

Amplify Math **TENNESSEE**

Grade 6

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today’s students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Unit 1 Area and Surface Area

Unit Narrative:
A Place for Space

Geometry is the mathematics of space and all the shapes and sizes within it, and even dimensions. You know the names of many special two-dimensional and three-dimensional figures, and have worked with the area of very basic shapes before. But, now, it is time to cover anything and everything, literally.



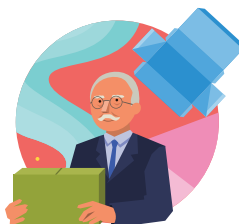
LAUNCH

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Can a sum ever really be greater than its parts?
Polygons are shapes whose sides are all line segments, and they can be decomposed and rearranged without changing their area.



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How did a misplaced ruler change the way you shop?
Polyhedra are three-dimensional figures composed of polygon faces. Their surfaces can be decomposed.



CAPSTONE

1.20	Designing a Suspended Tent	125
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Unit 2 Introducing Ratios

A little bit of this and a little bit of that. Well, maybe a lot of that? Wait, I think a ratio can help with this dilemma! Ratios help us see the relationship between one number and another, so when we make guacamole, it doesn't taste awful.

Unit Narrative:
Sensing a Ratio



Note: Lessons in gray are recommended to be omitted.
● = Tennessee-specific lessons



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Sub-Unit 1 What Are Ratios?

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How does an eggplant become a plum?

Ratios represent comparisons between quantities by multiplication or division. First, you must first learn the language of ratios and how quantities “communicate.”



Sub-Unit 2 Equivalent Ratios

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How do you put your music where your mouth is?

Equivalent ratios involve relationships between ratios themselves. They speak to each other through music and rhythm, beats and time.



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Who brought Italy to India and back again?

Now it is your turn to choose the information to represent and compare ratios.



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2.20 More Fermi Problems	264
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Unit 3 Rates and Percentages

One black truffle costs how much?! A hummingbird flaps its wings how many times in one minute?! Unit rates — how much per one — are useful ratios. And sometimes how much per one hundred, a percentage, is useful too — if you want to know: Who should take the technical foul shot with no time on the clock? Do people really like dogs better than cats? Who won the election?

Unit Narrative:
Stand and Be
Counted

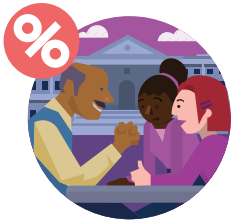


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How did student governments come to be?

Rates describe relationships between quantities like price and speed. Unit rates reveal which is a better deal or who is faster.



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What can a corpse teach us about governing?

Percentages are rates per 100. They can compare relationships between parts and wholes, even when two quantities have different total amounts.



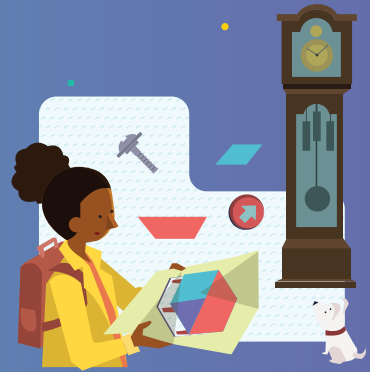
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Unit 4 Dividing Fractions

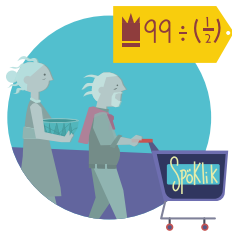
Division can be used to solve equal-sized groups problems, including when the size of a group and even the number of groups are represented by fractions. See how you can apply what you already know about multiplication and division to follow the mysteries within Spöklik Furniture and fraction division.

Unit Narrative:
Crossing the
Fractional Divide



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4.01 Seeing Fractions 382



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4.03 Relating Division and Multiplication 396
4.04 Size of Divisor and Size of Quotient 402

Which item costs between 100 and 1,000 spök-bucks?

Multiplication and division are related, and the relationship between fractions and division can be used to estimate quotients.



Sub-Unit 2 Division With Fractions 409

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4.11 Using an Algorithm to Divide Fractions 450
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How long is the bolt Samira needs?

To divide fractions, you can use multiplication, common denominators, or an algorithm. Apply these to determine the length of an oddly labeled bolt.



Sub-Unit 3 Fractions in Lengths, Areas, and Volumes 465

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How can Maya fit Penny in the box?

When you know an area or volume, but not every side length, you will often divide fractions.



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4.17 Now, Where Was That Bus? 491

Unit 5 Arithmetic in Base Ten

Decimals embody the numerical language of precision. And, because we use a base ten number system, and the world is a messy place, decimals are everywhere. Being able to add, subtract, multiply, and divide any numbers with any number of decimal places can help you determine and make sense of some astonishing facts and human accomplishments that are world records.

Unit Narrative:
Making Moves
With Decimals



LAUNCH

5.01 Precision and World Records 498



Sub-Unit 1 Adding and Subtracting Decimals 503

5.02 Speaking of Decimals 504

5.03 Adding and Subtracting Decimals 512

5.04 X Games Medal Results 519

How did a decimal decide an Olympic race?

Determine the results of high stakes competitions and identify record-setting moments by adding and subtracting decimals, as precisely as you need.



Sub-Unit 2 Multiplying Decimals 527

5.05 Decimal Points in Products 528

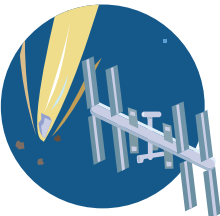
5.06 Methods for Multiplying Decimals 535

5.07 Representing Decimal Multiplication With Diagrams 542

5.08 Calculating Products of Decimals 548

What happens when you make a small change to a big bridge?

To reproduce something at large or small scales so it looks the same, you need decimals and multiplication.



Sub-Unit 3 Dividing Decimals 555

5.09 Exploring Division 556

5.10 Using Long Division 563

5.11 Dividing Numbers That Result in Decimals 571

5.12 Using Related Expressions to Divide With Decimals 578

5.13 Dividing Multi-digit Decimals 585

How do you dodge a piece of space junk?

Dividing whole numbers and decimals with many digits is the final set of operations you need to complete your trophy case.



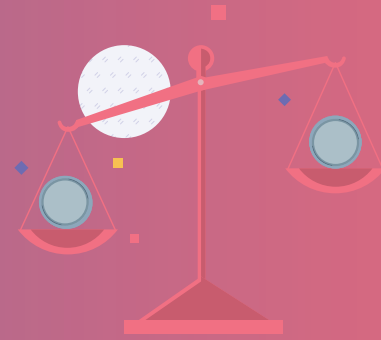
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5.14 The So-called World's "Littlest Skyscraper" 592

Unit 6 Expressions and Equations

Up until now, an equal sign meant you were being asked to calculate an answer. In this unit, you'll learn about its other meaning — balance. And when things are in balance, it becomes possible to know the unknown.

Unit Narrative:
The Power of
Balance

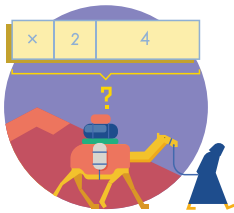


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Sub-Unit 1 Expressions and Equations in One Variable

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What's a bag of chips worth in Timbuktu?

Learn about the 14th century African salt trade, as you explore expressions and equations with tape diagrams and hanger diagrams.

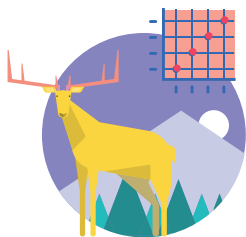


Sub-Unit 2 Equivalent Expressions

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6.16	Analyzing Exponential Expressions and Equations	699

How did a Welshman equalize England's upper crust with its common folk?

Extend the concept of equality as you investigate equivalent expressions, the all-important Distributive Property, and exponents.



Sub-Unit 3 Relationships Between Quantities

6.17	Two Related Quantities (Part 1)	706
6.18	Two Related Quantities (Part 2)	713

What's more dangerous: a pack of wolves or a gang of elk?

Balance is everywhere, especially in ecosystems. You'll look at systems that are in and out of balance.



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6.19	Creating a Class Mobile	719
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Unit 7 Rational Numbers

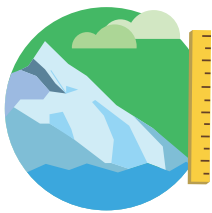
Think back to when you first learned about whole numbers and used them to count. Later, you saw there were numbers between them: fractions and decimals. Up until now, every number you've encountered has always been greater than 0. But no more. There is an entire set of numbers (just as many, in fact), lurking on the *other* side of every number line.

Unit Narrative:
Getting Where
We're Going



LAUNCH

7.01	How Far? Which Way?	728
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Sub-Unit 1 Negative Numbers and Absolute Value

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What's the tallest mountain in the world?

Consider the most extreme locations on Earth as you discover negative numbers, which lend new meaning to positive numbers and zero.



Sub-Unit 2 Inequalities

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How do you keep a quantity from wandering off?

A variable represents an unknown quantity. And sometimes it represents many possible values, which can be expressed as an inequality.



Sub-Unit 3 The Coordinate Plane

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How did Greenland get so big?

Armed with the opposites of positive rational numbers, it's time you expanded your coordinate plane. Welcome to the four quadrants!



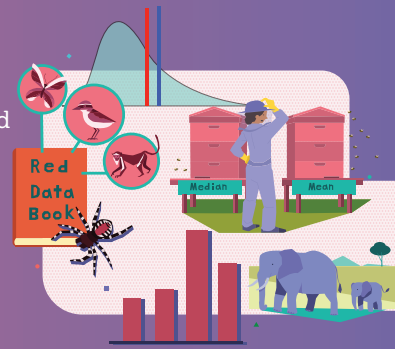
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7.19	Drawing on the Coordinate Plane	853
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Unit 8 Data Sets and Distributions

Statistics is the science of collecting and analyzing data. It is one of the most relevant aspects of mathematics in everyday life. And it is also used by researchers in many fields, such as zoologists identifying new species and studying populations of endangered species. In all cases, knowing what is typical is critical to understanding what is not.

Unit Narrative:
Walk on the Wild
Side with Data

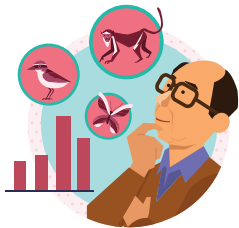


Note: Lessons in gray are recommended to be omitted.
● = Tennessee-specific lessons



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8.01 Plausible Variation or New Species? 860



Sub-Unit 1 Statistical Questions and Representing Data 867

8.02 Statistical Questions 868

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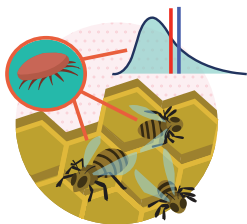
8.07 Describing Distributions With Histograms 902

8.07A Stem Plots TN-14

8.07B Pie Charts TN-21

How do you keep track of a disappearing animal?

When questions have more than one answer, it is helpful to visualize and describe a typical answer. For numbers, you can also identify the center and describe the spread of the numbers.



Sub-Unit 2 Measures of Center 909

8.08 Mean as a Fair Share 910

8.09 Mean as the Balance Point 917

8.10 Median 924

8.11 Comparing Mean and Median 930

What's the buzz on honey bees?

For numerical data, you can summarize an entire data set by a single value representing the center of the distribution. The mean and the median represent two ways you can do this.



Sub-Unit 3 Measures of Variability 937

8.12 Describing Variability 938

8.12A Measuring Variability TN-29

8.13 Variability and MAD 944

8.14 Variability and IQR 951

8.15 Box Plots 959

8.16 Comparing MAD and IQR 966

Where have the giant sea cows gone?

For numerical data, you can summarize an entire data set by a single value representing the variability of the distribution. The MAD, range, and IQR represent three ways you can do this.



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8.17 Asian Elephant Populations 972

Unit 2 | Tennessee Lesson 17A

Converting Units

Let's convert measurements to different units.

Warm-up Matching Metric Measurements

1. Write each amount from the table in the appropriate column to match equivalent distances.

10 mm	100 cm	0.01 m
1000 mm	100000 cm	1000 m

Distances equivalent to ...		
1 cm	1 m	1 km

2. Complete each blank.

a 3 m = _____ cm

b 0.5 cm = _____ mm

c 7 cm = _____ m

Activity 1 Cooking With a Tablespoon

Noah wants to make apple crisp using the following recipe, but he cannot find any measuring cups! He only has a tablespoon (tbsp) for measuring. Luckily, in the cookbook it says that 1 cup is equivalent to 16 tbsp, and 1 tbsp is equivalent to 3 teaspoons (tsp).

Apple crisp recipe

- 4 medium-sized apples, chopped
- $\frac{3}{8}$ c brown sugar
- $\frac{3}{4}$ c oats
- $\frac{1}{4}$ c butter
- $\frac{1}{2}$ c chopped pecans
- 2 tsp cinnamon
- 1 tsp vanilla extract

- 1. Complete the table to help Noah adjust the recipe so that all measurements are in tablespoons.

4 medium-sized apples, chopped

..... tbsp brown sugar

..... tbsp oats

..... tbsp butter

..... tbsp chopped pecans

..... tbsp cinnamon

..... tbsp vanilla extract

- 2. Noah decides to add in some dried cranberries to the recipe, and measures 10 tbsp. As he updates the original recipe he writes $\frac{2}{3}$ cups of cranberries. Did he write the correct amount? Show or explain your thinking using a double number line diagram, table, or any other representation.

Activity 2 Cooking for the Masses

Priya's grandmother has agreed to have the family over for dinner, all 48 members! She needs to increase her recipe for Chicken and Mushroom Pie to feed everyone, but also wants to ensure that she is using the most appropriate unit of measure (it is much easier to measure $\frac{1}{2}$ cup than 24 teaspoons!).

Chicken and Mushroom Pie

$1\frac{1}{2}$ c. of low-sodium chicken broth
 2 tbsp canola oil
 12 ounces of skinless boneless chicken thighs
 $1\frac{3}{4}$ c. chopped onion
 2 c. mushrooms
 $\frac{3}{4}$ c. of milk
 $\frac{1}{3}$ c. of flour
 1 egg
 1 package of puff pastry
 Yield: 6 servings

Helpful Conversions

4 tbsp = $\frac{1}{4}$ cup
 2 cups = 1 pint
 4 cups = 1 quart
 16 oz = 1 pound

You will be given two sets of cards:

- One with the amount of each ingredient in the original recipe (except eggs and puff pastry).
 - One with the amount of each ingredient in the increased recipe to feed Priya's extended family.
- 1. Work with your partner to match one card from each set for each ingredient using the conversions provided. You may use a calculator to perform the conversions.
- 2. Complete the table on the next page.
- Copy the recipe amount from the original recipe in the first column.
 - Copy the corresponding amount converted for the larger recipe in the second column.
 - Explain or show your thinking in the third column.

Activity 2 Cooking for the Masses (continued)

	Original	New	Show or explain your thinking
Chicken broth			
Canola oil			
Chicken			
Onions			
Mushrooms			
Milk			
Flour			



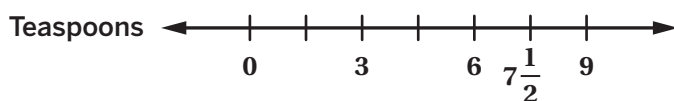
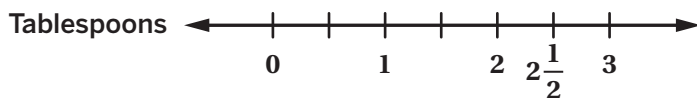
Summary

In today's lesson ...

You saw how when you measure the same attribute of two or more objects using the same two different units, the pairs of measurements are equivalent ratios. You can reason with these equivalent ratios to *convert* measurements from one unit to another.

Suppose you only have a tsp, but you are working with a recipe that has measurements in tbsp. If the recipe calls for $2\frac{1}{2}$ tbsp, you need to determine the number of tsp. Given that 1 tbsp is equivalent to 3 tsp, you can use the ratio of tbsp to tsp of 1 : 3 to determine an equivalent ratio for $2\frac{1}{2}$ tbsp. This can be done and represented in several ways.

Using a double number line diagram:



Using a ratio box or a table:

Tbsp	Tsp
1	3
$2\frac{1}{2}$	$7\frac{1}{2}$

➤ **Reflect:**



Practice

Name: _____ Date: _____ Period: _____

- 1. Place a check in the box when the quantity in the column is equivalent to the quantity in the row. Not all quantities will have a match.

	1 cm	1 liter	1 m
10 mm			
0.01 m			
0.001 km			
1000 ml			
10 ml			

- 2. Andre's catering company is making lemonade for a large party. Unfortunately, when he unpacked on site, he realized that he forgot his measuring cups and only has a pint-sized container. Andre knows that there are two cups in a pint and eight pints in a gallon. Convert each measurement in his recipe to pints.

	Original	New
Water	$\frac{1}{2}$ gallon	
Sugar	10 cups	
Lemon juice	7 cups	
Ice	$\frac{3}{4}$ gallon	

- 3. Priya's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Pesos	Dollars
4,250	250
	25
	1
	3
510	

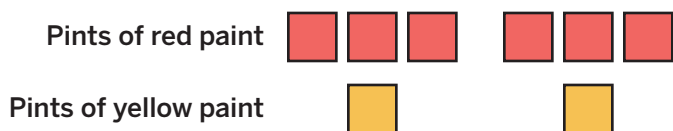


- 4. Elena mixes 5 cups of apple juice with 2 cups of sparkling water to make sparkling apple juice. She wants to make 35 cups of sparkling apple juice for a party. How much of each ingredient should Elena use? Show or explain your thinking.

- 5. Lin bought 3 hats for \$22.50. At this same rate, how many hats could she buy with \$60.00? Use the table to help with your thinking.

Number of hats	Price (\$)

- 6. Here is a diagram that represents the pints of red and yellow paint in a mixture.



Select *all* statements that accurately describe the diagram.

- A. The ratio of yellow paint to red paint is 2 to 6.
- B. For every 3 pints of red paint, there is 1 pint of yellow paint.
- C. For every pint of yellow paint, there are 3 pints of red paint.
- D. For every pint of yellow paint there are 6 pints of red paint.
- E. The ratio of red paint to yellow paint is 6 : 2.

Reasoning About Solving Equations

Let's solve more equations.

Warm-up Math Talk

Mentally solve each problem. Be prepared to explain your thinking.

> 1. $x + 2 = 4$

> 2. $x + 1 = 4$

> 3. $x + 0 = 4$

> 4. $x - 1 = 4$

> 5. $x - 2 = 4$

> 6. $4x = 8$

> 7. $2x = 8$

> 8. $1x = 8$

> 9. $\frac{1}{2}x = 8$

> 10. $\frac{x}{4} = 8$



Activity 1 Solving Equations With a Partner, Revisited

Work together to solve the following equations.

Equation	What I do to the variable side ...	=	... I do to the other side.	Solve and check
$\frac{x}{5} = 2$	$\frac{x}{5} \cdot 5$ Because this is dividing x by 5, I need to multiply by 5 to make it $1x$, or x .	=	$2 \cdot 5$ I need to multiply this side by 5, which equals 10.	Solution: $x = 10$ Check: $\frac{10}{5} = 2$
$\frac{x}{8} = 7$		=		Solution: Check:
$x - 6 = 11$		=		Solution: Check:
$\frac{3}{5} = x - \frac{1}{5}$		=		Solution: Check:
$8 = \frac{x}{0.25}$		=		Solution: Check:

Activity 2 Matching Equations With Scenarios

- 1. Match an equation with the scenario that it could represent. Be prepared to explain your thinking. Some of the equations may not have a match.

Scenario

- a** When a market stand first opens, 5 bolts of cloth are purchased. Now, 20 bolts of cloth remain. How many total bolts of cloth did the stand have before their first sale?
- b** A slab of salt is equally divided among 5 people. Each person receives 20 kg of salt. What is the total weight of the slab of salt?
- c** A caravan traveled 10 km, which was 4 km less than it traveled yesterday. How many kilometers did the caravan travel yesterday?
- d** After spice jars are placed equally on 10 shelves, there are 4 spice jars on each shelf. What is the total number of spice jars on the shelves?

Equation

..... $x - 4 = 10$

..... $\frac{x}{10} = 4$

..... $10 - 4 = x$

..... $x - 5 = 20$

..... $\frac{x}{5} = 20$

..... $x = 10 \cdot 4$

- 2. Select one match from Problem 1 and use the equation to calculate the solution for the scenario. Show your thinking.

- 3. For an equation that does not have a matching scenario, write a scenario that could be represented by the equation. State what quantity x represents, and the value for x that represents the solution.



Summary

In today's lesson ...

You further explored solving equations, using the Properties of Equality, to help you answer questions about mathematical scenarios.

In some scenarios, the two quantities can be related by subtraction, and you could write an equation like $x - 5 = 12$.

$$x - 5 = 12$$

$x - 5 + 5 = 12 + 5$ You can keep the equality by adding the same value, 5, to both sides.

$x = 17$ This isolates the variable x on one side and reveals the solution, 17.

In other scenarios, the two quantities can be related by division, and you could write an equation like $\frac{x}{2} = 9$.

$$\frac{x}{2} = 9$$

$\frac{x}{2} \cdot 2 = 9 \cdot 2$ You can keep the equality by multiplying both sides by the same value 2.

$x = 18$ This isolates the variable x on one side and reveals the solution, 18.

> Reflect:



Practice

Name: Date: Period:

- > 1. Solve each equation. Show your thinking. Be sure to check your solution by substituting it back into the original equation.

a $x - 7 = 24$

b $11 = \frac{n}{4}$

c $26 = r - 23$

d $w \div 8 = 2.5$

e $10 = c - \frac{2}{3}$

- > 2. After a box of crackers was equally distributed to 8 children, each child received 3 crackers. How many total crackers were in the box to begin with? Select the equation that represents the scenario, then determine the solution.

A. $\frac{x}{8} = 3$

B. $x - 8 = 3$

C. $x + 8 = 3$

D. $8x = 3$

$x = \dots\dots\dots$

Name: Date: Period:



Practice

- > 3. Write a scenario that could be represented by the equation $x - 3 = 15$. State what quantity x represents, and determine the value for x that represents the solution.
- > 4. Consider the equation $4n - 2 = 10$.
- a What is the variable?
 - b What is the coefficient of the variable?
 - c Which of these is a solution to the equation: 3, 4, 5, 6, n ?
- > 5. Lin's sister purchases 1.5 lb of almonds and 2.2 lb of cranberries. She mixes the almonds and cranberries to create a trail mix and then equally divides the mix into containers, so that each container weighs 0.5 lb. About how many containers can Lin's sister fill? Show or explain your thinking.
- > 6. Write an expression that can be used to determine how many times greater $4\frac{1}{2}$ is than $\frac{1}{8}$.

Stem Plots

Let's explore a new ("sideways") way to display a distribution.

Warm-up Same Data, Different Displays

As you know, there are several ways to display a set of data. Consider these three different displays of the same data, showing the lifespans of a sample of sea lions.



yykkaa/Shutterstock.com

Display A

33
21 27 31
24 19
27 30 23
21

Display B

19
21 21 23 24 27 27
30 31 33

Display C

19, 21, 21, 23, 24, 27, 27, 30, 31, 33

- 1. What is a typical value in this data set? Explain your thinking.
- 2. Which display do you find most helpful for identifying the typical value? Explain your thinking.



Activity 1 Stem Plots

This stem plot shows the number of floors in each of the 20 tallest buildings in Nashville.

Stem	Leaf
2	4 5 7 8 9
3	0 0 0 0 1 1 1 1 3 3 4 4 5
4	0 6

Key: 2 | 7 represents 27 floors

- > 1. Refer to the stem plot.
- a How many buildings have at least 30 floors?
 - b How many buildings have exactly 30 floors?
 - c How many buildings have at least 25 floors and less than 40 floors?
 - d How many floors does the building with the most floors have?
- > 2. What is a typical value for the data set? Explain your thinking.

Activity 2 Constructing a Stem Plot

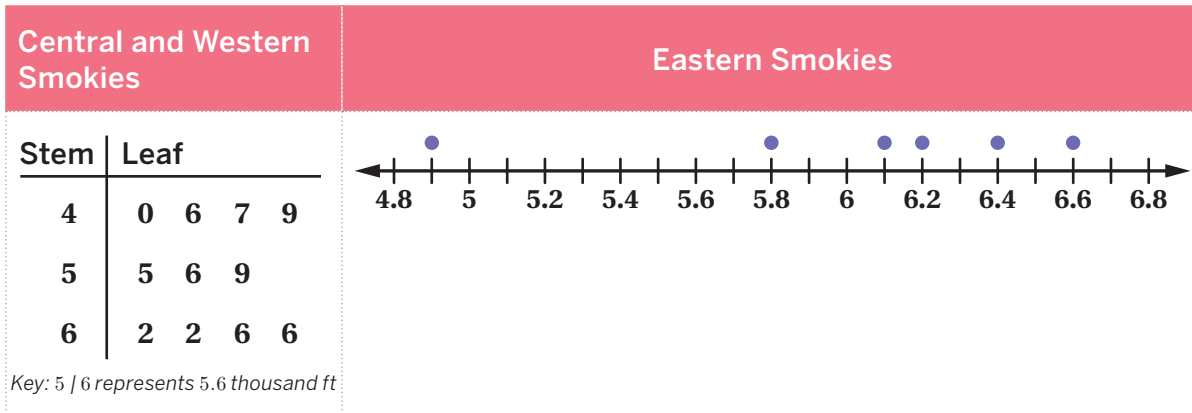
This data set shows the number of Tennessee shiners, a type of fish, caught and released in various waterways throughout the Great Smoky Mountains.

63, 63, 68, 75, 75, 91, 92, 92, 92, 109

1. Construct a stem plot to display the data.
2. What assumptions did you need to make while constructing your stem plot?

Activity 3 Comparing Different Displays

The plots shown display the data for the heights (in thousands of feet) of the tallest peaks in different regions of the Great Smoky Mountains.



- 1. Which region — Central and Western, or Eastern — tends to have higher peaks? Explain your thinking.

- 2. Which region has a greater variety of peak heights? Explain your thinking.

- 3. Which region's distribution is more symmetric? Explain your thinking.



Summary

In today's lesson ...

You explored and used a new way to visually represent data — the **stem plot**. The stem plot has both advantages and disadvantages compared to other types of displays you have made, such as tables, lists, and dot plots. Depending on the data and the questions you want to answer about the data, a stem plot may be a good way to display the data.

The conventions for constructing a stem plot involve drawing a vertical bar to separate the first digits of your data values, on the left, from the last digit of your data value on the right. A key is typically added to help the reader determine the place value of the digits.

The stem plot shown displays the following data set:

79 91 91 99 100 102 108 110

Stem	Leaf
7	9
8	
9	1 1 9
10	0 2 8
11	0

Key: 9 | 1 represents 91

> Reflect:



1. Consider the stem plot shown. Select *all* the true statements.

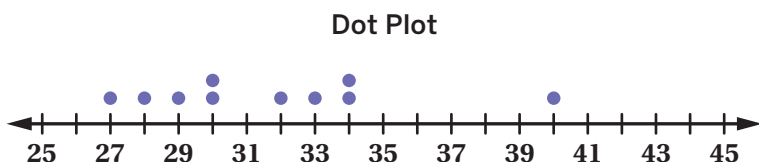
Stem	Leaf
2	4 7 8 8 9
3	0 0 1 2 3 3 4 5 5 5 5 5 6 6 7 7 7
4	0 0 0 0 1 1 2 2 3 5 7 8
5	1 5 7 9
6	3

- A. The center is near 37.
- B. The distribution is symmetric.
- C. The value 35 appears 5 times.
- D. 39 is the maximum value.

Key: 3 | 2 represents 32

2. Consider the dot plot shown.

- a Create a stem plot that displays the same data as the dot plot.



- b Do the data have any gaps? Which display, if any, did you use to identify any gaps in the data? Explain your thinking.

3. Consider the following data set:

1, 10, 92, 95, 105

If you needed to display the distribution visually, would you choose a dot plot or a stem plot? Explain your thinking.



Practice

Name: _____ Date: _____ Period: _____

4. Complete the table that shows the values of the expression $4x^3$ for given values of x .

x	$4x^3$
1	
2	
	$\frac{1}{2}$

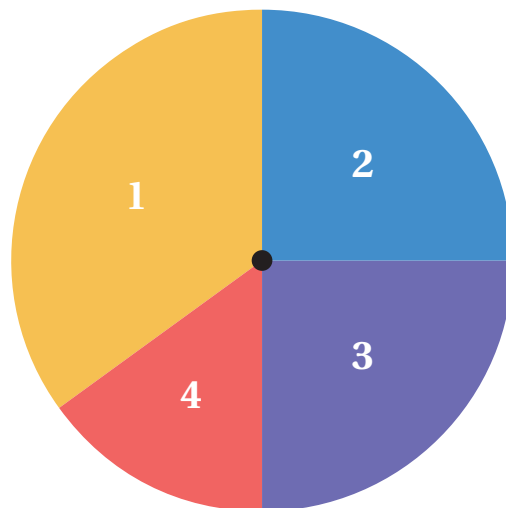
5. Jada is reading a book with 472 pages.

- a If Jada is currently on page 118, what percentage of the book has she already read?

- b How many more pages must Jada read to be 75% of the way through the book? Explain your thinking.

6. Consider the diagram shown. Select *all* the true statements.

- A. Region 4 represents about 25% of the whole.
- B. Region 1 is the largest section of the chart.
- C. Regions 2 and 3 combined represent about half of the total chart.
- D. Region 4 has the least area of any of the regions.
- E. Region 1 represents more than half of the total area of the chart.



Unit 8 | Tennessee Lesson 7B

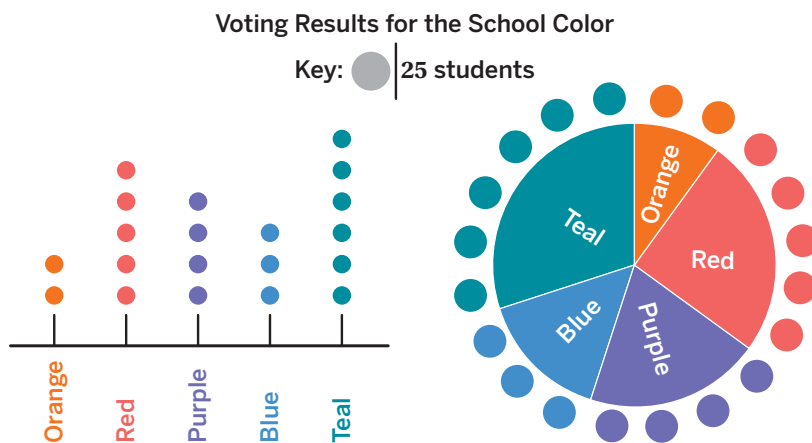
Pie Charts

Let's explore how pie charts represent data sets.

Warm-up Notice and Wonder

Diego and Mai are two students from different sixth-grade classes in a brand new school in Tennessee. All the students in the school voted for the spirit color of the school.

Diego used a dot plot to represent the results. Mai arranged the same dots in a circular chart.



Refer to their diagrams. What do you notice? What do you wonder?

> 1. I notice . . .

> 2. I wonder . . .



Log in to Amplify Math to complete this lesson online.

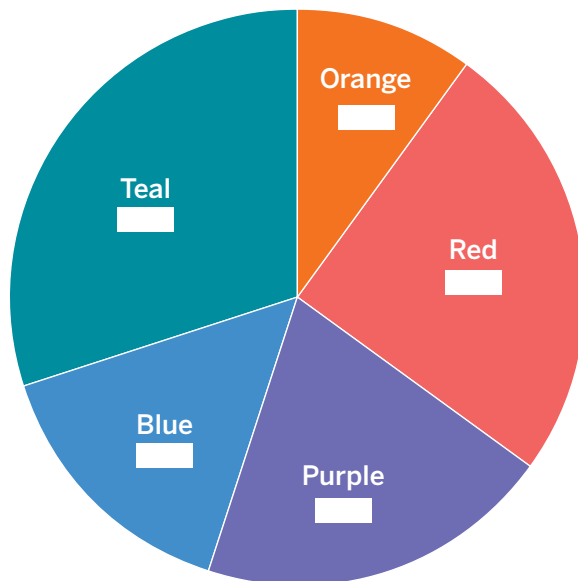
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Activity 1 Pie Charts

A pie chart (or circle graph) is a special chart that uses “pie slices” to show relative sizes of categories of data. The slices are called the sectors of the circle, where each sector represents a different category.

1. Refer to the frequency table for the results of the school spirit color survey. Calculate and record the percentages of each sector to complete the pie chart.

Colors	Orange	Red	Purple	Blue	Teal
Number of students	50	125	100	75	150



Activity 1 Pie Charts (continued)

- > 2. Do any of the categories receive more than one quarter of the votes? Explain your thinking.

- > 3. Diego thinks that half of the students voted for the colors teal and purple. Mai thinks that half of the students chose the colors red, blue and orange. Who is correct? Explain your thinking.

- > 4. Select *all* the correct statements that describe the pie chart.
 - A. The number of students who voted for red and purple are the same.
 - B. The least preferred color is orange.
 - C. 70% of the students did not choose teal as the color of their school.
 - D. More than half of the students in the school voted for the colors teal, blue and purple.

- > 5. Suppose 10 new students are enrolled in the school. What colors are most likely to be voted among these 10 students? Explain your thinking.

Activity 2 Constructing a Pie Chart

The school organizes several fundraisers for class trips. The table shown displays the amount of money raised from each fundraiser.

- 1. What is the total amount of money raised from all of the fundraisers?

Money raised by the fundraising activities

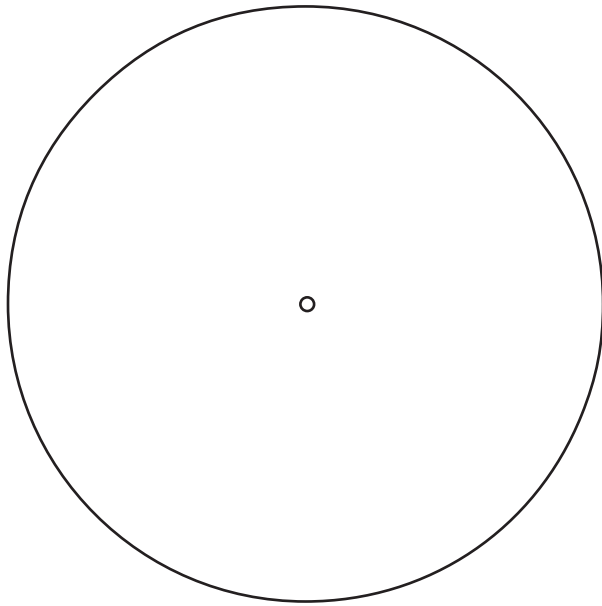
Activity	Money raised (\$)
Raffle	500
Bake sale	250
Donations	750
Fun run	375
Book fair	125

- 2. What portion of the total amount of money is raised from the raffle?

- 3. Mark and shade a sector on the empty pie chart on the next page to represent the money raised from the raffle. Explain your thinking.

- 4. Partition the circle according to the proportion among the categories to determine the size of the remaining sectors. Explain your thinking.

Activity 2 Constructing a Pie Chart (continued)



- 5. On the last day of the fundraising activities, an additional \$1000 in donations is received from the local businesses. Can the same pie chart be used only by increasing the sector size and the percentage of the donations category? Explain your thinking.



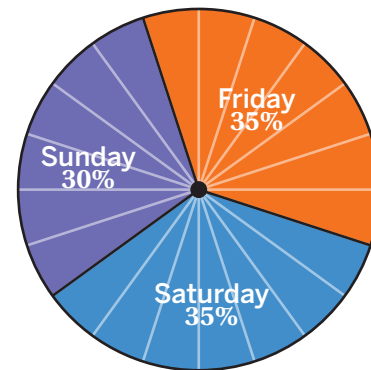
Summary

In today's lesson . . .

You saw that **pie charts** (pie graphs, circle graphs) are used to show a part-to-whole relationship for a data set.

Refer to the frequency table and the pie chart showing the survey results of the favorite day of the week for the sixth grade class at a local school.

Day	Number of Students	Percentage	Part-to-whole ratio
Friday	42	35%	7 : 20
Saturday	42	35%	7 : 20
Sunday	36	30%	6 : 20
<i>Total</i>	120	100%	20 : 20



*Favorite Day of the Week
in our Class*

To create a pie chart . . .

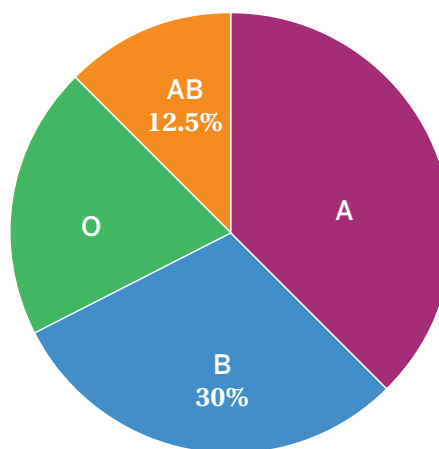
- Determine the total of all the data values.
- Determine the percentage of each category. Use these to label the sectors.
- Determine each part-to-whole ratio. Use these to determine the size of each sector and partition the pie chart.

> Reflect:



1. The pie chart displays the blood types of 360 people who were surveyed for a medical study.

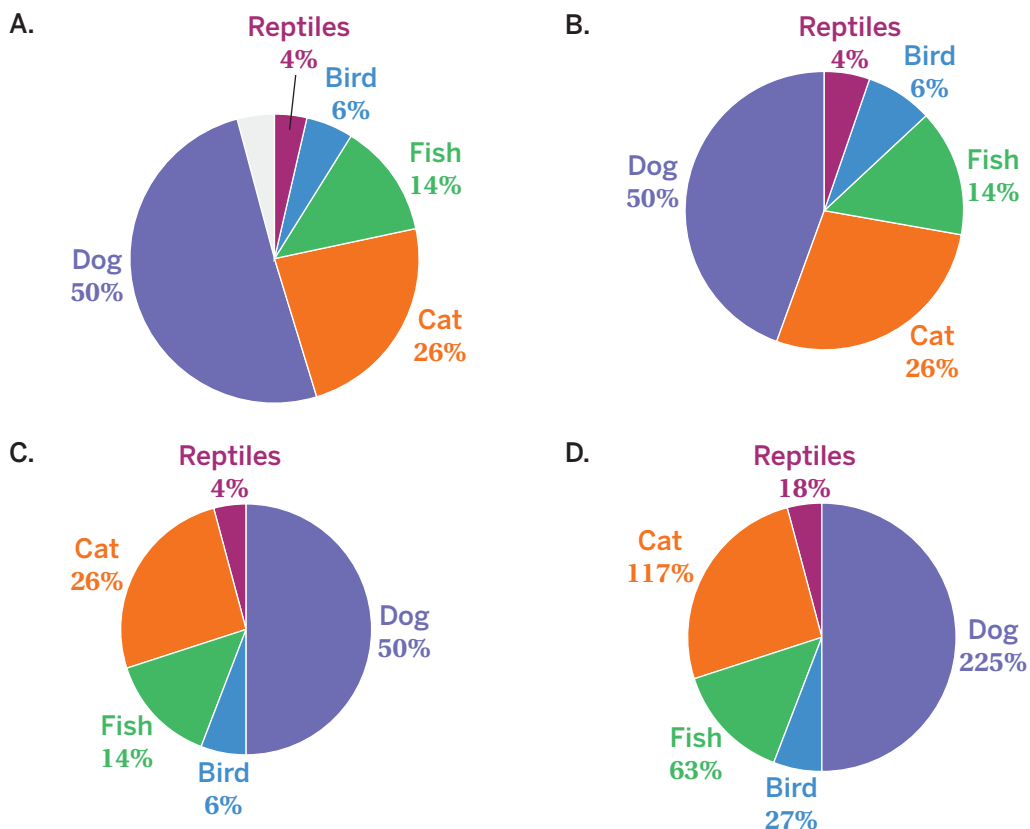
- a What percentage of the people have blood type A, if 135 people have blood type A?
- b What percentage of the people have blood type O?
- c What are the two rarest blood types?
- d How many people in this group have blood type A or AB? Explain your thinking.



2. Han surveyed 450 students in his school to determine the first pet they had. Refer to the frequency table Han created.

First pets	Reptiles	Bird	Fish	Cat	Dog
Number of students	18	27	63	117	225

Which pie chart displays the correct data based on his survey?

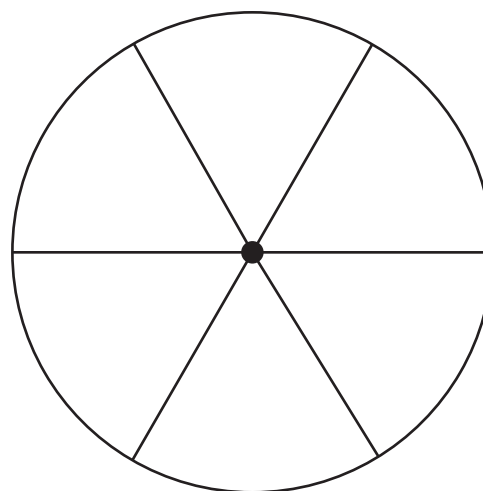




Practice

Name: _____ Date: _____ Period: _____

- 3. The word *Euouae* is a musical term from the Middle Ages. It is the longest English word composed exclusively of vowels, and it also has the most consecutive vowels of any word. Use the partitioned circle to create a pie chart to show the distribution of the vowels in the word *Euouae*.



- 4. Refer to the stem plot shown. Select *all* the true statements.

- A. The data value that occurs most frequently is 65.
- B. The distribution is symmetric.
- C. The data value 80 appears 2 times.
- D. 53 is the minimum data value.
- E. There are eight data values in which the ones digit is 8.

Stem	Leaf
5	3 5 7 8 8
6	3 4 4 5 5 5 5 7 8 8 8 9 9
7	0 2 4 5 6 6 7 7 8 8 9 9
8	0 0 1 1 2 3 5 6
9	3 5

Key: 2 | 5 represents 25

- 5. How many feet are in $5\frac{3}{4}$ yards? How many inches? Explain your thinking.

- 6. Bard calculated the quotient of 133 and 7 and got 19. Use Bard's result to determine each of the following:

- a $1330 \div 7$
- b $13.3 \div 7$
- c $1.33 \div 7$

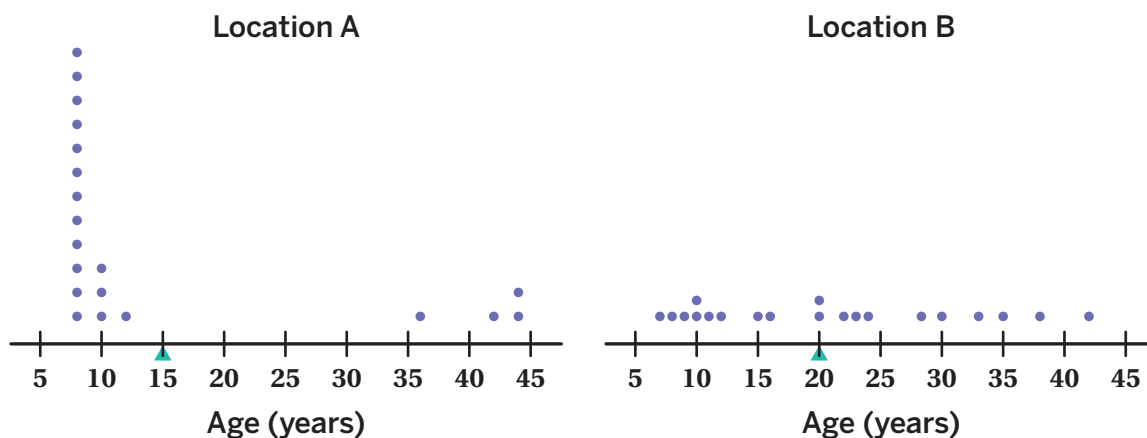
Unit 8 | Tennessee Lesson 12A

Measuring Variability

Let's explore two ways to describe the variability of data sets.

Warm-up Notice and Wonder

The two dot plots show the ages of 20 manatees from 2 different locations. The mean of each data set is marked with a triangle. What do you notice? What do you wonder?



> 1. I notice . . .

> 2. I wonder . . .



Log in to Amplify Math to complete this lesson online.

Activity 1 The Five-Number Summary

You have seen data sets that are not symmetric, have a wide spread, or have outliers. For those data sets, the median is an appropriate measure of center. But how about describing and summarizing variability for those types of distributions? And what are some reasons behind why the data may look like that?

Statisticians deal with these questions and issues all the time, because the reality is, reality is messy! Statistician Mary C. Christman, who has served as an advisor to the Florida Fish and Wildlife Commission Research Institute, has spent part of her career addressing exactly this question. Collecting environmental data, such as about manatees, is not easy and has many challenges.

Here are the ages of twenty manatees from Location B, ordered from least to greatest. Use the data set to complete the problems, and think about how your work is related to the relatively wide spread of values in this data set.

7 8 9 10 10 11 12 15 16 20 20 22 23 24 28 30 33 35 38 42

- 1. Circle the least data value and label it *Minimum*. Then circle the greatest data value and label it *Maximum*.
- 2. Determine the following values in the table. Mark the position of each value in the data and label each as indicated in the table.

	Value	Mark	Label
Median		◆	Q2
Middle value of the <i>lower half</i> of the data		↑	Q1
Middle value of the <i>upper half</i> of the data		↑	Q3



Featured Mathematician



Mary C. Christman

Mary C. Christman holds a BS in Biology from the University of Pennsylvania, an MS in Marine Biology and Physical Oceanography from the University of Delaware, and a PhD in Mathematical Statistics from George Washington University. She is currently the owner of MCC Statistical Consulting, which specializes in collecting and representing environmental and ecological data. She has advised the Florida Fish and Wildlife Commission Research Institute on the coastal ecosystems of Florida and the effects of the “red tide” phenomenon on both humans and sea life, including manatees.

Activity 1 The Five-Number Summary (continued)

Look back at the ordered list of data on the previous page, now with the marks and labels. The data set has been divided into four equal parts from the minimum to the maximum. The three values labeled Q1, Q2, and Q3 that divide the data are called **quartiles**.

- The *first quartile* (Q1) represents an upper bound for the lowest 25% of the data. It is also referred to as the *25th percentile*. Q1 is also a lower bound for the highest 75% of the data.
- The *second quartile* (Q2) corresponds to the *median*, and it represents an upper bound for the lowest 50% of the data. It is also referred to as the *50th percentile*. Q2 is also a lower bound for the highest 50% of the data.
- The *third quartile* (Q3) represents an upper bound for the lowest 75% of the data. It is also referred to as the *75th percentile*. Q3 is also a lower bound for the highest 25% of the data.

Together, these five numbers — minimum, Q1, Q2, Q3, maximum — make up what is called the **five-number summary** for a data set.

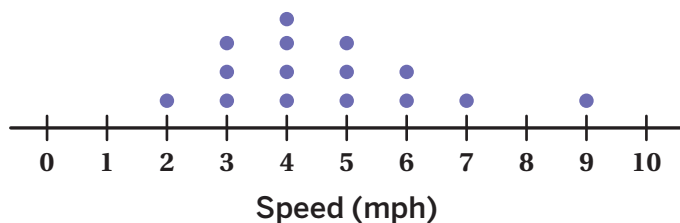
- 3. Record the five-number summary for data representing the ages of the manatees.

Minimum: Q1: Q2: Q3: Maximum:

- 4. What does the value of the third quartile (Q3) tell you about the ages of the manatees at this location?

Activity 2 Range

Refer to this dot plot that shows 15 recorded speeds of a manatee, in miles per hour.



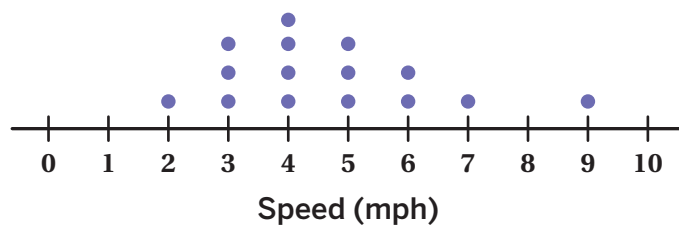
- 1. Determine the five-number summary for this data set. Show your thinking.

Minimum: Q1: Q2: Q3: Maximum:

- 2. One way to describe the *spread* of values in a data set is to look at the difference between the maximum and minimum values. This is called the **range**. What is the range of the speeds of the manatee?

- 3. The same dot plot is shown again here.

- a** Add one additional dot to represent a new data value so that the range increases. Determine the new range.



- b** Determine the median of the new data set.



Summary

In today's lesson ...

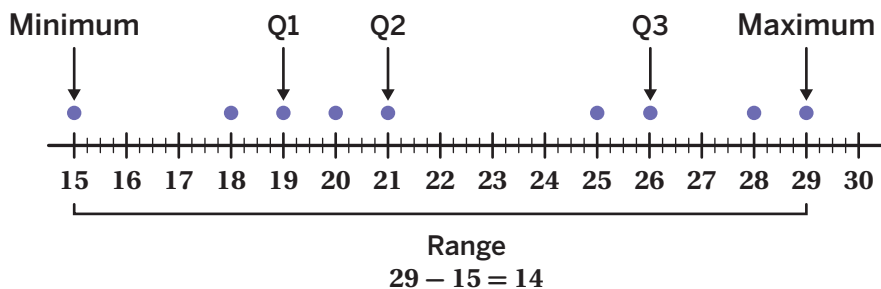
You saw how to calculate the **five-number summary** for a data set, which can be used to summarize its distribution. The *five-number summary* consists of the minimum, maximum, and the three quartiles, Q1, Q2, and Q3.

The **first quartile (Q1)** is the median of the lower half of the data.

The **second quartile (Q2)** is the median of the entire data set.

The **third quartile (Q3)** is the median of the upper half of the data.

The minimum and maximum can be used to calculate a measure of *variability* called the **range**. The range gives you a basic overall sense of how spread out the data is, but it does not tell you how the data is distributed between the minimum and maximum values.



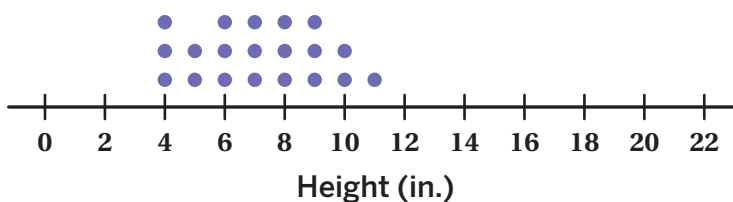
> Reflect:



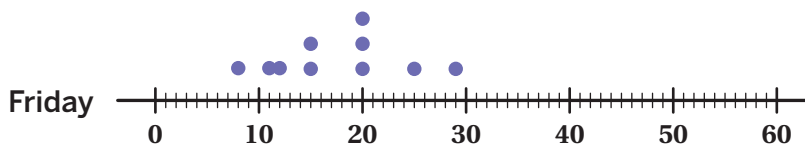
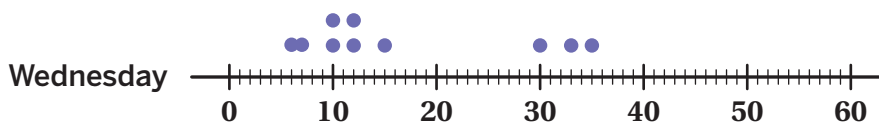
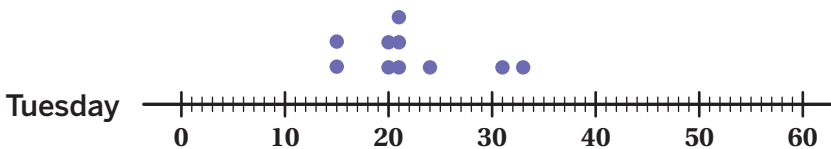
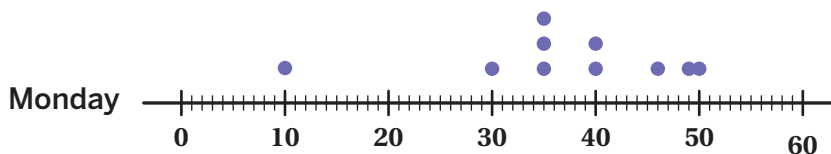
Practice

Name: _____ Date: _____ Period: _____

- 1. The physical education teacher at a school records how high each 6th grader in her class can jump off the ground. This dot plot shows the heights 20 students jumped.



- a What is the median height jumped?
 - b What is the first quartile (Q1)?
 - c What is the third quartile (Q3)?
 - d What is the range?
- 2. The 5 dot plots show the amount of time it took 10 sixth graders to complete their homework each day. Match each day's dot plot with the correct Q1 and Q3 values by writing the day next to each set of statistics in parts a-e.

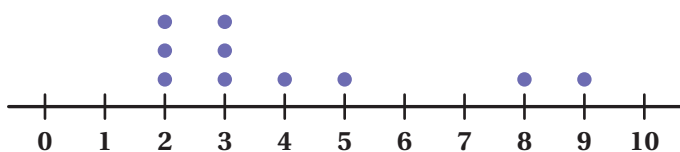


- a Q1: 20, Q3: 24
- b Q1: 10, Q3: 20
- c Q1: 35, Q3: 46
- d Q1: 12, Q3: 20
- e Q1: 35, Q3: 40

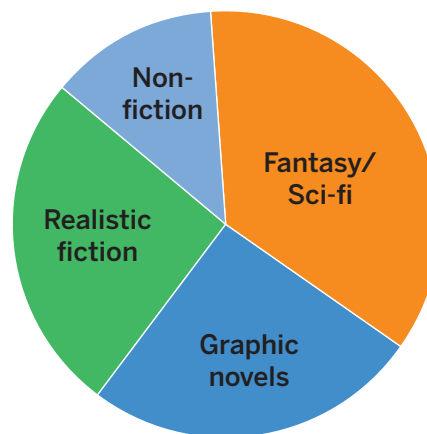


- 3. Clare and Han each played 10 games of darts and recorded their scores. Clare's median score was 100, with a Q1 of 80 and Q3 of 105. Han's median score was 98, with Q1 of 75 and Q3 of 120. Is it possible to know which person had the greater range of scores? Explain how you know.

- 4. The dot plot shown represents a data set. Explain why the mean of the data set is greater than its median.



- 5. The pie chart below displays the results of a survey given to a group of 200 sixth graders about their favorite book genre. Determine reasonable values for each of the following genres based on the chart.



Genre	Number of students
Non-fiction	
Fantasy/Sci-fi	
Realistic fiction	
Graphic novels	

- 6. Write a scenario that could be represented by this number summary.

Minimum: 5 Median: 7.5 Maximum: 9

