

Amplify Math TENNESSEE

Grade 8

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today’s students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Unit 1 Rigid Transformations and Congruence

Shapes are all around us. We find them in art, architecture, and even in animated dancing frogs. In this unit, you will find out what happens when you slide, flip, and turn figures of all shapes and sizes. Plus, you may even create a masterpiece artwork along the way.

Unit Narrative:
The Art of Transformation



Note: Lessons in gray are recommended to be omitted.



LAUNCH

1.01 Tessellations 4

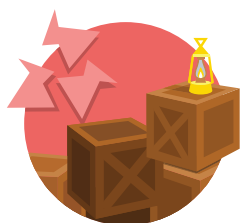


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 1.06 Coordinate Moves (Part 1) 40
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How do you make a piece of cardboard come alive?

Pack your geometry toolkits for a transformational journey into the movement of figures.



Sub-Unit 2 Rigid Transformations and Congruence 61

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How can a crack make a piece of art priceless?

Something special happens when you perform rigid transformations on a figure.



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What's got 10 billion galaxies and goes great with maple syrup?

Construct a triangle from a straight angle and cut two parallel lines to see what angle relationships you notice.



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1.18 Creating a Border Pattern Using Transformations 125

Unit 2 Dilations and Similarity

The way our brain interprets how objects appear — how big or small they are, how near or far — comes back to dilation. Learn to dilate figures and uncover the magic of this special type of transformation.

Unit Narrative:
More than
Meets the Eye



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Sub-Unit 1 Dilations

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Would you put poison in your eye?

Shrink and stretch objects on and off the plane and study the characteristics of the figures you dilate.



Sub-Unit 2 Similarity

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Do you really get what you pay for?

Learn how some companies use dilations to create similar, and slightly smaller, sized packaging, in a process called “shrinkflation.”



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2.12	Optical Illusions	212
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Unit 3 Linear Relationships

How many cups tall is your teacher? Find out in this unit as you make connections between proportional and linear relationships.

Unit Narrative:
A Straight
Change



● = Tennessee-specific lessons



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Sub-Unit 1 Proportional Relationships

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How fast is a geography teacher?

On your mark, get set, go! Use your understanding of slope to show how a geography teacher shocked the world with her record-setting speed.



Sub-Unit 2 Linear Relationships

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How did a coal mine help build America's most famous amusement park?

Use linear relationships to collect as many coins as you can at Honest Carl's Funtime World amusement park.



Sub-Unit 3 Linear Equations

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How did a 16-year-old take down a Chicago Bull?

Create equations from linear relationships and find how a 16-year-old was able to beat Michael Jordan in a game of basketball.



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Unit 4 Linear Equations and Systems of Linear Equations

In this unit, you will learn the algebraic skills you need to find a solution for x . But wait, what do you mean there is no solution?

Unit Narrative:
The Path the
Mind Takes



Note: Lessons in gray are recommended to be omitted.



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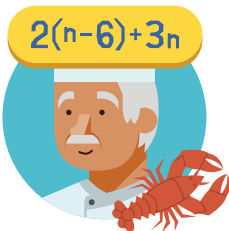
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Who was the Father of Algebra?

When traders in 9th century Baghdad needed a better system for solving problems, a mathematician developed a new method he called "al-jabr" or algebra.



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How is anesthesia like buying live lobsters?

Now that you have practiced solving equations, take a closer look at how you can use linear equations to solve everyday problems.



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4.17 Pay Gaps 465

Unit 5 Functions and Volume

By studying functional relationships in this unit, you will soon be able to explain how height affects the volume of a sphere, calculate how the hare outran the tortoise, and produce your own version of the Happy Birthday song using a graph.

Unit Narrative:
Pumping up
the Volume
on Functions



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Who has the better camera: you or your grandparents?

Learn how functions can help you tell stories.



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Who invented the waffle cone?

Use your prior knowledge about finding the volume of rectangular prisms to derive formulas for finding the volumes of cylinders, cones, and spheres.



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5.21 Packing Spheres 605

Unit 6 Exponents and Scientific Notation

Imagine the *smallest* number you can think of. Now imagine the *largest* number you can think of. How can you write these numbers? How can you work with these numbers? In this unit, you'll learn about the power of exponents (pun intended), and how you can use them to work with numbers as small as the mass of a bacterium or as large as the number of atoms in the Universe.

Unit Narrative:
From Teeny-Tiny
to Downright
Titanic



LAUNCH

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How many carbs are in a game of chess?

You probably already know a thing or two about exponents, but what happens when you multiply or divide expressions with exponents?



Sub-Unit 2 Scientific Notation 669

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6.14 Adding and Subtracting With Scientific Notation 703

Who should we call when we run out of numbers?

You'll work with numbers that are super small and incredibly large. But you won't waste your time writing pesky zeros!



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6.15 Is a Smartphone Smart Enough to Go to the Moon? 710

Unit 7 Irrationals and the Pythagorean Theorem

Discover how three squares can prove something radical about triangles that has captivated mathematicians for centuries.

Unit Narrative:
The Mystery of
the Pythagoreans



LAUNCH

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How rational were the Pythagoreans?

Find out if every number can be represented by a fraction.



$$a^2 + b^2 = c^2$$

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What do the President of the United States and Albert Einstein have in common?

Uncover a special property of right triangles when you explore one of the nearly 500 proofs of the Pythagorean Theorem.



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7.16	Pythagorean Triples	818
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Unit 8 Associations in Data

Data literacy — being able to tell and interpret stories using data — is one of the most important skills you will ever need. In this unit, you will make sense of data in the world around you, represented in different forms. By the end of the unit, you will put your new data literacy skills to the test by examining the accuracy of newspaper headlines.

Unit Narrative:
Data and the
Ozone Layer

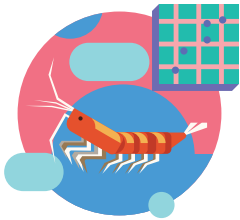


Note: Lessons in gray are recommended to be omitted.
● = Tennessee-specific lessons



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8.01 Creating a Scatter Plot 826



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8.08 Looking for Associations 879

Who is the biggest mover and shaker in the Antarctic Ocean?

Explore the ozone hole using scatter plots, while learning about the different kinds of associations data can have.



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8.09 Using Data Displays to Find Associations 887

Unit 3 | Tennessee Lesson 17A

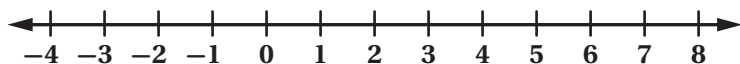
Solutions to Linear Inequalities

Let's use graphs to represent solutions of linear inequalities.

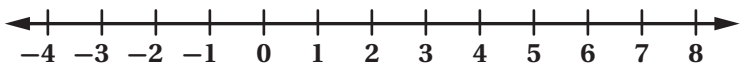
Warm-up Which Side has the Solutions?

Solve each inequality and graph the solution on the number line.

➤ 1. $2x < -4$



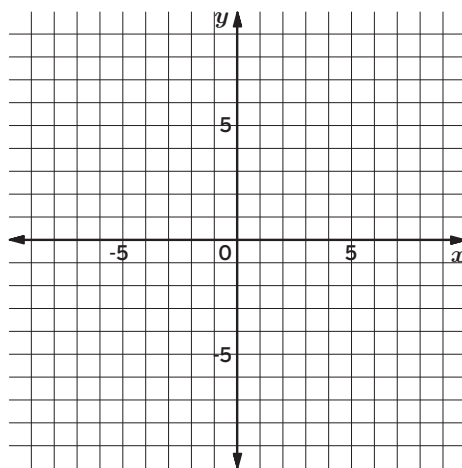
➤ 2. $-5x + 2 \leq 17$



Activity 1 Solutions and Non-solutions

Refer to the inequality $2x + 3y < 12$.

- 1. Choose as many ordered pairs that make the inequality true and plot these ordered pairs on the graph with a dot. Then choose as many ordered pairs that make the inequality false and plot these ordered pairs on the graph with an “x.”



- 2. What do you notice or wonder about the solutions and the non-solutions of the inequality?

Four inequalities are shown on the next page. Your group will be assigned one or more inequalities.

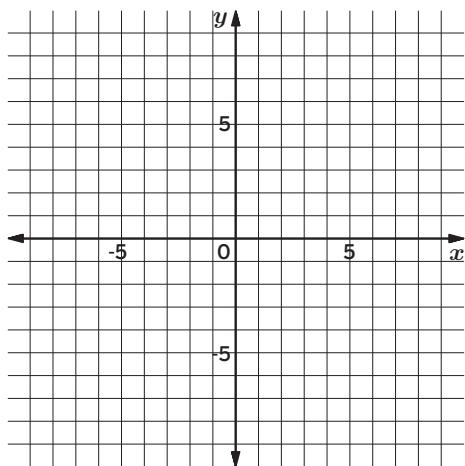
- 3. For each inequality assigned to your group:
- Choose three points from each quadrant and one point on each axis that you will test in your inequality:

Quadrant I	Quadrant II	Quadrant III	Quadrant IV	x-axis	y-axis

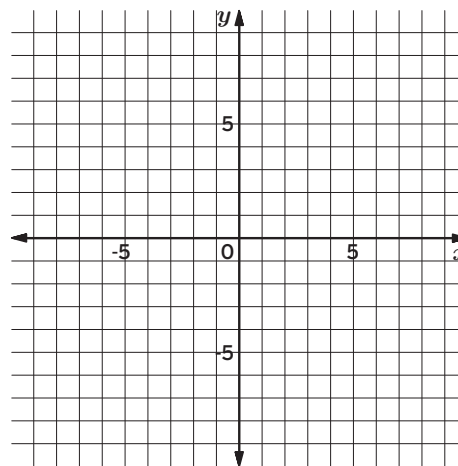
- Determine which coordinate pairs represent solutions to the inequality and which coordinate pairs do not.
- Plot points that are solutions with a point. Plot points that are non-solutions with an X.
- Continue plotting enough points until you start to see the region that contains solutions and the region that contains non-solutions.
- Look for a pattern to help determine the region of solutions.

Activity 1 Solutions and Non-solutions (continued)

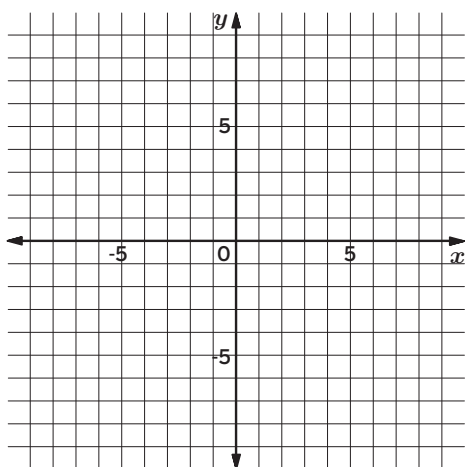
Inequality 1: $y \leq x$



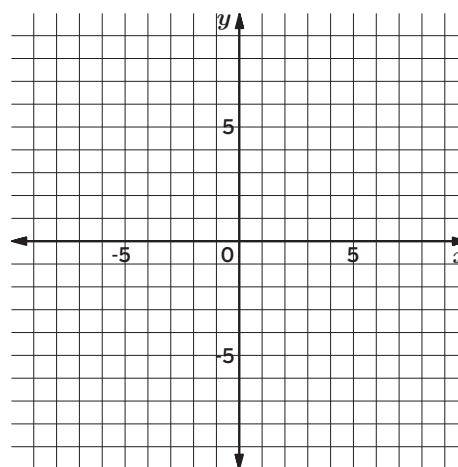
Inequality 3: $3x < 0$



Inequality 2: $-2y \leq -4$



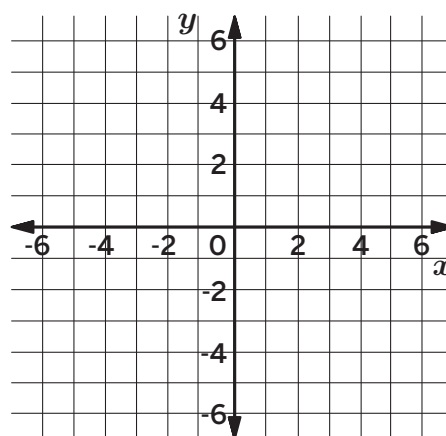
Inequality 4: $x + y > 10$



- > 4. What points are solutions to $y \leq x$, but not $y < x$? Explain your thinking.
- > 5. How could you show all the possible solutions of a linear inequality in two variables without plotting individual points?
- > 6. How could you use the inequalities to determine the equation for the boundary line that separates the two regions of solutions and non-solutions?
- > 7. Sketch the *boundary line* for your assigned inequality.

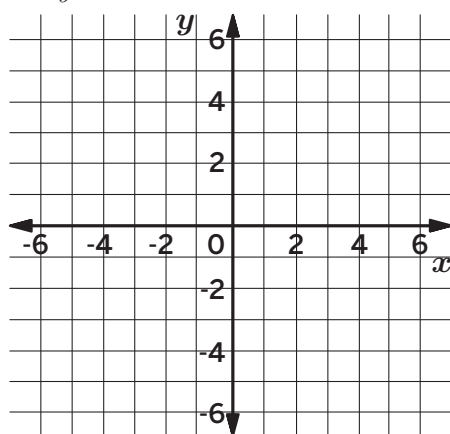
Activity 2 Sketching Solutions to Inequalities

- 1. Graph $x - y = 5$. What do the points on the line represent?

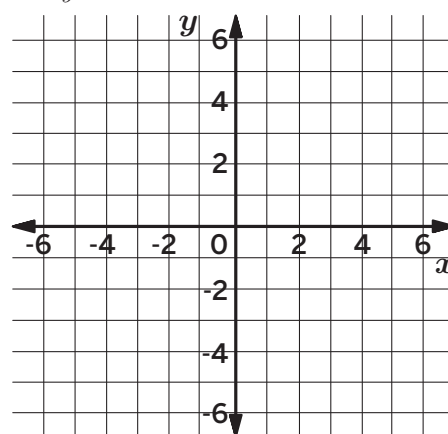


- 2. Sketch the following graphs representing the solutions to each of these inequalities:
- Make the boundary line solid if it is part of the solution and dashed if it is not part of the solution.
 - Shade in the region containing the solutions.

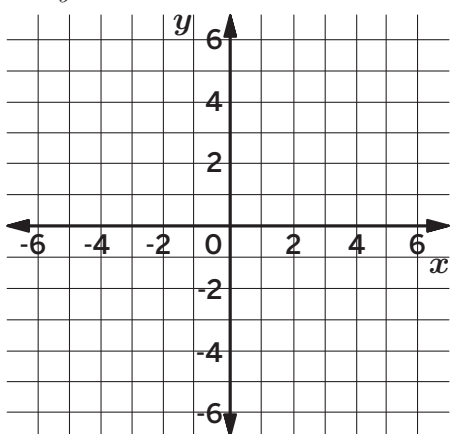
a $x - y < 5$



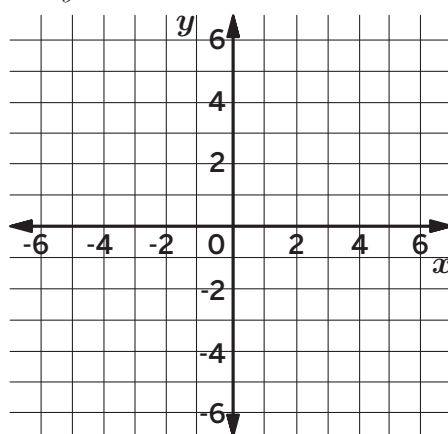
b $x - y \leq 5$



c $x - y > 5$

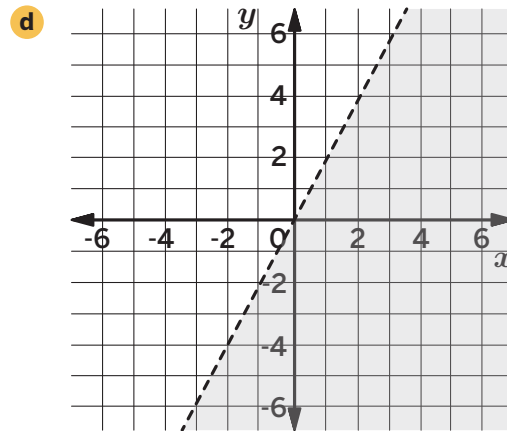
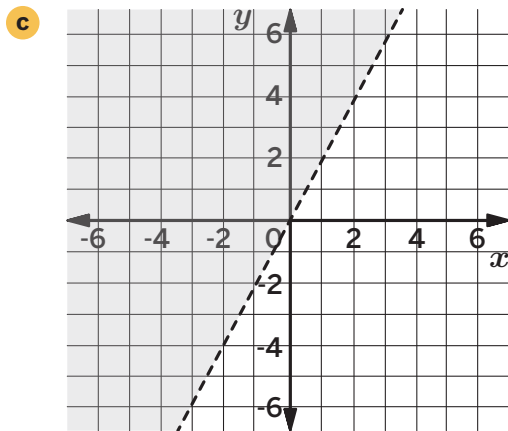
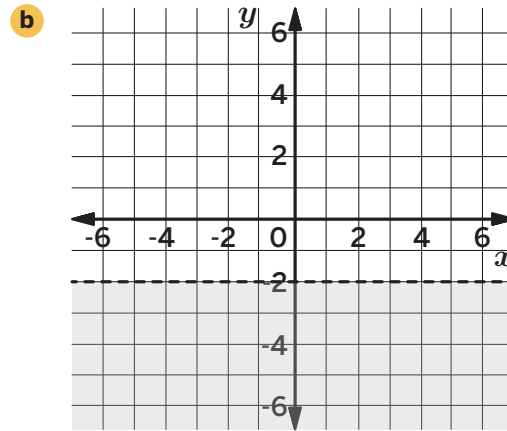
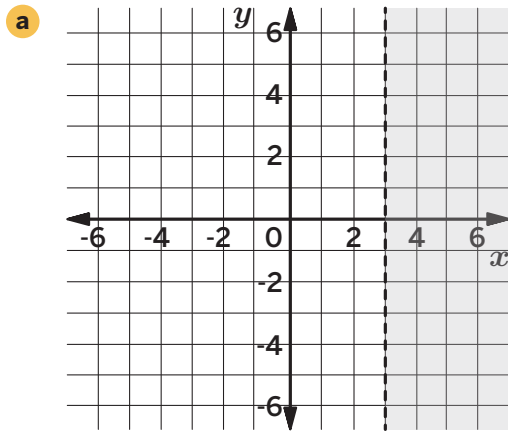


d $x - y \geq 5$



Activity 2 Sketching Solutions to Inequalities (continued)

3. For each graph, write an inequality whose solutions are represented by the shaded part of the graph.



Summary

In today's lesson ...

You examined how to determine the solutions for a two-variable linear inequality, and how to graphically indicate all the points that are part of the solution.

Here are the steps you can follow:

- **Graph the boundary line:**
The **boundary line** represents the boundary between the region containing solutions and the region containing non-solutions. You graphed this line by changing the inequality symbol to an equal sign and graphing the line represented by this equation.
- **Determine whether the line is dashed or solid:**
If the points that lie on the line are solutions, the line should be solid (\geq or \leq inequalities). If the points along the line are not solutions, the line should be dashed ($>$ or $<$ inequalities).
- **Test points to determine the solution region and where to shade:**
You can choose a point on either side of the line and substitute its coordinates into the inequality to determine whether it is a solution. This will help you determine which side of the line should be shaded.

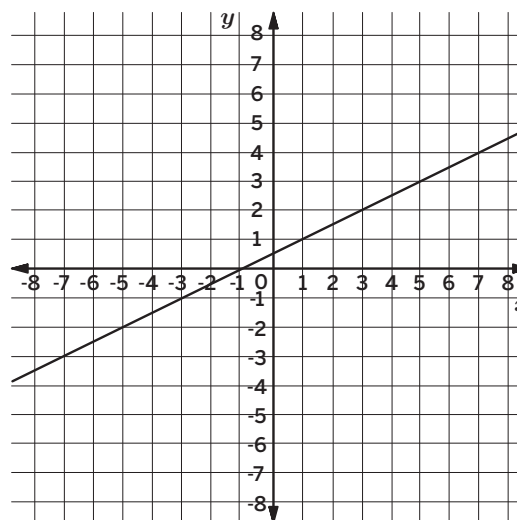
> Reflect:



- > 1. For the expression $3x - 4(2y - 1)$, which of the following ordered pairs makes the value of the expression greater than 28?
- A. $(0, -5)$
 - B. $(8, 0)$
 - C. $(3, 1)$
 - D. $(-2, 2)$

- > 2. Refer to the graph of the equation $2y - x = 1$.

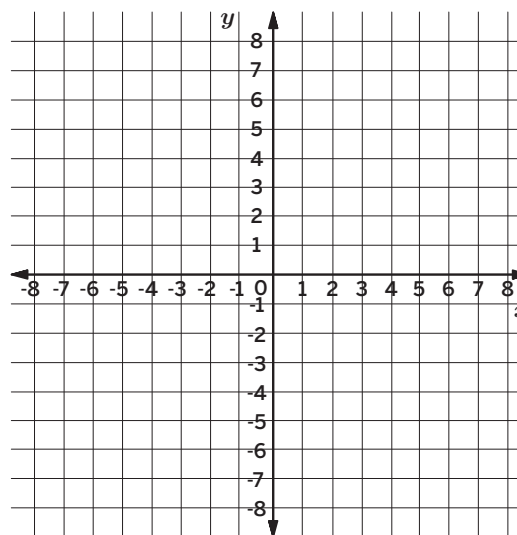
a Are the points $(0, 0.5)$ and $(-7, -3)$ solutions to the equation? Explain your thinking.



b Select *all* points that are solutions to the inequality $2y - x > 1$.

- A. $(0, 2)$
- B. $(8, 0.5)$
- C. $(-6, 3)$
- D. $(-7, -3)$

c Revise the original graph, and shade the region that represents the solution set to the inequality $2y - x > 1$.



d Are the points on the line included in the solution set? Explain your thinking.



Practice

Name: _____ Date: _____ Period: _____

- > 3. Point $(3, -5)$ was transformed using different transformations. Match each transformation described with the coordinates of its image.

Transformation

Image Coordinates

- | | |
|---|------------------|
| a Translated 2 units up and 4 units to the left | $(-3, 5)$ |
| b Reflected across the x -axis | $(5, -9)$ |
| c Rotated 90° counterclockwise about the origin | $(-3, 5)$ |
| d Reflected across the y -axis | $(3, 5)$ |
| e Rotated 180° about the origin | $(-3, -5)$ |
| f Translated 4 units down and 2 units to the right. | $(-1, -3)$ |

- > 4. Solve each equation. Show your thinking.

a $\frac{1}{2}x = 7$

b $a \div \frac{1}{3} = 3$

c $x - 5 = 7$

d $9 - y = 12$

- > 5. A set of pens cost \$2.99. A set of erasers cost \$3.99. You have \$15 to spend on x sets of pens and y sets of erasers. Write an equation to represent the relationship.

Unit 8 | Tennessee Lesson 7A

Keeping Track of All Possible Outcomes

Let's represent sample spaces using organized lists, tables, and tree diagrams.

Warm-up Ordering Spirit Wear

Diego is filling out a spirit wear order form. The options are shown.

Item

Short sleeve shirt

Long sleeve shirt

Sweatshirt

Color

Navy

Yellow

What are all the possible options that may be ordered?



Activity 1 Lists, Tables, and Tree Diagrams

A teacher is deciding on the day, Monday through Friday, and time, 4 p.m. or 5 p.m., to host the first STEM club meeting.

STEM club members Elena, Kiran, and Priya each use a different method to determine the number of options they have to host the meeting.

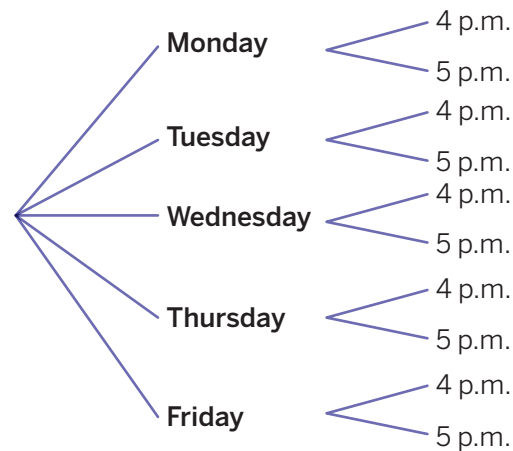
Organized list: Elena carefully creates an organized list of all the options.

Monday 4 p.m., Tuesday 4 p.m., Wednesday 4 p.m., Thursday 4 p.m., Friday 4 p.m.,
Monday 5 p.m., Tuesday 5 p.m., Wednesday 5 p.m., Thursday 5 p.m., Friday 5 p.m.

Table: Kiran creates a table.

	Monday	Tuesday	Wednesday	Thursday	Friday
4 p.m.	Monday 4 p.m.	Tuesday 4 p.m.	Wednesday 4 p.m.	Thursday 4 p.m.	Friday 4 p.m.
5 p.m.	Monday 5 p.m.	Tuesday 5 p.m.	Wednesday 5 p.m.	Thursday 5 p.m.	Friday 5 p.m.

Tree diagram: Priya draws a tree diagram with branches in which each complete path represents a different outcome.



Compare the three methods.

- 1. What is the same about each method?
- 2. What is different about the methods?
- 3. Why does each method show all the different outcomes without repeating any outcome?
- 4. Which method would you choose to show the sample space? Why?

Activity 2 Sample Spaces and Total Number of Outcomes

With your group, decide who will select Experiment A, who will select Experiment B, and who will select Experiment C.

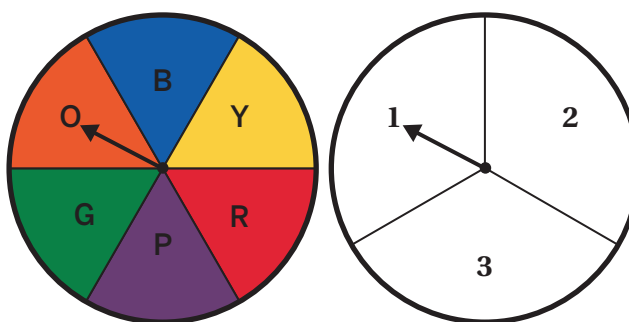
- 1. For the experiment you select, complete the following tasks.
- Use any method to find the sample space. Make sure you list all of the possible outcomes without repeating an outcome.
 - Determine the total number of outcomes for your chosen experiment.

Experiment A: Roll a standard number cube and then toss a penny. Record the number and whether the penny lands heads facing up or tails facing up.

Experiment B: Select one marble from Bag 1 and then one marble from Bag 2.



Experiment C: Spin and land on one color and then spin and land on one number.



- 2. For each experiment you selected, determine the number of outcomes for each event. Then study the relationship between the number of outcomes for each event and the total number of outcomes in the sample space. What do you notice?

Activity 3 How Many Options?

It is estimated that the average adult makes about 35,000 decisions per day! Let's look at a few decisions that someone might make throughout the day.

- 1. Elena's closet contains 15 shirts, 5 pair of pants, and 3 pairs of shoes. If one outfit consists of one shirt, one pair of pants, and one pair of shoes, how many different outfits are possible? Show or explain your thinking.

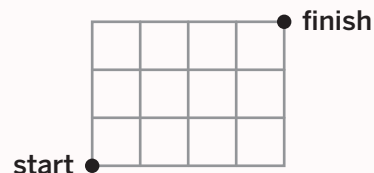
- 2. Elena's school cafeteria offers the items shown for lunch. If one lunch consists of one item from each category, how many different meals are possible? Show or explain your thinking.

Main	Side	Beverage	Dessert
Sandwich	Salad	Water	Zucchini bread
Pasta	Soup	Milk	Pudding
Veggie pizza	Baked potato	Seltzer	Fruit
Black bean burger	Carrots	Orange juice	
	Pretzels	Apple juice	
	Veggie crisps		

- 3. Elena registers as a new user for an online game, where she is asked to create a five letter password. If the letters are not case sensitive and can be repeated, how many passwords are possible? Show or explain your thinking.

Are you ready for more?

Suppose a mail carrier delivers mail using the map shown, where each grid square represents a block. How many different routes can the mail carrier travel, from start to finish, if she does not backtrack and only travels north and east?



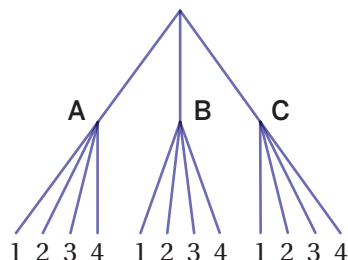
Summary

In today's lesson ...

You explored how to find the sample space for an experiment with multiple events. An event that consists of more than one event is called a **multi-step event**.

Suppose a multi-step event consists of choosing a letter from A, B, or C, and then choosing a number from 1, 2, 3, or 4. Sometimes, it is helpful to use a systematic way to count the number of outcomes which are possible. You can use **tree diagrams**, tables, and organized lists to determine and count the possible outcomes of a multi-step event.

Tree diagram



The total number of outcomes is $3 \cdot 4 = 12$.

Table

	1	2	3	4
A	A-1	A-2	A-3	A-4
B	B-1	B-2	B-3	B-4
C	C-1	C-2	C-3	C-4

The total number of outcomes is $3 \cdot 4 = 12$.

Organized list

A-1, A-2, A-3, A-4, B-1, B-2, B-3, B-4, C-1, C-2, C-3, C-4

The total number of outcomes is 12.

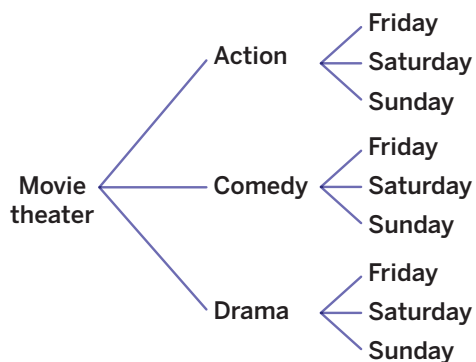
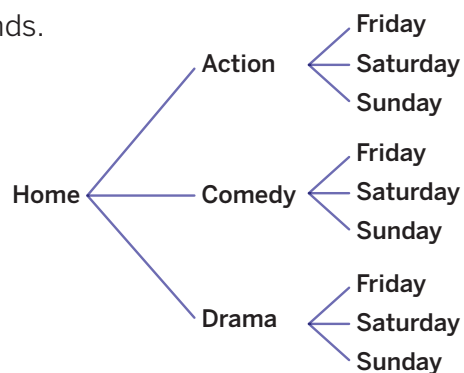
> Reflect:



Practice

Name: _____ Date: _____ Period: _____

1. Kiran is planning to host a movie party with his friends. Here is a tree diagram showing all of the possible locations, movie genres, and days of the week Kiran is considering.



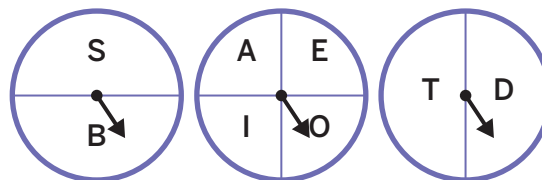
- a How many locations is Kiran considering?
- b How many movie genres is Kiran considering?
- c How many days of the week is Kiran considering?
- d One possibility is streaming an action movie at home on Sunday. Write two other possible parties Kiran is considering.
- e How many different possible outcomes are in the sample space?

2. For each event, write the sample space and determine the number of outcomes.

a Lin selects one frozen yogurt flavor and one topping.

Flavor	Topping
Vanilla	Granola
Strawberry	Coconut Flakes
	Almonds
	Fruit

b Spin each of the three spinners shown once.

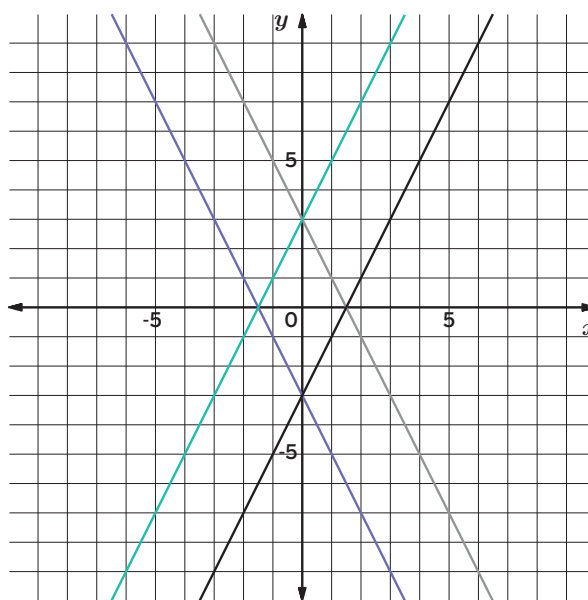




- 3. Jada spins a spinner that is divided into equal sections and labeled 1 to 10 and then she tosses a coin. Determine the number of outcomes. Show or explain your thinking.

- 4. Label each line with its corresponding equation.

- a** $y = 2x + 3$
- b** $y = -2x + 3$
- c** $y = 2x - 3$
- d** $y = -2x - 3$



- 5. Clare rolls two standard number cubes and records the two numbers. Write the sample space for this experiment.

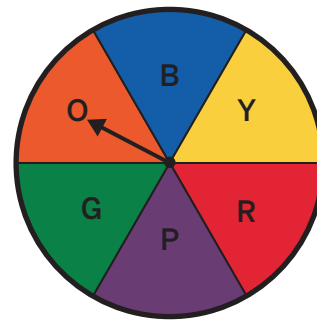
Probabilities of Multi-step Events

Let's determine the probabilities of multi-step events.

Warm-up Spinning a Spinner

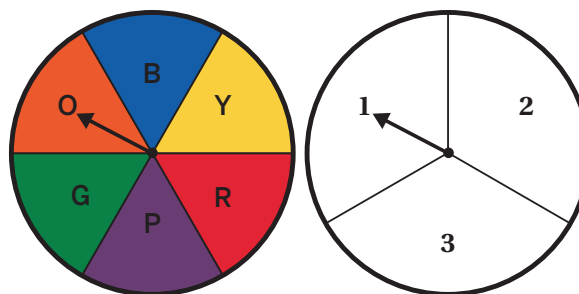
The spinner shown is divided into equal sections.

- > 1. What is the probability of landing on Y?
- > 2. What is the probability of *not* landing on Y?
- > 3. What is the probability of landing on R?
- > 4. What is the probability of landing on Y or R?



Activity 1 Spinning a Color and a Number

In the previous lesson, you wrote the sample space for spinning each of these spinners once. Let's see what happens when you create a multi-step event and spin both of them at the same time.



What is the probability of landing on:

- 1. B and 1? Explain your thinking.

$$P(\text{B and 1}) =$$

- 2. Y and any odd number? Explain your thinking.

$$P(\text{Y and odd number}) =$$

- 3. Not B and not 1? Explain your thinking.

$$P(\text{not B and not 1}) =$$

Activity 2 How Likely?

Consider the following games, and ways to win by rolling two standard number cubes. Order the games from the *most likely* to *least likely* to win for one round. Be prepared to explain your thinking.

Double trouble: *Both* number cubes show the same number.

No nonsense: *No* number cubes show the same number.

Even Steven: *At least* one number cube shows an even number.

Nifty nine: A sum of 9.

Make 15: A sum of 15.

	Most likely
	Least likely



Name: Date: Period:

Summary

In today's lesson ...

You explored how to determine probabilities for multi-step events. You can determine the probability for a multi-step event by finding the ratio of favorable outcomes to the total number of possible outcomes.

Writing the sample space using an organized list, a table, or a tree diagram can help you determine the number of favorable outcomes and the total number of possible outcomes. Another way to determine the number of possible outcomes is to multiply together the number of outcomes for each event.

> Reflect:



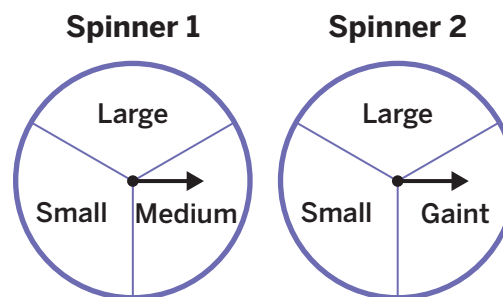
Practice

Name: Date: Period:

- 1. A carnival game lets you spin two spinners that are each divided into three equal sections, each with a word. The rules are shown.

Rules:

- If you land on matching words on Spinners 1 and 2, you win a prize.
 - If you do *not* land on matching words on Spinners 1 and 2, you do *not* win a prize.
- a Use any method to determine the sample space. Show your thinking.

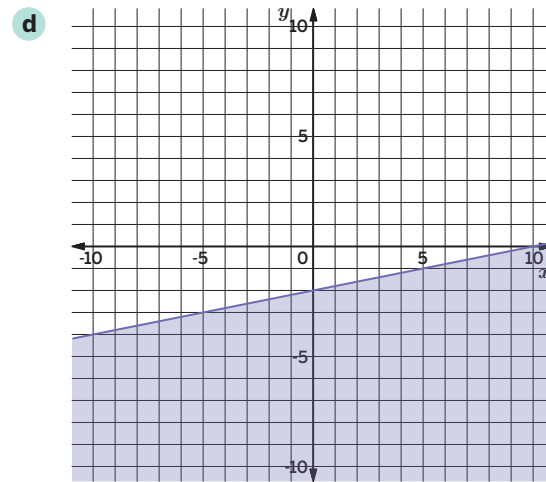
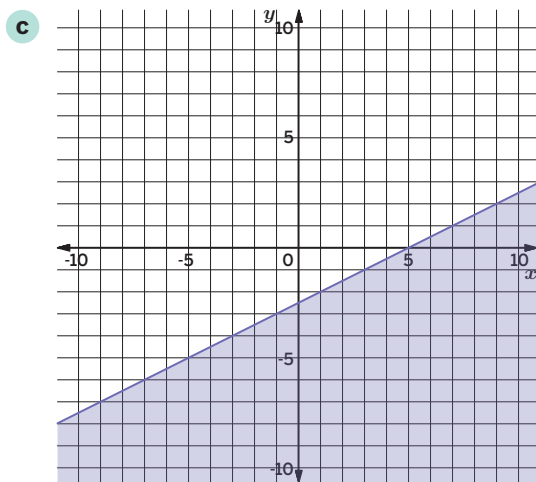
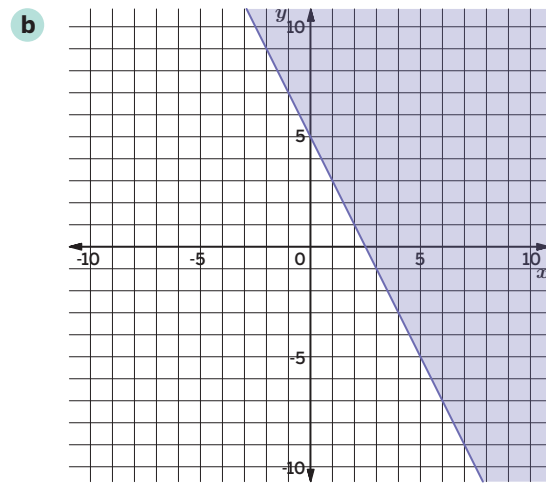
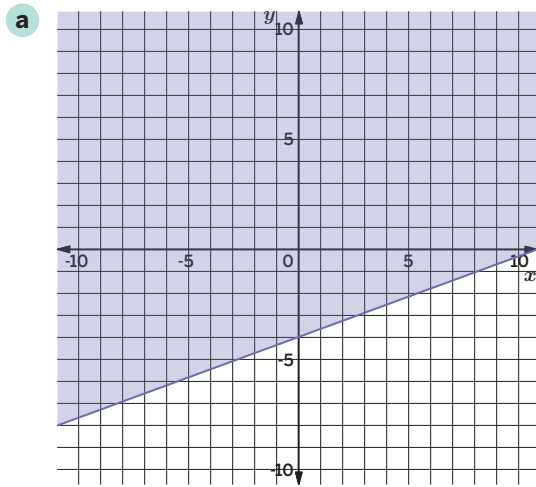


- b What is the probability of winning a prize after you spin both spinners? Show or explain your thinking.

- 2. A vending machine has 5 colors of gumballs (white, red, green, blue, and yellow). A second machine has 4 different animal-shaped rubber bands (lion, elephant, horse, and alligator). Each machine randomly dispenses one item for every purchase. If you buy one item from each machine, what is the probability of getting a yellow gumball and a lion band? Explain your thinking.



3. Match each inequality to the graph of its solution.



..... $4x - 8y \geq 20$ $8x + 4y \geq 20$ $2x - 10y \geq 20$ $2x - 5y \leq 20$

4. Noah is deciding which snack to eat: an apple, a banana, or a granola bar. Describe how Noah could use each of the following tools to make his decision.

a A spinner equally divided into three sections

b A standard number cube

Simulating Multi-step Events

Let's simulate multi-step events.

Warm-up Graphington Slopes (Part 1)

Graphington Slopes is a ski business. To make money over spring break, it needs to snow at least 4 out of the 10 days of spring break. The weather forecast indicates a $\frac{1}{3}$ chance it will snow on each day during spring break.

1. Describe how a spinner could be used to model an experiment to determine the probability of snow on the first day of spring break.
2. Describe how a standard number cube could be used to model the probability of snow on the first day of spring break.



Activity 1 Graphington Slopes (Part 2)

Recall the ski business, Graphington Slopes, from the Warm-up. To make money over spring break, it needs to snow at least 4 out of the 10 days. The weather forecast indicates a $\frac{1}{3}$ chance it will snow on each day during spring break.

- 1. How could a simulation be used to determine whether Graphington Slopes will make money?

- 2. Run your simulation for ten days to see if Graphington Slopes will make money over spring break. Record your results in the first row (Simulation 1) of the table.

Simulation	Did it snow? (✓ or X)									
1										
2										
3										
4										
5										

- 3. Complete the simulation four more times and record your results in the table (Simulations 2–5).

- 4. For each simulation, determine the frequency of days with snow and whether or not Graphington Slopes made money. Record your responses in the table.

Simulation	Frequency of days with snow	Did they make money? (Yes or No)
1		
2		
3		
4		
5		

Activity 1 Graphington Slopes (Part 2) (continued)

- 5. Based on your simulation results, estimate the probability that Graphington Slopes makes money over spring break.

$$P(\text{Graphington Slopes makes money}) = \frac{\text{number of simulations resulting in "yes"}}{\text{number of completed simulations}}$$

$$= \frac{\boxed{}}{\boxed{}}$$

Pause here and wait for further directions while your teacher collects class data.

- 6. Based on the class simulation results, estimate the probability that Graphington Slopes makes money over spring break.

Activity 2 Simulation Nation

Consider Simulation A, B, C, and D. For Problems 1–4, which simulation could be used to determine the indicated probability?

Simulation A

- Toss a number cube 2 times and record the outcomes.
- Repeat this process as many times as needed.
- Determine the ratio of the simulated results in which a 1 or 2 appeared both times.

Simulation B

- Create a spinner with four equal-sized sections labeled 1, 2, 3, and 4. Spin the spinner 5 times and record the outcomes.
- Repeat this process as many times as needed.
- Determine the ratio of the simulated results in which a 4 appears three or more times.

Simulation C

- Toss a fair coin 4 times and record the outcomes.
- Repeat this process as many times as needed.
- Determine the ratio of the simulated results in which the coin lands heads facing up exactly 3 times.

Simulation D

- Place 8 blue chips and 2 red chips in a bag. Shake the bag, randomly select a chip, record its color, and then place it back in the bag.
- Repeat this 4 more times to obtain one outcome. Then repeat this process as many times as needed.
- Determine the ratio of the simulated results in which exactly 4 blue chips are selected.

- 1. In a small lake, 25% of the fish are female. Suppose you catch a fish, record whether it is male or female, and toss the fish back into the lake. If you repeat this process 5 times, what is the probability at least 3 of the 5 fish you catch are female?
- 2. Elena makes about 80% of her free throws. Based on this, what is the probability she will make exactly 4 out of the 5 free throws in her next basketball game?
- 3. On a game show, a contestant will randomly select one of three doors. There are two rounds. In each round, one of the three doors contains a prize. In Round 1, the prize is a vacation. In Round 2, the prize is a new car. What is the probability of winning a vacation and a car?
- 4. Diego's choir is singing in 4 concerts. Diego and one of his other classmates both learned the solo. Before each concert, the choir director will randomly select Diego or his classmate to sing the solo. What is the probability Diego will be selected to sing the solo in exactly 3 of the 4 concerts?



Summary

In today's lesson . . .

You saw the more complex an experiment is, the more challenging it can be to estimate the probability of a particular event. Well-designed simulations are ways to estimate a probability in a complex experiment, especially when it would be challenging or impossible to determine the probability from reasoning alone.

To design a good **simulation** — an experiment to model a real-world event — you need to know or be able to determine the probability of the individual events you wish to determine. These probabilities can help you design the simulation. For example, if an event has the probability of $\frac{1}{2}$, you can use a coin toss to simulate the experiment. You can also use a standard number cube, in which rolling three out of the six possible outcomes is favorable.

As the number of trials of the simulation increases, the experimental (observed) probability should approach the theoretical (expected) probability.

> Reflect:



- 1. Based on prior orders, a customer notices that when ordering takeout food from a particular restaurant, napkins were not included in the bag 50% of the time. Design a simulation you can use to estimate the probability of napkins not being included on every one of your next three takeout orders.

- 2. Priya’s cat is pregnant with a litter of 5 kittens. Each kitten has a 30% chance of being chocolate brown. Priya wants to know the probability that at least two of the kittens will be chocolate brown. To estimate this probability, Priya designed and conducted a simulation. The table shows her results. The letter *W* represents selecting a white cube and the letter *G* represents selecting a green cube.

Simulation	
•	Place 3 white cubes and 7 green cubes in a bag.
•	Randomly select a cube, record its color, and place it back in the bag.
•	Repeat this 5 times to simulate one trial representing a litter of 5 kittens.
•	Perform 12 trials.

Trial	Outcome
1	GGGGG
2	GGGWG
3	WGWGW
4	GWGGG
5	GGGWG
6	WWGGG
7	GWGGG
8	GGWGW
9	WWWGG
10	GGGGW
11	WGGWG
12	GGGWG

- a** How many successful trials were there? Describe how you determined whether a trial was a success.
- b** Based on this simulation, estimate the probability that *exactly* two kittens will be chocolate brown.
- c** Based on this simulation, estimate the probability that *at least* two kittens will be chocolate brown.
- d** Write and answer another question Priya could estimate using this simulation.
- e** How could Priya increase the accuracy of her predictions?



Practice

Name: Date: Period:

- 3. Based on prior games played, a team's record shows they have won 75% of the games they have played so far this season. They will play 3 games this week. To estimate the probability that the team will win at least two of these games, Clare designs a simulation shown in the table. She obtains a result of "win," "win," "lose."

What is the estimated probability that the team will win at least two games? Can Clare trust this estimate?

Simulation

- Place 4 cards in a bag, 3 of which are labeled "win" and 1 of which is labeled "lose."
- Randomly select a card, record the result, and place the card back in the bag. This represents one trial of playing a game.
- Perform three trials to simulate playing three games.

- 4. Six coins are tossed. Determine the probability of each event. Show or explain your thinking.

a All of the coins land tails facing up.

b At least 1 coin landing heads facing up.

- 5. Suppose you roll two standard number cubes. What is the probability of rolling a sum of 7? Explain your thinking.

