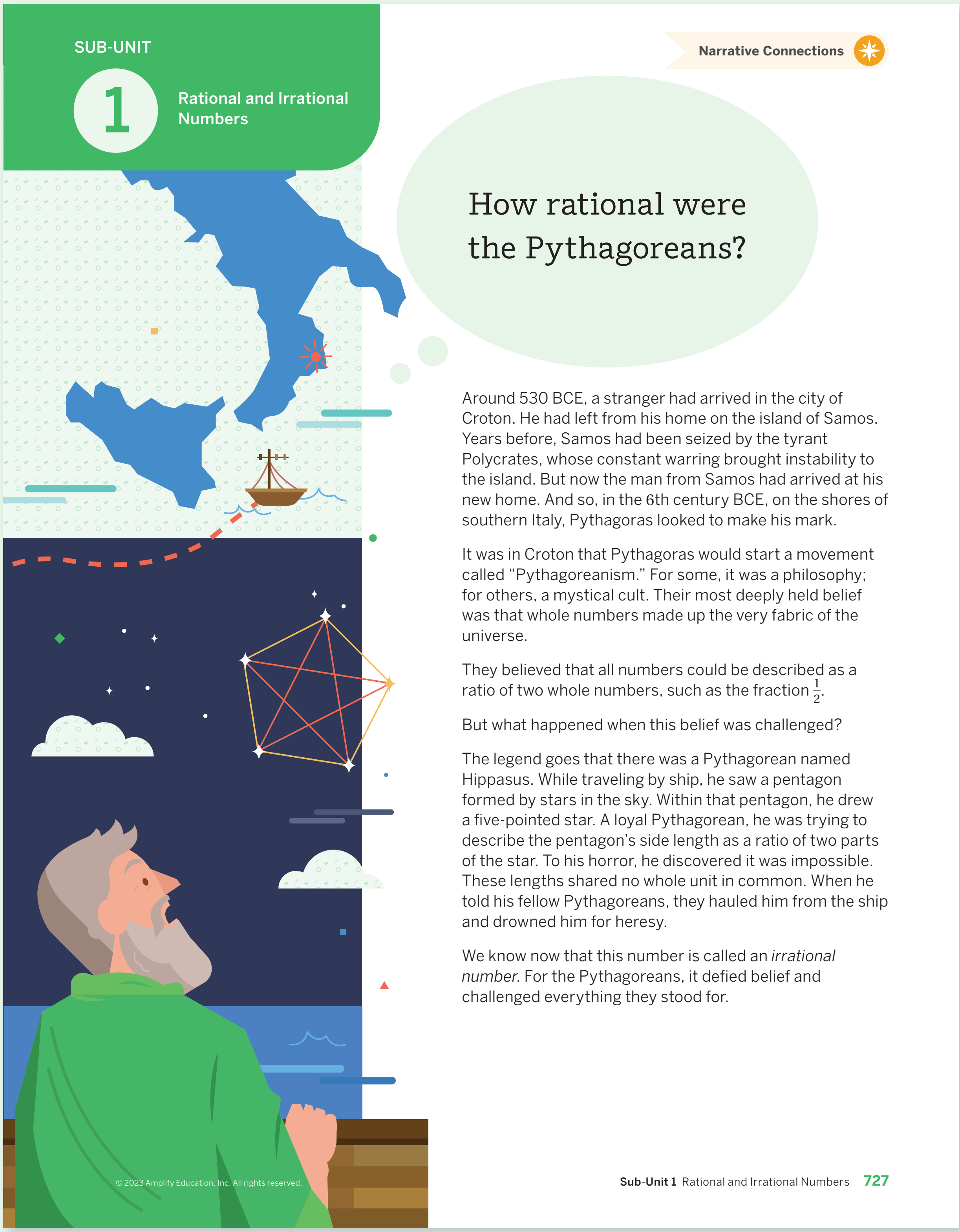
For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Sub-Unit 1

Rational and Irrational Numbers

Whole Class

In this Sub-Unit, students are introduced to the differences between rational and irrational numbers. Students build their understanding by solving equations of the form $x^2 = p$ or $x^3 = p$.



SUB-UNIT

1 Rational and Irrational Numbers

Narrative Connections

How rational were the Pythagoreans?

Around 530 BCE, a stranger had arrived in the city of Croton. He had left from his home on the island of Samos. Years before, Samos had been seized by the tyrant Polycrates, whose constant warring brought instability to the island. But now the man from Samos had arrived at his new home. And so, in the 6th century BCE, on the shores of southern Italy, Pythagoras looked to make his mark.

It was in Croton that Pythagoras would start a movement called “Pythagoreanism.” For some, it was a philosophy; for others, a mystical cult. Their most deeply held belief was that whole numbers made up the very fabric of the universe.

They believed that all numbers could be described as a ratio of two whole numbers, such as the fraction $\frac{1}{2}$.

But what happened when this belief was challenged?

The legend goes that there was a Pythagorean named Hippasus. While traveling by ship, he saw a pentagon formed by stars in the sky. Within that pentagon, he drew a five-pointed star. A loyal Pythagorean, he was trying to describe the pentagon’s side length as a ratio of two parts of the star. To his horror, he discovered it was impossible. These lengths shared no whole unit in common. When he told his fellow Pythagoreans, they hauled him from the ship and drowned him for heresy.

We know now that this number is called an *irrational number.* For the Pythagoreans, it defied belief and challenged everything they stood for.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore irrational numbers in the following places:

- **Lesson 3, Activities 1–3:** Comparing Squares, Sorting Square Roots
- **Lesson 4, Activities 1–2:** Estimating Square Roots, Ordering Square Roots on a Number Line
- **Lesson 5, Activities 1–2:** Determining the Value of x and x^3, Cube Roots
- **Lesson 6, Activities 1–3:** Ratio of Integers, Is It Irrational?, Is It Rational or Irrational?

UNIT 7 | **LESSON 2**

The Square Root

Let's learn about square roots.

Focus

Goals

1. **Language Goal:** Comprehend the term *square root of* p and the notation $\sqrt{p}$ to mean a solution to $x^2 = p$, where p is a positive rational number. **(Speaking and Listening)**
2. Evaluate square roots of perfect squares.

Coherence

Today

Students try to determine a solution to the equation $x^2 = 2$ and recognize the need for square root notation. Students model Pythagorean thinking as they dive into a geometrical exploration of a perfect square.

Previously

In Lesson 1, students explored the diagonal length of a rectangle and began relating the side lengths of a rectangle to the diagonal of the rectangle.

Coming Soon

In Lesson 3, students will explore the relationship between the side length of a square, given its area, and use the area of squares to compare square roots.

Rigor

- Students build **conceptual understanding** of a square root as the solution to an equation of the form $x^2 = p$.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	10 min	15 min	5 min	5 min
Whole Class	Pairs	Independent	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Perfect Squares*
- calculators
- dot grid paper

Math Language Development

New words

- ***perfect square***
- ***square root***

Review word

- *exponent*
- *integer*

Amps | Featured Activity

Activity 1

Spirit of Competition

Students face off in a friendly competition to see who can guess the closest value of x for the equation $x^2 = 2$.

Building Math Identity and Community

Connecting to Mathematical Practices

As students try to determine the solution to an equation involving a quadratic term, they might not understand the process of becoming more precise when the square root is not a whole number. Explain that they will need to be organized with their guesses. Have students record all of their guesses and determine whether each was too low or too high. Point out that the more decimal places the answer has, the more precise it is, but that the most precise answer is a value with a radical symbol.

Modifications to Pacing

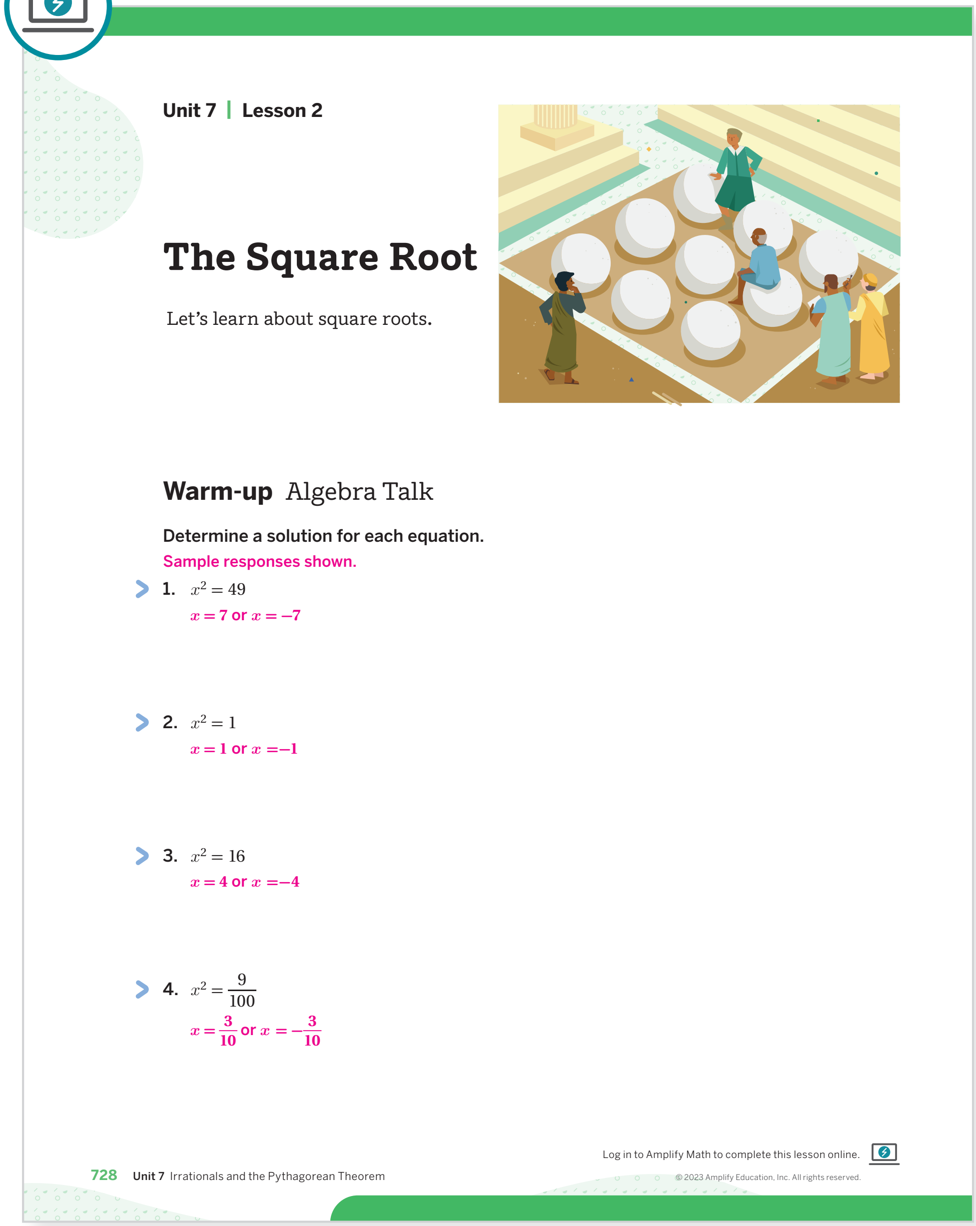
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 3 and 4 may be omitted.
- In **Activity 2**, have students only complete 3 rows in the table.
- Optional **Activity 3** may be omitted.

Whole Class | 5 min

Warm-up Algebra Talk

Students solve equations of the form $x^2 = p$ to prepare them in estimating solutions to the equation $x^2 = 2$.



Unit 7 | Lesson 2

The Square Root

Let's learn about square roots.

Warm-up Algebra Talk

Determine a solution for each equation.

Sample responses shown.

1. $x^2 = 49$

   $x = 7$ or $x = -7$

2. $x^2 = 1$

   $x = 1$ or $x = -1$

3. $x^2 = 16$

   $x = 4$ or $x = -4$

4. $x^2 = \frac{9}{100}$

   $x = \frac{3}{10}$ or $x = -\frac{3}{10}$

Log in to Amplify Math to complete this lesson online.

728 Unit 7 Irrationals and the Pythagorean Theorem

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1 Launch

Conduct the ***Algebra Talk*** routine.

2 Monitor

Help students get started by asking, "Could 8 be a solution to $x^2 = 49$? Why or why not?"

Look for points of confusion:

- **Struggling to determine a solution for Problem 4.** Ask, "What fraction multiplied by itself is equal to $\frac{9}{100}$?" If students still struggle to determine a solution, consider asking them to solve the equations $x^2 = 90$ and $x^2 = 100$.

Look for productive strategies:

- Providing a positive or negative value for x.

3 Connect

Have students share their strategies for each problem. Record and display their responses for all to see. Select students who selected different strategies for solving each problem.

Highlight that, to determine a solution to the equation in the form $x^2 = p$ (where p is a positive rational number), students can determine a power of 2 that equals p. **Note:** You may discuss the negative solutions at this time, depending on the readiness level of students.

Ask, "How can you check whether a number is a solution to the equation?" Substitute the value for x and see whether the equation is true.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, draw their attention to the structure of each equation and how they are similar or different. Ask:

- "What is similar about these equations?" Highlight how they are written in the form $x^2 = p$, where p is a positive, *rational number.* You may wish to review the term rational number.
- "How is the equation in Problem 4 different from the others?" It is written as a fraction.
- "What do you notice about the solutions?" Each solution squared (or raised to the second power) equals p. Solutions can be positive or negative.

Power-up

To power up students' ability to ordering decimal values, have students complete:

Align the following values vertically, and then circle the greatest value.

$3.02, 30, 3.005, 3.52, 30.5$

3.02
30
3.005
3.52
$\boxed{30.5}$

Use: Before Activity 1

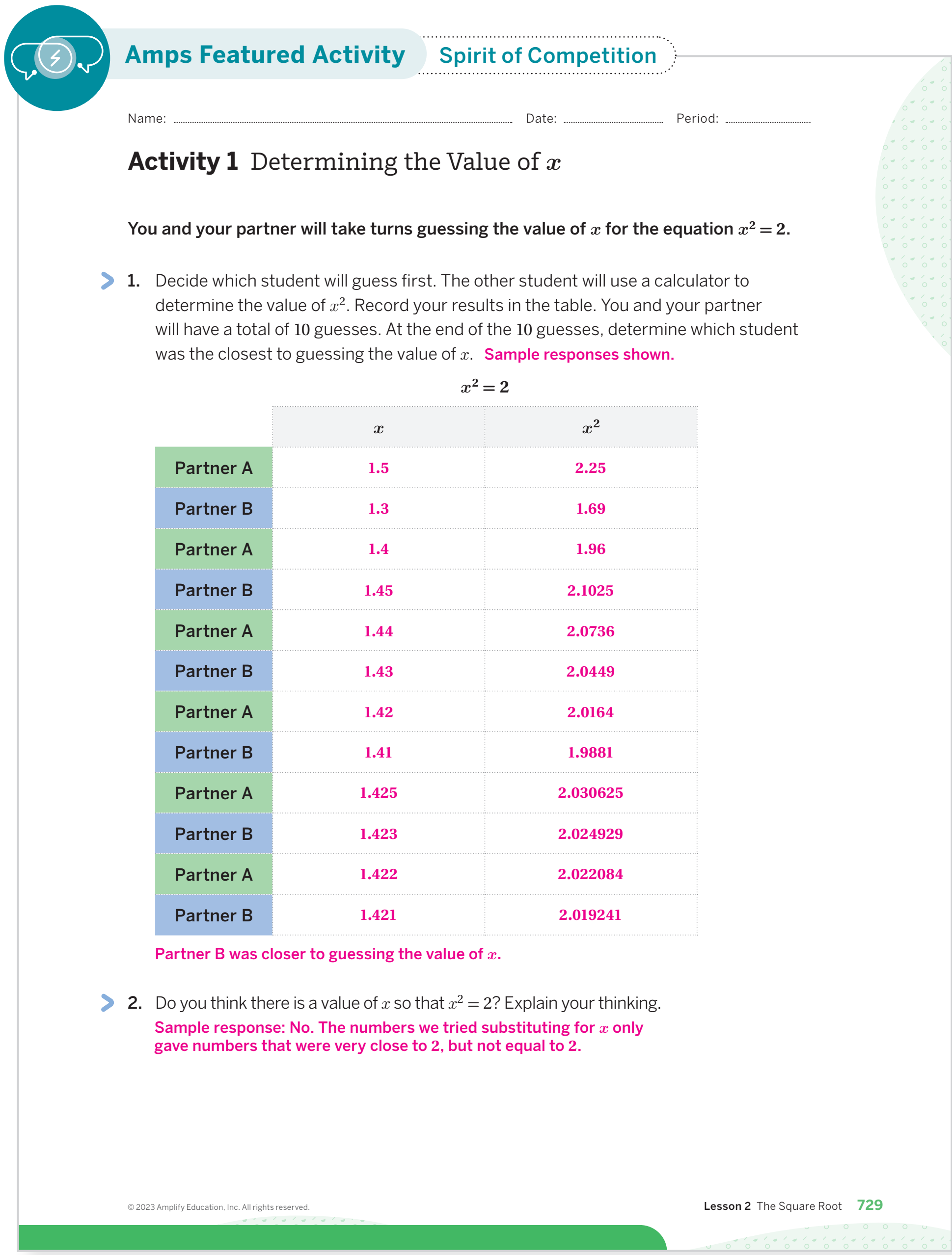
Informed by: Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Pairs | 20 min

Activity 1 Determining the Value of x

Students investigate the solution to $x^2 = 2$ as an introduction to square root notation.

Amps Featured Activity Spirit of Competition



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Activity 1 Determining the Value of x

You and your partner will take turns guessing the value of x for the equation $x^2 = 2$.

1. Decide which student will guess first. The other student will use a calculator to determine the value of x^2. Record your results in the table. You and your partner will have a total of 10 guesses. At the end of the 10 guesses, determine which student was the closest to guessing the value of x. Sample responses shown.

$x^2 = 2$

	x	x^2
Partner A	1.5	2.25
Partner B	1.3	1.69
Partner A	1.4	1.96
Partner B	1.45	2.1025
Partner A	1.44	2.0736
Partner B	1.43	2.0449
Partner A	1.42	2.0164
Partner B	1.41	1.9881
Partner A	1.425	2.030625
Partner B	1.423	2.024929
Partner A	1.422	2.022084
Partner B	1.421	2.019241

Partner B was closer to guessing the value of x.

2. Do you think there is a value of x so that $x^2 = 2$? Explain your thinking.
Sample response: No. The numbers we tried substituting for x only gave numbers that were very close to 2, but not equal to 2.

Lesson 2 The Square Root 729

1 Launch

Review the directions for the activity and answer any questions students may have. **Note:** Provide access to calculators and collect at the end of this activity.

2 Monitor

Help students get started by having them choose a value for x, and then determining the value of x^2.

Look for points of confusion:

- **Guessing values for x that are not getting closer to 2.** Have students start by guessing decimals to the nearest tenth and asking if their next guess should be higher or lower. Repeat for several numbers to the nearest tenth. Then have students guess numbers to the hundredth and thousandths to determine a more precise solution.

3 Connect

Have pairs of students share their closest guess for x. Then have students share their responses for Problem 2.

Highlight the exact solution could be expressed using a *square root symbol*.

Define the term ***square root*** of p as a solution to an equation of the form $x^2 = p$. Say, "You can represent the exact solution to the equation $x^2 = 2$ by using the square root notation, $\sqrt{2}$, read as "the square root of two."
Note: While a square root can be either positive or negative, students will primarily explore positive square roots in this unit.

Ask students how they could represent the exact solution to the equations $x^2 = 10$ and $x^2 = 16$ to check for understanding.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can engage in a digital competition to see who can guess the closest value of x for the equation $x^2 = 2$.

Accessibility: *Guide Processing and Visualization*

Consider demonstrating how to guess a value for x and determine the value of x^2 using that guess.

Extension: *Math Enrichment*

Have students guess and check to determine the solution to the equation $x^2 = 2$ to the nearest thousandth. 1.414

Math Language Development

MLR2: Collect and Display

As students share their guesses and discuss how to find the exact solution, invite them to suggest language or diagrams to add to the class display that will support their understanding of the term *square root*. Continue adding to the display in Activity 2 as students work with *perfect squares*.

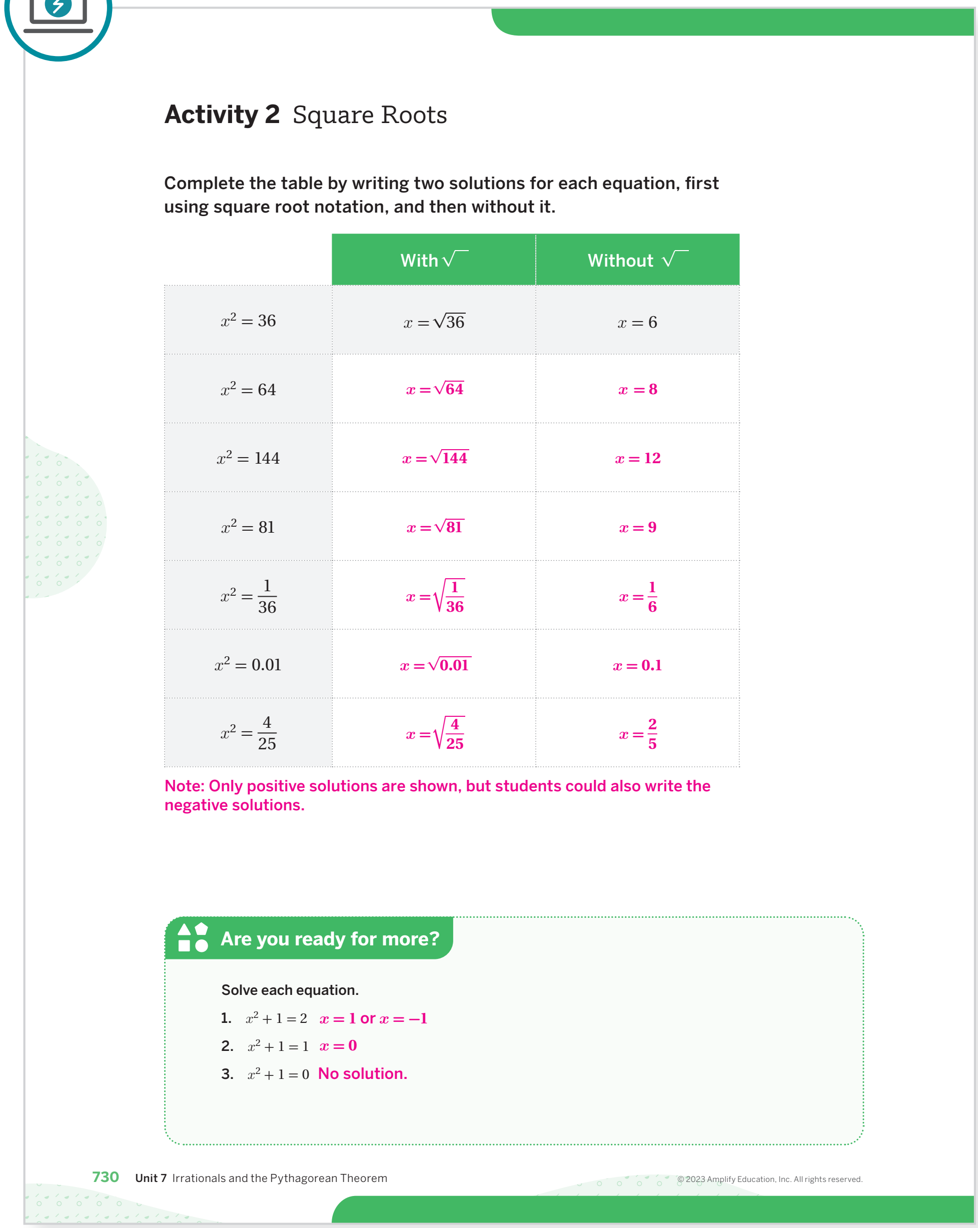
English Learners

Clarify the meaning of the word *square* in *square root*. Provide examples, such as a picture or diagram, to help students connect the definition to a visual representation of the side length of a square representing the square root of the square's area.

Independent | 10 min

Activity 2 Square Roots

Students write an expression using square root notation and evaluate the expression to build fluency in evaluating square roots.



Activity 2 Square Roots

Complete the table by writing two solutions for each equation, first using square root notation, and then without it.

	With $\sqrt{}$	Without $\sqrt{}$
$x^2 = 36$	$x = \sqrt{36}$	$x = 6$
$x^2 = 64$	$x = \sqrt{64}$	$x = 8$
$x^2 = 144$	$x = \sqrt{144}$	$x = 12$
$x^2 = 81$	$x = \sqrt{81}$	$x = 9$
$x^2 = \frac{1}{36}$	$x = \sqrt{\frac{1}{36}}$	$x = \frac{1}{6}$
$x^2 = 0.01$	$x = \sqrt{0.01}$	$x = 0.1$
$x^2 = \frac{4}{25}$	$x = \sqrt{\frac{4}{25}}$	$x = \frac{2}{5}$

Note: Only positive solutions are shown, but students could also write the negative solutions.

Are you ready for more?

Solve each equation.

1. $x^2 + 1 = 2$ $x = 1$ or $x = -1$
2. $x^2 + 1 = 1$ $x = 0$
3. $x^2 + 1 = 0$ No solution.

730 **Unit 7** Irrationals and the Pythagorean Theorem

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity. **Note:** Students will be introduced to evaluating square roots using a calculator in Lesson 4.

2 Monitor

Help students get started by reviewing the example with the class. Ensure that students understand that $\sqrt{36}$ and 6 are equivalent and both numbers represent the solution to the equation $x^2 = 36$.

Look for points of confusion:

- **Struggling with any equations that include fractions or decimals.** Suggest that students use a guess-and-check method using different fractions. Revisit these students during the Connect.

3 Connect

Display student work showing correct responses.

Have students share their strategies for determining a solution for any equation with a fraction or decimal.

Highlight:

- To determine the square root of a fraction, students can calculate the square root of the numerator and the square root of the denominator. Write $\sqrt{\frac{4}{25}}$ as $\frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$ for all to see.
- For Problem 5, students could write the fraction $\sqrt{\frac{1}{100}}$ before determining their response.

Define the term ***perfect square***. Say, "A perfect square is a number that is the square of an integer. For example 16 is a perfect square because $4^2 = 16$, but 8 is not a perfect square."

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Chunk this task by having students first complete the first three rows of the table for whole number squares. Pause for a brief discussion of their responses and strategies. Then have them complete the next three rows, one at a time, pausing between each row to discuss their responses and strategies.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you define the term *perfect square*, draw students' attention to the differences between perfect squares and numbers that are not perfect squares. Review the term *integer*, as needed. Consider adding a table similar to the following to the class display.

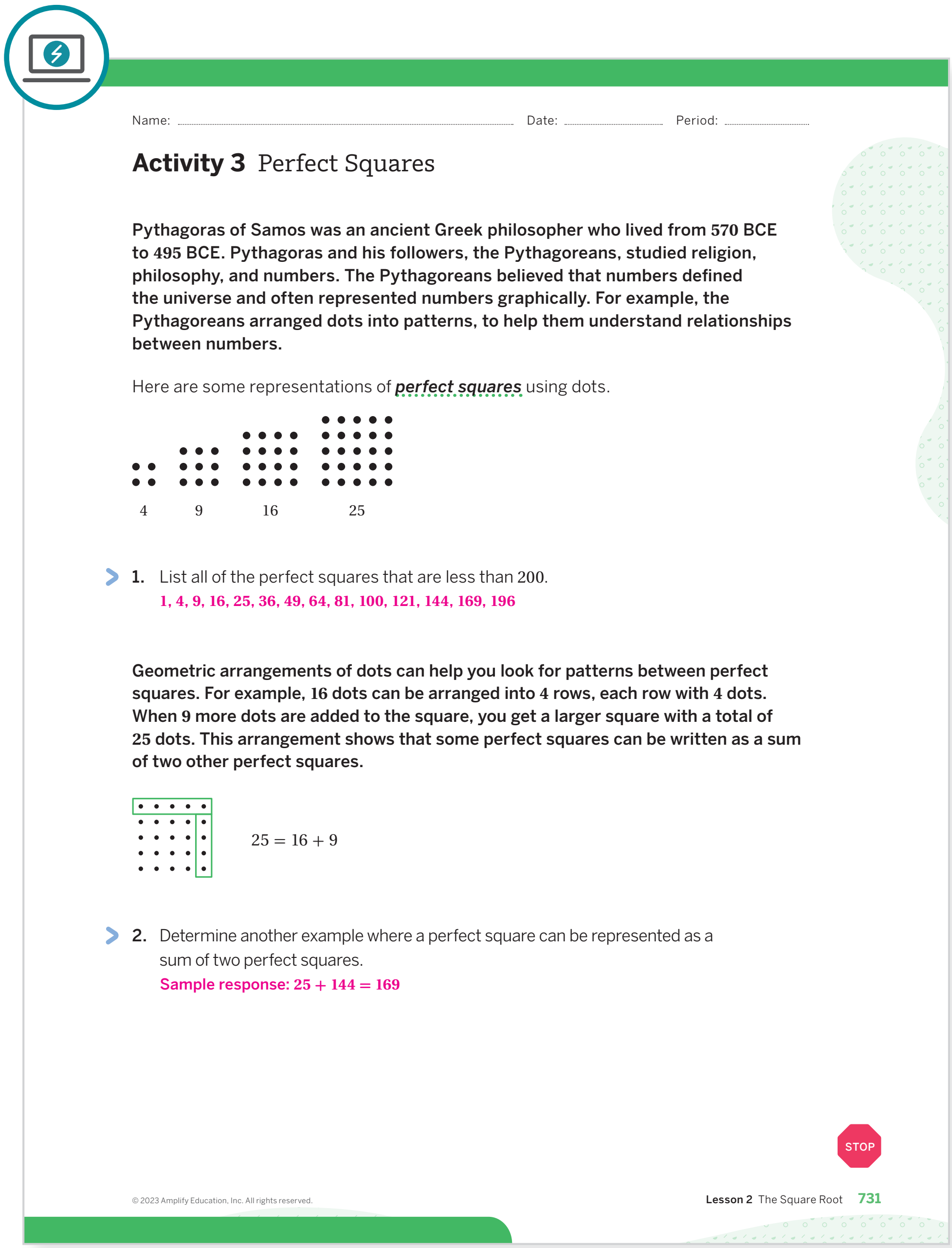
Examples of perfect squares	Examples of numbers that are not perfect squares
1 because $1^2 = 1$ 4 because $2^2 = 4$ 9 because $3^2 = 9$	2 because there is no integer that when multiplied by itself equals 2. 5 because there is no integer that when multiplied by itself equals 2.

Optional

Independent | 15 min

Activity 3 Perfect Squares

Students use a visual representation to identify perfect squares and determine a perfect square that could be the sum of two other perfect squares.



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Activity 3 Perfect Squares

Pythagoras of Samos was an ancient Greek philosopher who lived from 570 BCE to 495 BCE. Pythagoras and his followers, the Pythagoreans, studied religion, philosophy, and numbers. The Pythagoreans believed that numbers defined the universe and often represented numbers graphically. For example, the Pythagoreans arranged dots into patterns, to help them understand relationships between numbers.

Here are some representations of ***perfect squares*** using dots.

4 9 16 25

1. List all of the perfect squares that are less than 200.
   1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196

Geometric arrangements of dots can help you look for patterns between perfect squares. For example, 16 dots can be arranged into 4 rows, each row with 4 dots. When 9 more dots are added to the square, you get a larger square with a total of 25 dots. This arrangement shows that some perfect squares can be written as a sum of two other perfect squares.

$25 = 16 + 9$

2. Determine another example where a perfect square can be represented as a sum of two perfect squares.
   Sample response: $25 + 144 = 169$

STOP

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Lesson 2 The Square Root 731

1 Launch

Ask, "How can the dot representation help you determine another perfect square?" I can draw the same number of dots in each row and column to determine a perfect square. Distribute dot grid paper.

2 Monitor

Help students get started by having them draw the next perfect square using dots.

Look for points of confusion:

- **Struggling to complete Problem 2.** Provide a shorter list of perfect squares from Problem 1, such as 36, 49, 64, 81, and 100. From this list, students may determine $100 = 36 + 64$.

Look for productive strategies:

- Using the dot grid paper to determine perfect squares.
- Determining more than one example for Problem 2.

3 Connect

Have students share their responses for Problem 2 and display the visual representation that depicts their response. Select students who determined different examples.

Ask, "Do you think any perfect square could be written as a sum of two perfect squares? Explain your thinking." No; For example, 9 cannot be written as the sum of two perfect squares because 4 and 1 are the only two perfect squares less than 9.

Highlight that only some perfect squares can be written as a sum of two other perfect squares. These special numbers are called *Pythagorean triples*. Students will explore these numbers further in Lesson 16.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on listing perfect squares that are less than 100 in Problem 1, not 200. Then have them move on to complete Problem 2.

Extension: *Math Enrichment*

Challenge students to write all of perfect squares less than 200 that can be represented as the sum of two perfect squares. Tell them that the two perfect-square addends must be different numbers.

$9 + 16 = 25$; $36 + 64 = 100$; $25 + 144 = 169$

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the four perfect squares. Have students write 2–3 questions they could ask about the diagrams. Have them share their questions with a partner and generate one list of 2–3 questions. Ask volunteers to share their questions with the class. Sample questions shown.

- How are the number of dots on each side changing each time?
- How are the total number of dots changing each time?
- Is the dot pattern growing at a linear or nonlinear rate?

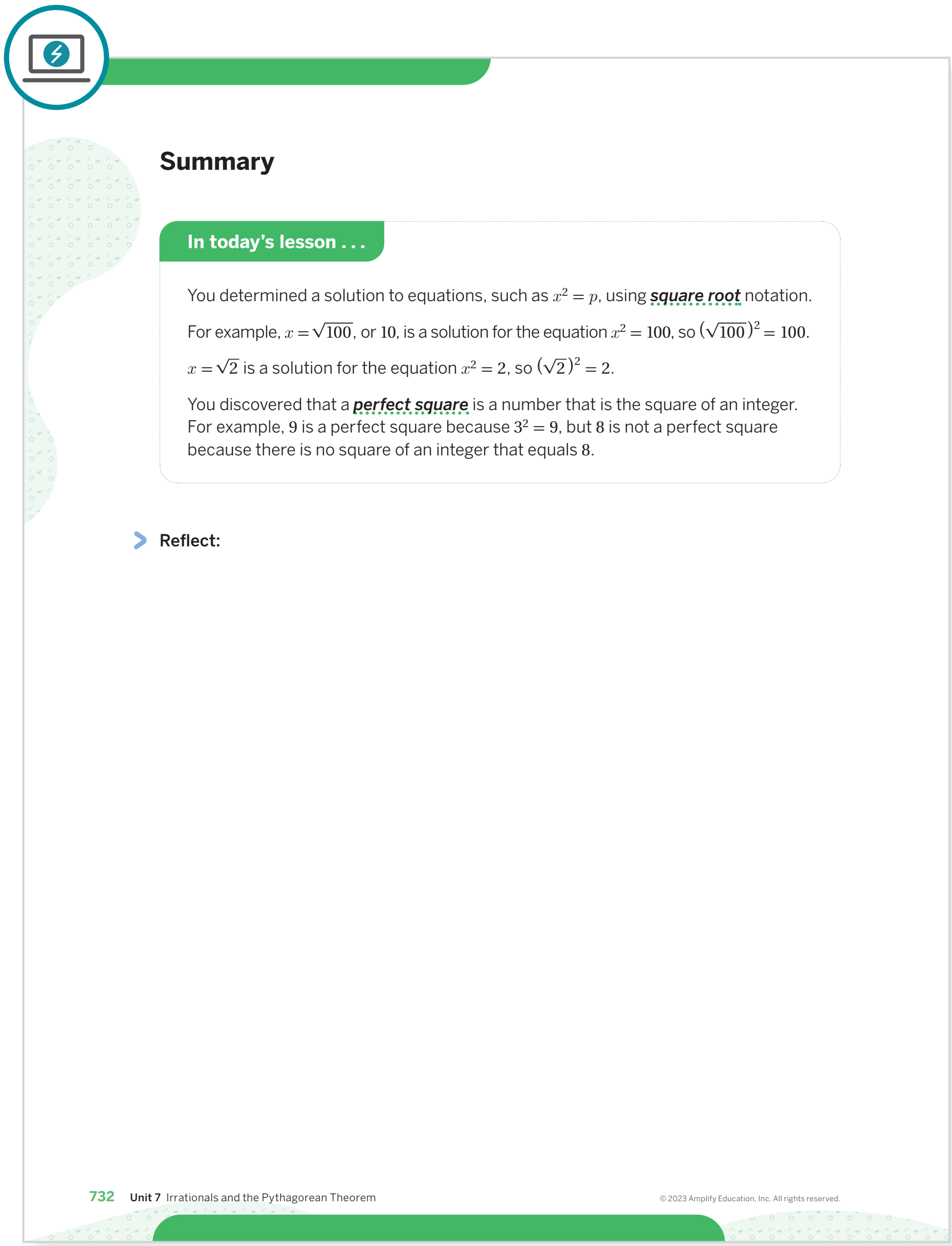
English Learners

Consider displaying one of the sample questions for students to use as a reference in crafting their own.

Whole Class | 5 min

Summary

Review and synthesize solutions to equations in the form $x^2 = p$ and square root notation.



Summary

In today's lesson . . .

You determined a solution to equations, such as $x^2 = p$, using ***square root*** notation.

For example, $x = \sqrt{100}$, or 10, is a solution for the equation $x^2 = 100$, so $(\sqrt{100})^2 = 100$.

$x = \sqrt{2}$ is a solution for the equation $x^2 = 2$, so $(\sqrt{2})^2 = 2$.

You discovered that a ***perfect square*** is a number that is the square of an integer. For example, 9 is a perfect square because $3^2 = 9$, but 8 is not a perfect square because there is no square of an integer that equals 8.

Reflect:

Synthesize

Display the Anchor Chart PDF, *Perfect Squares*, and use it as reference, as needed. Remind students that the middle column represents numbers that are perfect squares.

Have students share how they can determine the solution to equations written in the form $x^2 = p$.

Highlight that students could determine the solution to equations like $x^2 = 100$ using square root notation. For these equations, if the solution is a rational number, then x^2 is considered a perfect square.

Ask, "What is $(\sqrt{9})^2$? $(\sqrt{5})^2$? 9, 5

Formalize vocabulary:

- ***perfect square***
- ***square root***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does a square root of a number represent?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *square root* and *perfect square* that were added to the display during the lesson. Add the following statement to the display and ask students to complete it, to help them distinguish between the terms *square root* and *perfect square*.

"The _____ of a _____ is always an integer." square root; perfect square

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining solutions to x and x^2 given one of those values.

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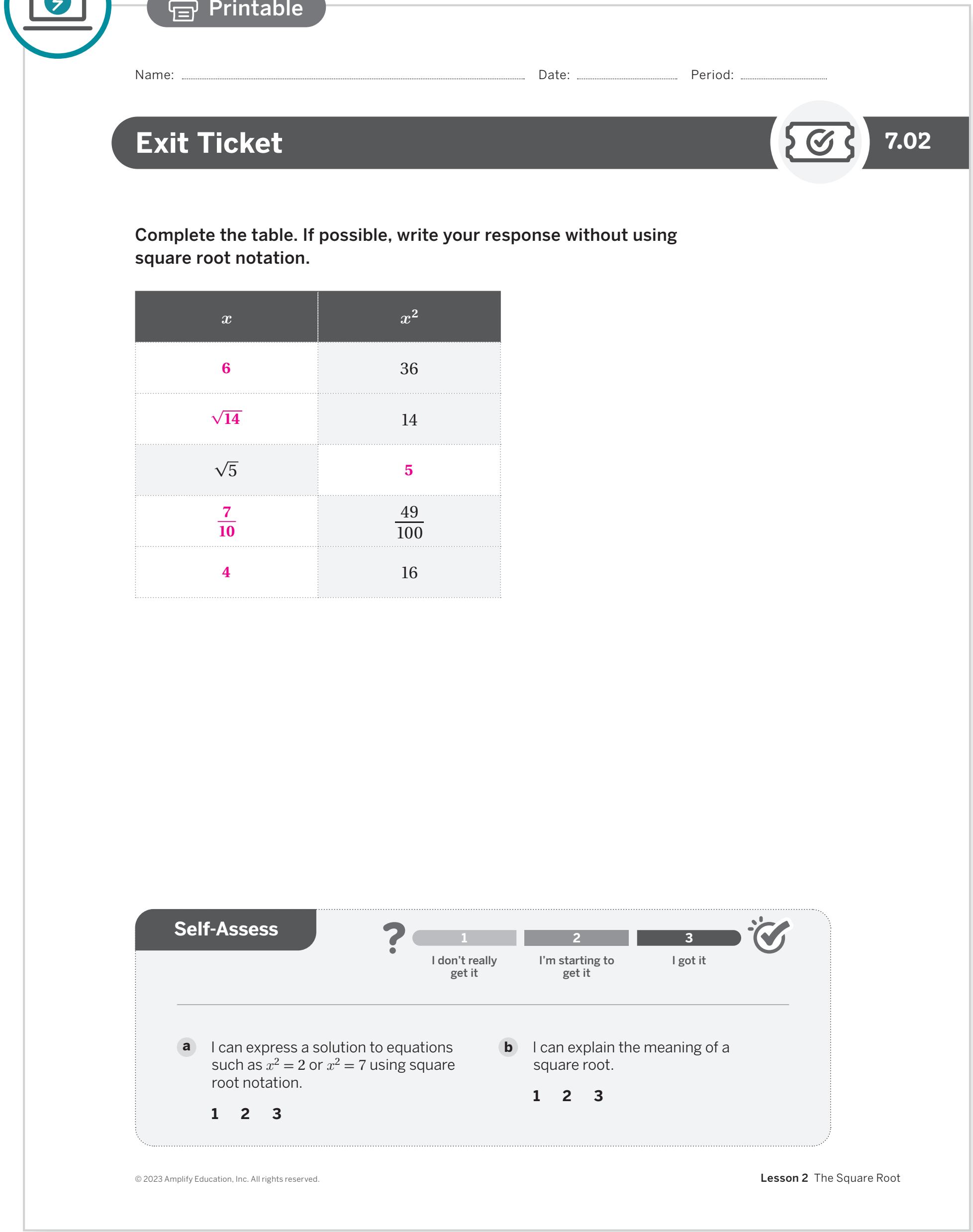

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Exit Ticket 7.02

Complete the table. If possible, write your response without using square root notation.

x	x^2
6	36
$\sqrt{14}$	14
$\sqrt{5}$	5
$\frac{7}{10}$	$\frac{49}{100}$
4	16

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can express a solution to equations such as $x^2 = 2$ or $x^2 = 7$ using square root notation.

1 2 3

b I can explain the meaning of a square root.

1 2 3

Lesson 2 The Square Root

Success looks like . . .

- **Language Goal:** Comprehending the term "square root of p" and the notation $\sqrt{p}$ to mean a solution to $x^2 = p$, where p is a positive rational number. **(Speaking and Listening)**
  - » Understanding how to determine the square root of $36, 14, \frac{49}{100},$ and 16.
- Evaluating square roots of perfect squares.
  - » Evaluating the square roots of $36, \frac{49}{100},$ and 16.

Suggested next steps

If students do not correctly complete the table, consider:

- Reviewing Activity 2.
- Assigning Practice Problems 1 and 2.

Professional Learning

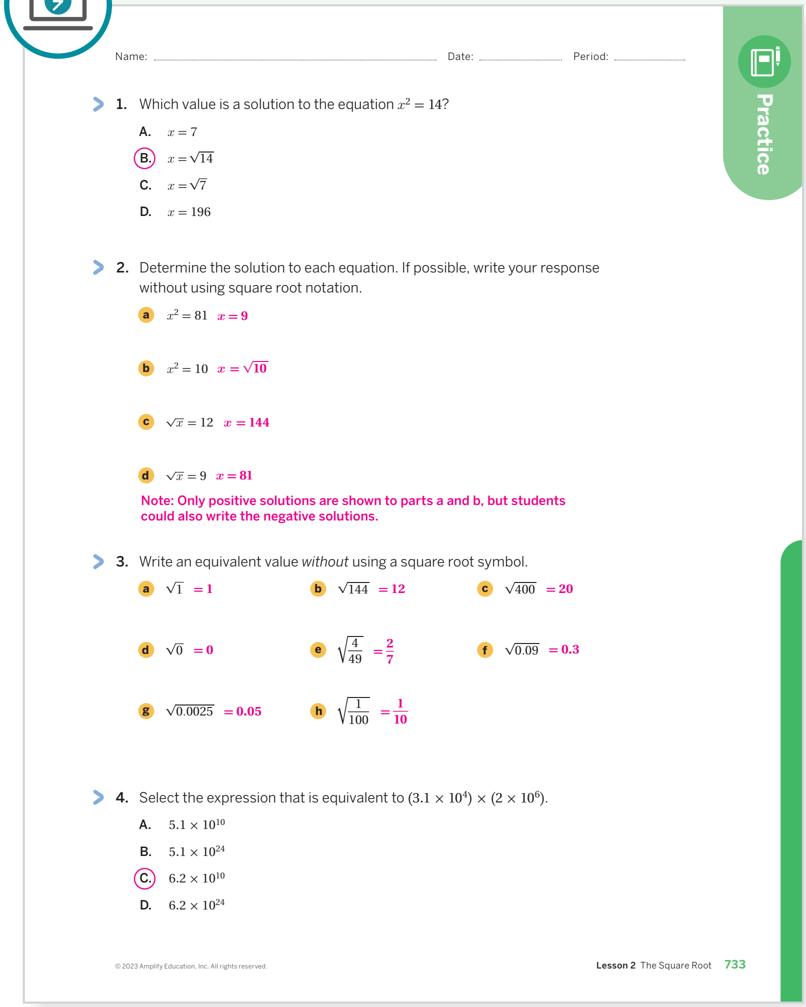
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Which students' ideas were you able to highlight during Activity 1?
- What routines enabled all students to do math in today's lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

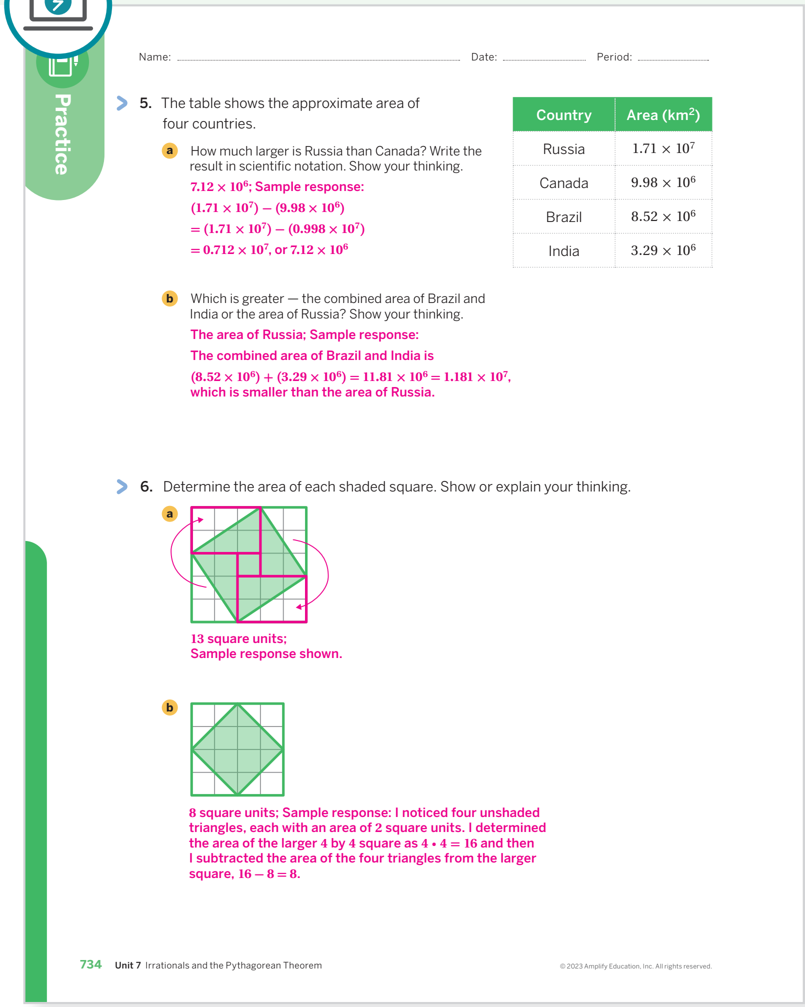
1. Which value is a solution to the equation $x^2 = 14$?
   - A. $x = 7$
   - [✓ B.] $x = \sqrt{14}$
   - C. $x = \sqrt{7}$
   - D. $x = 196$

2. Determine the solution to each equation. If possible, write your response without using square root notation.
   - a $x^2 = 81$ $x = 9$
   - b $x^2 = 10$ $x = \sqrt{10}$
   - c $\sqrt{x} = 12$ $x = 144$
   - d $\sqrt{x} = 9$ $x = 81$

   Note: Only positive solutions are shown to parts a and b, but students could also write the negative solutions.

3. Write an equivalent value *without* using a square root symbol.
   - a $\sqrt{1} = 1$
   - b $\sqrt{144} = 12$
   - c $\sqrt{400} = 20$
   - d $\sqrt{0} = 0$
   - e $\sqrt{\frac{4}{49}} = \frac{2}{7}$
   - f $\sqrt{0.09} = 0.3$
   - g $\sqrt{0.0025} = 0.05$
   - h $\sqrt{\frac{1}{100}} = \frac{1}{10}$

4. Select the expression that is equivalent to $(3.1 \times 10^4) \times (2 \times 10^6)$.
   - A. 5.1×10^{10}
   - B. 5.1×10^{24}
   - [✓ C.] 6.2×10^{10}
   - D. 6.2×10^{24}



Name: ______ Date: ______ Period: ______

Practice

5. The table shows the approximate area of four countries.

Country	Area (km²)
Russia	1.71×10^7
Canada	9.98×10^6
Brazil	8.52×10^6
India	3.29×10^6

   a How much larger is Russia than Canada? Write the result in scientific notation. Show your thinking.

   7.12×10^6; Sample response:

   $(1.71 \times 10^7) - (9.98 \times 10^6)$

   $= (1.71 \times 10^7) - (0.998 \times 10^7)$

   $= 0.712 \times 10^7$, **or** 7.12×10^6

   b Which is greater — the combined area of Brazil and India or the area of Russia? Show your thinking.

   The area of Russia; Sample response:

   The combined area of Brazil and India is $(8.52 \times 10^6) + (3.29 \times 10^6) = 11.81 \times 10^6 = 1.181 \times 10^7$, which is smaller than the area of Russia.

6. Determine the area of each shaded square. Show or explain your thinking.

   a **13 square units; Sample response shown.**

   b **8 square units; Sample response: I noticed four unshaded triangles, each with an area of 2 square units. I determined the area of the larger 4 by 4 square as $4 \cdot 4 = 16$ and then I subtracted the area of the four triangles from the larger square, $16 - 8 = 8$.**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	1
Spiral	4	Unit 6 Lesson 14	2
	5	Unit 6 Lesson 14	2
Formative ⚡	6	Unit 7 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 3**

The Areas of Squares and Their Side Lengths

Let's investigate the squares and their side lengths.

Focus

Goals

1. **Language Goal:** Determine the exact side length of a square and express it using square root notation. **(Writing)**
2. Use the area and side length of different squares to compare square roots.
3. **Language Goal:** Determine the whole numbers that a square root lies between, and explain the reasoning. **(Speaking and Listening)**

Coherence

Today

Students calculate the area of squares on a grid and explore the relationship between the side length of a square given its area. They represent the side length of a square using square root notation and use the area and side lengths to compare square roots.

Previously

In Lesson 2, students were introduced to the term *square root* and used square root notation to represent a solution to $x^2 = p$.

Coming Soon

In Lesson 4, students will use a number line to order and estimate the values of the square roots.

Rigor

- Students build **conceptual understanding** of square roots as they explore side lengths and areas of squares.
- Students **apply** square roots to determine the exact side length of a square.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	20 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per student
- Anchor Chart PDF, *Perfect Squares*

Math Language Development

Review words

- *area*
- *perfect square*
- *square root*

Amps | Featured Activity

Activity 1
Formative Feedback

Instead of just being told whether they are right or wrong, students view the immediate consequences of their response and correct any errors on their own.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not think that they have enough background knowledge to be able to compare square roots. In order to help them be a bit more optimistic, explain that the quantitative reasoning involved with comparing square roots is similar to comparing whole numbers. As a class, have students describe how comparing square roots is like comparing whole numbers and how it is different.

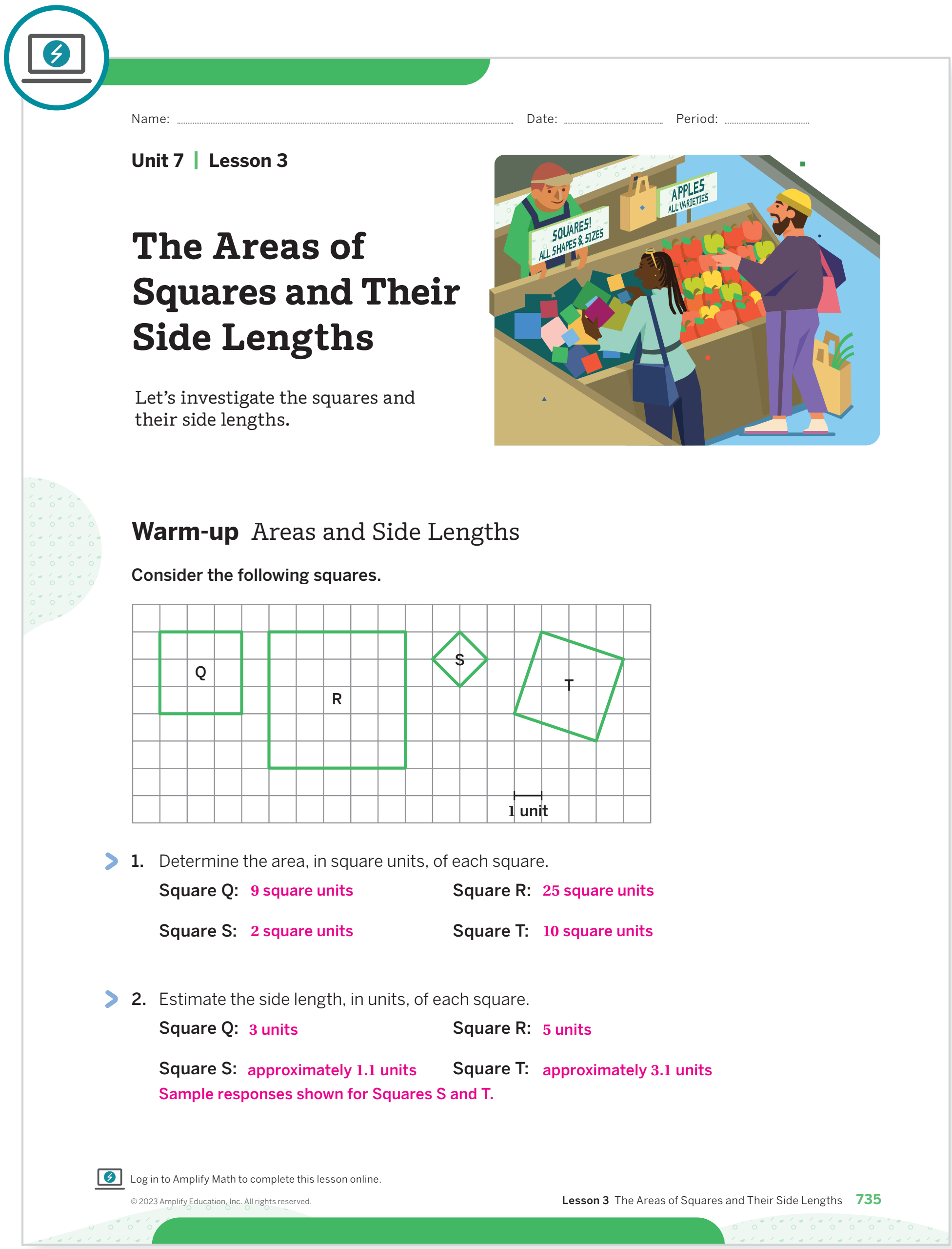
Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Square R and Square S may be omitted.
- In **Activity 1**, provide students with the area of each square and have them complete Problems 2–4.

Warm-up Areas and Side Lengths

Students approximate the area and side length of a square to review how to calculate the area of a square and express the exact side length of a square using square root notation.



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Unit 7 | Lesson 3

The Areas of Squares and Their Side Lengths

Let's investigate the squares and their side lengths.

Warm-up Areas and Side Lengths

Consider the following squares.

1. Determine the area, in square units, of each square.

   Square Q: 9 square units **Square R:** 25 square units

   Square S: 2 square units **Square T:** 10 square units

2. Estimate the side length, in units, of each square.

   Square Q: 3 units **Square R:** 5 units

   Square S: approximately 1.1 units **Square T:** approximately 3.1 units

   Sample responses shown for Squares S and T.

Log in to Amplify Math to complete this lesson online.

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Lesson 3 The Areas of Squares and Their Side Lengths 735

1 Launch

Activate students' prior knowledge by asking them what strategies they could use to determine the area of each square. Use a formula or count each grid square.

2 Monitor

Help students get started by having them count the number of grid squares for each given square.

Look for points of confusion:

- **Struggling to determine the area of Square S or Square T.** Consider assigning the Power-up.
- **Estimating a side length that is not close for Square S or Square T.** At this time, allow for different responses. Revisit these students during the Connect and Activity 1 to ensure they have a better understanding.

Look for productive strategies:

- Using the equation $x^2 = p$ to determine the side length of a square where p represents the area of the square and x represents the side length.

3 Connect

Have students share their strategies for determining the area and side length of each square. Select students that used different strategies to determine the area of each square.

Highlight that they can represent the exact side length of each square using square root notation. Say, "The side length of a square is equal to the square root of its area." Label the side lengths:

Square Q: $\sqrt{9} = 3$, Square R: $\sqrt{25} = 5$,
Square S: $\sqrt{2}$, Square T: $\sqrt{10}$.

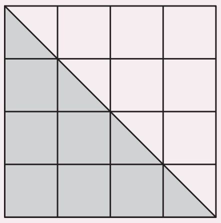
Ask, "Do you think the side length of Square T is less than or greater than the side length of Square Q? Explain your thinking." Because Square T has a greater area than Square Q, the side length for Square T is also greater.

Power-up

To power up students' ability to determine the area of square on a grid that is rotated 45°, have students complete:

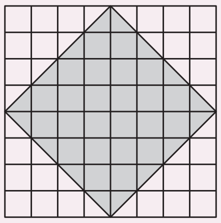
Determine the area of the shaded region. Be prepared to explain your thinking.

a.



8 square units

b.



32 square units

Use: Before the Warm-up

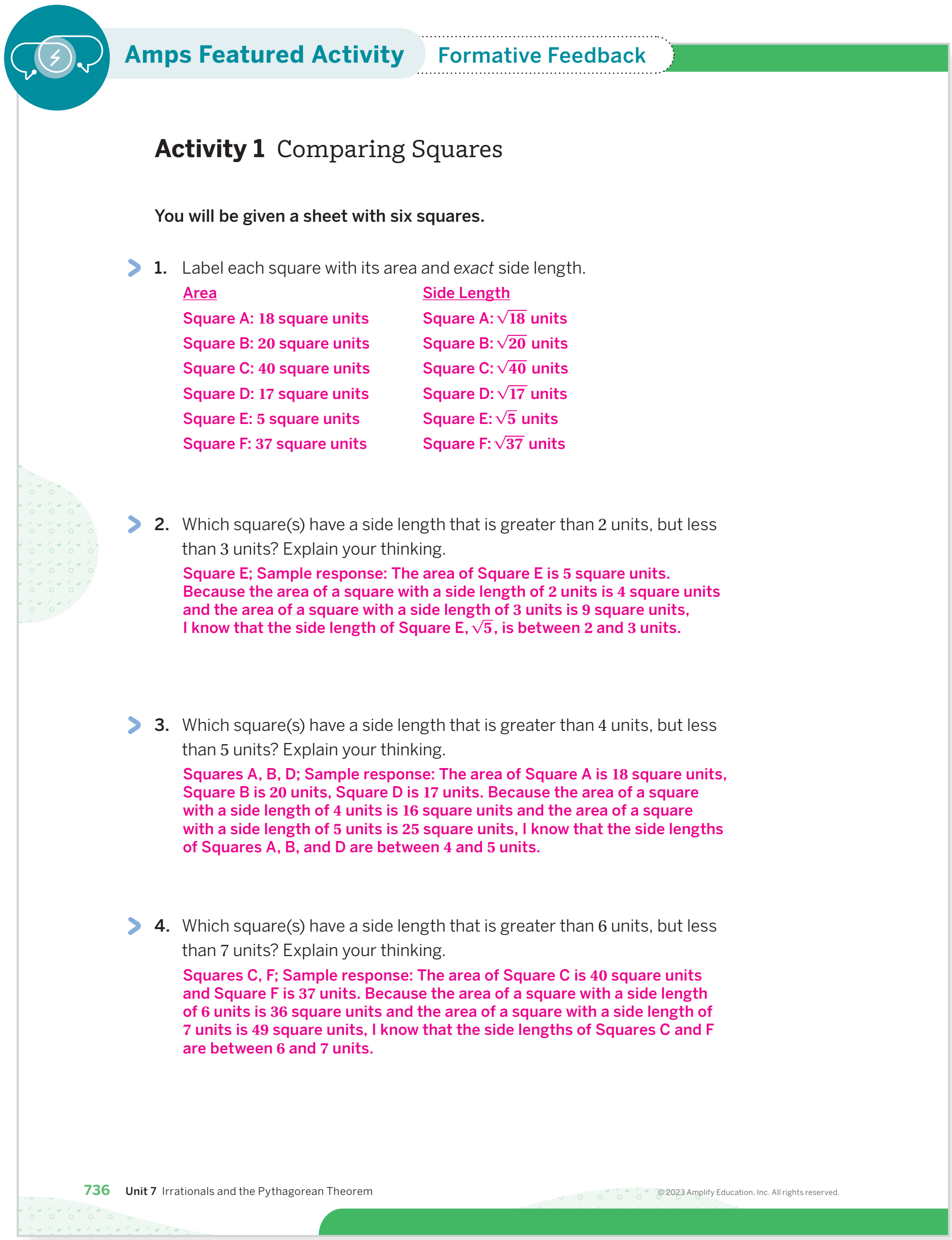
Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6

Pairs | 20 min

Activity 1 Comparing Squares

Students determine the area and side length of different squares to compare the square roots of numbers.

Amps Featured Activity Formative Feedback



Activity 1 Comparing Squares

You will be given a sheet with six squares.

1. Label each square with its area and *exact* side length.

Area	Side Length
Square A: 18 square units	Square A: $\sqrt{18}$ units
Square B: 20 square units	Square B: $\sqrt{20}$ units
Square C: 40 square units	Square C: $\sqrt{40}$ units
Square D: 17 square units	Square D: $\sqrt{17}$ units
Square E: 5 square units	Square E: $\sqrt{5}$ units
Square F: 37 square units	Square F: $\sqrt{37}$ units

2. Which square(s) have a side length that is greater than 2 units, but less than 3 units? Explain your thinking.
Square E; Sample response: The area of Square E is 5 square units. Because the area of a square with a side length of 2 units is 4 square units and the area of a square with a side length of 3 units is 9 square units, I know that the side length of Square E, $\sqrt{5}$, is between 2 and 3 units.

3. Which square(s) have a side length that is greater than 4 units, but less than 5 units? Explain your thinking.
Squares A, B, D; Sample response: The area of Square A is 18 square units, Square B is 20 units, Square D is 17 units. Because the area of a square with a side length of 4 units is 16 square units and the area of a square with a side length of 5 units is 25 square units, I know that the side lengths of Squares A, B, and D are between 4 and 5 units.

4. Which square(s) have a side length that is greater than 6 units, but less than 7 units? Explain your thinking.
Squares C, F; Sample response: The area of Square C is 40 square units and Square F is 37 units. Because the area of a square with a side length of 6 units is 36 square units and the area of a square with a side length of 7 units is 49 square units, I know that the side lengths of Squares C and F are between 6 and 7 units.

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1 Launch

Distribute the Activity 1 PDF to each student. Give students 10 minutes and have students determine the area of as many squares as time allows. After 10 minutes, record and display the actual area of all of the squares and have students adjust or add to their responses as needed. Have students complete the remainder of the activity in pairs.

2 Monitor

Help students get started by having them "decompose and arrange" or "surround and subtract" to determine the area of each square.

Look for points of confusion:

- **Forgetting how to determine the exact side length of a square.** Write and display *side length of square* $= \sqrt{area}$ for all to see.
- **Struggling to complete Problems 2–4.** Remind students that they can compare the areas of squares, including squares that represent perfect squares that are not shown, to help them determine their responses.

3 Connect

Have pairs of students share their responses and strategies for Problems 2–4.

Highlight that, if the area of a square is greater than another square, then the side length of the square with the greater area will be greater than the side length of the other square.

Ask, "Which is greater, $-\sqrt{5}$ or $\sqrt{12}$? $\sqrt{12}$

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view the immediate consequences of their response and correct any errors on their own.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, assign pairs to examine two different squares. After each pair has determined the areas and side lengths for their two assigned squares, combine the class results and display them for all six squares. Have pairs continue with the rest of the activity.

Math Language Development

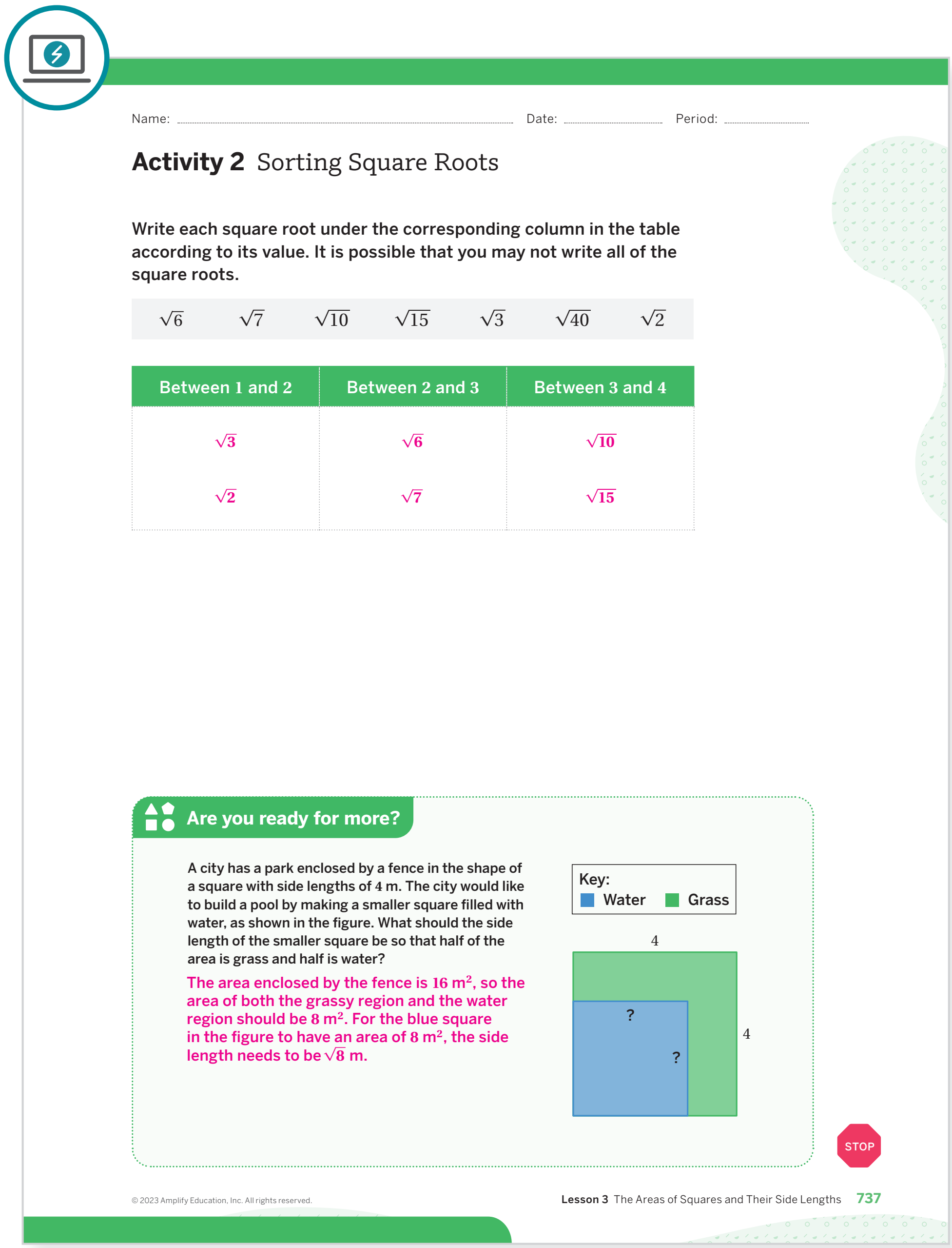
MLR8: Discussion Supports

After a student shares their strategies for Problems 2-4, consider asking students to list perfect squares that are less than 50 and annotate them with their corresponding square roots. Then ask them to determine where each of the side lengths would be placed between these perfect squares. For example, the side length for Square A would be placed between 4 and 5 because 18 is greater than 16 (the square of 4) and less than 25 (the square of 5). Encourage the use of precise mathematical language by asking students to use the terms *square*, *perfect square*, *side length*, or *area*.

Pairs | 10 min

Activity 2 Sorting Square Roots

Students compare the value of square roots, without squares on grids, to deepen their understanding of square root values.



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Activity 2 Sorting Square Roots

Write each square root under the corresponding column in the table according to its value. It is possible that you may not write all of the square roots.

$\sqrt{6}$ $\sqrt{7}$ $\sqrt{10}$ $\sqrt{15}$ $\sqrt{3}$ $\sqrt{40}$ $\sqrt{2}$

Between 1 and 2	Between 2 and 3	Between 3 and 4
$\sqrt{3}$	$\sqrt{6}$	$\sqrt{10}$
$\sqrt{2}$	$\sqrt{7}$	$\sqrt{15}$

Are you ready for more?

A city has a park enclosed by a fence in the shape of a square with side lengths of 4 m. The city would like to build a pool by making a smaller square filled with water, as shown in the figure. What should the side length of the smaller square be so that half of the area is grass and half is water?

The area enclosed by the fence is 16 m^2, so the area of both the grassy region and the water region should be 8 m^2. For the blue square in the figure to have an area of 8 m^2, the side length needs to be $\sqrt{8}$ m.

1 Launch

Tell students that they will categorize the value of each square root without the use of a grid. **Note:** Students will further explore how to estimate the values of square roots in Lesson 4.

2 Monitor

Help students get started by asking them what strategies they could use to categorize each square root.

Look for points of confusion:

- **Having trouble categorizing each square root.** Consider relating each square root to the side length of a square. For the header "Between 1 and 2" have students identify the area of a square that has a side length of 1 and the area of a square that has a side length of 2. Have students make annotations to help with the problem. Additionally, consider having students annotate "Between 1 and 2" as "Between $\sqrt{1}$ and $\sqrt{4}$."

3 Connect

Display student work showing the completed table.

Have pairs of students share their strategies for categorizing the square roots. Select different students that highlight different strategies.

Highlight that although students may not know the value of a square root, they can determine which two whole consecutive numbers it lies between.

Ask students to provide a square root that has a value between 5 and 6 to check for understanding. A square root of any number between 25 and 36.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the Anchor Chart PDF, *Perfect Squares*, to use as a reference. Consider also using a think-aloud to demonstrate how to reason about the location of one of the numbers, such as $\sqrt{6}$. Consider saying the following during the think-aloud.

- "The $\sqrt{6}$ is greater than 2 because $2^2 = 4$ and $6 > 4$."
- "The $\sqrt{6}$ is less than 3 because $3^2 = 9$ and $6 < 9$."

Math Language Development

MLR3: Critique, Correct, Clarify

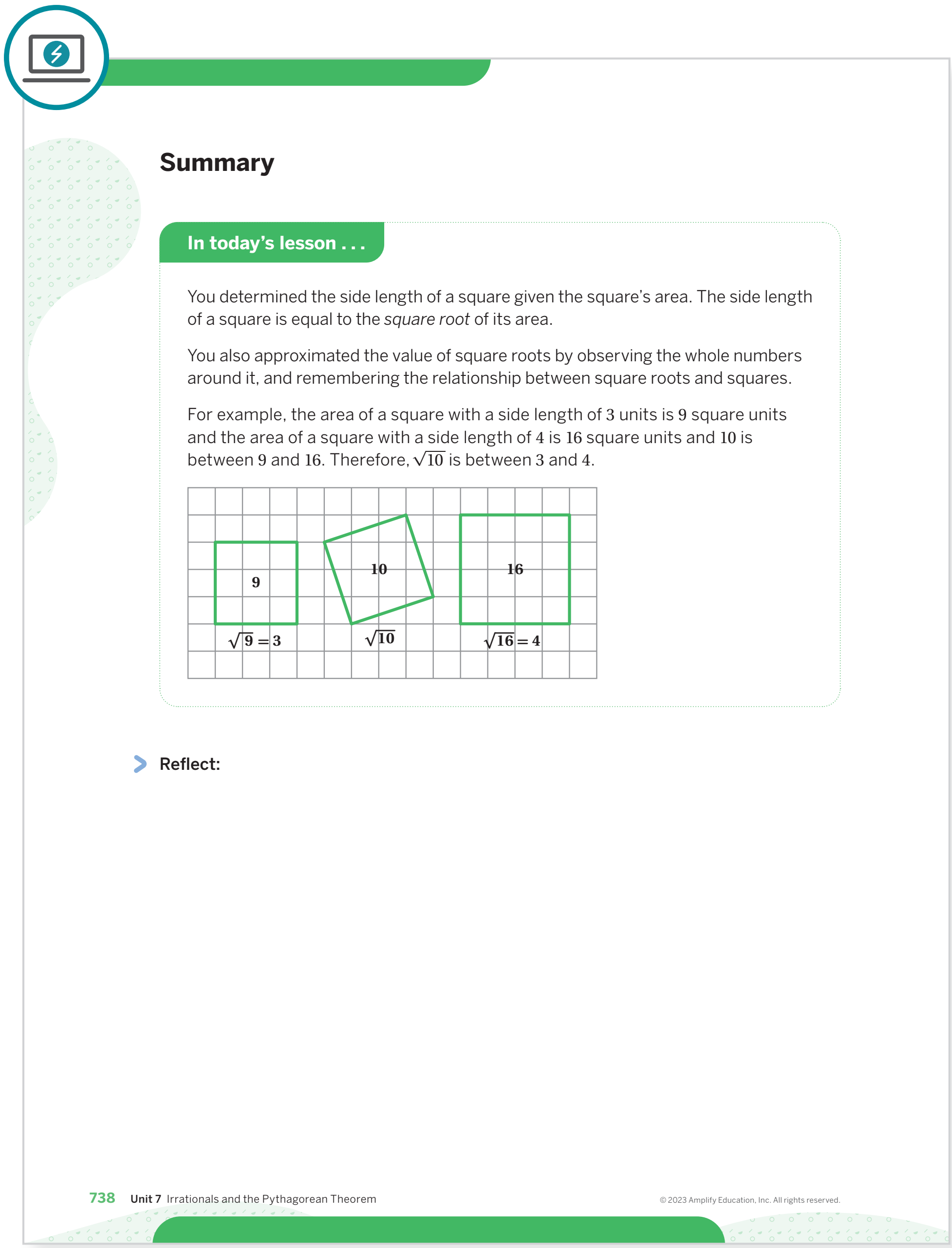
During the Connect, before students share their strategies, present an incorrect solution based on a common misunderstanding. For example, "$\sqrt{7}$ is between 3 and 4, because $3^2 = 6$, and $4^2 = 16$." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

Whole Class | 5 min

Summary

Review and synthesize how to use square root notation to represent the side length of a square given its area and how to compare the value of square roots.



Summary

In today's lesson . . .

You determined the side length of a square given the square's area. The side length of a square is equal to the *square root* of its area.

You also approximated the value of square roots by observing the whole numbers around it, and remembering the relationship between square roots and squares.

For example, the area of a square with a side length of 3 units is 9 square units and the area of a square with a side length of 4 is 16 square units and 10 is between 9 and 16. Therefore, $\sqrt{10}$ is between 3 and 4.

Reflect:

Synthesize

Ask, "How can you determine the exact side length of a square given the square's area?"

Have students share their strategies for determining which two whole consecutive numbers a square root is between.

Highlight that students can determine the exact side length of a square by expressing it as the square root of the square's area. Also highlight that students can use the areas and side lengths of squares to compare the values of square roots.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

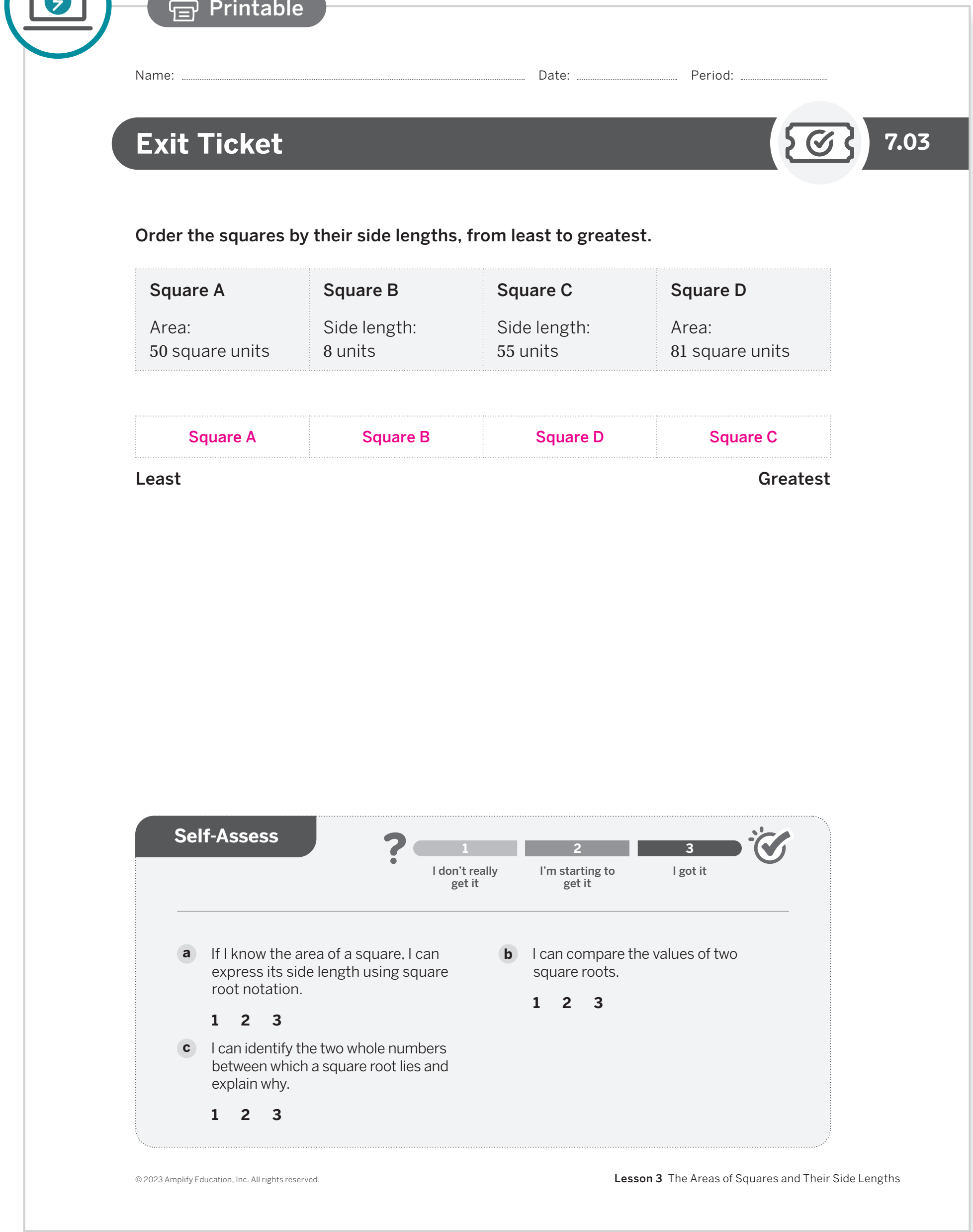
- "What does a square root of a number represent?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by ordering squares by length size given a square's area or side length.

Printable



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Exit Ticket 7.03

Order the squares by their side lengths, from least to greatest.

Square A	Square B	Square C	Square D
Area: 50 square units	Side length: 8 units	Side length: 55 units	Area: 81 square units

Square A	Square B	Square D	Square C

Least **Greatest**

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a If I know the area of a square, I can express its side length using square root notation.

1 2 3

b I can compare the values of two square roots.

1 2 3

c I can identify the two whole numbers between which a square root lies and explain why.

1 2 3

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Lesson 3 The Areas of Squares and Their Side Lengths

Success looks like . . .

- **Language Goal:** Determining the exact side length of a square and expressing it using square root notation. **(Writing)**
  - » Determining the side lengths of Squares A and D.
- **Goal:** Using the area and side length of different squares to compare square roots.
  - » Determining the side lengths of Squares A and D by determining the square root of the area and then comparing the side lengths.
- **Language Goal:** Determining the whole numbers that a square root lies between, and explaining the reasoning. **(Speaking and Listening)**

Suggested next steps

If students incorrectly order the squares, consider:

- Reviewing Activity 1.
- Having students determine the area and side length for each square before ordering the squares by length.

Professional Learning

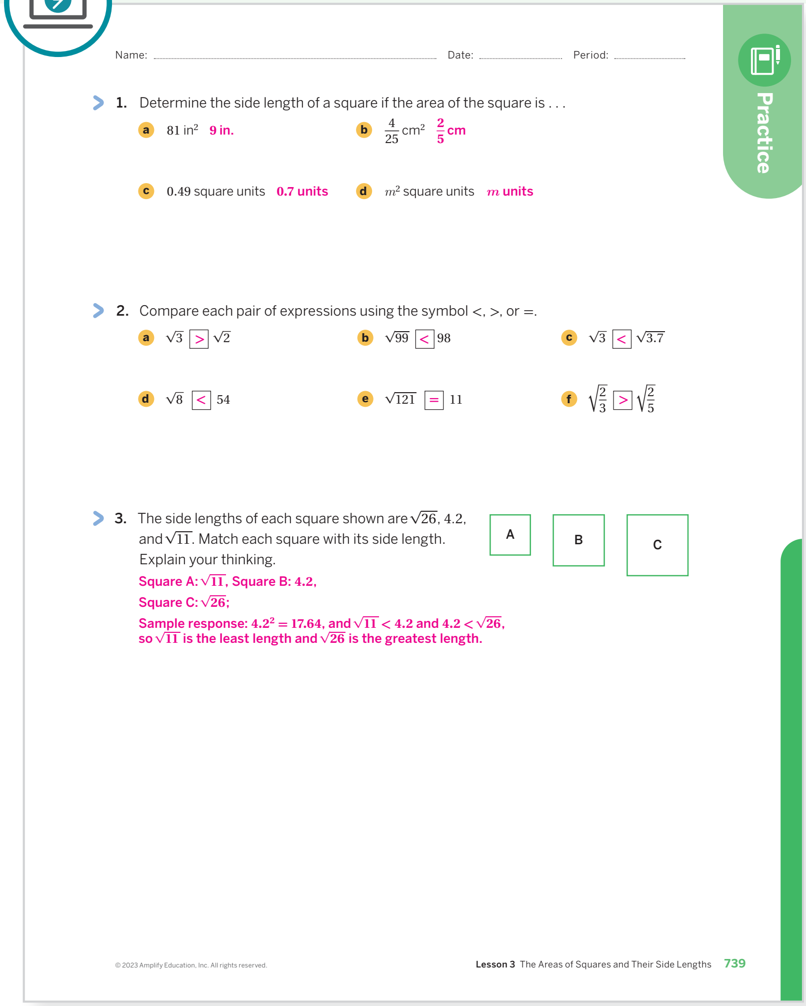
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What did students find frustrating about Activity 1? What helped them work through this frustration?
- During the discussion in Activity 2, how did you encourage each student to listen to one another's strategies?

Practice

Independent



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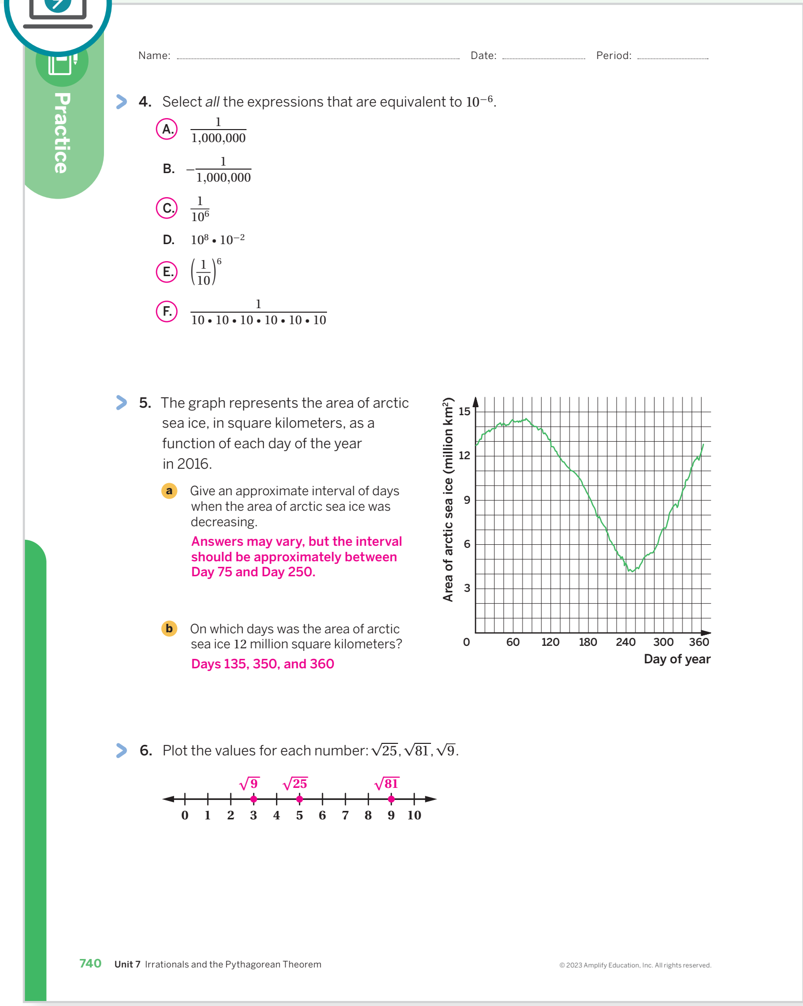
1. Determine the side length of a square if the area of the square is . . .
   - a 81 in^2 **9 in.**
   - b $\frac{4}{25}\text{ cm}^2$ **$\frac{2}{5}$ cm**
   - c 0.49 square units **0.7 units**
   - d m^2 square units **m units**

2. Compare each pair of expressions using the symbol <, >, or =.
   - a $\sqrt{3}\ \boxed{>}\ \sqrt{2}$
   - b $\sqrt{99}\ \boxed{<}\ 98$
   - c $\sqrt{3}\ \boxed{<}\ \sqrt{3.7}$
   - d $\sqrt{8}\ \boxed{<}\ 54$
   - e $\sqrt{121}\ \boxed{=}\ 11$
   - f $\sqrt{\frac{2}{3}}\ \boxed{>}\ \sqrt{\frac{2}{5}}$

3. The side lengths of each square shown are $\sqrt{26}$, 4.2, and $\sqrt{11}$. Match each square with its side length. Explain your thinking.

   Square A: $\sqrt{11}$, Square B: 4.2,
   Square C: $\sqrt{26}$;
   Sample response: $4.2^2 = 17.64$, and $\sqrt{11} < 4.2$ and $4.2 < \sqrt{26}$, so $\sqrt{11}$ is the least length and $\sqrt{26}$ is the greatest length.



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4. Select *all* the expressions that are equivalent to 10^{-6}.
   - [✓ A.] $\frac{1}{1{,}000{,}000}$
   - B. $-\frac{1}{1{,}000{,}000}$
   - [✓ C.] $\frac{1}{10^6}$
   - D. $10^8 \cdot 10^{-2}$
   - [✓ E.] $\left(\frac{1}{10}\right)^6$
   - [✓ F.] $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

5. The graph represents the area of arctic sea ice, in square kilometers, as a function of each day of the year in 2016.
   - a Give an approximate interval of days when the area of arctic sea ice was decreasing.

     Answers may vary, but the interval should be approximately between Day 75 and Day 250.
   - b On which days was the area of arctic sea ice 12 million square kilometers?

     Days 135, 350, and 360

6. Plot the values for each number: $\sqrt{25}$, $\sqrt{81}$, $\sqrt{9}$.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 5	2
	5	Unit 5 Lesson 5	2
Formative ⚡	6	Unit 7 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

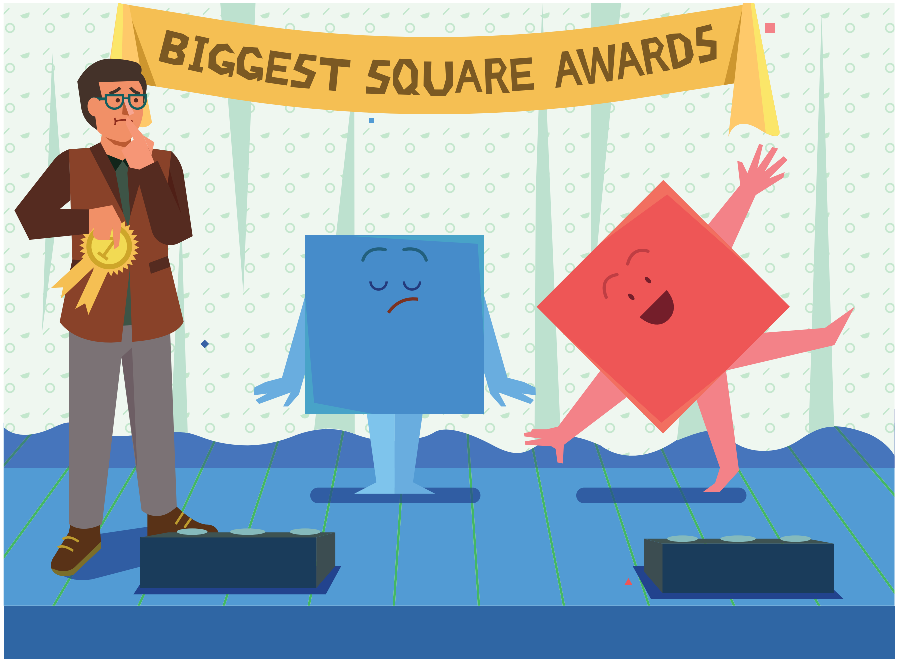
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 4**

Estimating Square Roots

Let's approximate square roots.



Focus

Goals

1. **Language Goal:** Approximate the value of a square root to the nearest tenth and explain the reasoning used. **(Speaking and Listening)**
2. **Language Goal:** Represent a square root as a point on a number line and describe how the point was placed. **(Speaking and Listening)**

Coherence

Today

Students are encouraged to reason about square roots and to reinforce the idea that square roots are numbers on a number line. This lesson continues to connect algebraic and geometric characterizations of square roots as students determine how to make a more precise estimate for square roots.

Previously

In Lesson 3, students represented the side length of a square using square root notation and used the area and side lengths to compare square roots.

Coming Soon

In Lesson 5, students will recognize and use cube root notation to represent the edge length of a cube given its volume.

Rigor

- Students build **procedural fluency** estimating square roots.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 2 PDF, pre-cut cards, one card per group
- Activity 2 PDF (answers)
- calculators
- sticky notes

Math Language Development

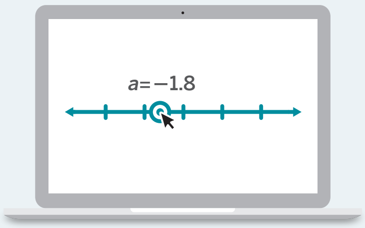
Review word

- *square root*

Amps | **Featured Activity**

Activity 1
Collaborative Number Lines

Students can collaborate with other students to use interactive number lines to order a set of numbers.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not be motivated to find a more precise value for a square root because they think approximating it with a whole number is good enough. Ask partners to help provide the motivation to stay on task and be precise with the placement of the square root on a number line. Explain that they are going to provide positive peer pressure so that both partners can be successful.

Modifications to Pacing

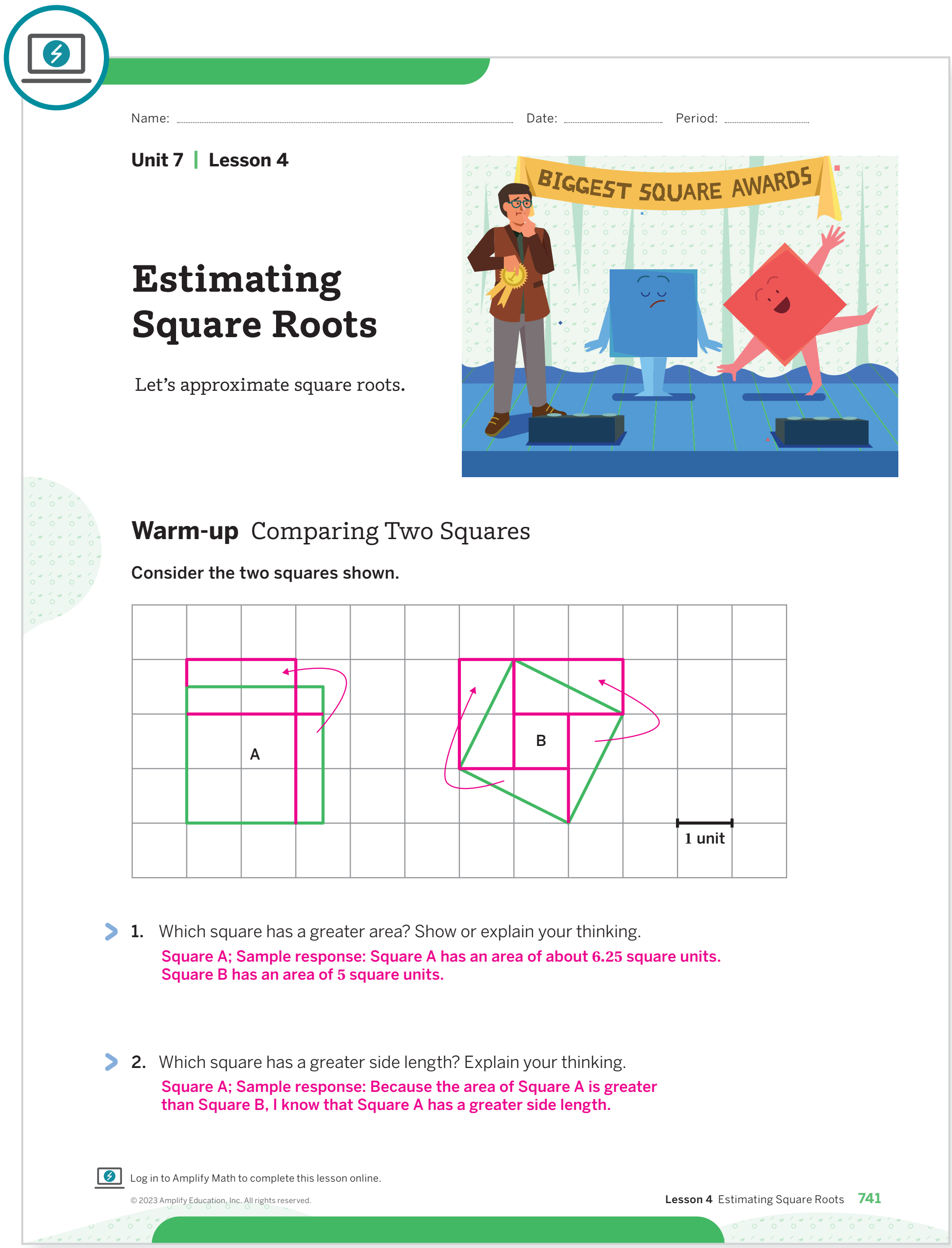
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, limit the amount of numbers students plot.

Pairs | 8 min

Warm-up Comparing Two Squares

Students compare the area and side lengths of two squares to see that $\sqrt{5}$ is greater than 2, but less than 2.5.



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Unit 7 | Lesson 4

Estimating Square Roots

Let's approximate square roots.

Warm-up Comparing Two Squares

Consider the two squares shown.

1. Which square has a greater area? Show or explain your thinking.
   Square A; Sample response: Square A has an area of about 6.25 square units. Square B has an area of 5 square units.

2. Which square has a greater side length? Explain your thinking.
   Square A; Sample response: Because the area of Square A is greater than Square B, I know that Square A has a greater side length.

Log in to Amplify Math to complete this lesson online.

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Lesson 4 Estimating Square Roots 741

1 Launch

Complete the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by having them use any strategy to determine the area of each square.

Look for points of confusion:

- **Not determining the correct area for either square.** Encourage students to compare their responses and discuss their strategies for determining the area. Remind them about strategies such as "decompose and arrange" or "surround and subtract."

Look for productive strategies:

- Recognizing that Square A will have a greater side length because the area of Square A is larger than the area of Square B.

3 Connect

Display student work showing different strategies for determining the area of each square.

Have pairs of students share which square had a greater side length.

Ask, "Do you think $\sqrt{5}$ is greater or less than 2.5?"

Highlight that the area of Square A is more than 5 (actually it is 6.25), so its side length, which is 2.5, should be greater than the side length of Square B.

Power-up

To power up students' ability to plot square roots on a number line:

Provide students with a copy of the Power-up PDF.

Use: Before Activity 1

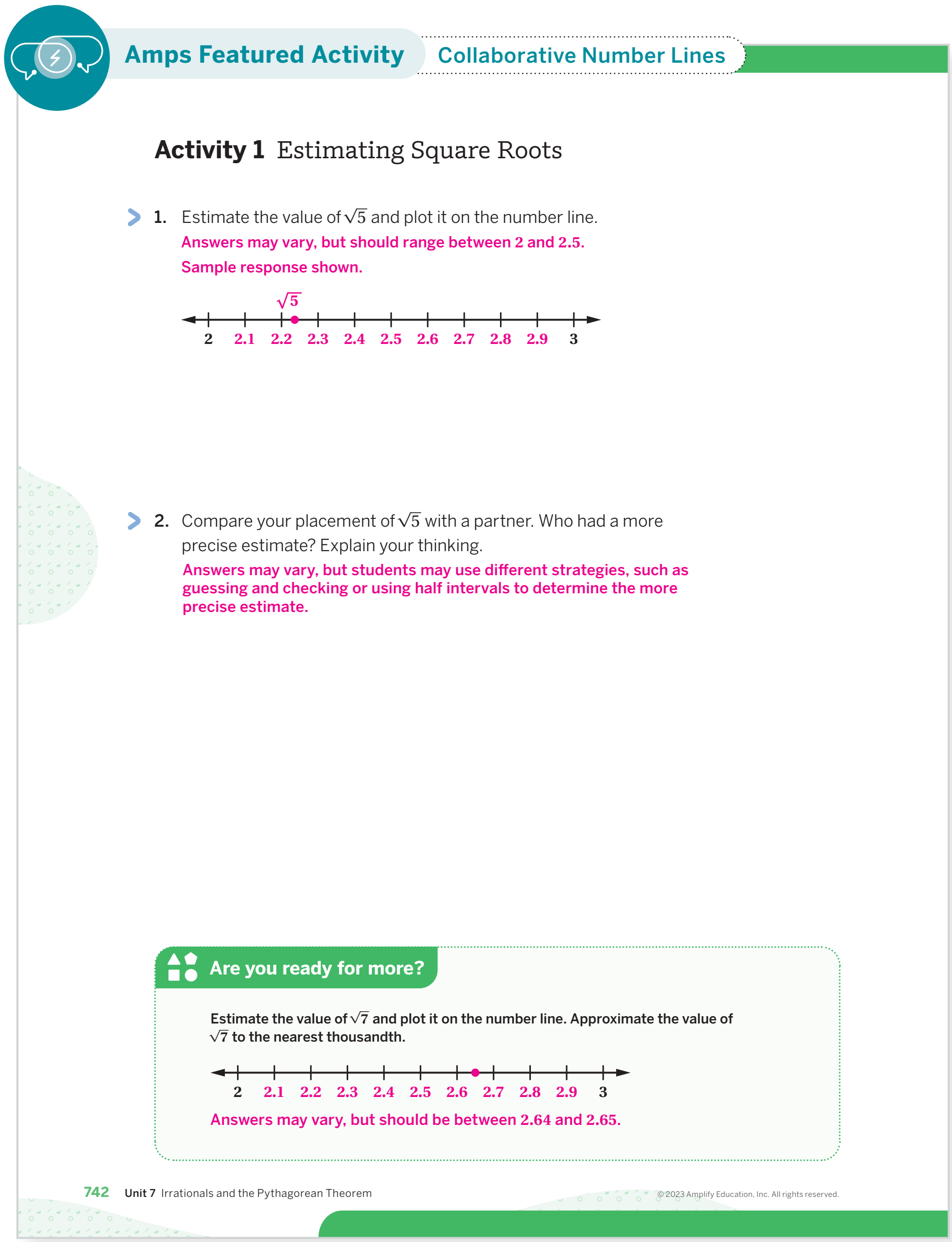
Informed by: Performance on Lesson 3, Practice Problem 6

Pairs | 10 min

Activity 1 Estimating Square Roots

Students continue investigating the value of $\sqrt{5}$ to explore strategies on how to estimate a square root to the nearest tenth.

Amps Featured Activity Collaborative Number Lines



Activity 1 Estimating Square Roots

1. Estimate the value of $\sqrt{5}$ and plot it on the number line.
   Answers may vary, but should range between 2 and 2.5.
   Sample response shown.

2. Compare your placement of $\sqrt{5}$ with a partner. Who had a more precise estimate? Explain your thinking.
   Answers may vary, but students may use different strategies, such as guessing and checking or using half intervals to determine the more precise estimate.

Are you ready for more?

Estimate the value of $\sqrt{7}$ and plot it on the number line. Approximate the value of $\sqrt{7}$ to the nearest thousandth.

Answers may vary, but should be between 2.64 and 2.65.

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1 Launch

Remind students that from the Warm-up, they determined that $\sqrt{5}$ is greater than 2, but less than 2.5. **Note:** As students test different values, allow access to calculators for students.

2 Monitor

Help students get started by having them label the tick marks on the number line.

Look for points of confusion:

- **Struggling to estimate $\sqrt{5}$.** Have students test different numbers between 2 and 2.5. Encourage them to adjust their number to find an even more precise estimate.

Look for productive strategies:

- Not plotting a point exactly on a tick mark, but instead plotting a point in between two tick marks for a more precise estimate.

3 Connect

Display student work showing their estimates on the number line. Select several students with different points.

Have students share how they can determine which point is the most precise.

Highlight that students could estimate the value of a square root by determining the two whole numbers it lies between, use the halfway point between the whole numbers as a benchmark, and then test different numbers to determine their estimate.

Ask students how they could determine an even more precise estimate for $\sqrt{5}$. Test values between 2.2 and 2.3.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can collaborate with other students to use interactive number lines to order a set of numbers.

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Provide students with a pre-labeled number line for Problem 1.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share how they can determine which point is the most precise, display these sentence frames they can use to organize their thinking.

- "_____is more precise than _____because . . ."
- "The value of _____is greater/less than _____, so I know that . . ."

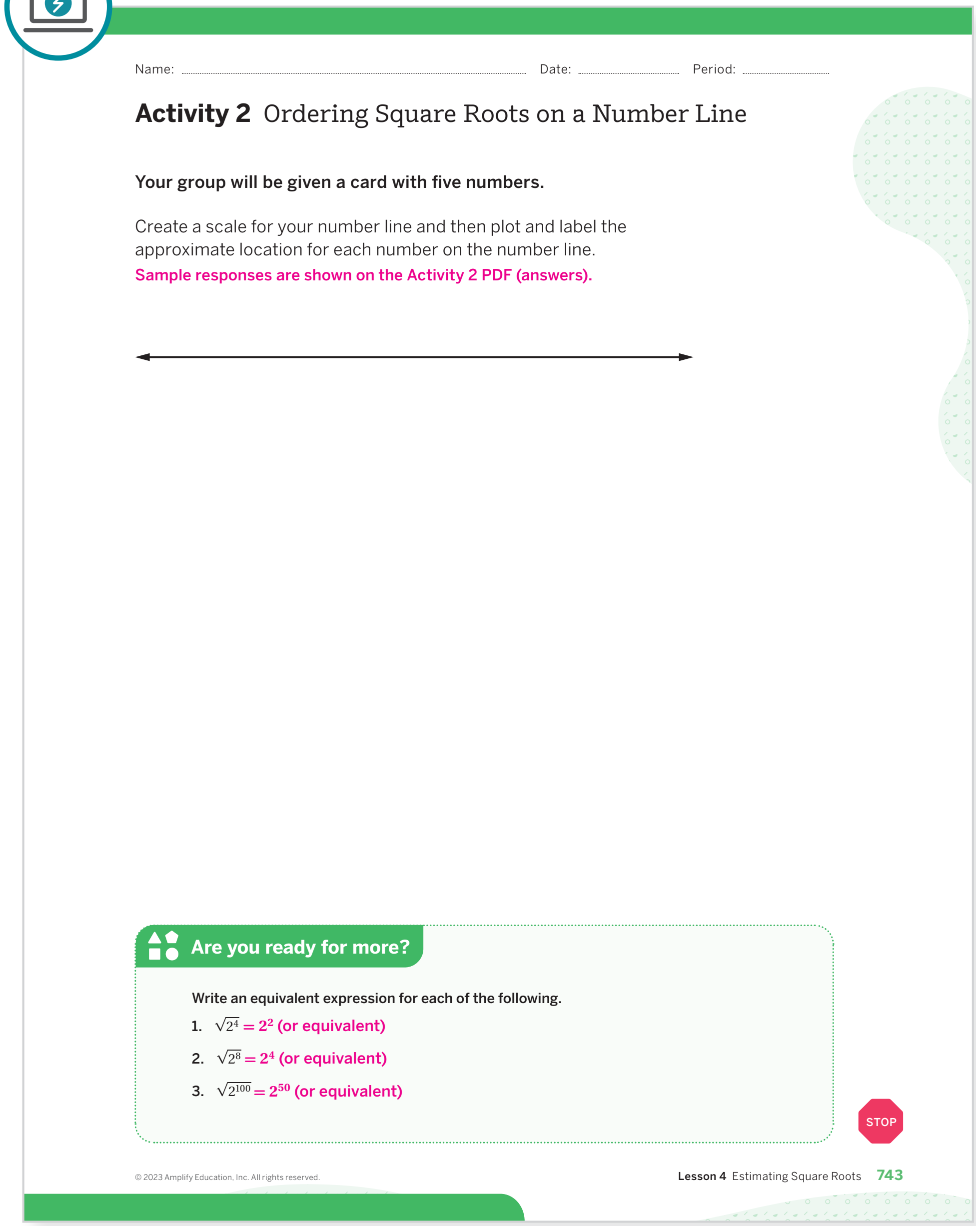
English Learners

Allow students to formulate a response with their partner before sharing with the whole class.

Small Groups | 15 min

Activity 2 Ordering Square Roots on a Number Line

Students order square roots on a number line to build procedural fluency.



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Activity 2 Ordering Square Roots on a Number Line

Your group will be given a card with five numbers.

Create a scale for your number line and then plot and label the approximate location for each number on the number line.
Sample responses are shown on the Activity 2 PDF (answers).

Are you ready for more?

Write an equivalent expression for each of the following.

1. $\sqrt{2^4} = 2^2$ (or equivalent)
2. $\sqrt{2^8} = 2^4$ (or equivalent)
3. $\sqrt{2^{100}} = 2^{50}$ (or equivalent)

STOP

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Lesson 4 Estimating Square Roots 743

1 Launch

Group students by three or four, and distribute one card from the Activity 2 PDF to each group. **Note:** Provide access to calculators during the Connect.

2 Monitor

Help students get started by having them label the number line based on the whole numbers shown on their card. Encourage students to be as precise as possible when plotting each point on the number line.

Look for points of confusion:

- **Incorrectly ordering the numbers on the number line.** Have students create tick marks on the number line and check the placement of each number using the tick marks.

Look for productive strategies:

- Creating additional tick marks to plot a precise estimate.

3 Connect

Display student work by having them create a collective number line on a wall in the classroom. Provide sticky notes to allow them to write and post their numbers.

Have groups of students share their strategies for ordering the numbers. Select students with different strategies to share with the class.

Highlight that students can use a number line to help them estimate and order square roots. At this time, provide access to calculators and tell students that a calculator could help them determine a more precise estimate. Demonstrate how students can enter $\sqrt{5}$ on their calculator.

Ask students to enter each square root shown on their card and have them compare this number to their estimate on the number line. Collect calculators before students complete the Exit Ticket.

Math Language Development

MLR7: Compare and Connect

During the Connect, have pairs of students share their strategies for ordering the numbers on the number line. Ask their classmates to identify what is the same and different about the various strategies used.

For example, for Card 1:

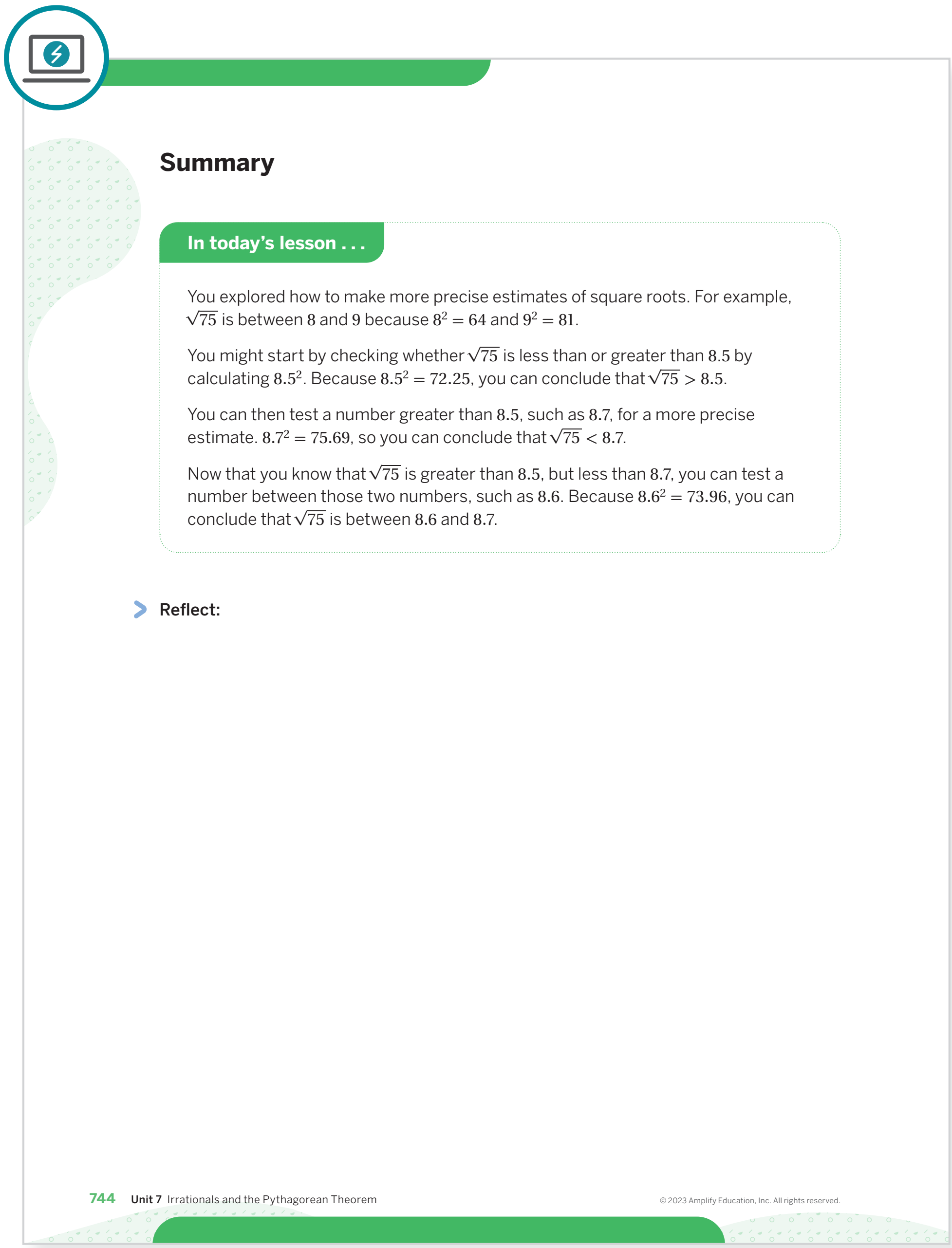
- Some students may have plotted the values 1, 2, and 1.3 first and then reasoned about the square roots before determining the rest of the scale on their number line.
- Other students may have reasoned about the square root values first to determine that $\sqrt{3}$ and $\sqrt{2}$ are each between 1 and 2 and then marked tick marks using tenths between 1 and 2.

Ask students whether they felt it was advantageous to determine the scale and tick marks for the number line after first reasoning about the square root values.

Whole Class | 5 min

Summary

Review and synthesize how to estimate square roots.



Summary

In today's lesson . . .

You explored how to make more precise estimates of square roots. For example, $\sqrt{75}$ is between 8 and 9 because $8^2 = 64$ and $9^2 = 81$.

You might start by checking whether $\sqrt{75}$ is less than or greater than 8.5 by calculating 8.5^2. Because $8.5^2 = 72.25$, you can conclude that $\sqrt{75} > 8.5$.

You can then test a number greater than 8.5, such as 8.7, for a more precise estimate. $8.7^2 = 75.69$, so you can conclude that $\sqrt{75} < 8.7$.

Now that you know that $\sqrt{75}$ is greater than 8.5, but less than 8.7, you can test a number between those two numbers, such as 8.6. Because $8.6^2 = 73.96$, you can conclude that $\sqrt{75}$ is between 8.6 and 8.7.

Reflect:

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Synthesize

Have students share their strategies on how to estimate a square root.

Highlight any strategies that students found helpful to estimate square roots.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

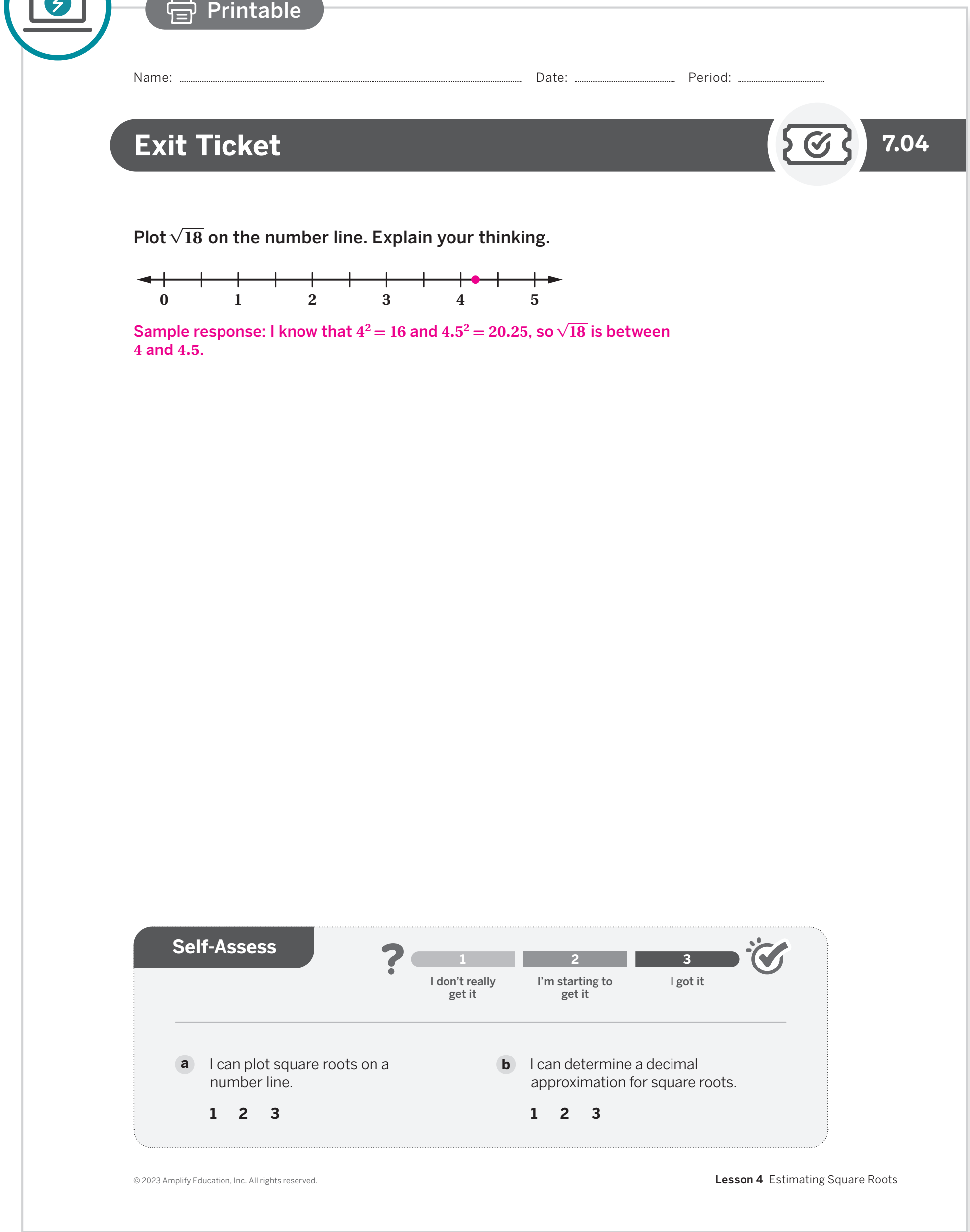
- "How can you estimate the square root of a number?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by estimating the value of a square root using a number line.

Printable



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Exit Ticket 7.04

Plot $\sqrt{18}$ on the number line. Explain your thinking.

Sample response: I know that $4^2 = 16$ and $4.5^2 = 20.25$, so $\sqrt{18}$ is between 4 and 4.5.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can plot square roots on a number line.

1 2 3

b I can determine a decimal approximation for square roots.

1 2 3

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Lesson 4 Estimating Square Roots

Success looks like . . .

- **Language Goal:** Approximating the value of a square root to the nearest tenth and explaining the reasoning used. **(Speaking and Listening)**
  - » Approximating $\sqrt{18}$ by determining 18 to be between two values and then taking the square roots of those two values.
- **Language Goal:** Representing a square root as a point on a number line and describing how the point was placed. **(Speaking and Listening)**
  - » Plotting $\sqrt{18}$ on the number line and explaining how to determine its location.

Suggested next steps

If students do not plot $\sqrt{18}$ between 4 and 5, consider:

- Reviewing perfect squares.

If students do not plot $\sqrt{18}$ between 4 and 4.5, consider:

- Reviewing Activity 1.

Professional Learning

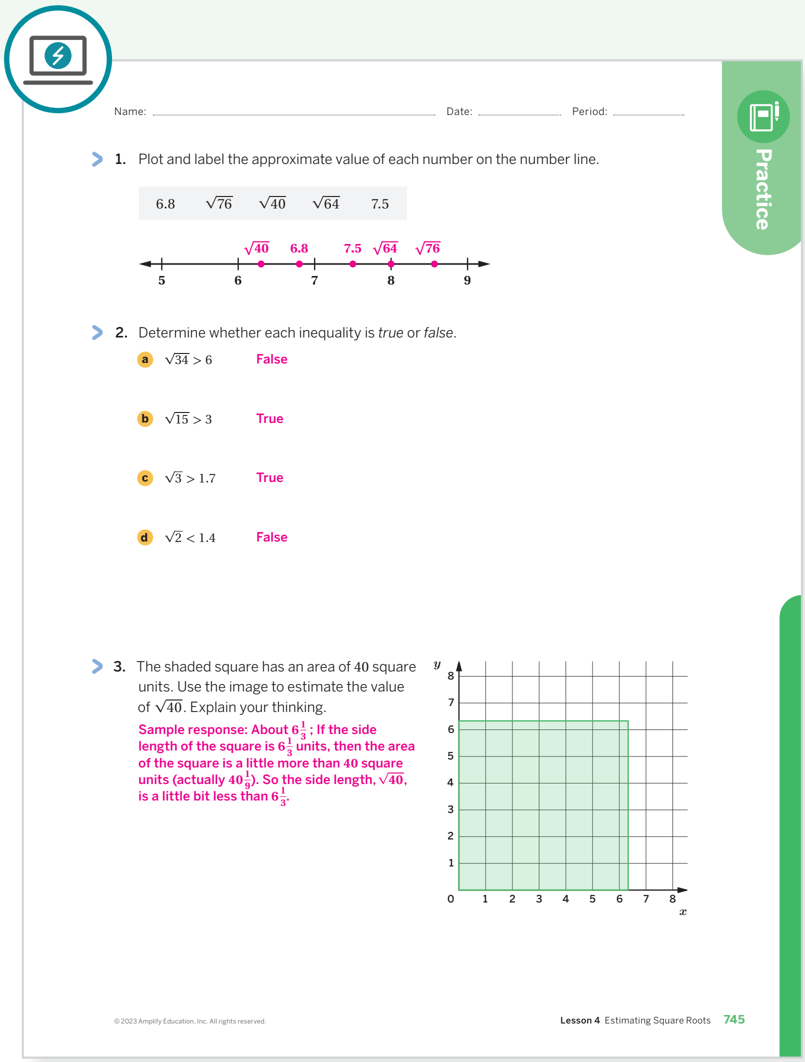
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Who participated and who didn't participate in Activity 2 today? What trends do you see in participation?
- What challenges did students encounter as they worked on Activity 2? How did they work through them?

Practice

Independent



Name: Date: Period:

Practice

1. Plot and label the approximate value of each number on the number line.

6.8 $\sqrt{76}$ $\sqrt{40}$ $\sqrt{64}$ 7.5

2. Determine whether each inequality is *true* or *false*.
   - a $\sqrt{34} > 6$ **False**
   - b $\sqrt{15} > 3$ **True**
   - c $\sqrt{3} > 1.7$ **True**
   - d $\sqrt{2} < 1.4$ **False**

3. The shaded square has an area of 40 square units. Use the image to estimate the value of $\sqrt{40}$. Explain your thinking.

Sample response: About $6\frac{1}{3}$; If the side length of the square is $6\frac{1}{3}$ units, then the area of the square is a little more than 40 square units (actually $40\frac{1}{9}$). So the side length, $\sqrt{40}$, is a little bit less than $6\frac{1}{3}$.

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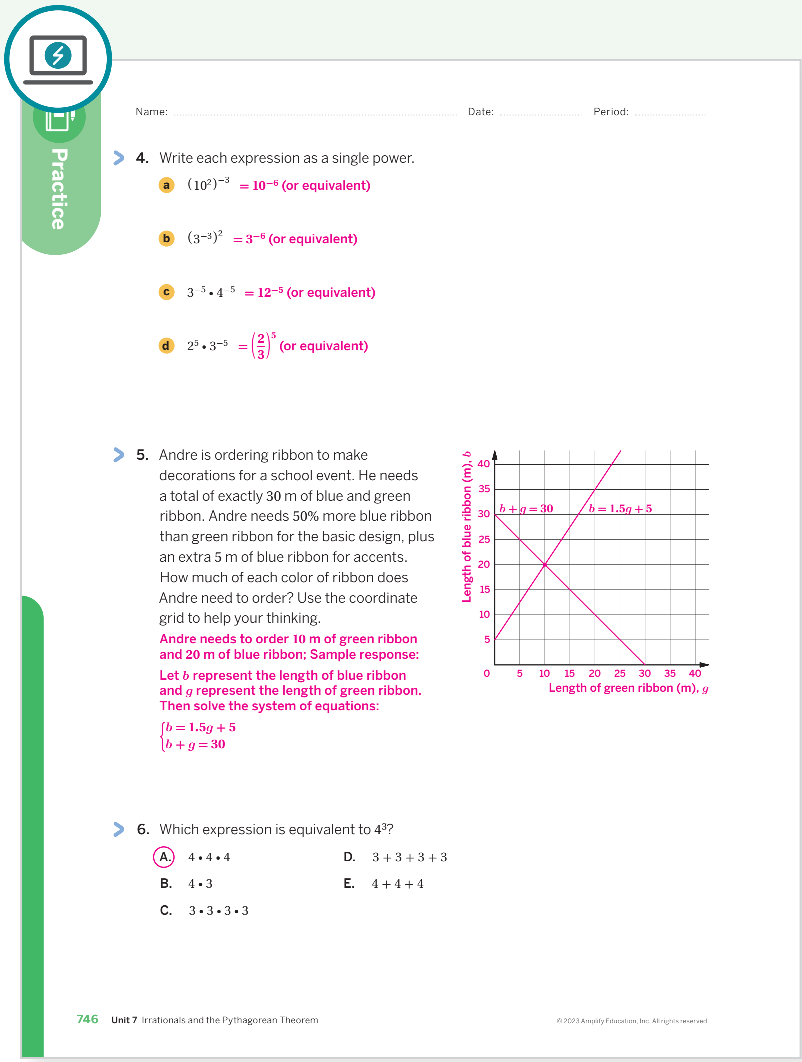
Lesson 4 Estimating Square Roots 745



Name: Date: Period:

Practice

4. Write each expression as a single power.
   - a $(10^2)^{-3}$ **$= 10^{-6}$ (or equivalent)**
   - b $(3^{-3})^2$ **$= 3^{-6}$ (or equivalent)**
   - c $3^{-5} \cdot 4^{-5}$ **$= 12^{-5}$ (or equivalent)**
   - d $2^5 \cdot 3^{-5}$ **$= \left(\frac{2}{3}\right)^5$ (or equivalent)**

5. Andre is ordering ribbon to make decorations for a school event. He needs a total of exactly 30 m of blue and green ribbon. Andre needs 50% more blue ribbon than green ribbon for the basic design, plus an extra 5 m of blue ribbon for accents. How much of each color of ribbon does Andre need to order? Use the coordinate grid to help your thinking.

Andre needs to order 10 m of green ribbon and 20 m of blue ribbon; Sample response:

Let b represent the length of blue ribbon and g represent the length of green ribbon. Then solve the system of equations:

$$\begin{cases} b = 1.5g + 5 \\ b + g = 30 \end{cases}$$

6. Which expression is equivalent to 4^3?

[✓ A.] $4 \cdot 4 \cdot 4$
B. $4 \cdot 3$
C. $3 \cdot 3 \cdot 3 \cdot 3$
D. $3 + 3 + 3 + 3$
E. $4 + 4 + 4$

746 **Unit 7** Irrationals and the Pythagorean Theorem

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 6 Lesson 5	2
	5	Unit 4 Lesson 13	3
Formative	6	Unit 7 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 5**

The Cube Root

Let's learn about cube roots.

Focus

Goals

1. **Language Goal:** Comprehend the term *cube root of* p and the notation $\sqrt[3]{p}$ to mean the solution to the equation $x^3 = p$, where p is a positive rational number. **(Speaking and Listening)**
2. Recognize and use cube root notation to represent the edge length of a cube, given its volume.
3. **Language Goal:** Approximate the value of a cube root and explain the reasoning used. **(Speaking and Listening)**

Coherence

Today

Students determine solutions to equations of the form $x^3 = p$, and discover that they can represent the solution using cube root notation. Students explore the relationship between edge length and volume as they learn about ***cube roots*** and construct viable arguments when they approximate the values of the cube roots on a number line.

Previously

In Lessons 2 and 3, students explored solutions to equations of the form $x^2 = p$ and related the areas and side length of squares to learn about square roots. In Lesson 4, students approximated the values of square roots on a number line.

Coming Soon

In Lesson 6, students will revisit the equation $x^2 = 2$ to learn about irrational numbers.

Rigor

- Students build **conceptual understanding** of cube roots as they explore edge lengths and volumes of cubes.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Perfect Cubes*
- calculators

Math Language Development

New words

- ***cube root***
- ***perfect cube***

Review words

- *area*
- *perfect square*
- *square root*
- *volume*

Amps | Featured Activity

Activity 1
Spirit of Competition

Students face off in a friendly competition among classmates to see who can determine the closest solution to the equation $x^3 = 100$.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not communicate clearly about which estimate they think is the most precise. Remind students that clear communication includes the volume with which they speak, how well they enunciate words, the precision of their words, as well as the mathematical accuracy of their explanation. Encourage students to justify their reasoning for plotting an approximate location on the number line, and to listen to others share possible corrections of those approximations.

Modifications to Pacing

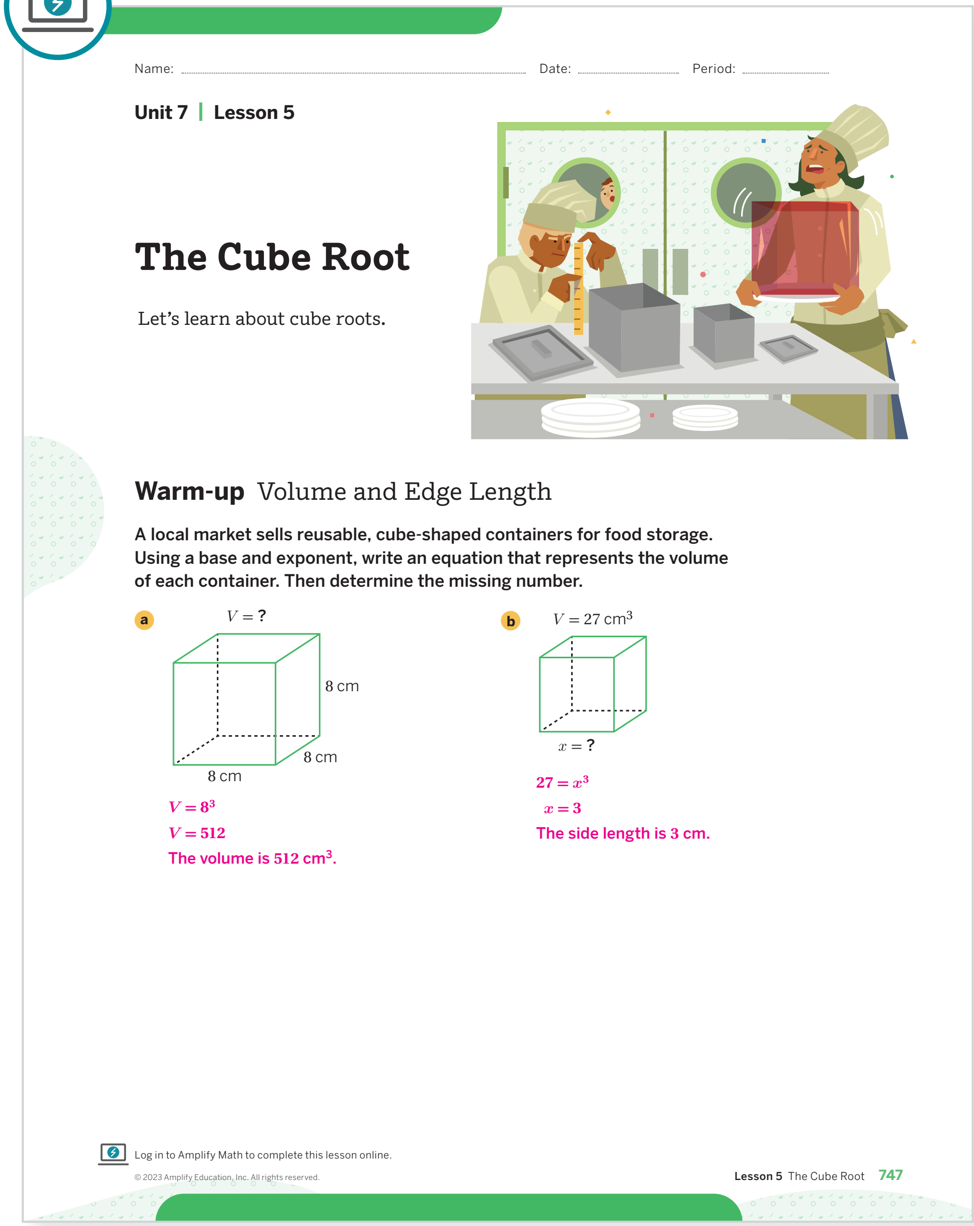
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, omit the first and third row in Problem 1.
- In **Activity 2**, have students choose one cube to complete the problem.

Pairs | 5 min

Warm-up Volume and Edge Length

Students determine an unknown edge length or volume to review the relationship between the edge length and the volume of cubes and the equation that represents this relationship.



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Unit 7 | Lesson 5

The Cube Root

Let's learn about cube roots.

Warm-up Volume and Edge Length

A local market sells reusable, cube-shaped containers for food storage. Using a base and exponent, write an equation that represents the volume of each container. Then determine the missing number.

a $V = ?$

8 cm

8 cm

8 cm

$V = 8^3$

$V = 512$

The volume is 512 cm^3.

b $V = 27 \text{ cm}^3$

$x = ?$

$27 = x^3$

$x = 3$

The side length is 3 cm.

Log in to Amplify Math to complete this lesson online.

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Lesson 5 The Cube Root 747

1 Launch

Activate students' prior knowledge by asking them how to determine the volume of a cube.

2 Monitor

Help students get started by displaying the equation *volume* = *edge*^{3}.

Look for points of confusion:

- **Writing $8 \cdot 3$ instead of 8^3 to solve part a.** Remind students that $8 \cdot 3 = 8 + 8 + 8$ and $8^3 = 8 \cdot 8 \cdot 8$.
- **Struggling to solve part b.** Suggest that students solve the problem using a guess-and-check method.

3 Connect

Display the Warm-up.

Have students share their equations and their strategies for determining the missing value for each cube. Select students that used different strategies to determine the edge length of the cube.

Highlight that the equation $V = x^3$ represents the volume of a cube, where V represents the volume and x represents the edge length of the cube.

Ask students to compare the equation that represents the area of a square with the equation that represents the volume of a cube. Have them discuss any similarities and differences of each equation.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

Remind students they have previously learned about the volume of cubes in prior grades. Clarify the formula for the volume V of a cube with edge length s as $V = s^3$. Ask, "What does the exponent of 3 mean?"

Power-up

To power up students' ability to evaluate expressions with an exponent, have students complete:

Recall that an exponent represents repeated multiplication. For example $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$. Match the equivalent expressions.

a. 3^2 ....b........ $2 \cdot 2 \cdot 2$

b. 2^3 ....c........ $4 \cdot 4 \cdot 4$

c. 4^3 ....a........ $3 \cdot 3$

d. $4 \cdot 3$ ....d........ $4 + 4 + 4$

Use: Before Activity 1

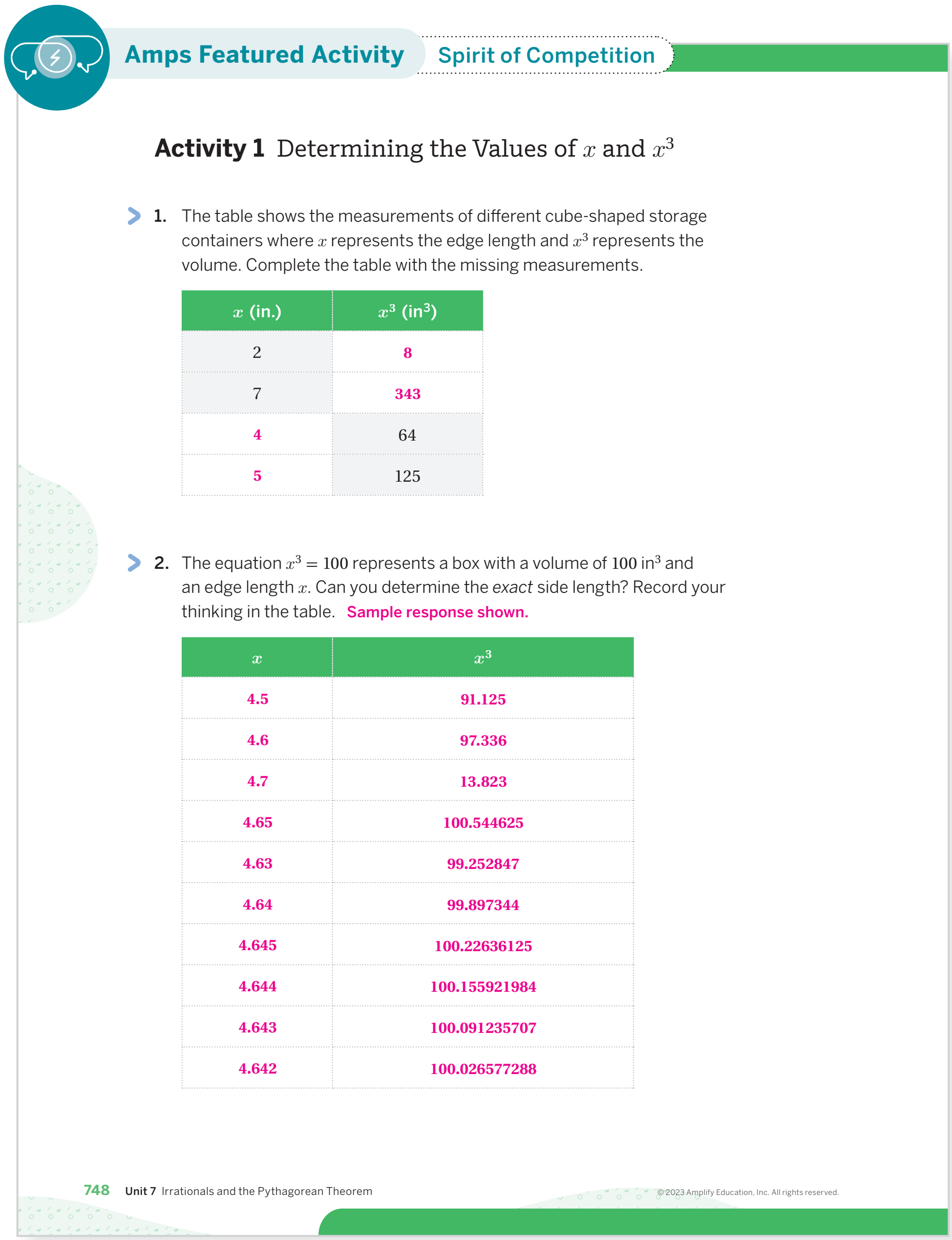
Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2

Independent | 20 min

Activity 1 Determining the Values of x and x^3

Students investigate solutions to $x^3 = p$ as an introduction to cube roots and their notation.

Amps Featured Activity Spirit of Competition



Activity 1 Determining the Values of x and x^3

1. The table shows the measurements of different cube-shaped storage containers where x represents the edge length and x^3 represents the volume. Complete the table with the missing measurements.

x (in.)	x^3 (in^3)
2	**8**
7	**343**
4	64
5	125

2. The equation $x^3 = 100$ represents a box with a volume of 100 in^3 and an edge length x. Can you determine the *exact* side length? Record your thinking in the table. **Sample response shown.**

x	x^3
4.5	**91.125**
4.6	**97.336**
4.7	**13.823**
4.65	**100.544625**
4.63	**99.252847**
4.64	**99.897344**
4.645	**100.22636125**
4.644	**100.155921984**
4.643	**100.091235707**
4.642	**100.026577288**

748 Unit 7 Irrationals and the Pythagorean Theorem

1 Launch

Have students complete Problem 1 individually. Then have them share responses with a partner before completing Problem 2. For Problem 2, demonstrate how students can determine a solution to the equation $x^3 = 100$ by providing different values for x and using that value to evaluate x^3.

2 Monitor

Help students get started by reminding students that $x \cdot x \cdot x = x^3$.

Look for points of confusion:

- **Struggling to complete the third and fourth rows in Problem 1.** Suggest that students use a "guess-and-check" method using whole numbers.

3 Connect

Have students share their strategies for determining their responses to Problem 2.

Define the term ***cube root*** as a solution to an equation of the form $x^3 = p$. Say, "You can determine the exact solution to $x^3 = 100$ by using the cube root notation, $\sqrt[3]{100}$. Demonstrate how to write each row in Problem 1 using cube root notation. $\sqrt[3]{8} = 2, \sqrt[3]{343} = 7, \sqrt[3]{64} = 4, \sqrt[3]{125} = 5$

Highlight that a ***perfect cube*** is a number that is the cube of an integer. For Problem 1, tell students that the values in the x^3 column represent perfect cubes. Point out that 64 is a perfect cube, but 100 is not.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can engage in a digital competition to see who can guess the closest value of x for the equation $x^3 = 100$.

Accessibility: *Activate Prior Knowledge*

Remind students they completed a similar table in Lesson 2 when they guessed the value of x for the equation $x^2 = 3$. Ask them what strategies they used in the prior activity and if those same strategies can be used in this activity.

Math Language Development

MLR2: Collect and Display

As students share their guesses and discuss how to find the exact solution, invite them to suggest language or diagrams to add to the class display that will support their understanding of the term *cube root*.

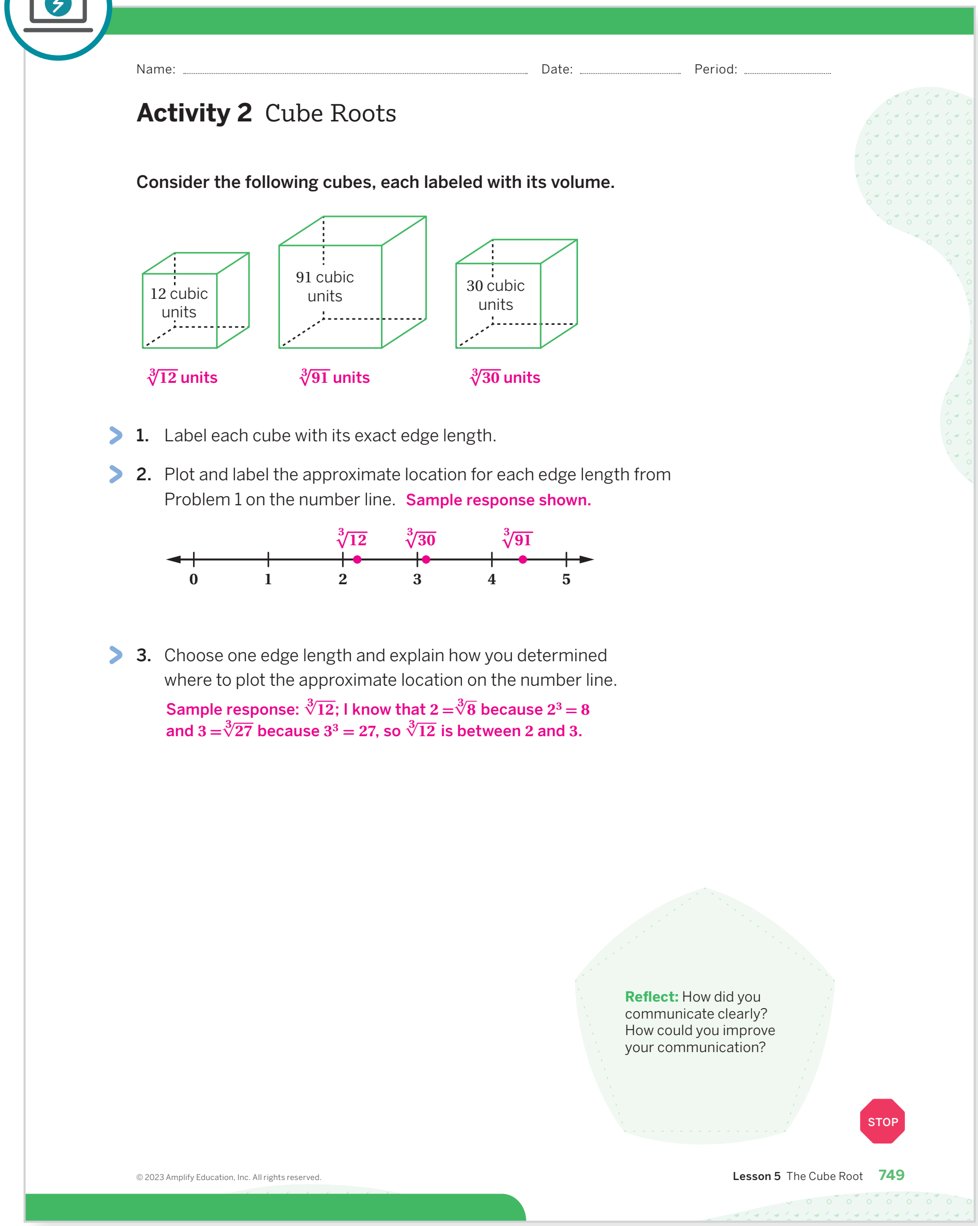
English Learners

Clarify the meaning of the word *cube* in *cube root*. Provide examples, such as a picture or diagram, to help students connect the definition to a visual representation of the edge length of a cube representing the cube root of the cube's volume.

Pairs | 10 min

Activity 2 Cube Roots

Students use cube root notation to represent the edge length of a cube, given its volume and approximate the value of a cube root.



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Activity 2 Cube Roots

Consider the following cubes, each labeled with its volume.

12 cubic units — $\sqrt[3]{12}$ units

91 cubic units — $\sqrt[3]{91}$ units

30 cubic units — $\sqrt[3]{30}$ units

1. Label each cube with its exact edge length.
2. Plot and label the approximate location for each edge length from Problem 1 on the number line. Sample response shown.

$\sqrt[3]{12}$ $\sqrt[3]{30}$ $\sqrt[3]{91}$

0 1 2 3 4 5

3. Choose one edge length and explain how you determined where to plot the approximate location on the number line.

Sample response: $\sqrt[3]{12}$; I know that $2 = \sqrt[3]{8}$ because $2^3 = 8$ and $3 = \sqrt[3]{27}$ because $3^3 = 27$, so $\sqrt[3]{12}$ is between 2 and 3.

Reflect: How did you communicate clearly? How could you improve your communication?

STOP

Lesson 5 The Cube Root 749

1 Launch

Activate students' prior knowledge of square roots by asking what strategies they used when comparing square roots. Sample responses may include comparing the areas of squares or using the perfect squares on a number line. Then tell students that they will use a similar strategy to compare the value of cube roots. **Note:** Provide access to calculators during the Connect.

2 Monitor

Help students get started by reminding them that they can represent the edge length of each cube using cube root notation.

Look for points of confusion:

- **For Problem 2, struggling to plot the approximate location of each cube root.** Have students label each marked interval on the number line using cube root notation. For example, students can label 2 as $\sqrt[3]{8}$ and 5 as $\sqrt[3]{125}$. Allow for different placements of cube roots between the correct consecutive whole numbers.

3 Connect

Have students share where they plotted each cube root and their strategies for plotting the cube root. Select students with different approximations and ask the class how they can determine which student's placement of the cube root is more precise. At this time, provide access to calculators and tell students that a calculator could help them determine a more precise estimate. Demonstrate how students can enter a cube root on a calculator, and then have them use the calculator to compare their placement of each cube root with the number displayed on the calculator.

Highlight that the edge length of a cube with a volume of p can be written as $\sqrt[3]{p}$. Also, highlight that, if the volume of a cube is greater than another cube, the edge length of the cube with the greater volume will also be greater.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide students with the Anchor Chart PDF, *Perfect Cubes*, to use as a reference.
If students need more processing time, have them focus on completing Problems 1–3 for one or two of the cubes, instead of all three cubes.

Math Language Development

MLR8: Discussion Supports

During the Connect, display the following sentence frames to support students as they justify their placement of each cube on the number line.

- "The cube root of _____ is between _____ and _____ because . . ."
- "The cube root of _____ is greater than _____ because . . ."
- "The cube root of _____ is less than _____ because . . ."
- "The cube root of _____ is greater than the cube root of _____ because . . ."

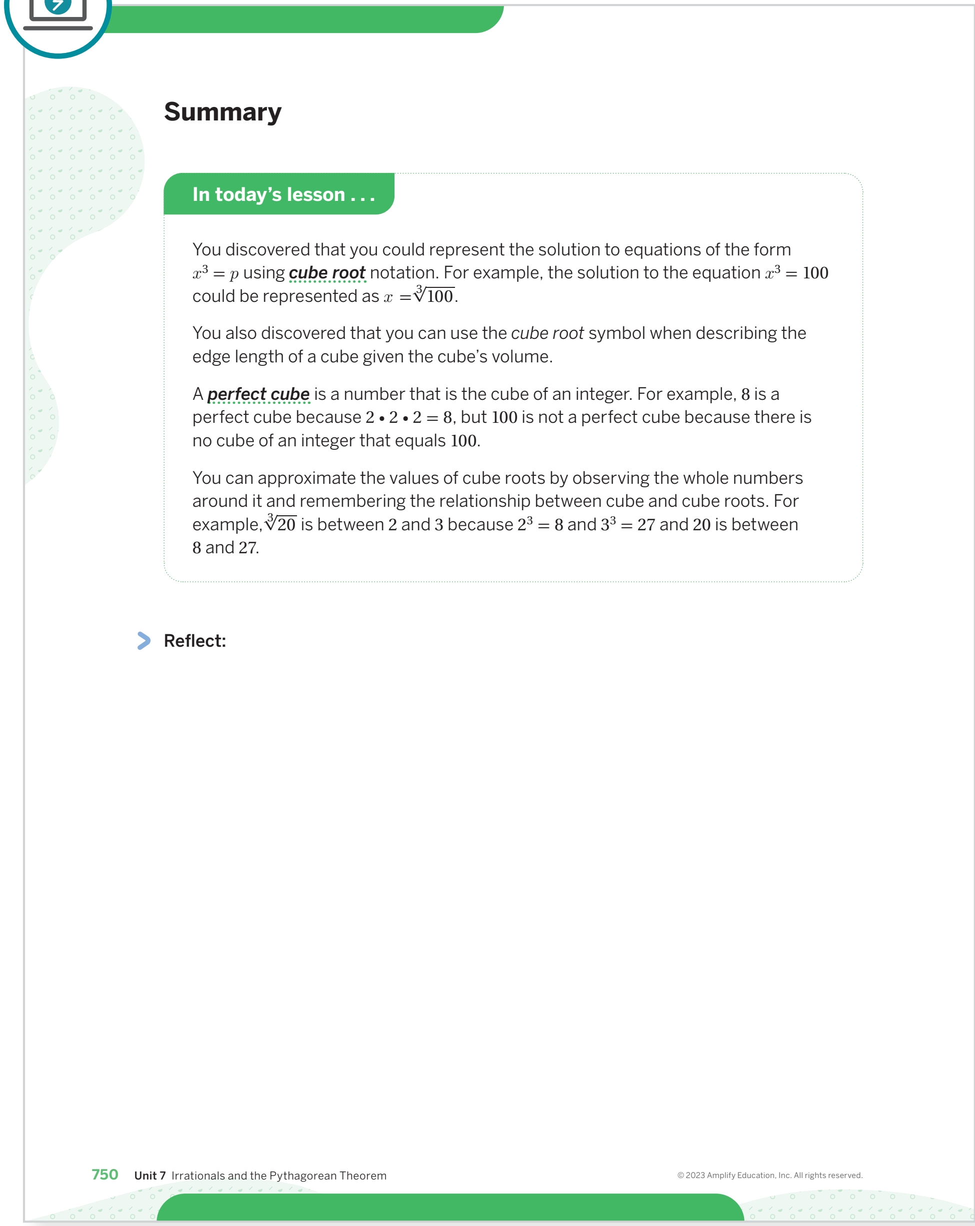
English Learners

Provide students time to rehearse and formulate what they will say with a partner before sharing with the class.

Whole Class | 5 min

Summary

Review and synthesize how to use cube root notation to represent the edge length of a cube given its volume and how to approximate the value of cube roots.



Summary

In today's lesson . . .

You discovered that you could represent the solution to equations of the form $x^3 = p$ using ***cube root*** notation. For example, the solution to the equation $x^3 = 100$ could be represented as $x = \sqrt[3]{100}$.

You also discovered that you can use the *cube root* symbol when describing the edge length of a cube given the cube's volume.

A ***perfect cube*** is a number that is the cube of an integer. For example, 8 is a perfect cube because $2 \bullet 2 \bullet 2 = 8$, but 100 is not a perfect cube because there is no cube of an integer that equals 100.

You can approximate the values of cube roots by observing the whole numbers around it and remembering the relationship between cube and cube roots. For example, $\sqrt[3]{20}$ is between 2 and 3 because $2^3 = 8$ and $3^3 = 27$ and 20 is between 8 and 27.

Reflect:

750 **Unit 7** Irrationals and the Pythagorean Theorem

Synthesize

Formalize vocabulary:

- *cube root*
- *perfect cube*

Display the Anchor Chart PDF, *Perfect Cubes*, and use it as reference, as needed. Remind students that the numbers in the middle column are examples of perfect cubes.

Highlight that students can determine the exact edge length of a cube by expressing it as the cube root of the cube's volume.

Have students share their strategies for determining between which two whole consecutive numbers a cube root falls.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does a cube root of a number represent and how can you estimate the cube root of a number?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started for this unit. Ask them to review and reflect on any terms and phrases related to the terms *cube root* and *perfect cube* that were added to the display during the lesson. Add visual examples of cubes annotated with the terms *volume*, *cube* (or *perfect cube*, if applicable), *cube root*, and *edge*.

Be sure the display clearly distinguishes square roots from cube roots. Consider adding the following statements to the display and have students complete them.

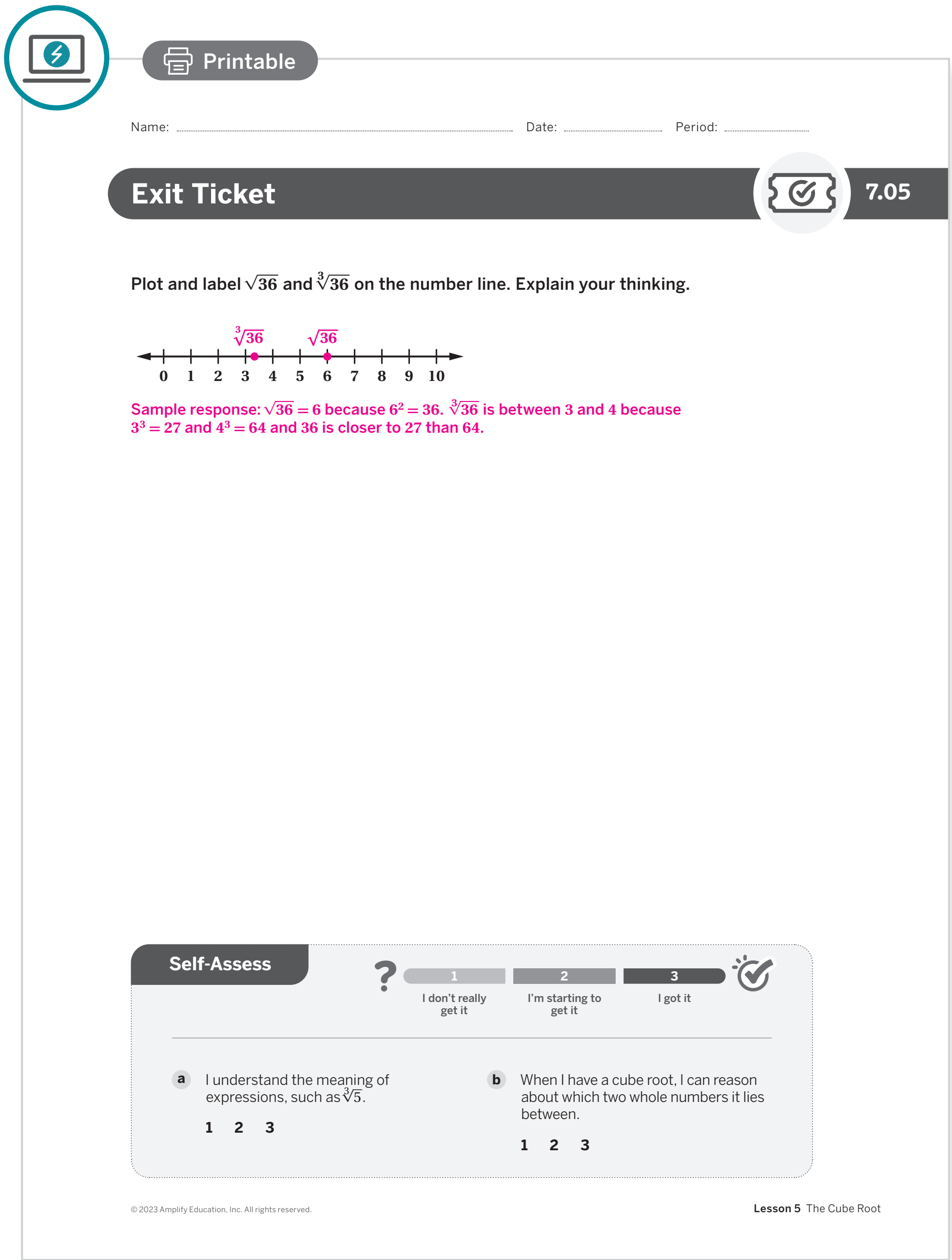
- "The side length of a square is the _____ of the square's _____."
  square root; area
- "The edge length of a cube is the _____ of the cube's _____."
  cube root; volume

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by approximating square roots and cube roots on a number line.

Printable



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Exit Ticket 7.05

Plot and label $\sqrt{36}$ and $\sqrt[3]{36}$ on the number line. Explain your thinking.

Sample response: $\sqrt{36} = 6$ because $6^2 = 36$. $\sqrt[3]{36}$ is between 3 and 4 because $3^3 = 27$ and $4^3 = 64$ and 36 is closer to 27 than 64.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I understand the meaning of expressions, such as $\sqrt[3]{5}$.

1 2 3

b. When I have a cube root, I can reason about which two whole numbers it lies between.

1 2 3

Lesson 5 The Cube Root

Success looks like . . .

- **Language Goal:** Comprehending the term *cube* root of p and the notation $\sqrt[3]{p}$ to mean the solution to the equation $x^3 = p$, where p is a positive rational number. **(Speaking and Listening)**
- **Goal:** Recognizing and using cube root notation to represent the edge length of a cube, given its volume.
- **Language Goal:** Approximating the value of a cube root and explaining the reasoning used. **(Speaking and Listening)**
  - » Approximating the location of the cube root of 36 on a number line and explaining how it was approximated.

Suggested next steps

If students incorrectly plot $\sqrt{36}$, consider:

- Reviewing Lesson 2.

If students incorrectly plot $\sqrt[3]{36}$, consider:

- Reviewing perfect cubes, and then reviewing Activity 2.

Professional Learning

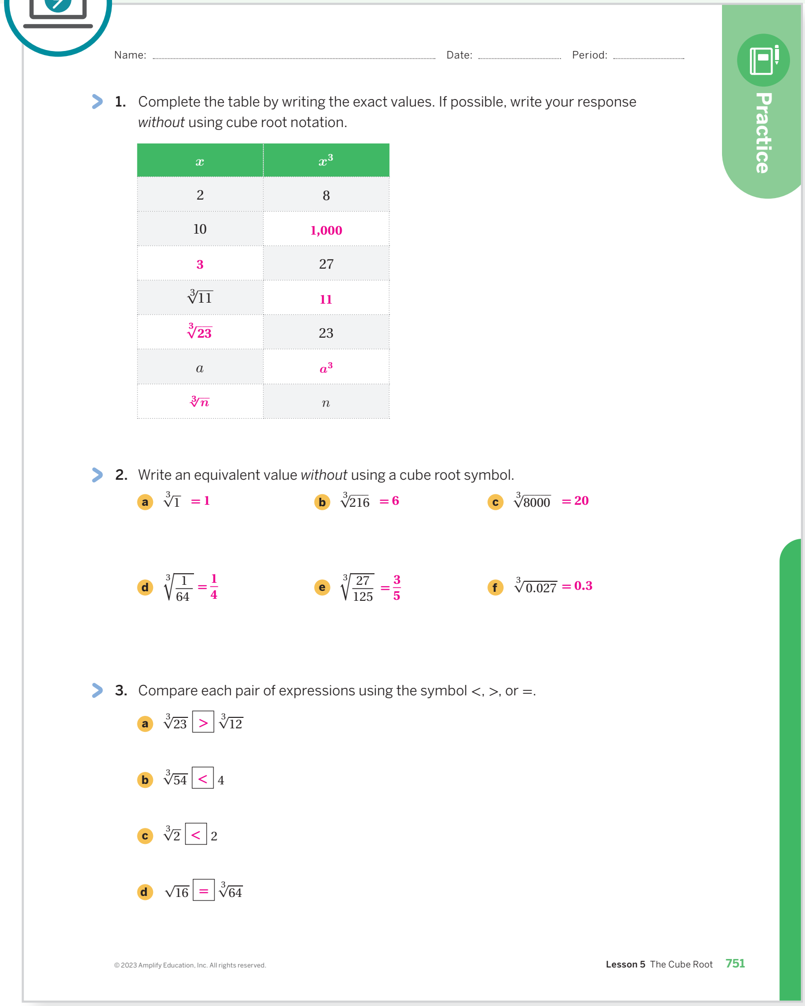
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How was Activity 2 similar to or different from Activity 2 from Lesson 2?
- What different ways did students approach estimating cube roots on a number line? What does that tell you about similarities and differences among your students?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Complete the table by writing the exact values. If possible, write your response *without* using cube root notation.

x	x^3
2	8
10	**1,000**
3	27
$\sqrt[3]{11}$	**11**
$\sqrt[3]{23}$	23
a	a^3
$\sqrt[3]{n}$	n

2. Write an equivalent value *without* using a cube root symbol.

a $\sqrt[3]{1}$ $= 1$

b $\sqrt[3]{216}$ $= 6$

c $\sqrt[3]{8000}$ $= 20$

d $\sqrt[3]{\frac{1}{64}} = \frac{1}{4}$

e $\sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

f $\sqrt[3]{0.027} = 0.3$

3. Compare each pair of expressions using the symbol <, >, or =.

a $\sqrt[3]{23}$ $\boxed{>}$ $\sqrt[3]{12}$

b $\sqrt[3]{54}$ $\boxed{<}$ 4

c $\sqrt[3]{2}$ $\boxed{<}$ 2

d $\sqrt{16}$ $\boxed{=}$ $\sqrt[3]{64}$

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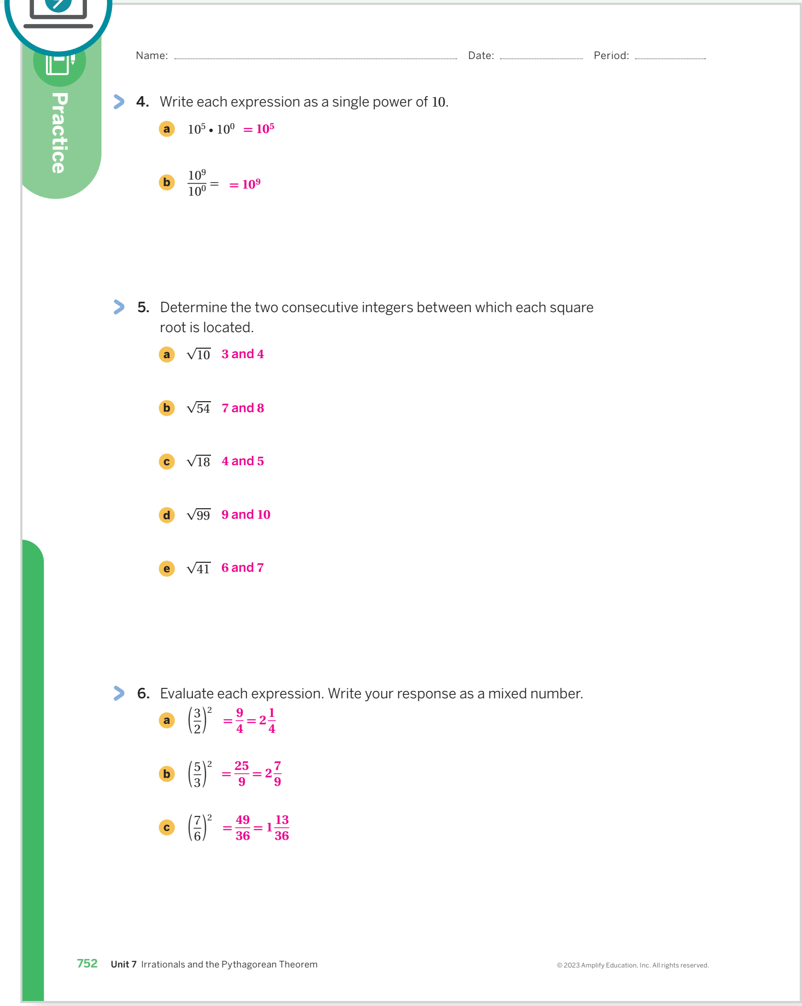
Lesson 5 The Cube Root 751



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Practice

4. Write each expression as a single power of 10.

a $10^5 \bullet 10^0$ $= 10^5$

b $\frac{10^9}{10^0} =$ $= 10^9$

5. Determine the two consecutive integers between which each square root is located.

a $\sqrt{10}$ 3 and 4

b $\sqrt{54}$ 7 and 8

c $\sqrt{18}$ 4 and 5

d $\sqrt{99}$ 9 and 10

e $\sqrt{41}$ 6 and 7

6. Evaluate each expression. Write your response as a mixed number.

a $\left(\frac{3}{2}\right)^2$ $= \frac{9}{4} = 2\frac{1}{4}$

b $\left(\frac{5}{3}\right)^2$ $= \frac{25}{9} = 2\frac{7}{9}$

c $\left(\frac{7}{6}\right)^2$ $= \frac{49}{36} = 1\frac{13}{36}$

752 Unit 7 Irrationals and the Pythagorean Theorem

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 6 Lessons 3 and 4	1
	5	Unit 7 Lesson 3	2
Formative ⚡	6	Unit 7 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 6**

Rational and Irrational Numbers

Let's learn about rational and irrational numbers.

Focus

Goals

1. **Language Goal:** Understand that a rational number is defined as a number that can be written as a fraction that represents the ratio of two integers. **(Speaking and Listening, Writing)**
2. **Language Goal:** Comprehend that numbers that are not rational are called *irrational*. **(Speaking and Listening)**

Coherence

Today

Students revisit Pythagorean thinking as they try to determine if a fraction could represent the solution to the equation $x^2 = 2$. Students discover that some numbers, such as $\sqrt{2}$ are called *irrational numbers* and construct viable arguments as they provide examples of irrational numbers.

Previously

In Grades 6 and 7, students discovered that rational numbers can be expressed as fractions. In Lesson 2, students first discovered numbers expressed as square roots.

Coming Soon

In Lessons 7 and 8, students will explore the relationship between repeating decimals and fractions.

Rigor

- Students build their **conceptual understanding** of irrational numbers.
- Students are introduced to irrational numbers to build **procedural skills**.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	8 min	8 min	5 min	5 min
Small Groups	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, (for display)
- Anchor Chart PDF, *Perfect Squares*
- Anchor Chart PDF, *Perfect Cubes*

Math Language Development

New words

- ***irrational number***
- ***rational number***

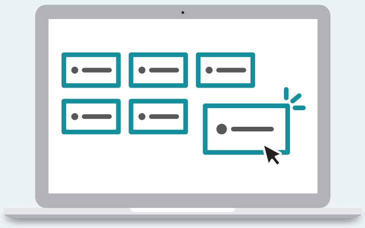
Review words

- *integer*

Amps | Featured Activity

Activity 3
Digital Number Sort

Students sort rational and irrational numbers by dragging and connecting them on screen.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might forget that the point of a critique is to help someone, not to harm them. Discuss with students what a positive critique sounds like and why they are good. Have students imagine being the person whose work will be critiqued, and have them describe how unkind or insensitive words would feel. Then have them describe how to communicate logic or reasoning to others in a way that would support their thinking and encourage improved arguments.

Modifications to Pacing

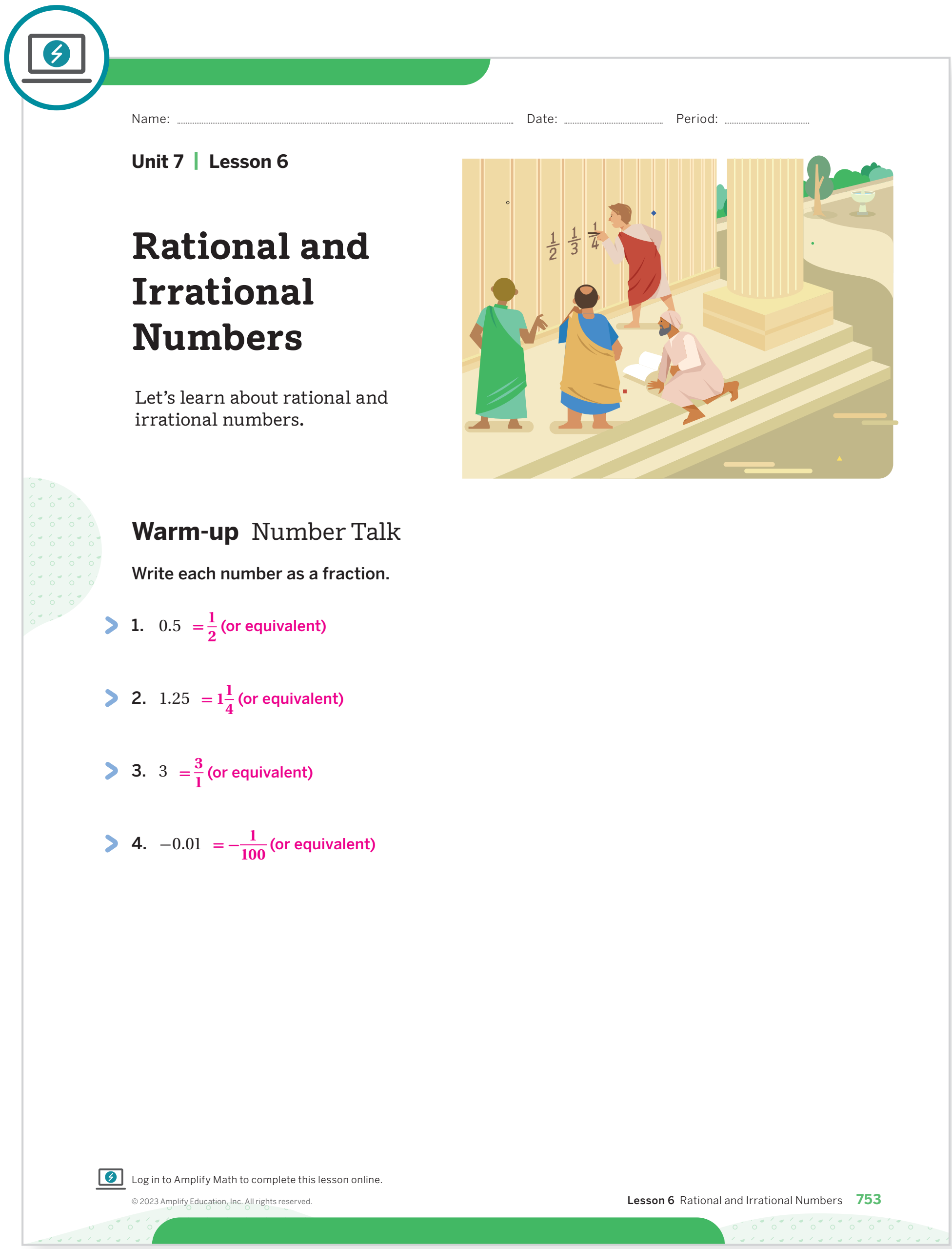
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 2 and 4 may be omitted.
- In **Activity 1**, have students complete as many rows as time allows.
- Consider assigning **Activity 3** as practice.

Small Groups | 5 min

Warm-up Number Talk

Students write decimals as equivalent fractions as a reminder that any decimal number can be written as a fraction.



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Unit 7 | Lesson 6

Rational and Irrational Numbers

Let's learn about rational and irrational numbers.

Warm-up Number Talk

Write each number as a fraction.

1. 0.5 $= \frac{1}{2}$ (or equivalent)
2. 1.25 $= 1\frac{1}{4}$ (or equivalent)
3. 3 $= \frac{3}{1}$ (or equivalent)
4. -0.01 $= -\frac{1}{100}$ (or equivalent)

Log in to Amplify Math to complete this lesson online.

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Lesson 6 Rational and Irrational Numbers 753

1 Launch

Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by activating their prior knowledge about the relationship between place value positions and fractions.

Look for points of confusion:

- **Struggling to write 3 as a fraction.** Encourage students to look for any ratio of integers that is equivalent to 3, such as $\frac{6}{2}$ or $\frac{3}{1}$.
- **Not understanding why 3 can be written as the fraction $\frac{3}{1}$, but 0.5 cannot be written as the fraction $\frac{0.5}{1}$.** Remind students that a fraction is a ratio of integers.

Look for productive strategies:

- Writing an equivalent fraction not in simplest form.

3 Connect

Have students share their responses.

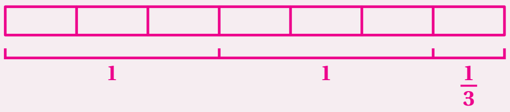
Highlight that ***rational numbers*** are numbers that can be written as a fraction, or ratio of integers.

Ask students if they think all numbers could be represented as a fraction to transition to Activity 1.

Power-up

To power up students' ability to write improper fractions as mixed numbers, have students complete:

1. Draw a tape diagram that represents $\frac{7}{3}$.



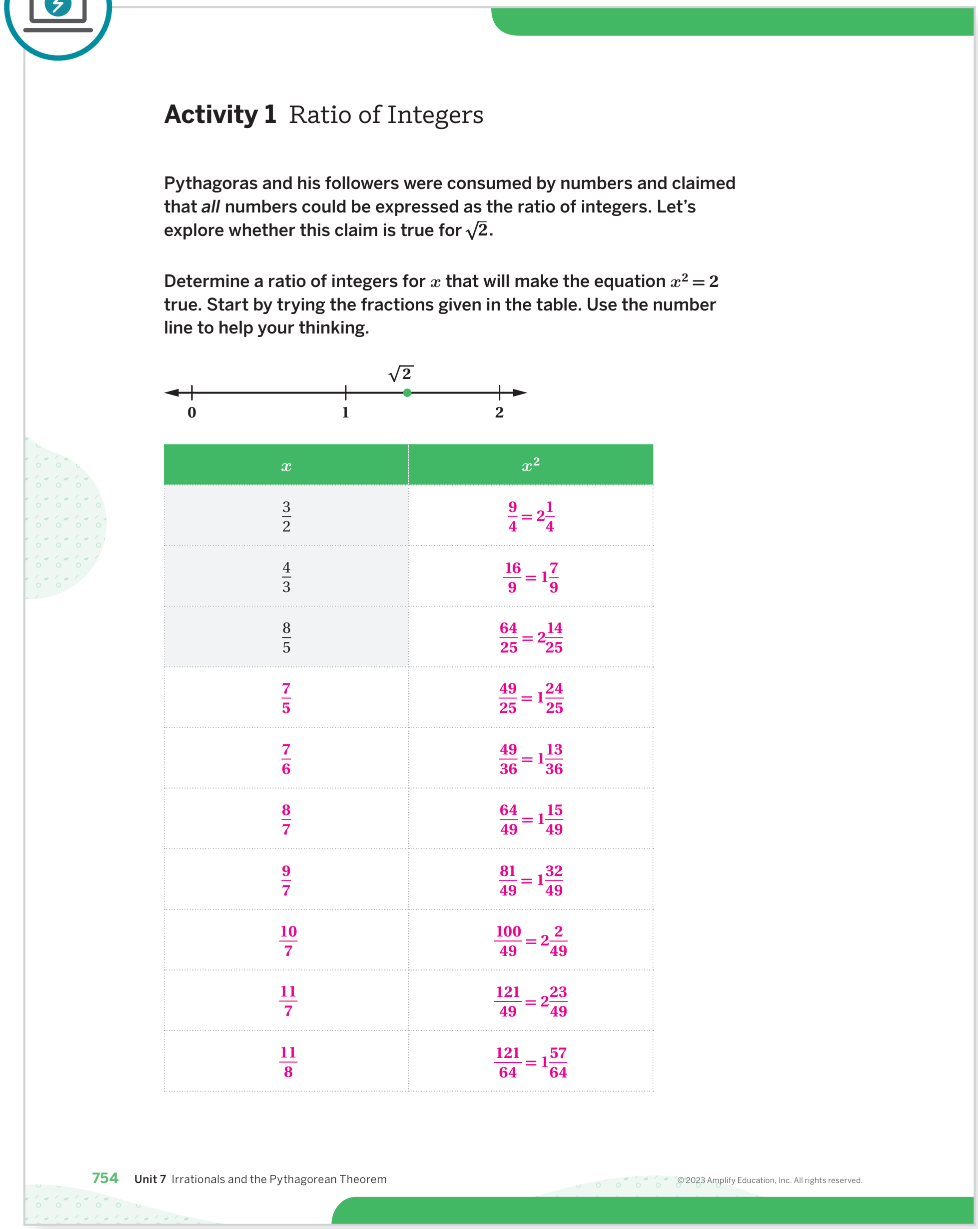
2. Use your tape diagram to rewrite $\frac{7}{3}$ as a mixed number. $2\frac{1}{3}$

Use: Before the Warm-up
Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7

Independent | 15 min

Activity 1 Ratio of Integers

Students determine if $\sqrt{2}$ could be represented as a ratio of integers as an introduction to irrational numbers.



Activity 1 Ratio of Integers

Pythagoras and his followers were consumed by numbers and claimed that *all* numbers could be expressed as the ratio of integers. Let's explore whether this claim is true for $\sqrt{2}$.

Determine a ratio of integers for x that will make the equation $x^2 = 2$ true. Start by trying the fractions given in the table. Use the number line to help your thinking.

x	x^2
$\frac{3}{2}$	$\frac{9}{4} = 2\frac{1}{4}$
$\frac{4}{3}$	$\frac{16}{9} = 1\frac{7}{9}$
$\frac{8}{5}$	$\frac{64}{25} = 2\frac{14}{25}$
$\frac{7}{5}$	$\frac{49}{25} = 1\frac{24}{25}$
$\frac{7}{6}$	$\frac{49}{36} = 1\frac{13}{36}$
$\frac{8}{7}$	$\frac{64}{49} = 1\frac{15}{49}$
$\frac{9}{7}$	$\frac{81}{49} = 1\frac{32}{49}$
$\frac{10}{7}$	$\frac{100}{49} = 2\frac{2}{49}$
$\frac{11}{7}$	$\frac{121}{49} = 2\frac{23}{49}$
$\frac{11}{8}$	$\frac{121}{64} = 1\frac{57}{64}$

754 **Unit 7** Irrationals and the Pythagorean Theorem

1 Launch

Activate students' background knowledge by asking what they remember about Pythagoras from Lesson 2. Tell students they will revisit the equation $x^2 = 2$, but, this time, they will determine if the solution could be represented by a ratio of integers. Display the Activity 1 PDF. Model how students could use the number line to choose estimate fractions that are close to $\sqrt{2}$. Tell students that the denominator of the fraction helps them create intervals, while the numerator shows the number of intervals.

2 Monitor

Help students get started by completing the first three rows together. Have students write each improper fraction as a mixed number to see how close the number is to 2.

Look for points of confusion:

- **Struggling to determine a ratio of integers that is close to 2.** Have students choose the same denominator and adjust the numerator.

3 Connect

Have students share their closest guess for x.

Ask students if they think there is a ratio of integers equal to x so that $x^2 = 2$.

Highlight that $\sqrt{2}$ cannot be represented as a ratio of integers. Therefore, Pythagoras's claim was not correct.

Define the term ***irrational number*** as a number that is not rational. That is, an irrational number cannot be written as a fraction representing the ratio of two integers. $\sqrt{2}$ is an example of an irrational number. Emphasize that numbers do not have to be written as fractions in order to be rational numbers.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously learned about rational numbers. Rational numbers can be written as a fraction, where the fraction represents the ratio of two integers. Ask students to generate examples of rational numbers. Sample responses: -3, -1.77, 0, $\frac{2}{3}$, $4\frac{5}{8}$

Extension: *Math Enrichment*

Ask students whether they agree with the statement, "Twice the square root of 2 is equal to $\sqrt{2}$." Have them explain their thinking. Sample response: I do not agree. The square root of 4 is 2 and if twice the square root of 2 was 2, then one square root of 2 would be 1, which is not true because 1^2 does not equal 2.

Math Language Development

MLR2: Collect and Display

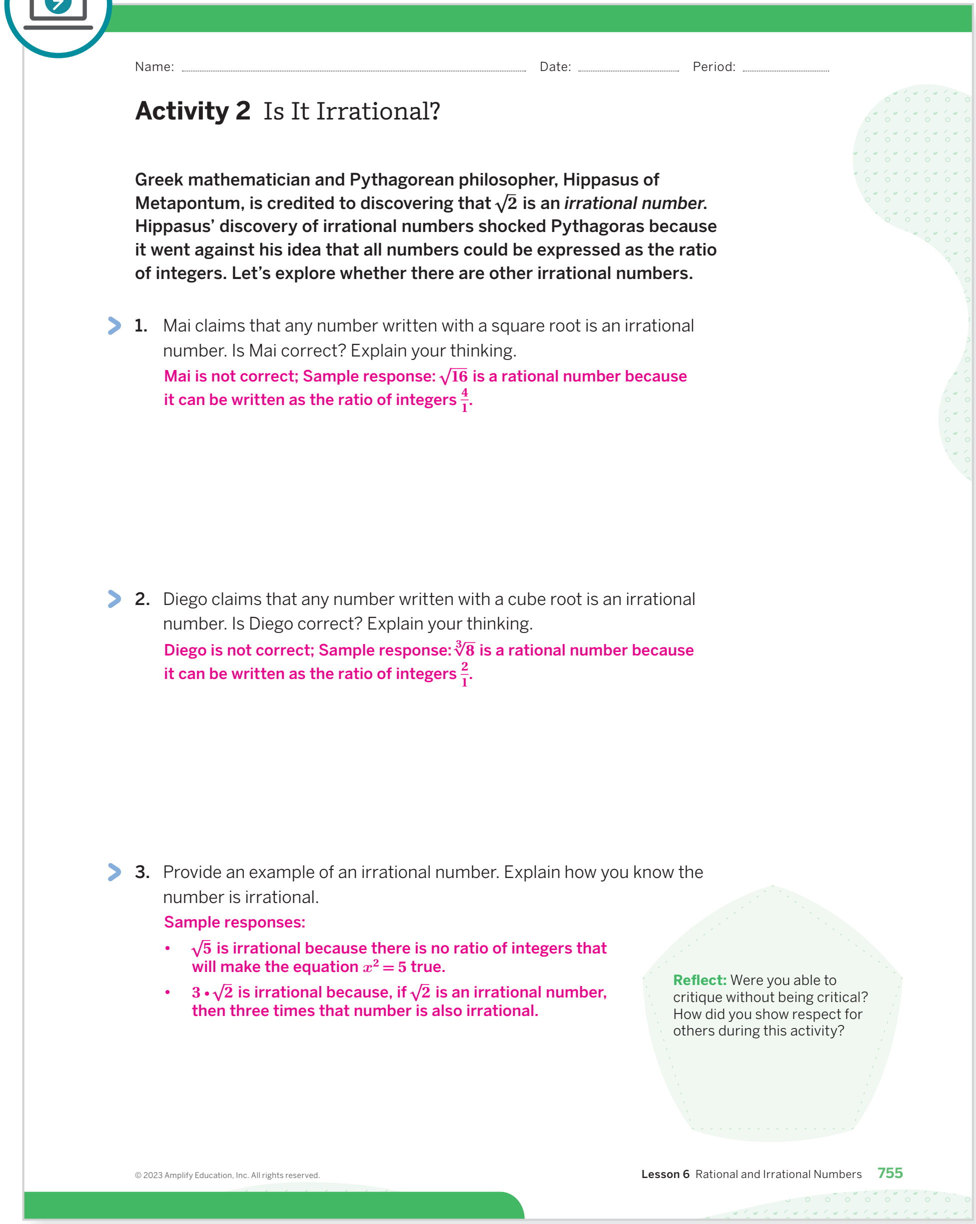
During the Connect, as you define the term irrational number, consider decomposing it to help students make sense of its definition. Add the following to the class display.

Rational	Irrational
Ratio of two integers	"Ir" + "rational" "Ir" means "not" Not the ratio of two integers

Pairs | 8 min

Activity 2 Is It Irrational?

Students critique the reasoning of others to gain a better understanding of irrational numbers.



Name: ______ Date: ______ Period: ______

Activity 2 Is It Irrational?

Greek mathematician and Pythagorean philosopher, Hippasus of Metapontum, is credited to discovering that $\sqrt{2}$ is an *irrational number*. Hippasus' discovery of irrational numbers shocked Pythagoras because it went against his idea that all numbers could be expressed as the ratio of integers. Let's explore whether there are other irrational numbers.

1. Mai claims that any number written with a square root is an irrational number. Is Mai correct? Explain your thinking.
   Mai is not correct; Sample response: $\sqrt{16}$ is a rational number because it can be written as the ratio of integers $\frac{4}{1}$.

2. Diego claims that any number written with a cube root is an irrational number. Is Diego correct? Explain your thinking.
   Diego is not correct; Sample response: $\sqrt[3]{8}$ is a rational number because it can be written as the ratio of integers $\frac{2}{1}$.

3. Provide an example of an irrational number. Explain how you know the number is irrational.
   Sample responses:
   - **$\sqrt{5}$ is irrational because there is no ratio of integers that will make the equation $x^2 = 5$ true.**
   - **$3 \cdot \sqrt{2}$ is irrational because, if $\sqrt{2}$ is an irrational number, then three times that number is also irrational.**

Reflect: Were you able to critique without being critical? How did you show respect for others during this activity?

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Lesson 6 Rational and Irrational Numbers 755

1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by having students write several examples of square roots and cube roots to use as an example.

Look for points of confusion:

- **Thinking that Mai and Diego are correct.** Activate students' knowledge of perfect squares and perfect cubes. Provide the numbers $\sqrt{16}$ and $\sqrt[3]{8}$ to help them with their thinking.
- **Struggling to determine an example of an irrational number.** Remind students that, in Activity 1, they saw that $\sqrt{2}$ was an irrational number. Encourage students to make connections to multiples of $\sqrt{2}$ or a number that would represent the opposite of $\sqrt{2}$.

3 Connect

Have students share their responses. Use the ***Poll the Class*** routine for Problems 1 and 2. Have students share their thinking.

Highlight that the square root of a whole number that is not a perfect square is irrational, and the cube root of a whole number that is not a perfect cube is irrational.

Ask, "Do you think there are other examples of irrational numbers?" Sample response: Yes; π is an irrational number.

Note: In future grades, students will explore proofs showing why a number is irrational.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

To support students as they respond to Problems 1 and 2, provide sample numbers they could reason about as they examine Mai's and Diego's claims. For example, provide the following numbers: $\sqrt{2}, \sqrt{3}, \sqrt{9}, \sqrt[3]{5}, \sqrt[3]{6}$, and $\sqrt[3]{8}$.

Extension: *Math Enrichment*

Have students determine whether the number $\sqrt[3]{\frac{8}{64}}$ is rational or irrational and explain their thinking. Rational; $\sqrt[3]{\frac{8}{64}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

Math Language Development

MLR8: Discussion Supports

During the Connect, as you highlight the relationship between perfect squares, perfect cubes, and irrational numbers, add these statements to the class display and have students complete them. Have students brainstorm examples.

- "The square root of a whole number that is not a perfect square is _____" irrational (Examples: $\sqrt{2}$ and $\sqrt{6}$)
- "The cube root of a whole number that is not a perfect cube is _____." irrational (Examples: $\sqrt[3]{5}$ and $\sqrt[3]{9}$)

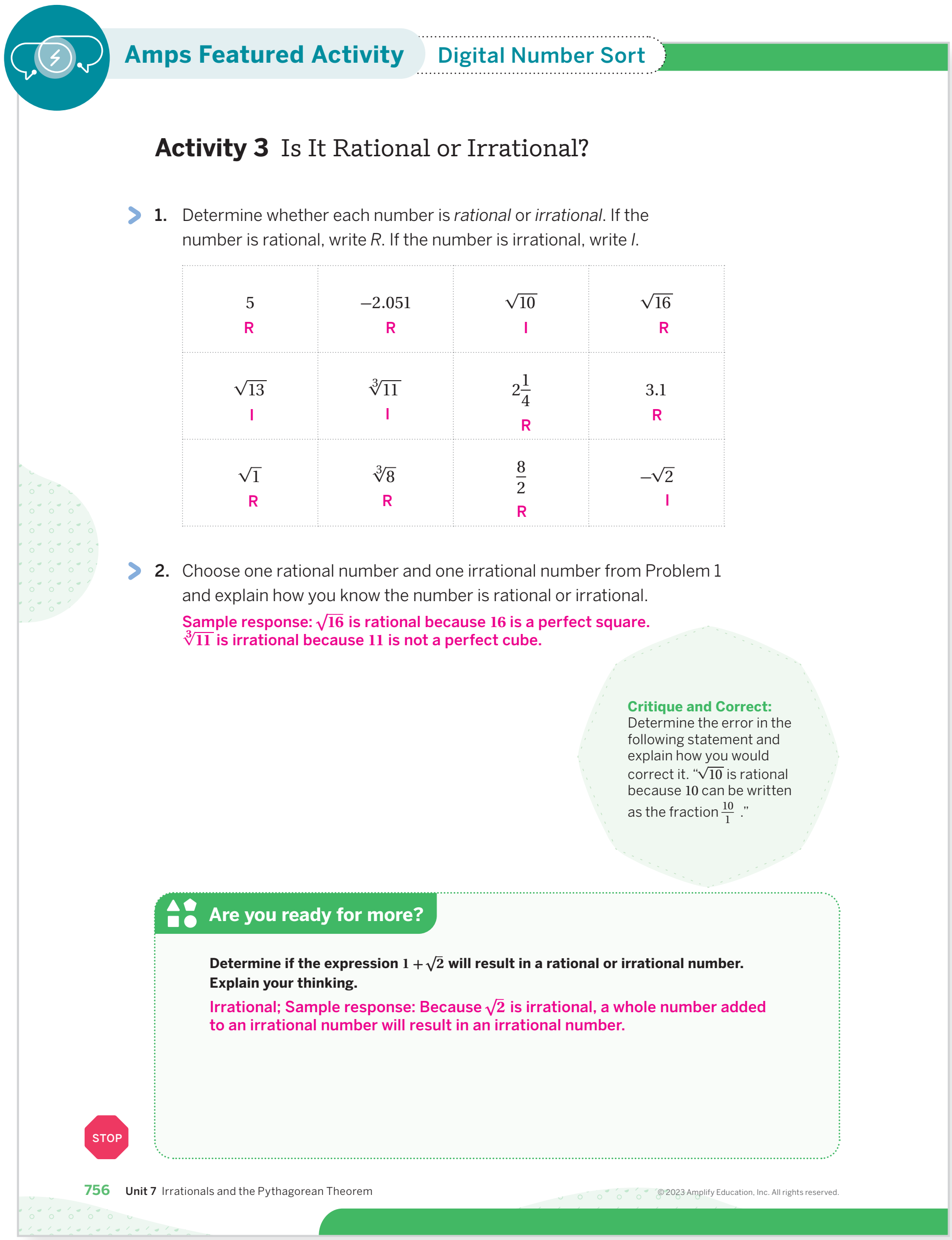
Emphasize that $\sqrt[3]{9}$ is irrational because, while 9 is a perfect square, it is not a perfect cube, and the given notation is cube root notation.

Pairs | 8 min

Activity 3 Is It Rational or Irrational?

Students classify a number as rational or irrational to build procedural fluency.

Amps Featured Activity Digital Number Sort



Activity 3 Is It Rational or Irrational?

1. Determine whether each number is *rational* or *irrational*. If the number is rational, write *R*. If the number is irrational, write *I*.

5 R	-2.051 R	$\sqrt{10}$ I	$\sqrt{16}$ R
$\sqrt{13}$ I	$\sqrt[3]{11}$ I	$2\frac{1}{4}$ R	3.1 R
$\sqrt{1}$ R	$\sqrt[3]{8}$ R	$\frac{8}{2}$ R	$-\sqrt{2}$ I

2. Choose one rational number and one irrational number from Problem 1 and explain how you know the number is rational or irrational.

Sample response: $\sqrt{16}$ is rational because 16 is a perfect square. $\sqrt[3]{11}$ is irrational because 11 is not a perfect cube.

Critique and Correct: Determine the error in the following statement and explain how you would correct it. "$\sqrt{10}$ is rational because 10 can be written as the fraction $\frac{10}{1}$."

Are you ready for more?

Determine if the expression $1 + \sqrt{2}$ will result in a rational or irrational number. Explain your thinking.

Irrational; Sample response: Because $\sqrt{2}$ is irrational, a whole number added to an irrational number will result in an irrational number.

STOP

756 **Unit 7** Irrationals and the Pythagorean Theorem

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1 Launch

Activate students' prior knowledge about rational numbers.

2 Monitor

Help students get started by asking them to provide examples of rational numbers.

Look for points of confusion:

- **Trying to determine if a number is rational by looking for an equivalent fraction.** Have students focus on identifying irrational numbers first by looking for any square roots or cube roots that do not produce a rational number.
- **Mislabeling $-\sqrt{2}$ as rational.** Have students refer to Activity 1 where they saw that $\sqrt{2}$ is irrational. Ask them to reason abstractly or quantitatively to determine why $-\sqrt{2}$ is irrational.

3 Connect

Have students share their responses, discussing any discrepancies.

Ask students to explain why they think $-\sqrt{2}$ is a rational or irrational number. Explain that if $\sqrt{2}$ cannot be written as a rational number, $\frac{a}{b}$, then $-\sqrt{2}$ cannot be written as $-\frac{a}{b}$.

Highlight that when looking for irrational numbers, students can focus on square roots and cube roots. They can look for square roots or cube roots that do not produce a rational number, and identify these numbers as irrational. Remind students that, while these are not the only irrational numbers, using this method may help them determine whether a number is rational or irrational.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can sort rational and irrational numbers by dragging and connecting them on screen.

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

Provide students with copies of the Anchor Chart PDFs, *Perfect Squares* and *Perfect Cubes*.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present an incorrect statement that reflects a possible point of confusion from the class. For example, "$\sqrt{10}$ is rational because $\sqrt{10}$ can be written as the fraction $\frac{\sqrt{10}}{7}$." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking." Listen for students who reason that while 10 is rational, $\sqrt{10}$ is not rational because 10 is not a perfect square.
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

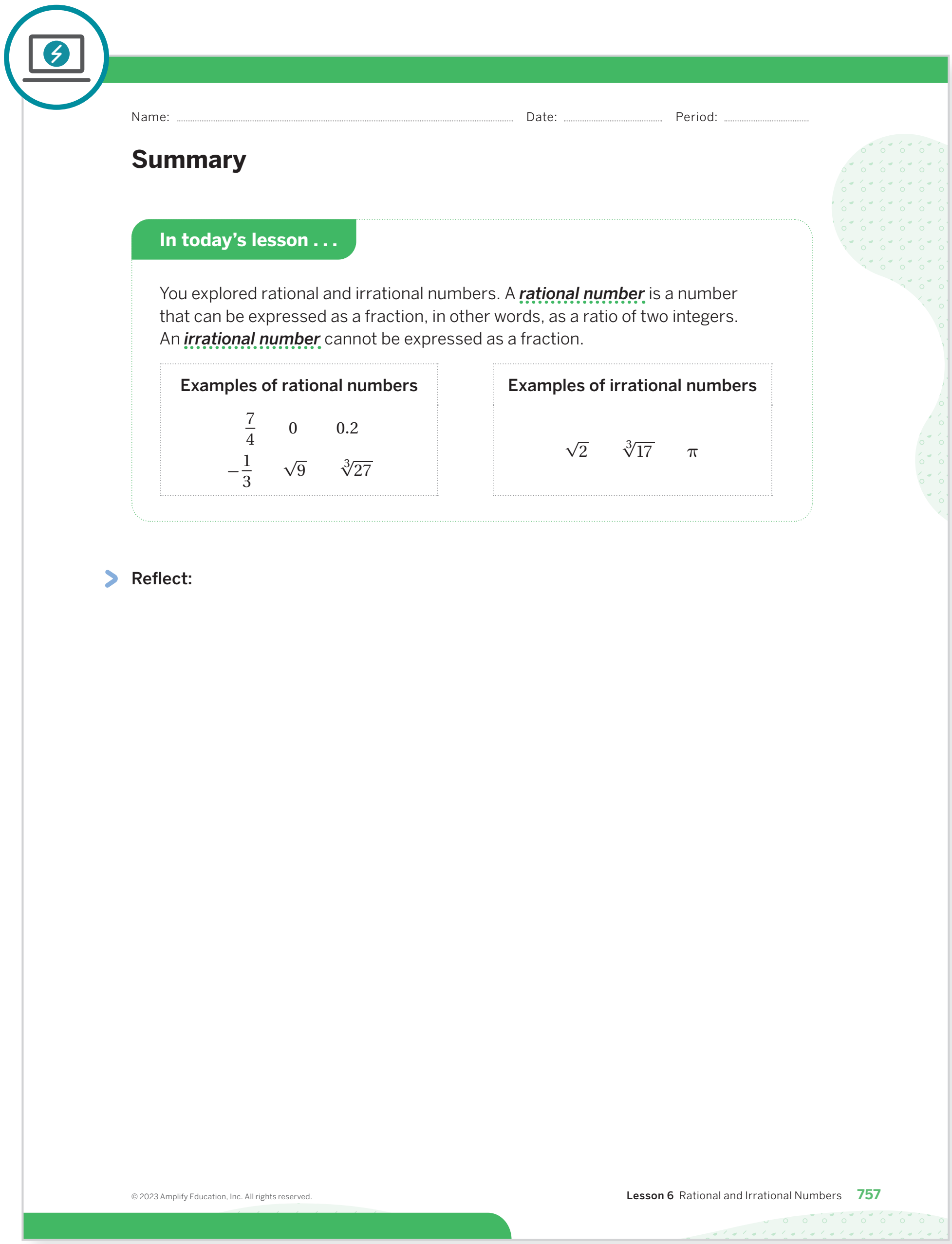
English Learners

Have students correct the statement by first writing "$\sqrt{10}$ is irrational because 10 is/is not a perfect _____."

Whole Class | 5 min

Summary

Review and synthesize rational and irrational numbers.



Name: __________ Date: __________ Period: __________

Summary

In today's lesson . . .

You explored rational and irrational numbers. A ***rational number*** is a number that can be expressed as a fraction, in other words, as a ratio of two integers. An ***irrational number*** cannot be expressed as a fraction.

Examples of rational numbers	Examples of irrational numbers
$\frac{7}{4}$ $\quad 0 \quad 0.2$ $-\frac{1}{3}$ $\quad \sqrt{9} \quad \sqrt[3]{27}$	$\sqrt{2} \quad \sqrt[3]{17} \quad \pi$

Reflect:

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Lesson 6 Rational and Irrational Numbers 757

Synthesize

Have students share how they know whether a number is rational or irrational in their own words.

Ask students to provide an example of a rational number and an irrational number.

Highlight that rational numbers can be expressed as fractions, while irrational numbers cannot be expressed as fractions.

Formalize vocabulary:

- *irrational number*
- *rational number*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What is the difference between a rational number and an irrational number?"

Math Language Development

MLR2: Collect and Display

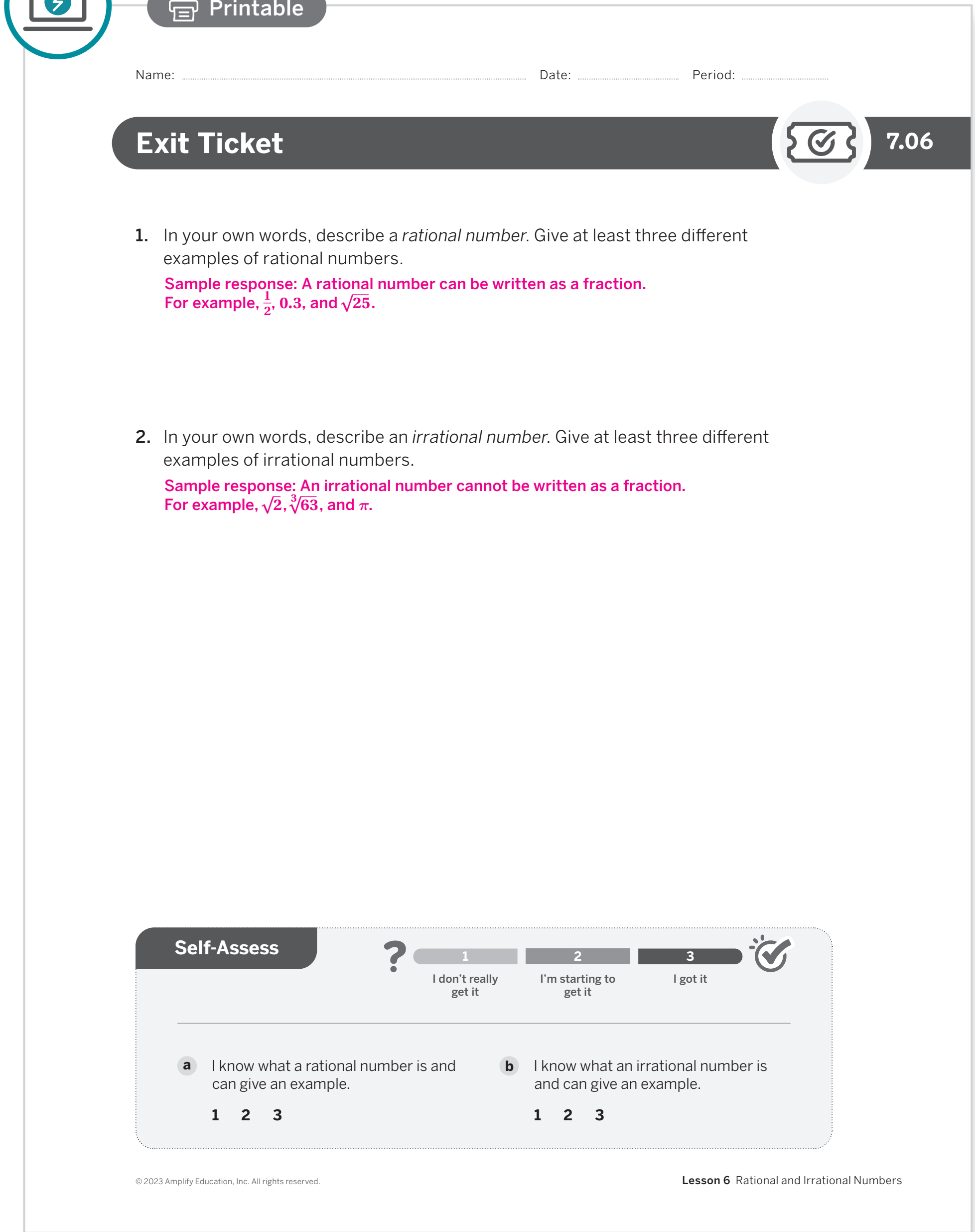
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started for this unit. Ask them to review and reflect on any terms and phrases related to the term *irrational number* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing a rational and irrational number and providing examples of each.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

7.06

1. In your own words, describe a *rational number*. Give at least three different examples of rational numbers.

   Sample response: A rational number can be written as a fraction. For example, $\frac{1}{2}$, 0.3, and $\sqrt{25}$.

2. In your own words, describe an *irrational number*. Give at least three different examples of irrational numbers.

   Sample response: An irrational number cannot be written as a fraction. For example, $\sqrt{2}$, $\sqrt[3]{63}$, and π.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I know what a rational number is and can give an example.
1 2 3

b. I know what an irrational number is and can give an example.
1 2 3

Lesson 6 Rational and Irrational Numbers

Success looks like . . .

- **Language Goal:** Understanding that a rational number is defined as a number that can be written as a fraction that represents the ratio of two integers. **(Speaking and Listening, Writing)**
  - » Describing a rational number in Problem 1.
- **Language Goal:** Comprehending that numbers that are not rational are called *irrational*. **(Speaking and Listening)**
  - » Describing an irrational number in Problem 2.

Suggested next steps

If students do not correctly describe a rational or irrational number, but provide correct examples, consider:

- Reviewing the definitions of a rational or irrational number.

If students do not provide correct examples of rational numbers or irrational numbers, consider:

- Reviewing Activity 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was for students to learn about irrational numbers. How well do you think your students understand the concept of irrational numbers and are they able to distinguish them from rational numbers?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Math Language Development

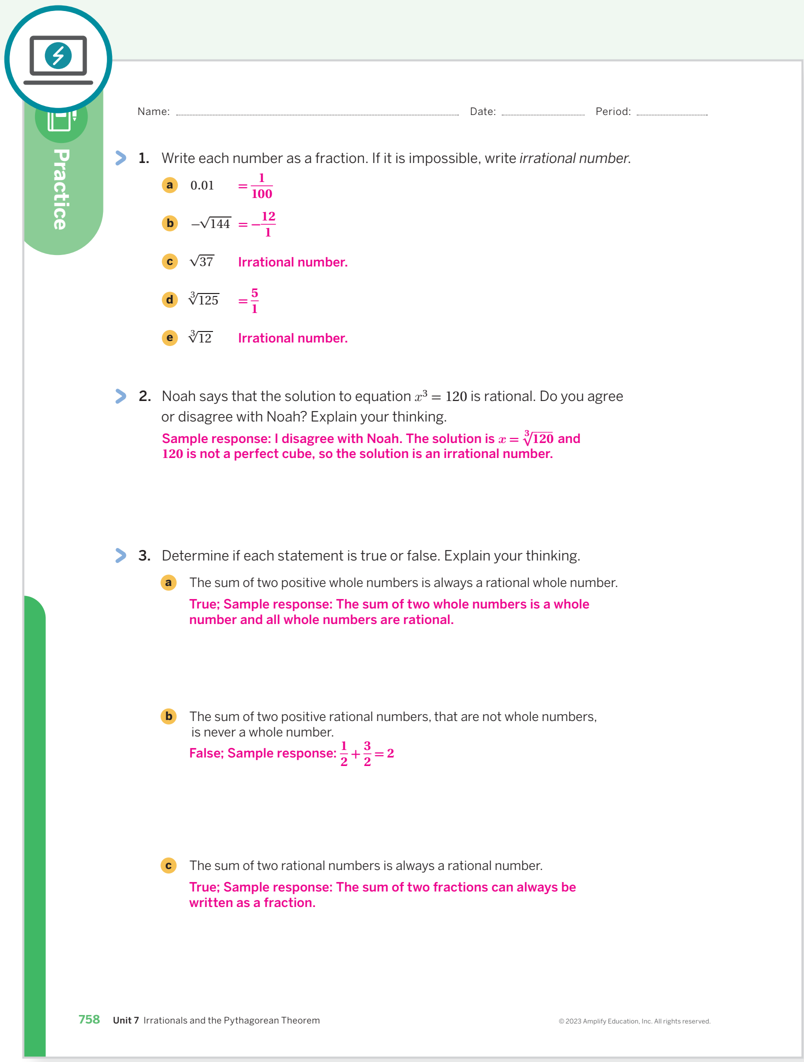
Language Goal: Understanding that a rational number is defined as a number that can be written as a fraction that represents the ratio of two integers.

Reflect on students' language development toward this goal.

- Do students' responses to the Exit Ticket problems indicate that they understand the meaning of a rational number or what makes a number irrational?
- What mathematical language are they using in their descriptions? Do they use terms such as *fraction* or *ratio*?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Write each number as a fraction. If it is impossible, write *irrational number.*
   - a 0.01 $= \frac{1}{100}$
   - b $-\sqrt{144}$ $= -\frac{12}{1}$
   - c $\sqrt{37}$ **Irrational number.**
   - d $\sqrt[3]{125}$ $= \frac{5}{1}$
   - e $\sqrt[3]{12}$ **Irrational number.**

2. Noah says that the solution to equation $x^3 = 120$ is rational. Do you agree or disagree with Noah? Explain your thinking.

   Sample response: I disagree with Noah. The solution is $x = \sqrt[3]{120}$ and 120 is not a perfect cube, so the solution is an irrational number.

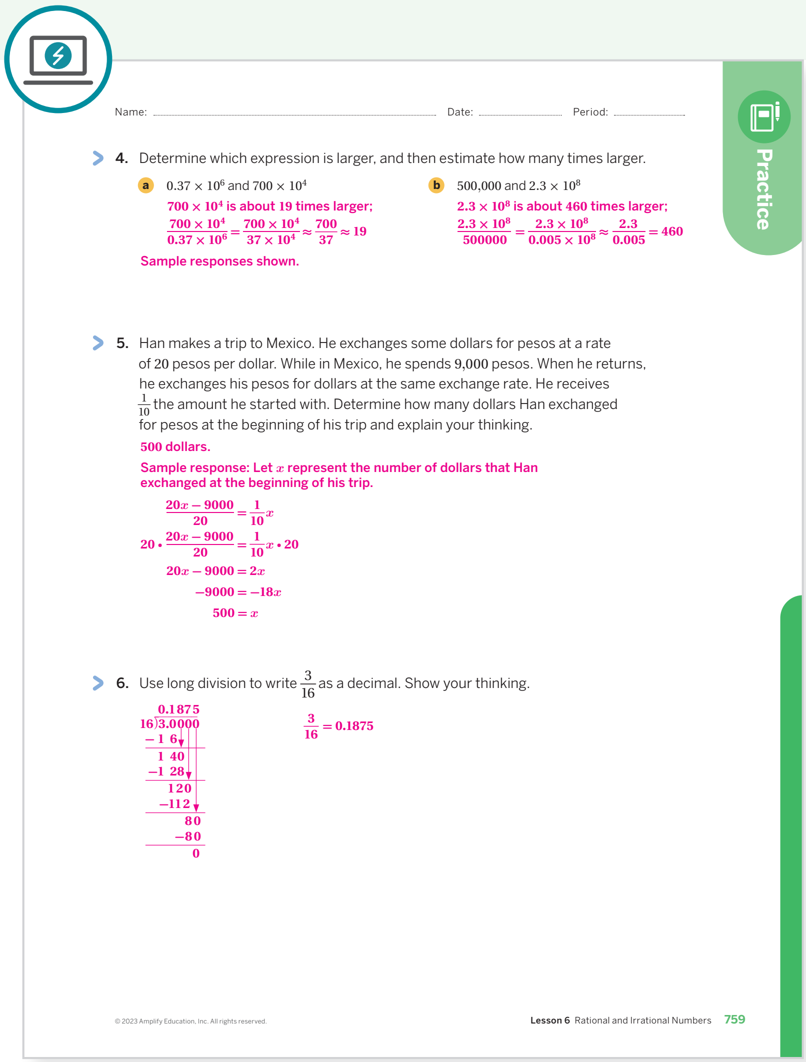
3. Determine if each statement is true or false. Explain your thinking.
   - a The sum of two positive whole numbers is always a rational whole number.

     True; Sample response: The sum of two whole numbers is a whole number and all whole numbers are rational.
   - b The sum of two positive rational numbers, that are not whole numbers, is never a whole number.

     False; Sample response: $\frac{1}{2} + \frac{3}{2} = 2$
   - c The sum of two rational numbers is always a rational number.

     True; Sample response: The sum of two fractions can always be written as a fraction.

758 **Unit 7** Irrationals and the Pythagorean Theorem



Name: ______ Date: ______ Period: ______

Practice

4. Determine which expression is larger, and then estimate how many times larger.
   - a 0.37×10^6 and 700×10^4

     700×10^4 is about 19 times larger;

     $$\frac{700 \times 10^4}{0.37 \times 10^6} = \frac{700 \times 10^4}{37 \times 10^4} \approx \frac{700}{37} \approx 19$$
   - b $500{,}000$ and 2.3×10^8

     2.3×10^8 is about 460 times larger;

     $$\frac{2.3 \times 10^8}{500000} = \frac{2.3 \times 10^8}{0.005 \times 10^8} \approx \frac{2.3}{0.005} = 460$$

   Sample responses shown.

5. Han makes a trip to Mexico. He exchanges some dollars for pesos at a rate of 20 pesos per dollar. While in Mexico, he spends 9,000 pesos. When he returns, he exchanges his pesos for dollars at the same exchange rate. He receives $\frac{1}{10}$ the amount he started with. Determine how many dollars Han exchanged for pesos at the beginning of his trip and explain your thinking.

   500 dollars.

   Sample response: Let x represent the number of dollars that Han exchanged at the beginning of his trip.

   $$\begin{aligned} \frac{20x - 9000}{20} &= \frac{1}{10}x \\ 20 \cdot \frac{20x - 9000}{20} &= \frac{1}{10}x \cdot 20 \\ 20x - 9000 &= 2x \\ -9000 &= -18x \\ 500 &= x \end{aligned}$$

6. Use long division to write $\frac{3}{16}$ as a decimal. Show your thinking.

   $$\begin{array}{r} 0.1875 \\ 16\overline{)3.0000} \\ -1\,6 \\ \hline 1\,40 \\ -1\,28 \\ \hline 120 \\ -112 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

   $$\frac{3}{16} = 0.1875$$

Lesson 6 Rational and Irrational Numbers 759

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	1
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Unit 6 Lesson 13	2
	5	Unit 4 Lesson 6	2
Formative ⚡	6	Unit 7 Lesson 7	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 7**

Decimal Representations of Rational Numbers

Let's learn more about how rational numbers can be represented.

Focus

Goals

1. Express a fraction as either a repeating or a terminating decimal.
2. Use bar notation to represent decimals that repeat.
3. **Language Goal:** Understand whether a unit fraction will repeat or terminate and explain the reason why. **(Speaking and Listening)**

Coherence

Today

Students look for and make use of structure to explore connections between fractions and their decimal representations. To support this exploration, students use long division to write fractions as decimals and use bar notation to write repeating decimals.

Previously

In Grade 7, students used long division to represent fractions as decimals. In Lesson 6, students saw that irrational numbers cannot be expressed as a ratio of integers.

Coming Soon

In Lesson 8, students will convert repeating decimals into fractions.

Rigor

- Students develop **procedural fluency** in identifying terminating and repeating decimals.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Small Groups	Small Groups	Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- calculators (optional)

Math Language Development

New words

- ***bar notation***
- ***repeating decimal***
- ***terminating decimal***

Review words

- *rational*
- *irrational*

Amps | Featured Activity

Activity 2
See Student Thinking

Students are asked to explain their thinking when determining which student's work is correct, and these explanations are available to you digitally, in real time.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Repeating decimals might cause students' stress levels to rise because they are infinitely long. Students might not be able to imagine how to work with such numbers. Explain that mathematicians have a beautiful structure by which to record repeating decimals. As they learn about bar notation, remind students to put the bar over only the part that repeats. This simple notation communicates a big concept.

● Modifications to Pacing

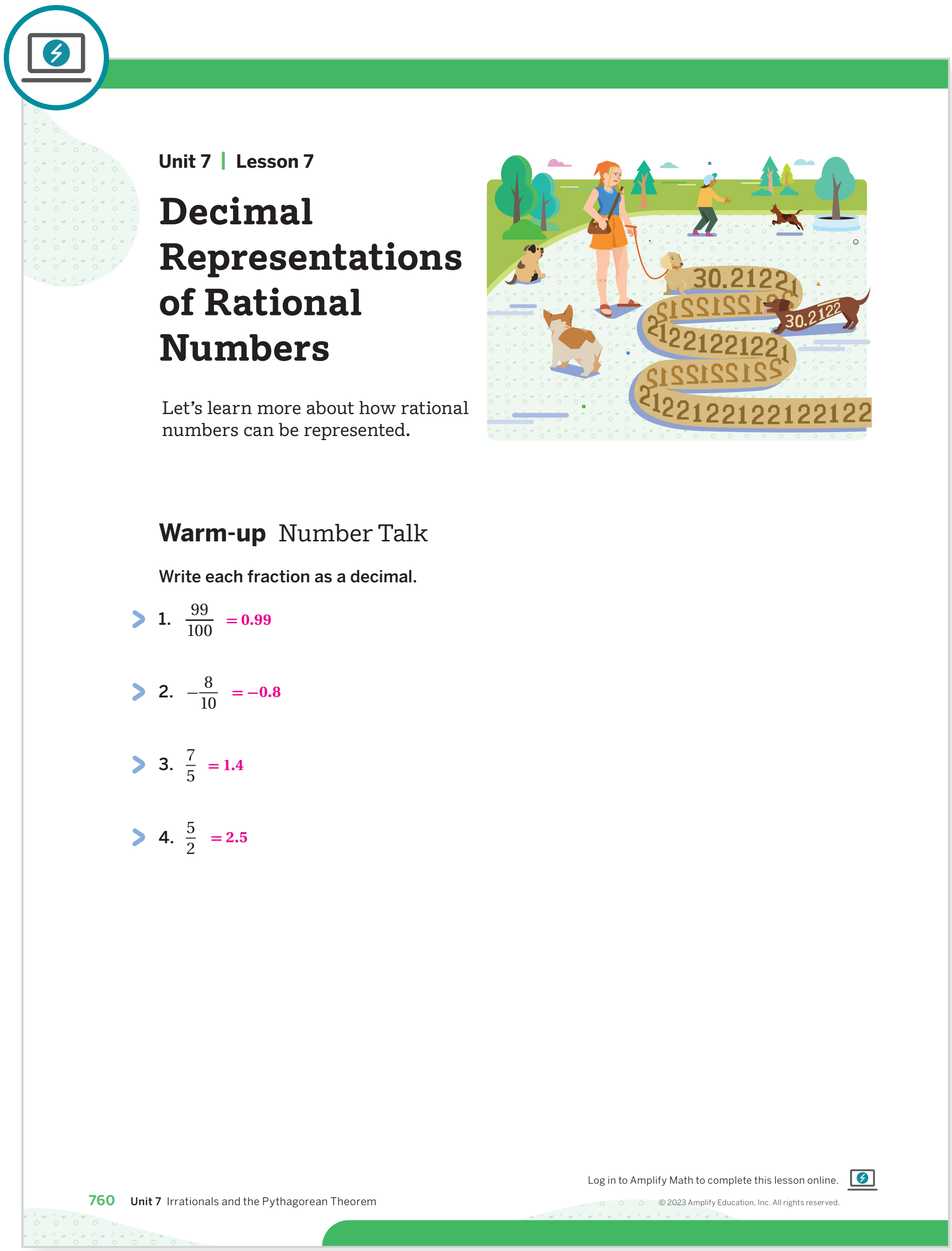
You may want to consider this additional modification if you are short on time.

- In **Activities 1 and 3**, consider allowing students to use calculators.

Small Groups | 5 min

Warm-up Number Talk

Students write fractions as decimals to review different strategies to write fractions as decimals.



Unit 7 | Lesson 7

Decimal Representations of Rational Numbers

Let's learn more about how rational numbers can be represented.

Warm-up Number Talk

Write each fraction as a decimal.

1. $\frac{99}{100}$ $= 0.99$

2. $-\frac{8}{10}$ $= -0.8$

3. $\frac{7}{5}$ $= 1.4$

4. $\frac{5}{2}$ $= 2.5$

Log in to Amplify Math to complete this lesson online.

760 **Unit 7** Irrationals and the Pythagorean Theorem

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1 Launch

Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by having them brainstorm the different ways a fraction could be written as a decimal with their group before the Warm-up is completed.

Look for productive strategies:

- For Problems 1 and 2, using place values of a decimal to help write the fraction as a decimal.
- For Problems 3 and 4, writing an equivalent fraction with a base of 10 or 100 before writing a decimal.
- Using long division to write a fraction as a decimal.

3 Connect

Ask students whether the number in each problem represents a rational or irrational number and why. All of the numbers represent a rational number because they are written as a ratio of integers.

Have students share their strategies for writing each fraction as a decimal and why that strategy worked.

Highlight the different strategies for writing fractions as decimals such as using place value and using long division. Also, highlight that students can use long division to write *any* fraction as a decimal, so any fraction can be written as a decimal.

Power-up

To power up students' ability to use long division to write a fraction as a decimal, have students complete:

Determine which of the following correctly show how to use long division to rewrite $\frac{5}{8}$ as a decimal.

A. $$\begin{array}{r} 1.6 \\ 5\overline{)8.0} \\ -5 \\ \hline 30 \\ -30 \\ \hline 0 \end{array}$$

[✓ B.] $$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -0 \\ \hline 50 \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

C. $$\begin{array}{r} 0625 \\ 8\overline{)5000} \\ -0 \\ \hline 50 \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

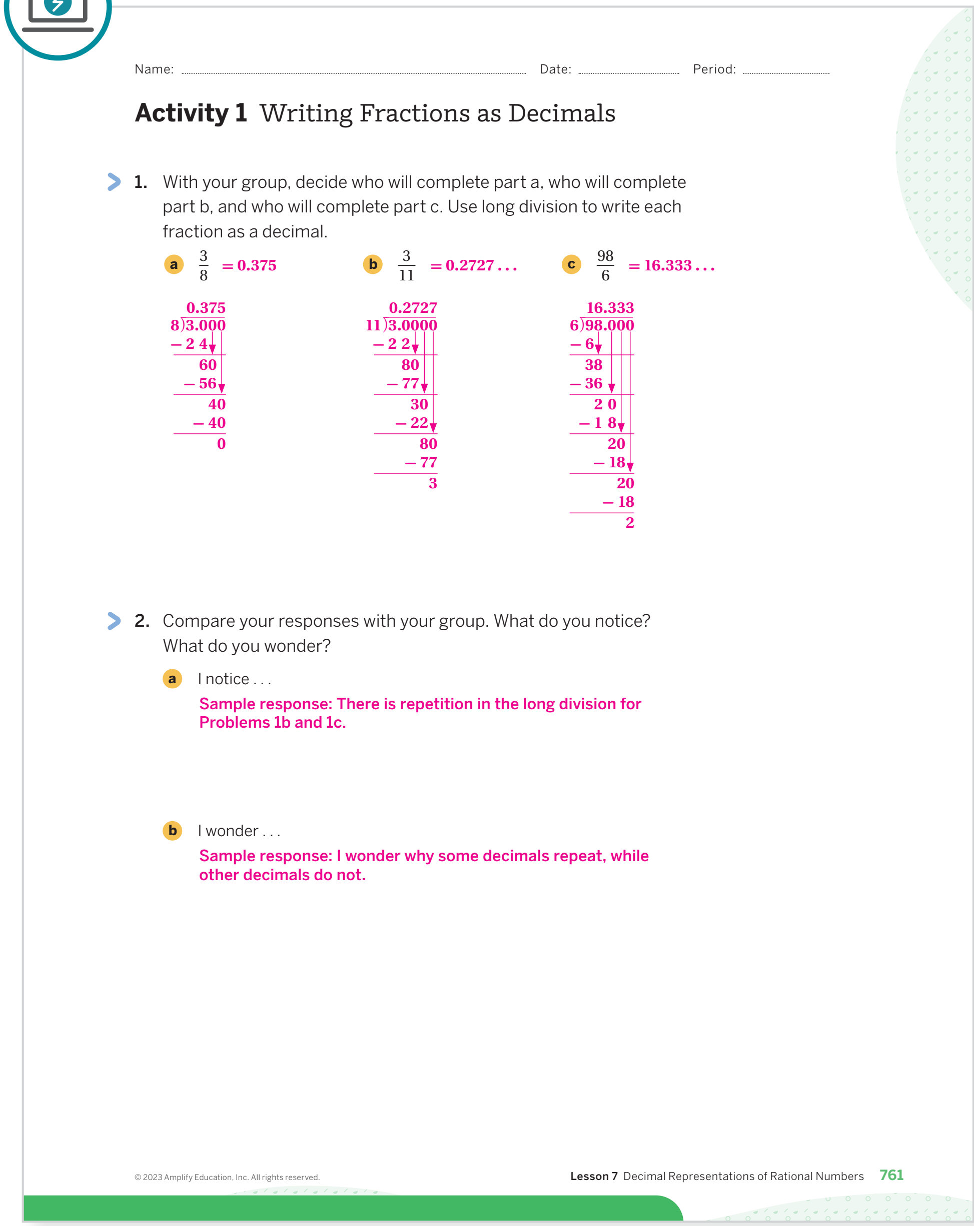
Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5

Small Groups | 10 min

Activity 1 Writing Fractions as Decimals

Students write fractions as decimals as an introduction to terminating and repeating decimals.



Name: ______ Date: ______ Period: ______

Activity 1 Writing Fractions as Decimals

1. With your group, decide who will complete part a, who will complete part b, and who will complete part c. Use long division to write each fraction as a decimal.

a $\frac{3}{8} = 0.375$

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\,4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

b $\frac{3}{11} = 0.2727\ldots$

$$\begin{array}{r} 0.2727 \\ 11\overline{)3.0000} \\ -2\,2 \\ \hline 80 \\ -77 \\ \hline 30 \\ -22 \\ \hline 80 \\ -77 \\ \hline 3 \end{array}$$

c $\frac{98}{6} = 16.333\ldots$

$$\begin{array}{r} 16.333 \\ 6\overline{)98.000} \\ -6 \\ \hline 38 \\ -36 \\ \hline 2\,0 \\ -1\,8 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

2. Compare your responses with your group. What do you notice? What do you wonder?

a I notice . . .

Sample response: There is repetition in the long division for Problems 1b and 1c.

b I wonder . . .

Sample response: I wonder why some decimals repeat, while other decimals do not.

1 Launch

Have students choose one fraction from Problem 1 to complete, and then have them share their response with their group.

2 Monitor

Help students get started by reminding students how to set up each problem as a long division problem.

Look for points of confusion:

- **Not rewriting the dividend as a decimal.** Remind students that a whole number could be written as a decimal with an infinite amount of zeros.

Look for productive strategies:

- Noticing that there is repetition in two of the problems, and no repetition in the other problem.

3 Connect

Display correct student work showing each completed problem. Then conduct the *Notice and Wonder* routine.

Highlight that all rational numbers have a decimal expansion that repeats. Some decimals, such as one in part a, repeat 0, so they are called ***terminating decimals***.

Define the term ***repeating decimal***. Say, "A repeating decimal has the same sequence of digits that repeat indefinitely." The repeating digits are marked using ***bar notation***, which shows a line above the repeating digits." Write the answers for parts b and c as $0.\overline{27}$ and $16.\overline{3}$ for all to see. Be careful in the use of the word *pattern* as it can be ambiguous. For example, there is a *pattern* to the digits of the number $0.12112111211112\ldots$, but the number is not rational.

Ask, "How is $0.\overline{27}$ different than $0.2\overline{7}$?" 0.27 repeats in $0.\overline{27}$, but only 7 repeats in $0.2\overline{7}$.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously used long division to write fractions as decimals. Ask, "What number represents the dividend and where should it be placed when setting up the long division?" Repeat for the divisor.

Extension: *Math Enrichment*

Have students use calculators to determine the decimal expansions for fractions with denominators of 9. Have them describe what they notice. Sample response: All of the digits repeat and the repeating digit is the same digit as the numerator of the fraction. Some students may notice that if the pattern were to continue, $\frac{9}{9}$ would be $0.\overline{9}$, however $\frac{9}{9} = 1$. This can make for a rich class discussion.

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to describe and compare terminating and repeating decimals in their own words. Listen for and amplify language used to indicate that a repeating decimal repeats non-zero digits and a terminating decimal repeats 0s, which essentially means it ends (terminates). Add the language students use to describe these terms, along with examples, to the class display.

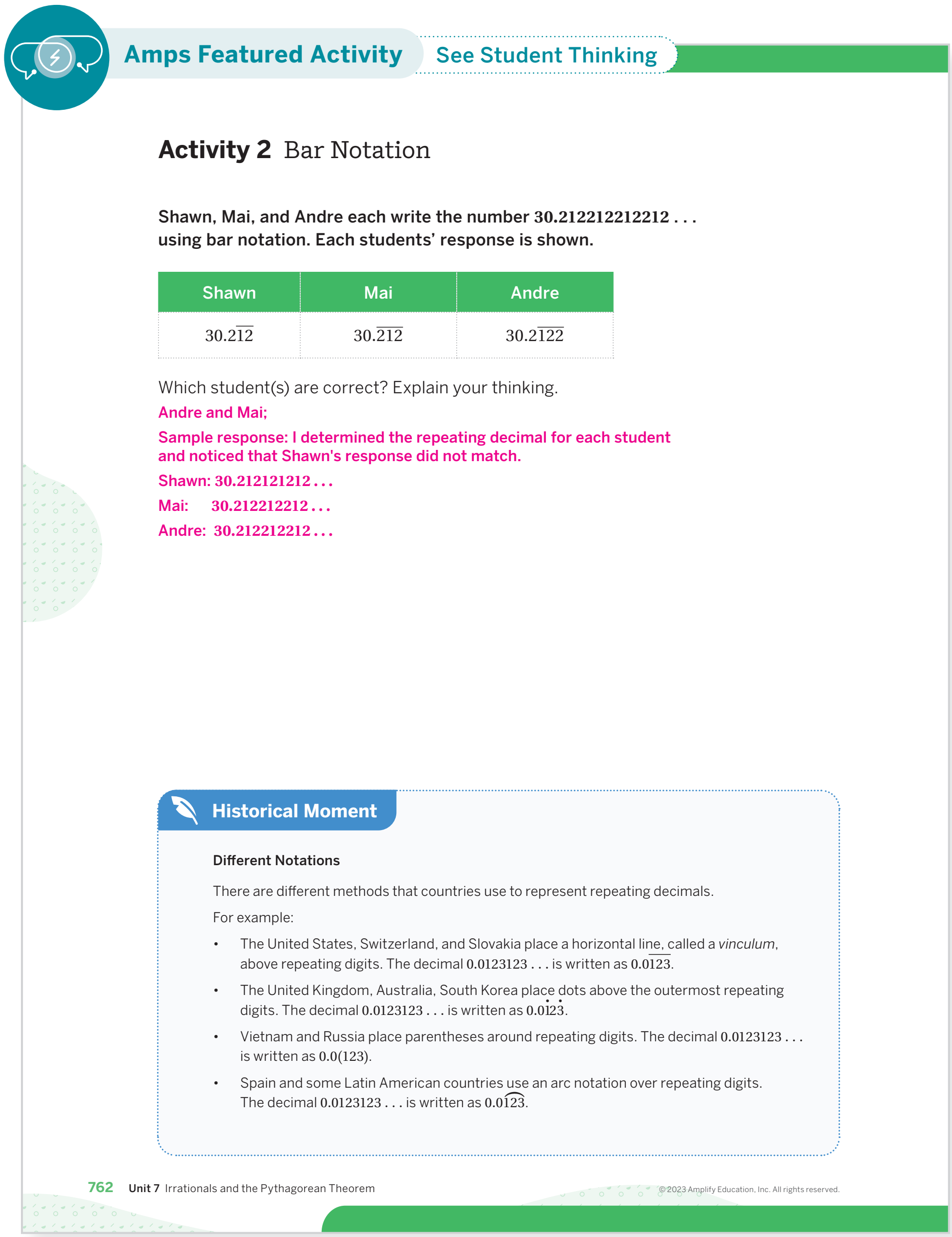
Terminating decimals	Repeating decimals
Repeats 0s. Terminates (ends) because 0s do not have to be written.	Repeats non-zero digits. Repeats forever.

Independent | 10 min

Activity 2 Bar Notation

Students attend to precision critique the reasoning of others to build fluency in writing repeating decimals using bar notation.

Amps Featured Activity See Student Thinking



Activity 2 Bar Notation

Shawn, Mai, and Andre each write the number 30.212212212212 . . . using bar notation. Each students' response is shown.

Shawn	Mai	Andre
$30.2\overline{12}$	$30.\overline{212}$	$30.2\overline{122}$

Which student(s) are correct? Explain your thinking.

Andre and Mai;

Sample response: I determined the repeating decimal for each student and noticed that Shawn's response did not match.

Shawn: 30.212121212 . . .

Mai: 30.212212212 . . .

Andre: 30.212212212 . . .

Historical Moment

Different Notations

There are different methods that countries use to represent repeating decimals.

For example:

- The United States, Switzerland, and Slovakia place a horizontal line, called a *vinculum*, above repeating digits. The decimal 0.0123123 . . . is written as $0.0\overline{123}$.
- The United Kingdom, Australia, South Korea place dots above the outermost repeating digits. The decimal 0.0123123 . . . is written as $0.0\dot{1}2\dot{3}$.
- Vietnam and Russia place parentheses around repeating digits. The decimal 0.0123123 . . . is written as 0.0(123).
- Spain and some Latin American countries use an arc notation over repeating digits. The decimal 0.0123123 . . . is written as $0.0\overset{\frown}{123}$.

762 **Unit 7** Irrationals and the Pythagorean Theorem

1 Launch

Draw students' attention to the placement of the line above each number.

2 Monitor

Help students get started by having them write Shawn's, Mai's, and Andre's numbers as repeating decimals.

Look for points of confusion:

- **Thinking that Shawn is correct.** Have students write at least 9 digits after the decimal point to help them identify patterns that do not match. Encourage students to group numbers by three to help them look for patterns.

Look for productive strategies:

- Noticing that more than one student is correct.

3 Connect

Have students share their responses. Use the *Poll the Class* routine to determine the students responses. Have students share anything that made it difficult to identify the repeating decimal using bar notation.

Ask, "How are each of Shawn's, Mai's, and Andre's numbers written as a repeating decimal?"

Highlight that the same decimal could be represented using a different ***bar notation***. Also, highlight that students should carefully assess each decimal, as the repeated numbers may not start immediately after the decimal point, and may have one or more repeating digits.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write their response, have them meet with 2–3 partners to both give and receive feedback on their responses and explanations. Encourage reviewers to ask clarifying questions such as:

- "Why do you think ___ is not correct? What error or mistake did they make?"
- "What is the repeating group of digits in the number 30.212212212212 . . .?"
- "What is the repeating group of digits in ___ response?"

Have students revise their responses, as needed.

English Learners

Group students together who speak the same primary language. This will give students an opportunity to participate in conversation and written feedback with peers from the same linguistic background.

Historical Moment

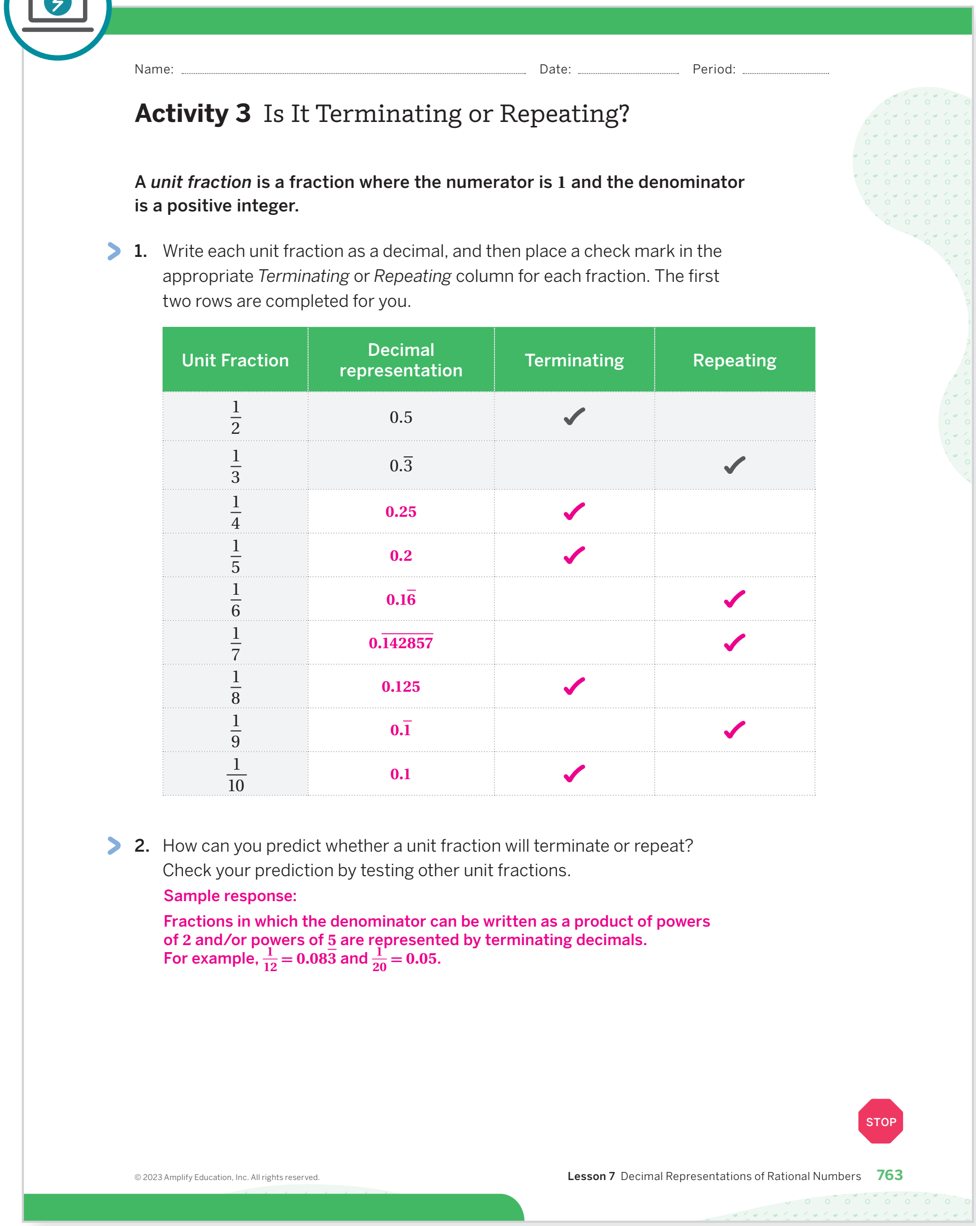
Different Notations

Have students read the Historical Moment to learn about the different ways countries represent repeating decimals.

Small Groups | 15 min

Activity 3 Is it Terminating or Repeating?

Students explore connections between unit fractions and their decimal representations to determine which unit fractions terminate or repeat.



Name: ______ Date: ______ Period: ______

Activity 3 Is It Terminating or Repeating?

A *unit fraction* is a fraction where the numerator is 1 and the denominator is a positive integer.

1. Write each unit fraction as a decimal, and then place a check mark in the appropriate *Terminating* or *Repeating* column for each fraction. The first two rows are completed for you.

Unit Fraction	Decimal representation	Terminating	Repeating
$\frac{1}{2}$	0.5	✓	
$\frac{1}{3}$	$0.\overline{3}$		✓
$\frac{1}{4}$	0.25	✓	
$\frac{1}{5}$	0.2	✓	
$\frac{1}{6}$	$0.1\overline{6}$		✓
$\frac{1}{7}$	$0.\overline{142857}$		✓
$\frac{1}{8}$	0.125	✓	
$\frac{1}{9}$	$0.\overline{1}$		✓
$\frac{1}{10}$	0.1	✓	

2. How can you predict whether a unit fraction will terminate or repeat? Check your prediction by testing other unit fractions.

Sample response:

Fractions in which the denominator can be written as a product of powers of 2 and/or powers of 5 are represented by terminating decimals. For example, $\frac{1}{12} = 0.08\overline{3}$ and $\frac{1}{20} = 0.05$.

STOP

Lesson 7 Decimal Representations of Rational Numbers 763

1 Launch

Have students choose 2–3 rows of the table each to complete in small groups, and then have them share their responses with a partner. Provide access to additional paper to allow students to show their work.

2 Monitor

Help students get started by having them use any strategy to write each fraction as a decimal. Allow access to calculators.

Look for points of confusion:

- **Struggling with Problem 2.** Have students focus on the fractions they marked as terminating. Then have them write each denominator in factored form to look for any patterns.

3 Connect

Display a student's completed table.

Have groups of students share their prediction for whether a unit fraction will terminate or repeat.

Highlight that for unit fractions, if the factors of the denominator consists of only 2s and 5s, then the decimal representation will terminate. Otherwise, it will repeat.

Ask students to think of another unit fraction that will terminate. Sample response: $\frac{1}{20} = \frac{1}{5 \cdot 2 \cdot 2}$ **Note:** some students may think that some repeating fractions terminate because the result on a calculator shows a finite number of digits. Encourage them to be critical thinkers by comparing the decimal representation for $\frac{2}{3}$ using long division $(0.666\ldots)$ with the calculator display (0.6666667).

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they learned about unit fractions in prior grades. Ask:
"Is $\frac{1}{3}$ a unit fraction? Why or why not?"
"Is $\frac{2}{3}$ a unit fraction? Why or why not?"
"What is true about all of the fractions listed in the table?"

Extension: *Math Enrichment*

Have students determine whether the unit fraction $\frac{1}{2408}$ will terminate or repeat. Ask them to explain their thinking. Terminate; Sample response: The denominator 2,408 can be written as 2^{11}, which are factors of 2.

Math Language Development

MLR8: Discussion Supports

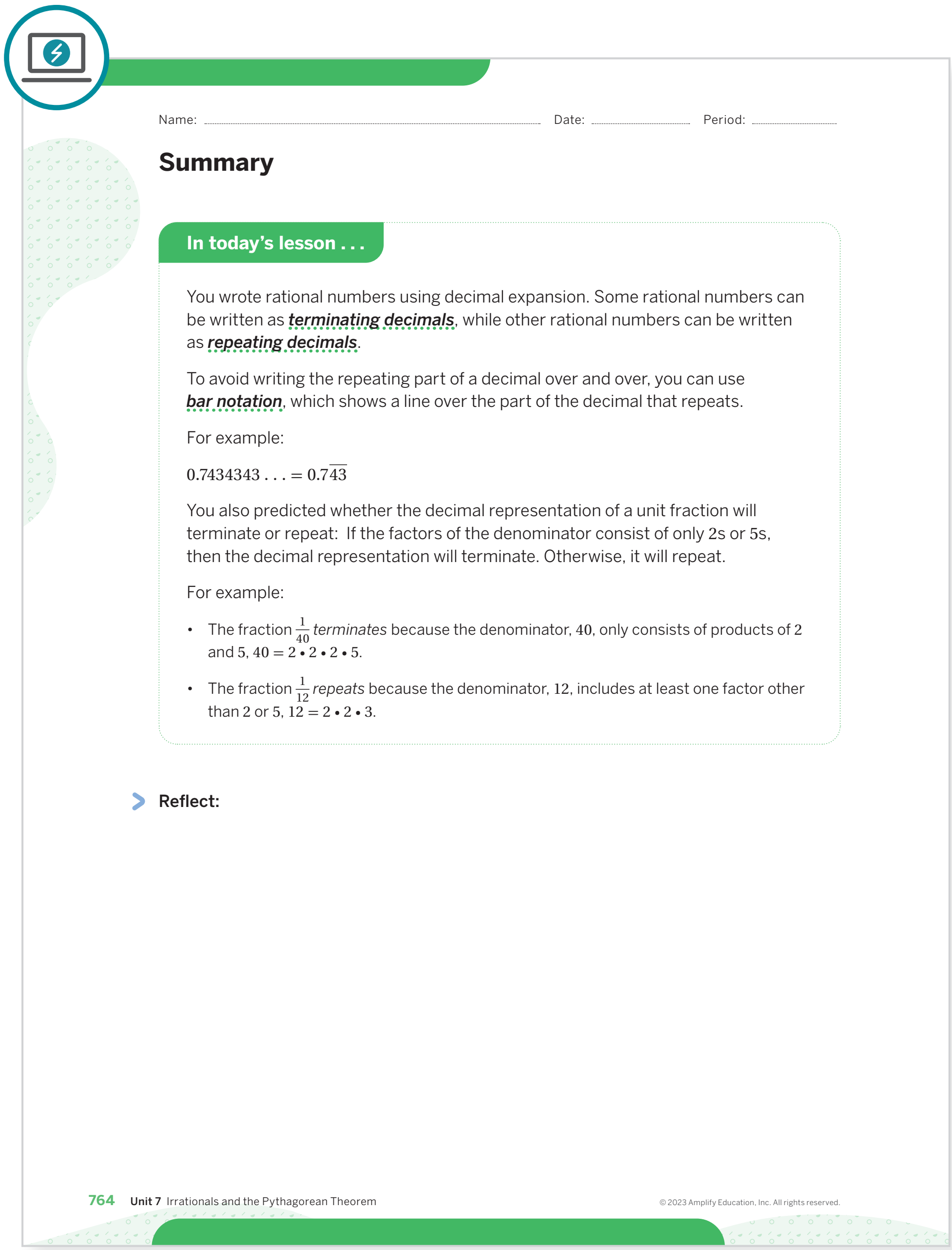
During the Connect, as you highlight the relationship between unit fractions and their corresponding decimal expansions, add these statements to the class display and have students complete them. Have students brainstorm examples that correspond with each statement.

- "If the denominator of a unit fraction can be written as powers of 2 and/or 5, the decimal expansion will ___." terminate
- "If the denominator of a unit fraction contains factors other than 2s or 5s, the decimal expansion will ___." repeat

Whole Class | 5 min

Summary

Review and synthesize how to write fractions as terminating and repeating decimals.



Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You wrote rational numbers using decimal expansion. Some rational numbers can be written as ***terminating decimals***, while other rational numbers can be written as ***repeating decimals***.

To avoid writing the repeating part of a decimal over and over, you can use ***bar notation***, which shows a line over the part of the decimal that repeats.

For example:

$0.7434343\ldots = 0.7\overline{43}$

You also predicted whether the decimal representation of a unit fraction will terminate or repeat: If the factors of the denominator consist of only 2s or 5s, then the decimal representation will terminate. Otherwise, it will repeat.

For example:

- The fraction $\frac{1}{40}$ *terminates* because the denominator, 40, only consists of products of 2 and 5, $40 = 2 \bullet 2 \bullet 2 \bullet 5$.
- The fraction $\frac{1}{12}$ *repeats* because the denominator, 12, includes at least one factor other than 2 or 5, $12 = 2 \bullet 2 \bullet 3$.

Reflect:

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Synthesize

Have students share which strategy they prefer when determining whether the decimal representation of a fraction will terminate or repeat and why.

Highlight that students could use the structure of a unit fraction to determine whether it will terminate or repeat.

Formalize vocabulary:

- *bar notation*
- *repeating decimal*
- *terminating decimal*

Ask students how they think they could determine whether a fraction that is not a unit fraction terminates or repeats.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What are some strategies to determine whether a fraction will terminate or repeat?"

Math Language Development

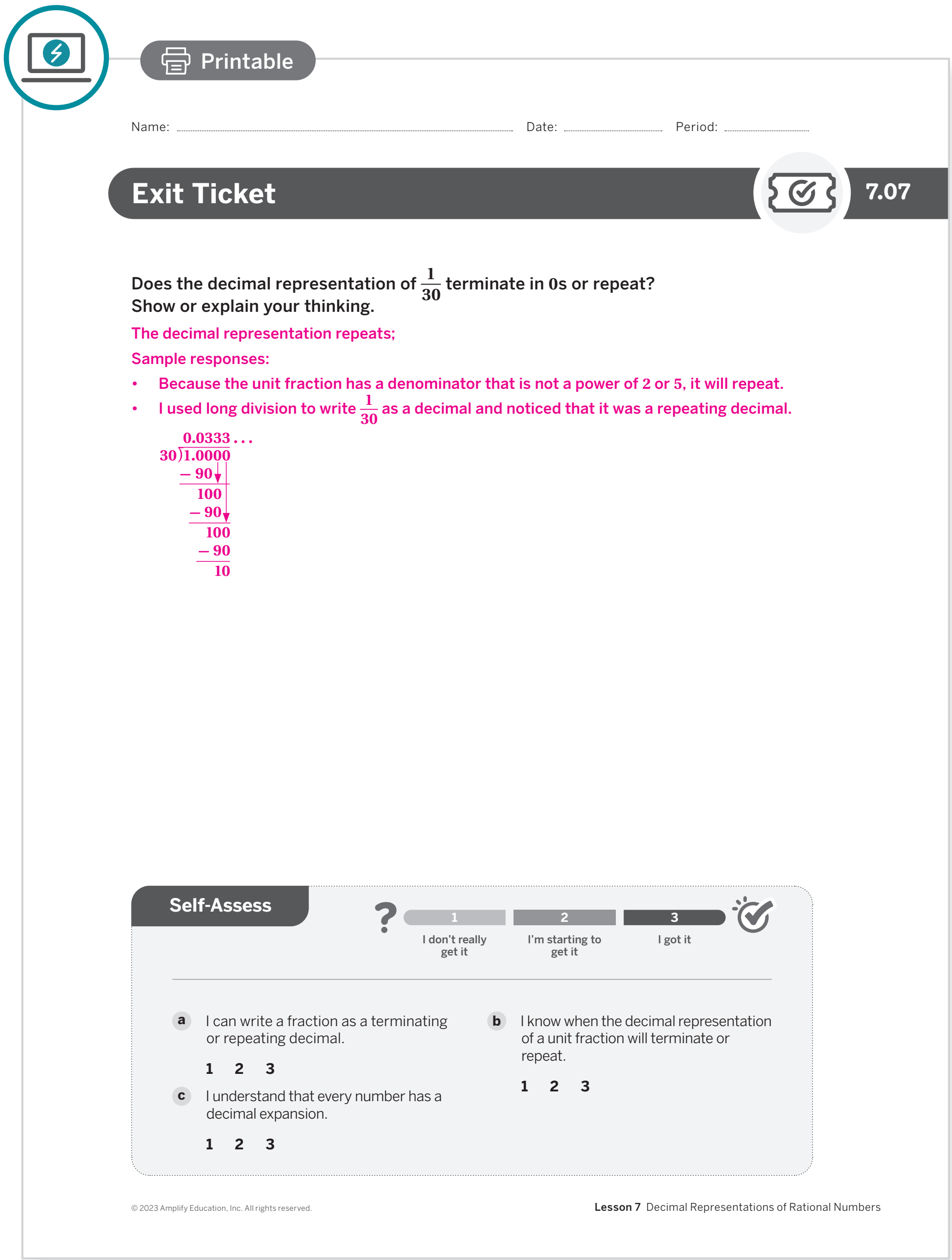
MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started for this unit. Ask them to review and reflect on any terms and phrases related to the terms *bar notation*, *repeating decimal*, and *terminating decimal* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining whether the given fraction will terminate or repeat.



Printable

Name: ______ Date: ______ Period: ______

Exit Ticket

7.07

Does the decimal representation of $\frac{1}{30}$ terminate in 0s or repeat? Show or explain your thinking.

The decimal representation repeats;

Sample responses:

- Because the unit fraction has a denominator that is not a power of 2 or 5, it will repeat.
- I used long division to write $\frac{1}{30}$ as a decimal and noticed that it was a repeating decimal.

$$\begin{array}{r} 0.0333\ldots \\ 30\overline{)1.0000} \\ -\,90 \\ \hline 100 \\ -\,90 \\ \hline 100 \\ -\,90 \\ \hline 10 \end{array}$$

Self-Assess

a I can write a fraction as a terminating or repeating decimal.

1 2 3

b I know when the decimal representation of a unit fraction will terminate or repeat.

1 2 3

c I understand that every number has a decimal expansion.

1 2 3

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Lesson 7 Decimal Representations of Rational Numbers

Success looks like . . .

- **Goal:** Expressing a fraction as either a repeating or a terminating decimal.
  - » Determining the decimal representation of $\frac{1}{30}$ as repeating.
- **Goal:** Using bar notation to represent decimals that repeat.
- **Language Goal:** Understanding whether a unit fraction will repeat or terminate and explaining the reason why. **(Speaking and Listening)**
  - » Showing why $\frac{1}{30}$ can be expressed as a repeating decimal.

Suggested next steps

If students use long division to determine their response, consider:

- Reviewing Activity 3.
- Asking, "How could you use the structure of the unit fraction to determine whether the decimal representation will terminate or repeat?"

Professional Learning

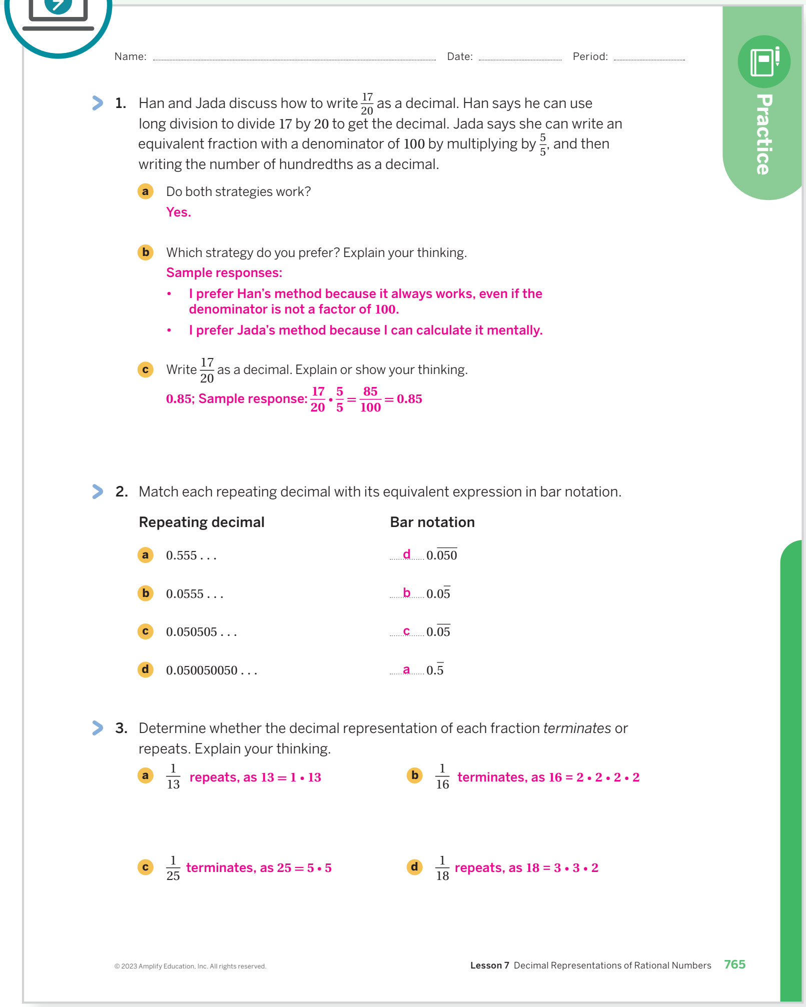
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- During the discussion in Activity 3, how did you encourage each student to listen to one another's strategies?

Practice

Independent



Practice

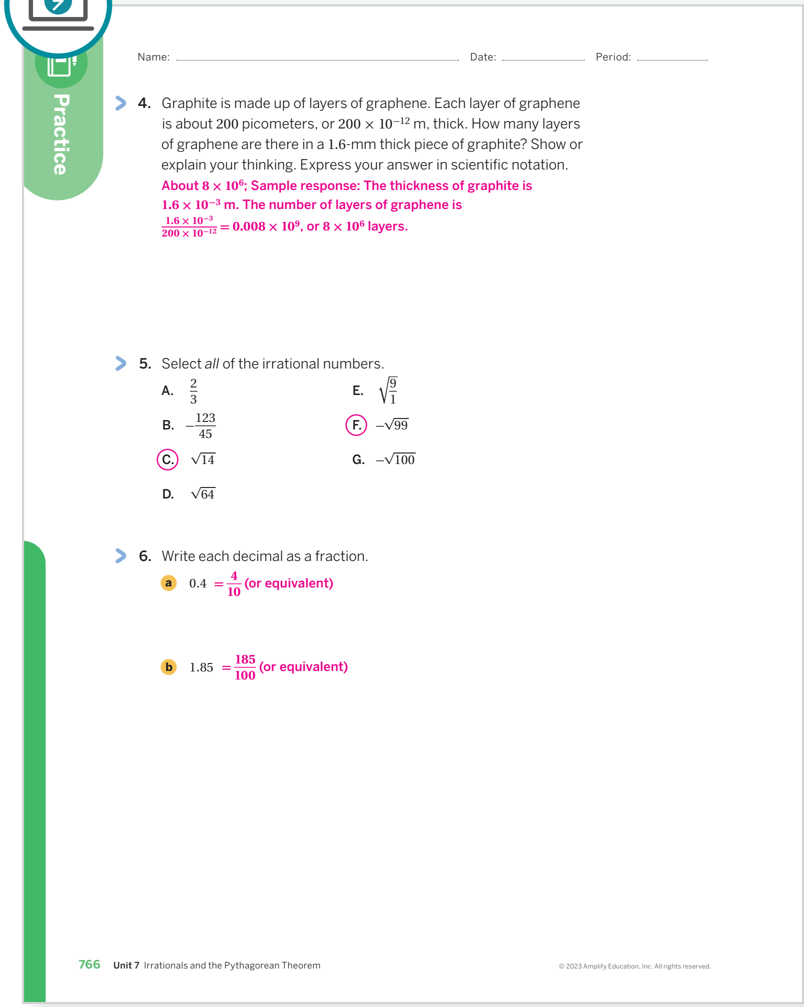
Name: Date: Period:

1. Han and Jada discuss how to write $\frac{17}{20}$ as a decimal. Han says he can use long division to divide 17 by 20 to get the decimal. Jada says she can write an equivalent fraction with a denominator of 100 by multiplying by $\frac{5}{5}$, and then writing the number of hundredths as a decimal.
   - a Do both strategies work?
     Yes.
   - b Which strategy do you prefer? Explain your thinking.
     Sample responses:
     - **I prefer Han's method because it always works, even if the denominator is not a factor of 100.**
     - **I prefer Jada's method because I can calculate it mentally.**
   - c Write $\frac{17}{20}$ as a decimal. Explain or show your thinking.
     0.85; Sample response: $\frac{17}{20} \cdot \frac{5}{5} = \frac{85}{100} = 0.85$

2. Match each repeating decimal with its equivalent expression in bar notation.

Repeating decimal	Bar notation
a $0.555\ldots$	d $0.\overline{050}$
b $0.0555\ldots$	b $0.0\overline{5}$
c $0.050505\ldots$	c $0.\overline{05}$
d $0.050050050\ldots$	a $0.\overline{5}$

3. Determine whether the decimal representation of each fraction *terminates* or repeats. Explain your thinking.
   - a $\frac{1}{13}$ **repeats, as $13 = 1 \cdot 13$**
   - b $\frac{1}{16}$ **terminates, as $16 = 2 \cdot 2 \cdot 2 \cdot 2$**
   - c $\frac{1}{25}$ **terminates, as $25 = 5 \cdot 5$**
   - d $\frac{1}{18}$ **repeats, as $18 = 3 \cdot 3 \cdot 2$**



Practice

Name: Date: Period:

4. Graphite is made up of layers of graphene. Each layer of graphene is about 200 picometers, or 200×10^{-12} m, thick. How many layers of graphene are there in a 1.6-mm thick piece of graphite? Show or explain your thinking. Express your answer in scientific notation.
   About 8×10^6; Sample response: The thickness of graphite is 1.6×10^{-3} m. The number of layers of graphene is $\frac{1.6 \times 10^{-3}}{200 \times 10^{-12}} = 0.008 \times 10^9$, or 8×10^6 layers.

5. Select *all* of the irrational numbers.
   - A. $\frac{2}{3}$
   - B. $-\frac{123}{45}$
   - [✓ C.] $\sqrt{14}$
   - D. $\sqrt{64}$
   - E. $\sqrt{\frac{9}{1}}$
   - [✓ F.] $-\sqrt{99}$
   - G. $-\sqrt{100}$

6. Write each decimal as a fraction.
   - a $0.4 = \frac{4}{10}$ **(or equivalent)**
   - b $1.85 = \frac{185}{100}$ **(or equivalent)**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 6 Lesson 13	2
	5	Unit 7 Lesson 6	1
Formative ⚡	6	Unit 7 Lesson 8	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

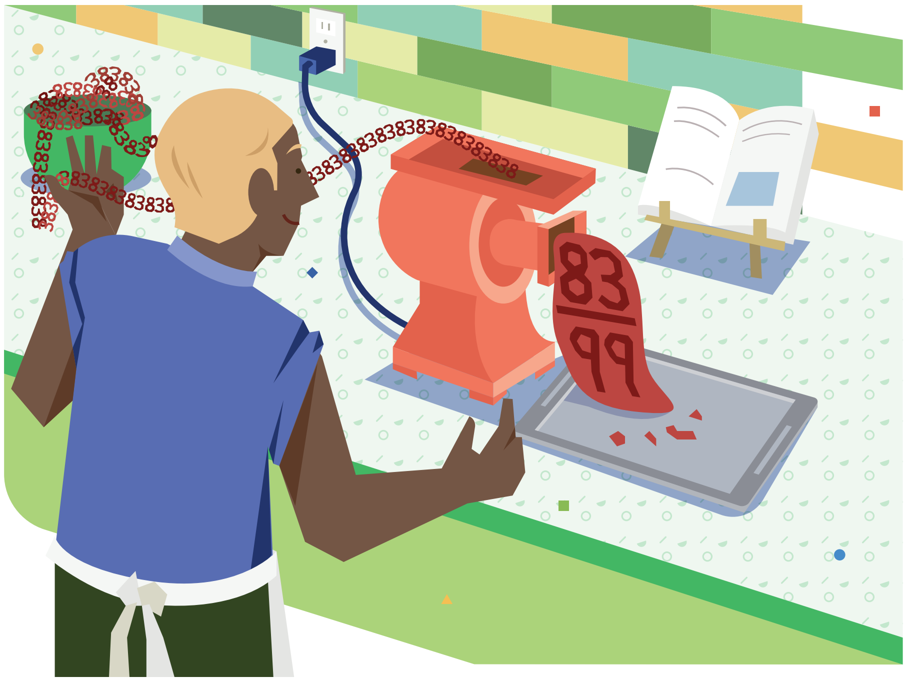
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 8**

Converting Repeating Decimals Into Fractions

Let's convert repeating decimals into fractions.



Focus

Goals

1. **Language Goal:** Compare and contrast decimal expansions for rational and irrational numbers. **(Speaking and Listening)**
2. **Language Goal:** Coordinate repeating decimal expansions and rational numbers that represent the same number. **(Speaking and Listening, Writing)**

Coherence

Today

Students are introduced to an algorithm for rewriting repeating decimals as fractions. Students see that the decimal expansion of an irrational number must be non-repeating and non-terminating.

Previously

In Lesson 7, students reviewed long division and used bar notation to write repeating decimals.

Coming Soon

In the second Sub-Unit, students will explore the Pythagorean Theorem and solve problems utilizing it.

Rigor

- Students develop the **procedural skills** necessary to convert repeating decimals to fractional representations.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart, *Representing Repeating Decimals as Fractions*
- calculators (optional)

Math Language Development

Review words

- *bar notation*
- *irrational number*
- *long division*
- *rational number*
- *repeating decimal*
- *terminating decimal*

Amps | Featured Activity

Exit Ticket
Real-Time Exit Ticket

Check in real time whether your students can express a repeating decimal as a fraction using a digital Exit Ticket that is automatically scored.

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Building Math Identity and Community

Connecting to Mathematical Practices

Converting repeating decimals into a fraction might seem like an impossible task to some students. Help students regulate their emotions by providing a step-by-step guide that explains the quantitative reasoning behind each step in the algorithm. As a whole class, invite students to contribute to this "cheat sheet" where they can be reminded what they have to do.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, omit Problems 1 and 4.
- In **Activity 2**, have students only complete two problems.

Independent | 5 min

Warm-up Working With Decimals

Students perform operations with decimals including repeating decimals to prepare for converting repeating decimals into fractions.

Name: Date: Period:

Unit 7 | Lesson 8

Converting Repeating Decimals Into Fractions

Let's convert repeating decimals into fractions.

Warm-up Working With Decimals

Without using a calculator, evaluate each expression.

1. $100.321 - 99.321$ $= 1$
2. $3.\overline{45} \cdot 100$ $= 345.\overline{45}$
3. $18.\overline{33} - 1.3\overline{3}$ $= 17$
4. $18.\overline{33} - 1.8\overline{3}$ $= 16.5$

Compare and Connect: If you subtract the repeating part of a decimal, what happens? Be ready to share your thoughts during the class discussion.

Log in to Amplify Math to complete this lesson online.

Lesson 8 Converting Repeating Decimals Into Fractions 767

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by having them write a few iterations of the repeating decimals and write the subtraction problems vertically lining up the place values.

Look for points of confusion:

- **Thinking that Problem 2 is equivalent to 345.** Ask students to divide 345 by 100 to see if they get the original factor.
- **Thinking they cannot subtract in Problem 4.** Have students write a few iterations of the repetition such as $18.3\overline{33} - 1.83\overline{3}$.

Look for productive strategies:

- Writing multiple iterations of the repetition to make sense of the problem

3 Connect

Display the Warm-up problems and have students share their responses and strategies.

Ask, "If you multiplied by 10 instead of 100 in Problem 2, how would that change the answer? Multiplied by 1,000?

Highlight the structure of the numbers and what the repeating bar signifies in each number. The skills from Problems 2 and 3 will be used in the following activities. Consider providing more examples to ensure students understand how multiplying by factors of 10 changes the number and how subtraction can eliminate the repeating decimal values.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses and strategies, ask them to also share their response to the question posed to them in their Student Edition, "If you subtract the repeating part of a decimal, what happens?" Consider chunking this question into smaller parts by asking:

- "In Problem 3, what repeating digits were subtracted? What digits remained and in which place value positions?"
- "How is Problem 4 similar to or different from Problem 3? Can you set the problem up vertically to help you?"

Power-up

To power up students' ability to write terminating decimals as fractions, have students complete:

Recall that terminating decimals can be rewritten as fractions using the place value of the final digit. For example $1.04 = \frac{104}{100}$ because the final digit 4 is in the hundredths place.

Write each decimal as a fraction.

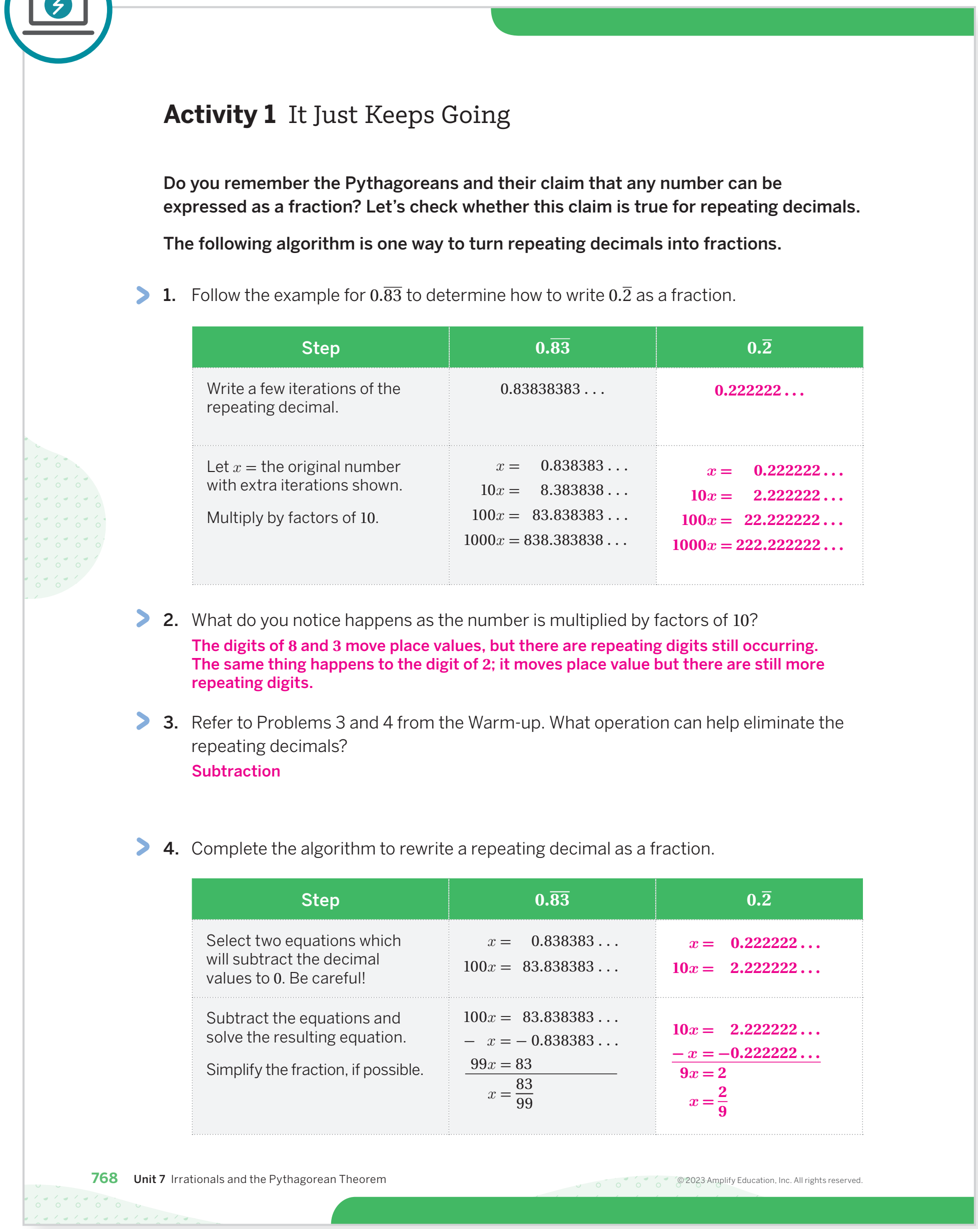
1. $0.02 = \frac{2}{100}$ or $\frac{1}{50}$ **2.** $0.18 = \frac{18}{100}$ or $\frac{9}{50}$ **3.** $2.005 = \frac{2005}{1000}$ or $2\frac{1}{200}$

Use: Before the Warm-up
Informed by: Performance on Lesson 7, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 15 min

Activity 1 It Just Keeps Going

Students review an example of one algorithm for turning repeating decimals into fractions and practice one with a partner.



Activity 1 It Just Keeps Going

Do you remember the Pythagoreans and their claim that any number can be expressed as a fraction? Let's check whether this claim is true for repeating decimals.

The following algorithm is one way to turn repeating decimals into fractions.

1. Follow the example for $0.\overline{83}$ to determine how to write $0.\overline{2}$ as a fraction.

Step	$0.\overline{83}$	$0.\overline{2}$
Write a few iterations of the repeating decimal.	$0.83838383\ldots$	$0.222222\ldots$
Let $x =$ the original number with extra iterations shown. Multiply by factors of 10.	$x = 0.838383\ldots$ $10x = 8.383838\ldots$ $100x = 83.838383\ldots$ $1000x = 838.383838\ldots$	$x = 0.222222\ldots$ $10x = 2.222222\ldots$ $100x = 22.222222\ldots$ $1000x = 222.222222\ldots$

2. What do you notice happens as the number is multiplied by factors of 10?

The digits of 8 and 3 move place values, but there are repeating digits still occurring. The same thing happens to the digit of 2; it moves place value but there are still more repeating digits.

3. Refer to Problems 3 and 4 from the Warm-up. What operation can help eliminate the repeating decimals?

Subtraction

4. Complete the algorithm to rewrite a repeating decimal as a fraction.

Step	$0.\overline{83}$	$0.\overline{2}$
Select two equations which will subtract the decimal values to 0. Be careful!	$x = 0.838383\ldots$ $100x = 83.838383\ldots$	$x = 0.222222\ldots$ $10x = 2.222222\ldots$
Subtract the equations and solve the resulting equation. Simplify the fraction, if possible.	$100x = 83.838383\ldots$ $-\ \ x = -\,0.838383\ldots$ $99x = 83$ $x = \frac{83}{99}$	$10x = 2.222222\ldots$ $-x = -0.222222\ldots$ $9x = 2$ $x = \frac{2}{9}$

768 **Unit 7** Irrationals and the Pythagorean Theorem

1 Launch

Activate prior knowledge of rational numbers as numbers that can be written as terminating or repeating decimals. Display the number $0.\overline{83}$ and ask students for ways to write this number as a fraction. Record ideas on the board to reference during the class discussion.

2 Monitor

Help students get started by encouraging them to read the step and reference the example before beginning to write. Allow access to calculators.

Look for points of confusion:

- **Having difficulty comparing the decimal place values.** Encourage students to line up the numbers at the decimal point.

Look for productive strategies:

- Wanting to use different equations, such as $10x = 2.222\ldots$ and $100x = 22.2222\ldots$ and noticing that the end result is the same.

3 Connect

Have students share their responses.

Ask:

- "Because these repeating decimals can be written as fractions, what type of numbers are they?" Rational
- Do you think there are repeating decimals which cannot be written as a fraction using this algorithm?

Highlight that any repeating or terminating decimal can be written as a fraction; therefore, any repeating or terminating decimal is a rational number. On the other hand, irrational numbers cannot be written as fractions; therefore, they are non-terminating, non-repeating decimals. In the algorithm shown, there is not a way to subtract the decimal portions to 0 in an irrational number.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Ask the following questions to help students make sense of the example given in Problems 1 and 4.

- "What happens when $0.\overline{83}$ is multiplied by 10?" By 100? By 1,000?"
- "Why do you think the equation $100x = 83.838383\ldots$ was chosen in Problem 1? What is it about 10 that helps to eliminate the repeating digits?"

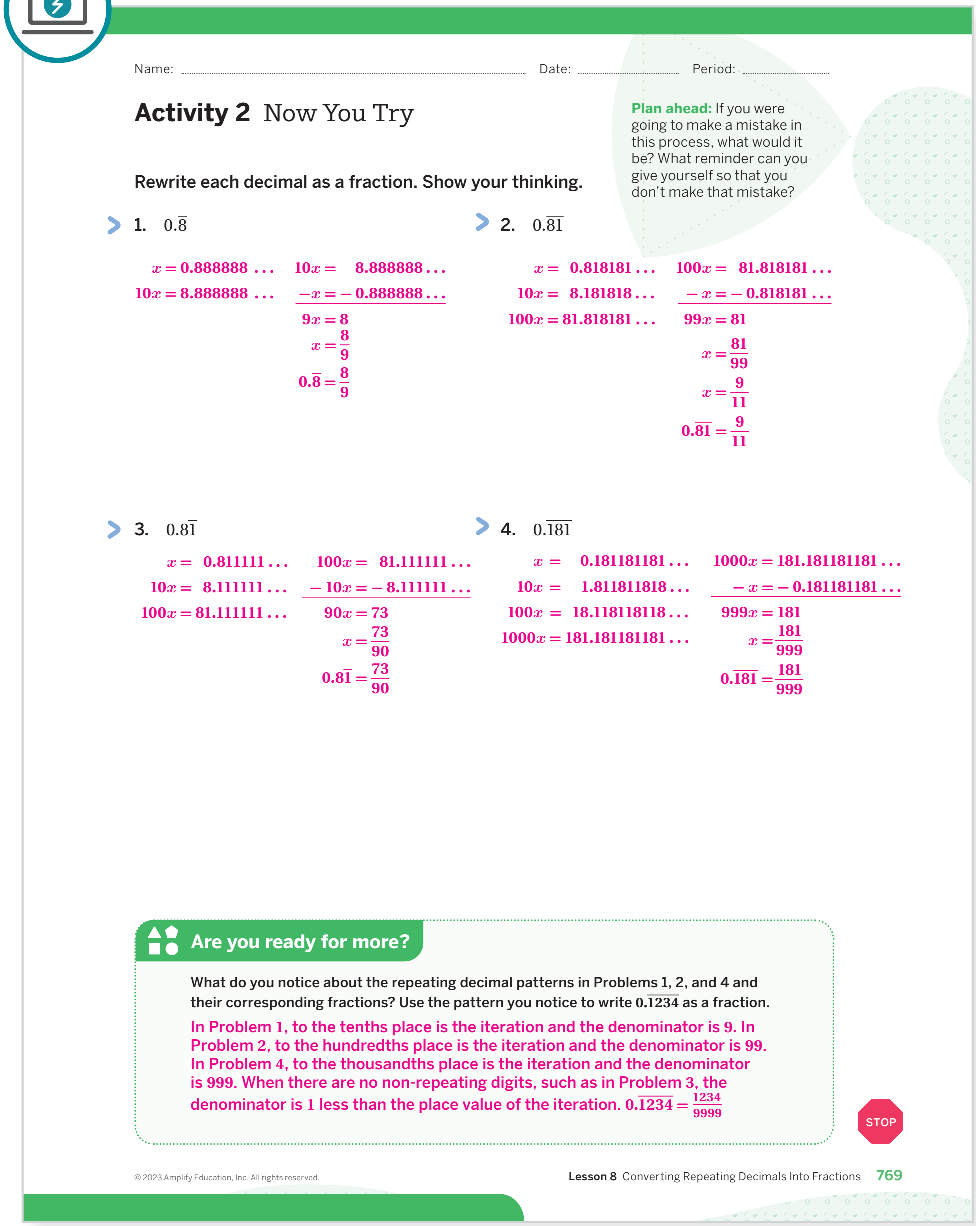
Extension: *Math Enrichment*

Challenge students to use the algorithm in this activity to write $2.1\overline{4}$ as a fraction. $\frac{193}{90}$; Multiply by 10 and 100 and write the equations $10x = 21.44\ldots$ and $100x = 214.44\ldots$ and subtract them to get the equation $90x = 193$.

Pairs | 15 min

Activity 2 Now You Try

Students practice to build fluency rewriting repeating decimals as fractions.



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Activity 2 Now You Try

Plan ahead: If you were going to make a mistake in this process, what would it be? What reminder can you give yourself so that you don't make that mistake?

Rewrite each decimal as a fraction. Show your thinking.

1. $0.\overline{8}$

$$x = 0.888888\ldots \qquad 10x = 8.888888\ldots$$
$$10x = 8.888888\ldots \qquad -x = -0.888888\ldots$$
$$9x = 8$$
$$x = \frac{8}{9}$$
$$0.\overline{8} = \frac{8}{9}$$

2. $0.\overline{81}$

$$x = 0.818181\ldots \qquad 100x = 81.818181\ldots$$
$$10x = 8.181818\ldots \qquad -x = -0.818181\ldots$$
$$100x = 81.818181\ldots \qquad 99x = 81$$
$$x = \frac{81}{99}$$
$$x = \frac{9}{11}$$
$$0.\overline{81} = \frac{9}{11}$$

3. $0.8\overline{1}$

$$x = 0.811111\ldots \qquad 100x = 81.111111\ldots$$
$$10x = 8.111111\ldots \qquad -10x = -8.111111\ldots$$
$$100x = 81.111111\ldots \qquad 90x = 73$$
$$x = \frac{73}{90}$$
$$0.8\overline{1} = \frac{73}{90}$$

4. $0.\overline{181}$

$$x = 0.181181181\ldots \qquad 1000x = 181.181181181\ldots$$
$$10x = 1.811811818\ldots \qquad -x = -0.181181181\ldots$$
$$100x = 18.118118118\ldots \qquad 999x = 181$$
$$1000x = 181.181181181\ldots \qquad x = \frac{181}{999}$$
$$0.\overline{181} = \frac{181}{999}$$

Are you ready for more?

What do you notice about the repeating decimal patterns in Problems 1, 2, and 4 and their corresponding fractions? Use the pattern you notice to write $0.\overline{1234}$ as a fraction.

In Problem 1, to the tenths place is the iteration and the denominator is 9. In Problem 2, to the hundredths place is the iteration and the denominator is 99. In Problem 4, to the thousandths place is the iteration and the denominator is 999. When there are no non-repeating digits, such as in Problem 3, the denominator is 1 less than the place value of the iteration. $0.\overline{1234} = \frac{1234}{9999}$

STOP

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Lesson 8 Converting Repeating Decimals Into Fractions 769

1 Launch

Let students know they will be practicing the algorithm presented in Activity 1. Encourage students to reference their work in Activity 1, as needed. Provide access to calculators.

2 Monitor

Help students get started by reminding them the first step is to write a few iterations of the repeating digits.

Look for points of confusion:

- **Thinking Problem 2 and 3 are the same.** Remind students the repetition bar only is above the digits which repeat. In Problem 2, the 8 and 1 repeat, but in Problem 3, only the 1 repeats.

Look for productive strategies:

- Being careful when writing their equations and ensuring decimals are inline.

3 Connect

Display any problems necessary to facilitate class discussion and have students share their solutions and thinking.

Ask, "Were you able to know which equations would be most helpful before writing the multiples of 10?" Sample response: I noticed that if the repetition went to the hundredths place, I would need the $100x$ equation.

Highlight the equations students selected to subtract. Consider showing a few examples to determine whether they produce equivalent fractions.

Differentiated Support

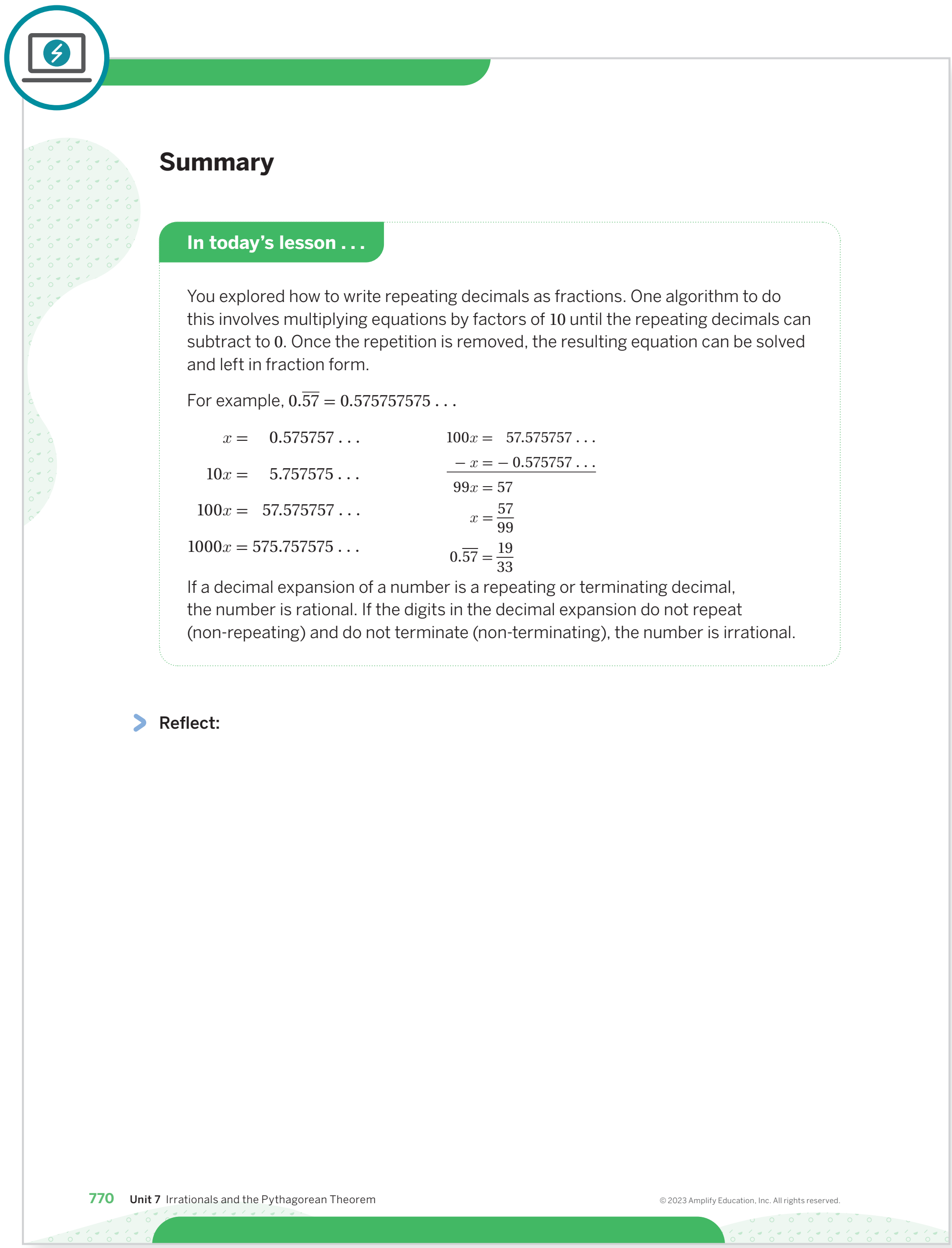
Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on Problems 1 and 2. As time allows, they can choose another problem to complete.

Whole Class | 5 min

Summary

Review and synthesize how to write repeating decimals as fractions.



Summary

In today's lesson . . .

You explored how to write repeating decimals as fractions. One algorithm to do this involves multiplying equations by factors of 10 until the repeating decimals can subtract to 0. Once the repetition is removed, the resulting equation can be solved and left in fraction form.

For example, $0.\overline{57} = 0.575757575\ldots$

$$\begin{aligned} x &= 0.575757\ldots \\ 10x &= 5.757575\ldots \\ 100x &= 57.575757\ldots \\ 1000x &= 575.757575\ldots \end{aligned}$$

$$\begin{aligned} 100x &= 57.575757\ldots \\ -\,x &= -\,0.575757\ldots \\ \hline 99x &= 57 \\ x &= \frac{57}{99} \\ 0.\overline{57} &= \frac{19}{33} \end{aligned}$$

If a decimal expansion of a number is a repeating or terminating decimal, the number is rational. If the digits in the decimal expansion do not repeat (non-repeating) and do not terminate (non-terminating), the number is irrational.

Reflect:

770 Unit 7 Irrationals and the Pythagorean Theorem

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Synthesize

Display the Anchor Chart PDF, *Representing Repeating Decimals as Fractions,* and discuss an example.

Ask, "What do you need to keep in mind when converting a repeating decimal to a fraction?"

Highlight the algorithm used in Activity 1 is just one method of writing repeating decimals as fractions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What is the difference between a rational number and an irrational number?"

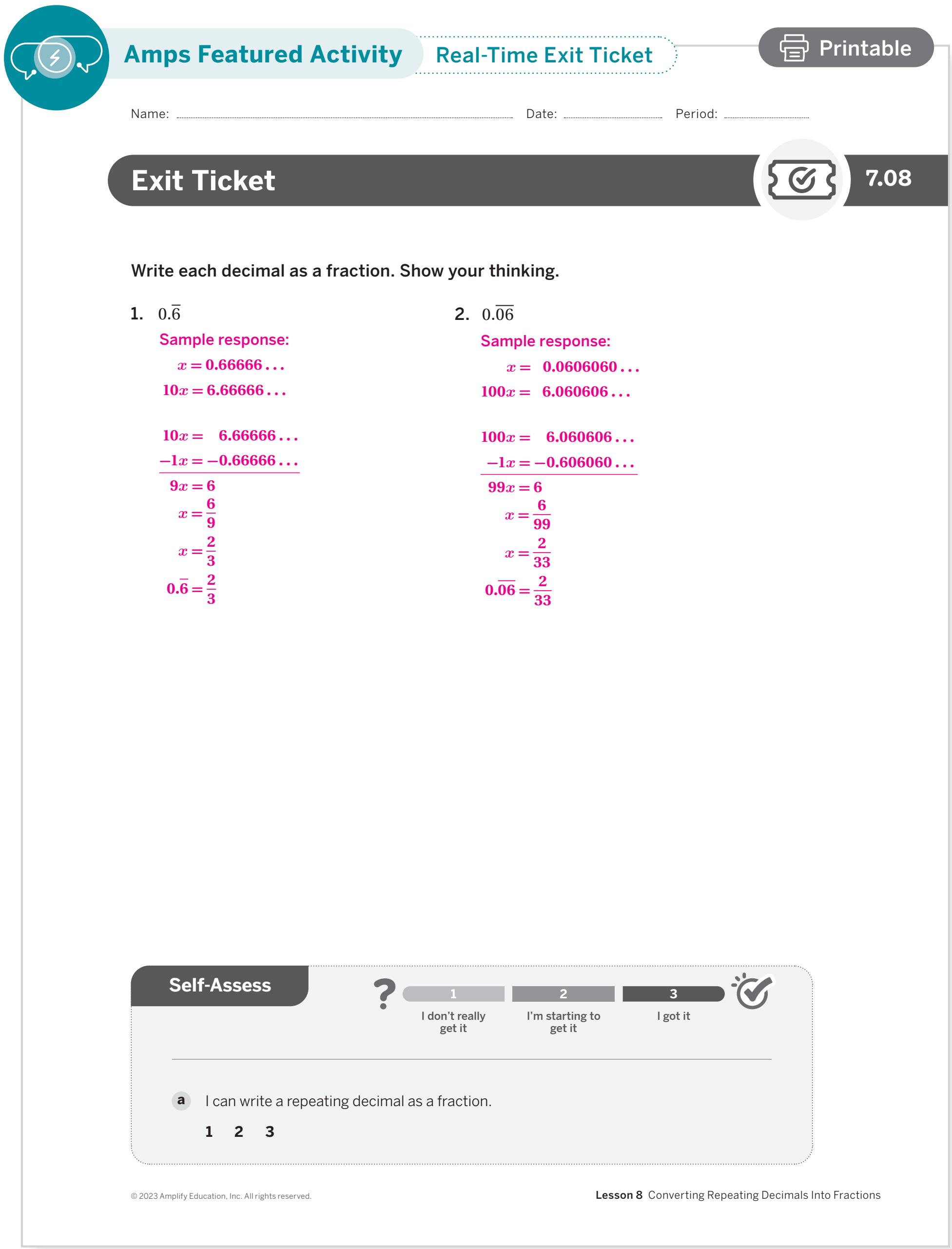
Independent | 5 min

Exit Ticket

Students demonstrate their understanding by rewriting repeating decimals as fractions.

Amps Featured Activity Real-Time Exit Ticket

Printable



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Exit Ticket 7.08

Write each decimal as a fraction. Show your thinking.

1. $0.\overline{6}$

Sample response:

$$\begin{aligned} x &= 0.66666\ldots \\ 10x &= 6.66666\ldots \end{aligned}$$

$$\begin{array}{r} 10x = 6.66666\ldots \\ \underline{-1x = -0.66666\ldots} \\ 9x = 6 \\ x = \frac{6}{9} \\ x = \frac{2}{3} \\ 0.\overline{6} = \frac{2}{3} \end{array}$$

2. $0.\overline{06}$

Sample response:

$$\begin{aligned} x &= 0.0606060\ldots \\ 100x &= 6.060606\ldots \end{aligned}$$

$$\begin{array}{r} 100x = 6.060606\ldots \\ \underline{-1x = -0.606060\ldots} \\ 99x = 6 \\ x = \frac{6}{99} \\ x = \frac{2}{33} \\ 0.\overline{06} = \frac{2}{33} \end{array}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can write a repeating decimal as a fraction.

1 2 3

Lesson 8 Converting Repeating Decimals Into Fractions

Success looks like . . .

- **Language Goal:** Comparing and contrasting decimal expansions for rational and irrational numbers. **(Speaking and Listening)**
- **Language Goal:** Coordinating repeating decimal expansions and rational numbers that represent the same number. **(Speaking and Listening, Writing)**
  - » Converting each repeating decimal into its equivalent fraction using algebraic thinking.

Suggested next steps

If students recognize the decimal in Problem 1 is equivalent to $\frac{2}{3}$ without showing work, consider:

- Accepting that response as it shows they have met the goal of the lesson.

If students rewrite the decimal in Problem 2 as $0.06666\ldots$, consider:

- Reviewing Lesson 7, Activity 2.
- Assigning Practice Problems 1 and 2.

Professional Learning

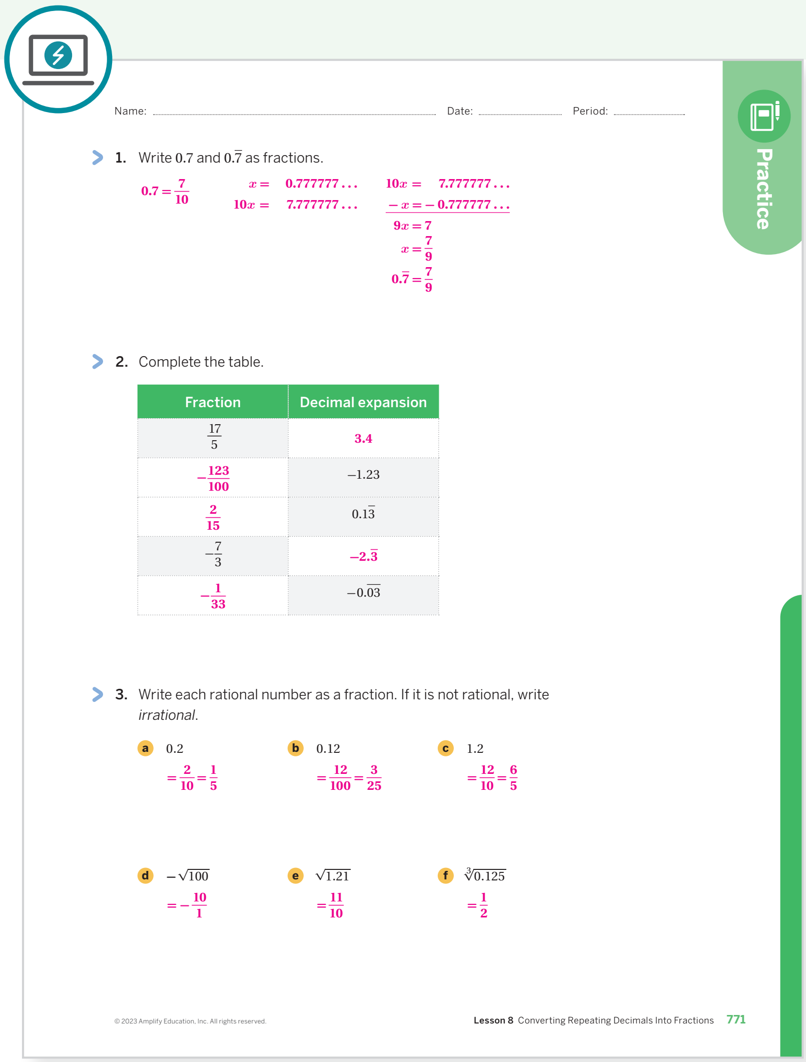
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did your students' reaction to the algorithm in Activity 1 reveal about your students as learners?
- What surprised you as your students worked on Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Write 0.7 and $0.\overline{7}$ as fractions.

$0.7 = \frac{7}{10}$

$x = 0.777777\ldots$

$10x = 7.777777\ldots$

$$\begin{aligned} 10x &= 7.777777\ldots \\ -x &= -0.777777\ldots \\ \hline 9x &= 7 \\ x &= \frac{7}{9} \\ 0.\overline{7} &= \frac{7}{9} \end{aligned}$$

2. Complete the table.

Fraction	Decimal expansion
$\frac{17}{5}$	3.4
$-\frac{123}{100}$	-1.23
$\frac{2}{15}$	$0.1\overline{3}$
$-\frac{7}{3}$	$-2.\overline{3}$
$-\frac{1}{33}$	$-0.\overline{03}$

3. Write each rational number as a fraction. If it is not rational, write *irrational*.

a. 0.2 $= \frac{2}{10} = \frac{1}{5}$

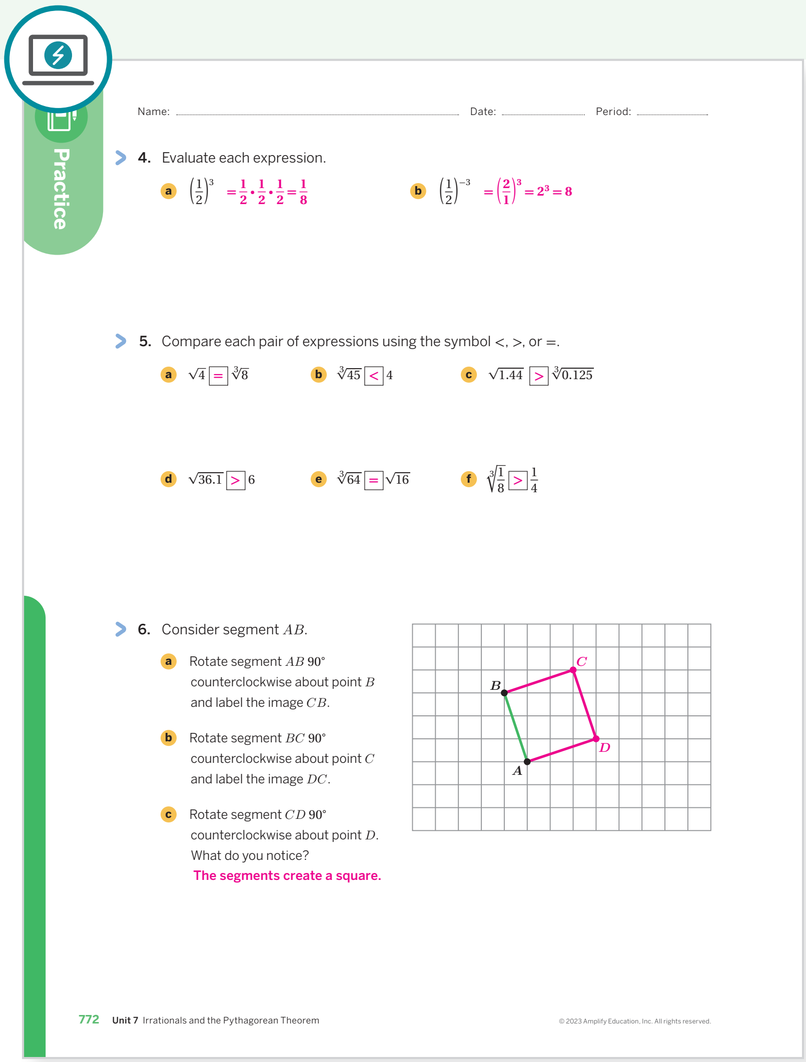
b. 0.12 $= \frac{12}{100} = \frac{3}{25}$

c. 1.2 $= \frac{12}{10} = \frac{6}{5}$

d. $-\sqrt{100}$ $= -\frac{10}{1}$

e. $\sqrt{1.21}$ $= \frac{11}{10}$

f. $\sqrt[3]{0.125}$ $= \frac{1}{2}$



Practice

Name: ________ Date: ________ Period: ________

4. Evaluate each expression.

a. $\left(\frac{1}{2}\right)^3$ $= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

b. $\left(\frac{1}{2}\right)^{-3}$ $= \left(\frac{2}{1}\right)^3 = 2^3 = 8$

5. Compare each pair of expressions using the symbol <, >, or =.

a. $\sqrt{4}\ \boxed{=}\ \sqrt[3]{8}$

b. $\sqrt[3]{45}\ \boxed{<}\ 4$

c. $\sqrt{1.44}\ \boxed{>}\ \sqrt[3]{0.125}$

d. $\sqrt{36.1}\ \boxed{>}\ 6$

e. $\sqrt[3]{64}\ \boxed{=}\ \sqrt{16}$

f. $\sqrt[3]{\frac{1}{8}}\ \boxed{>}\ \frac{1}{4}$

6. Consider segment AB.

a. Rotate segment AB 90° counterclockwise about point B and label the image CB.

b. Rotate segment BC 90° counterclockwise about point C and label the image DC.

c. Rotate segment CD 90° counterclockwise about point D. What do you notice?

The segments create a square.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	1
	3	Unit 7 Lesson 7	1
Spiral	4	Unit 6 Lesson 5	1
	5	Unit 7 Lesson 5	1
Formative ⚡	6	Unit 7 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Sub-Unit 2

The Pythagorean Theorem

Whole Class

In this Sub-Unit, students are introduced to the Pythagorean Theorem and a few of its proofs. Students build their understanding by determining missing side lengths of right triangles, both in and out of context.

SUB-UNIT

2 The Pythagorean Theorem

Narrative Connections

What do the President Garfield and Albert Einstein have in common?

There is evidence many cultures have grappled with the Pythagorean Theorem well before Pythagoras. The earliest record came from the ancient Mesopotamians. In 1945, archaeologists unearthed tablets that showed calculations for the lengths of right triangles a thousand years before Pythagoras was even born.

In around the 8th century BCE, the theorem appeared in India in the Baudhāyana Śulbasûtra. The text described rules for calculating a diagonal produced by a stretched rope using its vertical and horizontal lengths. The theorem appeared again in China, around 700 BCE in the Bi Suan Jing.

In the centuries since, the Pythagorean Theorem has not exactly fallen out of style. Even to this day, math enthusiasts are still searching for new proofs. In 1928, math teacher Elisha Loomis published *The Pythagorean Proposition*. His book collected 370 different methods for proving the theorem. Even a young Albert Einstein and President James A. Garfield have authored their own proofs.

In 1995, *The Guinness Book of World Records* declared the Pythagorean Theorem "the most proven theorem of all time." Today, there are nearly 500 different proofs for this humble theorem.

But why?

Well, for one, the theorem is practical. It helps us calculate distances. This makes navigation a breeze. The theorem is also useful for engineers and architects. Knowing the lengths for a right triangle guarantees your constructions are built at right angles. But there's another reason beyond the practical. There's also the simple pleasure in discovering a proof. A well-crafted proof can be a delight — one of those aha moments. Math is not just about solving problems. It's also about training the mind to find solutions that are creative and elegant.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore the history of right triangle relationships in the following places:

- **Lesson 10, Warm-up, Activities 1–2:** Notice and Wonder, Arranging Shapes, Any Right Triangle
- **Lesson 12, Activity 2:** Is This a Right Triangle?
- **Lesson 15, Activity 1:** Navigating the Seas

UNIT 7 | **LESSON 9**

Observing the Pythagorean Theorem

Let's determine the side lengths of triangles.

Focus

Goals

1. **Language Goal:** Identify and describe patterns in the relationships between the side lengths of triangles. **(Speaking and Listening)**
2. **Language Goal:** Understand that the relationship between the side lengths in a right triangle represents the Pythagorean Theorem. **(Speaking and Listening, Writing)**

Coherence

Today

Students investigate relationships between the side lengths of right and non-right triangles leading to the Pythagorean Theorem. Students systematically look at the side lengths of right triangles for patterns. By the end of this lesson, they see that, for right triangles with legs a and b and hypotenuse c, the side lengths are related by $a^2 + b^2 = c^2$.

Previously

In Grade 7, students studied properties of triangles. Previously in this unit, students learned that the diagonal of a rectangular figure is the longest cut that can be made.

Coming Soon

In Lesson 10, students will prove the Pythagorean Theorem, before learning how to use the theorem to find unknown sides in Lesson 11. In Lesson 12, students learn the converse of the Pythagorean Theorem is true.

Rigor

- Students develop a **conceptual understanding** for how the Pythagorean Theorem is observed in right triangles.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Pairs	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per group
- Anchor Chart PDF, *The Pythagorean Theorem*
- Anchor Chart PDF, *The Pythagorean Theorem* (answers)

Math Language Development

New words

- ***hypotenuse***
- ***legs***
- ***Pythagorean Theorem***

Review words

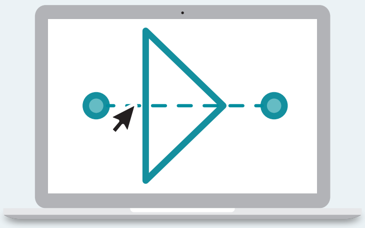
- *area*

Amps | Featured Activity

Activity 1

Real-Time Feedback

Students determine the squares of side lengths of triangles, and are able to check their work in real time, allowing them to make observations about the relationships between their values.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not be comfortable speculating about the Pythagorean Theorem in Activity 1. Point out that through repeated reasoning, students will be able to observe patterns which they can then apply to hypothesize about the Pythagorean Theorem. Remind students that it is ok to have to revise a hypothesis because, as part of a growth mentality, they should recognize that they are not finished learning about it *yet*.

Modifications to Pacing

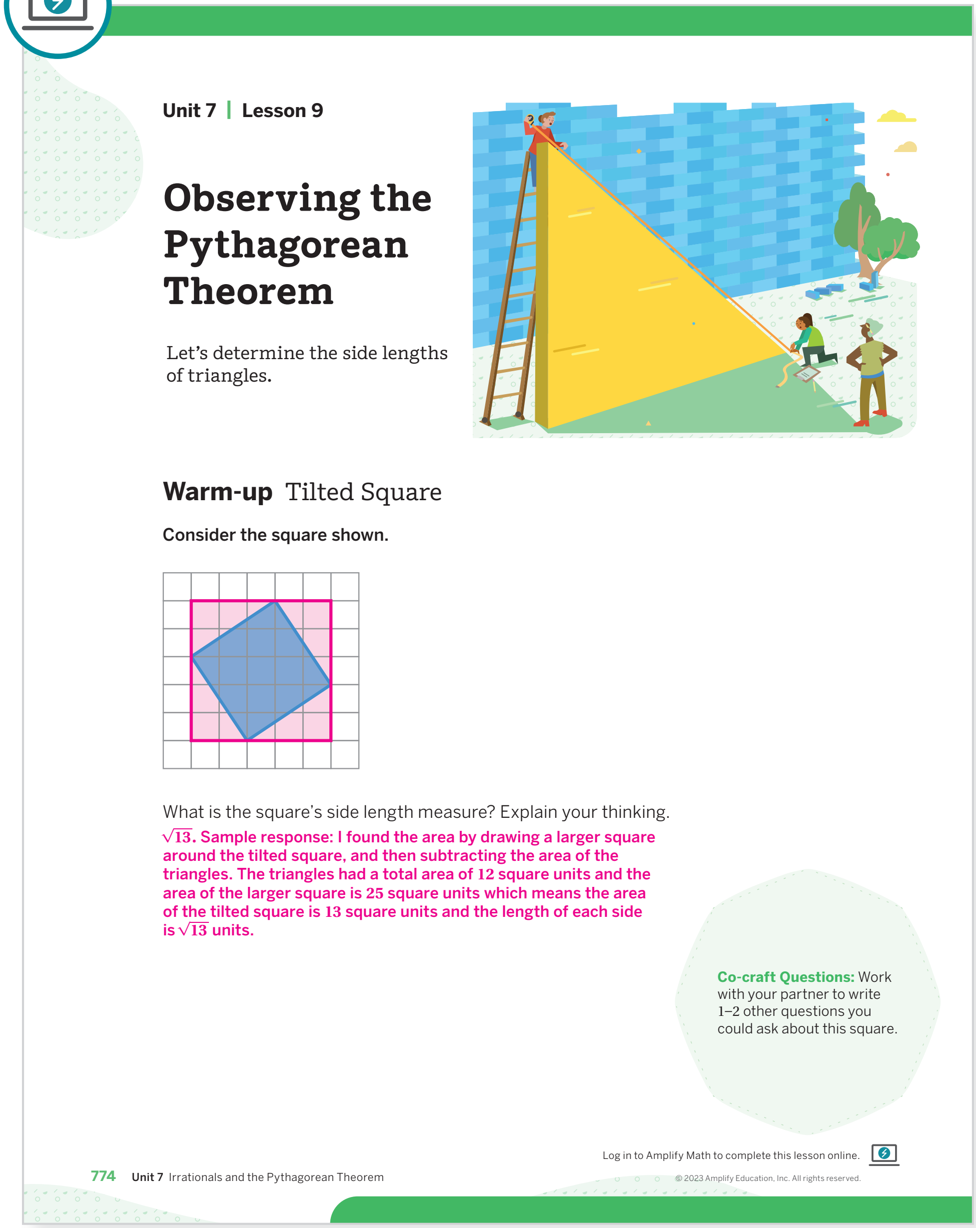
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, provide students with triangles that already have square lengths drawn, so students can focus on the patterns observed in the squared lengths of the triangles.
- **Activity 2** may be omitted, or shortened. The points of confusion addressed with Problems 1 and 2 can be addressed in Activity 1.

Pairs | 5 min

Warm-up Tilted Square

Students apply their prior knowledge of the areas of squares to determine the lengths of diagonals.



Unit 7 | Lesson 9

Observing the Pythagorean Theorem

Let's determine the side lengths of triangles.

Warm-up Tilted Square

Consider the square shown.

What is the square's side length measure? Explain your thinking.

$\sqrt{13}$. Sample response: I found the area by drawing a larger square around the tilted square, and then subtracting the area of the triangles. The triangles had a total area of 12 square units and the area of the larger square is 25 square units which means the area of the tilted square is 13 square units and the length of each side is $\sqrt{13}$ units.

Co-craft Questions: Work with your partner to write 1–2 other questions you could ask about this square.

Log in to Amplify Math to complete this lesson online.

774 Unit 7 Irrationals and the Pythagorean Theorem

1 Launch

Activate students' prior knowledge by asking them how they can determine the length of a square's side if they know the square's area. Conduct the *Think-Pair-Share* routine. Have students discuss strategies for how to determine the area of the square before beginning the activity.

2 Monitor

Help students get started by asking whether they can determine the area of a square drawn along the grid around the shaded square.

Look for points of confusion:

- **Not being sure how to determine the area of the square.** After students draw a square around the shaded square, ask students what shapes they see and how they can use the area of the triangle parts to determine the area of the shaded square.

Look for productive strategies:

- Drawing a surrounding square and subtracting areas or drawing triangles that can be arranged to determine the area of the shaded square.

3 Connect

Have students share how they can determine the area and side length of the square.

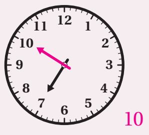
Ask, "What steps can you take to determine the length of a diagonal?"

Highlight that, to determine the length of a diagonal using a grid, draw a square that shares a side length with the diagonal. Then determine the area of the square, and the diagonal length will be the square root of the area.

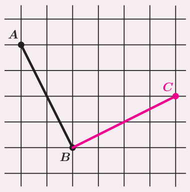
Power-up

To power up students' ability to rotate a line segments on grid, have students complete:

1. The minute hand of a clock is pointing at 7. If the hand rotates clockwise 90° at which number will it be pointed?



2. Rotate segment AB clockwise 90° about point B. Label this segment BC.



Use: Before the Warm-up

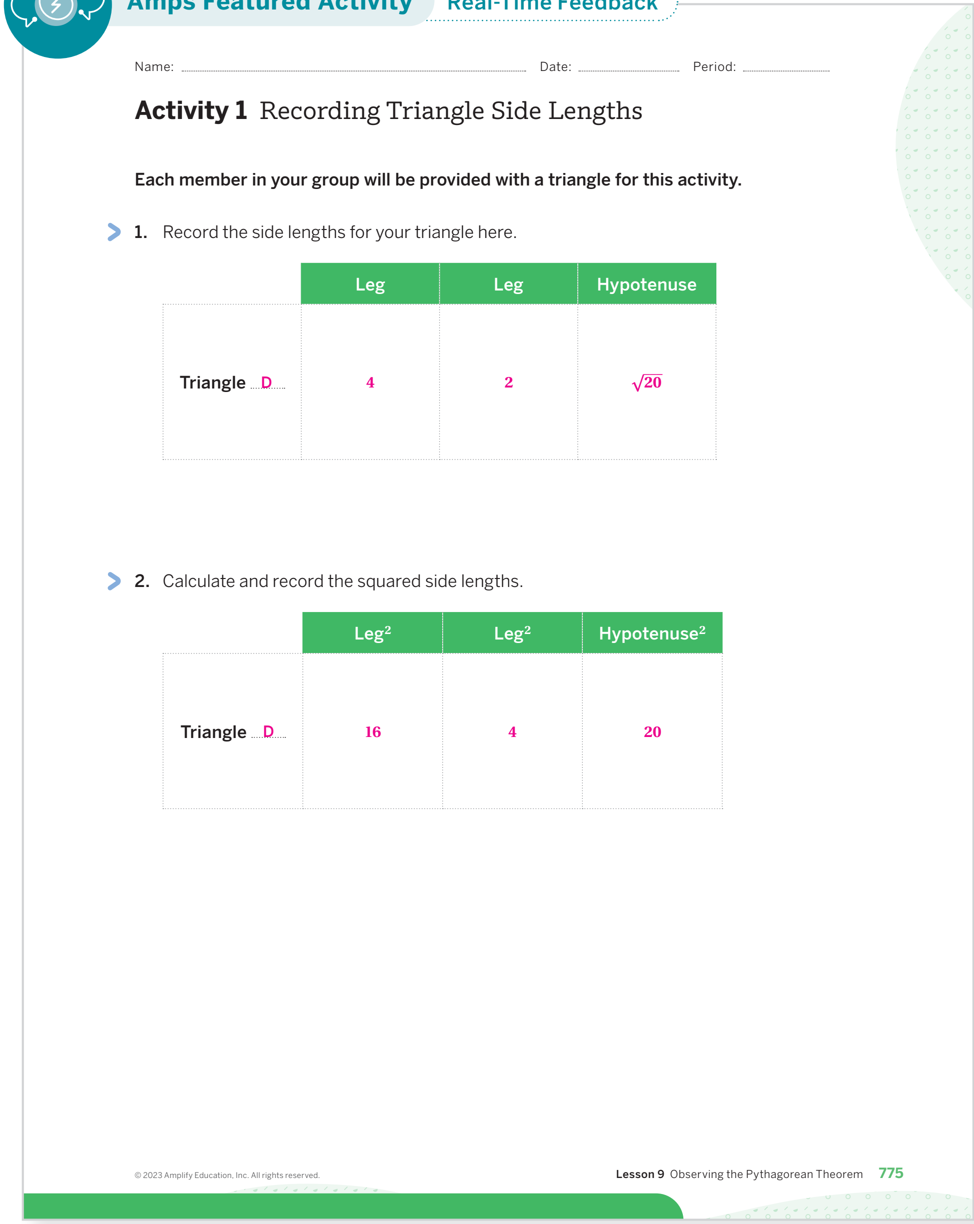
Informed by: Performance on Lesson 8, Practice Problem 6

Small Groups | 20 min

Activity 1 Recording Triangle Side Lengths

Students calculate the side lengths of the triangles by both drawing in tilted squares and reasoning about segment lengths to observe patterns predicted by the Pythagorean Theorem.

Amps Featured Activity Real-Time Feedback



Name: Date: Period:

Activity 1 Recording Triangle Side Lengths

Each member in your group will be provided with a triangle for this activity.

1. Record the side lengths for your triangle here.

	Leg	Leg	Hypotenuse
Triangle D	4	2	$\sqrt{20}$

2. Calculate and record the squared side lengths.

	Leg^2	Leg^2	Hypotenuse^2
Triangle D	16	4	20

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Lesson 9 Observing the Pythagorean Theorem 775

1 Launch

Distribute the pre-cut cards from the Activity 1 PDF to groups of 5. Give the cards showing Triangles P and Q to students who will benefit from an added challenge.

As soon as each student has their own triangle, activate students' prior knowledge by asking them what type of triangle they have — acute, right, or obtuse. Tell them they all have right triangles of different sizes.

Define ***legs*** and ***hypotenuse*** of a right triangle. Have students apply the definitions by naming which sides are the legs and which side is the hypotenuse.

Tell students that they will use strategies similar to the Warm-Up to determine the side lengths of the right triangle they have. Ask students when they think they will need to draw a square to determine a side length, and when they do not need to draw a square.

2 Monitor

Help students get started by having them label the known sides of their triangle and demonstrate how to draw one side of the square for determining the length of a diagonal.

Look for points of confusion:

- **Having difficulty determining diagonal side lengths.** Help students see how to repeatedly rotate the diagonal to create a square. Review strategies from the Warm-Up for determining the area and the measure of a side.
- **Not seeing a pattern in the squared sides.** Ask students what they notice about the squared length of the hypotenuse in relation to the squared length of the legs.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use digital geometry tools to determine the squares of side lengths of triangles, allowing them to make observations about the relationships between their values.

Math Language Development

MLR8: Discussion Supports

While students work, circulate and listen as they discuss the patterns they notice after they have recorded the results from their groups members in the table for Problem 3. During the Connect, encourage students to clarify the meaning of a word or phrase they used. For example:

If a student says . . .	Ask . . .
"The sum of the legs is the same as the hypotenuse."	"Can you show me how this works on your triangle?" Press students to clarify their language by talking about the squares of these lengths.

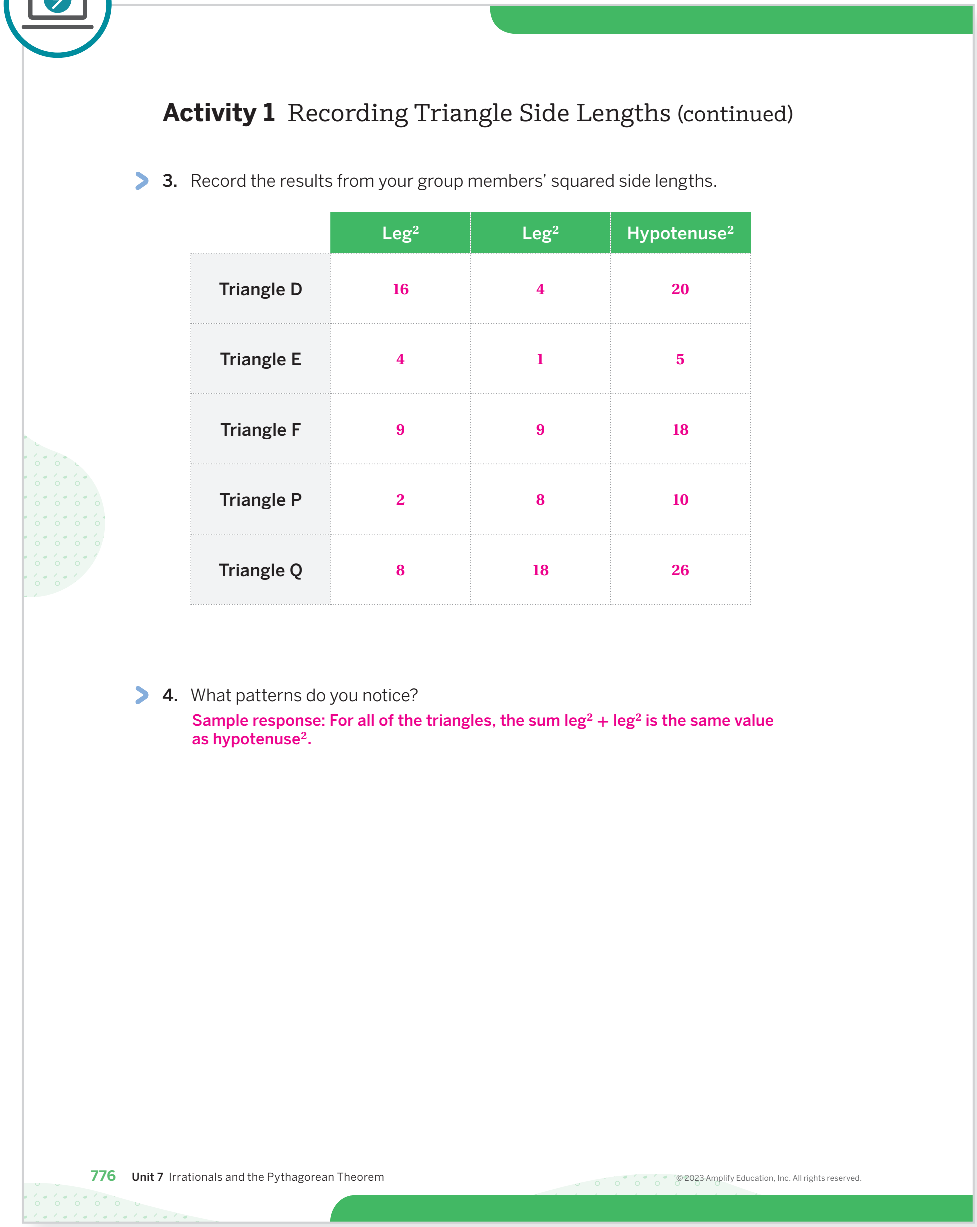
English Learners

Sketch examples of the triangles from the Activity 1 PDF on the class display as students discuss what they notice. Annotate the sketches to make connections between the words and phrases students use and the triangles.

Small Groups | 20 min

Activity 1 Recording Triangle Side Lengths (continued)

Students calculate the side lengths of the triangles by both drawing in tilted squares and reasoning about segment lengths to observe patterns predicted by the Pythagorean Theorem.



Activity 1 Recording Triangle Side Lengths (continued)

3. Record the results from your group members' squared side lengths.

	Leg^2	Leg^2	Hypotenuse^2
Triangle D	16	4	20
Triangle E	4	1	5
Triangle F	9	9	18
Triangle P	2	8	10
Triangle Q	8	18	26

4. What patterns do you notice?
Sample response: For all of the triangles, the sum $\text{leg}^2 + \text{leg}^2$ is the same value as hypotenuse^2.

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3 Connect

Display a student's table showing the correct squared side lengths for each triangle.

Have pairs of students share what they noticed about the relationships in the squared sides of the triangles. Discuss whether this pattern was true for every triangle they measured.

Define the ***Pythagorean Theorem***: If a and b are the lengths of the legs and c is the length of the hypotenuse of a right triangle, then $a^2 + b^2 = c^2$.

Ask:

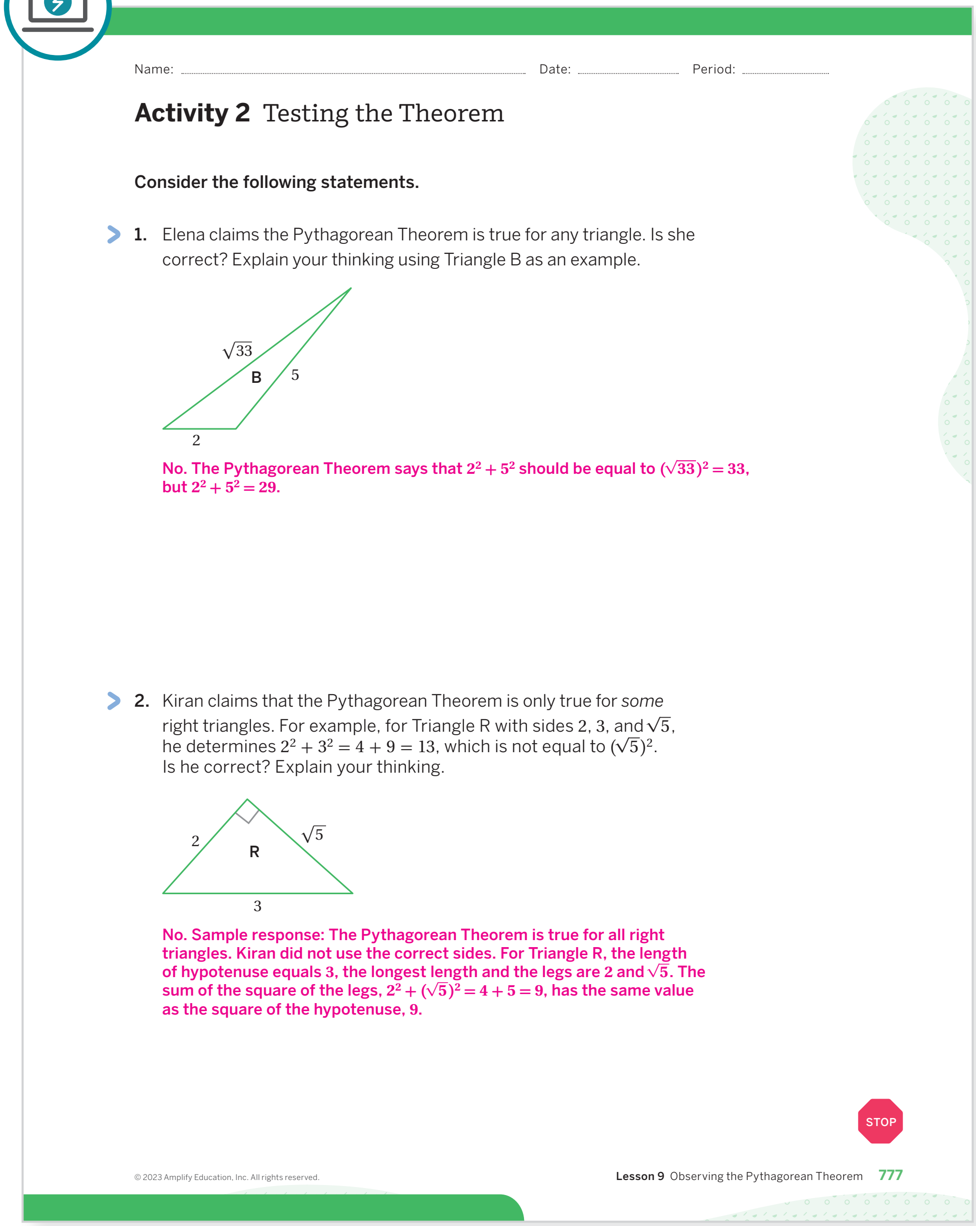
- "Which side, the hypotenuse or the leg, is the longest side in a right triangle?"
- "How can you identify if a triangle makes the Pythagorean Theorem true?" Check whether the equation $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ is true.

Highlight that students have just observed that the Pythagorean Theorem appears to be true for right triangles. Emphasize for students that they will now test this observation.

Pairs | 15 min

Activity 2 Testing the Theorem

Students evaluate two statements to apply their understanding of the Pythagorean Theorem.



Name: ________ Date: ________ Period: ________

Activity 2 Testing the Theorem

Consider the following statements.

1. Elena claims the Pythagorean Theorem is true for any triangle. Is she correct? Explain your thinking using Triangle B as an example.

No. The Pythagorean Theorem says that $2^2 + 5^2$ should be equal to $(\sqrt{33})^2 = 33$, but $2^2 + 5^2 = 29$.

2. Kiran claims that the Pythagorean Theorem is only true for *some* right triangles. For example, for Triangle R with sides 2, 3, and $\sqrt{5}$, he determines $2^2 + 3^2 = 4 + 9 = 13$, which is not equal to $(\sqrt{5})^2$. Is he correct? Explain your thinking.

No. Sample response: The Pythagorean Theorem is true for all right triangles. Kiran did not use the correct sides. For Triangle R, the length of hypotenuse equals 3, the longest length and the legs are 2 and $\sqrt{5}$. The sum of the square of the legs, $2^2 + (\sqrt{5})^2 = 4 + 5 = 9$, has the same value as the square of the hypotenuse, 9.

STOP

1 Launch

Set an amount of time for students to work in pairs on the activity.

2 Monitor

Help students get started by having them label which side lengths they think would correspond with *leg*, *leg*, and *hypotenuse* in Problem 1.

Look for points of confusion:

- **Thinking Elena's claim is true.** Make sure students are substituting values into the equation $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ and correctly evaluating to see that the equation is not true for the side lengths of the triangle.
- **Thinking Kiran's claim is true.** Have students go back to Activity 1 and check which sides were named *legs* and which side was named *hypotenuse*.

3 Connect

Have students share their responses to Problems 1 and 2 by conducting the ***Poll the Class*** routine.

Ask:

- "How can you tell whether a triangle satisfies the equation for the Pythagorean Theorem? How can you tell if it does not satisfy the theorem?"
- "Why does it not matter which leg is listed first in the equation? "
- "When you check Elena's claim using Triangle B, how can you tell that, no matter how you name your sides, the Pythagorean Theorem will not be satisfied?"

Highlight that the Pythagorean Theorem only works for right triangles, and then only when the hypotenuse is correctly identified. Tell students they will prove that this is always true for any right triangle in the next lesson.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problem 1.

Accessibility: *Guide Processing and Visualization*

Help students make sense of the triangle in Problem 1 by asking, "Which side do you think would be the hypotenuse if the Pythagorean Theorem worked for this triangle? Why do you think so?"

Math Language Development

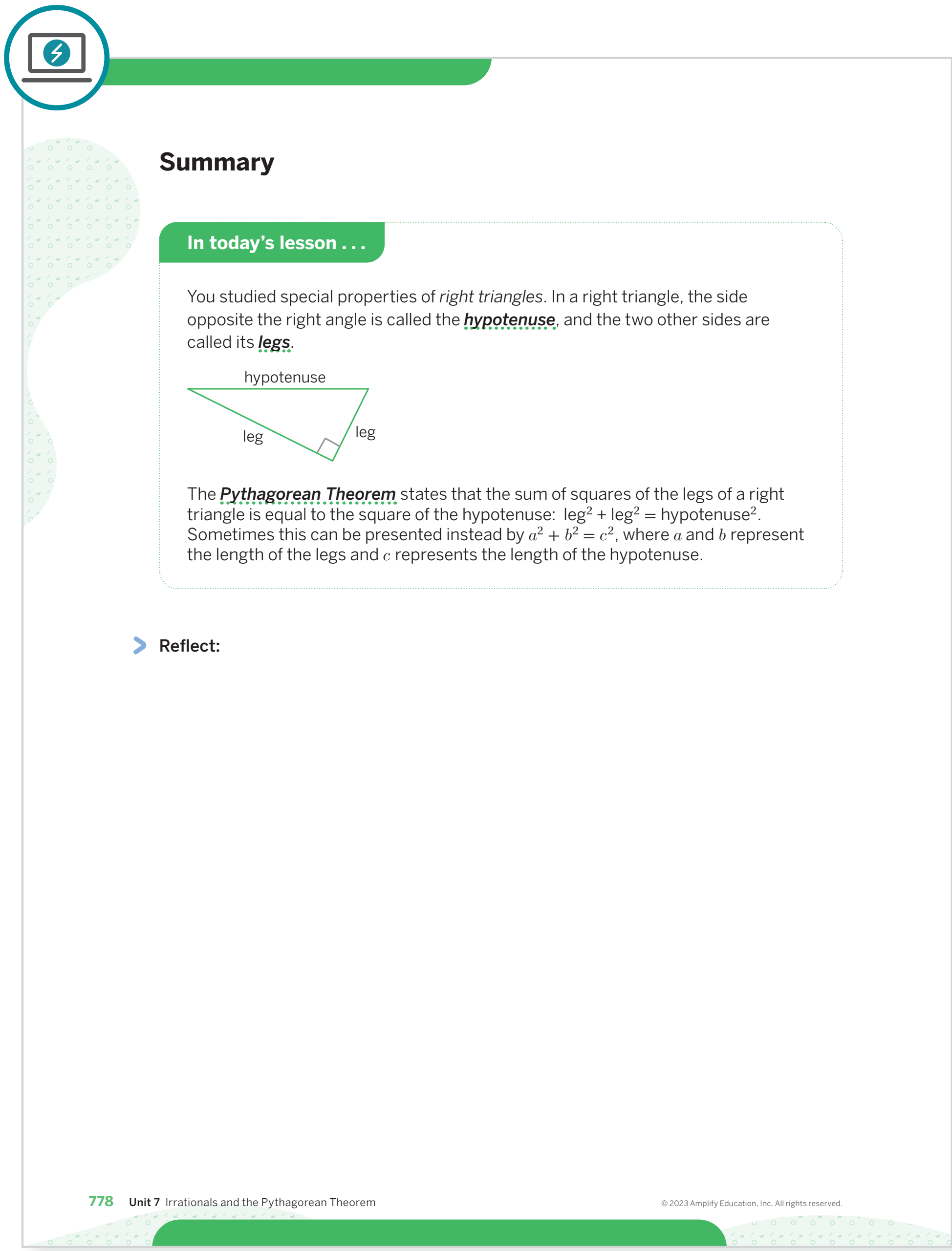
MLR8: Discussion Supports

During the Connect, as students share their responses and respond to the Ask questions, display the following sentence frames to help them organize their thinking.

- "For the Pythagorean Theorem to be true for a triangle, the equation ___ must be true." $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
- "The hypotenuse is always the ___ side of a right triangle." longest
- "It *does/does not* matter which leg is listed first in the equation." does not

Summary

Review and synthesize the relationship between the side lengths of right triangles.



Summary

In today's lesson . . .

You studied special properties of *right triangles*. In a right triangle, the side opposite the right angle is called the ***hypotenuse***, and the two other sides are called its ***legs***.

The ***Pythagorean Theorem*** states that the sum of squares of the legs of a right triangle is equal to the square of the hypotenuse: $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$. Sometimes this can be presented instead by $a^2 + b^2 = c^2$, where a and b represent the length of the legs and c represents the length of the hypotenuse.

Reflect:

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Synthesize

Display the Anchor Chart PDF, *The Pythagorean Theorem*.

Have students share which sides are the legs and which is the hypotenuse as you fill in the first section of the Anchor Chart together.

Ask:

- "What was the relationship between the side lengths of any right triangle?"
- "How can you check to see whether the Pythagorean Theorem is true for any triangle?"

Formalize vocabulary:

- *hypotenuse*
- *legs*
- *Pythagorean Theorem*

Highlight that when using the Pythagorean Theorem, the lengths of the legs must be squared first before adding. Tell students in the next lesson, they will actually prove that the relationships between the sides is true for any right triangle and never true for triangles that are not right triangles.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "In the next lesson, you will prove why the Pythagorean Theorem works for all right triangles. How is proving something is true different from the work you did today?"

Math Language Development

MLR2: Collect and Display

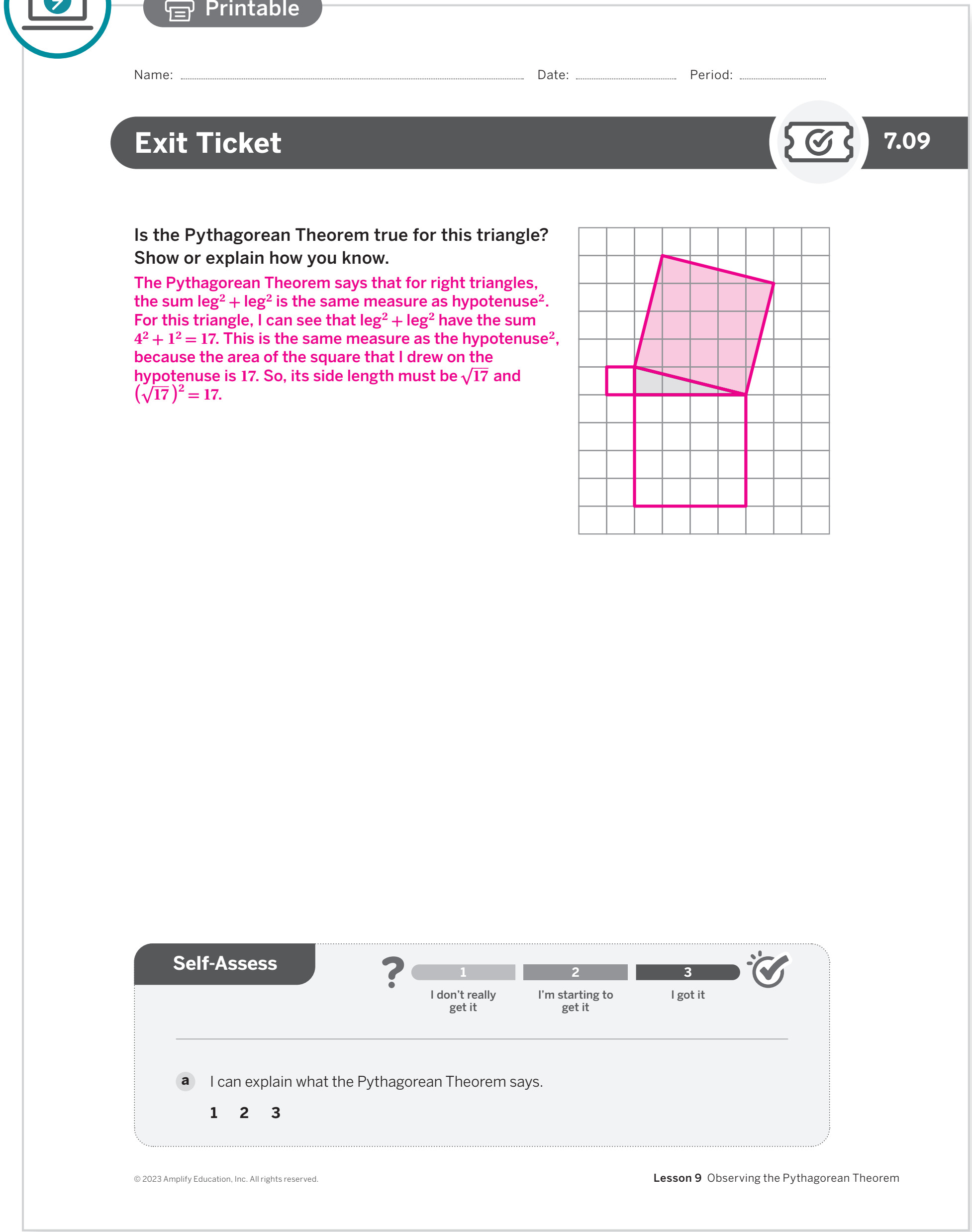
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started for this unit. Ask them to review and reflect on any terms and phrases related to the terms *Pythagorean Theorem*, *legs*, and *hypotenuse* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by evaluating whether a given triangle satisfies the Pythagorean Theorem.

Printable



Name: Date: Period:

Exit Ticket

7.09

Is the Pythagorean Theorem true for this triangle? Show or explain how you know.

The Pythagorean Theorem says that for right triangles, the sum $\text{leg}^2 + \text{leg}^2$ is the same measure as hypotenuse^2. For this triangle, I can see that $\text{leg}^2 + \text{leg}^2$ have the sum $4^2 + 1^2 = 17$. This is the same measure as the hypotenuse^2, because the area of the square that I drew on the hypotenuse is 17. So, its side length must be $\sqrt{17}$ and $(\sqrt{17})^2 = 17$.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain what the Pythagorean Theorem says.

1 2 3

Lesson 9 Observing the Pythagorean Theorem

Success looks like . . .

- **Language Goal:** Identifying and describing patterns in the relationships between the side lengths of triangles. **(Speaking and Listening)**
- **Language Goal:** Understanding that the relationship between the side lengths in a right triangle represents the Pythagorean Theorem. **(Speaking and Listening, Writing)**
  - » Explaining whether the side lengths of the triangle represent the Pythagorean Theorem.

Suggested next steps

If students are unable to determine if the Pythagorean Theorem is true for the triangle, consider:

- Reviewing strategies from Activity 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today?
- What might you change for the next time you teach this lesson?

Math Language Development

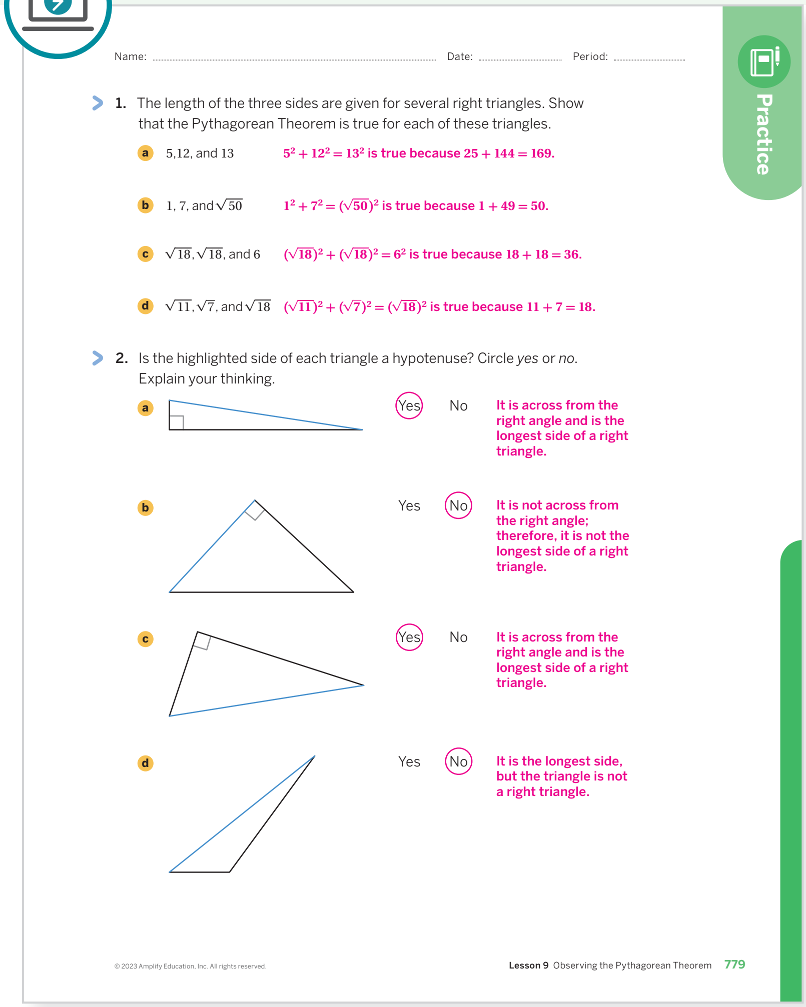
Language Goal: Understanding that the relationship between side lengths in a right triangle represents the Pythagorean Theorem.

Reflect on students' language development toward this goal.

- How did using the ***Discussion Supports*** routine in Activity 1 help students clarify the meaning of the statements they use to describe this relationship?
- How did using the ***Discussion Supports*** routine in Activity 2 help students use correct mathematical language, such as *hypotenuse* and *legs*, and precisely describe the relationship among the side lengths of a right triangle?

Practice

Independent



Practice

Name: Date: Period:

1. The length of the three sides are given for several right triangles. Show that the Pythagorean Theorem is true for each of these triangles.
   - a $5, 12$, and 13 — $5^2 + 12^2 = 13^2$ is true because $25 + 144 = 169$.
   - b $1, 7$, and $\sqrt{50}$ — $1^2 + 7^2 = (\sqrt{50})^2$ is true because $1 + 49 = 50$.
   - c $\sqrt{18}, \sqrt{18}$, and 6 — $(\sqrt{18})^2 + (\sqrt{18})^2 = 6^2$ is true because $18 + 18 = 36$.
   - d $\sqrt{11}, \sqrt{7}$, and $\sqrt{18}$ — $(\sqrt{11})^2 + (\sqrt{7})^2 = (\sqrt{18})^2$ is true because $11 + 7 = 18$.

2. Is the highlighted side of each triangle a hypotenuse? Circle *yes* or *no*. Explain your thinking.
   - a [✓ Yes] No — It is across from the right angle and is the longest side of a right triangle.
   - b Yes [✓ No] — It is not across from the right angle; therefore, it is not the longest side of a right triangle.
   - c [✓ Yes] No — It is across from the right angle and is the longest side of a right triangle.
   - d Yes [✓ No] — It is the longest side, but the triangle is not a right triangle.

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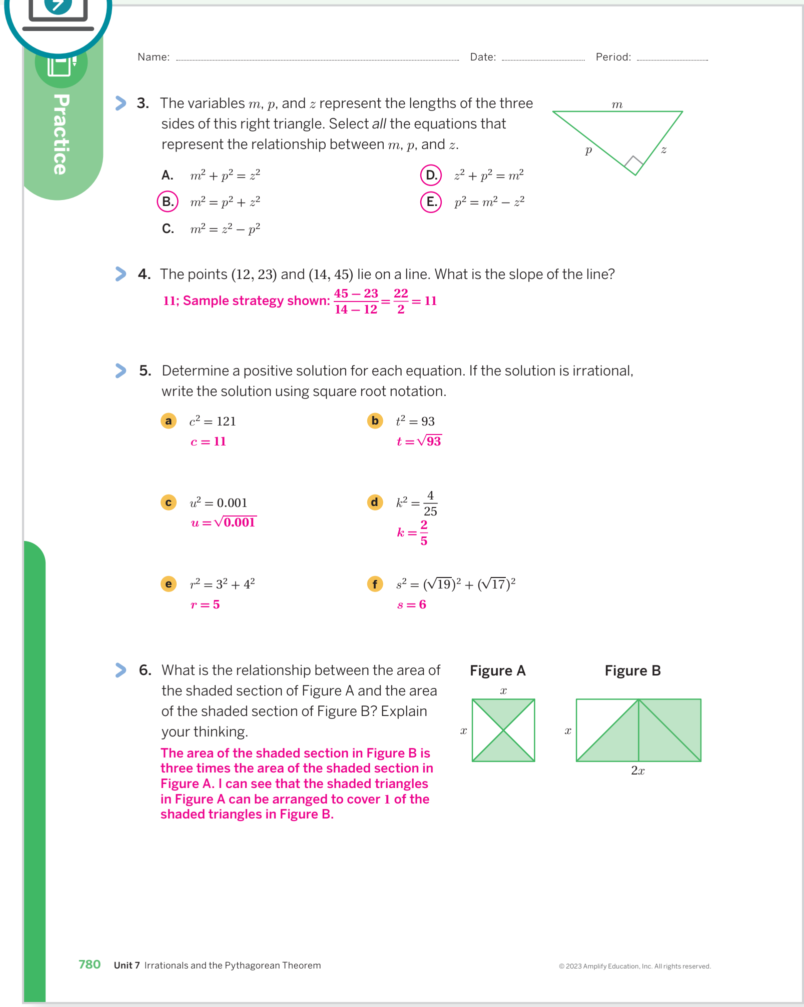
Lesson 9 Observing the Pythagorean Theorem 779



Practice

Name: Date: Period:

3. The variables m, p, and z represent the lengths of the three sides of this right triangle. Select *all* the equations that represent the relationship between m, p, and z.

A. $m^2 + p^2 = z^2$
[✓ B.] $m^2 = p^2 + z^2$
C. $m^2 = z^2 - p^2$
[✓ D.] $z^2 + p^2 = m^2$
[✓ E.] $p^2 = m^2 - z^2$

4. The points $(12, 23)$ and $(14, 45)$ lie on a line. What is the slope of the line?

   11; Sample strategy shown: $\frac{45-23}{14-12} = \frac{22}{2} = 11$

5. Determine a positive solution for each equation. If the solution is irrational, write the solution using square root notation.
   - a $c^2 = 121$ — $c = 11$
   - b $t^2 = 93$ — $t = \sqrt{93}$
   - c $u^2 = 0.001$ — $u = \sqrt{0.001}$
   - d $k^2 = \frac{4}{25}$ — $k = \frac{2}{5}$
   - e $r^2 = 3^2 + 4^2$ — $r = 5$
   - f $s^2 = (\sqrt{19})^2 + (\sqrt{17})^2$ — $s = 6$

6. What is the relationship between the area of the shaded section of Figure A and the area of the shaded section of Figure B? Explain your thinking.

The area of the shaded section in Figure B is three times the area of the shaded section in Figure A. I can see that the shaded triangles in Figure A can be arranged to cover 1 of the shaded triangles in Figure B.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	1
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 4	1
	5	Unit 7 Lesson 2	1
Formative ⚡	6	Unit 7 Lesson 10	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 10**

Proving the Pythagorean Theorem

Let's prove the Pythagorean Theorem.

Focus

Goals

1. Understand that the Pythagorean Theorem is true for every right triangle.
2. **Language Goal:** Explain a proof of the Pythagorean Theorem. **(Speaking and Listening)**

Coherence

Today

Students explore an area-based proof of the Pythagorean Theorem. While there are many proofs of the Pythagorean Theorem similar to the one in this activity, they often rely on $(a+b)^2 = a^2 + 2ab + b^2$, which is material beyond the scope of Grade 8. For this proof, students reason about the areas of the two squares with the same dimensions. Each square is divided into smaller regions in different ways and by using the equality of the total area of each square, students uncover the Pythagorean Theorem.

Previously

In Lesson 9, students measured side lengths of right triangles and found patterns in the data that suggested the Pythagorean Theorem is true for right triangles.

Coming Soon

In Lesson 11, students will find the missing side lengths of a right triangle using the Pythagorean Theorem.

Rigor

- Students build **conceptual understanding** for how the area of squares can be used to show that the Pythagorean Theorem is true for every right triangle.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Arranging Squares*, one per pair
- scissors

Math Language Development

Review words

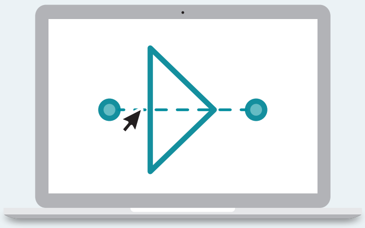
- *area*
- *hypotenuse*
- *legs*
- *Pythagorean Theorem*
- *square root*

Amps | Featured Activity

Activity 1

Arranging Squares

Working in pairs, students digitally rearrange right triangles as squares to work through a proof of the Pythagorean Theorem.



Building Math Identity and Community

Connecting to Mathematical Practices

Being asked to explain a proof in either Activity 1 or 2 can be intimidating because making sense of the proof can be a big task. Encourage students to take advantage of working in pairs, drawing on each person's strengths and helping each other through their personal limitations.

Modifications to Pacing

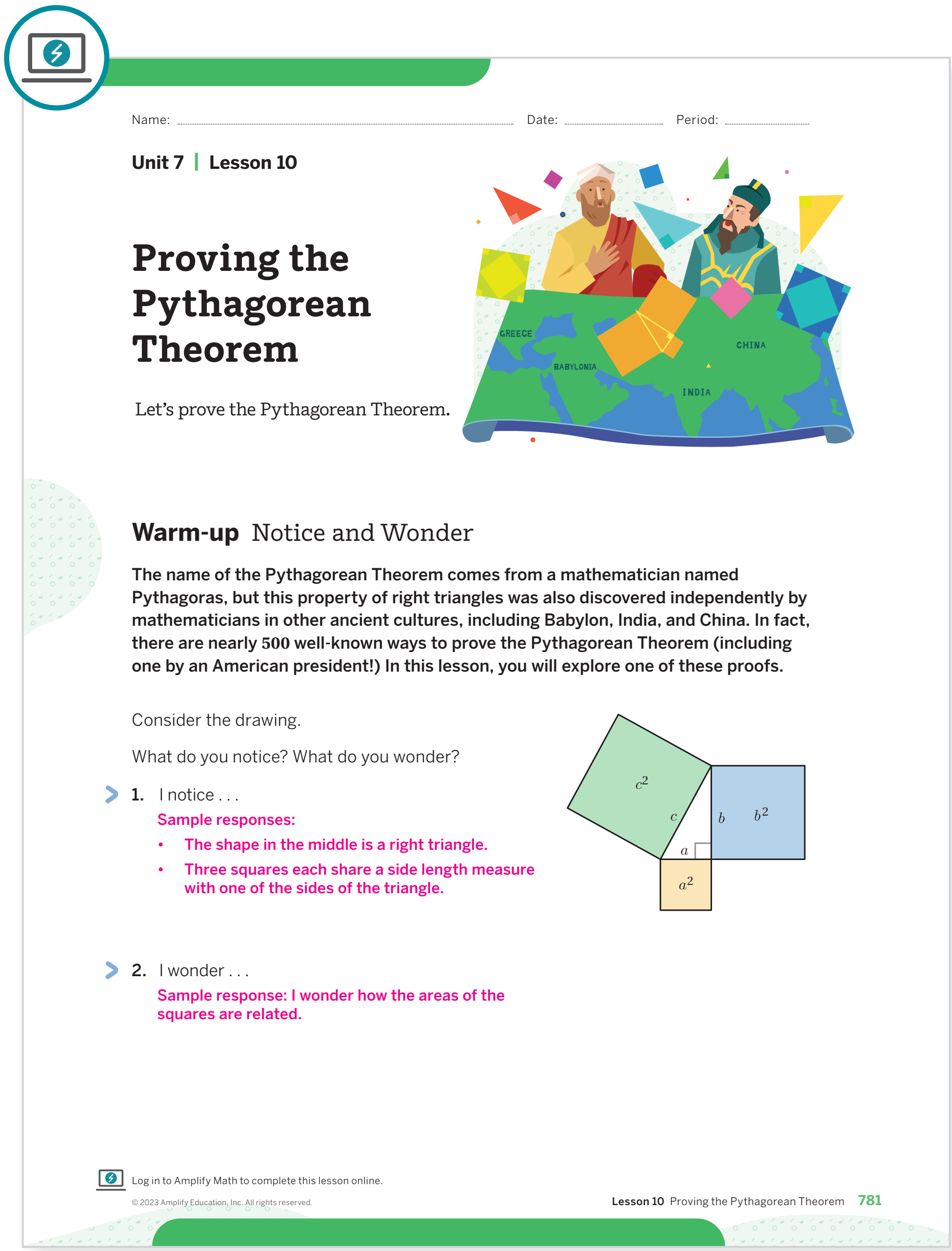
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be shortened. Instead, review the diagram in the Launch to Activity 1.
- In **Activity 1**, omit distributing the Activity 1 PDF and instead play the animation before launching the activity. Consider providing a demonstration of the animation using the PDF.

Pairs | 5 min

Warm-up Notice and Wonder

Students study a diagram to understand a relationship they will need for an upcoming proof of the Pythagorean Theorem.



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Unit 7 | Lesson 10

Proving the Pythagorean Theorem

Let's prove the Pythagorean Theorem.

Warm-up Notice and Wonder

The name of the Pythagorean Theorem comes from a mathematician named Pythagoras, but this property of right triangles was also discovered independently by mathematicians in other ancient cultures, including Babylon, India, and China. In fact, there are nearly 500 well-known ways to prove the Pythagorean Theorem (including one by an American president!) In this lesson, you will explore one of these proofs.

Consider the drawing.

What do you notice? What do you wonder?

1. I notice . . .

   Sample responses:
   - The shape in the middle is a right triangle.
   - Three squares each share a side length measure with one of the sides of the triangle.

2. I wonder . . .

   Sample response: I wonder how the areas of the squares are related.

Log in to Amplify Math to complete this lesson online.

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Lesson 10 Proving the Pythagorean Theorem 781

1 Launch

Activate students' prior knowledge by asking them to recall the Pythagorean Theorem they observed in Lesson 9. Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking them about the relation between the side lengths and the areas of the squares.

Look for productive strategies:

- Noticing or wondering about connections between the area of a^2, b^2, and c^2.

3 Connect

Have students share what they noticed and wondered about the drawing.

Display student responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class whether they agree or disagree and to explain alternative ways of thinking, referring back to the images each time.

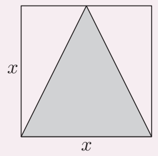
Ask, "How does this drawing relate to the Pythagorean Theorem?"

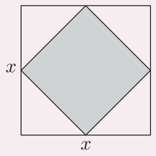
Highlight that, in order to show $a^2 + b^2 = c^2$, it will be sufficient to show the areas of the two squares are equal to the area of the larger square. Students will next explore how to use this diagram to show $a^2 + b^2 = c^2$.

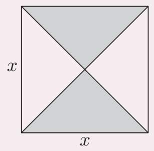
Power-up

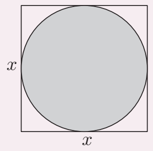
To power up students' ability to compare areas of related figures, have students complete:

Determine which figures have a shaded area of $\frac{1}{2}x^2$. Select *all* that apply.

[✓ A.]


[✓ B.]


[✓ C.]


D.


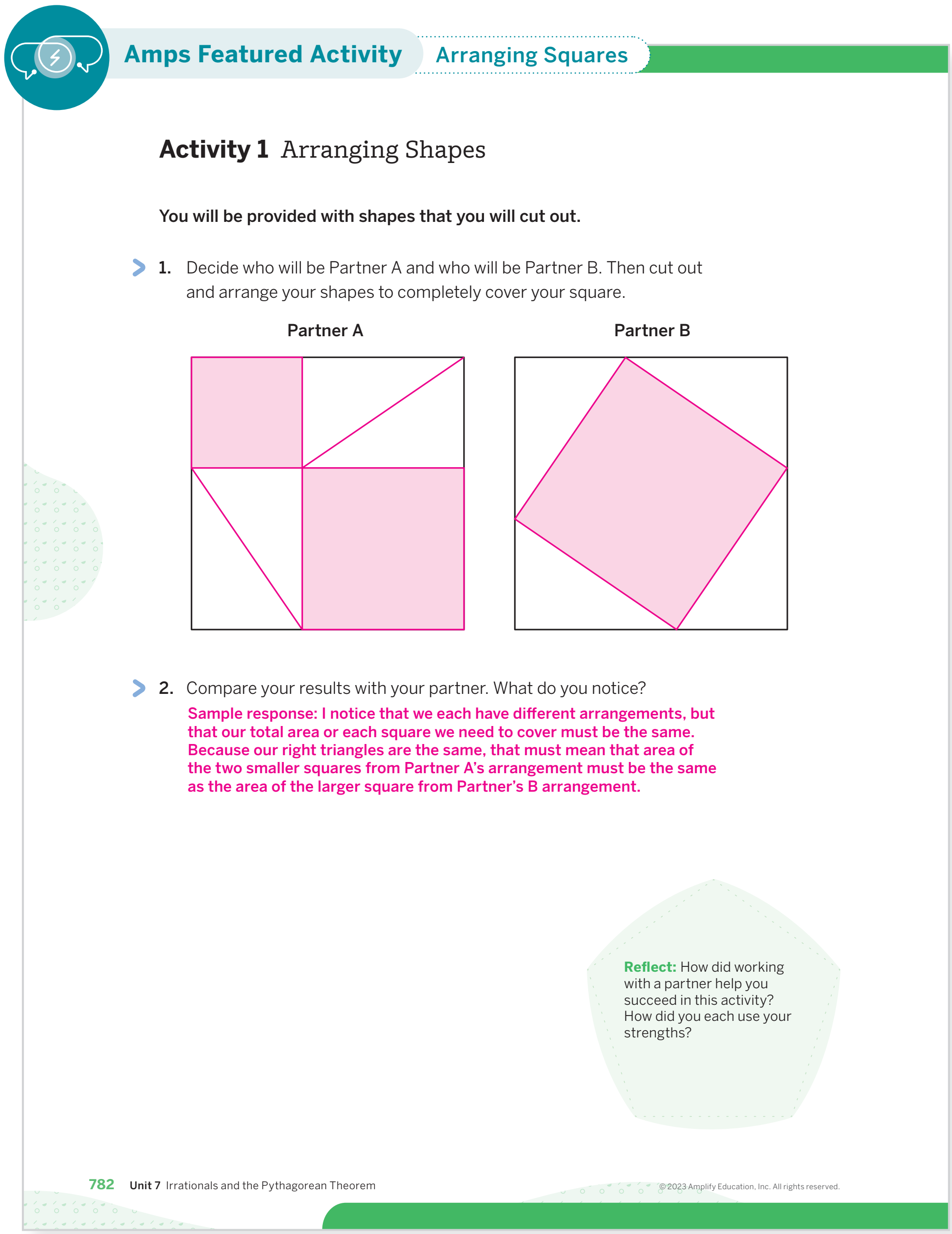
Use: Before Activity 1
Informed by: Performance on Lesson 9, Practice Problem 6

Pairs | 20 min

Activity 1 Arranging Shapes

Students rearrange triangles and squares to notice a relationship between the areas of the shapes in two congruent figures.

Amps Featured Activity Arranging Squares



Activity 1 Arranging Shapes

You will be provided with shapes that you will cut out.

1. Decide who will be Partner A and who will be Partner B. Then cut out and arrange your shapes to completely cover your square.

Partner A **Partner B**

2. Compare your results with your partner. What do you notice?

Sample response: I notice that we each have different arrangements, but that our total area or each square we need to cover must be the same. Because our right triangles are the same, that must mean that area of the two smaller squares from Partner A's arrangement must be the same as the area of the larger square from Partner's B arrangement.

Reflect: How did working with a partner help you succeed in this activity? How did you each use your strengths?

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1 Launch

Tell students that they will use the same squares as in the Warm-up, divided between partners. Each partner will also receive four right triangles. Distribute the Activity 1 PDF and a pair of scissors to each pair of students.

2 Monitor

Help students get started by helping them decide how to arrange the shaded squares in the provided area.

Look for points of confusion:

- **Thinking the area of the squares in each figure is different.** Help students identify that while the arrangement may be different, the four right triangles are the same area. Ask them what this means for the remaining area of the large square and the two smaller squares.

3 Connect

Have students share their arrangements and what they noticed about them. If none of the pairs got both arrangements correct, consider playing the animation from the Activity 1 Amps slides.

Ask:

- "What can you tell about the side lengths of the squares you made?" They are equal because each side is equal to the sum of the legs.
- "What can you tell about the areas of both squares?" They are equal because the sides have the same lengths.
- "What can you tell about the shaded areas?" They are equal, because the areas of the big squares are the same and the total areas of the four triangles are the same, so the remaining areas are the same.

Highlight that the area of the shaded areas on the left is the same as the shaded area on the right.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can digitally arrange right triangles as they work through a proof of the Pythagorean Theorem.

Extension: *Math Enrichment*

Ask students to show how they know the shapes they made are actually squares. Sample response: The right triangles in the shape on the right are congruent and the sum of the non-right angles in a right triangle is 90°. This means the shape I made is a rectangle. One side of the shape is labeled $a + b$ and the other side is labeled $a + b$. These expressions are equivalent, so the side lengths are congruent, which means the shape is a square.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask questions, display these sentence frames to help them organize their thinking.

- "The side lengths of the squares we made are ____ because . . ."
- "The areas of the squares are ____ because . . ."
- "The shaded areas are ____ because . . ."

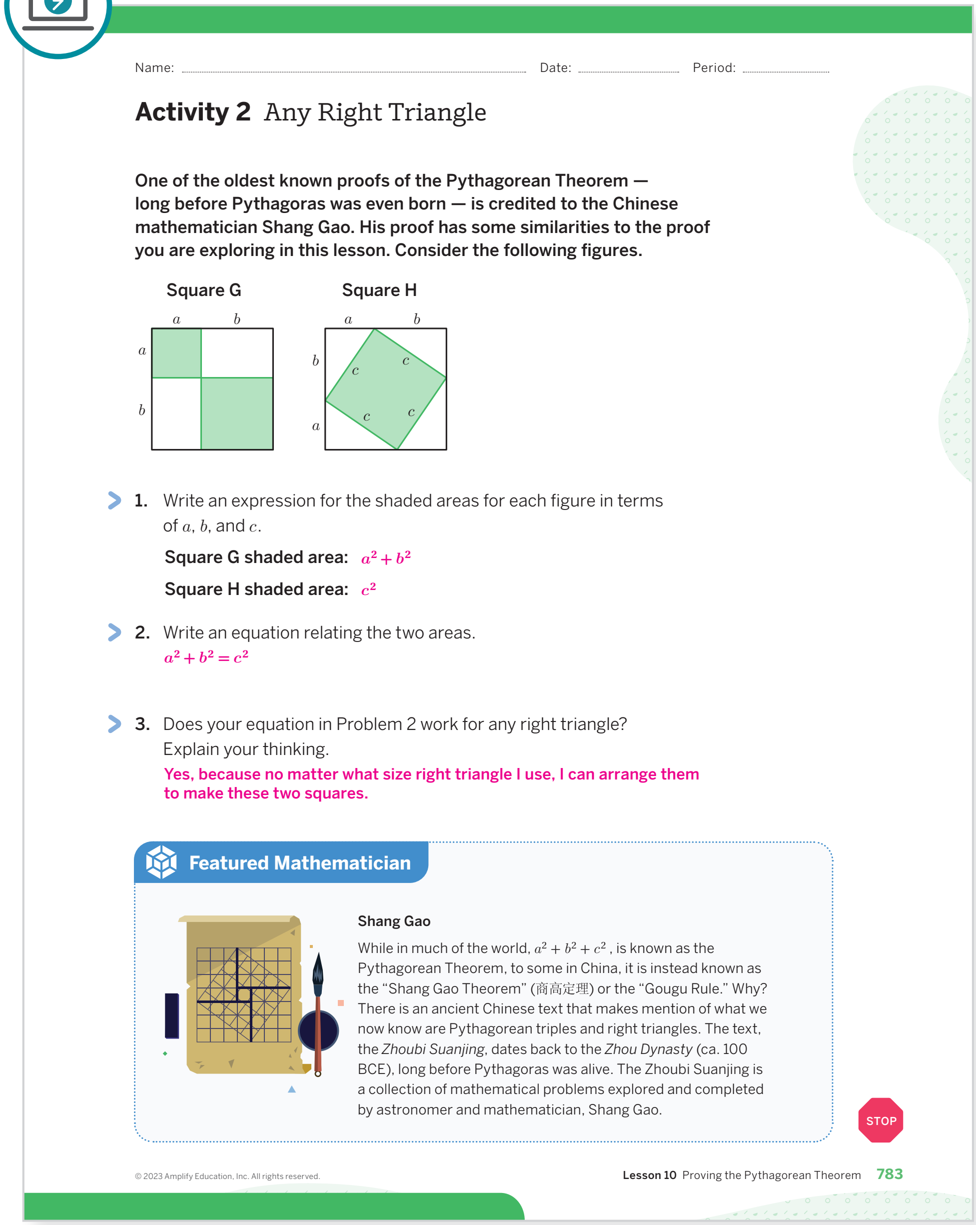
English Learners

Encourage students to use language from the class display related to right triangles.

Pairs | 10 min

Activity 2 Any Right Triangle

Students study the areas of two figures to understand an area-based proof of the Pythagorean Theorem.



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Activity 2 Any Right Triangle

One of the oldest known proofs of the Pythagorean Theorem — long before Pythagoras was even born — is credited to the Chinese mathematician Shang Gao. His proof has some similarities to the proof you are exploring in this lesson. Consider the following figures.

1. Write an expression for the shaded areas for each figure in terms of a, b, and c.

   Square G shaded area: a^2+b^2

   Square H shaded area: c^2

2. Write an equation relating the two areas.

   $a^2+b^2=c^2$

3. Does your equation in Problem 2 work for any right triangle? Explain your thinking.

   Yes, because no matter what size right triangle I use, I can arrange them to make these two squares.

Featured Mathematician

Shang Gao

While in much of the world, $a^2+b^2+c^2$, is known as the Pythagorean Theorem, to some in China, it is instead known as the "Shang Gao Theorem" (商高定理) or the "Gougu Rule." Why? There is an ancient Chinese text that makes mention of what we now know are Pythagorean triples and right triangles. The text, the *Zhoubi Suanjing*, dates back to the *Zhou Dynasty* (ca. 100 BCE), long before Pythagoras was alive. The Zhoubi Suanjing is a collection of mathematical problems explored and completed by astronomer and mathematician, Shang Gao.

STOP

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Lesson 10 Proving the Pythagorean Theorem 783

1 Launch

Tell students they will now examine the same arrangement of triangles and squares as in Activity 1.

2 Monitor

Help students get started by asking them what is the area of a square with side lengths of a.

Look for points of confusion:

- **Thinking that the proof is not true for any right triangle.** Have students sketch a figure similar to Square G, but with different side lengths, and ask whether they think they could make and rearrange right triangles to make a figure such as Square H.

3 Connect

Have students share how they found the expressions for each shaded area.

Ask:

- "Why is the expression a^2+b^2 equal to c^2?
- "Does this prove the Pythagorean Theorem is true for any right triangle? For any triangle?"

Highlight that the transformation of right triangles from Square G to Square H can be duplicated using right triangles with any side length, which means that $a^2+b^2=c^2$ will be true for any right triangle. Show the animation from the Activity 2 Amps slide. Discuss how this animation supports the idea that the Pythagorean Theorem works for any right triangle.

Math Language Development

MLR5: Co-craft Questions

Before revealing the questions in this activity, display the image of the squares and have students work with their partner to write 2–3 mathematical questions they could ask about them. Sample questions shown.

- How do the total areas of the two figures compare?
- How do the shaded areas compare between the two figures?
- How do the unshaded areas compare between the two figures?

English Learners

Use wait time to support students in thinking and formulating mathematical questions.

Featured Mathematician

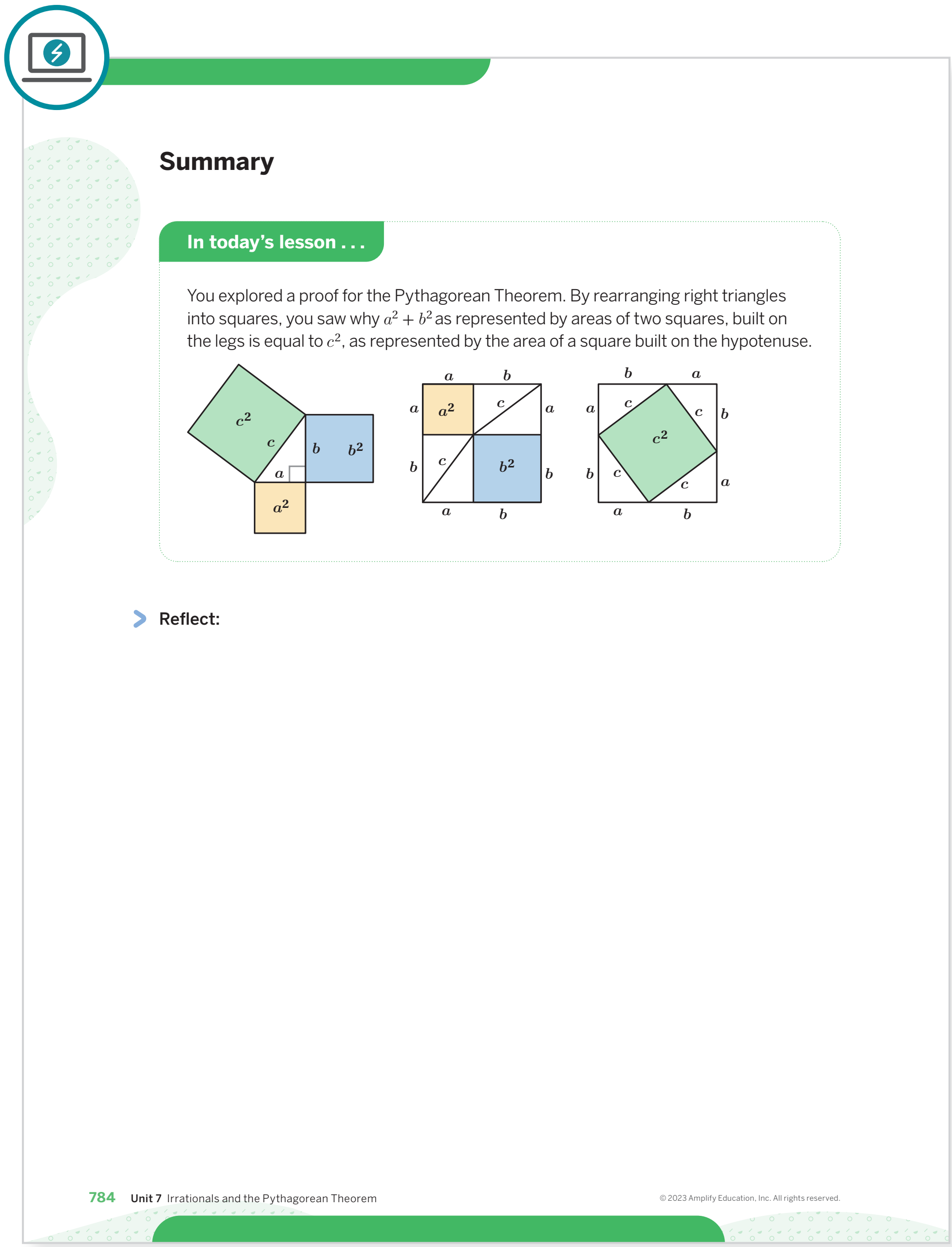
Shang Gao

Have students read about featured matematician, Shang Gao, who may have been the first to discover the proof for what we know now as the Pythagorean Theorem.

Whole Class | 5 min

Summary

Review and synthesize an area-based proof of the Pythagorean Theorem.



Summary

In today's lesson . . .

You explored a proof for the Pythagorean Theorem. By rearranging right triangles into squares, you saw why $a^2 + b^2$ as represented by areas of two squares, built on the legs is equal to c^2, as represented by the area of a square built on the hypotenuse.

Reflect:

784 Unit 7 Irrationals and the Pythagorean Theorem

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Synthesize

Display the figures from the Summary.

Have students share how the figures are related and how they show that $a^2 + b^2 = c^2$.

Ask:

- "How do you think the Pythagorean Theorem might be useful in the real world?"
- "How could you use the Pythagorean Theorem to determine unknown dimensions?"
- "Do you think the converse is true — if a triangle is not a right triangle, then it will not satisfy the Pythagorean Theorem?"

Highlight that students will study applications of the Pythagorean Theorem in future lessons, including an exploration for whether the converse is true. Tell students there are other ways to prove the Pythagorean Theorem and encourage them to research other proofs online.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- Is it true that $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ for all right triangles? If so, can you prove it?

Differentiated Support

Extension: ***Math Enrichment, Interdisciplinary Connections***

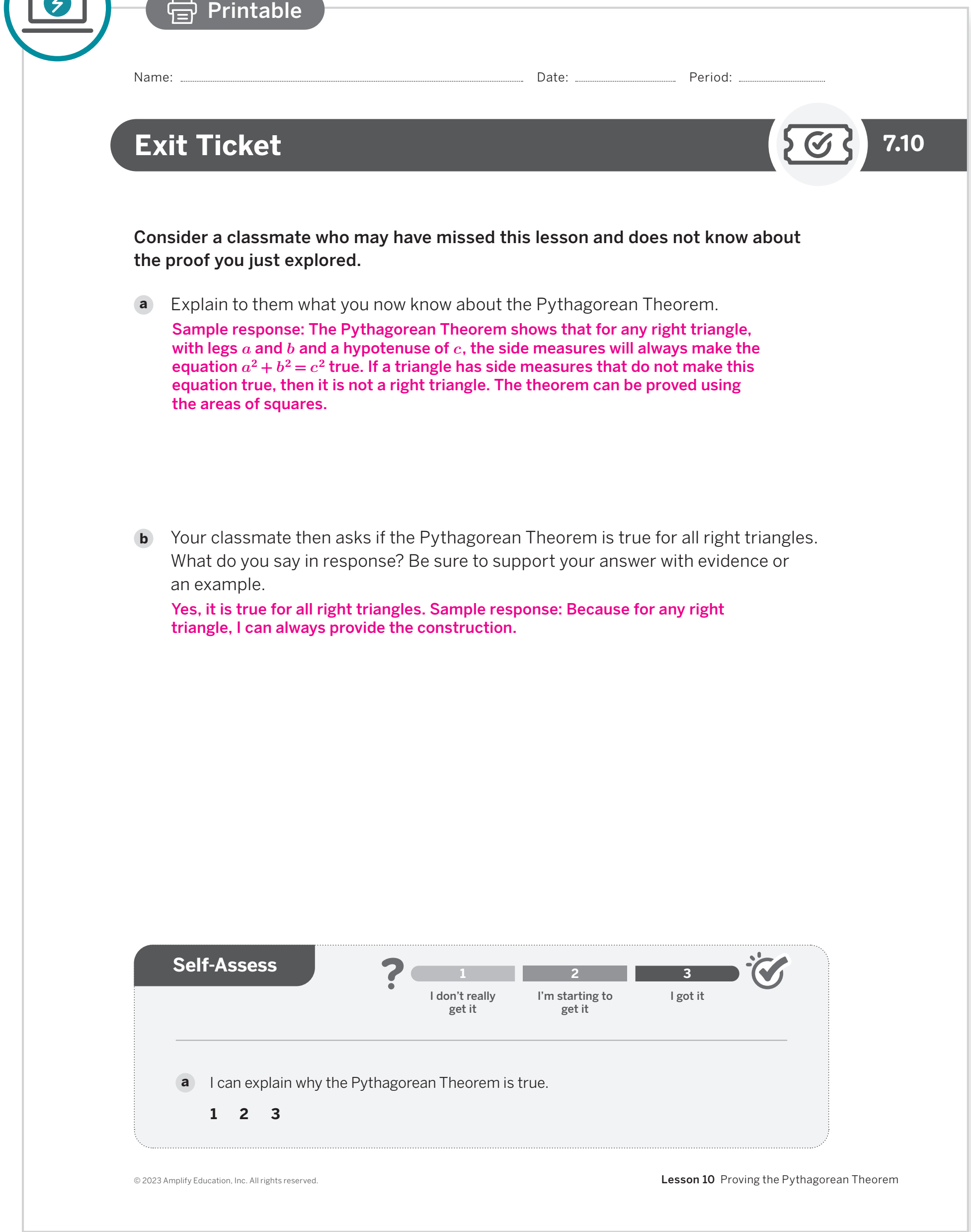
As stated in the Sub-Unit 2 narrative, there are over 500 proofs of the Pythagorean Theorem today. Tell students that Albert Einstein even developed a proof when he was 12 years old. Einstein's proof was based on the properties of similar triangles. He later used the Pythagorean Theorem in both the Special Theory of Relativity and the General Theory of Relativity. In these, the Pythagorean Theorem graphically relates energy, momentum, and mass. Have students research Einstein's proof of the Pythagorean Theorem, and then describe it in their own words, using diagrams or illustrations. **(History, Science)**

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by explaining a proof of the Pythagorean Theorem.

Printable



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Exit Ticket 7.10

Consider a classmate who may have missed this lesson and does not know about the proof you just explored.

a Explain to them what you now know about the Pythagorean Theorem.

Sample response: The Pythagorean Theorem shows that for any right triangle, with legs a and b and a hypotenuse of c, the side measures will always make the equation $a^2 + b^2 = c^2$ true. If a triangle has side measures that do not make this equation true, then it is not a right triangle. The theorem can be proved using the areas of squares.

b Your classmate then asks if the Pythagorean Theorem is true for all right triangles. What do you say in response? Be sure to support your answer with evidence or an example.

Yes, it is true for all right triangles. Sample response: Because for any right triangle, I can always provide the construction.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain why the Pythagorean Theorem is true.

1 2 3

Lesson 10 Proving the Pythagorean Theorem

Success looks like . . .

- **Goal:** Understanding that the Pythagorean Theorem is true for every right triangle.
  - » Explaining that a construction can be provided in part b.
- **Language Goal:** Explaining a proof of the Pythagorean Theorem. **(Speaking and Listening)**
  - » Explaining that the theorem can be proved using areas of squares in part a.

Suggested next steps

If students have a difficult time explaining the Pythagorean Theorem, consider

- Referring them to Activities 1 and 2 and having them use either activity as an example to support their thinking.

Professional Learning

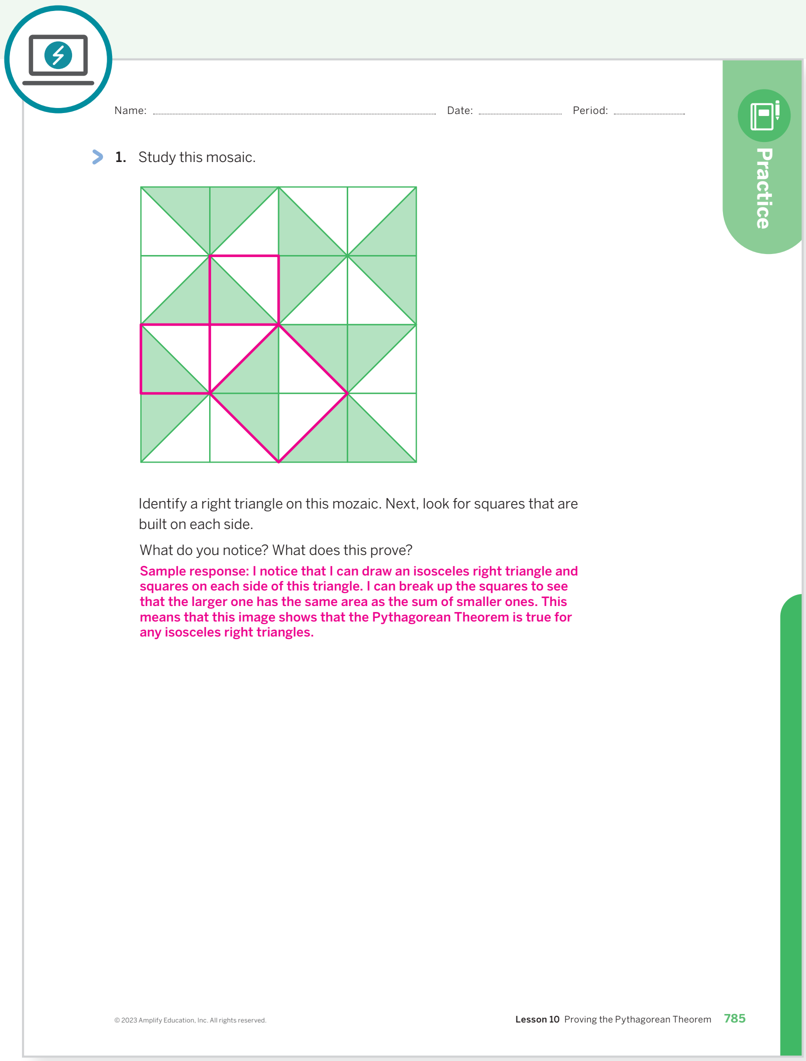
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What was especially satisfying about seeing students prove the Pythagorean Theorem?
- The instructional goal for this lesson was to understand the Pythagorean Theorem is true for all right triangles. How well did students accomplish this? What did you specifically do to help students accomplish it?

Practice

Independent



Name: Date: Period:

Practice

1. Study this mosaic.

Identify a right triangle on this mozaic. Next, look for squares that are built on each side.

What do you notice? What does this prove?

Sample response: I notice that I can draw an isosceles right triangle and squares on each side of this triangle. I can break up the squares to see that the larger one has the same area as the sum of smaller ones. This means that this image shows that the Pythagorean Theorem is true for any isosceles right triangles.

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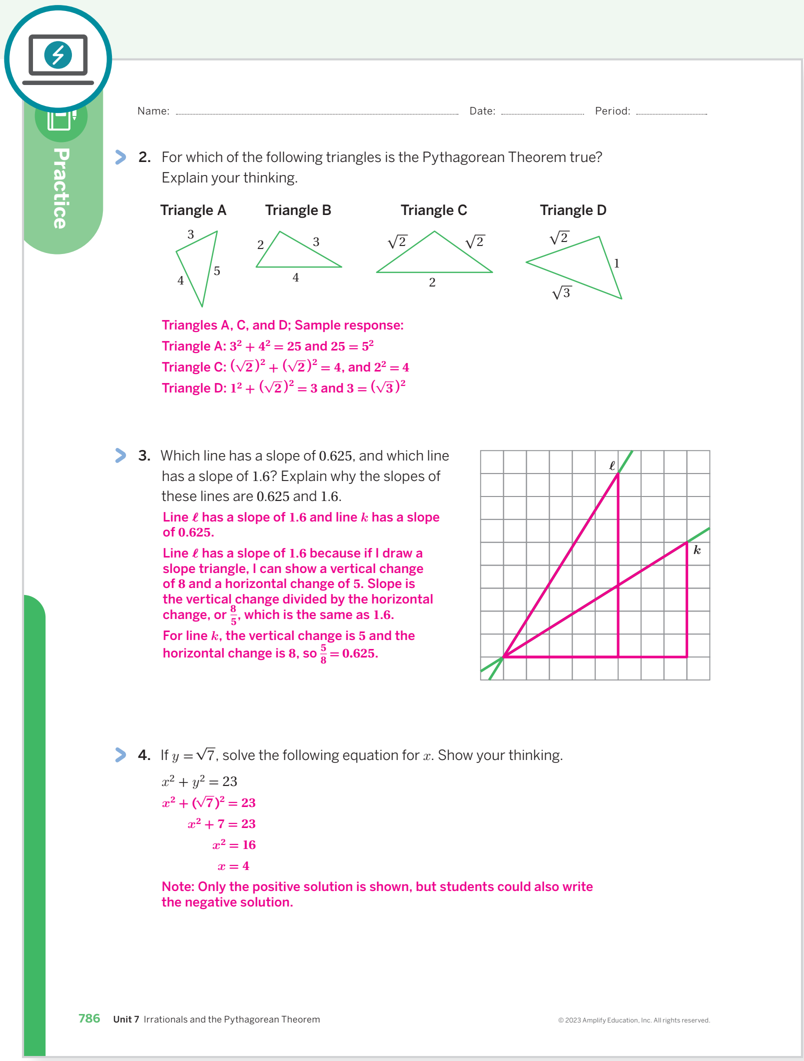
Lesson 10 Proving the Pythagorean Theorem 785



Name: Date: Period:

Practice

2. For which of the following triangles is the Pythagorean Theorem true? Explain your thinking.

Triangles A, C, and D; Sample response:

Triangle A: $3^2 + 4^2 = 25$ and $25 = 5^2$

Triangle C: $(\sqrt{2})^2 + (\sqrt{2})^2 = 4$, and $2^2 = 4$

Triangle D: $1^2 + (\sqrt{2})^2 = 3$ and $3 = (\sqrt{3})^2$

3. Which line has a slope of 0.625, and which line has a slope of 1.6? Explain why the slopes of these lines are 0.625 and 1.6.

Line ℓ has a slope of 1.6 and line k has a slope of 0.625.

Line ℓ has a slope of 1.6 because if I draw a slope triangle, I can show a vertical change of 8 and a horizontal change of 5. Slope is the vertical change divided by the horizontal change, or $\frac{8}{5}$, which is the same as 1.6.

For line k, the vertical change is 5 and the horizontal change is 8, so $\frac{5}{8} = 0.625$.

4. If $y = \sqrt{7}$, solve the following equation for x. Show your thinking.

$$x^2 + y^2 = 23$$
$$x^2 + (\sqrt{7})^2 = 23$$
$$x^2 + 7 = 23$$
$$x^2 = 16$$
$$x = 4$$

Note: Only the positive solution is shown, but students could also write the negative solution.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	3
	2	Activity 2	2
Spiral	3	Unit 2 Lesson 11	2
Formative ⚡	4	Unit 7 Lesson 11	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 11**

Determining Unknown Side Lengths

Let's determine missing side lengths of right triangles.

Focus

Goal

1. **Language Goal:** Calculate unknown side lengths of a right triangle by using the Pythagorean Theorem, and explain the solution method. **(Speaking and Listening, Writing)**

Coherence

Today

Students apply the Pythagorean Theorem to determine unknown side lengths of a right triangle.

Previously

In the first Sub-Unit of this unit, students learned how to use square root notation to represent exact values of irrational numbers. In Lesson 9, students observed patterns found in the side lengths of right triangles and were formally introduced to the Pythagorean Theorem. In Lesson 10, students proved why the Pythagorean Theorem is true for any right triangle.

Coming Soon

In Lesson 12, students will see that the converse of the Pythagorean is also true — if a triangle does not have side lengths that make the equation $a^2 + b^2 = c^2$ true, then it is not a right triangle.

Rigor

- Students **apply** the Pythagorean Theorem to determine unknown side lengths.
- Students build **fluency** working with the Pythagorean Theorem in mathematical and real-world contexts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Small Groups	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *The Pythagorean Theorem*
- Anchor Chart PDF, *The Pythagorean Theorem* (answers)
- calculators
- colored pencils (as needed)

Math Language Development

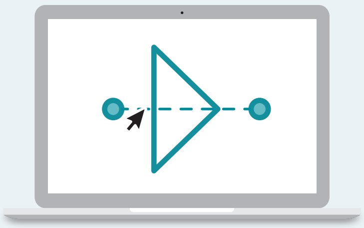
Review words

- *hypotenuse*
- *legs*
- *Pythagorean Theorem*
- *square root*

Amps | **Featured Activity**

Activity 2
Interactive Geometry

Students can explore the dimensions of a 3D prism digitally, as they try to determine the measure of its diagonal.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively go off script and not apply what they have been taught to determine the lengths of the diagonals in Activity 2. Remind students that the first thing they must do is make sense of the problem by determining what the edge lengths of the prism represent in a right triangle. Then they can determine the lengths of the diagonals in a new context.

Modifications to Pacing

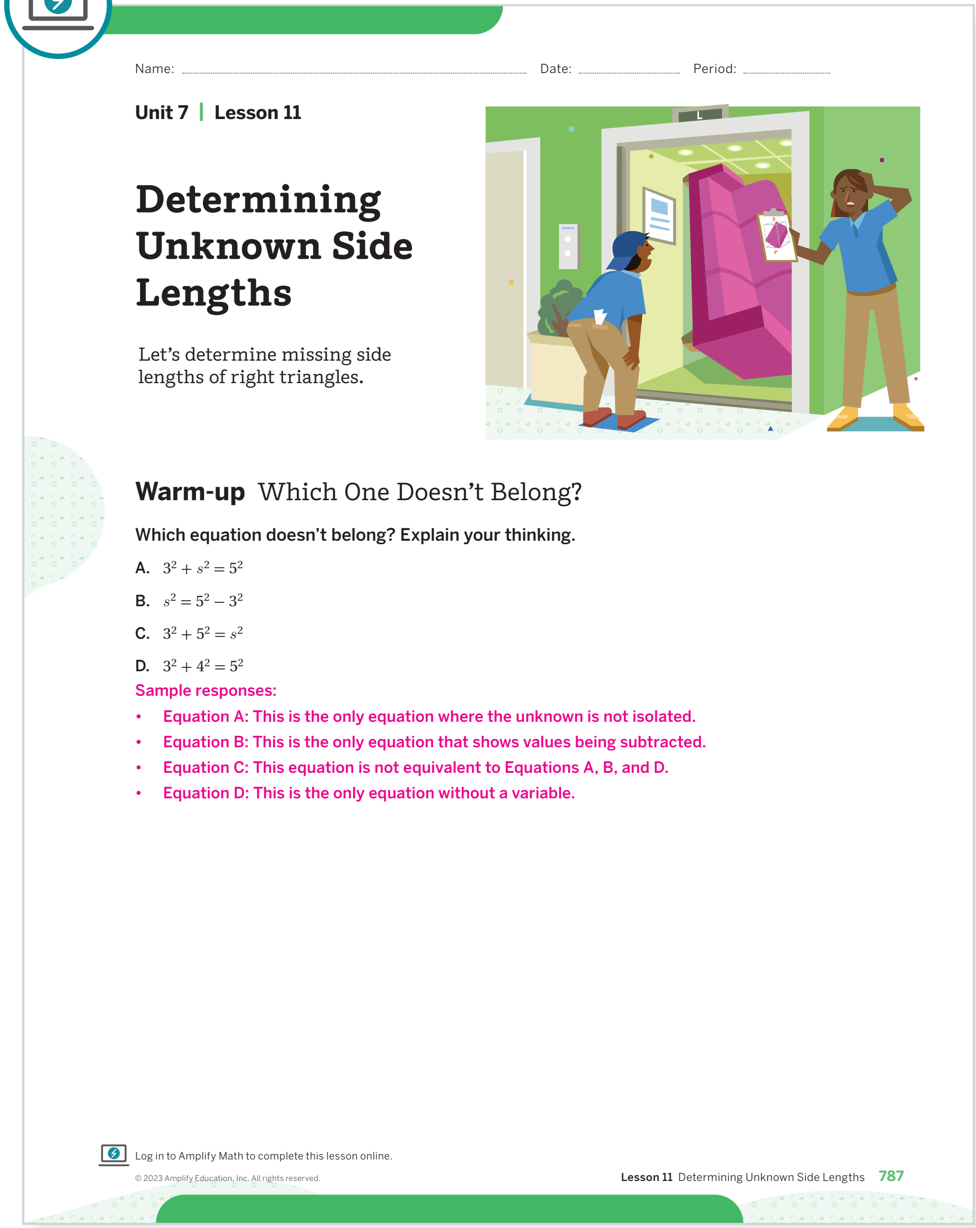
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students solve only Problems 1 and 2.
- **Activity 2** may be omitted and instead used as additional practice.

Small Groups | 5 min

Warm-up Which One Doesn't Belong?

Students study examples of equations to prime students for solving equations that arise while using the Pythagorean Theorem.



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Unit 7 | Lesson 11

Determining Unknown Side Lengths

Let's determine missing side lengths of right triangles.

Warm-up Which One Doesn't Belong?

Which equation doesn't belong? Explain your thinking.

A. $3^2 + s^2 = 5^2$

B. $s^2 = 5^2 - 3^2$

C. $3^2 + 5^2 = s^2$

D. $3^2 + 4^2 = 5^2$

Sample responses:

- Equation A: This is the only equation where the unknown is not isolated.
- Equation B: This is the only equation that shows values being subtracted.
- Equation C: This equation is not equivalent to Equations A, B, and D.
- Equation D: This is the only equation without a variable.

Log in to Amplify Math to complete this lesson online.

Lesson 11 Determining Unknown Side Lengths 787

1 Launch

Activate students' prior knowledge by asking them to recall the formula for the Pythagorean Theorem and to which types of triangles it applies.

Conduct the ***Which One Doesn't Belong*** routine. Arrange students in groups of 2–4. In their small groups, tell each student to share their reason why a particular equation does not belong and to determine together at least one reason why each equation doesn't belong

2 Monitor

Help students get started by asking if they see any similarities between the equations in A and B.

Look for points of confusion:

- **Thinking all the equations are different or all the equations are equal.** Ask students what they would need to do to the equation in Choice A to isolate s^2 to help them see its relation to the equation in Choice B.

Look for productive strategies:

- Noticing that Choice C shows the only equation that is not equivalent to the others.

3 Connect

Have students share why each equation does not belong. Record student thinking for all to see. After each response, ask the class whether they agree or disagree.

Highlight that the equation in Choice C is the only equation that is not equivalent to the other equations.

Ask:

- "How can you show that the equations in Choices A, B, and D are all equivalent?"
- "Do you notice any connections between these equations and what you learned yesterday?"

Power-up

To power up students' ability to solve equations involving square roots, have students complete:

Recall that $(\sqrt{x})^2 = x$, for any $x \geq 0$. Therefore, $(\sqrt{3})^2 = 3$. Given that $y = \sqrt{3}$, solve each equation for x.

a. $x - y^2 = 1$

$x - (\sqrt{3})^2 = 1$

$x - 3 = 1$

$x = 4$

b. $x^2 - y^2 = 1$

$x^2 - (\sqrt{3})^2 = 1$

$x^2 - 3 = 1$

$x^2 = 4$

$x = 2$

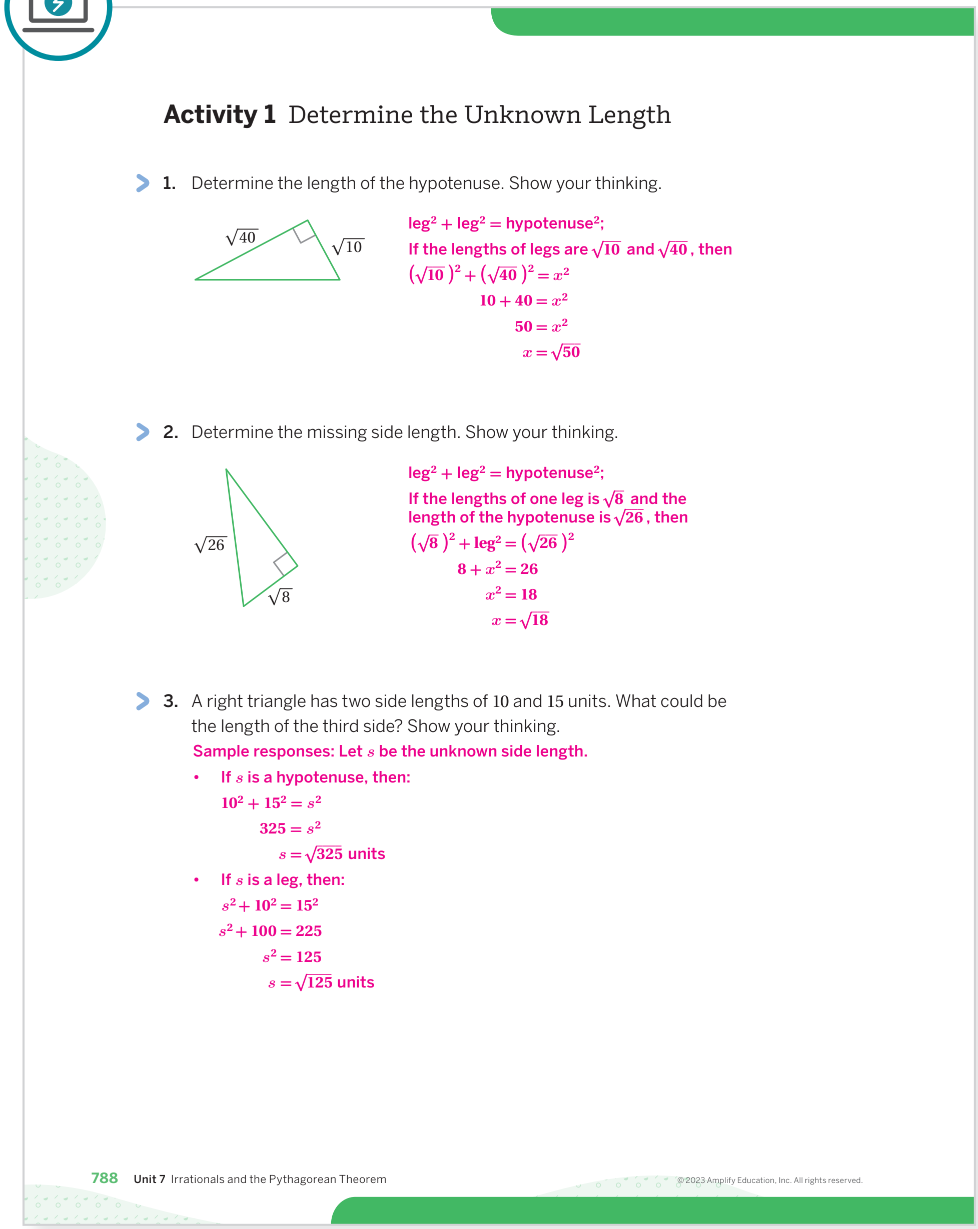
Use: Before Activity 1

Informed by: Performance on Lesson 10, Practice Problem 4

Pairs | 20 min

Activity 1 Determine the Unknown Length

Students apply the Pythagorean Theorem to determine unknown side lengths.



Activity 1 Determine the Unknown Length

1. Determine the length of the hypotenuse. Show your thinking.

leg^2 + leg^2 = hypotenuse^2;
If the lengths of legs are $\sqrt{10}$ and $\sqrt{40}$, then
$$(\sqrt{10})^2 + (\sqrt{40})^2 = x^2$$
$$10 + 40 = x^2$$
$$50 = x^2$$
$$x = \sqrt{50}$$

2. Determine the missing side length. Show your thinking.

leg^2 + leg^2 = hypotenuse^2;
If the lengths of one leg is $\sqrt{8}$ and the length of the hypotenuse is $\sqrt{26}$, then
$$(\sqrt{8})^2 + \text{leg}^2 = (\sqrt{26})^2$$
$$8 + x^2 = 26$$
$$x^2 = 18$$
$$x = \sqrt{18}$$

3. A right triangle has two side lengths of 10 and 15 units. What could be the length of the third side? Show your thinking.

Sample responses: Let s be the unknown side length.

- If s is a hypotenuse, then:
  $$10^2 + 15^2 = s^2$$
  $$325 = s^2$$
  $$s = \sqrt{325} \text{ units}$$
- If s is a leg, then:
  $$s^2 + 10^2 = 15^2$$
  $$s^2 + 100 = 225$$
  $$s^2 = 125$$
  $$s = \sqrt{125} \text{ units}$$

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1 Launch

Arrange students in groups of 2. Give students 10 minutes to work quietly and then have them compare with a partner. If partners disagree about any of their answers, ask them to explain their reasoning to one another until they reach agreement. Consider providing access to calculators for the duration of the lesson.

2 Monitor

Help students get started by referencing the Anchor Chart PDF, *The Pythagorean Theorem*, writing leg^2 + leg^2 = hypotenuse^2, and identifying which values they can substitute for Problem 1.

Look for points of confusion:

- **Not being sure how to solve the equation in Problem 1.** Remind students that $(\sqrt{(x)})^2 = x$.
- **Not being sure how to solve for the length of the leg in Problem 2.** Ensure students have substituted correctly and ask them which variable they are trying to isolate.

Look for points of confusion:

- Drawing and labeling a triangle with the given side lengths in Problem 3.

3 Connect

Have students share how they used the Pythagorean Theorem to determine the missing side lengths in Problems 1 and 2.

Display student work for Problem 3 showing $s = \sqrt{325}$ units and student work showing $s = \sqrt{125}$ units, with drawn triangles, if available.

Ask, "How can both values be true for Problem 3?"

Highlight that when two side lengths of a right triangle are known, students can always determine the third side length by using the Pythagorean Theorem. Remind them that it is important to keep track of which side is the hypotenuse, but the order in which the legs are listed in the equation does not matter.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1 and 2. As time permits, they can work on Problem 3.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students use color coding to annotate the legs of each right triangle in one color and the hypotenuse in another color.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect solution for Problem 3 such as, "The side lengths of the triangle are 9, 15, and $\sqrt{306}$, where the hypotenuse has a length of 15 units." Ask:

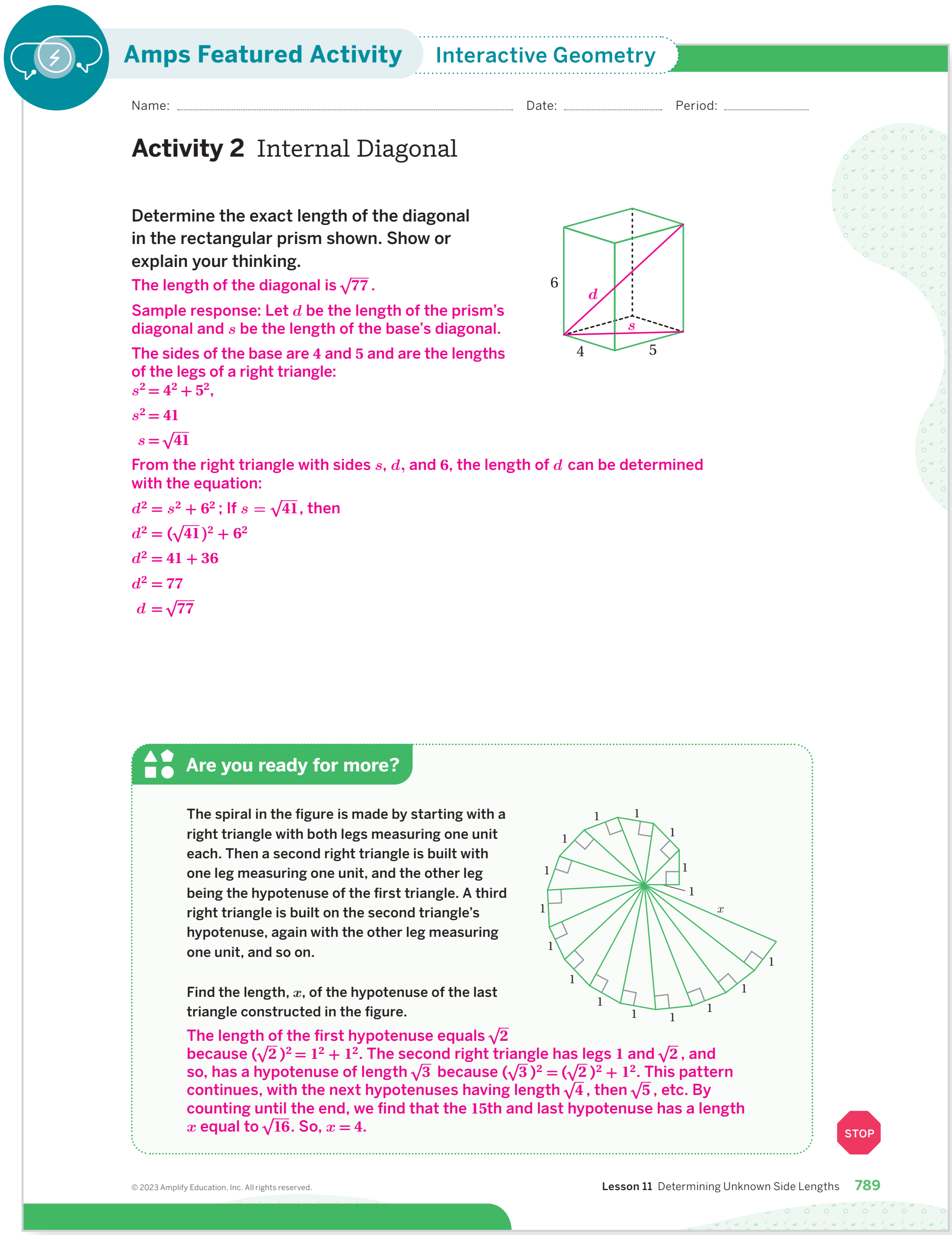
- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking." Listen for students who recognize that such a triangle cannot be formed because 9 + 15 is not greater than $\sqrt{306}$.
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

Pairs | 10 min

Activity 2 Internal Diagonal

Students repeatedly apply the Pythagorean Theorem to determine unknown diagonal lengths in a new context and build fluency using the theorem.

Amps Featured Activity | Interactive Geometry



Name: Date: Period:

Activity 2 Internal Diagonal

Determine the exact length of the diagonal in the rectangular prism shown. Show or explain your thinking.

The length of the diagonal is $\sqrt{77}$.

Sample response: Let d be the length of the prism's diagonal and s be the length of the base's diagonal.

The sides of the base are 4 and 5 and are the lengths of the legs of a right triangle:

$s^2 = 4^2 + 5^2$,

$s^2 = 41$

$s = \sqrt{41}$

From the right triangle with sides s, d, and 6, the length of d can be determined with the equation:

$d^2 = s^2 + 6^2$; If $s = \sqrt{41}$, then

$d^2 = (\sqrt{41})^2 + 6^2$

$d^2 = 41 + 36$

$d^2 = 77$

$d = \sqrt{77}$

Are you ready for more?

The spiral in the figure is made by starting with a right triangle with both legs measuring one unit each. Then a second right triangle is built with one leg measuring one unit, and the other leg being the hypotenuse of the first triangle. A third right triangle is built on the second triangle's hypotenuse, again with the other leg measuring one unit, and so on.

Find the length, x, of the hypotenuse of the last triangle constructed in the figure.

The length of the first hypotenuse equals $\sqrt{2}$ because $(\sqrt{2})^2 = 1^2 + 1^2$. The second right triangle has legs 1 and $\sqrt{2}$, and so, has a hypotenuse of length $\sqrt{3}$ because $(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2$. This pattern continues, with the next hypotenuses having length $\sqrt{4}$, then $\sqrt{5}$, etc. By counting until the end, we find that the 15th and last hypotenuse has a length x equal to $\sqrt{16}$. So, $x = 4$.

STOP

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Lesson 11 Determining Unknown Side Lengths 789

1 Launch

Activate students' prior knowledge from Lesson 1 by asking students to draw the longest diagonal length. Display a correct example so all students can be sure they are locating the correct length.

2 Monitor

Help students get started by asking them to draw a triangle using the internal diagonal as a hypotenuse, and then identifying the necessary leg lengths.

Look for points of confusion:

- **Not being sure how to find the measure of the diagonal on the base.** Have students draw an additional 2D sketch of the base and ask them to label the sides of the right triangle created by the diagonal drawn from one corner to the other.
- **Not being sure how to use the diagonal drawn on the base to find the diagonal, d.** Have students draw a third sketch of the right triangle created by the diagonal, s and have students label the sides.

Look for productive strategies:

- Noticing that the diagonal length of the rectangular prism is the square root of the sum of the squares of the three edge lengths.

3 Connect

Have students share how they calculated the diagonal of the rectangular prism. Discuss how the answer will be the same no matter which way they drew their diagonals.

Highlight that the diagonal length of the rectangular prism is the square root of the sum of the squares of the three edge lengths.

Ask, "What is different about the value you found today compared to the value you found in Lesson 1?" In Lesson 1, I was only able to estimate a length, but today I can determine the exact length represented by an irrational number using the square root sign.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can explore the dimensions of a 3D prism digitally, as they try to determine the measure of its diagonal.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students draw the diagonal of the base in one color, and then the diagonal of the entire prism in another color. Ask them how they can use the Pythagorean Theorem to determine the diagonal length of the base first.

Math Language Development

MLR3: Critique, Correct, Clarify

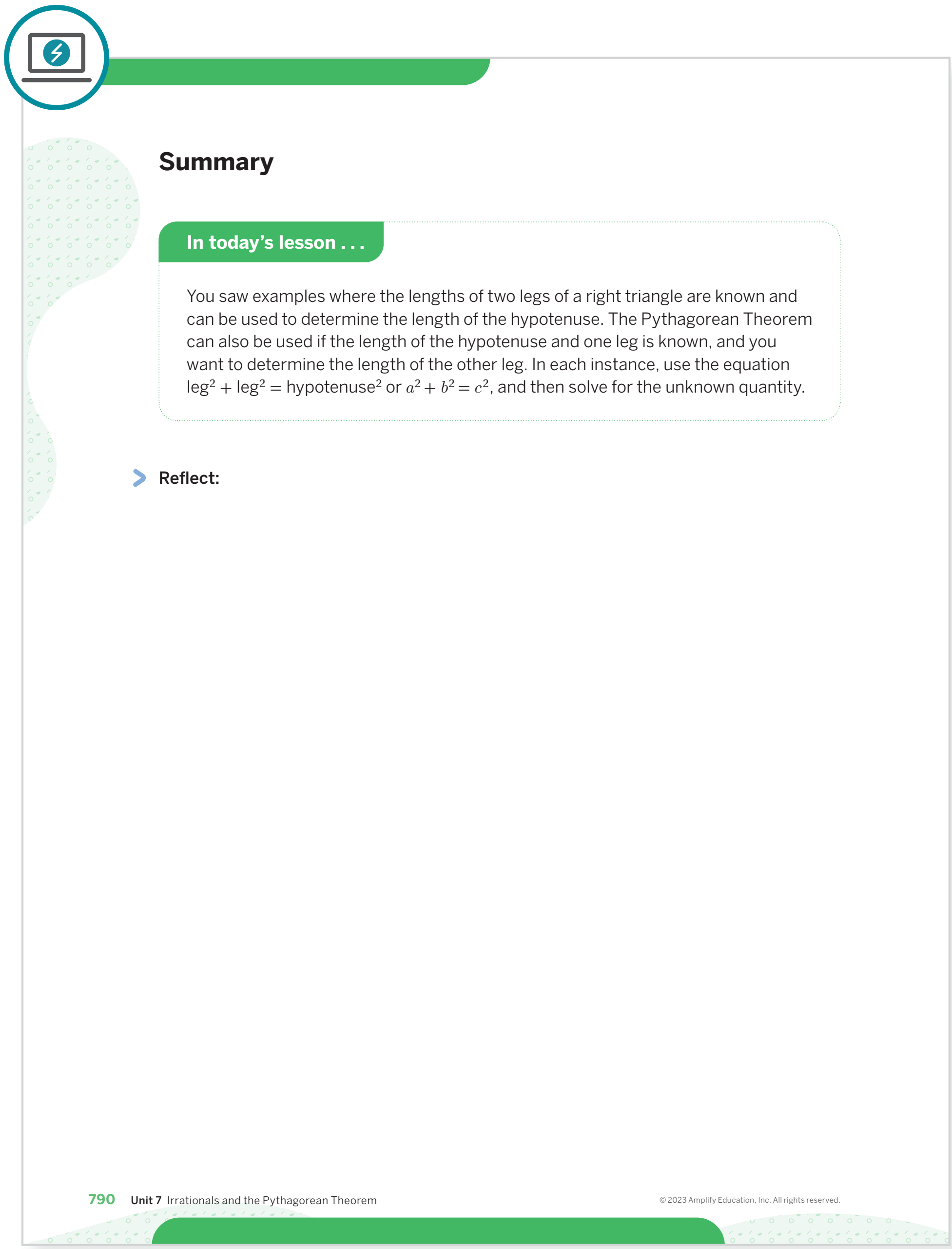
During the Connect, present an incorrect solution based on a common misunderstanding of not distinguishing between the diagonal of the base and the diagonal of the entire prism. For example, "The length of the diagonal is $\sqrt{41}$ units because the side lengths of the base are 4 and 5." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
- ***Correct and Clarify:*** "Write a corrected statement. What mathematical language can you use to explain why your statement is correct?"

Whole Class | 5 min

Summary

Review and synthesize how to apply the Pythagorean Theorem to determine an unknown side length, given the measures of two of the side lengths.



Summary

In today's lesson . . .

You saw examples where the lengths of two legs of a right triangle are known and can be used to determine the length of the hypotenuse. The Pythagorean Theorem can also be used if the hypotenuse and one leg is known, and you want to determine the length of the other leg. In each instance, use the equation leg^2 + leg^2 = hypotenuse^2 or $a^2 + b^2 = c^2$, and then solve for the unknown quantity.

Reflect:

Synthesize

Have students share how they can use the Pythagorean Theorem to determine the length of an unknown side.

Ask:

- "What steps did you have to take to determine unknown side measures of a right triangle?"
- "What are some situations that involve solving for the length of a leg or hypotenuse of a right triangle?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

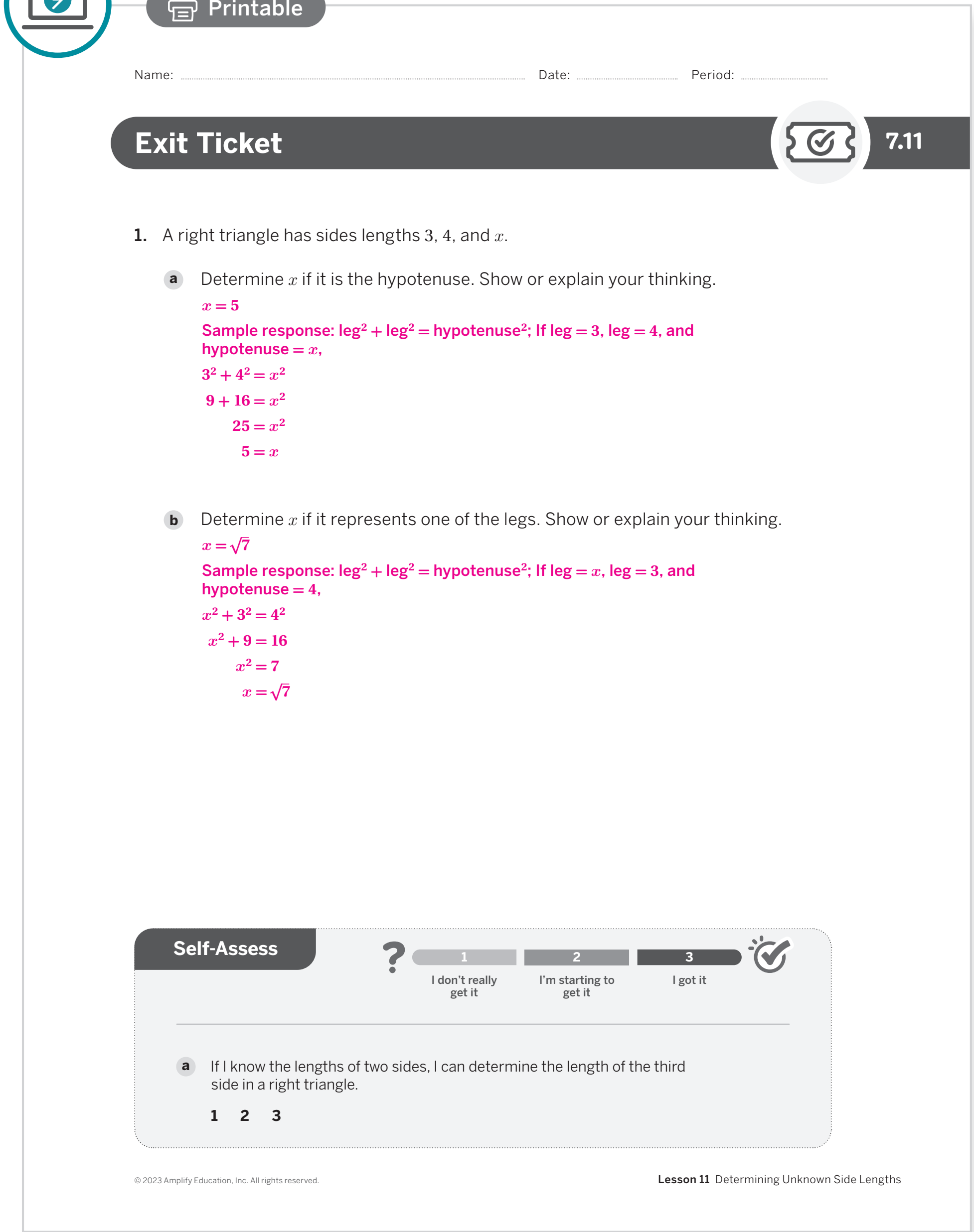
- "When using the Pythagorean Theorem, what fluency skills did you find most important to master?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the unknown side of a right triangle.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket

7.11

1. A right triangle has sides lengths 3, 4, and x.

a Determine x if it is the hypotenuse. Show or explain your thinking.

$x = 5$

Sample response: leg^2 + leg^2 = hypotenuse^2; If leg = 3, leg = 4, and hypotenuse = x,

$3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$

$5 = x$

b Determine x if it represents one of the legs. Show or explain your thinking.

$x = \sqrt{7}$

Sample response: leg^2 + leg^2 = hypotenuse^2; If leg = x, leg = 3, and hypotenuse = 4,

$x^2 + 3^2 = 4^2$

$x^2 + 9 = 16$

$x^2 = 7$

$x = \sqrt{7}$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a If I know the lengths of two sides, I can determine the length of the third side in a right triangle.

1 2 3

Lesson 11 Determining Unknown Side Lengths

Success looks like . . .

- **Language Goal:** Calculating unknown side lengths of a right triangle by using the Pythagorean Theorem, and explaining the solution method. **(Speaking and Listening, Writing)**
  - » Determining whether a side length is the hypotenuse or leg.

Suggested next steps

If students have difficulty determining the unknown hypotenuse, consider:

- Reviewing strategies for applying the Pythagorean Theorem from Activity 1.
- Assigning Practice Problem 1.

If students have difficulty determining the unknown leg, consider:

- Reviewing strategies for applying the Pythagorean Theorem from Activity 1.
- Assigning Practice Problem 1.

Professional Learning

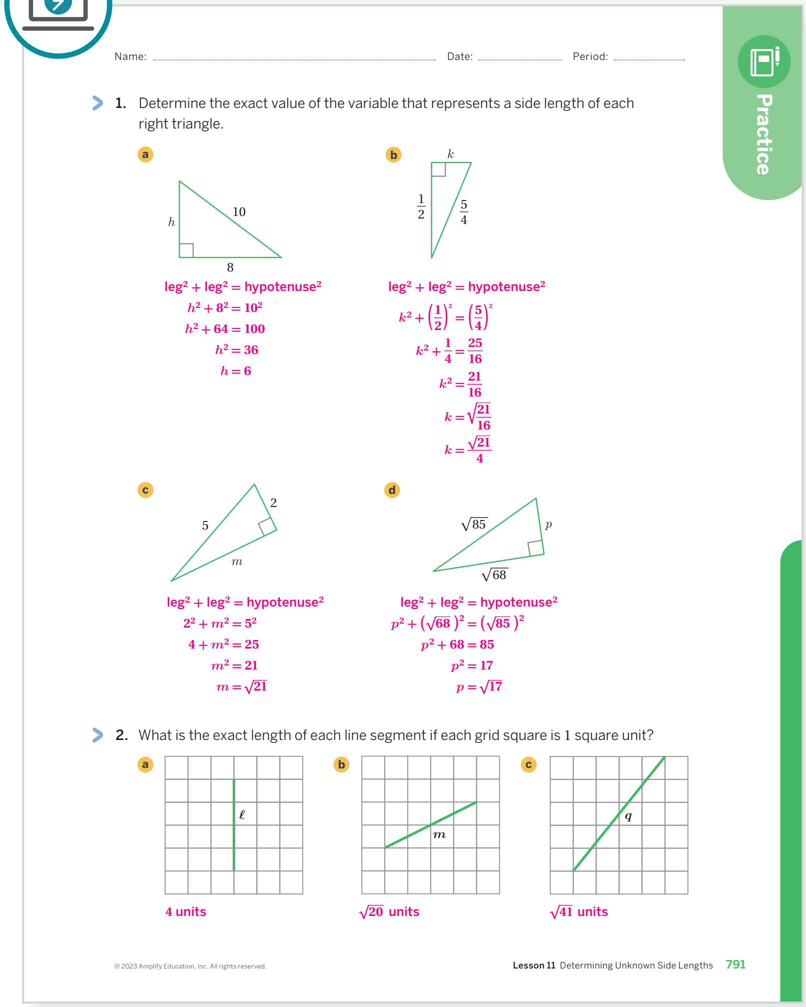
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In earlier lessons, students studied squares and square roots. How did that support students in solving for the unknown side length?
- In what ways have your students gotten better at solving equations over the course of the year?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Determine the exact value of the variable that represents a side length of each right triangle.

a

leg² + leg² = hypotenuse²
$h^2 + 8^2 = 10^2$
$h^2 + 64 = 100$
$h^2 = 36$
$h = 6$

b

leg² + leg² = hypotenuse²
$k^2 + \left(\frac{1}{2}\right)^2 = \left(\frac{5}{4}\right)^2$
$k^2 + \frac{1}{4} = \frac{25}{16}$
$k^2 = \frac{21}{16}$
$k = \sqrt{\frac{21}{16}}$
$k = \frac{\sqrt{21}}{4}$

c

leg² + leg² = hypotenuse²
$2^2 + m^2 = 5^2$
$4 + m^2 = 25$
$m^2 = 21$
$m = \sqrt{21}$

d

leg² + leg² = hypotenuse²
$p^2 + (\sqrt{68})^2 = (\sqrt{85})^2$
$p^2 + 68 = 85$
$p^2 = 17$
$p = \sqrt{17}$

2. What is the exact length of each line segment if each grid square is 1 square unit?

a 4 units

b $\sqrt{20}$ units

c $\sqrt{41}$ units

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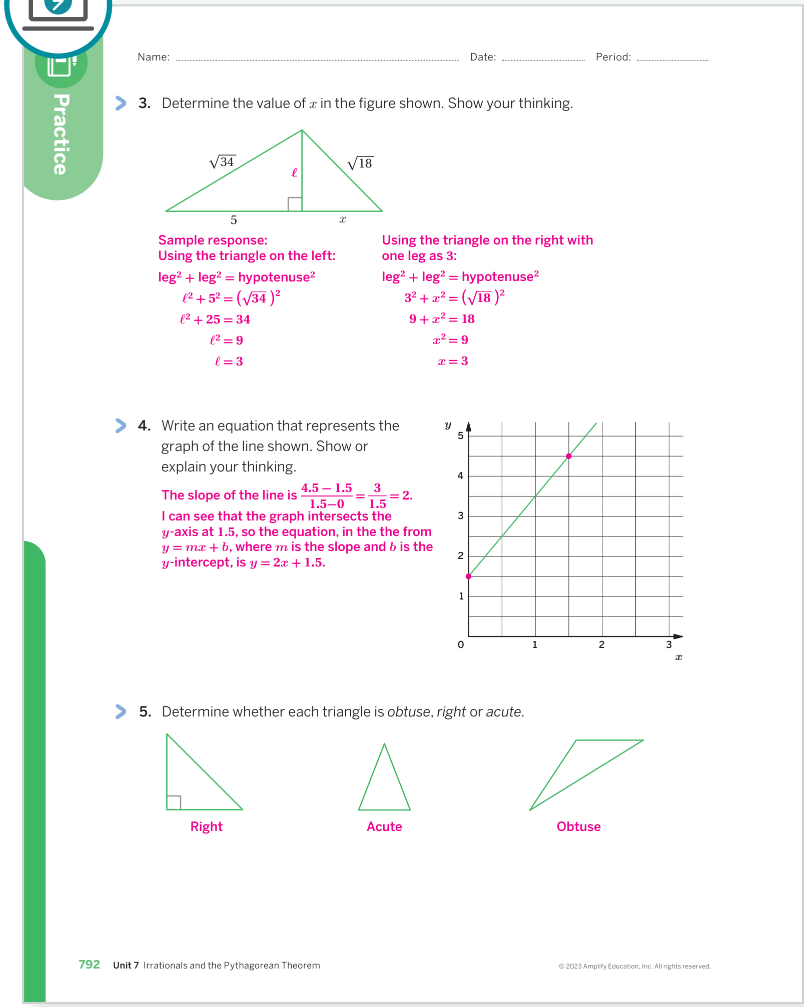
Lesson 11 Determining Unknown Side Lengths 791



Name: ______ Date: ______ Period: ______

3. Determine the value of x in the figure shown. Show your thinking.

Sample response:
Using the triangle on the left:
leg² + leg² = hypotenuse²
$\ell^2 + 5^2 = (\sqrt{34})^2$
$\ell^2 + 25 = 34$
$\ell^2 = 9$
$\ell = 3$

Using the triangle on the right with one leg as 3:
leg² + leg² = hypotenuse²
$3^2 + x^2 = (\sqrt{18})^2$
$9 + x^2 = 18$
$x^2 = 9$
$x = 3$

4. Write an equation that represents the graph of the line shown. Show or explain your thinking.

The slope of the line is $\frac{4.5 - 1.5}{1.5 - 0} = \frac{3}{1.5} = 2.$ I can see that the graph intersects the y-axis at 1.5, so the equation, in the the from $y = mx + b$, where m is the slope and b is the y-intercept, is $y = 2x + 1.5$.

5. Determine whether each triangle is *obtuse*, *right* or *acute*.

Right

Acute

Obtuse

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 9	1
Formative	5	Unit 7 Lesson 12	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 12**

Converse of the Pythagorean Theorem

Let's determine whether a triangle is a right triangle.

Focus

Goals

1. Understand the converse of the Pythagorean Theorem.
2. **Language Goal:** Determine whether a triangle with given side lengths is an acute, right, or obtuse triangle using the converse of the Pythagorean Theorem. **(Speaking and Listening, Writing)**

Coherence

Today

In this lesson, students show that the converse of the Pythagorean Theorem is true. They have an opportunity to decide whether a triangle with three given side lengths is or is not a right triangle.

Previously

In Lessons 9 and 10, students were introduced to the Pythagorean Theorem. In Lesson 11, students applied the Pythagorean Theorem to find unknown side lengths.

Coming Soon

In Lessons 13 and 14, students will use the Pythagorean Theorem to determine distances on the coordinate plane. In Lesson 15, students will apply the Pythagorean Theorem to solve real-world problems.

Rigor

- Students **apply** the converse of the Pythagorean Theorem to determine if a triangle is a right triangle.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
8 min	15 min	10 min	8 min	5 min	5 min
Pairs	Small Groups	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *The Pythagorean Theorem*
- Anchor Chart PDF, *The Pythagorean Theorem* (answers)
- plain sheets of paper
- rulers

Math Language Development

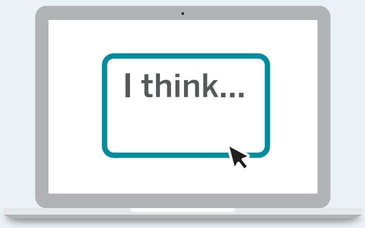
Review words

- *acute angle*
- *congruent*
- *hypotenuse*
- *legs*
- *obtuse angle*
- *Pythagorean Theorem*
- *square root*

Amps | Featured Activity

Activity 1
See Student Thinking

Students can use digital geometry tools to create triangles. You can view their thinking in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might have a tendency to get off track, lacking the self-discipline to focus on the task at hand. Have students think of ways they can recall previous concepts to help focus on the task at hand. As students work to determine whether a triangle is acute, obtuse, or right, have them study the structure of each triangle, and draw a connection to how the Pythagorean Theorem can be used to determine the type of triangle without knowing the angle measures.

Modifications to Pacing

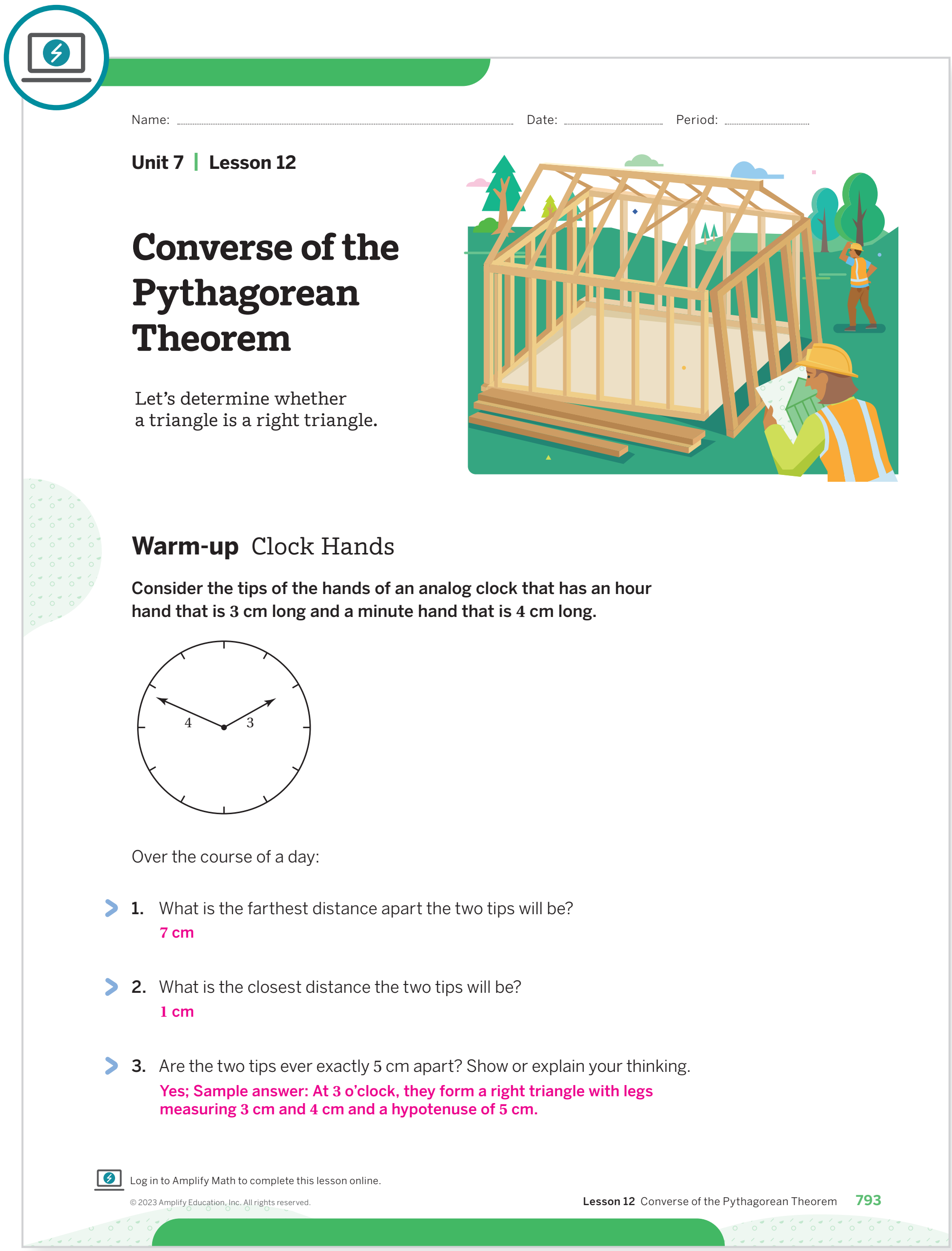
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 1 and 2 may be omitted.
- In **Activity 1**, have students work in groups of three, each drawing one of the triangles, rather than having each student draw three triangles. Have groups compare their triangles with the triangles from another group to look for similarities and differences.
- **Activity 3** may be omitted and used instead as additional practice.

Pairs | 8 min

Warm-up Clock Hands

Students use the hands on a clock to explore the relationships between the sides of a triangle.



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Unit 7 | Lesson 12

Converse of the Pythagorean Theorem

Let's determine whether a triangle is a right triangle.

Warm-up Clock Hands

Consider the tips of the hands of an analog clock that has an hour hand that is 3 cm long and a minute hand that is 4 cm long.

Over the course of a day:

1. What is the farthest distance apart the two tips will be?
   7 cm
2. What is the closest distance the two tips will be?
   1 cm
3. Are the two tips ever exactly 5 cm apart? Show or explain your thinking.
   Yes; Sample answer: At 3 o'clock, they form a right triangle with legs measuring 3 cm and 4 cm and a hypotenuse of 5 cm.

Log in to Amplify Math to complete this lesson online.

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Lesson 12 Converse of the Pythagorean Theorem 793

1 Launch

Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by encouraging them to use two pencils to mimic the motion of the hands of the clock to solve Problem 1.

Look for points of confusion:

- **Thinking the solution to Problem 2 is 0.** Have students sketch what this would look like and ask them to consider the difference in the distance from one hand tip to the other).

Look for productive strategies:

- Drawing the hands of the clock and noticing they can make a 3–4–5 right triangle.

3 Connect

Have students share their responses to Problems 1 and 2.

Display students work showing the hands of the clock making a 3–4–5 right triangle as a response for Problem 3.

Ask:

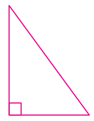
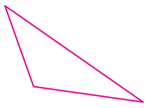
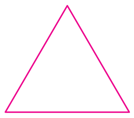
- "How can you show the hands are 5 cm apart?"
- "What are the dimensions of this triangle?"
- "How do you know it is a right triangle?"
- "What about other triangles that have side lengths 3, 4, and 5 — will they also be right triangles? Why or why not?" Because all the sides lengths are the same, the triangles are all congruent.

Highlight that, if the side lengths a, b, and c of a triangle are known, and given that $a^2 + b^2 = c^2$, then the triangle must be a right triangle. This is called the *converse* of the Pythagorean Theorem. Ask students what they think the term *converse* means in this context.

Power-up

To power up students' ability to classify triangles as acute, right, or obtuse, have students complete:

Recall that a *right triangle* has one right angle, an *obtuse triangle* has one obtuse angle, and an *acute triangle* has no right or obtuse angles (all angles are acute). Sketch an example of each type of triangle. Sample responses shown.

Right	Obtuse	Acute
		

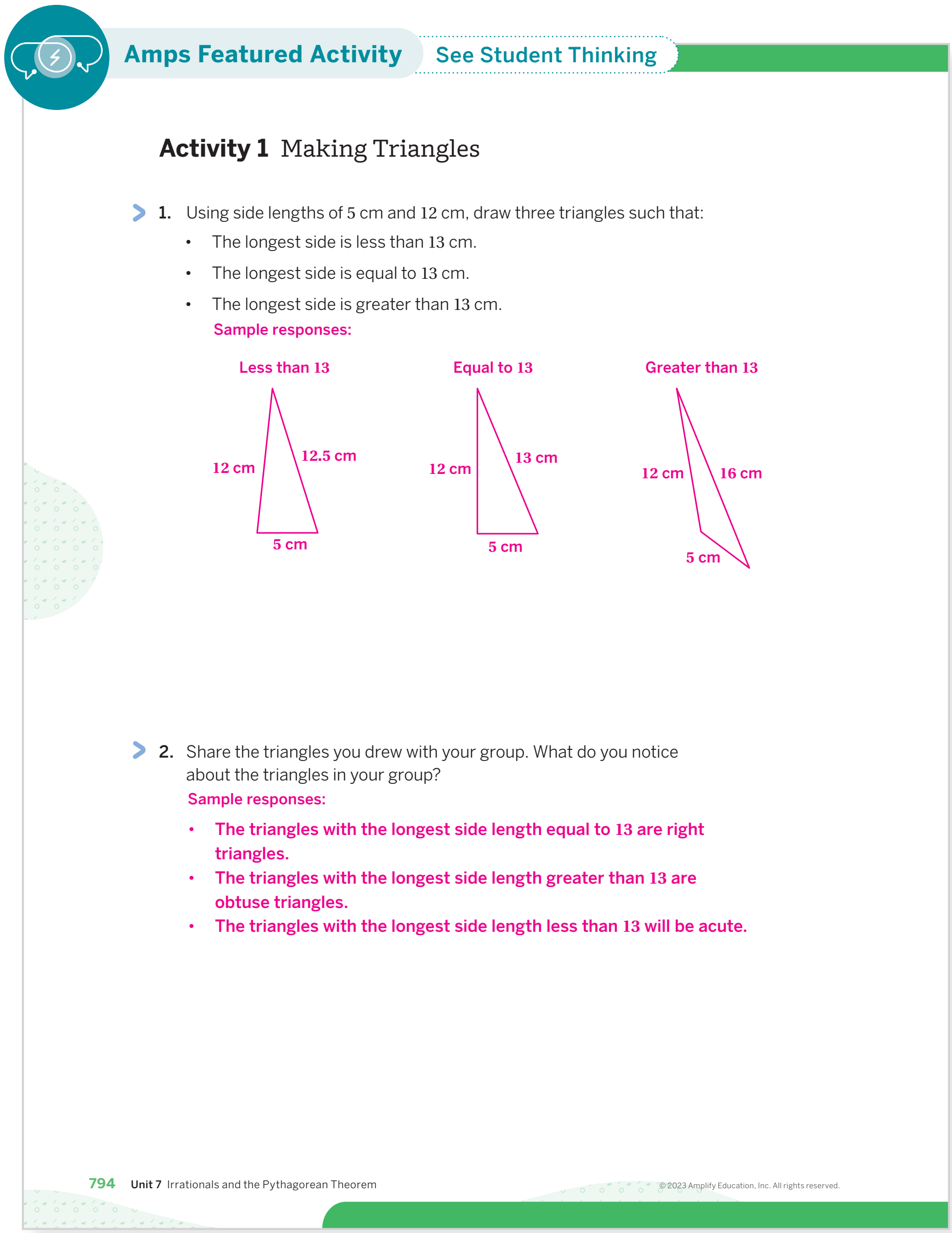
Use: Before Activity 1
Informed by: Performance on Lesson 11, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 3

Small Groups | 15 min

Activity 1 Making Triangles

Students create three triangles with two shared side lengths and one changing side length to develop strategies for identifying when a triangle is acute, right, or obtuse.

Amps Featured Activity See Student Thinking



Activity 1 Making Triangles

1. Using side lengths of 5 cm and 12 cm, draw three triangles such that:
   - The longest side is less than 13 cm.
   - The longest side is equal to 13 cm.
   - The longest side is greater than 13 cm.

   Sample responses:

2. Share the triangles you drew with your group. What do you notice about the triangles in your group?

   Sample responses:
   - The triangles with the longest side length equal to 13 are right triangles.
   - The triangles with the longest side length greater than 13 are obtuse triangles.
   - The triangles with the longest side length less than 13 will be acute.

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1 Launch

Distribute rulers and plain sheets of paper to groups of 4. Ask each student to draw and label their own triangles on a separate sheet of paper before sharing with the group.

2 Monitor

Help students get started by having them draw two side lengths of 5 and 12 cm and measure the unknown side that completes the triangle. Ask, "How can you change the position of the two sides to make the third side longer or shorter?"

Look for points of confusion:

- **Being unsure how to draw each triangle.** Help them use their ruler to use trial and error for each triangle.

3 Connect

Have groups of students share what they notice about each other's triangles.

Ask:

- "What was the same about all the triangles in your group with side lengths of 5, 12 and 13?"
- "How can you show this is a right triangle?"
- "What was true about all the triangles with a third side length less than 13? Greater than 13?
- "If you know three sides of a triangle, how can you determine whether it is acute, right, or obtuse?"

Highlight that the Pythagorean Theorem can be used to determine whether the triangle is an acute, right or obtuse triangle. If the two shorter sides are a and b and $a^2 + b^2 = c^2$, then the triangle is a right triangle. If $a^2 + b^2 < c^2$, then the triangle is obtuse. If $a^2 + b^2 > c^2$, the triangle is acute.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with pre-drawn examples of each triangle in Problem 1. Have them focus on identifying what they notice about the triangles in Problem 2. This will allow them to still focus on the targeted goal of the activity without the requirement of measuring and drawing the triangles.

Extension: *Math Enrichment*

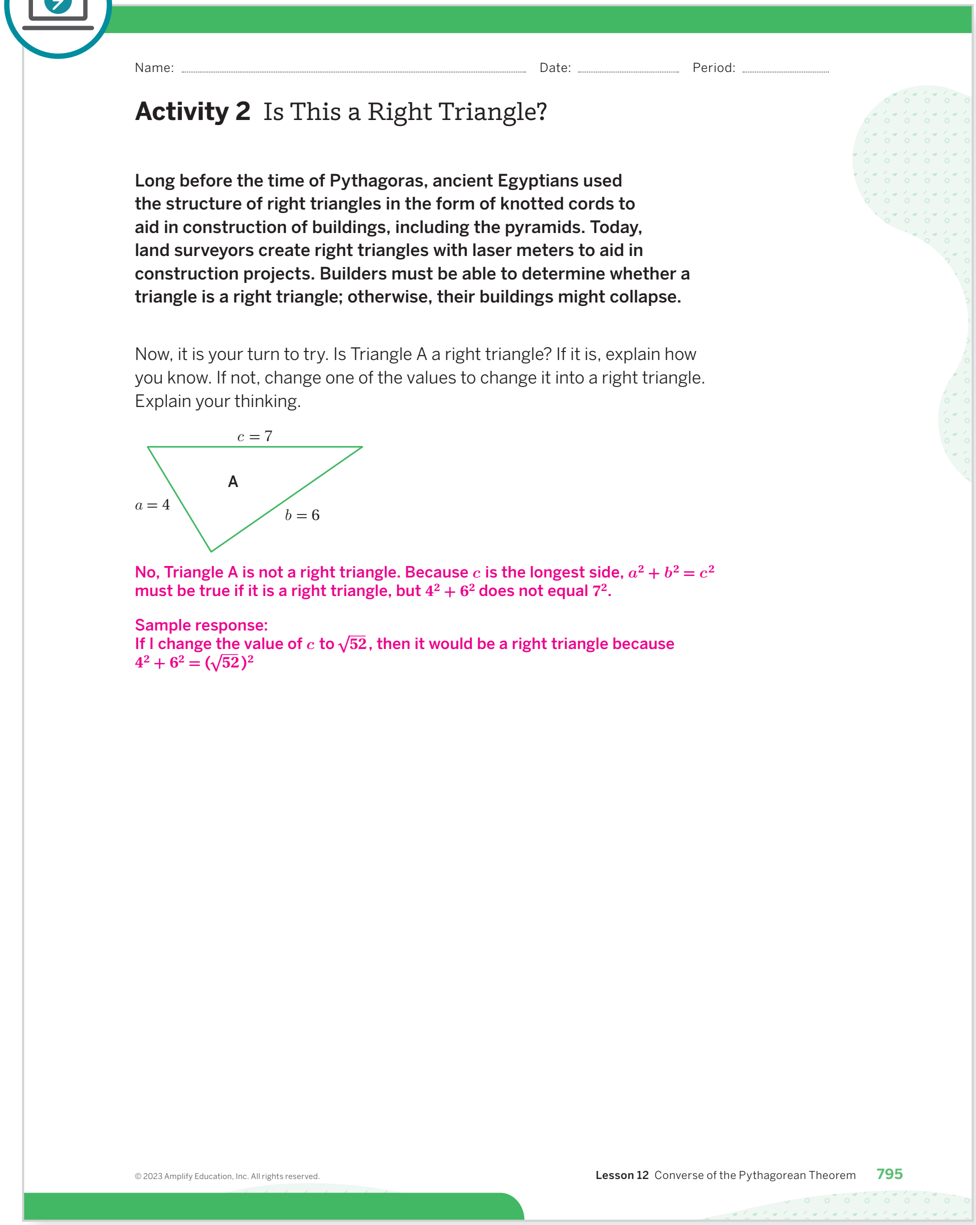
Without sketching triangles, have students sort the following set of triangle side lengths by whether they would form a right triangle, an acute triangle, or an obtuse triangle.

- 6, 8, 10 Right triangle
- 6, 8, 12 Obtuse triangle
- 6, 8, 9 Acute triangle
- 3, 4, 5 Right triangle
- 3, 4, 6.5 Obtuse triangle
- 3, 4, 4.5 Acute triangle

Independent | 10 min

Activity 2 Is This a Right Triangle?

Students consider a triangle with three known sides to apply the Pythagorean Theorem and its converse in a mathematical context.



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Activity 2 Is This a Right Triangle?

Long before the time of Pythagoras, ancient Egyptians used the structure of right triangles in the form of knotted cords to aid in construction of buildings, including the pyramids. Today, land surveyors create right triangles with laser meters to aid in construction projects. Builders must be able to determine whether a triangle is a right triangle; otherwise, their buildings might collapse.

Now, it is your turn to try. Is Triangle A a right triangle? If it is, explain how you know. If not, change one of the values to change it into a right triangle. Explain your thinking.

No, Triangle A is not a right triangle. Because c is the longest side, $a^2 + b^2 = c^2$ must be true if it is a right triangle, but $4^2 + 6^2$ does not equal 7^2.

Sample response:
If I change the value of c to $\sqrt{52}$, then it would be a right triangle because $4^2 + 6^2 = (\sqrt{52})^2$

1 Launch

Ask students to predict, without calculating, whether the triangle is acute, right, or obtuse.

2 Monitor

Help students get started by having them reference the Anchor Chart PDF, *The Pythagorean Theorem*.

Look for points of confusion:

- **Thinking Triangle A is a right triangle based on inspection only.** Remind students to use the Pythagorean Theorem to show if it is a right triangle or not.
- **Not being sure how to change one of the sides to make it a right triangle.** Ask students to suppose two of the sides are the legs a and b of a right triangle. Have them substitute those values into the equation $a^2 + b^2 = c^2$ and have them describe what the value of c must represent and how they can solve for the value of c.

3 Connect

Have students share why Triangle A is not a right triangle.

Display a list of all the possible changes students suggested making and have them discuss their process for determining these new side length measures.

Ask:

- "What kind of triangle is Triangle A? How do you know?"
- "Given the side length changes that have been suggested, how can you confirm that they would make Triangle A a right triangle?"

Highlight that there were nine possible changes that could make Triangle A a right triangle. Have students work in pairs to find the other solutions, if time permits.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write their response, have them meet with 2–3 partners to both give and receive feedback on their responses and explanations. Encourage reviewers to ask clarifying questions such as:

- "How did you use the Pythagorean Theorem to determine whether Triangle A is a right triangle?"
- "Which value did you select to change? Why did you select this value?"

Have students revise their responses, as needed.

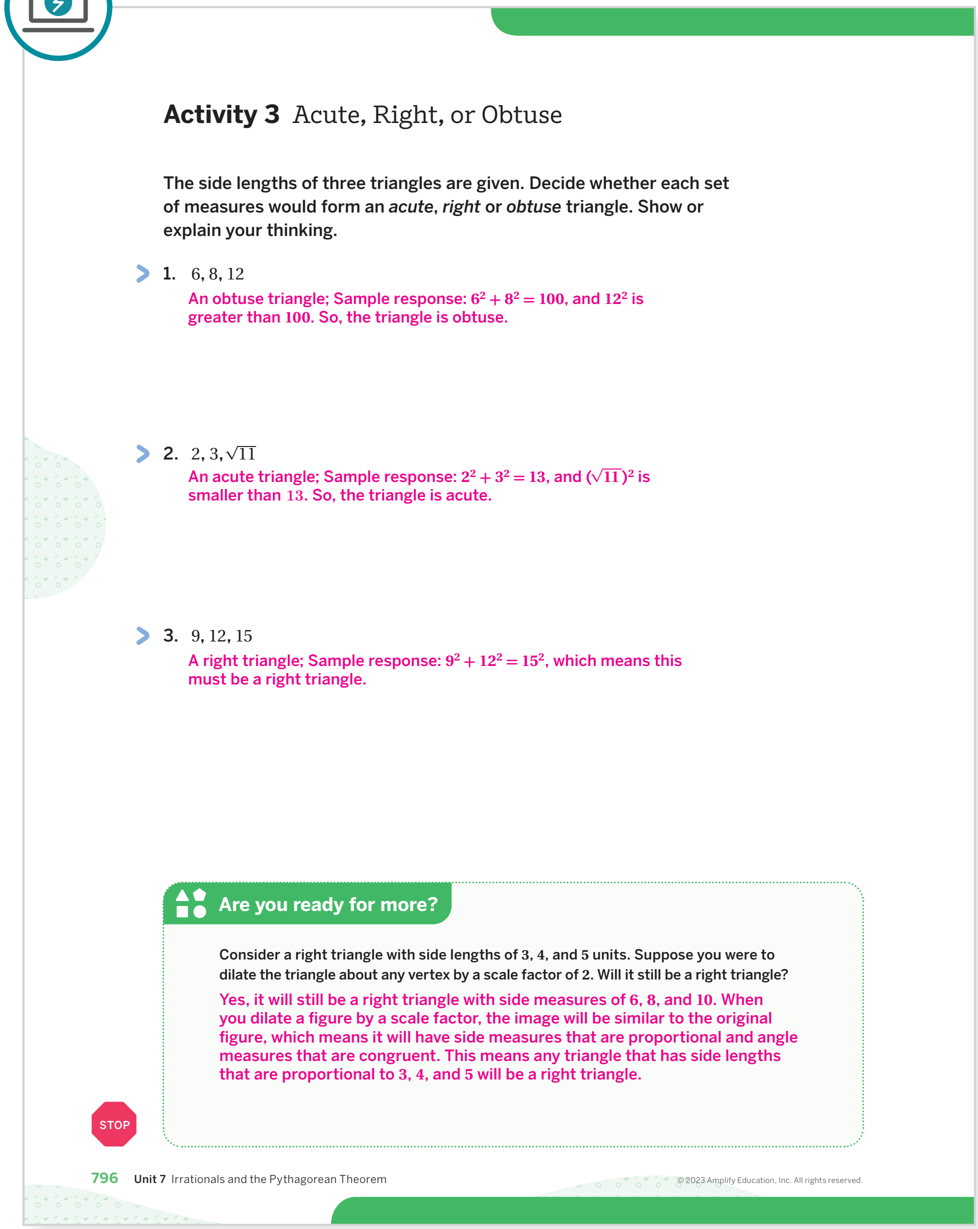
English Learners

Provide sentence frames for students to use to complete their responses, such as "Triangle A is/is not a right triangle because . . ." and "If I change the value ___ to ___, then Triangle A would be a right triangle because . . ."

Pairs | 8 min

Activity 3 Acute, Right, or Obtuse

Students apply the Pythagorean Theorem and its converse to determine whether triangles are acute, right, or obtuse, based on their side length measures.



Activity 3 Acute, Right, or Obtuse

The side lengths of three triangles are given. Decide whether each set of measures would form an *acute*, *right* or *obtuse* triangle. Show or explain your thinking.

1. $6, 8, 12$
   An obtuse triangle; Sample response: $6^2 + 8^2 = 100$, and 12^2 is greater than 100. So, the triangle is obtuse.

2. $2, 3, \sqrt{11}$
   An acute triangle; Sample response: $2^2 + 3^2 = 13$, and $(\sqrt{11})^2$ is smaller than 13. So, the triangle is acute.

3. $9, 12, 15$
   A right triangle; Sample response: $9^2 + 12^2 = 15^2$, which means this must be a right triangle.

Are you ready for more?

Consider a right triangle with side lengths of 3, 4, and 5 units. Suppose you were to dilate the triangle about any vertex by a scale factor of 2. Will it still be a right triangle?

Yes, it will still be a right triangle with side measures of 6, 8, and 10. When you dilate a figure by a scale factor, the image will be similar to the original figure, which means it will have side measures that are proportional and angle measures that are congruent. This means any triangle that has side lengths that are proportional to 3, 4, and 5 will be a right triangle.

STOP

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1 Launch

Set an amount of time for students to work in pairs on the activity.

2 Monitor

Help students get started by asking how they can test whether a triangle is acute, right, or obtuse.

Look for points of confusion:

- **Substituting values incorrectly for $a^2 + b^2 = c^2$.** Help students identify the longest side of the triangle to substitute for c.
- **Misidentifying any of the triangles as acute, right, or obtuse.** If their substitution is correct, but students are having difficulty identifying whether a triangle is obtuse or acute, have students sketch the triangle. Ask, "If the longest side is longer than c, would the angle need to be greater than or less than a right angle?"

3 Connect

Display student work showing correct solutions.

Have students share how they determined whether each triangle was acute, right, or obtuse.

Ask:

- "What has to be true in order to be sure a triangle is a right triangle?"
- "How can you tell if a triangle is an obtuse triangle?"
- "How can you tell if a triangle is an acute triangle?"
- "If you only know two sides of the triangle, can you determine whether it is acute, right, or obtuse?"

Highlight that all three side lengths of the triangle must be known to determine whether the triangle is acute, right, or obtuse. Complete the section of the Anchor Chart, *The Pythagorean Theorem* for Lesson 12.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students color code which side length measures would represent the legs in one color and which side length measure would represent the hypotenuse in another color, if each set of measures formed a right triangle.

Math Language Development

MLR8: Discussion Supports

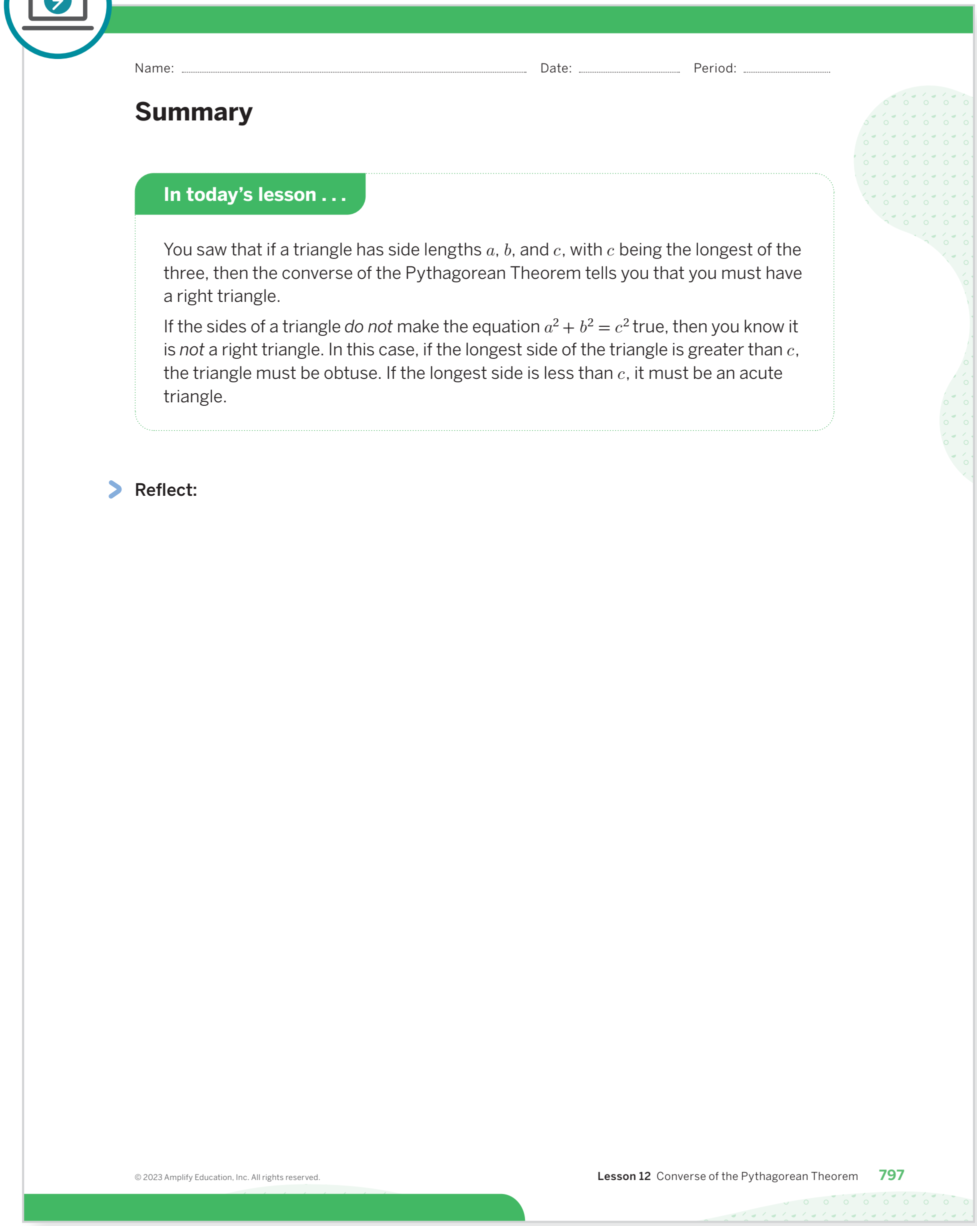
While students work, display sentence frames they can use to help organize their thinking, such as:

- "The side lengths of ___ would form an acute triangle because . . ."
- "The side lengths of ___ would form a right triangle because . . ."
- "The side lengths of ___ would form an obtuse triangle because . . ."

Whole Class | 5 min

Summary

Review and synthesize how to apply the Pythagorean Theorem and its converse.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that if a triangle has side lengths a, b, and c, with c being the longest of the three, then the converse of the Pythagorean Theorem tells you that you must have a right triangle.

If the sides of a triangle *do not* make the equation $a^2 + b^2 = c^2$ true, then you know it is *not* a right triangle. In this case, if the longest side of the triangle is greater than c, the triangle must be obtuse. If the longest side is less than c, it must be an acute triangle.

Reflect:

Synthesize

Have students share what the converse of the Pythagorean Theorem states.

Ask:

- "How can you tell whether a triangle is acute? Right? Obtuse?"
- "Are all triangles with side lengths 3, 4, and 5, right triangles? How do you know? What about all triangles with side lengths 5, 12, and 13?"

Highlight that, if the three side lengths of a triangle are known, the converse of the Pythagorean Theorem can be applied to determine whether it is a right triangle. If it is not a right triangle, the measures of $a^2 + b^2$ in relation to c^2 can be considered to determine whether the triangle is acute or obtuse.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What makes the converse of the Pythagorean Theorem true? Can you think of an example, even not related to mathematics, where the converse is not true?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by applying the converse of the Pythagorean Theorem to determine if a triangle is a right triangle.

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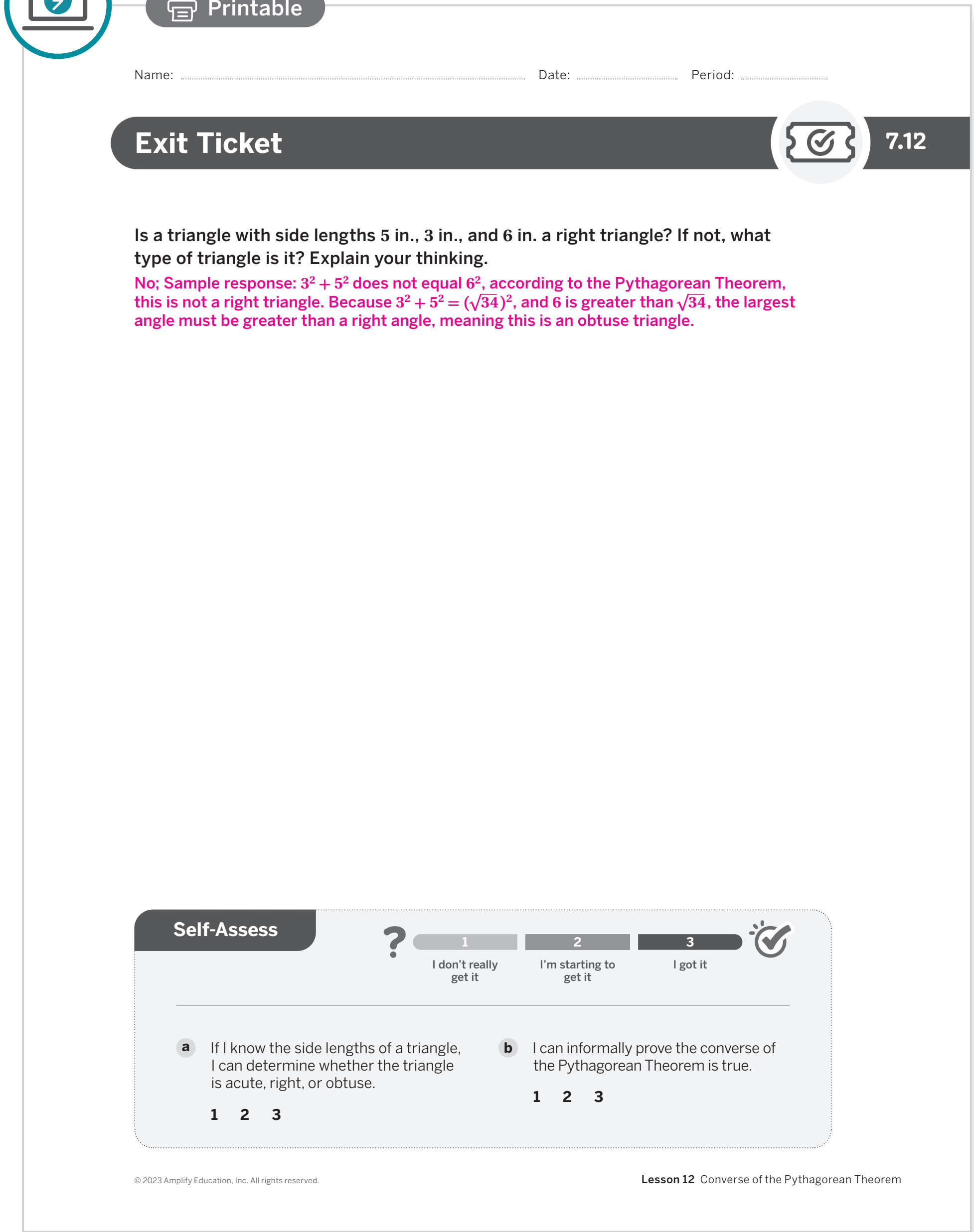

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Exit Ticket 7.12

Is a triangle with side lengths 5 in., 3 in., and 6 in. a right triangle? If not, what type of triangle is it? Explain your thinking.

No; Sample response: $3^2 + 5^2$ does not equal 6^2, according to the Pythagorean Theorem, this is not a right triangle. Because $3^2 + 5^2 = (\sqrt{34})^2$, and 6 is greater than $\sqrt{34}$, the largest angle must be greater than a right angle, meaning this is an obtuse triangle.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. If I know the side lengths of a triangle, I can determine whether the triangle is acute, right, or obtuse.

1 2 3

b. I can informally prove the converse of the Pythagorean Theorem is true.

1 2 3

Success looks like . . .

- **Goal:** Understanding the converse of the Pythagorean Theorem.
- **Language Goal:** Determining whether a triangle with given side lengths is an acute, right, or obtuse triangle using the converse of the Pythagorean Theorem. **(Speaking and Listening, Writing)**
  - » Determining that the triangle is an obtuse triangle.

Suggested next steps

If students think the triangle is a right triangle, consider:

- Reviewing how to use the converse of the Pythagorean Theorem from Activity 2.
- Assigning Practice Problem 1.

If students think the triangle is an acute triangle, consider:

- Reviewing how to classify triangles using the converse of the Pythagorean Theorem from Activity 3.
- Assigning Practice Problem 2.
- Asking students to draw a quick sketch of the triangle.

Professional Learning

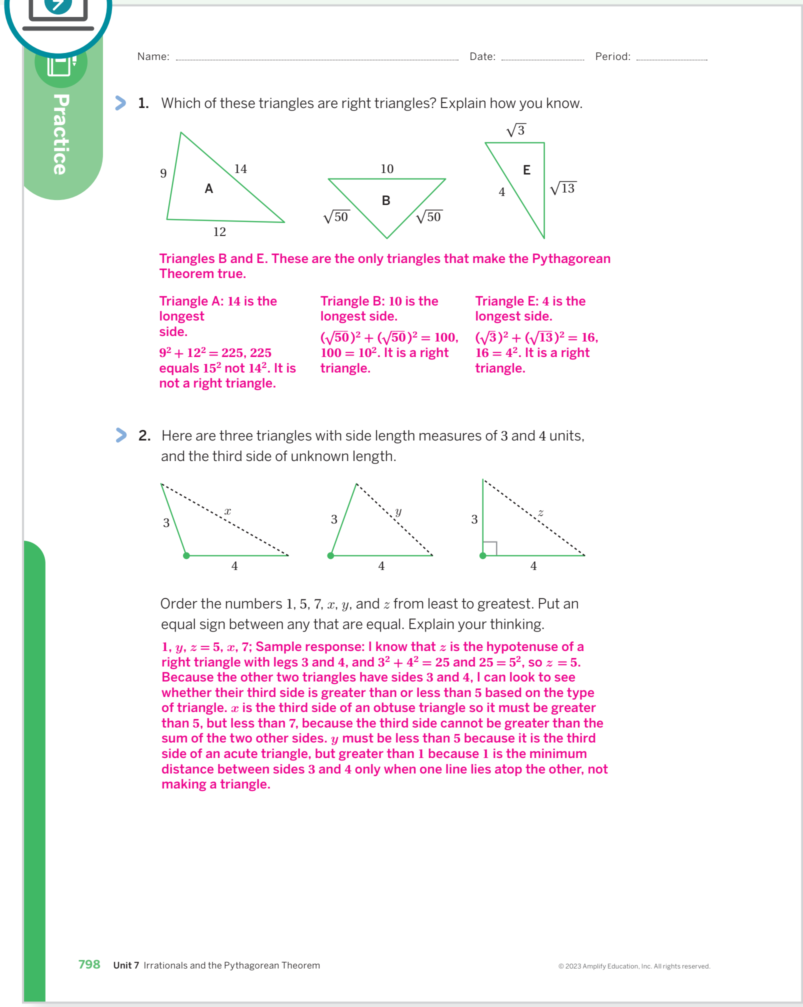
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students apply the converse of the Pythagorean Theorem. How did that build on the earlier work students did with the Pythagorean Theorem?
- In what ways in Activity 1 did things happen that you did not expect?

Practice

Independent



Name: .... Date: .... Period: ....

1. Which of these triangles are right triangles? Explain how you know.

Triangles B and E. These are the only triangles that make the Pythagorean Theorem true.

Triangle A: 14 is the longest side. $9^2 + 12^2 = 225$, 225 equals 15^2 not 14^2. It is not a right triangle.

Triangle B: 10 is the longest side. $(\sqrt{50})^2 + (\sqrt{50})^2 = 100$, $100 = 10^2$. It is a right triangle.

Triangle E: 4 is the longest side. $(\sqrt{3})^2 + (\sqrt{13})^2 = 16$, $16 = 4^2$. It is a right triangle.

2. Here are three triangles with side length measures of 3 and 4 units, and the third side of unknown length.

Order the numbers 1, 5, 7, x, y, and z from least to greatest. Put an equal sign between any that are equal. Explain your thinking.

$1, y, z = 5, x, 7$; Sample response: I know that z is the hypotenuse of a right triangle with legs 3 and 4, and $3^2 + 4^2 = 25$ and $25 = 5^2$, so $z = 5$. Because the other two triangles have sides 3 and 4, I can look to see whether their third side is greater than or less than 5 based on the type of triangle. x is the third side of an obtuse triangle so it must be greater than 5, but less than 7, because the third side cannot be greater than the sum of the two other sides. y must be less than 5 because it is the third side of an acute triangle, but greater than 1 because 1 is the minimum distance between sides 3 and 4 only when one line lies atop the other, not making a triangle.

798 **Unit 7** Irrationals and the Pythagorean Theorem

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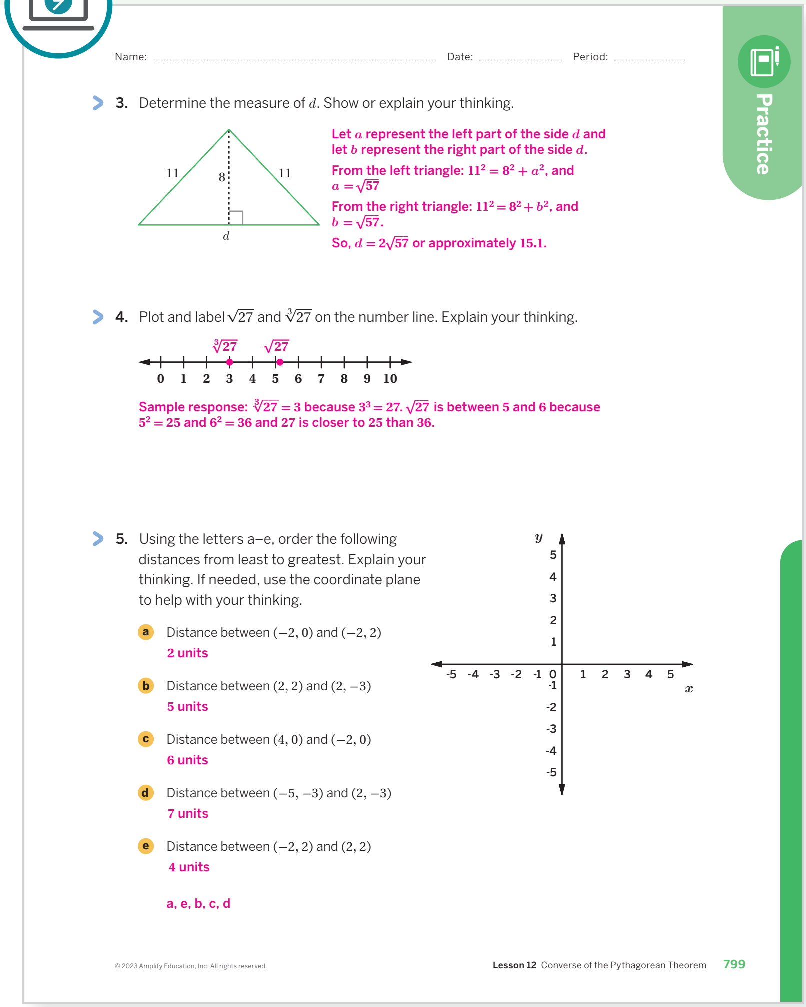

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3. Determine the measure of d. Show or explain your thinking.

Let a represent the left part of the side d and let b represent the right part of the side d.

From the left triangle: $11^2 = 8^2 + a^2$, and $a = \sqrt{57}$

From the right triangle: $11^2 = 8^2 + b^2$, and $b = \sqrt{57}$.

So, $d = 2\sqrt{57}$ or approximately 15.1.

4. Plot and label $\sqrt{27}$ and $\sqrt[3]{27}$ on the number line. Explain your thinking.

Sample response: $\sqrt[3]{27} = 3$ because $3^3 = 27$. $\sqrt{27}$ is between 5 and 6 because $5^2 = 25$ and $6^2 = 36$ and 27 is closer to 25 than 36.

5. Using the letters a–e, order the following distances from least to greatest. Explain your thinking. If needed, use the coordinate plane to help with your thinking.

- a Distance between $(-2, 0)$ and $(-2, 2)$
  2 units
- b Distance between $(2, 2)$ and $(2, -3)$
  5 units
- c Distance between $(4, 0)$ and $(-2, 0)$
  6 units
- d Distance between $(-5, -3)$ and $(2, -3)$
  7 units
- e Distance between $(-2, 2)$ and $(2, 2)$
  4 units

a, e, b, c, d

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Lesson 12 Converse of the Pythagorean Theorem 799

Practice Problem Analysis

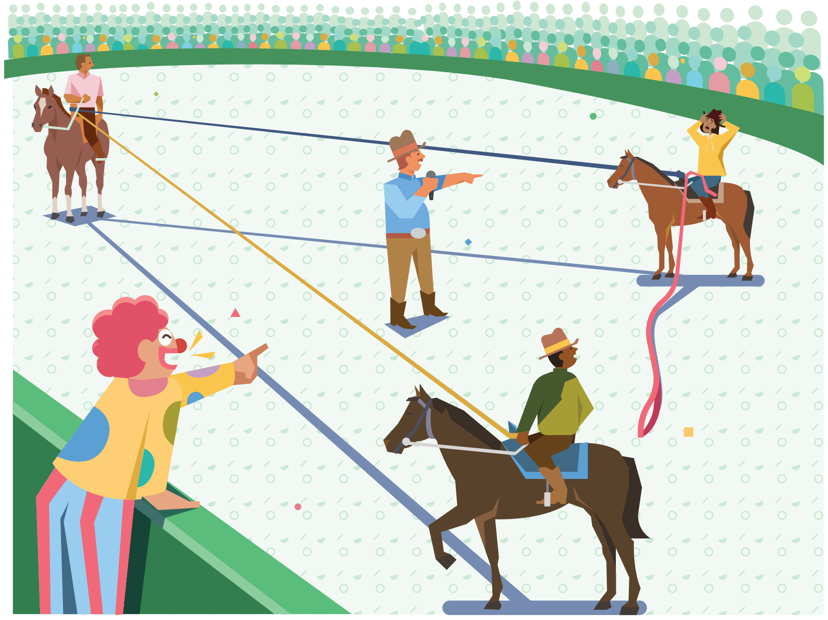
Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 3	3
Spiral	3	Unit 7 Lesson 11	2
	4	Unit 7 Lesson 5	2
Formative ⚡	5	Unit 7 Lesson 13	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Distances on the Coordinate Plane (Part 1)



Let's determine the distance between two points on the coordinate plane.

Focus

Goals

1. **Language Goal:** Calculate the distance between two points in the coordinate plane by using the Pythagorean Theorem and explain the solution method. **(Speaking and Listening)**
2. **Language Goal:** Generalize a method for calculating the length of a line segment in the coordinate plane using the Pythagorean Theorem. **(Speaking and Listening)**

Coherence

Today

In this lesson, students continue to apply the Pythagorean Theorem to determine distances between points in the coordinate plane. Students use the structure of the coordinate plane to draw right triangles, an example of looking for and making use of structure in the coordinate plane.

Previously

In Lesson 12, students determined whether side lengths yielded a right triangle by using the converse of the Pythagorean Theorem.

Coming Soon

In Lesson 14, students will continue their work determining the distance between two points on a coordinate plane using the Pythagorean Theorem.

Rigor

- Students **apply** the Pythagorean Theorem to find the length of a segment on the coordinate plane.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
8 min	15 min	15 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 2 PDF (as needed)
- Anchor Chart PDF, *The Pythagorean Theorem*
- Anchor Chart PDF, *The Pythagorean Theorem* (answers)
- calculators
- index cards (optional)

Math Language Development

Review words

- *hypotenuse*
- *leg*
- *Pythagorean Theorem*

Amps | **Featured Activity**

Activity 1
See Student Thinking

Students are asked to explain their thinking behind determining the distance between points on a coordinate plane, and these explanations are available to you digitally, in real time.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be distracted by the others in the group or might be focused on their own agendas in Activity 1. Remind students that they must work as a team, offering and seeking help when needed. Point out that when providing help, precision of language is important. Specific terms, rather than vague, general terms, will help guide students and, thus, the whole group, to success.

Modifications to Pacing

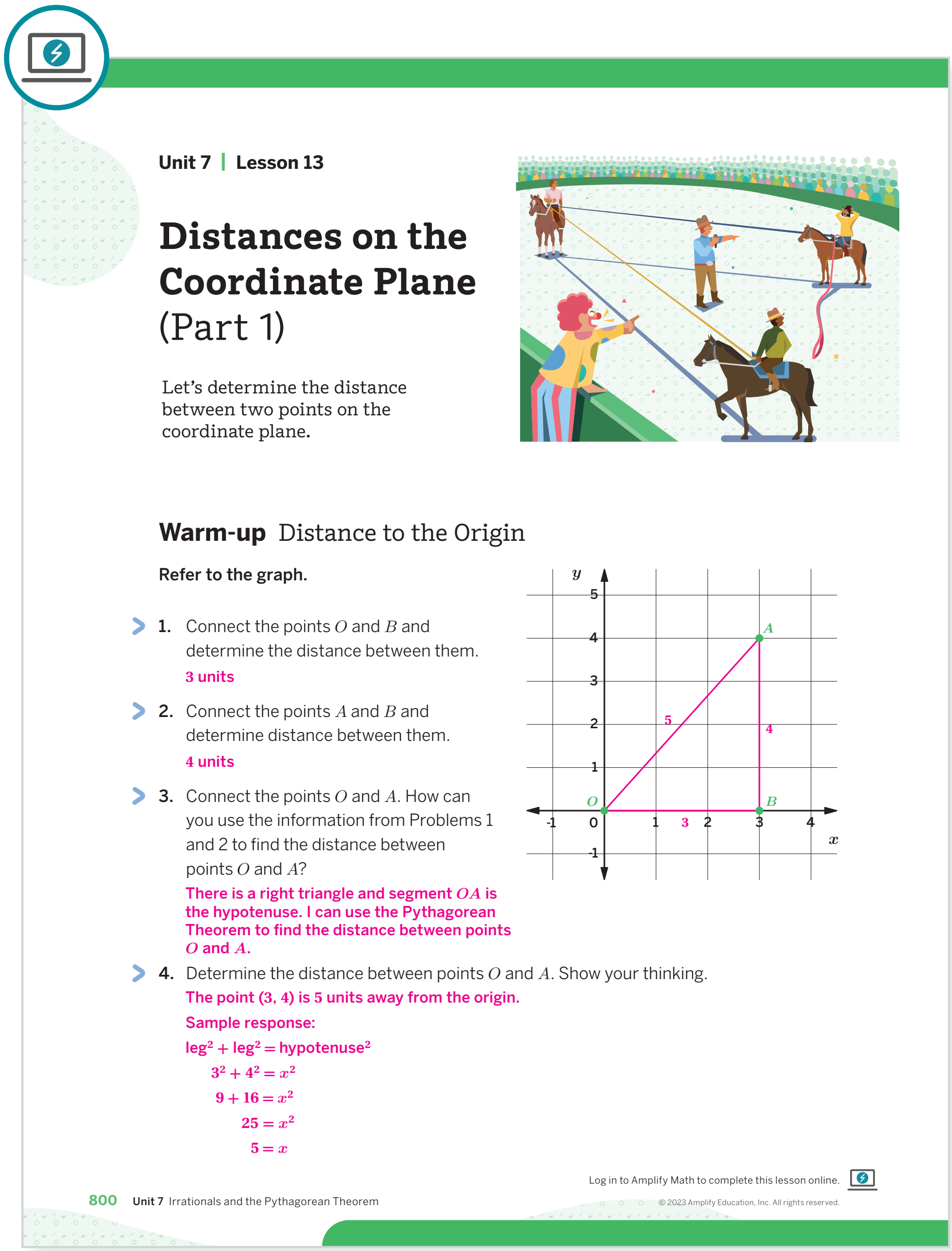
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, consider assigning Problems 1–3 to different group members and have them share answers before completing the activity.

Pairs | 8 min

Warm-up Distance to the Origin

Students build a right triangle on the coordinate plane to determine the distance between two points using the Pythagorean Theorem.



Unit 7 | Lesson 13

Distances on the Coordinate Plane (Part 1)

Let's determine the distance between two points on the coordinate plane.

Warm-up Distance to the Origin

Refer to the graph.

1. Connect the points O and B and determine the distance between them.
   3 units
2. Connect the points A and B and determine distance between them.
   4 units
3. Connect the points O and A. How can you use the information from Problems 1 and 2 to find the distance between points O and A?
   There is a right triangle and segment OA is the hypotenuse. I can use the Pythagorean Theorem to find the distance between points O and A.
4. Determine the distance between points O and A. Show your thinking.
   The point (3, 4) is 5 units away from the origin.
   Sample response:
   $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
   $3^2 + 4^2 = x^2$
   $9 + 16 = x^2$
   $25 = x^2$
   $5 = x$

Log in to Amplify Math to complete this lesson online.

1 Launch

Ask, "Is it possible to determine the length of segment OA by only counting squares?" Have students use the *Think-Pair-Share* routine.

2 Monitor

Help students get started by having them think about what shape they have drawn once all three points are connected.

Look for points of confusion:

- **Not knowing how to approach Problem 3.** Have students think about the shape they have been studying for the second Sub-Unit and ask if they can see that shape on the coordinate plane.
- **Incorrectly using the Pythagorean Theorem (i.e., substituting the value of a leg for the hypotenuse or forgetting to take the square root).** Have students review the Anchor Chart PDF, *The Pythagorean Theorem*.

Look for productive strategies:

- Completing the problem accurately without any prompting. Make note of students who understand this idea to help explain it during the Connect portion.

3 Connect

Display the graph and have students explain their thinking for Problem 3.

Ask, "Why is Triangle AOB a right triangle?"

Highlight that the vertical and horizontal lines always form right angles and produce a right triangle when drawn from a diagonal line segment. Using the structure of the legs and the Pythagorean Theorem allows students to determine the precise measurement of the length of the segment.

Power-up

To power up students' ability to determine vertical and horizontal distance on a coordinate plane:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

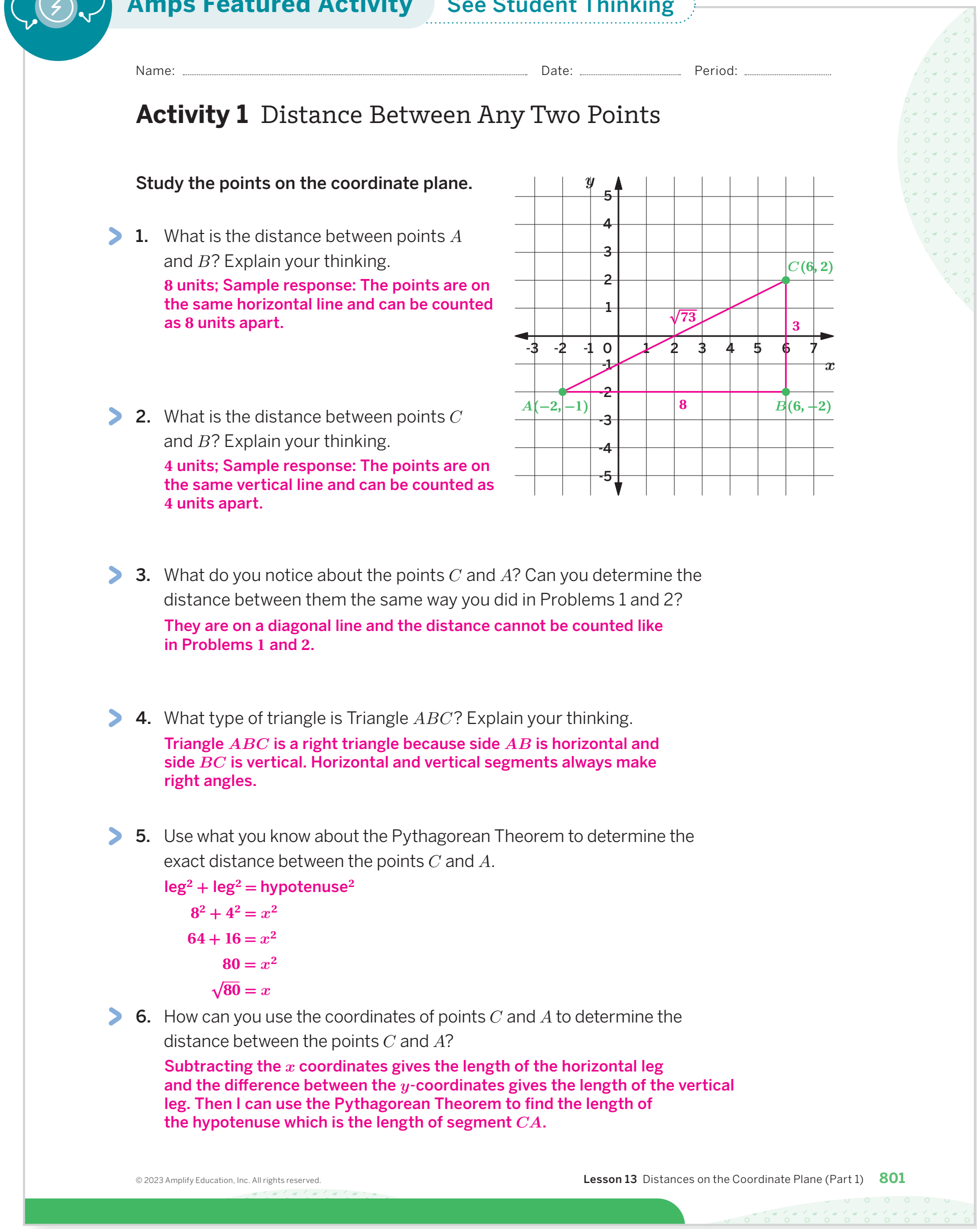
Informed by: Performance on Lesson 12, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 1

Small Groups | 15 min

Activity 1 Distance Between Any Two Points

Students use the Pythagorean Theorem to determine the distance between two points on a coordinate plane that do not have the same vertical or horizontal coordinates.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 1 Distance Between Any Two Points

Study the points on the coordinate plane.

1. What is the distance between points A and B? Explain your thinking.
   8 units; Sample response: The points are on the same horizontal line and can be counted as 8 units apart.

2. What is the distance between points C and B? Explain your thinking.
   4 units; Sample response: The points are on the same vertical line and can be counted as 4 units apart.

3. What do you notice about the points C and A? Can you determine the distance between them the same way you did in Problems 1 and 2?
   They are on a diagonal line and the distance cannot be counted like in Problems 1 and 2.

4. What type of triangle is Triangle ABC? Explain your thinking.
   Triangle ABC is a right triangle because side AB is horizontal and side BC is vertical. Horizontal and vertical segments always make right angles.

5. Use what you know about the Pythagorean Theorem to determine the exact distance between the points C and A.
   $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
   $8^2 + 4^2 = x^2$
   $64 + 16 = x^2$
   $80 = x^2$
   $\sqrt{80} = x$

6. How can you use the coordinates of points C and A to determine the distance between the points C and A?
   Subtracting the x coordinates gives the length of the horizontal leg and the difference between the y-coordinates gives the length of the vertical leg. Then I can use the Pythagorean Theorem to find the length of the hypotenuse which is the length of segment CA.

① Launch

Have your students work in groups of at least three for the reminder of the lesson.

② Monitor

Help students get started by having them review the Warm-up for ideas on how to approach this problem.

Look for points of confusion:

- **Wanting to count the length of segment AC.** Have students mark the length on an index card and transfer it to the x-axis to see whether their length was correct. Let them know that the steps they will take in Problem 5 will make sure they get the exact distance and not an estimate.

Look for productive strategies:

- Recognizing the coordinates can help determine the lengths.

③ Connect

Display the graph.

Have students share their reasoning as why this triangle is a right triangle followed by their ideas for Problem 5. Start with students who use vague language, such as "subtract them," and end with students using more precise language, such as "the absolute value of the difference between the x-coordinates is the length of the horizontal leg".

Ask, "How is the length of the diagonal line different from the lengths of the vertical and horizontal lines? How are those lines useful in finding the length of the diagonal line?"

Highlight that a horizontal line and a vertical line can be drawn from the ends of a diagonal line segment. These lines can become the legs of a right triangle and the Pythagorean Theorem can be used to find the length of the diagonal segment.

Differentiated Support

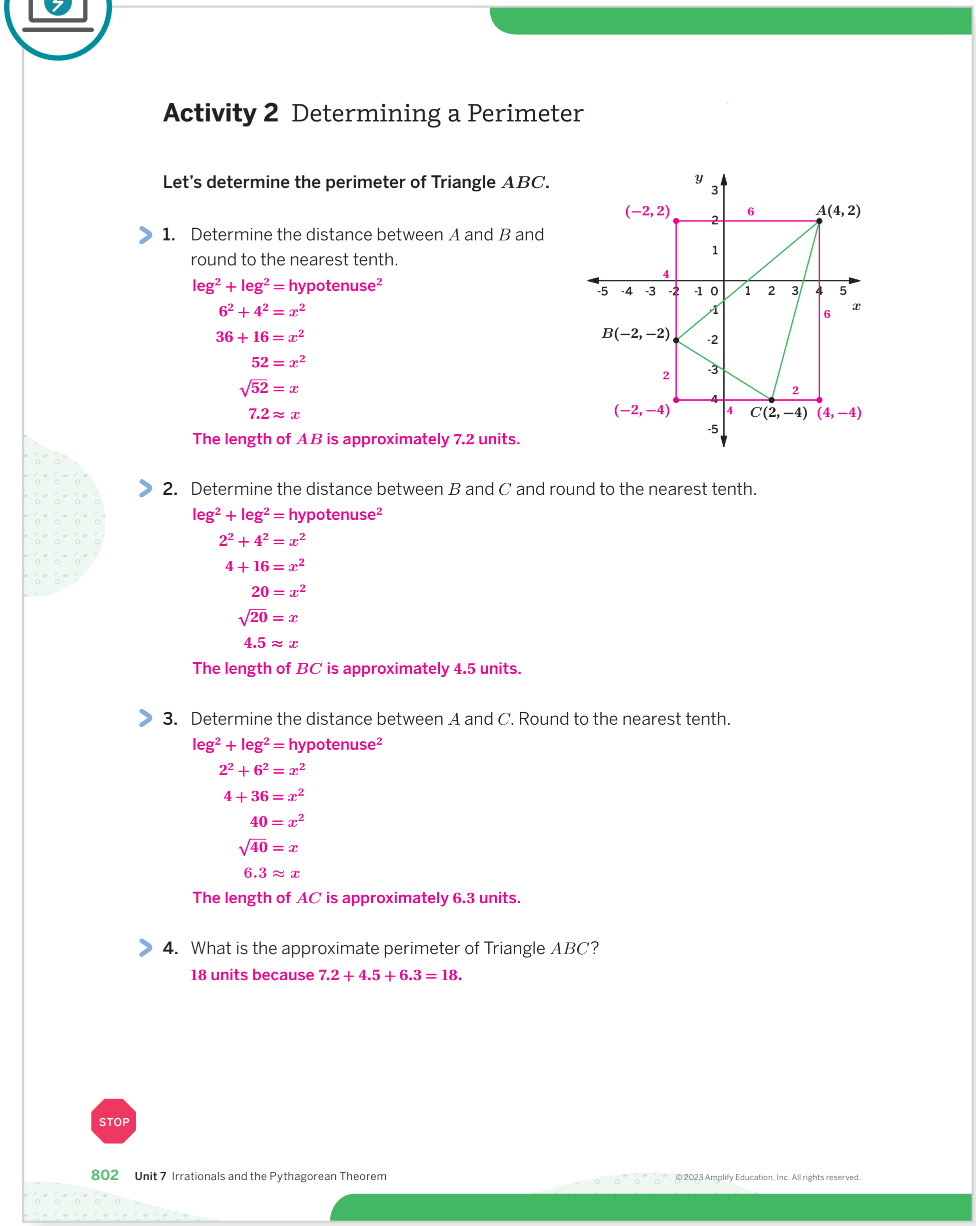
Accessibility: *Guide Processing and Visualization*

After students complete Problems 1–4, pause for a brief class discussion and suggest they draw a square connecting points A, B, C and a new point (–2, 2). Ask them how determining the length of the hypotenuse is similar to determining the length of a diagonal of a rectangle. Then have them proceed with Problems 5 and 6.

Small Groups | 15 min

Activity 2 Determining the Perimeter

Students use the Pythagorean Theorem to calculate the perimeter of a triangle.



Activity 2 Determining a Perimeter

Let's determine the perimeter of Triangle ABC.

1. Determine the distance between A and B and round to the nearest tenth.

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
$6^2 + 4^2 = x^2$
$36 + 16 = x^2$
$52 = x^2$
$\sqrt{52} = x$
$7.2 \approx x$

The length of AB is approximately 7.2 units.

2. Determine the distance between B and C and round to the nearest tenth.

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
$2^2 + 4^2 = x^2$
$4 + 16 = x^2$
$20 = x^2$
$\sqrt{20} = x$
$4.5 \approx x$

The length of BC is approximately 4.5 units.

3. Determine the distance between A and C. Round to the nearest tenth.

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
$2^2 + 6^2 = x^2$
$4 + 36 = x^2$
$40 = x^2$
$\sqrt{40} = x$
$6.3 \approx x$

The length of AC is approximately 6.3 units.

4. What is the approximate perimeter of Triangle ABC?

18 units because $7.2 + 4.5 + 6.3 = 18$.

STOP

802 Unit 7 Irrationals and the Pythagorean Theorem

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1 Launch

Have students continue working in groups of at least three. Allow access to calculators.

2 Monitor

Help students get started by covering up point C so they can focus on determining the distance between points A and B.

Look for points of confusion:

- **Thinking that they cannot find the distance because they do not have a right triangle.** Have students plot the point (–2, 2) to show the vertical and horizontal legs for one of the segments.
- **Drawing a triangle that does not appear to be a right triangle.** If it is causing students to misunderstand the problem, consider providing the Activity 2 PDF where grid paper is provided. If students are still unable to determine the lengths of the legs, consider allowing the lack of precision because they are utilizing the structure of the coordinates.

Look for productive strategies:

- Drawing the horizontal and vertical legs for each diagonal segment.

3 Connect

Display the Activity 2 PDF, as needed, to help students explain their strategies.

Highlight that a diagonal line on a coordinate plane can be treated as the hypotenuse of a right triangle. Drawing the horizontal and vertical legs and counting or finding the difference in the coordinates are valid ways to determine the leg lengths of the right triangle.

Ask, "How did not having the grid to count change the way you approached this problem?"

Differentiated Support

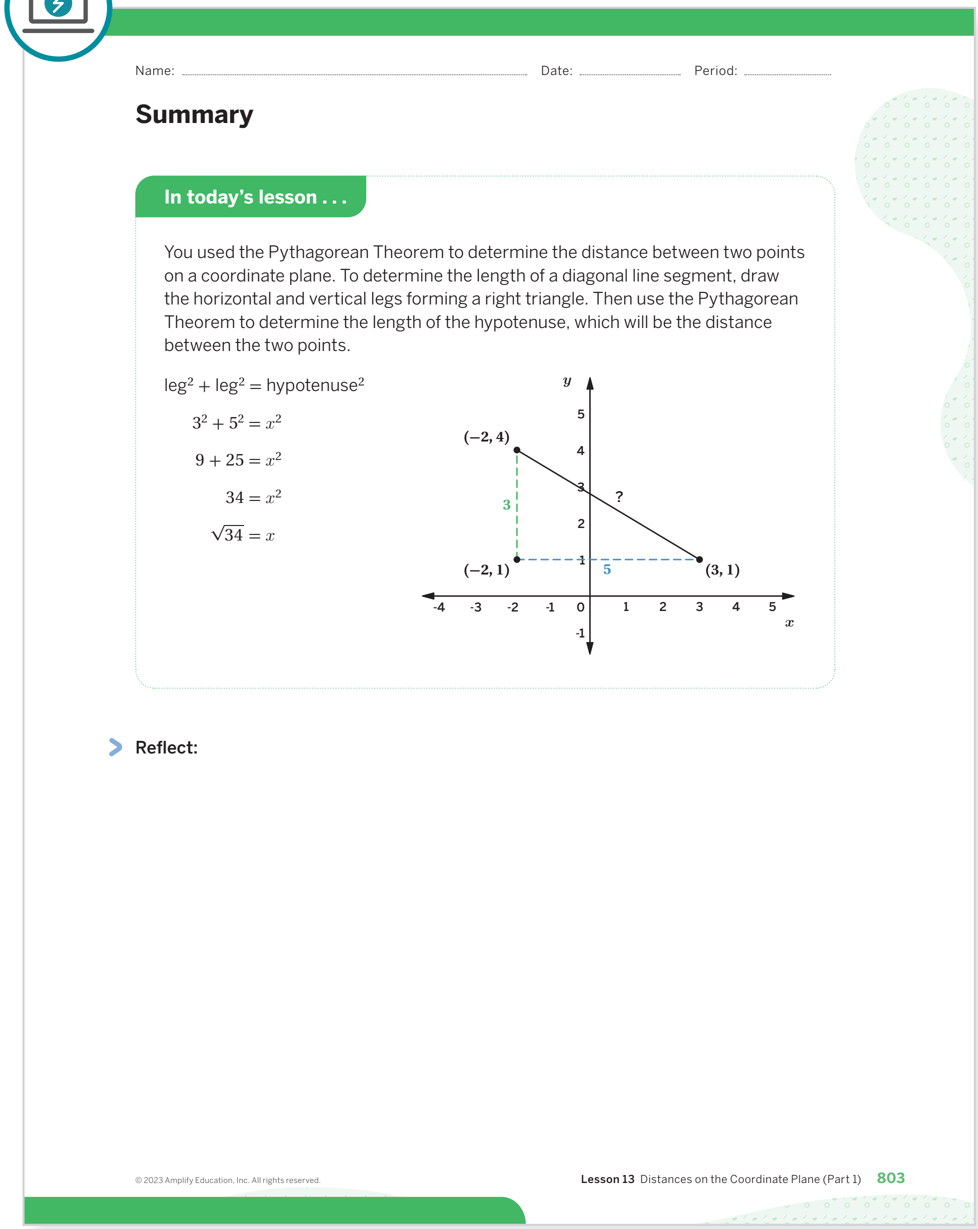
Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have each student in the group complete one of the problems. Then have group members check each other's work and determine the approximate perimeter together.

Whole Class | 5 min

Summary

Review and synthesize how to determine the distance between points on a coordinate plane using the Pythagorean Theorem.



Name: Date: Period:

Summary

In today's lesson . . .

You used the Pythagorean Theorem to determine the distance between two points on a coordinate plane. To determine the length of a diagonal line segment, draw the horizontal and vertical legs forming a right triangle. Then use the Pythagorean Theorem to determine the length of the hypotenuse, which will be the distance between the two points.

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$

$3^2 + 5^2 = x^2$

$9 + 25 = x^2$

$34 = x^2$

$\sqrt{34} = x$

Reflect:

Synthesize

Display the Anchor Chart PDF, *The Pythagorean Theorem,* and complete the bottom portion.

Highlight that diagonal distances on a coordinate plane cannot be counted like vertical and horizontal distances. However, creating a right triangle by drawing the vertical and horizontal legs allows students to use the Pythagorean Theorem to find the distance.

Note: In future math courses, students will apply the Pythagorean Theorem to the distance formula to find the distance between ordered pairs.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

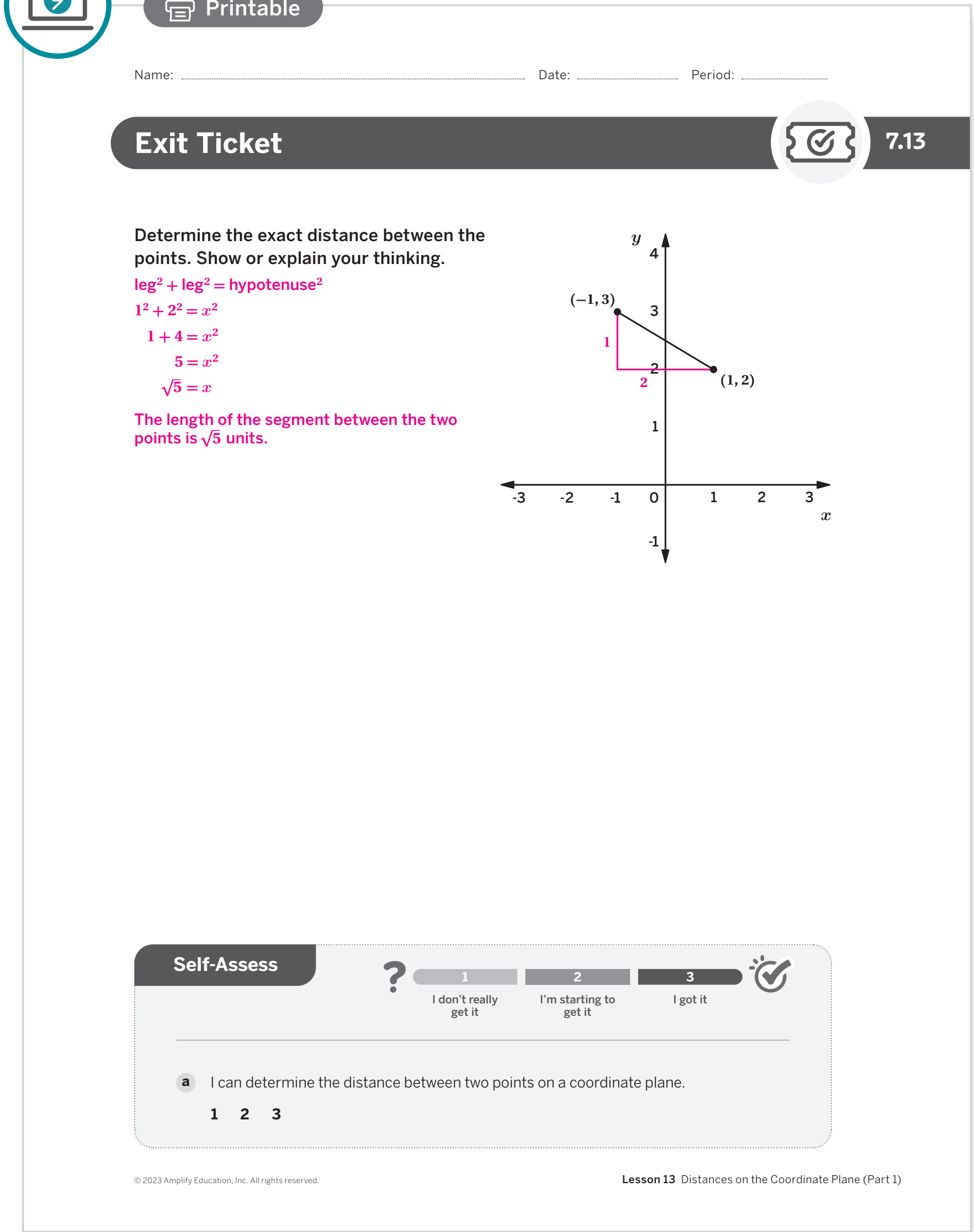
- "How is determining a distance between two points similar to determining the length of the hypotenuse?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the distance between two points on a coordinate plane.

Printable



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Exit Ticket

7.13

Determine the exact distance between the points. Show or explain your thinking.

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$

$1^2 + 2^2 = x^2$

$1 + 4 = x^2$

$5 = x^2$

$\sqrt{5} = x$

The length of the segment between the two points is $\sqrt{5}$ units.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine the distance between two points on a coordinate plane.

1 2 3

Lesson 13 Distances on the Coordinate Plane (Part 1)

Success looks like . . .

- **Language Goal:** Calculating the distance between two points in the coordinate plane by using the Pythagorean Theorem and explaining the solution method. **(Speaking and Listening)**
    - » Determining the distance between $(-1,3)$ and $(1,2)$.
- **Language Goal:** Generalizing a method for calculating the length of a line segment in the coordinate plane using the Pythagorean Theorem. **(Speaking and Listening)**

Suggested next steps

If students determine the length is 5 units, consider:

- Reviewing the Anchor Chart PDF, *The Pythagorean Theorem*.
- Reminding them they have determined the square of the hypotenuse and need to determine the length of the hypotenuse.
- Assigning Practice Problem 1.

Professional Learning

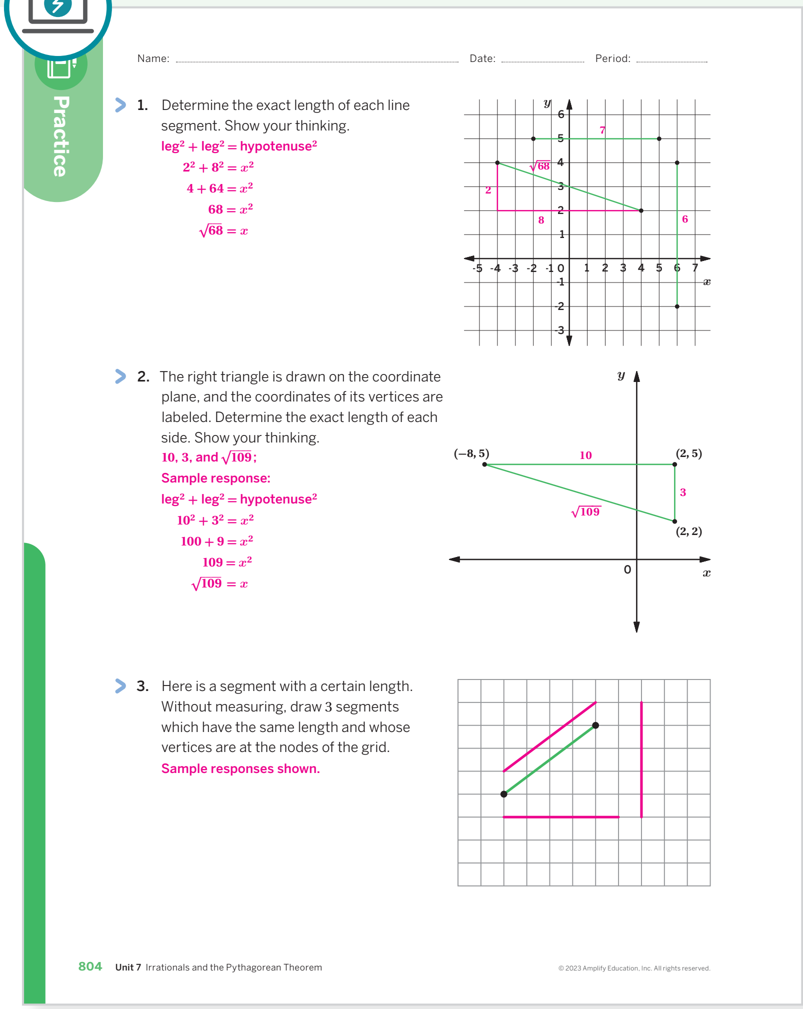
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier in this unit using the Pythagorean Theorem, what similarities and differences do you see?
- What did you see in the way some students approached finding the distance between two points on a coordinate plane that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Determine the exact length of each line segment. Show your thinking.
   leg^2 + leg^2 = hypotenuse^2
   $2^2 + 8^2 = x^2$
   $4 + 64 = x^2$
   $68 = x^2$
   $\sqrt{68} = x$

2. The right triangle is drawn on the coordinate plane, and the coordinates of its vertices are labeled. Determine the exact length of each side. Show your thinking.
   10, 3, and $\sqrt{109}$;
   Sample response:
   leg^2 + leg^2 = hypotenuse^2
   $10^2 + 3^2 = x^2$
   $100 + 9 = x^2$
   $109 = x^2$
   $\sqrt{109} = x$

3. Here is a segment with a certain length. Without measuring, draw 3 segments which have the same length and whose vertices are at the nodes of the grid.
   Sample responses shown.

804 Unit 7 Irrationals and the Pythagorean Theorem

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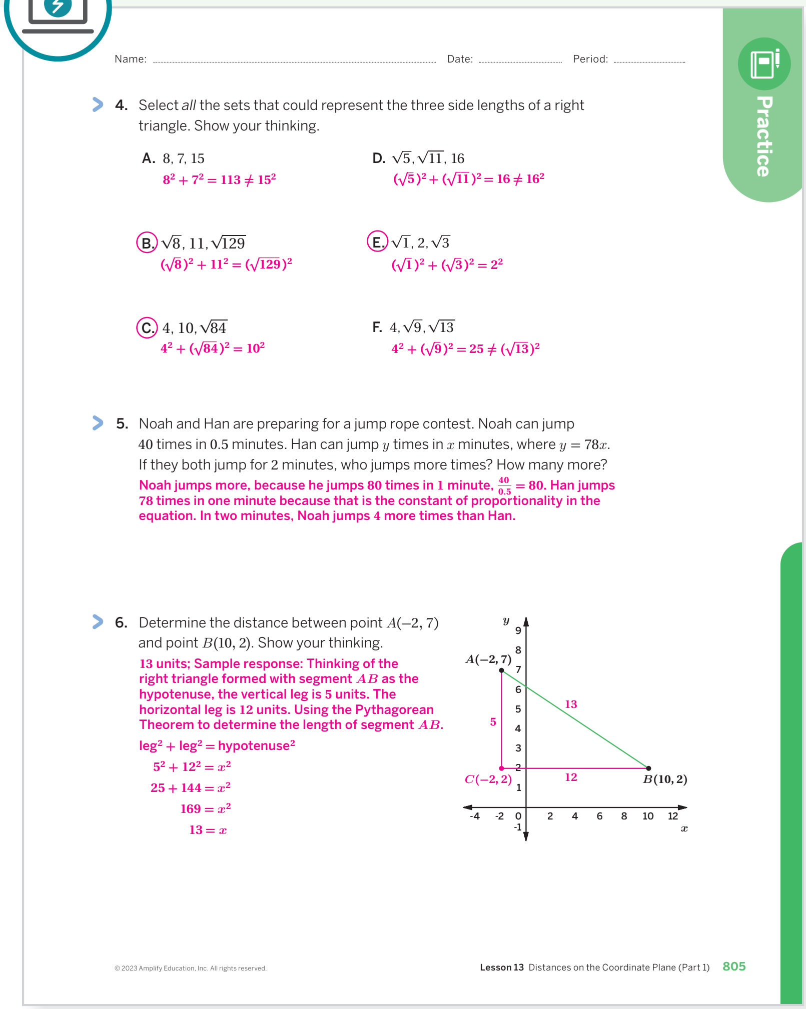

Name: Date: Period:

4. Select *all* the sets that could represent the three side lengths of a right triangle. Show your thinking.

A. 8, 7, 15 $8^2 + 7^2 = 113 \neq 15^2$	D. $\sqrt{5}, \sqrt{11}, 16$ $(\sqrt{5})^2 + (\sqrt{11})^2 = 16 \neq 16^2$
[✓ B.] $\sqrt{8}, 11, \sqrt{129}$ $(\sqrt{8})^2 + 11^2 = (\sqrt{129})^2$	[✓ E.] $\sqrt{1}, 2, \sqrt{3}$ $(\sqrt{1})^2 + (\sqrt{3})^2 = 2^2$
[✓ C.] $4, 10, \sqrt{84}$ $4^2 + (\sqrt{84})^2 = 10^2$	F. $4, \sqrt{9}, \sqrt{13}$ $4^2 + (\sqrt{9})^2 = 25 \neq (\sqrt{13})^2$

5. Noah and Han are preparing for a jump rope contest. Noah can jump 40 times in 0.5 minutes. Han can jump y times in x minutes, where $y = 78x$. If they both jump for 2 minutes, who jumps more times? How many more?
   Noah jumps more, because he jumps 80 times in 1 minute, $\frac{40}{0.5} = 80$. Han jumps 78 times in one minute because that is the constant of proportionality in the equation. In two minutes, Noah jumps 4 more times than Han.

6. Determine the distance between point $A(-2, 7)$ and point $B(10, 2)$. Show your thinking.
   13 units; Sample response: Thinking of the right triangle formed with segment AB as the hypotenuse, the vertical leg is 5 units. The horizontal leg is 12 units. Using the Pythagorean Theorem to determine the length of segment AB.
   leg^2 + leg^2 = hypotenuse^2
   $5^2 + 12^2 = x^2$
   $25 + 144 = x^2$
   $169 = x^2$
   $13 = x$

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Lesson 13 Distances on the Coordinate Plane (Part 1) 805

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 7 Lesson 12	1
	5	Unit 3 Lesson 6	2
Formative ⚡	6	Unit 7 Lesson 14	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 14**

Distances on the Coordinate Plane (Part 2)

Let's determine more distances between two points.

Focus

Goal

1. Determine the distance between two points.

Rigor

- Students **apply** their work of the Pythagorean Theorem to determine the distance between two points.

Coherence

Today

Students continue their work determining the distance between two points. They begin their work without graphing the points to determine the connection between the coordinates and the lengths of the legs of the right triangle produced by any diagonal distance.

Previously

In Lesson 13, students used the Pythagorean Theorem to determine the distance between two points on a coordinate plane.

Coming Soon

In Lesson 15, students will see how the Pythagorean Theorem can be applied to solve real-world problems involving speed, distance, and time.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
8 min	25 min	5 min	7 min
Independent	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, pre-cut cards, one set per group
- Anchor Chart PDF, *The Pythagorean Theorem*
- Anchor Chart PDF, *The Pythagorean Theorem* (answers)
- calculators (optional)
- graph paper (optional)

Math Language Development

Review words

- *absolute value*
- *circle*
- *hypotenuse*
- *leg*
- *Pythagorean Theorem*

Amps | Featured Activity

Activity 1
Interactive Graphs

Students digitally plot their points to determine the shape presented in Activity 1.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might not understand the connections among the distances between points on the coordinate plane and the shape created. Point out that the structure of the coordinate plane provides lengths to the sides of the triangles and the radius of the circle. Encourage students to break the task into smaller, more-manageable tasks for the group. Rather than use the divide-and-conquer method, have all students work together to complete each part.

Modifications to Pacing

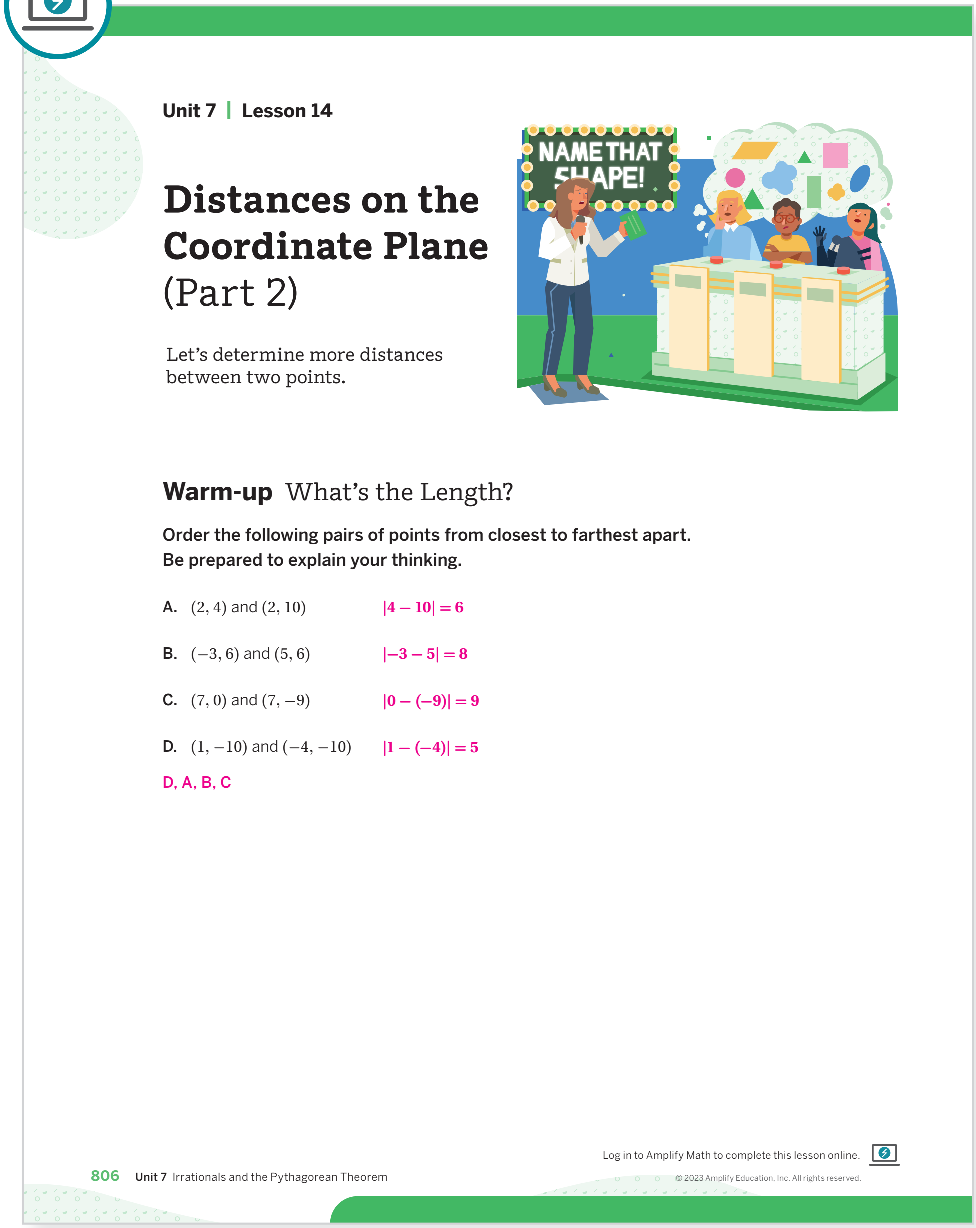
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, omit Problem 3.

Independent | 8 min

Warm-up What's the Length?

Students review how to determine the distance between two points on vertical or horizontal lines.



Unit 7 | Lesson 14

Distances on the Coordinate Plane

(Part 2)

Let's determine more distances between two points.

Warm-up What's the Length?

Order the following pairs of points from closest to farthest apart. Be prepared to explain your thinking.

A. $(2, 4)$ and $(2, 10)$ $|4 - 10| = 6$

B. $(-3, 6)$ and $(5, 6)$ $|-3 - 5| = 8$

C. $(7, 0)$ and $(7, -9)$ $|0 - (-9)| = 9$

D. $(1, -10)$ and $(-4, -10)$ $|1 - (-4)| = 5$

D, A, B, C

Log in to Amplify Math to complete this lesson online.

806 **Unit 7** Irrationals and the Pythagorean Theorem

1 Launch

Remind students it does not matter which point they begin with because they can take the absolute value to determine the length of the segments. Have students conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by having them determine whether the segment is horizontal or vertical. Provide access to calculators and graph paper.

Look for points of confusion:

- **Miscalculating the lengths of the segments.** Remind students to find the differences carefully.
- **Having negative values representing the distance.** Remind students of absolute value to determine the distance.

3 Connect

Display the Warm-up problems and have students share their thinking. Start with students who plotted the ordered pairs precisely on graph paper, followed by students who sketched a coordinate plane to get an idea of where the points would be, and end with students who subtracted the coordinates without using a graph.

Highlight the multiple methods used by the students to determine the lengths of the segments. Remind students they can subtract the coordinates and use the absolute value to ensure the length is a positive value.

Ask, "If you subtracted the coordinates, what you must remember to do to ensure you have found the proper length?" Determine the absolute value of the difference.

Power-up

To power up students' ability to use the Pythagorean Theorem to determine the distance between two points on the coordinate plane:

Provide students with a copy of the Power-up PDF.

Use: Before Activity 1

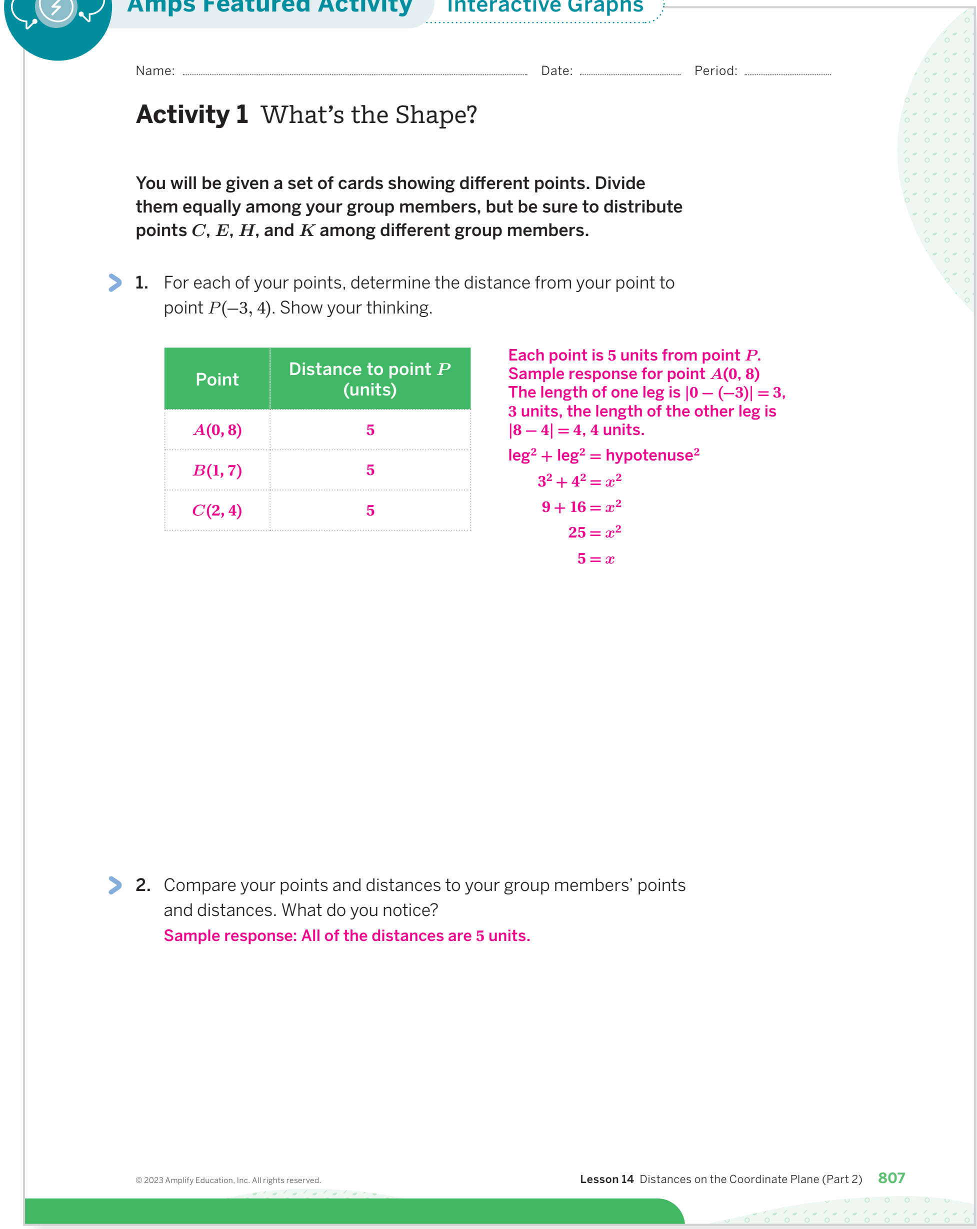
Informed by: Performance on Lesson 13, Practice Problem 6

Small Groups | 25 min

Activity 1 What's the Shape?

Students determine the distance between a series of points to create a circle with radius of 5 units.

Amps Featured Activity Interactive Graphs



Name: ______ Date: ______ Period: ______

Activity 1 What's the Shape?

You will be given a set of cards showing different points. Divide them equally among your group members, but be sure to distribute points C, E, H, and K among different group members.

1. For each of your points, determine the distance from your point to point $P(-3, 4)$. Show your thinking.

Point	Distance to point P (units)
$A(0, 8)$	5
$B(1, 7)$	5
$C(2, 4)$	5

Each point is 5 units from point P.
Sample response for point $A(0, 8)$
The length of one leg is $|0-(-3)|=3$, 3 units, the length of the other leg is $|8-4|=4$, 4 units.

leg^2 + leg^2 = hypotenuse^2

$$3^2+4^2=x^2$$
$$9+16=x^2$$
$$25=x^2$$
$$5=x$$

2. Compare your points and distances to your group members' points and distances. What do you notice?
Sample response: All of the distances are 5 units.

1 Launch

Distribute pre-cut cards from the Activity 1 PDF so that each group receives points A through K.

2 Monitor

Help students get started by having them start with point C, E, H, or K which either make horizontal or vertical line segments with point P.

Look for points of confusion:

- **Not realizing their point is in a vertical or horizontal line with point P.** Have students visualize where their points would be located on a coordinate plane, or if needed, plot them on graph paper.
- **Struggling to determine the distance with a point diagonal from point P.** Have students revisit Lesson 13 and plot their points on graph paper.

Look for productive strategies:

- Realizing the connected points form congruent triangles, thus resulting in only using the Pythagorean Theorem one time.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally plot their points to determine the shape.

Accessibility: *Vary Demands to Optimize Challenge*

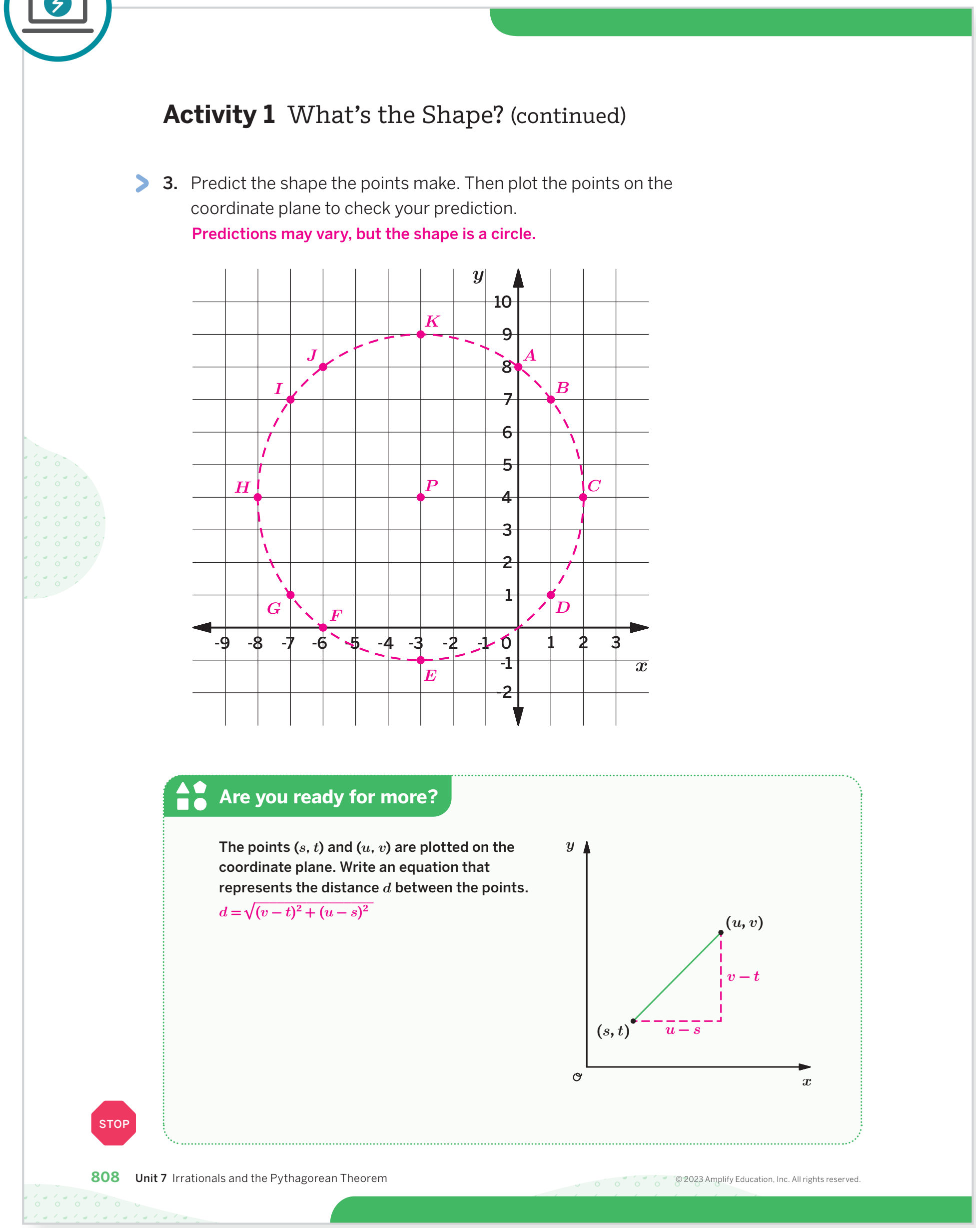
Provide access to graph paper should students choose to use it to plot their points. Consider assigning fewer points to each student if they would benefit from more processing time.

Extension: *Math Enrichment*

If students complete the *Are you ready for more?* problem, tell them that the equation they wrote to represent the distance d between the points is actually the distance formula. Have them use their equation to determine the distance between the points $(-3, -1)$ and $(-8, 6)$ without graphing the points. $\sqrt{101}$, or about 8.6 units

Activity 1 What's the Shape? (continued)

Students determine the distance between a series of points to create a circle with radius of 5 units.



Activity 1 What's the Shape? (continued)

3. Predict the shape the points make. Then plot the points on the coordinate plane to check your prediction.
Predictions may vary, but the shape is a circle.

Are you ready for more?

The points (s, t) and (u, v) are plotted on the coordinate plane. Write an equation that represents the distance d between the points.

$d = \sqrt{(v - t)^2 + (u - s)^2}$

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3 Connect

Have students share how they determined the distance between two points and what their predictions were for Problem 3. Then display a student's work for graphing the points.

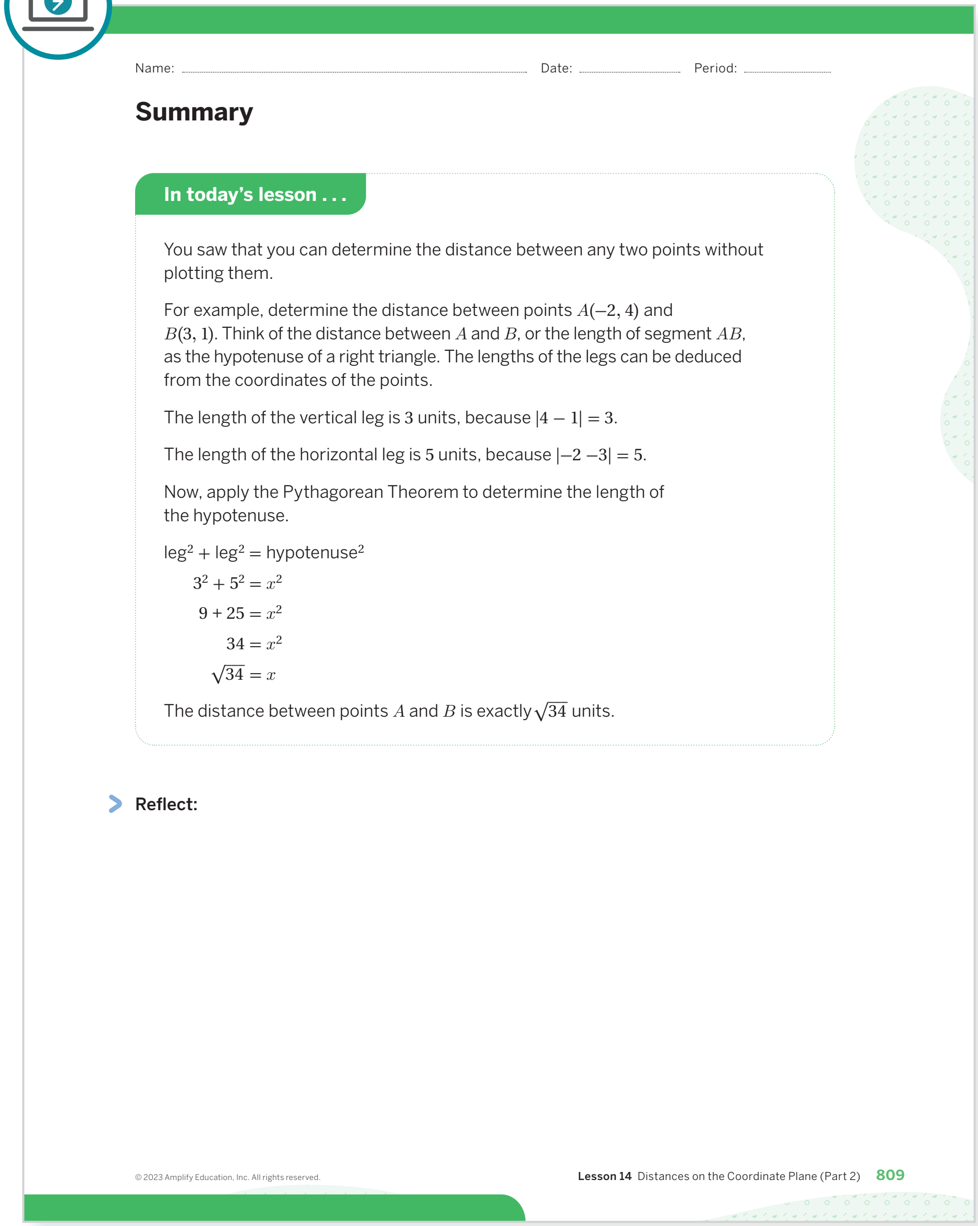
Ask, "Why is this shape a circle? Identify another point with integer coordinates which will be located on the circle." All points are 5 units from the center. $(0, 0)$ is another point on the circle.

Highlight that, in high school, students will learn how to write a formula which describes the coordinates of every point on a circle.

Whole Class | 5 min

Summary

Review and synthesize how the Pythagorean Theorem can be used to determine the distance between any two points.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that you can determine the distance between any two points without plotting them.

For example, determine the distance between points $A(-2, 4)$ and $B(3, 1)$. Think of the distance between A and B, or the length of segment AB, as the hypotenuse of a right triangle. The lengths of the legs can be deduced from the coordinates of the points.

The length of the vertical leg is 3 units, because $|4 - 1| = 3$.

The length of the horizontal leg is 5 units, because $|-2 - 3| = 5$.

Now, apply the Pythagorean Theorem to determine the length of the hypotenuse.

$$\begin{aligned} \text{leg}^2 + \text{leg}^2 &= \text{hypotenuse}^2 \\ 3^2 + 5^2 &= x^2 \\ 9 + 25 &= x^2 \\ 34 &= x^2 \\ \sqrt{34} &= x \end{aligned}$$

The distance between points A and B is exactly $\sqrt{34}$ units.

Reflect:

Synthesize

Have students share their strategies for determining the distance between any two points. Start with students who graph the points, draw the right triangle, and count to determine the lengths of the legs and end with students who subtract corresponding coordinates to determine the lengths of the legs.

Ask, "When, in real life, is it helpful to determine distances between two objects or places?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

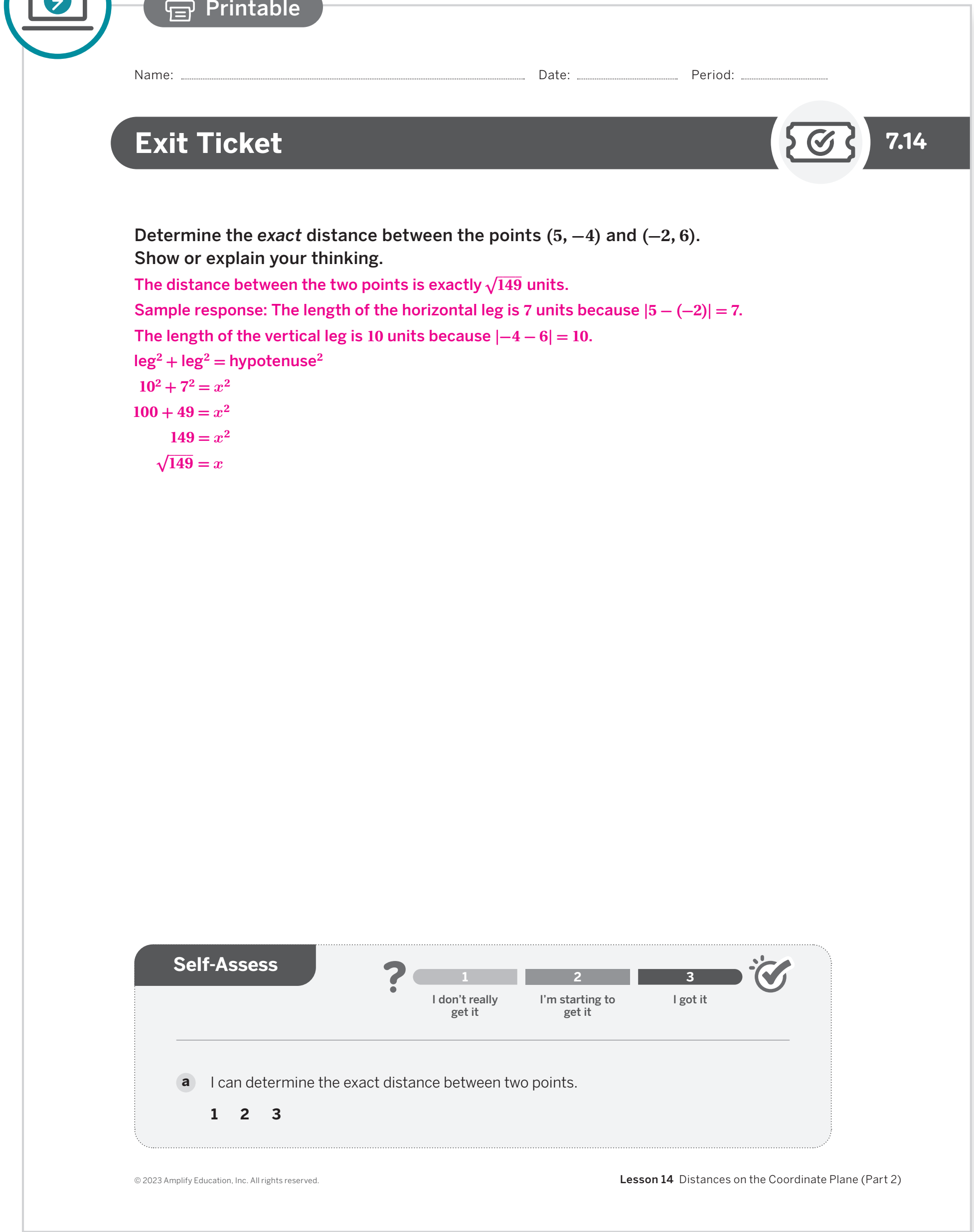
- "How can the Pythagorean Theorem help you determine the distance between any two points?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by determining the distance between two points.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 7.14

Determine the *exact* distance between the points $(5, -4)$ and $(-2, 6)$. Show or explain your thinking.

The distance between the two points is exactly $\sqrt{149}$ units.

Sample response: The length of the horizontal leg is 7 units because $|5-(-2)|=7$.

The length of the vertical leg is 10 units because $|-4-6|=10$.

$$\begin{aligned} \text{leg}^2+\text{leg}^2&=\text{hypotenuse}^2\\ 10^2+7^2&=x^2\\ 100+49&=x^2\\ 149&=x^2\\ \sqrt{149}&=x \end{aligned}$$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine the exact distance between two points.

1 2 3

Lesson 14 Distances on the Coordinate Plane (Part 2)

Success looks like . . .

- **Goal:** Determining the distance between two points.
  - » Determining the exact distance between the points $(5, -4)$ and $(-2, 6)$.

Suggested next steps

If students struggle to determine the distance, consider:

- Providing graph paper to help with their thinking.

If students determine the incorrect distance, consider:

- Reviewing the Warm-up.
- Assigning Practice Problems 1 and 2.

Professional Learning

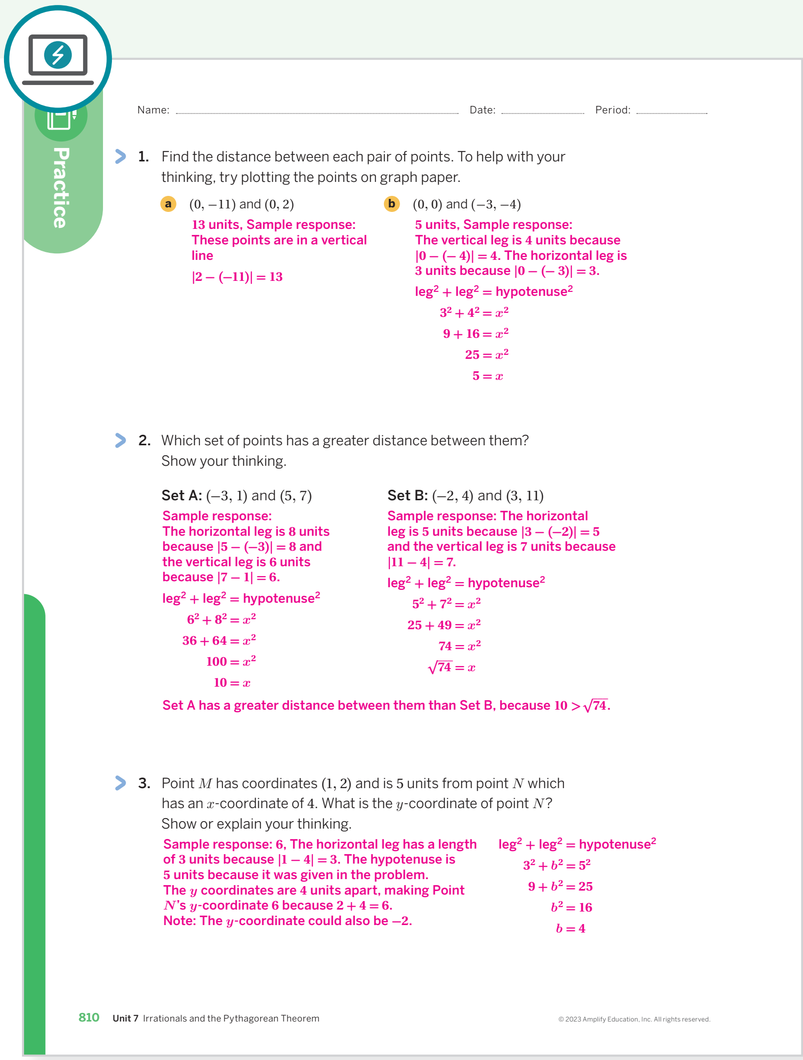
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did Activity 1 go as planned?
- Did students find today's activity engaging? Why do you think that is? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. Find the distance between each pair of points. To help with your thinking, try plotting the points on graph paper.
   - a $(0, -11)$ and $(0, 2)$

     13 units, Sample response: These points are in a vertical line

     $|2-(-11)|=13$
   - b $(0, 0)$ and $(-3, -4)$

     5 units, Sample response: The vertical leg is 4 units because $|0-(-4)|=4$. The horizontal leg is 3 units because $|0-(-3)|=3$.

     $\text{leg}^2+\text{leg}^2=\text{hypotenuse}^2$

     $3^2+4^2=x^2$

     $9+16=x^2$

     $25=x^2$

     $5=x$

2. Which set of points has a greater distance between them? Show your thinking.

   Set A: $(-3, 1)$ and $(5, 7)$

   Sample response: The horizontal leg is 8 units because $|5-(-3)|=8$ and the vertical leg is 6 units because $|7-1|=6$.

   $\text{leg}^2+\text{leg}^2=\text{hypotenuse}^2$

   $6^2+8^2=x^2$

   $36+64=x^2$

   $100=x^2$

   $10=x$

   Set B: $(-2, 4)$ and $(3, 11)$

   Sample response: The horizontal leg is 5 units because $|3-(-2)|=5$ and the vertical leg is 7 units because $|11-4|=7$.

   $\text{leg}^2+\text{leg}^2=\text{hypotenuse}^2$

   $5^2+7^2=x^2$

   $25+49=x^2$

   $74=x^2$

   $\sqrt{74}=x$

   Set A has a greater distance between them than Set B, because $10>\sqrt{74}$.

3. Point M has coordinates $(1, 2)$ and is 5 units from point N which has an x-coordinate of 4. What is the y-coordinate of point N? Show or explain your thinking.

   Sample response: 6, The horizontal leg has a length of 3 units because $|1-4|=3$. The hypotenuse is 5 units because it was given in the problem. The y coordinates are 4 units apart, making Point N's y-coordinate 6 because $2+4=6$. Note: The y-coordinate could also be -2.

   $\text{leg}^2+\text{leg}^2=\text{hypotenuse}^2$

   $3^2+b^2=5^2$

   $9+b^2=25$

   $b^2=16$

   $b=4$

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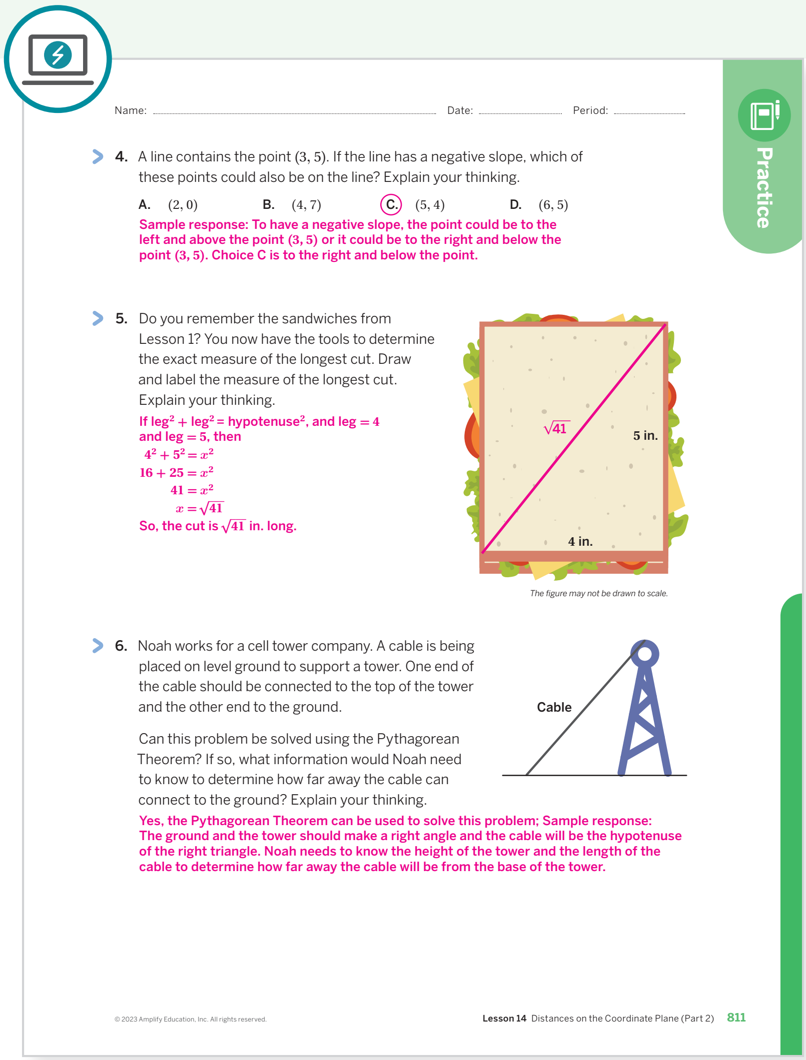

Name: ________ Date: ________ Period: ________

Practice

4. A line contains the point $(3, 5)$. If the line has a negative slope, which of these points could also be on the line? Explain your thinking.

   A. $(2, 0)$ B. $(4, 7)$ [✓ C.] $(5, 4)$ D. $(6, 5)$

   Sample response: To have a negative slope, the point could be to the left and above the point $(3, 5)$ or it could be to the right and below the point $(3, 5)$. Choice C is to the right and below the point.

5. Do you remember the sandwiches from Lesson 1? You now have the tools to determine the exact measure of the longest cut. Draw and label the measure of the longest cut. Explain your thinking.

   If $\text{leg}^2+\text{leg}^2=\text{hypotenuse}^2$, and $\text{leg}=4$ and $\text{leg}=5$, then

   $4^2+5^2=x^2$

   $16+25=x^2$

   $41=x^2$

   $x=\sqrt{41}$

   So, the cut is $\sqrt{41}$ in. long.

   The figure may not be drawn to scale.

6. Noah works for a cell tower company. A cable is being placed on level ground to support a tower. One end of the cable should be connected to the top of the tower and the other end to the ground.

   Can this problem be solved using the Pythagorean Theorem? If so, what information would Noah need to know to determine how far away the cable can connect to the ground? Explain your thinking.

   Yes, the Pythagorean Theorem can be used to solve this problem; Sample response: The ground and the tower should make a right angle and the cable will be the hypotenuse of the right triangle. Noah needs to know the height of the tower and the length of the cable to determine how far away the cable will be from the base of the tower.

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Lesson 14 Distances on the Coordinate Plane (Part 2) 811

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	3
Spiral	4	Unit 3 Lesson 13	1
	5	Unit 7 Lesson 11	2
Formative ⚡	6	Unit 7 Lesson 15	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 7 | **LESSON 15**

Applications of the Pythagorean Theorem

Let's solve problems using the Pythagorean Theorem.

Focus

Goals

1. **Language Goal:** Describe situations that use right triangles, and explain how the Pythagorean Theorem could help solve problems in those situations. **(Speaking and Listening)**
2. **Language Goal:** Use the Pythagorean Theorem to solve problems within a context, and explain the reasoning used. **(Speaking and Listening)**

Coherence

Today

Students use the Pythagorean Theorem as a tool to solve application problems. They consider real-world contexts that can be modeled with right triangles with two known sides and one unknown side. Students use the Pythagorean Theorem to determine an unknown side of a right triangle and interpret that number in the context of problems involving speed, time, and distance.

Previously

In Lesson 11, students used the Pythagorean Theorem to determine unknown side lengths.

Coming Soon

In high school, students will continue their study of triangles. When students study trigonometry, they will learn that angles can be used to determine side lengths, which allows them to apply properties of triangles to solve more complex problems.

Rigor

- Students **apply** the Pythagorean Theorem to solve real-world problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *The Pythagorean Theorem*
- Anchor Chart PDF, *The Pythagorean Theorem* (answers)
- calculators
- rulers

Math Language Development

Review words

- *hypotenuse*
- *legs*
- *Pythagorean Theorem*
- *square root*

Amps | Featured Activity

Activity 2
See Student Thinking

Students use digital tools to compare routes and speeds, and you can review their thinking as they work.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Before beginning Activity 2, students might not understand what the problem is or how they are going to solve it. As students develop a plan, make sure they take steps such as identifying the problem, analyzing the situation to determine what information they do have, and then determining what strategies they can use to solve the problem. Afterward, they can evaluate the results, reflecting on the effectiveness of decisions made along the way.

Modifications to Pacing

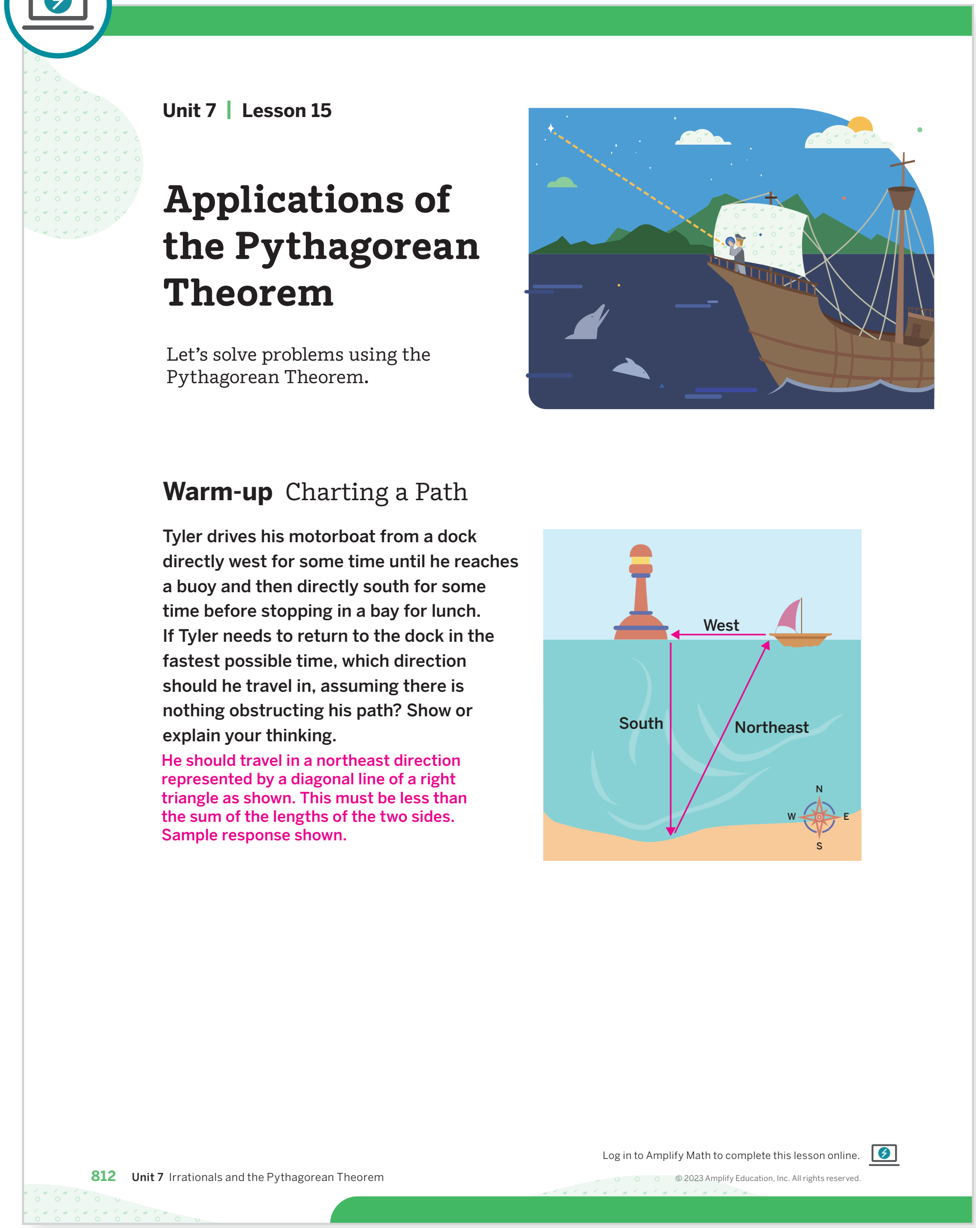
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, Problem 2 may be omitted.

Pairs | 5 min

Warm-up Charting a Path

Students reason about a real-world context to explore an example of applying a right triangle to a real-world problem.



Unit 7 | Lesson 15

Applications of the Pythagorean Theorem

Let's solve problems using the Pythagorean Theorem.

Warm-up Charting a Path

Tyler drives his motorboat from a dock directly west for some time until he reaches a buoy and then directly south for some time before stopping in a bay for lunch. If Tyler needs to return to the dock in the fastest possible time, which direction should he travel in, assuming there is nothing obstructing his path? Show or explain your thinking.

He should travel in a northeast direction represented by a diagonal line of a right triangle as shown. This must be less than the sum of the lengths of the two sides. Sample response shown.

Log in to Amplify Math to complete this lesson online.

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1 Launch

To activate prior knowledge, ask students whether they have ever had an experience trying to determine the best route using a map, a compass, or other navigation tools. Provide access to rulers.

2 Monitor

Help students get started by suggesting they draw a diagram.

Look for points of confusion:

- **Thinking a path other than the diagonal is the fastest route.** Ensure students have a sketch. Be prepared to return to these students during the connect when comparing their route to the route of a straight line in the northeast direction
- **Being unsure which direction to call their diagonal line.** Ask students which direction is opposite south, and which is opposite west.

3 Connect

Display a student's drawing of a right triangle with the diagonal as the most direct route and ask students to explain why they think it is the shortest.

Ask:

- "What shape do you see? How do you know it is a right triangle?"
- "How could we show this diagonal is the shortest distance?"
- "Would this always be the best route to take? What could be changed or added to the context that might make it not be the best route?"

Highlight that students can say this is a right triangle based on the language of the problem. South and west are directionally perpendicular to each other, meaning they form a right angle. This means they can model this context with a right triangle.

Power-up

To power up students' ability to connect the Pythagorean Theorem to real-world problems, have students complete:

Recall that in order to use the Pythagorean Theorem you must have a right triangle. Determine which scenarios describe right angles. Select *all* that apply.

[✓ **A.**] The angle between the side of a building and the ground.

B. Traveling South and then turning and traveling North.

[✓ **C.**] Traveling South and then turning and traveling West.

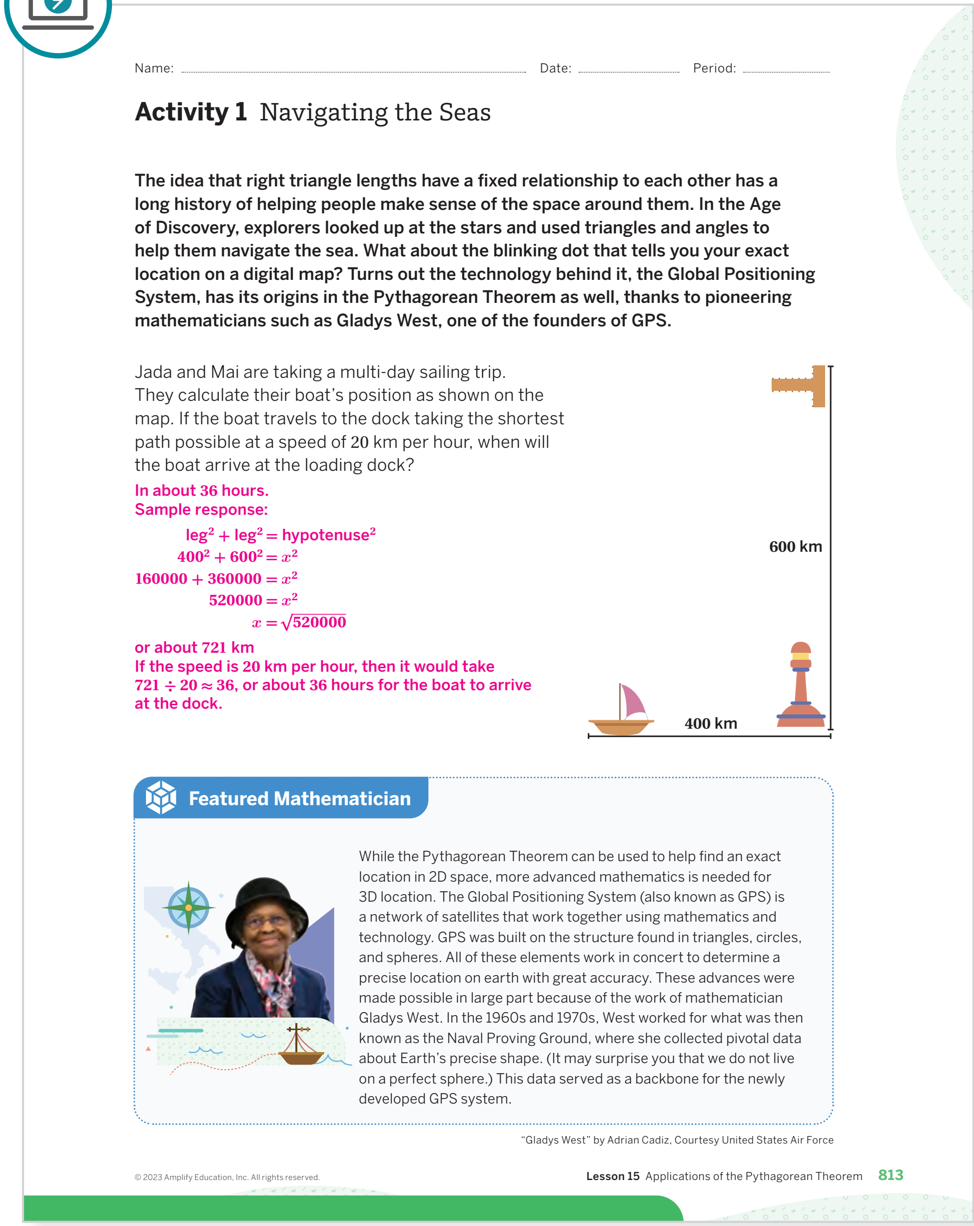
D. The angle between the ground and a ladder leaning against a building.

Use: Before the Warm-up
Informed by: Performance on Lesson 14, Practice Problem 6

Pairs | 15 min

Activity 1 Navigating the Seas

Students apply the Pythagorean Theorem to a real-world problem to determine the time it would take to travel the fastest route between two points.



Name: ________ Date: ________ Period: ________

Activity 1 Navigating the Seas

The idea that right triangle lengths have a fixed relationship to each other has a long history of helping people make sense of the space around them. In the Age of Discovery, explorers looked up at the stars and used triangles and angles to help them navigate the sea. What about the blinking dot that tells you your exact location on a digital map? Turns out the technology behind it, the Global Positioning System, has its origins in the Pythagorean Theorem as well, thanks to pioneering mathematicians such as Gladys West, one of the founders of GPS.

Jada and Mai are taking a multi-day sailing trip. They calculate their boat's position as shown on the map. If the boat travels to the dock taking the shortest path possible at a speed of 20 km per hour, when will the boat arrive at the loading dock?

In about 36 hours.
Sample response:

$$\begin{aligned} \text{leg}^2 + \text{leg}^2 &= \text{hypotenuse}^2 \\ 400^2 + 600^2 &= x^2 \\ 160000 + 360000 &= x^2 \\ 520000 &= x^2 \\ x &= \sqrt{520000} \end{aligned}$$

or about 721 km
If the speed is 20 km per hour, then it would take $721 \div 20 \approx 36$, or about 36 hours for the boat to arrive at the dock.

Featured Mathematician

While the Pythagorean Theorem can be used to help find an exact location in 2D space, more advanced mathematics is needed for 3D location. The Global Positioning System (also known as GPS) is a network of satellites that work together using mathematics and technology. GPS was built on the structure found in triangles, circles, and spheres. All of these elements work in concert to determine a precise location on earth with great accuracy. These advances were made possible in large part because of the work of mathematician Gladys West. In the 1960s and 1970s, West worked for what was then known as the Naval Proving Ground, where she collected pivotal data about Earth's precise shape. (It may surprise you that we do not live on a perfect sphere.) This data served as a backbone for the newly developed GPS system.

"Gladys West" by Adrian Cadiz, Courtesy United States Air Force

1 Launch

Provide access to calculators for the duration of the lesson.

2 Monitor

Help students get started by asking them to draw the path of the ship and identify the shape made by the boat's path.

Look for points of confusion:

- **Not being able to determine the distance the boat needs to travel.** Ensure students recognize the right triangle made by the path of the boat. Have students use the Anchor Chart PDF, *The Pythagorean Theorem*, to help them label the legs of the triangle, and then identify how they can use the Pythagorean Theorem to solve for the unknown distance.
- **Not being able to solve for the time it will take.** After students have found the distance, ask them what information they have (speed and distance), and what they will need to know (time), and how they can write an equation using distance, speed, and time to solve for the unknown variable.

3 Connect

Have students share their strategies. If applicable, have students compare responses that are estimates written in radical form.

Ask:

- "What about this problem indicated you could apply the Pythagorean Theorem to help you solve?"
- "How did you know that the boat's path made a right triangle?"
- "What are the limitations of using the Pythagorean Theorem to solve real world problems?"

Highlight that the Pythagorean Theorem can be used to solve a problem that involves a right triangle with two known sides.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1**: Students should understand that Jada and Mai are in the boat and want to determine when the boat will arrive at the loading dock.
- **Read 2**: Ask students to name or highlight the given quantities and relationships, such as the boat is traveling at a speed of 20 km per hour.
- **Read 3**: Ask students to annotate the map as they plan how they will approach this task.

English Learners

Chunk this routine by pausing after each read and giving students quiet, independent think time to process what they just read, as well as ask questions to a partner and/or the class.

Featured Mathematician

Gladys West

Have students read about Featured Mathematician Gladys West, whose research served as the backbone for the Global Positioning System, or GPS.

Pairs | 15 min

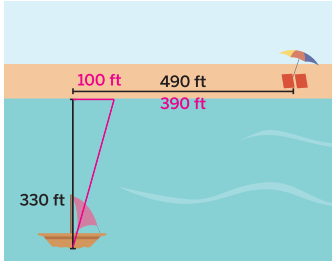
Activity 2 Fastest Route

Students use the Pythagorean Theorem to solve problems involving speeds and distances.

Amps Featured Activity | See Student Thinking

Activity 2 Fastest Route

Jada and Mai want to jump off their boat anchored in a lake and swim back to their towels and umbrella set up on the beach. They decide to race to the umbrella.



1. Jada and Mai decide to take separate routes. They can each swim 3 ft per second. Their speed on the sand is 5 ft per second. Mai decides to swim directly to the umbrella and Jada decides to swim directly to shore and then run to the umbrella. Who will reach the umbrella first?

   Mai will reach the umbrella first, about 11 seconds before Jada.

   Mai swam the distance of $\sqrt{490^2 + 330^2} \approx 591$, 591 ft and she needed $591 \div 3 = 197$, 197 seconds to cover this distance.

   Jada's time can be found by evaluating the expression: $\frac{330}{3} + \frac{490}{5}$, which is equal to 208 seconds.

2. Is there a path the person who finished second could have taken to reach the umbrella first? Sketch your path and explain your thinking.

   Yes; Sample response shown on diagram: If Jada follows a diagonal line from the boat to the shore 390 ft from their belongings, she will reach the umbrella before Mai. She would get there in about 193 seconds, or about 4 seconds before Mai.

   $$\frac{\sqrt{330^2 + 100^2}}{3} + \frac{390}{5} \approx 193$$

Stronger and Clearer: Share your responses to Problems 1 and 2 with another pair of students to give and receive feedback. Use the feedback you receive to improve your responses.

STOP

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1 Launch

Read the task aloud with students. Use the *Poll the Class* routine to see who they think will win without performing any calculations.

2 Monitor

Help students get started by encouraging them to label the relevant distances and information on the diagram.

Look for points of confusion:

- **Not using Jada's two different speeds to correctly determine her time.** Ask students for what distance Jada is traveling 5 ft per second and for what distance she is traveling 3 ft per second. Ensure students have two separate expressions for each section she travels.
- **Drawing a path for Problem 2 for which they are not able to determine a distance.** If students draw, for example, a curved line, remind them that a straight line is the fastest route if the speed is the same.

3 Connect

Have students share how they determined who arrived first in Problem 1.

Display different suggestions for the paths Jada could take in Problem 2. Ask students what they notice about the paths.

Ask:

- "What about the shape of the path taken allowed you to use the Pythagorean Theorem?"
- "How could you write one Mai's time as one expression? Jada's time?"
- "Will Jada always win if she takes a different path? Why or why not?"

Highlight that, in high school, they will learn how to find the optimum path.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problem 1.

Extension: *Math Enrichment*

Challenge students to determine a speed by which Jada could travel on land to reach the umbrella first. Jada's equation would be $\frac{330}{3} + \frac{490}{x} = 197$. Solving the equation for x means if Jada traveled at a speed greater than 5.63 ft per second, she would reach the umbrella before Mai.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students respond to Problems 1 and 2, have them meet with 2–3 partners to both give and receive feedback on their responses and explanations. Encourage reviewers to ask clarifying questions such as:

- "Does your response include calculations and evidence to show the time it takes for both Mai and Jada to reach the umbrella?"
- "How did you know the Pythagorean Theorem could help you solve this problem?"

Have students revise their responses, as needed.

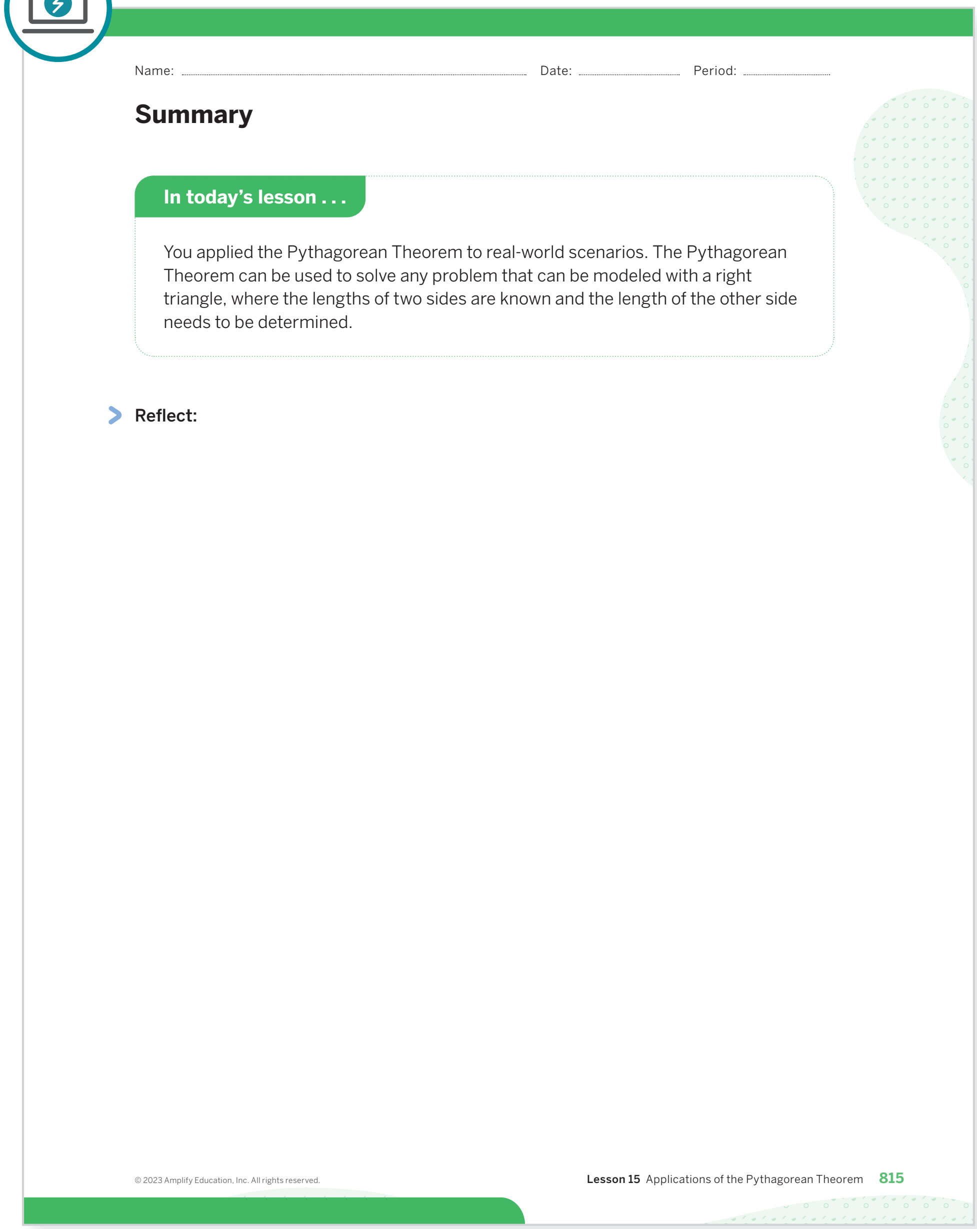
English Learners

Provide sentence frames for students to use to complete their responses, such as "_____ will reach the umbrella first because . . ." and "If _____ changes her path by _____, then she will reach the umbrella first because . . ."

Whole Class | 5 min

Summary

Review and synthesize how to apply the Pythagorean Theorem to solve real-world problems.



Name: __________ Date: __________ Period: __________

Summary

In today's lesson . . .

You applied the Pythagorean Theorem to real-world scenarios. The Pythagorean Theorem can be used to solve any problem that can be modeled with a right triangle, where the lengths of two sides are known and the length of the other side needs to be determined.

Reflect:

Synthesize

Have students share how they applied the Pythagorean Theorem to solve real-world problems.

Ask:

- "What other types of problems could the Pythagorean Theorem help you solve?"
- "If you know the lengths of the two legs of a right triangle, how can you determine the length of the hypotenuse? If you know the lengths of the hypotenuse and one leg, how can you determine the length of the other leg?"

Highlight that the Pythagorean Theorem can be used to solve any problem that can be modeled with a right triangle where two sides are known and one is unknown.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

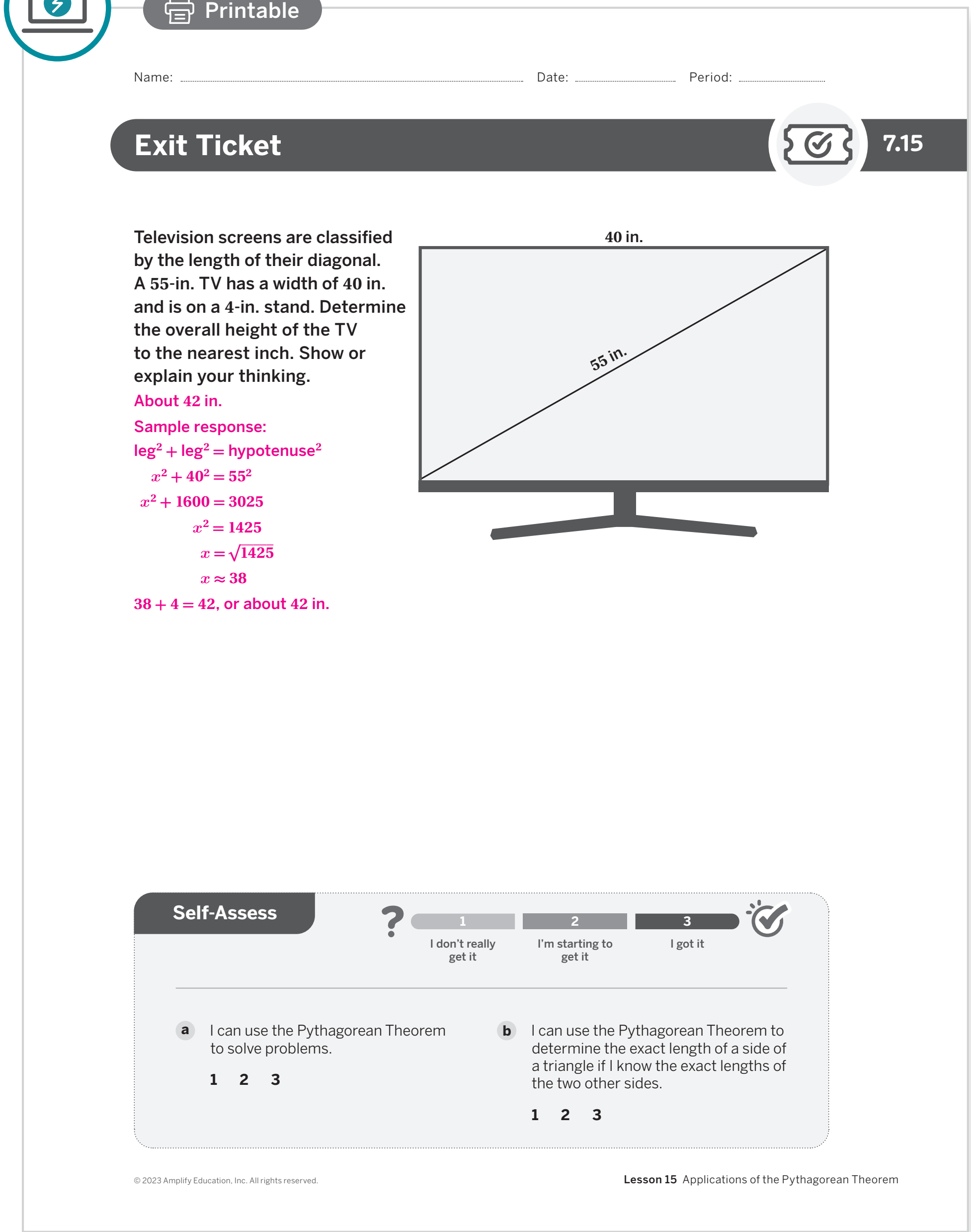
- "In what context(s) is the Pythagorean Theorem useful in solving real-world problems?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by applying the Pythagorean Theorem to solve a problem in a real-world situation.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

7.15

Television screens are classified by the length of their diagonal. A 55-in. TV has a width of 40 in. and is on a 4-in. stand. Determine the overall height of the TV to the nearest inch. Show or explain your thinking.

About 42 in.

Sample response:

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$

$x^2 + 40^2 = 55^2$

$x^2 + 1600 = 3025$

$x^2 = 1425$

$x = \sqrt{1425}$

$x \approx 38$

$38 + 4 = 42$, or about 42 in.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a) I can use the Pythagorean Theorem to solve problems.

1 2 3

b) I can use the Pythagorean Theorem to determine the exact length of a side of a triangle if I know the exact lengths of the two other sides.

1 2 3

Success looks like . . .

- **Language Goal:** Describing situations that use right triangles, and explaining how the Pythagorean Theorem could help solve problems in those situations. **(Speaking and Listening)**
- **Language Goal:** Using the Pythagorean Theorem to solve problems within a context, and explaining the reasoning used. **(Speaking and Listening)**
  - » Applying the Pythagorean theorem to determine the height of the TV.

Suggested next steps

If students do not recognize that the height is represented by a leg and that they know the measure of a leg and the hypotenuse of a right triangle, consider:

- Having them label the sides of the triangle with *leg*, *leg*, and *hypotenuse* and ensure they can correctly substitute the values in the equation $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$.

If students have difficulty solving the equation correctly, consider:

- Reviewing solving strategies from Activities 1 and 2.
- Having students substitute their solution into the Pythagorean Theorem to see if it makes the equation true.

If students do not recognize that they must add the height of the TV to the height of the base to determine the overall height, consider:

- Reviewing Activity 2 for how labeling a diagram with knowns and unknowns can be useful.
- Asking students what it means when the text asks for the *overall* height of the TV.

Professional Learning

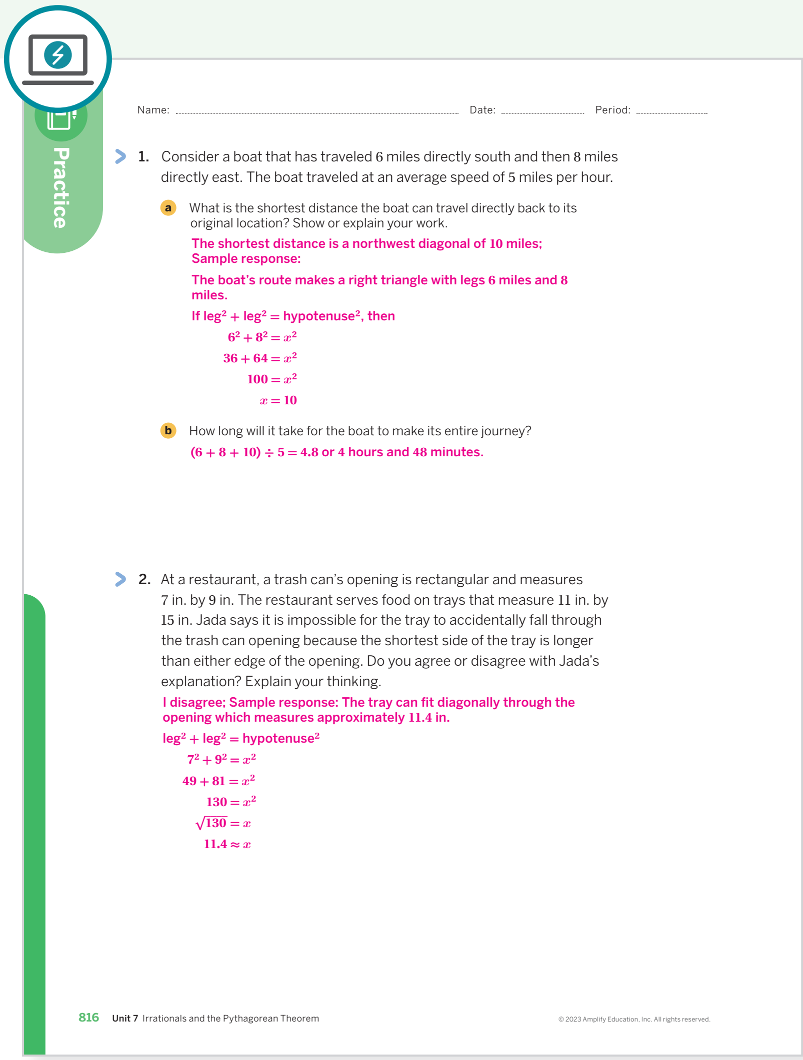
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What resources did students use as they worked on applying the Pythagorean Theorem to solve problems? Which resources were especially helpful?
- What other ways are there to show how to apply the Pythagorean Theorem?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Consider a boat that has traveled 6 miles directly south and then 8 miles directly east. The boat traveled at an average speed of 5 miles per hour.

a What is the shortest distance the boat can travel directly back to its original location? Show or explain your work.

The shortest distance is a northwest diagonal of 10 miles; Sample response:

The boat's route makes a right triangle with legs 6 miles and 8 miles.

If $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$, then

$$\begin{aligned} 6^2 + 8^2 &= x^2 \\ 36 + 64 &= x^2 \\ 100 &= x^2 \\ x &= 10 \end{aligned}$$

b How long will it take for the boat to make its entire journey?

$(6 + 8 + 10) \div 5 = 4.8$ or 4 hours and 48 minutes.

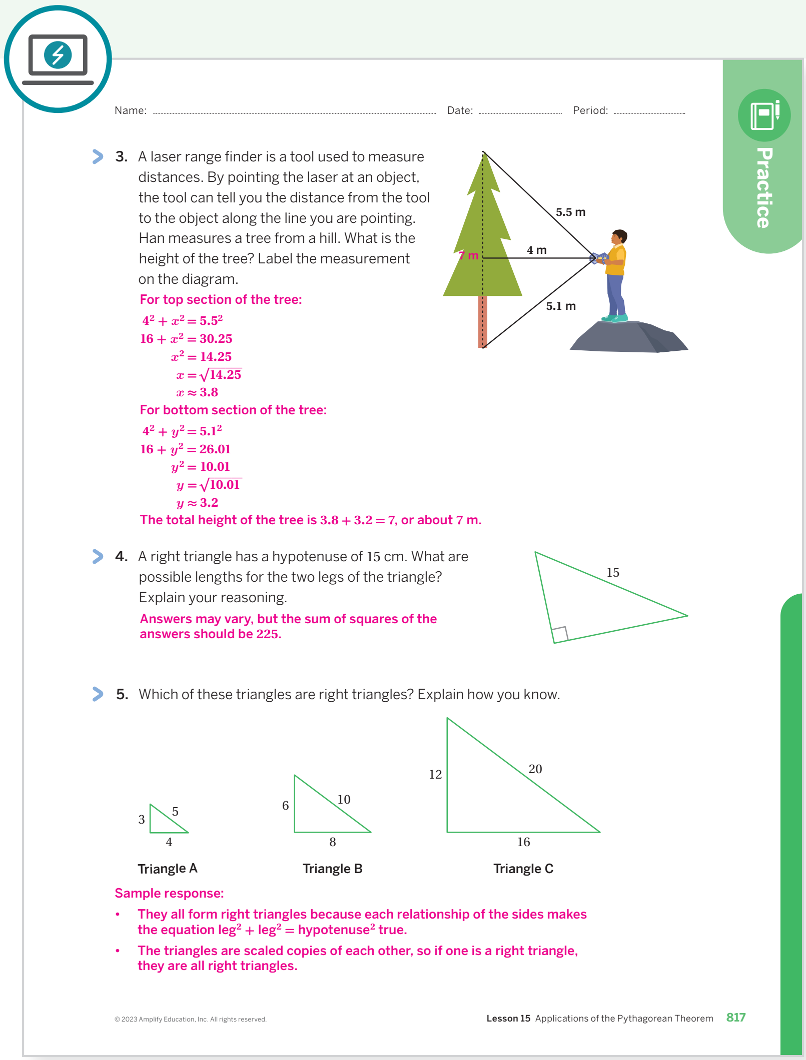
2. At a restaurant, a trash can's opening is rectangular and measures 7 in. by 9 in. The restaurant serves food on trays that measure 11 in. by 15 in. Jada says it is impossible for the tray to accidentally fall through the trash can opening because the shortest side of the tray is longer than either edge of the opening. Do you agree or disagree with Jada's explanation? Explain your thinking.

I disagree; Sample response: The tray can fit diagonally through the opening which measures approximately 11.4 in.

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$

$$\begin{aligned} 7^2 + 9^2 &= x^2 \\ 49 + 81 &= x^2 \\ 130 &= x^2 \\ \sqrt{130} &= x \\ 11.4 &\approx x \end{aligned}$$

816 **Unit 7** Irrationals and the Pythagorean Theorem



Practice

Name: ________ Date: ________ Period: ________

3. A laser range finder is a tool used to measure distances. By pointing the laser at an object, the tool can tell you the distance from the tool to the object along the line you are pointing. Han measures a tree from a hill. What is the height of the tree? Label the measurement on the diagram.

For top section of the tree:

$$\begin{aligned} 4^2 + x^2 &= 5.5^2 \\ 16 + x^2 &= 30.25 \\ x^2 &= 14.25 \\ x &= \sqrt{14.25} \\ x &\approx 3.8 \end{aligned}$$

For bottom section of the tree:

$$\begin{aligned} 4^2 + y^2 &= 5.1^2 \\ 16 + y^2 &= 26.01 \\ y^2 &= 10.01 \\ y &= \sqrt{10.01} \\ y &\approx 3.2 \end{aligned}$$

The total height of the tree is $3.8 + 3.2 = 7$, or about 7 m.

4. A right triangle has a hypotenuse of 15 cm. What are possible lengths for the two legs of the triangle? Explain your reasoning.

Answers may vary, but the sum of squares of the answers should be 225.

5. Which of these triangles are right triangles? Explain how you know.

Triangle A Triangle B Triangle C

Sample response:

- **They all form right triangles because each relationship of the sides makes the equation $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ true.**
- **The triangles are scaled copies of each other, so if one is a right triangle, they are all right triangles.**

Lesson 15 Applications of the Pythagorean Theorem 817

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 7 Lesson 11	1
Formative ⚡	5	Unit 7 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Pythagorean Triples

Let's try to recognize patterns that will make Pythagorean triples.

Focus

Goals

1. **Language Goal:** Know small-integer Pythagorean triples and explain patterns among them. **(Speaking and Listening, Writing)**
2. Understand there are infinitely many Pythagorean triples.

Coherence

Today

Students look for a pattern among a series of Pythagorean triples. They build on their understanding of $a^2 + b^2 = c^2$ to grapple with Fermat's Last Theorem (the equation $a^n + b^n = c^n$ does not have positive integer solutions for any n greater than 2) and learn how it took mathematicians hundreds of years to prove it.

Previously

In Lesson 15, students used the Pythagorean Theorem to solve real-world problems.

Coming Soon

In the last unit of the year, students will explore bivariate data displays, such as scatter plots and two-way frequency tables.

Rigor

- Students **apply** their knowledge of the Pythagorean Theorem to determine connections between various Pythagorean triples.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	20 min	7 min	5 min	7 min
Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- calculators

Math Language Development

New words

- ***Pythagorean triple***

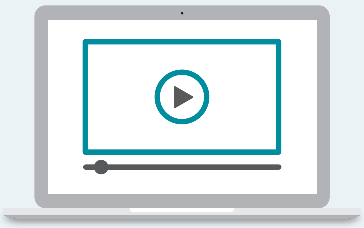
Review words

- *hypotenuse*
- *leg*
- *Pythagorean Theorem*

Amps | Featured Activity

Warm-up
Pythagorean Triples

Students can sketch over of nested squares to discover the connection between specific Pythagorean triples, while you can overlay their work.



Building Math Identity and Community

Connecting to Mathematical Practices

Fermat's Last Theorem might be intimidating to students as they begin Activity 2. They might doubt their ability to ever understand the theorem. By holding to their self-efficacy, students believe that if they persevere, they will be able to make sense of the problem.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

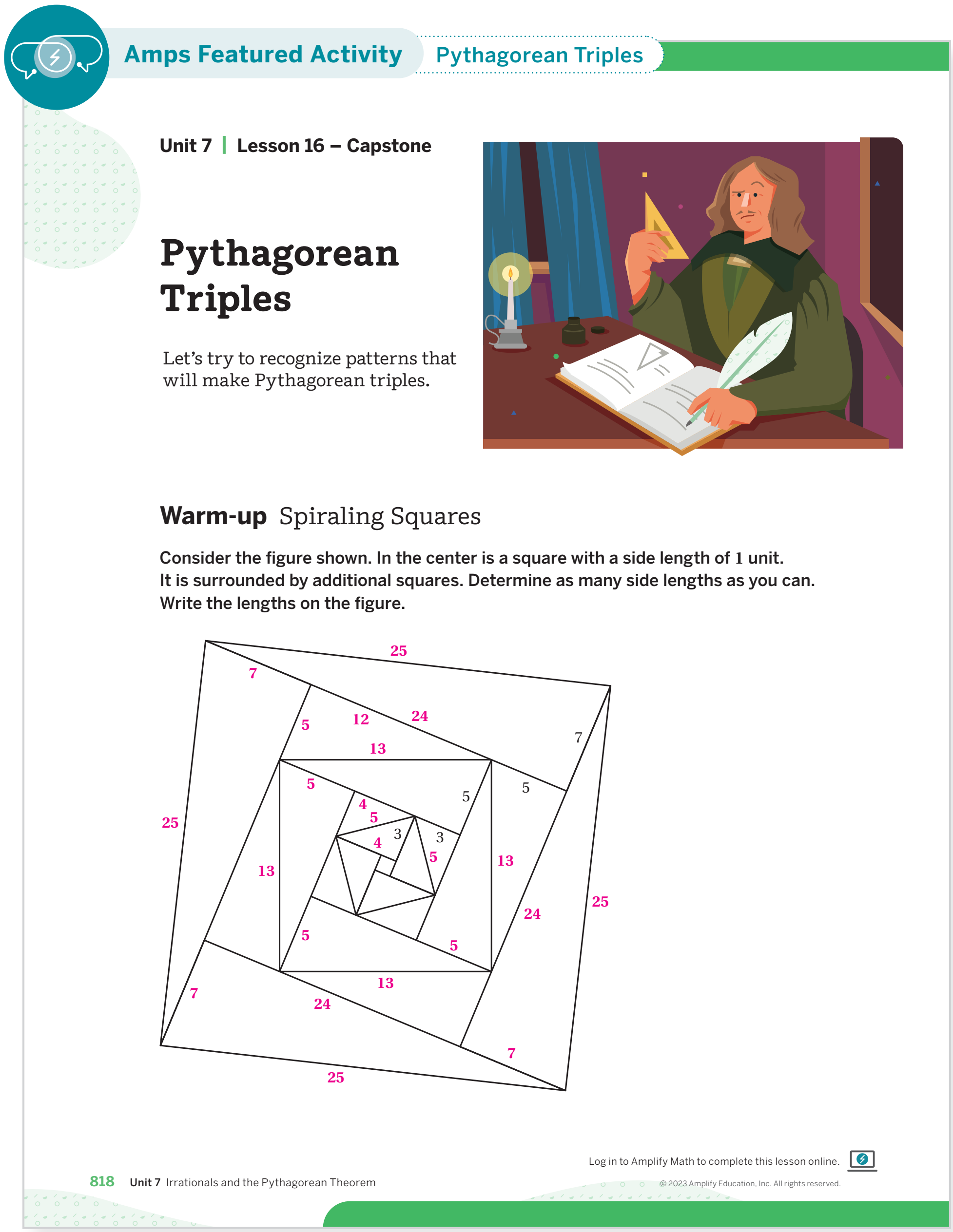
- Consider assigning **Activity 1** or **Activity 2**, but not both.

Small Groups | 10 min

Warm-up Spiraling Squares

Students generate Pythagorean triples using spiraling squares to prepare for Activity 1.

Amps Featured Activity Pythagorean Triples



Unit 7 | Lesson 16 – Capstone

Pythagorean Triples

Let's try to recognize patterns that will make Pythagorean triples.

Warm-up Spiraling Squares

Consider the figure shown. In the center is a square with a side length of 1 unit. It is surrounded by additional squares. Determine as many side lengths as you can. Write the lengths on the figure.

Log in to Amplify Math to complete this lesson online.

818 Unit 7 Irrationals and the Pythagorean Theorem

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1 Launch

Display the Pythagorean Triple animation from the Amps slides. Conduct the ***Notice and Wonder*** routine. Then set an expectation for the amount of time students will have to work on the activity.

2 Monitor

Help students get started by asking them to write the length of the center square and how that helps them determine the side length of the smallest right triangle.

Look for points of confusion:

- **Being overwhelmed by the picture.** Have students only focus on the triangle or square they are using by covering the rest.

Look for productive strategies:

- **Using more than looks to determine if the triangles are congruent.** Students may use addition and the Pythagorean Theorem to determine the missing side lengths.

3 Connect

Display the Warm-up PDF and use the first figure to discuss why the length of the red segment is $1 + 3$ and how to determine the length of the blue segment using the Pythagorean Theorem.

Have students share how they can determine the lengths of the red segment in the second figure on the Warm-up PDF $(3 + 4 + 5)$ and the blue segment.

Highlight the inner triangles are used to determine the side length of the next triangle and the Pythagorean Theorem can be used to determine the remaining side length.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view an animation of nested squares to discover the connection between specific Pythagorean triples.

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Chunk this task into smaller, more manageable parts and provide students with a copy of the Warm-up PDF to help organize and focus their thinking.

Power-up

To power up students' ability to determine whether a triangle is a right triangle, have students complete:

Recall that a triangle is a right triangle if it satisfies $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$. Determine whether each set of side lengths results in a right triangle.

1. $(3, 4, 5)$ Yes
2. $(5, 3, 8)$ No
3. $(5, 10, 15)$ No
4. $(6, 8, 10)$ Yes

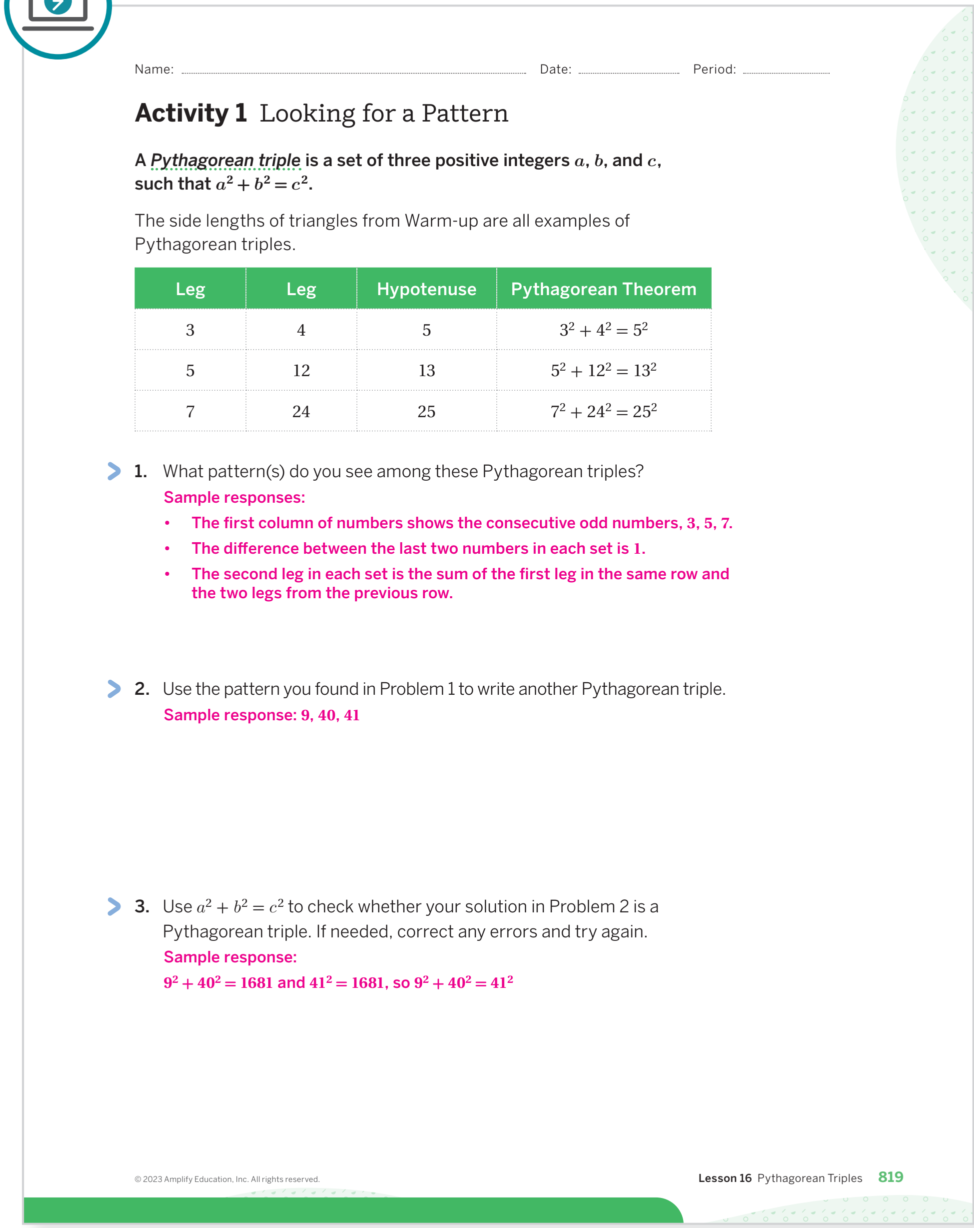
Use: Before Activity 1

Informed by: Performance on Lesson 15, Practice Problem 5

Small Groups | 20 min

Activity 1 Looking for a Pattern

Students use the numbers from the Warm-up to determine a pattern among a series of Pythagorean triples.



Name: ______ Date: ______ Period: ______

Activity 1 Looking for a Pattern

A *Pythagorean triple* is a set of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$.

The side lengths of triangles from Warm-up are all examples of Pythagorean triples.

Leg	Leg	Hypotenuse	Pythagorean Theorem
3	4	5	$3^2 + 4^2 = 5^2$
5	12	13	$5^2 + 12^2 = 13^2$
7	24	25	$7^2 + 24^2 = 25^2$

1. What pattern(s) do you see among these Pythagorean triples?
   Sample responses:
   - The first column of numbers shows the consecutive odd numbers, 3, 5, 7.
   - The difference between the last two numbers in each set is 1.
   - The second leg in each set is the sum of the first leg in the same row and the two legs from the previous row.

2. Use the pattern you found in Problem 1 to write another Pythagorean triple.
   Sample response: 9, 40, 41

3. Use $a^2 + b^2 = c^2$ to check whether your solution in Problem 2 is a Pythagorean triple. If needed, correct any errors and try again.
   Sample response:
   $9^2 + 40^2 = 1681$ and $41^2 = 1681$, so $9^2 + 40^2 = 41^2$

Lesson 16 Pythagorean Triples 819

1 Launch

Define a ***Pythagorean triple*** as a set of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$.

2 Monitor

Help students get started by asking them if they notice a pattern among the first number of each set.

Look for points of confusion:

- **Not being able to find a pattern.** Have students work together to see if anyone can discover a connection between the numbers. Have students look across the sets and between the sets of numbers.

Look for productive strategies:

- Creating additional Pythagorean triples.

3 Connect

Have students share their patterns and display them on the board.

Ask:

- "Can you explain any of these patterns using the diagram from the Warm-up?" The second number in each set is the sum of the first number in the same row and the first and second numbers from the previous row. For instance, the segments 5, 12, and 7 make up the leg of 24.
- "Using these patterns, what would the next three Pythagorean triples be?" 9, 40, 41; 11, 60, 61; 13, 84, 85

Highlight these patterns can continue and produce infinitely many Pythagorean Triples. In the next activity, students will see a related mathematical equation.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing students with the first number 9 as they attempt to write another Pythagorean triple in Problem 2.

Extension: Math Enrichment

Have students response to the following question:
If you have values a, b, and c, such that they satisfy the Pythagorean Theorem, show or explain why the values $2a$, $2b$, and $2c$ will also satisfy the Pythagorean Theorem. Sample response:

$(2a)^2 + (2b)^2 = (2c)^2$

$4a^2 + 4b^2 = 4c^2$ Simplify using exponent rules.

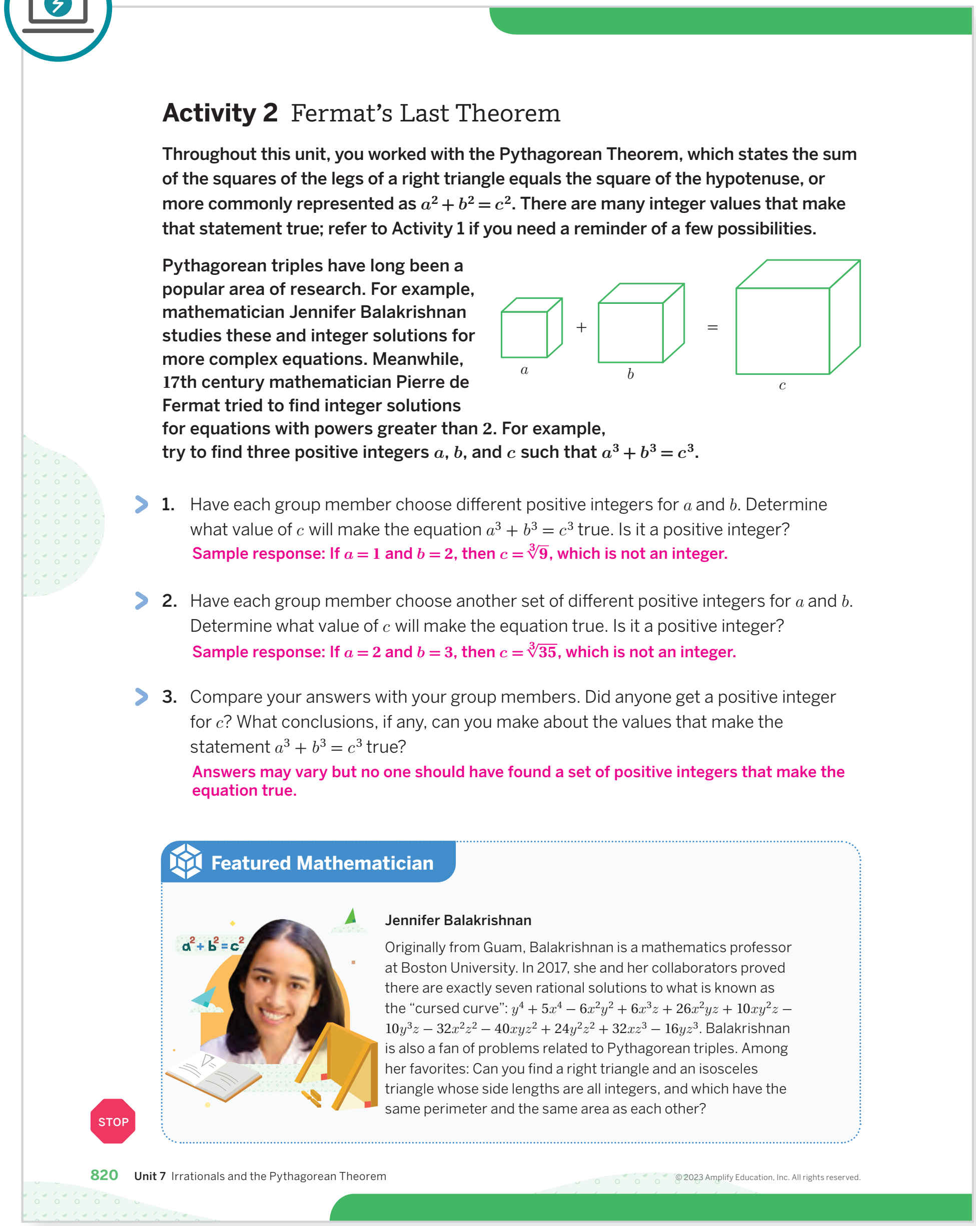
$4(a^2 + b^2) = 4c^2$ Distributive Property

$a^2 + b^2 = c^2$ Divide both sides by 4.

Small Groups | 7 min

Activity 2 Fermat’s Last Theorem

Students are introduced to Fermat’s Last Theorem to try their hand at determining whether there is any set of integers for which the equation $a^3 + b^3 = c^3$ is true.



Activity 2 Fermat’s Last Theorem

Throughout this unit, you worked with the Pythagorean Theorem, which states the sum of the squares of the legs of a right triangle equals the square of the hypotenuse, or more commonly represented as $a^2 + b^2 = c^2$. There are many integer values that make that statement true; refer to Activity 1 if you need a reminder of a few possibilities.

Pythagorean triples have long been a popular area of research. For example, mathematician Jennifer Balakrishnan studies these and integer solutions for more complex equations. Meanwhile, 17th century mathematician Pierre de Fermat tried to find integer solutions for equations with powers greater than 2. For example, try to find three positive integers a, b, and c such that $a^3 + b^3 = c^3$.

1. Have each group member choose different positive integers for a and b. Determine what value of c will make the equation $a^3 + b^3 = c^3$ true. Is it a positive integer?
   Sample response: If $a = 1$ and $b = 2$, then $c = \sqrt[3]{9}$, which is not an integer.

2. Have each group member choose another set of different positive integers for a and b. Determine what value of c will make the equation true. Is it a positive integer?
   Sample response: If $a = 2$ and $b = 3$, then $c = \sqrt[3]{35}$, which is not an integer.

3. Compare your answers with your group members. Did anyone get a positive integer for c? What conclusions, if any, can you make about the values that make the statement $a^3 + b^3 = c^3$ true?
   Answers may vary but no one should have found a set of positive integers that make the equation true.

Featured Mathematician

Jennifer Balakrishnan

Originally from Guam, Balakrishnan is a mathematics professor at Boston University. In 2017, she and her collaborators proved there are exactly seven rational solutions to what is known as the “cursed curve”: $y^4 + 5x^4 - 6x^2y^2 + 6x^3z + 26x^2yz + 10xy^2z - 10y^3z - 32x^2z^2 - 40xyz^2 + 24y^2z^2 + 32xz^3 - 16yz^3$. Balakrishnan is also a fan of problems related to Pythagorean triples. Among her favorites: Can you find a right triangle and an isosceles triangle whose side lengths are all integers, and which have the same perimeter and the same area as each other?

STOP

820 **Unit 7** Irrationals and the Pythagorean Theorem

1 Launch

Have a student read the prompt aloud. Encourage students to choose different numbers than their group members so the class gets a variety of equations. Provide access to calculators and remind students how to find the cube root on their calculators.

2 Monitor

Help students get started by asking them to select two integers for a and b and substitute them into the equation.

Look for points of confusion:

- **Calculating the square root instead of the cube root.** Remind students this equation uses cubes and they need to undo the cubing operation.

Look for productive strategies:

- Attempting more than two sets of numbers.

3 Connect

Have students share the numbers they tried and display the sets of numbers on the board.

Ask, “Do you think it is possible to test every possible integer combination? Do you think Fermat was able to try every combination of numbers? Why or why not?”

Display the Summary. Have students read the Summary or have a student volunteer read it aloud.

Highlight that it took mathematicians over 300 years to prove this theorem to be true. They did not test every number combination but used mathematics which was not available to Fermat during the time in which he lived.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate, using a think-aloud and two integer values for a and b, how to determine the value of c and show how the value of c is not an integer. Then have students choose different integer values for a and b, continuing with the activity.

Extension: *Math Enrichment*

Have students repeat the activity, this time using the equation $a^4 + b^4 = c^4$. Students should discover that there are no integer solutions to this equation.

Featured Mathematician

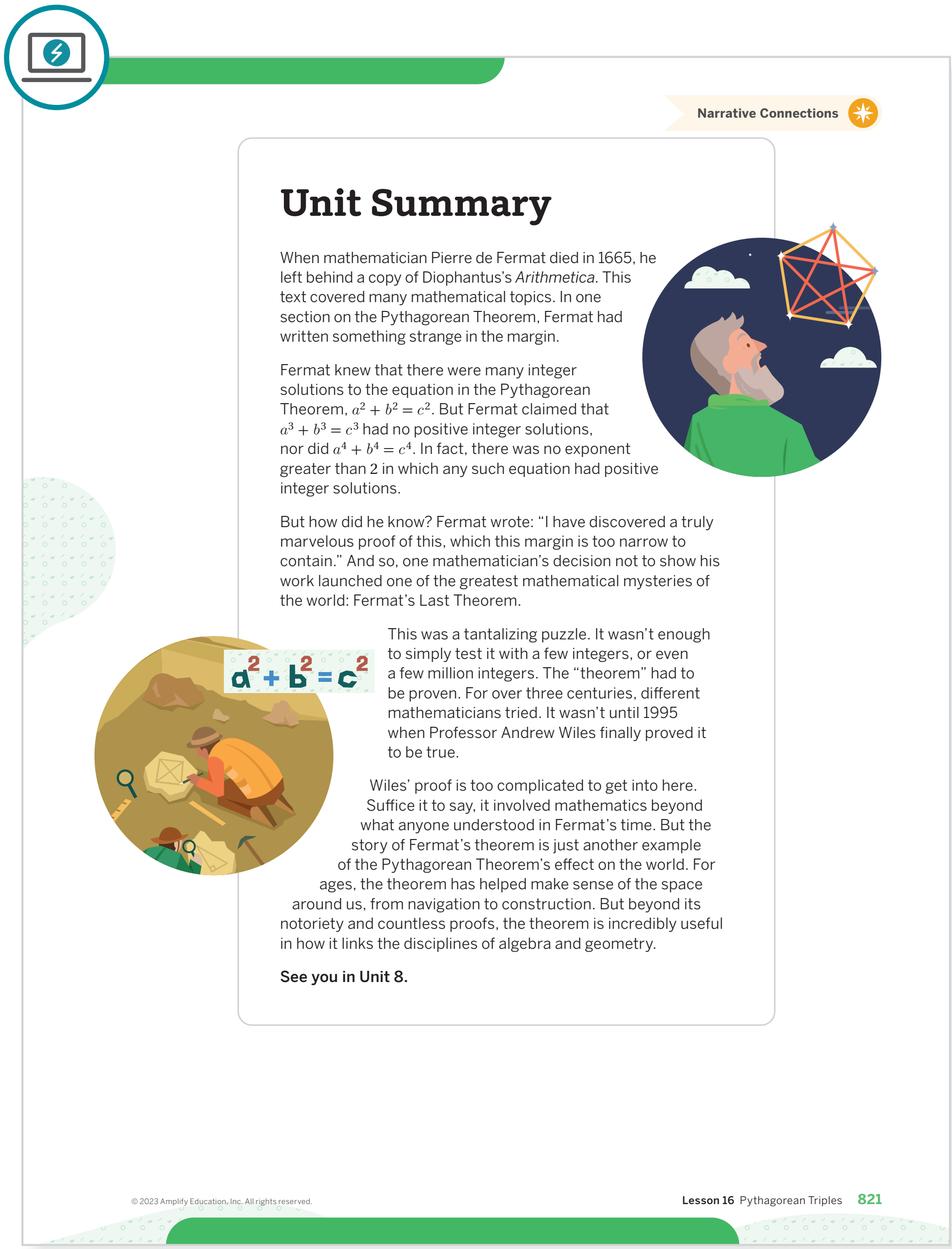
Jennifer Balakrishnan

Have students read about featured mathematician Jennifer Balakrishnan who was leading a team that solved the problem of the “cursed curve,” a special type of Diophantine equation.

Whole Class | 5 min

Unit Summary

Review and synthesize the concepts of the unit.



Narrative Connections

Unit Summary

When mathematician Pierre de Fermat died in 1665, he left behind a copy of Diophantus's *Arithmetica*. This text covered many mathematical topics. In one section on the Pythagorean Theorem, Fermat had written something strange in the margin.

Fermat knew that there were many integer solutions to the equation in the Pythagorean Theorem, $a^2 + b^2 = c^2$. But Fermat claimed that $a^3 + b^3 = c^3$ had no positive integer solutions, nor did $a^4 + b^4 = c^4$. In fact, there was no exponent greater than 2 in which any such equation had positive integer solutions.

But how did he know? Fermat wrote: "I have discovered a truly marvelous proof of this, which this margin is too narrow to contain." And so, one mathematician's decision not to show his work launched one of the greatest mathematical mysteries of the world: Fermat's Last Theorem.

This was a tantalizing puzzle. It wasn't enough to simply test it with a few integers, or even a few million integers. The "theorem" had to be proven. For over three centuries, different mathematicians tried. It wasn't until 1995 when Professor Andrew Wiles finally proved it to be true.

Wiles' proof is too complicated to get into here. Suffice it to say, it involved mathematics beyond what anyone understood in Fermat's time. But the story of Fermat's theorem is just another example of the Pythagorean Theorem's effect on the world. For ages, the theorem has helped make sense of the space around us, from navigation to construction. But beyond its notoriety and countless proofs, the theorem is incredibly useful in how it links the disciplines of algebra and geometry.

See you in Unit 8.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Formalize vocabulary: ***Pythagorean triple***

Have students share their reflections from their work in this unit.

Ask:

- "What are your biggest takeaways from this unit?"
- "What are your biggest questions you still have about this unit?"

Highlight that students will continue to explore the Pythagorean Theorem and the relationships between the side lengths of right triangles in high school.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Math Language Development

MLR2: Collect and Display

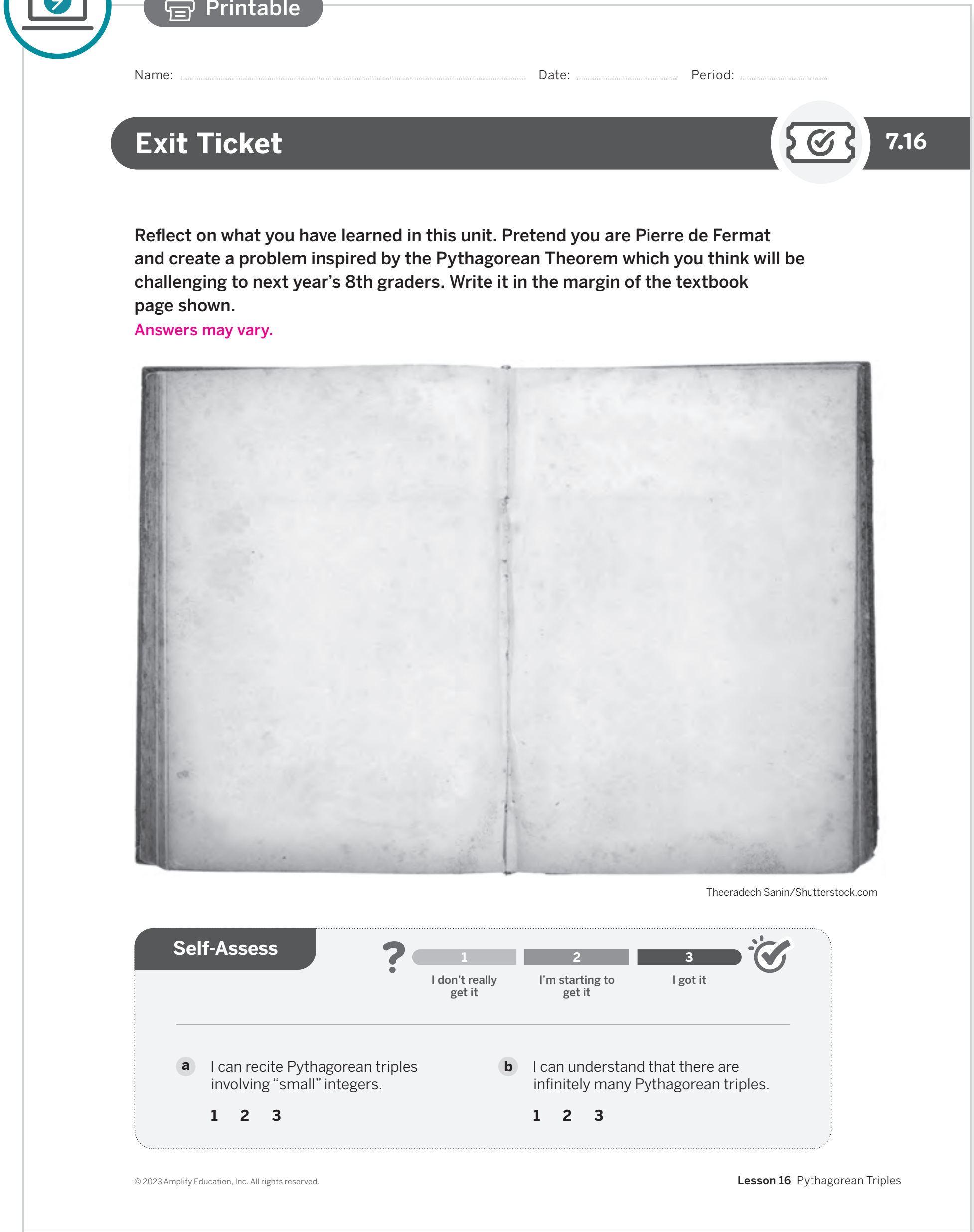
As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started for this unit. Ask them to review and reflect on any terms and phrases related to the term *Pythagorean triple* that were added to the display during the lesson.

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by reflecting on their work in this lesson and unit.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket

7.16

Reflect on what you have learned in this unit. Pretend you are Pierre de Fermat and create a problem inspired by the Pythagorean Theorem which you think will be challenging to next year's 8th graders. Write it in the margin of the textbook page shown.

Answers may vary.

Theeradech Sanin/Shutterstock.com

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can recite Pythagorean triples involving "small" integers.
1 2 3

b I can understand that there are infinitely many Pythagorean triples.
1 2 3

Lesson 16 Pythagorean Triples

Success looks like . . .

- **Language Goal:** Knowing small-integer Pythagorean triples and explaining patterns among them. **(Speaking and Listening, Writing)**
- **Goal:** Understanding there are infinitely many Pythagorean triples.

Suggested next steps

If students are unsure what to write, consider:

- Activating students' prior knowledge by having them reference the Anchor Charts for this unit.
- Encouraging students to provide a solution and consider keeping these problems to give to next year's students.

Professional Learning

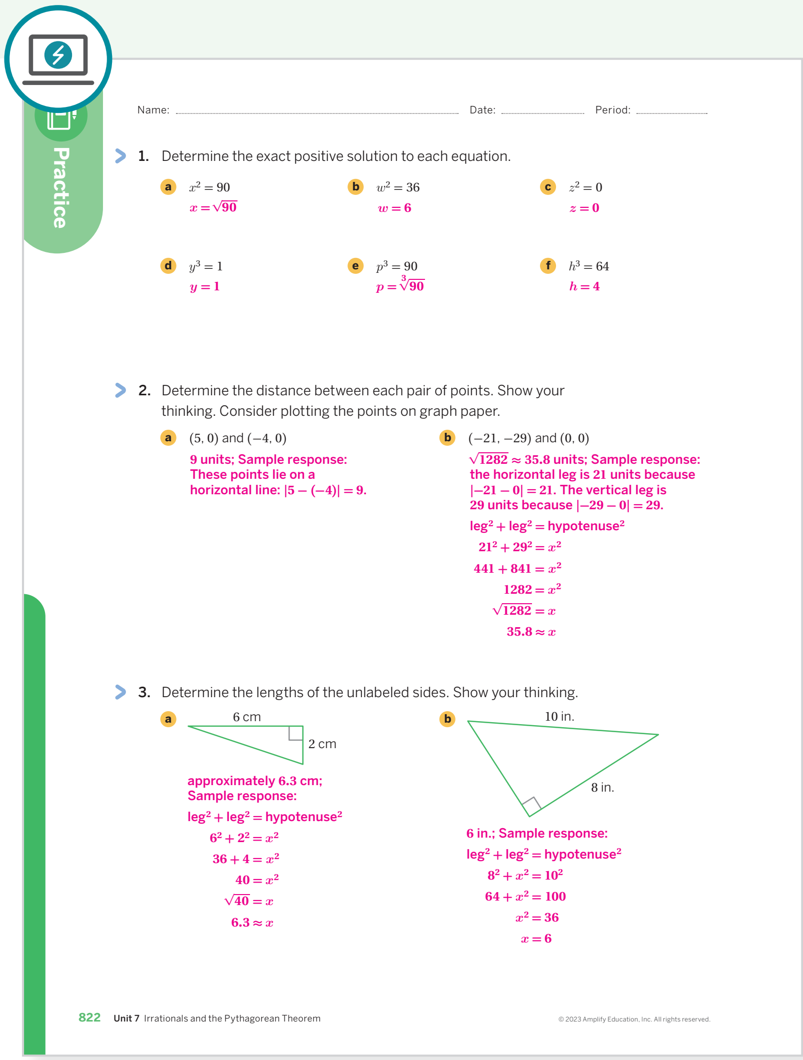
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? This lesson asked students to determine a pattern among a series of numbers. Where in your students' work today did you see or hear evidence of them doing this?
- What challenges did students encounter as they worked on Activity 2? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

Practice

1. Determine the exact positive solution to each equation.

a $x^2 = 90$
$x = \sqrt{90}$

b $w^2 = 36$
$w = 6$

c $z^2 = 0$
$z = 0$

d $y^3 = 1$
$y = 1$

e $p^3 = 90$
$p = \sqrt[3]{90}$

f $h^3 = 64$
$h = 4$

2. Determine the distance between each pair of points. Show your thinking. Consider plotting the points on graph paper.

a $(5, 0)$ and $(-4, 0)$

9 units; Sample response: These points lie on a horizontal line: $|5 - (-4)| = 9$.

b $(-21, -29)$ and $(0, 0)$

$\sqrt{1282} \approx 35.8$ units; Sample response: the horizontal leg is 21 units because $|-21 - 0| = 21$. The vertical leg is 29 units because $|-29 - 0| = 29$.

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
$21^2 + 29^2 = x^2$
$441 + 841 = x^2$
$1282 = x^2$
$\sqrt{1282} = x$
$35.8 \approx x$

3. Determine the lengths of the unlabeled sides. Show your thinking.

a

approximately 6.3 cm; Sample response:

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
$6^2 + 2^2 = x^2$
$36 + 4 = x^2$
$40 = x^2$
$\sqrt{40} = x$
$6.3 \approx x$

b

6 in.; Sample response:

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
$8^2 + x^2 = 10^2$
$64 + x^2 = 100$
$x^2 = 36$
$x = 6$

822 **Unit 7** Irrationals and the Pythagorean Theorem

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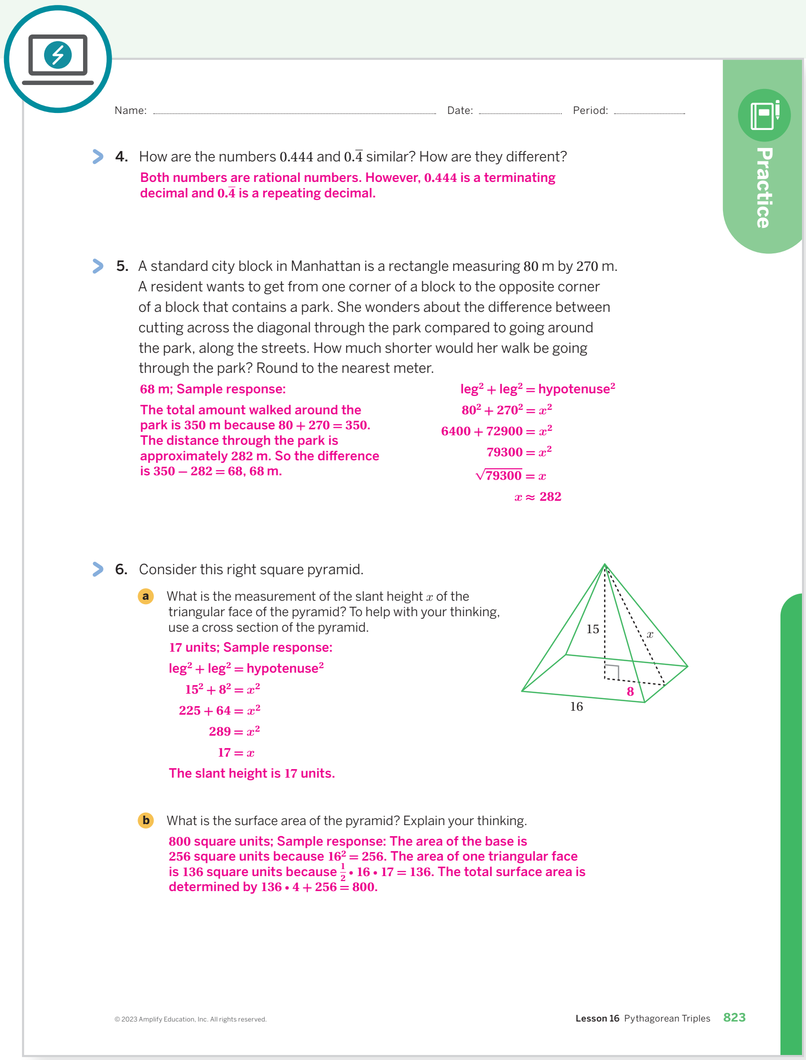

Name: .... Date: .... Period: ....

Practice

4. How are the numbers 0.444 and $0.\overline{4}$ similar? How are they different?

Both numbers are rational numbers. However, 0.444 is a terminating decimal and $0.\overline{4}$ is a repeating decimal.

5. A standard city block in Manhattan is a rectangle measuring 80 m by 270 m. A resident wants to get from one corner of a block to the opposite corner of a block that contains a park. She wonders about the difference between cutting across the diagonal through the park compared to going around the park, along the streets. How much shorter would her walk be going through the park? Round to the nearest meter.

68 m; Sample response:

The total amount walked around the park is 350 m because $80 + 270 = 350$. The distance through the park is approximately 282 m. So the difference is $350 - 282 = 68$, 68 m.

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
$80^2 + 270^2 = x^2$
$6400 + 72900 = x^2$
$79300 = x^2$
$\sqrt{79300} = x$
$x \approx 282$

6. Consider this right square pyramid.

a What is the measurement of the slant height x of the triangular face of the pyramid? To help with your thinking, use a cross section of the pyramid.

17 units; Sample response:

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
$15^2 + 8^2 = x^2$
$225 + 64 = x^2$
$289 = x^2$
$17 = x$

The slant height is 17 units.

b What is the surface area of the pyramid? Explain your thinking.

800 square units; Sample response: The area of the base is 256 square units because $16^2 = 256$. The area of one triangular face is 136 square units because $\frac{1}{2} \cdot 16 \cdot 17 = 136$. The total surface area is determined by $136 \cdot 4 + 256 = 800$.

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Lesson 16 Pythagorean Triples 823

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 7 Lesson 5	1
	2	Unit 7 Lesson 14	1
	3	Unit 7 Lesson 11	1
	4	Unit 7 Lesson 8	2
	5	Unit 7 Lesson 15	2
	6	Unit 7 Lesson 15	3

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

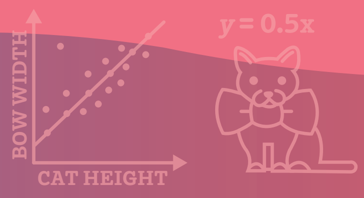
UNIT 8

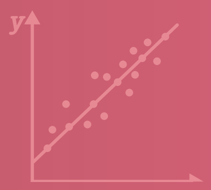
Associations in Data

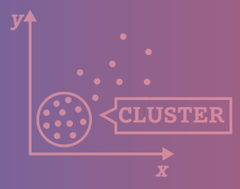
What makes a cat logo consumer friendly? Is there a relationship between eye distance and height for a species of krill? Are adults just as likely to ride a bike as kids? Did the hole in the ozone layer have an association with skin cancer rates in Australia? In this unit, students will grapple with these questions and more, as they discover new ways to represent associations in data and build their data literacy.

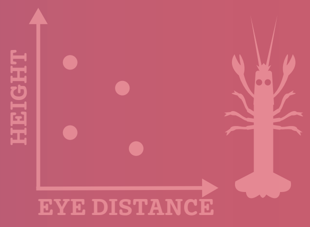
Essential Questions

- What is a scatter plot? And what can it tell you?
- How can you model data in a scatter plot? And what does that model tell you?
- What associations can you find, if any, in bivariate data?
- *(By the way, how can you use data to check the accuracy of news headlines?)*









Key Shifts in Mathematics

Focus

In this unit . . .

Students graph and analyze bivariate data on scatter plots. This analysis is descriptive at first, focusing on associations (positive vs. negative, linear vs. nonlinear, clusters, and potential outliers), and becomes increasingly quantitative as students use linear models to make predictions. Finally, students explore categorical data using two-way tables and bar charts.

Coherence

Previously . . .

In Grade 6, students represented the distribution of a single statistical variable using dot plots, histograms, and box plots. Also in Grade 6, students began exploring ratios represented as percentages, which will help them as they calculate relative frequencies to look for associations in data. Earlier in Grade 8, students studied slope and linear functions, skills they will use when creating and analyzing linear models.

Coming soon . . .

In future grades, students will continue exploring associations in data, as well as the differences between causation and correlation. They will further use linear models to make predictions. In Algebra 1, students will study the correlation coefficient of a line of fit, along with more sophisticated measures of center to describe associations and patterns in data.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

As some of the notions in this unit are subjective, conceptual understanding is extremely important. Students make connections between the overall shape of a scatter plot, the slope of a fitted line, and the trends in the data. They debate, perhaps inconclusively, about what represents an outlier and what represents a cluster. Students examine the difference between predicted and real data points. In the final lessons of the unit, students consider what associations they can claim about a set of data and consider which representation(s) best describe the story in a data set.

Procedural Fluency

Students begin the unit by developing their ability to represent data in scatter plots and tables, with ample opportunities in Practice and Additional Practice. In high school, students will learn how to draw a line of fit with precision, while in this unit they practice fitting a line informally, on paper or with the aid of technology (Lessons 4–6). By the end of the unit, students coordinate representations of the same data by creating their own bar graphs and two-way tables.

Application

The final lessons of the unit allow students to apply what they have learned about representations of data. In particular, students determine if newspaper headlines tell an accurate story about associations in data represented by scatter plots, two-way tables, and segmented bar graphs (Lesson 9).

Data and the Ozone Layer

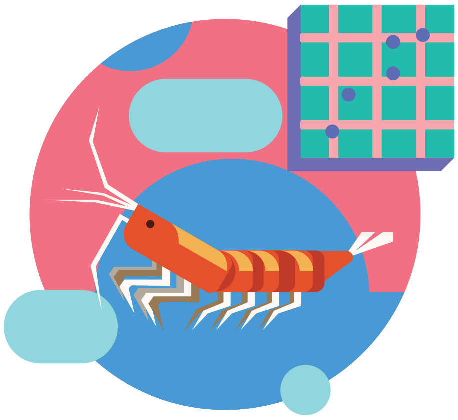
SUB-UNIT

1

Lessons 2–8

Associations in Data

In the sole Sub-Unit of this unit, students explore bivariate data. They begin by creating and interpreting ***scatter plots***, identifying patterns in the data, such as whether there is an association between a krill's eye distance and its height. They move toward describing these associations with more formal language — ***positive***, ***negative***, ***linear***, or ***nonlinear***. Students move on to fit ***linear models*** to data and use those models to make predictions. They then transition from looking at bivariate quantitative data sets to bivariate categorical data sets. They use various representations — ***two-way tables***, double bar graphs, relative frequency tables, and ***segmented bar graphs*** — to present and analyze data.



Narrative: Understanding statistics can help us study and preserve the balance of ecosystems.

Launch

Lesson 1

Creating a Scatter Plot

Students begin the unit by looking for associations between the area of the ozone hole and the number of skin cancer cases in Australia. Students are presented with tabular data, from which it can be difficult to see trends and make predictions. Finally, students create their own ***scatter plot*** and interpret the results, discovering the power of visualizing data sets in two dimensions. The story of the hole in the ozone layer will be woven into the remaining lessons in the unit, including the Montreal Protocol and its effect.

Capstone

Lesson 9

Using Data Displays to Find Associations

Students end the unit by calculating relative frequencies to determine whether there is an association between categorical data. They create ***segmented bar*** graphs to display the relative frequencies among row totals and the relative frequencies among column totals.

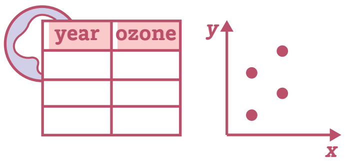
Unit at a Glance

Spoiler Alert: Data in a scatter plot that can be represented using a straight line show a linear association. Students can use this straight line as a linear model to make predictions and describe associations in the data.

Assessment

A Pre-Unit Readiness Assessment

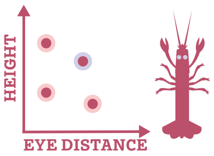
Launch Lesson



1 Creating a Scatter Plot

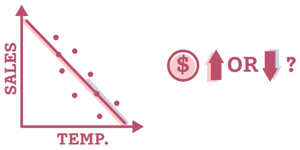
Make predictions about the hole in the ozone using real data, and then create a scatter plot and informally describe the trend.

Sub-Unit 1: Associations in Data



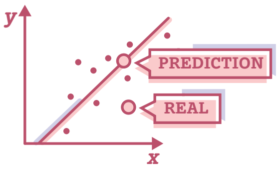
2 Interpreting Points on a Scatter Plot

Make connections between points on a scatter plot and what they represent in a real-world context.



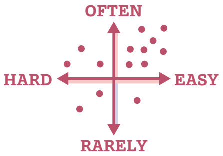
6 Interpreting Slope and y-intercept

Interpret the slope and y-intercept of a linear model in context.



7 Analyzing Bivariate Data

Describe the relationship between two variables using a line fit to data on a scatter plot, and interpret points that do and do not lie on a line fit to the data.



8 Looking for Associations

Coordinate two-way tables and double bar graphs representing the same categorical data to look for associations.

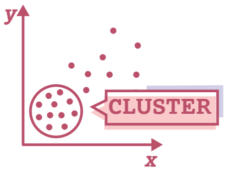
Key Concepts

Lesson 3: Scatter plots can show positive, negative, or no association. They can also show linear or nonlinear association.
Lesson 5: When applicable, using a linear model to represent data in a scatter plot helps to interpret trends and make predictions.
Lesson 9: Different representations of bivariate categorical data — two-way tables and segmented bar graphs — can show relative frequencies, which can be used to determine any associations in the data.

Pacing

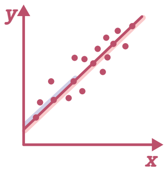
9 Lessons: 45 min each	**Full Unit:** 11 days
2 Assessments: 45 min each	● **Modified Unit:** 9 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



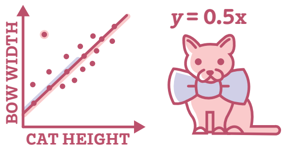
3 Observing Patterns in Scatter Plots

Categorize and describe features of data on scatter plots, including linear and nonlinear association, positive and negative association, and clustering using informal language.



4 Fitting a Line to Data

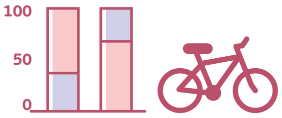
The plot thickens. Fit a straight line to best describe the trend in data represented in a scatter plot.



5 Using a Linear Model

Interpret a linear model in context and use the model to make predictions. Don't skip this!

Capstone Lesson



9 Using Data Displays to Find Associations

Calculate relative frequencies, and describe associations between variables using relative frequency tables, double bar graphs, and segmented bar graphs.

Assessment

A End-of-Unit Assessment

● Modifications to Pacing

Lessons 1–2: Use the data set about the ozone layer, modified as needed, in Lesson 1 and incorporate the learning goals from Lesson 2 to ensure students are able to interpret points on a scatter plot.

Lessons 6–7: Interpreting and using linear models is best taught over two lessons, but Lessons 6 and 7 can be combined by including Activity 1 from Lesson 6 as an example for students to interpret before making and interpreting their own linear models in a modified version of Lesson 7.

Unit Supports

Math Language Development

Lesson	New vocabulary
1	scatter plot
3	cluster linear association negative association nonlinear association positive association
5	linear model outlier
8	two-way table
9	relative frequency segmented bar graph

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
2, 4, 5, 9	**MLR1:** Stronger and Clearer Each Time
1, 3, 5	**MLR2:** Collect and Display
1, 4, 8	**MLR5:** Co-craft Questions
1, 6	**MLR6:** Three Reads
1–4, 6, 8	**MLR7:** Compare and Connect
5	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Materials
8, 9	colored pencils
3–7	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
4–7	rulers

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
3	Card Sort
1–3	Notice and Wonder
4, 7	Poll the Class
2–4, 9	Think-Pair-Share
6	Two Truths and a Lie
4	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 9

Amps

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Social & Collaborative Digital Moments

Featured Activity

Survival of the Fittest

Put on your student hat and work through **Lesson 4, Activity 1:**

Activity 1: Survival of the Fittest

Drag the blue points to fit the line to the data

Try to make your line as precise as you can.

Then describe how to get a high score on the

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities

- Create a Scatter Plot **(Lesson 1)**
- Changing Krill **(Lesson 2)**
- Measuring Chemy Cat **(Lesson 5)**
- Relative Frequencies **(Lesson 9)**
- Segmented Bar Graphs **(Lesson 9)**
- Frequency Tables and Segmented Bar Graphs **(Lesson 9)**

Unit Study

Professional Learning

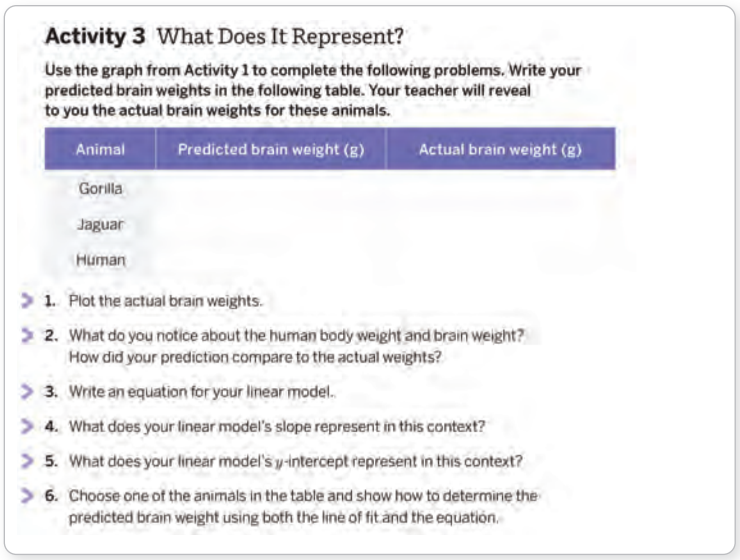
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

The sole Sub-Unit of this unit introduces the idea of analyzing a scatter plot and its line of good fit. Students are asked to examine associations and trends, including noticing for any potential outliers and clusters. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 7, Activity 3:



Activity 3 What Does It Represent?

Use the graph from Activity 1 to complete the following problems. Write your predicted brain weights in the following table. Your teacher will reveal to you the actual brain weights for these animals.

Animal	Predicted brain weight (g)	Actual brain weight (g)
Gorilla		
Jaguar		
Human		

1. Plot the actual brain weights.
2. What do you notice about the human body weight and brain weight? How did your prediction compare to the actual weights?
3. Write an equation for your linear model.
4. What does your linear model's slope represent in this context?
5. What does your linear model's y-intercept represent in this context?
6. Choose one of the animals in the table and show how to determine the predicted brain weight using both the line of fit and the equation.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

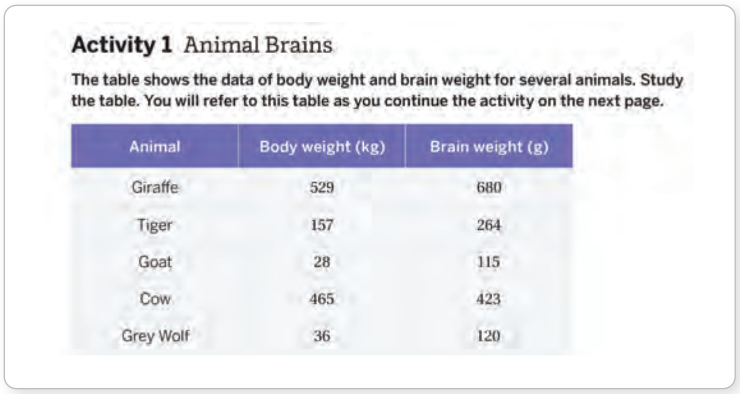
- What was it like to engage in this problem as a learner?
- How did you find the equation (Problem 3)? Did you consider or use a graphing tool?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Poll the Class

Rehearse . . .

How you'll facilitate the *Poll the Class* instructional routine in Lesson 7, Activity 1:



Activity 1 Animal Brains

The table shows the data of body weight and brain weight for several animals. Study the table. You will refer to this table as you continue the activity on the next page.

Animal	Body weight (kg)	Brain weight (g)
Giraffe	529	680
Tiger	157	264
Goat	28	115
Cow	465	423
Grey Wolf	36	120

Point to Ponder . . .

- What systems or routines will you want to have in place to facilitate the poll so that you are collecting and displaying the data efficiently and effectively?

This routine . . .

- Gets students thinking about the relationship between the two variables, animal body weight and animal brain weight.
- Provides you with a way to collect data on how your students are initially thinking about the activity.
- Establishes a low-floor entry to the activity to build engagement for all students.
- Creates an opportunity for you to call on students who you may not hear from as often.
- Increases investment as students are likely to want to find out how their response compares with responses of other students.

Anticipate . . .

- What do you think the data from the poll will show?
- How will you use the data to facilitate discourse and launch the activity?
- If you *haven't* used this routine before, do you want to share the data with students? If so, how?
- If you *have* used this routine before, how will you respond if a student wants to opt out of the poll? What does that tell you and what can you do to support that student?

Strengthening your Effective Teaching Practices

Elicit and use evidence of student thinking.

This effective teaching practice . . .

- Helps you assess student progress toward the mathematical goals and objectives of the lessons and units. By knowing where your students are at, you can help them get to where they need to be!
- Allows you to adjust your instruction, based upon student responses, so that you can support your students and extend their learning.

Points to Ponder . . .

- How and when will I use student responses from this unit's assessments to help adjust my instruction?
- Where are there moments during the lessons' activities in which I can informally elicit evidence of my students' thinking? How will I approach these moments? desciption.

Math Language Development

MLR1: Stronger and Clearer Each Time

MLR1 appears in Lessons 2, 4, 5, and 9.

- In these lessons, opportunities are provided to have students first craft an initial draft of their response to a particular problem. Students then share their responses with 2–3 partners to receive feedback and then revise or refine their original response.
- Often, specific suggestions are provided to help reviewing partners look for clarity in the responses. For example:
  - In Lesson 5, reviewers are encouraged to ask whether students used the graph or the equation to respond to the problem and why they chose the representation they did.
  - In Lesson 9, display the suggested questions so that reviewers look for whether the response included mention of relative frequencies or other mathematical language.

Point to Ponder . . .

- How can you help your students grow in both giving and receiving feedback? How will you structure your classroom culture so that there is an expected norm in which your students feel supported, not criticized?

Unit Assessments

- Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional support?
- How might you support your students with representing and interpreting bivariate data throughout the unit? Do you think your students will generally:
  - Understand the difference between quantitative data and categorical data?
  - Struggle with using linear models to represent bivariate quantitative data in an attempt to better understand real-world problems?

Differentiated Support

Accessibility: *Optimize Access to Technology*

Opportunities that support the use of technology (through the Amps slides or other forms of technology), appear in Lessons 1–9.

- In Lessons 2, students can manipulate a point on the graph to see how its placement changes a krill's eye distance and height.
- In Lesson 4, students can manipulate the graph of a line and observe how a score meter changes in real time, based on the positioning of the line and how well it fits the data.
- In Lesson 8, students can see their classmates' data added to a collaborative scatter plot as the class response to the questions.
- In Lesson 9, students can adjust the heights of bars in a segmented bar graph to match two-way tables and relative frequency tables

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology to deepen student understanding?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

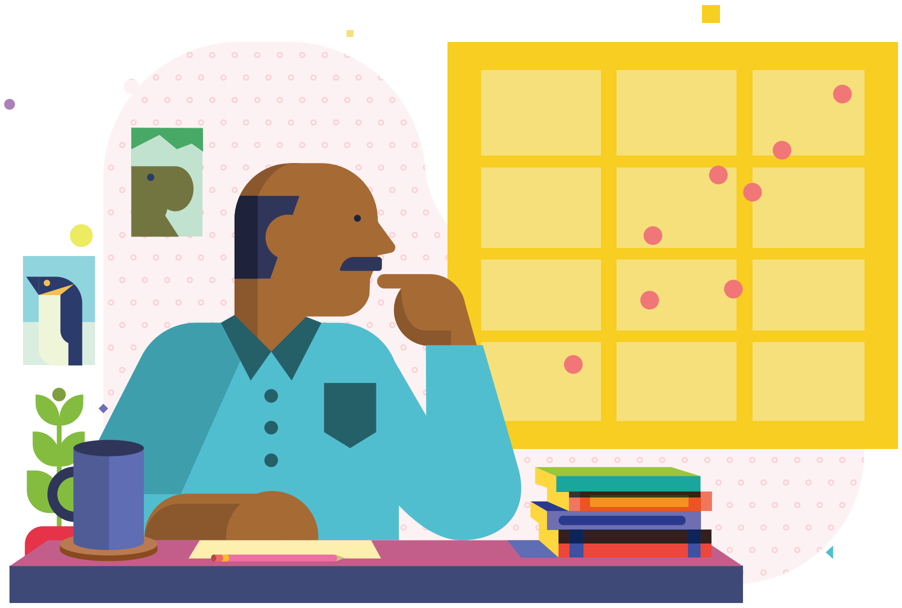
- In this unit, pay particular attention to supporting students in building their social awareness and self-management.

Points to Ponder . . .

- Are students able to anticipate how their interpretations of data will be perceived by others, preparing to justify their conclusions through clear communication?
- Do students reflect on their results within the context from which the data was taken to see if they make sense or if there is a way to improve the models used?

Creating a Scatter Plot

Let's find ways to show patterns in data.



Focus

Goals

1. Understand that a scatter plot represents data with two variables and does not represent a function.
2. **Language Goal:** Create a scatter plot from a table of data, and informally describe the trend of the data. **(Speaking and Listening, Writing)**
3. **Language Goal:** Create a table of collected data, and explain how to organize the data. **(Speaking and Listening)**

Coherence

Today

Students see the association between the area of the ozone hole and the number of skin cancer cases in Australia. They read data from a table to make predictions, and then create a scatter plot to visualize patterns in the data to make connections between two quantities.

Previously

In Grade 6, students represented the distribution of a single statistical variable using dot plots, histograms, and box plots.

Coming Soon

In Lesson 3, students will classify associations in data (and discover that the area of the ozone hole has started showing signs of recovery). In Lesson 7, students will learn about the reasons for the recovery.

Rigor

- Students build **conceptual understanding** of how scatter plots can show patterns in data.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Independent	Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

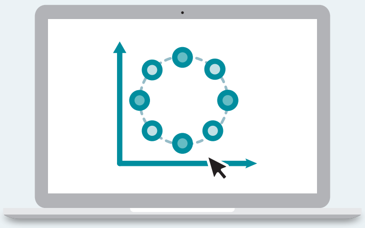
New words

- *scatter plot*

Amps | **Featured Activity**

Activity 2
Interactive Scatter Plots

Students create a scale and plot points on a graph. You can check all students' answers at once by overlaying student screens, checking for accuracy and addressing any concerns immediately.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overwhelmed when faced with quantitative data that will be represented abstractly by a scatter plot in Activity 1. Encourage them to analyze the situation, finding a simpler display of the data that helps them interpret the data more readily.

Modifications to Pacing

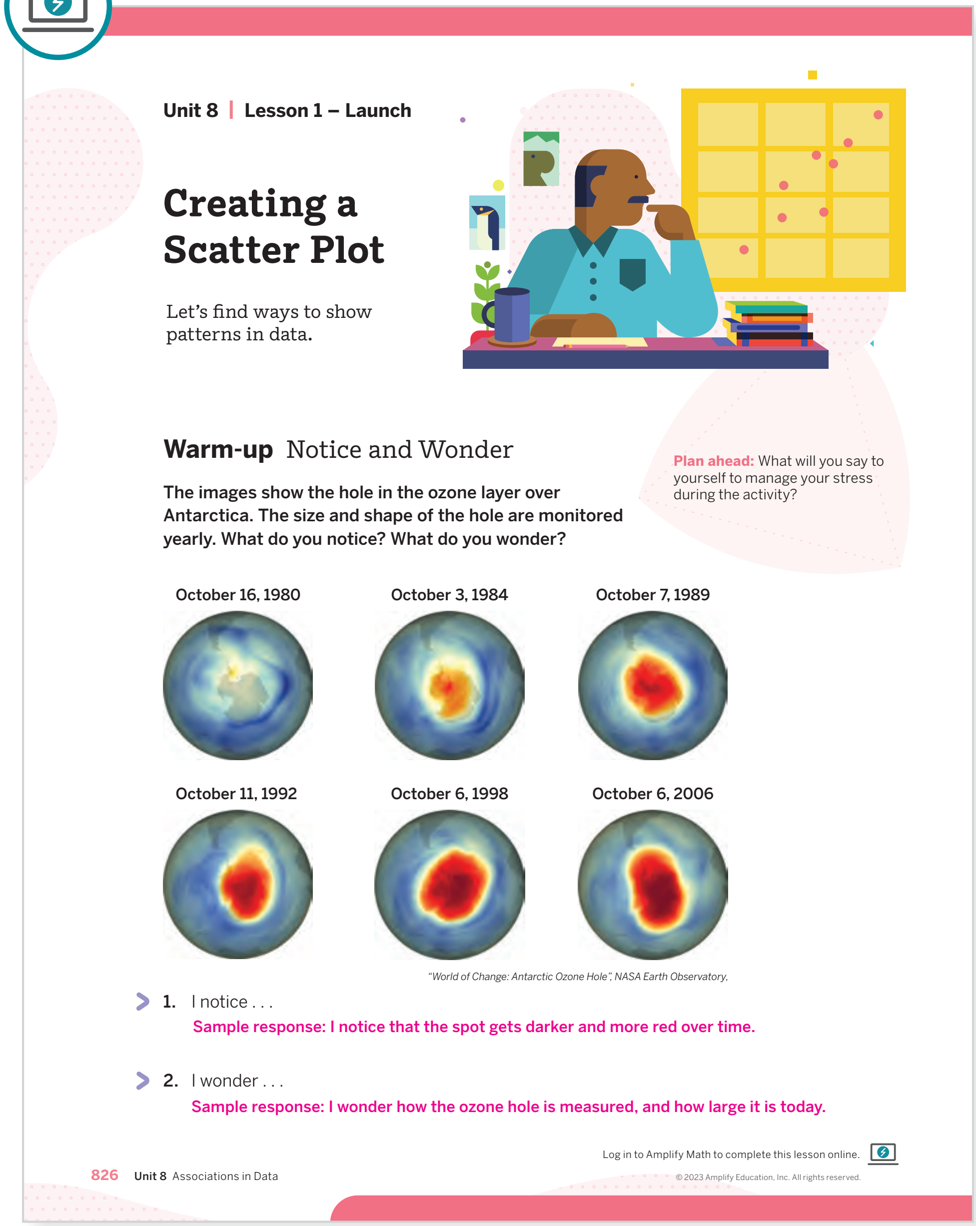
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 1 part b may be omitted.
- In **Activity 3**, Problems 3 and 4 may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students study infographics showing changes in the ozone hole to discover that the ozone hole increased from 1980 to 2006.



Unit 8 | **Lesson 1 – Launch**

Creating a Scatter Plot

Let's find ways to show patterns in data.

Warm-up Notice and Wonder

Plan ahead: What will you say to yourself to manage your stress during the activity?

The images show the hole in the ozone layer over Antarctica. The size and shape of the hole are monitored yearly. What do you notice? What do you wonder?

October 16, 1980 | October 3, 1984 | October 7, 1989

October 11, 1992 | October 6, 1998 | October 6, 2006

"World of Change: Antarctic Ozone Hole", NASA Earth Observatory,

1. I notice . . .

   Sample response: I notice that the spot gets darker and more red over time.

2. I wonder . . .

   Sample response: I wonder how the ozone hole is measured, and how large it is today.

Log in to Amplify Math to complete this lesson online.

826 **Unit 8** Associations in Data

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1 Launch

Conduct the *Notice and Wonder* routine. The Warm-up should be used to introduce the ozone layer. Say, "Ozone is a gas found in the atmosphere. It protects the Earth from ultraviolet radiation. Here, we see the area of the ozone hole in different years."

Note: In this lesson, students will use scatter plots to determine that, as the ozone hole area increases, the number of skin cancer cases also increases. Embrace student's curiosity about the ozone layer, but do not address the signs of recovery as they will discuss this in more detail in Lessons 3 and 7.

In Lesson 3, students will learn about associations in scatter plots and reexamine ozone hole data to find that there is a recovery in the ozone hole area after 1990. In Lesson 7, students learn about the Montreal Protocol, an international treaty introduced in 1987, that was planned to protect the ozone layer.

2 Monitor

Help students get started by asking about the changes they see in the images.

Look for points of confusion:

- **Noticing changes in the image, but not noticing the changes in time.** Have students read the date above each image. Then ask them how the time is changing and how it might relate to the red spot.

3 Connect

Have students share what they noticed and wondered. Record their responses for all to see.

Highlight the color changes. Darker red represents lower levels of ozone. The ozone hole area is increasing over time.

Ask, "What do you think makes the area of the ozone hole change?" Sample response: the amount of pollution made by humans.

Math Language Development

MLR5: Co-craft Questions

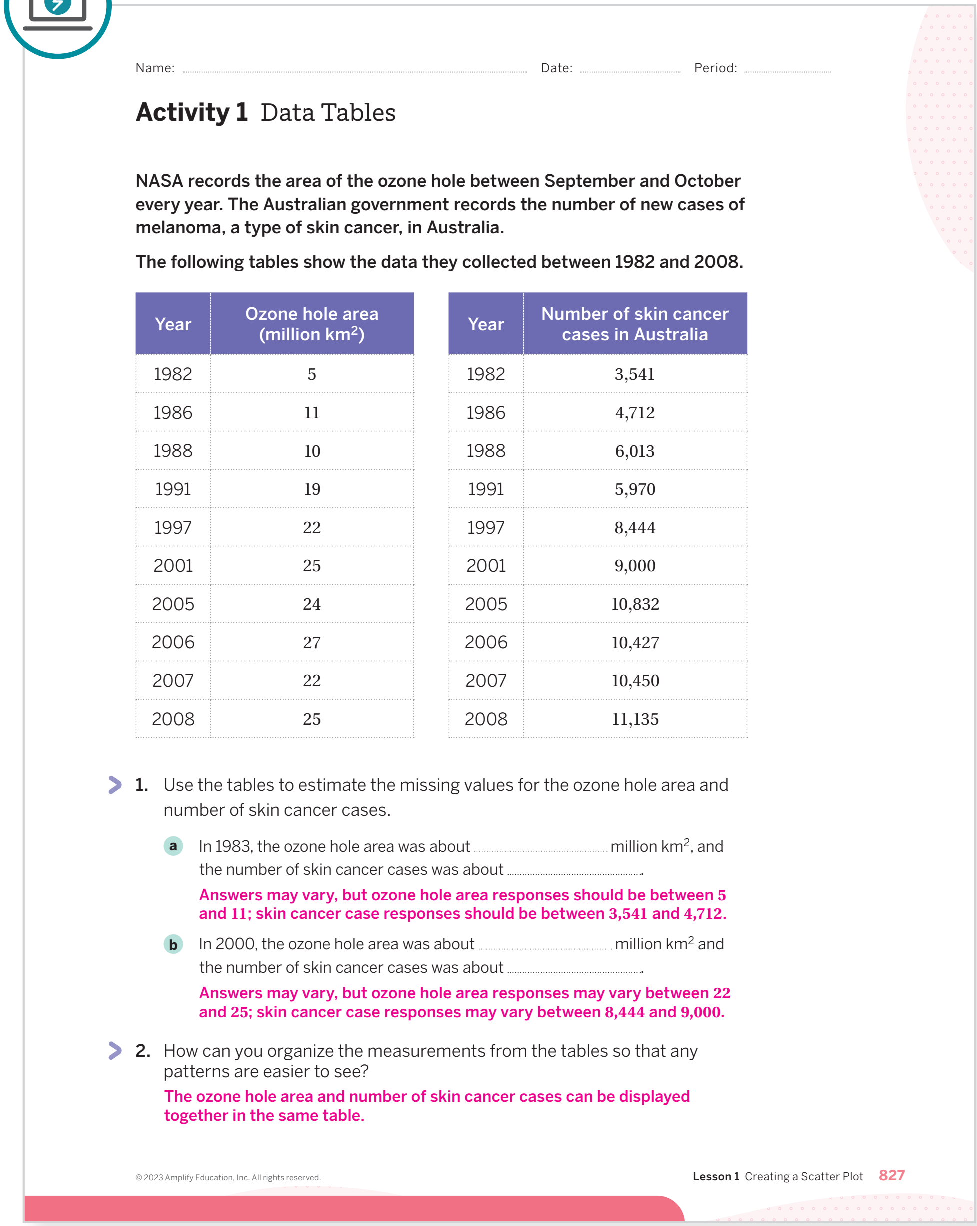
After students respond to Problems 1 and 2, have them meet with a partner to share their responses. Then ask partners to decide on 2–3 questions they could ask about the images and record them.

Clarify the term *ozone* and explain to students they will be studying the ozone layer throughout this unit.

Pairs | 10 min

Activity 1 Data Tables

Students analyze data, presented in tables, about the ozone hole area and number of skin cancer cases in Australia to look for better ways to organize data.



Name: ________ Date: ________ Period: ________

Activity 1 Data Tables

NASA records the area of the ozone hole between September and October every year. The Australian government records the number of new cases of melanoma, a type of skin cancer, in Australia.

The following tables show the data they collected between 1982 and 2008.

Year	Ozone hole area (million km^2)
1982	5
1986	11
1988	10
1991	19
1997	22
2001	25
2005	24
2006	27
2007	22
2008	25

Year	Number of skin cancer cases in Australia
1982	3,541
1986	4,712
1988	6,013
1991	5,970
1997	8,444
2001	9,000
2005	10,832
2006	10,427
2007	10,450
2008	11,135

1. Use the tables to estimate the missing values for the ozone hole area and number of skin cancer cases.

   a. In 1983, the ozone hole area was about ________ million km^2, and the number of skin cancer cases was about ________.

   Answers may vary, but ozone hole area responses should be between 5 and 11; skin cancer case responses should be between 3,541 and 4,712.

   b. In 2000, the ozone hole area was about ________ million km^2 and the number of skin cancer cases was about ________.

   Answers may vary, but ozone hole area responses may vary between 22 and 25; skin cancer case responses may vary between 8,444 and 9,000.

2. How can you organize the measurements from the tables so that any patterns are easier to see?

   The ozone hole area and number of skin cancer cases can be displayed together in the same table.

Lesson 1 Creating a Scatter Plot 827

1 Launch

Read the prompt aloud with students and answer questions they may have about the context.

2 Monitor

Help students get started by asking, "In 1983, do you think the ozone hole area was closer to 5 or 11 million square kilometers? How do you know?"

Look for productive strategies:

- Writing an estimate that is any number between adjacent data. Ask students to make a more precise estimate by comparing which year the question is closer to, before writing an estimate for the ozone hole area or the number of skin cancer cases.

3 Connect

Have students share how they used data in the table to make a prediction.

Highlight that the ozone hole area and the number of skin cancer cases are from the same years. Since we are looking at the same years, we can see how the area and the number of cases might be related.

Ask, "Did anyone find it challenging to make connections based on the table alone?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students draw a circle around the rows in the tables that correspond with the years 1983 and 2000 to help them complete Problem 1.

Extension: *Math Enrichment*

Ask students to make a prediction about the ozone hole area in the current year. Then have them use the internet, or another source, to research the area of the ozone hole over Antarctica for the current year. Consider providing this value to students after they have made their prediction. For example, the area of the ozone hole on September 20, 2020 was about 24.8 million square kilometers.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the two scenarios.

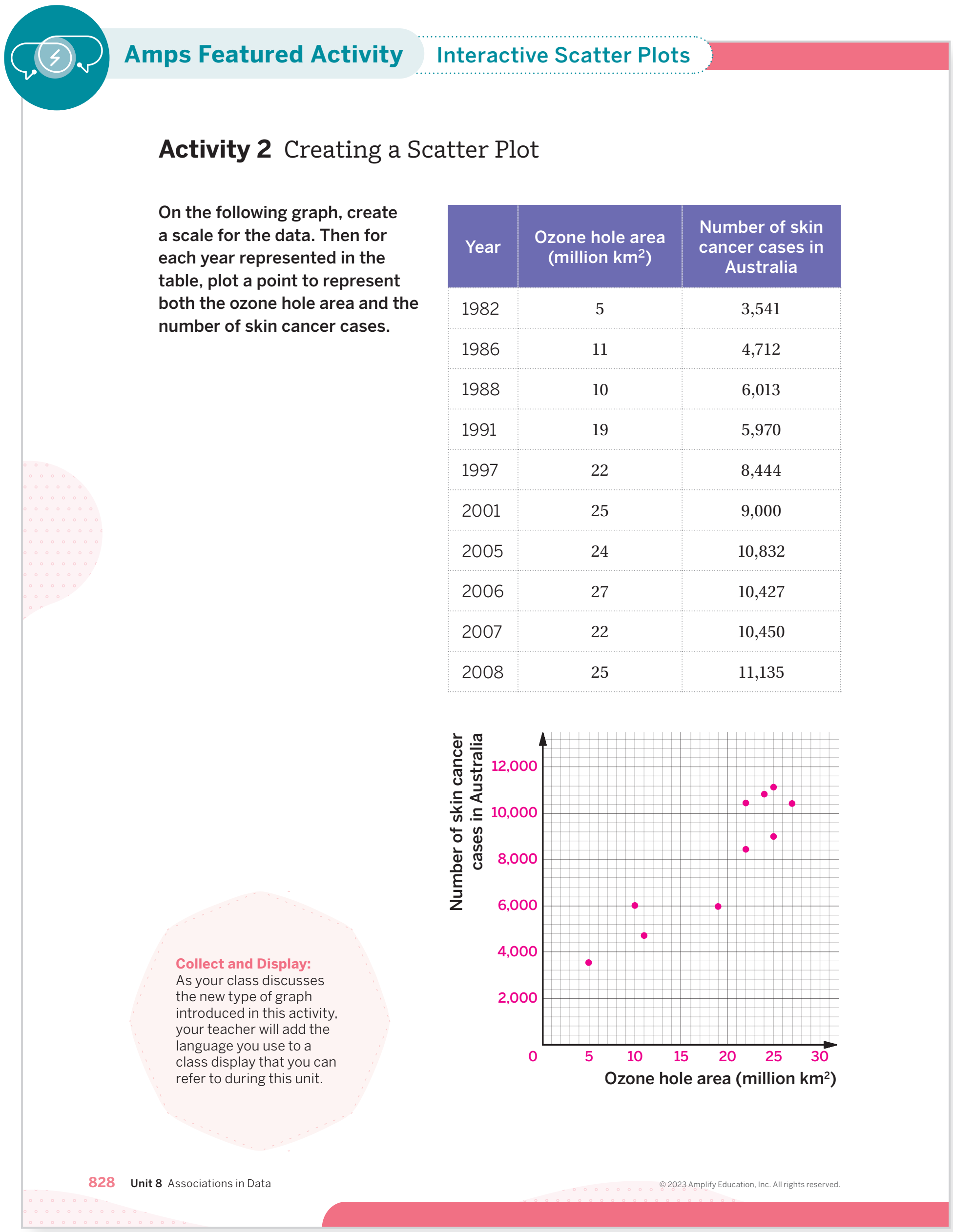
- **Read 1:** Students should understand that they will be looking at data that gives the area of the ozone hole and the number of skin cancer cases recorded every year.
- **Read 2:** Ask students to name or highlight given quantities in the table, such as the fact that both tables provide data for the same years.
- **Read 3:** Ask students to preview Problem 2 and brainstorm strategies for other ways to organize the data.

Independent | 10 min

Activity 2 Creating a Scatter Plot

Students plot data from a table and create a scatter plot to explore a new way of representing data.

Amps Featured Activity Interactive Scatter Plots



Activity 2 Creating a Scatter Plot

On the following graph, create a scale for the data. Then for each year represented in the table, plot a point to represent both the ozone hole area and the number of skin cancer cases.

Year	Ozone hole area (million km^2)	Number of skin cancer cases in Australia
1982	5	3,541
1986	11	4,712
1988	10	6,013
1991	19	5,970
1997	22	8,444
2001	25	9,000
2005	24	10,832
2006	27	10,427
2007	22	10,450
2008	25	11,135

Collect and Display: As your class discusses the new type of graph introduced in this activity, your teacher will add the language you use to a class display that you can refer to during this unit.

1 Launch

Tell students they will be plotting points using the data from Activity 1 to look for more patterns.

2 Monitor

Help students get started by explaining how to plot a point with the coordinates $(5, 3541)$.

Look for points of confusion:

- **Not knowing how to create scales for the axes.** Ask students to identify the range of each data set.
- **Reversing the coordinates, and plotting the points as (number of skin cancer cases, ozone hole area).** Write x next to the ozone hole area and y next to the number of skin cancer cases in the table and color code the table titles to the axes labels.

3 Connect

Display student scatter plots representing the same information as the matching table.

Highlight that this graph is different from the ones they have seen so far this year. There are several points scattered on the graph, and it does not represent a function. For example, points $(22, 8444)$ and $(22, 10450)$ have the same x-coordinate but different y-coordinates.

Define the term ***scatter plot*** as a graphical representation of data. A scatter plot is created when two numerical variables are graphed by using one variable as the x-coordinate and the other as the y-coordinate. Data pairs are represented as plotted points.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use the sketch feature to create a scale and plot points on a graph. You can check all students' responses at once by overlaying student screens, checking for accuracy and addressing any concerns immediately.

Accessibility: *Guide Processing and Visualization*

Demonstrate how to plot the first two ordered pairs from the table and have students plot the remaining pairs.

Math Language Development

MLR2: Collect and Display

During the Connect, as you highlight how the graph is different from other graphs that students have examined this year, collect and display key features of the graph, such as the fact that it is not a function. Add the term *scatter plot* to the display along with its definition. Continue to add to the display throughout the unit. Invite students to add to and use language from the display during class discussions.

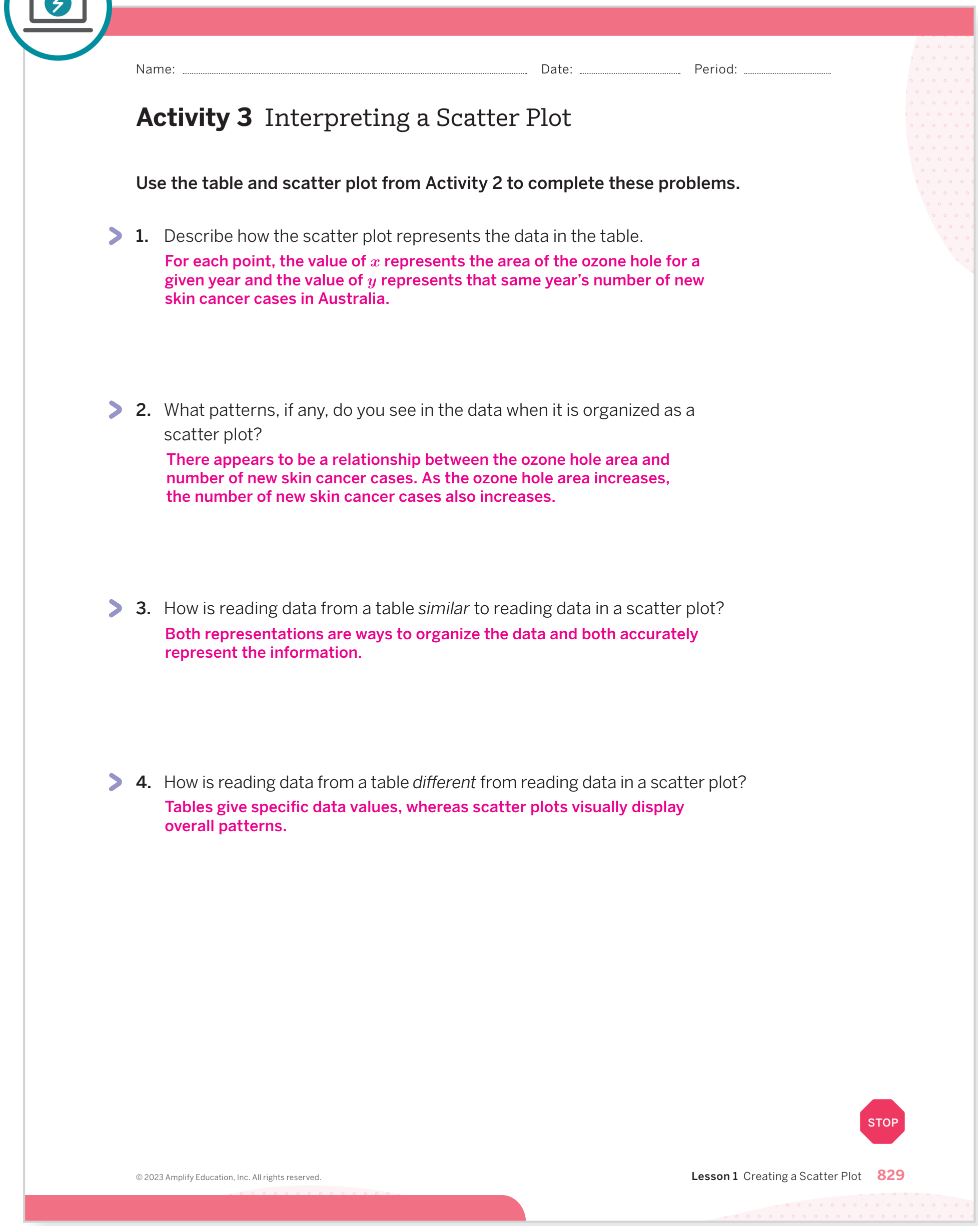
English Learners

Add a quick visual sketch of an example of a scatter plot to the class display.

Pairs | 10 min

Activity 3 Interpreting a Scatter Plot

Students look for patterns in the data that might not have been visible in the table to informally recognize the association between the size of the ozone hole area and the number of cancer cases in Australia.



Name: ______ Date: ______ Period: ______

Activity 3 Interpreting a Scatter Plot

Use the table and scatter plot from Activity 2 to complete these problems.

1. Describe how the scatter plot represents the data in the table.
   For each point, the value of x represents the area of the ozone hole for a given year and the value of y represents that same year's number of new skin cancer cases in Australia.
2. What patterns, if any, do you see in the data when it is organized as a scatter plot?
   There appears to be a relationship between the ozone hole area and number of new skin cancer cases. As the ozone hole area increases, the number of new skin cancer cases also increases.
3. How is reading data from a table *similar* to reading data in a scatter plot?
   Both representations are ways to organize the data and both accurately represent the information.
4. How is reading data from a table *different* from reading data in a scatter plot?
   Tables give specific data values, whereas scatter plots visually display overall patterns.

STOP

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Lesson 1 Creating a Scatter Plot 829

1 Launch

Ask students to look at the scatter plot from Activity 2 to complete these problems.

2 Monitor

Help students get started by asking, "How does the scatter plot help organize data differently than the table does?" It creates a visual of the overall patterns.

Look for points of confusion:

- **Not recognizing any patterns.** Ask students what happens to the number of skin cancer cases as the ozone hole area increases. In Lesson 3, students will learn about associations more formally.

3 Connect

Display a scatter plot completed by a student from Activity 2.

Have students share the advantages of representing data using a scatter plot.

Ask, "Based on the data, do you think there is an association between ozone hole area and number of skin cancer cases?" Yes, they appear to increase at the same time.

Highlight that scatter plots help students to investigate possible associations between two attributes. Based on the data, as the ozone hole area increases, the number of skin cancer cases also increases. Moreover, scientists found that the increase in ozone hole area increases the UV rays that actually cause skin cancer.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to verbally discuss their responses to each problem and record notes that will prepare them to share their responses with the class during the Connect discussion.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share the advantages and disadvantages to representing data using a scatter plot, help them make connections between the table and the scatter plot by asking:

- "What patterns did you see in the table?"
- "How do these patterns appear in the scatter plot?"
- "If you wanted to see overall patterns and trends in the data, which representation would you choose to use? Why?"
- "If you wanted to determine a specific data value, which representation would you choose to use? Why?"

This will help students reason about the ways to create a scatter plot from a table, and to identify patterns using both representations.

Whole Class | 5 min

Summary Data and the Ozone Layer

Review and synthesize how scatter plots can be used to organize data relating two variables.

Narrative Connections

Unit 8 Associations in Data

Data and the Ozone Layer

It's easy to run your eyes down a table of numbers or a bunch of graphs and not feel very much. But numbers give us direction. They can tell us stories about where we are, and where we still need to go.

Here, we'll be looking at one such story — a story, told through data, about how our actions matter.

It is a story with billions of characters — heroes and villains, winners and losers. It's a race against time, and in the balance, the fate of an entire planet.

This is the story of our ozone layer. This dense shield of gas in the upper reaches of the stratosphere that protects us from the sun's radiation. It is a story of how that shield weakened and cracked, and what it took to put it back together.

In these next lessons, we'll tell this story through representations like scatter plots. These graphs show the values of two different variables on a coordinate plane. We'll use them to see how the different pieces of that story fit together, and what we had to do to pull our planet from the brink.

Welcome to Unit 8.

830 **Unit 8** Associations in Data

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Have students share what a scatter plot is in their own words, and how interpreting data on a scatter plot compares to interpreting data from a table.

Ask, "How is reading data from a scatter plot different from reading data from a table?"

Formalize vocabulary: ***scatter plot***

Highlight that a scatter plot helps students visualize data and identify any overall trends in data.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "What is a scatter plot? And what can it tell you?"

Math Language Development

MLR2: Collect and Display

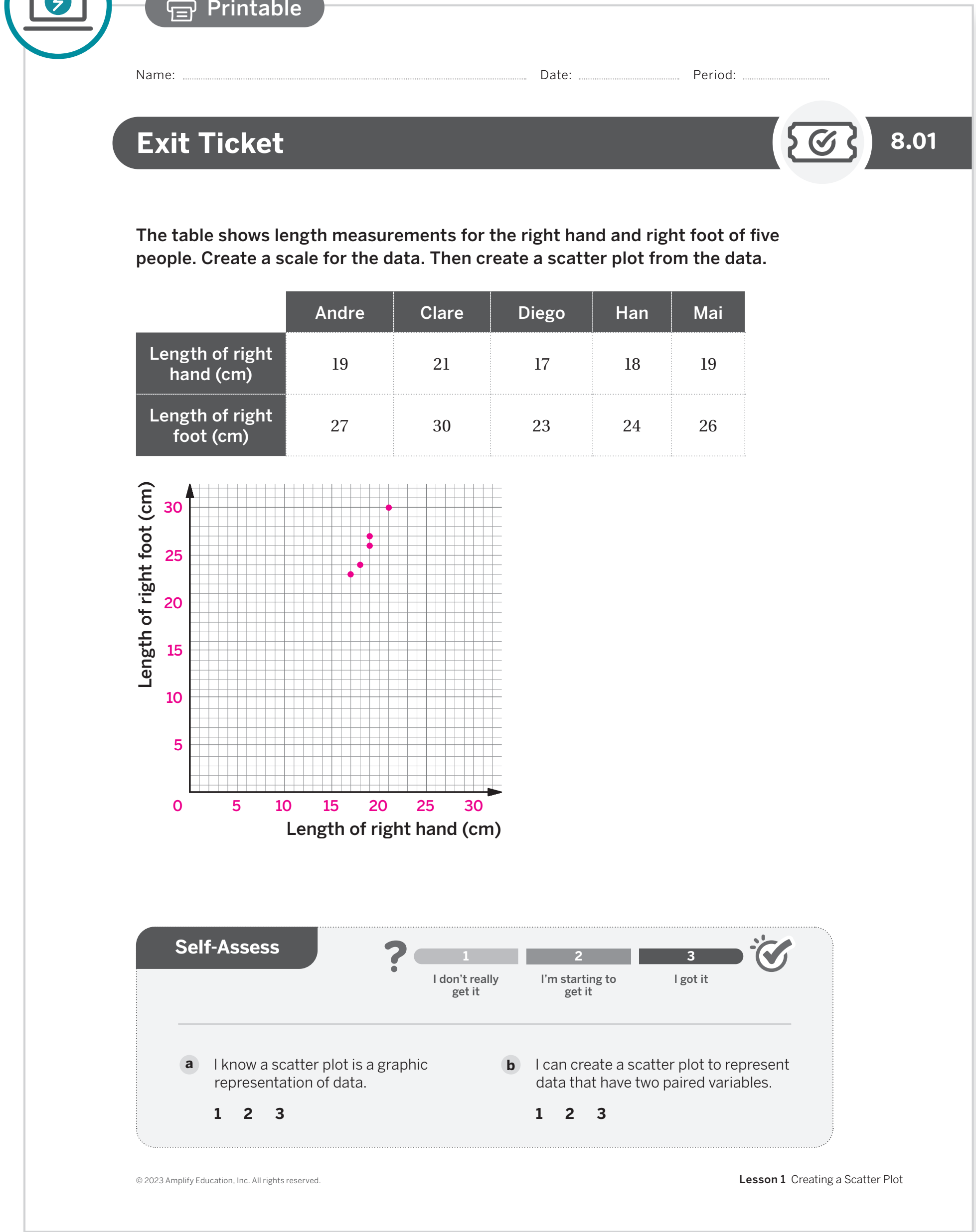
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on the terms and phrases related to the term *scatter plot* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by creating a scatter plot, given a table.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 8.01

The table shows length measurements for the right hand and right foot of five people. Create a scale for the data. Then create a scatter plot from the data.

	Andre	Clare	Diego	Han	Mai
Length of right hand (cm)	19	21	17	18	19
Length of right foot (cm)	27	30	23	24	26

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I know a scatter plot is a graphic representation of data.
1 2 3

b I can create a scatter plot to represent data that have two paired variables.
1 2 3

Success looks like . . .

- **Goal:** Comprehending that a scatter plot represents data with two variables and does not represent a function.
- **Language Goal:** Creating a scatter plot from a table of data, and describing the trend of the data. **(Speaking and Listening, Writing)**
- **Language Goal:** Creating a table of collected data, and explaining how to organize the data. **(Speaking and Listening)**

Suggested next steps

If students write a scale that is not appropriate for the data set, consider:

- Reviewing strategies to create a scale from Activity 2.
- Assigning Practice Problem 1.

If students reverse the coordinates, plotting the data as (*right hand length*, *right foot length*), consider:

- Labeling the table and graph with x and y next to the corresponding variables.
- Assigning Practice Problem 1.

Professional Learning

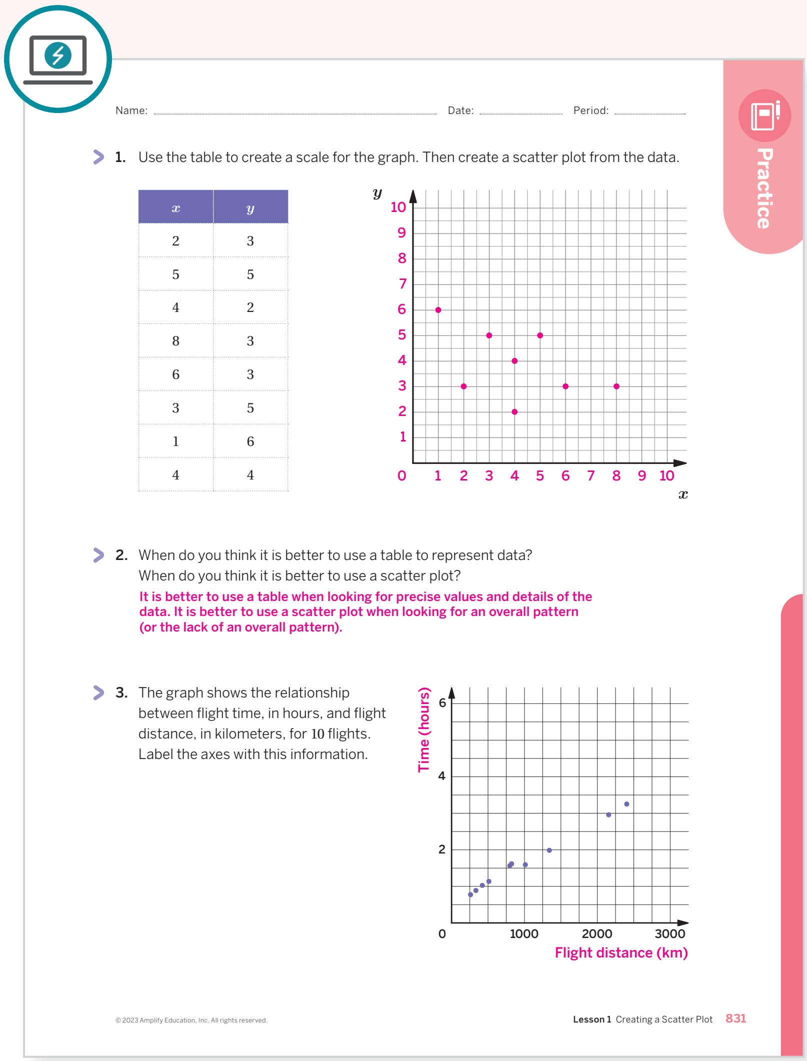
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to create a scatter plot and informally describe the trend of the data. How well did students accomplish this? What did you specifically do to help students accomplish it? What might you change the next time you teach this lesson?
- Which students' ideas were you able to highlight during Activity 3?

Practice

Independent



Name: Date: Period:

1. Use the table to create a scale for the graph. Then create a scatter plot from the data.

x	y
2	3
5	5
4	2
8	3
6	3
3	5
1	6
4	4

2. When do you think it is better to use a table to represent data? When do you think it is better to use a scatter plot?

It is better to use a table when looking for precise values and details of the data. It is better to use a scatter plot when looking for an overall pattern (or the lack of an overall pattern).

3. The graph shows the relationship between flight time, in hours, and flight distance, in kilometers, for 10 flights. Label the axes with this information.

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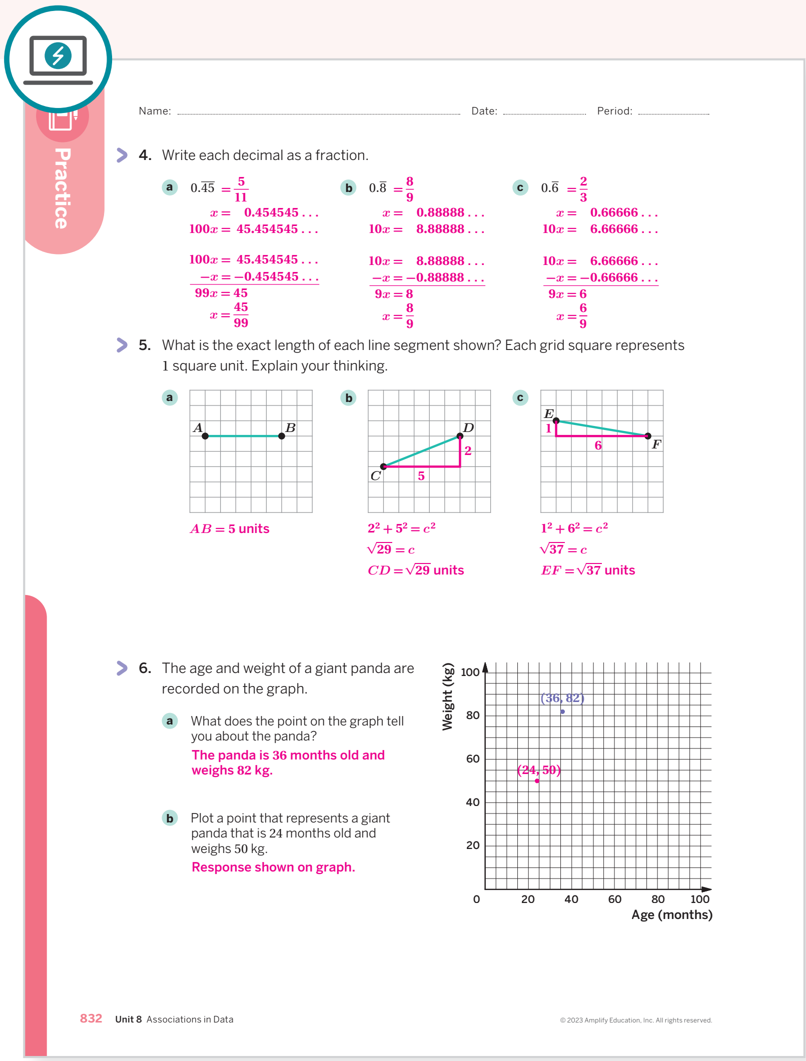
Lesson 1 Creating a Scatter Plot 831



Name: Date: Period:

4. Write each decimal as a fraction.

a. $0.\overline{45} = \frac{5}{11}$

$$\begin{aligned} x &= 0.454545\ldots \\ 100x &= 45.454545\ldots \\ \\ 100x &= 45.454545\ldots \\ -x &= -0.454545\ldots \\ \hline 99x &= 45 \\ x &= \frac{45}{99} \end{aligned}$$

b. $0.\overline{8} = \frac{8}{9}$

$$\begin{aligned} x &= 0.88888\ldots \\ 10x &= 8.88888\ldots \\ \\ 10x &= 8.88888\ldots \\ -x &= -0.88888\ldots \\ \hline 9x &= 8 \\ x &= \frac{8}{9} \end{aligned}$$

c. $0.\overline{6} = \frac{2}{3}$

$$\begin{aligned} x &= 0.66666\ldots \\ 10x &= 6.66666\ldots \\ \\ 10x &= 6.66666\ldots \\ -x &= -0.66666\ldots \\ \hline 9x &= 6 \\ x &= \frac{6}{9} \end{aligned}$$

5. What is the exact length of each line segment shown? Each grid square represents 1 square unit. Explain your thinking.

a. $AB = 5$ units

b. $2^2 + 5^2 = c^2$
$\sqrt{29} = c$
$CD = \sqrt{29}$ units

c. $1^2 + 6^2 = c^2$
$\sqrt{37} = c$
$EF = \sqrt{37}$ units

6. The age and weight of a giant panda are recorded on the graph.

a. What does the point on the graph tell you about the panda?
The panda is 36 months old and weighs 82 kg.

b. Plot a point that represents a giant panda that is 24 months old and weighs 50 kg.
Response shown on graph.

832 Unit 8 Associations in Data

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 3	3
	3	Activity 1	1
Spiral	4	Unit 7 Lesson 8	1
	5	Unit 7 Lesson 13	2
Formative ⚡	6	Unit 8 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Sub-Unit 1

Associations in Data

Whole Class

In the sole Sub-Unit of this unit, students will build their data literacy skills and explore bivariate data as they create and interpret scatter plots, fit linear models to data, and study two-way tables.

SUB-UNIT

1 Associations in Data

Narrative Connections

Who is the biggest mover and shaker in the Antarctic Ocean?

The humble krill!

They may be just a few inches long, but these pint-sized crustaceans play a gigantic role in the ecosystem.

Before we get into why they're so important, let's talk about another life form: phytoplankton — tiny, single-celled plants that float along the ocean's surface. Like all plants, they absorb sunlight and convert it to energy. They're also really good at reproducing. While you would need a microscope to see an individual phytoplankton, phytoplankton "blooms" can get so large that they can be seen from space.

Their ability to convert sunlight into energy and reproduce quickly should make them an ideal food source for many animals. The problem is that they're too small for most animals to eat. And that's where the krill comes in.

Krill are small enough to eat plankton, but big enough to make a worthwhile meal for fish, whales, seals, squid, penguins, and birds. In fact, they are *the* main source of food in the Antarctic Ocean, making them a vital part of the food chain. Krill are so important that they are considered a "keystone species," providing support and balance to the entire Antarctic ecosystem.

But what happens when something comes along and disrupts that balance? Mathematicians and scientists around the world use data and statistics to study ecosystems. They look for patterns and any associations in the data which can then be used to make predictions as well as generate ideas for ways to restore balance.

Narrative Connections

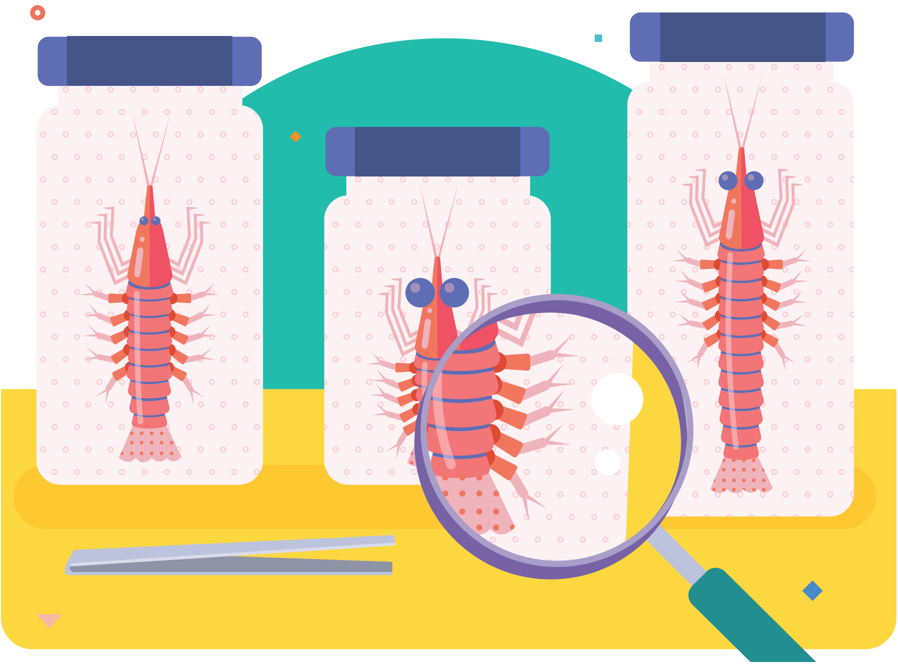
Read the narrative aloud as a class or have students read it individually. Students continue to explore how math can be used to study the balance of ecosystems in the following places:

- **Lesson 1, Activities 1–3:** Matching Krill, Adding a Point, What's the Point?
- **Lesson 6, Activity 3:** Two Truths and a Lie
- **Lesson 7, Warm-up:** Making a Prediction
- **Lesson 8, Activity 3:** Other Forms of Transformation

UNIT 8 | **LESSON 2**

Interpreting Points on a Scatter Plot

Let’s investigate points on a scatter plot.



Focus

Goals

1. **Language Goal:** Coordinate data in a table and points on a scatter plot. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret a point on a scatter plot in context. **(Speaking and Listening, Writing)**

Coherence

Today

Students interpret and compare a point in a scatter plot in context. Students add points to a scatter plot, given information about an individual in the population.

Previously

In Grade 5, students represented real-world problems by graphing points in the first quadrant, and interpreted the value of points, given the context of the situation. So far in this unit, students have focused on plotting and organizing data.

Coming Soon

In Lesson 4, students will create and assess a linear model to judge the closeness of data points to a line. In Lesson 5, students will use the linear model to solve problems, given a real-world context.

Rigor

- Students build **conceptual understanding** by interpreting points on a scatter plot.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	8 min	10 min	5 min	7 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)

Math Language Development

Review

- *scatter plot*

Amps | Featured Activity

Activity 1

Changing Krill

Students manipulate a point on a graph to see how its placement changes a krill's eye distance and height.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated as they try to interpret the points on a scatter plot in Activity 1. Remind students that the scatter plot helps them be organized in their approach, as they can apply the process of elimination by focusing on one axis at a time.

Modifications to Pacing

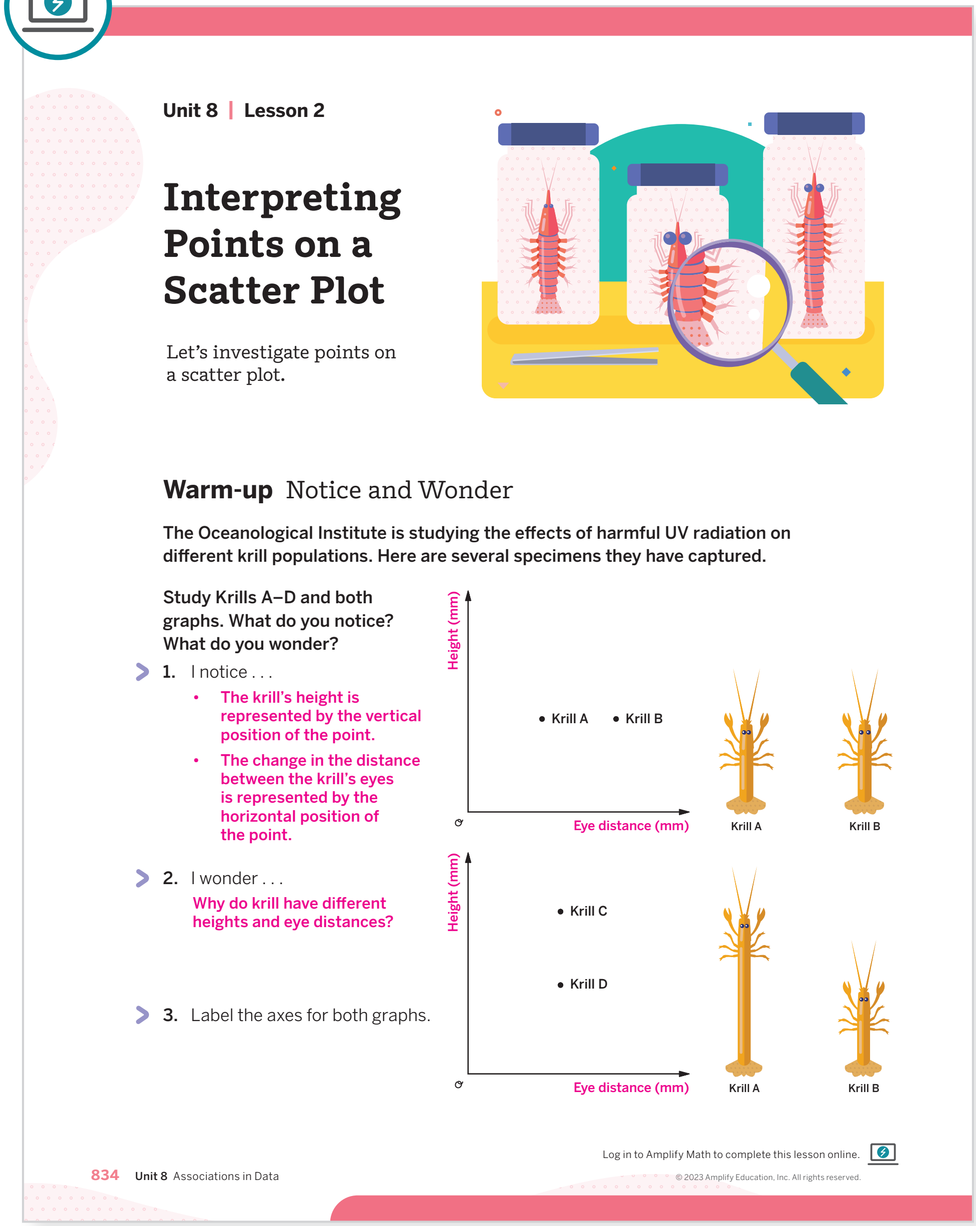
You may want to consider this additional modification if you are short on time.

- **Activity 1** may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students observe patterns to make connections between the placement of points and the effects on the krill's height and eye distance.



Unit 8 | **Lesson 2**

Interpreting Points on a Scatter Plot

Let's investigate points on a scatter plot.

Warm-up Notice and Wonder

The Oceanological Institute is studying the effects of harmful UV radiation on different krill populations. Here are several specimens they have captured.

Study Krills A–D and both graphs. What do you notice? What do you wonder?

1. I notice . . .
    - The krill's height is represented by the vertical position of the point.
    - The change in the distance between the krill's eyes is represented by the horizontal position of the point.
2. I wonder . . .
    Why do krill have different heights and eye distances?
3. Label the axes for both graphs.

Log in to Amplify Math to complete this lesson online.

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① Launch

Tell students that ultraviolet radiation is not only harmful to humans, but also to marine life, such as small, shrimp-like crustaceans called krill. Then conduct the ***Notice and Wonder*** routine.

② Monitor

Help students get started by having them closely analyze the krills' eye distances and heights.

Look for productive strategies:

- Noticing the krill's eye distance changes as the point moves horizontally.
- Noticing the krill's height changes as the point moves vertically.

③ Connect

Have students share what they notice about the placement of the point and the krill's height and eye distance.

Highlight how the point movement changes the krill's features. If the point is higher, the krill's height changes. If the point is farther right, the krill's eye distance changes.

Ask, "How did you determine the labels for the axes?"

Power-up

To power up students' ability to determine the x- and y-coordinates from points on a coordinate grid:

Provide students with a copy of the Power-up PDF.

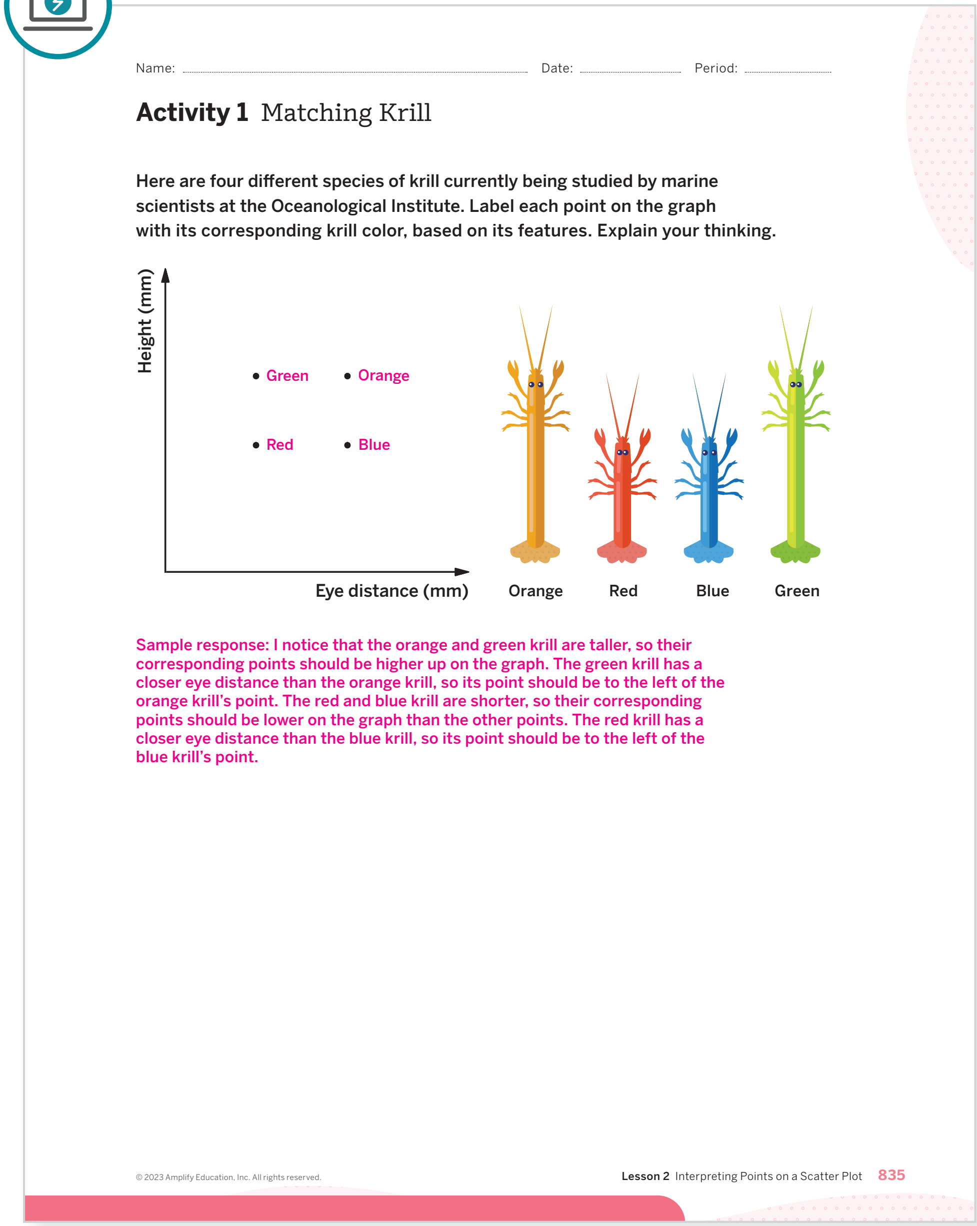
Use: Before Activity 1

Informed by: Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Pairs | 10 min

Activity 1 Matching Krill

Students continue analyzing the placement of a point on a graph to see that krill with the same eye distance are represented by points on the same vertical line and krill of the same height are represented by points on the same horizontal line.



Name: ______ Date: ______ Period: ______

Activity 1 Matching Krill

Here are four different species of krill currently being studied by marine scientists at the Oceanological Institute. Label each point on the graph with its corresponding krill color, based on its features. Explain your thinking.

Sample response: I notice that the orange and green krill are taller, so their corresponding points should be higher up on the graph. The green krill has a closer eye distance than the orange krill, so its point should be to the left of the orange krill's point. The red and blue krill are shorter, so their corresponding points should be lower on the graph than the other points. The red krill has a closer eye distance than the blue krill, so its point should be to the left of the blue krill's point.

1 Launch

Conduct the *Think-Pair-Share routine*.

2 Monitor

Help students get started by asking, "Which krill do you think the two higher points represent?"

Look for points of confusion:

- **Having difficulty supporting their response.** Have students reread the axis labels and relate them to point placement.

Look for productive strategies:

- Knowing where to place krill based on height, but not eye distance. Ask students to compare the eye distances of the krill, and then look at the x-axis. Ask if a closer eye distance would be plotted left or right horizontally.

3 Connect

Ask, "How did you decide which point corresponded with each krill?"

Have students share their methods used to identify each krill, such as the height of the orange and green krill corresponding with the two higher points, and points closer to the y-axis corresponding with a smaller eye distance.

Highlight that the same vertical distance means the same height, and the same horizontal distance means the same eye distance.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Have students compare only the orange and red krill first by having students cover up the blue and green krill using an index card or a slip of paper.

Extension: *Math Enrichment*

Ask students to add another point representing a krill that has a different eye distance and is taller than the current krill shown.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students label each point and explain their thinking, have pairs meet with one other pair of students to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "Which krill did you begin with and why did you choose that krill?"
- "How did you decide which of these four points corresponded with the orange krill?"

Have students write a final response, based on the feedback they received.

English Learners

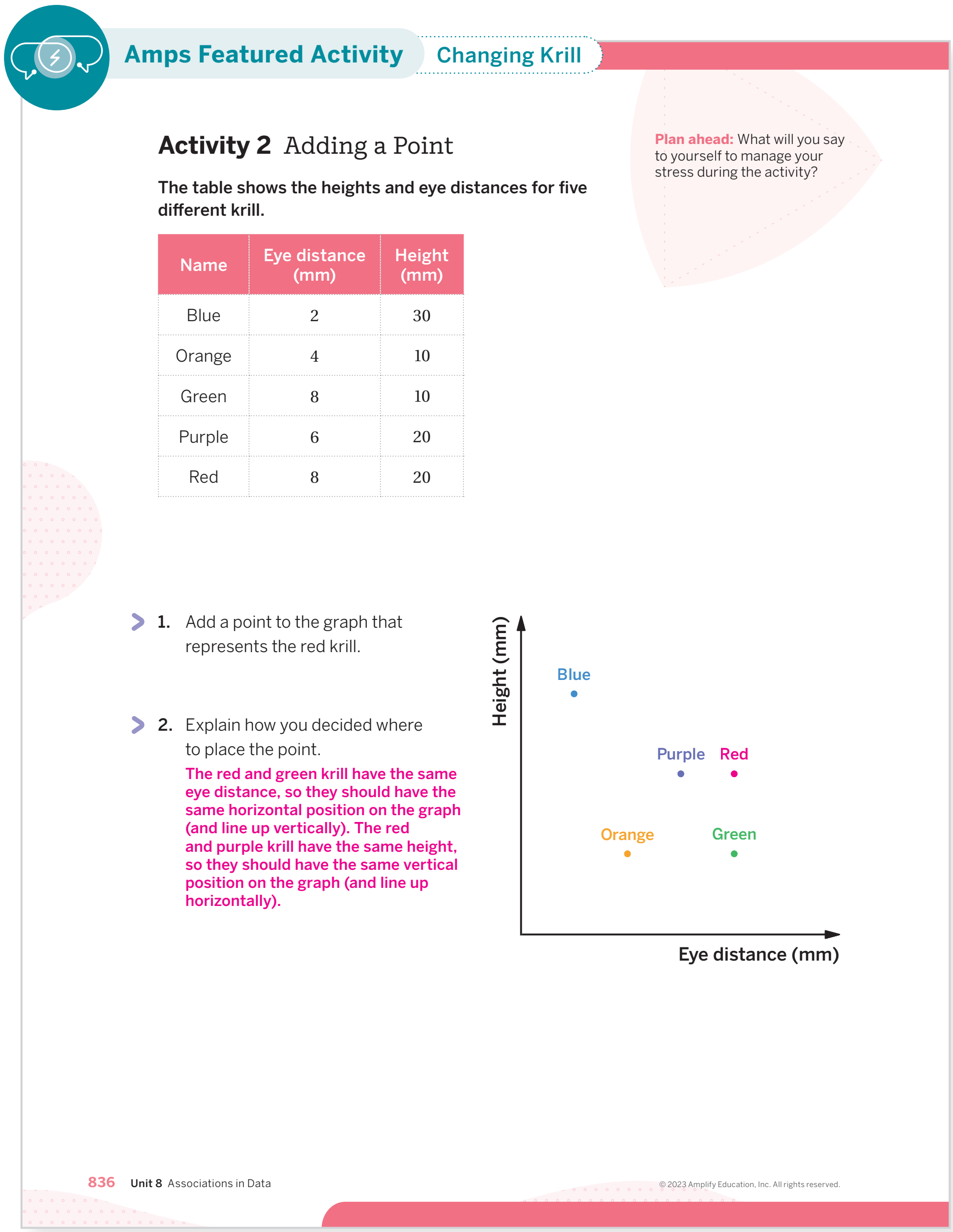
Encourage students to use hand gestures as they explain their thinking. For example, they could point to the orange and green krill's height (to the eyes) and then point to the two dots on the graph corresponding to the tallest height.

Independent | 8 min

Activity 2 Adding a Point

Students connect points on a scatter plot with data in a table to see the connection between exact data values and placement of values on a graph.

Amps Featured Activity Changing Krill



Activity 2 Adding a Point

The table shows the heights and eye distances for five different krill.

Name	Eye distance (mm)	Height (mm)
Blue	2	30
Orange	4	10
Green	8	10
Purple	6	20
Red	8	20

Plan ahead: What will you say to yourself to manage your stress during the activity?

1. Add a point to the graph that represents the red krill.
2. Explain how you decided where to place the point.
   The red and green krill have the same eye distance, so they should have the same horizontal position on the graph (and line up vertically). The red and purple krill have the same height, so they should have the same vertical position on the graph (and line up horizontally).

836 Unit 8 Associations in Data

1 Launch

Introduce this activity by telling students that you are curious to see how the graph can help them notice and compare the features of four different krill.

2 Monitor

Help students get started by asking what they know about the red krill based on the table.

Look for points of confusion:

- **Unsure where to place the red krill.** Tell them to look at the table. Ask, "Which krill has the same eye distance as the red krill? Which krill has the same height as the red krill?"
- **Having difficulty organizing information.** Write the coordinates next to each krill.
- **Placing the red and green krill on vertically instead of horizontally.** Emphasize the axis labels. Because the red and green krill both have an eye distance of 8 mm, they should both have the same horizontal distance.

3 Connect

Have students share their process, such as what data points they used in the table and how they correspond to the horizontal or vertical position in the graph.

Ask, "How did you decide where to plot a point for the red krill? Which krill has the same eye distance? Which krill has the same height? How did this information help you know where to plot the point?"

Highlight the red and green krill's equal eye distance, as their corresponding points should have the same horizontal position. The red and purple krill have the same height, so their points should have the same vertical position.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate a point on the graph to see how its placement changes a krill's eye distance and height.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, simplify the table and graph by removing the blue and orange krill data.

Math Language Development

MLR7: Compare and Connect

During the Connect, help support students' sense-making by pressing them to make connections between point placement and krill features. Amplify the language students use to describe the horizontal and vertical positions between corresponding points, such as "they have the same eye distance" or "they have the same height."

English Learners

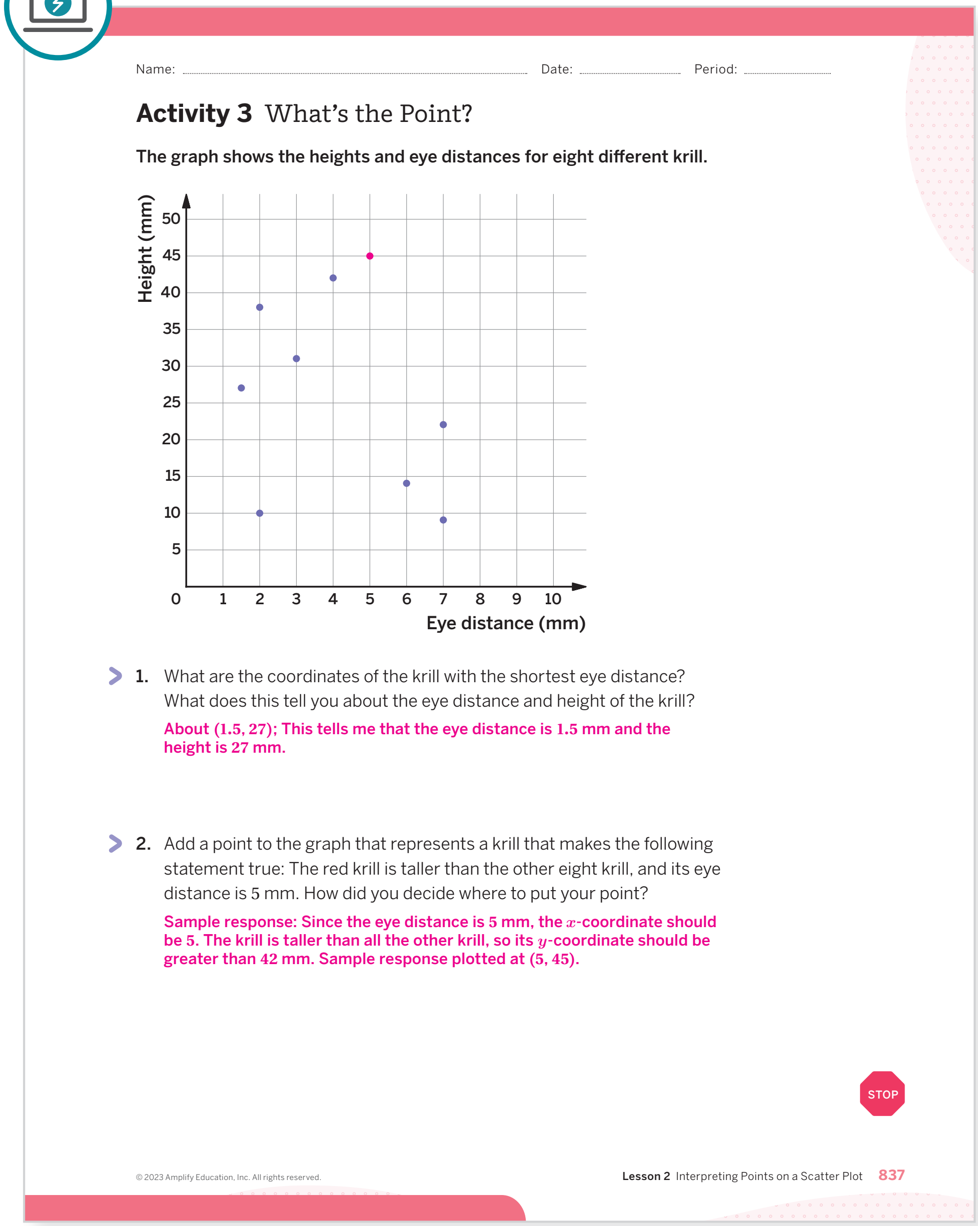
Display these sentence frames to help students make connections between point placement and krill features.

- "The red krill has the same eye distance as ___, so ___ is the same."
- "The red krill has the same height as ___, so ___ is the same."

Pairs | 10 min

Activity 3 What's the Point?

Students explain the meaning of a point in context to compare one data value to the entire data set.



Name: ____________ Date: ________ Period: ________

Activity 3 What's the Point?

The graph shows the heights and eye distances for eight different krill.

1. What are the coordinates of the krill with the shortest eye distance? What does this tell you about the eye distance and height of the krill?

   About $(1.5, 27)$; This tells me that the eye distance is 1.5 mm and the height is 27 mm.

2. Add a point to the graph that represents a krill that makes the following statement true: The red krill is taller than the other eight krill, and its eye distance is 5 mm. How did you decide where to put your point?

   Sample response: Since the eye distance is 5 mm, the x-coordinate should be 5. The krill is taller than all the other krill, so its y-coordinate should be greater than 42 mm. Sample response plotted at $(5, 45)$.

Lesson 2 Interpreting Points on a Scatter Plot 837

1 Launch

Tell students that, in addition to an image or a table, they can also use a written description to add points to a scatter plot.

2 Monitor

Help students get started by asking, "Where are the points that represent the shortest eye distances?" and "How do we write an ordered pair as coordinates?"

Look for points of confusion:

- **Unsure what the point $(1.5, 27)$ represents.** Emphasize that eye distance represents the first number and height represents the second number in the coordinates of a point.

Look for productive strategies:

- Writing a number without units. Ask students what units are used to measure the height and eye distance. Have them record the units with their response.
- Thinking of multiple places to plot a point for Problem 2. Since there are multiple answers, ask students to highlight a region of the graph that can represent the answer.

3 Connect

Have students share what strategies they used to determine where to place the red krill.

Highlight the meaning of zero eye distance.

Ask, "Can there be multiple points that represent the red krill? Why or why not?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students label each point as Krill 1, Krill 2, etc. and suggest that they create a table relating eye distance and height if they choose to analyze the data using a different representation.

Krill	Eye distance (mm)	Height (mm)
Krill 1		
Krill 2		

Extension: *Math Enrichment*

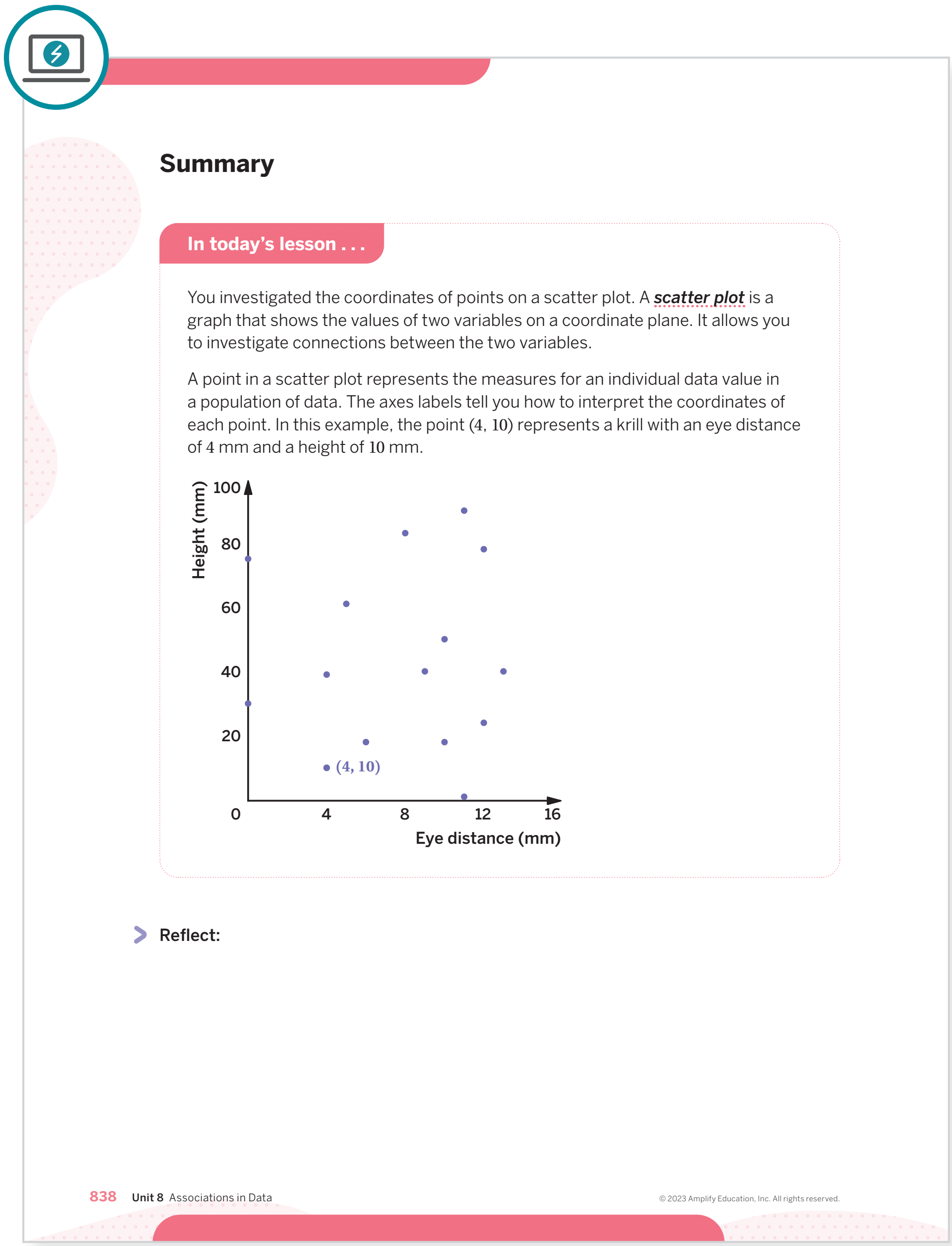
Challenge students to add 4 new points to the graph such that:

- The first point, Krill A, represents the tallest krill.
- The second point, Krill B, represents a krill that has the greatest eye distance, but that has a shorter height than Krill A.
- The third point, Krill C, represents a krill whose height is a whole number multiple of its eye distance.
- The fourth point, Krill D, represents the shortest krill, with the same eye distance as one of the other krill shown.

Whole Class | 5 min

Summary

Review and synthesize how to interpret points on a scatter plot within the context of a real-world problem.



Summary

In today's lesson . . .

You investigated the coordinates of points on a scatter plot. A ***scatter plot*** is a graph that shows the values of two variables on a coordinate plane. It allows you to investigate connections between the two variables.

A point in a scatter plot represents the measures for an individual data value in a population of data. The axes labels tell you how to interpret the coordinates of each point. In this example, the point $(4, 10)$ represents a krill with an eye distance of 4 mm and a height of 10 mm.

Reflect:

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Synthesize

Display the graph from the Summary and highlight the point $(4, 10)$.

Highlight the placement of points on a scatter plot based on context.

Ask:

- "How do you know what a point represents on a scatter plot?"
- "What do the axes labels tell you about how to interpret a point on a scatter plot?"

Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

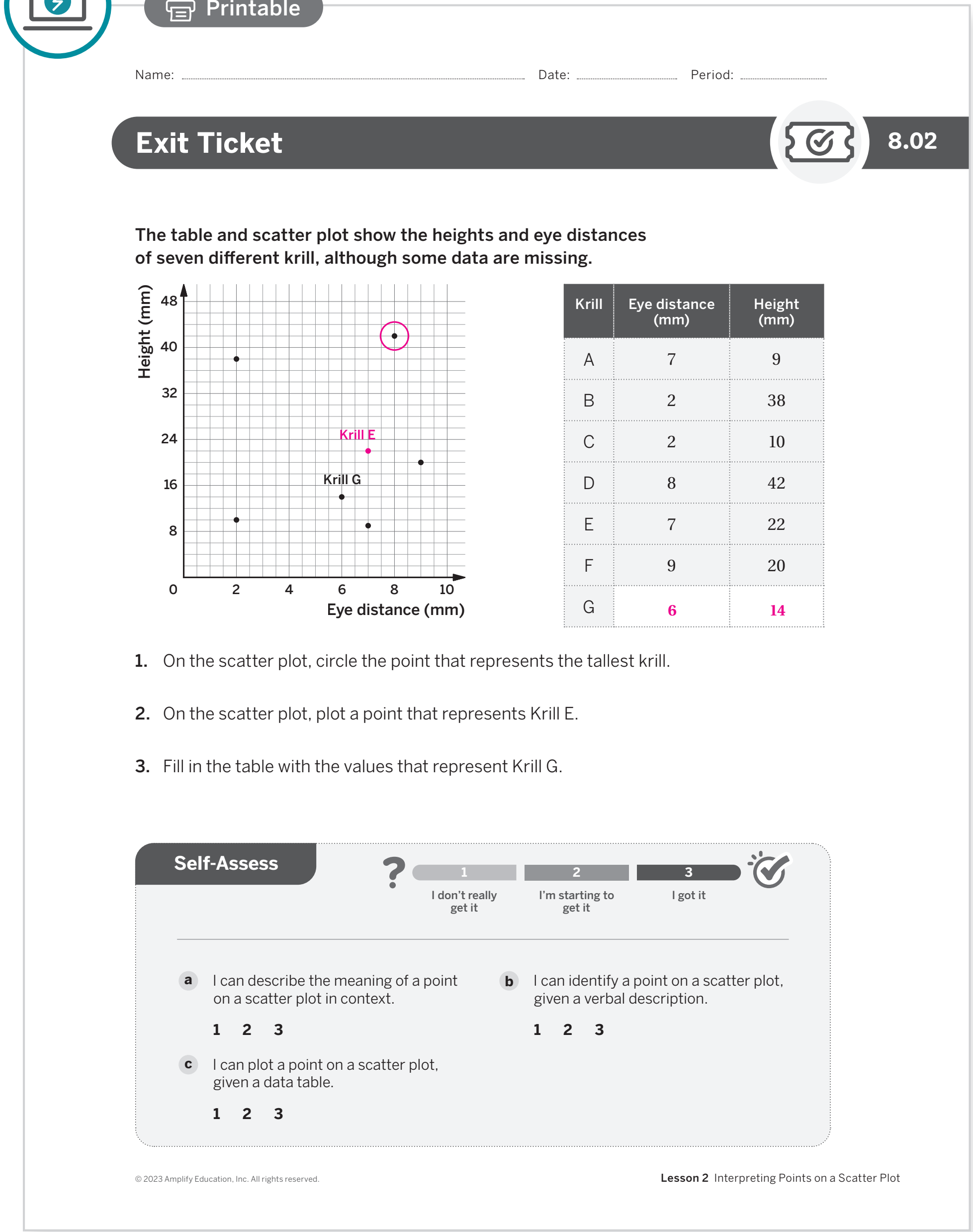
- "What helped you interpret the context of a point on a scatter plot?"

Independent | 7 min

Exit Ticket

Students demonstrate their understanding by identifying a point on a scatter plot, given a verbal description and table of data.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 8.02

The table and scatter plot show the heights and eye distances of seven different krill, although some data are missing.

Krill	Eye distance (mm)	Height (mm)
A	7	9
B	2	38
C	2	10
D	8	42
E	7	22
F	9	20
G	6	14

1. On the scatter plot, circle the point that represents the tallest krill.
2. On the scatter plot, plot a point that represents Krill E.
3. Fill in the table with the values that represent Krill G.

Self-Assess

? 1 I don't really get it · 2 I'm starting to get it · 3 I got it

a I can describe the meaning of a point on a scatter plot in context.
1 2 3

b I can identify a point on a scatter plot, given a verbal description.
1 2 3

c I can plot a point on a scatter plot, given a data table.
1 2 3

Lesson 2 Interpreting Points on a Scatter Plot

Success looks like . . .

- **Language Goal:** Coordinating data in a table and points on a scatter plot. **(Speaking and Listening, Writing)**
- **Language Goal:** Interpreting a point on a scatter plot in context. **(Speaking and Listening, Writing)**

Suggested next steps

If students circle the point farthest to the right, instead of the highest point for Problem 1, consider:

- Asking students which label identifies the krill height. Ask them to identify the tallest and shortest krill.
- Assigning Practice Problem 1.

If students write coordinates as (y, x), consider:

- Reviewing how to write coordinates from Activity 3.

Professional Learning

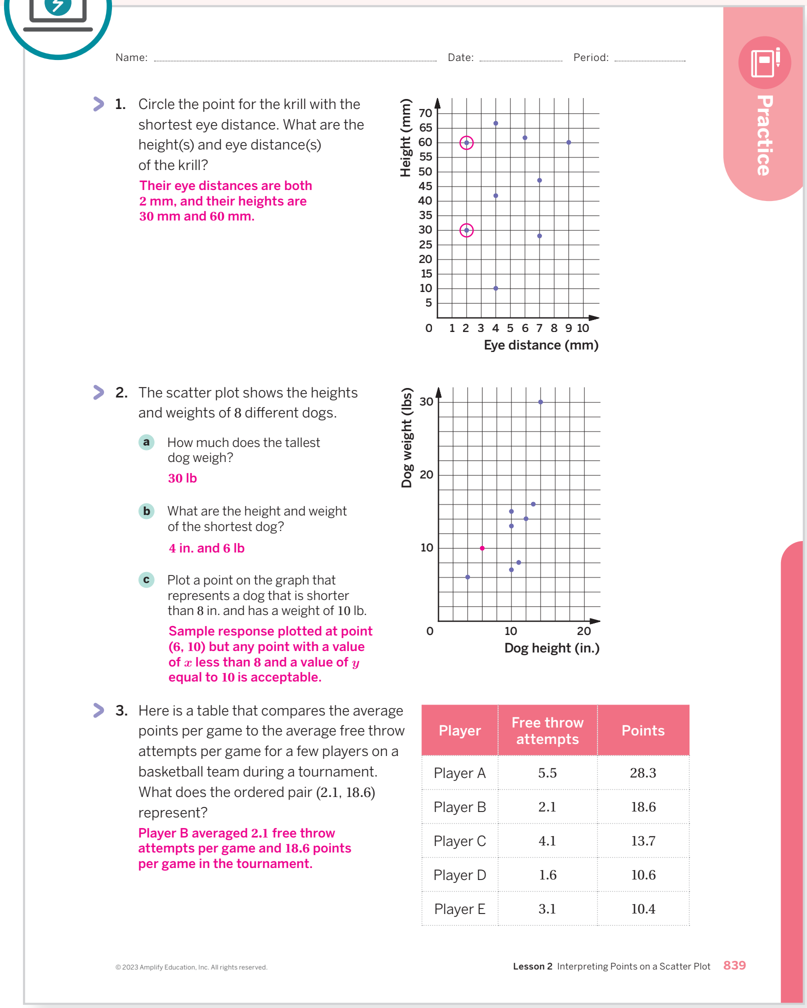
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did the *Notice and Wonder* routine enable all students to do math in today's lesson?
- During the discussion during Activity 3, how did you encourage each student to share their understandings?

Practice

Independent



Name: Date: Period:

Practice

1. Circle the point for the krill with the shortest eye distance. What are the height(s) and eye distance(s) of the krill?

   Their eye distances are both 2 mm, and their heights are 30 mm and 60 mm.

2. The scatter plot shows the heights and weights of 8 different dogs.
   - **a** How much does the tallest dog weigh?

     30 lb
   - **b** What are the height and weight of the shortest dog?

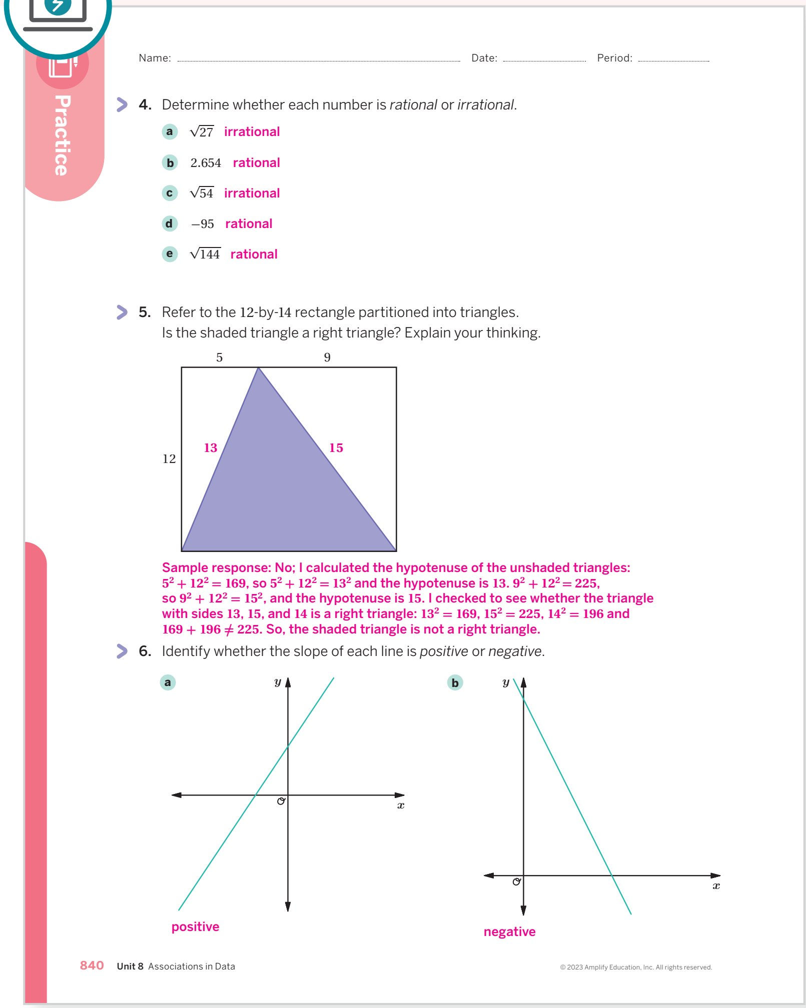
     4 in. and 6 lb
   - **c** Plot a point on the graph that represents a dog that is shorter than 8 in. and has a weight of 10 lb.

     Sample response plotted at point (6, 10) but any point with a value of x less than 8 and a value of y equal to 10 is acceptable.

3. Here is a table that compares the average points per game to the average free throw attempts per game for a few players on a basketball team during a tournament. What does the ordered pair (2.1, 18.6) represent?

   Player B averaged 2.1 free throw attempts per game and 18.6 points per game in the tournament.

Player	Free throw attempts	Points
Player A	5.5	28.3
Player B	2.1	18.6
Player C	4.1	13.7
Player D	1.6	10.6
Player E	3.1	10.4



Name: Date: Period:

Practice

4. Determine whether each number is *rational* or *irrational*.
   - **a** $\sqrt{27}$ **irrational**
   - **b** 2.654 **rational**
   - **c** $\sqrt{54}$ **irrational**
   - **d** -95 **rational**
   - **e** $\sqrt{144}$ **rational**

5. Refer to the 12-by-14 rectangle partitioned into triangles. Is the shaded triangle a right triangle? Explain your thinking.

Sample response: No; I calculated the hypotenuse of the unshaded triangles: $5^2 + 12^2 = 169$, so $5^2 + 12^2 = 13^2$ and the hypotenuse is 13. $9^2 + 12^2 = 225$, so $9^2 + 12^2 = 15^2$, and the hypotenuse is 15. I checked to see whether the triangle with sides 13, 15, and 14 is a right triangle: $13^2 = 169$, $15^2 = 225$, $14^2 = 196$ and $169 + 196 \neq 225$. So, the shaded triangle is not a right triangle.

6. Identify whether the slope of each line is *positive* or *negative*.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 3	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 6	1
	5	Unit 7 Lesson 12	2
Formative ⚡	6	Unit 8 Lesson 3	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 8 | **LESSON 3**

Observing Patterns in Scatter Plots

Let's look for patterns in scatter plots.

Focus

Goals

1. **Language Goal:** Categorize data sets, and describe the properties used to create categories. **(Speaking and Listening)**
2. **Language Goal:** Describe features of data on scatter plots, including *linear* and *nonlinear association*, *positive* and *negative association*, and *clustering*, using informal language. **(Speaking and Listening)**
3. **Language Goal:** Explain what might cause a nonlinear association or clustering of data points in context. **(Speaking and Listening)**

Coherence

Today

Students look at scatter plots as a whole. They determine whether a scatter plot has a linear or nonlinear association, a positive or negative association, and identify any clusters in the data. They learn how to use appropriate vocabulary and precise language to describe patterns.

Previously

In Lesson 1, students created a scatter plot to see patterns more clearly. In Lesson 2, students described the meaning of a point on a scatter plot in context.

Coming Soon

In Lessons 4–5, students will draw a line to fit to model data on a scatter plot with linear association. Students will use and assess the line of fit to judge the closeness of data points and to solve problems, given context.

Rigor

- Students build their **conceptual understanding** of how scatter plots can show associations and clusters in data.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Activity 4	Summary	Exit Ticket
5 min	10 min	8 min	5 min	7 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activities 1 & 2 PDF, pre-cut cards, one set of cards per pair of students
- Anchor Chart PDF, *Clusters in Scatter Plots*
- Anchor Chart PDF, *Types of Associations in Scatter Plots*

Math Language Development

New words

- ***cluster***
- ***linear association***
- ***negative association***
- ***nonlinear association***
- ***positive association***

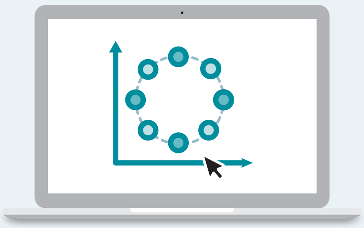
Review words

- *scatter plot*

Amps | Featured Activity

Activity 4
Digitally Identify Clusters

Students circle clusters using the sketch tool, which you can overlay to assess student work and give immediate feedback.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may be frustrated with all of the terms that can be used to sort the cards in Activity 2. Students will build confidence in their abilities as you provide opportunities to practice using the terms throughout the lesson and the rest of the unit.

Modifications to Pacing

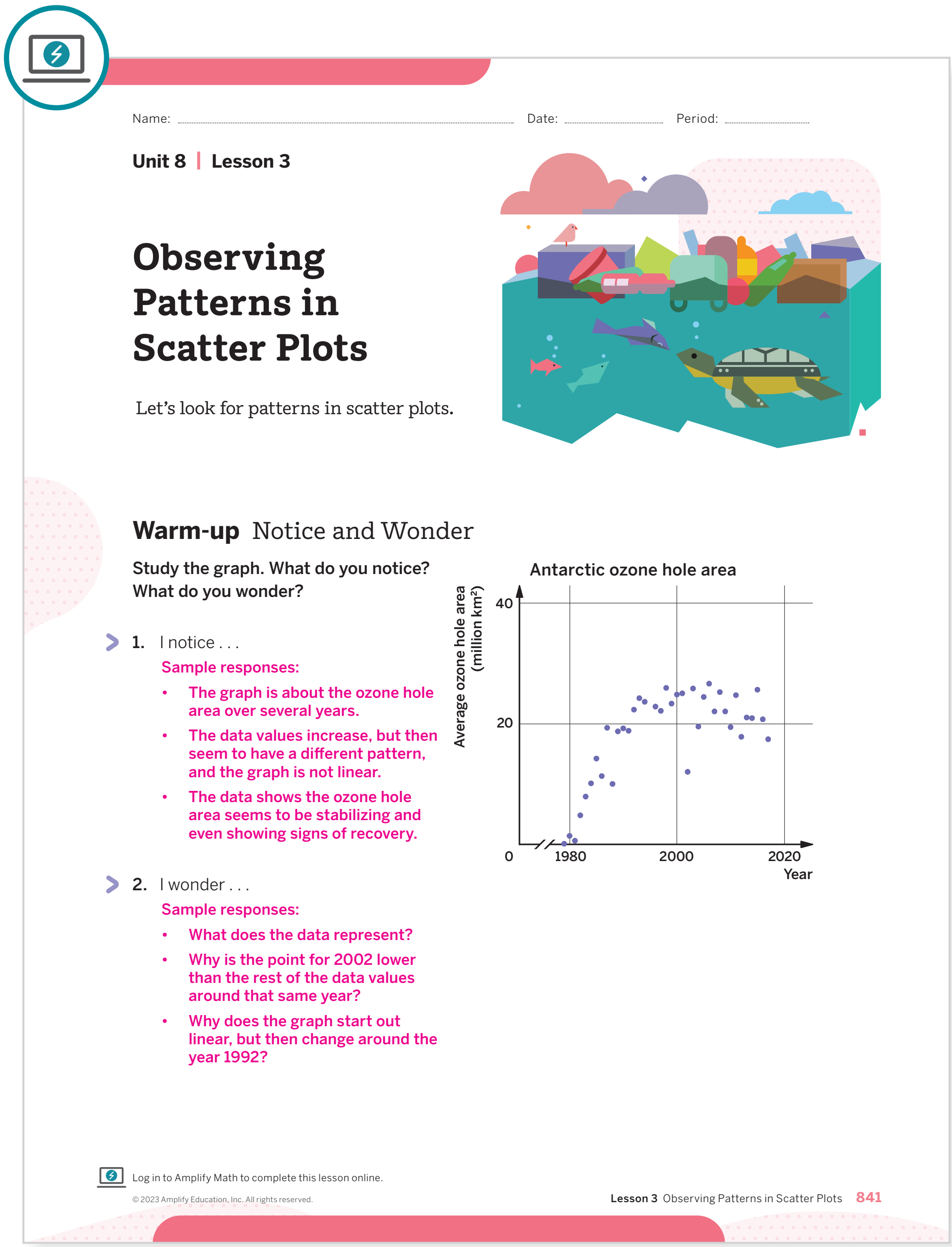
You may want to consider this additional modification if you are short on time.

- In **Activity 4**, Problem 2 may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students analyze a scatter plot that shows how the Antarctic ozone hole area changes over time to see an example of nonlinear association.



Name: Date: Period:

Unit 8 | Lesson 3

Observing Patterns in Scatter Plots

Let's look for patterns in scatter plots.

Warm-up Notice and Wonder

Study the graph. What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - The graph is about the ozone hole area over several years.
   - The data values increase, but then seem to have a different pattern, and the graph is not linear.
   - The data shows the ozone hole area seems to be stabilizing and even showing signs of recovery.
2. I wonder . . .
   Sample responses:
   - What does the data represent?
   - Why is the point for 2002 lower than the rest of the data values around that same year?
   - Why does the graph start out linear, but then change around the year 1992?

Log in to Amplify Math to complete this lesson online.

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Lesson 3 Observing Patterns in Scatter Plots 841

1 Launch

Conduct the *Notice and Wonder* routine.

Note: In Lesson 7, students will learn why the ozone shows signs of recovery.

2 Monitor

Help students get started by asking how the shape of this graph is different than the ones they have seen so far.

Look for points of confusion:

- **Struggling with describing the pattern.** Ask, "Are the values increasing or decreasing? Does the graph look curved or straight?"
- **Having trouble understanding the context.** Ask, "What do the axes represent? What does the point $(1998, 10)$ represent?"

Look for productive strategies:

- Noticing the pattern is not linear. Ask students to describe how the data changes.

3 Connect

Ask, "How do the data change? What does this tell you about the ozone hole area?" At first, the data increased at a steady rate, and then it changed, slowing down. This tells me the ozone hole area became smaller and showed improvement after 1990.

Highlight the different patterns shown in this scatter plot. It is different from the previous ones, because it shows a curve rather than a straight line.

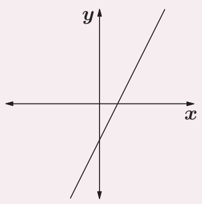
Define linear and nonlinear association. Say, "If a straight line can model the data, the data have a ***linear association***. If a straight line cannot model the data, the data have a ***nonlinear association***."

Power-up

To power up students' ability to identify when the graph of a line has a positive or a negative slope, have students complete:

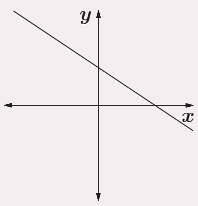
Recall that a line with a positive slope is *increasing* from left to right while a line with a negative slope is *decreasing* from left to right. Identify whether the slope of each line is *positive* or *negative*.

a.



Positive

b.



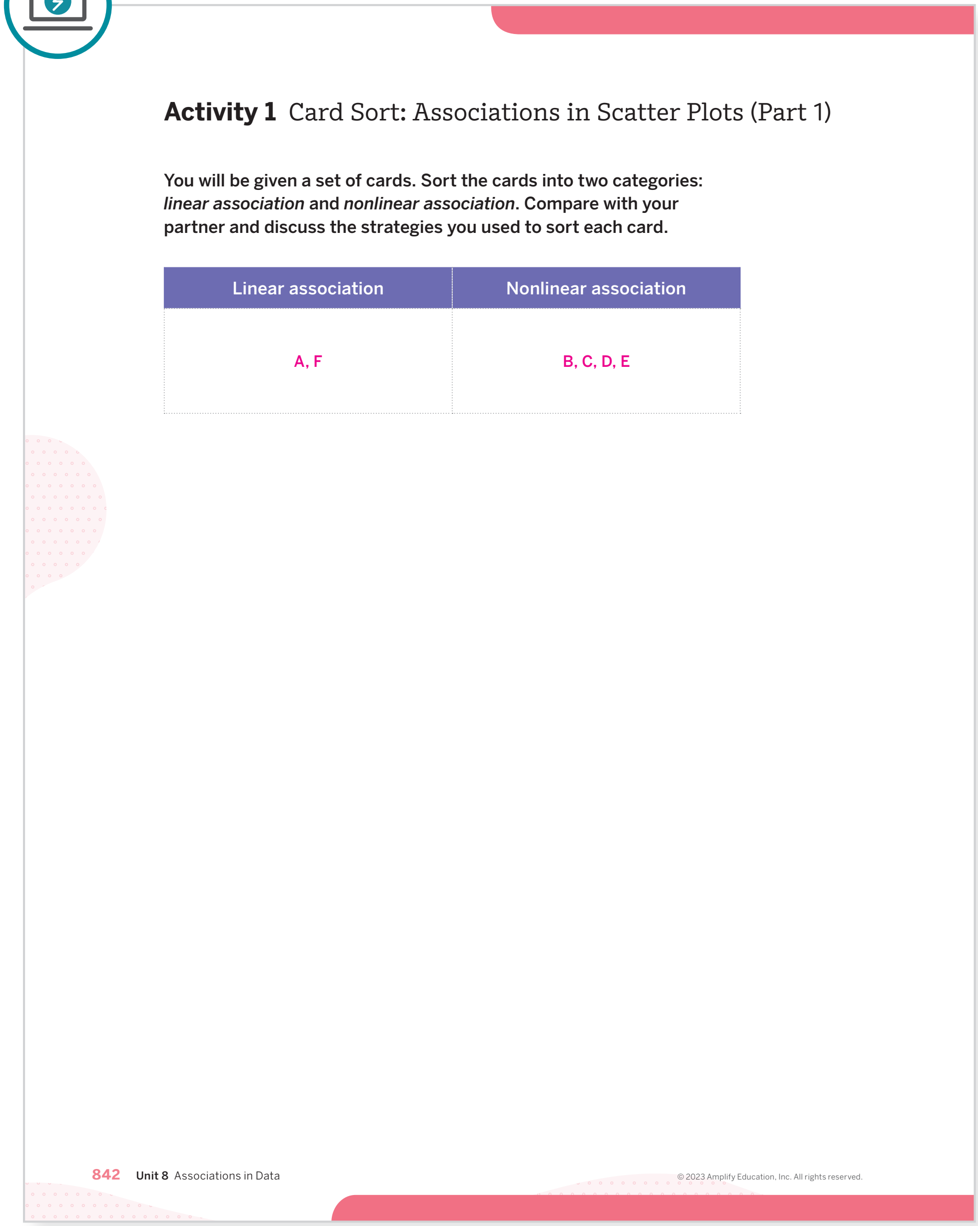
Negative

Use: Before Activity 2
Informed by: Performance on Lesson 2, Practice Problem 6

Pairs | 10 min

Activity 1 Card Sort: Associations in Scatter Plots (Part 1)

Students sort cards based on linearity to explore what it means to have a *positive association* or *negative association.*



Activity 1 Card Sort: Associations in Scatter Plots (Part 1)

You will be given a set of cards. Sort the cards into two categories: *linear association* and *nonlinear association*. Compare with your partner and discuss the strategies you used to sort each card.

Linear association	Nonlinear association
A, F	B, C, D, E

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1 Launch

Distribute one set of cards to each pair of students, from the Activities 1 & 2 PDFs. Then conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by asking what they know about linear associations and which cards appear to have that type of association.

Look for points of confusion:

- **Thinking linear associations can only be positive (or only negative).** Remind students that linear association, which can be positive or negative, just means the data can be modeled with a straight line.
- **Thinking Cards D and E are linear.** Tell students that although parts of the graph look linear, they should look at the entire data set before identifying the association.

3 Connect

Have students share their reasons for card placement.

Highlight the differences between linear and nonlinear association.

Ask, "What do the linear association cards have in common?" Have students draw lines on Cards A and F to reinforce their linear association.

Define the terms ***positive*** and ***negative associations***. At the end of the activity, have students compare the difference between Cards A and F to introduce these terms. Card A has a negative association because the values of y decrease as the values of x increase. Card F has a positive association because the values of y increase as the values of x increase.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously learned about *linear* and *nonlinear* relationships. Ask them to describe these two types of relationships using their own words before distributing the cards.

Extension: *Math Enrichment*

Ask students to sketch their own scatter plot that shows a positive, nonlinear association. Their scatter plot should look different than Card D.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you introduce the terms *positive* and *negative association*, draw students' attention to the similarities and differences between graphs that show each type of association. Ask:

- "How are the associations on Cards A and F similar?" They are both linear.
- "How are they different?" Card A: negative association, Card F: positive association

Ask similar questions about Cards C, D, and E.

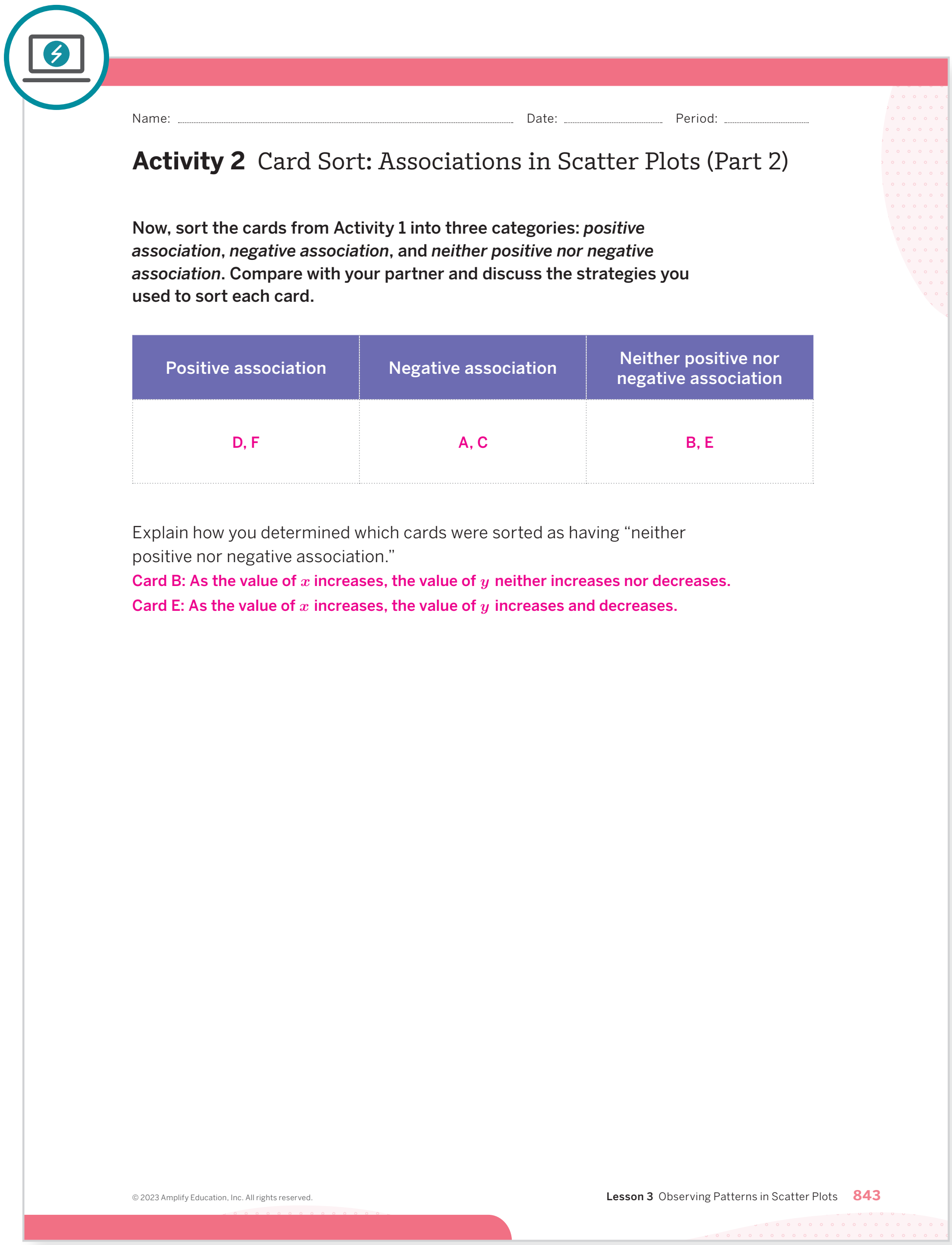
English Learners

Display these sentence frames to help students organize their thinking.

- "Cards _____ and _____ are similar because . . ."
- "Cards _____ and _____ are different because . . ."

Activity 2 Card Sort: Associations in Scatter Plots (Part 2)

Students sort cards based on positive, negative, or neither positive nor negative association to identify similarities and differences.



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Activity 2 Card Sort: Associations in Scatter Plots (Part 2)

Now, sort the cards from Activity 1 into three categories: *positive association*, *negative association*, and *neither positive nor negative association*. Compare with your partner and discuss the strategies you used to sort each card.

Positive association	Negative association	Neither positive nor negative association
D, F	A, C	B, E

Explain how you determined which cards were sorted as having "neither positive nor negative association."

Card B: As the value of x increases, the value of y neither increases nor decreases.

Card E: As the value of x increases, the value of y increases and decreases.

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Lesson 3 Observing Patterns in Scatter Plots 843

1 Launch

Keep students in pairs and conduct the *Card Sort* routine. Then have students sort the same cards from Activity 1 into categories: *positive*, *negative*, or *neither positive nor negative* association.

2 Monitor

Help students get started by asking them to describe positive and negative associations in their own words. Record student answers with a sketch of the description.

Look for points of confusion:

- **Thinking Card F is negative.** Ask, "As the values of x increase, what do you notice about the values of y?"
- **Thinking Card E can be both positive and negative.** Remind students to look at the entire data set before determining the association.

Look for productive strategies:

- Only sorting nonlinear cards. Tell students positive and negative associations can be used to describe both linear and nonlinear relationships, as long as the values of y increase/decrease as the values of x increase.

3 Connect

Display the different ways students sorted their cards.

Have students share their thinking for each card placement. Record their reasoning, attending to appropriate vocabulary and precise use of language.

Highlight that as the values of x increase, sometimes the values of y also increase (positive association), and sometimes the values of y decrease (negative association).

Ask, "What makes Card B and Card E different from the rest of the cards?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight how the values of y change as the values of x increase, consider displaying a table (or adding it to the class display) similar to the following to help students make this connection.

	Positive association	Negative association
As x increases . . .	y also increases	y decreases

As students respond to the Ask question about Cards B and E, be sure they understand that the data in each of these cards show neither a positive nor a negative association, because the association should be for the entirety of the data.

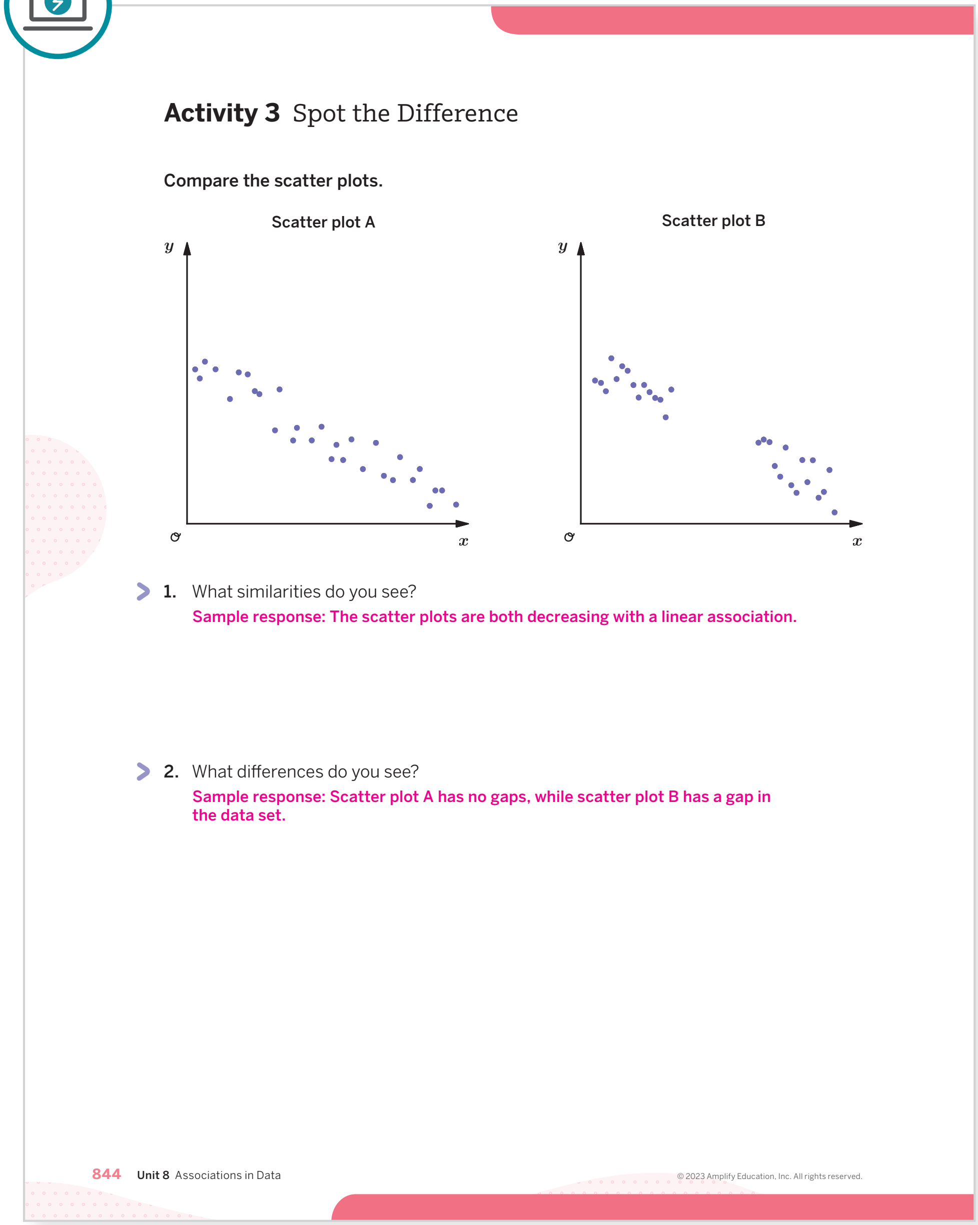
English Learners

Use hand gestures, such as pointing, to illustrate how y increases or decreases as x increases.

Independent | 5 min

Activity 3 Spot the Difference

Students compare scatter plots to learn about clustering of data values.



Activity 3 Spot the Difference

Compare the scatter plots.

Scatter plot A

Scatter plot B

1. What similarities do you see?
   Sample response: The scatter plots are both decreasing with a linear association.

2. What differences do you see?
   Sample response: Scatter plot A has no gaps, while scatter plot B has a gap in the data set.

844 Unit 8 Associations in Data

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1 Launch

Have students compare the scatter plots individually before sharing with the class.

2 Monitor

Help students get started by asking what they notice about the patterns in each scatter plot.

Look for points of confusion:

- **Having trouble seeing similarities.** Ask, "As the values of x increase, what happens to the values of y in both scatter plots?"

3 Connect

Have students share the similarities and differences they noticed between the two scatter plots.

Highlight the two separate groupings of data in Scatter plot B.

Define the term ***cluster*** as a grouping of points around the same value or a distinct separate grouping of points, as seen in Scatter plot B. Because the number of data values that make up a cluster can be subjective, reinforce the idea that clusters are used to look for trends in data.

Ask, "How can you identify whether there is a cluster in a scatter plot?"

Math Language Development

MLR2: Collect and Display

During the Connect, as you define the term *cluster*, draw circles around the two distinct groupings of data in Scatter plot B and add this visual example of clustering to the class display.

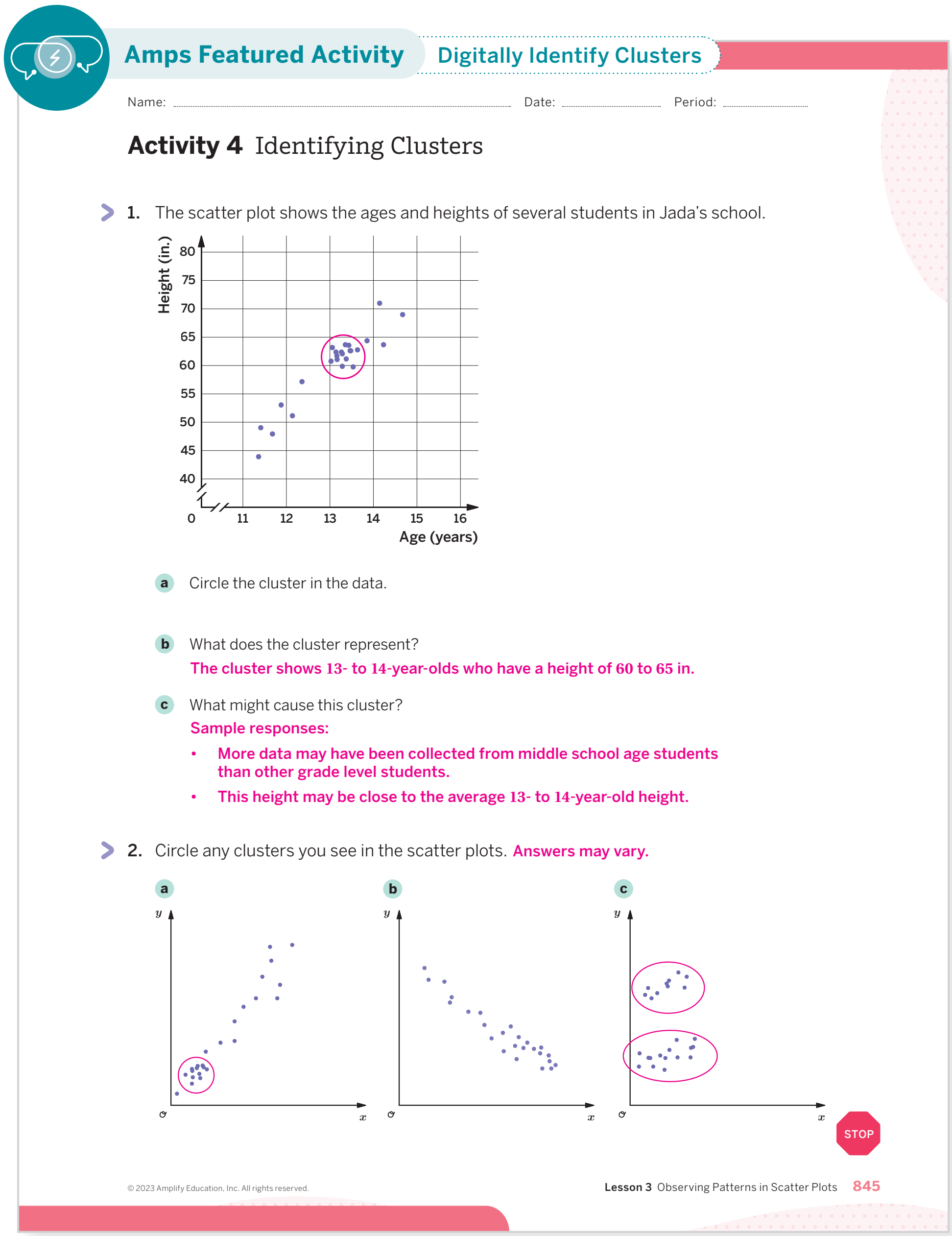
English Learners

Annotate the two distinct groupings in Scatter plot B by labeling them with the term "cluster." Use hand gestures to illustrate the gap in data between the two clusters.

Pairs | 7 min

Activity 4 Identifying Clusters

Students identify clusters and interpret what a cluster represents in the context of a situation.



Amps Featured Activity Digitally Identify Clusters

Name: ______ Date: ______ Period: ______

Activity 4 Identifying Clusters

1. The scatter plot shows the ages and heights of several students in Jada's school.

a Circle the cluster in the data.

b What does the cluster represent?
The cluster shows 13- to 14-year-olds who have a height of 60 to 65 in.

c What might cause this cluster?
Sample responses:
- More data may have been collected from middle school age students than other grade level students.
- This height may be close to the average 13- to 14-year-old height.

2. Circle any clusters you see in the scatter plots. Answers may vary.

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Lesson 3 Observing Patterns in Scatter Plots 845

1 Launch

Conduct the *Think-Pair-Share* routine.

Note: Because the number of data values that make up a cluster can be subjective, ask students to explain their thinking during discussions.

2 Monitor

Help students get started by having them explain the term *cluster* in their own words.

Look for points of confusion:

- **Not knowing what caused the cluster for Problem 1c**. Have students refer to their answer in Problem 1b and ask, "Why do you think you're seeing a cluster at that age and height?"
- **Not knowing if Problem 2b has clusters**. Some students identify a cluster, while other students may not. Take this time to discuss subjectivity; there may not always be a single correct answer in math. You may also want to discuss how the context, not given in this problem, may help give meaning to clusters.
- **Thinking a cluster is limited to one grouping**. Have students circle any and all cluster groupings.

3 Connect

Have students share their answers. Take turns displaying different student answers. Ask for input from peers to see if they had similar answers.

Highlight that a cluster can be one or more grouping of points. Because the number of data points in a cluster can vary, identifying a cluster can be subjective.

Ask, "What might cause clustering?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they use the sketch tool to circle clusters in the data. You can overlay student responses to assess their work and provide immediate feedback.

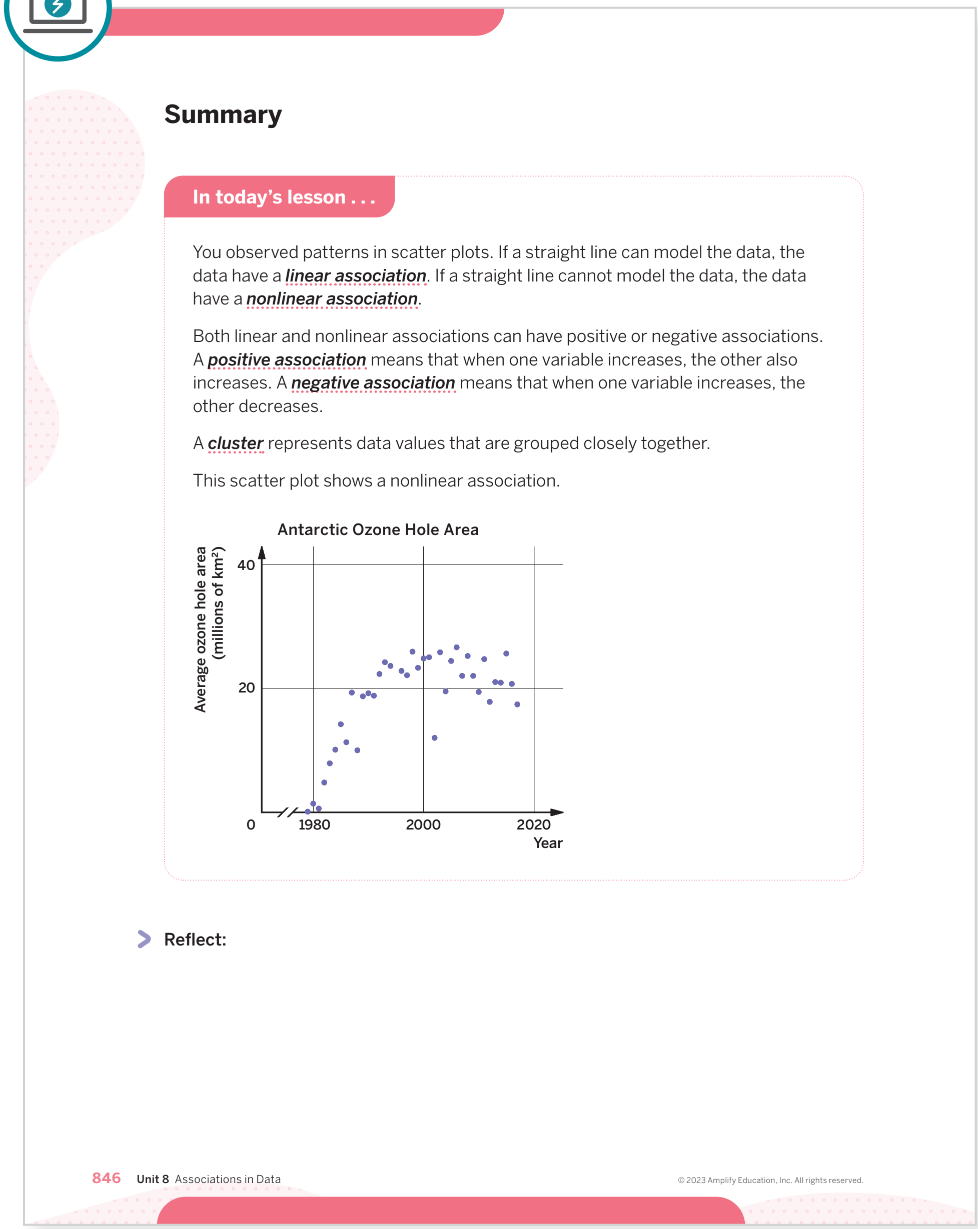
Extension: *Math Enrichment*

Have students think of two real-world quantities that could be compared on a scatter plot, in which clustering of the data is likely. Have them draw a sketch of a scatter plot that they think would represent the quantities. Answers will vary.

Whole Class | 5 min

Summary

Review and synthesize how a scatter plot can show associations between two variables and clustering of data.



Summary

In today's lesson . . .

You observed patterns in scatter plots. If a straight line can model the data, the data have a ***linear association***. If a straight line cannot model the data, the data have a ***nonlinear association***.

Both linear and nonlinear associations can have positive or negative associations. A ***positive association*** means that when one variable increases, the other also increases. A ***negative association*** means that when one variable increases, the other decreases.

A ***cluster*** represents data values that are grouped closely together.

This scatter plot shows a nonlinear association.

Reflect:

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Synthesize

Formalize vocabulary:

- *cluster*
- *linear association*
- *negative association*
- *nonlinear association*
- *positive association*

Have students share each new vocabulary word learned today using precise language.

Display the Anchor Chart PDFs, *Types of Associations in Scatter Plots* and *Clusters in Scatter Plots*.

Ask:

- "What associations, if any, do you see in the Antarctic ozone hole area data? What does this tell you about the ozone hole size?"
- "What clusters, if any, do you see in the data? What does this tell you about the ozone hole size? What might cause the clusters?"

Highlight that the data appear to have a positive linear association until the 1990s at which point the data no longer follow the same linear pattern. Tell students they will learn more about what caused this change in Lesson 7.

Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "How can the type of association help you interpret the data?"
- "What does a cluster represent in data?"

Math Language Development

MLR2: Collect and Display

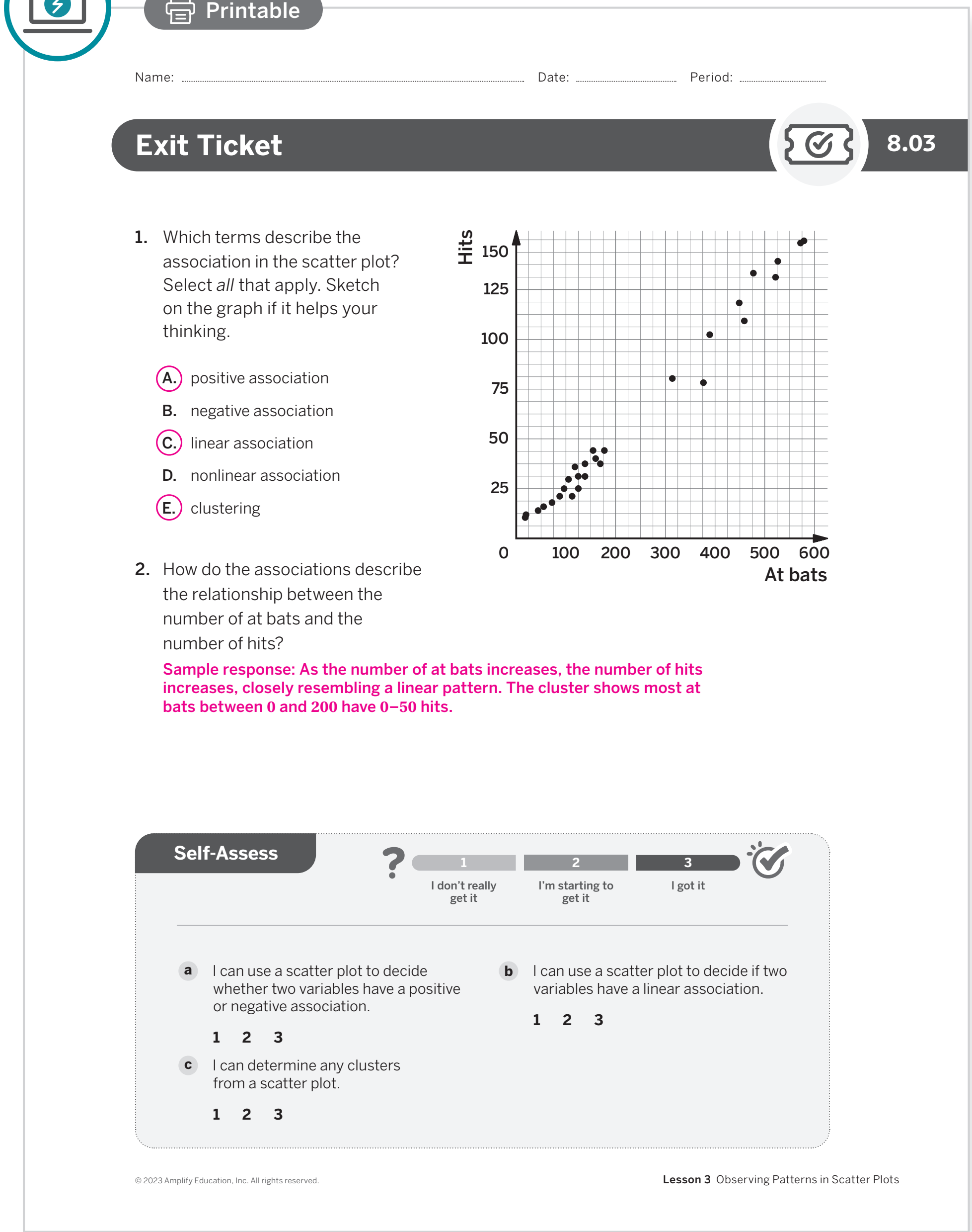
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on the terms and phrases related to the terms *linear association*, *nonlinear association*, *positive association*, *negative association*, and cluster that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying associations and clustering in a scatter plot.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 8.03

1. Which terms describe the association in the scatter plot? Select *all* that apply. Sketch on the graph if it helps your thinking.

[✓ A.] positive association
B. negative association
[✓ C.] linear association
D. nonlinear association
[✓ E.] clustering

2. How do the associations describe the relationship between the number of at bats and the number of hits?

Sample response: As the number of at bats increases, the number of hits increases, closely resembling a linear pattern. The cluster shows most at bats between 0 and 200 have 0–50 hits.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use a scatter plot to decide whether two variables have a positive or negative association.
1 2 3

b I can use a scatter plot to decide if two variables have a linear association.
1 2 3

c I can determine any clusters from a scatter plot.
1 2 3

Lesson 3 Observing Patterns in Scatter Plots

Success looks like . . .

- **Language Goal:** Categorizing data sets, and describing the properties used to create categories. **(Speaking and Listening)**
- **Language Goal:** Describing features of data on scatter plots, including *linear* and *nonlinear association*, *positive* and *negative association*, and *clustering*, using informal language. **(Speaking and Listening)**
- **Language Goal:** Explaining what might cause a nonlinear association or clustering of data points in context. **(Speaking and Listening)**

Suggested next steps

If students do not correctly identify a positive, or linear association, consider:

- Showing diagrams of a linear graph in place of a scatter plot and reviewing positive and negative associations, before repeating with positive and negative scatter plots.
- Showing a diagram of a linear graph highlighted with descriptive words, such as "straight line" or "can increase or decrease." Then present a nonlinear graph and have students state differences from a linear graph.

If students do not identify clusters, consider:

- Displaying three similar graphs, one with clustering and two without clustering. Ask students to choose the graph that has a distinct grouping of points.
- Assigning Practice Problem 2.

Professional Learning

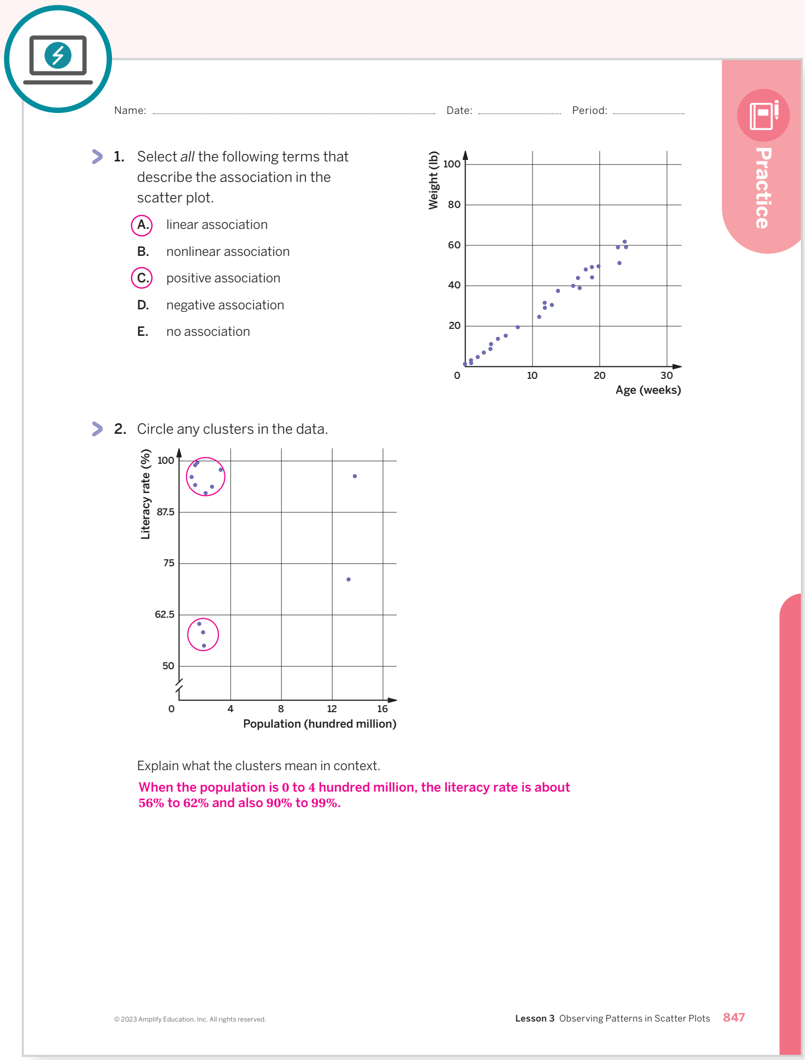
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In what ways did the card sort go as planned?
- Were there any unexpected occurrences during the card sort activity?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Select *all* the following terms that describe the association in the scatter plot.
   - [✓ A.] linear association
   - B. nonlinear association
   - [✓ C.] positive association
   - D. negative association
   - E. no association

2. Circle any clusters in the data.

Explain what the clusters mean in context.

When the population is 0 to 4 hundred million, the literacy rate is about 56% to 62% and also 90% to 99%.

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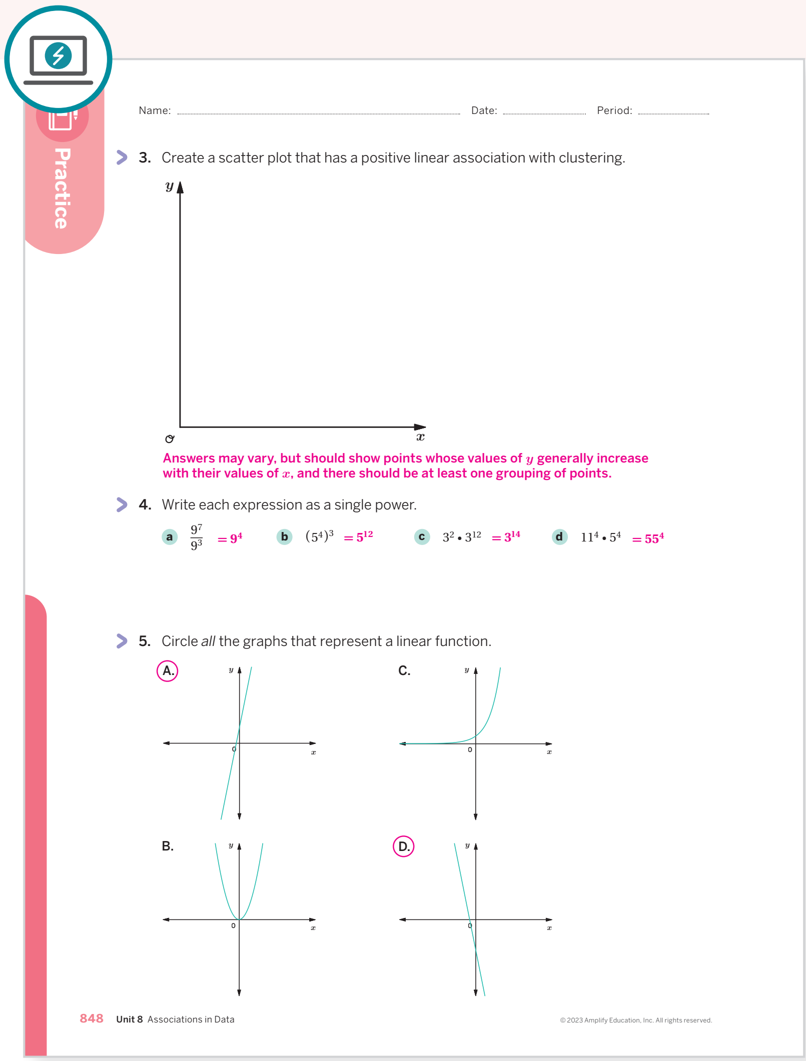
Lesson 3 Observing Patterns in Scatter Plots 847



Name: ______ Date: ______ Period: ______

Practice

3. Create a scatter plot that has a positive linear association with clustering.

Answers may vary, but should show points whose values of y generally increase with their values of x, and there should be at least one grouping of points.

4. Write each expression as a single power.

   a $\frac{9^7}{9^3}$ $= 9^4$ b $(5^4)^3$ $= 5^{12}$ c $3^2 \cdot 3^{12}$ $= 3^{14}$ d $11^4 \cdot 5^4$ $= 55^4$

5. Circle *all* the graphs that represent a linear function.

   [✓ A.] C. B. [✓ D.]

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 2	1
	2	Activity 4	2
	3	Activities 1–4	3
Spiral	4	Unit 6 Lesson 8	1
Formative ⚡	5	Unit 8 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 8 | **LESSON 4**

Fitting a Line to Data

Let's draw a line to fit data.

Focus

Goals

1. **Language Goal:** Critique a given line of fit on a scatter plot, and draw a different linear model of the same data. **(Speaking and Listening, Writing)**
2. **Language Goal:** Draw a linear model to fit data in a scatter plot, and describe features of a line that fits data well. **(Writing)**

Coherence

Today

This lesson kicks off the dive into linear models. Students evaluate how well different lines fit a given scatter plot, and begin modeling the relationship between two variables with a line.

Previously

In Lesson 3, students identified a positive and negative association, linear and nonlinear association, and clustering in scatter plots.

Coming Soon

In Lesson 5, students will use the graph and equation of a linear model to make predictions and solve problems in the context of bivariate measurement data.

Rigor

- Students build **conceptual understanding** of what makes a line of fit a good fit for bivariate data.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Activity 4 (optional)	Summary	Exit Ticket
5 min	8 min	12 min	10 min	7 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- rulers

Math Language Development

Review

- *cluster*
- *linear association*
- *nonlinear association*
- *negative association*
- *positive association*
- *scatter plot*

Amps | Featured Activity

Activity 1

Interactive Lines of Fit

Given a scatter plot, students manipulate a line and observe how a score meter changes in real time, based on the positioning of the line.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

At first, students might feel stressed that others give a different answer in Activity 3. Remind students to listen actively to the arguments of others in order to better understand the solution and possibly even modify their own.

Modifications to Pacing

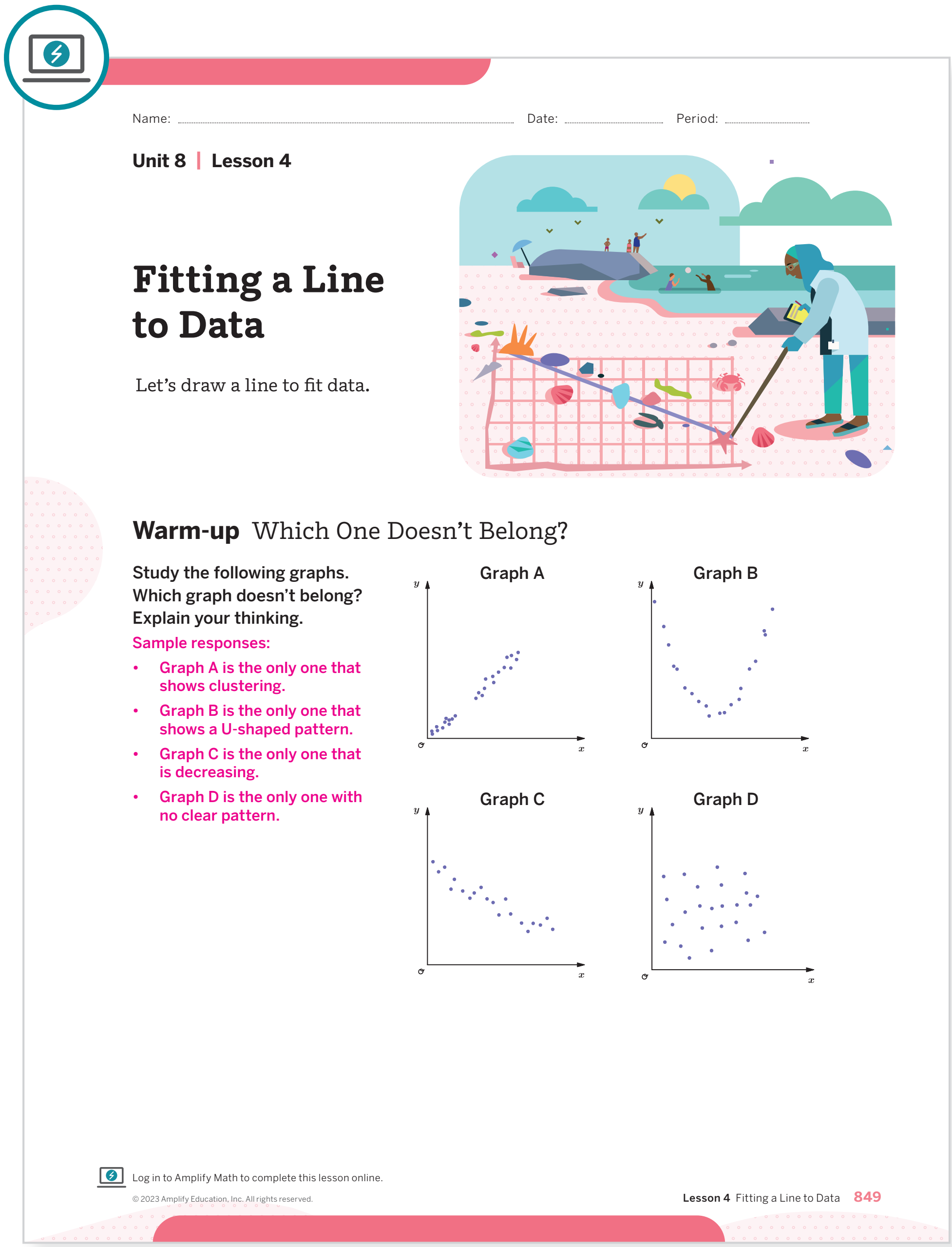
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- **Activity 3** may be omitted.
- Optional **Activity 4** may be omitted.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students analyze linear and nonlinear scatter plots to create a need for a more efficient way to check whether a scatter plot shows linear association.



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Unit 8 | Lesson 4

Fitting a Line to Data

Let's draw a line to fit data.

Warm-up Which One Doesn't Belong?

Study the following graphs. Which graph doesn't belong? Explain your thinking.

Sample responses:

- Graph A is the only one that shows clustering.
- Graph B is the only one that shows a U-shaped pattern.
- Graph C is the only one that is decreasing.
- Graph D is the only one with no clear pattern.

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the ***Which One Doesn't Belong?*** routine. Be sure students understand that there can be more than one correct response. Encourage them to find a reason why each graph might not belong with the others.

2 Monitor

Help students get started by asking them to choose one graph and identify what makes it different from the other graphs.

Look for productive strategies:

- Noticing clusters in Graph A.
- Noticing that Graph D has neither a positive nor negative association.
- Identifying any one graph that is different. Each graph has a feature that makes it different from the others. Challenge students to find the feature in each graph that makes it different from the others.

3 Connect

Have students share which graph they chose. Conduct the ***Poll the Class*** routine to see who chose Graphs A and C. Have a student share aloud with the class why their chosen graph does not belong. Ask them to be as precise as possible. Discuss Graphs A and C last to transition into Activity 1.

Highlight that Graphs B and D do not represent a linear association.

Ask, "What can you do to determine whether the scatter plot has a linear pattern?" Sample response: Draw a straight line on the graph to see if the line passes through most of the data points.

Power-up

To power up students' ability to identify linear functions, have students complete:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

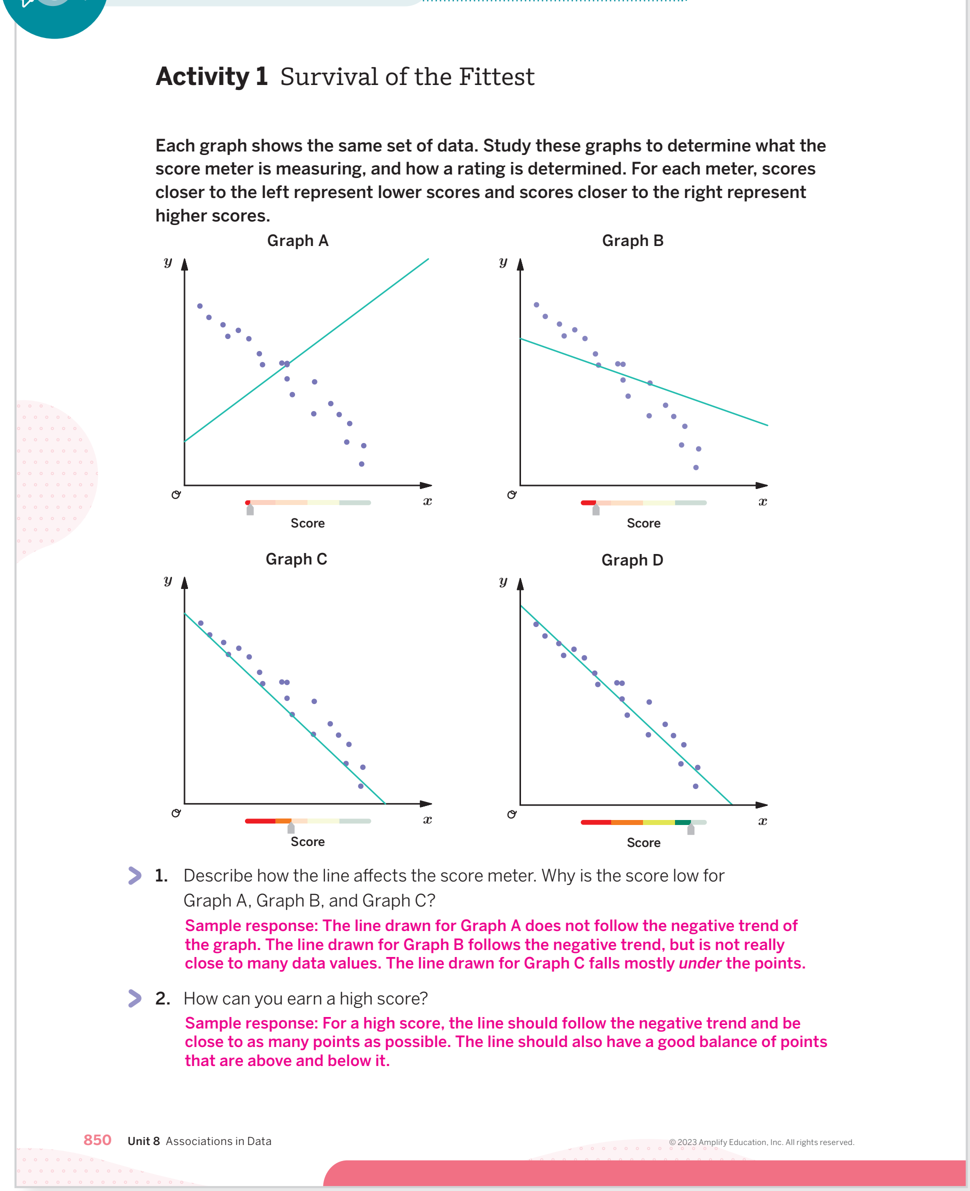
Informed by: Performance on Lesson 3, Practice Problem 5

Pairs | 8 min

Activity 1 Survival of the Fittest

Students analyze several scatter plots and lines to determine what makes a good line of fit.

Amps Featured Activity Interactive Lines of Fit



Activity 1 Survival of the Fittest

Each graph shows the same set of data. Study these graphs to determine what the score meter is measuring, and how a rating is determined. For each meter, scores closer to the left represent lower scores and scores closer to the right represent higher scores.

1. Describe how the line affects the score meter. Why is the score low for Graph A, Graph B, and Graph C?

   Sample response: The line drawn for Graph A does not follow the negative trend of the graph. The line drawn for Graph B follows the negative trend, but is not really close to many data values. The line drawn for Graph C falls mostly *under* the points.

2. How can you earn a high score?

   Sample response: For a high score, the line should follow the negative trend and be close to as many points as possible. The line should also have a good balance of points that are above and below it.

850 Unit 8 Associations in Data

1 Launch

Activate students' background knowledge by asking them when a score meter would be used and how a score meter works.

2 Monitor

Help students get started by asking, "How does the line compare to the points when the meter shows a low score? A high score?"

Look for points of confusion:

- **Thinking the meter gives a high score when the line passes through a lot of points.** Have students compare the Graphs C and D. Graph C's line passes through more points than Graph D's line, but does not have a higher score.

Look for productive strategies:

- Noticing that a high score has a line that follows the trend of the data and is close to many of the data values.

3 Connect

Have students share their responses. Ask for input from peers to see if they had similar conclusions.

Highlight that the score meter measures how well the line fits the data. When a scatter plot models a linear association, students can draw a line to model the data.

Ask, "What makes a good line of fit?"

Note: Sometimes called lines of best fit, students will be formally introduced to these lines as *linear models* in Lesson 5.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate the graph of a line and observe how a score meter changes in real time, based on the positioning of the line and how well it fits the data.

Accessibility: *Guide Processing and Visualization*

Help students interpret the score meter by annotating the left side of the score meters with "low score" and the right side with "high score."

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, reduce the number of tasks by having them only analyze Graphs A, C, and D.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, draw connections between the scores shown on the score meters with how well each line drawn fits the data. Ask:

- "What is different about each line?"
- "What do you notice about the line with the lowest score? The highest score?"

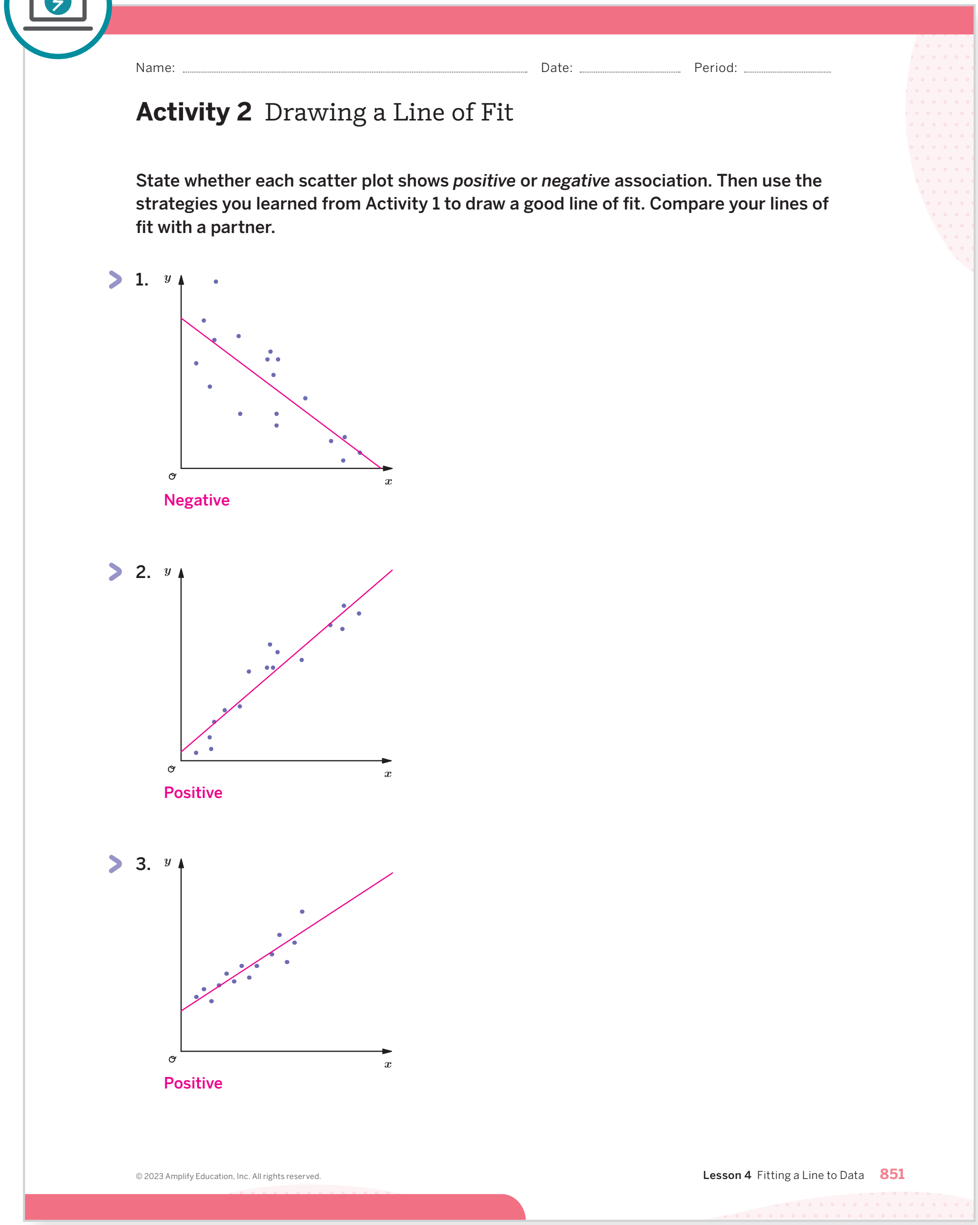
English Learners

Annotate the graphs by writing "low score" for Graphs A and B and "high score" for Graph D.

Pairs | 12 min

Activity 2 Drawing a Line of Fit

Students draw lines of fit and compare their lines with their partners.



Name: ______ Date: ______ Period: ______

Activity 2 Drawing a Line of Fit

State whether each scatter plot shows *positive* or *negative* association. Then use the strategies you learned from Activity 1 to draw a good line of fit. Compare your lines of fit with a partner.

1. Negative

2. Positive

3. Positive

Lesson 4 Fitting a Line to Data 851

1 Launch

Distribute rulers and conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by asking, "How can knowing the association help you draw a line of fit?"

Look for points of confusion:

- **Thinking their line must pass through as many points as possible.** Refer to Graphs C and D from Activity 1. Graph C has a line that passes through more points, but it does not have a higher score.

Look for productive strategies:

- Having the same number of points above and below the line they drew. Their line should follow the trend, while also balancing points above and below the line.
- Drawing an oval around the set of data and using it to help fit the line.

3 Connect

Display the different lines students drew.

Have students share the line they drew with their partner. Students can discuss which line is better and why.

Highlight that knowing the association helps students to draw a line of fit.

Ask, "How did you decide where to draw your line of fit?"

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to rulers, straightedges, or index cards that students can use to draw their lines of fit.

Accessibility: *Guide Processing and Visualization*

Suggest students refer to the scatter plots from Activity 1. Ask, "What do you think the lines drawn from each data point to the line drawn represent? Can you draw similar lines here in Activity 2 to help you draw a good line of fit?"

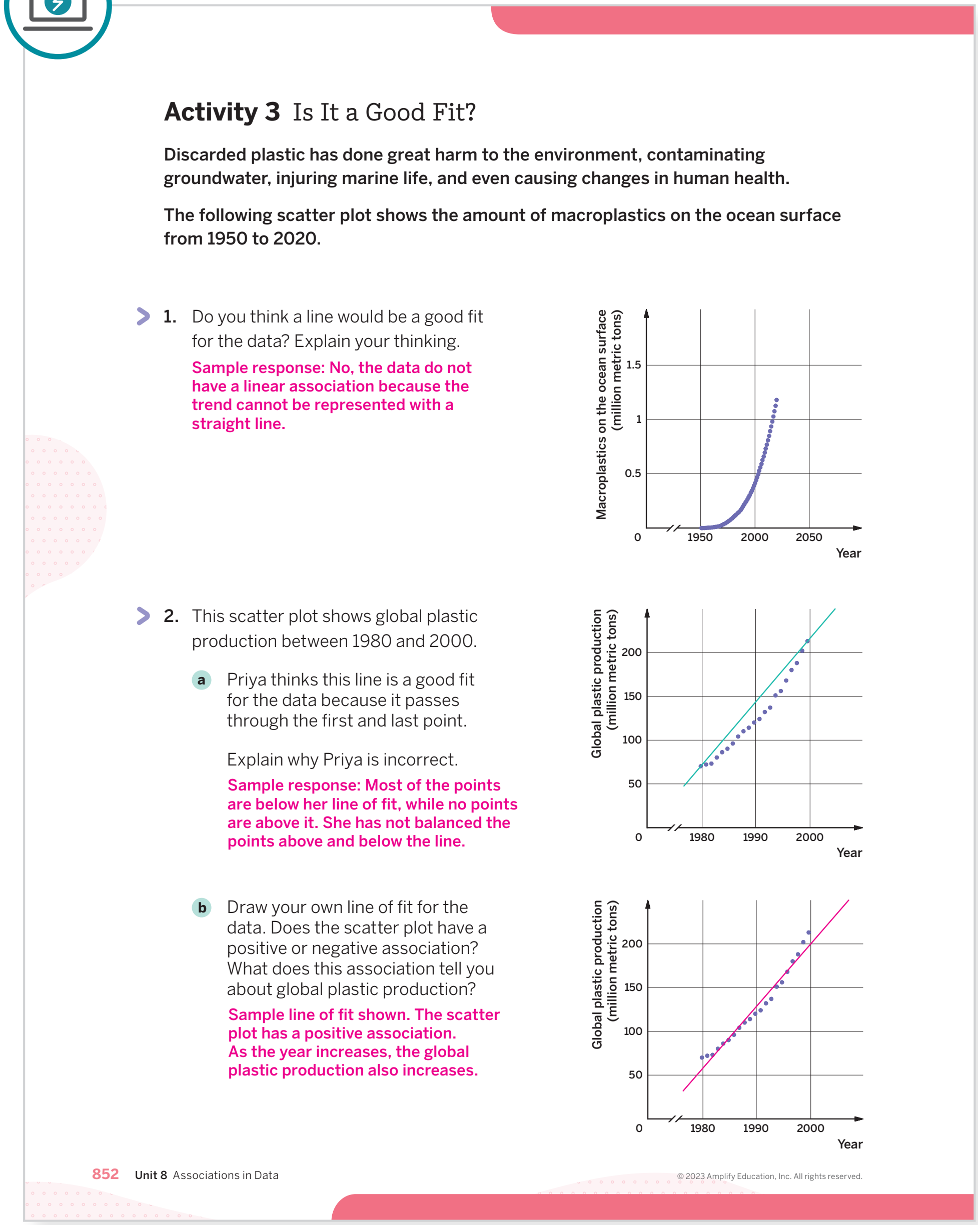
Extension: *Math Enrichment*

Challenge students to add 4 more data points to the scatter plot in Problem 1 so that the line of fit would still be the same. Answers will vary.

Pairs | 10 min

Activity 3 Is It a Good Fit?

Students critique two students' lines of fit, and draw their own line of fit, for the same data.



Activity 3 Is It a Good Fit?

Discarded plastic has done great harm to the environment, contaminating groundwater, injuring marine life, and even causing changes in human health.

The following scatter plot shows the amount of macroplastics on the ocean surface from 1950 to 2020.

1. Do you think a line would be a good fit for the data? Explain your thinking.

   Sample response: No, the data do not have a linear association because the trend cannot be represented with a straight line.

2. This scatter plot shows global plastic production between 1980 and 2000.

   a. Priya thinks this line is a good fit for the data because it passes through the first and last point.

   Explain why Priya is incorrect.

   Sample response: Most of the points are below her line of fit, while no points are above it. She has not balanced the points above and below the line.

   b. Draw your own line of fit for the data. Does the scatter plot have a positive or negative association? What does this association tell you about global plastic production?

   Sample line of fit shown. The scatter plot has a positive association. As the year increases, the global plastic production also increases.

1 Launch

Say, "Another environmental problem is the increase of global plastic production. You are looking at two graphs. The first graph shows the amount of *macroplastic* on the surface of the ocean. The second graph shows the global plastic production."

Note: Macroplastics are buoyant plastic materials with diameters greater than 0.5 cm.

2 Monitor

Help students get started by asking, "Do the data show a positive or negative association? Where should the line be compared to the points?"

Look for points of confusion:

- **Thinking that a line can or cannot model the data for Problem 1.** Allow for either answer. Have students explain their thinking, and revisit these students during the Connect.
- **Thinking Priya is correct.** Have students look back to Activity 1, to review what makes a good line of fit. Then ask, "Does a line of fit necessarily need to connect the first and last point?"

3 Connect

Have students share reasons why a line could and could not model the data in Problem 1. Then have students share why Priya is incorrect in Problem 2.

Highlight that when drawing a line of fit, it is important to consider the whole data set, not just a few points.

Ask, "Why might a line be added to a scatter plot?" Sample responses: To show a linear trend, to predict a certain value, to show an association between the variables.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they have ever seen discarded plastic wash up on the shores of a beach or be found in a river. Consider showing them images from the internet or another source that show discarded plastic in oceans, rivers, or on beaches.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory text and graph from Problem 1. Ask pairs of students to work together to craft 2–3 questions they could ask about this scenario or graph. Sample questions shown.

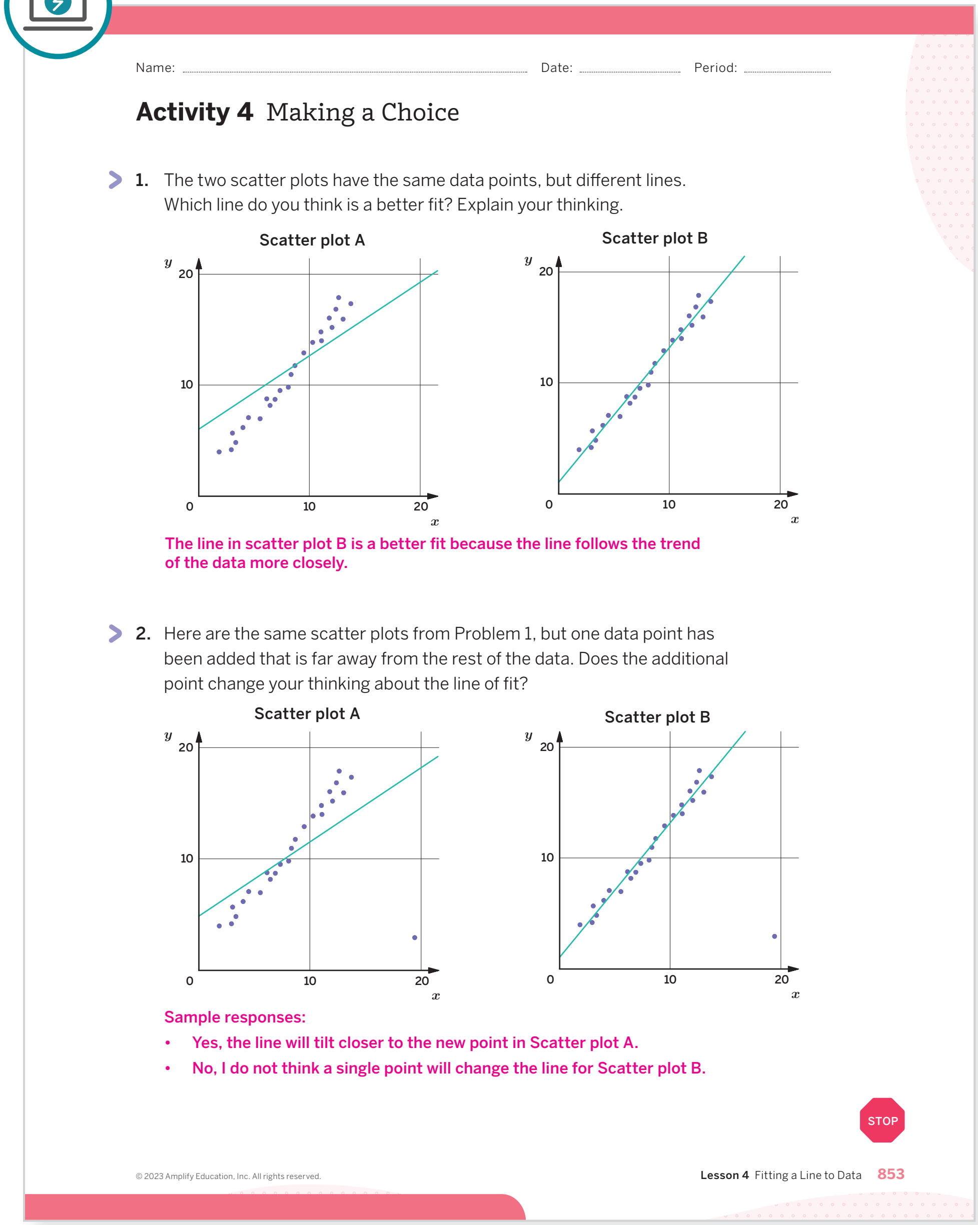
- Do the data show a linear or nonlinear association?
- Why is the graph increasing so rapidly since 1950?
- What can you do to decrease the amount of discarded plastic on the ocean's surface?

English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Activity 4 Making a Choice

Students choose a better line of fit when an outlier is present. This optional activity is meant for students to create viable arguments without being introduced to the term outlier.



Name: ______ Date: ______ Period: ______

Activity 4 Making a Choice

1. The two scatter plots have the same data points, but different lines. Which line do you think is a better fit? Explain your thinking.

The line in scatter plot B is a better fit because the line follows the trend of the data more closely.

2. Here are the same scatter plots from Problem 1, but one data point has been added that is far away from the rest of the data. Does the additional point change your thinking about the line of fit?

Sample responses:

- Yes, the line will tilt closer to the new point in Scatter plot A.
- No, I do not think a single point will change the line for Scatter plot B.

STOP

1 Launch

Tell students they will look at two lines of fit for the same data. They will decide which line is a better fit and support their response using mathematical reasoning.

2 Monitor

Help students get started by asking, "Which line would receive a higher score on the score meter from Activity 1?"

Look for points of confusion:

- **Having a difficult time deciding if the additional point in Problem 2 changes their thinking.** Have students list why each line is a good fit for each graph, and then choose the graph with more reasons.

Look for productive strategies:

- Explaining a line of fit is good because it balances the data. Remind students that balancing the data is not the only factor that makes a good line of fit.

3 Connect

Have students share which graph they chose for Problem 2. Have a volunteer share aloud with the class why they made their choice.

Highlight the point farthest from the rest of the data. This point makes it hard to decide which line is a better fit. In this case, they both can be a good line of fit, because the line that appears to not follow the trend of the other data values is slanted in a direction toward this point.

Ask, "Should you look at all the data points to fit a line to data?" Sample response: It depends on the context of the data.

Note: Students will be formally introduced to the term *outlier* in Lesson 5.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students respond to Problems 1 and 2, have them meet with 2–3 other students to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

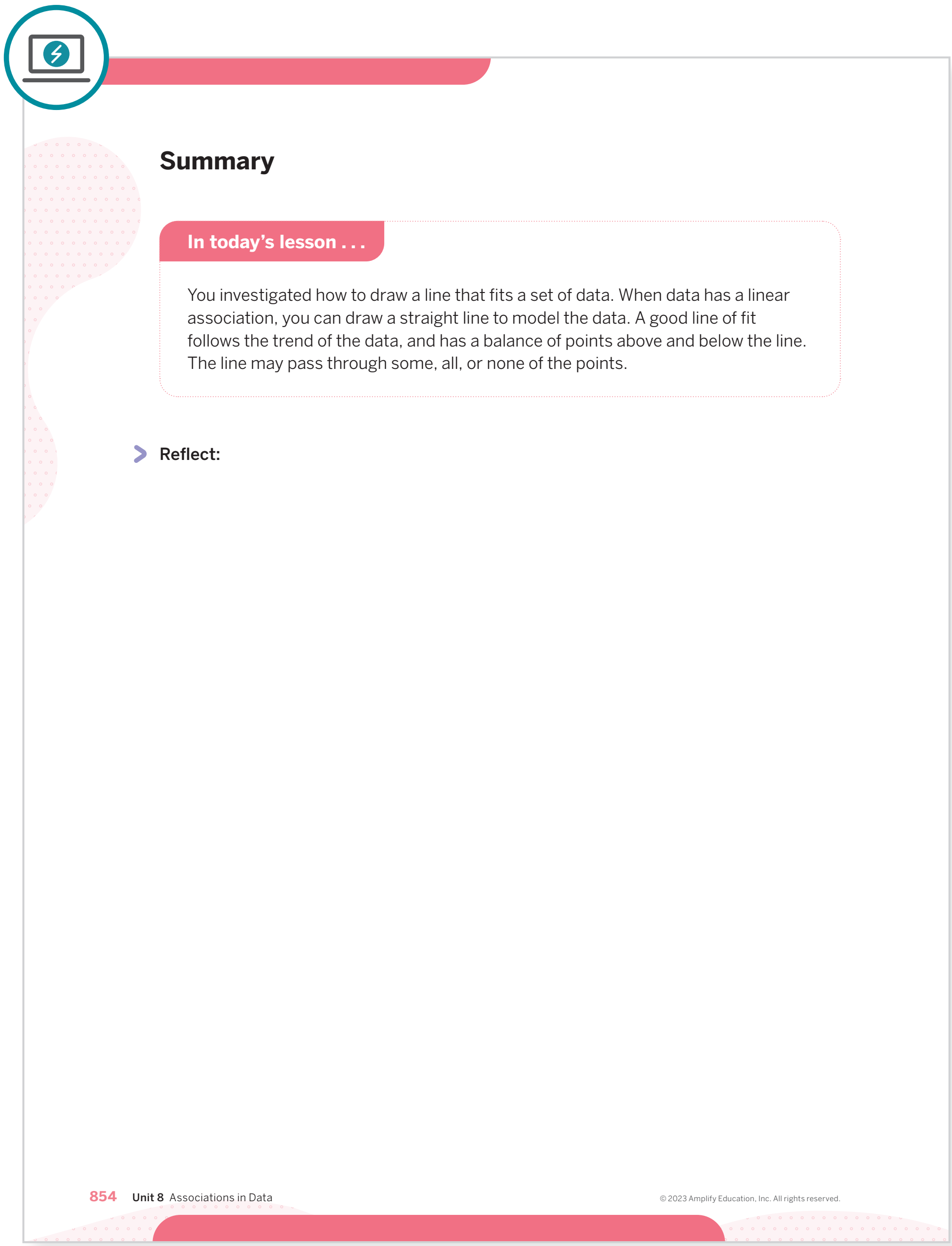
- "What math language can you use in your response to each problem?"
- "Can you explain to me why you think the line of fit in Problem 2 should change or not change?"

Have students write a final response, based on the feedback they received.

Whole Class | 5 min

Summary

Review and synthesize how a good line of fit follows the trend of the data, and balances the points above and below the line.



Summary

In today's lesson . . .

You investigated how to draw a line that fits a set of data. When data has a linear association, you can draw a straight line to model the data. A good line of fit follows the trend of the data, and has a balance of points above and below the line. The line may pass through some, all, or none of the points.

Reflect:

854 Unit 8 Associations in Data

Synthesize

Have students share what makes a good line of fit with their partner before sharing with the whole class.

Highlight that a line of fit should follow the trend of the data. It is important to consider the whole data set, not just a few points. On the Unit Anchor Chart PDF, *Types of Associations in Scatter Plots*, draw a line of fit for the scatter plots shown under linear association.

Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

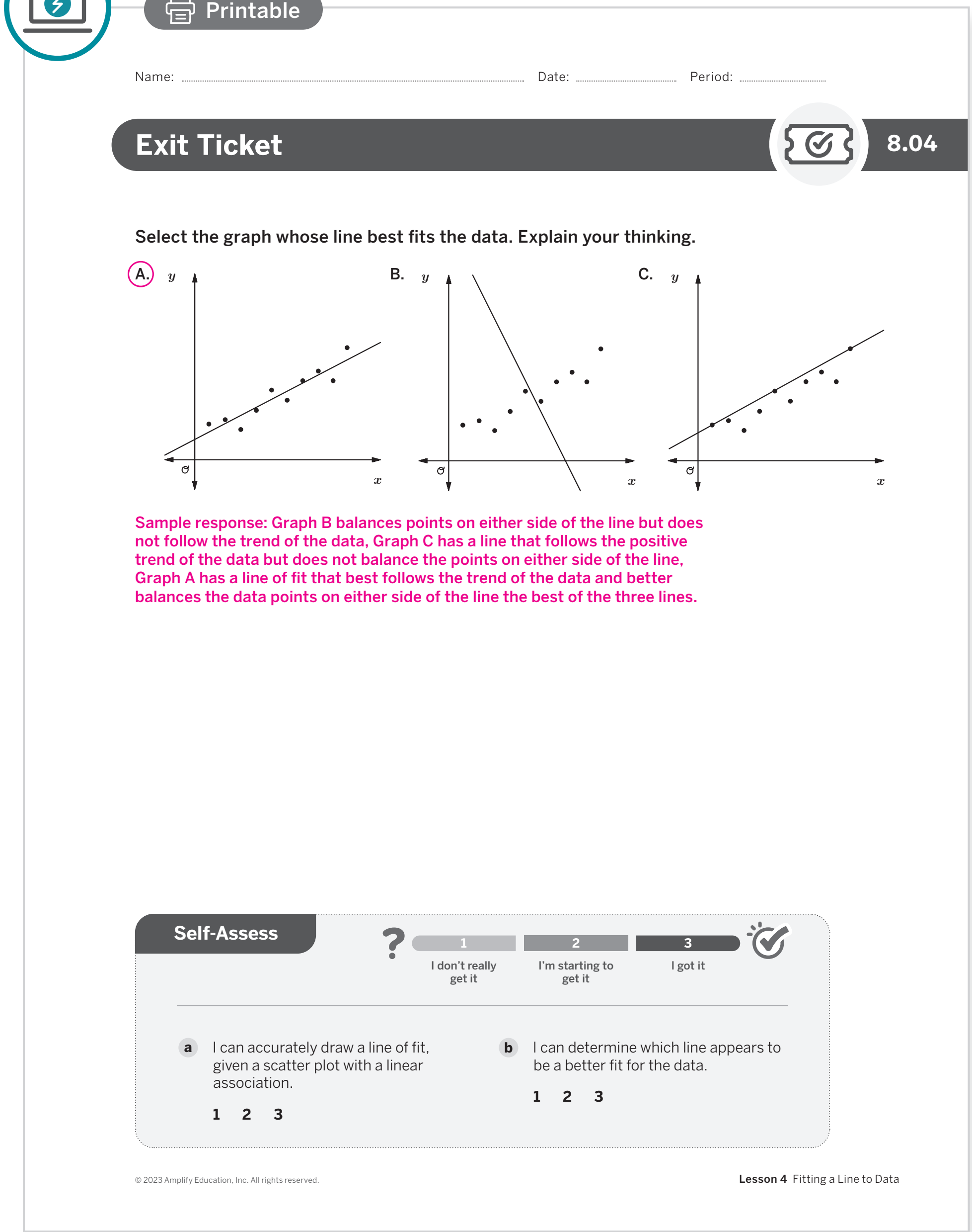
- "What makes a good line of fit?"
- "What makes a bad line of fit?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by comparing three different lines with the same data set to determine the best fit.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 8.04

Select the graph whose line best fits the data. Explain your thinking.

Sample response: Graph B balances points on either side of the line but does not follow the trend of the data, Graph C has a line that follows the positive trend of the data but does not balance the points on either side of the line, Graph A has a line of fit that best follows the trend of the data and better balances the data points on either side of the line the best of the three lines.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can accurately draw a line of fit, given a scatter plot with a linear association.
1 2 3

b. I can determine which line appears to be a better fit for the data.
1 2 3

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Lesson 4 Fitting a Line to Data

Success looks like . . .

- **Language Goal:** Critiquing a given line of fit on a scatter plot, and drawing a different linear model of the same data. **(Speaking and Listening, Writing)**
  - » Selecting the graph with the line that best fits the data.
- **Language Goal:** Drawing a linear model to fit data in a scatter plot, and describing features of a line that fits data well. **(Writing)**

Suggested next steps

If students select Graph B, consider:

- Showing multiple scatter plots without a line of fit. Ask students if the scatter plot shows a positive or negative association.
- Reviewing Graph A's low meter rating from Activity 1.

If students select Graph C, consider:

- Telling students to look closely at the points. Students may find it helpful to think of the line as a hot dog and the group of data values as a hot dog bun.
- Reviewing Graph C's meter rating from Activity 1.

Professional Learning

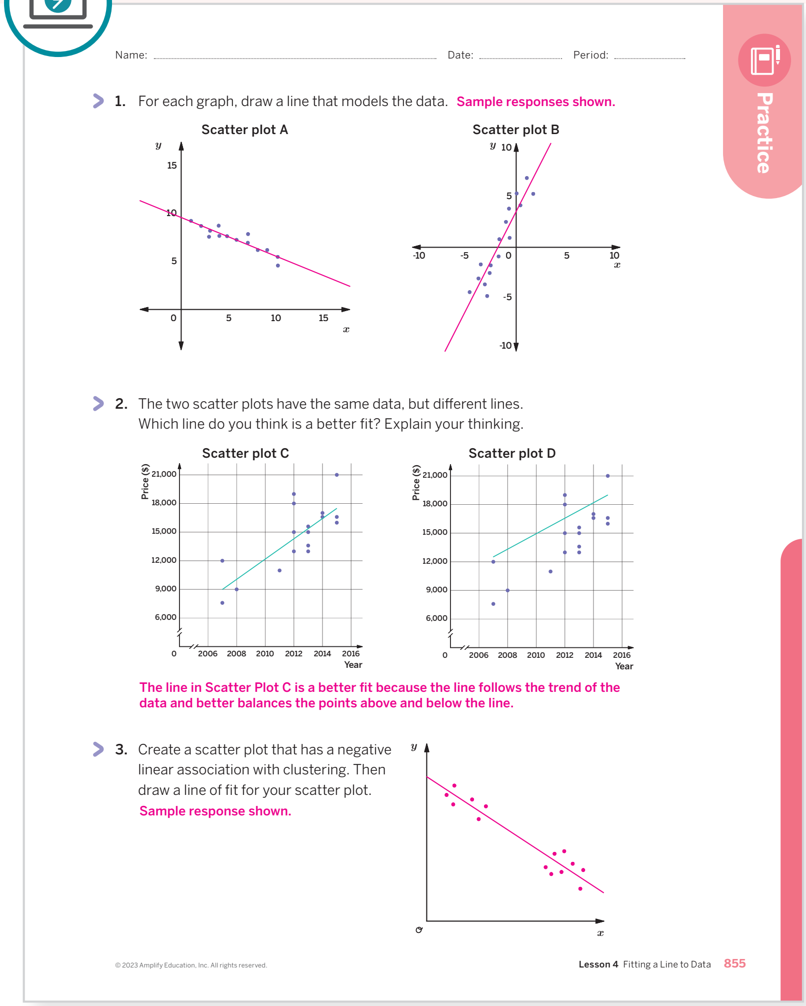
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Which students' ideas were you able to highlight during Activity 1?
- What challenges did students encounter as they worked on Activity 3? How did they work through them?

Practice

Independent



Name: Date: Period:

1. For each graph, draw a line that models the data. **Sample responses shown.**

2. The two scatter plots have the same data, but different lines. Which line do you think is a better fit? Explain your thinking.

The line in Scatter Plot C is a better fit because the line follows the trend of the data and better balances the points above and below the line.

3. Create a scatter plot that has a negative linear association with clustering. Then draw a line of fit for your scatter plot.
Sample response shown.

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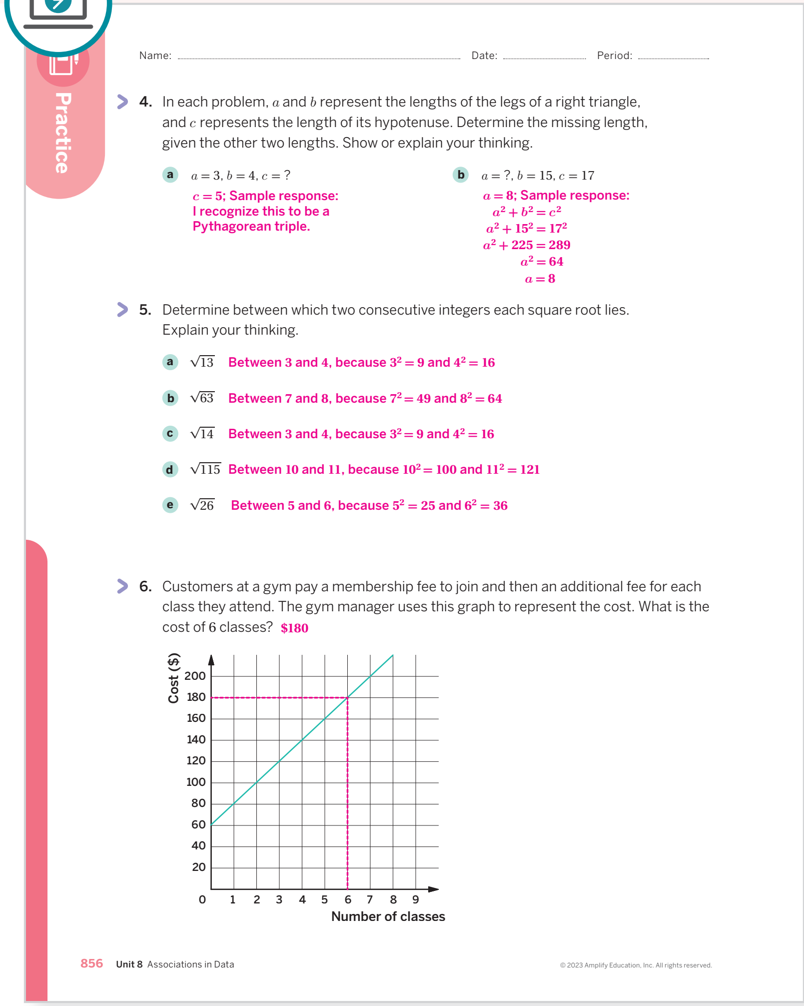
Lesson 4 Fitting a Line to Data 855



Name: Date: Period:

4. In each problem, a and b represent the lengths of the legs of a right triangle, and c represents the length of its hypotenuse. Determine the missing length, given the other two lengths. Show or explain your thinking.

a $a = 3, b = 4, c = ?$

$c = 5$; Sample response: I recognize this to be a Pythagorean triple.

b $a = ?, b = 15, c = 17$

$a = 8$; Sample response:

$$a^2 + b^2 = c^2$$
$$a^2 + 15^2 = 17^2$$
$$a^2 + 225 = 289$$
$$a^2 = 64$$
$$a = 8$$

5. Determine between which two consecutive integers each square root lies. Explain your thinking.

a $\sqrt{13}$ **Between 3 and 4, because $3^2 = 9$ and $4^2 = 16$**

b $\sqrt{63}$ **Between 7 and 8, because $7^2 = 49$ and $8^2 = 64$**

c $\sqrt{14}$ **Between 3 and 4, because $3^2 = 9$ and $4^2 = 16$**

d $\sqrt{115}$ **Between 10 and 11, because $10^2 = 100$ and $11^2 = 121$**

e $\sqrt{26}$ **Between 5 and 6, because $5^2 = 25$ and $6^2 = 36$**

6. Customers at a gym pay a membership fee to join and then an additional fee for each class they attend. The gym manager uses this graph to represent the cost. What is the cost of 6 classes? **$180**

856 **Unit 8** Associations in Data

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 3	2
	3	Activity 2	2
Spiral	4	Unit 7 Lesson 11	1
	5	Unit 7 Lesson 4	2
Formative ⚡	6	Unit 8 Lesson 5	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

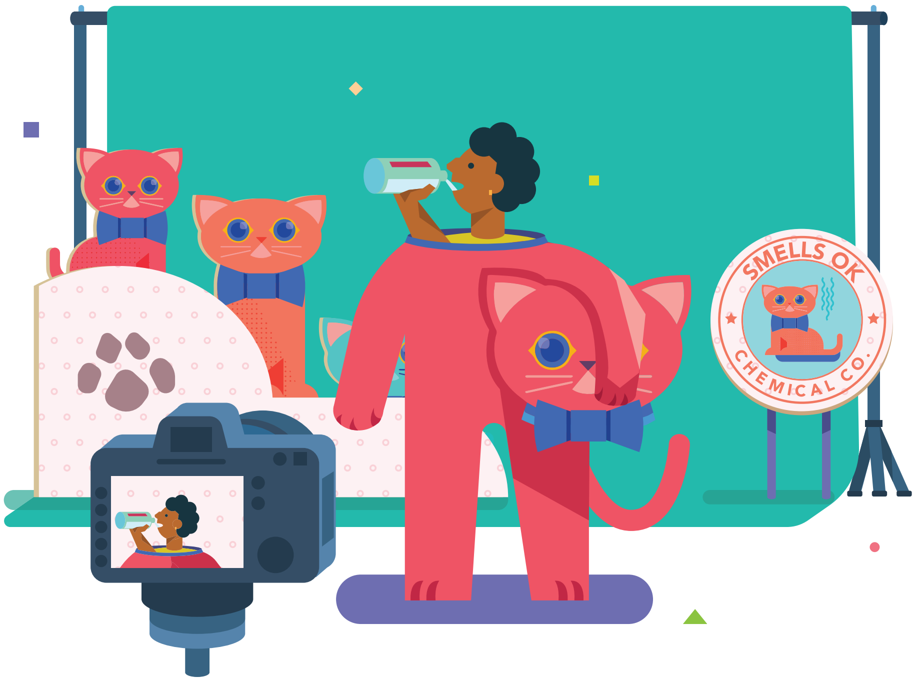
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 8 | **LESSON 5**

Using a Linear Model

Let's identify potential outliers and use a linear model to predict values.



Focus

Goals

1. **Language Goal:** Compare and contrast values in a data set with predictions made using a given line. **(Speaking and Listening)**
2. Comprehend that a model of data, such as a line of fit, can be used to predict values that are not given in the data.
3. **Language Goal:** Identify potential outliers on a scatter plot. **(Speaking and Listening)**

Rigor

- Students develop **conceptual understanding** of how linear models can be used to represent bivariate data and predict values.
- Student build **fluency** skills in writing and using linear models to predict values.

Coherence

Today

Students compare Chemy Cat logo designs and identify potential outliers in a scatter plot. Students describe patterns in the scatter plot, find a need to create a linear model, and use the linear model to predict values that are not given in the data.

Previously

In Lesson 4, students reasoned about the fit of lines for a given scatter plot and drew their own lines to fit data in a scatter plot.

Coming Soon

In Lesson 6, students will move from general descriptions of association to more detailed descriptions of what the slope and y-intercept of a fitted line represent in context.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	7 min	8 min	5 min	5 min
Independent	Small Groups	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cats, one set per group
- Anchor Chart PDF, *Clusters in Scatter Plots*
- Anchor Chart PDF, *Representations of Linear Relationships* (from Unit 3)
- rulers

Math Language Development

New

- ***linear model***
- ***outlier***

Review

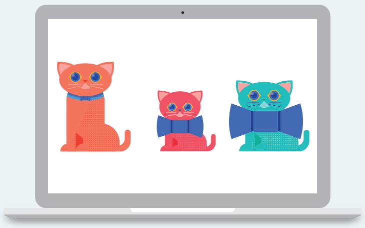
- *negative association*
- *positive association*
- *scatter plot*

Amps | Featured Activity

Activity 1

Interactive Chemy Cats

Students move a point on a graph to see how its placement changes a cat's bow-tie width and receive immediate feedback when they click the *Try It* button.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to motivate themselves to find patterns in the structure of a scatter plot in Activity 1. Ask them to shift their perspective from the scatter plot as a whole to individual points to identify points that do not fit the pattern.

Modifications to Pacing

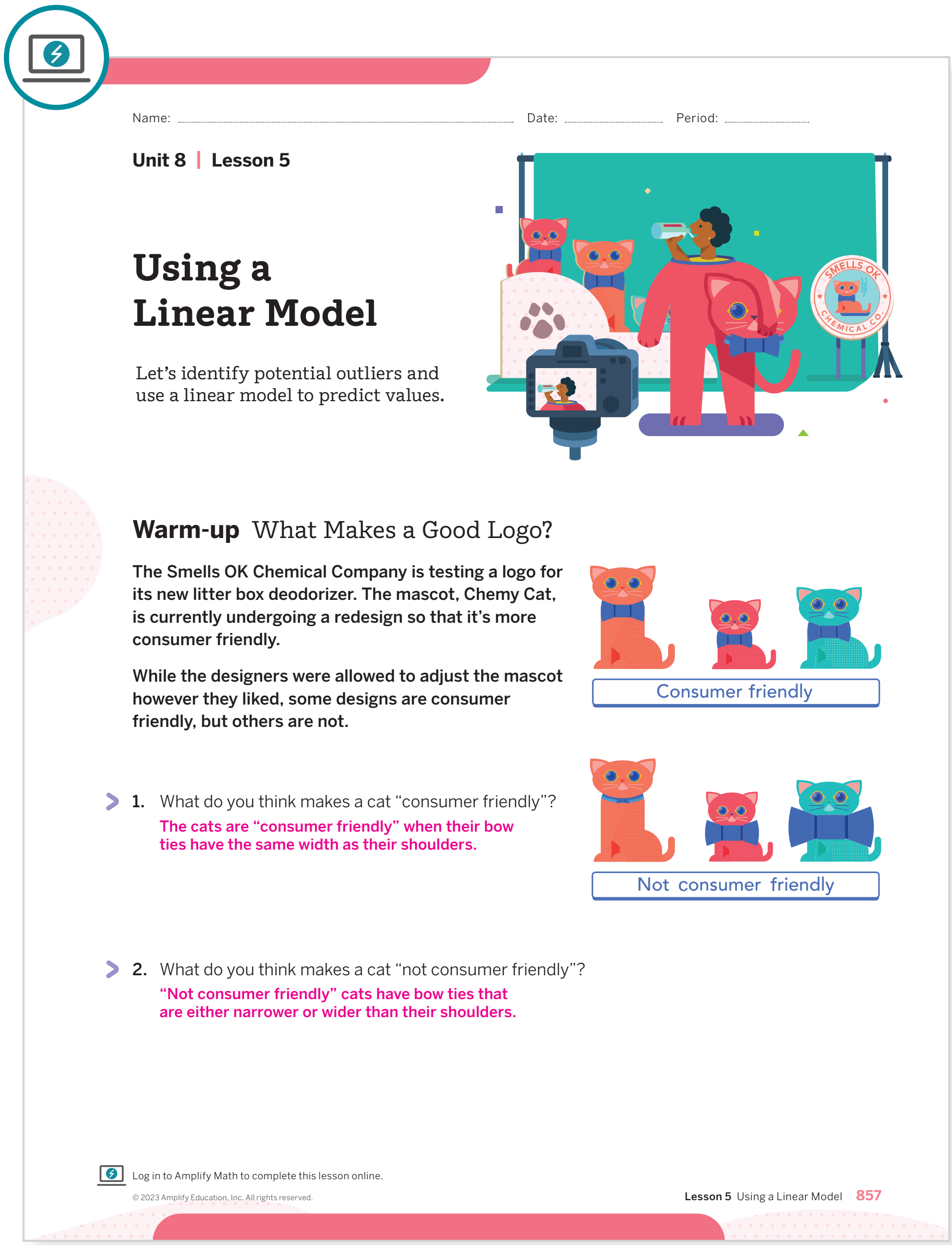
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, provide the measurements of each cat instead of having students measure them.
- **Activity 2** may be omitted.

Independent | 5 min

Warm-up What Makes a Good Logo?

Students informally analyze cat height and bow-tie width ratios to identify a logo design that is "consumer friendly."



Name: ______ Date: ______ Period: ______

Unit 8 | **Lesson 5**

Using a Linear Model

Let's identify potential outliers and use a linear model to predict values.

Warm-up What Makes a Good Logo?

The Smells OK Chemical Company is testing a logo for its new litter box deodorizer. The mascot, Chemy Cat, is currently undergoing a redesign so that it's more consumer friendly.

While the designers were allowed to adjust the mascot however they liked, some designs are consumer friendly, but others are not.

1. What do you think makes a cat "consumer friendly"?
   The cats are "consumer friendly" when their bow ties have the same width as their shoulders.

2. What do you think makes a cat "not consumer friendly"?
   "Not consumer friendly" cats have bow ties that are either narrower or wider than their shoulders.

Log in to Amplify Math to complete this lesson online.

① Launch

Activate students' background knowledge by asking them to think about a company logo they have seen and how that logo might represent the company.

② Monitor

Help students get started by asking how the width of each bow tie compares to the cat's height.

Look for points of confusion:

- **Thinking all the cat bow ties have the same width.** List measurements for each bow-tie width.

③ Connect

Ask, "What makes a cat 'consumer friendly' or 'not consumer friendly'?"

Have students share how the bow-tie width and height affect logo designs that are "consumer friendly" and "not consumer friendly."

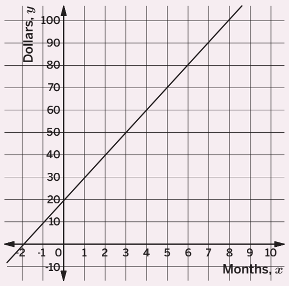
Highlight the ratio of the cat's height to its bow-tie width.

Power-up

To power up students' ability to determine a value given a line on a graph, have students complete:

Refer to the graph representing the cost y for x months at a local gym. Which of the following statements is true? Select *all* that apply.

A. The fee for 1 month is $30.
B. 70 months cost $5.
C. The cost for 6 months is $20.
[✓ **D.**] The starting fee (cost at 0 months) is $20.
[✓ **E.**] The cost for 4 months $60.



Use: Before Activity 1

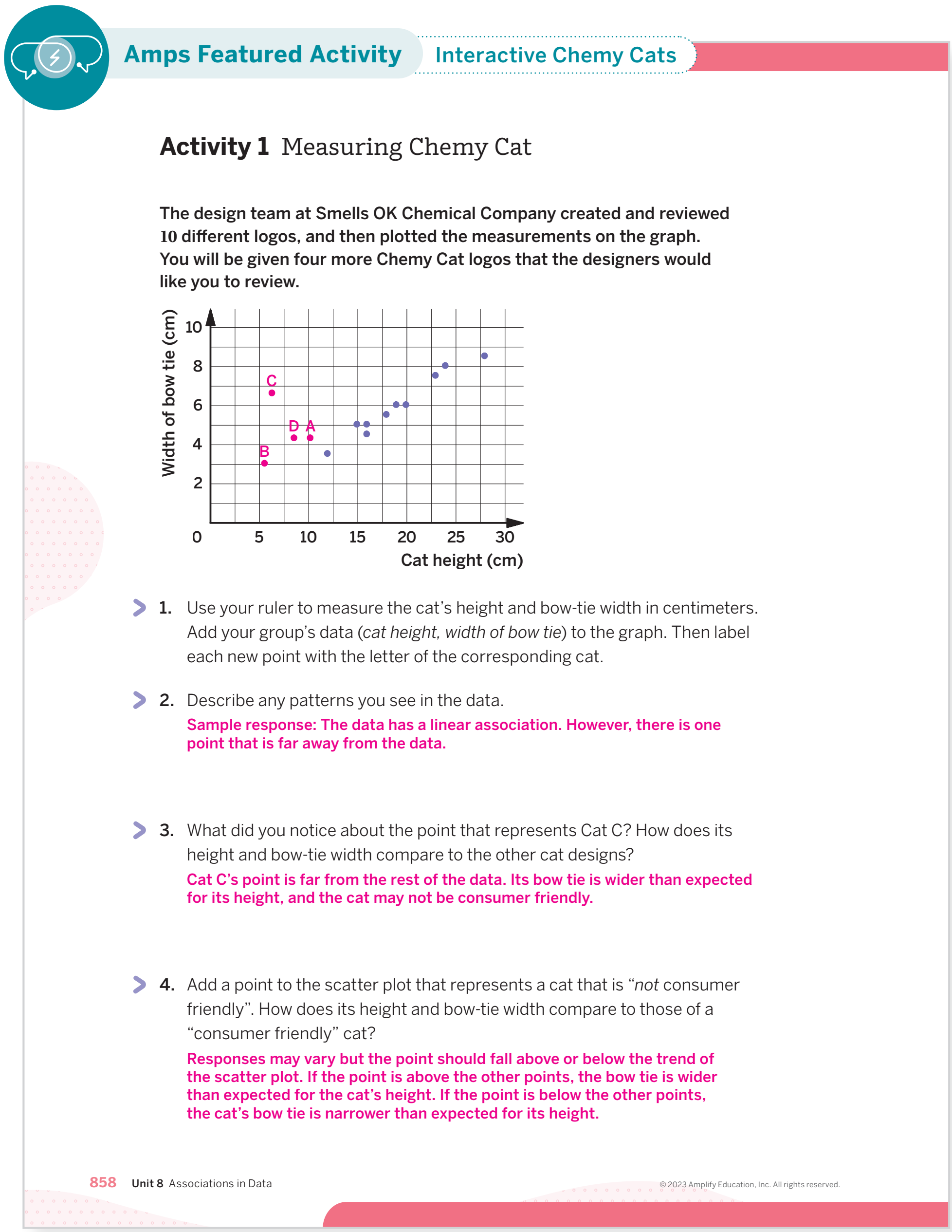
Informed by: Performance on Lesson 4, Practice Problem 6

Small Groups | 15 min

Activity 1 Measuring Chemy Cat

Students measure cat logo designs and analyze scatter plots to determine which designs are "consumer friendly".

Amps Featured Activity Interactive Chemy Cats



Activity 1 Measuring Chemy Cat

The design team at Smells OK Chemical Company created and reviewed 10 different logos, and then plotted the measurements on the graph. You will be given four more Chemy Cat logos that the designers would like you to review.

1. Use your ruler to measure the cat's height and bow-tie width in centimeters. Add your group's data (*cat height, width of bow tie*) to the graph. Then label each new point with the letter of the corresponding cat.

2. Describe any patterns you see in the data.
   Sample response: The data has a linear association. However, there is one point that is far away from the data.

3. What did you notice about the point that represents Cat C? How does its height and bow-tie width compare to the other cat designs?
   Cat C's point is far from the rest of the data. Its bow tie is wider than expected for its height, and the cat may not be consumer friendly.

4. Add a point to the scatter plot that represents a cat that is "*not* consumer friendly". How does its height and bow-tie width compare to those of a "consumer friendly" cat?
   Responses may vary but the point should fall above or below the trend of the scatter plot. If the point is above the other points, the bow tie is wider than expected for the cat's height. If the point is below the other points, the cat's bow tie is narrower than expected for its height.

858 **Unit 8** Associations in Data

1 Launch

Provide each group of students rulers and a set of cats from the Activity 1 PDF. Have them measure the cat's height and bow-tie width in centimeters. Demonstrate how to find the cat's height by measuring from its feet to the top of its head.

2 Monitor

Look for points of confusion:

- **Plotting the values of x and y incorrectly.** Write (x, y) and *(cat height, bow-tie width)* on the board for students to reference.
- **Not knowing where to place a point for Problem 4.** Ask students to sketch a "not consumer friendly" cat first, and then measure the height and bow-tie width before adding the data to the graph.

Look for productive strategies:

- Noticing Cat C is a "not consumer friendly" cat. Ask students how the placement of this point compares to the rest of the points.

3 Connect

Have students share their cat data with their group, and then add the points on their own scatter plot.

Define the term ***outlier***. At the end of this activity, say, "The point that represents Cat C has a bow tie that is much wider than its predicted width. This is called an outlier."

Highlight Cat C. Ask, "What makes Cat C an outlier?" Cat C is an outlier because the cat's bow tie is larger than what is expected for its height. It is a "not consumer friendly" cat.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can move a point on a graph to see how its placement changes a cat's bow-tie width and receive immediate feedback when they click the *Try It* button.

Accessibility: *Vary Demands to Optimize Challenge*

Provide the measurements of the cats so that students do not have to measure them. This will allow them to spend more time looking for patterns and analyzing the data.

Math Language Development

MLR8: Discussion Supports

During the Launch, clarify the meaning of *width* and *height* in the context of the scenario. Connect the terms multi-modally by utilizing different types of sensory inputs, such as demonstrating the width and height on the images of each cat using gestures.

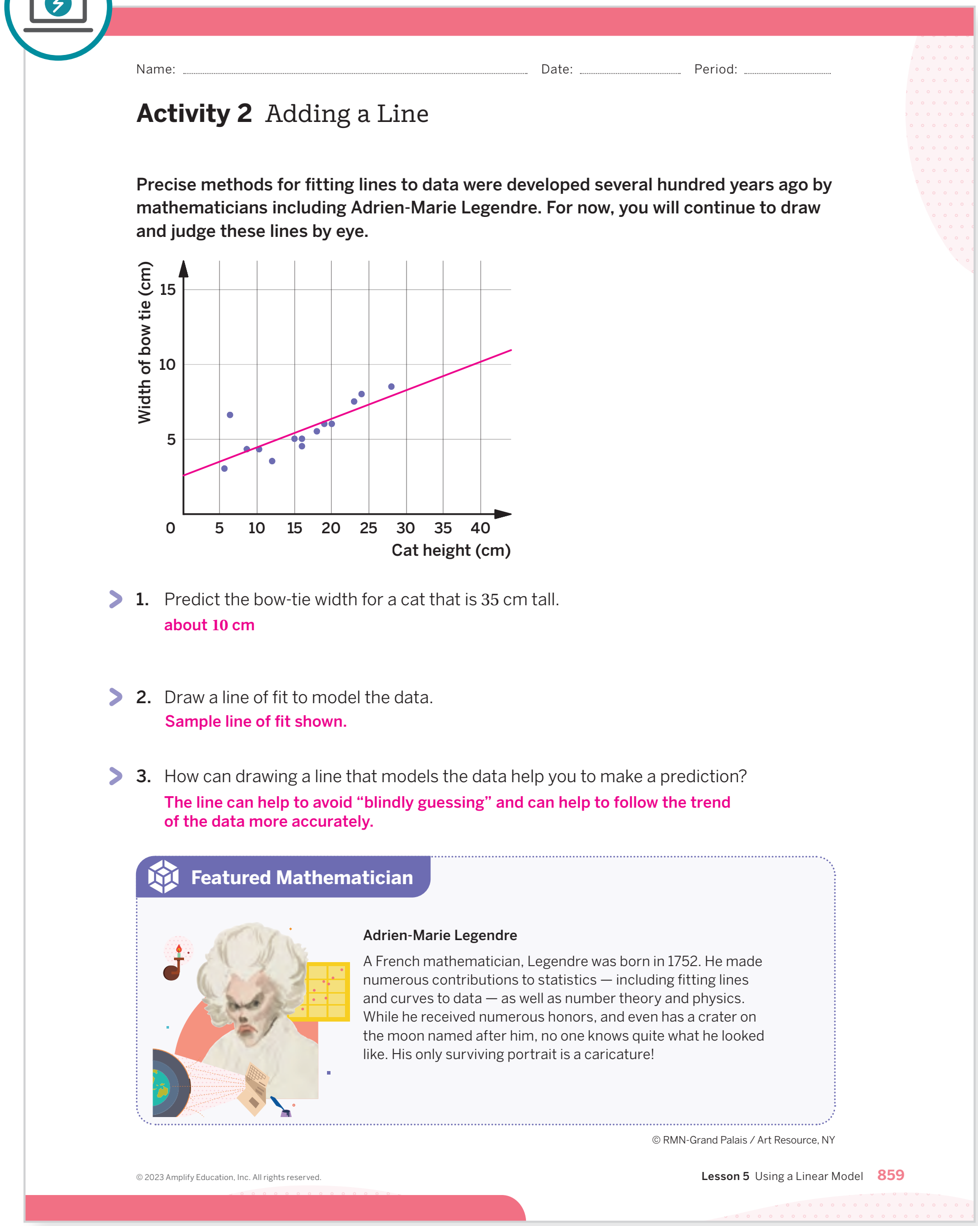
English Learners

Invite students to repeat phrases, such as "Here is the *width/height* of the bow tie/cat."

Pairs | 7 min

Activity 2 Adding a Line

Students model a relationship between two variables with a line. The term *linear model* is introduced, and students are asked to make a prediction.



Name: ________ Date: ________ Period: ________

Activity 2 Adding a Line

Precise methods for fitting lines to data were developed several hundred years ago by mathematicians including Adrien-Marie Legendre. For now, you will continue to draw and judge these lines by eye.

1. Predict the bow-tie width for a cat that is 35 cm tall.
   about 10 cm

2. Draw a line of fit to model the data.
   Sample line of fit shown.

3. How can drawing a line that models the data help you to make a prediction?
   The line can help to avoid "blindly guessing" and can help to follow the trend of the data more accurately.

Featured Mathematician

Adrien-Marie Legendre

A French mathematician, Legendre was born in 1752. He made numerous contributions to statistics — including fitting lines and curves to data — as well as number theory and physics. While he received numerous honors, and even has a crater on the moon named after him, no one knows quite what he looked like. His only surviving portrait is a caricature!

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1 Launch

Have students use the scatter plot from Activity 1 to complete the problems.

2 Monitor

Help students get started by asking, "What axes do you look at if you want to find a cat that is 40 cm tall?"

Look for productive strategies:

- Making a prediction close to the line they drew. Have this student compare their prediction and line to a student whose prediction was not close to the line. Have them discuss why drawing a line can be helpful in making a prediction.

3 Connect

Display a completed scatter plot from Activity 1 created by a student.

Have students share the bow-tie width they predicted for Problem 1 and discuss how it compares to the line of fit they drew for Problem 2.

Highlight that a line fit to data can help make a more accurate prediction.

Define linear model at the end of this activity. Say, "A ***linear model*** is useful because it helps you see the trend in the data more clearly so you can make predictions. The model may be represented as a line fit to data or as an equation."

Ask, "Does the linear model tell you that the consumer friendly cat *must* have a bow-tie width represented by a point on the line?" Circle a point that is close to, but not on the line so students can use the point as a reference. Sample response: No, the linear model helps you find a close approximation to make a prediction for the height or bow-tie width for a consumer friendly cat.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate to students how to use the scatter plot to make predictions. Before the line of fit is drawn, demonstrate how to predict the bow-tie width for a cat that is 35 cm tall by first asking:

- "What are some unreasonable values for the bow-tie width?"
- "What makes those values unreasonable?"

After students draw their line of fit, ask them to return to Problem 1 and determine if their first prediction was accurate or if they would like to revise it.

Featured Mathematician

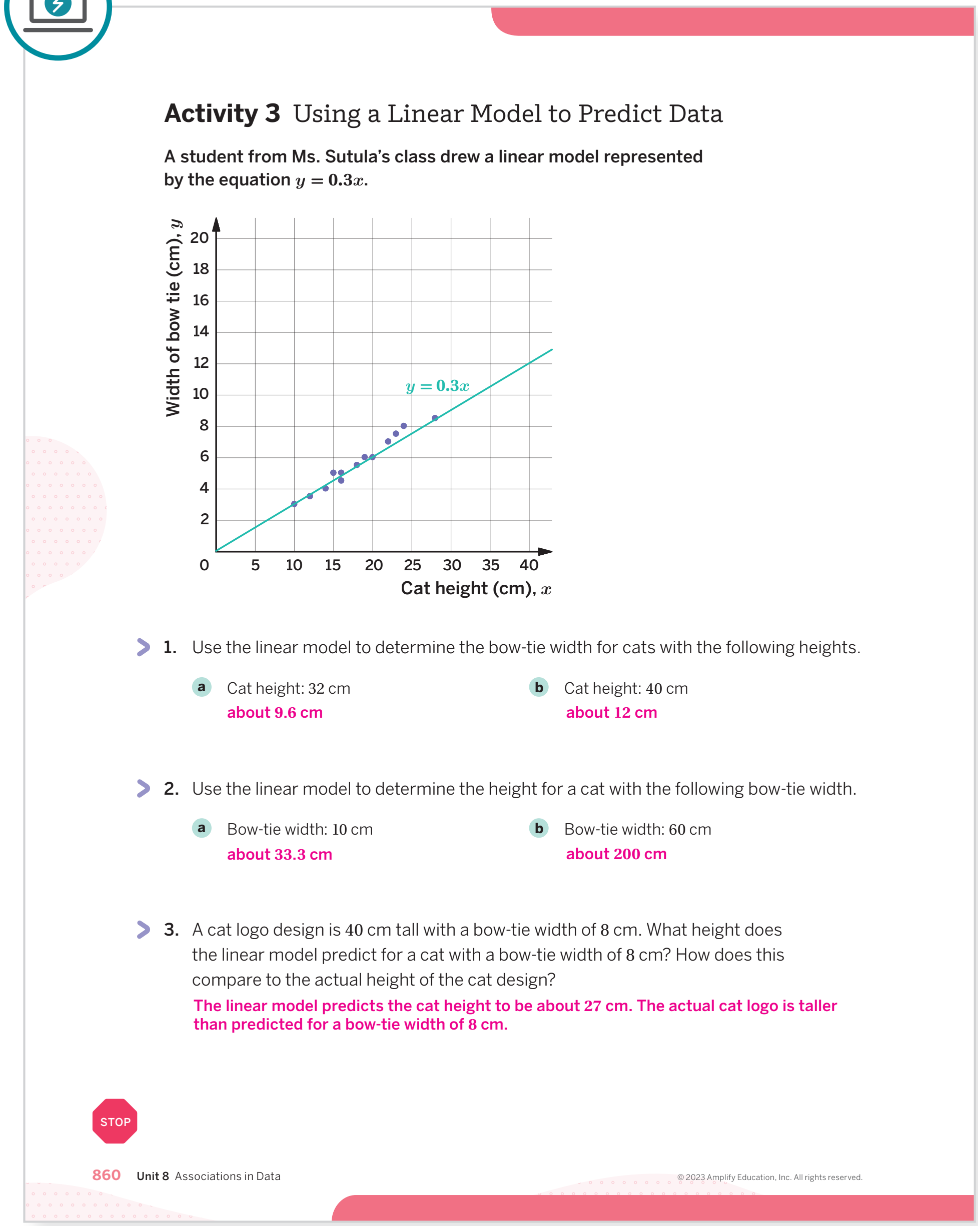
Adrien-Marie Legendre

Have students read about featured mathematician Adrien-Marie Legendre, a French mathematician who made numerous contributions to mathematics and physics.

Pairs | 8 min

Activity 3 Using a Linear Model to Predict Data

Students use a linear model to reason abstractly and quantitatively.



Activity 3 Using a Linear Model to Predict Data

A student from Ms. Sutula's class drew a linear model represented by the equation $y = 0.3x$.

1. Use the linear model to determine the bow-tie width for cats with the following heights.
   - a. Cat height: 32 cm
     about 9.6 cm
   - b. Cat height: 40 cm
     about 12 cm
2. Use the linear model to determine the height for a cat with the following bow-tie width.
   - a. Bow-tie width: 10 cm
     about 33.3 cm
   - b. Bow-tie width: 60 cm
     about 200 cm
3. A cat logo design is 40 cm tall with a bow-tie width of 8 cm. What height does the linear model predict for a cat with a bow-tie width of 8 cm? How does this compare to the actual height of the cat design?
   The linear model predicts the cat height to be about 27 cm. The actual cat logo is taller than predicted for a bow-tie width of 8 cm.

STOP

860 Unit 8 Associations in Data

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1 Launch

Reintroduce the term *linear model*. Say, "Since the data of the cats' height and bow-tie width show a linear association, you can model the data using a linear model. Here, the linear model is expressed by the line $y = 0.3x$."

2 Monitor

Help students get started by asking, "How can you identify the slope of a linear model from the equation?" Have students reference the Anchor Chart PDF, *Representations of Linear Relationships*.

Look for points of confusion:

- **Not understanding how to use the equation of the linear model to find the predicted height in Problem 3.** Ask students which variable represents the bow-tie width, and then tell them to substitute 60 for x.

3 Connect

Have students share how they used the line and the equation to make predictions and what outliers they identified during whole-group discussion.

Highlight Problem 2b. Here, students must use the equation, not the line.

Ask:

- "When is it better to use the graph/equation to make a prediction?"
- "What is the point called when it is far from the rest of the data?"

Differentiated Support

Accessibility: *Activate Prior Knowledge*

During the Launch, display the graph and the equation $y = 0.3x$. Ask students what they remember about linear relationships. Then ask:

- "Is this equation linear? How do you know?"
- "What is the y-intercept of this line? Where can you see it on the equation and on the graph?"
- "What is the slope of this line? Where can you see it on the equation and on the graph?"

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, eliminate Problem 3 until they are more comfortable using the linear model to make predictions.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have pairs meet with one other pair of students to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

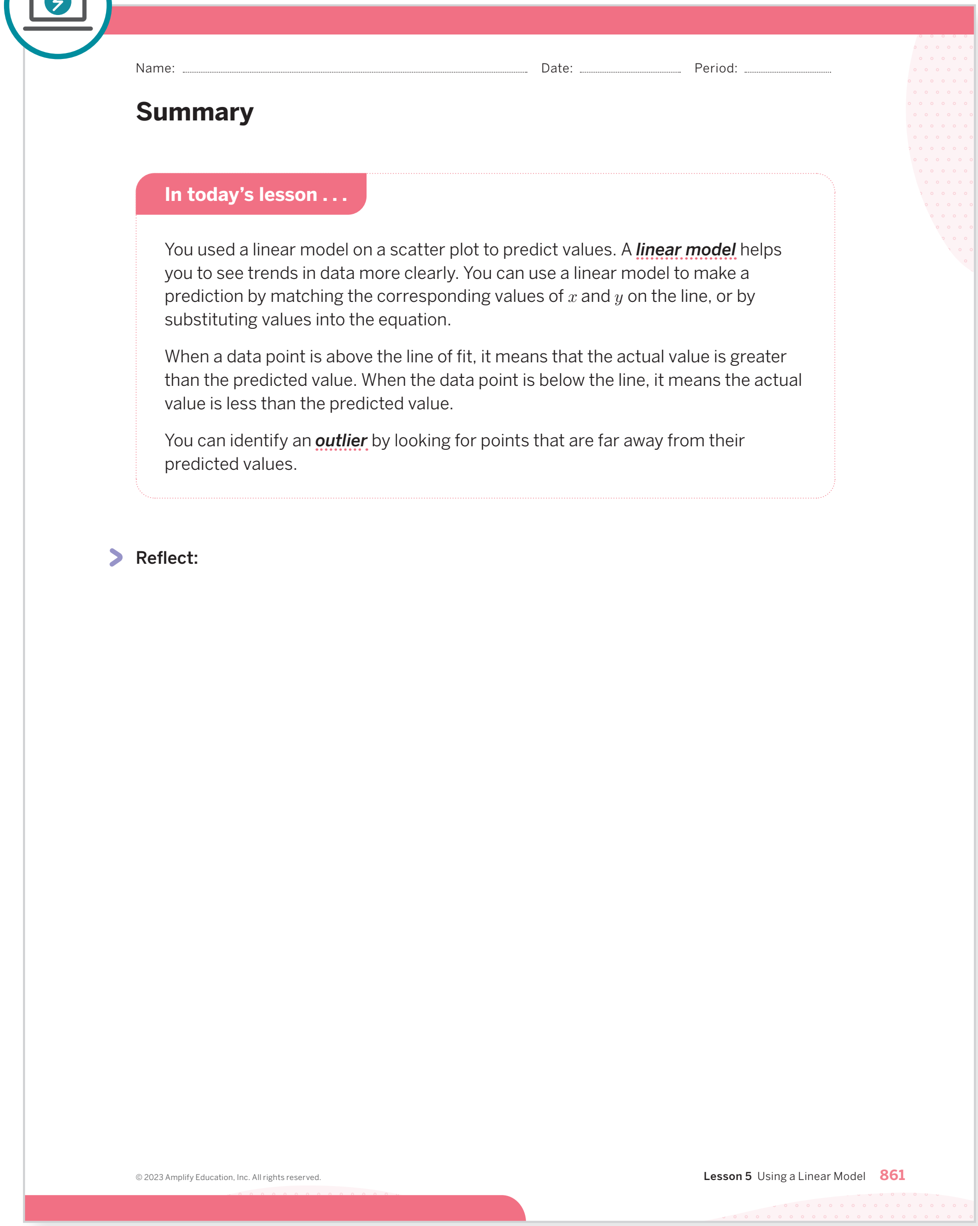
- "How did you predict the bow-tie width for a height of 32 cm?"
- "Did you use the graph or the equation? Was there a reason why you chose the representation that you did?"
- "What mathematical language did you use in your response?"

Have students write a final response, based on the feedback they received.

Whole Class | 5 min

Summary

Review and synthesize the advantages of creating and using a linear model and how to identify an outlier in a scatter plot.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You used a linear model on a scatter plot to predict values. A ***linear model*** helps you to see trends in data more clearly. You can use a linear model to make a prediction by matching the corresponding values of x and y on the line, or by substituting values into the equation.

When a data point is above the line of fit, it means that the actual value is greater than the predicted value. When the data point is below the line, it means the actual value is less than the predicted value.

You can identify an ***outlier*** by looking for points that are far away from their predicted values.

Reflect:

Synthesize

Formalize vocabulary:

- *linear model*
- *outlier*

Have students share why a linear model is useful and how they can identify potential outliers from a scatter plot. On the Anchor Chart PDF, *Clustering in Scatter Plots*, plot a point at $(20, 20)$ and label this point as "outlier".

Highlight that a linear model is useful because it helps students see the trend in the data more clearly, so they can make predictions.

Ask:

- "How can you use a line to predict a value?"
- "How can you use an equation to predict a value?"
- "How can you identify an outlier?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you model data in a scatter plot? And what does that model tell you?"

Math Language Development

MLR2: Collect and Display

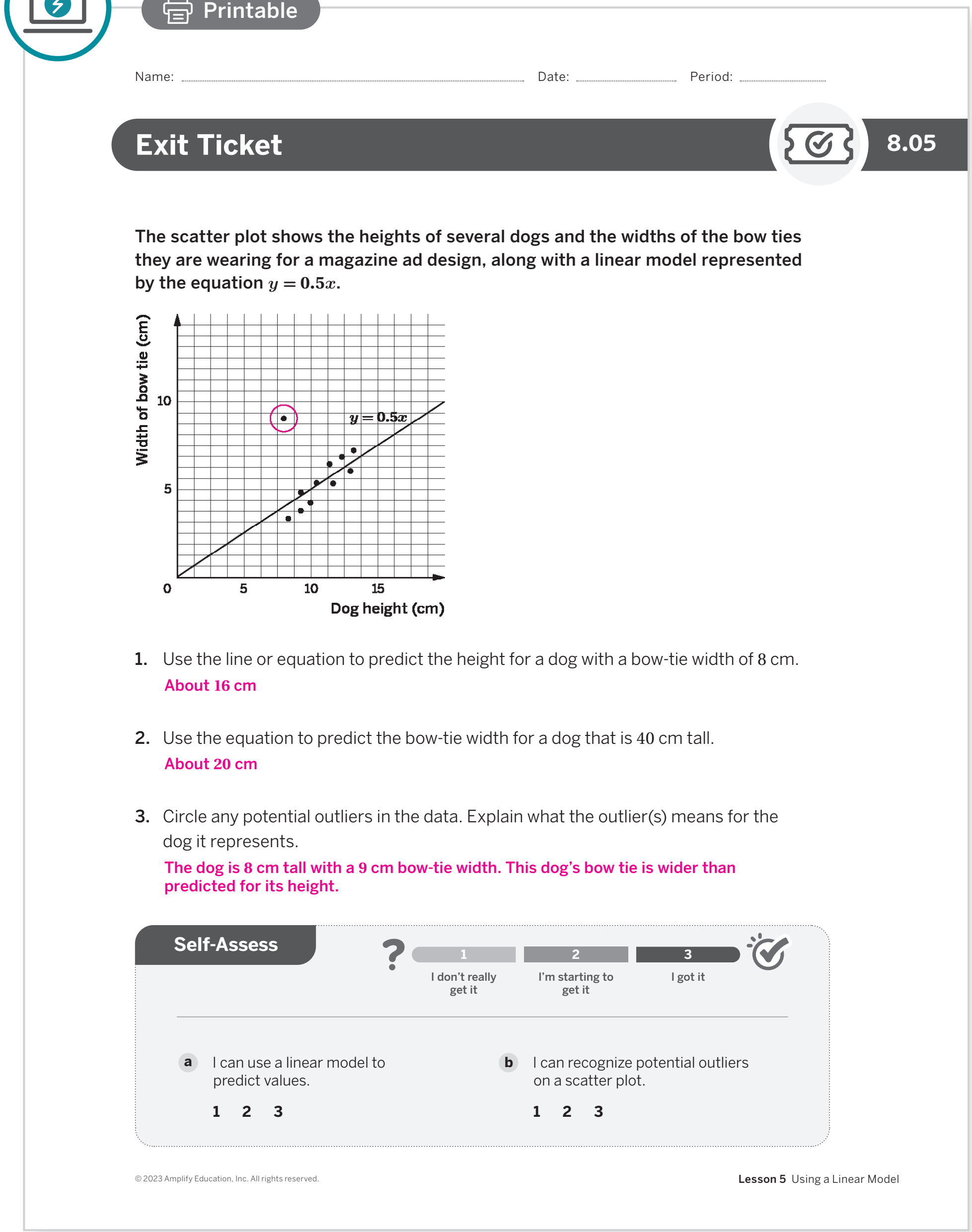
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on the terms and phrases related to the term *linear model* and *outlier*. If these terms have not yet been added to the class display, use this time to add them to the class display.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using a linear model to make predictions and identifying outliers.

Printable



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Exit Ticket 8.05

The scatter plot shows the heights of several dogs and the widths of the bow ties they are wearing for a magazine ad design, along with a linear model represented by the equation $y = 0.5x$.

1. Use the line or equation to predict the height for a dog with a bow-tie width of 8 cm.
   About 16 cm
2. Use the equation to predict the bow-tie width for a dog that is 40 cm tall.
   About 20 cm
3. Circle any potential outliers in the data. Explain what the outlier(s) means for the dog it represents.
   The dog is 8 cm tall with a 9 cm bow-tie width. This dog's bow tie is wider than predicted for its height.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use a linear model to predict values.
1 2 3

b I can recognize potential outliers on a scatter plot.
1 2 3

Lesson 5 Using a Linear Model

Success looks like . . .

- **Language Goal:** Comparing and contrasting values in a data set with predictions made using a given line. **(Speaking and Listening)**
- **Goal:** Comprehending that a model of data, such as a line of fit, can be used to predict values that are not given in the data.
  - » Predicting the height of the dog using the line in Problem 1.
- **Language Goal:** Identifying potential outliers on a scatter plot. **(Speaking and Listening)**
  - » Circling potential outliers on the graph in Problem 3.

Suggested next steps

If students look at the x-axis for bow-tie width, consider:

- Highlighting the x-axis to show this represents the dog height.
- Assigning Practice Problem 1.

If students do not know how to use the equation to predict the bow-tie width, consider:

- Writing the variables next to each label on the axes. Have students color code the variables on the graph and equation.
- Assigning Practice Problem 2.

If students do not know how to identify a potential outlier, consider:

- Having students write the definition of an outlier.
- Giving different examples of scatter plots with distinct outliers and asking students to identify the outlier.
- Reviewing what makes Cat C an outlier from Activity 1.

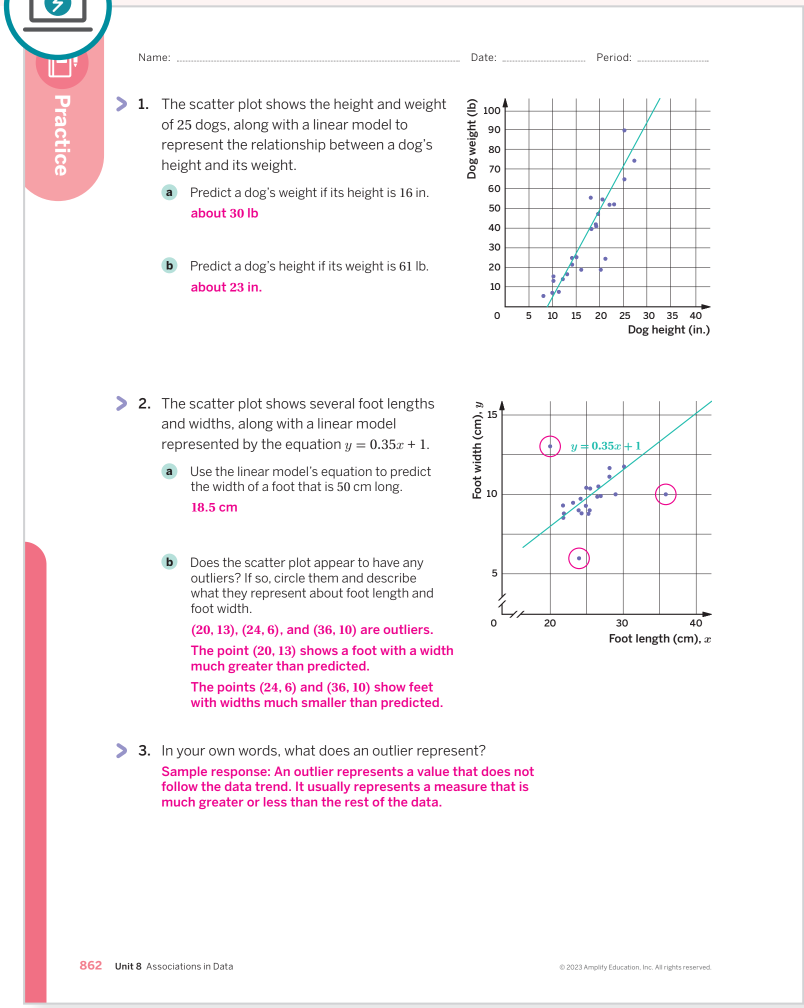
Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- During the discussion in Activity 1, how did you encourage each student to share their understandings?
- What did students find frustrating about using the graph or equation of a linear model? What helped them work through this frustration?

Practice



Name: ______ Date: ______ Period: ______

1. The scatter plot shows the height and weight of 25 dogs, along with a linear model to represent the relationship between a dog's height and its weight.

   a Predict a dog's weight if its height is 16 in.

   about 30 lb

   b Predict a dog's height if its weight is 61 lb.

   about 23 in.

2. The scatter plot shows several foot lengths and widths, along with a linear model represented by the equation $y = 0.35x + 1$.

   a Use the linear model's equation to predict the width of a foot that is 50 cm long.

   18.5 cm

   b Does the scatter plot appear to have any outliers? If so, circle them and describe what they represent about foot length and foot width.

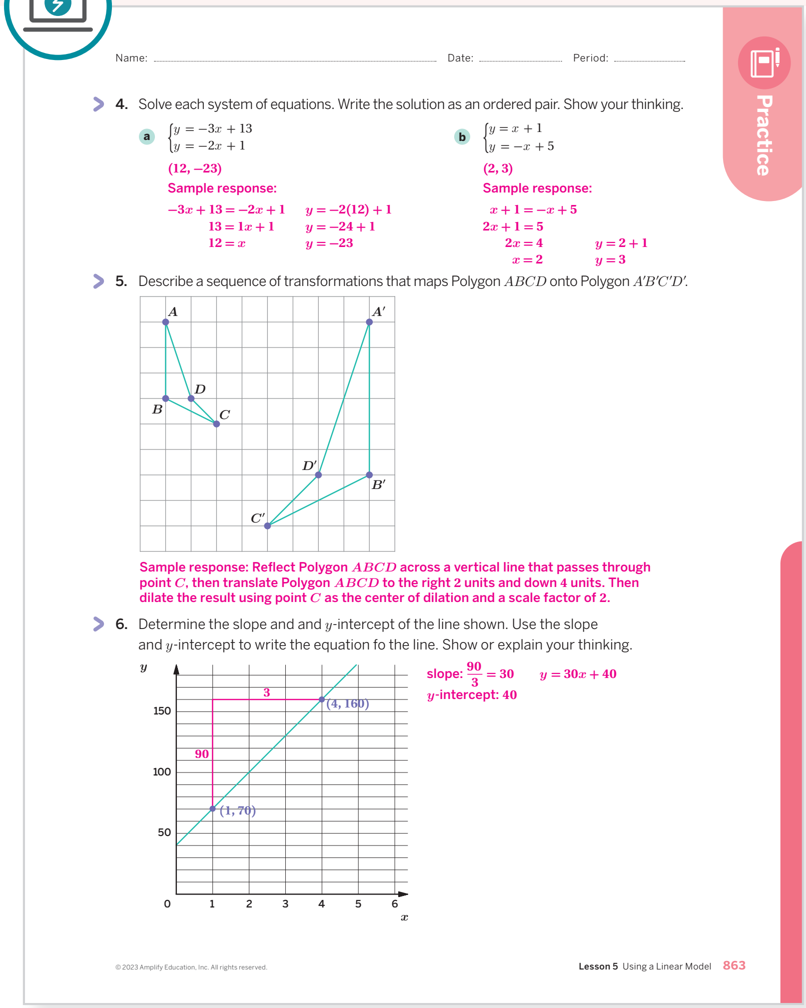
   (20, 13), (24, 6), and (36, 10) are outliers.

   The point (20, 13) shows a foot with a width much greater than predicted.

   The points (24, 6) and (36, 10) show feet with widths much smaller than predicted.

3. In your own words, what does an outlier represent?

   Sample response: An outlier represents a value that does not follow the data trend. It usually represents a measure that is much greater or less than the rest of the data.



Name: ______ Date: ______ Period: ______

4. Solve each system of equations. Write the solution as an ordered pair. Show your thinking.

   a $\begin{cases} y = -3x + 13 \\ y = -2x + 1 \end{cases}$

   (12, –23)

   Sample response:

   $-3x + 13 = -2x + 1$ $\quad y = -2(12) + 1$

   $13 = 1x + 1$ $\quad y = -24 + 1$

   $12 = x$ $\quad y = -23$

   b $\begin{cases} y = x + 1 \\ y = -x + 5 \end{cases}$

   (2, 3)

   Sample response:

   $x + 1 = -x + 5$

   $2x + 1 = 5$

   $2x = 4$ $\quad y = 2 + 1$

   $x = 2$ $\quad y = 3$

5. Describe a sequence of transformations that maps Polygon $ABCD$ onto Polygon $A'B'C'D'$.

   Sample response: Reflect Polygon $ABCD$ across a vertical line that passes through point C, then translate Polygon $ABCD$ to the right 2 units and down 4 units. Then dilate the result using point C as the center of dilation and a scale factor of 2.

6. Determine the slope and and y-intercept of the line shown. Use the slope and y-intercept to write the equation fo the line. Show or explain your thinking.

   slope: $\frac{90}{3} = 30$ $\quad$ **$y = 30x + 40$**

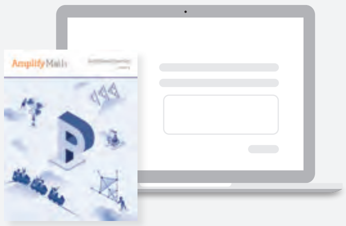
   y-intercept: 40

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1 and 3	2
	2	Activity 3	2
	3	Activity 2	3
Spiral	4	Unit 4 Lesson 15	2
	5	Unit 2 Lesson 4	1
Formative	6	Unit 8 Lesson 6	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

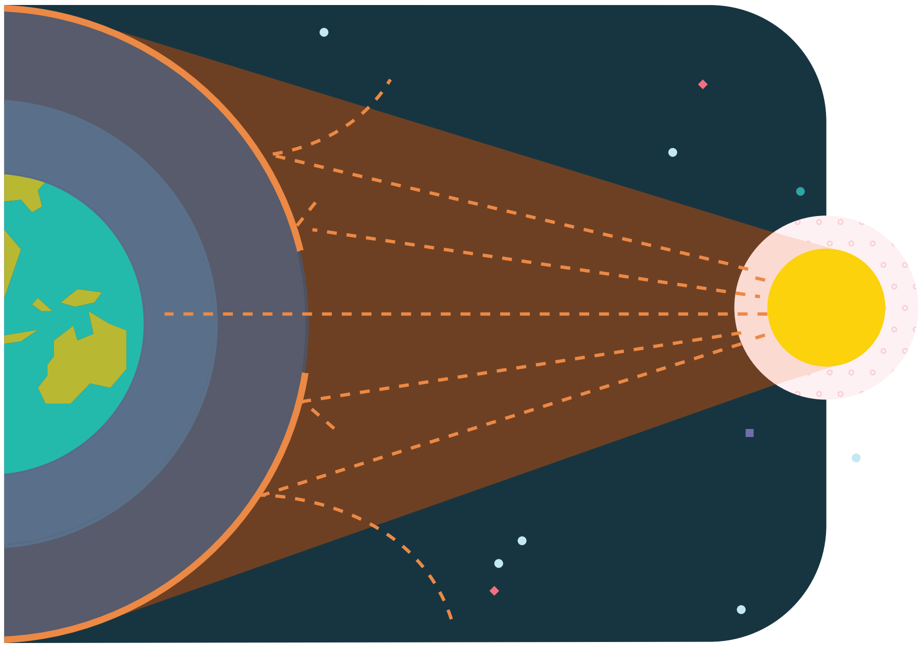


For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 8 | **LESSON 6**

Interpreting Slope and y-intercept

Let's see what the slope and y-intercept represent in context.



Focus

Goals

1. **Language Goal:** Describe the relationship between two variables using a line fit to data on a scatter plot. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret the slope and y-intercept of a line fit to data in context. **(Speaking and Listening, Writing)**

Coherence

Today

Students use the equation of a linear model to determine a positive or negative association. They draw a line to model data, write the equation of the line, and interpret the slope and y-intercept in context.

Previously

In Lesson 3, students looked at scatter plots to determine whether there was a positive or negative association. In Lesson 5, students used a linear model to predict values.

Coming Soon

In Lesson 8, students will continue to look for association in data. Students will use a scatter plot to construct a two-way frequency table.

Rigor

- Students strengthen their **fluency** in applying a line of fit to a set of data, and interpreting the slope and points that do not lie on the line in context.
- Students **apply** their knowledge of slope and y-intercept to linear data.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Representations of Linear Relationships* (from Unit 3)
- Anchor Chart PDF, *Writing Linear Equations in $y = mx + b$ Form* (from Unit 3)
- rulers

Math Language Development

Review words

- *linear model*
- *negative association*
- *outlier*
- *positive association*
- *scatter plot*
- *slope*
- *y-intercept*

Amps | **Featured Activity**

Activity 2

Interactive Linear Models

Students manipulate a line to create a linear model. Students can click any point on the line to see its coordinates and use this information to help them write an equation.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might rush to judgment and respond negatively if their partner has a different line than they do in Activity 2. Encourage students to show respect for their partner by listening carefully and working together to evaluate both answers for accuracy. Have students think about why it is possible for there to be two different lines that represent the data well.

Modifications to Pacing

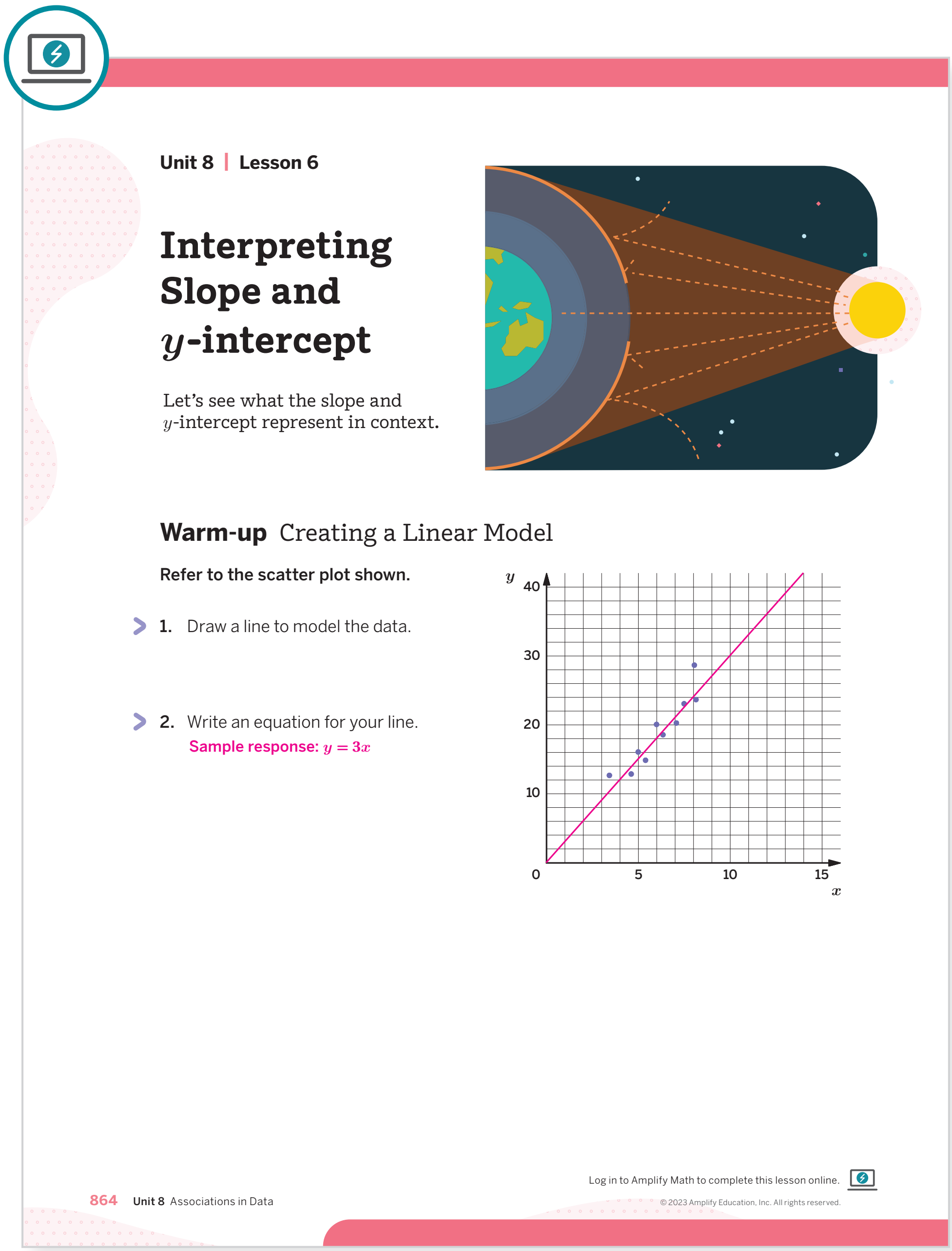
You may want to consider this additional modification if you are short on time.

- **Activity 3** may be omitted.

Independent | 10 min

Warm-up Creating a Linear Model

Students draw a line to model data, and then write an equation to strengthen their understanding of linear models prior to today's lesson.



Unit 8 | Lesson 6

Interpreting Slope and y-intercept

Let's see what the slope and y-intercept represent in context.

Warm-up Creating a Linear Model

Refer to the scatter plot shown.

1. Draw a line to model the data.

2. Write an equation for your line.
   Sample response: $y = 3x$

Log in to Amplify Math to complete this lesson online.

1 Launch

Have students use a ruler to draw a line to model the data, compare their line with a partner, and then write an equation for their line. Remind students that their equation may be different than their partner's equation, depending on the line drawn.

2 Monitor

Help students get started by activating their prior knowledge about equations of the from $y = mx + b$. Have students reference the Anchor Chart PDF, *Writing Linear Equations in* $y = mx + b$ *Form*.

Look for points of confusion:

- **Struggling to write an equation for their line.** Have students find two points and label each with the coordinates. Ask them to use a formula to solve for the slope.

Look for productive strategies:

- Drawing a slope triangle to determine the slope.
- Estimating the slope visually. Encourage students to use additional methods for accuracy.

3 Connect

Have students share their line with their partner. Have them discuss how the line they drew affects the slope and y-intercept of the equation.

Ask, "How did you determine how to write the equation for your line?"

Highlight different lines and equations. Although students' equations differ based on the line they drew, the slope should be close to 3 and the y-intercept should be close to 0.

Power-up

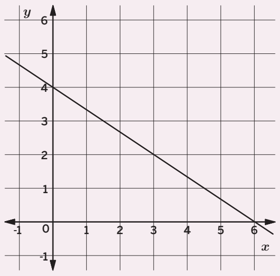
To power up students' ability to write linear equations, have students complete:

Recall that the equation $y = mx + b$ can be written to represent a line where m represents the slope and b represents the vertical intercept.

a. What is the slope of the line shown? $\frac{2}{3}$

b. What is the vertical intercept of the line shown? $(0,4)$

c. What is the equation of the line shown? $y = -\frac{2}{3}x + 4$



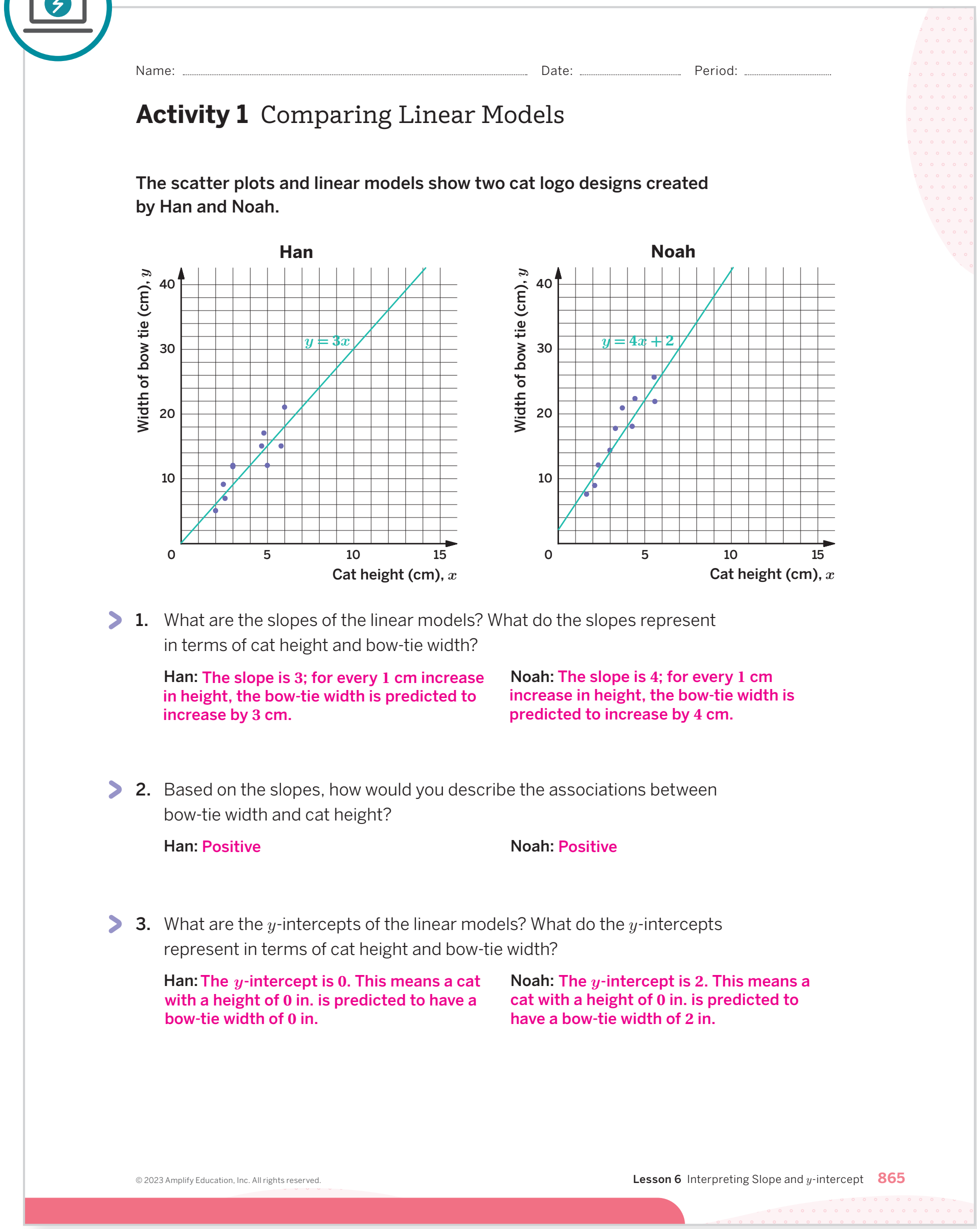
Use: Before the Warm-up

Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 1, 2, and 3

Pairs | 10 min

Activity 1 Comparing Linear Models

Students move from general descriptions of association to more detailed descriptions of a linear model. The numerical value of the slope and y-intercept is interpreted within the context of the problem.



Name: Date: Period:

Activity 1 Comparing Linear Models

The scatter plots and linear models show two cat logo designs created by Han and Noah.

1. What are the slopes of the linear models? What do the slopes represent in terms of cat height and bow-tie width?

Han: **The slope is 3; for every 1 cm increase in height, the bow-tie width is predicted to increase by 3 cm.**

Noah: **The slope is 4; for every 1 cm increase in height, the bow-tie width is predicted to increase by 4 cm.**

2. Based on the slopes, how would you describe the associations between bow-tie width and cat height?

Han: Positive

Noah: Positive

3. What are the y-intercepts of the linear models? What do the y-intercepts represent in terms of cat height and bow-tie width?

Han: **The y-intercept is 0. This means a cat with a height of 0 in. is predicted to have a bow-tie width of 0 in.**

Noah: **The y-intercept is 2. This means a cat with a height of 0 in. is predicted to have a bow-tie width of 2 in.**

1 Launch

Tell students they are looking at more data of Chemy Cat logos from the previous lesson.

2 Monitor

Look for points of confusion:

- **Not knowing what the y-intercept represents.** Ask students to write the y-intercept as a coordinate, and then write *(cat height, bow-tie width)*.

Look for productive strategies:

- Identifying the slope from the line. Encourage students to also look at the equation and see if that is a more useful representation in this situation. Remind them they are looking at the coefficient of x.

3 Connect

Have students share their responses by using the *Turn and Talk* routine described in the Math Language Development section.

Highlight the slope and y-intercept in context. The slope represents the change in bow-tie width for every additional centimeter increase in height. The y-intercept represents the bow-tie width for a cat height of 0 cm. Note that while it may not make sense for there to be a bow-tie height of 2 cm in Noah's model, his line of fit still can represent a good fit and be used to make predictions.

Ask, "If you are only given the equation of the linear model, how can you tell whether the data have a positive/negative association?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, assign different pairs of students to complete the activity for either Han's scatter plot or Noah's scatter plot. During the Connect, all students can listen and participate in the discussion.

Extension: *Math Enrichment*

Ask students if they think that the y-intercept makes sense for either Han's or Noah's models and have them explain their thinking. Sample response: I don't think that the y-intercept in Noah's model makes sense because a cat with no height should not have a bow-tie width, since that cat would not exist.

Math Language Development

MLR8: Discussion Supports

During the Connect, have pairs use the *Turn and Talk* routine to share their responses with another pair of students. Provide the following sentence frames for students to complete during their discussion, which will help scaffold their language when describing the meaning of the slope in this context.

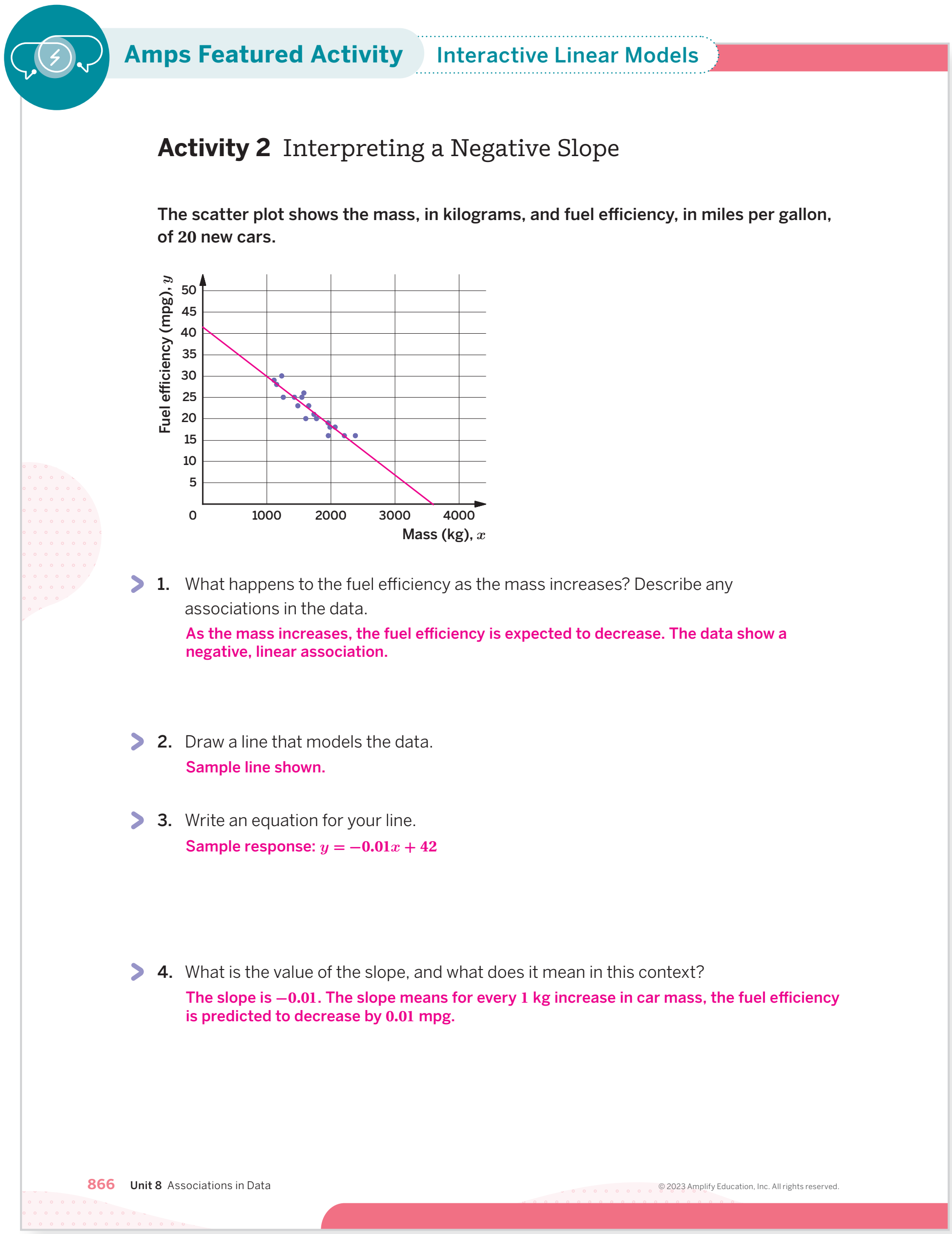
- "As the cat's height increases by 1 cm, Han's linear model predicts the width will increase by _____ cm."
- "As the cat's height increases by 1 cm, Noah's linear model predicts the width will increase by _____ cm."

Pairs | 15 min

Activity 2 Interpreting a Negative Slope

Students create a linear model for a scatter plot with a negative association to realize the linear model has a negative slope.

Amps Featured Activity Interactive Linear Models



Activity 2 Interpreting a Negative Slope

The scatter plot shows the mass, in kilograms, and fuel efficiency, in miles per gallon, of 20 new cars.

1. What happens to the fuel efficiency as the mass increases? Describe any associations in the data.
   As the mass increases, the fuel efficiency is expected to decrease. The data show a negative, linear association.

2. Draw a line that models the data.
   Sample line shown.

3. Write an equation for your line.
   Sample response: $y = -0.01x + 42$

4. What is the value of the slope, and what does it mean in this context?
   The slope is -0.01. The slope means for every 1 kg increase in car mass, the fuel efficiency is predicted to decrease by 0.01 mpg.

866 Unit 8 Associations in Data

1 Launch

Tell students the scatter plot measures a car's mass and fuel efficiency. Activate students' background knowledge by saying that fuel efficiency measures how far a vehicle can travel per gallon of gas.

2 Monitor

Help students get started by asking, "Does the scatter plot have the same or different association from Activity 1?"

Look for points of confusion:

- **Writing a positive number for the slope in the equation.** Ask students if that matches the negative association.
- **Not knowing what the slope represents.** Point out the labels on each axis. Ask, "As the mass of the car increases, what is predicted to happen to the fuel efficiency?"

Look for productive strategies:

- Drawing an oval around the data to help create a line of fit which follows the trend and balances the data above and below the line.

3 Connect

Have students share their line with responses. Have them discuss how the line they drew relates to the slope and y-intercept of the equation.

Ask, "If the car has a greater mass, is the fuel efficiency expected to increase or decrease?"

Highlight the connection between the association and slope. If a scatter plot has a negative association, the slope of the linear model will also be negative.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate the graph of a line to create a linear model. They can click on any point on the line to see its coordinates and use this information to help them write an equation.

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Provide a word bank that students can use to help complete Problem 1. A sample word bank is shown.

mass	fuel efficiency	increases	decreases
positive	negative	linear	nonlinear

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, draw connections between the type of association (positive or negative) and the slope of the linear model. Consider adding the following to the class display.

Association	Slope of linear model	As the x-value increases, . . .
Positive, linear	Positive	The y-value increases.
Negative, linear	Negative	The y-value decreases.

English Learners

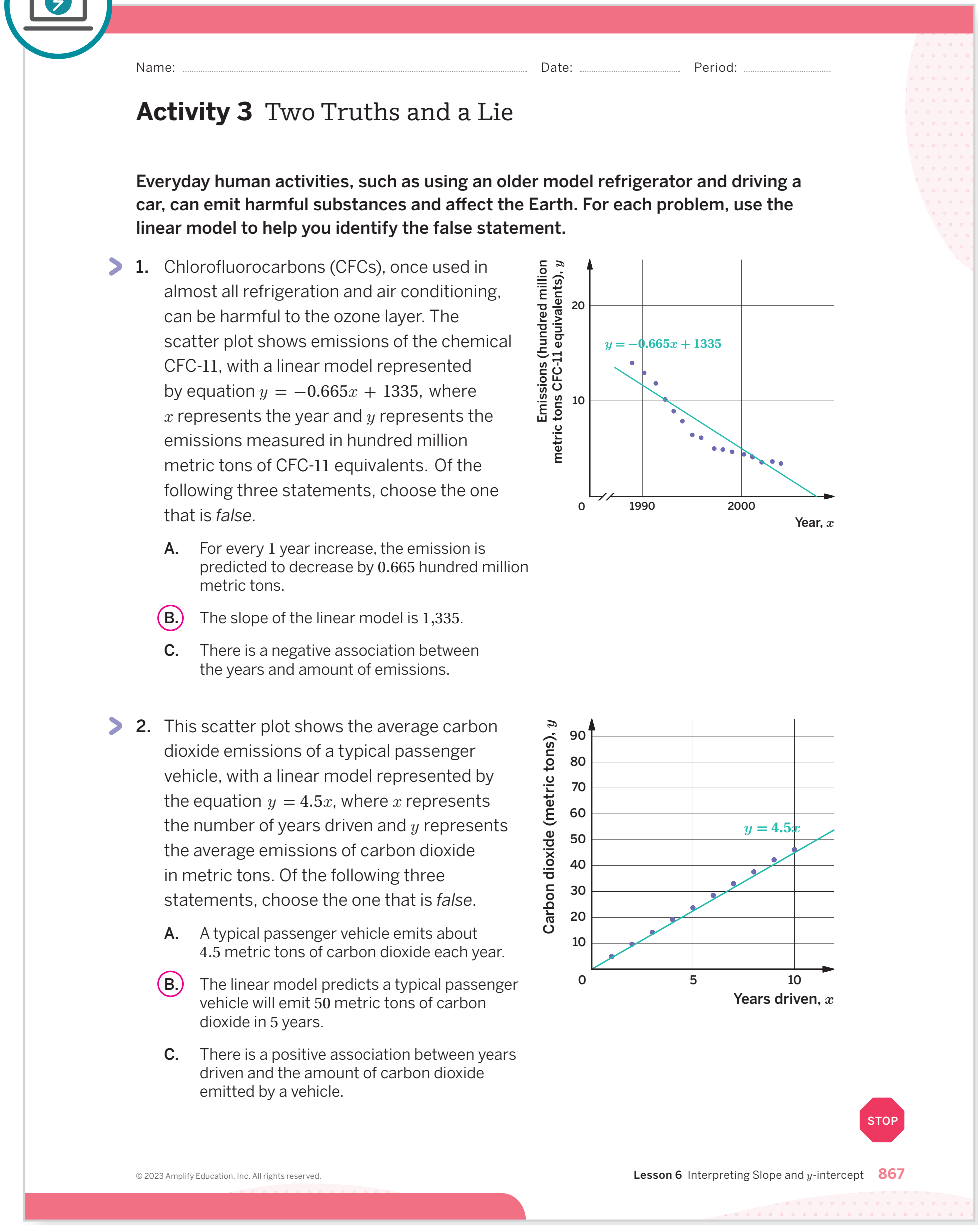
Add visual examples of scatter plots, along with their linear models, to highlight the information above.

Optional

Pairs | 10 min

Activity 3 Two Truths and a Lie

Students analyze bivariate measurement data to determine which of three statements describing the data is false.



Name: ______________________ Date: __________ Period: __________

Activity 3 Two Truths and a Lie

Everyday human activities, such as using an older model refrigerator and driving a car, can emit harmful substances and affect the Earth. For each problem, use the linear model to help you identify the false statement.

1. Chlorofluorocarbons (CFCs), once used in almost all refrigeration and air conditioning, can be harmful to the ozone layer. The scatter plot shows emissions of the chemical CFC-11, with a linear model represented by equation $y = -0.665x + 1335$, where x represents the year and y represents the emissions measured in hundred million metric tons of CFC-11 equivalents. Of the following three statements, choose the one that is *false*.

   A. For every 1 year increase, the emission is predicted to decrease by 0.665 hundred million metric tons.

   [✓ B.] The slope of the linear model is 1,335.

   C. There is a negative association between the years and amount of emissions.

2. This scatter plot shows the average carbon dioxide emissions of a typical passenger vehicle, with a linear model represented by the equation $y = 4.5x$, where x represents the number of years driven and y represents the average emissions of carbon dioxide in metric tons. Of the following three statements, choose the one that is *false*.

   A. A typical passenger vehicle emits about 4.5 metric tons of carbon dioxide each year.

   [✓ B.] The linear model predicts a typical passenger vehicle will emit 50 metric tons of carbon dioxide in 5 years.

   C. There is a positive association between years driven and the amount of carbon dioxide emitted by a vehicle.

Lesson 6 Interpreting Slope and y-intercept 867

1 Launch

Introduce the activity by telling students there are everyday activities that contribute to changes in the environment and ozone level. Conduct the ***Two Truths and a Lie*** routine.

2 Monitor

Help students get started by identifying the slope and y-intercept of the linear model. Ask, "What does it mean if a slope is negative?"

Look for points of confusion:

- **Thinking the slope is** $\frac{\textbf{horizontal change}}{\textbf{vertical change}}$. Have students reference the Anchor Chart PDF, *Representations of Linear Relationships* and review the definition of slope as $\frac{\text{vertical change}}{\text{horizontal change}}$. Have students match the slope in context with the definition.

3 Connect

Have students share the slope and y-intercept in each model. Have students discuss what the slope means in context with a partner before selecting the false statement.

Highlight the meaning of the slope's value in each context.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, provide Problem 1 with only two answer choices — one truth and one lie — and present the directions as "choose the lie."

Extension: *Math Enrichment*

Have students refer back to one of the scatter plots and linear models they studied in this unit so far and write three statements. One statement should be a lie, while the other two statements should be true. Have them trade statements with a partner and each partner should discern the lie.

Math Language Development

MLR6: Three Reads

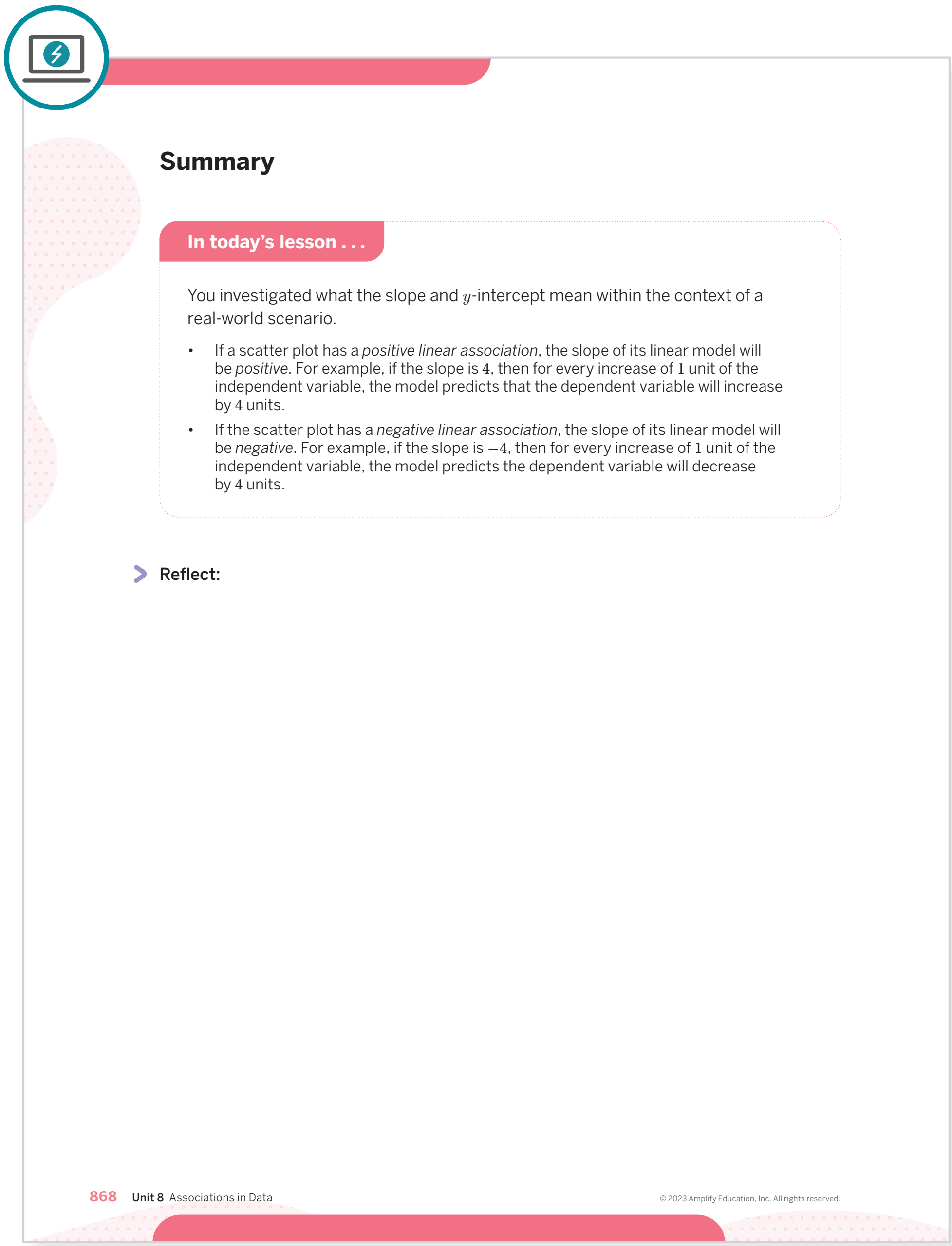
Use this routine to help students make sense of the text for each scenario.

- **Read 1:** Have pairs of students read each scenario to make sense of the context. Consider using structured pairing so that students with differing English language proficiencies can interact.
- **Read 2:** Ask students to study the graph and linear model for each scenario to understand the two variables and any associations they see.
- **Read 3:** Ask students to plan their solution strategy by identifying what they are being tasked to do.

Whole Class | 5 min

Summary

Review and synthesize how to interpret the slope of a linear model within context, and how the slope is related to the association of the data.



Summary

In today's lesson . . .

You investigated what the slope and y-intercept mean within the context of a real-world scenario.

- If a scatter plot has a *positive linear association*, the slope of its linear model will be *positive*. For example, if the slope is 4, then for every increase of 1 unit of the independent variable, the model predicts that the dependent variable will increase by 4 units.
- If the scatter plot has a *negative linear association*, the slope of its linear model will be *negative*. For example, if the slope is -4, then for every increase of 1 unit of the independent variable, the model predicts the dependent variable will decrease by 4 units.

Reflect:

Synthesize

Ask:

- "How can you determine if a scatter plot has a positive or negative association from an equation?"
- "How can you use the slope of a linear model to say how the variables are connected in a scenario?"

Have students share how they can identify the slope of a linear model and how they can interpret what the slope means in context.

Highlight how to find the slope and y-intercept from an equation. Reinforce that the sign of the slope tells them whether the data have a positive or negative association.

Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

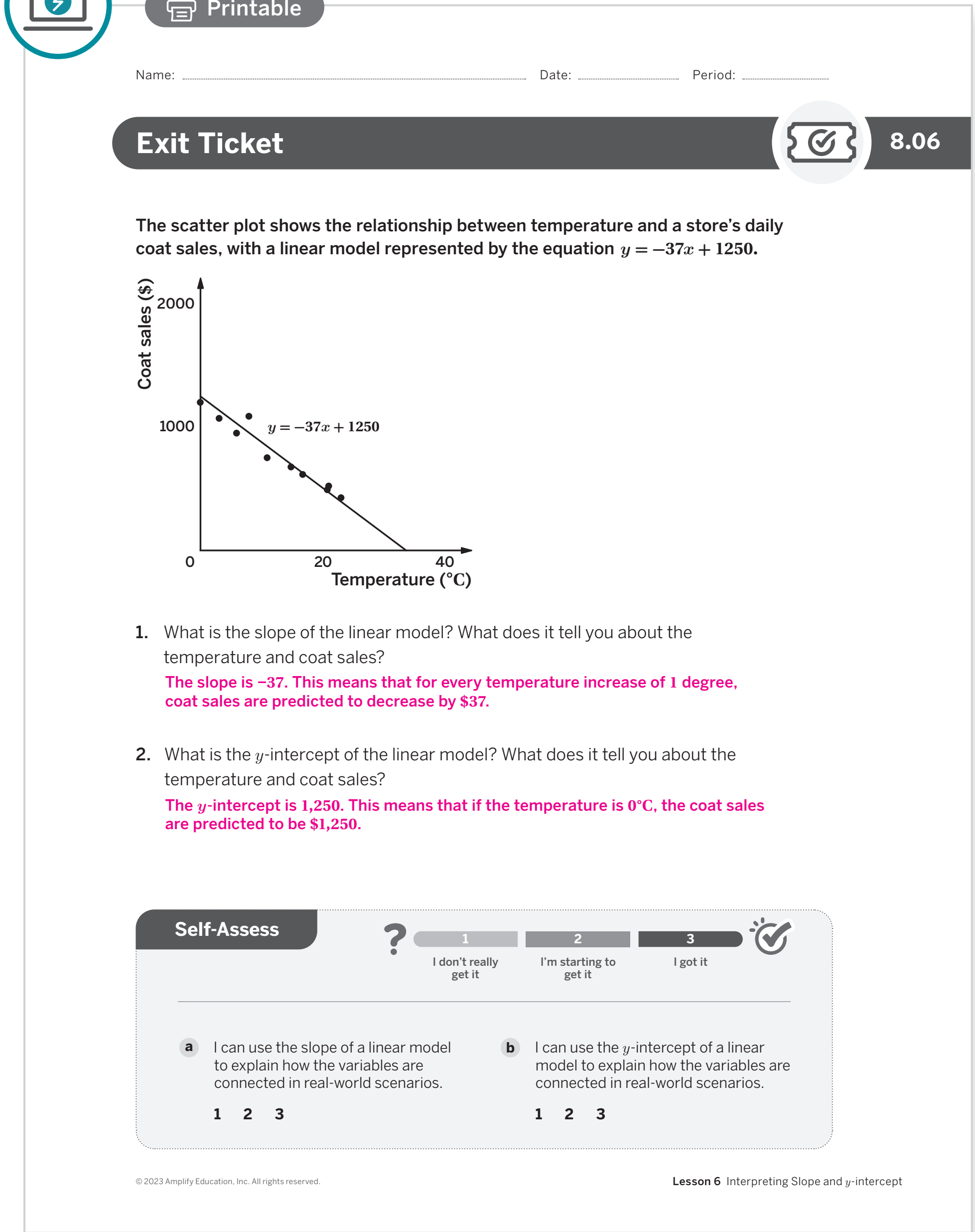
- "What strategies or tools did you find helpful today when interpreting the equation of the line of fit? How were they helpful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying the slope and y-intercept of a linear model and explaining what each represents in context.

Printable



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Exit Ticket 8.06

The scatter plot shows the relationship between temperature and a store's daily coat sales, with a linear model represented by the equation $y = -37x + 1250$.

1. What is the slope of the linear model? What does it tell you about the temperature and coat sales?
   The slope is −37. This means that for every temperature increase of 1 degree, coat sales are predicted to decrease by $37.

2. What is the y-intercept of the linear model? What does it tell you about the temperature and coat sales?
   The y-intercept is 1,250. This means that if the temperature is 0°C, the coat sales are predicted to be $1,250.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can use the slope of a linear model to explain how the variables are connected in real-world scenarios.
1 2 3

b I can use the y-intercept of a linear model to explain how the variables are connected in real-world scenarios.
1 2 3

Lesson 6 Interpreting Slope and y-intercept

Success looks like . . .

- **Language Goal:** Describing the relationship between two variables using a line fit to data on a scatter plot. **(Speaking and Listening, Writing)**
- **Language Goal:** Interpreting the slope and y-intercept of a line fit to data in context. **(Speaking and Listening, Writing)**
  - » Interpreting the slope and y-intercept in terms of temperature and coat sales in Problems 1 and 2.

Suggested next steps

If students do not correctly identify the slope or y-intercept, consider:

- Having students reference the Anchor Chart PDF, *Representations of Linear Relationships* and identify each variable in $y = mx + b$.
- Reviewing how to identify associations from Activity 1.
- Assigning Practice Problem 1.

If students identify the slope, but cannot relate it in context, consider:

- Providing the following visual:

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$\frac{-37}{1} = \frac{\text{coat sales}}{\text{temperature}}$$

If students identify the y-intercept, but cannot relate it in context, consider:

- Writing $(0, 1250) =$ (*temperature*, *coat sales*).
- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- This lesson asks students to solve problems in the context of bivariate data. Where in your students' work today did you see or hear evidence of them doing this?
- In earlier lessons, students used a linear model to reason abstractly and quantitatively. How did that support their interpretations of slope and y-intercepts today?

Math Language Development

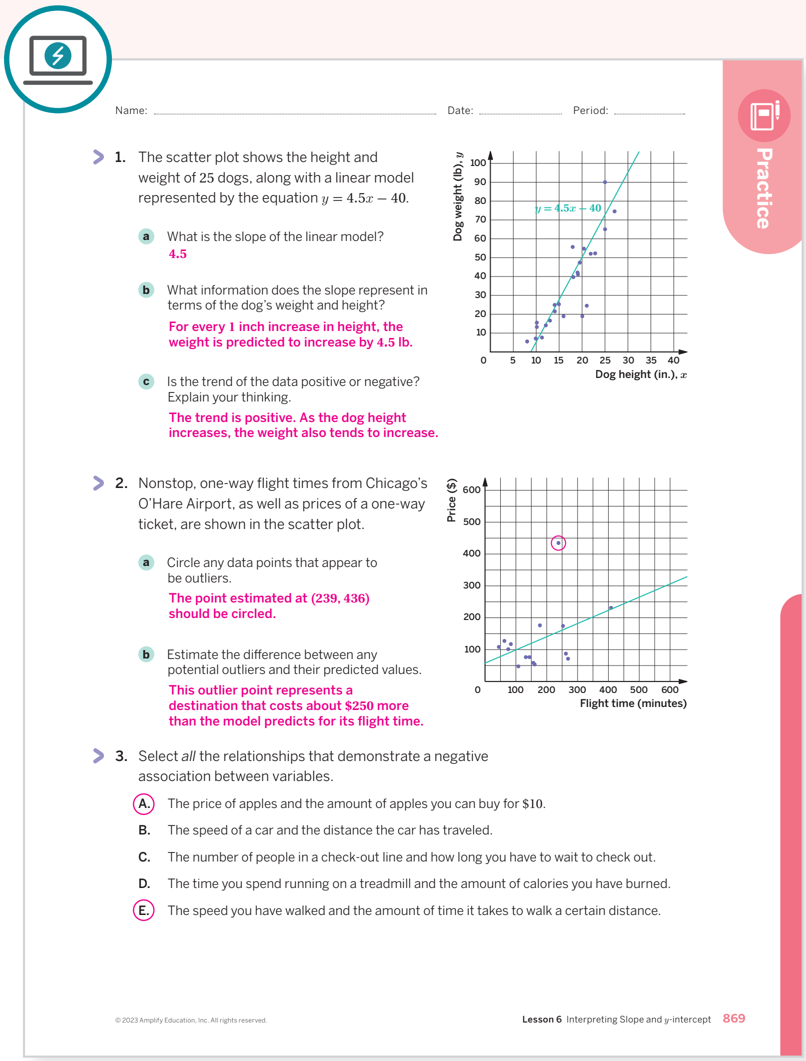
Language Goal: Interpreting the slope and y-intercept of a line fit to data in context.

Reflect on students' language development toward this goal.

- Do students' responses to Problem 1 of the Exit Ticket indicate they understand how to interpret the slope of a line of fit within context? Are they using mathematical language such as "for every temperature increase of 1 degree, . . ."?
- Do students' responses to Problem 2 of the Exit Ticket indicate they understand how to interpret the y-intercept of a line of fit within context? What mathematical language are they using?

Practice

Independent



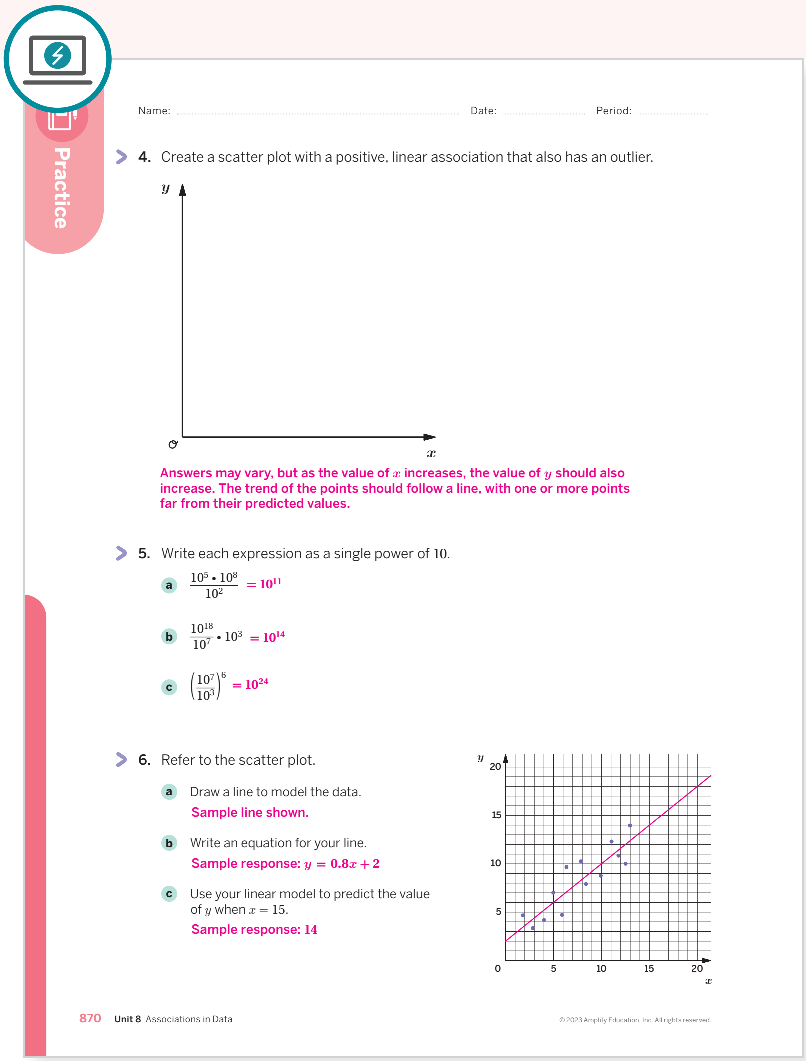
Name: Date: Period:

1. The scatter plot shows the height and weight of 25 dogs, along with a linear model represented by the equation $y = 4.5x - 40$.
   - **a** What is the slope of the linear model?
     4.5
   - **b** What information does the slope represent in terms of the dog's weight and height?
     For every 1 inch increase in height, the weight is predicted to increase by 4.5 lb.
   - **c** Is the trend of the data positive or negative? Explain your thinking.
     The trend is positive. As the dog height increases, the weight also tends to increase.

2. Nonstop, one-way flight times from Chicago's O'Hare Airport, as well as prices of a one-way ticket, are shown in the scatter plot.
   - **a** Circle any data points that appear to be outliers.
     The point estimated at (239, 436) should be circled.
   - **b** Estimate the difference between any potential outliers and their predicted values.
     This outlier point represents a destination that costs about $250 more than the model predicts for its flight time.

3. Select *all* the relationships that demonstrate a negative association between variables.
   - [✓ A.] The price of apples and the amount of apples you can buy for $10.
   - B. The speed of a car and the distance the car has traveled.
   - C. The number of people in a check-out line and how long you have to wait to check out.
   - D. The time you spend running on a treadmill and the amount of calories you have burned.
   - [✓ E.] The speed you have walked and the amount of time it takes to walk a certain distance.

Lesson 6 Interpreting Slope and y-intercept 869



Name: Date: Period:

4. Create a scatter plot with a positive, linear association that also has an outlier.

   Answers may vary, but as the value of x increases, the value of y should also increase. The trend of the points should follow a line, with one or more points far from their predicted values.

5. Write each expression as a single power of 10.
   - **a** $\frac{10^5 \cdot 10^8}{10^2}$ **$= 10^{11}$**
   - **b** $\frac{10^{18}}{10^7} \cdot 10^3$ **$= 10^{14}$**
   - **c** $\left(\frac{10^7}{10^3}\right)^6$ **$= 10^{24}$**

6. Refer to the scatter plot.
   - **a** Draw a line to model the data.
     Sample line shown.
   - **b** Write an equation for your line.
     Sample response: $y = 0.8x + 2$
   - **c** Use your linear model to predict the value of y when $x = 15$.
     Sample response: 14

870 Unit 8 Associations in Data

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 8 Lesson 3	2
	4	Unit 8 Lesson 4	1
	5	Unit 6 Lesson 8	1
Formative	6	Unit 8 Lesson 7	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 8 | **LESSON 7**

Analyzing Bivariate Data

Let's analyze data like a pro.

Focus

Goals

1. **Language Goal:** Describe the relationship between two variables using a line fit to data on a scatter plot. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret points on the scatter plot, including points that do and do not lie on a line fit to the data. **(Speaking and Listening, Writing)**
3. **Language Goal:** Interpret the slope of a line fit to data in context. **(Speaking and Listening, Writing)**

Coherence

Today

Students bring everything they have studied in the unit so far to analyze and interpret bivariate data in the context of brain and body weight. They look at data in a scatter plot, create a linear model, and compare the actual data values to the values predicted by the linear model.

Previously

Throughout this unit, students organized data in scatter plots and determined associations, outliers, and clusters. Students created a linear model and interpreted the slope in context.

Coming Soon

In Lesson 8, students will use a scatter plot to create two-way tables and bar graphs to connect associations in data.

Rigor

- Students **apply** their knowledge of bivariate data to interpret and analyze the relationship between the weight of animals and the weight of their brains.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	5 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Representations of Linear Relationships* (from Unit 3)
- rulers

Math Language Development

Review words

- *cluster*
- *linear association*
- *linear model*
- *negative association*
- *nonlinear association*
- *outlier*
- *positive association*
- *scatter plot*

Amps | Featured Activity

Activity 2
Formative Feedback for Students

Students move points to estimate the body and brain weights of animals. They compare their prediction to the actual data when they click the *Check* button.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively draw a line of fit without taking the time to draw a good line of fit in Activity 2. Remind them to discipline themselves to use what they know to aim for the line of best fit so that their predictions are more accurate.

Modifications to Pacing

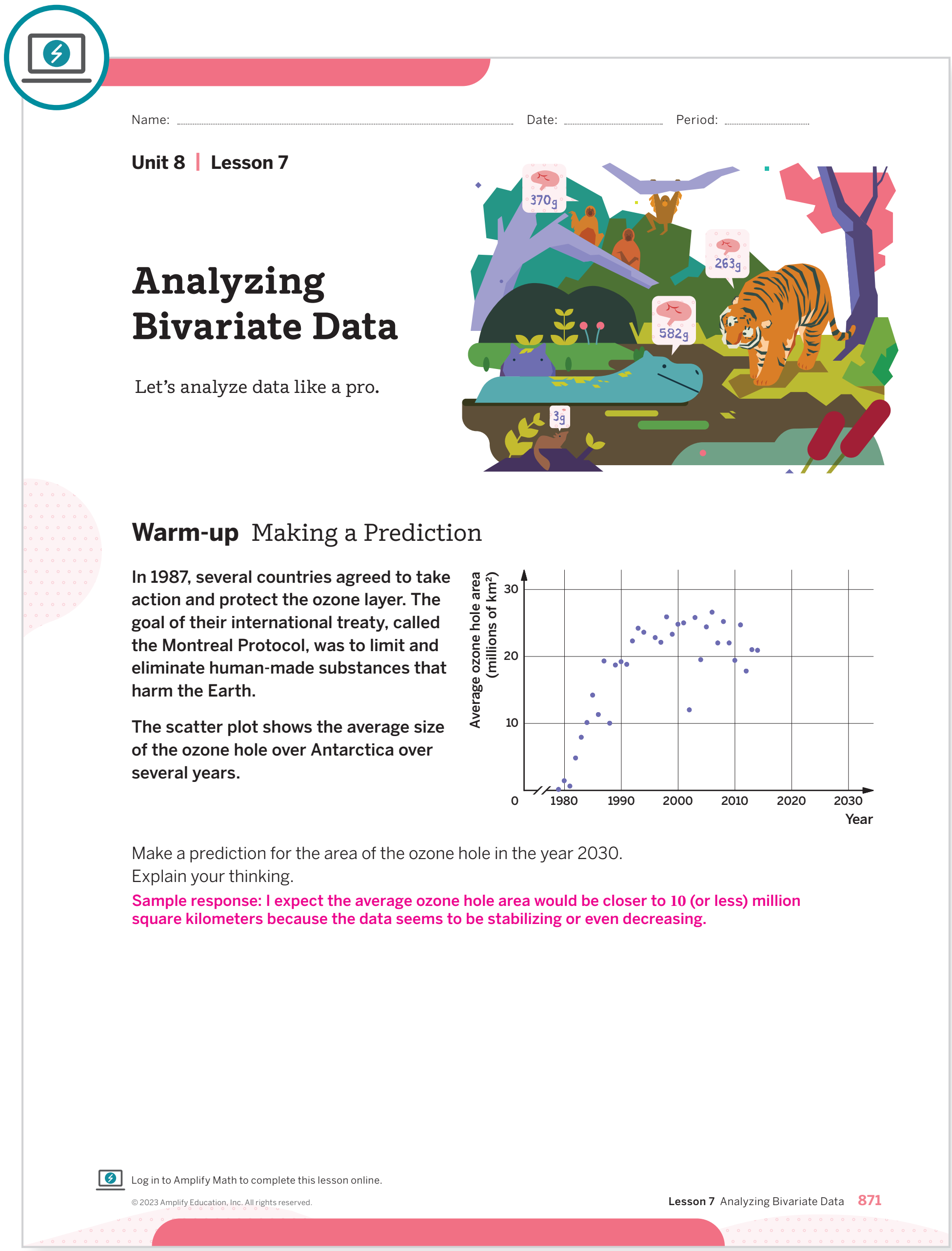
You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Independent | 5 min

Warm-up Making a Prediction

Students interpret a scatter plot to make a prediction and to recognize signs of ozone recovery.



Name: Date: Period:

Unit 8 | Lesson 7

Analyzing Bivariate Data

Let's analyze data like a pro.

Warm-up Making a Prediction

In 1987, several countries agreed to take action and protect the ozone layer. The goal of their international treaty, called the Montreal Protocol, was to limit and eliminate human-made substances that harm the Earth.

The scatter plot shows the average size of the ozone hole over Antarctica over several years.

Make a prediction for the area of the ozone hole in the year 2030. Explain your thinking.

Sample response: I expect the average ozone hole area would be closer to 10 (or less) million square kilometers because the data seems to be stabilizing or even decreasing.

Log in to Amplify Math to complete this lesson online.

Lesson 7 Analyzing Bivariate Data 871

1 Launch

Tell students they are looking at the average Antarctic ozone hole area from the years 1980 to 2015.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "What trends do you see in the scatter plot? How can you use these trends to make your prediction?"

Look for points of confusion:

- **Thinking the answer is closer to 30 million km^2.** Ask, "What patterns do you see in the size of the ozone hole since the year 1990?"

Look for productive strategies:

- Knowing the answer is closer to 18 million km^2, but having trouble providing an explanation. Provide a word bank to use, such as *negative association* or *increasing/decreasing*.

3 Connect

Ask, "How does the Montreal Protocol change the association in the scatter plot?"

Have students share the types of trends they see in the data and what it represents in terms of the ozone hole area and year. Listen for precise vocabulary and recognize students who use these terms.

Highlight that trends in the data show signs of ozone hole recovery. The actual Antarctic ozone hole area in 2019 was 9.3 million km^2.

Power-up

To power up students' ability to make predictions from a scatter plot, have students complete:

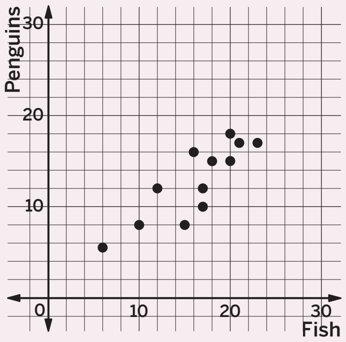
Refer to the graph to determine which predictions match the data given. Select *all* that apply.

[✓ A.] For 10 penguins you would need 12 fish.

B. For 20 penguins you would need 10 fish.

[✓ C.] If you have 24 fish you could give a snack to about 20 penguins.

D. If you have 2 fish you could give a snack to about 10 penguins.



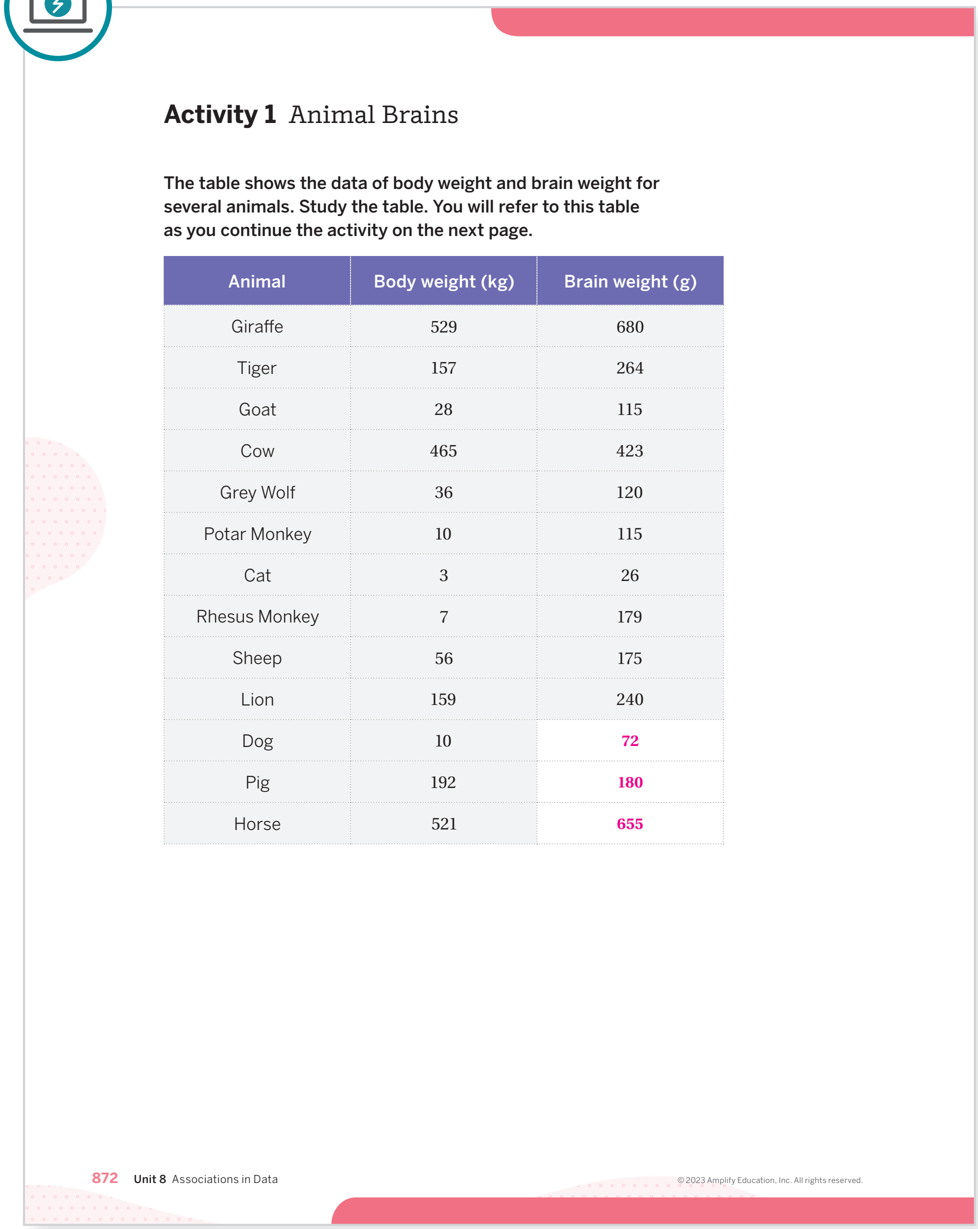
Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 6.

Pairs | 15 min

Activity 1 Animal Brains

Students create and interpret a scatter plot to look for patterns of association in bivariate measurement data.



Activity 1 Animal Brains

The table shows the data of body weight and brain weight for several animals. Study the table. You will refer to this table as you continue the activity on the next page.

Animal	Body weight (kg)	Brain weight (g)
Giraffe	529	680
Tiger	157	264
Goat	28	115
Cow	465	423
Grey Wolf	36	120
Potar Monkey	10	115
Cat	3	26
Rhesus Monkey	7	179
Sheep	56	175
Lion	159	240
Dog	10	72
Pig	192	180
Horse	521	655

872 **Unit 8** Associations in Data

1 Launch

Use the *Poll the Class* routine and ask, “Do heavier animals have heavier brains?” Then ask how they can explore this question. Provide access to rulers.

2 Monitor

Help students get started by having them label the axes and checking their work.

Look for points of confusion:

- **Forgetting how to graph points from the table.** Display coordinates of points written as (x, y), and then write x next to body weight and y next to brain weight in the table.
- **Not knowing what labels to write for the x- and y-axes.** Ask students to identify the dependent and independent variables from the table.

Look for productive strategies:

- Using the associations in the data to make a prediction for Problem 3. Ask these students to share their strategies with a partner.

Activity 1 continued ›

Differentiated Support

Accessibility: ***Activate Background Knowledge***

Ask students if they have ever considered how different animals have different brain weights. Ask them to compare the body weight and brain weight of two different animals listed in the table and describe what they notice. This will help give them context about the activity.

Accessibility: ***Vary Demands to Optimize Challenge***

Provide a pre-completed graph for Problem 1 so that students can focus on analyzing the association(s) shown in Problem 2 and make their predictions in Problem 3.

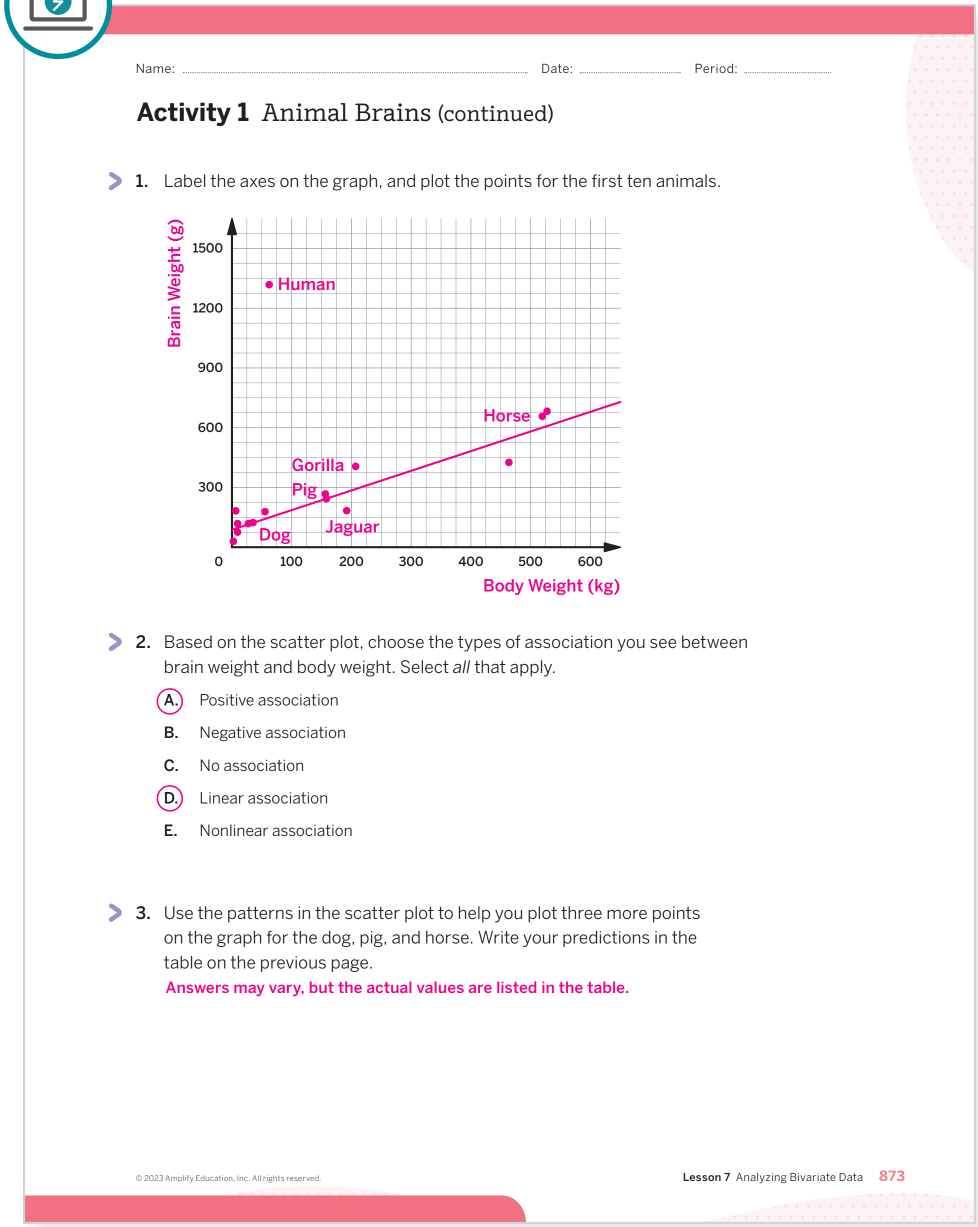
Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask questions, revoice their responses demonstrating appropriate and precise mathematical language. For example, if a student says, “I drew a straight line to help me make predictions,” revise their response by saying, “The data appear to have a positive, linear association, so you drew a straight line to model the data. Is that correct?”

Activity 1 Animal Brains (continued)

Students create and interpret a scatter plot to look for patterns of association in bivariate measurement data.



Name: ______ Date: ______ Period: ______

Activity 1 Animal Brains (continued)

1. Label the axes on the graph, and plot the points for the first ten animals.

2. Based on the scatter plot, choose the types of association you see between brain weight and body weight. Select *all* that apply.

[✓ A.] Positive association
B. Negative association
C. No association
[✓ D.] Linear association
E. Nonlinear association

3. Use the patterns in the scatter plot to help you plot three more points on the graph for the dog, pig, and horse. Write your predictions in the table on the previous page.
Answers may vary, but the actual values are listed in the table.

3 Connect

Display student work showing all points correctly plotted from the table.

Have students share how they made their predictions for Problem 3.

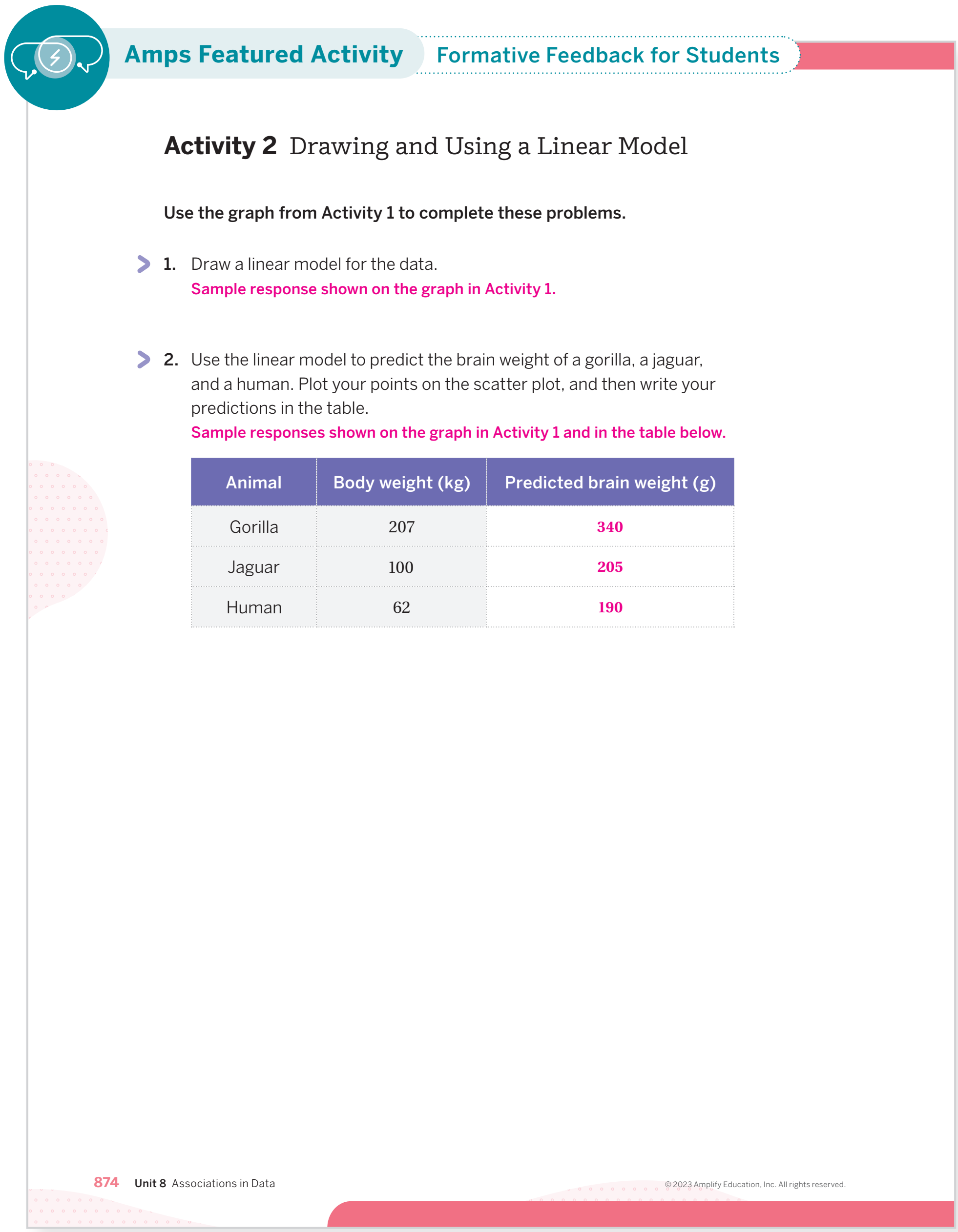
Highlight the actual data values for Problem 3. Have students talk to their partner about how their answer compared to these data values.

Ask, "Once you determine the types of association, how can you then use this information? Do heavier animals always have heavier brains?"

Pairs | 5 min

Activity 2 Drawing and Using a Linear Model

Students look at the same data set from Activity 1, but use a linear model to compare actual data with predictions.



Amps Featured Activity Formative Feedback for Students

Activity 2 Drawing and Using a Linear Model

Use the graph from Activity 1 to complete these problems.

1. Draw a linear model for the data.
   Sample response shown on the graph in Activity 1.

2. Use the linear model to predict the brain weight of a gorilla, a jaguar, and a human. Plot your points on the scatter plot, and then write your predictions in the table.
   Sample responses shown on the graph in Activity 1 and in the table below.

Animal	Body weight (kg)	Predicted brain weight (g)
Gorilla	207	340
Jaguar	100	205
Human	62	190

874 **Unit 8** Associations in Data

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1 Launch

Remind students to use a ruler when drawing their linear model.

2 Monitor

Help students get started by asking them to think back to Lesson 4. Ask, "How did the line fit the data, so that it had a high score?"

Look for points of confusion:

- **Not knowing how to make a prediction using the line.** Tell students to find the axis that represents body weight, and then use the trends in the data to estimate the brain weight.

Look for productive strategies:

- Drawing a line that does not start at $(0, 0)$. In this instance, the best y-intercept for this model may be different from a realistic y-intercept of 0 for the data.

3 Connect

Display the different linear models students created.

Ask, "How did you use the line of fit to make your prediction?"

Have students share their predictions with their partner.

Highlight that a linear model can help students make more accurate predictions.

Differentiated Support

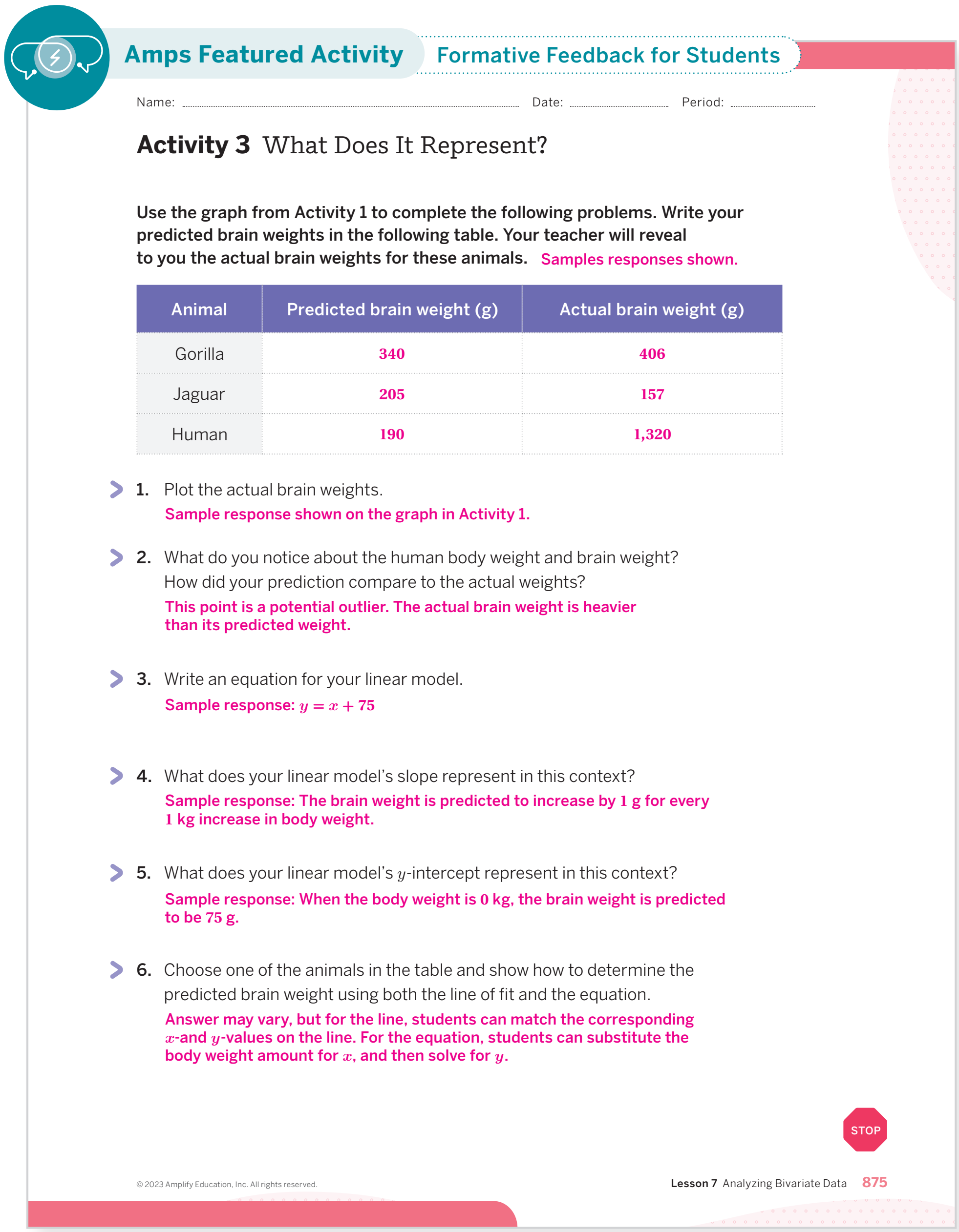
Accessibility: ***Guide Processing and Visualization***

Once students have drawn their linear models in Problem 1, ask a student volunteer to demonstrate how to use their linear model to predict the brain weight of a gorilla.

Pairs | 10 min

Activity 3 What Does It Represent?

Students read a linear model to solve problems in context and discover that sometimes predictions made by a the linear model are not true.



Amps Featured Activity Formative Feedback for Students

Name: ______________ Date: ______ Period: ______

Activity 3 What Does It Represent?

Use the graph from Activity 1 to complete the following problems. Write your predicted brain weights in the following table. Your teacher will reveal to you the actual brain weights for these animals. Samples responses shown.

Animal	Predicted brain weight (g)	Actual brain weight (g)
Gorilla	340	406
Jaguar	205	157
Human	190	1,320

1. Plot the actual brain weights.
   Sample response shown on the graph in Activity 1.

2. What do you notice about the human body weight and brain weight? How did your prediction compare to the actual weights?
   This point is a potential outlier. The actual brain weight is heavier than its predicted weight.

3. Write an equation for your linear model.
   Sample response: $y = x + 75$

4. What does your linear model's slope represent in this context?
   Sample response: The brain weight is predicted to increase by 1 g for every 1 kg increase in body weight.

5. What does your linear model's y-intercept represent in this context?
   Sample response: When the body weight is 0 kg, the brain weight is predicted to be 75 g.

6. Choose one of the animals in the table and show how to determine the predicted brain weight using both the line of fit and the equation.
   Answer may vary, but for the line, students can match the corresponding x-and y-values on the line. For the equation, students can substitute the body weight amount for x, and then solve for y.

STOP

1 Launch

Have students fill in their predicted brain weight from Activity 2 in the table. Write the actual brain weights of the animals on the board, Gorilla: 406 g, Jaguar: 157 g, Human: 1,320 g, and have students write the numbers under the "Actual brain weight" column.

2 Monitor

Help students get started by having them plot the actual brain weight data.

Look for points of confusion:

- **Having trouble finding the slope of their line.** Have students reference the Anchor Chart PDF, *Representations of Linear Relationships*.
- **Not knowing how to use the equation to predict brain weight.** Label the variable x in the equation with "body weight" and variable y with "brain weight."

Look for productive strategies:

- Noticing the predicted brain weight for the human is far from its actual brain weight. Ask students to describe this point using precise vocabulary (*outlier*) and call on these students during the Connect in this activity.

3 Connect

Display student work showing correctly plotted points of actual body and brain weights.

Have students share how they made their predictions with their partner.

Ask, "How does the actual data value of the human body and brain weight compare to the linear model? Why are they different? Will they be different for other animals? "

Highlight that a linear model does not always make a correct prediction, but it provides a good result for data that follow the trend.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can move points to estimate the body and brain weights of various animals and compare their prediction to the actual data when they click the *Check My Work* button.

Math Language Development

MLR8: Discussion Supports

While students work, display the following sentence frames for them to use to respond to Problems 4 and 5.

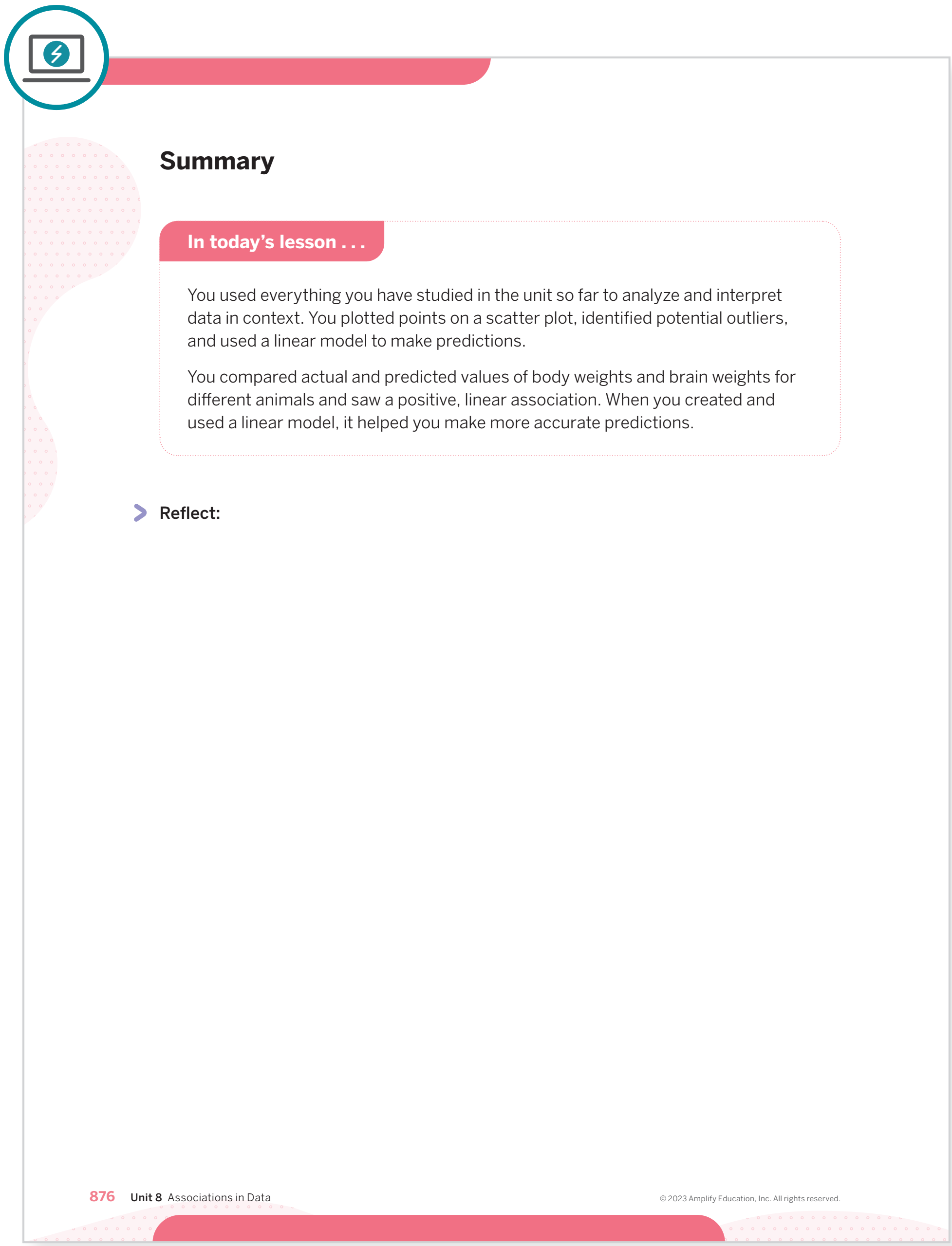
- "For every 1 kg increase in body weight, the brain weight is predicted to . . ." (Problem 4)
- "For a body weight of 0 kg, the brain weight is predicted to be _____." (Problem 5)

Ask students how they know which sentence frame to use for the slope and which sentence frame to use for the y-intercept, based on the phrases used in the sentence frame.

Whole Class | 5 min

Summary

Review and synthesize all the concepts students have learned about bivariate data in this unit.



Summary

In today's lesson . . .

You used everything you have studied in the unit so far to analyze and interpret data in context. You plotted points on a scatter plot, identified potential outliers, and used a linear model to make predictions.

You compared actual and predicted values of body weights and brain weights for different animals and saw a positive, linear association. When you created and used a linear model, it helped you make more accurate predictions.

Reflect:

876 **Unit 8** Associations in Data

Synthesize

Have students share their reflections on creating and interpreting a scatter plot to look for associations and make predictions during whole-class discussion.

Highlight that data is often collected in two variables to investigate possible associations between two numerical variables. We can use associations to make predictions.

Ask:

- "How does organizing data in a scatter plot help you identify trends and make predictions?"
- "What are your takeaways about the story of the ozone hole area?"
- "How has what you have learned been useful in your understanding of the hole in the ozone?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

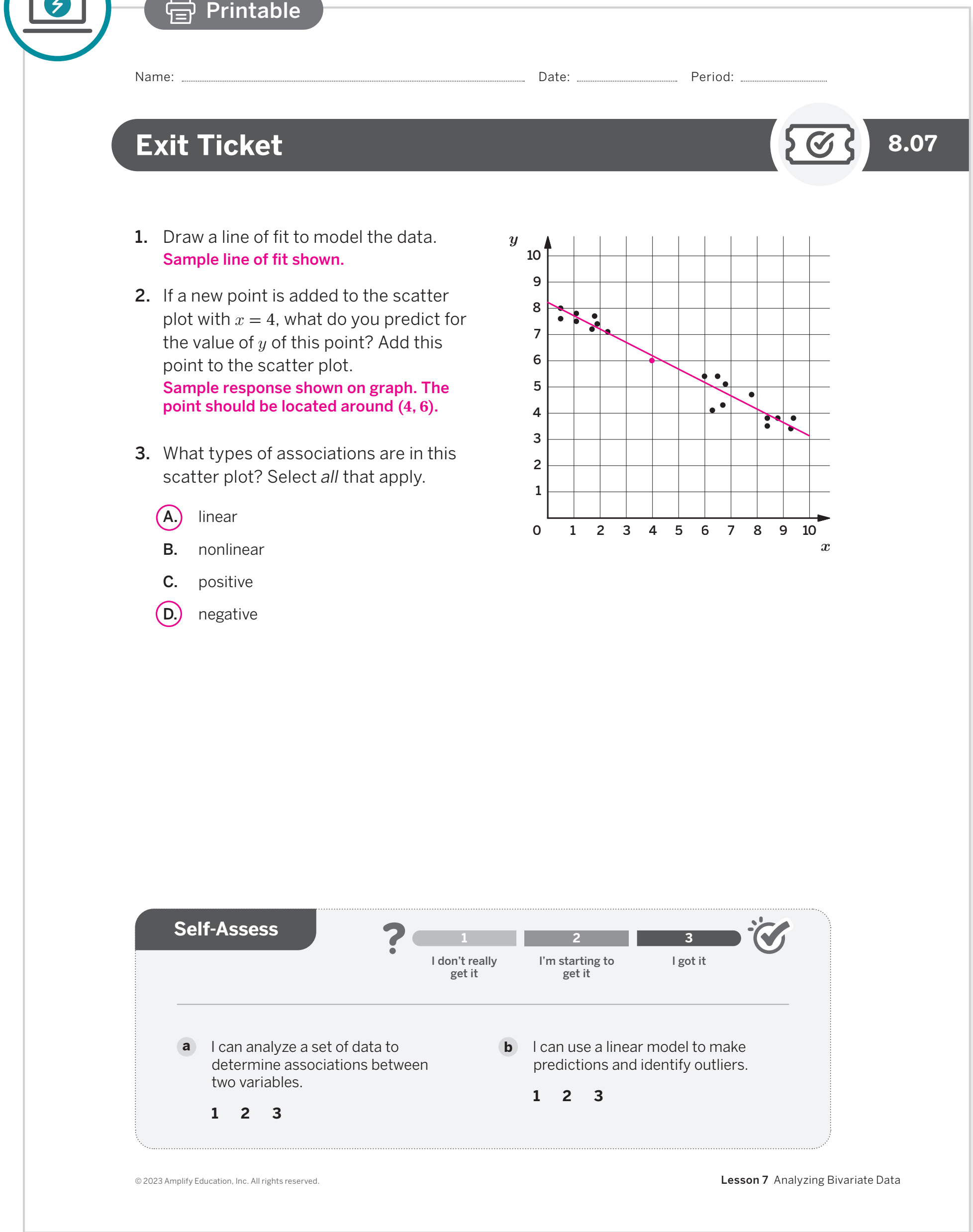
- "What associations can you find, if any, in bivariate data?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by drawing a linear model for a scatter plot, making a prediction, and identifying any associations in the data.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 8.07

1. Draw a line of fit to model the data.
   Sample line of fit shown.
2. If a new point is added to the scatter plot with $x = 4$, what do you predict for the value of y of this point? Add this point to the scatter plot.
   Sample response shown on graph. The point should be located around $(4, 6)$.
3. What types of associations are in this scatter plot? Select *all* that apply.
   - [✓ A.] linear
   - B. nonlinear
   - C. positive
   - [✓ D.] negative

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can analyze a set of data to determine associations between two variables.
1 2 3

b I can use a linear model to make predictions and identify outliers.
1 2 3

Lesson 7 Analyzing Bivariate Data

Success looks like . . .

- **Language Goal:** Describing the relationship between two variables using a line fit to data on a scatter plot. **(Speaking and Listening, Writing)**
  - » Describing the associations between the two variables in the graph in Problem 3.
- **Language Goal:** Interpreting points on the scatter plot, including points that do and do not lie on a line fit to the data. **(Speaking and Listening, Writing)**
- **Language Goal:** Interpreting the slope of a line fit to data in context. **(Speaking and Listening, Writing)**

Suggested next steps

If students do not draw a good line of fit, consider:

- Asking students to identify the association before drawing a line of fit.

If students do not correctly identify the association, consider:

- Drawing several lines that are positive and negative. Ask students to write what is similar about the positive graphs and then the negative graphs.
- Reviewing associations in Lesson 3.

If students do not predict the correct value of y when $x = 4$, consider:

- Giving a linear graph in place of a scatter plot and asking students to make the same prediction. Once students have mastered this concept, repeat using a scatter plot.
- Assigning Practice Problem 3.

Professional Learning

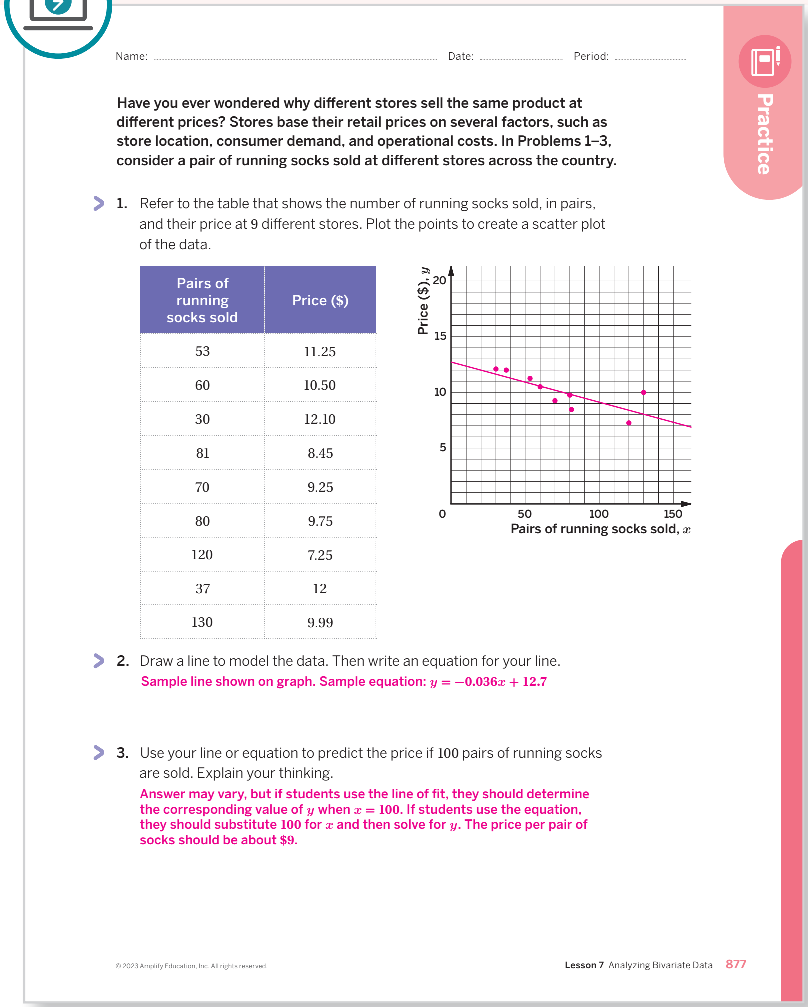
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did students look for and make use of structure today? How are you helping students become aware of how they are progressing in this area?
- During the discussion about Activity 3, how did you encourage each student to share their understandings?

Practice

Independent



Name: Date: Period:

Have you ever wondered why different stores sell the same product at different prices? Stores base their retail prices on several factors, such as store location, consumer demand, and operational costs. In Problems 1–3, consider a pair of running socks sold at different stores across the country.

1. Refer to the table that shows the number of running socks sold, in pairs, and their price at 9 different stores. Plot the points to create a scatter plot of the data.

Pairs of running socks sold	Price ($)
53	11.25
60	10.50
30	12.10
81	8.45
70	9.25
80	9.75
120	7.25
37	12
130	9.99

2. Draw a line to model the data. Then write an equation for your line.
Sample line shown on graph. Sample equation: $y = -0.036x + 12.7$

3. Use your line or equation to predict the price if 100 pairs of running socks are sold. Explain your thinking.
Answer may vary, but if students use the line of fit, they should determine the corresponding value of y when $x = 100$. If students use the equation, they should substitute 100 for x and then solve for y. The price per pair of socks should be about $9.

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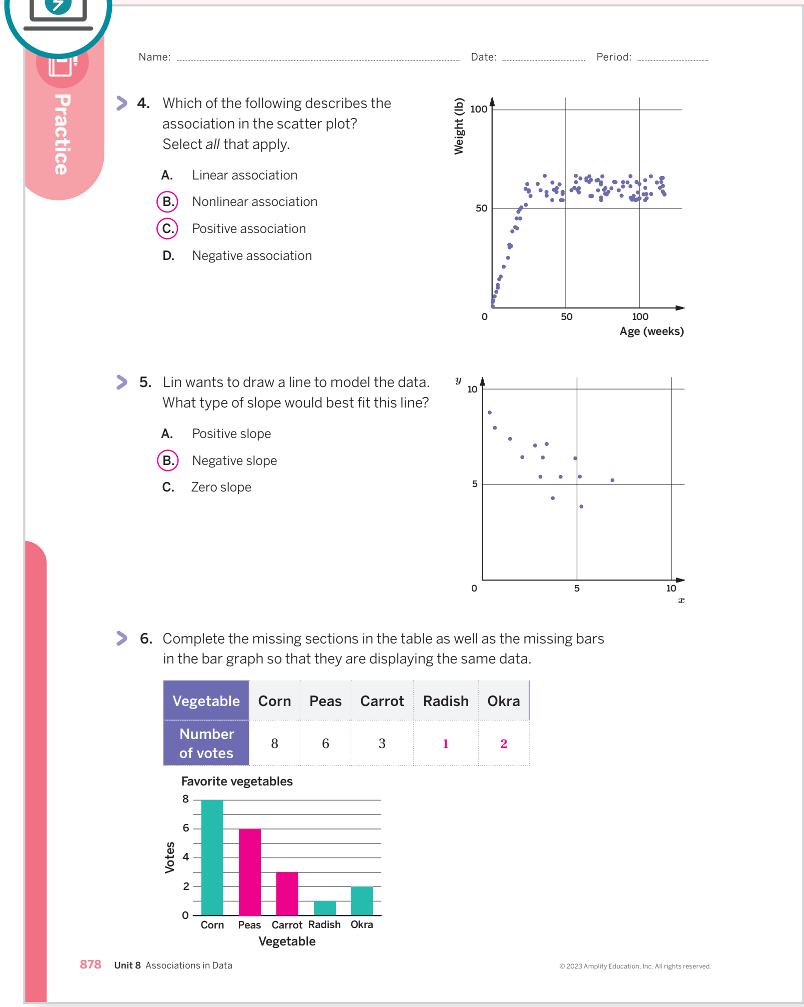
Lesson 7 Analyzing Bivariate Data 877



Name: Date: Period:

4. Which of the following describes the association in the scatter plot? Select *all* that apply.
   - **A.** Linear association
   - [✓ **B.**] Nonlinear association
   - [✓ **C.**] Positive association
   - **D.** Negative association

5. Lin wants to draw a line to model the data. What type of slope would best fit this line?
   - **A.** Positive slope
   - [✓ **B.**] Negative slope
   - **C.** Zero slope

6. Complete the missing sections in the table as well as the missing bars in the bar graph so that they are displaying the same data.

Vegetable	Corn	Peas	Carrot	Radish	Okra
Number of votes	8	6	3	**1**	**2**

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 3	2
	3	Activity 1	3
Spiral	4	Unit 8 Lesson 3	1
	5	Unit 8 Lesson 4	1
Formative ⚡	6	Unit 8 Lesson 8	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 8 | **LESSON 8**

Looking for Associations

Let's look for associations in data.

Focus

Goals

1. **Language Goal:** Coordinate two-way tables and double bar graphs representing the same data. **(Speaking and Listening, Writing)**
2. Use two-tables, scatter plots, and bar graphs to make sense of data.

Coherence

Today

Today, students will encounter various bivariate data sets. Students will explore different ways of representing this data, coordinating between two-way tables, scatter plots, and bar graphs. Students will begin to draw conclusions based on the data sets before conducting more detailed analysis in the following lesson.

Previously

In prior courses, students studied different ways of representing sets of data.

Coming Soon

In Lesson 9, students will take what they have learned about representing bivariate data, and perform analysis on the data, calculating and comparing relative frequencies.

Rigor

- Students build **conceptual understanding** of bivariate data by coordinating between two-way tables, scatter plots, and bar graphs.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	5 min	12 min	5 min	8 min
Pairs	Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- colored pencils

Math Language Development

New words

- ***two-way table***

Review words

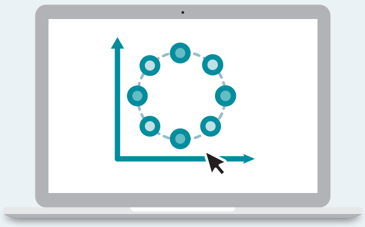
- *bar graph*
- *scatter plot*

Amps | Featured Activity

Warm-up

Collaborative Scatter Plot

Students respond to questions about bicycle riding and see their classmates' data added to a scatter plot.



Building Math Identity and Community

Connecting to Mathematical Practices

At first, students might become stressed as they find another way to represent the categorical data in Activity 2. Have students draw comparisons between the two-way table from Activity 1 and the bar graph in Activity 2, identifying the similarities and differences of how the same data is displayed in each way.

Modifications to Pacing

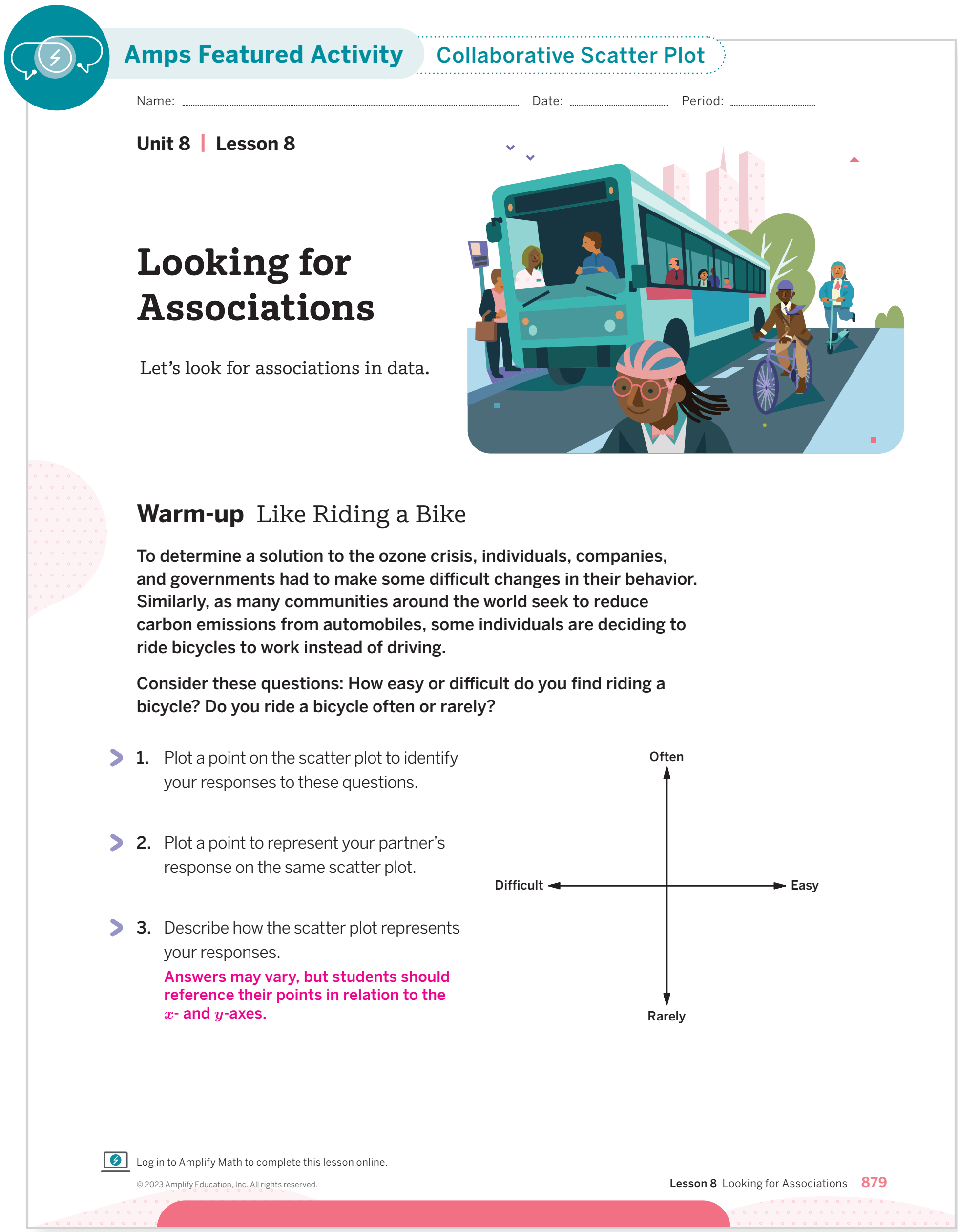
You may want to consider this additional modification if you are short on time.

- **Activity 3** may be omitted.

Warm-up Like Riding a Bike

Students generate categorical data to help them explore the relationship between the difficulty and frequency of riding a bicycle.

Amps Featured Activity Collaborative Scatter Plot



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Unit 8 | Lesson 8

Looking for Associations

Let's look for associations in data.

Warm-up Like Riding a Bike

To determine a solution to the ozone crisis, individuals, companies, and governments had to make some difficult changes in their behavior. Similarly, as many communities around the world seek to reduce carbon emissions from automobiles, some individuals are deciding to ride bicycles to work instead of driving.

Consider these questions: How easy or difficult do you find riding a bicycle? Do you ride a bicycle often or rarely?

1. Plot a point on the scatter plot to identify your responses to these questions.
2. Plot a point to represent your partner's response on the same scatter plot.
3. Describe how the scatter plot represents your responses.
   Answers may vary, but students should reference their points in relation to the x- and y-axes.

Log in to Amplify Math to complete this lesson online.

Lesson 8 Looking for Associations 879

1 Launch

Activate students' prior knowledge about the ozone hole area from previous lessons. Make the connection that today, we face a new environmental concern, the rise in global temperatures, that will require a new set of trade-offs to solve the problem. For example, some individuals are switching to transportation methods that have lower emissions than driving a car, such as riding a bike.

Ask:

- "Have you ever made changes to your behavior out of concern for the environment?"
- "Can you think of any examples of rules, restrictions, or suggestions you know about that seek to solve an environmental concern?"

Give students 1 minute to independently plot their opinions about riding a bicycle on the scatter plot before working with a partner.

2 Monitor

Help students get started by saying, "Look at the label on the x-axis first. Point to how you feel about riding a bicycle. Now, look at the y-axis."

Look for points of confusion:

- **Being unsure of where to plot a point.** Ask, "If you really like riding a bicycle, where would you plot your point?"

3 Connect

Display a blank scatter plot.

Have students share where they placed their points.

Highlight students who describe both variables or use mathematical vocabulary, such as *scatter plot*, x-*axis*, and y-*axis*.

Ask students to share what they wrote. Then ask, "How is this scatter plot a useful way to describe the two variables you wanted to represent in the data? What limitations does it have?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see their classmates' data added to a collaborative scatter plot as the class response to the questions.

Power-up

To power up students' ability to create and interpret bar graphs:

Provide students with a copy of the Power-up PDF.

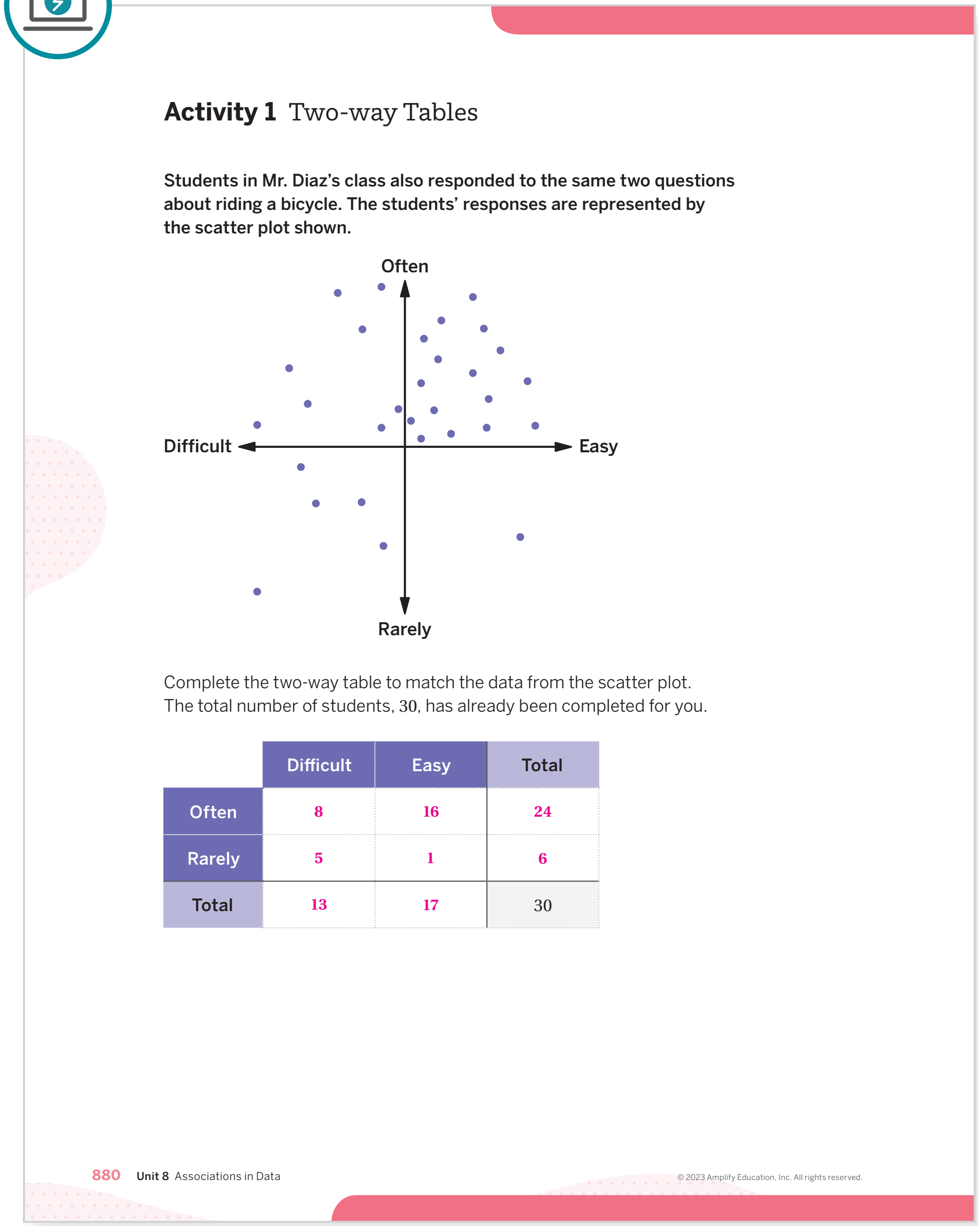
Use: Before Activity 2

Informed by: Performance on Lesson 7, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 5,–7

Independent | 10 min

Activity 1 Two-way Tables

Students use data represented on a scatter plot to complete a matching two-way table.



Activity 1 Two-way Tables

Students in Mr. Diaz's class also responded to the same two questions about riding a bicycle. The students' responses are represented by the scatter plot shown.

Complete the two-way table to match the data from the scatter plot. The total number of students, 30, has already been completed for you.

	Difficult	Easy	Total
Often	8	16	24
Rarely	5	1	6
Total	13	17	30

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1 Launch

Define a ***two-way table*** as a table that represents categorical data with two variables.

Tell students that the two-way table is incomplete and that the total number of points in the scatter plot matches the total, 30, in the two-way table. Have students complete the two-way table independently before sharing with the whole class.

2 Monitor

Help students get started by pointing to the first cell (Often, Difficult) and asking students where they see the data related to this cell on the scatter plot.

Look for points of confusion:

- **Misreading the table.** Point to the cell and ask students what it represents. Narrow the focus by having students find the number of all the students who ride a bicycle often and write the value in the table. Ask them what they can find next.
- **Miscounting numbers on the scatter plot.** Have students check their work by finding the totals for each row and column together. They should get matching totals of 30.

3 Connect

Display student work with a correct two-way table that matches the scatter plot.

Have students share how they know the two-way table matches the scatter plot.

Ask, "What are the pros and cons of each method of representation?"

Highlight that both representations show the same data. The scatter plot shows visual trends, such as a potential outlier of someone who thinks riding a bike is easy but does not ride often. The two-way table shows the exact numerical data.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to complete one of the cells in the two-way table. For example, demonstrate that the value that goes in the Easy column and Rarely cell should be 1 because there is only one data value corresponding to both of those terms on the scatter plot.

Extension: *Math Enrichment*

Have students predict, without calculating, the total sum for all of the values in the two-way table. Then have them explain why this total sum is equal to 4 • 30.

Math Language Development

MLR7: Compare and Connect

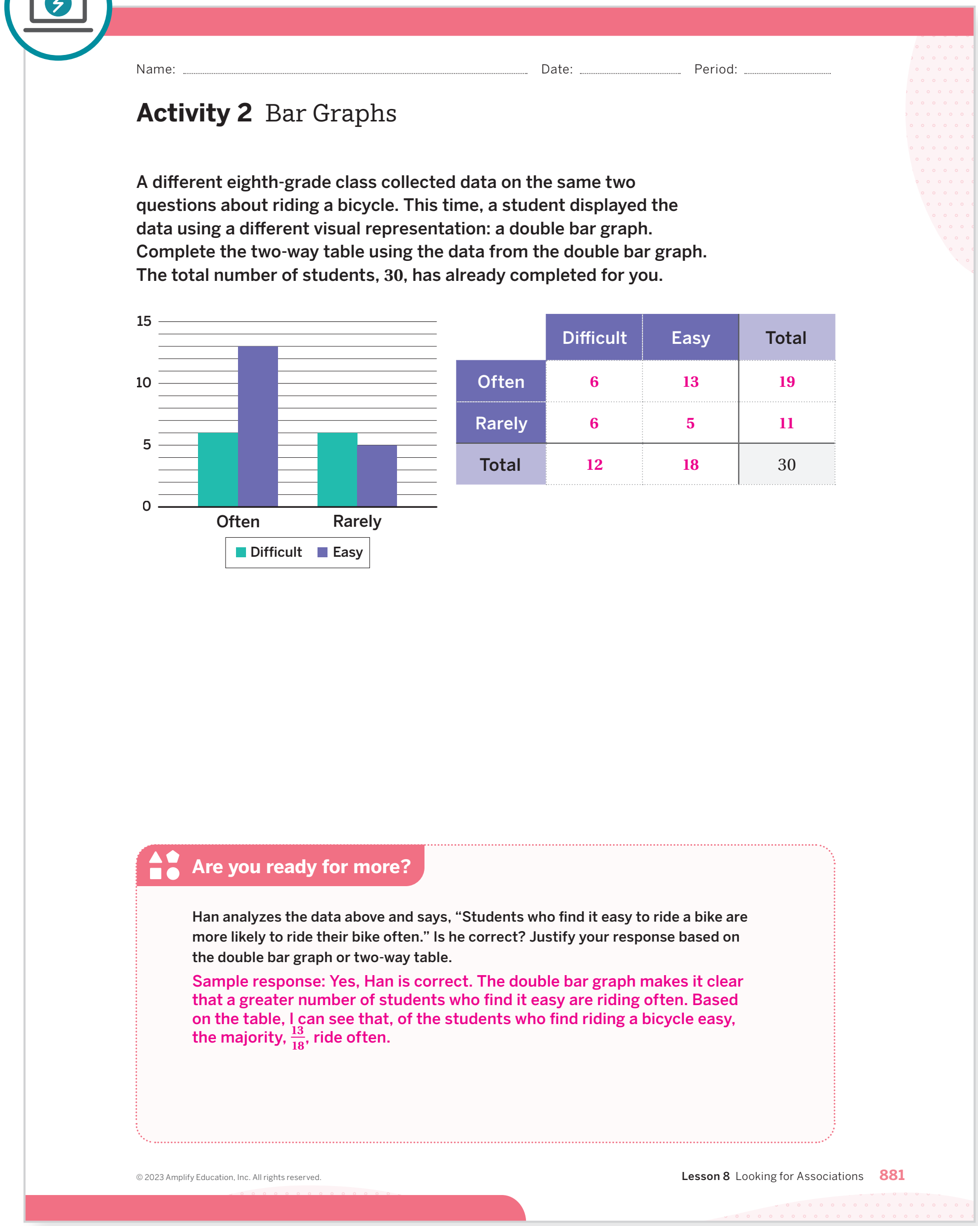
During the Connect, as students respond to the Ask question, draw their attention to the connections between the scatter plot and the two-way table. Ask:

- "Where do you see the total number of students surveyed on the scatter plot? In the two-table?"
- "Which representation do you prefer to use to determine the number of students who chose 'difficult' and 'rarely'? Why?"
- "Which representation do you prefer to see overall trends and whether there might be an association in the data? Why?"

Independent | 5 min

Activity 2 Bar Graphs

Students look at a new data set in order to coordinate a two-way table and double bar graph.



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Activity 2 Bar Graphs

A different eighth-grade class collected data on the same two questions about riding a bicycle. This time, a student displayed the data using a different visual representation: a double bar graph. Complete the two-way table using the data from the double bar graph. The total number of students, 30, has already completed for you.

	Difficult	Easy	Total
Often	6	13	19
Rarely	6	5	11
Total	12	18	30

Are you ready for more?

Han analyzes the data above and says, “Students who find it easy to ride a bike are more likely to ride their bike often.” Is he correct? Justify your response based on the double bar graph or two-way table.

Sample response: Yes, Han is correct. The double bar graph makes it clear that a greater number of students who find it easy are riding often. Based on the table, I can see that, of the students who find riding a bicycle easy, the majority, $\frac{13}{18}$, ride often.

1 Launch

Ask students to review the double bar graph and two-way table. Note for students the key and axis labels. Give students 3 minutes to coordinate the two representations independently before discussing with the class.

2 Monitor

Help students get started by asking, “What labels do you see on the double bar graph? What does the key tell you? Where do we see this information in our two-way table? ”

Look for points of confusion:

- **Misreading the double bar graph.** Point out one bar at a time, and ask what each bar represents.
- **Incorrectly matching the data from the double bar graph in the two-way table.** Ask students how they can use the totals to check their answers. Direct students back to the double bar graph to revise.

3 Connect

Display student work showing the correct two-way table and double bar graph.

Have students share how they know the double bar graph matches the two-way table. Have students share strategies for using the row and column totals to check that the data is accurate.

Ask, “What are the pros and cons of each method of representation?”

Highlight student observations on how the double bar graph represents the two variables.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to complete one of the cells in the two-way table. For example, demonstrate that the value that goes in the Difficult column and Rarely cell should be 6 because the bar height for Rarely and Difficult is 6.

Math Language Development

MLR7: Compare and Connect

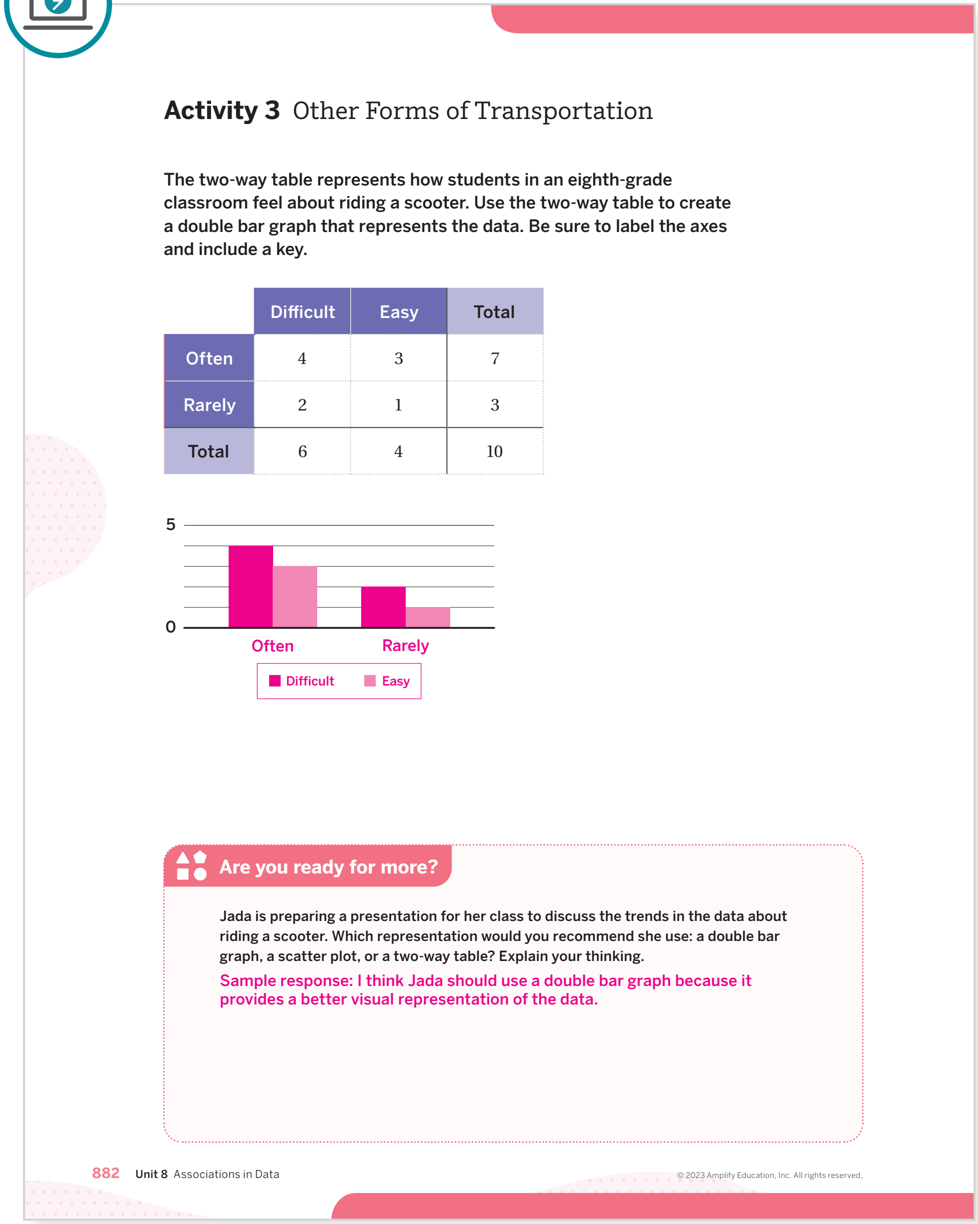
During the Connect, as students respond to the Ask question, draw their attention to the connections between the bar graph and the two-way table. Ask:

- “Where do you see the total number of students surveyed on the bar graph? In the two-table?”
- “Which representation do you prefer to use to determine the number of students who chose ‘difficult’ and ‘often’? Why?”
- “Which representation do you prefer to see overall trends and whether there might be an association in the data? Why?”

Pairs | 12 min

Activity 3 Other Forms of Transportation

Using two new data sets describing alternative forms of transportation, students create double bar graphs and draw conclusions based on any associations in the data.



Activity 3 Other Forms of Transportation

The two-way table represents how students in an eighth-grade classroom feel about riding a scooter. Use the two-way table to create a double bar graph that represents the data. Be sure to label the axes and include a key.

	Difficult	Easy	Total
Often	4	3	7
Rarely	2	1	3
Total	6	4	10

Are you ready for more?

Jada is preparing a presentation for her class to discuss the trends in the data about riding a scooter. Which representation would you recommend she use: a double bar graph, a scatter plot, or a two-way table? Explain your thinking.

Sample response: I think Jada should use a double bar graph because it provides a better visual representation of the data.

1 Launch

Ask students to recall the double bar graph from Activity 2 as an example. Have students review the new data in the two-way table. Give students the option to use colored pencils. Remind students to complete their key and label the axes.

After discussing this example about riding scooters, have students make new bar graphs and conclusions for the data set describing public transportation use.

2 Monitor

Help students get started by asking, "Look back at the bar graph in Activity 2. How should you label your axes here? What information should go in your key?"

Look for points of confusion:

- **Having difficulty labeling all parts of the double bar graph.** While most students will likely label the x-axis "Often" and "Rarely" to match Activity 2, labeling "Easy" and "Difficult" is also acceptable. In this case, encourage students to use "Often" and "Rarely" as in Activity 2 for consistency and clarity.
- **Having difficulty drawing associations from the data.** Give students a sentence frame to complete, such as, "Students who use bicycles often also found bicycles ___ to use."

Activity 3 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide a double bar graph partially completed for students and ask them to complete certain information on the bar graph. For example, provide the bar heights for "Often" and ask students to complete the graph by adding the bar heights for "Rarely."

Math Language Development

MLR5: Co-craft Questions

During the Launch, display the introductory text and two-way table. Ask pairs of students to work together to craft 2-3 questions they could ask about this scenario or graph. Sample questions shown.

- "How many total students were surveyed?
- Do more students think it is difficult or easy to ride a scooter?
- Do students who find it easy to ride a scooter ride it more often than students who find it difficult?

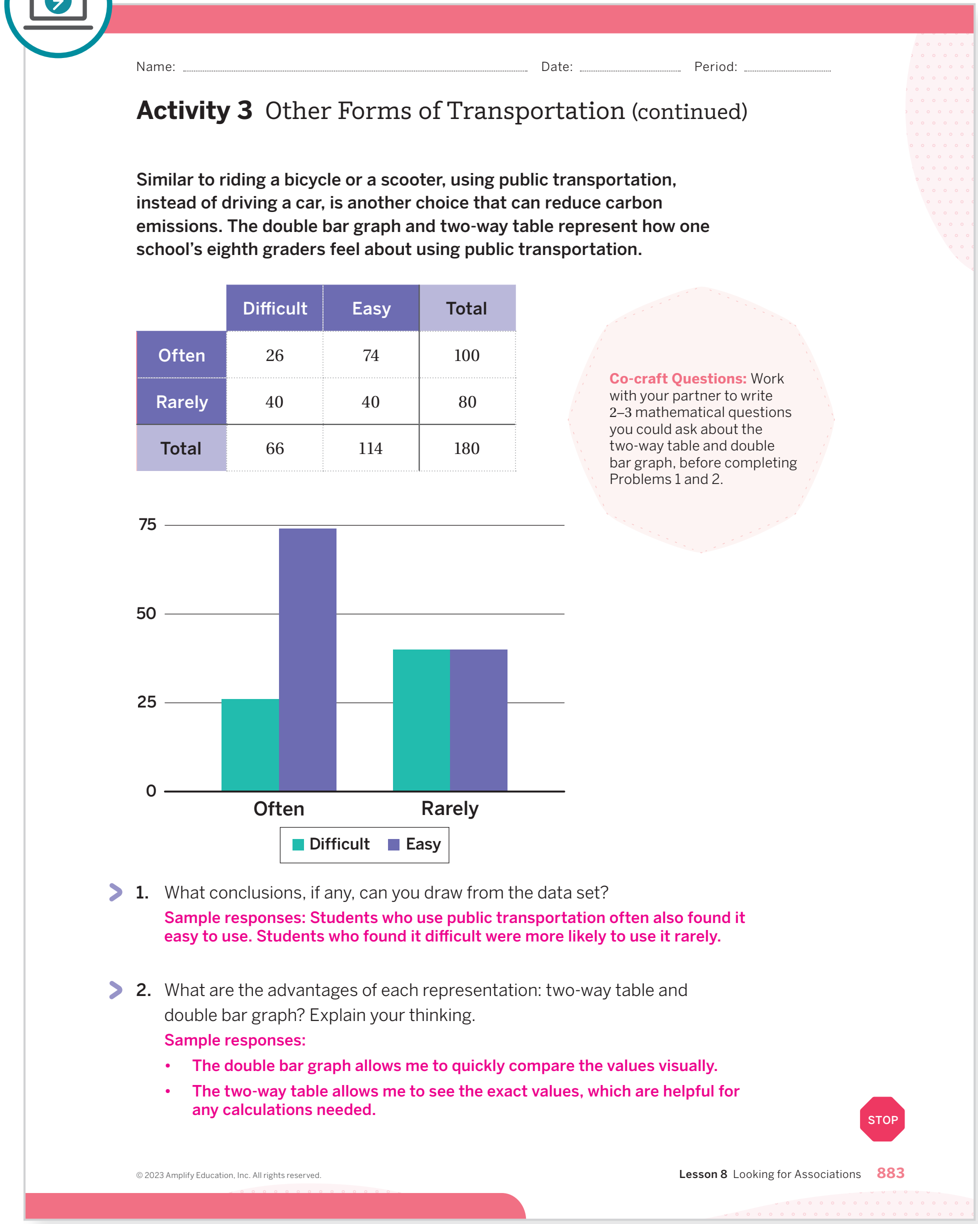
English Learners

To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 12 min

Activity 3 Other Forms of Transportation (continued)

Using two new data sets describing alternative forms of transportation, students create double bar graphs and draw conclusions based on any associations in the data.



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Activity 3 Other Forms of Transportation (continued)

Similar to riding a bicycle or a scooter, using public transportation, instead of driving a car, is another choice that can reduce carbon emissions. The double bar graph and two-way table represent how one school's eighth graders feel about using public transportation.

	Difficult	Easy	Total
Often	26	74	100
Rarely	40	40	80
Total	66	114	180

Co-craft Questions: Work with your partner to write 2–3 mathematical questions you could ask about the two-way table and double bar graph, before completing Problems 1 and 2.

1. What conclusions, if any, can you draw from the data set?
   Sample responses: Students who use public transportation often also found it easy to use. Students who found it difficult were more likely to use it rarely.

2. What are the advantages of each representation: two-way table and double bar graph? Explain your thinking.
   Sample responses:
   - **The double bar graph allows me to quickly compare the values visually.**
   - **The two-way table allows me to see the exact values, which are helpful for any calculations needed.**

STOP

3 Connect

Display student work showing a correct double bar graph.

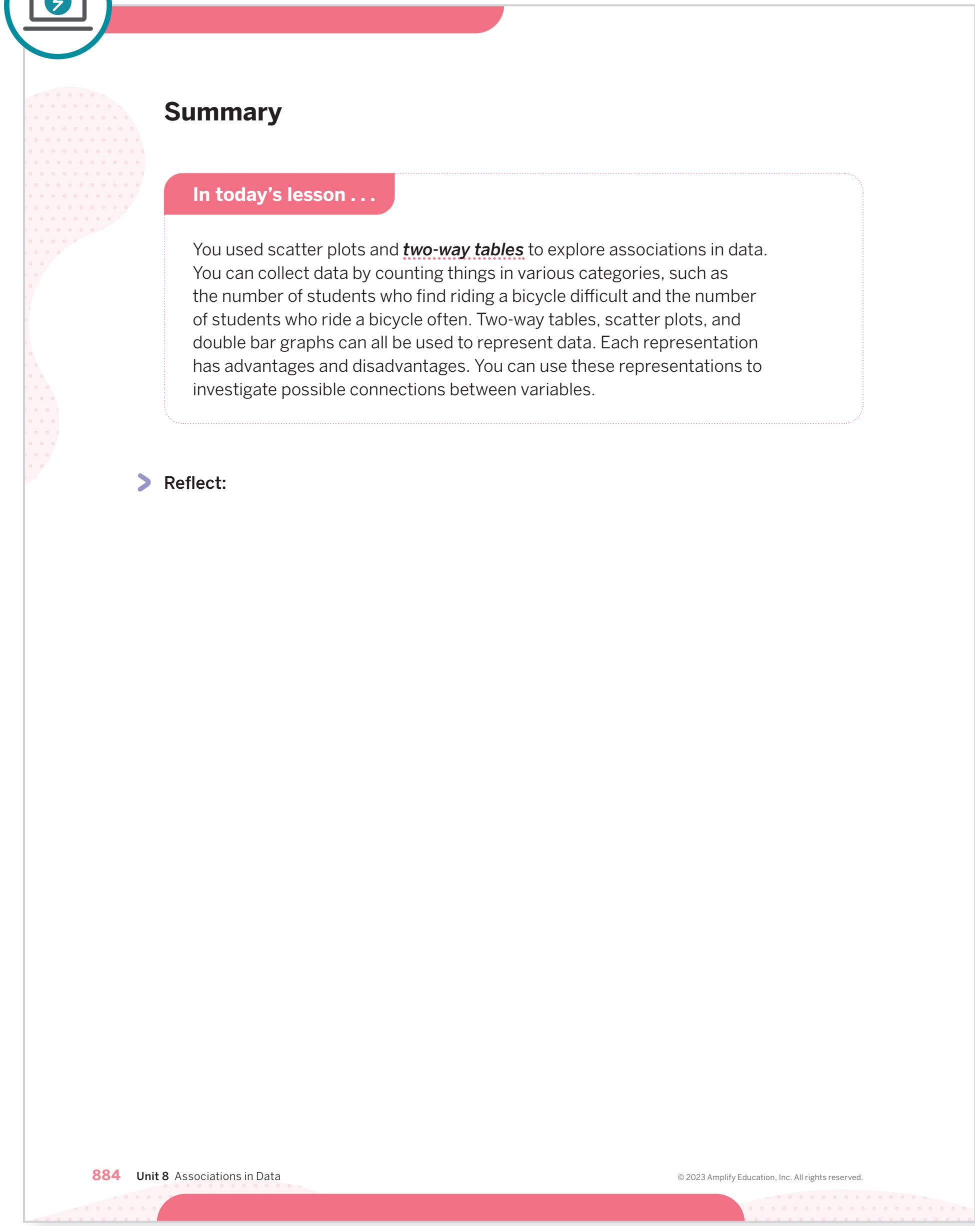
Have students share how they know the double bar graph matches the two-way table.

Highlight any reference to the key or axes labels, and how students were able to determine row and column totals based on the parts of the double bar graph. For the public transportation bar graph, highlight various conclusions based on the data. Announce that the class will be discussing more ways in which they can analyze bivariate data sets in Lesson 9. As students discuss, point out those who use math vocabulary and strong evidence. This will help elevate the quality of student responses.

Whole Class | 5 min

Summary

Review and synthesize the different representations used thus far to display bivariate data: scatter plots, two-way tables, and double bar graphs.



Summary

In today's lesson . . .

You used scatter plots and ***two-way tables*** to explore associations in data. You can collect data by counting things in various categories, such as the number of students who find riding a bicycle difficult and the number of students who ride a bicycle often. Two-way tables, scatter plots, and double bar graphs can all be used to represent data. Each representation has advantages and disadvantages. You can use these representations to investigate possible connections between variables.

Reflect:

884 **Unit 8** Associations in Data

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Synthesize

Display the double bar graph and the two-way table from Activity 3.

Have students share their opinions on the different representations of the data.

Highlight similarities and differences in each representation.

Ask, "How could you determine when to use each representation going forward?"

Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

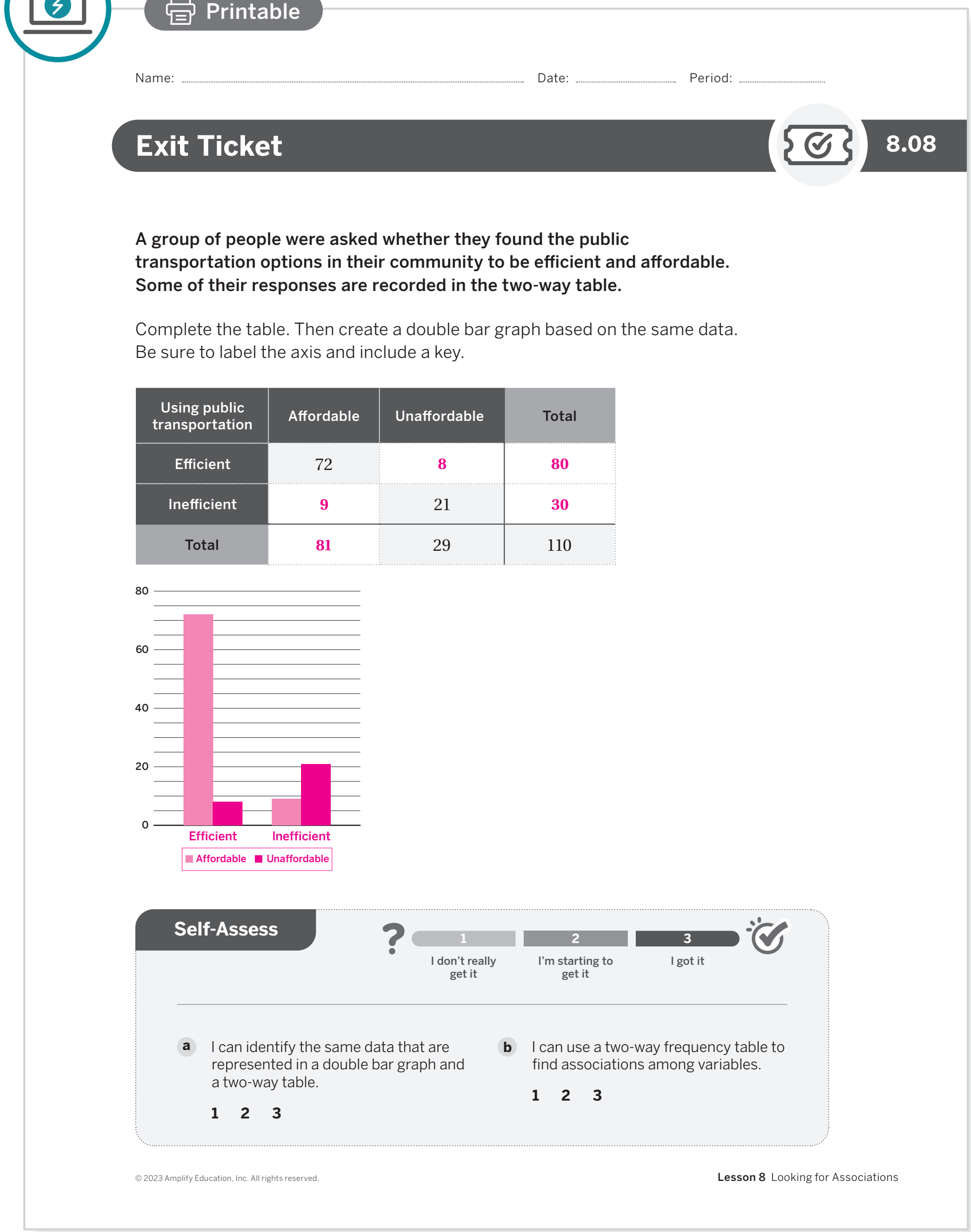
- "What ways of representing data did you learn about today? How are they helpful?"
- "What does it mean for there to be an association in the data?"

Independent | 8 min

Exit Ticket

Students demonstrate their understanding by completing a double graph and two-way table to represent the same data set.

Printable



Name: Date: Period:

Exit Ticket

8.08

A group of people were asked whether they found the public transportation options in their community to be efficient and affordable. Some of their responses are recorded in the two-way table.

Complete the table. Then create a double bar graph based on the same data. Be sure to label the axis and include a key.

Using public transportation	Affordable	Unaffordable	Total
Efficient	72	8	80
Inefficient	9	21	30
Total	81	29	110

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can identify the same data that are represented in a double bar graph and a two-way table.

1 2 3

b I can use a two-way frequency table to find associations among variables.

1 2 3

Lesson 8 Looking for Associations

Success looks like . . .

- **Language Goal:** Coordinating two-way tables and double bar graphs representing the same data. **(Speaking and Listening, Writing)**
  - » Creating a double bar graph for the two-way table on public transportation.
- **Goal:** Using two-way tables, scatter plots, and bar graphs to make sense of data.

Suggested next steps

If students are missing labels or keys in their double bar graph, consider:

- Reviewing Activity 2 for an exemplar version of a double bar graph.
- Assigning Practice Problem 3.

If students create two-way tables that do not match the double bar graph, consider:

- Reviewing strategies from Activity 3.
- Asking, "How can you verify that your two-way table and double bar graph represent the same data?"

Professional Learning

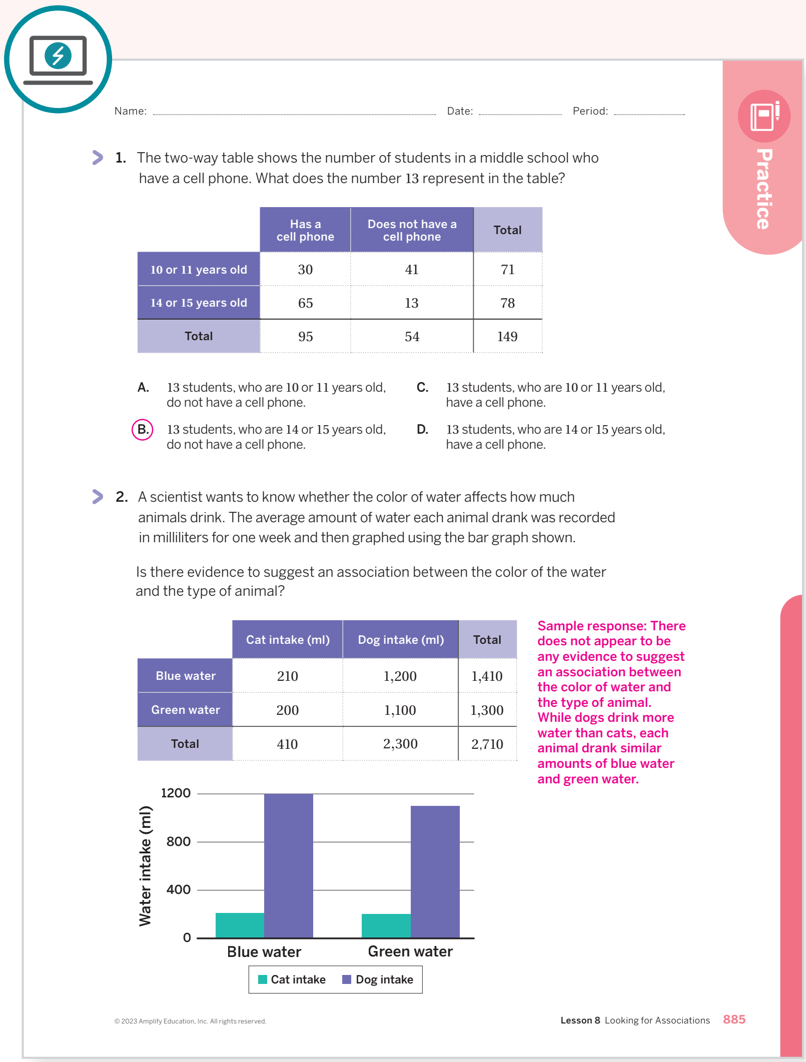
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How did coordinating two-way tables and bar graphs set students up to develop their data literacy?
- What worked and didn't work today? What might you need to better support students in the next lesson on representing data?

Practice

Independent



Name: Date: Period:

1. The two-way table shows the number of students in a middle school who have a cell phone. What does the number 13 represent in the table?

	Has a cell phone	Does not have a cell phone	Total
10 or 11 years old	30	41	71
14 or 15 years old	65	13	78
Total	95	54	149

A. 13 students, who are 10 or 11 years old, do not have a cell phone.

[✓ B.] 13 students, who are 14 or 15 years old, do not have a cell phone.

C. 13 students, who are 10 or 11 years old, have a cell phone.

D. 13 students, who are 14 or 15 years old, have a cell phone.

2. A scientist wants to know whether the color of water affects how much animals drink. The average amount of water each animal drank was recorded in milliliters for one week and then graphed using the bar graph shown.

Is there evidence to suggest an association between the color of the water and the type of animal?

	Cat intake (ml)	Dog intake (ml)	Total
Blue water	210	1,200	1,410
Green water	200	1,100	1,300
Total	410	2,300	2,710

Sample response: There does not appear to be any evidence to suggest an association between the color of water and the type of animal. While dogs drink more water than cats, each animal drank similar amounts of blue water and green water.

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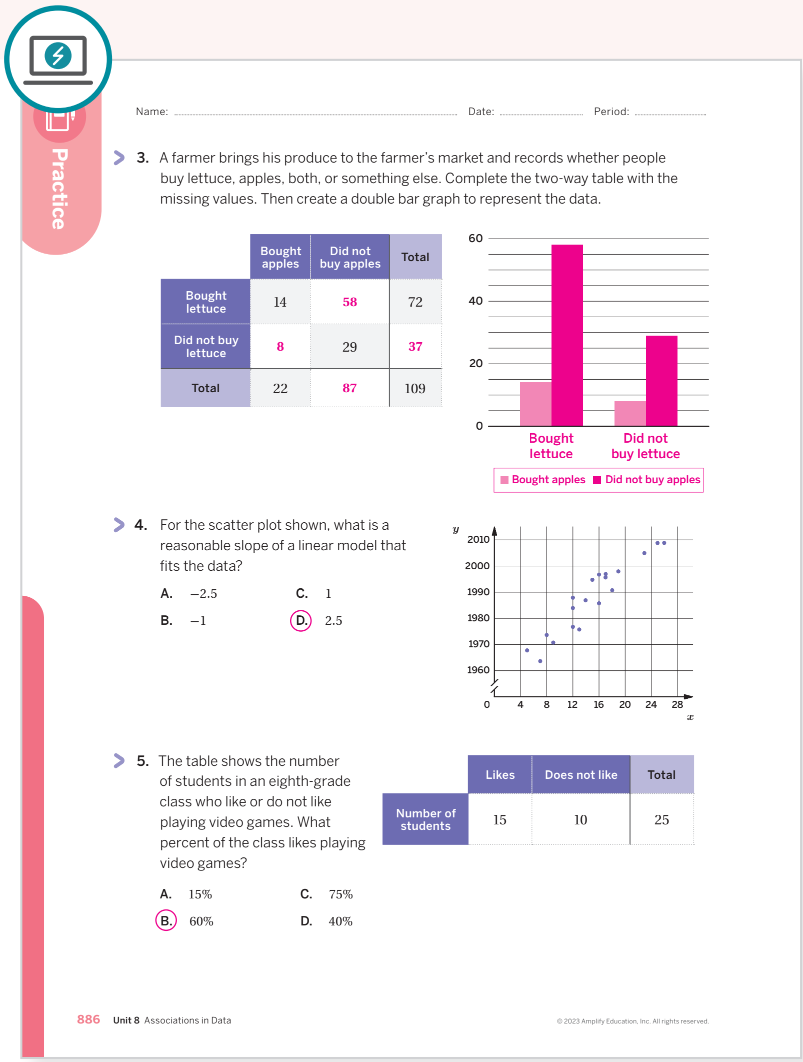
Lesson 8 Looking for Associations 885



Name: Date: Period:

3. A farmer brings his produce to the farmer's market and records whether people buy lettuce, apples, both, or something else. Complete the two-way table with the missing values. Then create a double bar graph to represent the data.

	Bought apples	Did not buy apples	Total
Bought lettuce	14	**58**	72
Did not buy lettuce	**8**	29	**37**
Total	22	**87**	109

4. For the scatter plot shown, what is a reasonable slope of a linear model that fits the data?

A. −2.5

B. −1

C. 1

[✓ D.] 2.5

5. The table shows the number of students in an eighth-grade class who like or do not like playing video games. What percent of the class likes playing video games?

	Likes	Does not like	Total
Number of students	15	10	25

A. 15%

[✓ B.] 60%

C. 75%

D. 40%

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 3	2
	3	Activity 3	2
Spiral	4	Unit 8 Lesson 5	2
Formative ⚡	5	Unit 8 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

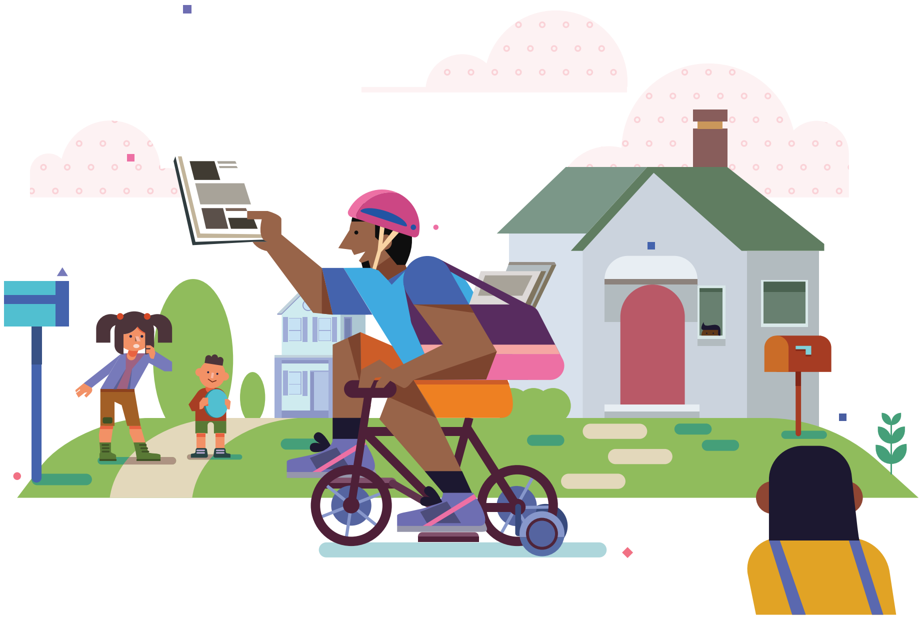
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 8 | **LESSON 9 – CAPSTONE**

Using Data Displays to Find Associations

Let's use data displays to find associations.



Focus

Goals

1. **Language Goal:** Calculate relative frequencies and describe associations between variables using a relative frequency table. **(Speaking and Listening, Writing)**
2. **Language Goal:** Coordinate two-way tables, bar graphs, and segmented bar graphs representing the same data. **(Speaking and Listening, Writing)**

Coherence

Today

Students use two-way tables, bar graphs, and segmented bar graphs to decide whether there is evidence of an association in categorical data. Students will calculate frequencies relative to the column totals and relative to the row totals.

Previously

In Lesson 8, students used two-way tables and bar graphs to represent categorical data with two variables. Students used these representations to begin to make connections in the data.

Coming Soon

In future grades, students will continue exploring associations in data. Students will deepen their knowledge of using linear models to make predictions and assess lines of fit. Students will continue discussing associations and will explore more explicitly the difference between causation and correlation.

Rigor

- Students **apply** what they have learned about coordinating between two-way tables and bar graphs to determine possible associations among bivariate categorical data.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	8 min	10 min	12 min	5 min	5 min
Independent	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils
- rulers

Math Language Development

New words

- ***relative frequency***
- ***segmented bar graph***

Review words

- *two-way table*

Amps | Featured Activity

Activity 2
Interactive Segmented Bar Graphs

Students adjust the heights of bars in a segmented bar graph to match two-way tables and relative frequency tables.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students learn about evaluating the accuracy of statements made about two-way tables and segmented bar graphs in Activity 3, they might choose to gather the correct information from the displays. Remind students of the ethical responsibility to draw conclusions that can be supported by data.

Modifications to Pacing

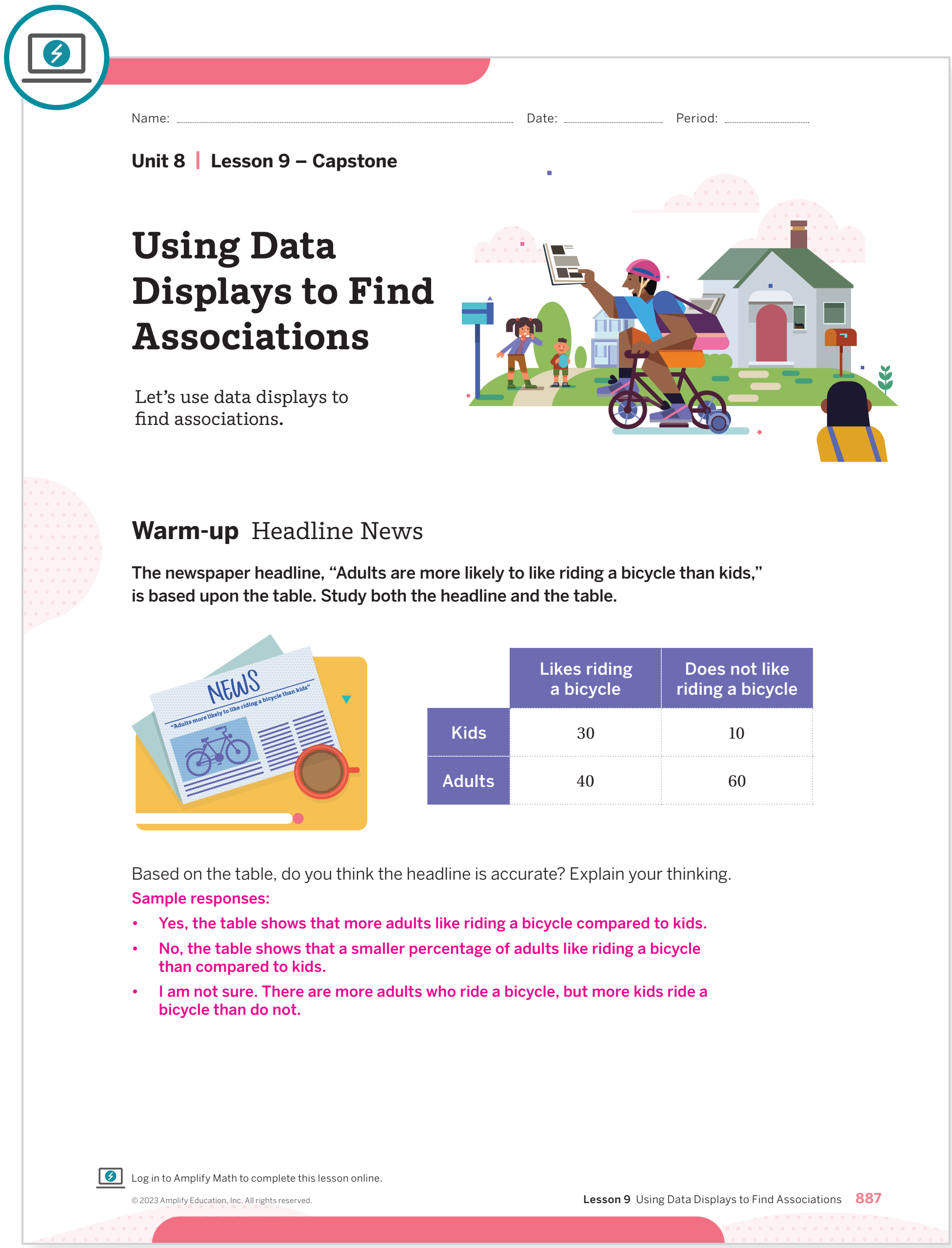
You may want to consider this additional modification if you are short on time.

- **Activity 3** may be omitted.

Independent | 5 min

Warm-up Headline News

Students use data in a two-way table to determine if a headline is accurate.



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Unit 8 | Lesson 9 – Capstone

Using Data Displays to Find Associations

Let's use data displays to find associations.

Warm-up Headline News

The newspaper headline, "Adults are more likely to like riding a bicycle than kids," is based upon the table. Study both the headline and the table.

	Likes riding a bicycle	Does not like riding a bicycle
Kids	30	10
Adults	40	60

Based on the table, do you think the headline is accurate? Explain your thinking.

Sample responses:

- **Yes, the table shows that more adults like riding a bicycle compared to kids.**
- **No, the table shows that a smaller percentage of adults like riding a bicycle than compared to kids.**
- **I am not sure. There are more adults who ride a bicycle, but more kids ride a bicycle than do not.**

Log in to Amplify Math to complete this lesson online.

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Lesson 9 Using Data Displays to Find Associations 887

1 Launch

Have students review the headline and respond to the reflection question independently before discussing with the class. Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by asking, "How many adults like riding a bike? How many adults were there in all? How does this compare to the number of kids?"

Look for points of confusion:

- **Relying on only their background knowledge to answer.** Tell students to base their answer on the data only.

Look for productive strategies:

- Saying the headline is accurate because more adults like riding a bicycle than kids.
- Saying the headline is inaccurate because a greater proportion of kids like riding a bicycle than adults.
- Expressing uncertainty based on the table.

3 Connect

Have students share their thinking. Sequence answers to hear from a student who says it is accurate first, then a student who says it is inaccurate, and finally a student who is unsure.

Highlight the variety of student responses and tell students that more tools or data might be necessary for them to be clearer about any associations in the data.

Power-up

To power up students' ability to determine percentages, have students complete:

Recall that in order to calculate a percentage you can use the formula $\frac{part}{whole} \cdot 100 = percent$. Determine each percentage based on the scenario given.

1. 180 out of 210 students say they wish they had less homework. About 85.7%.
2. 200 out of 240 students say they wish they could sleep in later. About 83.3%.
3. 35 out of 36 teachers have been counting down the number of days until summer break since March. About 97.2%.

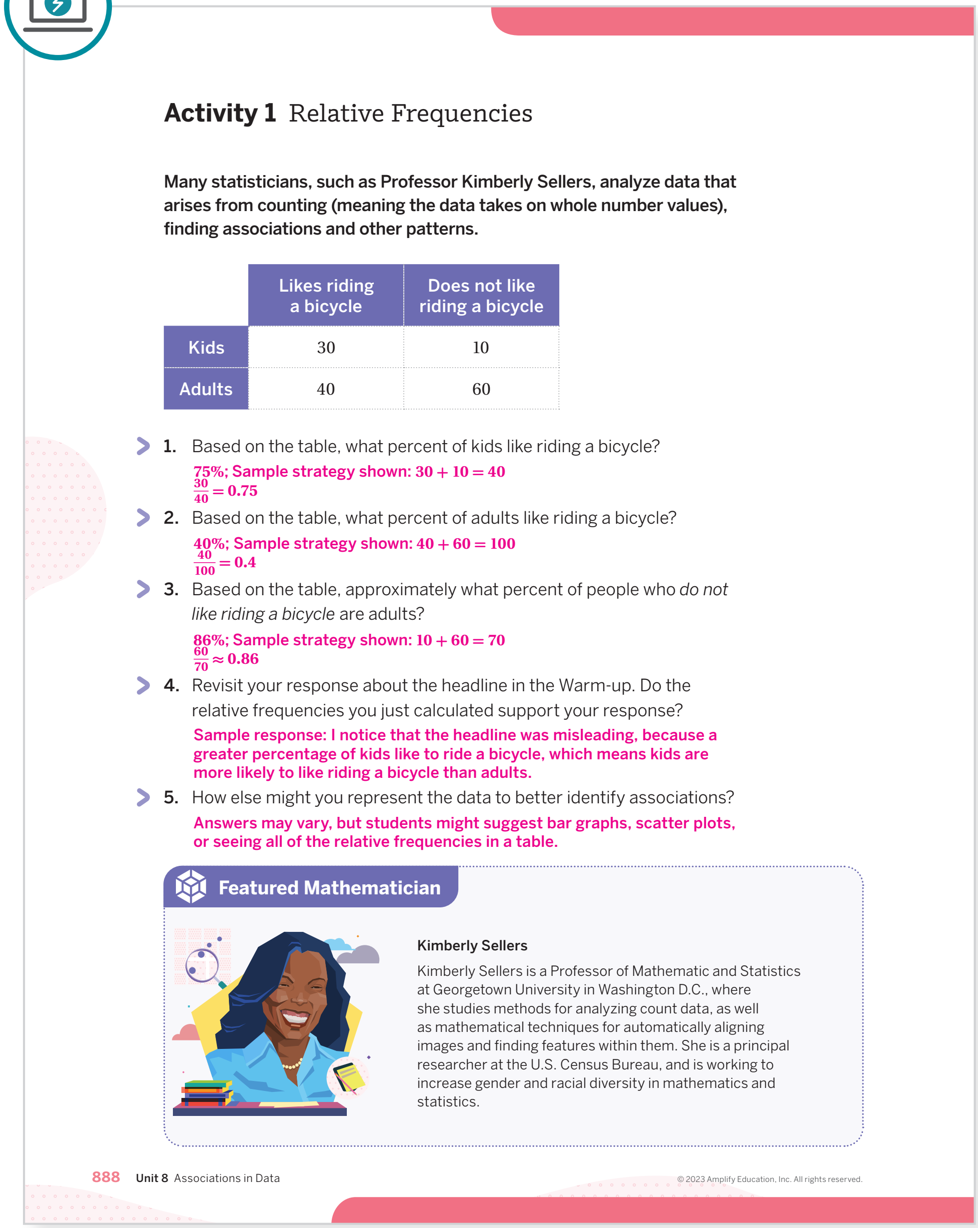
Use: Before Activity 1.

Informed by: Performance on Lesson 8, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 4

Independent | 8 min

Activity 1 Relative Frequencies

Students calculate relative frequencies using the data from the Warm-up, before re-examining their responses to the Warm-up.



Activity 1 Relative Frequencies

Many statisticians, such as Professor Kimberly Sellers, analyze data that arises from counting (meaning the data takes on whole number values), finding associations and other patterns.

	Likes riding a bicycle	Does not like riding a bicycle
Kids	30	10
Adults	40	60

1. Based on the table, what percent of kids like riding a bicycle?

   75%; Sample strategy shown: $30 + 10 = 40$ $\frac{30}{40} = 0.75$

2. Based on the table, what percent of adults like riding a bicycle?

   40%; Sample strategy shown: $40 + 60 = 100$ $\frac{40}{100} = 0.4$

3. Based on the table, approximately what percent of people who *do not like riding a bicycle* are adults?

   86%; Sample strategy shown: $10 + 60 = 70$ $\frac{60}{70} \approx 0.86$

4. Revisit your response about the headline in the Warm-up. Do the relative frequencies you just calculated support your response?

   Sample response: I notice that the headline was misleading, because a greater percentage of kids like to ride a bicycle, which means kids are more likely to like riding a bicycle than adults.

5. How else might you represent the data to better identify associations?

   Answers may vary, but students might suggest bar graphs, scatter plots, or seeing all of the relative frequencies in a table.

Featured Mathematician

Kimberly Sellers

Kimberly Sellers is a Professor of Mathematic and Statistics at Georgetown University in Washington D.C., where she studies methods for analyzing count data, as well as mathematical techniques for automatically aligning images and finding features within them. She is a principal researcher at the U.S. Census Bureau, and is working to increase gender and racial diversity in mathematics and statistics.

888 **Unit 8** Associations in Data

1 Launch

Review strategies for finding percentages. Remind students that they need a total in order to find a percentage, and note that the totals are not given in the table.

2 Monitor

Help students get started by asking, "How many total kids are represented by the data?"

Look for points of confusion:

- **Using the incorrect total to find percentages.** Remind students they need to find the totals for each row first. Ask students which group they need to know the total for based on the question.

Look for productive strategies:

- Creating and using a total column to find percentages.

3 Connect

Display correct student work for Problems 1–3.

Have students share how they calculated each percentage.

Define that the ***relative frequency*** of a category tells them the proportion at which the category occurs in the data set.

Highlight that students need to think about proportions as a part relative to the total indicated or implied by the text. Use Problem 2 and Problem 3 to highlight the different totals they would need to use to find the different relative percentages.

Ask, "How do these relative frequencies change or support your opinions about the headline in the Warm-up?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students add a column to the two-way table for totals and ask students to identify totals for each row.

Accessibility: *Activate Prior Knowledge*

Remind students they previously solved problems involving percentages in Grade 7. Demonstrate how to determine a percent, when given a part and a total.

Featured Mathematician

Kimberly Sellers

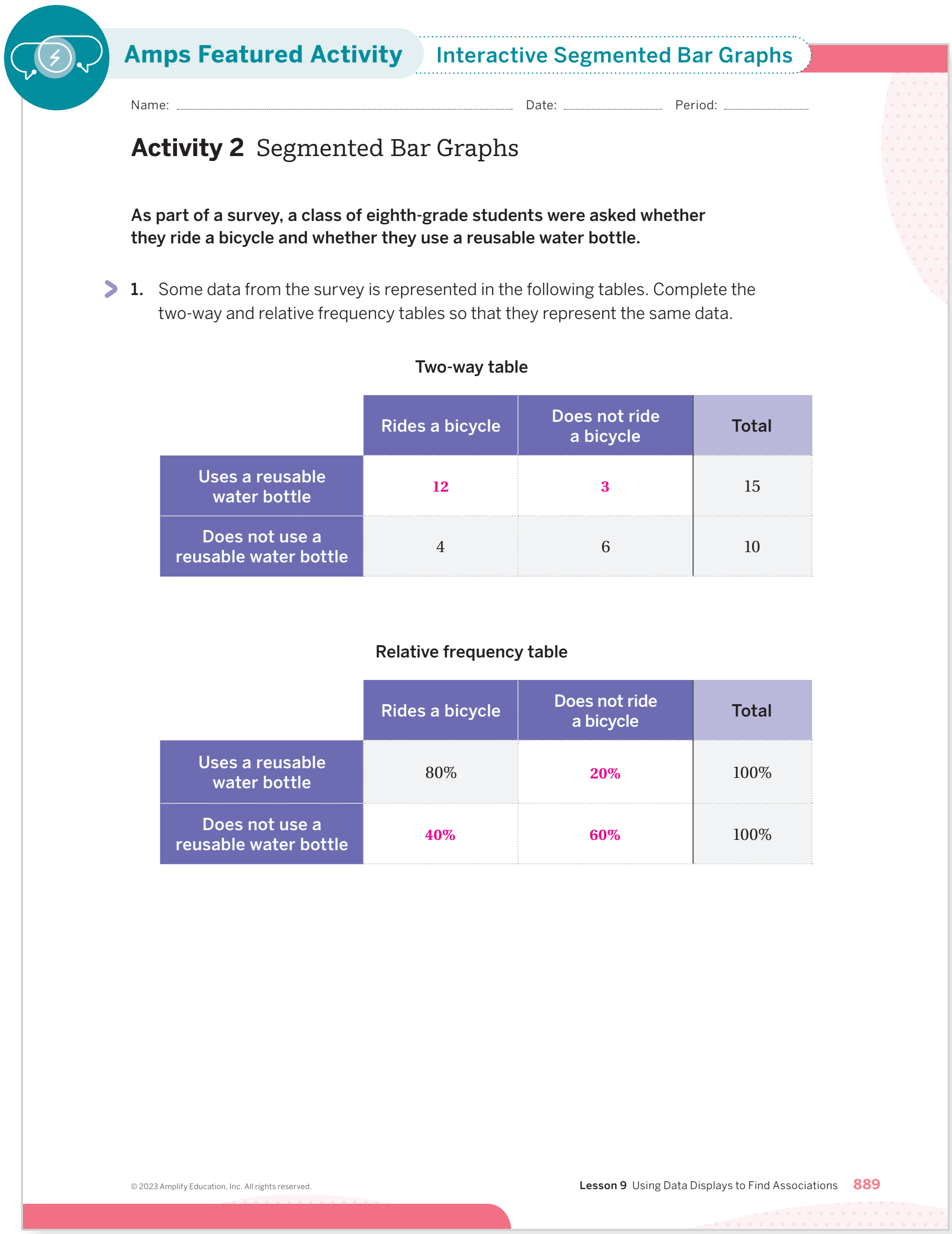
Have students read about featured mathematician Kimberly Sellers, who studied methods for analyzing count data, as well as mathematical techniques for automatically aligning images and finding features within them.

Pairs | 10 min

Activity 2 Segmented Bar Graphs

Students interpret relative frequencies and segmented bar graphs to better identify associations in a data set.

Amps Featured Activity Interactive Segmented Bar Graphs



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Activity 2 Segmented Bar Graphs

As part of a survey, a class of eighth-grade students were asked whether they ride a bicycle and whether they use a reusable water bottle.

1. Some data from the survey is represented in the following tables. Complete the two-way and relative frequency tables so that they represent the same data.

Two-way table

	Rides a bicycle	Does not ride a bicycle	Total
Uses a reusable water bottle	12	3	15
Does not use a reusable water bottle	4	6	10

Relative frequency table

	Rides a bicycle	Does not ride a bicycle	Total
Uses a reusable water bottle	80%	20%	100%
Does not use a reusable water bottle	40%	60%	100%

Lesson 9 Using Data Displays to Find Associations 889

1 Launch

Direct students to complete the tables first without looking at the segmented bar graph in Problem 2. Once students have completed the tables, introduce the segmented bar graph and have students complete Problems 2 and 3.

Define that a ***segmented bar graph*** compares two categories within a data set. The whole bar represents all the data within one category. Each bar is separated into parts (segments) that show the percentage of each part in the second category.

2 Monitor

Help students get started by asking, "Look at the missing percentage for 'Does not ride a bicycle'. What percentage must that be to have a total of 100%? Now cross-reference with the other table to find the rest of the missing cells."

Look for points of confusion:

- **Choosing any pair of numbers that add to 15.** Point out that there is a connection between the tables. The number of students in the first cell needs to be 80% of the total, 15, for that row.
- **Finding incorrect percentages.** Point out that the percentages in each row need to add to 100% and help students identify the correct part and total for each calculation.
- **Being unable to determine an association.** Ask students what they notice about the relative frequency of students who use a reusable water bottle and ride a bicycle compared to students who use a reusable water bottle and do not ride a bicycle.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can adjust the heights of bars in a segmented bar graph to match two-way tables and relative frequency tables.

Extension: *Math Enrichment*

Challenge students to complete the two-way table and relative frequency table without looking at the graph, providing them only with a few cells pre-completed that they can use as clues for the other cells.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, have pairs meet with another pair of students to share responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "Why do you think there is a positive/negative association?"
- "How do the relative frequencies help to answer the question posed in this problem?"
- "What mathematical language did you use in your response?"

Have students write a final response, based on the feedback they received.

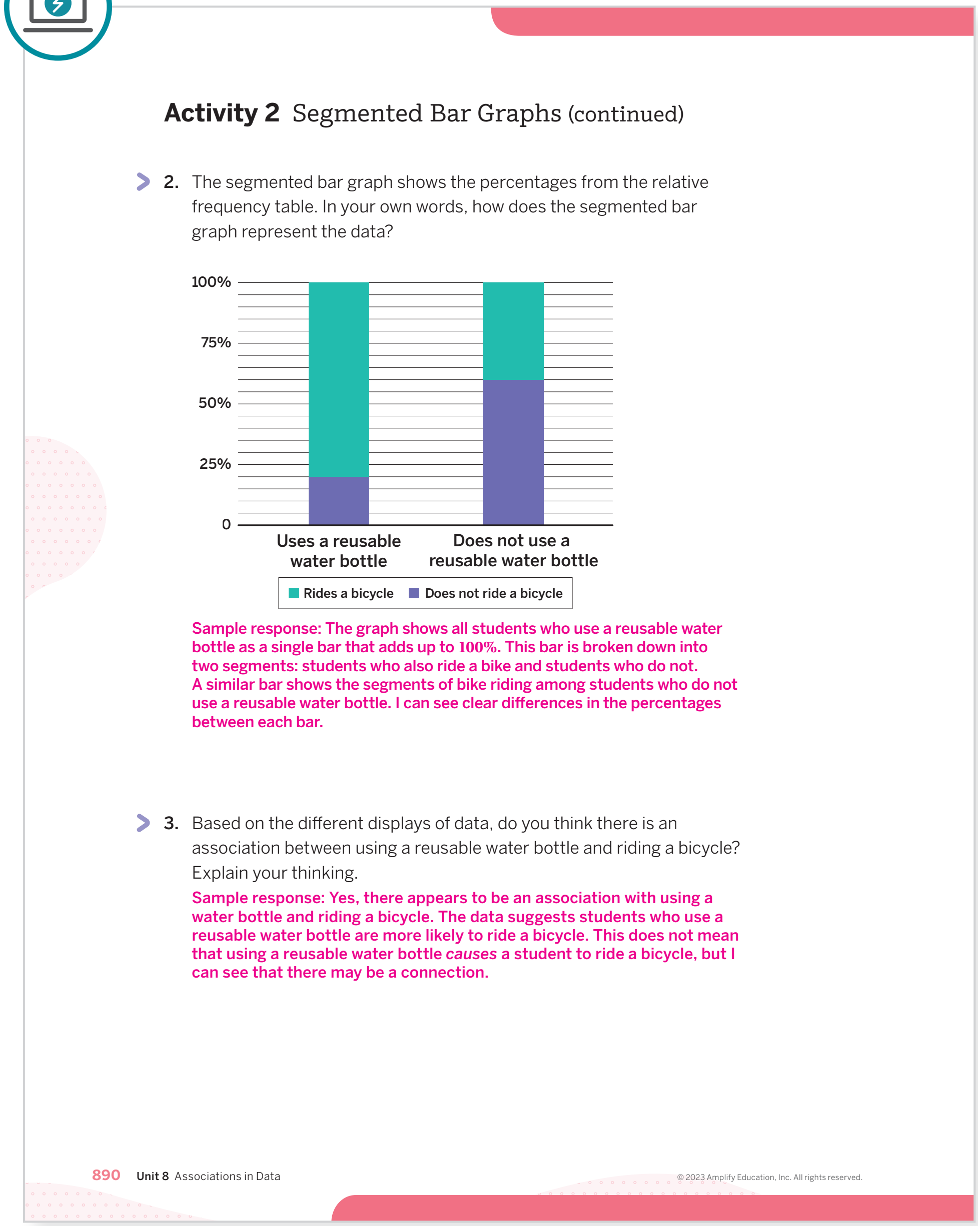
English Learners

Encourage students to refer to and use language from the class display to support their use of appropriate mathematical language while giving feedback to their peers.

Pairs | 10 min

Activity 2 Segmented Bar Graphs (continued)

Students interpret relative frequencies and segmented bar graphs to better identify associations in a data set.



Activity 2 Segmented Bar Graphs (continued)

2. The segmented bar graph shows the percentages from the relative frequency table. In your own words, how does the segmented bar graph represent the data?

Sample response: The graph shows all students who use a reusable water bottle as a single bar that adds up to 100%. This bar is broken down into two segments: students who also ride a bike and students who do not. A similar bar shows the segments of bike riding among students who do not use a reusable water bottle. I can see clear differences in the percentages between each bar.

3. Based on the different displays of data, do you think there is an association between using a reusable water bottle and riding a bicycle? Explain your thinking.

Sample response: Yes, there appears to be an association with using a water bottle and riding a bicycle. The data suggests students who use a reusable water bottle are more likely to ride a bicycle. This does not mean that using a reusable water bottle *causes* a student to ride a bicycle, but I can see that there may be a connection.

3 Connect

Display student work showing the correct relative frequency table.

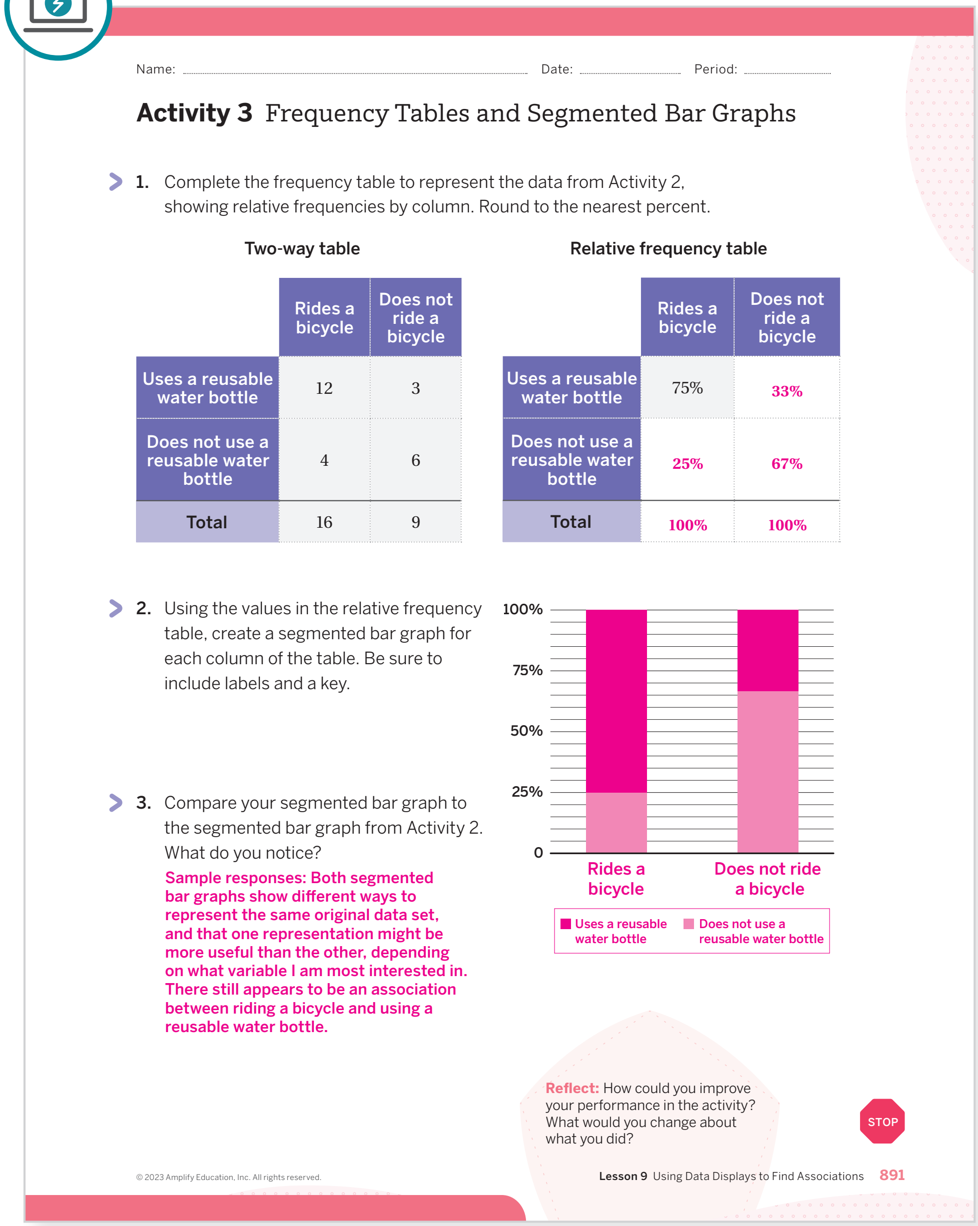
Have students share how the segmented bar graph represents the data and what associations they see in the data.

Highlight the positive association between students who use a reusable water bottle and ride a bicycle, noting that students cannot determine if one *causes* the other, just that there appears to be a connection.

Pairs | 12 min

Activity 3 Frequency Tables and Segmented Bar Graphs

Students use relative frequency tables to create segmented bar graphs.



Name: Date: Period:

Activity 3 Frequency Tables and Segmented Bar Graphs

1. Complete the frequency table to represent the data from Activity 2, showing relative frequencies by column. Round to the nearest percent.

Two-way table

	Rides a bicycle	Does not ride a bicycle
Uses a reusable water bottle	12	3
Does not use a reusable water bottle	4	6
Total	16	9

Relative frequency table

	Rides a bicycle	Does not ride a bicycle
Uses a reusable water bottle	75%	33%
Does not use a reusable water bottle	25%	67%
Total	100%	100%

2. Using the values in the relative frequency table, create a segmented bar graph for each column of the table. Be sure to include labels and a key.

3. Compare your segmented bar graph to the segmented bar graph from Activity 2. What do you notice?

Sample responses: Both segmented bar graphs show different ways to represent the same original data set, and that one representation might be more useful than the other, depending on what variable I am most interested in. There still appears to be an association between riding a bicycle and using a reusable water bottle.

Reflect: How could you improve your performance in the activity? What would you change about what you did?

STOP

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Lesson 9 Using Data Displays to Find Associations 891

1 Launch

Have students recall that relative frequency can be determined by row or by column. They will now look at the same data set, but this time, they will calculate the relative frequencies by column. Distribute colored pencils or explain how to shade the sections differently with a pencil.

2 Monitor

Help students get started by asking, "How many students do not use a reusable water bottle and ride a bicycle? How many total students ride a bicycle? How can you find the percentage?"

Look for points of confusion:

- **Segmented bar graphs showing frequencies instead of relative frequencies.** Remind students that segmented bar graphs should represent relative frequencies (percentages) and should add up to 100% for each category.

3 Connect

Display student work showing the correct segmented bar graph.

Have students share how the segmented bar graph matches the data and how it compares to the segmented bar graph in Activity 2.

Highlight that segmented bar graphs will look different depending on the relative totals — columns or rows. Both represent the same original set of data values, but one representation might be more useful in determining associations. In this case, both show a similar positive association between riding a bicycle and using a reusable water bottle.

Differentiated Support

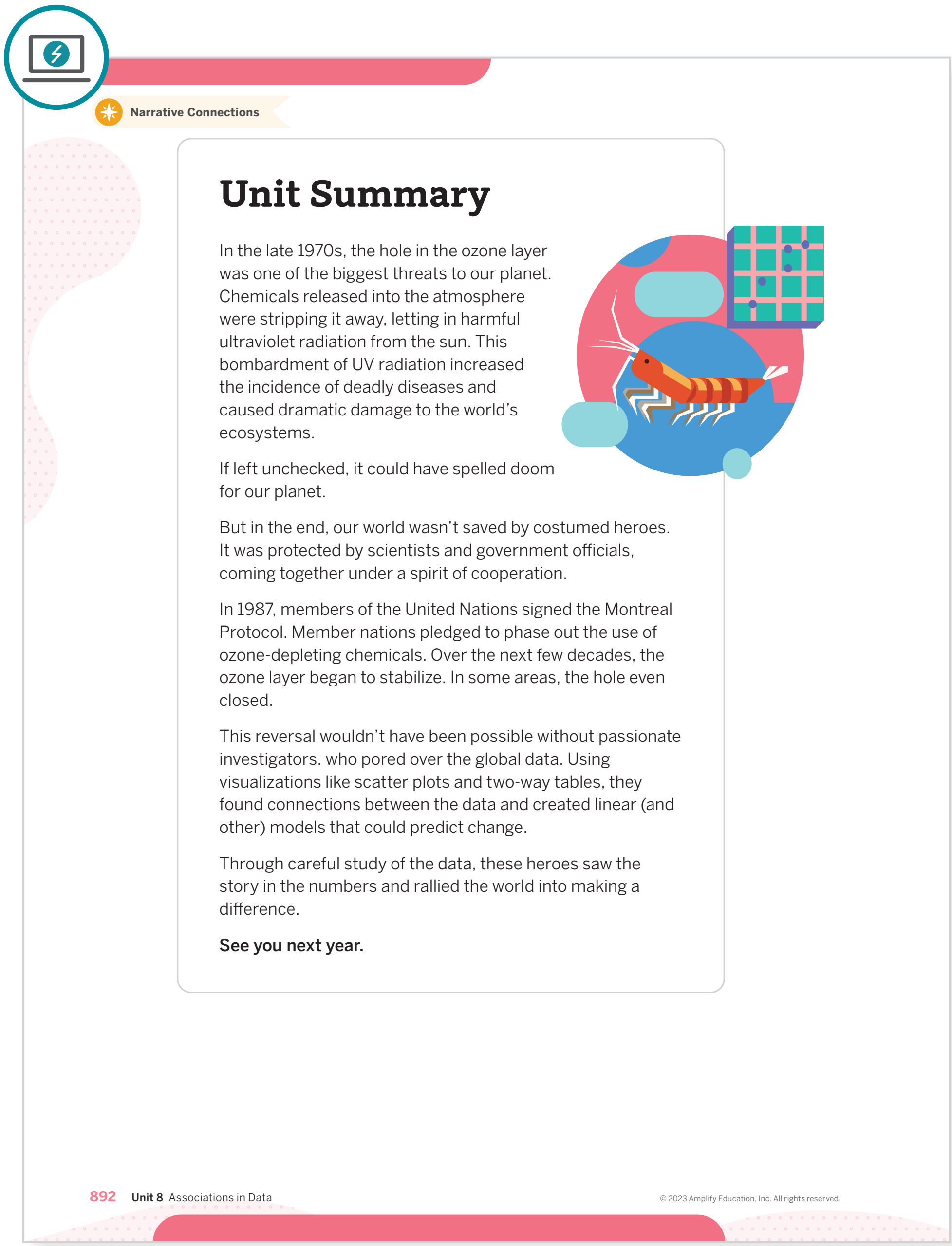
Accessibility: *Guide Processing and Visualization*

Provide students with a blank graph they can use to create their segmented bar graph in Problem 2. Consider providing the labels for them and have them draw the bar heights.

Whole Class | 5 min

Unit Summary

Review and synthesize the differences between frequency tables and relative frequency tables when looking for possible associations in data sets.



Narrative Connections

Unit Summary

In the late 1970s, the hole in the ozone layer was one of the biggest threats to our planet. Chemicals released into the atmosphere were stripping it away, letting in harmful ultraviolet radiation from the sun. This bombardment of UV radiation increased the incidence of deadly diseases and caused dramatic damage to the world's ecosystems.

If left unchecked, it could have spelled doom for our planet.

But in the end, our world wasn't saved by costumed heroes. It was protected by scientists and government officials, coming together under a spirit of cooperation.

In 1987, members of the United Nations signed the Montreal Protocol. Member nations pledged to phase out the use of ozone-depleting chemicals. Over the next few decades, the ozone layer began to stabilize. In some areas, the hole even closed.

This reversal wouldn't have been possible without passionate investigators. who pored over the global data. Using visualizations like scatter plots and two-way tables, they found connections between the data and created linear (and other) models that could predict change.

Through careful study of the data, these heroes saw the story in the numbers and rallied the world into making a difference.

See you next year.

892 **Unit 8** Associations in Data

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

Ask, "What new tools have you learned since our Warm-up that help you determine any associations in the data?" "Is it easier to see evidence of an association in a frequency table or a relative frequency table?"

Highlight ideas that suggest all tools might be useful depending on the data. Remind students that they have been looking for associations in categorical data, and that there is evidence of an association if the relative frequencies of some characteristic are very different from each other in the different groups.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

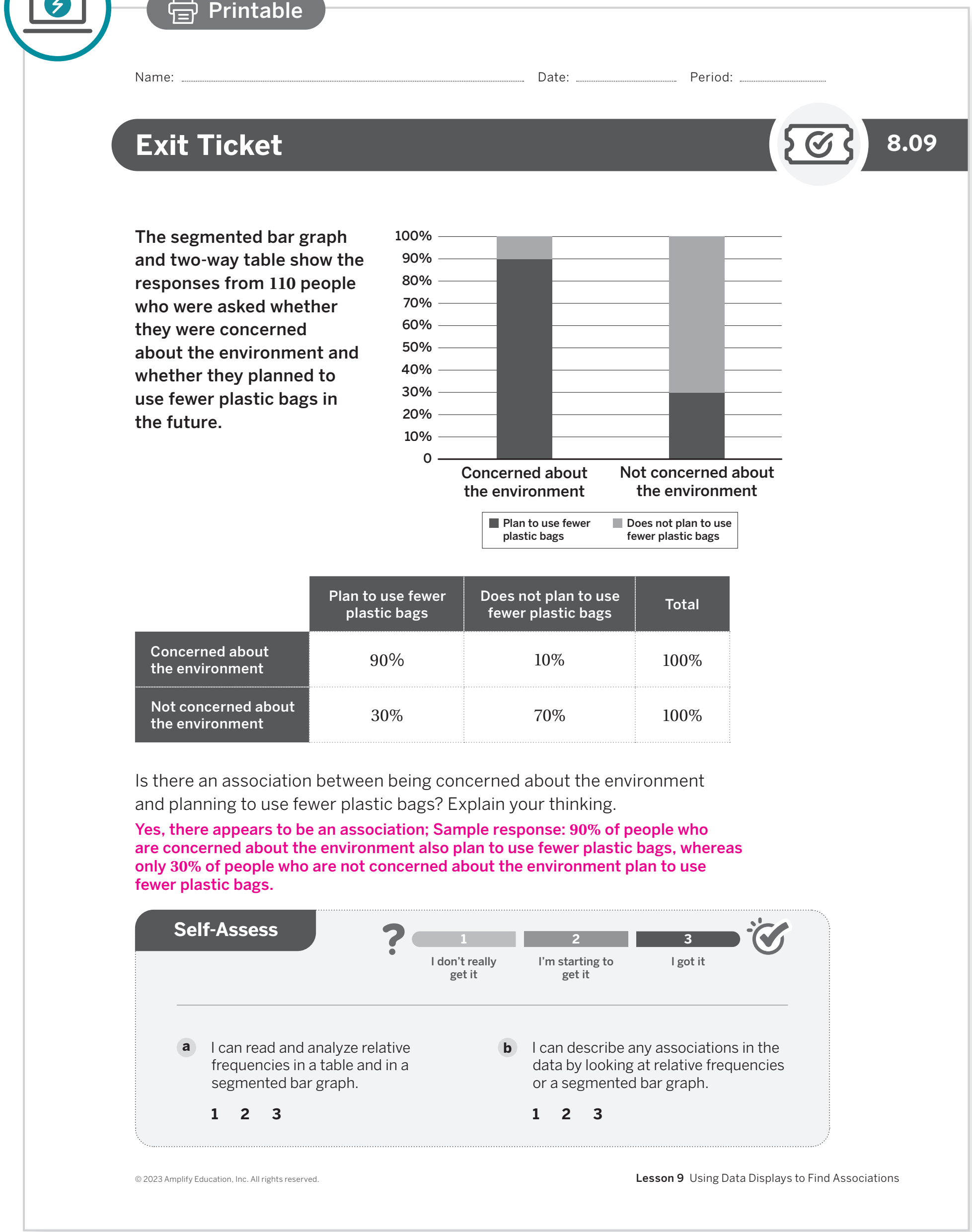
- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using a segmented bar graph and a relative frequency table to determine any associations in the data.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 8.09

The segmented bar graph and two-way table show the responses from 110 people who were asked whether they were concerned about the environment and whether they planned to use fewer plastic bags in the future.

	Plan to use fewer plastic bags	Does not plan to use fewer plastic bags	Total
Concerned about the environment	90%	10%	100%
Not concerned about the environment	30%	70%	100%

Is there an association between being concerned about the environment and planning to use fewer plastic bags? Explain your thinking.

Yes, there appears to be an association; Sample response: 90% of people who are concerned about the environment also plan to use fewer plastic bags, whereas only 30% of people who are not concerned about the environment plan to use fewer plastic bags.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can read and analyze relative frequencies in a table and in a segmented bar graph.

1 2 3

b I can describe any associations in the data by looking at relative frequencies or a segmented bar graph.

1 2 3

Lesson 9 Using Data Displays to Find Associations

Success looks like . . .

- **Language Goal:** Calculating relative frequencies and describing associations between variables using a relative frequency table. **(Speaking and Listening, Writing)**
  - » Determining whether there is an association between concern about the environment and planning to use fewer plastic bags.
- **Language Goal:** Coordinating two-way tables, bar graphs, and segmented bar graphs representing the same data. **(Speaking and Listening, Writing)**

Suggested next steps

If students see no association in the data, consider:

- Reminding students that there is evidence of an association if the relative frequencies of some characteristic are very different from each other in the different groups.
- Referring students to Activity 2 to review an example which showed an association in the data.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to coordinate different representations of the same data. How well did students accomplish this? What did you specifically do to help students accomplish it?
- What did students seem to best understand about topics in this unit? What would you teach differently if you could do it over again?

Math Language Development

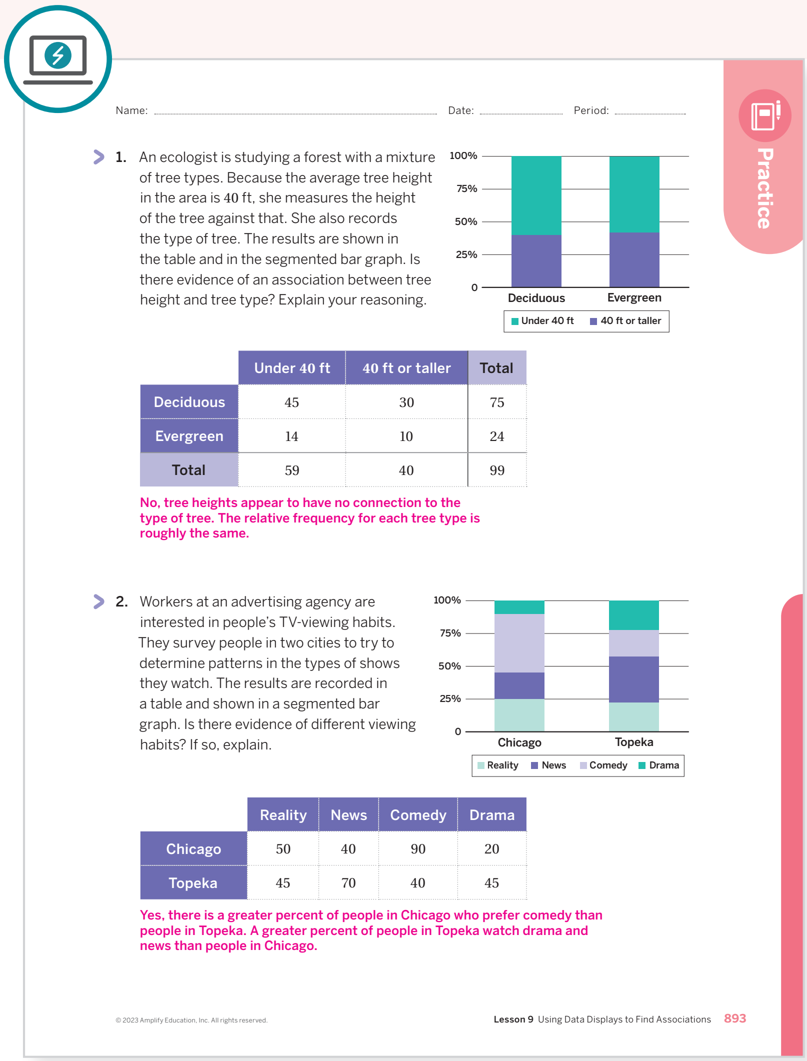
Language Goal: Calculating relative frequencies and describing associations between variables using a relative frequency table.

Reflect on students' language development toward this goal.

- Do students' responses to the Exit Ticket problem indicate they can interpret the relative frequency table in order to make a prediction about whether there appears to be an association between the variables?
- Do their explanations include specific values from the relative frequency table and what they mean in context?

Practice

Independent



Name: .... Date: .... Period: ....

Practice

1. An ecologist is studying a forest with a mixture of tree types. Because the average tree height in the area is 40 ft, she measures the height of the tree against that. She also records the type of tree. The results are shown in the table and in the segmented bar graph. Is there evidence of an association between tree height and tree type? Explain your reasoning.

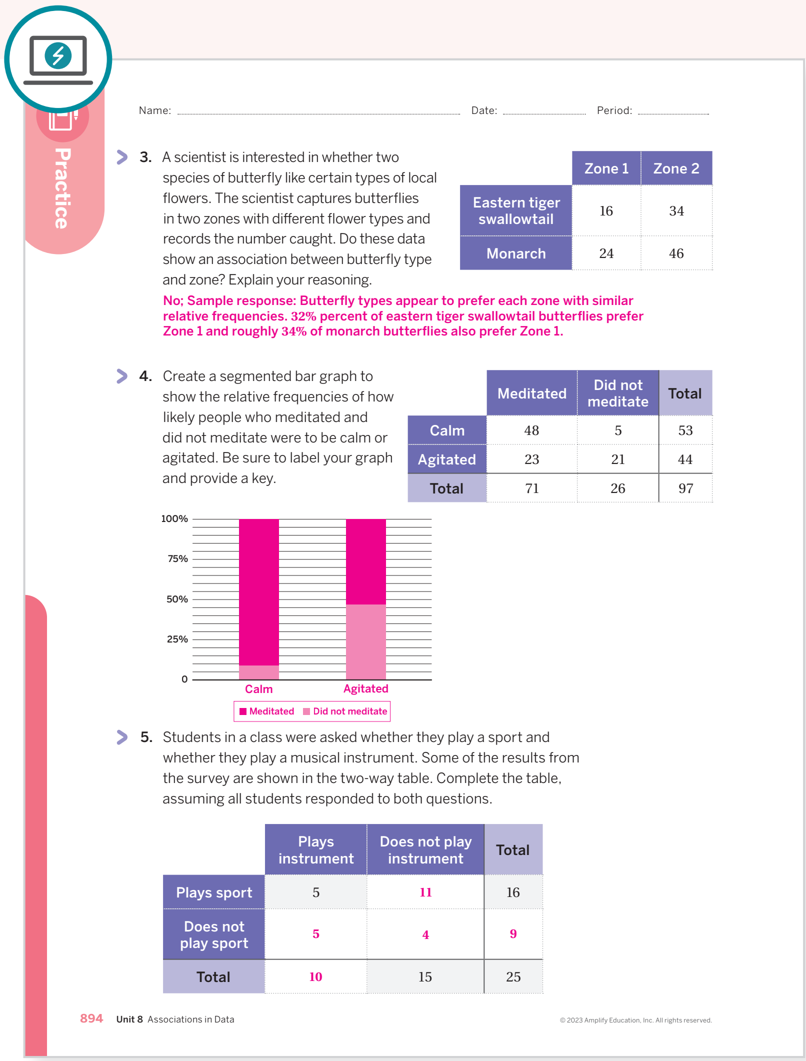
	Under 40 ft	40 ft or taller	Total
Deciduous	45	30	75
Evergreen	14	10	24
Total	59	40	99

No, tree heights appear to have no connection to the type of tree. The relative frequency for each tree type is roughly the same.

2. Workers at an advertising agency are interested in people's TV-viewing habits. They survey people in two cities to try to determine patterns in the types of shows they watch. The results are recorded in a table and shown in a segmented bar graph. Is there evidence of different viewing habits? If so, explain.

	Reality	News	Comedy	Drama
Chicago	50	40	90	20
Topeka	45	70	40	45

Yes, there is a greater percent of people in Chicago who prefer comedy than people in Topeka. A greater percent of people in Topeka watch drama and news than people in Chicago.



Practice

Name: .... Date: .... Period: ....

3. A scientist is interested in whether two species of butterfly like certain types of local flowers. The scientist captures butterflies in two zones with different flower types and records the number caught. Do these data show an association between butterfly type and zone? Explain your reasoning.

	Zone 1	Zone 2
Eastern tiger swallowtail	16	34
Monarch	24	46

No; Sample response: Butterfly types appear to prefer each zone with similar relative frequencies. 32% percent of eastern tiger swallowtail butterflies prefer Zone 1 and roughly 34% of monarch butterflies also prefer Zone 1.

4. Create a segmented bar graph to show the relative frequencies of how likely people who meditated and did not meditate were to be calm or agitated. Be sure to label your graph and provide a key.

	Meditated	Did not meditate	Total
Calm	48	5	53
Agitated	23	21	44
Total	71	26	97

5. Students in a class were asked whether they play a sport and whether they play a musical instrument. Some of the results from the survey are shown in the two-way table. Complete the table, assuming all students responded to both questions.

	Plays instrument	Does not play instrument	Total
Plays sport	5	**11**	16
Does not play sport	**5**	**4**	**9**
Total	**10**	15	25

Practice Problem Analysis

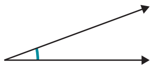
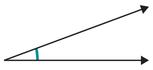
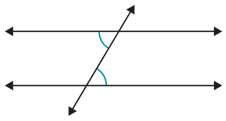
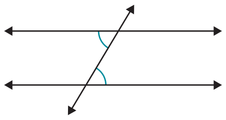
Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
	4	Activity 3	2
Spiral	5	Unit 8 Lesson 8	1

Additional Practice Available

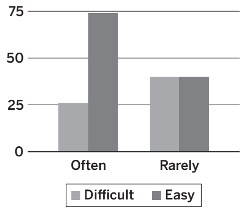
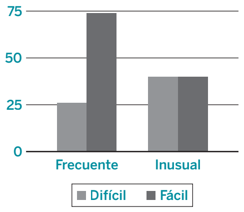
For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Glossary/Glosario

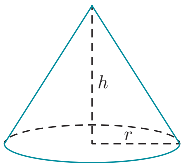
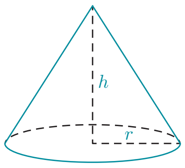
A

English	Español
absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $\lvert -3 \rvert = 3$.	**valor absoluto** Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $\lvert -3 \rvert = 3$.
acute angle An angle whose measure is less than 90 degrees. 	**ángulo agudo** Ángulo cuya medida es menor que 90 grados. 
alternate interior angles Alternate interior angles are created when a pair of parallel lines are intersected by a transversal. These angles lie inside the parallel lines and on opposite (alternate) sides of the transversal. 	**ángulos interiores alternos** Se crean ángulos interiores alternos cuando un par de líneas paralelas son intersecadas por una transversal. Estos ángulos están dentro de las líneas paralelas y en lados opuestos (alternos) de la transversal. 
angle of rotation See the definition for *rotation*.	**ángulo de rotación** Ver *rotación*.
area The number of unit squares needed to fill a two-dimensional shape without gaps or overlaps.	**área** Número de unidades cuadradas necesario para llenar una forma bidimensional sin dejar espacios vacíos ni superposiciones.

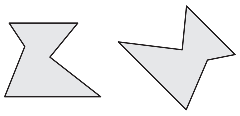
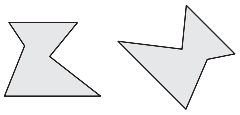
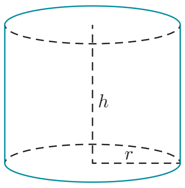
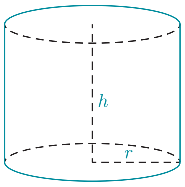
B

English	Español
bar graph A graph that presents data using rectangular bars that have heights proportional to the values that they represent. 	**gráfica de barras** Gráfica que presenta datos por medio de barras con alturas proporcionales a los valores que representan. 
bar notation Notation that indicates the repeated part of a repeating decimal. For example, $0.\overline{6} = 0.66666\ldots$.	**notación de barras** Notación que indica la parte repetida de un número decimal periódico. Por ejemplo, $0.\overline{6} = 0.66666\ldots$.
base The number that is raised to a power. The power indicates the number of times the base is multiplied by itself.	**base** Número que se eleva a una potencia. La potencia indica el número de veces que la base debe multiplicarse por sí mismo.

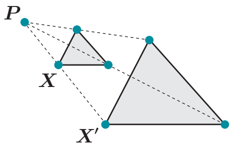
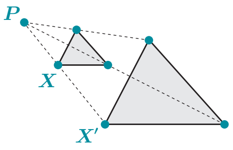
C

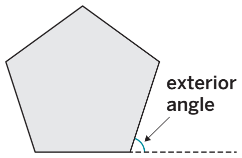
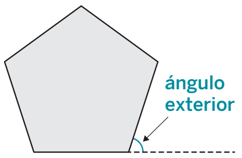
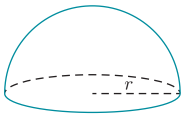
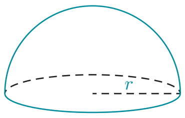
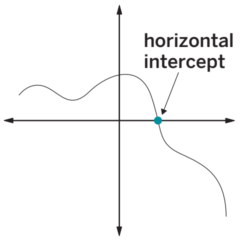
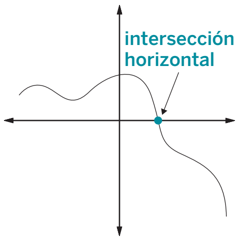
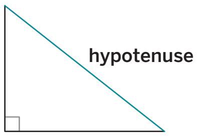
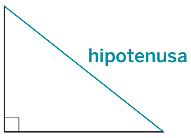
English	Español
center of dilation See the definition for *dilation*.	**centro de dilatación** Ver *dilatación*.
center of rotation See the definition for *rotation*.	**centro de rotación** Ver *rotación*.
circle A shape that is made up of all of the points that are the same distance from a given point.	**círculo** Forma constituida por todos los puntos que están a la misma distancia de un punto dado.
circumference The distance around a circle.	**circunferencia** Distancia alrededor de un círculo.
clockwise A rotation in the same direction as the way hands on a clock move is called a *clockwise* rotation.	**en el sentido de las agujas del reloj** Una rotación en la misma dirección en que se mueven las agujas de un reloj es llamada una rotación *en el sentido de las agujas del reloj*.
cluster A cluster represents data values that are grouped closely together.	**agrupación** Una agrupación representa valores de datos que se agrupan de manera cercana entre ellos.
coefficient A constant by which a variable is multiplied, written in front of the variable. For example, in the expression $3x + 2y$, 3 is the coefficient of x.	**coeficiente** Constante por la cual una variable es multiplicada, escrita frente a la variable. Por ejemplo, en la expresión $3x + 2y$, 3 es el coeficiente de x.
cone A three-dimensional solid that consists of a circular base connected by a curved surface to a single point. 	**cono** Sólido tridimensional compuesto de una base circular conectada a un solo punto por medio de una superficie curva. 
congruent Two figures are "congruent" to each other if one figure can be mapped onto the other by a sequence of rigid transformations.	**congruente** Dos figuras son "congruentes" si una de las figuras puede mapearse con la otra mediante una secuencia de transformaciones rígidas.

Glossary/Glosario

English	Español
congruent Two figures are *congruent* to each other if one figure can be mapped onto the other by a sequence of rigid transformations. 	**congruente** Dos figuras son *congruentes* entre sí, si una figura puede adquirir la forma de la otra figura mediante una secuencia de transformaciones rígidas. 
constant A value that does not change, meaning it is not a variable.	**constante** Valor que no cambia, lo que significa que no es una variable.
constant of proportionality The number in a proportional relationship that the value of one quantity is multiplied by to get the value of the other quantity.	**constante de proporcionalidad** En una relación proporcional, el número por el cual el valor de una cantidad es multiplicado para obtener el valor de otra cantidad.
coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both represent on values that are positive, negative, or zero.	**plano de coordenadas** Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.
corresponding parts Parts of two scaled copies that match up, or "correspond" with each other. These corresponding parts could be points, segments, angles, or lengths.	**partes correspondientes** Partes de dos copias a escala que coinciden, o "se corresponden", entre sí. Estas partes correspondientes pueden ser puntos, segmentos, ángulos o longitudes.
counterclockwise A rotation in the opposite direction as the way hands on a clock move is called a *counterclockwise* rotation.	**en el sentido contrario a las agujas del reloj** Una rotación en la dirección opuesta a la forma en que las agujas de un reloj se mueven es llamada una rotación *en el sentido contrario a las agujas del reloj*.
cube root The cube root of a positive number p is a positive solution to equations of the form $x^3 = p$. Write the cube root of p as $\sqrt[3]{p}$.	**raíz cúbica** La raíz cúbica de un número positivo p es una solución positiva a las ecuaciones de la forma $x^3 = p$. Escribimos la raíz cúbica p como $\sqrt[3]{p}$.
cylinder A three-dimensional solid that consists of two parallel, circular bases joined by a curved surface. 	**cilindro** Sólido tridimensional compuesto por dos bases paralelas y circulares unidas por una superficie curva. 

D

English	Español
dependent variable The dependent variable represents the output of a function.	**variable dependiente** La variable dependiente representa el resultado, o salida, de una función.
diagonal A line segment connecting two vertices on different sides of a polygon or polyhedra.	**diagonal** Segmento de línea que conecta dos vértices que están en lados diferentes de un polígono o de un poliedro.
diameter The distance across a circle through its center. The line segment with endpoints on the circle, that passes through its center.	**diámetro** Distancia que atraviesa un círculo por su centro. El segmento de línea cuyos extremos se ubican en el círculo y que pasa a través de su centro.
dilation A transformation defined by a fixed point P (called the *center of dilation*) and a scale factor k. The dilation moves each point X to a point X' along ray PX, such that its distance from P changes by the scale factor. 	**dilatación** Transformación definida por un punto fijo P (llamado *centro de dilatación*) y un factor de escala k. La dilatación mueve cada punto X a un punto X' a lo largo del rayo PX, de manera tal que su distancia con respecto a P es cambiada por el factor de escala. 
Distributive Property A property relating addition and multiplication: $a(b + c) = ab + ac$.	**Propiedad distributiva** Propiedad que relaciona la suma con la multiplicación: $a(b + c) = ab + ac$.

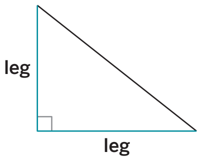
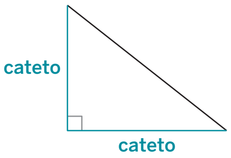
English	Español
E	**E**
equation A mathematical statement that two expressions are equal.	**ecuación** Declaración matemática de que dos expresiones son iguales.
equivalent If two mathematical objects (especially fractions, ratios, or expressions) are equal in any form, then they are equivalent.	**equivalente** Si dos objetos matemáticos (especialmente fracciones, razones o expresiones) son iguales de cualquier manera, entonces son equivalentes.
equivalent equations Equations that have the same solution or solutions.	**ecuaciones equivalentes** Ecuaciones que tienen la misma solución o soluciones.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
exponent The number of times a factor is multiplied by itself.	**exponente** Número de veces que un factor es multiplicado por sí mismo.
expression A quantity that can include constants, variables, and operations.	**expresión** Cantidad que puede incluir constantes, variables y operaciones.
exterior angle An angle between a side of a polygon and an extended adjacent side. 	**ángulo exterior** Ángulo que se encuentra entre un lado de un polígono y un lado extendido adyacente. 
F	**F**
function A function is a rule that assigns exactly one output to each possible input.	**función** Una función es una regla que asigna exactamente un resultado, o salida, a cada posible entrada.
H	**H**
hanger diagram A model in which quantities are represented as weights attached to either side of a hanger. When the hanger is balanced, the sum of the quantities on either side must be equal.	**diagrama de colgador** Modelo en el cual ciertas cantidades son representadas como pesos sujetos a cada lado de un colgador. Cuando el colgador está en equilibrio, la suma de las cantidades en cualquiera de los lados debe ser igual.
hemisphere Half of a sphere. 	**hemisferio** La mitad de una esfera. 
horizontal Running straight from left to right (or right to left).	**horizontal** Que corre en línea recta de izquierda a derecha (o de derecha a izquierda).
horizontal intercept A point where a graph intersects the horizontal axis. Also known as the x-intercept, it is the value of x when y is 0. 	**intersección horizontal** Punto en que una gráfica se interseca con el eje horizontal. Conocida también como intersección x, se trata del valor de x, cuando y es 0. 
hypotenuse In a right triangle, the side opposite the right angle is called the hypotenuse. 	**hipotenusa** En un triangulo rectángulo, el lado opuesto al ángulo recto se llama la hipotenusa. 

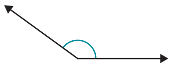
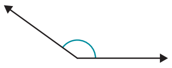
Glossary/Glosario

I

English	Español
image A new figure that is created from an original figure (called the *preimage*) by a transformation.	**imagen** Nueva figura que se crea a partir de una figura original (llamada la *preimagen*) por medio de una transformación.
independent variable The independent variable represents the input of a function.	**variable independiente** La variable independiente representa la entrada de una función.
initial value The starting amount in a context.	**valor inicial** Monto inicial en un contexto.
input The independent variable of a function.	**entrada** La variable independiente de una función.
integers Whole numbers and their opposites. For example, -4, 0, and 15 are whole numbers.	**enteros** Números completos y sus opuestos. Por ejemplo, -4, 0 y 15 son números enteros.
interior angle An angle between two adjacent sides of a polygon.	**ángulo interior** Ángulo que se encuentra entre dos lados adyacentes de un polígono.
irrational number A number that is not rational. That is, an irrational number cannot be written as a fraction.	**número irracional** Número que no es racional. Es decir, un número irracional no puede ser escrito como fracción.

L

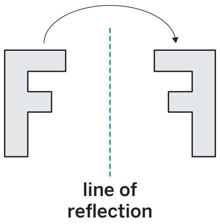
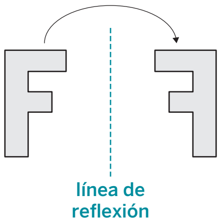
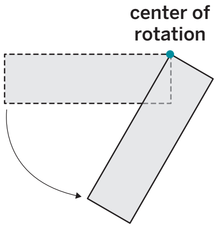
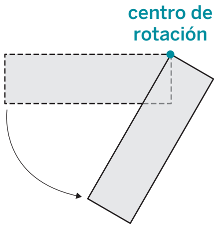
English	Español
legs The two sides of a right triangle that form the right angle. 	**catetos** Los dos lados de un triángulo rectángulo que componen el ángulo recto. 
like terms Parts of an expression that have the same variables and exponents. *Like terms* can be added or subtracted into a single term.	**términos similares** Partes de una expresión que tienen las mismas variables y exponentes. Los *términos similares* pueden ser reducidos a un solo término mediante su suma o resta.
line of reflection See the definition for *reflection*.	**línea de reflexión** Ver *reflexión*.
linear association If a straight line can model the data, the data have a linear association.	**asociación lineal** Si una línea recta puede modelar los datos, los datos tienen una asociación lineal.
linear function A linear relationship which assigns exactly one output to each possible input.	**función lineal** Relación lineal que asigna exactamente un resultado, o salida, a cada entrada posible.
linear model A linear equation that models a relationship between two quantities.	**modelo lineal** Ecuación lineal que modela una relación entre dos cantidades.
linear relationship A relationship between two quantities in which there is a constant rate of change. When one quantity increases by a certain amount, the other quantity increases or decreases by a proportional amount.	**relación lineal** Relación entre dos cantidades en la cual existe una tasa de cambio constante. Cuando una cantidad aumenta un cierto monto, la otra cantidad aumenta o disminuye en un monto proporcional.
long division A way to show the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right. $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\ 4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$	**división larga** Forma de mostrar los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha. $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\ 4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$

English	Español
N	
negative association A negative association is a relationship between two quantities where one tends to decrease as the other increases.	**asociación negativa** Una asociación negativa es una relación entre dos cantidades, en la cual una tiende a disminuir a medida que la otra aumenta.
nonlinear association If a straight line cannot model the data, the data have a nonlinear association.	**asociación no lineal** Si una línea recta no puede modelar los datos, los datos tienen una asociación no lineal.
nonlinear function A function that does not have a constant rate of change. Its graph is not a straight line.	**función no lineal** Función que no tiene un índice constante de cambio. Su gráfica no es una línea recta.
nonproportional relationship A relationship between two quantities in which the corresponding values do not have a constant ratio. (In other words, a relationship that is not a proportional relationship.)	**relación no proporcional** Relación entre dos cantidades, en la cual los valores correspondientes no tienen una razón constante. (En otras palabras, una relación que no es una relación proporcional.)
O	
obtuse angle An angle that measures more than 90 degrees. 	**ángulo obtuso** Ángulo que mide más de 90 grados. 
order of operations When an expression has multiple operations, they are applied in a consistent order (the *order of operations*) so that the expression is evaluated the same way by everyone.	**orden de las operaciones** Cuando una expresión tiene múltiples operaciones, estas se aplican en cierto orden consistente (el *orden de las operaciones*) de manera que la expresión sea evaluada de la misma manera por todas las personas.
ordered pair Two values x and y, written as (x, y), that represent a point on the coordinate plane.	**par ordenado** Dos valores x y y, escritos como (x, y), que representan un punto en el plano de coordenadas.
orientation The arrangement of the vertices of a figure before and after a transformation. A figure's orientation changes when it is reflected across a line.	**orientación** El arreglo de los vertices de una figura antes y después de una transformación. La orientación de una figura cambia cuando esta es reflejada con respecto de una línea.
origin The point represented by the ordered pair $(0, 0)$ on the coordinate plane. The *origin* is where the x- and y-axes intersect.	**origen** Punto representado por el par ordenado $(0, 0)$ en el plano de coordenadas. El *origen* es donde los ejes x y y se intersecan.
outlier Outliers are points that are far away from their predicted values.	**valor atípico** Los valores atípicos son puntos que están muy lejos de sus valores predichos.
output The dependent variable of a function.	**resultado o salida** Variable dependiente de una función.

Glossary/Glosario

English	Español
P	**P**
perfect cube A number that is the cube of an integer. For example, 8 is a perfect cube because $2^3 = 8$.	**cubo perfecto** Número que es el cubo de un número entero. Por ejemplo, 8 es un cubo perfecto porque $2^3 = 8$.
perfect square A number that is the square of an integer. For example, 16 is a perfect square because $4^2 = 16$.	**cuadrado perfecto** Número que es el cuadrado de un número entero. Por ejemplo, 16 es un cuadrado perfecto porque $4^2 = 16$.
pi The ratio of the circumference of a circle to its diameter. It is usually represented by π.	**pi** Razón entre la circunferencia y el diámetro de un círculo. Usualmente se representa como π.
piecewise function A function that is defined by two or more equations. Each equation is valid for some interval.	**función por partes** Función definida por dos o más ecuaciones. Cada ecuación es válida para alguno de los intervalos.
polygon A closed, two-dimensional shape with straight sides that do not cross each other.	**polígono** Forma cerrada y bidimensional de lados rectos que no se entrecruzan.
positive association A positive association is a relationship between two quantities where one tends to increase as the other increases.	**asociación positiva** Una asociación positiva es una relación entre dos cantidades, en la cual una tiende a aumentar a medida que la otra disminuye.
preimage See the definition of *image*.	**preimagen** Ver *imagen*.
prime notation A labeling notation that uses a tick mark. *Prime notation* is typically applied to an image, to tell it apart from its preimage.	**notación prima** Notación para etiquetar que usa un signo de prima. Una *notación prima* usualmente se aplica a una imagen, para distinguirla de su preimagen.
Properties of Equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that if an equation is true, then applying the same operation to both sides will give a new equation that is also true.	**Propiedades de igualdad** Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al aplicar la misma operación a ambos lados se obtendrá una nueva ecuación que también es verdadera.
proportional relationship A relationship in which the values for one quantity are each multiplied by the same number (the *constant of proprtionality*) to get the values for the other quantity.	**relación proporcional** Relación en la que los valores de una cantidad se multiplican cada uno por el mismo número (la *constante de proporcionalidad*) para obtener los valores de la otra cantidad.
Pythagorean Theorem The Pythagorean Theorem states that, for any right triangle, leg^2 + leg^2 = hypotenuse^2. Sometimes this can be presented as $a^2 + b^2 = c^2$, where a and b represent the length of the legs and c represents the length of the hypotenuse.	**Teorema de Pitágoras** El Teorema de Pitágoras establece que para todo triángulo rectángulo: cateto^2 + cateto^2 = hipotenusa^2. A veces puede ser también presentado como $a^2 + b^2 = c^2$, donde a y b representan las longitudes de los catetos y c representa la longitud de la hipotenusa.
Pythagorean triple Three positive integers a, b, and c, such that $a^2 + b^2 = c^2$.	**Triplete pitagórico** Tres enteros positivos a, b y c, tales como $a^2 + b^2 = c^2$.
Q	**Q**
quadrilateral A polygon with exactly four sides.	**cuadrilátero** Polígono de exactamente cuatro lados.

R

English	Español
radius A line segment that connects the center of a circle with a point on the circle. The term can also refer to the length of this segment.	**radio** Segmento de línea que conecta el centro de un círculo con cualquier punto del círculo. El término puede también referirse a la longitud de este segmento.
rate of change The amount one quantity (often y) changes when the value of another quantity (often x) increases by 1. The *rate of change* in a linear relationship is also the slope of its graph.	**tasa de cambio** Monto en que una cantidad (usualmente y) cambia cuando el valor de otra cantidad (usualmente x) aumenta en un factor de 1. La *tasa de cambio* en una relación lineal es también la pendiente de su gráfica.
ratio A comparison of two quantities by multiplication or division.	**razón** Comparación de dos cantidades a través de una multiplicación o una división.
rational numbers The set of all the numbers that can be written as positive or negative fractions.	**números racionales** Conjunto de todos los números que pueden ser escritos como fracciones positivas o negativas.
rectangular prism A polyhedron with two congruent and parallel bases, whose faces are all rectangles.	**prisma rectangular** Poliedro con dos bases congruentes y paralelas, cuyas caras son todas rectángulos.
reflection A transformation that flips each point on a preimage across a *line of reflection* to a point on the opposite side of the line.  	**reflexión** Transformación que hace girar cada punto de una preimagen a lo largo de una *línea de reflexión* hacia un punto en el lado opuesto de la línea. 
relative frequency The relative frequency is the ratio of the number of times an outcome occurs in a set of data. It can be written as a fraction, a decimal, or a percentage.	**frecuencia relativa** La frecuencia relativa es la razón del número de veces que ocurre un resultado en un conjunto de datos. Se puede escribir como una fracción, un decimal o un porcentaje.
repeating decimal A decimal in which there is a sequence of non-zero digits that repeat indefinitely.	**número decimal periódico** Decimal que tiene una secuencia de dígitos diferentes de cero que se repite de manera indefinida.
rigid transformation A move that does not change any of the measurements of a figure. Translations, rotations, and reflections are all examples of *rigid transformations* (as well as any sequence of these).	**transformación rígida** Movimiento que no cambia medida alguna de una figura. Traslaciones, rotaciones y reflexiones son ejemplos de *transformaciones rígidas* (como también cualquier secuencia de estas transformaciones).
rotation A transformation that turns a figure a certain angle (called the *angle of rotation*) about a point (called the *center of rotation*).  	**rotación** Transformación que hace girar una figura en cierto ángulo (llamado *ángulo de rotación*) alrededor de un punto (llamado *centro de rotación*). 

Glossary/Glosario

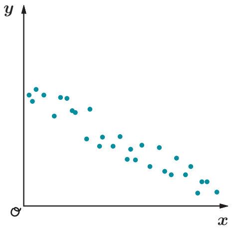
S

English

scale factor The value that side lengths are multiplied by to produce a certain scaled copy.

scaled copy A copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy.

scatter plot A scatter plot is a graph that shows the values of two variables on a coordinate plane. It allows us to investigate connections between the two variables.

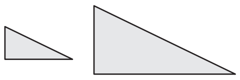


scientific notation A way of writing very large or very small numbers. When a number is written in scientific notation, the first factor is a number greater than or equal to one, but less than ten. The second factor is an integer power of ten. For example, $23000 = 2.3 \times 10^4$ and $0.00023 = 2.3 \times 10^{-4}$.

segmented bar graph A segmented bar graph compares two categories within a data set. The whole bar represents all the data within one category. Then, each bar is separated into parts (segments) that show the percentage of each part in the second category.

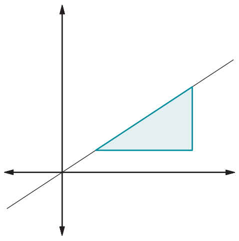
sequence of transformations Two or more transformations that are performed in a particular order.

similar Two figures are *similar* if they can be mapped onto each other by a sequence of transformations, including dilations.



slope The numerical value that represents the ratio of the vertical side length to the horizontal side length in a slope triangle. The rate of change in a linear relationship is also the slope of its graph.

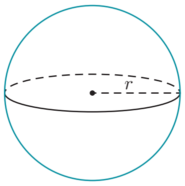
slope triangle A right triangle whose longest side is part of a line, and whose other sides are horizontal and vertical. *Slope triangles* can be used to calculate the slope of a line.



solution A value that makes an equation true.

solution to a system of equations An ordered pair that makes every equation in a system of equations true.

sphere A three-dimensional figure that consists of the set of points, in space, that are the same distance from a given point called the center.



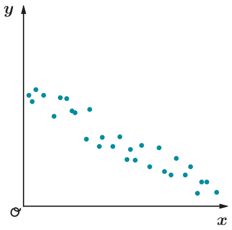
square root The square root of a positive number p is a positive solution to equations of the form $x^2 = p$. Write the square root of p as $\sqrt{p}$.

Español

factor de escala Valor por el cual las longitudes de cada lado son multiplicadas para producir una cierta copia a escala.

copia a escala Copia de una figura donde cada longitud de la figura original es multiplicada por el mismo valor, para producir longitudes correspondientes en la copia.

diagrama de dispersión Un diagrama de dispersión es una gráfica que muestra los valores de dos variables en un plano de coordenadas. Nos ayuda a investigar relaciones entre las dos variables.

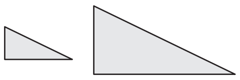


notación científica Manera de escribir números muy grandes o números muy pequeños. Cuando un número es escrito en notación científica, el primer factor es un número mayor o igual a uno, pero menor que diez. El segundo factor es un número entero que es potencia de diez. Por ejemplo, $23000 = 2.3 \times 10^4$ y $0.00023 = 2.3 \times 10^{-4}$.

gráfica de barras segmentada Una gráfica de barras segmentada compara dos categorías dentro de una serie de datos. La barra completa representa la totalidad de los datos dentro de una categoría. Entonces, cada barra es separada en partes (llamadas segmentos) que muestran el porcentaje de cada parte en la segunda categoría.

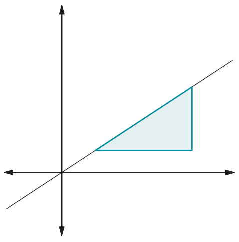
secuencia de transformaciones Dos o más transformaciones que se llevan a cabo en un orden particular.

similar Dos figuras son *similares* si pueden ser imagen la una de la otra, mediante una secuencia de transformaciones que incluyen las dilataciones.



pendiente El valor numérico que representa la razón entre la longitud del lado vertical y la longitud del lado horizontal en un triángulo de pendiente. Dada una línea, todo triángulo de pendiente tiene la misma pendiente.

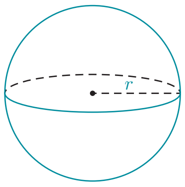
triángulo de pendiente Triángulo rectángulo cuyo lado más largo es parte de una línea, y cuyos otros lados son horizontales y verticales. Los *triángulos de pendiente* pueden ser usados para calcular la pendiente de una línea.



solución Valor que hace verdadera a una ecuación.

solución al sistema de ecuaciones Par ordenado que hace verdadera cada ecuación de un sistema de ecuaciones.

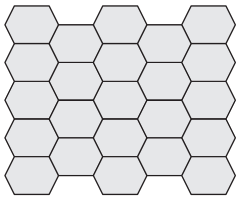
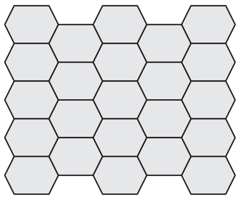
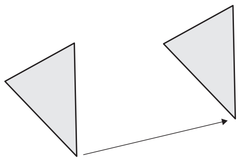
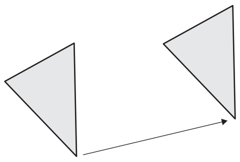
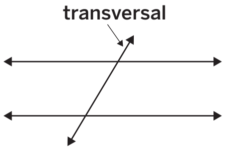
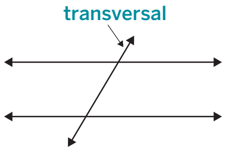
esfera Figura tridimensional que consiste en una serie de puntos en el espacio que están a la misma distancia de un punto específico, llamado centro.



raíz cuadrada La raíz cuadrada de un número positivo p es una solución positiva a las ecuaciones de la forma $x^2 = p$. Escribimos la raíz cuadrada de p como $\sqrt{p}$.

English	Español
straight angle An angle that forms a straight line. A straight angle measures 180 degrees.	**ángulo llano** Ángulo que forma una línea recta. Un ángulo llano mide 180 grados.
substitution Replacing an expression with another expression that is known to be equal.	**sustitución** Reemplazo de una expresión por otra expresión que se sabe es equivalente.
supplementary angles Two angles whose measures add up to 180 degrees.	**ángulos suplementarios** Dos ángulos cuyas medidas suman 180 grados.
symmetry When a figure can be transformed in a certain way so that it returns to its original position, it is said to have *symmetry*, or be *symmetric*.	**simetría** Cuando una figura puede ser transformada de manera tal que regrese a su posición original, se dice que tiene *simetría* o que es *simétrica*.
system of equations A set of two equations with two variables. (In a later course, you will see systems with more than two equations and variables.)	**sistema de ecuaciones** Conjunto de dos ecuaciones con dos variables. (En un curso posterior verán sistemas con más de dos ecuaciones y variables.)

T

term An expression with constants or variables that are multiplied or divided.	**término** Expresión con constantes o variables que son multiplicadas o divididas.
terminating decimal A decimal that ends in 0s.	**decimal exacto** Un decimal que termina en ceros.
tessellation A pattern made of repeating shapes that completely covers a plane, without any gaps or overlaps. 	**teselado** Patrón compuesto por formas repetidas que cubren por completo un plano, sin dejar espacios vacíos ni superposiciones. 
transformation A rule for moving or changing figures on the plane. Transformations include translations, reflections, and rotations.	**transformación** Regla que se aplica al movimiento o al cambio de figuras en el plano. Traslaciones, rotaciones y reflexiones son ejemplos de transformaciones.
translation A transformation that slides a figure without turning it. In a *translation*, each point of the figure moves the same distance in the same direction. 	**traslación** Transformación que desliza una figura sin hacerla girar. En una *traslación* cada punto de la figura se mueve la misma distancia en la misma dirección. 
transversal A line that intersects two or more other lines. 	**transversal** Línea que se interseca con dos o más líneas distintas. 
Triangle Sum Theorem A theorem that states the sum of of the three interior angles of any triangle is 180 degrees.	**Teorema de la suma del triángulo** Teorema que afirma que la suma de los tres ángulos interiores de cualquier triángulo es 180 grados.
two-way table A two-way table provides a way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.	**tabla de dos entradas** Una tabla de dos entradas provee una forma de comparar dos variables categóricas. Muestra una de las variables de forma horizontal y la otra de forma vertical. Cada entrada en la tabla es la frecuencia o frecuencia relativa de la categoría mostrada en los encabezados de la columna y la fila.

U

unit rate How much one quantity changes when the other changes by 1.	**tasa unitaria** Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.

Glossary/Glosario

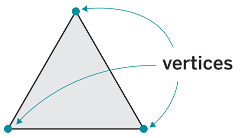
English

Español

V

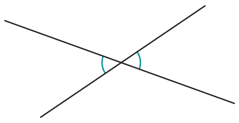
variable A quantity that can take on different values, or that has a single unknown value. Variables are typically represented using letters.

vertex A point where two sides of a two-dimensional shape or two or more edges of a three-dimensional figure intersect. (The plural of *vertex* is *vertices*.)

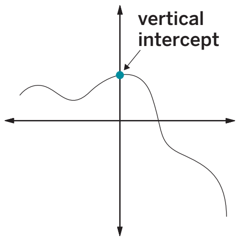


vertical Running straight up or down.

vertical angles Opposite angles that share the same vertex, formed by two intersecting lines. Vertical angles have equal measures.



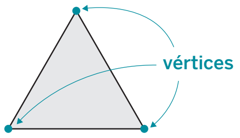
vertical intercept A point where a graph intersects the vertical axis. Also known as the y-intercept, it is the value of y when x is 0.



volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.

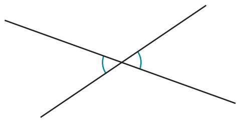
variable Cantidad que puede asumir diferentes valores o que tiene un solo valor desconocido. Las variables usualmente son representadas por letras.

vértice Punto donde se intersecan dos lados de una forma bidimensional, o dos o más aristas de una figura tridimensional.

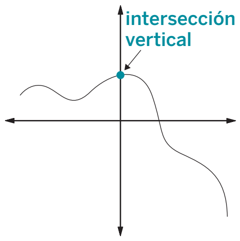


vertical Que corre en línea recta hacia arriba o hacia abajo.

ángulos verticales Ángulos opuestos que comparten el mismo vértice, conformado por dos líneas que se intersecan. Los ángulos verticales tienen las mismas medidas.



intersección vertical Punto en que una gráfica se interseca con el eje vertical. También conocida como intersección y, se trata del valor de y cuando x es 0.



volumen Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

X

x-intercept See the definition for *horizontal intercept*.

intersección x Ver *intersección horizontal*.

Y

y-intercept See the definition for *vertical intercept*.

intersección y Ver *intersección vertical*.

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