Amplify Math TENNESSEE

Grade 7

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Volume 1

Unit 1 Scale Drawings

Certain objects in our universe exist at sizes and distances that are impossible for our eyes to see (such as a red blood cell, or Jupiter). In this unit, students harness the power of scaling — bringing large and small objects to a manageable size without distorting them.

Unit Narrative:
Life in the Little Big City

PRE-UNIT READINESS ASSESSMENT

LAUNCH

1.01 Scale-y Shapes ..... 4A — MP1, 2, 8

Sub-Unit 1 Scaled Copies 11

Sub-Unit Narrative:
How do you get the perfect fit?
If we are making a larger or smaller copy of something, it needs to look right. The key is the scale factor.

1.02 What Are Scaled Copies? ..... 12A — **7.G.A.1,** MP6, 7

1.03 Corresponding Parts and Scale Factors ..... 19A — **7.G.A.1, 7.RP.A.2,** MP6, 7

1.04 Making Scaled Copies ..... 26A — **7.G.A.1, 7.EE.B.3b,** MP3, 5, 7

1.05 The Size of the Scale Factor ..... 32A — **7.G.A.1,** MP7, 8

1.06 Scaling Area ..... 39A — **7.G.A.1,** MP3, 7, 8

Sub-Unit 2 Scale Drawings 47

Sub-Unit Narrative:
Who was the King of Monsters?
We use maps and other scale drawings to help simplify large, complex places. Interpreting them is about knowing the scale and how to measure.

1.07 Scale Drawings ..... 48A — **7.G.A.1,** MP5, 7

1.08 Creating Scale Drawings ..... 54A — **7.G.A.1, 7.EE.B.3b,** MP7

1.09 Scale Drawings and Maps ..... 61A — **7.G.A.1,** MP1

1.10 Changing Scales in Scale Drawings ..... 67A — **7.G.A.1,** MP2, 7

1.11 Scales Without Units ..... 74A — **7.G.A.1,** MP6, 7

1.12 Units in Scale Drawings ..... 80A — **7.G.A.1,** MP3

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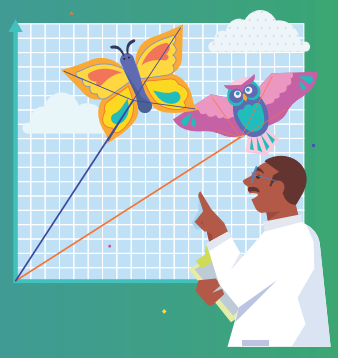
1.13 Build Your Brand ..... 86A — **7.G.A.1,** MP1, 6

END-OF-UNIT ASSESSMENT

Unit 2 Introducing Proportional Relationships

When we exchange money from one currency to another, there is a rate that helps us find the amount of one currency equal in value to the other. Students see that a rate is at the heart of every proportional relationship as they encounter problems across cultures where two quantities are directly related.

Unit Narrative:
The World in Proportion



PRE-UNIT READINESS ASSESSMENT

LAUNCH

2.01 Making Music ..... 94A — MP7

Sub-Unit 1 Representing Proportional Relationships With Tables and Equations 101

2.02 Introducing Proportional Relationships With Tables ..... 102A — **7.RP.A.2, 7.RP.A.2a, 7.RP.A.2b,** MP7

2.03 More About the Constant of Proportionality ..... 108A — **7.RP.A.2b, 7.RP.A.2,** MP8

2.04 Comparing Relationships With Tables ..... 114A — **7.RP.A.2a, 7.RP.A.2b,** MP1, 3, 6

2.05 Proportional Relationships and Equations ..... 121A — **7.RP.A.2c,** MP7, 8

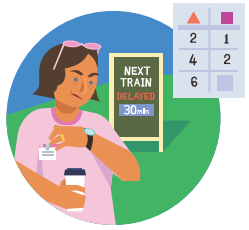
2.06 Speed and Equations ..... 127A — **7.RP.A.2c, 7.RP.A.1, 7.RP.A.2b, 7.EE.B.3a,** MP2, 8

2.07 Two Equations for Each Relationship ..... 133A — **7.RP.A.2c, 7.RP.A.2b,** MP2, 8

2.08 Using Equations to Solve Problems ..... 140A — **7.RP.A.2, 7.RP.A.2c, 7.EE.B.3b,** MP2

2.09 Comparing Relationships With Equations ..... 146A — **7.RP.A.2a, 7.RP.A.2, 7.RP.A.2b, 7.G.B.5,** MP7, 8

2.10 Solving Problems About Proportional Relationships ..... 154A — **7.RP.A.2, 7.RP.A.2c,** MP1, 2, 6



Sub-Unit 2 Representing Proportional Relationships With Graphs 161

2.11 Introducing Graphs of Proportional Relationships ..... 162A — **7.RP.A.2,** MP3, 7

2.12 Interpreting Graphs of Proportional Relationships ..... 168A — **7.RP.A.2d, 7.RP.A, 7.RP.A.2, 7.RP.A.2b,** MP2, 4, 6

2.13 Using Graphs to Compare Relationships ..... 176A — **7.RP.A.2,** MP1, 2

2.14 Two Graphs for Each Relationship ..... 183A — **7.RP.A.2, 7.RP.A.2b, 7.RP.A.2c, 7.RP.A.2d,** MP6, 7

2.15 Four Ways to Tell One Story (Part 1) ..... 189A — **7.RP.A.2, 7.RP.A.2b, 7.RP.A.2c, 7.RP.A.2d,** MP2, 6, 7

2.16 Four Ways to Tell One Story (Part 2) ..... 196A — **7.RP.A.2,** MP1, 8

CAPSTONE

2.17 Welcoming Committee ..... 202A — **7.RP.A.2, 7.RP.A.2c,** MP1, 5

END-OF-UNIT ASSESSMENT

Sub-Unit Narrative:
Who was the original globetrotter?

Tables help keep us organized, but equations tell an entire story with just a few symbols. We'll use both of them to represent proportional relationships.

Sub-Unit Narrative:
Narrative: What good is a graph?

We turn to drawing, interpreting, and comparing proportional relationships in graphs, and notice what is particular to these types of graphs.

Unit 3 Measuring Circles

Identifying a circle may be straightforward, but measuring it is decidedly not. Students experience both the usefulness and challenges presented by this "perfect" shape.

Unit Narrative:
'Round and 'Round We Go

PRE-UNIT READINESS ASSESSMENT

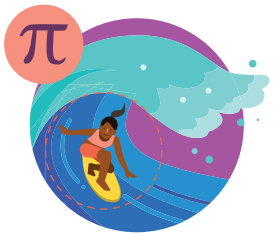
LAUNCH

3.01 The Wandering Goat ..... 212A — MP1, 4, 5

Sub-Unit 1 Circumference of Circles 219

Sub-Unit Narrative:
Why do aliens love circles?
Circles are famously difficult to measure precisely, but that won't stop us from trying. Let's see how close we can get.

3.02	Exploring Circles	220A	**7.G.A, 7.G.A.2,** MP3, 6
3.03	How Well Can You Measure?	227A	**7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2c, 7.G.A.1,** MP6, 7, 8
3.04	Exploring Circumference	234A	**7.G.A, 7.G.B.3, 7.RP.A.2, 7.RP.A.2a,** MP2, 8
3.05	Understanding π	242A	**7.G.B.3,** MP1, 3, 6
3.06	Applying Circumference	248A	**7.G.B.3, 7.EE.B.4,** MP1, 6
3.07	Circumference and Wheels	254A	**7.G.B.3, 7.RP.A.2b, 7.RP.A.2c, 7.RP.A.3, 7.EE.B.4,** MP3, 7, 8



Sub-Unit 2 Area of Circles 261

Sub-Unit Narrative:
What makes a circle so perfect?
Squares and circles may not have much in common, but we'll need both to measure a circle's area.

3.08	Exploring the Area of a Circle	262A	**7.G.A, 7.G.B,** MP2, 3
3.09	Relating Area to Circumference	268A	**7.G.B.3,** MP2
3.10	Applying Area of Circles	275A	**7.G.B.3, 7.EE.B.4,** MP1
3.11	Distinguishing Circumference and Area	281A	**7.G.B.3, 7.EE.B.4,** MP1, 2, 3

CAPSTONE

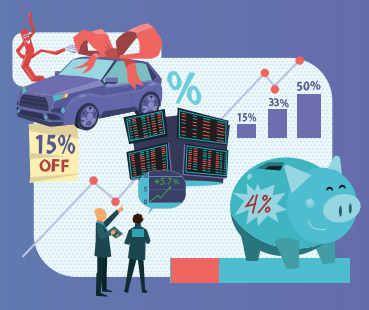
3.12 Capturing Space ..... 287A — **7.G.B.3,** MP7

END-OF-UNIT ASSESSMENT

Unit 4 Percentages

From the supermarket to the stock market, percents are relied on to communicate quickly about how much something has changed. Students build on their experience with proportional relationships while using percentages to compare quantities within the friendly confines of the number 100.

Unit Narrative:
Keepin' it 100



PRE-UNIT READINESS ASSESSMENT

LAUNCH

4.01 (Re)Presenting the United States ..... 296A — MP1, 2, 4, 6

Sub-Unit 1 Percent Increase and Decrease 303

4.02 Understanding Percentages Involving Decimals ..... 304A — **7.RP.A.3,** MP1, 2, 6, 7

4.03 Percent Increase and Decrease ..... 310A — **7.RP.A.3,** MP1, 2

4.04 Determining 100% ..... 317A — **7.RP.A.3,** MP1, 2

4.05 Determining Percent Change ..... 323A — **7.RP.A.3,** MP1, 3

4.06 Percent Increase and Decrease With Equations ..... 331A — **7.RP.A.3, 7.EE.A.2,** MP1, 3, 8

4.07 Using Equations to Solve Percent Problems ..... 338A — **7.RP.A.3, 7.EE.A.2,** MP1, 2, 3

Sub-Unit Narrative:
Is there truth in numbers?

Numbers never lie, but should we always believe them? Percentages can show how something changes – if we pay careful attention to the original amount.

Sub-Unit 2 Applying Percentages 345

4.08 Tax and Tip ..... 346A — **7.RP.A.3,** MP2, 8

4.09 Percentage Contexts ..... 352A — **7.RP.A.3,** MP1, 2

4.10 Determining the Percentage ..... 360A — **7.RP.A.3,** MP1, 7

4.11 Measurement Error ..... 367A — **7.RP.A.3,** MP1, 6

4.12 Error Intervals ..... 373A — **7.RP.A.3,** MP1, 3, 6

CAPSTONE

4.13 Writing Better Headlines ..... 379A — **7.RP.A.3,** MP3

END-OF-UNIT ASSESSMENT

Sub-Unit Narrative:
Did a quarantined U.S. keep a healthy economy?

See why percentages are used to calculate taxes, tips, interest, and other amounts when spending or saving money.

Volume 2

Unit 5 Rational Number Arithmetic

Students discover the need to work with both positive and negative values to describe the vastness of the world around them. With the entire set of rational numbers and all four operations now at their disposal, the sky (or the sea floor) is the limit.

Unit Narrative:
A World of Opposites

PRE-UNIT READINESS ASSESSMENT

LAUNCH

5.01	Target: Zero	388A	MP3, 7

Sub-Unit 1 Adding and Subtracting Rational Numbers ... 395

Sub-Unit Narrative:
What was Jeanne Baret's big secret?
Sure, you've probably been adding and subtracting for many years, but have you ever tried to take something away when you had less than zero to start with?

5.02	Interpreting Negative Numbers	396A	**7.NS.A.1a,** MP3, 7
5.03	Changing Temperatures	402A	**7.NS.A.1b, 7.NS.A.1,** MP1, 2, 3
5.04	Adding Rational Numbers	409A	**7.NS.A.1, 7.NS.A.1a,** MP2, 7
5.05	Money and Debts	417A	**7.NS.A.1, 7.NS.A.3,** MP4, 7
5.06	Representing Subtraction	423A	**7.NS.A.1b, 7.NS.A.1, 7.NS.A.1a,** MP1, 6, 7
5.07	Subtracting Rational Numbers (Part 1)	429A	**7.NS.A.1b, 7.NS.A.1,** MP1, 2, 6, 8
5.08	Subtracting Rational Numbers (Part2)	435A	**7.NS.A.1b, 7.NS.A.1,** MP1, 2
5.09	Adding and Subtracting Rational Numbers	442A	**7.NS.A.1, 7.NS.A.1c, 7.NS.A.3,** MP1, 2, 3

MID-UNIT ASSESSMENT

Sub-Unit 2 Multiplying and Dividing Rational Numbers ... 451

Sub-Unit Narrative:
Who was the toughest Grandma to ever hike the Appalachian Trail?
Travel forwards and backwards in time to help make sense of multiplication and division of negative numbers.

5.10	Position, Speed, and Time	452A	**7.NS.A.2a, 7.NS.A.2, 7.NS.A.2c,** MP1, 2, 4, 7, 8, 3
5.11	Multiplying Rational Numbers	458A	**7.NS.A.2a, 7.NS.A.2, 7.NS.A.2c,** MP1, 4, 6, 8
5.12	Multiply!	465A	**7.NS.A.2c, 7.NS.A.2, 7.NS.A.2a,** MP2, 3, 7, 8
5.13	Dividing Rational Numbers	471A	**7.NS.A.2b, 7.NS.A.2, 7.NS.A.2a, 7.NS.A.2c,** MP7, 8
5.14	Negative Rates	477A	**7.RP.A.2, 7.NS.A.3, 7.EE.B.3,** MP2, 4

Sub-Unit 3 Four Operations With Rational Numbers ... 485

Sub-Unit Narrative:
How do you climb the world's most dangerous mountain?
Put it all together — adding, subtracting, multiplying, and dividing with rational numbers — while exercising your algebraic thinking muscles in a sneak preview of Unit 6.

5.15	Expressions With Rational Numbers	486A	**7.NS.A.3,** MP7
5.16	Say It With Decimals	492A	**7.NS.A.2d, 7.NS.A.2c,** MP8
5.17	Solving Problems With Rational Numbers	499A	**7.NS.A.3, 7.NS.A.1c, 7.NS.A.2c,** MP3, 6, 8
5.18	Solving Equations With Rational Numbers	506A	**7.EE.B.4, 7.NS.A.2, 7.NS.A.1b,** MP1, 6, 7
5.19	Representing Contexts With Equations	514A	**7.EE.B.4a, 7.NS.A.3, 7.NS.A,** MP1, 2

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5.20	Summiting Everest	522A	**7.EE.B.3, 7.RP.A.2c,** MP1, 2, 3, 4

END-OF-UNIT ASSESSMENT

Unit 6 Expressions, Equations, and Inequalities

Students return to the study of algebra and focus on how representation plays such a large role in communicating mathematical ideas. In this unit, the symbols, language, and drawings students use will help them tell the stories they see in the numbers.

Unit Narrative:
Solving One Step at a Time

PRE-UNIT READINESS ASSESSMENT

LAUNCH

6.01 Keeping the Balance ..... 532A — **7.EE.B.4,** MP2, 4

Sub-Unit 1 Solving Two-Step Equations 541

6.02 Balanced and Unbalanced ..... 542A — **7.EE.B.4,** MP2

6.03 Reasoning About Solving Equations (Part 1) ..... 549A — **7.EE.B.4, 7.EE.B.4a,** MP2, 7

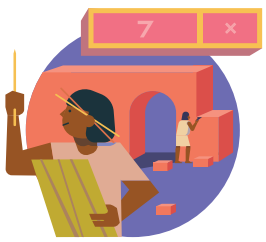
6.04 Reasoning About Solving Equations (Part 2) ..... 555A — **7.EE.B.4a,** MP1, 6, 7

6.05 Dealing With Negative Numbers ..... 562A — **7.EE.B.4a,** MP2, 6

6.06 Two Ways to Solve One Equation ..... 568A — **7.EE.B.4a,** MP3, 7

6.07 Practice Solving Equations ..... 574A — **7.EE.B.4a,** MP6, 7

Sub-Unit Narrative:
What are the first words you learn in "Caveman"?
Dog walking, tools of early civilization, and hangers all come together to help you explore new ways of solving equations.



Sub-Unit 2 Solving Real-World Problems Using Two-Step Equations 581

6.08 Reasoning With Tape Diagrams ..... 582A — **7.EE.B.4,** MP2, 4, 7

6.09 Reasoning About Equations and Tape Diagrams (Part 1) ..... 589A — **7.EE.B.3, 7.EE.B.4a,** MP2, 6, 7

6.10 Reasoning About Equations and Tape Diagrams (Part 2) ..... 595A — **7.EE.B.3, 7.EE.B.4a,** MP1, 2, 6, 7

6.11 Using Equations to Solve Problems ..... 601A — **7.RP.A.2, 7.RP.A.2c,** MP2

6.12 Solving Percent Problems in New Ways ..... 608A — **7.EE.A.2, 7.EE.B.4, 7.EE.B.3, 7.EE.B.4a,** MP1, 4, 5

Sub-Unit Narrative:
Who were the VIPs of ancient Egypt?
Solving word problems is about making meaning of the quantities, and tape diagrams return to help.

MID-UNIT ASSESSMENT

Sub-Unit 3 Inequalities 615

6.13 Reintroducing Inequalities ..... 616A — **7.EE.B.4,** MP1, 2

6.14 Solving Inequalities ..... 623A — **7.EE.B.4,** MP3, 6, 7

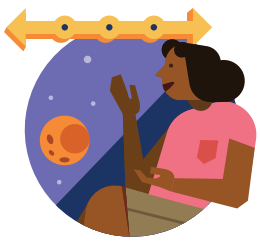
6.15 Finding Solutions to Inequalities in Context ..... 631A — **7.EE.B.4b,** MP1, 2

6.16 Efficiently Solving Inequalities ..... 637A — **7.EE.B.4b,** MP1, 6

6.17 Interpreting Inequalities ..... 644A — **7.EE.B.4b,** MP2, 8

6.18 Modeling With Inequalities ..... 650A — **7.EE.B.4b,** MP2, 4

Sub-Unit Narrative:
Did a member of the School of Night infiltrate your math class?
Expressions are not always equal, so we must reckon with inequalities. Thankfully, finding their solutions will feel familiar.



Sub-Unit 4 Equivalent Expressions 657

6.19 Subtraction in Equivalent Expressions ..... 658A — **7.NS.A.1b, 7.EE.A.1, 7.NS.A.1,** MP3, 6, 7

6.20 Expanding and Factoring ..... 665A — **7.EE.A.1,** MP7

6.21 Combining Like Terms (Part 1) ..... 672A — **7.EE.A.1,** MP6, 7

6.22 Combining Like Terms (Part 2) ..... 679A — **7.EE.A.1,** MP3, 6

Sub-Unit Narrative:
Which three blockheads did NASA send into space?
Find efficiencies for simplifying expressions like the Distributive Property and combining like terms.

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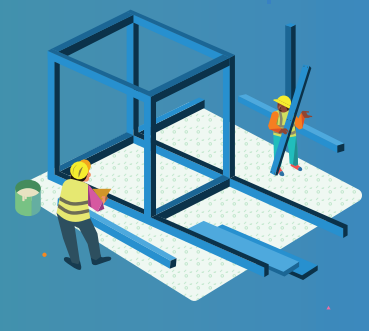
6.23 Pattern Thinking ..... 685A — **7.EE.A.2, 7.EE.B.3.a,** MP7, 8

END-OF-UNIT ASSESSMENT

Unit 7 Angles, Triangles, and Prisms

This unit is about the math of what can be seen and what can be held. Through constructing and drawing, students explore relationships among angles, lines, surfaces, and solids.

Unit Narrative:
Journey to the Third Dimension



Note: *Lessons in gray are recommended to be omitted.*

PRE-UNIT READINESS ASSESSMENT

LAUNCH

7.01 Shaping Up ..... 694A — **7.G.A,** MP3, 5, 6

Sub-Unit 1 Angle Relationships 701

Sub-Unit Narrative:
Did radio kill the aviation star?
As you'll see, some angles were just meant to go together. Here, you'll be introduced to complementary, supplementary, and vertical angles.

7.02	Relationships of Angles	702A	**7.G.A, 7.G.B,** MP3
7.03	Supplementary and Complementary Angles (Part 1)	708A	**7.G.B, 7.G.B.4,** MP3, 6
7.04	Supplementary and Complementary Angles (Part 2)	715A	**7.G.B.4, 7.EE.A,** MP6, 7
7.05	Vertical Angles	722A	**7.G.B, 7.G.B.4,** MP6, 8
7.06	Using Equations to Solve for Unknown Angles	728A	**7.G.B.4, 7.EE.B.4,** MP6
7.07	Like Clockwork	734A	**7.RP.A.2c, 7.EE.B.4, 7.RP.A.1,** MP1, 2, 4

Sub-Unit 2 Drawing Polygons With Given Conditions 741

Sub-Unit Narrative:
How did triangles help win a war?
In this Sub-Unit, you will find that constructing polygons with specific lengths and angle measures can have dramatically different results.

7.08	Building Polygons (Part 1)	742A	**7.G.A.2, 7.NS.A.1,** MP2, 3, 5
7.09	Building Polygons (Part 2)	749A	**7.G.A.2,** MP2, 8
7.10	Triangles With Three Common Measures	756A	**7.G.A.2,** MP3
7.11	Drawing Triangles (Part 1)	763A	**7.G.A, 7.G.A.2,** MP7
7.12	Drawing Triangles (Part 2)	769A	**7.G.A, 7.G.A.2,** MP5, 7

MID-UNIT ASSESSMENT

Sub-Unit 3 Solid Geometry 777

Sub-Unit Narrative:
This machine will slice, but will it dice?
You've studied the surfaces of three-dimensional figures and the spaces inside them. Now, let's see what happens when we slice them open.

7.13	Slicing Solids	778A	
7.14	Volume of Right Prisms	785A	**7.G.B.5,** MP7
7.15	Decomposing Bases for Area	791A	**7.G.B.5,** MP1, 7
7.16	Surface Area of Right Prisms	798A	**7.G.B.5,** MP3, 7
7.17	Distinguishing Surface Area and Volume	805A	**7.G.B.5,** MP4

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7.18 Applying Volume and Surface Area ..... 812A — **7.G.B, 7.G.B.5,** MP4

END-OF-UNIT ASSESSMENT

Unit 8 Probability and Sampling

For the first time, students encounter how to quantify the chances of something happening. Though the future is unwritten, probability and statistics help us make better predictions and thus better decisions.

Unit Narrative:
Winning Chance

Note: *Lessons in gray are recommended to be omitted.*
● = *Tennessee-specific lessons*

PRE-UNIT READINESS ASSESSMENT

LAUNCH

8.01 The Invention of Fairness ..... 820A — **7.SP.C.6,** MP2, 3

Sub-Unit 1 Probabilities of Single-Step Events 827

Sub-Unit Narrative:
How did the women of Bletchley Park save the free world?
Welcome to probability, the math of games and chance. Discover how probability can reveal hidden information, even secret codes.

8.02 Chance Experiments ..... 828A — **7.SP.C.5,** MP1, 2, 6

8.03 What Are Probabilities? ..... 835A — **7.SP.C.5, 7.SP.C.7, 7.SP.C.7a, 7.SP.C.7b,** MP2, 3

8.04 Estimating Probabilities Through Repeated Experiments ..... 841A — **7.SP.C.6a, 7.SP.C.6b, 7.SP.C.6c, 7.SP.C.7, 7.SP.C.5, 7.RP.A, 7.SP.C.7b,** MP6, 8

8.05 Code Breaking (Part 1) ..... 847A — **7.SP.C.6, 7.SP.C.6a, 7.SP.C.5, 7.SP.C.7,** MP3, 7

8.06 Code Breaking (Part 2) ..... 854A — **7.SP.A.1, 7.SP.C.5, 7.SP.C.6, 7.SP.C.6a, 7.RP.A,** MP7

Sub-Unit 2 Probabilities of Multi-Step Events 861

Sub-Unit Narrative:
How did a blazing shoal bring the Philadelphia Convention Center to its feet?
When predicting the chances gets complicated, a simulation can help make predictions.

8.07 Keeping Track of All Possible Outcomes 862A

8.08 Experiments With Multi-step Events 869A

8.09 Simulating Multi-step Events 876A

8.10 Designing Simulations 883A

Sub-Unit 3 Sampling 889

Sub-Unit Narrative:
What's on your mind?
Not all data is created equal. It is important to know how to identify when a sample is representative of a population.

8.10A Variability and IQR ..... TN-1A — **7.SP.D.8a, 7.SP.D.8b,** MP2

8.10B Comparing Distributions ..... TN-8A — **7.SP.B.3,** MP3

8.10C Larger Populations ..... TN-15A — **7.SP.A.1, 7.SP.B,** MP1, 3

8.11 Comparing Two Populations 890A

8.12 Larger Populations 897A

8.13 What Makes a Good Sample? ..... 903A — **7.SP.A.1, 7.SP.B.4, 7.SP.A.2, 7.SP.B.3,** MP2, 7

8.14 Sampling in a Fair Way ..... 910A — **7.SP.A.1, 7.SP.A.2,** MP3, 6, 7

8.14A Which Measure of Center Is Better? ..... TN-21A — **7.SP.D.8b, 7.SP.B.4, 7.SP.D.8,** MP2, 3

8.15 Estimating Population Measures of Center 916A

8.16 Estimating Population Proportions ..... 922A — **7.SP.A.2, 7.SP.B.4, 7.RP.A.2,** MP3, 6

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8.17 Presentation of Findings ..... 928A — **7.SP.A.1, 7.SP.A.2,** MP3

END-OF-UNIT ASSESSMENT

Tennessee Mathematics Standards, Grade 7

7.RP	Ratios and Proportional Relationships	Lesson(s)
7.RP.A	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**	
7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. For example, if a person walks $1/2$ mile in each 15 minutes, compute the unit rate as the complex fraction $(1/2)/(1/4)$ miles per hour, equivalently 2 miles per hour.	**Unit 2, Lesson** 6 **Unit 7, Lesson** 7
7.RP.A.2	Recognize and represent proportional relationships between quantities.	**Unit 1, Lesson** 3 **Unit 2, Lessons** 2, 3, 8–17 **Unit 3, Lesson** 4 **Unit 5, Lesson** 14 **Unit 6, Lesson** 11 **Unit 8, Lesson** 16
7.RP.A.2a	Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).	**Unit 2, Lessons** 2, 4, 9 **Unit 3, Lessons** 3, 4
7.RP.A.2b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	**Unit 2, Lessons** 2–4, 6, 7, 9, 12, 14, 15 **Unit 3, Lessons** 3, 7
7.RP.A.2c	Use the concept of equality to represent proportional relationships with equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.	**Unit 2, Lessons** 5–8, 10, 14, 15, 17 **Unit 3, Lessons** 3, 7 **Unit 5, Lesson** 20 **Unit 6, Lesson** 11 **Unit 7, Lesson** 7
7.RP.A.2d	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.	**Unit 2, Lessons** 12, 14, 15
7.RP.A.3	Use proportional relationships to solve multi-step ratio and percent problems. Examples: batting averages, recipes, simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error, etc.	**Unit 3, Lesson** 7 **Unit 4, Lessons** 2–13

Tennessee Mathematics Standards, Grade 7

7.NS	The Number System	Lesson(s)
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**	
7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.	**Unit 5, Lessons** 3–9 **Unit 6, Lesson** 19 **Unit 7, Lesson** 8
7.NS.A.1a	Understand $p + q$ as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	**Unit 5, Lessons** 2, 4, 6
7.NS.A.1b	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.	**Unit 5, Lessons** 3, 6–8, 18 **Unit 6, Lesson** 19
7.NS.A.1c	Apply properties of operations as strategies to add and subtract rational numbers.	**Unit 5, Lessons** 9, 17
7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.	**Unit 5, Lessons** 10–13, 18
7.NS.A.2a	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	**Unit 5, Lessons** 10–13
7.NS.A.2b	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real world contexts.	**Unit 5, Lesson** 13
7.NS.A.2c	Apply properties of operations as strategies to multiply and divide rational numbers.	**Unit 5, Lessons** 10–13, 16, 17
7.NS.A.2d	Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in zeros or eventually repeats.	**Unit 5, Lesson** 16
7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)	**Unit 5, Lessons** 5, 9, 14, 15, 17, 19

7.EE	Expressions and Equations	Lesson(s)
7.EE.A	**Use properties of operations to generate equivalent expressions.**	
7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**Unit 6, Lessons** 19–22
7.EE.A.2	Rewrite and connect equivalent expressions in different forms in a contextual problem to provide multiple ways of interpreting the problem and how the quantities in it are related. For example, shoes are on sale at a 25% discount. How is the discounted price P related to the original cost C of the shoes? $C - 0.25C = P$. In other words, P is 75% of the original cost since $C - 0.25C$ can be written as $0.75C$.	**Unit 4, Lessons** 6, 7 **Unit 6, Lessons** 12, 23
7.EE.B	**Solve real-world and mathematical problems using numerical and algebraic expressions and equations and inequalities.**	
7.EE.B.3	Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers presented in any form (whole numbers, fractions, and decimals).	**Unit 5, Lessons** 14, 20 **Unit 6, Lessons** 9, 10, 12
7.EE.B.3a	Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate.	**Unit 2, Lesson** 6
7.EE.B.3b	Assess the reasonableness of answers using mental computation and estimation strategies.	**Unit 1, Lessons** 4, 8 **Unit 2, Lesson** 8
7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	**Unit 3, Lessons** 6, 7, 10, 11 **Unit 5, Lesson** 18 **Unit 6, Lessons** 1–3, 8, 12–14 **Unit 7, Lessons** 6, 7
7.EE.B.4a	Solve real-world and mathematical problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?	**Unit 5, Lesson** 19 **Unit 6, Lessons** 3–7, 9, 10, 12
7.EE.B.4b	Solve real-world and mathematical problems leading to inequalities of the form $px + q > r$, $px + q < r$, $px + q \geq r$, and $px + q \leq r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality on a number line and interpret it in the context of the problem. For example: As a salesperson, you are paid $\$50$ per week plus $\$3$ per sale. This week you want your pay to be at least $\$100$. Write an inequality for the number of sales you need to make, and describe the solutions.	**Unit 6, Lessons** 15–18

Tennessee Mathematics Standards, Grade 7

7.G	Geometry	Lesson(s)
7.G.A	**Draw, construct, and describe geometrical figures and describe the relationships between them.**	
7.G.A.1	Solve problems involving scale drawings of congruent and similar geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**Unit 1, Lessons** 2–13 **Unit 3, Lesson** 3
7.G.A.2	Draw triangles with given conditions: three angle measures or three side measures. Notice when the conditions determine a unique triangle, more than one triangle, or no triangle.	**Unit 3, Lesson** 2 **Unit 7, Lessons** 8–12
7.G.B	**Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.**	
7.G.B.3	Know the formulas for the area and circumference of a circle and use them to solve problems. Explore the relationships between the radius, the circumference, and the area of a circle, and the number π.	**Unit 3, Lessons** 4–7, 9–12
7.G.B.4	Know and use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**Unit 7, Lessons** 3–6
7.G.B.5	Solve real-world and mathematical problems involving area of two-dimensional objects composed of triangles, quadrilaterals, and polygons, and volume and surface area of three-dimensional objects composed of cubes and right prisms.	**Unit 2, Lesson** 9 **Unit 7, Lessons** 14–18

7.SP	Statistics and Probability	Lesson(s)
7.SP.A	**Use random sampling to draw inferences about a population.**	
7.SP.A.1	Explore how statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	**Unit 8, Lessons** 6, 10C, 13, 14, 17
7.SP.A.2	Collect and use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.	**Unit 8, Lessons** 13, 14, 16, 17
7.SP.B	**Draw informal comparative inferences about two populations.**	
7.SP.B.3	Informally compare the measures of center (mean, median, mode) of two numerical data distributions with similar variabilities. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team; on a dot plot or box plot, the separation between the two distributions of heights is noticeable.	**Unit 8, Lessons** 10B, 13

7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a 7th grade science book are generally longer than the words in a chapter of a 4th grade science book.	**Unit 8, Lessons** 13, 14A, 16
7.SP.C	**Investigate chance processes and develop, use, and evaluate probability models.**	
7.SP.C.5	Recognize that the probability of a chance event is a number between 0 and 1 and interpret the likelihood of the event occurring.	**Unit 8, Lessons** 2–6
7.SP.C.6	Calculate theoretical and experimental probability of simple events.	**Unit 8, Lessons** 1, 5, 6
7.SP.C.6a	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	**Unit 8, Lessons** 4–6
7.SP.C.6b	Calculate the theoretical probability of a simple event.	**Unit 8, Lesson** 4
7.SP.C.6c	Compare theoretical probabilities to experimental probabilities; explain any possible sources of discrepancy. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.	**Unit 8, Lesson** 4
7.SP.C.7	Develop a probability model and use it to find experimental or theoretical probabilities of events.	**Unit 8, Lessons** 3–5
7.SP.C.7a	Use a uniform probability model, with equal probability assigned to all outcomes, to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.	**Unit 8, Lesson** 3
7.SP.C.7b	Develop a probability model, including non-uniform models, by observing frequencies in data generated from a chance process. Use the model to estimate the probabilities of events. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?	**Unit 8, Lessons** 3, 4
7.SP.D	**Summarize and describe numerical data sets.**	
7.SP.D.8	Summarize a numerical data set in relation to its context.	**Unit 8, Lesson** 14A
7.SP.D.8a	Give quantitative measures of center (median and/or mean) and variability (range and/or interquartile range), as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	**Unit 8, Lesson** 10A
7.SP.D.8b	Relate and understand the choice of measures of center (median and/or mean) and variability (range and/or interquartile range) to the shape of the data distribution and the context in which the data were gathered.	**Unit 8, Lessons** 10A, 14A

Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Unit 1, Lessons 1, 9, 13

Unit 2, Lessons 4, 10, 13, 16, 17

Unit 3, Lessons 1, 5, 6, 10, 11

Unit 4, Lessons 1–7, 9–12

Unit 5, Lessons 3, 6–11, 18–20

Unit 6, Lessons 4, 10, 12, 13, 15, 16

Unit 7, Lessons 7, 15

Unit 8, Lessons 2, 10C

MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

Unit 1, Lessons 1, 10

Unit 2, Lessons 6–8, 10, 12, 13, 15

Unit 3, Lessons 4, 8, 9, 11

Unit 4, Lessons 1–4, 7–9

Unit 5, Lessons 3, 4, 7–10, 12, 14, 19, 20

Unit 6, Lessons 1–3, 5, 8–11, 13, 15, 17, 18

Unit 7, Lessons 7–9

Unit 8, Lessons 1–3, 10A, 13, 14A

MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Unit 1, Lessons 4, 6, 12

Unit 2, Lessons 4, 11

Unit 3, Lessons 2, 5, 7, 8, 11

Unit 4, Lessons 5–7, 12, 13

Unit 5, Lessons 1–3, 9, 10, 12, 17, 20

Unit 6, Lessons 6, 14, 19, 22

Unit 7, Lessons 1–3, 8, 10, 16

Unit 8, Lessons 1, 3, 5, 10B, 10C, 14, 14A, 16, 17

MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Unit 1, Lessons
Unit 2, Lessons
Unit 3, Lessons
Unit 4, Lessons
Unit 5, Lessons
Unit 6, Lessons
Unit 7, Lessons
Unit 8, Lessons

MP5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Unit 1, Lessons 4, 7
Unit 2, Lesson 17
Unit 3, Lesson 1
Unit 6, Lesson 12
Unit 7, Lessons 1, 8, 12

MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

Unit 1, Lessons 2, 3, 11, 13
Unit 2, Lessons 4, 10, 12, 14, 15
Unit 3, Lessons 2, 3, 5, 6
Unit 4, Lessons 1, 2, 11, 12
Unit 5, Lessons 6, 7, 11, 17, 18
Unit 6, Lessons 4, 5, 7, 9, 10, 14, 19, 21, 22
Unit 7, Lessons 1, 3–6
Unit 8, Lessons 2, 4, 14, 16

Standards for Mathematical Practice

MP7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Unit 1, Lessons 2–11

Unit 2, Lessons 1, 2, 5, 9, 11, 14, 15

Unit 3, Lessons 3, 7, 12

Unit 4, Lessons 2, 10

Unit 5, Lessons 1, 2, 4–6, 10, 12, 13, 15, 18

Unit 6, Lessons 3, 4, 6-10, 14, 19–21, 23

Unit 7, Lessons 4, 11, 12, 14, 15, 16

Unit 8, Lessons 5, 6, 13, 14

MP8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Unit 1, Lessons 1, 5, 6

Unit 2, Lessons 3, 5–7, 9, 16

Unit 3, Lessons 3, 4, 7

Unit 4, Lessons 6, 8

Unit 5, Lessons 7, 10–13, 16, 17

Unit 6, Lessons 17, 23

Unit 7, Lessons 5, 9

Unit 8, Lesson 4

Practice Problem Analysis

Teachers may omit the following Practice Problems from the indicated lessons as they address topics beyond the scope of the Tennessee Mathematics Standards.

Unit 7: Angles, Triangles, and Prisms

Lesson	Problem(s)
12	6
16	3

Unit 8: Probability and Sampling

Lesson	Problem(s)
6	5
13	4, 5
14	4—6
16	4, 5
17	2, 3, 6

Variability and IQR

Let's describe variability using the median and the interquartile range (IQR).

Focus

Goals

1. **Language Goal:** Calculate the range and interquartile range (IQR) of a data set and interpret what they tell about a scenario. **(Speaking and Listening, Writing)**
2. **Language Goal:** Comprehend the interquartile range (IQR) as another measure of variability, which describes the span of the middle half of the data. **(Writing)**
3. **Language Goal:** Explain the effect of an extreme value on the measures of variability of a data set. **(Writing)**

Coherence

Today

Students are reminded of how to divide a data set into *quartiles* and relate the three quartiles to the 25th, 50th, and 75th percentiles, which are useful in describing a distribution. They also identify the maximum and minimum values of the data set, and combining those with the quartiles, they identify the *five-number summary*. Finally, students explore two measures of variability for a distribution – the range and the interquartile range (IQR) – as ways to describe its spread and consider the effect of an extreme value on both measures. Students interpret what the IQR, as the middle half of the data, tells them about a scenario **(MP2)**.

Previously

In Grade 6, students calculated the mean, median, mode and range of data sets and represented data using box plots.

Coming Soon

In Tennessee Lesson 10B, students will calculate and interpret interquartile range and compare data sets using their measures of center and variability.

Rigor

- Students further their **conceptual understanding** of measures of variability.
- Students build **procedural skills** for identifying the five-number summary for a data set and calculating IQR.

Standards

Addressing

7.SP.D.8a

Give quantitative measures of center (median and/or mean) and **variability (range and/or interquartile range), as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.**

Also Addressing: **7.SP.D.8b**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent
MP2	MP2			
7.SP.D.8a	7.SP.D.8a, 7.SP.D.8b	7.SP.D.8a	7.SP.D.8a, 7.SP.D.8b	7.SP.D.8a

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Five-number Summary* (from Grade 6)

Math Language Development

New words

- ***interquartile range***

Review words

- *five-number summary*
- *median*
- *outlier*
- *quartile*
- *range*
- *variability*

Amps | Featured Activity

Activity 1

Soccer Practice Formative Feedback

Students calculate the range and interquartile range of different data sets and get immediate feedback about their calculations so they can compare the variability of the data sets.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may get frustrated having to determine three different medians for one data set **(MP2)**. Encourage students to be careful and deliberate when counting to find the middle of the upper half and lower half of the data.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

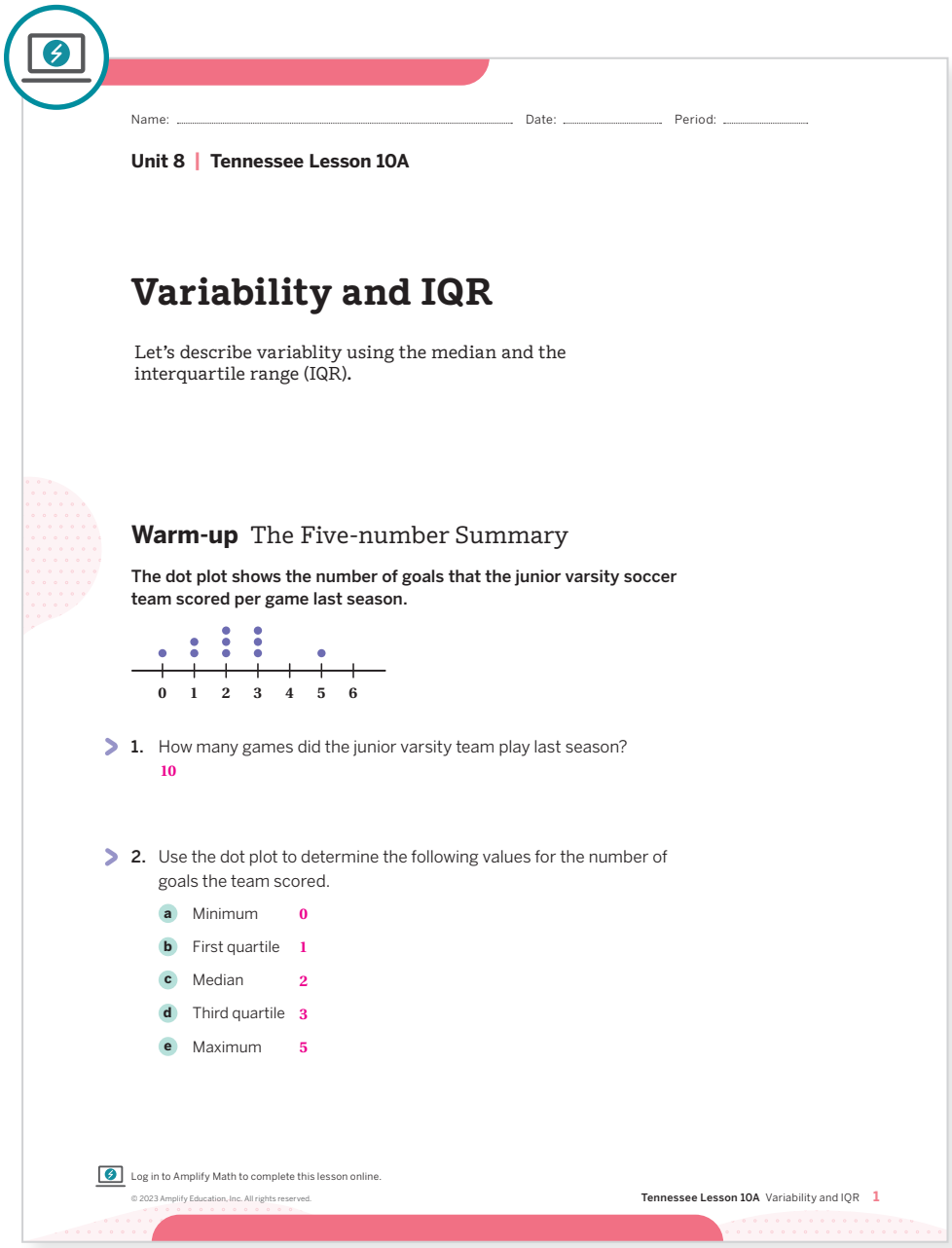
- In **Activity 1**, discuss Problems 4–6 as a whole class, providing the ordered data sets (or even the five-number summary) for students in Problem 6.

Pairs | 10 min

Warm-up The Five-number Summary

MP2
7.SP.D.8A

Students review quartiles and the *five-number summary* for a data set, identifying and interpreting those values, and preparing for IQR in the next activity **(MP2)**.



Name: ..... Date: ..... Period: .....

Unit 8 | Tennessee Lesson 10A

Variability and IQR

Let's describe variablity using the median and the interquartile range (IQR).

Warm-up The Five-number Summary

The dot plot shows the number of goals that the junior varsity soccer team scored per game last season.

1. How many games did the junior varsity team play last season?
   10

2. Use the dot plot to determine the following values for the number of goals the team scored.
   a. Minimum 0
   b. First quartile 1
   c. Median 2
   d. Third quartile 3
   e. Maximum 5

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate prior knowledge of calculating the five-number summary by asking, "What does it mean to break something down into quartiles, or quarters?" Review the Anchor Chart PDF, ***Five-number Summary***, reminding students that the first quartile represents 25% of the data, the median represents 50% of the data, and the third quartile represents 75% of the data.

2 Monitor

Help students get started by saying "How can you determine the median with this set of data?"

Look for points of confusion:

- **Not knowing how to calculate Q1 or Q3 (Problem 2).** Say, "The median divides the data in half. The first quartile is the middle value of the lower half of the data and the third quartile is the middle value of the upper half of the data."
- **Not knowing whether to include the 2s when calculating Q1 and Q3 (Problem 2).** Explain to students that when the median is the average of the middle two values, the two values are included with their lower and upper halves.

Look for productive strategies:

- Recognizing that the minimum and maximum are included in the lower and upper sections of data values.
- Understanding that Q1 and Q3 are determined as if they are the medians of the two halves of data around Q2.
- Analyzing and interpreting what the data between each of the quartiles represents.

3 Connect

Have students share how they determined the answer to Problem 1 and the values they found for Problem 2.

Highlight that the five-number summary helps describe a data set without listing or showing every value. It summarizes the data by dividing it into quartiles with the median determining the middle point of the data. The five-number summary for a data set summarizes a distribution by its minimum, first quartile, median, third quartile, and maximum. Remind students that a *quartile* is one of three numbers (Q1, Q2, Q3) that divide a data set into 4 sections so that each contains the same number of data values. The closer the values that bound a section of the data, the more of a cluster the data values in that range represent.

Power-up

To power up students' ability to determine the possible outcomes of an event, have students complete:

List all the possible outcomes for pressing a single button on the following keypad:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0

Use: Before Warm-up

Informed by: Performance on Lesson 6, Practice Problem 5

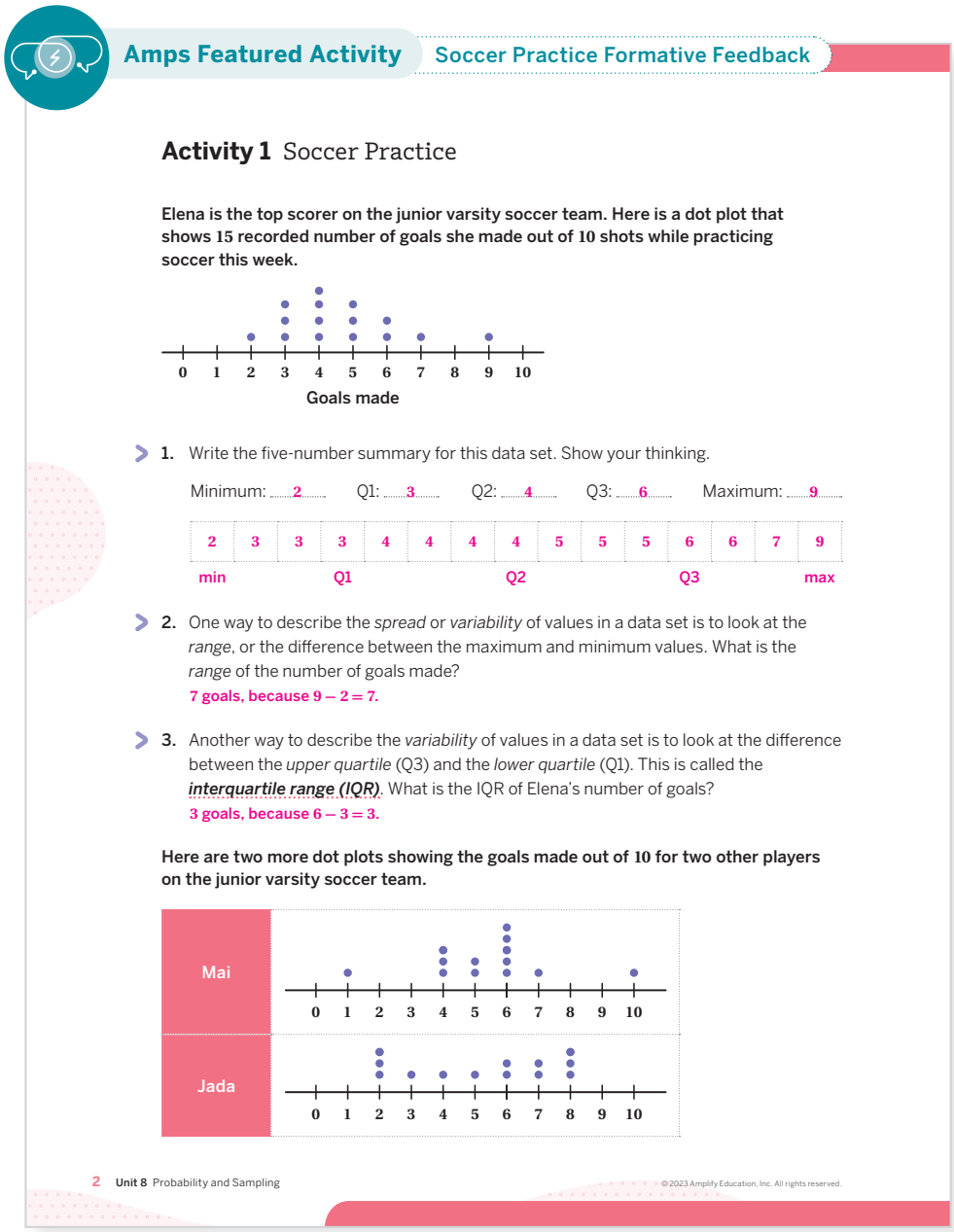
Pairs | 15 min

Activity 1 Soccer Practice

MP2
7.SP.D.8A, 7.SP.D.8B

Students determine two measures of variability for data sets – range and interquartile range (IQR) – and they use the IQR to describe variability in context **(MP2).**

Amps Featured Activity Soccer Practice Formative Feedback



Activity 1 Soccer Practice

Elena is the top scorer on the junior varsity soccer team. Here is a dot plot that shows 15 recorded number of goals she made out of 10 shots while practicing soccer this week.

1. Write the five-number summary for this data set. Show your thinking.

Minimum: 2 Q1: 3 Q2: 4 Q3: 6 Maximum: 9

2	3	3	3	4	4	4	4	5	5	5	6	6	7	9
min			Q1				Q2				Q3			max

2. One way to describe the *spread* or *variability* of values in a data set is to look at the *range*, or the difference between the maximum and minimum values. What is the *range* of the number of goals made?
7 goals, because 9 – 2 = 7.

3. Another way to describe the *variability* of values in a data set is to look at the difference between the *upper quartile* (Q3) and the *lower quartile* (Q1). This is called the ***interquartile range (IQR)***. What is the IQR of Elena's number of goals?
3 goals, because 6 – 3 = 3.

Here are two more dot plots showing the goals made out of 10 for two other players on the junior varsity soccer team.

1 Launch

Activate prior knowledge about variability by reminding students that variability measures how spread out the values in a data set are. Ask, "How is this data set different from the one in the Warm-up?" There is an odd number of data values.

2 Monitor

Help students get started by asking "How would you identify the quartiles?" List the values of all the data values in order and then count off to determine the median, and then do the same for Q1 and Q3.

Look for points of confusion:

- **Not knowing when to include certain values when determining quartiles.** Have students review the data set from Activity 1 and ask, "For Q1 and Q3, did you include the two values used to determine the median? Why?" Then have them look at the current data set and ask, "Is the median an average of two points or is it the data value?" Consider having students mark the median in the data set, whether it is a value in the set or is between two values, in order to visually separate it from the lower and upper halves.

Look for productive strategies:

- Accurately identifying the five-number summaries, and using those to calculate ranges and IQRs.
- Associating the range of values around the median between Q1 and Q3 with typical values and with 50% of the data.
- Recognizing that a dot plot with less spread and more points clustered around the center will have a lesser IQR; and vice versa.

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider displaying or providing a checklist similar to the one shown to help students organize their thinking around determining the five-number summary.

- ☐ Determine the minimum and maximum.
- ☐ Determine the median (Q2) - the middle of the data set.
- ☐ Determine Q1 - halfway between the minimum and Q2.
- ☐ Determine Q3 - halfway between the maximum and Q2.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the dot plots from Problem 4 and annotate which dot plot shows the lesser range and which dot plot shows the lesser IQR. Draw students' attention to the connections between the visual distribution of the data values on the dot plots and the comparisons of their ranges and IQRs. Have students complete these statements.

- "A distribution with a _____ range means the entire data set will be more spread out than a data set with a _____ range."
- "A distribution with a _____ IQR means the middle half of the data values will be closer together than a data set with a _____ IQR."
- "The IQR is/is not affected by extreme values or outliers because . . ."

Pairs | 15 min

Activity 1 Soccer Practice (continued)

MP2
7.SP.D.8A, 7.SP.D.8B

Students determine two measures of variability for data sets – range and interquartile range (IQR) – and they use the IQR to describe variability in context **(MP2).**

Name: _____ Date: _____ Period: _____

Activity 1 Soccer Practice (continued)

4. Using the dot plots given for Mai and Jada, and without doing any calculations, predict:

a. Which player has the lesser range?
Jada

b. Which player has the lesser IQR?
Mai

5. Check your predictions by calculating the values of the ranges and IQRs for the data in each dot plot.

Mai:

1	4	4	4	5	5	6	6	6	6	6	7	10
			Q1			Median (Q2)			Q3			

Minimum: 1 Q1: 4 Q2: 15 Q3: 6 Maximum: 10

Range: 9 IQR: 2

Jada:

2	2	2	3	4	5	6	6	7	7	8	8	8
			Q1			Median (Q2)			Q3			

Minimum: 2 Q1: 2.5 Q2: 6 Q3: 7.5 Maximum: 8

Range: 6 IQR: 5

6. Which player would you say has a greater *variability* in their goals made? Explain your thinking.
Jada; Sample response: Her data values are more spread out and have a higher IQR. Mai's goals are mostly clustered in the middle.

3 Connect

Have pairs of students share their responses, briefly ensuring that students know how to determine range and IQR. Then focus the discussion on how these measures relate to the shape of a distribution, such as on a dot plot, and interpreting the information they provide in context.

Define:

- The ***interquartile range*** *(IQR) of a data set* as a measure of variability that is calculated as the difference between the third quartile (Q3) and the first quartile (Q1).

Ask: (as many as time permits)

- "What does a range of 7 goals tell you about Elena's expected number of goals out of 10?" The minimum and maximum number of goals differed by 7. The greatest difference in goals was 7.
- "What does an IQR of 3 goals tell you about the spread of the number of goals?" The number of goals of the most typical half of tries are all within 3 goals.
- "In general, what does a greater range tell you? Greater IQR?" A wider overall spread in the data. More variability in the data set.
- "Can a data set have a greater range and a lesser IQR?" Yes,this can happen for a data set with most of the points in one big cluster but a few points very far away from the cluster.

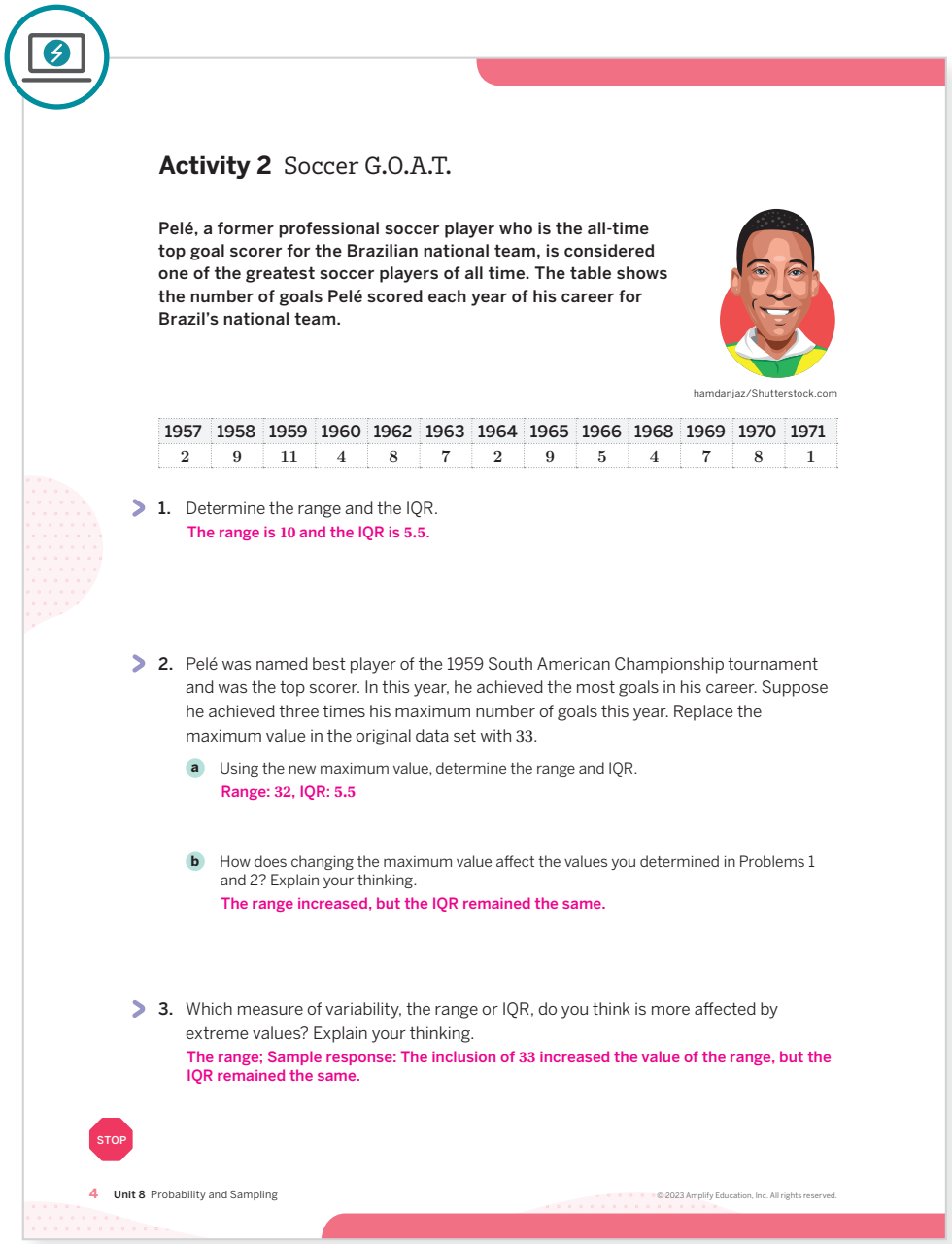
Highlight how the range and IQR are represented in a dot plot and the five-number summary, and that the range encompasses 100% of the data and the IQR encompasses 50% of the data.

Pairs | 10 min

Activity 2 Soccer G.O.A.T.

7.SP.D.8A

Students investigate how extreme data affects measures of variability to build understanding for when each measure is appropriate.



Activity 2 Soccer G.O.A.T.

Pelé, a former professional soccer player who is the all-time top goal scorer for the Brazilian national team, is considered one of the greatest soccer players of all time. The table shows the number of goals Pelé scored each year of his career for Brazil's national team.

hamdanjaz/Shutterstock.com

1957	1958	1959	1960	1962	1963	1964	1965	1966	1968	1969	1970	1971
2	9	11	4	8	7	2	9	5	4	7	8	1

1. Determine the range and the IQR.
   The range is 10 and the IQR is 5.5.

2. Pelé was named best player of the 1959 South American Championship tournament and was the top scorer. In this year, he achieved the most goals in his career. Suppose he achieved three times his maximum number of goals this year. Replace the maximum value in the original data set with 33.

   a. Using the new maximum value, determine the range and IQR.
   Range: 32, IQR: 5.5

   b. How does changing the maximum value affect the values you determined in Problems 1 and 2? Explain your thinking.
   The range increased, but the IQR remained the same.

3. Which measure of variability, the range or IQR, do you think is more affected by extreme values? Explain your thinking.
   The range; Sample response: The inclusion of 33 increased the value of the range, but the IQR remained the same.

STOP

4 Unit 8 Probability and Sampling

1 Launch

Ask, "What is your favorite sport? Who, in your opinion, is the G.O.A.T.?" If students are unfamiliar with the term G.O.A.T., explain that it stands for "Greatest of All Time."

2 Monitor

Help students get started by prompting them to list the data values in order from least to greatest.

Look for points of confusion:

- **Adding 33 to the data set instead of replacing the maximum value in Problem 3.** Prompt students to create separate data sets so that they can compare them with one another.

Look for productive strategies:

- Comparing two separate data sets.
- Recognizing that range is more affected by an extreme value than IQR is.

3 Connect

Have pairs of students share how replacing the maximum value affected (or did not affect) the range and IQR.

Ask:

- "Which measure of variability is more affected by an extreme value? Explain your thinking." Range; The range is calculated using the maximum and minimum so changing the maximum value changes the range. However, changing the maximum value does not affect the middle half of the data.

Highlight that the maximum value was 33 was an *outlier* because it was far away from the rest of the values in the set . The range is more sensitive to outliers while the IQR resists the effects of outliers.

Differentiated Support

Extension: *Are you ready for more?*

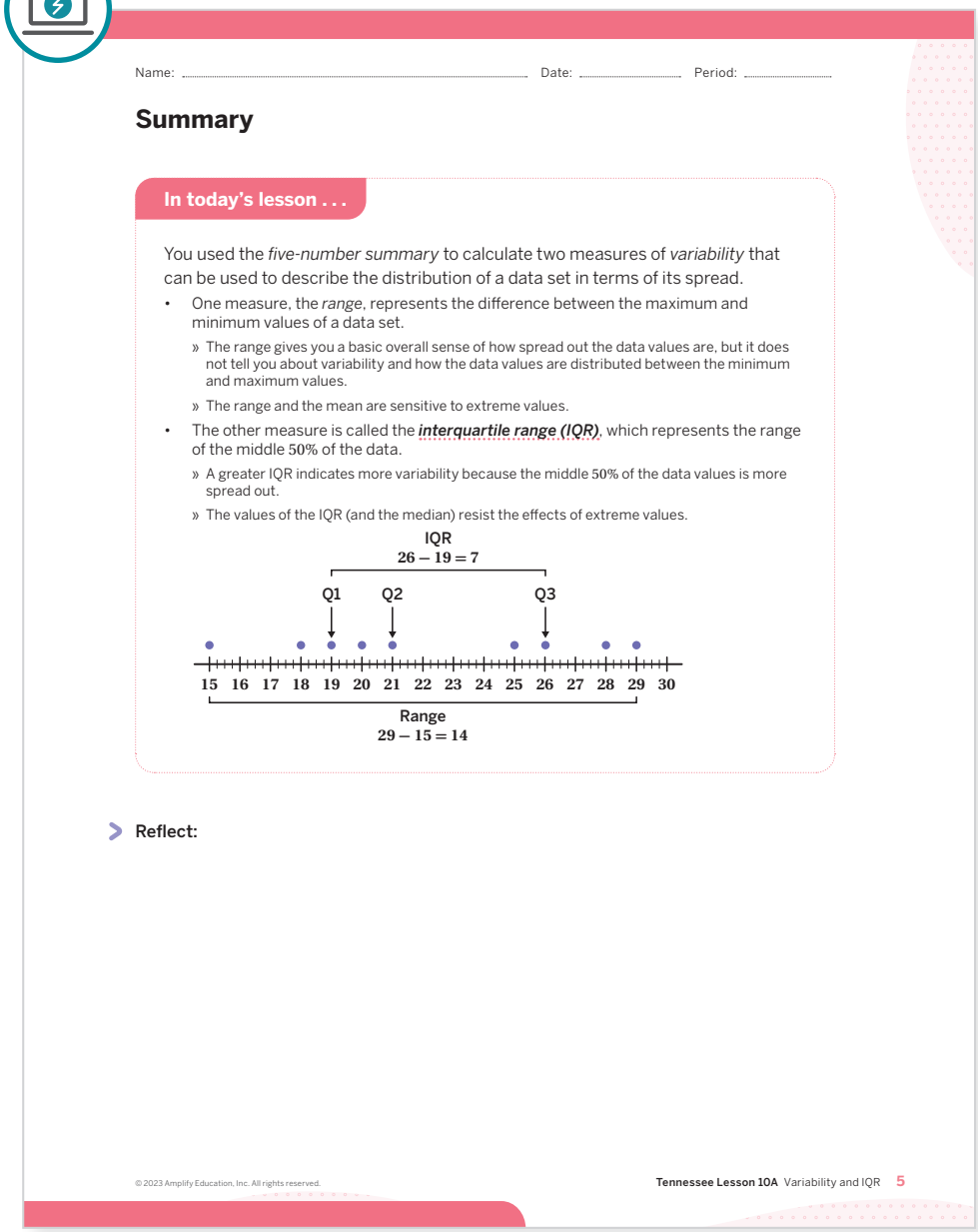
What changes to the data set must occur in order for the IQR value to change? Have students add values or remove them from the original data set so that its IQR changes.

Whole Class | 5 min

Summary

7.SP.D.8A, 7.SP.D.8B

Review and synthesize that the range and IQR are both measures of variability that are centered around the median.



Name: ..... Date: ..... Period: .....

Summary

In today's lesson . . .

You used the *five-number summary* to calculate two measures of *variability* that can be used to describe the distribution of a data set in terms of its spread.

- One measure, the *range*, represents the difference between the maximum and minimum values of a data set.
  - » The range gives you a basic overall sense of how spread out the data values are, but it does not tell you about variability and how the data values are distributed between the minimum and maximum values.
  - » The range and the mean are sensitive to extreme values.
- The other measure is called the ***interquartile range (IQR)***, which represents the range of the middle 50% of the data.
  - » A greater IQR indicates more variability because the middle 50% of the data values is more spread out.
  - » The values of the IQR (and the median) resist the effects of extreme values.

> Reflect:

Synthesize

Ask

- "What are the quartiles for a numerical data set?" Numbers that show where you can divide the data set so that the data values are in quarters or fourths.
- "What is the relationship between the quartiles and the median?" The second quartile is also the median.
- "What is the interquartile range (IQR)? What does it mean?" The IQR is the difference between the third and first quartile. It is a measure of the variability or spread of the data. It tells how much "space" the middle half of the data occupies.
- "When might IQR be a better measure of variability than the range?" When the data set includes an extreme value.

Highlight that the range and IQR are measures of variability, but each provides different information. The range is sensitive to extreme values, or outliers, but IQR is not. The IQR tells about typical values but only represents the spread of 50% of the data.

Formalize vocabulary:

- ***interquartile range***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can measures of variability be used to analyze data?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *quartile*, *five-number summary*, *range*, and *interquartile range* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

7.SP.D.8A

Students demonstrate their understanding by determining the median and IQR and what it means in context.

Printable

Name: ..... Date: ..... Period: .....

Exit Ticket 8.10A

Diego wanted to know how far seventh-grade students could throw a dodgeball, so he asked 10 seventh-graders to each throw a dodgeball as far as they could. He measured the distances, in feet, from where the ball was thrown to where it first landed. Here are the data he recorded.

40 76 40 63 47 57 49 55 50 53

1. Determine the median and IQR of the data set.

40 40 47 49 50 53 55 57 63 76
Q1 51.5 Q3

Median: 51.5 ft

IQR: 10 ft; 57 − 47 = 10

2. On another day, he asked the same group of students to throw a basketball, and he collected the data in the same way as he did for the dodgeball. The median distance of the basketball throws was 49 ft with an IQR of 6 ft.

   a. Overall, did this group of seventh graders throw the basketball farther than the dodgeball? Explain how you know.

   No, they did not throw the basketball farther. I know this because the median is less, which means there were more shorter throws pulling down the typical distance of a throw.

   b. Overall, for which type of ball were the distances thrown by this group of seventh graders more consistent? Explain how you know.

   The distances for the basketball were less variable because the IQR is less, which means the throws were less spread out or more consistent.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I know what quartiles and interquartile range (IQR) measure and what each tells me about the data. 1 2 3

b. When given a list of data values or a dot plot, I can determine the quartiles and interquartile range (IQR) for the data. 1 2 3

c. I can explain the effect of an extreme value on the measures of variability of a data set. 1 2 3

Success looks like . . .

- **Language Goal:** Calculating the range and interquartile range (IQR) of a data set and interpreting what they tell about a scenario. **(Speaking and Listening, Writing)**
  - » Calculating the correct IQR of the data set.
- **Language Goal:** Comprehending the interquartile range (IQR) as another measure of variability, which describes the span of the middle half of the data. **(Writing)**
  - » Using the IQR of both data sets to compare their variability.
- **Language Goal:** Explaining the effect of an extreme value on the measures of center and variability of a data set. **(Writing)**

Suggested next steps

If students do not put the data values in order first, consider:

- Asking, "To determine the median, what must be true about the data set?"

If students have difficulty determining the median, consider:

- Asking, "How many data values are in this set? 10 "If you divide the data values in half, how many points should be on each side? 5 "What number is between the lower five and the upper five?" You need to find the average of the fifth and sixth data values, which are 50 and 53.

If students think that Q1 and Q3 are averages of two numbers because they think that the 50 and 53 are not used to calculate them, consider:

- Color coding the data sets in Activity 1, Problem 6.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did determining the five-number summary and IQR reveal about your students as procedural learners?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

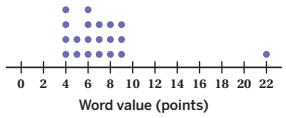
Practice

Independent

Practice

Name: Date: Period:

1. In a word game, 1 letter is worth 1 point. This dot plot shows the scores for 20 words.



a. What is the median score?
6.5 points; (6 + 7) ÷ 2 = 6.5

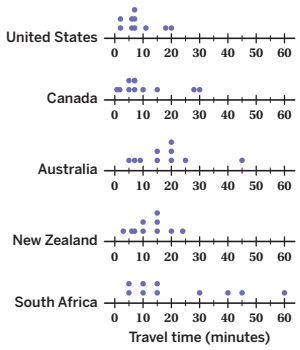
b. What is the first quartile (Q1)?
5 points

c. What is the third quartile (Q3)?
8 points

d. What is the interquartile range (IQR)?
3 points; 8 − 3 = 5

e. What is the range?
18 points; 22 − 4 = 18

2. The 5 dot plots show the travel times to school of 10 seventh graders in 5 countries. Match each dot plot with the correct median and IQR by writing the country's name next to each set of statistics.



a. Median: 17.5, IQR: 11 **Australia**

b. Median: 15, IQR: 30 **South Africa**

c. Median: 8, IQR: 4 **United States**

d. Median: 7, IQR: 10 **Canada**

e. Median: 12.5, IQR: 8 **New Zealand**

Name: Date: Period:

Practice

3. Mai and Priya each bowled 10 games and recorded their scores. Mai's median score was 120 with an IQR of 5. Priya's median score was 118 with an IQR of 15. Whose scores had less variability? Explain how you know.

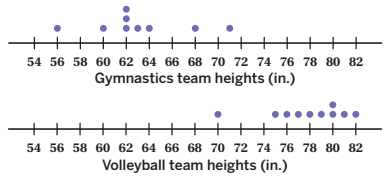
Mai's IQR is 5 and Priya's IQR is 15, so Mai's scores probably had less variability.

4. To make a specialty pizza, you need $4\frac{3}{4}$ cups of cheese, $\frac{3}{4}$ cups of olives, and $2\frac{1}{4}$ cups of sausage. How much of each ingredient is needed to make 10 specialty pizzas? Show or explain your thinking.

Sample response:

Number of pizzas	Cheese (cups)	Olives (cups)	Sausage (cups)
1	$\frac{19}{4}$ or $4\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{4}$ or $2\frac{1}{4}$
10	$\frac{190}{4}$ or $47\frac{1}{2}$	$\frac{30}{4}$ or $7\frac{1}{2}$	$\frac{90}{4}$ or $22\frac{1}{2}$

5. The dot plots show the heights of the members of a gymnastics team and the members of a volleyball team.



Which team has taller members? Explain your thinking.

Sample response: The volleyball team has a taller members because almost all of the gymnastics team members' heights are less than 70 in. and all the volleyball team members' heights are greater than or equal to 70 in.

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 1	7.SP.D.8a	1
	2	Activity 2	7.SP.D.8a	2
	3	Activity 2	7.SP.D.8a	2
Spiral	4	Unit 2 Lesson 2	7.RP.A.2	1
Formative	5	Unit 8 Lesson 10B	7.SP.B.3	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Comparing Distributions

Let's compare two distributions.

Focus

Goals

1. Calculate measures of center and variability for data sets represented in different ways.
2. **Language Goal:** Compare and contrast distributions in terms of their center, spread, variability, and visual overlap. **(Speaking and Listening, Writing)**

Coherence

Today

Students determine the three measures of center for distributions as well as their range and IQR. They also construct box plots on the same number line to compare distributions and reason about the data sets **(MP3)**.

Previously

In Tennessee Lessons 10A, students identified the five-number summary of a data set and used the range and IQR to describe variability.

Coming Soon

In Tennessee Lesson 10C, students will describe the sample of different populations and determine whether their class represents an adequate sample.

Rigor

- Students demonstrate **fluency** for determining the two measures of center: mean, and median.
- Students **apply** their knowledge of five-number summaries and box plots to compare distributions.

Standards

Addressing

7.SP.B.3
Informally compare the measures of center (mean, median, mode) of two numerical data distributions with similar variabilities.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent
	MP3	MP3		
7.SP.B.3	7.SP.B.3	7.SP.B.3	7.SP.B.3	7.SP.B.3

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 2 PDF (for display)
- Anchor Chart PDF, *Creating Box Plots*
- calculators
- rulers

Math Language Development

Review words

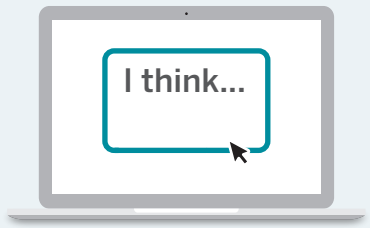
- *box plot*
- *five-number summary*
- *interquartile range (IQR)*
- *mean*
- *median*
- *mode*
- *outlier*
- *range*
- *sample*
- *variability*

Amps | Featured Activity

Activity 1

See Student Thinking

Students are asked to explain their thinking as they compare two box plots, and these explanations are available to you digitally, in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may become frustrated if they cannot clearly compare data distributions. Encourage students to use the data sets, the measures they calculated, and the displays they created to support their conclusions **(MP3)**.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

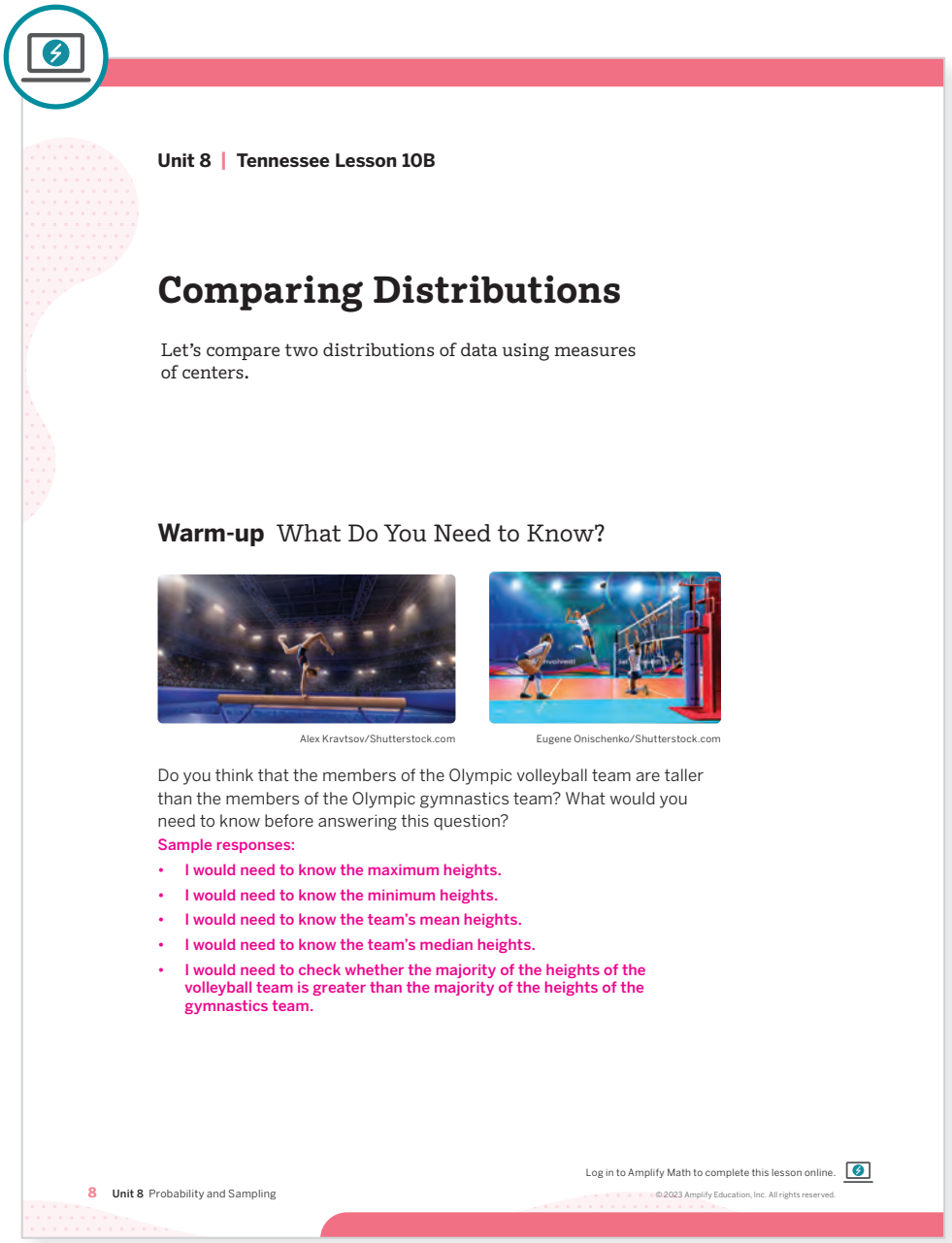
- The **Warm-up** may be omitted.

Pairs | 5 min

Warm-up What Do You Need to Know?

7.SP.B.3

Students think about what information is needed in order to compare the heights of two Olympic teams.



Unit 8 | Tennessee Lesson 10B

Comparing Distributions

Let's compare two distributions of data using measures of centers.

Warm-up What Do You Need to Know?

Alex Kravtsov/Shutterstock.com

Eugene Onischenko/Shutterstock.com

Do you think that the members of the Olympic volleyball team are taller than the members of the Olympic gymnastics team? What would you need to know before answering this question?

Sample responses:

- I would need to know the maximum heights.
- I would need to know the minimum heights.
- I would need to know the team's mean heights.
- I would need to know the team's median heights.
- I would need to check whether the majority of the heights of the volleyball team is greater than the majority of the heights of the gymnastics team.

Log in to Amplify Math to complete this lesson online.

1 Launch

Conduct the *Think-Pair-Share* routine.

2 Monitor

Help students get started by activating background knowledge and asking what they know about the heights of gymnastics and volleyball players.

Look for points of confusion:

- **Thinking they are only comparing one person from each team.** Let students know they are comparing the heights of the two entire teams.

Look for productive strategies:

- Writing multiple answers for things they would need to know to determine which team was taller.
- Recalling the data from Lesson 10A, Practice Problem 5 to infer that volleyball players are generally taller than gymnasts.

3 Connect

Have students share their responses and reasoning to the prompt.

Highlight that comparing two individuals is straightforward but when comparing two data sets, more information is needed. For instance, knowing the measures of center and variability, whether there are any outliers, and whether there is overlap can help compare the data sets.

Ask, "Which team would you expect to be taller?"

Differentiated Support

Accessibility: *Activate Background Knowledge*

Use the *Poll the Class* routine to determine what sport your students would choose to add to their school, from the ones shown in the Warm-up. Consider using the digital poll provided in the Amps slides.

Power-up

To power up students' ability to determine and compare the typical values of two different data sets, ask:

The data sets show the heights of the members of Clare's family and the heights of the members of Diego's family.

Clare's family heights (in.)	Diego's family heights (in.)
28, 39, 43, 52, 63	49, 60, 69, 70, 77

Whose family, on average, is taller? Show or explain your thinking.

Diego's family; Sample response: The mean of the heights of Diego's family is greater than the mean of the heights of Clare's family. The mean height of Clare's family is 45 in., and the mean height of Diego's family is 65 in.

Use: Before Activity 1

Informed by: Performance on Tennessee Lesson 10A, Practice Problem 5

Pairs | 15 min

Activity 1 Team Heights

MP3
7.SP.B.3

Students use measures of center and variability to draw conclusions about the data sets and then use box plots to compare the data to determine whether they were correct **(MP3).**

Amps Featured Activity See Student Thinking

Name: _____ Date: _____ Period: _____

Activity 1 Team Heights

Let's compare the heights of the Olympic gymnastics team, volleyball team, and softball team. Choose two teams to compare and circle those data sets.

Gymnastic team heights (in.)	Softball team heights (in.)	Volleyball team heights (in.)
56 57 58 60 60 62 62 63 65	63 64 66 66 67 67 67 67 68 68 68 69 70 74 75	67 69 70 70 71 72 73 74 74 74 75 76 77

1. For the teams you chose, determine the mean of the heights for each team.
   Gymnastics team: 60.3 in. Softball team: 67.9 in. Volleyball team: 72.5 in.

2. For the teams you chose, determine the median of the heights for each team.
   Gymnastics team: 60 in. Softball team: 67 in. Volleyball team: 73 in.

3. For the teams you chose, determine the mode of the heights for each team.
   Gymnastics team: 60 in. and 62 in. Softball team: 67 in. Volleyball team: 74 in.

4. For the teams you chose, can you conclude that one team is taller than the other? Explain your thinking.
   Answers may vary.

Now, let's determine which team has more variability.

5. For the teams you chose, determine the range of the heights for each team.
   Gymnastics team: 9 in. Softball team: 12 in. Volleyball team: 10 in.

6. For the teams you chose, determine the interquartile range (IQR) of the heights for each team.
   Gymnastics team: 5 in. Softball team: 3 in. Volleyball team: 4.5 in.

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Provide access to calculators for the remainder of the lesson. Remind students that a box plot is created using the values from the five-number summary of a data set. **Note:** If students are having difficulty creating their box plots, pause the class and demonstrate how to plot the values from the five-number summary and review how to draw each section of the box plot.

2 Monitor

Help students get started by activating prior knowledge of calculating the mean, median, and mode of a data set.

Look for points of confusion:

- **Switching the values for mean and median.** Remind students to carefully calculate each one.
- **Drawing the box plots so they overlap each other.** Let students know that the box plots should be parallel and one should be above the other, but they should use the same number line.

Look for productive strategies:

- Using words, such as *quartile* and/or *percentages*, to describe why one team is taller than the other.
- Using the range or IQR to describe and compare the variability of the data sets

Activity 1 continued

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Ensure that the Anchor Chart PDF, *Creating Box Plots* is displayed so that students can refer to the process for creating a box plot in Problem 7.

Accessibility: *Vary Demands to Optimize Challenge*

Consider one of the alternative approaches to this activity:

- Providing the calculations for the mean in Problem 1 so that students can spend more time comparing the means.
- Providing the five-number summary for each set of data so that students can spend more time comparing the distributions of each data set.

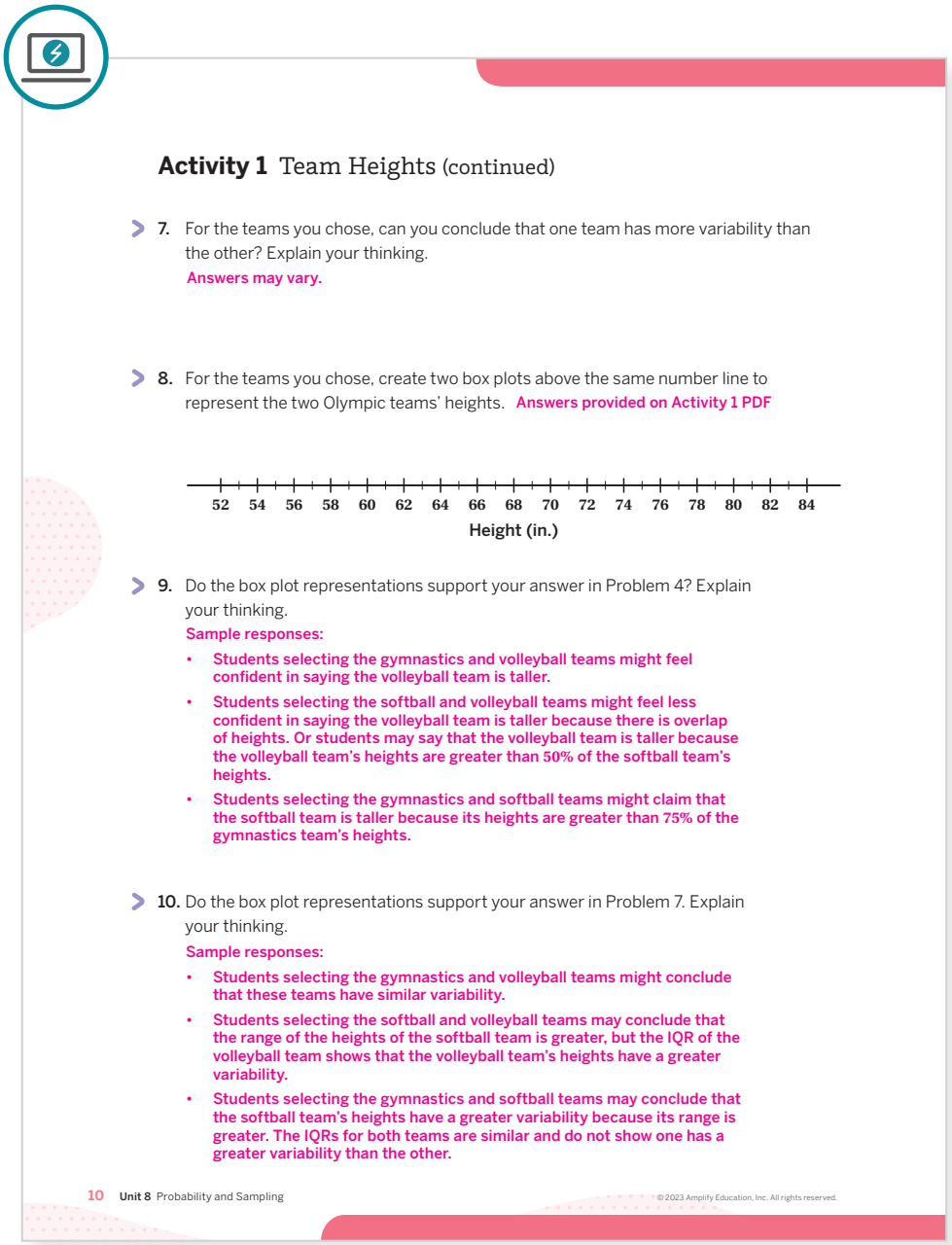
Extension: *Math Enrichment*

Tell students that the coach of each team has added an additional player. Ask them to describe how the means might be affected. If the additional player's height is close to the mean height, the mean will likely not vary much. If it is far away from the mean height, the mean will likely change.

Activity 1 Team Heights (continued)

MP3
7.SP.B.3

Students use measures of center and variability to draw conclusions about the data sets and then use box plots to compare the data to determine whether they were correct **(MP3).**



Activity 1 Team Heights (continued)

7. For the teams you chose, can you conclude that one team has more variability than the other? Explain your thinking.

Answers may vary.

8. For the teams you chose, create two box plots above the same number line to represent the two Olympic teams' heights. Answers provided on Activity 1 PDF

9. Do the box plot representations support your answer in Problem 4? Explain your thinking.

Sample responses:

- Students selecting the gymnastics and volleyball teams might feel confident in saying the volleyball team is taller.
- Students selecting the softball and volleyball teams might feel less confident in saying the volleyball team is taller because there is overlap of heights. Or students may say that the volleyball team is taller because the volleyball team's heights are greater than 50% of the softball team's heights.
- Students selecting the gymnastics and softball teams might claim that the softball team is taller because its heights are greater than 75% of the gymnastics team's heights.

10. Do the box plot representations support your answer in Problem 7. Explain your thinking.

Sample responses:

- Students selecting the gymnastics and volleyball teams might conclude that these teams have similar variability.
- Students selecting the softball and volleyball teams may conclude that the range of the heights of the softball team is greater, but the IQR of the volleyball team shows that the volleyball team's heights have a greater variability.
- Students selecting the gymnastics and softball teams may conclude that the softball team's heights have a greater variability because its range is greater. The IQRs for both teams are similar and do not show one has a greater variability than the other.

3 Connect

Display the Activity 1 PDF to show all three box plots and have students share their reasoning for Problems 9 and 10 **(MP3)**.

Highlight that the measures of center give one value to represent the data but a visual display, such as a box plot, helps show the spread of the data. Having two box plots on the same number line allows for comparison between data sets.

Ask:

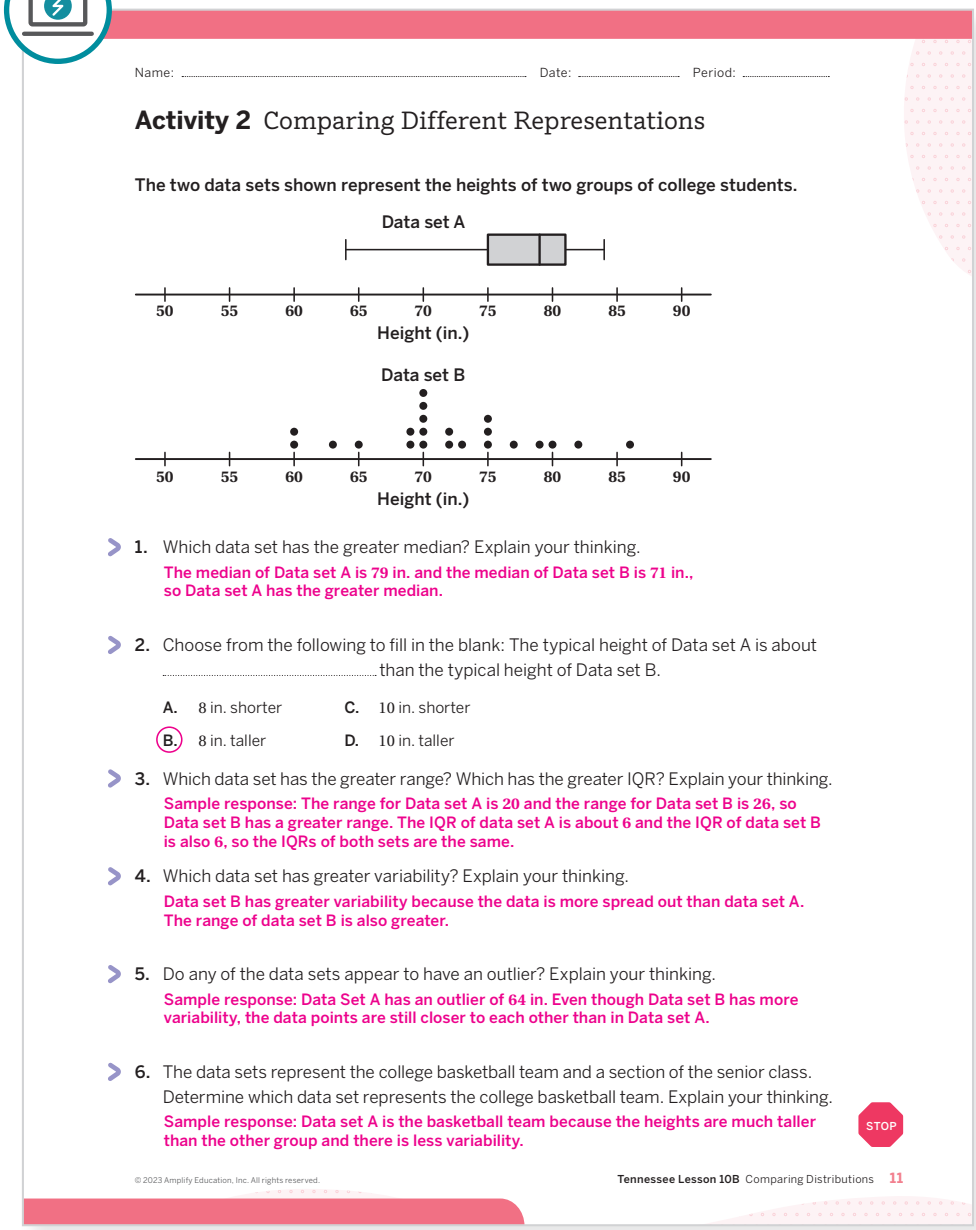
- "How did seeing the data displayed in the box plot clarify your understanding of the data sets?"
- "What should be taken into consideration when comparing two distributions?"

Pairs | 15 min

Activity 2 Comparing Different Representations

MP3
7.SP.B.3

Students use different representations of data to compare distributions and draw conclusions about the groups they represent **(MP3).**



Name: _____ Date: _____ Period: _____

Activity 2 Comparing Different Representations

The two data sets shown represent the heights of two groups of college students.

Data set A

Data set B

1. Which data set has the greater median? Explain your thinking.
   The median of Data set A is 79 in. and the median of Data set B is 71 in., so Data set A has the greater median.

2. Choose from the following to fill in the blank: The typical height of Data set A is about _____ than the typical height of Data set B.

   A. 8 in. shorter
   C. 10 in. shorter
   (B.) 8 in. taller
   D. 10 in. taller

3. Which data set has the greater range? Which has the greater IQR? Explain your thinking.
   Sample response: The range for Data set A is 20 and the range for Data set B is 26, so Data set B has a greater range. The IQR of data set A is about 6 and the IQR of data set B is also 6, so the IQRs of both sets are the same.

4. Which data set has greater variability? Explain your thinking.
   Data set B has greater variability because the data is more spread out than data set A. The range of data set B is also greater.

5. Do any of the data sets appear to have an outlier? Explain your thinking.
   Sample response: Data Set A has an outlier of 64 in. Even though Data set B has more variability, the data points are still closer to each other than in Data set A.

6. The data sets represent the college basketball team and a section of the senior class. Determine which data set represents the college basketball team. Explain your thinking.
   Sample response: Data set A is the basketball team because the heights are much taller than the other group and there is less variability.

STOP

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking them what measure could be used to determine the typical value in a set.

Look for points of confusion:

- **Trying to compare the box plot of Data set A to the dot plot of Data set B.** Ask, "What can you do to the data to make it easier to compare?"

Look for productive strategies:

- Creating similar data displays for both sets of data.
- Identifying the five-number summary of Data Set B.
- Using the range or IQR to describe variability.

3 Connect

Display the Activity 2 PDF and have students share a sentence comparing the box plots using the percentages from the box plot. Example: Most of Data set A falls within the last 50%of Data set B.

Ask:

- "How is the displayed box plot for Data Set A different from the one in your workbook ?" It has a plotted point at 64; the left whisker is shorter.
- "How many students are in each data set?" Set A is unknown and Set B has 22

Highlight that each representation is helpful to see different aspects of the data, but it might be helpful to represent the data in the same way to compare. Note the point on the displayed box plot of Data set A, which indicates that the value of that data point is an outlier.

Ask, "Could you represent both data sets as dot plots? Why or why not?"

Differentiated Support

Extension: *Math Enrichment*

Have students complete the following problem:

Create a set of data with at least 10 values that could produce the box plot for Data set A and have a mean of 77.5 in. ?

Sample response: 64, 75, 75, 76, 79, 79, 80, 81, 82, 84

Math Language Development

MLR2: Collect and Display

While students respond to Problem 6, circulate and collect any language they use to describe how they determine which set of data matched each population. Add this language to a class display for this unit, and invite students to add to and refer to this class display throughout the rest of this unit.

English Learners

Allow students to work with a partner who shares the same primary language.

Whole Class | 5 min

Summary

7.SP.B.3

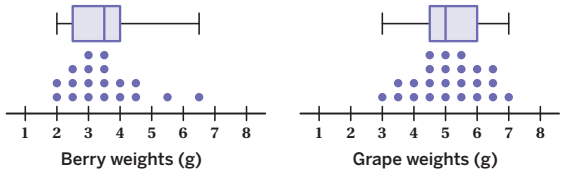
Review and synthesize how to informally identify the amount of visual overlap between two distributions.

Summary

In today's lesson . . .

You decided whether two sets of data were very different from each other. Comparing two individuals or objects is fairly straightforward. The question, "Which piece of fruit weighs more?" can be answered by measuring the weights of two pieces of fruit and comparing them directly. However, comparing two data sets requires some additional analysis.

Here are two box plots, and their corresponding dot plots (for reference), showing the weights of some berries and grapes.



Notice how the box plots make it possible to compare the minimums, maximums, medians, ranges, and IQRs of the two groups at a glance. In this case, you can clearly see that the maximum and minimum weights are approximately the same, but grape weights are more consistent, and most grapes typically weigh more than most berries.

Reflect:

Synthesize

Highlight comparing distributions is possible but requires analyzing more than one piece of information. Measures of center, variability, and visual overlap can be used to help compare the data sets.

Ask:

- "What are some measures of center, and how are they calculated?" The mean is calculated by summing the data and dividing the sum by the total number of data values in the set. The median is calculated by finding the middle of the data. The mode is the value that appears most in the data set.
- "Why are measures of center useful for comparing two groups?" They help compare how the data sets are similar or different.
- "What are some measures of variability, and how are they calculated?" The range is the difference between the maximum and minimum value. IQR is the difference between the first and third quartiles.
- "Why are measures of variability also helpful when comparing two groups?" Measures of variability help describe the distance between data values, or how spread out the data sets are.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can measures of center and variability be used to analyze and compare distributions?"

Independent | 5 min

Exit Ticket

7.SP.B.3

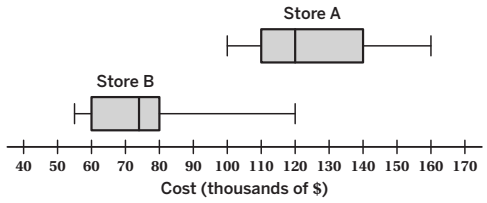
Students demonstrate their understanding by comparing two distributions.

Printable

Name: ..... Date: ..... Period: .....

Exit Ticket 8.10B

Priya's parents are interested in buying a recreational vehicle (RV) for traveling and camping. They researched the prices of several RVs for sale at two different stores and recorded the prices in thousands of dollars.



For each statement about the price of RVs, state whether you agree or disagree. Explain your thinking.

1. More than half of the RVs at Store A cost $140,000.

   Disagree; Sample response: 140 thousand is equal to Q3, so no more than 25% would cost more than $140,000.

2. About 50% of the RVs at Store A cost more than the RVs at Store B.

   Agree; Sample response: The median of Store A is equal to the maximum value of Store B which means 50% of the data from Store A is greater than or equal to the maximum value of Store B.

3. The prices at Store B vary more than the prices at Store A.

   Sample responses:
   - **Agree; The range for Store B is about $65,000, while the range for Store A is $60,000.**
   - **Disagree; The IQR for Store B is $20,000 but the IQR for Store A is $30,000.**

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can compare data sets using their measures of center. 1 2 3

b. I can compare data sets using measures of variability, such as the range and IQR. 1 2 3

Success looks like . . .

- **Goal:** Calculating measures of center and variability for data sets represented in different ways.
- **Language Goal:** Comparing and contrasting distributions in terms of their center, spread, variability, and visual overlap. **(Speaking and Listening, Writing)**
  - » Providing an explanation for each statement using the values of the data sets' quartiles, measures of center, or measures of variability.

Suggested next steps

If students do not support their thinking, consider:

- Reviewing Activity 1 Problems 9 and 10.
- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- During the discussion about comparing distributions, how did you encourage each student to listen to one another's ideas? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

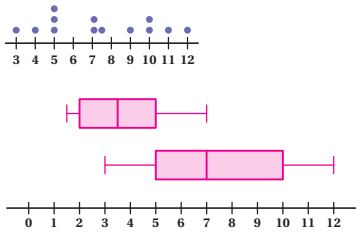
Practice

1. The data show the number of hours per week that sixth and seventh grade students spent doing homework. Create two box plots above the same number line to represent both sets of data.

Sixth graders:

Minimum: 1.5 Q1: 2 Median: 3.5 Q3: 5 Maximum: 7

Seventh graders:



a. How are the distributions of the time spent doing homework by the students in the two grades most alike? Explain your thinking.

Sample responses:

- **About 50% of the students in each grade spent between 3 and 7 hours doing homework per week.**
- **The students in each grade who spent the most hours doing homework per week only spent up to 2 hours more time than typical students in their grade.**
- **A typical seventh grader spent as much time doing homework per week as the sixth graders who spent the most time doing homework per week.**

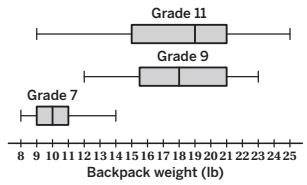
b. Which grade level of students corresponds to the data with the greatest variability in time spent on homework? Explain your thinking with measures of data.

The seventh graders' data have the greatest variability. Sample response: For the seventh grade, the range is 9 hours and the IQR is 5 hours. For the sixth grade, the range is 5.5 hours and the IQR is 3 hours. The range and IQR are greater for seventh graders.

Practice

Name: Date: Period:

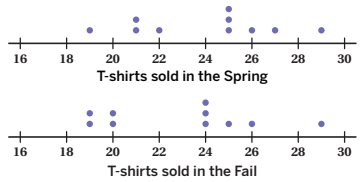
2. The box plots show the weights of backpacks for several students in seventh grade, ninth grade, and eleventh grade.



Compare the weights of the backpacks for the students in these three grades.

Sample response: The backpacks of the Grade 7 students tend to weigh less than the backpacks of the Grade 9 and Grade 11 students. A typical weight for a Grade 7 students' backpack is about 10 lb compared to a typical weight of about 18 lb for Grade 9. The distributions for the Grade 9 and Grade 11 students are similar, but the distribution for the Grade 11 students has a larger spread.

3. A school's art club held a fundraiser to raise money for art supplies. The dot plots show the number of t-shirts sold each week during the fall and spring seasons.



For each statement about the number of t-shirts sold, state whether you agree or disagree. Explain your thinking.

a. In general, the art club sold more t-shirts in the spring than in the fall.

Agree; Sample response: The median number of t-shirts sold in the spring is greater than the median number of t-shirts sold in the fall.

b. There is more variability in the number of t-shirts sold in the fall than in the spring.

Disagree; Sample response: The range and the IQR for the number of shirts sold in both seasons are the same, so they have the same variability.

4. Solve each inequality:

a. $4x + 7 < 32$

$4x + 7 = 32$

$x = 6.25$

For $x = 0 : 4(0) + 7 < 32$ is true

$x < 6.25$

b. $4(x + 7) \geq 32$

$4(x + 7) = 32$

$x = 1$

For $x = 0 : 4(0 + 7) \geq 32$ is not true

$x \geq 1$

5. Write a statistical question about your local weather.

Sample response: What is the typical amount of rainfall during the month of July in Nashville?

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 1	7.SP.B.3	3
	2	Activity 1	7.SP.B.3	2
	3	Activity 2	7.SP.B.3	2
Spiral	4	Unit 6 Lesson 16	7.EE.B.4b	2
Formative	5	Unit 8 Tennessee Lesson 10C	6.SP.A.1	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.

Larger Populations

Let's compare larger populations of data.

Focus

Goals

1. **Language Goal:** Comprehend that the terms *population* and *sample* refer to the whole group and a part of the whole group under consideration, respectively. **(Speaking and Listening, Writing)**
2. **Language Goal:** Describe a sample for a given population. **(Speaking and Listening, Writing)**
3. **Language Goal:** Explain that a sample may be used when it is unreasonable to gather data about an entire population. **(Speaking and Listening)**

Coherence

Today

Students are introduced to the idea of using data from a sample of a population when it is impractical or impossible to gather data from every individual in the population. Students consider whether the people in their class would be an adequate sample for several different questions and associated populations **(MP3)**. **Note:** Throughout the last lessons, Practice Problems, labeled as *Capstone project helper*, will aid students in their statistical Capstone project for Lesson 17.

Previously

In Tennessee Lesson 10B, students calculated the measures of center (mean, median, and mode) and measures of variability (range and IQR) for two data sets and compared them using different representations.

Coming Soon

In Lesson 13, students will learn what makes some samples more representative of a population than others. Students will also explore the best ways to obtain such samples.

Rigor

- Students build **conceptual understanding** of how a population and a sample of that population are related.

Standards

Addressing

7.SP.A.1

Explore how statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Also Addressing: **7.SP.B**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent
MP1		MP3		
7.SP.A.1, 7.SP.B	7.SP.A.1	7.SP.A.1, 7.SP.B	7.SP.A.1	7.SP.A.1

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket PDF
- Additional Practice Book
- Activity 1 PDF, pre-cut cards, one set per pair
- class list of first and last names

Math Language Development

New words

- ***population***
- ***sample***

Review words

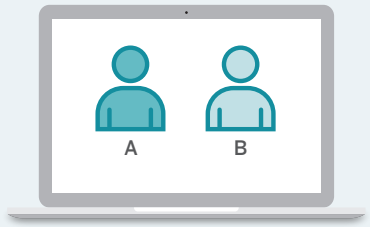
- *IQR*
- *mean*
- *range*
- *statistical question*
- *variability*

Amps | Featured Activity

Activity 2

Aggregate Class Data

Collect and share class data quickly using the aggregation feature.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may rush to a conclusion that they have enough information about the population they are considering **(MP3)**. Encourage students to engage in metacognitive functions, i.e., thinking about their own thinking process to make sure their conclusions make sense given the context of the data.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

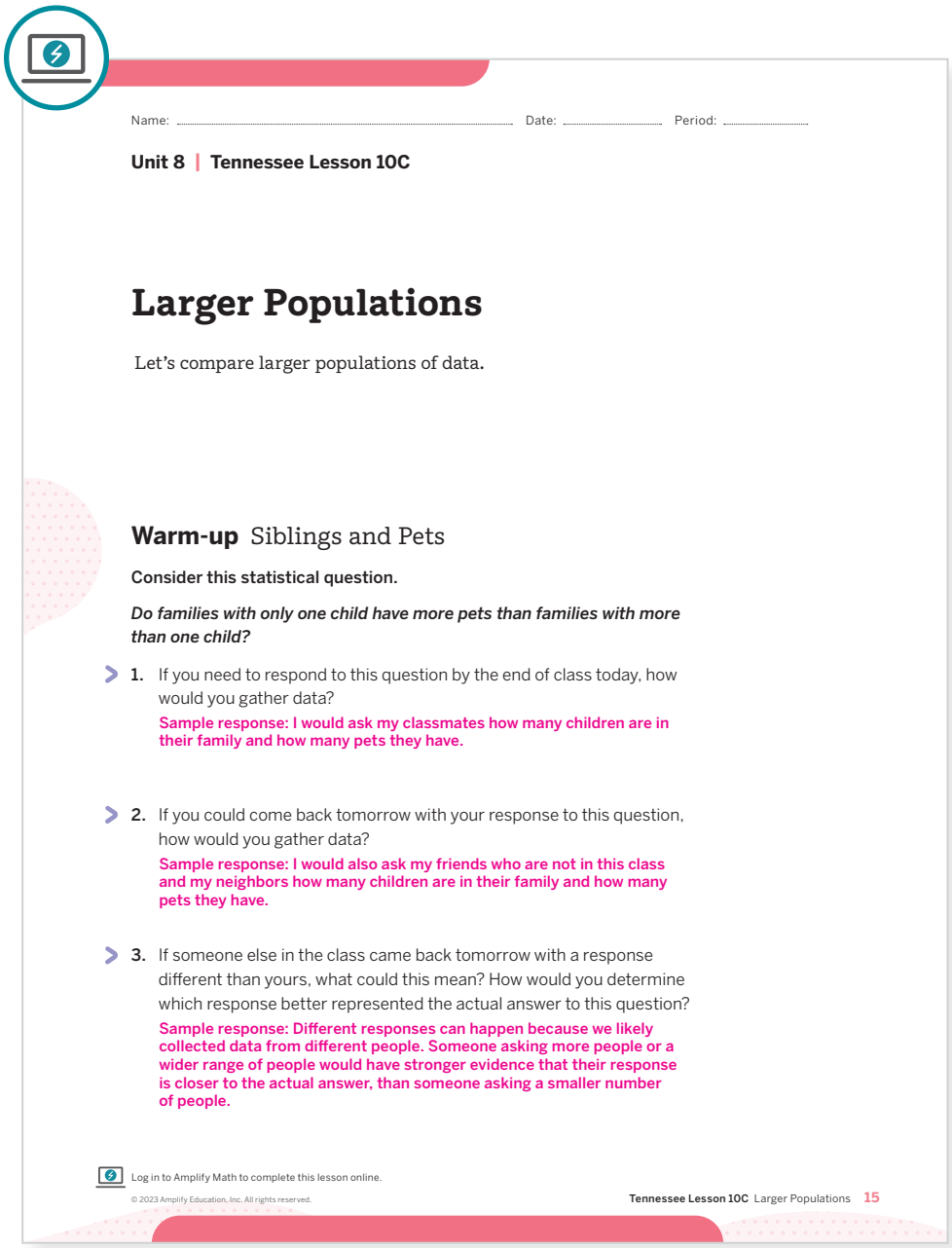
- In the **Warm-up**, omit Problem 3.
- For **Activity 2**, provide a class list with the letters in each name already counted for. To save more time, provide the mean for the class data.

Pairs | 10 min

Warm-up Siblings and Pets

MP1
7.SP.A.1, 7.SP.B

Students consider ways they gather data to realize it is unreasonable to collect data from everyone addressed by the question.



Name: _____ Date: _____ Period: _____

Unit 8 | Tennessee Lesson 10C

Larger Populations

Let's compare larger populations of data.

Warm-up Siblings and Pets

Consider this statistical question.

Do families with only one child have more pets than families with more than one child?

1. If you need to respond to this question by the end of class today, how would you gather data?

   Sample response: I would ask my classmates how many children are in their family and how many pets they have.

2. If you could come back tomorrow with your response to this question, how would you gather data?

   Sample response: I would also ask my friends who are not in this class and my neighbors how many children are in their family and how many pets they have.

3. If someone else in the class came back tomorrow with a response different than yours, what could this mean? How would you determine which response better represented the actual answer to this question?

   Sample response: Different responses can happen because we likely collected data from different people. Someone asking more people or a wider range of people would have stronger evidence that their response is closer to the actual answer, than someone asking a smaller number of people.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate students' prior knowledge by asking them to explain the term *statistical question*.

2 Monitor

Help students get started by having them think about who should be included in answering the question.

Look for points of confusion:

- **Thinking that families with only one child are the population.** Remind students that they need to ask all the family units with at least one child.

3 Connect

Have pairs of students share how they would gather data to answer the question.

Highlight the difference between the data you would like to have to answer the question and the data you have available **(MP1)**. Note that the populations in the previous lesson were manageable for considering all members of the population, but sometimes a population is too large to use or from which to collect all the necessary data.

Define the term ***population*** as the set of people or things from which the data is taken. A ***sample*** is the part of the population from which the data are actually collected.

Ask:

- "What is the population for the statistical question?" Every family unit in the world with at least one child.
- "What might be a sample you could use to answer the question?" Students in our class.
- "Why is it unreasonable to actually collect all the necessary data to answer the question?" There are too many people to collect data from.

Math Language Development

MLR2: Collect and Display

To help students make sense of the terms *sample* and *population*, draw a diagram of a few circles inside a larger circle. Label the larger circle with *population* and each smaller circle with "*sample*.

Accessibility: *Representation: Develop Language and Symbols*

Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include to support their understanding of the terms *population* and *sample*.

Power-up

To power up students' ability to identify a statistical question, ask:

Select *all* the questions that are statistical questions

A. Why do you like to listen to music?

B. How many songs does the class usually listen to each day?

C. How many songs did you listen to today?

D. How long does it typically take for 7th graders to get to school?

Use: Before the Warm-up

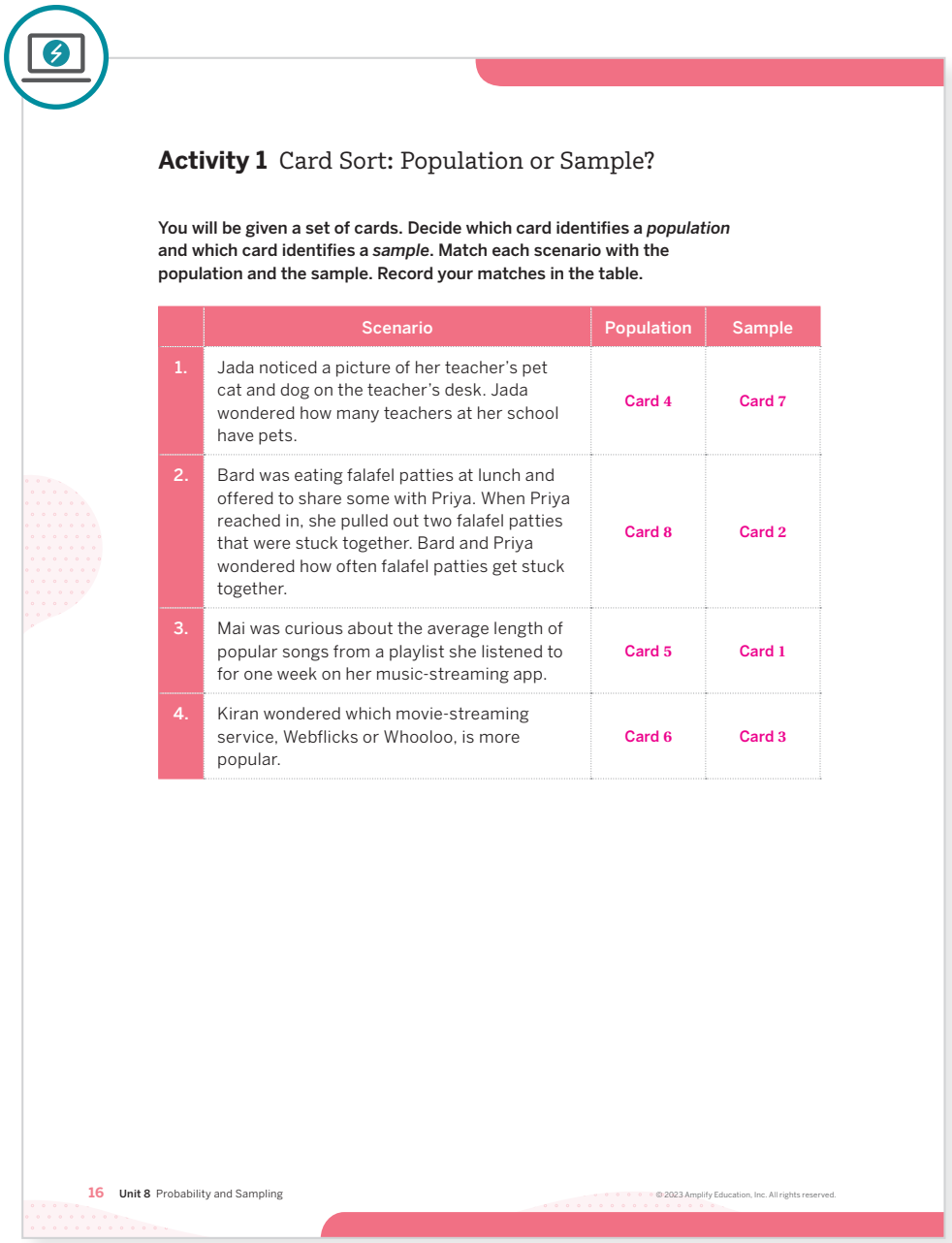
Informed by: Performance on Tennessee Lesson 10B, Practice Problem 5

Pairs | 10 min

Activity 1 Card Sort: Population or Sample?

7.SP.A.1

Students practice identifying populations and samples based on several scenarios.



Activity 1 Card Sort: Population or Sample?

You will be given a set of cards. Decide which card identifies a *population* and which card identifies a *sample*. Match each scenario with the population and the sample. Record your matches in the table.

	Scenario	Population	Sample
1.	Jada noticed a picture of her teacher's pet cat and dog on the teacher's desk. Jada wondered how many teachers at her school have pets.	Card 4	Card 7
2.	Bard was eating falafel patties at lunch and offered to share some with Priya. When Priya reached in, she pulled out two falafel patties that were stuck together. Bard and Priya wondered how often falafel patties get stuck together.	Card 8	Card 2
3.	Mai was curious about the average length of popular songs from a playlist she listened to for one week on her music-streaming app.	Card 5	Card 1
4.	Kiran wondered which movie-streaming service, Webflicks or Whooloo, is more popular.	Card 6	Card 3

16 Unit 8 Probability and Sampling

1 Launch

Distribute the pre-cut cards from the Activity 1 PDF. Conduct the ***Card Sort*** routine.

2 Monitor

Help students get started by having them match pairs of cards together before reading the scenarios.

Look for points of confusion:

- **Switching the terms *population* and *sample*.** Have students think about which card represents the larger group (population) and which card represents the smaller group (sample).

3 Connect

Have pairs of students share the populations and samples for the scenarios.

Ask:

- "For each scenario, could there be another population other than the ones given?" No, The scenario should describe the population you want to research.
- "For each scenario, could there be another sample other than the ones given?" Yes. A sample refers to a few of the individuals from the data that will be collected and there can be many different sets of individuals.
- "What are some of the advantages and disadvantages for using the samples in this activity?" Some samples are more convenient but might miss large sections of the population and not be an accurate representation.

Highlight that well phrased questions should have known populations. A question that is not well phrased should be reconsidered so that the purpose of the question is clear. However, there are usually many ways to find samples within the populations.

Differentiated Support

Accessibility: *Students With Disabilities*

Allow students to tape the cards on their pages for reference during later lessons.

Accessibility: *Students Who Need Help*

Provide students with cards 2, 4, 7, and 8. Have them focus on answering Problems 1 and 2. If time permits, give them the remaining cards.

Extension: *Enrichment*

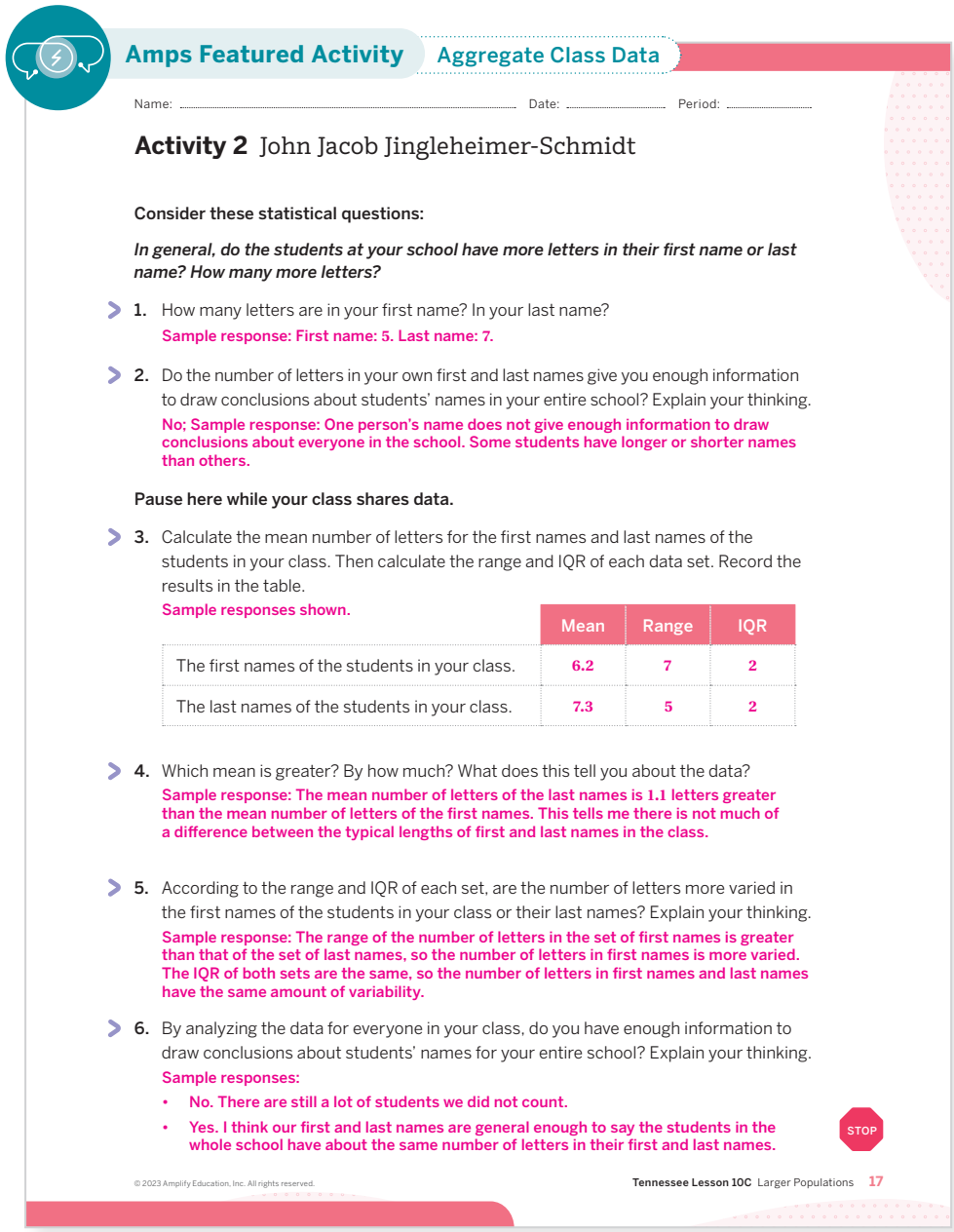
Have students write another scenario and give a population and sample for that scenario.

Pairs | 15 min

Activity 2 John Jacob Jingleheimer-Schmidt

MP3
7.SP.A.1, 7.SP.B

Students compare two groups by collecting data to draw a conclusion about a larger group.



Amps Featured Activity Aggregate Class Data

Name: _____ Date: _____ Period: _____

Activity 2 John Jacob Jingleheimer-Schmidt

Consider these statistical questions:

In general, do the students at your school have more letters in their first name or last name? How many more letters?

1. How many letters are in your first name? In your last name?
   Sample response: First name: 5. Last name: 7.

2. Do the number of letters in your own first and last names give you enough information to draw conclusions about students' names in your entire school? Explain your thinking.
   No; Sample response: One person's name does not give enough information to draw conclusions about everyone in the school. Some students have longer or shorter names than others.

Pause here while your class shares data.

3. Calculate the mean number of letters for the first names and last names of the students in your class. Then calculate the range and IQR of each data set. Record the results in the table.
   Sample responses shown.

	Mean	Range	IQR
The first names of the students in your class.	6.2	7	2
The last names of the students in your class.	7.3	5	2

4. Which mean is greater? By how much? What does this tell you about the data?
   Sample response: The mean number of letters of the last names is 1.1 letters greater than the mean number of letters of the first names. This tells me there is not much of a difference between the typical lengths of first and last names in the class.

5. According to the range and IQR of each set, are the number of letters more varied in the first names of the students in your class or their last names? Explain your thinking.
   Sample response: The range of the number of letters in the set of first names is greater than that of the set of last names, so the number of letters in first names is more varied. The IQR of both sets are the same, so the number of letters in first names and last names have the same amount of variability.

6. By analyzing the data for everyone in your class, do you have enough information to draw conclusions about students' names for your entire school? Explain your thinking.
   Sample responses:
   - **No. There are still a lot of students we did not count.**
   - **Yes. I think our first and last names are general enough to say the students in the whole school have about the same number of letters in their first and last names.**

STOP

1 Launch

Ask students why knowing the length of names would be helpful (i.e., printing name cards for an event, diplomas, etc.) Have students answer Problems 1—2. Then have students share their name letter counts.

2 Monitor

Help students get started by providing the class list with the names already counted.

Look for points of confusion:

- **Thinking that their own name is a reasonable sample for the school population.** Ask, "Is your name an example of a random sample? Is a single name a large enough sample for making an inference about the population?"
- **Calculating the median instead of the mean.** Ask, "How is the mean, or average, calculated?"

3 Connect

Have individual students share their conclusions about the entire school's data based on the class data.

Highlight how the data they have might relate to a larger group. A sample might give some estimate of a larger population, but the estimate should not be assumed to be exact **(MP3)**.

Ask:

- "Do you expect the mean length of first names for the school to be exactly the same as the mean length for the class?"
- "How would a really long name, such as Jingleheimer-Schmidt, affect the value of the mean?"
- "Why might IQR be a better measure of variability than range in this context?"

Highlight that a long name, such as Jingleheimer-Schmidt, might be an outlier in the set of last names because it has so many letters. Including an outlier in the set would affect the mean and the range, which are both sensitive to outliers.

Differentiated Support

Accessibility: *Students Who Need Help*

Provide students with the mean for the class data. Have students focus on comparing the differences.

Extension: *Enrichment*

Have students count the letters in their middle names (or an additional name) and have them compare that data set to those of the first and last names.

Math Language Development

MLR8: Discussion Supports

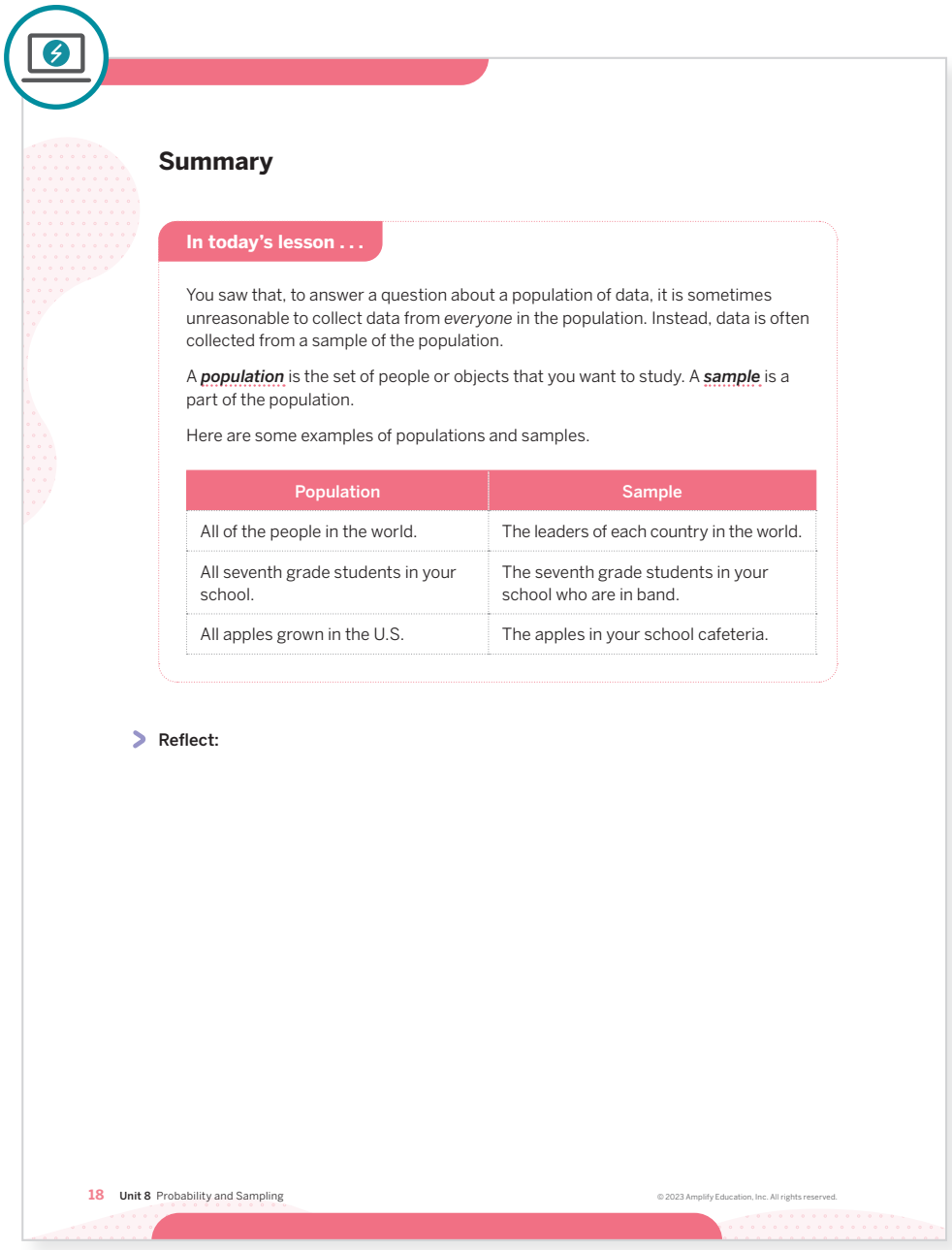
Display the sentence frame, "The mean length of first names for the school will not be exactly the same as the mean length for the class because ....."

Whole Class | 5 min

Summary

7.SP.A.1

Review and synthesize why collecting data from a population is not always reasonable or efficient and why using samples can help answer statistical questions about the population.



Summary

In today's lesson . . .

You saw that, to answer a question about a population of data, it is sometimes unreasonable to collect data from *everyone* in the population. Instead, data is often collected from a sample of the population.

A ***population*** is the set of people or objects that you want to study. A ***sample*** is a part of the population.

Here are some examples of populations and samples.

Population	Sample
All of the people in the world.	The leaders of each country in the world.
All seventh grade students in your school.	The seventh grade students in your school who are in band.
All apples grown in the U.S.	The apples in your school cafeteria.

> **Reflect:**

18 **Unit 8** Probability and Sampling

Synthesize

Have students share a question they might be curious about and identify the population and sample they could use in their study.

Highlight that the size of the population, by definition, is always larger than the size of the sample for a survey. A sample is the specific part of the population from which data are collected.

Formalize vocabulary:

- ***population***
- ***sample***

Ask:

- "When the group you are interested in becomes too large, how can you obtain some data to begin answering a question about the group?" By taking a sample.
- "What are some drawbacks of using samples instead of the entire population?" Some groups may not be included in the sample.
- "What are some reasons samples are necessary?" "If you wanted to know which breed of a dog is most popular as a pet in this state, think about different samples you could use. Would asking all the teachers at this school be a good sample? The people at a dog park? A few dog owners from around the state?

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What characteristics or key words did you look for when completing your card sort today?"
- "What questions do you still have about the difference between a population and a sample?"

Independent | 5 min

Exit Ticket

7.SP.A.1

Students demonstrate their understanding of populations and samples by gathering data related to a population and a sample.

Printable

Name: ..... Date: ..... Period: .....

Exit Ticket 8.10C

Shawn wants to know how many games teenagers in the United States typically have on their phones.

1. What is the population for Shawn's question?
   All teenagers in the United States who have a phone.

2. Explain why collecting data for this population might be difficult.
   Sample response: There are too many people to collect data from every teenager in the United States. It would require a lot of time, energy, and, potentially, money to collect the data from everyone.

3. Give an example of a sample that could be used to help answer Shawn's question.
   Sample response: Ask 20 teens at Shawn's school how many games they have on their phones.

Self-Assess

	1	2	3
	I don't really get it	I'm starting to get it	I got it
a	I can explain why it may be useful to gather data on a sample of a population.		
b	When I read or hear a statistical question, I can name the population of interest and give an example of a sample for that population.		

Success looks like . . .

- **Language Goal:** Comprehending that the terms *population* and *sample* refer to the whole group and a part of the whole group under consideration, respectively. **(Speaking and Listening, Writing)**
  - » Identifying the population for Shawn's question.
- **Language Goal:** Describing a sample for a given population. **(Speaking and Listening, Writing)**
  - » Naming an appropriate sample for the population in Shawn's question.
- **Language Goal:** Explaining that a sample may be used when it is unreasonable to gather data about an entire population. **(Speaking and Listening)**
  - » Giving an appropriate explanation for why collecting data for the given population might be difficult.

Suggested next steps

If students struggle to identify the population or sample, consider:

- Reviewing Activity 3.
- Assigning Practice Problems 1—2.

If students struggle to explain why collecting data for a population would be difficult, consider

- Reviewing Activity 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about Activity 1, how did you encourage each student to listen to one another's strategies?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ..... Date: ..... Period: .....

Practice

1. For each sample, describe two possible populations to which it could belong.

Sample responses shown.

Sample	Possible Population 1	Possible Population 2
The prices for apples at two stores near your house.	Prices for apples at all stores in your state.	Prices for all fruit at the same two stores.
The daily high temperatures for the capital cities of all 50 U.S. states over the past year.	The daily high temperatures for all the cities in the U.S. over the past year.	The daily high temperatures for the capital cities of all 50 U.S. states over the past 10 years.

2. Identify the population and a possible sample for the following statistical question.

What is the median salary for teachers in North America?

Sample response:

- Population: The salary of each teacher in North America.
- Sample: The salary of each teacher at your school.

The last lesson of this unit is a Capstone project. In this project, you will:

1. **Pose a statistical question.**
2. **Create a survey.**
3. **Decide on a sample.**
4. **Collect and analyze data from the sample.**
5. **Present your findings.**

In the remaining lessons in this unit, there are problems marked "*Capstone project helper.*" These problems will help you prepare for the project.

3. *Capstone project helper.* Think of a statistical question you are interested in studying. Be sure it meets the criteria shown in the table. An example of a statistical question is: *Do students in my school who have purchased a new phone within the last 3 months tend to have more apps on their phone?*

Answers may vary.

Criteria for Statistical Questions

- Can it be answered by collecting data?
- Will there be some variability in the data?
- Is it possible to actually collect this data?

Practice

Name: ..... Date: ..... Period: .....

4. Match each expression in the first list with an equivalent expression from the second list.

a	$(8x + 6y) + (2x + 4y)$	a	$10(x + y)$
b	$(8x + 6y) - (2x + 4y)$	d	$10(x - y)$
c	$(8x + 6y) - (2x - 4y)$	e	$8x + 6y + 2x - 4y$
d	$8x - 6y + 2x - 4y$	c	$8x + 6y - 2x + 4y$
e	$8x - (-6y - 2x + 4y)$	b	$8x - 2x + 6y - 4y$

5. A school is selling candles for a fundraiser. The school keeps 40% of the total sales as their commission, and they pay the rest to the candle company. The table shows the price and number sold of each candle size. How much money will the school pay to the candle company? Show or explain your thinking.

Candle size	Price of candle ($)	Number of candles sold
Small	11	68
Medium	18	45
Large	25	21

The school paid $1,249.80 to the candle company; Sample response: $68 \cdot 11 + 45 \cdot 18 + 21 \cdot 25 = 2083$, so the school sold $2,083 worth of candles. 60% is paid to the company, so $0.6 \cdot 2083 = 1249.80$.

6. Describe what the terms *shape*, *center*, and *spread* mean in your own words. Use the following dot plot, which shows the ages of the first 20 people surveyed at a movie theater, as an example in your explanation.

Sample responses:

- Shape describes the overall form of the data, including whether any symmetry is displayed. Most of the data values are around 8-10 years. If the values below 7 are excluded, the plot would show some symmetry. With all the data values included, the data are not symmetric.
- The center describes a value that is typical of the data. For this data, the center is around 9 years.
- The spread of the data describes the variability of the data. For this data, the ages go from 4 to 11, which is a range of 7 years.

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 2	7.SP.A.1	2
	2	Activity 2	7.SP.A.1	2
	3	Activity 2	7.SP.A.1	2
Spiral	4	Unit 6 Lesson 22	7.EE.A.1	1
	5	Unit 4 Lesson 9	7.RP.A.3	2
Formative	6	Unit 8 Lesson 13	7.SP.B	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, see the **Grade 7 Additional Practice**.

Which Measure of Center Is Better?

Let's determine when the mean is an appropriate measure of center and when it is not.

Focus

Goals

1. **Language Goal:** Determine which measure of center is best for a given distribution, including one with outliers. **(Speaking and Listening, Writing)**
2. **Language Goal:** Analyze and describe the shape and characteristics of a data distribution. **(Reading and Writing, Speaking and Listening.)**

Coherence

Today

Students revisit dot plots and the three measures of centers to see which is most affected by outliers. They use quantifiable measures to determine which movie should be recommended to a certain age group **(MP2)**.
Note: This lesson's Practice contains a milestone for the Capstone project.

Previously

In Lesson 14, students selected a sample and critiqued different sampling methods as to their benefits and drawbacks.

Coming Soon

In Lesson 16, students will determine whether a sample is representative of the population, and then use proportional reasoning to make predictions about the population.

Rigor

- Students build **conceptual understanding** of the effects that outliers could have on the measures of center.

Standards

Addressing

7.SP.D.8b
Relate and understand the choice of measures of center (median and/or mean) and variability (range and/or interquartile range) **to the shape of the data distribution and the context in which the data were gathered.**

Also Addressing: **7.SP.B.4, 7.SP.D.8**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent
	MP2, MP3	MP2, MP3		
7.SP.B.4	7.SP.D.8, 7.SP.D.8b	7.SP.D.8, 7.SP.D.8b	7.SP.D.8b	7.SP.D.8b

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- Anchor Chart PDF, *Measures of Center and Spread*
- Power-up PDF
- calculators

Math Language Development

Review words

- *box plot*
- *five-number summary*
- *histogram*
- *interquartile range (IQR)*
- *mean*
- *median*
- *mode*
- *outlier*

Amps | Featured Activity

Activity 1

See Student Thinking

Students are asked to explain their thinking behind matching dot plots with data sets, and these explanations are available to you digitally, in real time.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel disorganized as they determine the three measures of center. Encourage students to find ways to keep track of the data and their calculations as this will also organize their thinking for proper quantitative reasoning while problem solving **(MP2)**.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

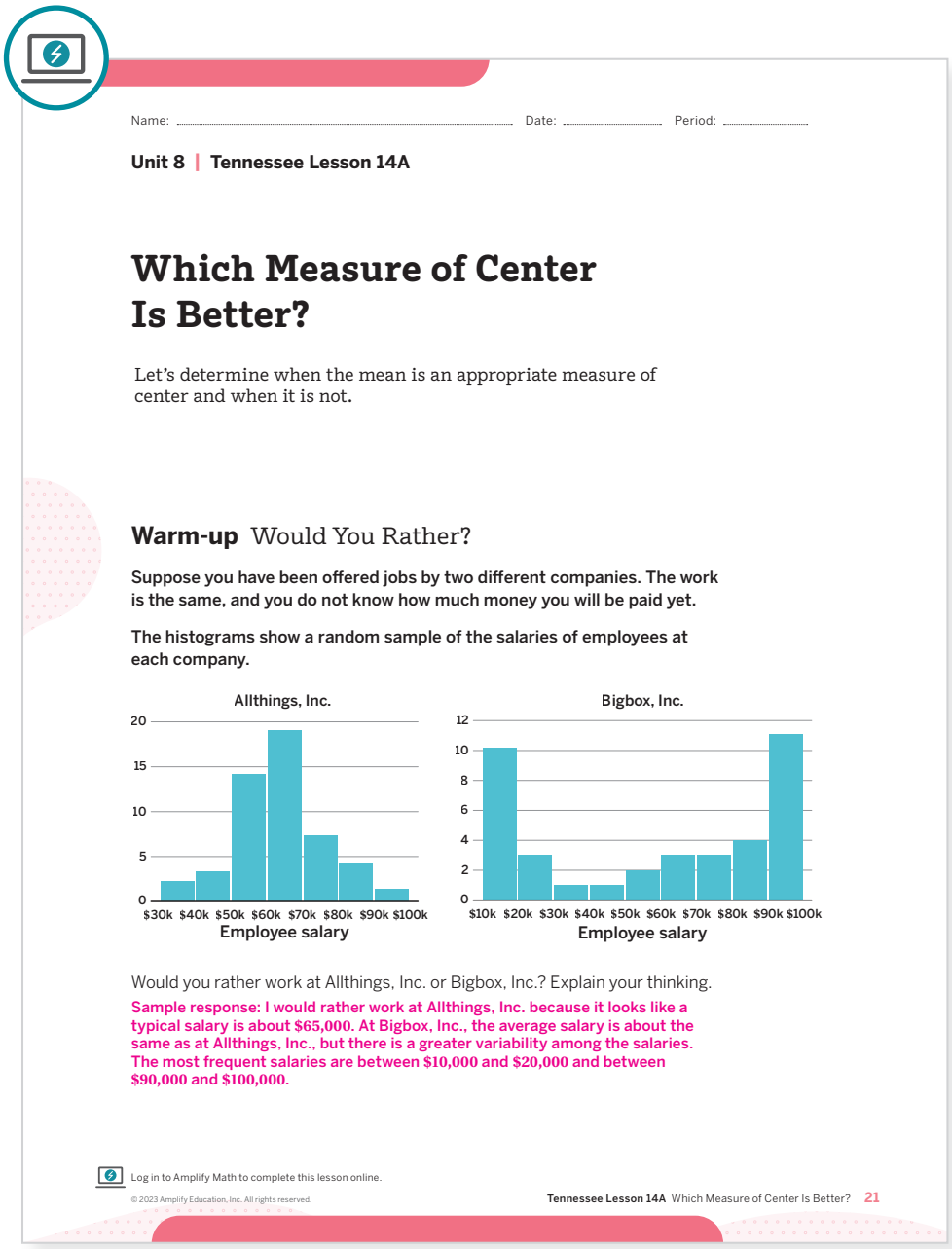
- The **Warm-up** may be omitted.

Pairs | 5 min

Warm-up Would You Rather?

7.SP.B.4

Students compare the distributions of a random sample of salaries at two different companies to spur discussion about average and typical values.



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Unit 8 | Tennessee Lesson 14A

Which Measure of Center Is Better?

Let's determine when the mean is an appropriate measure of center and when it is not.

Warm-up Would You Rather?

Suppose you have been offered jobs by two different companies. The work is the same, and you do not know how much money you will be paid yet.

The histograms show a random sample of the salaries of employees at each company.

Would you rather work at Allthings, Inc. or Bigbox, Inc.? Explain your thinking.

Sample response: I would rather work at Allthings, Inc. because it looks like a typical salary is about $65,000. At Bigbox, Inc., the average salary is about the same as at Allthings, Inc., but there is a greater variability among the salaries. The most frequent salaries are between $10,000 and $20,000 and between $90,000 and $100,000.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate students' background knowledge by asking them what they know about the term salary. Describe how a *salary* differs from an hourly wage. Have students use the ***Think-Pair-Share*** routine. Provide them 1 minute of individual think time. Then have them complete the Warm-up with a partner.

2 Monitor

Help students get started by sharing that the median salary for both companies is about $65,000.

Look for points of confusion:

- **Thinking that the average salary is the middle value on the horizontal axis.** Have students explain what the height of each bar represents.

Look for productive strategies:

- Identifying a typical value for each data set.
- Estimating the mean salary for each company.

3 Connect

Have students share observations they made about the data — particularly focusing on the center, shape, and spread of the data.

Highlight that the shape of the data can tell students as much — and sometimes more — about a data set as the measures of center.

Ask, "Can you think of any additional information you would like to know before selecting a company?"

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their observations, collect the language they use that describe the center, shape, and spread of the data. For example, they may use these words and phrases: *typical salary*, *average salary*, *greater variability*, and *most frequent salaries*. Invite students to add to the display during the lesson and encourage students to refer back to the display during class discussions.

English Learners

Annotate the distributions with the terms and phrases students use to describe them. For example, annotate the peaks of each distribution with the phrase *most frequent*.

Power-up

To power up students' ability to interpret data from dot plots:

Provide students with a copy of the Power-up PDF.

Use: Before Activity 1

Informed by: Performance on Lesson 14, Practice Problem 6

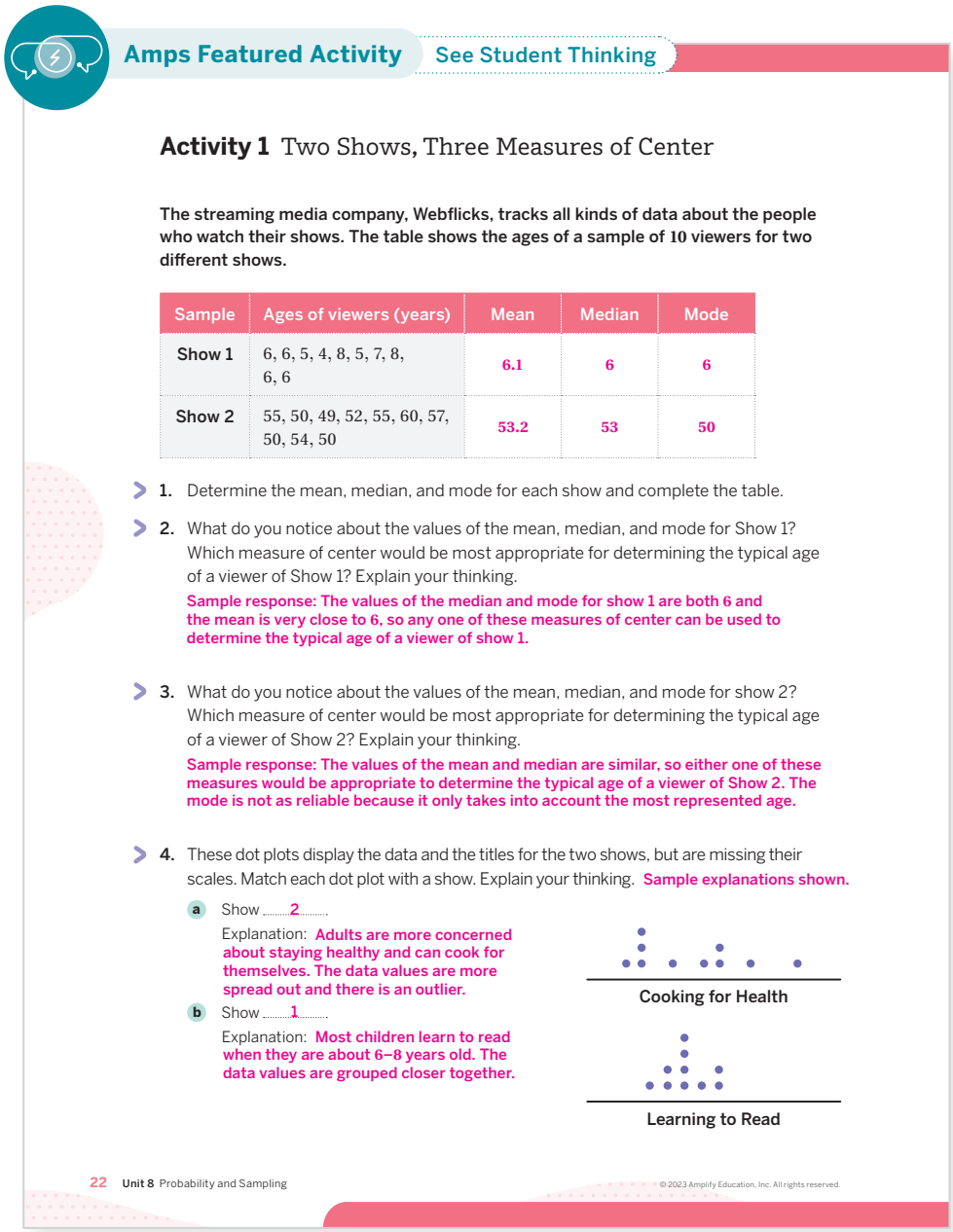
Pairs | 15 min

Activity 1 Two Shows, Three Measures of Center

MP2, MP3
7.SP.D.8, 7.SP.D.8b

Students analyze data from samples of viewers for different media streaming shows to better understand the population of viewers **(MP2)**.

Amps Featured Activity See Student Thinking



Activity 1 Two Shows, Three Measures of Center

The streaming media company, Webflicks, tracks all kinds of data about the people who watch their shows. The table shows the ages of a sample of 10 viewers for two different shows.

Sample	Ages of viewers (years)	Mean	Median	Mode
Show 1	6, 6, 5, 4, 8, 5, 7, 8, 6, 6	6.1	6	6
Show 2	55, 50, 49, 52, 55, 60, 57, 50, 54, 50	53.2	53	50

1. Determine the mean, median, and mode for each show and complete the table.
2. What do you notice about the values of the mean, median, and mode for Show 1? Which measure of center would be most appropriate for determining the typical age of a viewer of Show 1? Explain your thinking.
   Sample response: The values of the median and mode for show 1 are both 6 and the mean is very close to 6, so any one of these measures of center can be used to determine the typical age of a viewer of show 1.
3. What do you notice about the values of the mean, median, and mode for show 2? Which measure of center would be most appropriate for determining the typical age of a viewer of Show 2? Explain your thinking.
   Sample response: The values of the mean and median are similar, so either one of these measures would be appropriate to determine the typical age of a viewer of Show 2. The mode is not as reliable because it only takes into account the most represented age.
4. These dot plots display the data and the titles for the two shows, but are missing their scales. Match each dot plot with a show. Explain your thinking. Sample explanations shown.
   a. Show 2
      Explanation: Adults are more concerned about staying healthy and can cook for themselves. The data values are more spread out and there is an outlier.
   b. Show 1
      Explanation: Most children learn to read when they are about 6–8 years old. The data values are grouped closer together.

1 Launch

Activate prior knowledge of the mean, median, and mode as measures of center. Remind students that measures of center attempt to identify the center of the data set. Provide access to calculators for the remainder of the lesson.

2 Monitor

Help students get started by prompting them to count the number of values in the data set that are greater than and less than the measure of center in question to determine how centered it is within the set.

- **Switching values for mean, median, and/or mode.** Review how to determine each measure of center.

Look for productive strategies:

- Using background knowledge, the titles of the shows, and the data displays to answer Problem 4.

3 Connect

Have students share their responses for the table and display the distributions from the Student Edition.

Ask:

- "Which data set has the least variability? How can that be seen in the measures of center and in the dot plot?"
- "Could the five-number summary help us determine which show represents each dot plot?"

Highlight how close the values of the mean, median, and mode are in the data set for Show 1. In a perfectly symmetric distribution, the measures of centers all have the same value. Note that the measures of center only give one piece of information. In order to draw conclusions about the data students need more information **(MP3)**. In Activity 2, students will see how the measures of center can misinterpret the data set.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with the calculations for the mean, median, and mode for each show so that they can focus more time thinking about which show corresponds to each dot plot, as opposed to spending a lot of time on calculations.

Accessibility: *Activate Prior Knowledge*

Display the Anchor Chart PDF, *Measures of Center and Spread* so that students can refer to it when determining appropriate measures of center in Problems 2 and 3.

Math Language Development

MLR8: Discussion Supports – Revoicing

During the Connect, as students share their matches and their thinking, demonstrate mathematical language use by restating a student statement as a question in order to clarify, apply appropriate language, and involve more students. For example, if a student says, "The *Cooking for Health* dot plot shows dots that are farther apart," revoice this by asking, "What do you mean by farther apart? Is there a measure that can describe this? Is it a measure of center or variation?"

English Learners

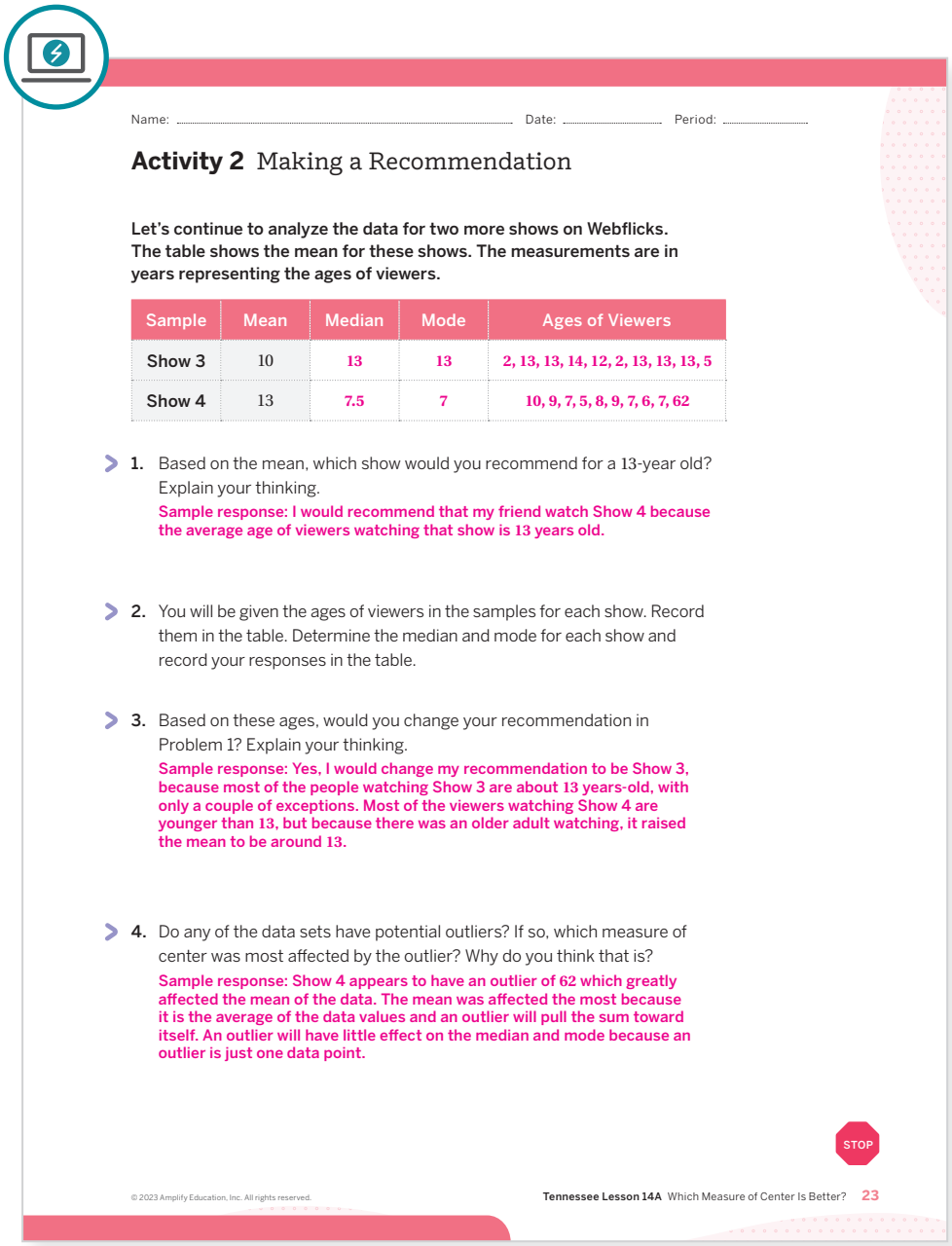
Model how to use the class display to apply appropriate mathematical language to the discussion.

Pairs | 15 min

Activity 2 Making a Recommendation

MP2, MP3
7.SP.D.8, 7.SP.D.8b

Students see how an outlier can affect the value of the mean making the mean a less reliable measure of center.



Name: _____ Date: _____ Period: _____

Activity 2 Making a Recommendation

Let's continue to analyze the data for two more shows on Webflicks. The table shows the mean for these shows. The measurements are in years representing the ages of viewers.

Sample	Mean	Median	Mode	Ages of Viewers
Show 3	10	13	13	2, 13, 13, 14, 12, 2, 13, 13, 13, 5
Show 4	13	7.5	7	10, 9, 7, 5, 8, 9, 7, 6, 7, 62

1. Based on the mean, which show would you recommend for a 13-year old? Explain your thinking.
   Sample response: I would recommend that my friend watch Show 4 because the average age of viewers watching that show is 13 years old.

2. You will be given the ages of viewers in the samples for each show. Record them in the table. Determine the median and mode for each show and record your responses in the table.

3. Based on these ages, would you change your recommendation in Problem 1? Explain your thinking.
   Sample response: Yes, I would change my recommendation to be Show 3, because most of the people watching Show 3 are about 13 years-old, with only a couple of exceptions. Most of the viewers watching Show 4 are younger than 13, but because there was an older adult watching, it raised the mean to be around 13.

4. Do any of the data sets have potential outliers? If so, which measure of center was most affected by the outlier? Why do you think that is?
   Sample response: Show 4 appears to have an outlier of 62 which greatly affected the mean of the data. The mean was affected the most because it is the average of the data values and an outlier will pull the sum toward itself. An outlier will have little effect on the median and mode because an outlier is just one data point.

STOP

1 Launch

Activate students' background knowledge by asking students why media companies might want to know the ages of the viewers watching their shows. Give students a few moments to answer Problem 1 and then display the Activity 2 PDF. Have students continue to work in pairs to complete Problems 3 and 4.

2 Monitor

Help students get started by asking, "What does the mean tell you about a data set?"

Look for points of confusion:

- **Thinking that Show 4 is a better recommendation because the mean is closer to 13, even after seeing the ages of viewers.** Ask students to create a dot plot or box plot representing the data so they can get a visual of the data.

Look for productive strategies:

- Being wary from the start of only using the mean age to make a recommendation.

3 Connect

Have students share their original recommendations and if they decided to change their recommendation when they were given more information **(MP3)**.

Highlight that, because Show 4 has a very great outlier, the mean is pulled toward that value, making it a less reliable measure of center. In the Show 3 data values, the younger ages pull the mean age lower. This makes the median and mode better measures of center when data has potential outliers **(MP2)**.

Ask:

- "Who do you think is represented by the data value 62?"
- "What would you expect to happen to the value of the mean if the outlier was less than the majority of the data?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

After sharing the ages of viewers for Shows 3 and 4, suggest that students arrange the values in order from least to greatest. Then suggest they circle the values they think are close to the age of 13 to help them better make sense of the data set.

Math Language Development

MLR1: Stronger and Clearer Each Time

During the Connect, after students record their response for Problem 2, invite them to meet with another pair of students to give and receive feedback on their responses to both Problems 3 and 4. Provide these prompts for feedback to help strengthen ideas and clarify language.

- "How would you describe the ages of the viewers for Shows 3 and 4?"
- "Do you think the means, medians, or modes of these data sets best describe the ages of the viewers? Why?"
- "What mathematical language can you use in your responses?"

English Learners

Allow students time to formulate with their partner how they will improve their final draft before proceeding with the Connect discussion.

Whole Class | 5 min

Summary

7.SP.D.8b

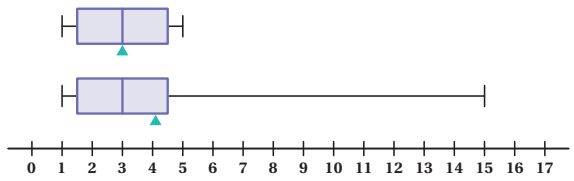
Review and synthesize the mean, median, and mode as measures of centers and how outliers affect them.

Summary

In today's lesson . . .

You saw the effect that outliers have on the mean of a data set. The median and mode are better measures of center if the data set has a potential outlier.

The following boxplots show the same data except for one value. One has an outlier and one does not have the outlier. The mean of each is marked with a triangle.



The potential outlier of 15 pulls the mean toward itself, making the mean less reliable.

> **Reflect:**

Synthesize

Display the images from the Student Edition.

Highlight how the extra long whisker may show an outlier and the mean is moved toward that potential outlier.

Ask, "Which measure of center is most reliable when there is an outlier in the data set? Explain your thinking."

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which measures of center can be affected by outliers?"

Independent | 5 min

Exit Ticket

7.SP.D.8b

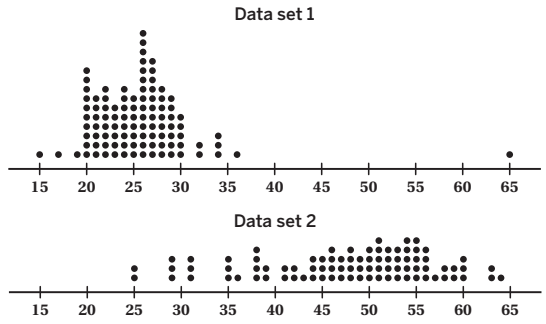
Students demonstrate their understanding by explaining which measure of center could be used for distributions, including one with an outlier.

Printable

Name: ..... Date: ..... Period: .....

Exit Ticket 8.14A

Here are dot plots representing two different data sets.



1. For which data set(s) would you use the median as the measure of center? Explain why that is your choice and explain why you would not use it for the other data set(s).
   Sample response: The median could be used for either Data set 1 or Data set 2, because the middle value could be a good representation of either data set.
2. For which data set(s) would you use the mean as the measure of center? Explain why that is your choice and explain why you would not use it for the other data set(s).
   Sample response: The mean could be used for Data set 2 because the balancing point of about 47 could be a good representation of the data set. However, the mean should not be used for Data set 1 because there is a potential outlier at 65, which could skew the value towards itself.
3. For which data set(s) would you use the mode as the measure of center? Explain why that is your choice and explain why you would not use it for the other data set(s).
   Sample response: The mode could be used for Data set 1 because 26 could be a good measure of center. For Data set 2, there is no mode.

Self-Assess

	1	2	3
	I don't really get it	I'm starting to get it	I got it

a. I can determine which measure of center is a better representation of a data set. **1 2 3**

b. I can interpret the shape of a distribution and how it affects the measures of center. **1 2 3**

Success looks like . . .

- **Language Goal:** Determining which measure of center is best for a given distribution including one with outliers. **(Speaking and Listening, Writing)**
  - » Providing an explanation for how they decided which of the two data sets should use the mean, median, or mode as its measure of center.
- **Language Goal:** Analyzing and describing the shape and characteristics of a data distribution. **(Reading and Writing, Speaking and Listening.)**

Suggested next steps

If students do not explain their reasoning, consider:

- Reviewing Activity 2.
- Assigning Practice Problems 1 and 2.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- What surprised you as your students worked? What might you change for the next time you teach this lesson?

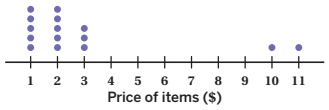
Practice

Independent

Name: ..... Date: ..... Period: .....

Practice

1. A random sample of 15 items were selected at a grocery store. The dot plot shows their prices. Is the mean of this sample likely to be representative of the population mean? Explain your thinking.



No; Sample response: The data values appear to have outliers at $10 and $11, which can have an effect on the mean.

2. If a data set had an outlier added which is less than the majority of the data, how would you expect each measure to change?

a. Mean The new mean would be less than the original mean.

b. Median The median most likely will not change but may move down slightly because a new data point is added.

c. Mode The mode should not be affected at all.

d. Range The range would be made larger because the spread of the data will be larger.

e. Interquartile range Most likely the IQR will not be affected, but might change slightly because the Q1 could move down by adding another piece of data.

3. A high school plans to take all of its students to see a documentary on climate change at a large movie theater. The school has 1,325 students. Each screen has enough seats for 250 students. How many screens are needed? Write and solve an inequality and explain what the solution means in context.

$250x \geq 1325$

$250x = 1325$
$250x \div 250 = 1325 \div 250$
$x = 5.3$

Test a value less than 5.3.
$250(5) \geq 1325$
$1250 \geq 1325$ is not true, so $x \geq 5.3$.

At least 5.3 screens are needed to show the documentary. Because it is not possible to have a fraction of a screen, at least 6 screens are needed (an integer value where $x \geq 6$).

Name: ..... Date: ..... Period: .....

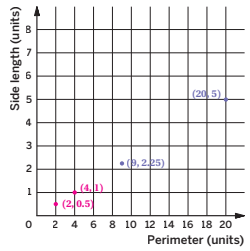
Practice

4. The graph of the relationship between the perimeter of a square and its side length are shown. Two points are plotted.

a. Plot and label two more points that represent the relationship.

b. Is there a proportional relationship between the perimeter of a square and its side length? Explain your thinking.

Yes, there is a proportional relationship; Sample response: For every ordered pair, the side length divided by the perimeter is 0.25.



5. *Capstone project helper.* Organize the data you collected in the previous lesson for your survey. Use a separate sheet of paper to display your collected data in a table or chart.

Answers may vary.

Important items to consider

- Your visual representation could be a dot plot, histogram, or table.
- Choose a display that you think best shows your collected data.

6. Han and Clare want to know how students in their classes travel to school.

a. In Han's class, 3 out of 17 students take the bus to get to school. What fraction of the students take the bus?

$\frac{3}{17}$

b. In Clare's class, $\frac{2}{5}$ of the students walk to school. If there are 35 students in Clare's class, how many students walk to school?

$\frac{2}{5} \cdot 35 = 14$, so 14 students walk to school.

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 2	7.SP.D.8b	2
	2	Activity 2	7.SP.D.8b	2
Spiral	3	Unit 6 Lesson 15	7.EE.B.4b	1
	4	Unit 2 Lesson 11	7.RP.A.2	2
	5	Grade 6	6.SP.B.4	1
Formative	6	Unit 8 Lesson 16	7.SP.A.2	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 7 Additional Practice**.