Amplify Math TENNESSEE

Grade 6

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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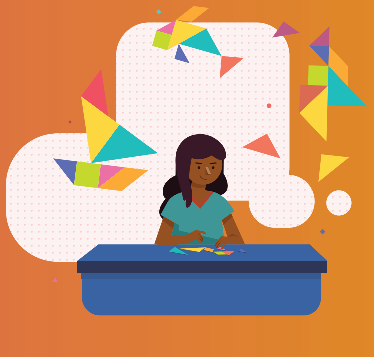
Volume 1

Unit 1 Area and Surface Area

Unit Narrative:
A Place for Space



Students extend their elementary understanding of area as compositions and decompositions for covering, shifting from limited experiences with rectangles and unit-square thinking to more general formulas for parallelograms and triangles. They leverage these in working with three-dimensional figures as well, recognizing surface area as a different measure than volume.

PRE-UNIT READINESS ASSESSMENT

LAUNCH

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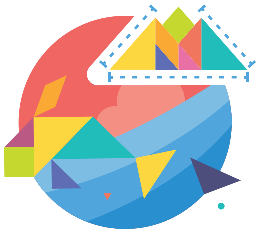
1.02	Exploring the Tangram	10A	MP2

Sub-Unit 1 Area of Special Polygons 17



1.03	Tiling the Plane	18A	**6.G.A.1,** MP1, 3, 5, 7
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1.12	From Triangles to Trapezoids	76A	**6.G.A.1,** MP7
1.13	Polygons	82A	**6.G.A.1,** MP5, 7, 8

Sub-Unit Narrative:
Can a sum ever really be greater than its parts?

Polygons are shapes whose sides are all line segments, and they can be decomposed and rearranged without changing their area.

MID-UNIT ASSESSMENT

Sub-Unit 2 Nets and Surface Area 89

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1.20	Designing a Suspended Tent	125A	**6.G.A.1, 6.G.A.4,** MP3, 4, 7

END-OF-UNIT ASSESSMENT

Sub-Unit Narrative:
How did a misplaced ruler change the way you shop?

Polyhedra are three-dimensional figures composed of polygon faces. Their surfaces can be decomposed.

Unit 2 Introducing Ratios

Students understand ratios using three of their five senses. They use written and visual representations to learn the language of ratios, and scale up (with multiplication) or down (with division) to calculate equivalent ratios. Ratios are also used for thinking about constant rates or occurrences happening at the same rate.

Unit Narrative:
Sensing a Ratio

Note: *Lessons in gray are recommended to be omitted.*
● = *Tennessee-specific lessons*

PRE-UNIT READINESS ASSESSMENT

LAUNCH

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Sub-Unit 1 What Are Ratios? 141

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Sub-Unit Narrative:
How does an eggplant become a plum?

Ratios represent comparisons between quantities by multiplication or division. First, you must first learn the language of ratios and how quantities "communicate."

Sub-Unit 2 Equivalent Ratios 171

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2.13	Tempo and Double Number Lines	217A	**6.RP.A.3**, MP2, 6, 7

Sub-Unit Narrative:
How do you put your music where your mouth is?

Equivalent ratios involve relationships between ratios themselves. They speak to each other through music and rhythm, beats and time.

Sub-Unit 3 Solving Ratio Problems 225

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Sub-Unit Narrative:
Who brought Italy to India and back again?

Now it is your turn to choose the information to represent and compare ratios.

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2.20	More Fermi Problems	264A	**6.RP.A**, MP1, 4, 6

END-OF-UNIT ASSESSMENT

Unit 3 Rates and Percentages

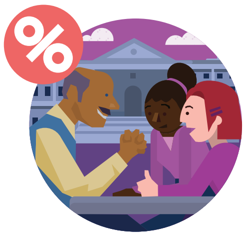
Students understand the concept of unit rate in the contexts of constant price and speed, recognizing that equivalent ratios have the same unit rates. They use several visual and algebraic representations of percentages to determine missing percentages, parts, and wholes.

Unit Narrative:
Stand and Be Counted

Note: *Lessons in gray are recommended to be omitted.*

PRE-UNIT READINESS ASSESSMENT

LAUNCH
3.01 Choosing Representation for Student Council 274A — 6.RP.A.1, 6.RP.A.3, 6.RP.A.3c, MP1, 2, 3



Sub-Unit 1 Rates

Sub-Unit 1 Rates 281
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3.06 Comparing Rates 310A — 6.RP.A.2, 6.RP.A.3b, MP6, 7
3.07 Solving Rate Problems 317A

Sub-Unit Narrative:
How did student governments come to be?

Rates describe relationships between quantities like price and speed. Unit rates reveal which is a better deal or who is faster.

Sub-Unit 2 Percentages

Sub-Unit 2 Percentages 323
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3.13 Solving Percentage Problems 357A — 6.RP.A.3c, MP4, 5
3.14 If Our Class Were the World 364A — 6.RP.A, MP1, 2, 6

Sub-Unit Narrative:
What can a corpse teach us about governing?

Percentages are rates per 100. They can compare relationships between parts and wholes, even when two quantities have different total amounts.

CAPSTONE
3.15 Voting for a School Mascot 371A — 6.RP.A.3, 6.RP.A.3c, MP1, 2, 3, 6

END-OF-UNIT ASSESSMENT

Unit 4 Dividing Fractions

Students extend their understanding of partitive and quotitive division from whole numbers to fractions. They use this along with the relationship between multiplication and division to construct models and develop an algorithm for dividing fractions, and they apply it to problems involving lengths, areas, and volumes.

Unit Narrative:
Crossing the Fractional Divide

PRE-UNIT READINESS ASSESSMENT

LAUNCH
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Sub-Unit 1 Interpreting Division Scenarios

Sub-Unit 1 Interpreting Division Scenarios 389
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Sub-Unit Narrative:
Which item costs between 100 and 1,000 spök-bucks?
Multiplication and division are related, and the relationship between fractions and division can be used to estimate quotients.

Sub-Unit 2 Division With Fractions

Sub-Unit 2 Division With Fractions 409
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4.12 Related Quotients .... 457A — **6.NS.A.1,** MP1, 6, 7

Sub-Unit Narrative:
How long is the bolt Samira needs?
To divide fractions, you can use multiplication, common denominators, or an algorithm. Apply these to determine the length of an oddly labeled bolt.

Sub-Unit 3 Fractions in Lengths, Areas, and Volumes

Sub-Unit 3 Fractions in Lengths, Areas, and Volumes 465
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4.17 Now, Where Was That Bus? .... 491A — **6.NS.A.1,** MP1, 2

END-OF-UNIT ASSESSMENT

Sub-Unit Narrative:
How can Maya fit Penny in the box?
When you know an area or volume, but not every side length, you will often divide fractions.

Volume 2

Unit 5 Arithmetic in Base Ten

Students synthesize previous learning of place value, properties of operations, and relationships between operations to complete their understanding of both the “whys” and “hows” of the four operations with positive rational numbers. They develop general algorithms for working with whole numbers and decimals, containing any arbitrary number of digits.

Unit Narrative:
Making Moves With Decimals

PRE-UNIT READINESS ASSESSMENT

LAUNCH
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Sub-Unit 1 Adding and Subtracting Decimals 503
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5.03 Adding and Subtracting Decimals .... 512A — **6.NS.B.3,** MP7
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Sub-Unit Narrative:
How did a decimal decide an Olympic race?
Determine the results of high stakes competitions and identify record-setting moments by adding and subtracting decimals, as precisely as you need.

Sub-Unit 2 Multiplying Decimals 527
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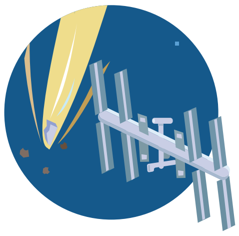
5.07 Representing Decimal Multiplication With Diagrams .... 542A — **6.NS.B.3,** MP7, 8
5.08 Calculating Products of Decimals .... 548A — **6.NS.B.3,** MP2, 6, 8

Sub-Unit Narrative:
What happens when you make a small change to a big bridge?
To reproduce something at large or small scales so it looks the same, you need decimals and multiplication.



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5.11 Dividing Numbers That Result in Decimals .... 571A — **6.NS.B.2,** MP2, 6, 7
5.12 Using Related Expression to Divide With Decimals .... 578A — **6.NS.B.3,** MP7
5.13 Dividing Multi-digit Decimals .... 585A — **6.NS.B.3, 6.EE.A.4,** MP7, 8

Sub-Unit Narrative:
How do you dodge a piece of space junk?
Dividing whole numbers and decimals with many digits is the final set of operations you need to complete your trophy case.

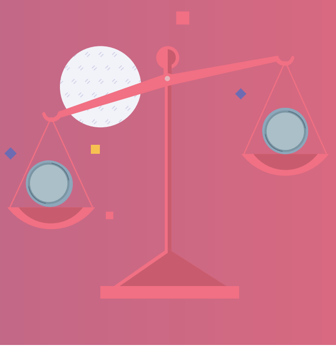
CAPSTONE
5.14 The So-called World’s “Littlest Skyscraper” .... 592A — **6.NS.B.3,** MP1, 3, 7

END-OF-UNIT ASSESSMENT

Unit 6 Expressions and Equations

Students discover that the equal sign is more than a prompt, it's also a way to indicate balance — a critical understanding that allows them to move beyond the strictly numeric world and into the realm of algebra.

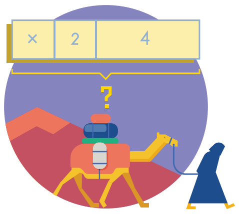
Unit Narrative:
The Power
of Balance



● = *Tennessee-specific lessons*

PRE-UNIT READINESS ASSESSMENT

LAUNCH	**6.01** Detecting Counterfeit Coins	600A	**6.EE.B.6,** MP1, 8



Sub-Unit 1 Expressions and Equations in One Variable 607

6.02	Write Expressions Where Letters Stand for Numbers	608A	**6.EE.B.6, 6.EE.A.2b,** MP2, 8
6.03	Tape Diagrams and Equations	614A	**6.EE.B.6, 6.EE.A.2b,** MP2, 4, 7
6.04	Truth and Equations	620A	**6.EE.B.5,** MP1, 2, 3, 8
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6.09	Revisiting Percentages	654A	**6.RP.A.3c, 6.EE.B.6, 6.EE.B.7,** MP2, 8

Sub-Unit Narrative:
What's a bag of chips worth in Timbuktu?

Learn about the 14th century African salt trade, as you explore expressions and equations with tape diagrams and hanger diagrams.

MID-UNIT ASSESSMENT

Sub-Unit 2 Equivalent Expressions 661

6.10	Equal and Equivalent (Part 1)	662A	**6.EE.A.2, 6.EE.A.4,** MP1, 2, 7
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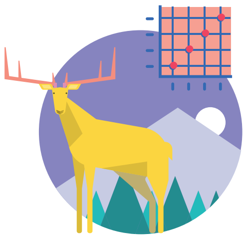
6.16	Analyzing Exponential Expressions and Equations	699A	**6.EE.A.1, 6.EE.A.2c,** MP3, 6, 7

Sub-Unit Narrative:
How did a Welshman equalize England's upper crust with its common folk?

Extend the concept of equality as you investigate equivalent expressions, the all-important Distributive Property, and exponents.



Sub-Unit 3 Relationships Between Quantities 705

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CAPSTONE	**6.19** Creating a Class Mobile	719A	**6.EE.A.1, 6.EE.A.2, 6.EE.A.4, 6.EE.B.5, 6.EE.B.6, 6.EE.B.7,** MP2, 4, 6

Sub-Unit Narrative:
What's more dangerous: a pack of wolves or a gang of elk?

Balance is everywhere, especially in ecosystems. You'll look at systems that are in and out of balance.

END-OF-UNIT ASSESSMENT

Unit 7 Rational Numbers

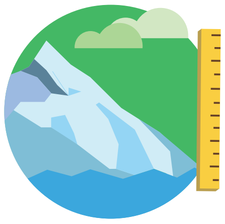
Students recognize a need to expand their concept of number to represent both magnitude and direction, extending the number line and coordinate plane to include negative rational numbers. They compare these numbers, as well as their absolute values, and write inequality statements using variables.

Unit Narrative:
Getting Where We're Going

PRE-UNIT READINESS ASSESSMENT

LAUNCH

7.01 How Far? Which Way? .... 728A — MP6



Sub-Unit 1 Negative Numbers and Absolute Value 735

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7.08 Comparing Numbers and Distances From Zero .... 776A — **6.NS.C.6, 6.NS.C.6a, 6.NS.C.7, 6.NS.C.7c,** MP2, 3, 4, 6

Sub-Unit Narrative:
What's the tallest mountain in the world?
Consider the most extreme locations on Earth as you discover negative numbers, which lend new meaning to positive numbers and zero.

Sub-Unit 2 Inequalities 783

7.09 Writing Inequalities .... 784A — **6.EE.B.8, 6.NS.C.7b,** MP2
7.10 Graphing Inequalities .... 790A — **6.EE.B.8, 6.NS.C.7b,** MP2, 7
7.11 Solutions to One or More Inequalities .... 796A — **6.EE.B.5, 6.EE.B.8,** MP2, 3, 6
7.12 Interpreting Inequalities .... 803A — **6.EE.B.5, 6.EE.B.6, 6.EE.B.8,** MP2, 3

Sub-Unit Narrative:
How do you keep a quantity from wandering off?
A variable represents an unknown quantity. And sometimes it represents many possible values, which can be expressed as an inequality.

Sub-Unit 3 The Coordinate Plane 811

7.13 Extending the Coordinate Plane .... 812A — **6.NS.C.6b, 6.NS.C.6c, 6.NS.C.8,** MP7
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7.18 Lost and Found Puzzles .... 844A — **6.G.A.3, 6.NS.C.8,** MP1, 2, 4, 5

Sub-Unit Narrative:
How did Greenland get so big?
Armed with the opposites of positive rational numbers, it's time you expanded your coordinate plane. Welcome to the four quadrants!

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7.19 Drawing on the Coordinate Plane .... 853A — **6.G.A.3, 6.NS.C.8,** MP6, 7

END-OF-UNIT ASSESSMENT

Unit 8 Data Sets and Distributions

In this unit, students learn about populations and study variables associated with a population, focusing on populations of animal species and their respective endangerment classifications. They distinguish numerical and categorical data, relative to survey and statistical questions, and represent and describe the distributions of response data. Students first interpret frequency tables, dot plots, and histograms, before calculating measures of center — mean and median — and measures of variability — mean absolute deviation (MAD), range, and interquartile range (IQR). They then construct box plots in addition to interpreting these measures in context, and relating the shape and features of a distribution to the best choice of measures.

Unit Narrative:
Walk on the Wild Side With Data

Note: *Lessons in gray are recommended to be omitted.*
● = *Tennessee-specific lessons*

PRE-UNIT READINESS ASSESSMENT

LAUNCH	**8.01** Plausible Variation or New Species?	860A	**6.SP.A,** MP1, 3

Sub-Unit 1 Statistical Questions and Representing Data 867

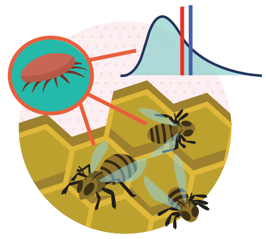
8.02 Statistical Questions	868A	**6.SP.A.1, 6.SP.B.5b,** MP3, 6, 7
8.03 Interpreting Dot Plots	874A	**6.SP.A.1, 6.SP.A.2, 6.SP.B, 6.SP.B.4, 6.SP.B.5a,** MP2, 3, 7
8.04 Using Dot Plots to Answer Statistical Questions	881A	**6.SP.A.2, 6.SP.B,** MP2, 3, 7
8.05 Interpreting Histograms	888A	**6.SP.A.1, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5b,** MP2, 7
8.06 Using Histograms to Interpret Statistical Data	895A	**6.SP.A.1, 6.SP.A.2, 6.SP.B, 6.SP.B.4, 6.SP.B.5b,** MP2, 7
8.07 Describing Distributions With Histograms	902A	**6.SP.A.2, 6.SP.B.4, 6.RP.A.3c,** MP3, 6, 7
8.07A Stem Plots	TN-14A	**6.SP.B.4, 6.SP.A.3,** MP3, 7
8.07B Pie Charts	TN-21A	**6.SP.B.4, 6.RP.A.3, 6.RP.A.3c,** MP3, 5

Sub-Unit Narrative:
How do you keep track of a disappearing animal?

When questions have more than one answer, it is helpful to visualize and describe a typical answer. For numbers, you can also identify the center and describe the spread of the numbers.

Sub-Unit 2 Measures of Center 909

8.08 Mean as a Fair Share	910A	**6.SP.A.3, 6.SP.B.5c,** MP2, 3, 7
8.09 Mean as the Balance Point	917A	**6.SP.A.3, 6.SP.B.5c,** MP2, 3, 7, 8
8.10 Median	924A	**6.SP.A.3, 6.SP.B.5c,** MP2, 7
8.11 Comparing Mean and Median	930A	**6.SP.B.5b, 6.SP.B.5c, 6.SP.B.5d,** MP1, 2, 3, 7



Sub-Unit Narrative:
What's the buzz on honey bees?

For numerical data, you can summarize an entire data set by a single value representing the center of the distribution. The mean and the median represent two ways you can do this.

Sub-Unit 3 Measures of Variability 937

8.12 Describing Variability	938A	**6.SP.A.2, 6.SP.A.3,** MP1, 3
8.12A Measuring Variability	TN-29A	**6.SP.B.5c,** MP2
8.13 Variability and MAD	944A	
8.14 Variability and IQR	951A	
8.15 Box Plots	959A	**6.SP.B, 6.SP.B.4, 6.SP.B.5,** MP1, 2, 7
8.16 Comparing MAD and IQR	966A	

Sub-Unit Narrative:
Where have the giant sea cows gone?

For numerical data, you can summarize an entire data set by a single value representing the variability of the distribution. The MAD, range, and IQR represent three ways you can do this.

CAPSTONE	**8.17** Asian Elephant Populations	972A	**6.SP.A, 6.SP.B.5c, 6.SP.B.5d,** MP1, 3, 5

END-OF-UNIT ASSESSMENT

Tennessee Mathematics Standards, Grade 6

6.RP	Ratios and Proportional Relationships	Lesson(s)
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**	
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. Make a distinction between ratios and fractions. For example, the ratio of wings to beaks in a bird house at the zoo was $2{:}1$, because for every 2 wings there was 1 beak. Another example could be for every vote candidate A received, candidate C received nearly three votes.	**Unit 2, Lessons** 2–6 **Unit 3, Lesson** 1 **Unit 6, Lesson** 17
6.RP.A.2	Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a{:}b$ with $b \neq 0$. Use rate language in the context of a ratio relationship. For example, this recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar. Also, we paid $\$75$ for 15 hamburgers, which is a rate of $\$5$ per hamburger. (Expectations for unit rates in 6th grade are limited to non-complex fractions).	**Unit 3, Lessons** 4–6
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).	**Unit 2, Lessons** 11–15 **Unit 3, Lessons** 1, 15 **Unit 8, Lesson** 7B
6.RP.A.3a	Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	**Unit 2, Lessons** 7, 11, 12, 16, 17 **Unit 3, Lesson** 4 **Unit 6, Lessons** 17, 18
6.RP.A.3b	Solve unit rate problems including those involving unit pricing and constant speed. For example, if a runner ran 10 miles in 90 minutes, running at that speed, how long will it take him to run 6 miles? How fast is he running in miles per hour?	**Unit 3, Lessons** 2–6 **Unit 6, Lessons** 17, 18
6.RP.A.3c	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.	**Unit 3, Lessons** 1, 8–13, 15 **Unit 6, Lesson** 9 **Unit 8, Lesson** 7, 7B
6.RP.A.3d	Use ratio reasoning to convert customary and metric measurement units (within the same system); manipulate and transform units appropriately when multiplying or dividing quantities.	**Unit 2, Lesson** 17A

Tennessee Mathematics Standards, Grade 6

6.NS	**The Number System**	**Lesson(s)**
6.NS.A	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**	
6.NS.A.1	Interpret and compute quotients of fractions and solve contextual problems involving division of fractions by fractions (e.g., connecting visual fraction models and equations to represent the problem is suggested). For example, create a story context for $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $\Big(\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3} \left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{ad}{bc}\Big)$. Further example: How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{3}{4}$ cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ mile and area $\frac{1}{2}$ square mile?	**Unit 4, Lessons** 5–14, 17
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**	
6.NS.B.2	Fluently divide multi-digit numbers using a standard algorithm.	**Unit 5, Lessons** 9–11
6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm and making connections to previous conceptual work with each operation.	**Unit 5, Lessons** 1–8, 12–14
6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers $1–100$ with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.	**Unit 2, Lessons** 9, 10
6.NS.C	**Apply and extend previous understandings of numbers to the system of rational numbers.**	
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of zero in each situation as well as describing situations in which opposite quantities can combine to make 0.	**Unit 7, Lessons** 2, 6
6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	**Unit 7, Lessons** 2, 3, 5, 8
6.NS.C.6a	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself. For example, $-(-3) = 3$, and that 0 is its own opposite.	**Unit 7, Lessons** 3, 5, 8
6.NS.C.6b	Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	**Unit 7, Lesson** 13
6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	**Unit 7, Lessons** 3, 13–17

6.NS.C.7	Understand ordering and absolute value of rational numbers.	**Unit 7, Lessons** 5, 8
6.NS.C.7a	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.	**Unit 7, Lesson** 4
6.NS.C.7b	Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3^\circ C > -7^\circ C$ to express the fact that $-3^\circ C$ is warmer than $-7^\circ C$.	**Unit 7, Lessons** 4, 9, 10
6.NS.C.7c	Understand the absolute value of a rational number as its distance from 0 on the number line and distinguish comparisons of absolute value from statements about order in a real-world context. For example, an account balance of -24 dollars represents a greater debt than an account balance -14 dollars because -24 is located to the left of -14 on the number line.	**Unit 7, Lessons** 7, 8, 15
6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**Unit 7, Lessons** 13, 15–19
6.EE	**Expressions and Equations**	**Lesson(s)**
6.EE.A	**Apply and extend previous understandings of arithmetic to algebraic expressions.**	
6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	**Unit 1, Lesson** 19 **Unit 6, Lessons** 14–16, 19
6.EE.A.2	Write, read, and evaluate expressions in which variables stand for numbers.	**Unit 1, Lesson** 18 **Unit 6, Lessons** 10, 13, 19
6.EE.A.2a	Write expressions that record operations with numbers and with variables. For example, express the calculation “Subtract y from 5” as $5 - y$.	**Unit 1, Lessons** 7, 11
6.EE.A.2b	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.	**Unit 6, Lessons** 2, 3, 12–15
6.EE.A.2c	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	**Unit 1, Lessons** 7, 8, 11, 19 **Unit 6, Lessons** 15, 16
6.EE.A.3	Apply the properties of operations (including, but not limited to, commutative, associative, and distributive properties) to generate equivalent expressions. (The distributive property of multiplication over addition is prominent here. Negative coefficients are not an expectation at this grade level.) For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.	**Unit 6, Lessons** 11–13

Tennessee Mathematics Standards, Grade 6

6.EE.A.4	Identify when expressions are equivalent (i.e., when the expressions name the same number regardless of which value is substituted into them). For example, the expression $5b + 3b = (5 +3)b = 8b$.	**Unit 5, Lesson** 13 **Unit 6, Lessons** 10–13, 15, 19
6.EE.B	**Reason about and solve one-variable equations and inequalities.**	
6.EE.B.5	Understand that a solution to an equation or inequality is the value(s) that makes that statement true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**Unit 6, Lessons** 4–6, 8, 19 **Unit 7, Lessons** 11, 12
6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.	**Unit 6, Lessons** 1–3, 6–9, 19 **Unit 7, Lesson** 12
6.EE.B.7	Solve real-world and mathematical problems by writing and solving one-step equations of the form $x + p = q$, $px = q$, $x - p = q$, and $\frac{x}{p} = q$ for cases in which p, q, and x are all nonnegative rational numbers and $p \neq 0$. (Complex fractions are not an expectation at this grade level.)	**Unit 6, Lessons** 5–9, 19
6.EE.B.8	Interpret and write an inequality of the form $x > c$, $x < c$, $x \leq c$, or $x \geq c$ which represents a condition or constraint in a real-world or mathematical problem. Recognize that inequalities have infinitely many solutions; represent solutions of inequalities on number line diagrams.	**Unit 7, Lessons** 9–12
6.EE.C	**Represent and analyze quantitative relationships between dependent and independent variables.**	
6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another. For example, Susan is putting money in her savings account by depositing a set amount each week ($50). Represent her savings account balance with respect to the number of weekly deposits ($s = 50w$, illustrating the relationship between balance amount s and number of weeks w).	**Unit 6, Lessons** 17, 18
6.EE.C.9a	Write an equation in the form of $y = px$ where y, p, and x are all non-negative and $p \neq 0$, to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.	**Unit 6, Lesson** 17
6.EE.C.9b	Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	**Unit 6, Lesson** 18
6.G	**Geometry**	**Lesson(s)**
6.G.A	**Solve real-world and mathematical problems involving area, surface area, and volume.**	
6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**Unit 1, Lessons** 3–13, 20 **Unit 4, Lesson** 14
6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ where B is the area of the base to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**Unit 4, Lessons** 15, 16

6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side that joins two vertices (vertical or horizontal segments only). Apply these techniques in the context of solving real-world and mathematical problems.	**Unit 7, Lessons** 17–19
6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**Unit 1, Lessons** 15–18, 20
6.SP	**Statistics and Probability**	**Lesson(s)**
6.SP.A	**Develop understanding of statistical variability.**	
6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.	**Unit 8, Lessons** 2, 3, 5, 6
6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its measures of center (mean, median, mode), measures of variation (range only), and overall shape.	**Unit 8, Lessons** 3, 4, 6, 7, 12
6.SP.A.3	Recognize that a measure of center (mean, median, mode) for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	**Unit 8, Lessons** 5, 7A, 8–10, 12
6.SP.B	**Summarize and describe distributions.**	
6.SP.B.4	Display a single set of numerical data using dot plots (line plots), box plots, pie charts and stem plots.	**Unit 8, Lessons** 3, 5–7, 7A, 15
6.SP.B.5	Summarize numerical data sets in relation to their context.	**Unit 8, Lesson** 15
6.SP.B.5a	Report the number of observations.	**Unit 8, Lesson** 3
6.SP.B.5b	Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.	**Unit 8, Lessons** 2, 5, 6, 11
6.SP.B.5c	Give quantitative measures of center (median and/or mean) and variability (range) as well as describing any overall pattern with reference to the context in which the data were gathered.	**Unit 8, Lessons** 8–11, 12A, 17
6.SP.B.5d	Relate the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.	**Unit 8, Lessons** 11, 17

Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Unit 1, Lessons 1, 3, 8, 14

Unit 2, Lessons 1, 14, 15, 17, 17A, 20

Unit 3, Lessons 1, 14, 15

Unit 4, Lessons 9, 12, 14–17

Unit 5, Lessons 1, 4, 10, 14

Unit 6, Lessons 1, 4, 5, 7A, 10

Unit 7, Lessons 14, 18

Unit 8, Lessons 1, 11, 12, 15, 17

MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

Unit 1, Lesson 2

Unit 2, Lessons 3, 6, 10, 12, 13, 15, 17A

Unit 3, Lessons 1, 3, 14, 15

Unit 4, Lessons 2, 3, 6, 8, 9, 17

Unit 5, Lessons 8, 9, 11

Unit 6, Lessons 2–10, 12, 13, 15, 17–19

Unit 7, Lessons 2–4, 6–12, 15–18

Unit 8, Lessons 3–6, 8–11, 12A, 15

MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Unit 1, Lessons 3, 5, 10, 16, 20

Unit 2, Lesson 16

Unit 3, Lessons 1, 5, 15

Unit 4, Lessons 1, 7, 14–16

Unit 5, Lesson 14

Unit 6, Lessons 4, 5, 15, 16

Unit 7, Lessons 4, 5, 8, 11, 12, 17

Unit 8, Lessons 1–4, 7–9, 11, 12, 17

MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	**Unit 1, Lesson** 20 **Unit 2, Lessons** 1, 14, 20 **Unit 3, Lesson** 13 **Unit 4, Lesson** 13 **Unit 5, Lessons** 3, 17, 19 **Unit 6, Lessons** 6, 8 **Unit 7, Lessons** 17, 18

MP5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	**Unit 1, Lessons** 3, 13 **Unit 3, Lesson** 13 **Unit 5, Lesson** 2 **Unit 7, Lesson** 18 **Unit 8, Lesson** 7B, 17

MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.	**Unit 1, Lessons** 1, 5, 7, 14, 16, 18, 19 **Unit 2, Lessons** 2, 3, 6, 9, 13, 14, 17A, 20 **Unit 3, Lessons** 2, 6, 8, 10, 12, 14, 15 **Unit 4, Lessons** 1, 6, 12, 13 **Unit 5, Lessons** 1, 2, 8, 11 **Unit 6, Lessons** 12, 16, 19 **Unit 7, Lessons** 1, 5, 7, 8, 11, 15, 19 **Unit 8, Lessons** 2, 7

Standards for Mathematical Practice

MP7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Unit 1, Lessons 3, 5, 6, 9, 10–17, 19, 20

Unit 2, Lessons 4–11, 13, 17, 17A

Unit 3, Lessons 2, 4–6, 9, 10, 12

Unit 4, Lessons 4, 5, 7, 8, 10–14

Unit 5, Lessons 3–7, 11–14

Unit 6, Lessons 3, 7, 7A, 8, 10, 11, 13–16

Unit 7, Lessons 2–5, 7, 10, 13, 14, 16, 17, 19

Unit 8, Lessons 2–11, 15

MP8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Unit 1, Lessons 4, 6, 7, 11, 13, 18, 19

Unit 2, Lessons 5, 7, 8, 10, 11

Unit 3, Lessons 4, 9, 11, 12

Unit 4, Lessons 4, 10, 11, 15

Unit 5, Lessons 4, 5, 7, 8, 13

Unit 6, Lessons 1, 2, 4–9, 14, 17

Unit 7, Lesson 16

Unit 8, Lesson 9

Practice Problem Analysis

Teachers may omit the following Practice Problems from the indicated lessons as they address topics beyond the scope of the Tennessee Mathematics Standards.

Unit 2: Introducing Ratios

Lesson	Problem(s)
17	6
20	6

Unit 3: Rates and Percentages

Lesson	Problem(s)
6	4, 6
9	5
11	4
14	5

Unit 7: Rational Numbers

Lesson	Problem(s)
16	5

Unit 8: Data Sets and Distributions

Lesson	Problem(s)
12	6
15	5
17	4

Converting Units

Let’s convert measurements to different units.

Focus

Goals

1. Choose and create a double number line diagram or table to solve problems involving unit conversion.
2. **Language Goal:** Explain how to use a “rate per 1” to solve problems involving unit conversion. **(Speaking and Listening)**
3. Recognize that two measurements of the same object in different units form equivalent ratios.

Coherence

Today

Students work to convert units using ratio reasoning and their choice of representations and strategies, such as double number lines, tables, or multiplication or division to determine equivalent ratios and missing values **(MP1, MP7)**. They practice these skills, checking for accuracy, and think about how to use different tools in some real-world scenarios of measuring out recipe ingredients **(MP2)**.

Previously

In Lesson 17, students compared ratios in which the quantities had different total parts, requiring multiple steps and determining equivalent ratios with common values.

Coming Soon

In Lesson 20, students will revisit Fermi problems from Lesson 1, now fully equipped to reason about them using ratio reasoning.

Rigor

- Students develop **procedural fluency** to convert between units.
- Students **apply** equivalent ratios to converting measurements.

Standards

Addressing

6.RP.A.3d

Use ratio reasoning to convert customary and metric measurement units (within the same system); manipulate and transform units appropriately when multiplying or dividing quantities.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent
	MP2	MP1, MP7, MP6		
6.RP.A.3d	6.RP.A.3d	6.RP.A.3d	6.RP.A.3d	6.RP.A.3d

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For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

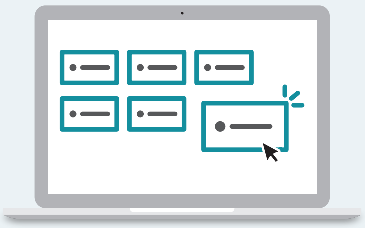
Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- Calculators

Amps | **Featured Activity**

Activity 1
Digital Card Sort

Students match metric distance measurements to their equivalent values by dragging and connecting them on screen.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel confused about the significance of knowing how to convert measurements within the same system as they struggle to reason through conversions **(MP2)**. Ask them to engage in metacognitive functions (thinking about their own thinking process) by asking themselves, "Why are conversion strategies important? Why might one way not be able to be used all the time? How have I been able to overcome difficulties and mental blocks like this in the past?"

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

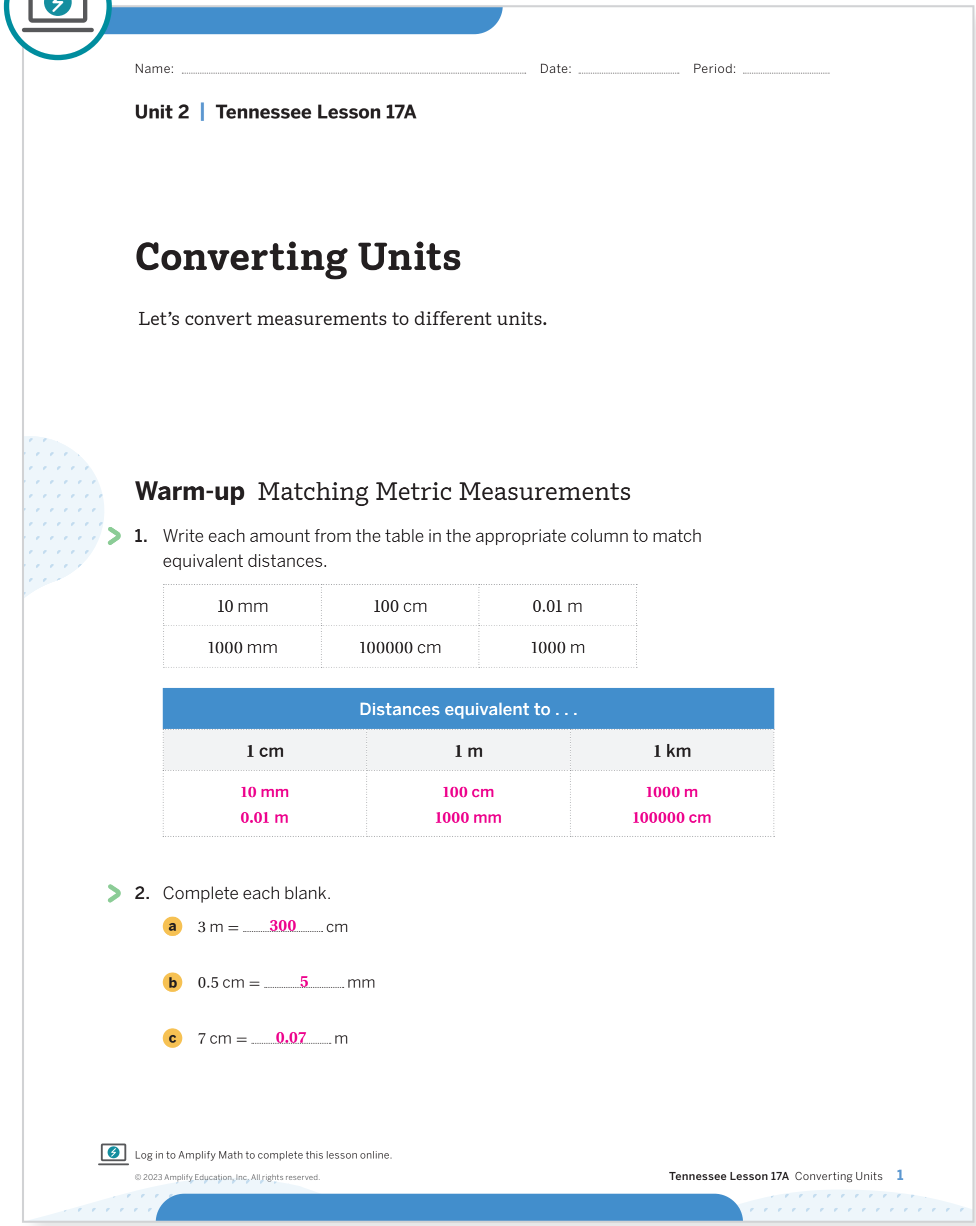
- In **Activity 1**, have pairs divide the workload so that each student converts only half of the ingredients in Problem 1.
- In **Activity 2**, have pairs divide the workload so each student converts only half of the ingredients.

Independent | 5 min

Warm-up Matching Metric Measurements

6.RP.A.3d

Students consider the relationship between metric distance units to match equivalent lengths and convert measurements within the metric system.



Name: ______________ Date: ________ Period: ________

Unit 2 | Tennessee Lesson 17A

Converting Units

Let's convert measurements to different units.

Warm-up Matching Metric Measurements

1. Write each amount from the table in the appropriate column to match equivalent distances.

10 mm	100 cm	0.01 m
1000 mm	100000 cm	1000 m

Distances equivalent to . . .		
1 cm	**1 m**	**1 km**
10 mm 0.01 m	100 cm 1000 mm	1000 m 100000 cm

2. Complete each blank.

a 3 m = 300 cm

b 0.5 cm = 5 mm

c 7 cm = 0.07 m

Log in to Amplify Math to complete this lesson online.

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Tennessee Lesson 17A Converting Units 1

1 Launch

Consider creating pre-cut slips containing each unit for students to paste into the table.

2 Monitor

Help students get started by asking them to provide examples of different contexts where mm, cm, m, and km are used to measure distances.

Look for points of confusion:

- **Writing measurements under the wrong column.** Give an example of an object that would be measured using the given unit. For example, the width of a paperclip for mm, the width of a pencil for cm, the width of two desks for m, and about 2 laps around a football field for km.

Look for productive strategies:

- Using known relationships to help with unknown relationships. For example, knowing there are 10 mm in 1 centimeter and 100 cm in 1 meter to reason that there are 1000 mm in 1 m.

3 Connect

Have students share which measurements match with each column, discussing one unit at a time.

Ask, "How can knowing the unit rate for each length conversion help in Problem 2?" Sample response: If you know the unit rate, you can set up equivalent ratios, a ratio table, or another representation to determine the missing amount.

Highlight that different units can be used to measure the same real-world distance or length. Knowing how to use equivalent ratios can help students to convert from one unit to another.

Power-up

To power up students' ability to categorize or identify appropriate units to measure length, weight, or volume:

Complete each blank using the terms "meters", "grams", or "liters".

1. The length of a classroom can be measured in meters.
2. The volume of water in a tub can be measured using liters.
3. The mass or weight of a pencil can be measured using grams.

Use: Before the Warm-up

Informed by: Performance on Lesson 17, Practice Problem 6 and the Pre-Unit Readiness Assessment, Problems 7 and 8.

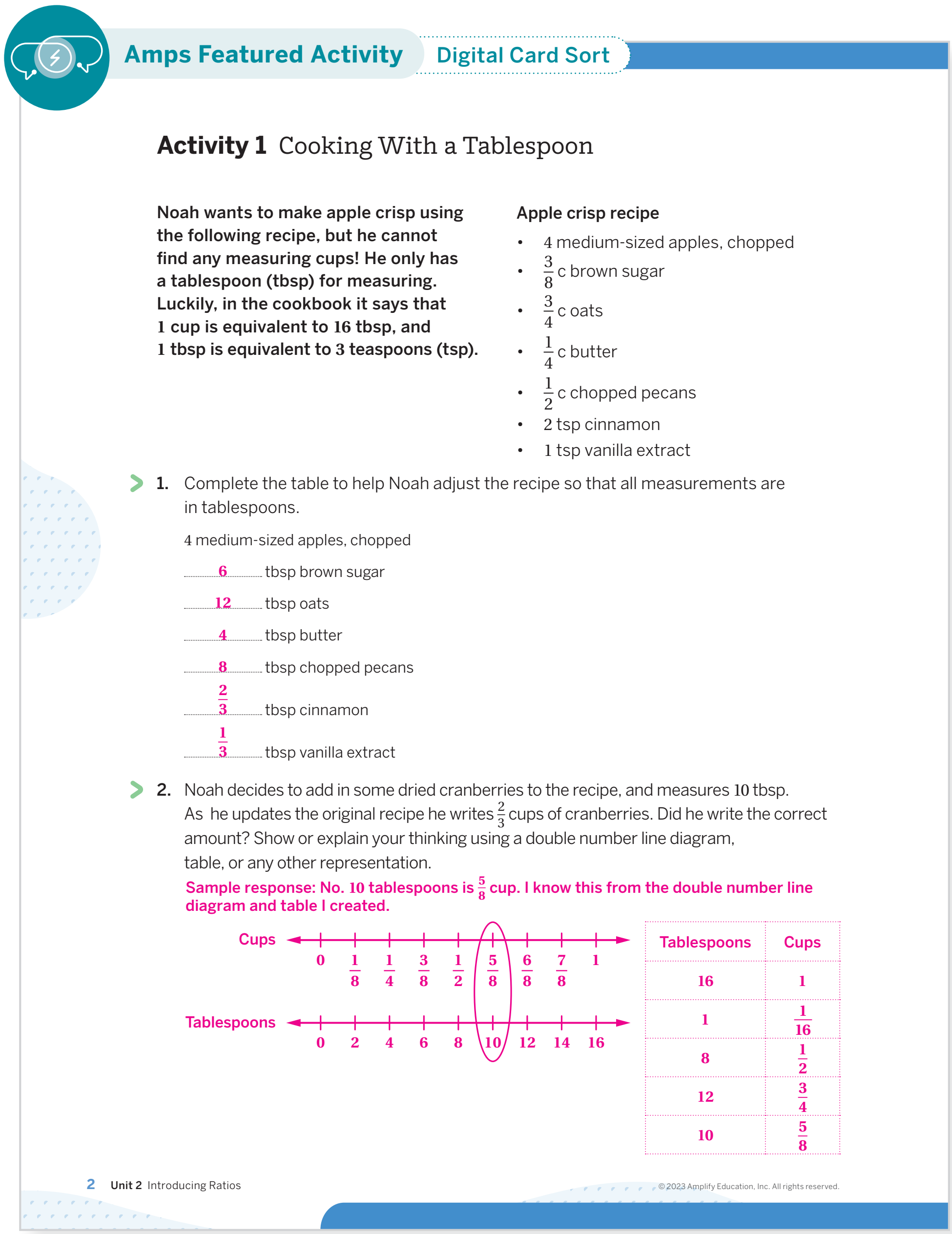
Pairs | 15 min

MP2
6.RP.A.3d

Activity 1 Cooking With a Tablespoon

Students relate measurement conversions within the same measurement system (cups and tablespoons) to equivalent ratios.

Amps Featured Activity Digital Card Sort



Activity 1 Cooking With a Tablespoon

Noah wants to make apple crisp using the following recipe, but he cannot find any measuring cups! He only has a tablespoon (tbsp) for measuring. Luckily, in the cookbook it says that 1 cup is equivalent to 16 tbsp, and 1 tbsp is equivalent to 3 teaspoons (tsp).

Apple crisp recipe

- 4 medium-sized apples, chopped
- $\frac{3}{8}$ c brown sugar
- $\frac{3}{4}$ c oats
- $\frac{1}{4}$ c butter
- $\frac{1}{2}$ c chopped pecans
- 2 tsp cinnamon
- 1 tsp vanilla extract

1. Complete the table to help Noah adjust the recipe so that all measurements are in tablespoons.

   4 medium-sized apples, chopped

   6 tbsp brown sugar

   12 tbsp oats

   4 tbsp butter

   8 tbsp chopped pecans

   $\frac{2}{3}$ tbsp cinnamon

   $\frac{1}{3}$ tbsp vanilla extract

2. Noah decides to add in some dried cranberries to the recipe, and measures 10 tbsp. As he updates the original recipe he writes $\frac{2}{3}$ cups of cranberries. Did he write the correct amount? Show or explain your thinking using a double number line diagram, table, or any other representation.

   Sample response: No. 10 tablespoons is $\frac{5}{8}$ cup. I know this from the double number line diagram and table I created.

Tablespoons	Cups
16	1
1	$\frac{1}{16}$
8	$\frac{1}{2}$
12	$\frac{3}{4}$
10	$\frac{5}{8}$

2 **Unit 2** Introducing Ratios

1 Launch

Use the *Think-Pair-Share* routine to have students work together on the problems.

2 Monitor

Help students get started by asking, "What information do you need to know? What information do you know? How can you use that?"

Look for points of confusion:

- **Saying 0 tbsp for cinnamon and vanilla extract because they are less than one tablespoon.** Ask, "Could a fraction of a tablespoon be used? How could you determine that fraction?"
- **Having trouble explaining the conversion method used for Problem 2.** Ask, "How might you use a double number line or table?"

Look for productive strategies:

- Recognizing when a measurement would be less than one tablespoon and when it would be more.
- Writing the given conversions as unit ratios and using those to determine necessary equivalent ratios **(MP2)**.

3 Connect

Have students share their conversion strategies, focusing on when they converted smaller units to larger units and vice versa. Record examples as you find them helpful.

Ask, "How did you use ratios specifically in your conversions?"

Highlight that within the same measurement system, it is generally true that each larger unit corresponds to a whole number of smaller units. If students can determine the unit ratio, they can then set up equivalent ratios to determine larger or smaller amounts of a given quantity, even when those values themselves might be fractional or decimal amounts.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Consider bringing in a set of measuring cups that show how 1 cup, 1 tbsp, and 1 tsp compare in size to one another. Consider demonstrating, using water or another substance, how 3 tbsp is equivalent to 1 tsp, and how 1 cup is equivalent to 16 tbsp.

Accessibility: *Math Enrichment*

Have students complete the following problem:
How could you adjust the table you created in Problem 1 so that the measurements in tablespoons for every ingredient are whole numbers?
Sample response: Triple the recipe.

Math Language Development

MLR3: Critique, Correct, Clarify

Present an incorrect solution and explanation. For example, "Noah used zero cups of cinnamon because 2 tsp is less than 1 tbsp." Ask students to critique the solution and reasoning, propose a corrected solution, and clarify the reasoning they use.

English Learners

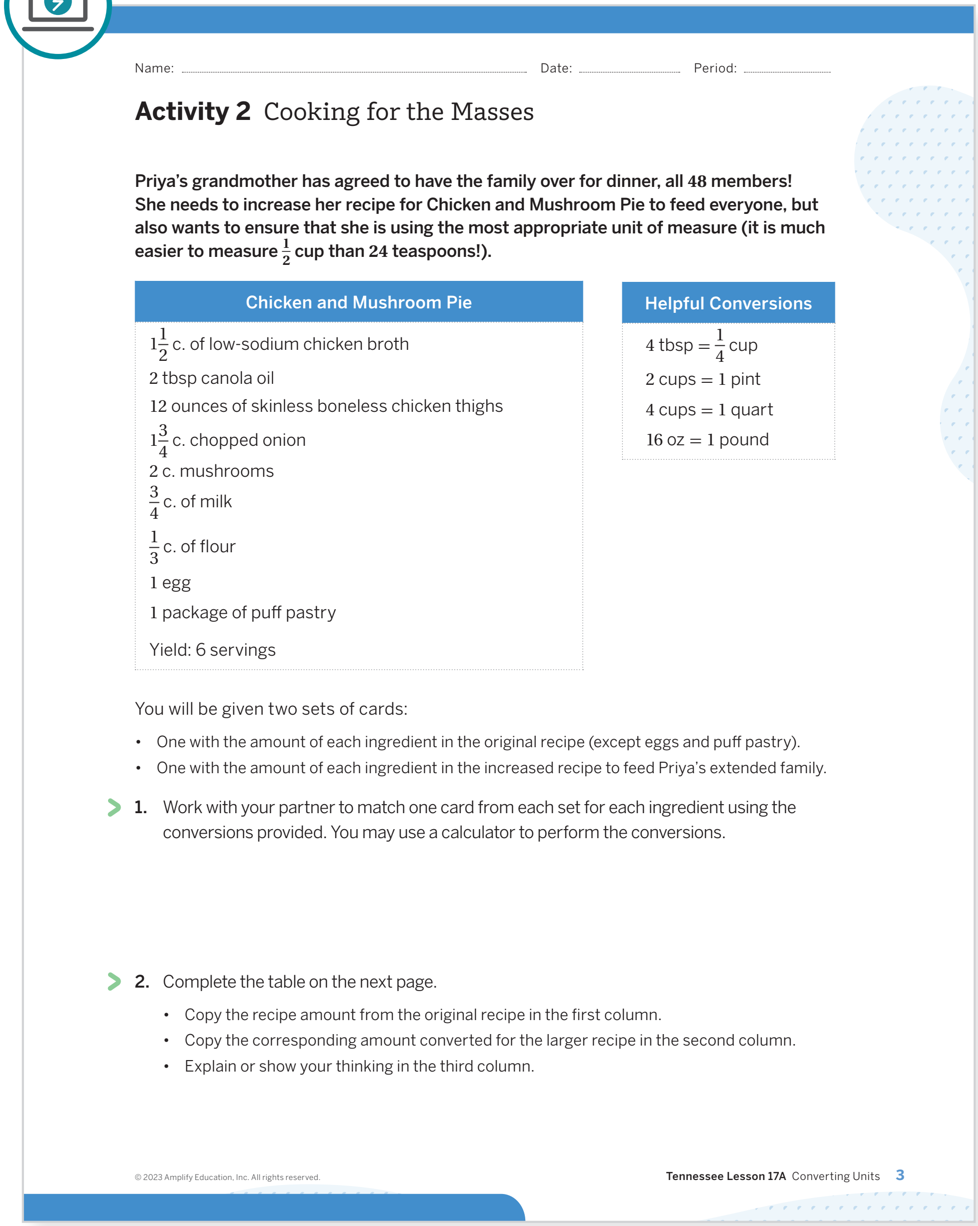
Encourage students to refer to the class anchor chart to support their use of appropriate mathematical language in their improved response.

Independent | 15 min

MP1, MP7, MP6
6.RP.A.3d

Activity 2 Cooking for the Masses

Students extend ratio reasoning to convert a recipe, reinforcing their understanding of “how much per 1.”



Name: ______ Date: ______ Period: ______

Activity 2 Cooking for the Masses

Priya's grandmother has agreed to have the family over for dinner, all 48 members! She needs to increase her recipe for Chicken and Mushroom Pie to feed everyone, but also wants to ensure that she is using the most appropriate unit of measure (it is much easier to measure $\frac{1}{2}$ cup than 24 teaspoons!).

Chicken and Mushroom Pie
$1\frac{1}{2}$ c. of low-sodium chicken broth
2 tbsp canola oil
12 ounces of skinless boneless chicken thighs
$1\frac{3}{4}$ c. chopped onion
2 c. mushrooms
$\frac{3}{4}$ c. of milk
$\frac{1}{3}$ c. of flour
1 egg
1 package of puff pastry
Yield: 6 servings

Helpful Conversions

4 tbsp = $\frac{1}{4}$ cup
2 cups = 1 pint
4 cups = 1 quart
16 oz = 1 pound

You will be given two sets of cards:

- One with the amount of each ingredient in the original recipe (except eggs and puff pastry).
- One with the amount of each ingredient in the increased recipe to feed Priya's extended family.

1. Work with your partner to match one card from each set for each ingredient using the conversions provided. You may use a calculator to perform the conversions.

2. Complete the table on the next page.
   - Copy the recipe amount from the original recipe in the first column.
   - Copy the corresponding amount converted for the larger recipe in the second column.
   - Explain or show your thinking in the third column.

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Tennessee Lesson 17A Converting Units 3

1 Launch

Keep students in the same pairs and distribute pre-cut cards from the Activity 2 PDF to every pair. Note that ounces is a unit of weight here, not volume (which would be fluid ounces). Consider also discussing why some of the recipe measurements are in ounces and some are in cups. Provide access to calculators as needed.

2 Monitor

Help students get started by asking, “How much of each ingredient will Priya's grandmother need to feed 48 people?”

Look for points of confusion:

- **Focusing more on matching than converting.** Explain that using estimation can be helpful in some examples, and may narrow options, but calculations should be done to check or determine final actual matches **(MP6)**.
- **Not knowing how to make the conversions when values are not factors or multiples.** Ask, “Could you set up a ratio box for two equivalent ratios? What operation do you need to do?” Then remind them they can use a calculator.

Look for productive strategies:

- Using estimation strategies to eliminate unreasonably large or small amounts **(MP1)**.
- Knowing which measurements to multiply and which to divide, and recognizing the same operation can be applied to every same type of conversion **(MP7)**.
- Being able to convert amounts in multiple ways — using a calculator, setting up a double number line or table, or using mental math strategies.

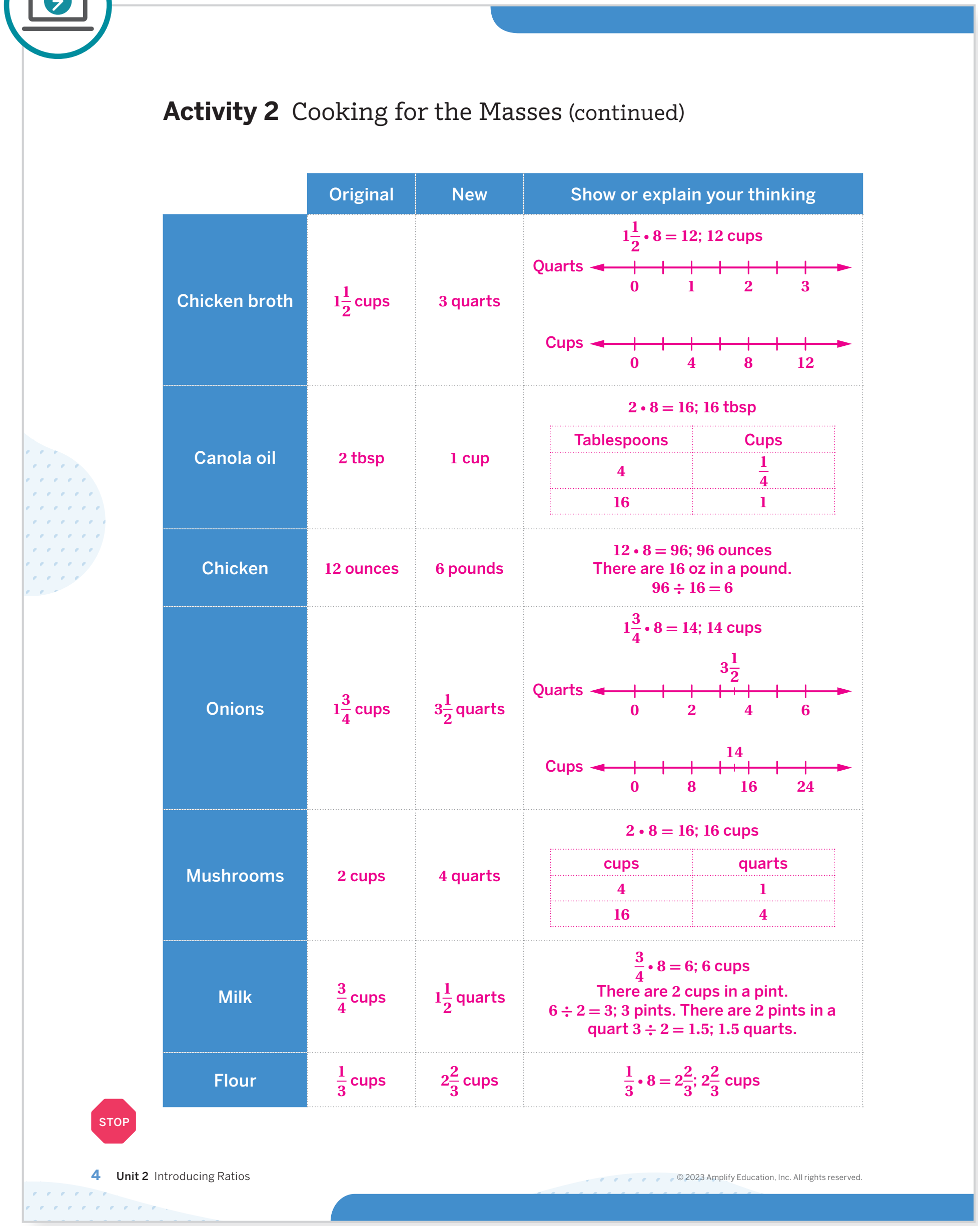
Activity 2 continued ›

Pairs | 15 min

Activity 2 Cooking for the Masses (continued)

MP1, MP7, MP6
6.RP.A.3d

Students extend ratio reasoning to convert a recipe, reinforcing their understanding of “how much per 1.”



Activity 2 Cooking for the Masses (continued)

	Original	New	Show or explain your thinking
Chicken broth	$1\frac{1}{2}$ cups	3 quarts	$1\frac{1}{2} \cdot 8 = 12$; 12 cups Quarts: 0, 1, 2, 3 Cups: 0, 4, 8, 12
Canola oil	2 tbsp	1 cup	$2 \cdot 8 = 16$; 16 tbsp Tablespoons: 4, 16 Cups: $\frac{1}{4}$, 1
Chicken	12 ounces	6 pounds	$12 \cdot 8 = 96$; 96 ounces There are 16 oz in a pound. $96 \div 16 = 6$
Onions	$1\frac{3}{4}$ cups	$3\frac{1}{2}$ quarts	$1\frac{3}{4} \cdot 8 = 14$; 14 cups Quarts: 0, 2, 4, 6; $3\frac{1}{2}$ Cups: 0, 8, 16, 24; 14
Mushrooms	2 cups	4 quarts	$2 \cdot 8 = 16$; 16 cups cups: 4, 16 quarts: 1, 4
Milk	$\frac{3}{4}$ cups	$1\frac{1}{2}$ quarts	$\frac{3}{4} \cdot 8 = 6$; 6 cups There are 2 cups in a pint. $6 \div 2 = 3$; 3 pints. There are 2 pints in a quart $3 \div 2 = 1.5$; 1.5 quarts.
Flour	$\frac{1}{3}$ cups	$2\frac{2}{3}$ cups	$\frac{1}{3} \cdot 8 = 2\frac{2}{3}$; $2\frac{2}{3}$ cups

STOP

4 Unit 2 Introducing Ratios

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3 Connect

Display a blank table for showing correct matches.

Have students share one match at a time and the strategies or representations they used to make the conversions. If time allows, have others share different thinking or representations for the same result.

Ask:

- “Did you use the same conversion strategy for each ingredient? Why or why not?” Answers may vary.
- “How do you know whether to multiply or divide?” I used the unit ratios to see which quantity had a 1 and compared that to what I was given and needed to know.

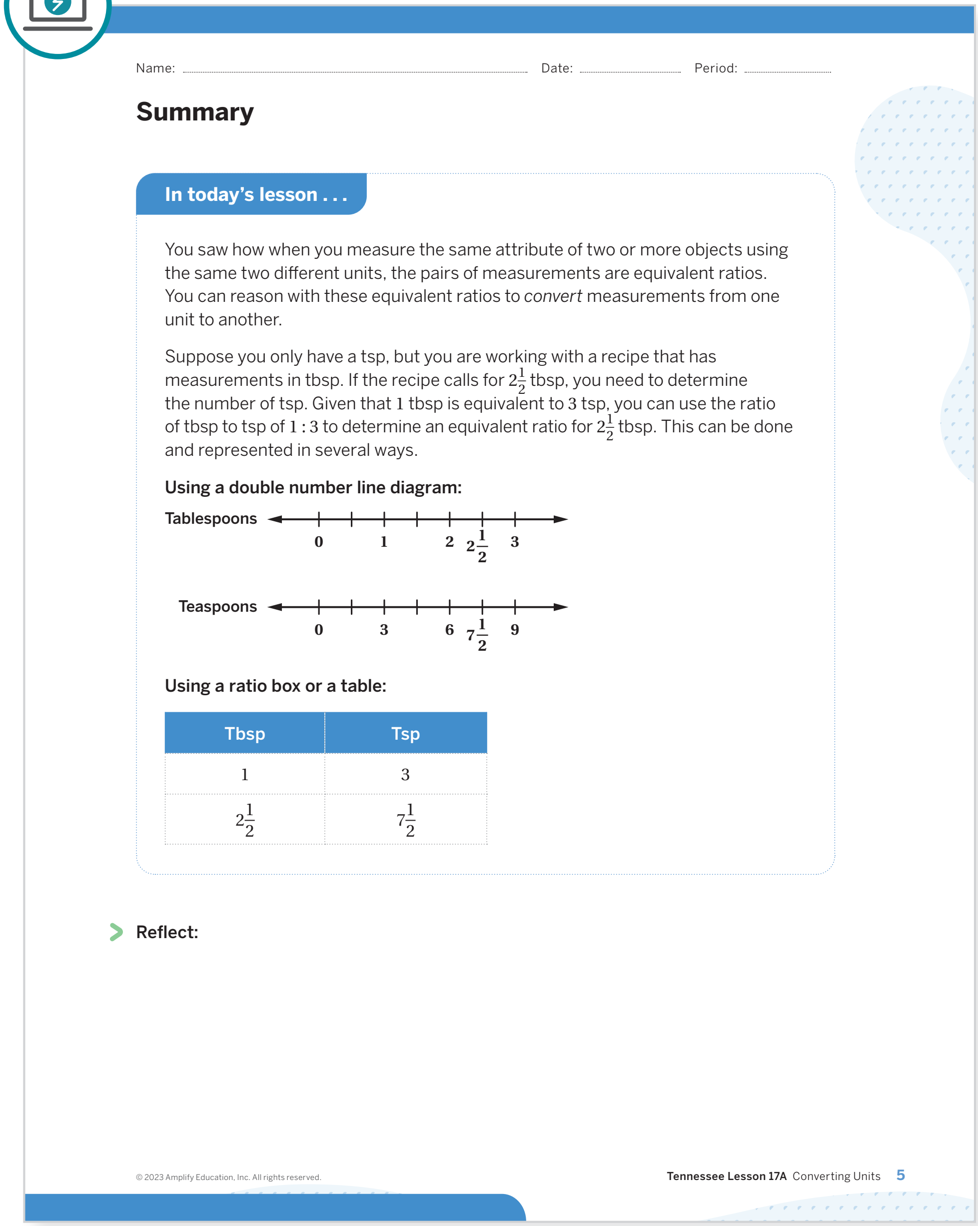
Highlight that the conversions given were unit ratios telling students “how much per 1,” which they have seen are useful tools in determining any equivalent ratio. However, in this case, the unit ratios did not have a 1 corresponding to the same units that were given in the recipe, so students could not just multiply to determine the equivalent conversions. All of the tools and strategies developed in this unit could be helpful in visualizing the relationships, and once students determined the calculation necessary (division, or multiplication by a unit fraction or decimal), then the same calculation could be used for every conversion between the same two units.

Whole Class | 5 min

Summary

6.RP.A.3d

Review and synthesize how using a given conversion for “how much per 1” to write equivalent ratios relates to converting within the same system of measurements.



Name: ______ Date: ______ Period: ______

Summary

In today’s lesson . . .

You saw how when you measure the same attribute of two or more objects using the same two different units, the pairs of measurements are equivalent ratios. You can reason with these equivalent ratios to *convert* measurements from one unit to another.

Suppose you only have a tsp, but you are working with a recipe that has measurements in tbsp. If the recipe calls for $2\frac{1}{2}$ tbsp, you need to determine the number of tsp. Given that 1 tbsp is equivalent to 3 tsp, you can use the ratio of tbsp to tsp of $1:3$ to determine an equivalent ratio for $2\frac{1}{2}$ tbsp. This can be done and represented in several ways.

Using a double number line diagram:

Using a ratio box or a table:

Tbsp	Tsp
1	3
$2\frac{1}{2}$	$7\frac{1}{2}$

Reflect:

Tennessee Lesson 17A Converting Units 5

Synthesize

Display the table relating tablespoons and teaspoons from the Summary.

Ask:

- “How does knowing ‘how much per 1’ help you convert between units of measurement?” $1:3$ is the unit ratio, so I can divide or multiply depending upon whether I need a larger or smaller amount.
- “How do the pairs of numbers in the table represent equivalent ratios? How can you use equivalent ratios to convert between units of measurement?” I can use the unit ratio to multiply 11 to get 2.5 and 3 to get 7.5.
- “Are any of the conversion strategies you saw today more efficient? Less efficient? Explain.” Answers may vary.

Highlight that two measurements of the same object in different units form equivalent ratios, and students can use all of their familiar tools (tables, double number line diagrams) when thinking about converting units of measure. If they know a rate of “how much per 1” that relates the two units, they can use it to convert one measurement to the other by multiplication or division, regardless of the values or whether the units come from the same measurement system or different measurement systems.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- “What is a strategy you can use to convert measurements from one unit to another?”
- “How do equivalent ratios help you when converting measurements?”

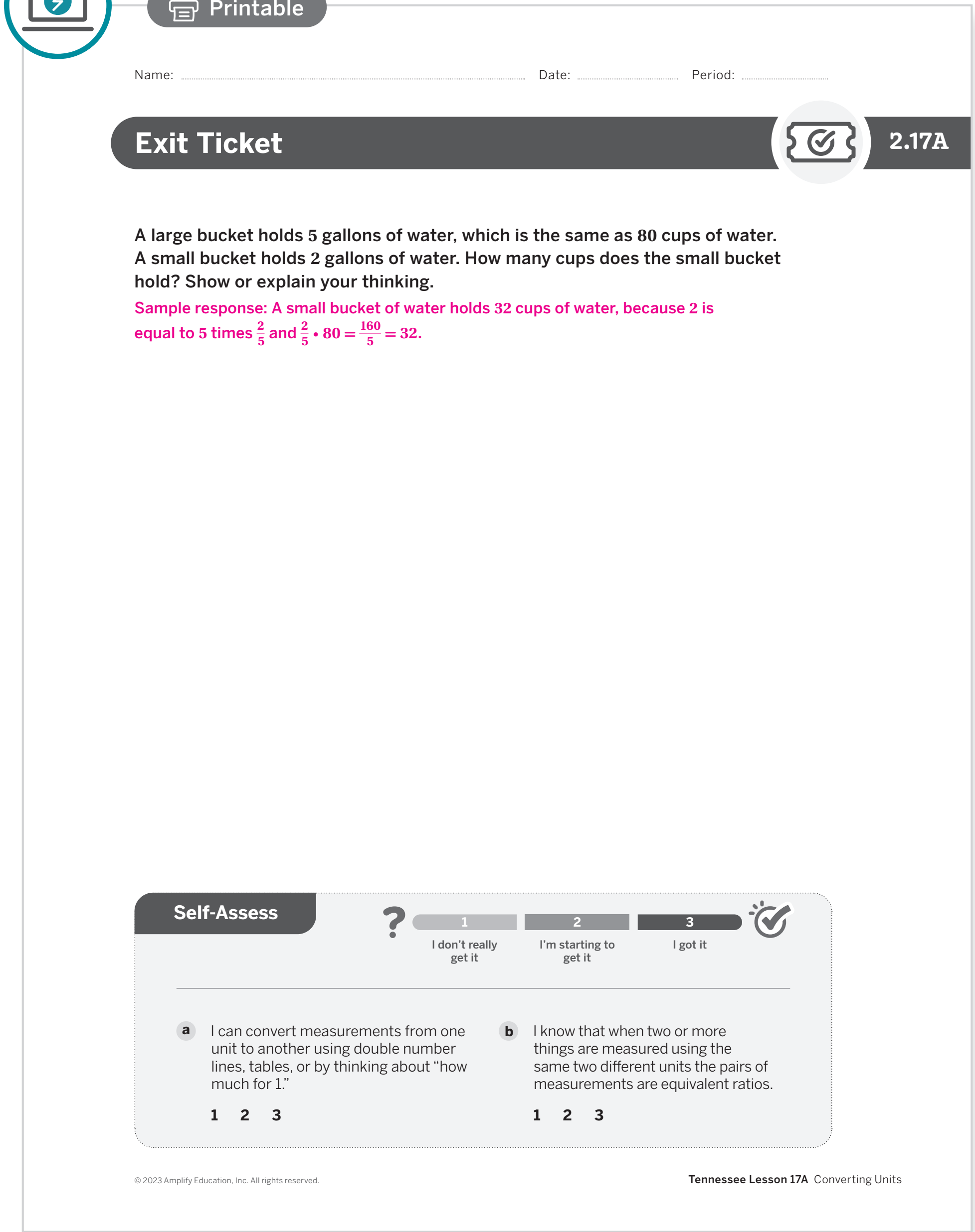
Independent | 5 min

Exit Ticket

6.RP.A.3d

Students demonstrate their understanding of unit conversions by using equivalent ratios to convert gallons to cups.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.17A

A large bucket holds 5 gallons of water, which is the same as 80 cups of water. A small bucket holds 2 gallons of water. How many cups does the small bucket hold? Show or explain your thinking.

Sample response: A small bucket of water holds 32 cups of water, because 2 is equal to 5 times $\frac{2}{5}$ and $\frac{2}{5} \cdot 80 = \frac{160}{5} = 32$.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can convert measurements from one unit to another using double number lines, tables, or by thinking about "how much for 1."

1 2 3

b. I know that when two or more things are measured using the same two different units the pairs of measurements are equivalent ratios.

1 2 3

Tennessee Lesson 17A Converting Units

Success looks like . . .

- **Goal:** Choosing and creating a double number line diagram or table to solve problems involving unit conversion.
  - » Representing comparison between cups and gallons using a double number line.
- **Language Goal:** Explaining how to use a "rate per 1" to solve problems involving unit conversion. **(Speaking and Listening)**
  - » Determining the unit rate of 16 cups per gallon.
- **Goal:** Recognizing that two measurements of the same object in different units form equivalent ratios.

Suggested next steps

If students are unsure how to make the conversion, consider:

- Having students create a double number line or a table, and asking, "How can you use multiplication or division to create an equivalent ratio?"
- Reviewing Problem 2 of Activity 1.
- Assigning Practice Problem 2.

If students are stalled thinking they need to know "how much per 1" for cups or gallons, consider:

- Asking, "Why do you think that? What information can you use from the problem to help you determine that equivalency?"
- Reviewing the Summary from Lesson 14 about the information necessary to determine a missing value, and how to use that to set up equivalent ratios.

Professional Learning

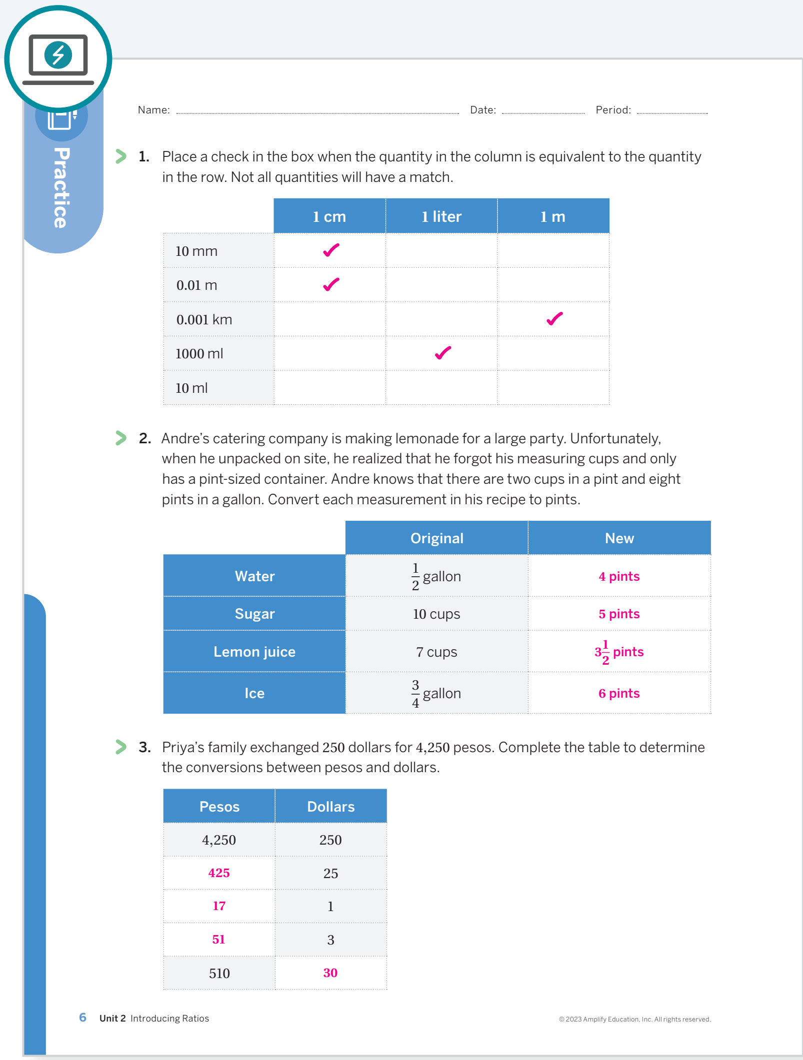
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on the recipe conversions?
- During the discussions, how did you encourage each student to share their understanding? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Place a check in the box when the quantity in the column is equivalent to the quantity in the row. Not all quantities will have a match.

	1 cm	1 liter	1 m
10 mm	✓		
0.01 m	✓		
0.001 km			✓
1000 ml		✓	
10 ml			

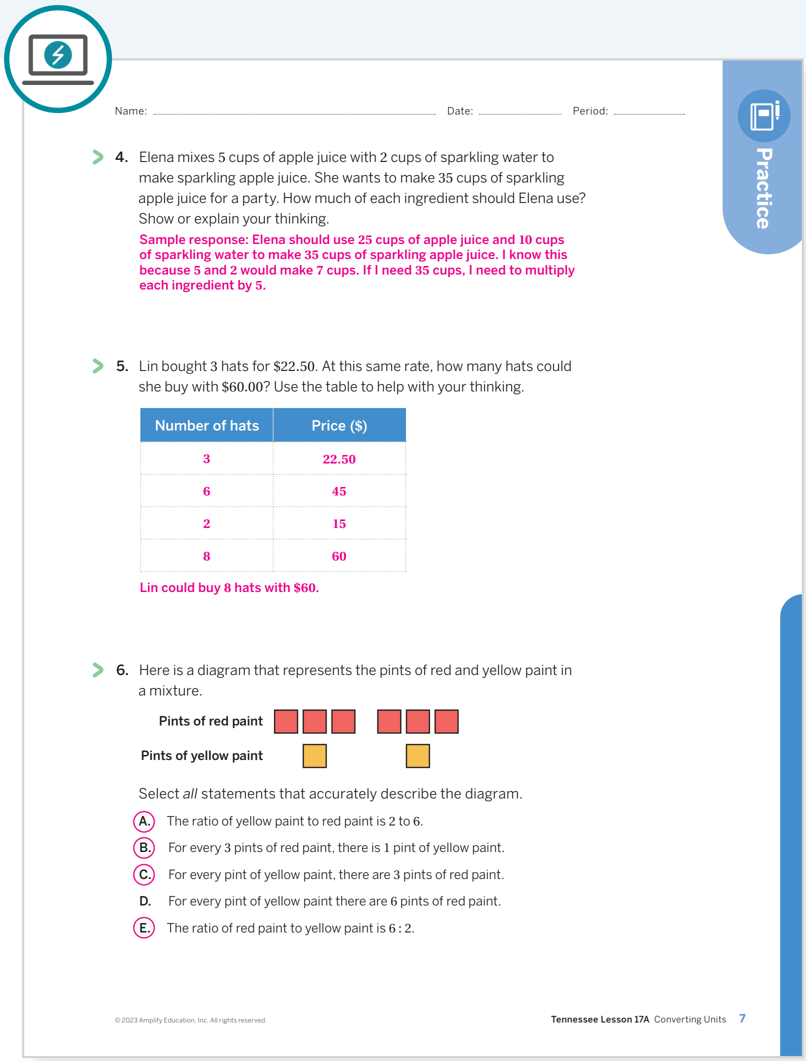
2. Andre's catering company is making lemonade for a large party. Unfortunately, when he unpacked on site, he realized that he forgot his measuring cups and only has a pint-sized container. Andre knows that there are two cups in a pint and eight pints in a gallon. Convert each measurement in his recipe to pints.

	Original	New
Water	$\frac{1}{2}$ gallon	**4 pints**
Sugar	10 cups	**5 pints**
Lemon juice	7 cups	**$3\frac{1}{2}$ pints**
Ice	$\frac{3}{4}$ gallon	**6 pints**

3. Priya's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Pesos	Dollars
4,250	250
425	25
17	1
51	3
510	**30**

6 **Unit 2** Introducing Ratios



Practice

Name: ________ Date: ________ Period: ________

4. Elena mixes 5 cups of apple juice with 2 cups of sparkling water to make sparkling apple juice. She wants to make 35 cups of sparkling apple juice for a party. How much of each ingredient should Elena use? Show or explain your thinking.

Sample response: Elena should use 25 cups of apple juice and 10 cups of sparkling water to make 35 cups of sparkling apple juice. I know this because 5 and 2 would make 7 cups. If I need 35 cups, I need to multiply each ingredient by 5.

5. Lin bought 3 hats for $22.50. At this same rate, how many hats could she buy with $60.00? Use the table to help with your thinking.

Number of hats	Price ($)
3	**22.50**
6	**45**
2	**15**
8	**60**

Lin could buy 8 hats with $60.

6. Here is a diagram that represents the pints of red and yellow paint in a mixture.

Pints of red paint 🟥🟥🟥 🟥🟥🟥

Pints of yellow paint 🟨 🟨

Select *all* statements that accurately describe the diagram.

[✓ **A.**] The ratio of yellow paint to red paint is 2 to 6.
[✓ **B.**] For every 3 pints of red paint, there is 1 pint of yellow paint.
[✓ **C.**] For every pint of yellow paint, there are 3 pints of red paint.
D. For every pint of yellow paint there are 6 pints of red paint.
[✓ **E.**] The ratio of red paint to yellow paint is 6 : 2.

Tennessee Lesson 17A Converting Units 7

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Warm-up	6.RP.A.3d	1
	2	Activity 1	6.RP.A.3d	2
	3	Activity 1	6.RP.A.3d	2
Spiral	4	Unit 2 Lesson 15	6.RP.A.3	2
	5	Unit 2 Lesson 11	6.RP.A.3	2
Formative ⚡	6	Unit 2 Lesson 20	6.RP.A.1	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Reasoning About Solving Equations

Let's solve more equations.

Focus

Goals

1. **Language Goal:** Solve equations of the form $x - p = q$ or $\frac{x}{p} = q$ and explain the solution method. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret problems in context and explain how an equation and its solution corresponds to the context. **(Speaking and Listening)**

Coherence

Today

Students solve equations of the form $x - p = q$ or $\frac{x}{p} = q$. They reason about unknown quantities by applying the Properties of Equality and solve real-world problems by solving equations. Students make sense of problems as they create a scenario that corresponds with an equation, and interpret their solution in context **(MP1)**.

Previously

In Lesson 7, students solved equations of the forms $x + p = q$ and $px = q$. They developed procedural fluency in representing and solving equations that involve whole numbers, fractions, and decimal values.

Coming Soon

In Lesson 8, students will further explore solving equations of the form $px = q$ with fractional values, extending their understanding of fractions as division.

Rigor

- Students build **procedural fluency** writing and solving equations with variables.

Standards

Addressing

6.EE.B.7

Solve real-world and mathematical problems by writing and solving one-step equations of the form $x + p = q$, $px = q$, $x - p = q$, and $\frac{x}{p} = q$ for cases in which p, q, and x are all nonnegative rational numbers and $p \neq 0$.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent
MP7	MP7	MP1		
6.EE.B.7	6.EE.B.7	6.EE.B.7	6.EE.B.7	6.EE.B.7

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For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

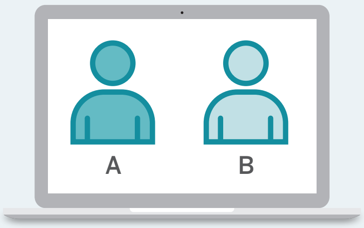
- *equation*
- *variable*

Amps | **Featured Activity**

Activity 1

Interactive Table

Students can enter values into an interactive table and solve equations simultaneously.



Building Math Identity and Community

Connecting to Mathematical Practices

Students who are more confident with the mathematical topic of this lesson may be able to lead discussions within their groups in Activity 2 **(MP1)**. Remind students to "step up" if they have something to add to the conversation, but also to "step back" to give other voices a chance to share.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

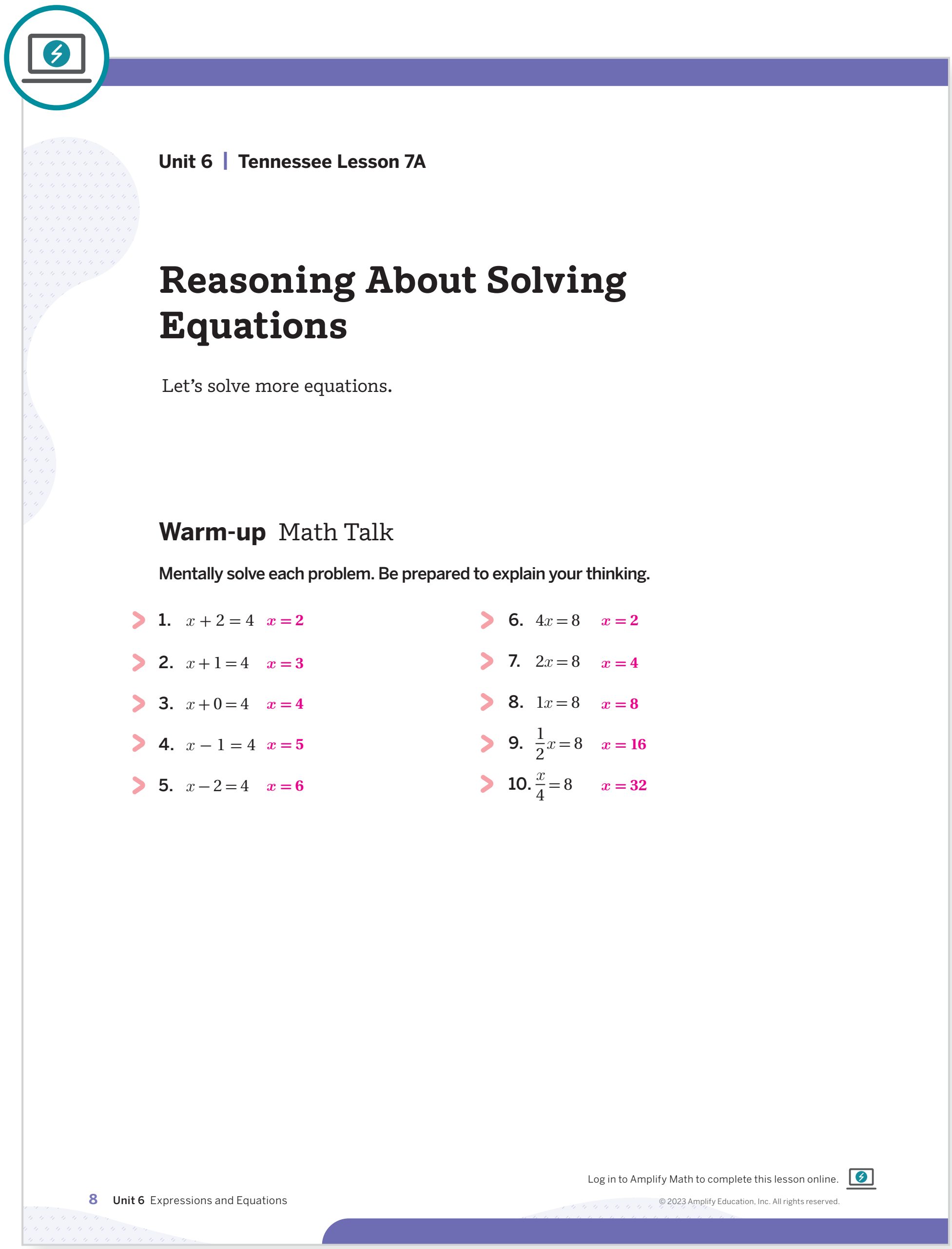
- In the **Warm-up**, the odd numbered problems may be omitted.
- In **Activity 1**, the last row may be omitted.
- In **Activity 2**, consider having students match two equations, and omit Problem 3.

Independent | 5 min

MP7
6.EE.B.7

Warm-up Math Talk

Students mentally solve equations to see how the structure of an equation affects the solution of the equation **(MP7)**.



Unit 6 | **Tennessee Lesson 7A**

Reasoning About Solving Equations

Let's solve more equations.

Warm-up Math Talk

Mentally solve each problem. Be prepared to explain your thinking.

1. $x+2=4$ $x=2$
2. $x+1=4$ $x=3$
3. $x+0=4$ $x=4$
4. $x-1=4$ $x=5$
5. $x-2=4$ $x=6$
6. $4x=8$ $x=2$
7. $2x=8$ $x=4$
8. $1x=8$ $x=8$
9. $\frac{1}{2}x=8$ $x=16$
10. $\frac{x}{4}=8$ $x=32$

Log in to Amplify Math to complete this lesson online.

8 **Unit 6** Expressions and Equations

1 Launch

Conduct the *Math Talk* routine.

2 Monitor

Help students get started by having them use any strategy, such as using a guess-and-check method or applying the Properties of Equality, to determine a solution.

Look for points of confusion:

- **Struggling to solve any problem.** Remind students of inverse operations and how to isolate the variable.
- **Struggling to complete Problems 4 and 5.** Say, "One/two less than a number is four. What is that number?"
- **Struggling to complete Problems 9 and 10.** Say, "Half of a number is 8. What is that number?" or "A number divided by four is eight. What is that number?"

Look for productive strategies:

- For Problems 1–5, noticing that the solution is one more than the previous solution because the number added to the variable is one number less than the previous problem.
- For Problems 6–10, noticing that the solution is double the previous solution because the number being multiplied by the variable is half of the previous coefficient.

3 Connect

Have students share their responses, strategies, and what they noticed about the problems. Remind students that, for Problem 10, $\frac{x}{4}$ can be written as $x \div 4$.

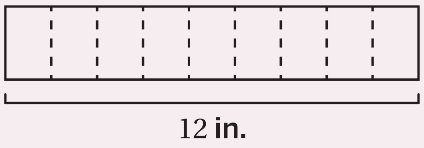
Ask, "How do you think you can solve equations with subtraction or division, such as Problems 4, 5, and 10?"

Highlight that for any equation, students can use inverse operations and the Properties of Equality to isolate the variable and determine the solution to the equation. Write and display Problem 5 and 10. Model how these equations can be solved step by step. Demonstrate how students can check the solution by substituting the answer into the original equation.

Power-up

To power up students' ability to use a diagram to assist them in writing an expression to represent a real-world scenario, have students complete:

Lin is making mini loaves and wants to divide a dough with a length of 12 in. into 9 equal-sized pieces, as shown by the diagram.



1. Write an expression that can be used to determine the length of each piece. $12 \div 9$
2. What should the length of each smaller piece? $1\frac{1}{3}$ in.

Use: Before the Warm-up

Informed by: Performance on Lesson 7, Practice Problem 6

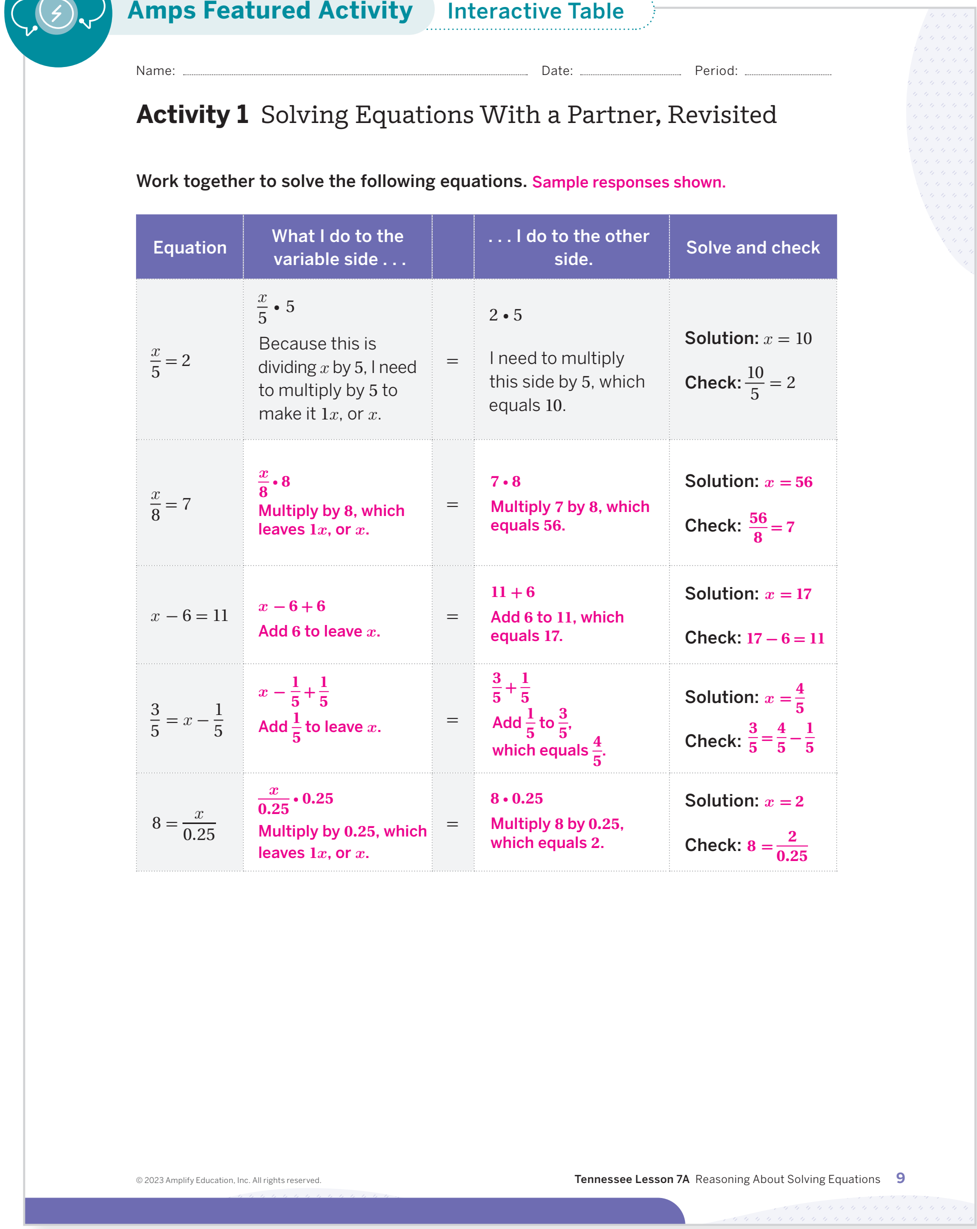
Pairs | 15 min

MP7
6.EE.B.7

Activity 1 Solving Equations With a Partner, Revisited

Students solve equations of the forms $x - p = q$ and $\frac{x}{p} = q$, making use of structure when solving equations of different forms.

Amps Featured Activity Interactive Table



Name: Date: Period:

Activity 1 Solving Equations With a Partner, Revisited

Work together to solve the following equations. Sample responses shown.

Equation	What I do to the variable side . . .		. . . I do to the other side.	Solve and check
$\frac{x}{5} = 2$	$\frac{x}{5} \cdot 5$ Because this is dividing x by 5, I need to multiply by 5 to make it $1x$, or x.	$=$	$2 \cdot 5$ I need to multiply this side by 5, which equals 10.	Solution: $x = 10$ Check: $\frac{10}{5} = 2$
$\frac{x}{8} = 7$	$\frac{x}{8} \cdot 8$ Multiply by 8, which leaves $1x$, or x.	$=$	$7 \cdot 8$ Multiply 7 by 8, which equals 56.	Solution: $x = 56$ Check: $\frac{56}{8} = 7$
$x - 6 = 11$	$x - 6 + 6$ Add 6 to leave x.	$=$	$11 + 6$ Add 6 to 11, which equals 17.	Solution: $x = 17$ Check: $17 - 6 = 11$
$\frac{3}{5} = x - \frac{1}{5}$	$x - \frac{1}{5} + \frac{1}{5}$ Add $\frac{1}{5}$ to leave x.	$=$	$\frac{3}{5} + \frac{1}{5}$ Add $\frac{1}{5}$ to $\frac{3}{5}$, which equals $\frac{4}{5}$.	Solution: $x = \frac{4}{5}$ Check: $\frac{3}{5} = \frac{4}{5} - \frac{1}{5}$
$8 = \frac{x}{0.25}$	$\frac{x}{0.25} \cdot 0.25$ Multiply by 0.25, which leaves $1x$, or x.	$=$	$8 \cdot 0.25$ Multiply 8 by 0.25, which equals 2.	Solution: $x = 2$ Check: $8 = \frac{2}{0.25}$

Tennessee Lesson 7A Reasoning About Solving Equations 9

1 Launch

Tell students that this activity is similar to the previous lesson and review the routine for completing the activity in pairs. Remind students that one partner will complete the “What I do to the variable side” column, while the other partner completes the “I do to the other side” column.

2 Monitor

Help students get started by asking how the equations differ from the previous lesson. Then ask students how they can isolate the variable x.

Look for points of confusion:

- **Struggling to solve an equation.** Remind students of inverse operations. Tell students that multiplication and division, and addition and subtraction, are inverse operations.
- **Struggling to solve the equations in the first or last rows.** Encourage students to rewrite each equation using the symbol ÷.

Look for productive strategies:

- Using mental math to check the reasonableness of a solution.

3 Connect

Have students share their strategies and solutions for each equation, and how the solution check shows the relationship between multiplication and division, and addition and subtraction **(MP7)**.

Highlight that to solve equations of the forms $x - p = q$ and $\frac{x}{p} = q$, students can use inverse operations and the Properties of Equality to keep the equation balanced and isolate the variable. The operation performed on the variable side is also done to the other side to maintain equality.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity in which they can enter values into an interactive table and solve equations simultaneously.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing only the first two rows of the table.

Extension: *Math Enrichment*

Have students choose one equation and create a word problem that could be represented by that equation. For an added challenge, ask them to choose an equation that includes fractional or decimal values.

Math Language Development

MLR8: Discussion Supports

During the Launch, display these prompts that partners coils ask each other as they progress through the activity.

- “Can you tell me why you _____ by _____?” (use for multiplication/division)
- “Can you tell me why you _____ to/from each side?” (use for addition and/subtraction)

English Learners

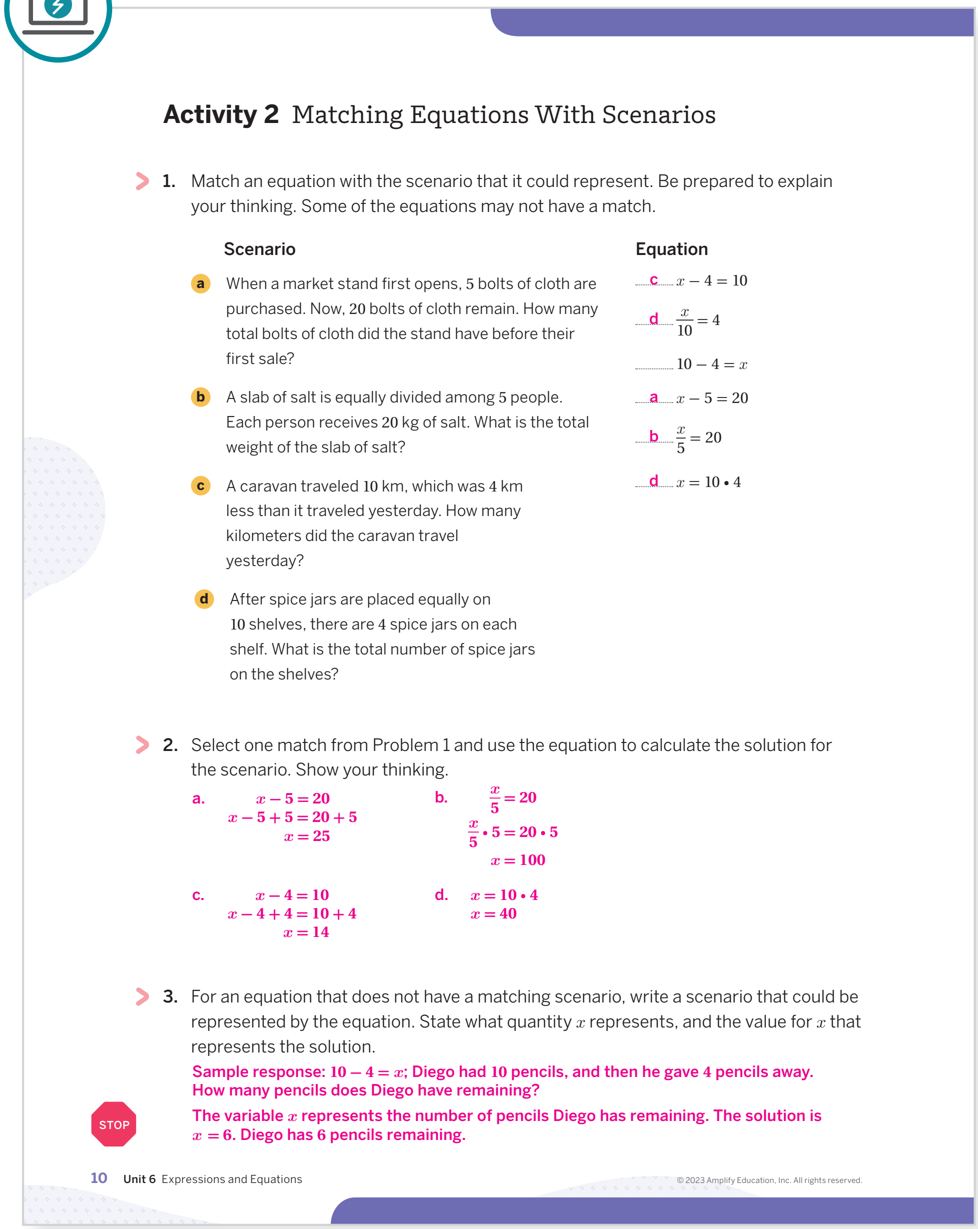
Annotate the first equation $\frac{x}{5} = 2$ by writing “variable side” next to the side that contains the variable so that students can connect this phrase to the algebraic representation. Draw an arrow that points to the variable x.

Pairs | 15 min

MP1
6.EE.B.7

Activity 2 Matching Equations With Scenarios

Students match equations to scenarios to see how equations can help determine a solution to problems in context.



Activity 2 Matching Equations With Scenarios

1. Match an equation with the scenario that it could represent. Be prepared to explain your thinking. Some of the equations may not have a match.

Scenario

- **a** When a market stand first opens, 5 bolts of cloth are purchased. Now, 20 bolts of cloth remain. How many total bolts of cloth did the stand have before their first sale?
- **b** A slab of salt is equally divided among 5 people. Each person receives 20 kg of salt. What is the total weight of the slab of salt?
- **c** A caravan traveled 10 km, which was 4 km less than it traveled yesterday. How many kilometers did the caravan travel yesterday?
- **d** After spice jars are placed equally on 10 shelves, there are 4 spice jars on each shelf. What is the total number of spice jars on the shelves?

Equation

c $x - 4 = 10$

d $\frac{x}{10} = 4$

___ $10 - 4 = x$

a $x - 5 = 20$

b $\frac{x}{5} = 20$

d $x = 10 \cdot 4$

2. Select one match from Problem 1 and use the equation to calculate the solution for the scenario. Show your thinking.

a. $$\begin{aligned} x - 5 &= 20 \\ x - 5 + 5 &= 20 + 5 \\ x &= 25 \end{aligned}$$

b. $$\begin{aligned} \frac{x}{5} &= 20 \\ \frac{x}{5} \cdot 5 &= 20 \cdot 5 \\ x &= 100 \end{aligned}$$

c. $$\begin{aligned} x - 4 &= 10 \\ x - 4 + 4 &= 10 + 4 \\ x &= 14 \end{aligned}$$

d. $$\begin{aligned} x &= 10 \cdot 4 \\ x &= 40 \end{aligned}$$

3. For an equation that does not have a matching scenario, write a scenario that could be represented by the equation. State what quantity x represents, and the value for x that represents the solution.

Sample response: $10 - 4 = x$; Diego had 10 pencils, and then he gave 4 pencils away. How many pencils does Diego have remaining?

The variable x represents the number of pencils Diego has remaining. The solution is $x = 6$. Diego has 6 pencils remaining.

STOP

10 **Unit 6** Expressions and Equations

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Remind students that not all the equations may be matched.

2 Monitor

Help students get started by asking them to underline any values and key words that will help them determine the matching equation.

Look for points of confusion:

- **Struggling to match an equation, or matching the wrong equation.** Encourage students to draw a diagram representing the scenario.

Look for productive strategies:

- Noticing that Scenario d can be matched with two equations and solving for the total using either equation.

3 Connect

Ask students which key words, phrases, or quantities in the scenario helped them match the corresponding equations **(MP1)**. Then ask students what the variable x and solution in Problem 2 tell them about the scenario.

Have students share their scenarios for Problem 3. As students share their scenarios, ask the class how they know whether the scenario matches the equation.

Highlight that students may identify quantities and key words, such as less than, equally divided, or total, to help them determine an equation and solution to a problem in context.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2. Have them complete Problem 3 if time allows.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of each scenario.

- **Read 1:** Ask, “What is this scenario about? Describe it in your own words, without using the numbers.”
- **Read 2:** Ask, “What are the quantities or relationships in this scenario? Tell me about one of them.”
- **Read 3:** Ask students to brainstorm possible strategies to connect the scenario with the appropriate equation.

English Learners

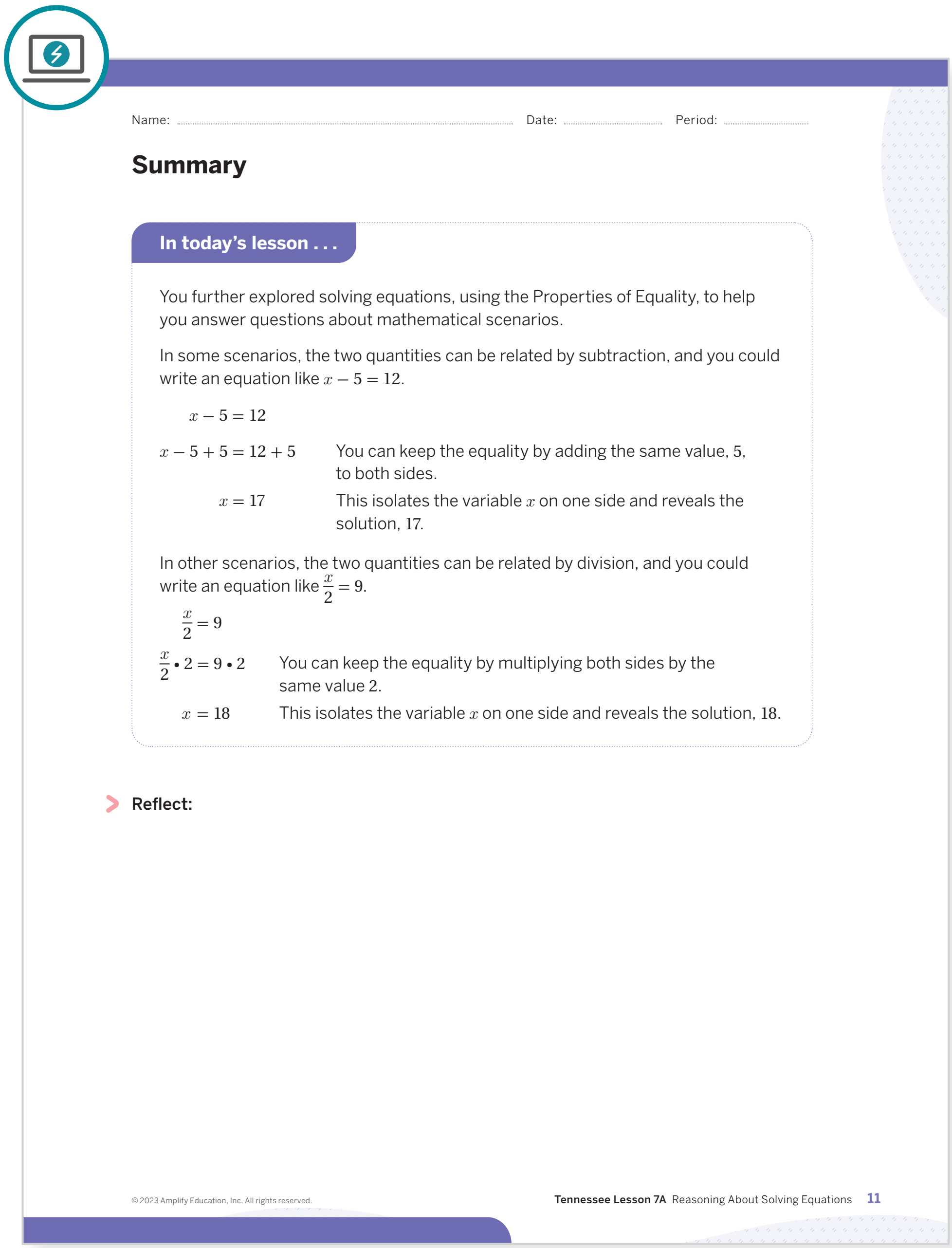
Annotate key words and phrases in the text, such as *total*, *equally divided*, and *less than*.

Whole Class | 5 min

Summary

6.EE.B.7

Review and synthesize how to solve equations of the forms $x - p = q$ and $\frac{x}{p} = q$.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You further explored solving equations, using the Properties of Equality, to help you answer questions about mathematical scenarios.

In some scenarios, the two quantities can be related by subtraction, and you could write an equation like $x - 5 = 12$.

$x - 5 = 12$

$x - 5 + 5 = 12 + 5$ You can keep the equality by adding the same value, 5, to both sides.

$x = 17$ This isolates the variable x on one side and reveals the solution, 17.

In other scenarios, the two quantities can be related by division, and you could write an equation like $\frac{x}{2} = 9$.

$\frac{x}{2} = 9$

$\frac{x}{2} \cdot 2 = 9 \cdot 2$ You can keep the equality by multiplying both sides by the same value 2.

$x = 18$ This isolates the variable x on one side and reveals the solution, 18.

Reflect:

Synthesize

Have students share their strategies for solving equations of the forms $x - p = q$ and $\frac{x}{p} = q$.

Highlight that, although the forms of equations may differ, students can use the Properties of Equality to solve the equations.

Ask students how they would describe the steps to solving an equation to a student who was absent from class.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are equations helpful in solving a problem in context?"

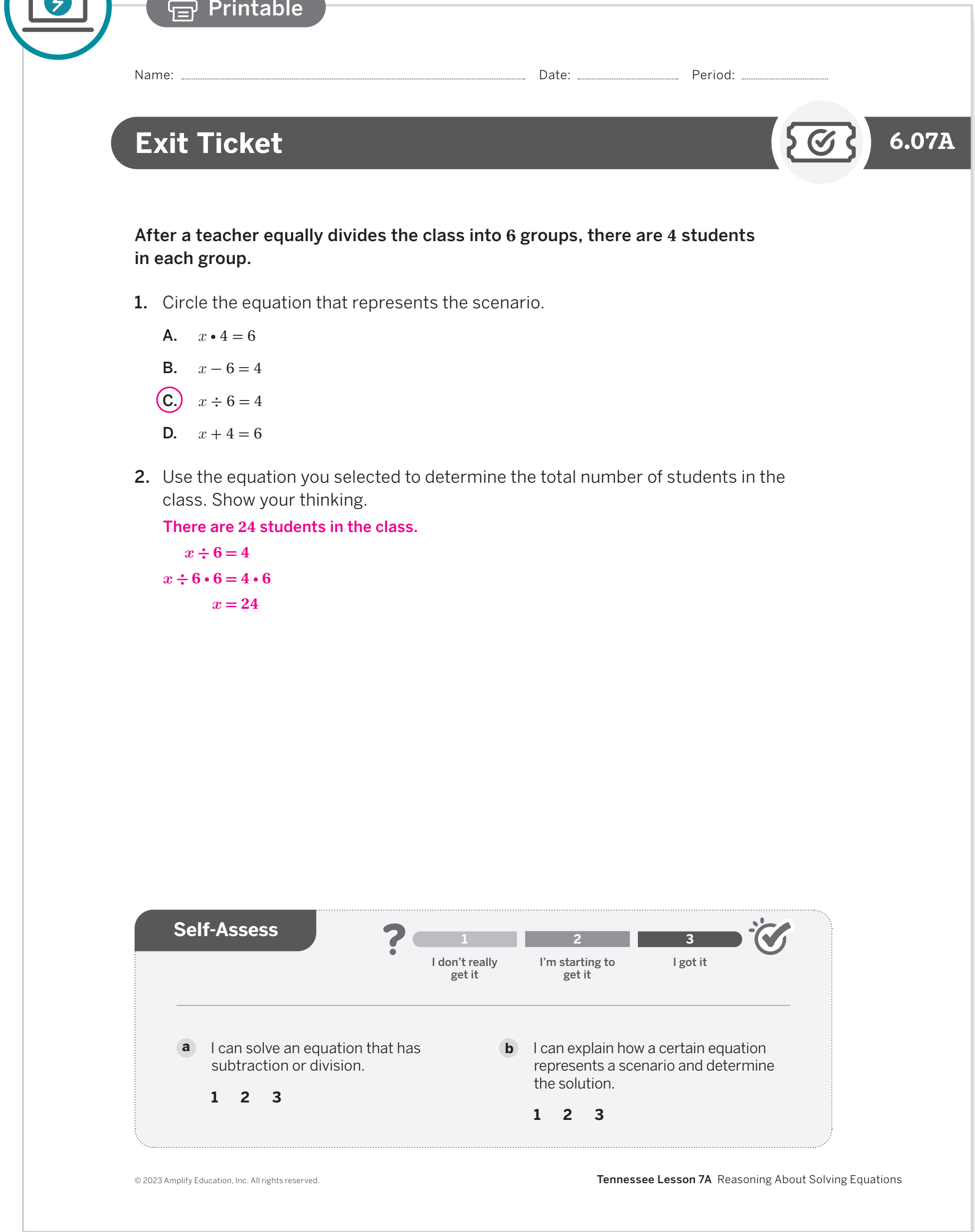
Independent | 5 min

Exit Ticket

6.EE.B.7

Students demonstrate their understanding by matching an equation to a scenario and then solving the equation.

Printable



Name: __________ Date: __________ Period: __________

Exit Ticket

6.07A

After a teacher equally divides the class into 6 groups, there are 4 students in each group.

1. Circle the equation that represents the scenario.
   - A. $x \cdot 4 = 6$
   - B. $x - 6 = 4$
   - [✓ C.] $x \div 6 = 4$
   - D. $x + 4 = 6$
2. Use the equation you selected to determine the total number of students in the class. Show your thinking.

   There are 24 students in the class.

   $$x \div 6 = 4$$
   $$x \div 6 \cdot 6 = 4 \cdot 6$$
   $$x = 24$$

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I can solve an equation that has subtraction or division.

1 2 3

b I can explain how a certain equation represents a scenario and determine the solution.

1 2 3

Tennessee Lesson 7A Reasoning About Solving Equations

Success looks like . . .

- **Language Goal:** Solving equations of the form $x - p = q$ and $\frac{x}{p} = q$ and explaining the solution method. **(Speaking and Listening, Writing)**
  - » Showing the correct work for Problem 2.
- **Language Goal:** Interpreting problems in context and explaining how an equation and its solution corresponds to the context. **(Speaking and Listening).**
  - » Selecting the corresponding equation for Problem 1.

Suggested next steps

If students do not select the correct choice for Problem 1, consider:

- Underlining the term *equally divides* and the numbers 6 and 4.
- Reviewing Activity 2.

If students do not show the correct work for Problem 2, consider:

- Reviewing Activity 1.

Professional Learning

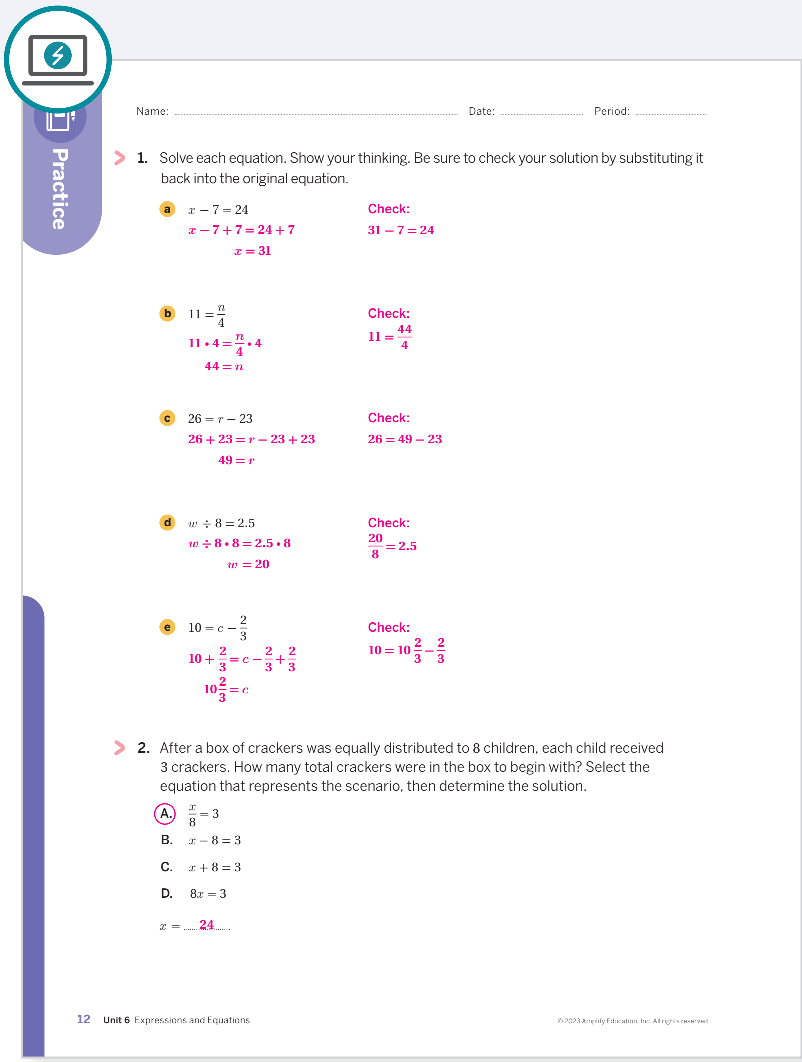
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What worked and didn't work today? The focus of this lesson was to solve equations of the form $x - p = q$ and $\frac{x}{p} = q$. How did it go?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Solve each equation. Show your thinking. Be sure to check your solution by substituting it back into the original equation.

   a $x - 7 = 24$
   $x - 7 + 7 = 24 + 7$
   $x = 31$
   Check:
   $31 - 7 = 24$

   b $11 = \frac{n}{4}$
   $11 \cdot 4 = \frac{n}{4} \cdot 4$
   $44 = n$
   Check:
   $11 = \frac{44}{4}$

   c $26 = r - 23$
   $26 + 23 = r - 23 + 23$
   $49 = r$
   Check:
   $26 = 49 - 23$

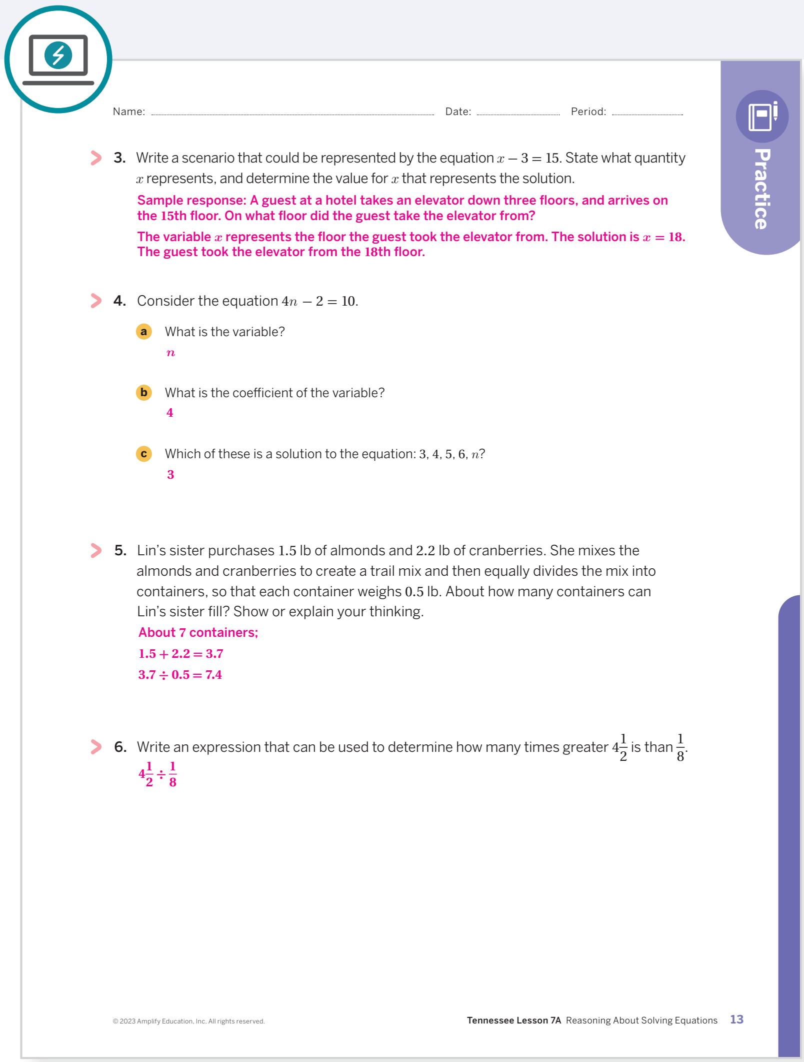
   d $w \div 8 = 2.5$
   $w \div 8 \cdot 8 = 2.5 \cdot 8$
   $w = 20$
   Check:
   $\frac{20}{8} = 2.5$

   e $10 = c - \frac{2}{3}$
   $10 + \frac{2}{3} = c - \frac{2}{3} + \frac{2}{3}$
   $10\frac{2}{3} = c$
   Check:
   $10 = 10\frac{2}{3} - \frac{2}{3}$

2. After a box of crackers was equally distributed to 8 children, each child received 3 crackers. How many total crackers were in the box to begin with? Select the equation that represents the scenario, then determine the solution.

   [✓ A.] $\frac{x}{8} = 3$
   B. $x - 8 = 3$
   C. $x + 8 = 3$
   D. $8x = 3$

   $x =$ 24



Practice

Name: ______ Date: ______ Period: ______

3. Write a scenario that could be represented by the equation $x - 3 = 15$. State what quantity x represents, and determine the value for x that represents the solution.

   Sample response: A guest at a hotel takes an elevator down three floors, and arrives on the 15th floor. On what floor did the guest take the elevator from?

   The variable x represents the floor the guest took the elevator from. The solution is $x = 18$. The guest took the elevator from the 18th floor.

4. Consider the equation $4n - 2 = 10$.

   a What is the variable?
   n

   b What is the coefficient of the variable?
   4

   c Which of these is a solution to the equation: $3, 4, 5, 6, n$?
   3

5. Lin's sister purchases 1.5 lb of almonds and 2.2 lb of cranberries. She mixes the almonds and cranberries to create a trail mix and then equally divides the mix into containers, so that each container weighs 0.5 lb. About how many containers can Lin's sister fill? Show or explain your thinking.

   About 7 containers;
   $1.5 + 2.2 = 3.7$
   $3.7 \div 0.5 = 7.4$

6. Write an expression that can be used to determine how many times greater $4\frac{1}{2}$ is than $\frac{1}{8}$.
   $4\frac{1}{2} \div \frac{1}{8}$

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 1	6.EE.B.7	1
	2	Activity 2	6.EE.B.7	2
	3	Activity 2	6.EE.B.7	2
Spiral	4	Unit 6 Lesson 4	6.EE.A.2b, 6.EE.B.5	2
	5	Unit 5 Lesson 11	6.NS.B.2	2
Formative ⚡	6	Unit 7 Lesson 8	6.EE.B.7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice.**

Stem Plots

Let's explore a new ("sideways") way to display a distribution.

Focus

Goals

1. Interpret the shape and characteristics of a data distribution displayed in a stem plot
2. Create a stem plot to represent a set of data.
3. **Language Goal:** Critique the benefits of using a stem plot to display a set of data. **(Speaking and Listening)**

Coherence

Today

Students explore a new type of data display, building on their understanding of distributions and familiarity with dot plots. They are first introduced to the benefit of the organization in a stem plot. Students then analyze and interpret the data represented in a stem plot before constructing stem plots of their own. Finally, they compare the data represented both in a stem plot and a dot plot to notice the benefits of each type of display.

Previously

In Lessons 3 and 4, students explored the characteristics of data distributions using dot plots.

Coming Soon

In Tennessee Lesson 7B, students will create pie charts as another type of visual display to represent a set of data.

Rigor

- Students build conceptual understanding of the benefits of using a particular display for a set of data.

Standards

Addressing

6.SP.B.4
Display a single set of numerical data using dot plots (line plots), box plots, pie charts and **stem plots.**

Also Addressing: **6.SP.A.3**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Independent	Whole Class	Independent
MP3	MP7				
6.SP.B.4	6.SP.B.4	6.SP.B.4	6.SP.B.4, 6.SP.A.3	6.SP.B.4	6.SP.B.4

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart, *Stem Plot*

Math Language Development

New words

- ***stem plot***

Review words

- *center*
- *dot plot*
- *spread*

Amps | Featured Activity

Activity 2
Formative Feedback

Use your teacher tools to save time while checking stem plots for accuracy.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated by having to interpret numbers that are split apart when looking at a stem plot **(MP7)**. Consider having students develop a method for making this connection for themselves, whether that involves rewriting the values or drawing lines to visually represent the connection.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

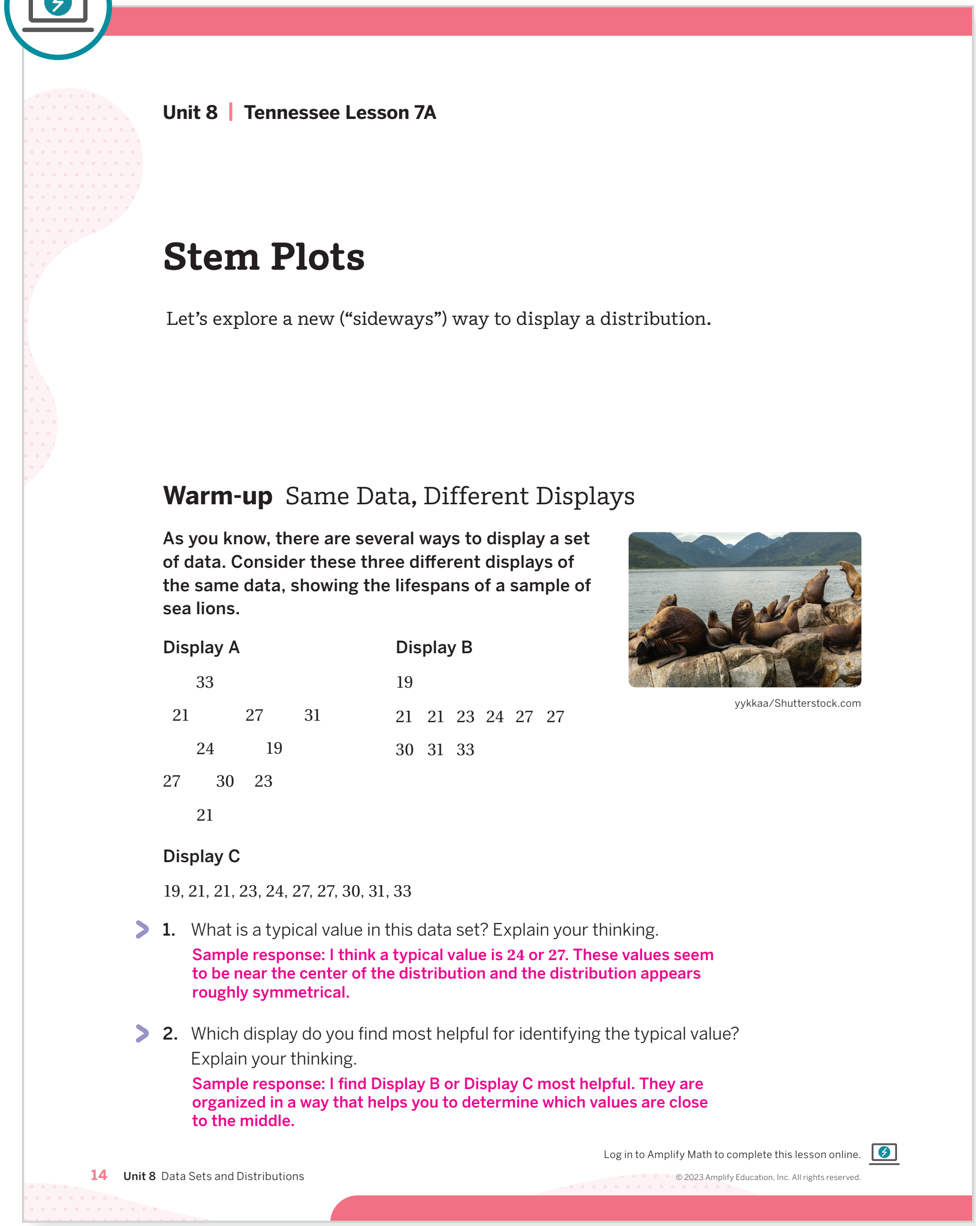
- The **Warm-up may** be omitted. Then, introduce the features of a stem plot using the Anchor Chart, *Stem Plot*, in the Launch of Activity 1.
- In **Activity 3**, discuss the responses to Problems 1–3 as a whole class.

Pairs | 5 min

MP3
6.SP.B.4

Warm-up Same Data, Different Displays

Students consider the same data, organized in three different ways, introducing them to an informal stem plot.



Unit 8 | **Tennessee Lesson 7A**

Stem Plots

Let's explore a new ("sideways") way to display a distribution.

Warm-up Same Data, Different Displays

As you know, there are several ways to display a set of data. Consider these three different displays of the same data, showing the lifespans of a sample of sea lions.

yykkaa/Shutterstock.com

Display A

33
21 27 31
24 19
27 30 23
21

Display B

19
21 21 23 24 27 27
30 31 33

Display C

19, 21, 21, 23, 24, 27, 27, 30, 31, 33

1. What is a typical value in this data set? Explain your thinking.
   Sample response: I think a typical value is 24 or 27. These values seem to be near the center of the distribution and the distribution appears roughly symmetrical.

2. Which display do you find most helpful for identifying the typical value? Explain your thinking.
   Sample response: I find Display B or Display C most helpful. They are organized in a way that helps you to determine which values are close to the middle.

Log in to Amplify Math to complete this lesson online.

14 **Unit 8** Data Sets and Distributions

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1 Launch

Activate prior knowledge by asking students about the different ways they already know in which data can be organized and displayed.

2 Monitor

Help students get started by asking, "What does it mean for a value to be 'typical' of a data set?"

Look for points of confusion:

- **Thinking that only 21 or 27 can be considered typical values.** Ask, "Where would you consider the center of the data set to be?"

Look for productive strategies:

- Noticing the data are roughly symmetric and do not contain any outliers.
- Noticing that Display B is organized according to the tens place of the data.

3 Connect

Have students share their responses to Problem 2.

Display the Anchor Chart, *Stem Plot*.

Define *stem plot* as a type of table used to organize and display numerical data.

Highlight the steps for creating a stem plot, focusing on identifying the place values of the first and last digits of each number in the set. A key is written below the plot to help clarify the place values. Tell students that stem plots are also sometimes referred to as stem-and-leaf plots, because the "stem" value to the left of the bar corresponds with each "leaf" value to the right of the bar.

Ask, "How is a stem plot similar to a dot plot? How is it different?"

Math Language Development

Accessibility: *MLR1: Stronger and Clearer Each Time*

After students complete Problem 2, have pairs meet with 1–2 other pairs of students to share their responses. Encourage reviewers to ask clarifying questions such as:

- "What about the display you chose do you find helpful?"
- "What about the other displays did you not find helpful?

Have students revise their responses, as needed.

English Learners

Use intentional grouping so that students with different English language proficiency levels can interact and have an opportunity to listen to peers with more advanced proficiency.

Power-up

To power up students' ability to determine the quotient when dividing with decimal values, have students complete:

1. $55 \div 11 = 5$
2. $5.5 \div 11 = 0.5$
3. $5.5 \div 1.1 = 5$

Use: Before the Warm-up.

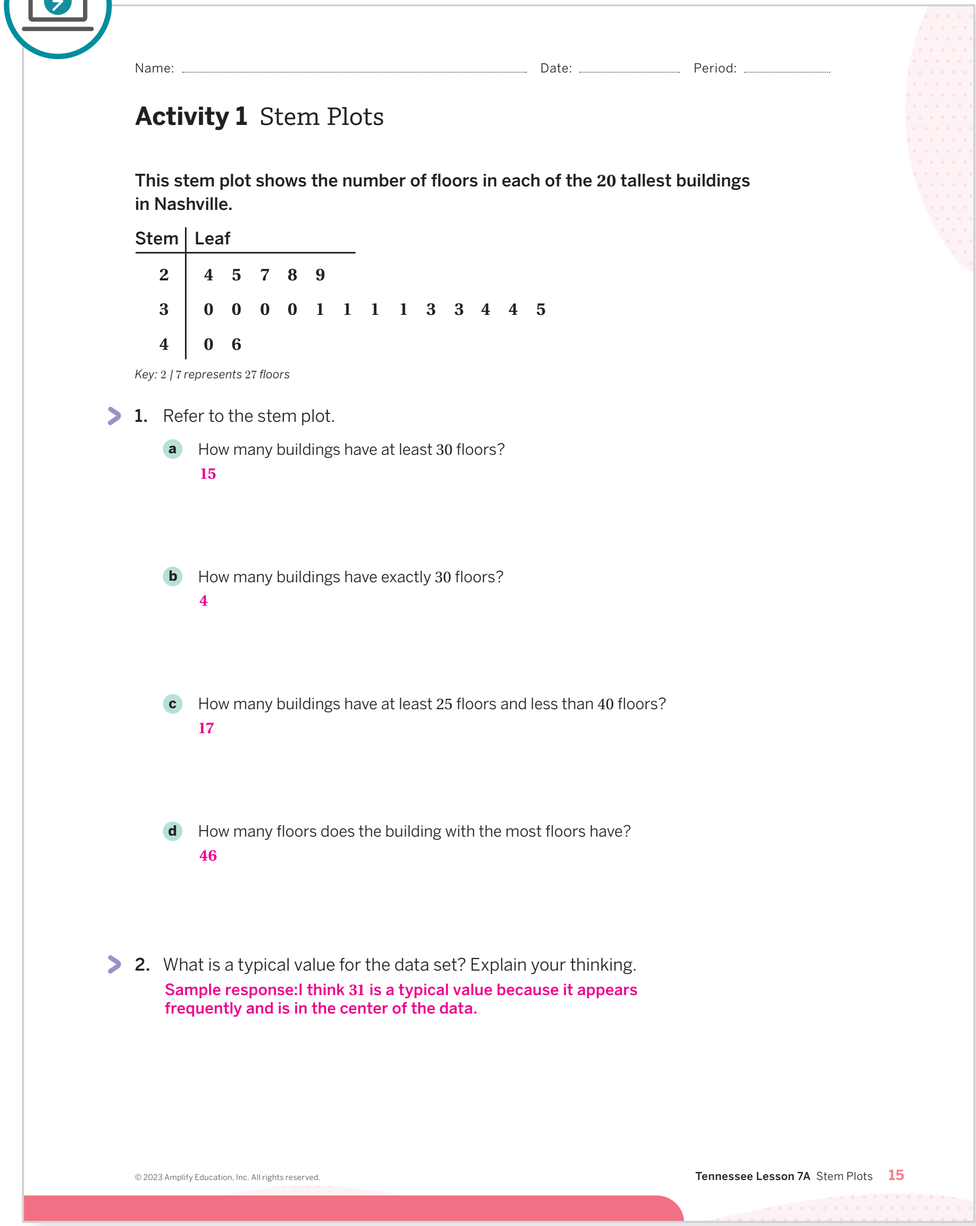
Informed by: Performance on Lesson 7, Practice Problem 6

Pairs | 10 min

Activity 1 Stem Plots

MP7
6.SP.B.4

Students examine a stem plot to interpret characteristics of the data represented.



Name: ______ Date: ______ Period: ______

Activity 1 Stem Plots

This stem plot shows the number of floors in each of the 20 tallest buildings in Nashville.

Stem	Leaf
2	4 5 7 8 9
3	0 0 0 0 1 1 1 1 3 3 4 4 5
4	0 6

Key: 2 | 7 represents 27 floors

1. Refer to the stem plot.
   - a How many buildings have at least 30 floors?
     15
   - b How many buildings have exactly 30 floors?
     4
   - c How many buildings have at least 25 floors and less than 40 floors?
     17
   - d How many floors does the building with the most floors have?
     46
2. What is a typical value for the data set? Explain your thinking.
   Sample response:I think 31 is a typical value because it appears frequently and is in the center of the data.

1 Launch

Ask, “What is the least number of floors in any of the buildings?” If students say “2”, remind them to check the key to identify the place value position of the digit to the left of the bar.

2 Monitor

Help students get started by asking them to read or write all of the data values in the 20s.

Look for points of confusion:

- **Thinking the value to the left of the bar only connects with the first digit to the right.** Have students draw a line from the value on the left to each of the digits on the right.

Look for productive strategies:

- In Problem 2, counting toward the center value of the data set appropriately.
- Using different interpretations of what a typical value means, e.g., using the number that appears most often.

3 Connect

Have students share their methods for determining a typical value.

Ask:

- “How does the structure of a stem plot help you to determine a typical value?” **(MP7)**
- “How is a stem plot like a bar graph? How is it different?”

Highlight that a stem plot helps to see where most of the data in a set exists, similar to a dot plot. In some cases, it may be quicker to create a stem plot than to create a dot plot.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students rewrite the values from the stem plot in an organized list.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share how a stem plot is like or unlike a bar graph, revoice their ideas in the form of a question using appropriate mathematical language or language from the context. For example:

If a student says . . .	**Revoice their ideas by asking . . .**
“The stem plot is like a bar graph because it shows how long the different categories are.”	“When you refer to the length of the category, what are you actually referring to? What makes it longer or shorter?”

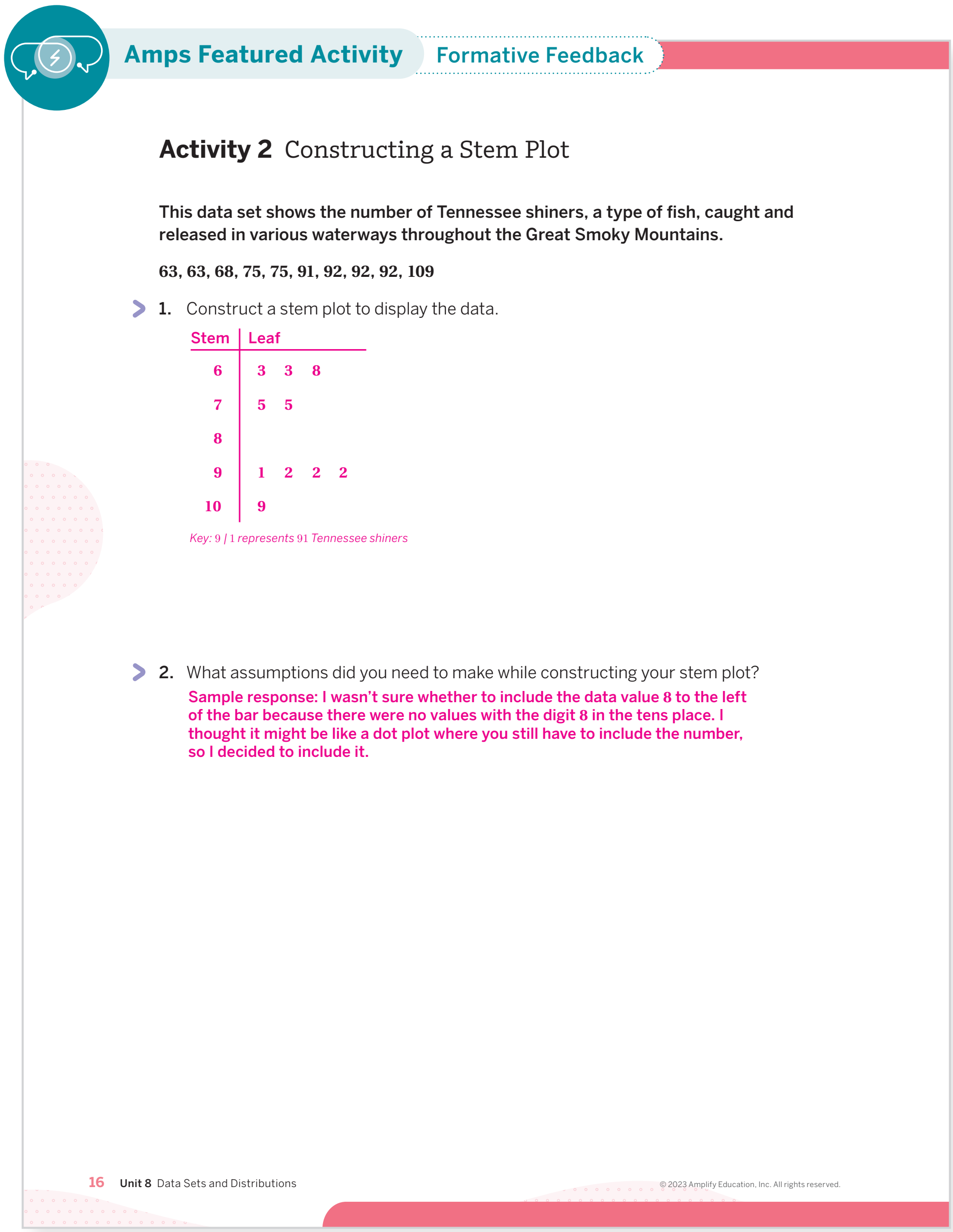
Pairs | 10 min

Activity 2 Constructing a Stem Plot

6.SP.B.4

Students create a stem plot from a data set to learn about the conventions of the stem plot format.

Amps Featured Activity Formative Feedback



Activity 2 Constructing a Stem Plot

This data set shows the number of Tennessee shiners, a type of fish, caught and released in various waterways throughout the Great Smoky Mountains.

63, 63, 68, 75, 75, 91, 92, 92, 92, 109

1. Construct a stem plot to display the data.

Stem	Leaf
6	3 3 8
7	5 5
8	
9	1 2 2 2
10	9

Key: 9 | 1 *represents* 91 *Tennessee shiners*

2. What assumptions did you need to make while constructing your stem plot?
Sample response: I wasn't sure whether to include the data value 8 to the left of the bar because there were no values with the digit 8 in the tens place. I thought it might be like a dot plot where you still have to include the number, so I decided to include it.

16 **Unit 8** Data Sets and Distributions

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Remind students that the vertical bar in the stem plot represents a dividing line between two place value positions. Suggest that students write the place value position above each section for their stem plot.

1 Launch

Let students know that as they create their stem plot, they will likely come across situations they have not yet encountered. Encourage them to do their best and reassure them that their questions will be answered during the class discussion.

2 Monitor

Help students get started by having them draw a vertical bar and asking, "Which should be the first value on the left of the bar?"

Look for points of confusion

- **Not being sure about whether to include 8 on the left side since there are no values in the 80s.** Ask, "Would you find it helpful, if you were looking at the data, to see no numbers to the right of the bar next to the 8? What might this tell you about the data?"
- **Not knowing whether to write 1 or 10 to the left of the bar for 109.** Ask, "Would the shape of the data to the right of the bar be affected by your decision?"

Note: Each of the above could also be considered a productive strategy as these are open questions about the conventions of the stem plot format.

3 Connect

Display the completed stem plot for Problem 1.

Have students share the questions they generated while creating their stem plot.

Highlight that one convention of stem plots is to include consecutive values to the left of the bar even when there are no data value that correspond to them. This is similar to how, on a dot plot, the number line is consistent and shows values for which there is no data. Seeing that there is no data in a particular place is useful when considering the distribution of the data.

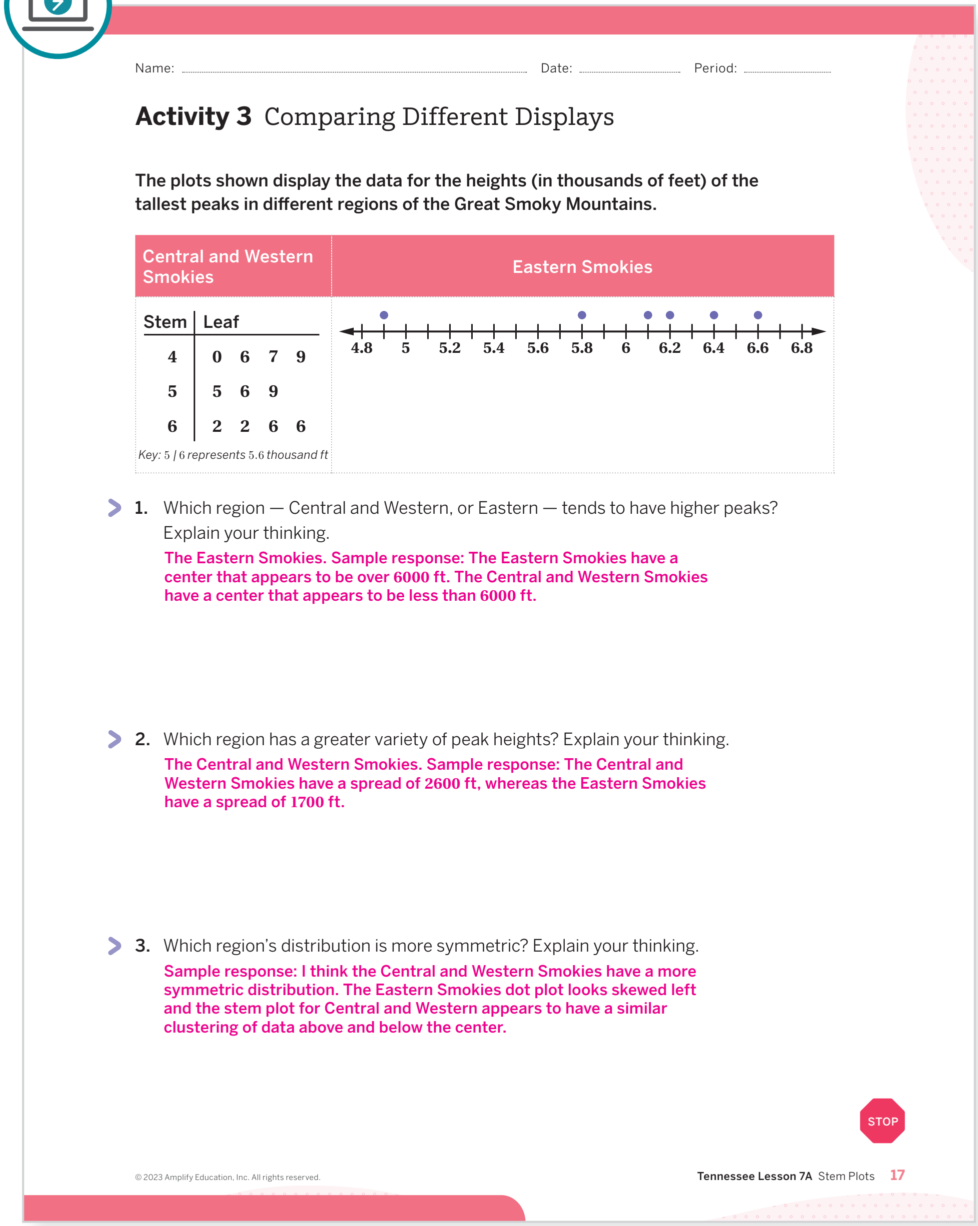
Ask, "Can you think of a data set that might be challenging to display with a stem plot? What about that data set would make it challenging?" I think a data set with values that are very spread out might be challenging, because there would need to be a lot of empty rows in the stem plot.

Independent | 10 min

Activity 3 Comparing Different Displays

6.SP.B.4, 6.SP.A.3

Students consider a side-by-side display of different data represented in a stem plot and a dot plot and use what they know about each display to make comparisons.



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Activity 3 Comparing Different Displays

The plots shown display the data for the heights (in thousands of feet) of the tallest peaks in different regions of the Great Smoky Mountains.

Central and Western Smokies

Stem	Leaf
4	0 6 7 9
5	5 6 9
6	2 2 6 6

Key: 5 | 6 represents 5.6 thousand ft

Eastern Smokies

1. Which region — Central and Western, or Eastern — tends to have higher peaks? Explain your thinking.
   The Eastern Smokies. Sample response: The Eastern Smokies have a center that appears to be over 6000 ft. The Central and Western Smokies have a center that appears to be less than 6000 ft.

2. Which region has a greater variety of peak heights? Explain your thinking.
   The Central and Western Smokies. Sample response: The Central and Western Smokies have a spread of 2600 ft, whereas the Eastern Smokies have a spread of 1700 ft.

3. Which region's distribution is more symmetric? Explain your thinking.
   Sample response: I think the Central and Western Smokies have a more symmetric distribution. The Eastern Smokies dot plot looks skewed left and the stem plot for Central and Western appears to have a similar clustering of data above and below the center.

STOP

Tennessee Lesson 7A Stem Plots 17

Math Language Development

MLR7: Compare and Connect

During the Connect, draw students' attention to the connections between the dot plot and the stem plot. After students respond to the Ask questions, consider displaying a second version of each display in the other form.

1 Launch

Tell students that choosing the best display for a set of data can be subjective. Suggest, as they complete the problems, that they consider which display they find more helpful for analyzing the data.

2 Monitor

Help students get started by asking what they know about the typical height of mountain peaks. Ask, "Are they tens, hundreds, or thousands of feet tall?"

Look for points of confusion:

- **Not noticing that the vertical bar in the stem plot represents the place value of a decimal point.** Ask students, "Looking at the key, how would you write the least value in the stem plot?"

Look for productive strategies:

- Writing each of the values from each data set in similar formats to more efficiently make comparisons.
- Reorganizing one of the data distributions to use the same display as the other.

3 Connect

Display both data distributions.

Have students share their responses for Problems 1–3. If students do not use terms such as *center*, *typical* and *spread* in their responses, ask, "Which problem required you to identify a typical value? Which problem required you to think about the spread of the data?"

Ask:

- "For which display were you more readily able to determine the typical value?"
- "For which display were you more readily able to determine the spread?"
- "For which display were you more readily able to determine how symmetric the distribution is?"

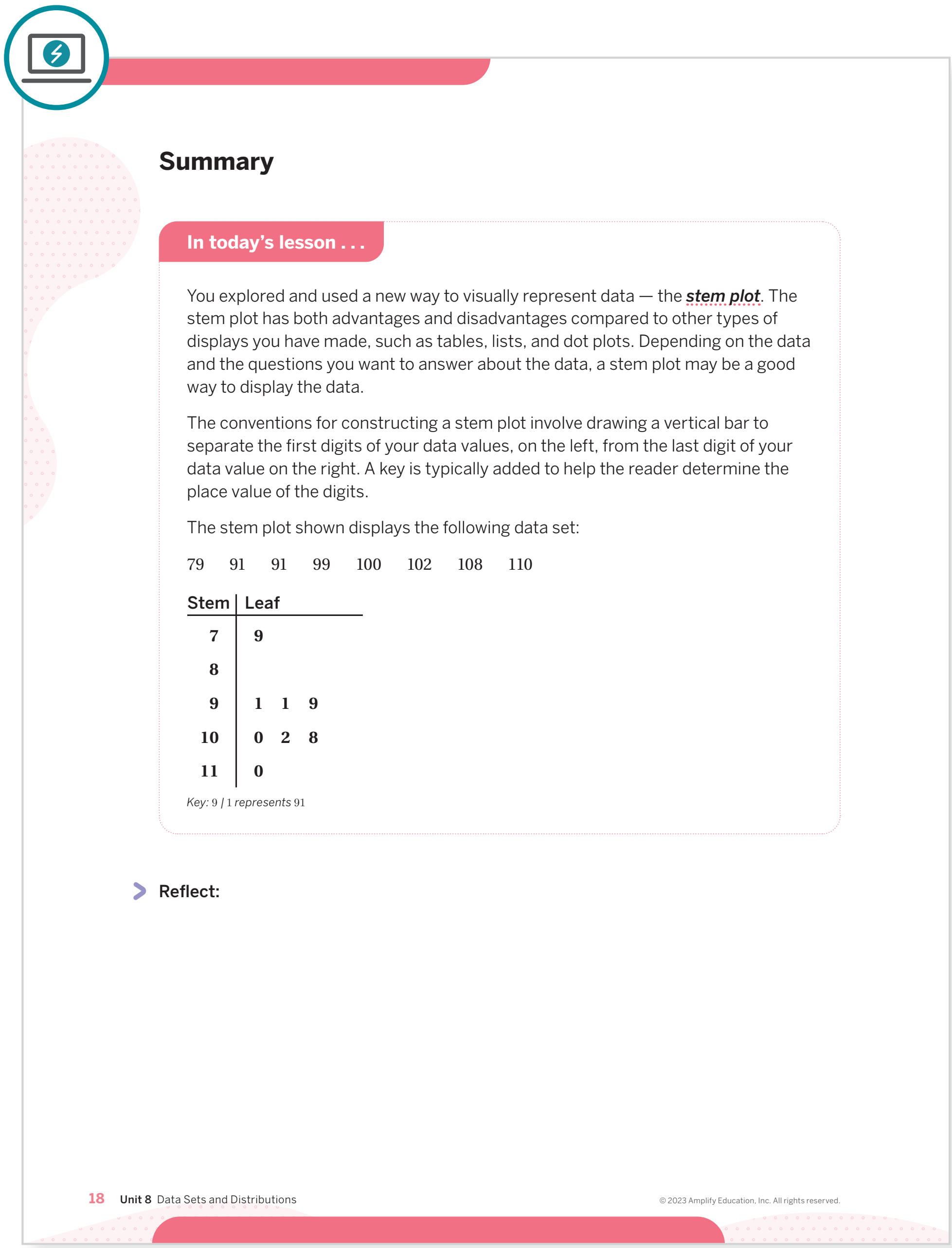
Highlight that each type of display has advantages and disadvantages. Though it is a subjective choice, some students may find themselves preferring one type of display over another.

Whole Class | 5 min

Summary

6.SP.B.4

Review and synthesize how to interpret and construct stem plot displays.



Summary

In today's lesson . . .

You explored and used a new way to visually represent data — the ***stem plot***. The stem plot has both advantages and disadvantages compared to other types of displays you have made, such as tables, lists, and dot plots. Depending on the data and the questions you want to answer about the data, a stem plot may be a good way to display the data.

The conventions for constructing a stem plot involve drawing a vertical bar to separate the first digits of your data values, on the left, from the last digit of your data value on the right. A key is typically added to help the reader determine the place value of the digits.

The stem plot shown displays the following data set:

79 91 91 99 100 102 108 110

Stem	Leaf
7	9
8	
9	1 1 9
10	0 2 8
11	0

Key: 9 | 1 represents 91

Reflect:

18 Unit 8 Data Sets and Distributions

Synthesize

Display the stem plot from the Summary.

Have students share the steps for creating a stem plot from a set of data.

Highlight how it is important to first consider the types of numbers in the data set. Remind them that the key helps readers to understand the place value of the digits in the display.

Formalize vocabulary:

- *stem plot*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are stem plots and dot plots similar? How are they different?"

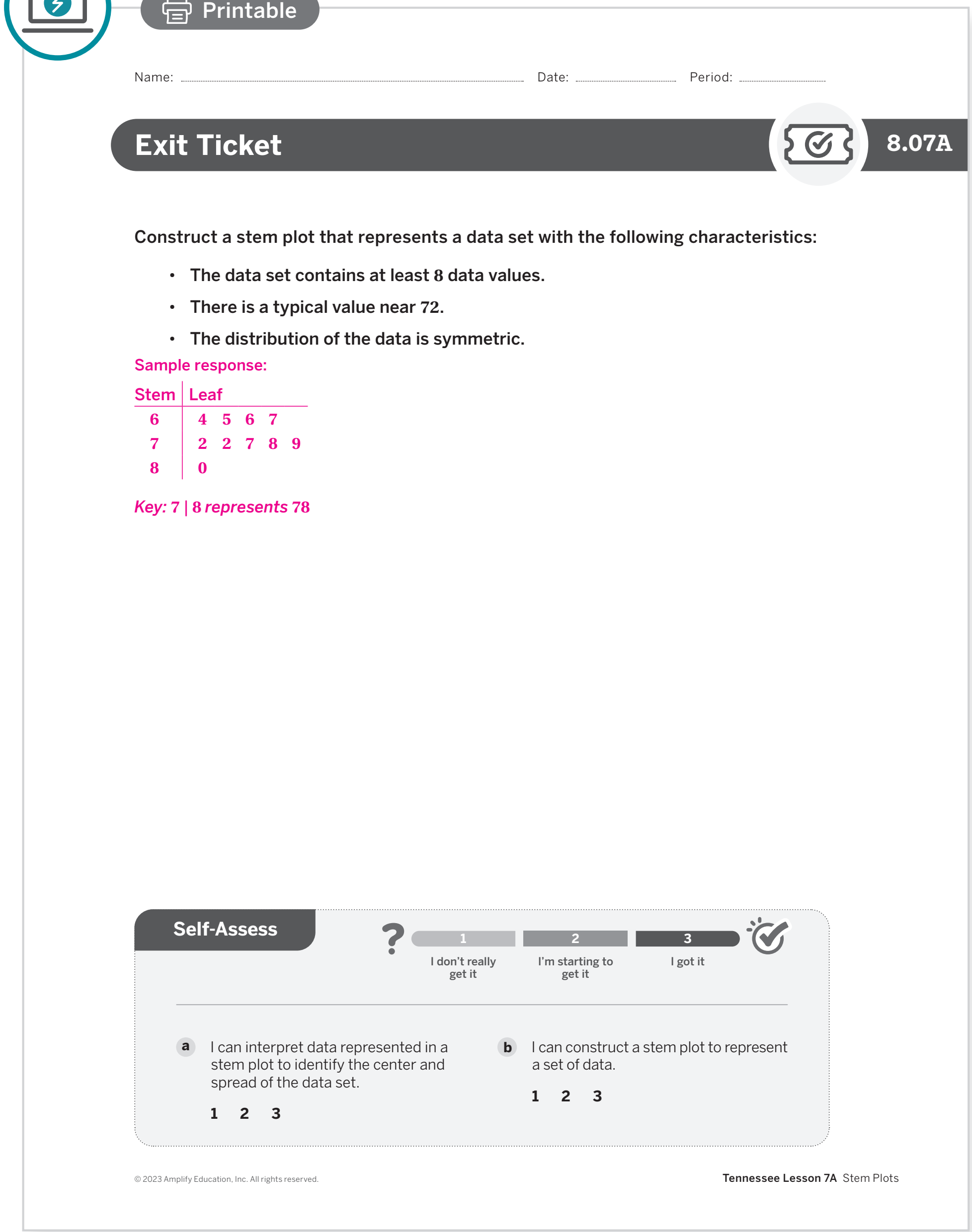
Independent | 5 min

Exit Ticket

6.SP.B.4

Students demonstrate their understanding of constructing and interpreting stem plots.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 8.07A

Construct a stem plot that represents a data set with the following characteristics:

- The data set contains at least 8 data values.
- There is a typical value near 72.
- The distribution of the data is symmetric.

Sample response:

Stem	Leaf
6	4 5 6 7
7	2 2 7 8 9
8	0

Key: 7 | 8 represents 78

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can interpret data represented in a stem plot to identify the center and spread of the data set.

1 2 3

b I can construct a stem plot to represent a set of data.

1 2 3

Tennessee Lesson 7A Stem Plots

Success looks like . . .

- **Goal:** Interpreting the shape and characteristics of a data distribution displayed in a stem plot.
- **Goal:** Creating a stem plot to represent a set of data.
  - » Choosing a set of values for the data set that are not too spread out.
- **Language Goal:** Critiquing the benefits of using a stem plot to display a set of data. **(Speaking and Listening)**

Suggested next steps

If students choose values that appear symmetric but are not actually symmetric, consider:

- Reviewing Problem 3 in Activity 3.
- Assigning Practice Problem 1.
- Asking, "If you represented these values on a dot plot, would they still appear symmetric?"

If students choose values that have a center less than 70 or greater than 74, consider

- Reviewing Problem 1 in Activity 3.
- Asking, "How could you adjust your values to obtain the typical value closer to 72?"

Professional Learning

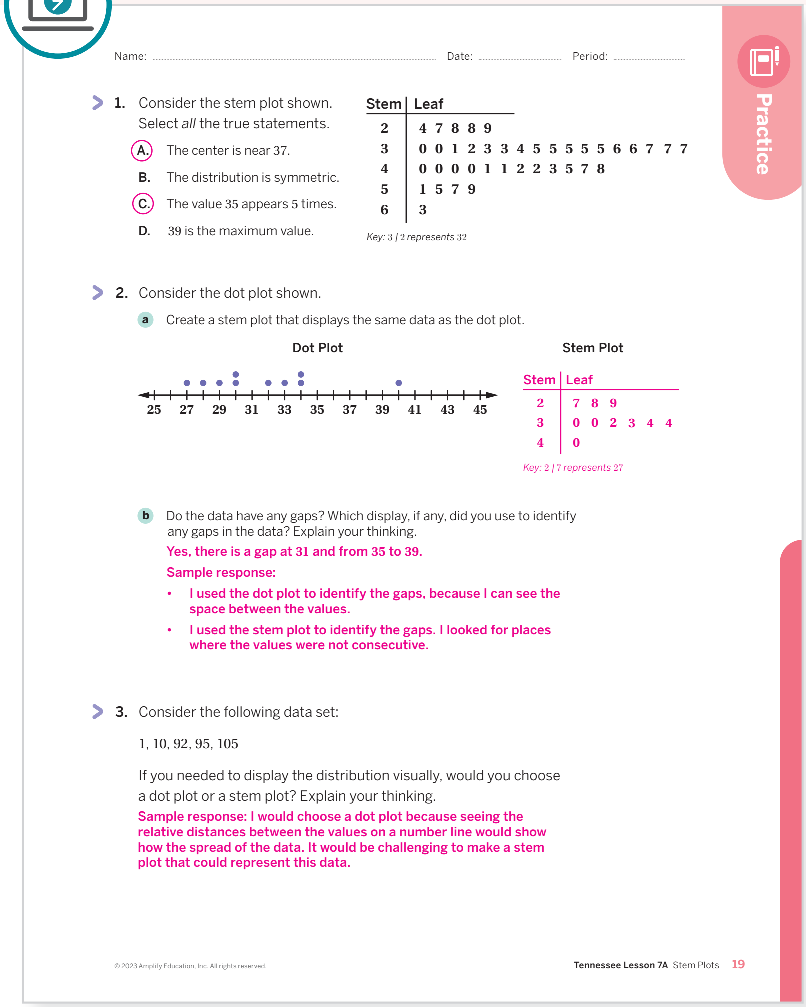
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did creating the stem plot in Activity 2 go as planned?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Consider the stem plot shown. Select *all* the true statements.

[✓ A.] The center is near 37.

B. The distribution is symmetric.

[✓ C.] The value 35 appears 5 times.

D. 39 is the maximum value.

Stem	Leaf
2	4 7 8 8 9
3	0 0 1 2 3 3 4 5 5 5 5 5 6 6 7 7 7
4	0 0 0 0 1 1 2 2 3 5 7 8
5	1 5 7 9
6	3

Key: 3 | 2 represents 32

2. Consider the dot plot shown.

a Create a stem plot that displays the same data as the dot plot.

Stem Plot

Stem	Leaf
2	7 8 9
3	0 0 2 3 4 4
4	0

Key: 2 | 7 represents 27

b Do the data have any gaps? Which display, if any, did you use to identify any gaps in the data? Explain your thinking.

Yes, there is a gap at 31 and from 35 to 39.

Sample response:

- **I used the dot plot to identify the gaps, because I can see the space between the values.**
- **I used the stem plot to identify the gaps. I looked for places where the values were not consecutive.**

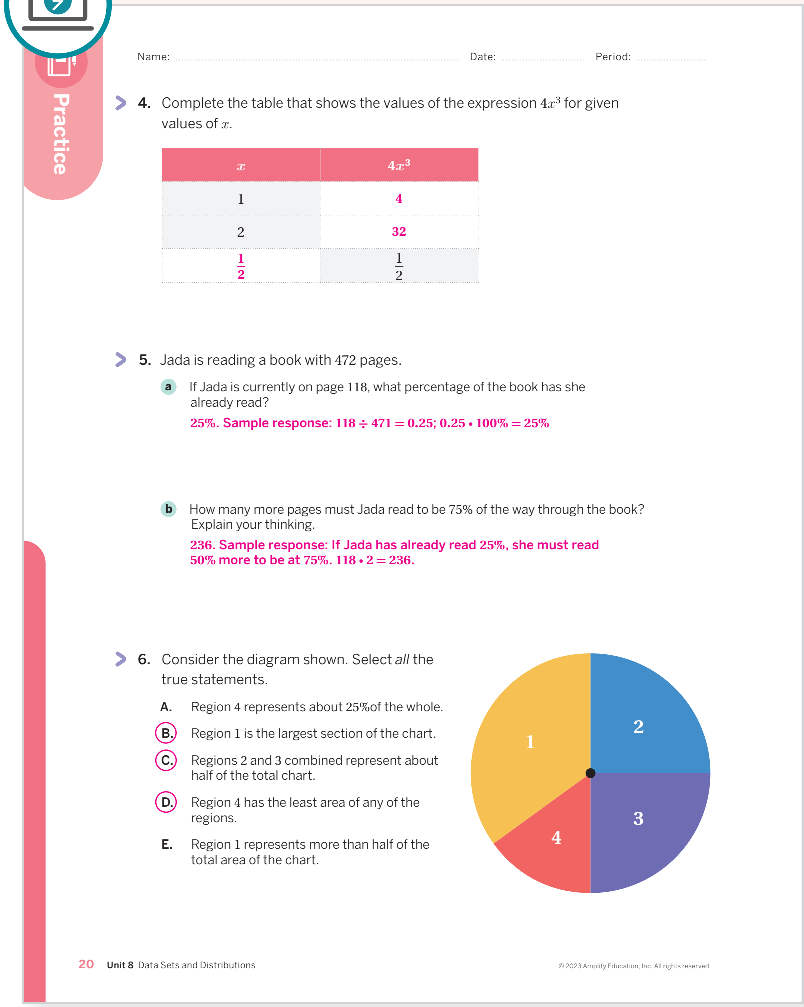
3. Consider the following data set:

1, 10, 92, 95, 105

If you needed to display the distribution visually, would you choose a dot plot or a stem plot? Explain your thinking.

Sample response: I would choose a dot plot because seeing the relative distances between the values on a number line would show how the spread of the data. It would be challenging to make a stem plot that could represent this data.

Tennessee Lesson 7A Stem Plots 19



Name: Date: Period:

4. Complete the table that shows the values of the expression $4x^3$ for given values of x.

x	$4x^3$
1	**4**
2	**32**
$\frac{1}{2}$	$\frac{1}{2}$

5. Jada is reading a book with 472 pages.

a If Jada is currently on page 118, what percentage of the book has she already read?

25%. Sample response: 118 ÷ 471 = 0.25; 0.25 • 100% = 25%

b How many more pages must Jada read to be 75% of the way through the book? Explain your thinking.

236. Sample response: If Jada has already read 25%, she must read 50% more to be at 75%. 118 • 2 = 236.

6. Consider the diagram shown. Select *all* the true statements.

A. Region 4 represents about 25%of the whole.

[✓ B.] Region 1 is the largest section of the chart.

[✓ C.] Regions 2 and 3 combined represent about half of the total chart.

[✓ D.] Region 4 has the least area of any of the regions.

E. Region 1 represents more than half of the total area of the chart.

20 **Unit 8** Data Sets and Distributions

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 1	6.SP.B.4	2
	2	Activity 2	6.SP.B.4	2
	3	Activity 3	6.SP.B.4	3
Spiral	4	Unit 6 Lesson 15	6.EE.A.1	2
	5	Unit 3 Lesson 13	6.RP.A.3c	2
Formative ⚡	6	Unit 8 Tennessee Lesson 7B	6.SP.B.4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Pie Charts

Let's explore how pie charts represent data sets.

Focus

Goals

1. **Language Goal:** Describe each category in a data set as a portion of the whole using a pie chart. **(Speaking and Writing)**
2. Calculate equivalent ratios to help create pie charts for a set of data.
3. **Language Goal:** Compare pie charts to other visual data displays. **(Speaking and Listening)**

Coherence

Today

Students explore a new type of data display, building on their understanding of dot plots. Students are first introduced to when and why pie charts are used and practice calculating percentages of each data set, Then, they analyze and interpret the data represented in a pie chart. Finally, they construct a pie chart by partitioning a circle.

Previously

Students explored how to describe data using dot plots in Lessons 3 and 4 and stem plots in Tennessee Lesson 7A.

Coming Soon

In Lesson 8, students will determine and interpret the mean of a distribution as the amount each member of the group would receive if all items are distributed equally.

Rigor

- Students build **conceptual understanding** of the benefits of using pie charts to represent data.
- Students draw the pie chart for a given set of data to build **procedural** skills in calculating percentages and partitioning a circle.

Standards

Addressing

6.SP.B.4

Display a single set of numerical data using dot plots (line plots), box plots, **pie charts** and stem plots.

Also Addressing: **6.RP.A.3c, 6.RP.A.3**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Small Groups	Whole Class	Independent
MP3	MP3	MP5		
6.SP.B.4	6.SP.B.4, 6.RP.A.3c	6.SP.B.4, 6.RP.A.3	6.SP.B.4	6.SP.B.4

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems - Math Talk*
- Activity 2 PDF, one per student (as needed)
- Ruler, protractor
- Colored pencils
- Calculators

Math Language Development

New words

- ***pie chart***
- ***sector***

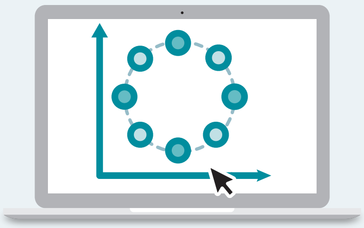
Review words

- *dot plot*
- *frequency*
- *percents*

Amps | Featured Activity

Activity 2
Creating Pie Charts

Students create the pie chart in Activity 2 to obtain a deeper visual understanding of the data in order to better make connections and comparisons. The pie chart has draggable points to help students adjust the size of their sectors.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may have trouble listening to their partner's ideas about what they think is true and what they think is false during the Warm-up **(MP3)**. Ask students to fully listen to their partner's comments without interrupting. Then have them restate what their partner shared in their own words and think carefully about how to respond as they provide feedback or critique given statements.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

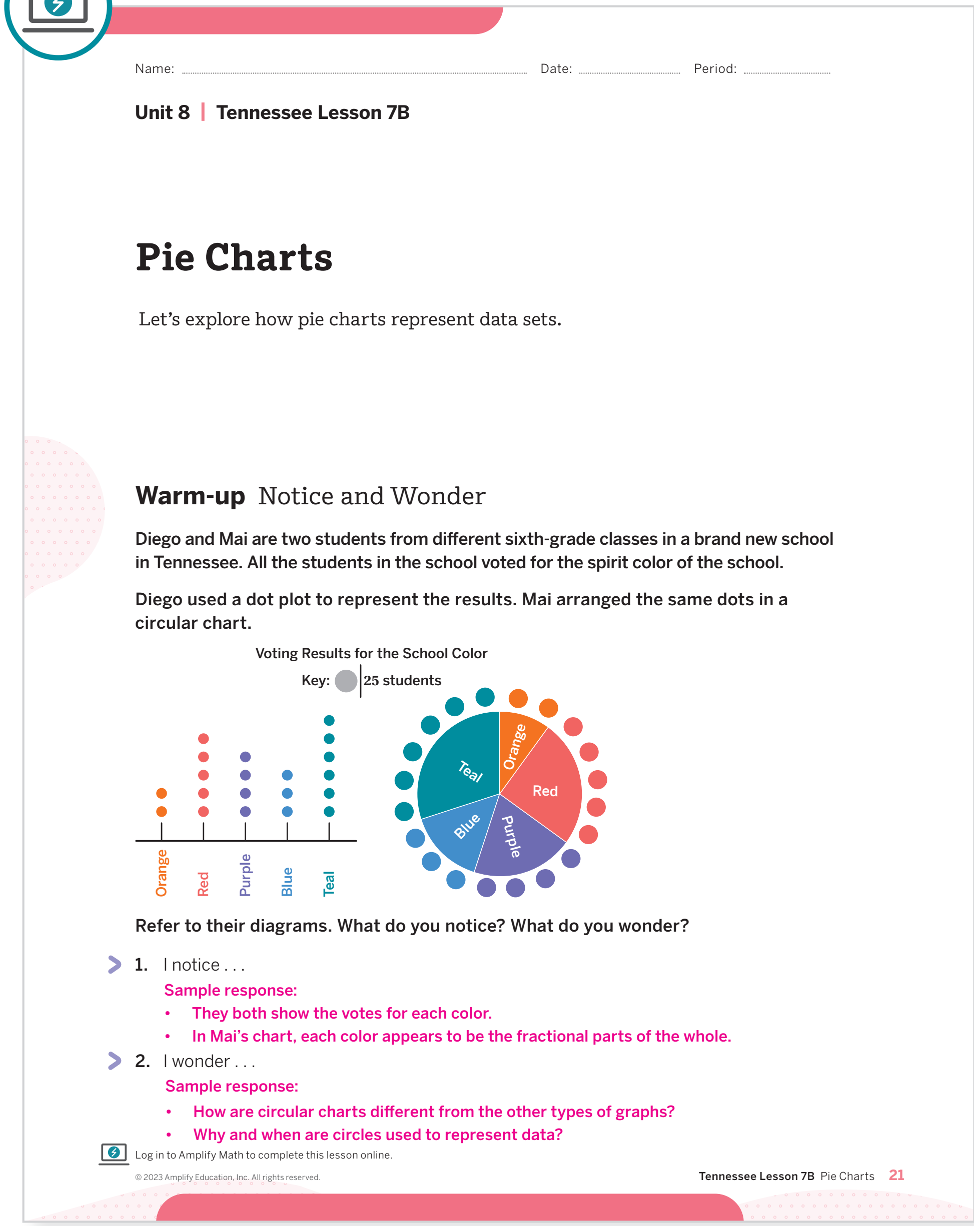
- The **Warm-up** may be omitted. Then use the chart in Activity 1 to introduce the features of a pie chart.
- In **Activity 2**, the Activity 2 PDF, *Partitioned Circle Template* can be given to all students instead of asking students to partition a blank circle for Problem 4.

Pairs | 5 min

MP3
6.SP.B.4

Warm-up Notice and Wonder

Students consider the same data, organized in two different data displays, introducing the pie chart.



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Unit 8 | Tennessee Lesson 7B

Pie Charts

Let's explore how pie charts represent data sets.

Warm-up Notice and Wonder

Diego and Mai are two students from different sixth-grade classes in a brand new school in Tennessee. All the students in the school voted for the spirit color of the school.

Diego used a dot plot to represent the results. Mai arranged the same dots in a circular chart.

Refer to their diagrams. What do you notice? What do you wonder?

1. I notice . . .
   Sample response:
   - They both show the votes for each color.
   - In Mai's chart, each color appears to be the fractional parts of the whole.
2. I wonder . . .
   Sample response:
   - How are circular charts different from the other types of graphs?
   - Why and when are circles used to represent data?

Log in to Amplify Math to complete this lesson online.

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Tennessee Lesson 7B Pie Charts 21

1 Launch

Conduct the *Notice and Wonder* routine followed by a whole class discussion about the similarities and differences between the dot plot and the pie chart. Review the terms *frequency* and *distribution* as needed.

2 Monitor

Help students get started by asking, "How is Mai's circular display of data similar to Diego's dot plot? How is it different?"

Look for points of confusion:

- **Not recognizing that the displays represent the same data.** Have them count the number of dots for each category.

Look for productive strategies:

- Using the pie chart to reason about the total number of responses.

3 Connect

Have students share what they noticed and wondered.

Define a ***pie chart*** as a specific type of data display which uses a circle divided into slices to illustrate numerical proportions of different categories. In a pie chart, the area of each slice is proportional to the quantity it represents.

$$\frac{\text{area of a slice}}{\text{total area of circle}} = \frac{\text{amount of the slice's category}}{\text{total amount}}$$

Highlight that each slice is a fractional part of the whole.

Ask, "What are the advantages of arranging data in a pie chart?" **(MP3)**

Differentiated Support

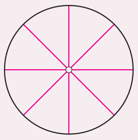
Accessibility: *Discussion Supports*

Display or provide students with the Anchor Chart PDF, *Sentence Stems - Math Talk* to support them when they explain their strategy. Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Power-up

To power up students' ability to partition a circle evenly, have students complete:

a. Divide the circle into 4 equal parts.
b. Use your response from part a to divide the circle into 8 equal parts.



Use: Before Activity 1

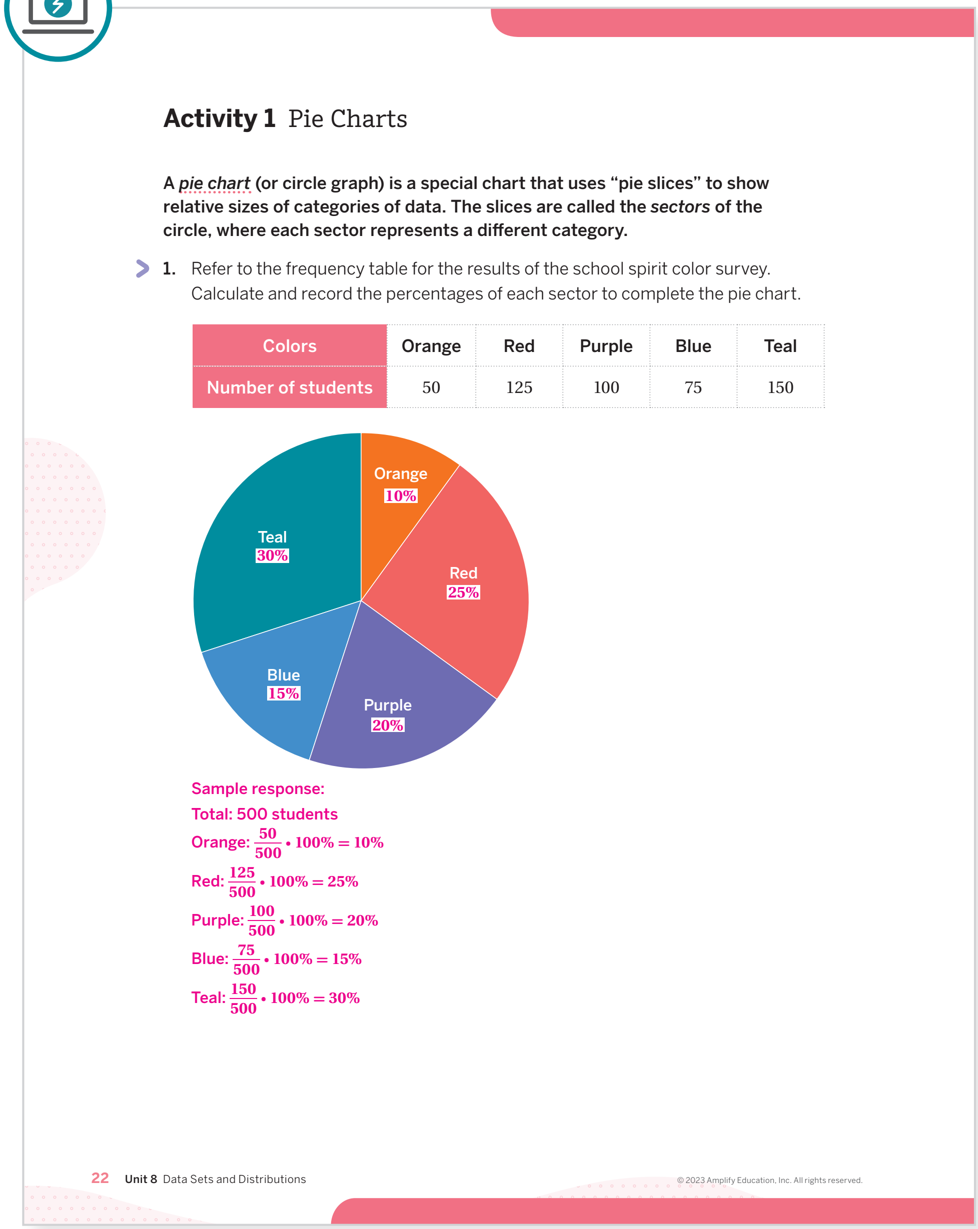
Informed by: Performance on Tennessee Lesson 7A, Practice Problem 6

Pairs | 10 min

MP3
6.SP.B.4, 6.RP.A.3c

Activity 1 Pie Charts

Students examine a pie chart and determine the percentage of each sector, connecting the percentages with part-to-whole relationships.



Activity 1 Pie Charts

A *pie chart* (or circle graph) is a special chart that uses "pie slices" to show relative sizes of categories of data. The slices are called the *sectors* of the circle, where each sector represents a different category.

1. Refer to the frequency table for the results of the school spirit color survey. Calculate and record the percentages of each sector to complete the pie chart.

Colors	Orange	Red	Purple	Blue	Teal
Number of students	50	125	100	75	150

Sample response:
Total: 500 students
Orange: $\frac{50}{500} \cdot 100\% = 10\%$
Red: $\frac{125}{500} \cdot 100\% = 25\%$
Purple: $\frac{100}{500} \cdot 100\% = 20\%$
Blue: $\frac{75}{500} \cdot 100\% = 15\%$
Teal: $\frac{150}{500} \cdot 100\% = 30\%$

22 **Unit 8** Data Sets and Distributions

1 Launch

Display a blank pie chart from a student's page and ask. "By looking at the chart only, can you tell which color received more votes, orange or red? Red and teal?" Discuss reasons for determining and labeling the slices with percentages.

2 Monitor

Help students get started by asking,

- "What is the total number of students that responded to the survey for the school spirit color?"
- "125 students voted for the color red. What portion of the students voted for red?"
- "What is the percentage of red among all the colors?"

Look for points of confusion:

- **Writing percentages to label the sectors without using the percent sign.** Ask students whether the number on the sector shows the number of votes or the percentages of votes.

Look for productive strategies:

- Writing the ratio of each category in the simplest form before calculating the percentages.
- Using the number of dots instead of the number of students to determine the percentages.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide options for students to calculate the percentages using a table of ratios, a tape diagram, or a double number line.

Extension: *Math Enrichment*

Ask, "If each of the categories had triple the number of votes as they do now, how would that change the pie chart?" Sample response: It would not change the pie chart because the part-to-whole ratios would remain the same.

Math Language Development

MLR1: Stronger and Clearer Each Time

Ask student partners to individually write their responses to each problem, share their responses, and revise their responses together to clarify their oral and written language.

English Learners

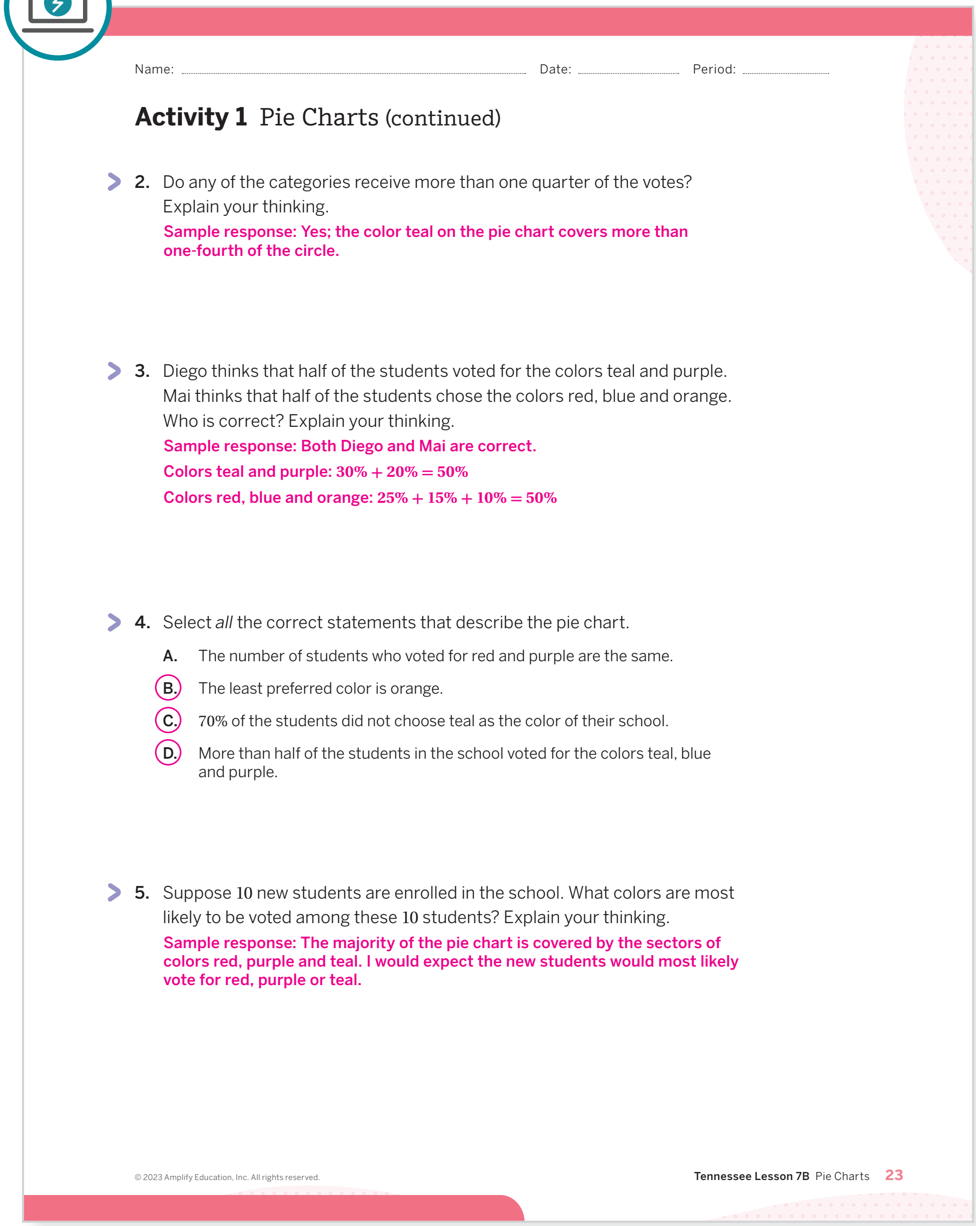
Use fractional representations to support students' understanding of the terms *half* and *quarter*.

Pairs | 10 min

MP3
6.SP.B.4, 6.RP.A.3c

Activity 1 Pie Charts (continued)

Students examine a pie chart and determine the percentage of each sector, connecting the percentages with part-to-whole relationships.



Name: ______ Date: ______ Period: ______

Activity 1 Pie Charts (continued)

2. Do any of the categories receive more than one quarter of the votes? Explain your thinking.
Sample response: Yes; the color teal on the pie chart covers more than one-fourth of the circle.

3. Diego thinks that half of the students voted for the colors teal and purple. Mai thinks that half of the students chose the colors red, blue and orange. Who is correct? Explain your thinking.
Sample response: Both Diego and Mai are correct.
Colors teal and purple: $30\% + 20\% = 50\%$
Colors red, blue and orange: $25\% + 15\% + 10\% = 50\%$

4. Select *all* the correct statements that describe the pie chart.

A. The number of students who voted for red and purple are the same.
[✓ B.] The least preferred color is orange.
[✓ C.] 70% of the students did not choose teal as the color of their school.
[✓ D.] More than half of the students in the school voted for the colors teal, blue and purple.

5. Suppose 10 new students are enrolled in the school. What colors are most likely to be voted among these 10 students? Explain your thinking.
Sample response: The majority of the pie chart is covered by the sectors of colors red, purple and teal. I would expect the new students would most likely vote for red, purple or teal.

Tennessee Lesson 7B Pie Charts 23

3 Connect

Display the completed pie chart with labels and percentages.

Have students share their methods for determining the percentages and their responses for Problems 2–6.

Define ***sector*** as the mathematical term that represents the "slices" of the pie chart.

Ask:

- "Which problem(s) can you more efficiently answer using a pie chart and which problem(s) can you more efficiently answer using other representations?"
  Sample response: When the data is represented with a circular design, it is more efficient to comment on the proportional relations such as one half and one quarter. This makes it easier to answer Problems 2 and 4.
- "What is the total of all the percentages?"
  100%.
- "What are the advantages of labeling sectors with the percentages?"
  Sample response: When there are many categories, it becomes more challenging for the eye to distinguish the relative size of each sector.
- "Why are percentages sometimes preferred to fractions to represent part-to-whole relationships?"
  Sample response: Percentages can help aid the comparison between the sectors by having a common denominator, 100.

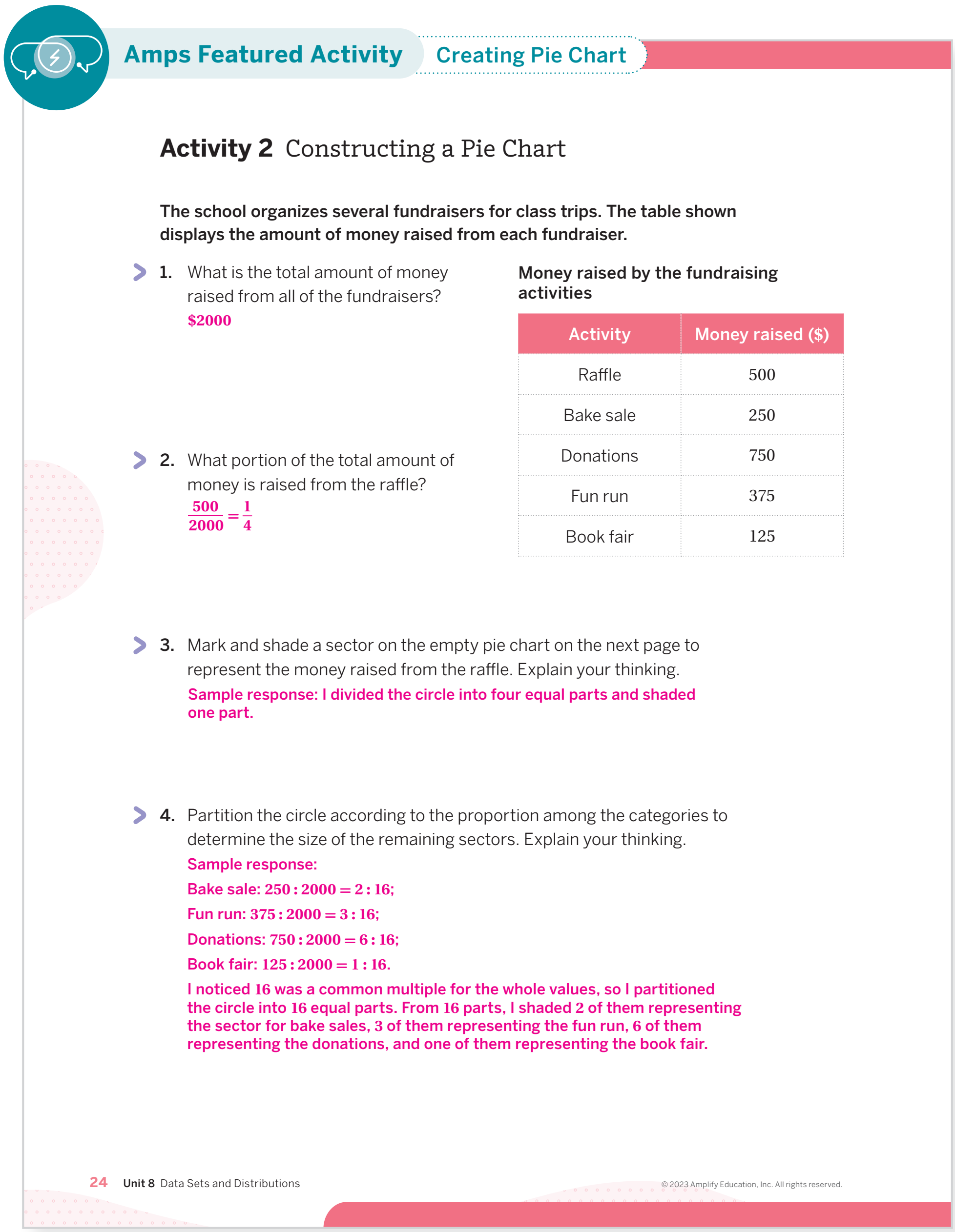
Highlight that pie charts are especially useful to show a part-to-whole relationship for each category.

Small Groups | 20 min

Activity 2 Constructing a Pie Chart

MP5
6.SP.B.4, 6.RP.A.3

Students partition a circle based on the given data set to create a pie chart.



Amps Featured Activity Creating Pie Chart

Activity 2 Constructing a Pie Chart

The school organizes several fundraisers for class trips. The table shown displays the amount of money raised from each fundraiser.

Money raised by the fundraising activities

Activity	Money raised ($)
Raffle	500
Bake sale	250
Donations	750
Fun run	375
Book fair	125

1. What is the total amount of money raised from all of the fundraisers?
   \$2000

2. What portion of the total amount of money is raised from the raffle?
   $\frac{500}{2000} = \frac{1}{4}$

3. Mark and shade a sector on the empty pie chart on the next page to represent the money raised from the raffle. Explain your thinking.
   Sample response: I divided the circle into four equal parts and shaded one part.

4. Partition the circle according to the proportion among the categories to determine the size of the remaining sectors. Explain your thinking.
   Sample response:
   Bake sale: $250 : 2000 = 2 : 16$;
   Fun run: $375 : 2000 = 3 : 16$;
   Donations: $750 : 2000 = 6 : 16$;
   Book fair: $125 : 2000 = 1 : 16$.
   I noticed 16 was a common multiple for the whole values, so I partitioned the circle into 16 equal parts. From 16 parts, I shaded 2 of them representing the sector for bake sales, 3 of them representing the fun run, 6 of them representing the donations, and one of them representing the book fair.

24 **Unit 8** Data Sets and Distributions

1 Launch

Set an expectation for the amount of time that students will have to work individually for Problems 1–3. Then, arrange students in small groups to complete the activity.

2 Monitor

Help students get started by asking students to determine the ratio of the money raised by each activity to the whole amount.

Look for points of confusion:

- **Thinking that using percentages can help them to divide the circle evenly.** Have them think about the challenges of dividing a circle into 100 sectors.
- **Not knowing how to divide the circle into 16 equal parts.** Present the Power-up problem and use it to discuss methods of dividing a circle into 2, 4, 8, and 16 parts.

Look for productive strategies:

- Noticing the total amount of the donations and bake sales categories is the half of the total money raised by all the fundraisers, and using this information to divide the circle into halves for a more efficient method of partition.
- Noticing the total amount of the fun run and book fair categories is equal to the money raised by the raffle, which is one quarter of the circle. Then, using this information to create another quarter sector for the total of the fun run and book fair categories.
- Using manipulatives or tools such as a string or fingers, or geometry tools such as a protractor and a ruler to partition the circle evenly. **(MP5)**

Activity 2 continued >

Differentiated Support

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

For Problem 4, provide students with a physical copy of Activity 2 PDF, *Partitioned Circle Template* to use.

Extension: *Math Enrichment*

Ask, "What is the new percentage of the donations category after the additional donations?" Sample response: The total amount will be \$3000, and the total amount of the donations will be \$1750.
The percentage of the donations will be $1750 \div 3000 \cong 0.583$, 58.3%

Math Language Development

MLR2: Collect and Display

Collect different examples of student graphs. Display the various examples and ask students to compare the charts. Listen for and amplify the mathematical language students use to support their thinking.

English Learners

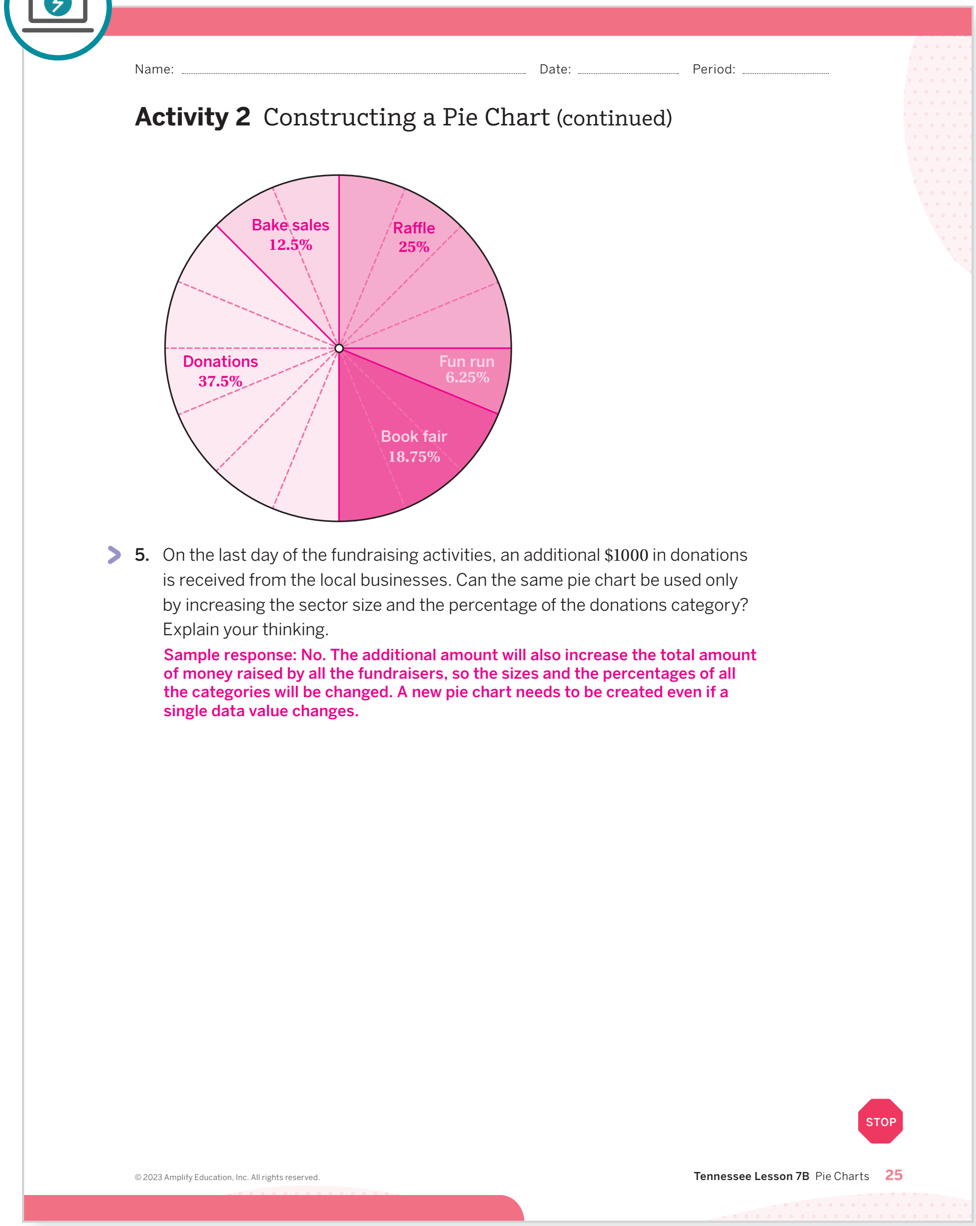
After the Connect discussion, clearly annotate the correct and incorrect parts of the statements.

Small Groups | 20 min

Activity 2 Constructing a Pie Chart (continued)

MP5
6.SP.B.4, 6.RP.A.3

Students partition a circle based on the given data set to create a pie chart.



Name: ______ Date: ______ Period: ______

Activity 2 Constructing a Pie Chart (continued)

5. On the last day of the fundraising activities, an additional $1000 in donations is received from the local businesses. Can the same pie chart be used only by increasing the sector size and the percentage of the donations category? Explain your thinking.

Sample response: No. The additional amount will also increase the total amount of money raised by all the fundraisers, so the sizes and the percentages of all the categories will be changed. A new pie chart needs to be created even if a single data value changes.

STOP

Tennessee Lesson 7B Pie Charts 25

3 Connect

Have students share their strategies they used to create their pie charts.

Highlight that determining the least common denominator of ratios is important to partition the circle accurately.

Ask:

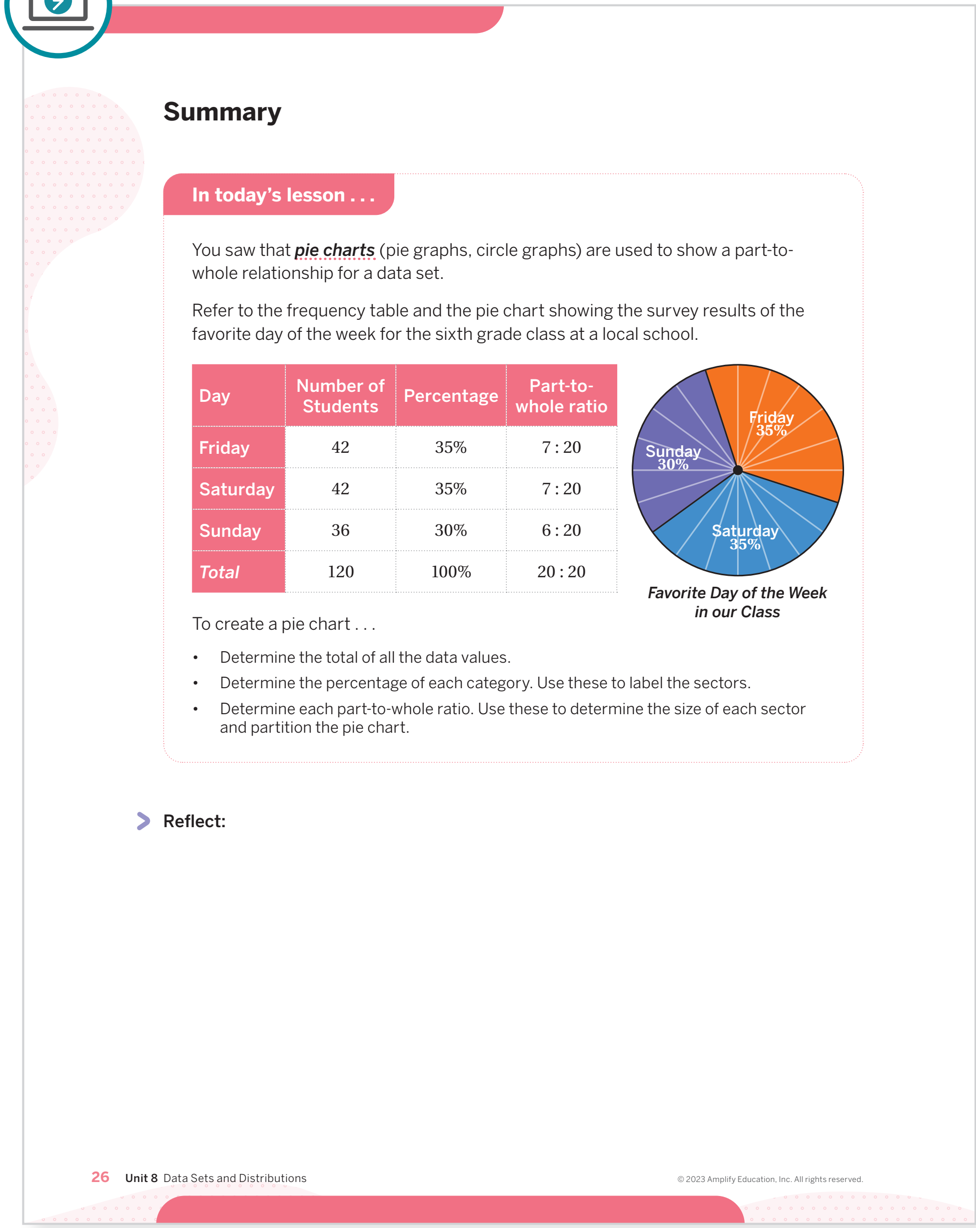
- "How can benchmark percentages such as 25% and 50% help partition the pie chart?"
  Sample response: I can start partitioning by shading one half and one quarter of the circle using a ruler.
- "What is your strategy to divide the circle into 16 sectors evenly?" **(MP5)**
  Sample response: I divided the circle into four equal sectors and then I divided each sector into halves two times.
- "What tools could be helpful in partitioning the circle?"
  Sample response: ruler, index cards or protractors
- "Can the same pie chart be used if only one of the values is changed? Why or why not?"
  No. Sample response: When one of the values changes, the total number of values also change. Therefore, the size of all the other sectors will be affected.
- "Will the size of each sector increase or decrease in the pie chart that will be created after receiving the additional donation amount?"
  Sample response: When the amount of donations increases, the donations category will have a greater percentage and a larger sector. The sizes of the remaining sectors will decrease.

Whole Class | 5 min

Summary

6.SP.B.4

Review and synthesize how to represent a set of data in a pie chart by examining the part-to-whole relationships in the data set.



Summary

In today's lesson . . .

You saw that ***pie charts*** (pie graphs, circle graphs) are used to show a part-to-whole relationship for a data set.

Refer to the frequency table and the pie chart showing the survey results of the favorite day of the week for the sixth grade class at a local school.

Day	Number of Students	Percentage	Part-to-whole ratio
Friday	42	35%	7 : 20
Saturday	42	35%	7 : 20
Sunday	36	30%	6 : 20
Total	120	100%	20 : 20

Favorite Day of the Week in our Class

To create a pie chart . . .

- Determine the total of all the data values.
- Determine the percentage of each category. Use these to label the sectors.
- Determine each part-to-whole ratio. Use these to determine the size of each sector and partition the pie chart.

Reflect:

Synthesize

Display the table and pie chart from the Summary.

Highlight how benchmark fractions and percentages can help to reason about the data displayed in a pie chart. Using common relationships such as halves, quarters, fifths, or tenths can help when partitioning a pie chart. They can also help a viewer to quickly orient themselves to the relative sizes of each sector.

Formalize vocabulary:

- *pie chart*
- *sector*

Ask:

- "What is the part in the part-to-whole relationship in a data set? What is the *whole*?"
  Sample response: The whole is the total number of values. The ratio of the value of each category to the total determines the part-to-whole relationship.
- "How do you decide how many partitions the pie chart should have?"
  Sample response: I can use the least common denominator of each ratio to represent the number of partitions.
- "Is it possible to create a pie chart without first partitioning it into equal sectors?"
  Yes. Sample response: I can use benchmark fractions such as one half and one quarter, if they exist or I can calculate the size of each sector separately.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What information does a pie chart display that a dot plot does not?"

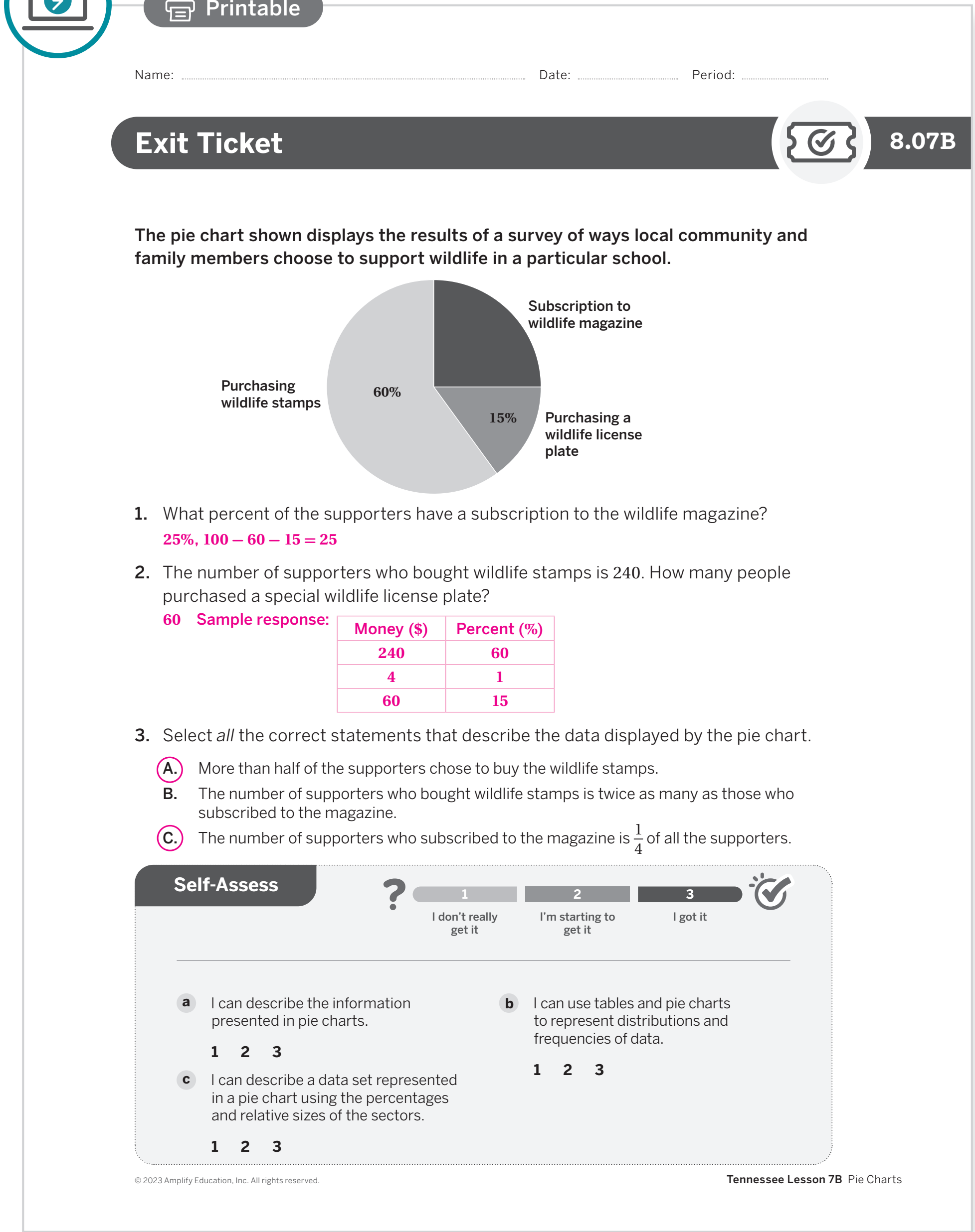
Independent | 5 min

Exit Ticket

6.SP.B.4

Students demonstrate their understanding by analyzing the data displayed in a pie chart.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

8.07B

The pie chart shown displays the results of a survey of ways local community and family members choose to support wildlife in a particular school.

Subscription to wildlife magazine

Purchasing wildlife stamps 60%

Purchasing a wildlife license plate 15%

1. What percent of the supporters have a subscription to the wildlife magazine?

   25%, 100 − 60 − 15 = 25

2. The number of supporters who bought wildlife stamps is 240. How many people purchased a special wildlife license plate?

   60 Sample response:

Money ($)	Percent (%)
240	60
4	1
60	15

3. Select *all* the correct statements that describe the data displayed by the pie chart.

   [✓ A.] More than half of the supporters chose to buy the wildlife stamps.

   B. The number of supporters who bought wildlife stamps is twice as many as those who subscribed to the magazine.

   [✓ C.] The number of supporters who subscribed to the magazine is $\frac{1}{4}$ of all the supporters.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can describe the information presented in pie charts.

1 2 3

b I can use tables and pie charts to represent distributions and frequencies of data.

1 2 3

c I can describe a data set represented in a pie chart using the percentages and relative sizes of the sectors.

1 2 3

Success looks like . . .

- **Language Goal:** Describing each category in a data set as a portion of the whole using a pie chart. **(Speaking and Writing)**
  - » Finding the missing percentage in Problem 1.
  - » Identifying correct statements in Problem 3.
- **Goal:** Calculating equivalent ratios to help create pie charts for a set of data.
  - » Using the ratio between the sectors of stamps and license plates to determine the number of supporters who purchased license plates in Problem 2.
- **Language Goal:** Comparing pie charts to other visual data displays. **(Speaking and Listening)**

Suggested next steps

If students add the percents for the other two categories for Problem 1, consider:

- Reviewing Activity 1.
- Assigning Practice Problem 1.
- Asking, "What fraction of the pie do the subscriptions seem to cover?"

If students select choice B, consider:

- Asking, "Is the stamps sector exactly twice as big as the magazine sector?"

Professional Learning

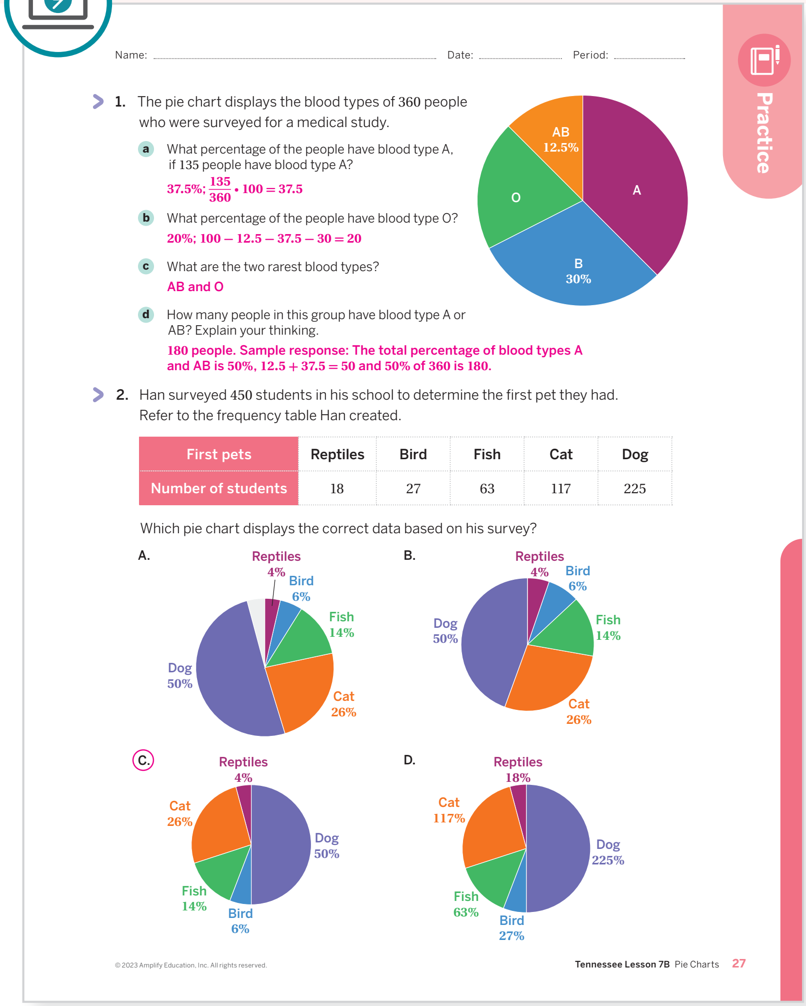
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier this year on ratios and percentages, what similarities and differences do you see?
- In this lesson, students used pie charts to display data. How did that build on the earlier work students did with ratios and percentages? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. The pie chart displays the blood types of 360 people who were surveyed for a medical study.

a What percentage of the people have blood type A, if 135 people have blood type A?

$37.5\%; \frac{135}{360} \cdot 100 = 37.5$

b What percentage of the people have blood type O?

$20\%; 100 - 12.5 - 37.5 - 30 = 20$

c What are the two rarest blood types?

AB and O

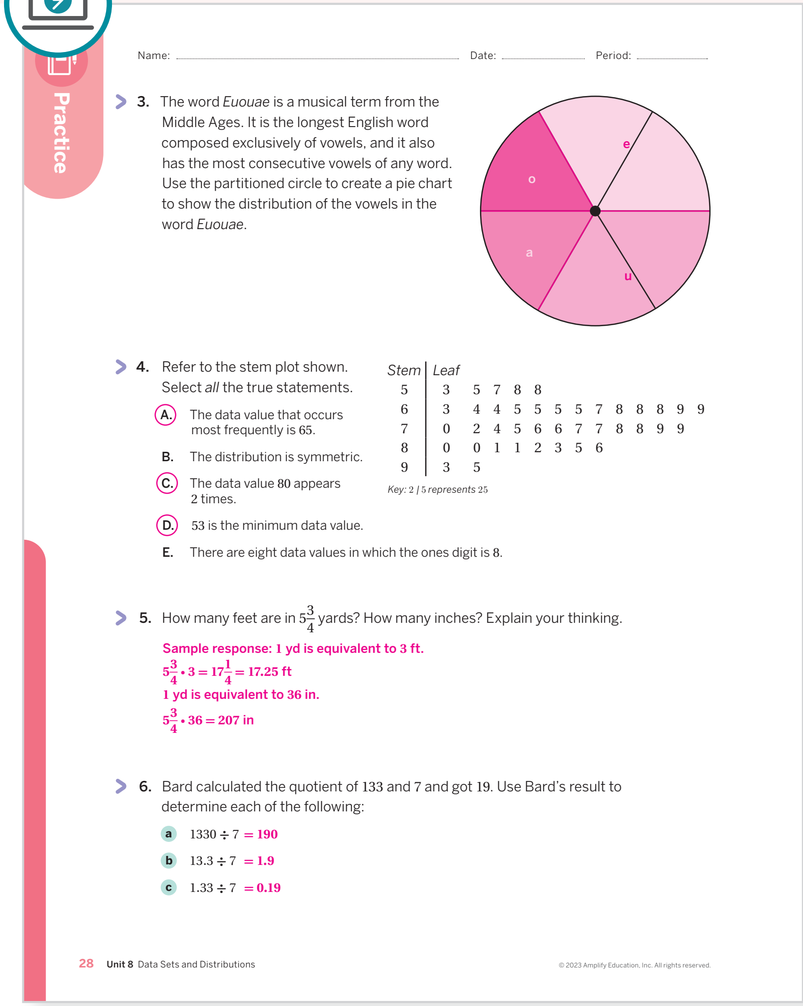
d How many people in this group have blood type A or AB? Explain your thinking.

180 people. Sample response: The total percentage of blood types A and AB is 50%, $12.5 + 37.5 = 50$ and 50% of 360 is 180.

2. Han surveyed 450 students in his school to determine the first pet they had. Refer to the frequency table Han created.

First pets	Reptiles	Bird	Fish	Cat	Dog
Number of students	18	27	63	117	225

Which pie chart displays the correct data based on his survey?



Name: Date: Period:

3. The word *Euouae* is a musical term from the Middle Ages. It is the longest English word composed exclusively of vowels, and it also has the most consecutive vowels of any word. Use the partitioned circle to create a pie chart to show the distribution of the vowels in the word *Euouae*.

4. Refer to the stem plot shown. Select *all* the true statements.

[✓ A.] The data value that occurs most frequently is 65.

B. The distribution is symmetric.

[✓ C.] The data value 80 appears 2 times.

[✓ D.] 53 is the minimum data value.

E. There are eight data values in which the ones digit is 8.

Stem	Leaf
5	3 5 7 8 8
6	3 4 4 5 5 5 5 7 8 8 8 9 9
7	0 2 4 5 6 6 7 7 8 8 9 9
8	0 0 1 1 2 3 5 6
9	3 5

Key: 2 | 5 *represents* 25

5. How many feet are in $5\frac{3}{4}$ yards? How many inches? Explain your thinking.

Sample response: 1 yd is equivalent to 3 ft.

$5\frac{3}{4} \cdot 3 = 17\frac{1}{4} = 17.25$ ft

1 yd is equivalent to 36 in.

$5\frac{3}{4} \cdot 36 = 207$ in

6. Bard calculated the quotient of 133 and 7 and got 19. Use Bard's result to determine each of the following:

a $1330 \div 7 = 190$

b $13.3 \div 7 = 1.9$

c $1.33 \div 7 = 0.19$

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 1	6.SP.B.4	2
	2	Activity 1	6.SP.B.4	2
	3	Activity 2	6.SP.B.4	2
Spiral	4	Unit 8 Tennessee Lesson 7A	6.SP.B.4	2
	5	Unit 2 Tennessee Lesson 17A	6.RP.A.3d	2
Formative	6	Unit 8 Lesson 8	6.NS.B.3	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Measuring Variability

Let’s explore two ways to describe the variability of data sets.

Focus

Goals

1. **Language Goal:** Calculate the range of a data set and interpret what it tells about a scenario. **(Speaking and Listening, Writing)**
2. **Language Goal:** Comprehend the range as a measure of variability, which describes the span of the data. **(Writing)**
3. **Language Goal:** Identify and interpret the numbers in the five-number summary for a data set: the minimum, first quartile (Q1), median (Q2), third quartile (Q3), and the maximum. **(Writing)**

Coherence

Today

Students expand on their understanding of median to split data into quarters by determining three values called *quartiles*. They relate the three quartiles to the 25th, 50th, and 75th percentiles, which are useful in describing a distribution. Students also identify the maximum and minimum values of the data set, and combining those with the quartiles, they can identify the five-number summary. Students also explore the range as a way to describe a data set’s spread and summarize its variability with a single number.

Previously

In Lesson 10, students decomposed a data set into two halves by identifying the median.

Coming Soon

In Lesson 15, students will use the five-number summary of a data set to construct another representation of the distribution — a box plot.

Rigor

- Students further their **conceptual understanding** of measures of variability.
- Students build **procedural skills** for constructing box plots.

Standards

Addressing

6.SP.B.5c

Give quantitative measures of center (median and/or mean) and variability (range) as well as describing any overall pattern with reference to the context in which the data were gathered.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent
		MP2		
6.SP.B.5c	6.SP.B.5c	6.SP.B.5c	6.SP.B.5c	6.SP.B.5c

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (answers, for display)
- Anchor Chart, *Five Number Summary*

Math Language Development

New words

- ***five-number summary***
- ***range****

Review words

- *median*
- *variability*

*Students may confuse the statistical term *range* with the various everyday uses of the term. Be ready to address the similarities and differences between them.

Amps | Featured Activity

Activity 1
Five-Number Formative Feedback

Students receive immediate feedback about their calculations so that they can move forward with correct values.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might not adequately analyze the situation in order to generate a five-number summary. The process requires both quantitative and abstract reasoning **(MP2)**. Remind students that generating the five-number summary is a matter of calculating the quartiles. Encourage them to identify the problem by listing what they need to find for the five-number summary. Then the process of interpreting those values requires processing of a more abstract nature. By beginning with the end in mind, students can discern the information they need to identify and interpret the five-number summary of a data set.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

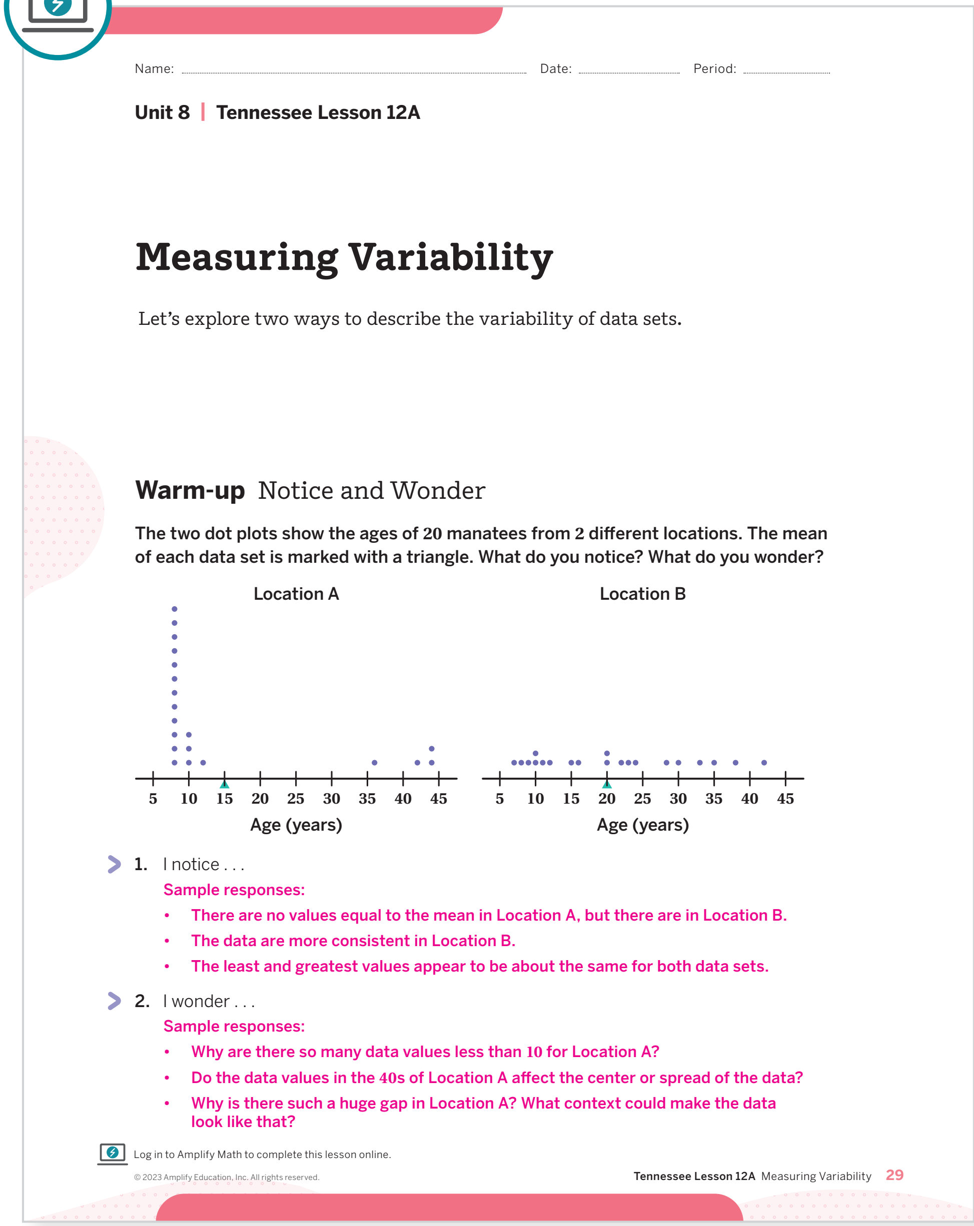
- The **Warm-up** may be omitted.
- **Activity 1** can be done as a whole class, and Problem 4 may also be omitted.

Pairs | 5 min

Warm-up Notice and Wonder

6.SP.B.5c

Students study two distributions that look very different but have a similar mean, demonstrating the need for a way to quantify and compare variability.



Name: ______ Date: ______ Period: ______

Unit 8 | Tennessee Lesson 12A

Measuring Variability

Let's explore two ways to describe the variability of data sets.

Warm-up Notice and Wonder

The two dot plots show the ages of 20 manatees from 2 different locations. The mean of each data set is marked with a triangle. What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - There are no values equal to the mean in Location A, but there are in Location B.
   - The data are more consistent in Location B.
   - The least and greatest values appear to be about the same for both data sets.
2. I wonder . . .
   Sample responses:
   - Why are there so many data values less than 10 for Location A?
   - Do the data values in the 40s of Location A affect the center or spread of the data?
   - Why is there such a huge gap in Location A? What context could make the data look like that?

Log in to Amplify Math to complete this lesson online.

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Tennessee Lesson 12A Measuring Variability 29

1 Launch

Use the *Notice and Wonder* routine.

2 Monitor

Help students get started by asking "How would you describe the distribution for Location A? and Location B?"

Look for points of confusion:

- **Thinking that one of the means must be incorrect.** Remind students how to calculate the mean.

Look for productive strategies:

- Using the mathematical language of the unit in their observations, such as *peak, gap, cluster, typical, center, spread, variability,* and *mean.*
- Noticing that both sets of data have similar minimum and maximum values, but the data points between them are distributed very differently.
- Connecting or comparing the two locations in their observations and questions.

3 Connect

Display the two dot plots.

Have students share their responses, recording them around the plots if possible. Have others share agreement or disagreement, and alternative ways of thinking.

Ask,

- "Does knowing the mean help you to understand the distribution at all?"
- "Can you think of any ways to use the data to describe how spread out it is?"

Highlight that there are ways to quantify and describe the variability of different distributions.

Power-up

To power up students' ability to calculate sums and differences with decimal values, have students complete:

1. $3.0 + 4.1$ $= 7.1$
2. $4.2 + 4.1$ $= 8.3$
3. $4.2 - 3.8$ $= 0.4$
4. $5 - 3.8$ $= 1.2$

Use: Before the Warm-up.

Informed by: Performance on Lesson 12, Practice Problem 6.

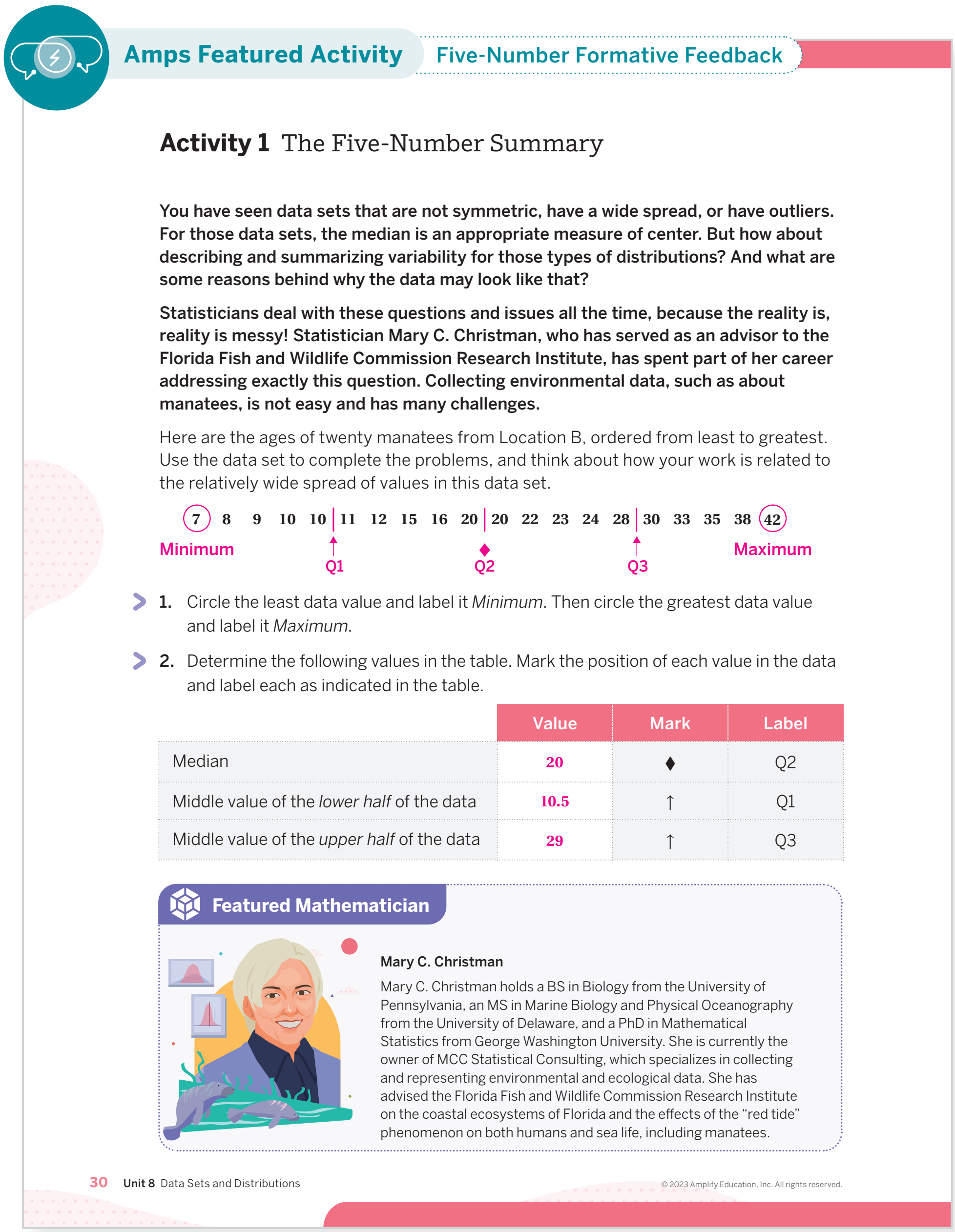
Pairs | 15 min

Activity 1 The Five-Number Summary

6.SP.B.5c

Students are introduced to quartiles and the five-number summary for a data set while identifying and interpreting those values.

Amps Featured Activity Five-Number Formative Feedback



Activity 1 The Five-Number Summary

You have seen data sets that are not symmetric, have a wide spread, or have outliers. For those data sets, the median is an appropriate measure of center. But how about describing and summarizing variability for those types of distributions? And what are some reasons behind why the data may look like that?

Statisticians deal with these questions and issues all the time, because the reality is, reality is messy! Statistician Mary C. Christman, who has served as an advisor to the Florida Fish and Wildlife Commission Research Institute, has spent part of her career addressing exactly this question. Collecting environmental data, such as about manatees, is not easy and has many challenges.

Here are the ages of twenty manatees from Location B, ordered from least to greatest. Use the data set to complete the problems, and think about how your work is related to the relatively wide spread of values in this data set.

⑦ 8 9 10 10 | 11 12 15 16 20 | 20 22 23 24 28 | 30 33 35 38 ㊷

Minimum ↑ Q1 ◆ Q2 ↑ Q3 Maximum

1. Circle the least data value and label it *Minimum*. Then circle the greatest data value and label it *Maximum*.

2. Determine the following values in the table. Mark the position of each value in the data and label each as indicated in the table.

	Value	Mark	Label
Median	20	◆	Q2
Middle value of the *lower half* of the data	10.5	↑	Q1
Middle value of the *upper half* of the data	29	↑	Q3

Featured Mathematician

Mary C. Christman

Mary C. Christman holds a BS in Biology from the University of Pennsylvania, an MS in Marine Biology and Physical Oceanography from the University of Delaware, and a PhD in Mathematical Statistics from George Washington University. She is currently the owner of MCC Statistical Consulting, which specializes in collecting and representing environmental and ecological data. She has advised the Florida Fish and Wildlife Commission Research Institute on the coastal ecosystems of Florida and the effects of the "red tide" phenomenon on both humans and sea life, including manatees.

30 **Unit 8** Data Sets and Distributions

1 Launch

Say, "You will work with the data from Location B in the Warm-up to determine alternative ways of summarizing the spread with numbers. Be sure to think about what each value represents both in the data and in context." Give pairs 8–10 minutes to complete the activity.

2 Monitor

Help students get started by asking "How can you determine the median of this set of data?"

Look for points of confusion:

- **Not knowing whether to include the data values in their 20s when calculating Q1 and Q3 (Activity 1).** Explain to students that when the median is not one of the data values, i.e., when it is the average of the middle two values, the two values are included with their lower and upper halves.
- **Not knowing what to do when there is an even number of values to the left or right of the mean.** Remind students that Q1 and Q3 are the medians of that half of the data. Consider covering one of the halves so students can see only the half of the data they are working with.

Look for productive strategies:

- Recognizing that the minimum and maximum values are included in the lower and upper sections of data points.
- Understanding that Q1 and Q3 are determined as if they are the medians of the two halves of data around Q2.
- Analyzing and interpreting what the data between each of the quartiles represents, and, specifically understanding that, while 29 is not a data value, it is included in statements about both the lower 75% and upper 25% of data (and, possibly, noting the same is true for 20 being part of both the upper and lower halves).

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

In Problem 2, annotate the value marking Q1 with "one quarter of the manatees are 10.5 years old or younger" to help students visualize the data set divided into fourths.

Featured Mathematician

Mary C. Christman

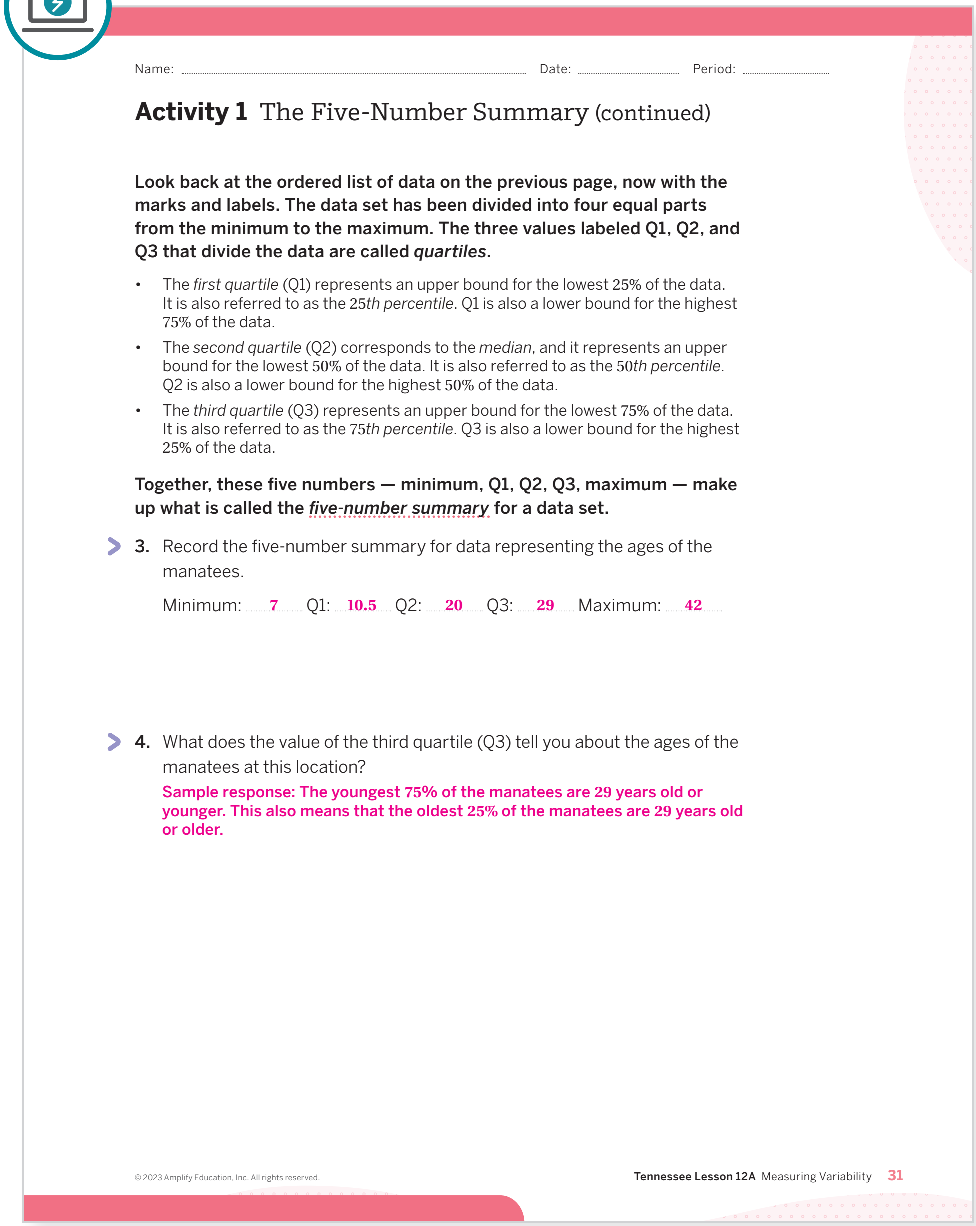
Have students read about Mary C. Christman, who uses statistics to collect and represent environmental and ecological data.

Pairs | 15 min

Activity 1 The Five-Number Summary (continued)

6.SP.B.5c

Students are introduced to quartiles and the five-number summary for a data set while identifying and interpreting those values.



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Activity 1 The Five-Number Summary (continued)

Look back at the ordered list of data on the previous page, now with the marks and labels. The data set has been divided into four equal parts from the minimum to the maximum. The three values labeled Q1, Q2, and Q3 that divide the data are called *quartiles*.

- The *first quartile* (Q1) represents an upper bound for the lowest 25% of the data. It is also referred to as the 25*th percentile*. Q1 is also a lower bound for the highest 75% of the data.
- The *second quartile* (Q2) corresponds to the *median*, and it represents an upper bound for the lowest 50% of the data. It is also referred to as the 50*th percentile*. Q2 is also a lower bound for the highest 50% of the data.
- The *third quartile* (Q3) represents an upper bound for the lowest 75% of the data. It is also referred to as the 75*th percentile*. Q3 is also a lower bound for the highest 25% of the data.

Together, these five numbers — minimum, Q1, Q2, Q3, maximum — make up what is called the *five-number summary* for a data set.

3. Record the five-number summary for data representing the ages of the manatees.

Minimum: 7 Q1: 10.5 Q2: 20 Q3: 29 Maximum: 42

4. What does the value of the third quartile (Q3) tell you about the ages of the manatees at this location?

Sample response: The youngest 75% of the manatees are 29 years old or younger. This also means that the oldest 25% of the manatees are 29 years old or older.

3 Connect

Display the Activity 1 PDF for students to check their responses from Problems 1–3.

Ask:

- "How did you determine where to mark Q1, Q2, and Q3 for this data set?" Note: This discussion should focus on how to work with an even number of data values and particularly what to do with the two numbers used to determine the median when determining Q1 and Q3. Activity 2 will present problems that are the opposite of this (i.e., the median is the middle value because it is an odd number of data values).
- "How do these five numbers help you understand the distribution and spread of the data?" Because each section of the data contains (at least) 25% of the values, the closer together a pair of numbers in the summary is, the more clustered the data values in that range; and vice versa.

Define:

- A ***quartile*** as one of three numbers (Q1, Q2, Q3) that divide a data set into 4 sections so that each contains the same number of data values.
- The ***five-number summary*** for a data set summarizes a distribution by five specific values: its minimum, first quartile, median, third quartile, and maximum.

Have students share their responses to Problem 4.

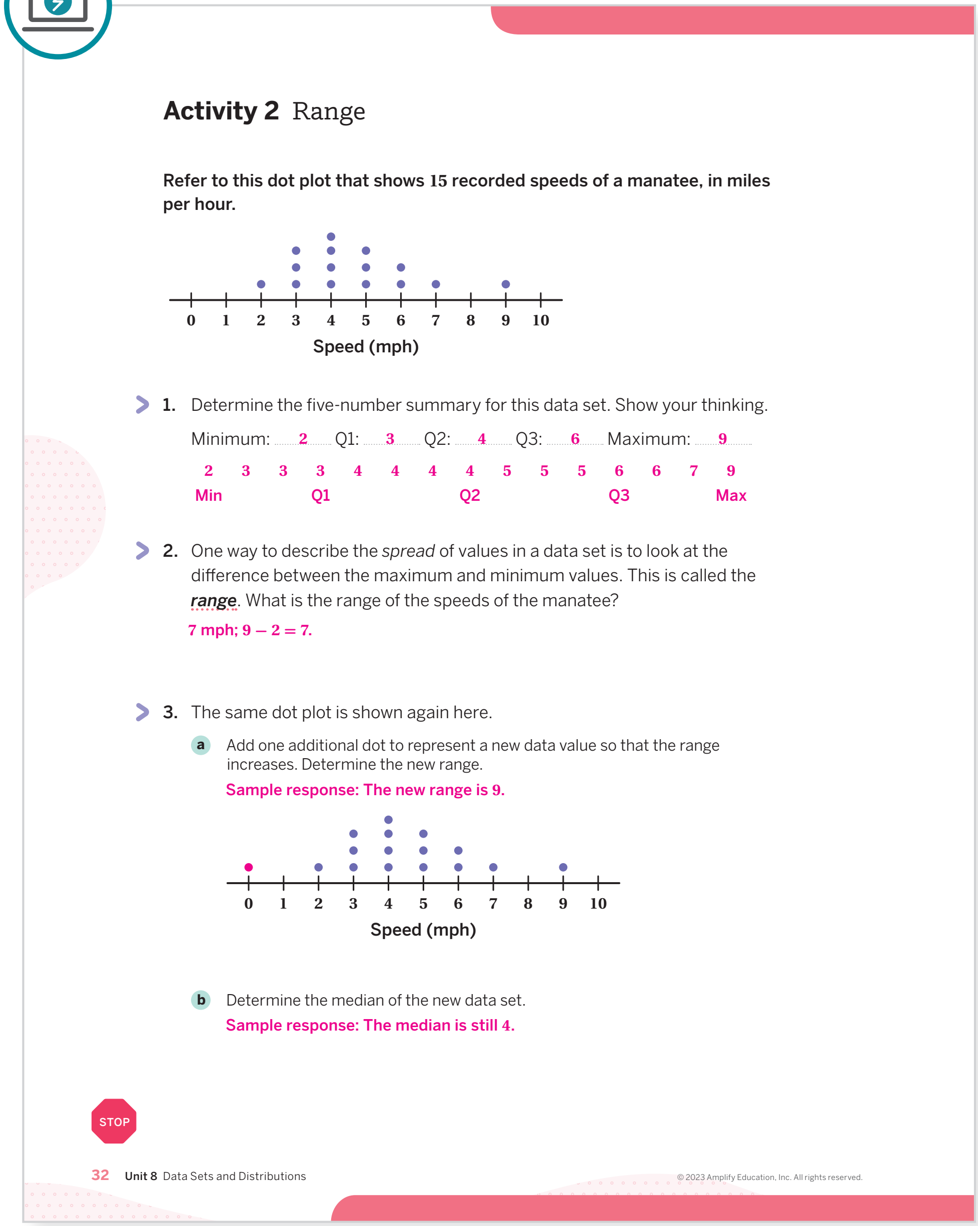
Highlight that the five-number summary helps describe a data set without listing or showing every value. It summarizes the data by dividing it into four equal parts, or quartiles, with the median determining the middle point of the data. The closer the values that bound a section of the data, the more of a cluster the data points in that range represent.

Pairs | 15 min

MP2
6.SP.B.5c

Activity 2 Range

Students determine another measure of variability for data sets – the range – and use it to describe variability in context **(MP2).**



Activity 2 Range

Refer to this dot plot that shows 15 recorded speeds of a manatee, in miles per hour.

1. Determine the five-number summary for this data set. Show your thinking.

Minimum: 2 Q1: 3 Q2: 4 Q3: 6 Maximum: 9

2	3	3	3	4	4	4	4	5	5	5	6	6	7	9
Min			Q1				Q2				Q3			Max

2. One way to describe the *spread* of values in a data set is to look at the difference between the maximum and minimum values. This is called the ***range***. What is the range of the speeds of the manatee?

7 mph; 9 – 2 = 7.

3. The same dot plot is shown again here.

   a. Add one additional dot to represent a new data value so that the range increases. Determine the new range.

   Sample response: The new range is 9.

   b. Determine the median of the new data set.

   Sample response: The median is still 4.

STOP

32 **Unit 8** Data Sets and Distributions

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider displaying or providing a checklist similar to the one shown to help students organize their thinking around determining the five number summary.

- Determine the minimum and maximum.
- Determine the median (Q2) — the middle of the data set.
- Determine Q1 — halfway between the minimum and Q2.
- Determine Q3 — halfway between the maximum and Q2.

1 Launch

Activate background knowledge by asking, "How is this data set different?" There is an odd number of data values. Display the Anchor Chart, *Five Number Summary*.

2 Monitor

Help students get started by asking "How would you identify the quartiles?" List the values of all the data values in order and then count off to determine the median, and then do the same for Q1 and Q3.

Look for points of confusion:

- **Not knowing when to include certain values when determining quartiles.** Have students review the data set from Activity 1 and ask, "For Q1 and Q3, did you include the two values used to determine the median? Why?" Then have them look at the current data set and ask, "Is the median an average of two points or is it the data value?"

Look for productive strategies:

- Accurately identifying the five-number summaries, and using those to calculate the range.
- Associating the range of values around the median between Q1 and Q3 with typical values and with 50% of the data.

3 Connect

Define the ***range*** of a data set as a measure of variability that is calculated as the difference between the maximum and minimum values in the data set.

Ask:

- "What does a range of 7 mph tell you about the speeds of these manatees?" The slowest and fastest manatees' speeds differed by 7 mph. The greatest difference in speeds was 7 mph.
- "In general, what does a greater range tell you?" There is a wider overall spread in the data.
- "What effect, if any, did adding a new dot have on the median?" It did not have any effect.

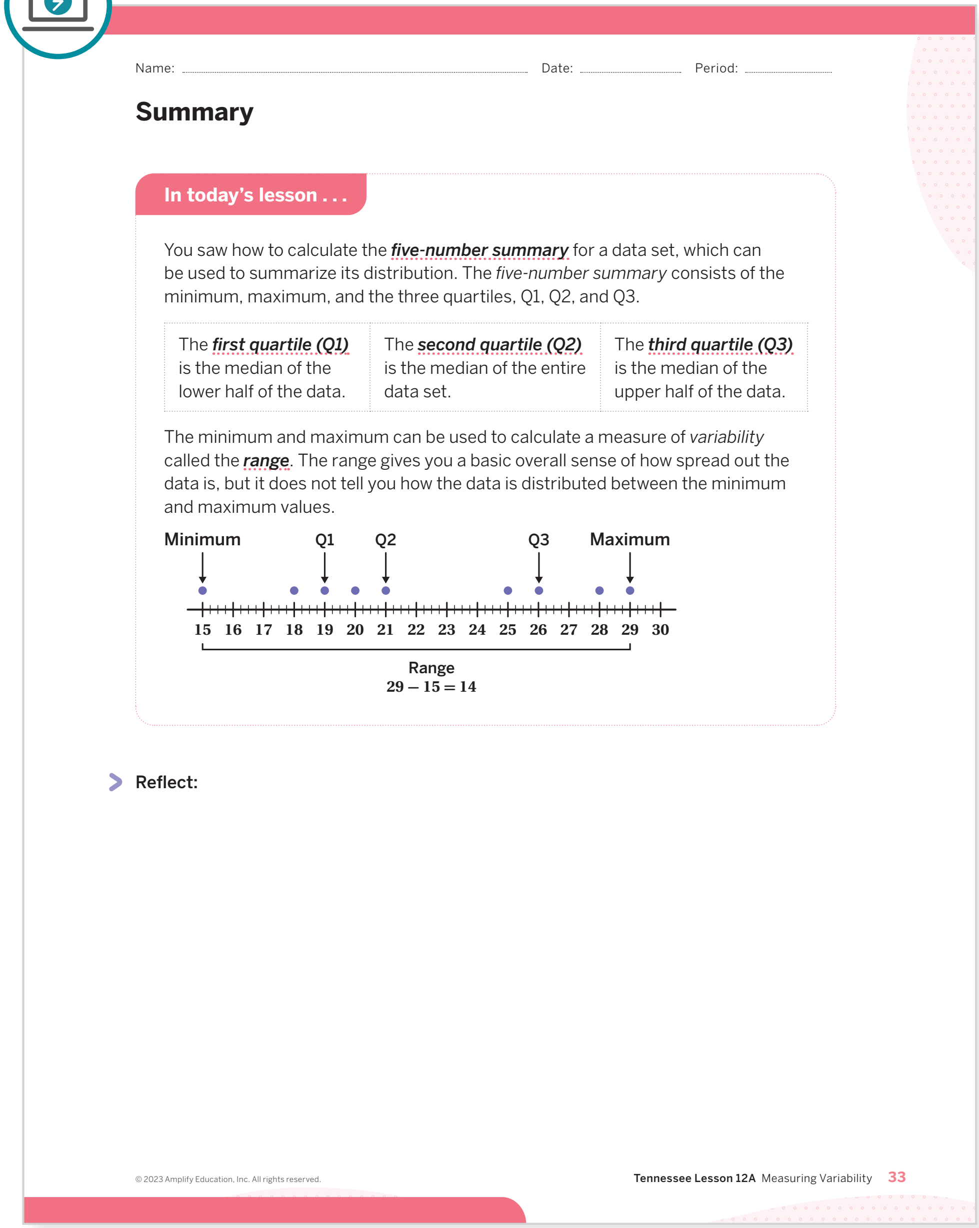
Highlight how the range and five number summary are represented in a dot plot, and that the range encompasses 100% of the data.

Whole Class | 5 min

Summary

6.SP.B.5c

Review and synthesize how the five-number summary and the range both help to describe the variability of a data set.



Name: Date: Period:

Summary

In today's lesson . . .

You saw how to calculate the ***five-number summary*** for a data set, which can be used to summarize its distribution. The *five-number summary* consists of the minimum, maximum, and the three quartiles, Q1, Q2, and Q3.

The ***first quartile (Q1)*** is the median of the lower half of the data.	The ***second quartile (Q2)*** is the median of the entire data set.	The ***third quartile (Q3)*** is the median of the upper half of the data.

The minimum and maximum can be used to calculate a measure of *variability* called the ***range***. The range gives you a basic overall sense of how spread out the data is, but it does not tell you how the data is distributed between the minimum and maximum values.

Reflect:

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Tennessee Lesson 12A Measuring Variability 33

Synthesize

Highlight that the range is a measure of variability that describes the span of the data, but it does not tell us much about the distribution of the data. The five-number summary will be used again when students work with box plots in the next lesson.

Formalize vocabulary:

- ***five-number summary***
- ***range***
- ***variability***

Ask:

- "What are the quartiles for a numerical data set?" Numbers that show where you can divide the data set, so that the data are in quarters or fourths.
- "What is the relationship between the quartiles and the median?" The second quartile is also the median.
- "What is the relationship between the five-number summary and the range?" The five-number summary includes the minimum and maximum. The range is the difference between these values.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is thinking about measures of variability different from thinking about measures of center?"

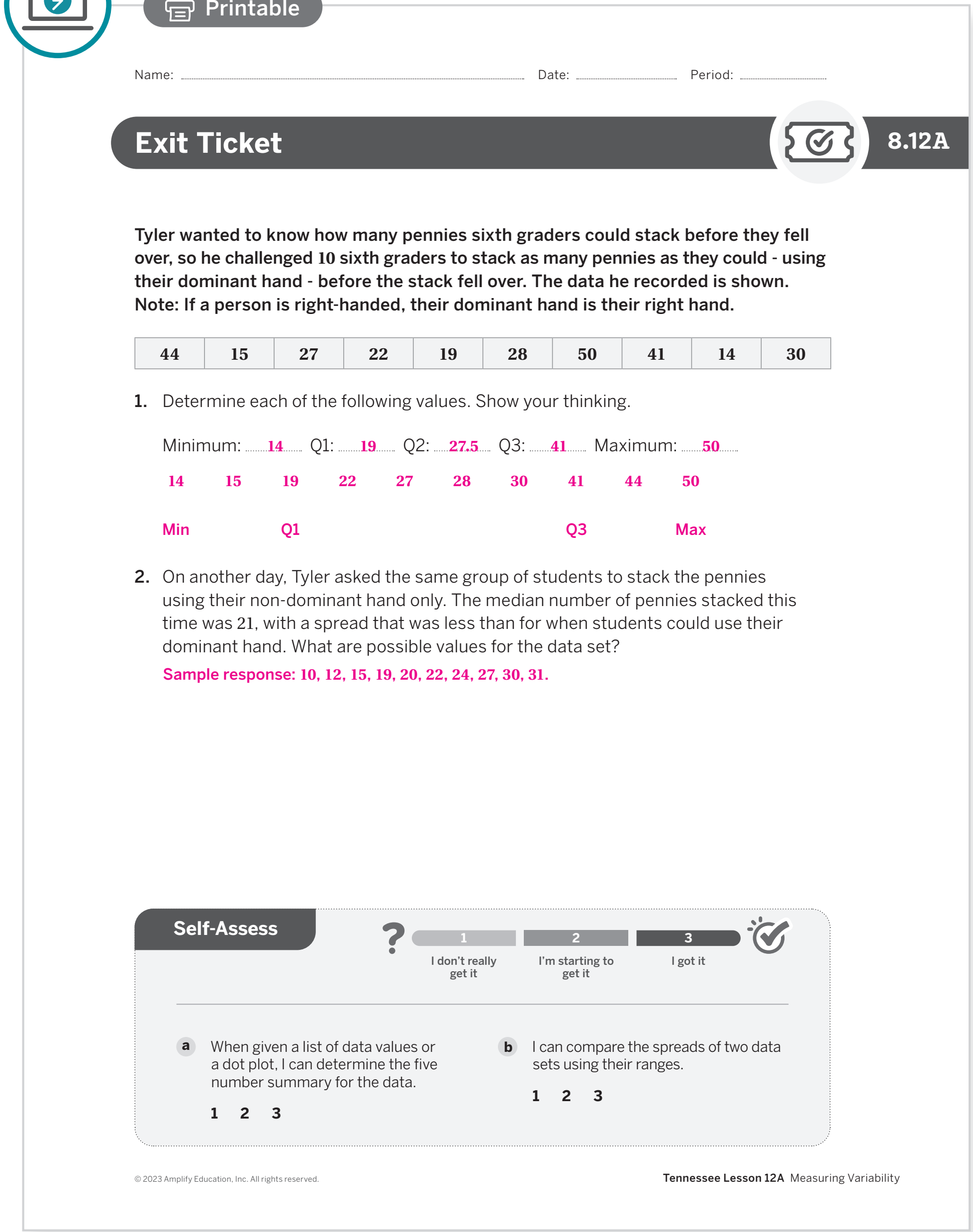
Independent | 5 min

Exit Ticket

6.SP.B.5c

Students demonstrate their understanding by determining the five-number summary and creating a data set with a certain median and range.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

8.12A

Tyler wanted to know how many pennies sixth graders could stack before they fell over, so he challenged 10 sixth graders to stack as many pennies as they could - using their dominant hand - before the stack fell over. The data he recorded is shown. Note: If a person is right-handed, their dominant hand is their right hand.

44	15	27	22	19	28	50	41	14	30

1. Determine each of the following values. Show your thinking.

Minimum: 14 Q1: 19 Q2: 27.5 Q3: 41 Maximum: 50

14	15	19	22	27	28	30	41	44	50
Min		Q1					Q3		Max

2. On another day, Tyler asked the same group of students to stack the pennies using their non-dominant hand only. The median number of pennies stacked this time was 21, with a spread that was less than for when students could use their dominant hand. What are possible values for the data set?

Sample response: 10, 12, 15, 19, 20, 22, 24, 27, 30, 31.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

(a) When given a list of data values or a dot plot, I can determine the five number summary for the data.

1 2 3

(b) I can compare the spreads of two data sets using their ranges.

1 2 3

Tennessee Lesson 12A Measuring Variability

Success looks like . . .

- **Language Goal:** Calculating the range of a data set and interpreting what it tells about a scenario. **(Speaking and Listening, Writing)**
- **Language Goal:** Comprehending the range as a measure of variability, which describes the span of the data. **(Writing)**
  - » In Problem 2, choosing maximum and minimum values that have a difference of less than 36.
- **Language Goal:** Identifying and interpreting the numbers in the five-number summary for a data set: the minimum, first quartile (Q1), median (Q2), third quartile (Q3), and the maximum. **(Writing)**
  - » Correctly identifying each of the values in the five-number summary in Problem 1.

Suggested next steps

If students do not first place the data in numerical order, consider:

- Asking, "To determine the median, what must be true about the data set?"

If students have difficulty determining the quartiles, consider asking:

- "How many data values are in this set?" 10
- "If you divide it in half, how many values should be in each half?" 5
- "What value is between the lower five values and the upper five values?" I need to find the average of the fifth and sixth data values, which are 50 and 53.

Professional Learning

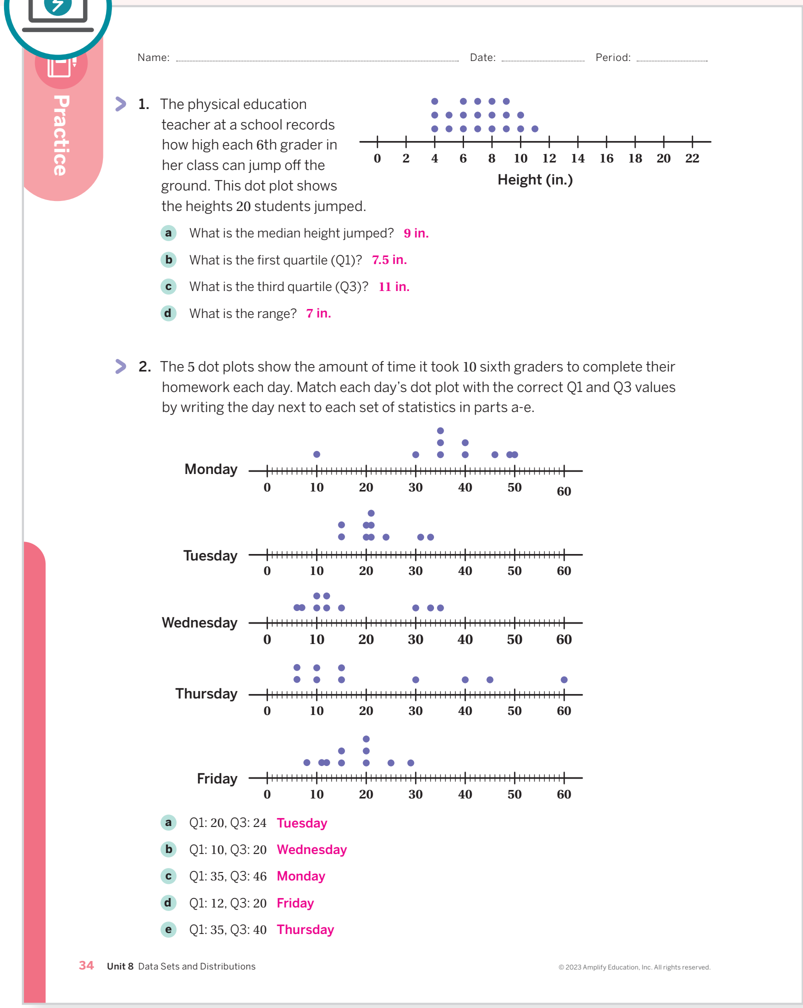
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did determining the five-number summary reveal about your students as procedural learners?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________________ Date: ________ Period: ________

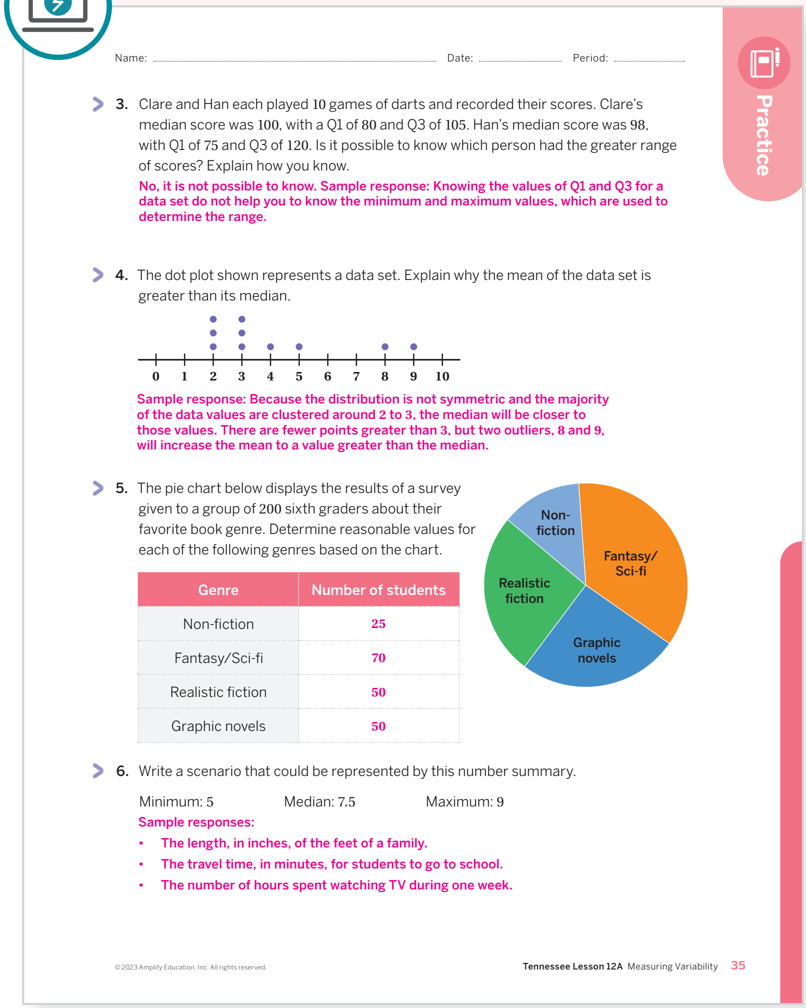
Practice

1. The physical education teacher at a school records how high each 6th grader in her class can jump off the ground. This dot plot shows the heights 20 students jumped.

   a. What is the median height jumped? **9 in.**
   b. What is the first quartile (Q1)? **7.5 in.**
   c. What is the third quartile (Q3)? **11 in.**
   d. What is the range? **7 in.**

2. The 5 dot plots show the amount of time it took 10 sixth graders to complete their homework each day. Match each day's dot plot with the correct Q1 and Q3 values by writing the day next to each set of statistics in parts a-e.

   a. Q1: 20, Q3: 24 **Tuesday**
   b. Q1: 10, Q3: 20 **Wednesday**
   c. Q1: 35, Q3: 46 **Monday**
   d. Q1: 12, Q3: 20 **Friday**
   e. Q1: 35, Q3: 40 **Thursday**



Name: ________________ Date: ________ Period: ________

Practice

3. Clare and Han each played 10 games of darts and recorded their scores. Clare's median score was 100, with a Q1 of 80 and Q3 of 105. Han's median score was 98, with Q1 of 75 and Q3 of 120. Is it possible to know which person had the greater range of scores? Explain how you know.
   No, it is not possible to know. Sample response: Knowing the values of Q1 and Q3 for a data set do not help you to know the minimum and maximum values, which are used to determine the range.

4. The dot plot shown represents a data set. Explain why the mean of the data set is greater than its median.

   Sample response: Because the distribution is not symmetric and the majority of the data values are clustered around 2 to 3, the median will be closer to those values. There are fewer points greater than 3, but two outliers, 8 and 9, will increase the mean to a value greater than the median.

5. The pie chart below displays the results of a survey given to a group of 200 sixth graders about their favorite book genre. Determine reasonable values for each of the following genres based on the chart.

Genre	Number of students
Non-fiction	**25**
Fantasy/Sci-fi	**70**
Realistic fiction	**50**
Graphic novels	**50**

6. Write a scenario that could be represented by this number summary.

   Minimum: 5 Median: 7.5 Maximum: 9

   Sample responses:
   - **The length, in inches, of the feet of a family.**
   - **The travel time, in minutes, for students to go to school.**
   - **The number of hours spent watching TV during one week.**

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activities 1	6.SP.B.5c	1
	2	Activities 2	6.SP.B.5c	2
	3	Activities 2	6.SP.B.5c	2
Spiral	4	Unit 8 Lesson 11	6.SP.B.5c	2
	5	Unit 8 Tennessee Lesson 7B	6.SP.A.3d	3
Formative	6	Unit 8 Lesson 13	6.SP.B.5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.